Workbooks

WORKBOOKS TABLE OF CONTENTS

WORKBOOKS INTRODUCTION

There are many opportunities for the student to work individually during the course of the lessons described in the other content strands. In the Workbooks strand, however, it is this individualized work which becomes the chief end of the majority of lessons. The goal in this strand is to provide students with opportunities

- to review many of the ideas they have met in other content strands;
- to apply their acquired knowledge to new situations requiring various kinds of strategic thinking; and
- to learn how to read and use mathematics workbooks.

The following workbooks are provided:

- *Collection of Problems #1*
- *Collection of Problems #2*
- *Collection of Problems #3*
- *Collection of Problems #4*
- *Collection of Problems #5*

Two story-workbooks are provided:

- *A Strange Country*
- *Halloween Puzzles*

Each workbook contains problems of varying levels of difficulty. Approximately the first ten pages of each workbook are easy problems, the next ten to twelve pages are average level difficulty, and the last ten pages are more challenging problems. For each workbook, we suggest that all students start work at the easiest level (i.e., on page 2) and then work through as many pages as they can handle during the two lessons scheduled for that workbook. We estimate that, in a typical class, about two-thirds of the students will correctly finish the first ten pages, about one-third will finish the first twenty pages, and a few will finish all or most of the workbook. These proportions will vary from class to class.

This guide contains an answer key for each workbook. The key follows an introduction to the workbook and a suggested collective lesson. The lesson either presents the workbook to the whole class or provides a warm-up activity on a problem similar to one found in the workbook.

The story-workbooks *A Strange Country* and *Halloween Puzzles* combine the motivation of a storybook and the problem-solving opportunities of a workbook. These two booklets allow students to become deeply involved in an appealing fantasy as they struggle with difficult mathematics problems. The situations support topics and strategies developed in other strands.

Use of the Workbooks Strand for Evaluation Purposes

The workbooks provide an excellent instrument to assess the progress of your students on a regular basis. You may not feel it is necessary to check every page and problem for each student, but you should develop a procedure for checking students' work with which you are comfortable. This may include checking one or more specific pages, discussing some particular mistakes with individual students and letting them correct their work, or just looking carefully at a few pages to be sure the students have understood the general idea of the problems in that particular book.

In the Blacklines, you will find a record-keeping tool for each workbook to help you assess student progress in the various strands. This tool may also assist you in parent conferences and in filling out periodic progress reports.

Here are some important points to bear in mind for workbooks.

- Always read the introductory material for each workbook and give the short introductory collective lesson(s).
- All students should start at the beginning of each workbook and progress as far as they can.
- All students should begin a new title on the same day, even if some students have not finished work on the previous title.
- Not all students should be expected to complete a given workbook. Only some students will reach the most challenging problems. Other students may succeed only in doing the easiest problems, although you should not assume this automatically—surprises are not at

Content Overview

Workbooks

The five *Collection of Problems* workbooks both review and extend many of the ideas introduced in the content strands. The extensions occur through problems which require students to apply the mathematics to new situations or to synthesize their knowledge in new ways.

Lessons: W1, 2, 4, 5, 8, 9, 10, 11, 14, and 15

Halloween Puzzles

The story-workbook *Halloween Puzzles* presents several intriguing numerical mysteries, compliments of favorite Halloween characters. All the puzzles require students to label dots in a given arrow picture. With the aid of composition, students discover that some arrow pictures have an infinity of solutions, some have a unique solution (a "bat" picture), and still others are self-contradictory having no solution (a "witch" picture). This story-workbook presents a useful application of composition while illustrating that not all mathematical situations have the traditional unique solution.

Lessons: W6 and 7

A Strange Country

The story-workbook *A Strange Country* takes students to a curious country inhabited by numbers. In this country, a numerical rule determines which numbers' houses are connected by paths. The rule involves two relations, doubling and doubling plus one. Using the language of cords, students explore these relations as they solve problems, such as, "Find a route from 30's house to 50's house." Students' maps of various parts of this strange country may result in artistic pictures. To conclude, a story about two friends leaving 1's house on different paths, never to meet again, suggests a subtle connection with binary numbers.

This story-workbook illustrates how a simple concept can be imaginatively expanded into a rich mathematical experience. And all students can participate since all are comfortable with the underlying operation, doubling.

Lessons: W12 and 13

Capsule Lesson Summary

Use the calculator to do some mental arithmetic. Begin the workbook *Collection of Problems #1*. (This is the first of two lessons using this workbook.)

Materials

Teacher		Student	
	• Overhead calculator (optional)		• Calculator • *Collection of Problems #1* Workbook • Colored pencils, pens, or crayons • Metric ruler

Description of Lesson

Exercise 1: Mental Arithmetic

Pair students and distribute calculators, one to each pair. Explain to the class that in this activity one person in each pair will use the calculator, and the other will do calculations in his or her head. The people without calculators may not look at the calculators for a few minutes.

T: ***We are going to have a little contest to see which is faster, the calculator or the brain. Half of you have calculators, and you must do my problems on the calculator. You must press the right keys and show the result on the calculator. Half of you do not have calculators, and you must do the calculations in your head or with paper and pencil. You cannot use the calculator.***

You may need to discuss the rules for a moment. If you like, suggest that students in pairs take turns using the calculators.

T: ***I will give you a calculation. As soon as you know the result, raise your hand. When I say stop, leave your hand up if it is up or down if it is down. Let's try it.***

Give the class a simple calculation such as 2 x 100; say it clearly but not too slowly. Watch hands go up, and when several hands are up say stop. You should purposely try to say, "Stop," before those with calculators have time to press all the appropriate keys.

T: ***Look around. Do more people using calculators or more using their brains have their hands up?***

S: ***Brains; but it wasn't fair. You didn't give us enough time to do the problem on the calculator. I knew the answer, but I had to press the keys on the calculator and that was slower.***

Repeat this contest several times with mental calculations and with more difficult calculations such as 17 x 56 or 208 ÷ 16. Make some observations with the class about how some calculations are easier and faster to do in our heads, while others can be done more efficiently on the calculator.

Conclude the contest, and tell student pairs that now they will use both the calculator and their heads to do some calculations. You may want to use an overhead calculator during this activity.

T: ***Press* C *and make sure 0 is on the display. Cover the display with a hand (finger), but be careful not to cover the light panel (energy source). Now I will tell you which keys to press. Try to do the calculations in your head, just the way you think the calculator is doing the calculations.***

Slowly announce which keys to press, giving students an opportunity to do the calculations mentally. Cover the display and press the same keys on the overhead calculator, if you are using one.

T: ***Press* 4 × 5 (pause) + 6 =.**
Don't look. What number do you think will be on the display?

S: ***26.***

Allow several students to answer before letting students check the display of their calculators. Also, check the display of the overhead calculator. If some students have other than 26 on the display, ask those students to press C and then to put 26 on the display.

T: ***Cover the display again. Now press* + 4 = = =.**
Don't look. What number do you think will be on the display?

S: ***38.***

Again, let several students respond before letting them check the display of their calculators. Help student pairs correct the number on the display as necessary.

Continue this activity with the following (or a similar) sequence of calculations:

- Start with 38 on the display: press − 2 = = = = (30)
- Start with 30 on the display: press ÷ 2 × 3 = (45)
- Start with 45 on the display: press + 1 0 = =† (65)

Exercise 2

Distribute copies of the workbook *Collection of Problems #1* and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem. Your students have solved problems similar to the ones in this workbook in their earlier *CSMP* work. The topics and concepts will continue to be reviewed and extended during this year.

At the end of the lesson, collect the workbooks for your review. They will be used again in Lesson W2.

†Read as "plus ten, equal, equal."

Capsule Lesson Summary

Given a true addition statement, solve some related addition problems and observe the patterns. Continue individual work in the workbook *Collection of Problems #1*. (This is the second of two lessons using this workbook.)

Materials

Teacher	• None	Student	• *Collection of Problems #1* Workbook • Colored pencils, pens, or crayons • Metric ruler

Description of Lesson

Write this boxed addition statement on the board.

36 + 67 = 103

Ask students to check the calculation, preferably without pencil and paper, and invite students to explain their checking techniques.

S: ***30 + 60 = 90, 6 + 7 = 13, and 90 + 13 = 103.***

S: ***36 + 60 = 96 and 96 + 7 = 103.***

S: ***36 + 4 = 40 and 40 + 63 = 103.***

Write this problem on the board below the boxed addition statement.

38 + 67

T: ***Try to solve this problem without doing much work. The addition statement 36 + 67 = 103 can help you.***

S: ***38 + 67 = 105. We are adding 38 to 67 instead of 36 to 67. 38 is 2 more than 36 and 105 is 2 more than 103.***

Continue the activity with the following problems. Answers are in the boxes. Let students explain how they use an earlier problem to solve a new problem. Keep up a rather brisk pace.

35 + 67 = 102	46 + 67 = 113	37 + 166 = 203
36 + 66 = 102	56 + 67 = 123	37 + 366 = 403
36 + 69 = 105	36 + 57 = 93	137 + 66 = 203
35 + 66 = 101	36 + 47 = 83	1 037 + 66 = 1 103
37 + 66 = 103	46 + 57 = 103	237 + 266 = 503

Invite students to comment on patterns they observe.

W2

Distribute students' copies of the workbook *Collection of Problems #1*. Ask students first to correct or complete pages from the previous week's work, and then to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

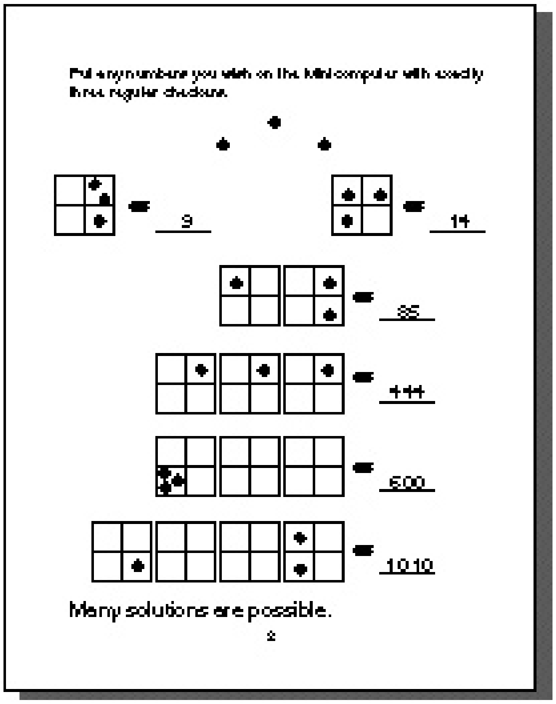
At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in their workbooks at school during a study time or at home as an assignment.

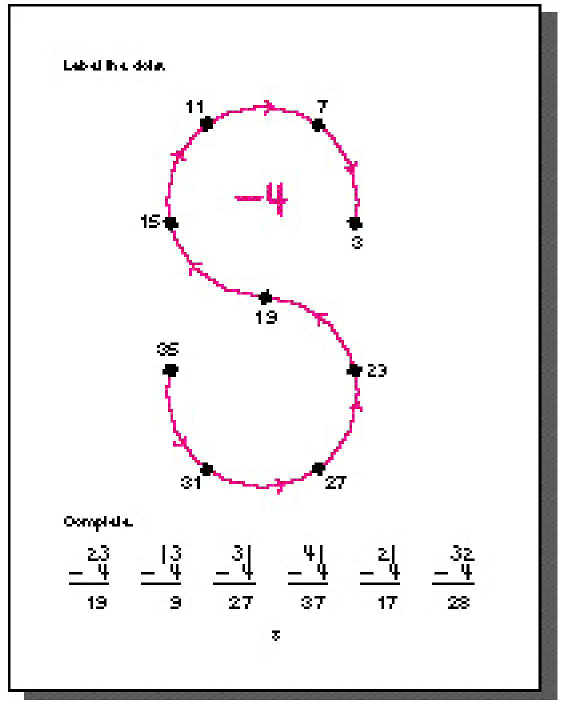
Assessment Activity

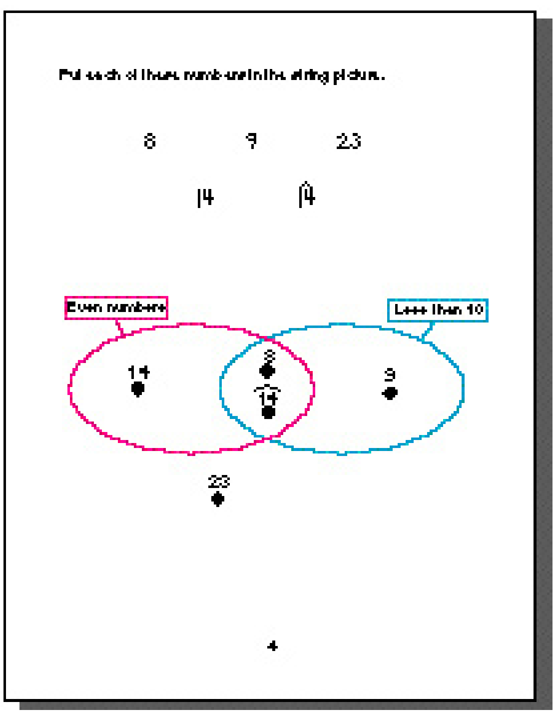
An individual student progress record for the workbook is available on Blackline W2(a). You may like to use this form to monitor student work.

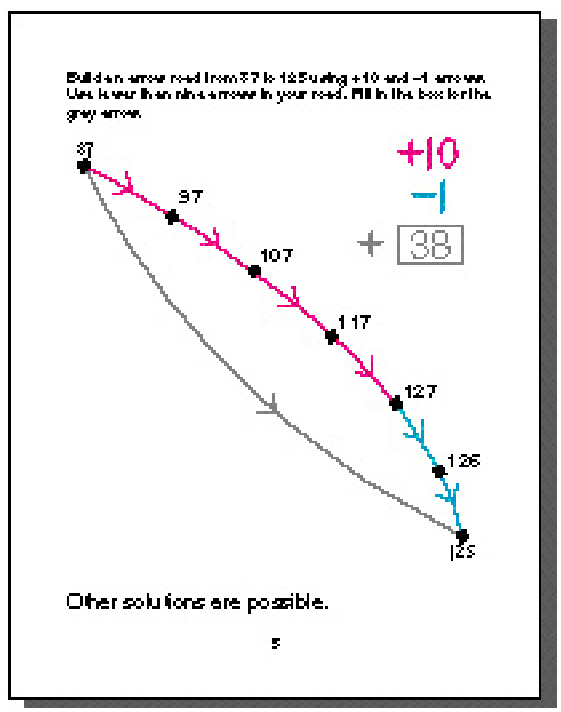
Home Activity

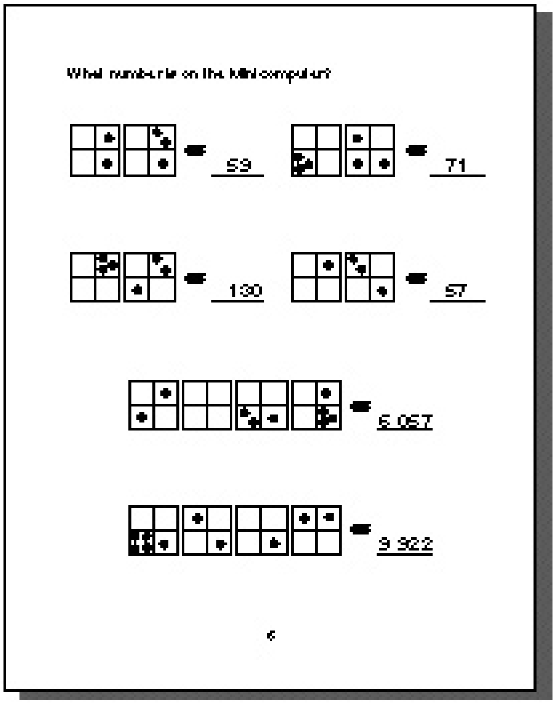
If you choose to send workbooks home with students, you may want to include a letter to parents/guardians with this workbook. Blackline W2(b) has a sample letter.


Put any numbers you wish on the Minicomputer with exactly three regular checkers.
= 9
= 14
= 35
= 444
= 600
= 1010
Many solutions are possible.
2


Label the dots.
−4
11
7
15
3
19
35
23
31
27
Complete.
23 − 4 = 19
13 − 4 = 9
31 − 4 = 27
41 − 4 = 37
21 − 4 = 17
32 − 4 = 28
3


Put each of these numbers in the string picture.
8
9
23
14
14
Even numbers
Less than 10
14
8
14
9
23
4


Build an arrow road from 87 to 125 using +10 and −1 arrows.
Use fewer than nine arrows in your road. Fill in the box for the grey arrow.
+10
−1
+ 38
87
97
107
117
127
126
125
Other solutions are possible.
5



What numbers are on the Minicomputer?

= 59

= 71

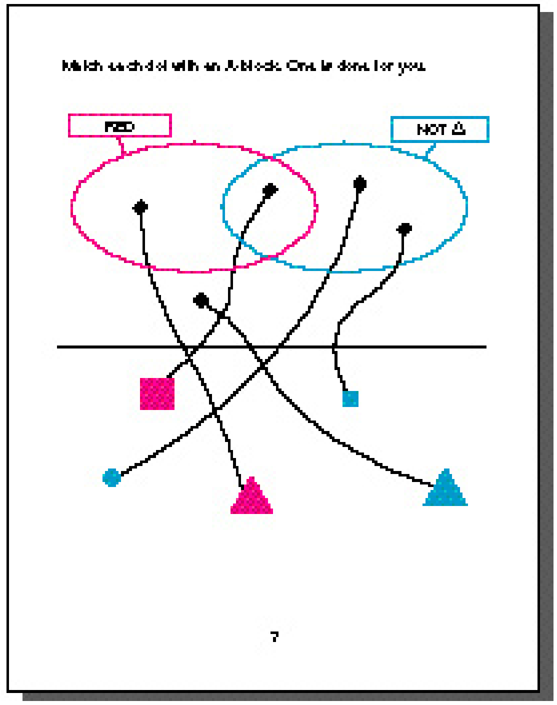
= 130

= 57

= 6 057

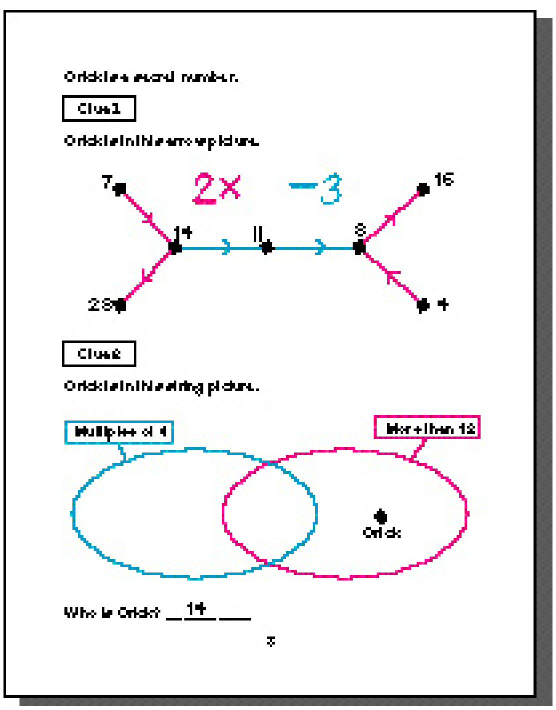
= 9 922

6



Match each dot with an A-block. One is done for you.

7



Grick is a secret number.

Clue 1

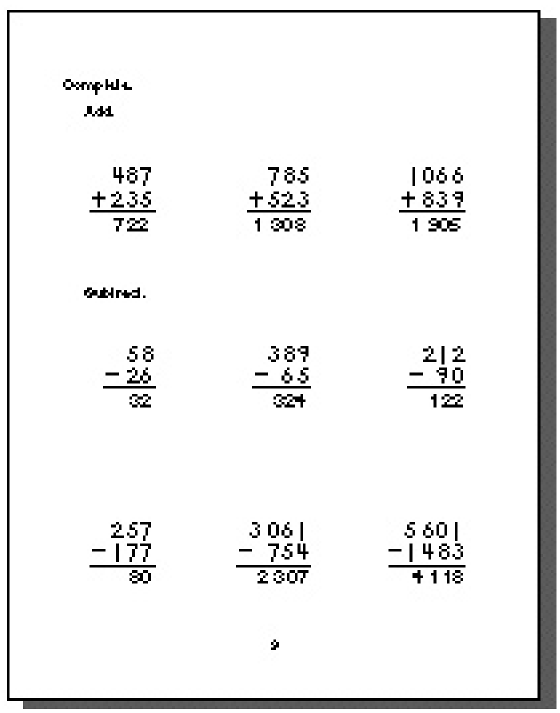
Grick is in this arrow picture.

Clue 2

Grick is in this string picture.

Who is Grick? 14

8



Complete.

Add.

487 + 235 = 722

785 + 523 = 1 308

1066 + 839 = 1 905

Subtract.

58 − 26 = 32

389 − 65 = 324

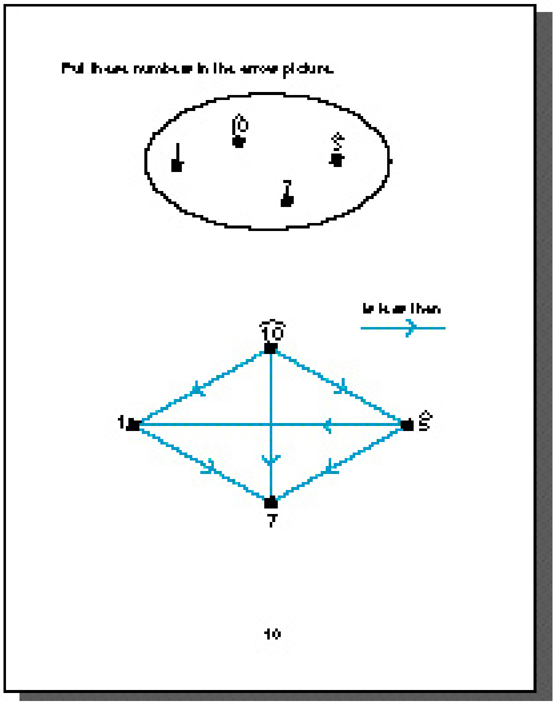
212 − 90 = 122

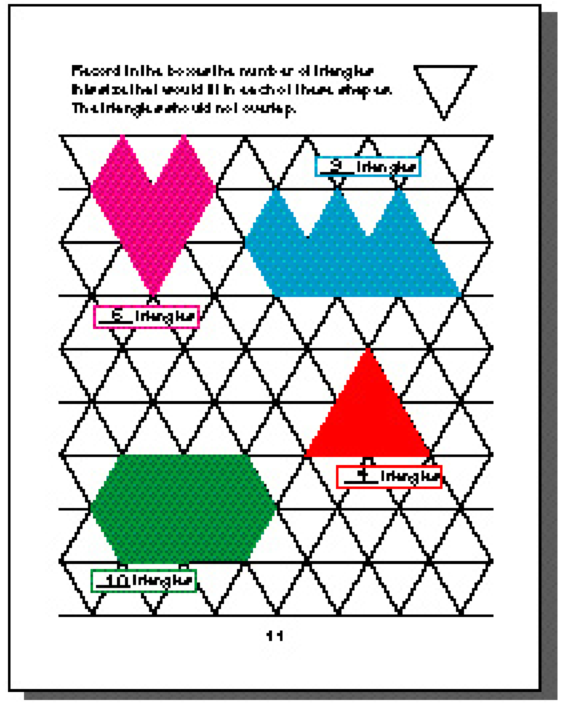
257 − 177 = 80

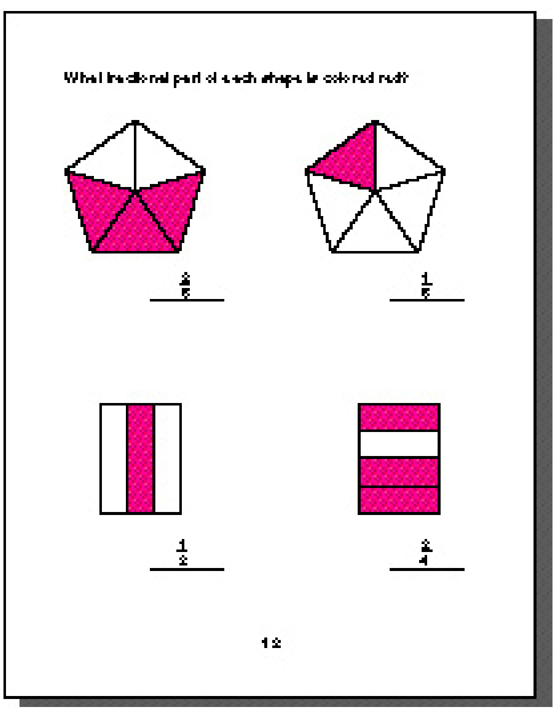
3061 − 754 = 2 307

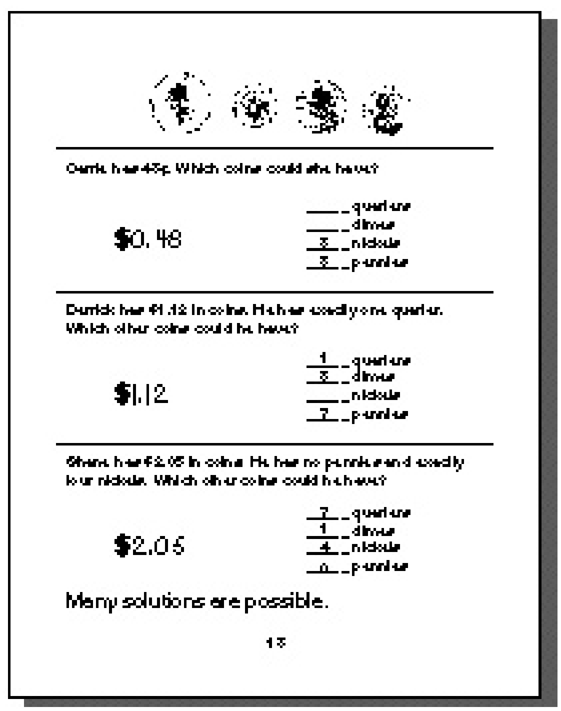
5601 − 1483 = 4 118

9

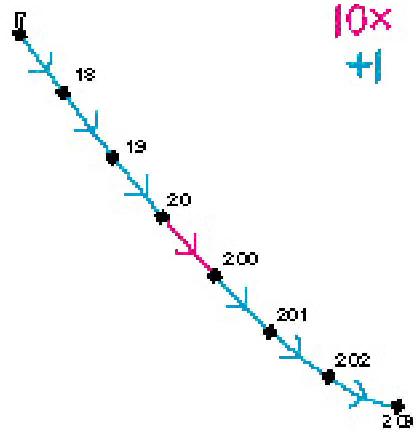

10
1
7
10


10 triangles
11


12


$0.48
$1.12
$2.05
Many solutions are possible.
13

Build an arrow road from 17 to 203 using 10x and +1 arrows. Use fewer than ten arrows in your road.

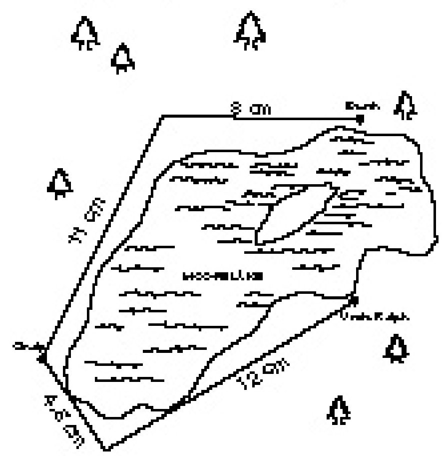


This is the only solution that uses fewer than ten arrows.

14

Measure the length of the zigzag from Cindy's house to the beach. 19 cm

Draw a zigzag from Cindy's house to Uncle Ralph's cabin without going through the lake. Measure its length. 16.5 cm



Answers will vary.

15

Ramona has a punch recipe. He remembers that to serve six people, he uses 2 cups orange juice. To serve 12 people, he uses 2 liters ginger ale. And to serve 24 people, he uses 2 quarts ice cream.

Complete the recipe for serving 6, 12, or 24 people.

Punch	
2 cups	Orange Juice
1 liter	Ginger Ale
1/2 quart	Ice Cream
Serves 6	

Punch	
4 cups	Orange Juice
2 liters	Ginger Ale
1 quart	Ice Cream
Serves 12	

Punch	
8 cups	Orange Juice
4 liters	Ginger Ale
2 quarts	Ice Cream
Serves 24	

Ramona wants to make enough punch to serve 18 people. Write the recipe for Ramona.

Punch	
6 cups	Orange Juice
3 liters	Ginger Ale
1 1/2 quarts	Ice Cream
Serves 18	

16

Gig is a secret number.

Clue 1

Gig is one of these numbers.

$(3 + 6) \times 4 =$ 36

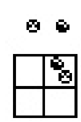
$(3 \times 6) + 4 =$ 22

$3 + (6 \times 4) =$ 27

$3 \times (6 + 4) =$ 30

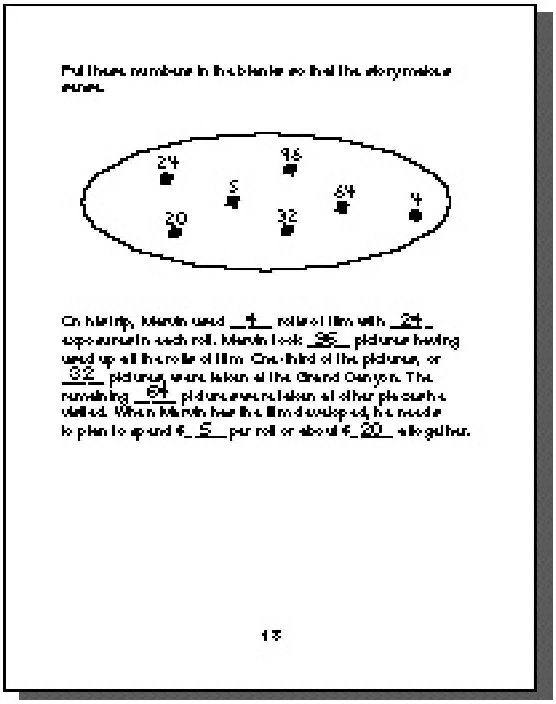
Clue 2

Gig can be put on this Minicomputer with exactly one negative and one ● checker.



Who is Gig? 36

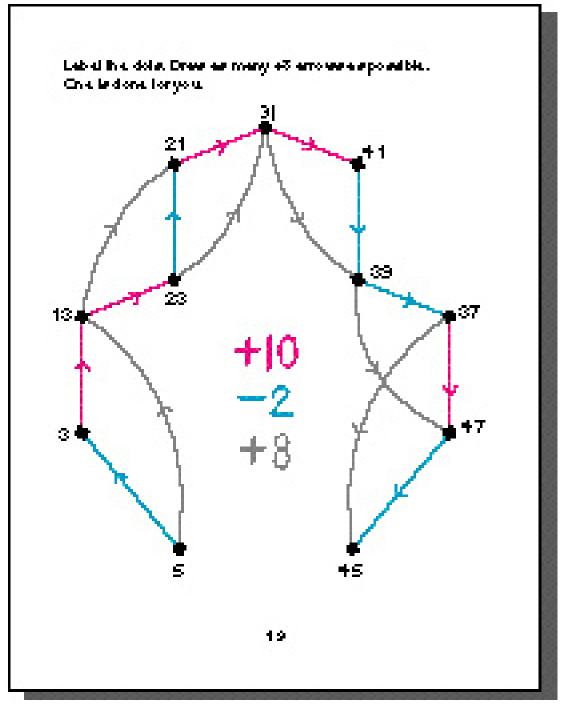
17



Put these numbers in the blanks so that the story makes sense.

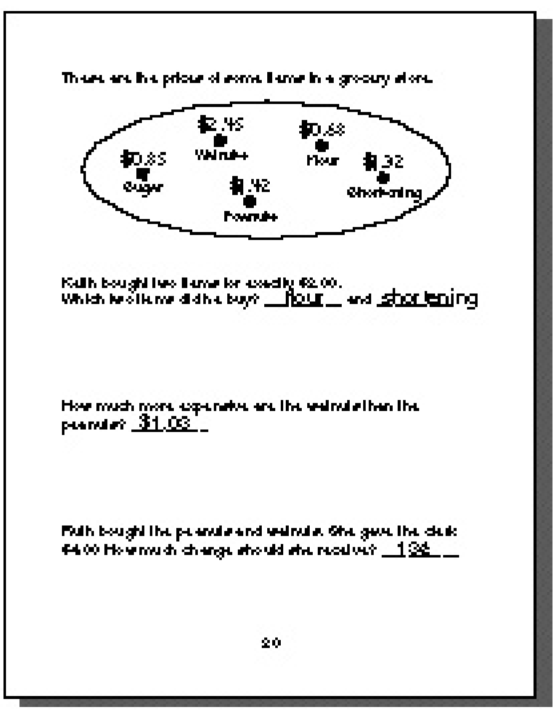
On his trip, Marvin used __4__ rolls of film with __24__ exposures in each roll. Marvin took __96__ pictures having used up all the rolls of film. One-third of the pictures, or __32__ pictures were taken at the Grand Canyon. The remaining __64__ pictures were taken at other places he visited. When Marvin has the film developed, he needs to plan to spend $__5__ per roll or about $__20__ altogether.

18



Label the dots. Draw as many +8 arrows as possible. One is done for you.

19



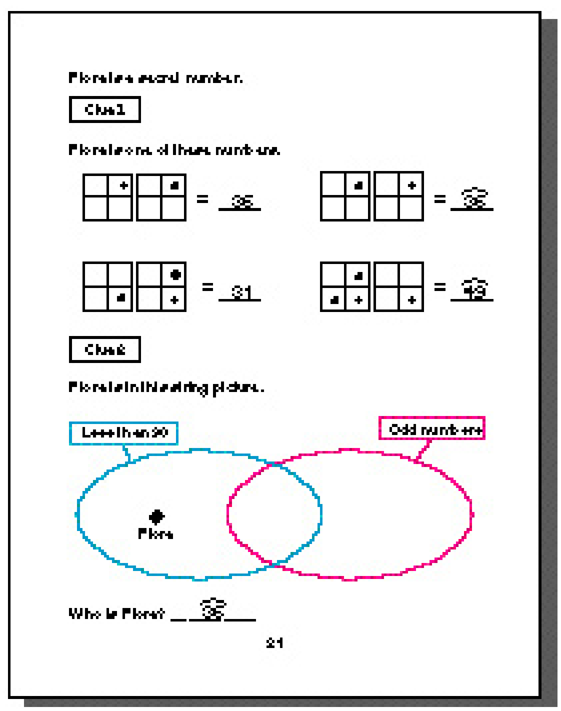
These are the prices of some items in a grocery store.

