Geometry & Measurement

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GEOMETRY & MEASUREMENT INTRODUCTION

Geometry is useful and interesting, an integral part of mathematics. Geometric thinking permeates mathematics and relates to most of the applications of mathematics. Geometric concepts provide a vehicle for exploring the rich connections between arithmetic concepts (such as numbers and calculations) and physical concepts (such as length, area, symmetry, and shape). Even as children experience arithmetic notions in their daily lives, they encounter geometric notions. Common opinion recognizes that each child should begin developing a number sense early. There is no sound argument to postpone the development of geometric sense. These are compelling reasons why the systematic development of geometric thinking should be a substantial component of an elementary school curriculum.

A benefit of early development of geometric thinking is that it provides opportunities for all learners. Children develop concepts at individual rates. We are careful to allow students a great deal of time to explore quantitative relationships among physical objects before studying arithmetic. In the same spirit, students should be allowed to explore geometric relationships through free play and guided experiences. Before studying geometric properties of lines and shapes, they should have free play with models and with drawing instruments, using both to create new figures and patterns. Before considering parallelism and perpendicularity as relations on geometric lines, they should have free play in creating patterns with parallel and perpendicular lines. Before learning length and area formulas, they should have informal experiences to establish and sharpen intuitive notions of length and area. These early experiences are aesthetically rewarding, and just as stimulating and suggestive of underlying concepts as early experiences leading to arithmetic. Such experiences are a crucial first step in the development of a geometric sense.

A variety of constructions forms the basis for the geometry of *CSMP Mathematics for the Intermediate Grades*. Students use tools to explore geometric concepts, directly discovering their properties and interrelationships. The tools include a straightedge, a compass, mirrors, angle templates, and a new tool called a *translator*, used for drawing parallel lines. The constructions foster insights into the properties of shapes, independent of the measurement of those properties. Only after students are familiar with the shapes do they begin to use rulers and protractors to measure lengths and angles. In this sequential development of geometric ideas, the measurement is viewed as the intersection of geometric concepts and arithmetic concepts.

The focus of this strand is experience. The measurement activities guide the students to refine their ability to accurately measure lengths of line segments and areas of polygons. Another sequence of lessons allows students to explore parallelism through use of a translator. As a natural consequence of their involvement in these activities, the students develop their knowledge and skills in geometry. The effects of this informal approach should be judged by the long-term effects on the students' knowledge, confidence, intuition, and interest in the world of geometry and measurement.

Content Overview

Measurement

Consensus calls for measurement activities in the elementary curriculum, but with no agreement on the form or scope of these activities. Rather than stress mastery of formulas for area and perimeter, or for comparison of standard units, the lessons of this strand provide open-ended experiences within rich problem-solving situations. Measurement becomes a means for investigating problems and developing concepts, rather than an end in itself. Direct experiences with the concepts and tools is central. Mastery of measurement skills will develop naturally from these experiences.

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Several lessons of this strand extend earlier experiences with linear and area measure. Students review certain measurements on their own bodies (for example, arm length) which approximate one centimeter, ten centimeters, and one meter. This activity increases the students' familiarity with the metric system and allows them to estimate the lengths of objects without use of a measuring device. In another activity, students use the length of one line segment to estimate the length of other segments or zigzags. Other lessons and worksheets focus on situations that require the accurate use of rulers. A project involving cutting a label for a can introduces a need for a finer unit than a centimeter; consequently, the millimeter is introduced.

To determine the areas of given shapes, students cut grid paper to cover the given shape and count the number of centimeter squares used. This simple method of finding area leads quickly to more efficient techniques used to determine areas of rectangular shapes or polygons constructed from rectangles. In particular, students dissect and rearrange areas while preserving the original size. Besides determining the areas of given shapes, students also construct regions of specified areas. Because area is viewed as "the amount of cover," this strand makes extensive use of centimeter grid paper. The activities allow students to make insights concerning the concept of area, and to check their ideas by counting the squares. Also, continued exposure to half-squares provides preparation for later work with triangles and parallelograms.

Lessons: G1, 2, 3, 4, 10, 11, 13, and 14

Parallelism

Suggestions of parallel lines are everywhere: railroad tracks across the Great Plains, the flight of a pair of mallards, the opposite sides of a desk, the ruled lines on notebook paper. The architectural embodiments of parallelism indicate its role even in the earliest geometry. From its formal debut as a central concept of Euclid's *Elements*, a rich and fascinating study of parallel lines and related objects has developed over many centuries.

Although traditionally postponed to secondary school, the notions of parallelism appeal directly to one's intuition. Equipped with a rolling straightedge called a *translator* and a compelling dynamic definition of parallel line segments, students perform accurate constructions freely and openly, based on parallel line segments and parallelograms. The lessons provide an opportunity to develop familiarity with the translator, using it to construct families of parallel line segments, as well as to begin to understand the geometric relation of parallelism. The first lesson introduces the basic constructions of the parallelogram and parallelogram train. These constructions appear in each lesson and provide the principal problem-solving tools.

As students construct parallelogram trains, certain sets of line segments stand out. The line segments within a set are not only parallel, but also appear to have the same length. This observation is a natural adjunct of the parallelogram construction and serves to introduce *equipollence*, a relation of couples of points. Roughly speaking, two couples of points are equipollent if they define opposite sides of a parallelogram. Several lessons focus on constructing families of equipollent couples of points under a variety of constraints. Students create a visually stimulating representation for equipollent families of couples by using colored arrows and line segments. For example, a colored arrow connecting the points of a couple emphasizes the relationship; equipollent couples are indicated by the same color arrows.

GEOMETRY & MEASUREMENT INTRODUCTION

During this sequence of lessons, the equal lengths of opposite sides of a parallelogram becomes apparent to students. Students' insights will probably involve the concept of length, as well as parallelism. Yet length is never explicitly mentioned in these lesson; equipollence is defined only in terms of parallelism. Do not discourage student comments about segments of equal length, but emphasize that they must use only a translator for the constructions, rulers or marked straightedges are not allowed. The geometry lessons in the Intermediate Grades curriculum illustrate the variety of geometrical concepts that can be developed using only parallelism.

The introductory work with the translator in this semester prepares the students for constructions that appear regularly in later parts of *CSMP Mathematics for the Intermediate Grades*. The subsequent constructions lead to geometrical insights and provide links to other mathematical topics. Already in this strand, the lessons provide an overt connection with the World of Numbers strand. The equipollence relation together with a properly directed parallelogram train construction yields a number line. Constructing number lines and locating points on them foster the students' developing understanding of the interrelations among numbers. Remember, the focus of this strand is experience. Encourage your students' development both of their physical adeptness with the translator and of their concept of parallelism.

Before using the translator in the classroom, practice using it yourself. Take some time to practice using a translator at a desk as well as at the chalkboard (occasionally a construction at the board requires an odd posture). Provide a few small pieces of masking tape to each student so that paper can be anchored to the desk. Instruct the class in the proper manipulation of the translator: lightly hold the plastic tube at the center and gently push or pull to move it across the paper or desk.

Lessons: G5, 6, 8, and 9

Networks

Two lessons during the semester deal with situations from an area of mathematics called *graph theory* or *network analysis*. The setting for these situations is the two-dimensional plane, and the problems depend on planar characteristics.

The notion of a graph, a collection of points connected by segments (arrows), is one that can depict a variety of interesting, real situations. Graphs are used in a wide range of disciplines; for example, psychology (sociograms), economics (organizational charts), anthropology (family trees), chemistry (structure), engineering (circuit diagrams).

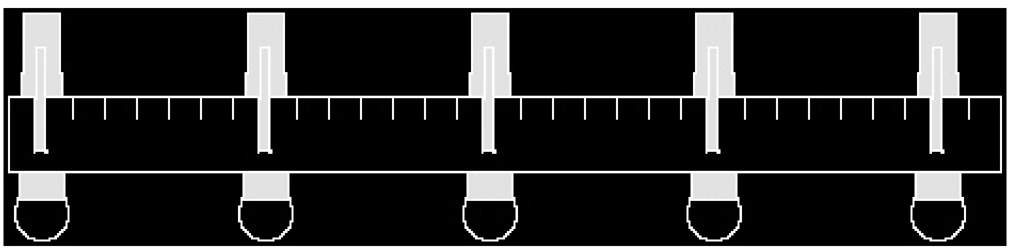
The pictorial nature of much of graph theory and its association with many intriguing puzzles and problems, suggest its appropriateness at the elementary school level. The inductive nature of some of the work provides useful experiences for students. The problem-solving techniques used in analysis of these simple situations is important for students in many areas of study.

Lessons: G7 and 12

Note on Grids

Several lesson call for a demonstration on the grid board. A portable, commercially made, permanent grid chalkboard is ideal. However, if such a grid board is not available, try one of these alternatives.

- Make a grid transparency for use on an overhead projector and use erasable overhead markers. The Blackline section includes several size grids to use in making a grid transparency.
- Printed grids on large sheets of paper are provided in classroom sets of materials for *CSMP*. Tape one of these sheets to the board, use thick felt-tipped pens for drawings, and discard the sheet after use. You may prefer to laminate or mount one of the sheets on cardboard and cover it with clear contact paper to provide a protective surface. Then use erasable markers or wax crayons for drawings. These grid sheets also are useful as graphing mats.
- For each lesson, carefully draw a grid on the chalkboard using an easy-to-make tool. Either (1) use a music staff drawer with chalk in the first, third, and fifth positions; or (2) clip five pen caps, each having the diameter to hold a piece of chalk tightly, to a ruler at intervals of 4–8 centimeters, and secure them with tape.



Capsule Lesson Summary

Practice using the ruler to measure the length of line segments to the nearest centimeter. Knowing the length of particular line segments, estimate the length of other line segments nearby. Draw zigzags of specified lengths within the boundaries of closed curves.

Materials

Teacher	• Meter stick	**Student**	• Metric ruler • Unlined paper • Worksheets G1 (no star), *, and **

Description of Lesson

Before the lesson begins, ask each student to draw about five line segments of different lengths on their paper.

Hold a meter stick between your hands.

T: ***This length is a meter. How many centimeters is a meter?***

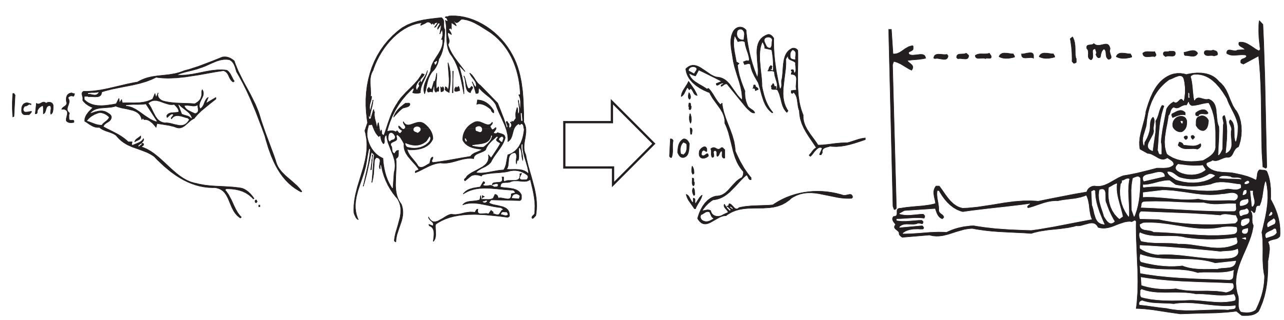
S: ***100 centimeters.***

Record this equivalence on the board, pointing out the abbreviations for centimeter and meter.

100 cm = 1 m

T: ***About how long is a centimeter? Show me with your fingers.***

Also ask for approximations of 10 cm and 1 m in terms of body measurements. Mention that 10 centimeters = 1 decimeter.



The following dialogue assumes your students have 30 cm length rulers. If necessary, adjust the discussion to the length of their rulers.

T: ***How many centimeters are there between the first and last marks on these rulers?***

S: ***30 cm.***

T: ***When measuring with a ruler, what are some important things to remember?***

S: ***First look for the mark for 0 on the ruler. On some rulers, it is not at one of the ends. Then line up the mark for 0 with one end of whatever you're measuring.***

T: ***Now let's measure some line segments to the nearest centimeter. That is, we will find the centimeter mark nearest to the endpoint of a line segment when we measure its length.***

But we need to make an agreement. If the endpoint of a line segment falls exactly halfway between two of the centimeter marks, then we will use the larger measurement. We could make a similar agreement if we were measuring to the nearest meter, decimeter, or any other unit.

Ask students to trade papers with a partner and to measure the line segments drawn by their partner. When they finish the measurements, suggest they trade papers back and check each other's measurements. As this activity is going on, check that students are using good measurement techniques.

Ask the students to put their rulers away while you distribute copies of Worksheet G1 (no star).

T: ***On the front side of this worksheet there are segments of four different colors. I'll tell you the length to the nearest centimeter of the first segment of each color. Using this information, you estimate the lengths of the other segments or zigzags of the same color.***

The first black segment measures about 2 cm. About how long do you think the second black segment is?

Pause to let students record their estimates on their worksheets. Do not reveal the segment's length.

Likewise continue with the other segments and the zigzag. Then make a list of some of the students' estimates on the board. Invite students to share their estimation techniques. Perhaps your list will look similar to the one below.

First black segment: 2 cm
Second black segment? 5 cm, 6 cm, 7 cm

First red segment: 4 cm
Second red segment? 10½ cm, 11 cm, 12 cm
Third red segment? 9 cm, 10 cm, 11 cm

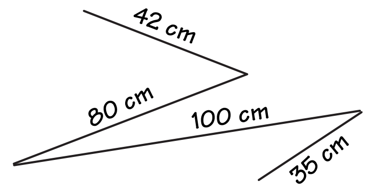
First blue segment: 3 cm
Blue zigzag? 10 cm, 15 cm, 17 cm

First gray segment: 7 cm
Second gray segment? 8 cm, 9 cm
Third gray segment? 6 cm, 7 cm, 8 cm

Ask students to use their rulers to measure the segments to the nearest centimeter. When the class agrees on a segment's length, circle the correct length in the list of estimates.

Invite students to complete the back of Worksheet G1 (no star). Emphasize that they should first compare the segments without measuring. At the end they can measure to check themselves.

While students are working, draw a zigzag on the board. Invite students to measure the segments to the nearest centimeter. Perhaps you will have drawn a zigzag similar to this one.



T: ***How can we find the length of this zigzag?***

S: ***Add the lengths of the pieces.***

$$\begin{array}{r} 42\text{ cm} \\ 80\text{ cm} \\ 100\text{ cm} \\ +\ 35\text{ cm} \\ \hline 257\text{ cm} \end{array}$$

Worksheets G1* and ** are available for individual work.

Extension Activity

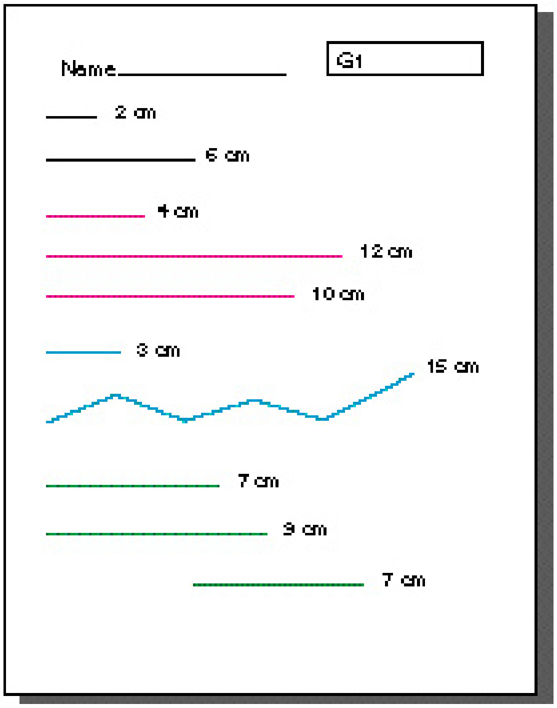
Make classroom lists of objects that measure about 1 cm, 1 dm, and 1 m.

Writing Activity

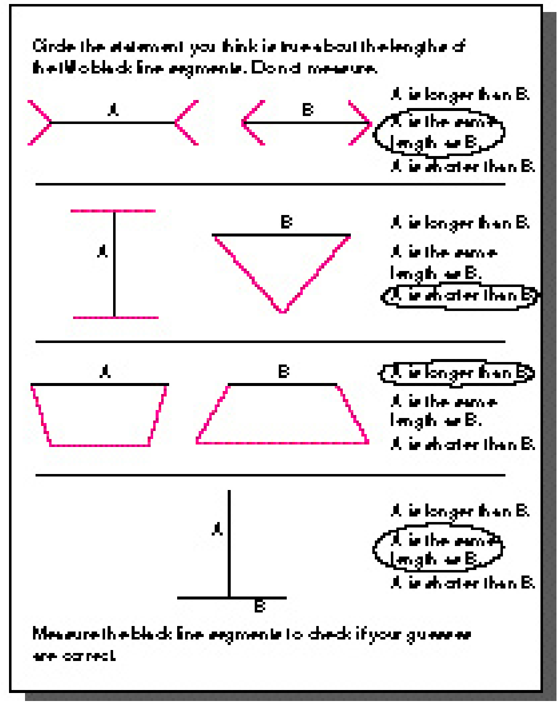
You may like students to take lesson notes on some, most, or even all their math lessons. The "Lesson Notes" section in Notes to the Teacher gives some suggestions and refers to forms in the Blacklines you may provide to students for this purpose. In this lesson, for example, students may note metric length equivalences such as 100 cm = 1 m and 10 cm = 1 dm. They may also describe some things that are about 1 cm, 1 dm, and 1 m long.

Home Activity

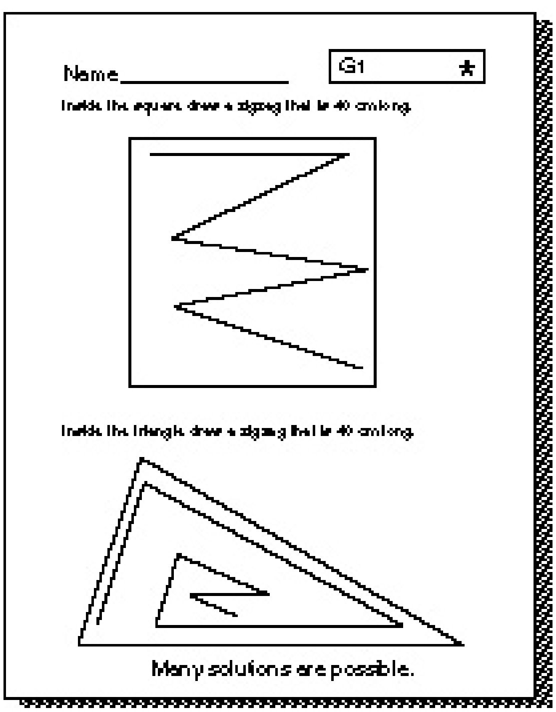
Suggest that students ask a family member to help them find things at home that are 1 cm, 1 dm, and 1 m long.



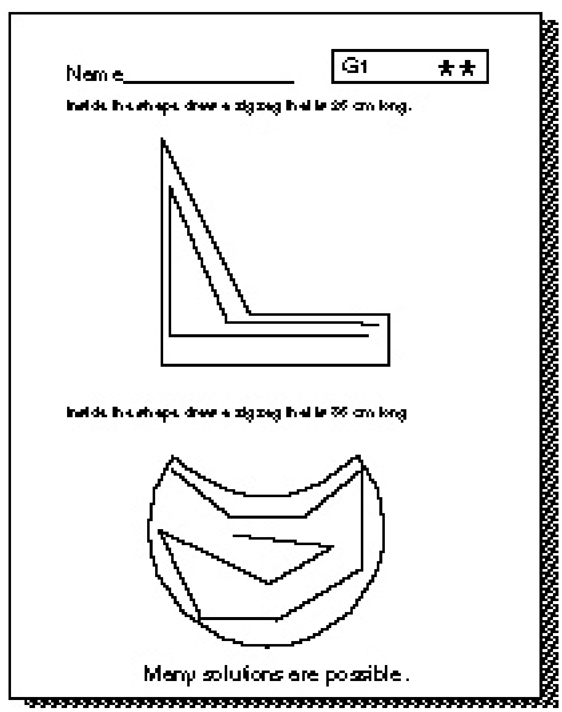
Name
G1
2 cm
6 cm
4 cm
12 cm
10 cm
3 cm
15 cm
7 cm
9 cm
7 cm



Circle the statement you think is true about the lengths of the two black line segments. Do not measure.
A
B
A is longer than B.
A is the same length as B.
A is shorter than B.
A
B
A is longer than B.
A is the same length as B.
A is shorter than B.
A
B
A is longer than B.
A is the same length as B.
A is shorter than B.
A
B
A is longer than B.
A is the same length as B.
A is shorter than B.
Measure the black line segments to check if your guesses are correct.



