Probability & Statistics

PROBABILITY AND STATISTICS TABLE OF CONTENTS

PROBABILITY AND STATISTICS INTRODUCTION

In today's world, probabilistic and statistical methods have become a part of everyday life. They are an important part of industry, business, and both the physical and social sciences. Just being an intelligent consumer requires an awareness of basic techniques of descriptive statistics, for example, the use of averages and graphs. Probability and statistics are key to the present day modeling of our world in mathematical terms. The lessons in this strand utilize both fictional stories and real data as vehicles for presenting problems and applications.

Probability stories fascinate most students and encourage their personal involvement in the situations. They often relate the probability activities, such as marble games, to games they have encountered outside the classroom. This personal involvement builds students' confidence and encourages them to rely on their intuition and logical thinking to analyze the situations. The questions and paradoxes that arise focus attention on key concepts of probability such as randomness, equally likely events, and prediction.

Some applications of mathematics are more easily investigated than others. The probability and statistics applications in this strand lean towards the easy end of the spectrum, both because of the small amount of theoretical knowledge required, and because the link between the situation and the mathematical model is easily perceived. In particular, the development of pictorial models for

s the ease of solutions.

Content Overview

Probability

The story-workbook *The Island of Tam-Tam* introduces a variety of probability concepts using humor and fantasy. In the story, the voracious bugs of Tam-Tam select marbles from bags to determine if they will eat the leaves of a coconut tree. Knowing the mixture of marbles, students use ratios to predict the number of trees a particular type of bug will destroy. Experimentation then tends to either confirm or cast doubt on their predictions.

As the story continues, the islanders of Tam Tam discover a repellent spray which inhibits but does not prevent the bugs from eating the leaves. The bugs then alter the mixture of marbles in their bags and, using analysis and experimentation, students investigate the effects and economic feasibility of using the spray.

The area of probability also produces many paradoxical situations. Answers to simple questions may seem obvious, yet experimentation or analysis may suggest significantly different solutions. Such paradoxes can be either very motivating or very frustrating for students. To be an effective teaching tool, paradoxes should be presented in clearly understood problems, the doubt and the "obvious" should arise from students' experiences, and a clear explanation or analysis must be accessible to students.

PROBABILITY AND STATISTICS INTRODUCTION

The lessons in this strand introduce paradoxes in probability through stories about Bruce, a boy who invents games of chance, using marbles, to play with his friends. The marble games appear to be fair, but after playing a few games, students become suspicious of Bruce because most of the games strongly favor him. The unexpected results produce a desire for an explanation. Dot and cord pictures, an application of networks, provide a means for students to analyze the marble games quickly and convincingly, and thereby discover the roots of the paradoxes. After unmasking a paradox, students most likely will want to make the game fair. This quest for a fair game motivates students to apply the dot and cord techniques in analyzing variations of the games. The series of lessons on Bruce's games reviews and extends students' knowledge of several key probability concepts: randomness, equally likely versus unequally likely events, the role of symmetry, and the effect of selecting marbles with replacement versus without replacement.

Lessons: P1, 2, 4, 5, and 6

Statistics

One branch of statistics involves the prediction of future events, given some knowledge of the current situation. *The Island of Tam-Tam* and Bruces's games introduce this concept. In both, students determine the probability of certain outcomes occurring, predict the expected ratio of outcomes when the activity is repeated many times, and then use marbles to test their predictions by simulating the situation. The predictions and experimental results are rarely equal, but usually are close to each other. Through these experiences, students learn that randomness causes all predictions to be only approximations. On the other hand, randomness is not entirely chaotic, the predictions are usually quite close to reality.

Several other lessons in this strand focus on descriptive statistics—the use of numerical and graphical techniques to summarize and compare sets of data. The activities continue to develop the students' abilities to read, draw, and interpret bar graphs, and introduce scatter plot graphs and concepts of averages. The goal is to increase the students' familiarity with these topics through rich experiences rather than to drill the techniques of computing an average or drawing a graph. Therefore students learn to compute the mean of a set of numbers by "balancing" the numbers. This process emphasizes the role of the mean as a "central number" and later the process will lead to an efficient way to compute the mean: add and divide.

The pedagogical choice to focus on the overall picture rather than specific techniques allows students to consider and discuss complex graphs of the monthly rainfall and mean temperature of selected U.S. cities. Not only do the students read these graphs, but they also interpret them relative to their knowledge of climate and geography.

Besides summarizing a set of data, another goal of statistics is to compare two or more sets of data. In fact, the comparisons are often the primary motivation for studying the data. The lessons in this strand present several simple techniques for comparing data: analysis and experimentation, averages, and graphing. No confidence intervals are calculated, but the experiences certainly augment the students' confidence in using statistics as a tool to understand their world.

Lessons: P1, 2, 3, 4, 5, 6, 7, and 8

Capsule Lesson Summary

Present the story of *The Island of Tam-Tam* and examine the effect that two different kinds of bugs have on groves of coconut trees. Discuss *chance* and *best prediction* in a very intuitive manner. Practice calculations involving $\frac{1}{2}x$ and $\frac{1}{3}x$.

Materials

Teacher
- *Island of Tam-Tam* Story-Workbook
- Marbles (two blue and one red)
- Paper cup
- Colored chalk

Student
- Worksheets P1 (no star), *, **, ***, and ****

Advance Preparation: Although students do not begin using *The Island of Tam-Tam* Story-Workbook until Lesson P2, you will want to preview the story yourself before beginning this lesson.

Description of Lesson

Tell the class the story of *The Island of Tam-Tam*.

T: ***Tam-Tam is a beautiful island in the South Pacific, and life is very pleasant there. The sun shines almost all year round, the sea is warm, and the people are very friendly. What kind of work do you suppose the people on the island of Tam-Tam do?***

S: ***Fish.***

S: ***Make swimwear.***

After students make a few suggestions, continue with the story.

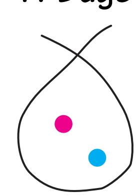
T: ***There are two main lines of work for the Tam-Tam islanders, fishing and growing coconuts. The Ambas have a grove of coconut trees that are just beginning to grow and they are worried. You see, the only bad thing about the island of Tam-Tam is that there are a large number of bugs. These bugs like to eat the leaves of young coconut trees. Why do you suppose that would worry coconut growers?***

Let students express some thoughts, but bring out the idea that when a coconut tree loses its leaves, it will not produce coconuts.

T: ***There are different kinds of bugs, some more harmful than others. The islanders call them A-bugs, B-bugs, C-bugs, and D-bugs. As we go on with the story of Tam-Tam you'll learn more about these bugs.***

Record the pictured information on the board as you tell the class about A-bugs.

A-Bugs



T: ***This morning the Ambas saw A-bugs coming into a grove of 22 young coconut trees. The Ambas know that the A-bugs don't eat the leaves of every coconut tree. Rather the chief A-bug carries a bag with one red marble and one blue marble in it. How might the A-bugs use this bag with marbles to decide when to eat the leaves of a coconut tree?***

P1

Show the class that you put one red and one blue marble in a cup while students give suggestions. Act out the following explanation. You may like to invite a student to pick the marble—like the chief A-bug.

T: ***Suppose I cover the cup with my hand, shake it, reach in without looking, and take out one marble. What could I get?***

S: ***The blue marble or the red marble.***

T: ***Yes, there are two possibilities. The chief A-bug does the same sort of thing each time the A-bugs approach a coconut tree. If the chief gets a red marble, the A-bugs eat all the leaves on the tree. If the chief gets a blue marble, the A-bugs move on to the next tree. About how often do you think the A-bugs will eat the leaves of a tree?***

S: ***About one-half of the time.***

T: ***The chance of getting a red marble is one out of two, so the chief A-bug should get the red marble about one-half of the time.***

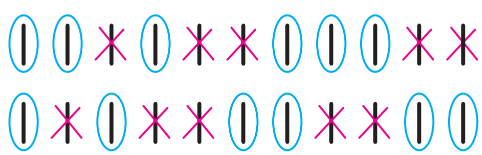
The A-bugs approach each tree in the Amba's grove of 22 young coconut trees just once. What might the Ambas expect to happen?

S: ***About one-half of their trees will have the leaves eaten.***

The discussion may involve actual predictions of numbers. Encourage the class to observe that although it is possible that all or none of the trees will have the leaves eaten, such possibilities are highly unlikely. Perhaps the best prediction for the Ambas to make is that 11 trees will have the leaves eaten and 11 will survive.

T: ***Let's see what might happen to the Ambas' 22 coconut trees. We'll take turns being the chief A-bug.***

Draw 22 strokes on the board to represent the 22 trees. Pass the cup with marbles around the class letting each of 22 students take a turn shaking the cup and picking a marble. If a red marble is chosen, draw a red x through one of the strokes to indicate a tree has its leaves eaten. If a blue marble is drawn, draw a blue loop around a stroke to indicate a tree is left untouched. Your picture will look similar to the one below after the 22 turns.



10 destroyed
12 survive

T: ***How many trees did we expect to be destroyed (have their leaves eaten)?***

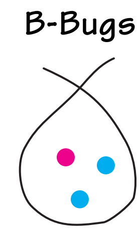
S: ***11, because*** $\frac{1}{2} \times 22 = 11$***.***

T: ***Our results are pretty close to your predictions.***

Distribute copies of Worksheet P1 (no star) and let students work independently or with a partner on the front side for only a few minutes. Then collectively discuss the worksheet. Bring out the idea that in each case the best prediction is that one-half of the total number of trees will lose their leaves and one-half will survive. In the last example of Dr. Namba who has 61 trees, the best prediction is that 30 or 31 trees will lose their leaves and the other 31 or 30 will survive. Here the theoretical result, $30\frac{1}{2}$, cannot happen.

T: ***Now, remember I told you that there were several kinds of bugs on the island of Tam-Tam. So far we have only seen what A-bugs do. The B-bugs behave in the same way as A-bugs except that the chief B-bug has one red and two blue marbles in his bag.***

Show the class that you add one blue marble to the cup so that there are two blue marbles and one red marble in it now. Picture the information on the board.



T: ***When the B-bugs enter a grove of young coconut trees and approach a tree, the chief selects one marble from the bag. What could the chief get?***

S: ***The red marble or one of the blue marbles.***

T: ***Yes, again there are two possibilities, red or blue. Just like the A-bugs, if the chief gets a red marble, the B-bugs eat all the leaves on the tree. If the chief gets a blue marble, they leave the tree alone and move on. What are the chances of getting a red marble and eating the leaves on a tree?***

S: ***One out of three.***

T: ***About how often do you think the B-bugs will eat the leaves of a tree?***

S: ***About one-third of the time.***

T: ***What are the chances of the chief B-bug getting a blue marble?***

S: ***Two out of three.***

T: ***About how often do you think the B-bugs will pass by a tree and let it survive?***

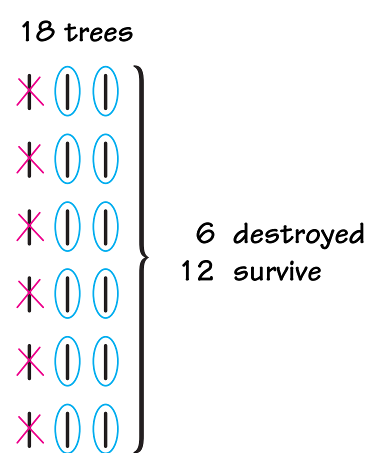
S: ***About two-thirds of the time.***

T: ***Ms. Caramba has a grove of 18 young trees and the B-bugs are coming into it. They approach each tree just once. About how many trees can Ms. Caramba expect to lose?***

S: ***About one-third of them;*** $\frac{1}{3} \times 18 = 6$.

S: ***One out of every three; there are 6 groups of 3 trees each in 18, so she can expect 6 trees to lose their leaves.***

Let students discuss what might happen and, if appropriate, picture their ideas on the board. For example, the second suggestion above could be pictured as shown.



Such an illustration may be very useful, especially for those students who have difficulty finding one-third of a number.

Some of your students' predictions might be numbers other than 6. Accept these, but suggest that perhaps the best prediction is $\frac{1}{3} \times 18$ or 6.

T: ***About how many trees can Ms. Caramba expect to survive?***

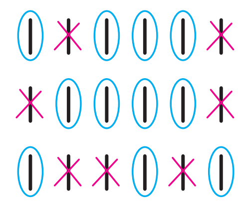
S: ***About two-thirds of them;*** $\frac{2}{3} \times 18 = 12$.

T: ***12, because 18 – 6 = 12.***

T: ***Let's see what happens to Ms. Caramba's 18 coconut trees. We'll take turns being the chief B-bug.***

P1

As before, continue to act out the story and to keep a record of the results. Your record will look similar to this one.

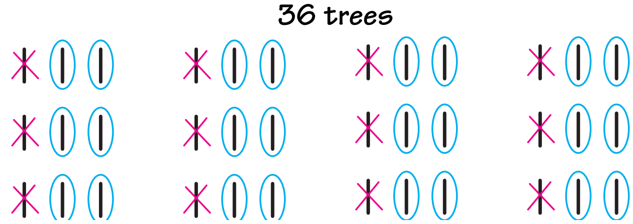


T: ***How many trees did we expect to be destroyed (have their leaves eaten)? How many trees did we expect to survive?***

S: ***6 destroyed because $^1/_3$ x 18 = 6, and 12 survive because $^2/_3$ x 8 = 12 (or 18 – 6 = 12).***

T: ***Our results are close to the best prediction.***

Ask students to turn over the worksheet and to work independently or with a partner on these problems about B-bugs for a few minutes. Then collectively discuss the worksheet. In your discussion, you might want to include analyzing one of the problems with a picture. For example, in the second problem with 36 trees, find that there are 12 groups of 3 trees, and in each group we expect one to lose its leaves and two to survive.



T: ***Which kind of bugs, A-bugs or B-bugs, appears to be more harmful? Why?***

S: ***A-bugs, because they usually destroy about one-half of the trees in a grove, but B-bugs only destroy about one-third of the trees.***

First let students use their intuition to answer the above questions, and then examine the question in the context of an example.

T: ***Ms. Zamba and Mr. Samba each have a grove of 60 young coconut trees. Ms. Zamba's grove is visited by A-bugs, and Mr. Samba's grove is visited by B-bugs. About how many of Ms. Zamba's trees will lose their leaves and how many will survive?***

S: ***About one-half of each; 30 lose their leaves and 30 survive.***

T: ***About how many of Mr. Samba's trees will lose their leaves and how many will survive?*** (Refer to the first problem about B-bugs on the worksheet.)