Keith bought two items for exactly $2.00.
Which two items did he buy? __flour__ and __shortening__

How much more expensive are the walnuts than the peanuts? __$1.03__

Ruth bought the peanuts and walnuts. She gave the clerk $4.00 How much change should she receive? __13¢__

20



Flora is a secret number.

Clue 1

Flora is one of these numbers.

Clue 2

Flora is in this string picture.

Who is Flora? __38__

21

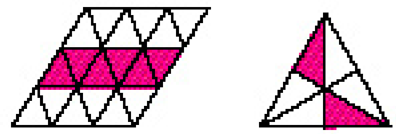
Use the true addition statement in the box to help complete the other addition problems.

48 + 26 = 74

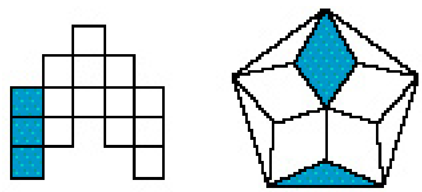
47 + 26 = 73	48 + 36 = 84
48 + 29 = 77	38 + 26 = 64
148 + 26 = 174	48 + 126 = 174
46 + 25 = 71	50 + 24 = 74
24 + 13 = 37	96 + 52 = 148

22

Color one-third of each shape red.



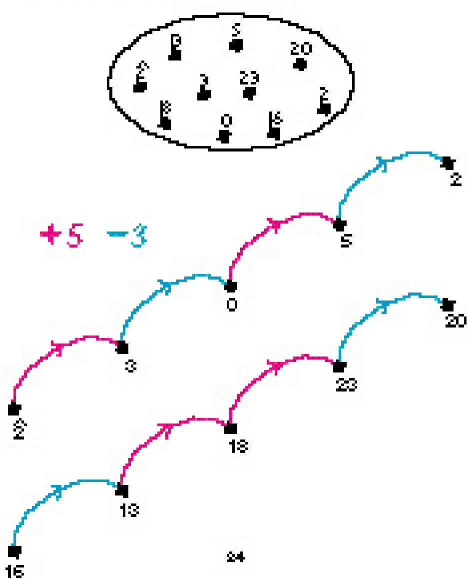
Color one-fifth of each shape blue.



Many colorings are possible.

23

Put these numbers in the arrow picture.



24

Share 51 cherries among Sandy, Jill, and Bob.

For Sandy	For Jill	For Bob
10	10	10
5	5	5
2	2	2
17		

Write a number sentence about this sharing.

51 ÷ 3 = 17

Share 85 pennies among Andrea, Joel, Kim, Randy, and Mark.

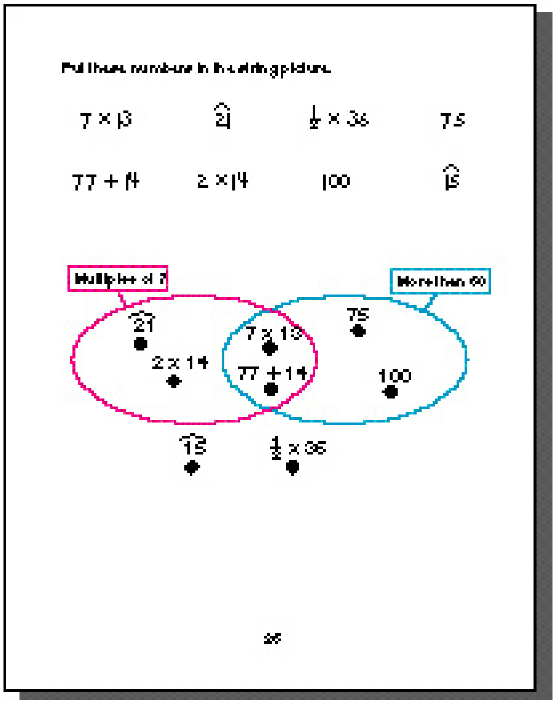
For Andrea	For Joel	For Kim	For Randy	For Mark
15	15	15	15	15
2	2	2	2	2
17				

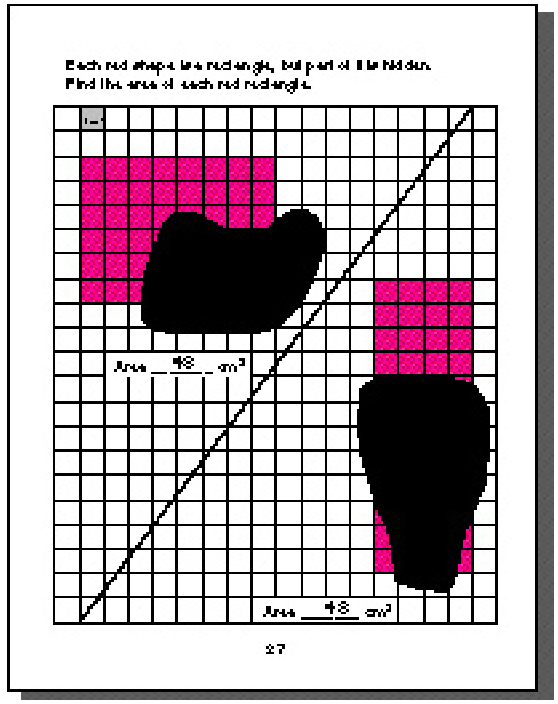
Write a number sentence about this sharing.

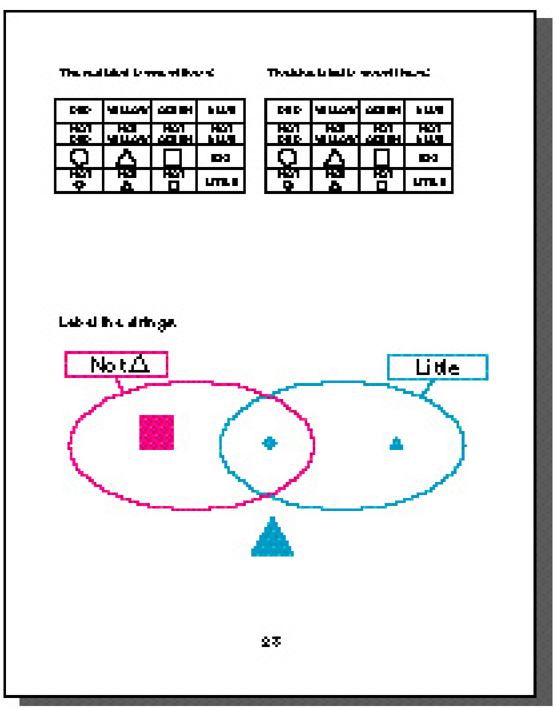
5 x 17 = 85

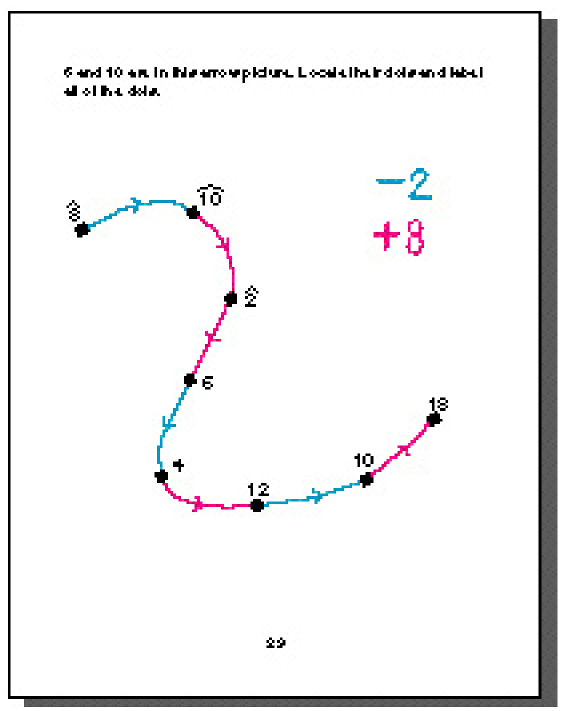
Other ways of doing the sharing and other number sentences can be used.

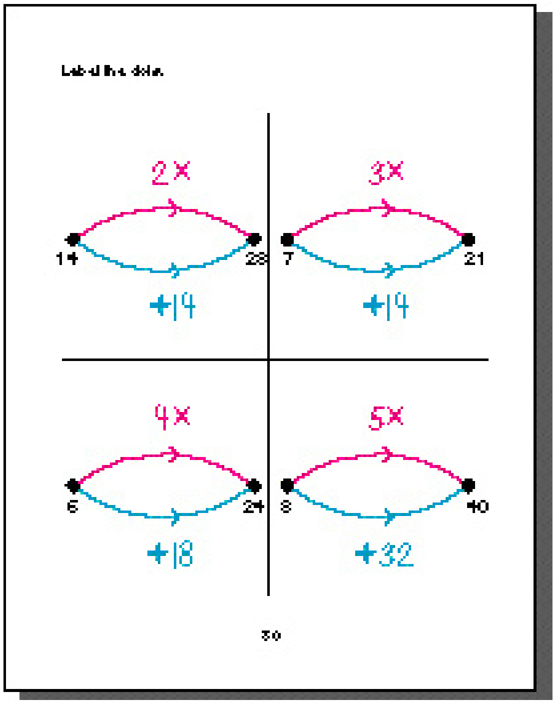
25

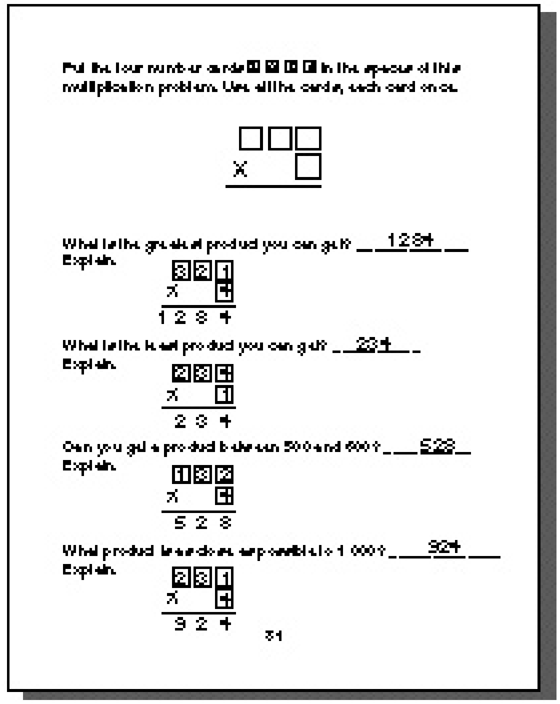

Put these numbers in the string picture.
7 × 13
21
½ × 36
75
77 + 14
2 × 14
100
13
Multiples of 7
More than 60

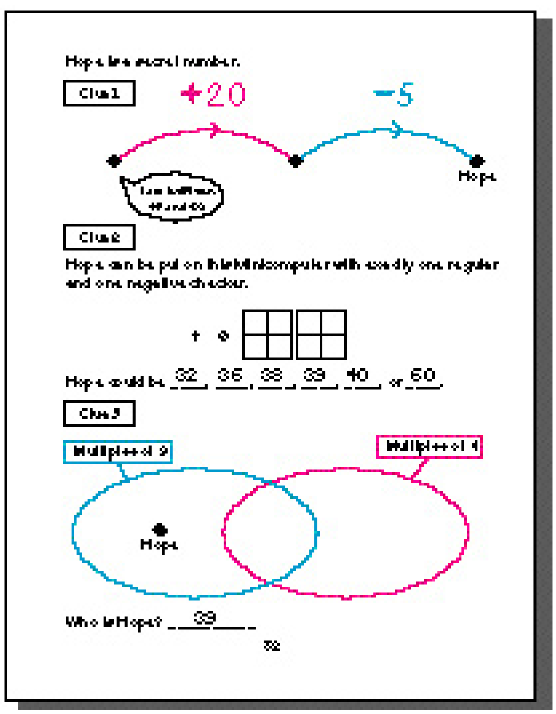

Each red shape is a rectangle, but part of it is hidden.
Find the area of each red rectangle.
Area 48 cm²
Area 48 cm²


Label the strings.
Not △
Little


6 and 10 are in this arrow picture. Locate the hidden dot and label all of the dots.
−2
+8


Label the dots.
2×
3×
14
28
7
21
+14
+14
4×
5×
6
24
8
40
+18
+32
30


Put the four number cards in the spaces of this multiplication problem. Use all the cards, each card once.
What is the greatest product you can get? 1284
Explain.
What is the least product you can get? 234
Explain.
Can you get a product between 500 and 600? 528
Explain.
What product is as close as possible to 1 000? 924
Explain.
31


Hope is a secret number.
Clue 1
+20
−5
Hope
Clue 2
Hope can be put on this Minicomputer with exactly one regular and one negative checker.
Hope could be 32, 36, 38, 39, 40, or 60.
Clue 3
Multiple of 3
Multiple of 4
Hope
Who is Hope? 39
32

Capsule Lesson Summary

Use a calculator to explore sequences of numbers generated by the relations [+] [3] [=] … and [+] [5] [=] … and their inverses. Solve a detective story with sequential clues involving the numbers that can be put on the Minicomputer under restrictions on the kinds and placement of checkers, the numbers in a certain region of a string picture, and the numbers related to 573 in the [−] [5] [=] … relation.

Materials

Teacher
- Overhead calculator (optional)
- Minicomputer set
- Colored chalk
- Blackline W3(a)

Student
- Calculator
- Minicomputer record sheet
- Paper
- Colored pencils, pens, or crayons

Advance Preparation: Use Blackline W3(a) to make copies of the Minicomputer record sheet for use in Exercise 3.

Description of Lesson

Arrange for students to work with partners for this lesson. Each pair of students should have a calculator.

Exercises 1 and 2 are good activities to model with an overhead calculator.

Exercise 1

Give these instructions slowly and clearly. Require that students stay with you.

T: ***Turn on your calculators and check that 0 is on the display.***
Press [+] [3] [=] [=] [=] [=] [=] [=]. What is happening?

S: ***The calculator is counting by threes.***

S: ***Multiples of 3 appear on the display.***

T: ***What number is on the display now?***

S: ***18.***

T: ***Do not press any keys for a moment. What numbers will appear on the display if we press [=] four more times?***

S: ***21, 24, 27, and 30.***

T: ***Do it and check.***
Everyone should have 30 on the display.
Now hide the display with your hand. Press [=] [=] [=] [=]. (Pause.)
What number is on the display? Why do you think so?

S: ***42. I counted by threes from 30: 33, 36, 39, 42.***

S: ***4*** x ***3 = 12 and 30 + 12 = 42.***

T: *Check and see.*
Everyone should have 42 on the display.
Do not press any key for a moment. What is the first number greater than 100 that will appear if we continue to press [=]*?*

S: *102.*

T: *How many more times do we have to press* [=] *before 102 will appear?*

S: *20 times. 20 x 3 = 60 and 42 + 60 = 102.*

T: *Let's check. Press* [=] *until you get a number greater than 100. Count how many times you press* [=].

Everyone should have 102 on the display. Let's try to go back to 0 and see all of the same numbers in reverse order. What could we do?

S: *Press* [−] [3] [=] [=] [=] *and so on.*

T: *Try it and see.*

Exercise 2

T: *Clear the calculator and put 17 on the display. Watch the display as you press* [+] [5] [=] [=] [=] [=] [=]*. What do you notice?*

S: *Each time we press* [=]*, the calculator adds 5.*

S: *The ones digit of the number on the display alternates between 2 and 7.*

T: *Everyone should have 42 on the display. Do not press any key for a moment.*
If we continue pressing [=]*, do you think 197 will ever appear?*

S: *Yes; 197 is 155 more than 42 and you can add 155 by adding 5's.*

S: *Yes, the ones digit is 7.*

T: *Will 275 ever appear?*

S: *No; 272 and 277 will appear, but not 275.*

S: *The ones digit of 275 is not 2 or 7, so it will not appear.*

T: *What is the greatest number less than 1,000 that will appear?*

S: *997.*

T: *What is the least number more than 1,000 that will appear?*

S: *1,002.*

T: *Press* [=] *until you have a number greater than 200. Stop at the first number you reach that is more than 200. Watch the numbers that appear along the way and see if you can detect some patterns.*

Students should stop when 202 is on the display. If students did not observe the alternating 2 and 7 ones digit before, they should do so now.

T: *Now, suppose we want to go back and see the same numbers appear in reverse order. What could we do?*

S: *Press* [−] [5] [=] [=] [=] *and so on.*

Instruct students to press [−] [5] [=] [=] [=] and so on until they get back to 17. Let them comment on the fact that indeed they are seeing the same numbers in reverse order—that they do see 42 and do get back to 17.

T: ***If we continue to press [=], what is the least positive number we will see?***

S: ***2.***

T: ***Will we see $^{-}22$?***

Some students may predict yes based on the alternating 2 and 7 pattern. By continuing to press [=], the class should find that $^{-}22$ does not appear. Students may observe that the greatest negative number that appears is $^{-}3$, then $^{-}8$, $^{-}13$, $^{-}18$, $^{-}23$, and so on. In this case, the pattern of the ones digits with negative numbers is that they alternate between 3 and 8.

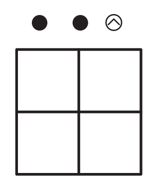
Put aside calculators for awhile.

Exercise 3

Tell the class that you have a secret number and that you will give them clues about the number. They are to be detectives and try to discover the secret number by following the clues.

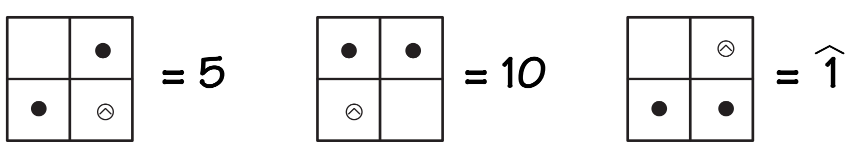
Clue 1

Display one Minicomputer board and three checkers, one negative checker and two regular (positive) checkers.

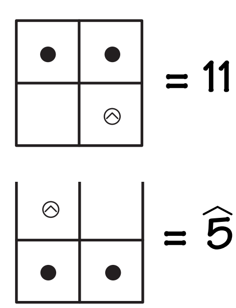


T: ***Let's call the secret number Boris. Boris can be put on the ones board of the Minicomputer using exactly these three checkers. The three checkers must all be on different squares.***

Invite students to put numbers on the Minicomputer using just the three given checkers, and ask them to identify which numbers they are going to put on before doing so. Call on other students to check which numbers are displayed. For example:



After two or three possibilities for Boris have been given, ask,



T: ***What is the greatest number Boris could be?***

S: ***11.***

T: ***What is the least number Boris could be?***

S: ***$\hat{5}$.***

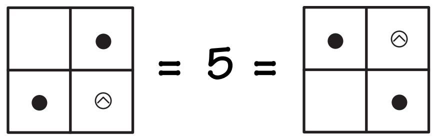
T: ***Could Boris be any integer from $\hat{5}$ to 11?*** (Point out where these numbers are on the number line.)

S: ***No, Boris cannot be 0.***

Do not be concerned if students cannot answer this last question. It is intended only to get them thinking about numbers from $\hat{5}$ to 11.

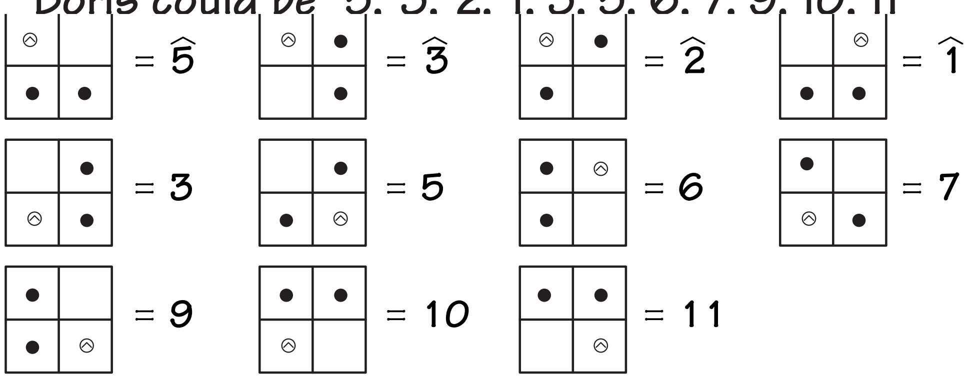
Instruct students to find all of the possibilities for Boris and record them on the Minicomputer record sheet (Blackline W3(a)). Tell them there are more Minicomputers than they need on the record sheet, and that they only need display a number once.

There are 11 possibilities (see below); however, students may announce that they have found more than this because they have duplicate configurations or because they have more than one configuration for the same number. For example:



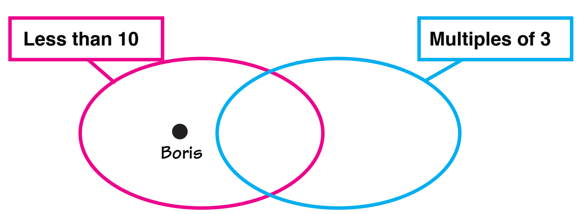
When many students have found all 11 possibilities, make a list on the board with the help of students. Be sure that at least one configuration is given for every possibility.

Boris could be $\hat{5}$. $\hat{3}$. $\hat{2}$. $\hat{1}$. 3. 5. 6. 7. 9. 10. 11



Clue 2

Draw this string picture on the board, and ask students to copy it on their papers.



T: ***Boris belongs here in this string picture. What does this clue tell you about Boris?***

S: ***Boris is less than 10.***

S: ***Boris is not a multiple of 3.***

T: ***Which of these numbers*** (point to the list) ***could Boris be?***

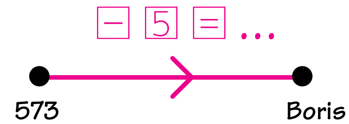
Direct the class to check each number in the list by locating it in the string picture. Erase numbers from the list that are not possibilities for Boris. There are five remaining possibilities for Boris.

Boris could be: $\hat{5}$, $\hat{2}$, $\hat{1}$, 5, 7

Clue 3

Allow students to use calculators while working with this clue.

Draw this arrow on the board as you announce the clue.



T: ***Put 573 on the display of your calculator. Press [−] [5] [=] [=] [=] and so on. Boris is a number that will eventually appear on the display.***

Let students experiment until they find the secret number. Tell them not to announce it aloud, but rather to write it on their papers or whisper it to you. When most everyone has found that Boris is $\hat{2}$, let students describe how they found Boris. Encourage shortcut methods.

S: ***I just kept pressing [=], and I found that $\hat{2}$ appeared but none of the other possibilities for Boris.***

S: ***I pressed [=] for awhile and saw that the numbers on the display always ended in 3 or 8. So I knew that 5 and 7 would not appear. I put 3 on the calculator and pressed [−] [5] [=] and found $\hat{2}$. The next number would be $\hat{7}$, so I knew $\hat{1}$ and $\hat{5}$ would not appear.***

S: ***$\hat{2}$ is 575 less than 573, so you can get to $\hat{2}$ by subtracting fives from 573.***

Writing Activity

Suggest students try to write a detective story that has two or three clues.

Home Activity

This is a good time to send a letter to parents/guardians about detective stories. Blackline W3(b) has a sample letter.

Capsule Lesson Summary

Count the number of small squares in a four by four square that have either 2, 1, or 0 sides in the boundary of the larger square. Begin the workbook *Collection of Problems #2*. (This is the first of two lessons using this workbook.)

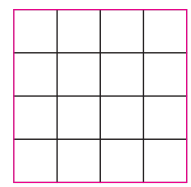
Materials

Teacher
- Colored chalk

Student
- *Collection of Problems #2* Workbook
- Colored pencils, pens, or crayons
- Metric ruler

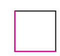
Description of Lesson

Draw this figure from page 9 of the workbook on the board.



T: ***Imagine cutting this square*** (trace the black lines) ***into small squares. How many small squares are there?*** (16)

Draw this figure on the board.

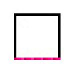


T: ***How many pieces like this would we have?***

S: ***Four.***

Ask a student to mark the four pieces in your drawing with an **x** (see below).

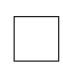
Draw this figure on the board and ask how many of the pieces would be like it.



S: ***Eight.***

Invite a student to mark the eight pieces with an **o**.

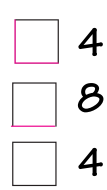
Draw this figure on the board and ask how many pieces would be like it.



S: ***Four.***

Ask a student to mark the four pieces with a ✓.

Tell the students that this activity is part of a problem that occurs in the workbook.



x	o	o	x
o	✓	✓	o
o	✓	✓	o
x	o	o	x

Distribute copies of the workbook *Collection of Problems #2* and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of the lesson, collect the workbooks for your review. They will be used again in Lesson W5.

Capsule Lesson Summary

Solve a calculator puzzle in which 100 is put on the calculator using only a limited set of keys. Continue individual work in the workbook *Collection of Problems #2*. (This is the second of two lessons using this workbook.)

Materials

Teacher
- None

Student
- *Collection of Problems #2* Workbook
- Colored pencils, pens, or crayons
- Metric ruler

Description of Lesson

Arrange for every student to have access to a calculator. You may like to let students work with a partner.

T: ***Today I have a calculator puzzle for you. The puzzle requires you only use a few of the keys on the calculator.***

Write these key symbols on the board: [3], [7], [+], [−], [×], [÷], [=].

T: ***You may use just these keys*** ([3], [7], [+], [−], [×], [÷], [=])***, but you may use them in any way you like. Start with 0 on your calculator display, and then try to put 100 on the display.***

You may need to remind students that they may use only the keys in the list on the board. To help them remember, suggest students record the sequence of keys they press to get 100.

When many students have found at least one solution, begin to record some of their suggestions on the board. For example:

100: [7][7] [+] [3] [+] [7] [+] [3] [+] [7] [+] [3] [=]
[3][7] [+] [7][3] [−] [7] [−] [3] [=]
[3][3] [×] [3] [+] [7] [−] [3] [=] [=]
[3] [×] [7] [=] [=] [+] [3][7] [=]
[7] [−] [3] [×] [7] [=] [=] [−] [3] [=] [=] [=] [=]
[3][7] [+] [7] [=] [=] [=] [=] [=] [=] [=] [=] [=]
[7] [+] [3] [×] [=]
[3][3] [+] [7][3] [−] [3] [=] [=]

Note: Some of these solutions depend on special features of the calculator. You may want to read “Role and Use of Calculators” in Section One: Notes to the Teacher to learn more about the features used here.

Try to get a variety of solutions. Sometimes one student's solution will result in several similar solutions from other students. One way to put a little additional challenge in this activity is to announce that it costs a dollar (penny) for each key they press. Challenge students to make their solutions as cheap as possible.

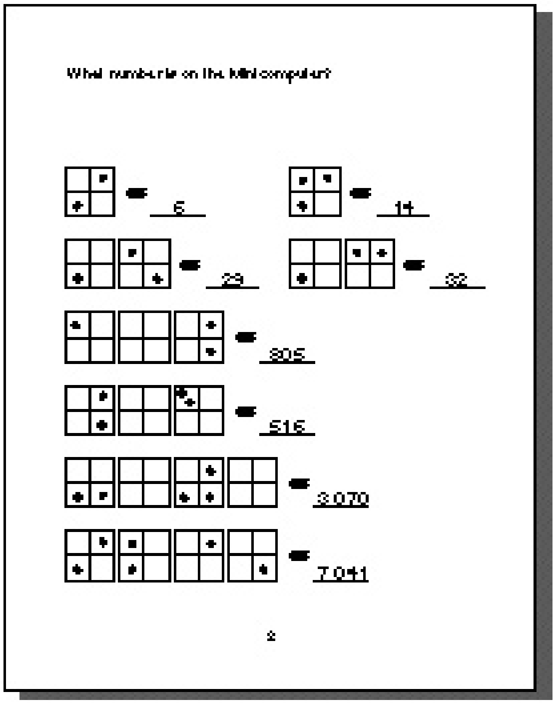
Distribute students' copies of the workbook *Collection of Problems #2*. Ask students first to correct or complete pages from their previous week's work in this workbook.

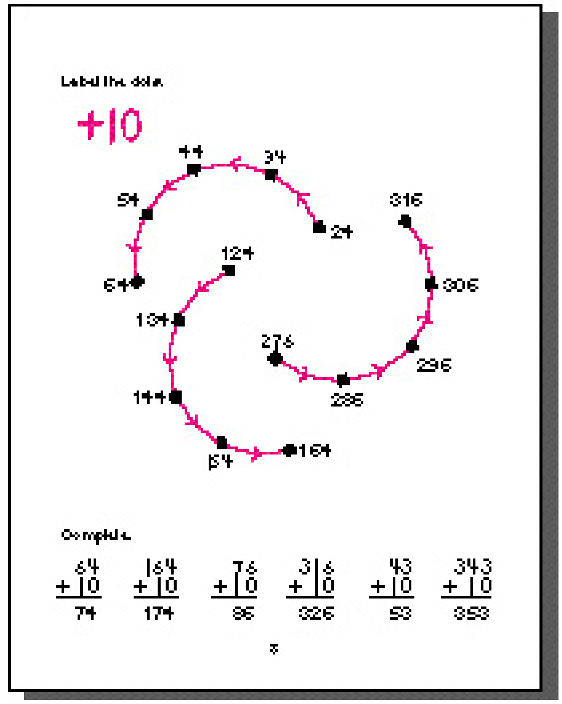
Your review of the workbook may indicate that a short collective discussion about a particular page is needed.

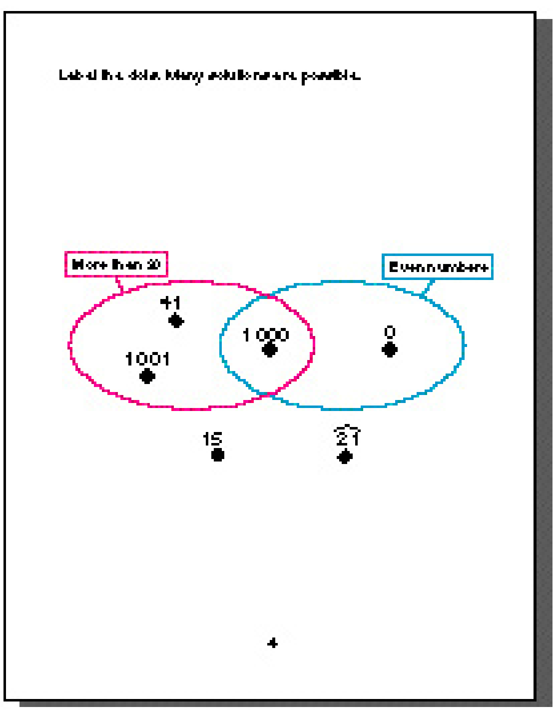
At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in their workbooks at school during a study time or at home as an assignment.

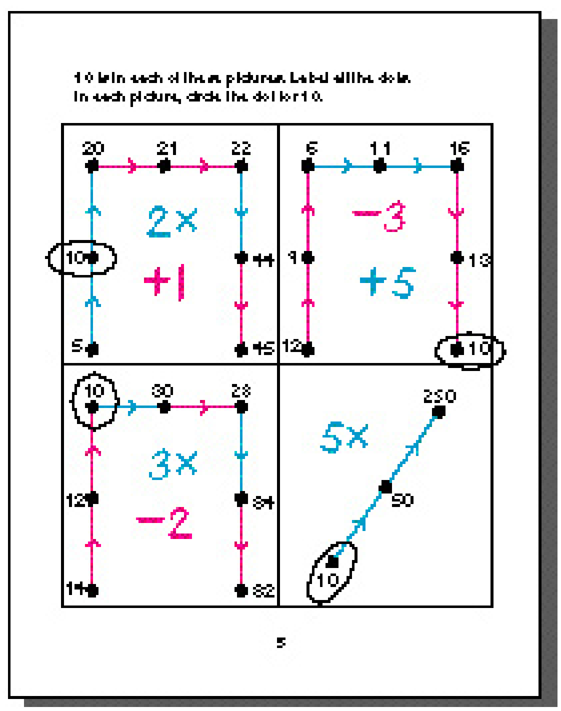
Assessment Activity

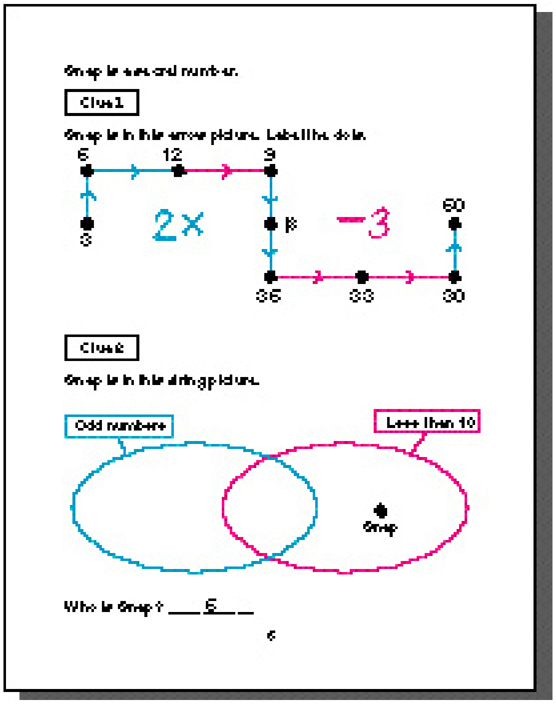
An individual student progress record for the workbook is available on Blackline W5. You may like to use this form to monitor student work.

