The World of Numbers

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By now, veteran *CSMP* students have had a rich variety of experiences in the World of Numbers. They have met and become familiar with various kinds of numbers, and with operations and relations on them. They have encountered positive and negative integers, decimal numbers, fractions, numerical functions (such as 5x, +3, ÷10, –5, ½x), order relations (such as < and >), and the notion of multiples of a given number. They have been introduced to paper-and-pencil algorithms for addition of whole numbers and decimal numbers, and for subtraction of whole numbers. Students have also had extensive experience using systematic methods for multiplication and division of whole numbers in preparation for algorithms.

In *CSMP Mathematics for the Intermediate Grades*, *Part I*, these earlier numerical experiences will be revisited, extended, and deepened through familiar games and activities, as well as in fascinating new situations. As always, *CSMP* stresses the unity and continuity of growth of mathematical ideas and concepts. The program's spiral approach does not require mastery of each lesson, but rather allows students to encounter the elements of each content strand in different situations throughout the year. It is important to recognize this approach consciously. If you strive for mastery of each single lesson, you will find yourself involved in a great deal of redundancy as the year progresses.

Further, *CSMP* presents the content in a situational framework. That is, a "pedagogy of situations" engages students in rich problem-solving activities as they construct mathematical ideas. These situations offer opportunities both to develop necessary numerical skills and to gain deeper understanding of mathematical concepts in the world of numbers. At the same time, the situations presented encourage students to develop patterns of logical thinking and strategies for attacking problems.

Perhaps the most important embodiments of the *CSMP* approach are the non-verbal languages and tools used throughout the program. These are vehicles that allow students to investigate the contexts in which the content is presented and to explore new mathematical ideas. It is hard to overstate the value of developing languages and tools that are not confined to one area of mathematical content or to one level of the development of content; that aid in attacking problems as well as in representing situations. Armed with the universally applicable languages of the *CSMP* curriculum, students grow more and more familiar with the syntax of these languages and are free to explore new content as extensions rather than think of each new mathematical idea as tied to a certain new language. This is not to say that *CSMP* students do not learn the usual descriptive language of mathematics; naturally, they do. However, in the *CSMP* approach the usual descriptive language is not a requisite for learning new concepts, but only a means for succinctly describing those ideas as they are being explored.

Standard Algorithms of Arithmetic

CSMP seeks to develop basic numerical skills as well as an understanding of the underlying mathematical ideas. We are fully in agreement with the thesis that, along with the growth of understanding of the world of numbers, there must be a concomitant growth of familiarity and facility with numbers and operations on them. But facility should not be confused with understanding; they are partners in the growth of mathematical maturity. A balanced growth of each must be maintained, neither being sacrificed for the other.

Students must eventually learn mechanical algorithms for the basic operations (addition, subtraction, multiplication, division). However, the development of these methods should occur only after students have had many experiences with prerequisite concepts. Premature presentation of these algorithms may actually inhibit a student's desire and ability to develop alternative algorithms, to do mental arithmetic, or to estimate.

CSMP believes that students should be able to solve a problem such as 672 ÷ 4 using models, pictures, or mental arithmetic before being introduced to a division algorithm. Even after students have mastered an algorithm, they should be aware that alternative methods are often more appropriate. For example, consider the problem of calculating 699 x 9. Rather than using a standard multiplication algorithm, it may be easier and more efficient to note that 700 x 9 = 6 300, so that 699 x 9 = 6 300 – 9 = 6 291. Indeed, built into this way of approaching the problem is an excellent estimate (6 300) of the product. To insist on a mechanistic response to such a problem would be to encourage inefficiency and might also inhibit the development of the flexibility necessary for problem solving. On the other hand, a rich array of situations in which students interact with numbers provides them with opportunities to gain the necessary facility with standard algorithmic procedures while retaining the openness required to respond creatively to new situations in the world

Content Overview

Subtraction

In *CSMP Mathematics for the Upper Primary Grades*, students learned to solve many subtraction problems through mental arithmetic. Arrow pictures suggested ways to change difficult problems into a sequence of simpler subtractions. For example, to calculate 63 – 28 students learned they could use any of these calculations: 63 – 20 – 8; 63 – 30 + 2; or 63 – 23 – 5. Mental arithmetic activities and patterns (for example, 63 – 28 = 65 – 30 = 35) eventually led to the development of a standard paper-and-pencil algorithm for subtraction, the method of compensation or equal additions.

$$\begin{array}{r} {}^{10} \\ 63 \\ {}^{1} \\ -\,28 \\ \hline 35 \end{array} \qquad \begin{array}{r} {}^{10\,10} \\ 406 \\ {}^{1\,1} \\ -\,279 \\ \hline 127 \end{array}$$

If you are unfamiliar with this method of subtraction, refer to *CSMP Mathematics for the Upper Primary Grades, Part IV* or to a mathematics methods textbook for elementary school teachers.

A goal of this strand is to both reinforce and broaden the students' experiences with subtraction. To review subtraction, students will encounter the operation in many contexts: the *Minicomputer Golf* game, *Guess My Rule* activities, story applications, and arrow problems. This variety of activities provides for practice with an algorithm, subtraction facts, and subtraction patterns. Other lessons extend the concept of subtraction to include decimal numbers.

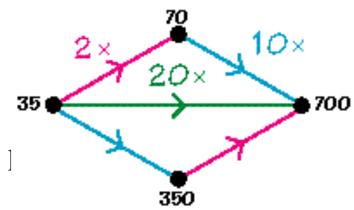
Lessons: N3, 4, 5, 6, 7, 8, 10, 13, 18, 19, 21, 22, 25, 29, 31, 32, and 35

Multiplication

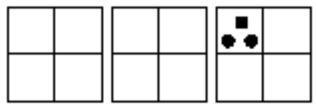
The lessons on multiplication in this strand illustrate how arrows, the Minicomputer, the calculator, and strings all aid the development of one concept. Situations like “Tag the Arrows” provide practice with basic multiplication facts within problem-solving situations.

Arrows also support mental arithmetic techniques, as illustrated here:

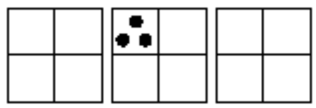
This example shows 20 x 35 calculated using composition:
2 x 35 = 70 and 10 x 70 = 700, or 10 x 35 = 350 and 2 x 350 = 700.



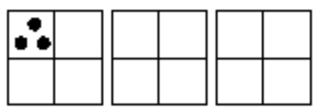
In this semester, students use the Minicomputer to review multiplication



$= 3 \times 8 = 24$



$= 3 \times 80 = 240$



$= 3 \times 800 = 2\,400$

There are many opportunities for students to use number classifications, such as “multiples of 4,” and thereby work with multiplication facts. Further, *Guess My Rule* and calculator activities include use of multiplication facts.

The above experiences provide support for students to gain proficiency in both a long and a compact form of a pencil-and-paper multiplication algorithm:

$$\begin{array}{r} 568 \\ \times\ 4 \\ \hline 32 \\ 240 \\ 2\,000 \\ \hline 2\,272 \end{array} \qquad \begin{array}{r} {\scriptstyle 2\ 3} \\ 568 \\ \times\ 4 \\ \hline 2\,272 \end{array}$$

Besides reinforcing multiplication patterns and developing an algorithm, lessons in this strand also extend the students’ understanding of multiplication. Story situations provide applications for multiplication and patterns such as these: 14 x 15 = 210 therefore 15 x 15 = 210 + 15 = 225 and 13 x 15 = 210 – 15 = 195.

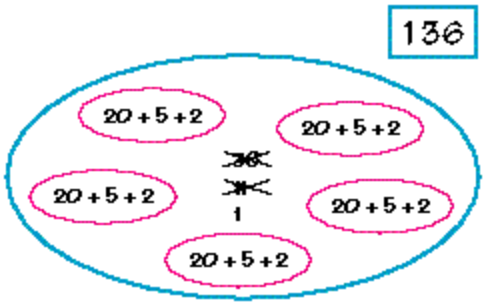
Lessons: N1, 2, 3, 5, 7, 9, 11, 14, 15, 16, 18, 21, 23, 26, 27, 29, 32, 33, and 35

Division

WORLD OF NUMBERS INTRODUCTION

The lessons in this strand provide a rich variety of situations involving division that reinforce and deepen earlier experiences, broadening them to include new applications and to involve decimal numbers.

Having already been introduced to division both as a sharing process (for example, Share 108 storybooks equally among three classes) and as a repeated subtraction process (for example, How many nines are in 200?) students continue to develop systematic techniques for division. Strings and charts naturally support the concepts of sharing, as illustrated here.



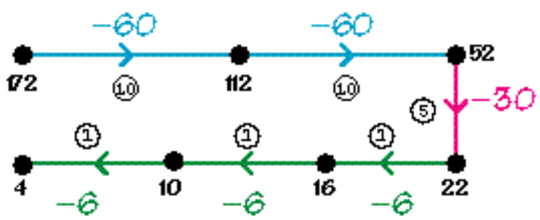
$$\begin{array}{rl} 136 & \\ -100 & (5 \times 20) \\ \hline 36 & \\ -25 & (5 \times 5) \\ \hline 11 & \\ -10 & (5 \times 2) \\ \hline 1 & \end{array}$$

First Row	Second Row	Third Row	Fourth Row	Fifth Row
20	20	20	20	20
5	5	5	5	5
2	2	2	2	2

$$5\overline{)136} \quad 27\ R = 1$$

$$(5 \times 27) + 1 = 136$$

Arrow roads may better illustrate division as repeated subtraction.



$$6\overline{)172} \quad 28\ R = 4$$

These systematic techniques of solving division problems eventually lead to the development of a standard division algorithm. But for now, these methods allow students to gain insight into the concept of division as they gradually learn more efficient means to solve the problems.

Besides the above applications and algorithm development, the lessons in this strand also introduce the concept of *divisors* of a number.

Lessons: N5, 7, 12, 16, 18, 20, 23, 28, 29, 30, 32, and 34

Negative Integers

CSMP introduces negative integers in first grade through a story about Eli the Elephant and magic peanuts. The story leads to a model for adding integers, first by pictures, then also on the Minicomputer. By the end of third grade, *CSMP* students have encountered negative numbers in games, in reading outdoor temperatures, in applications such as elevator problems, and in recording statistics. These experiences extend the concept of order from whole numbers to negative numbers.

The activities in this strand increase the students' familiarity with negative numbers in many contexts. The goal is to portray negative integers not as a strange new set of numbers, but as a natural and necessary extension of counting numbers. Therefore, few lessons focus on negative numbers but many lessons include them.

Negative numbers appear regularly on the Minicomputer, in *Minicomputer Golf,* and in detective stories. Arrow roads and *Guess My Rule* activities often include negative numbers. Students encounter the negative multiples of a number. Use of the number line is reinforced through midpoint problems and in a lesson about the maximum and minimum elevations of continents. Students use calculators to explore patterns with repeated subtractions of 10 (for example, 27 – 10 = 17, 17 – 10 = 7, 7 – 10 = $\widehat{3}$, $\widehat{3}$ – 10 = $\widehat{13}$, ...). Through such a variety of situations, students gradually learn the characteristics of negative numbers and accept them as "real" numbers.

CSMP employs a special notation for representing negative numbers. Traditional approaches to arithmetic often make no distinction on the printed page between the function "subtract 3" and the number "negative 3"; both are denoted by "–3." Only by context can a person discern the intended meaning of "–3." In *CSMP*, negative numbers are distinguished from subtraction in the following ways:

- The minus sign "–" is reserved for subtraction. Thus, for example, "–14" denotes the function "subtract 14."
- The ⌒ symbol denotes a negative number. Thus, "$\widehat{14}$" denotes the number "negative 14." The ⌒ symbol was introduced first in the story about Eli the Elephant.
- In context, other notations for negative numbers are recognized. For example, in calculator situations, a raised "–" is used to denote a negative number, for example, $^{-}14$.

We recommend that you continue to use the ⌒ notation for negative numbers and gradually recognize alternative notations as students begin to encounter them in other contexts (calculators, temperature, tests, and so on).

Lessons: N1, 4, 5, 8, 14, 19, 21, 25, and 31

Decimal Numbers

CSMP Mathematics for the Upper Primary Grades introduces decimal numbers, motivated both by problems involving our decimal system of money and by solving problems on the Minicomputer, such as 7 ÷ 2 = 3.5. The lessons in this strand review these activities and proceed to extend the students' experiences. The strong visual portrayal of place value on the Minicomputer makes it an appropriate tool to support the extension from whole numbers to decimal numbers. The same trades that allow students to calculate 387 – 169 and ½ x 268 on the Minicomputer now empower them to calculate 38.7 – 16.9 and ½ x 19. Through such situations, students learn to represent decimal numbers using the "dimes (tenths) board" and the "pennies (hundreths) board." The Minicomputer supports distinguishing 0.5 from 0.05 and noting that 0.5 = 0.50. The Minicomputer also strongly suggests the extension of the effect of 10x on whole numbers (10 x 37 = 370) to decimal numbers (10 x 1.68 = 16.8).

Relying heavily on a money model, students label numbers between 0 and 1 on the number line and build arrow roads involving decimal numbers; for example they use 2x arrows and –0.1 arrows to draw a road from 0.7 to 10. To further develop the students' sense of the relative magnitude of decimal numbers, one activity invites students to use trial and error techniques with calculators to solve problems such as 4 x ☐ = 149.

The goal of all of these activities is to gradually build a strong foundational understanding of decimal numbers as preparation for further concepts and computational skills. The students' abilities will progress as they become more familiar with the subtleties of our decimal place-value system.

WORLD OF NUMBERS INTRODUCTION

Lessons: N12, 14, 22, 27, 31, and 35

Fractions

CSMP Mathematics for the Upper Primary Grades introduces fractions through activities involving the sharing of equal quantities; for example:

- When three children share a cake fairly, each child receives $\frac{1}{3}$ of the cake.
- If a class of 28 students divides into two equal-sized groups, each group has $\frac{1}{2}$ x 28 or 14 students.

The Workbook strand of this volume reviews and extends students' skill in shading fractional parts of a region. The emphasis shifts from unit fractions (for example, $\frac{1}{4}$) to other fractions (for example, $\frac{3}{4}$). Similarly, in this strand, a story about a greedy monkey who eats more that his share of bananas, supports the extension from calculating $\frac{1}{7}$ x 28 to calculating $\frac{3}{7}$ x 28. Students use string pictures to do these calculations and later to solve problems about two children painting a fence.

Besides continuing the above developments, a further goal of this semester is to introduce new situations in which fractions occur. The various models will lead students to a broader understanding of the role of fractions in mathematics and in the real world.

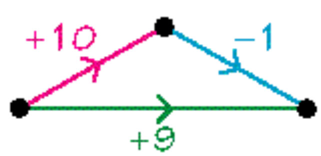
To expand the students' exposure to fractions, two lessons this semester employ a story about the legendary monster, Sasquatch, to locate fractions on a number line. The story engenders the need to find equivalent fractions and common denominators.

To support this development in the World of Numbers strand, the Probability and Statistics strand employs fractions to record probabilities. Fractions not only allow the comparison of possible events in probabilistic situations, but also facilitate the prediction of the outcome of a series of games through calculations such as $\frac{1}{3}$ x 120 = 40 and $\frac{2}{3}$ x 120 = 80.

Lessons: N17, 24, 30, and 34

Composition of Functions

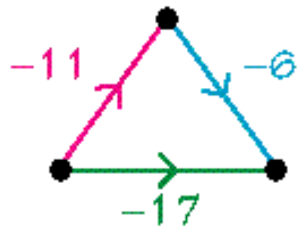
Several lessons in this strand deal with what happens when you compose a sequence of functions, that is, when you apply the functions in order, one at a time. These compositions lead to many general, powerful insights into the properties of numbers and operations.
Arrow diagrams provide a concrete means to study this abstract but practical concept. For example, it is clear that if you add 10 to any number and then you subtract 1 from the result, the net effect is to add 9 to the original number.

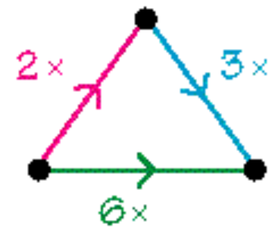


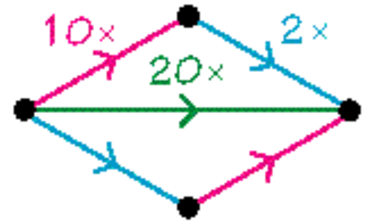
Besides succinctly depicting the composition, this arrow picture also su mentally add 9 to any number is to add 10 and then subtract 1. (For exa 96 – 1 = 95, so 86 + 9 = 95.)

Your students' extensive experience with the composition of functions in the Upper Primary Grades curriculum led them to many insights that involved the development of algorithms, the discovery of number patterns, and efficient mental arithmetic techniques. A goal in this strand is to review these discoveries and to apply composition to new situations and problems.

In conjunction with stories and the Minicomputer, several compositions are reviewed, for example:

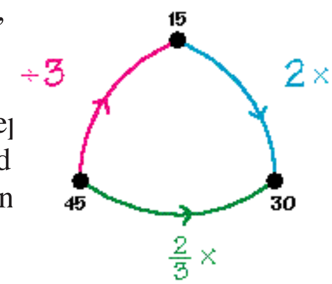




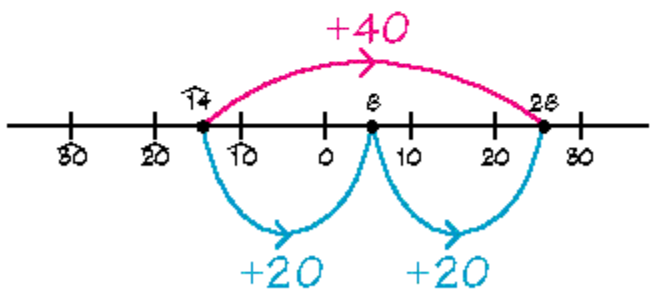


In the lessons on fractions, the “monkeys and bananas” stories suggest that composition be used to calculate, for example, $\frac{2}{3}$ x 45.

Many pairs of functions, for example, 10x and 2x, *commute*; that is, they produce the same effect regardless of which order they are applied. +10 and –1, –11 and –6, and 2x and 3x also all commute. But in the Assorted Problems there are examples that do not commute, for example, 2x and +1.



As students become familiar with the composition of functions, the concep problem-solving tool. In the game of *Minicomputer Golf*, composition aid move winning combinations. The following illustration depicts an efficien midpoint of two points (for example, $\widehat{14}$ and 26) on a number line.



The compositions of functions exemplifies how the language of arrows is able to visually highlight rich and practical mathematical concepts and techniques. A series of lessons in the Languages of Strings and Arrows strand also focuses on the composition of arrows, which supports this component of the World of Numbers strand.

Lessons: N2, 5, 6, 7, 8, 11, 13, 18, 21, 25, 26, 28, 29, 30, 31, 33, and 34

Capsule Lesson Summary

Decode various configurations of checkers on the Minicomputer. Put various numbers on the Minicomputer using both positive and negative checkers. Solve a detective story with sequential clues that involve an arrow road, a string picture, and the Minicomputer.

Materials

Teacher
- Minicomputer set†
- Colored chalk
- ⑩-checkers

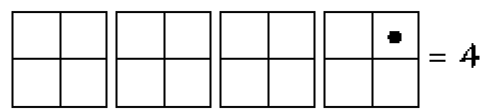
Student
- Minicomputer set†
- Worksheets N1*, **, ***, and ****

Description of Lesson

The first exercise provides a brief review of the Minicomputer and should be done at a fairly brisk pace.

Exercise 1

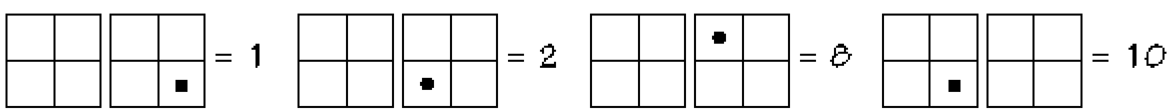
Display four Minicomputer boards and put one checker on the 4-square.



T: ***What number is this?***

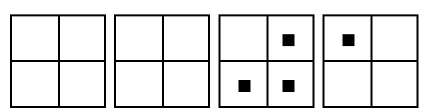
S: ***4.***

In the same way review the 1-, 2-, 8-, and 10-squares.



Continue with 20, 40, 80; 100, 200, 400, 800; and so on.

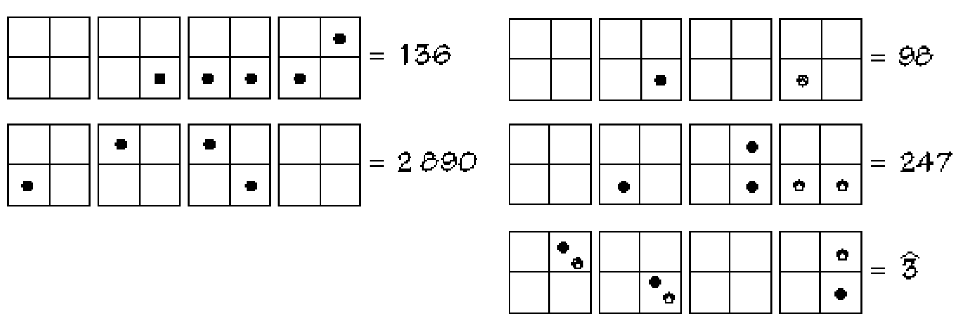
Clear the Minicomputer and put on 78.



T: ***What number is this?***

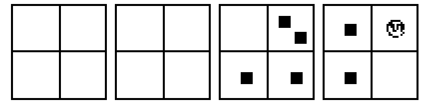
S: ***78.***

Continue this activity by asking students to decode the following configurations.



†A teacher's Minicomputer set consists of four demonstration Minicomputer boards and a sufficient number of demonstration Minicomputer checkers. A student's Minicomputer set consists of two sheets of Minicomputer boards (two boards per sheet) and cardboard checkers.

Put this configuration on the Minicomputer.

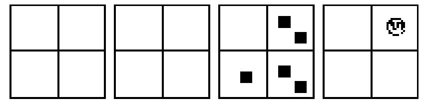


T: ***What number is this?***

If students have difficulty, ask,

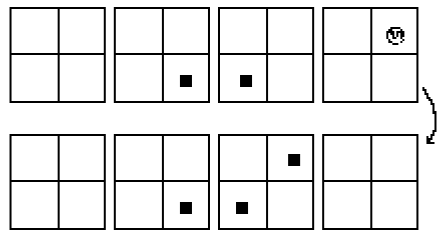
T: ***Can you show the same number with one less checker?***

Invite a student who is having difficulty to make a trade.
For example:



S: ***8 + 2 = 10.***

Continue to invite various students to make trades until an easy to read configuration is obtained.



T: ***What number is this?***

S: ***160.***

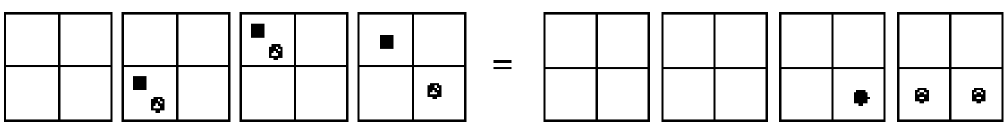
Exercise 2

This would be a good exercise for students to work in pairs. Provide each student or student pair with a desk Minicomputer set.

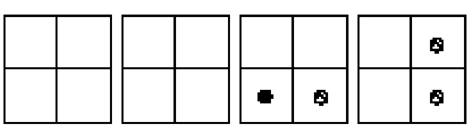
Display four Minicomputer boards.

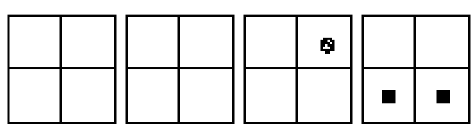
T: ***Put 7 on the Minicomputer using both positive and negative checkers.***

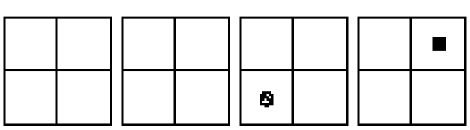
Give students a minute to respond on their desk Minicomputers before you call on someone to come to the board. There are many possible answers. Accept any correct configuration.

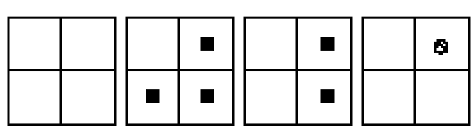
 = 7

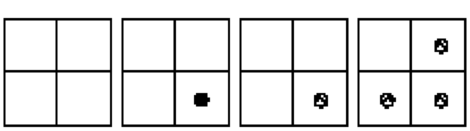
Continue this activity using the numbers suggested below or others more appropriate for the numerical abilities of your students. Only one configuration for each number is given here.

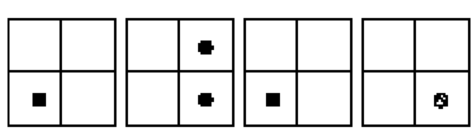
 = 5

 = $\widehat{37}$

 = $\widehat{16}$

 = 746

 = 83

 = 2 519

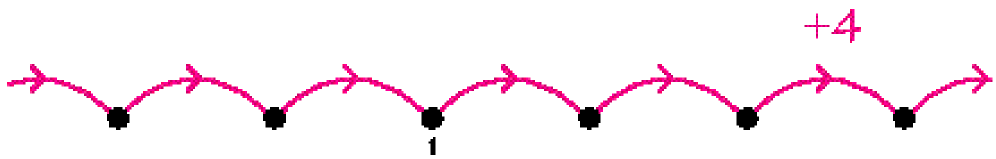
Exercise 3

You may like to begin this exercise with a discussion about detectives and using clues to discover something hidden or a secret.

Tell the class that they are going to be detectives who are trying to discover a secret number named Zap, and that you will give them clues.

Clue 1

Draw this part of a +4 arrow road on the board.



T: ***Zap is on the same +4 arrow road as 1. What numbers could Zap be?***

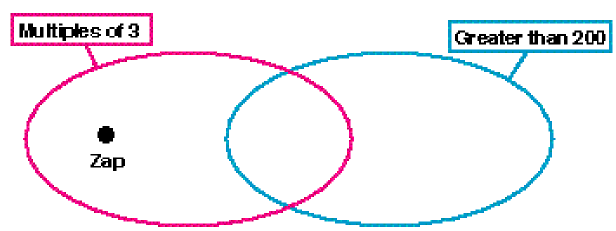
S: ***Zap could be 5, because 1 + 4 = 5.***

List the numbers that Zap could be in order, even though your students may not suggest them in order. Encourage students to find both positive and negative numbers for Zap. Also, encourage students to observe patterns in the list of possibilities for Zap. Try to put the list somewhere on the board where it can be referred to easily.

..., $\widehat{15}$, $\widehat{11}$, $\hat{7}$, $\hat{3}$, 1, 5, 9, 13, 17, 21, 25, ..., 33, ..., 93, 97, 101, ...

Clue 2

Draw this string picture on the board.



T: ***The second clue is given by this string picture. What new information does this clue give us about Zap?***

Allow a minute or two for students to study the string picture.

S: ***Zap is a multiple of 3 and is not greater than 200.***

T: ***Can you name some multiples of 3?***

Let the students name multiples of 3, both positive and negative, until it is obvious that most students understand the clue. It may be helpful to some students to remind them that the multiples of 3 are the numbers on the +3 arrow road that includes the number 0.

T: ***What numbers could Zap be?***

Circle numbers in your list that Zap could be as the students suggest them.

…, $\widehat{15}$, $\widehat{11}$, $\hat{7}$, $\hat{3}$, 1, 5, 9, 13, 17, 21, 25, …, 33, …, 93, 97, 101, …

T: ***Look at the numbers that Zap could be. Do you notice anything interesting?***

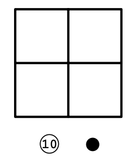
S: ***Every third number in the list is a multiple of 3.***

S: ***Two consecutive numbers that Zap could be are 12 apart.***

Students should conclude that Zap could be … , $\widehat{15}$, $\hat{3}$, 9, 21, 33, 45, ….

Clue 3

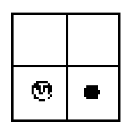
Display one Minicomputer board and these checkers.

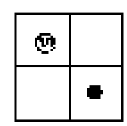


T: ***Zap can be put on this Minicomputer using exactly one ⑩-checker and one regular checker. What numbers could Zap be?***

Suggest students first put a number for Zap on their desk Minicomputers, and then invite students to put the number on the demonstration Minicomputer. As necessary, remind students that any number that Zap could be must be in the list of numbers generated by the previous clue. Be prepared to check any number that is disputed by extending the list.

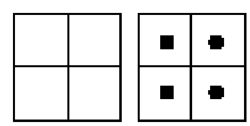
The class should conclude that Zap could be 21 or 81.


= 21


= 81

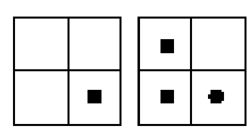
Clue 4

Display two Minicomputer boards and this configuration of checkers.



T: ***Zap can be put on the Minicomputer by moving exactly one checker from the ones board to the tens board. Who is Zap?***

Allow enough time for students to think about possible moves before calling on a student. You may ask students to make the move first on their desk Minicomputers.

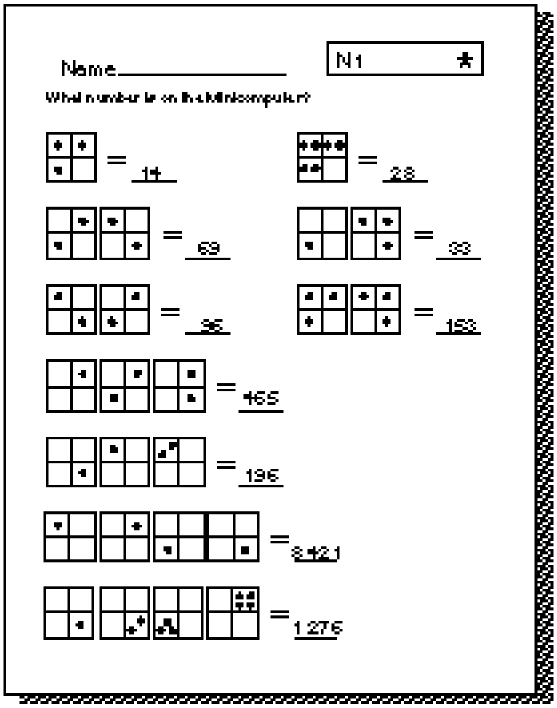

= 21

The class should conclude that Zap is 21 and that Zap cannot be 81.

Worksheets N1*, **, ***, and **** are available for individual work.

Writing Activity

You may like students to take lesson notes on some, most, or even all their math lessons. The "Lesson Notes" section in Notes to the Teacher gives some suggestions and refers to forms in the Blacklines you may provide to students for this purpose. In this lesson, for example, students may note what they should remember about the Minicomputer review. Or they may write their own detective stories to challenge you, other students, or another class.


Name________________ N1 ★

What number is on the Minicomputer?

= 14

= 28

= 69

= 33

= 96

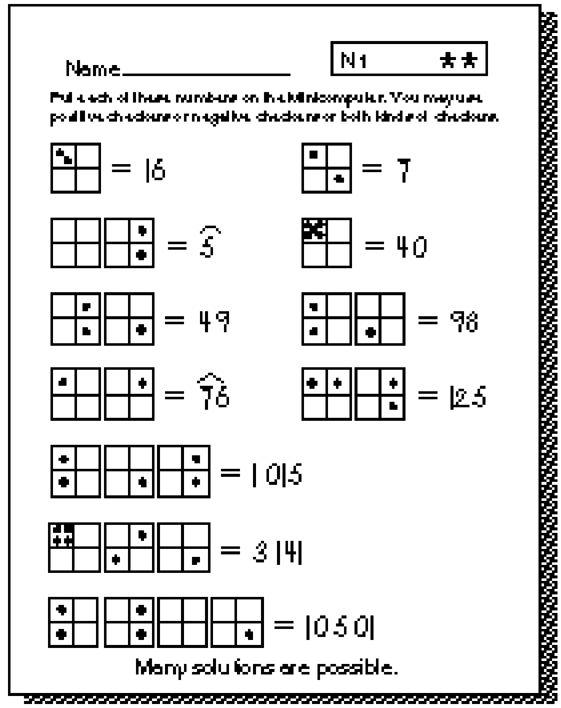
= 153

= 465

= 196

= 8421

= 1276


Name________________ N1 ★★

Put each of these numbers on the Minicomputer. You may use positive checkers or negative checkers or both kinds of checkers.

= 16

= 7

= $\hat{5}$

= 40

= 49

= 98

= $\hat{76}$

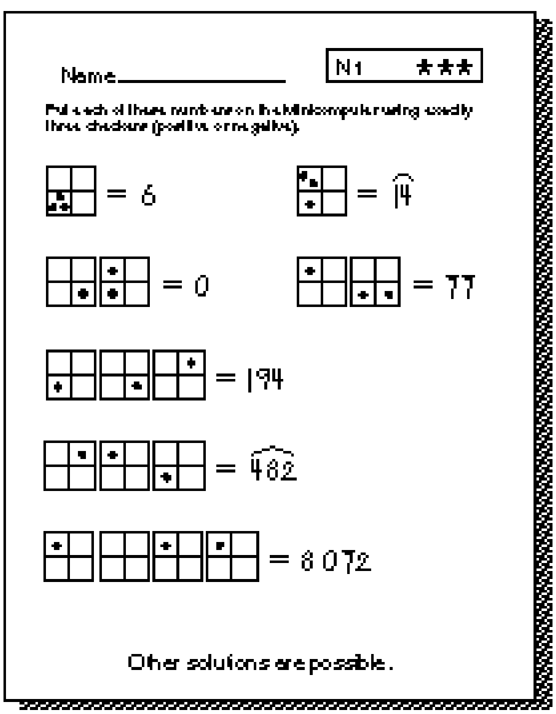
= 125

= 1015

= 314

= 10501

Many solutions are possible.


Name________________ N1 ★★★

Put each of these numbers on the Minicomputer using exactly three checkers (positive or negative).

= 6

= $\hat{14}$

= 0

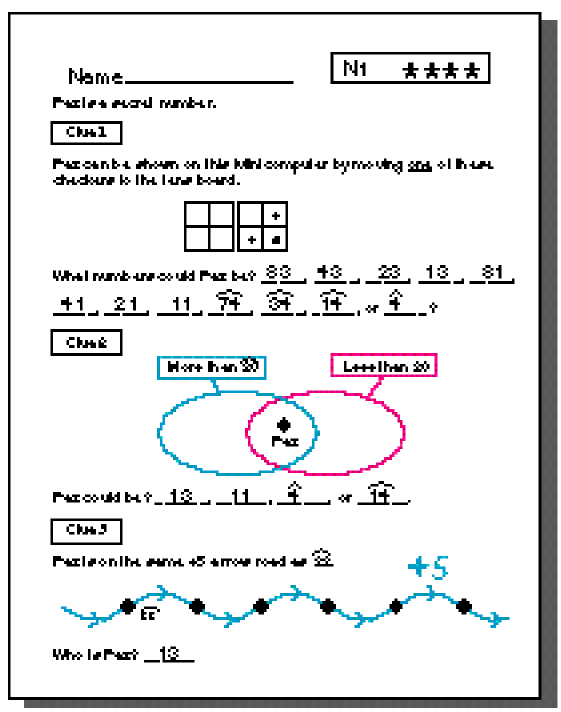
= 77

= 194

= $\hat{482}$

= 8072

Other solutions are possible.


Name________________ N1 ★★★★

Paz is a secret number.

Clue 1

Paz can be shown on this Minicomputer by moving one of these checkers to the tens board.

What numbers could Paz be? 83, 43, 23, 13, 81, 41, 21, 11, $\hat{74}$, $\hat{34}$, $\hat{14}$, or $\hat{4}$?

Clue 2

More than 20

Less than 20

Paz

Paz could be? 13, 11, $\hat{4}$, or $\hat{14}$.

Clue 3

Paz is on the same +5 arrow road as $\hat{22}$.

+5

Who is Paz? 13

Capsule Lesson Summary

Using arrow diagrams and the Minicomputer, investigate the composition of certain numerical functions; for example, +10 followed by +2, and 3x followed by 2x.

Materials

Teacher
- Colored chalk
- A bag and a box of any type of counters
- Minicomputer set

Student
- Worksheets N2* and **

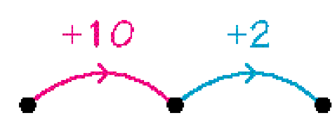
Advance Preparation: You may like to prepare multiple copies (10) of a paper with many stars on it for use in Exercise 2.

Description of Lesson

Begin the lesson with some mental arithmetic involving the functions +10, –10, +20, –20, 2x, and 3x.
Choose calculations similar to the ones below, appropriate for your students' abilities.

18 + 10	90 + 10	17 + 20	2 x 15
72 + 10	95 + 10	354 + 20	2 x 8
3 + 10	195 + 10	61 + 20	2 x 24
103 + 10	46 + 10	792 + 20	2 x 50
16 – 10	24 – 10	50 – 20	3 x 4
26 – 10	14 – 10	57 – 20	3 x 10
96 – 10	514 – 10	390 – 20	3 x 12
106 – 10	504 – 10	601 – 20	3 x 20

Exercise 1



Draw this arrow picture on the board.

Show the class your bag of counters and the box of extra counters.

T: ***I have a bag of counters*** (shake the bag)***, but I'm not going to tell you how many are in it. Imagine that the number of counters in this bag is at the beginning of our arrow picture.***

Give a student the box of extra counters.

T: ***The red arrow is for +10, so you are to add ten counters to my bag.***

The student should count out ten counters from the box and put them in the bag all at once.

T: ***Now I have ten more counters in my bag than I had at the start.***

What does the blue arrow tell us to do?

S: ***Add two more counters.***

N2

Ask a student to add two more counters from the box to your bag.

T: ***What can you tell me about the number of counters in the bag?***

S: ***We don't know for sure how many there are.***

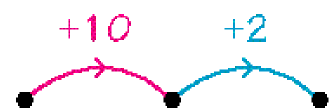
S: ***There are twelve more counters in the bag than when you started.***

T: ***If I had 24 counters in the bag at the start, how many do I have now?***

S: ***36.***

Repeat the question for 88 counters, for 140 counters, and for 256 counters at the start.

T: ***Each of you pick some number. Ready?***
Add 10 (trace the red arrow) ***to your number.*** (Pause.)
Now add 2 (trace the blue arrow) ***to that number.***



Starting number	Ending number
6	18
10	22
5	7
89	101
20	8

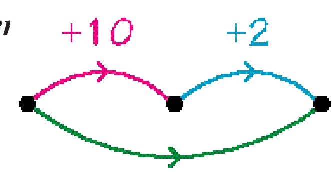
Draw a chart below your arrow picture.

Call on several students to tell the class their starting numbers and their ending numbers. Record these in your chart. You may like to ask if anyone can choose a starting number less than 0 or greater than 50. Vary the activity by asking one student to tell his or her starting number, and ask another student to figure out the ending number. Then ask one student to tell his or her ending number, and ask another student to determine the starting number.

T: ***Look closely at this chart. What patterns do you notice?***

S: ***An ending number is always 12 greater than the starting number.***

S: ***If you start with an even number, you end with an even numbe number, you end with an odd number.***

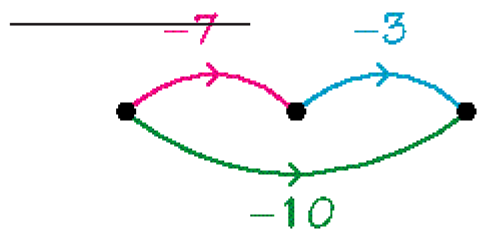


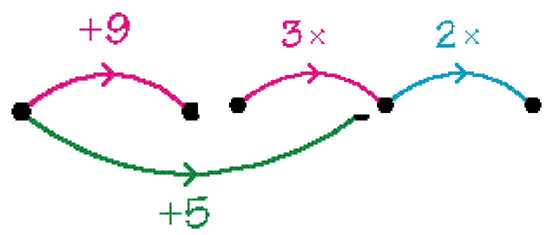
Draw this green arrow.

T: ***What could the green arrow be for?***

S: ***+12.***

Repeat this exercise with other arrow pictures involving composition. Ask students to think of a number each time and to follow the instructions of the arrows. For each picture, fill in a chart and label the green arrow. Use the bag of counters for support.





Exercise 2

Draw this arrow picture on the board.

Show the class a piece of paper with stars on it.

T: ***This piece of paper has lots of stars on it, but for the moment let's not worry about how many. Imagine that the number of stars on this paper is at the beginning of our arrow picture.***

Place the piece of paper near the starting dot of the arrow picture.

T: ***The red arrow is for 3x. How could we get three times as many stars as this?***

S: ***Count the stars and multiply by 3. Then draw that many stars.***

S: ***Draw two more stars next to each one already on the paper.***

S: ***Make three copies of the paper.***

Show the class three copies of the paper taped together, and place it near the middle dot of the arrow picture. Trace the 3x arrow as you say,

T: ***This big sheet has 3x as many stars as the paper at the starting dot.***

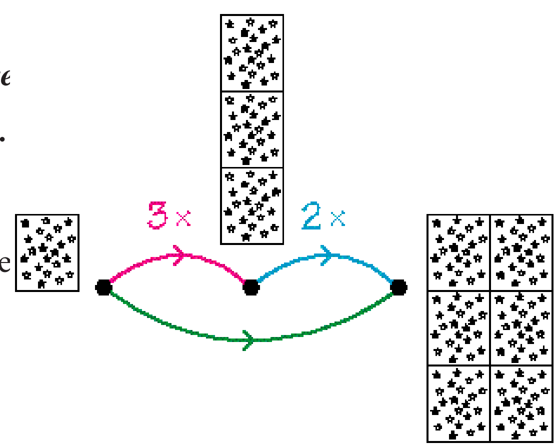
What does the blue arrow tell us to do?

S: ***Double or multiply by 2.***

T (pointing to the paper at the middle dot)**: *How can we ge***

S: ***Draw another star next to each one already there.***

S: ***Make two copies.***



Show the class two copies of the middle paper and tape the near the ending dot of the arrow picture.

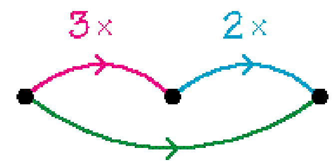
T: ***What can you tell me about the number of stars on this large piece of paper*** (point to the paper at the ending dot)***?***

S: ***We don't know for sure how many stars there are.***

S: ***There are a lot more stars than when we started.***

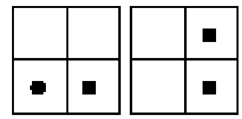
S: ***There are six times as many stars as when we started.***

T: ***If there were 100 stars on this first paper*** (point to the starting dot) ***how many would there be on this last paper*** (point to the ending dot)***?***



S: ***600.***

Repeat the question for 50 stars or 200 stars at the start.



Draw this green arrow.

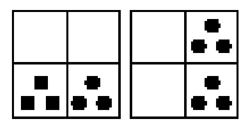
T: ***What could the green arrow be for?***

S: ***6*x.**

Display two Minicomputer boards, give a student four regular checkers, and ask that he or she put on a two-digit number. For example:

T: ***What number is on the Minicomputer?***

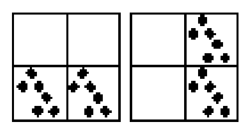
S: ***35.***



Write this number (for example, 35) at the starting dot of the arrow picture.

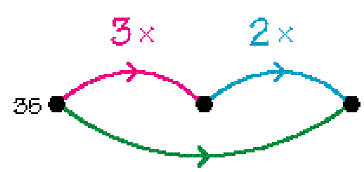
T: ***The red arrow says to multiply by 3. Who can add some checkers so that 3*** x ***35 is on the Minicomputer?***

If necessary, emphasize that you can see 3 x 35 on the Minicomputer when you put three checkers in each position where there was one checker.



Trace the red 3x arrow as you say,

T: ***We now have 3*** x ***35 on the Minicomputer. What does the blu***



S: ***Double.***

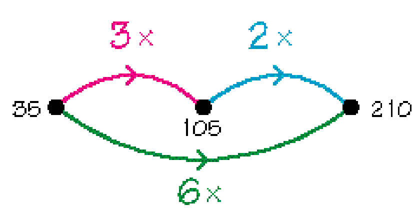
T: ***Who can double the number on the Minicomputer?***
Now how many 35's are on the Minicomputer?

S: ***6.***

Trace the green arrow in the arrow picture.

T: ***What could the green arrow be for?***

S: ***6*x.**



Label the green arrow 6x. Trace the 3x arrow as you ask,

T: ***What is 3*** x ***35? What is 35 + 35 + 35?***

S: ***105.***

Label the middle dot 105. Trace the 2x arrow as you ask,

T: ***What is 2*** x ***105?***

S: ***210.***

T (tracing the green arrow)**:** ***What is 6*** x ***35?***

S: ***210.***

Erase the dot labels in the arrow picture. Suggest to the class that they use the arrow picture to help do these calculations.

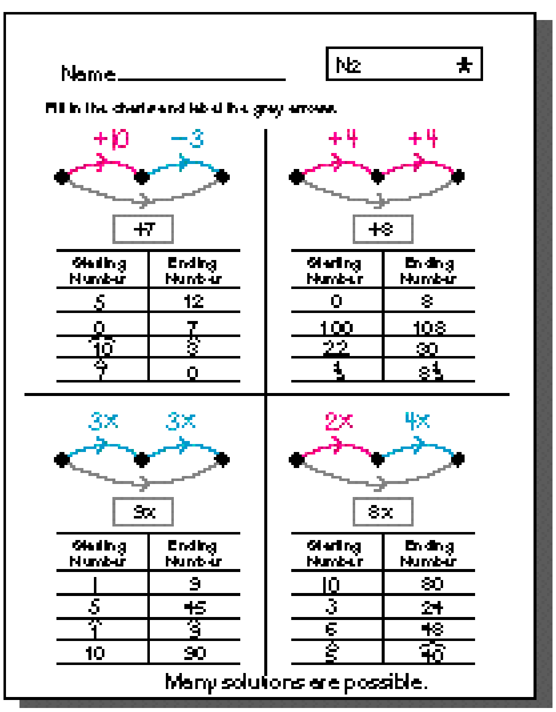
6 x 9   6 x 15   6 x 12   6 x 25

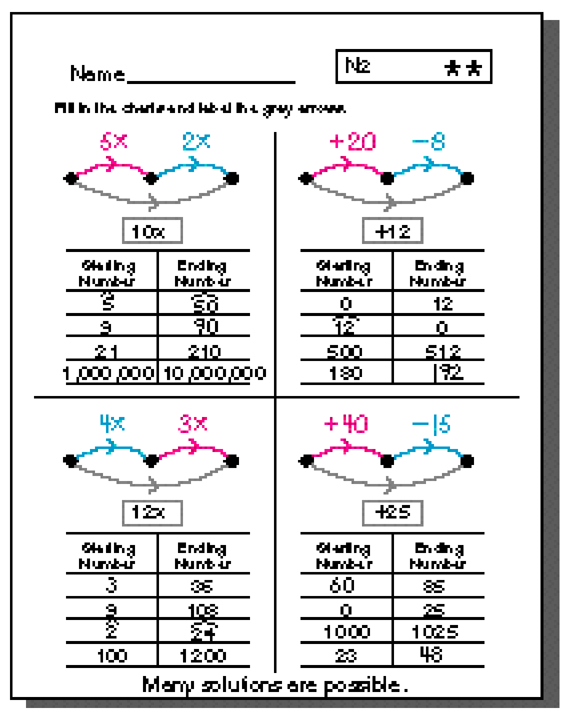
ible for individual work.

Additional Practice

Use composition in mental arithmetic giving practice with facts. For example:

17 + 10	35 – 5	46 + 20	2 x 7	4 x 7	3 x 5
27 + 3	30 – 3	66 – 1	2 x 14	2 x 28	3 x 15
17 + 13	35 – 8	46 + 19	4 x 7	8 x 7	9 x 5





N3 ROAD MAP OF ILLINOIS

N3

Capsule Lesson Summary

Find shortest routes between cities in Illinois. Decide which of two cities is closer to a third. Plan a trip that starts at a particular city and is approximately 1 000 km long.

Materials

Teacher
- Overhead projector (optional)
- Blackline N3(a)

Student
- Map of Illinois
- Paper
- Worksheets N3(a) and (b)
- Calculator

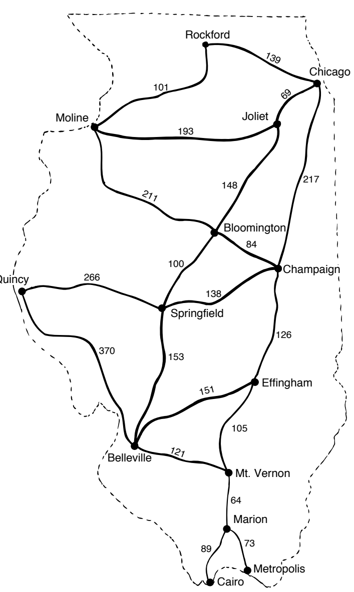
Advance Preparation: Use Blackline N3(a) to make copies of the Illinois map for display and for students. You may want to make the display copy on an overhead transparency for projection.

Description of Lesson

Display an Illinois map, and provide copies of the Illinois map on Blackline N3(a) for students.

Note: You may prefer to do one problem collectively, using a projected Illinois map before giving students their copies of the map.

(If you and your students are in Illinois, adjust the following dialogue so that it is appropriate.)



T: ***This is a map of one of the states in the United States of America. What is the name of this state? Do you know any of these cities?***

You may need to tell your class that this state is Illinois. Encourage students to share what they know about Illinois and the cities indicated on this map. Point out that the numbers on the map show how far it is in kilometers from one city to another. Trace the road from Moline to Joliet.

T: ***How far is it from Moline to Joliet according to this map?*** (193 kilometers)
How far is it from Mt. Vernon to Effingham? (105 kilon
How far is it from Effingham to Marion? (169 kilomete

Exercise 1

Abbreviate a statement of this problem on the board.

Quincy to Champaign?

T: ***What is the shortest route from Quincy to Champaign?***

S: ***Go from Quincy to Springfield, and then from Springfield to Champaign.***

Ask a student to trace this route on the map; then record it on the board.

T: ***How long is this route from Quincy to Champaign? Write your answer on a piece of paper.***

When most students have responded on paper, ask,

T: ***How did you calculate the distance from Quincy to Champaign?***

S: ***I added 266 + 138.***

Quincy to Champaign?
Quincy to Springfield to Champaign is 404

You may like to write the addition problem on the board, and to call on a volunteer to solve this problem at the board, explaining each step.

T: ***Is this the shortest route from Quincy to Champaign? How do you know?***

Let students give explanations, comparing the length of this route to other possible routes.

Erase the board before going on to Exercise 2. Exercise 2 is on Worksheet N3(a).

Exercise 2

Abbreviate this problem on the board.

Chicago to

T: ***What is the shortest route from Chicago to Belleville?***

Suggest that students work on this problem individually for a few minutes. As necessary, encourage students to calculate two different routes, one through Joliet, Bloomington, and Springfield, and the other through Champaign and Effingham. The lengths of these two routes then can be compared. Insist that students convince you which route is shorter.

After a short while invite two students to solve the respective addition problems at the board.

Conclude that the shortest route from Chicago to Belleville is 470 km, that is the one through Joliet, Bloomington, and Springfield.

$$\begin{array}{r} 69 \\ 148 \\ 100 \\ +153 \\ \hline 470 \end{array} \qquad \begin{array}{r} 217 \\ 126 \\ +151 \\ \hline 494 \end{array}$$

T: ***How much longer is the route from Chicago to Belleville through Champaign and Effingham than the route through Joliet, Bloomington, and Springfield?***

S: ***24 km.***

Erase the board before going on to Exercise 3. Exercise 3 is on Worksheet N3(b).

Exercise 3

T: ***Which city is closer to Springfield — Chicago or Marion?***

Abbreviate the question on the board.

Springfield to Chicago? Springfield

Encourage students to decide on an answer to this question by calculating both distances. Allow several minutes for independent work. Then ask students to describe the shortest routes from Springfield to Chicago, and from Springfield to Marion, and to tell the length of each of these routes.

Record the information on the board.

Springfield to Chicago?
Springfield to Bloomington to Joliet to Chicago is <u>317</u> km.

$(100 \times 148 \times 69 = 317)$

Springfield to Marion?
Springfield to Belleville to Mt. Vernon to Marion is <u>338</u> km.

$(153 \times 121 \times 64 = 338)$

T: ***Which distance is shorter?***

S: ***317 km is less than 338 km.***

T: ***Is Springfield closer to Chicago or Marion?***

S: ***Chicago.***

T: ***How much closer?***

S: ***21 km.***

Erase the board before going on to Exercise 4.

Exercise 4: A Trip Starting at Bloomington

T: ***I want everyone to plan a trip that starts at Bloomington and is approximately 1 000 km long. Try to make the length of your trip as close to 1 000 km as possible. You may end your trip in any of the cities on the map. Keep a record of your trip by writing down the names of the cities you go through.***

Allow several minutes for independent or group work. Then choose a student's trip to record on the board. Try to choose a trip that is at least 50 km longer or shorter than 1 000 km for the collective discussion. An example is given here.

$$\begin{array}{r} 148 \\ 69 \\ 217 \\ 126 \\ 151 \\ +370 \\ \hline 1\,081 \end{array}$$

T: ***Terry has a trip from Bloomington to Joliet to Chicago to Champaign to Effingham to Belleville to Quincy. How long is this trip?***

S: ***1 081 km.***

T: ***Is this trip longer or shorter than 1 000 km?***

S: ***81 km longer.***

T: ***Is there another trip we could take starting at Bloomington that is close to 1 000 km long?***

Perhaps a student will suggest this trip.

$$\begin{array}{r} 100 \\ 266 \\ 370 \\ 153 \\ +100 \\ \hline 989 \end{array}$$

S: ***Bloomington to Springfield to Quincy to Belleville to Springfield and back to Bloomington. It is 989 km.***

T: ***Is this trip more or less than 1 000 km long?***

S: ***Less.***

T: ***How much less?***

S: ***11 km.***

Conclude that 989 is closer to 1 000 than is 1 081.

T: ***Is there another trip we could take starting at Bloomington that is still closer to 1 000 km long?***

Continue letting students suggest trips closer to 1 000 km. Compare all the trips you have recorded on the board, and decide which of them has a length closest to 1 000 km. There are some trips exactly 1 000 km in length such as Bloomington to Springfield to Belleville to Effingham to Mt. Vernon to Belleville to Quincy (100 + 153 + 151 + 105 + 121 + 370 = 1 000), but do not expect students to find such a trip.

Extension Activity

Use an actual road map for your state and investigate the distances between cities. Students may pose problems of their own.

Writing Activity

Direct students to write a letter to a friend about one of the trips they planned in this lesson. They should include the names of the cities they will go through, the distances between cities, and how they found the total length of the trip.

Home Activity

Send home the Illinois map with one or two problems similar to Exercise 3 and 4. For example:

- Which city is closer to Quincy—Joliet or Effingham? How much closer?
- Plan a trip starting at Chicago that is approximately 750 km long.

This would be a good time to send a letter to parents/guardians about addition and subtraction practice. Blackline N3(b) has a sample letter.

N4 SUBTRACTION #1

Capsule Lesson Summary

Subtract whole numbers from 26 and decide which yield answers less than 10 and which yield answers greater than 10. Examine subtraction properties in patterns generated by related subtraction statements. Label the arrows of an extensive arrow road with addition and subtraction functions, and with dots for both positive and negative integers.

Materials

Teacher
- Colored chalk

Student
- Colored pencils, pens, or crayons
- Paper
- Worksheets N4*, **, and ***

Description of Lesson

Begin the lesson with some mental arithmetic involving subtraction patterns. For example:

7 – 3	12 – 4	50 – 5
17 – 3	22 – 4	50 – 15
27 – 3	32 – 4	50 – 25
37 – 3	52 – 4	50 – 35
37 – 13	52 – 14	60 – 35
37 – 23	52 – 24	80 – 35
47 – 33	62 – 34	90 – 45

Exercise 1

Ask students to name pairs of numbers such that the difference between the numbers in each pair is 10. If necessary, prompt students to include numbers other than multiples of 10.

Write this number sentence on the board.

$$26 - \square = 10$$

T: ***26 minus what number equals 10?***

S: ***16.***

Put 16 in the box and set up a chart as in this illustration.

$$26 - \boxed{16} = 10$$

$26 - \square < 10$	$26 - \square > 10$

Point to the left side of the chart as you ask,

T: ***26 – 16 = 10. Can you think of a whole number that we could put in the box so that 26 minus that number is less than 10?***

S: ***25; 26 – 25 = 1 and 1 < 10.***

S: ***20; 26 – 20 = 6 and 6 < 10.***

Let many students respond, and record correct answers in the chart.

Point to the right side of the chart as you ask,

T: ***Can you think of a whole number that we could put in the box so that 26 minus that number is greater than 10?***

S: ***1; 26 – 1 = 25 and 25 > 10.***

S: ***6; 26 – 6 = 20 and 20 > 10.***

S: ***0; 26 – 0 = 26.***

Let many students respond, and record correct answers in the chart. Point to the left side of the chart as you ask,

T: ***What do you think is the least whole number we can put in the box so that 26 minus that number is less than 10?***

Perhaps one or two numbers will be suggested before the class concludes that 17 is the least such number. If someone suggests a number that gives an answer greater than 10 when subtracted from 26, record it in the right column of the chart. Use a real situation to check some numbers. For example:

S: ***18.***

T: ***Let's check. 26 – 16 = 10. So if you had 26 marbles and you gave me 16 of them, you would have 10 left. What if you gave me 18 instead? Would you have more or less than 10 left?***

S: ***Less.***

T: ***So 26 – 18 < 10.***

Similarly, decide that 15 is the greatest whole number that can be put in the box so that 26 minus that number is greater than 10.