S: ***About one-third will lose their leaves and two-thirds will survive; $^1/_3$ x 60 = 20 and $^2/_3$ x 60 = 40, so 20 trees will lose their leaves and 40 will survive.***

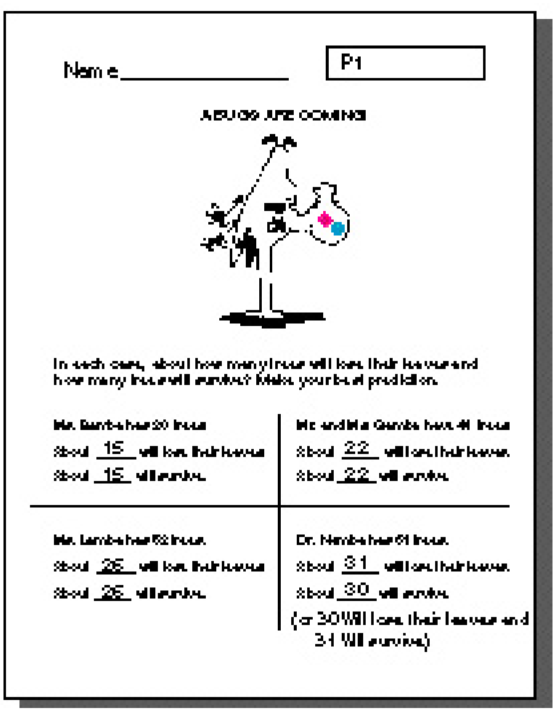
T: ***Which kind of bugs would we predict to be more harmful?***

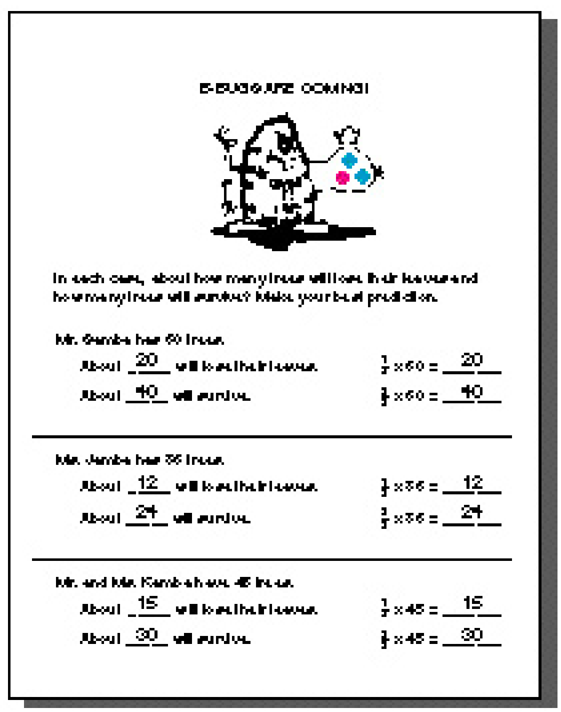
S: ***A-bugs, because of the 60 trees, A-bugs destroy 30 trees and B-bugs destroy only 20 trees.***

Worksheets P1*, **, ***, and **** are available for individual work.

Writing Activity

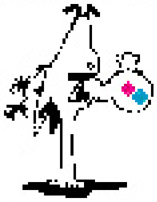
You may like students to take lesson notes on some, most, or even all their math lessons. The "Lesson Notes" section in Notes to the Teacher gives some suggestions and refers to forms in the Blacklines you may provide to students for this purpose. In this lesson, for example, students may recall something about the Island of Tam-Tam story and describe the effect of A-bugs or B-bugs in a grove of young coconut trees.





Name________________ P1 ★

A-BUGS ARE COMING!



In each case, about how many trees will lose their leaves and how many trees will survive? Make your best prediction.

28 Trees — About 14 will lose their leaves. About 14 will survive. $\frac{1}{2} \times 28 =$ 14

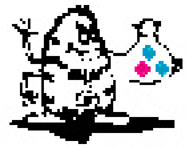
300 Trees — About 150 will lose their leaves. About 150 will survive. $\frac{1}{2} \times 300 =$ 150

342 Trees — About 171 will lose their leaves. About 171 will survive. $\frac{1}{2} \times 342 =$ 171

405 Trees — About 202 will lose their leaves. About 203 will survive. $\frac{1}{2} \times 405 =$ 202 $\frac{1}{2}$
(or 203 will lose their leaves and 202 will survive)

Name________________ P1 ★★

B-BUGS ARE COMING!



In each case, about how many trees will lose their leaves and how many trees will survive? Make your best prediction.

69 Trees — About 23 will lose their leaves. $\frac{1}{3} \times 69 =$ 23. About 46 will survive. $\frac{2}{3} \times 69 =$ 46

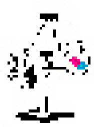
123 Trees — About 41 will lose their leaves. $\frac{1}{3} \times 123 =$ 41. About 82 will survive. $\frac{2}{3} \times 123 =$ 82

135 Trees — About 45 will lose their leaves. $\frac{1}{3} \times 135 =$ 45. About 90 will survive. $\frac{2}{3} \times 135 =$ 90

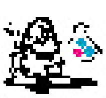
321 Trees — About 107 will lose their leaves. $\frac{1}{3} \times 321 =$ 107. About 214 will survive. $\frac{2}{3} \times 321 =$ 214

Name________________ P1 ★★★

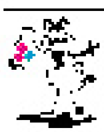
In each case, about how many trees will lose their leaves and how many trees will survive? Make your best prediction.



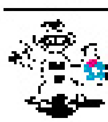
38 Trees — About 19 will lose their leaves. About 19 will survive. $\frac{1}{2} \times 38 =$ 19



42 Trees — About 14 will lose their leaves. $\frac{1}{3} \times 42 =$ 14. About 28 will survive. $\frac{2}{3} \times 42 =$ 28



48 Trees — About 32 will lose their leaves. $\frac{2}{3} \times 48 =$ 32. About 16 will survive. $\frac{1}{3} \times 48 =$ 16

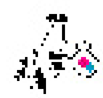


35 Trees — About 7 will lose their leaves. $\frac{1}{5} \times 35 =$ 7. About 28 will survive. $\frac{4}{5} \times 35 =$ 28

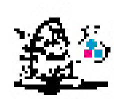
Name________________ P1 ★★★★

In each case, about how many trees will lose their leaves and how many trees will survive? Make your best prediction.

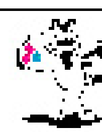
67 Trees — About 34 will lose their leaves. About 33 will survive.
(or 33 will lose their leaves and 34 will survive)



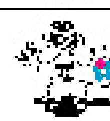
67 Trees — About 22 will lose their leaves. About 45 will survive.



67 Trees — About 45 will lose their leaves. About 22 will survive.



67 Trees — About 13 will lose their leaves. About 54 will survive.



Capsule Lesson Summary

Read and complete the problems in the story-workbook *The Island of Tam-Tam*. Use the story to examine some questions related to magnitude of fractions. Compare fractions in the context of the story.

Materials

Teacher
- *The Island of Tam-Tam* Story-Workbook
- Colored chalk

Student
- *The Island of Tam-Tam* Story-Workbook

Description of Lesson

Recall some of the story of *The Island of Tam-Tam* and then distribute copies of the story-workbook.

Pages 2–4

Read these pages together and discuss the question at the bottom of page 4. C-bugs are expected to be the most harmful. You might want to examine the question in a specific case. For example:

60 trees:
- A-bugs are expected to destroy about $\frac{1}{2} \times 60$ or 30 trees.
- B-bugs are expected to destroy about $\frac{1}{3} \times 60$ or 20 trees.
- C-bugs are expected to destroy about $\frac{2}{3} \times 60$ or 40 trees.
- D-bugs are expected to destroy about $\frac{1}{5} \times 60$ or 12 trees.

Ask other questions related to the relative harm the bugs are expected to do.

T: ***Which type of bug, A or B or D, is expected to be most harmful?***

S: ***A-bugs; one-half of some trees destroyed would be more than either one-third or one-fifth of those trees.***

T: ***Compare B-bugs with C-bugs. How much more harmful are C-bugs expected to be?***

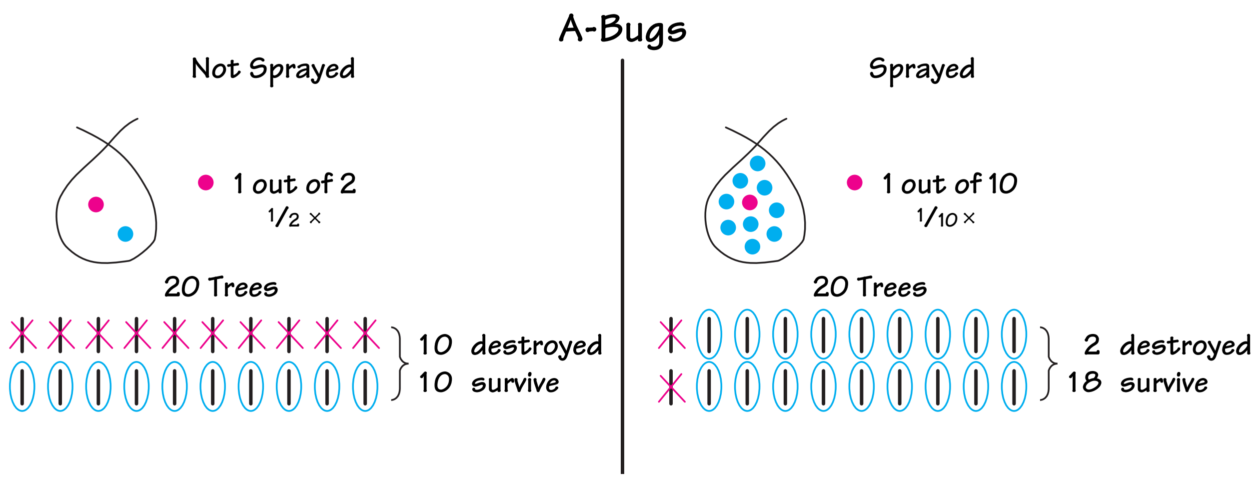
S: ***C-bugs are twice as harmful as B-bugs.***

Pages 5–8

Read these pages together and discuss the question at the bottom of page 8. Putting more blue marbles in the bag increases the chances that the bugs will pass up a tree (the bugs won't eat the leaves).

Page 9

Read and answer the questions on this page collectively. You might wish to analyze these problems using pictures similar to those used in Lesson P1.



Another question that you might pose is this:

T: ***How much more harmful would the A-bugs be to a grove that has not been sprayed than to a grove that has been sprayed?***

S: ***Five times more harmful—we would expect them to eat the leaves of five times as many trees if they were not sprayed.***

S: ***When 20 trees have been sprayed, A-bugs destroy about 2 trees; when they are not sprayed, they destroy about 10 trees; 10 = 5 x 2.***

This question may be difficult to consider except in the case of a specific example, such as on page 9. Do not expect a general response. You might examine the question again after solving the problems on page 10.

Page 10

Let students read and answer the questions on this page individually or with a partner. Then check the answers collectively.

Page 11

Read and answer the question on this page collectively. Again, you might pose this question:

T: ***How much more harmful would the B-bugs be to a grove that has not been sprayed than to a grove that has been sprayed?***

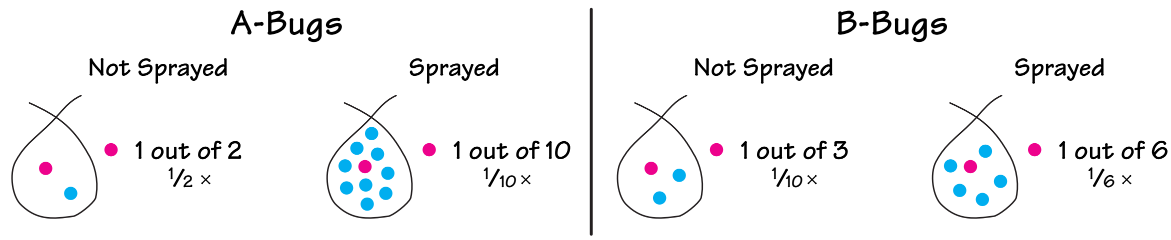
S: ***Twice as harmful; when a grove is not sprayed B-bugs destroy about one-third of the trees, and when a grove is sprayed they destroy about one-sixth of the trees. One-third is twice as much as one-sixth.***

Do not expect an accurate or well-phrased answer to this difficult question. Be content with some intuitive idea. Examine the question after page 12 in the context of the solution to the problems on that page.

Page 12

Let students read and answer the questions on this page individually or with a partner. Then check the answers collectively.

Compare A-bugs to B-bugs. First organize the information on the board.



T: ***When a grove is not sprayed, which type of bug would we expect to be more harmful?***

S: ***A-bugs.***

T: ***When a grove is sprayed, which type of bug would we expect to be more harmful?***

S: ***B-bugs.***

Pages 13–15

Read these pages together and then discuss the problems on pages 14 and 15.

T: ***What information would help Ms. Pajamba decide whether to have her coconut trees sprayed?***

S: ***The number to trees she expects to survive if they are not sprayed.***

S: ***The number of trees she expects to survive if they are sprayed.***

S: ***But if they are sprayed, she has to pay Mr. Tamba. So from the trees she expects to survive, she should take away eight—she would have to give the coconuts from eight trees to Mr. Tamba.***

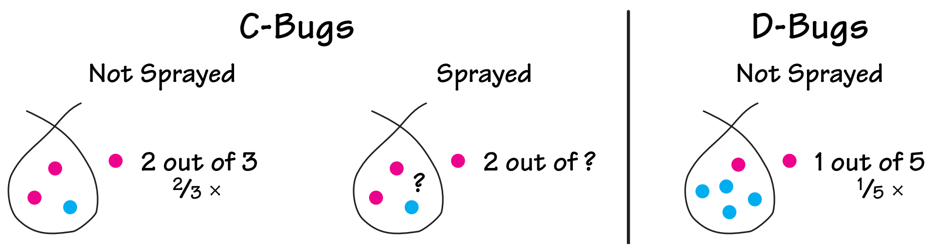
When it appears students understand the problems, let them work on them individually or with a partner for a short while. Ask students who finish quickly to solve the same problems with the A-bugs coming. Check the students work on pages 14–15 collectively (see the solution key following this lesson description).

Extension Activity

If time remains, present the following problem to the class.

T: ***Previously, we saw that C-bugs were expected to be the most harmful. Just like for A-bugs and B-bugs, the chief C-bug puts more marbles in the bag when a grove of coconut trees has been sprayed. In fact, if a grove has been sprayed, we expect C-bugs to be just like D-bugs in a grove that has not been sprayed.***

Write the information on the board.



T: ***What marbles does the chief C-bug add to the bag when coconut trees have been sprayed? Remember, C-bugs in a grove that has been sprayed are like D-bugs in a grove that has not been sprayed.***

Let students take some time to understand the problem. Hold off a student who wishes to respond immediately until most students have an idea.