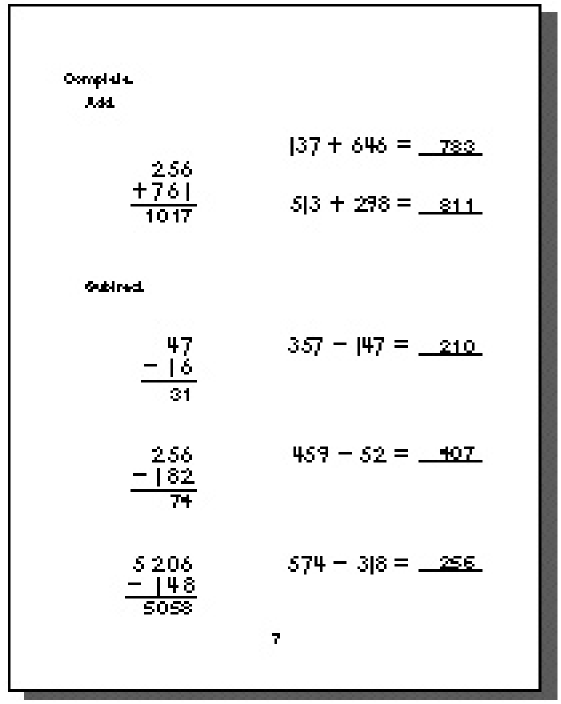

What numbers are on the Minicomputer?
6
14
29
32
305
516
3 070
7 041
2

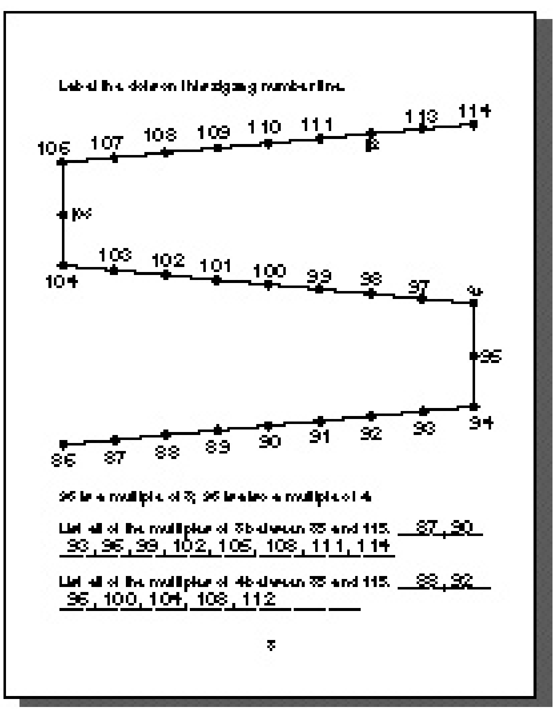

Label the dots.
+10
44
34
54
24
316
124
64
306
134
276
296
144
286
164
154
Complete.
64 + 10 = 74
164 + 10 = 174
76 + 10 = 86
316 + 10 = 326
43 + 10 = 53
343 + 10 = 353
3

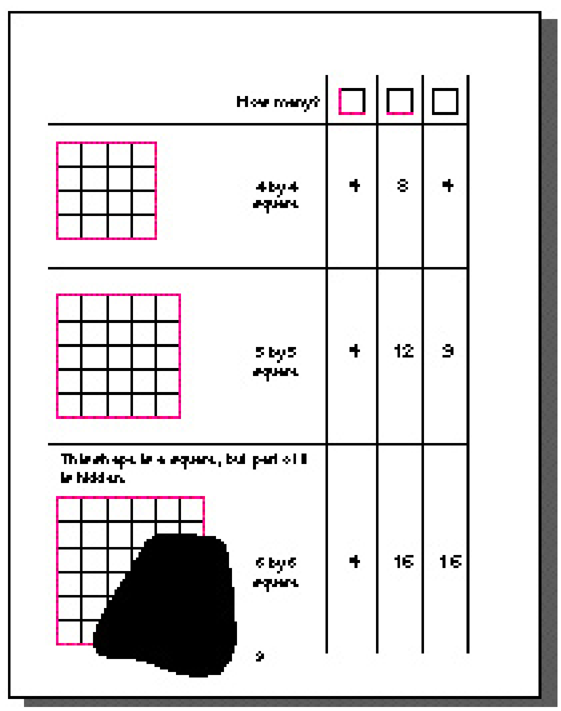

Label the dots. Many solutions are possible.
More than 20
Even numbers
41
1001
1 000
0
15
27
4


10 is in each of these pictures. Label all the dots.
In each picture, circle the dot for 10.
20
21
22
2x
+1
10
44
5
45
6
11
16
−3
+5
9
13
12
10
10
30
28
3x
−2
12
84
14
82
250
5x
50
10
5

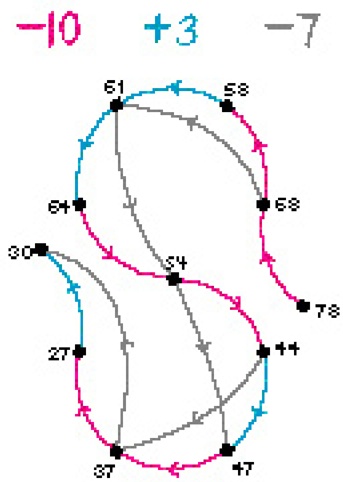

Snap is a secret number.
Clue 1
Snap is in this arrow picture. Label the dots.
6
12
9
3
2x
−3
60
36
33
30
Clue 2
Snap is in this string picture.
Odd numbers
Less than 10
Snap
Who is Snap? 5
6


Complete.
Add.
256
+761
1017
137 + 646 = 783
513 + 298 = 811
Subtract.
47
− 16
31
357 − 147 = 210
256
− 182
74
459 − 52 = 407
5 206
− 148
5058
574 − 318 = 256
7


Label the dots on this zigzag number line.
106
107
108
109
110
111
113
114
103
102
101
100
99
98
97
104
95
86
87
88
89
90
91
92
93
94
8


How many?
4 by 4 square
4
8
4
5 by 5 square
4
12
9
This shape is a square, but part of it is hidden.
6 by 6 square
4
16
16
9

Label the dots. Draw as many −7 arrows as possible in this picture.



10

Write at least five more number sentences about 60. Some examples are given here.

$\frac{1}{2} \times 120 = 60$

$60 \times 1 = 60$

$120 \div 2 = 60$

$57 + 3 = 60$

60

$2 \times 30 = 60$

$91 - 31 = 60$

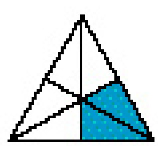
$(2 \times 25) + (2 \times 5) = 60$

$59 + 1 = 60$

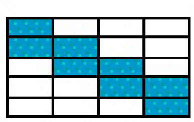
Many solutions are possible.

11

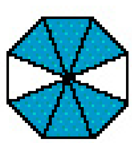
What fractional part of each shape is colored blue?



$\frac{2}{6}$ or $\frac{1}{3}$

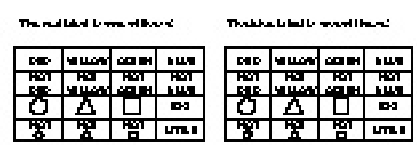


$\frac{8}{20}$ or $\frac{4}{10}$ or $\frac{2}{5}$

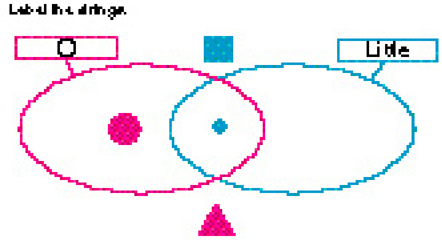


$\frac{6}{8}$ or $\frac{3}{4}$

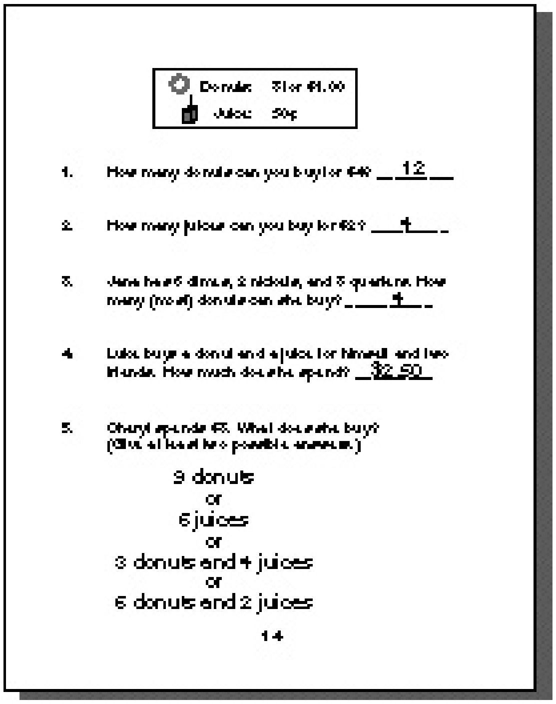
12



Label the strings.



13

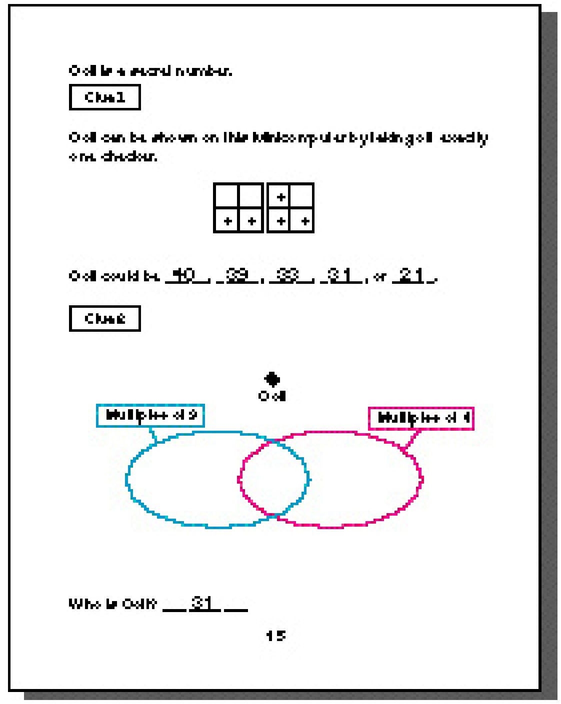


Donuts 3 for $1.00
Juice 50¢

1. How many donuts can you buy for $4? 12
2. How many juices can you buy for $2? 4
3. Jane has 6 dimes, 2 nickels, and 3 quarters. How many (most) donuts can she buy? 4
4. Luke buys a donut and a juice for himself and two friends. How much does he spend? $2.50
5. Cheryl spends $3. What does she buy? (Give at least two possible answers.)

9 donuts
or
6 juices
or
3 donuts and 4 juices
or
6 donuts and 2 juices

14



Odd is a secret number.

Clue 1

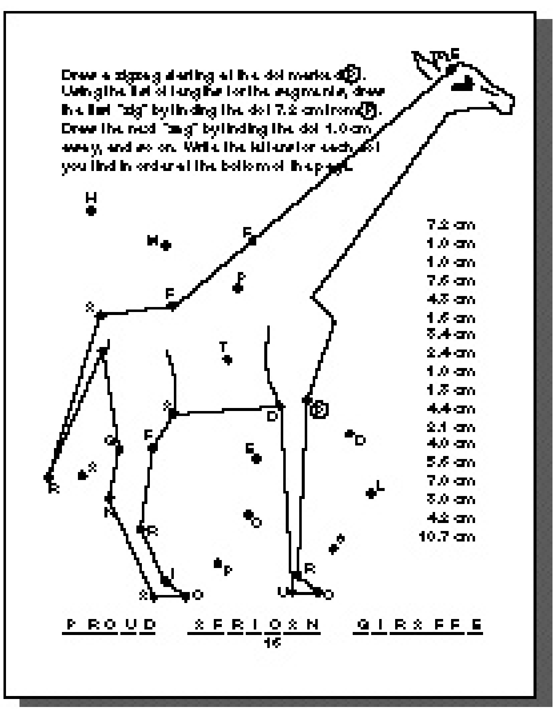
Odd can be shown on this Minicomputer by taking off exactly one checker.

Odd could be 40, 39, 33, 31, or 21.

Clue 2

Who is Odd? 31

15

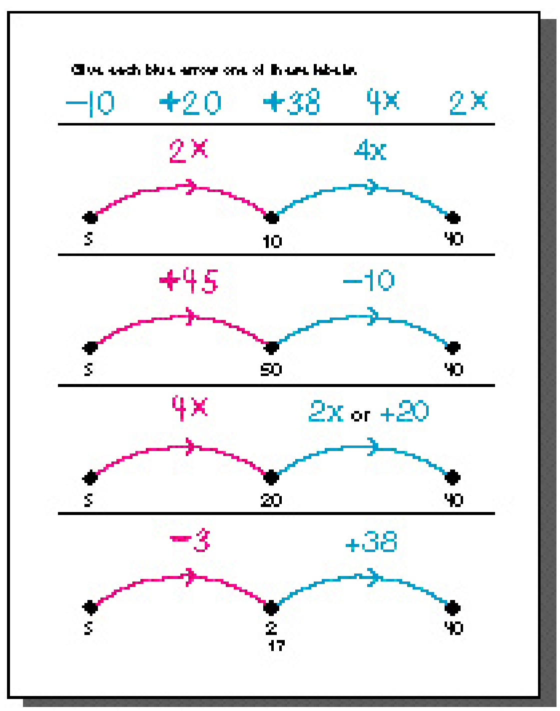


Draw a zigzag starting at the dot marked (S). Using the list of lengths for the segments, draw the first "zig" by finding the dot 7.2 cm from (S). Draw the next "zag" by finding the dot 1.0 cm away, and so on. Write the letter for each dot you find in order at the bottom of the page.

7.2 cm
1.0 cm
1.0 cm
7.6 cm
4.3 cm
1.6 cm
3.4 cm
2.4 cm
1.0 cm
1.3 cm
4.4 cm
2.1 cm
4.0 cm
5.6 cm
7.0 cm
3.0 cm
4.2 cm
10.7 cm

P R O U D   A F R I C A N   G I R A F F E

16



Give each blue arrow one of these labels.

−10   +20   +38   4×   2×

17

Use the true subtraction statement in the box to help complete the other subtraction problems.

57 − 19 = 38

67 − 29 = 38	58 − 20 = 38
157 − 19 = 138	50 − 12 = 38
47 − 19 = 28	57 − 24 = 33
47 − 9 = 38	57 − 14 = 43

18

Sports Shorts

Solve each problem. Show your work.

1. In a basketball game, Patton School beat Lawton School 63-57. Patton's best two players are Angela and Marcel. In this game, Angela scored 29 points and Marcel scored 17 points. How many of Patton's points were not scored by Angela or Marcel?
 __ 17 __ points

$$\begin{array}{r} 29 \\ +\ 17 \\ \hline 46 \end{array}$$ points scored by Angela and Marcel

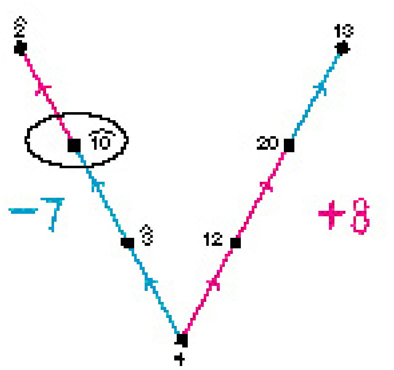
$$\begin{array}{r} 63 \\ -\ 46 \\ \hline 17 \end{array}$$ points not scored by Angela and Marcel

2. At the end of the first half of a football game, the score was Patton 21, Lawton 7. However, in the second half, Lawton scored 24 points while Patton scored only 17 points.
 Patton 21 + 17 = 38
 Lawton 7 + 24 = 31
 Which team won the game? Patton
 What was the final score? Patton 38, Lawton 31
 How many more points did the winning team score than the losing team? __ 7 __ points

19

20 is the greatest number in this picture. Where is 20? Label the dot.

Label the other dots.



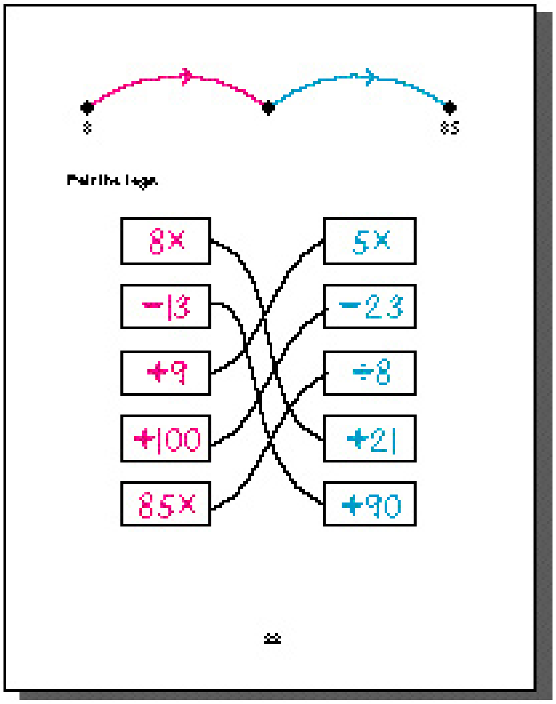
Circle the dot for the least number in this picture.

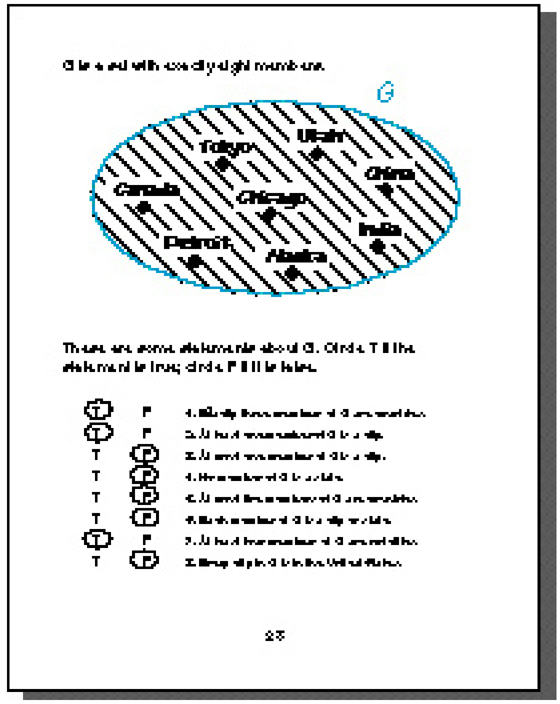
20

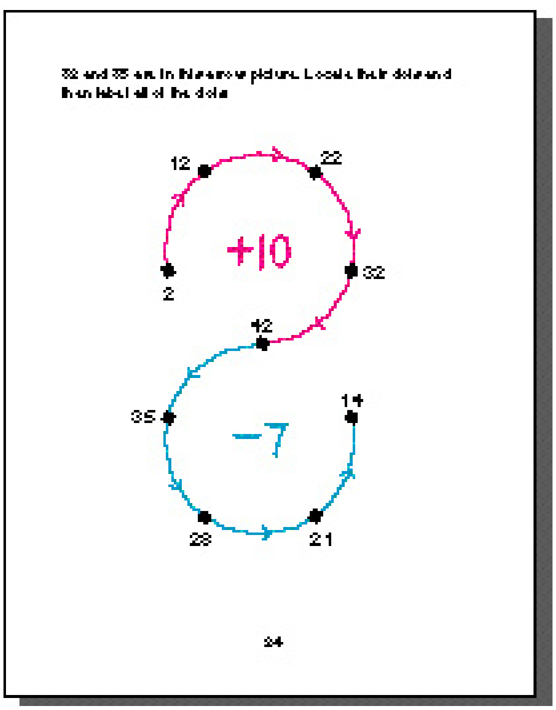
The air distance from Chicago to Denver is 1 450 km. Match the pairs of cities with the distance between them. One is done for you.

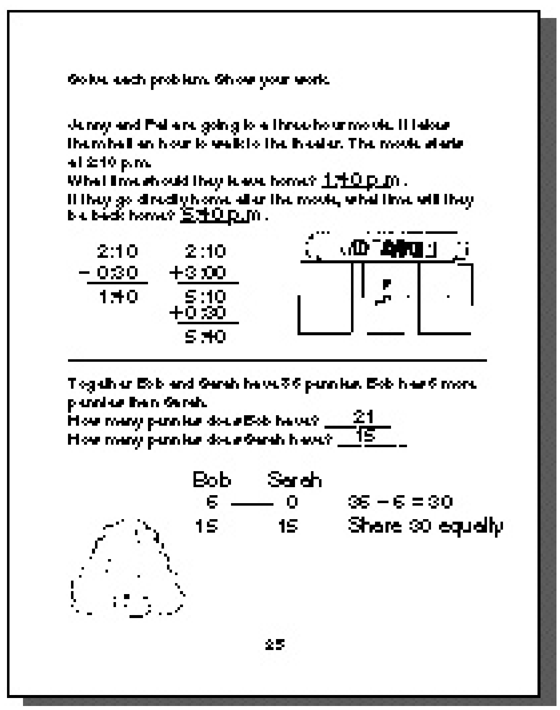
- Chicago – Denver — 425 km
- Chicago – Minneapolis — 530 km
- Chicago – Boston — 1 330 km
- Chicago – St. Louis — 1 450 km
- Chicago – Houston — 1 540 km
- Chicago – San Francisco — 2 300 km
- Chicago – Seattle — 2 970 km

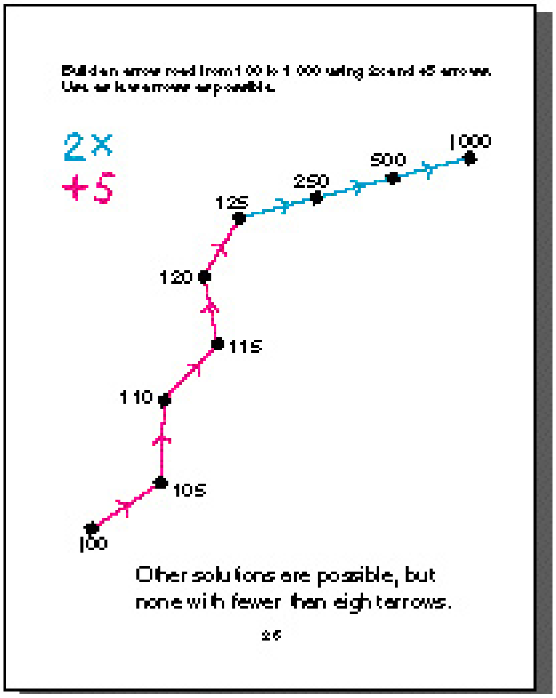
21


8
85
8×
−13
+9
+100
85×
5×
−23
÷8
+21
+90
22


G
Tokyo
Utah
Canada
China
Chicago
India
Detroit
Alaska
23

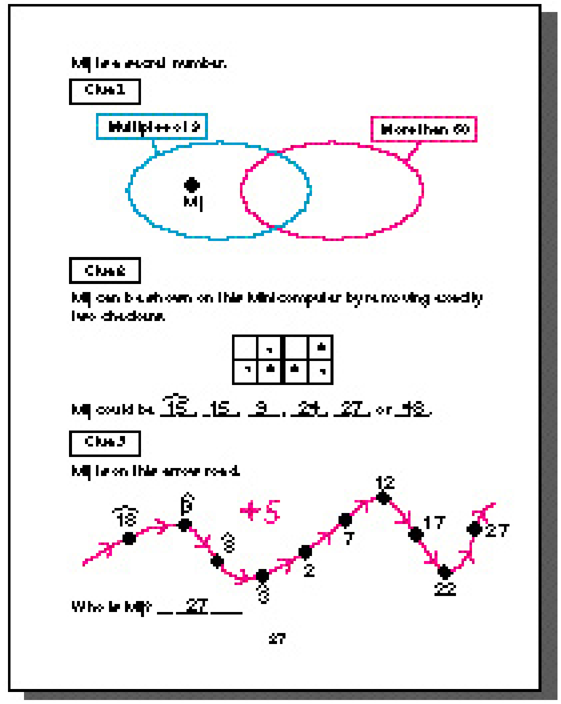

32 and 35 are in this arrow picture. Locate their dots and then label all of the dots.
12
22
+10
2
32
42
35
−7
14
28
21
24


Solve each problem. Show your work.
Jenny and Pat are going to a three-hour movie. It takes them half an hour to walk to the theater. The movie starts at 2:10 p.m.
What time should they leave home? 1:40 p.m.
If they go directly home after the movie, what time will they be back home? 5:40 p.m.
2:10 − 0:30 = 1:40
2:10 + 3:00 = 5:10 + 0:30 = 5:40
Together Bob and Sarah have 36 pennies. Bob has 6 more pennies than Sarah.
How many pennies does Bob have? 21
How many pennies does Sarah have? 15
Bob Sarah
6 0
15 15
36 − 6 = 30
Share 30 equally
25


Build an arrow road from 100 to 1 000 using 2x and +5 arrows. Use as few arrows as possible.

Other solutions are possible, but none with fewer than eight arrows.

26


MJ is a secret number.

Clue 1

Clue 2

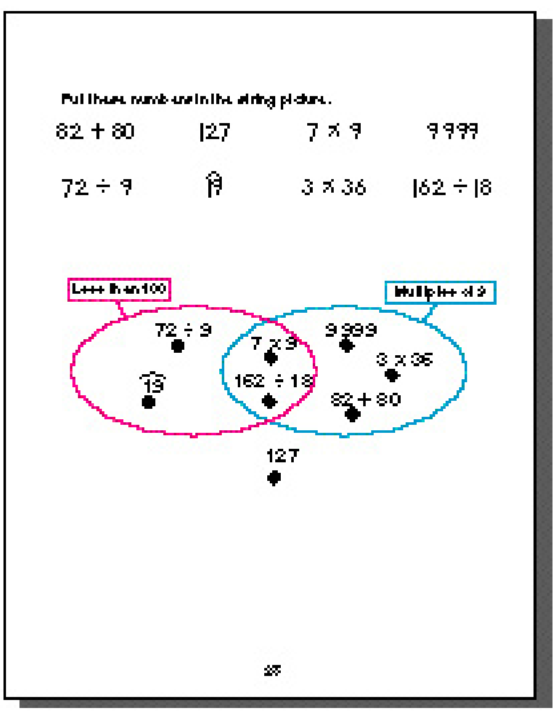
MJ can be shown on this Minicomputer by removing exactly two checkers.

Clue 3

MJ is on this arrow road.

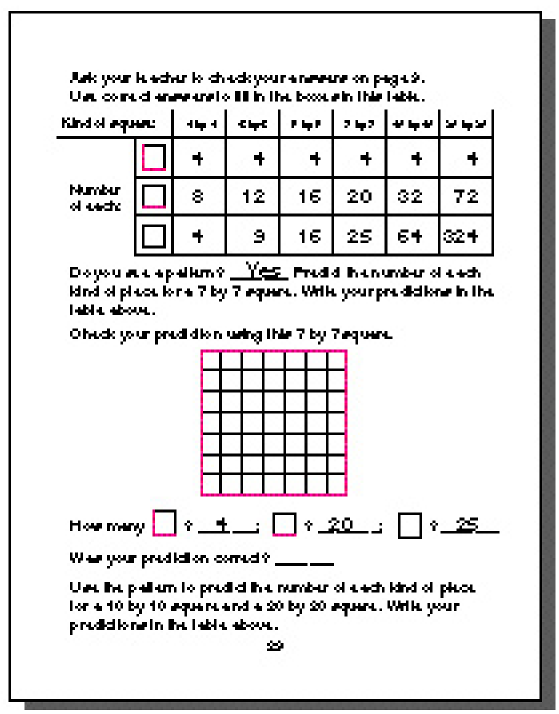
Who is MJ? 27

27


Put these numbers in the string picture.

82 + 80	127	7 × 9	9 999
72 ÷ 9	19	3 × 36	162 ÷ 18

28


Ask your teacher to check your answers on page 9. Use correct answers to fill in the boxes in this table.

Kind of square:		4 by 4	5 by 5	6 by 6	7 by 7	10 by 10	20 by 20
Number of each	□	4	4	4	4	4	4
	□	8	12	16	20	32	72
	□	4	9	16	25	64	324

Do you see a pattern? Yes. Predict the number of each kind of piece for a 7 by 7 square. Write your predictions in the table above.

Check your prediction using this 7 by 7 square.

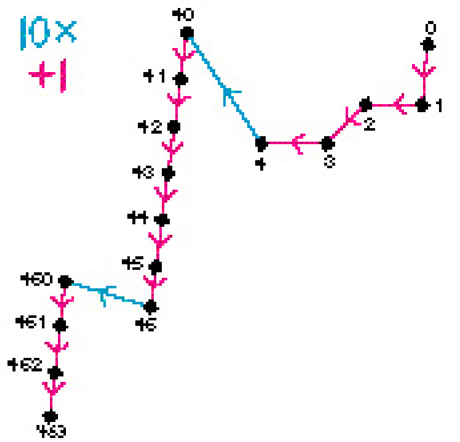
How many □ ? 4 ; □ ? 20 ; □ ? 25

Was your prediction correct? ____

Use the pattern to predict the number of each kind of piece for a 10 by 10 square and a 20 by 20 square. Write your predictions in the table above.

29

Build an arrow road from 0 to 463 using 10x and +1 arrows. Use fewer than 20 arrows.

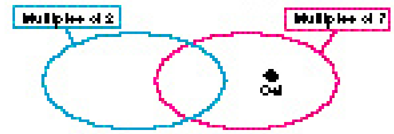


30

Oal and Yel are secret whole numbers.

Clue 1

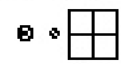
Oal is less than 100 and is in this string picture.



Oal could be 7, 21, 35, 49, 63, 77, or 91.

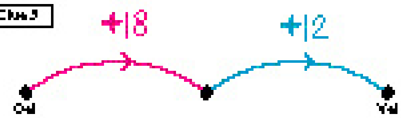
Clue 2

Yel is greater than 55 and can be put on this Minicomputer with exactly these two checkers.



Yel could be 79, 78, 75, 72, 39, 38, or 35.

Clue 3



Who is Oal? 49 Who is Yel? 79

31

20 35 37

59 91 99 252

$\widehat{29}$ $\widehat{12}$ $\widehat{8}$

A name for each of the above numbers can be written following these rules:

- Use each of these digits exactly once: 4, 7, 9
- Use any of these symbols as often as you wish: +, −, x, (,)

Write names for all of the numbers above. One is done for you.

20 = 9 + (4 + 7)	99 = (4 + 7) x 9
35 = (9 − 4) x 7	252 = 4 x 7 x 9
37 = (4 x 7) + 9	$\widehat{29}$ = 7 − (4 x 9)
59 = (7 x 9) − 4	$\widehat{12}$ = (4 − 7) − 9
91 = (9 + 4) x 7	$\widehat{8}$ = (7 − 9) x 4

32

Capsule Lesson Summary

Investigate two arrow pictures and find that any number can belong in one of them and no number can belong in the other. (This is the fist of two lessons using the story-workbook *Halloween Puzzles*.)

Materials

Teacher		Student	
	• *Halloween Puzzles* Story-Workbook • Colored chalk		• *Halloween Puzzles* Story-Workbook

Description of Lesson

Distribute copies of the story-workbook *Halloween Puzzles*.

Pages 2–7

Copy the picture from page 4 onto the board. Read pages 3 and 5 together. After reading the third paragraph on page 5, invite a student to come to the board and point to where 2, 8, 14, 20, and 11 would be in the picture.

Instruct students to fill in the blanks on page 5. You may need to warn them that a blue arrow subtracts 9, not adds 9. Ask students who finish quickly to try other numbers under the sheet. Check the answers on page 5 collectively.

Invite a student to read page 7 aloud. In the picture on the board, replace the ghost with a box (see the picture on page 8 of the story-workbook).

Pages 8–12

Invite a student to read page 9 aloud. Let students have a minute or two to record in their books which numbers play in the picture if 13 jumps into the box.

Ask everyone to think of a number to put in the box. Starting at the dot next to the box, trace the arrows in the picture as you ask the students to do +6 and –9 calculations using their particular starting numbers.

T: ***What number did you start with in the box?***

S: ***10.***

T: ***And what number did you end with?***

S: ***10.***

T: ***Did each of you end with the same number that you started with in the box?***

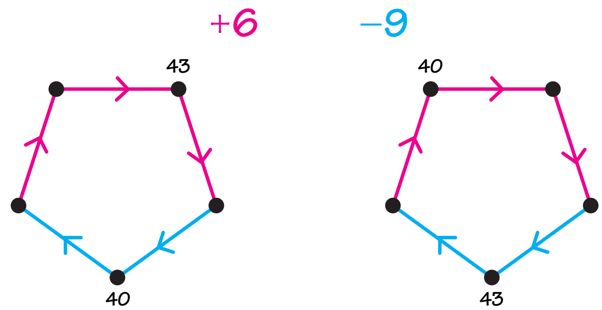
Check the calculations of students who say they did not start and end with the same number, and help them find their calculation errors.

T: ***Why do you think that we always return to our starting number in this picture?***

S: ***Because we add three 6's, which is the same as adding 18. Then we subtract two 9's, which is the same as subtracting 18. We subtract the same amount as we add.***

Do not insist that students make this precise observation at this time. The situation will be examined again in the second *Halloween Puzzles* lesson.

Invite students to read pages 10, 11, and 12 aloud. On page 12, instruct students to find where 40 and 43 can be in the picture. There are two solutions; your class should find both of them.



Pages 13–15

Read page 13 aloud. Copy the picture on page 14 onto the board.

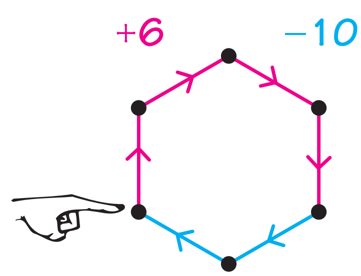
Invite a student to read page 15 aloud. On page 14 of the story-workbook, ask students to point to the dot where 0 is going to play. Quickly check that they are pointing to the correct dot, and then label this dot in the picture on the board.

T: ***If 0 could play here, which other numbers would play with 0? What is wrong? Write your answers on page 15.***

Allow several minutes for students to complete page 15. Then let students share their answers. They should notice that as you follow the arrows from 0, the last blue arrow does not end at 0, but instead ends at 4. Begin a chart on the board.

Starting Number	Ending Number †
0	4

T: ***0 can't play here. Do you think there is a number that could play here?***

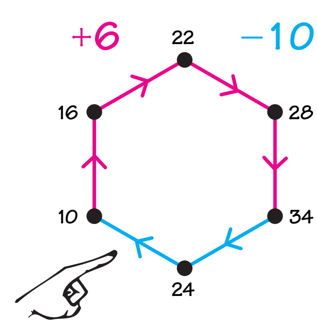


Let students make suggestions.

S: ***Maybe 10 could play there.***

T: ***Let's check.***

With the class's help, label the lower left dot 10 and then label the other dots in the order that they occur as you follow the arrows.



T: ***Does a –10 arrow start at 24 and end at 10?***

S: ***No, 24 – 10 = 14.***

†In this chart, pair numbers so that starting with the first number at the lower left dot you end with the second number at this same dot.

T: ***So if we start here with 10*** (point to the lower left dot)***, we end with 14 here.***

Starting Number	Ending Number
0	4
10	14

Record the information in the chart.

