# The World of Numbers

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By now, veteran *CSMP* students have had a rich variety of experiences in the World of Numbers. They have met and become familiar with various kinds of numbers, and with operations and relations on them. They have encountered positive and negative integers, decimal numbers, fractions, numerical functions (such as 5x, +3, ÷10, –5, ½x), order relations (such as < and >), and the notions of multiples and divisors of a given number. They have been introduced to paper-and-pencil algorithms for addition and subtraction of whole numbers and decimal numbers, and for multiplication of whole numbers. Students have also had extensive experience using systematic methods for division of whole numbers in preparation for algorithms.

In *CSMP Mathematics for the Intermediate Grades*, *Part II*, these earlier numerical experiences will be revisited, extended, and deepened through familiar games and activities, as well as in fascinating new situations. As always, *CSMP* stresses the unity and continuity of growth of mathematical ideas and concepts. The program's spiral approach does not require mastery of each lesson, but rather allows students to encounter the elements of each content strand in different situations throughout the year. It is important to recognize this approach consciously. If you strive for mastery of each single lesson, you will find yourself involved in a great deal of redundancy as the year progresses.

Further, *CSMP* presents the content in a situational framework. That is, a "pedagogy of situations" engages students in rich problem-solving activities as they construct mathematical ideas. These situations offer opportunities both to develop necessary numerical skills and to gain deeper understanding of mathematical concepts in the world of numbers. At the same time, the situations presented encourage students to develop patterns of logical thinking and strategies for attacking problems.

Perhaps the most important embodiments of the *CSMP* approach are the non-verbal languages and tools used throughout the program. These are vehicles that allow students to investigate the contexts in which the content is presented and to explore new mathematical ideas. It is hard to overstate the value of developing languages and tools that are not confined to one area of mathematical content or to one level of the development of content; that aid in attacking problems as well as in representing situations. Armed with the universally applicable languages of the *CSMP* curriculum, students grow more and more familiar with the syntax of these languages and are free to explore new content as extensions rather than think of each new mathematical idea as tied to a certain new language. This is not to say that *CSMP* students do not learn the usual descriptive language of mathematics; naturally, they do. However, in the *CSMP* approach the usual descriptive language is not a requisite for learning new concepts, but only a means for succinctly describing those ideas as they are being explored.

## Standard Algorithms of Arithmetic

*CSMP* seeks to develop basic numerical skills as well as an understanding of the underlying mathematical ideas. We are fully in agreement with the thesis that, along with the growth of understanding of the world of numbers, there must be a concomitant growth of familiarity and facility with numbers and operations on them. But facility should not be confused with understanding; they are partners in the growth of mathematical maturity. A balanced growth of each must be maintained, neither being sacrificed for the other.

# WORLD OF NUMBERS INTRODUCTION

Students must eventually learn mechanical algorithms for the basic operations (addition, subtraction, multiplication, division). However, the development of these methods should occur only after students have had many experiences with prerequisite concepts. Premature presentation of these algorithms may actually inhibit a student's desire and ability to develop alternative algorithms, to do mental arithmetic, or to estimate.

*CSMP* believes that students should be able to solve a problem such as 672 ÷ 4 using models, pictures, or mental arithmetic before being introduced to a division algorithm. Even after students have mastered an algorithm, they should be aware that alternative methods are often more appropriate. For example, consider the problem of calculating 699 x 9. Rather than using a standard multiplication algorithm, it may be easier and more efficient to note that 700 x 9 = 6 300, so that 699 x 9 = 6 300 – 9 = 6 291. Indeed, built into this way of approaching the problem is an excellent estimate (6 300) of the product. To insist on a mechanistic response to such a problem would be to encourage inefficiency and might also inhibit the development of the flexibility necessary for problem solving. On the other hand, a rich array of situations in which students interact with numbers provides them with opportunities to gain the necessary facility with standard algorithmic procedures while retaining the openness required to respond creatively to new situations in the world

## Content Overview

### Subtraction

In *CSMP Mathematics for the Upper Primary Grades*, students learned to solve many subtraction problems through mental arithmetic. Arrow pictures suggested ways to change difficult problems into a sequence of simpler subtractions. For example, to calculate 63 – 28 students learned they could use any of these calculations: 63 – 20 – 8; 63 – 30 + 2; or 63 – 23 – 5. Mental arithmetic activities and patterns (for example, 63 – 28 = 65 – 30 = 35) eventually led to the development of a standard paper-and-pencil algorithm for subtraction, the method of compensation or equal additions.

$$\begin{array}{r} 6\,{}^{10}3 \\ -\ {}^{1}2\,8 \\ \hline 35 \end{array} \qquad \begin{array}{r} 4\,{}^{10}0\,{}^{10}6 \\ -\ {}^{1}2\,{}^{1}7\,9 \\ \hline 127 \end{array}$$

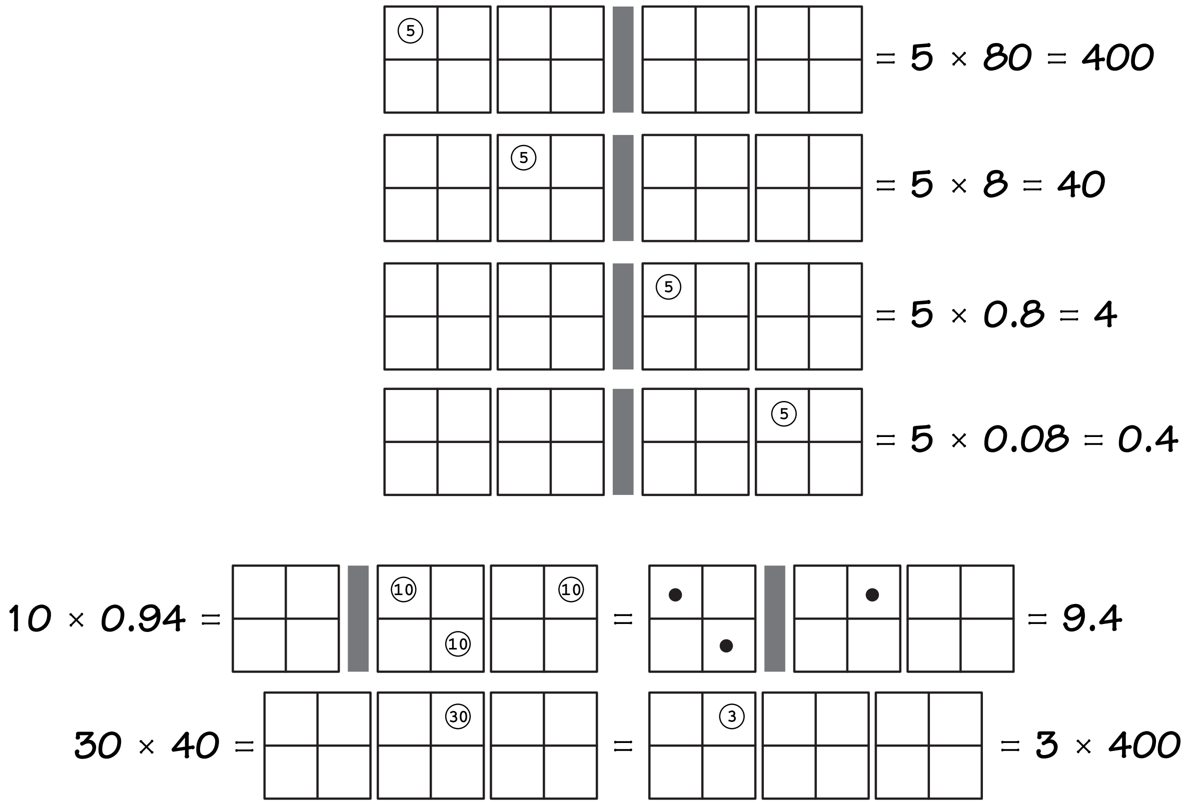
If you are unfamiliar with this method of subtraction, refer to *CSMP Mathematics for the Upper Primary Grades, Part IV* or to a mathematics methods textbook for elementary school teachers.

A goal of this strand is to both reinforce and broaden the students' experiences with subtraction. To review subtraction, students will encounter the operation in many contexts: the *Minicomputer Golf* game, *Guess My Rule* activities, real-world applications, calculator problems, and arrow pictures. The variety of activities provide for practice with an algorithm, subtraction facts, and number patterns. Students' increased familiarity with the operation allows the development of repeated subtraction as a technique for solving introductory division problems. The lessons in *IG-II* continue the extension of subtraction to include decimals, fractions, and negative numbers.

Lessons: N1, 6, 7, 8, 12, 14, 15, 17, 20, 21, 22, 24, 26, 27, 32, 33, 34, and 35

### Multiplication

The multiplication lessons in this strand demonstrate the power of the Minicomputer with its inherent decimal place-value to introduce and reinforce fundamental multiplication patterns. The introduction of "weighted checkers" facilitates the portrayal of multiplication on the Minicomputer. For example:




The presentation of such patterns helps to extend the students' capabilities with a paper-and-pencil algorithm to include problems such as 43 x 36, to further develop mental arithmetic abilities, and to provide more experiences with the multiplication of decimal numbers by whole numbers.

Besides the work on the Minicomputer, other activities provide opportunities to apply multiplication to new situations. Calculator and division lessons employ multiplication as a shortcut for investigating problems that involve repeated additions or subtractions. A story about Bobo the monkey continues the development of methods for multiplying a whole number by a fraction. There are many opportunities for students to use number classifications such as "multiples of 6" and thereby work with multiplication facts.

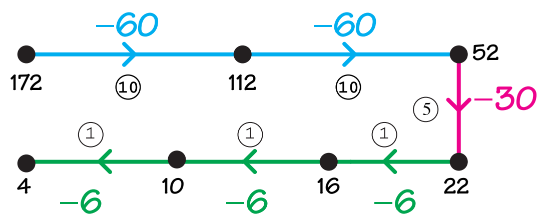
Lessons: N1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 15, 17, 18, 19, 20, 22, 23, 24, 25, 27, 28, 29, 30, 32, 33, 34, and 35

## Division

*CSMP* students have already experienced division both as a sharing process (sharing 108 books equally among three classes) and as repeated subtraction (finding how many 12s are in 200). The lessons in *IG-II* continue the development toward an efficient paper-and-pencil algorithm for division and extend students' experiences with division to new patterns and applications.

Arrow roads provide an ideal model for illustrating division as repeated subtraction. The problem 172 ÷ 6 can be interpreted as, "How many sixes are there in 172?" With arrows, the equivalent problem is, "How many –6 arrows are there in a road from 172 to a positive number less than 6?" But students quickly realize that not only –6 arrows are useful.

> To divide 172 by 6, a student may use –60 arrows, –30 arrows, and –6 arrows to determine that there are 28 sixes in 172.




$$\begin{array}{r} 28 \text{ R} = 4 \\ 6\overline{)172} \end{array}$$

Allowing students flexibility in choosing which arrows to use acknowledges that at any one time students differ dramatically in ability and confidence. Some students prefer using only –6,–60, and –600 arrows. A few students creatively use a variety of arrows, –30, –24, –120, –12, ... . The choice of arrows reflects various stages of development in progressing toward a standard division algorithm.

In order to build a broader understanding of division than the algorithm alone provides, students investigate and apply many patterns involving division. Patterns often suggest mental arithmetic techniques for solving division problems.

| | |
|---|---|
| $56 \div 7 = 8$ | $170 \div 5 = 34$ |
| $560 \div 7 = 80$ | $175 \div 5 = 35$ |
| $5\,600 \div 7 = 800$ | $185 \div 5 = 37$ |

$$\begin{array}{r} 6 \text{ R}=0 \\ 7\overline{)42} \end{array} \quad \begin{array}{r} 6 \text{ R}=1 \\ 7\overline{)43} \end{array} \quad \begin{array}{r} 6 \text{ R}=2 \\ 7\overline{)44} \end{array} \quad \begin{array}{r} 6 \text{ R}=3 \\ 7\overline{)45} \end{array}$$

$$\begin{array}{r} 6 \text{ R}=4 \\ 7\overline{)46} \end{array} \quad \begin{array}{r} 6 \text{ R}=5 \\ 7\overline{)47} \end{array} \quad \begin{array}{r} 6 \text{ R}=6 \\ 7\overline{)48} \end{array} \quad \begin{array}{r} 7 \text{ R}=0 \\ 7\overline{)49} \end{array}$$

In the lessons, the division activities described above occur within a variety of appealing stories about bicycling cross-country, packing softballs into boxes, and arranging the members of a marching band into lines. Such applications both illustrate when division is appropriate and provide insights into the concept of division.

Lessons: N1, 4, 6, 9, 10, 12, 13, 17, 19, 23, 24, 32, and 33

# WORLD OF NUMBERS INTRODUCTION

## Negative Integers

*CSMP* introduces negative integers in first grade through a story about Eli the Elephant and magic peanuts. The story leads to a model for adding integers, first by pictures, then also on the Minicomputer. By the end of third grade, *CSMP* students have encountered negative numbers in games, in reading outdoor temperatures, in applications such as elevator problems, and in recording statistics. These experiences extend the concept of order from whole numbers to negative numbers.

The activities in this strand increase the students' familiarity with negative numbers in many contexts. The goal is to portray negative integers not as a strange new set of numbers, but as a natural and necessary extension of counting numbers. Therefore, few lessons focus on negative numbers but many lessons include them.

Negative numbers appear regularly on the Minicomputer, in *Minicomputer Golf,* in *The String Game* with numbers, and in detective stories. *Guess My Rule* activities, arrow roads, and calculator activities often include negative numbers. Through these experiences, students gradually learn the characteristics of negative numbers and accept them as "real" numbers.

*CSMP* employs a special notation for representing negative numbers. Traditional approaches to arithmetic often make no distinction on the printed page between the function "subtract 3" and the number "negative 3"; both are denoted by "–3." Only by context can a person discern the intended meaning of "–3." In *CSMP*, negative numbers are distinguished from subtraction in the following ways:

- The minus sign "–" is reserved for subtraction. Thus, for example, "–14" denotes the function "subtract 14."
- The $\frown$ symbol denotes a negative number. Thus, "$\overset{\frown}{14}$" denotes the number "negative 14." The $\frown$ symbol was introduced first in the story about Eli the Elephant.
- In context, other notations for negative numbers are recognized. For example, in calculator situations, a raised "–" is used to denote a negative number, for example, $^{-}14$.

We recommend that you continue to use the $\frown$ notation for negative numbers and recognize alternative notations as students encounter them in other contexts (calculators, temperature, tests, and so on).

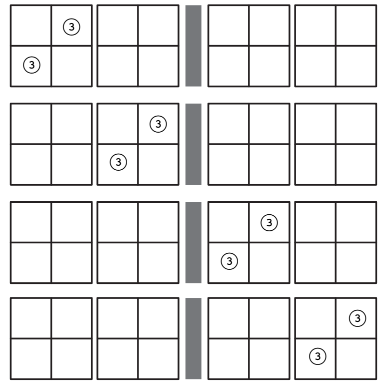
Lessons: N6, 7, 8, 14, 17, 22, 26, 27, 32, 33, 34, and 35

## Decimal Numbers

*CSMP Mathematics for the Upper Primary Grades* introduces decimal numbers, motivated both by problems involving our decimal system of money and by solving problems on the Minicomputer, such as 7 ÷ 2 = 3.5. The lessons in this strand continue to build a strong understanding of decimal numbers as preparation for further concepts and computational skills.

The lesson this semester utilize three interrelated models for decimal numbers: money, the Minicomputer, and the number line. Money reflects students' most frequent exposure to decimal numbers in everyday life and provides support for calculations such as 0.8 + 0.4 (8 dimes + 4 dimes) equals 1.2, not 0.12.

# WORLD OF NUMBERS INTRODUCTION



$= 3 \times 60 = 180$

$= 3 \times 6 = 18$

$= 3 \times 0.6 = 1.8$

$= 3 \times 0.06 = 0.18$

Many decimal number patterns on the Minicomputer are simply extensions of whole number patterns that students have already encountered. The Minicomputer is an ideal tool, with which they are already familiar, for demonstrating the consistency of the decimal place value system.

The various models for decimal numbers compliment each other. Whereas the Minicomputer highlights patterns, the number line and money focus on order and relative magnitude of decimal numbers. For example, the game *Intervals* forces students to find numbers between two given numbers such as 4.77 and 4.78.

The goal of all these activities is to provide opportunities for students to discover and become familiar with the subtleties of decimal numbers, foregoing a too early reliance on rules and mechanical manipulation of the numbers.

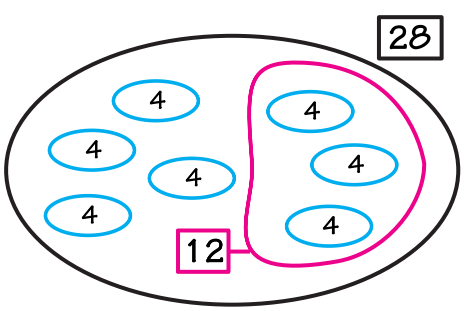
Lessons: N2, 3, 4, 8, 10, 11, 13, 15, 20, 21, 22, 28, 29, and 31

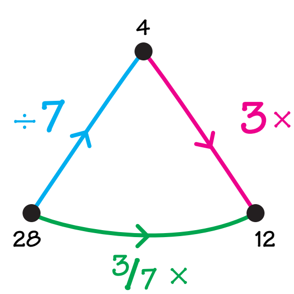
## Fractions

*CSMP Mathematics for the Upper Primary Grades* introduces fractions through activities involving the sharing of equal quantities; for example:

- When three children share a cake fairly, each child receives ⅓ of the cake.
- If a class of 28 students divides into two equal-sized groups, each group has ½ x 28 or 14 students.

In the intermediate grades the emphasis shifts from unit fractions (for example, ¼) to other fractions (for example, ¾). A story about a greedy monkey who eats more that his share of bananas, supports the extension from calculating $\frac{1}{7}$ x 28 to calculating $\frac{3}{7}$ x 28. Students use both string pictures and arrow pictures to do these calculations.




Both pictures demonstrate that $\frac{3}{7}$ x 28 = 12. The string picture clearly portrays both the fair sharing of the 28 bananas among the seven monkeys and what happens when Bobo snatches three shares. The arrow picture succinctly records the calculations and suggests a general composition rule for multiplying an integer by a fraction.

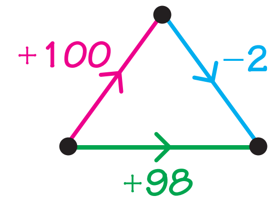
# WORLD OF NUMBERS INTRODUCTION

An additional goal this semester is further development of the number line as a model for fractions. Students mark ½'s, ⅓'s, ¼'s, ⅙'s and ⅛'s on fraction strips and then use the strips to compare fractions and to identify equivalent fractions; revisiting the story about Sasquatch motivates consideration of fractions greater that 1; and several mental arithmetic activities involve counting forward and backward by fractions.

Lessons N5, 16, 19, 20, 22, 23, and 30

## Composition of Functions

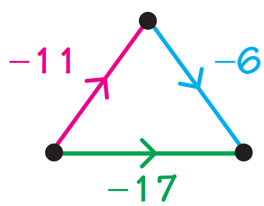
Several lessons in this strand deal with what happens when you compose a sequence of functions, that is, when you apply the functions in order, one at a time. These compositions lead to many general, powerful insights into the properties of numbers and operations. Arrow diagrams provide a concrete means to study this abstract but practical concept. For example, it is clear that if you add 100 to any number and then you subtract 2 from the result, the net effect is to add 98 to the original number.

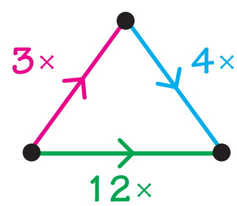


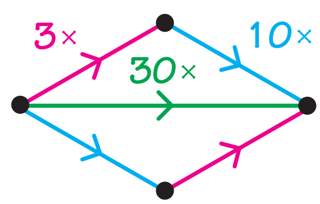

Besides succinctly depicting the composition, this arrow picture also suggests that an easy way to mentally add 98 to any number is to add 100 and then subtract 2. (For example, 86 + 100 = 186 and 186 – 2 = 184, so 86 + 98 = 184.)

Your students' extensive experience with the composition of functions in the Upper Primary Grades curriculum led them to many insights that involved the development of algorithms, the discovery of number patterns, and efficient mental arithmetic techniques. A goal in this strand is to review these discoveries and to apply composition to new situations and problems.

In conjunction with stories and the Minicomputer, several compositions are reviewed, for example:

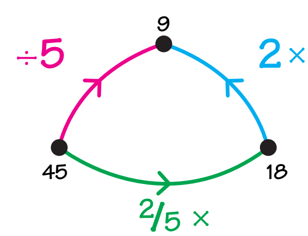



Many pairs of functions, for example, 3x and 10x, *commute*; that is, they produce the same effect regardless of which order they are applied. +100 and –2, –11 and –6, and 3x and 4x also all commute. But in the *Guess My Rule* activities there are examples that do not commute, for example, 2x and +1.

In the lessons on fractions, the "monkeys and bananas" stories suggest that composition be used to calculate, for example, ⅖ x 45.




As students become familiar with the composition of functions, the concept becomes useful as a problem-solving tool. In the game of *Minicomputer Golf*, composition aids in the finding of two-move winning combinations. The compositions of functions exemplifies how the language of arrows is able to visually highlight rich and practical mathematical concepts and techniques.

Lessons: N8, 10, 12, 13, 17, 18, 19, 23, and 27

# N1 DIVISION #1

## Capsule Lesson Summary

Practice subtracting 9, 99, and powers of 10 from large numbers. Tell a story about Nabu packing softballs into cartons of a specified capacity. Use repeated subtraction in arrow pictures to determine the number of boxes that Nabu fills with a given number of softballs.

### Materials

**Teacher**
- Colored chalk
- Props (optional)

**Student**
- Paper
- Colored pencils, pens, or crayons
- Worksheets N1*, **, ***, and ****

**Advance Preparation:** You may like to use props to introduce the packing problems in Exercise 2.

## Description of Lesson

### Exercise 1: Mental Arithmetic

**Note:** Spend about ten minutes on this exercise. Adjust the difficulty of the suggested mental arithmetic to the abilities of your students. This exercise can be repeated with similar problems whenever you have a few minutes available.

Write this number on the board and ask students to read it.

835 342

**S:** *Eight hundred thirty-five thousand, three hundred forty-two.*

**T:** *What is 835 342 – 10? You do not need pencil and paper to do this calculation.*

**S:** *835 332.*

**T:** *What is 835 332 – 10?*

**S:** *835 322.*

Draw –10 arrows and record the answers.

$$\begin{array}{l} 835\,342 \\ \quad \downarrow -10 \\ 835\,332 \\ \quad \downarrow -10 \\ 835\,322 \end{array}$$

**T:** *What is 835 322 – 100?*

**S:** *835 222.*

**T:** *Why?*

**S:** *Because you take away one of the three hundreds. The rest of the number stays the same.*

You may wish to write the subtraction problem in a standard format on the board to highlight that only one digit changes.

$$\begin{array}{r} 835\,322 \\ -100 \\ \hline 835\,222 \end{array}$$

Present the following problems in a similar manner, occasionally stopping to let students explain their answers. (Answers are in boxes.)

835 222
$\downarrow -10\,000$
$\boxed{825\,222}$
$\downarrow -100\,000$
$\boxed{725\,222}$
$\downarrow -100\,000$
$\boxed{625\,222}$
$\downarrow -1\,000$
$\boxed{624\,222}$
$\downarrow -10$
$\boxed{624\,212}$
$\downarrow -10$
$\boxed{624\,202}$

624 202
$\downarrow -10$
$\boxed{624\,192}$
$\downarrow -1\,000$
$\boxed{623\,192}$
$\downarrow -10\,000$
$\boxed{613\,192}$
$\downarrow -10$
$\boxed{613\,182}$
$\downarrow -100$
$\boxed{613\,082}$
$\downarrow -100$
$\boxed{612\,982}$

Erase the board and begin another sequence of problems. Record the subtractions with –9 arrows as you get responses.

**T:** ***Let's try a new pattern. What is 496 – 9?***

**S:** ***487.***

**T:** ***What is 487 – 9?***

**S:** ***478.***

**T:** ***Does anyone know an easy way to subtract 9?***

**S:** ***Subtract 10 and then add 1.***

**S:** ***Add 1 and then subtract 10.***

496
$\downarrow -9$
487
$\downarrow -9$
478

Mention one of the above techniques for subtracting 9 yourself if neither is suggested by your students. Illustrate it by referring to the two problems just solved.

**T:** ***496 – 10 = 486, so 496 – 9 = 486 + 1 = 487.***
***487 – 10 = 477, so 487 – 9 = 477 + 1 = 478.***

Present these problems in a similar manner.

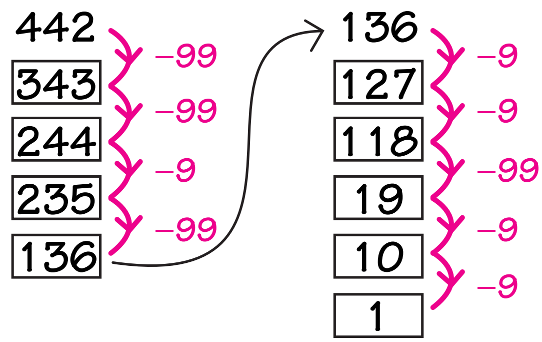
478
$\downarrow -9$
$\boxed{469}$
$\downarrow -9$
$\boxed{460}$
$\downarrow -9$
$\boxed{451}$
$\downarrow -9$
$\boxed{442}$

**T:** ***Do you see any patterns in this list as we subtract 9s?***

**S:** ***Each time the ones digit increases by 1 while the tens digit decreases by 1.***

**T:** ***That's usually true, but not always. For example, 469 – 9 = 460.***

Present the following problems. Encourage students to subtract 99 by subtracting 100 and then adding 1, or by adding 1 and then subtracting 100.




## Exercise 2

Ask the class what they remember about the story of Nabu working in a factory.

**T:** ***What was his job?***

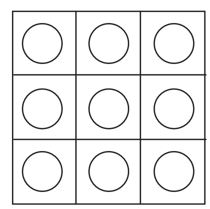
**S:** ***One time he worked in a pencil factory and packed pencils into boxes.***

**S:** ***He also worked at a bottle recycling plant and put bottles into cartons.***

**T:** ***Nabu has changed jobs again. Now he works in a factory making softballs. What do you think Nabu's job is?***

**S:** ***Packing softballs into boxes.***

Draw this picture on the board.



**T:** ***This is a picture of the type of box Nabu uses. How many softballs does he put in each box?***

**S:** ***Nine softballs.***

**T:** ***One day Nabu is given 132 softballs to pack into boxes like this*** (point to the picture on the board)***. About how many boxes will he be able to fill?***

List students' estimates on the board.

**T:** ***Will he fill at least ten boxes?***

**S:** ***Yes, 90 balls fit in ten boxes since 10*** x ***9 = 90.***

**T:** ***Will he fill 100 boxes?***

**S:** ***No, 900 softballs fit in 100 boxes since 100*** x ***9 = 900.***

**T:** ***Will Nabu fill 20 boxes?***

**S:** ***No, 180 balls fit into 20 boxes since 20*** x ***9 = 180.***

**T:** ***Nabu can use an arrow road to help calculate the number of boxes he will be able to fill.***

On the board, draw an arrow road starting at 132 with five red arrows for –9.

**T:** ***Nabu has 132 softballs and will put 9 in the first box. How any softballs will he have left to pack?***

**S:** ***123 softballs, since 132 – 9 = 123.***

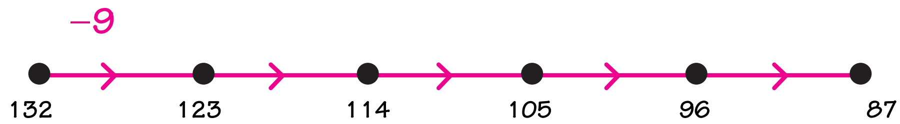
# N1

Label the dot for 123.

**T:** ***He will fill a second box with nine balls. How many balls will he have left?***

**S:** ***114 softballs, since 123 – 9 = 114.***

Label the dot for 114. Similarly, invite students to label the next several dots.

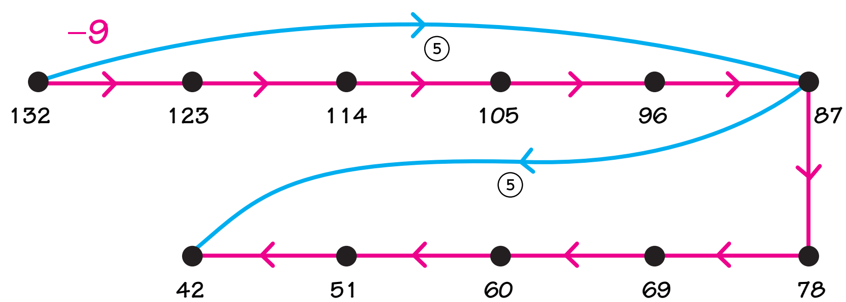



**T:** ***How many boxes has Nabu filled so far?***

**S:** ***Five boxes; there are five red arrows.***

**T:** ***Every time he counts five boxes Nabu draws a blue arrow.***
***I'll add more red arrows. Help me label the dots and tell me when Nabu has filled five more boxes.***

Add one arrow at a time. When you reach 42, students should suggest drawing a blue arrow from 87 to 42.

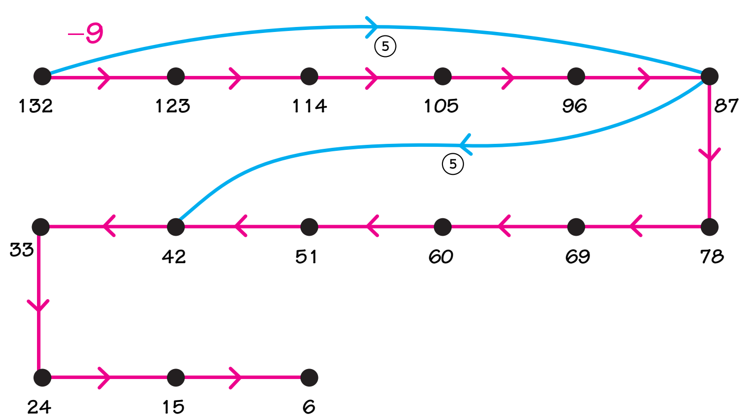



**T:** ***How many boxes has Nabu filled so far?***

**S:** ***Ten boxes; there are ten red arrows from 132 to 42.***

**S:** ***Ten boxes; there are two blue arrows from 132 to 42, and each blue arrow represents five boxes. 2 x 5 = 10.***

Add red arrows until you reach 6.




**T:** ***How many boxes can Nabu fill?***

**S:** ***14 boxes. There will be 6 softballs left over.***

Follow the two blue arrows and four red arrows as you count the number of boxes Nabu fills.

**T:** ***5, 10, 11, 12, 13, 14.***
***Yes, Nabu can fill 14 boxes.***

Some students may suggest that Nabu could put the extra six balls in another box. Accept this as a possibility, but tell students that Nabu always fills boxes and leaves any extra softballs unpacked.

**T:** ***What are the blue arrows for?***

**S:** ***–45; each blue arrow represents five boxes of nine softballs and 5 x 9 = 45.***

**S:** ***–45, since 132 – 45 = 87, and 87 – 45 = 42.***

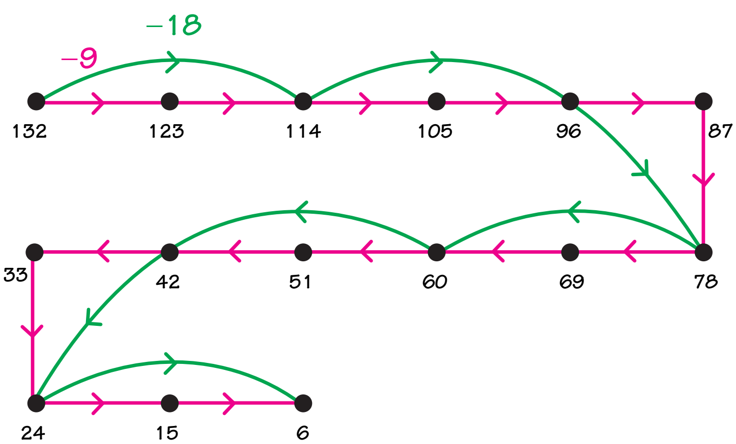
Erase the blue arrows.

**T:** ***Nabu's company decides to put two layers in a box, so now it will take 18 softballs to fill each box. How many of these larger boxes could Nabu fill with 132 softballs?***

**S:** ***Seven boxes. He could fill half as many boxes since each large box holds twice as many softballs. ½ x 14 = 7. There would still be six softballs left over.***

**T:** ***Let's check by drawing –18 arrows in green to represent filling the larger boxes. Start at 132 and draw –18 arrows.***

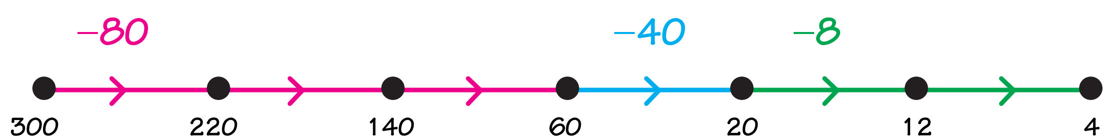
Invite students to draw the arrows from 132 to 114, from 114 to 96, and so on to 6. Observe that two –9 arrows is the same as one –18 arrow. Count the –18 arrows to check that seven large boxes are filled.




## Exercise 3

**T:** ***One day Nabu is given 300 softballs to put into boxes that hold eight softballs each. He starts drawing a –8 arrow road from 300, but realizes the arrow road will have a lot of arrows. Instead, he cleverly draws this arrow road. Try to figure out the shortcuts Nabu is using.***

Draw this arrow road on the board.




**T:** ***How many boxes of eight does Nabu fill with 300 softballs?***

Perhaps a student will suggest six boxes since there are six arrows. In this case, observe that six boxes would be the answer if all of the arrows were for –8, but here Nabu used arrows for –80 and –40 also.

**S:** ***37 boxes.***

Ask a student to show the class how to use the arrow road to count the needed boxes, or do it yourself if necessary.

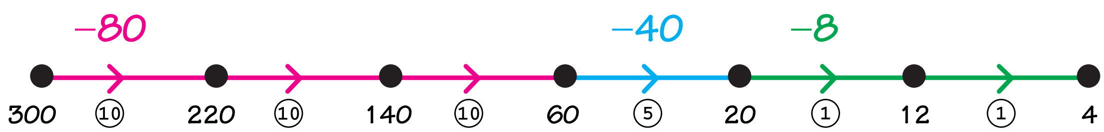
**S:** ***Each –80 (red) arrow represents packing ten boxes of eight since 10 x 8 = 80. Three red arrows show filling 30 boxes.***

**T:** ***After Nabu fills 30 boxes, there are 60 softballs left to pack. He cannot use another –80 arrow.***

**S:** ***The –40 (blue) arrow represents packing five boxes of eight since 5 x 8 = 40. After Nabu fills 35 boxes, he has 20 softballs left.***

**S:** ***Each –8 (green) arrow represents filling one box.***

Write the number of boxes filled—10, 5, or 1—near each arrow as mentioned.




Point to the appropriate arrows as you count the number of boxes Nabu fills.

**T:** ***10, 20, 30, 35, 36, 37. Nabu needs 37 boxes. How many softballs will be left over?***

**S:** ***Four softballs.***

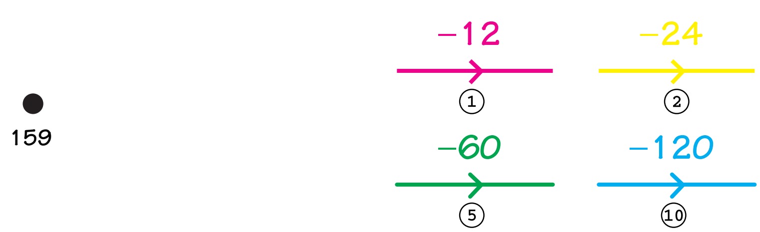
**T:** ***By using –80, –40, and –8 arrows, Nabu draws a fairly short arrow road to calculate the number of boxes that he needs.***

## Exercise 4

Pose another packing problem for students to solve individually or with a partner.

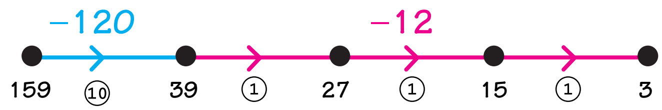
**T:** ***Nabu has new boxes that hold 12 softballs each. He must pack 159 softballs. He could draw an arrow road starting from 159 and use only –12 arrows, but what other arrows could he use in order to draw a shorter road?***

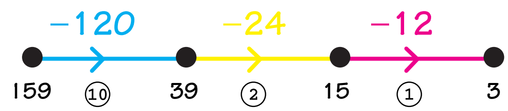
Many answers are possible. As they are mentioned, draw keys on the board.

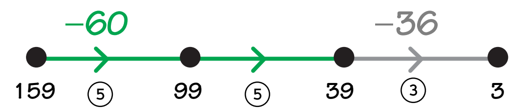


**T:** ***Draw an arrow road to determine the number of boxes Nabu will fill. Start at 159 and end at a number less than 12. You may use any of these arrows that you wish, but be sure to label your arrows.***

As students find solutions, invite some to draw their arrow roads on the board.



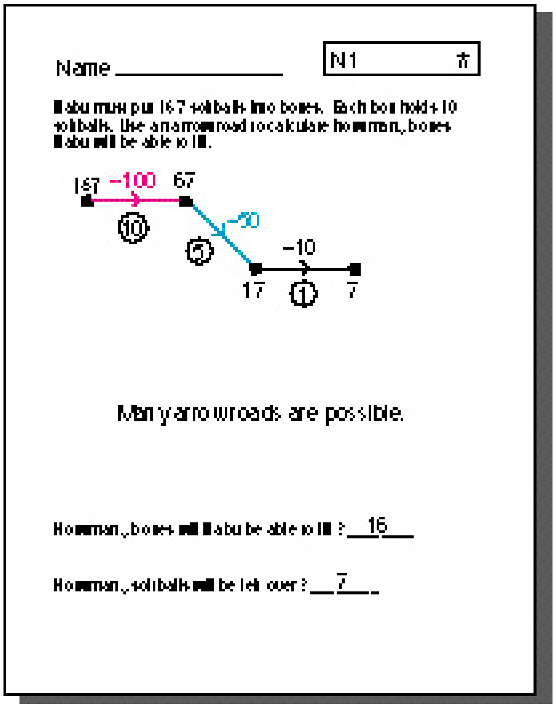



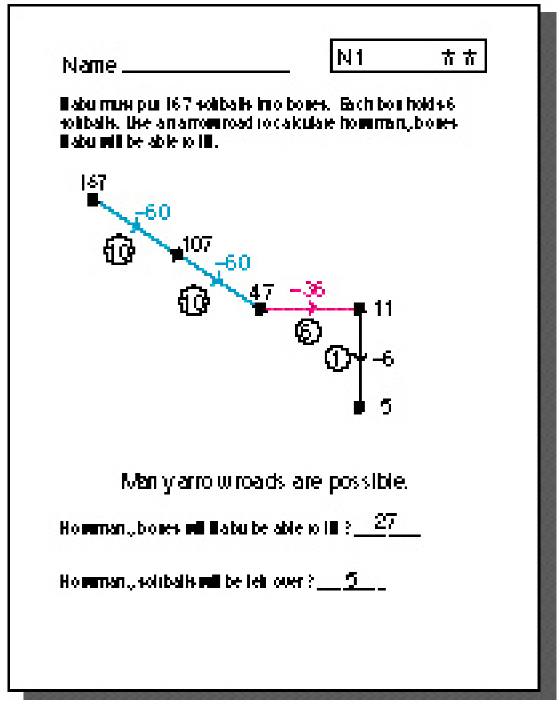

Refer to the arrow roads on the board and show how each road indicates that 13 boxes will be filled and three softballs are left over.

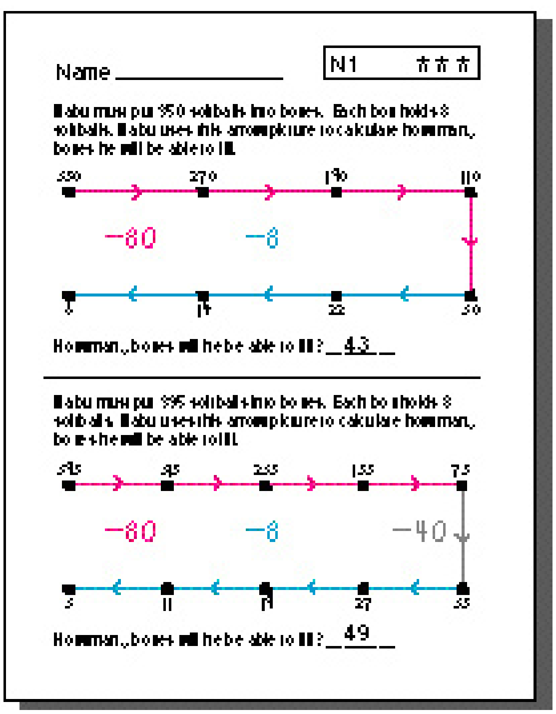
Worksheets N1*, **, ***, and **** are available for individual work.

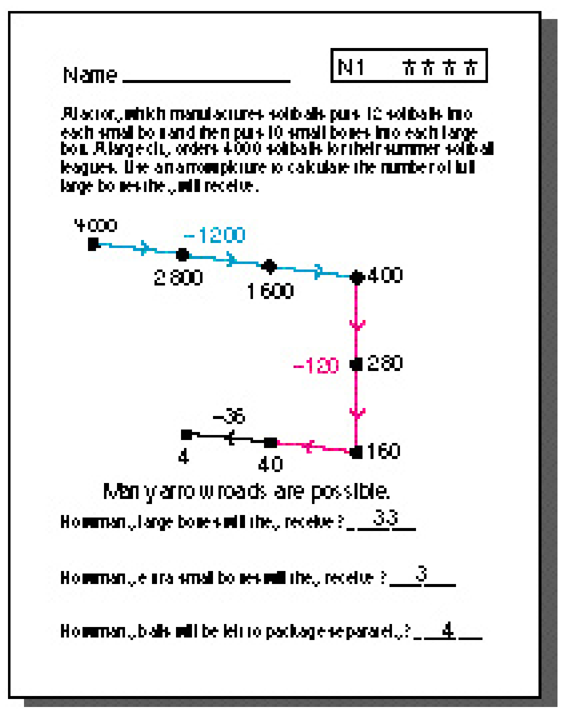
## Writing Activity

You may like students to take lesson notes on some, most, or even all their math lessons. The "Lesson Notes" section in Notes to the Teacher gives suggestions and refers to forms in the Blacklines you may provide to students for this purpose. In this lesson, for example, students may note how arrow roads are used to solve Nabu's packing problems. They may like to describe a new packing job for Nabu.


Name
N1 ☆
Babu must put 167 softballs into boxes. Each box holds 10 softballs. Use an arrow road to calculate how many boxes Babu will be able to fill.
167
−100
67
10
5
−50
−10
17
1
7
Many arrow roads are possible.
How many boxes will Babu be able to fill? 16
How many softballs will be left over? 7


Name
N1 ☆☆
Babu must put 167 softballs into boxes. Each box holds 6 softballs. Use an arrow road to calculate how many boxes Babu will be able to fill.
167
−60
10
107
−60
10
47
−36
11
6
1
−6
5
Many arrow roads are possible.
How many boxes will Babu be able to fill? 27
How many softballs will be left over? 5


Name
N1 ☆☆☆
Babu must put 350 softballs into boxes. Each box holds 8 softballs. Babu uses this arrow picture to calculate how many boxes he will be able to fill.
350
270
190
110
−80
−8
6
14
22
30
How many boxes will he be able to fill? 43
Babu must put 395 softballs into boxes. Each box holds 8 softballs. Babu uses this arrow picture to calculate how many boxes he will be able to fill.
395
315
235
155
75
−80
−8
−40
3
11
19
27
35
How many boxes will he be able to fill? 49


Name
N1 ☆☆☆☆
A factory, which manufactures softballs, puts 12 softballs into each small box and then puts 10 small boxes into each large box. A large city orders 4000 softballs for their summer softball leagues. Use an arrow picture to calculate the number of full large boxes they will receive.
4000
−1200
2800
1600
400
−120
280
−36
160
4
40
Many arrow roads are possible.
How many large boxes will they receive? 33
How many extra small boxes will they receive? 3
How many balls will be left to package separately? 4

# N2 NEW CHECKERS ON THE ____________

## Capsule Lesson Summary

Introduce these weighted checkers: ②  ③  ④  ⑤  ⑥  ⑦  ⑧  ⑨

Put various whole and decimal numbers on the Minicomputer using exactly one of these weighted checkers. Use weighted checkers to emphasize parts of a multiplication calculation; for example, 5 x 484 = (5 x 400) + (5 x 80) + (5 x 4 )

### Materials

**Teacher**
- Minicomputer set
- Weighted checker set
- Worksheets N2*, **, ***, and ****

**Student**
- Minicomputer set

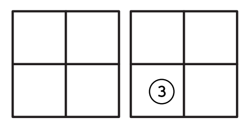
## Description of Lesson

### Exercise 1

Display two Minicomputer boards and these weighted checkers: ②, ③, ④, ⑤, ⑥, ⑦, ⑧, ⑨.

**T:** ***Today we have some new checkers. We've used checkers with 10s on them before. A ⑩-checker on a square shows the same number as ten regular checkers on the square.***

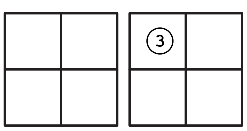
Put a ③-checker on the 2-square.



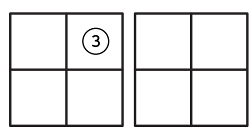
**T:** ***A ③-checker on a square shows the same number as three regular checkers on that square. What number is this?***

**S:** ***6; 3 x 2 = 6.***

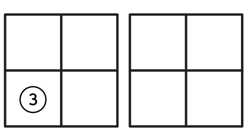
Move the ③-checker to other squares, each time asking for the number.



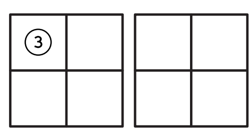
$= 3 \times 8 = 24$



$= 3 \times 40 = 120$



$= 3 \times 20 = 60$

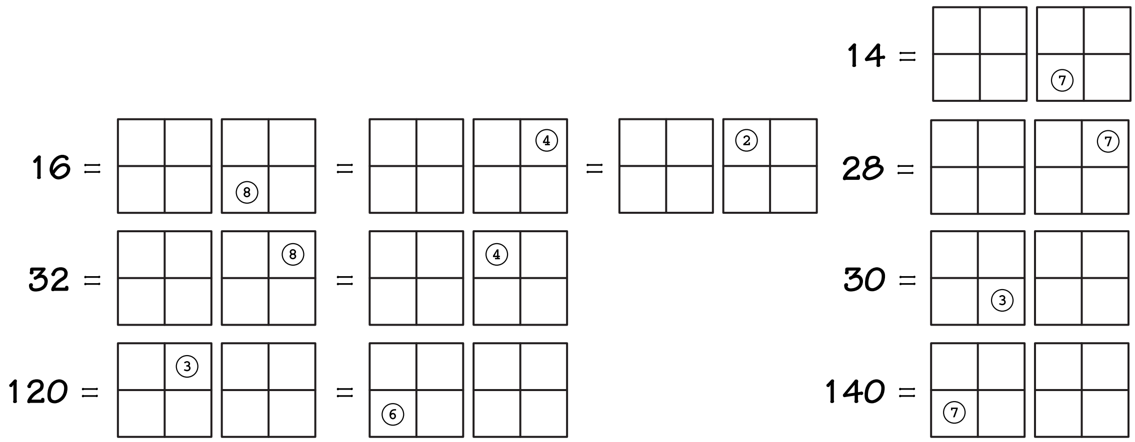


$= 3 \times 80 = 240$

Repeat the activity using a ⑤-checker and then using an ⑧-checker.

## Exercise 2

Ask students how to put these numbers on the Minicomputer, using exactly one of the new checkers for each number. Your class should be able to find all of the possible ways to display a number with one of the given checkers.

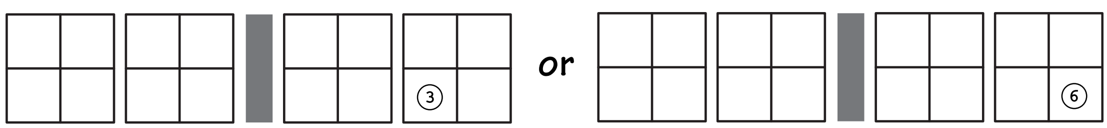



## Exercise 3

Display four Minicomputer boards with a bar between the second and third boards.

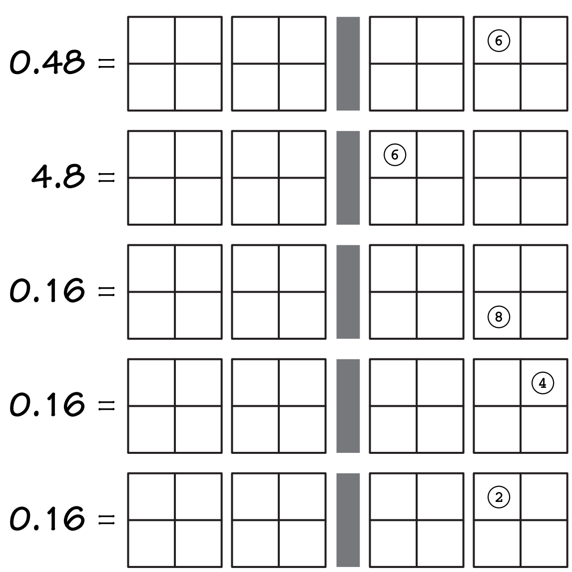
**T:** ***How can we put on 0.06*** (read as "zero point zero six" or "six cents") ***using exactly one of these new checkers?***

Record the corresponding multiplication facts.




$0.06 = 3 \times 0.02$ $\quad$ $0.06 = 6 \times 0.01$

Ask students how to put on the following numbers using exactly one of the new checkers for each. Record the corresponding multiplication facts.




$6 \times 0.08 = 0.48$

$6 \times 0.8 = 4.8$

$8 \times 0.02 = 0.16$

$4 \times 0.04 = 0.16$

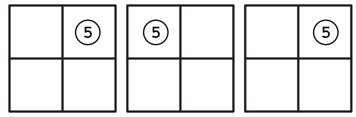
$2 \times 0.08 = 0.16$

## Exercise 4

Display three Minicomputer boards and have several of each weighted checker available.

**T:** ***Let's put 5 x 484 on the Minicomputer using the new checkers. How can we do this?***

**S:** ***Use ⑤-checkers.***

 = 5 × 484

Ask students what number is one each board of the Minicomputer. Record the corresponding multiplication facts on the board.

**S:** ***5 x 4 = 20 (on the ones board).***

**S:** ***5 x 80 = 400 (on the tens board).***

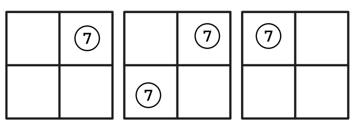
**S:** ***5 x 400 = 2 000 (on the hundreds board).***

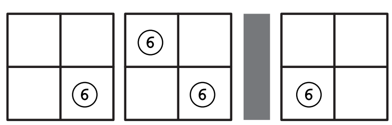
$$\begin{array}{rcrcr} 5 & \times & 4 & = & 20 \\ 5 & \times & 80 & = & 400 \\ 5 & \times & 400 & = & 2\,000 \\ \hline 5 & \times & 484 & = & 2\,420 \end{array}$$

You may like to let a student do the multiplication calculation in the usual compact way.

$$\begin{array}{r} {\scriptstyle 4\ 2\ \ } \\ 484 \\ \times\ 5 \\ \hline 2\,420 \end{array}$$

Repeat the activity with 7 x 468 and 6 x 19.2.

 = 7 × 468 = 3 276

 = 6 × 19.2 = 115.2

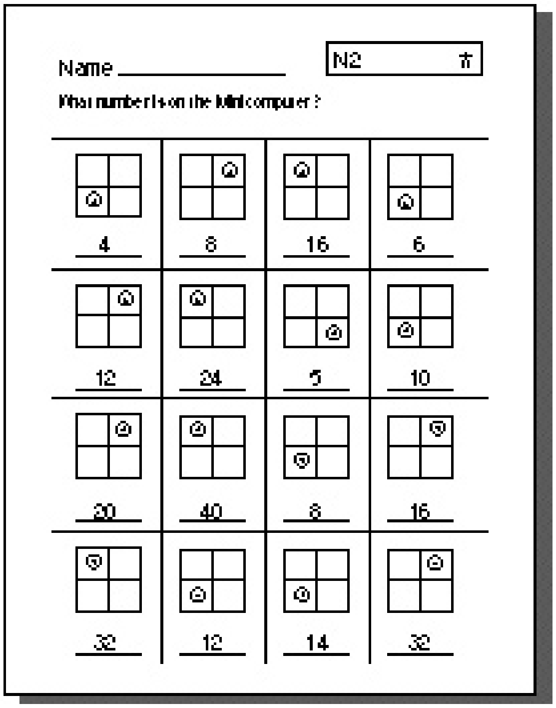
Worksheets N2*, **, ***, and **** are available for individual work. Allow students to use individual Minicomputers.

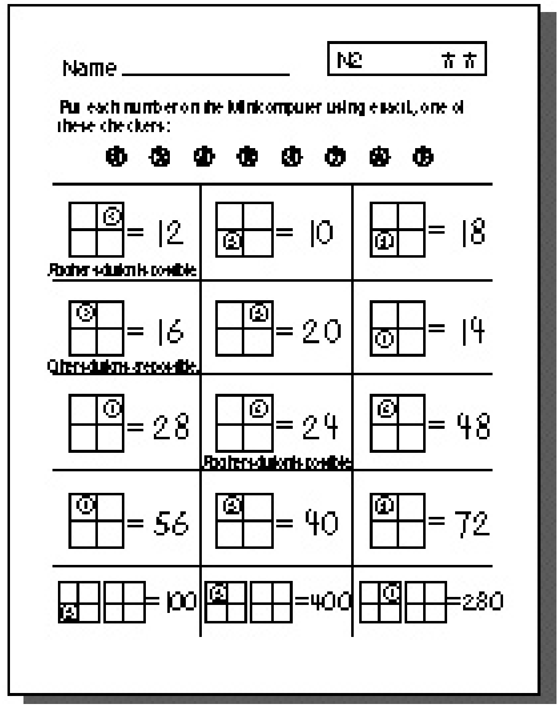
### Writing Activity

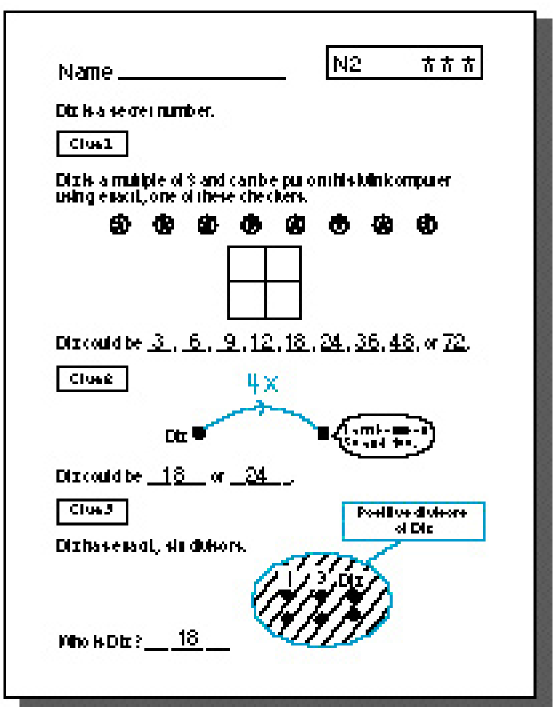
Ask students to list all the two-digit numbers that they can put on the Minicomputer using exactly one weighted checker ②…⑨.

