The Languages of Strings and Arrows

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THE LANGUAGES OF STRINGS & ARROWS INTRODUCTION

Two fundamental modes of thought for understanding the world around us are the classification of objects into sets and the study of relationships among objects. In everyday life, we classify cars by brand (Ford, Chevrolet, Toyota, and so on) and we study relationships among people (Sally is Mark's sister, Nancy is Mark's cousin). Chemists classify elements by properties, and zoologists study predator-prey relationships. Similarly, mathematicians sort numbers by primeness, and they employ functions to model predicted relationships, for example, between inflation and unemployment.

Many of a child's earliest learning experiences involve attempts to classify and to discern relationships. A child classifies people by roles (the teacher, the doctor), and creates relationships between the smell and taste of foods. Part of language development depends on a child's repeated attempts to sort objects by function, and to relate words with things or events.

The role of sets and relations is so pervasive in mathematics, that perhaps the simplest definition of mathematics is "the study of sets and relations principally involving numbers and geometrical objects." Given the equally pervasive presence of these two notions in everyday life and in a child's experiences, it is natural that they should play a key role in an elementary mathematics curriculum. Yet the inclusion of classification and the study of relations require an appropriate language for representing and studying them. For that reason, *CSMP* develops the non-verbal classification and relations languages of strings and of arrows.

The pictorial language of strings represents the grouping together of objects into sets. The pictorial language of arrows represents relations among objects of the same or different sets. Each of these languages permeates the different content strands of the *CSMP* curriculum, providing unity both pedagogically and mathematically. With continual use, the languages become versatile student tools for modeling situations, for posing and solving problems, and for investigating mathematical concepts.

The general aim of this strand is to present situations that are inherently interesting and thought provoking, and that involve classification or the analysis of relations. The activities emphasize the role of logical thinking in problem solving rather than the development of specific problem-solving techniques.

Classification: The Language of Strings

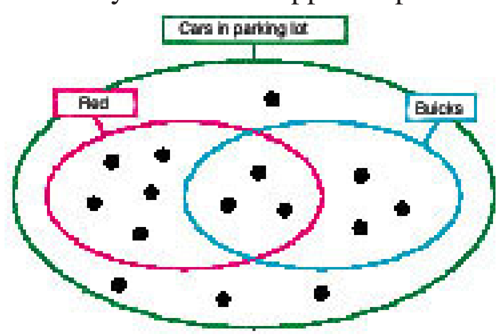
As the word implies, classifying means putting things into classes, or as the mathematician says, sets. The mathematics of sets can help students to understand and use the ideas of classification. The basic idea is simple: Given a set S and any object x, either x belongs to S (x is in S) or x does not belong to S (x is not in S). We represent this simple act of sorting—in or out—by using pictures to illustrate in and out in a dramatic way. Objects to be sorted are represented by dots, and the sets into which they are sorted are represented by drawing strings around dots. A dot inside the region delineated by a set's string is for an object in the set, and a dot outside a set's string is for an object not in the set.

This language of strings and dots provides a precise (and nonverbal) way of recording and communicating thoughts about classification. The ability to classify, to reason about classification, and to extract information from a classification are important skills for everyday life, for intellectual activity in general, and for the pursuit and understanding of mathematics in particular. The unique quality of the language of strings is that it provides a nonverbal language that is particularly suited to the mode of thinking involved in classification. It frees young minds to think logically and creatively about classes, and to report their thinking long before they have extensive verbal skills.

In this strand we present situations and ask carefully phrased questions to continue to advance skills in classification, always remembering that the skills grow out of such experiences. To be able to draw strings and dots is not an objective in itself; to develop the mode of thinking involved is the objective. Thus it is important for us to construct the situations carefully. The sets into which we ask students to classify objects must be determined by well-defined attributes; otherwise, there is the added problem of deciding whether a certain object does or does not have a certain attribute. For this reason we make extensive use of A-blocks (attribute blocks) and sets of numbers in designing classification situations. Students can immediately say whether or not a block is red, whether or not it is a square, and so on. They know whether or not a given number is less than 15, whether or not it is odd, and so on.

One reason for classifying objects is to count the objects that have a certain attribute. Suppose, for example, there are eight red cars and six Buicks in a parking lot. If four cars in the lot are neither red nor Buicks, must there be 18 cars in the lot? Suppose we count the cars in the parking lot and find there are exactly 15. Is this possible? A string picture immediately settles this apparent paradox.

There are three red Buicks.



Relations: The Language of Arrows

Relations are interesting and important to us in our everyday lives, in our careers, in school, and in scientific pursuits. We are always trying to establish, explore, and understand relations. In mathematics it is the same; to study mathematics means to study the relations among mathematical objects like numbers or geometric shapes. The tools we use to understand the everyday world are useful to understand the world of mathematics. Conversely, the tools we develop to help us think about mathematical things often serve us in non-mathematical situations.

A serious study of anything requires a language for representing the things under investigation. The language of arrows provides an apt language for studying and talking about relations. Arrow diagrams are a handy graphic representation of a relation, somewhat the same way that a blueprint is a handy graphic representation of a house. By means of arrow diagrams, we can represent important facts about a given relation in a simple, suggestive, pictorial way—usually more conveniently than the same information could be presented in words.

The convenience of arrow diagrams has important pedagogical consequences for introducing children to the study of relations in the early grades. A child can read—and also draw—an arrow diagram of a relation long before he or she can read or present the same information in words. The difficulty of presenting certain ideas to children lies not in their intellectual inability to grasp the ideas; rather, the limitations are often mechanical. Arrow diagrams have all the virtues of a good notation: they present information in a clear, natural way; they are attractive, colorful things to look at; they are easy and fun for children to draw. Students may use arrow pictures to study, test, and explain their thinking about concepts or situations under consideration. Discussion about an arrow picture often aids the teacher in clarifying a student's solution or misunderstanding of a problem.

Another educational bonus occurs when an arrow diagram spurs students' curiosity to investigate variations or extensions of the original problem. A minor change in an arrow picture sometimes reinforces a pattern already discussed and at other times suggests new problems to explore.

One of the purposes of this strand is to use the power of the language of arrows to help children think logically about relations. Again it must be remembered that our goals concern the thinking process and not the mechanism. The ability to draw prescribed arrows is not the objective in itself, nor is viewing an arrow diagram just another format for drill problems in arithmetic.

The general aim of the Languages of Strings and Arrows strand is to suggest situations that are inherently interesting and thought provoking, and to give children modes of thinking and appropriate languages with which they can organize, classify, and analyze. In addition to a varied assortment of lessons concerning sets and relation, this strand includes lessons involving systematic methods for problems.

Content Overview

The String Game

By this time, your students are quite comfortable playing even sophisticated versions of *The String Game* with A-blocks. They have played games that involve three strings and/or "not-cards" (cards indicating that a set may consist of all pieces that are "not square," for example). They have gained experience classifying numbers by order (for example, "less than 50") and as multiples of some numbers.

Two lessons in this semester review *The String Game* with A-blocks and present situations that require logical analysis in order to use clues to determine the strings. This review paves the way for a shift from *The String Game* played with A-blocks to *The String Game* played with numbers. The first games with numbers involve only attributes referring to order and multiples. Students will develop playing strategies as they gradually gain familiarity with the numerical version; analysis is not forced. This respects individual learning rates and also keeps the game fun. Premature insistence on sophisticated strategies can create tensions and frustrations in some students.

Activities in the World of Numbers strand develop the concept of the positive divisors of a whole number. Building on this idea, the last string game in *IG-I* (Lesson L10) includes the full set of numerical attributes (order, multiples, and divisors). This semester's introduction to the numerical version of *The String Game* provides the background experience students will need to logically analyze the game later in the *CSMP* curriculum.

Lessons: L1, 3, 6, 9, and 10

Logical Thinking

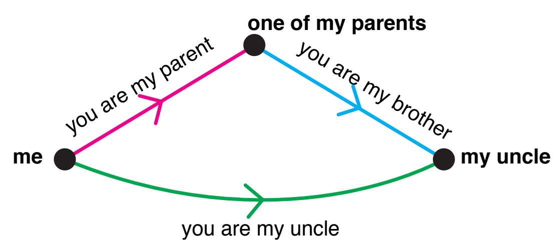
THE LANGUAGES OF STRINGS & ARROWS INTRODUCTION

The role of the Language of Strings in the *CSMP* curriculum is not limited to string games. Two lessons in this strand use string pictures as a vehicle to present problems that stimulate logical thinking. For example, given a set of objects, students must determine the truth or falsity of series of statements about the set. The statements place particular emphasis on the precise meaning of terms such as *every*, *at least*, *at most*, and *none*. Disagreements often lead to lively student discussions that result in a clearer understanding of the terms involved. Such lessons encourage reasoning and clear explanations on the part of the students, and they draw upon the students' numerical and non-numerical knowledge.

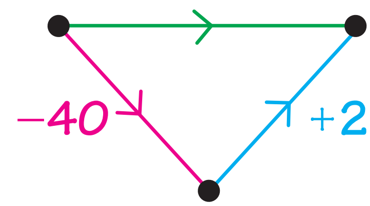
Lessons: L4 and 8

Composition of Relations

Composition of relations is the study of what occurs when two or more relations are applied in sequence. Many common ideas and ways of thinking outside the classroom implicitly involve this concept. For example the relation, "you are my uncle," is the composite of "you are my parent" and "you are my brother." An arrow picture succinctly conveys the composition.



In their *CSMP* mathematics classes, students have frequently encountered both non-numerical and numerical examples of composition. The numerical situations often lead to insights into the properties of numbers and operations. These insights, in turn, might lead to mental arithmetic techniques. For example, *CSMP* students realize that subtracting 38 is easily accomplished by subtracting 40 and adding 2. An arrow picture reinforces that the technique is general, that is, it works regardless of the starting number.



Other lessons in the World of Numbers strand will use the composition of numerical relations in applied situations. In the Languages of Strings and Arrows strand, the lessons concentrate on an explicit study of composition, removed from its applications. Instead of dealing with specific relations, such as "you are my uncle" or ⁻38, students focus on the idea itself, where the relations are identified only by unlabeled, colored arrows. This more abstract approach reveals the unity among the many examples of compositions the students have experienced.

Lessons: L2 and 5

Combinatorics

One lesson this semester uses a tree diagram to help solve a counting problem. This kind of graphic has other uses in the Probability and Statistics strand, but here it serves to organize a systematic method of counting—a multiplication method.

Lesson: L7

L1 STRING GAME WITH A-BLOCKS #1

Capsule Lesson Summary

Review the pieces in a set of A-blocks and the possible string labels used in *The String Game* with A-blocks. Play a two-string version of the game. In a game situation, present a sequence of clues that give sufficient information for the class to determine the labels for the strings.

Materials

Teacher
- A-Blocks String Game kit
- Small box
- Colored chalk
- Red and blue crayons or markers

Student
- Worksheets L1* and **

Advance Preparation: Before this lesson, you will need to prepare some materials from the A-Blocks String Game kit. These materials will be used again in future lessons on *The String Game*, so keep them together in the envelope with the kit.

1. Punch out one set of shapes (24). If you have a magnetic board, magnetize each A-block by sticking a small piece of the magnetic material to the back. If there is no magnetic board available, you can still use a regular chalkboard. In this case, have loops of masking tape ready to stick to the back of each A-block so that they can be stuck to the board. Masking tape loses its stickiness quickly, so be prepared to reinforce the A-blocks regularly with new loops of tape.
2. Prepare the string cards in the same manner as the A-blocks.
3. Prepare a team board as pictured on the next page. This board should be metallic if you have magnetized A-blocks. Post a list of attributes (found in the A-Blocks String Game kit) above the team board . You may want to laminate the A-Block pieces and cards to make them more durable.

Description of Lesson

Put the 24 A-block pieces in a box (a greeting cards box is a good size).

T: ***In this box I have some pieces called A-blocks. Do you know how we distinguish the blocks?***

S: ***By color, shape, and size.***

As an attribute of the A-blocks is mentioned, show the class a piece having that attribute.

T: ***What different shapes are there?***

S: ***Square, circle, and triangle.***

T: ***What colors are there?***

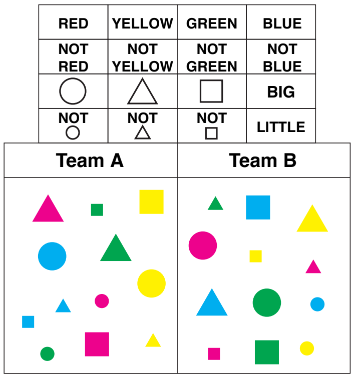
S: ***Red, blue, green, and yellow.***

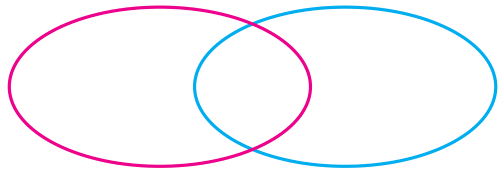
T: ***What sizes are there?***

S: ***Big and little.***

Prepare to play *The String Game* with A-blocks by setting up a team board and taping a list of the 16 possible string labels above it. Divide the class into two or more teams and distribute the game pieces on the team board. Draw overlapping red and blue strings on the chalkboard near the team board.

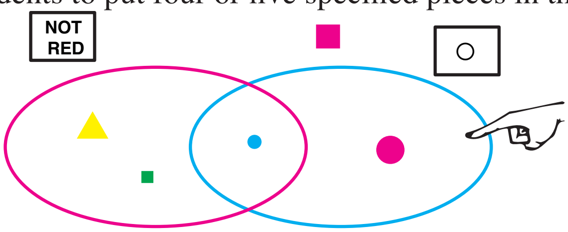
Note: If you want to have more than two teams or to let the teams choose their own names, prepare the team board accordingly.





Exercise 1

Give the red and blue strings visible labels, and then ask where various pieces go in the picture. For example, label the strings as shown here and ask students to put four or five specified pieces in the picture. Then point to a region.