S: ***The chief C-bug puts seven blue marbles in the bag; 2 out of 10 would have the same effect***

Writing Activity

Suggest that students write some problems similar to those on pages 10 and 12 of the story-workbook
for their partners or other students to solve. If you have done the Extension Activity in class, they
the C-bugs.

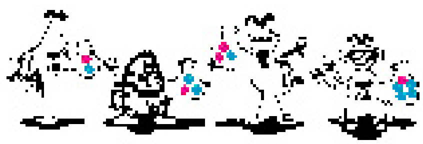
Home Activity

Students may enjoy taking this story-workbook home to tell the story to family members.

When bugs of any type arrive at a tree, their child takes a marble from the bag.

If it is RED, the bugs eat all the leaves of the tree.
If it is BLUE, the bugs pass by the tree without eating any leaves.

Which type of bug would you expect to be most harmful to a grove of young coconut trees? C-bugs



4

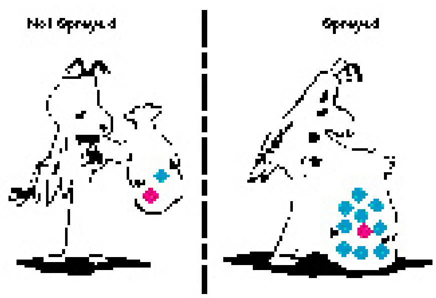
"Yuck! These leaves smell of fish!" said the A-bugs in disgust. "Do we have to eat them?"

"You have to eat some," replied the child A-bug. "People expect it of us."

"Well, okay. We are hungry," said the A-bugs. "But at least put some more blue marbles in the bag."

Why did the A-bugs want the child to put more blue marbles in the bag?
That would increase the chance of picking a blue marble and, therefore, passing by a tree without eating any leaves.

5



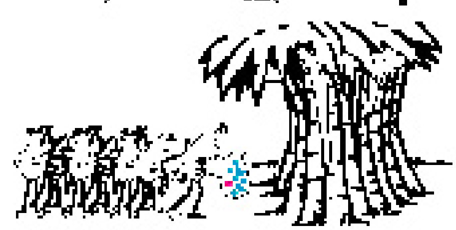
How many blue marbles does the child A-bug add to the bag if trees have been sprayed? 8

Mr. Tembe has 20 trees. They have been sprayed.
About how many of them will the A-bugs destroy? 2 ($\frac{1}{10}$ × 20 = 2)

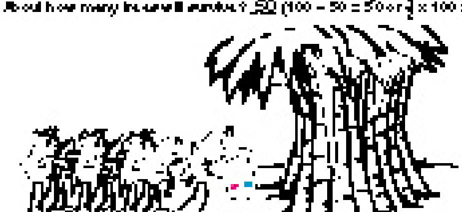
Mr. Zambe also has 20 trees. They have not been sprayed.
About how many of them will the A-bugs destroy? 10 ($\frac{1}{2}$ × 20 = 10)

9

Mr. and Mrs. Gumbe have 100 coconut trees. They asked Mr. Tembe to spray their trees.
About how many trees will the A-bugs destroy? 10 ($\frac{1}{10}$ × 100 = 10)
About how many trees will survive? 90 (100 − 10 = 90 or $\frac{9}{10}$ × 100 = 90)



Dr. Yembe also has 100 coconut trees, but they have not been sprayed.
About how many trees will the A-bugs destroy? 50 ($\frac{1}{2}$ × 100 = 50)
About how many trees will survive? 50 (100 − 50 = 50 or $\frac{1}{2}$ × 100 = 50)



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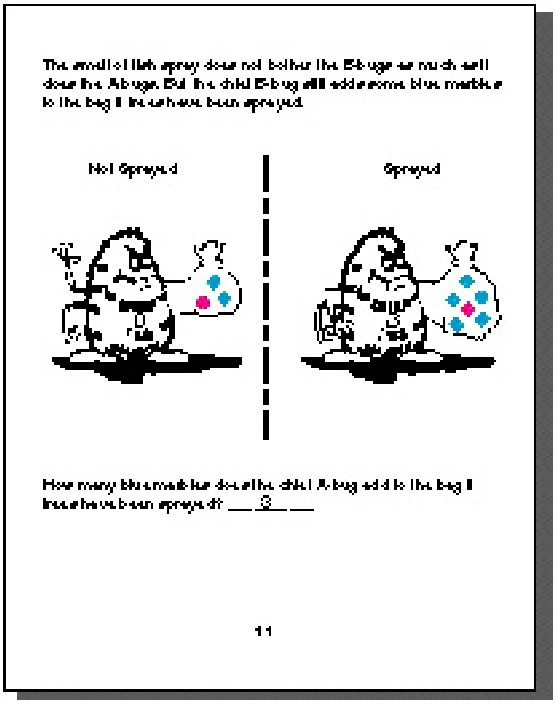


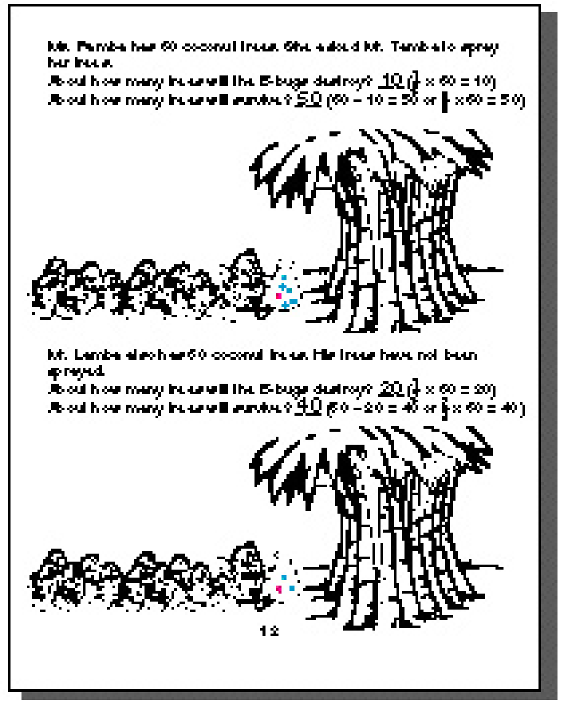
The smell of fish spray does not bother the B-bugs as much as it does the A-bugs. But the child B-bug still adds some blue marbles to the bag if trees have been sprayed.

Not Sprayed

Sprayed

How many blue marbles does the child A-bug add to the bag if trees have been sprayed? 3

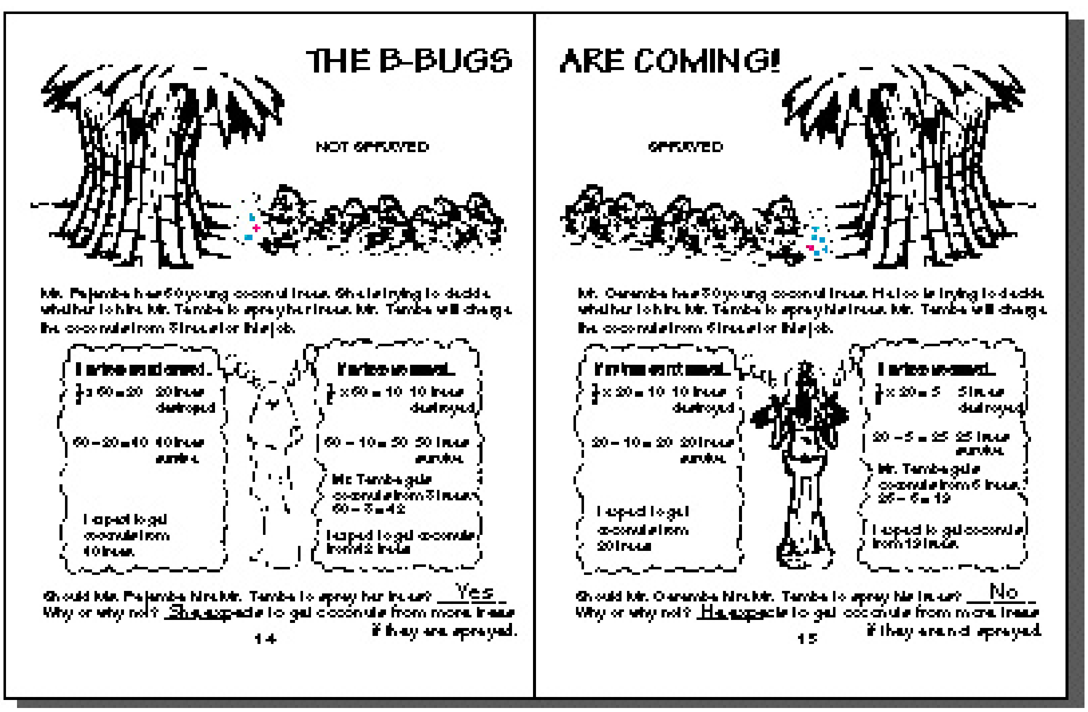
11



Ms. Pamba has 60 coconut trees. She asked Mr. Tembo to spray her trees.
About how many trees will the B-bugs destroy? 10 ($\frac{1}{6} \times 60 = 10$)
About how many trees will survive? 50 ($60 - 10 = 50$ or $\frac{5}{6} \times 60 = 50$)

Mr. Lamba also has 60 coconut trees. His trees have not been sprayed.
About how many trees will the B-bugs destroy? 20 ($\frac{1}{3} \times 60 = 20$)
About how many trees will survive? 40 ($60 - 20 = 40$ or $\frac{2}{3} \times 60 = 40$)

12



THE B-BUGS ARE COMING!

NOT SPRAYED

Ms. Pajamba has 60 young coconut trees. She is trying to decide whether to hire Mr. Tembo to spray her trees. Mr. Tembo will charge the coconuts from 8 trees for this job.

If trees not sprayed.
$\frac{1}{3} \times 60 = 20$ 20 trees destroyed
$60 - 20 = 40$ 40 trees survive.
I expect to get coconuts from 40 trees.

If trees sprayed.
$\frac{1}{6} \times 60 = 10$ 10 trees destroyed
$60 - 10 = 50$ 50 trees survive.
Mr. Tembo gets coconuts from 8 trees. $50 - 8 = 42$
I expect to get coconuts from 42 trees.

Should Ms. Pajamba hire Mr. Tembo to spray her trees? Yes
Why or why not? She expects to get coconuts from more trees if they are sprayed.

14

SPRAYED

Mr. Garamba has 30 young coconut trees. He too is trying to decide whether to hire Mr. Tembo to spray his trees. Mr. Tembo will charge the coconuts from 6 trees for this job.

If trees aren't sprayed.
$\frac{1}{3} \times 30 = 10$ 10 trees destroyed
$30 - 10 = 20$ 20 trees survive.
I expect to get coconuts from 20 trees.

If trees sprayed.
$\frac{1}{6} \times 30 = 5$ 5 trees destroyed
$30 - 5 = 25$ 25 trees survive.
Mr. Tembo gets coconuts from 6 trees. $25 - 6 = 19$
I expect to get coconuts from 19 trees.

Should Mr. Garamba hire Mr. Tembo to spray his trees? No
Why or why not? He expects to get coconuts from more trees if they aren't sprayed.

15

Capsule Lesson Summary

Use a graph to compare the hitting performances of Babe Ruth in 1920 and Hank Aaron in 1971. Base the comparison on several criteria, including home runs, total hits, and total bases. Do a similar analysis of a popular contemporary baseball player.

Materials

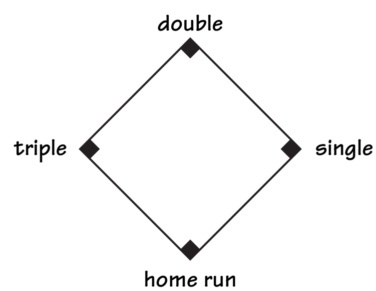
Teacher
- Colored chalk
- Blacklines P3(a) and (b)

Student
- Graphs of hits for Babe Ruth and Hank Aaron
- Statistics for other players
- Straightedge
- Colored pencils, pens, or crayons
- Worksheets P3(a) and (b)

Advance Preparation: Use Blacklines P3(a) and (b) to prepare graphs and statistics for students. In Exercise 2, baseball statistics from 1980 for Mike Schmidt of the Philadelphia Phillies and from 1993 for Juan Gonzalez of the Texas Rangers are given. You may substitute the statistics of popular local baseball players. These statistics are available from the sports desk at a newspaper, radio station, or TV station.

Description of Lesson

Draw a picture of a baseball diamond on the board.



T: ***Who can explain what singles, doubles, triples, and home runs are in baseball?***

Let students define these four types of hits. Emphasize the number of bases the batter runs for each hit: single—one base; double—two bases; triple—three bases; and home run—four bases.

Invite students to name a few famous baseball players.

T: ***Perhaps two of the best baseball players of all time were Babe Ruth and Hank Aaron. Today we'll compare the hitting performances of Babe Ruth in 1920 and Hank Aaron in 1971.***

Distribute copies of the graphs on Blackline P3.

T: ***What do these two graphs tell us about the hitting performances of Babe Ruth in 1920 and Hank Aaron in 1971?***

Encourage both descriptive statements (Aaron had 90 singles) and comparative statements (Ruth had more homeruns than Aaron.) In any reading of the graphs, students should come within one or two of the actual data (see the table). This table is for your reference only; you need not draw it on the board.

	Babe Ruth: 1920	Hank Aaron: 1971
Singles	73	90
Doubles	36	22
Triples	9	3
Home Runs	54	47

T: ***Who do you think had the better year as far as hits, Babe Ruth in 1920 or Hank Aaron in 1971? Why?***

S: ***Aaron, because he had more singles.***

S: ***Ruth had more doubles, triples, and home runs.***

Students should agree that it is not sufficient to use just one of these statistics to determine the better hitter. Also, they may realize that other information could be useful, for example, the number of times at bat, walks, and strikeouts.

T: ***One way to compare the hitting performances of two baseball players is to calculate the total number of bases for each.***

Record this information in a table on the board.

T: ***Suppose a player hits one double, one triple, and two home runs in one game. What is the player's total number of bases for this game?***

S: ***The player has two bases for the double, three bases for the triple, and eight bases for the two home runs.***

One Game

	Hits	Number of Bases
Singles	0	0
Doubles	1	2
Triples	1	3
Home Runs	2	8
Totals	4	13

S: ***2 + 3 + 8 = 13. Therefore, the player has 13 bases in total.***

T: ***What is the player's total number of hits in this game?***

S: ***Four.***

Complete the following two tables in a similar manner. (Answers are in parentheses.)