Note: Any integer less than 16 gives an answer greater than 10 when subtracted from 26. Should a student make this observation, acknowledge its correctness. Otherwise, to avoid calculations such as $26 - \widehat{8} = 26 + 8 = 34$, do not bring up negative integers unless a student does.

Exercise 2

T: ***What number is 93 – 73?***

S: ***20.***

Record these number sentences on the board.

$$93 - 73 = 20$$
$$93 - 75 = \square$$

T: ***93 – 73 = 20. Is 93 – 75 greater or less than 93 – 73?***

S: ***Less.***

T: ***Why?***

S: ***You are subtracting a greater number.***

Use a real example, such as the marbles from Exercise 1, whenever you think it would be helpful.

T: *How much less?*

S: *2 less, because you take away 73 and then take away 2 more.*

T: *So what number is 93 – 75?* (18)

Record 18 and add a third number sentence to the list.

$$93 - 72 = \boxed{}$$
$$93 - 73 = 20$$
$$93 - 75 = \boxed{18}$$

T: *Is 93 – 72 greater or less than 93 – 73?*

S: *Greater.*

T: *Why?*

S: *The number you are subtracting is less.*

T: *How much greater?*

S: *1 more.*

T: *So what number is 93 – 72?* (21)

Continue this exercise with the following problems. (Answers are in boxes.)

$$102 - 50 = \boxed{52} \qquad 145 - 70 = \boxed{75}$$
$$102 - 47 = \boxed{55} \qquad 145 - 68 = \boxed{77}$$

Exercise 3

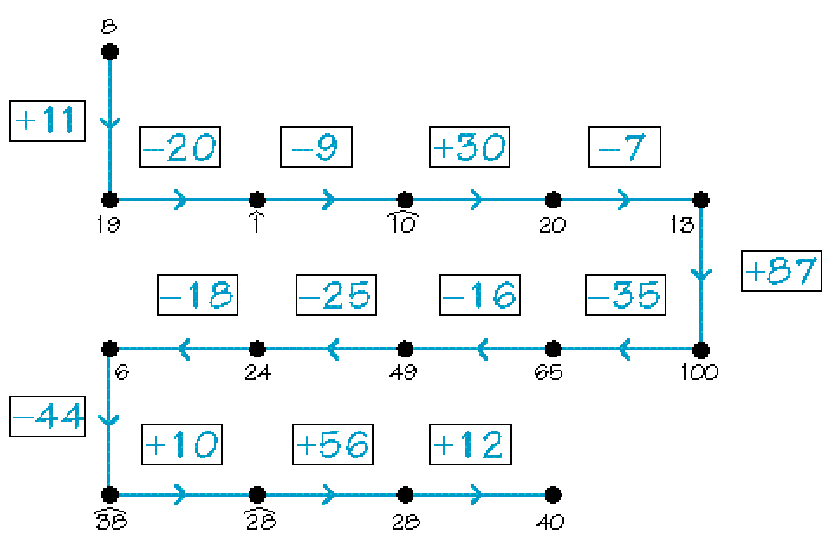
T: *I'm going to draw an arrow road on the board. Each arrow is for plus some whole number or for minus some whole number. Tell me what each arrow is for after I draw it.*

Instruct students to copy the arrow picture as you draw it on the board and to label the arrows in their pictures before giving answers to the class.

Draw one arrow at a time with both the starting and ending numbers given. Ask students to label the arrows. Continue until your road has 12 to 15 arrows. Through your choice of dot labels, give appropriate problems for the abilities of your students. You may refer to a number line when negative numbers are involved. (Answers are in boxes.)

Note: In this picture, arrows of the same color are used for many different functions, however, every arrow is labeled.

Worksheets N4*, **, and *** are av



Extension/Home Activity

Ask students to draw their own arrow road and answer key, similar to N4**, to exchange with a partner or to take home for a family member.

Name_______________ N4 ★

Fill in the boxes with whole numbers, a different number in each box.

74 − [54] = 20

74 − ▢ < 20	74 − ▢ > 20
74 − [55] < 20	74 − [0] > 20
74 − [100] < 20	74 − [53] > 20
74 − [74] < 20	74 − [20] > 20
74 − [89] < 20	74 − [4] > 20

Explain any rule or pattern you used to fill in the boxes.

On the left side, the numbers in the boxes are greater than 54.

On the right side, the numbers in the boxes are less than 54.

Many solutions are possible.

Fill in the boxes with whole numbers, a different number in each box.

146 − [46] = 100

146 − ▢ < 100	146 − ▢ > 100
146 − [50] < 100	146 − [45] > 100
146 − [105] < 100	146 − [6] > 100
146 − [1000] < 100	146 − [40] > 100
146 − [146] < 100	146 − [0] > 100

Explain any rule or pattern you used to fill in the boxes.

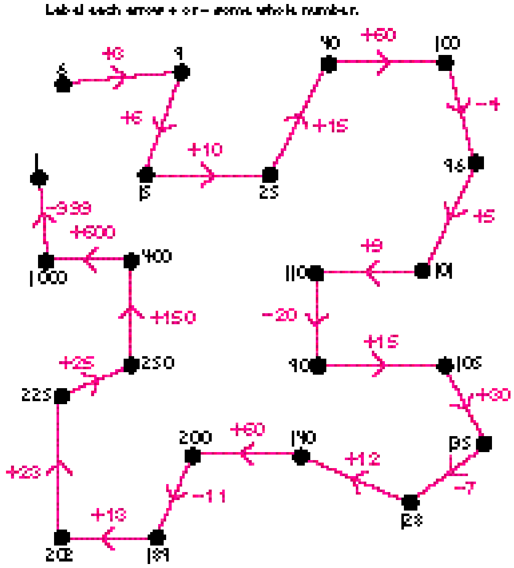
On the left side, the numbers in the boxes are greater than 46.

On the right side, the numbers in the boxes are less than 46.

Many solutions are possible.

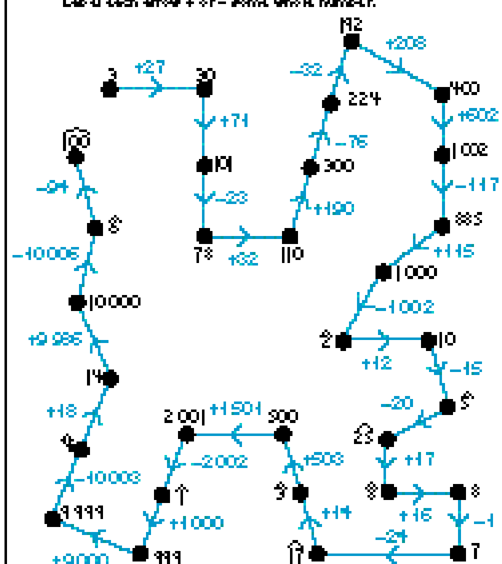
Name_______________ N4 ★★

Label each arrow + or − some whole number.



Name_______________ N4 ★★★

Label each arrow + or − some whole number.



N5 GUESS MY RULE #1

Capsule Lesson Summary

Introduce the *Guess My Rule* activities with rules applied to one number at a time. Begin by using simple functions like –5 and 3**x** for rules, and progress to two-step rules such as “double and then subtract 1.” Use arrow pictures to display the rules and pose problems. Invite students to invent their own similar rules.

Materials

Teacher
- Overhead calculator (optional)
- Colored chalk

Student
- Paper
- Colored pencils, pens, or crayons

Description of Lesson

When playing *Guess My Rule*, encourage students to think about what the rule could be without announcing it to the rest of the class. This will give other students a chance to discover the rule on their own. Let students who think that they know the rule test it on numbers given by you or other students. Each time, confirm or deny the result.

Exercise 1

Draw this arrow picture on the board.

Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: ***I have a secret rule. You give me a number to put here (at a) and I will use my secret rule*** (trace the arrow) ***on it. Then I will give you the resulting number (at b).***

For this first exercise, choose simple rules like –5 or 3**x**. You may like to say that you have taught the calculator your secret number. Students can then put the number (at **a**) on the calculator and press $\boxed{=}$. The calculator uses the secret rule and shows the result (at **b**).

Note: You can prepare the calculator to add, subtract, or multiply by any number *n* as follows:

- +*n* (Press $\boxed{-}$ $\boxed{n}$ $\boxed{+}$ $\boxed{n}$ $\boxed{=}$.)
- –*n* (Press $\boxed{n}$ $\boxed{-}$ $\boxed{n}$ $\boxed{=}$.)
- *n***x** (Press $\boxed{n}$ $\boxed{\times}$ $\boxed{0}$ $\boxed{=}$.)

Invite several students to give you a number to put at **a** (or to put on the calculator). Use your secret rule on the number (or press $\boxed{=}$), and announce the result at **b**.

After using your secret rule two or three times, let students who think they know the rule test it on numbers given by other students. Each time, confirm or deny the result, using the calculator if you like. When most students know the rule, let one student announce the rule and label the arrow.

Ask students to use the rule on several numbers given by you. Each time, label the dots in the arrow picture. Here, for example, the secret rule is –5.

$$273 \xrightarrow{-5} 268 \quad \text{or} \quad \widehat{39} \xrightarrow{-5} \widehat{44}$$

Reverse the process by asking students what number they would have to give in order for you or the calculator to respond, for example, with 117 or $\widehat{4}$.

$$122 \xrightarrow{-5} 117 \quad \text{or} \quad 1 \xrightarrow{-5} \widehat{4}$$

Repeat the exercise with another rule such as 3x.

Exercise 2

T: ***So far my rules have been pretty easy for you to guess. Now I'm going to make it a little more difficult. In fact, I won't be able to teach the calculator my next rule. This time my secret rule will not be just one step like "subtract 5" or "multiply by 3."***

For example, choose a rule like "double and then subtract 1." Use the rule on numbers that your students suggest, and tell them the results as you did in Exercise 1. When most students know the rule, ask a student to describe the rule, but do not attempt to label the arrow. Rather, repeat the rule as you label the dots in the arrow picture as shown below.

T: ***When you give me a number — let's call it* b *— I multiply it by 2 (or double it) and then subtract 1.***

$$\mathbf{b} \longrightarrow (2 \times \mathbf{b}) - 1$$

Check that there is general understanding of the rule by asking students to use the rule on numbers given by you. For example:

$$9 \longrightarrow 17 \;\; (2 \times 9) - 1 \qquad 100 \longrightarrow 199 \;\; (2 \times 100) - 1$$

$$20 \longrightarrow 39 \;\; (2 \times 20) - 1 \qquad \widehat{3} \longrightarrow \widehat{7} \;\; (2 \times \widehat{3}) - 1$$

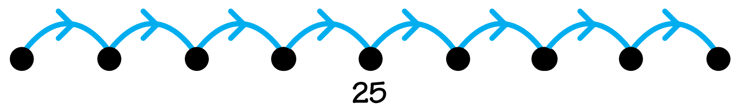
$$0 \longrightarrow \widehat{1} \;\; (2 \times 0) - 1 \qquad 8.5 \longrightarrow 16 \;\; (2 \times 8.5) - 1$$

Reverse the process by asking students what number they would have to give you in order to obtain a specific number as a result of using this rule. For example:

$$\boxed{11} \longrightarrow 21 \qquad \boxed{52} \longrightarrow 103$$

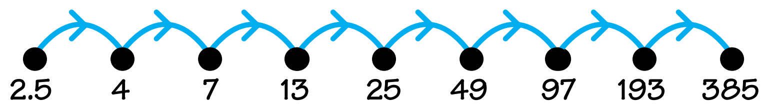
$$\boxed{40} \longrightarrow 79 \qquad \boxed{1/2 \text{ or } 0.5} \longrightarrow 0$$

Draw this arrow picture on the board and ask students to copy it on their papers.



T: ***The blue arrows are for the rule we have been using, "double and then subtract 1." The middle dot in this arrow road is labeled. Use the rule for the blue arrows and label the other dots.***

Allow a few minutes for individual work. Ask students who finish quickly to continue the arrow road in one or both directions. When several students have completed labeling the dots in the original arrow road, check the work collectively.



Exercise 3

Invite students to invent their own rules for the class to guess. Be sure you know the students' rules so that you can judge their responses. Also, you might want to discourage a rule that is too complicated, for example, one that has three or more steps to process, or one that involves operations ivision, addition, and subtraction.

Home Activity

This would be a good time to send a letter to parents/guardians about basic facts. Blackline N5 has a sample letter.

N6 MINICOMPUTER GOLF #1

Capsule Lesson Summary

By moving exactly one checker, change a number on the Minicomputer by a specified amount. Play *Minicomputer Golf*.

Materials

Teacher
- Minicomputer set
- Colored chalk

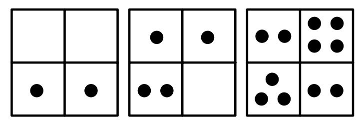
Student
- Paper
- Worksheets N6*, **, ***, and ****

Description of Lesson

Exercise 1

T: ***I am going to put a number on the Minicomputer. See if you can figure out what it is.***

Gradually put this configuration on the Minicomputer, starting with the checkers on the squares of largest value. Pause frequently so your students can do the mental calculations.

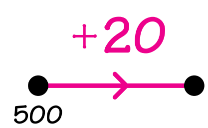


You may like to ask students to write the number on their papers or whisper it to you before letting someone announce it.

S: ***500.***

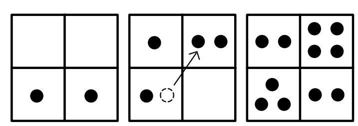
T: ***As I draw an arrow picture, follow the arrows by moving checkers on the Minicomputer.***

Draw this arrow on the board.



T: ***500 is on the Minicomputer. The red arrow is for +20. Can you make the number on the Minicomputer 20 more by moving just one checker?***

Invite a student to make a move.



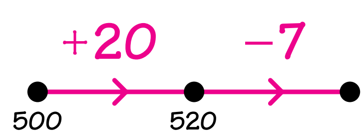
S: ***Move a checker from the 20-square to the 40-square.***

S: ***We could also move a checker from the 80-square to the 100-square.***

T: ***What number is on the Minicomputer now?***

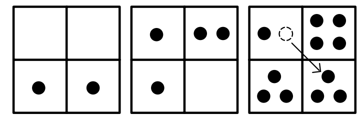
S: ***500 + 20 = 520.***

Label the dot 520 and extend your arrow picture.



T: ***What does this arrow tell us to do?***

S: ***Decrease the number by 7.***



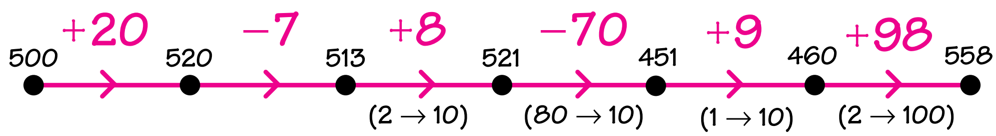
T: ***Who can move just one checker and make the number 7 less?***

S: ***Move a checker from the 8-square to the 1-square.***

T: ***What number is on the Minicomputer now?***

S: ***513; 520 – 7 = 513.***

Continue this activity by extending the arrow road, one arrow at a time, until 558 is reached. A possible road is shown below. The ($x \rightarrow y$) notation indicates a move from the x-square to the y-square.



Exercise 2

Begin this exercise with some discussion about the game of golf.

T: ***What do you know about playing golf?***

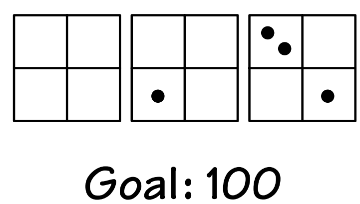
S: ***You play with a ball and clubs.***

S: ***You try to get the ball in a hole.***

S: ***Sometimes you drive the ball a long distance; sometimes you putt.***

T: ***We are going to learn a game called Minicomputer Golf.***

Put this configuration on the Minicomputer.



T: ***What number is this?***

S: ***37.***

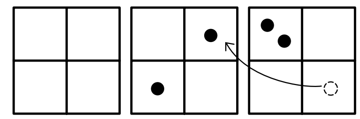
T: ***Our goal is to reach 100. When we get exactly 100 on the Minicomputer, it is like getting the ball in the hole in golf.***

Draw and label a dot for 37 and another dot for 100.

T: ***Do we need to make the number on the Minicomputer more or less?*** (More)
The way we play in this game is to move a checker. We cannot put on more checkers or take off checkers.

Invite a student to move exactly one checker from any square to another square on the Minicomputer. After moving a checker, ask the student how much more or less the new number is. You may like to require students to be able to tell how much change was made, or otherwise to make a different move. Also, you may require that when the number on the Minicomputer is less than the goal, a move must increase the number, or vice-versa. Continue in this way until the goal is reached. The move that reaches the goal is the winning move. A sample game is described below:

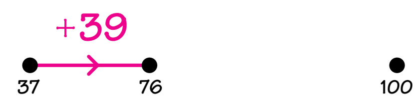
The first volunteer moves a checker from the 1-square to the 40-square.



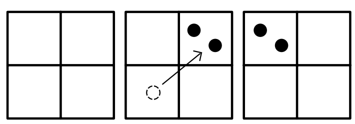
T: ***How did your move change the number?***

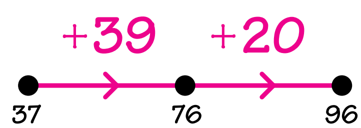
S: ***It increased it by 39. The new number is 76.***

Draw an arrow to record the move.

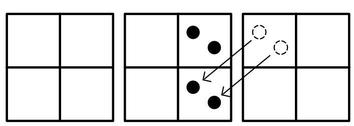


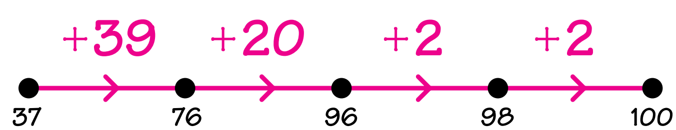
The next volunteer moves a checker from the 20-square to the 40-square and tells the class that the number is now 20 greater; the new number on the Minicomputer is 96.





The next two moves are the checkers from the 8-square to the 10-square.

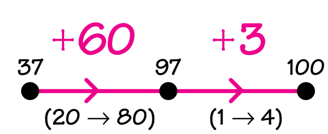




Return the checkers to their original positions (for 37).

T: ***In this game we took four shots. Do you think we could reach the goal with fewer moves?***

Some students might discover the two-move solution illustrated here. But if no shorter solution is forthcoming, do not insist on one.

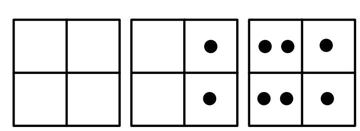


Exercise 3

Put this configuration on the Minicomputer.

T: ***What number is on the Minicomputer?***

S: *75.*



N6

T: ***Let's play Minicomputer Golf again, using this number as the starting number. But this time let's play a team game.***

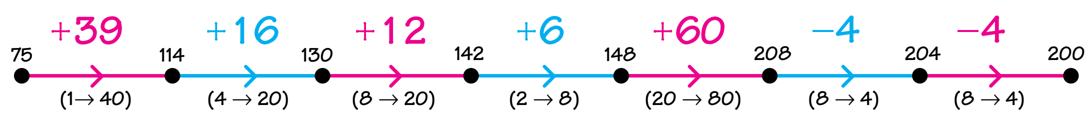
Divide your class into two groups, a Red Team and a Blue Team.

Write this information on the board.

T: ***In this game, the starting number is 75 and the goal is 200. Students from the Red Team and from the Blue Team play in turn. When it is your turn, you may move one checker from the square it's on to any other square. The first team to reach the goal is the winner.***

Note: In a competitive game it is a good idea to require that a move must take the number towards the goal. That is, when the number on the Minicomputer is below the goal, a move should increase the number, and when it is above the goal, a move should decrease the number.

A sample game is recorded in the arrow picture below.



The Red Team wins!

Worksheets N6*, **, ***, and **** are available for individual work.

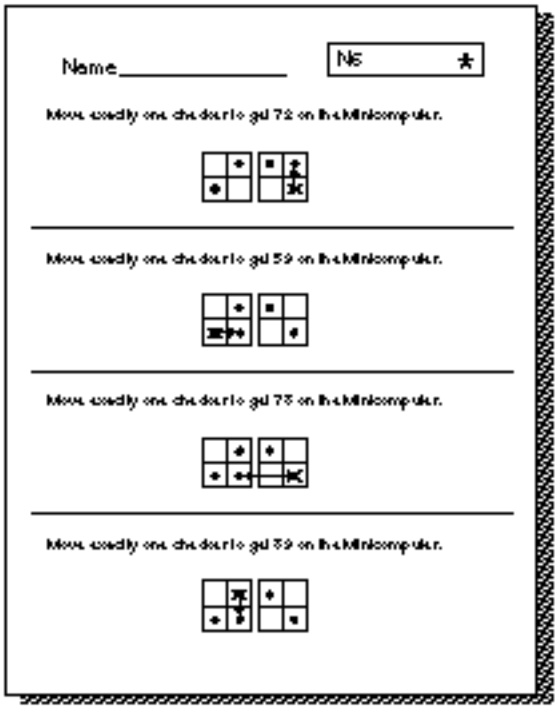
Extension Activity

Use Blackline N6(d) to review the game of *Minicomputer Golf*. Let students play the game cooperatively in pairs (trying to reach the goal in few moves), or competitively in teams of two or three.

Home Activity

This would be a good time to send a letter to parents/guardians about the Minicomputer. Blacklines N6(a), (b), and (c) have a sample letter together with a "home" Minicomputer.

You may also send home Blackline N6(d) to acquaint parents with *Minicomputer Golf*, and suggest they play it with their child.

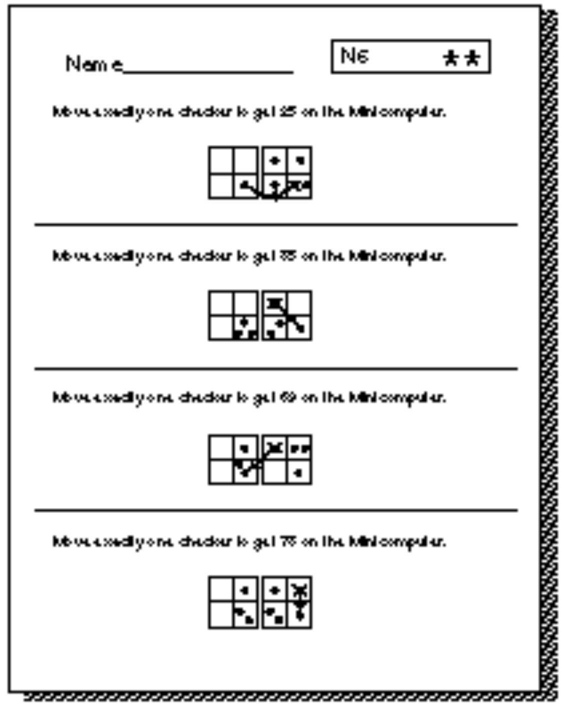

Name______________ N6 ★

Move exactly one checker to get 72 on the Minicomputer.

Move exactly one checker to get 59 on the Minicomputer.

Move exactly one checker to get 73 on the Minicomputer.

Move exactly one checker to get 39 on the Minicomputer.

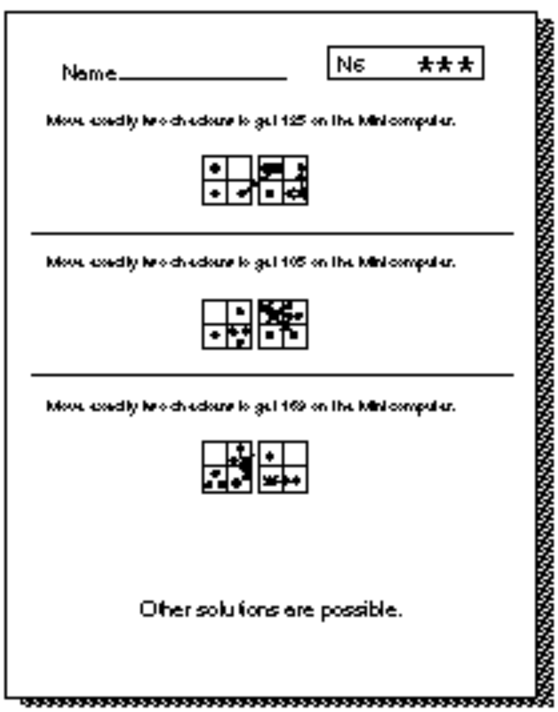

Name______________ N6 ★★

Move exactly one checker to get 25 on the Minicomputer.

Move exactly one checker to get 35 on the Minicomputer.

Move exactly one checker to get 69 on the Minicomputer.

Move exactly one checker to get 75 on the Minicomputer.

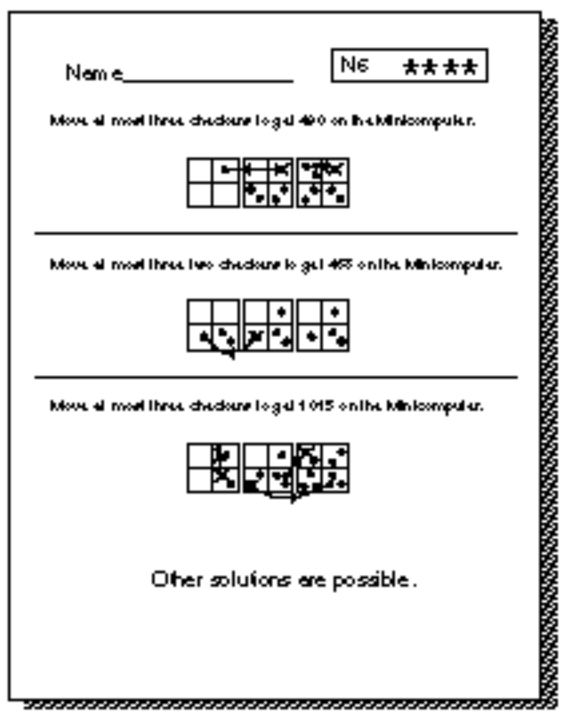

Name______________ N6 ★★★

Move exactly two checkers to get 125 on the Minicomputer.

Move exactly two checkers to get 105 on the Minicomputer.

Move exactly two checkers to get 169 on the Minicomputer.

Other solutions are possible.


Name______________ N6 ★★★★

Move at most three checkers to get 490 on the Minicomputer.

Move at most three two checkers to get 465 on the Minicomputer.

Move at most three checkers to get 1 015 on the Minicomputer.

Other solutions are possible.

N7 TAG THE ARROWS #1

Capsule Lesson Summary

Given a road with two arrows from one number to another, sort through lists of possibilities for each arrow to find pairs of labels for the arrows. Do a related activity with another road, filling in a chart with possible arrow labels. In some lines of the chart, name possible partners for given functions; in other lines, name any pairs of functions that could be used in the road.

Materials

Teacher
- Colored chalk
- Index cards
- Red and blue markers

Student
- Paper
- Colored pencils, pens, or crayons
- Worksheets N7*, **, *** and ****

Advance Preparation: Write the possibilities for red and blue arrows in Exercise 2 on index cards (magnetized, if you have a magnetic board). If you choose to do Exercise 3 in cooperative groups, prepare decks of cards and use Blacklines N7(a) and (b) to prepare arrow roads.

Description of Lesson

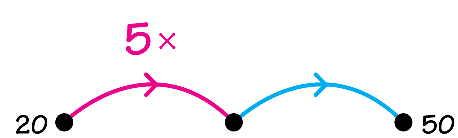
Begin the lesson with some mental arithmetic such as in the sequences of problems below.

10 x 1	3 + 3	2 x 4	7 + 6
10 x 2	2 x 3	2 x 40	70 + 60
10 x 3	7 + 7	2 x 400	700 + 600
10 x 4	2 x 7	2 x 4,000	7,000 + 6,000
10 x 8		2 x 4,000,000	7,000,000 + 6,000,000

100 + 100	3 x 1	5 x 5	3 x 8	4 x 6
2 x 100	3 x 3	5 x 6	24 ÷ 3	24 ÷ 4
6 + 6 + 6	3 x 7	5 x 7	24 ÷ 8	24 ÷ 6
3 x 6		5 x 9		

Exercise 1

Draw this arrow picture on the board.



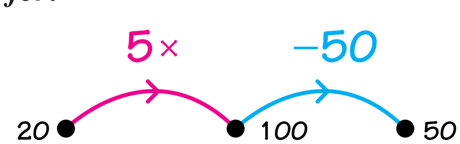
T: ***This arrow road starts at 20 and ends at 50. The red arrow is for 5x. What number is here*** (point to the middle dot)***?***

S: ***100.***

Label the middle dot.

T: ***The blue arrow goes from 100 to 50. What could it be for?***

S: ***–50.***



T: ***Let's check.*** (Trace the blue arrow.) ***What number is 100 – 50?***

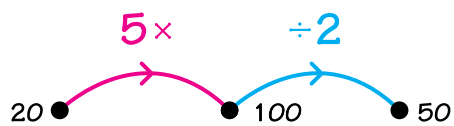
S: ***50.***

Erase the label for the blue arrow and ask for another possibility.

T: ***The blue arrow could be for –50. Is there anything else it could be for?***

S: ***÷2, because 100 ÷ 2 = 50.***

Label the blue arrow and let the class check the solution.



S: ***The blue arrow also could be for "is greater than."***

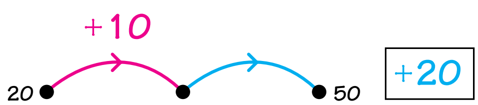
T: ***Yes, 100 is greater than 50.***

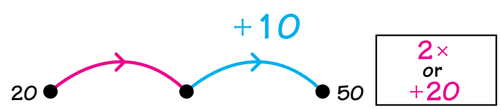
S: ***The blue arrow could be for "is a multiple of."***

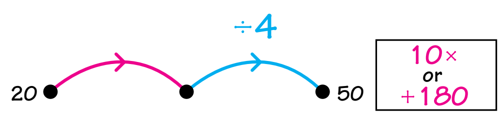
T: ***Yes, 100 is a multiple of 50.***

Do not press the students for solutions like the last two given in the sample dialogue, but consider them if they are offered.

Erase the arrow labels and repeat the exercise using other labels for either the red or blue arrow. Three examples are given below; the solutions your students are most likely to find are given in boxes.

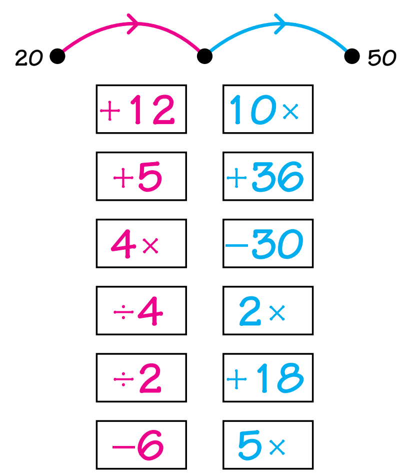






Exercise 2

Erase the arrow labels and put up several possibilities each for the red and blue arrows. If you put the possible arrow labels on index cards, students can move them around to test combinations and match tags.



T: ***For each red tag there is one blue tag that goes with it to build a road from 20 to 50. Likewise, for each blue tag there is one red tag that goes with it. Let's pair the tags.***

Most likely students will match tags in no particular order. Perhaps the first pair to be matched will be +5 with 2x.

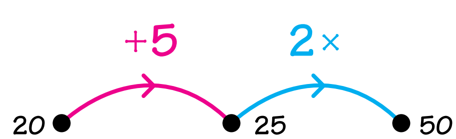
S: ***+5 for the red arrow and 2*****x** ***for the blue arrow.***

T: ***Let's check.***

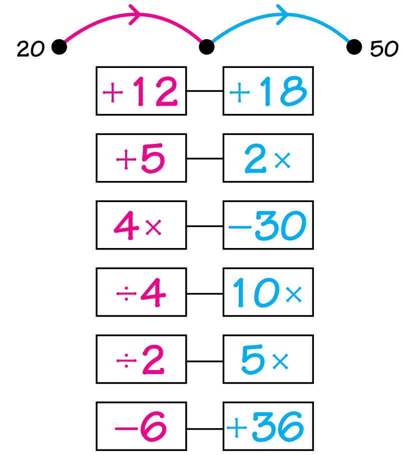
Trace the appropriate arrow as you ask,

T: ***What number is 20 + 5?*** (25)

Does 2 x ***25 = 50?*** (Yes)



To pair up the tags continue in this way. If your list has nonmoveable tags, draw line segments to show the matches.

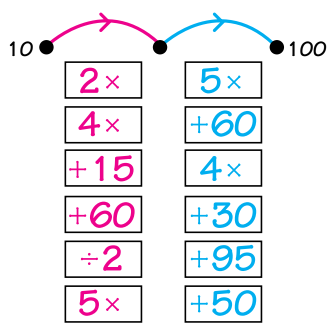


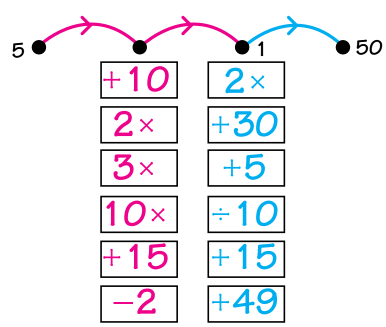
If students need help getting started, select the first red tag, for example, +12. Label the middle dot accordingly (20 + 12 = 32) and find the correct tag for the blue arrow, which goes from 32 to 50.

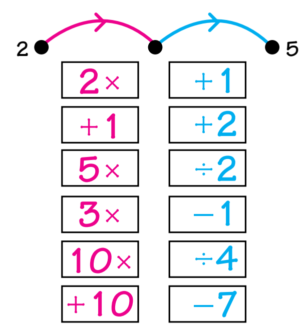
Exercise 3 (optional)

Organize the class into cooperative groups. Provide each group with an arrow road and a deck of cards (possible labels for the red and blue arrows). Direct the group to match the cards (labels) in their decks for the arrow road. When a group finishes one problem, they can change problems (deck of cards and arrow road) with another group.

Several possible problem situations are given below. Choose problem situations that vary in difficulty to involve all your students.

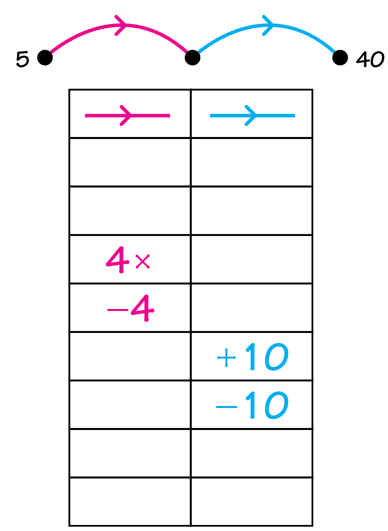






Exercise 4

Draw this picture on the board. Students should have paper for doing calculations.



T (tracing the red arrow)**:** ***What could the red arrow be for? You are free to label it any way you wish.***

Suppose that a student suggests +5. Put +5 in red on the first line of the chart.

T: ***If the red arrow were for +5, what number would be here*** (point to the middle dot)***?***

S: ***10, because 5 + 5 = 10.***

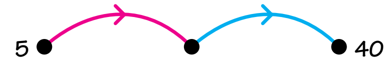
Label the middle dot and trace the blue arrow.

T: ***What could the blue arrow be for?***

S: ***+30, because 10 + 30 = 40.***

S: ***4*x*, because 4* x *10 = 40.***

Write the suggestions given by your class, +30 or 4x in the above dialogue, in blue on the first line of the chart.



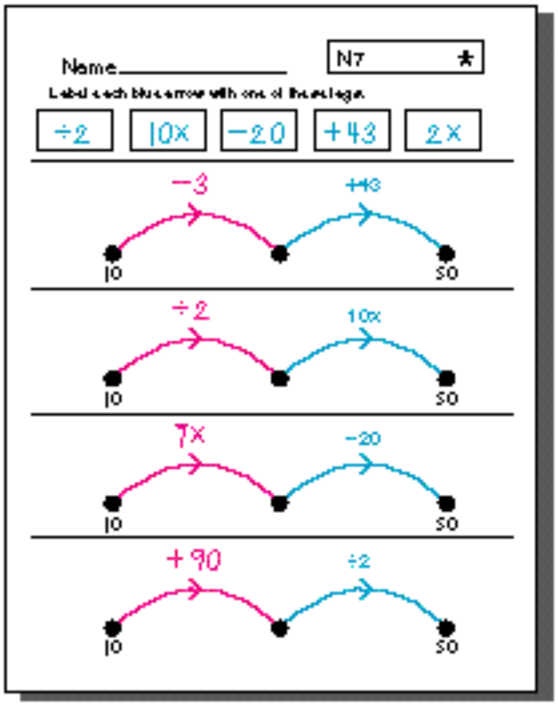
To complete the chart continue in this way. Encourage the class to find solutions that assign negative numbers (or even decimal numbers) to the middle dot. A sample chart is given here.

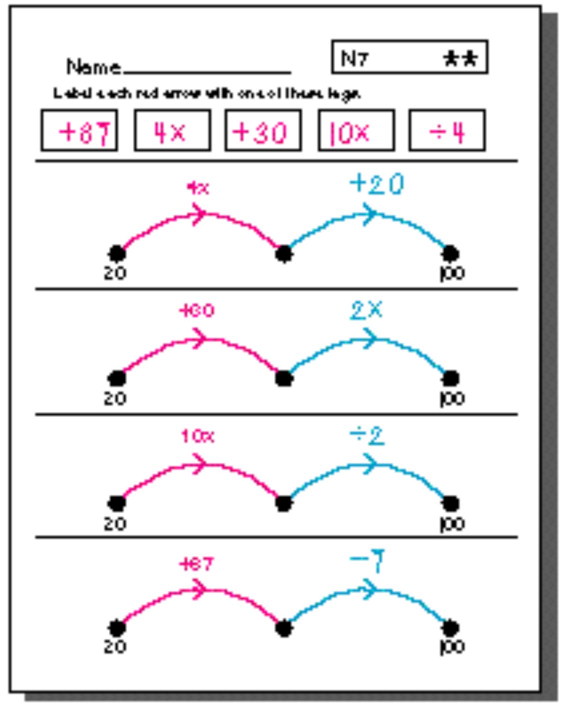
→	→
+5	+30 or 4×
+40	−5
4×	2× or +20
−4	+39 or 40×
+25 or 6×	+10
+45 or 10×	−10
−6	+41
−25	+60

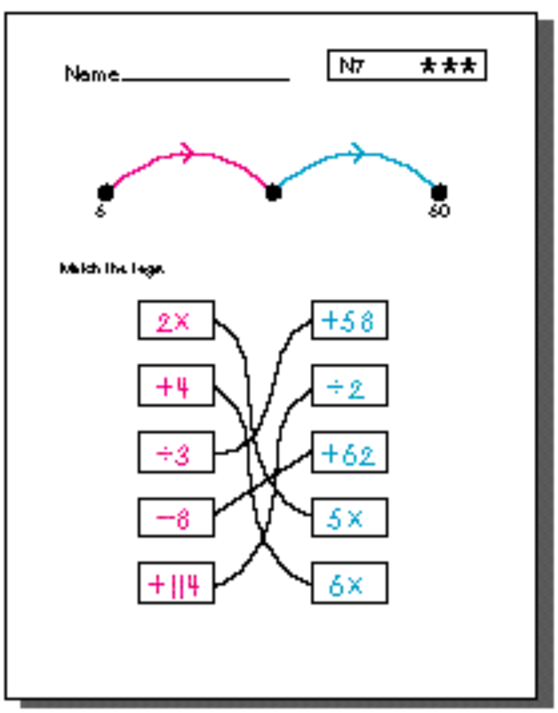
Worksheets N7*, **, ***, and **** are available for individual work. You may like to make cards (as described in Exercises 2 and 3) available to students to manipulate as they work on

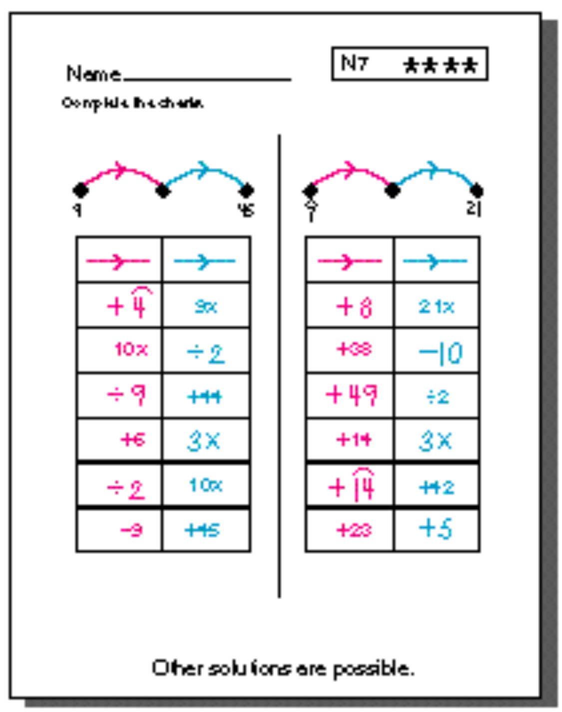
Home Activity

This would be a good time to send a letter to parents/guardians about mental arithmetic. Blackline N7(c) has a sample letter.


Name
N7 ★
Label each blue arrow with one of these legs.
÷2
10x
−20
+43
2x
−3
+43
10
50
÷2
10x
7x
−20
+90
÷2


Name
N7 ★★
Label each red arrow with one of these legs.
+87
4x
+30
10x
÷4
4x
+20
20
100
+30
2x
10x
÷2
+87
−7


Name
N7 ★★★
6
60
Match the legs.
2x
+4
÷3
−8
+114
+58
÷2
+62
5x
6x


Name
N7 ★★★★
Complete the charts.
9
45
7
21
+4
9x
10x
÷2
÷9
+44
+6
3x
÷2
10x
−9
+45
+8
21x
+38
−10
+49
÷2
+14
3x
+14
+42
+23
+5
Other solutions are possible.

Capsule Lesson Summary

Count by sixes starting at 0 and relate the numbers in this sequence to 6x facts. Use a pairing method and arrow diagrams to find the number halfway between two given numbers on the number line.

Materials

Teacher	• Colored chalk • Meter stick	Student	• Worksheets N8*, **, *** and ****

Description of Lesson

Exercise 1

Invite several students to count by sixes from 0 to 78; then ask the class to do so in unison. List the sequence of numbers on the board as each is given.

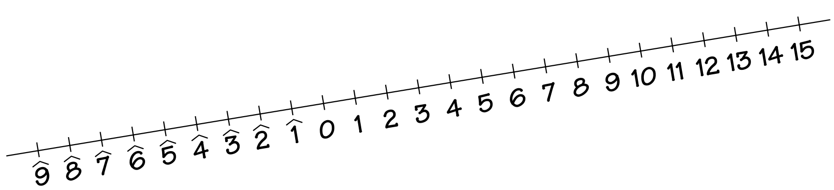
0 6 12 18 24 30 36 42 48 54 60 66 72 78

Ask the class to identify 2 x 6, 7 x 6, 8 x 6, 12 x 6, and 0 x 6 in this list of numbers. Then reverse the identification by asking for a 6x fact for 24, 60, and 36.

0	6	12	18	24	30	36	42	48	54	60	66	72	78
0 × 6		2 × 6		4 × 6		6 × 6	7 × 6	8 × 6		10 × 6		12 × 6	

Exercise 2

Draw this number line on the board.



T: ***Which number is greater, $\hat{3}$ or $\hat{8}$?*** ($\hat{3}$)
How much greater? (5 more)

Draw a red arrow from $\hat{8}$ to $\hat{3}$ on the number line.

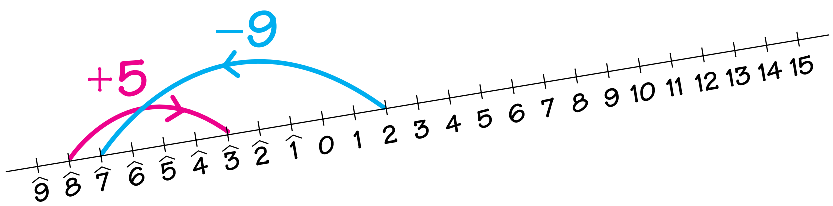
T: ***What could this red arrow be for?***

S: ***+5.***

Label the red arrow and record the related number sentence.

Repeat the activity with $\hat{7}$ and 2, only this time ask which number is less.

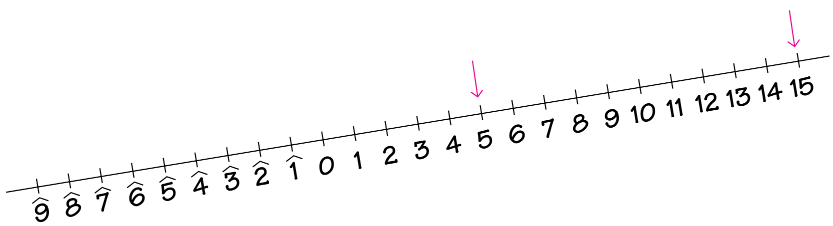
$\hat{8} + 5 = \hat{3}$ $\qquad\qquad$ $2 - 9 = \hat{7}$



Erase the arrows and the number sentences.

T: ***What number is halfway between $\hat{9}$ and 9 on the number line?*** (0)
… halfway between $\hat{4}$ and 4? (0)
… halfway between 100 and $\widehat{100}$? (0)

On the number line, draw an arrow pointing to 5 and another pointing to 15.



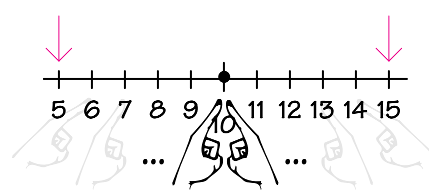
T: ***What number is halfway between 5 and 15?***

S: ***10.***

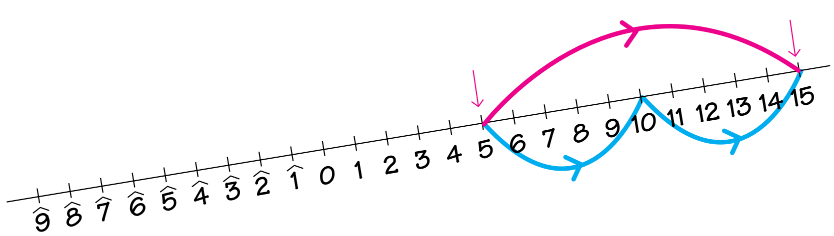
T: ***How do you know?***

S: ***10 is 5 more than 5 and 5 less than 15.***

Demonstrate that 10 is halfway between 5 and 15 by pointing to each simultaneously and then moving in one space at a time until both index fingers are pointing to 10.



Add these arrows to your picture.



T: ***What could the red arrow be for?*** (+10)
What could the blue arrow be for? (+5)

Label the arrows.

Put your left index finger on the mark for 5. With your right index finger trace the blue arrows and then the red arrow as you say,

T: ***+5 followed by +5 is the same as +10.***

Repeat this activity for other pairs of numbers such as those suggested below. Erase arrows before proceeding from one problem to the next. (Answers are in parentheses.)

- 7 and 13 (midpoint 10, +6, +3)
- $\widehat{2}$ and 12 (midpoint 5, +14, +7)
- $\widehat{3}$ and 14 (midpoint 5½, +17, +8½)

Note: In the last problem, if you use your index fingers to come in from $\widehat{3}$ and 14 to the number halfway between them, stop at 5 and 6 and ask what to do next. (Move in a half-space.)

T: ***Have you noticed a relationship between the red and blue arrows?***

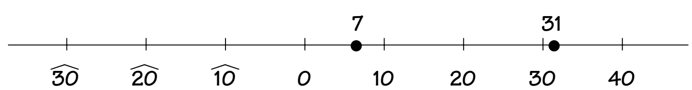
S: ***In each case, the blue arrow adds half what the red arrow adds.***

S: ***Two blue arrows are the same as a red.***

Erase the board and then draw a number line graduated by multiplies of 10.

T: ***Who can locate about where 7 is on this number line?***
Who can locate about where 31 is on this number line?

Let volunteers draw dots for 7 and 31.



T: ***How much greater is 31 than 7?***

S: ***24 more.***

T: ***How do you know?***

S: ***31 is 21 more than 10 and 24 more than 7.***

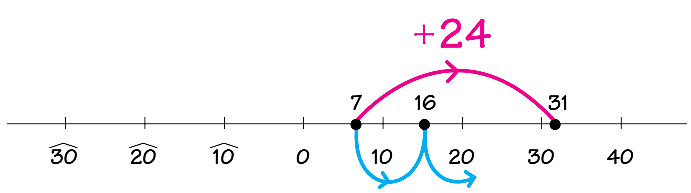
Draw a red arrow from 7 to 31.

T: ***What could the red arrow be for?***

S: ***+24.***

T: ***Which number is halfway between 7 and 31?***

Suppose a student incorrectly answers "16." Locate 16 on the number line, draw a blue arrow from 7 to 16, and start a blue arrow at 16.



T: ***A blue arrow starts at 7 and ends at 16. What could it be for?***

S: ***+9.***

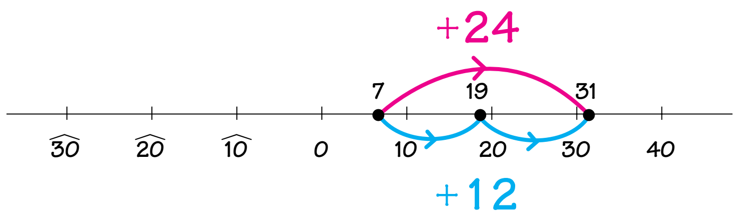
N8

T: ***Can we draw another +9 arrow from 16 to 31?***

S: ***No. 16 + 9 = 25, not 31.***

T: ***So 16 must not be halfway between 7 and 31.***

Erase the blue arrows and the dot for 16. Ask for another suggestion. As in the preceding example, check suggestions until your class has verified that 19 is halfway between 7 and 31. Perhaps a student will explain ½ x 24 = 12, so the blue arrows should be for +12. 7 + 12 = 19, so 19 is the number halfway between 7 and 31.

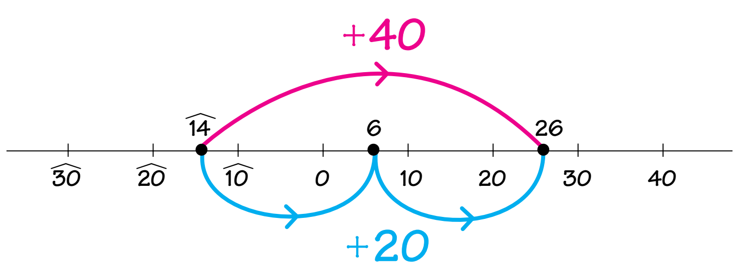


T: ***19 is halfway between 7 and 31. The red arrow is for +24 and the blue arrows are for +12.***

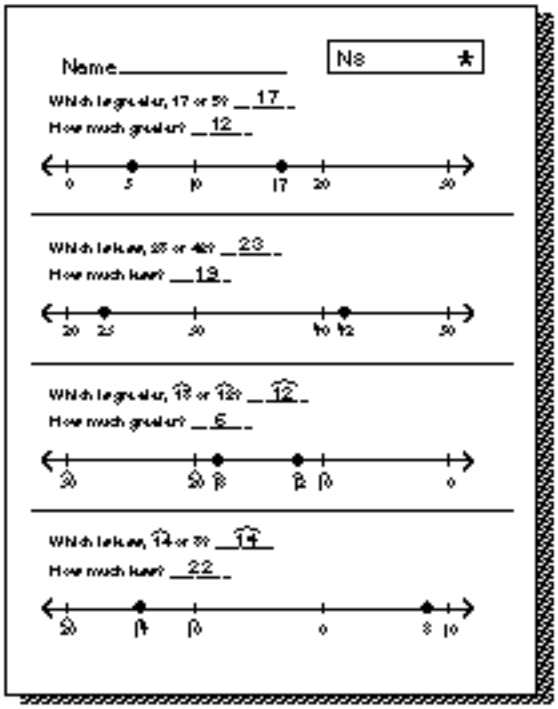
Put your left index finger on the dot for 7. With your right index finger, trace the blue arrows and then the red arrow as you say,

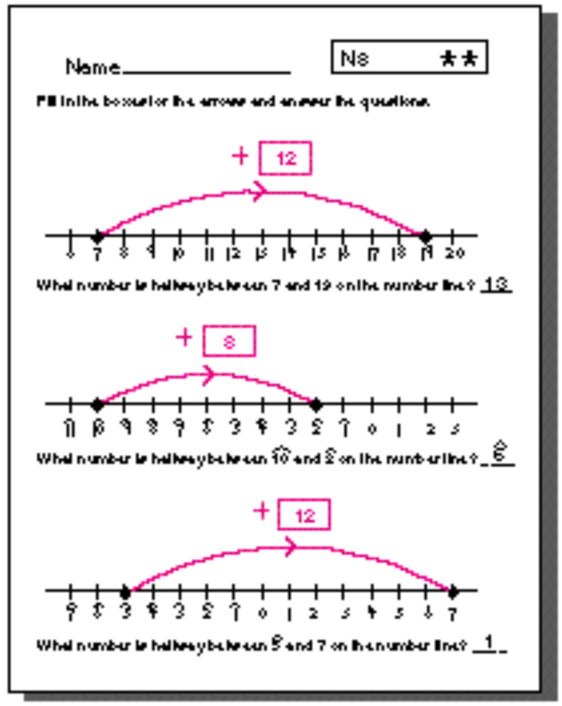
T: ***+12 followed by +12 is the same as +24.***

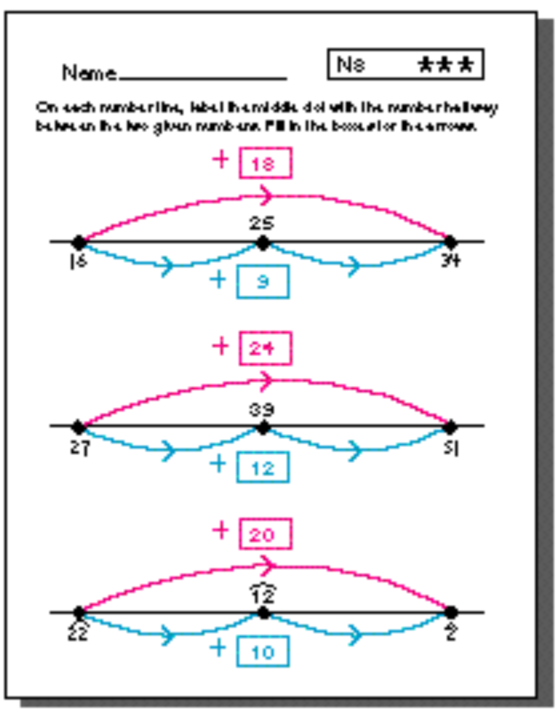
Similarly, find the number halfway between $\widehat{14}$ and 26.

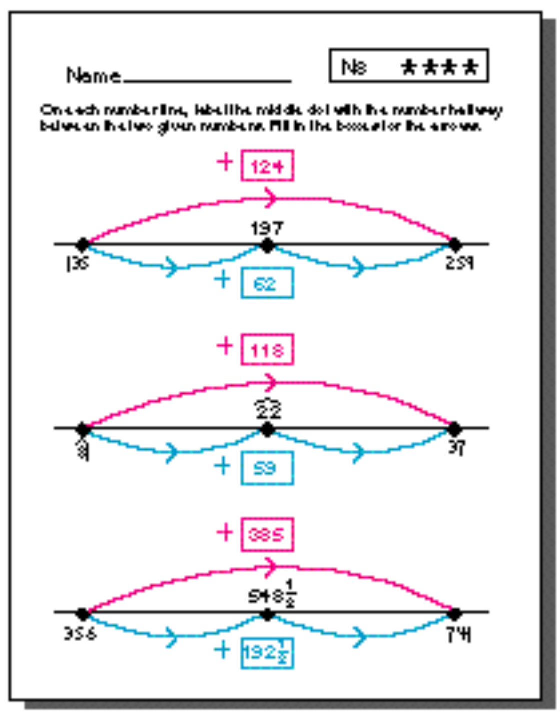


Worksheets N8*, **, ***, and **** are available for individual work.


Name
N8 ★
Which is greater, 17 or 5? 17
How much greater? 12
Which is less, 23 or 42? 23
How much less? 19
How much greater? 6
How much less? 22


Name
N8 ★★
Fill in the boxes for the arrows and answer the questions.
+ 12
What number is halfway between 7 and 19 on the number line? 13
+ 8
+ 12
1


Name
N8 ★★★
On each number line, label the middle dot with the number halfway between the two given numbers. Fill in the boxes for the arrows.
+ 18
25
16
34
+ 9
+ 24
39
27
51
+ 12
+ 20
+ 10


Name
N8 ★★★★
On each number line, label the middle dot with the number halfway between the two given numbers. Fill in the boxes for the arrows.
+ 124
197
135
259
+ 62
+ 118
37
+ 59
+ 385
356
741

N9 THE FUNCTION 10X #1

Capsule Lesson Summary

Review the effect of multiplying numbers by 10 using the Minicomputer and arrow pictures. Explore a situation that uses addition and 10x functions alternately.