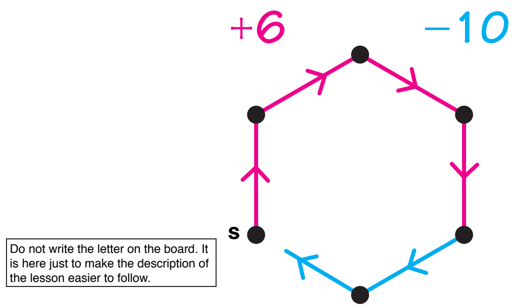
Erase the dot labels and check other suggestions. Perhaps your chart will look similar to this one.

T: ***What do you notice about this situation?***

S: ***We always end with a number 4 greater than the one we start with in the picture.***

T: ***So it appears that this blue arrow*** (break the cycle of arrows as in the picture below) ***…***

Starting Number	Ending Number
0	4
10	14
15	19
8	12
4	8
$\hat{4}$	0



…cannot end at this dot (point to **s**)***. Let's find out what the numbers do with the picture.***

Pages 16 and 17

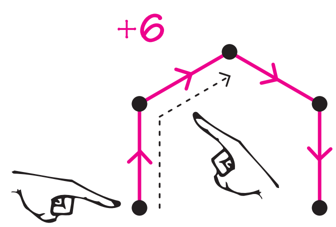
Invite students to read pages 16 and 17 aloud. Solicit suggestions for what the green arrow could be for in the picture on page 17. Then systematically lead the class to find that it could be for +24 as in the dialogue that follows. First, erase the blue arrows in the picture on the board.

T: ***How much more is this number*** (top dot) ***than this number*** (lower left dot)***?***

S: ***12 more.***

T: ***What is +6 followed by +6?***

S: ***+12.***



Note: For additional manipulative help, you can pose the question, "If you added six counters to a bag and then again added six counters, how many more counters would you have added to the bag?"

Trace the first three arrows in succession as you ask,

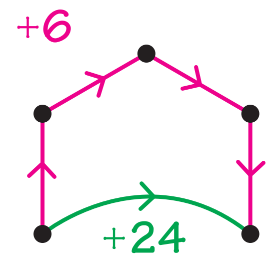
T: ***What is +6 followed by +6 followed by +6?***

S: ***+18.***

Trace the four arrows in succession as you ask,

T: ***What is +6 followed by +6 followed by +6 followed by +6?***

S: ***+24.***

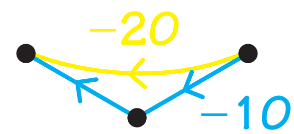


Draw a green arrow from the starting dot to the ending dot of the red arrow road. Label it +24.

Draw two blue arrows for –10 in a road.

T: ***The numbers also looked at the blue arrows. What is –10 followed by –10?***

S: ***–20.***



Draw a yellow arrow connecting the starting and ending dots of the blue arrow road. Label it –20.

Pages 18–22

Invite students to read pages 18, 19, 20, and 21 aloud. Instruct students to write their comments in the boxed space on page 21. Then let the class discuss the problem with the picture.

S: ***–20 is not the return (opposite) of +24.***

S: ***If you add 24 to a number and then subtract 20 from the result, you do not get the starting number. You always end with a number that is 4 more than the starting number, because +24 followed by –20 is +4.***

S: ***Maybe the blue arrows were incorrectly labeled. They should be labeled –12.***

S: ***Maybe the red arrows should be +5 arrows rather than +6 arrows.***

Ask a student to read page 22 aloud.

You may like to return to page 14 and discuss how to change the picture so that it is not a "witch picture." Changing the blue arrows to –12 or the red arrows to +5 are both good suggestions. There are, of course, many other ways to change the picture such as changing both kinds of arrows. Collect the story-workbooks and have them ready for use in Lesson W7.

Capsule Lesson Summary

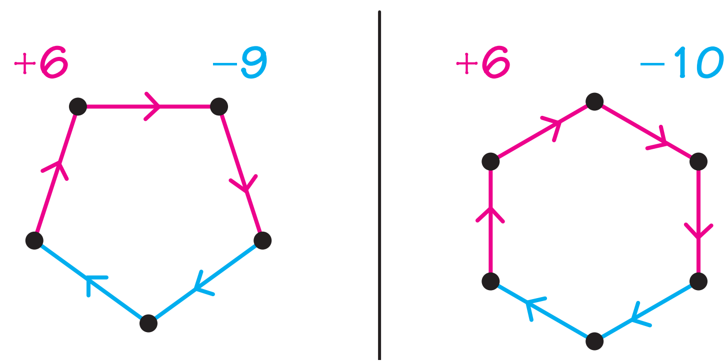
Review the first part of the story in *Halloween Puzzles*. Find labels for arrows in a picture so that any number can play in it. Investigate an arrow picture in which there is only one possible labeling of the dots. (This is the second of two lessons using the story-workbook *Halloween Puzzles*.)

Materials

Teacher		Student	
	• *Halloween Puzzles* Story-Workbook • Colored chalk		• *Halloween Puzzles* Story-Workbook • Worksheets W7* and **

Description of Lesson

Draw the pictures from pages 12 and 14 of *Halloween Puzzles* on the board.

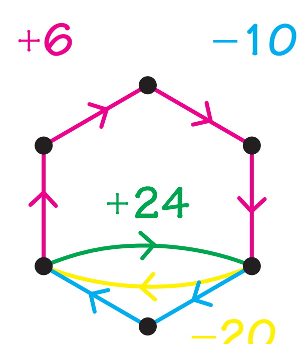


With the class, recall the story of *Halloween Puzzles*. When someone mentions a "witch picture," ask which picture on the board is a witch picture and why.

S: ***The one on the right is a witch picture. The four +6 arrows add 24 altogether, but the two –10 arrows subtract 20. So the picture does not work for any numbers.***

Draw these green and yellow arrows in the picture on the right.

T: ***+6 followed by +6 followed by +6 followed by +6 is +24. –10 followed by –10 is –20. Is there any number that adding 24 to it*** (trace the green arrow) ***and then subtracting 20 from the result*** (trace the yellow arrow) ***gets you back to the starting number?***



S: ***No, you would have to subtract 24, not 20.***

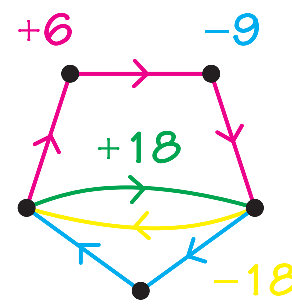
T: ***So no number can play in this picture. What about the "ghost" picture*** (point to the picture on the left)***?***

S: ***Any number can play in that picture.***

T: ***Why?***

S: ***The three +6 arrows add 18 and the two –9 arrows subtract 18, so you start and end at the same number.***

As necessary, prompt your students to look at the composition of the red arrows and the composition of the blue arrows in the picture.

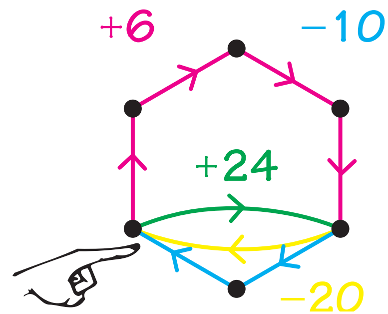


Draw these green and yellow arrows in the picture on the left.

Trace the appropriate arrows as you say,

T: ***+6 followed by +6 followed by +6 is +18. –9 followed by –9 is –18. If you add 18 to a number and then subtract 18 from the result, do you get back to the starting number no matter what that number is?***

S: ***Yes, +18 and –18 are opposites of each other.***



T: ***Is there any way we could change this picture*** (point to the picture on the right) ***so that it isn't a witch picture—so that the numbers could take turns playing in it?***

S: ***Maybe we could add more arrows of each color.***

S: ***We could add a dot and a –4 arrow here.***

T: ***But suppose we want to keep four red arrows and two blue arrows.***

S: ***We could make the red arrows be for +5.***

Change the label accordingly for the red arrows and then for the green arrow.

T (tracing the red arrows)**:** ***Let's check. What is +5 followed by +5 followed by +5 followed by +5?***

S: ***+20.***

Label the green arrow +20.

T: ***If you add 20 to a number and then subtract 20 from the result, do you get back to the starting number no matter what that number is?***

S: ***Yes, +20 and –20 are opposites of each other.***

T: ***So any number can play if the red arrows are for +5 and the blue arrows are for –10.***

Erase the labels for the green and yellow arrows and change the label for the red arrows back to +6.

T: ***Is there another change that we could make so that the numbers could take turns playing in this picture?***

S: ***Make the blue arrows be for –12.***

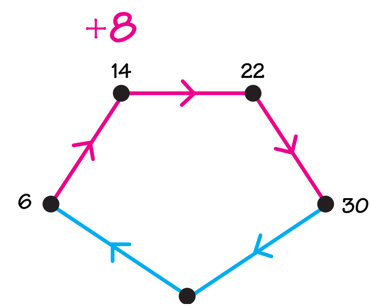
As before, check the composition of the red and blue arrows.

Distribute students' copies of *Halloween Puzzles*.

Pages 23–24

Instruct students to work on these pages individually or with a partner. Encourage students who have trouble getting started to try numbers in a picture. On page 23, for example, knowing that the red arrows are for +8 enables students to label four dots easily.

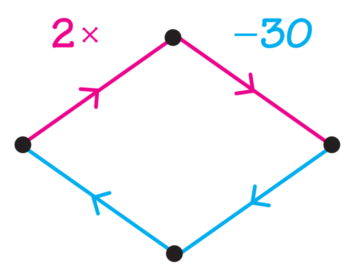
Suppose a student starts by putting 6 at the dot on the far left as illustrated here. The student should then try to figure out how to get from 30 back to 6 using two of the same kind of arrows.



Some students may not need to label dots and will solve the problem using composition. On page 23, the composition of three red arrows is +24, so the composition of two blue arrows must be –24. –12 followed by –12 is –24. On page 24, the composition of four blue arrows is +12, so the composition of three red arrows must be –12. The red arrows are for –4.

Pages 25–27

Invite students to read pages 25, 26, and 27 aloud. Draw the bat's picture on the board.



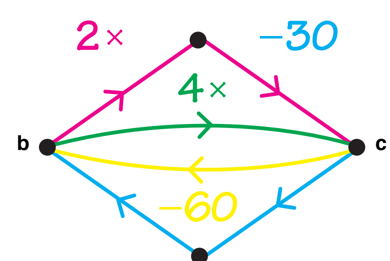
T (tracing the red arrows)**:** ***What is 2x followed by 2x?***

S: ***4x.***

T (tracing the blue arrows)**:** ***What is –30 followed by –30?***

S: ***–60.***

Draw these green and yellow arrows in the picture on the board.



T: ***If you multiply a number by 4 and then subtract 60 from the result, do you get back to the starting number no matter what that number is?***

S: ***No, I tried 5. 4 x 5 = 20 and 20 – 60 = $\widehat{40}$.***

S: ***Not always, because 4x and –60 are not opposites of each other.***

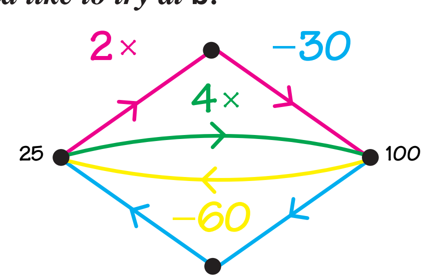
T: ***The bat did say that some numbers can play in this picture. Let's find out what the numbers do with the picture. Turn to page 28.***

Pages 28–29

Invite students to read pages 28 and 29 aloud.

T: ***10 cannot play at b. Is there another number that you would like to try at b?***

Check suggestions as in the following dialogue.



S: ***Let's try 25.***

T: ***What number is 4 x 25?***

S: ***100.***

T: ***What number is 100 – 60?***

S: ***40.***

T: ***So we don't get back to our starting number. 25 cannot play at* b.**

Allow several minutes for students to search for numbers that can play in the picture.

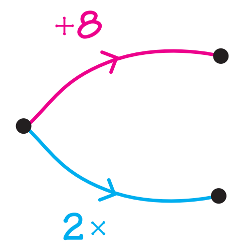
Pages 30 and 31

Invite students to read pages 30 and 31 aloud. Instruct students to complete page 31, and observe that 20 and 80 solve the bat's riddle.

Distribute copies of Worksheets W7* and **.

T: ***Let's look at the first problem on the * worksheet together. I'll show one way to think about it. Suppose that you go to a carnival. At this particular carnival, you get to play one of two games just before you leave. In one game, you have a chance to win eight more dollars. In the other game, you have a chance to double your winnings.***

Draw this arrow picture on the board.



T: ***Which of the two games would you choose to play?***

Let students discuss the situation. At some point, they should decide that their choice would depend upon their winnings so far.

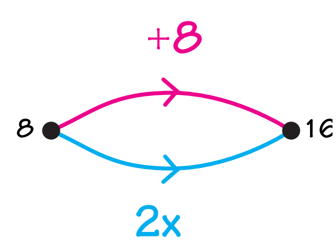
T: ***Let's make a chart. We'll use* w *for the amount of our winnings before playing the final game.***

w	2 × w	w + 8

Let students fill in several lines of the chart. Observe when a choice for **w** yields a big difference between 2 x **w** and **w** + 8, and when a choice yields very little difference.

T: ***Do you think that in some case it wouldn't matter which game you played—that 2 x* w *would equal* w *+ 8?***

Change the arrow picture.



Add suggestions to your chart until your class discovers that it would not matter which of the two games you played if your previous winnings were $8 because 2 x 8 = 8 + 8.

It's a number! But which number?

"If we knew, then we'd know who else can play this Halloween game," answer the numbers.

"If I were under the sheet," says 2, "Then 8, 14, 20, and 11 would play with me because 2 + 6 = 8; 8 + 6 = 14; 20 − 9 = 11; and 11 − 9 = 2."

"If I were under the sheet," says 10, "Then __16__, __22__, __28__, and __19__ would play with me."

"If I were under the sheet," says 15, "Then __21__, __27__, __33__, and __24__ would play with me."

"If I were under the sheet," says 37, "Then __43__, __49__, __55__, and __46__ would play with me."

"If I were under the sheet," says 125, "Then __131__, __137__, __143__, and __134__ would play with me."

5

"I'll jump into the box," says 0, "Then 6, 12, 18, and 9 play with me."

"I'll jump into the box," says 13, "Then __19__, __25__, __31__, and __22__ would play with me."

"Any one of us can jump into the box!" observe the numbers. "When one of us gets into the box, then we can figure out who else plays."

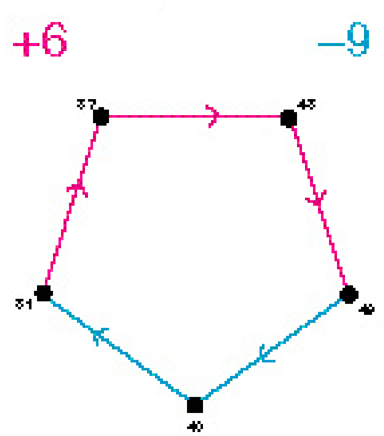
"We can take turns playing this game," the numbers agree.

9

"But I would like to play a game with 43."

"Can we play this game together?" asks 40.

If they can, show them in the picture below.



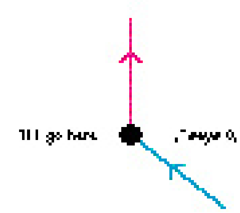
Another solution is possible.

12

The numbers gather around the picture and wonder if this is a new Halloween game for them to play.

"There are places for six of us in the picture," murmurs 0.

0 looks at the picture and becomes worried.



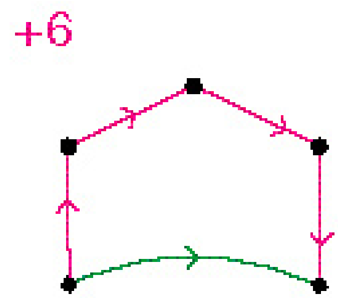
"Then who plays with me?"

Why is it strange? Write what you think below.

If this dot is for 0, then by following the arrows, we see that it also has to be for 4. But one dot cannot represent two different numbers at the same time.

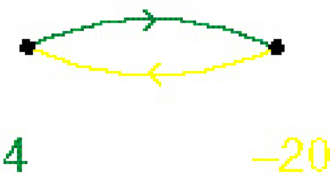
15

Looking at the red arrows, the numbers decide to add a green arrow.



What could the green arrow be for? +24

Calculate this part of the arrow picture.



"Look," says 0. "I have a feeling that the witch was fooling us."

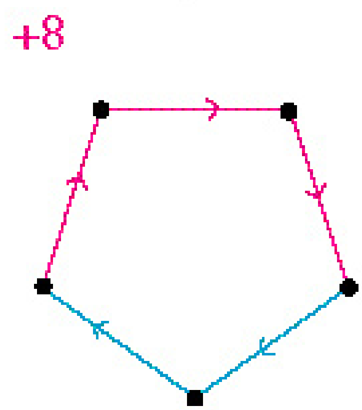
Use this space to explain why.

The return arrow for a +24 arrow is a –24 arrow, not a –20 arrow.

The numbers would like to take turns playing in this picture.

What could the blue arrows be for? –12

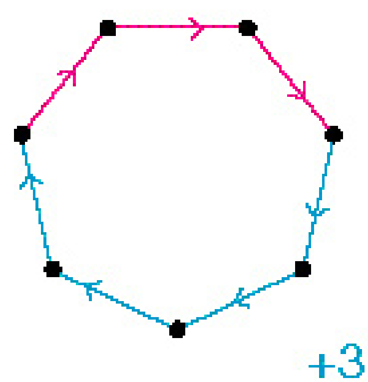
Be careful not to make a "witch" picture.

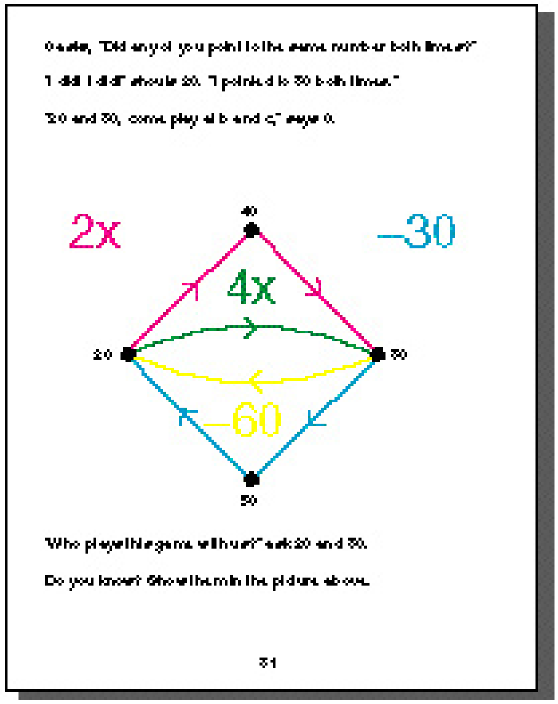


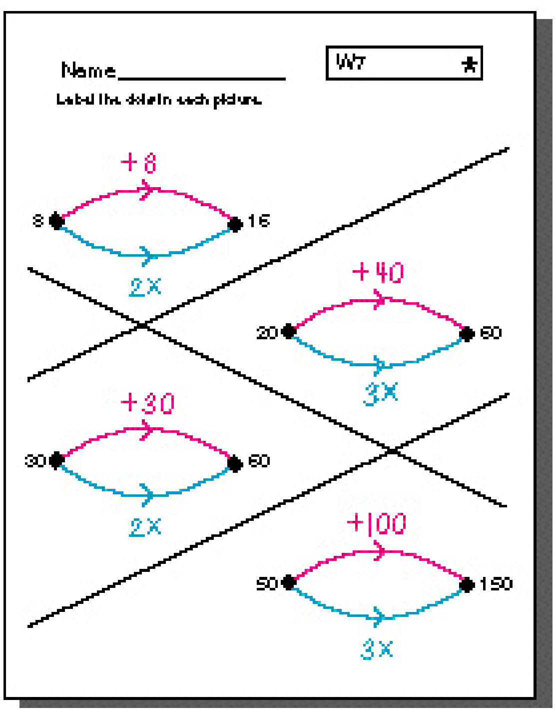
The numbers would like to take turns playing in this picture.

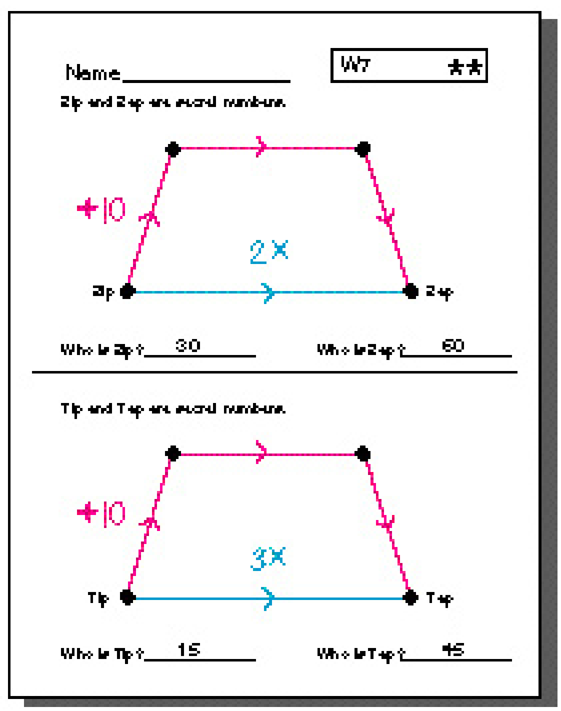
What could the red arrows be for? –4

Be careful not to make a "witch" picture.




O asks, "Did any of you point to the same number both times?"
"I did! I did!" shouts 20. "I pointed to 80 both times."
"20 and 80, come play a b and c," says O.
2x
40
−30
4x
20
80
−60
50
"Who played the game with us?" ask 20 and 80.
Do you know? Show them in the picture above.
31


Name
W7 ★
Label the dots in each picture.
+8
8
16
2X
+40
20
60
3X
+30
30
60
2X
+100
50
150
3X


Name
W7 ★★
Zip and Zap are secret numbers.
+10
2X
Zip
Zap
Who is Zip? 30
Who is Zap? 60
Tip and Tap are secret numbers.
+10
3X
Tip
Tap
Who is Tip? 15
Who is Tap? 45

Capsule Lesson Summary

Introduce a calculator game called Calculator Golf in which players go from a starting number to a target number using the operations +, –, x, or ÷, and one-digit positive integers. Begin the workbook *Collection of Problems #3*. (This is the first of two lessons using this workbook.)

Materials

Teacher
- Overhead calculator (optional)
- Colored chalk

Student
- Calculator
- Paper
- Colored pencils, pens, or crayons
- *Collection of Problems #3* Workbook
- Metric ruler

Description of Lesson

Exercise 1

Display an overhead calculator, if available, and provide each student or pair of students with a calculator.

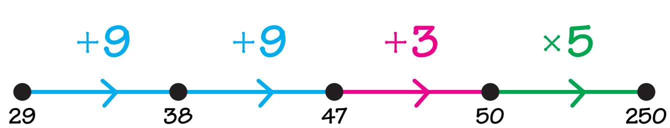
Ask students to recall some of what they remember about golf and the Minicomputer Golf game.

T: ***Today we are going to play a game called Calculator Golf. We start with a number on the display of the calculator and then set a goal.***

Draw two dots on the board. Label one of them 29 and the other 250.

T: ***We might start with 29*** (put 29 on the display) ***and make 250 be the goal. When you play this golf game, you can press any operation key (⊞, ⊟, ⊠, or ⊡) followed by a one-digit number (1 through 9), and then ⊟. Play continues until 250 is on the display.***

Invite students to take turns playing the game on the overhead calculator or a classroom calculator. Record the play in an arrow picture on the board. For example, the following picture is for a game with four turns (steps).



T: ***Put 29 on your display. You can add, subtract, multiply, or divide by any of the one-digit numbers 1 through 9. As in golf, try to reach 250 in fewer steps than we did here.***

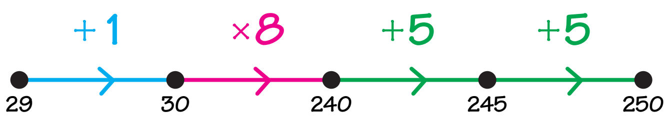
Suggest that students press ⊟ after pressing a number key so that they can see the result before deciding which operation to use next.

W8

Note: If your students have calculators, you need not require that they record their steps on paper. Some will want to keep track of their steps while others will find working with pencil and calculator simultaneously awkward and inhibiting.

Allow a few minutes for students to work on this problem before asking students to share solutions with the class. As a student describes a solution, draw the corresponding arrow road on the board. For example:

S: ***I pressed [+] [1] [=], then [×] [8] [=], and then [+] [5] [=] [=].***

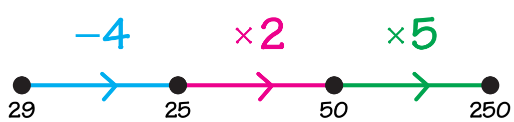


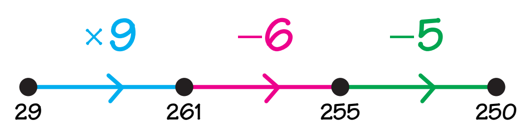
T: ***That took four steps (arrows). Did anyone get to 250 with fewer steps?***

S: ***I pressed [−] [4] [=] and then [×] [1][0] [=].***

T: ***But 10 is not a one-digit number. Try again.***

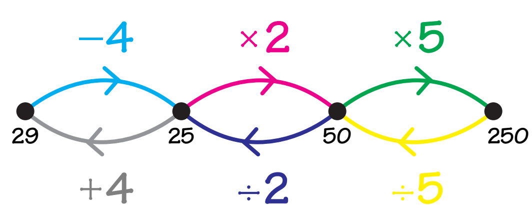
Continue this activity until several solutions are on the board. Try to include at least one solution with three steps (arrows). Two of the many three-step solutions are shown here for your information.





Encourage students to try to find other solutions that use fewer steps (arrows) than those on the board. Perhaps your class will see that it is not possible to go from 29 to 250 using only one or two such arrows.

Do not erase the arrow pictures. Ask students to go from 250 to 29 with the same restrictions; that is, play the game starting at 250 and make 29 the goal. Allow students to work independently on this problem for a few minutes before sharing solutions with the class. As a solution is described, draw the corresponding arrow picture on the board. Encourage students to find solutions that use as few steps (arrows) as possible. Students should notice that to build an arrow road from 250 to 29, you only need to go backward on a road from 29 to 250. For example, if this arrow road was suggested from 29 to 250, then you can reverse the arrows and the return road goes from 250 to 29.



Distribute copies of the workbook *Collection of Problems #3* and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of the lesson, collect the workbooks for your review. They will be used again in Lesson W9.

Capsule Lesson Summary

Add two lengths and determine that 12.6 + 15.7 = 28.3. Continue individual work in the workbook *Collection of Problems #3*. (This is the second of two lessons using this workbook.)

Materials

Teacher
- Two rectangular cards of lengths 12.6 cm and 15.7 cm
- Tape

Student
- Metric ruler
- *Collection of Problems #3* Workbook
- Colored pencils, pens, or crayons
- Translator

Advance Preparation: Before you teach this lesson, cut two rectangles out of cardboard or poster board. The widths of the rectangles should be the same; the length of one should be 12.6 cm, and the length of the other should be 15.7 cm.

Description of Lesson

Show the class your two rectangular cards. Invite three students to measure the length of the smaller card. Each of them should record the measurement on a slip of paper and not reveal it to the class. Collect the data from the three students, announce the measurements to the class, and if they are not the same, let the three students reach a consensus. Perhaps they will want to remeasure. Record the length on the board: 12.6 cm.

Ask the rest of the students to find the mark for 12.6 cm on a centimeter ruler.

T: ***Between which two marks is it?***

S: ***12 and 13 cm.***

T: ***What are the small subdividing marks for?***

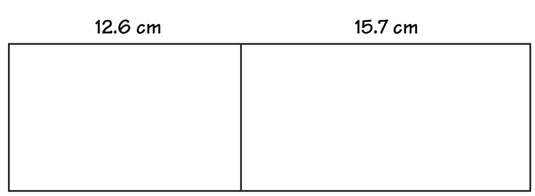
S: ***Millimeters.***

T: ***So where is the mark for 12.6 cm?***

S: ***6 millimeters past the mark for 12 cm.***

Similarly, let three students arrive at a consensus on the length of the bigger card. Record the length on the board.

Carefully tape the two cards together as in the illustration below. Do not overlap them.



12.6 cm
15.7 cm

T: ***How long do you think this rectangle is?***

You are likely to get these two responses: 27.13 cm and 28.3 cm.

Record the students' answers on the board. Then invite a few students to measure the rectangle.

S: ***The length is 28.3 cm!***

Write this addition problem on the board.

$$\begin{array}{r} 12.6 \text{ cm} \\ +\ 15.7 \text{ cm} \\ \hline 28.3 \text{ cm} \end{array}$$

Let students comment on the result. If no one mentions that 6 millimeters + 7 millimeters = 13 millimeters, and that 13 millimeters = 1 centimeter and 3 millimeters, mention it yourself.

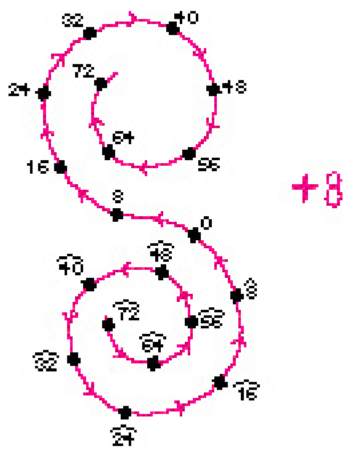
Return students' copies of the workbook *Collection of Problems #3*. Ask students first to correct or complete pages they worked on the previous week and then to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in their workbooks at school during a study time or at home as an assignment.

Assessment Activity

An individual student progress record for the workbook is available on Blackline W9. You may like to use this form to monitor student work.

Label the dots.



Complete.

38 − 16 = 22

39 − 17 = 22

40 − 18 = 22

41 − 19 = 22

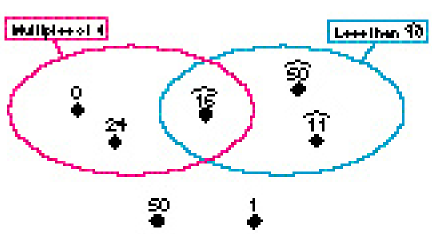
42 − 20 = 22

45 − 23 = 22

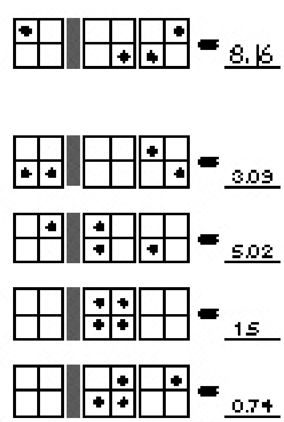
47 − 25 = 22

57 − 35 = 22

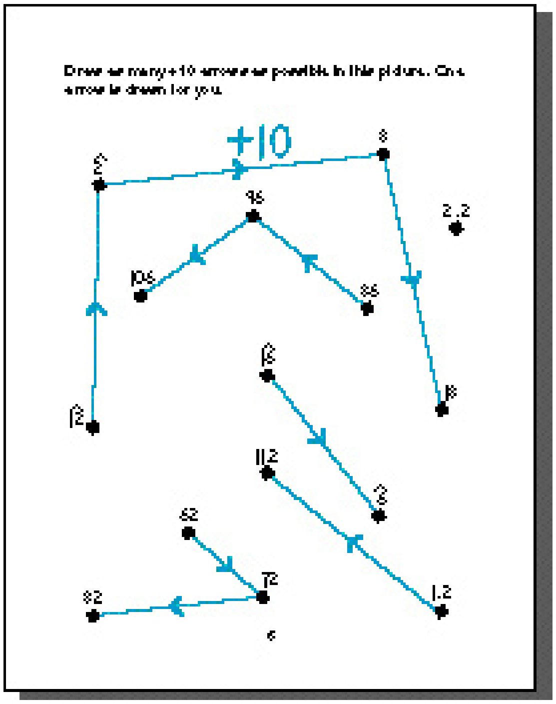
Label the dots. There are many possibilities.

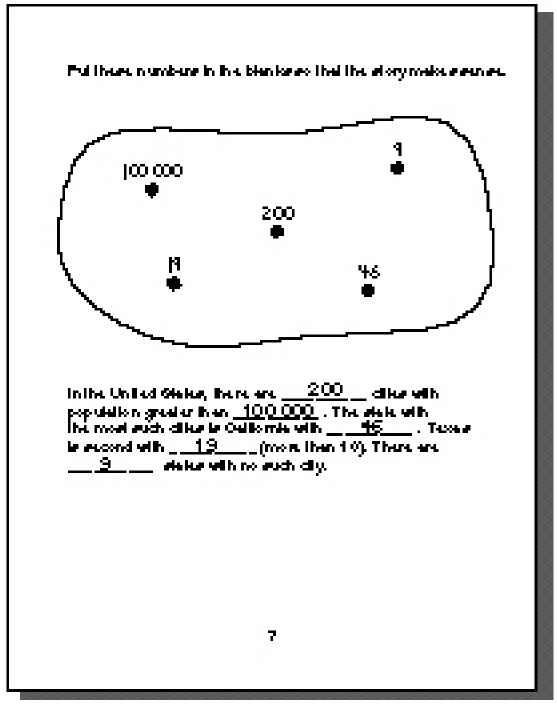


Put any numbers you wish on the Minicomputer with exactly four regular checkers. One solution is given for you.



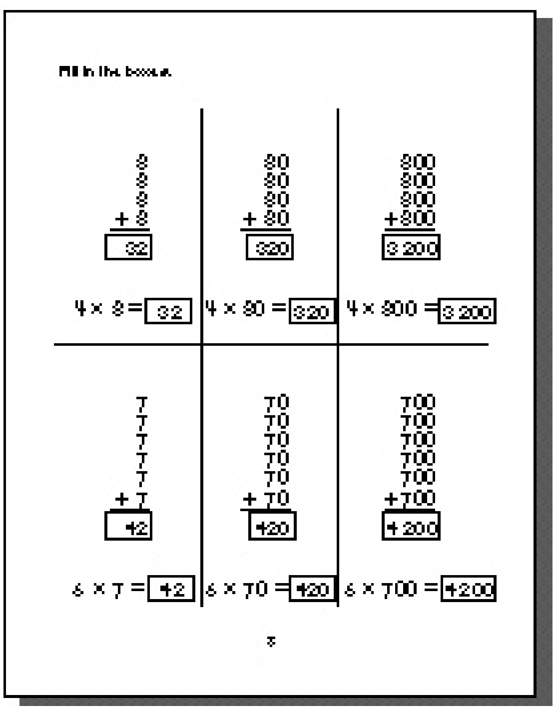
Many solutions are possible.