T: ***Describe another piece that goes in this region.***

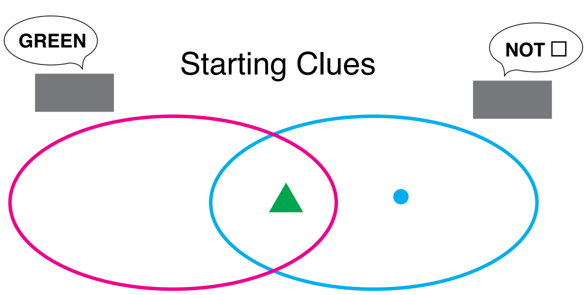
S: ***The little red circle.***

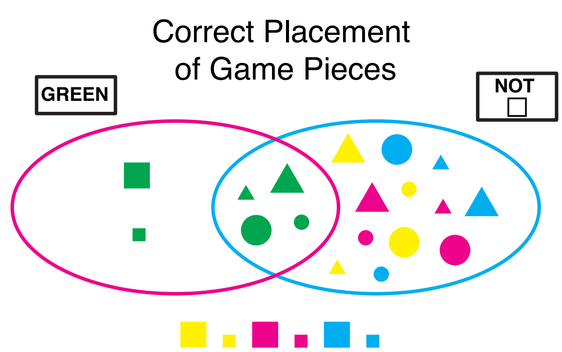
With the same string labels, discuss the other three regions of the string picture by asking students to describe pieces that could go in each of them.

Change the labels a couple of times and repeat the exercise. This activity is a review of the different string labels and the basic ideas in *The String Game* with A-blocks. Repeat it only as often as you feel is appropriate for your class.

Exercise 2

Play a two-string version of *The String Game* in the usual way with facedown string labels.† A game is suggested below. The illustration on the left shows two possible starting clues in a starting situation; the one on the right shows correct placement of all 24 A-block pieces.





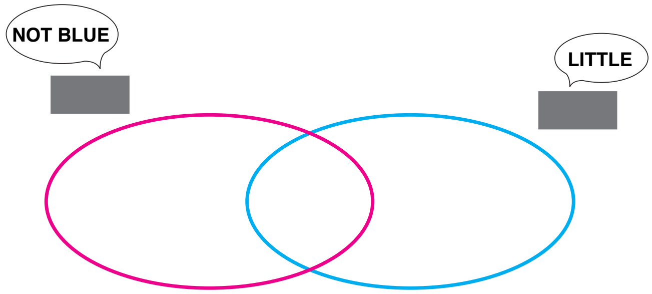
†See Appendix D on *The String Game* for a description of various versions of the game with A-blocks. The appendix also includes rules for the game.

Exercise 3

Reset the board for *The String Game* as illustrated below. Tape two A-Blocks String Game posters (version B) near the strings.

Red

RED	YELLOW	GREEN	BLUE
NOT RED	NOT YELLOW	NOT GREEN	NOT BLUE
○	△	□	BIG
NOT ○	NOT △	NOT □	LITTLE

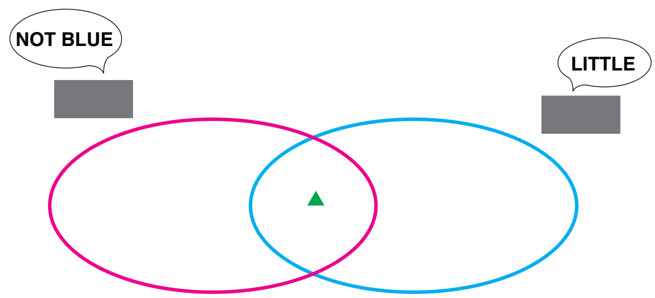


Blue

RED	YELLOW	GREEN	BLUE
NOT RED	NOT YELLOW	NOT GREEN	NOT BLUE
○	△	□	BIG
NOT ○	NOT △	NOT □	LITTLE

T: ***Instead of playing the game again right now, we are going to look at what information we get from knowing where some of the A-block pieces belong in the picture. I will place some pieces correctly and you can use these as clues to decide what labels the strings can and cannot have.***

Clue 1



T: ***The little green triangle belongs in the middle region.***

What does this tell us about the strings? Are there some labels (point to one of the lists) ***that the strings cannot have?***

S: **RED.**

T: ***We can cross off* RED *for which string?***

S: ***Both, because there is a green piece inside both strings.***

Use red and blue crayons or markers to cross off **RED** from both lists. In the same manner, let students eliminate as many labels as they can from both lists. Each time a label is crossed off from one list, the class should see that also it can be crossed off from the other list. A piece in the center region gives the same information about both strings.

A student may suggest incorrectly that some label be crossed off the lists. For example:

S: ***Cross off* NOT RED *from both lists.***

T: ***But this piece*** (point to the little green triangle) ***is not red.***

When your class has exhausted the information from this clue, they should find that it eliminates half of the possibilities for each string.

Red

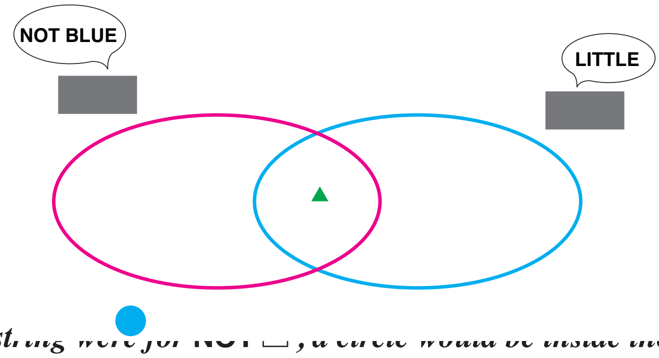
~~RED~~	~~YELLOW~~	GREEN	~~BLUE~~
NOT RED	NOT YELLOW	~~NOT GREEN~~	NOT BLUE
~~○~~	△	~~□~~	~~BIG~~
NOT ○	~~NOT △~~	NOT □	LITTLE

Blue

~~RED~~	~~YELLOW~~	GREEN	~~BLUE~~
NOT RED	NOT YELLOW	~~NOT GREEN~~	NOT BLUE
~~○~~	△	~~□~~	~~BIG~~
NOT ○	~~NOT △~~	NOT □	LITTLE

Clue 2

T: ***Here is another clue. The big blue circle is outside both strings.***



Let students eliminate labels from both lists giving explanations. Since the big blue circle is outside both strings, its placement gives the same information about both strings.

S: ***Cross off* NOT □ *from both lists. If a string were for* NOT □, *a circle would be inside the string.***

S: ***Cross off* NOT RED *from both lists. If a string were for* NOT RED, *a blue piece would be inside the string.***

Continue until your class finds that there are five remaining possibilities for each string.

Red

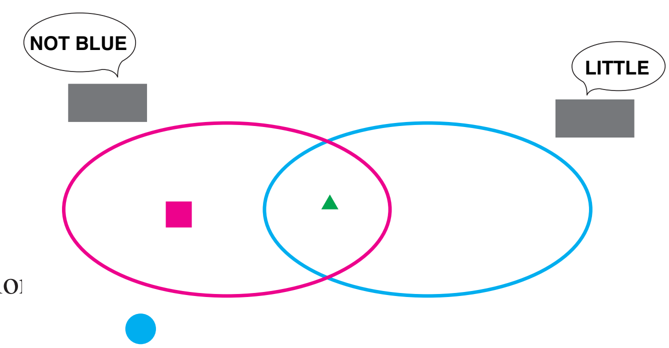
~~RED~~	~~YELLOW~~	GREEN	~~BLUE~~
~~NOT RED~~	~~NOT YELLOW~~	~~NOT GREEN~~	NOT BLUE
~~○~~	△	~~□~~	~~BIG~~
NOT ○	~~NOT △~~	~~NOT □~~	LITTLE

Blue

~~RED~~	~~YELLOW~~	GREEN	~~BLUE~~
~~NOT RED~~	~~NOT YELLOW~~	~~NOT GREEN~~	NOT BLUE
~~○~~	△	~~□~~	~~BIG~~
NOT ○	~~NOT △~~	~~NOT □~~	LITTLE

Clue 3

T: ***The next clue is that the big red square is inside the red string but outside the blue string.***



As before, cross off appropriate labels from the two lists. Students might observe that since the big red square is inside one string and outside the other, its placement gives opposite information about the two strings.

S: ***Cross off* GREEN *from the Red list. There is a red piece inside the red string, so it cannot be for* GREEN.**

S: ***The blue string could still be for* GREEN; *the piece inside the blue string is green and the pieces outside the blue string are not green.***

Continue until your class finds that there are two remaining possibilities for the red string and three remaining possibilities for the blue string.

Red

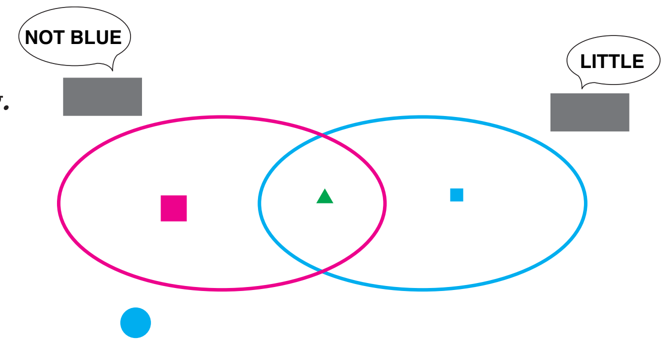
~~RED~~	~~YELLOW~~	~~GREEN~~	~~BLUE~~
~~NOT RED~~	~~NOT YELLOW~~	~~NOT GREEN~~	NOT BLUE
~~○~~	~~△~~	~~□~~	~~BIG~~
NOT ○	~~NOT △~~	~~NOT □~~	~~LITTLE~~

Blue

~~RED~~	~~YELLOW~~	GREEN	~~BLUE~~
~~NOT RED~~	~~NOT YELLOW~~	~~NOT GREEN~~	~~NOT BLUE~~
~~○~~	△	~~□~~	~~BIG~~
~~NOT ○~~	~~NOT △~~	~~NOT □~~	LITTLE

Clue 4

T: ***The little blue square is inside the blue string but outside the red string.***



From this clue your class should determine the strings.

- The red string must be for **NOT BLUE**. (The little blue square is not a circle, so **NOT** ○ can be crossed off from the Red list.)
- The blue string must be for **LITTLE**. (The little blue square is not green and it is not a triangle, so **GREEN** and △ can be crossed off the Blue list.)

Worksheets L1* and ** are available for individual work.

Writing Activity

You may like students to take lesson notes on some, most, or even all their math lessons. The "Lesson Notes" section in Notes to the Teacher gives some suggestions and refers to forms in the Blacklines you may provide to students for this purpose. In this lesson, for example, students may describe a set of A-blocks and then explain how to use the placement of an A-block piece to gain information about possible string labels. This could be done with an example.

Home Activity

This is a good time to send home a letter to parents/guardians about the use of strings (Venn diagrams) in mathematics. Blackline L1 has a sample letter.

Capsule Lesson Summary

Complete an arrow picture according to a composition rule.

Materials

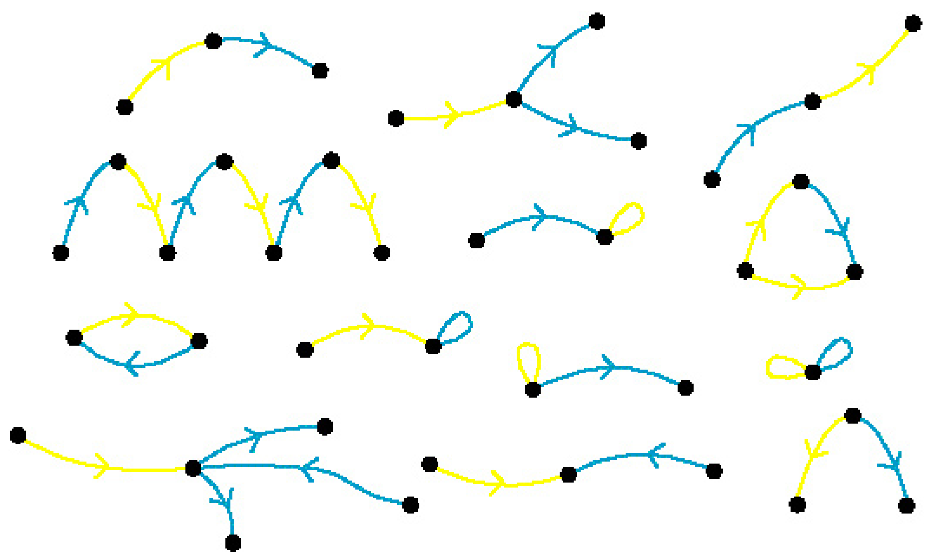
Teacher	• Tape • Colored chalk or markers	• Worksheets L2*, **, ***, and ****
Student	• Colored pencils	

Advance Preparation: Before the lesson begins, draw the arrow picture from Exercise 1 on the board or prepare it on a transparency. Give the picture plenty of space, and draw the arrows in colors that can be distinguished easily. Practice the motions described in the first activity . Use two hands, one to point at a starting dot and the other to trace arrows.

Description of Lesson

Exercise 1

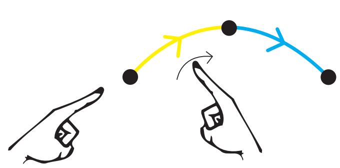
Invite students to comment on this arrow picture.



T: ***We are going to play a game with yellow and blue arrows. Some of you will remember the game from last year. The object of the game is to draw red arrows, but there is a rule for drawing red arrows.***

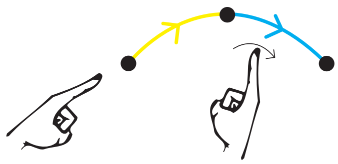
Use the upper left corner of the picture for your explanation. As you explain the rule stated in the left column below, make the motions described in the right column.

T: ***Each time there is a yellow arrow…***



(Point and hold your left forefinger on a dot at which a yellow arrow starts. Follow the yellow arrow with your right forefinger in the direction of the arrowhead.)