One Game

	Hits	Number of Bases
Singles	2	(2)
Doubles	2	(4)
Triples	0	(0)
Home Runs	1	(4)
Totals	(5)	(10)

One Week

	Hits	Number of Bases
Singles	5	(5)
Doubles	1	(2)
Triples	2	(6)
Home Runs	3	(12)
Totals	(11)	(25)

Erase the entries in the table on the board, and distribute copies of Worksheet P3(a). Instruct students to fill in the table on the worksheet along with the class.

T: ***Let's calculate Babe Ruth's total number of bases for 1920. How many of each type of hit did he have?***

In any reading of the graph for Babe Ruth, students should come within one or two of the actual data (see the table). Invite students to complete the table. The entries in your table may vary slightly from those here depending on your students' graph readings.

Babe Ruth: 1920

	Hits	Number of Bases
Singles	73	73
Doubles	36	72
Triples	9	27
Home Runs	54	216
Totals	172	388

T: ***Do you think Hank Aarons's total number of bases in 1971 was more or less than Babe Ruth's in 1920?***

Let students explain their answers.

T: ***Use the graph to complete the table for Hank Aaron. Then answer the questions.***

As the students are working, put the information from Blackline P3(b) on the board. You may prefer to substitute data for some popular local baseball players (see the note at the beginning of the lesson).

Distribute copies of the statistics for two baseball players and direct students to compare the hitting of these two players. Encourage use of information such as total number of hits and number of bases.

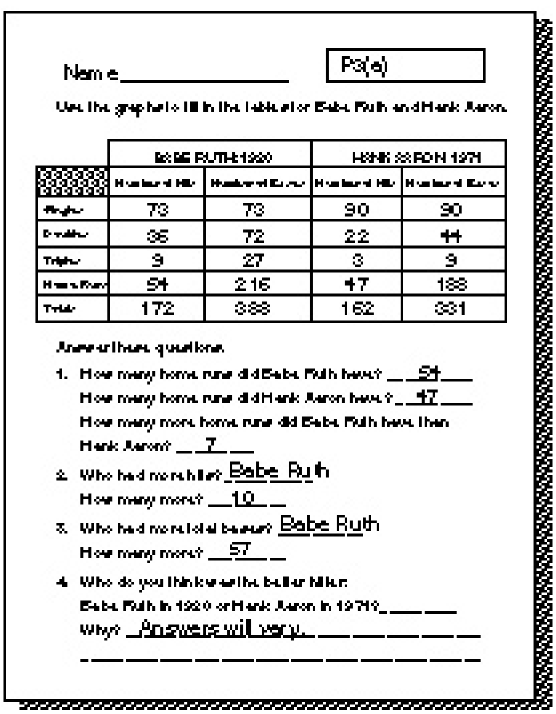
Students can graph the hitting records of the two players on Worksheet P3(b).

Extension Activity

Collect and compare other baseball statistics, or statistics from another sport.

Home Activity

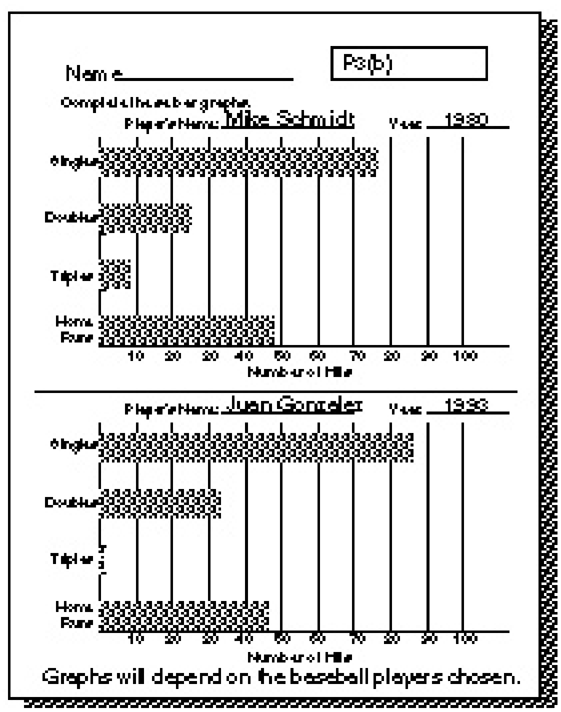
Suggest parents/guardians work with their child to collect data on their favorite baseball player and compare it to Babe Ruth and Hank Aaron.



Name______________ P3(a)

	[illegible]		[illegible]	
	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	73	73	90	90
[illegible]	36	72	22	44
[illegible]	9	27	3	9
[illegible]	54	216	47	188
[illegible]	172	388	162	331

1. 54
47
7
2. Babe Ruth
10
3. Babe Ruth
57
4. Answers will vary.



P4 SAME OR DIFFERENT #1

Capsule Lesson Summary

Using a set of marbles of two colors, select two marbles at random and note whether the two marbles are of the same color or different colors. Use cord pictures to determine the probabilities and expected frequencies of same and different. Compare expected frequencies to students' experimental results.

Materials

Teacher
- Colored chalk
- Marbles
- Paper cup
- Paper
- Worksheets P4* and **

Student
- Marbles (two blue, one red)

Description of Lesson

Exercise 1

Initiate a class discussion centered on the following questions.

T: ***What games do you play? Are all your games fair? What do you mean by a "fair" game?***

Which games that you play are fair? Which are unfair? How are they unfair? How can you tell if a game is fair or unfair?

We will examine some marble games to see if they are fair.

Show the class that you have one red and two blue marbles. Tell the class that you will demonstrate how to play a game.

1. Put the marbles in a cup and shake them.
2. Invite a student to take two marbles from the cup without looking.
3. Note whether the two marbles are of the same color (both blue) or of different colors (one red and one blue).

Draw a table on the board and make a tally to indicate the result.

Same	Different
I	III

Repeat the game two or three times so that the students understand the procedure.

T: ***Do you think it's more likely that the two marbles will be of the same color or that they will be of different colors?***

Let students express their opinions. Put the question in terms of a fair game: if one player gets a point for same and another gets a point for different, would it be a fair game? If not, who is favored?

Divide the class into pairs. Give each pair one red and two blue marbles in a cup. You may need to let one student be your partner or form one group of three. One student in each pair should be scorekeeper.

T: ***Each pair will play the game ten times. How many games will we play altogether?***

P4

Adapt the following discussion according to the number of students in your class.

S: ***We have 26 students, or 13 pairs. 13* x *10 = 130, so we will play 130 games.***

T: ***About how many times in 130 games do you think we will get two marbles of the same color? Of different colors?***

Record students' predictions on the board. Encourage students to give explanations, but do not insist that they do so.

Let students play the game. When two students finish their ten games, record the results in a table on the board. Total the number of same and different for all the games.

	Same	Different
	4	6
	2	8
	3	7
	4	6
	1	9
	5	5
	2	8
	6	4
	3	7
	3	7
	5	5
	4	6
	4	6
Totals	46	84

Check that the total for same plus the total for different equals the total number of games played; in this sample data, 46 + 84 = 130.

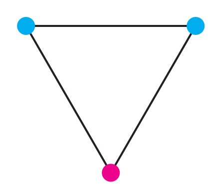
T: ***Which is favored, same or different?***

S: ***Different occurred more often than same.***

Compare the totals, 46 and 84 in the sample, to students' predictions.

T: ***Your predictions varied a lot. Here's a way to make better predictions.***

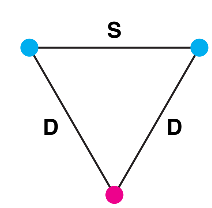
Represent the marbles by drawing one red and two blue dots on the board. Connect the corresponding colored dots with a cord as you say,



T: ***We could pick the two blue marbles.***
… or we could pick this blue marble and the red marble.
… or we could pick the other blue marble and the red marble.

So there are three ways to select two marbles. Of these three ways, how many result in two marbles of the same color?

S: ***One way, only when you select both blue marbles.***



T: ***Of these three ways, how many result in two marbles of different colors?***

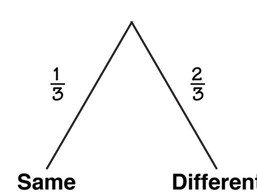
S: ***Two ways.***

T: ***Which is more likely, same or different?***

S: ***Different, because there are two ways to get marbles of different colors and only one way to get marbles of the same color.***

T: ***Here's another way to record what we found out about this game.***

Draw this tree diagram on the board, explaining that $\frac{1}{3}$ means there is one chance out of three of getting same and that $\frac{2}{3}$ means there are two chances out of three of getting different.



T: ***The probability of getting two marbles of the same color is $^1/_3$.***
What is the probability of getting two marbles of different colors? ($^2/_3$)

This picture also tells us that in three games we might expect one same and two different.

Record three games in a table on the board.

Same	Different
I	II
I	II
I	II
I	II
I	II

T: ***In three more games, we might again expect one same and two different.***

Continue adding three games at a time to the table until you have about 15 games recorded.

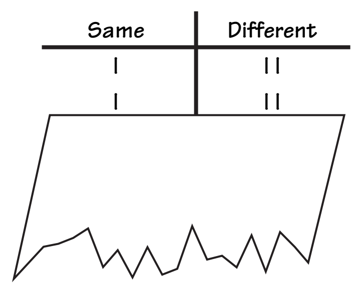
With a piece of paper, cover all but the first line of tallies in the table.

T: ***In three games, we expect same to occur once and different to occur twice.***

Record this in another table on the board.

Number of Games	Same	Different
3	1	2

T: ***In six games, we expect same to occur twice and different to occur four times.***



Record this information. Invite students to continue the table to make predictions for nine, 12, and 15 games.

Number of Games	Same	Different
3	1	2
6	2	4
9	3	6
12	4	8
15	5	10

T: ***What patterns do you notice?***

S: ***For every three games, same increases by 1 and different increases by 2.***

S: ***Different is always double same.***

S: ***Same is always one-third of the total number of games.***

Invite students to use these patterns to make predictions for 30, 60, 90, and 120 games.

Number of Games	Same	Different
30	10	20
60	20	40
90	30	60
120	40	80

Adapt the following discussion to the total number of games played in your class.

T: ***We played 130 games in our class. Using this method, what would have been good predictions for same and for different?***

S: ***43 for same, because*** $\frac{1}{3} \times 130 = 43\frac{1}{3}$.

S: ***43 for same, because we except 40 same in 120 games. We expect 3 same in the next 10 games since*** $\frac{1}{3} \times 10 = 3\frac{1}{3}$. $40 + 3 = 43$.

S: ***87 for different, because*** $130 - 43 = 87$.

S: ***87 for different, because*** $2 \times 43\frac{1}{3} = 86\frac{2}{3}$, ***almost 87.***

Compare this theoretical prediction to the actual result in your classroom. Discuss what it means for something to be the best prediction in a situation—that even though the predicted results for a particular set of games does not correspond exactly to the actual results, it is likely that they will come close, especially if many games are played.

Note: If the game is played 130 times, there is a 70% chance that same will occur between 38 and 48 times.

Exercise 2

T: ***When Anita and Bruce play this game, Anita gets one point each time*** same ***occurs and Bruce gets one point each time different occurs. Is this a fair game?***

S: ***No, Bruce will usually win.***

T: ***Anita and Bruce would like to make the game fair. Anita suggests changing the number of red marbles and blue marbles. How many red marbles and blue marbles do you think they should use to make this a fair game?***

Let students suggest and discuss various distributions (for example, two blue, two red; three blue, two red). It is likely that many students will suggest an equal distribution of red and blue marbles; that is, one and one, two and two, three and three, and so on.

Draw two red dots and two blue dots on the board.

T: ***Do you think playing with two red marbles and two blue marbles is a fair game?***

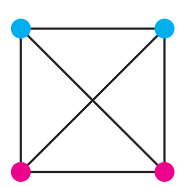
S: ***Yes, there are the same number of marbles of each color.***

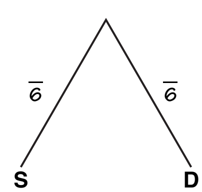
S: ***No, I think that it is more likely to select two marbles of different colors.***

T: ***Let's check it. How many ways can we select two marbles?***

S: ***Six ways.***

Invite a student to draw the six cords. Draw a tree diagram near the cord picture.



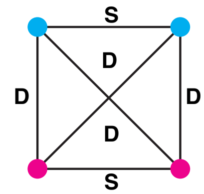


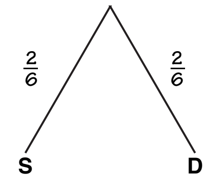
T: ***Of the six ways to select two marbles, how many ways are there to select marbles of the same color?***

S: ***Two ways.***

T: *Of different colors?*

S: *Four ways.*





Record the information.

T: *Is this a fair game?*

S: ***No, there are four chances out of six of getting different and only two chances out of six of getting same.***

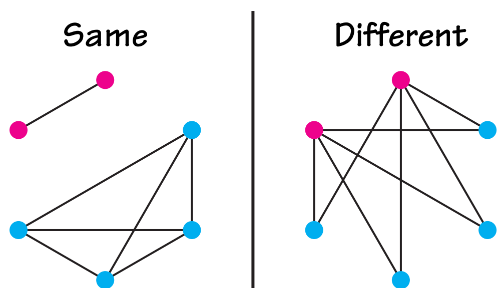
S: ***No, the probability of getting different is greater than the probability of getting same. $^4/_6$ is more than $^2/_6$.***

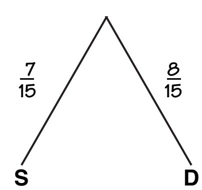
T: ***We thought that using two red marbles and two blue marbles might make this a fair game, but it does not.***

Let's continue looking for a fair same or different game with red marbles and blue marbles. What combination of marbles should we try? (Limit the possibilities to seven or fewer marbles; otherwise, pictures get very complicated.)

Note: The only combinations of seven or fewer marbles that result in a fair game are one red and three blue marbles or one blue and three red marbles. Analyze one of these combinations only if several students strongly insist. Preferably, select a different combination to analyze in this lesson and thereby allow all students to search again for a fair game in the next probability lesson, P5.

Select a combination of marbles favored by several students and analyze the game collectively in a manner similar to the preceding one. If you analyze a game with five, six, or seven marbles, suggest using two identical pictures of marbles, one for same and one for different. For example, suppose you are analyzing a game with two red marbles and four blue marbles.





Comment that the use of two pictures is optional, but that it makes it easier to draw and count all of the cords. Refer to the end of Lesson P5 for a list of solutions for all distributions of between four and seven marbles of two colors.

Optional: Complete a table of best predictions as was done in Exercise 1.