Draw as many +10 arrows as possible in this picture. One arrow is drawn for you.


Put these numbers in the blanks so that the story makes sense.

In the United States, there are __200__ cities with population greater than __100 000__. The state with the most such cities is California with __46__. Texas is second with __19__ (more than 10). There are __9__ states with no such city.

7


Fill in the boxes.

8 + 8 + 8 + 8 = 32	80 + 80 + 80 + 80 = 320	800 + 800 + 800 + 800 = 3 200
4 × 8 = 32	4 × 80 = 320	4 × 800 = 3 200
7 + 7 + 7 + 7 + 7 + 7 = 42	70 + 70 + 70 + 70 + 70 + 70 = 420	700 + 700 + 700 + 700 + 700 + 700 = 4 200
6 × 7 = 42	6 × 70 = 420	6 × 700 = 4 200

8

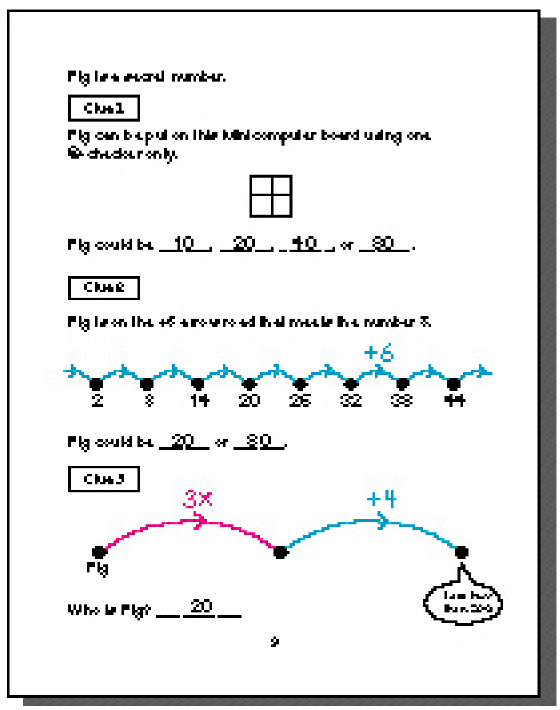


Fig is a secret number.

Clue 1

Fig can be put on this Minicomputer board using one checker only.

Fig could be __10__, __20__, __40__, or __80__.

Clue 2

Fig is on the +6 arrow road that leads the number 2.

Fig could be __20__ or __80__.

Clue 3

Who is Fig? __20__

9

Complete the calculations.

236 + 384 + 192 = 812

170 − 98 = 72

805 − 549 = 256

195 + 242 = 437

219 + 347 = 566

83 × 5 = 415

42 × 7 = 294

135 × 4 = 540

7 × [4] = 28

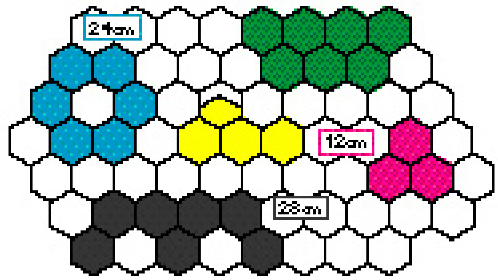
6 × [6] = 36

28 ÷ 7 = [4]

36 ÷ 6 = [6]

10

The length of each line segment in this honeycomb is 1 cm. Find the perimeter of each shape and record it in the box of the same color.



Can you color a shape with perimeter 20 cm? Yes
Explain.

See the green shape.

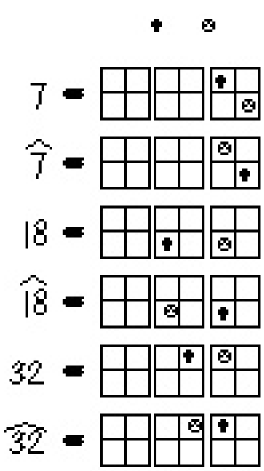
Can you color a shape with perimeter 15 cm? No
Explain.

Any shape that includes whole hexagons has perimeter an even number.

(Yes, if the shape can include parts of hexagons—see the yellow shape.)

11

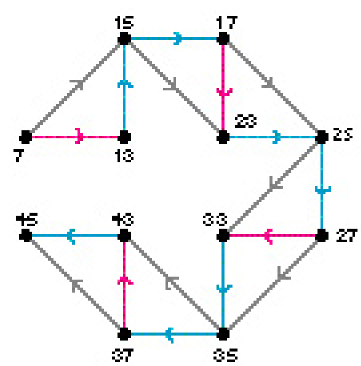
Put each number on the Minicomputer using exactly one regular checker and one negative checker.



12

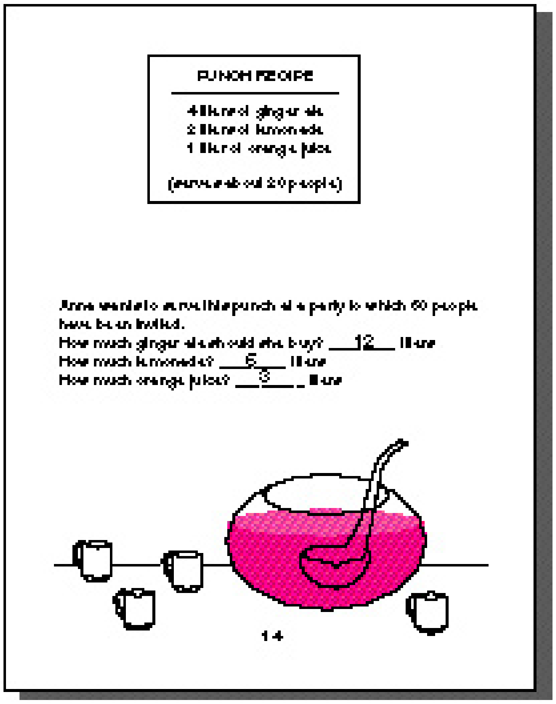
Label the dots. Draw as many +3 arrows as possible in the picture.

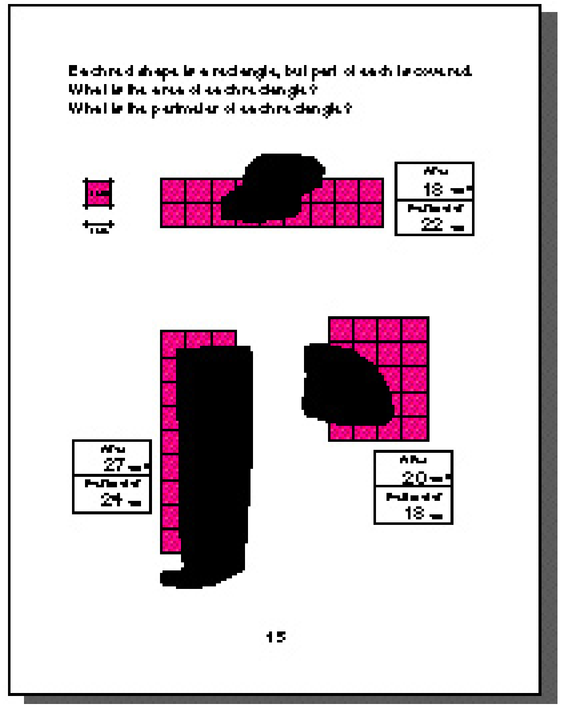
+6 +2 +8

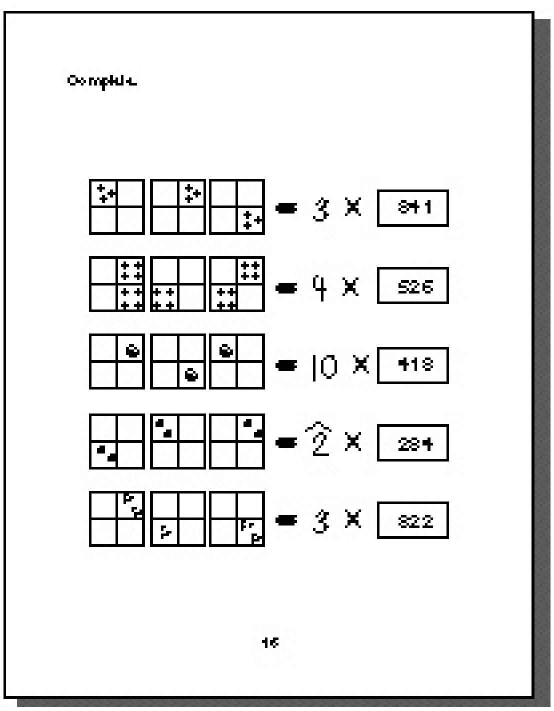


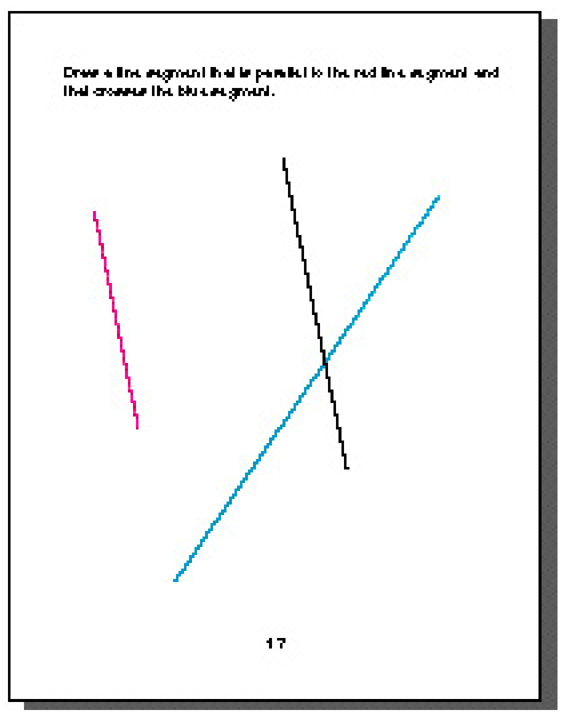
Are there seven +3 arrows in your picture? Yes

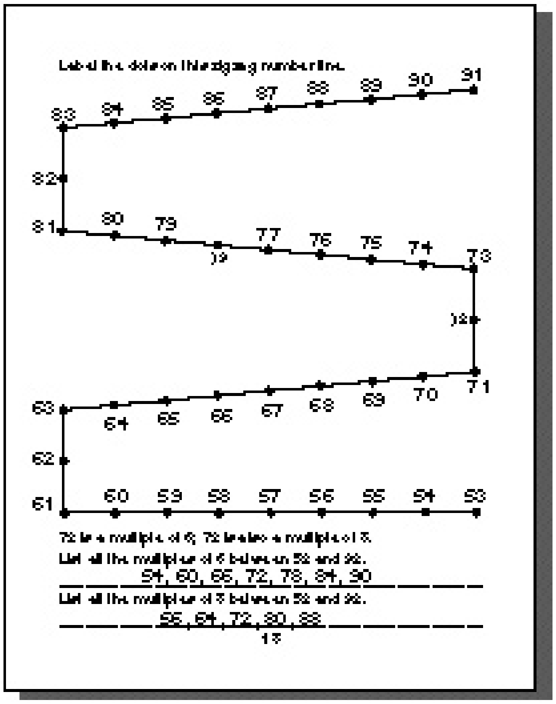
13


PUNCH RECIPE
4 liters of ginger ale
2 liters of lemonade
1 liter of orange juice
(serves about 20 people)
Anne wants to serve this punch at a party to which 60 people have been invited.
How much ginger ale should she buy? 12 liters
How much lemonade? 6 liters
How much orange juice? 3 liters
14


Each red shape is a rectangle, but part of each is covered.
What is the area of each rectangle?
What is the perimeter of each rectangle?
Area
18
Perimeter
22
Area
27
Perimeter
24
Area
20
Perimeter
18
15


Complete.
= 3 × 341
= 4 × 526
= 10 × 418
= 2 × 284
= 3 × 322
16


Draw a line segment that is parallel to the red line segment and that crosses the blue segment.
17



Label the dots on this zigzag number line.

72 is a multiple of 6; 72 is also a multiple of 8.

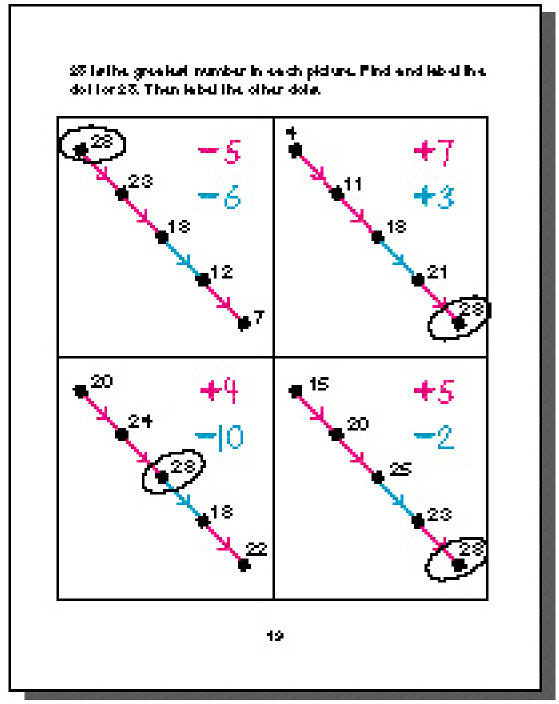
List all the multiples of 6 between 52 and 92.

54, 60, 66, 72, 78, 84, 90

List all the multiples of 8 between 52 and 92.

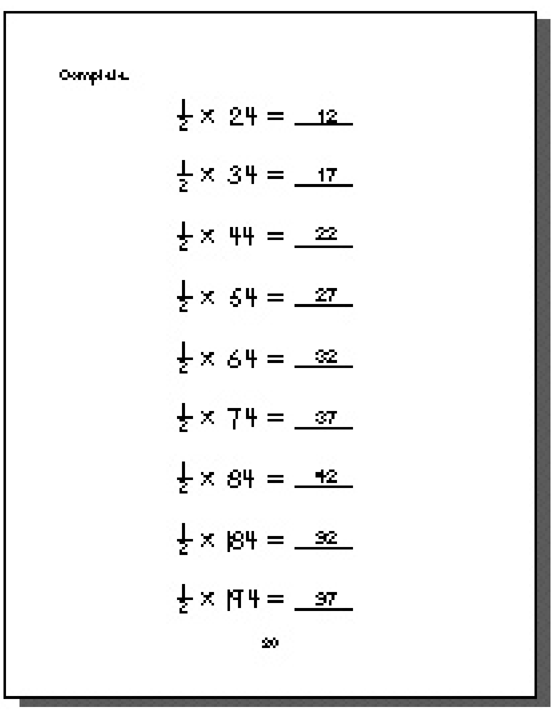
56, 64, 72, 80, 88

18



28 is the greatest number in each picture. Find and label the dot for 28. Then label the other dots.

19



Complete.

$\frac{1}{2} \times 24 =$ 12

$\frac{1}{2} \times 34 =$ 17

$\frac{1}{2} \times 44 =$ 22

$\frac{1}{2} \times 54 =$ 27

$\frac{1}{2} \times 64 =$ 32

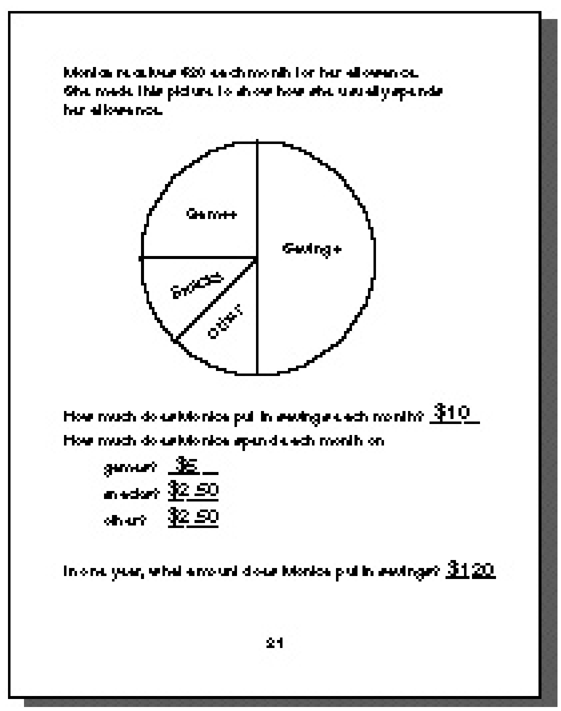
$\frac{1}{2} \times 74 =$ 37

$\frac{1}{2} \times 84 =$ 42

$\frac{1}{2} \times 184 =$ 92

$\frac{1}{2} \times 194 =$ 97

20



Monica receives $20 each month for her allowance. She made this picture to show how she usually spends her allowance.

How much does Monica put in savings each month? $10

How much does Monica spend each month on

games? $5

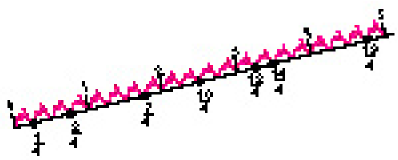
snacks? $2.50

other? $2.50

In one year, what amount does Monica put in savings? $120

21

Jim is playing in some of Gasquatch's tracks. The red arrows are for his steps. Jim takes four steps to get to Gasquatch's next step. Label the dots.



How many steps does it take Jim to reach Gasquatch's 5th step? 20

What is another way we could label the mark for 5? $\frac{20}{4}$ or an equivalent fraction

How many steps does it take Jim to reach Gasquatch's 7th step? 28

What is another way we could label the mark for 7? $\frac{28}{4}$ or an equivalent fraction

22

Put an operation (+, −, ×, or ÷) in each box to make true number sentences.

(25 [+] 35) [−] 15 = 45

20 [−] (16 [÷] 2) = 12

(12 [×] 6) [+] 28 = 100

(36 [−] 10) [×] 4 = 104

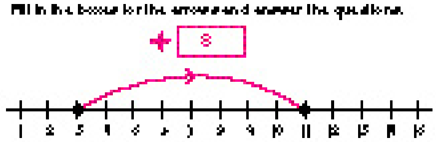
328 [−] (13 [×] 6) = 250

(475 [+] 256) [−] 341 = 390

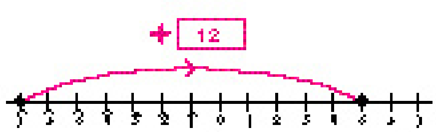
(50 [÷] 10) [×] 12 = 60

23

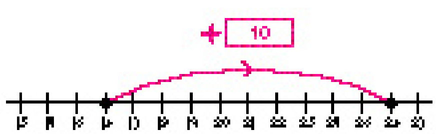
Fill in the boxes for the arrows and answer the questions.



What number is halfway between 3 and 11 on the number line? 7



What number is halfway between −5 and 6 on the number line? 1



What number is halfway between 16 and 26 on the number line? 21

24

June

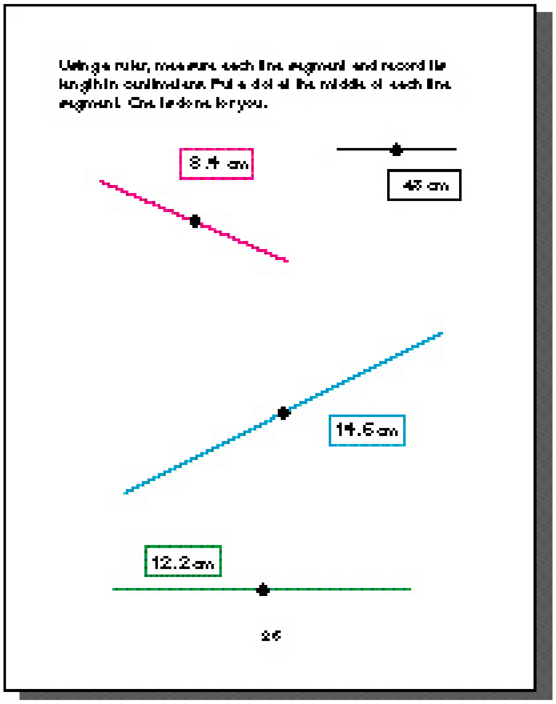
S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

July

S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

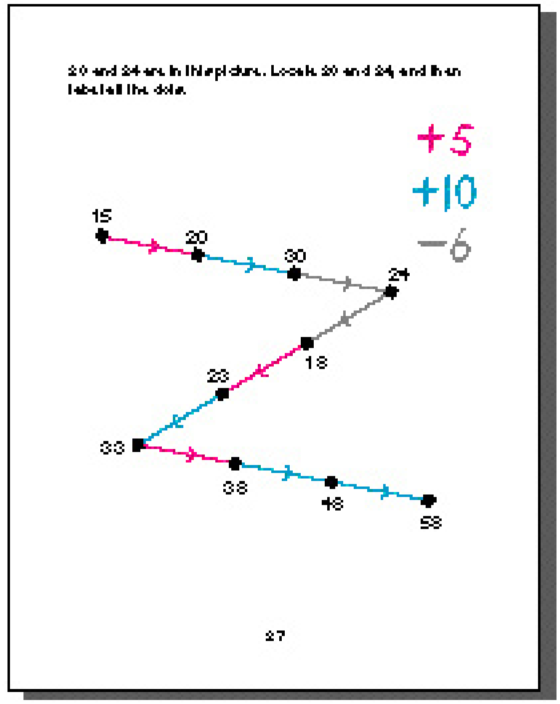
Dan is a camp director. He wants to set up four-day camp sessions during the months of June and July. No two sessions can overlap, and Dan cannot use July 3, 4 or 5 as camp days. What is the most number of sessions Dan can schedule? 14

25



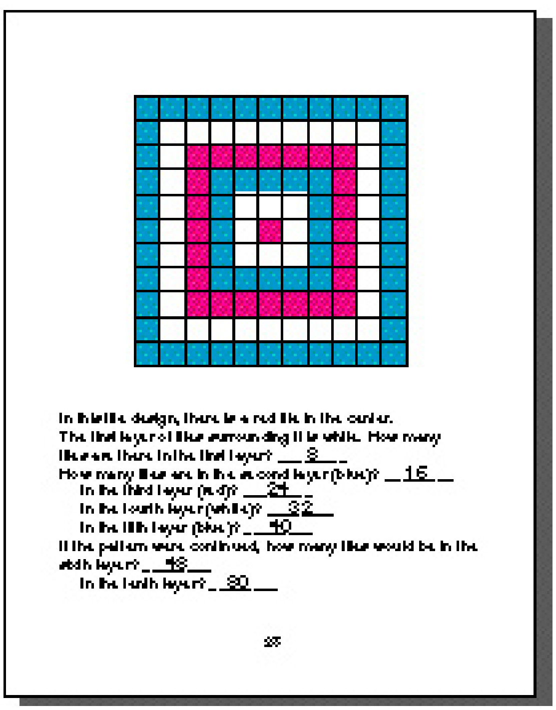
Using a ruler, measure each line segment and record its length in centimeters. Put a dot at the middle of each line segment. One is done for you.

26



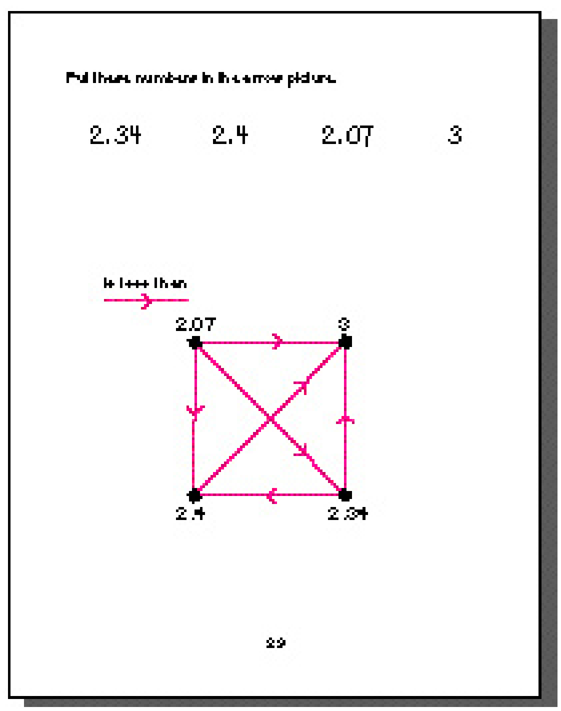
20 and 24 are in this picture. Locate 20 and 24 and then label all the dots.

27



In this tile design, there is a red tile in the center.
The first layer of tiles surrounding it is white. How many tiles are there in the first layer? __8__
How many tiles are in the second layer (blue)? __16__
In the third layer (red)? __24__
In the fourth layer (white)? __32__
In the fifth layer (blue)? __40__
If the pattern were continued, how many tiles would be in the sixth layer? __48__
In the tenth layer? __80__

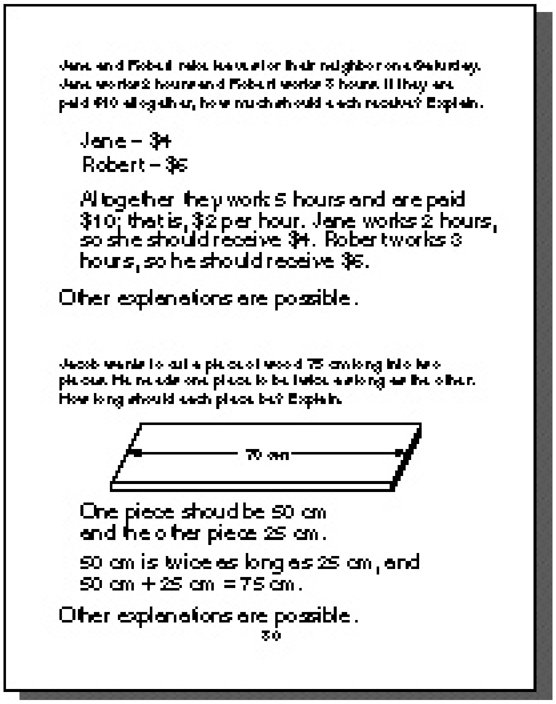
28



Put these numbers in the arrow picture.

2.34 2.4 2.07 3

29



Jane – $4
Robert – $6

Altogether they work 5 hours and are paid $10; that is, $2 per hour. Jane works 2 hours, so she should receive $4. Robert works 3 hours, so he should receive $6.

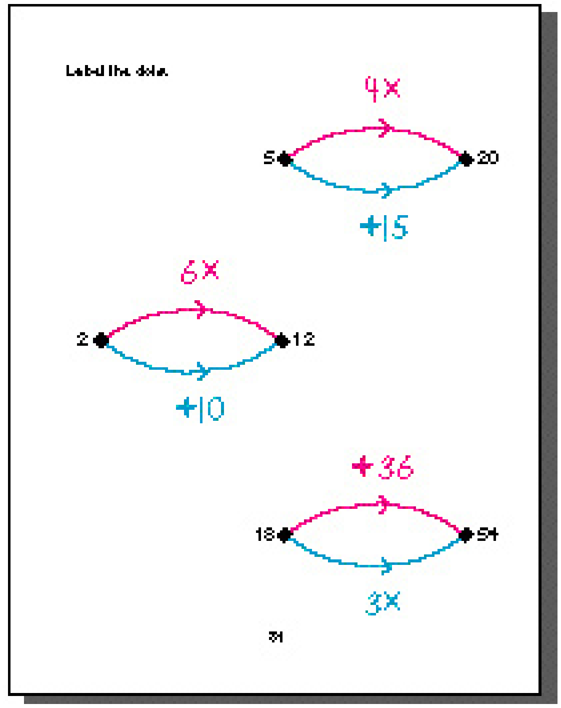
Other explanations are possible.

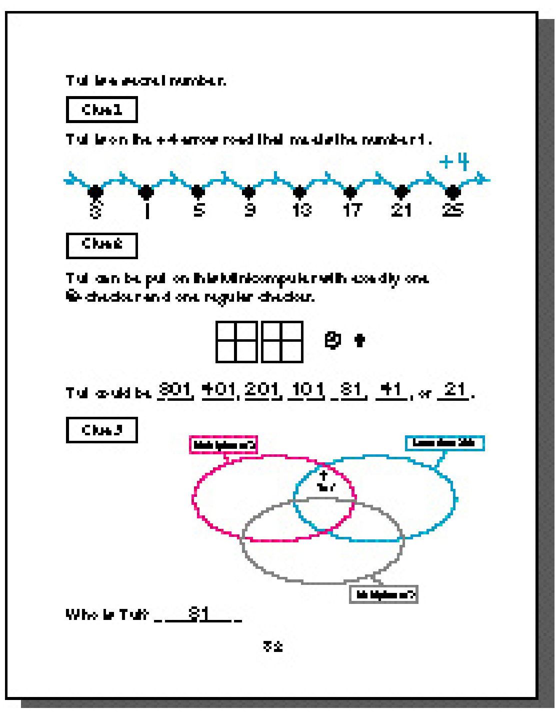
One piece should be 50 cm and the other piece 25 cm.

50 cm is twice as long as 25 cm, and 50 cm + 25 cm = 75 cm.

Other explanations are possible.

30





Capsule Lesson Summary

Given a true subtraction statement, solve some related subtraction problems and observe the patterns. Begin the workbook *Collection of Problems #4*. (This is the first of two lessons using this workbook.)

Materials

Teacher
- None

Student
- *Collection of Problems #4* Workbook
- Colored pencils, pens, or crayons
- Metric ruler

Description of Lesson

Write this boxed subtraction statement on the board.

53 – 37 = 16

T: ***Is this subtraction calculation correct?***

Allow time for students to check the calculation, and encourage explanations that use different checking techniques.

S: ***56 – 40 = 16, so 53 – 37 = 16.***

S: ***53 – 30 = 23 and 23 – 7 = 16.***

S: ***53 – 33 = 20 and 20 – 4 = 16.***

S: ***37 + 16 = 53, so 53 – 37 = 16.***

Write this problem on the board below the subtraction statement. 153 – 37

T: ***Try to solve this problem without doing much work. The subtraction statement 53 – 37 = 16 can help you.***

S: ***153 – 37 = 116.***

Continue the activity with the following problems. Answers are in the boxes. Keep up a rather brisk pace.

153 – 37 = 116	63 – 37 = 26
453 – 37 = 416	63 – 47 = 16
453 – 237 = 216	63 – 57 = 6
753 – 737 = 16	63 – 27 = 36
54 – 37 = 17	53 – 38 = 15
55 – 37 = 18	53 – 39 = 14
56 – 37 = 19	53 – 49 = 4
57 – 37 = 20	53 – 50 = 3

Encourage students to comment on how they use an earlier fact to solve a problem and on the patterns they observe.

Distribute copies of the workbook *Collection of Problems #4* and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collection discussion about that problem.

At the end of the class period, collect the workbooks for your review. This workbook will be used again in Lesson W11.

Capsule Lesson Summary

Put decimal numbers on the Minicomputer, and solve a subtraction problem involving these numbers as amounts of money. Read some numbers that are on the Minicomputer with only ⑩-checkers. Continue individual work in the workbook *Collection of Problems #4*. (This is the second of two lessons using this workbook.)

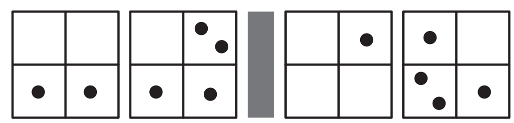
Materials

Teacher
- Minicomputer set
- ⑩-checkers

Student
- Paper
- *Collection of Problems #4* Workbook
- Colored pencils, pens, or crayons
- Centimeter ruler

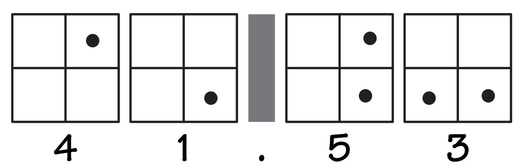
Description of Lesson

Display four Minicomputer boards with this configuration of checkers on it.



T: ***Write this number on your paper.***

Check the responses of several students. After a minute or two, continue by asking students to make trades to get a configuration of checkers that is easier to read. When the standard configuration for 41.53 is on the Minicomputer, ask a student to write the number below the boards.



Leave the checkers on the Minicomputer, but erase the number written under the boards.

T: ***Before going to the grocery story, I had $41.53. Suppose I spent $28.49. How much money would I have left? How can I solve this problem?***

Let students estimate your remaining amount of money (do not expect exact calculations at this time) and suggest solution techniques. The class should observe that this is a subtraction problem.

Write it on the board.

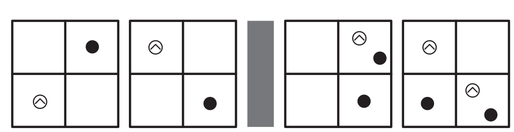
$41.53 – $28.49

T: ***How can we use the Minicomputer to help us solve this problem?***

Follow the lead of your students as there is more than one way to use the Minicomputer.

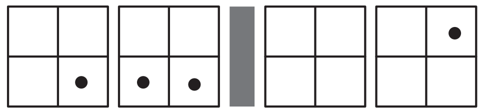
S: ***Subtract $28.49 by putting $\widehat{28.49}$ on the Minicomputer with negative checkers.***

S: ***Make backward trades until we get checkers in position to take away 28.49.***



W11

Call on students to make trades until the result of 41.53 – 28.49 is in standard configuration on the Minicomputer. Complete the subtraction problem, announcing that you would have $13.04 left.



= 13.04

$41.53 – $28.49 = $13.04

Clear the Minicomputer and hold up a ⑩-checker.

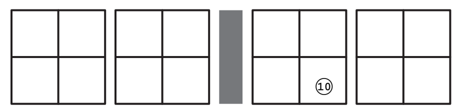
T: ***What are some numbers that we can put on the Minicomputer using just this ⑩-checker?***

Invite students to display some of the numbers as they are mentioned.

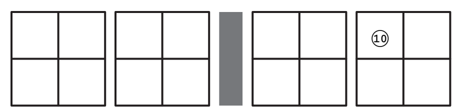
S: ***10, 20, 40, and 80.***

S: ***Also 100, 200, 400, and 800.***

S: ***We can put on 1. Ten dimes is one dollar.***



S: ***We can put 0.8.***



Repeat this activity using two ⑩-checkers and then using three ⑩-checkers. Spend only one or two minutes on each problem.

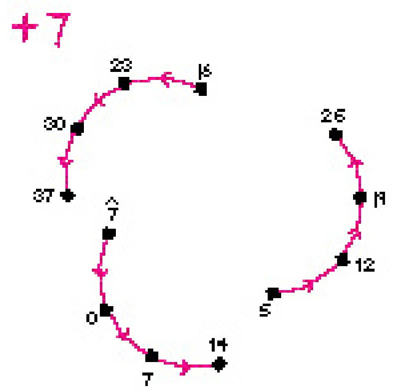
Return students' copies of the workbook *Collection of Problems #4*. Ask students first to correct or complete pages they worked on the previous week and then to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in their workbooks at school during a study time or at home as an assignment.