T: ***… followed by a blue arrow …***



(Stop the motion of your right forefinger at the middle dot; tap the dot, and then follow the blue arrow. Hold your right forefinger at the ending dot of the blue arrow.)

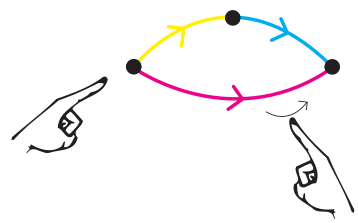
T: ***…then we can draw a red arrow from the dot where the yellow arrow starts…***

(Tap this dot several times with your left forefinger.)

…to the dot where the blue arrow ends.

(Tap this dot several times with your right forefinger.)

First trace and then draw the red arrow. Emphasize that your left forefinger marks the start of the red arrows and that your right forefinger is at the end of the red arrow.

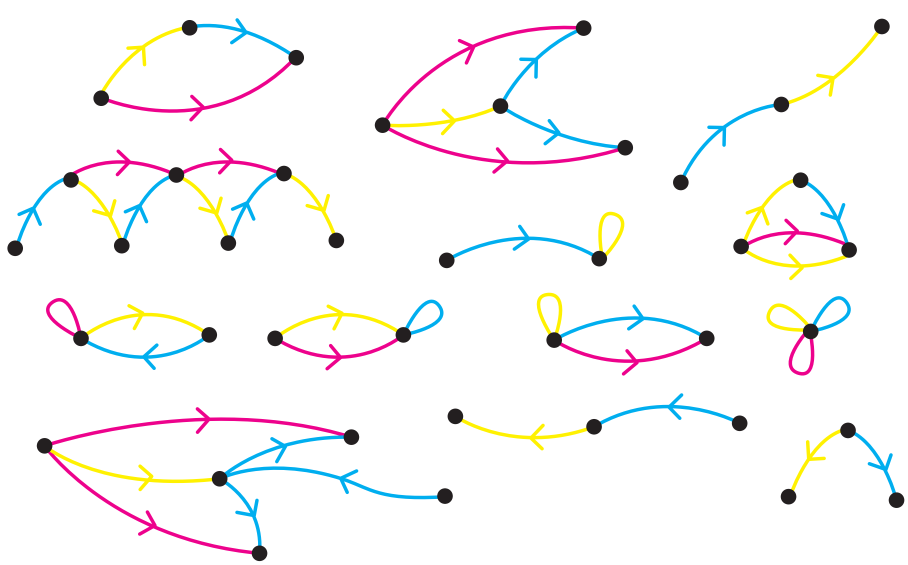


Repeat the motions as you summarize the rule.

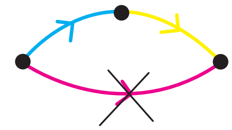
T: ***Yellow followed by blue is red.***
Where can we draw other red arrows?

Invite students to the board, one at a time, to show where other red arrows can be drawn (see the answer key below). Ask students to first trace a yellow arrow and a blue arrow following it, and then to trace how a red arrow can be drawn. Stop a student who starts to trace against the direction of an arrow, and emphasize that the direction of an arrow must be followed. Encourage the class to help you check for mistakes. Let a student draw a red arrow if it has been traced correctly. You may insist that students verbalize the rule "yellow followed by blue is red" each time they find a place to draw a red arrow. Common mistakes and difficulties are discussed following the answer key.

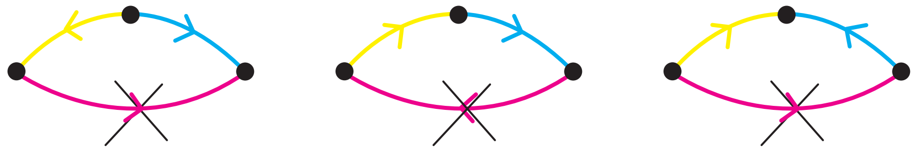
Answer key



Common Errors and Difficulties: The most common mistake at first will occur when students confuse the rule "yellow followed by blue'' with "blue followed by yellow."

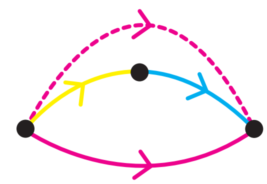


Other mistakes might occur when the directions of the arrows are not correctly followed.



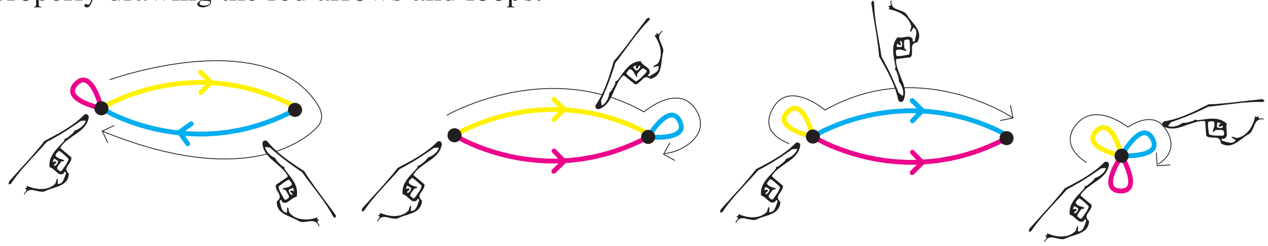
Remind students that arrows have directions like one-way streets, and they are not allowed to go the wrong way. Encourage other students to catch and explain such errors.

If it happens that a student proposes to draw a red arrow such as the one indicated by the dotted arrow in this illustration …



…another student will probably remark that the red arrow has already been drawn. There is nothing wrong with the dotted arrow; it is merely redundant. That portion of the picture is complete without it.

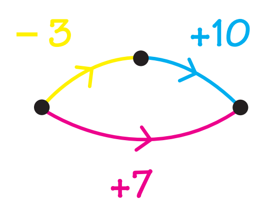
Students may have difficulty with compositions that involve loops. Remind students that they can trace or draw loops as well as arrows. The following pictures indicate how using both hands aids in properly drawing the red arrows and loops.



Worksheets L2* and ** are available for students to work on individually, using the yellow followed by blue composition to draw red arrows. You may like to allow about five minutes now for individual work. Then continue with Exercise 2.

Exercise 2

Erase the board except for the upper left portion of your picture.



T: ***Suppose the yellow arrow is for –3 and the blue arrow is for +10. What could the red arrow be for?***

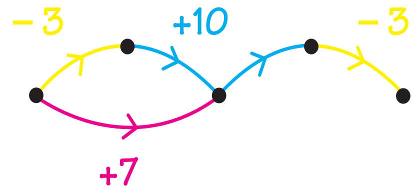
S: ***+7.***

T: ***Why?***

Let students check +7 for the red arrow with several examples of assigning numbers to the dots. Some students may want to reverse the order of the arrows and find +7 by calculating 10 – 3 = 7. You may like to use a bag and ask the class to pretend it contains a lot of something (beans, marbles, coins, and so on). Think about first taking out three objects and then putting in ten objects. The effect is a gain of seven objects in the bag.

Add a blue followed by a yellow arrow to your picture.

Trace the new blue arrow followed by the yellow arrow as you ask,

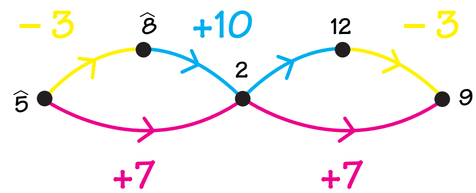


T: ***What is +10 followed by –3?***

S: ***Also +7.***

After checking, use some examples of assigning numbers to the dots or refer to your bag of objects. Observe that you can draw another red arrow for +7.

Ask students to check to see if the picture works with negative numbers as well as positive. For example:



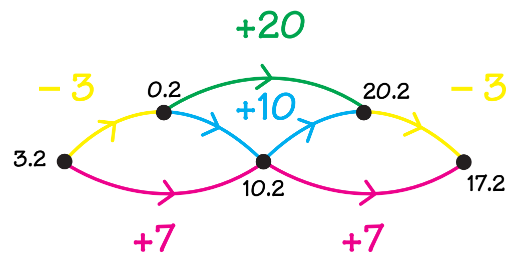
Trace the two blue arrows in turn as you ask.

T: ***What is +10 followed by +10?***

S: ***+20.***

Draw and label an arrow for +20.

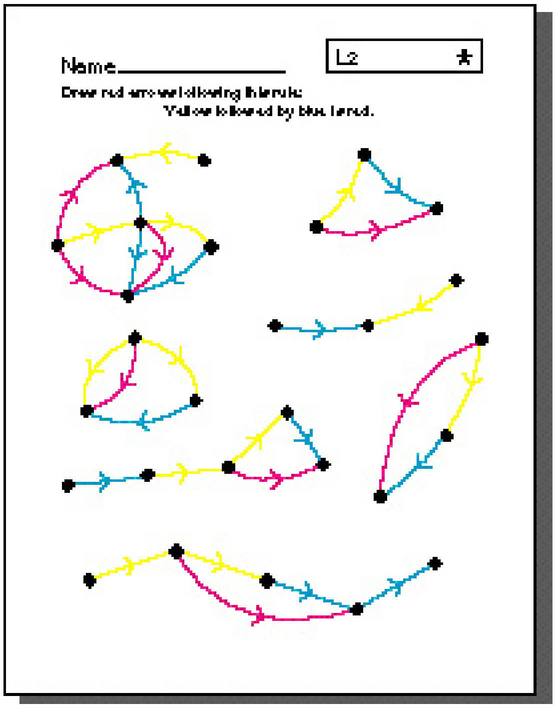
Then ask students to check to see if the picture works for decimal numbers as well. For example:

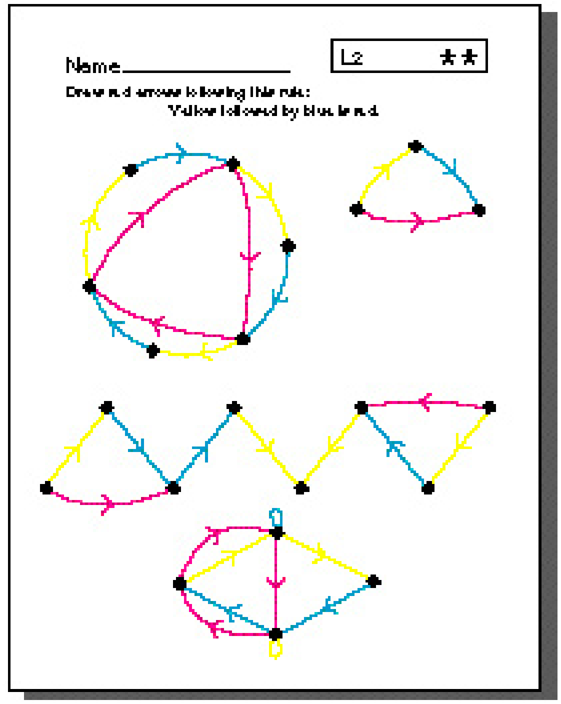


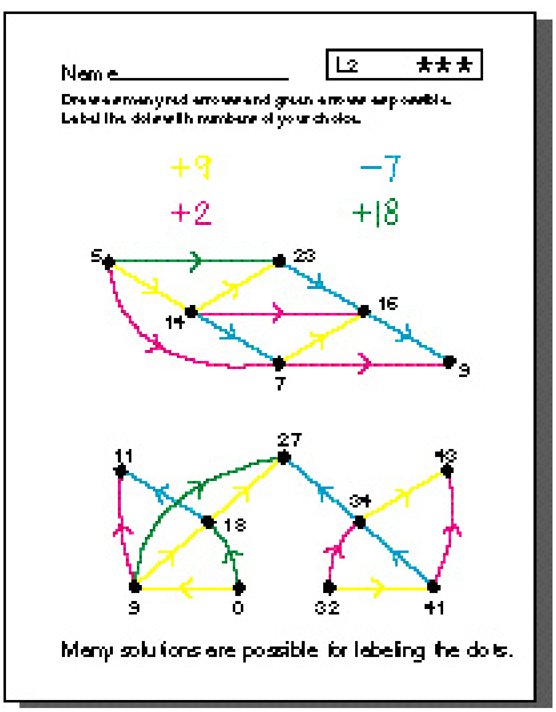
Worksheets L2*** and **** are available for individual wo

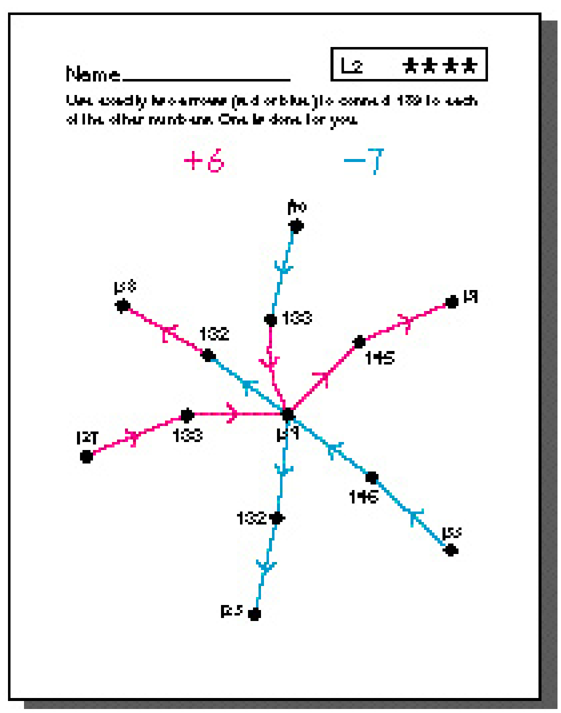
Home Activity

This is a good time to send home a letter to parents/guardians about the use of arrows. Blackline L2 has a sample letter. Suggest that parents/guardians practice using composition by doing some mental arithmetic with their child. For example, practice adding 95 by adding 100 and then subtracting 5, or practice subtracting 13 by subtracting 10 and then subtracting 3.


Name
L2 ★


Name
L2 ★★


Name
L2 ★★★
+9
−7
+2
+18
5
23
14
16
7
9
11
27
43
18
34
9
0
32
41
Many solutions are possible for labeling the dots.