Materials

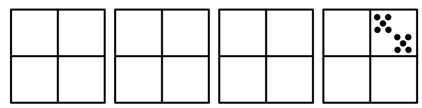
Teacher
- Minicomputer set
- Colored chalk
- ⑩-checkers

Student
- Paper
- Colored pencils, pens, or crayons
- Worksheets N9*, **, ***, and ****

Description of Lesson

Exercise 1

Put this configuration on the Minicomputer.



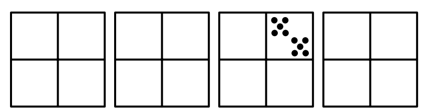
T: ***What number is this?***

S: ***40; 10 x 4 = 40.***

Invite students to make trades until one checker remains on the 40-square.

T: ***Ten checkers on the 4-square is the same number as one checker on the 40-square.***

Put ten checkers on the 40-square.



T: ***What number is this?***

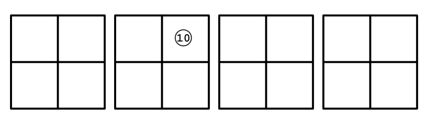
S: ***400; 10 x 40 = 400.***

T: ***Can we make just one big trade?***

S: ***Ten checkers on the 40-square is the same number as one checker on the 400-square.***

Remove the checkers on the 40-square and put one checker on the 400-square.

Put a ⑩-checker on the 400-square.

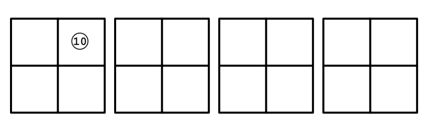


T: ***Rather than put ten checkers on the Minicomputer, I used a ⑩-checker. What number is this?***

S: ***4 000; 10 x 400 = 4 000.***

S: ***Ten checkers on the 400-square is the same number as one checker on the 4 000-square.***

Trade the ⑩-checker on the 400-square for one checker on the 4 000-square. Then take the regular checker off the 4 000-square and put on a ⑩-checker.

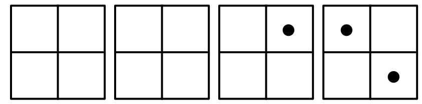


T: *What number is this?*

S: *40 000; 10* x *4 000 = 40 000.*

With your class, imagine a trade of the ⑩-checker on the 4 000-square for one checker on the 40 000-square.

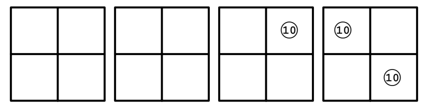
Put 49 on the Minicomputer.



T: *What number is this?*

S: *49.*

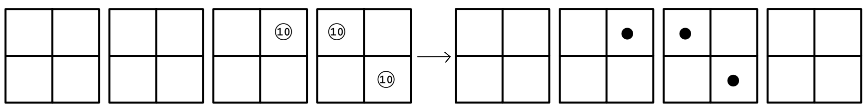
Replace each regular checker with a ⑩-checker.



T: *Now what number?*

S: *490; 10* x *49 = 490.*

Ask students to make the appropriate trades of ⑩-checkers for regular checkers.



Repeat this activity with 10 x 63; 10 x 630; 10 x 187; and 10 x 305. Emphasize that the effect of multiplying by 10 is that digits move over a place to the left.

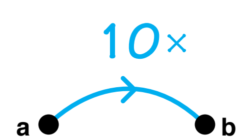
Exercise 2

Write a set of 10x problems on the board, and ask students to solve the problems on their papers. (Answers are in the boxes.)

$10 \times 5 = \boxed{50}$	$10 \times 4 = \boxed{40}$	$10 \times 5 = \boxed{50}$
$10 \times 70 = \boxed{700}$	$10 \times 20 = \boxed{200}$	$10 \times 10 = \boxed{100}$
$10 \times 75 = \boxed{750}$	$10 \times 100 = \boxed{1\,000}$	$10 \times 2\,000 = \boxed{20\,000}$
	$10 \times 124 = \boxed{1\,240}$	$10 \times 2\,015 = \boxed{20\,150}$

Exercise 3

Draw this arrow picture on the board. Label the dots as you say,



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

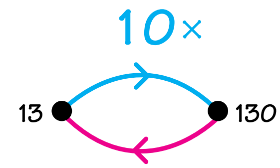
T: *If 13 is here (at* **a***), then what number is here (at* **b***)?*

S: *130, because 10* x *13 = 130.*

Draw a return arrow in red.

T: *What could the red arrow be for?*

S: *÷ 10.*



Label the red arrow ÷10.

Continue the activity, using the values in the chart below for **a** and **b**. (Answers are in boxes.) Or you may prefer to let students choose a label for either **a** or **b** and then ask the class for the other number.

Note: Sometimes start with a number at **a** and ask for the number at **b,** and sometimes start with a number at **b** and ask for the number at **a**.

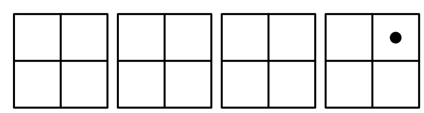
a	b
24	$\boxed{240}$
125	$\boxed{1250}$
$\boxed{67}$	670
20	$\boxed{200}$

a	b
$\boxed{55}$	550
$\boxed{450}$	4 500
8 900	$\boxed{89000}$
1 356	$\boxed{13560}$

Exercise 4

Display four Minicomputer boards with nothing on them. Announce that 0 is on the Minicomputer and draw a dot for 0.

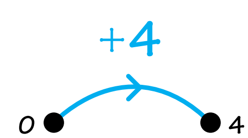
Put one checker on the 4-square.



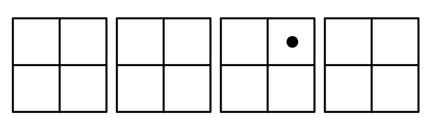
T: *What number is on the Minicomputer?*

S: *4.*

Announce that you added 4 on the Minicomputer and record this information on the board. Suggest that students copy the arrow picture as it develops during this exercise.



Move the checker from the 4-square to the 40-square and extend the arrow picture with a red arrow.



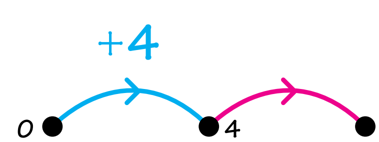
T: *What number is on the Minicomputer?*

S: *40.*

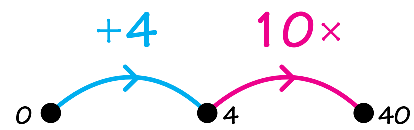
T: *What could the red arrow be for?*

S: *10*x.

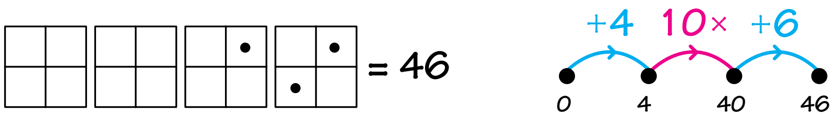
S: *+36.*

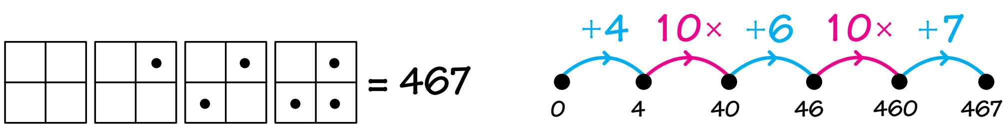


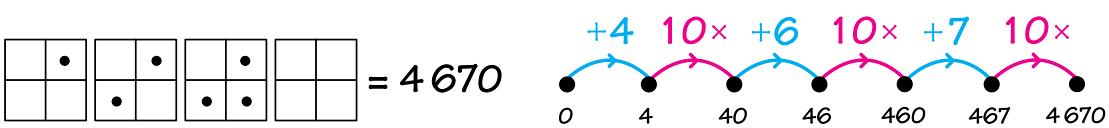
T: ***Both are correct. I will make it 10x because I multiplied by 10 when I moved the checker over to the tens board of the Minicomputer.***

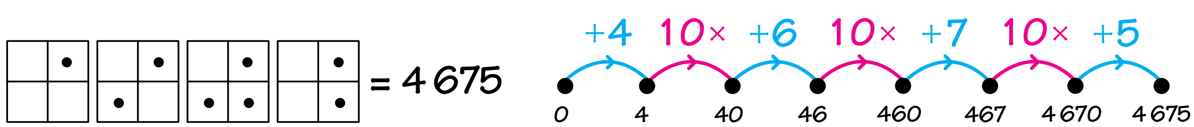


Continue in this manner by adding single-digit numbers with checkers on the ones board, and then multiplying by ten by moving all the checkers over one to the left. A possible sequence is shown below. You may prefer to let your students decide which one-digit numbers to add. Students may be copying the arrow picture and announcing what to do next.









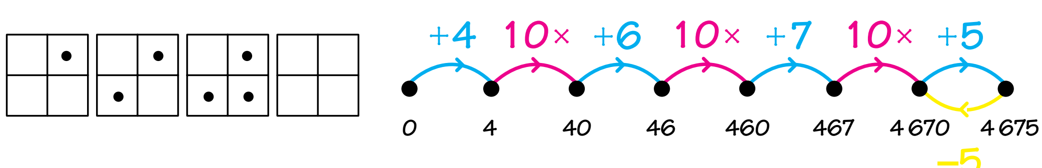
T: ***Now I would like to "undo" what we have done on the Minicomputer by following the arrow picture backwards. What should we do first?***

S: ***Take away 5.***

Remove the appropriate checkers from the Minicomputer.

T: ***How can we show this in the arrow picture?***

S: ***Draw a –5 return arrow.***



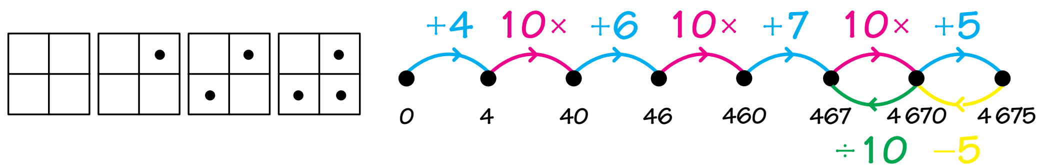
T: ***What should we do next?***

S: ***Divide by 10.***

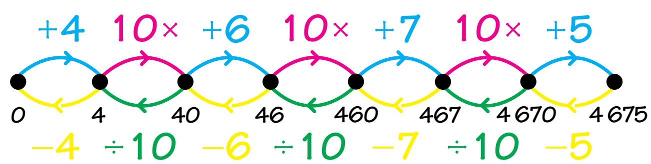
T: ***How can we divide by 10 on the Minicomputer?***

S: ***Move each checker from the square it's on to the same color square on the next board to the right.***

Invite a student to move the checkers on the Minicomputer while you or another student draws the ÷10 return arrow.



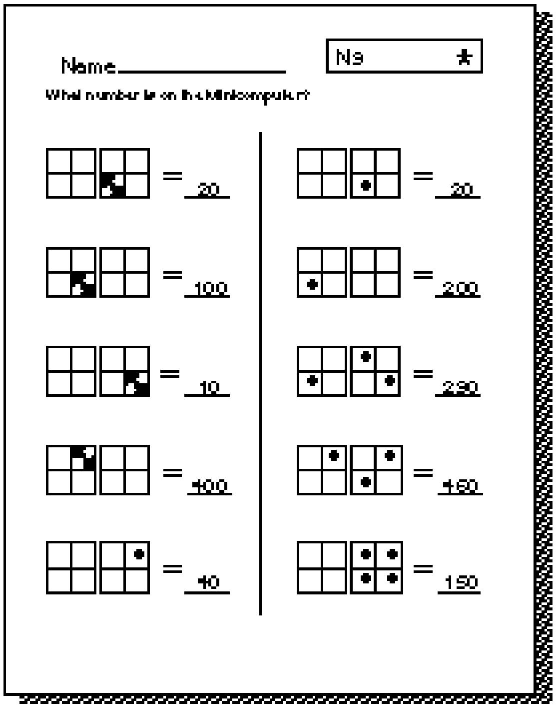
Continue in this manner until 0 is on the Minicomputer and your arrow picture looks like the one below.

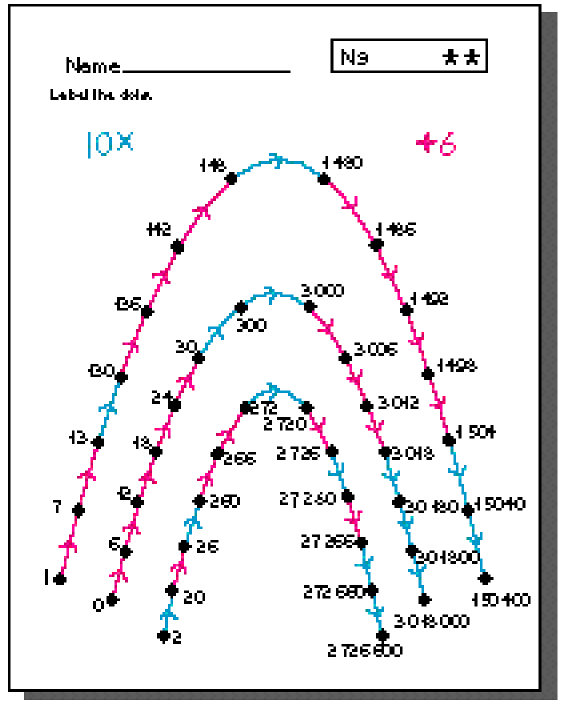


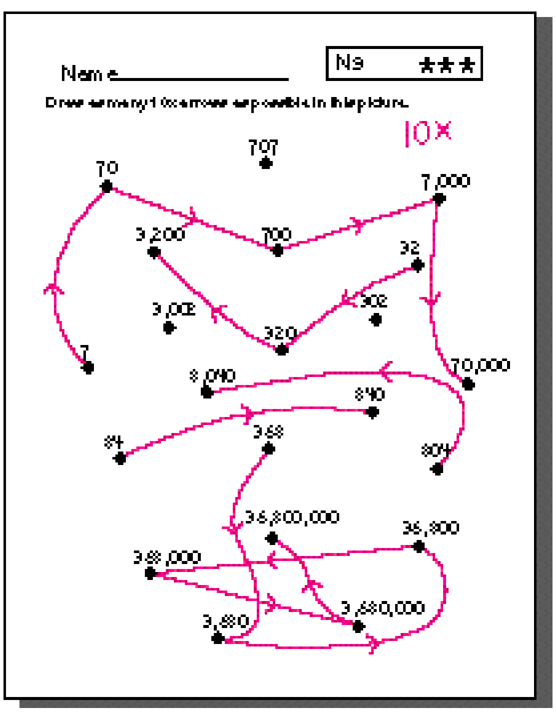
nd **** are available for individual work.

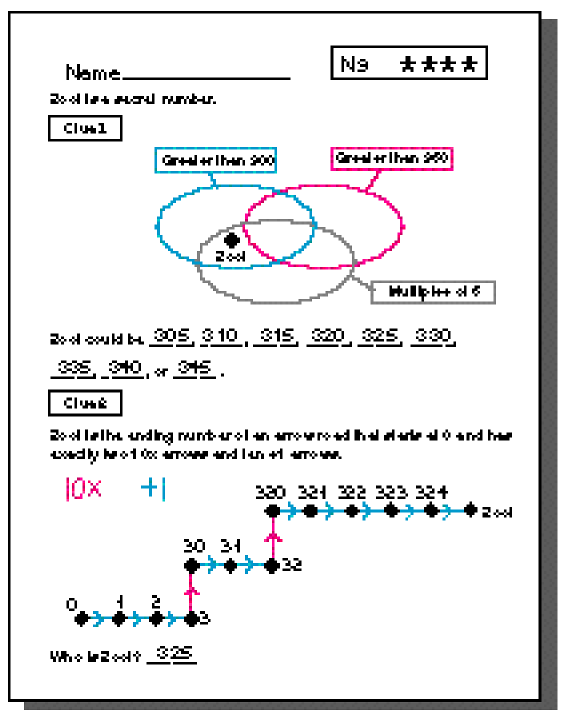
Home Activity

This would be a good time to send a letter to parents/guardians about multiplying by 10. Blackline N9 has a sample letter.


Name
N9 ★
What number is on the Minicomputer?
= 20
= 100
= 10
= 400
= 40
= 20
= 200
= 230
= 460
= 150


Name
N9 ★★
Label the dots.
10X
+6
1
7
13
130
136
142
148
1 480
1 486
1 492
1 498
1 504
15 040
150 400
0
6
12
18
24
30
300
3 000
3 006
3 012
3 018
30 180
301 800
3 018 000
2
20
26
260
266
272
2 720
2 726
27 260
27 266
272 660
2 726 600


Name
N9 ★★★
Draw as many 10x arrows as possible in this picture.
10X
70
707
7,000
3,200
700
32
3,002
302
320
7
8,040
70,000
840
84
368
804
36,800,000
36,800
368,000
3,680,000
3,680


Name
N9 ★★★★
Zool is a secret number.
Clue 1
Greater than 300
Greater than 300
Zool
Multiples of 5
Zool could be: 305, 310, 315, 320, 325, 330, 335, 340, or 345.
Clue 2
Zool is the ending number of an arrow road that starts at 0 and has exactly two 10x arrows and ten +1 arrows.
10X
+1
0
1
2
3
30
31
32
320
321
322
323
324
Zool
Who is Zool? 325

N10 SUBTRACTION #2

Capsule Lesson Summary

Examine subtraction properties in patterns generated by related subtraction statements. Review a subtraction algorithm and use it to do several calculations.

Materials

Teacher
- None

Student
- Paper
- Worksheets N10*, **, and ***

Description of Lesson

This lesson reviews a method of subtraction that was introduced in *CSMP Mathematics for the Upper Primary Grades, Part IV* (Third Grade). Even if your students do not use this method, the patterns and properties of subtraction used here are valuable.

Exercise 1

Begin this exercise with a comparison subtraction problem.

T: ***Darren wants to compare his weight (in pounds) to his little sister's. Darren weighs 58 pounds and his sister weighs 30 pounds. What calculation would Darren do to determine how much more he weighs?***

S: ***Subtract 58 – 30.***

T: ***What number is 58 – 30?***

S: ***28.***

Write the calculation on the board.

$58 - 30 = 28$

T: ***Suppose both Darren and his sister each gain one pound. Then, what calculation would Darren do?***

S: ***59 – 31.***

T: ***What number is 59 – 31?***

$59 - 31 = 28$

S: ***28.***

T: ***If they each gain another pound.… What number is 60 – 32?***

S: ***Still 28.***

$60 - 32 = 28$

T: ***Why do you think that these problems all have the same answer?***

S: ***Each time both Darren and his sister gain one pound.***

S: ***Because we added 1 to each number in the problem to get the next problem.***

Continue with 62 – 34 and 67 – 39. Students should observe that when both numbers increase by 2 (both children gain 2 pounds) or both numbers increase by 5 (both children gain 5 pounds), the difference is still 28.

$$\begin{aligned} 58 - 30 &= 28 \\ 59 - 31 &= 28 \\ 60 - 32 &= 28 \\ 62 - 34 &= 28 \\ 67 - 39 &= 28 \end{aligned}$$

T: ***I am going to write some more subtraction problems on the board. In each problem the difference is still 28. Find what number is missing from each. Use patterns like you found here*** (point to the list) ***to help you.***

Direct students to copy and solve these problems individually.

$\boxed{} - 40 = 28$	$\boxed{} - 53 = 28$	$293 - \boxed{} = 28$
$70 - \boxed{} = 28$	$\boxed{} - 153 = 28$	$300 - \boxed{} = 28$
$80 - \boxed{} = 28$	$291 - \boxed{} = 28$	$\boxed{} - 283 = 28$

After about five minutes, check the problems collectively, and occasionally ask students to explain their answers.

$\boxed{68} - 40 = 28$	$\boxed{81} - 53 = 28$	$293 - \boxed{265} = 28$
$70 - \boxed{42} = 28$	$\boxed{181} - 153 = 28$	$300 - \boxed{272} = 28$
$80 - \boxed{52} = 28$	$291 - \boxed{263} = 28$	$\boxed{311} - 283 = 28$

Exercise 2

T: ***What number is 3 + 5?*** (8)
30 + 50? (80)
130 + 50? (180)
139 + 50? (189)

My friend Nick would write the problem 139 + 50 like this.

On the board write 139 with a small 5 above the 3.

$$1\overset{5}{3}9$$

T: ***Why does he write a 5 above the 3 in 139?***

S: ***Because 50 is 5 tens. Adding 139 + 50 is the same as adding 5 tens to 139.***

T: ***How many tens is 3 tens + 5 tens?***

S: ***8 tens.***

$$1\overset{5}{3}9 = 189$$

T: ***What number is 237 + 8?***

S: ***245.***

$$23\overset{8}{7} = 245$$

Continue with this set of problems. (Answers are in boxes.)

$$7\overset{9}{0}8 = \boxed{798} \qquad 5\overset{6}{4}2 = \boxed{602} \qquad 1\,\overset{5}{4}8\overset{2}{2} = \boxed{1\,984}$$

$$46\overset{7}{1} = \boxed{468} \qquad \overset{7}{3}65 = \boxed{1\,065} \qquad \overset{2}{8}3\overset{4}{7} = \boxed{1\,041}$$

T: ***What number is 245 + 10?***

On the board write 245 with a small 10 above the 5.

$$24\overset{10}{5}$$

S: ***245 + 10 = 255.***

T: ***Could Nick write the problem in a different way?***

S: ***Nick could write a 1 above the 4 instead.***

Record the equivalent expression and answer.

$$24\overset{10}{5} = 2\overset{1}{4}5 = \boxed{255}$$

T: ***What about this problem?***

$$6\overset{10}{7}3$$

S: ***10 tens is the same as 1 hundred, so Nick could write a 1 above the 6 instead.***

T: ***What number is 673 + 100?***

$$6\overset{10}{7}3 = \overset{1}{6}73 = \boxed{773}$$

S: ***773.***

Exercise 3

Write this problem on the board, and invite a student to do the subtraction.

$$\begin{array}{r} 346 \\ -132 \\ \hline 214 \end{array}$$

T: ***In this case, Nick solves the problem in the same way.***

Write another problem on the board.

$$\begin{array}{r} 87\overset{10}{1} \\ -5\overset{1}{4}5 \\ \hline \end{array}$$

T: ***Here, Nick solves this problem by adding 10 to both numbers. He adds 10 ones to the top number, and he adds 1 ten to the bottom number.***

10 + 1 = 11, and 11 – 5 = 6.

$$\begin{array}{r} 87\overset{10}{1} \\ -5\overset{1}{4}5 \\ \hline 6 \end{array}$$

4 + 1 = 5, and 7 tens – 5 tens = 2 tens.

$$\begin{array}{r} 87\overset{10}{1} \\ -5\overset{1}{4}5 \\ \hline 26 \end{array}$$

T: ***8 hundreds – 5 hundreds = 3 hundreds.***

$$\begin{array}{r} 87\overset{10}{1} \\ -5\overset{1}{4}5 \\ \hline 326 \end{array}$$

Invite a student to use Nick's method to subtract 429 – 136.

$$\begin{array}{r} 4\overset{10}{2}9 \\ -\overset{1}{1}36 \\ \hline 293 \end{array}$$

This exercise reviews Nick's method for students who learned it in third grade. Let students share other ways of doing the subtraction, especially if you have students who insist on a "borrowing" method. For example,

$$\begin{array}{r} \overset{3}{\cancel{4}}\overset{10}{2}9 \\ -136 \\ \hline 293 \end{array} \qquad \begin{array}{r} 429 \\ -136 \\ \hline \end{array} \xrightarrow[\text{both numbers}]{+70} \begin{array}{r} 499 \\ -206 \\ \hline 293 \end{array}$$

Ask students to copy and solve these problems individually. Check answers collectively. (Answers are in boxes.)

$$\begin{array}{r} 792 \\ -345 \\ \hline \boxed{447} \end{array} \qquad \begin{array}{r} 780 \\ -248 \\ \hline \boxed{532} \end{array} \qquad \begin{array}{r} 2\,549 \\ -1\,869 \\ \hline \boxed{680} \end{array}$$

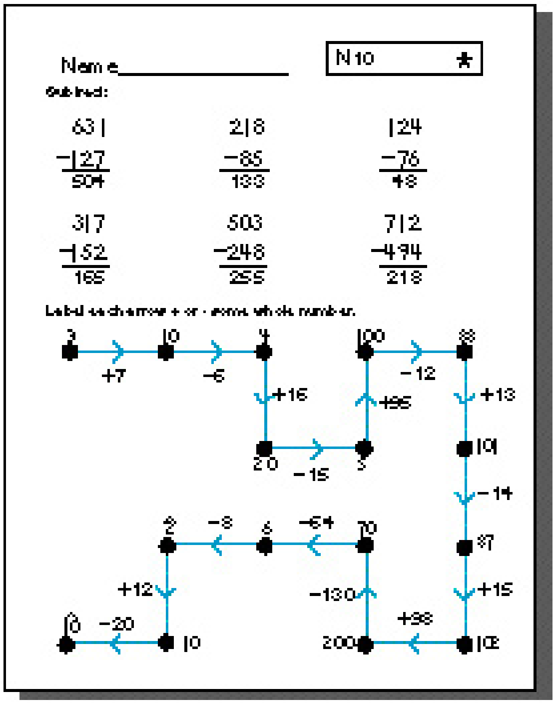
Worksheets N10*, **, and *** are available for individual work.

Writing Activity

Invite students to write an explanation of how they would solve the problem 106 – 38.

Home Activity

This would be a good time to send a letter to parents/guardians about subtraction methods. Blackline N10 has a sample letter.

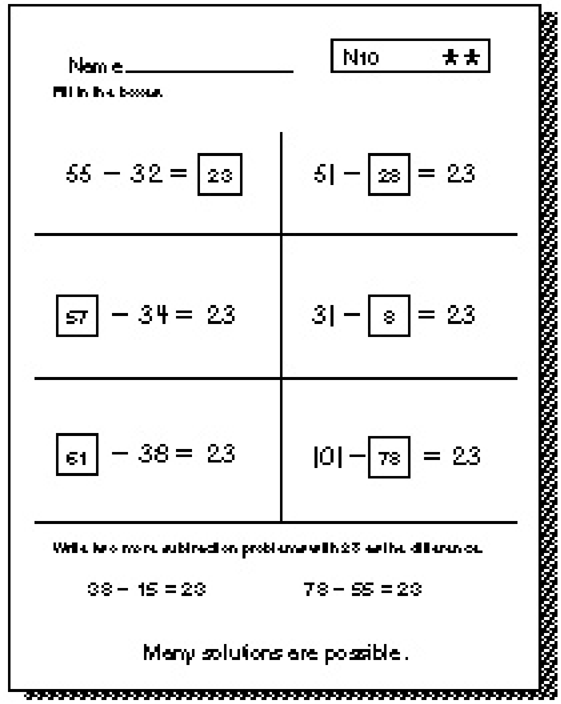


Name____________ N10 ★

Subtract:

631 − 127 = 504	218 − 85 = 133	124 − 76 = 48
317 − 152 = 165	503 − 248 = 255	712 − 494 = 218

Label each arrow + or − some whole number.



Name____________ N10 ★★

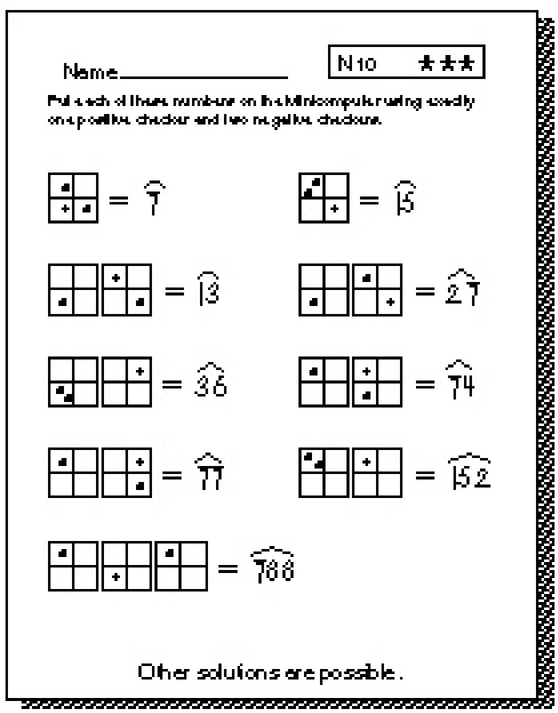
Fill in the boxes.

55 − 32 = $\boxed{23}$	51 − $\boxed{28}$ = 23
$\boxed{57}$ − 34 = 23	31 − $\boxed{8}$ = 23
$\boxed{61}$ − 38 = 23	101 − $\boxed{78}$ = 23

Write two more subtraction problems with 23 as the difference.

38 − 15 = 23 78 − 55 = 23

Many solutions are possible.

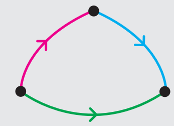


Name____________ N10 ★★★

Put each of these numbers on the Minicomputer using exactly one positive checker and two negative checkers.

Other solutions are possible.

Capsule Lesson Summary



In a composition arrow diagram ask what the green arrow could be for, given labels for the red and blue arrows; then ask what the red and blue arrows could be for, given a label for the green arrow. Use the composition of relations to label the dots in an arrow picture. Find possible locations of pairs of numbers in an arrow picture with only the arrows labeled.

Materials

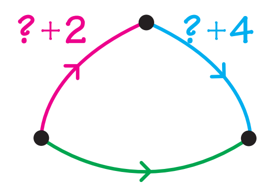
Teacher
- Colored chalk

Student
- Paper
- Colored pencils, pens, or crayons
- Worksheets N11*, **, ***, and ****

Description of Lesson

Exercise 1

Draw this arrow picture on the board.



Trace the appropriate arrows with motions as described in the Lesson L2 *Composition Games #1* as you ask,

T: ***A +2 arrow followed by a +4 arrow is the same as what kind of arrow*** (trace the green arrow)***?***

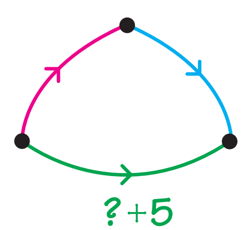
S: ***+6.***

Repeat this activity with other choices for the red and blue arrows. Some suggestions are given below.

?+8 followed by −5 ?(+3) ?−11 followed by −6 ?(−17) ?2× followed by 3× ?(6×)

Occasionally check the label for the green arrow by choosing a starting number and then labeling the other dots following the red and blue arrows.

Vary the activity. Instead of labeling the red and blue arrows, label the green arrow only.



T: ***If the green arrow were for +5, what could the red and blue arrows be for?***

S: ***The red arrow could be for +4, and the blue arrow could be for +1.***

S: ***The red arrow could be for +10, and the blue arrow could be for –5.***

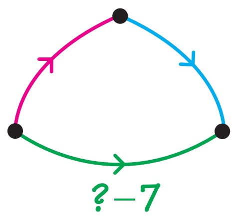
N11

Make a chart of possibilities on the board. Continue until many are found, and until your students begin to sense that there are infinitely many solutions involving one arrow for plus a whole number and the other arrow for minus a whole number. Perhaps someone will give a solution involving the addition or subtraction of fractions or decimals.

→	→
+4	+1
+10	−5
+2	+3
+1	+4
+0	+5†
−10	+15
+8	−3
−270	+275
$+3\frac{1}{2}$	$+1\frac{1}{2}$

Repeat this activity for a –7 green arrow. The list on the right shows only some of the infinitely many possibilities.

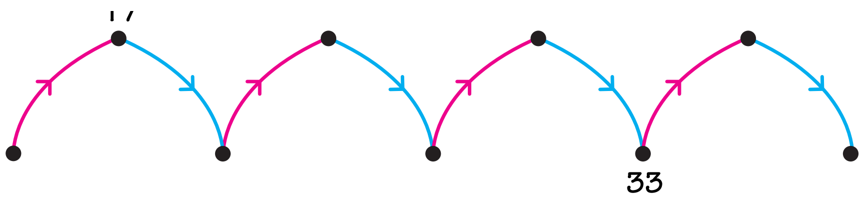
You may like to allow students to work with partners during Exercises 2 and 3.



→	→
−3	−4
−6	−1
−10	+3
+1	−8
+100	−107
−2	−5
−17	+10
$-2\frac{1}{2}$	$-4\frac{1}{2}$

Exercise 2

Draw this arrow picture on the board and instruct students to copy it on their papers.

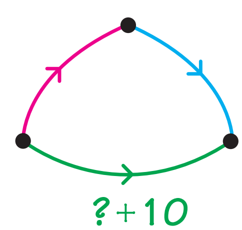


T: ***I've labeled two dots. Do you know what numbers the other dots are for?***

S: ***We need more information, like what the arrows are for.***

T: ***I'm not going to tell you what the arrows are for, but I will tell you that a red arrow followed by a blue arrow is for +10.***

Near the picture above, draw a composition arrow picture showing red followed by blue.

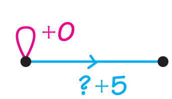


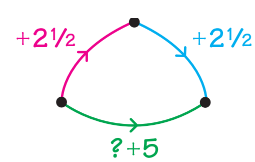
Instruct students to try to use this composition information to help label the other dots in the arrow picture. You may want to remind students to draw as many +10 (green) arrows as they can, and to try to use green arrows to help label other dots.

After a while, discuss the labeling of dots collectively.

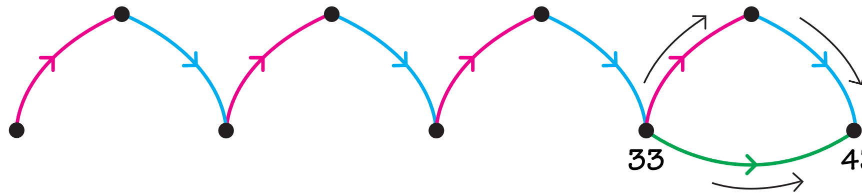
---

†You may wish to show how the picture would look if one of the arrows were for +0. There is no need to have two dots for the same number nor to have two different colored arrows for the same relation, for $+2\frac{1}{2}$ in this picture.

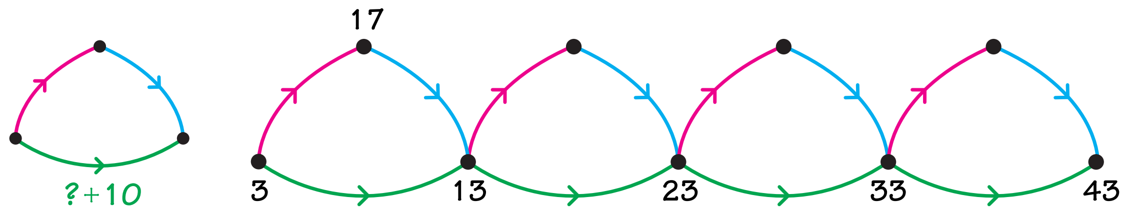




S (tracing arrows)**:** ***Red followed by blue is green, and 33 + 10 = 43.***



By using composition, students should find +10 arrows across the bottom of the picture. Following green (+10) arrows in reverse they should be able to label the dots across the bottom.



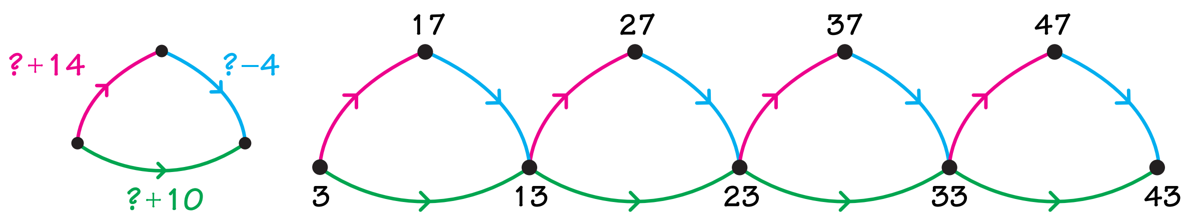
T: ***We still have three dots to label. Does anyone have a suggestion?***

S: ***Maybe we could figure out what the red and blue arrows are for.***

S: ***There are lots of possibilities that would give us a +10 arrow: +1 and +9, +3 and +7, +20 and –10, and so on.***

S: ***But we know that a red arrow goes from 3 to 17 and that a blue arrow goes from 17 to 13. So the red arrows are for +14 and the blue arrows are for –4.***

After this discovery is made, let students finish labeling the dots.

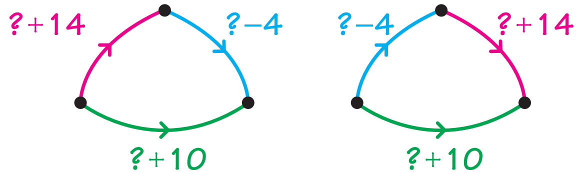


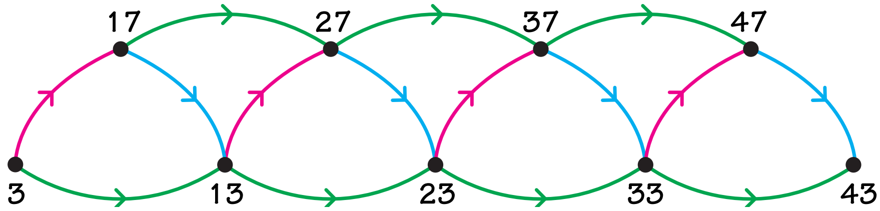
T: ***Can we draw any more green arrows?***

S: ***From 17 to 27 because 17 + 10 = 27, from 27 to 37 because 27 + 10 = 37; and so on.***

T: ***–4*** (trace the blue arrow from 17 to 13) ***followed by +14*** (trace the red arrow from 13 to 27) ***is +10.***

Some students may have used this fact to draw the +10 arrows across the top of the picture, and then labeled the top dots using these +10 arrows.

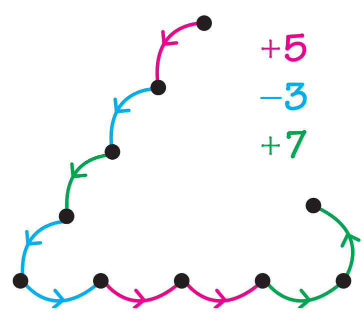




Erase the board before proceeding to Exercise 3.

Exercise 3

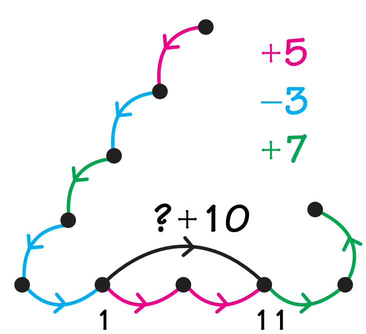
Draw this arrow picture on the board, and instruct students to copy it on their papers.



T: ***Suppose that 1 and 11 are in this picture. Where could they be?***

Let students experiment with locating 1 and 11 in the picture. You may suggest they think about how 1 and 11 are related; i.e., how much more than 1 is 11? This clue should be sufficient to enable most students to locate 1 and 11.

Draw an arrow from 1 to 11.



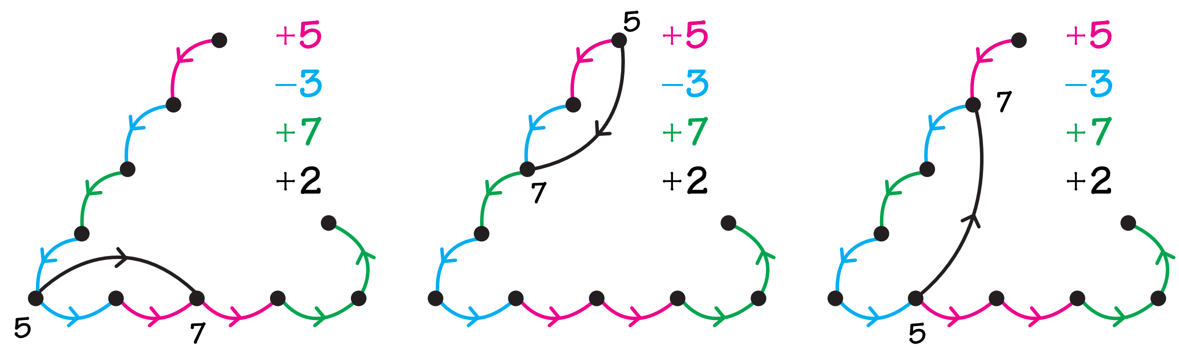
T: ***What could this arrow be for?***

S: ***+10.***

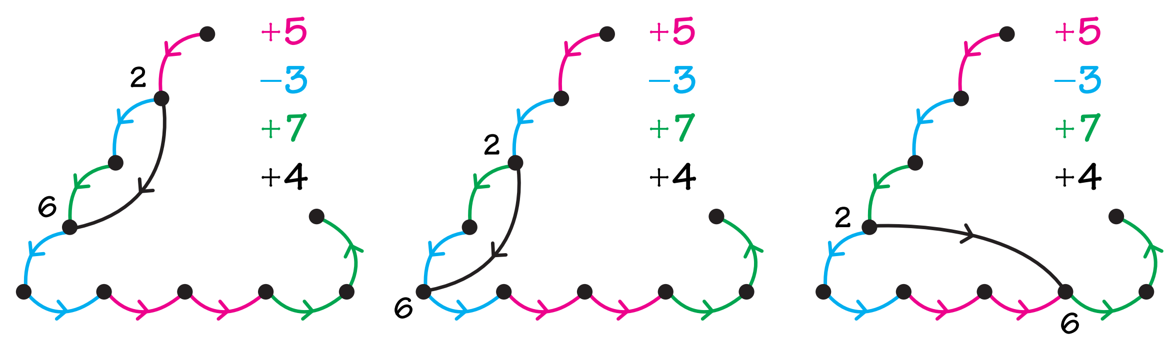
T: ***+5*** (trace the red arrow starting at 1) ***followed by +5*** (trace the red arrow ending at 11) ***is +10*** (trace the new arrow)***.***

Erase the dot labels and the +10 arrow. Continue this activity with other pairs of numbers to locate in the picture, such as ones suggested below. Do not insist that your class find all of the possible locations for each pair.

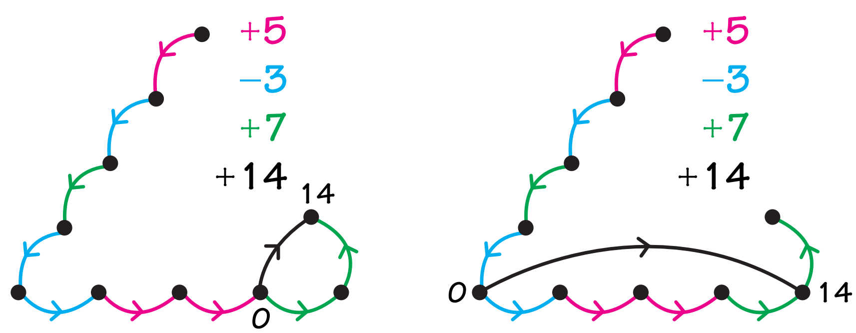
5 and 7



2 and 6



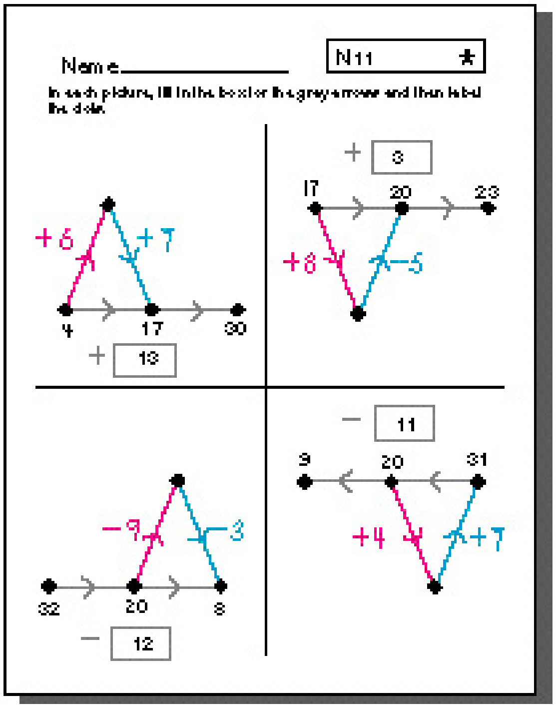
0 and 14

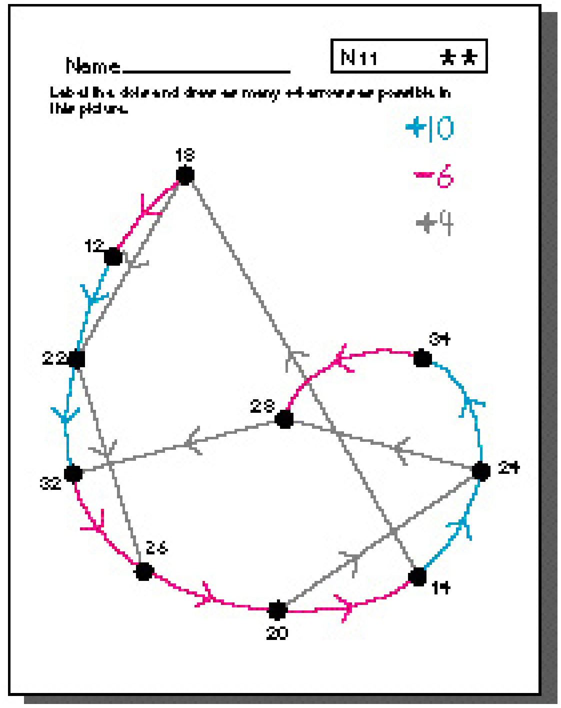


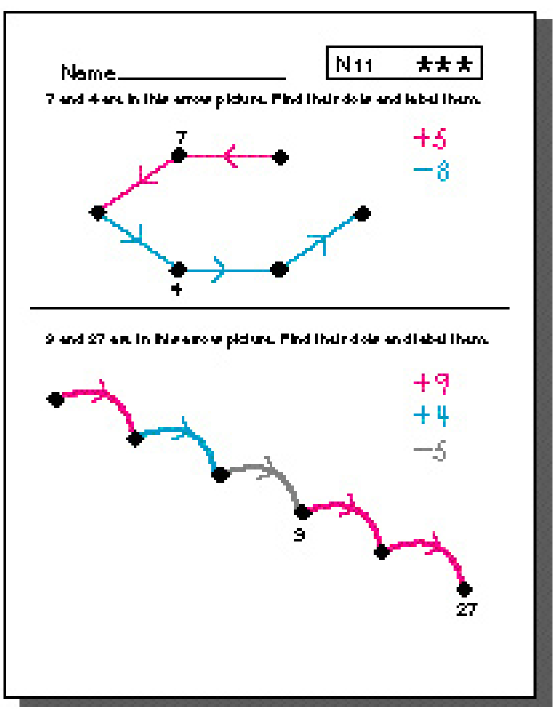
Worksheets N11*, **, ***, and **** are available for individual work.

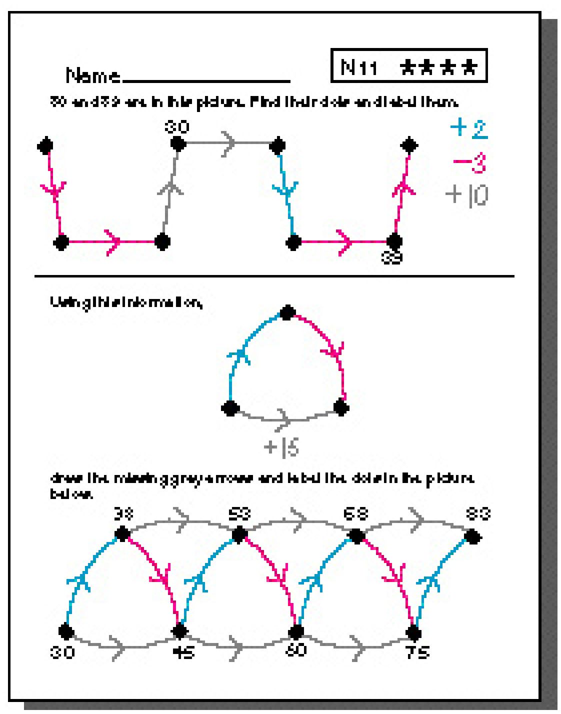
Home Activity

You may like to send the *** and/or **** worksheets home for students to do with a family member.


Name
N11 ★
In each picture, fill in the box for the grey arrows and then label the dots.
+6
+7
4
17
30
+ 13
+ 3
17
20
23
+8
−5
− 11
9
20
31
+4
+7
−9
−3
32
20
8
− 12


Name
N11 ★★
Label the dots and draw as many +4 arrows as possible in this picture.
+10
−6
+4
18
12
22
28
34
24
32
26
20
14


Name
N11 ★★★
7 and 4 are in this arrow picture. Find their dots and label them.
7
4
+5
−8
9 and 27 are in this arrow picture. Find their dots and label them.
+9
+4
−5
9
27


Name
N11 ★★★★
30 and 39 are in this picture. Find their dots and label them.
30
39
+2
−3
+10
Using this information,
+15
draw the missing grey arrows and label the dots in the picture below.
38
53
68
83
30
45
60
75

Capsule Lesson Summary

Review $\frac{1}{2}$x in string pictures and on the Minicomputer. Review decimals on the Minicomputer.

Materials

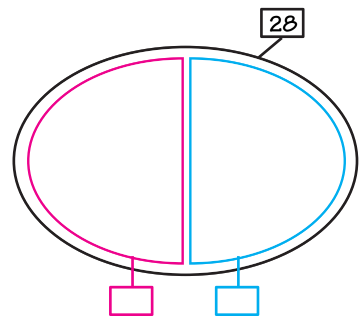
Teacher
- Minicomputer set
- Colored chalk

Student
- Paper
- Colored pencils, pens, or crayons

Description of Lesson

Exercise 1

Draw the string picture on the board as you tell this story.



T: ***There is a class of 28 students. Exactly half of the students in the class are girls and half are boys. How many girls and how many boys are in the class?***

S: ***14 girls and 14 boys.***

Write 14 in both the red and the blue tags.

T: ***Tell me a number sentence we could write about this situation.***

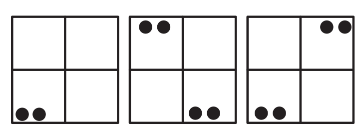
On the board, write the number sentences suggested by the students. For example:

$$14 + 14 = 28 \qquad \tfrac{1}{2} \times 28 = 14$$
$$2 \times 14 = 28 \qquad 28 \div 2 = 14$$

Repeat this activity with other simple stories using numbers such as 48; 70; 110; 300; 1 000; and 1 050. Adjust the difficulty of the problems to suit the numerical abilities of your students.

Exercise 2

Gradually put this configuration on the Minicomputer.



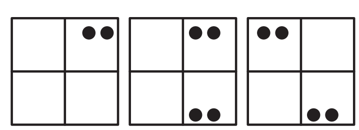
T: ***What number is on the Minicomputer?***

S: ***592.***

T: ***By looking at this configuration, can you tell what*** $\frac{1}{2}$ x ***592*** (read as "one-half of 592") ***is?***

S: ***2*** x ***296 is on the Minicomputer, so*** $\frac{1}{2}$ x ***592 = 296.***

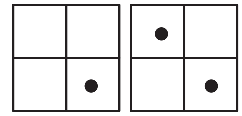
Repeat this activity with 918.


= 918

The students should conclude that 918 = 2 x 459 and $\frac{1}{2}$ x 918 = 459.

Exercise 3

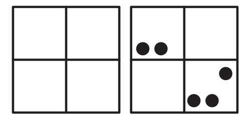
T: ***Suppose I have $19 and I want to share it equally between two friends. How much do I give to each of them?***



Ask students to write the answer on a sheet of paper. Then model this problem on the Minicomputer.
First put on 19.

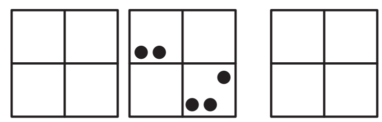
T: ***19 is on the Minicomputer. How can we show sharing two ways and calculating*** $^{1}/_{2}$ x ***19?***

S: ***Make backward trades to get checkers in pairs.***



Invite students to make trades until the checkers are in pairs, except for an extra checker on the 1-square. Move the pairs of checkers into the corners of the squares they are on and refer to the extra checker on the 1-square.

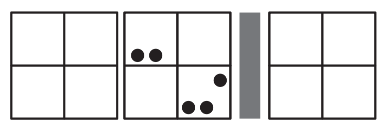
T: ***We can share two ways with the pairs of checkers, but we still have one checker left over. What should we do now?***



Encourage students to express their ideas. Perhaps someone will suggest adding another Minicomputer board to the right, or suggest it yourself. Place an extra board to the right of the ones board, but leave more space than usual.

At least one student should object and say that now it looks like the checkers are on the tens board (the number is 190 rather than 19). If no one objects, you should make this observation yourself.

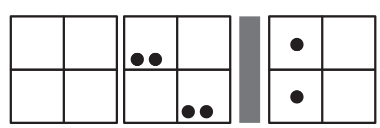
T: ***In order not to forget that this number is 19 (i.e., the checkers are on the ones board), let's draw a bar between the ones board and this new board. The bar will help us locate the ones place.***



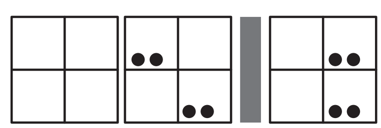
What should we do next?

Some students may be confused and suggest some wrong trades. It's natural for the bar to disturb them for a short while. Perhaps you will need to make one trade yourself.

T(pointing to the appropriate squares)**:** ***One checker on the white square (one dollar) is the same as one checker on the brown square (eight dimes) and one checker on the red square (two dimes).***

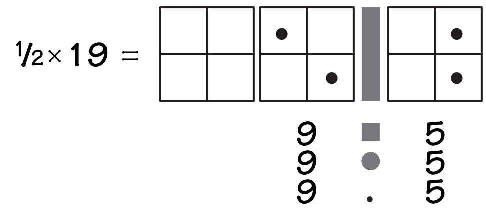


Generally students will make the next two trades without any trouble.



Invite a student to show $^{1}/_{2}$ x 19 by removing one checker from each pair.

T: ***How might we write this answer?***



Your students may make several suggestions.

T: ***Which method of writing do you like best?***

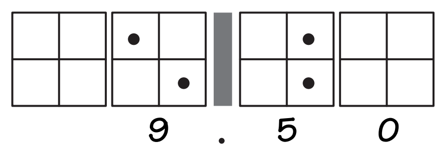
Let the students express their ideas. They will probably agree on the "dot" method because it is the easiest and the one commonly used for writing amounts of money.

T: ***What is*** $^1/_2$ x ***$19?***

S: ***Nine dollars and fifty cents.***

S: ***You need another 0 for 9.50 (or $9.50).***

Add another board on the right.



T: ***There are no checkers on the new board. We can write 0 under it.***

Emphasize that 9.5 = 9.50 and conclude that

$$^1/_2 \times 19 = 9.5 = 9.50$$

You may like to show students how to use a calculator to do the same calculation. On a simple four function calculator you will need to use ÷2 rather than $^1/_2$x. Even if you put $19 in as 19.00, when you press ÷ 2 = you will see 9.5 rather than 9.50.

Invite students to show several amounts of money as decimals on the Minicomputer.

Exercise 4

Write these three series of problems on the board and ask students to solve as many of them as they can in the time remaining for the lesson. (Answers are in boxes.)

$\frac{1}{2} \times 24 = \boxed{12}$	$\frac{1}{2} \times 86 = \boxed{43}$	$\frac{1}{2} \times 570 = \boxed{285}$
$\frac{1}{2} \times 40 = \boxed{20}$	$\frac{1}{2} \times 860 = \boxed{430}$	$\frac{1}{2} \times 57 = \boxed{28.5}$
$\frac{1}{2} \times 46 = \boxed{23}$	$\frac{1}{2} \times 34 = \boxed{17}$	$\frac{1}{2} \times 5.7 = \boxed{2.85}$
$\frac{1}{2} \times 30 = \boxed{15}$	$\frac{1}{2} \times 434 = \boxed{217}$	
	$\frac{1}{2} \times 3 = \boxed{1.5}$	

Reading Activity

Your students might enjoy reading a story involving money problems such as *Alexander, Who Used to Be Rich Last Sunday* by Judith Viorst.

Home Activity

This would be a good time to send a letter to parents/guardians about decimals on the Minicomputer. Blackline N12 has a sample letter.

N13 MINICOMPUTER GOLF #2

Capsule Lesson Summary

By moving exactly one checker, change a number on the Minicomputer by a specified amount. Play *Minicomputer Golf*.

Materials

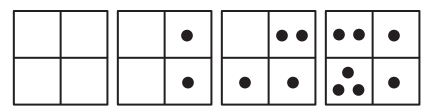
Teacher
- Minicomputer set
- Colored chalk

Student
- Paper
- Worksheets N13*, **, ***, and ****

Description of Lesson

Exercise 1

Gradually put this configuration on the Minicomputer, starting with the checkers on the squares of greatest value. Pause frequently so your students can do mental calculations.



T: ***What number is on the Minicomputer?***

S: ***637.***

T: ***I am going to change the number on the Minicomputer by moving one checker to another square. Tell me if the new number is more than or less than the number on the Minicomputer now.***

Move a checker from the 2-square to the 8-square.

T: ***Did I increase or decrease the number?***

S: ***Increase.***

T: ***How much more is this new number?***

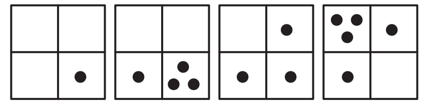
S: ***6 more, because 2 + 6 = 8.***

Repeat this activity several times as suggested below. Do not return checkers to their original positions. Each move starts from a new number on the Minicomputer.