Assessment Activity

An individual student progress record for the workbook is available on Blackline W11. You may like to use this form to monitor student work.

Label the dots.



Complete.

16	19	14	12	37	16	26
+7	−7	+7	−7	+7	−7	+7
23	12	21	5	44	9	33

2

Complete.

45 − 24 = 21

145 − 24 = 121

345 − 24 = 321

345 − 124 = 221

50 − 24 = 26

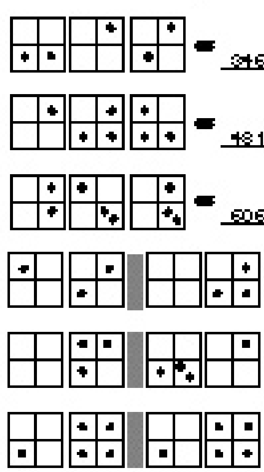
51 − 24 = 27

52 − 24 = 28

53 − 24 = 29

3

What numbers are on the Minicomputer?



346

481

606

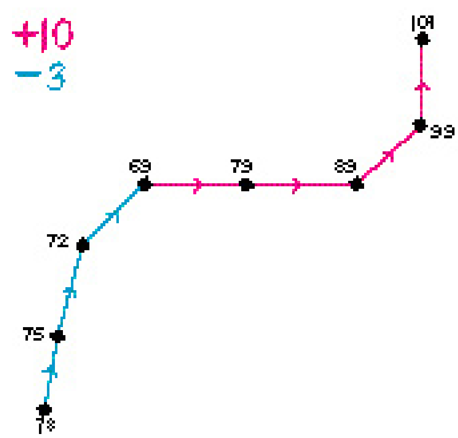
86.07

14.44

35.35

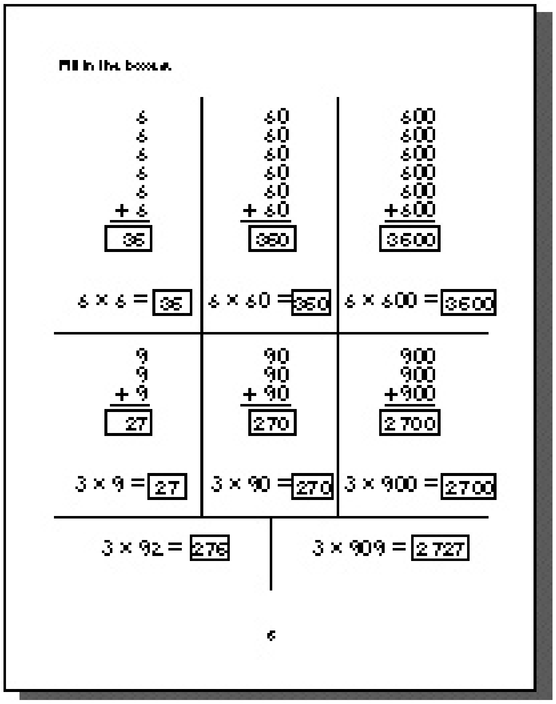
4

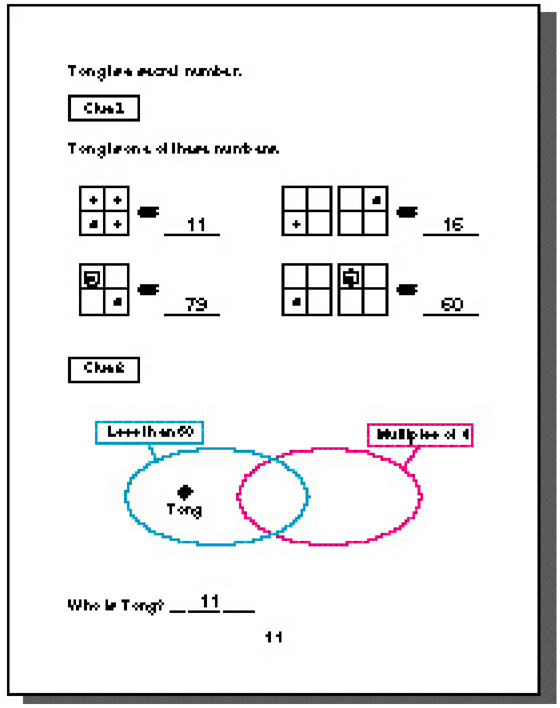
Build an arrow road from 78 to 109 using +10 and −3 arrows.

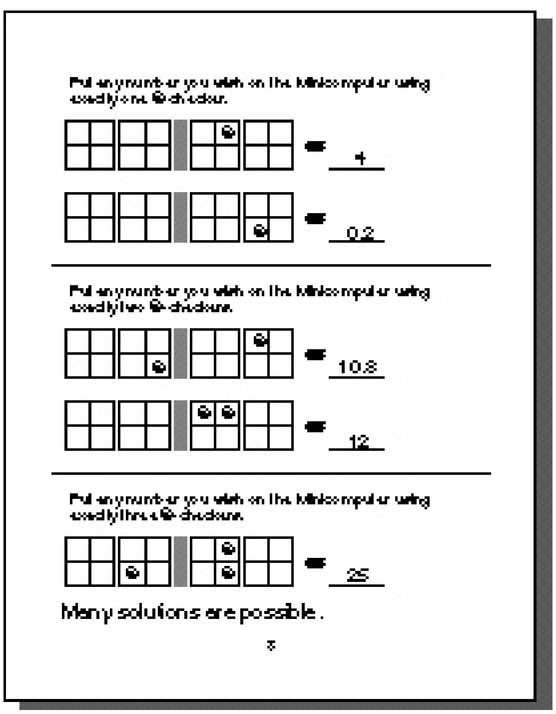


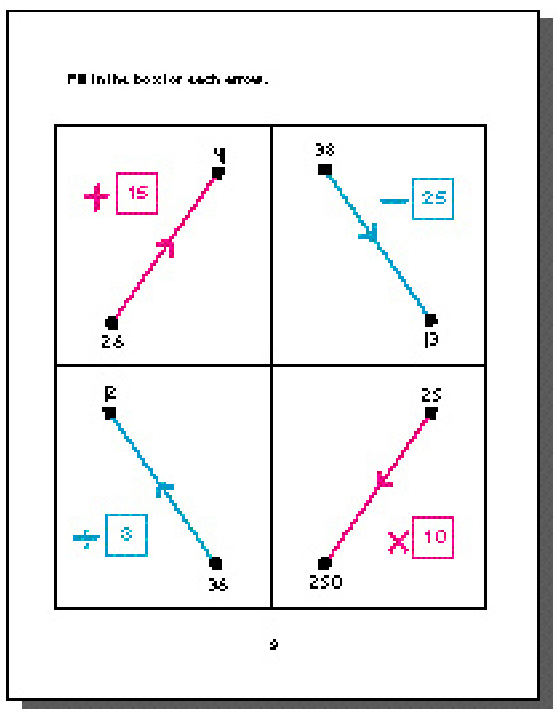
Other solutions are possible.

5


Fill in the boxes.
6 6 6 6 6 + 6 = 36
60 60 60 60 60 + 60 = 360
600 600 600 600 600 + 600 = 3600
6 × 6 = 36
6 × 60 = 360
6 × 600 = 3600
9 9 + 9 = 27
90 90 + 90 = 270
900 900 + 900 = 2 700
3 × 9 = 27
3 × 90 = 270
3 × 900 = 2700
3 × 92 = 276
3 × 909 = 2 727
6


Clue 1
11
16
79
60
Clue 2
Less than 60
Multiples of 4
Tong
Who is Tong? 11
11


4
0.2
10.8
12
25
Many solutions are possible.
8


4
+ 15
26
38
− 25
13
12
÷ 3
36
25
× 10
250
9

Complete the calculations.

$$\begin{array}{r} 259 \\ 142 \\ +345 \\ \hline 746 \end{array}$$

$482 + 219 = \underline{701}$

$$\begin{array}{r} 137 \\ -55 \\ \hline 82 \end{array} \qquad \begin{array}{r} 906 \\ -329 \\ \hline 577 \end{array}$$

$73 - 37 = \underline{36}$

$$\begin{array}{r} 125 \\ \times\ 4 \\ \hline 500 \end{array} \qquad \begin{array}{r} 63 \\ \times\ 8 \\ \hline 504 \end{array}$$

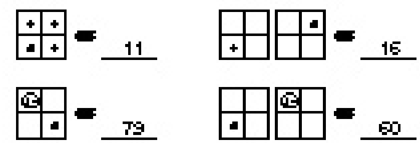
$12 \div 4 = \underline{3}$

10

Tong is a secret number.

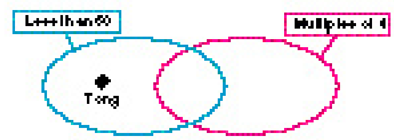
Clue 1

Tong is one of these numbers.



= 11
= 16
= 79
= 60

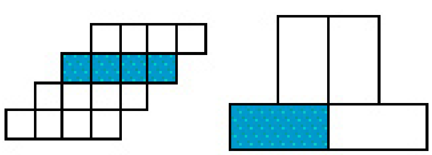
Clue 2



Who is Tong? 11

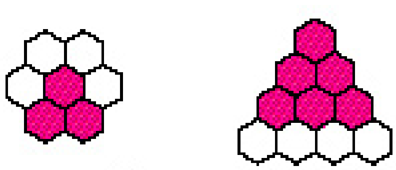
11

Color one-fourth of each shape blue.



Other solutions are possible.

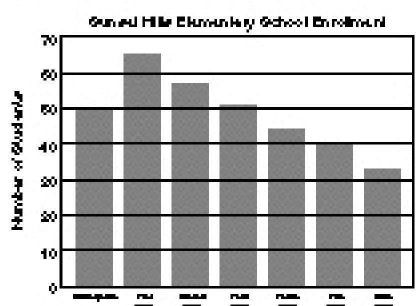
What fractional part of each shape is colored red?



$\frac{4}{7}$ $\qquad$ $\frac{6}{10}$ or $\frac{3}{5}$

12

This graph shows the number of students in each grade level at the Gwned Hills Elementary School.

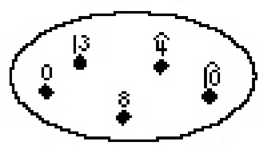


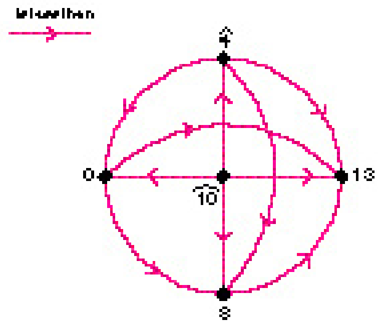
1. How many students are there in each grade level?
   50 65 57 51 44 40 32
2. How many students are in the Intermediate Grades (Fourth, Fifth, and Sixth)? 116
3. How many students are in the Primary Grades (Kindergarten, First, Second, and Third)? 223
4. How many students are there altogether at Gwned Hills Elementary School? 339

Answers may vary slightly.

13

Put these numbers in the arrow picture.





14

Jason and Carla are making Mardi Gras necklaces for some of their friends.

Materials for a Mardi Gras necklace	
6	large beads
15	small beads
8	charms
52 cm	string
1	fastener

1. Jason wants to make necklaces for three friends. How much of each item does he need?

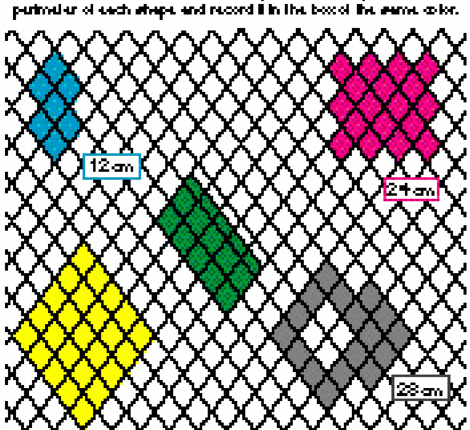
 18 large beads — 156 cm string
 45 small beads — 3 fasteners
 24 charms

2. Carla wants to make necklaces for five friends. How much of each item does she need?

 30 large beads — 260 cm string
 75 small beads — 5 fasteners
 40 charms

15

Each side of a small diamond in this grid is 1 cm long. Find the perimeter of each shape and record it in the box of the same color.



Can you color a shape with perimeter 20 cm? Yes Explain.

See the yellow shape.

Can you color a shape with perimeter 15 cm? No Explain.

All shapes that include full diamonds have perimeter an even number.

(Yes, if the shape can include half diamonds—see the green shape.)

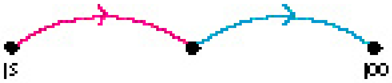
16

Complete the table.

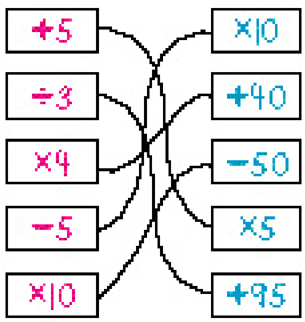
25¢	10¢	5¢	1¢	Total Amount
0	4	3	3	$0.58
5	0	1	2	$1.32
1	4	3	4	$0.84
6	5	1	2	$2.07
3	3	3	0	$1.20

Other solutions are possible.

17



Pair the legs. One is done for you.



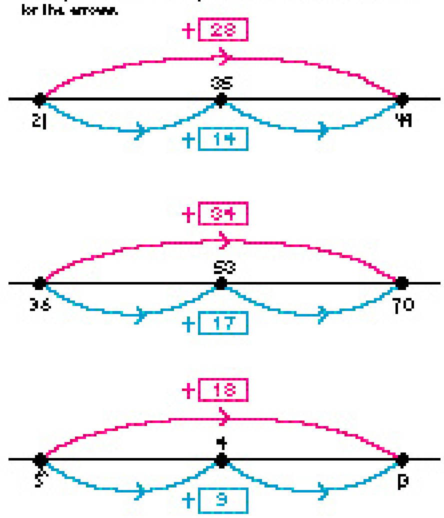
18

Fill in the boxes to complete these calculations.

4 × 7 = 28	10 × 15 = 150
5 × 7 = 35	11 × 15 = 165
6 × 7 = 42	12 × 15 = 180
7 × 7 = 49	13 × 15 = 195
20 × 7 = 140	12 × 12 = 144
20 × 8 = 160	12 × 13 = 156
20 × 9 = 180	12 × 14 = 168
20 × 10 = 200	12 × 15 = 180

19

On each number line, label the middle dot with the number halfway between the two given numbers. Fill in the boxes for the arrows.



20

Tom's Popcorn
Package (2 cups) of kernels gives 80 cups of popped corn.

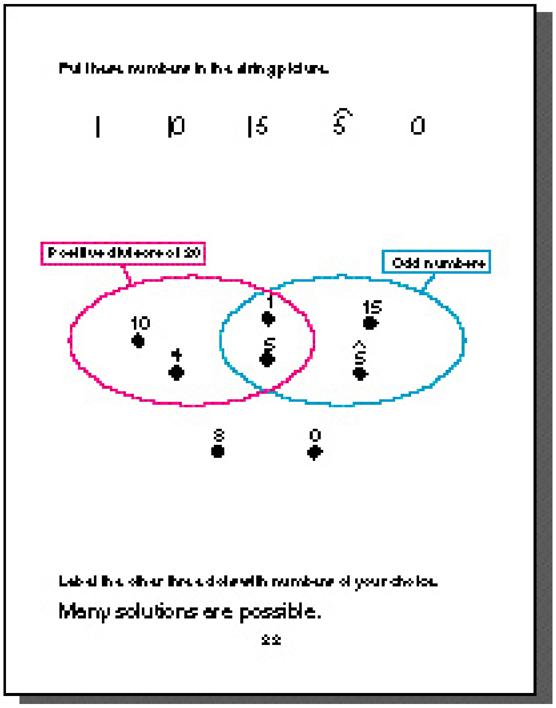
1. About how much popped corn do you get from ¼ cup of kernels? 20 cups

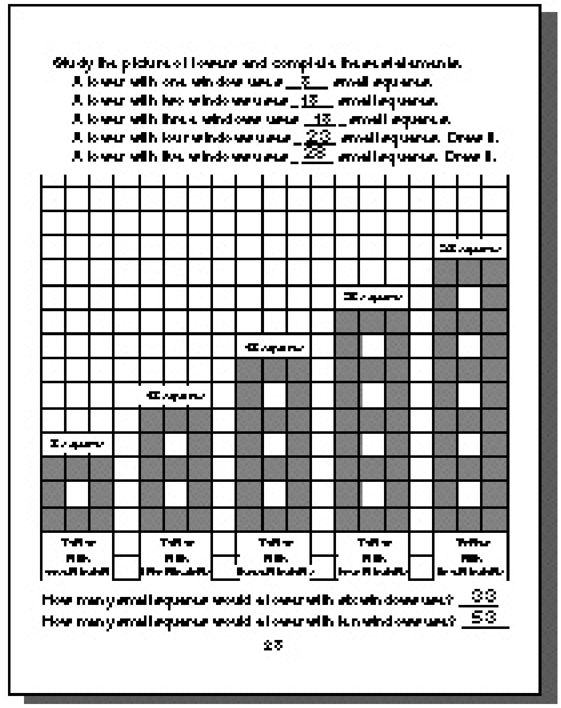
2. Marcie popped a cup of Tom's popcorn kernels. She wants to share it with the 10 students on her volleyball team. About how much popped corn could each team member get? 4 cups

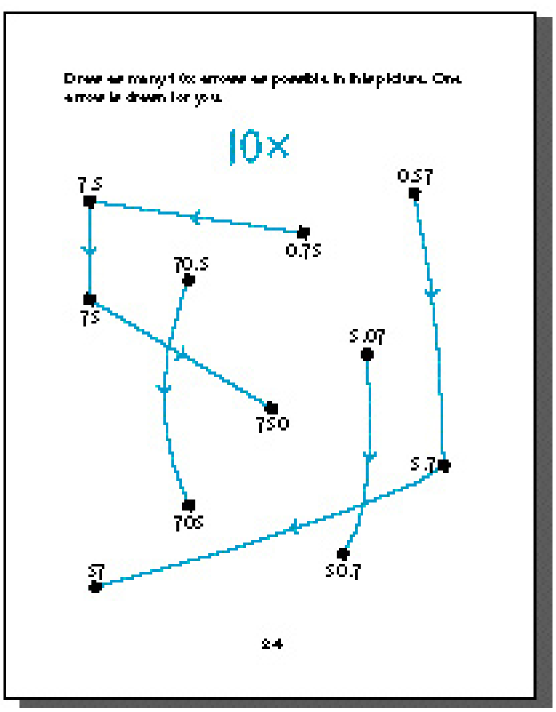
3. Ms. Trent wants about 300 cups of popped corn for the school party. How many packages of kernels should she buy? 4 packages

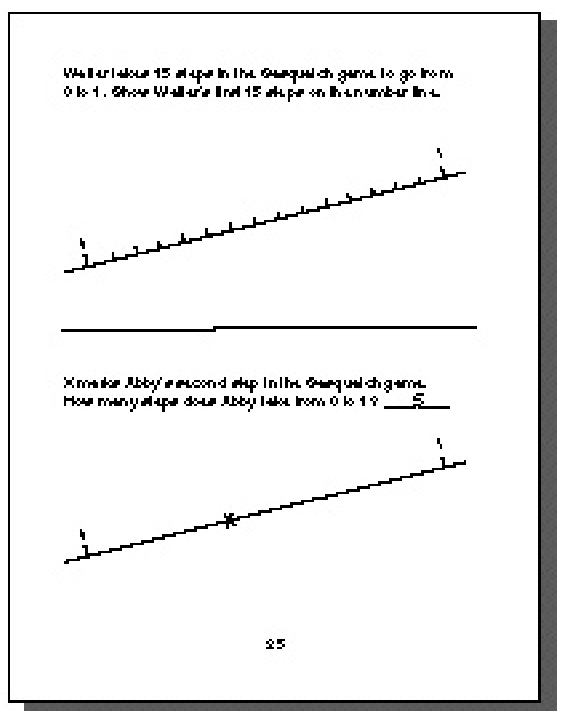
4. You need 10 cups of popped corn to make 3 popcorn balls. How many popcorn balls could you make from 1 cup of kernels? 32 popcorn balls

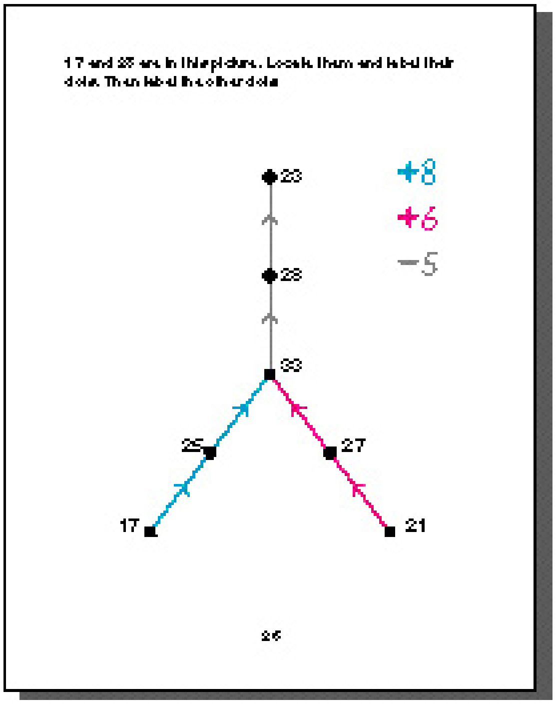
21

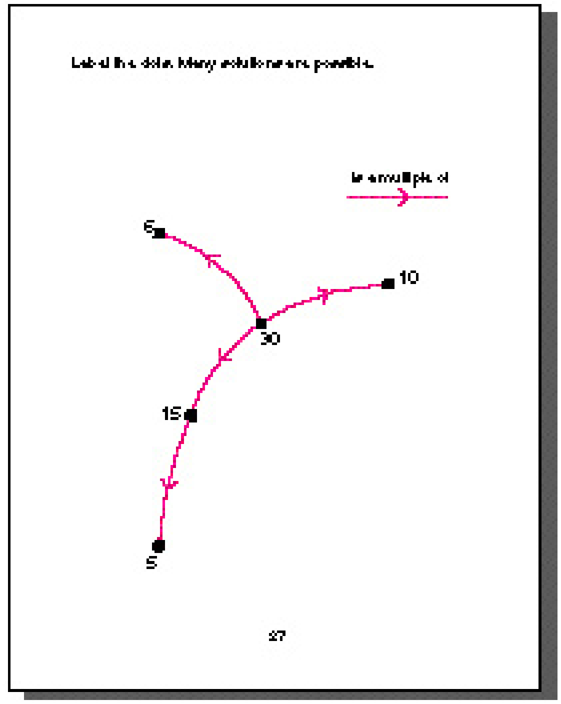

1 10 15 5 0
Odd numbers
10
1
15
5
5
8
0
Many solutions are possible.
22

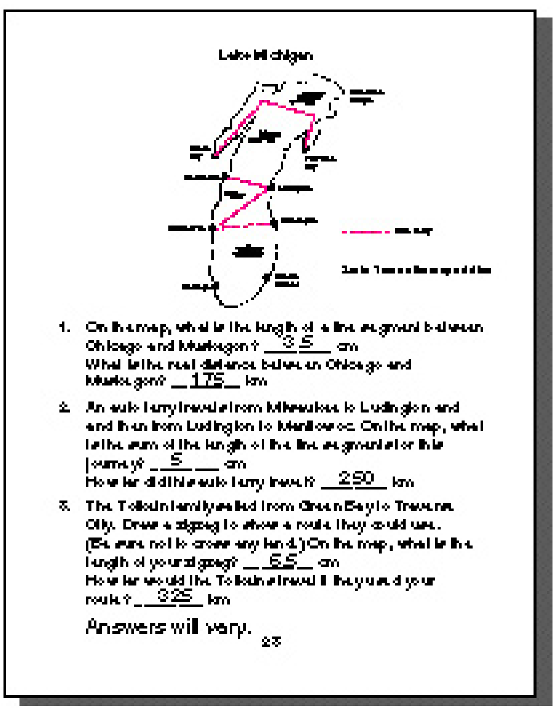

23

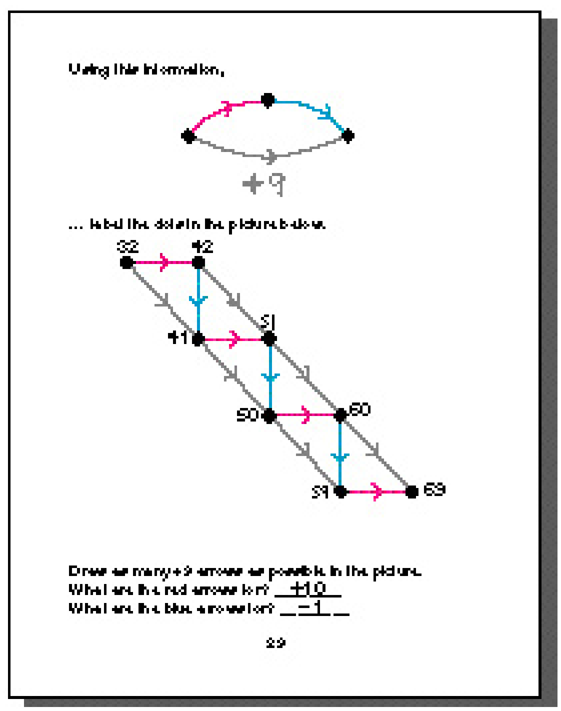

10x
24

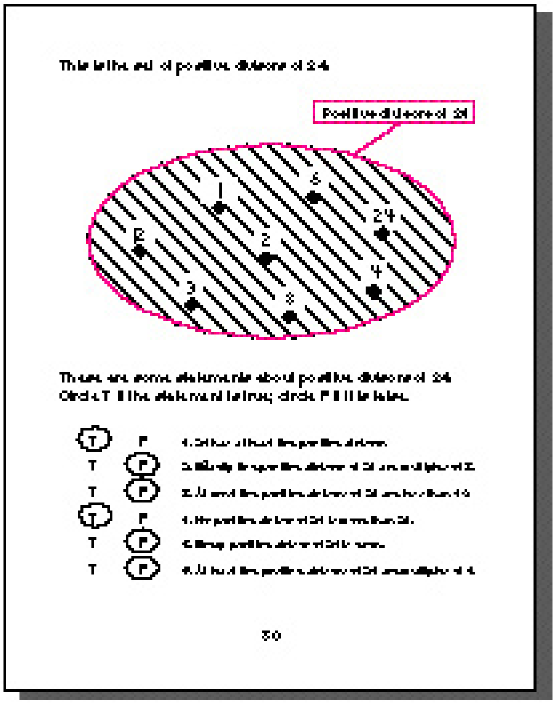

5
25

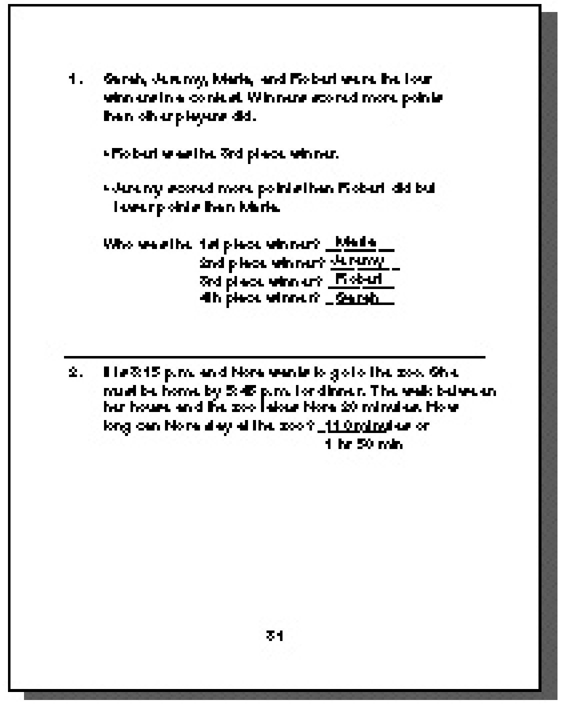

17 and 23 are in this picture. Locate them and label their dots. Then label the other dots.
+8
+6
−5
23
28
33
25
27
17
21
26

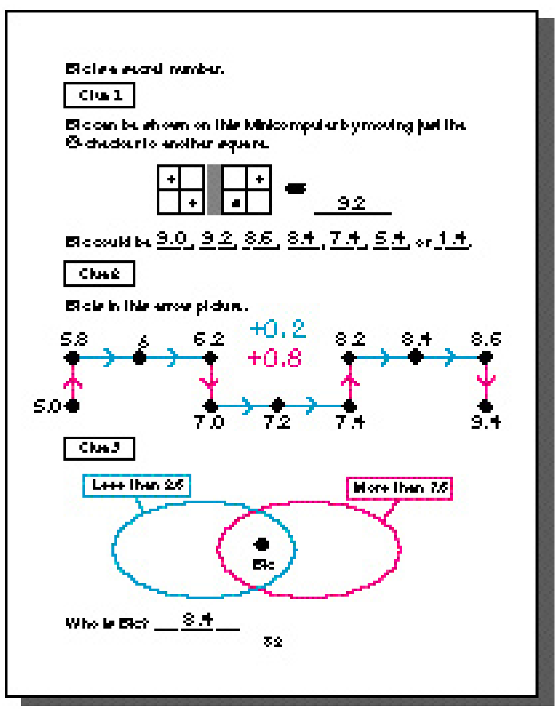

Label the dots. Many solutions are possible.
is a multiple of
6
10
30
15
5
27


Lake Michigan
1. On the map, what is the length of the line segment between Chicago and Muskegon? 3.5 cm
What is the real distance between Chicago and Muskegon? 175 km
2. An auto ferry travels from Milwaukee to Ludington and and then from Ludington to Manitowoc. On the map, what is the sum of the length of the line segments for this journey? 5 cm
How far did the auto ferry travel? 250 km
3. The Tolkein family sailed from Green Bay to Traverse City. Draw a zigzag to show a route they could use. (Be sure not to cross any land.) On the map, what is the length of your zigzag? 6.5 cm
How far would the Tolkeins travel if they used your route? 325 km
Answers will vary.
28


Using this information,
+9
... label the dots in the picture below.
32
42
51
41
50
60
59
69
Draw as many +9 arrows as possible in the picture.
What are the red arrows for? +10
What are the blue arrows for? −1
29


This is the set of positive divisors of 24.
Positive divisors of 24
1
6
24
2
2
4
3
8
These are some statements about positive divisors of 24.
Circle T if the statement is true; circle F if it is false.
T F
T F
T F
T F
T F
T F
30


1. Sarah, Jeremy, Maria, and Robert were the four winners in a contest. Winners scored more points than other players did.
• Robert was the 3rd place winner.
• Jeremy scored more points than Robert did but fewer points than Maria.
Who was the 1st place winner? Maria
2nd place winner? Jeremy
3rd place winner? Robert
4th place winner? Sarah
2. It is 3:15 p.m. and Nora wants to go to the zoo. She must be home by 5:45 p.m. for dinner. The walk between her house and the zoo takes Nora 20 minutes. How long can Nora stay at the zoo? 110 minutes or 1 hr 50 min
31


Eric is a secret number.
Clue 1
Eric can be shown on this Minicomputer by moving just the checker to another square.
= 9.2
Eric could be 9.0, 9.2, 8.6, 8.4, 7.4, 5.4, or 1.4.
Clue 2
Eric is in this arrow picture.
+0.2
+0.8
5.8
6
6.2
8.2
8.4
8.6
5.0
7.0
7.2
7.4
9.4
Clue 3
Eric
Who is Eric? 8.4
32

Capsule Lesson Summary

Begin reading the story-workbook *A Strange Country*, in which two whole numbers are joined by a cord if and only if one number equals the double of the other, or one number equals one more than the double of the other. Draw pictures showing cord neighborhoods of some number. Notice that each whole number except 0 is connected by cords to three other whole numbers, and that 0 is only connected to itself and to 1. (This is the first of two lessons using this story-workbook.)

Materials

Teacher
- *A Strange Country* Story-Workbook
- Colored chalk

Student
- *A Strange Country* Story-Workbook
- Colored pencils, pens, or crayons
- Unlined paper

Description of Lesson

Distribute copies of the story-workbook *A Strange Country*. Tell the class that this story takes place in the world of whole numbers, and review that whole numbers are 0, 1, 2, 3, 4, and so on.

Pages 3–5

Invite students to read pages 3 and 4 aloud.

T: ***Look at page 5. What could 0's rule be?***

Check that any rule suggested works for all of the numbers in the picture. For example, suppose a student suggests that the rule is 2x.

S: ***2 x 12 = 24, 2 x 6 = 12, 2 x 25 = 50, and 2 x 3 = 6. The rule could be 2x.***

T: ***But 6's and 13's houses are connected by a path. 2 x 6 ≠ 13 and 2 x 13 ≠ 6.***

Perhaps a student will guess the rule (see page 7 of the story-workbook).

Pages 6–8

Ask a student to read aloud the rule in the red box on page 7. Then explain how to apply the rule using the three examples on page 6.

Allow a few minutes for students to answer the questions on the lower half of page 7. Check the work collectively, asking students to give reasons for their answers.

T: ***Are the houses of 12 and 25 directly connected?*** (Yes)
Why?

S: ***2 x 12 = 24 and 24 + 1 = 25.***

T: ***Good. What about 7 and 5?*** (No)
Why?

S: ***The double of 5 is 10 and the double plus 1 is 11.***

T: ***And certainly the double of 7 does not equal 5 nor does its double plus 1 equal 5.***

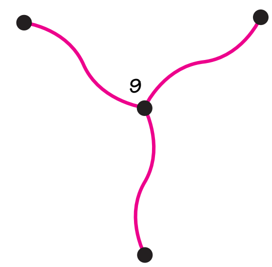
Note: With the rule for the red cords, there are four checks that one must make before deciding whether or not two numbers **b** and **c** can be connected:

- Does **b** equal the double of **c**?
- Does **b** equal one more than the double of **c**?
- Does **c** equal the double of **b**?
- Does **c** equal one more than the double of **b**?

But if **b** and **c** are non-negative numbers (in this lesson we are concerned only with whole numbers), and if **b** is greater than **c**, then the answer to each of the last two questions is no. Students will intuitively see this, but you may want to point out early in the lesson that there are really four checks.

Continue until all six answers have been discussed. Then ask students to look at page 8, where the six answers are given pictorially.