Name
L2 ★★★★
+6
−7
133
132
145
133
146
132

Capsule Lesson Summary

Examine a string game situation with three strings by placing specified pieces in the correct regions and by describing pieces that go in designated regions. Play a three-string version of *The String Game*. Decide what information is given by three starting clues (correctly placed pieces) for a string game. Play additional pieces until the strings are determined.

Materials

Teacher	• A-Blocks String Game kit • Colored chalk • Crayons	**Student**	• None

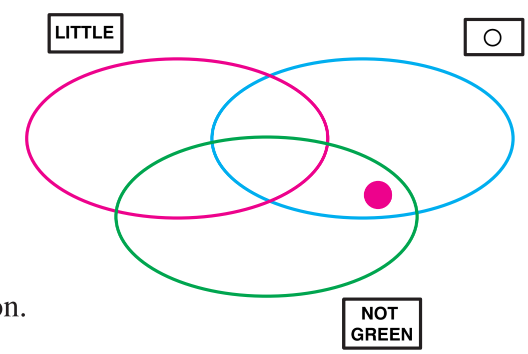
Description of Lesson

Prepare to play *The String Game* with A-blocks by setting up a team board and taping a list of the 16 possible string labels above it.

Exercise 1

Draw a three-string picture on the board and label the strings as shown below.

Hold up an A-block piece; for example, the big red circle.

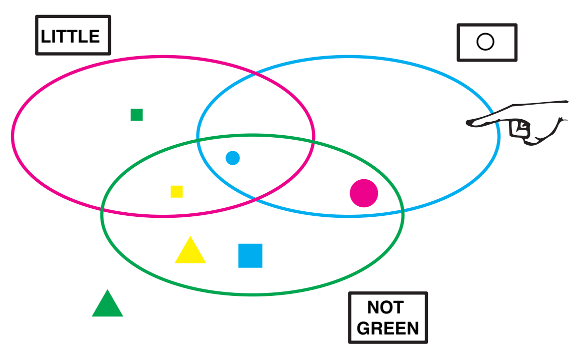


T: ***Who can place this piece correctly in the picture?***

S: ***It is a circle, so it goes inside the blue string; it is not green, so it goes inside the green string; and it is big, so it goes outside the red string.***

Do not expect such precise explanations, but do encourage students to explain why they put a piece in a particular region. Ask the class to check the placement.

Continue in the same manner with several other pieces. Then point to a region and ask,



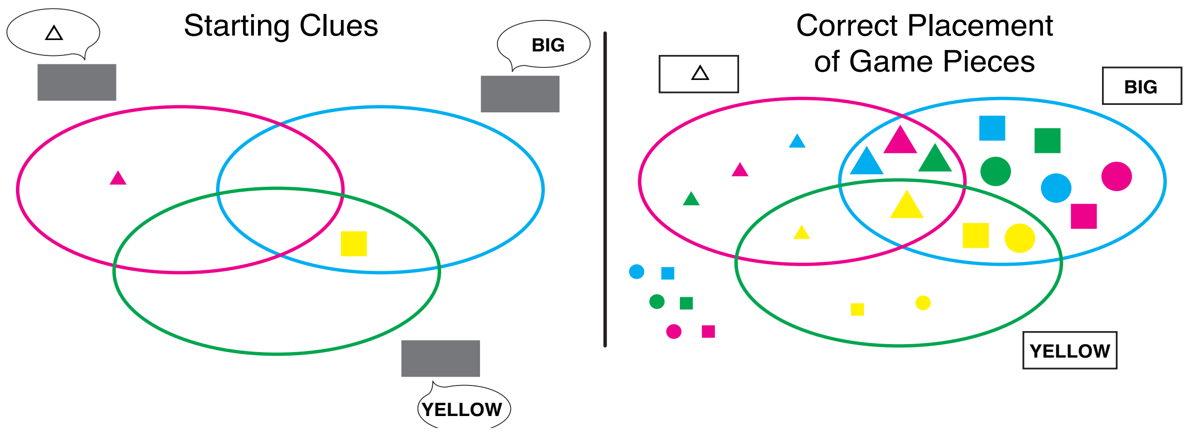
T: ***Can you describe a piece that belongs here?***

S: ***The big green circle.***

Ask for pieces that belong in other specified regions. Try to get at least one piece in each region and two or more pieces in some regions.

Exercise 2

Distribute all the A-block pieces on the team board, and prepare to play a three-string version of *The String Game*. The illustration on the left shows two possible starting clues. The illustration on the right shows correct placement of all the A-block pieces, and can be used as a crib sheet during the play of the game.

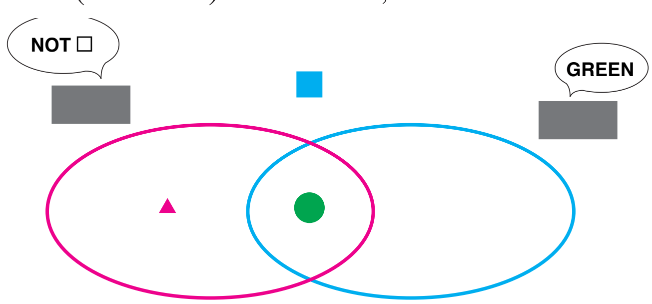


Exercise 3

Clear the board and set up a two-string version of *The String Game* as illustrated below. Tape two A-Blocks String Game posters (version B) to the board, one next to each of the strings.

Red

RED	YELLOW	GREEN	BLUE
NOT RED	NOT YELLOW	NOT GREEN	NOT BLUE
○	△	□	BIG
NOT ○	NOT △	NOT □	LITTLE



Blue

RED	YELLOW	GREEN	BLUE
NOT RED	NOT YELLOW	NOT GREEN	NOT BLUE
○	△	□	BIG
NOT ○	NOT △	NOT □	LITTLE

T: ***Instead of playing the game in the usual way, we are going to work together to see what information we get from knowing where some of the A-block pieces belong in the picture. Just like in a regular game, we are trying to figure out what labels the strings have.***

Several pieces are placed correctly in this picture. What does their placement tell us about the strings? Are there labels the strings cannot have, so we can cross them off the lists?

When explaining why labels can be crossed off, students should identify which pieces give the information. The following dialogue indicates how some labels are eliminated from the lists.

S: ***The red string cannot be for*** ○ ***because there is a triangle inside it.***

S: ***The blue string cannot be for*** **LITTLE** ***because there is a big piece inside it. Also, the red string cannot be for*** **LITTLE.**

S: ***Neither string can be for*** **NOT YELLOW** ***because the big blue square is not yellow and it is outside of both strings.***

Cross off labels that are correctly suggested by students. You may need to ask about some labels or to explain how they are eliminated yourself. For example:

T: ***Could the red string be for* NOT RED*?***

S: ***No, there is a red piece inside the red string.***

T: ***What about the blue string; could it be for* NOT RED*?***

S: ***The big green circle inside the blue string is not red, so that's okay.***

S: ***But the big blue square is not red and it is outside the blue string. If the blue string were for* NOT RED*, the big blue square would be inside the blue string.***

Continue in this way until each list has all but two labels crossed off, as shown below.

Red

~~RED~~	~~YELLOW~~	~~GREEN~~	~~BLUE~~
~~NOT RED~~	~~NOT YELLOW~~	~~NOT GREEN~~	NOT BLUE
~~○~~	~~△~~	~~□~~	~~BIG~~
~~NOT ○~~	~~NOT △~~	NOT □	~~LITTLE~~

Blue

~~RED~~	~~YELLOW~~	GREEN	~~BLUE~~
~~NOT RED~~	~~NOT YELLOW~~	~~NOT GREEN~~	~~NOT BLUE~~
○	~~△~~	~~□~~	~~BIG~~
~~NOT ○~~	~~NOT △~~	~~NOT □~~	~~LITTLE~~

T: ***There are only two possibilities remaining for each string.***
The red string could be for* NOT BLUE *or for* NOT □ *.
The blue string could be for* GREEN *or for* ○*.

Now you choose a piece and I'll show you where it belongs in the picture. Try to select a piece whose placement will help you figure out what the strings are for.

Several types of responses could occur here.

1. A student might choose a piece whose placement is already determined.

S: ***The little green circle.***

Hold up the piece as you ask,

T: ***Do we know anything about where this piece belongs in the picture?***

S: ***It belongs in the blue string because it is green and a circle.***

S: ***If the red string is for* NOT BLUE*, it belongs inside the red string.***

S: ***If the red string is for* NOT □ *, it belongs inside the red string.***

S: ***It must be inside of both strings—in the middle.***

T: ***Knowing where the little green circle belongs gives us no new information.***

Place the piece and ask again for a piece whose placement will help to determine the strings.

2. A student might choose a piece whose placement determines one of the strings.

S: ***The little green triangle.***

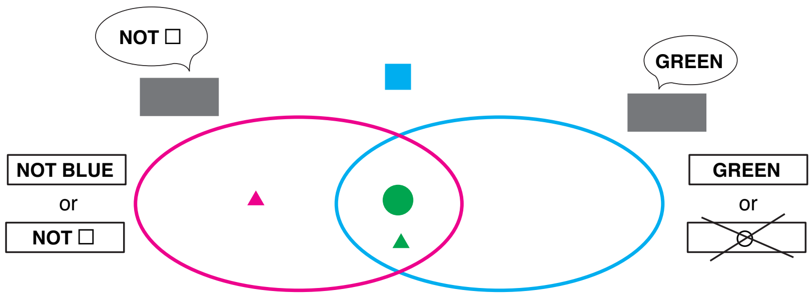
Hold up the piece as you ask,

T: ***Do we know anything about where this piece belongs in the picture?***

S: ***It is not blue and it is not square, so it goes inside the red string.***

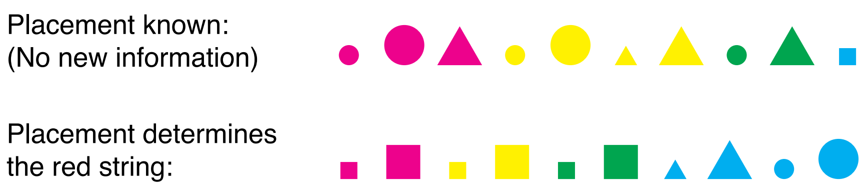
S: ***If the blue string is for* GREEN*, it goes inside the blue string. But if the blue string is for* ○*, it goes outside the blue string.***

Place the piece correctly in the picture. Let the class discover that the blue string is determined but that there are still two possibilities for the red string.



T: ***After seeing where the little green triangle belongs in the picture, we now know that the blue string is for* GREEN*. We still need to figure out what the red string is for. Which piece should I play next to determine the red string?***

At this point, half of the unplaced pieces can be placed in the picture giving no new information. Placement of any one of the other pieces will determine the red string.

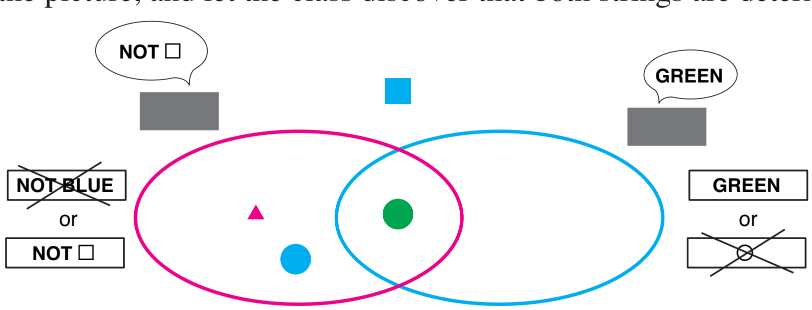


Let students suggest pieces to put in the picture until one is chosen whose placement determines the red string.

3. A student might initially choose a piece whose placement determines both strings.

S: ***The big blue circle.***

Place the piece in the picture, and let the class discover that both strings are determined.



L4 LOGICAL THINKING #1

Capsule Lesson Summary

Decide the truth or falsity of statements involving these logical connectives: *at most*, *at least*, *every*, *none*, *all*, *each*, and *every*.

Materials

Teacher
- Colored chalk

Student
- Paper
- Colored pencils, pens, or crayons
- Worksheets L4* and **

Description of Lesson

The discussion in this lesson will center on recognizing various attributes of class members and of numbers, and on sorting according to those attributes. For the first exercise, choose some well-defined and easily verified attribute such as "wearing a sweater." The lesson will be more easily managed if the attribute is possessed by five to ten of those present in your room. In any event, base the discussion on your own class, their names, and one attribute of some of your students.

Exercise 1

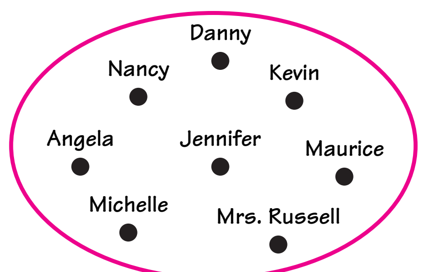
T: ***Who is wearing a sweater today?***

S: ***Michelle, Angela, Nancy, Maurice, Kevin, Jennifer, Danny, and you Mrs. Russell.***

T: ***How many of us are there?***

S: ***Eight.***

Draw a large string on the board and put dots inside it for those wearing a sweater.



T: ***This string is for the people in our room who are wearing a sweater today. I will make some statements about this set of people. You tell me whether each statement is true or false. My first statement is this: Every member of the group is male.***

S: ***False, because only Kevin, Danny, and Maurice are male.***

T: ***Now consider this statement: At least one member of the group is male.***

S: ***True.***

T: ***What does* at least one *mean?***

S: ***One or more than one. Kevin, Danny and Maurice are male; there are three males, so there is at least one member who is male.***

T: ***At least half of the group is male.***

S: ***False. We just said that there are three males in the group. Half of the group would be four and* at least four *means four or more than four.***

T: ***At most three members of the group are female.***

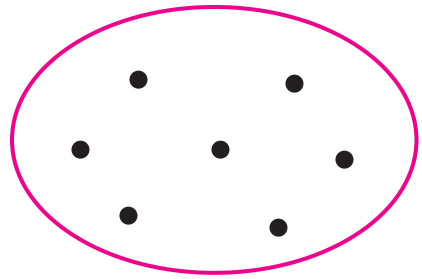
S: ***False.***

T: **At most three** ***means three or less than three.***

Note: *At most* is generally more difficult for students than *at least*. Your students will become familiar with these ideas through usage over time and exposure to them in several contexts. You may find it appropriate to remind students of the meanings when opportunities to use these phrases arise.