T: ***In 15 games, the best prediction of the results is seven same and eight different. In 30 games, what is the best prediction of the outcome?***

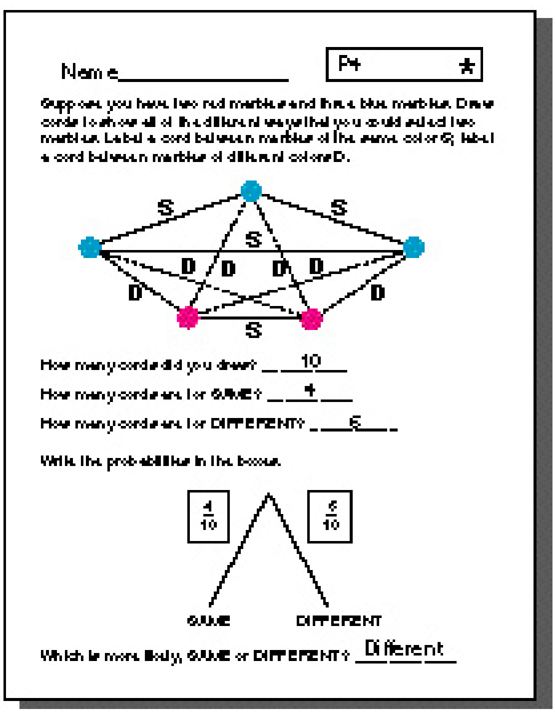
Number of Games	Same	Different
15	7	8
30	14	16
45	21	24
60	28	32
120	56	64

S: ***14 same and 16 different. In another set of 15 games, the most likely result is again seven same and eight different. 7 + 7 = 14 and 8 + 8 = 16.***

Record those results and complete the table for 45, 60, and 120 games.

Worksheets P4* and ** are available for individual work. If your class has already analyzed one of the combinations of marbles in a problem on the worksheet, you may want to direct students to change the marbles in that problem.

You may wish to challenge students who finish P4** to find and analyze combinations of marbles that favor Anita, that is, favor different.


Name____________ P4 *

Suppose you have two red marbles and three blue marbles. Draw cords to show all of the different ways that you could select two marbles. Label a cord between marbles of the same color S; label a cord between marbles of different colors D.

How many cords did you draw? 10

How many cords are for SAME? 4

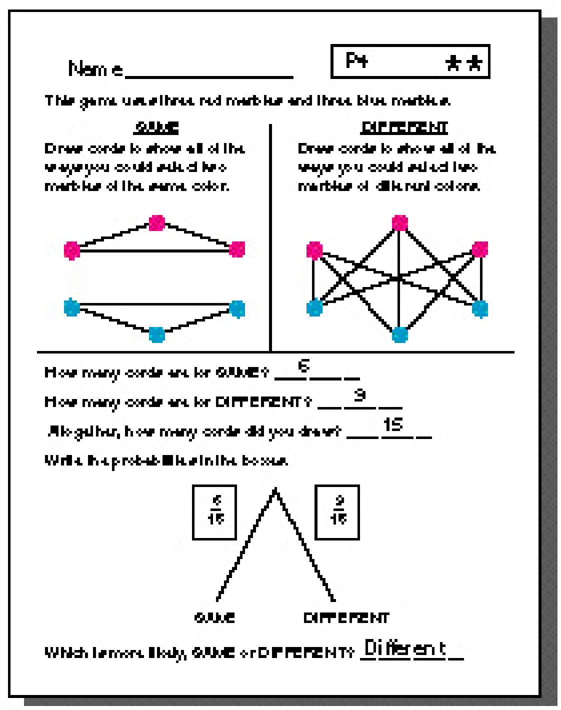
How many cords are for DIFFERENT? 6

Write the probabilities in the boxes.

$\frac{4}{10}$ $\frac{6}{10}$

SAME DIFFERENT

Which is more likely, SAME or DIFFERENT? Different


Name____________ P4 **

This game uses three red marbles and three blue marbles.

SAME

Draw cords to show all of the ways you could select two marbles of the same color.

DIFFERENT

Draw cords to show all of the ways you could select two marbles of different colors.

How many cords are for SAME? 6

How many cords are for DIFFERENT? 9

Altogether, how many cords did you draw? 15

Write the probabilities in the boxes.

$\frac{6}{15}$ $\frac{9}{15}$

SAME DIFFERENT

Which is more likely, SAME or DIFFERENT? Different

Capsule Lesson Summary

Discover the relationship between a number of dots and the number of cords that can be drawn between pairs of those dots. Review the *Same or Different?* game. Analyze several distributions of marbles of two colors in an attempt to find a fair game.

Materials

Teacher
- Colored chalk
- Marbles

Student
- Colored pencils, pens, or crayons
- Paper
- Worksheet P5

Description of Lesson

Exercise 1

Draw four dots on the board.

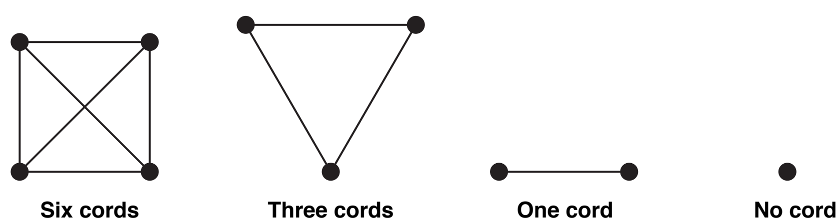
T: ***If we have four marbles, how many different ways can we select two marbles at random?***

S: ***Six ways.***

T: ***How can we check?***

S: ***Draw a cord between each pair of dots and count the total number of cords.***

Invite a student to draw the cords. Similarly, determine the number of cords for three dots, two dots, and one dot.



Record the results in a table. Draw the table in a place where you can save it for use later in the lesson.

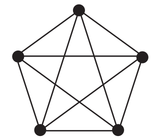
T: ***Can you predict the number of cords that can be drawn given five dots? … given six dots?***

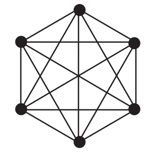
Ask students to explain how they arrive at their predictions.

Instruct students to draw first five dots and all the cords between pairs of dots, then six dots and all the cords between pairs of dots.

Number of Dots	Number of Cords
1	0
2	1
3	3
4	6
5	
6	

P5





T: ***How many cords are there for five dots?*** (10)
…six dots? (15)

Record the results in the table.

T: ***We won't draw the cord picture for seven dots, but can you predict the number of cords for seven dots?***

S: ***21.***

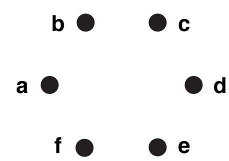
T: ***That's correct. Why did you predict 21?***

S: ***I noticed that the number of cords first increased by 1 (0 + 1 = 1), then 2 (1 + 2 = 3), then by 3 (3 + 3 = 6), and so on. 15 + 6 = 21.***

Number of Dots	Number of Cords	
1	0	
2	1	+1
3	3	+2
4	6	+3
5	10	+4
6	15	+5
7	21	+6
8	$\boxed{28}$	+$\boxed{7}$
9	$\boxed{36}$	+$\boxed{8}$
10	$\boxed{45}$	+$\boxed{9}$

Draw the following arrows to highlight this pattern. Extend the table to include eight, nine, and ten dots. Ask the students for the numbers that are in the boxes before recording them. Save the table for use later in the lesson.

Note: Your students may pose other (perhaps more sophisticated) ways to count the number of cords. For example, suppose there are six dots.



Method 1: Dot **a** connects to dots **b**, **c**, **d**, **e**, and **f** (five cords); dot **b** connects to dots **c**, **d**, **e**, and **f** (four new cords—we already counted the cord between **a** and **b**); dot **c** connects to dots **d**, **e**, and **f** (three new cords); and so on. 5 + 4 + 3 + 2 + 1 = 15, so the total number of cords is 15.

Method 2: Each of the six dots connects to five other dots by cords; 5 x 6 = 30. But every cord is counted twice this way, once at each end. 30 ÷ 2 = 15, so there are 15 cords.

You may wish to explain one of these methods to your class.

Exercise 2

T: ***Who remembers the game we played recently with red and blue marbles?***

Let students describe the game. They should mention that in the game you select two marbles and note whether they are of the same color or of different colors. Also, some students may remember that you can analyze the game by drawing cord pictures.

On the board, draw dots for two red and two blue marbles.

T: ***Many of you thought that the probability of same and the probability of different would be equal if we used the same number of marbles of each color. Was that true when we used two red and two blue marbles?***

S: ***No, different was more likely.***

T: ***Let's check that again. With four marbles, how many ways can we select two marbles at random?***

S: ***Six ways. I used the table on the board.***

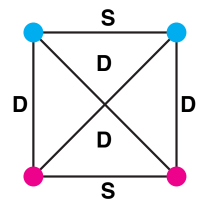
S: ***Six ways. I imagined drawing a cord between each pair of dots.***

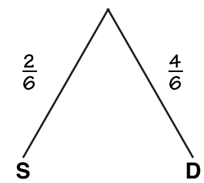
Invite students to draw the six cords, to label each of them **S** or **D**, and to draw and label a tree diagram with the probabilities for same and different.

T: ***Which is more likely, same or different?***

S: ***Different; there are four ways to get different and only two ways to get same.***

S: ***The probability for different is greater than the probability for same. $\frac{4}{6}$ is more than $\frac{2}{6}$.***





You may need to adjust the following dialogue slightly if your class or some individuals in the class found a fair game in Lesson P4.

T: ***Last time we checked a few other combinations of red and blue marbles, but we did not find a fair game. We'll try again to find a fair game, but first I have a couple of questions.***

What marbles could we use if we want same to occur always?

S: ***Use two or more marbles, all of the same color. For example, use five red marbles.***

T: ***What marbles could we use if we want different to occur always?***

S: ***One red marble and one blue marble.***

S: ***If you have several colors of marbles, use exactly one marble of each color.***

T: ***Let's try to find a fair game. What combination of red marbles and blue marbles should we try?***

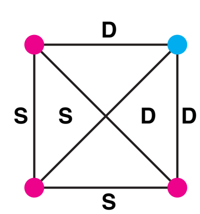
Select one of the combinations suggested by a student and analyze the game in a manner similar to that used in the beginning of this exercise and in Lesson P4. Solutions for all distributions using between four and seven marbles are provided at the end of this lesson.

After analyzing one combination collectively, challenge students to try to find a fair game if they have not already done so. You may like to organize students in cooperative groups to do this. Also, you may want to suggest using at most seven marbles, but allow adventurous students to use more marbles if they wish. Encourage students to use the table from Exercise 1 to check that the total number of cords for each situation is correct.

After about ten minutes, let students or groups present their findings to the class.

T: ***Has anyone found a game that is fair or almost fair?***

It is likely that some students or groups will find that using one blue and three red marbles (or one red and three blue) produces a fair game. If no one has found this solution, give the hint that there is a fair game with only four marbles.



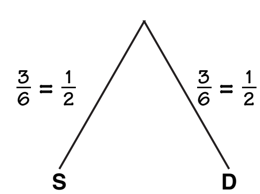


table on the board in which to record expected results of many games.

In 30 games using one blue and three red marbles , how many times should we expect same to occur? How many times should we expect different to occur?

Fifteen each, because it's a fair game.

Who found a game that is almost fair?

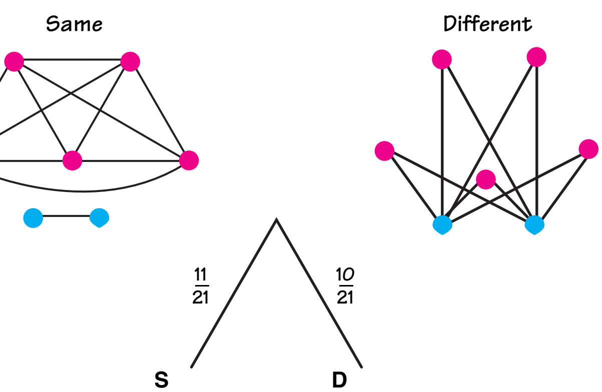
Number of Games	Same	Different
30	15	15
50	25	25
100	50	50
150	75	75
500	250	250

re several games that are almost fair, for example:

- four red, two blue (7/15 same and 8/15 different)
- five red, two blue (11/21 same and 10/21 different)
- six red, two blue (8/14 same and 6/14 different)

Doubling the number of both colors of marbles in a fair game does not produce another fair Another fair game results if you use six red and three blue (or six blue and three red).

udent or group present a game they believe is nearly fair. For that game, make a table to expected results of many games. For example:



Number of Games	Same	Different
21	11	10
42	22	20
63	33	30
210	110	100
2100	1100	1000

You may observe that students need to play a lot of times before they recognize mentally) that this game is not fair.

se 3

ute copies of Worksheet P5.

Anita and Bruce decide to play the* Same or Different?** ***game with three colors of marbles**:** ***red, blue, and white. Find the probabilities for same and different in each case on the worksheet.***

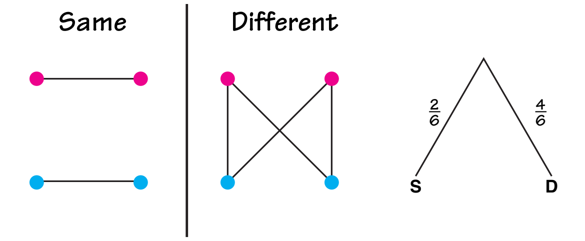
nts finish quickly, suggest they try to find a fair game using marbles of three colors.