Name
G1 ★
Inside the square draw a zigzag that is 40 cm long.
Inside the triangle draw a zigzag that is 40 cm long.
Many solutions are possible.



Name
G1 ★★
Inside the shape draw a zigzag that is 26 cm long.
Inside the shape draw a zigzag that is 36 cm long.
Many solutions are possible.

Capsule Lesson Summary

Make labels for cans. Discuss how the problem of getting accurately-sized labels might be reduced by measuring to the nearest millimeter instead of to the nearest centimeter.

Materials

Teacher
- A can with a label that can be easily removed
- Cord with little stretch or a metric tape measure†
- Metric ruler
- Scissors
- Construction paper
- Tape

Student
- An empty, medium- or small-size can with no label
- Cord with little stretch or a metric tape measure
- Metric ruler
- Scissors
- Construction paper
- Tape
- Unlined paper

Advance Preparation: Request that students bring a can with its label removed from home. Collect the cans from the students prior to the lesson to insure that everyone has one. Do not distribute them until they are needed in the lesson.

Description of Lesson

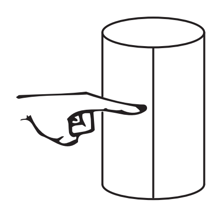
Exercise 1

Begin the lesson by reviewing centimeters, decimeters, and meters as at the beginning of Lesson G1.

Exercise 2

Note: This is a good activity for students to work in pairs. Each pair of students can make labels for two cans.

Hold up a can with a label that can be easily removed. Trace straight down the side of the can; the seam is a natural guide.



T: ***If I cut the label straight down the side and then flatten it, what shape do you think the label will have?***

S: ***A circle.***

S: ***A rectangle.***

S: ***An oval.***

Remove the can's label. Hold it outstretched between your hands and thus reveal that it is rectangular in shape.

T: ***Today we are going to make labels for the cans you brought. We'll try to make the labels so that they just fit the cans—so that they do not overlap themselves. What measurements of the cans will we need to make?*** (Hold up the label that you removed earlier from a can.) ***How long should the label for a can be? How wide?***

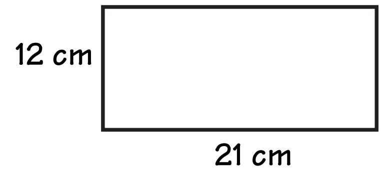
†You might borrow a set of centimeter tape measures, provided with the second and third grade *CSMP* materials, from a teacher in your building. Otherwise, cord with little stretch can be used together with a ruler.

Allow students to replace and remove the label from your can as they describe which measurements to make.

S: ***We need to measure how far it is around our cans and how tall they are.***

Call on students to measure (to the nearest centimeter) the circumference of your can and its height. Let them use a metric tape measure, or cut pieces of string in the appropriate lengths and then measure the string with a metric ruler. Sample measurements are given here.

T: ***My can measures about 12 cm high and about 21 cm around.***



Tape your can's label to the board and invite students to indicate a 12 cm side and a 21 cm side.

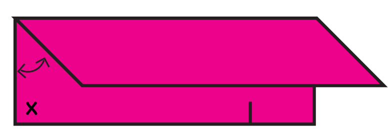
T (pointing to the other two sides)**:** ***What should these sides measure?***

S: ***12 cm and 21 cm.***

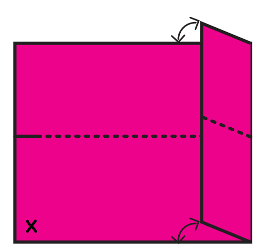
Distribute the unlabeled cans. Direct students first to draw rectangles on a paper and to record the measurements of their cans near the sides of the rectangles. Students should measure the cans by using metric tape measures, or by cutting pieces of string the appropriate lengths and then measuring the string. Then they should cut appropriately-sized rectangles out of construction paper to use as can labels. Suggest to students that they can decorate the labels if they wish and tape them on their cans.

Students who need help cutting rectangles the right size (as indicated on their papers) may need directions like the following.

- Use a corner of the construction paper as one corner of the label. Mark it with an x.
- Measure the height of the can from that corner along one side of the paper and make a mark.
- Measure the distance around the can also from that corner along the other edge of the paper and make a mark.
- Fold the paper at one of the marks so that side edges line up.



- Unfold the paper, and then fold it at the other mark so that side edges line up.
- Cut along the fold lines.



T: ***Some of you found that your labels did not fit quite right. The label might have been too wide, too short, or too long. You may or may not have tried to make an adjustment. What might have been some reasons for the size differences?***

Accept reasonable comments. If no one suggests or hints at size problems caused by measuring to the nearest centimeter, mention it yourself.

T: ***Suppose that the distance around your can is a little more than 21 cm and that you cut the label at 21 cm. What happens?***

S: ***The label does not quite fit around the can.***

T: ***What if you cut the label at 22 cm?***

S: ***The label would overlap some.***

T: ***To get a better measurement, we can use the marks between the centimeter marks on the ruler. Look at your rulers. Do you see the short marks between the centimeter marks?***

Perhaps some of your students used those marks when making their labels.

T: ***Those marks are for millimeters. On our rulers, every centimeter is subdivided into how many millimeters?***

S: ***10.***

Write the equivalence on the board and point out that *mm* is the abbreviation for millimeter.

100 cm = 1 m
10 mm = 1 cm
1000 mm = 1 m

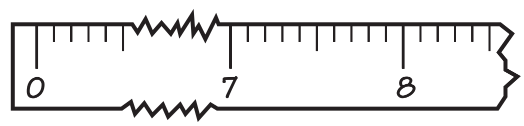
T: ***100 centimeters is 1 meter.***
How many millimeters is 1 meter?

S: ***10 000 millimeters.***

T: ***How do you know?***

S: ***100* x *10 = 1000.***

Draw this enlargement of a ruler on the board.



T: ***Why do you think I drew zigzags*** (point to them) ***in my picture?***

S: ***To indicate that you didn't draw all of the ruler between 0 and 7.***

Point to the appropriate marks between the marks for 7 cm and 8 cm as you say,

T: ***This mark is for 7.1 cm; this mark, 7.2 cm; this mark, 7.3 cm; this mark, 7.9 cm.***
10 millimeters = 1 centimeter; so how many millimeters is 7.1 cm?

S: ***71 mm.***

T: ***How do you know?***

S: ***7* x *10 = 70, and 1 more is 71.***

T: ***If you measure your can to the nearest millimeter instead of to the nearest centimeter, you might get a label that better fits your can.***

If there is time, let students continue working on labels for their cans, either making more accurate labels or making adjustments to their first labels.

Additional Practice

Set out a collection of objects to be measured to the nearest millimeter.

Capsule Lesson Summary

Cover shapes with 1 cm by 1cm squares to find their areas. Find the perimeters of these shapes.

Materials

Teacher
- Centimeter grid paper
- 1 cm by 1 cm square
- String with little stretch
- Scissors
- Grid board†

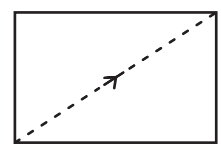
Student
- Scissors
- Centimeter grid paper
- Worksheets G3 (no star), *, **, ***, and ****

Description of Lesson

Use a bulletin board or some other board in your classroom for the following discussion.

T: ***Suppose we want to decorate this bulletin board. We want to cover it with a piece of cloth, and we want to make a divider out of ribbon that goes from one corner to the corner diagonally across from it.***

Trace one of the diagonals of the board.



T: ***We need to find a length for the ribbon—how far is it from one corner of the bulletin board to the diagonally opposite corner?***

S: ***Use a meter stick and measure its length. We need to find an area for the cloth—how much surface*** (wave your hand over the board) ***does the bulletin board have?***

Pick up a piece of paper and flip-flop it over the bulletin board a few times.

T: ***We could use this paper to measure how much cloth is needed. We could use an index card, a Post-it® note, or any number of things.***

Hold up a centimeter grid and one little centimeter square (1 cm by 1 cm).

T: ***But we will use this little square measuring 1 cm on each edge because it is a common unit for area. We say that its area is 1 square centimeter.***

Show students the abbreviation for square centimeter.

1 square centimeter
1 cm^2

Walk around the room and show the students how big the centimeter square is. Then distribute copies of Worksheet G3 (no star).

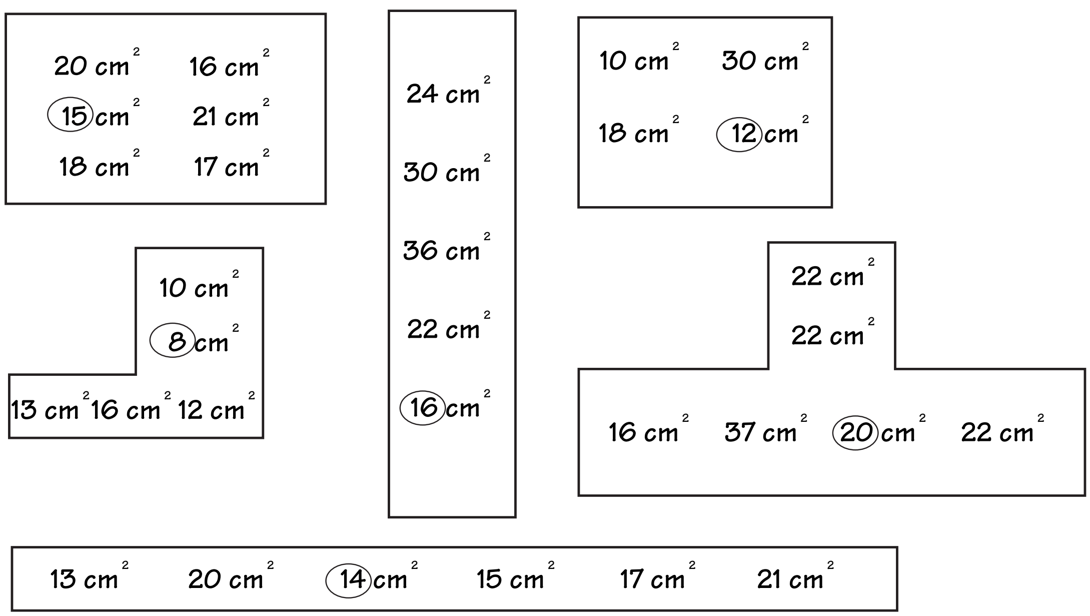
†See the "Note on Grids" section in the introduction to this strand.

T: ***Rather than find the area of the bulletin board today, we will find the areas of some smaller shapes like those on this worksheet. First I'd like you to estimate areas in square centimeters—estimate how many squares like this*** (hold up the centimeter square) ***it would take to cover each of the shapes.***

While students work on estimates, draw enlarged copies of the shapes on the board. After a few minutes, let students share their estimates and record some of them inside the shapes on the board. See the next illustration.

Provide students with centimeter grid paper and scissors. Direct students to find the areas of the shapes on the worksheet by cutting them out and covering them with centimeter squares.

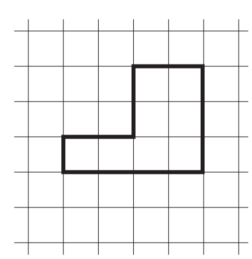
Check the answers collectively, and record or circle the correct areas among your estimates on the board.



Display a grid.

T: ***Let's pretend that these squares are the same size as those on your grid paper, namely 1 cm by 1 cm. Would someone draw one of the shapes from the worksheet on this grid?***

Suppose a volunteer draws this one.



With the help of a student, trace along the edges of the shape with string. Then cut the string to get a piece the same length as the border of the shape.

T (holding the string outstretched between two hands)**:** ***The length of this string is the same as the length of the border of the shape. We call this length the*** **perimeter** ***of the shape.***

Lay the string out along a grid line with a student's help.

T: ***This string measures approximately 14 of these lengths*** (indicate the length of one of the grid edges) ***. The string might stretch a little. If the grid edges are 1 cm each, then what is the perimeter of this shape?*** (14 cm) ***If you measure along the edge of this shape on your worksheet, you will find that the perimeter is 14 cm.***

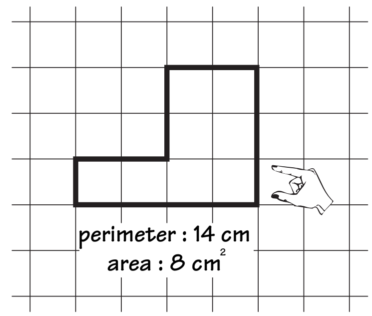
Record the perimeter and area near the shape.

T: ***The area of this shape is 8 square centimeters. Its perimeter is 14 centimeters. Perimeter of a shape is a length—the length of the border of the shape.***

How could we use the grid instead of string to find the perimeter of this shape?

S: ***We could just count the grid edges along the border of the shape.***

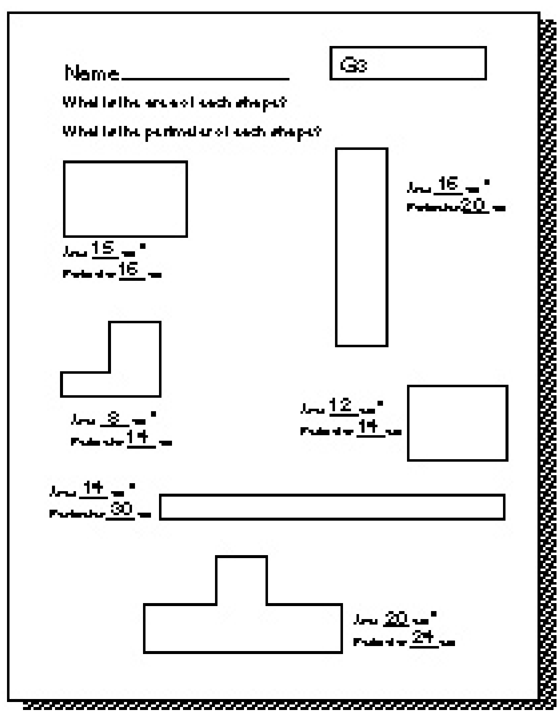
Invite the student to demonstrate this method at the board.

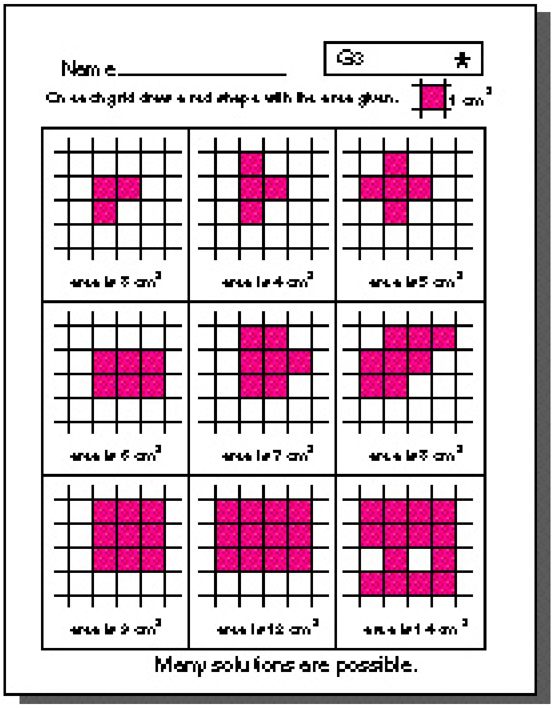


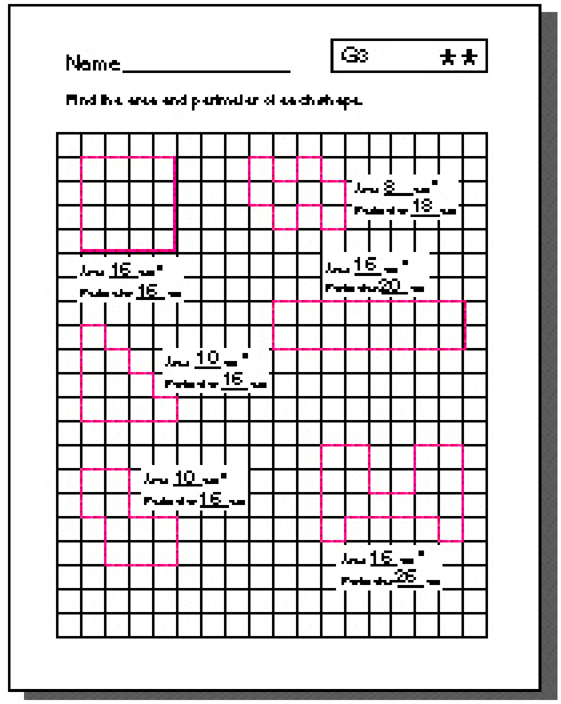
You may suggest that students mark the edge that they start with when using this method.

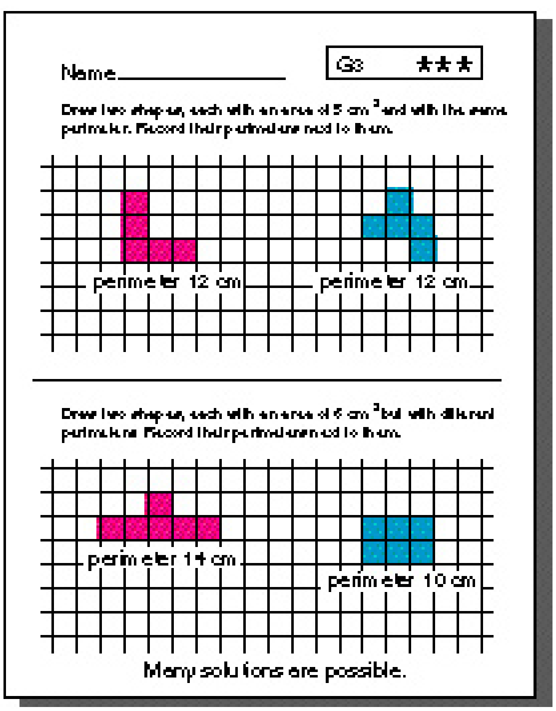
Use this method to find the perimeter of one of the other shapes on Worksheet G3 (no star). Then let the students individually find the perimeters of the other four shapes.

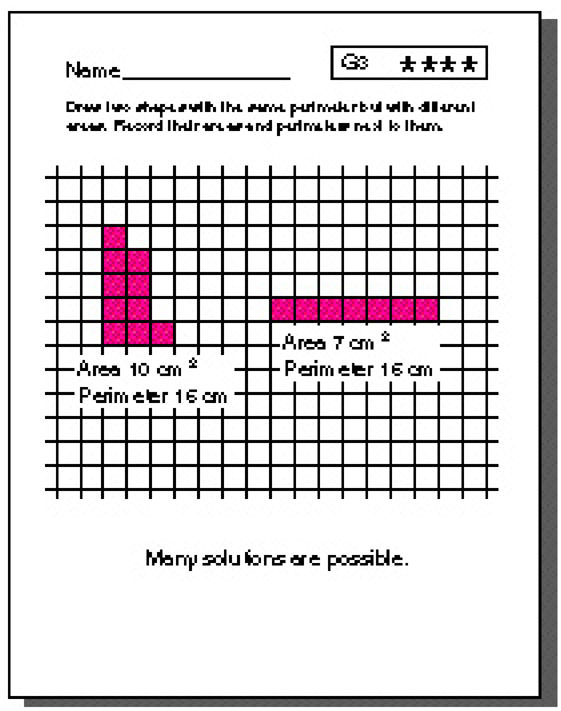
Worksheets G3*, **, ***, and **** are available for individual work.




Name
G3 ★
1 cm²
area is 3 cm²
area is 4 cm²
area is 5 cm²
area is 6 cm²
area is 7 cm²
area is 8 cm²
area is 9 cm²
area is 12 cm²
area is 14 cm²
Many solutions are possible.


Name
G3 ★★
Area 8
Perimeter 18
Area 16
Perimeter 16
Area 16
Perimeter 20
Area 10
Perimeter 16
Area 10
Perimeter 16
Area 16
Perimeter 26


Name
G3 ★★★
perimeter 12 cm
perimeter 12 cm
perimeter 14 cm
perimeter 10 cm
Many solutions are possible.


Name
G3 ★★★★
Area 10 cm ²
Perimeter 16 cm
Area 7 cm ²
Perimeter 16 cm
Many solutions are possible.

Capsule Lesson Summary

Given three squares, construct regions of specified areas and perimeters. By overlapping two squares, expose specified areas of the larger square.

Materials

Teacher
- Three squares
- Overhead projector (optional)
- Tape
- Grid board
- Colored chalk

Student
- Worksheet G4
- Colored pencils, pens, or crayons
- Scissors
- Centimeter grid paper

Advance Preparation: Prepare three squares for demonstration. If you use an overhead projector, prepare a blue, a green, and a red square as on Worksheet G4, for projection. Otherwise, prepare larger versions of these squares to use on the board. Grid posters in the classroom set have 8 cm grid squares and can be used for this purpose.