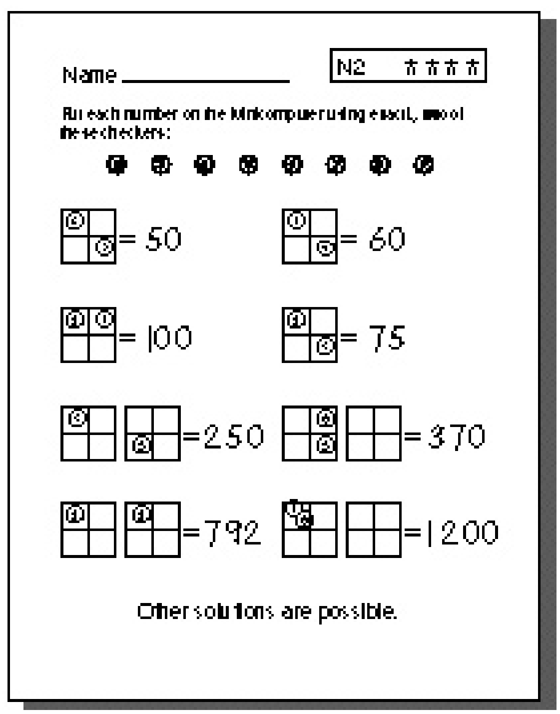
### Home Activity

Suggest that parents/guardians use weighted checkers on the home Minicomputer to practice multiplication facts and to look at parts of a multiplication calculation.


Name
N2 ☆
What number is on the Minicomputer?
4
8
16
6
12
24
5
10
20
40
8
16
32
12
14
32


Name
N2 ☆☆
Put each number on the Minicomputer using exactly one of these checkers:
= 12
= 10
= 18
= 16
= 20
= 14
= 28
= 24
= 48
= 56
= 40
= 72
= 100
=400
=280


Name
N2 ☆☆☆
Diz is a secret number.
Clue 1
Diz is a multiple of 3 and can be put on this Minicomputer using exactly one of these checkers.
Diz could be 3, 6, 9, 12, 18, 24, 36, 48, or 72.
Clue 2
4x
Diz
Diz could be 18 or 24.
Clue 3
Positive divisors of Diz
Diz has exactly six divisors.
1
3
Diz
Who is Diz? 18


Name
N2 ☆☆☆☆
Put each number on the Minicomputer using exactly two of these checkers:
= 50
= 60
= 100
= 75
=250
=370
=792
=1200
Other solutions are possible.

## Capsule Lesson Summary

Review decimal numbers using the Minicomputer, the number line, and the calculator.

### Materials

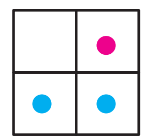
**Teacher**
- Minicomputer set
- Calculator
- Colored chalk

**Student**
- Paper
- Worksheets N3*, **, and ***
- Calculator

## Description of Lesson

### Exercise 1

Display one Minicomputer board and put on this configuration of red and blue checkers.



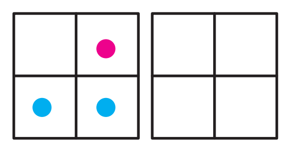
**T:** ***Tell me an addition problem that you see with red and blue checkers on the Minicomputer.***

**S:** *4 + 3 = 7.*

**S:** *3 + 4 = 7.*

Write these addition facts on the chalkboard.

Add one board to the right.



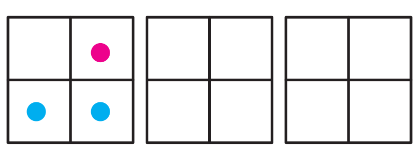
**T:** ***Tell me an addition problem that you see now with red and blue checkers.***

Write the addition sentences on the chalkboard as students give them.

**S:** *40 + 30 = 70.*

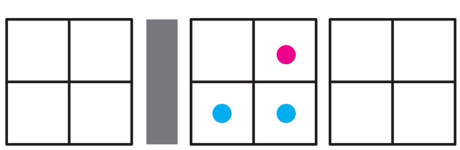
**S:** *30 + 40 = 70.*

Continue in this manner, first adding one more board to the right …



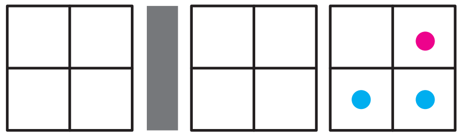
400 + 300 = 700
300 + 400 = 700

… and then moving the boards to this position.



0.4 + 0.3 = 0.7
0.3 + 0.4 = 0.7

Next, reverse the dimes (tenths) and pennies (hundredths) boards.



$$0.04 + 0.03 = 0.07$$
$$0.03 + 0.04 = 0.07$$

Similarly, generate this sequence of addition sentences with the help of the Minicomputer.

Perhaps 0.8 + 0.4 = 1.2 will give the class some trouble. A frequent response is 0.12. You may want to rephrase the problem in terms of money (for example, 8 dimes plus 4 dimes, or 80 cents plus 40 cents) to help students see that 0.8 + 0.4 = 1.2.

$$\begin{aligned} 8 &+ 4 &= 12 \\ 80 &+ 40 &= 120 \\ 800 &+ 400 &= 1\,200 \\ 0.8 &+ 0.4 &= 1.2 \\ 0.08 &+ 0.04 &= 0.12 \end{aligned}$$

Write the following problems on the board and ask students to do the calculations on their papers. Encourage the use of patterns like the ones just seen with the Minicomputer. (Answers are in boxes.)

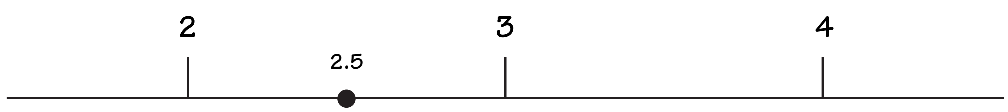
$$37 + 25 = \boxed{62} \qquad 3.7 + 2.5 = \boxed{6.2}$$
$$370 + 250 = \boxed{620} \qquad 0.37 + 0.25 = \boxed{0.62}$$

### Exercise 2

Draw part of a number line with marks evenly spaced for 2, 3, and 4.

**T:** ***Where is 2.5 on the number line?***

Let a student put a dot in the approximate location.




**T:** ***Yes, 2.5 is halfway between 2 and 3. It is the midpoint of the segment with endpoints 2 and 3. Where is 2.7?***

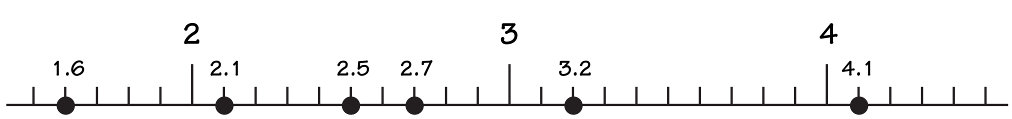
Let a student point to the approximate location.

**T:** ***How could we locate 2.7 more accurately?***

Perhaps students will suggest you divide the segment between 2 and 3 into ten smaller segments of the same length. Do so, and continue subdividing until you have the number line as pictured below.

**T:** ***Where is 2.1? 3.2? 4.1?***
***Where is 1.6?***

Encourage students to count backward from 2 to locate 1.6.

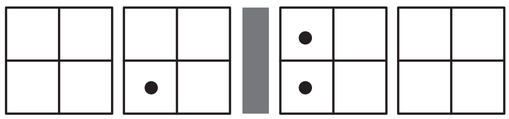


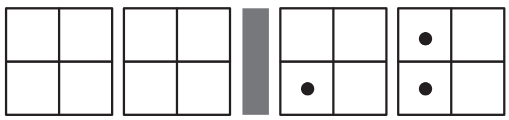
## Exercise 3

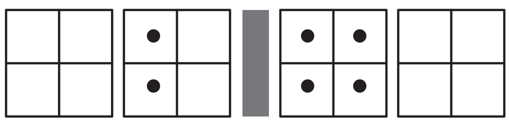
Arrange that every student has access to a calculator. Return to the Minicomputer.

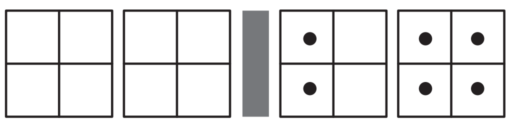
**T:** ***I will put a number on the Minicomputer. Try to put the number on the display of your calculator.***

Let the class decode the following configurations. (The answers at the right should be written on the board after students give them.) If some students have difficulty decoding a configuration, invite those students to make trades before announcing the number.


= 3 = 3.0

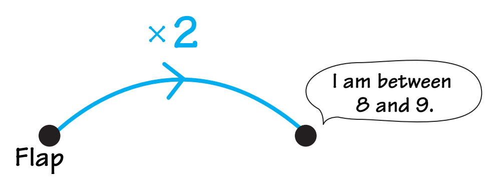

= 0.3 = 0.30


= 11.50 = 11.5


= 1.15

## Exercise 4

Draw a dot and label it Flap.




**T:** ***Flap is a secret number. If I put Flap on the display of the calculator and press*** $\boxed{\times}$ $\boxed{2}$ $\boxed{=}$***, the new number I get is between 8 and 9.***

Add this information with an arrow.

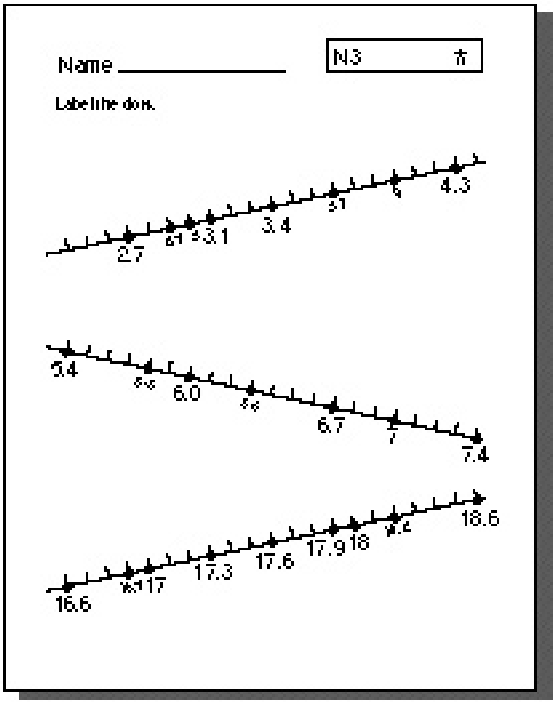
**T:** ***On your paper write some numbers that Flap could be.***

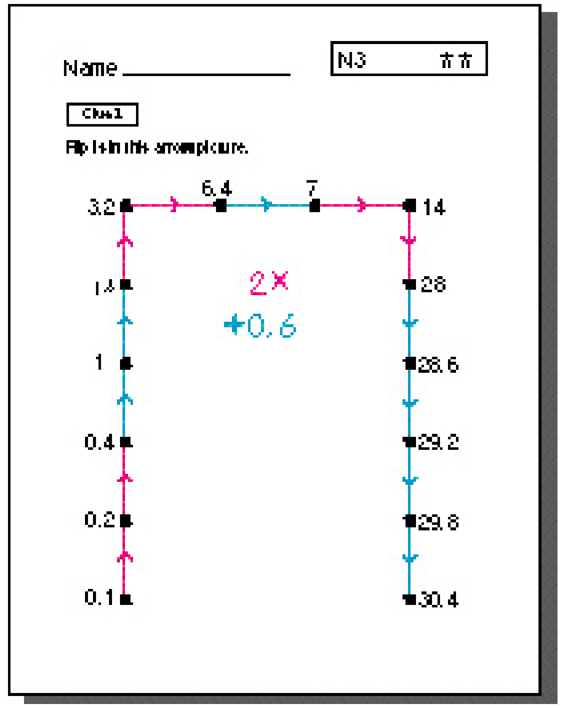
Let students use their calculators to find numbers that Flap could be. You may be surprised by the variety of numbers that they find; for example, 4.1; 4.2; 4.4; 4.31; 4.33; and 4.45. Flap could be any number between 4 and 4.5.

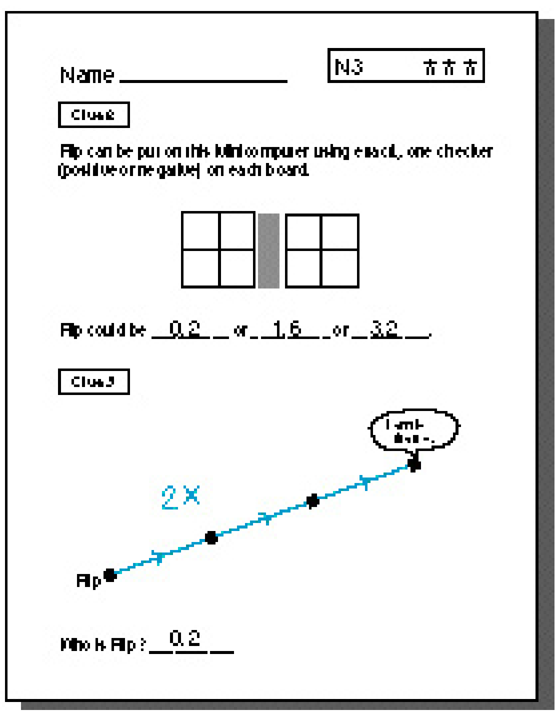
Worksheets N3*, **, and *** are available for individual work.

## Extension Activity

Introduce a calculator game called *Home on the Range*. In this game, two players (or teams) use one calculator. Select a starting number to put on the display and a range (interval) as the goal. Players take turns pressing $\boxed{\times}$ some number $\boxed{=}$. The first player to put the number on the display in the range wins.


Name
N3 ☆
Label the dots.
2.7
3.1
3.4
4.3
5.4
6.0
6.7
7
7.4
16.6
17
17.3
17.6
17.9
18
18.6


Name
N3 ☆☆
Flip is in this arrow picture.
3.2
6.4
7
14
2x
+0.6
28
1
28.6
0.4
29.2
0.2
29.8
0.1
30.4


Name
N3 ☆☆☆
Flip can be put on this Minicomputer using exactly one checker (positive or negative) on each board.
Flip could be 0.2 or 1.6 or 3.2.
2x
Flip
Who is Flip? 0.2

## Capsule Lesson Summary

Name multiples of 10 and multiples of 100 that are between two given numbers. Find 10x a number by making trades with ⑩-checkers on the Minicomputer. Multiply and divide decimals by 10.

### Materials

**Teacher**
- Minicomputer set
- Weighted checker set
- Colored chalk

**Student**
- Colored pencils, pens, or crayons
- Worksheets N4 (no star), N4*, **, and ***

## Description of Lesson

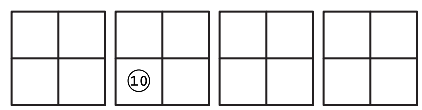
Begin the lesson with some mental arithmetic involving multiples of 10 and multiples of 100. Ask students to name the following:

Multiples of 10
- between 100 and 200
- between 500 and 700
- between 1 000 and 1 500

Multiples of 100
- between 400 and 1 000
- between 2 000 and 4 000
- between 11 000 and 15 000

### Exercise 1

Display four Minicomputer boards and put a ⑩-checker on the 200-square.

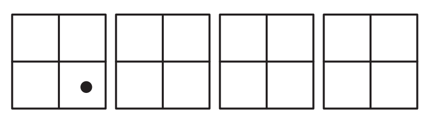


**T:** ***10x what number is on the Minicomputer?*** (200)
***What number is 10 x 200?*** (2 000)

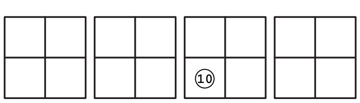
$$10 \times 200 = 2\,000$$

Record the 10x calculation on the board.

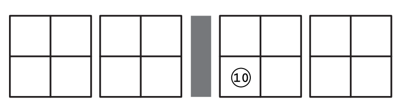
Invite a student to trade the ⑩-checker for a regular checker.



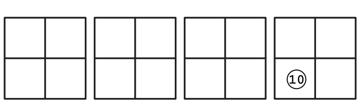
Continue the activity with each of the following configurations, and record the corresponding 10x calculations in a list.



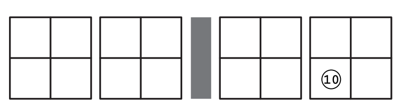
$10 \times 20 = 200$



$10 \times 0.2 = 2$



$10 \times 2 = 20$



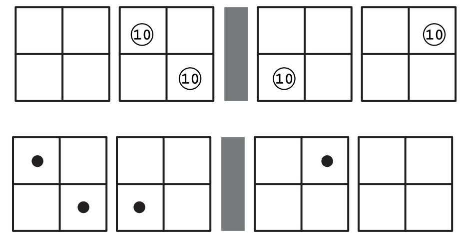
$10 \times 0.02 = 0.2$

Repeat the activity with the following sequences.

10 x 80
10 x 8
10 x 0.8
10 x 0.08

10 x 5
10 x 0.5
10 x 0.05

T: ***Who can put 10* x *9.24 on the Minicomputer using ⑩-checkers? Who can make a trade?***



Invite students to make trades until you have the standard configuration for 92.4.

T: ***What number is 10* x *9.24?***

S: ***92.4 (or 92.40).***

$10 \times 9.24 = 92.4$

Record the 10x calculation.

Similarly calculate 10 x 43.6 on the Minicomputer and record the number sentence: 10 x 43.6 = 436.

Distribute copies of Worksheet N4 (no star) and go over the directions with the class. Give students a few minutes to work individually and then check answers collectively.

T: ***In the first problem, 10*x *what number is on the Minicomputer?***

S: ***0.28.***

T: ***What number is 10* x *0.28?***

S: ***2.80 (or 2.8).***

Record the number sentences from the worksheet on the board.

$$\begin{aligned} 10 \times 0.28 &= 2.8 \\ 10 \times 2.45 &= 24.5 \\ 10 \times 4.29 &= 42.9 \\ 10 \times 9.07 &= 90.7 \\ 10 \times 0.36 &= 3.6 \end{aligned}$$

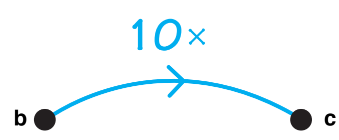
T: ***Do you notice any patterns?***

Let students comment freely, but encourage them to look at the change in position of the decimal point.

Erase the board before going on to Exercise 2.

## Exercise 2

Draw this arrow picture on the board.




Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

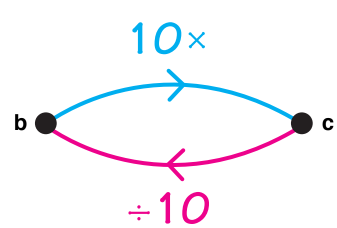
T: ***If 34 is here*** (point to **b**)***, what number is here*** (point to **c**)***?*** (340)

***If 60 is here*** (point to **c**)***, what number is here*** (point to **b**)***?*** (6)

| | |
|---|---|
| 34 | 340 |
| 6 | 60 |

Record pairs of numbers for **b** and **c** in a chart.

Draw a red arrow from **c** to **b**.




Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: ***What could this red arrow be for?***

S: ***÷10.***

S: ***$^1/_{10}$x.***

Label the red arrow ÷10.

**T:** *If this number* (at **c**) *is 2 000, what number is this* (at **b**)*?*

**S:** *200.*

Record 200 and 2 000 in the chart.

**T:** *If this number* (at **c**) *is 57, what number is this* (at **b**)*?*

You should expect some hesitation.

**T:** *About how big is this number (*at **b***)?*

**S:** *10* x *5 = 50, so it must be greater than 5.*

**S:** *10* x *6 = 60, so it must be less than 6.*

**S:** *5.7.*

You may need to help your class see that if **c** is 57, then **b** is between 5 and 6. Record 5.7 and 57 in the chart.

**T:** *If this number* (at **c**) *is 8, what number is this* (at **b**)*?*

**S:** *10* x *1 = 10, so it must be less than 1.*

Some students may think that the corresponding number at **b** is negative. If so, calculate 10x a negative number, for example, 10 x $\widehat{2} = \widehat{2} + \widehat{2} + \ldots + \widehat{2} = \widehat{20}$, and observe that 10x a negative number is a negative number.

**T:** *10* x *0 = 0 and 10* x *1 = 10, so we are looking for a number between 0 and 1.*

**S:** *0.8.*

Record 0.8 and 8 in the chart.

| | |
|---|---|
| 34 | 340 |
| 6 | 60 |
| 200 | 2 000 |
| 5.7 | 57 |
| 0.8 | 8 |
| 0.19 | 1.9 |

**T:** *If this number* (at **c**) *is 1.9, what number is this* (at **b**)*?*
*10*x *how much money is $1.90?*

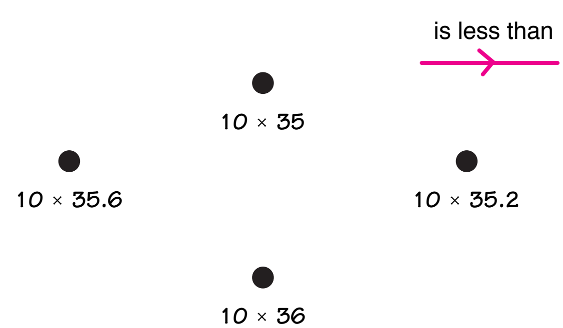
**S:** *19 cents.*

**T:** *What decimal number do we write for 19 cents?*

**S:** *0.19.*

## Exercise 3

Distribute copies of Worksheet N4*.
Copy the picture onto the board.



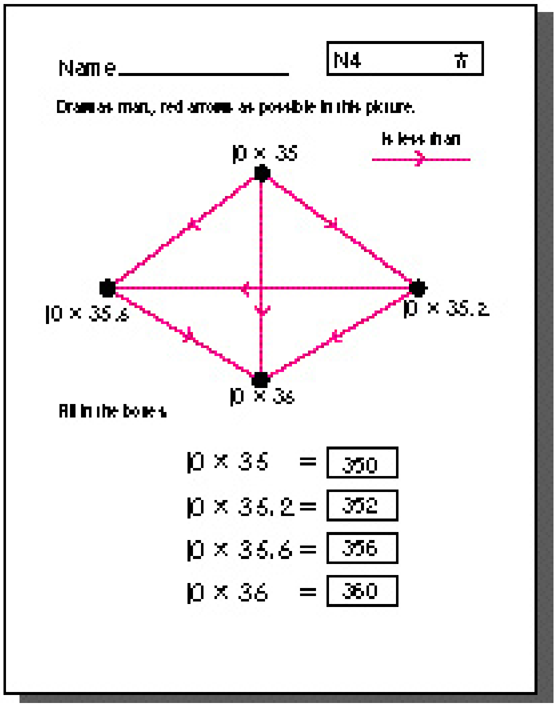

**T:** *Let's start this worksheet together.*
*Which is greater, 10* x *$35* (point to 10 x 35)
*or 10* x *$35.20* (point to 10 x 35.2)*?*

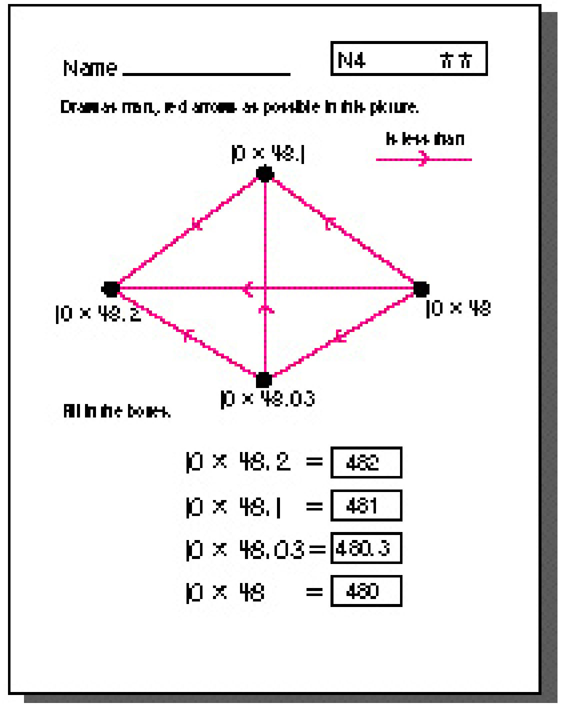
**S:** *10* x *$35.20.*

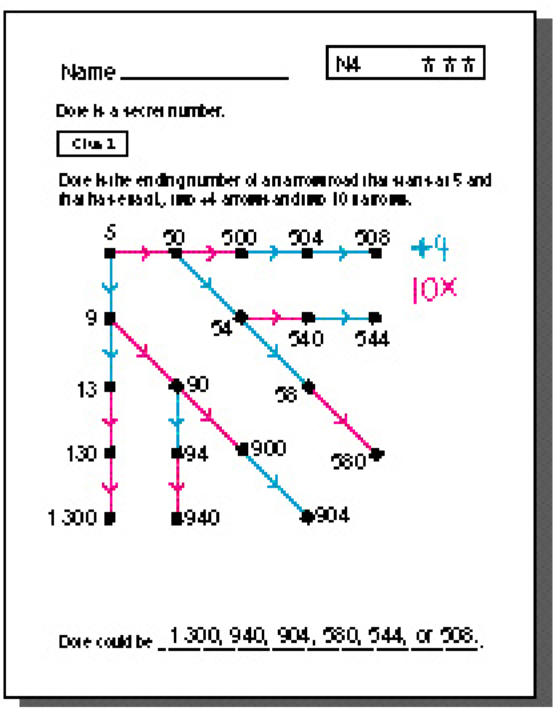
**T:** ***So where can we draw a red arrow?***

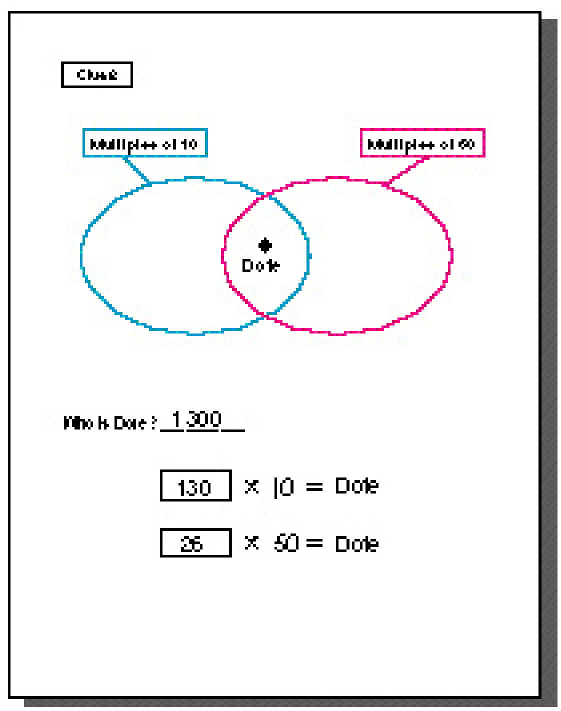
**S:** ***From 10 x 35 to 10 x 35.2 because 10 x 35 is less than 10 x 35.2.***

Let students complete the worksheets individually. Worksheets N4** and *** are also available for individual work.



# N5 FRACTIONS ON THE NUMBER LINE #1

## Capsule Lesson Summary

As a mental arithmetic warm-up, count by threes and by fifths. Fold and measure strips of paper to construct fraction number lines involving halves, fourths, eighths, and sixths. Use the number lines to order fractions and to find equivalent fractions.

### Materials

**Teacher**
- Meter stick
- Colored chalk
- Blackline N5

**Student**
- Paper strips
- Metric ruler
- Scissors
- Colored pencils, pens, or crayons
- Worksheets N5*, **, ***, and ****

**Advance Preparation:** Use Blackline N5 to make copies of paper strips for students.

## Description of Lesson

### Exercise 1: Mental Arithmetic

Go around the class counting by threes. You may start the count at 0 and then call on students in an order natural to their seating arrangement to continue: 3, 6, 9, 12, and so on to about 45.

**T:** ***Let's count by threes again, but this time we'll start at 2.***

Start the count yourself at 2 and then let students continue: 5, 8, 11, 14, and so on to about 50.

**T:** ***Now, let's start at 0 and count by fifths. I'll start: 0, $\frac{1}{5}$.***

Let students continue: $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{5}{5}$.

**T:** ***Do you know another name for $\frac{5}{5}$?***

**S:** ***1.***

**T:** ***What comes next?***

**S:** ***$\frac{6}{5}$ or $1\frac{1}{5}$.***

Accept both responses, but encourage $1\frac{1}{5}$. Let students continue: $1\frac{2}{5}$, $1\frac{3}{5}$, $1\frac{4}{5}$, 2, $2\frac{1}{5}$, … until all students have responded at least once. Always accept equivalent names for numbers, for example, $\frac{10}{5}$ for 2, or $2\frac{6}{5}$ for $3\frac{1}{5}$. When a student gives an "uncommon" name, ask for a "common" name.

### Exercise 2

Distribute copies of Blackline N5, metric rulers, and scissors. Ask students to carefully cut out five strips of paper as indicated on the blackline.

**T:** ***Today we will use these paper strips to locate and compare fractions on a number line. Take one of your strips of paper. The ends are for 0 and 1. How can we locate $\frac{1}{2}$?***

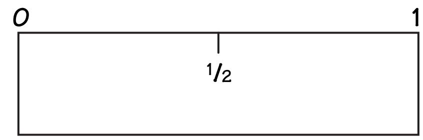
**S:** ***Fold the strip in half. Mark the fold ½.***

**S:** ***Measure the strip. It is 24 cm long. Measure 12 cm from the 0 end and make a mark for ½.***

Tell students to use either way to locate and label the place for ½ on one of their strips. While students do this, draw a large copy of a paper strip on the board, say 120 cm long.

**T:** ***Who can locate ½ on this number line*** (point to the picture on the board)***?***

**S:** ***The strip on the board is 120 cm. ½ x 120 = 60. Measure 60 cm from 0; make a mark there for ½.***




Locate ½ on the number line on the board following the students' directions.

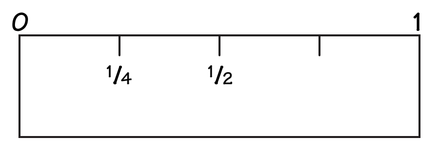
**T:** ***Take another strip of paper. How could we locate ¼?***

**S:** ***We could fold it in half and then in half again.***

**S:** ***We could measure it (24 cm) and find ¼ of its length (¼ x 24 = 6).***

Let students use either method to locate a mark for ¼.

**T:** ***You found ½ on one strip of paper and ¼ on another strip, but on the board we'll use just one number line. How could we locate ¼?***




Following students' suggestions, locate ¼ by measuring. Also, make a mark 30 cm from the edge for 1.

**T:** ***What is this unlabeled mark on the number line for?***

**S:** ***¾.***

You may refer to the Sasquatch story from *IG-I* Lessons N17 and N24 to explain why the mark is for ¾. Then indicate the mark for ½ as you ask,

**T:** ***Does this suggest another name for ½?***

**S:** ***2/4.***

**T:** ***Does this suggest another name for 1?***

**S:** ***4/4 or 2/2.***

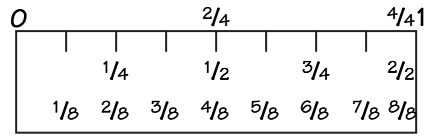
**Note:** There are, of course, infinitely many names for ½ and 1.

Write these equivalent names for ½ and 1 on the number line on the board.

Tell students to locate marks for 2/4, ¾, and 4/4 on the strip where they have a mark for ¼.

**T:** ***Take a third strip of paper. How could we find the place for ⅝?***

Lead students to label a paper strip for eighths, and let them tell you how to locate the marks on the number line on the board. Students might use measurement or folding in half repeatedly to locate eighths on their own number lines.




**T:** ***Take another strip of paper. How could we locate $\frac{1}{3}$?***

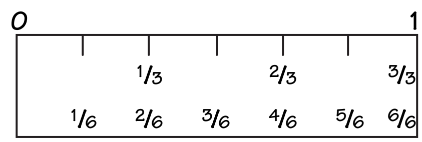
This can be done by folding or measuring. Let students work on their own. Students who try to locate $\frac{1}{3}$ by folding are likely to find it quite difficult to be accurate. When this occurs, encourage them to use measurement.

**T:** ***How can we locate $\frac{1}{3}$ by measuring?***

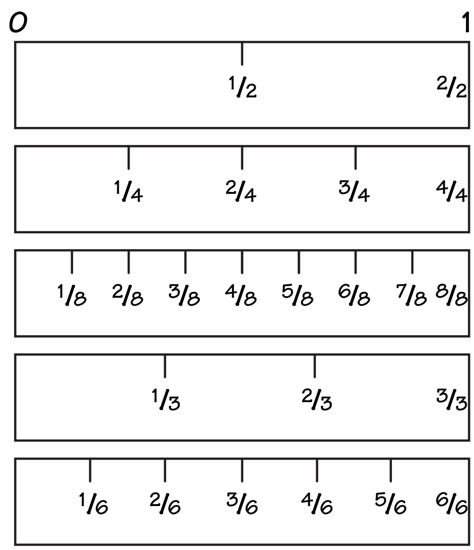
**S:** ***The paper strip is 24 cm long. $\frac{1}{3} \times 24 = 8$. So 8 cm from 0, make a mark for $\frac{1}{3}$; and 16 cm from 0, make a mark for $\frac{2}{3}$. The mark for 1 is also $\frac{3}{3}$.***

While students mark thirds on one of their paper strips, draw another paper strip of the same length on the board, under the one already there.

Let a student direct you to use measurement to locate $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{3}{3}$ on the number line on the board. In a similar manner, direct students to label a strip of paper for sixths and label the number line on the board.




Now each student should have five labeled strips of paper. Ask them to align the strips on their desks.




**T:** ***You have these five copies of a number line from 0 to 1 lined up. What do you notice?***

Let students comment. Encourage ideas that suggest equivalence or comparison of certain fractions.

**T:** ***Can we use your number lines to name some equal fractions?***

**S:** ***$\frac{3}{4}$ and $\frac{6}{8}$.***

**T:** ***How do you know that $\frac{3}{4} = \frac{6}{8}$?***

**S:** ***When I line up the strips, the mark for $\frac{6}{8}$ is directly below the mark for $\frac{3}{4}$.***

**S:** ***$\frac{3}{4}$ and $\frac{6}{8}$ are both the same distance from 0 (or from 1).***

**Note:** This method of using number lines and measurement to find equivalent fractions or to compare fractions depends on the accuracy with which the students labeled their number lines.

Encourage students to name other sets of equivalent fractions, for example:

$\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{3}{6}$ $\quad\quad$ $\frac{2}{3} = \frac{4}{6}$ $\quad\quad$ $1 = \frac{2}{2} = \frac{4}{4} = \frac{8}{8} = \frac{3}{3} = \frac{6}{6}$

## Exercise 3

Write this information on the board.

$<$ $\quad$ $=$ $\quad$ $>$

$5 \ \square \ 9$ $\quad\quad$ $\frac{1}{2} \ \square \ \frac{1}{3}$ $\quad\quad$ $\frac{7}{8} \ \square \ \frac{2}{3}$

$\frac{1}{8} \ \square \ \frac{1}{6}$ $\quad\quad$ $\frac{2}{8} \ \square \ \frac{1}{4}$ $\quad\quad$ $\frac{4}{6} \ \square \ \frac{5}{8}$

Review that < is read "is less than" and > is read "is more than." Invite a student to write one of the three relation symbols in the box to make 5 □ 9 a true number sentence.

$5 \ \boxed{<} \ 9$

**T:** ***Who can fill in another box to make a true number sentence?***

**S:** ***$\frac{1}{8} < \frac{1}{6}$.***

**T:** ***Why?***

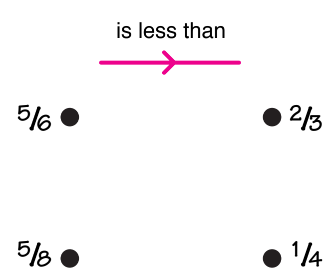
**S:** ***When I line up my number lines for eighths and sixths, the mark for $\frac{1}{8}$ is further to the left than the mark for $\frac{1}{6}$. So $\frac{1}{8}$ is less than $\frac{1}{6}$.***

You may want to discuss other students' explanations of why $\frac{1}{8} < \frac{1}{6}$ is true, but be sure to include the method of using the number lines. Continue until all the boxes are filled in.

$5 \ \boxed{<} \ 9$ $\quad\quad$ $\frac{1}{2} \ \boxed{>} \ \frac{1}{3}$ $\quad\quad$ $\frac{7}{8} \ \boxed{>} \ \frac{2}{3}$

$\frac{1}{8} \ \boxed{<} \ \frac{1}{6}$ $\quad\quad$ $\frac{2}{8} \ \boxed{=} \ \frac{1}{4}$ $\quad\quad$ $\frac{4}{6} \ \boxed{>} \ \frac{5}{8}$

Draw this picture on the board.



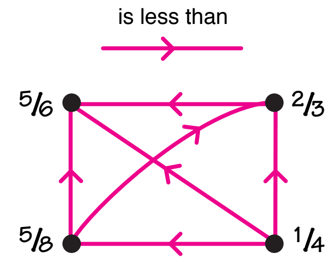

**T:** ***Which of these four fractions is the greatest? Why?***

**S:** ***$\frac{5}{6}$. When I line up my number lines, the mark for $\frac{5}{6}$ is further to the right than the mark for any other number.***

**S:** ***The mark for $\frac{5}{6}$ is closer to the mark for 1 than any of the marks for other fractions.***

**T:** ***Since $\frac{5}{6}$ is the greatest of the four numbers, which arrows can we draw in this picture?***

Invite students to draw the three red arrows ending at $\frac{5}{6}$.

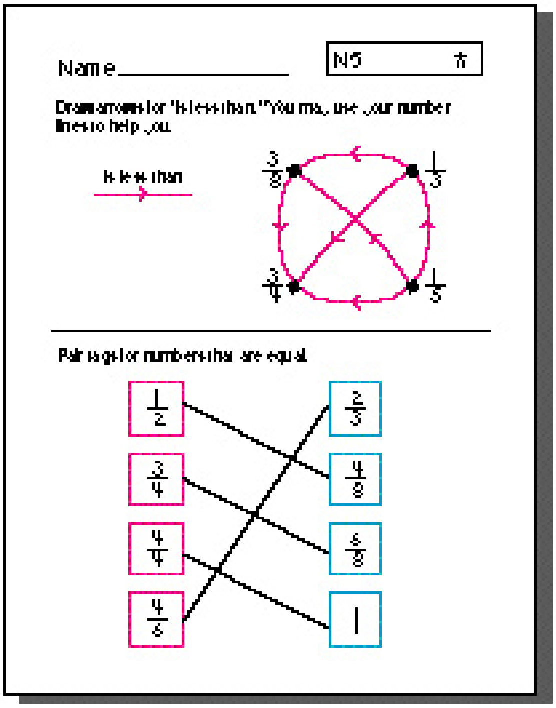



Similarly, let students determine that $\frac{1}{4}$ is the least of the four fractions and draw two more red arrows starting at $\frac{1}{4}$.

**T:** ***Can we draw an arrow between $\frac{5}{8}$ and $\frac{2}{3}$? Which way should it point?***

**S:** ***Draw the arrow from $\frac{5}{8}$ to $\frac{2}{3}$. $\frac{5}{8}$ is less than $\frac{2}{3}$ since $\frac{5}{8}$ is to the left of $\frac{2}{3}$ when I line up the number lines.***

Worksheets N5*, **, ***, and **** are available for individual work.



Name ____________ N5 ☆

Draw arrows for "is less than." You may use your number lines to help you.

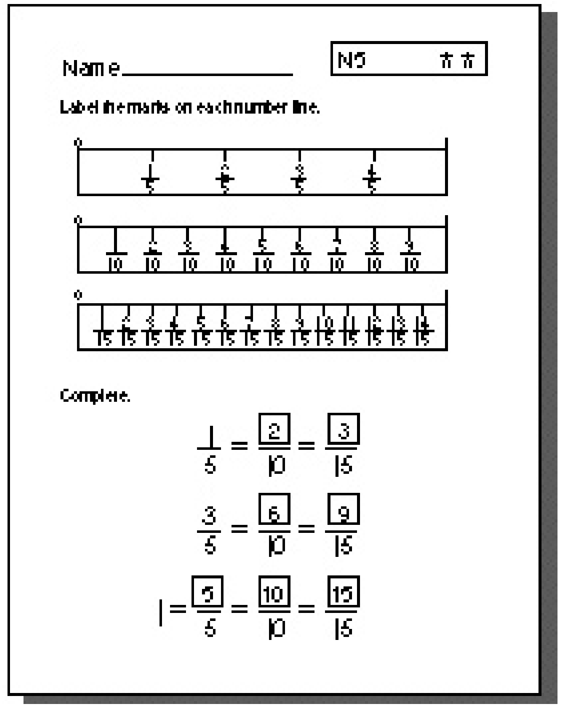
is less than

$\frac{3}{8}$ $\frac{1}{3}$ $\frac{3}{4}$ $\frac{1}{5}$

Pair tags for numbers that are equal.

$\frac{1}{2}$ $\frac{3}{4}$ $\frac{4}{4}$ $\frac{4}{6}$

$\frac{2}{3}$ $\frac{4}{8}$ $\frac{6}{8}$ 1



Name ____________ N5 ☆☆

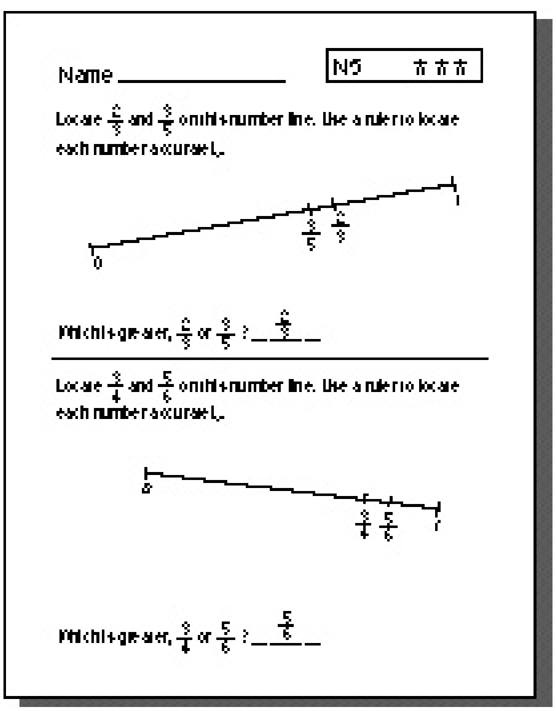
Label the marks on each number line.

Complete.

$\frac{1}{5} = \frac{\boxed{2}}{10} = \frac{\boxed{3}}{15}$

$\frac{3}{5} = \frac{\boxed{6}}{10} = \frac{\boxed{9}}{15}$

$1 = \frac{\boxed{5}}{5} = \frac{\boxed{10}}{10} = \frac{\boxed{15}}{15}$



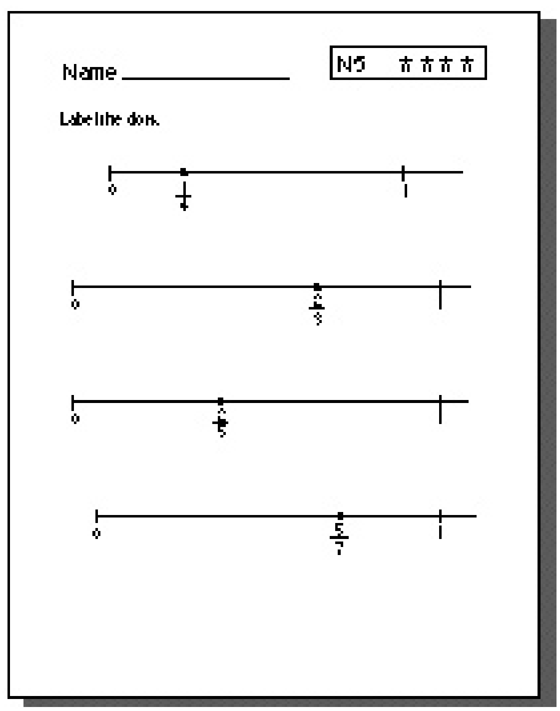
Name ____________ N5 ☆☆☆

Locate $\frac{2}{3}$ and $\frac{3}{5}$ on this number line. Use a ruler to locate each number accurately.

Which is greater, $\frac{2}{3}$ or $\frac{3}{5}$? $\frac{2}{3}$

Locate $\frac{3}{4}$ and $\frac{5}{6}$ on this number line. Use a ruler to locate each number accurately.

Which is greater, $\frac{3}{4}$ or $\frac{5}{6}$? $\frac{5}{6}$



Name ____________ N5 ☆☆☆☆

Label the dots.

## Capsule Lesson Summary

Play *Guess My Rule* where the rules are for operations applied to pairs of numbers. Use $*$ for a new operation and complete number sentences involving $*$. Label dots in an arrow road where the arrows are for $*4$.

### Materials

**Teacher**
- Colored chalk

**Student**
- Paper
- Colored pencils, pens, or crayons

## Description of Lesson

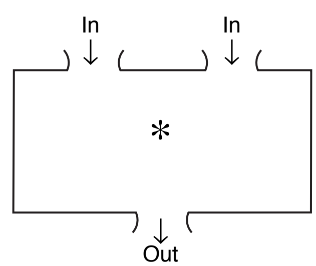
When playing *Guess My Rule*, encourage students to think about what the rule could be without announcing it to the rest of the class. This will allow other students a chance to discover the rule on their own. Let students who think they know the rule test it on numbers given by you or other students. Each time, confirm or deny the result.

### Exercise 1

Choose a secret rule such as $a * b = (a \times b) + 1$ or "multiply two numbers and then add one."

**T:** ***I have a secret rule. I use my rule on a pair of numbers to get a resulting number.***

You may like to draw a "machine" picture on the board to further explain how an operation rule works.




**T:** ***This secret rule is like a machine. We put two (a pair of) numbers into the machine. The machine operates on the numbers and sends one number out.***

***Let's see if you can guess my secret rule. I'll give you some clues using a star (*) for the operation; you try to figure out the secret rule of *.***

Write clues on the board. You may let students give you two numbers to operate on.

$$3 * 7 = 22$$
$$7 * 3 = 22$$
$$5 * 4 = 21$$
$$8 * 6 = 49$$
$$3 * 3 = 10$$

If you use a machine model, show how you put 3 and 7 into the machine and 22 comes out. Observe that although sometimes a machine cares what order you put in the numbers, in this case it does not.

Do not give away the rule. Let students discover it from your clues. Often there are several rules that students think could be the secret rule. Make it a practice when doing this type of activity to tell the class that there might be more than one rule that works for the pairs of numbers tried thus far; however, you are thinking of just one rule and they must discover it.

# N6

When some students think they know the rule, let them give the results for some problems using the rule. For example, continue with these problems letting students provide the boxed numbers.

$$5 * 3 = \boxed{16} \qquad 5 * 5 = \boxed{26} \qquad 9 * 1 = \boxed{10}$$

$$4 * 4 = \boxed{17} \qquad 0 * 7 = \boxed{1} \qquad 9 * 9 = \boxed{82}$$

When many students know the rule, let one explain it to the class. Insist on a clear description. As the student explains, write this expression on the board.

multiply the numbers then add one

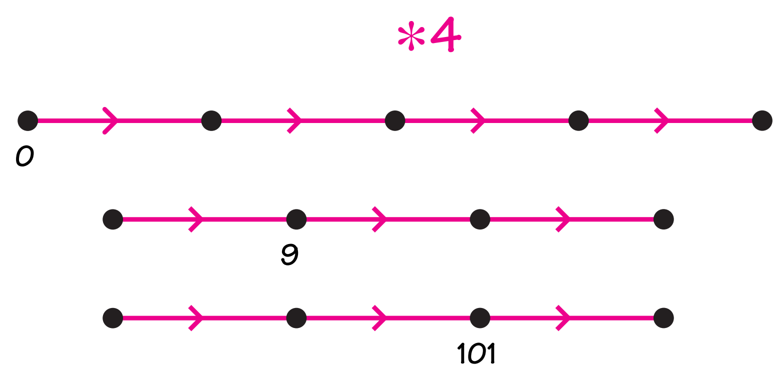
$$a * b = (a \times b) + 1$$

Check that everyone understands how to apply the rule by solving some more problems involving $*$. (Answers are in boxes.)

$$10 * 8 = \boxed{81} \qquad 7 * 6 = \boxed{43} \qquad 9 * 0 = \boxed{1}$$

$$8 * 10 = \boxed{81} \qquad 5 * 15 = \boxed{76} \qquad 12 * 9 = \boxed{109}$$

Erase the board except for the statement of the rule for $*$, and then draw this arrow picture. Ask students to copy the picture on their papers and to note the rule for $*$ at the top.




**T:** ***The red arrows are for *4. Here is 0*** (point to 0 and trace the red arrow starting at 0)***;*** ***0 * 4 is what number?***

**S:** ***1, because (0 x 4) + 1 = 1.***

Label the ending dot of the arrow starting at 0.

**T:** ***Who can label another dot in this picture?***

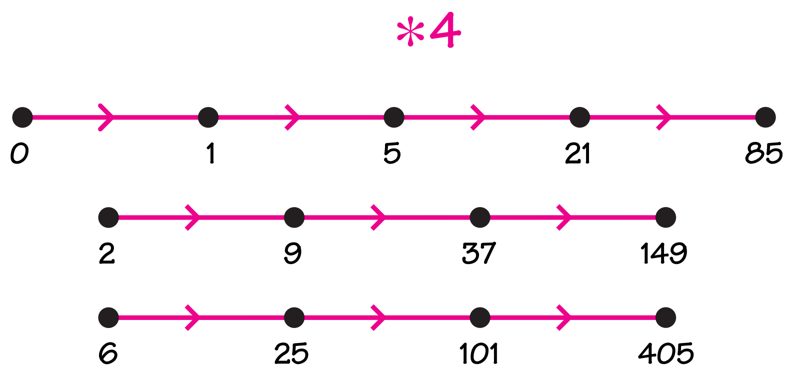
As a class, label one or two more dots in the picture. Then ask the students to go on by themselves to complete the labeling of the dots in all three parts of the arrow picture.

If students have difficulty labeling a dot to the left of a previously labeled dot, draw a box (□) under the dot and say, for example,

**T:** *□ ∗ 4 = 9. What number could be in the box?*

**S:** *2, because (2 x 4) + 1 = 9.*

When several students have correctly labeled all of the dots, collectively complete the picture on the board.




Erase the board completely before starting Exercise 2.

## Exercise 2

**T:** ***Now I am thinking of a new rule. I'll give you some clues and you try to discover the new rule for ∗.***

Choose a secret rule such as $a * b = \frac{a+b}{2}$ or "add the numbers and then divide by 2."

Another way to describe this rule is to think about the number line; a ∗ b is the midpoint of a and b. Do not give away the rule; let students discover it from your clues. If students find it very difficult to discover your rule, you might suggest they think about the number line.

Write clues on the board, or let students choose two numbers for you to operate on.

| | | |
|---|---|---|
| $4 * 10 = 7$ | $0 * 12 = 6$ | $1 * 19 = 10$ |
| $3 * 5 = 4$ | $\widehat{12} * 0 = \widehat{6}$ | $\widehat{2} * 2 = 0$ |

When some students think they know the rule, let them give the results for some problems using the rule. For example, continue with these problems letting students provide the boxed numbers.

| | | |
|---|---|---|
| $10 * 20 = \boxed{15}$ | $\widehat{3} * 7 = \boxed{2}$ | $31 * 55 = \boxed{43}$ |
| $5 * 15 = \boxed{10}$ | $0 * 5 = \boxed{2.5}$ | $\widehat{21} * \widehat{15} = \boxed{\widehat{18}}$ |

When many students know the rule, let several explain it to the class. Try to get at least two descriptions: one that suggests finding the midpoint of the two numbers on the number line, and one that involves adding the two numbers and dividing the sum by 2.

Write these descriptions of the rule on the board.

$$a * b = \frac{a+b}{2} \qquad a * b \text{ is the midpoint of } a \text{ and } b$$

Illustrate both descriptions of the rule with two or three examples from your list of clues.

Give some problems for students to do independently involving this rule for ∗. Check that everyone understands how to apply the rule. (Answers are in boxes.)

| | | |
|---|---|---|
| $0 * 32 = \boxed{16}$ | $8 * 15 = \boxed{11.5}$ | $\boxed{40} * 54 = 47$ |
| $10 * 26 = \boxed{18}$ | $\widehat{16} * \hat{4} = \boxed{\widehat{10}}$ | $\hat{7} * \boxed{15} = 4$ |
| $\hat{5} * 9 = \boxed{2}$ | $110 * 158 = \boxed{134}$ | $\boxed{\widehat{30}} * \widehat{10} = \widehat{20}$ |
| | $7 * \boxed{23} = 15$ | |

When everyone has completed at least half of these problems, check the work collectively.

Erase the board except for the descriptions of the rule.
Write this expression on the board below the rule.

$$\square * \triangle = 20$$

**T:** ***We are still using the same rule for ∗. What are some numbers that could be in □ and △ to make this expression true?***

Let students suggest pairs of numbers and make a list of the possibilities.

When your list has five or more entries, you might wish to encourage students to look for patterns; for example, that both numbers are odd or both numbers are even.

$\square * \triangle = 20$

| $\square$ | $\triangle$ |
|---|---|
| 15 | 25 |
| 10 | 30 |
| 19 | 21 |
| 0 | 40 |
| 8 | 32 |
| $\hat{3}$ | 43 |
| ⋮ | ⋮ |

If there is time, give the students a couple of similar problems to work on individually. For example:

| $\square * \triangle = 7$ | | $\square * \triangle = \hat{8}$ | | $\square * \triangle = 0$ | |
|---|---|---|---|---|---|
| | | | | | |

## Home Activity

Suggest that parents/guardians play *Guess My Rule* with their child.

## Capsule Lesson Summary

Do some mental arithmetic comparing numbers and generating patterns useful in doing subtraction calculations. Examine subtraction properties to solve pairs of related subtraction problems and suggest shortcuts in calculation. Find many subtraction calculations all of which have 19 as their result. Using addition and subtraction functions, label the arrows in an extensive arrow road in which the dots are labeled with positive or negative integers.

### Materials

| Teacher | | Student | |
|---|---|---|---|
| | • Colored chalk | | • Paper<br>• Colored pencils, pens, or crayons<br>• Worksheets N7*, **, and *** |

## Description of Lesson

### Exercise 1: Mental Arithmetic

This exercise should proceed at a brisk pace.

**T:** ***4 is how much less than 10?***

**S:** ***6.***

Continue with the following questions.

**T:** ***4 is how much less than 20?*** (16)
***… than 50?*** (46)
***… than 100*** (96)
***… than 1 000?*** (996)
***… than 5 000?*** (4 996)
***… than 10 000?*** (9 996)

***23 is how much less than 30?*** (7)
***… than 50?*** (27)
***… than 100*** (77)
***… than 700?*** (677)
***… than 1 000?*** (977)
***… than 10 000?*** (9 977)

### Exercise 2

Write this pair of subtraction calculations on the board.

33 – 9 33 – 10

**T:** ***Which is less, 33 – 9 or 33 – 10? Why?***

**S:** ***33 – 10, because you subtract a greater number from 33.***

**T:** ***How much less is 33 – 10 than 33 – 9?***

**S:** ***1 less.***

Ask students which calculation is easier to do in their heads. Then ask them to do that calculation and to use the result to do the other calculation.

**S:** ***33 – 10 = 23, so 33 – 9 = 24 because it is 1 greater.***

Continue this exercise with the following pairs of subtraction problems. (Answers are in boxes.)

$45 - 18 = \boxed{27}$
$45 - 20 = \boxed{25}$

$76 - 30 = \boxed{46}$
$76 - 29 = \boxed{47}$

$275 - 100 = \boxed{175}$
$275 - 97 = \boxed{178}$

$2\,472 - 995 = \boxed{1\,477}$
$2\,472 - 1\,000 = \boxed{1\,472}$

### Exercise 3

Write these subtraction calculations on the board.

$39 - 20$
$40 - 21$
$44 - 25$
$46 - 27$
$56 - 37$
$76 - 57$

**T:** ***What do all of these calculations have in common?***

**S:** ***The result is always 19.***

**T:** ***Do you see any pattern in these problems?***

Accept any reasonable comment, but encourage the class to observe that you can add the same thing to both numbers in a subtraction problem to get another problem with the same result.