There is no fair game using ten or fewer marbles of three colors. Several distributions e games that are nearly fair, for example:

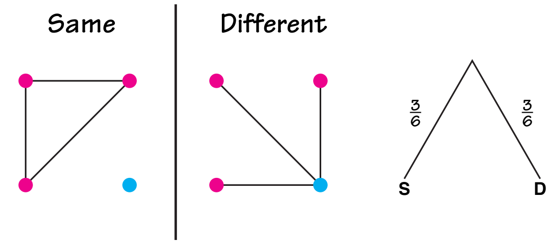
- one red, one blue, five white (10/21 same and 11/21 different)
- one red, one blue, six white (15/28 same and 13/28 different)
- one red, two blue, seven white (22/45 same and 23/45 different)

Solutions for marbles of two colors with 4, 5, 6, or 7 marbles

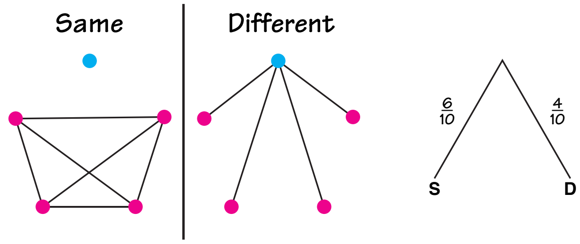
2 red, 2 blue



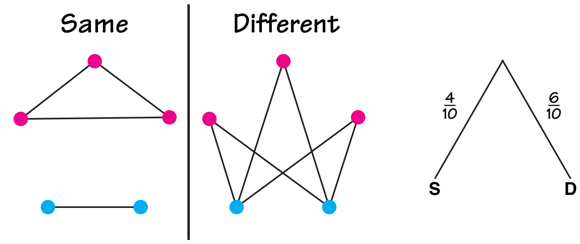
3 red, 1 blue
(equivalent to 1 red, 3 blue)



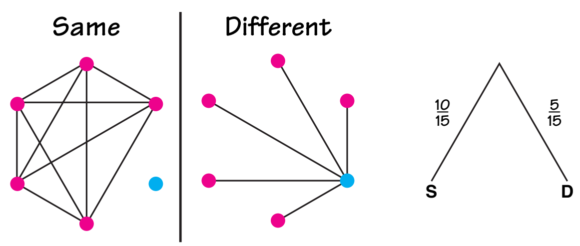
4 red, 1 blue
(equivalent to 1 red, 4 blue)



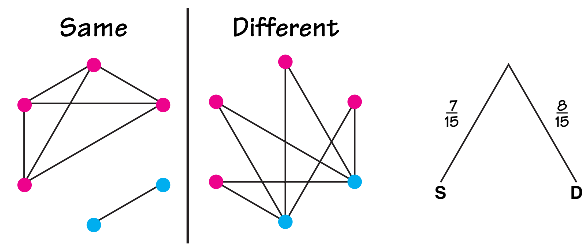
3 red, 2 blue
(equivalent to 2 red, 3 blue)



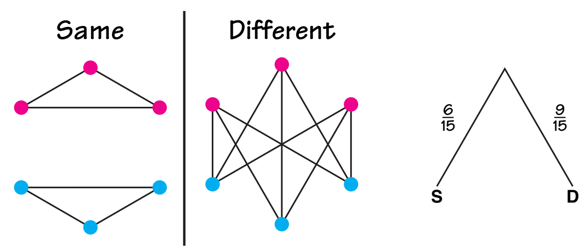
5 red, 1 blue
(equivalent to 1 red, 5 blue)



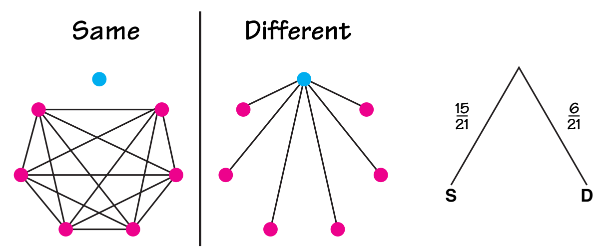
4 red, 2 blue
(equivalent to 2 red, 4 blue)



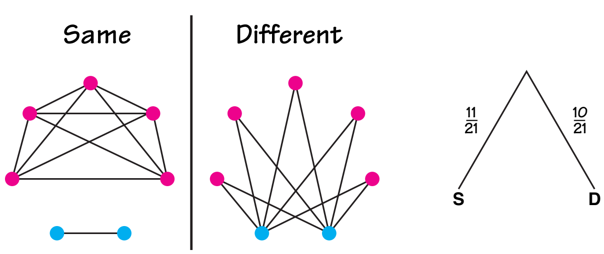
3 red, 3 blue



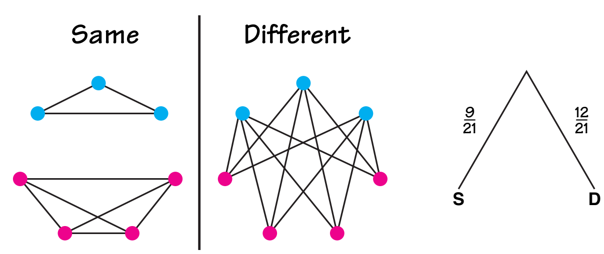
6 red, 1 blue
(equivalent to 1 red, 6 blue)

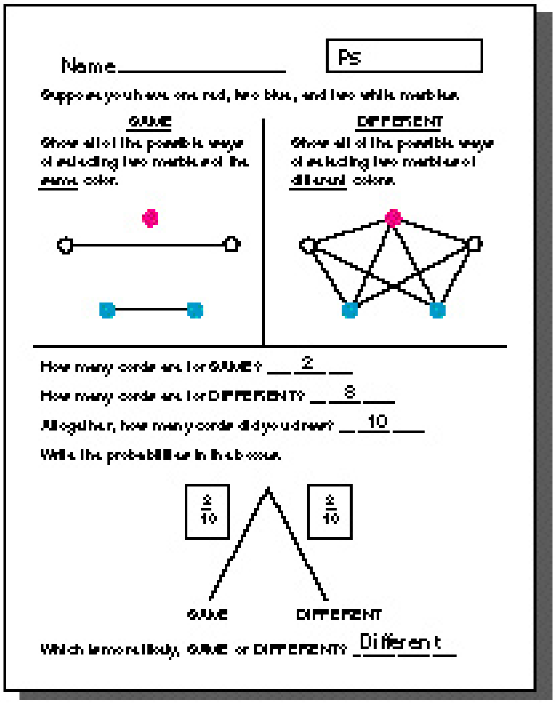


5 red, 2 blue
(equivalent to 2 red, 5 blue)



4 red, 3 blue
(equivalent to 3 red, 4 blue)





Name________________ P5

Suppose you have one red, two blue, and two white marbles.

SAME	DIFFERENT
Show all of the possible ways of selecting two marbles of the same color.	Show all of the possible ways of selecting two marbles of different colors.

How many cords are for SAME? ___2___

How many cords are for DIFFERENT? ___8___

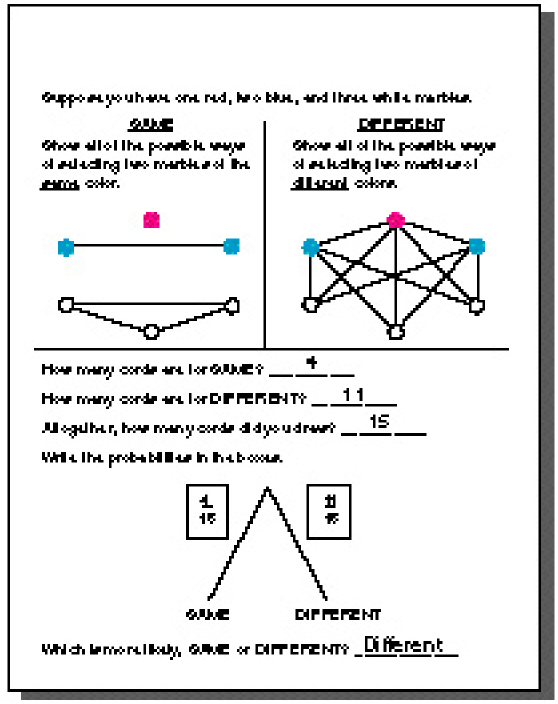
Altogether, how many cords did you draw? ___10___

Write the probabilities in the boxes.

$\frac{2}{10}$ $\frac{2}{10}$

SAME DIFFERENT

Which is more likely, SAME or DIFFERENT? ___Different___



Suppose you have one red, two blue, and three white marbles.

SAME	DIFFERENT
Show all of the possible ways of selecting two marbles of the same color.	Show all of the possible ways of selecting two marbles of different colors.

How many cords are for SAME? ___4___

How many cords are for DIFFERENT? ___11___

Altogether, how many cords did you draw? ___15___

Write the probabilities in the boxes.

$\frac{4}{15}$ $\frac{11}{15}$

SAME DIFFERENT

Which is more likely, SAME or DIFFERENT? ___Different___

Capsule Lesson Summary

Discover the relationship between a number of dots and the number of arrows that can be drawn between pairs of those dots. Play the *Same or Different?* game but change one rule: select a marble at random, replace it, and select another marble at random. Investigate what effect the change of rule has on the probability for same and for different with various combinations of marbles.

Materials

Teacher		Student	
	• Colored chalk • Marbles		• Paper • Worksheets P6* and **

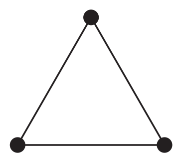
Description of Lesson

Exercise 1

Draw one group of three dots and one group of four dots on the board.

T: ***These dots represent marbles. In an earlier lesson, we counted the number of different ways two marbles could be selected from a collection of marbles. If there are three marbles, how many different ways can we select two marbles?***

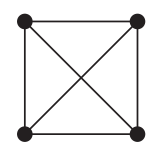
S: ***Three ways.***



Invite students to draw cords to represent selecting two marbles at a time.

T: ***With four marbles, how many ways can we select two marbles?***

S: ***Six ways.***

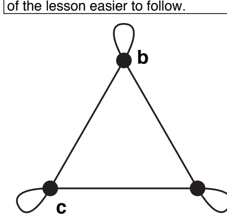


T: ***Today we are going to change the method of selecting the two marbles. We will select one marble, put it back, and then select another marble.***

Use three marbles to demonstrate this replacement method. Emphasize the difference between the original way of selecting two marbles (two at a time) and the new way (select one marble, replace it, and then select another marble).

Refer to the cord picture with three dots.

Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.



T: ***How can we change this picture to indicate the new way of selecting two marbles?***

S: ***Add a loop at each dot. You could get the same marble twice.***

Students might not recognize the need for the following change. Use two different colors of marbles to demonstrate the difference in order of selecting two marbles using this replacement method.

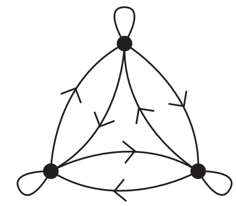
T: ***We must make one more change in this picture.*** (Point to dots **b** and **c**.) ***What are two ways we could select these marbles using the new (replacement) method?***

P6

S: ***We might select this marble* (b) *first and then the other marble* (c)*, or vice-versa.***

T: ***Yes, the order of selecting the two marbles is now important. To show the two ways of selecting each pair of marbles, we'll replace each cord with two arrows.***

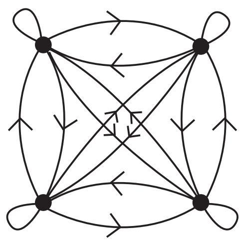
With three marbles, how many ways can we select two marbles using the new method?



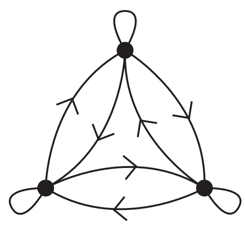
S: ***Nine ways. There are six arrows and three loops.***

Instruct students to draw four dots to represent four marbles on their papers. Then they should draw arrows and loops to show all the ways of selecting two marbles using the new (replacement) method. The picture on the board can then be altered accordingly.

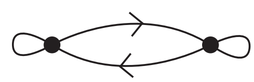
In a similar manner, determine the number of ways to select two marbles with replacement given two marbles, and given one marble.



16 ways



9 ways



4 ways

1 way

Enter the results in a table.

Number of Dots	Number of Arrows and Loops
1	1
2	4
3	9
4	16
5	25
6	

T: ***Can you predict the number of ways to select two marbles from a collection of five marbles?***

S: ***25 ways.***

Suggest students draw five dots and then the arrows and loops to confirm the prediction. Enter the results and extend the table.

T: ***What is your prediction for six dots?***

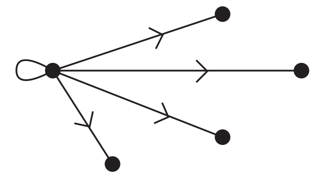
S: ***36 ways.***

T: ***Why?***

S: ***There's a pattern. The numbers in the second column first increase by 3 (1 + 3 = 4), then by 5 (4 + 5 = 9), then by 7 (9 + 7 = 16), and so on. 25 + 11 = 36.***

S: ***There's another pattern. If you multiply the number of dots by itself, you get the number of arrows and loops. 6 x 6 = 36***

Note: You may wish to use a picture on the board to explain why the number of ways (arrows and loops) is the square of the number of marbles (dots). For example, given five dots, consider one of the dots. Exactly one arrow is drawn from that dot to each of the five dots, including itself. This is done at each dot. 5 x 5 = 25, so there are 25 arrows and loops.



Illustrate students' patterns in the table and invite students to extend the table as shown here.

Save the table for use later in the lesson.

Number of Dots	Number of Arrows and Loops	
1	1	
2	4	+3
3	9	+5
4	16	+7
5	25	+9
6	36	+11
7	$\boxed{49}$	+$\boxed{13}$
8	$\boxed{64}$	+$\boxed{15}$
9	$\boxed{81}$	+$\boxed{17}$
10	$\boxed{100}$	+$\boxed{19}$

Exercise 2

With the class, review the *Same or Different?* game from Lessons P4 and P5.

T: ***Let's play the* Same or Different?** ***game again. What do you remember about the game?***

S: ***You select two marbles from a collection of marbles without looking and note whether they are the same color or different colors.***

S: ***We used cord pictures to analyze a game to determine whether same or different was more likely. We tried to find fair games.***

Draw a red dot and a blue dot on the board.

T: ***One day Anita and Bruce want to play, but they only have one red marble and one blue marble. Would it be a fair game if they selected two marbles at a time?***

S: ***No, they will always get different, one red and one blue marble.***

T: ***They decide to try the replacement method of selecting marbles. They select one marble at random, put it back, and then select another marble at random. With this method is it possible to get two marbles of the same color?***

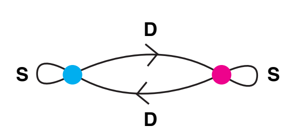
S: ***Yes, you can choose the red (blue) marble twice.***

Refer to the table from Exercise 1 as you ask,

T: ***How many different ways are there to select two marbles using the replacement method?***

S: ***Four ways.***

Invite a student to draw the arrows and loops. Label them **S** and **D** as students observe which represent same and which represents different.



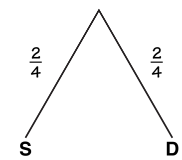
T: ***There are four ways to select the two marbles. How many ways are there to get same?***

S: ***Two ways.***

T: ***Different?***

S: ***Two ways.***

Complete a tree diagram as you announce,



T: ***There are two chances out of four for same and two chances out of four for different. So it is a fair game.***

The probability for same and probability for different are both $2/4$ or $1/2$.

Under the old rules, using one blue and three red marbles produced a fair game. Do you think it will be a fair game with the new rules?

Let your students express their opinions. Refer to the table from Exercise 1 as you ask,

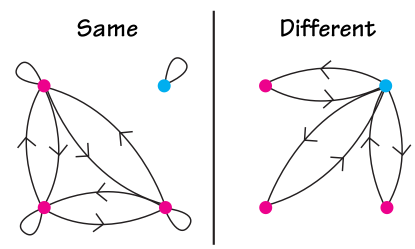
T: ***With four marbles, how many ways can we select two marbles with replacement?***

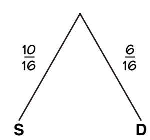
S: ***16 ways.***

Draw two groups of one blue and three red dots on the board.