Description of Lesson

Note: This lesson describes two exercises: one finding all the possible areas of regions that can be constructed with three squares, and the other finding all the possible perimeters. You may prefer to combine these exercises (1 and 2) so students check both the area and perimeter of a constructed region at one time.

Exercise 1

Distribute scissors and Worksheet G4. Ask students to cut out the three colored squares carefully.

T: ***What size is each of these squares?***

Let students respond with their notion of size, and comment on references to area and perimeter. Include in the discussion that the grid squares are 1 cm by 1 cm and have area 1 cm^2. Write this information on the board.

Area □ = 1 cm^2
Length |—| = 1 cm

Read 1 cm^2 as "one square centimeter."

S: ***The area of the red square is 4 cm^2.***
The perimeter of the red square is 8 cm.

S: ***The area of the green square is 9 cm^2. Its perimeter is 12 cm.***

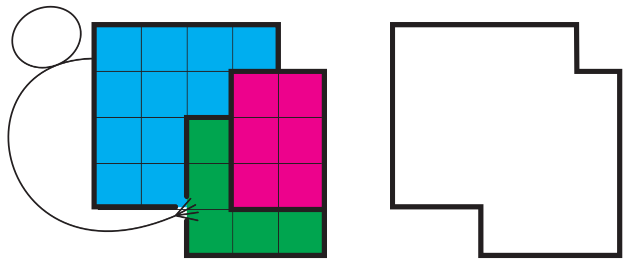
S: ***The area of the blue square is 16 cm^2. Its perimeter is 16 cm.***

Display your three demonstration squares.

T: ***I also have a red, a green, and a blue square. Let's pretend that they are the same size as your squares. I'll cover a region of the board.***

With the grid lines showing, place the three squares in some configuration like the following. Draw the boundary of the covered region. You may need to tape down the squares or ask students to help you hold them.

Lift the squares and hold them next to the outlined region.



T: ***We covered this region with the squares. What is its area?***

S: ***22 cm²***.

T: ***How did you calculate that?***

S: ***I counted the number of little squares.***

S: ***11 blue squares, 7 green squares, and 4 red squares are showing. 11 + 7 + 4 = 22.***

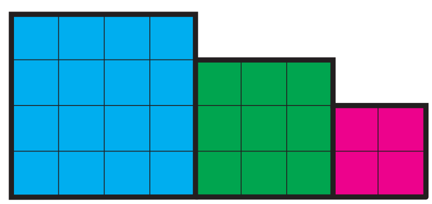
T: ***With these three squares, I was able to cover 22 cm² of the board. What is the area of a largest region we could cover with these three squares? Try it using your squares and grid paper.***

S: ***29 cm²***.

T: ***How did you cover a region with area 29 cm²?***

S: ***I put the three squares next to each other without overlapping.***

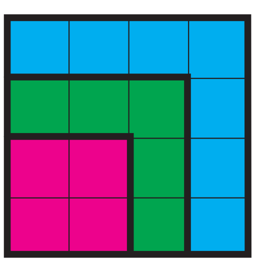
Show a student's solution, for example:



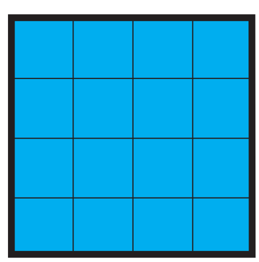
T: ***What is the area of a smallest region we could cover using all three squares?***

S: ***16 cm²***.

Invite a student to display a solution. Two possible solutions are shown below.



or



(The red and green squares are placed underneath the blue square.)

Record the whole number areas from 16 cm^2 to 29 cm^2 on the board. Circle those that the class has already found.

16 cm^2	21 cm^2	26 cm^2
17 cm^2	22 cm^2	27 cm^2
18 cm^2	23 cm^2	28 cm^2
19 cm^2	24 cm^2	29 cm^2
20 cm^2	25 cm^2	30 cm^2

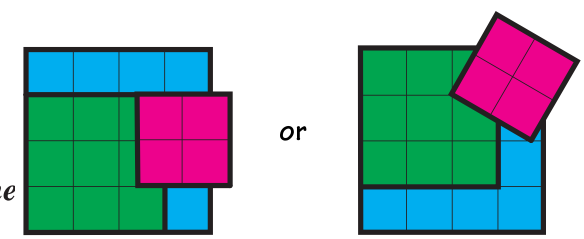
T: ***Using these three squares, the area of the smallest region we can cover is 16 cm^2 and the area of the largest region we can cover is 29 cm^2. Also, we were able to cover a region with area 22 cm^2. Do you think we could cover regions with any of these other areas using these three squares?***

Accept any suggestions between 16 cm^2 and 29 cm^2.

T: ***On your worksheet, find the areas of regions that you can cover with your three squares. There are two rules:***

1. You must use all three squares each time to cover a region.
2. The grid lines of the squares must line up.

Use examples to demonstrate the second rule. The grid lines must line up as in the previous examples. They should not look like this:



T: ***After you construct a region, record it on your grid paper as follows:***

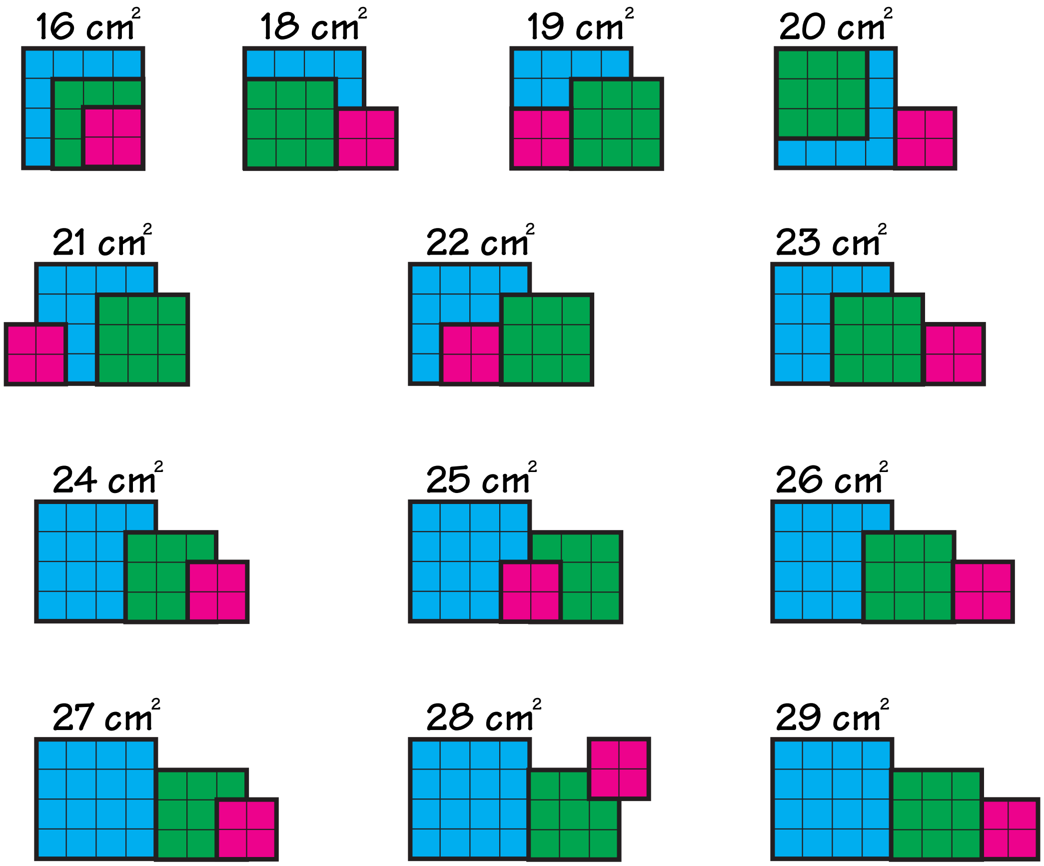
1. Cover a region of your grid paper with the squares with lines on the grid paper.

2. Draw the boundary of the region.

3. Calculate and record the area of the region.

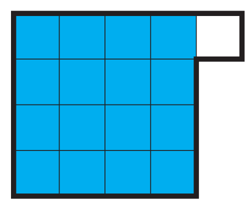
Let the students work individually or with a partner for about ten minutes. When a student draws a region with an area not yet in the list, check the student's solution and include that area in the list on the board. Acknowledge all correct drawings, but challenge students to find regions with areas not yet in the list. Be prepared to provide students with more centimeter grid paper.

It is possible to cover regions with all whole number areas betweeen 16 cm^2 and 29 cm^2, except for 17 cm^2. Challenge the class to find solutions for all these areas. A set of solutions follows.



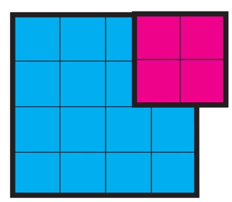
T: ***It appears to be impossible to construct a region with area 17 cm². Can anyone convince the class of that?***

S: ***The area of the blue square is 16 cm². To form a region with area 17 cm², you need to add one small square; for example:***



But it is impossible to do this with the green or red square.

A student may suggest the following solution for 17 cm². Point out that it looks good (has area 17 cm²) but does not follow the rule that grid lines must line up.

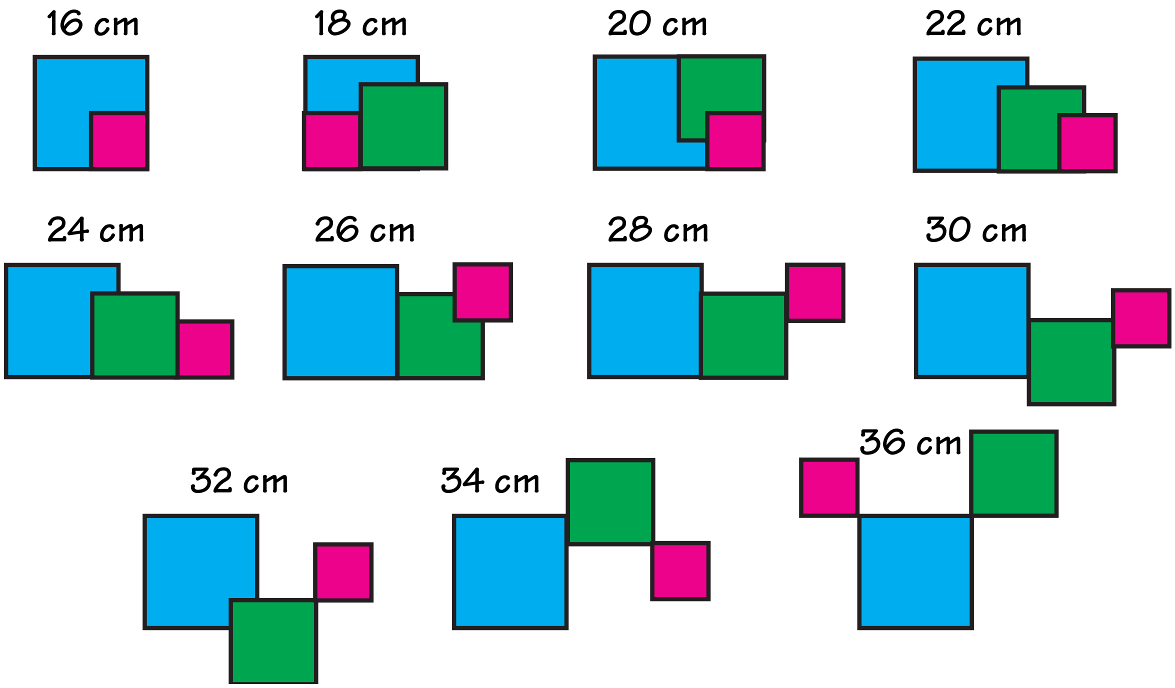


Exercise 2

Repeat Exercise 1 looking for regions with perimeters between 16 cm (least) and 36 cm (greatest).

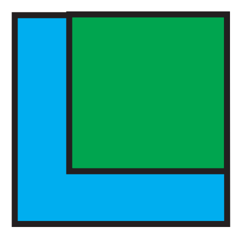
In this case, only regions with even perimeters can be found, but every even perimeter from 16 cm to 36 cm is possible. A complete set of solutions is given on the next page.

Students may begin by finding the perimeters of the regions they found in Exercise 1. This will likely only give solutions for perimeters less than 30 cm.



Exercise 3

Set aside the red square. Place the green square on the blue square so that the grid lines are not showing, or draw this picture on the board.



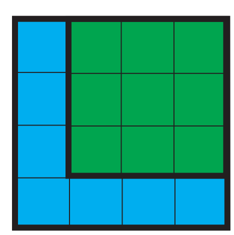
T: ***We can see only part of the blue square. What is the area of that part?***

S: ***7 cm^2.***

T: ***Why?***

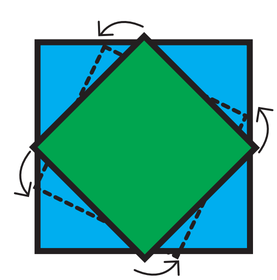
S: ***If the grid lines were showing, there would be 4 small blue squares down the side and 3 more blue squares along the bottom.***

If no one gives the correct area or if no one can confirm that the area is 7 cm^2, set up the same configuration with grid lines showing or draw in the grid lines.



Determine that 7 cm^2 of the blue square is uncovered and note the equivalence of the two problems.

Without showing the grid lines, rotate the green square as you place it on the blue square, or draw this picture.



T: ***We can see only part of the blue square. What is the area of that part?***

If no one answers 7 cm^2, ask the following series of questions.

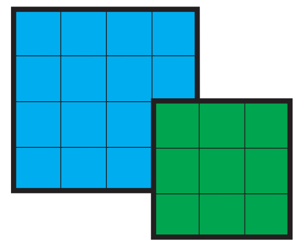
T: ***What is the area of the blue squares?*** (16 cm^2)

How much of the blue square is covered by the green square? (9 cm^2)
So how much of the blue square is uncovered? (7 cm^2, since 16 – 9 = 7)

Display this configuration.

T: ***We can see only part of the blue square. What is the area of that part?***

S: ***14 cm^2. I counted the little blue squares.***



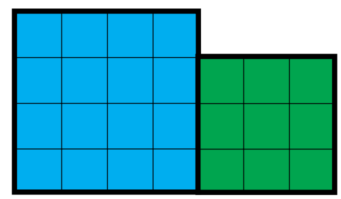
S: ***14 cm^2. The area of the blue square is 16 cm^2 and 2 cm^2 are covered.***

T: ***Using the green square and the blue square, what is the area of the smallest blue region that I can leave uncovered?***

S: ***7 cm^2. You already did it.***

T: ***What is the area of the largest blue region that I can leave uncovered?***

S: ***16 cm^2, place the green square next to the blue square.***

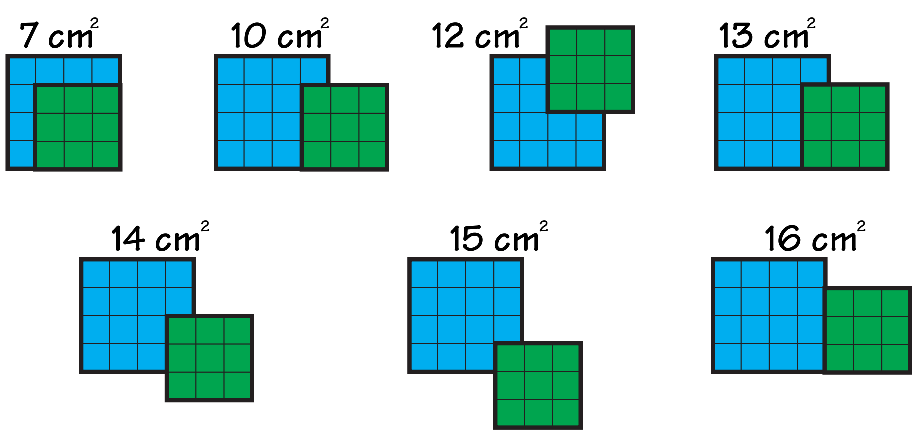


Record whole number blue areas from 7 cm^2 to 16 cm^2 on the board, and circle them. Those the class has found can be left uncovered.

7 cm^2	12 cm^2
8 cm^2	13 cm^2
9 cm^2	14 cm^2
10 cm^2	15 cm^2
11 cm^2	16 cm^2

T: ***Using your blue square and green square, try to find blue regions for all of the whole number areas between 7 cm^2 and 16 cm^2.***

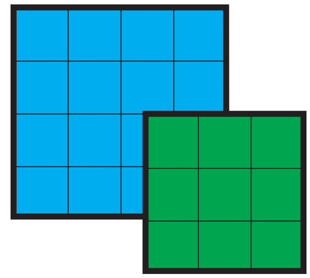
As students suggest areas not yet circled in your list, check their solutions and then use your demonstration squares to show the whole class. If students follow the rule of keeping the grid lines aligned, the following areas are possible:



If your students have found all of the solutions and there is sufficient time and interest, continue the activity in the following way.

T: ***Following our earlier rules, we cannot find solutions for 8 cm², 9 cm², or 11 cm². Suppose we ignore the rule that grid lines must line up.***

Display this configuration.

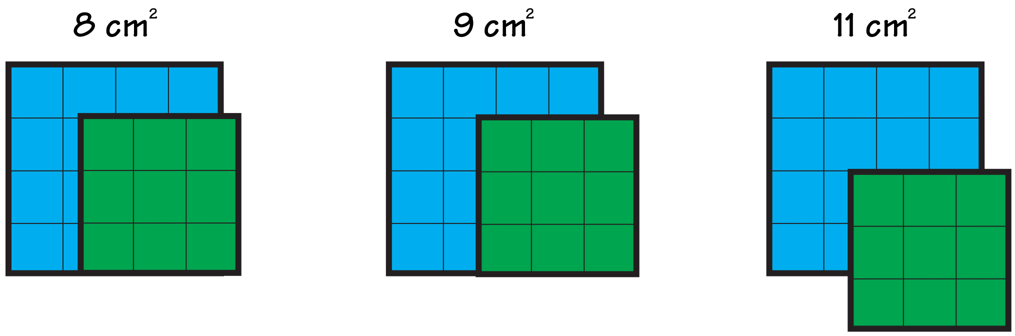


T: ***How much of the blue square is uncovered?***

S: ***13 cm², 12 whole squares and 2 half squares.***

T: ***We already had 13 cm², but maybe you can find solutions for 8 cm², 9 cm², and 11 cm² with this method (no grid lines rule).***

Possible solutions are shown below. The solutions for 8 cm² and 9 cm² cannot be done using half squares only.



Capsule Lesson Summary

Introduce the translator, parallelism, and a basic parallelogram construction. Practice using the translator to draw parallelogram trains.

Materials

Teacher
- Translator
- Colored chalk
- Tape
- 40–50 cm piece of wood
- Colored pencils, pens, or crayons
- Large unlined paper
- Worksheets G5* and **

Student
- Translator

Advance Preparation: Read the section on parallelism in the introduction to this strand. Using the translator, practice drawing some line segments and some parallelograms on the board or on paper before you teach this lesson. You may want to extend the straight edge portion of your translator in order to draw longer line segments on the board. Do this by taping a 40–50 cm straight piece of wood on top of the straightedge part of the translator.

Description of Lesson

This lesson introduces the translator, parallelism, and a basic parallelogram construction. Familiarity with the concept of parallel lines, and dexterity with the translator in performing the constructions, will grow over the course of the sequence of lessons on parallelism. Do not be alarmed if at first students have difficulty using the translator.

An important distinction for you to bear in mind during these lessons is that between *lines* and *line segments*. While you will probably use whatever terminology is customary with you, the necessity for making the distinction may arise. Your students will benefit from learning the distinction, and we refer to both in these lessons. Occasionally, indicate to the class that a segment is only part of a line and that the line can be imagined as the extension of both ends of the line segment "to infinity" or "forever."

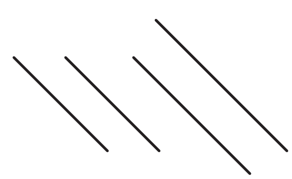
Display a translator.

T: ***This is a translator. We can use it to draw line segments.***

Draw a long line segment on the board.

T: ***The translator has wheels so that it will roll across the board or across your paper.*** (Demonstrate rolling the translator.) ***We can draw a line segment, roll the translator, and then draw another line segment. We can do this as many times as we want.***

Start at your original segment, roll the translator to new positions, and draw several more segments, each time rolling the translators 15–20 centimeters.



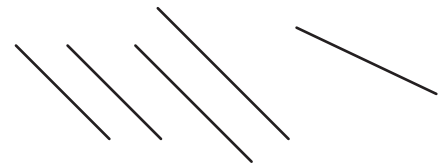
T: ***What do you notice about these line segments?***

S: ***They all go in the same direction.***

S: ***They do not cross.***

S: ***They look like the rows in a garden (or a marching band).***

With several pairs of line segments on the board, demonstrate aligning the translator with one of the line segments and rolling to the other. Then draw a segment at an angle to those already drawn.