Write this open sentence on the board.

$\boxed{\phantom{00}} - 30 = 19$

**T:** ***What number should we put in the box so that the result of the subtraction calculation is still 19?***

**S:** ***49.***

Continue with these problems. (Student responses are in boxes.) Encourage students to use patterns to help them solve these problems.

$\boxed{49} - 30 = 19$
$59 - \boxed{40} = 19$
$70 - \boxed{51} = 19$
$\boxed{73} - 54 = 19$

$\boxed{79} - 60 = 19$
$\boxed{119} - 100 = 19$
$100 - \boxed{81} = 19$
$104 - \boxed{85} = 19$

## Exercise 4

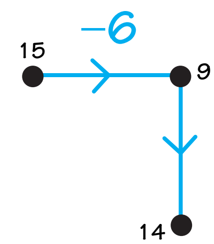
**T:** ***I am going to draw a long arrow road on the board. Each time I draw an arrow, tell me what it is for; it will always be for plus some number or minus some number.***

Start the road with this arrow.


**T:** ***What is this arrow for?***

**S:** ***–6.***

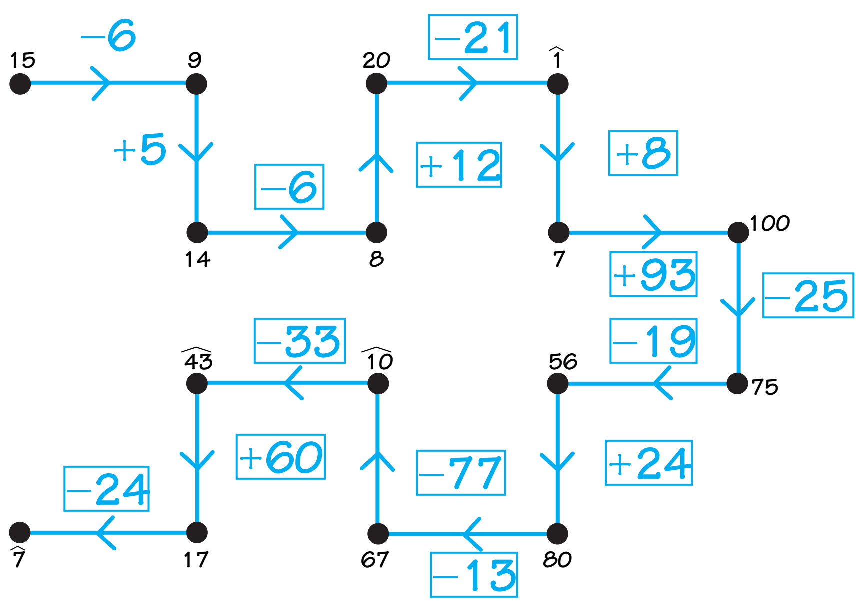
Label the arrow and proceed with another arrow.



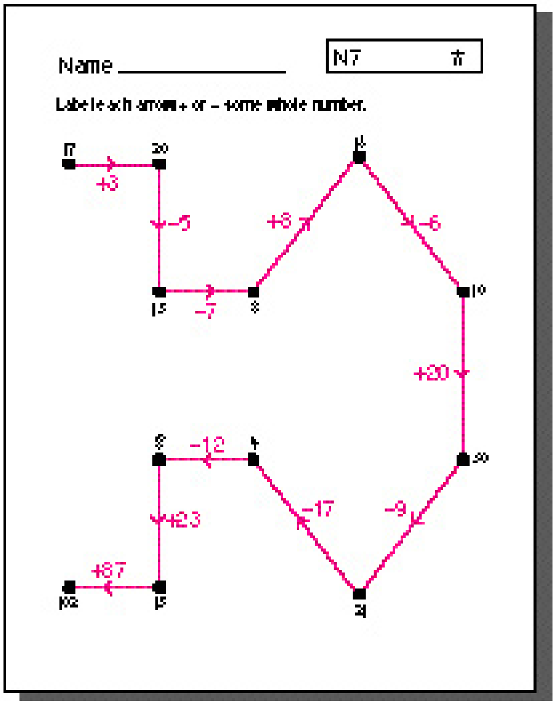

**T:** ***What is this next arrow for?***

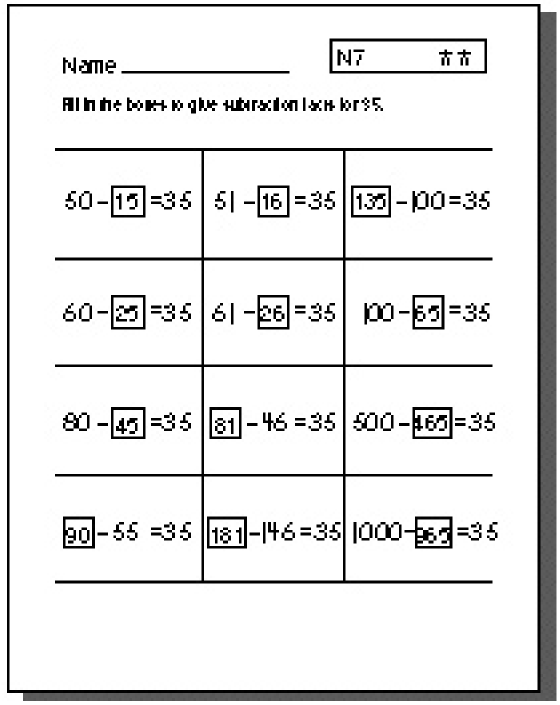
**S:** ***+5.***

Label the arrow and continue with another. Progressively build the road, one arrow at a time, with students telling you how to label the arrows. The length of the road and the difficulty of the problems can be adjusted to suit the abilities of your class. An arrow road that you might build is illustrated and labeled here. (Student responses are in boxes.)

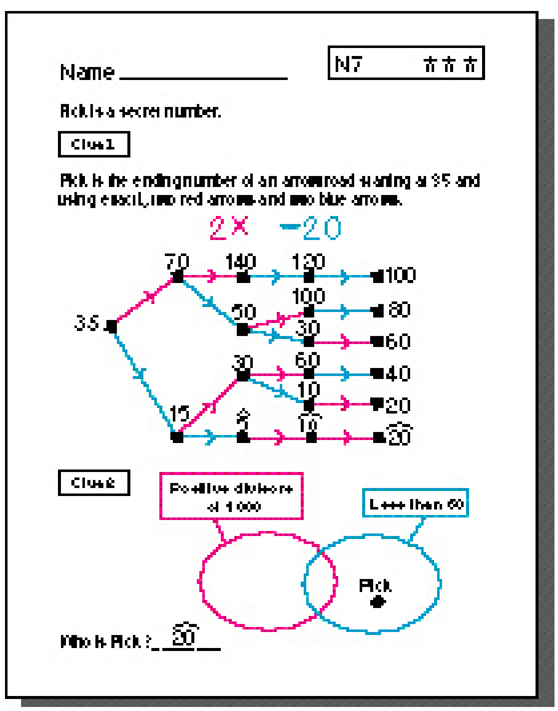



Worksheets N7*, **, and *** are available for individual work.





Name
N7 ☆☆
Fill in the boxes to give subtraction facts for 35.

| | | |
|---|---|---|
| 50 − $\boxed{15}$ = 35 | 51 − $\boxed{16}$ = 35 | $\boxed{135}$ − 100 = 35 |
| 60 − $\boxed{25}$ = 35 | 61 − $\boxed{26}$ = 35 | 100 − $\boxed{65}$ = 35 |
| 80 − $\boxed{45}$ = 35 | $\boxed{81}$ − 46 = 35 | 500 − $\boxed{465}$ = 35 |
| $\boxed{90}$ − 55 = 35 | $\boxed{181}$ − 146 = 35 | 1000 − $\boxed{965}$ = 35 |



# N8 MINICOMPUTER GOLF #1

## Capsule Lesson Summary

Decode numbers on the Minicomputer. By moving exactly one checker, change a number by a given amount. Play *Minicomputer Golf*.

### Materials

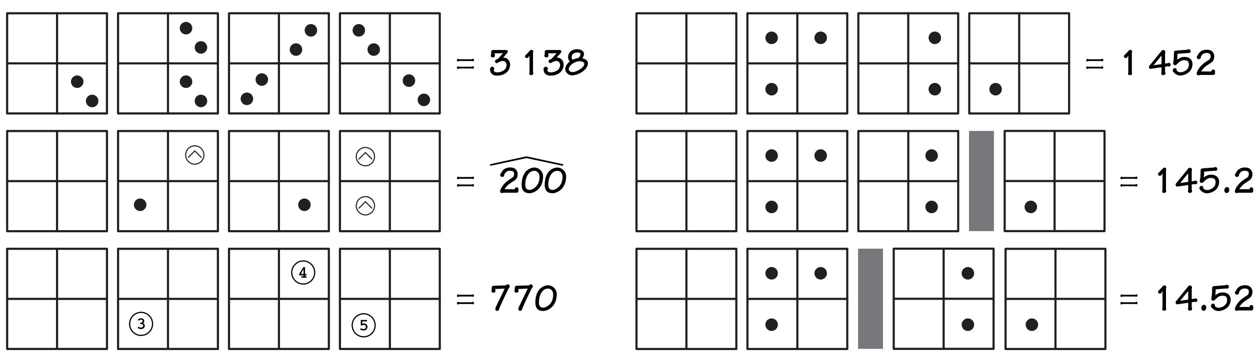
**Teacher**
- Minicomputer set
- Colored chalk

**Student**
- Paper
- Worksheets N8*, **, and ***

## Description of Lesson

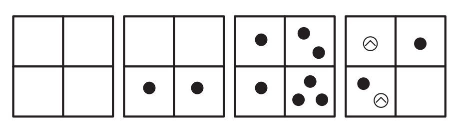
### Exercise 1

Display four Minicomputer boards. Ask students to decode several configurations such as the ones below, choosing configurations appropriate for the numerical abilities of your class. Ask students to first write their answers on paper; then ask someone to answer aloud.




### Exercise 2

Gradually put this configuration of checkers on the Minicomputer.



**T:** ***What number is on the Minicomputer?***

**S:** ***506.***

**T:** ***I am going to change the number on the Minicomputer by moving one checker to another square. Each time, tell me if the new number is more than or less than 506, the number on the Minicomputer now.***

Move a checker from the 10-square to the 200-square.

**T:** ***Did I increase or decrease the number?***

**S:** ***Increase.***

**T:** ***How much more is this new number?***

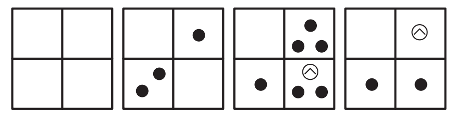
**S:** ***190 more, because 10 + 190 = 200.***

Continue in this manner, making the following moves. Do not return the checkers to their original positions. Each move starts from the new configuration on the Minicomputer.

Move a checker
- from the 100-square to the 80-square (20 less)
- from the 4-square to the 40-square (36 more)
- from the 80-square to the 2-square (78 less)
- from the 2-square to the 400-square (398 more)
- from the 80-square to the 1-square (79 less)

Move a negative checker
- from the 8-square to the 4-square (4 more)
- from the 2-square to the 10-square (8 less)

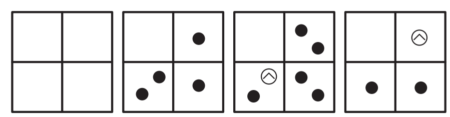
Check that this configuration is on the Minicomputer.



**T:** ***Who can move just one checker and increase the number by 60, make it 60 more than it is now?***

A student should move a checker from the 40-square to the 100-square, or a checker from the 20-square to the 80-square.

Continue the activity by asking volunteers to make other changes such as those suggested below. Feel free to adjust the level of difficulty to the numerical abilities of your students.



Make a change that is
- 4 less (Move the negative checker from the 4-square to the 8-square.)
- 12 less (Move the negative checker from the 8-square to the 20-square, or move the regular checker from the 20-square to the 8-square.)
- 6 more (Move the negative checker from the 10-square to the 4-square, or move a regular checker from the 2-square to the 8-square.)

**T:** ***Can you move one checker and make this number 3 less?***

Let the students have a minute or two to consider this question.

**S:** ***It cannot be done; we need a regular checker on the 4-square or a negative checker on the 1-square.***

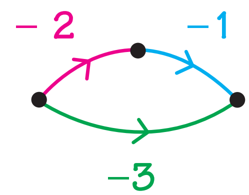
**T:** ***Can we make the number 3 less by moving exactly two checkers?***

Invite one student to make a move and then another student to make a second move.

**S:** ***Move a checker from the 10-square to the 8-square; that will make the number 2 less.***

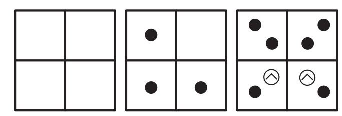
**S:** ***Move the checker on the 2-square to the 1-square.***

Summarize the two moves† by drawing this arrow picture.




## Exercise 3

Put this configuration on the Minicomputer.



**T:** ***What number is on the Minicomputer?***

**S:** ***134.***

**T:** ***Let's play Minicomputer Golf using 134 as the starting number and 500 as the goal.***

Organize the class into two groups, a Red Team and a Blue Team. Write this information on the board.


Play a game of *Minicomputer Golf* in the usual manner (see Lesson N6 in *IG-I*).

Worksheets N8*, **, and *** are available for individual work.

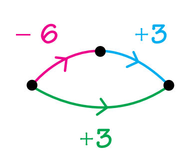
## Extension Activity

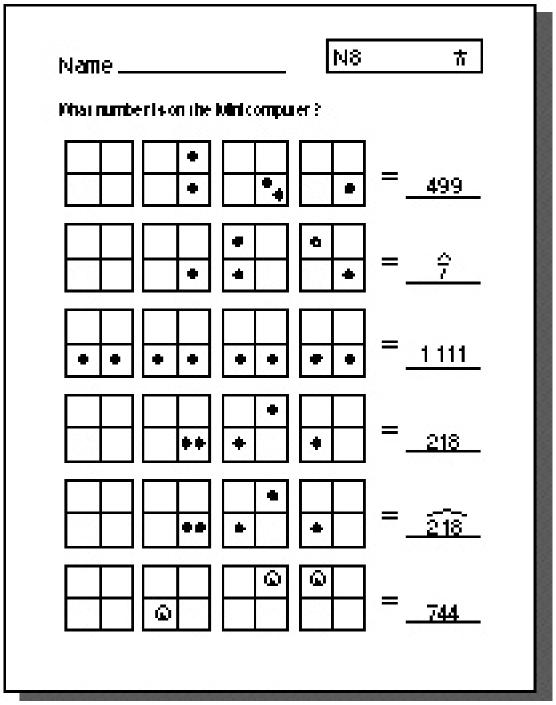
Organize a *Minicomputer Golf* tournament.

## Home Activity

Send home a description of the *Minicomputer Golf* game and suggest that parents/guardians play this game with their child

†Depending on previous moves, checkers may not be in position for these two moves. They may, however, be in position to move a regular checker from the 10-square to the 4-square, and a regular checker from the 1-square to the 1-square to the 4-square.





Name ______________ N8 ☆

What number is on the Minicomputer?

= 499

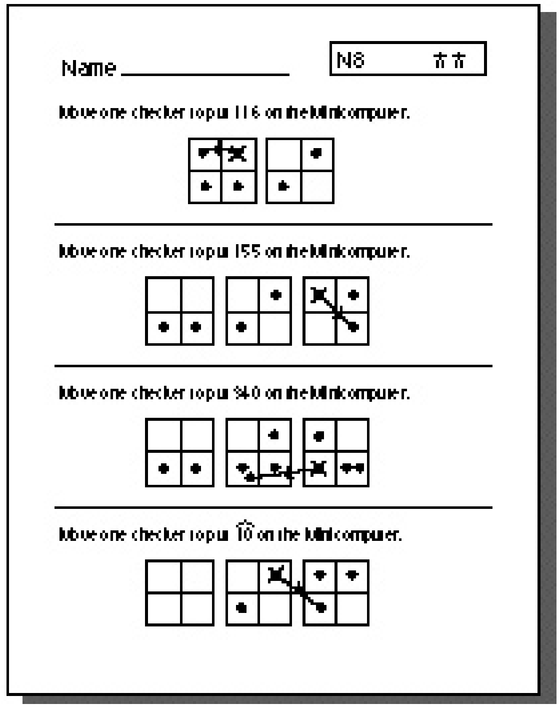
= $\hat{7}$

= 1 111

= 218

= $\widehat{218}$

= 744



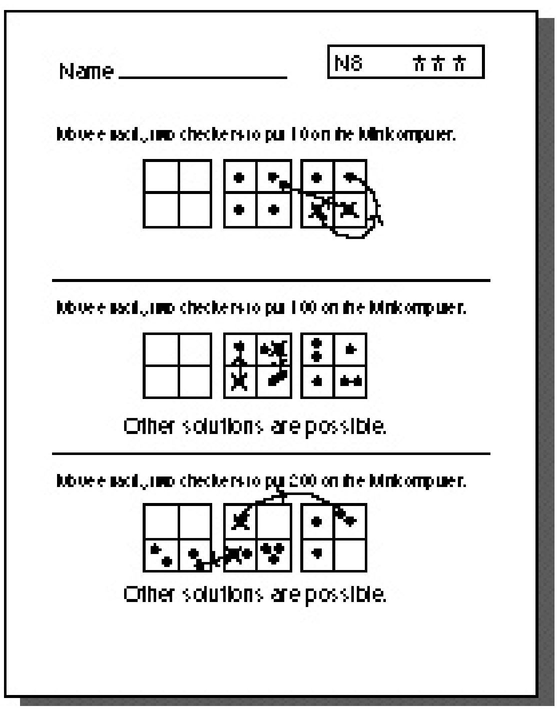
Name ______________ N8 ☆☆

Move one checker to put 116 on the Minicomputer.

Move one checker to put 155 on the Minicomputer.

Move one checker to put 340 on the Minicomputer.

Move one checker to put $\hat{10}$ on the Minicomputer.



Name ______________ N8 ☆☆☆

Move exactly two checkers to put 10 on the Minicomputer.

Move exactly two checkers to put 100 on the Minicomputer.

Other solutions are possible.

Move exactly two checkers to put 200 on the Minicomputer.

Other solutions are possible.

# N9 POSITIVE DIVISORS

## Capsule Lesson Summary

Investigate the relation “is a positive divisor of” with several different arrow pictures. Use these situations to find numbers with more than seven positive divisors and to find common multiples of some numbers.

### Materials

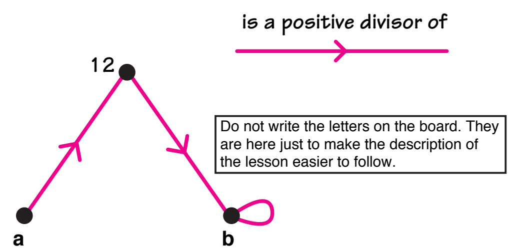
**Teacher**
- Colored chalk

**Student**
- Paper
- Colored pencils, pens, or crayons
- Worksheets N9* and **

## Description of Lesson

### Exercise 1

Draw this arrow picture on the board.




**T:** ***The red arrow is for “is a positive divisor of.” What could be here*** (point to **a**)***?***

**S:** ***4.***

Check the response by tracing the arrow from **a** to 12 as you say,

**T:** ***4 is a positive divisor of 12. Is that true?*** (Yes) ***Could another number be here*** (at **a**)***?***

**S:** ***1, 2, 3, or 6 (any positive divisor of 12 except 12).***

If a student says 12, observe that 12 is already in the picture and ask,

**T:** ***How can we show that 12 is a positive divisor of 12?***

**S:** ***Draw a loop at 12.***

**T:** ***What number could be here*** (point to **b**)***?***

**S:** ***36.***

Check the response by tracing the arrow from 12 to **b** as you say,

**T:** ***12 is a positive divisor of 36. Is that true?*** (Yes) ***Could another number be here*** (at **b**)***?***

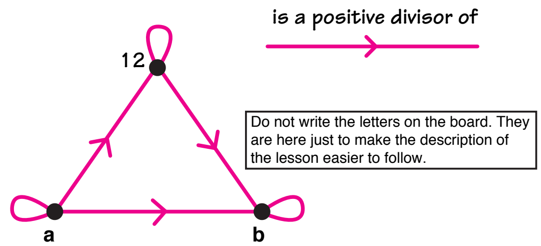
**S:** ***24, 48, 60, 72 … (any positive† multiple of 12 except 12).***

**T:** ***There are some red arrows and loops missing from the picture. Where can we draw red arrows or loops for sure?***

---

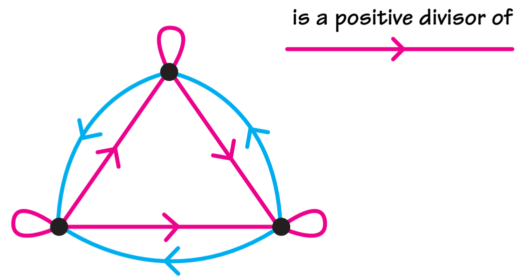
†The loop at **b** means that **b** is a positive divisor of **b**, hence positive.

Invite students to trace and then draw missing arrows and loops. Call on others in the class to check that a new arrow added to the picture is correct. In doing this, you may find it necessary to refer to some possible numbers at **a** and **b**. The completed picture is shown here.




Replace 12 by 15 in the picture and repeat the exercise of finding numbers that could be at **a** and **b**. In this case, check that all of the red arrows and loops are correct.

Erase all the numbers from the picture and draw some return arrows in blue.




**T:** ***What could the blue arrows be for?***

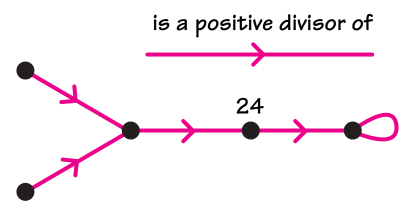
**S:** ***"Is a multiple of."***

Check the response with an example, and then observe that if the blue arrows are for "is a multiple of," you can draw blue loops as well.

Erase the picture before going on to Exercise 2.

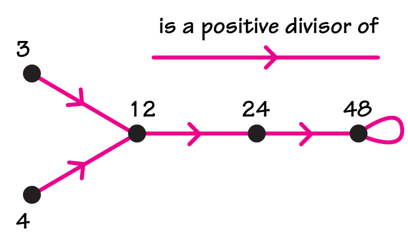
## Exercise 2

Draw this arrow picture on the board and ask students to copy it on their papers.



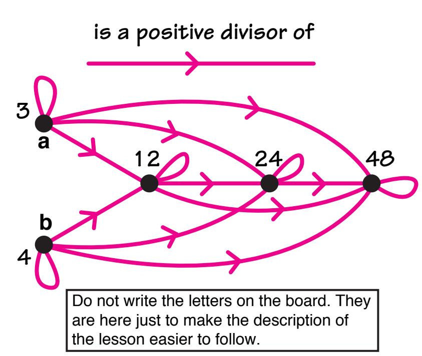

**T:** ***Label the dots in this picture and then draw all of the missing red arrows and loops. There are many solutions to labeling the dots.***

Allow several minutes for students to work individually or with a partner on the problem. Some students will need help getting started; others may find a solution very quickly. In the latter case, ask the student to find another solution and to draw a new arrow picture for it. When many students have at least one solution, call on one student to label the dots in the picture on the board. For example, one possible solution is shown here.




**T:** ***Very good. Let's draw the missing red arrows and loops in this case.***

Call on students to draw red arrows and loops. For this example, the completed picture looks like this:




**T:** ***Does everyone's arrow picture have exactly the same arrows as this one?***

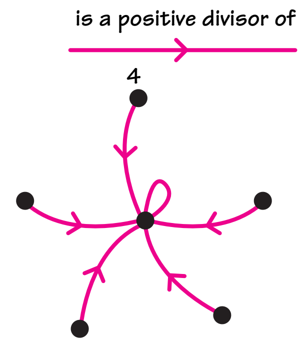
**S:** ***No, mine has an arrow from top to bottom*** (**a** to **b**). ***I had 6 at the bottom instead of 4.***

**S:** ***I had 2 at the bottom (b) and 6 at the top (a), so my picture has a red arrow from bottom to top (b to a).***

**T:** ***An arrow from top*** (point to **a**) ***to bottom*** (point to **b**) ***depends on how you labeled the dots; however, you should all have the rest of these arrows.***

Erase the picture before starting Exercise 3.

## Exercise 3

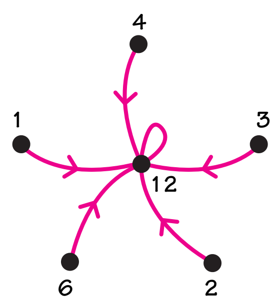


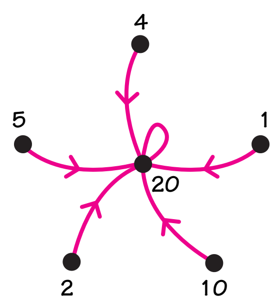

Draw this arrow picture on the board and ask students to copy it on their papers.

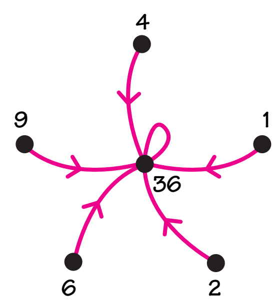
**T:** ***Label the dots in this picture. All of the dots must be for different numbers. I omitted the loops, but we know there would be a loop at every dot. Again, there are many solutions.***

Allow several minutes for students to work individually or with a partner. Monitor students' progress and encourage those who finish quickly to find other solutions. If necessary, suggest that students having difficulty label the center dot first. If some label it 8, ask them to count the positive divisors of 8 (there are four: 1, 2, 4, 8) and remind them that a number with at least five positive divisors besides itself is needed. When many students have at least one solution, call on two or three students to put their solutions on the board. Some possibilities are given below.

**Note:** Any positive multiple of 4 except 4 and 8 could be at the center dot. Some of these, like 12 and 20, will have all of their positive divisors in the picture; others, like 36, will not because they have more than six positive divisors.



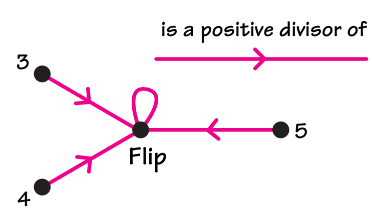




Erase the pictures from the board before starting Exercise 4.

**Note:** Another approach to this problem is to label all but the center dot with any numbers that you like. Then label the center dot with a common multiple of these numbers.

## Exercise 4

Draw this arrow picture on the board.




**T:** ***What information does this picture give us about Flip?***

**S:** ***Flip has 3, 4, and 5 as positive divisors.***

**S:** ***Flip is a multiple of 3, of 4, and of 5.***

# N9

**S:** ***Flip is a positive number because it is a positive divisor of itself.***

**T:** ***What are some numbers that Flip could be?***

Encourage students to find several possibilities, letting the class check each suggestion to be sure that 3, 4, and 5 are positive divisors.

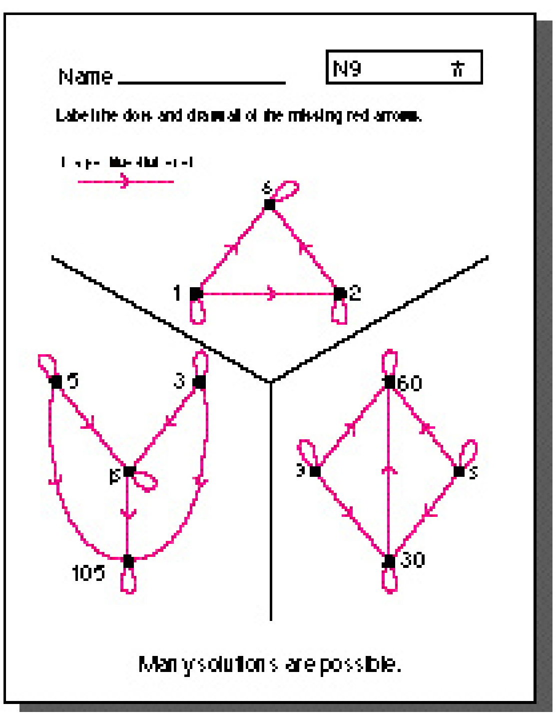
**S:** ***120.***

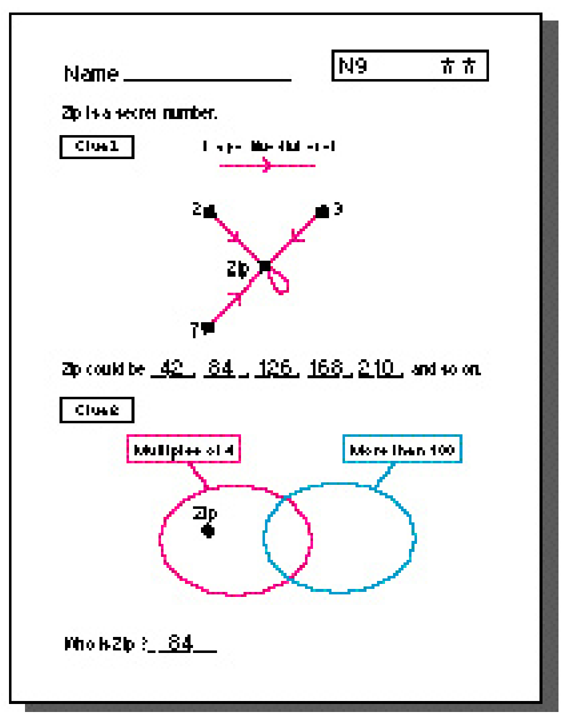
**T:** ***60.***

**S:** ***600.***

The class should conclude that 60 is the least number Flip could be and that any positive multiple of 60 could be Flip (i.e., 60, 120, 180, 240, …).

Worksheets N9* and ** are available for individual work.



## Capsule Lesson Summary

Multiply numbers by 20, 30, and so on by using the compositions of 10x with 2x, with 3x, and so on.

### Materials

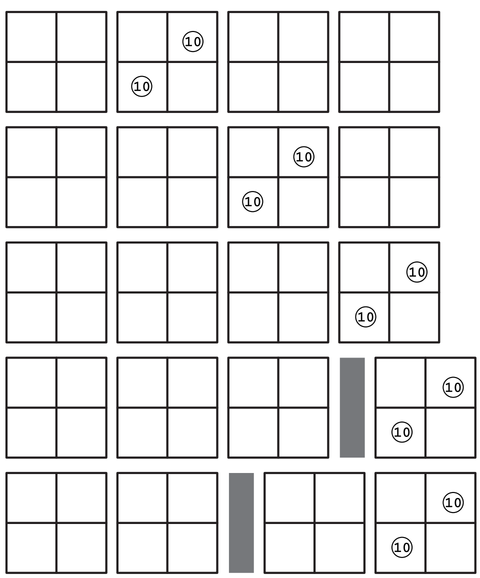
**Teacher**
- Minicomputer set
- Colored chalk
- Weighted checker set

**Student**
- Colored pencils, pens, or crayons
- Worksheets N10*, **, and ***

## Description of Lesson

### Exercise 1

Put these 10x configurations on the Minicomputer, one at a time, and ask students for the numbers. You may check by making trades with the ⑩-checkers to get standard configurations. Write the sequence of number sentences on the board.



$10 \times 600 = 6\,000$

$10 \times 60 = 600$

$10 \times 6 = 60$

$10 \times 0.6 = 6$

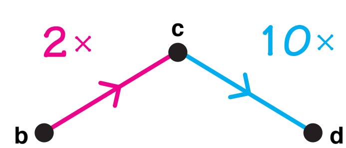
$10 \times 0.06 = 0.6$

**T:** ***What patterns do you notice?***

Accept any correct patterns, but emphasize the change in position of the decimal point. Erase the board.

### Exercise 2

Draw this arrow picture on the board.




Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

**T:** ***Suppose 900 is here*** (point to **c**), ***what number is here*** (point to **d**)***?***

**S:** ***9 000.***

**T:** ***What number is here*** (point to **b**)***?***

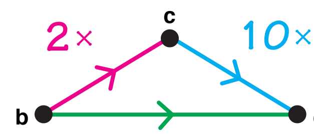
**S:** ***450.***

# N 10

Repeat this activity several times. Some suggestions for choices of labels at **c** are given here. (Answers are in boxes.) Adjust the level of difficulty to the abilities of your students.

| b | c | d |
|---|---|---|
| $\boxed{45}$ | 90 | $\boxed{900}$ |
| $\boxed{4.5}$ | 9 | $\boxed{90}$ |
| $\boxed{0.45}$ | 0.9 | $\boxed{9}$ |

Add this green arrow to your picture.

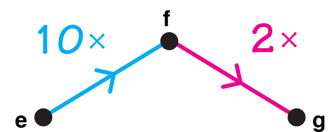



**T:** ***What could this green arrow be for,*** x ***what whole number?***

**S:** ***20*x *.***

Label the green arrow 20x. A common error is to say that the green arrow could be for 12x. By putting 1 or 2 at **b** and then finding the numbers that are at **c** and **d**, one can easily check that the green arrow is not for 12x, but is for 20x.

Draw this arrow picture on the board.



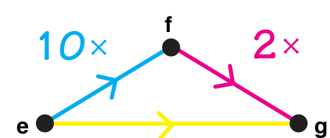

Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

Label one of the three dots and then ask the class what numbers are at the other dots. Repeat the activity several times. Suggested problems are given here. (Answers are in boxes.)

| e | f | g |
|---|---|---|
| $\boxed{24}$ | $\boxed{240}$ | 480 |
| $\boxed{0.16}$ | 1.6 | $\boxed{3.2}$ |
| $\boxed{5.05}$ | $\boxed{50.5}$ | 101 |
| $\boxed{4.3}$ | $\boxed{43}$ | 86 |

Some students might have difficulty finding **e** when it is for a decimal number. In such cases, write on the board what 10x each suggested number is. For example, if someone suggests **e** is 16 when **f** is 1.6, say "no" and write 10 x 16 = 160 on the board. Continue until the correct number at **e** is given; in this case **e** is for 0.16.

Add this yellow arrow to your picture.



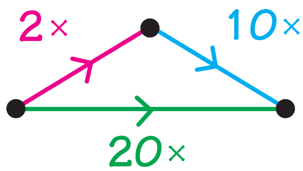

**T:** ***What could this yellow arrow be for,*** x ***what whole number?***

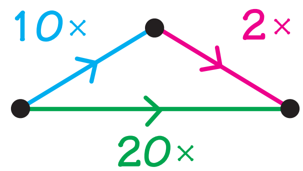
**S:** ***20*x.**

Do not be surprised if 12x is suggested. Simply check that it cannot be for 12x by putting 1 or 2 at **e** and finding the numbers at the other two dots.

**T:** ***Yes, 10*x *followed by 2*x *is 20*x*, so this could also be a green arrow.***

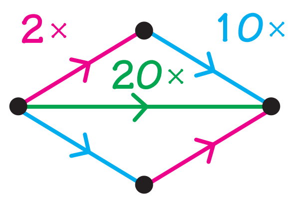
Replace the yellow arrow with a green arrow and label it 20x. You will have these two arrow pictures on the board.




**T:** ***Red followed by blue is green, and blue followed by red is green. We can combine these two pictures into one.***

Draw this picture and then erase the other two.

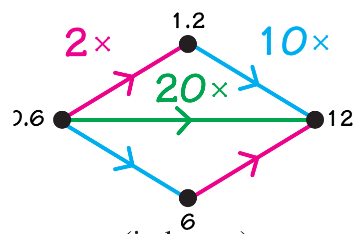



Write these four numbers near the arrow picture on the board.

0.6 1.2 12 6

**T:** ***Try to locate these four numbers in the picture.***

Let volunteers come to the board and label the dots.



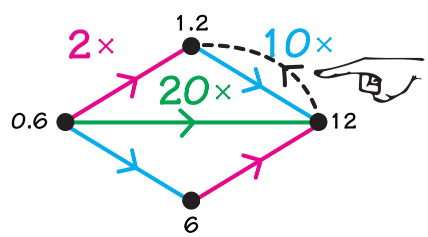

**T:** ***Use this picture to help you solve these problems.***

Write these problems on the board. After a few minutes, discuss the answers (in boxes).

$12 \div 10 = \boxed{1.2}$ $6 \div 10 = \boxed{0.6}$ $12 \div 20 = \boxed{0.6}$

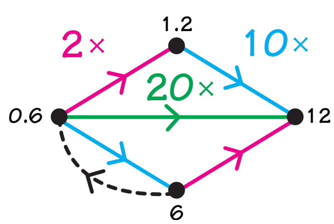
**T:** ***How can the arrow picture help us to do the first calculation?***

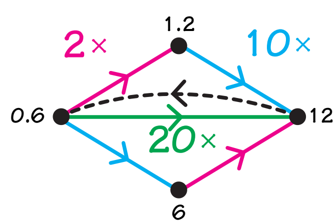
**S** (tracing an arrow from 12 to 1.2)**:** ***÷10 is the opposite (return) arrow for 10*x*. 12 ÷ 10 = 1.2.***




Let students explain how to do the other two calculations.

$6 \div 10 = 0.6$




$12 \div 20 = 0.6$

Keep the arrow picture on the board, but erase the dot labels.

## Exercise 3

**T:** ***To multiply a number by 20, we can multiply by 2 and then by 10, or we can multiply by 10 and then by 2. What number is 20 x 31?***

**S:** ***620.***

**T:** ***How did you do the calculation?***

**S:** ***I multiplied 2 x 31 = 62 and then 10 x 62 = 620.***

**S:** ***I multiplied 10 x 31 = 310 and then doubled 310 (2 x 310 = 620).***

Continue with these problems.

$20 \times 14 = \boxed{280}$ $\qquad 20 \times 70 = \boxed{1\,400}$

$20 \times 106 = \boxed{2\,120}$ $\qquad 20 \times 45 = \boxed{900}$

**T:** ***How could we multiply a number by 30?***

**S:** ***Multiply by 3 and then by 10, or multiply by 10 and then by 3.***

**T:** ***What number is 30 x 15?***

**S:** ***450; multiply 3 x 15 = 45 and then 10 x 45 = 450.***

**T:** ***What number is 30 x 41?***

**S:** ***1 230; multiply 10 x 41 = 410 and then 3 x 410 = 1 230.***

**T:** ***How could we multiply a number by 50?***

**S:** ***Multiply by 5 and then by 10, or multiply by 10 and then by 5.***

**T:** ***How could we multiply a number by 70?***

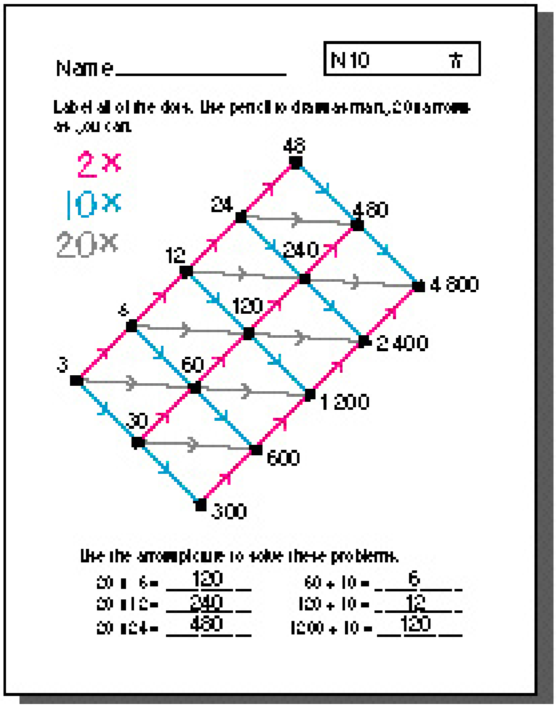
**S:** ***Multiply by 7 and then by 10, or multiply by 10 and then by 7.***

**Note:** There are many ways to multiply a number by a multiple of 10 that is less than 100. For example, to multiply a number by 70, you can multiply by 2 and then by 35, or multiply by 5 and then by 14. But in this lesson, the focus is on using 10x and *n*x (*n* being a whole number).

Worksheets N10*, **, and *** are available for individual work. Remind students to label the dots in the arrow pictures first and then to use the arrow pictures to solve the problems in the lower right-hand corners.

## Home Activity

Suggest that parents/guardians practice with their child multiplying numbers by 20, or by 30, using composition.

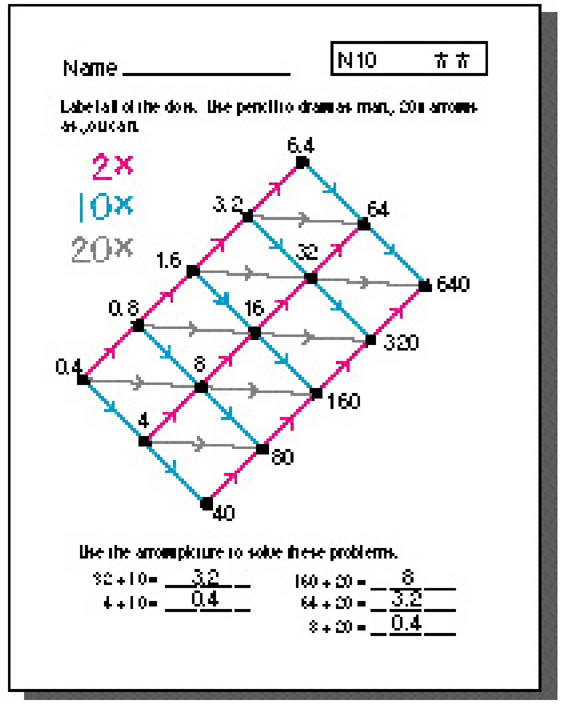

Name ______ N10 ☆

Label all of the dots. Use pencil to draw as many 20x arrows as you can.



Use the arrow picture to solve these problems.

20 x 6 = 120
20 x 12 = 240
20 x 24 = 480
60 ÷ 10 = 6
120 ÷ 10 = 12
1200 ÷ 10 = 120

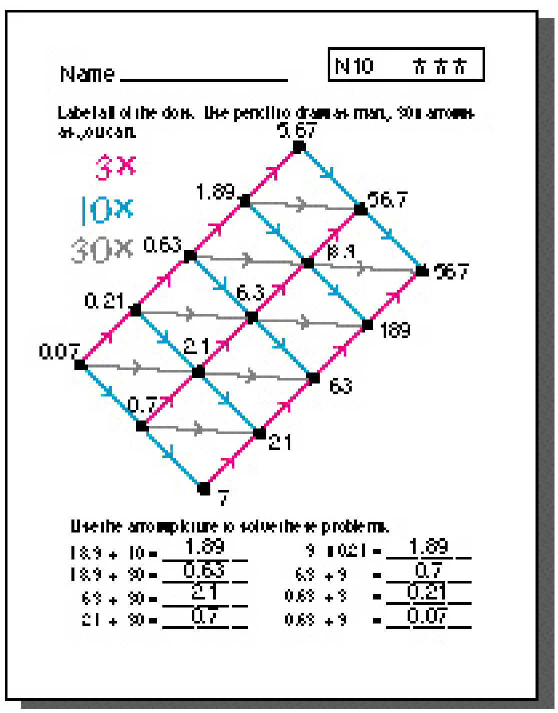

Name ______ N10 ☆☆

Label all of the dots. Use pencil to draw as many 20x arrows as you can.



Use the arrow picture to solve these problems.

32 ÷ 10 = 3.2
4 ÷ 10 = 0.4
160 ÷ 20 = 8
64 ÷ 20 = 3.2
8 ÷ 20 = 0.4


Name ______ N10 ☆☆☆

Label all of the dots. Use pencil to draw as many 30x arrows as you can.



Use the arrow picture to solve these problems.

18.9 ÷ 10 = 1.89
18.9 ÷ 30 = 0.63
63 ÷ 30 = 2.1
21 ÷ 30 = 0.7
9 x 0.21 = 1.89
6.3 ÷ 9 = 0.7
0.63 ÷ 3 = 0.21
0.63 ÷ 9 = 0.07

## Capsule Lesson Summary

Solve a detective story for a secret number with clues involving the calculator, a string picture, an arrow picture, and the Minicomputer. Find that the secret number is a non-integer decimal.

### Materials

**Teacher**
- Calculator
- Minicomputer set
- Colored chalk

**Student**
- Calculator
- Worksheets N11(a), (b), (c), and (d)

## Description of Lesson

Arrange that all students have access to a calculator. Announce that today Zip is a secret number and students are to be detectives trying to discover Zip.

**Clue 1**

**T:** ***If I put Zip on the calculator display and press + = = = and so on, 12 will appear on the display. What numbers could Zip be? Zip does not have to be a whole number.***

Give students time to explore the numbers that Zip could be. When a number for Zip is suggested, let students enter the number on their calculator display and check to see if it could be Zip.

**S:** ***Zip could be 3.***

**T:** ***Let's check. Put 3 on your display and press + = = and so on. Does 12 appear?***

**S:** ***Yes.***

**T:** ***How many times do you press = to get 12?***

**S:** ***Four times.***

Continue in this manner to find many numbers that Zip could be, making a list of the numbers on the board. Among the numbers that students might find for Zip are 12, 6, 4, 3, 2, 1.5, 1, 0.8, 0.75, 0.6, 0.3, 0.2, 0.04, 0.02, 0.01, and so on. Try to allow enough time for 1.5 as well as several other decimals to be found, as this will make the rest of the detective story more interesting.

When a student suggests a decimal for Zip, for example 0.2, occasionally use the decimal in a counting exercise as follows.

**S:** ***Zip could be 0.2.***

**T:** ***Put 0.2 on your display. Press + = = = = = and watch what happens. What number is on the display?***

**S:** ***1.***

**T:** ***Suppose we press = five more times. What number do you predict will be on the display?***

**S:** *2.*

**T:** *Try it and see if you are correct. Now hide the display. Press ⊟ until you think you have 4 on the display. How many times do you press ⊟?*

**S:** *Ten times.*

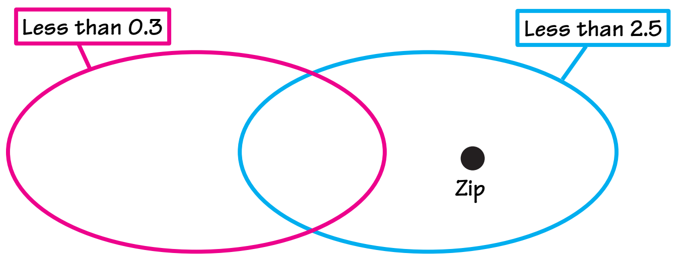
**T:** *Suppose you keep pressing ⊟. Will 12 appear?*

**S:** *Yes.*

Using other decimal numbers, for example, 0.75, examine the idea of counting how many times you press ⊟ until another whole number appears. In the case of 0.75, you press ⊟ four times to get 3, four more times to get 6, and so on until 12 appears. Once this method has been suggested, students may use it to find other numbers that Zip could be.

Clue 2

Draw this string picture on the board.




**T:** *Zip is in this string picture. What new information about Zip does the picture give us?*

**S:** *Zip is more than or equal to 0.3, and Zip is less than 2.5.*

**T:** *Is there a number in our list that Zip cannot be?*

Accept any number that is less than 0.3 or more than or equal to 2.5, for example, 12, 6, 4, 3, 0.2, 0.05, and so on, and cross it off your list.

**T:** *Is there a number Zip could be?*

**S:** *2 (or 1 or 1.5 or …).*

Accept any correct answers. Be sure to check each of the numbers you found from the first clue.

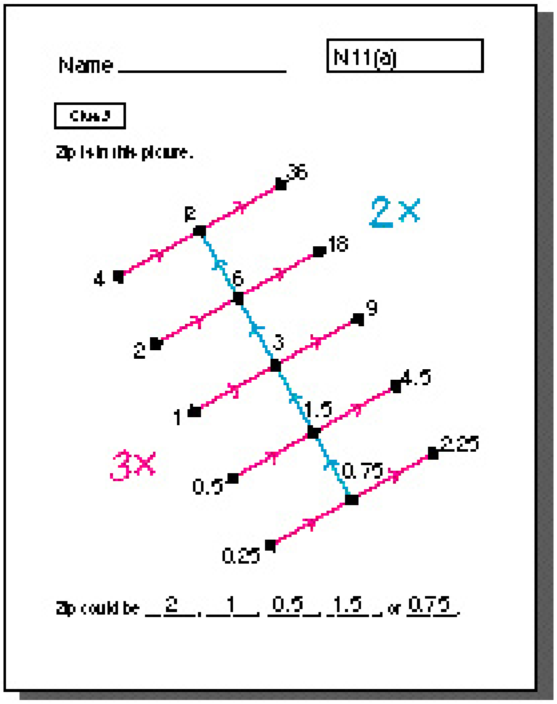
Clue 3

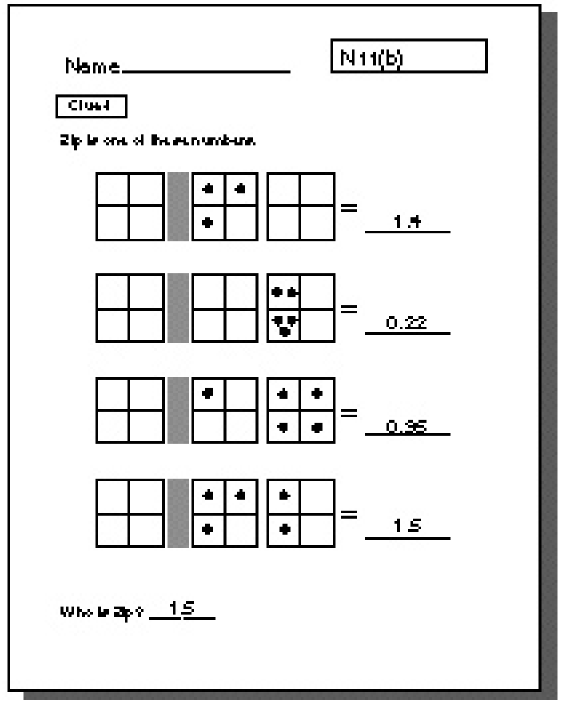
Distribute copies of Worksheet N11(a) and note that this worksheet gives the next clue. Instruct students to label dots in their pictures while you draw the picture on the board. When several students have completed the worksheet, ask volunteers to label the dots in the picture on the board. The completed picture can be found at the end of the lesson description. The class should conclude that Zip could be 2, 1.5, 1, 0.75, or 0.5.

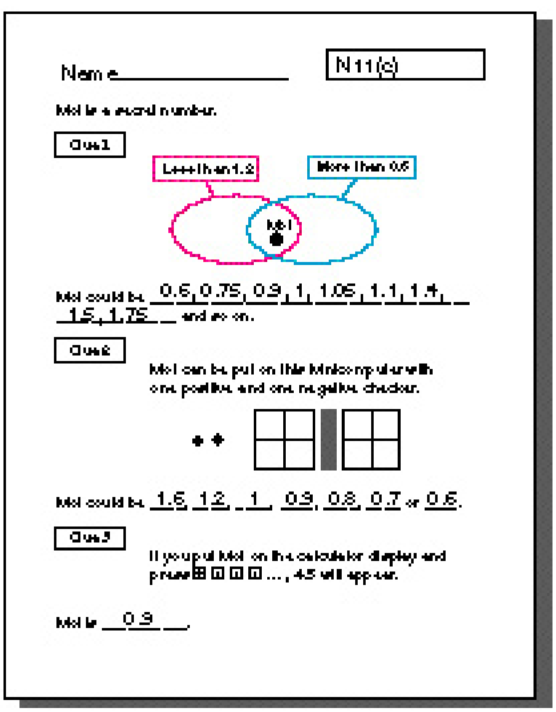
Clue 4

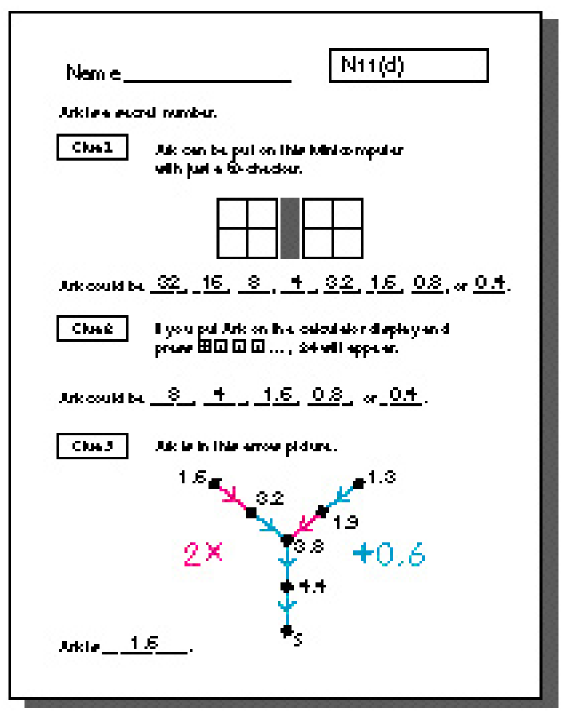
Direct students to the fourth clue on Worksheet N11(b). Some students may complete this worksheet independently, while others may benefit from putting the configurations on the demonstration Minicomputer and making trades to get standard configurations. The class should conclude that Zip is 1.5.

Organize the class into cooperative groups to work on the detective stories on Worksheets N11(c) and (d).


Name
N11(a)
Clue 3
Zip is in this picture.
36
2x
18
4
6
9
2
3
4.5
1
1.5
2.25
3x
0.75
0.5
0.25
Zip could be 2, 1, 0.5, 1.5, or 0.75.


Name
N11(b)
Clue 4
Zip is one of these numbers.
= 1.4
= 0.22
= 0.35
= 1.5
Who is Zip? 1.5


Name
N11(c)
Mel is a secret number.
Clue 1
Less than 1.2
More than 0.6
Mel
Mel could be 0.6, 0.75, 0.9, 1, 1.05, 1.1, 1.4, 1.5, 1.75 and so on.
Clue 2
Mel can be put on this Minicomputer with one positive and one negative checker.
Mel could be 1.6, 1.2, 1, 0.9, 0.8, 0.7 or 0.6.
Clue 3
If you put Mel on the calculator display and press … , 4.5 will appear.
Mel is 0.9.


Name
N11(d)
Ark is a secret number.
Clue 1
Ark can be put on this Minicomputer with just a ⑩-checker.
Ark could be 32, 16, 8, 4, 3.2, 1.6, 0.8, or 0.4.
Clue 2
If you put Ark on the calculator display and press … , 24 will appear.
Ark could be 8, 4, 1.6, 0.8, or 0.4.
Clue 3
Ark is in this arrow picture.
1.6
1.3
3.2
1.9
3.8
2x
+0.6
4.4
5
Ark is 1.6.

## Capsule Lesson Summary

Create a list of division problems, each with 6 as its solution. Investigate patterns in division problems such as 56 ÷ 7 = 8; 560 ÷ 7 = 80; and 5 600 ÷ 7 = 800. Present a series of division problems in order to study the effect of adding 1 to a dividend and of subtracting 1 from a dividend. Label dots joined by ÷3 and ÷4 arrows. In preparation for a division algorithm, build an arrow road using –600, –60, and –6 arrows.

### Materials

| Teacher | | Student | |
|---|---|---|---|
| | • Colored chalk | | • Paper<br>• Colored pencils, pens, or crayons<br>• Worksheets N12*, **, and *** |

## Description of Lesson

The first three exercises of this lesson are mostly mental arithmetic using division patterns. They should all be done at a brisk pace.

### Exercise 1

One at a time, ask students to solve these division problems and write the results on the board.

$24 \div 4 = \boxed{6}$

$54 \div 9 = \boxed{6}$

$18 \div 3 = \boxed{6}$

**T:** ***The solution to each of these problems is 6. What are some other division problems that have 6 as the answer?***

List students' suggestions on the board. Encourage them to use both basic facts and patterns to create new problems. Comment on any patterns that emerge. Examples of such division problems are given below.

| 30 ÷ 5 = 6 | 36 ÷ 6 = 6 | 6 ÷ 1 = 6 |
|---|---|---|
| 12 ÷ 2 = 6 | 42 ÷ 7 = 6 | 60 ÷ 10 = 6 |
| 24 ÷ 4 = 6 | 420 ÷ 70 = 6 | 66 ÷ 11 = 6 |
| 48 ÷ 8 = 6 | 4 200 ÷ 700 = 6 | 72 ÷ 12 = 6 |
| 96 ÷ 16 = 6 | 42 000 ÷ 7 000 = 6 | 78 ÷ 13 = 6 |

### Exercise 2

Erase the board. Pose the following problems, one at a time. (Answers are in boxes.)