Exercise 2

Draw this string picture on the board.



T: ***Let's label the seven dots in this string with whole numbers that are greater than 20 and are multiples of 3.***

Write this requirement on the board.

Each number is greater than 20 and is a multiple of 3.

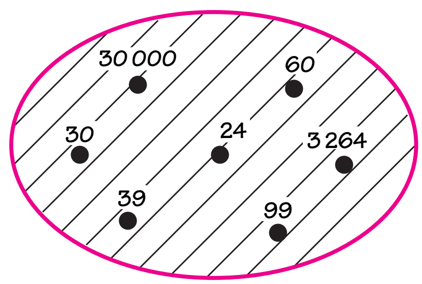
T: ***What are some numbers we could put in this string?***

S: ***24; 24 is greater than 20 and is a multiple of 3.***

S: ***How about 99? It is both a multiple of 3 and greater than 20.***

S: ***30; 300; 3,000; 30,000; and so on.***

This is a good opportunity to practice naming multiples of 3. Invite students to label the dots in the picture at the board. When all the dots are labeled, hatch the region inside the string. Your picture will be similar to this one.



Adjust the following dialogue according to which numbers are in your picture.

T: ***We now have the seven numbers in the string, and I have hatched the region inside the string to show that no other numbers are in it.***

I will make some statements about the numbers in this string, and you tell me if they are true or false. The first statement is this: All numbers in the string are even.

S: ***False; 39 and 99 are both odd, not even.***

T: ***No number in the string is even.***

S: ***False; 60 is even and so are several others.***

T: ***At least three numbers in the string are even.***

S: ***True; there are five even numbers in the string, and*** **at least three** ***means three or more than three.***

T: ***At least six numbers in the strings are even.***

S: ***False.***

T: ***At least three numbers in the string are odd.***

S: ***False; only two of the numbers are odd.***

T: ***At most one number in the string is odd.***

S: ***False;* at most one *means one or zero, but there are two odd numbers in the string.***

T: ***At most four numbers in the string are odd.***

S: ***That's true;* at most four *means four or less than four. There are two odd numbers; that's less than four.***

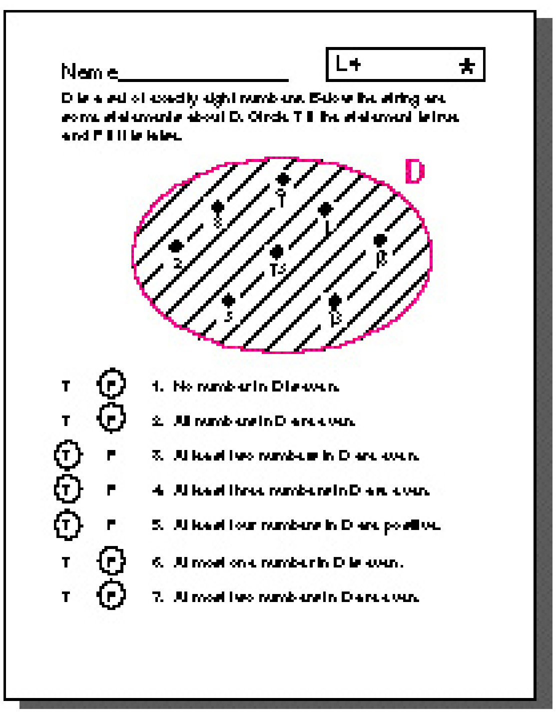
There are two worksheets for this lesson. Distribute copies of Worksheet L4* and do the first three problems collectively. Then let the students work individually on the other problems. Caution them to think carefully before answering. You may want to have discussions of problems with individual students to reinforce the language. As students complete Worksheet L4*, direct them to do Worksheet L4**.

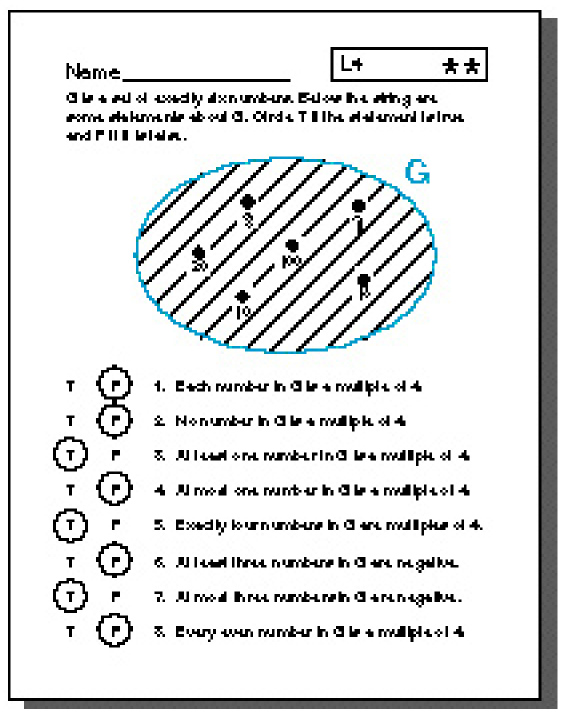
Writing Activity

Ask students to write sentences using the terms *at most*, *at least*, *all*, and *none*. Then ask that they write explanations about the meanings of those terms in their sentences.

Extension Activity

Suggest that students create a Worksheet like L4* or ** to challenge the teacher or to give to another student.





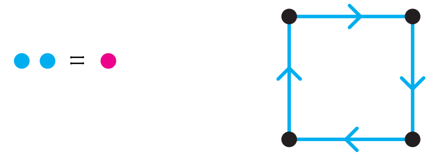
Capsule Lesson Summary

Explore compositions when a relation is composed with itself several times.

Materials

Teacher	• Colored chalk	• Colored pencils, pens, or crayons
Student	• Unlined paper	

Description of Lesson

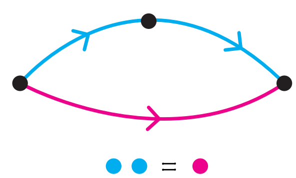


Draw this picture on the board.

Point to the color rule to the side of the arrow picture.

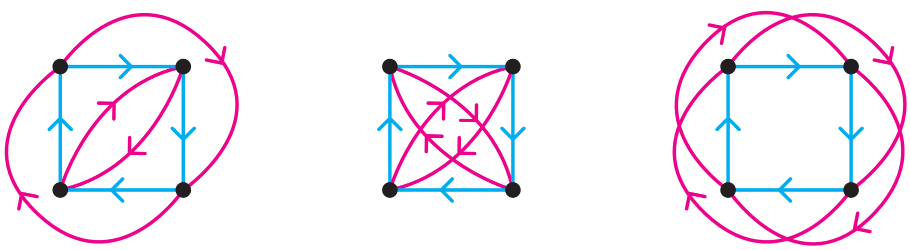
T: ***What could this mean?***

S: ***Two blues is the same as one red.***



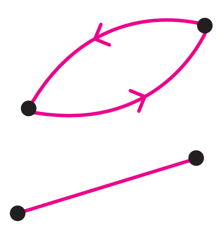
Accept any answers of this sort. When referring to the color rule you should say, “Blue followed by blue is red.” Illustrate the ruleby drawing an arrow picture above it. With your forefingers, trace the blue arrows in succession, indicating that a red arrow can be drawn from the starting dot of the first blue arrow to the ending dot of the second blue arrow.

Instruct students to copy the square arrow picture with four blue arrows. Then ask that they use the color rule “blue followed by blue is red” to draw as many red arrows as they can find in their picture. As you observe individual work, you might see a variety of correct pictures such as these:



Invite some students to explain how they knew to draw an arrow and then to draw it in the picture on the board. When all four red arrows are in the picture on the board, explain each by starting at each dot, tracing the blue arrows, and then the red. Say, “Blue, blue, RED,” each time.

Simplify the picture on the board by replacing two red arrows with a red cord as follows.

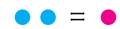


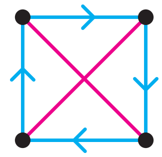
T: ***We can make this picture simpler. Whenever two dots are connected by arrows of the same color in opposite directions …***

…we can draw a cord instead.

Erase the red arrows and draw red cords as you say,

T: ***We can draw two red cords in this picture.***



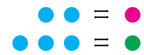


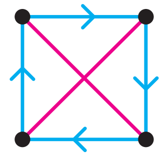
You may like to suggest students redraw the picture on their papers.

Add another color rule to the key.

T (pointing to the new color rule)**:** ***What does this mean?***

S: ***Three blues is one green.***

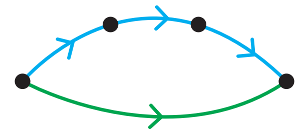




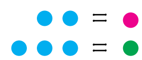
Again, accept any answer of this sort. Illustrate this new color rule with an arrow picture

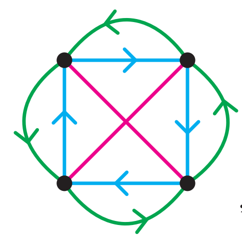
Trace the blue arrows and then the green arrow as you say,

T: ***Blue followed by blue followed by blue is green.***



Ask students to draw as many green arrows as they can find in their pictures. Invite some students to green arrows in the picture on the board.

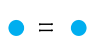


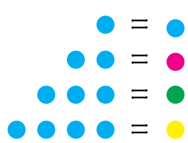


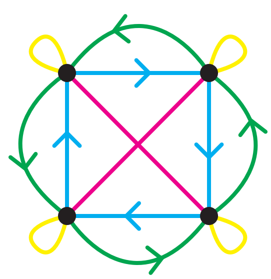
As you explain how each green arrow is obtained, trace arrow ”

Add another color rule to the key.

Perhaps some students will see immediately that there will be four yellow loops. As you explain how a yellow loop is obtained, trace arrows and say, “Blue, blue, blue, blue, YELLOW LOOP.”

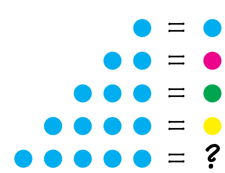
Add  to the list of color rules. This will emphasize the importance of the number 4 in the construction. Your picture on the board should look similar to this.



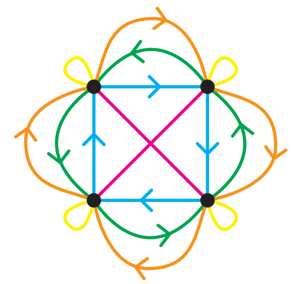


Now add another row to the list of color rules. Draw five blue dots and ask the class for color suggestions.

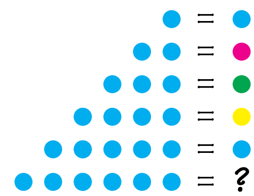
Your students may suggest yet another color, such as orange. Go through the process of adding the appropriate orange arrows to the picture.



The picture will now be complex. Ask for observations about the arrows. The key response is that every orange arrow duplicates a blue arrow and vice versa. Hence the orange arrows are for the same relation as the blue.



After making this observation, erase the orange arrows and replace the **?** with a blue dot. Then go to the next step: six blue dots.

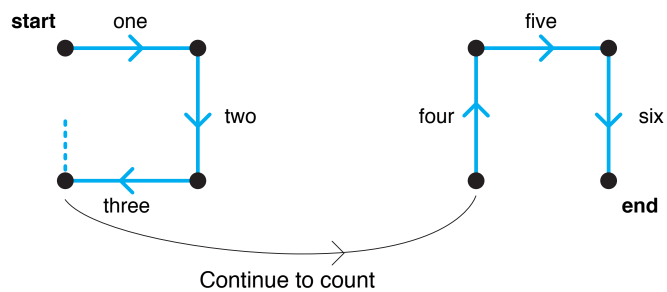


T: ***What color?***

S: ***Red.***

S: ***Two blues.***

After some discussion, place your left forefinger on one of the dots and follow the arrows while the class counts.



T: ***We started here.*** (Tap the starting dot with your left forefinger.) ***We ended here.*** (Tap the ending dot with your right forefinger.) ***Then we can draw an arrow from this dot*** (tap the start) ***to this dot*** (tap the end)***. But what arrow has already been drawn from this dot*** (tap the start) ***to this dot*** (tap the end)***?***

S: ***Red.***

Quickly repeat the procedure using each of the other three dots as starting dots. Finally, ask a student to complete the color rule.

T: ***What will seven blue arrows make?***

S: ***Three blues.***

S: ***Green!***

T: ***What will eleven blue arrows make? How can you decide?***

S: ***Green again.***

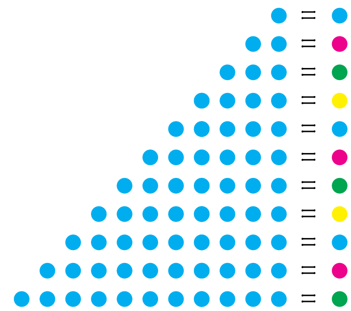
S: ***I guessed, but I can't explain it.***

S: ***After four blue arrows, you come back to the starting dot. After four more, you come back to the starting dot again. That's eight. There are still three blues left, and three blues is green.***

You may need to give the preceding explanation again. Have the class count as you start from a dot and move along the blue arrows.

L5

Add several more rows of blue dots to your color key and ask students to provide the color.