T: ***Is this line segment parallel to the others?***

S: ***No.***

Demonstrate that aligning the translator with one of the first four line segments and rolling it results in the line of the translator crossing the last line segment drawn.

T: ***Two lines are not parallel if the translator can be aligned with one and then rolled so that the line of the translator crosses the other.***

Note: You may like to use a chant to accompany and reinforce the use of the translator.

“Parallel, parallel, parallel, roll.”

Emphasize and extend the last word. Such a chant may serve as a warm-up in subsequent lessons on parallelism and stimulate students to remember the dynamics of the translator.

T: ***We can also use the translator to draw a special shape.***

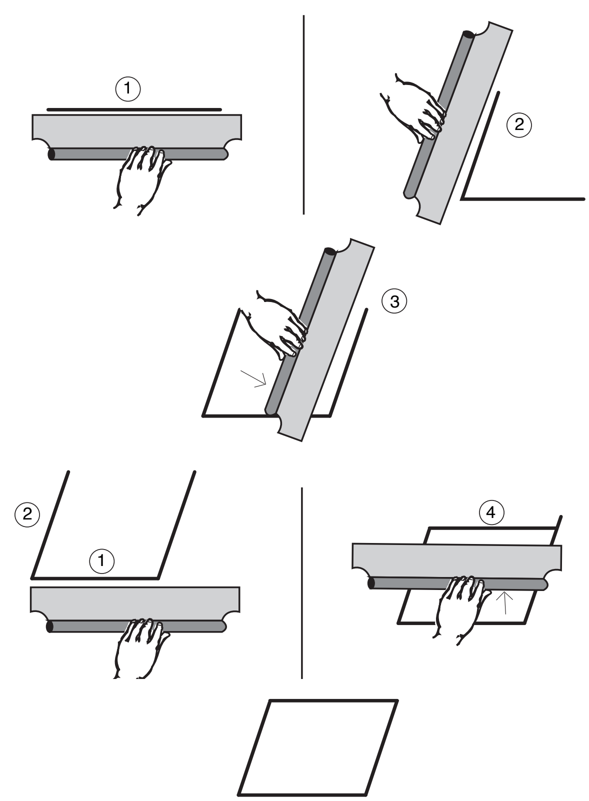
The construction that follows is easier to perform than to describe. Describe your actions while performing the construction, and emphasize the verbs. The circled numbers refer to line segments so indicated in the illustrations.

T: ***Watch me draw a shape with the translator.***
I draw a line segment 1.
Then I draw another line segment 2
starting at one end point of my first segment.

Now I roll the translator to the other end of the first line segment and draw another line segment 3.

I align the translator with the first line segment 1, and roll the translator to the endpoint of the second line segment 2, where I draw the last segment 4.

I tidy the corners by erasing any pieces of lines that extend beyond the corners.



T: ***What do you notice about this shape?***

S: ***It has four sides.***

S: ***Opposite sides are parallel.***

S: ***It looks like a tilted rectangle.***

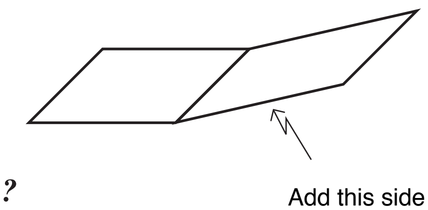
T: ***This shape is called a parallelogram because it has four sides and opposite sides are parallel.*** (Demonstrate again that the left side is parallel to the right side, and that the top side is parallel to the bottom side.)

Direct students to practice with their translators, first drawing sets of parallel line segments, and then drawing a parallelogram. After a few minutes, call the class's attention back to the board.

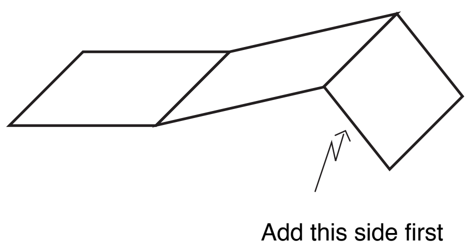
T: ***I'm going to draw another parallelogram attached to one on the board.***

Again, describe the construction while doing it.

T: ***Could we draw another parallelogram attached to the one I just drew? Who can trace another parallelogram for us that is built on this side*** (point to the side furthest to the right in the drawing)***?***



Invite a student to trace a parallelogram on the board. After someone has given a rough tracing, perform the construction yourself and again describe the steps. Be sure to emphasize aligning the translator with a line segment, rolling, and then drawing a parallel line segment. Your drawing may now look like the one below.



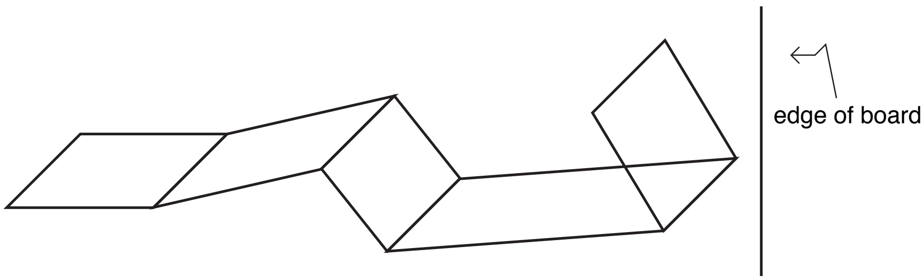
Repeat this activity, attaching a parallelogram to the last one drawn a couple more times. Solicit comments from students about the resulting picture. They may say it looks like a train of parallelograms. If not, use the word *train* yourself to describe adding another parallelogram.

Continue drawing parallelograms that students trace until the parallelogram train reaches the edge of the board.

Instruct students to draw a parallelogram train on their papers. You may want to suggest students tape their papers to their tables. Provide technical assistance as necessary.

When you notice a student stop his or her train at the edge of the paper, ask if there is a way to add another parallelogram car to the train (perhaps other than a very small one). After many students have drawn several parallelograms on a train, call the class's attention back to the board.

Again pose the problem of constructing another parallelogram using the right side of the last parallelogram constructed. This problem may be difficult for many students. They could suggest that a parallelogram be drawn in the small space between the end of the drawing and the edge of the board. Indicate that it may be possible, but there is not much room to maneuver the translator and it would be an awfully small parallelogram car on the train. Encourage discussion until a solution is found that reverses the direction of the construction as in the drawing below.

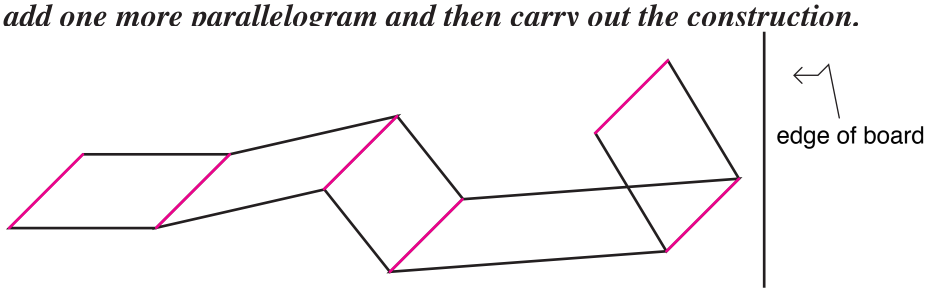


Trace over the left-most line segment in red.

T: ***Look at the drawing. Who can find several line segments that are all parallel to the red line segment? Are there more?***

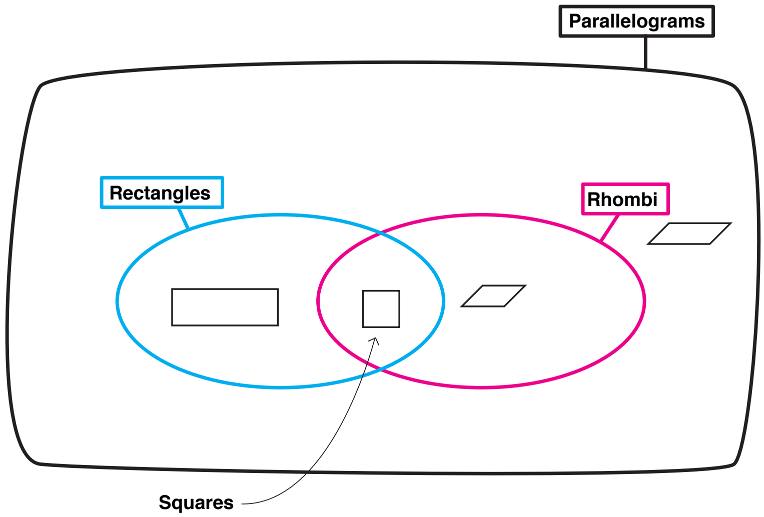
Invite students to point out the parallel line segments in the drawing on the board. As they are identified, retrace them with red chalk. This will emphasize the special collection of parallel line segments that arise in the construction. Your drawing should look similar to the one below.

T: ***We'll add one more parallelogram and then carry out the construction.***



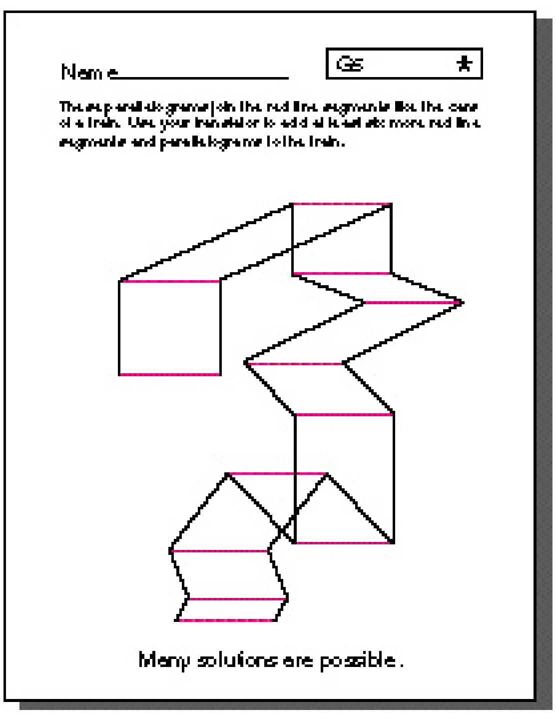
Worksheets G5* and ** are available for individual work. Do not be concerned if students have difficulty with the constructions. At this time they are getting used to the translator and how it works. You may make a few extra copies of the worksheets for students who want to start over, or suggest they do so on their papers.

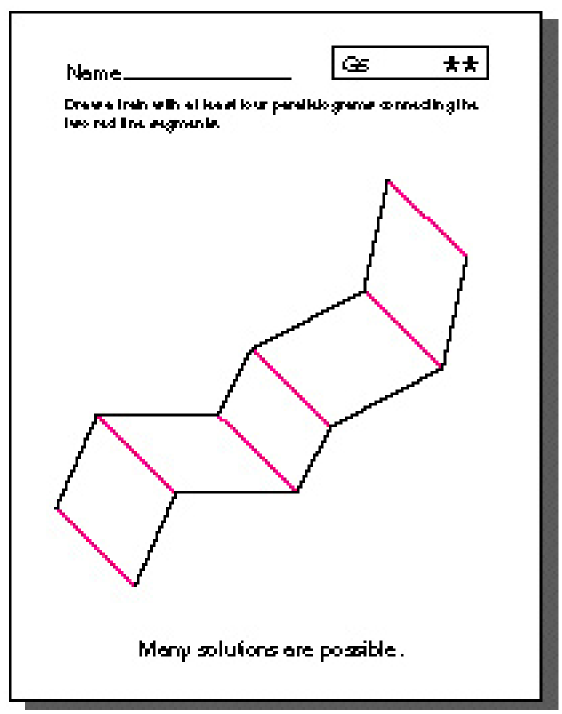
Note on Terminology: The *parallelogram* is the principal shape encountered in these lessons. A parallelogram is a four-sided figure with opposite sides parallel but with no condition on the corner angles. There are several ways to specialize the definition: a parallelogram with four angles of equal measure is called a *rectangle*. A parallelogram with four sides of equal length and four angles of equal measure is called a *square*. Thus, every square is a rectangle and every rectangle is a parallelogram. Also, every square is a *rhombus* and every rhombus is a parallelogram. In particular, every square is a parallelogram.



Additional Practice

Place translators and large unlined paper out for student exploration.





Capsule Lesson Summary

Draw parallelogram trains. Introduce *couples* and *equipollence*. Draw more trains to construct families of equipollent couples.

Materials

Teacher
- Translator
- Colored chalk
- Masking tape
- Colored pencils, pens, or crayons
- Large unlined paper
- Worksheets G6* and **

Student
- Translator

Advance Preparation: Before teaching the lesson, reread the section on parallelism in the introduction to this strand.

Description of Lesson

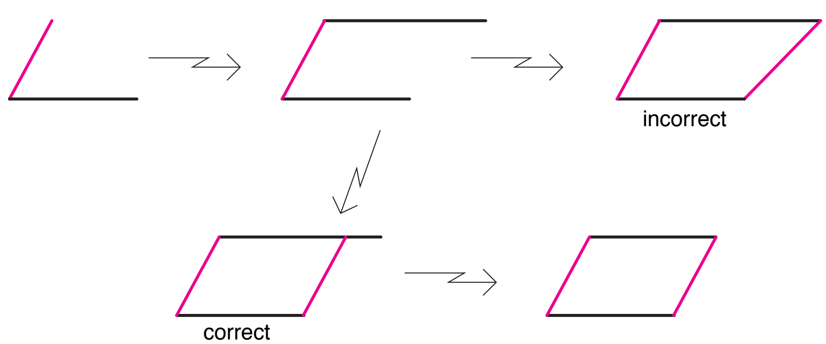
Display your translator and demonstrate its use by drawing several mutually parallel line segments. Then draw a line segment at an angle to the others and use the translator to show that it is not parallel to the others. All the while, describe your movements and ask which segments are parallel to which other segments. Review the basic parallelogram construction. You may like to ask students to follow the construction by doing it as well on their papers.

T: ***Help me draw a parallelogram. I will draw two sides.***

Draw two sides as illustrated. Using two different colors of chalk will help to emphasize the construction process of rolling the translator to build opposite sides of the parallelogram.

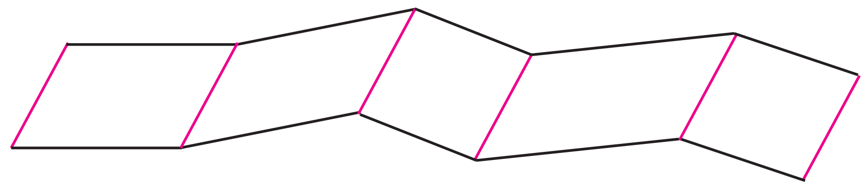
T: ***Remember that we use a translator to draw the other two sides. Who can trace where those sides should be?***

When a student has given a rough tracing of a third side, use the translator to locate it precisely. Then invite another student to locate the fourth side. If you drew the third side too long or too short (which is to be expected), many students will simply join the endpoints of the segments (see the incorrect completion in the illustration) rather than indicate a segment that is parallel to the opposite side. Request several tracings until a student is close to accurate. Then perform the construction with your translator, all the while describing your actions. In order to emphasize the construction process, say “red, roll, roll, red” while rolling the translator when you start at the red edge, and say “black, roll to black” when starting at the black edge. Tidy the drawing by erasing extra long segments.



G6

Continue to draw a train of parallelograms across the board. Each time before you add another parallelogram to the train, ask a student to trace how it will look.



T: ***Who can find two parallel line segments in the drawing?***

Invite students to trace pairs of parallel line segments in the drawing.

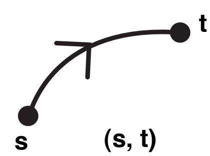
T: ***Who can find a set of more than two line segments that are all parallel?***

Continue the discussion until the full family of six (in this illustration) mutually† parallel red line segments is found.

On another part of the board draw two dots and label them **s** and **t**.

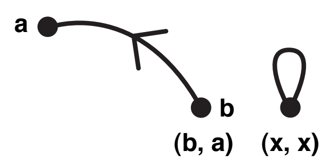
T: ***Here are two points, s and t. We can join two points together and call them a couple. To make a couple from two points, we need to choose one to be the starting point. The other will, of course, be the ending point. An arrow between the points will help us remember which point is first and which is second in the couple.***

Draw an arrow from **s** to **t**.



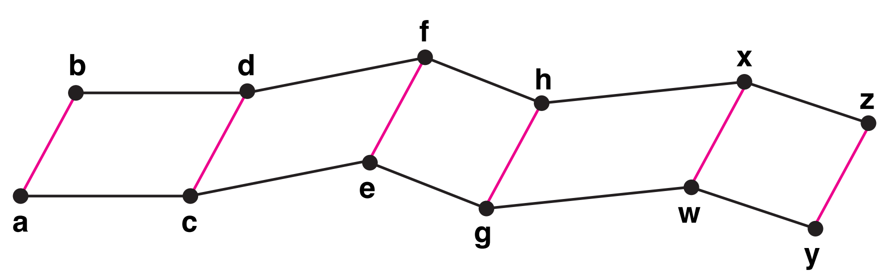
T: ***Another way of remembering is to write the name of the couple in a special way, using parentheses. If s is the first point and t is the second, we write (s, t) for the couple.***

Repeat the explanation for one or two other examples.
Return your class's attention to the parallelogram train on the board.



T: ***Let's give names to the corner points of this parallelogram train.***

Emphasize each point by highlighting it with a dot. Label the dots with letters.



T: ***Who can name some couples in our drawing?***

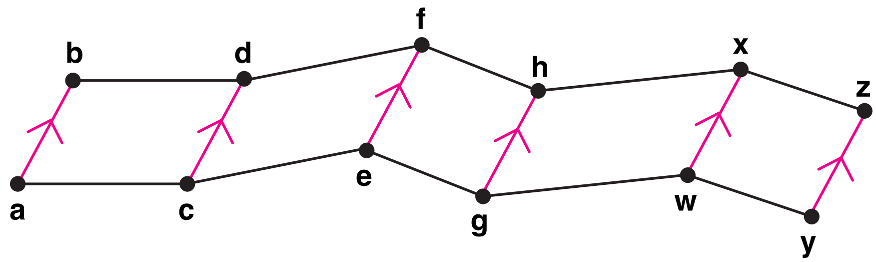
†A technical adverb denoting, in this case, that any two of the six line segments are parallel.

To correctly name a couple, all a student needs to do is to name any two points of the drawing. There are 144 couples in the illustration. However, the line segments of the parallelogram train will tend to limit students' perception of couples to those that are connected by one of the line segments. There are 32 such couples in the illustration.

After several couples have been named, continue.

T: ***Let's draw red arrows for some of the couples. We already have some red line segments.***

Draw a red arrow for each of the couples (**a**, **b**), (**c**, **d**), (**e**, **f**), (**g**, **h**), (**w**, **x**), (**y**, **z**).

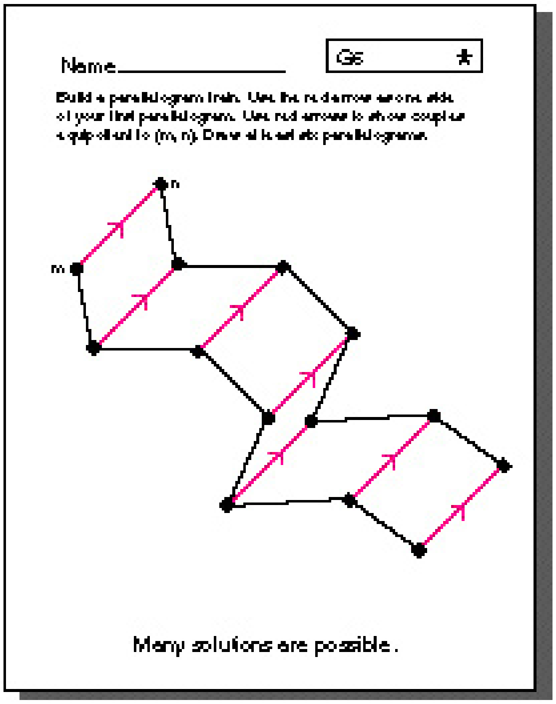


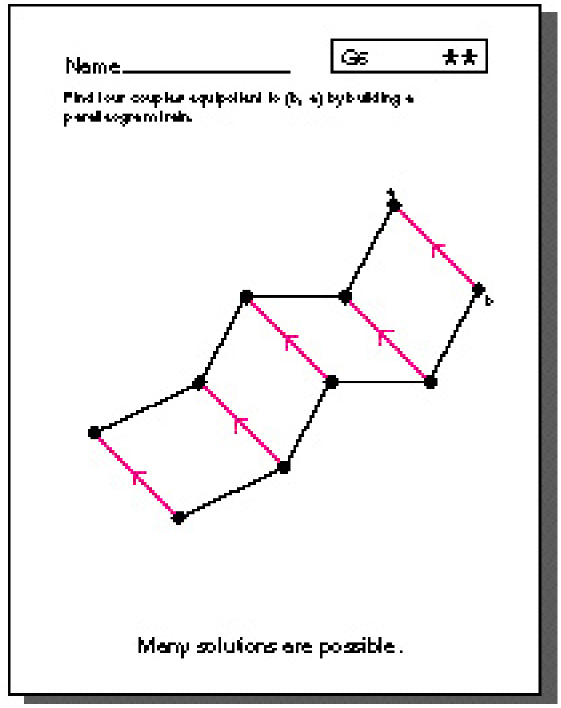
T: ***The couples marked with red arrows are related. The line segments that connect each of these couples are parallel and are connected by a parallelogram train, and all of the arrows are pointing in the same direction. The starting point of an arrow*** (point to **a**) ***is connected to the starting point of the next arrow*** (point to **c**)***. The ending point of an arrow*** (point to **b**) ***is connected the ending point of the next arrow*** (point to **d**)***. We say that such couples are* equipollent.** (Write this word on the board and ask the students to read it aloud several times.) ***All of the couples marked with red arrows are equipollent.***

Now it is your turn to draw a parallelogram train and to find equipollent couples. Remember to connect starting points to starting points and ending points to ending points.