Move a checker
- from the 20-square to the 4-square; (16 less)
- from the 1-square to the 20-square; (19 more)
- from the 40-square to the 100-square; (60 more)
- from the 4-square to the 2-square; (2 less)
- from the 400-square to the 200-square; (200 less)
- from the 2-square to the 100-square; (98 more)
- from the 2-square to the 1 000-square. (998 more)

N13

Check that this configuration is on the Minicomputer.



T: ***Who can move just one checker and increase the number by 20, make it 20 more than it is now?***

A student should move a checker from the 20-square to the 40-square. Continue the activity by asking volunteers to make other changes such as those suggested below. Feel free to adjust the level of difficulty to the numerical abilities of your students.

Make a change that is
- 60 less (Move a checker from the 100-square to the 40-square)
- 7 less (Move a checker from the 8-square to the 1-square)
- 6 more (Move a checker from the 2-square to the 8-square,
  or from the 4-square to the 10-square.)

You may want to ask for a change that cannot be made because there is no checker in a position to do so; for example, 10 more when there is no checker on the 10-square.

T: ***Who can move one checker and decrease the number by 5— make it 5 less than it is now.***

Let the students have a minute or two to consider this question.

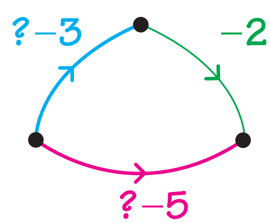
S: ***It can't be done. You cannot move one checker and make the number 5 less.***

T: ***Could we make the number 5 less by moving exactly two checkers?***

Invite one student to make the first move, and then another to make the second move.

Record the two moves in an arrow picture. For example,

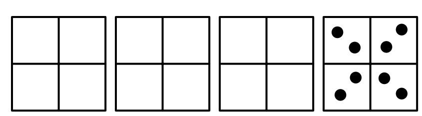
Move a checker from the 4-square to the 1-square.



Move a checker from the 4-square to the 2-square.

Exercise 2

Put this configuration on the Minicomputer.



T: ***What number is on the Minicomputer?***

S: ***30; 2 x 15 = 30.***

Write this information on the board.

Goal: 300 • 30

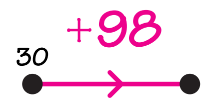
T: ***By moving some of these checkers one at a time let's try to reach 300. Who would like to make the first move?***

Suppose a student moves a checker from the 2-square to the 100-square.

T: ***How much more is the number now?***

S: ***98 more.***

Draw an arrow to record the move.

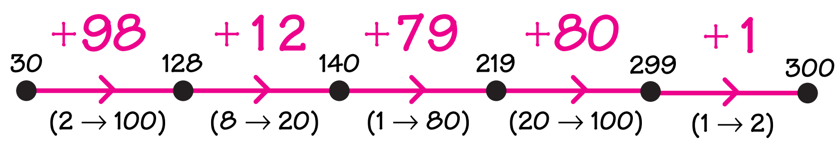


T: ***What number is on the Minicomputer?***

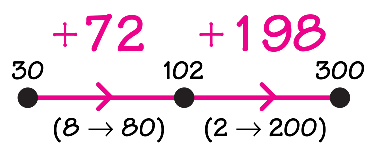
S: ***128; 30 + 98 = 128.***

Label the ending dot.

Let students make moves until the goal of 300 is reached. Be sure to record the moves as the activity progresses. A sample arrow road is given below.



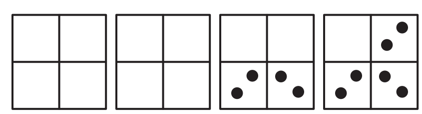
Return the checkers to their original positions (for 30) and ask the class if they can reach 300 in fewer than five moves. Some students might discover this two-move solution.



If no shorter solution is forthcoming, do not insist that one be found.

Exercise 3

Put this con~~figuration on the~~ Minicomputer.



T: ***What number is on the Minicomputer?***

S: ***74.***

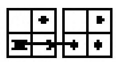
T: ***Let's play a game of Minicomputer Golf, using this number as the starting number and 400 as the goal.***

Play a *Minicomputer Golf* game with teams as described in Lesson N6 *Minicomputer Golf #1*.

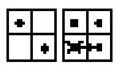
Worksheets N13*, **, ***, and **** are available for individual work.

Name______________ N13 ★

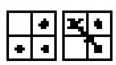
Move exactly one checker to get 47 on the Minicomputer.



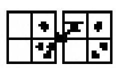
Move exactly one checker to get 103 on the Minicomputer.



Move exactly one checker to get 75 on the Minicomputer.

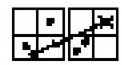


Move exactly one checker to get 76 on the Minicomputer.

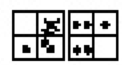


Name______________ N13 ★★

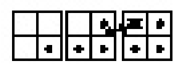
Move exactly one checker to get 34 on the Minicomputer.



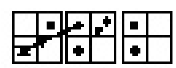
Move exactly one checker to get 64 on the Minicomputer.



Move exactly one checker to get 217 on the Minicomputer.

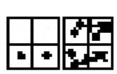


Move exactly one checker to get 590 on the Minicomputer.

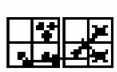


Name______________ N13 ★★★

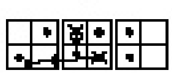
Move exactly two checkers to get 54 on the Minicomputer.



Move exactly two checkers to get 172 on the Minicomputer.



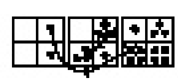
Move exactly two checkers to get 370 on the Minicomputer.



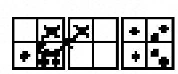
Other solutions are possible.

Name______________ N13 ★★★★

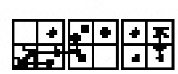
Move at most three checkers to get 650 on the Minicomputer.



Move at most three checkers to get 620 on the Minicomputer.



Move at most three checkers to get 751 on the Minicomputer.



Other solutions are possible.

Capsule Lesson Summary

Solve equations such as 4 x $\square$ = 149 by estimation and successive approximations. Investigate patterns among the integers generated by repeated addition of 10's and repeated subtraction of 10's.

Materials

Teacher
- Calculator
- Colored chalk

Student
- Calculator
- Colored pencils, pens, or crayons
- Worksheets N14* and **

Description of Lesson

Exercise 1

Make calculators available for this exercise. Write this open sentence on the board.

$$4 \times \square = 149$$

T: ***4* x *some number equals 149. Try to find which number belongs in the box. You may use your calculators.***

Note: Some students may know immediately to calculate 149 ÷ 4 = 37.25 on the calculator, and then put 37.25 in the box. Let those students confirm their method by doing the calculation 4 x 37.25 = 149. Continue to discuss the following process of successive approximations as it leads into patterns in multiplication calculations.

Let the students try various numbers individually or with partners.

T: ***Who got a number close to 149 when they multiplied by 4?***

S: ***4* x *34 = 136.***

Write the number sentence 4 x 34 = 136 on the board.

T: ***Will the number in the box be more or less than 34?***

S: ***More. 4* x *34 = 136 and 149 is greater than 136.***

After several more attempts, students should observe that 4 x 37 = 148 and that 4 x 38 = 152. Write these number sentences on the board.

$$4 \times 37 = 148$$
$$4 \times \square = 149$$
$$4 \times 38 = 152$$

S: ***37 is less than the number in the box and 38 is more. Maybe there is no solution.***

T: ***There is no* whole number *solution. The solution is between 37 and 38. Do you know any numbers between 37 and 38?***

S: ***37.5 (or $37\frac{1}{2}$).***

T: ***Calculate 4*** x ***37.5 on your calculator.***

S: ***4*** x ***37.5 = 150. The solution is between 37 and 37.5.***

Let students continue to use such approximations until they find the solution, 37.25.

T: ***Now, I would like you to use the calculator*** **only** ***when I say to check an answer.***
4 x ***37 = 148. 4*** x ***38 = 152. Who can predict what 4*** x ***39 equals?***

S: ***156.***

T: ***Check it on your calculator.***

Continue asking for 4x the next greater whole number until you have these number sentences on the board.

$4 \times 37 = 148$
$4 \times 38 = 152$
$4 \times 39 = 156$
$4 \times 40 = 160$
$4 \times 41 = 164$
$4 \times 42 = 168$

T: ***What patterns do you see?***

S: ***The answers increase by 4 each time.***

S: ***The numbers that you multiply by 4 increase by 1 each time.***

T: ***Can anyone explain these patterns?***

S: ***It would be like having four groups of 42 people instead of four groups of 41 people; there would be four more people since each group has one more person.***

A real context such as the above example may help students see the pattern.

Erase the board, and then write this open sentence.

$3 \times \boxed{} = 516$

Invite students to use their calculators to find what number to put in the box. They should find that 3 x 172 = 516.

T: ***3*** x ***172 = 516. What number is 3*** x ***173?***

S: ***519.***

Continue asking for 3x the next greater whole number until you have these number sentences on the board.

$3 \times 172 = 516$
$+1 \qquad +3$
$3 \times 173 = 519$
$+1 \qquad +3$
$3 \times 174 = 522$
$+1 \qquad +3$
$3 \times 175 = 525$

When students observe the +1 and +3 patterns occurring, use arrows to highlight them.

T: ***When the numbers we multiply by 3 increase by 1, the answers increase by 3. What if we increase the numbers by 2 each time? How do you think the answers will increase? What do you predict 3*** x ***177 equals?***

Let several students answer before asking for a calculator check.

Indicate the pattern in your list of number sentences.

In a similar manner compute 3 x 179, 3 x 181, and 3 x 184.

$3 \times 172 = 516$
$3 \times 173 = 519$
$3 \times 174 = 522$
$3 \times 175 = 525$
$3 \times 177 = 531$
$3 \times 179 = 537$
$3 \times 181 = 543$
$3 \times 184 = 552$

Exercise 2

This would be a good exercise in which to use an overhead calculator. To begin, write these symbols on the board. Then give calculator instructions, slowly and clearly.

$\boxed{+}\ \boxed{10}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ldots$

T: ***Put 9 on the display of your calculators. Press $\boxed{+}$ $\boxed{1}\boxed{0}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and keep on pressing $\boxed{=}$. Do it slowly so that you can observe the numbers that appear on the display. What is the calculator doing? Do you see any patterns?***

S: ***The calculator is counting by tens.***

S: ***It adds 10 each time I press $\boxed{=}$.***

S: ***All the numbers end in 9: 9, 19, 29, 39, 49, and so on.***

Instruct students to clear the display and then to follow your instructions carefully. For the moment they should not go ahead on their own.

T: ***Put 8 on the display. Press $\boxed{+}$ $\boxed{1}\boxed{0}$ $\boxed{=}$ and then stop. What number is on the display?*** (18)
Press $\boxed{=}$ again. (28) ***Press $\boxed{=}$.*** (38)

Hide the display by placing one finger over it. Now predict each answer and don't look at the answer until I say, "Calculator check." Press $\boxed{=}$. What number do you predict is on the display?

S: ***48.***

T: ***Calculator check.***

S: ***Yes, it's 48.***

T: ***Hide the display. Press $\boxed{=}$ $\boxed{=}$.***

S: ***I predict 68.***

T: ***Calculator check.*** (68)
Hide the display. Press $\boxed{=}$ $\boxed{=}$ $\boxed{=}$.

S: ***I predict 98.***

T: ***Calculator check.***

Instruct students who do not have 98 on the display to press $\boxed{8}$ $\boxed{+}$ $\boxed{1}\boxed{0}$ $\boxed{=}$ $\boxed{=}$ …. until they reach 98. Use this method throughout the lesson to allow students who make key mistakes to catch up.

Use the following instructions to continue the activity from 78. For each instruction, ask students to hide their displays, to predict the answer, and to check it. Occasionally ask a student to explain a prediction. (Resulting numbers are in parentheses.)

$\boxed{=}$	(108)
$\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$	(148)
$\boxed{=}$ $\boxed{=}$ $\boxed{=}$	(178)
$\boxed{=}$ $\boxed{=}$ $\boxed{=}$	(208)

T: ***Everyone should have 208 on the calculator. Now I'd like to count backward by tens. How can we do that on the calculator?***

S: ***Press*** $\boxed{-}$ $\boxed{1}\boxed{0}$ $\boxed{=}$ $\boxed{=}$ ***….***

T: ***Try it.***

S: ***Yes, it's subtracting tens.***

T: ***Keep pressing*** $\boxed{=}$ ***until you get to 128. Stop at 128. Hide the display. Press*** $\boxed{=}$ ***once. What number do you predict is on the display?*** (118) ***Calculator check.***

Continue in a similar manner from 118 with the following instructions.

$\boxed{=}$	(108)
$\boxed{=}$ $\boxed{=}$ $\boxed{=}$	(78)
$\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$	(38)
$\boxed{=}$ $\boxed{=}$ $\boxed{=}$	(8)
$\boxed{=}$	($^-2$)[†]

Discuss with your class why $^-2$ is in the calculators' display. Refer to a number line to show that $8 - 10 = {}^-2$. Continue the activity by giving the following instructions.

$\boxed{=}$	($^-12$)
$\boxed{=}$ $\boxed{=}$	($^-32$)
$\boxed{=}$ $\boxed{=}$ $\boxed{=}$	($^-62$)

T: ***Clear the display of your calculator. Who can put $^-56$ on the calculator?***

Let students suggest several methods; for example, $\boxed{0}$ $\boxed{-}$ $\boxed{5}\boxed{6}$ $\boxed{=}$ or $\boxed{1}$ $\boxed{-}$ $\boxed{5}\boxed{7}$ $\boxed{=}$. If a calculator has a "change sign" key (usually marked $\boxed{+/-}$), $\boxed{5}\boxed{6}$ $\boxed{+/-}$ also works.

T: ***With $^-56$ on your display, press*** $\boxed{+}$ $\boxed{1}\boxed{0}$ $\boxed{=}$***. What number do you have?*** ($^-46$)
Press $\boxed{=}$***. What number is on the display?*** ($^-36$)

T: ***Hide your display. Press*** $\boxed{=}$***. What number do you predict is on the display?***

S: ***$^-26$.***

T: ***Calculator check.***

[†]Note that here in a calculator context we use "$^-2$" rather than "$\hat{2}$." Students should begin to recognize alternative notations for negative numbers.

Continue in a similar manner with the following instructions.

$\boxed{=}$ $\boxed{=}$ ($^-6$)
$\boxed{=}$ (4)

Refer to a number line to show that $^-6 + 10 = 4$. Continue with these instructions.

$\boxed{=}$ $\boxed{=}$ $\boxed{=}$ (34)
$\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ (84)

T: ***What patterns do you notice when you add tens or subtract tens?***

S: ***If you add or subtract tens from some number, the last digit (ones) of the positive numbers you get are all the same, and the last digit of the negative numbers you get are all the same.***

S: ***When we subtracted tens from 208, all of the positive numbers ended in 8 and then all of the negative numbers ended in 2.***

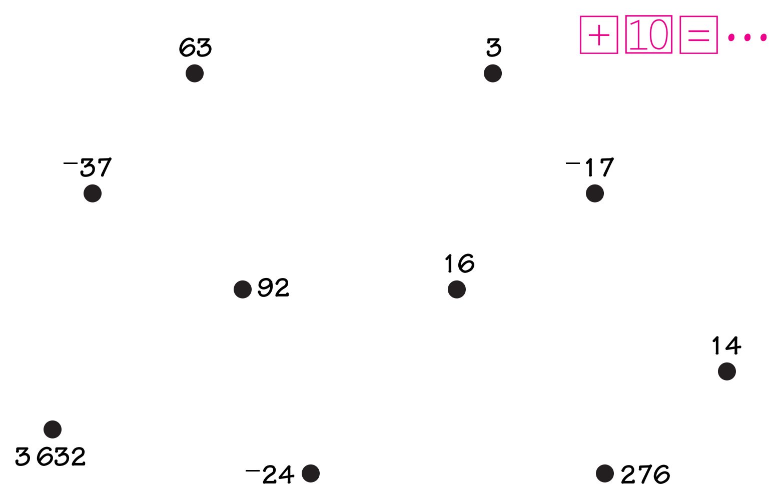
S: ***When we added tens to $^-56$, all of the negative numbers ended in 6 and all of the positive numbers ended in 4.***

S: ***8 + 2 = 10. 6 + 4 = 10. The sum of the two ending digits in each pattern is 10.***

If any student makes the last observation, you may wish to briefly explore it further by determining what happens when you subtract tens from other numbers such as 25, 47, or 19.

Exercise 3

Draw the following picture on the board.

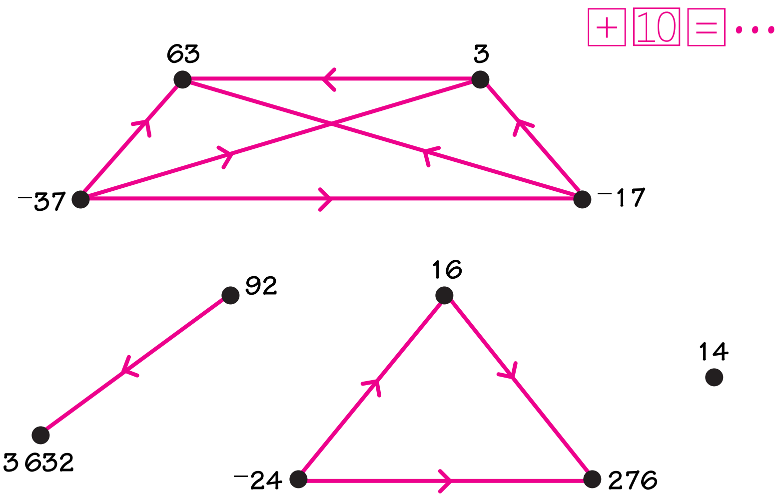


T: ***Let's draw a red arrow between two numbers if we can put one of the numbers on the calculator and eventually reach the other number by pressing*** $\boxed{+}$ $\boxed{1}\boxed{0}$ $\boxed{=}$ $\boxed{=}$ ...

Invite students to find places to draw red arrows. Encourage them to recall and look for +10 patterns.

N14

Your class should find all of the possible red arrows. You may need to stop and consider what happens when you put a negative number on the display and press [+] [1][0] [=] ….

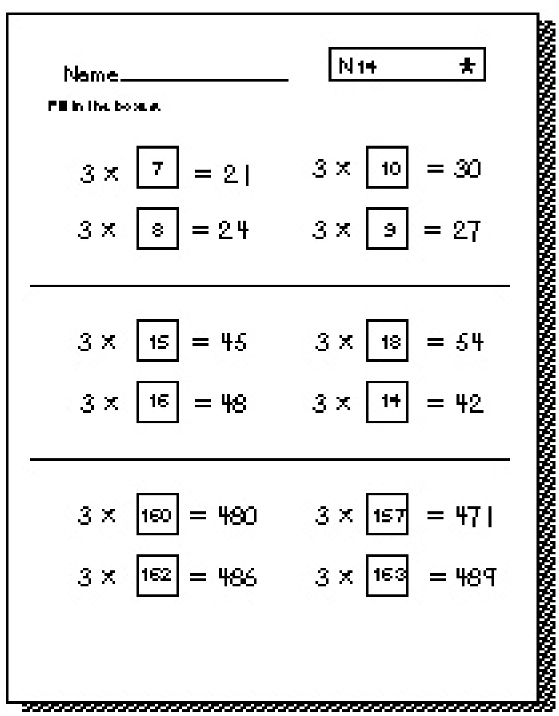


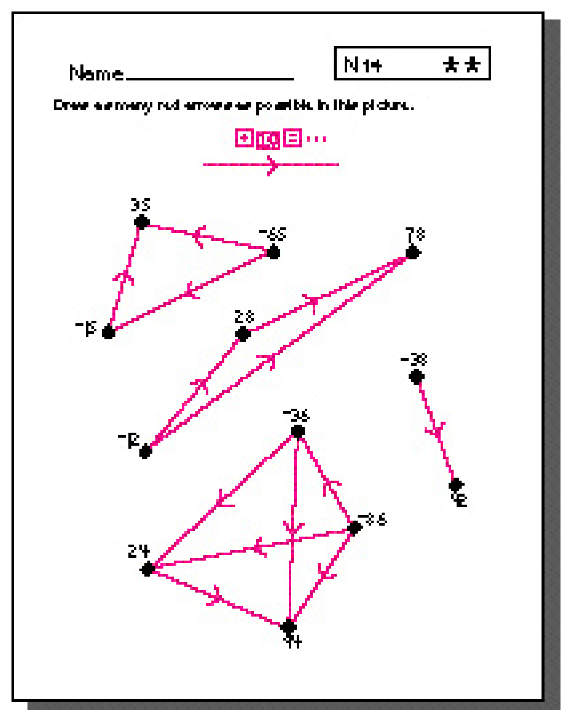
Students will probably use +10 patterns to determine where to draw some arrows, especially the one from 92 to 3 632.

Worksheets N14* and ** are available for individual work.

Home Activity

This would be a good time to send a letter to parents/guardians about the use of calculators. Blackline N14 has a sample letter.





Capsule Lesson Summary

Review earlier experiences involving multiplication with whole and decimal numbers. Using the Minicomputer, develop a systematic approach that leads to written forms such as the one illustrated here. Introduce the usual compact form (standard "carrying" notation). In a story problem context, add multiples of 6, 10, 25, and 15 to get sums of 250.

$$\begin{array}{r} 789 \\ \times\ 4 \\ \hline 36 \\ 320 \\ 2\,800 \\ \hline 3\,156 \end{array}$$

Materials

Teacher
- Minicomputer set
- Blackline N15(a)

Student
- Paper
- Record-keeping chart

Advance Preparation: Use Blackline N15(a) to make copies of the record-keeping chart for the problem in Exercise 5.

Description of Lesson

Exercise 1: Mental Arithmetic

In this exercise write the number sentences on the board.

T: ***What number is 5* x *8?*** (40) — $5 \times 8 = 40$

What number is 2* x *8? (16) — $2 \times 8 = 16$

Five 8's are 40 and two 8's are 16, so how much are seven 8's?

S: ***56.***

T: ***How did you do the calculation?***

S: ***I added 40 + 16.***

S: ***I remembered that 7* x *8 = 56.***

$$5 \times 8 = 40$$
$$2 \times 8 = 16$$
$$7 \times 8 = 56$$

Continue with these calculations. (Answers are in boxes.)

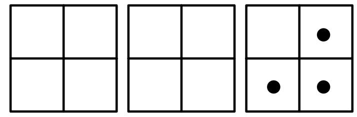
$5 \times 4 = \boxed{20}$ $5 \times 7 = \boxed{35}$ $5 \times 6 = \boxed{30}$

$2 \times 4 = \boxed{8}$ $2 \times 7 = \boxed{14}$ $2 \times 6 = \boxed{12}$

$7 \times 4 = \boxed{28}$ $7 \times 7 = \boxed{49}$ $7 \times 6 = \boxed{42}$

Exercise 2

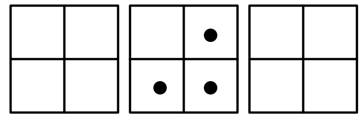
Put 7 on the Minicomputer.



T: ***What number is on the Minicomputer?***

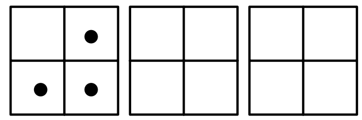
S: ***7.***

Move the checkers to the tens board.



S: ***70.***

Move the checkers to the hundreds board.



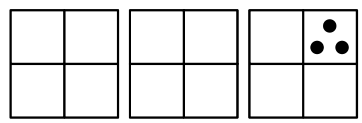
S: ***700.***

T: ***When all the checkers are moved from one board to the next, how does the number change?***

S: ***It is multiplied by 10.***

S: ***It has another 0 (in the ones place).***

Put this configuration on the Minicomputer.



T: ***3x what number is on the Minicomputer?***

S: ***4.***

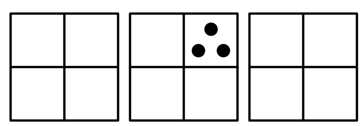
T: ***What number is 3 x 4?***

S: ***12.***

Record the multiplication fact on the board.

$$3 \times 4 = 12$$

Move the checkers from the 4-square to the 40-square.



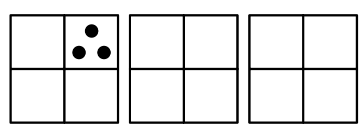
$$3 \times 4 = 12$$
$$3 \times 40 = 120$$

T: ***3x what number is on the Minicomputer?***

S: ***40.***

S: ***3 x 40 = 120.***

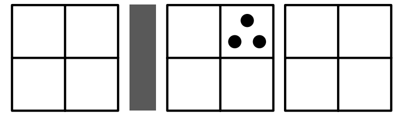
Move the checkers from the 40-square to the 400-square.



S: ***3 x 400 = 1 200.***

$$3 \times 4 = 12$$
$$3 \times 40 = 120$$
$$3 \times 400 = 1\,200$$

Put this configuration on the Minicomputer.



T: ***Each checker on this square is for four dimes. If we had three stacks of four dimes, how many dimes would we have?***

S: ***12 dimes.***

T: ***How much money is 12 dimes?***

S: ***$1.20.***

T: ***What decimal number corresponds to $1.20?***

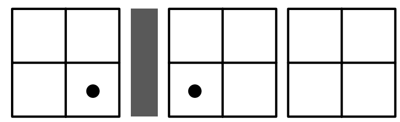
S: ***1.2*** (read as "one point two" or "one and two tenths").

S: ***1.20*** (read as "one point two zero" or "one and twenty hundreths").

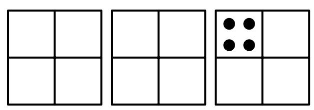
Record the appropriate number sentence as you read it.

$3 \times 0.4 = 1.2$
or
$3 \times 0.40 = 1.20$

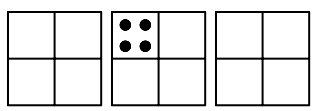
You may like to invite students to make trades on the Minicomputer to get the standard configuration for 1.2.



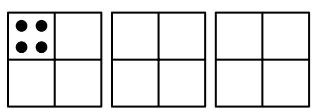
Do a similar activity by moving four checkers from a brown square to another brown square to arrive at these number sentences.



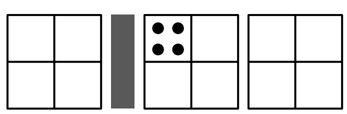
$4 \times 8 = 32$



$4 \times 80 = 320$



$4 \times 800 = 3200$

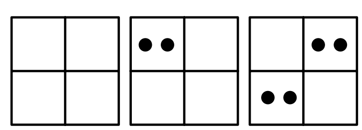


$4 \times 0.8 = 3.2$
or
$4 \times 0.80 = 3.20$

Exercise 3

Suggest tha~~t students record t~~he calculations on their papers while you write them on the board.

Put this configuration on the Minicomputer.



2×86

T: ***The double of what number is on the Minicomputer?***

S: ***86.***

T (pointing to the ones board)**:** ***What number is 2 x 6?***

S: ***12.***

T (pointing to the tens board)**:** ***What number is 2*** x ***80?***

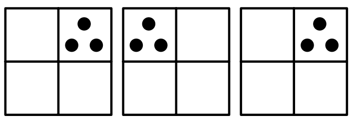
S: ***160.***

T: ***So what number is 2*** x ***86?***

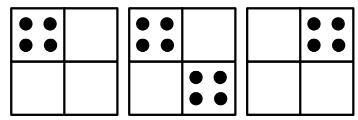
S: ***172.***

$$\begin{array}{r} 2 \times 6 = 12 \\ 2 \times 80 = 160 \\ \hline 2 \times 86 = 172 \end{array}$$

Repeat the activity with each of the following problems.

 $= 3 \times 484$

$$\begin{array}{r} 3 \times 4 = 12 \\ 3 \times 80 = 240 \\ 3 \times 400 = 1\,200 \\ \hline 3 \times 484 = 1\,452 \end{array}$$

 $= 4 \times 894$

$$\begin{array}{r} 4 \times 4 = 16 \\ 4 \times 90 = 360 \\ 4 \times 800 = 3\,200 \\ \hline 4 \times 894 = 3\,576 \end{array}$$

Exercise 4

Suggest that students do the problems in this exercise on their papers while you write them on the board. Begin with this problem on the board.

$$\begin{array}{r} 376 \\ \times \;\; 2 \\ \hline \end{array}$$

T: ***Let's do this problem by just thinking about the Minicomputer but not actually using it.***

Record the steps of the problem as they are mentioned.

T: ***On the ones board, what number do we double?***

S: ***6. 2*** x ***6 = 12.***

T: ***And on the tens board?***

S: ***70. 2*** x ***70 = 140.***

T: ***And on the hundreds board?***

S: ***300. 2*** x ***300 = 600.***

T: ***So what is 2*** x ***376?***

S: ***752.***

$$\begin{array}{r} 376 \\ \times \;\; 2 \\ \hline 12 \\ 140 \\ 600 \\ \hline 752 \end{array}$$

Let a student do the addition at the board.

Similarly, do these calculations with your class. You may want to introduce the more usual compact form.

compact writing

$$\begin{array}{r} 476 \\ \times\ 3 \\ \hline 18 \\ 210 \\ 1\,200 \\ \hline 1\,428 \end{array} \qquad \begin{array}{r} 568 \\ \times\ 4 \\ \hline 32 \\ 240 \\ 2\,000 \\ \hline 2\,272 \end{array} \qquad \begin{array}{r} {\scriptstyle 2\ 3} \\ 568 \\ \times\ 4 \\ \hline 2\,272 \end{array}$$

Ask your students to do these calculations individually on their papers. (Answers are in boxes.)

$$\begin{array}{r} 275 \\ \times\ 2 \\ \hline \boxed{550} \end{array} \qquad \begin{array}{r} 487 \\ \times\ 3 \\ \hline \boxed{1\,461} \end{array} \qquad \begin{array}{r} 789 \\ \times\ 4 \\ \hline \boxed{3\,156} \end{array}$$

Exercise 5

This would be a good exercise to let students work in groups. Draw this chart on the board and provide copies to the groups (Blackline N15(a)).

Tapes $ 6	Tennis Rackets $ 10	Electronic Games $ 25	Baseball Gloves $ 15

T: ***Mr. and Mrs. Raves spent $250 buying gifts for their family. They chose from four types of items; tapes at $6 each, tennis rackets at $10 each, electronic games at $25 each, and baseball gloves at $15 each. What could they have bought?***

S: ***Ten electronic games.***

S: ***Nine electronic games, one tennis racket, and one baseball glove.***

Allow students to give you one or two solutions and record them in the chart. Verify with the class that the total cost of the items suggested is $250.

Let the groups work cooperatively to find as many solutions as they can.

After a while you might like to let some students record their solutions in the class chart. Since there are close to 200 solutions, you should expect to get a variety but nowhere near all. Many solutions are given below.

Tapes $6	Tennis Rackets $10	Electronic Games $25	Baseball Gloves $15
0	0	10	0
0	1	9	1
0	20	2	0
0	5	5	5
0	2	8	2
0	25	0	0
5	2	8	0
5	4	6	2
0	10	6	0
0	15	4	0
10	5	2	6
5	6	4	4
20	3	4	0
10	0	7	1
15	9	1	3

Writing Activity

Ask students to write about any strategies they find for generating solutions to the problem in

Home Activity

This would be a good time to send a letter to parents/guardians about the multiplication algorithm. There is a sample letter on Blackline N15(b).

Capsule Lesson Summary

Study the concepts of multiple and divisor as inverse relations. Practice recognizing multiples and divisors of small numbers. Introduce prime numbers.

Materials

Teacher		Student	
	• Colored chalk		• Paper • Colored pencils, pens, or crayons

Description of Lesson

Begin the lesson with a brief warm-up and review of the idea of multiples. Whenever necessary, remind the class of the definition of multiples that comes from an arrow picture. For example, the multiples of 3 are the numbers on the +3 arrow road that meets the number 0, the multiples of 7 are the numbers on the +7 arrow road that meets the number 0, and so on.

T: ***Today we will only talk about whole numbers. What are some multiples of 3?***

S: ***Count by threes: 0, 3, 6, 9, and so on.***

S: ***21.***

T: ***How many threes make 21?***

S: ***Seven 3's.***

If it would be helpful, refer to the +3 arrow road that meets the number 0, and count the arrows from 0 to the number suggested (21 in the dialogue) to find how many threes in the number.

S: ***603 is a multiple of 3, because 600 is a multiple of 3 and 600 + 3 = 603.***

T: ***What are some multiples of 7?***

S: ***0, 7, 14, 21, 28, and so on.***

S: ***77.***

S: ***700.***

Ask students to name some multiples of other numbers, such as 2, 4, and 13. Observe that multiples of 2 are even numbers.

T: ***What number is a multiple of every number?***

S: ***0.***

T: ***If I ask for a multiple of some number, what is another easy choice besides 0?***

S: ***The number itself.***

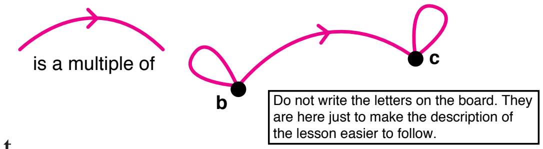
T: ***Right! 7 is a multiple of 7; 3 is a multiple of 3.***

Exercise 1

N16

Draw this arrow picture on the board.

T: ***One number*** (point to **b**) ***is a multiple of another*** (point to **c**)***. What could these numbers be?***



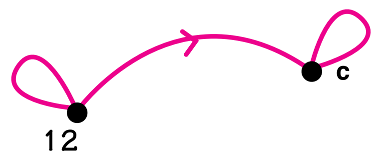
Accept several solutions from the class, each time reading t

T (putting 12 at **b**)**:** ***Now what could this dot*** (**c**) ***be for?***

S: ***3.***

T (putting 3 at **c**)**:** ***Let's read the arrow picture. 12 is a multiple of 3. How do you know?***

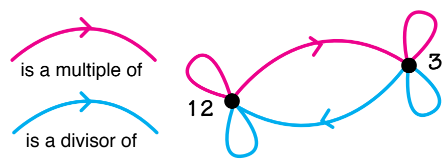
S: ***I counted by threes from 0: 0, 3, 6, 9, 12.***



Continue in this manner until all of the possibilities for **c** (1, 2, 3, 4, 6) have been found. Note that 12 is not included as a possibility for **c** because 12 is at **b**. However, the loop at 12 indicates that 12 is a multiple of itself. Leave one possibility for **c** in the picture; for example, 3.

T: ***Let's look at the opposite (return) arrow for a red arrow. The opposite of "is a multiple of" is "is a divisor† of"***

Let's read the blue arrow together. "3 is a divisor of 12."



Replace 3 by a succession of divisors of 12, each time leading the class in reading the picture.

T: ***What are the divisors of 12?***

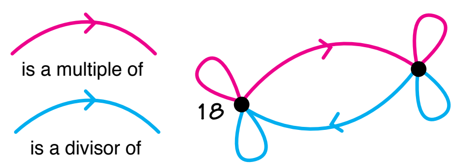
S: ***1, 2, 3, 4, 6, and 12.***

T: ***12 is a divisor of itself just like 12 is a multiple of itself. We show that in the arrow picture with a loop.***

Again emphasize the two concepts by paired readings of the arrow picture: 12 is a multiple of 3 and 3 is a divisor of 12; 12 is a multiple of 6 and 6 is a divisor of 12; and so on.

Erase the numbers in the picture and put 18 at **b**.

T: ***Let's find the divisors of 18. Write them on a piece of paper.***

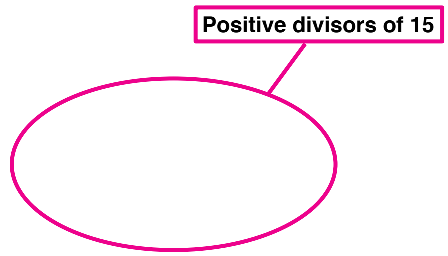


As you check students' answers, ask students who quickly find all of the divisors of 18 (1, 2, 3, 6, 9, 18) to find all of the divisors of 30 (1, 2, 3, 5, 6, 10, 15, 30).

Exercise 2

†You may prefer to use the term *factor* rather than *divisor* as it is somewhat more common. We like the word *divisor* better and will use it until we begin working on prime factorization. Feel free to interchange these terms.

Leave the arrow picture from Exercise 1 on the board for reference. Draw this string picture on the board.



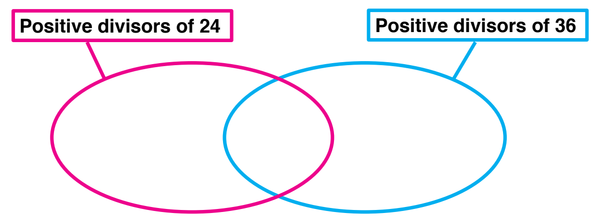
T: ***How many positive divisors does 15 have? Can you find all of them? Copy this picture on your paper and put the divisors of 15 in your picture. After you find the positive divisors of 15, do the same for several other numbers.***

On the board, make lists of the divisors for some of the numbers students try.

	positive divisors
15	1, 3, 5, 15
9	1, 3, 9
10	1, 2, 5, 10
7	1, 7
24	1, 2, 3, 4, 6, 8, 12, 24

Exercise 3

Draw this two-string picture on the board.



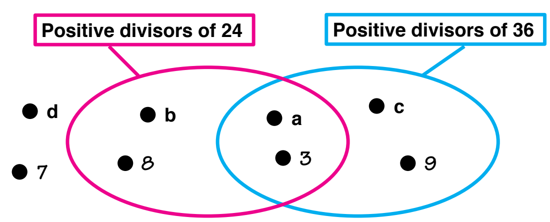
T: ***Where does 8 belong in this picture?***

S: ***In the red string, but not in the blue string.***

T: ***Why?***

S: ***Because we can count by eights from 0 and get to 24, but we skip over 36: 0, 8, 16, 24, 32, 40.***

Invite students to place 7, 3, and 9 in the picture also. Add four unlabeled dots to the picture, one in each region.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: ***What number could be here (at a)?***

The following dialogue suggests a variety of correct arguments that students might give to justify answers.

S: ***4, because we can count by fours from 0 and get both to 24 and to 36.***

S: ***2, because 24 and 36 are both even numbers.***

S: ***12; two 12's are 24 and three 12's are 36.***

S: ***1 is a divisor of any integer.***

S: ***30 is a multiple of 6, so 30 + 6 is a multiple of 6 and 30 – 6 is a multiple of 6.***

T: ***What number could be here (at b)?***

S: ***24, because 8 and 24 are the only divisors of 24 that are not divisors of 36.***

T: ***What number could be here*** (at **c**)***?***

S: ***18 or 36.***

T: ***What number could be here*** (at **d**)***?***

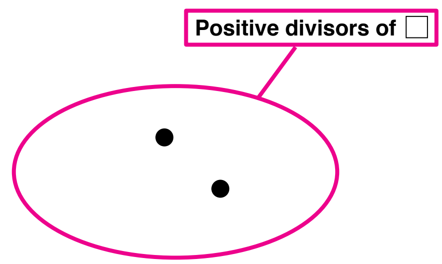
S: ***5, 7, 21, 117, 123; there are a lot of choices.***

If there is sufficient time, continue with Exercise 4.

Exercise 4

Draw this string picture on the board and ask students to copy it.

T: ***There are exactly two numbers in this string. We could use hatching to show that just two numbers are inside the string. The picture tells us that the number in the box has exactly two positive divisors. Try to find some numbers we could put in the box.***



You will note that a number for the box is a prime number. After a few minutes, collect students' results in a list on the board. The first twelve positive prime numbers are listed below for your reference.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, ...

T: ***Numbers with exactly two positive divisors are called*** **prime numbers.** ***They are very special numbers and we will do a lot with them.***

Capsule Lesson Summary

Introduce the story of *Sasquatch*, or *Big Foot*, and a game children play on Big Foot's tracks. Use this situation to locate rational numbers on the number line and to order a set of rational numbers. Acquire many names for ½ during the discussion.

Materials

Teacher
- Colored chalk
- Chalkboard compass (optional)
- Straightedge

Student
- Paper
- Worksheets N17*, **, ***, and ****

Advance Preparation: Put the table from Exercise 2 on the board before starting the lesson.

Description of Lesson

Exercise 1

Write **Sasquatch** at the top of the board.

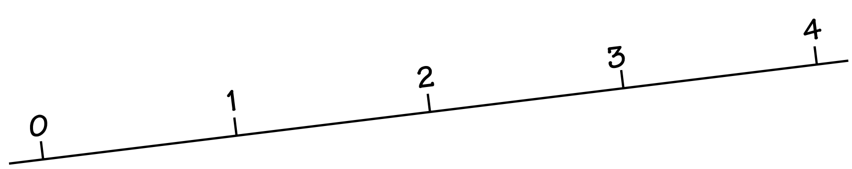
T: ***Have you ever heard about Sasquatch?***

S: ***Sasquatch is another name for Big Foot.***

Let the students discuss the legend of Sasquatch for a few minutes.

T: ***According to the legend, Sasquatch or Big Foot is a primitive creature living in the northwestern part of the United States. It has never been proven that Big Foot actually exists, but many people believe that such a creature does.***

On the board, draw this part of the number line with marks spaced approximately 50 cm apart.



T: ***One morning some children discover a set of footprints in the snow. The footprints are very large and very far apart. The children are sure the tracks are Big Foot's.***

Pretend that the marks I have drawn on this number line represent Big Foot's tracks.

The children invent a game using the tracks. Taking turns, they each start walking from here (0 in our picture); they are careful to take steps of the same length. Each child tries to find a step size so that he or she lands exactly in Big Foot's tracks (at 1, 2, 3, and so on in our picture). No child is big enough to have a step size as big at Big Foot's.

Provide a selection of objects of different lengths (for example, an eraser, pencils, pieces of chalk, paper clips, and so on), or invite students to look around the room for objects to simulate a step size. Then, using the object or length of their choice, instruct them to play the game, each time starting at 0 and trying to land on 1. (An alternative to using objects is to use a chalkboard compass. With this tool students can choose a step size by opening the compass suitably.)

This experience provides opportunity for students to use, talk about, and develop some intuitive measurement concepts.

Exercise 2

Refer to this table on the board.

Child	A	B	C	D	E	F	G	H	J
Number of steps to reach 1	4	12	15	2	5	10	9	3	99

T: ***All the children eventually find good step sizes. Here is some information about how many steps it takes each child to land exactly on 1. Which child takes the longest steps?***

S: ***D; D only needs two steps to go from 0 to 1.***

T: ***Which child takes the shortest steps?***

S: ***J; J needs ninety-nine steps to reach 1.***

T: ***Who takes bigger steps, H or E?***

S: ***H; H needs three steps to reach 1, while E needs five steps. H takes fewer steps, so they must be bigger steps.***

T: ***Who takes smaller steps, A or D?***

S: ***A; A needs four steps to reach 1, and D only needs two steps. The more steps needed, the smaller the step.***

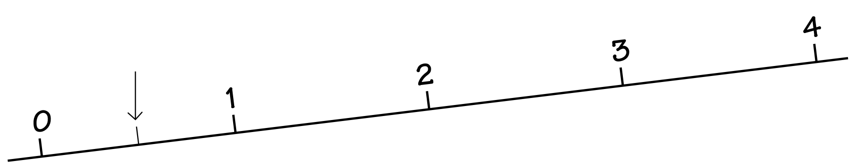
T: ***How does E's step size compare to F's step size?***

S: ***F takes two steps for every one of E's, so F's steps are half the size of E's.***

Ask students to compare several other step sizes.

T: ***Consider child D. This child takes two steps to go from 0 to 1. Point to the place where D's first step lands.***

Invite a student to locate **D**'s first step.



S: ***D's first step is in the middle between 0 to 1.***

T: ***How could we label this mark?***

S: ***½.***

Use red chalk to label the mark for **D**'s first step. Point to the numerator and denominator, respectively, of the fraction ½ as you say,

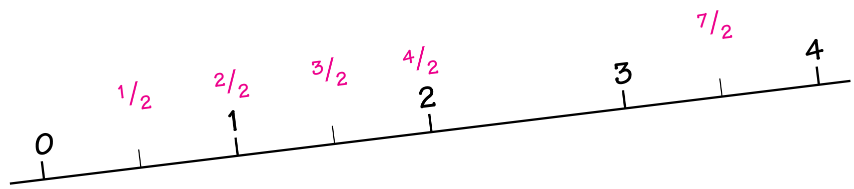
T: ***1 tells us that this is D's first step, and 2 tells us that D takes two steps to go from 0 to 1. Point to the place where D's second step lands.***

S: ***At 1.***

T: ***How could we label this mark to show D's second step lands here?***

S: ***2/2.***

Continue in this way to label marks for **D**'s third, fourth, and seventh steps.



T: ***Now look at A. A takes four steps from 0 to 1. Who takes longer steps, A or D?***

$\frac{1}{4} < \frac{1}{2}$

S: ***D; D only takes two steps, while A takes four.***

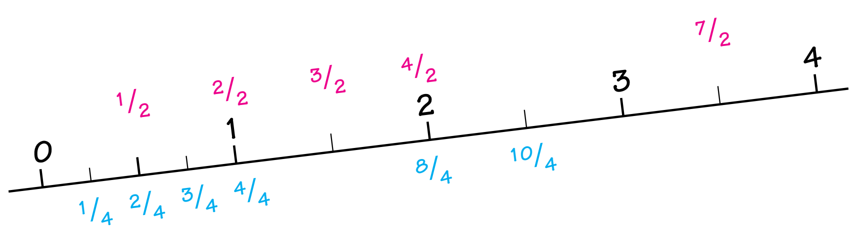
T: ***One step of A's is smaller than one step of D's. How many steps does A take for each step that D takes?***

S: ***A takes two steps for each step D takes.***

T: ***Where is A's first step?***

S: ***In the middle, between 0 and ½.***

Invite a student to locate the mark and to label it ¼ with blue chalk. Continue by asking students to label marks for **A**'s second, third, fourth, eighth, and tenth steps.



As a challenge, ask where **A**'s 29th step would be. Refer to the number line displayed in your room.

S: ***One step past 7 on the number line.***

Exercise 3

Add this row of information to the table.

Child	A	B	C	D	E	F	G	H	J
Number of steps to reach 1	4	12	15	2	5	10	9	3	99
First step lands on (step size)	$1/4$			$1/2$					

Invite students to complete the table by writing a fraction for each child's step size. A completed table follows.

Child	A	B	C	D	E	F	G	H	J
Number of steps to reach 1	4	12	15	2	5	10	9	3	99
First step lands on (step size)	$1/4$	$1/12$	$1/15$	$1/2$	$1/5$	$1/10$	$1/9$	$1/3$	$1/99$

T: ***Which step size is biggest; which of these numbers is the greatest?***

S: ***$1/2$.***

T: ***Which is the smallest?***

S: ***$1/99$.***

T: ***On a piece of paper, arrange these numbers ($1/2$, $1/4$, $1/12$, …, $1/99$) from greatest to least.***

When some students finish this task, ask them to list the fractions from greatest to least on the board.

$$1/2 > 1/3 > 1/4 > 1/5 > 1/9 > 1/10 > 1/12 > 1/15 > 1/99$$

Exercise 4

Refer back to the table and to the number line, as it was labeled in Exercise 2.

T: ***Did you notice how A lands not only on all of the Big Foot's steps, but also on all of D's steps? Is there any other child like A that will land on all of D's steps?***

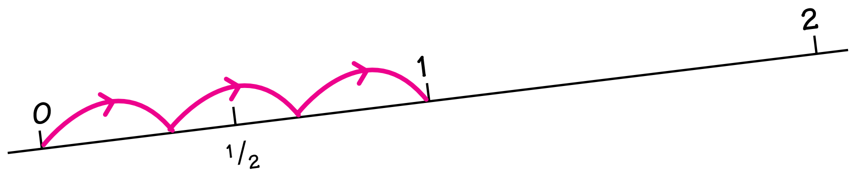
S: ***B; B takes six steps and lands on $1/2$ and six more steps to land on 1.***

S: ***F; F takes five steps for every one of D's.***

Write this information on the board indicating that you are abstracting from the discussion.

$$1/2 = 2/4 = 6/12 = 5/10$$

Use the number line to settle any disputes. **H**'s steps jump over ½, so there is no "thirds-name" for ½.



T: ***What are some other names for ½ that are not suggested by the children in the table?***

Let the students suggest many other names for ½.

$$1/2 = 2/4 = 6/12 = 5/10 = 10/20 = 50/100 = 200/400$$

T: ***Do you notice anything interesting about the names for ½?***

S: ***The bottom number is always double the top number.***

S: ***The bottom number is always even.***

Accept any reasonable comments.

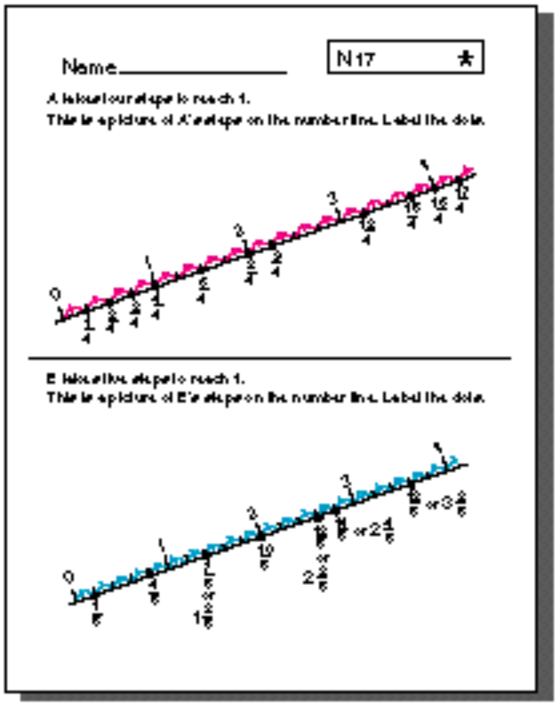
**** are available for individual work.

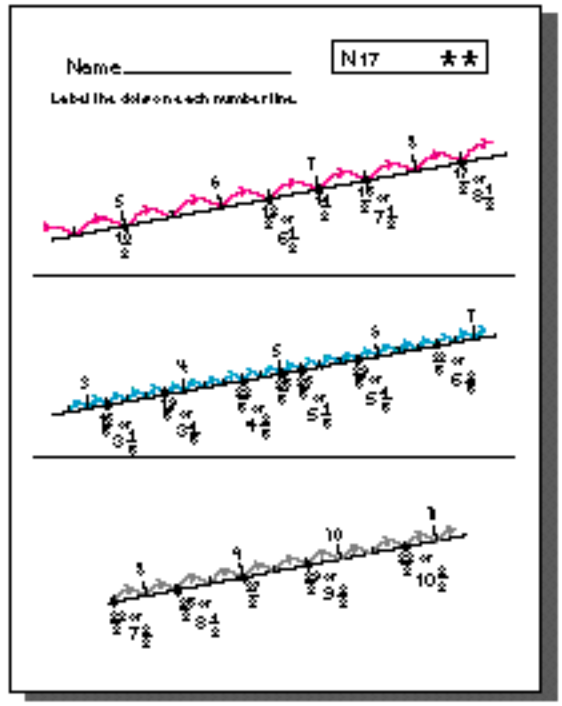
Extension Activity

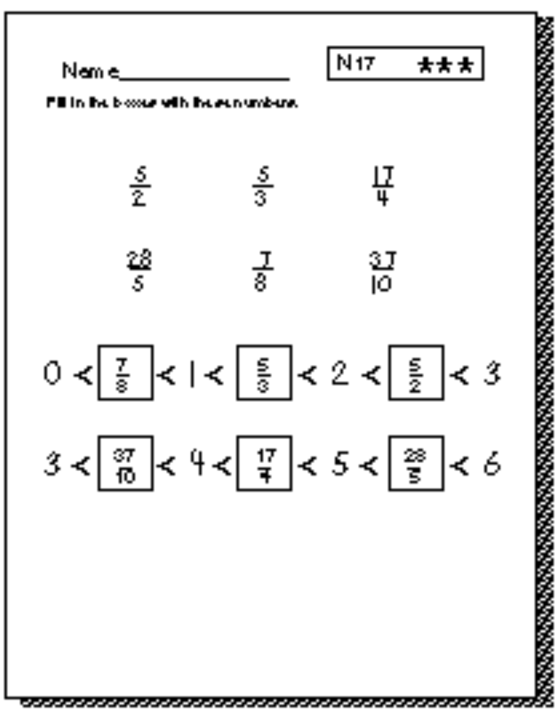
Invite students to find out some information about Sasquatch or Big Foot, and to write a mathematics

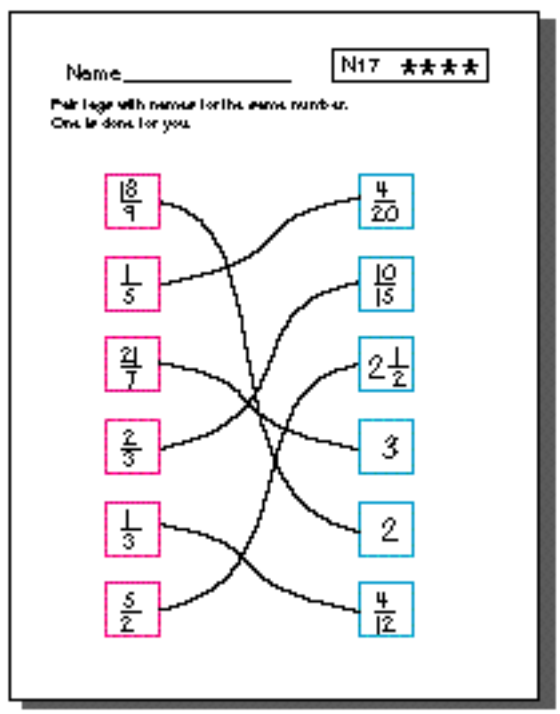
Home Activity

Suggest that students compare the step sizes of family members. Does anyone in the family have a step size one-half that of another person in the family?


Name
N17 ★
A takes four steps to reach 1.
This is a picture of A's steps on the number line. Label the dots.
B takes five steps to reach 1.
This is a picture of B's steps on the number line. Label the dots.


Name
N17 ★★
Label the dots on each number line.


Name
N17 ★★★
Fill in the boxes with these numbers.
5/2 5/3 17/4
28/5 7/8 37/10
0 < 7/8 < 1 < 5/3 < 2 < 5/2 < 3
3 < 37/10 < 4 < 17/4 < 5 < 28/5 < 6


Name
N17 ★★★★
Pair tags with names for the same number.
One is done for you.
18/9
1/5
21/7
2/3
1/3
5/2
4/20
10/15
2 1/2
3
2
4/12

N18 ASSORTED PROBLEMS #1

Capsule Lesson Summary

Discuss ways to compute 6 x 7 if one does not recall the fact. Make up story problem situations in which to use a particular calculation. Present several problems for group problem solving and discuss their solutions.

Materials

Teacher
- Colored chalk

Student
- Paper
- Colored pencils, pens, or crayons
- Calculator
- Assorted problems sheets

Advance Preparation: Select two or three pages from the assorted problems section of the Blacklines for use in Exercise 3. Make copies for groups.

Description of Lesson

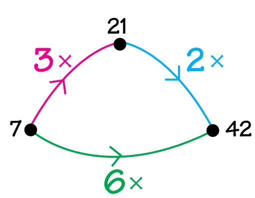
Adjust the time you spend on Exercises 1 and 2 to allow 20–30 minutes for Exercise 3.

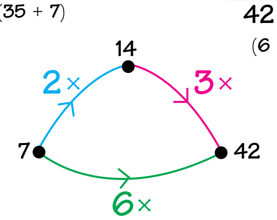
Exercise 1

Ask students to suggest various ways to compute 6 x 7 for someone who does not remember the fact. Display several methods on the board. Some of the many possible ways are shown below.

$$\begin{array}{r} 5 \times 7 = 35 \\ \underline{1 \times 7 = 7} \\ 6 \times 7 = 42 \end{array} \qquad \begin{array}{r} 4 \times 7 = 28 \\ \underline{2 \times 7 = 14} \\ 6 \times 7 = 42 \end{array}$$

$$\begin{array}{rl} 7 & \\ 7 & \rangle\ 14 \\ 7 & \\ 7 & \rangle\ 14 \\ 7 & \\ \underline{+7} & \rangle\ 14 \\ 42 & \end{array} \qquad \begin{array}{r} \left.\begin{array}{r} 7 \\ 7 \\ 7 \\ 7 \\ 7 \end{array}\right\} 35 \\ \underline{+7} \\ 42 \\ (35 + 7) \end{array} \qquad \begin{array}{rl} 6 & \\ 6 & \rangle\ 12 \\ 6 & \\ 6 & \rangle\ 12 \\ 6 & \\ 6 & \rangle\ 12 \\ \underline{+6} & -\ 6 \\ 42 & \\ (6 \times 7 = 7 \times 6) & \end{array}$$





Note: The arrow pictures are ways to record a composition method: multiply 3 x 7 = 21 and 2 x 21 = 42, or 2 x 7 = 14 and 3 x 14 = 42.

Exercise 2

Write this number sentence on the board.

$$20 - 14 = 6$$

T: *Tell us a story in which you would subtract 20 – 14 to get the answer to a problem.*

S: *I have 20 racing cars. I gave 14 of them to my friend Larry.*

T: *What is the question?*

S: *How many racing cars do I have left?*

Continue inviting several students to tell stories. If no story involves comparison, proceed with these comments.

T: ***Your stories all involve a group of things or people and something happens to part of them. Then you ask how many are left. Can you think of a different kind of story?***

You may need to tell one yourself.