$56 \div 7 = \boxed{8}$ $\quad$ $54 \div 9 = \boxed{6}$

$560 \div 7 = \boxed{80}$ $\quad$ $540 \div 9 = \boxed{60}$

$5\,600 \div 7 = \boxed{800}$ $\quad$ $5\,400 \div 9 = \boxed{600}$

**T:** ***What patterns do you notice?***

**S:** ***When you multiply the first number in a division problem by 10, the answer is multiplied by 10.***

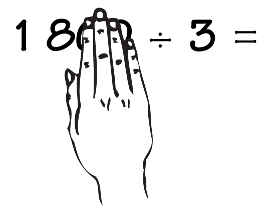
**T:** ***Let's use this pattern to solve some more problems.***

Write these problems on the board, one at a time, and ask for the answers.

| | |
|---|---|
| $120 \div 4 = \boxed{30}$ | $420 \div 6 = \boxed{70}$ |
| $350 \div 5 = \boxed{70}$ | $3\,200 \div 8 = \boxed{400}$ |
| $3\,500 \div 5 = \boxed{700}$ | $320 \div 8 = \boxed{40}$ |
| $1\,800 \div 3 = \boxed{600}$ | $56\,000 \div 7 = \boxed{8\,000}$ |

If students have difficulty, you may find the following strategy useful. For example, suppose students cannot solve the problem 1 800 ÷ 3.

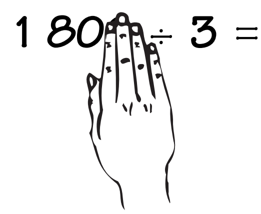
Cover the two 0s of 1 800.




**T:** ***What number is 18 ÷ 3?***

**S:** ***6.***

Cover one 0.




**T:** ***What number is 180 ÷ 3?***

**S:** ***60.***

Return to the original problem.

$1\,800 \div 3 =$

**T:** ***What number is 1 800 ÷ 3?***

**S:** ***600.***

You can repeat this exercise with similar problems whenever you have a few minutes available for mental arithmetic.

### Exercise 3

Write this problem on the board.

$7\overline{)45}$

**T:** ***Before we solve this problem, can you suggest a story problem for which you would divide 45 by 7?***

Accept several stories; if most situations involve sharing, suggest the following problem.

**T:** ***Here's a story involving 45 ÷ 7 that is not about sharing. A Girl Scout leader has a 45-meter length of rope. Each scout needs 7 meters of rope. How many 7-meter pieces can the leader cut from the 45-meter rope?***

Encourage students to find stories similar to the preceding one.

Use the response to one of the student's stories or the rope problem when completing the problem on the board. For example:

**T:** ***The scout leader can cut 6 pieces of rope (7-meter pieces) and will have 3 meters of rope left over.***

$$\begin{array}{r} 6 \text{ R} = 3 \\ 7\overline{)45} \end{array}$$

Write these problems on the board, one at a time, and solve them as a class. Refer to a story line to help explain the solutions. (Answers are in boxes.)

$$\begin{array}{r} \boxed{6 \text{ R} = 4} \\ 7\overline{)46} \end{array} \qquad \begin{array}{r} \boxed{6 \text{ R} = 6} \\ 7\overline{)48} \end{array} \qquad \begin{array}{r} \boxed{7 \text{ R} = 1} \\ 7\overline{)50} \end{array}$$

$$\begin{array}{r} \boxed{6 \text{ R} = 5} \\ 7\overline{)47} \end{array} \qquad \begin{array}{r} \boxed{7} \\ 7\overline{)49} \end{array}$$

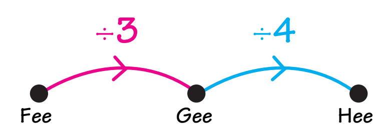
Refer back to the original problem and present these problems in a similar manner.

$$\begin{array}{r} \boxed{6 \text{ R} = 2} \\ 7\overline{)44} \end{array} \qquad \begin{array}{r} \boxed{6} \\ 7\overline{)42} \end{array} \qquad \begin{array}{r} \boxed{5 \text{ R} = 5} \\ 7\overline{)40} \end{array}$$

$$\begin{array}{r} \boxed{6 \text{ R} = 1} \\ 7\overline{)43} \end{array} \qquad \begin{array}{r} \boxed{5 \text{ R} = 6} \\ 7\overline{)41} \end{array}$$

## Exercise 4

Draw this arrow picture and table on the board.




| Fee | Gee | Hee |
|---|---|---|
| 24 | | |
| 60 | | |
| 600 | | |

**T:** ***If Fee is 24, what numbers are Gee and Hee?***

**S:** ***Gee is 8 because 24 ÷ 3 = 8, and Hee is 2 because 8 ÷ 4 = 2.***

Complete the columns for Gee and Hee in the table. Then add several more rows to the table, giving values for Gee.

| Fee | Gee | Hee |
|---|---|---|
| 24 | 8 | 2 |
| 60 | 20 | 5 |
| 600 | 200 | 50 |
| | 12 | |
| | 32 | |
| | 320 | |

**T:** ***If Gee is 12, what number is Fee?***

**S:** ***36.***

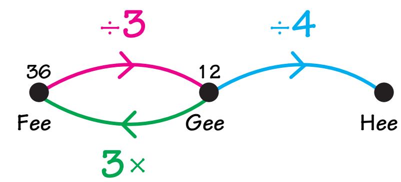
**T:** ***Why?***

**S:** ***36 ÷ 3 = 12.***

**S:** ***The opposite of ÷3 is 3x. 3 x 12 = 36.***

# N12

Draw a return arrow from Gee to Fee and label it 3x.




**T:** ***What number is Hee?***

**S:** ***3, since 12 ÷ 4 = 3.***

Complete these rows in the table. Then add a couple more rows, giving values for Hee.

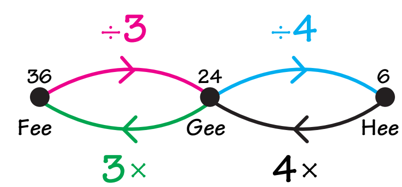
| Fee | Gee | Hee |
|---|---|---|
| 36 | 12 | 3 |
| 96 | 32 | 8 |
| 960 | 320 | 80 |
| | | 6 |
| | | 60 |

**T:** ***If Hee is 6, what number is Gee?***

**S:** ***24, since 24 ÷ 4 = 6.***

**S:** ***24. The opposite of ÷4 is 4x. 4 x 6 = 24.***

Draw a return arrow from Hee to Gee and label it 4x.




**T:** ***What number is Fee?***

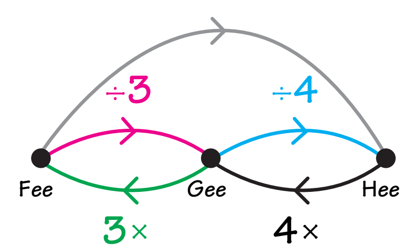
**S:** ***72, since 3 x 24 = 72.***

Complete these rows in the table.

Then add three more rows to the table as shown here, leaving out the boxed numbers. Solve the problems in a similar manner as above. You may want to refer to money to solve problems involving decimals.

| Fee | Gee | Hee |
|---|---|---|
| 72 | 24 | 6 |
| 720 | 240 | 60 |
| $\boxed{30}$ | 10 | $\boxed{2.5}$ |
| $\boxed{39}$ | $\boxed{13}$ | 3.25 |
| 18 | $\boxed{6}$ | $\boxed{1.5}$ |

Draw an arrow from Fee to Hee.




**T:** ***What could this (gray) arrow be for?***

**S:** ***÷12, since ÷3 followed by ÷4 is the same as ÷12.***

**S:** ***÷12. In the table, Fee divided by 12 always equals Hee. For example, 24 ÷ 12 = 2 and 72 ÷ 12 = 6.***

Label the arrow ÷12 and then trace an arrow from Hee to Fee.

**T:** ***If I drew an arrow from Hee to Fee, what could it be for?***

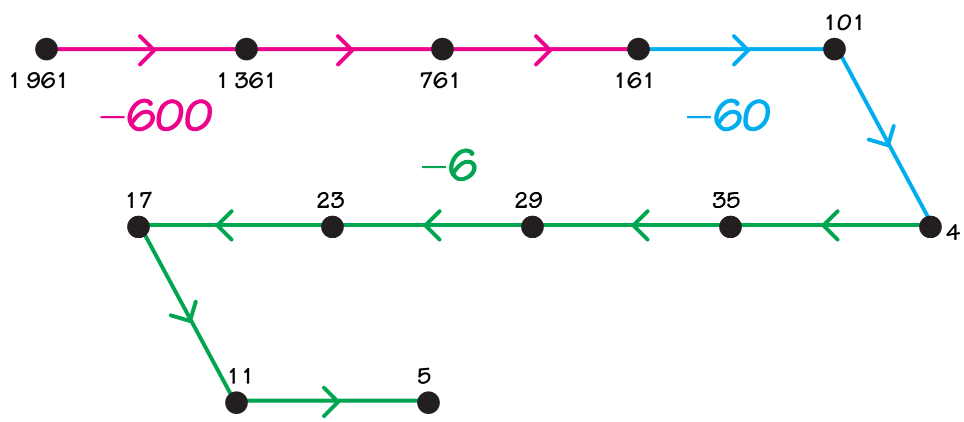
**S:** ***12x, since the opposite of ÷12 is 12x.***

**S:** ***12x, because 4x followed by 3x is 12x.***

## Exercise 5

Write this information on the board and suggest students copy it on their papers.

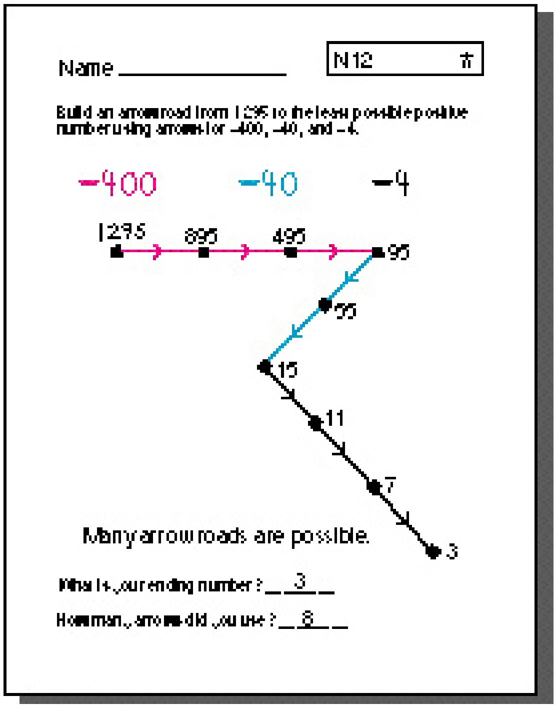

**T:** ***Build an arrow road from 1 961 to the least positive number you can. Try to use as few arrows as possible; use as many –600 arrows as you can, then as many –60 arrows, and finally as many –6 arrows.***




Let students build roads on their own. As students complete this problem, use Worksheets N12*, **, and *** for additional individual work.

### Practice Activity

Use mental arithmetic exercises like Exercises 1, 2, and 3 of this lesson to give division practice.



Name ____________ N12 ☆

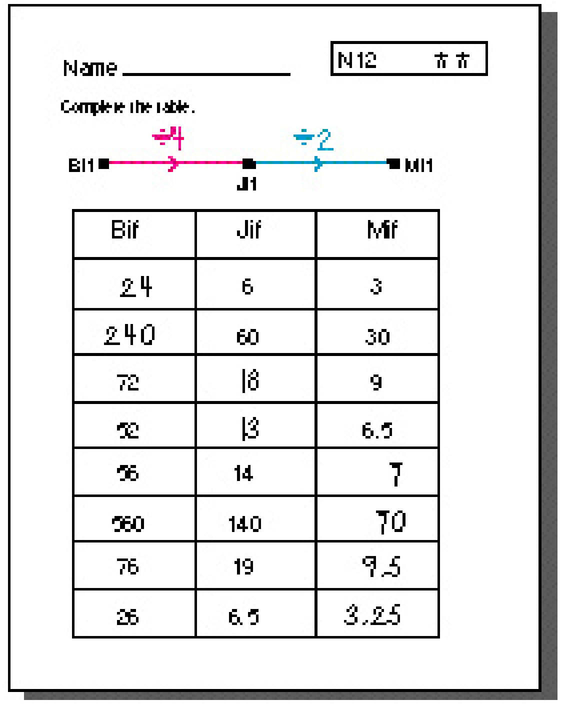
Build an arrowroad from 1295 to the least possible positive number using arrows for -400, -40, and -4.



Many arrow roads are possible.

What is your ending number? 3

How many arrows did you use? 8

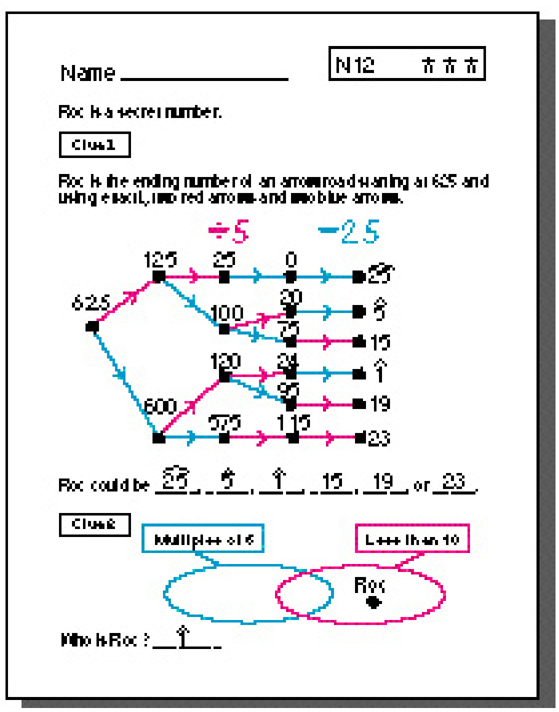


Name ____________ N12 ☆☆

Complete the table.



| Bif | Jif | Mif |
|---|---|---|
| 24 | 6 | 3 |
| 240 | 60 | 30 |
| 72 | 18 | 9 |
| 52 | 13 | 6.5 |
| 56 | 14 | 7 |
| 560 | 140 | 70 |
| 76 | 19 | 9.5 |
| 26 | 6.5 | 3.25 |



Name ____________ N12 ☆☆☆

Roc is a secret number.

Clue 1

Roc is the ending number of an arrowroad starting at 625 and using exactly two red arrows and two blue arrows.



Roc could be $\hat{25}$, $\hat{5}$, $\hat{1}$, 15, 19, or 23.

Clue 2



Who is Roc? $\hat{1}$

# N13 MULTIPLICATION ALGORITHM #1



## Capsule Lesson Summary

Using the Minicomputer, review how to multiply decimals by 10. Label each arrow in a road with x some number or with ÷ some number; stop occasionally to ask for the composite of two arrows. Find the composite of 10x followed by 10x. Calculate 42 x 45 by adding the results of 2 x 45 and 40 x 45. Do other calculations similarly.

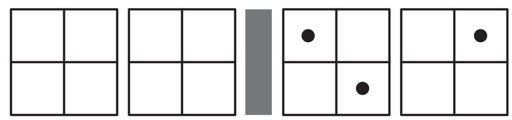
### Materials

Teacher
- Minicomputer set
- Weighted checker set
- Colored chalk
- *Multiplication Problems* Booklet

## Description of Lesson

### Exercise 1

Put this configuration on the Minicomputer.



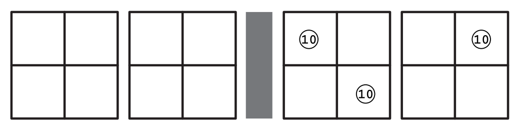
**T:** ***What number is on the Minicomputer?***

**S:** ***0.94.***

Begin a list of numbers on the board starting with 0.94.

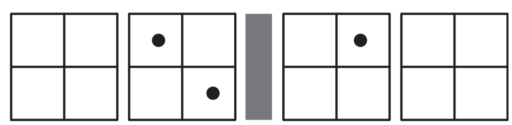
**T:** ***How can we put 10* x *0.94 on the Minicomputer?***

**S:** ***Replace each of the regular checkers with a ⑩-checker.***



Invite students to make trades with the ⑩-checkers. Check the meaning of a trade. For example:

**T:** ***We trade a ⑩-checker on the 4-pennies square for a regular checker on the 4-dimes square because 10 times 4 pennies is 40 cents or 4 dimes.***



***What did you notice about these trades?***

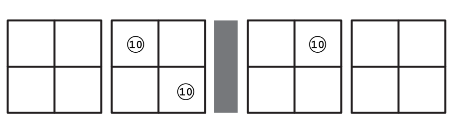
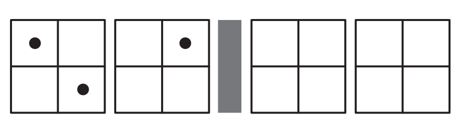
**S:** ***We always trade a ⑩-checker for a regular checker in the same position but one board to the left.***

**T:** ***What number is 10* x *0.94?***

**S:** ***9.40.***

0.94
9.40

Replace the regular checkers with ⑩-checkers and repeat this activity with 10 x 9.4.

10 × 9.4 =  =  = 94

**T:** *What patterns do you notice?*

**S:** *Each number has a 9 and a 4.*

**S:** *Each time we multiply by 10 the digits move over to the left.*

**S:** *The decimal point moves one place to the right each time we multiply by 10.*

0.94
9.40
94.00

**T:** *What could be the next number in the list?*

**S:** *940; 10* x *94 = 940.*

**T:** *10*x *what number is 0.94?*

**S:** *0.094.*

0.094
0.94
9.40
94.00
940.00

### Exercise 2

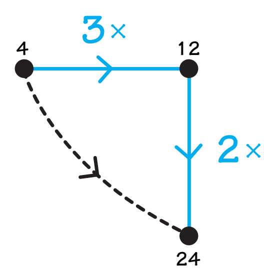
Announce to the class that in the arrow road you are going to draw on the board, each arrow will be x some number or ÷ by some number.

Start with this arrow.


**T:** *What could the arrow be for?*

**S:** *3*x*.*

Extend the road to 24.




**T:** *What could this arrow be for?*

**S:** *2*x*.*

Trace a dotted arrow from 4 to 24.

**T:** *What could an arrow from 4 to 24 be for?*

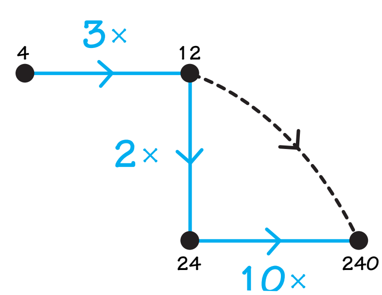
**S:** *6*x*.*

Extend the road with an arrow to 240.

**T:** *What could this arrow be for?*

**S:** *10*x*.*

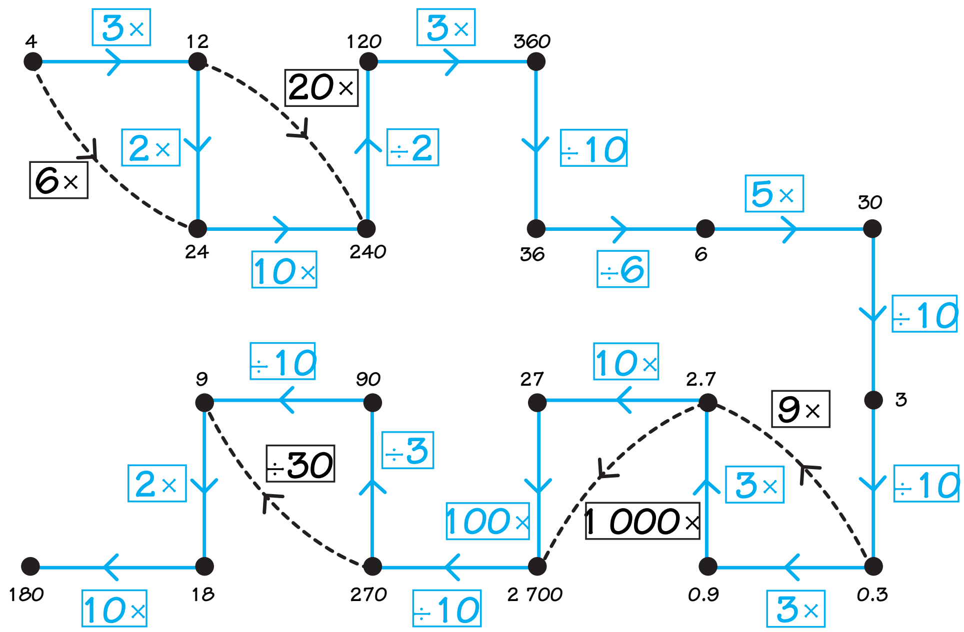
Trace a dotted arrow from 12 to 240.




**T:** *What could an arrow from 12 to 240 be for?*

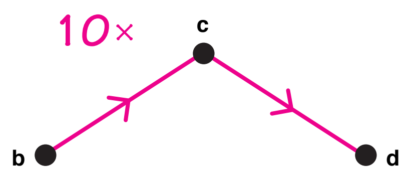
**S:** *20*x*.*

Continue extending the arrow road as long as there is interest. You may use the arrow picture below or choose other arrows. A few dotted arrows have been included and correct responses have been boxed. Feel free to adjust the level of this exercise to your class. Be careful to choose numbers so that x or ÷ arrows are appropriate.




## Exercise 3

Draw this arrow picture on the board. Put 7 at **b**.




Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

**T:** *If 7 is here* (point to **b**)*, what number is here* (at **c**)*?*

**S:** *70; 10* x *7 = 70.*

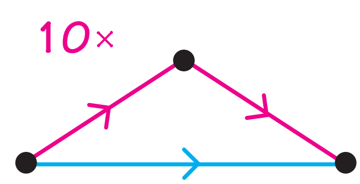
**T:** *What number is here* (at **d**)*?*

**S:** *700; 10* x *70 = 700.*

| b | c | d |
|---|---|---|
| 7 | $\boxed{70}$ | $\boxed{700}$ |
| $\boxed{90}$ | 900 | $\boxed{9\,000}$ |
| $\boxed{4.5}$ | $\boxed{45}$ | 450 |
| $\boxed{0.68}$ | 6.8 | $\boxed{68}$ |

Repeat the activity several times.
Suggested labels are given here with answers in boxes.

Add this blue arrow to your picture.




**T:** *What could this blue arrow be for?*

**S:** *100*x.

If students think that 10x followed by 10x could be 20x, label **b** with a number such as 2 or 3; then let the class check to find that the number at **d** is not 20x the number at **b**. Label the blue arrow 100x.

**T:** *What number is* *100* x *51?* (5 100)
*100* x *32?* (3 200)
*100* x *129?* (12 900)
*100* x *7.3?* (730)

### Exercise 4

Record the corresponding number sentences on the board as students respond to these questions.

**T:** ***What number is ten 45s?***

**S:** ***450.***

**T:** ***What number is two 45s?***

**S:** ***90.***

**T:** ***What number is twelve 45s?***

**S:** ***540.***

**T:** ***How did you do the calculation?***

**S:** ***I added 450 + 90.***

$$10 \times 45 = 450$$
$$2 \times 45 = 90$$
$$12 \times 45 = 540$$

Continue in this manner to generate a list of number sentences such as the ones here. Students should solve each problem mentally, using a multiplication fact already on the board.

**T:** ***What number is 20 x 45? Try to use one of the facts on the board.***

**S:** ***10 x 45 = 450, so 20 x 45 = 900. We just need to double 450.***

**S:** ***2 x 45 = 90, so I calculated 10 x 90.***

$$20 \times 45 = 900$$
$$4 \times 45 = 180$$
$$24 \times 45 = 1080$$
$$40 \times 45 = 1800$$
$$42 \times 45 = 1890$$

Ask students to calculate 12 x 72 on their papers. Look for different ways that students use to do the calculation. These two methods may be tried.

$$\begin{array}{r} 10 \times 72 = 720 \\ 2 \times 72 = 144 \\ \hline 12 \times 72 = 864 \end{array} \qquad \begin{array}{r} 12 \times 2 = 24 \\ 12 \times 70 = 840 \\ \hline 12 \times 72 = 864 \end{array} \ (12 \times 7 = 84, \text{ so } 12 \times 70 = 840)$$

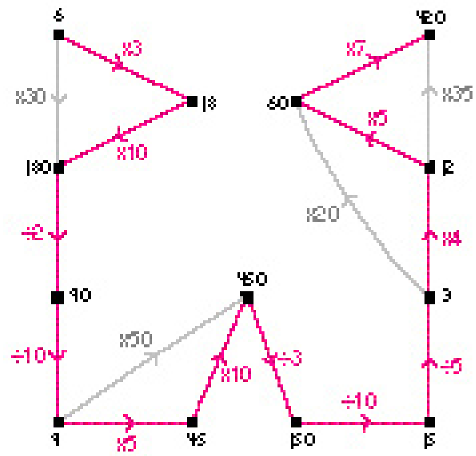
A 16-page booklet, *Multiplication Problems*, contains worksheets to use with a sequence of lessons (N13, N18, and N25) involving the multiplication algorithm. Allow about 10–15 minutes for independent work in this booklet. At the end of the lesson, collect the booklets, check them, and store them for future use in the remaining lessons on the multiplication algorithm.

Multiply.

| | | |
|---|---|---|
| 54 × 4 = 216 | 39 × 2 = 78 | 62 × 3 = 186 |
| 76 × 5 = 380 | 121 × 4 = 484 | 143 × 3 = 429 |
| 275 × 2 = 550 | 487 × 3 = 1 461 | 789 × 4 = 3 156 |

2

Label each arrow as × or ÷ some whole number.



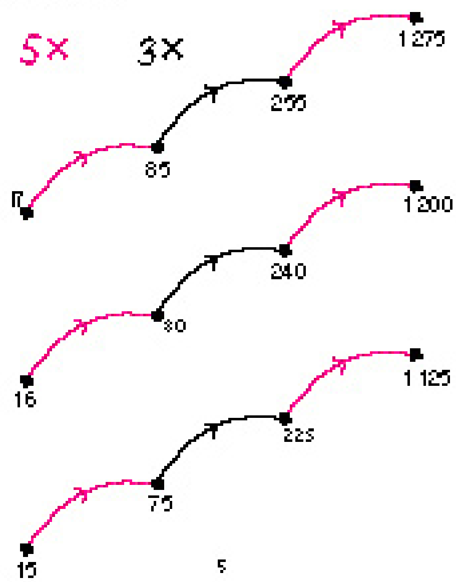

3

Complete.

| | |
|---|---|
| 6 × 8 = 48 | 7 × 4 = 28 |
| 6 × 80 = 480 | 7 × 40 = 280 |
| 6 × 800 = 4 800 | 7 × 400 = 2 800 |
| 9 × 10 = 90 | 5 × 20 = 100 |
| 9 × 11 = 99 | 5 × 22 = 110 |
| 9 × 12 = 108 | 5 × 24 = 120 |
| 9 × 13 = 117 | 5 × 26 = 130 |
| 8 × 10 = 80 | 4 × 200 = 800 |
| 8 × 3 = 24 | 4 × 30 = 120 |
| 8 × 13 = 104 | 4 × 230 = 920 |

4

Label the dots.




5

Complete.

10 × 25 = 250

2 × 25 = 50

12 × 25 = 300

20 × 25 = 500

22 × 25 = 550

24 × 25 = 600

48 × 25 = 1 200

6

Complete.

2 × 12 = 24

6 × 12 = 72

10 × 12 = 120

16 × 12 = 192

20 × 12 = 240

26 × 12 = 312

18 × 12 = 216

36 × 12 = 432

7

Circle the greatest number. Draw a box around the least number.

$\boxed{3 \times 56}$ 3 × 58 3 × 56.4

Complete.

| | | |
|---|---|---|
| 56 | 58 | 56.4 |
| × 3 | × 3 | × 3 |
| 168 | 174 | 169.2 |

Circle the greatest number. Draw a box around the least number.

8 × 4 8 × 3.72 $\boxed{8 \times 3}$

Complete.

| | | |
|---|---|---|
| 4 | 3.72 | 3 |
| × 8 | × 8 | × 8 |
| 32 | 29.76 | 24 |

8

Multiply.

| | | | |
|---|---|---|---|
| 382 | 38.2 | 247 | 2.47 |
| × 7 | × 7 | × 6 | × 6 |
| 2 674 | 267.4 | 1 482 | 14.82 |

| | | | |
|---|---|---|---|
| 2 365 | 23.65 | 8 459 | 845.9 |
| × 9 | × 9 | × 4 | × 4 |
| 21 285 | 212.85 | 33 836 | 3383.6 |

9

Suppose Aleta does 15 push-ups every day.

How many push-ups would she do in a week? 105

How many push-ups would she do in a 31-day month? 465

How many push-ups would she do in a year? 5 475 or 5 490
(365 days or 366 days)

Lionel needs 24 bottles to fill a carton.

How many bottles does he need to fill 8 cartons? 192

to fill 23 cartons? 552

to fill 105 cartons? 2 520

10

Complete the multiplication calculation.
Then write a story problem in which you would use the calculation.

36
× 7
252

Story problems will vary.

65
× 31
2015

Story problems will vary.

11

Multiply.

58
× 23
3 × 58 = 174
20 × 58 = 1 160
1 334

74
× 32
2 × 74 = 148
30 × 74 = 2 220
2 368

93
× 57
7 × 93 = 651
50 × 93 = 4 650
5 301

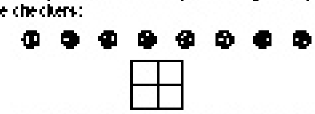
86
× 49
9 × 86 = 774
40 × 86 = 3 440
4 214

12

Cal is a secret number.

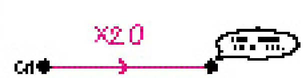
Clue 1

Cal can be put on this Minicomputer using exactly one of these checkers:



Cal could be 2, 4, 8, 16, 3, 6, 12, 24, 32, 5, 10, 20, 40, 48, 7, 14, 28, 56, 64, 9, 18, 36, 72

Clue 2




Cal could be 16, 24, 32, 20, 28, or 18.

Clue 3

Cal is a multiple of 7.

Who is Cal? 28

13

Multiply.

```
     74                  138
  × 123                ×  66
3 × 74 =    222       6 × 138 =   690
20 × 74 =  1480      60 × 138 =  8280
100 × 74 = 7400                  8970
           9102

    132                   156
   × 49                 × 214
9 × 132 = 1188        4 × 156 =   624
40 × 132 = 5280      10 × 156 =  1560
           6468     200 × 156 = 31200
                                33384
```

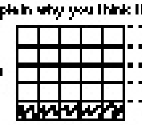
14

$5 \times 5 = 25$ $9 \times 9 = 81$ $20 \times 20 = 400$

$6 \times 4 = 24$ $10 \times 8 = 80$ $21 \times 19 = 399$

Do you see a pattern? Can you explain why you think the pattern works?

When you add a column and take away a row from a square, the result is one less.



Make up your own examples using this pattern.

$8 \times 8 = 64$ $12 \times 12 = 144$ Many examples

$9 \times 7 = 63$ $13 \times 11 = 143$ are possible.

Use the pattern to do the calculations below.

$25 \times 25 = 625$ $50 \times 50 = 2\,500$

$26 \times 24 = \underline{624}$ $51 \times 49 = \underline{2\,499}$

$39 \times 39 = \underline{1521}$ $99 \times 99 = \underline{9\,801}$

$40 \times 38 = 1520$ $100 \times 98 = 9\,800$

15

Jo is selling cards to earn money for the band. She buys the cards for $1.18 each and puts them into packages of 3. Then she sells a 3-pack for $5.00. How much does she earn if she sells 25 packages of 3? $36.50

3-pack costs $3.54 and sells for $5.00.
Jo earns $1.46 on each 3-pack.
25 x $1.46 = $36.50

---

One package of chocolate weighs 171.4 grams. A recipe that makes 50 fudge brownies calls for 3 packages. How many grams of chocolate are needed to make 300 brownies? 3085.2 grams

3 x 171.4 grams = 514.2 grams
One recipe takes 514.2 grams chocolate.
6 x 50 = 300
Six recipes make 300 brownies.
6 x 514.2 grams = 3 085.2 grams

16

# N14 CALCULATOR RELATIONS #1

## Capsule Lesson Summary

Explore the calculator relation $\boxed{+}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ … (constant mode of $\boxed{+}$) and do some mental arithmetic. Introduce new relations involving the constant mode operation of $\boxed{+}$ and $\boxed{-}$ on a calculator. Begin to investigate some patterns in sequences of numbers and in arrow pictures with these new relations.

## Materials

| Teacher | | Student | |
|---|---|---|---|
| | • Class/overhead calculator<br>• Colored chalk | | • Calculator<br>• Worksheets N14*, **, ***, and **** |

## Description of Lesson

Arrange that every student has access to a calculator. You may prefer to do Exercise 1 with an overhead calculator, letting students use their own calculators starting with Exercise 2.

### Exercise 1

When giving calculator instructions, do so slowly and clearly.

**T:** ***Turn on the calculator. Put 13 on the display. Slowly press*** $\boxed{+}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ ***and so on. Read the numbers aloud that appear on the display as we press*** $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ ***… .***

**S:** ***16, 19, 22, 25, 28, 31, 34, … .***

**T:** ***What is the calculator doing?***

**S:** ***Counting by threes.***

Let students continue reading the numbers that appear on the display until 70 is reached.

**T:** ***Now listen carefully to my instructions. 70 is on the display. Press*** $\boxed{=}$ $\boxed{=}$ $\boxed{=}$***.***
***What number is on the display?*** (79)
***Press*** $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$***. What number is on the display?*** (91)

***Now hide the display until I ask for a calculator check. Press*** $\boxed{=}$ $\boxed{=}$ $\boxed{=}$***.***
***What number should be on the display?***

**S:** ***100.***

**T:** ***Calculator check.***
***Okay, now hide the display again. Press*** $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$***.***
***What number should be on the display?*** (112) ***Check.***
***Hide the display and press*** $\boxed{=}$ $\boxed{=}$***. What two numbers should have appeared?***

**S:** ***115 and 118.***

Ask the class how many times they would need to press $\boxed{=}$ to go from 118 to 133. (Five times) After students respond, perform a calculator check.

**T:** ***What is the closest number to 200 that should appear?***

**S:** ***199.***

**T:** ***Let's have a calculator check.***

Ask students to press $\boxed{=}$ to see that 199 does indeed appear and that the next number is 202.

**T:** ***Now 202 is on the display of our calculator. How could we go backward and see all of the same numbers appear in reverse order?***

**S:** ***Press*** $\boxed{-}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ ***and so on.***

**T:** ***Let's try it. Press*** $\boxed{-}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ ***… .***

Invite students to read the numbers that appear on the display (199, 196, 193, 190, 187, …). Stop at around 175.

**T:** ***Without using the calculator, pretend we are still pressing*** $\boxed{=}$***.***
***What are some numbers that would appear?***

**S:** ***115.***

**S:** ***100.***

**S:** ***13, the starting number.***

If any answer is disputed, ask for a calculator check.

**T:** ***Are there some positive numbers less than 13 that would appear?***

**S:** ***10 or 7.***

**T:** ***What is the least positive number that would appear?***

**S:** ***1.***

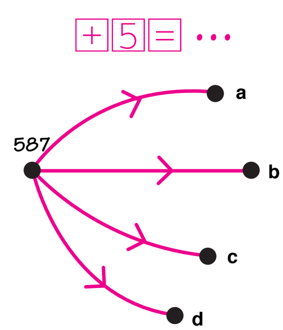
**T:** ***If we keep pressing*** $\boxed{=}$***, what negative numbers would appear?***

**S:** $^{-}2$.[†]

**S:** $^{-}5$, $^{-}8$, $^{-}11$, ***and so on.***

## Exercise 2

Draw this arrow picture on the board.




Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

**T:** ***The red arrow is for*** $\boxed{+}$ $\boxed{5}$ $\boxed{=}$ $\boxed{=}$ ***… as many times as you want.*** (Trace the arrow from 587 to **a**.) ***We start with 587 on the display and press*** $\boxed{+}$ $\boxed{5}$ $\boxed{=}$ $\boxed{=}$ ***and so on. What number could be here*** (point to **a**)***?***

**S:** ***602.***

Label **a** with any correct response.

[†]Negative numbers are displayed on calculators in a variety of ways, but usually with some placement of a "–" symbol. Therefore, in pictures and discussions with calculator relations, we use $^{-}2$ rather than $\widehat{2}$. Your students should begin to recognize different notations for negative numbers.

**T:** ***Continue pressing*** $\boxed{=}$ ***and watch the numbers that appear on the display. What number could be here*** (point to **b**)***?***

Get answers from several students and then label **b** with one that is correct.

**T:** ***Can you find a number between 850 and 860 to put here*** (at **c**)***?***
***Can you find a number greater than 1 000 to put here*** (at **d**)***?***

Label **c** and **d** with any correct answers.

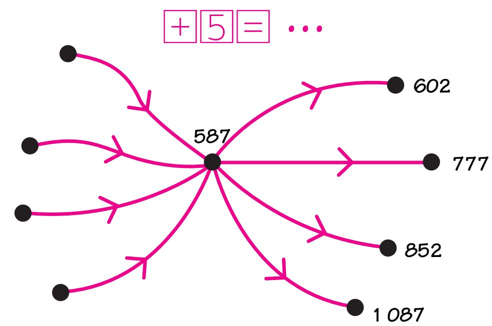
**T:** ***Do you notice any pattern to the numbers that appear on the display?***

**S:** ***The last digit is always 2 or 7.***

**T:** ***What is the number closest to 3 000 that could be at the end of an arrow in our picture?***

**S:** ***3 002.***

Add arrows to the left side of the picture as shown here.



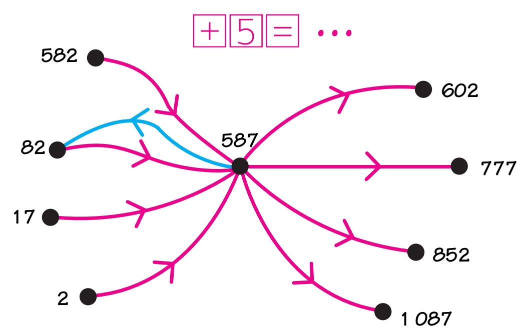

**T** (pointing to the unlabeled dots)**:** ***What numbers could we start with on the calculator so that when we press*** $\boxed{+}$ $\boxed{5}$ $\boxed{=}$ ***..., we eventually see 587?***

Let student try numbers and, as correct answers are given, record three of them in your arrow picture.

**T:** ***What is the least positive number that we could put in our picture?***

**S:** ***2.***

Add this return arrow to the picture.




**T:** ***What could the blue arrow be for?***

**S:** $\boxed{-}$ $\boxed{5}$ $\boxed{=}$ ***... .***

**T:** ***What negative numbers could be in the picture?***

Allow a few minutes for the class to consider the question. If students have trouble finding any negative numbers, suggest the following:

**T:** ***Put 2 on the display of your calculator; then press*** $\boxed{-}$ $\boxed{5}$ $\boxed{=}$ $\boxed{=}$ ***and so on. What numbers appear?***

**S:** ***$^{-}3$ (or $^{-}8$, $^{-}13$, $^{-}18$, $^{-}23$, ...).***

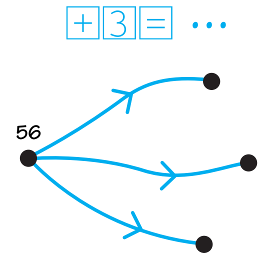
**T:** ***Do you notice a pattern to the numbers that appear?***

**S:** ***The last digit is always 3 or 8.***

## Exercise 3

# N14

Erase the board and draw this arrow picture.




**T:** ***What numbers could we put at the end of one of these arrows?***

Let several students respond, and then label two of the dots with two correct answers that you receive. Point to the remaining unlabeled dot.

**T:** ***What number greater than 300 could we put here?***

**S:** ***356.***

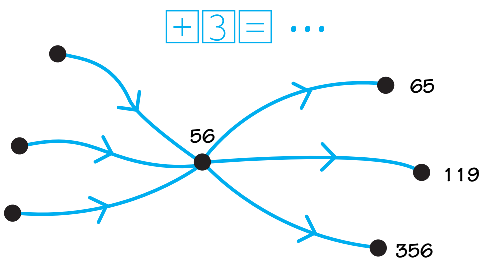
**T:** ***How much greater is 356 than 56?***

**S:** ***300.***

**T:** ***Is 300 a multiple of 3?***

**S:** ***Yes, 3*** x ***100 = 300.***

Label the dot and add arrows to the left side of the picture.




**T** (pointing to the unlabeled dots)**:** ***What numbers could be here?***

Suggest students use their calculators to find some numbers. Label two of the dots with correct answers that you receive from the class.

**T** (pointing to the remaining unlabeled dot)**:** ***What is the least positive number that we could put here?*** (2)
***What is the greatest possible negative number that we could put here?*** ($^-1$)

If necessary, suggest looking at return arrows to find negative numbers.

**T:** ***What would the return arrows for*** $\boxed{+}$ $\boxed{3}$ $\boxed{=}$ ***. . . be for?***

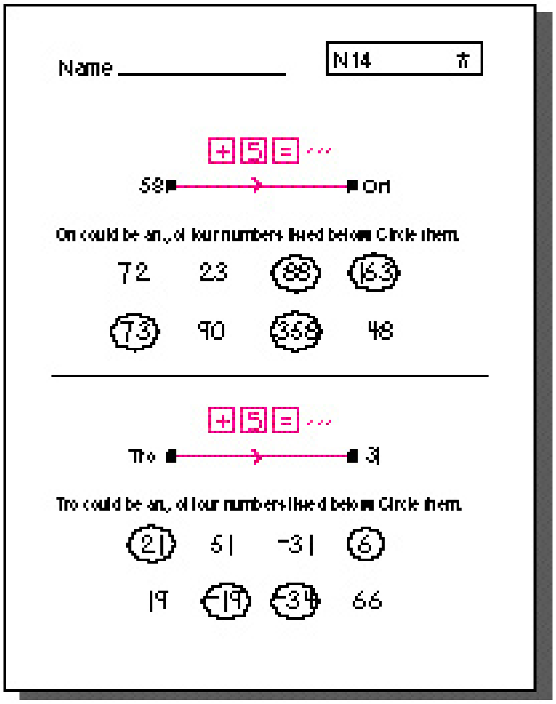
**S:** $\boxed{-}$ $\boxed{3}$ $\boxed{=}$ ***. . . .***

**T:** ***We can get to 2 by pressing*** $\boxed{-}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ ***and so on. Suppose we continue pressing*** $\boxed{=}$***. What numbers would we see displayed?***

**S:** ***$^-1$.***

**S:** ***$^-4$, $^-7$, $^-10$, and so on.***

Worksheets N14*, **, ***, and **** are available for individual work.


Name ______ N14 ☆

+ 5 = ...

58 → Ori

Ori could be any of four numbers listed below. Circle them.

72 23 88 163

73 90 358 48

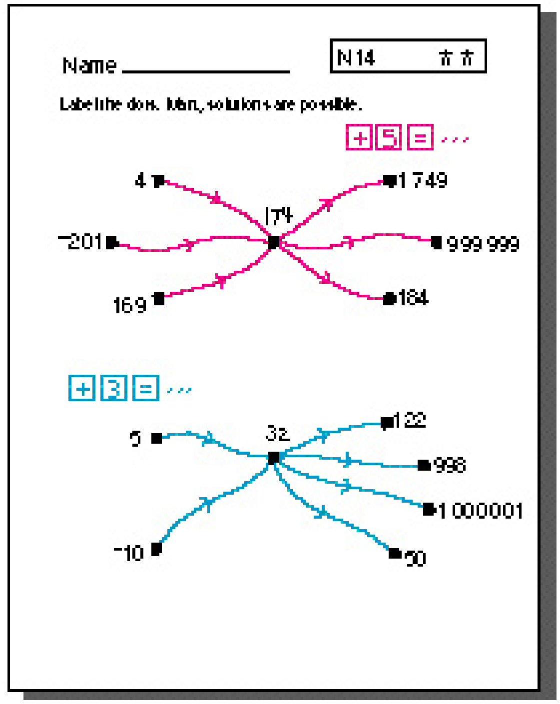
+ 5 = ...

Tro → 31

Tro could be any of four numbers listed below. Circle them.

21 51 −31 6

19 −19 −34 66


Name ______ N14 ☆☆

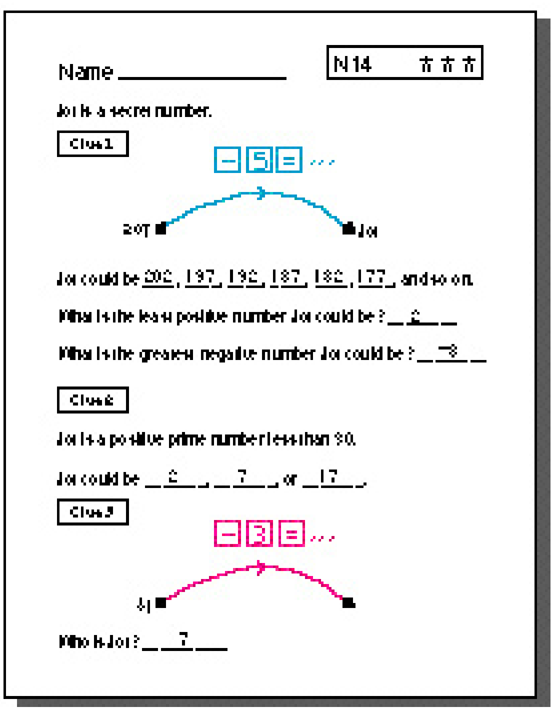
Label the dots. Many solutions are possible.

+ 5 = ...

4, 1 749, 174, −201, 999 999, 169, 184

+ 3 = ...

5, 32, 122, 998, 1 000 001, −10, 50


Name ______ N14 ☆☆☆

Joi is a secret number.

Clue 1

− 5 = ...

207 → Joi

Joi could be 202, 197, 192, 187, 182, 177, and so on.

What is the least positive number Joi could be? 2

What is the greatest negative number Joi could be? −3

Clue 2

Joi is a positive prime number less than 30.

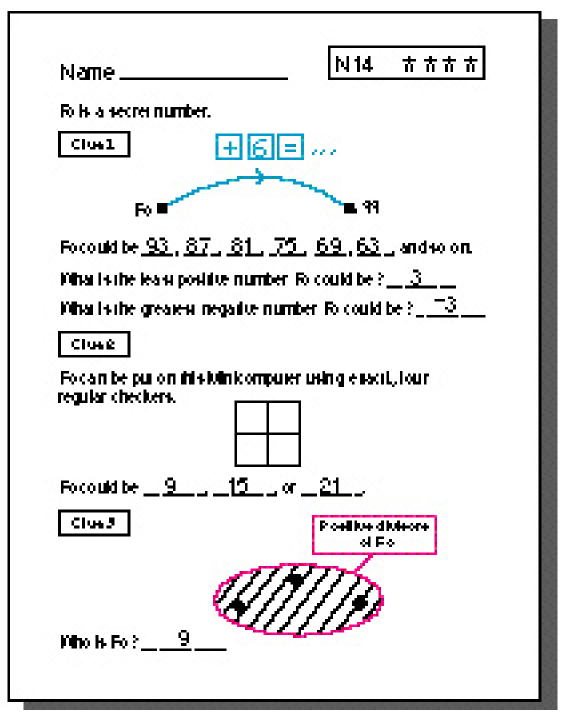
Joi could be 2, 7, or 17.

Clue 3

− 3 = ...

4

Who is Joi? 7


Name ______ N14 ☆☆☆☆

Fo is a secret number.

Clue 1

+ 6 = ...

Fo → 99

Fo could be 93, 87, 81, 75, 69, 63, and so on.

What is the least positive number Fo could be? 3

What is the greatest negative number Fo could be? −3

Clue 2

Fo can be put on this Minicomputer using exactly four regular checkers.

Fo could be 9, 15, or 21.

Clue 3

Positive divisors of Fo

Who is Fo? 9

## Capsule Lesson Summary

Do sequences of calculations such as 5 x 800; 5 x 80; 5 x 8; 5 x 0.8; and 5 x 0.08. Calculate 3 x 4.65 after noting that the answer is between 3 x 4 and 3 x 5. Compare 3 x 4, 3 x 4.65, and 3 x 5 each to 13.88.

### Materials

| | | | |
|---|---|---|---|
| Teacher | • Minicomputer set | Student | • Paper<br>• Worksheets N15* and ** |

## Description of Lesson

### Exercise 1

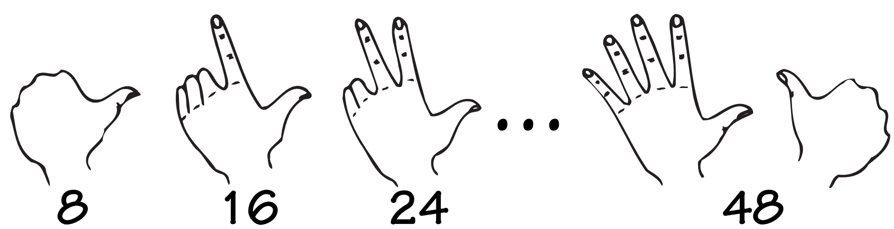
Invite several students, one at a time, to count by eights from 8 to 80.

**T:** ***Suppose you forgot what 9*** x ***5 is. How would you figure it out?***

Perhaps students will suggest counting fives on their fingers.

**T:** ***We could do something similar to figure out 6*** x ***8. We could count by eights on our fingers.***

Indicate this method as you count by eights from 0 to 48.




Perhaps a student will notice that one could also count by sixes and use eight fingers.

### Exercise 2

Record the corresponding number sentences on the board as students respond to these questions.

**T:** ***What number is 2*** x ***3?*** (6)
***What number is 2*** x ***40?*** (80)
***What number is 2*** x ***43?*** (86)

$$\begin{array}{r}2 \times 3 = 6 \\ 2 \times 40 = 80 \\ \hline 2 \times 43 = 86\end{array}$$

Continue with these problems. (Answers are in boxes.)

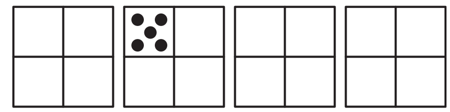
$$\begin{array}{r}5 \times 7 = \boxed{35} \\ 5 \times 30 = \boxed{150} \\ \hline 5 \times 37 = \boxed{185}\end{array} \qquad \begin{array}{r}7 \times 7 = \boxed{49} \\ 7 \times 10 = \boxed{70} \\ \hline 7 \times 17 = \boxed{119}\end{array}$$

Invite two students to do these problems at the board and to explain to the class how they did them. (Answers are in boxes.)

$$\begin{array}{r} 384 \\ \times\ 9 \\ \hline \boxed{3\,456} \end{array} \qquad \begin{array}{r} 706 \\ \times\ 8 \\ \hline \boxed{5\,648} \end{array}$$

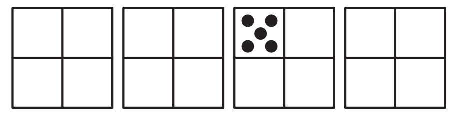
**Exercise 3**

Put 5 x 800 on the Minicomputer.

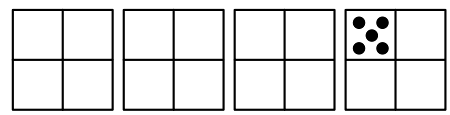


**T:** ***5** x **what number is on the Minicomputer?*** (800)
***What number is 5** x **800?*** (4 000)

Record the corresponding sentence on the board. Continue with checkers on the 80-square, and then on the 8-square.



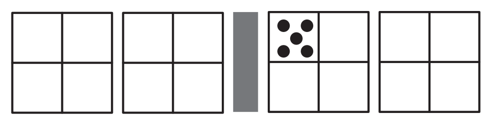
**S:** ***5** x **80 = 400.***



**S:** ***5** x **8 = 40.***

$$\begin{array}{rcr} 80 & = & 400 \\ 8 & = & 40 \end{array}$$

Put 5 x 0.8 on the Minicomputer as follows.



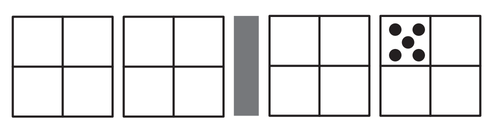
**T:** ***What number is 5** x **0.8? If you think about money, how much would five stacks of eight dimes be?***

**S:** ***$4.00.***

**T:** ***So 5** x **0.8 = 4 or 4.00.***

Add this number sentence to the list on the board.

Move the five checkers to the 0.08 square.



**T:** ***How much money would five stacks of eight pennies be?***

**S:** ***40¢.***

**T:** ***What decimal number do we write for 40¢?***

**S:** ***0.40 or 0.4.***

$$\begin{array}{rrcrcl} 5 \times & 800 & = & 4\,000 & & \\ 5 \times & 80 & = & 400 & & \\ 5 \times & 8 & = & 40 & & \\ 5 \times & 0.8 & = & 4 & = & 4.00 \\ 5 \times & 0.08 & = & 0.4 & = & 0.40 \end{array}$$

Record the number sentence in the list.

Ask the class to do these calculations, only this time do not put them on the Minicomputer. Refer to money whenever it is helpful.

$$90 = \boxed{360}$$
$$9 = \boxed{36}$$
$$0.9 = \boxed{3.6}$$
$$0.09 = \boxed{0.36}$$

Write these problems on the board and ask students to solve them individually. (Answers are in boxes.)

| | |
|---|---|
| $3 \times 700 = \boxed{2\,100}$ | $6 \times 600 = \boxed{3\,600}$ |
| $3 \times 70 = \boxed{210}$ | $6 \times 60 = \boxed{360}$ |
| $3 \times 7 = \boxed{21}$ | $6 \times 6 = \boxed{36}$ |
| $3 \times 0.7 = \boxed{2.1}$ | $6 \times 0.6 = \boxed{3.6}$ |
| $3 \times 0.07 = \boxed{0.21}$ | $6 \times 0.06 = \boxed{0.36}$ |
| | $6 \times 0.006 = \boxed{0.036}$ |

When you check answers with the class, ask what patterns students notice. Especially observe the pattern involving the change in position of the decimal point.

## Exercise 4

**T:** ***Leslie, Dorenne, and Marianne each want to get their own kickball by the time summer starts, which is in about three months. The kickball they like best costs $13.88.***

Record the price of the kickball on the board.

Kickball: $13.88

**T:** ***Leslie can earn $4 a month doing chores; Dorenne can earn $4.65 a month; and Marianne can earn $5 a month.***

Record the information on the board.

| | Leslie | Dorenne | Marianne |
|---|---|---|---|
| In one month | $4 | $4.65 | $5 |

**T:** ***Who will earn the most money in three months?***

**S:** ***Marianne. 3 x 5 = 15, so she will earn $15 in three months.***

**T:** ***Who will earn the least amount in three months?***

**S:** ***Leslie. 3 x 4 = 12, so she will earn $12 in three months.***

| | Leslie | Dorenne | Marianne |
|---|---|---|---|
| In one month | $4 | $4.65 | $5 |
| In three months | 3 × 4 = 12<br>$12 | | 3 × 5 = 15<br>$15 |

**T:** *About how much money will Dorenne have earned in three months?*

**S:** *She'll have earned more than Leslie but less than Marianne; between $12 and $15.*

**T:** *How could we find out exactly how much money Dorenne will have earned?*

**S:** *We could add: 4.65 + 4.65 + 4.65.*

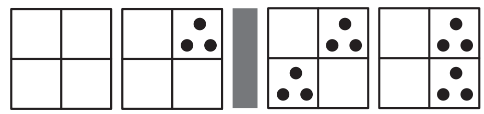
**S:** *We could multiply: 3 x 4.65.*

Record these two methods.

$$\begin{array}{r} 4.65 \\ 4.65 \\ +\ 4.65 \\ \hline \end{array} \qquad \begin{array}{r} 4.65 \\ \times\ 3 \\ \hline \end{array}$$

**T:** *How would we put this on the Minicomputer?*

**S:** *Put 4.65 on three times.*



Invite students to do this.