Ask students to close their story-workbooks for awhile.
Draw this picture on the board.



T: ***Who are the next*** [†] ***neighbors of 9?***

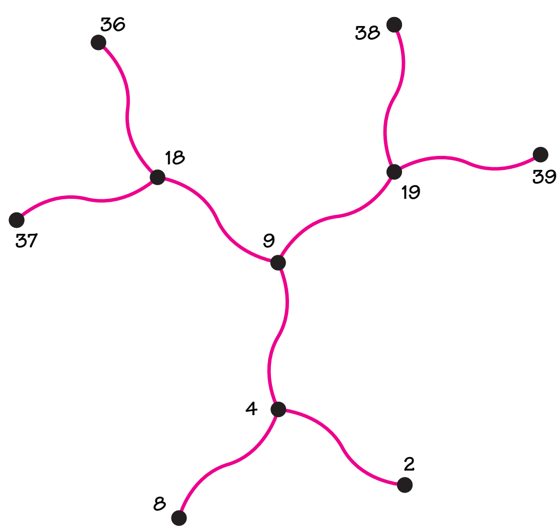
S: ***18 and 19, 2 x 9 = 18 and 18 + 1 = 19.***

S: ***But 9 is not the double of any whole number.***

T: ***That's right. But is 9 equal to one more than the double of some whole number?***

S: ***4. The double of 4 is 8 and 8 + 1 = 9.***

Label the dots and then expand the neighborhood. Let students identify the new neighbors.

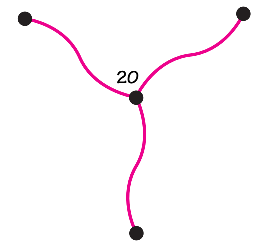


Direct students to open their story-workbooks to page 9.

Page 9

[†] You may wish to call these "next-door neighbors."

Read the first paragraph on page 9 together.
Then draw the cord picture on the board.

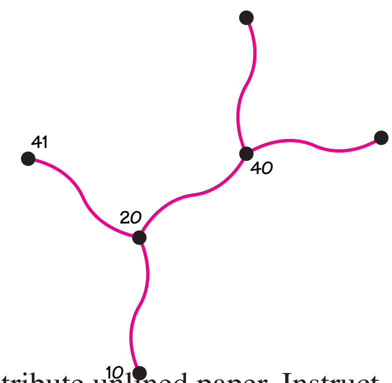


T: ***Who are 20's next neighbors?***

S: ***40 and 41.***

S: ***10.***

Label the three dots for these neighbors. Students should do the same on page 9 of their story-workbooks. Expand one branch of the picture.



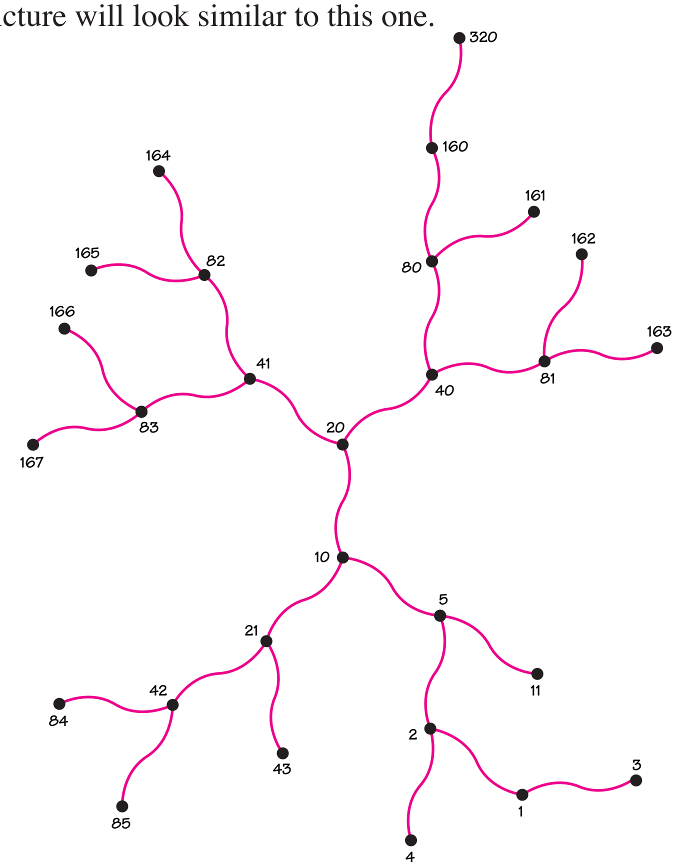
T: ***One of 40's next neighbors is 20. What are the other two?***

S: ***80 and 81.***

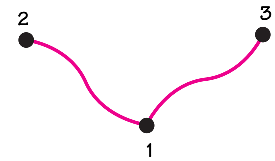
Label the dots for 80 and 81. The students should do the same on the bottom of page 9.

Tell students to close their story-workbooks and put them aside. Distribute unlined paper. Instruct students to copy the drawing on the board and to continue expanding the neighborhood. When drawing a cord from a number, suggest that they label the other dot immediately. This will help prevent students from drawing pictures with more than three cords at any one dot and becoming confused.

While the students are working, invite some of them (one at a time) to expand the picture on the board. Perhaps your picture will look similar to this one.



Take the opportunity to discuss the neighborhood of 0 when the cord picture on the board is expanded to include 1. Direct students' attention to this part of the picture.



T: ***Two of 1's neighbors are 2 and 3. Does 1 have another neighbor? Is 1 the double of some whole number?***

S: ***No, but it's the double of ½.***

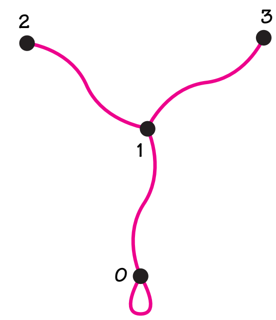
T: ***Is it one more than the double of some whole number?***

S: ***Yes, 0. 2 x 0 = 0 and 0 + 1 = 1.***

Expand the picture to include 0.

T: ***One of 0's next neighbors is 1. Does 0 have any other next neighbors?***

S: ***The double of 0 is 0.***



T: ***So we have a path from 0's house back to 0's house. Does 0 have any other next neighbors beside 1?***

S: ***No. 0 is not one more than the double of some whole number.***

Perhaps a student will figure out that 0 equals one more than the double of $\widehat{1/2}$. However, $\widehat{1/2}$ is not a whole number.

T: ***So it appears that all of the whole numbers except 0 have three next neighbors. 0 is special.***

Direct students to open their story-workbooks to page 10.

Pages 10 – 13

Ask a student to read page 10 aloud.

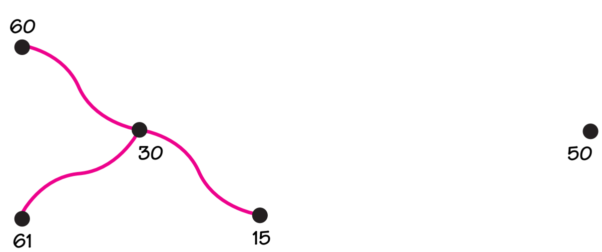
T: ***On page 10, trace the walk from 20's house to 649's house.*** (Pause.) ***Now trace a walk from 163's house to 10's house.*** (Pause.) ***Which numbers did you visit going between 163's and 10's houses?***

S: ***81, 40, and 20.***

Ask a student to read page 11 aloud while you draw two dots for 30 and 50 on the board.

T: ***Where do you want to start, at 30 or at 50?***

Suppose the class chooses to start the walk at 30.



T: ***Who are the next neighbors of 30?***

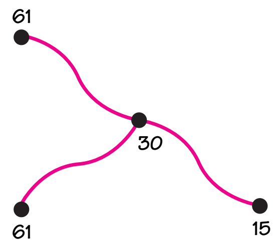
S: ***60, 61, and 15.***

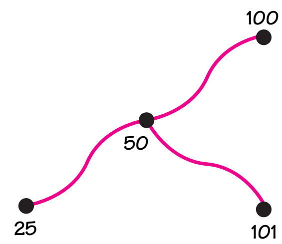
T: ***Which way should we try going?***

Follow the suggestion of the class without comment. In this situation there is exactly one way of going between any two numbers. The walk between 30 and 50 visits 15, so any attempt to build the walk by visiting 60 to 61 would at sometime need to be abandoned.

T: ***Let's look at a neighborhood of 50. Who are 50's next neighbors?***

S: ***100, 101, and 25.***





T: ***Which one should we try visiting?***

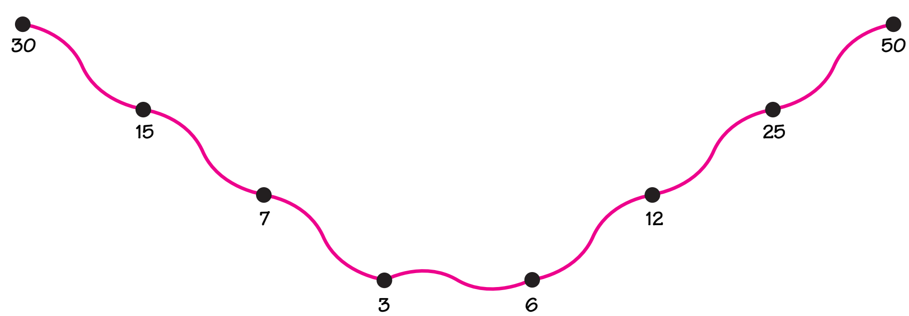
Again follow the suggestion of the class without comment. The walk between 30 and 50 visits 25, so any attempt to build the walk by visiting 100 or 101 would at sometime need to be abandoned.

In the next question use the numbers that your students have chosen to visit so far.

T: ***Our walk goes from 30 to 15 and from 25 to 50. We need to try to connect 15 and 25 with a walk. What should we try?***

Let the class continue by trial and error. Do not be afraid to let students make incorrect choices, but do encourage them to build the walk from both sides rather than from only one.

Once the walk is complete, erase any extraneous cords that were not used. There is only one solution.



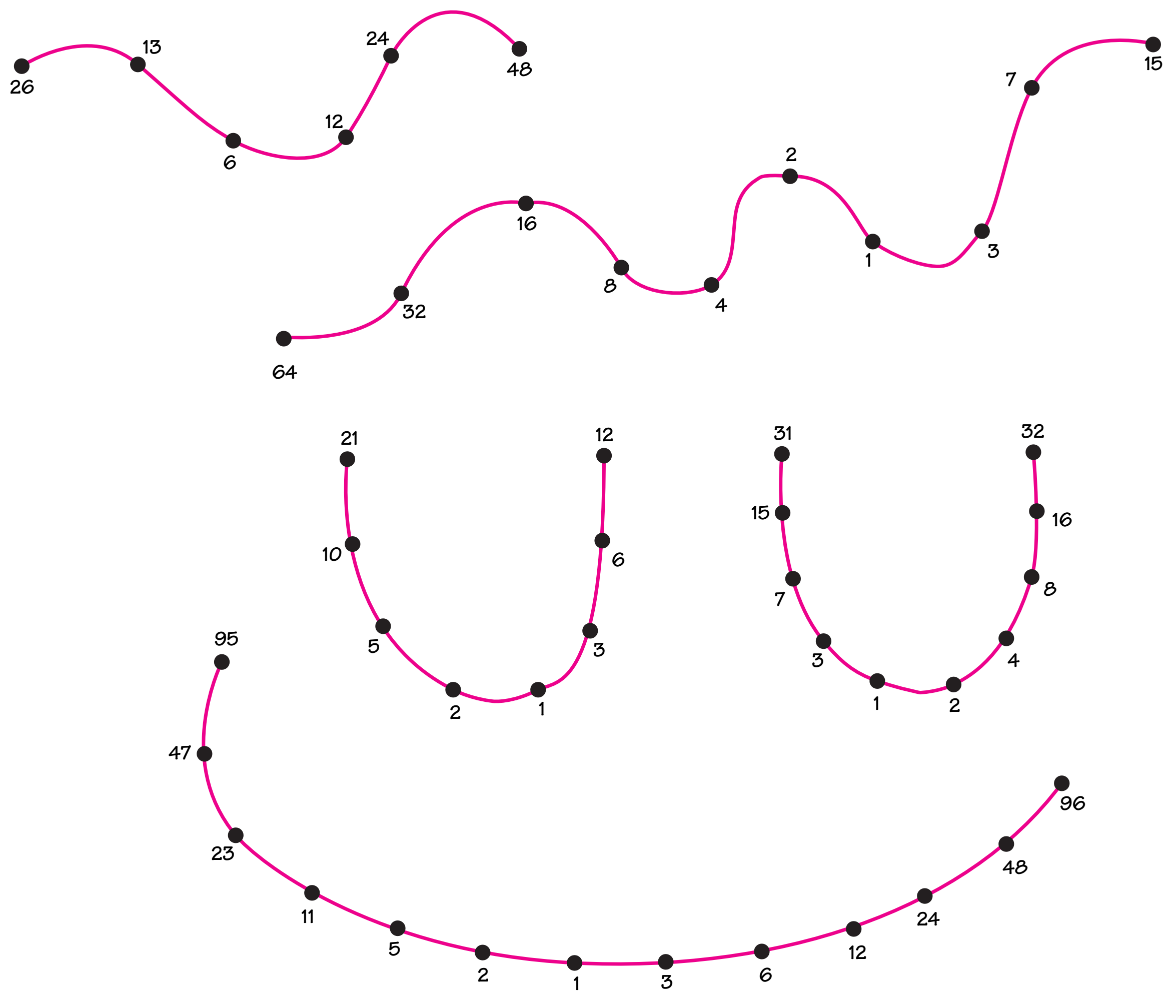
On page 11, students should record their solution and then observe the same solution on page 12.

Ask students to work individually or with a partner on the two problems on page 13. Students should use pencil so they can make erasures easily; once they find solutions they can record them in color. Give those who complete the walks quickly all or some of the following problems.

- Build a walk between 21 and 12 (*).

- Build a walk between 31 and 32 (**).
- Build a walk between 95 and 96 (***).

Collectively discuss solutions to the problems on page 13. Encourage students to share techniques that they find useful. Solutions are shown below.



Pages 14–16

Let students work individually or in pairs on the problems presented on pages 14 and 15. A solution to the problem on page 15 is shown on page 16 of the story-workbook.

Collect the story-workbooks and have then ready for use in Lesson W14.

Capsule Lesson Summary

Review the situation described in the story-workbook *A Strange Country*. Draw pictures of neighborhoods for 1,000 and 0. Use the picture of 0's neighborhood to find walks from 0 to other numbers in fairly close proximity. Show that a walk starting at 0 and visiting 2, and a walk starting at 0 and visiting 3 can never meet again. (This is the second of two lessons using this story-workbook.)

Materials

Teacher
- *A Strange Country* Story-Workbook
- *IG-I Workbook Poster #1*
- Tape
- Red marker
- Colored chalk

Student
- *A Strange Country* Story-Workbook
- Unlined paper
- Colored pencils, pens, or crayons

Advance Preparation: If you wish to make the poster reusable, laminate it.

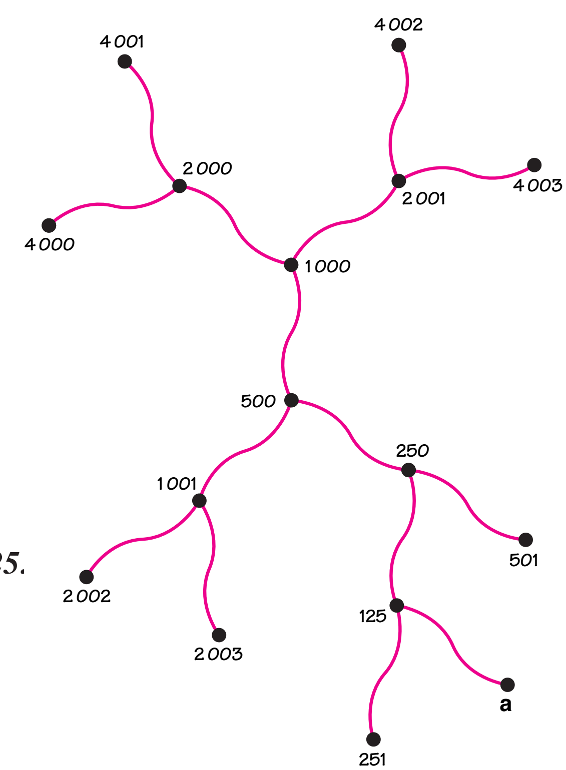
Description of Lesson

Distribute the students' copies of the story-workbook and unlined paper. Ask the class to recall the story of *A Strange Country*. Especially recall the rule on page 7 and use it to find the next neighbors of a few different whole numbers.

Page 17

Draw the picture from page 17 on the board.

T: ***Copy this drawing of the neighborhood of 1,000 on your paper. We will label these dots on the board, and while we are doing that, you should expand the neighborhood on your paper.***



Do not write the letter on the board. It is here just to make the description of the lesson easier to follow.

While students are working, invite individuals to label dots, two at a time, in the picture on the board. This should provide you with a good opportunity to help some students who are having trouble getting started. Expand the picture on the board and continue to invite students to label dots. When your picture looks similar to the one here, initiate some discussion.

T (pointing to **a**)**:** ***250 and 251 are next neighbors of 125. Which other number is a next neighbor of 125?***

S: ***62.***

T: ***How do you know?***

S: ***I subtracted 1 from 125 and halved 124.***

S: ***(2 x 62) + 1 = 125.***

T: ***Look at your picture. Tell me a number that is in your picture and that is greater than any of the numbers in the picture on the board.***

S: ***8,000.***

T: ***Do any of you have a number greater than 8,000 in your picture?***

S: ***16,014.***

T: ***Now tell me a number in your picture that is less than any of the numbers in the picture on the board.***

S: ***31.***

T: ***Does anyone have a number less than 31?***

S: ***3 (1 or 0).***

T: ***Would you agree that every whole number is directly connected to three other whole numbers?***

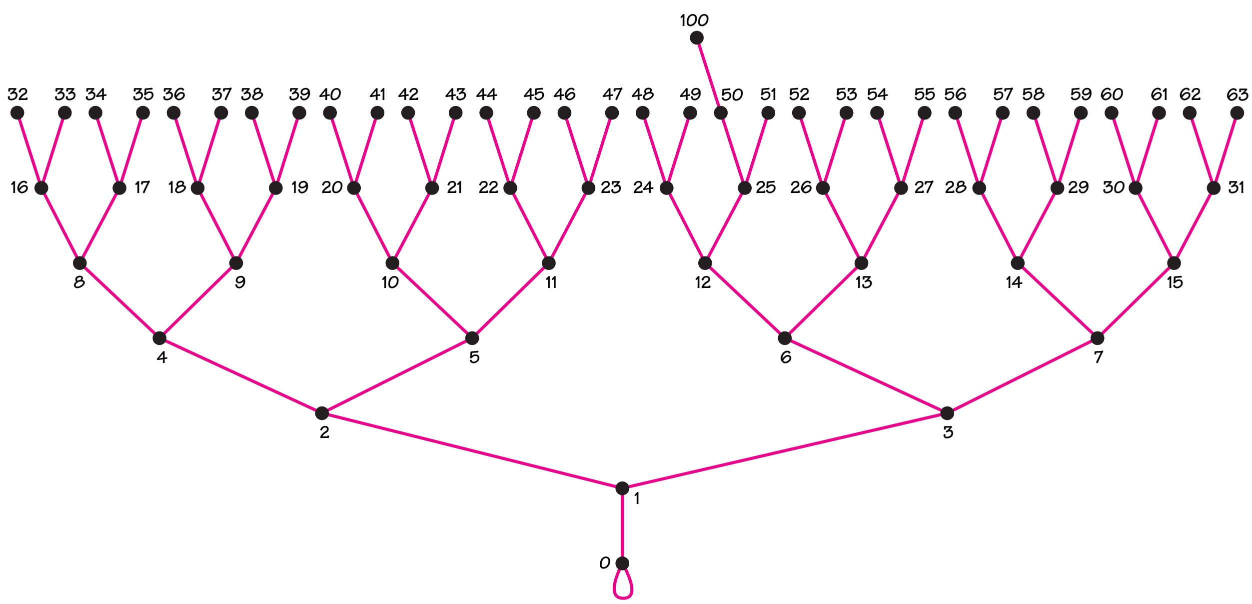
S: ***That's true for every whole number except 0. 0 is connected to 1 and itself.***

T: ***For a whole number other than 0, how many of its next neighbors are more than it? How many are less?***

S: ***Two are more and one is less.***

Pages 18–20

Read pages 18 and 19. Ask students to draw a picture of a neighborhood of 0 on a piece of paper instead of on page 19 and to try to extend it far enough to include 100. When many students have 100 in their pictures, tape *Workbook Poster* #1 to the board.



T: ***Look at your pictures. How can we go from 0's house to 7's house following paths? Who do we visit along the way?***

S: ***1 and 3.***

Trace the walk from 0 to 7 on the poster.

Repeat this activity asking how to go from 0's house to 20's house and how to go from 0's house to 100's house. Do not expect all students to have 7, 20, and 100 in their pictures. Trace each walk on the poster as students name the numbers it visits along the way.

Page 20 shows a neighborhood of 0 but it is not as extensive as that on the poster. Keep the poster on the board. You may want to suggest that students locate the whole numbers in sequence (0, 1, 2, 3, 4, 5, …) on the poster.

Pages 21–22

Let students work on pages 21 and 22 individually or with a partner. Ask students who finish quickly to build a walk between 26 and 118 (the walk visits 13, 6, 3, 7, 14, 29, and 59).

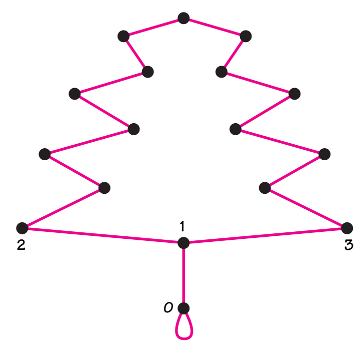
Review answers to the problems on pages 21 and 22 collectively. (See answer key.)

Pages 23–27

Ask a student to read page 23 aloud. Invite comments on the pictures on pages 24 to 27. Students should notice the levels or rings of numbers in the pictures. You can ask students to predict which ring of numbers will include 1,000.

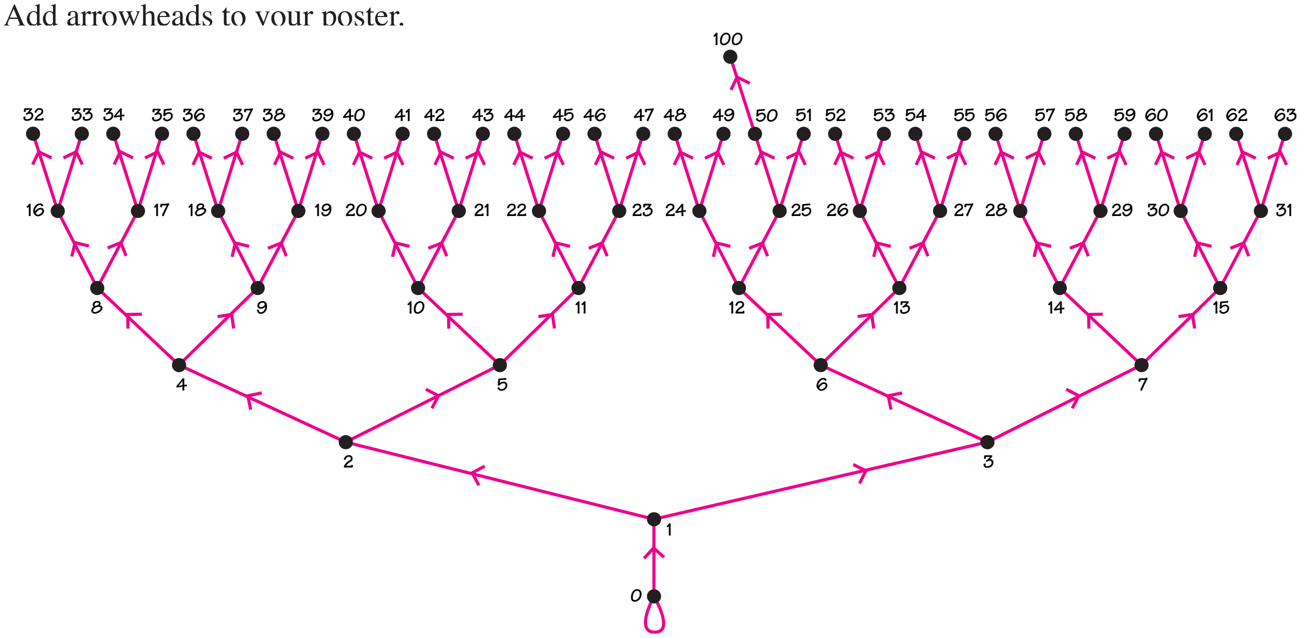
Pages 28–31

Invite students to read pages 28 and 29 aloud. Draw the picture from page 28 on the board close to the poster of 0's neighborhood.



T: ***Zero decided to add arrows to show the way to walk from home to other numbers' houses.***

Add arrowheads to your poster.



T: ***What do you notice about this picture?***

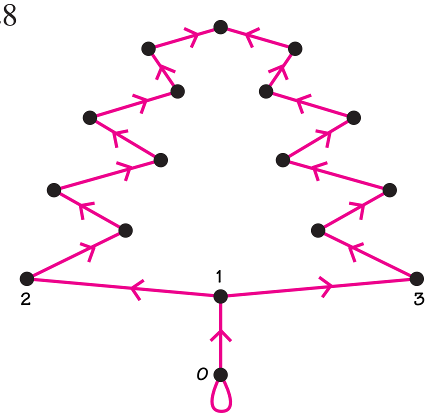
S: ***It looks like a tree.***

S: ***Each dot has one arrow ending at it and two arrows starting from it.***

Note: The loop at 0 is equivalent to an arrow that starts and ends at 0.

Accept other reasonable comments, but it is this last observation that will help in discussing the picture on page 28.

Point to the picture on the board that looks like the one on page 28

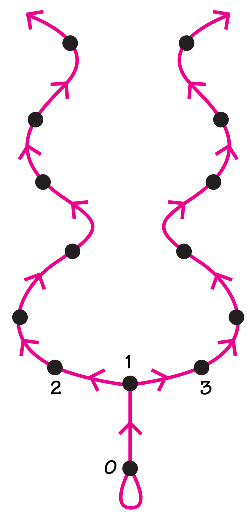


T: ***Zero could draw arrows to show the way from home to other numbers' houses in this picture as well. What's wrong with this picture?***

S: ***There are two arrows ending at the top dot.***

S: ***Earlier we found that each dot had exactly one arrow ending at it and exactly two arrows starting at it.***

Draw the picture from page 31 on the board and add arrowheads.



T: ***In this picture the arrows show the way Zero walked and the way Zero's friend walked. Will they ever meet?***

S: ***No, if they were to meet there would be a dot with two arrows ending at it, but each dot has only one arrow ending at it.***

The Rule

The houses of two numbers are directly connected by a path

if and only if

one number equals the double of the other

or

one number equals one more than the double of the other

Are the houses of these numbers directly connected?

12 and 25 ___Yes___

7 and 5 ___No___

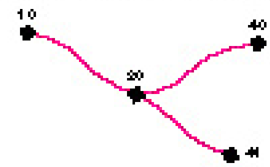
17 and 8 ___Yes___

28 and 14 ___Yes___

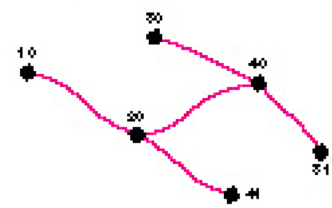
3 and 5 ___No___

27 and 13 ___Yes___

Come along. We will go for a walk in the whole numbers' country.
Let's start at 20's house. How many paths can we choose?
Who are the next neighbors of 20? Write their names in the picture.

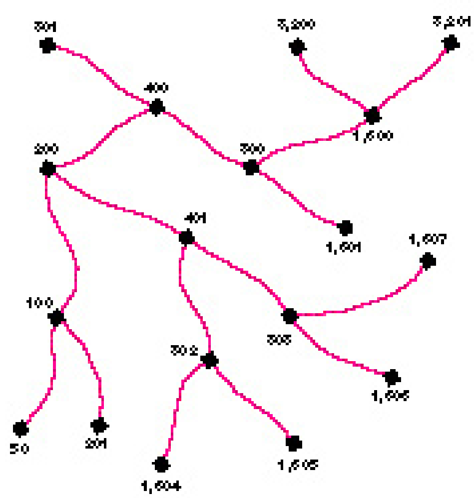


Suppose we go from 20's house to 40's house.
How can we continue our walk?
Who are the next neighbors of 40? Write their names in the picture.

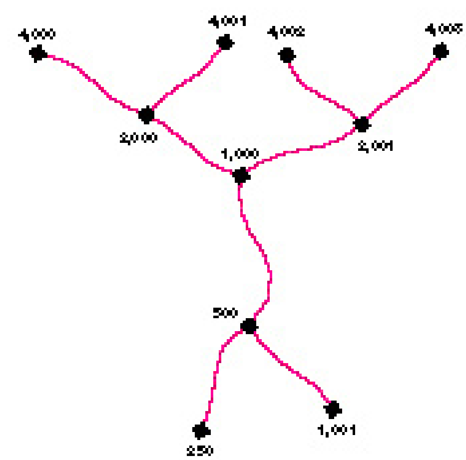


Before you turn the page, take a large sheet of paper and continue the walk along several more paths.

Here is part of a map of this fascinating country.
Can you find numbers who could live in these houses?
Many solutions are possible. Write one of them in the picture.



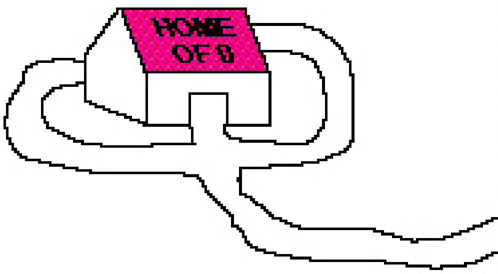
Do you know the numbers who live in this neighborhood of 1,000?
Write their names in the picture.



Do you remember that 0 is the leader of the whole numbers and that 0 invented the rule for connecting their houses?

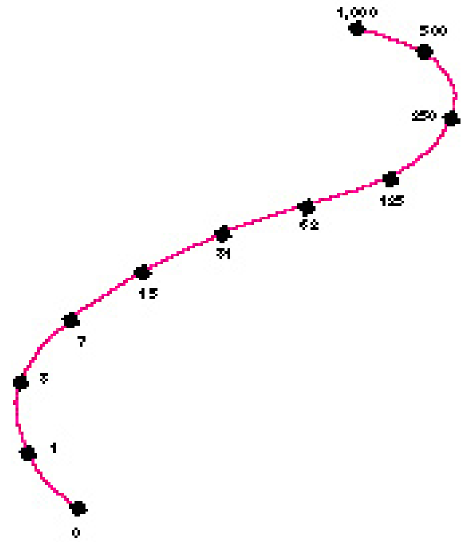
Do you know who are the next neighbors of 0?

How many are there? __One__



13

0 can easily walk from home to the house of any other number. Show below how 0 can walk to the house of 1,000.



21

There is a path from 0's house back to 0's house; a nice way for 0 to take a walk without being disturbed.

1 is the only other neighbor of 0.

$2 \times 0 = 0$ $(2 \times 0) + 1 = 1$

With the exception of 0, each number has exactly three next neighbors.

Who are the three next neighbors of 100?

__200__, __201__, __50__.

Who are the three next neighbors of 45?

__90__, __91__, __22__.

Who are the three next neighbors of 428?

__856__, __857__, __214__.

Who are the three next neighbors of 75?

__150__, __151__, __37__.

22

Capsule Lesson Summary

Ask what subtraction function each arrow could be for in an arrow road that starts at 3,786,049 and ends at 0. Begin working in the workbook *Collection of Problems #5*. (This is the first of two lessons using this workbook.)

Materials

Teacher
- Colored chalk

Student
- *Collection of Problems #5* Workbook
- Colored pencils, pens, or crayons
- Metric ruler

Description of Lesson

Draw a dot on the board and label it 3,786,049.

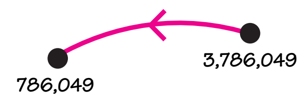
T: ***I'm going to start with this large number and take it down to 0 by subtracting. I'll tell you the result each time I do a subtraction; you tell me what number I subtracted.***

Who can read this number (3,786,049)?

S: ***Three million, seven hundred eighty-six thousand, forty-nine.***

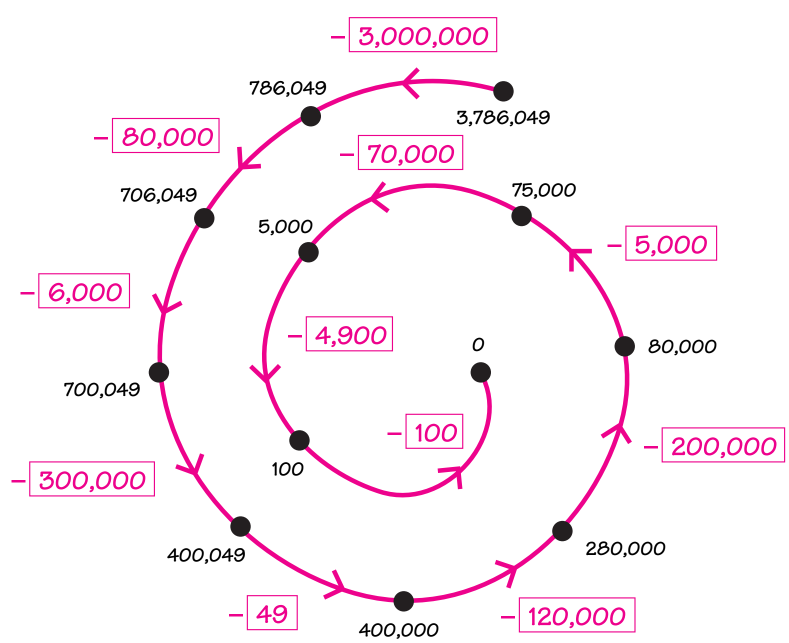
T: ***Now I'll subtract.***

What could this arrow be for? What subtraction did I do?



S: ***–3,000,000.***

Continue this activity, drawing one arrow at a time until you reach 0. Use this opportunity to let students practice reading large numbers.



W14

Distribute copies of the workbook *Collection of Problems #5* and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of the lesson, collect the workbooks for your review. The workbook will be used again in Lesson W15.

Capsule Lesson Summary

Find all of the [+] [1][0] [=] … arrows that can be drawn between pairs of given numbers. Continue individual work in the workbook *Collection of Problems #5*. (This is the second of two lessons using this workbook.)