T: ***Joshua listed 20 books that he read this school year. Last school year he listed 14 books. How many more books did Joshua list this year?***

S: ***I sold 20 boxes of cookies for our school fund-raising project; my brother sold 14. How many more boxes did I sell than my brother?***

S: ***Last Monday my plant was 14 cm high. Today it is 20 cm high. How much did my plant grow?***

Let several students give comparison stories.

Repeat this activity with a multiplication problem, for example:

$$5 \times 6 = 30$$

T: ***I have five envelopes. Each envelope has six picture postcards in it. How many postcards are there altogether?***

Can someone tell us another story in which we would multiply 5 x 6?

Let several students tell stories.

Exercise 3

Organize the class into small cooperative groups for a problem-solving activity. Provide the groups with supplies (paper, calculator, props or manipulatives) for acting out and solving problems. Choose two or three problems from the assorted problems section of the Blacklines to give to the groups. Here we present two examples, first with a suggested class discussion of a problem and then with possible solution methods. Groups should prepare to present their solutions to the class.

Problem #1

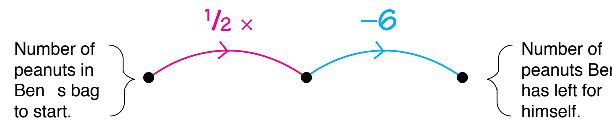
Ben has a bag of peanuts and two friends. He promises to give six peanuts to one friend and to share his peanuts equally with the other friend. Ben cannot decide what to do first. Ben would like as many peanuts as possible for himself. What advice would you give Ben?

Hold a collective discussion to be sure students understand the problem: Should Ben first give six peanuts and then share, or first share and then give six peanuts? Does it matter how many peanuts Ben has to start? As groups are working, you may like to ask some other questions related to the problem and to their advice to Ben; for example, if Ben wants to have 15 peanuts left for himself, how many peanuts should he start with?

As groups present their advice to Ben, you may want to draw arrow pictures on the board to record what happens.

S: ***We think Ben should first share his peanuts.***

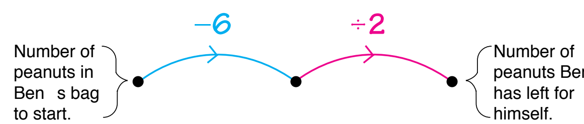
When Ben shares his peanuts he gives half to his friend and he keeps half. Then Ben gives six peanuts to the other friend so we subtract 6.



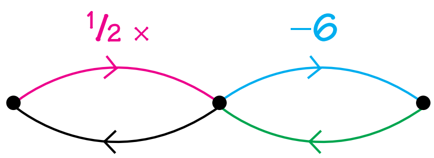
Check with the group what Ben has left for himself with various starting numbers, or how many he would have to start with to have a given number left for himself. Students may suggest using return arrows to find the starting number when they are given what Ben wants to have left.

S: ***We decided that Ben should first give six peanuts to a friend and then share the rest of the peanuts with the other friend. He gets more for himself this way.***

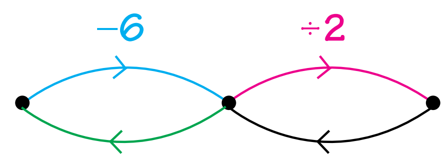
Ben gives six peanuts to a friend so we subtract 6. Then he shares the rest so we divide by 2.



Again, check what Ben has left for himself with various starting numbers (use the same starting numbers as before), or what he would have to start with to have a given number left for himself.



	2×		+6	
50		25		19
100		50		44
20		10		4
42		21		15



	+6		2×	
50		44		22
100		94		47
20		14		7
36		30		15

The class should observe that Ben always has more left (three more) if he first gives a friend six and then shares the rest.

Problem #2

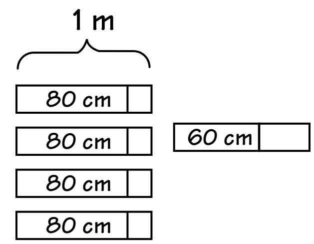
Ms. Lambert needs four boards 80 cm long and one board 60 cm long for a project. The boards must all be 20 cm wide. At the lumberyard, the salesperson tells Ms. Lambert that 20 cm wide boards only come in meter lengths: 1 m, 2 m, 3 m, 4 m, 5 m, or 6 m. Ms. Lambert will have to cut longer pieces down to the lengths she needs. What should Ms. Lambert buy?

While posing the problem, record the lengths of wood on the board.

Needs	Available board lengths
four 80 cm boards and one 60 cm board	1 m 2 m 3 m 4 m 5 m 6 m

Hold a short collective discussion to be sure students understand the problem. As groups are working, you may want to ask that they try to find a solution for Ms. Lambert that results in a minimum of leftover board.

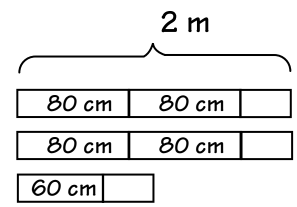
As groups present their solutions, illustrate or draw arrow pictures on the board to record them.



S: ***We decided she could buy five 1-meter boards. She can cut each board she needs from a 1 m board.***

T: ***How much would be left over?***

S: ***120 cm.***



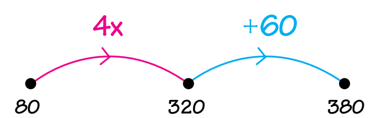
S: ***We thought she could buy two 2-meter boards and one 1-meter board. She could cut two 80 cm boards from each of the 2-meter boards. There would be two pieces 40 cm long left. Then she could cut the 60 cm board from the 1-meter board, leaving another board 40 cm long.***

T: ***How much left over?***

S: ***Still 120 cm.***

S: ***We found the total length of boards she needs so she could buy one long board. She needs at least 380 cm length of board.***

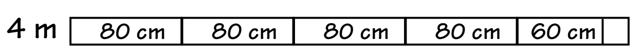
Four 80 cm boards is 4 x 80 = 320 plus one 60 cm board: 320 + 60 = 380. She should buy a 4 m board.



T: ***How much would be left over?***

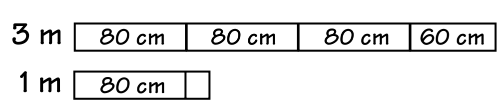
S: ***20 cm.***

T: ***If Ms. Lambert cut the pieces of wood from one 4-meter piece, the dividing marks would look like this.***



Note: Students may think that Ms. Lambert could buy two 2 m boards because that would give her as much length as one 4 m board. However, they should discover that all five pieces could not be cut from two 2-meter boards.

Perhaps someone will observe that all five pieces could be cut from one 3-meter board and one 1-meter board, again with a 20 cm piece left over.



Extension Activity

Use other problems from the assorted problems section of the Blacklines for group or individual work.

N19 NUMBER LINE #2

Capsule Lesson Summary

Determine how close you can get to one number by adding tens to another number.
Compare two numbers and ask what an arrow from one to the other could be for.
Compare the elevations of some of the Earth's lowest and highest points.

Materials

Teacher
- Meter stick
- Colored chalk
- A world map or globe (optional)
- Blacklines N19(a) and (b)

Student
- Elevation charts
- Worksheets N19*, **, and ***

Advance Preparation: Use Blackline 19(a) to make copies of the elevation charts used in Exercise 3. Blackline N19(b) has the chart information in a number line form for students to use while doing the worksheets.

Description of Lesson

Exercise 1

Begin the lesson with some mental arithmetic involving adding tens. A suggested sequence of problems is given below.

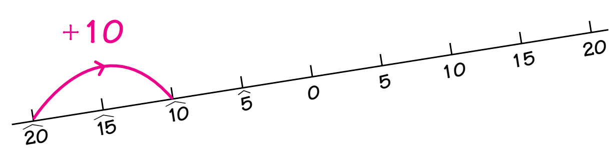
- If we start at 41 and add tens, what is the closest number we can get to 83? (81)
- If we start at 64 and add tens, what is the closest number we can get to 90? (94)
- If we start at 23 and add tens, what is the closest number we can get to 106? (103)
- If we start at 32 and add tens, what is the closest number we can get to 99? (102)
- If we start at 35 and add tens, what is the closest number we can get to 84? (85)
- If we start at 7 and add tens, what is the closest number we can get to 52? (47 or 57)

Exercise 2

Draw a number line graduated in fives from $\widehat{20}$ to 20 on the board.

T: ***Which number is greater, $\widehat{20}$ or $\widehat{10}$?*** ($\widehat{10}$)
How much greater? (10 more)

Draw a red arrow from $\widehat{20}$ to $\widehat{10}$.



T: ***What could the red arrow be for?***

S: ***+10.***

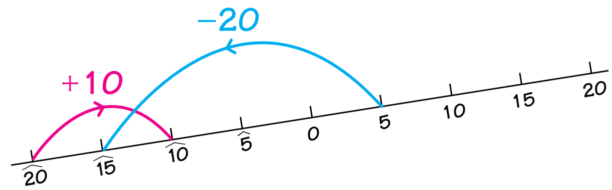
T: ***Which is less, $\widehat{15}$ or 5?*** ($\widehat{15}$)
How much less? (20 less)
How do you know?

S: ***0 is 5 less than 5, and $\widehat{15}$ is 15 less than 0, so $\widehat{15}$ is 20 less than 5.***

S: ***I counted by fives.***

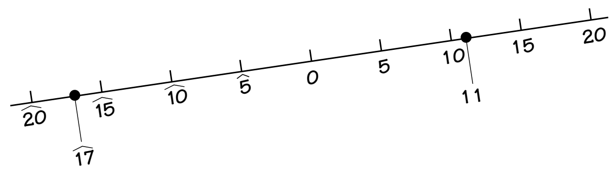
Draw a blue arrow from 5 to $\widehat{15}$ and ask what it could be for.

Erase the arrows.



T: ***Who can locate about where 11 is on this number line?***
Who can locate about where $\widehat{17}$ is on this number line?

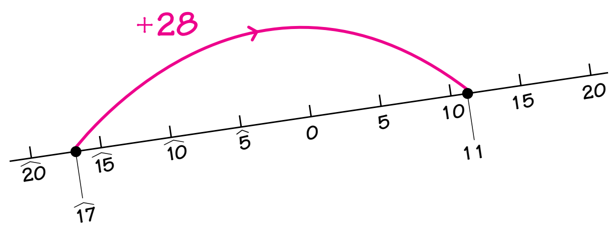
Call on volunteers to draw dots for 11 and $\widehat{17}$.



Point to the dots for 11 and $\widehat{17}$ and the mark for 0 as you ask the following questions.

T: ***Which is greater, 11 or $\widehat{17}$?*** (11)
How much greater? (28)
How much greater is 0 than $\widehat{17}$? (17)
How much greater is 11 than 0? (11)
So how much greater is 11 than $\widehat{17}$? (28)

Draw a red arrow from $\widehat{17}$ to 11 and ask what it could be for.



Exercise 3

Distribute copies of the chart on Blackline N19(a) giving highest and lowest points on the Earth's surface by continent.

T: ***What information is in this chart?***

S: ***Highest and lowest points on the Earth's surface.***

S: ***Highest and lowest points on each continent.***

T: ***That's right. The continents are listed in the first column. How many continents are there?*** (Seven)

Read the names of the continents with your students and then refer to middle column.

T: ***Where do you think the highest points on the Earth are?***

S: ***Tops of mountains.***

T: ***Let's look at the data for Europe. The highest point in Europe is the top of Mount El'Brus. What is the elevation of that mountain?***

S: ***5 642 meters above sea level.***

T: ***Yes. The small* m *is an abbreviation for meters. Where is Mount El'Brus?***

S: ***In Russia.***

T: ***What is the highest point in Africa?***

S: ***Kibo.***

T: ***Kibo is one peak of Mount Kilimanjaro. Where is that mountain?***

S: ***Tanzania.***

T: ***What is the elevation of Kibo?***

S: ***5 895 meters above sea level.***

T: ***The third column of this chart tells the lowest points of each continent. Where do you think the lowest points on the continents are?***

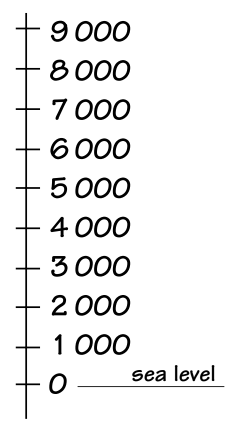
S: ***In valleys.***

S: ***At lakes and rivers.***

Point out to the class that Salinas Grandes is a dry salt lake, or salt flat, in Valdes Peninsula, and that Antarctica has places at sea level but not below.

Ask for the lowest points of two of the continents and for their elevations (which are below sea level). Note that the given measurement for a lake or sea is from sea level to the surface of the water, not from sea level to the bottom of the lake or sea.

T: ***All of these measurements are relative to sea level. If we draw a number line*** (draw a vertical line on the board) ***and locate the elevations of places named in this chart, what number do you think should correspond to sea level?***



S: ***0.***

T: ***Should we make marks for 0, 1, 2, 3, and so on?***

S: ***No, you'll run out of room on the board.***

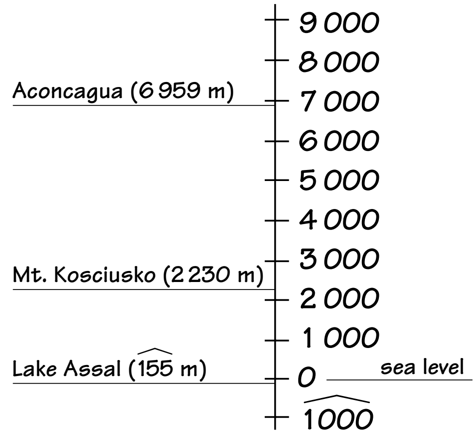
T: ***What do you think would be a good scale—marks for what numbers?***

Encourage the class to observe that the number line needs to go almost as high as 9 000, so maybe marking it in thousands would be a good start.

Invite students to locate some of the highest points for different continents on your line. For example:

T: ***Aconcagua is 6 959 m above sea level. Who can locate about where 6 959 is on this number line?***

Then refer to the lowest points.



T: ***Where do the marks for the elevations of the continents' lowest points belong on this number line?***

S: ***Below 0, because they are below sea level.***

T: ***How far below 0 do we need?***

If you have marks every 1 000 m above 0, extend the number line below 0 to $\widehat{1\,000}$. Then ask where to put a mark for Lake Assal's elevation.

T: ***We could say that Lake Assal's elevation is $\widehat{155}$ m.***

Distribute copies of the number line on Blackline N19(b). Point out that all the information in the chart is on this number line. Hold up a copy of the number line as you say,

T: ***If we tried to put marks on one number line for all of the elevations of the highest and lowest points of the continents, either the marks would get very crowded between 0 and $\widehat{1000}$ or the number line would have to be very long. So on this worksheet, the number line on the right*** (point to it) ***is a magnification, or a blow-up, of the section between 0 and $\widehat{400}$ of this number line*** (point to the one on the left).

Worksheets N19*, **, and *** are available for individual work. You may prefer to do the first three questions on N19* collectively. Refer students to the number lines on Blackline N19(b) to easily identify the highest and lowest points on the Earth's surface. Some students might look at the third question as a "missing addend" problem; others, as a subtraction problem. Discuss any method that students suggest.

$$\begin{array}{r} 6\,959 \\ -\ 2\,230 \\ \hline \end{array} \quad \text{or} \quad \begin{array}{r} 2\,230 \\ + \\ \hline 6\,959 \end{array}$$

If there is sufficient time, you may like to discuss answers to the questions on the * and ** ıave attempted some of the problems on the ** worksheet.

Extension Activity

Invite students to find the highest and lowest points in your state and the elevation of your school's isons with the data for the continents.

Writing/Home Activity

Suggest students write a problem or two about the data to take home for family members.

Name_______________ N19 ★

1. Where is the highest point on the Earth's surface? __Asia__
   What is its elevation? __8 848 m__ above sea level
2. Where is the lowest point on the Earth's surface? __Asia__
   What is its elevation? __396 m__ below sea level
3. How much higher is Mount Aconcagua than Mount Kosciusko?
   __4 729 m__ (Show work.)

$$\begin{array}{r}6\,959\\ -\,2\,230\\ \hline 4\,729\end{array}\quad \text{or} \quad \begin{array}{r}2\,230\\ +\,4\,729\\ \hline 6\,959\end{array}$$

4. How much higher is Mount Everest than Mount McKinley?
   __2 654 m__ (Show work.)

$$\begin{array}{r}8\,848\\ -\,6\,194\\ \hline 2\,654\end{array}\quad \text{or} \quad \begin{array}{r}6\,194\\ +\,2\,654\\ \hline 8\,848\end{array}$$

5. Which is lower: the surface of the Dead Sea or the surface of Lake Eyre? __Dead Sea__
   How much lower? __379 m__ (Show work.)

$$\begin{array}{r}395\\ -\,16\\ \hline 379\end{array}\quad \text{or} \quad \begin{array}{r}379\\ +\,16\\ \hline 395\end{array}$$

Name_______________ N19 ★★

6. What is the difference in elevation between the surface of the Dead Sea and the top of Mount Everest? __9 243 m__
   (Show work.)

$$\begin{array}{r}8\,848\\ +\,395\\ \hline 9\,243\end{array}$$

7. What is the difference in elevation between Europe's highest and lowest points? __5 670 m__ (Show work.)

$$\begin{array}{r}5\,642\\ +\,28\\ \hline 5\,670\end{array}$$

8. Is it possible that Europe has a mountain twice as high as Mount Kosciusko? __Yes__
   If so how high would it be? __4 460 m__
   If not, why not? ______________________________
   ____

Name_______________ N19 ★★★

9. What is the elevation of a point halfway between the top of Elbrus and the top of Mount McKinley? __5 918 m__ (Show work.)

$$\begin{array}{r}6\,194\\ -\,5\,642\\ \hline 552\end{array}\qquad \begin{array}{l}\frac{1}{2}\times 500 = 250\\ \frac{1}{2}\times 50 = 25\\ \frac{1}{2}\times 2 = 1\\ \hline \frac{1}{2}\times 552 = 276\end{array}\qquad \begin{array}{r}5\,642\\ +\,276\\ \hline 5\,918\end{array}$$

10. What is the elevation of a point halfway between the top of Kibo and the surface of Lake Assal? __2 870 m__ (Show work.)

$$\begin{array}{r}5\,895\\ +\,155\\ \hline 6\,050\end{array}\qquad \begin{array}{l}\frac{1}{2}\times 6000 = 3\,000\\ \frac{1}{2}\times 50 = 25\\ \hline \frac{1}{2}\times 6050 = 3\,025\end{array}\qquad \begin{array}{r}3\,025\\ +\,155\\ \hline 2\,870\end{array}$$

Capsule Lesson Summary

Present situations in which a given number of objects are to be shared equally among a specified number of children. Use string pictures to determine the number of objects that each child will receive. Using a table, solve similar problems concerning the planting of a given number of trees in rows.

Materials

Teacher	• Colored chalk	Student	• Paper • Colored pencils, pens, or crayons • Worksheets N20*, **, ***, and ****

Description of Lesson

Begin this lesson with a short mental arithmetic activity involving multiplication patterns. Use the following problems or similar ones more appropriate for the abilities of your students.

3 x 7	(21)	7 x 4	(28)	9 x 600	(5 400)
3 x 70	(210)	7 x 40	(280)	9 x 6	(54)
3 x 700	(2 100)	7 x 400	(2 800)	9 x 6 000	(54 000)
3 x 7 000	(21 000)	7 x 4 000	(28 000)	9 x 60	(540)
½ x 18	(9)	⅓ x 18	(6)	⅓ x 210	(70)
½ x 180	(90)	⅓ x 180	(60)	⅓ x 21	(7)
½ x 1 800	(900)	⅓ x 1 800	(600)	⅓ x 2 100	(700)

In Exercises 1 and 2, students should follow the class solution to problems on their papers.

Exercise 1

T: ***What things do children collect to earn prizes and money?***

Let students name some items (for example, labels and box tops) that can be collected and turned in for prizes or money.

T: ***Three children, André, Lisa, and Tom, form a club to collect aluminum cans. Each child hopes to earn enough money to buy a bicycle. Each Saturday they get together and share equally all of the cans that they have collected during the week.***

Suppose they collect 80 cans in one week and share them equally. About how many cans do you think each child receives?

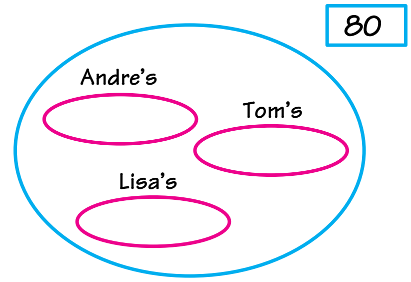
Write some students' estimates on the board.

T: ***How could we find out how many cans they each get?***

Let students discuss possible methods before solving the problem with the help of a string picture.

N20

Draw this string picture on the board.



T: ***How many cans do you want to give to each person?***

S: ***10.***

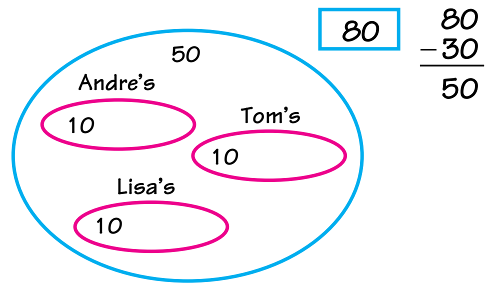
Write 10 in each red string. This distribution is only a possibility. You may want to encourage students to use multiples of 5 or 10 for initial shares as the calculations are usually simpler.

T: ***How many of the 80 cans have we distributed?***

S: ***30 cans; 3 x 10 = 30.***

T: ***How many cans are left?***

S: ***50 cans; 80 – 30 = 50.***



Record the computation on the board.

T: ***How many more cans could we give to each person?***

S: ***10 cans.***

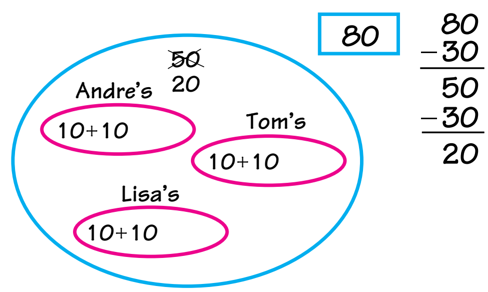
Indicate the suggestion in each child's string.

T: ***If we give 10 more cans to each child, how many more cans did we distribute?***

S: ***30 cans.***

T: ***How many are left?***

S: ***20 cans; 50 – 30 = 20.***



Record the computation on the board.

T: ***We still have 20 cans to distribute. How many could we give to each child?***

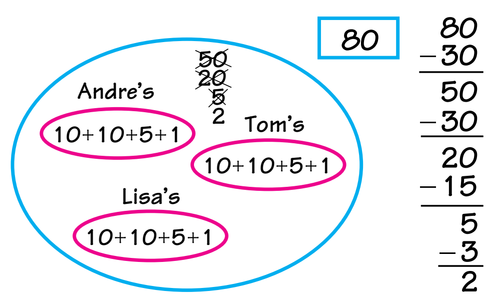
Continue this process until only 2 cans are left. For example:

T: ***How many cans does each child receive?***

S: ***26 (10 + 10 + 5 + 1) cans.***

T: ***How many cans are left over?***

S: ***2 cans. They could save those 2 cans for the next week.***



Compare the solution, 26 cans, with the students' estimates.

T: ***What number sentences could we write about this situation?***

On the board, list appropriate number sentences that students suggest. You may need to suggest sentences involving division yourself.

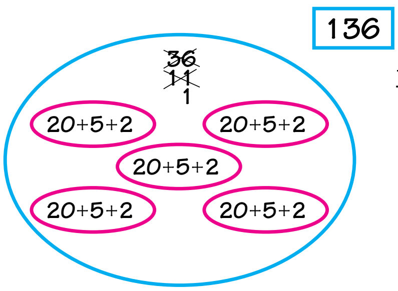
$\frac{1}{3} \times 78 = 26$ $\qquad$ $26 + 26 + 26 = 80 - 2$

$(3 \times 26) + 2 = 80$ $\qquad$ $3\overline{)80}$ → 26 R=2

Present the following two sharing problems in a similar manner.

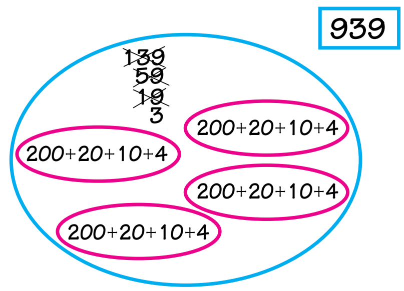
Share 136 labels among 5 children.
Share 939 box tops among 4 children.

Do not insist that your students find solutions in the most efficient ways. Encourage them to share in multiples of 10 or 100 whenever possible. The following pictures reflect solution strategies your students might suggest.



$$\begin{array}{r} 136 \\ -100 \\ \hline 36 \\ -25 \\ \hline 11 \\ -10 \\ \hline 1 \end{array}$$

$(5 \times 27) + 1 = 136$ $\qquad$ $5\overline{)136}$ → 27R=1



$$\begin{array}{r} 939 \\ -800 \\ \hline 139 \\ -80 \\ \hline 59 \\ -40 \\ \hline 19 \\ -16 \\ \hline 3 \end{array}$$

$(4 \times 234) + 3 = 939$ $\qquad$ $4\overline{)939}$ → 234R=3

Exercise 2

T: ***The principal of a school decides to plant rows of trees along the side of the school. Why ~~might he do that~~?***

S: ***For decoration.***

S: ***To save energy by shading the school and protecting it from the wind.***

Draw a table on the board as you explain the problem. The dialogue is a possible class solution.

T: ***The principal buys 110 seedlings and wants the gardener to plant them in three rows. About how many trees should be planted in each row?***

Write some students' estimates on the board.

T: ***As a start, how many trees could the gardener plant in each row?***

$$\begin{array}{r} 110 \\ -60 \\ \hline 50 \end{array}$$

First Row	Second Row	Third Row
20	20	20

S: ***20 trees.***

T: ***If 20 trees are planted in each row, how many trees are planted altogether?***

S: ***60 trees; 3 x 20 = 60.***

T: ***How many trees still must be planted?***

S: ***50 trees; 110 – 60 = 50.***

Continue in a similar manner until only 2 trees remain to be planted.

T: ***How many trees are planted in each row?***

S: ***36 trees, and there are 2 trees left over.***

T: ***What number sentences could we write about this situation?***

$$\begin{array}{r} 110 \\ \underline{-60} \\ 50 \\ \underline{-30} \\ 20 \\ \underline{-18} \\ 2 \end{array}$$

First Row	Second Row	Third Row
20	20	20
10	10	10
6	6	6

Accept any appropriate number sentences. Be sure to include examples involving multiplication and division.

$$3\overline{)110}\ \ 36\ R=2 \qquad (3 \times 36) + 2 = 110$$

Present the following two problems in a similar manner. (Ans

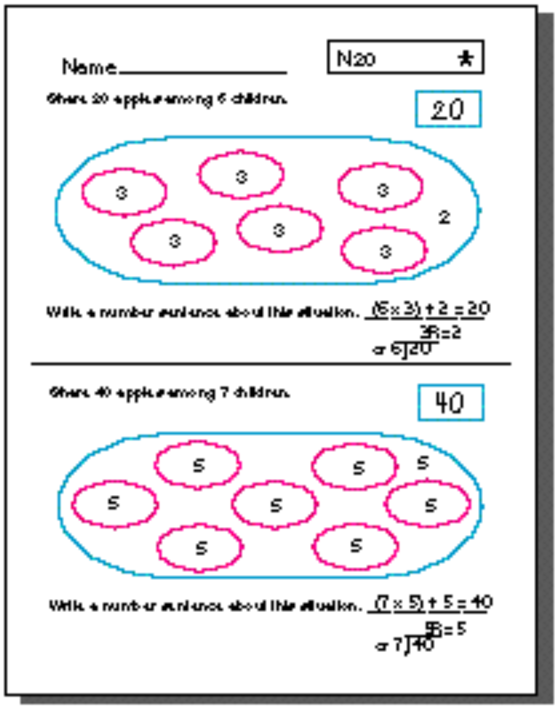
Plant 110 trees in 4 rows. (27 trees in each row; 2 trees left over)

Plant 1 600 trees in 6 rows. (266 trees in each row; 4 trees left over)

Worksheet N20*, **, ***, and **** are available for individual work.

Home Activity

This would be a good time to send a letter to parents/guardians about division. A sample letter is available on Blackline N20.



Name____________ N20 ★

Share 20 apples among 6 children. 20

3 3 3 3 3 3 2

Write a number sentence about this situation. (6 × 3) + 2 = 20

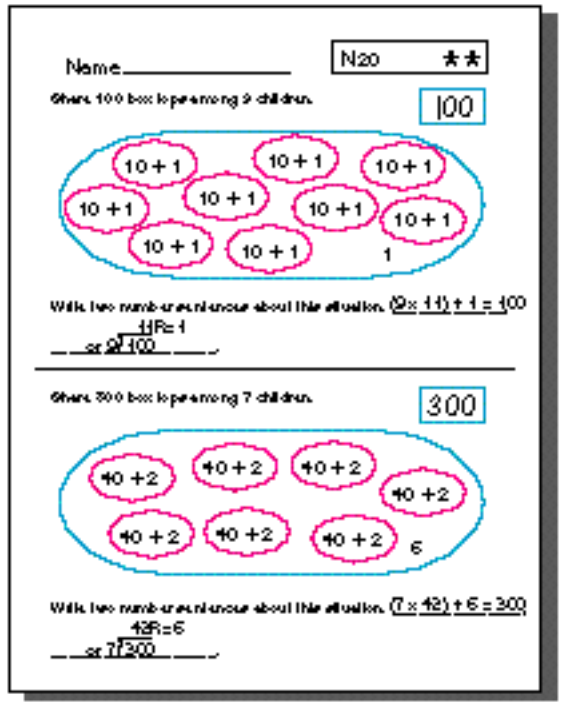
or 6)20 3R=2

Share 40 apples among 7 children. 40

5 5 5 5 5 5 5 5

Write a number sentence about this situation. (7 × 5) + 5 = 40

or 7)40 5R=5



Name____________ N20 ★★

Share 100 balloons among 9 children. 100

10 + 1 10 + 1 10 + 1 10 + 1 10 + 1 10 + 1 10 + 1 10 + 1 10 + 1 1

Write two number sentences about this situation. (9 × 11) + 1 = 100

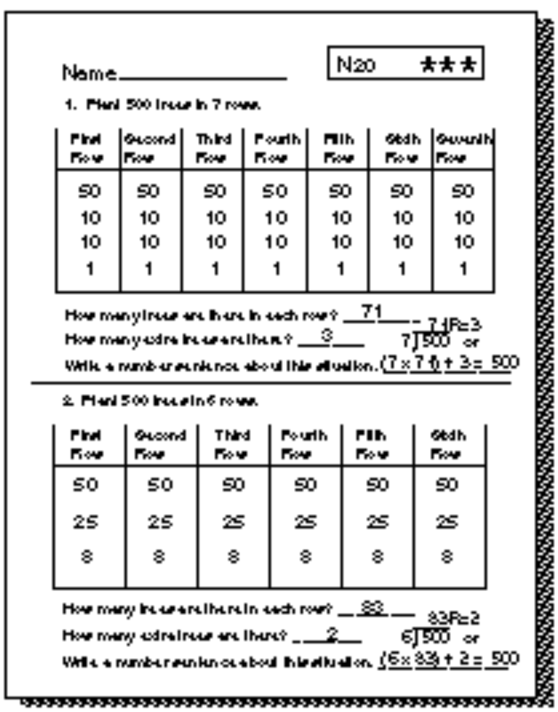
or 9)100 11R=1

Share 300 balloons among 7 children. 300

40 + 2 40 + 2 40 + 2 40 + 2 40 + 2 40 + 2 40 + 2 6

Write two number sentences about this situation. (7 × 42) + 6 = 300

or 7)300 42R=6



Name____________ N20 ★★★

1. Plant 500 trees in 7 rows.

First Row	Second Row	Third Row	Fourth Row	Fifth Row	Sixth Row	Seventh Row
50 10 10 1	50 10 10 1	50 10 10 1	50 10 10 1	50 10 10 1	50 10 10 1	50 10 10 1

How many trees are there in each row? 71

How many extra trees are there? 3

7)500 71R=3 or

Write a number sentence about this situation. (7 × 71) + 3 = 500

2. Plant 500 trees in 6 rows.

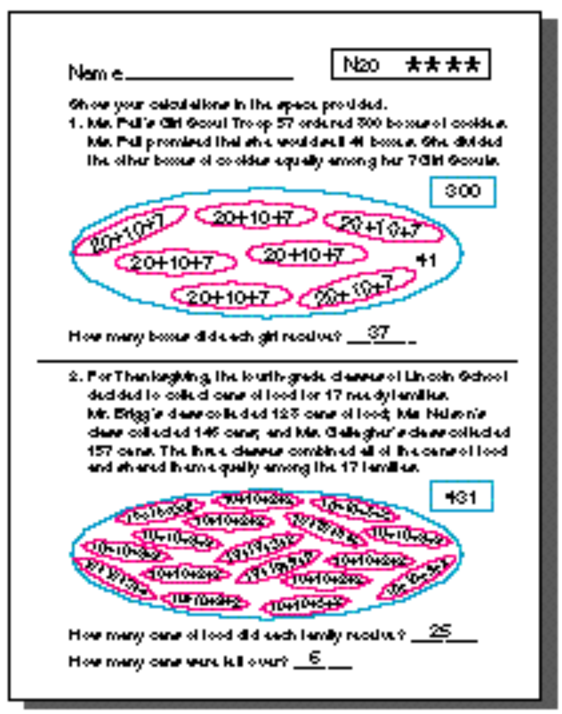
First Row	Second Row	Third Row	Fourth Row	Fifth Row	Sixth Row
50 25 8	50 25 8	50 25 8	50 25 8	50 25 8	50 25 8

How many trees are there in each row? 83

How many extra trees are there? 2

6)500 83R=2 or

Write a number sentence about this situation. (6 × 83) + 2 = 500



Name____________ N20 ★★★★

Show your calculations in the space provided.

1. Ms. Pell's Girl Scout Troop 57 ordered 300 boxes of cookies. Ms. Pell promised that she would sell 41 boxes. She divided the other boxes of cookies equally among her 7 Girl Scouts.

300

20+10+7 20+10+7 20+10+7 20+10+7 20+10+7 20+10+7 20+10+7 41

How many boxes did each girl receive? 37

2. For Thanksgiving, the fourth-grade classes of Lincoln School decided to collect cans of food for 17 needy families. Mr. Brigg's class collected 128 cans of food; Ms. Nelson's class collected 146 cans; and Ms. Gallagher's class collected 157 cans. The three classes combined all of the cans of food and shared them equally among the 17 families.

431

How many cans of food did each family receive? 25

How many cans were left over? 6

N21 GUESS MY RULE #2

Capsule Lesson Summary

Play *Guess My Rule* where the rules are for operations applied to pairs of numbers. Begin with simple operations like "add the two numbers" and "subtract the first number from the second"; then progress to the rule "double the first number and then add the second number." Use $*$ for this new operation and complete number stories involving $*$. Label the dots in an arrow road where the arrows are for $*5$.

Materials

Teacher
- Colored chalk

Student
- Paper
- Worksheets N21*, **, and ***

Description of Lesson

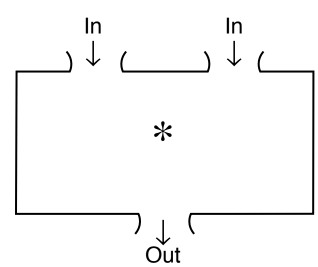
When playing *Guess My Rule*, encourage students to think about what the rule could be without announcing it to the rest of the class. This will allow other students a chance to discover a rule on their own. Let students who think that they know the rule test it on numbers given by you or other students. Each time, confirm or deny the result.

Exercise 1

For this first exercise, choose simple operations for a secret rule; for example, "add the two numbers" or "subtract the first number from the second."

T: ***I have a secret rule. I use my secret rule on a pair of numbers to get a resulting number.***

You may like to draw a "machine" picture on the board to further explain how an operation rule works.



T: ***This secret rule is like a machine. We put two (a pair of) numbers into the machine. The machine operates on the numbers and sends one number out.***

Let's see if you can guess my secret rule. I'll give you some clues using a star ($*$) for the operation; you try to figure out the secret rule of $*$.

Write clues on the board. You may let students give you the two numbers to operate on.

$$\begin{aligned} 9 * 7 &= 16 \\ 7 * 9 &= 16 \\ 19 * 17 &= 36 \\ 14 * 8 &= 22 \\ 4 * 18 &= 22 \end{aligned}$$

If you use a machine model, show how you put 9 and 7 into the machine and 16 comes out. Observe that although sometimes a machine cares what order you put in the numbers (which number goes in which entry), in this case it does not.

N21

Often there are several rules that students think could be the secret rule. In this case, it should be quickly obvious to most of the class that the rule simply involves the addition operation. However, you might make it a practice when doing this kind of activity to tell the class that there might be more than one rule that works for the pairs of numbers tried thus far, although you are thinking of just one rule and they must discover it.

Continue by letting students use the rule on pairs of numbers. Each time, confirm or deny the result. When most students know the rule, call on one student to announce it. For example, extend the list of sentences using $*$ with students giving the result shown boxed below. Then ask for a verbal description of the rule.

$$6 * 5 = \boxed{11} \qquad 67 * 16 = \boxed{83}$$
$$16 * 35 = \boxed{51} \qquad 57 * 26 = \boxed{83}$$
$$116 * 235 = \boxed{351} \qquad 57 * 46 = \boxed{103}$$
$$357 * 246 = \boxed{603}$$

Erase the board and explain that now you are thinking of a new rule. Repeat the exercise using the rule: $a * b = a - b$. For this rule, give several initial clues and observe that this time order makes a difference (the machine cares which number goes in which entry). Again, you may let students give you the numbers to operate on. In this case, ask students to specify the first and second number.

$$15 * 7 = 8$$
$$7 * 15 = \hat{8}$$
$$21 * 12 = 9$$
$$12 * 21 = \hat{9}$$
$$50 * 30 = 20$$

When you believe most students know the rule, present several problems for students to solve. You may, of course, adjust the difficulty of the problems to the abilities of your students.

$$24 * 17 = \boxed{7} \qquad 32 * 28 = \boxed{4}$$
$$17 * 24 = \boxed{\hat{7}} \qquad 132 * 128 = \boxed{4}$$
$$97 * 102 = \boxed{\hat{5}} \qquad 532 * 128 = \boxed{404}$$
$$102 * 97 = \boxed{5}$$

Exercise 2

T: ***Now I'm going to think of a more difficult rule. I'll give you some clues.***

Choose a secret rule such as $a * b = (2 \times a) + b$ or "double the first number and then add the second number." Do not give away the rule; let students discover it from your clues. Encourage the class to think of rules more complicated than just addition, subtraction, multiplication, or division.

Write clues on the board, or let students choose two numbers for you to operate on. Here, observe that order is important.

$$4 * 2 = 10 \qquad 1 * 5 = 7 \qquad 5 * 3 = 13$$
$$2 * 4 = 8 \qquad 5 * 1 = 11 \qquad 3 * 5 = 11$$

At this point, some students may think they know the rule. If so, let them provide results for some problems using the rule. For example, continue with these problems letting students provide the boxed numbers.

$8 * 5 = \boxed{21}$ $\qquad$ $3 * 0 = \boxed{6}$ $\qquad$ $9 * 9 = \boxed{27}$

When many students know the rule, let one explain it to the class. Insist on a clear description. As the student explains, write this expression on the board.

$$a * b = \overbrace{(2 \times a)}^{\text{double the first number}} \;\text{then}\; \overbrace{+\; b}^{\text{add the second number}}$$

Give some problems for students to do independently involving this rule for $*$. Check that everyone understands how to apply the rule. (Answers are in boxes.)

$7 * 1 = \boxed{15}$ $\qquad$ $3 * \hat{2} = \boxed{4}$ $\qquad$ $\boxed{3} * 4 = 10$

$1 * 7 = \boxed{9}$ $\qquad$ $\hat{2} * 3 = \boxed{\hat{1}}$ $\qquad$ $\boxed{6} * 8 = 20$

$20 * 5 = \boxed{45}$ $\qquad$ $8 * \boxed{4} = 20$ $\qquad$ $\boxed{5} * \boxed{5} = 15$

$5 * 20 = \boxed{30}$

When everyone has completed at least half of these problems, check the work collectively.

Erase the board except for the statement of the rule: a $*$ b = (2 x a) + b.
Write this expression on the board below the rule.

$\square * \triangle = 15$

T: ***We're still using the same rule for $*$. What are some numbers that could be in $\square$ and $\triangle$ to make this expression true?***

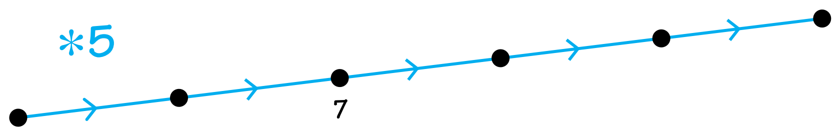
Let students suggest pairs of numbers and make a list of the possibilities. Be sure the students understand that with different shaped frames the numbers can be different, but that the same number in both $\square$ and $\triangle$ is one possibility, as illustrated here.

When your list has five or more entries, you might wish to encourage students to look for patterns. For example, if you add something to the first number ($\square$), you must subtract twice as much from the second number ($\triangle$).
Or, the second number ($\triangle$) is always odd.

$\square * \triangle = 15$

4	7
7	1
5	5
10	$\hat{5}$
8	$\hat{1}$
⋮	⋮

Erase the board except for the statement of the rule for ∗ and then draw this arrow picture.



T: ***The blue arrows are for ∗5. Who can label another dot in this picture?***

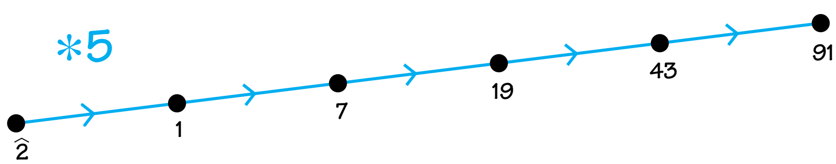
As necessary point out or let students observe that the starting dot of an arrow is the first number (a) and that 5 is always the second number. The ending dot of an arrow is the result of a ∗ 5. You could do this by observing that 7 is the starting number of an arrow; the ending number of that arrow is 7 ∗ 5. Point to the dot for 7 as you say, "7," and trace the blue arrow as you say, "∗5." Students should then be able to label the three dots to the right of 7.

If no student finds how to label a dot to the left of 7, point on the first such dot and draw a □ under it.

T: ***We want to find this number; □ ∗ 5 = 7.***

Write □ ∗ 5 = 7 on the board near the arrow picture. Then students should be able to figure out which number goes in the box.

A completely labeled arrow picture is shown below.



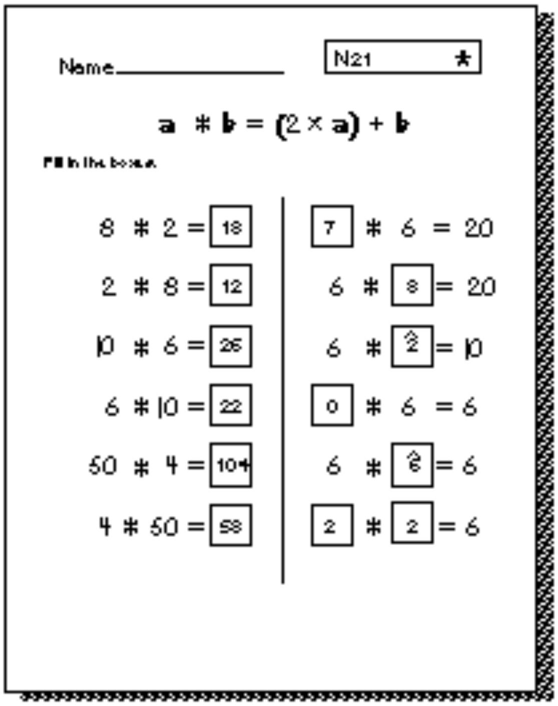
Worksheets N21*, **, and *** are available for individual work.

Extension Activity

Use computer games (for example, *Teasers by Tobbs* or *King's Rule* from Sunburst) or other resources to work with other variations of *Guess My Rule* activities. Invite students to write a *Guess My Rule* book. Students' *Guess My Rule* variations could challenge classmates or students in another class.

Home Activity

Send home a description of *Guess My Rule* activities and invite parents/guardians to play *Guess My Rule* with their child at home.



Name____________ N21 *

$a * b = (2 \times a) + b$

Fill in the boxes.

$8 * 2 = \boxed{18}$

$2 * 8 = \boxed{12}$

$10 * 6 = \boxed{26}$

$6 * 10 = \boxed{22}$

$50 * 4 = \boxed{104}$

$4 * 50 = \boxed{58}$

$\boxed{7} * 6 = 20$

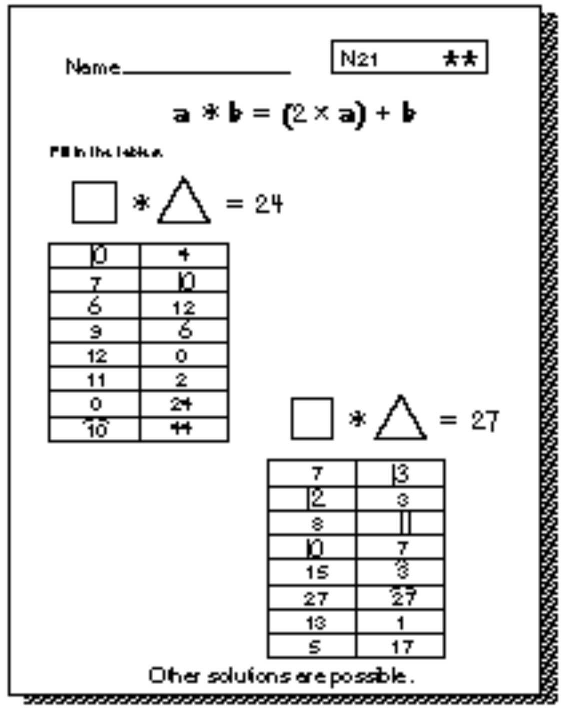
$6 * \boxed{8} = 20$

$6 * \boxed{\hat{2}} = 10$

$\boxed{0} * 6 = 6$

$6 * \boxed{\hat{6}} = 6$

$\boxed{2} * \boxed{2} = 6$



Name____________ N21 **

$a * b = (2 \times a) + b$

Fill in the tables.

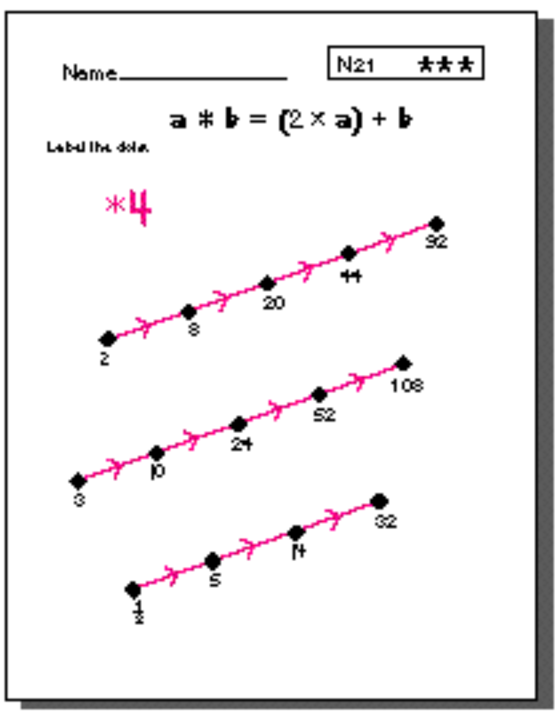
$\square * \triangle = 24$

10	4
7	10
6	12
9	6
12	0
11	2
0	24
$\hat{10}$	44

$\square * \triangle = 27$

7	13
12	3
8	11
10	7
15	$\hat{3}$
27	$\hat{27}$
13	1
5	17

Other solutions are possible.



N22 DECIMALS #2

N22

Capsule Lesson Summary

Represent various amounts of money on the Minicomputer. Using monetary situations as motivation, perform subtraction calculations with decimal numbers. Locate positions of various decimal numbers on the number line.

Materials

Teacher
- Minicomputer set
- Colored chalk
- Blackline N22

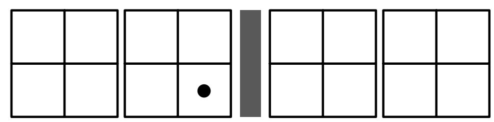
Student
- Calculator
- $ number line

Advance Preparation: Use Blackline N22 to make copies of the $ number line for students to use in Exercise 3.

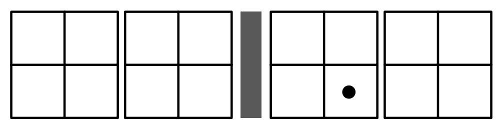
Description of Lesson

Exercise 1

Put this configuration on the Minicomputer.



T: ***This is one dollar. Can you put one dime on the Minicomputer? How do we write this number?***



0 . 1 0

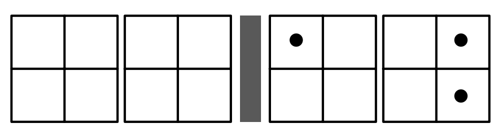
S: ***0.10.***

Emphasize that 0.10 = 0.1 and record the equivalence on the board.

0.10 = 0.1

Observe that ten dimes make a dollar, so a dime is one-tenth of a dollar. We can read 0.1 as "one dime" or "one tenth" or "zero point one." Continue this activity by asking for one nickel and one penny to be put on the Minicomputer.

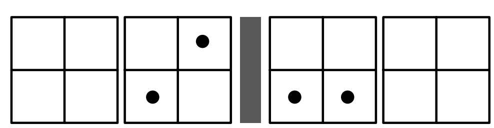
Ask a student to put 85 cents on the Minicomputer. You may need to ask for the simplest configuration.



T: ***We have 85 cents on the Minicomputer as how many dimes and how many pennies?***

S: ***8 dimes and 5 pennies.***

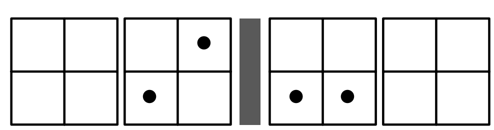
Put this configuration on the Minicomputer.



T: ***What number is on the Minicomputer?***

S: ***6 dollars and 30 cents (or 6.30).***

T: ***How do we write this number?***



6 . 3 0

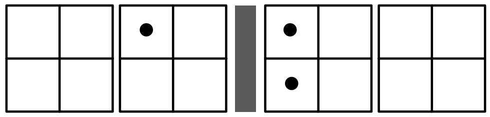
6.30 = 6.3

S: ***6.3.***

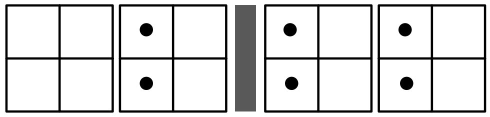
S: ***6.30.***

N22

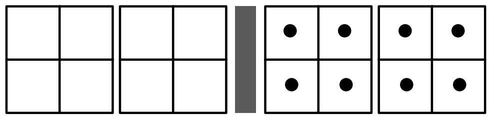
Continue this activity by putting these numbers on the Minicomputer.



$= 9.00 = 9.0 = 9$



$= 11.1 = 11.10$



$= 1.65$

If some students find decoding some of these configurations difficult, ask them to make trades until standard configurations are reached.

Exercise 2

T: ***Suppose you have $5, and you spend $2.85. How much money is left? What calculation could we do?***

S: ***$5.00 – $2.85.***

Write this calculation on the board.

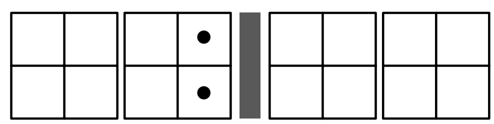
$5.00 - 2.85$

T: ***How do you suggest we do the calculation?***

Let students discuss how to do the calculation and why $2.15 is left. It is likely that many students will solve this problem mentally, while some will use pencil-and-paper methods. Others may ask that the problem be done using the Minicomputer.

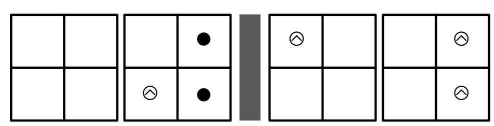
T: ***How can we use the Minicomputer?***

S: ***First put on $5.***



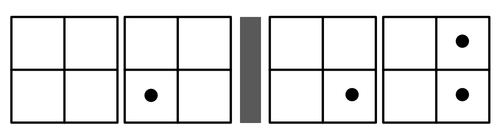
S: ***$5.00 - 2.85 = 5.00 + \widehat{2.85}$. Put $\widehat{2.85}$ on the Minicomputer.***

If someone suggests making backward trades until 2.85 can be taken off, accept and use that method.



S: ***We need to make some trades.***

Invite students to make trades until only regular checkers remain on the Minicomputer. Summarize the work on the board.



$5.00 - 2.85 = 5.00 + \widehat{2.85} = 2.15$

Do a similar problem starting with $10 and then spending $6.67. The students should find that $3.33 is left.

Exercise 3

Distribute copies of the $ number line (Blackline N22) and draw a similar number line on the board.

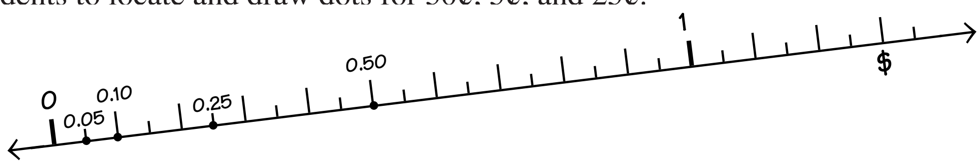
T: ***This mark is for 0 dollars*** (point to it)***, and this mark is for 1 dollar*** (point to it)***. What do you think these marks*** (point to several marks in succession) ***are for?***

S: ***Counting by nickels. There are twenty nickels in a dollar.***

T: ***Let's put a dot for one dime on the number line. Locate and draw a dot for 0.10.***

Let a student draw the dot on the board and label it 0.10.

Also ask students to locate and draw dots for 50¢, 5¢, and 25¢.



T: ***Where would a dot for one penny be on this number line?***

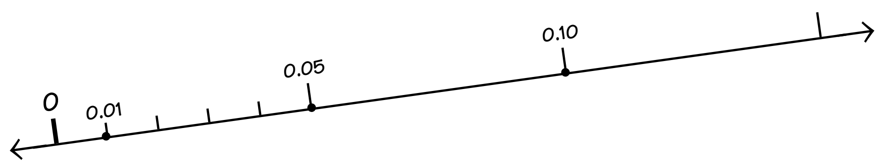
Invite a student to show the approximate location.

T: ***How could we locate this point more accurately?***

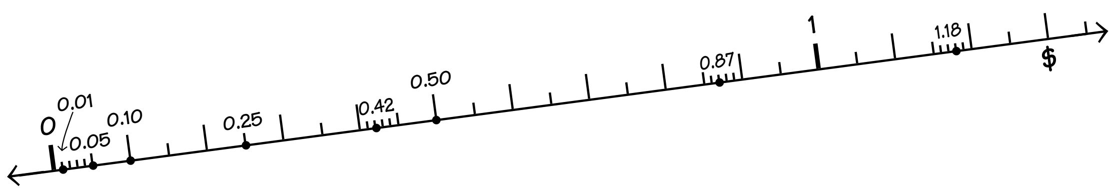
S: ***Divide the segment from 0 to 0.05 into five smaller segments of the same length.***

T: ***Yes. Imagine that we magnify part of the number line.***

Draw this part of the number line (magnification) to the side of the original number line. Ask a student to locate and draw a dot for one penny.



Ask students to locate and draw dots for $0.87, $0.42, and $1.18.



Practice Activity

Put students in groups of four, each group having a number line, a student Minicomputer set, a calculator, and paper and pencil. When you call out a decimal number, each group should display it in four ways. Read decimals both as money amounts, and as tenths and hundredths.

Extension Activity

Provide grocery tapes with amounts for students to locate on a number line.

Capsule Lesson Summary

Identify multiples and divisors of numbers less than 30. Build an arrow road between two numbers using “is a divisor of” arrows. Introduce prime numbers.

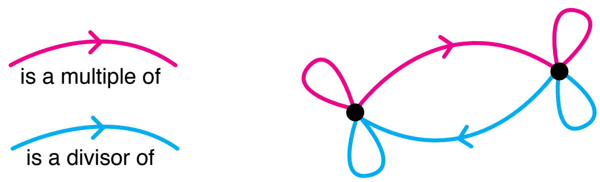
Materials

Teacher
- Colored chalk

Student
- Paper
- Colored pencils, pens, or crayons

Description of Lesson

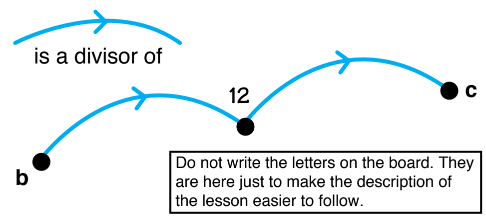
Lead a warm-up similar to that which began Lesson N16. In an arrow picture review the relations “is a multiple of” and “is a divisor of” as opposites of each other. Tell the class that in this lesson they will only use positive integers.



Ask the class to name the positive divisors of several numbers: 14, 15, 6, 8, 12, and 5.

Exercise 1

Draw this arrow picture on the board.