**T:** *How much money is on the pennies board? What is 3* x *5¢* (15¢)
*What decimal do we write for 15¢?*

**S:** *0.15.*

**T:** *How much money is on the dimes board? What is 3* x *60¢?*

**S:** *$1.80.*

**T:** *How much money is on the one-dollars board?*

**S:** *$12.*

$$\begin{array}{r} 4.65 \\ \times\ 3 \\ \hline 0.15 \\ 1.80 \\ +12.00 \\ \hline 13.95 \end{array}$$

Invite a student to do the addition required to complete the calculation. Then invite a student to do the repeated addition problem.

Point to the appropriate numbers in the repeated addition problem as you say,

**T:** *5 pennies + 5 pennies + 5 pennies = 15 pennies.*
*15 pennies is the same as 1 dime and 5 pennies.*
*6 dimes + 6 dimes + 6 dimes + the extra dime = 19 dimes.*
*19 dimes is the same as 1 dollar and 9 dimes.*
*4 dollars + 4 dollars + 4 dollars + the extra dollar = 13 dollars.*

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ } \\ 4.65 \\ 4.65 \\ +\ 4.65 \\ \hline 13.95 \end{array}$$

Point to the multiplication problem.

**T:** *Do you think that there is another way to do this multiplication problem?*

Perhaps a student will suggest multiplying 3 x 465 and then adding decimal points to the problem and to the answer.

**T:** *If we didn't already know the answer, how would we know where the decimal point belongs in the answer?*

**S:** ***We know that the answer is between 12 (3 x 4) and 15 (3 x 5).***

You may need to remind the class that Dorenne will earn more than Leslie but less than Marianne. Demonstrate that there is only one place to put the decimal point in 1 395 to arrive at a number between 12 and 15.

**T:** ***These two problems are related.***
***Let's look at why this is so.***

$$\begin{array}{r} 465 \\ \times 3 \\ \hline 1\,395 \end{array} \qquad \begin{array}{r} 4.65 \\ \times 3 \\ \hline 13.95 \end{array}$$

Point to appropriate numbers in the problems as you ask these questions and students respond.

**T:** ***If we had $4.65 all in pennies, how many pennies would we have?***

**S:** ***465 pennies.***

**T:** ***If we had three stacks of 465 pennies, how many pennies would we have?***

**S:** ***1 395 pennies.***

**T:** ***How much money is 1 395 pennies? How many dollars and how many cents?***

**S:** ***$13.95.***

On the board, record that Dorenne will earn $13.95.

| | Leslie | Dorenne | Marianne |
|---|---|---|---|
| In one month | $4 | $4.65 | $5 |
| In three months | 3 × 4 = 12<br>$12 | 3 × 4.65 = 13.95<br>$13.95 | 3 × 5 = 15<br>$15 |

**T:** ***Who will have earned enough money for a kickball?***

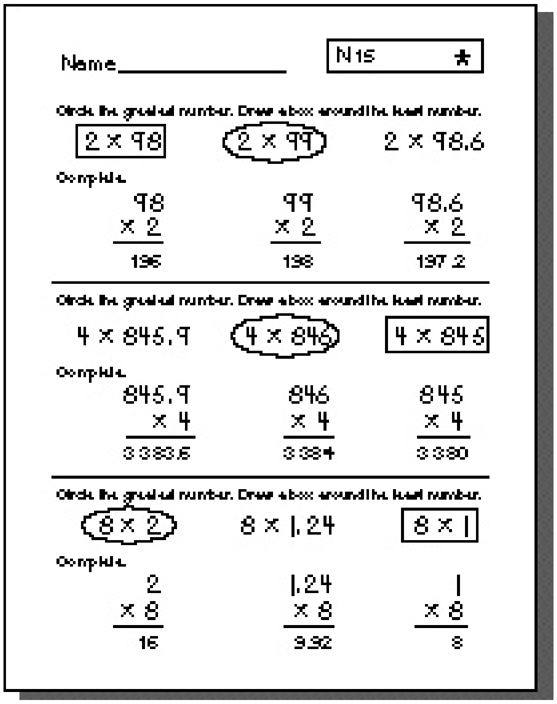
**S:** ***Dorenne and Marianne.***

**T:** ***Leslie will be short by how much?*** ($1.88)
***Dorenne will have how much money left over?*** (7¢)
***Marianne will have how much money left over?*** ($1.12)

Worksheets N15* and ** are available for individual work.

## Writing Activity

Suggest that students write another problem, similar to the one in Exercise 4, for a friend to solve.



Name____________ N15 ★

Circle the greatest number. Draw a box around the least number.

$\boxed{2 \times 98}$ 2 × 99 2 × 98.6

Complete.

| | | |
|---|---|---|
| 98 × 2 = 196 | 99 × 2 = 198 | 98.6 × 2 = 197.2 |

Circle the greatest number. Draw a box around the least number.

4 × 845.9 4 × 846 $\boxed{4 \times 845}$

Complete.

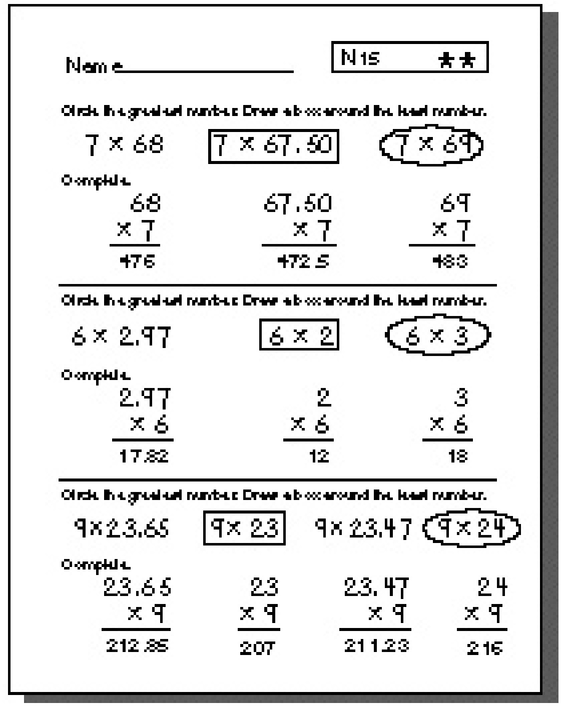
| | | |
|---|---|---|
| 845.9 × 4 = 3383.6 | 846 × 4 = 3384 | 845 × 4 = 3380 |

Circle the greatest number. Draw a box around the least number.

8 × 2 8 × 1.24 $\boxed{8 \times 1}$

Complete.

| | | |
|---|---|---|
| 2 × 8 = 16 | 1.24 × 8 = 9.92 | 1 × 8 = 8 |



Name____________ N15 ★★

Circle the greatest number. Draw a box around the least number.

7 × 68 $\boxed{7 \times 67.50}$ 7 × 69

Complete.

| | | |
|---|---|---|
| 68 × 7 = 476 | 67.50 × 7 = 472.5 | 69 × 7 = 483 |

Circle the greatest number. Draw a box around the least number.

6 × 2.97 $\boxed{6 \times 2}$ 6 × 3

Complete.

| | | |
|---|---|---|
| 2.97 × 6 = 17.82 | 2 × 6 = 12 | 3 × 6 = 18 |

Circle the greatest number. Draw a box around the least number.

9 × 23.65 $\boxed{9 \times 23}$ 9 × 23.47 9 × 24

Complete.

| | | | |
|---|---|---|---|
| 23.65 × 9 = 212.85 | 23 × 9 = 207 | 23.47 × 9 = 211.23 | 24 × 9 = 216 |

# N16 FRACTIONS ON THE NUMBER LINE

## Capsule Lesson Summary

Count by thirds and by sevenths. Review finding fractional parts of a rectangle by considering how to cut up a sheet of plywood. Refer to the *Sasquatch* story to locate fractions greater than 1 on a number line. Use a number line to compare fractions.

### Materials

**Teacher**
- Meter stick
- Colored chalk

**Student**
- Metric ruler
- Worksheets N16*, **, ***, and ****

## Description of Lesson

### Exercise 1

**T:** ***Let's start at 0 and count by thirds. I'll start: 0, $\frac{1}{3}$.***

Call on students in an order natural to their seating arrangement to continue.

**S:** ***$\frac{2}{3}$.***

**S:** ***$\frac{3}{3}$.***

**T:** ***Give us another name for $\frac{3}{3}$.***

**S:** ***1.***

Call on students to continue: $1\frac{1}{3}$, $1\frac{2}{3}$, 2, $2\frac{1}{3}$, … until every student has responded exactly one time. As necessary, explain alternative names for the same number, for example, $\frac{4}{3}$ names the same number as $1\frac{1}{3}$. Write the last student's number on the board.

**T:** ***Let's start again and count by sevenths, starting at 0. Do you think our ending number will be more or less?***

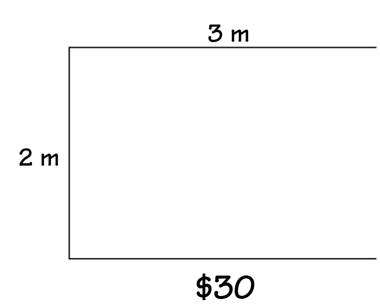
Let several students give their opinions, but do not reveal the correct answer yet.

**T:** ***Let's check. I'll start: 0, $\frac{1}{7}$.***

Call on students to continue $\frac{2}{7}$, $\frac{3}{7}$, … until every student has responded exactly one time. The class should notice that the ending number is less when they count by sevenths than when they count by thirds.

### Exercise 2

Record this information on the board. A convenient size for the rectangle is 90 cm by 60 cm.



# N16

Ask your students if any of their parents do carpentry at home. Let students briefly describe any recent carpentry projects in their homes.

Point to the rectangle on the board.

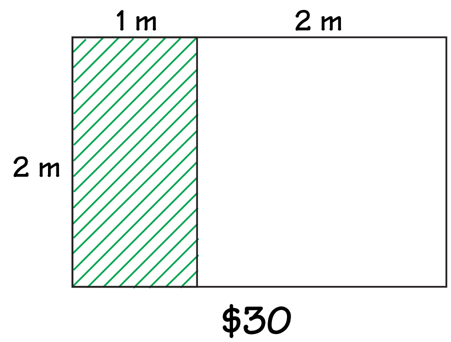
**T:** ***A hardware store owner has one large 3 m by 2 m sheet of plywood for sale for $30. His first customer is a carpenter who needs a 1 m by 2 m piece of plywood for a door. About how large is a 1 m by 2 m door?***

A 1 m by 2 m door is about the same height but somewhat wider than doors in most modern homes. You may wish to ask some students to measure the classroom door.

**T:** ***The hardware store owner doesn't have the correct size piece of plywood and reluctantly agrees to cut up the large sheet. Who can show how the plywood could be cut?***

**S:** ***Measure one-third of the rectangle's length and cut it there.***

There are other ways to cut off a 1 m by 2 m piece. Allow students to show other ways, but draw this picture on the board.



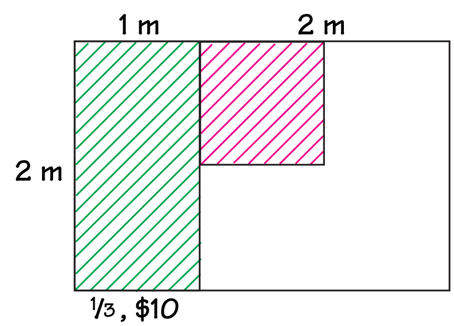

**T:** ***What fraction of the entire sheet did the carpenter buy?*** (⅓)
***How much should the hardware store owner charge?***

**S:** ***$10. ⅓ of the rectangle is green, and ⅓ of $30 is $10.***

**S:** ***You could cut exactly three 1 m by 2 m pieces from the original sheet of plywood; 3 x $10 = $30.***

If anyone suggests that the owner should charge more for cutting up a large sheet of plywood, agree that this would be reasonable, but explain that in fact the owner decides to charge $10.

Add the following information to the picture.




**T:** ***The next customer buys the piece I've shaded red. What is its size?***

**S:** ***1 m by 1 m.***

**T:** ***What fraction of the original sheet is this square?***

**S:** ***⅙. Six small red squares would fit in the large rectangle.***

**T:** ***How much should this piece cost?***

**S:** ***$5, since ⅙ of $30 = $5.***

**S:** ***$5. The red piece is half of the size of the green piece. ½ of $10 = $5.***

**T:** ***What fraction of the whole sheet has been sold to the two customers?***

**S:** ***½. The unsold piece is the same size and shape as the two pieces that have been sold.***

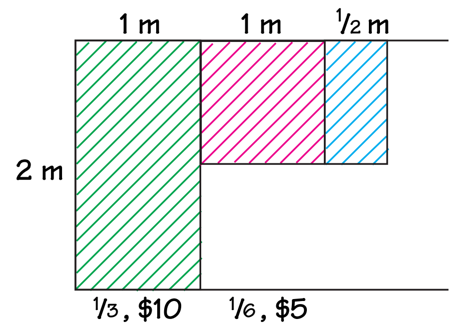
Another likely response is 3/6, but do not insist on this answer.

**T:** ***The two customers bought $\frac{1}{3}$ and $\frac{1}{6}$ of the sheet, respectively; $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$.***

If students seem surprised at this equation, use the illustration to assure that it is correct. Do not worry about usual methods to add fractions at this time; they will be considered later in the *CSMP* curriculum.

**T:** ***The next customer buys another rectangular piece of the sheet of plywood and pays $2.50. Who can show a piece this person could buy?***

Many answers are possible. Accept any rectangle that is half the size of the red square. Color the new piece blue, as in this example:




**T:** ***What size is the blue piece?***

**S:** ***$\frac{1}{2}$ m by 1 m.***

**S:** ***1 m by 50 cm.***

**T:** ***What fraction of the original sheet is the blue rectangle?***

**S:** ***$\frac{1}{12}$. Twelve of the blue rectangles would fit in the large rectangle.***

**S:** ***$\frac{1}{12}$. Two of the blue rectangles would fit in the red square, and six red squares would fill the large rectangle. 2 x 6 = 12.***

**T:** ***The next customer buys the rest of the sheet of plywood. What is the cost? Why?***

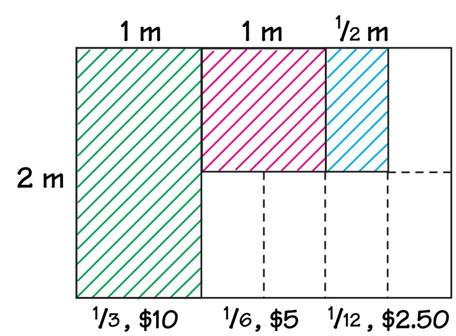
**S:** ***$12.50. The remaining piece is the same size as one green piece and one blue piece; $10 + $2.50 = $12.50.***

**S:** ***$12.50. The pieces sold cost $10 + $5 + $2.50, which is $17.50. $30 – $17.50 = $12.50.***

**T:** ***What fraction of the whole sheet did this person buy?***

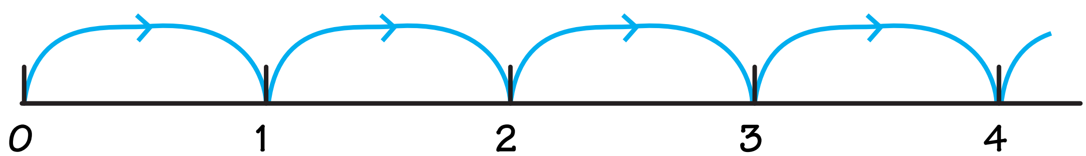
This is a difficult question. You may need to suggest subdividing that region.

**S:** ***$\frac{5}{12}$. Five blue pieces would fit in that region, and each blue piece is $\frac{1}{12}$ of the rectangle.***




## Exercise 3

Erase the board. Then draw this number line with a 50 cm distance between marks.



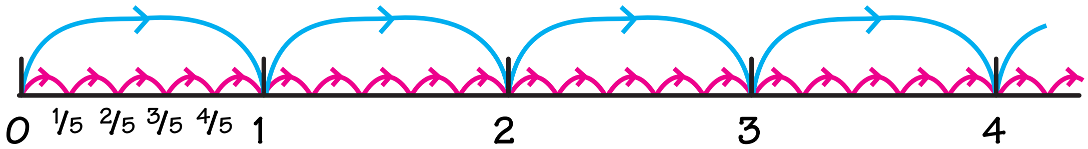

**T:** ***Do you remember the story about Sasquatch? The arrows show Sasquatch's large steps, from 0 to 1, 1 to 2, 2 to 3, and so on. Of course, the children who play in Sasquatch's footprints take smaller steps. One child takes five steps to walk from 0 to 1. How could we locate and show these steps?***

**S:** ***Measure the distance from 0 to 1 and divide it into five equal parts.***

**T:** ***The distance from 0 to 1 is 50 centimeters. Where should we mark the child's footprints?***

**S:** ***50 ÷ 5 = 10, so make marks at 10 cm, 20 cm, 30 cm, and 40 cm from 0.***

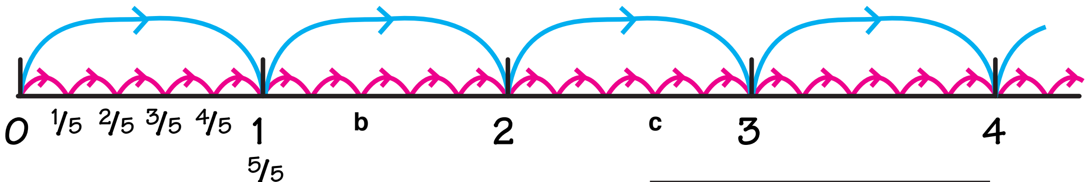
Use a ruler to accurately mark the child's footprints; then draw red arrows to show the child's steps. Invite students to label the marks.




**T:** ***Does this suggest another name for 1?***

**S:** ***$\frac{5}{5}$.***

Continue with red arrows between 1 and 2, 2 and 3, and so on.




Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

**T:** ***The child keeps walking beyond 1. How would we label this footprint*** (point to **b**)***?***

**S:** ***$\frac{7}{5}$; it is the seventh step from 0.***

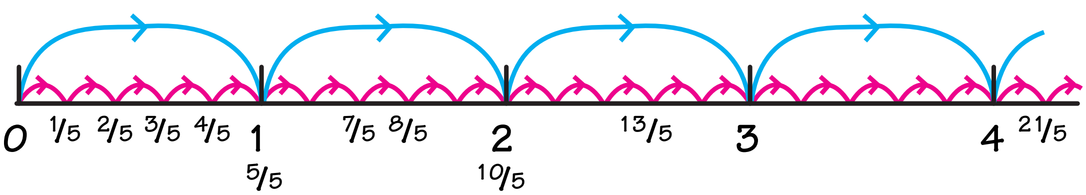
**S:** ***$1\frac{2}{5}$; it is two steps beyond 1.***

Put $\frac{7}{5}$ at **b**, but be sure that $\frac{7}{5}$ and $1\frac{2}{5}$ have both been mentioned as possible labels.

**T** (pointing to **c**)**:** ***How would we label this footprint?***

**S:** ***$2\frac{3}{5}$ or $\frac{13}{5}$.***

Ask students to locate $\frac{8}{5}$, $\frac{21}{5}$, and $\frac{10}{5}$.




**T:** ***We've found that $1 = \frac{5}{5}$ and $2 = \frac{10}{5}$. What are other names for 3 and 4?***

**S:** ***$3 = \frac{15}{5}$.***

**S:** ***$4 = \frac{20}{5}$.***

Write this inequality on the board.

$2 < \text{Sid} < 3$

**T:** ***What numbers could Sid be?***

**S:** ***$2\frac{3}{5}$.***

**S:** ***$\frac{14}{5}$.***

**T:** ***Both answers are correct. For the moment let's just find answers in the form like $\frac{14}{5}$. What other numbers could Sid be?***

**S:** ***$\frac{11}{5}$, $\frac{12}{5}$, and $\frac{13}{5}$.***

**T:** ***Are there other numbers Sid could be?***

As students think about this, they may suggest numbers in decimal notationsuch as 2.3, or complex fractions such as $\frac{12\frac{1}{2}}{5}$. Both are correct. Prompt the class, if necessary, to find fractions between 2 and 3 with denominators other than 5. For example, write this expression on the board.

$$\frac{\square}{4}$$

**T:** ***What fractions like this could Sid be?***

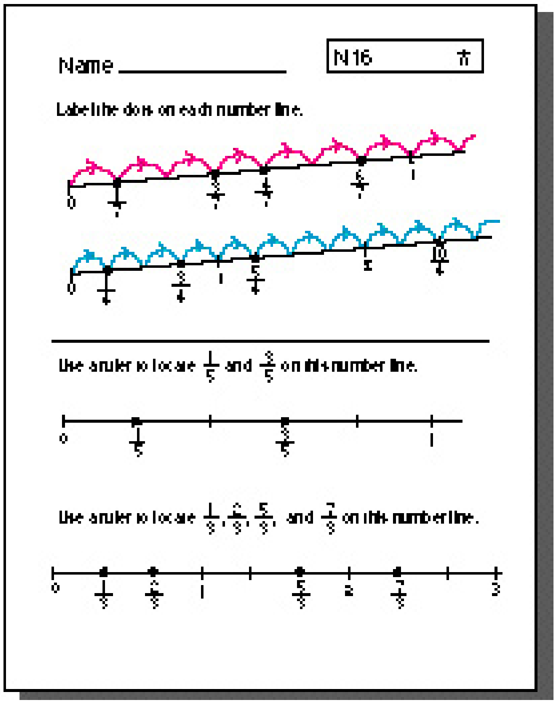
**S:** ***$\frac{9}{4}$, $\frac{10}{4}$, or $\frac{11}{4}$.***

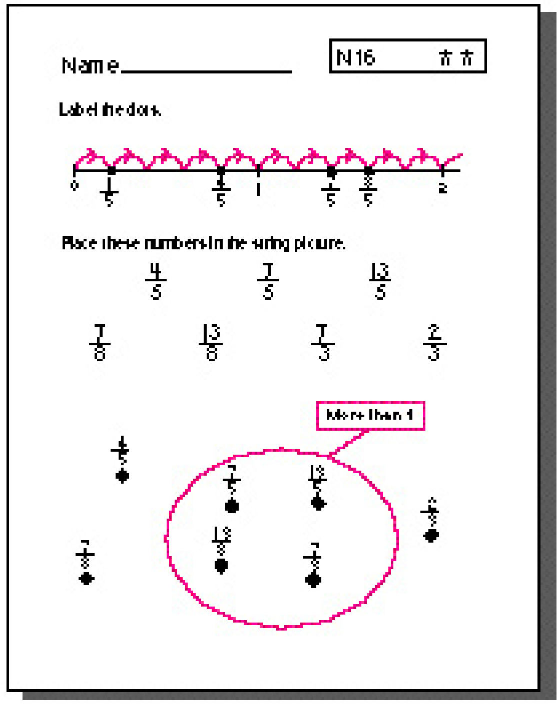
**T:** ***Why?***

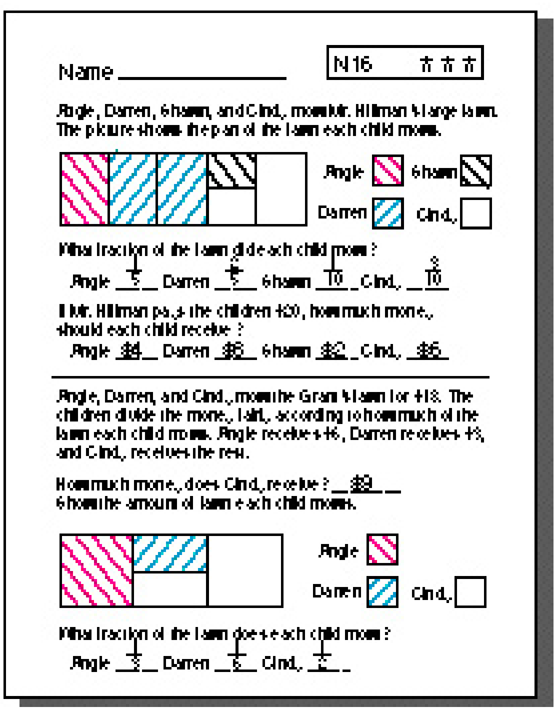
**S:** ***$\frac{4}{4} = 1$, $\frac{8}{4} = 2$, $\frac{12}{4} = 3$, so Sid must be between $\frac{8}{4}$ and $\frac{12}{4}$.***

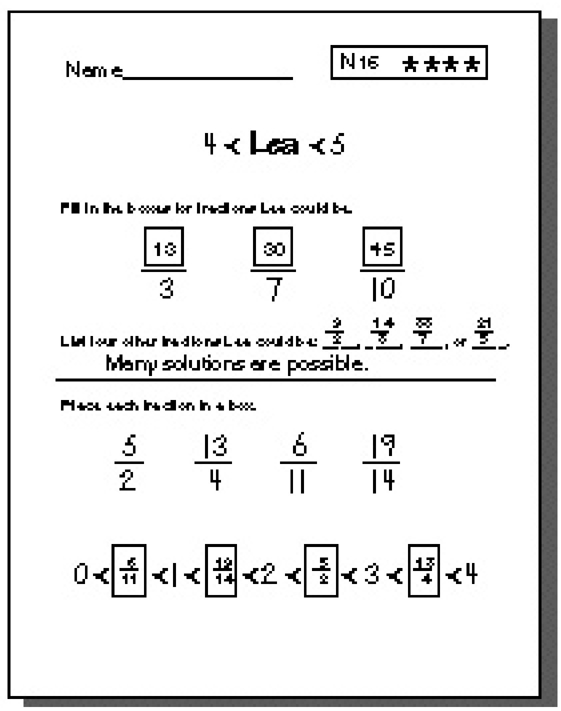
Let students suggest possibilities for Sid that involve sevenths or elevenths, for example, $\frac{18}{7}$ and $\frac{30}{11}$.

Worksheets N16*, **, ***, and **** are available for individual work.


Name
N 16 ☆
Label the dots on each number line.
Use a ruler to locate 1/5 and 3/5 on this number line.
Use a ruler to locate 1/3, 2/3, 5/3, and 7/3 on this number line.


Name
N 16 ☆☆
Label the dots.
Place these numbers in the string picture.
4/5
7/5
13/5
7/8
13/8
7/3
2/3
More than 1


Name
N 16 ☆☆☆
Angie, Darren, Shawn, and Cindy mow Mr. Hillman's large lawn. The picture shows the part of the lawn each child mows.
Angie
Shawn
Darren
Cindy
What fraction of the lawn did each child mow?
Angie 1/5 Darren 2/5 Shawn 1/10 Cindy 3/10
If Mr. Hillman pays the children $20, how much money should each child receive?
Angie $4 Darren $8 Shawn $2 Cindy $6
Angie, Darren, and Cindy mow the Grant's lawn for $18. The children divide the money fairly, according to how much of the lawn each child mows. Angie receives $6, Darren receives $3, and Cindy receives the rest.
How much money does Cindy receive? $9
Show the amount of lawn each child mows.
Angie
Darren
Cindy
What fraction of the lawn does each child mow?
Angie 1/3 Darren 1/6 Cindy 1/2


Name
N 16 ★★★★
4 < Lea < 5
Fill in the boxes for fractions Lea could be.
13/3
30/7
45/10
List four other fractions Lea could be: 9/2, 14/3, 33/7, or 21/5.
Many solutions are possible.
Place each fraction in a box.
5/2
13/4
6/11
19/14

## Capsule Lesson Summary

In the context of finding a secret number, use arrow pictures to solve linear equations such as $\frac{(N \times 4) + 13}{2} - 7 = 192$. Introduce a *Guess My Rule* activity where the rule is for arrows defined by the composition 2x followed by –3.

## Materials

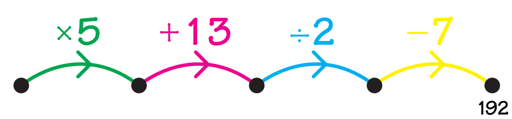
| | | | |
|---|---|---|---|
| Teacher | • Colored chalk<br>• Class/overhead calculator | Student | • Paper<br>• Colored pencils, pens, or crayons<br>• Worksheets N17*, **, ***, and **** |

## Description of Lesson

### Exercise 1

Invite a student to choose a secret number between 50 and 100. Direct the student to write the secret number on a piece of paper, and then to put it on the display of the overhead or class calculator without revealing it to the other students. As you carry out the following sequence of calculations, you may like to ask the rest of the class to copy the arrow picture that you create on the board.

**T:** ***Press*** $\boxed{\times}$ $\boxed{5}$.
***Now press*** $\boxed{+}$ $\boxed{1}\boxed{3}$ (read as "plus thirteen").
***Press*** $\boxed{\div}$ $\boxed{2}$.
***Finally, press*** $\boxed{-}$ $\boxed{7}$ $\boxed{=}$.
***What number is on the display?***



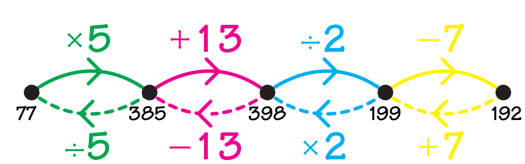

Label the ending dot with the number given; for example, 192.

Instruct students (possibly working with a partner) to try to figure out what the secret (starting) number is. They should write it on their papers. As students work on the problem, call on someone to suggest that return (opposite) arrows might be helpful. Check papers to see if anyone knows the starting number (in this example, 77), and let the student who chose the secret number confirm that 77 is it.

**T:** ***Let's label all the dots in this arrow picture.***

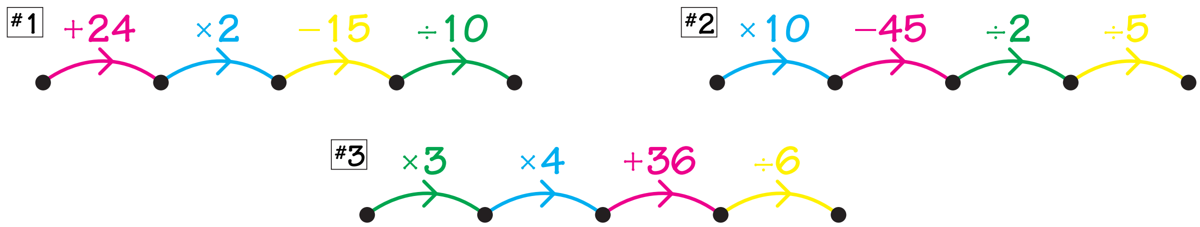
Call on students to draw return (opposite) arrows and label dots until the picture is complete.

Check that the starting (secret) number is the same as the number written on the student's paper.



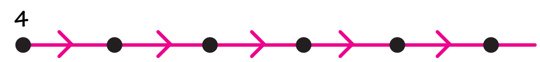
# N17

Erase the board and repeat this activity with these sequences of calculations. Let several students have a turn at choosing a secret number. You may want to encourage a student to choose a non-whole number (for example, 58.3) as the secret number.




## Exercise 2

Draw this arrow road on the board.

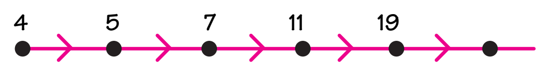



**T:** ***I have a secret rule for red arrows. Let's see if you can guess my secret rule.***

**Note:** Choose a rule based on a two-step composition such as "double the number and subtract 3."

When playing *Guess My Rule*, encourage students to think about what the rule could be without announcing it to the rest of the class. This will allow other students a chance to discover a rule on their own. Let students who think they know the rule test it by telling the class how to label the next unlabeled dot in the arrow road. Each time, confirm or deny the result.

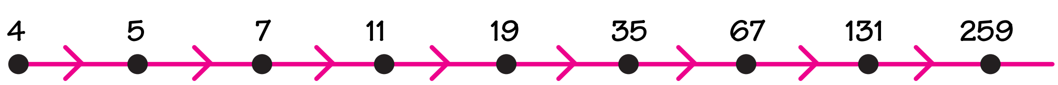
**T:** ***I'll give you some clues by labeling a few dots in this arrow road using the secret rule for red arrows.***



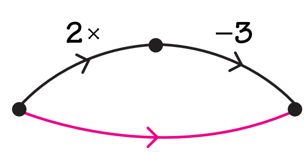

Do not give away the rule. Let students try to discover it from your clues.

Often there are several rules that the arrows could be for. Make it a practice to tell the class that there might be more than one rule that works for red arrows, but that they should try to discover your particular secret rule.

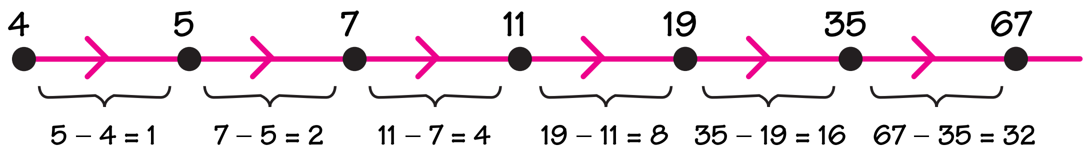
If some students think they know the rule, let them use it to label the next dot in the arrow road. Extend the arrow road and continue, letting students who think they know the rule provide labels for dots.




When many students know the rule, let one of them explain it to the class. Insist on a clear description. As a student explains, write a key for the red arrows on the board.

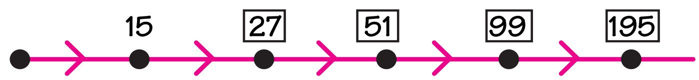


**Note:** Another correct description for this rule is "the difference between the starting number and ending number of an arrow doubles with each successive arrow in the road."




Such a description actually fits many rules (in fact, it fits any composition of 2x followed by + or – some number), so solicit the composition description or give it yourself.

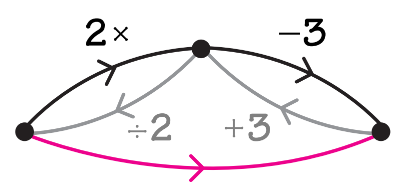
Check that everyone understands how to apply the rule by labeling the dots in a second arrow road. For example, draw this arrow road and call on students to provide the boxed numbers.




**T** (pointing to the left-most dot)**:** ***How can we find what number is here?***

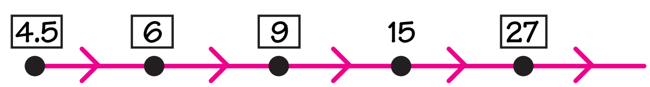
**S:** ***Go backward; add 3 and then divide by 2.***

Indicate return arrows in your key for the red arrows.




**T:** ***If we know the ending number for a red arrow, we can use opposite arrows to go backward. First +3 (the return of –3) and then ÷2 (the return of 2x) gets the starting number.***

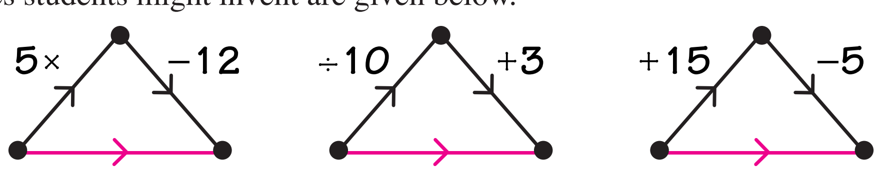
Label the dot for 9, and draw a couple more red arrows with one ending at 9. Repeat the use of return arrows to label the starting dots of the arrows.



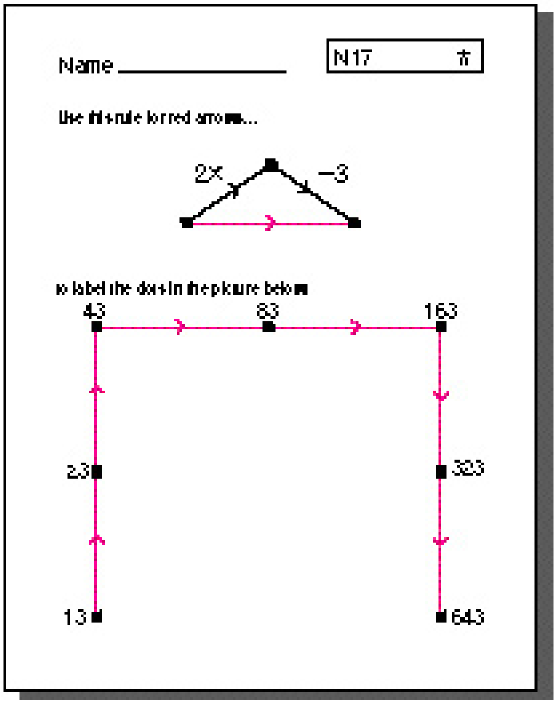

Worksheets N17*, **, ***, and **** are available for individual work.

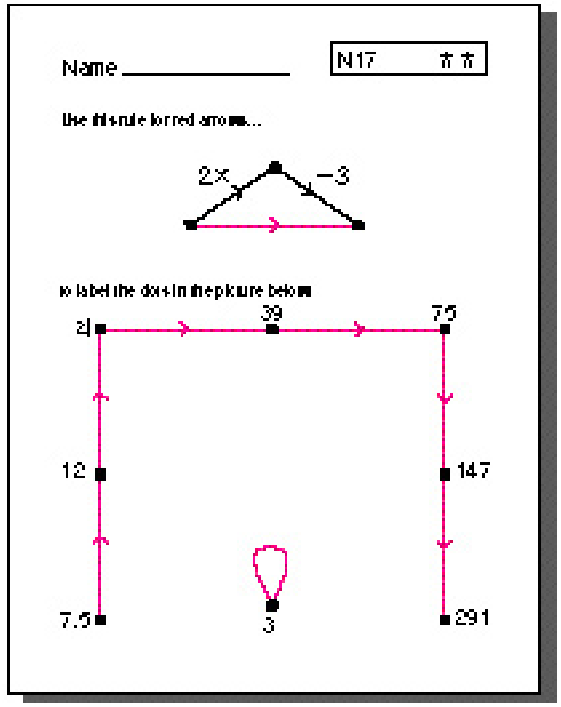
## Extension Activity

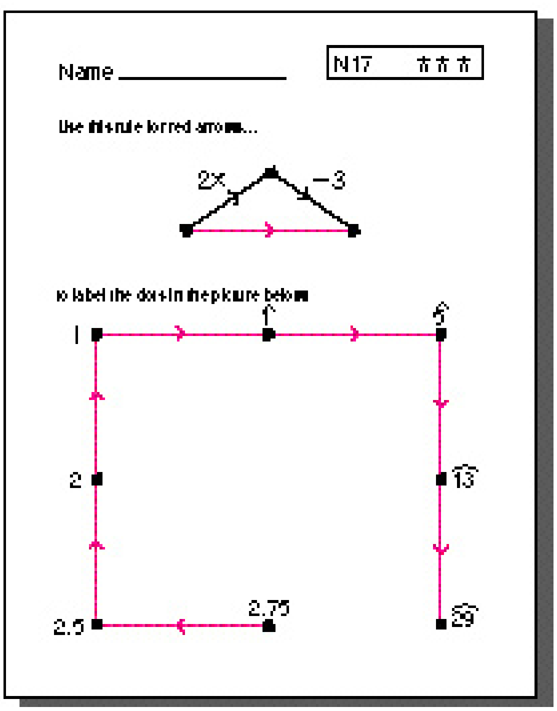
If your students enjoy *Guess My Rule* activities, you might let a student invent an arrow rule for the class to guess. Use arrow pictures to record the results of applying the rule. This is a very open activity with many possibilities. You will need to monitor the choice and application of rules. A few examples of rules students might invent are given below.

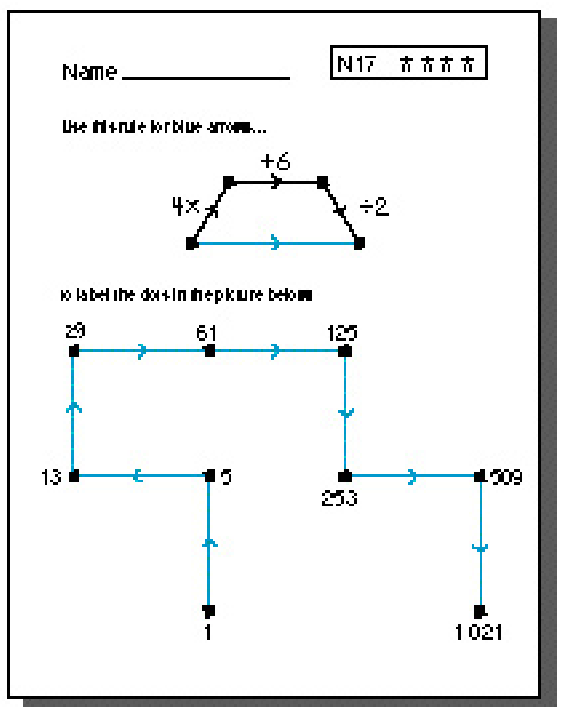



In the third example, students should discover quickly that this rule is the same as +10.


Name
N 17 ☆
Use this rule for red arrows...
2x
−3
to label the dots in the picture below
43
83
163
23
323
13
643


Name
N 17 ☆☆
Use this rule for red arrows...
2x
−3
to label the dots in the picture below
21
39
75
12
147
7.5
3
291


Name
N 17 ☆☆☆
Use this rule for red arrows...
2x
−3
to label the dots in the picture below
1
2
2.5
2.75


Name
N 17 ☆☆☆☆
Use this rule for blue arrows...
+6
4x
÷2
to label the dots in the picture below
29
61
125
13
5
253
509
1
1021

# N18 MULTIPLICATION ALGORITHM #2



## Capsule Lesson Summary

Review 30x as the composite of 3x and 10x, and of 10x and 3x. Using weighted checkers, reinforce equivalences such as 30 x 21 = 3 x 210, and 90 x 80 = 9 x 800. Present an algorithm for doing calculations, such as 36 x 42.

## Materials

**Teacher**
- Colored chalk
- Minicomputer set
- Weighted checker set

**Student**
- *Multiplication Problems* Booklet

## Description of Lesson

### Exercise 1

Draw this arrow on the board.

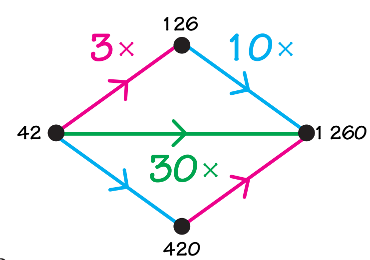

**T:** ***How could we multiply a number by 30?***

**S:** ***Multiply by 3 and then by 10.***

**S:** ***Multiply by 10 and then by 3.***

**Note:** There are other ways to multiply a number by 30, for example, multiply by 5 and then by 6, but in the development of a multiplication algorithm we are most interested in the ways given above.

Draw arrows in the picture to show these two methods, and then calculate 30 x 42 in both ways.

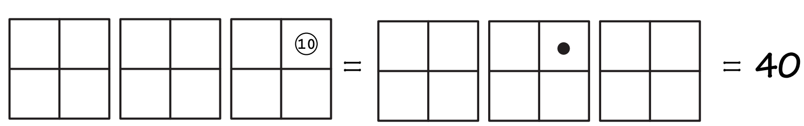



### Exercise 2

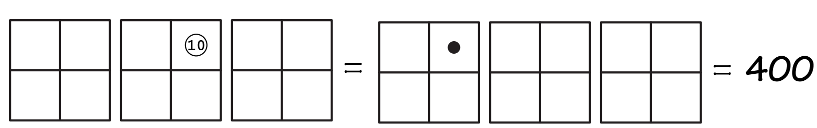
Display three Minicomputer boards. Put a ⑩-checker on the 4-square.

**T:** ***What number is this?*** (40)
***Can we show the same number using just one regular checker?***

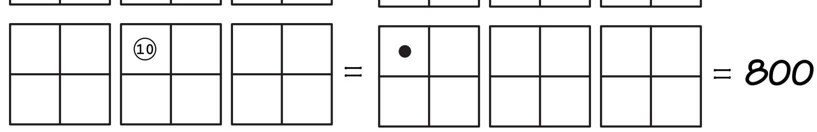
Let a student make the trade.




Continue with a couple more examples using the ⑩-checker.



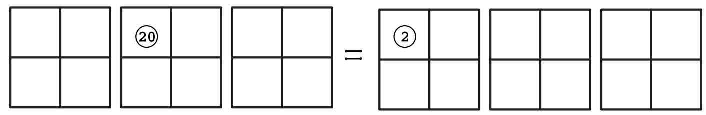

Put a ⑳-checker on the 80-square.



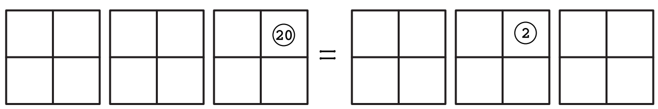
**T:** ***What number is this?*** (1 600)
***Can we show the same number using this ②-checker?***

**S:** ***Put a ②-checker on the 800-square.***

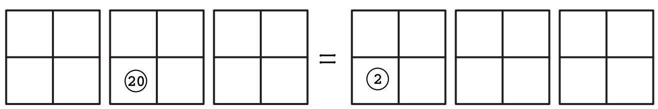
**T:** ***What number is 2 x 800?*** (1 600)



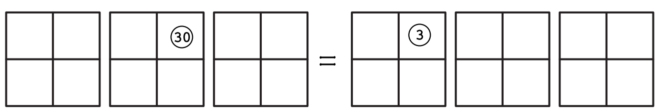
Likewise, let students make the following trades. Reinforce the trades by writing the corresponding number sentences on the board.



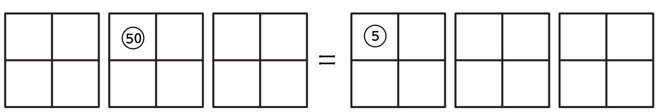
= 80 $\quad 20 \times 4 = 2 \times 40 = 80$



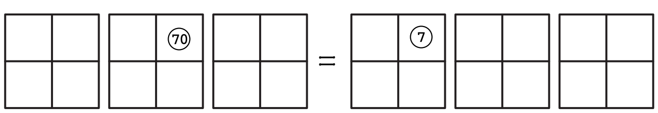
= 400 $\quad 20 \times 20 = 2 \times 200 = 400$



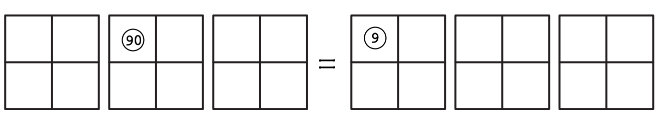
= 1 200 $\quad 30 \times 40 = 3 \times 400 = 1\,200$



= 4 000 $\quad 50 \times 80 = 5 \times 800 = 4\,000$

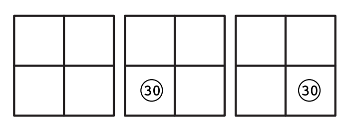


= 2 800 $\quad 70 \times 40 = 7 \times 400 = 2\,800$



= 7 200 $\quad 90 \times 80 = 9 \times 800 = 7\,200$

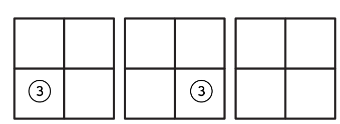
Put this configuration on the Minicomputer.



**T:** ***30x what number is on the Minicomputer?***

**S:** ***21.***

**T:** ***How can we show the same number using ③-checkers?***
***3x what number is on the Minicomputer?***



**S:** ***210.***

**T:** ***What number is 3 x 210?***

**S:** ***630.***

Record the number sentence.

$$30 \times 21 = 3 \times 210 = 630$$

Write these problems on the board, leaving the boxes and blanks empty. Ask students to copy and solve them on their papers. You may use the Minicomputer with students who have difficulty getting started.

$$20 \times 40 = 2 \times \boxed{400} = \underline{800}$$
$$20 \times 800 = 2 \times \boxed{8\,000} = \underline{16\,000}$$
$$20 \times 64 = 2 \times \boxed{640} = \underline{1\,280}$$
$$30 \times 42 = 3 \times \boxed{420} = \underline{1\,260}$$

Ask students what patterns they notice.

## Exercise 3

Record the corresponding number sentences on the board as students respond to these questions.

$$\begin{array}{r} 3 \times 62 = 186 \\ 20 \times 62 = 1\,240 \\ \hline 23 \times 62 = 1\,426 \end{array}$$

**T:** ***What number is 3 x 62?***

**S:** ***186.***

**T:** ***What number is 20 x 62?***

**S:** ***2 x 62 = 124, so 20 x 62 = 1 240.***

**T:** ***What number is 23 x 62?***

**S:** ***1 426; 186 + 1 240 = 1 426.***

**T:** ***Here is a way to record this way of doing the calculation.***
***Write what 3 x 62 is …***
***then write what 20 x 62 is …***
***then add 186 and 1 240.***

$$\begin{array}{rr} & 62 \\ & \times\ 23 \\ \hline 3 \times 62 = & 186 \\ 20 \times 62 = & 1\,240 \\ \hline & 1\,426 \end{array}$$

**Note:** We write 3 x 62 and 20 x 62 off to the side of the problem to emphasize the steps. Your students most likely will record them at first and then stop doing so after they become more familiar with the algorithm.

**T:** ***Let's do another problem.***
***What multiplication should we do first?***

$$\begin{array}{r} 87 \\ \times\ 54 \\ \hline \end{array}$$

**S:** ***4 x 87.***

You may like to cover the 5 in 54 with your hand to look at just 4 x 87.

**T:** ***Can you multiply 4 x 87?***

**S:** ***4 x 7 = 28.***
***4 x 80 = 320 and 20 more is 340.***

$$\begin{array}{rr} & \overset{2}{87} \\ & \times 54 \\ \hline 4 \times 87 = & 348 \\ 50 \times 87 = & \end{array}$$

**T:** ***To avoid confusion, let's erase the 2 tens that we carried.***
***Now what multiplication do we do next?***

**S:** ***50 x 87.***

**S:** ***Multiply by 5 and then by 10, or by 10 and then by 5.***

Invite a student to multiply 50 x 87 in the problem on the board. The student might note that 50 x 87 = 5 x 870 and then multiply 5 x 870, or the student might multiply 5 x 87 and then multiply the result by 10. If the latter method is used, it is helpful to record 0 first to insure that the answer gets lined up properly with 348.

$$\begin{array}{rr} & 87 \\ & \times\ 54 \\ \hline 4 \times 87 = & 348 \\ 50 \times 87 = & 4\,350 \\ \hline & 4\,698 \end{array}$$

**T:** ***How do we complete the calculation?***

**S:** ***Add 348 + 4 350.***

**Note:** This exercise can be modified and repeated as a warm-up activity to be used whenever time permits.

Distribute students' copies of the *Multiplication Problems* Booklet and allow 10 to 15 minutes for individual work. Encourage students to correct errors and to complete unfinished pages before starting new pages. At the end of the lesson, collect the booklets, check them, and have them ready for use in Lesson N25. An answer key for the *Multiplication Problems* Booklet follows Lesson N13.

## Home Activity

This is a good time to send a letter to parents/guardians about multiplication. Blackline N18 has a sample letter.

## Capsule Lesson Summary

Investigate patterns that emerge in a series of division problems. Review multiplying by a fraction in the story of the zookeeper who feeds bananas to monkeys and of the greedy monkey, Bobo, who eats several shares of the bananas. Represent $\frac{3}{5}$x as the composition of ÷5 and 3x in an arrow picture. Present a detective story involving the relation $\frac{2}{5}$x.

## Materials

| | | |
|---|---|---|
| Teacher | • Colored chalk | • Worksheets N19*, **, and *** |
| Student | • Colored pencils, pens, or crayons | |

## Description of Lesson

### Exercise 1

Write this problem on the board.

$3\overline{)69}$

**T:** ***Can you think of a story problem in which you would divide 69 by 3?***

Encourage students to tell a variety of story problems, for example:

**S:** ***69 students are divided into three classes of the same size. How many students are in each class?***

**S:** ***A tennis player buys 69 balls one summer. If there are three balls in each can, how many cans of balls did the tennis player buy?***

Select one of the students' stories that involves 69 people, places, or things being divided into three groups of the same size. (Preferably do not choose a situation in which each of the 69 things could be divided into thirds and the pieces shared.) Refer to your story in the following discussion.

**T:** ***If 69 students were divided into three classes of the same size, how many students would be in each class?***

**S:** ***23 students, since 3 x 23 = 69.***

Record the solution, and write another problem.

$$\begin{array}{r}23\\3\overline{)69}\end{array} \qquad 3\overline{)70}$$

**T:** ***What if there were 70 students?***

**S:** ***23 students could be put in each class, but there would be one extra student. That student could be put in any one of the three classes.***

**S:** ***70 divided by 3 is 23 with a remainder of 1.***

Refer to the story to solve two more similar problems. (Answers are in the boxes.)

$$\begin{array}{r}\boxed{23\ R{=}2}\\3\overline{)71}\end{array} \qquad \begin{array}{r}\boxed{24}\\3\overline{)72}\end{array}$$

If students suggest that 72 ÷ 3 equals 23 with a remainder of 3, accept this as true, but note that each class would get one more student, so 72 ÷ 3 = 24.

**T:** ***72 ÷ 3 has no remainder. If we continue to increase the number of students, one at a time, what is the next problem without a remainder?***

**S:** ***75 ÷ 3, since there are three more students. 75 ÷ 3 = 25.***

Check this answer by considering as well 73 ÷ 3 and 74 ÷ 3.

$\begin{array}{r}\boxed{24}\\3\overline{)72}\end{array}$ $\begin{array}{r}\boxed{24\ R=1}\\3\overline{)73}\end{array}$ $\begin{array}{r}\boxed{24\ R=2}\\3\overline{)74}\end{array}$ $\begin{array}{r}\boxed{25}\\3\overline{)75}\end{array}$

**T:** ***Tell me the next few division problems without a remainder.***

**S:** ***78 ÷ 3 = 26. 81 ÷ 3 = 27. 84 ÷ 3 = 28.***

**S:** ***When the number of students increases by three, the number of students in each class increases by one.***

## Exercise 2

Write this problem on the board.

$\square \times 18 = 12$

Perhaps students will not quickly find the solution, ⅔.
You may need to remind the class of the story about Bobo.

**T:** ***You have already solved this type of problem when we talked about the zookeeper and the greedy monkey Bobo. Who remembers that story?***

Be sure students include the following details:

- Each day, the zookeeper divides a certain number of bananas into equal shares for the monkeys.
- Bobo always eats more than his own share.

Draw a string picture on the board as you tell the story.

**T:** ***One day the zookeeper shares 18 bananas fairly among three monkeys. How many bananas does each monkey receive?***

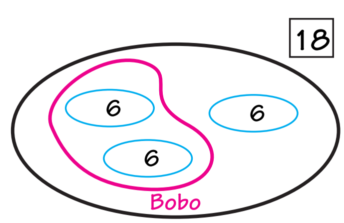
**S:** ***Six bananas; 18 ÷ 3 = 6 (or ⅓ x 18 = 6).***

**T:** ***Bobo eats two shares, his own and one other. How many bananas does he eat?***

**S:** ***12 bananas; 2 x 6 = 12.***

**T:** ***What number sentence would we write?***

**S:** ***⅔ x 18 = 12, since Bobo eats two of the three shares.***




$\boxed{2/3} \times 18 = 12$

You may want to emphasize that ⅔ x 18 is sometimes read "⅔ of 18."