T: ***Will the list ever end?***

S: ***No.***

S: ***We only use four colors.***

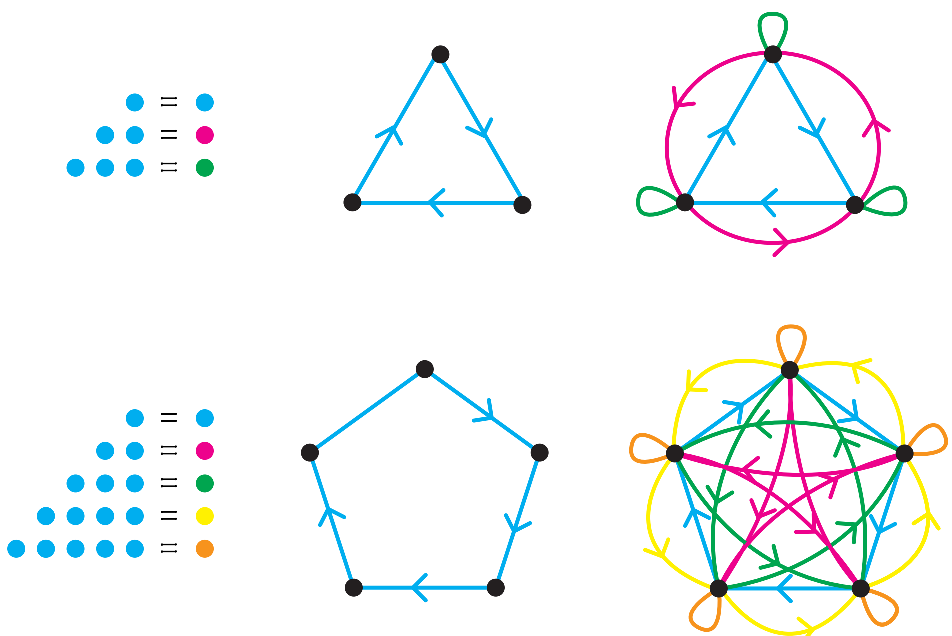
T: ***What color is 33 blues?***

Let the students discuss this until it is decided that 33 blues is blue.

T: ***60 blues?*** (Yellow)
62 blues? (Red)
100 blues? (Yellow)

If there is time left at the end of the class period, ask students to copy one of the following arrow pictures and color rules, to draw all of the arrows following color rules, and to give more color rules if they can.

Starting Picture Resulting Picture



L6 STRING GAME WITH NUMBERS #1

Capsule Lesson Summary

Review classification of numbers greater than $\widehat{10}$ and less than $\widehat{10}$. Locate given numbers and name other numbers in specific regions of a string picture with the strings labeled. Play *The String Game* with numbers with a limited set of attributes (multiples and order).

Materials

Teacher
- Colored chalk
- Numerical String Game kit

Student
- Unlined paper
- Colored pencils, pens, or crayons

Advance Preparation: Before this lesson , you will need to prepare some materials from the Numerical String Game kit. These materials will be used again in all future lessons on *The String Game*, so keep them together in the envelope with the kit. You may want to laminate the game pieces and cards to make them more durable.

1. Cut out one set of game pieces (30 numbers). If you have a magnetic board, magnetize each piece by sticking a small piece of the magnetic material to the back. If there is no magnetic board available, you can still use a regular chalkboard. In this case, have loops of masking tape ready to stick to the back of each piece so that they can be stuck to the board. Masking tape loses its stickiness quickly, so be prepared to reinforce the pieces regularly with new loops of tape.
2. Prepare the string cards in the same manner as the game pieces.
3. Prepare a team board as pictured on the next page. This board should be metallic if you have magnetized the game pieces.

Description of Lesson

Since this lesson will make use of only a limited set of attributes, cut one Numerical String Game poster so only the attributes shown below remain.

MULTIPLES OF 2	**MULTIPLES OF 3**	**MULTIPLES OF 4**	**MULTIPLES OF 5**
MULTIPLES OF 10	**LESS THAN 50**	**LESS THAN $\widehat{10}$**	**ODD NUMBERS**
	GREATER THAN 50	**GREATER THAN $\widehat{10}$**	

This abbreviated poster will be used in Exercise 3 of this lesson and again in Lesson L9.

Exercise 1

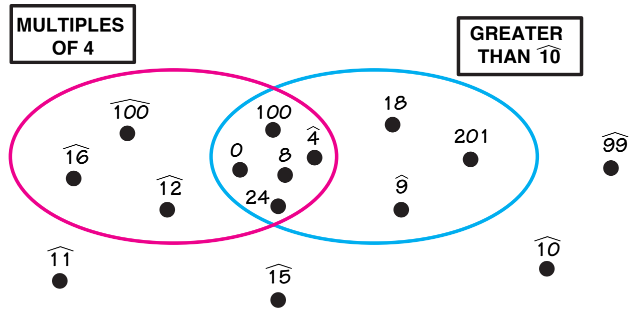
Ask students to name some numbers greater than $\widehat{10}$ and then to name some numbers less than $\widehat{10}$. When appropriate, locate the numbers on the number line and point out their position relative to $\widehat{10}$. Observe that $\widehat{10}$ is neither greater nor less than $\widehat{10}$.

Exercise 2

Draw a two-string picture on the board and label the strings **MULTIPLES OF 4** and **GREATER THAN $\widehat{10}$.**

Ask students to locate 8, 18, $\widehat{12}$, $\widehat{15}$, 24, $\widehat{4}$, and $\widehat{10}$ in the picture. Then point to each region of the picture and ask, in turn, for two or three more numbers that belong in that region.

Your string picture might look similar to this illustration.



Exercise 3

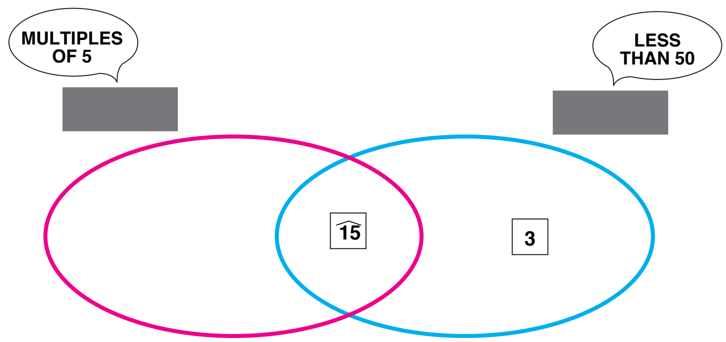
Prepare to play *The String Game* with numbers by setting up the team board and taping the abbreviated Numerical String Game poster above it. Distribute game pieces evenly among the teams.

Note: If you prefer, the game can be played with more than two teams and the team members can choose names for their teams (rather than A or B). For example, you might use three of four cooperative groups as teams. Prepare the team board accordingly.

Choose **MULTIPLES OF 5** and **LESS THAN 50** as string labels for the red and blue strings, respectively. Place these cards face down in the picture. Select one playing piece from each side of the team board—or allow the teams to each select one piece for you—to place correctly in the picture as starting clues.

MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4	MULTIPLES OF 5
MULTIPLES OF 10	LESS THAN 50	LESS THAN $\widehat{10}$	ODD NUMBERS
	GREATER THAN 50	GREATER THAN $\widehat{10}$	

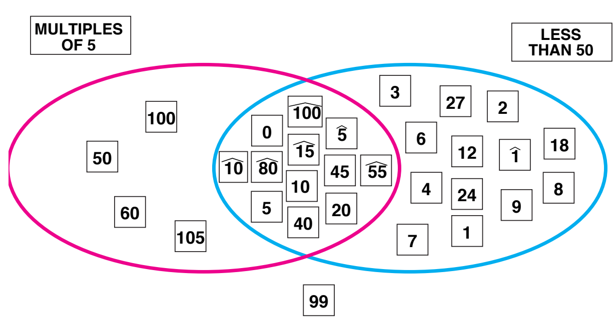
Team A			Team B		
20	$\widehat{100}$	12	$\widehat{80}$	10	4
0	27	$\widehat{1}$	$\widehat{5}$	1	7
60	45	40	9	8	24
$\widehat{55}$	100	$\widehat{10}$	18	99	2
5		6	50		105



T: ***We are going to play The String Game with numbers. It is played like the game with A-Blocks except that the playing pieces are numbers and the string labels can be these*** (point to the poster)**.**

Play this two-string game in the usual way.†

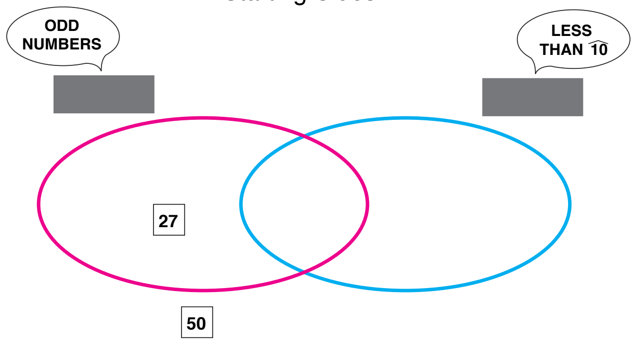
This illustration shows the correct placement of all 30 numbers and can be used as a crib sheet.



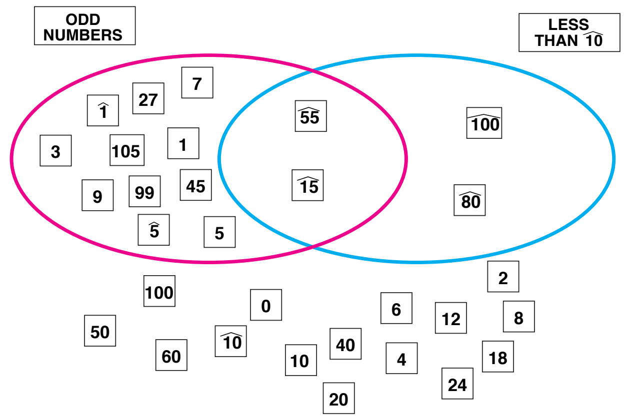
†See Appendix D on *The String Game* for a description of various versions of the game with numbers. The appendix also provides an expanation of how the game is played.

If the class finishes playing this game quickly, play a second game, such as the one suggested below. Save five to ten minutes for Exercise 4.

Starting Clues



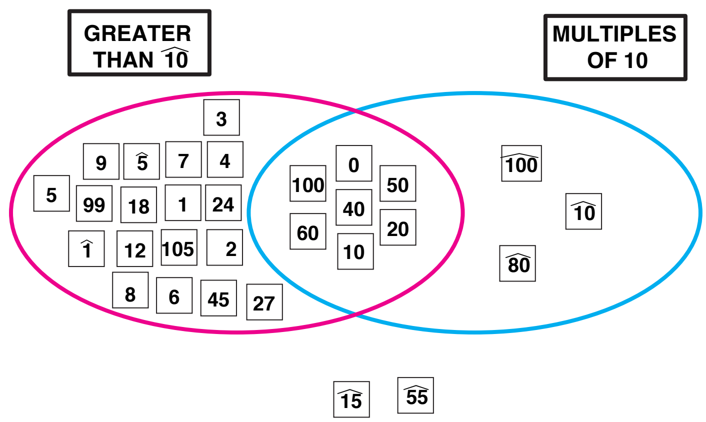
Correct Placement of Game Pieces



Exercise 4

Instruct students to draw a two-string picture on their papers, labeling the strings **GREATER THAN $\widehat{10}$** and **MULTIPLES OF 10**.

Leave the game pieces for *The String Game* with numbers visible in the room, and ask students to work independently to locate as many of these numbers as possible in their pictures. This exercise need not continue very long; expect some students to locate only a few numbers while others will quickly locate most of the numbers. A complete picture is shown below.



Home Activity

Suggest that parents work with their child to make lists of multiples of 2, 3, 4, 5, and 10. Ask them to include both positive and negative numbers in the lists and to look for patterns.

L7 HOW MANY DISGUISES?

Capsule Lesson Summary

Use a tree diagram to calculate how many different disguises can be made from two choices of hairstyles, two choices of eyebrows, three choices of mustaches, and two choices of beards. A disguise must include one choice from each category.

Materials

Teacher • Disguise kit

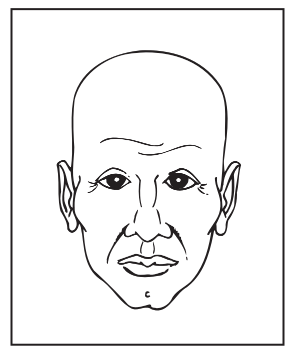
Student • Worksheets L7* and **

Advance Preparation: The materials in the disguise kit need some preparation before the lesson.Cut out the large disguise pieces and the disguise cards. Attach tape or magnetic material to both the disguise pieces and the cards so that they can be put up on your board at appropriate times during the lesson.

Description of Lesson

Exercise 1

Tape the poster from the disguise kit, picturing the detective Spike, to the board.



T: ***This is Spike the detective. Frequently Spike must disguise himself before he goes out on a mission.***

He has these disguises to choose from:

(Put the large disguise pieces on the board as you refer to them.)

T: ***Two hairstyles, two styles of eyebrows, three styles of mustaches, and two styles of beards.***

T: ***Each time Spike puts on a disguise, he wears exactly one hairpiece, one pair of eyebrows, one mustache, and one beard. Come and disguise Spike.***

Let several students disguise Spike as they please. If necessary, remind your students that it is not necessary to change all the attributes to have a different disguise.

For example, the disguises pictured below are different with only a change of beard.

T: ***We have seen three or four different disguises that Spike could wear. How many different disguises do you think Spike has?***

Write students' estimates on the board for future reference.

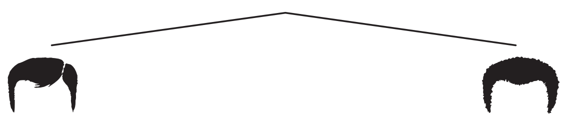
T: ***How might we solve this problem?***

Let the students express any ideas they have, and then direct them in making a tree diagram.

T: ***Let's start with hairstyles. From how many different hairstyles can Spike choose?***

S: ***Two.***

Begin a tree on the board, using the disguise cards to show possibilities at each step. Allow plenty of room for the tree that will be constructed.