T: *Which positive numbers could be here (at* **b***)?*

S: *1, 2, 3, 4, or 6.*

S: *12 is a divisor of 12, but we show that with a loop at 12.*

T: *Yes, this (***b***) could be any divisor of 12 other than 12 itself. Which positive numbers could be here (at* **c***)?*

S: *24 or 36 or 48.*

S: *120.*

T: *What do you know about those numbers?*

S: *12 is a divisor of each of them.*

S: *They are multiples of 12.*

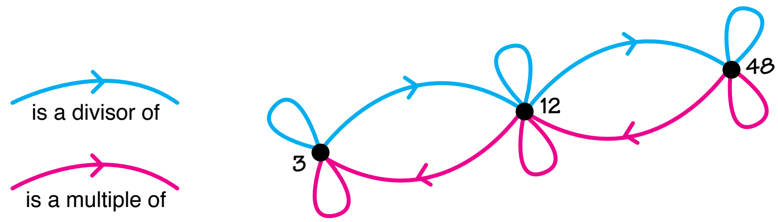
T: *Yes, that is what the arrow tells us. What is the return arrow for?*

S: *“Is a multiple of.”*

T: *Let’s draw the return arrows in the picture.*

S: *May we add loops also?*

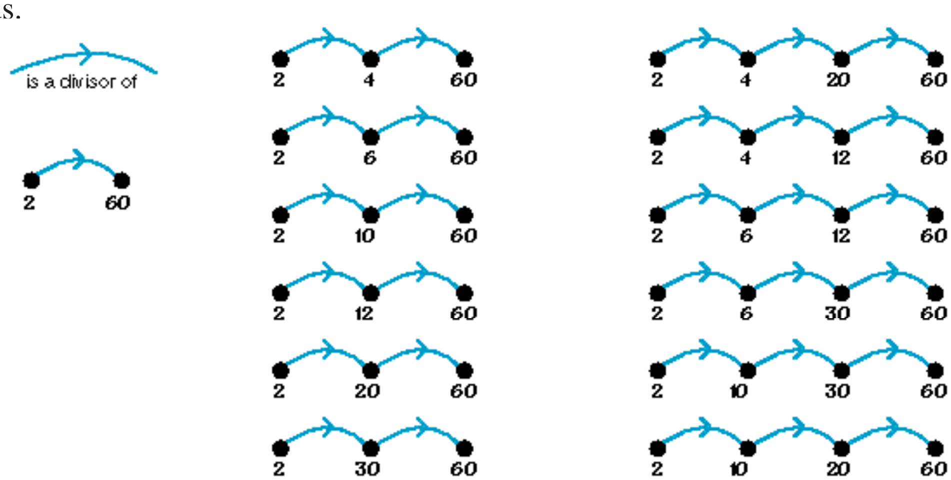
Invite students to draw the return arrows and loops.



Exercise 2

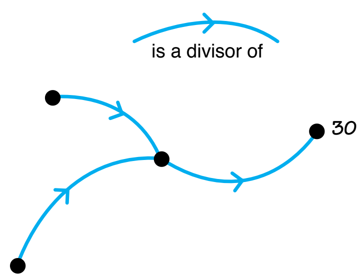
T: ***This is a short arrow road with two blue arrows*** (refer the picture of Exercise 1)***. Can we build other arrow roads with "is a divisor of" arrows? Try to build a blue arrow road starting at 2 and ending at 60.***

Draw dots for 2 and for 60 on the board and let students work with partners on this problem. Ask those who finish quickly to find shorter or longer solutions, and perhaps to find several different solutions. When many have found a road from 2 to 60, lead a collective discussion of the solutions and invite students with different solutions to draw them on the board. A collection of solutions is shown below. Observe that there are three different lengths. Ask the class to find longest and shortest roads.



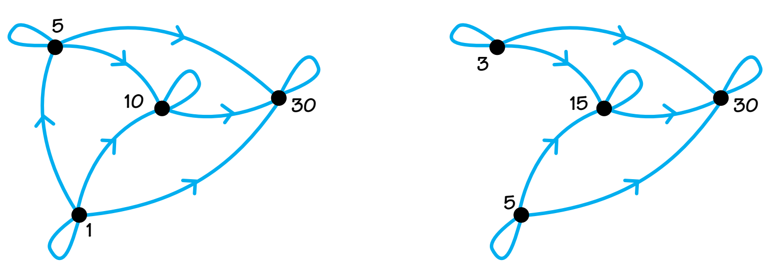
Exercise 3

Draw this arrow picture on the board.



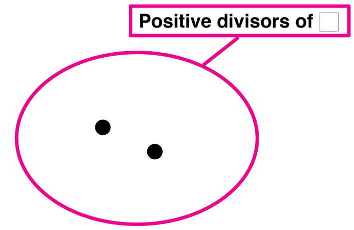
T: ***Copy this picture on your paper. Label the dots, and then add any missing blue arrows and loops.***

Encourage students to find several solutions to this problem. Invite students with different solutions to draw them on the board. Two solutions are shown below.



Exercise 4

If you were able to include Exercise 4 of Lesson N16, adjust the dialogue here to be a review of that exercise. Draw this string picture on the board, and ask students to copy it.



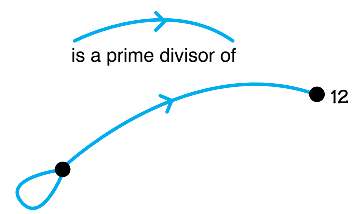
T: ***There are exactly two numbers in this string. We could use hatching to show that just two numbers are inside the string. The picture tells us that the number in the box has exactly two positive divisors. Try to find some numbers we could put in the box.***

You will note that a number for the box is a prime number. After a few minutes collect students' results in a list on the board. The first twelve positive prime numbers are listed below for your reference.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, ...

T: ***Numbers with exactly two positive divisors are called* prime numbers. *They are very special numbers and we will do a lot with them.***

Draw this arrow picture on the board. Point to the unlabeled dot as you ask,



T: ***Can you tell me a positive prime number that this could be?***

S: ***2 or 3.***

Replace 12 with 30 and ask the same question.

S: ***2 or 3 or 5.***

Repeat the activity, asking for the positive prime divisors of 14, 9, 4, 8, 7, 3, and 25. Recognition of the prime divisors of a number will be important in future lessons.

Note: Since 3 (or 7) is a prime number, its only prime divisor is the number itself. The picture would be a loop.

Home Activity

Send home a definition of prime numbers. Ask parents/guardians to list with their child the prime numbers less than 30 (or 50).

Capsule Lesson Summary

Review the Sasquatch story and the game (from Lesson N17) that the children play using footprints. Locate various rational numbers on the number line and observe, for example, that $\frac{2}{6} = \frac{1}{3}$ and $\frac{12}{6} = \frac{6}{3} = 2$. In this situation, compare rational numbers and determine which numbers can occur as denominators of fractions for $\frac{1}{3}$ and for $\frac{1}{4}$. For example, observe that $\frac{1}{3} = \frac{8}{24} = \frac{2}{6} = \frac{4}{12} = \ldots$ and that $\frac{1}{4} = \frac{6}{24} = \frac{3}{12} = \frac{5}{20} = \ldots$.

Materials

Teacher
- Colored chalk
- Meter stick
- Chalkboard compass (optional)
- Overhead projector (optional)
- Blackline N24

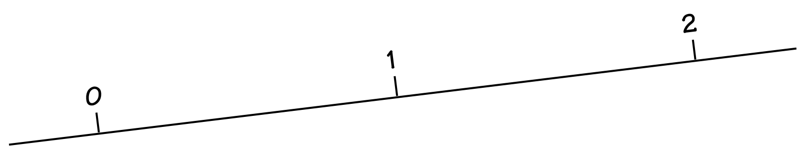
Student
- Information chart

Advance Preparation: Use Blackline N24 to make copies of the chart in Exercises 1 and 2 for students, and possibly for use on the overhead.

Description of Lesson

Exercise 1

Draw this part of a number line on the board, making the distance between marks 48 cm.



T: ***Do you remember the Sasquatch story?***

Let students recall as much about the previous Sasquatch lesson as they can. Include in the review an explanation of the game that some children play with Big Foot's tracks.

S: ***The children play a game using the tracks. Each child starts walking from here (at 0), takes steps of the same length, and tries to land on all the Big Foot tracks.***

If you like, invite students to play the game as they did in Lesson N17.

Display this table.

Child	A	B	C	D	G	H	K	L	M	N
Number of steps to reach 1	4	12	15	2	9	3	24	6	20	18
First step lands on (step size)	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{15}$	$\frac{1}{2}$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{1}{24}$	$\frac{1}{6}$	$\frac{1}{20}$	$\frac{1}{18}$

T: ***Which child takes the longest steps?***

S: **D; D** ***takes only two steps to go from 0 to 1. The other children take more than two steps to go from 0 to 1.***

T: ***Which child takes the shortest steps?***

S: **K**. ***K takes 24 steps to go from 0 to 1.***

T: ***One of the children, H, takes three steps from 0 to 1. Who can show on the number line where this child's first step lands?***

Most likely a student will point to an approximate location for **H**'s first step.

T: ***How can we locate this point more accurately?***

Give the class time to consider your question.

S: ***Cut a string the length from 0 to 1 and fold it to make 3 equal pieces.***

S: ***Measure the distance from 0 to 1, and divide by 3.***

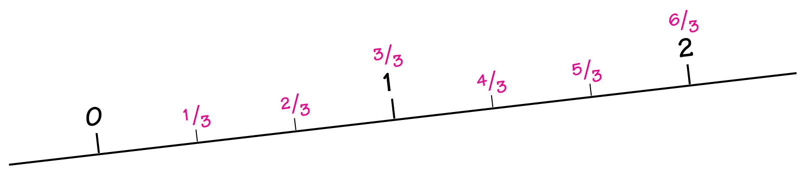
Invite students to measure the segment from 0 to 1; they should find it is 48 cm.

T: ***The length from 0 to 1 is 48 cm. How will that help us to locate the point for ⅓?***

S: ***H takes 3 steps from 0 to 1. 3 x 16 = 48, so measure 16 cm from 0 to locate the first step.***

Invite a student to locate the point, using a meter stick or centimeter ruler.

Ask students to locate ⅔, 3⁄3, 4⁄3, 5⁄3, and 6⁄3 on the number line.



T: ***Where does H's 15th step land on the number line?***

Give the class some time to think about the question. Encourage students who have difficulty to use the classroom number line.

S: ***5 x 3 = 15. H's 15th step lands on 5.***

T: ***What about H's 30th step?***

S: ***H's 30th step lands on 10, because 10 x 3 = 30.***

S: ***The 15th step is at 5, so the 30th step is at 10.***

T: ***What about H's 43rd step?***

Encourage use of the classroom number line.

S: ***It is between 14 and 15 on the number line; one step past 14, because 14 x 3 = 42.***

T: ***Consider child L. How many steps from 0 to 1 does this child take?***

S: ***Six steps.***

T: ***Is L's step size larger or smaller than H's step size?***

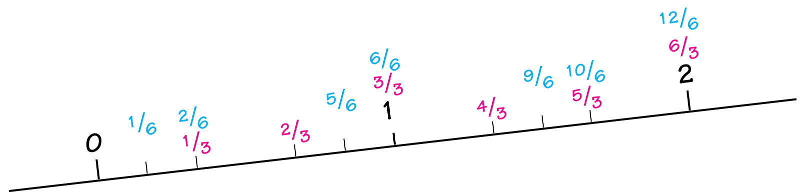
S: *Smaller;* **L** *takes two steps to equal one of* **H***'s steps.*

T: *Where on the number line is* **L***'s first step?*

S: *Halfway between 0 and ⅓.*

S: *8 cm to the right of 0.*

Let a student label the point for ⅙. Continue by asking students to locate and label points for 2/6, ⅚, 6/6, 9/6, 10/6, and 12/6.

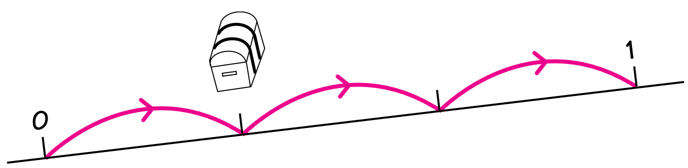


As this activity progresses, students should note that 2/6 = ⅓, 6/6 = 3/3 = 1, and 12/6 = 6/3 = 2.

Ask students where **L**'s 24th step and 49th step are on the number line.

Exercise 2

Add another row to your table and draw part of the number line as illustrated below. Make the distance from 0 to 1 measure 96 cm.



T: ***One day some children discover that a treasure is buried in the snow path between the Sasquatch tracks.*** (Point to the illustrated treasure chest.) ***Where is the treasure located?***

S: ***At ⅓.***

T: ***The children know their step sizes and decide that those children whose steps land on the treasure will share it. Which children get to share the treasure? Explain your answers.***

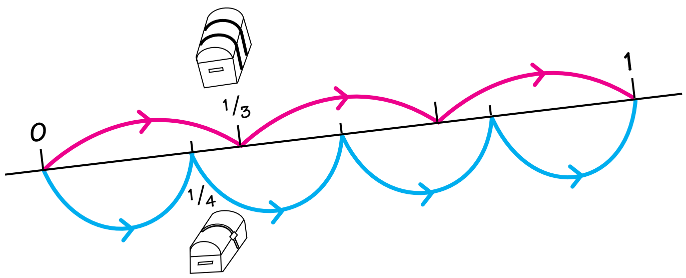
S: **D** ***will miss the treasure, because*** **D*****'s first step lands on ½ which is greater than ⅓.***

S: **L** ***will land on the treasure, because*** **L** ***takes two steps and lands on ⅓.***

Summarize each student's explanation in your table. When complete, your table should look like the one below.

Child	A	B	C	D	G	H	K	L	M	N
Number of steps to reach 1	4	12	15	2	9	3	24	6	20	18
First step lands on (step size)	1/4	1/12	1/15	1/2	1/9	1/3	1/24	1/6	1/20	1/18
Treasure at 1/3	X	4/12	5/15	X	3/9	1/3	8/24	2/6	X	6/18

Continue the story by introducing a buried treasure at $\frac{1}{4}$.



Ask which children get to share the second treasure. A completed table is shown below.

Child	A	B	C	D	G	H	K	L	M	N
Number of steps to reach 1	4	12	15	2	9	3	24	6	20	18
First step lands on (step size)	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{15}$	$\frac{1}{2}$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{1}{24}$	$\frac{1}{6}$	$\frac{1}{20}$	$\frac{1}{18}$
Treasure at $\frac{1}{3}$	X	$\frac{4}{12}$	$\frac{5}{15}$	X	$\frac{3}{9}$	$\frac{1}{3}$	$\frac{8}{24}$	$\frac{2}{6}$	X	$\frac{6}{18}$
Treasure at $\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{12}$	X	X	X	X	$\frac{6}{24}$	X	$\frac{5}{20}$	X

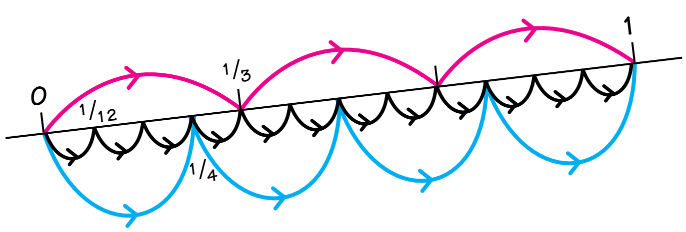
Observe with the class that the fractions in the row labeled **Treasure at** $\frac{1}{3}$ are names for $\frac{1}{3}$, and those in the row **Treasure at** $\frac{1}{4}$ are names for $\frac{1}{4}$.

T: ***Which children receive a share of both treasures?***

S: **B** ***and*** **K.**

T: **B*****'s step size is*** $\frac{1}{12}$***. Can you locate*** $\frac{1}{12}$ ***on the number line?***

Invite a student to locate $\frac{1}{12}$ approximately. Then locate $\frac{2}{12}$, $\frac{3}{12}$, … $\frac{12}{12}$ (**B**'s steps) on the number line.



S: **B** ***lands on*** $\frac{1}{4}$ ***after three steps and on*** $\frac{1}{3}$ ***after four steps.***

T: ***What about*** **K*****?*** **K*****'s step size is*** $\frac{1}{24}$***. Locate*** $\frac{1}{24}$ ***on the number line.***

S: $\frac{1}{24}$ ***is halfway between 0 and*** $\frac{1}{12}$***.***

S: **K** ***takes six steps to land on*** $\frac{1}{4}$ ***and eight steps to land on*** $\frac{1}{3}$***.***

T: ***Can you think of some other step sizes for children (besides those in our table) who would receive a share of both treasures if they were playing?***

S: $\frac{1}{36}$***.***

List correct suggestions on the board.

$$1/12,\ 1/24,\ 1/36,\ 1/48,\ 1/72,\ 1/60,\ \ldots,\ 1/600,\ \ldots$$

T: ***Do you notice anything interesting about these fractions?***

S: ***The bottom numbers are multiples of 3 and multiples of 4.***

S: ***The bottom number are multiples of 12.***

Accept any reasonable observations.

Extension Activity

Suggest students extend the chart to find who would share a treasure at ½, or at ⅕, or at ⅚.

N25 CALCULATOR RELATIONS #2

Capsule Lesson Summary

Review the patterns generated by subtracting tens. Investigate the effect of subtracting 10 a specified number of times (for example, 2, 3, 10, or 100 times). Use an arrow picture to illustrate problems that focus on the relation $\boxed{+}\boxed{10}\boxed{=}$….

Materials

Teacher
- Overhead calculator
- Colored chalk

Student
- Calculator
- Colored pencils, pens, or crayons
- Worksheets N25*, **, ***, and ****

Description of Lesson

Exercise 1

Begin the exercise with a mental arithmetic activity.

T: ***Let's start at 87 and count backward by tens. What is 87 – 10?***

S: ***77.***

T: ***What is 77 – 10?***

S: ***67.***

Select students, either in order or randomly, to continue subtracting tens to produce this sequence: 57, 47, 37, 27, 17, 7, $\widehat{3}$, $\widehat{13}$, $\widehat{23}$, $\widehat{33}$. Invite students to explain why 7 – 10 = $\widehat{3}$ and why $\widehat{3}$ – 10 = $\widehat{13}$. They may find referring to a number line helpful.

Repeat the activity, starting at 134 and subtracting tens until you reach $\widehat{26}$ or less. If students have difficulty, write part of the sequence on the board. Invite students to explain why 4 – 10 = $\widehat{6}$ and why $\widehat{6}$ – 10 = $\widehat{16}$.

T: ***What patterns do you notice when we subtract tens repeatedly?***

S: ***The ones digit of all of the positive numbers is the same. The ones digit of the negative numbers is the same.***

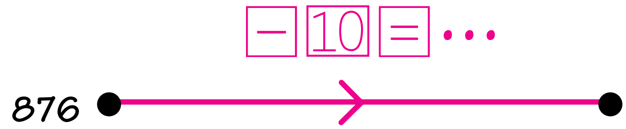
S: ***In the first sequence, each positive number ends in 7 and each negative number ends in 3. In the second sequence, each positive number ends in 4 and each negative number ends in 6. Also, the sum of the two possible ending digits in each sequence is 10: 7 + 3 = 10 and 4 + 6 = 10.***

S: ***The tens digit usually decreases by 1 each time (except in cases such as 7 – 10 = $\widehat{3}$, 104 – 10 = 94, 4 – 10 = $\widehat{6}$).***

Distribute calculators, and draw an arrow starting at 876 on the board.

T: ***How can we subtract tens on a calculator?***

S: *Press [−] [1][0] [=] and keep pressing [=].*



Label the red arrow.

T: *Put 876 on your calculator and follow my directions carefully. Do not go on ahead. Hide the display of your calculator and look at it only when I say, "Calculator check."*

Press [−] [1][0] [=]. What number do you predict is on the calculator?

S: *866.*

T: *Calculator check.*

For each of the following instructions, ask students to hide their displays, to predict the result, and to check it. Occasionally ask a student to explain a prediction. Stop now and then to allow students who have made a mistake to catch up. For example, if some students do not have 756 on their displays at the appropriate time, ask those students to press [C] [7][6][6] [−] [1][0] [=].

[=] [=]	(846)
[=] [=] [=]	(816)
[=] [=]	(796)
[=] [=] [=] [=]	(756)
[=] [=] [=] [=] [=]	(706)
[=] … [=] ((ten times)	(606)
[=] … [=] (ten times)	(506)
[=] [=] [=]	(476)

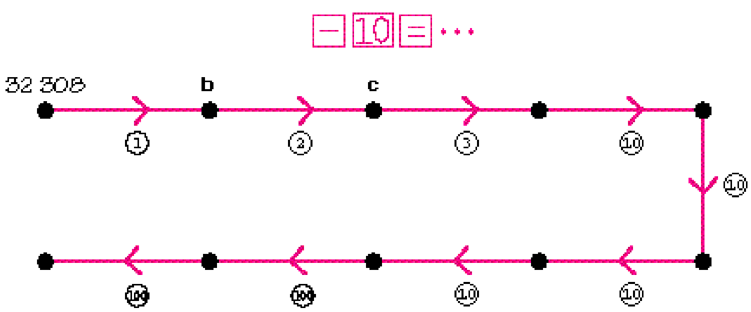
T: *If we keep going, what will be the least positive number that the calculator will display?*

S: *6.*

T: *What is the first negative number it will show?*

S: *$^-4$, since 6 – 10 = $^-4$.*

Draw this arrow road on the board.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: *The red arrows are for [−][10][=] …. The circled numbers near the red arrows tell how many times to press [=], that is, how many times to subtract 10.*

Put 32 308 on your calculator.

Some students may be concerned because the calculator has no comma or space. If you use a comma in 32,308, hold a brief discussion about the role of the comma in writing numbers.

T: ***Hide the display of your calculator. Press ⊟ 1 0 ⊜. What number do you predict is on the display?***

S: ***32,298.***

T: ***Calculator check.***

We subtracted 10 once, so we put 32,298 here (at* b*).

Label the dot and trace the arrow from **b** to **c** as you explain,

T: ***According to this arrow, we should press ⊜ twice. Without using your calculator, do you know which number this*** (point to **c**) ***will be?***

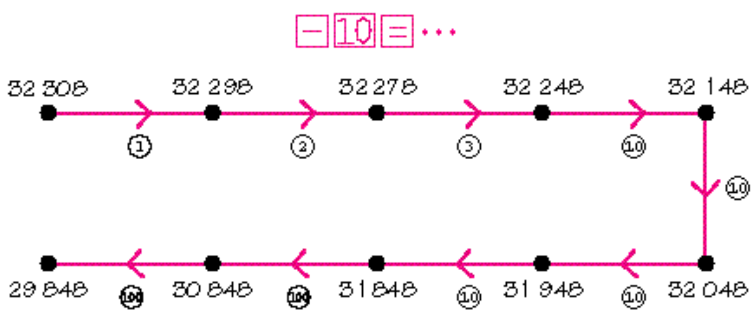
S: ***32,278.***

T: ***Why?***

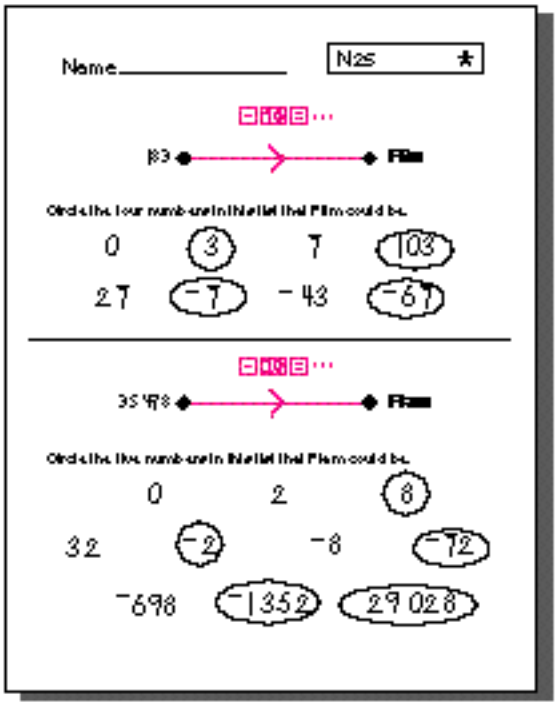
S: ***If you press ⊜ two times, you subtract two tens. 32,298 – 20 = 32,278.***

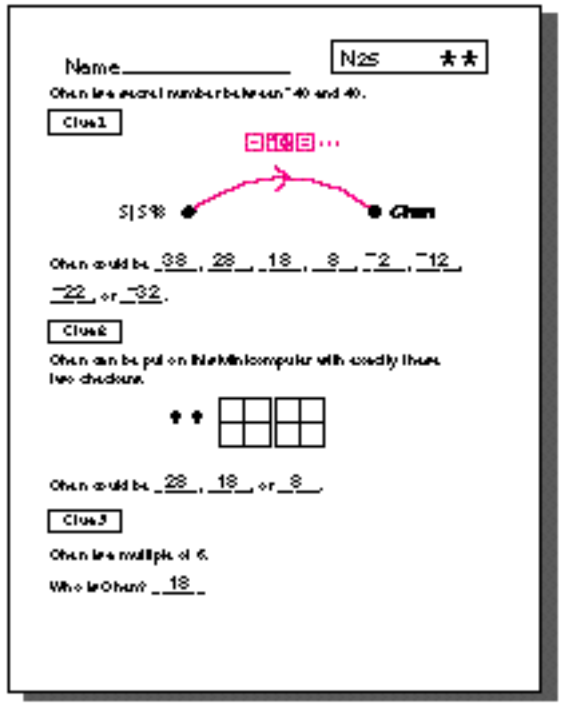
T: ***Calculator check.***

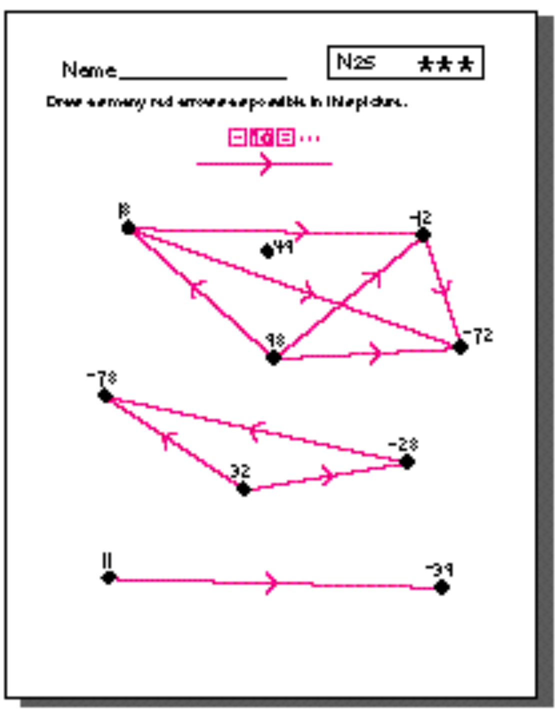
In a similar manner, invite students to label the other dots. Do not ask students to check the last two calculations on their calculators.

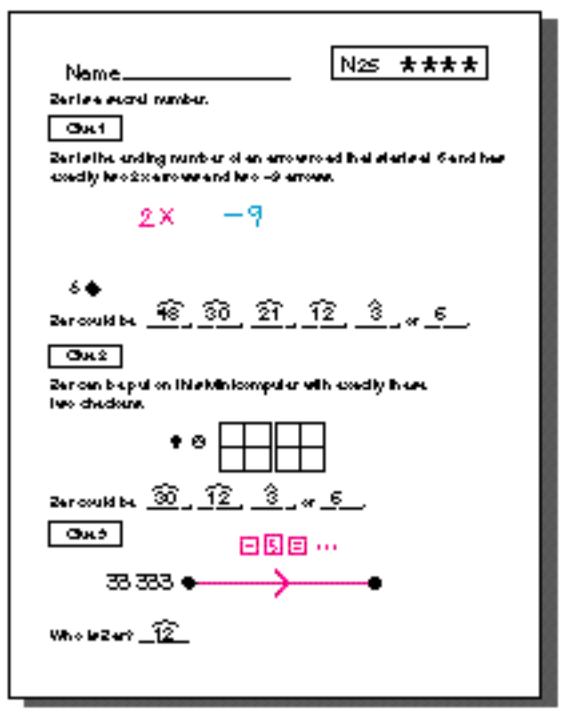


Worksheets N25*, **, ***, and **** are available for individual work. If students are having difficulty with the first clue on N25**, you may want to discuss it collectively.


Name
N25 ★
183
Flim
Circle the four numbers in this list that Flim could be.
0 3 7 103
27 ⁻7 ⁻43 ⁻67
35 478
Flam
Circle the five numbers in this list that Flam could be.
0 2 8
32 ⁻2 ⁻8 ⁻72
⁻698 ⁻1352 29 028


Name
N25 ★★
Chen is a secret number between ⁻40 and 40.
Clue 1
51 548
Chen
Chen could be 38, 28, 18, 8, ⁻2, ⁻12, ⁻22, or ⁻32.
Clue 2
Chen can be put on this Minicomputer with exactly these two checkers.
Chen could be 28, 18, or 8.
Clue 3
Chen is a multiple of 6.
Who is Chen? 18


Name
N25 ★★★
Draw as many red arrows as possible in this picture.
18
42
44
98
⁻72
⁻78
32
⁻28
11
⁻39


Name
N25 ★★★★
Zar is a secret number.
Clue 1
Zar is the ending number of an arrow road that starts at 6 and has exactly two 2x arrows and two −9 arrows.
2x
−9
6
Zar could be 48, 30, 21, 12, 3, or 6.
Clue 2
Zar can be put on this Minicomputer with exactly these two checkers.
Zar could be 30, 12, 3, or 6.
Clue 3
38 383
Who is Zar? 12

N26 MULTIPLICATION #2

Capsule Lesson Summary

Use related problems to recall some multiplication facts, and to complete some multiplication problems. For example, calculate 14 x 15 on a calculator and use the result to calculate 15 x 15 and 13 x 15.

Materials

Teacher
- Overhead calculator
- Colored chalk

Student
- Paper
- Worksheets N26*, **, ***, and ****

Description of Lesson

Exercise 1

T: ***What number is 5 x 8?***

S: ***40.***

Record the number fact on the board.

$5 \times 8 =$

40

T: ***Is 6 x 8 more or less than 5 x 8?***

S: ***More.***

T: ***How much more?***

S: ***8 more.***

T: ***So if you forget what 6 x 8 is, how can you easily find out?***

S: ***By adding 8 to 40.***

Ask for 7 x 8 in the same way.

$5 \times 8 = 40$
$6 \times 8 = 48$
$7 \times 8 = 56$

T: ***Is 4 x 8 more or less than 5 x 8?***

S: ***Less.***

T: ***How much less?***

S: ***8 less.***

T: ***What number is 4 x 8?***

S: ***32; 40 – 8 = 32.***

$4 \times 8 = 32$
$5 \times 8 = 40$
$6 \times 8 = 48$
$7 \times 8 = 56$

Exercise 2

Invite a student to calculate 6 x 135 at the board, while others do so on their papers.

Go over the steps of the problem carefully for the benefit of the class. Introduce the compact way of writing in multiplication.

$$\begin{array}{r} 135 \\ \times\ 6 \\ \hline 30 \\ 180 \\ +600 \\ \hline 810 \end{array} \qquad \begin{array}{r} {}^{2}\,{}^{3}\quad \\ 135 \\ \times\ 6 \\ \hline 810 \end{array}$$

N26

Record the result horizontally.

$6 \times 135 = 810$

T: ***How could we calculate 7 x 135 without doing too much work?***

S: ***We could add 135 seven times.***

S: ***Like we did 6 x 135.***

S: ***7 x 135 is 135 more than 6 x 135. So we could add 135 to 810.***

T: ***What number is 810 + 135?***

S: ***945.***

Accept all valid methods, but emphasize that 7 x 135 is 135 more than 6 x 135, and so 7 x 135 = 810 + 135. Record this number sentence on the board.

$$5 \times 135 = 675$$
$$6 \times 135 = 810$$
$$7 \times 135 = 945$$

Similarly, decide that 5 x 135 = 675. Your students should notice that 5 x 135 is 135 less than 6 x 135, and so 5 x 135 = 810 – 135.

Exercise 3

Take out a calculator and tell the class that you are going to use it to calculate 13 x 17. Record the result on the board.

$$13 \times 17 = 221$$

T: ***Can you think of an easy way to calculate 14 x 17 without using the calculator?***

Let students react to the question. Most likely some will suggest that 14 x 17 is 17 more than 13 x 17, so 14 x 17 = 221 + 17. Record the result on the board.

$$12 \times 17 = 204$$
$$13 \times 17 = 221$$
$$14 \times 17 = 238$$

Similarly, lead your class to calculate 12 x 17 by subtracting 17 from 221.

Use a calculator to calculate 27 x 45. Record the result on the board.

$$27 \times 45 = 1215$$

Pose the following problems, one at a time. (Answers are given in boxes.)

$$28 \times 45 = \boxed{1260} \qquad 26 \times 45 = \boxed{1170}$$
$$29 \times 45 = \boxed{1305} \qquad 25 \times 45 = \boxed{1125}$$
$$30 \times 45 = \boxed{1350} \qquad 24 \times 45 = \boxed{1080}$$

Ask students to write their answers on paper and check several students' answers to a problem before letting someone answer aloud. Emphasize the method of adding or subtracting 45 from one of the preceding answers. Review subtraction when the opportunity arises.

Exercise 4

In this exercise, you will ask the class to do a series of calculations. The first two calculations are simple; each of the others can be done using answers to calculations earlier in the series. Encourage students to do the calculations mentally, not on paper.

T: ***What number is 10 x 35?***

S: ***350.***

T: ***What number is 2 x 35?***

S: ***70.***

T: ***Now, what number is 12 x 35?***

S: ***420.***

T: ***How did you do the calculation?***

S: ***I added 350 + 70.***

$$\begin{array}{r} 10 \times 35 = 350 \\ 2 \times 35 = 70 \\ \hline 12 \times 35 = 420 \end{array}$$

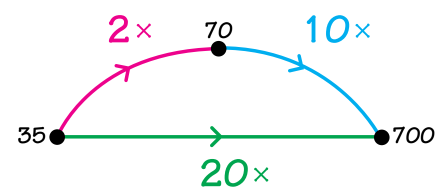
Write this problem near 10 x 35 = 350 and 2 x 35 = 70.

$$20 \times 35 =$$

T: ***Try to use some of the results we already have to do this calculation.***

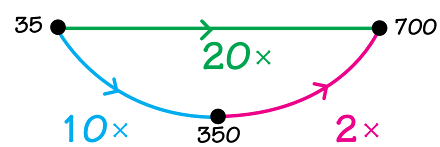
S: ***We know that 2 x 35 = 70.***
So we just need to multiply by 10. 10 x 70 = 700.

An arrow picture records this method nicely.



S: ***10 x 35 = 350, so 20 x 35 = 2 x 350 = 700.***

An arrow picture records this method nicely.



If both methods are suggested, the arrow pictures can be combined rather than drawn separately. Accept other correct explanations.

Add another problem.

$$\begin{array}{l} 20 \times 35 = 700 \\ 19 \times 35 \end{array}$$

T: ***Is 19 x 35 more or less than 700?***

S: ***Less.***

T: ***How much less?***

S: ***35 less.***

T: ***What number is 700 – 35?***

S: ***665.***

Record the result and add another problem.

$$\begin{array}{l} 20 \times 35 = 700 \\ 19 \times 35 = 665 \\ 22 \times 35 \end{array}$$

T: ***Could we use some of the results we have on the board to help us do this calculation?***

S: ***20 x 35 = 700 and 2 x 35 = 70, so 22 x 35 = 770.***

Record the result and put this problem nearby.

$$44 \times 35 =$$

T: ***How could we do this calculation?***

S: ***The easiest way is to double 770; 770 + 770 = 1 540.***

Record the result and add another problem.

$$44 \times 35 = 1540$$
$$45 \times 35 =$$

T: ***Is 45 x 35 more or less than 1 540?***

S: ***35 more.***

S: ***So 45 x 35 = 1 540 + 35 = 1 575.***

Record the result and add another problem.

$$44 \times 35 = 1540$$
$$45 \times 35 = 1575$$
$$43 \times 35 =$$

T: ***Is 43 x 35 more or less than 1 540?***

S: ***35 less.***

S: ***So 43 x 35 = 1 540 – 35 = 1 505.***

Exercise 5

T: ***What number is 10 x 38?***

S: ***380.***

T: ***What number is 2 x 38?***

S: ***76.***

Write these number sentences on the board.

$$10 \times 38 = 380 \qquad 2 \times 38 = 76$$
$$\boxed{} \times 38 = 380 + 76$$

T: ***What number belongs in the box?***

A common incorrect answer is 456.

T: ***380 + 76 = 456. But we still need to find which number belongs in the box.***

$$\boxed{} \times 38 = 380 + 76 = 456$$

S: ***Ten 38's are 380 and two 38's are 76, so 380 + 76 must be twelve 38's.***

Continue with these problems. (Answers are in the boxes.)

$$\boxed{11} \times 38 = 380 + 38 \qquad \boxed{14} \times 38 = 380 + 76 + 76$$
$$\boxed{8} \times 38 = 380 - 76 \qquad \boxed{9} \times 38 = 380 - 38$$

Worksheets N26*, **, ***, and **** are available for individual work.

Name______________ N26 ★

Fill in the boxes.

9
9
9
9
9
9
9
9
9
9
9
9
+9
117

13 × 9 = 117

9
9
9
9
9
9
9
9
9
9
9
9
9
+9

14 × 9 = $\boxed{126}$

9
9
9
9
9
9
9
9
9
9
9
9
9
9
9
+9

16 × 9 = $\boxed{144}$

9
9
9
9
9
9
9
9
9
9
9
9
9
9
9
9
9
9
9
+9

20 × 9 = $\boxed{180}$

Name______________ N26 ★★

Fill in the boxes.

5 × 9 = 45
6 × 9 = $\boxed{54}$
7 × 9 = $\boxed{63}$
8 × 9 = $\boxed{72}$

11 × 14 = $\boxed{154}$
12 × 14 = 168
13 × 14 = $\boxed{182}$
14 × 14 = $\boxed{196}$

33 × 21 = $\boxed{693}$
34 × 21 = 714
35 × 21 = $\boxed{735}$

16 × 32 = 512
18 × 32 = $\boxed{576}$
19 × 32 = $\boxed{608}$

21 × 55 = $\boxed{1155}$
22 × 55 = $\boxed{1210}$
23 × 55 = 1265

10 × 67 = $\boxed{670}$
20 × 67 = $\boxed{1340}$
40 × 67 = $\boxed{2680}$

Name______________ N26 ★★★

Complete:

10 × 54 = 540
3 × 54 = 162
13 × 54 = 702
30 × 54 = 1620
43 × 54 = 2 322
60 × 54 = 3 240
59 × 54 = 3 186
100 × 54 = 5 400
103 × 54 = 5 562
130 × 54 = 7 020
133 × 54 = 7 182

Name______________ N26 ★★★★

3 × 46 = 138 10 × 46 = 460

Fill in the boxes. The first one is done for you.

1) 460 + 138 = $\boxed{13}$ × 46
2) 460 − 138 = $\boxed{7}$ × 46
3) 460 + 138 + 138 = $\boxed{16}$ × 46
4) 460 + 460 + 460 = $\boxed{30}$ × 46
5) 3 × 460 = $\boxed{30}$ × 46
6) 460 + 46 = $\boxed{11}$ × 46
7) 460 − 46 = $\boxed{9}$ × 46
8) 4600 = $\boxed{100}$ × 46
9) 4600 + 138 = $\boxed{103}$ × 46
10) 4600 + 460 = $\boxed{110}$ × 46
11) 4600 + 460 + 138 = $\boxed{113}$ × 46

N27 THE FUNCTION 10X #2

Capsule Lesson Summary

Review the effect of multiplying numbers by 10 using the Minicomputer. Explore a situation that uses addition and 10x functions alternately.

Materials

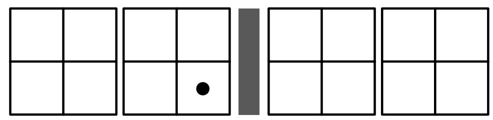
Teacher
- Minicomputer set
- Colored chalk
- ⑩-checkers

Student
- Colored pencils, pens, or crayons
- Worksheets N27*, **, ***, and ****

Description of Lesson

Exercise 1

Put a checker on the 1-square of the Minicomputer.



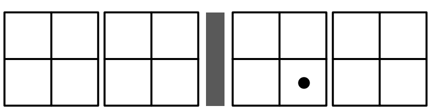
T: ***What number is this?***

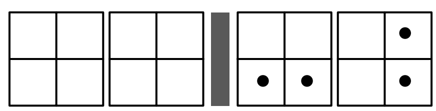
S: ***1.***

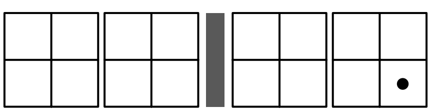
S: ***1.0 or 1.00.***

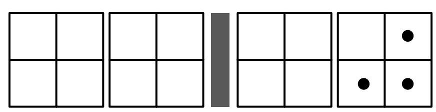
T: ***Yes, 1 = 1.0 = 1.00.***

Ask students to put on successively 0.1, 0.01, 0.35, and 0.07. Encourage students who have difficulty with this activity to think about money.

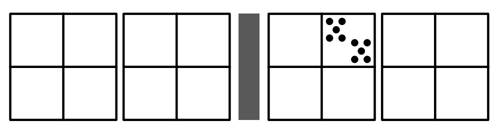

= 0.1


= 0.35


= 0.01


= 0.07

Put ten checkers on the 0.4-square.



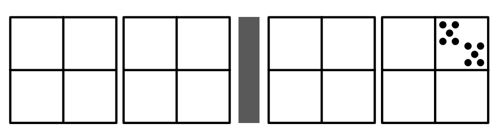
T: ***What number is this?***

S: ***4; 10 x 0.4 = 4.***

Invite students to make trades until one checker remains on the 4-square.

T: ***Ten checkers on the 0.4-square is the same as one checker on the 4-square.***

Put ten checkers on the 0.04-square.



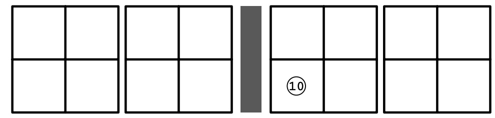
T: ***What number is this?***

S: ***0.4; 10 x 0.04 = 0.4.***

T: ***Can we make just one big trade?***

S: ***Ten checkers on the 0.04-square is the same as one checker on the 0.4-square.***

Put a ⑩-checker on the 0.2-square.

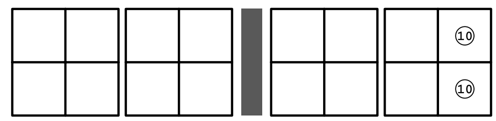


T: ***Rather than putting ten checkers on the Minicomputer, I used a ⑩-checker. What number is this?***

S: ***2; 10* x *0.2 = 2.***

S: ***Ten checkers on the 0.2-square is the same as one checker on the 2-square.***

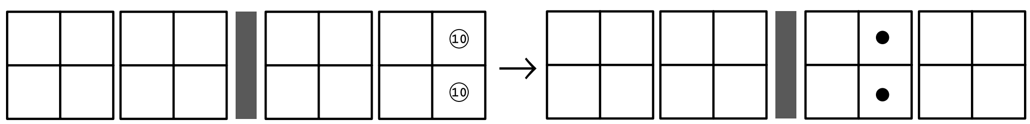
Next put 10 x 0.05 on the Minicomputer.



T: ***What number is this?***

S: ***0.5; 10* x *0.05 = 0.5.***

Ask a student to make the appropriate trades.



Repeat this activity with 10 x 0.25; 10 x 0.4; 10 x 27; 10 x 27.04.

Exercise 2

Write these problems on the board and ask students to do them on their papers. (Answers are in boxes.)

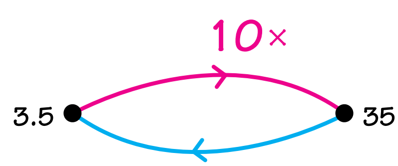
$10 \times 0.06 = \boxed{0.6}$	$10 \times 0.9 = \boxed{9.0}$	$10 \times 3.8 = \boxed{38}$
$10 \times 0.50 = \boxed{5.0}$	$10 \times 5.0 = \boxed{50}$	$10 \times 15.6 = \boxed{156}$
$10 \times 7.00 = \boxed{70}$	$10 \times 20.0 = \boxed{200}$	$10 \times 8.56 = \boxed{85.6}$
$10 \times 7.56 = \boxed{75.6}$	$10 \times 25.9 = \boxed{259}$	

Exercise 3

Draw this arrow on the board.

T: ***If 3.5 is here (at* a*)*** (label the dot)***, then what number is here (at* b*)?***

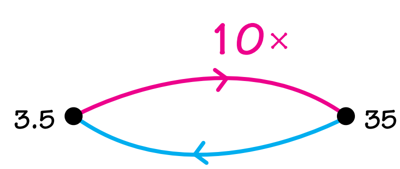
S: ***35; 10* x *3.5 = 35.***



Label the dot and draw a return arrow.

T: ***What could this blue arrow be for?***

S: ***÷10.***



Label the blue arrow.

Continue the activity using the values for **a** and **b** suggested here. (Answers are in boxes.)

a	b
3.5	$\boxed{35}$
$\boxed{2.17}$	21.7
0.56	$\boxed{5.6}$
$\boxed{0.4}$	4
31.06	$\boxed{310.6}$

If time and interest permit, invite a student to label either **a** or **b** and ask the class what number the other dot is for in the picture.

Exercise 4

Ask students to follow this exercise by drawing the arrow picture on their papers as it develops.

Display four Minicomputer boards.

T: ***What number is on the Minicomputer?***

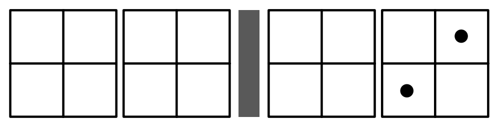
S: ***0.***

Draw a dot a label it 0.

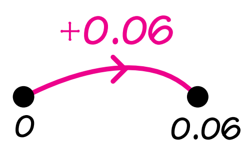
Put 0.06 on the Minicomputer.

T: ***What number is on the Minicomputer?***

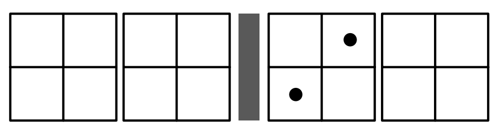
S: ***0.06.***



Record this information in an arrow.



Move the checkers on the Minicomputer to this configuration and add a blue arrow to your arrow picture.



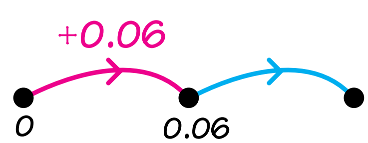
T: ***What number is on the Minicomputer?***

S: ***0.6.***

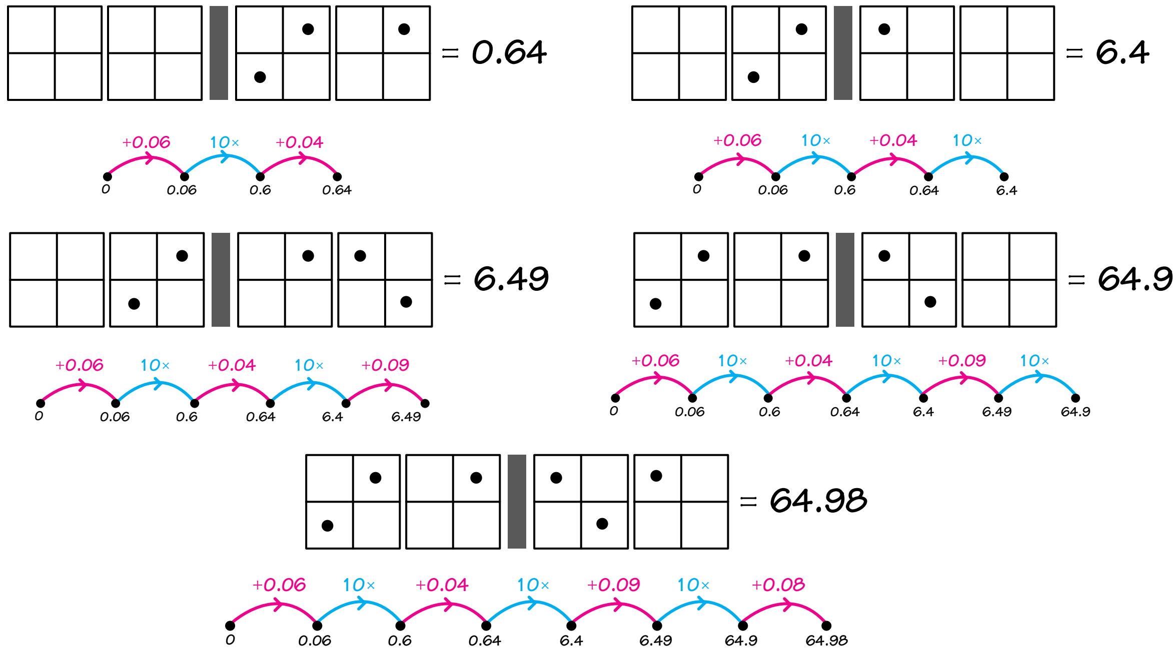
T: ***What could the blue arrow be for?***

S: ***10*x.**

T: ***Moving the checkers over a board (to the left) multiplies the number by 10.***



Continue in this manner, first manipulating checkers on the Minicomputer and then updating the arrow picture. Follow the sequence indicated below.



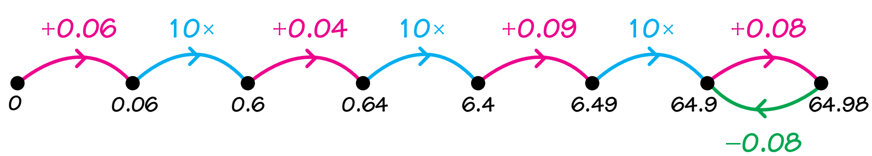
T: ***Now I would like to undo what we just did on the Minicomputer. What should I do?***

S: ***Take away 0.08.***

Remove the appropriate checkers from the Minicomputer.

T: ***How can we show this in the arrow picture?***

S: ***Draw a –0.08 return arrow.***



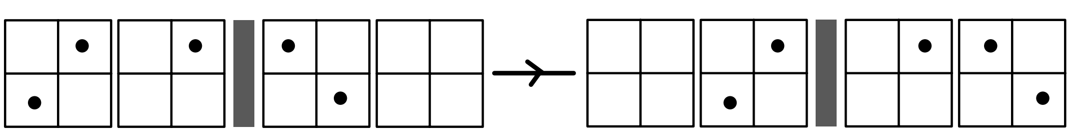
T: ***What should we do next?***

S: ***Divide by 10.***

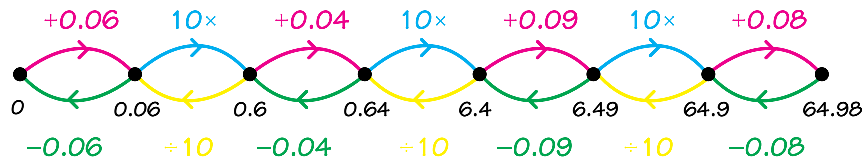
T: ***How can we divide by 10 on the Minicomputer?***

S: ***Move each checker from the square it's on to the same color square one board to the right.***

Invite a student to make the moves.

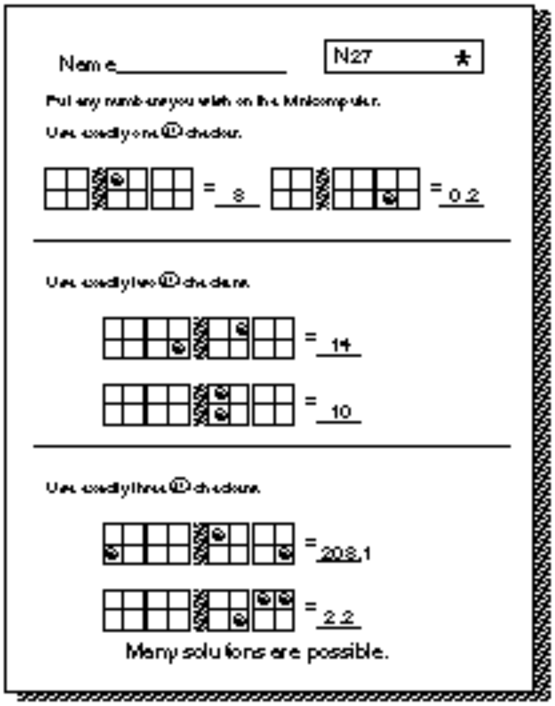


Draw a ÷10 return arrow in the picture to show the effect of this move. Continue in this manner until 0 is on the Minicomputer and your arrow picture looks like the one below.

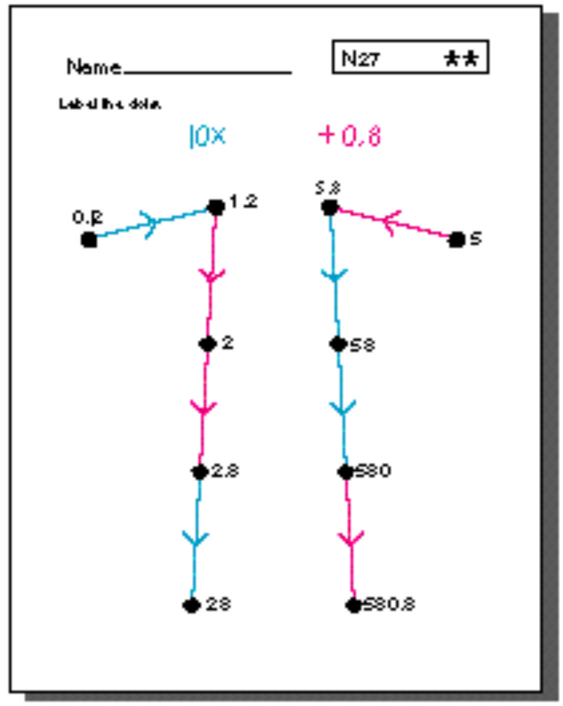


Worksheets N27*, **, ***, and **** are available for individual work.

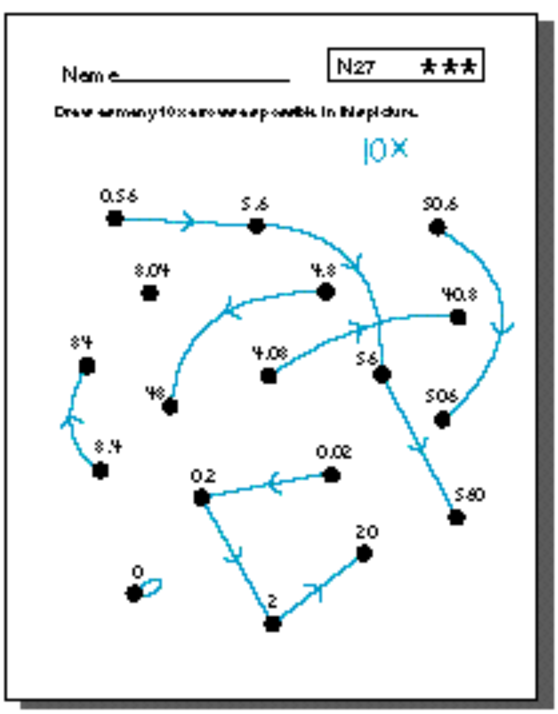
N27



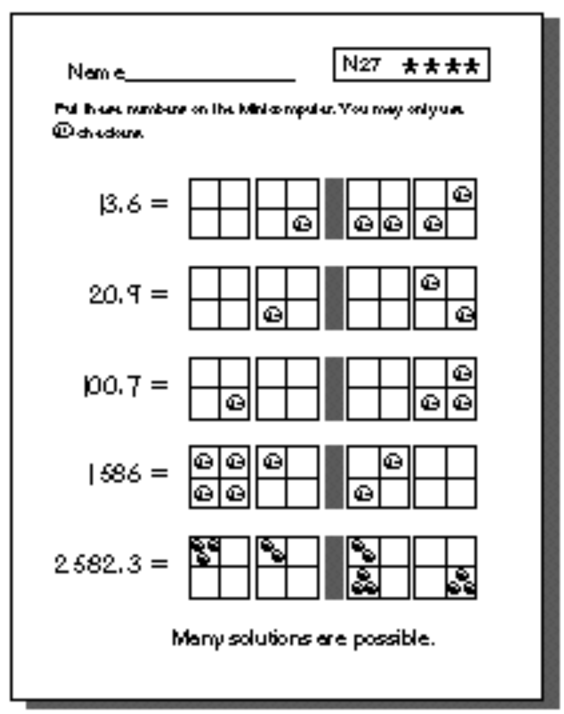
Name
N27 ★
Put any numbers you wish on the Minicomputer.
Use exactly one ⑩ checker.
= 8
= 0.2
Use exactly two ⑩ checkers.
= 14
= 10
Use exactly three ⑩ checkers.
= 208.1
= 2.2
Many solutions are possible.



Name
N27 ★★
Label the dots.
10x
+0.8
0.12
1.2
2
2.8
28
5.8
5
58
580
580.8



Name
N27 ★★★
Draw as many 10x arrows as possible in the picture.
10x
0.56
5.6
50.6
8.04
4.8
40.8
84
4.08
48
56
506
8.4
0.2
0.02
560
20
0
2



Name
N27 ★★★★
Put these numbers on the Minicomputer. You may only use ⑩ checkers.
13.6 =
20.9 =
100.7 =
1586 =
2582.3 =
Many solutions are possible.