**T:** ***The story about Bobo reminds us that number sentences like □ x 18 = 12 do have solutions, and that solutions can be fractions.***

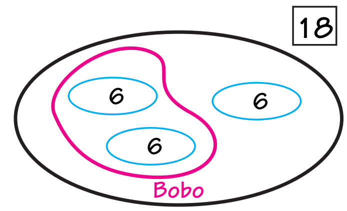
Write this problem on the board.

$$\frac{2}{3} \times 21 = \boxed{\phantom{0}}$$

**S:** *$\frac{2}{3}$ x 21 = 14.*

**T:** ***How should we change the string picture to show $\frac{2}{3}$ x 21?***

**S:** ***Put 21 in the tag for the bananas' string, since there are 21 bananas. $\frac{1}{3}$ x 21 = 7 (or 21 ÷ 3 = 7), so each share has seven bananas. Put 7 in each blue string.***




$$\boxed{\frac{2}{3}} \times 18 = 12$$

**S:** ***Bobo eats two of the three shares, or 14 bananas, since 2 x 7 = 14. So $\frac{2}{3}$ x 21 = 14.***

Invite students, one at a time, to relabel the string picture to solve the following problems in a similar manner. Point out that two calculations are necessary for each problem: divide the number of bananas by 3 (or multiply by $\frac{1}{3}$), and multiply that result by 2.

$$\frac{2}{3} \times 15 = \boxed{10} \qquad \frac{2}{3} \times 36 = \boxed{24} \qquad \frac{2}{3} \times 60 = \boxed{40}$$

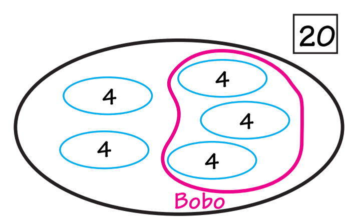
Write this problem on the board.

$$\frac{3}{5} \times 20 = \boxed{\phantom{0}}$$

**T:** ***What changes in the Bobo story does this problem suggest?***

Draw a corresponding string picture as students explain the changes in the story.

**S:** ***There are now 20 bananas and five monkeys. Draw five blue strings inside the large string.***




**S:** ***Each monkey receives four bananas, since 20 ÷ 5 = 4 (or $\frac{1}{5}$ x 20 = 4).***

**S:** ***Bobo eats three of the five shares.***

**S:** ***He eats 12 bananas since 3 x 4 = 12. So $\frac{3}{5}$ x 20 = 12.***

$$\frac{3}{5} \times 20 = \boxed{12}$$

Solve a couple more problems in a similar manner.

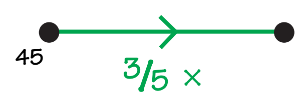
$$\frac{3}{5} \times 15 = \boxed{9} \qquad \frac{3}{5} \times 35 = \boxed{21}$$

Refer to the three problems on the board involving $\frac{3}{5}$x as you ask,

**T:** ***What two calculations did we do to solve each of these problems?***

**S:** ***We divided the total number of bananas by 5. Then we multiplied that result by 3.***

Set up the string picture for $\frac{3}{5}$ x 45 and draw this arrow.




**T:** ***Let's show the two steps we use to calculate $\frac{3}{5}$ x 45 with arrows. What do we do first?***

**S:** ***Divide 45 by 5 since there are five monkeys. 45 ÷ 5 = 9.***

**S:** ***Or take one-fifth of the number of bananas. $\frac{1}{5}$ x 45 = 9.***

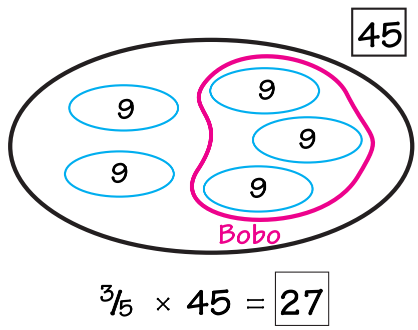
# N19

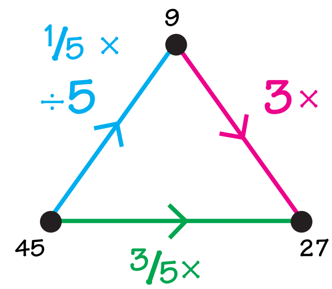
Show this calculation in both the string picture and the arrow picture.

**T:** ***This blue arrow for ÷5 or*** $\frac{1}{5}$x ***shows your first calculation. What is the second step?***

**S:** ***Multiply 9 by 3 since Bobo eats three shares. 3*** x ***9 = 27.***

**S:** ***So*** $\frac{3}{5}$ x ***45 = 27.***



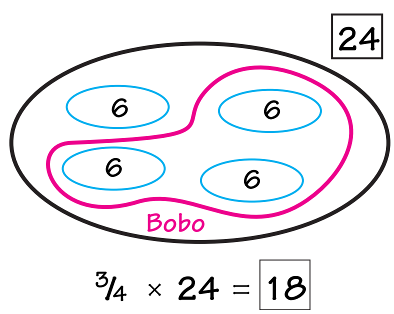



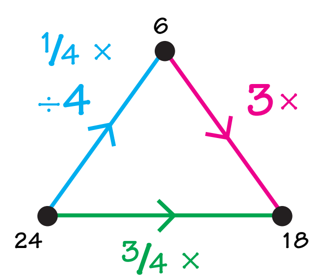

**T:** ***We can always use the string picture and the story about Bobo to solve problems like*** $\frac{3}{5}$ x ***45. It may be easier to use the detour (composition) suggested by the arrow picture.***

Trace the appropriate arrows as you describe how to multiply by $\frac{3}{5}$.

**T:** ***To multiply a number by*** $\frac{3}{5}$ (trace the green arrow)***, first divide the number by 5*** (trace the blue arrow)***, and then multiply that answer by 3*** (trace the red arrow)***.***

Repeat this activity to show $\frac{3}{4}$ x 24 in both a string picture and an arrow picture.



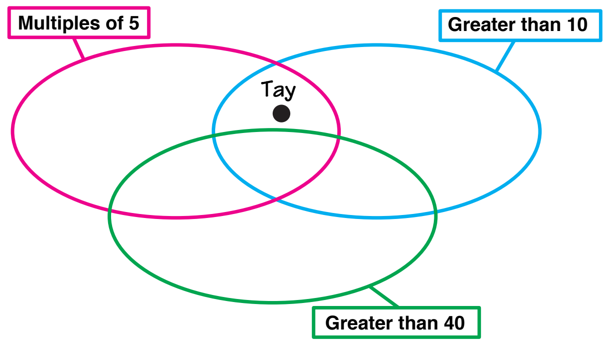

**Note:** Calculations such as $\frac{3}{4}$ x 24 may be performed by reversing the order of the two operations; that is, ÷4 followed by 3x is the same as 3x followed by ÷4. Mention this to the class if a student suggests this alternative.

## Exercise 3 (optional)

Use the following detective story about two secret numbers, Tay and Fay, if time allows.

**Clue 1**

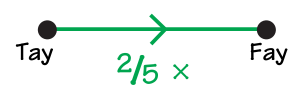
Draw this string picture on the board and let students find six possibilities for Tay as given in the table.



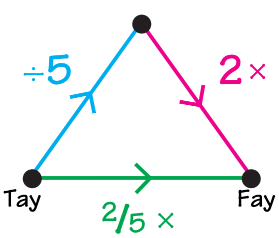

| Tay | Fay |
|---|---|
| 15 | |
| 20 | |
| 25 | |
| 30 | |
| 35 | |
| 40 | |

**Clue 2**

Draw this arrow picture on the board.




Invite students to draw a detour (composition) for ⅖x; then let them use the arrow picture to find the corresponding six possibilities for Fay. Refer to the Bobo story if necessary.

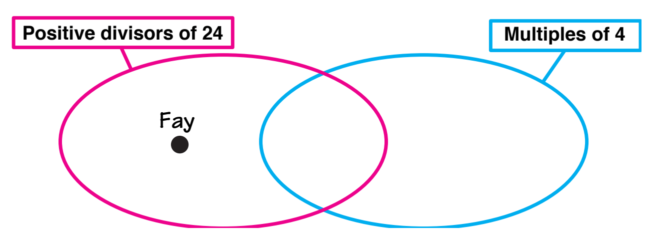



| Tay | Fay |
|---|---|
| 15 | 6 |
| 20 | 8 |
| 25 | 10 |
| 30 | 12 |
| 35 | 14 |
| 40 | 16 |

Observe that this second clue tells us that Tay and Fay come in pairs. For example, if Tay is 30, then Fay is 12.

**Clue 3**

Draw this string picture on the board.

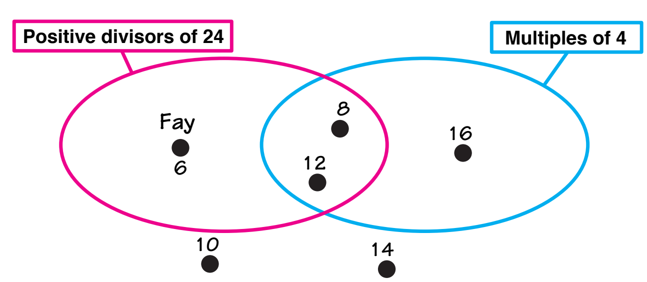



**T:** *What does this clue tell us about Fay?*

**S:** *Fay is a positive divisor of 24, but not a multiple of 4.*

**S:** *Fay is 6.*

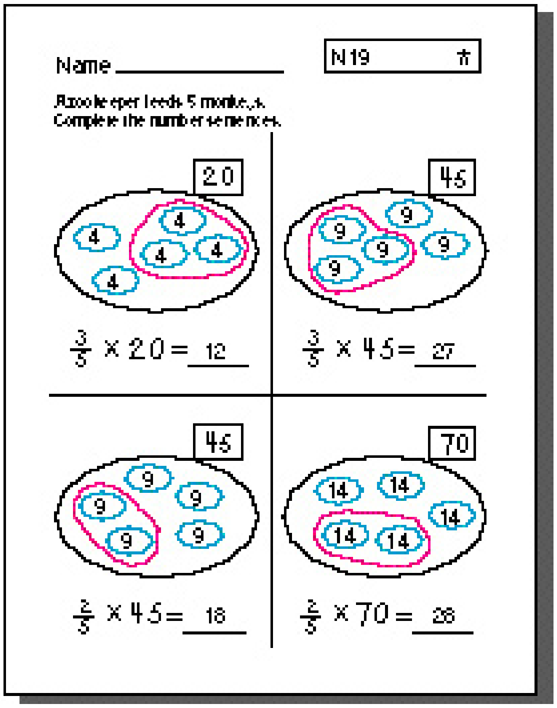
Invite students to place the six possibilities for Fay (from the second clue) in the string picture.




**T:** *Yes, Fay is 6, so who is Tay?*

**S:** *15. From the second clue we know that if Fay is 6, then Tay is 15.*

Worksheets N19*, **, and *** are available for individual work.



Name ____________ N19 ★

A zookeeper feeds 5 monkeys.
Complete the number sentences.

20

$\frac{3}{5} \times 20 =$ 12

45

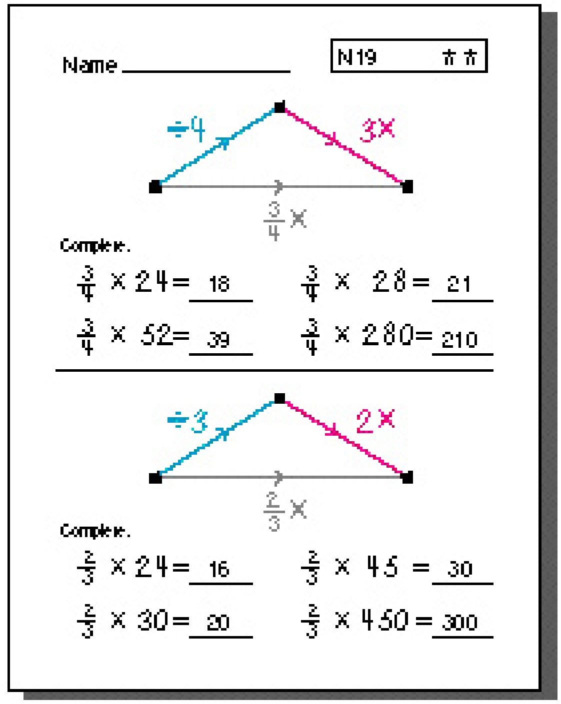
$\frac{3}{5} \times 45 =$ 27

45

$\frac{2}{5} \times 45 =$ 18

70

$\frac{2}{5} \times 70 =$ 28



Name ____________ N19 ★★

÷4 3× $\frac{3}{4}$×

Complete.

$\frac{3}{4} \times 24 =$ 18 $\frac{3}{4} \times 28 =$ 21

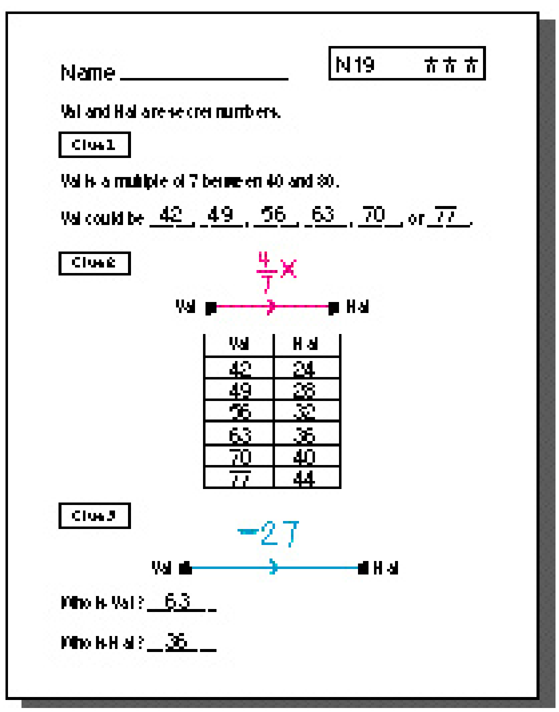
$\frac{3}{4} \times 52 =$ 39 $\frac{3}{4} \times 280 =$ 210

÷3 2× $\frac{2}{3}$×

Complete.

$\frac{2}{3} \times 24 =$ 16 $\frac{2}{3} \times 45 =$ 30

$\frac{2}{3} \times 30 =$ 20 $\frac{2}{3} \times 450 =$ 300



Name ____________ N19 ★★★

Val and Hal are secret numbers.

Clue 1

Val is a multiple of 7 between 40 and 80.

Val could be 42, 49, 56, 63, 70, or 77.

Clue 2

$\frac{4}{7}$× Val → Hal

| Val | Hal |
|---|---|
| 42 | 24 |
| 49 | 28 |
| 56 | 32 |
| 63 | 36 |
| 70 | 40 |
| 77 | 44 |

Clue 3

−27 Val → Hal

Who is Val? 63

Who is Hal? 36

## Capsule Lesson Summary

Find fraction and decimal equivalences with the help of the number line. Use arrow pictures and a calculator to do some decimal calculations.

### Materials

**Teacher**
- Class/overhead calculator
- Colored chalk

**Student**
- Paper
- Calculator
- Worksheets N20*, **, and ***

## Description of Lesson

### Exercise 1

Draw a section of the number line from 0 to 3 on the board. Make marks for whole numbers about 50 cm apart.

**T:** ***Do you remember the Sasquatch story?***

Let students recall the story briefly.

**T:** ***Where would we locate a dot for $\frac{1}{5}$ on the number line?***

**S:** ***Between 0 and 1.***

**T:** ***How could we locate $\frac{1}{5}$ accurately?***

**S:** ***Divide the segment between 0 and 1 into five smaller segments of the same length.***

Do this not only for the segment between 0 and 1, but also for the rest of the number line. Then let a student draw a dot for $\frac{1}{5}$. Ask students to locate and draw dots for $\frac{3}{5}$, $\frac{8}{5}$, and $\frac{13}{5}$.


Add tenths graduations to your number line, as illustrated below.

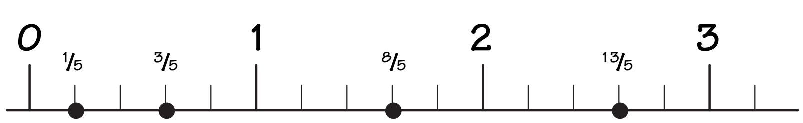
**T:** ***Who can give me another name for $\frac{1}{5}$?***

**S:** ***$\frac{2}{10}$.***

**S:** ***0.2.***

$$\frac{1}{5} = \frac{2}{10} = 0.2$$

Give several labels to the dots for $\frac{3}{5}$, $\frac{8}{5}$, and $\frac{13}{5}$ in a similar manner.




Students may suggest that $\frac{16}{10} = 1\frac{6}{10}$ and $\frac{26}{10} = 2\frac{6}{10}$, so you may want to label the dots accordingly.

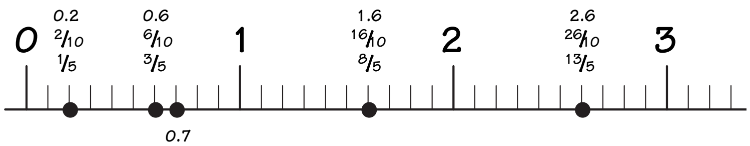
**T:** ***Where is 0.7 on the number line?***

**S:** ***Between 0 and 1.***

Let someone draw and label a dot for 0.7.

**T:** ***Can you think of a fraction for 0.7?***

**S:** ***7/10.***

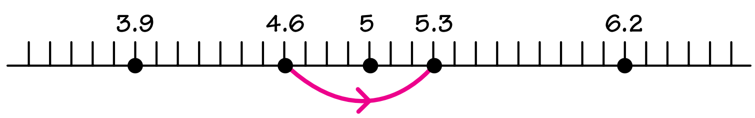



## Exercise 2

Draw this part of a number line on the board.


**T:** ***I have 4.6 and 5.3 on the number line. Who can point to 5? 3.9? 6.2?***

Let volunteers locate each number in turn. When a number causes difficulty, ask the class to count aloud (forward or backward) on the number line. Add this arrow to your picture.



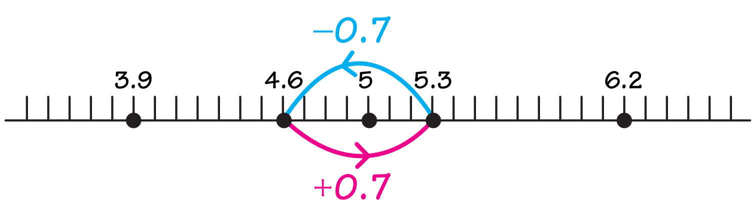

**T:** ***What could this red arrow be for?***

**S:** ***+0.7.***

$$\begin{array}{r} \scriptstyle 1 \\ 4.6 \\ +0.7 \\ \hline 5.3 \end{array}$$

**T:** ***Let's check the calculation. If we draw the opposite (return) arrow, what could it be for?***

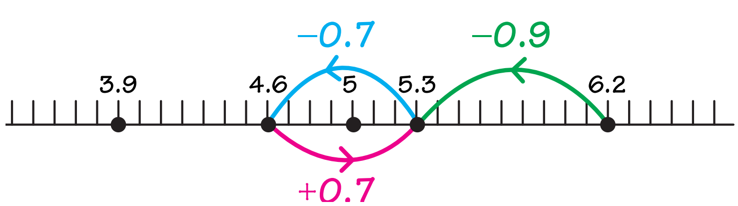
**S:** ***–0.7.***




Draw an arrow from 6.2 to 5.3 (in a third color) as you ask,

**T:** ***What could this arrow be for?***

**S:** ***–0.9.***



**T:** ***Let's check this calculation.***

Use whatever subtraction methods students suggest.

$$\begin{array}{r} \overset{\phantom{1}}{6}.\overset{10}{2} \\ -\overset{1}{0}.9 \\ \hline 5.3 \end{array} \quad \text{or} \quad \begin{array}{r} \overset{5}{\cancel{6}}.\overset{10}{2} \\ -0.9 \\ \hline 5.3 \end{array}$$

## Exercise 3

Arrange that all students have access to a calculator. You may prefer to do this exercise with a class or overhead calculator and wait to provide calculators for student use until Exercise 4.

With the class, discuss and model using a calculator to do some decimal calculations. For example:

6.2 – 0.9 = 5.3 (press $\boxed{6}\boxed{\cdot}\boxed{2}\ \boxed{-}\ \boxed{0}\boxed{\cdot}\boxed{9}\ \boxed{=}$)
3 x 1.4 = 4.2 (press $\boxed{3}\ \boxed{\times}\ \boxed{1}\boxed{\cdot}\boxed{4}\ \boxed{=}$)
7.5 ÷ 3 = 2.5 (press $\boxed{7}\boxed{\cdot}\boxed{5}\ \boxed{\div}\ \boxed{3}\ \boxed{=}$)

Tell students that now they have to do calculations in their heads just the way they think the calculator does them.

**T:** ***Clear the display. Cover the display and press*** $\boxed{2}\ \boxed{\times}\ \boxed{6}\ \boxed{+}\ \boxed{0}\boxed{\cdot}\boxed{7}\ \boxed{=}$.
***What number should be on the display?***

**S:** ***12.7.***

**T:** ***Check the display. Now clear and cover the display.***
***Press*** $\boxed{2}\ \boxed{\times}\ \boxed{0}\boxed{\cdot}\boxed{6}\ \boxed{+}\ \boxed{0}\boxed{\cdot}\boxed{3}\ \boxed{=}$.
***What number should be on the display?***

**S:** ***1.5.***

Continue in this way for a couple more mental arithmetic problems.

$\boxed{2}\ \boxed{\times}\ \boxed{1}\boxed{\cdot}\boxed{8}\ \boxed{-}\ \boxed{0}\boxed{\cdot}\boxed{5}\ \boxed{=}$ (3.1)
$\boxed{3}\ \boxed{\times}\ \boxed{2}\boxed{\cdot}\boxed{6}\ \boxed{+}\ \boxed{1}\boxed{\cdot}\boxed{2}\ \boxed{=}$ (9)

**T:** ***Clear the display and follow my directions carefully.***
***Press*** $\boxed{+}\ \boxed{0}\boxed{\cdot}\boxed{4}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}$.
***What number is on the display?*** (2.4) ***What does the calculator do each time we press*** $\boxed{=}$***?***

**S:** ***Adds 0.4.***

**T:** ***2.4 is on the display. Now cover the display. Press*** $\boxed{=}\ \boxed{=}$.
***What number should be on the display?*** (3.2) ***Check.***
***Cover the display. Press*** $\boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}$. ***What number should be on the display?*** (4.8) ***Check.***

***I would like to see 6 on the display. How many more times should we press*** $\boxed{=}$***?***

**S:** ***Three.***

**T:** ***Clear the display. Press*** $\boxed{+}\ \boxed{0}\boxed{\cdot}\boxed{6}\ \boxed{=}\ \boxed{=}\ \boxed{=}$.
***What number is on the display?***

**S:** ***1.8.***

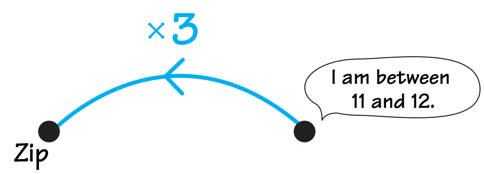
**T:** ***Now cover the display and press*** $\boxed{=}\ \boxed{=}\ \boxed{=}$.
***What number should be on the display?*** (3.6)

### Exercise 4

Draw an arrow picture while you explain to the class about a secret number, Zip.

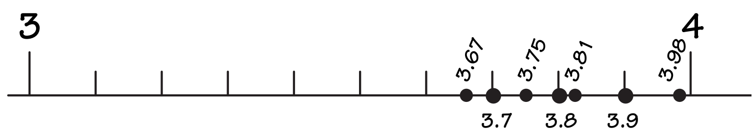
**T:** ***When I put this secret number, Zip, on the display of my calculator and press ⊠ ③ ⊟, the result on the display is a number between 11 and 12.***

***On your paper write some numbers that Zip could be.***




Let students use their calculators to find numbers that Zip could be. You may get a variety of possibilities; for example, 3.7; 3.8; 3.9; 3.75; 3.98; 3.67; and 3.81.

You might want to locate some of the numbers that Zip could be on the number line. For example:




It is not necessary to subdivide the number line any more than we have done in this lesson. For decimals such as 3.81 or 3.67, ask only that they be located in the right interval and at their approximate place.

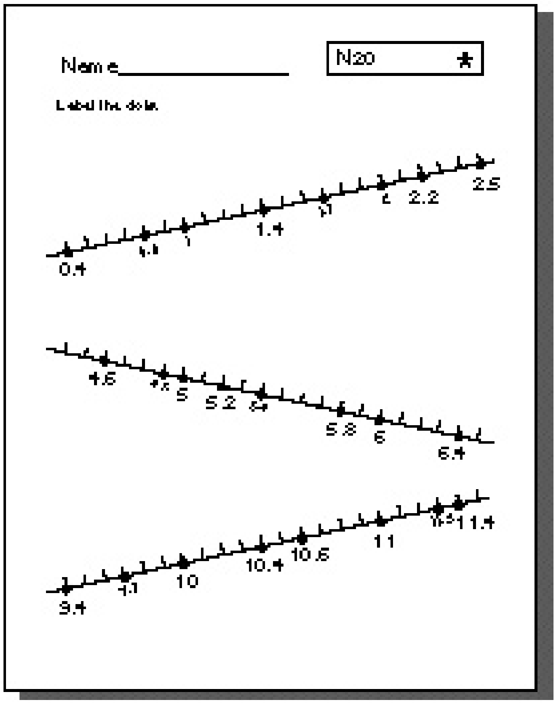
**T:** ***The number on my calculator display is 11.7. What number is Zip?***

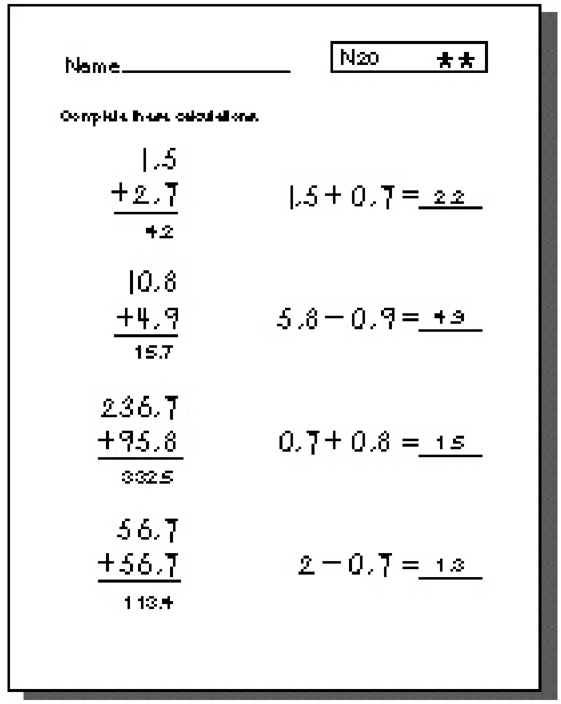
**S:** ***3.9.***

Worksheets N20*, **, and *** are available for individual work.

## Extension Activity

Play the calculator game *Home on the Range*. See the Extension Activity in Lesson N3.



Name

N20 ★★

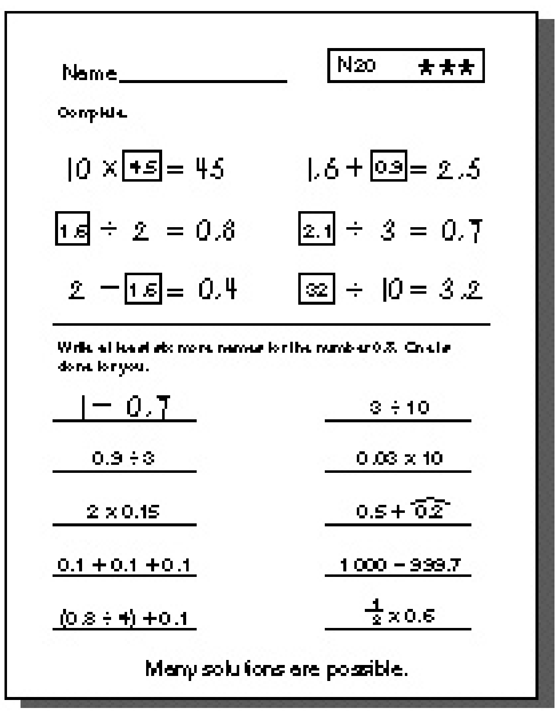
Complete these calculations.

$$\begin{array}{r} 1.5 \\ +2.7 \\ \hline 4.2 \end{array} \qquad 1.5 + 0.7 = \underline{2.2}$$

$$\begin{array}{r} 10.8 \\ +4.9 \\ \hline 15.7 \end{array} \qquad 5.8 - 0.9 = \underline{4.9}$$

$$\begin{array}{r} 236.7 \\ +95.8 \\ \hline 332.5 \end{array} \qquad 0.7 + 0.8 = \underline{1.5}$$

$$\begin{array}{r} 56.7 \\ +56.7 \\ \hline 113.4 \end{array} \qquad 2 - 0.7 = \underline{1.3}$$



Name

N20 ★★★

Complete.

$10 \times \boxed{4.5} = 45$ $\qquad$ $1.6 + \boxed{0.9} = 2.5$

$\boxed{1.6} \div 2 = 0.8$ $\qquad$ $\boxed{2.1} \div 3 = 0.7$

$2 - \boxed{1.6} = 0.4$ $\qquad$ $\boxed{32} \div 10 = 3.2$

Write at least six more names for the number 0.3. One is done for you.

| | |
|---|---|
| $1 - 0.7$ | $3 \div 10$ |
| $0.9 \div 3$ | $0.03 \times 10$ |
| $2 \times 0.15$ | $0.5 + 0.2$ |
| $0.1 + 0.1 + 0.1$ | $1000 - 999.7$ |
| $(0.8 \div 4) + 0.1$ | $\frac{1}{2} \times 0.6$ |

Many solutions are possible.

# N21 SUBTRACTION #2



## Capsule Lesson Summary

Do some mental arithmetic involving adding a number of ones, tens, hundreds, or tenths to a given number. Review subtraction methods in the context of solving related problems, one with whole numbers and one with decimals. Solve some subtraction problems related to a story about keeping sales records.

## Materials

| Teacher | | Student | |
|---|---|---|---|
| | • Chalk<br>• Blacklines N21(a) and (b) | | • Paper<br>• Sales record |

**Advance Preparation:** Use Blackline N21(a) to make student copies of the sales record for use in Exercise 3. Blackline N21(b) has bank account records for decimal subtraction problems.

## Description of Lesson

### Exercise 1

This first exercise should proceed like a mental arithmetic activity; move briskly and call on most students to participate.

Write this expression on the board.

$$23\overset{8}{6}$$

**T:** ***What number is this?***

**S:** ***244, because 236 + 8 = 244.***

If necessary, recall with the class that this expression shows that 8 ones are added to 236.

Continue with several more similar problems. (Answers are in boxes.)

$$23\overset{8}{6} = \boxed{244} \qquad 2\overset{9}{5}.7 = \boxed{34.7}$$

$$2\overset{7}{4}5 = \boxed{315} \qquad 47.\overset{6}{6} = \boxed{48.2}$$

$$\overset{5}{6}32 = \boxed{1\,132} \qquad 3\overset{10}{2}4 = \boxed{424}$$

Write these two expressions on the board.

$$2\,53\overset{10}{7} \qquad 2\,5\overset{1}{3}7$$

**T:** ***Compare these two numbers. Is one greater?***

**S:** ***They are equal because in the first one we add 10 ones and in the second we add 1 ten to 2 537.***

**S:** ***They both are 2 547.***

$$2\,53\overset{10}{7} = 2\,5\overset{1}{3}7 = 2\,547$$

Write another expression on the board.

$$4\overset{10}{3}5 =$$

**T:** ***Can we write this number in another way?***

**S:** ***Put a 1 above the 4 to show adding 1 hundred; adding 10 tens is the same as adding 1 hundred.***

Continue in this way with several more similar problems. (Answers are in boxes.)

$$4\overset{10}{3}5 = \boxed{\overset{1}{4}35} = 535 \qquad \overset{10}{5}21 = \boxed{\overset{1}{0}521} = 1\,521$$

$$2\,\overset{10}{1}30 = \boxed{\overset{1}{2}\,130} = 3\,130 \qquad 2\overset{10}{8}.3 = \boxed{2\overset{1}{8}.3} = 29.3$$

$$30\overset{10}{8} = \boxed{3\overset{1}{0}8} = 318 \qquad 57.\overset{10}{6}5 = \boxed{57.\overset{1}{6}5} = 57.75$$

### Exercise 2

Write these two problems on the board and ask students to do both calculations on their papers.

$$\begin{array}{r} 872 \\ -545 \\ \hline \end{array} \qquad \begin{array}{r} 8.72 \\ -5.45 \\ \hline \end{array}$$

After a few minutes, call on students to do the calculations at the board and to explain their methods. If more than one method (for example, borrowing and Nick's method) are used by students in your class, let students explain both methods. The class should observe that the second calculation is similar to the first; one only needs to place the decimal point correctly.

Repeat this activity with two more similar calculations.

$$\begin{array}{r} 601 \\ -328 \\ \hline \end{array} \qquad \begin{array}{r} 60.1 \\ -32.8 \\ \hline \end{array}$$

### Exercise 3

Distribute copies of the sales record on Blackline N21(a), and tell the class the following story.

**T:** ***Hans is the secretary for his school band. Band members are selling greeting cards to earn money for their spring concert tour. The goal is to sell 2 000 boxes of cards. Hans keeps a record of the weekly sales. Each week he announces the sales and reports how many more boxes they need to sell.***

Refer the class to Hans' sales records (Blackline N21(a)) and do the first week collectively to illustrate how the sales record is kept.

**T:** ***In the first week, how many boxes of cards did the band members sell?***

**S:** ***270 boxes.***

**T:** ***How many boxes do they still need to sell? What calculations should we do?***

**S:** ***2 000 – 270. They still need to sell 1 730.***

If appropriate, call on students to do the calculation at the board, explaining their methods. Show the class how to record the results; then ask them to go on completing the sales record individually.

Students who complete this sales record can continue with the record of Amelia's bank account on Blackline N21(b). When most students have completed at least one of these records, discuss the results collectively.

### Home Activity

Suggest that parents/guardians show their child how they keep a record of their bank account.

## Capsule Lesson Summary

Extend the experiences of Lesson N20 to include decimal names for numbers such as $1\frac{1}{5}$, and calculations such as 0.6 + 1.3. Play a decimal number line game with intervals to focus attention on decimal numbers with a hundredths place. Given a road with two arrows starting at 0.4 and ending at 2, find possible labels for the two arrows.

### Materials

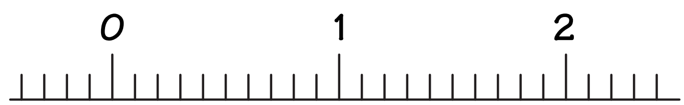
**Teacher**
- Meter stick
- Colored chalk

**Student**
- Colored pencils, pens, or crayons
- Worksheets N22*, **, and ***

## Description of Lesson

### Exercise 1

Draw this part of a number line on the board.



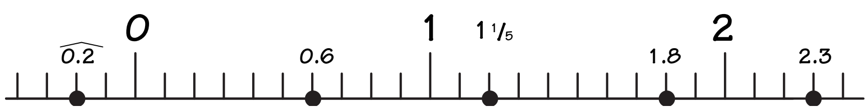

**T:** ***Where is 0.6 on the number line?***

Let a student locate and draw a dot for 0.6. Similarly ask students to locate 1.8, 2.3, and $\widehat{0.2}$.

**T:** ***Where is $1\frac{1}{5}$ on the number line?***

**S:** ***Between 1 and 2.***

Invite a student to locate and draw a dot for $1\frac{1}{5}$. If a student tries to put $1\frac{1}{5}$ halfway between 1 and 2, observe that this is the place for $1\frac{1}{2}$ or 1.5. To locate $1\frac{1}{5}$, divide the segment between 1 and 2 into five smaller segments of the same length.




**T:** ***What is a decimal name for $1\frac{1}{5}$?***

**S:** ***1.2.***

Draw a red arrow from 0.8 to 1.5 in the picture.

**T:** ***What could this red arrow be for?***

If given an incorrect answer, such as +7, check it by doing the indicated calculation on the board.

$$\begin{array}{r} 0.8 \\ +7.0 \\ \hline 7.8 \end{array}$$

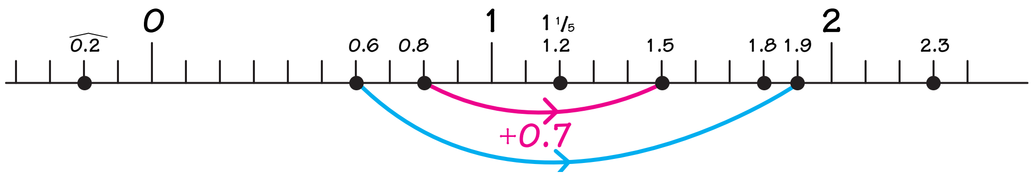
**T:** ***The red arrow cannot be for +7.***

**S:** ***+0.7.***

Ask students to check the calculation.

$$\begin{array}{r} 0.8 \\ +0.7 \\ \hline 1.5 \end{array}$$

Label the red arrow and draw a blue arrow from 0.6 to 1.9.




**T:** ***What could this blue arrow be for?***

Again, if given an incorrect answer such as +0.13, check it by doing the indicated calculation on the board.

$$\begin{array}{r} 0.60 \\ +0.13 \\ \hline 0.73 \end{array}$$

**T:** ***The blue arrow cannot be for +0.13. Do we need to add more or less than 0.13?***

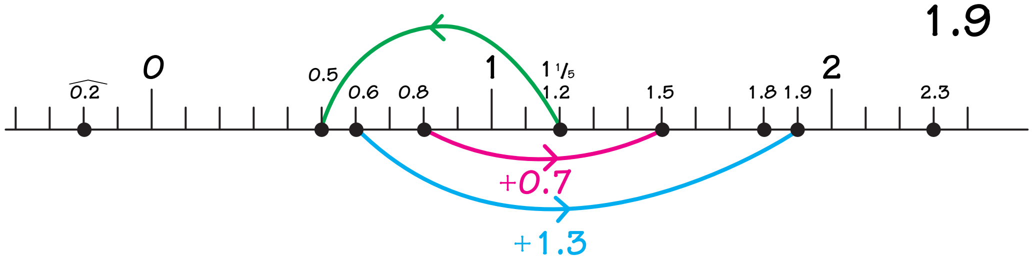
**S:** ***More.***

**S:** ***The blue arrow could be for +1.3.***

Check by doing the calculation on the board.

$$\begin{array}{r} 0.6 \\ +1.3 \\ \hline 1.9 \end{array}$$

Label the blue arrow and draw a green arrow from 1.2 to 0.5.

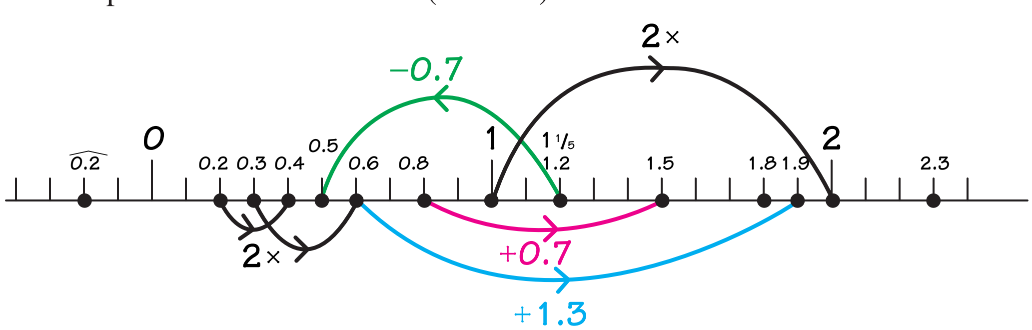



**T:** ***What could the green arrow be for?***

**S:** ***–0.7.***

**T:** ***I would like someone to draw a 2x arrow in our picture.***

A few possible responses are shown below (in black).



**T:** ***Could we draw a 2*x *arrow starting at 0.6?***

A student should indicate an arrow starting at 0.6 and ending at 1.2.

**T:** ***Could we draw a 2*x *arrow ending at 1.8?***

A student should indicate an arrow starting at 0.9 and ending at 1.8.

## Exercise 2

**Note:** This exercise introduces a decimal number line game called *Intervals*. In this lesson, the purpose of the game is to introduce the hundredths subgraduation. You need not choose the secret number until near the end of the game. As an example of how one may control the game and very quickly "home-in" on the hundredths subgraduations, the dialogue of a possible game is included below.

Draw a line on the board.

**T:** ***I am thinking of a secret number. Would someone like to guess what it is?***

**S:** ***10.***

Put 10 on your number line.

**T:** ***Each time you guess, I will respond to your guess with a number; the secret number is somewhere between your guess and my response.***
***My response to your guess is 50. This tells you than my secret number is between 10 and 50.***

Put 50 on the number line and draw this red line segment.


**T:** ***The red segment is a reminder that the secret number is between 10 and 50. Another guess?***

**S:** ***20.***

**T:** ***76.5.***


**T:** ***Now what do you know about my secret number?***

**S:** ***It is between 20 and 76.5.***

**S:** ***Yes, but it is also between 20 and 50.***

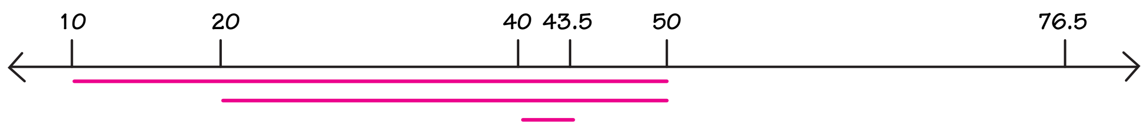
**T:** ***I will record this on the number line.***

**T:** ***Do you have another guess?***

**S:** ***43.5.***

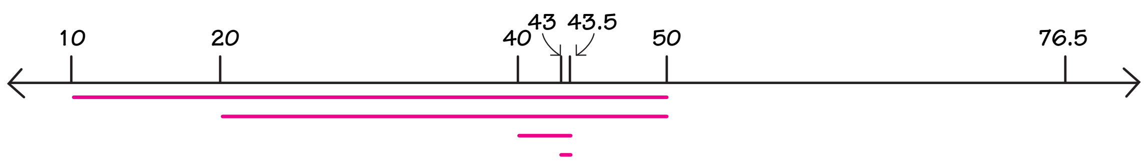
**T:** ***40.***

**S:** ***The secret number is between 40 and 43.5.***



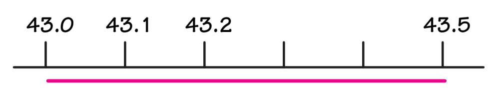

**S:** ***43.***

**T:** ***43.5.***

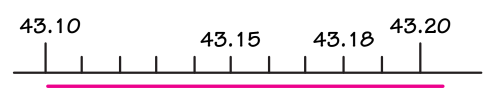



**S:** ***The secret number is between 43 and 43.5. I'll guess 43.1.***

**T:** ***43.2. Let's magnify the number line.***




Draw this magnified part of the number line.




**S:** ***43.15.***

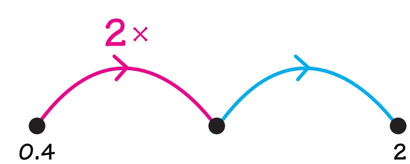
**T:** ***43.18. Let's magnify one more time.***

**S:** ***43.17.***

**T:** ***That's it!***

## Exercise 3

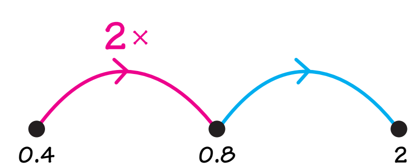
Draw this arrow picture on the board.




**T:** ***This arrow road starts at 0.4 and ends at 2.***
***The red arrow is for 2*x. *What number is here*** (point to the middle dot)***?***

**S:** ***0.8.***

Label the middle dot.




**T:** ***The blue arrow goes from 0.8 to 2.***
***What could it be for?***

As before, check an incorrect answer such as +0.12 by doing the indicated calculation at the board.

$$\begin{array}{r} 0.80 \\ +0.12 \\ \hline 0.92 \end{array}$$

**T:** ***The blue arrow cannot be for +0.12. What could it be for?***

**S:** ***+1.2.***

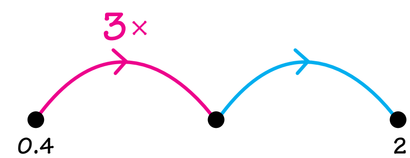
$$\begin{array}{r} 0.8 \\ +\,1.2 \\ \hline 2.0 \end{array}$$

**T:** ***Let's check.*** (Trace the blue arrow.)
***What number is 0.8 + 1.2?***

**S:** ***2.***

Do the calculation at the board and label the blue arrow +1.2.

Change your picture so that the red arrow is now for 3x.

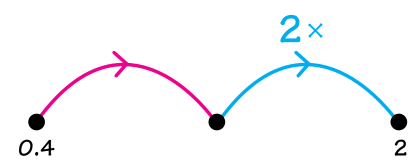



**T:** ***Suppose the red arrow is for 3x;***
***then what could the blue arrow be for?***

**S:** ***The middle dot is for 1.2.***

**S:** ***The blue arrow could be for +0.8, because 1.2 + 0.8 = 2.***

Change your picture so that the blue arrow is for 2x.



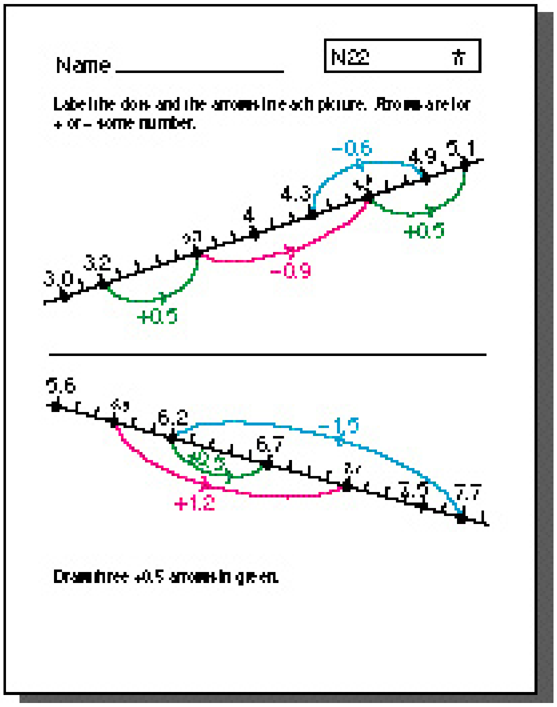

**T:** ***Suppose the blue arrow is for 2x;***
***then what could the red arrow be for?***

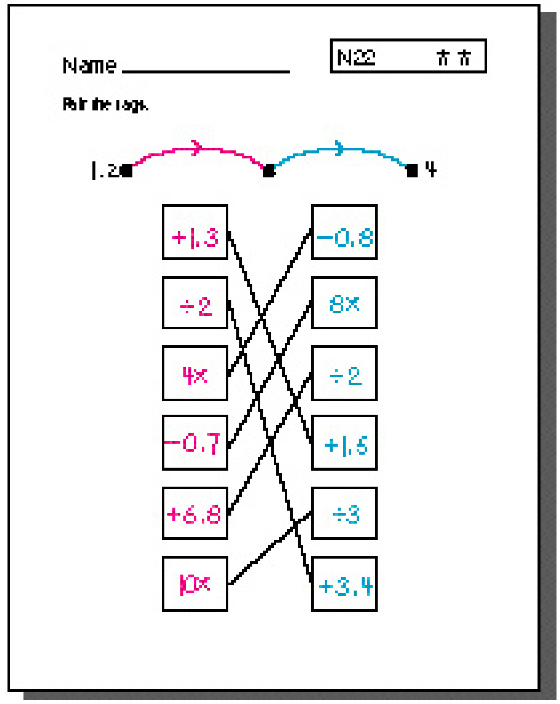
**S:** ***The middle dot is for 1.***

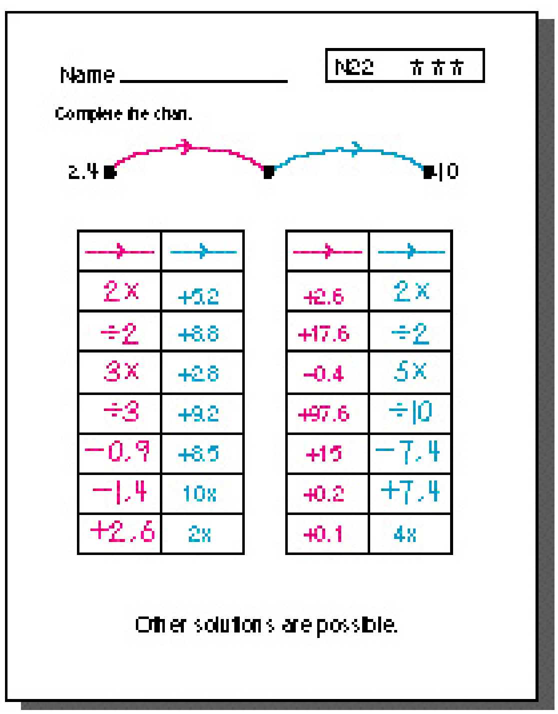
**S:** ***The red arrow could be for +0.6, because 0.4 + 0.6 = 1.***

Worksheets N22*, **, and *** are available for individual work. For Worksheet N22** you may like to make cards of the tags for students to manipulate. See *IG-1* Lesson N7 for suggestions about such cards.

# N22



Name

N22 ☆☆☆

Complete the chart.

2.4 → → 10

| → | → |
|---|---|
| 2x | +5.2 |
| ÷2 | +8.8 |
| 3x | +2.8 |
| ÷3 | +9.2 |
| −0.9 | +8.5 |
| −1.4 | 10x |
| +2.6 | 2x |

| → | → |
|---|---|
| +2.6 | 2x |
| +17.6 | ÷2 |
| −0.4 | 5x |
| +97.6 | ÷10 |
| +15 | −7.4 |
| +0.2 | +7.4 |
| +0.1 | 4x |

Other solutions are possible.

## Capsule Lesson Summary

Use both string pictures and arrow pictures to solve such problems as ¾ x 240 = $\square$ and ⅘ x $\square$ = 60. Interpret the function $\frac{n}{m}$x as the composition of ÷$m$ followed by $n$x. Build arrow roads from 24 to 36 to find functions equivalent to $\frac{3}{2}$x.

## Materials

| | | | |
|---|---|---|---|
| Teacher | • Colored chalk | Student | • Paper<br>• Colored pencils, pens, or crayons<br>• Worksheets N23*, **, ***, and **** |

## Description of Lesson

### Exercise 1

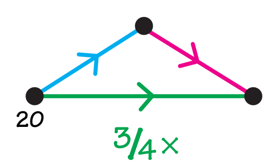
On the board, write this number sentence.

$3/4 \times 20 = \square$

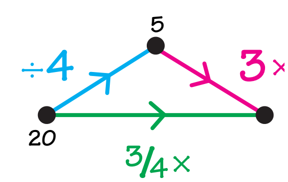
**T:** ***What number is ¾*** **x** ***20?*** (15)
***How do you get 15?***

Students may offer explanations based both on a composition arrow picture or on a string picture in the story about Bobo the monkey. As appropriate, review both methods.

Draw this arrow picture.




**T:** ***What "detour" could we use to find ¾*** **x** ***20?***




**S:** ***Let the blue arrow be for ÷4. 20 ÷ 4 = 5.***
***Then the red arrow is for 3*x. *3*** **x** ***5 = 15.***

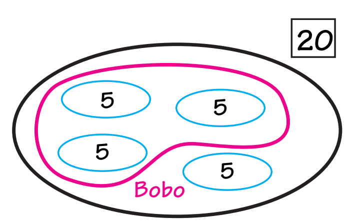
**S:** ***So ¾*** **x** ***20 = 15.***

$3/4 \times 20 = \boxed{15}$

Trace arrows as you describe this way to multiply by ¾.

**T:** ***To multiply a number by ¾*** (trace the green arrow)***, divide it by 4*** (trace the blue arrow) ***and multiply the result by 3*** (trace the red arrow)**.**

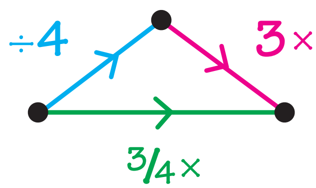
When a student tells a story about Bobo the monkey, draw the corresponding string picture according to the student's explanation.

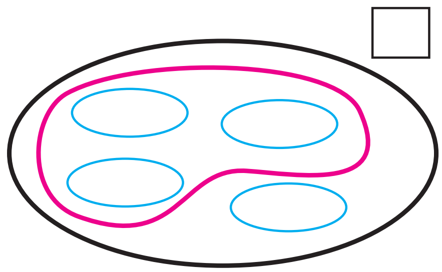



**S:** ***There are 20 bananas and four monkeys. Draw a string for 20 bananas and draw four blue strings inside it.***
***Each monkey receives five bananas, since 20 ÷ 4 = 5.***
***Bobo eats three of the four shares.***
***He eats 15 bananas, since 3*** **x** ***5 = 15.***

$3/4 \times 20 = \boxed{15}$

Erase the dot labels in the arrow picture and the numbers in the string picture so as to leave the following pictures on the board. Put several problems on the board for students to solve individually or with a partner. (Answers are in boxes.)



$\frac{3}{4} \times 12 = \boxed{9}$

$\frac{3}{4} \times 32 = \boxed{24}$

$\frac{3}{4} \times 40 = \boxed{30}$

Write this problem on the board.

$\frac{3}{4} \times \boxed{\phantom{00}} = 12$

**T:** ***¾ of some number is 12. What number belongs in the box?***

**S:** ***16.***

**T:** ***How did you get 16?***

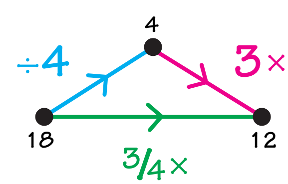
You may receive explanations referring to the arrow picture, or to the string picture and the Bobo story. Follow your students' instructions.

**S:** ***12 is at the end of the ¾x arrow because 12 is the answer this time.***

***4 is at the top dot because the opposite arrow for 3x is ÷3 and 12 ÷ 3 = 4 or (3 x 4 = 12).***

***16 is the starting number because 16 ÷ 4 = 4 (or the opposite of ÷4 is 4x and 4 x 4 = 16).***

***So, ¾ x 16 = 12.***



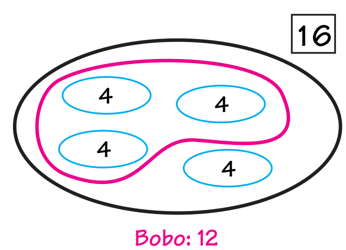

$\frac{3}{4} \times \boxed{16} = 12$

***In the Bobo story, we know that Bobo took 12 bananas. We want to know the total number of bananas.***

***Bobo took three shares, and 3 x 4 = 12. Each share must have four bananas.***

***The fourth share must also have four bananas. So there are 16 bananas total; 4 x 4 = 16.***

***Bobo took 12 of 16 bananas, so ¾ x 16 = 12.***




$\frac{3}{4} \times \boxed{16} = 12$

Individually or with a partner, ask students to solve the following problems. Students may use the arrow picture, the string picture, or patterns. For a problem involving decimals, suggest they think about money.

$\frac{3}{4} \times \boxed{24} = 18$

$\frac{3}{4} \times \boxed{240} = 180$

$\frac{3}{4} \times 2\,400 = \boxed{1\,800}$

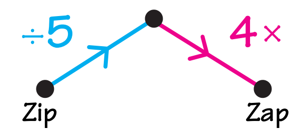
$\frac{3}{4} \times 10 = \boxed{7.5}$

$\frac{3}{4} \times 18 = \boxed{13.5}$

$\frac{3}{4} \times 2.4 = \boxed{1.8}$

## Exercise 2

Erase the board and then draw this arrow picture.



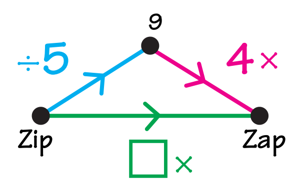

**T:** ***If Zip is 20, what number is Zap?***

**S:** ***16; 20 ÷ 5 = 4 and 4 x 4 = 16.***

Continue this activity to find many possibilities for Zip and Zap. (Student responses are in boxes.)

| Zip | Zap |
|---|---|
| 20 | 16 |
| 35 | $\boxed{28}$ |
| $\boxed{30}$ | 24 |
| $\boxed{25}$ | 20 |
| 31 | $\boxed{24.8}$ |
| $\boxed{17.5}$ | 14 |
| $\boxed{45}$ | 36 |

Draw a green arrow from Zip to Zap.




**T:** ***What could the green arrow be for?***

**S:** ***⅘x, since ÷5 followed by 4x is the same as ⅘x.***

Ask students to solve some other ⅘x problems individually or with a partner. They should notice that the first three problems are in the table.