Distribute translators, copies of Worksheets G6* and **, and colored pencils. Leave the parallelogram train on the board for the students to consult while they work. Students who finish quickly can make their own pictures with parallelograms or other translator constructions.

Note on Terminology: *Equipollence* is an equivalence relation on ordered pairs (couples) of points. Mathematics uses many equivalence relations such as congruence (for shapes) or equivalent fractions. When objects are related by an equivalence relation, they are essentially alike. The word equipollent comes from vector terminology where vectors (quantities with magnitude and direction) having equal effect are equipollent. For our purpose, this equivalence relation will make it easier to describe and carry out constructions as well as to highlight properties of parallel translations and projections.


Name
n
m
Many solutions are possible.


Name
Many solutions are possible.

Capsule Lesson Summary

Introduce the idea of a graph. Color dots in a graph so that dots connected by an edge are different colors; use as few colors as possible. Color countries in a map so that countries sharing a border are different colors; again, use as few colors as possible. Find a correspondence between a map and a graph relating the two color problems.

Materials

Teacher
- Colored chalk

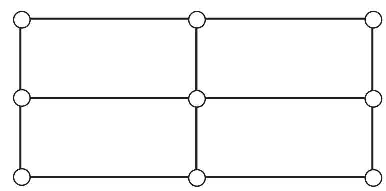
Student
- Unlined paper
- Colored pencils, pens, or crayons
- Worksheets G7(a) and (b)
- Colored counters (optional)

Description of Lesson

The two exercises of this lesson are related coloring problems. Try to do both exercises in one period, allowing about half the time for each.

Exercise 1

Draw this picture on the board.



T: ***What does this picture look like?***

S: ***A grid.***

S: ***Tinker toys.***

S: ***A game board.***

Allow students to express their ideas before introducing the following terminology.

T: ***This is an example of a graph. A graph has dots*** (point to some of the circles in the picture) ***and edges*** (point to some of the connectors between circles in the picture)***. In this picture the dots are not filled in because later we are going to color them. How many dots are in this graph?***

S: ***Nine.***

T: ***How many edges?***

S: ***Twelve.***

T: ***Now we are going to color in the dots, but there is a rule we must obey.***

Write this color rule on the board.

Dots connected by an edge must be different colors.

T: ***Before we color, let's decide how many colors to use. How many colors could we use?***

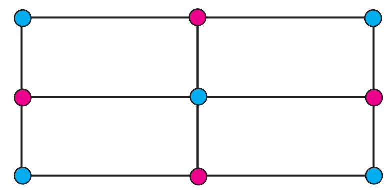
S: ***Nine; we could color every dot a different color.***

S: ***Five; color the middle dot one color and the four dots connected to it different colors. Color the dots not connected to the middle dot (the corner dots) the same color as the middle dot.***

Get several suggestions (answers can range from two to nine) before further restricting the problem.

T: ***What is the fewest number of colors we could use?***

S: ***Two.***

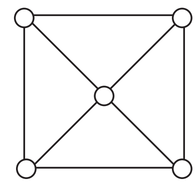


Whatever response is given, ask the student to show how to color the dots with that number of colors. Check that the rule is not broken. Encourage students to look for a coloring with fewer colors until they find that two is sufficient.

T: ***Could we use only one color?***

S: ***No, then edges would connect dots of the same color.***

Leave the first picture on the board and draw a second one next to it.

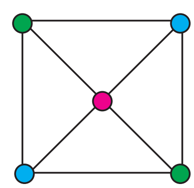


T: ***Here is another graph. How many dots and how many edges does it have?***

S: ***Five dots and eight edges.***

T: ***Copy this graph on your paper. Then color the dots following the rule, but try to use as few colors as you can.***

Note: You may like to suggest that students put colored disks (counters) on the dots to represent coloring. Then they can change or manipulate the colors easily. As you observe individual work, encourage those who use more than three colors to try to find a solution with fewer colors.



After a while call the class's attention back to the board. Invite a student to color the dots in the picture on the board; then ask whether anyone has a solution with fewer colors. The result of this collective exploration should be that three colors are needed. One possible solution is shown here.

T: ***Can anyone explain why we need three colors; why can't we use only two colors?***

S: ***Look at these three dots*** (the center and two connected outside dots)***. They are all connected to one another so they must all be different colors.***

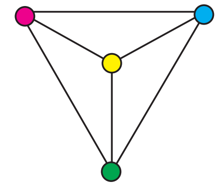
You may not expect such a sophisticated response, but at least students should convince themselves that three colors are needed.

T: ***Can you imagine a graph for which we would need only one color?***

S: ***Just a single dot. If there is an edge, we need two colors for the dots at the ends of this edge.***

T: ***Are there any graphs that need more than three colors? Try to find one.***

Allow several minutes for students to try to find such a graph. If one is found, put it on the board; otherwise leave the problem unsolved.



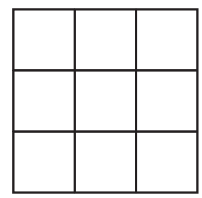
Note: For your information, here is a simple graph that requires four colors.

Distribute copies of Worksheet G9(a) and ask students to color the dots in the graphs following the color rule, using as few colors as possible.

When several students have completed Worksheets G9(a), check the results collectively. You may want to draw one or two of these graphs on the board and invite students to color the dots. An answer key follows this lesson.

Exercise 2

Erase the board and draw this picture.



T: ***What does this picture look like?***

S: ***A grid.***

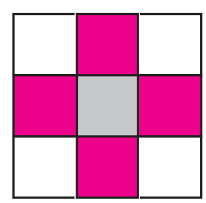
S: ***A checkerboard.***

S: ***A graph with the dots missing.***

T: ***This picture is a map of some countries; all of the countries are shown as squares. The line segments in the picture are the borders of the countries. How many countries are there?***

S: ***Nine.***

T: ***If you are going to color a map, it is helpful to use different colors for the different countries, especially when two countries share a border.***



In your picture, illustrate countries that share a border. For example, the middle country shares a border with each of four other countries.

Countries do not share a border if it is just one point, like the corner of a square in this example.

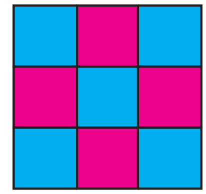
T: ***Suppose we try to conserve colors—when you print a map it is expensive to use a lot of colors—and still color the countries in the map so that those sharing a border are different colors.***

Write the color rule on the board.

Countries sharing a border must be different colors.

T: ***How can we color this map? Remember, we want to follow the rule and use as few colors as possible.***

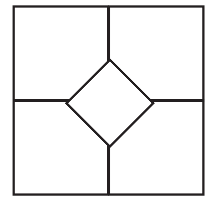
Allow students to experiment on their papers. They may use colored



counters to simulate coloring. Invite students to color the map on the board, checking that they have a definite coloring in mind. Keep asking for a solution with fewer colors until the class discovers that two are sufficient.

Some students may notice that this problem is similar to the graph coloring problem. Hold off discussion of the similarity until after the next example.

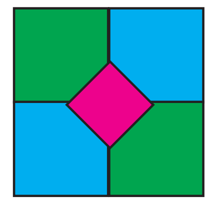
Leave the first map on the board and draw a second one next to it.



T: ***Here is another map. How many countries does it have?***

S: ***Five.***

T: ***Copy this map on your paper. Then color the countries, following the rule and using as few colors as possible.***



As before, allow time for individual work before asking a student to show a solution on the board. This time three colors are needed.

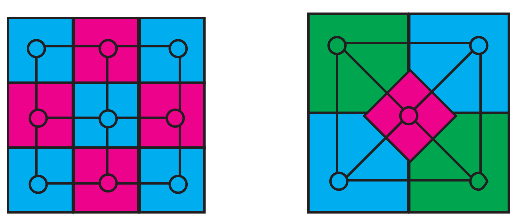
T: ***Do you notice any similarity between the graph-coloring problems and these map-coloring problems?***

Let students express their ideas. Direct the discussion as necessary to bring out these points.

1. The dots of a graph are like the countries of a map (that is what is being colored).
2. The edges of a graph are like shared boundaries in a map.

Dots connected by an edge are different colors.
Countries sharing a border are different colors.

Illustrate these ideas in the pictures at the board, as shown below.

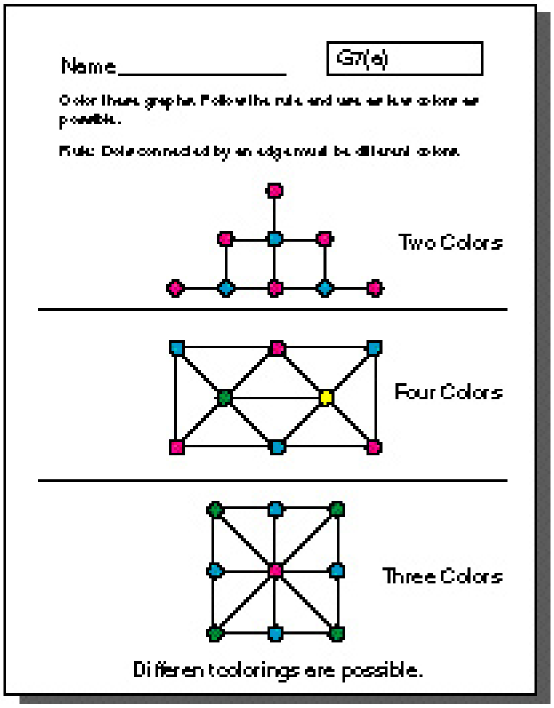


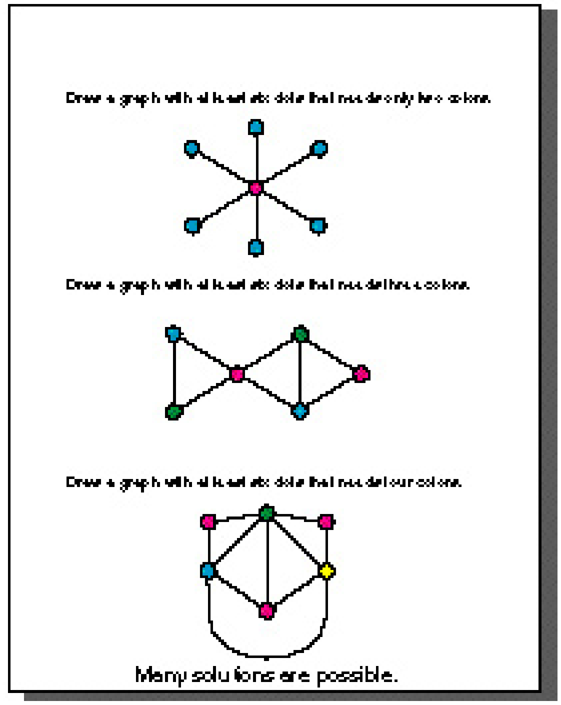
Students should observe the connection between the graphs in Exercise 1 and the maps in Exercise 2.

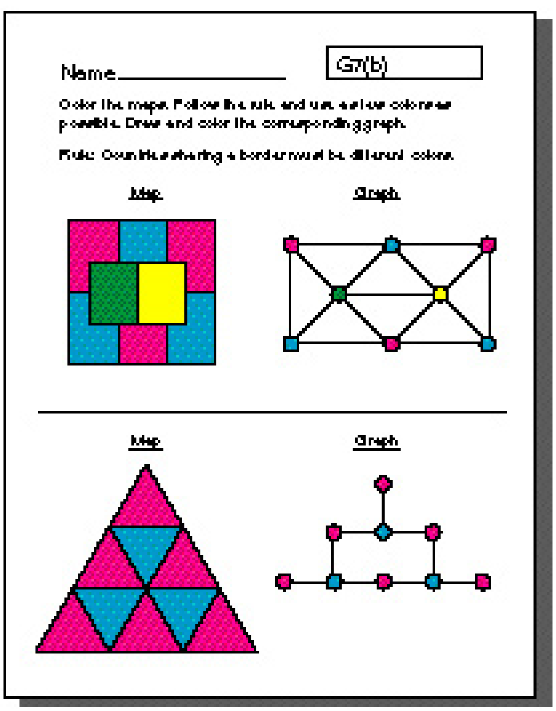
Distribute copies of Worksheet G9(b). Ask students to color the maps and to draw the corresponding graphs. You might suggest that they draw the graph for a map on top of it (before coloring) and then copy it to the right of the map. After coloring the map, students should color the dots on the graph in a corresponding manner.

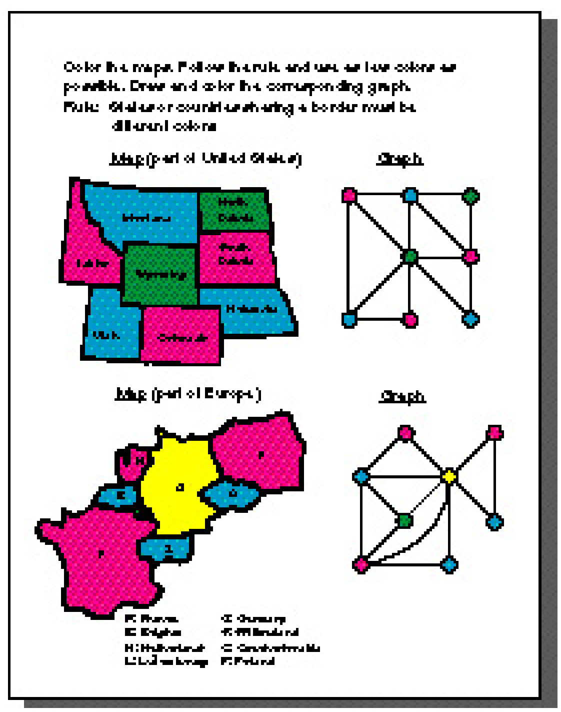
Extension Activity

Examine maps to see how many colors are used in the printing. Determine when fewer colors could be used and still have countries (states) sharing a border in different colors.


Name
G7(a)
Two Colors
Four Colors
Three Colors
Different colorings are possible.


Many solutions are possible.


Name
G7(b)
Map
Graph
Map
Graph


Graph
Graph

Capsule Lesson Summary

Practice recognizing equipollent couples. Construct families of equipollent couples with specified starting and ending points.

Materials

Teacher
- Translator
- Colored chalk
- *IG-I Geometry Posters* #1
- Masking tape

Student
- Translator
- Colored pencils, pens, or crayons
- Large unlined paper
- Worksheets G8*, **, ***, and ****

Advance Preparation: Draw Figure 1 (Exercise 2) on the board before the lesson begins. Note that the distance from **x** to **y** must be the same as the distance from **a** to **b**.

Description of Lesson

Exercise 1

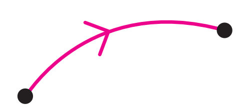
Begin the lesson by reviewing the notions of parallelogram, couple, and equipollence.

Draw two dots on the board.

T: ***How can we show a couple here?***

S: ***Draw an arrow to show the starting point and the ending point.***

Draw a red arrow from one dot to the other.



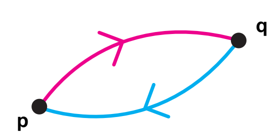
T: ***What could we name this couple?***

S: ***We could label the starting point* p *and the ending point* q*; then the couple is* (p, q).**

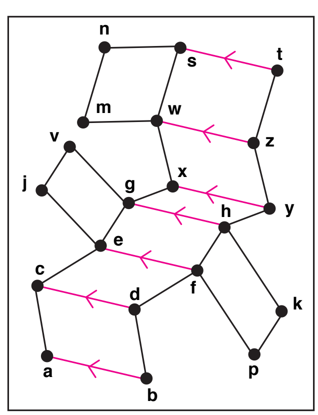
T: ***Can I indicate another couple with these dots?***

S: ***Yes,* (p, q)*. Draw an arrow from* q *to* p.**

Draw a blue arrow from **q** to **p**.



Display *IG-I Geometry Poster* #1.



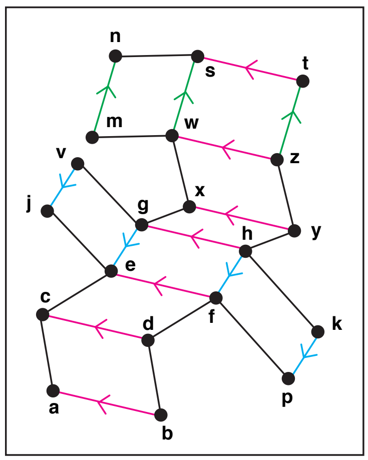
G8

Ask the class to name the couples that are identified by arrows. There are seven given by red arrows: **(b, a)**; **(d, c)**; **(f, e)**; **(h, g)**; **(y, x)**; **(z, w)**; **(t, s)**.

T: ***Are these couples equipollent?*** (Yes)
Are there other equipollent couples in this drawing?

S: ***Yes, we could add more arrows to show more equipollent couples.***

Accept all suggestions for equipollent couples if they are correct, but only draw arrows for the equipollent couples **(m, n)**, **(w, s)**, **(z, t)** and for the equipollent couples **(v, j)**, **(g, e)**, **(h, f)**, **(k, p)**. This will delineate two more equipollent families. Use a different color for each family. Color the first couple of each family that is mentioned. When another couple is suggested, ask what color to use and which direction to point the arrow. You may like to list equipollent couples on the board as you progress.



(m, n)
(w, s)
(z, t)

(v, j)
(g, e)
(h, f)
(k, p)

Exercise 2

Refer to your drawing of Figure 1 on the board.

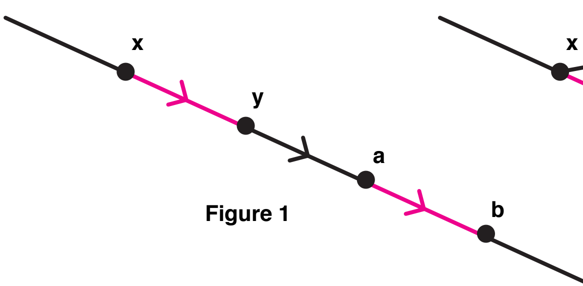


Figure 1

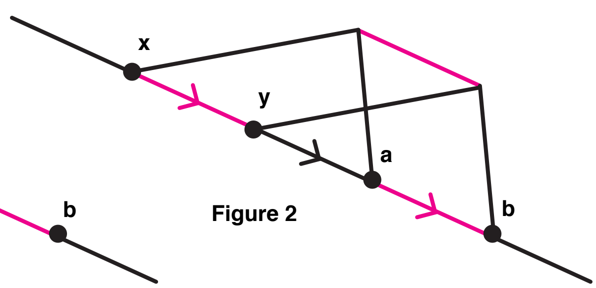


Figure 2

T: ***Here I have two couples* (x, y) *and* (a, b)*. Why do you think that I colored the arrows with the same color?***

S: ***Because they are equipollent. We always show equipollent couples with the same color arrow.***

T: ***How can we check that they are equipollent?***

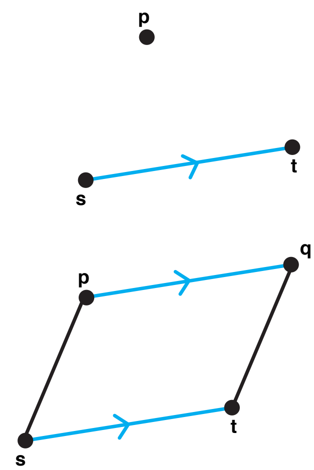
S: ***We need to build a parallelogram train from one couple to the other.***

T: ***Who can trace a parallelogram train from* (x, y) *to* (a, b)*?***

Encourage discussion of the problem and let the class attempt to find a solution by trial and error. If no one provides a solution now, leave the problem unsolved for further discussion. For your information, one possible solution is shown in Figure 2.