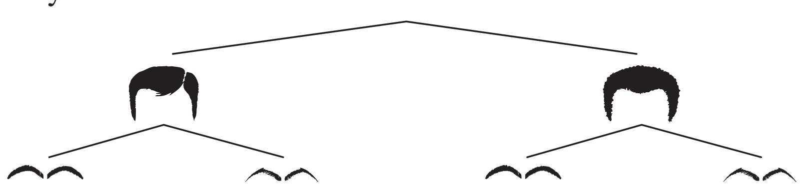
T (pointing to either of the hairstyles)***: Suppose Spike chooses to wear this hairstyle. How many choices does he have for eyebrows?***

S: ***Two choices.***

T: ***What if Spike chooses the other hairstyle?***

S: ***He still has two sets of eyebrows to choose from.***

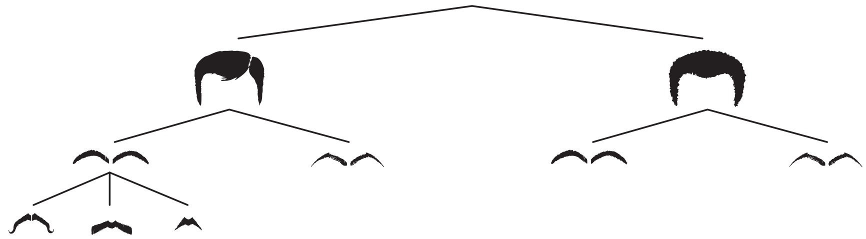
Add these branches to your tree.



T (pointing to the hairstyle on the left, then to the thicker set of eyebrows)**: *Suppose Spike chooses this hairstyle and these eyebrows. What next?***

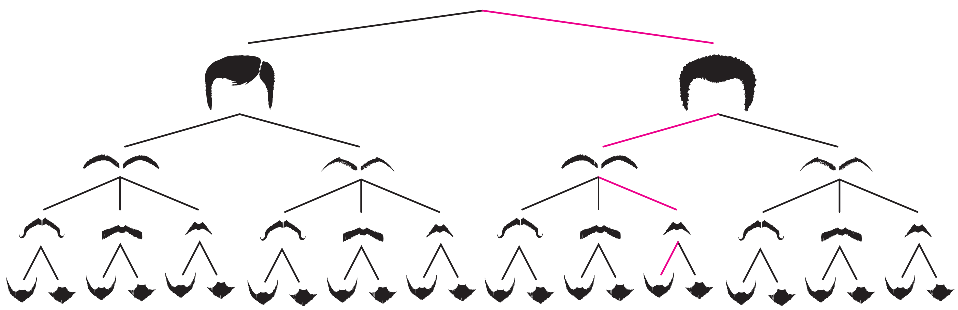
S: ***He has three mustaches to choose from.***

Add these three branches to your tree.



Continue to develop the tree in this manner to its completion. Your tree should look like the one below.

Invite a student to start at the top of the tree and trace a path through the tree. As the path is being traced, ask another student to put the corresponding disguise on Spike. For example, suppose a student traces the path shown in red here. Then the other student will equip Spike with the disguise on the right.



Ask different students to trace paths and disguise Spike a couple more times.

T: ***Each path in the tree corresponds to exactly one disguise.***

Reverse this activity by asking a student to put a disguise on Spike. As the disguise is being put on, let another student trace the corresponding path in the tree diagram. Repeat this activity a couple times.

T: ***For each disguise there is exactly one path in the tree, and for each path there is exactly one disguise.***

How many paths are there in the tree diagram?

Give students a minute or two to study the tree before answering.

S: ***Twenty-four paths. I counted the number of beards along the bottom of the tree.***

T: ***How many disguises for Spike?***

S: ***Twenty-four, one disguise for each path.***

Compare the correct number of disguises, 24, to the estimates on the board.

Worksheets L7* and ** are available for individual work.

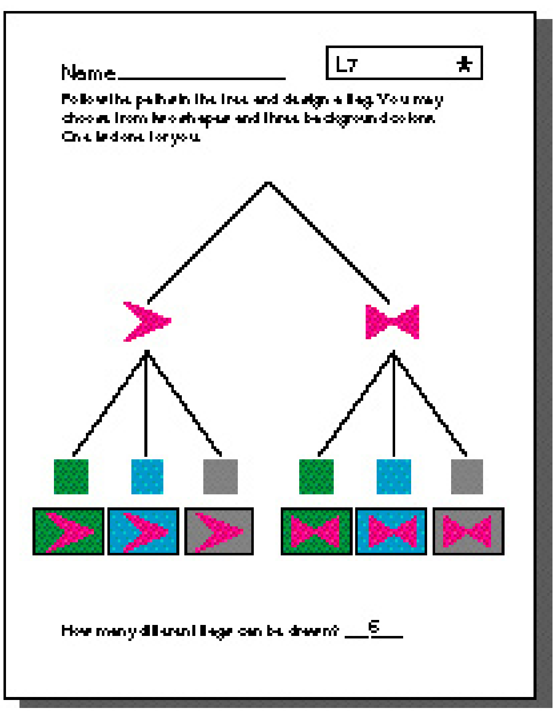
Extension Activity

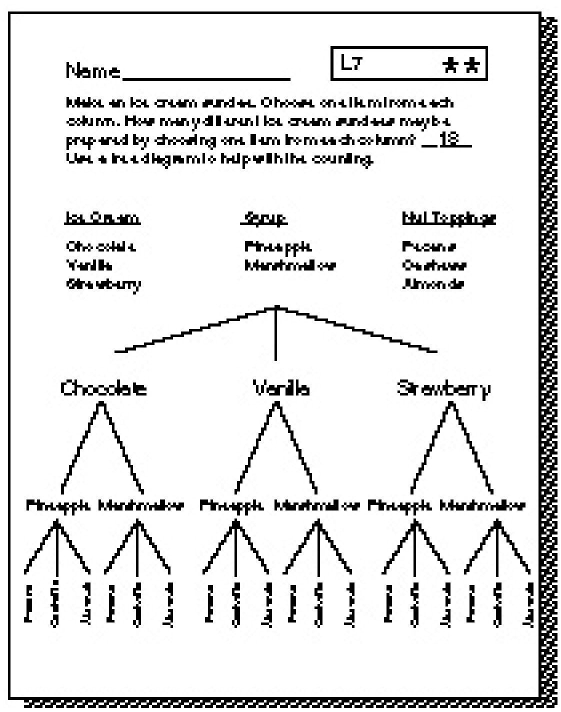
L7

Place the parts of the disguise kit out for students to investigate. Some students may enjoy making more disguises for Spike and counting, for example, how many disguises there would be if he had a third hairstyle.

Writing Activity

Invite students to write counting problems similar to the one in this lesson or on the worksheets. Suggest they write an explanation on how to use a tree diagram to solve such a counting problem.





Capsule Lesson Summary

Label dots in string pictures to meet several conditions. Solve a scheduling problem with several conditions.

Materials

Teacher
- Colored chalk

Student
- Unlined paper
- Colored pencils, pens, or crayons
- Worksheets L8*, **, ***, and ****

Description of Lesson

This is a good lesson for students to work on in cooperative groups or with partners.

Exercise 1

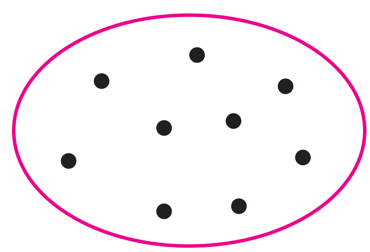
Begin with a brief warm-up in which you review the concepts of *at least* and *at most*. For example:

T: ***Everyone show me at least three fingers… at least five fingers… at least eight fingers. Everyone show me at most four fingers… at most two fingers… at most ten fingers.***

You can ascertain those who are hesitant with *at least* or *at most*; they will be helped by the responses of their peers. Meanwhile, explain the meaning of *at least three*, *at most four*, and so on as necessary.

Draw a string with nine dots on the board. Ask students to do the same on their papers.

T: ***This string contains exactly nine numbers. We could hatch it to show that there are no more numbers in it. The numbers in the string are all integers. I will tell you four true statements about these numbers.***

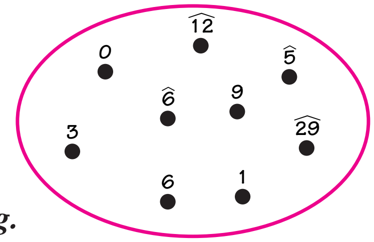


Write the following information on the board.

At least four multiples of 3 are in the string.
At most two even numbers are in the string.
Exactly five negative numbers are in the string.
No number in the string is greater than 10.

T: ***What numbers could these dots be for? Work with your group to label the nine dots so that all four of these statements are true. There are many solutions.***

When a group offers a solution, ask them to put it on the board for discussion. If the solution is not completely correct, such as the following, lead a discussion to modify it rather than start over.



T: ***What do you think of this solution?***

S: ***There are four even numbers, but the second statement says that there are at most two even numbers are in the string.***

T: ***Which of the numbers are even?***

S: ***6, $\widehat{6}$, $\widehat{12}$, and 0.***

S: ***Also, exactly five negative numbers are in the string, and here there are only four.***

T: ***Are there at least four multiples of 3 in the string?***

S: ***Yes. There are six multiples of 3: 0, 3, 6, $\widehat{6}$, $\widehat{12}$, and 9.***

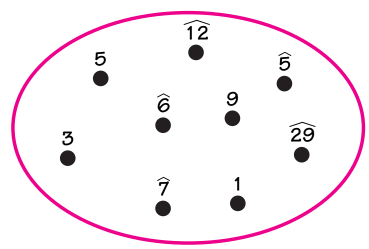
T: ***Is there any number greater than 10 in the string?***

S: ***No.***

T: ***Can we change a few of the numbers to make this solution correct?***

S: ***Change 6 to $\widehat{7}$, and 0 to 5.***

S: ***Now there are at least four multiples of 3: 3, $\widehat{6}$, 9, $\widehat{12}$. There are at most two even numbers: $\widehat{6}$, $\widehat{12}$. There are exactly five negative numbers: $\widehat{7}$, $\widehat{6}$, $\widehat{12}$, $\widehat{5}$, $\widehat{29}$. And 9 is the greatest number in the string. All of the statements are true.***



S: ***We have a different solution.***

Check and discuss one or two other solutions.

Exercise 2

Present the following situation to the class. Abbreviate the information on the board.

T: ***Alice has a very tight schedule this Friday. She has appointments at the dentist, the hair salon, the school, and the newspaper. She remembers that the times are noon, 2 p.m., 3 p.m., and 5 p.m., but she forgets which time is for which appointment.***

Here are some things that Alice knows:

> The dentist and the school do not take appointments after 4 p.m.
> The hair appointment is before the newspaper, but after the dentist.
> The dentist and the hair appointment are for more than 1 hour each.

T: ***Does Alice have enough information to decide each appointment time? With your group try to figure out Alice's schedule.***

Expect that students working in their groups will use guess and check methods. When a group finds a solution to share with the whole class, try to arrange their arguments in a chart on the board. For example:

S: ***The dentist and school appointments cannot be at 5 p.m.***

S: ***The dentist and hair appointments cannot be at 2 p.m.***

S: ***The hair appointment cannot be first (noon) because it is after the dentist. The hair appointment cannot be last (5 p.m.) because it is before the newspaper.***

	Noon	2 p.m.	3 p.m.	5 p.m.
Dentist		X		X
Hair	X	X		X
School				X
Newspaper				

The above observations lead to the conclusion that the hair appointment is at 3 p.m.

S: ***The newspaper appointment must be at 5 p.m. because it is after hair appointment.***

S: ***The dentist appointment must be at noon because it is before the hair appointment and cannot be at 2 p.m.***

S: ***That leaves 2 p.m. for the school appointment.***

	Noon	2 p.m.	3 p.m.	5 p.m.
Dentist	O	X		X
Hair	X	X	O	X
School		O		X
Newspaper				O

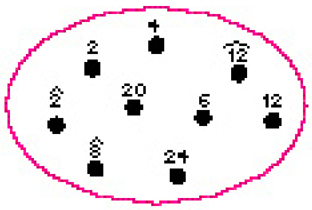
Worksheets L8*, **, ***, and **** are available for grou there may be many solutions on these worksheets.

Home Activity

You may like to send the worksheets home for students to work on with family members.

Name________________

Ls ★



Label the nine dots in this string with numbers so that

All the numbers are even,

and

at least two of the numbers are negative,

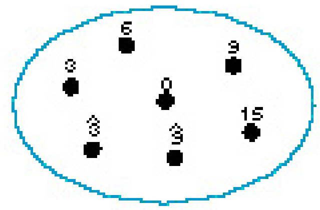
and

at most one of the numbers is a multiple of 5.

Many solutions are possible.

Name________________

Ls ★★



Label the seven dots in this string with numbers so that:

All the numbers are multiples of 3,

and

at most two of the numbers are greater than 7,

and

at least four of the numbers are odd.

Many solutions are possible.

Name________________

Ls ★★★

Allie, Brice, Guy and Hanna each have a different favorite number. Their favorite numbers are:

22 **12** **6** **10**

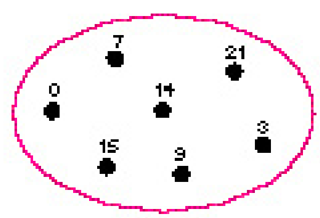
Use these clues to match the children with their favorite numbers.

- Allie's number is a multiple of 3.
- Brice's number is more than Allie's.
- Guy's number is less than one-half of Hanna's.

Allie	Brice	Guy	Hanna
6	12	10	22

Name________________

Ls ★★★★



Label the seven dots in this string with numbers so that:

At least four of the numbers are multiples of 7,

and

at least five of the numbers are multiples of 3,

and

at most two of the numbers are even,

and

all the numbers are less than 22.

Other solutions are possible using negative numbers.

L9 STRING GAME WITH NUMBERS #2

Capsule Lesson Summary

Locate given numbers and name other numbers in specific regions of a string picture with the strings labeled. Play *The String Game* with numbers and a limited set of attributes (multiples and order).