$4/5 \times 35 = \boxed{28}$ $\qquad$ $4/5 \times 55 = \boxed{44}$

$4/5 \times 31 = \boxed{24.8}$ $\qquad$ $4/5 \times \boxed{75} = 60$

$4/5 \times \boxed{25} = 20$ $\qquad$ $4/5 \times 1 = \boxed{0.8}$

You might suggest students think of \$1.00 when they do the calculation ⅘ x 1. Then they can put \$1.00 into five shares of \$0.20 each; four of the five shares is \$0.80.

**T:** ***What number is ⅘ x 1?***

**S:** ***0.8, since 1 ÷ 5 = 0.2 and 4 x 0.2 = 0.8.***

**T:** ***Yes. And what is true if we multiply any number by 1?***

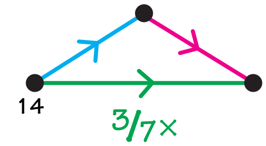
**S:** ***We get the same number.***

**S:** ***So ⅘ x 1 = ⅘.***

**T:** ***0.8 names the same number as ⅘; 0.8 = ⅘.***

Exercise 3

Put this problem on the board.




$\frac{3}{7} \times 14 = \boxed{\phantom{0}}$

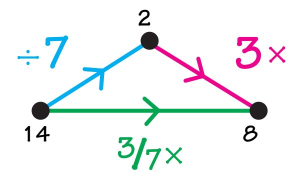
**T:** ***What is ³⁄₇ x 14?***

**S:** ***6.***

**T:** ***Why?***

**S:** ***For ³⁄₇x, use a detour of ÷7 followed by 3x. The blue arrow is ÷7 and the red arrow is 3x.***

**S:** ***14 ÷ 7 = 2; 3 x 2 = 6; so ³⁄₇ x 14 = 6.***



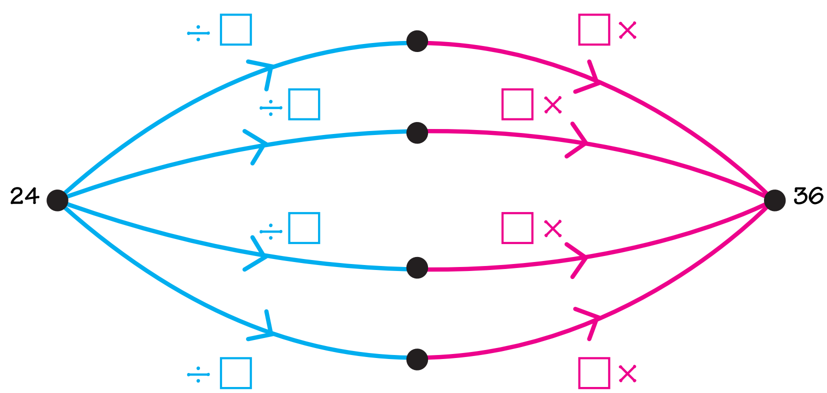

$\frac{3}{7} \times 14 = \boxed{6}$

In a similar manner, invite students to solve these problems. (Answers are in boxes.) Emphasize that to multiply by a fraction you can divide by the bottom number and then multiply by the top number. Introduce the terms *denominator* and *numerator* if you wish.

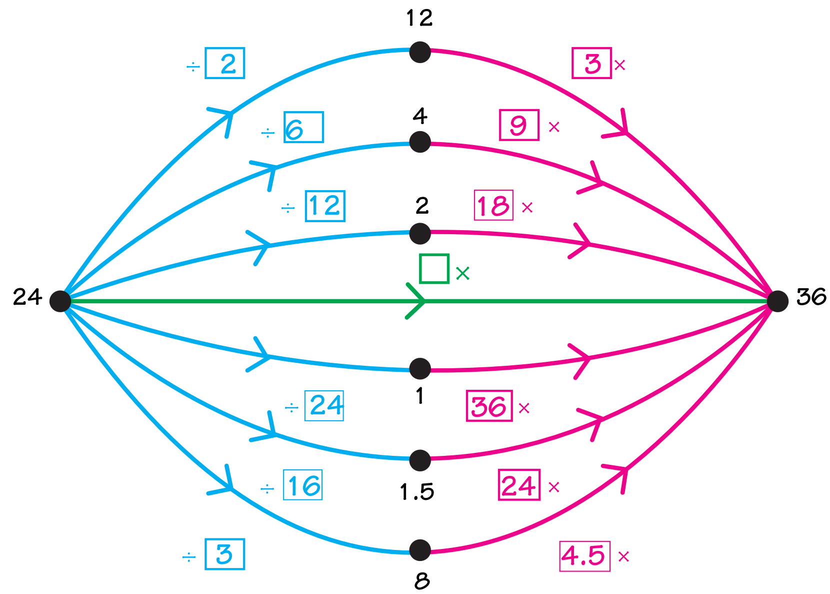
$\frac{2}{5} \times 20 = \boxed{?\ 8}$ $\qquad$ $\frac{5}{6} \times 18 = \boxed{15}$ $\qquad$ $\frac{2}{9} \times \boxed{54} = 12$

## Exercise 4

Draw the following arrow picture on the board.



Invite students to label the arrows and dots. Many solutions are possible. If students are finding many possibilities, add a few more blue arrows and red arrows to the picture. Then draw a green arrow from 24 to 36. The following picture includes some of the many possibilities for the blue and red arrows.




**T:** ***What could the green arrow be for? How could we label it?***

**S:** $\frac{3}{2}$x***, since one blue-red arrow road from 24 to 36 is ÷ 2 followed by 3*****x.**

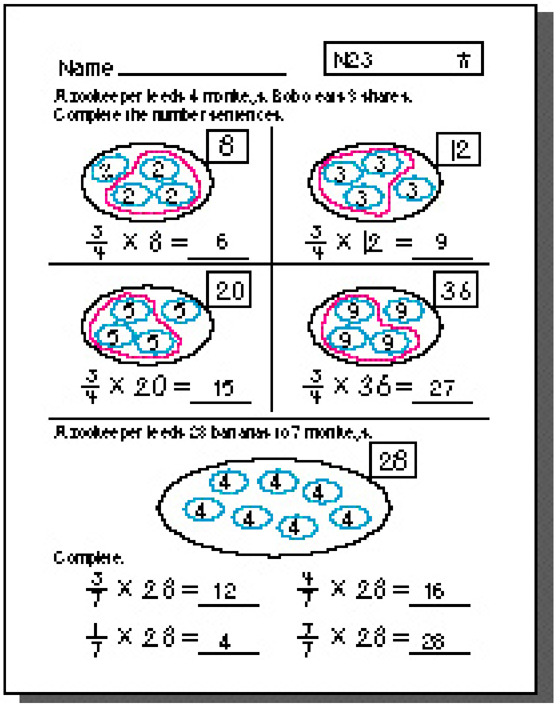
Accept other names for the green arrow. Students are likely to offer names suggested by the completed arrow picture, but accept other correct names as well. List students' suggestions on the board, for example:

$3/2\times$ $9/6\times$ $18/12\times$ $36/24\times$ $24/16\times$ $4.5/3\times$ $12/8\times$

**T:** ***These are all different names for the green arrow; they are equivalent.***

Worksheet N23*, **, ***, and **** are available for individual work.

# N23



Name ________ N23 ☆

A zookeeper feeds 4 monkeys. Bobo eats 3 shares.
Complete the number sentences.

8: $\frac{3}{4} \times 8 =$ 6

12: $\frac{3}{4} \times 12 =$ 9

20: $\frac{3}{4} \times 20 =$ 15

36: $\frac{3}{4} \times 36 =$ 27

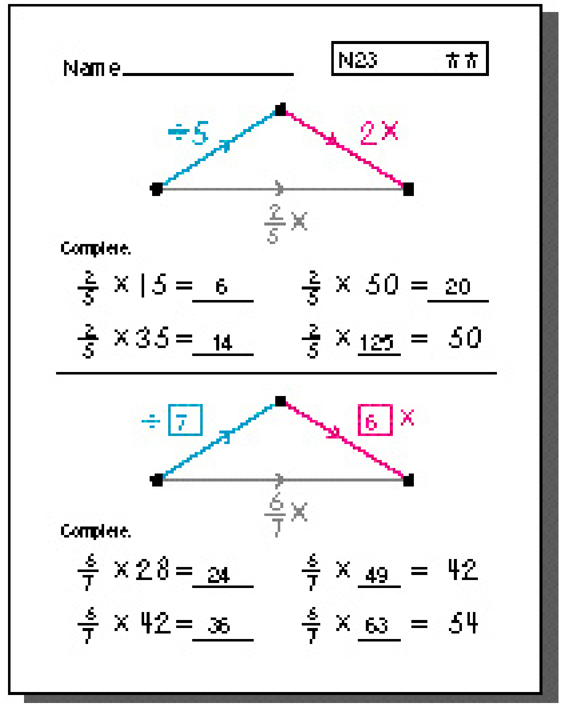
A zookeeper feeds 28 bananas to 7 monkeys.

28

Complete.

$\frac{3}{7} \times 28 =$ 12 $\quad \frac{4}{7} \times 28 =$ 16

$\frac{1}{7} \times 28 =$ 4 $\quad \frac{7}{7} \times 28 =$ 28



Name ________ N23 ☆☆

÷5 2×

$\frac{2}{5}$×

Complete.

$\frac{2}{5} \times 15 =$ 6 $\quad \frac{2}{5} \times 50 =$ 20

$\frac{2}{5} \times 35 =$ 14 $\quad \frac{2}{5} \times$ 125 $= 50$

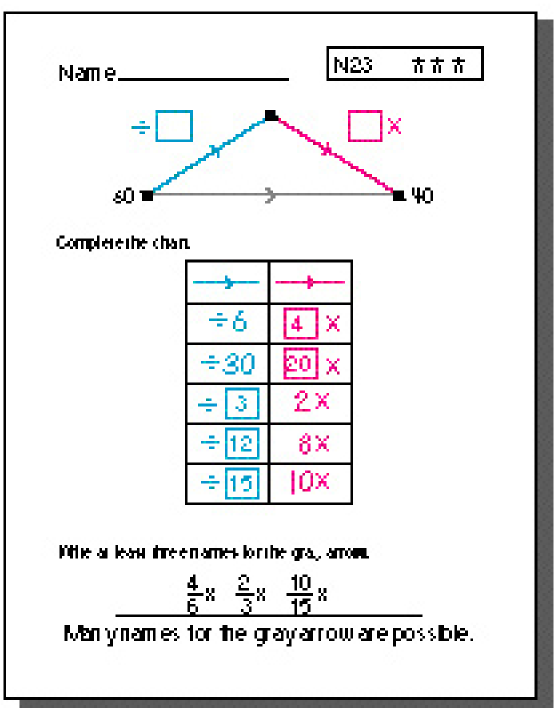
÷ 7 6 ×

$\frac{6}{7}$×

Complete.

$\frac{6}{7} \times 28 =$ 24 $\quad \frac{6}{7} \times$ 49 $= 42$

$\frac{6}{7} \times 42 =$ 36 $\quad \frac{6}{7} \times$ 63 $= 54$



Name ________ N23 ☆☆☆

÷ ☐ ☐ ×

60 40

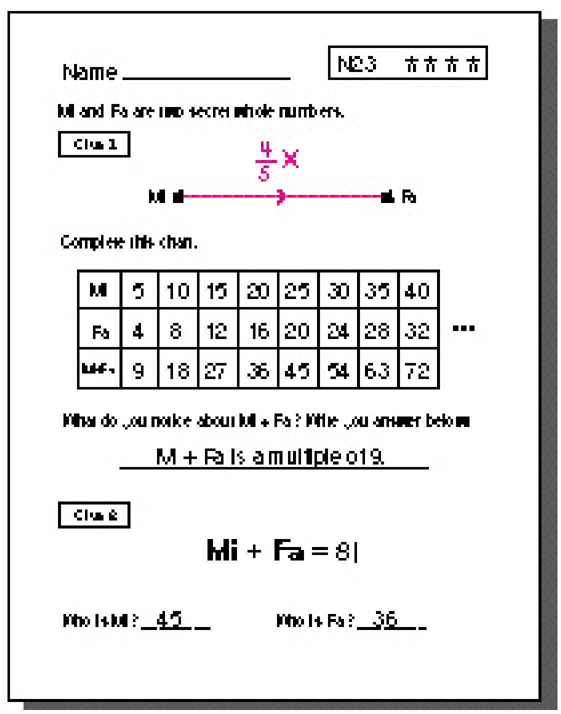
Complete the chart.

| → | → |
|---|---|
| ÷6 | 4 × |
| ÷30 | 20 × |
| ÷ 3 | 2× |
| ÷ 12 | 8× |
| ÷ 15 | 10× |

Write at least three names for the gray arrow.

$\frac{4}{6}$× $\frac{2}{3}$× $\frac{10}{15}$×

Many names for the gray arrow are possible.



Name ________ N23 ☆☆☆☆

Mi and Fa are two secret whole numbers.

Clue 1

$\frac{4}{5}$×

Mi → Fa

Complete this chart.

| Mi | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 |
|---|---|---|---|---|---|---|---|---|
| Fa | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 |
| Mi+Fa | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 |

...

What do you notice about Mi + Fa? Write your answer below.

M + Fa is a multiple of 9.

Clue 2

**Mi + Fa** = 81

Who is Mi? 45 Who is Fa? 36

# N24 DIVISION #3

## Capsule Lesson Summary

Use an array of dots representing a band marching in rows of five to suggest a sequence of related division problems. Present a story about hiking and another story about bicycling where the total distance of a trip and the average speed are known. To determine the total time needed for each trip, use repeated subtraction and arrow roads. (The solution method used for these problems is in preparation for a standard algorithm for division.)

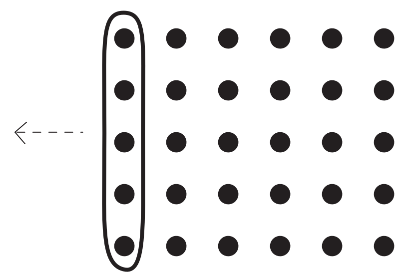
## Materials

| Teacher | | Student | |
|---|---|---|---|
| | • Colored chalk | | • Colored pencils, pens, or crayons<br>• Worksheets N24*, **, ***, and **** |

## Description of Lesson

### Exercise 1

Draw this dot picture on the board. Begin the lesson by talking with the class about marching bands and how they often march in rows.



**T:** ***This is a picture of part of a marching band. They are marching in this direction*** (trace an arrow going to the left)***. Each row has five band members. I've circled the first row. Right now, how many dots*** (band members) ***have I drawn?***

**S:** ***30 dots; there are six rows of five dots and 6 x 5 = 30.***

**S:** ***30 dots; I counted by fives.***

**T:** ***There are dots for only 30 band members, but this band has many more members, all marching in rows of five. In fact, this band has 170 members. How many rows are there?***

This is a fairly difficult question. Accept estimates and more exact answers with explanations until one or more students note that there are 34 rows.

**S:** ***34 rows. There are 150 band members in 30 rows, since 30 x 5 = 150. The extra 20 members (170 – 150 = 20) would stand in four rows of five. Altogether, there are 34 rows.***

**T:** ***What division fact does this suggest?***

**S:** ***170 ÷ 5 = 34.***

Write this information on the board.

$170 \div 5 = 34$
$175 \div 5 = \_\_\_$

**T:** ***When 170 band members march in rows of five, there are 34 rows. Five new members join the band; 175 members. Now how many rows are there?***

**S:** ***35 rows; the five new members form another row.***

# N24

Record the answer and add another problem to the list.

$$\begin{array}{rcl} & 170 \div 5 = 34 & \\ +5 & & + \\ & 175 \div 5 = 35 & \\ & 185 \div 5 = \underline{\qquad} & \end{array}$$

**T:** *Now there are 185 members in the band. How many new members joined?*

**S:** *Ten members, since 175 + 10 = 185.*

Draw a +10 arrow from 175 to 185.

**T:** *How many rows of five now?*

**S:** *37 rows. The ten new members form two new rows and 35 + 2 = 37.*

Record the solution and draw a +2 arrow from 35 to 37.

Emphasize that the red arrows indicate the number of new members and that the blue arrows indicate the number of new rows.

Add several more problems, one at a time, and present them in a similar manner. Solutions (in boxes), red arrows, and blue arrows are shown here.

$$\begin{array}{rcl} & 170 \div 5 = 34 & \\ +5 & & +1 \\ & 175 \div 5 = 35 & \\ +10 & & +2 \\ & 185 \div 5 = 37 & \\ +10 & & +2 \\ & 195 \div 5 = \boxed{39} & \\ +5 & & +1 \\ & 200 \div 5 = \boxed{40} & \\ +15 & & +3 \\ & 215 \div 5 = \boxed{43} & \\ +20 & & +4 \\ & 235 \div 5 = \boxed{47} & \end{array}$$

Begin another column of problems on the board.

$$\begin{array}{c} 170 \div 5 = 34 \\ 165 \div 5 = \underline{\qquad} \end{array}$$

**T:** *What happens if the original band begins losing members? How many rows of five would a band of 165 members have?*

**S:** *33 rows. Five members left (170 – 165 = 5), so there would be one less row. 34 – 1 = 33.*

Record the solution and appropriate arrows on the board. In a similar manner, present other such problems, one at a time.

$$\begin{array}{rcl} & 170 \div 5 = 34 & \\ -5 & & -1 \\ & 165 \div 5 = 33 & \\ -5 & & -1 \\ & 160 \div 5 = \boxed{32} & \\ -10 & & -2 \\ & 150 \div 5 = \boxed{30} & \\ -15 & & -3 \\ & 135 \div 5 = \boxed{27} & \\ -20 & & -4 \\ & 115 \div 5 = \boxed{23} & \end{array}$$

## Exercise 2

Invite students to talk briefly about any long hikes they have taken or have heard about.

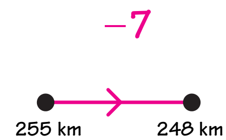
**T:** *Angie decides to take a 255 kilometer hike in the Appalachian Mountains. She wants to calculate how many days to allow for the hike. She knows that she hikes about seven kilometers per hour.*

**Note:** To give the students a feel for 255 km, refer to any city about 255 km from your own city. Seven kilometers per hour is a comfortably fast walking pace.

Draw this arrow road as you describe Angie's method of calculation.

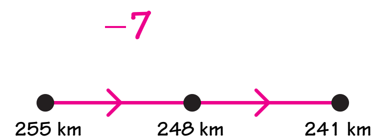
**T:** ***Angie uses –7 arrows to calculate the number of hours to plan for her hike. When she starts, she has 255 kilometers to walk.***

***She will walk 7 km in the first hour and will have 248 km left to go.***

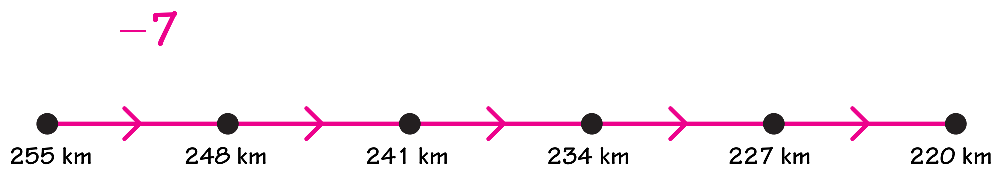



***In the next hour, she will hike 7 km more. How far will she have left to go?***

**S:** ***241 km, since 248 – 7 = 241.***




Continue to tell the story and build a road with two or three more arrows.




**T:** ***Angie will then have 220 km to go. How many hours are represented here?***

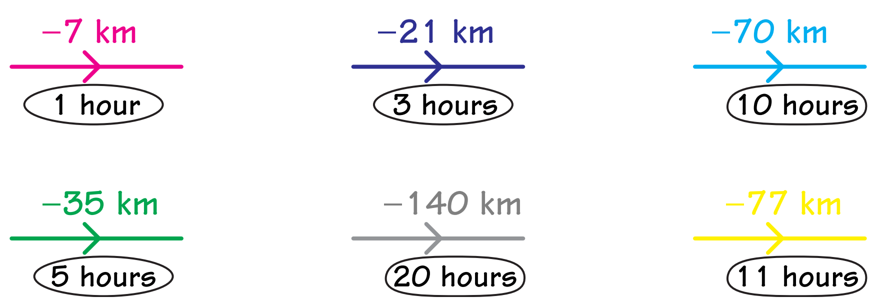
**S:** ***Five hours, since there are five red arrows.***

**T:** ***Angie realizes that it is taking a long time to build the arrow road with only –7 arrows. What other arrows might she find useful?***

**S:** ***–21 arrows. She hikes 21 km in three hours since 3 x 7 = 21.***

**S:** ***–70 arrows. She hikes 70 km in ten hours.***

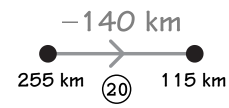
List students' suggestions on the board. Near each arrow, write the appropriate hiking time in hours, for example:




**T:** ***Since it is quite easy to subtract 140 or 70, Angie decides to start over with –140 arrows or –70 arrows.***

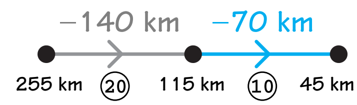
Erase the arrows on the board and then draw this arrow.

**T:** *After she hikes 140 km in 20 hours, how far will she have left to go?*




**S:** *115 km, since 255 – 140 = 115.*

**T:** *Will she need to hike at least 70 km more?* (Yes)
*So Angie draws a –70 arrow.*




**T:** *How many kilometers will be left?*

**S:** *45 km, since 115 – 70 = 45.*

**T:** *At this point, how many hours will she have walked?* (30)
*Will she have 70 km more to go?* (No) *What arrow could she use next?*

**S:** *She could use all –7 arrows.*

**S:** *She could use a –35 arrow. Angie could hike 35 km in 5 hours.*

Follow students' suggestions to complete the road to 3 km. For example:


**T:** *There are only 3 km left, which is less than a one-hour walk, so we'll stop here. How many hours will Angie's hike take altogether?*

**S:** *36 hours, since 10 + 19 + 10 + 5 + 1 = 36. She could then walk the last 3 km in about half an hour.*

**T:** *If she could walk non-stop, how many days would it take Angie to complete her hike?*

**S:** *1½ days, since there are 24 hours in a day. ½* x *24 = 12; 24 + 12 = 36.*

**T:** *But Angie only plans to hike for 6 hours each day. How many days will she hike?*

**S:** *6 days, since 6* x *6 = 36.*

**T:** *If she would plan to hike 9 hours a day, how many days would she hike?*

**S:** *4 days, since 4* x *9 = 36.*

**T:** *She'd like to complete the trip on a three-day weekend. How many hours would she have to be able to hike in a day?*

**S:** *12 hours per day, since 3* x *12 = 36.*

**T:** *Angie used these arrows* (point to the road) *because she is able to hike 7 km per hour. If she could only walk 6 km per hour, which arrows do you think she would use?*

**S:** *–6 arrows or –60 arrows, since she could walk 6 km in 1 hour and 60 km in 10 hours.*

**S:** *She could use arrows that subtract any multiple of 6.*

**T:** *Would it take her more hours or fewer hours if she walked at 6 km per hour?*

**S:** *More hours, because she would be walking slower.*

## Exercise 3

Discuss the possibility of bicycling across the United States. Let students estimate the number of days they think it would take.

**T:** ***Leonard is planning to bicycle 4 500 km across the United States. He knows he can average riding 21 km per hour and decides to bicycle 10 hours per day.***

***Let's use an arrow road to first calculate the number of hours his trip will take. What arrows would be convenient to use?***

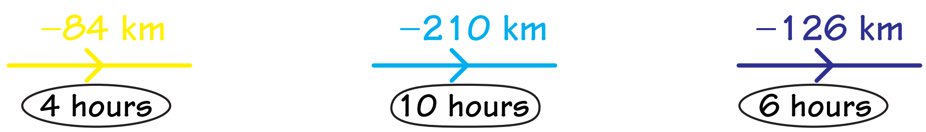
Accept any multiples of 21 as possibilities. For each suggested arrow, ask how many hours of bicycling it represents, for example:

**S:** ***We could use a –84 arrow.***

**T:** ***How long does it take Leonard to bicycle 84 km?***

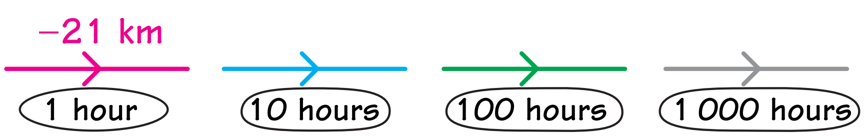
**S:** ***4 hours, since 4 x 21 = 84.***

For each possibility a student suggests, draw an appropriate arrow on the board, for example:




**T:** ***We could use any or all of these arrows, but Leonard decides to use the following arrows.***

Erase the board and draw these arrows.




**T:** ***Leonard can bicycle 21 km in 1 hour.*** (Point to the blue arrow.) ***How far can Leonard travel in 10 hours?***

**S:** ***210 km, since 10 x 21 = 210.***

**T** (point to the green arrow)**:** ***How far can Leonard bicycle in 100 hours?***

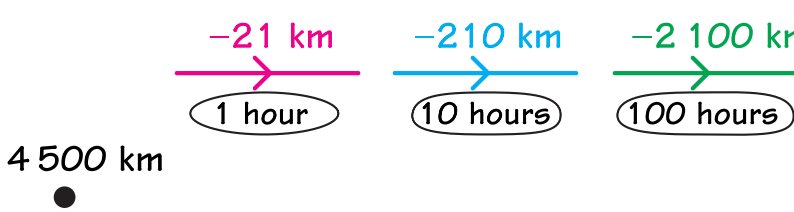
**S:** ***2 100 km, since 100 x 21 = 2 100.***

**T** (point to the gray arrow)**:** ***How far can Leonard bicycle in 1 000 hours?***

**S:** ***21 000 km.***

**S:** ***Leonard won't need the gray arrow because his trip is only 4 500 km long.***

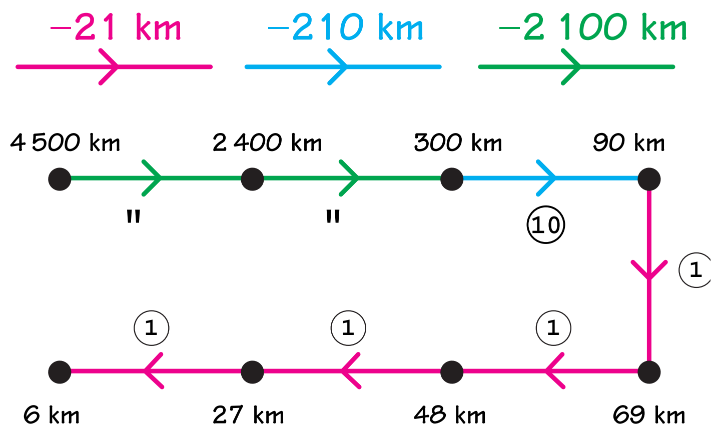
Erase the gray arrow. Draw a dot for 4 500 km.



# N24

**S:** ***4 500 – 2100 = 2 400.***

Continue following students' instructions to build the following arrow road.




**S:** ***Leonard will travel 214 hours and then will have 6 km left to bicycle.***

**T:** ***If he rides 10 hours per day, about how many days will his trip take?***

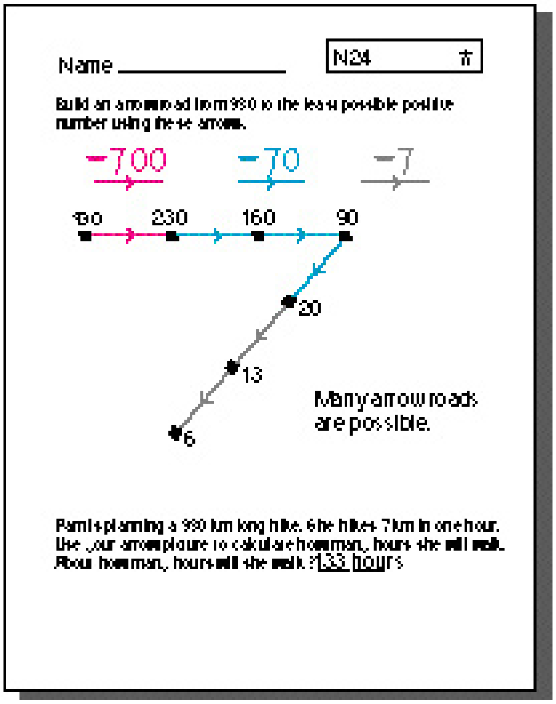
**S:** ***Between 21 and 22 days. 10* x *21 = 210, which is less than 214; 10* x *22 = 220, which is more than 214.***

Worksheets N24*, **, ***, and **** are available for individual work.

When many students have completed building the road on the * worksheet, you may like to hold a collective discussion about the question on the bottom of this worksheet.

## Home Activity

This would be a good time to send a letter to parents/guardians about division. A sample letter is available on Blackline N24.

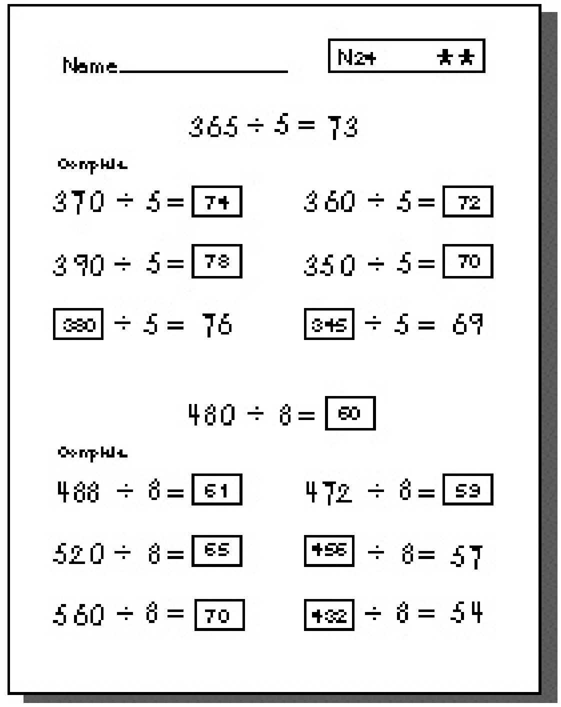


Name ____________ N24 ☆

Build an arrow road from 930 to the least possible positive number using these arrows.



Pam is planning a 930 km long hike. She hikes 7 km in one hour. Use your arrow picture to calculate how many hours she will walk. About how many hours will she walk? 133 hours



Name ____________ N24 ★★

365 ÷ 5 = 73

Complete.

370 ÷ 5 = $\boxed{74}$ 360 ÷ 5 = $\boxed{72}$

390 ÷ 5 = $\boxed{78}$ 350 ÷ 5 = $\boxed{70}$

$\boxed{380}$ ÷ 5 = 76 $\boxed{345}$ ÷ 5 = 69

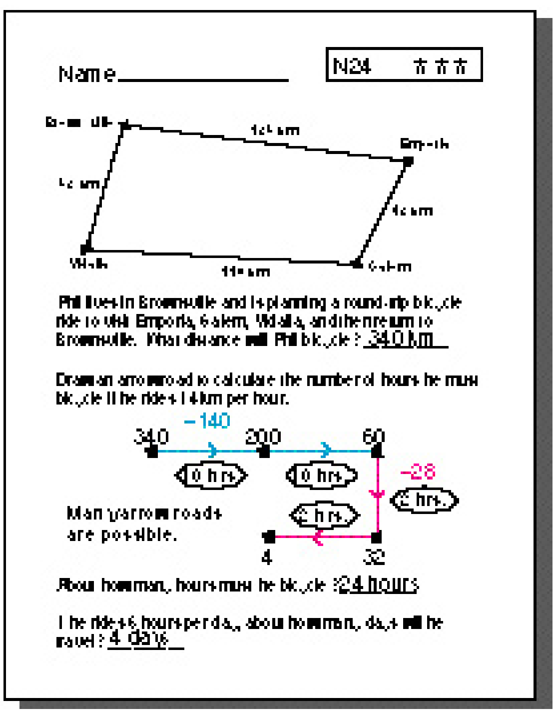
480 ÷ 8 = $\boxed{60}$

Complete.

488 ÷ 8 = $\boxed{61}$ 472 ÷ 8 = $\boxed{59}$

520 ÷ 8 = $\boxed{65}$ $\boxed{456}$ ÷ 8 = 57

560 ÷ 8 = $\boxed{70}$ $\boxed{432}$ ÷ 8 = 54



Name ____________ N24 ☆☆☆



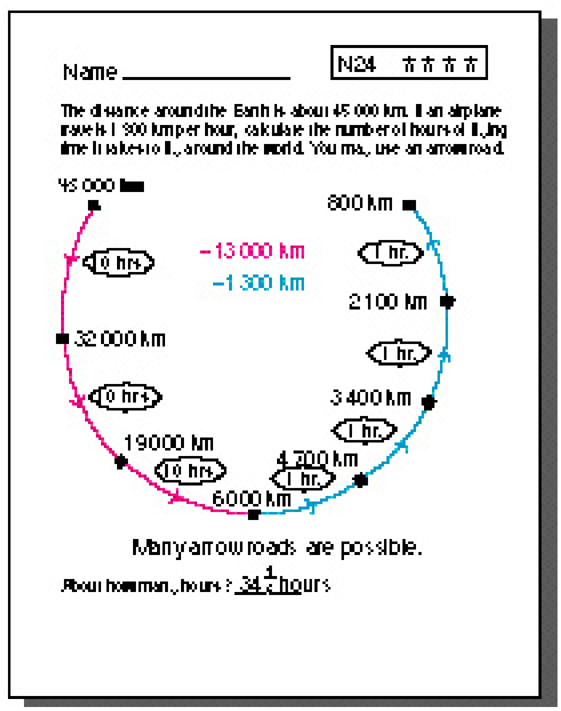
Phil lives in Brownsville and is planning a round-trip bicycle ride to visit Emporia, Salem, Vidalia, and then return to Brownsville. What distance will Phil bicycle? 340 km

Draw an arrow road to calculate the number of hours he must bicycle if he rides 14 km per hour.



About how many hours must he bicycle? 24 hours

He rides 6 hours per day, about how many days will he travel? 4 days



Name ____________ N24 ☆☆☆☆

The distance around the Earth is about 45 000 km. If an airplane travels 1 300 km per hour, calculate the number of hours of flying time it takes to fly around the world. You may use an arrow road.



About how many hours? $34\frac{1}{2}$ hours

# N25 MULTIPLICATION ALGORITHM #3

## Capsule Lesson Summary

Decode numbers on the Minicomputer with weighted checkers. Put numbers on the Minicomputer using two specific weighted checkers. Reinforce equivalences such as 60 x 40 = 6 x 400 with the use of weighted checkers. Use the multiplication algorithm presented in Lesson N18 to do calculations such as 47 x 34 and 211 x 352.

## Materials

| | | | |
|---|---|---|---|
| Teacher | • Minicomputer set | Student | • Weighted checker set<br>• *Multiplication Problems* Booklet |

## Description of Lesson

Begin the lesson with some mental arithmetic involving multiplication facts and patterns.

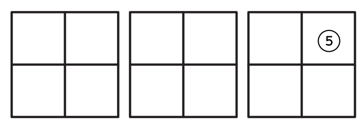
| | | |
|---|---|---|
| 2 x 13 = $\boxed{26}$ | 3 x 21 = $\boxed{63}$ | 5 x 82 = $\boxed{410}$ |
| 20 x 13 = $\boxed{260}$ | 30 x 21 = $\boxed{630}$ | 50 x 82 = $\boxed{4\ 100}$ |
| 200 x 13 = $\boxed{2\ 600}$ | 300 x 21 = $\boxed{6\ 300}$ | 500 x 82 = $\boxed{41\ 000}$ |

### Exercise 1

Display three Minicomputer boards and the weighted checkers ②, ③, …, ⑨.

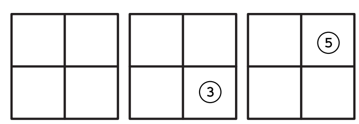
**T:** ***Each time I put some checkers on the Minicomputer, you tell me the number.***

Put a ⑤-checker on the 4-square.



**S:** ***20.***

Add a ③-checker on the 10-square.

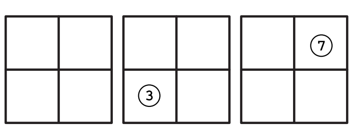


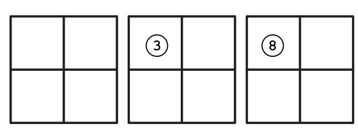
**S:** ***50.***

**T:** ***How did you get 50?***

**S:** ***5 x 4 = 20 and 3 x 10 = 30; 20 + 30 = 50.***

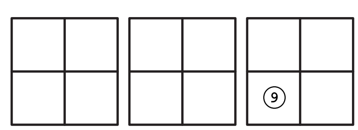
Continue this activity by decoding these configurations.

 = 88

 = 304

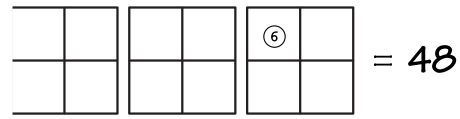
Clear the Minicomputer. Point to the displayed checkers.

**T:** ***Can we put on 18 with exactly one of these checkers?***

 = 18

# N25

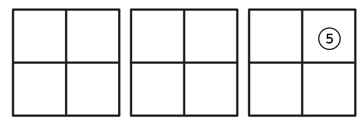
**T:** ***Can we put on 48 with exactly one of these checkers?***




## Exercise 2

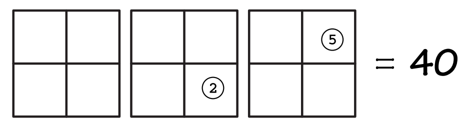
**T:** ***Can you put on 40 using only a ⑤-checker and a ②-checker?***

Let a student place one of the checkers.
Suppose the ⑤-checker is placed first.



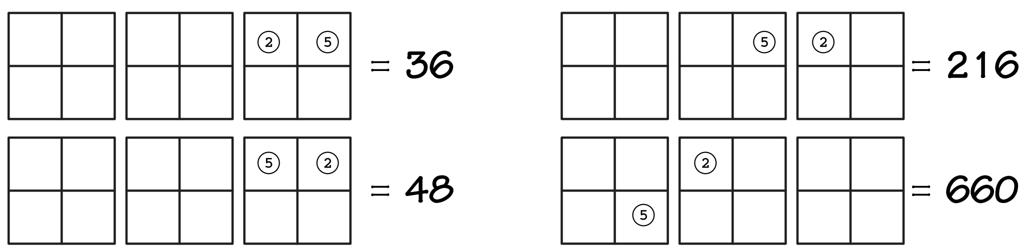

**T:** ***What number is on the Minicomputer?*** (20)
***Our goal is 40; how much more do we need?*** (20)

Invite a student to place the other checker.

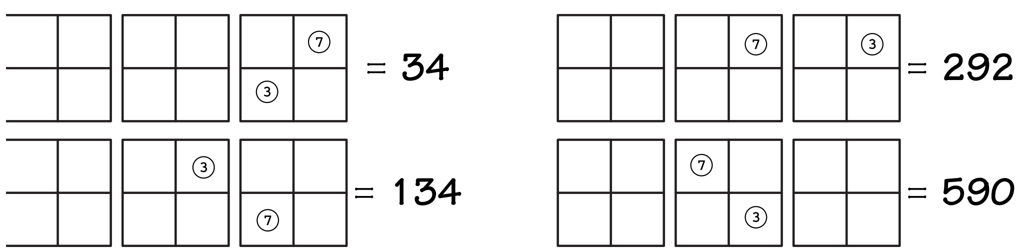



**Note:** If the placement of the first checker results in no solution, begin again.

Continue this activity by putting on 36, 48, 216, and 660, each with only a ②-checker and a ⑤-checker.




Repeat the exercise to put on the following numbers using only a ③-checker and a ⑦-checker.



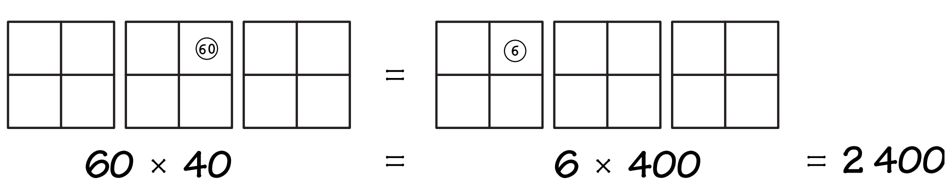

## Exercise 3

Display the weighted checkers ⑩, ⑳, …, (90), (100) along with ②, …, ⑨.

Put a (60)-checker on the 40-square.

**T:** ***Can we trade this (60)-checker for a ⑥-checker and not change the number?***

Let a student make the trade.



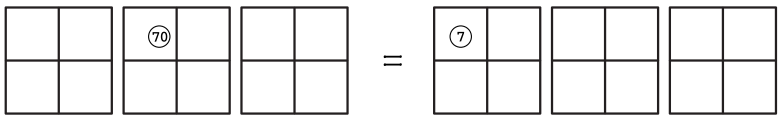
**T:** ***60* x *40 = 6* x *400. What number is on the Minicomputer?***

**S:** ***2 400.***

Emphasize the trade by writing the number sentence on the board.

Clear the Minicomputer; then put a ⑺-checker on the 80-square.

**T:** ***Can we trade this ⑺-checker for a ⑦-checker and not change the number?***



$70 \times 80 \quad = \quad 7 \times 800 \quad = 5\,600$

**T:** ***70* x *80 = 7* x *800. What number is on the Minicomputer?***

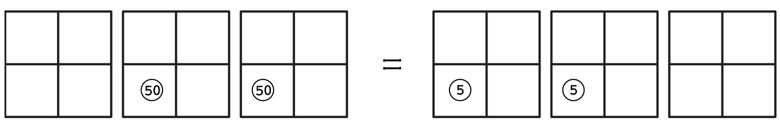
**S:** ***5 600.***

Clear the Minicomputer; then put ㊿-checkers on the 20-square and on the 2-square.

**T:** ***50*x *what number is on the Minicomputer?***

**S:** ***22.***

**T:** ***Can we trade these ㊿-checkers for ⑤-checkers and not change the number?***



$50 \times 22 \quad = \quad 5 \times 220 \quad = 1\,100$

**T:** ***5*x *what number is on the Minicomputer?***

**S:** ***220.***

**T:** ***What number is 5* x *220?***

**S:** ***1 100.***

Write these problems on the board and ask the class to solve them. (Answers are in the boxes and blanks.)

$$30 \times 25 = 3 \times \boxed{250} = \underline{\ 750\ }$$

$$40 \times 62 = 4 \times \boxed{620} = \underline{2\,480}$$

### Exercise 4

Write the problem 47 x 34 vertically on the board.

**T:** ***Let's use a standard method for doing this multiplication problem. First we multiply 4* x *47 and then 30* x *47. Who can multiply 4* x *47?***

$$\begin{array}{rr} & {}^{2}\phantom{0} \\ & 47 \\ & \times\ 34 \\ \hline 4 \times 47 = & 188 \\ 30 \times 47 = & \end{array}$$

**T:** ***To avoid confusion, let's erase the 2 that we carried. Now we need to multiply 30 x 47. How do we multiply a number by 30?***

**S:** ***Multiply by 3 and then by 10, or multiply by 10 and then by 3.***

Invite a student to calculate 30 x 47 at the board. The student might use the fact that 30 x 47 = 3 x 470 and multiply 3 x 470, or the student might multiply 3 x 47 and then multiply the result by 10. When the latter method is used, it is useful to record 0 (multiply by 10) first to insure that the result gets lined up properly with 188.

$$\begin{array}{rr} & 47 \\ & \times\ 34 \\ \hline 4 \times 47 = & 188 \\ 30 \times 47 = & 1\,410 \\ \hline \end{array}$$

**T:** ***How do we complete the calculation?***

**S:** ***Add 188 + 1 410.***

$$\begin{array}{rr} & 47 \\ & \times\ 34 \\ \hline 4 \times 47 = & 188 \\ 30 \times 47 = & 1\,410 \\ \hline & 1\,598 \end{array}$$

Ask students to do a couple more calculations on paper using the multiplication algorithm. Check answers collectively. For example:

$$\begin{array}{rr} & 175 \\ & \times\ 68 \\ \hline 8 \times 175 = & 1\,400 \\ 60 \times 175 = & 10\,500 \\ \hline & 11\,900 \end{array} \qquad \begin{array}{rr} & 211 \\ & \times\ 352 \\ \hline 2 \times 211 = & 422 \\ 50 \times 211 = & 10\,550 \\ 300 \times 211 = & 63\,300 \\ \hline & 74\,272 \end{array}$$

Write these problems on the board.

**T:** ***In 39 x 70, we are multiplying by 70. 70 is seven tens and no ones. So we just have one step. In 954 x 206, we are multiplying by 206. 206 is two hundreds, no tens, and six ones. So we only have two steps: multiply by 6 and multiply by 200.***

$$\begin{array}{r} 39 \\ \times\ 70 \\ \hline \end{array} \qquad \begin{array}{r} 954 \\ \times\ 206 \\ \hline \end{array}$$

Ask students to copy the problems and to complete the calculations. You may want to check the answers collectively, letting students put their solutions on the board.

$$\begin{array}{r} {\scriptstyle 6}\phantom{0} \\ 39 \\ \times\ 70 \\ \hline 2\,730 \end{array} \qquad \begin{array}{rr} & 954 \\ & \times\ 206 \\ \hline 6 \times 954 = & 5\,724 \\ 200 \times 954 = & 190\,800 \\ \hline & 196\,524 \end{array}$$

Distribute students' copies of the *Multiplication Problems* Booklet and allow 10 to 15 minutes for individual work. Encourage students to correct errors or complete unfinished pages before starting new pages. At the end of the lesson, collect the booklets for your review. This is the last lesson using the *Multiplication Problems* Booklet; however, you may want to use it again for practice. An answer key for the *Multiplication Problems* Booklet follows Lesson N13.

# N26 MINICOMPUTER GOLF #2

## Capsule Lesson Summary

Decode numbers on the Minicomputer. By moving exactly one checker, change a number on the Minicomputer by a specified amount. Play *Minicomputer Golf*.

## Materials

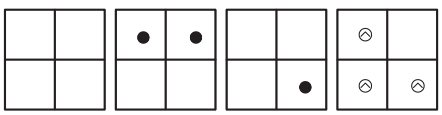
**Teacher**
- Minicomputer set
- Colored chalk

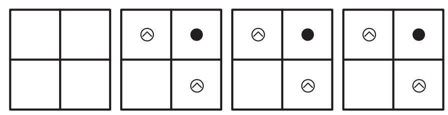
**Student**
- Paper

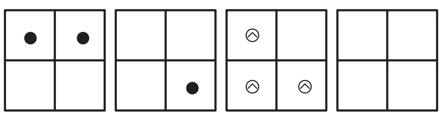
## Description of Lesson

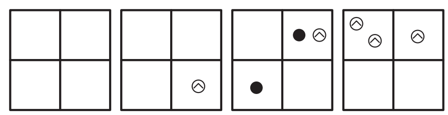
### Exercise 1

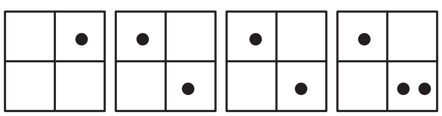
Display four Minicomputer boards. Ask students to decode several configurations, such as the ones below or others more appropriate for the numerical abilities of your class. Instruct students to first write their answers on paper; then call on students to answer aloud.

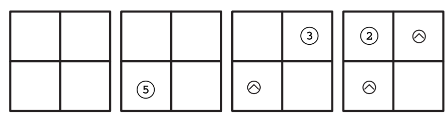

= 1 199


= $\widehat{555}$


= 11 990

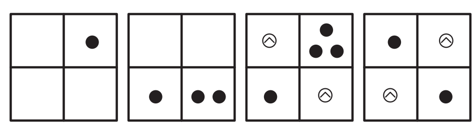

= $\widehat{100}$


= 5 000


= 1 110

### Exercise 2

Quickly put this configuration on the Minicomputer.



**T:** ***Don't try to tell me exactly what number this is. Is this number more than 1 000?***

**S:** ***Yes, and it is more than 4 000 because this is 4 000 on the thousands board.***

**S:** ***It is more than 4 400.***

**S:** ***It is less than 4 500.***

**T:** ***Good. The number is between 4 400 and 4 500. Now I am going to change the number by moving one checker from the square it is on to another square. Each time I move a checker, tell me if the new number is more or less than before. Also, tell me how much more or less it is.***

Move a positive checker from the 40-square to the 80-square.

**T:** ***Did I increase or decrease the number?***

**S:** ***Increase.***

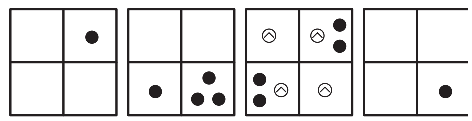
**T:** ***How much more is this number?***

**S:** ***40 more, because 40 + 40 = 80.***

Continue this activity, making the following moves. Do not return checkers to their original positions. Each move starts from a new number on the Minicomputer.

- Move the negative checker on the 80-square to the 40-square. (40 more; $\widehat{80} + 40 = \widehat{40}$)
- Move the positive checker on the 8-square to the 100-square. (92 more; $8 + 92 = 100$)
- Move the negative checker on the 4-square to the 20-square. (16 less; $\widehat{4} - 16 = \widehat{20}$)
- Move the positive checker on the 80-square to the 2-square. (78 less; $80 - 78 = 2$)
- Move the negative checker on the 20-square to the 80-square. (60 less; $\widehat{20} - 60 = \widehat{80}$)
- Move the positive checker on the 2-square to the 20-square. (18 more; $2 + 18 = 20$)
- Move the negative checker on the 2-square to the 20-square. (18 less; $\widehat{2} - 18 = \widehat{20}$)

If all the moves suggested above are made this configuration will be on the Minicomputer. You may need to adjust your request for moves if you have a different configuration.



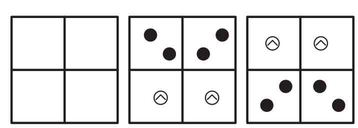
**T:** ***By moving a checker, can you increase the number by 2, that is, make the number 2 more than it is now?***

A student should move the negative checker from the 10-square to the 8-square. Continue this activity by asking for other similar changes. Feel free to adjust the level of difficulty of these questions to the numerical abilities of your class.

- Decrease the number by 2 000; make it 2 000 less. (Move the positive checker on the 4 000-square to the 2 000-square.)
- Increase the number by 39. (Move the positive checker on the 1-square to the 40-square, or move the negative checker on the 40-square to the 1-square.)
- Increase the number by 7. (Move the negative checker on the 8-square to the 1-square, or move the positive checker on the 1-square to the 8-square depending on how the number was increased by 39.)

## Exercise 3

Put this configuration on the Minicomputer.



**T:** ***What number is on the Minicomputer?***

**S:** ***204.***

**T:** ***Let's play a game of Minicomputer Golf using 204 as the starting number and 1 500 as the goal.***

Play a *Minicomputer Golf* game with teams, as described in Lesson N8.

## Extension Activity

Organize a *Minicomputer Golf* tournament.

## Capsule Lesson Summary

Continue to investigate patterns in sequences of numbers and in arrow pictures involving calculator relations such as $\boxed{+}\ \boxed{4}\ \boxed{=}$ … and $\boxed{-}\ \boxed{3}\ \boxed{=}$ … . Find that combining different calculator relations in an arrow picture can generate a sequence of numbers with a pattern related to the numbers involved in the relations.

## Materials

**Teacher**
- Calculator
- Colored chalk

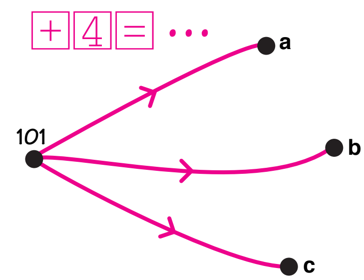
**Student**
- Calculator
- Paper
- Worksheets N27*, **, ***, and ****

## Description of Lesson

Arrange that every student has access to a calculator for this lesson.

### Exercise 1

Draw this arrow picture on the board.




Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

**T:** ***The red arrows are for*** $\boxed{+}\ \boxed{4}\ \boxed{=}\ \boxed{=}$ ***… as many times as you want.*** (Trace the arrow from 101 to **a**.) ***We start with 101 on the calculator and press*** $\boxed{+}\ \boxed{4}\ \boxed{=}\ \boxed{=}$ ***and so on. What number could be here*** (point to **a**)***?***

Label **a**, **b**, and **c** with correct answers from students; for example, 105, 125, 201, and so on. Each time you put a number in the picture, ask how much greater it is than 101. Erase the labels at **a**, **b**, and **c** before continuing.

**T** (pointing to **a**)**:** ***Could 200 be here? Why?***

**S:** ***No, 200 is 99 more than 101 and we cannot add 99 by adding fours.***

**S:** ***201 could be there, but not 200.***

**T:** ***Good. How many times would we have to press*** $\boxed{=}$ ***to get 201 on the display?***

**S:** ***25 times.***

**T** (pointing to **b**)**:** ***Could 325 be here? Why?***

**S:** ***Yes; 325 is 224 more than 101 and we can add 224 by adding fours. 224 is a multiple of 4.***

**S:** ***We had 125 as a possibility, so 325 could be there also.***

**T** (pointing to **c**)**:** ***What is the closest number to 1 000 that could be here?***

If students have difficulty with this question, suggest the following:

**T:** ***Rather than pressing*** $\boxed{=}$ ***many times until we get close to 1 000, let's just think about the situation. Remember, we found that starting at 101, pressing*** $\boxed{+}$ $\boxed{4}$***, and then pressing*** $\boxed{=}$ ***25 times gives us 201. What if we press*** $\boxed{=}$ ***25 more times?***

**S:** ***We'd get 301.***

Put these numbers in a list on the board.

201
301
401

**T:** ***What if we press*** $\boxed{=}$ ***25 more times?***

**S:** ***We'd get 401.***

Continue in this way until most students recognize the pattern.

**T** (pointing to **c**)**:** ***Now what is the closest number to 1 000 that could be here?***

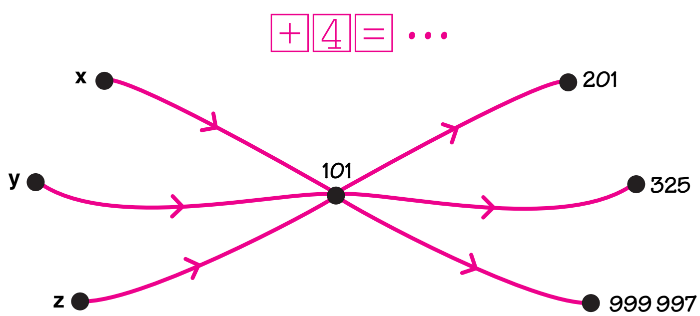
**S:** ***1 001.***

**T:** ***Good. What is the greatest possible number less than 1 000 that could be here (***at **c*****)?***

**S:** ***997 (1 001 – 4 = 997).***

Repeat the same two questions, replacing 1 000 with 1 000 000. (Answers: 1 000 001 and 999 997)

Add several arrows ending at 101 to the picture.




Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

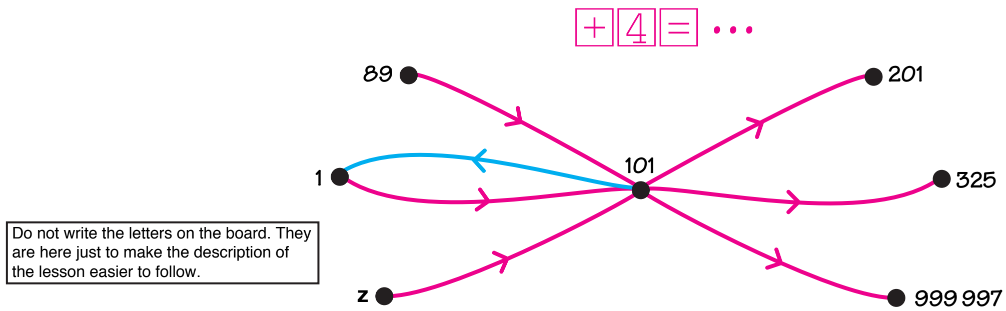
**T:** ***What number could be here*** (point to **x**)***?***

For each answer that a student gives, check how much less it is than 101. Observe that correct answers differ from 101 by a multiple of 4. Label **x** with any correct answer; for example, 97, 85, 41, and so on.