Exercise 3

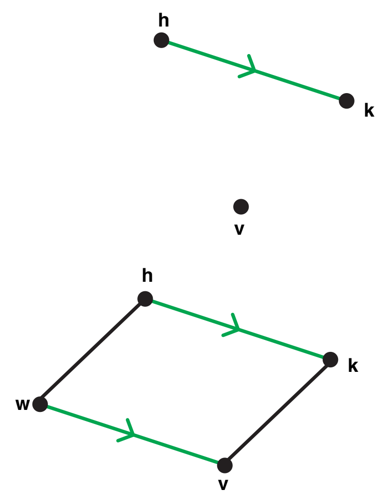
On the board, draw a point **p** and a couple (**s**, **t**) not equipollent to those in Figure 1. Use a different color than that used in Figure 1.



T: ***Let's find a couple equipollent to* (s, t) *using* p *as the starting point. How can we do it?***

Ask a student to trace the required parallelogram and then to perform the construction. Emphasize that the first step of the construction is to connect the starting point **s** to the starting point **p**.

Without disturbing the other drawings, draw another couple (**h**, **k**) and a point **v** that is not colinear with **h** and **k**.

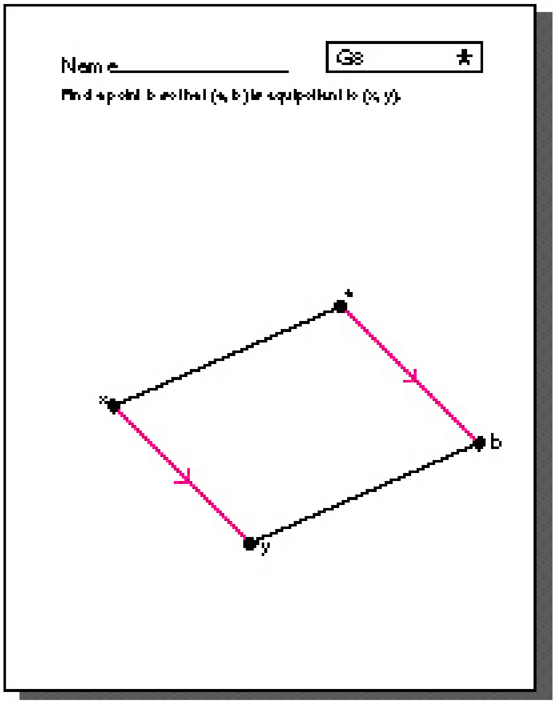


T: ***This time, let's find a couple equipollent to* (h, k) *using* v *as the ending point. How can we do it?***

Again, ask a student to trace the required parallelogram and then to perform the construction. Emphasize that the first step of the construction is to connect the ending point **k** to the ending point **v**.

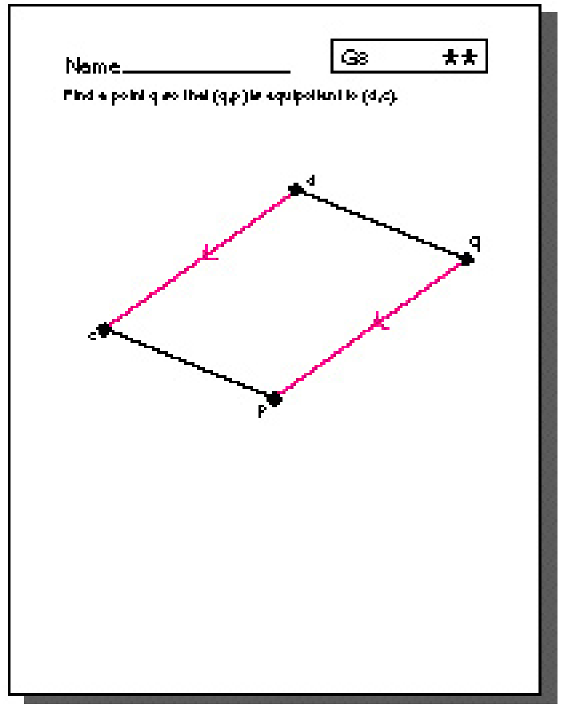
Worksheets G8*, **, ***, and **** are available for individual work. Students will need translators and colored pencils. Large sheets of unlined paper may be useful for students who need to start a construction over or want to have more space to work. The problems on the * and ** worksheets are similar to constructions done in Exercise 3.

If you left the problem in Exercise 2 unsolved, you may like to call the class back together and reopen the discussion. Some new ideas may come from the constructions in Exercise 3 and on the worksheets.



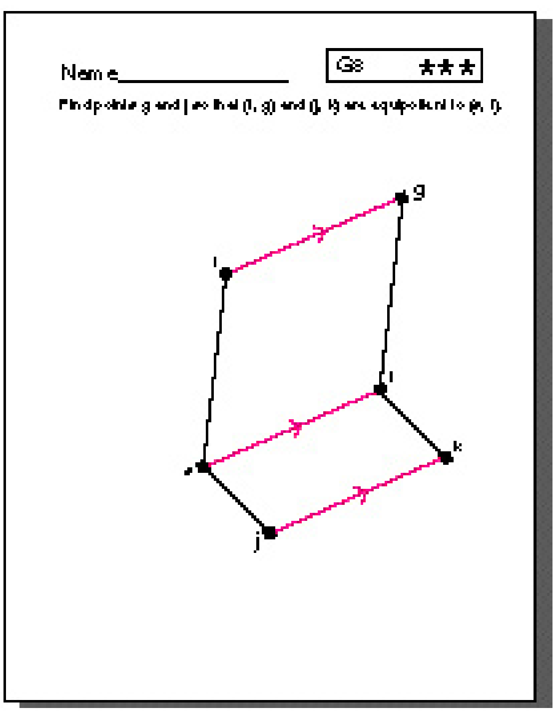
Name______________ G8 ★

Find a point b so that (a, b) is equipollent to (x, y).



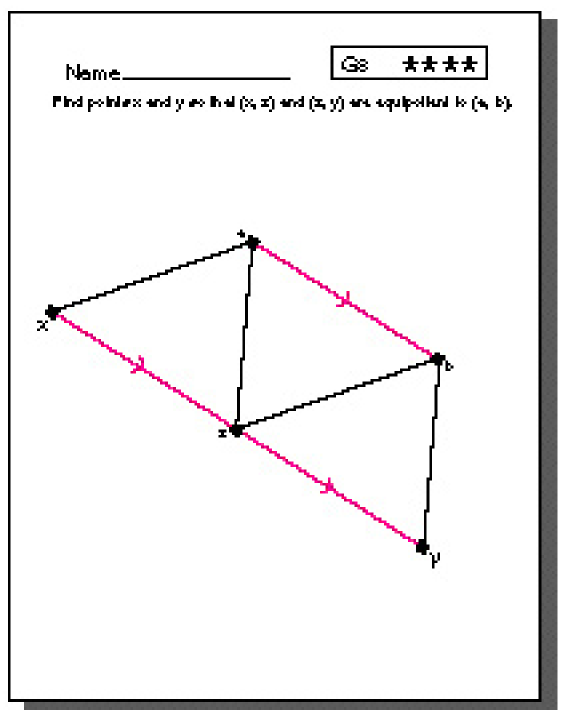
Name______________ G8 ★★

Find a point q so that (q,p) is equipollent to (d,c).



Name______________ G8 ★★★

Find points g and j so that (i, g) and (j, k) are equipollent to (e, f).



Name______________ G8 ★★★★

Find points x and y so that (x, z) and (z, y) are equipollent to (a, b).

Capsule Lesson Summary

Construct a family of equipollent couples with carefully prescribed starting points to achieve a number line. Solve problems of labeling points on the number line.

Materials

Teacher
- Translator
- Colored chalk
- Masking tape

Student
- Translator
- Colored pencils, pens, or crayons
- Large unlined paper
- Worksheets G9*, **, ***, and ****

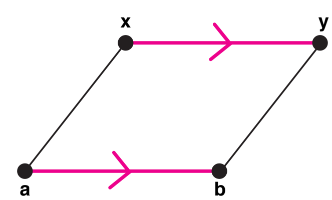
Description of Lesson

Provide students with translators, colored pencils, and large sheets of unlined paper so that they can follow your construction on the board with a similar one on their papers.

On the board, draw a couple (**a**, **b**) and a third point **x**, as shown here.

T: ***Let's find a couple equipollent to* (a, b) *that starts at* x.**

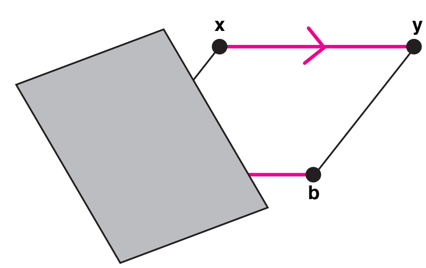
Invite a student to trace an appropriate parallelogram and then to perform the construction. Emphasize that the first step in the construction is to connect the starting point **a** to the starting point **x**.



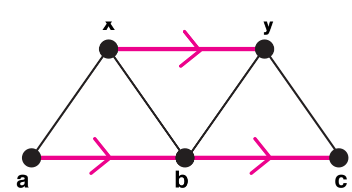
T: ***Now let's find a couple equipollent to* (a, b) *that starts at* b.**

This construction is difficult for many students. Encourage discussion and suggestions from the class. As necessary, suggest to students that they construct a parallelogram train. The couple (**x**, **y**) can be used since it is equipollent to (**a**, **b**). Remind the class that the starting points of equipollent couples are connected in a parallelogram train, so connecting **x** and **b** would be a good beginning for the construction. At each stage of the problem, reiterate this suggestion of beginning the construction by connecting starting points.

Covering a portion of your drawing as shown here may help students to ignore the elements of the drawing that are not used in this step.

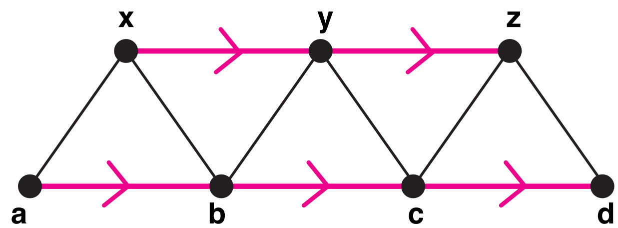


Continue until someone suggests a correct construction as shown here.



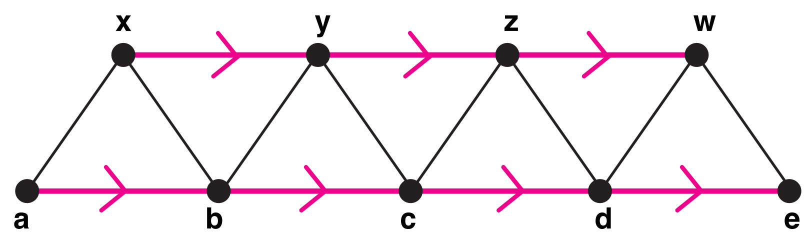
T: ***Now let's find a couple equipollent to* (a, b) *that starts at* c.**

This construction could use the "helping couple" (**x**, **y**) just as in the construction of the segment for (**b**, **c**); however, the picture quickly gets confusing. You might discuss a two-step construction, first constructing a couple starting at **y** and then one starting at **c**. Ask students to trace each parallelogram involved before you construct it.

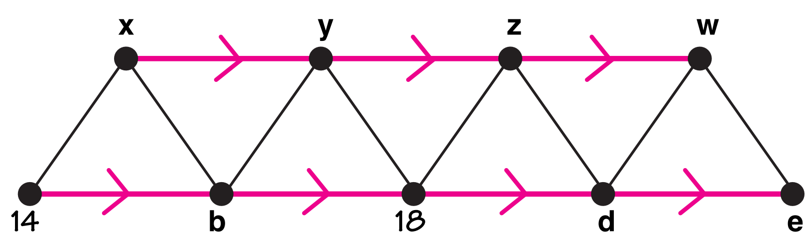


T: ***Let's continue the construction one more step and find a couple equipollent to* (a, b) *that starts at* d.**

Again encourage discussion until a student suggests a two-step construction and then completes it.



Ask students if the picture reminds them of anything—perhaps a number line? Put 14 at **a** and 18 at **c**.



T: ***What number is here*** (point to **b**)***?*** (16)
What numbers are here (point to **d** and **e**)***?***

Repeat this activity a few times with numbers appropriate to the numerical abilities of your class.

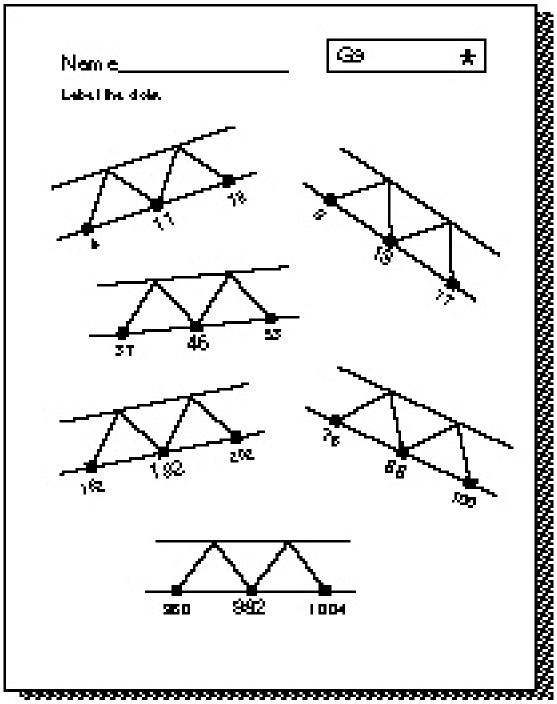
T: ***The midpoint between* a *and* c *is* b *when* (a, b) *and* (b, c) *are equipollent.***
***What is the midpoint of* c *and* e?** (**d**)
***What is the midpoint of* x *and* z?** (**y**)
***What is the midpoint of* b *and* d?** (**c**)

Solve several number line problems, such as those suggested in the following table. Answers are in boxes. Adjust the difficulty to fit the numerical abilities of your class.

a	b	c	d	e
$\boxed{34}$	37	$\boxed{40}$	43	$\boxed{46}$
$\boxed{24}$	$\boxed{44}$	64	$\boxed{84}$	104
5.8	$\boxed{6.3}$	6.8	$\boxed{7.3}$	$\boxed{7.8}$
20	$\boxed{25}$	$\boxed{30}$	35	$\boxed{40}$
$\boxed{29}$	36	$\boxed{43}$	$\boxed{50}$	57
50	$\boxed{56}$	62	68	74

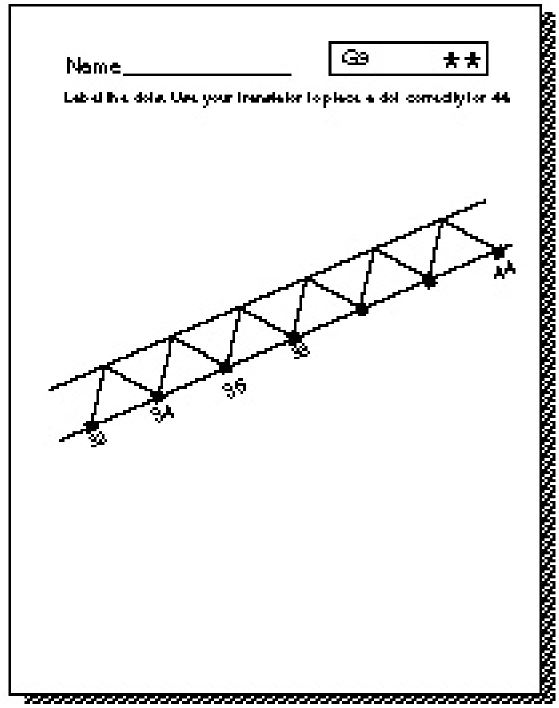
Instruct students to repeat the number line construction on a new sheet of paper. Encourage them to go with the construction—to construct longer number lines then you did on the board. Then suggest they put two numbers on their number line and challenge a neighbor to label the other dots as you did on the board.

Worksheets G9*, **, ***, and **** are available for individual work.



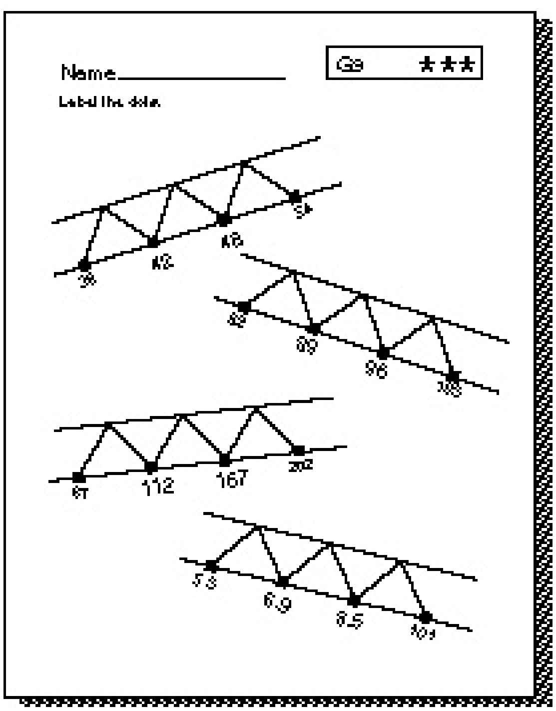
Name______________ G9 ★

Label the dots.



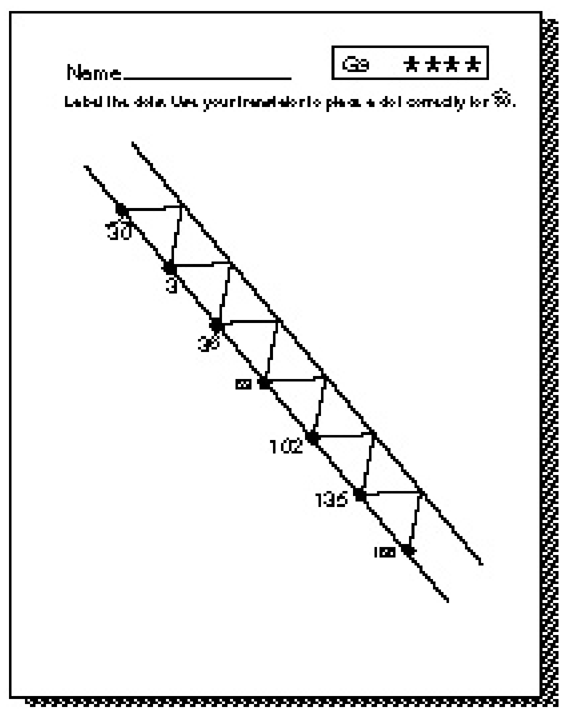
Name______________ G9 ★★

Label the dots. Use your translator to place a dot correctly for 44.



Name______________ G9 ★★★

Label the dots.



Name______________ G9 ★★★★

Label the dots. Use your translator to place a dot correctly for

G10 AREA AND PERIMETER #2

Capsule Lesson Summary

Find the areas and perimeters of shapes with whole square. Find areas of shapes with whole- and half-square units.

Materials

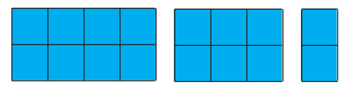
Teacher

- Grid board
- Scissors
- Blue and yellow construction paper

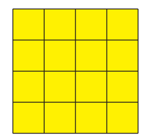
Student

- Colored pencils, pens, or crayons
- Worksheets G10*, **, ***, and ****

Advance Preparation: For Exercise 2, use blue construction paper to make three rectangles that cover a 4 by 2, a 3 by 2, and a 1 by 2 area of your grid board.



For Exercise 3, use yellow construction paper to make two squares that cover a 4 by 4 area of your grid board.



In Exercises 2 and 3 you may want to provide students with copies of these rectangles and the square. Use Blackline G10 to make such copies, and ask students to color them.

Description of Lesson

Exercise 1

Hold up a piece of centimeter grid paper and a 1 cm by 1 cm square cut from the paper.