N28 DIVISION #2

Capsule Lesson Summary

Present situations in which a given number of objects are to be packed into boxes, each box holding a specified number of objects. Use arrow roads to calculate the number of boxes to be filled.

Materials

Teacher
- Colored chalk

Student
- Paper
- Colored pencils, pens, or crayons
- Worksheets N28*, **, and ***

Advance Preparation: Draw the arrow picture for Exercise 1 on the board before starting the lesson.

Description of Lesson

Exercise 1

Tell the following or a similar story to the class.

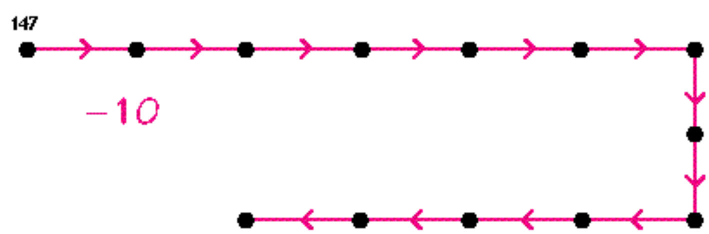
T: ***A young man, Nabu, works in a pencil factory. A machine there makes pencils, and Nabu's job is to pack the pencils into boxes. He puts ten pencils into a box.***

Nabu has 147 pencils to put into boxes of ten each. He must ask his supervisor for the exact number of boxes that he needs. How many boxes do you think he needs?

Accept students' estimates.

T: ***Nabu uses an arrow road to find how many boxes he needs.***

Refer to this arrow picture on the board.

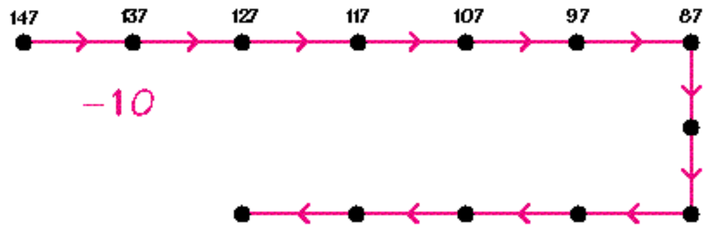


Trace the –10 arrows starting at 147 as you explain,

T: ***When Nabu fills the first box, he will use ten pencils so he will have how many pencils left?*** (137)
When he fills a second box he will use ten more pencils (trace the second arrow). ***How many pencils will he have left?***

S: ***127 pencils.***

Invite students to label dots until the road reaches 87.



T (pointing to 87)**:** ***So far, how many boxes does Nabu need?***

S: ***6 boxes; there are 6 arrows from 147 to 87.***

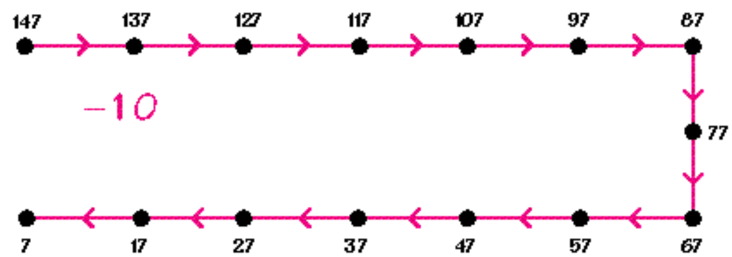
T: ***How many pencils will Nabu put in 6 boxes?***

S: ***60 pencils; 6*** x ***10 = 60.***

T: ***How many pencils will be left?***

S: ***87 pencils.***

Invite students to continue labeling dots. Add dots and red arrows until the arrow road reaches 7. Observe that Nabu cannot fill a box with only 7 pencils.



T: ***So how many boxes will Nabu need to pack 147 pencils?***

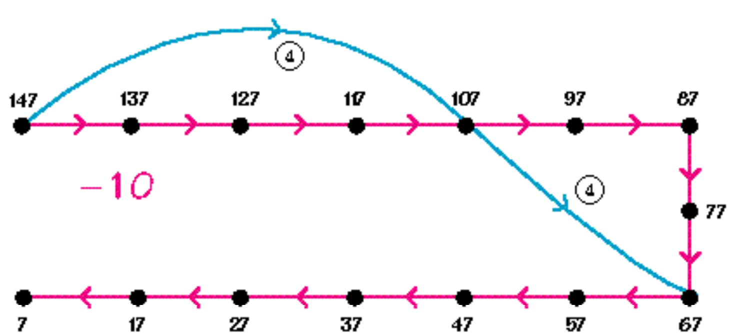
S: ***14 boxes. There are 14 arrows in the arrow road. There will be 7 pencils left over.***

T: ***Nabu will save the extra pencils until he can fill another box.***

Compare the solution, 14 boxes, to the students' estimates.

T: ***You counted the arrows one at a time. Nabu knows that the arrows can also be counted several at a time.***

Draw blue arrows as shown below and write ④ near each blue arrow.



T: ***For each blue arrow, Nabu needs four boxes. What could the blue arrows be for?***

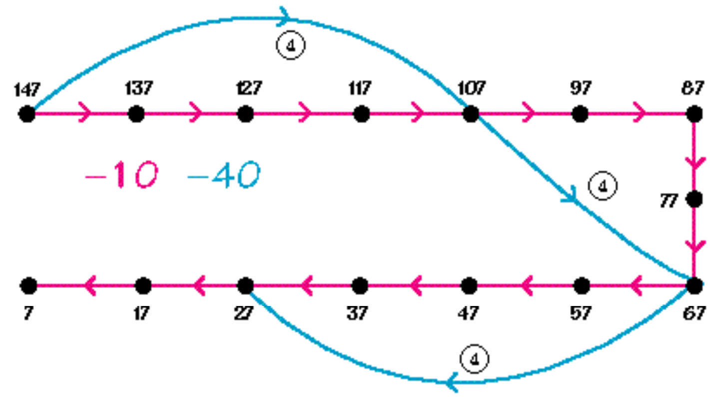
S: ***–40, because there are 40 pencils in four boxes.***

S: ***–40, because 147 – 40 = 107 and 107 – 40 = 67.***

T: ***Can we draw any more blue arrows from here (starting at 67)?***

S: ***From 67 to 27.***

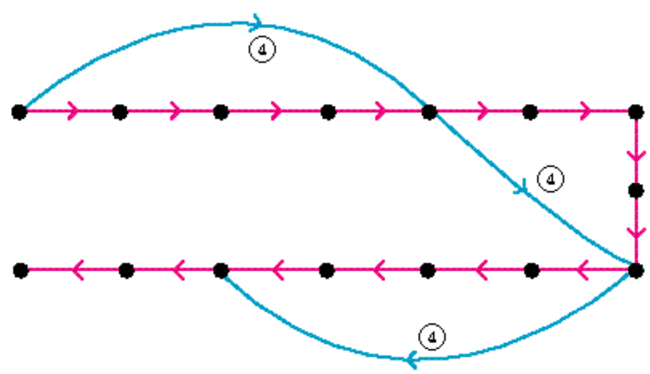
Add that blue arrow to the arrow road.



T: ***Let's use the blue arrows to help count the number of boxes Nabu needs.*** (Trace the blue arrows as you count) ***4, 8, 12.*** (Trace the last two red arrows) ***13, 14. Yes, Nabu needs 14 boxes.***

Note: Of course there are other blue arrows that could be drawn in the picture. If necessary, indicate that you only want the blue arrows that follow one another in a road starting at 147.

Erase the numbers and the labels for the arrows from the arrow picture. Do not erase the arrows.



T: ***The company Nabu works for has been selling boxes of 10 pencils for $1.00 each. But recently the cost of making the pencils has increased. What can the company do?***

S: ***Raise the price of a box of pencils.***

S: ***Put fewer pencils in a box.***

T: ***The company decides to put nine pencils in each box. Nabu once again has 147 pencils to pack, but now he has to put them into boxes of nine each. About how many boxes does he need?***

Accept students' estimates.

T: ***Does he need more or fewer than 14 boxes (the number he needed to put 147 pencils into boxes of ten each)?***

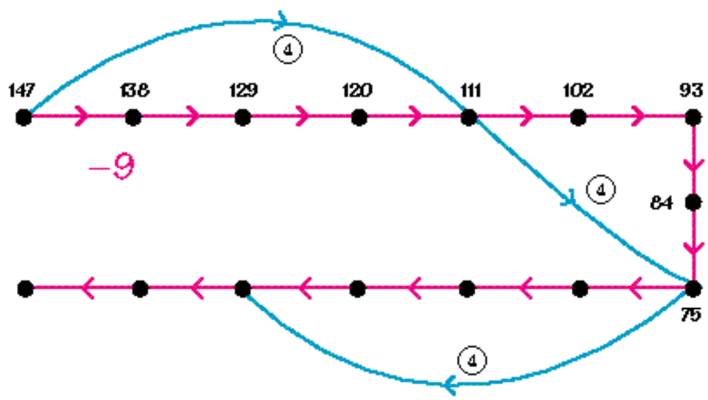
S: ***More. By putting fewer pencils in each box, Nabu will need more boxes in order to pack the 147 pencils.***

Label the first dot of the arrow road 147.

T: ***Let's use the arrow road to calculate how many boxes Nabu will need. What should the red arrow be for?***

S: ***–9; there are 9 pencils in each box.***

Write –9 in red on the board. Invite students to label the dots on the arrow road. Stop when the road reaches 75.



T (pointing to 75)**:** ***So far, how many boxes will Nabu need?***

S: ***8 boxes; there are 8 red arrows from 147 to 75.***

S: ***8 boxes; there are 2 blue arrows from 147 to 75 and 2 x 4 = 8.***

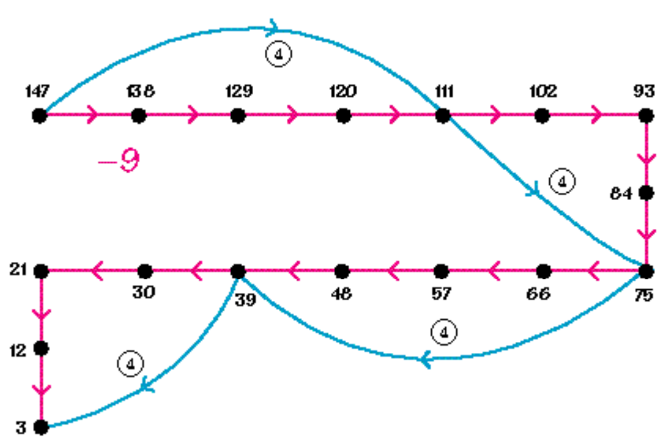
T: ***How many pencils will fill 8 boxes?***

S: ***72 pencils; 8 x 9 = 72.***

T: ***How many pencils will Nabu still need to put into boxes?***

S: ***75 pencils.***

Invite students to label the dots. Encourage students to explain any methods they use to subtract 9. Extend the arrow road until it reaches 3. Continue the blue arrow road.



T: ***How many boxes will Nabu need this time?***

S: ***16 boxes; he will need two more than last time.***

S: ***16 boxes; I counted the red arrows.***

S: ***16 boxes; I used the blue arrows and counted four for each.***

T: ***How many pencils will be left over?***

S: ***Three pencils.***

T: ***What could a blue arrow be for?***

S: ***–36; there are 36 pencils in 4 boxes.***

S: ***–36; four –9 arrows are the same as one –36 arrow.***

Write –36 in blue on the board. Draw a green arrow from 147 to 57.

T: ***What could this green arrow be for?***

S: ***–90; the green arrow is the same as ten red arrows and 9 x 10 = 90.***

T: ***Using the green arrow, Nabu could count the boxes ten at a time instead of three at a time.***

Exercise 2

T: ***Nabu decides to change jobs, and he goes to a bottle recycling factory. He hopes for a different kind of job but ends up with almost the same work. What do you think Nabu does now?***

S: ***He puts bottles into cartons.***

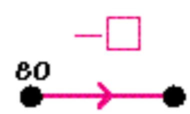
T: ***Yes, that's right. His first job is to put 80 bottles into 6-packs. About how many cartons will he need?***

Accept students' estimates.

T: ***How can Nabu calculate the number of cartons he needs?***

S: ***He could use an arrow road.***

Begin an arrow road on the board.



T: ***What could the red arrow be for?***

S: ***–6.***

T: ***It could be for –6, but Nabu realizes he would have to draw a long arrow road. What else could the red arrow be for that might be more efficient?***

A sample dialogue follows.

S: ***–18; each red arrow would show Nabu needs 3 cartons.***

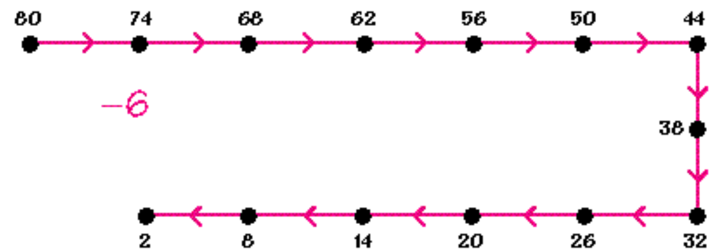
S: ***–60; each red arrow would show Nabu needs 10 cartons.***

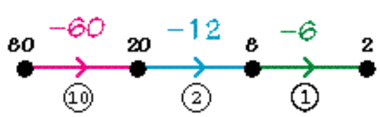
Accept other suggestions for the red arrow, for example, –12, –24, or –42.

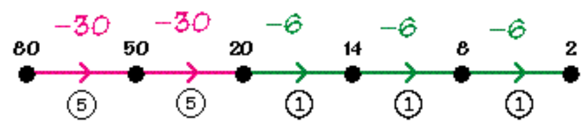
T: ***On your paper draw an arrow road to determine how many cartons Nabu will need for 80 bottles. You may use any arrows you wish, such as, –6, –18, –60. Next to an arrow indicate how many cartons.***

Observe students' individual work. Some students may use only –6 arrows. Accept that approach, but encourage them to try other arrows. Ask students who finish quickly to determine how many cartons are needed to put 320 bottles into 12-packs.

Invite students with different solutions to draw them on the board. For example:







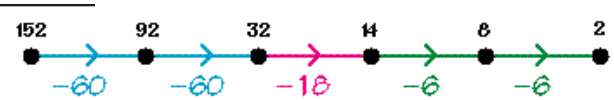
T: ***How many cartons will Nabu need?***

S: ***13 cartons; there will be 2 bottles left over.***

Use each arrow road on the board to confirm that 13 cartons are required.

Exercise 3

Draw this arrow picture on the board.



T: ***One day Nabu needs to put 152 bottles into 6-packs. He draws this arrow road to help him calculate the number of cartons he will need.***

How many cartons will Nabu need?

S: ***25 cartons.***

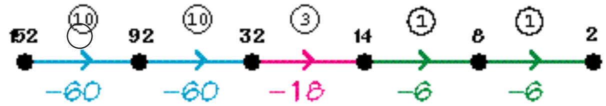
T: ***Why?***

S: ***Each blue arrow shows Nabu fills ten cartons.***

Write ⑩ near each blue arrow.

S: ***The –18 arrow represents filling three cartons. Each –6 arrow represents filling one carton.***

Write ③ near the red arrow and 1 near each green arrow.



S: ***25 or 10 + 10 + 3 + 1 + 1 cartons are needed altogether, and there will be two extra bottles.***

Distribute copies of Worksheet N28*. Ask a student to read the problem aloud.

T: ***What could the red arrow be for?***

A sample dialogue follows.

S: ***–8; there are eight bottles in each carton.***

S: ***–24; each arrow could represent filling three cartons since 3*** x ***8 = 24.***

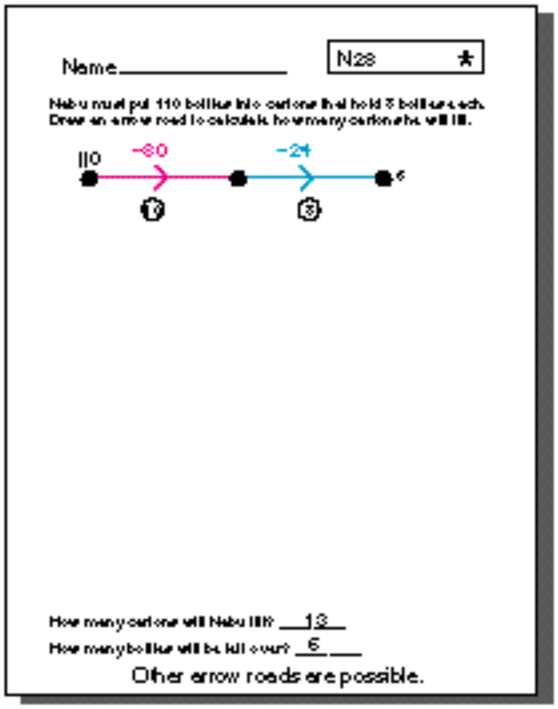
S: ***–80; each arrow could represent filling ten cartons.***

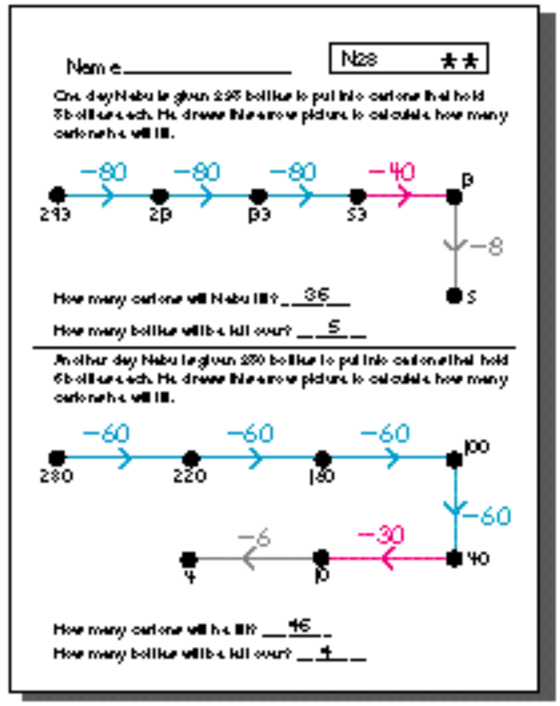
T: ***Use whichever arrows you wish to solve this problem. There are many possibilities.***

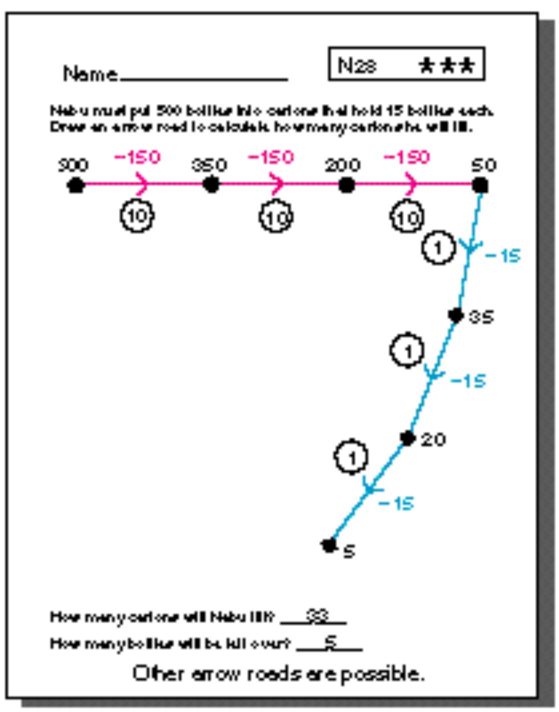
Let the students begin building an arrow road. Worksheets N28** and *** are also available for individual work.

Home Activity

Suggest that students show their parents/guardians how to use an arrow road to solve a division problem.


Name
N28 ★
Nabu must put 110 bottles into cartons that hold 8 bottles each.
Draw an arrow road to calculate how many cartons he will fill.
110
−80
−24
6
10
3
How many cartons will Nabu fill? 13
How many bottles will be left over? 6
Other arrow roads are possible.


Name
N28 ★★
One day Nabu is given 293 bottles to put into cartons that hold 8 bottles each. He draws this arrow picture to calculate how many cartons he will fill.
−80
−80
−80
−40
293
213
133
53
13
−8
5
How many cartons will Nabu fill? 36
How many bottles will be left over? 5
Another day Nabu is given 280 bottles to put into cartons that hold 6 bottles each. He draws this arrow picture to calculate how many cartons he will fill.
−60
−60
−60
280
220
160
100
−60
−30
−6
4
10
40
How many cartons will he fill? 46
How many bottles will be left over? 4


Name
N28 ★★★
Nabu must put 500 bottles into cartons that hold 15 bottles each.
Draw an arrow road to calculate how many cartons he will fill.
500
−150
350
−150
200
−150
50
10
10
10
1
−15
35
1
−15
20
1
−15
5
How many cartons will Nabu fill? 33
How many bottles will be left over? 5
Other arrow roads are possible.

N29 TAG THE ARROWS #2

Capsule Lesson Summary

Match arrow roads, involving one function each, with arrow labels. Find possible arrow labels for some roads involving one function each, and for an arrow road with three arrows, two for one function and one for another function.

Materials

Teacher
- Colored chalk
- Index cards (optional)

Student
- Paper
- Colored pencils, pens, or crayons
- Worksheets N29*, **, ***, and ****

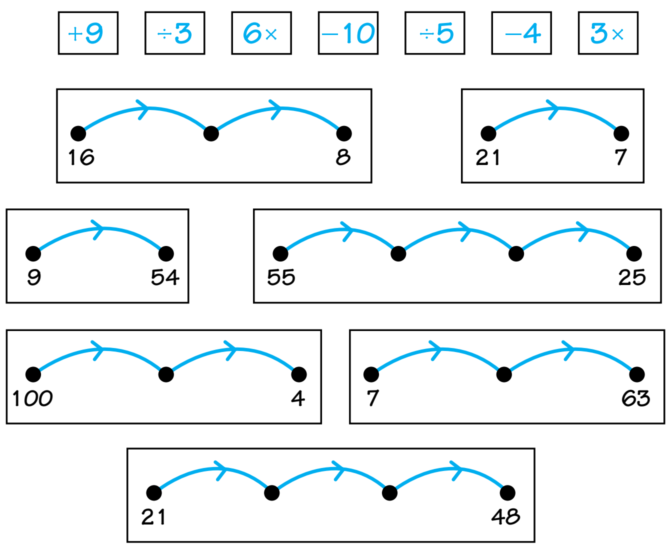
Advance Preparation: For Exercise 1, you may like to prepare the tags (possible arrow labels) on index cards so that they can be moved around. Prepare the arrow pictures on the board before the lesson.

Description of Lesson

Note: Throughout this lesson, make sure students have paper for doing and recording calculations.

Exercise 1

Display these arrow roads and tags on the board.



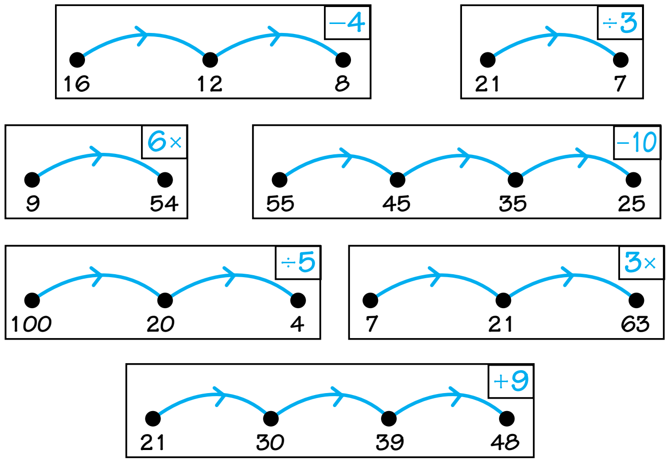
Collectively match the tags with the arrow pictures and finish labeling the dots. Often when considering an arrow road, trace it and ask, "Are the numbers increasing or decreasing?"

Since these pictures and tags only involve whole numbers, observe that

- arrow labels that divide or subtract match arrow roads with decreasing numbers; and
- arrow labels that multiply or add match arrow roads with increasing numbers.

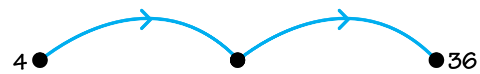
N29

The correct matching of tags to arrow roads is shown below.



Exercise 2

Draw this arrow road on the board.

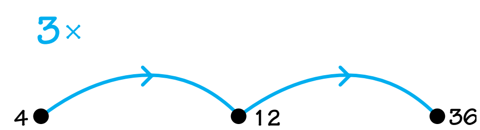


T: ***What could these blue arrows be for?***

Allow a few minutes; the problem is a little more difficult than Exercise 1 since there are no possibilities given for students to try. Suppose a student suggests 3x.

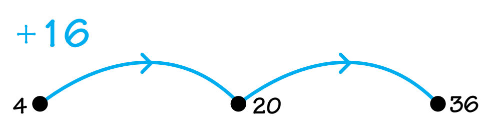
T: ***Let's check. What number is 3* x *4?*** (12)

Does 3* x *12 = 36? (Yes)



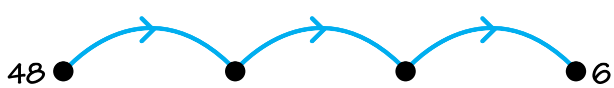
Erase 12 and the arrow label. Ask for other possibilities.

S: ***The blue arrow could be for +16.***

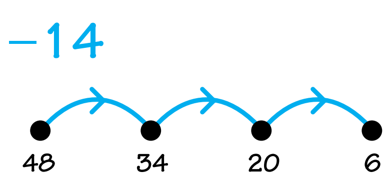


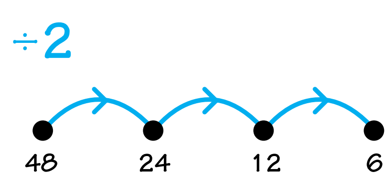
Check the solution as before.

Repeat the exercise with this arrow road.



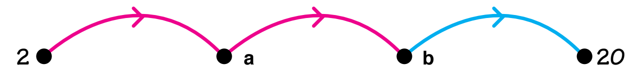
Most likely your students will find that the blue arrows could be for –14 or for ÷2.





Exercise 3

Draw this picture on the board.



→	→

Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: ***What could the red arrows be for? You are completely free to label them in any way you wish.***

Suppose that a student suggests +2. Write +2 in red in the first line of the chart.

T: ***If the red arrows were for +2, what numbers would these dots*** (point to **a** and **b**) ***be for?***

S: ***4 (at a) and 6 (at b).***

Label the dots.

T (tracing the blue arrow)**:** ***What could the blue arrow be for?***

S: ***+14.***

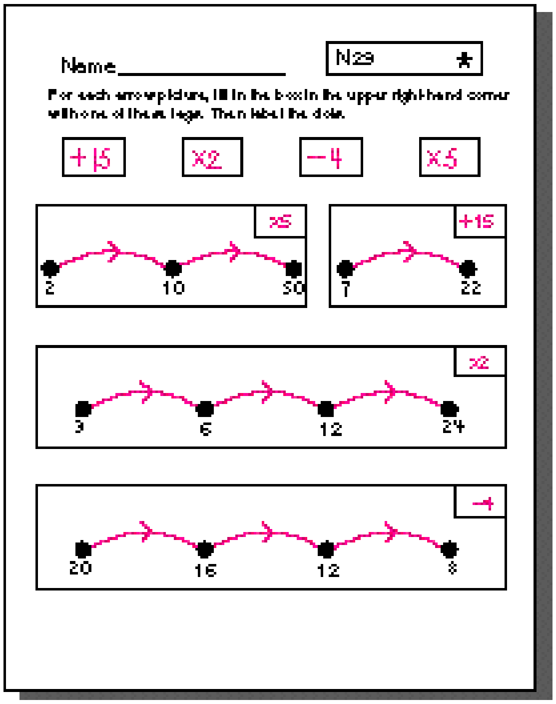
Write +14 in blue in the first line of the chart.

Collectively find several more entries for the chart. Encourage the class to find solutions that assign negative numbers (or even decimal numbers) to **a** and **b**.

Occasionally select a possibility for the red arrow yourself (for example, 2x or +10) and ask students to find the corresponding blue arrow label. Other times, select a possibility for the blue arrow (for example, +10 or –22) and ask students to find the corresponding red arrow label.

→	→
+2	+14
3×	+2
5×	−30
2×	+12
+10	−2
+4	+10
+20	−22
÷2	+19.5
−2	+22
+19	÷2
+50	−82
−10	+38

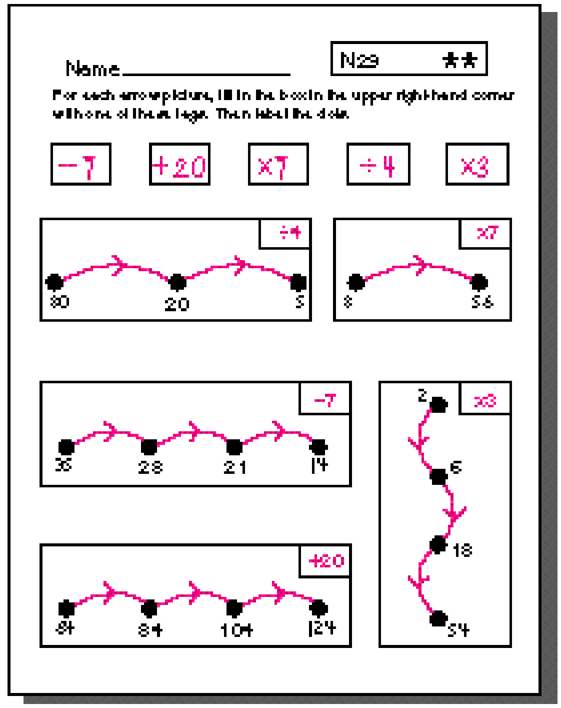
Worksheets N29*, **, ***, and **** are available for individual work.



Name____________ N29 ★

For each arrow picture, fill in the box in the upper right-hand corner with one of these tags. Then label the dots.

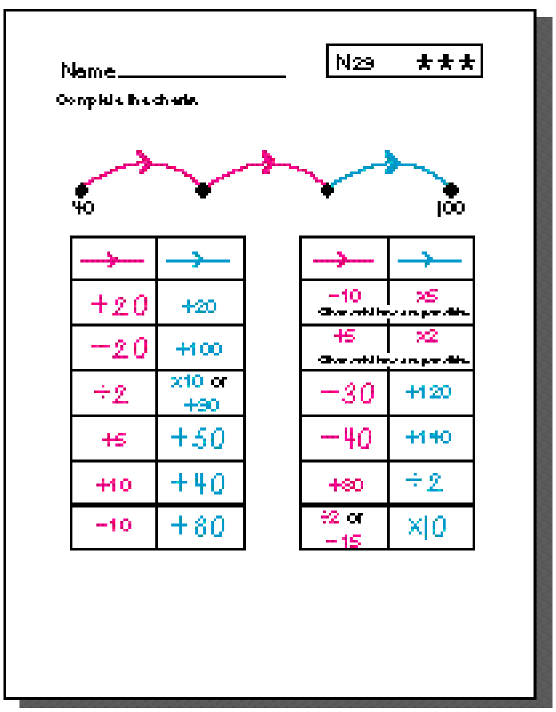
+15 x2 −4 x5



Name____________ N29 ★★

For each arrow picture, fill in the box in the upper right-hand corner with one of these tags. Then label the dots.

−7 +20 x7 ÷4 x3

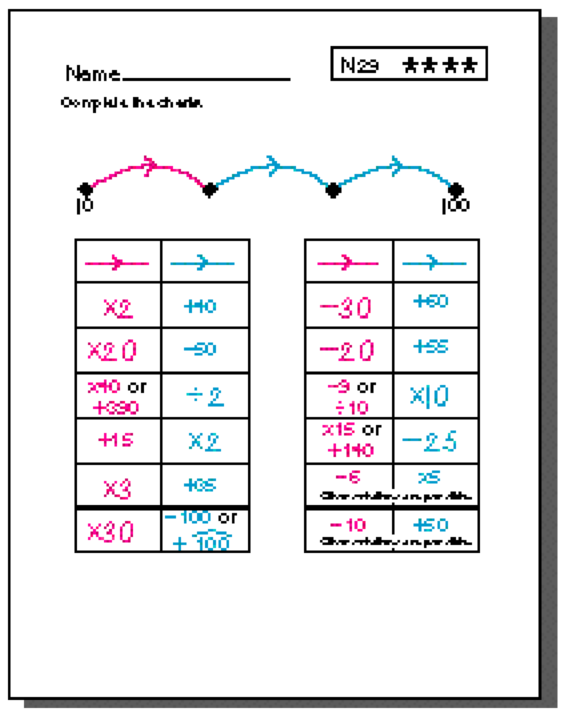


Name____________ N29 ★★★

Complete the charts.

→	→
+20	+20
−20	+100
÷2	x10 or +90
+5	+50
+10	+40
−10	+80

→	→
−10	x5
Other solutions are possible.	
+5	x2
Other solutions are possible.	
−30	+120
−40	+140
+80	÷2
÷2 or −15	x10



Name____________ N29 ★★★★

Complete the charts.

→	→
x2	+40
x20	−50
x40 or +390	÷2
+15	x2
x3	+65
x30	−100 or +100

→	→
−30	+60
−20	+55
−9 or ÷10	x10
x15 or +140	−25
−6	x5
Other solutions are possible.	
−10	+50
Other solutions are possible.	

Capsule Lesson Summary

Review the functions $\frac{1}{2}$x, $\frac{1}{3}$x, and $\frac{1}{5}$x. Determine how to share a cake fairly among several children. Use the cake model to order fractions. Tell a story about a zookeeper who feeds bananas to the monkeys. Use this model to solve problems such as $\frac{3}{4}$ of 28.

Materials

Teacher
- Colored chalk
- Metric stick

Student
- Metric ruler
- Colored pencils, pens, or crayons
- Worksheets N30*, **, ***, and ****

Description of Lesson

Exercise 1: Mental Arithmetic

Present a short mental arithmetic activity involving the functions $\frac{1}{2}$x, $\frac{1}{3}$x, and $\frac{1}{5}$x. Adjust the difficulty of the problems to the abilities of your students.

Note: The most natural way to read these functions is, for example, "one-half of 16" for "$\frac{1}{2}$ x 16."

$\frac{1}{2}$ x 16 (8)	$\frac{1}{2}$ x 12 (6)	$\frac{1}{2}$ x 36 (18)	$\frac{1}{2}$ x 114 (57)
$\frac{1}{3}$ x 12 (4)	$\frac{1}{3}$ x 15 (5)	$\frac{1}{3}$ x 36 (12)	$\frac{1}{3}$ x 105 (35)
$\frac{1}{5}$ x 30 (6)	$\frac{1}{5}$ x 15 (3)	$\frac{1}{5}$ x 70 (14)	$\frac{1}{5}$ x 115 (23)

During this activity, occasionally ask a student to explain an answer. For example:

S: ***$\frac{1}{3}$ x 12 = 4.***

T: ***Why?***

S: ***12 ÷ 3 = 4.***

S: ***3 x 4 = 12.***

Continue with sequences of related calculations. You may wish to write a sequence on the board to emphasize the pattern.

$\frac{1}{3}$ x 18 (6)	$\frac{1}{3}$ x 27 (9)	$\frac{1}{3}$ x 150 (50)	$\frac{1}{6}$ x 4 200 (700)
$\frac{1}{3}$ x 180 (60)	$\frac{1}{3}$ x 270 (90)	$\frac{1}{3}$ x 1 500 (500)	$\frac{1}{6}$ x 420 (70)
$\frac{1}{3}$ x 1 800 (600)	$\frac{1}{3}$ x 2 700 (900)	$\frac{1}{3}$ x 15 (5)	$\frac{1}{6}$ x 42 000 (7 000)

Exercise 2: Sharing a Cake

Draw a large rectangle on the board. Arrange that the measurements are multiples of 10 cm, for example, 40 cm by 60 cm.

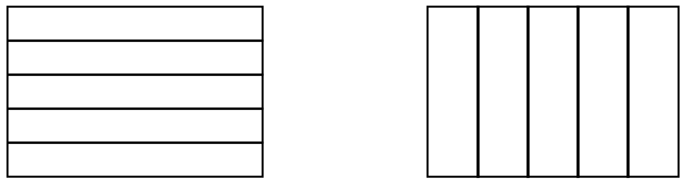
T: ***Pretend that this is an enlarged picture of a cake. There are five children who are going to share the cake fairly.***

N30

Ask your class to give you names for five children, or select names yourself. In this lesson description the five children are Amy, Brian, May, Joyce, and Ramon.

T: ***How could the children cut the cake so that they each get the same size piece?***

Accept any correct slicings, such as the following. Let students measure and make fairly accurate divisions.

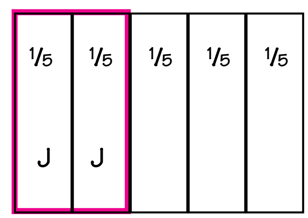


In the following dialogue, the illustrations use vertical cuts, but of course you could use horizontal cuts as well. Adapt the discussion if a horizontal slicing is used.

T: ***What fraction of the cake does each child receive?***

S: ***$\frac{1}{5}$, since each child gets one of the five equal-sized pieces.***

Label the pieces $\frac{1}{5}$.



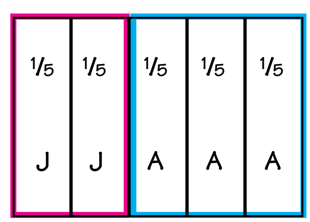
T: ***Brian is on a diet. So he gives his piece of cake to Joyce. What fraction of the cake does Joyce receive?***

S: ***$\frac{2}{5}$. She gets two of the five pieces.***

Invite a student to outline in red a section of the cake for Joyce, and record this information on the board.

Joyce: $\frac{1}{5} + \frac{1}{5} = \frac{2}{5}$

T: ***Joyce receives her $\frac{1}{5}$ and Brian's $\frac{1}{5}$. $\frac{1}{5} + \frac{1}{5} = \frac{2}{5}$.***



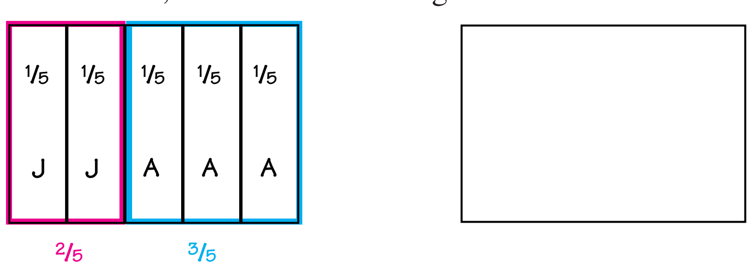
While May and Ramon are talking, Amy eats both of their pieces of cake as well as her own. What fraction of the cake does she eat?

S: ***$\frac{3}{5}$. She eats three of the five pieces.***

Invite a student to outline in blue the cake eaten by Amy, and record this information on the board.

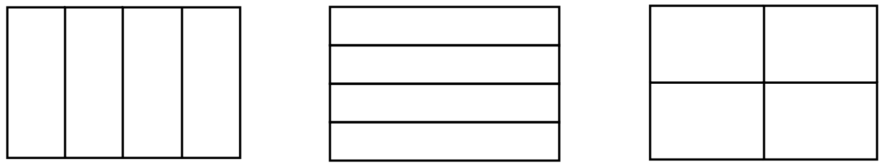
Joyce: $\frac{1}{5} + \frac{1}{5} = \frac{2}{5}$
Amy: $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5}$

Next to the rectangle on the board, draw another rectangle of the same size.



T: ***The next day the children get another cake of the same size. However, Amy is not with them. How could they cut the cake into four pieces of the same size?***

Accept any correct slicings, such as the following. Again, let students measure and make fairly accurate divisions.

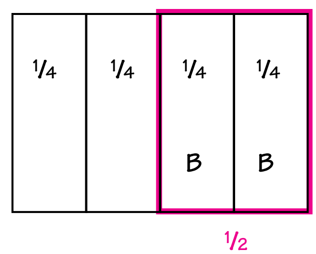


In this dialogue, the illustrations again are with vertical cuts. Adapt the discussion if a different slicing is used.

T: ***What fraction of the cake does each child receive?***

S: ***$\frac{1}{4}$. Each gets one of the four pieces.***

Label each piece $\frac{1}{4}$.

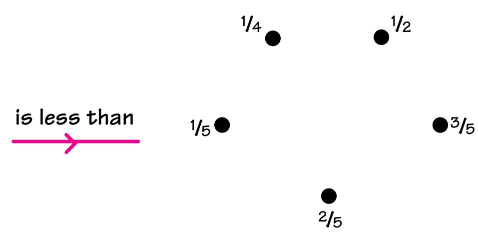


T: ***Brian decides to stop his diet, and this time Joyce gives him her piece of cake. What fraction of the cake does he receive?***

S: ***$\frac{2}{4}$. He gets two of the four pieces.***

Observe that $\frac{2}{4}$ is the same as $\frac{1}{2}$.

Keep both rectangles on the board and draw this picture nearby.



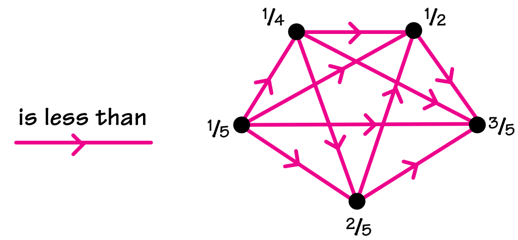
T: ***Where can we draw arrows for "is less than" in this picture? Give a reason for each arrow you suggest.***

Encourage the students to use the cake pictures to justify arrows. For example:

S: ***Draw an arrow from $\frac{1}{5}$ to $\frac{2}{5}$. One piece of a cake divided into five pieces is smaller than two pieces of the same cake.***

S: ***Draw an arrow from $\frac{1}{5}$ to $\frac{1}{4}$. One piece from a cake shared fairly among five children is smaller than one piece from a cake of the same size shared fairly among only four children.***

It may be difficult for students to explain why an arrow should be drawn from $\frac{1}{4}$ to $\frac{2}{5}$. They may be able to determine this from carefully divided rectangles. Here is a possible explanation: "Suppose $\frac{1}{4}$ and $\frac{2}{5}$ of the two cakes of the same size are eaten. Then $\frac{3}{4}$ and $\frac{3}{5}$ of the cakes remain. Three pieces of the cake cut into four pieces is more cake than three pieces of the cake cut into five pieces; $\frac{3}{4}$ is larger than $\frac{3}{5}$. Therefore, less of the cake divided into four parts has been eaten. So $\frac{1}{4}$ is less than $\frac{2}{5}$."



Your class should find all ten arrows.

Erase the board before proceeding to Exercise 3.

Exercise 3: Feeding the Monkeys

Tell the class to imagine they are going to the zoo. You may like to talk briefly about what animals they like to visit at the zoo. Direct their attention to monkeys, and ask what monkeys like to eat (bananas). Then tell them about a zookeeper who feeds the monkeys.

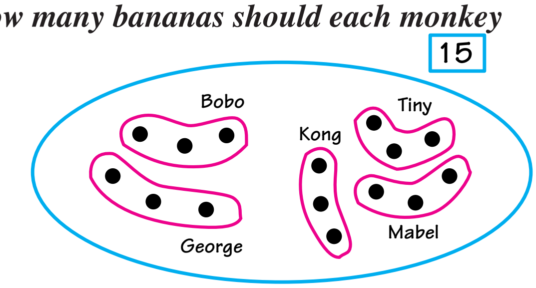
T: ***There are five monkeys at this zoo.***

Solicit names for five monkeys. Here the monkeys are Bobo, George, Mabel, Kong, and Tiny.

Draw a string with 15 dots inside as you explain,

T: ***The zookeeper has 15 bananas and wants to give the same share to each of the five monkeys. These 15 dots represent the bananas. How many bananas should each monkey receive?***

S: ***Three bananas, since 5 x 3 = 15.***



$\frac{1}{5}$ of 15 = 3

Let a student demonstrate dividing 15 into five groups of three. Give each monkey its share.

T: ***Yes, the zookeeper gives three bananas to each of the five monkeys. What fraction of the bananas does each monkey receive?***

S: ***$\frac{1}{5}$, since there are five monkeys.***

Write this number sentence on the board as you ask,

T: ***What is $\frac{1}{5}$ of 15?***

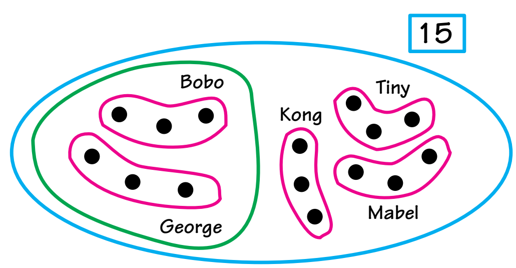
S: ***3; each monkey receives three bananas.***

T: ***George is sick, so Bobo eats both George's share and his own share. What fraction of the bananas does Bobo eat?***

S: ***$\frac{2}{5}$, since Bobo eats two of the five shares.***

T: ***Bobo eats his own share and George's share. $\frac{1}{5} + \frac{1}{5} = \frac{2}{5}$. What is $\frac{2}{5}$ of 15?***

S: ***6, since 3 + 3 = 6. Bobo eats six bananas.***



Bobo: $\frac{1}{5} + \frac{1}{5} = \frac{2}{5}$

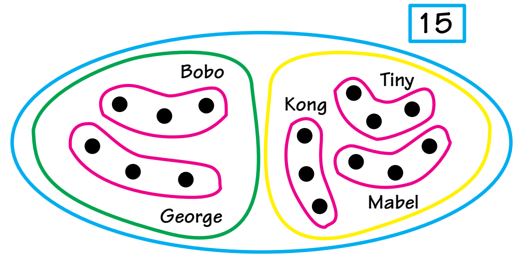
$\frac{2}{5}$ of 15 = 6

T: ***Kong is a bully and eats the rest of the bananas. What fraction of the bananas does Kong eat?***

S: ***³⁄₅, since Kong eats three of the five shares.***

T: ***Kong eats three shares. $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5}$. What is $\frac{3}{5}$ of 15?***

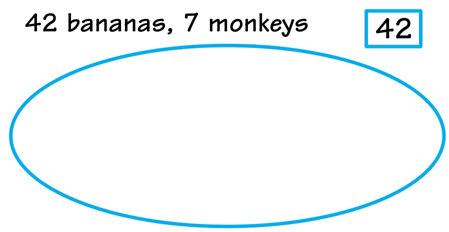
S: ***9, since 3 + 3 + 3 = 9 (or 3 x 3 = 9). Kong eats nine bananas.***



Kong: $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5}$

$\frac{3}{5}$ of 15 = 9

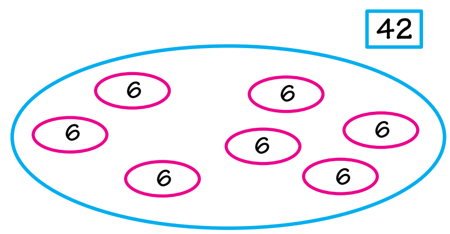
Erase the board and then draw this picture.



T: ***Another day the zookeeper has 42 bananas to give to seven monkeys. I won't draw dots this time; I'll record just numbers. How many red strings should I draw?***

S: ***Seven, because there are seven monkeys.***

Draw seven small red strings inside the blue string.



T: ***How many bananas does each monkey receive?***

S: ***Six bananas, since 7 x 6 = 42.***

Write 6 in each red string.

T: ***What fraction of the bananas does each monkey receive?***

S: ***$\frac{1}{7}$; each monkey receives one of the seven shares.***

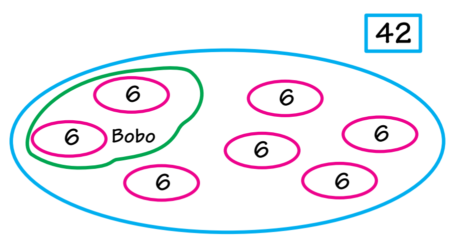
T: ***What is $\frac{1}{7}$ of 42?***

S: ***6; each monkey gets six bananas.***

$\frac{1}{7}$ of 42 = 6

T: ***Bobo takes two shares of the bananas. What fraction of the bananas does Bobo eat?***

S: ***$\frac{2}{7}$, since Bobo eats two of the seven shares.***



T: ***How many bananas does Bobo eat?***

S: ***Twelve bananas.***

T: ***What is $\frac{2}{7}$ of 42?***

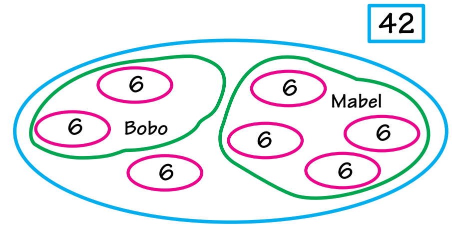
S: ***12.***

$\frac{2}{7}$ of 42 = 12

Repeat this series of questions, this time with Mabel taking four shares.

N30

Point to the number sentence $\frac{4}{7}$ of 42 = 24.



$\frac{1}{7}$ of 42 = 6
$\frac{2}{7}$ of 42 = 12
$\frac{4}{7}$ of 42 = 24

T: ***In terms of our story, what does each number in this number sentence tell us? What does the 42 tell us?***

S: ***There are 42 bananas.***

T: ***What does the 7 tell us?***

S: ***There are seven monkeys.***

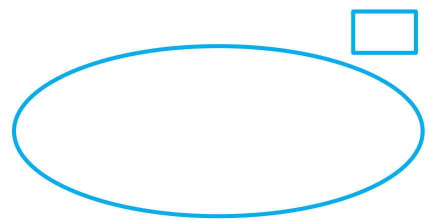
T: ***What does the 4 tell us?***

S: ***One monkey, Mabel, gets four shares.***

T: ***What does the 24 tell us?***

S: ***Mabel takes 24 bananas.***

Erase the board and then draw this picture.

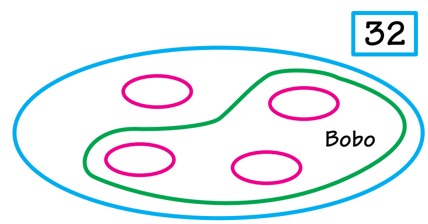


$\frac{3}{4}$ of 32 =____

T: ***In a "monkeys and bananas" story, what do these numbers tell us?***

S: ***There are 32 bananas and four monkeys. One monkey gets three shares.***

Add this information to the picture.



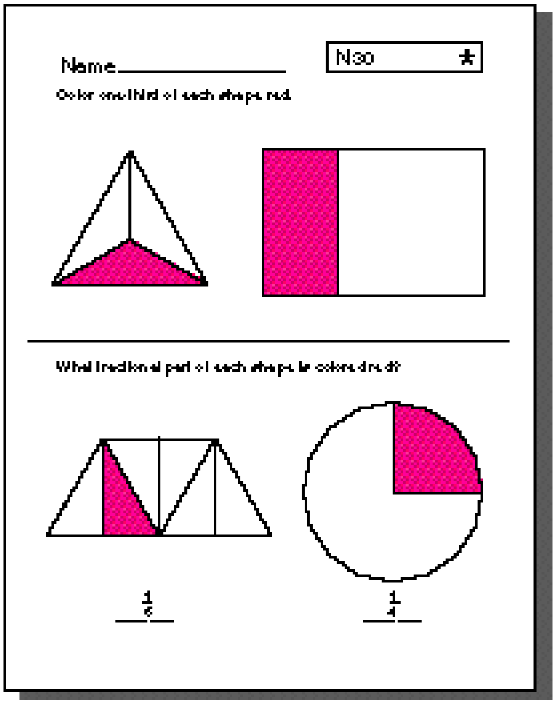
$\frac{3}{4}$ of 32 =____

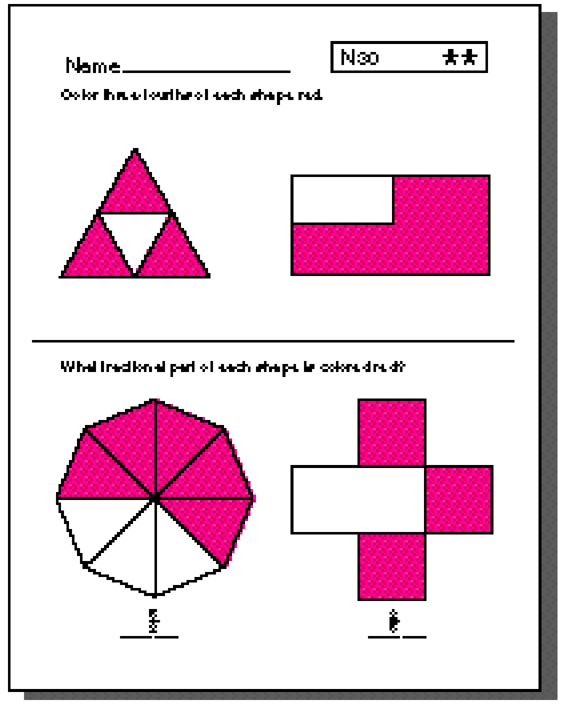
T: ***What is $\frac{3}{4}$ of 32?***

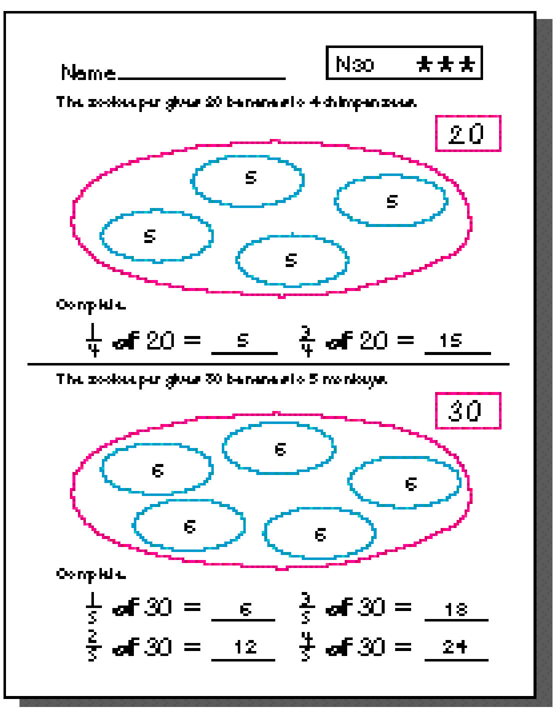
S: ***24. Each monkey should get eight bananas, but Bobo eats three shares. 3 x 8 = 24 (or 8 + 8 + 8 = 24).***

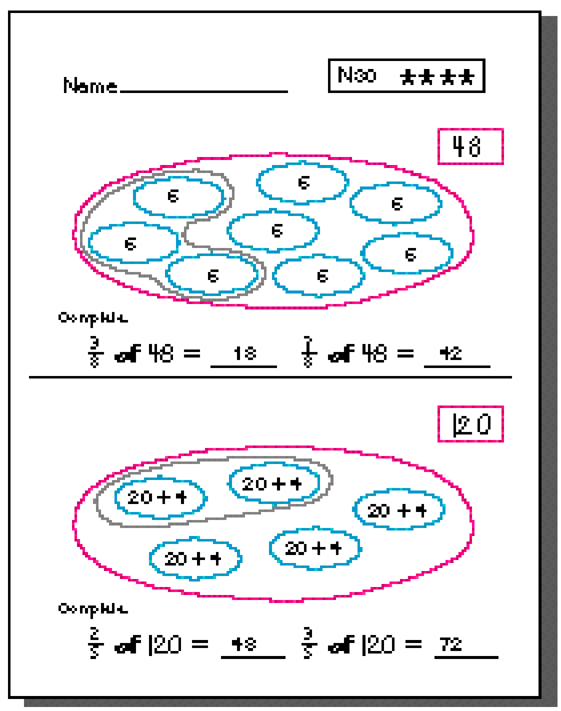
Write 8 in each red string, and complete the number sentence $\frac{3}{4}$ of 32 = 24.

Worksheets N30*, **, ***, and **** are available for individual work.


Name
N30 ★
Color one-third of each shape red.
What fractional part of each shape is colored red?
$\frac{1}{6}$
$\frac{1}{4}$


Name
N30 ★★
Color three-fourths of each shape red.
What fractional part of each shape is colored red?
$\frac{5}{8}$


Name
N30 ★★★
The zookeeper gives 20 bananas to 4 chimpanzees.
20
5
5
5
5
Complete.
$\frac{1}{4}$ of 20 = 5
$\frac{3}{4}$ of 20 = 15
The zookeeper gives 30 bananas to 5 monkeys.
30
6
6
6
6
6
Complete.
$\frac{1}{5}$ of 30 = 6
$\frac{3}{5}$ of 30 = 18
$\frac{2}{5}$ of 30 = 12
$\frac{4}{5}$ of 30 = 24


Name
N30 ★★★★
48
6
6
6
6
6
6
6
6
Complete.
$\frac{3}{8}$ of 48 = 18
$\frac{7}{8}$ of 48 = 42
120
20 + 4
20 + 4
20 + 4
20 + 4
20 + 4
Complete.
$\frac{2}{5}$ of 120 = 48
$\frac{3}{5}$ of 120 = 72

N31 MINICOMPUTER GOLF #3

Capsule Lesson Summary

By moving exactly one checker, change a number on the Minicomputer by a specified amount. Play *Minicomputer Golf*.

Materials

Teacher
- Minicomputer set
- Colored chalk
- ⑩ checkers

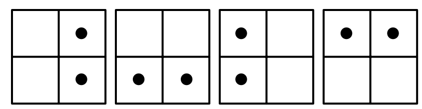
Student
- None

Description of Lesson

Exercise 1

T: ***I am going to put a number on the Minicomputer. See if you can figure out what number it is.***

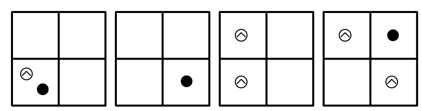
Starting with the checkers on the thousands board, gradually put this configuration on the Minicomputer.