Materials

Teacher
- Colored chalk
- Numerical String Game kit

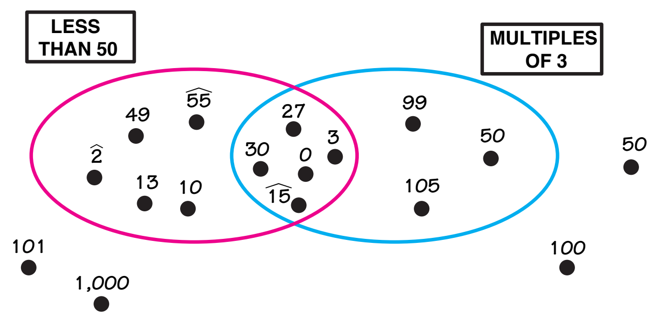
Student
- None

Description of Lesson

Exercise 1

Draw a two-string picture on the board and label the strings **LESS THAN 50** and **MULTIPLES OF 3**.

Ask students to locate 27, 10 $\widehat{55}$, 105, $\widehat{15}$, 50, 100, 0 and $\widehat{2}$ in the picture. Then point to each region of the picture and ask, in turn, for two or three more numbers that belong in that region. Your string picture might look similar to this illustration.



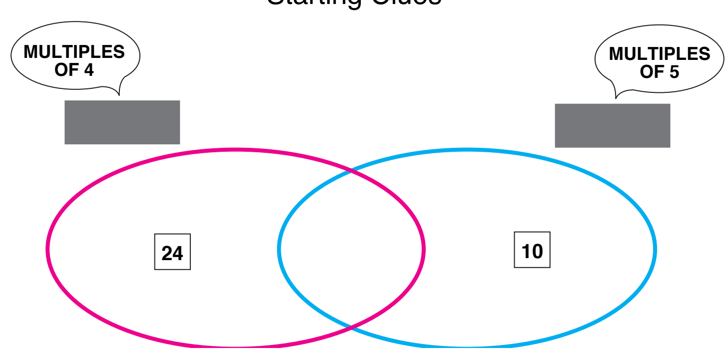
Exercise 2

Prepare to play *The String Game* with numbers by setting up a team board and taping the abbreviated Numerical String Game poster (see Lesson L6) above it. Divide the class into teams and distribute the game pieces on the team board. The first illustration below shows a possible game with one piece from each side of the team board placed correctly as starting clues. The second illustration shows correct placement of all 30 numbers and may be used by you as a crib sheet during the play of the game.

MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4	MULTIPLES OF 5
MULTIPLES OF 10	LESS THAN 50	LESS THAN $\widehat{10}$	ODD NUMBERS
	GREATER THAN 50	GREATER THAN $\widehat{10}$	

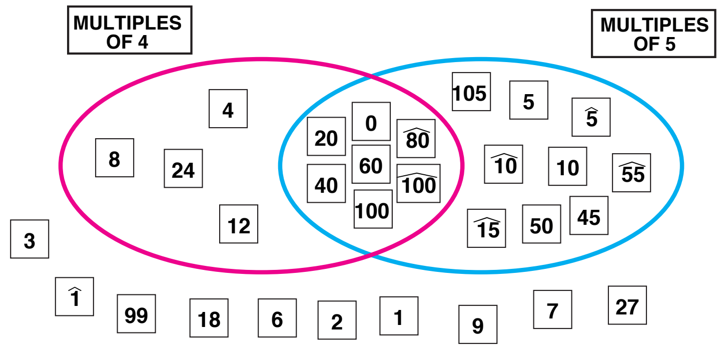
Team A	Team B
3	2
40	6
$\widehat{15}$	0
$\widehat{100}$	100
4	$\widehat{55}$
105	$\widehat{80}$
12	$\widehat{10}$
1	50
27	8
45	20
7	$\widehat{1}$
9	18
60	5
$\widehat{5}$	99

Starting Clues



Note: If you decide to play the game with three or more teams, create a team board with sections for more teams.

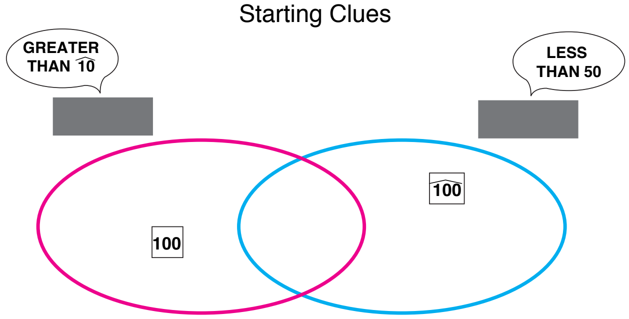
Correct Placement
of Game Pieces



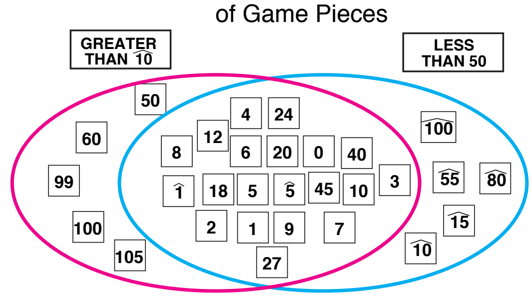
Exercise 3

Play another two-string version of *The String Game* with numbers, such as the one suggested below.

Starting Clues



Correct Placement
of Game Pieces



At the end of this game, it would be instructive to observe where the numbers in various regions are located on the number line and to discuss why the outside region is empty.

L10 STRING GAME WITH NUMBERS #3

Capsule Lesson Summary

Introduce new possibilities for string labels in *The String Game* with numbers so as to include a full set of 16 attributes. Name numbers in specific regions of a string picture with the strings labeled, and locate other numbers. Play *The String Game* with numbers using the full set of attributes.

Materials

Teacher		Student	
	• Colored chalk • Numerical String Game kit		• None

Description of Lesson

Exercise 1

Display a Numerical String Game poster.

MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4	MULTIPLES OF 5
MULTIPLES OF 10	LESS THAN 50	LESS THAN $\widehat{10}$	ODD NUMBERS
POSITIVE DIVISORS OF 12	GREATER THAN 50	GREATER THAN $\widehat{10}$	POSITIVE PRIME NUMBERS
POSITIVE DIVISORS OF 18	POSITIVE DIVISORS OF 20	POSITIVE DIVISORS OF 24	POSITIVE DIVISORS OF 27

T: ***We are going to play The String Game today, but first let's examine some new possibilities for the string labels.***

Draw a string on the board and label it **POSITIVE DIVISORS OF 18**.

T: ***Suppose this string is for the positive divisors of 18*** (point to this possibility on the poster)***. What numbers would be inside the string?***

S: ***All of the positive divisors of 18: 1, 2, 3, 6, 9, and 18.***

As necessary, discuss how to find the positive divisors of a number. Repeat this review for a couple other positive divisor labels.

T: ***Suppose this string is for positive prime numbers*** (point to this possibility on the poster)***. What are some numbers that would be inside the string?***

Invite students to name several prime numbers. Using their suggestions, review the definition of a prime number:

A positive prime number has exactly two positive divisors, 1 and itself.

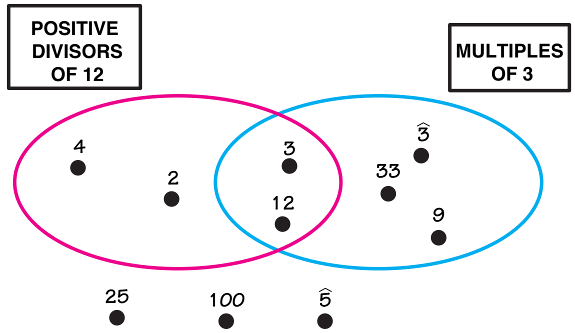
You might ask students to name prime numbers that also satisfy other characteristics; for example:

- prime numbers between 20 and 30 (23 and 29)
- prime numbers greater than 40 (53, 59, 61, 67, …)
- prime numbers that are also positive divisors of 20 (2 and 5)

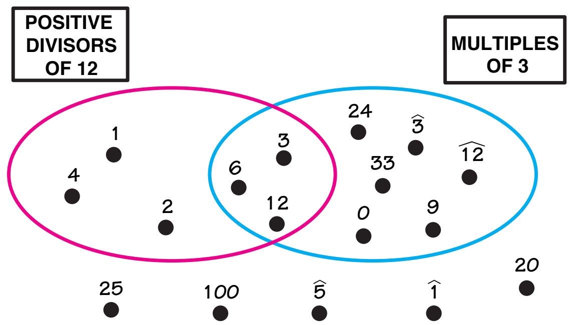
Exercise 2

Draw a two-string picture on the board and label the strings as shown below.

Point to each region of the picture and ask, in turn, for two or three numbers that belong in that region. Put these numbers in the picture; for example, your picture might look like this.



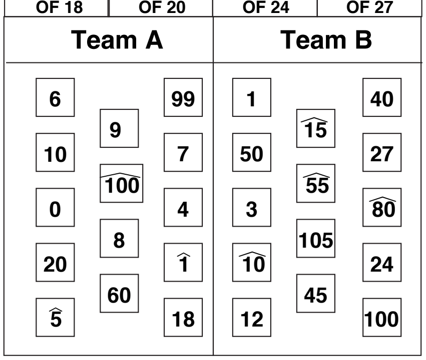
Ask where specific numbers would go in the picture, choosing numbers that have not already been located. For the above situation, you could ask students to locate 24, 1, $\hat{1}$, 0, 6, 20, and $\widehat{12}$ in the picture. Be sure to include 0, 1, $\hat{1}$, and $\widehat{12}$ in your picture.

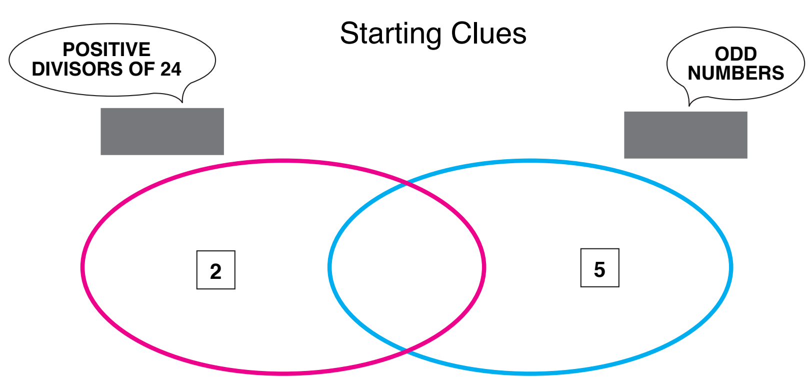


Exercise 3

Prepare to play *The String Game* with numbers by setting up a team board and taping the Numerical String Game poster above it. Divide the class into teams and distribute the game pieces on the team board. The illustration below shows a possible game with one piece from each side of the team board placed correctly as starting clues.

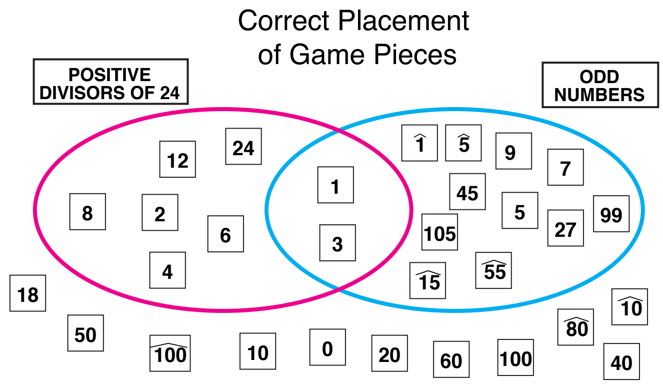
MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4	MULTIPLES OF 5
MULTIPLES OF 10	LESS THAN 50	LESS THAN $\widehat{10}$	ODD NUMBERS
POSITIVE DIVISORS OF 12	GREATER THAN 50	GREATER THAN $\widehat{10}$	POSITIVE PRIME NUMBERS
POSITIVE DIVISORS OF 18	POSITIVE DIVISORS OF 20	POSITIVE DIVISORS OF 24	POSITIVE DIVISORS OF 27





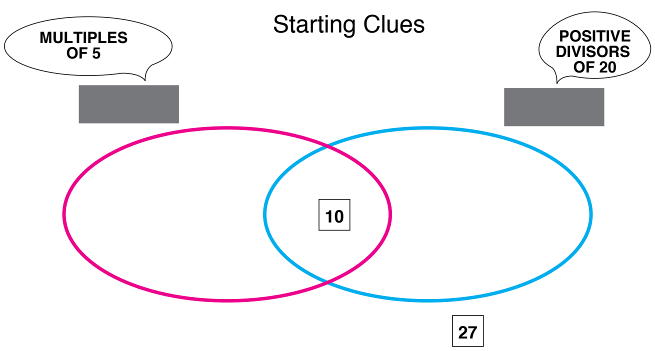
Note: If you decide to play the game with three or more teams, create a team board with sections for more teams.

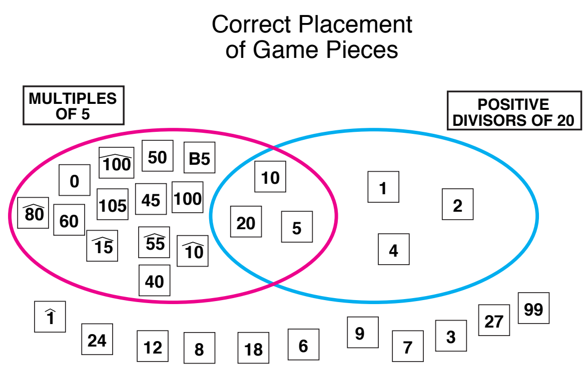
The following illustration shows correct placement of all 30 numbers and may be used by you as a crib sheet during the play of the game.



Exercise 4

If time is available, play another two-string version of *The String Game* with numbers, such as the one suggested below.





Home Activity

Suggest that parents work with their child to make lists of positive divisors of 12, 18, 20, 24 and 27. They can look for patterns or overlap.