T: ***What size do you think this square is?***

S: ***1 cm by 1 cm.***

S: ***About a half-inch.***

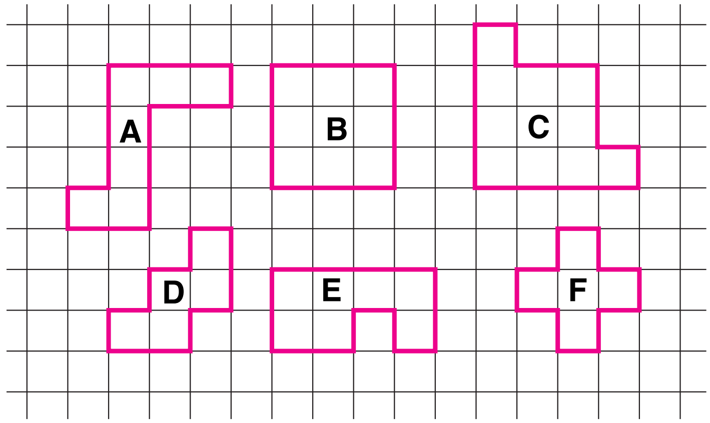
T: ***This square is 1 cm along an edge and is the size of the little squares on this grid paper. Earlier we used squares this size to find the area of some shapes. This little square is one square centimeter, and we found how many square centimeters were needed to cover a shape.***

Remind the class of how to abbreviate square centimeter.

cm^2

T: ***If it takes three of these little squares*** (hold up the 1 cm by 1 cm square) ***to cover a shape, then the area of the shape is 3 cm^2.***

Draw these shapes on a grid board. If necessary, tell the class to pretend that each grid square measures 1 cm by 1 cm.



Invite students to come to the board, and find the areas and perimeters of the shapes. Near each shape record both measurements.

A area: 7 cm^2 perimeter: 16 cm

B area: 9 cm^2 perimeter: 12 cm

C area: 11 cm^2 perimeter: 16 cm

D area: 5 cm^2 perimeter: 12 cm

E area: 7 cm^2 perimeter: 14 cm

F area: 5 cm^2 perimeter: 12 cm

T: *Are there two shapes on the board that have the same area?*

S: *Yes,* A *and* E.

S: *Also,* D *and* F.

T: *Are any of the perimeters the same?*

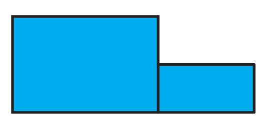
S: D *and* F *have the same perimeters, but* A *and* E *do not.*

T: *Are there two shapes on the board that have the same perimeters but have different areas?*

S: *Yes,* A *and* C *(or* B *and* D*, or* B *and* F*).*

Exercise 2

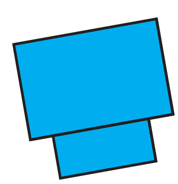
Show the students the blue rectangles one at a time. Let them place them on the grid board (again pretend the grid squares measure 1 cm by 1 cm) to find the areas of the rectangles (6 cm^2, 8 cm^2, and 2 cm^2). Place the rectangle of area 2 cm^2 next to but not overlapping the rectangle of area 6 cm^2. For example:



T: *What is the area of this shape? What is its perimeter?*

S: *8 cm^2 is the area and 14 cm is the perimeter.*

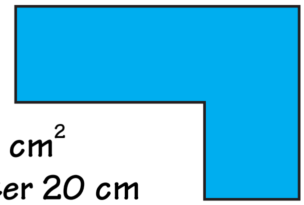
Move the rectangles into a different position, but again do not overlap them.



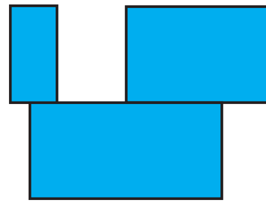
S: *Area is still 8 cm^2.*

S: ***The perimeter is 12 cm.***

Repeat the activity using all three of the blue rectangles. Let students create shapes with three rectangles. The rectangles should touch but not overlap. Students should observe that the area is always 16 cm^2 but the perimeter may vary. In fact, sometimes it may be hard to determine the perimeter exactly.

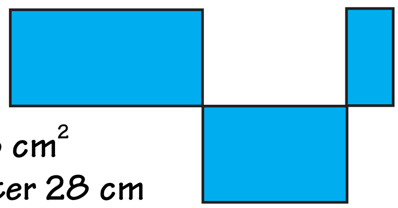


Area 16 cm^2
Perimeter 20 cm

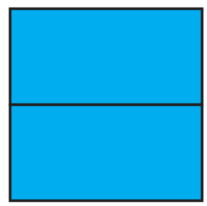


Area 16 cm^2
Perimeter 23 cm

Ask students to use the three rectangles to make a shape with as large a perimeter as possible, and one with as small a perimeter as possible.



Area 16 cm^2
Perimeter 28 cm



Area 16 cm^2
Perimeter 16 cm

Exercise 3

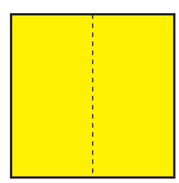
Show the class a yellow square the same size as the shape with the smallest perimeter made from the three blue rectangles.

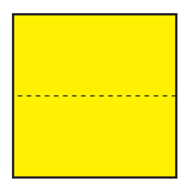
T: ***Suppose we want to cut this square into two pieces of the same size and shape. How could we do it?***

Ask students to trace where they would cut the yellow square.

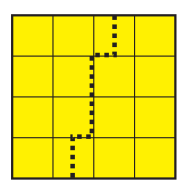
S: ***Cut it right down the middle.***

S: ***Cut it straight across in the middle.***





Some students might suggest other correct cuts such as the one shown here. Accept such cuttings as correct, but focus on the two mentioned above.



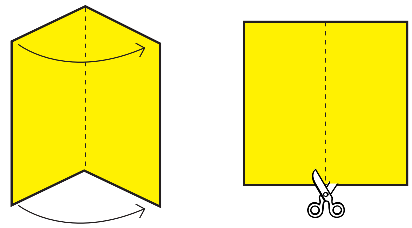
T: ***How can we locate the middle?***

S: ***Use a ruler.***

S: ***Place the square on the grid board and use the grid squares.***

S: ***Fold it in half.***

Follow this last suggestion—fold the paper in half vertically or horizontally, and then cut along the crease.



Show the class that the rectangles are the same size and shape by placing one on top of the other.

T: ***What do you think the area of one of these rectangles is?***

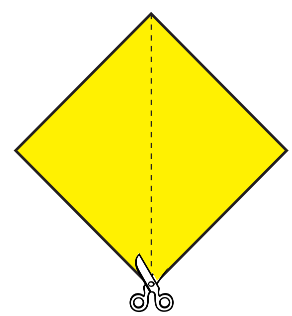
S: ***8 cm²; ½*** x ***16 = 8.***

Place one rectangle on the grid board and ask students to check that its area is 8 cm^2.

Display a second yellow square with area 16 cm^2.

T: ***How else could we cut the square to get two pieces with the same size and shape?***

When someone suggests folding the paper along a diagonal, do so, and then cut along the crease.

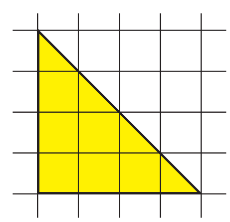


Show the class that the triangles are the same size and shape by placing one on top of the other.

T: ***What do you think the area of one of these triangles is?***

S: ***8 cm², because each triangle has half the area of the square.***

Place a triangle on the grid board and outline or color the shape it covers.



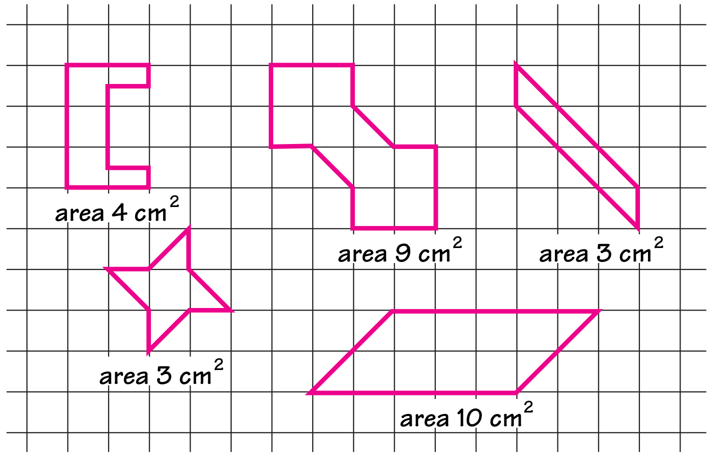
T: ***How can we find the area of this triangle?***

S: ***There are six whole squares and four half squares.***

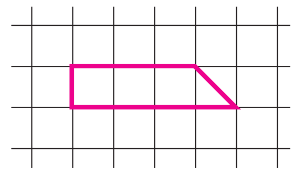
S: ***Two half squares make one whole square.***

S: ***The area is 6 + 1*** (point to two half squares) ***+ 1*** (point to the two other half squares), ***or 8 cm².***

Draw these shapes on the grid board. Invite students to find the areas of the shapes and to convince the class. Each of the shapes has an even number of half squares.



Draw this shape on the grid board.



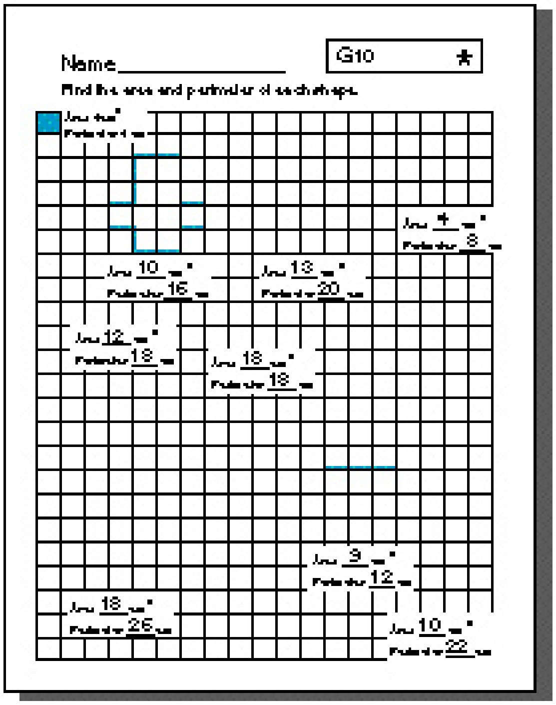
T: ***What is the area of this shape?***

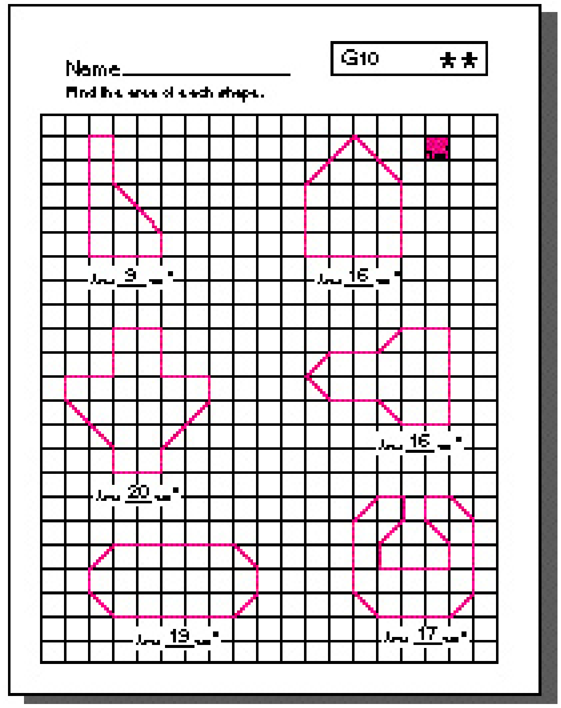
S: ***More than 3 cm²***.

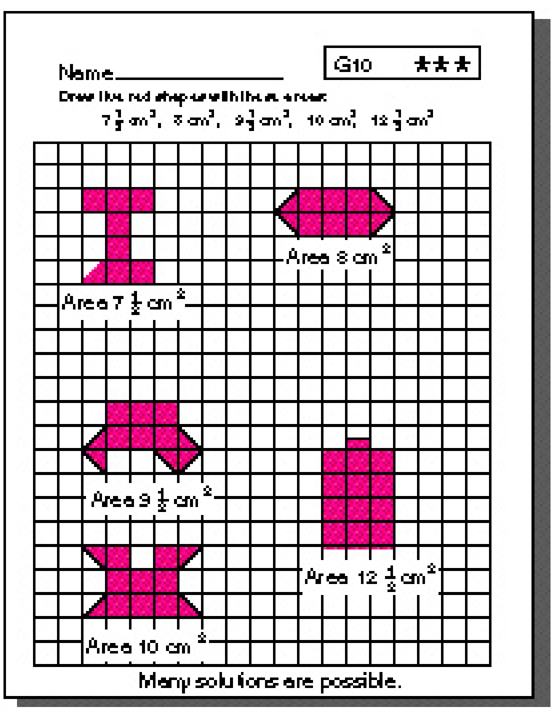
S: ***3½ cm²***.

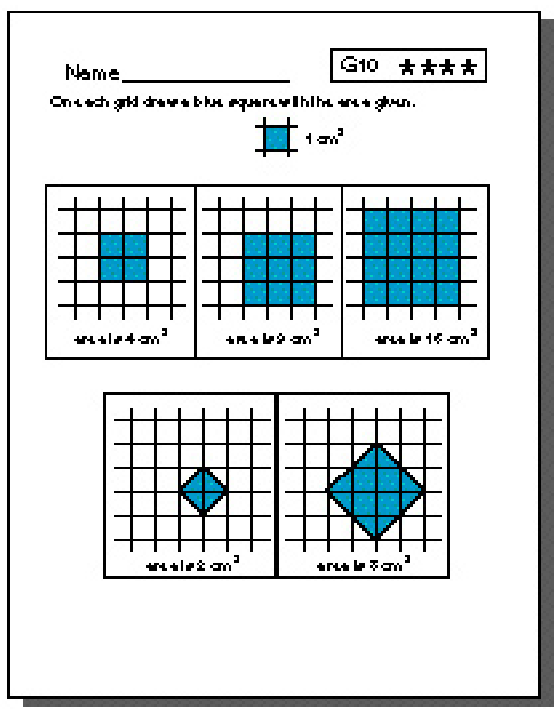
T: ***The area of the shape is 3 cm²*** (point to the three whole squares in the shape) ***plus ½ cm²*** (point to the half square) ***or 3½ cm².***

Worksheets G10*, **, ***, and **** are available for individual work.


Name
G10 ★


Name
G10 ★★


Name
G10 ★★★
Area 8 cm²
Area 7 ½ cm²
Area 9 ½ cm²
Area 12 ½ cm²
Area 10 cm²
Many solutions are possible.


Name
G10 ★★★★
1 cm²

Capsule Lesson Summary

After finding the areas of several rectangles drawn on a grid board, find the areas of some rectangles given only their measurements. To find the area of a right triangle, draw a rectangle around it that has twice its area, find the area of the rectangle, and then take half of that area.

Materials

Teacher
- Blue construction paper
- Scissors
- Grid board
- Colored chalk
- Meter stick

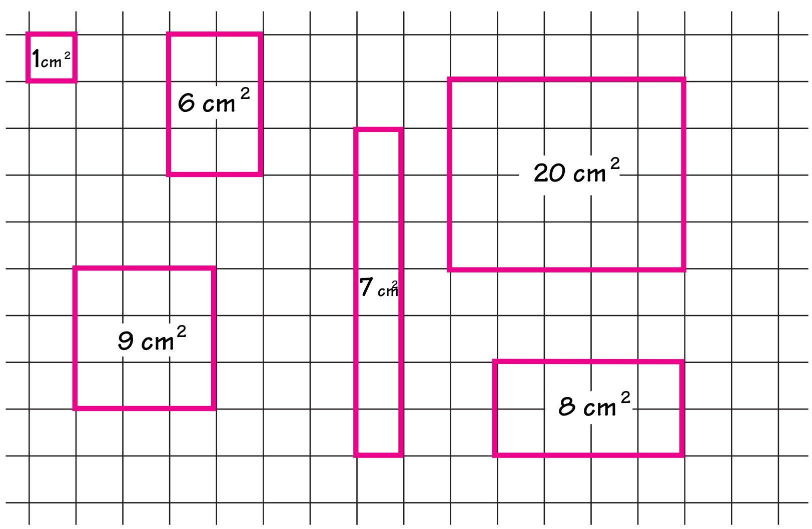
Student
- Worksheet G11
- Colored pencils, pens, or crayons
- Metric ruler

Description of Lesson

Note: As necessary tell students to pretend that the grid board has 1 cm by 1 cm little squares. Observe, then, that a square has area 1 cm^2.

Exercise 1

Draw several rectangles on the grid board. After you draw each one, ask for its width, length, and area. Include a square (a special rectangle) in your collection.



T: ***I'm thinking of a rectangle 3 cm wide and 20 cm*** [†] ***long. Can we find its area even though we cannot draw it on the grid board?***

S: ***It's area is 60 cm^2.***

T: ***How did you find its area?***

S: ***I multiplied: 3 x 20 = 60.***

S: ***I thought about three rows of twenty squares. 20 + 20 + 20 = 60.***

Draw a rectangle on the chalkboard and indicate three rows.

† The number of centimeters here should exceed the number of squares in a row of your grid board. Adjust the length of the rectangle as necessary.

T: ***Imagine the rectangle lined up with the grid lines on our board. It would contain three rows (3 cm wide) of 20 squares (20 cm long).*** (Point to each row of the picture on the chalkboard.) ***20 + 20 + 20 = 60. 3*** x ***20 = 60.***

If a student suggests adding 3 + 3 + … + 3 (twenty 3's), accept it as another good method for finding the area. You may want to draw a few vertical lines and form columns of three squares each to explain this method.

Repeat the activity for a few more rectangles, for example, 5 cm wide and 15 cm long; 4 cm wide and 32 cm long; and 10 cm wide and 18 cm long.

Exercise 2

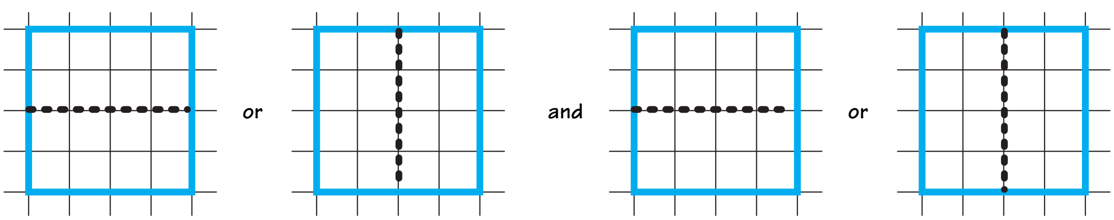
Erase the ~~grid board and then~~ draw a blue square 4 cm by 4 cm on it. Ask for its length, width, and area (16 cm^2).

T: ***Last week we looked at a square like this and cut it into pieces of the same size and shape. Do you remember how we did that?***

S: ***We folded a paper square and cut along the crease.***

S: ***We cut two squares in two different ways.***

T: ***Can you trace on this blue square that I've drawn on the grid board where the cuts were made?***

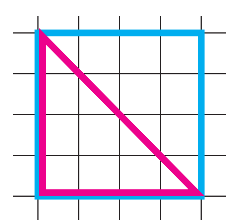


T: ***What is the area of each piece?***

S: ***8 cm^2.***

T: ***Why?***

S: ***The area of the square is 16 cm^2 and ½*** x ***16 = 8.***



Draw this red triangle.

T: ***What is the area of the blue square?*** (16 cm^2)
What is the area of the red triangle? (8 cm^2)

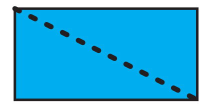
Hold up the blue rectangle and show the class how it covers a three by four rectangle on your grid board. Outline such a rectangle on the grid board.

T: ***What is the area of this rectangle?***

S: ***12 cm²***

T: ***Suppose we want to cut this rectangle into two pieces of the same size and shape. How could we do it?***

S: ***Fold it in half straight down the middle and then cut along the crease.***



S: ***Cut along a diagonal.***

In this lesson we are most interested in cutting the rectangle along the diagonal. Accept other methods such as ones using the grid lines or rules, but only demonstrate this one.

Draw one of the diagonals on the rectangle and cut along it. Show the class that the two triangles are the same size and shape by placing one on top of the other.

T: ***What is the area of each triangle?***

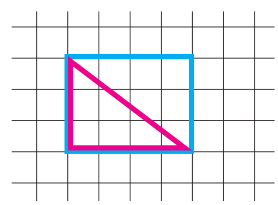
S: ***6 cm²***

T: ***Why?***

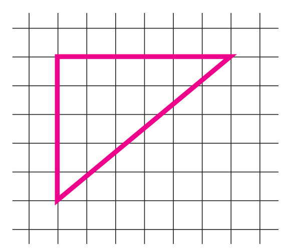
S: ***Because the area of the rectangle was 12 cm² and ½ x 12 = 6.***

Draw a red triangle inside the blue rectangle on the grid board.

T: ***The area of the red triangle is half the area of the blue rectangle. The area of the rectangle is 12 cm², so the area of the triangle is 6 cm².***



Draw this triangle on the grid board.