T: ***What number is on the Minicomputer?***

S: ***5 412.***

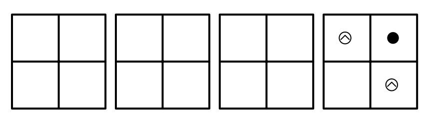
Repeat the activity with this configuration on the Minicomputer.



T: ***What number is this?***

S: $\widehat{5}$.

If some students have difficulty with this situation, ask for trades to make the number easier to read.



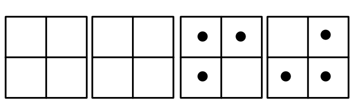
S: ***2 000 +*** $\widehat{2\,000}$ ***= 0.***

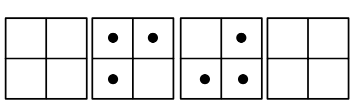
S: ***100 and*** $\widehat{100}$ ***= 0.***

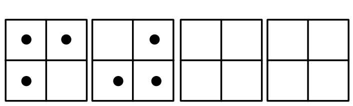
S: ***What number is on the Minicomputer?***

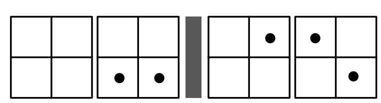
S: $\widehat{5}$***, because*** $\widehat{9}$ ***+ 4 =*** $\widehat{5}$.

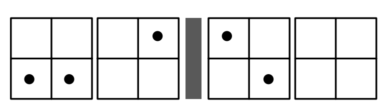
Continue this activity by asking the students to decode these configurations.


= 147


= 1 470

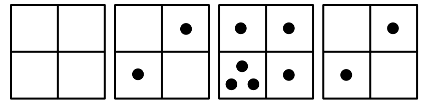

= 14 700


= 3.49


= 34.9

Exercise 2

Quickly put this configuration on the Minicomputer.



T: ***Don't try to tell me exactly what number this is. Estimate. About how many hundreds?***

Ask that students give estimations rather than the actual number.

S: ***The number is between 700 and 800.***

S: ***It's closer to 800.***

T: ***Good. Now I am going to change the number on the Minicomputer by moving one checker from the square it's on to another square. Each time I move a checker, tell me if the new number is greater or less than before. Also, I will ask you how much more or less it is.***

Move a checker from the 4-square to the 40-square.

T: ***Did I increase or decrease the number?***

S: ***Increase.***

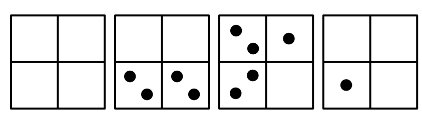
T: ***How much more is this number?***

S: ***36 more, because 40 = 4 + 36.***

Continue this activity, making the following moves. Do not return checkers to their original positions. Each move starts from a new number on the Minicomputer. Move a checker:

- from the 20-square to the 2-square (18 less)
- from the 10-square to the 200-square (190 more)
- from the 400-square to the 100-square (300 less)
- from the 40-square to the 100-square (60 more)
- from the 200-square to the 80-square (120 less)
- from the 2-square to the 200-square (198 more)

Check that this configuration is on the Minicomputer.



T: ***Who can move just one checker and increase the number by 38, that is, make the number 38 more than it is now?***

A student should move the checker on the 2-square to the 40-square. Continue this activity by asking for other similar changes.

T: ***Who can make the number 92 less?*** (Move a checker from the 100-square to the 8-square.) ***…20 more?*** (Move a checker from the 20-square to the 40-square, or from the 80-square to the 100-square.)

You may want to ask for a change that cannot be made because there is no checker in position to do so, for example, 30 more when there is no checker on the 10-square.

T: ***Who can move one checker and increase the number by 22, that is, make the number 22 more.***

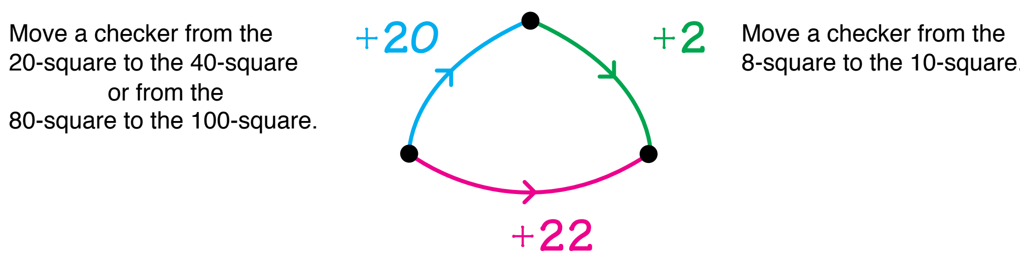
Let students have time to consider the request.

S: ***It can't be done by moving just one checker.***

T: ***Could we make the number 22 more by moving two checkers?***

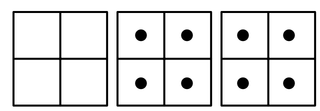
Invite one student to make the first move, and then another to make the second move.

Record the two moves in an arrow picture. For example:



Exercise 3

Put this configuration on the Minicomputer.



T: ***What number is on the Minicomputer?***

S: ***165.***

T: ***Let's play a game of Minicomputer Golf using 165 as the starting number and 500 as the goal.***

Play a *Minicomputer Golf* game with teams as described in Lesson N6 *Minicomputer Golf #1*.

If time and interest permit, let the class play a second game of *Minicomputer Golf,* using 165 as the starting number and 1001 as the goal.

N32 ASSORTED PROBLEMS #2

Capsule Lesson Summary

Find many possible solutions to a problem; then discuss why some solutions might be preferred over others within the context of the situation.

Materials

Teacher		Student	
	• Colored chalk		• Paper • Colored pencils, pens, or crayons • Calculator • Assorted problems sheets

Advance Preparation: Make copies of pages AP-9 and AP-10 of the Assorted Problems Blacklines for group work.

Description of Lesson

Organize the class into small cooperative groups for problem solving. Each group should have supplies such as paper, colored pencils, calculator, and props or manipulatives.

Exercise 1

In this exercise, groups will work on the problems on page AP-9 of the assorted problems section of the Blacklines.

Before giving groups the full problem, collectively discuss the following as a precursor.

T: ***Mr. Booker owns a bakery. He sells sugar cookies for 5¢ each and oatmeal cookies for 6¢ each. Of course, these cookies are very small at those prices.***

Record the prices on the board.

sugar cookies 5¢
oatmeal cookies

T: ***Suppose you buy four sugar cookies and three oatmeal cookies. How much will they cost?*** (38¢)

Record the order on the board as follows.

4s × 3o

T: ***Let's check. How much do four sugar cookies*** (point to 4s) ***cost?***

S: ***20¢, because 4 x 5 = 20.***

T (pointing to 3o)**:** ***How much do three oatmeal cookies cost?***

4s × 3o

S: ***18¢, because 3 x 6 = 18.***

T: ***20¢ + 18¢ = ?***

S: ***38¢.***

Find the cost of other orders in a similar manner. For example:

2s + 8o	9s + 10o
10¢ + 48¢ = $\boxed{58¢}$	45¢ + 60¢ = $\boxed{\$1.05}$

T: ***Mr. Booker wants to put together bags of cookies costing 75¢ each. How many oatmeal and sugar cookies could he put in a 75¢ bag?***

Invite students to work with their groups for a few minutes on this question. Then ask for some solutions; there are three possibilities and your students should find them all. You may like to record solutions in a chart.

75¢ bags

sugar	oatmeal
15	0
9	5
3	10

Give each group a copy of page AP-9 from the assorted problems section.

T: ***Seeing that there are only three possibilities, Mr. Booker decides to add a third variety of cookie—chocolate chip. Chocolate chip cookies cost 8¢ each. What could you get in a bag of cookies selling for 75¢?***

Direct the groups to work on this problem and to find as many solutions as they can. Some students may notice that they can get three solutions immediately from the similar problem with two varieties of cookies. That is, they just include zero chocolate chip cookies in the previous solutions.

75¢ bags

sugar	oatmeal	choc. chip
15	0	0
9	5	0
3	10	0

As you visit groups, you may like to ask some of the following questions to get them looking for other or particular solutions.

T: ***Suppose you like chocolate chip cookies best. What is the greatest number of chocolate chip cookies that could be in a bag? Could there be as many as ten chocolate chip cookies?***

S: ***No. 10 x 8 = 80, and 80¢ is more than 75¢.***

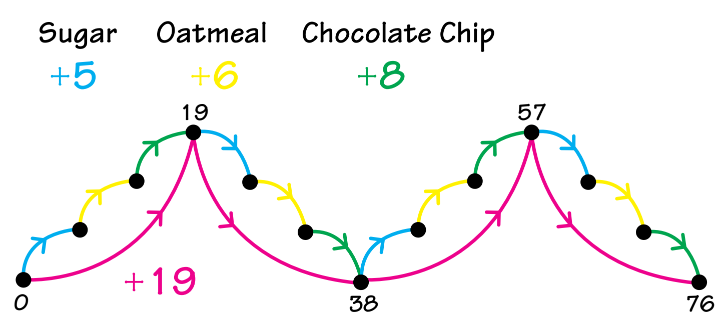
S: ***Nine chocolate chip cookies cost 72¢, and neither of the other kinds of cookies costs only 3¢ each.***

S: ***There could be eight chocolate chip cookies in a bag, together with one sugar cookie and one oatmeal cookie. (8 x 8 = 64 and 64 + 5 + 6 = 75)***

Similarly, ask for the greatest number of sugar cookies (15) and the greatest number of oatmeal cookies (10) that could be in a 75¢ bag.

T: ***Could you get a 75¢ bag with the same number of each kind of cookie?***

An arrow picture could help to answer this question.



T: ***This bag of cookies costs a penny too much. How many of each kind of cookie is in the bag?***

S: ***Four of each kind.***

T: ***What could you do to get a bag worth 75¢ instead of 76¢?***

S: ***Take out one oatmeal cookie and put in one more sugar cookie. That would bring the cost down a penny.***

T: ***Then how many sugar cookies would be in the bag?*** (5) ***Oatmeal cookies?*** (3) ***Chocolate chip cookies?*** (4)

Here are a few other good questions:

- Could you get a 75¢ bag with no sugar cookies?
- Could you get a 75¢ bag with no oatmeal cookies?
- Could you get a 75¢ bag with exactly one dozen (12) cookies?

Let groups present some of the solutions they find. Encourage students to comment on any patterns or methods they used to find solutions. A complete chart of solutions is given below for your reference. You may not expect that your class will find them all.

sugar	oatmeal	choc. chip
15	0	0
9	5	0
3	10	0
11	2	1
5	7	1
7	4	2
1	9	2

sugar	oatmeal	choc. chip
3	6	3
9	1	3
5	3	4
1	5	5
7	0	5
3	2	6
1	1	8

Exercise 2

In this exercise, groups will work on the problem on page AP-10 of the assorted problems section of the Blacklines. Begin with a discussion of the problem.

T: ***The Cruise A-Lot Club offers 2-day cruises and 5-day cruises off the coast of Jamaica. Why might a person prefer taking a 2-day cruise over taking a 5-day cruise?***

S: ***A 2-day cruise costs less.***

S: ***You may have only 2 days to spend on a cruise.***

S: ***If you are staying in Jamaica for just five or six days, a 2-day cruise would allow you to do other things as well.***

S: ***It could be your first cruise and you want to see if you like it before going on a longer trip.***

T: ***Why might a person prefer taking a 5-day cruise over taking a 2-day cruise?***

S: ***You get to see more.***

S: ***You may have come to Jamaica to take a cruise and want to spend as much time as possible on the cruise.***

S: ***5-day cruises might be a better deal — the cost per day might be less.***

T: ***The Cruise A-Lot Club gives Sam the job of scheduling spring cruises during the months of April and May. How many days are there altogether in April and May?***

S: ***61 days; April has 30 days and May has 31 days.***

T: ***Since the company has only one ship, it can schedule only one cruise at a time. How many 2-day cruises and how many 5-day cruises can Sam schedule during the 61 days?***

Let the groups work on the problem. Encourage them to find all six solutions. After a while invite groups to present their solutions and record them on the board.

2-day cruises	5-day cruises
28	1
23	3
18	5
13	7
8	9
3	11

You may like to continue with a discussion of other considerations Sam might make in scheduling.

T: ***There are six possibilities. What kinds of things might Sam consider when deciding how many 2-day and 5-day cruises to schedule?***

S: ***Sam should figure out which way (combination) gives the club the most money.***

S: ***If most people like 2-day cruises, Sam might choose twenty-eight 2-day cruises and one 5-day cruise.***

S: ***He might choose three 2-day cruises and eleven 5-day cruises if 5-day cruises are more popular.***

T: ***How could he know which is more popular?***

S: ***He could ask other cruise lines, or he could ask a lot of people thinking about taking a cruise.***

S: ***He could find out how many people signed up for each kind of cruise last year.***

T: ***Suppose that last year there were more requests for 2-day cruises, but not many more. What might Sam choose then?***

y cruises and seven 5-day cruises.

Writing Activity

Invite students to write their own problems similar to the cookie problem or the cruise problem. allenge other groups.

Extension Activity

Select other problems from the assorted problems section of the Blacklines to use in cooperative

Home Activity

Let students take home a copy of the cookie problem. Suggest they change it to a 50¢ bag of cookies, and then work with family members to find solutions.

Capsule Lesson Summary

Review how to multiply a whole number by a one-digit whole number. After observing a multiplication pattern in some arrow pictures involving the composition of two multiplication functions, conjecture that the composition of 10x and 2x is 20x. Then use the Minicomputer to support this conjecture. Discuss how one could multiply a number by 30, 70, and 90.

Materials

Teacher
- Colored chalk
- Minicomputer set
- ⑩-checkers

Student
- Paper
- Colored pencils, pens, or crayons
- Worksheets N33*, **, ***, and ****

Description of Lesson

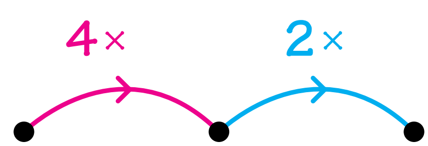
Exercise 1

Give students several multiplication problems to do individually on their papers. Check the answers collectively and let students explain their methods. You may like to ask for story situations in which you would do these multiplication problems.

$$\begin{array}{r} 28 \\ \times\ 4 \\ \hline \boxed{112} \end{array} \qquad \begin{array}{r} 392 \\ \times\ 8 \\ \hline \boxed{3136} \end{array} \qquad \begin{array}{r} 470 \\ \times\ 6 \\ \hline \boxed{2820} \end{array}$$

Exercise 2

Draw this arrow picture on the board.



T: ***Pick a number between 0 and 12.***
Don't tell anyone your number. Ready?

Multiply your number by 4 (trace the red arrow).
Multiply that result by 2 (trace the blue arrow).

Draw a chart for starting and ending numbers under your arrow picture.

T: ***Ameed, what number did you start with?***

S: ***4.***

Record 4 in the chart.

Starting Number	Ending Number
4	32

T: ***And what number did you end with?***

S: ***32.***

Record 32 in the chart.

N33

Call on several students to tell their starting and ending numbers. Ask the class to check pairs they think might be incorrect by multiplying the starting number by 4 and then by multiplying that answer by 2.

Starting Number	Ending Number
4	32
5	40
2	16
10	80

T: ***Now I want some of you to tell us just your ending number.***

S: ***72.***

T: ***Who knows Rebecca's starting number?***

S: ***9.***

T: ***Is that right, Rebecca?*** (Yes)

Record 72 and 9 in the chart. Call on several students to give only their ending numbers and let the class find the starting numbers. Return to the arrow picture to settle any disagreements or check for any errors.

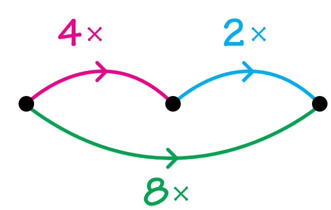
Starting Number	Ending Number
4	32
5	40
2	16
10	80
9	72
11	88

T: ***Look closely at this chart. Do you notice any patterns?***

S: ***The ending numbers are always even numbers.***

S: ***Each ending number is 8x the starting number.***

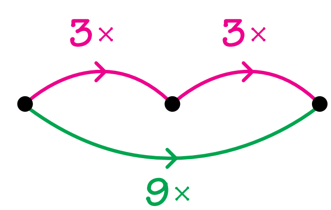
When someone gives this last response, add a green arrow to the arrow picture.



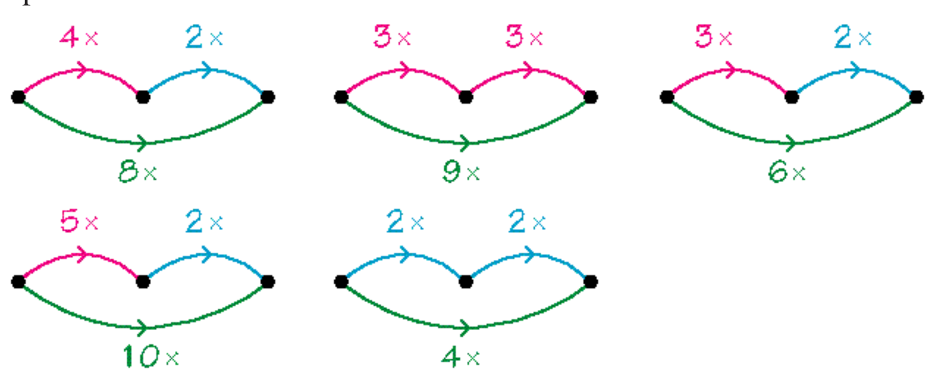
T: ***What is 4x followed by 2x?***

S: ***8x.***

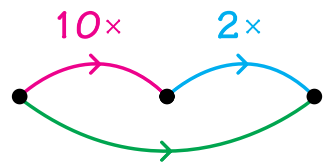
Leave the first arrow picture on the board, and repeat the activity with another arrow picture. Your students should conclude that 3x followed by 3x is 9x.



Put up three more arrow pictures: 3x followed by 2x, 2x followed by 2x, and 5x followed by 2x. You need not fill charts for these, but let students give several possible starting and ending numbers for each one before asking what the green arrow could be for. Let the pictures accumulate until you have all five of these pictures on the board.



Add this arrow picture. You may like to suggest students draw this picture on their papers.



T: ***What do you think 10x followed by 2x is?***

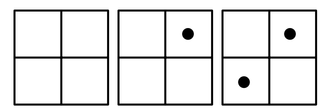
S: ***20x.***

T: ***Why?***

S: ***In the others, there is a pattern: 4 x 2 = 8, 3 x 3 = 9, 5 x 2 = 10, 2 x 2 = 4, and 2 x 3 = 6. So I think we just need to multiply 10 x 2 here.***

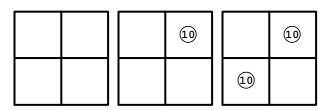
T: ***Let's see what happens on the Minicomputer when we put on a number, multiply it by 10, and then by 2.***

Invite a student to put on a number of his or her choice (you may want to limit the checkers and insist on standard configuration). Suppose the student puts on 46.



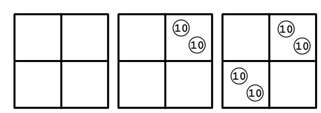
T: ***The first arrow is for 10x. How can we multiply 46 by 10 on the Minicomputer?***

Accept all correct methods, but use the one replacing each of the checkers with a ⑩-checker.



T: ***The second arrow is for 2x. How can we double this number on the Minicomputer?***

Accept all correct methods, but use one putting another ⑩-checker with each ⑩-checker already on a square. If necessary, review how you double a number in standard configuration on the Minicomputer by putting another checker with each checker already on a square.



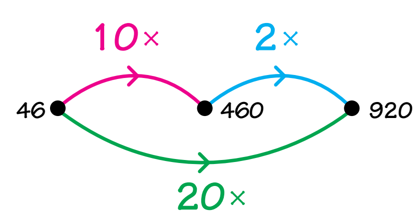
T: ***Now, how many 46's are on the Minicomputer?***

S: ***Twenty.***

T: ***So what is 10x followed by 2x?***

S: ***20x.***

Label the green arrow 20x. With the class, label the three dots, using the starting number your student put on the Minicomputer (46 in the lesson description).

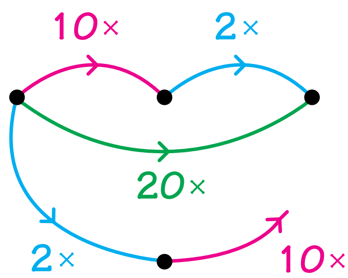


T: ***What number is 10 x 46?*** (460)
What number is 2 x 460? (920)
What number is 20 x 46? (920)

Note: If earlier someone thought that the green arrow could be for 12x, compute 12 x 46 in the following way:

$$\begin{array}{r} 10 \times 46 = 460 \\ 2 \times 46 = 92 \\ \hline 12 \times 46 = 552 \end{array}$$

Conclude that the green arrow cannot be for 12x.



Erase the dot labels and ask for another starting number.
Label the dots using other starting numbers, such as 14 and 28.

T: ***Suppose we start with a 2x arrow and then follow it with a 10x arrow.***

What do you think 2x followed by 10x is?

S: ***20x.***

T: ***Why?***

S: ***10 x 2 = 20 and 2 x 10 = 20.***

S: ***Suppose we start at 46 and follow the arrows. 2 x 46 = 92 and 10 x 92 = 920 —that's the same ending number we got when we multiplied by 10 and then by 2.***

Connect the red arrow to the ending dot.

Ask students to do several 20x calculations by first multiplying by 2 and then multiplying the answer by 10. For example, for 20 x 13 multiply 2 x 13 = 26 and 10 x 26 = 260; for 20 x 38 multiply 2 x 38 = 76 and 10 x 76 = 760.

T: ***This picture shows two ways to multiply a number by 20. We can multiply by 10 and then by 2*** (trace the upper arrow road), ***or we can multiply by 2 and then by 10*** (trace the lower arrow road)**.**

Students may want to express their preferences for one method over the other.

Continue with these or similar problems, asking students to tell how they did the calculations. (Answers are in boxes.)

$20 \times 51 = \boxed{1\,020}$ $\quad$ $20 \times 4.2 = \boxed{84}$

$20 \times 27 = \boxed{540}$ $\quad$ $20 \times 0.35 = \boxed{7}$

Erase everything on the board except the 20x arrow picture.

T: ***To multiply a number by 20, you could multiply it by 10 and the answer by 2, or vice versa. How do you think you could multiply a number by 30?***

S: ***Multiply by 3 and then by 10.***

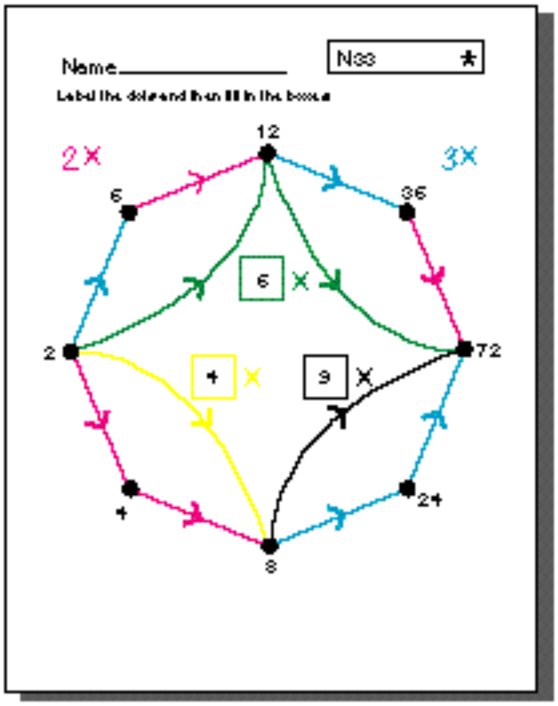
S: ***Multiply by 10 and then by 3.***

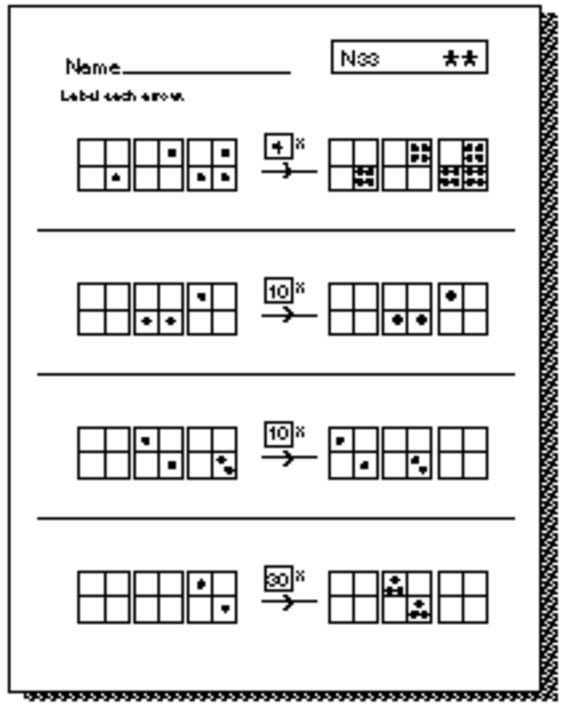
Also ask how you could multiply a number by 70 and how you could multiply a number by 90.

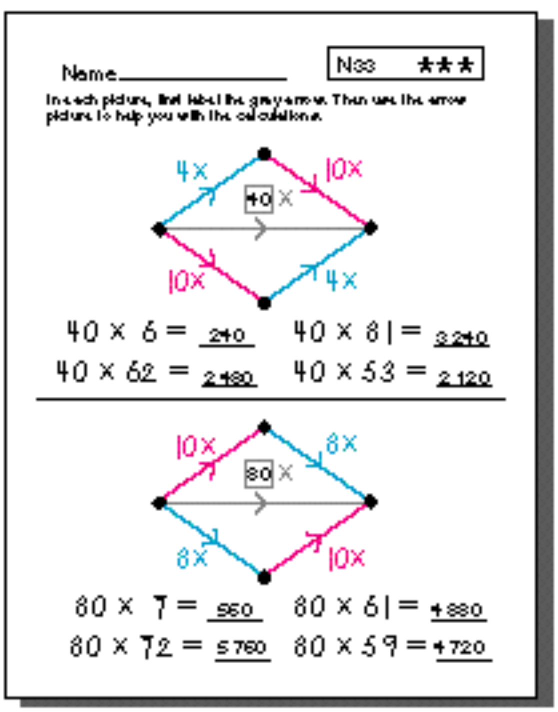
re available for individual work.

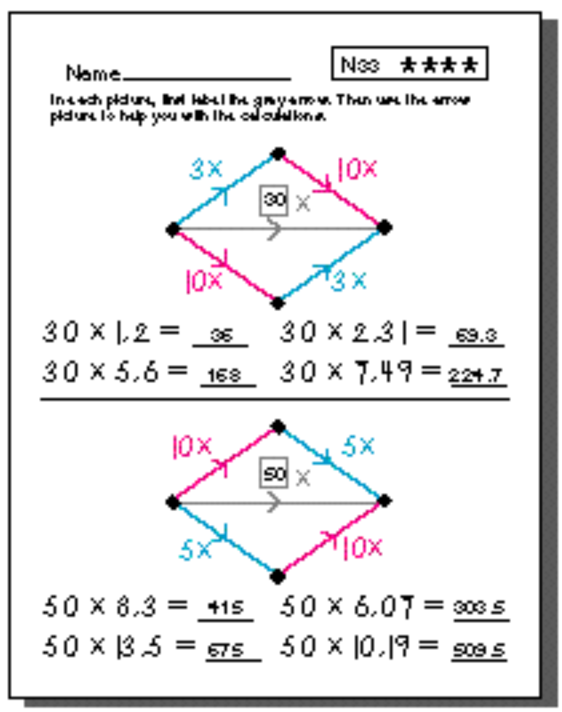
Writing/Home Activity

Suggest that students write a letter to their parents/guardians explaining how to multiply a number by 20 (or 30, 40, 50, and so on). Suggest that students illustrate their letters with arrow pictures.


Name
N33 ★
Label the dots and then fill in the boxes.
2×
3×
12
6
36
6 ×
2
4 ×
9 ×
72
4
24
8


Name
N33 ★★
Label each arrow.
4 ×
10 ×
10 ×
30 ×


Name
N33 ★★★
In each picture, first label the gray arrow. Then use the arrow picture to help you with the calculations.
4×
10×
40 ×
10×
4×
40 × 6 = 240
40 × 81 = 3 240
40 × 62 = 2 480
40 × 53 = 2 120
10×
8×
80 ×
8×
10×
80 × 7 = 560
80 × 61 = 4 880
80 × 72 = 5 760
80 × 59 = 4 720


Name
N33 ★★★★
In each picture, first label the gray arrow. Then use the arrow picture to help you with the calculations.
3×
10×
30 ×
10×
3×
30 × 1.2 = 36
30 × 2.31 = 69.3
30 × 5.6 = 168
30 × 7.49 = 224.7
10×
5×
50 ×
5×
10×
50 × 8.3 = 415
50 × 6.07 = 303.5
50 × 13.5 = 675
50 × 10.19 = 509.5

Capsule Lesson Summary

Review the "monkeys and bananas" story from Lesson N30. Solve problems such as $\frac{2}{7}$ of 28. Determine how two children painting a fence should share the work based on the number of hours that each can work.

Materials

Teacher
- Colored chalk
- Meter stick

Student
- Paper
- Colored pencils, pens, or crayons
- Metric ruler
- Worksheets N34*, **, ***, and ****

Description of Lesson

Exercise 1: Feeding the Monkeys

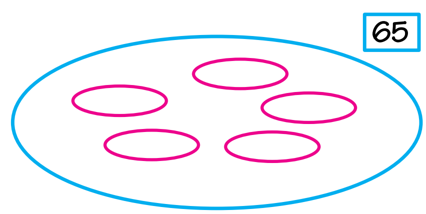
T: ***Not too long ago, I told you a story about a zookeeper feeding bananas to monkeys. Who can tell us the story? What kind of problems did we solve?***

Let students discuss the story.

T: ***Suppose the zookeeper has 65 bananas to share fairly among five monkeys. What could I draw to help decide how many bananas each monkey should get?***

S: ***Draw a blue string and label it 65. Draw five red strings inside the blue string.***

Following the student's instructions, draw this picture.



T: ***What fraction of the bananas does each monkey receive?***

S: ***$\frac{1}{5}$, since each monkey gets one of the five shares.***

T: ***How many bananas does each monkey receive?***

Accept estimates from students, not indicating, for the moment, if any are exact. It may be useful to solve the problem in two or more steps; for example:

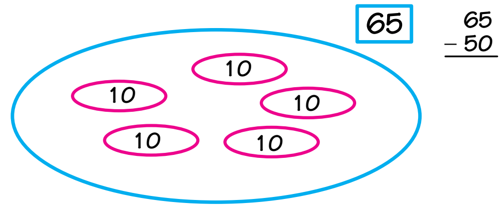
T: ***The zookeeper could begin by giving the same number of bananas to each monkey. How many bananas do you suggest?***

S: ***10 bananas.***

T: ***So far how many bananas would the zookeeper have given away?***

S: ***50 bananas, because 5 x 10 = 50.***

Add the information to the picture.



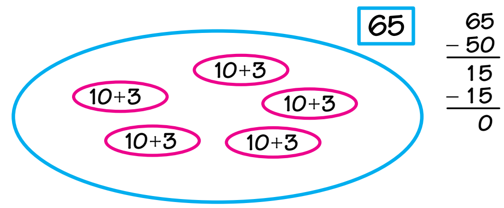
T: *How many bananas would the zookeeper have left?*

S: *15 bananas, 65 – 50 = 15.*

T: *With 15 bananas, how many could the zookeeper give to each monkey?*

S: *Three bananas, since 5 x 3 = 15. Then there would be none left.*

Add the information to the picture.



$1/5$ of 65 = ____

T: *How many bananas does each monkey receive?*

S: *13 bananas.*

T: *So what is ⅕ of 65?*

S: *13.*

Complete the number sentence: ⅕ of 65 = 13.

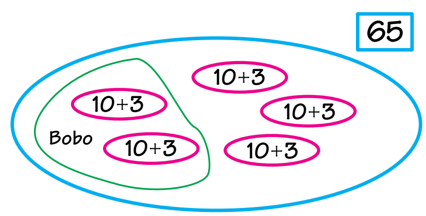
T: *What usually happens next in the "monkeys and bananas" story?*

S: *One monkey eats some other monkeys' bananas.*

T: *Remember Bobo? This time he eats his share and one other share. What fraction of the bananas does Bobo eat?*

S: *⅖, since he eats two of the five shares.*

T: *Yes, he eats his ⅕ and someone else's ⅕; ⅕ + ⅕ = ⅖.*



Bobo: $1/5 + 1/5 = 2/5$

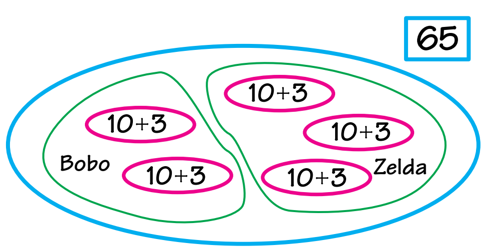
Write the number sentence on the board as you ask:

$2/5$ of 65 = 26

T: *What is ⅖ of 65?*

S: *26, since 13 + 13 = 26. Bobo eats 26 bananas.*

Repeat the preceding series of questions for a monkey who eats the other three shares, and determine that the monkey eats ⅗ of the bananas and that ⅗ of 65 = 39. Your picture might look similar to the one below.



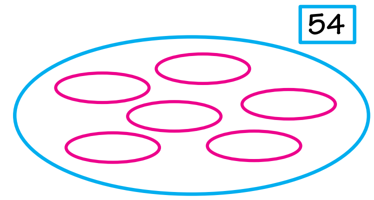
Bobo: $1/5 + 1/5 = 2/5$ Zelda: $1/5 + 1/5 + 1/5 = 3/5$

$2/5$ of 65 = 26 $3/5$ of 65 = 39

Erase the board.

T: ***The zoo obtains another monkey. It now has six monkeys. The zookeeper has 54 bananas. What picture could I draw to show this situation?***

Draw this picture following students' instructions.



T: ***How many bananas does each monkey receive?***

S: ***9 bananas, since 6 x 9 = 54.***

Put 9 in each red string. Write the following number sentences on the board, and invite students to use the story and string picture to complete them. (Answers are in boxes.)

$\frac{1}{6}$ of 54 = $\boxed{9}$ $\frac{3}{6}$ of 54 = $\boxed{27}$

$\frac{2}{6}$ of 54 = $\boxed{18}$ $\frac{5}{6}$ of 54 = $\boxed{45}$

Erase the board and then write this problem:

$\frac{2}{7}$ of 28 = ___

T: ***Can anyone solve this problem?***

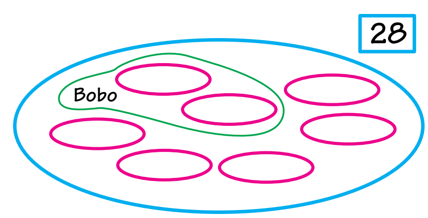
Accept the answers of several students without indicating if anyone is correct.

T: ***In terms of our story, what does each number is this problem tell us? What picture could I draw?***

S: ***There are 28 bananas and seven monkeys, so draw seven strings inside a large string.***

S: ***One monkey, perhaps Bobo, eats two shares of bananas.***

Following student instructions, draw this picture.



T: ***What is $\frac{2}{7}$ of 28?***

S: ***8. Each monkey receives four bananas. Bobo eats two shares and 2 x 4 = 8.***

Put 4 in each red string and complete the number sentence: $\frac{2}{7}$ of 28 = 8.

Exercise 2

Erase the board and then write these expressions.

$\frac{1}{3}$ of 72 $\frac{1}{4}$ of 72

T: ***Which is more, $\frac{1}{3}$ of 72 or $\frac{1}{4}$ of 72? Why?***

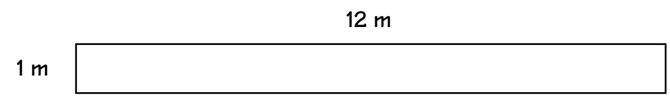
S: ***$\frac{1}{3}$ of 72. If 72 bananas are shared among three monkeys, each monkey receives more than if they are shared among four monkeys.***

Divide your class into two groups and partner the students within a group. Give a pair of problems to the student partners in each group as designated below. (Answers are in boxes.) Encourage students to draw and use a string picture. After a few minutes, invite student partners to announce their solutions. Note that ⅓ of 72 is more than ¼ of 72.

Group 1	Group 2
$\frac{1}{3}$ of 72 = $\boxed{24}$	$\frac{1}{4}$ of 72 = $\boxed{18}$
$\frac{2}{3}$ of 72 = $\boxed{48}$	$\frac{3}{4}$ of 72 = $\boxed{54}$

Exercise 3

Draw a long, narrow rectangle on the board.



T: ***This is the picture of a fence. It is 12 m long and 1 m tall. Show me about how tall 1 m is.***

After the students estimate a height of 1 m, use a meter stick to show them the exact height.

T: ***One afternoon Joanne and Rudi wish to paint one side of the fence. Rudi can work for three hours, but Joanne has only one hour before her Girl Scout meeting.***

Write the information on the board.

Joanne: 1 hour
Rudi: 3 hours

T: ***Would they divide the fence in half and each paint one-half of the fence?***

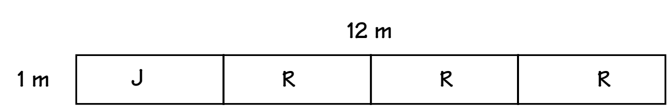
Let the students discuss the fairness of this suggestion.

T: ***Joanne did not like the idea. She would have to work extremely fast, while Rudi could work slowly. Where could they draw a line to divide the fence so that each person could work at the same rate?***

Let students come to the board and show approximately where they think a dividing line segment could be drawn on the fence. At first, students may just give a rough estimate, letting Rudi paint more than half. Ask students if they can be more precise and tell you exactly where to draw the line segment. After a while, ask how many parts the fence should be divided into and how many parts each person should paint.

S: ***Divide the fence into four parts of the same size. Rudi should paint three parts, and Joanne should paint one part.***

Follow this suggestion. There are several fair methods of dividing the fence. One example is shown below.



T: ***Why is this fair?***

S: ***Each person can take one hour to paint a section. Rudi can paint his three sections in three hours. Joanne can paint her one section in one hour.***

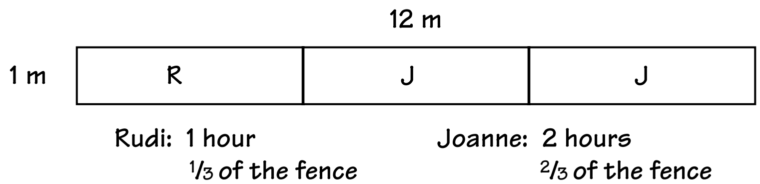
T: ***What fraction of the fence will each child paint?***

S: ***Joanne will paint ¼ of the fence, and Rudi will paint ¾ of it.***

Erase the board and redraw the rectangle.

T: ***Joanne and Rudi decide to paint the other side of the fence the next day. Joanne can work for two hours, and Rudi can only work for one hour before band practice. How should they divide the fence so that each can work at the same rate?***

In a similar manner as above, lead students to the following solution.



Exercise 4

T: ***Joanne and Rudi are such good fence painters that a neighbor asks them to paint her picket fence. Who knows what a picket fence is?***

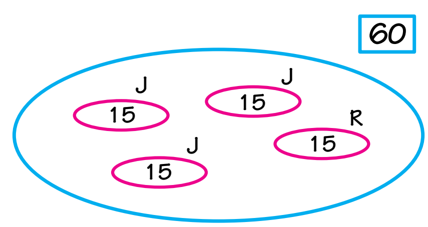
Let a student describe picket fences. Draw a picture if necessary.

Write this information on the board.

60 Pickets
Joanne: 3 hours

T: ***Joanne and Rudi decide to paint 60 pickets in one day. Joanne can work three hours and Rudi only one hour. How many pickets should each child paint?***

Lead students to the idea that since they can work a total of four hours, they can divide the fence into four sections with the same number of pickets in each. Encourage the use of a string picture.



T: ***What fraction of the pickets should each child paint?***

S: ***Joanne should paint three out of the four sections, so she paints ¾ of the pickets.***

S: ***Rudi should paint one out of the four sections, so he paints ¼ of the pickets.***

T: ***How many pickets should each person paint?***

S: ***Rudi should paint 15 pickets.***

S: ***Joanne should paint 45 pickets; 3 x 15 = 45.***

¼ of 60 = 15
¾ of 60 = 45

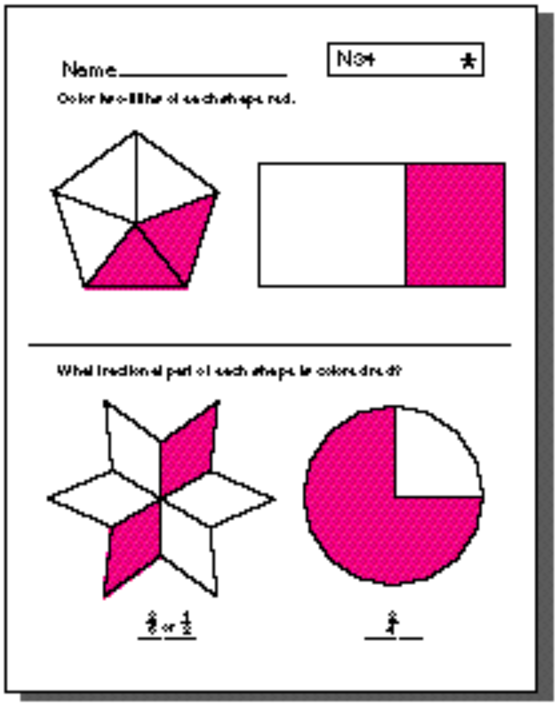
Record the number sentences on the board.

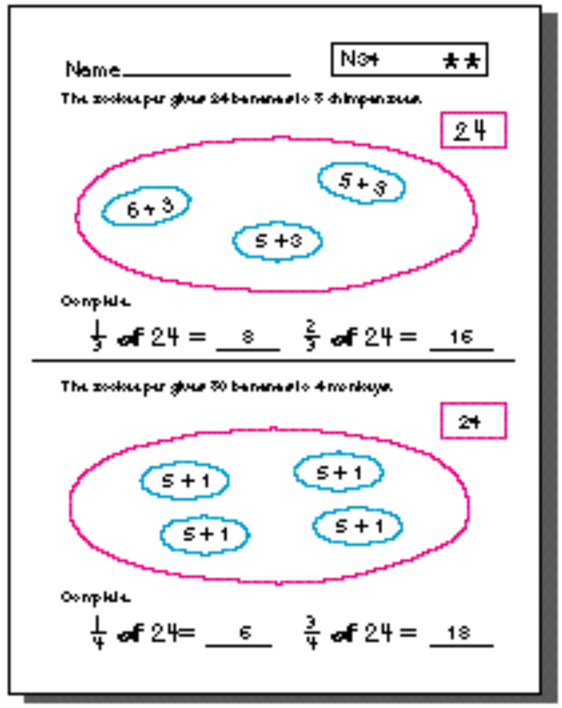
Worksheets N34*, **, ***, and **** are available for individual work.

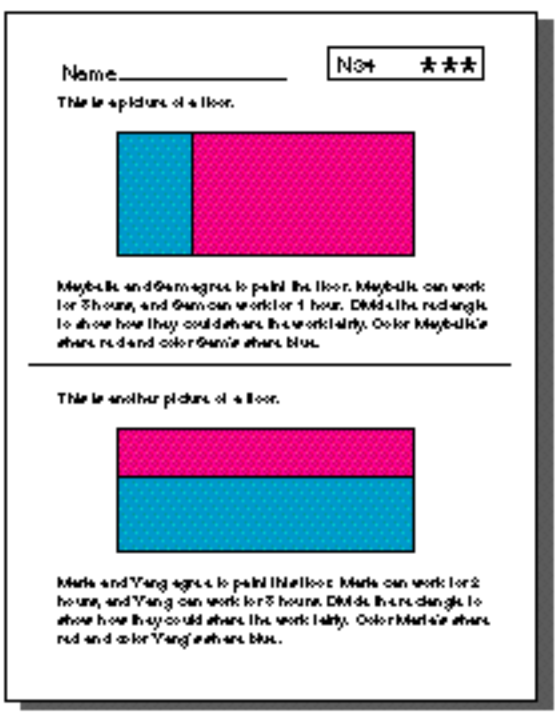
N34

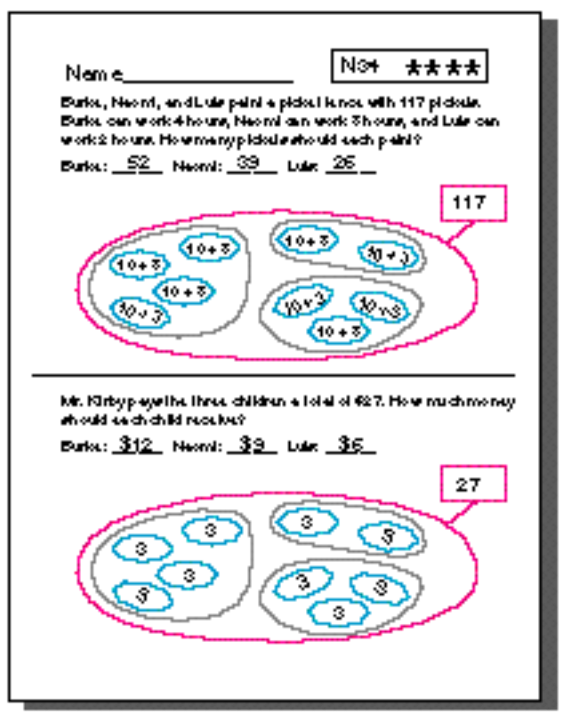
Writing Activity

Suggest that students write work problems similar to Exercises 3 or 4. Then they can write an explanation of how to solve their problems.









N35 DECIMALS #3

Capsule Lesson Summary

Multiply non-integer decimals by 10 on the Minicomputer. List the corresponding number sentences and look for patterns. Locate decimals between 0 and 1 on the number line. Build a road between two given numbers.

Materials

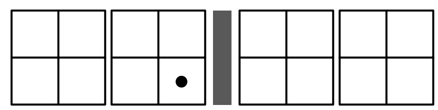
Teacher
- Minicomputer set
- ⑩-checkers
- Colored chalk

Student
- Paper
- Colored pencils, pens, or crayons

Description of Lesson

Exercise 1

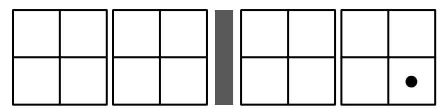
Put this configuration on the Minicomputer.



T: ***This is one dollar. Can you put one penny on the Minicomputer?***

How do we write this number?

S: ***0.01.***



Put nine more checkers on the 0.01-square.

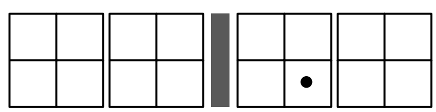
T: ***How many pennies are on the Minicomputer?***

S: ***Ten.***

Write the corresponding calculation to the right of the Minicomputer.

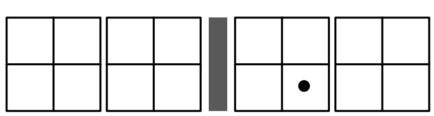
T: ***How much money is this?***

S: ***Ten cents, or one dime.***



$= 10 \times 0.01$

Ask students to make trades on the Minicomputer until the standard configuration for 0.1 is reached. Ask a student to write the number below the Minicomputer.



$= 10 \times 0.01 = 0.1$

T: ***Do you know another way to write this number?***

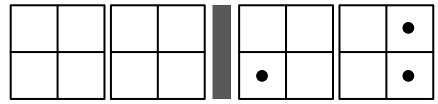
If necessary, suggest 0.1 yourself.

Begin a list of number sentences with 10x. Erase the board except for the number sentence.

$$10 \times 0.01 = 0.10 = 0.1$$

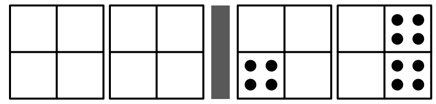
N35

Ask a student to put a quarter on the Minicomputer.



$= 0.25$

Then ask another student to put on three more quarters in the same way.



T: ***How many quarters are on the Minicomputer?***

S: ***Four quarters.***

T: ***How much money is four quarters?***

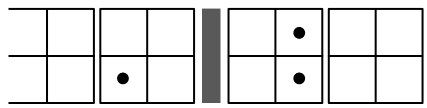
S: ***$1.***

$$4 \times 0.25 = 1$$

Students may want to make trades to check that four quarters on the Minicomputer is the same as $1.

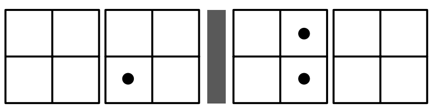
T: ***How much money is ten quarters?***

Invite a student to put 10 x 0.25 on the Minicomputer with ⑩-checkers.



$= 10 \times 0.25$

Then call on volunteers to make trades with the ⑩-checkers.



$= 10 \times 0.25 = 2.50 = 2.5$

T: ***What is another name for 2.50?***

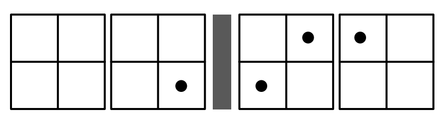
S: ***2.5.***

Add this sentence to the list.

Erase the board except for the list of number sentences.

$$10 \times 0.01 = 0.10 = 0.1$$
$$10 \times 0.25 = 2.50 = 2.5$$

Ask a student to put $1.68 on the Minicomputer and to write the number below the Minicomputer.



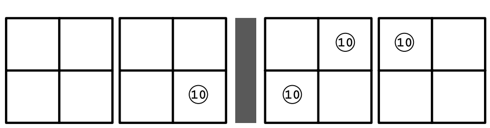
1 . 6 8

Write this problem on the board.

$$10 \times 1.68$$

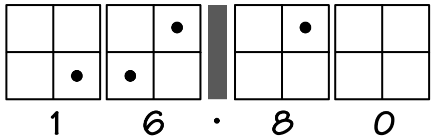
T: ***What number is 10* x *1.68?***

Invite a student to put 10 x 1.68 on the Minicomputer with ⑩-checkers and suggest that students write their answers on their papers.



$= 10 \times 1.68$

Call on volunteers to make trades with the ⑩-checkers.



$= 10 \times 1.68 = 16.80 = 16.8$

T: ***How much money is 10* x *$1.68?*** ($16.80)
10* x *1.68 = 16.80.
What is another name for 16.80?

S: ***16.8.***

Add this number sentence to the list on the board.

$10 \times 0.01 = 0.10 = 0.1$
$10 \times 0.25 = 2.50 = 2.5$
$10 \times 1.68 = 16.80 = 16.8$

T: ***Do you notice any patterns we could use when we multiply by 10?***

Encourage students to freely express any patterns they see, although they should be sure to notice the pattern that compares the positions of the decimal points before and after multiplying by 10.

Write these two additional problems on the board below your list.

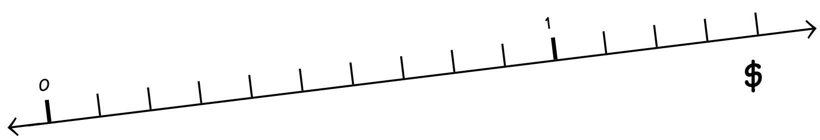
$10 \times 0.50 =$
$10 \times 0.43 =$

Ask students to do these problems first on their papers and then to complete the number sentences at the board. You should have this list on the board.

$10 \times 0.01 = 0.10 = 0.1$
$10 \times 0.25 = 2.50 = 2.5$
$10 \times 1.68 = 16.80 = 16.8$
$10 \times 0.50 = 5.00 = 5.0 = 5$
$10 \times 0.43 = 4.30 = 4.3$

Exercise 2

Draw this part of the $ number line on the board. Each segment should be approximately 10 cm in length. Explain to students that this is a "dollar" number line.



T: ***What could these marks be for?***

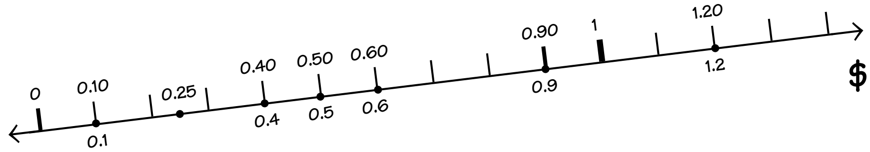
S: ***Dimes.***

T: ***Put a dot for one dime on the number line.***

Let a student draw the dot on the board and label it.

Ask students to locate and to label dots for the following amounts.

40¢ nine dimes **60¢ $1.20** **five dimes one quarter**



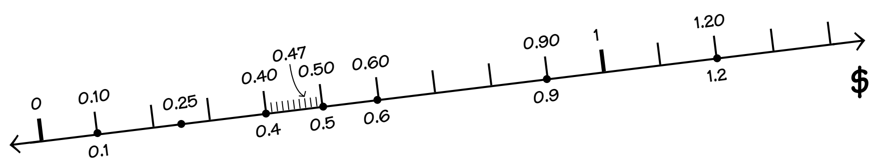
T: ***Where is 47¢ on this number line?***

Ask a student to show the approximate location.

T: ***How could we locate this point more accurately?***

S: ***Divide the segment from 0.40 to 0.50 into ten smaller segments of the same length.***

Perform the subdivisions and ask a student to draw and label the appropriate dot.



T: ***What part of a dollar is 50¢?***

S: ***Half of a dollar.***

Invite a student to point to the dot already labeled for half a dollar and label it ½ as well.

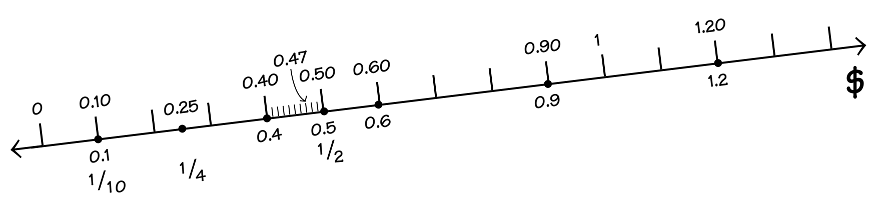
T: ***What part of a dollar is $0.25?***

S: ***A quarter of a dollar (or a fourth of a dollar).***

Again, call on a student to locate the dot labeled for a quarter and label it ¼ as well.

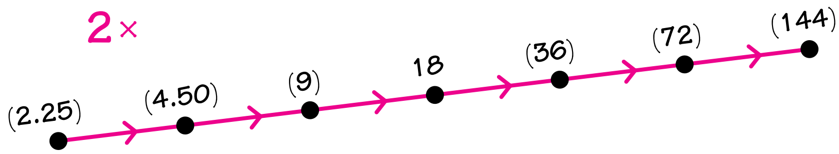
T: ***What part of a dollar is $0.10?***
How many dimes are there in one dollar? (Ten)
So one dime is one-tenth of a dollar.

Use ⅒ as a label for the dot already labeled for one dime.



Exercise 3

Draw this arrow road on the board. Ask students to copy the picture and to label the dots. Start with only 18 in the picture and let students provide the numbers in parentheses.



Encourage students having difficulty with the dots at the lower left to think about taking one-half of nine dollars, and then one-half of four dollars and fifty cents.

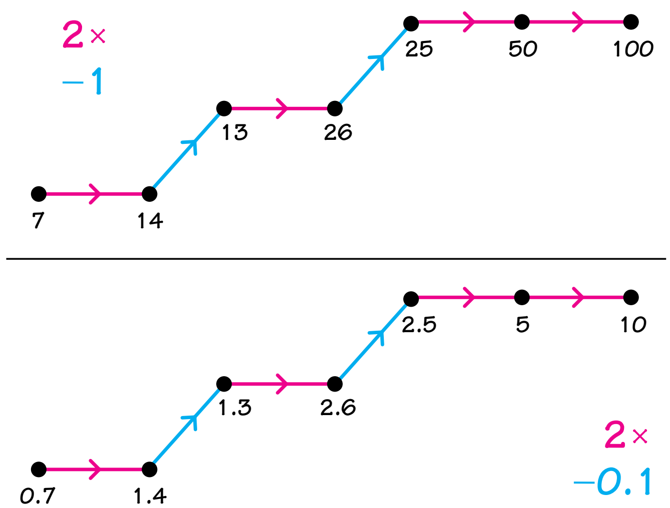
Exercise 4

On the board, draw dots for 7 and 100 spaced a distance apart. Draw keys for 2x (red) and –1 (blue) arrows.

T: ***Build an arrow road from 7 to 100 using 2x (red) and –1 (blue) arrows. Try to use as few arrows as possible.***

Let students work independently or with a partner for several minutes. Suggest to students who finish this road to try to build a road from 0.7 to 10 using 2x and –0.1 arrows. In this case you may wish to remind students that they can think about 0.7 (0.70) as seventy cents and 10 as ten dollars. Then they can use 2x and “minus ten cents” arrows.

Before the end of the lesson, ask students with six-arrow solutions to put them on the board. Invite comments on other possible solutions or comparisons between these two arrow roads.



Note: If they are thinking about money, students will most likely label all the dots in this second es, i.e., 1.40 rather than 1.4.

Writing Activity