Materials

Teacher

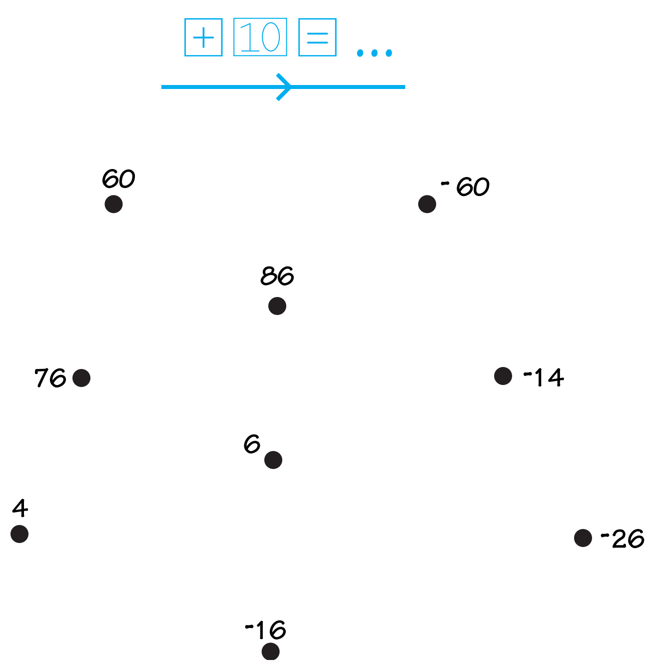
- Colored chalk

Student

- Calculator
- *Collection of Problems #5* Workbook
- Colored pencils, pens, or crayons
- Metric ruler

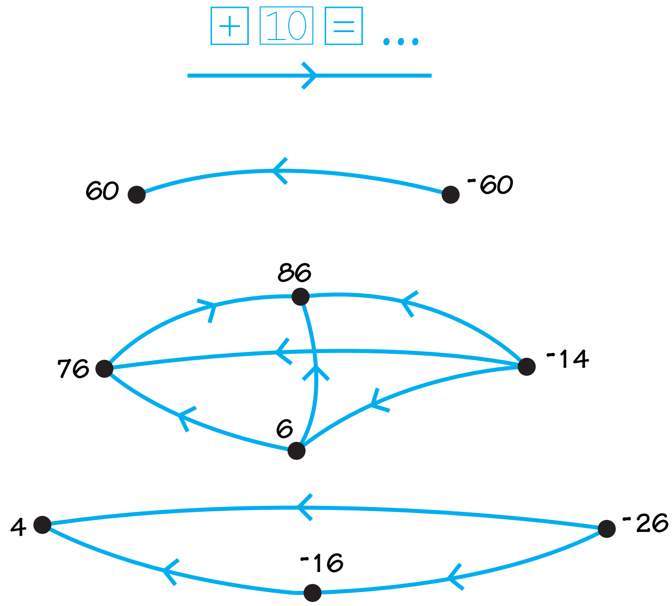
Description of Lesson

Distribute calculators and draw this picture on the board.



Let the class find all of the possible blue arrows that can be drawn between pairs of these numbers. When a student suggests that an arrow can be drawn, ask that student to convince the class. For example, a student might argue that the difference between 6 and 86 is 80, and 80 is a multiple of 10. For another example, a student might say that if you put $^{-}16$ on the calculator and press [+] [1][0] [=] [=], you will reach 4. If students disagree as to whether an arrow can be drawn, ask them to use calculators to find out.

You may need to review how to put a negative number on the calculator. To put ⁻16 on the calculator, press [0] [−] [1][6] [=] or [1][6] [+/−] if the calculator has a "change sign" key.

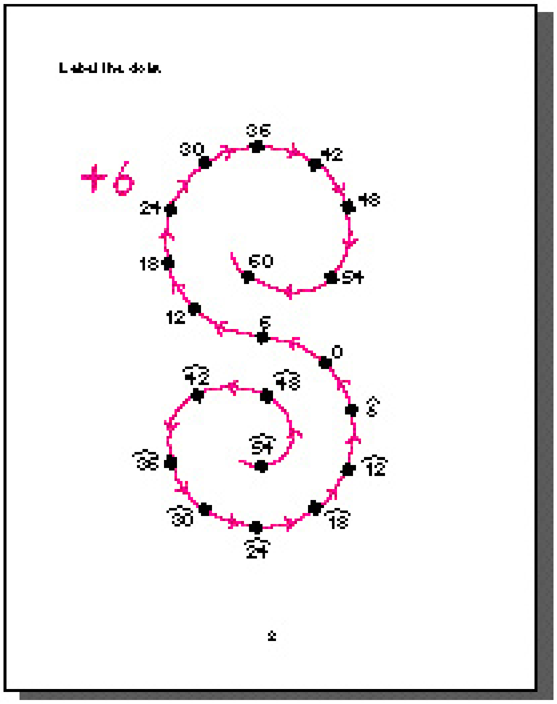


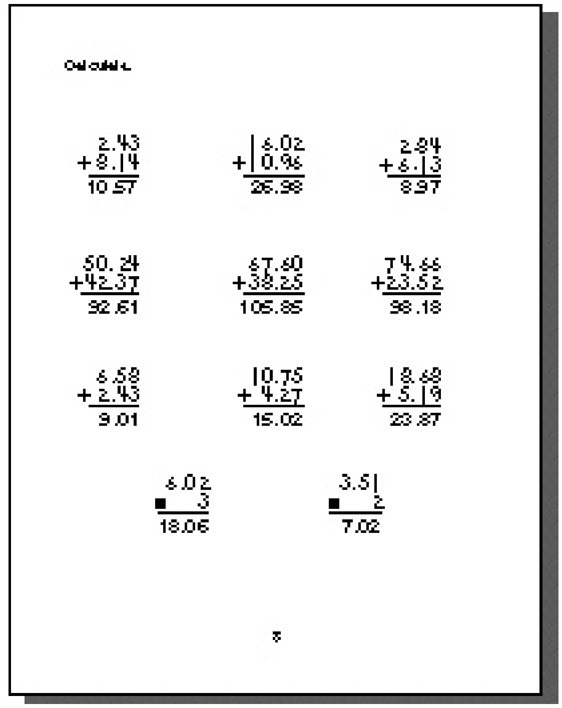
Return students' copies of the workbook *Collection of Problems #5*. Ask students first to correct or complete pages they worked on the previous week and then continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

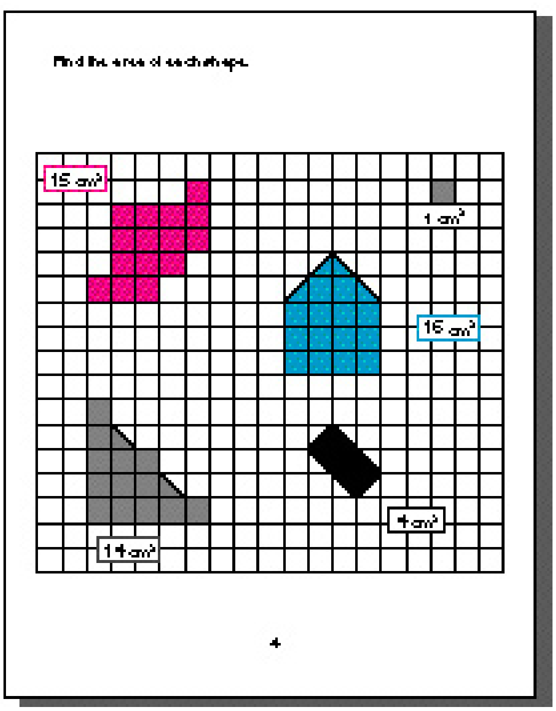
At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in their workbooks at school during a study time or at home as an assignment.

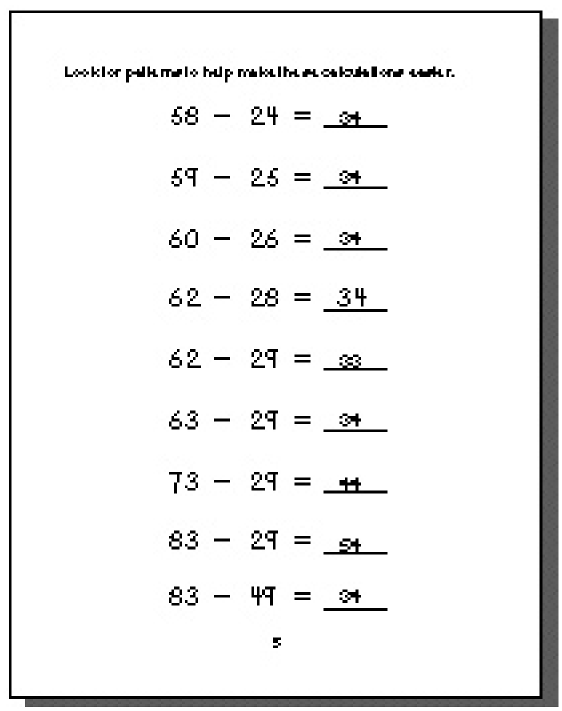
Assessment Activity

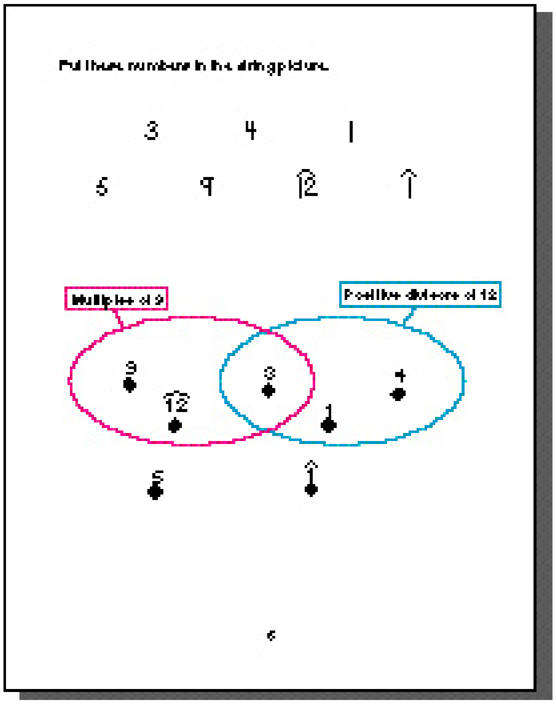
An individual student progress record for the workbook is available on Blackline W15. You may like to use this form to monitor student work.

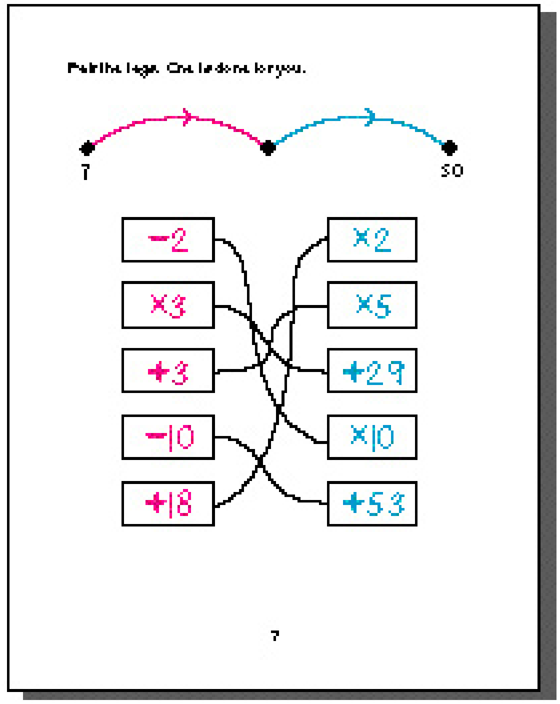

Label the dots.
+6
0
6
12
18
24
30
36
42
48
54
60
2

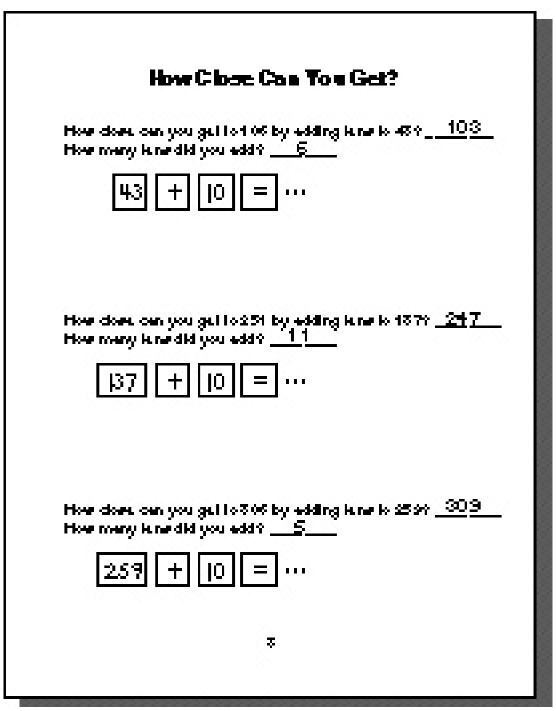

Calculate.
2.43 + 8.14 = 10.57
16.02 + 10.96 = 26.98
2.84 + 6.13 = 8.97
50.24 + 42.37 = 92.61
67.60 + 38.25 = 105.85
74.66 + 23.52 = 98.18
6.58 + 2.43 = 9.01
10.75 + 4.27 = 15.02
18.68 + 5.19 = 23.87
6.02 ■ 3 = 18.06
3.51 ■ 2 = 7.02
3

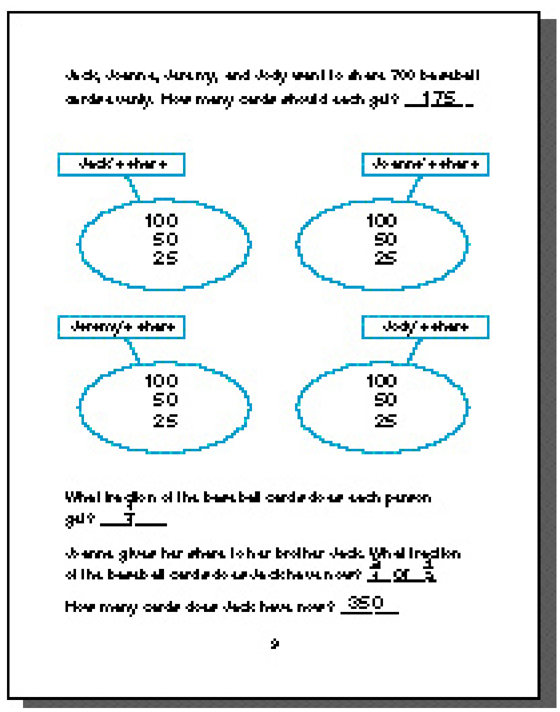

Find the area of each shape.
15 cm²
1 cm²
16 cm²
4 cm²
14 cm²
4

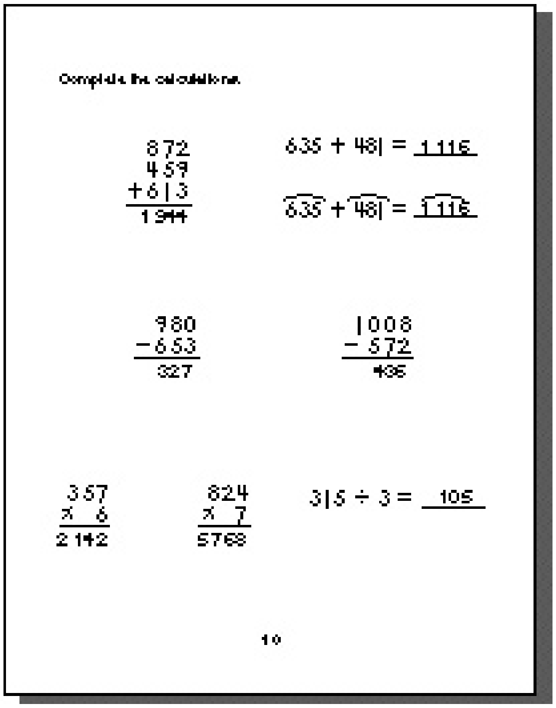

Look for patterns to help make these calculations easier.
58 − 24 = 34
59 − 25 = 34
60 − 26 = 34
62 − 28 = 34
62 − 29 = 33
63 − 29 = 34
73 − 29 = 44
83 − 29 = 54
83 − 49 = 34
5

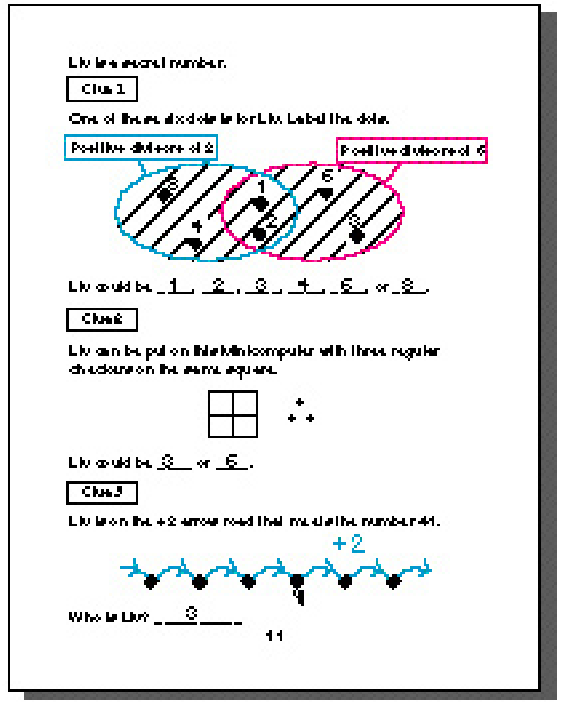

3
4
1
6
9
12
7
9
12
3
4
1
6
7
6

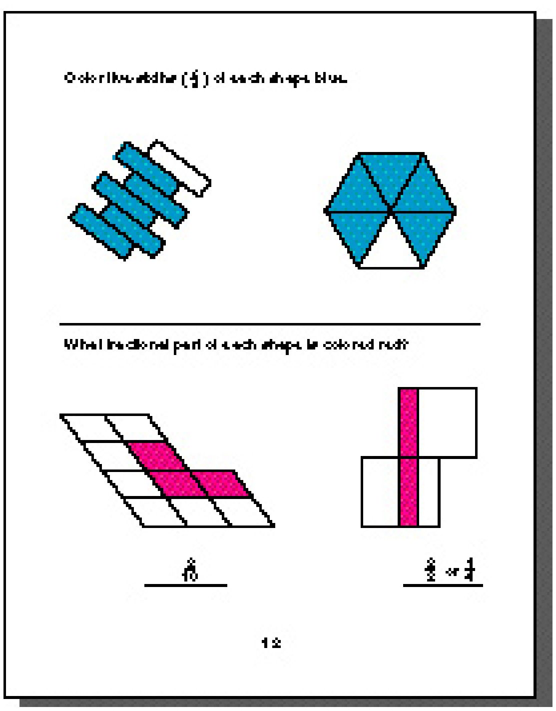

7
50
−2
×3
+3
−10
+18
×2
×5
+29
×10
+53
7

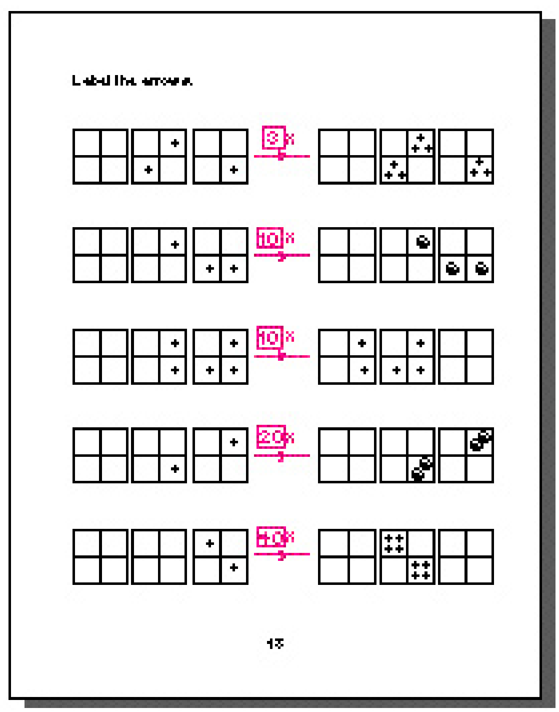

How Close Can You Get?
103
5
43 + 10 = …
247
11
137 + 10 = …
309
5
259 + 10 = …
8


175
100
50
25
100
50
25
100
50
25
100
50
25
350
9

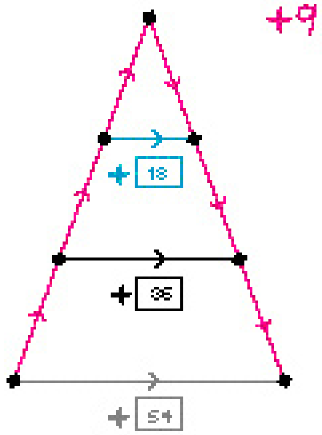

Complete the calculations.
872
459
+613
1944
635 + 481 = 1116
635 + 481 = 1116
980
−653
327
1008
− 572
436
357
× 6
2142
824
× 7
5768
315 ÷ 3 = 105
10


Liv is a secret number.
Clue 1
One of these dots is for Liv. Label the dots.
Positive divisors of 8
Positive divisors of 6
Liv could be 1, 2, 3, 4, 6, or 8.
Clue 2
Liv can be put on the Minicomputer with three regular checkers on the same square.
Liv could be 3 or 6.
Clue 3
Liv is on the +2 arrow road that meets the number 41.
+2
Who is Liv? 3
11

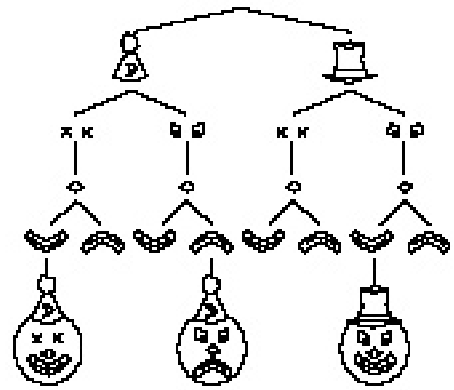

Color five-sixths (5/6) of each shape blue.
What fractional part of each shape is colored red?
4/16
2/8 or 1/4
12


Label the arrows.
3x
10x
10x
20x
40x
13

Label the blue, black, and grey arrows.



This tree shows some ways to design a clown's face. There are two choices of hats, two choices of eyes, one choice of nose, and two choices of mouths. Following the tree, draw three of the possible clown faces on the ovals provided.

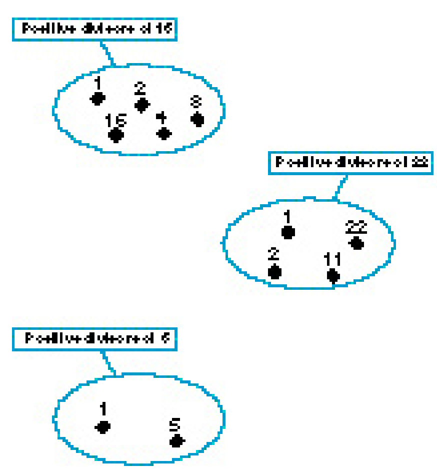


How many different clown faces could you draw with these choices? ___8___

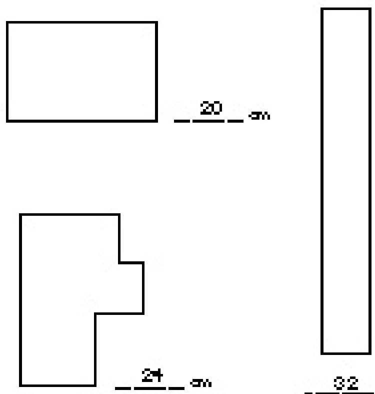
Fill in the boxes.

15 × 23 = 345	10 × 17 = 170
16 × 23 = 368	2 × 17 = 34
17 × 23 = 391	12 × 17 = 204
18 × 23 = 414	20 × 17 = 340
2 × 35 = 70	4 × 29 = 116
20 × 35 = 700	40 × 29 = 1160
7 × 8 = 56	6 × 9 = 54
7 × 80 = 560	6 × 90 = 540
70 × 8 = 560	60 × 9 = 540

Write all the positive divisors of each number in the appropriate strings.



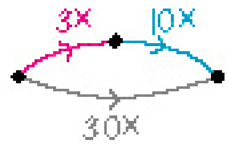
Use a ruler to find the perimeter of each shape.



20 cm

24 cm

32 cm



Do these calculations. This arrow picture can help you.

30 × 12 = 360

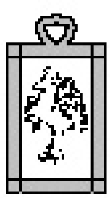
30 × 45 = 1350

30 × 91 = 2730

30 × 80 = 2400

30 × 0.70 = 21

Vanessa is making ribbon frames for some pictures. For each frame, she needs the following:



two pieces 4 cm long (top and bottom)
two pieces 6 cm long (sides)
one piece 3 cm long (loop)

Vanessa wants to make 14 frames. How many pieces of each length does she need to cut?

28 pieces 4 cm long
28 pieces 6 cm long
14 piece 3 cm long

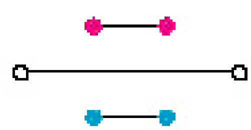
How many centimeters of ribbon does she need altogether? 322 cm
Show your work below.

One frame needs 8 cm + 12 cm + 3 cm = 23 cm ribbon

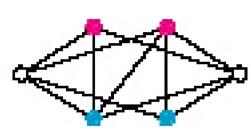
14 frames need 14 × 23 cm = 92 cm + 230 cm = 322 cm

Other methods of calculating the total amount of ribbon are possible.

How many ways are there to select two marbles of the same color from a set with two white, two red, and two blue marbles? 3 Show them below.



How many ways are there to select two marbles of different colors? 12 Show them below.

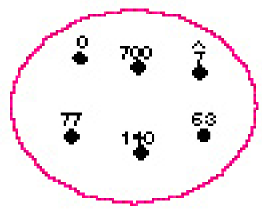


If two marbles are selected randomly, what is the probability that they will be the same color? 3/15 or 1/5

What is the probability that they will be different colors? 12/15 or 4/5

Label the circle with whole numbers so that

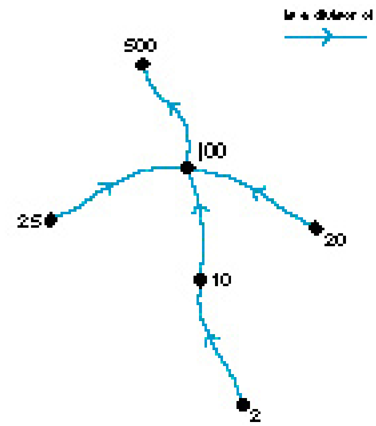
- all the numbers are multiples of 7;
- at least one number is greater than 100; and
- exactly two numbers are between 50 and 70.



Many solutions are possible.

22

Label the dots. Many solutions are possible.



23

Carnival Rides

Cynthia has $6.00 to spend on carnival rides. Most rides cost 75¢ and a few rides cost 45¢. How could she spend her money? Cynthia wants to have no more than 40¢ left over. Find several solutions.

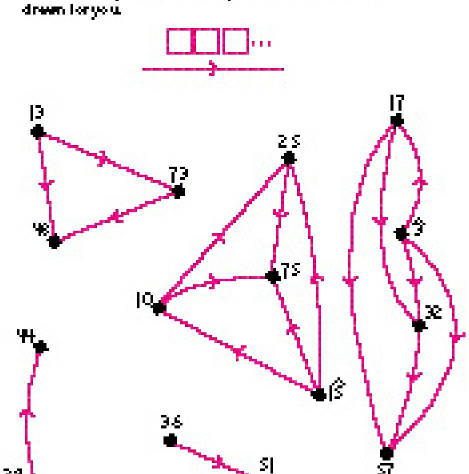
75¢ Rides	45¢ Rides	Money Left Over
8	0	0
7	1	30¢
6	3	15¢
5	5	0
4	6	30¢
3	8	15¢
2	10	0
1	11	30¢
0	13	15¢

Cynthia chooses the same number of 75¢ rides and 45¢ rides. How many of each does she ride? __5__

Does she have any money left over? __No__

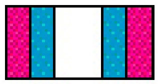
24

Draw as many red arrows as possible. One arrow is drawn for you.



25

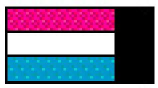
These are the two flags that the Around the World Club decided they liked best for their clubhouse. They will vote next week on which one to use.



One-third of this flag is white.

What fraction of the flag is blue? 1/3

What fraction of the flag is red? 1/3



One-fourth of this flag is black.

What fraction of the flag is blue? 1/4

What fraction of the flag is red? 1/4

What fraction of the flag is white? 1/4

26

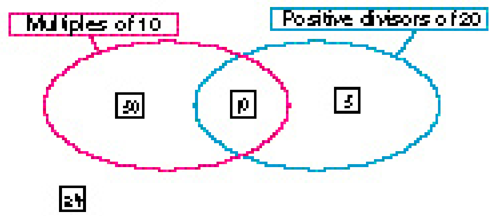
The red tag is one of these:

Multiples of 2
Multiples of 3
Multiples of 5
Multiples of 10
More than 10
Less than 60
Positive divisors of 12
Positive divisors of 20

The blue tag is one of these:

Multiples of 2
Multiples of 3
Multiples of 5
Multiples of 10
More than 10
Less than 60
Positive divisors of 12
Positive divisors of 20

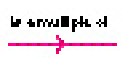
Label the strings.

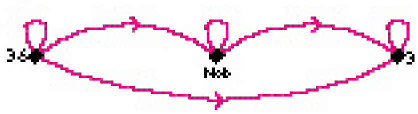


27

Nob is a secret positive number.

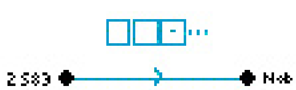
Clue 1





Nob could be 18, 12, 9, or 6.

Clue 2



Who is Nob? 18

28

The 1948 Presidential Election

The chart below gives the results of the 1948 election for the president of the United States.

Truman

Presidential Candidate	Party	Popular Votes	Electoral Votes
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	—

1. Who received the most popular votes? Truman
2. Who received the most electoral votes? Truman
3. Did Truman receive more than half of the popular vote? No

   Did he receive more than half of the electoral votes? Yes

   Explain your answers below.

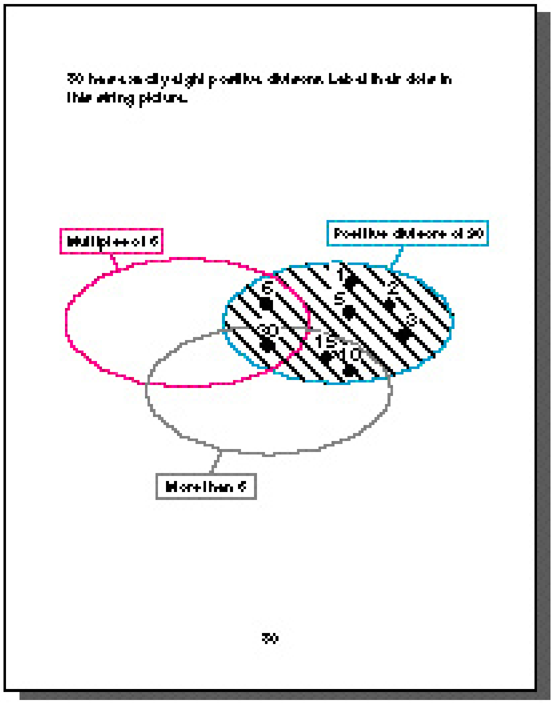
Total Popular Votes – 48 504 057

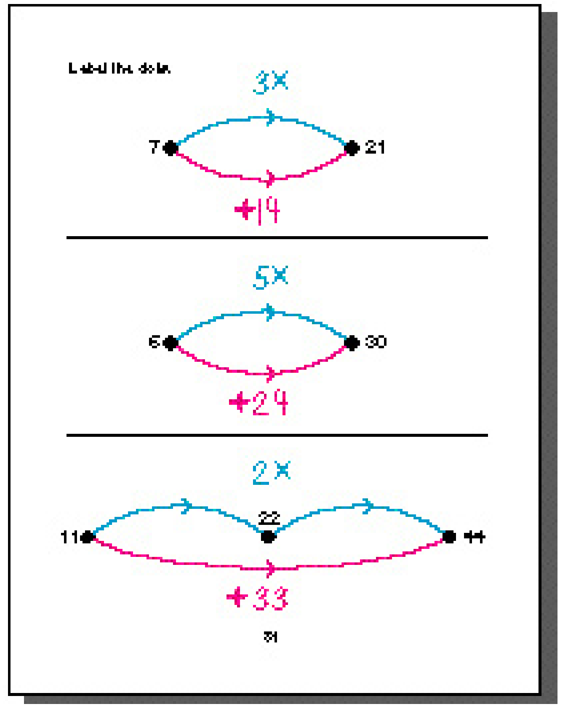
24 179 345 < ½ x 48 504 057

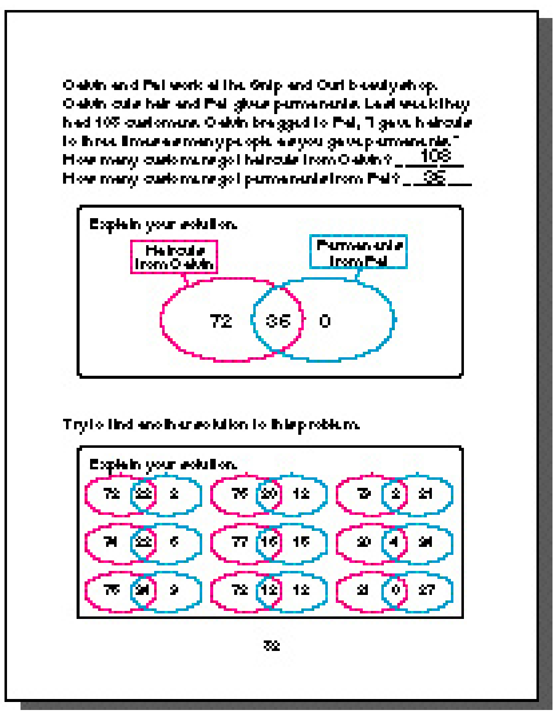
Total Electoral Votes – 531

303 > ½ x 531

29


Multiples of 6
Positive divisors of 30
More than 6
1
2
3
5
6
10
15
30


Label the dots.
3×
7
21
+14
5×
6
30
+24
2×
11
22
44
+33
31


108
36
Explain your solution.
72
36
0
Explain your solution.
72
12
21
77
15
4
12
12
0
27