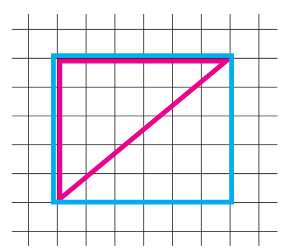


T: ***How could we find the area of this triangle?***

S: ***We could make it half of a rectangle.***

S: ***Draw a rectangle around it, find the area of the rectangle, and then take half of that area.***

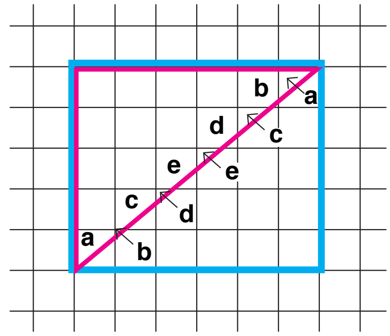
Invite a student to draw a rectangle with width 5 cm and length 6 cm around the triangle. This is the smallest rectangle that surrounds the triangle.



T: ***What is the area of the blue rectangle?*** (30 cm²)
What is the area of the red triangle? (15 cm²)

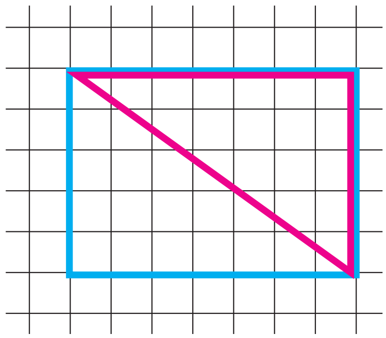
S: ***The area of the red triangle is half the area of the blue rectangle. ½ x 30 = 15.***

Note: Some students may suggest pairing pieces along the diagonal (as indicated by common lettering in this illustration) to give whole squares. This method is valid, but not easy to see.



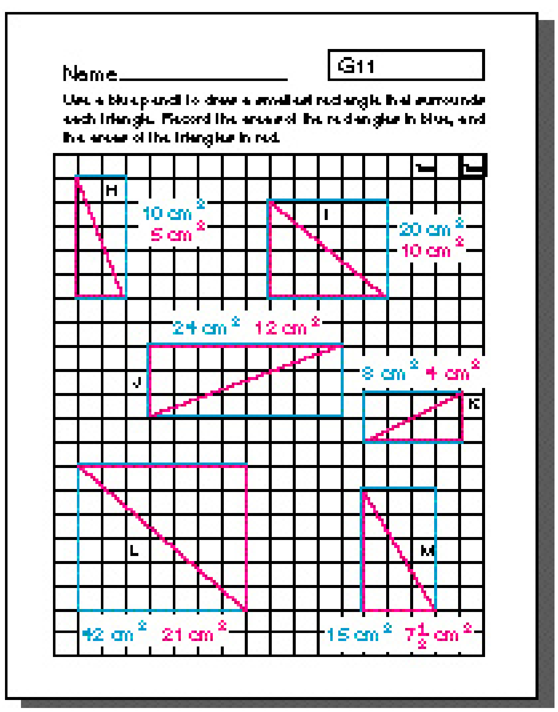
Beware of possible misuse of this pairing method in other situations; for example, students might try to pair pieces that do not form squares even thought they appear to. Estimation should be recognized as such.

Repeat the activity to find the area of another triangle. The example here shows a surrounding rectangle that is 3½ cm wide and 6 cm long; that is, it has area 21 cm^2.



area of the red triangle = 10.5 cm^2
($\frac{1}{2}$ x 21 = 10.5 or 10 $\frac{1}{2}$)

Worksheet G11 is available for individual work.



Capsule Lesson Summary

Review the idea of a graph. Introduce special paths in a graph—first, paths that go through each dot exactly once; and second, paths that start and end at the same dot and go through each other dot exactly once. Note some properties of the graphs in which you can find these paths.

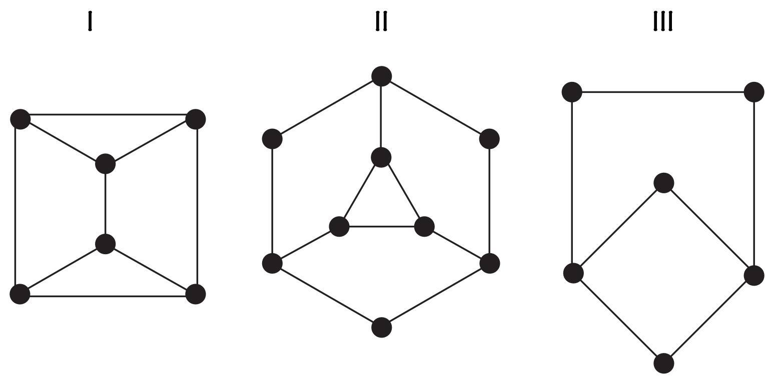
Materials

Teacher
- Colored Chalk

Student
- Paper
- Colored pencils, pens, or crayons
- Worksheets G12* and **

Description of Lesson

Draw three separate graphs on the board.



Review the idea of a graph as a collection of dots and edges. For each of these graphs, count the number of dots and the number of edges.

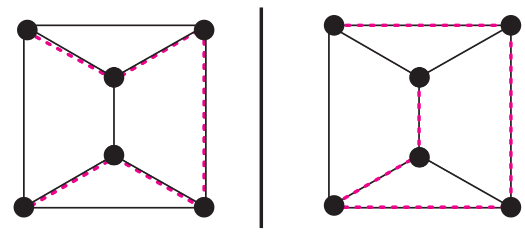
T: ***What's nice about graphs is that they might picture lots of different things. For example, we saw before how we could draw a graph for a map: the dots for countries, and edges between dots indicating countries that share a border.***

T: ***Do you have any ideas about what these graphs might picture?***

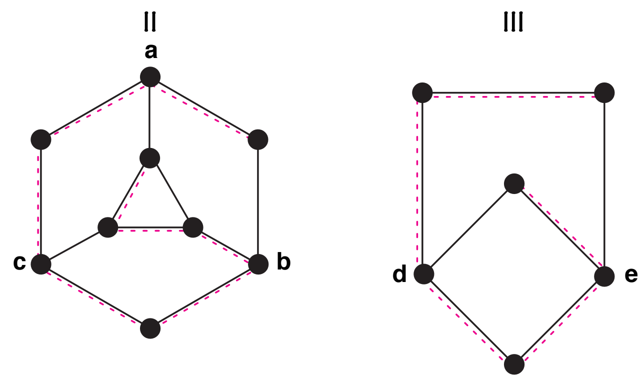
Let students express some ideas, and lead them to look at the graphs as picturing towns and roads between towns.

T: ***The dots in each of these graphs are for towns. The edges show roads between the towns.*** (Point to graph I.) ***Suppose we are touring the towns in this graph and we'd like to visit each town exactly once. Can we do it?***

Invite students to trace routes in the graph that pass through each town (dot) exactly once. Some things to note about each correct route a student traces are that it uses five roads (edges), it starts and ends at towns that are connected by a road, and choice of a starting or ending town is arbitrary. After several routes have been found, illustrate one possible solution in color. Two different solutions are shown below.



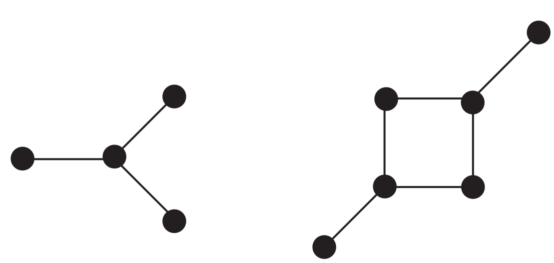
Check the other two graphs to see whether a route visiting each town exactly once can be found. Again let students find several solutions before illustrating one in color. Note that when there are nine towns to visit the route uses eight roads, and when there are six towns to visit the route uses five roads. Other things to observe in these graphs are that the starting and ending towns are not connected by a road, and the choice of a town at which to start a route is not completely arbitrary. For example, in graph II, we cannot start a route at **a**, **b**, or **c**, and in graph III, we cannot start or finish a route at **d** or **e**.



Do not write the letters on the board. They are here just to make the description of the

T: ***In all three of these graphs, we were able to find a route visiting each town exactly once. Do you think there are any graphs for which this would be impossible? Try to draw a graph on your paper*** (the dots for towns and the edges for roads between the towns) ***in which there is no way to trace a route visiting each town exactly once.***

Allow several minutes for students to explore; then invite two or three students to draw their graphs on the board. Two rather simple examples are given here; your students may find more complicated graphs.

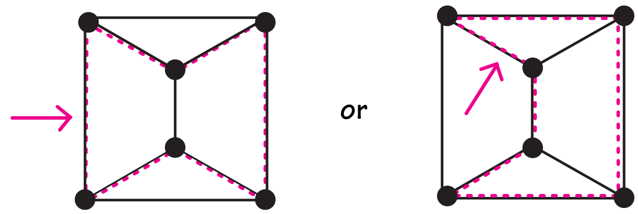


Return to the original three graphs on the board and put another condition on the touring problem.

T: ***I'd like to change the description of a route in these graphs a little bit. Suppose we want to start and finish at the same town but still visit each other town exactly once. In other words, we want to make a round trip visiting each other town exactly once.***

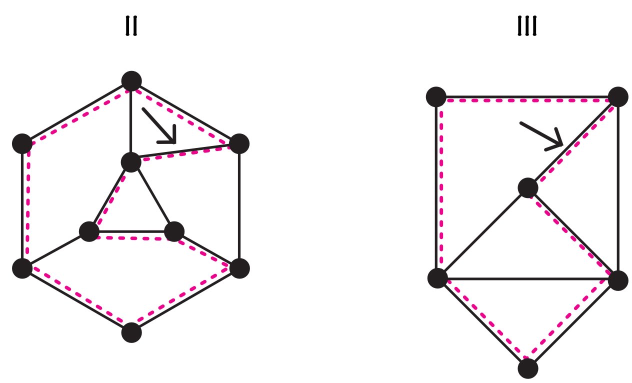
Can we make round trips in any of these graphs?

S (pointing to I)**:** ***Here we can. Just complete the trip along the road between the start and end of our previous route.***



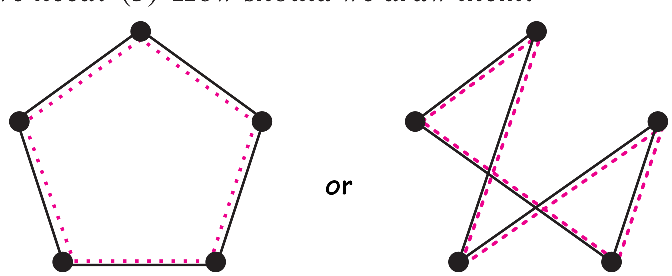
After some trials with graphs II and III, students should determine that round trips are not possible in these graphs. The fact that every route visiting each town exactly once starts and ends at towns not connected by a road precludes finding a round trip.

Next, ask students whether adding one new edge (road) to a graph would make it possible to find a round trip. They should discover that in graphs II and III one new edge is, indeed, enough to make finding a round trip possible. Just connect the starting and ending dots of a previous route that went through each town exactly once. The arrows in the next illustration show one way to add edges (based on routes previously displayed).



Draw five dots on the board.

T: ***Let's find a graph with as few edges as possible and connect these dots so that there is a round trip—a trip starts at one of the towns and visits each other town exactly once. How many edges do we need?*** (5) ***How should we draw them?***



If students suggest crossing the edges, that is fine. In this case you might say that these roads appear to cross, but really there is an overpass where one road goes over the other.

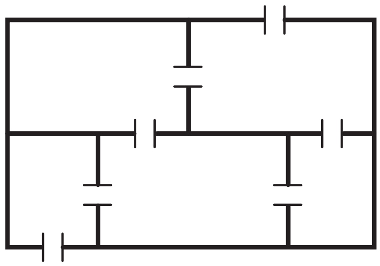
Worksheets G12 * and ** are available for individual work.

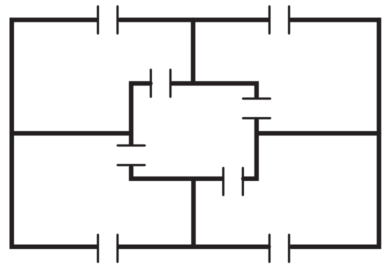
Reading Activity

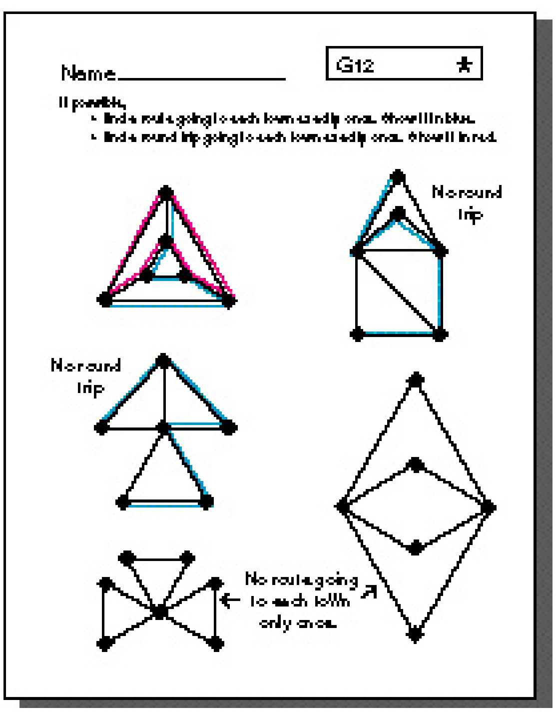
Read *Anno's Math Games III* by Mitsumasa Anno. Learn about the Königsberg Bridge Problem.

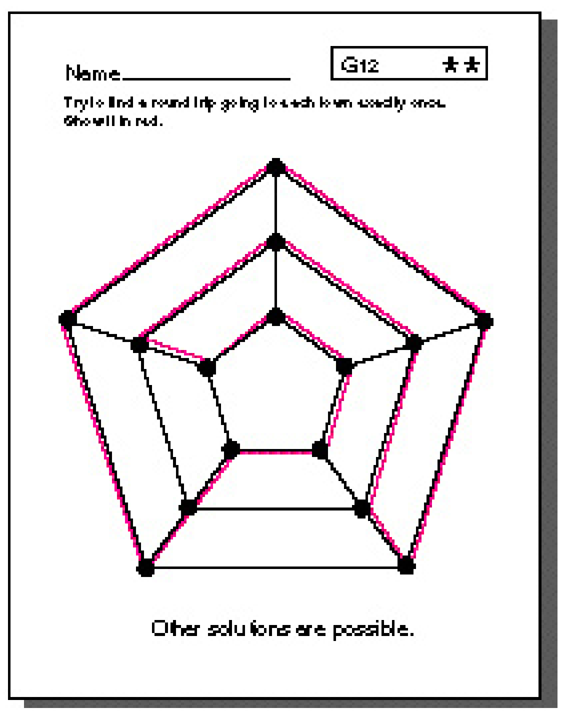
Extension Activity

Pose a variation of the touring or round trip problem in a house, such as illustrated below. Ask students to find a route (tour) of a house that visits each room exactly once. Ask if they can find a round trip route starting and ending outside the house.









Capsule Lesson Summary

Determine how much area a person can cover with one hand. Write a description of how to solve this problem and compare methods.

Materials

Teacher		Student	
	• Centimeter grid paper		• Centimeter grid paper • Colored pencils, pens, or crayons • Lined paper

Advance Preparation: Make centimeter grid paper using the grid provided in the unnumbered section of the Blacklines.

Description of Lesson

Begin the lesson with a short discussion of how we sometimes use a hand to cover something—our eyes, part of a picture, answers on our papers, the display of a calculator. Then ask students to estimate how much (greatest) area they can cover with one of their hands. Allow the students to compare hands, and then to predict who has the biggest hand (covers the most area) and who has the smallest (covers the least area). Include the idea of *units* in your discussion, and show students units such as square centimeters and square inches.

T: ***How could you figure out how much area one of your hands will cover?***

Let the class discuss methods, and then introduce them to centimeter grid paper.

T: ***Each of us is going to figure out how much area one of our hands will cover. You will use a piece of centimeter grid paper.***

Model the activity (without doing it completely) as you explain the steps to the class. You may want to let students work with a partner, but each student should figure out a close approximation for the area of one of his or her own hands.

T: ***First, trace around one hand on a centimeter grid paper. Next, try to figure out how much area your hand covers in square centimeters. You may approximate, but try to get as close as possible. Finally, when you think you have a good approximation for the area of your hand, write a description that you can share with the class about how you figured the area.***

Let students ask questions about what they are to do, but try not to be too directive about how they do it. Expect questions like the following:

- Which hand should we use? (Either one—it's probably easier to trace around your left hand if you are right-handed.)
- Should we keep our fingers apart of together? (Together may give an easier shape to measure area, but either way is okay.)
- How far up our arms should we trace? (Try to stop at the end of the palm or start of the wrist.)

Observe students working and, as necessary, respond to questions. Students may need reassurance, but they should realize that there is not just one best method. Students who finish quickly may be asked to check the area using another method, or to compare the area of their left hand with that of their right hand.

When many students have completed the activity, invite several to share their methods. Encourage other students to comment about how they used a similar counting method, or about whether a method is clear and makes sense. For example:

S: ***I counted the whole squares (square centimeters) my hand covered. Then I counted the half (or about half) squares and divided by two. The area was 110½ cm².***

S: ***I wrote 1, 2, 3, 4, and so on on the whole squares. I tried to match parts of squares to get more whole squares.***

S: ***I put a rectangle around my handprint. The area of the rectangle was 165 cm². Then I subtracted squares in the rectangle that were not in my handprint.***

Do not expect particularly well-formed written or oral descriptions of students' methods. You and other students may need to ask questions to get more information about what a student actually did. Encourage students to include enough information in their written descriptions to tell an absent classmate how they found the area covered by one of their hands.

Allow students to compare hands when the areas are close to the same or quite far apart. You may
handprints from smallest area to largest area.

Extension Activity

Your class may like to try to find the average area covered by their hands. That is, ask, "What is the
r's hand will cover?"

Home Activity

Send home centimeter grid paper and suggest a student work with his or her family members to find the areas covered by their hands. Then have the student compare these areas to the area of his or her hand.

Capsule Lesson Summary

Draw a handprint (for Michael Jordon) that has area approximately twice the average area of the class's hands.

Materials

Teacher
- Basketball (optional)

Student
- Centimeter grid paper
- Colored pencils, pens, or crayons
- Metric ruler or tape measure

Description of Lesson

Note: For this lesson you will need to know the average area covered by a student's hand, which you may have already calculated in Lesson G13. If not, use the information from that lesson to calculate an average now.

Begin the lesson with a quick review of what students did in Lesson G13. Then ask what would happen if Michael Jordon (former Chicago Bull's basketball player) were to do this activity. Let the students describe and guess how much area they think Michael Jordan's hand would cover. You might like to have a basketball available to let students use while describing Jordan's hands.

Announce the class average handprint and then pose this problem.

T: ***The average area covered by one of our hands is 105 cm^2. Suppose Michael Jordan's hand covered an area about twice the average hand area of our class? Draw a picture that could be Michael Jordan's hand.***

Allow students to work with a partner on this project. Each pair of students should have centimeter grid paper, unlined paper, and measuring tools if they wish. Initially, students may want to ask questions about what they are to do, but try not to be too directive. Expect questions like these:

- Does the picture have to look like a hand? (Yes, it should at least resemble a handprint.)
- Does it have to have area exactly 210 cm^2? (No, but try to get as close as possible.)
- Can we use the basketball? (Yes, if it will help you.)

Observe students working and, as necessary, respond to questions. Students may need some help getting started, but they should realize that there is not just one method to work on this activity. You may want to ask students to write a description of how they accomplished the task.

When many students have completed their pictures, invite several to display the pictures and to share their methods. Encourage other students to comment on the clarity of a method or to ask questions.

S: ***We took one of our handprints and added area all around it. First we added too much and then we took some away.***

S: ***We cut out our two handprints and put one above the other. We had to make a very long thumb and the hand looks very long.***

S: ***We drew a rectangle with area 210 cm^2. Then we tried to make it look like a hand by taking some off in one place and moving it to another place.***

Give students a chance to hear several different methods and to compare solutions. They may enjoy seeing the variety of different-looking hands.

T: ***Did anyone try making Michael Jordan's hand twice as long and twice as wide as their own hand? Would this be a good idea?***

Let students discuss this method. They may realize that doing this creates much too big a hand. In fact, if you cut out copies of an average handprint and model putting two next to each other (making it twice as wide) and one above the other (making it twice as long), you cover more than twice the ... es the area.

Extension Activity

Describe the following method for finding the area of a handprint:

> Use a piece of string to outline the handprint. Cut the string so it equals the perimeter of the hand. Then reshape the string into a square (rectangle) and find the area of the square (rectangle).

Ask students to decide whether or not this is a good method.