T: ***Let's use two pictures, one for same and one for different.***

Invite students to draw the loops and arrows for same and different, and to complete a tree diagram.





S: ***This is not a fair game. The probability for same is $10/16$ and the probability for different is only $6/16$. Same is more likely to occur.***

S: ***It's not a fair game. There are ten chances for same and only six chances for different.***

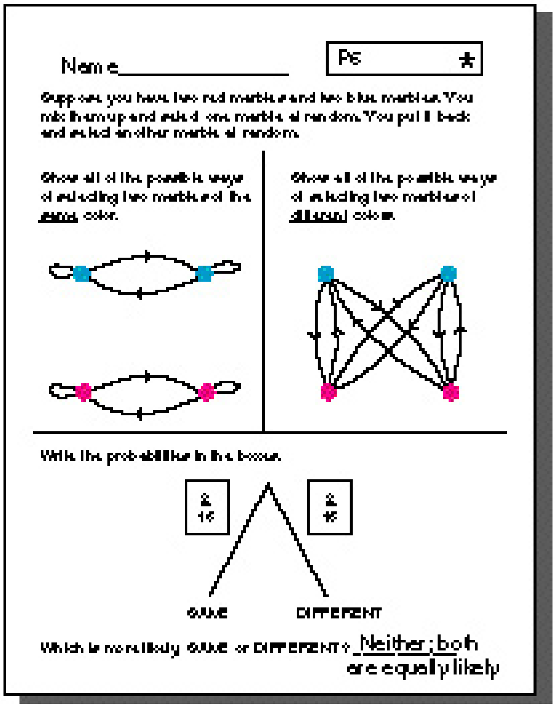
You may like to make a table to record expected results of many games for this situation. Allow time for students to do the worksheets.

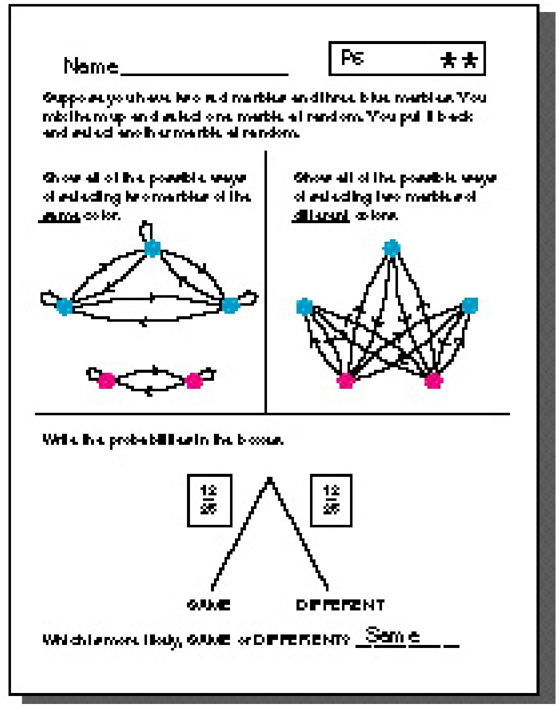
Number of Games	Same	Different
16	10	6
32	20	12
64	40	24
80	50	30
160	100	60

Worksheet P6* and ** are available for independent or group work. Ask students who finish quickly to find other collections of marbles that are fair for the *Same or* marbles with replacement.

Extension Activity

Suggest that students consider what would happen in a *Same or Different?* game with three colors of marbles when selecting two marbles with replacement.


Name
P6 ★
Suppose you have two red marbles and two blue marbles. You mix them up and select one marble at random. You put it back and select another marble at random.
Show all of the possible ways of selecting two marbles of the same color.
Show all of the possible ways of selecting two marbles of different colors.
Write the probabilities in the boxes.
SAME
DIFFERENT
Which is more likely, SAME or DIFFERENT? Neither; both are equally likely


Name
P6 ★★
Suppose you have two red marbles and three blue marbles. You mix them up and select one marble at random. You put it back and select another marble at random.
Show all of the possible ways of selecting two marbles of the same color.
Show all of the possible ways of selecting two marbles of different colors.
Write the probabilities in the boxes.
SAME
DIFFERENT
Which is more likely, SAME or DIFFERENT? Same

Capsule Lesson Summary

Use student-initiated methods to find the average[†] of a set of numbers in different contexts. Include a balancing process.

Materials

Teacher
- Colored chalk
- Meterstick

Student
- Paper
- Worksheets P7(a), (b), and (c)

Description of Lesson

Note: Different methods of averaging may be suggested in this lesson as students work on constructing an average concept. The balancing process will be used in other lessons in fourth grade. The more traditional total and divide method of finding an average is emphasized in fifth grade after students are more aware of what an average represents and division is more routine. If some of your students already know that method, feel free to let them use it.

Exercise 1

Organize the class in groups of four, and pose this problem. Choose a story and manipulatives that interest your class.

T: ***There are four friends who want to go to the fair together. Each friend has some tickets for the rides.***

Arrange that in each group, one person has 8 tickets, one has 14 tickets, one has 11 tickets, and one has 15 tickets.

T: ***Since they are best friends, they want to ride on all the same rides together. What can they do?***

Let students discuss the problem until they realize the job is to get an equal amount of tickets to all four friends. In their groups, direct students to make exchanges until everyone has the same number of tickets. Encourage groups to write about how they solved the problem so they can share their solutions with the class.

After a short while, encourage groups to explain how they solved the problem. Record actual solution methods on the board.

The following examples are two of many possible approaches. (Use student names rather than **A**, **B**, **C**, and **D**.)

S: ***First we wrote down how many tickets each person had. Then the person with the most tickets* (D) *gave some to the person with the least* (A).**

A	B	C	D
8	14	11	15

[†]Mean, median, and mode are three types of averages. In common usage, the word *average* often refers to mean. In this lesson, only this one type of average is referred to, but you may want to use *mean average* for emphasis.

T: ***How many?***

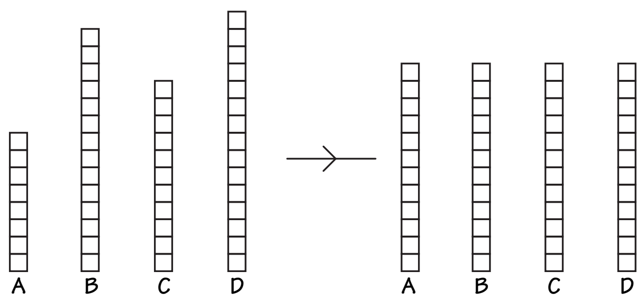
A	B	C	D
~~8~~	14	11	~~15~~
11			12

S: ***Three. Then B had the most tickets, so he gave 1 to A and 1 to C. Everyone ended up with 12 tickets.***

A	B	C	D
~~8~~	~~14~~	~~11~~	~~15~~
~~11~~	12	12	12
12			

Note: There are many variations of methods like this and you may want to look at more than one.

Also, in addition to recording the solution in a table, you can use counters (for example, tiles) on the overhead to represent the tickets. First, put the counters in columns for the starting situation, and then move them around until everyone has an equal stack.

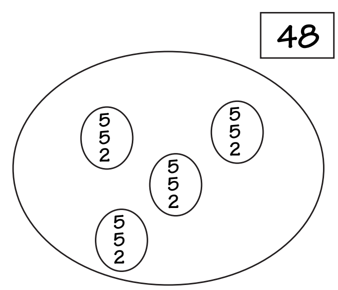


S: ***We put all the tickets in the middle of the table and then shared them equally.***

T: ***How many tickets did you have to share?*** (48)
How did you share them?

S: ***Each person took 5, then 5 more, then 2.***
Everyone ended up with 12 tickets.

T: ***Most of you found that each person gets 12 tickets .***
We say that 12 is the (mean) average number of tickets.



Exercise 2

Give the groups a second averaging problem, but this time with greater numbers. Perhaps this time you can give each person in the group a slip of paper with their starting number written on it. Groups may or may not want to use manipulatives to solve the problem.

T: ***The four best friends belong to a scout troop. The troop is selling fruit cakes in order to earn money for a trip. First they take orders for the fruit cakes, and later they deliver the fruit cakes. These are the numbers of orders each person took for fruit cakes.***

Give the four students in a group slips of paper with these numbers.

47	35	53	37

T: ***Being best friends, the children decide that they will share the work of delivering the fruit cakes. Each of them will deliver the same number of fruit cakes. About how many orders will each of them deliver?***

Let students explain their estimates.

Direct students to work in their groups to decide exactly how many orders each friend will deliver. Again, the groups should write about how they solve the problem.

Record actual solution methods used by groups on the board. Try to include at least one example where students made exchanges to even out (balance) the number of orders. For example:

	A	B	C	D
	47	35	53	37
C gives 8 to **B**	47	43	45	37
A gives 5 to **D**	42	43	45	42
C gives 1 each to **A** and **D**	43	43	43	43

or

	A	B	C	D
	47	35	53	37
C gives 3 to **D** and 5 to **B**	47	40	45	40
A gives 2 each to **B** and **D**	43	42	45	42
B gives 1 each to **B** and **D**	43	43	43	43

T: ***Most of you found that each person will need to deliver 43 orders for fruit cakes. The (mean) average number of orders is 43.***

At this time you may like to let the groups work on other averaging problems as presented on Worksheets P7(a) and (b). Alternatively, use Exercise 3 for group work.

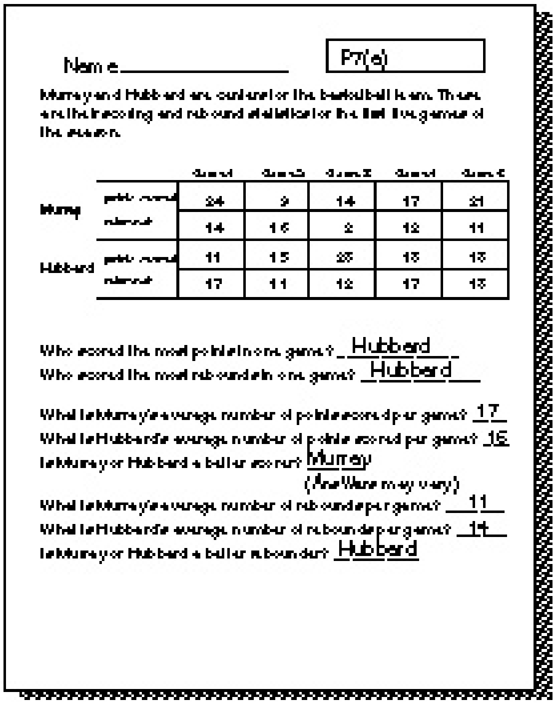
Exercise 3 (optional)

Provide each group with string or adding machine tape, scissors, and tape.

T: ***Here is another averaging problem for your group. This time I won't give you numbers. What I would like you to find is the average height of the four people in your group.***

Let the groups decide how to solve this problem, write about their solution, and present it to the class. You may like to compare the average heights of the groups and even find a class average.

Worksheet P7(c) has a height averaging problem with numbers for groups who finish quickly.



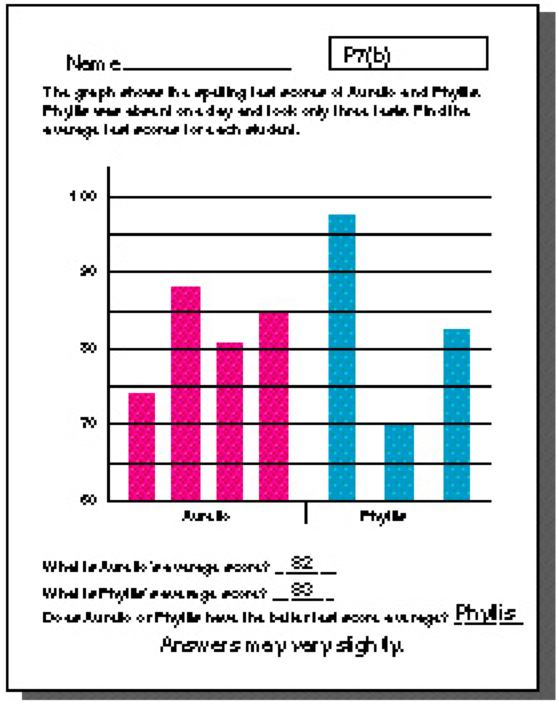
Name ____________ P7(a)

Murray and Hubbard are centers for the basketball team. These are the scoring and rebound statistics for the first five games of the season.

		Game 1	Game 2	Game 3	Game 4	Game 5
Murray	points scored	24	9	14	17	21
	rebounds	14	16	2	12	11
Hubbard	points scored	11	15	23	13	13
	rebounds	17	11	12	17	13

Who scored the most points in one game? Hubbard
Who scored the most rebounds in one game? Hubbard

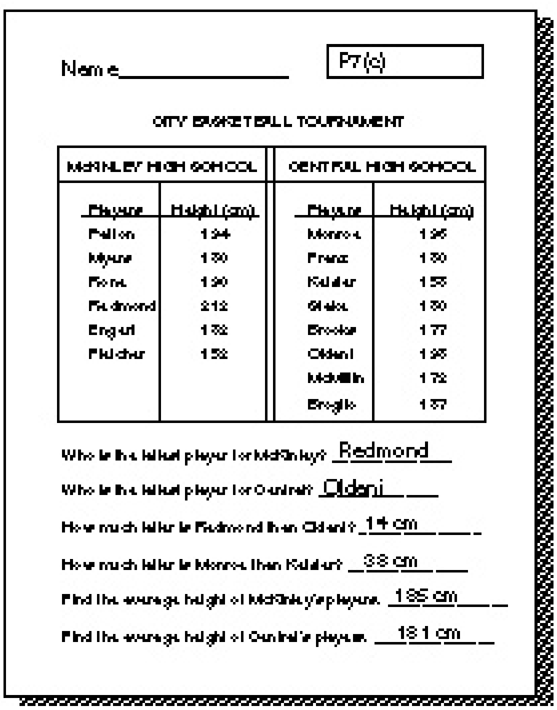
What is Murray's average number of points scored per game? 17
What is Hubbard's average number of points scored per game? 15
Is Murray or Hubbard a better scorer? Murray
(Answers may vary)
What is Murray's average number of rebounds per game? 11
What is Hubbard's average number of rebounds per game? 14
Is Murray or Hubbard a better rebounder? Hubbard



Name ____________ P7(b)

The graph shows the spelling test scores of Aurelio and Phyllis. Phyllis was absent one day and took only three tests. Find the average test scores for each student.

What is Aurelio's average score? 82
What is Phyllis' average score? 83
Does Aurelio or Phyllis have the better test score average? Phyllis
Answers may vary slightly.



Name ____________ P7(c)

CITY BASKETBALL TOURNAMENT

MCKINLEY HIGH SCHOOL		CENTRAL HIGH SCHOOL	
Players	Height (cm)	Players	Height (cm)
Patton	194	Monroe	196
Myers	180	Franz	180
Flora	190	Kalder	158
Redmond	212	Stein	180
Engart	182	Brooks	177
Fletcher	152	Oldeni	198
		McMillin	172
		Braglio	157

Who is the tallest player for McKinley? Redmond
Who is the tallest player for Central? Oldeni
How much taller is Redmond than Oldeni? 14 cm
How much taller is Monroe than Kalder? 38 cm
Find the average height of McKinley's players. 185 cm
Find the average height of Central's players. 181 cm

Capsule Lesson Summary

Review averaging while interpreting weather bureau information. Discuss Celsius temperature. Refer to a graph of the Number of Days of Precipitation versus Average Daily High Temperature to answer questions about the monthly climate of certain U.S. cities. Given a general description of a city's climate for a month, locate its dot on the graph.

Materials

Teacher		Student	
	• *IG-I Probability Poster* #1 • Chalk • Map of the United States or globe (optional)		• Straightedge • Worksheets P8 (no star), *, **, and ***

Description of Lesson

Exercise 1

The purpose of the following discussion of Celsius temperatures is to develop a general feeling for temperatures between 20° and 40°C. Adapt the comments about St. Louis weather to your geographical region. If your students are unfamiliar with the Celsius scale, provide much of the information yourself. Very little emphasis should be placed on Celsius-Fahrenheit equivalent temperatures.

T: ***0°C is a special temperature. What occurs at 0°C?***

S: ***Water freezes.***

S: ***Also, ice melts at 0°C.***

T: ***Is it ever that cold here?***

S: ***The daytime temperature in St. Louis*** (use your city or town here) ***is often near 0°C in December and January.***

T: ***What temperature in Fahrenheit corresponds to 0°C?***

S: ***32°F.***

Record this information on the board.

0°C   Water freezes   (December, January in St. Louis)   32°F

T: ***Is 10°C a comfortable temperature?***

S: ***10°C is cool, around 50°F.***

T: ***When is it near 10°C here?***

S: ***In March and November. It's good bicycling and football weather.***

Record this information on the board. Continue in a similar manner with other Celsius temperatures as suggested below.

0°C	Water freezes	(December, January)	32° F
10°C	Cool; Bicycling; Football	(March, November)	about 50°F
20°C	Warm; Picnics	(May, October)	about 70°F
30°C	Hot; Swimming	(July, August)	about 85°F
40°C	Very hot	(some days in summer)	about 105°F
$\widehat{10}$°C†	Cold; skating; Skiing	(some days in January)	about 15° F
$\widehat{20}$°C	Very cold	(some days in winter)	about $\widehat{5}$°F

Keep this information on the board to refer to later in the lesson.

Exercise 2

T: ***I'd like to visit New Orleans. I wonder if July is a good month. I called the weather bureau, and they told me that the average daily high temperature in New Orleans in July is 32°C.***

What does that mean? How do you think someone calculated that statistic?

S: ***It means that if you measure the high temperature in New Orleans every day in July, the average should be near 32°C.***

S: ***The weather bureau probably measured temperatures in New Orleans for many Julys.***

T: ***Can you suggest what the high temperatures might have been for five days in July so that the average was 32°C? Suppose the high temperature was different each day.***

Let students make several suggestions and review averaging as you check each one.

T: ***I plan to sail and swim. Is 32°C a good temperature for that?***

S: ***32°C is quite hot, but it might be good for swimming and sailing.***

T: ***What other weather information about New Orleans might I want to know?***

S: ***You should ask about how much it rains in New Orleans in July.***

T: ***The weather bureau told me that New Orleans averages 15 days of rain in July.***

Again discuss how the weather bureau got this statistic, and if you like, let students make up scenarios for the numbers of days of rain in several Julys to average 15 days.

Write the following information on the board.

New Orleans in July: 32° C, 15 days of rain

Display *IG-I Probability Poster* #1.

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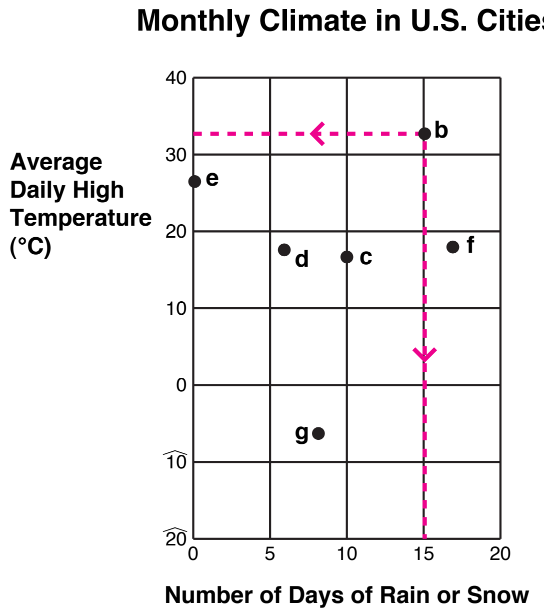
†To maintain consistency with *CSMP*, we write the temperature "negative ten degrees Celsius" as "$\widehat{10}$°C." Mention to your students that "$\widehat{10}$°C" is the same as "$^{-}$10°C," which is used on thermometers and temperature signs.

T: ***One of the black dots represents the climate of New Orleans in July. Which dot is it?***

S: ***This dot*** (**b**) ***is for New Orleans in July.***

As indicated in red in the illustration below, carefully trace horizontally from **b** to 32° C and vertically from **b** to 15 days to demonstrate that **b** represents the climate of New Orleans in July.



Do not write the letters or the dashed red lines on the poster. They are here just to make the description of the lesson easier to follow.

Label **b** "New Orleans in July."

T: ***New Orleans is too hot and rainy for me in July. I wonder if January would be a better month to visit New Orleans. In January, New Orleans averages 10 days of rain and its average daily high temperature is 17°C. Does that sound more comfortable?***

S: ***There are fewer rainy days, but it's much cooler. It might be too cold for swimming.***

T: ***On the graph, which dot is for New Orleans in January?***

S: ***This dot*** (**c**)***, since it represents 10 days of rain and 17°C.***

Label **c** "New Orleans in January."

T: ***Let's check out some other cities that I might want visit in January or July instead of New Orleans.***

Label **d** "Los Angeles in January."

T: ***This dot*** (point to **d**) ***represents the climate of Los Angeles in January. What is the average high temperature and average number of days of rain for Los Angeles in January?***

S: ***18°C and 6 days of rain.***

T: ***In July, Los Angeles is hot and very dry. Which of the unlabeled dots could be for Los Angeles in July?***

S: ***This dot*** (**e**)***, because it is for a city that has an average high temperature of 27°C and that averages 0 days of rain for some month.***

Label **e** “Los Angeles in July.”

T: ***Juneau, Alaska is warm and rainy in July. Which dot could be for Juneau in July?***

S: ***This dot* (f)*, because it is for a city that has an average high temperature of 18°C and that averages 17 days of rain for some month.***

Label **f** “Juneau in July.”

T (pointing to **g**)**:** ***Can you describe the climate in some month for the city represented by this dot?***

S: ***It is quite cold, 6°C, and there are about 8 days of snow.***

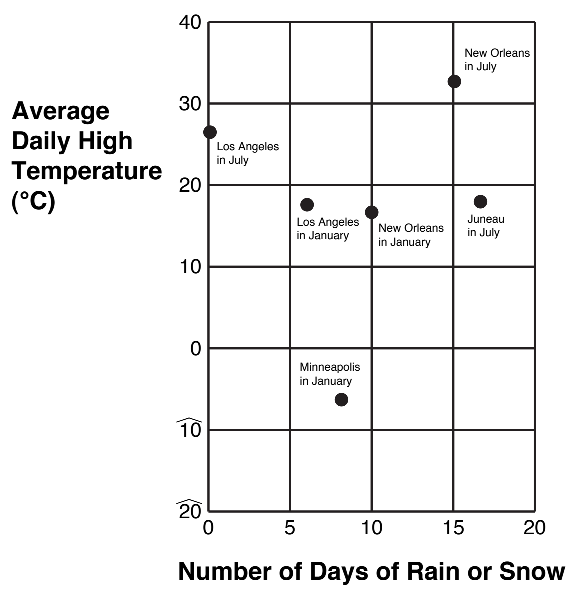
T: ***What U.S. city in which months could this dot* (g) *represent?***

Students are likely to suggest northern U.S. cities in the winter months.

T: ***This dot* (g) *represents the climate of Minneapolis in January.***

Label **g** “Minneapolis in January.”

Monthly Climate in U.S. Cities



T: ***Which city is warmer in January, New Orleans or Los Angeles?***

S: ***Los Angeles, because its dot for January is higher on the graph than the dot for New Orleans in January.***

S: ***Los Angeles, because in January its average daily high temperature is 18°C while New Orleans’s is 17°C.***

T: ***Which city averages more days of rain in July, Juneau or New Orleans?***

S: ***Juneau, because its dot for July is further to the right than the dot for New Orleans.***

S: ***Juneau, because Juneau averages 17 days of rain in July while New Orleans averages 15 days of rain in July.***

T: ***On this graph, which city in which month has the hottest weather?***

S: ***New Orleans in July. Its dot is the highest.***

T: ***Which is coldest?*** (Minneapolis in January)
Which is driest? (Los Angeles in July)
Which has the most days of rain or snow? (Juneau in July)
Which city in which month would be best for me to go swimming and sailing?
(Los Angeles in July)

On the poster, point to the upper right region of the graph.

T: ***Describe the climate of cities with dots in this part of the graph. What parts of the United States or the world might be represented here?***

S: ***The climate would be very hot and very rainy.***

S: ***Dots there could be for cities in Florida or Hawaii.***

S: ***They could be in tropical jungles.***

Ask similar questions about other regions of the graph:

Upper left: Very hot and dry
Texas, California, and Southwestern U.S.
Most desert regions

Lower right: Cold and much snow
Northeastern U.S. and parts of Alaska
Parts of Canada, Russia, and Northern Europe

(an average of more than 15 days of snow per month is rare)

Lower left: Very cold and dry
Northern Alaska
Northernmost Canada, Russia, and Europe; the Arctic and the Antarctic

Worksheets P8 (no star), *, **, and *** are available for individual work. Tell students to use the graph on P8 (no star) to answer the questions on P8*.

Near the end of the period you may wish to discuss some of the worksheets.

Writing Activity

Suggest that students write a letter to a cross-country skier explaining how to use the graph on Worksheet P8 (no star) to decide where and when to go for practice.

Extension Activity

Find some other statistics about U.S. cities that could be represented in graphs similar to those in this lesson.

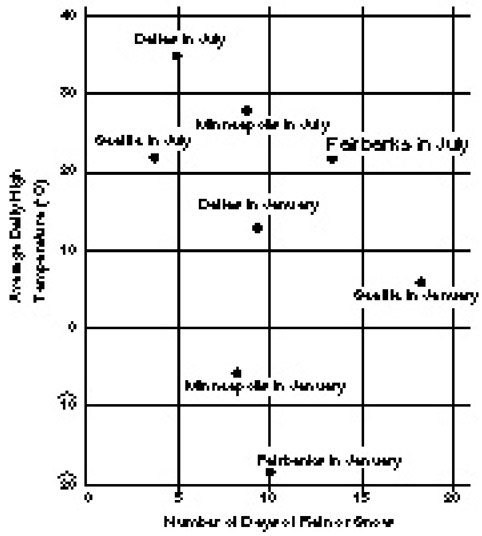
Home Activity

Suggest parents/guardians work with their child to record the high (or low) temperature each day for a week and find the average.

P8

Name________________ P8

MONTHLY CLIMATE OF U.S. CITIES



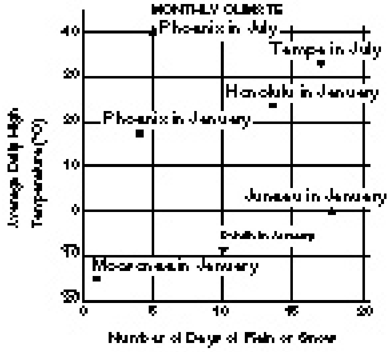
Name________________ P8 ★

Use the graph on Worksheet P8 (no stars) to answer these questions.

1. Which city is hottest in July? Dallas
   Which city is hottest in January? Dallas
   Which city is coldest in July? Seattle
   Which city is coldest in January? Minneapolis
2. Which city is driest in January? Minneapolis
   Which city is driest in July? Seattle
   Which city averages the most rainy days in January? Seattle
   Which city averages the most rainy days in July? Minneapolis
3. What is the average daily high temperature in Dallas in January? 13° C
   How many days of rain does Dallas average in January? 9
4. What is Minneapolis's average daily high temperature in January? -6° C
   How many days of rain or snow does Minneapolis average in January? 8
5. How much warmer is Dallas in July than Seattle in July? 13° C
   How much warmer is Seattle in July than in January? 16° C
   How much warmer is Minneapolis in July than in January? 34° C
6. On the graph, draw and label two dots for Fairbanks, Alaska.
   a. In July, Fairbanks averages 12 days of rain and the average daily high temperature is 22° C.
   b. In January, Fairbanks averages 10 days of snow and the average high temperature is -18° C.

Name________________ P8 ★★

MONTHLY CLIMATE



Using the following descriptions, label the dots on the graph. One is done for you.

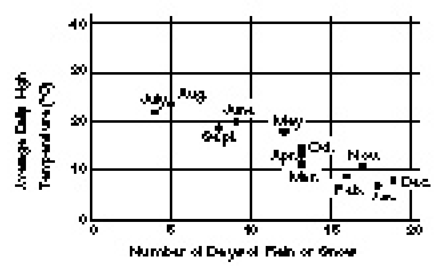
a. Duluth, Minnesota receives some snow and is very cold in January.
b. Phoenix, Arizona is dry and very hot in July.
c. In January, Phoenix is dry and has a pleasant temperature.
d. Juneau, Alaska is cold and very wet in January.
e. Tampa Bay, Florida is very hot and rainy in July.
f. Moosonee, Ontario, in Canada, is very cold and very dry in January.
g. In January, Honolulu is quite rainy, but has a pleasant temperature.

Name________________ P8 ★★★

Seattle, Washington

Month	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Days of Rain	12	15	12	12	12	9	4	5	8	12	17	19
Temp (°C)	7	9	11	14	12	20	22	22	19	15	11	8

Use the above data to draw a graph of Seattle's weather. Draw and label one dot for each month. January is done for you.



1. Which do you think is the best month to visit Seattle?________
   Why? ________________________________
2. Which do you think is the worst month to visit Seattle?________
   Why? ________________________________

Answers will vary.