**T:** ***What is the least positive number that could be here*** (point to **y**)***?***

**S:** ***1.***

Add this blue arrow to the picture.




**T:** ***What could this return arrow be for?***

**S:** $\boxed{-}\ \boxed{4}\ \boxed{=}$ ***… .***

**T:** ***Good. How many times must we press*** $\boxed{=}$ ***to go from 101 to 1 on the display?***

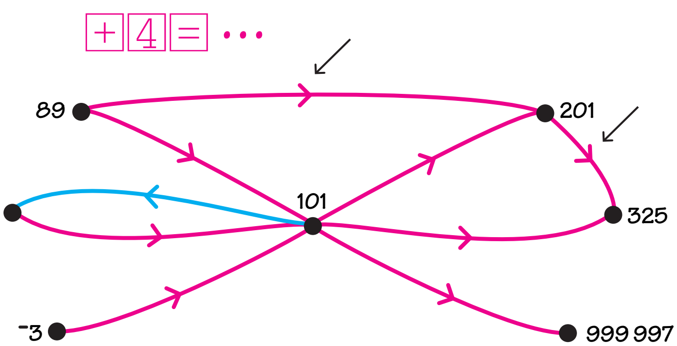
**S:** ***25 times.***

**T:** ***Let's label this dot*** (point to **z**) ***with a negative number.***

**S:** ***$^{-}3$ (or $^{-}11$, $^{-}35$, $^{-}99$ and so on).***

**T:** ***Can you trace some other red arrows that could be drawn in our picture?***

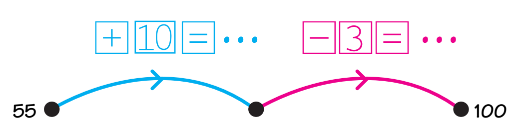
There are many possibilities; two are shown below.




In general, if a red arrow is followed by a red arrow, then a red arrow starting where the first one started and ending where the second one ended can be drawn. That is, red followed by red is still red. Also, a red arrow can be drawn between the beginning or ending points of the given red arrows, always from the lesser number to the greater number.

## Exercise 2

Erase the board and then draw this arrow picture.




**T:** ***What number could be here*** (point to the middle dot)***? There are many possibilities. Write some on your paper. Try to find a pattern.***

Allow a few minutes for individual work on this problem. Then solicit many possibilities from the class.

# N27

You may like to make a list of the numbers on the board. Although student responses may not be given in order, arrange your list on the board so that it presents solutions in numerical order. Students should soon see a +30 pattern.

**115, 145, 175, 205, 235 ...**

Check one or two possibilities by putting that number at the middle dot, tracing the blue arrow, and asking,

**T:** ***How many times do we press*** $\boxed{=}$***?***

**S:** ***Six times.***

Then trace the red arrow and ask,

**T:** ***How many times do we press*** $\boxed{=}$***?***

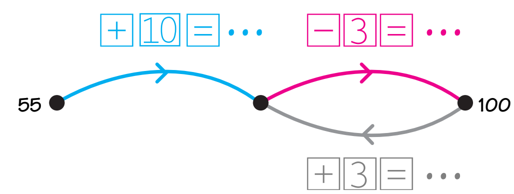
**S:** ***Five times.***

You may like to ask the class to explain the +30 pattern.

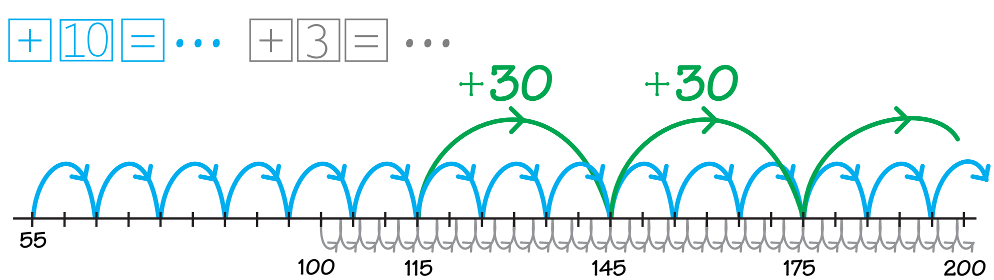
At this point accept any reasonable explanation, because the idea may be difficult to verbalize. A discussion of the +30 pattern is presented below for your information.

Consider the arrow picture with the addition of a $\boxed{+}$ $\boxed{3}$ $\boxed{=}$ ... return arrow.

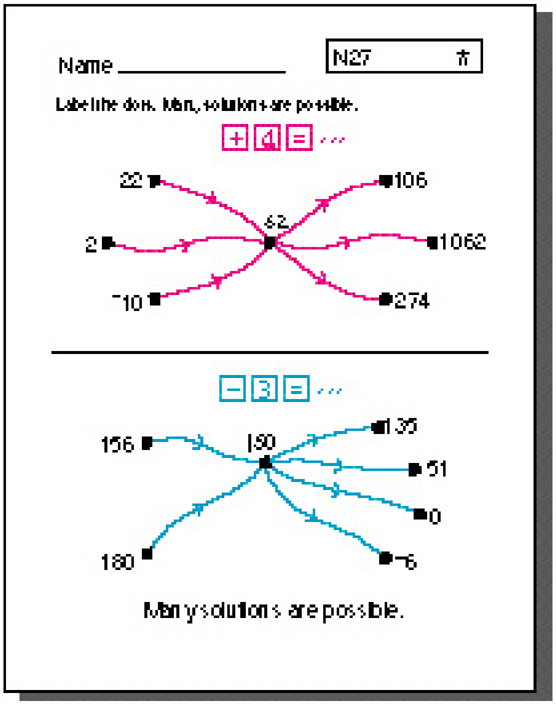
The blue arrow tells us that the middle number must have 5 in the ones place, and the gray arrow tells us that the middle number must be 100 plus a multiple of 3. A quick check shows that 115 is the least number that fulfills both conditions.




Now consider jumps on the number line.




Worksheets N27*, **, ***, and **** are available for individual work.

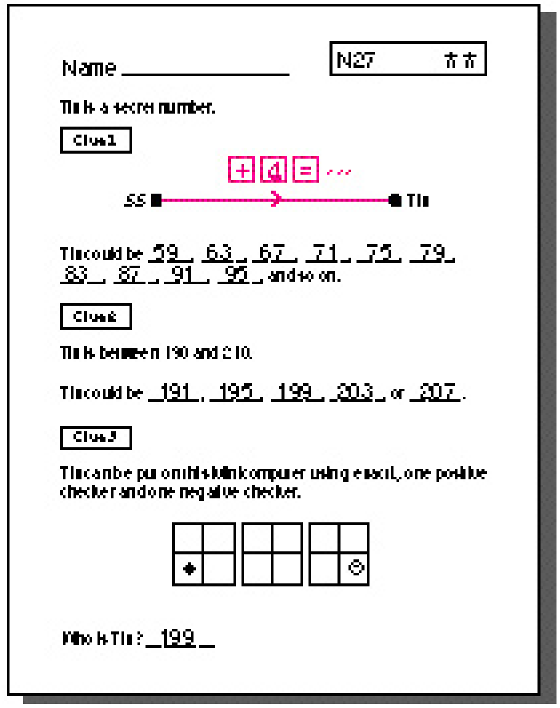


Name ____________ N27 ☆

Label the dots. Many solutions are possible.



Many solutions are possible.



Name ____________ N27 ☆☆

Tii is a secret number.

Clue 1



Tii could be 59, 63, 67, 71, 75, 79, 83, 87, 91, 95, and so on.

Clue 2

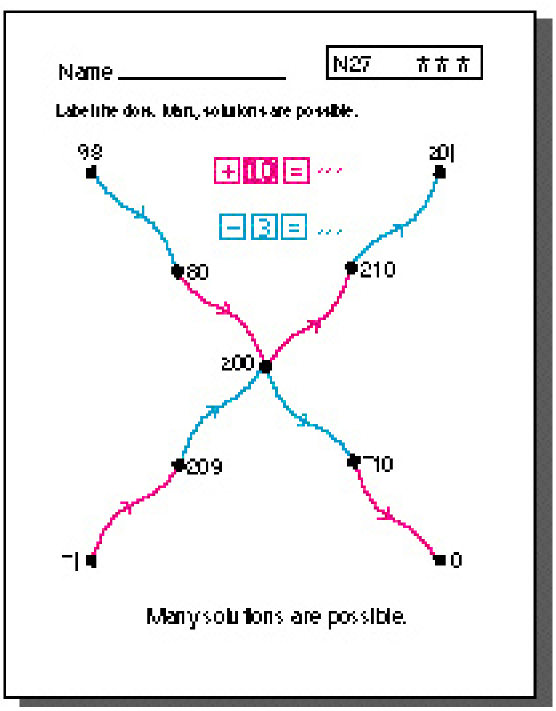
Tii is between 190 and 210.

Tii could be 191, 195, 199, 203, or 207.

Clue 3

Tii can be put on this Minicomputer using exactly one positive checker and one negative checker.

Who is Tii? 199

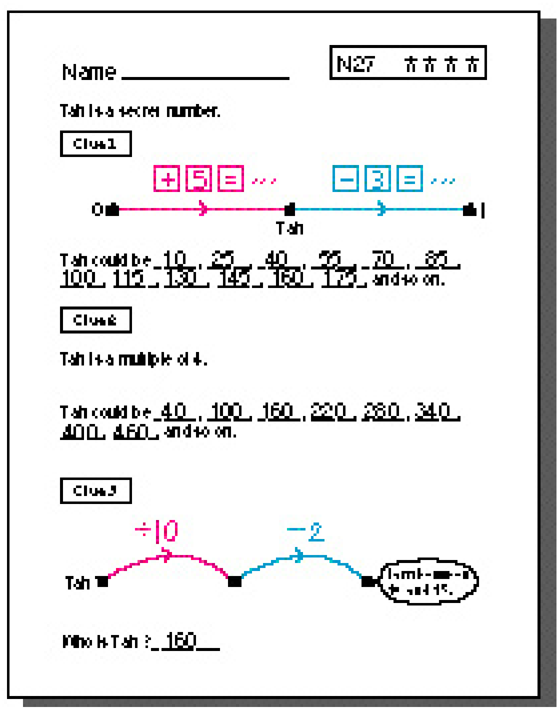


Name ____________ N27 ☆☆☆

Label the dots. Many solutions are possible.



Many solutions are possible.



Name ____________ N27 ☆☆☆☆

Tah is a secret number.

Clue 1



Tah could be 10, 25, 40, 55, 70, 85, 100, 115, 130, 145, 160, 175, and so on.

Clue 2

Tah is a multiple of 4.

Tah could be 40, 100, 160, 220, 280, 340, 400, 460, and so on.

Clue 3



Who is Tah? 160

## Capsule Lesson Summary

Use the result of a multiplication problem to quickly do related computations. Solve a detective story that involves the number of ways to have change for 50¢ in quarters, dimes, and nickels. Individually, find all of the ways to have change for 20¢ in dimes, nickels, and pennies using a tree diagram.

### Materials

**Teacher**
- 1 quarter and 5 nickels in a container
- Collection of coins

**Student**
- Unlined paper

## Description of Lesson

### Exercise 1

With student input, do this calculation on the board.

$$\begin{array}{r} 17 \\ \times\ 15 \\ \hline 5 \times 17 = \ \ 85 \\ 0 \times 17 = 170 \\ \hline 255 \end{array}$$

Record the result horizontally: 17 x 15 = 255.

**T:** ***How could we use this result to compute 18 x 15?***

**S:** ***We could add 15 to 255.***

Accept other correct methods, but emphasize this one.

**T:** ***Right! Eighteen 15s is one more 15 than seventeen 15s. What number is 255 + 15?***

**S:** ***270.***

Record the result on the board.

$$\begin{array}{l} 17 \times 15 = 255 \\ 18 \times 15 = 270 \end{array}$$

**T:** ***How could we compute 16 x 15?***

**S:** ***Subtract 15 from 255.***

Accept other correct methods, but emphasize this one.

$$\begin{array}{l} 16 \times 15 = 240 \\ 17 \times 15 = 255 \\ 18 \times 15 = 270 \end{array}$$

**T:** ***Sixteen 15s is one less 15 than seventeen 15s. What number is 255 – 15?***

**S:** ***240.***

Similarly arrive at the following three number sentences, starting with 24 x 61 = 1 464 and using this result to do the other two computations.

$$\begin{array}{l} 23 \times 61 = 1403 \\ 24 \times 61 = 1464 \\ 25 \times 61 = 1525 \end{array}$$

# N28

Write this problem on the board.

$1.5 \times 60$

**T:** ***What is one-and-a-half 60s?***

**S:** ***90; $\frac{1}{2} \times 60 = 30$ and $60 + 30 = 90$.***

$1.5 \times 60 = 90$

**T:** ***A moment ago, we used what 17 x 15 is to compute 18 x 15, and we used what 24 x 61 is to compute 25 x 61. Let's do the same here. What number is one more than 1.5?***

**S:** ***2.5.***

Record the problem on the board.

$1.5 \times 60 = 90$
$2.5 \times 60 =$

**T:** ***What number is 2.5 x 60?***

**S:** ***150.***

**T:** ***How did you do the calculation?***

**S:** ***I added 60 + 90.***

**S:** ***$2 \times 60 = 120$ and $\frac{1}{2} \times 60 = 30$, so $2\frac{1}{2} \times 60 = 120 + 30 = 150$.***

Accept both approaches. Continue with two more problems in sequence.

$1.5 \times 60 = 90$
$2.5 \times 60 = 150$
$3.5 \times 60 = 210$
$4.5 \times 60 = 270$

## Exercise 2: Detective Story

Display a container with one quarter and five nickels concealed within.

You may also like to have some coins available (quarters, dimes, and nickels), separate from those in the container, so that students can occasionally display responses with coins.

**T:** ***I have 50¢ in the container. Can you figure out which coins I have? I'll give you some clues.***

Clue 1

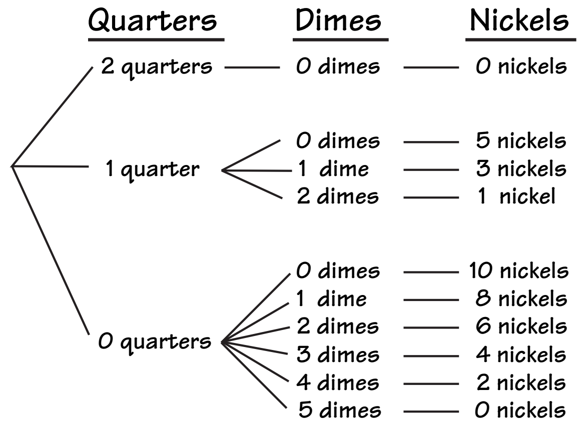
**T:** ***I do not have a half-dollar, and I do not have any pennies. What other kinds of coins might I have?***

**S:** ***Nickels, dimes, or quarters.***

Record the three kinds of coins on the board.

**T:** ***What coins might I have? How many quarters? How many dimes? How many nickels?***

As students suggest possibilities, list them in a tree as shown here. You can ask for the possible numbers of quarters (2, 1, or 0), draw the first column of the tree, and then consider each of the three cases in turn; or you can just list possibilities in a tree as offered and let students discover how the organization of the tree helps in finding all the possibilities. Study the spacing that the tree requires so that you do not run out of room on the board for the last rows of the tree.



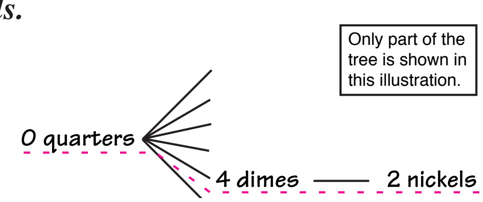

**T:** ***So how many ways are there to have 50¢ in quarters, dimes, and nickels?***

**S:** ***Ten ways.***

**T:** ***How did you count?***

**S:** ***I counted the lines on the right, the numbers of nickels.***

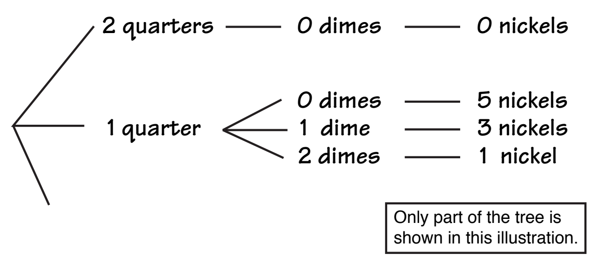
You may need to trace lines in the tree to show how they indicate ways to have 50¢. For example:




**T:** ***Of the ten ways, how many include at least one quarter?***

**S:** ***Four ways.***

Invite a student to trace the four ways in the tree.




Continue with the following questions.

- Of the ten ways, how many include at least two nickels? (Seven ways)
- Of the ten ways, how many include at most three nickels? (Five ways)
- Of the ten ways, how many include exactly one dime? (Two ways)

Clue 2

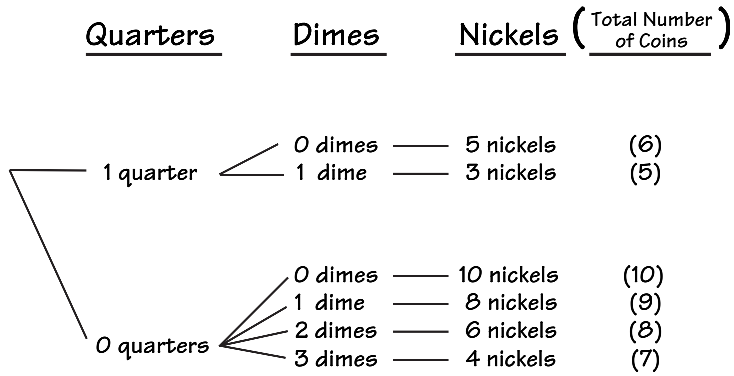
**T:** ***More than half of my coins are nickels. What coins could I have?***

**S:** ***You could have two dimes and six nickels. That's eight coins. $\frac{1}{2}$ x 8 = 4, and you would have more than four nickels.***

**S:** ***One quarter and five nickels.***

After a couple of possibilities have been given, lead the class through an exhaustive check. They should find that six of the ten possibilities satisfy this clue. Erase lines in the tree (possibilities from the first clue) that do not fit this second clue.

**Note:** When erasing ways that are no longer possibilities, be careful not to erase the entries with one quarter or zero quarters because some branches connecting to them will remain as possibilities. Your tree should look like the one below.



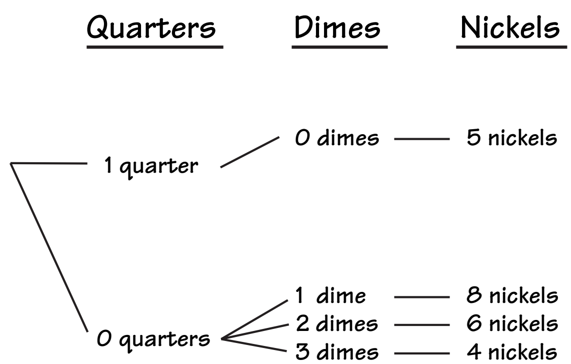

### Clue 3

**T:** ***I have exactly two types of coins in my container.***

**S:** ***You couldn't have one quarter, one dime, and three nickels.***

**S:** ***You couldn't have ten nickels.***

Erase the two ways that the clue rules out.




### Clue 4

Choose two students to star in this clue.

**T:** ***I owe Rob 15¢ and Nicole 20¢. I have enough money, but I do not have the correct change to repay both of them. What coins do I have?***

Let students whisper their answers to you, or ask them to write them on paper for you to check. Then let someone answer aloud. If there are disagreements, the class should settle them.

**S:** ***You have one quarter and five nickels.***

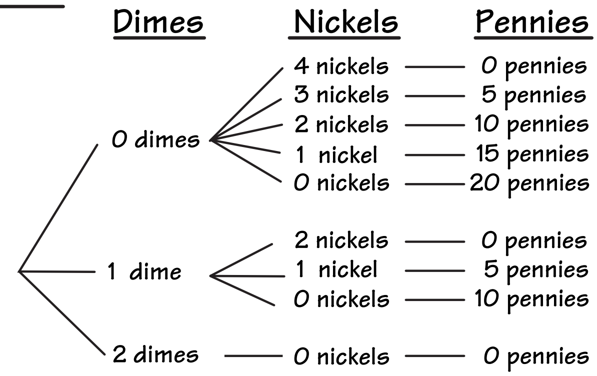
Give the box to a student to check its contents.

### Exercise 3

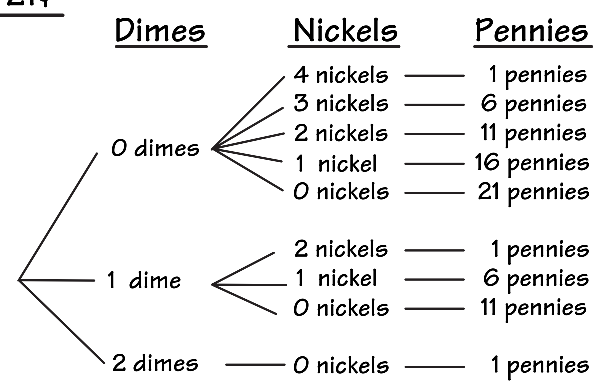
Instruct students to find all the possibilities to have change for 20¢ (that is, suppose there is 20¢ in your container). Note collectively that in this case you could have dimes, nickels, and pennies.

Then let students work individually or with a partner to list all the change possibilities, preferably in a tree. When a student finishes, ask how many ways there are to have change for 21¢. Does the student think that there will be the same number of ways as for 20¢, more ways, or fewer ways? Do not comment on the student's response. The student should then find the ways for 21¢ and list them in a new tree or make alterations to the tree for 20¢. If a student finishes a tree for 21¢, ask about 22¢, 23¢, 24¢, and 25¢. Perhaps the student will see that the number of ways to have change remains the same until you consider 25¢, for which there are more ways to have change than there were for 20¢.

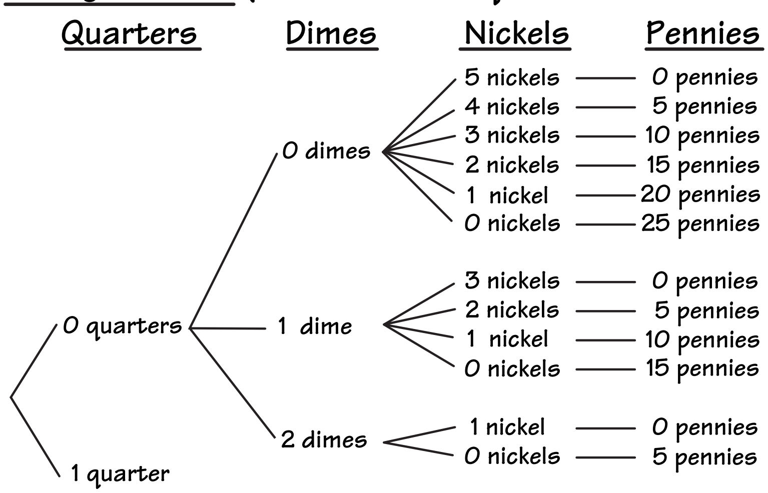
**Change for 20¢**




**Change for 21¢**




**Change for 25¢** (Coins could be quarters, dimes, nickels, and pennies.)



# N29 CALCULATOR RELATIONS #3



## Capsule Lesson Summary

Introduce a calculator relation $\boxed{+}$ $\boxed{=}$ … and use it to find multiples of a given number and then multiples of the multiples. Find numbers, including non-integer decimal numbers, that could be the starting point of a $\boxed{+}$ $\boxed{=}$ … arrow ending at 4.

### Materials

| | | | |
|---|---|---|---|
| Teacher | • Calculator<br>• Colored chalk | Student | • Calculator<br>• Worksheets N29*, **, and *** |

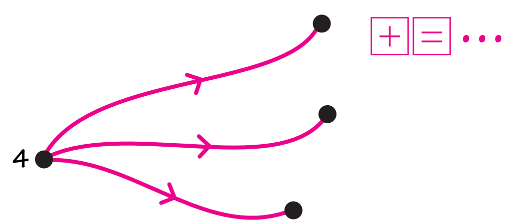
## Description of Lesson

Arrange that every student has access to a calculator for this lesson.

**Note:** The calculator relation introduced in this lesson is dependent on the constant feature of a calculator. See the section on Role and Use of Calculators in the "Notes to the Teacher" section. Here we assume calculators have a constant mode (or automatic constant) for addition, and that when you press $\boxed{4}$ $\boxed{+}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and so on you see 4, 8, 12, 16, … on the display . You may need to modify the description of this relation for your type of calculators.

### Exercise 1

Draw this arrow picture on the board.




**T:** ***Red arrows are for*** $\boxed{+}$ $\boxed{=}$ ***…, as many times as you want.*** (Trace an arrow starting at 4)***.*** ***We start with 4 on the calculator and press*** $\boxed{+}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ ***and so on. What number could one of these unlabeled dots be for?***

Let several students respond and label two of the dots with correct answers.

**T** (pointing to an unlabeled dot)**:** ***What is the greatest number less than 1 000 that could be here?***

**S:** ***996.***

**T:** ***When we start with 4, how many times would we press*** $\boxed{=}$ ***to get 1 000 on the display?***

**S:** ***250 times.***

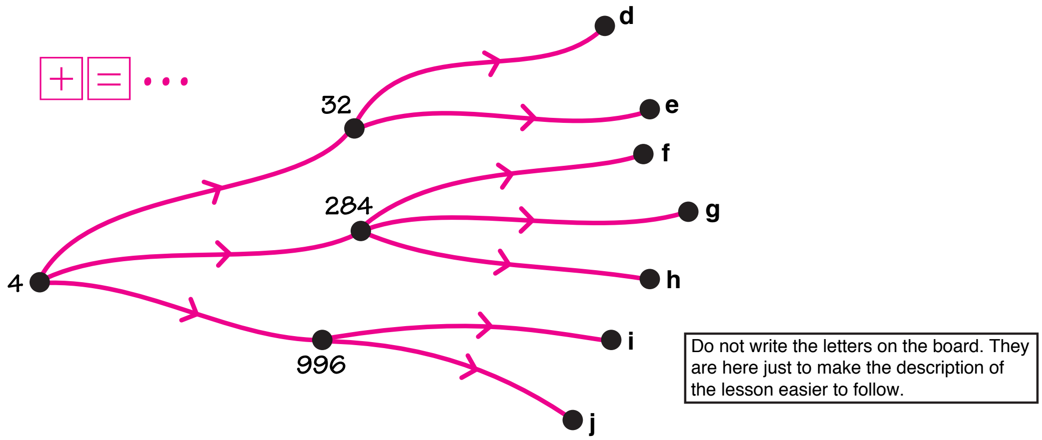
**T:** ***Why 250?***

**S:** ***250*** x ***4 = 1 000.***

**T:** ***Then how many times would we press*** $\boxed{=}$ ***to get 996?***

**S:** ***249 times, because 996 is just 4 less than 1 000. So, 249*** x ***4 = 996.***

Add these arrows to your drawing. For purposes of the lesson description, the dots labeled here have numbers that might have been given by a class.




**T** (pointing to 32)**:** ***Here we begin with 32. Put 32 on the display of your calculator and*** (trace the red arrow) ***press*** $\boxed{+}$ $\boxed{=}$ ***..., as many times as you like. What number could this dot (d) be for?***

Let several students suggest answers and label **d** and **e** with two of them.

**T:** ***Starting with 284 on your display, press*** $\boxed{+}$ ***and then*** $\boxed{=}$ ***exactly ten times. What number is on the display?***

**S:** ***2 840.***

Put 2 840 at **f**.

**T:** ***If we start with 284 and press*** $\boxed{+}$ ***and then*** $\boxed{=}$ ***one hundred times, what number would we get?***

**S:** ***28 400.***

Put 28 400 at **g**.

Repeat the question with pressing $\boxed{=}$ one thousand times to get 284 000 at **h**.

**T:** ***Here we start with 996. What number could this dot (i) be for?***

**S:** ***9 960 (for example).***

**T:** ***How many times would you press*** $\boxed{=}$ ***to get 9 960?***

**S:** ***10 times, because 10*** x ***996 = 9 960.***

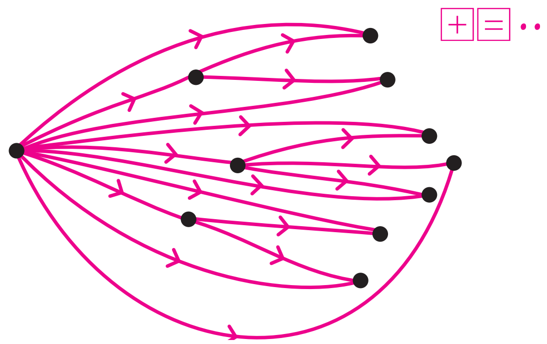
**T:** ***If we press*** $\boxed{+}$ ***and then*** $\boxed{=}$ ***one hundred times, what number would this dot (j) be for?***

**S:** ***99 600.***

Remove all the numbers from the picture.

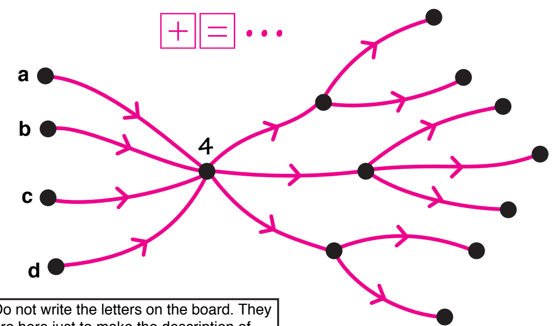
**T:** ***There are more red arrows that we can surely draw. Who can trace one of these arrows in our picture?***

Invite several students to the board to trace arrows. There are many possibilities. This picture shows all the arrows that we are certain can be added to the picture.



Put 4 back in the picture and expand it by drawing several arrows ending at 4.

**T:** ***What numbers could be here*** (at **a** or **b** or **c** or **d**)***?***




Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

1 and 2 are likely responses to this question. In any case, label **a** and **b** with two correct responses. Suppose a student suggests 0.5.

**T:** ***Let's check 0.5. Put 0.5 on the display and press*** $\boxed{+}$ $\boxed{=}$ $\boxed{=}$ ***and so on. Does 4 eventually appear on the display?***
***How many times did you press*** $\boxed{=}$ ***to see 4?***

**S:** ***Eight times, because 8*** x ***0.5 = 4.***

**S:** ***0.25 works.***

**T:** ***Put 0.25 on the display and press*** $\boxed{+}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$. ***What number is on the display?***

**S:** ***1.***

**T:** ***How many times 0.25 is 4?***

**S:** ***Sixteen times.***

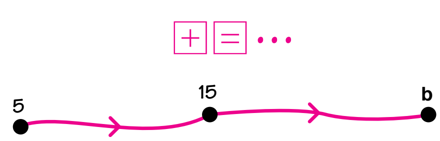
If students have difficulty with this question, ask,

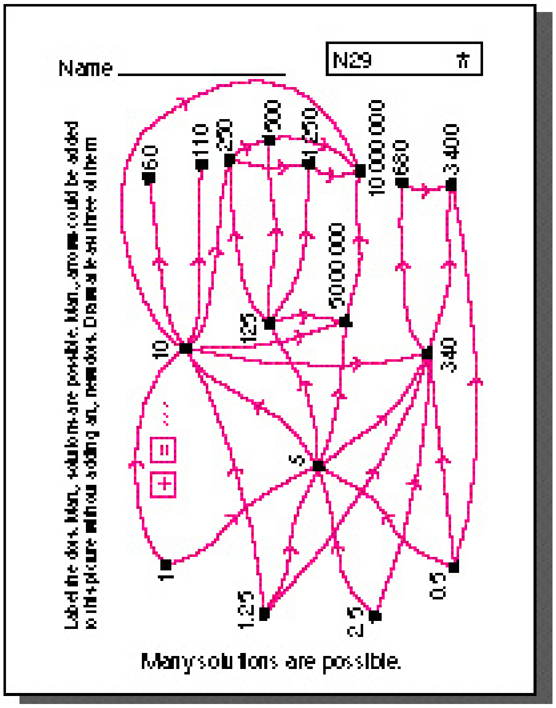
**T:** ***If we press*** $\boxed{=}$ ***four more times, what number will be on the display?*** (2)
***And if we press*** $\boxed{=}$ ***four more times?*** (3)
***And if we press*** $\boxed{=}$ ***four more times?*** (4)
***How many times 0.25 is 4?*** (16)

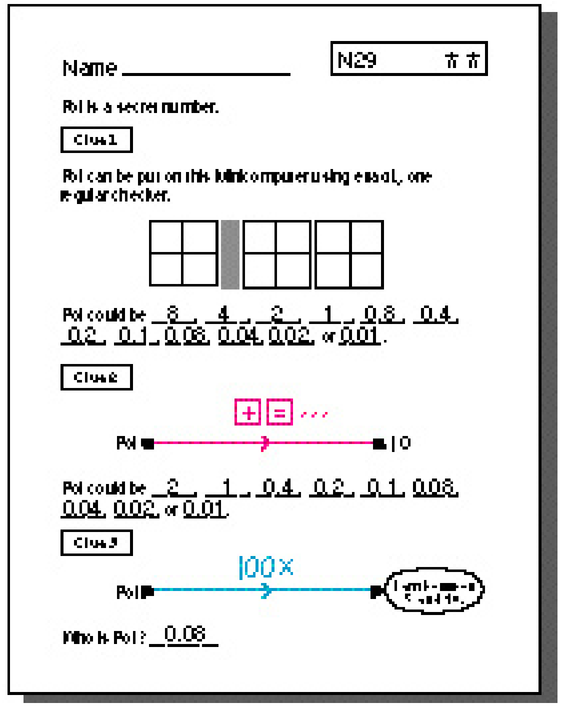
Continue in this manner with one or two more examples of decimal numbers that could be at **c** or **d**.

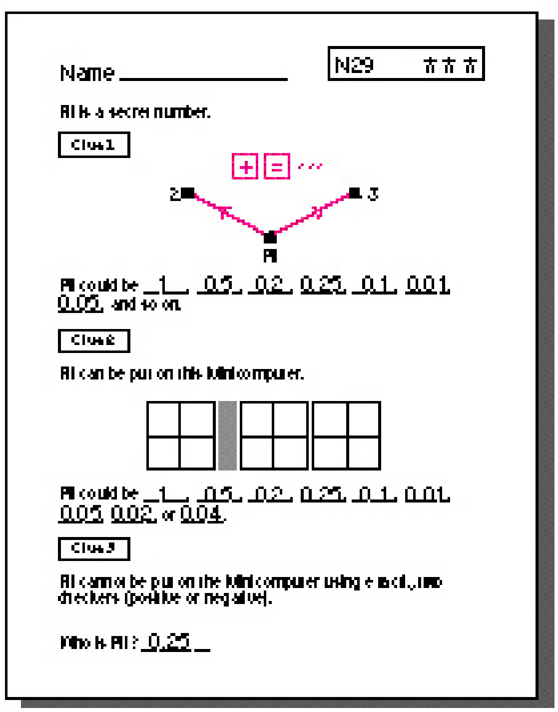
Worksheets N29 *, **, and *** are available for individual work.

**Note:** A common error students make labeling dots with the $\boxed{+}$ $\boxed{=}$ … relation is to forget that the starting number changes when we go from arrow to arrow. For example, after putting 15 at the end of the first arrow a frequent incorrect response for **b** is 20. But, **b** must be a multiple of 15.




Name
N29 ☆
Label the dots. Many solutions are possible. Many arrows could be added to this picture without adding any new dots. Draw at least three of them.
60
110
220
500
1,250
10 000 000
680
3 400
5 000 000
125
10
340
5
1
1.25
2.5
0.5
Many solutions are possible.


Name
N29 ☆☆
Pol is a secret number.
Clue 1
Pol can be put on this Minicomputer using exactly one regular checker.
Pol could be 8, 4, 2, 1, 0.8, 0.4, 0.2, 0.1, 0.08, 0.04, 0.02, or 0.01.
Clue 2
Pol
10
Pol could be 2, 1, 0.4, 0.2, 0.1, 0.08, 0.04, 0.02, or 0.01.
Clue 3
100x
Pol
Who is Pol? 0.08


Name
N29 ☆☆☆
Pil is a secret number.
Clue 1
2
3
Pil
Pil could be 1, 0.5, 0.2, 0.25, 0.1, 0.01, 0.05, and so on.
Clue 2
Pil can be put on this Minicomputer.
Pil could be 1, 0.5, 0.2, 0.25, 0.1, 0.01, 0.05, 0.02, or 0.04.
Clue 3
Pil cannot be put on the Minicomputer using exactly two checkers (positive or negative).
Who is Pil? 0.25

## Capsule Lesson Summary

As a class, count forward and backward by fourths. Use a number line model to locate fractions between two consecutive whole numbers. Find the greater of two fractions using measurement. Given the location of two numbers on a number line, make measurements to locate a third number.

### Materials

**Teacher**
- Colored chalk
- Meter stick
- Blackline N30 (optional)

**Student**
- Number lines (optional)
- Worksheets N30*, **, and ***
- Metric ruler

**Advance Preparation:** If you choose, use Blackline N30 to make student copies of number lines for Exercise 3.

## Description of Lesson

### Exercise 1

Explain to the class how you are going to count around the class by fourths.

**T:** ***I'll start the count with 0, $\frac{1}{4}$. Then each of you will give exactly one response (say the next number). What number do you think we will end at?***

Let students explain their answers. Do not acknowledge correct answers yet.

**T:** ***Let's count by fourths. I'll start: 0, $\frac{1}{4}$.***

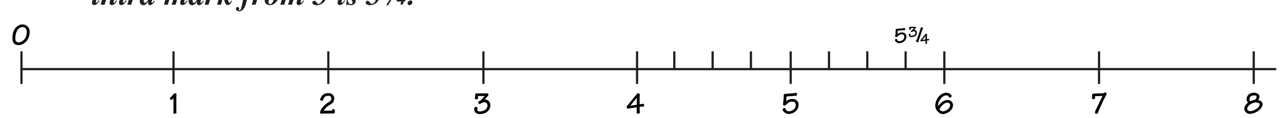
Call on students in an order natural to their seating arrangement. They should continue with $\frac{2}{4}$, $\frac{3}{4}$, … until each has responded exactly once. As appropriate, discuss alternative names for the same number, for example, $\frac{4}{4}$ and 1. On the board, write the last response and compare it to the students' predictions.

Draw a number line on the board with marks for whole numbers from 0 to 8. Make the distance between marks a length easily divisible by 4 (for example, 20 cm or 40 cm)

The following dialogue assumes that the last response was $5\frac{3}{4}$. Ask a student to locate $5\frac{3}{4}$ on the number line.

**S:** ***$5\frac{3}{4}$ is between 5 and 6.***

**S:** ***Use a ruler to divide the line segment between 5 and 6 into four parts of equal length. The third mark from 5 is $5\frac{3}{4}$.***




**T:** ***Let's count backward by fourths from $5\frac{3}{4}$. I'll call on you in the reverse order from when we counted forward. When it is your turn, state your number and tell how to find it on the number line.***

# N30

**S:** ***5¾.***

**S:** ***5²⁄4. It is at the mark just to the left of 5¾.***

**T:** ***What is another name for 5²⁄4?***

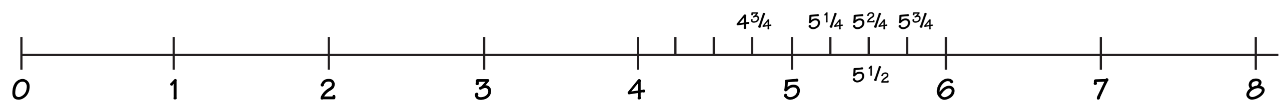
**S:** ***5½; it is halfway between 5 and 6.***

**S:** ***5¼; the next small mark to the left is 5¼.***

**S:** ***5; it's already labeled.***

**S:** ***4¾; divide the segment between 4 and 5 into four parts of equal length. The third mark is 4¾.***

Follow the students' instructions.




Let students continue to count backward. Label each number on the number line as instructed by your students. Students should notice that they are saying the same numbers they did when you were counting forward. The last student should say "²⁄4" (or ½) and you should say "¼, 0."

Erase all of the fractions and their marks on the number line. Put the following information on the board.

$$\frac{5}{2} \quad \frac{14}{3} \quad \frac{15}{4} \quad \frac{6}{7} \quad \frac{13}{8}$$

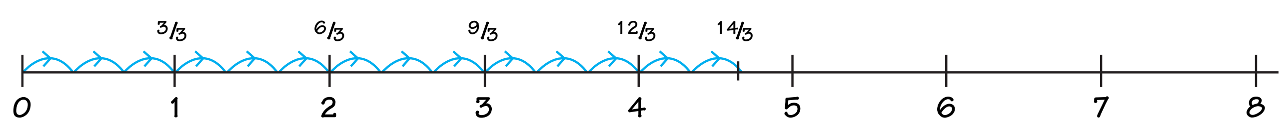
$$0 < \boxed{\phantom{00}} < 1 < \boxed{\phantom{00}} < 2 < \boxed{\phantom{00}} < 3 < \boxed{\phantom{00}} < 4 < \boxed{\phantom{00}} < 5$$

**T:** ***There are five fractions and five boxes. There is one fraction for each box. Select a fraction; tell me which box it goes in and why it goes there.***

Suppose a student chooses to locate ¹⁴⁄3 first.

**S:** ***¹⁴⁄3 is between 4 and 5. Draw three steps from 0 to 1, three more steps from 1 to 2, three more steps from 2 to 3, and three more steps from 3 to 4. That makes 12 steps; we need to go 14 steps, so two more steps. ¹⁴⁄3 is between 4 and 5.***

Draw the steps as described by the student. You need not be too careful in measuring the steps. Point out that ³⁄3 = 1, ⁶⁄3 = 2, ⁹⁄3 = 3, and ¹²⁄3 = 4.




In a similar manner, let students fill in the other boxes. They may use the number line to check their answers.

$$0 < \boxed{\frac{6}{7}} < 1 < \boxed{\frac{13}{8}} < 2 < \boxed{\frac{5}{2}} < 3 < \boxed{\frac{15}{4}} < 4 < \boxed{\frac{14}{3}} < 5$$

Refer to these answers and comment that neither the numerator alone nor the denominator alone determines order among fractions.

Erase the board before going on to Exercise 2.

## Exercise 2

Write these problems on the board. Invite students to put the appropriate symbol(<, =, or >) in each box to make true number sentences.

**Note:** If you prefer, start with a couple integer problems such as $2 < 8$ and $0 > \widehat{18}$ to review the symbols.

$<$ or $=$ or $>$

| | | | |
|---|---|---|---|
| $\frac{2}{7}\ \square\ \frac{2}{5}$ | | $\frac{9}{5}\ \square\ \frac{10}{7}$ | |
| $\frac{2}{5}\ \square\ \frac{6}{7}$ | | $2\ \square\ \frac{14}{7}$ | |
| $\frac{3}{7}\ \square\ \frac{2}{5}$ | | $\frac{8}{7}\ \square\ \frac{6}{5}$ | |

Let students discuss and explain answers to the problems. At this time do not confirm or deny answers. For each problem write the answer the class agrees upon (vote if necessary) regardless of whether it is right or wrong. Tell the class that in a moment you will show them how to check answers.

Draw a number line on the board with marks for 0, 1, and 2. Make the distance between marks 70 centimeters.

**T:** ***We'll use this number line to check our answers.***

***Remember the story of Sasquatch? Sasquatch takes one big step from 0 to 1 and another from 1 to 2, but the children take smaller steps.***

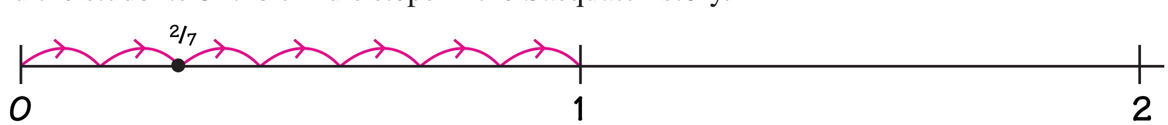
***Let's first check the problem by comparing $2/7$ and $2/5$. How can we locate $2/7$ on the number line?***

**S:** ***Divide the segment from 0 to 1 into seven equal parts. The second mark is for $2/7$.***

**T:** ***The distance from 0 to 1 is 70 cm, so how long should each segment be?***

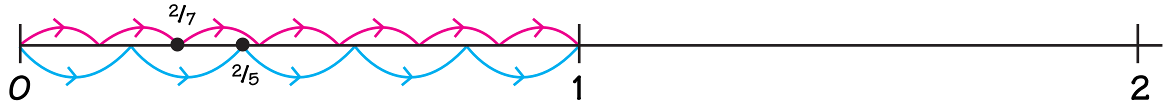
**S:** ***10 cm; 7 x 10 = 70.***

Following your students' instructions, use a meter stick to accurately locate $2/7$. Draw arrows to remind the students of the child's steps in the Sasquatch story.



**T:** ***Now let's locate ⅖ on the same number line.***

In a similar manner, let students explain how to use a meter stick to locate ⅖.




**T:** ***Which is greater, ⅖ or 2/7?***

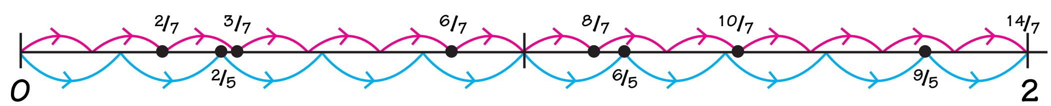
**S:** ***⅖, since ⅖ is to the right of 2/7 on the number line.***

**T:** ***Did we complete this number sentence correctly?***

$\frac{2}{7}\ \boxed{<}\ \frac{2}{5}$

You may want to compare this number line method of comparing fractions to other methods suggested earlier by students. For example, using pies or cakes, students may have reasoned that since ⅐ of a pie is smaller than ⅕ of a pie, then 2/7 of a pie is smaller than ⅖ of a pie.

Check the other problems in a similar manner. Extend the steps (arrows) when necessary. (Answers are given below.)




| | |
|---|---|
| $\frac{2}{7}\ \boxed{<}\ \frac{2}{5}$ | $\frac{9}{5}\ \boxed{>}\ \frac{10}{7}$ |
| $\frac{2}{5}\ \boxed{<}\ \frac{6}{7}$ | $2\ \boxed{=}\ \frac{14}{7}$ |
| $\frac{3}{7}\ \boxed{>}\ \frac{2}{5}$ | $\frac{8}{7}\ \boxed{<}\ \frac{6}{5}$ |

## Exercise 3

Draw a line on the board and locate marks for 0 and 1 about 30 cm apart.

**Note:** You may prefer to do this exercise with all students having copies of the lines. In this case, make copies of Blackline N30 for students and adjust the measurements in the dialogue accordingly.

**T:** ***How could we locate ⅗ on this number line?***

**S:** ***Measure the distance between 0 and 1. Divide that segment into five parts of equal length. The third mark is ⅗.***

**T:** ***So if we know the location of 0 and 1, we can locate ⅗.***

Replace 1 with 2/7.


**T:** ***Suppose we know the location of 0 and 2/7; how can we find 1?***

**S:** ***Measure the distance from 0 to 2/7 and divide it into two equal parts.***

**T:** ***Why two parts?***

**S:** ***In the Sasquatch story, 2/7 is the second step of a child taking seven steps to reach 1.***

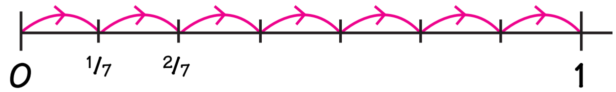
Measure the distance between 0 and $\frac{2}{7}$.

**T:** ***The distance from 0 to $\frac{2}{7}$ is 30 cm. Where should we make a mark for $\frac{1}{7}$?***

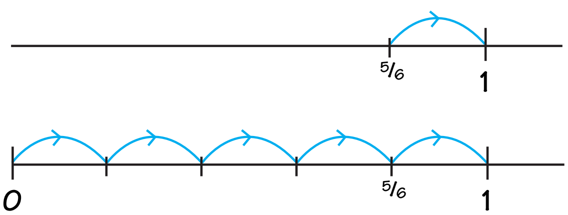
**S:** ***15 cm from 0, since $\frac{1}{2} \times 30 = 15$.***

**T:** ***How do we now locate 1?***

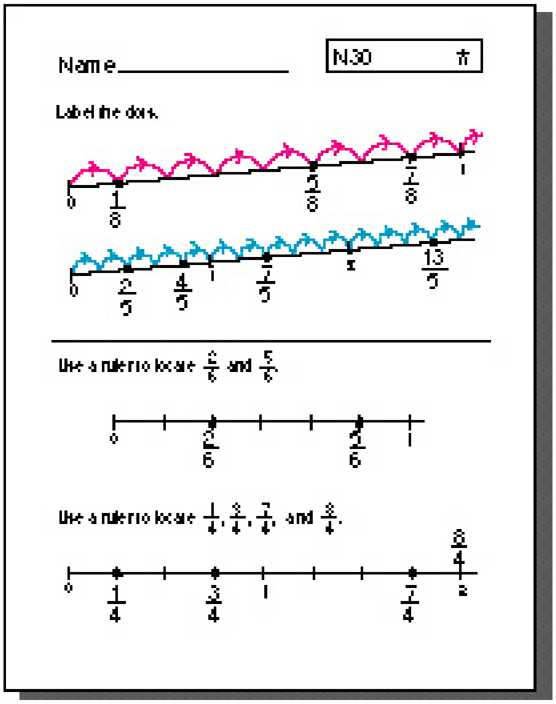
**S:** ***Continue on five more 15 cm-steps beyond $\frac{2}{7}$. Then we'll be at $\frac{7}{7}$ or 1.***

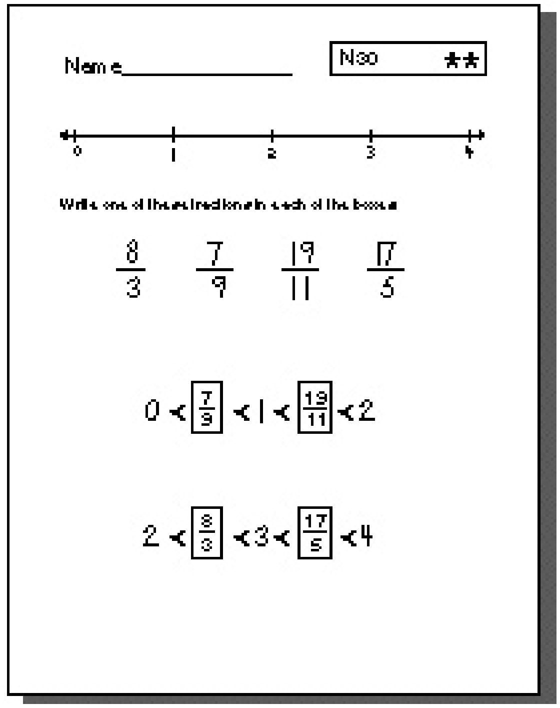


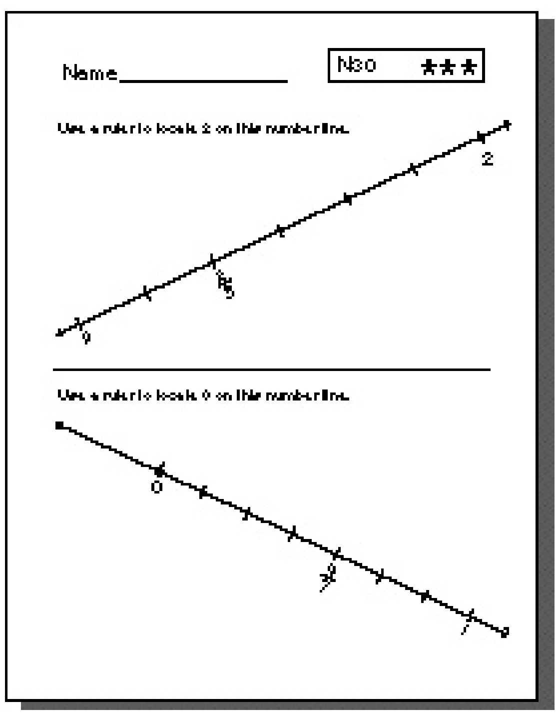

Repeat the exercise by locating 0 when you know the location of $\frac{5}{6}$ and 1.




Worksheets N30*, **, and *** are available for individual work.


Name
N30
Label the dots.
Use a ruler to locate 2/6 and 5/6.
Use a ruler to locate 1/4, 3/4, 7/4, and 8/4.


Name
N30
8/3 7/9 19/11 17/5
0 < 7/9 < 1 < 19/11 < 2
2 < 8/3 < 3 < 17/5 < 4


Name
N30
Use a ruler to locate 2 on this number line.
Use a ruler to locate 0 on this number line.

## Capsule Lesson Summary

Review the rule of a binary abacus, and decode numbers on the abacus using this rule. Determine values for the boards of a binary abacus both to the left and to the right of the bar, and put numbers on the binary abacus. Examine the relationship between a binary representation of a number and the subsets of a given set.

### Materials

**Teacher**
- Colored chalk
- Minicomputer checkers
- Blacklines N31(a) and (b)

**Student**
- Binary abacus
- Worksheets N31*, **, and ***

**Advance Preparation:** Use Blacklines N31(a) and (b) to make copies of a binary abacus for students to use in Exercises 2 and 3.

## Description of Lesson

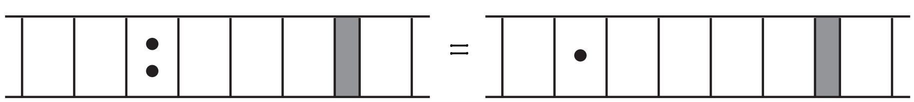
### Exercise 1

On the chalkboard, draw part of a binary abacus with eight to ten places represented.

**Note:** If your chalkboard is magnetic, use Minicomputer checkers on the abacus. Otherwise, draw and erase checkers as needed.

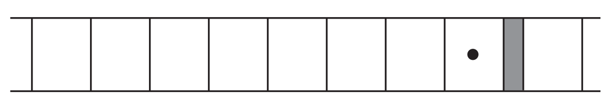
**T:** ***Do you remember the binary snake, or binary abacus, in* Clinton Street*? For a binary abacus, there is only one rule:***

Make this trade and write the rule on the board.



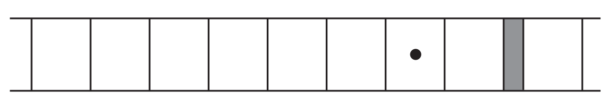
Two checkers on a board
trade for
one checker on the next board to the left.

Put 1 on the abacus.



**T:** ***This is the number 1.***

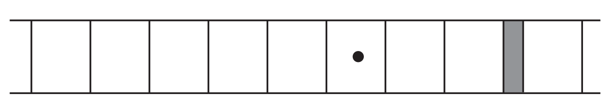
Move the checker one board to the left.



**T:** ***What number is this?*** (2)

***Suppose I move the checker one board to the left again.***

Show the move on the abacus.



**S:** ***4.***

**T:** ***And again.***

**S:** ***8.***

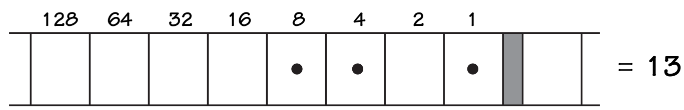
**T:** ***Again.***

**S:** ***16.***

Continue with 32, 64, 128, 256, 512, 1 024, … . Indicate to the students that the binary abacus goes on and on, and label the boards.

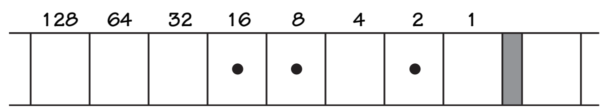
**T:** ***Who can put 13 on the binary abacus using at most one checker on each board?***

Ask a student to put on 13.




**T:** ***Good. Now I'll move each checker one board to the left.***

***What number is on the abacus now?***

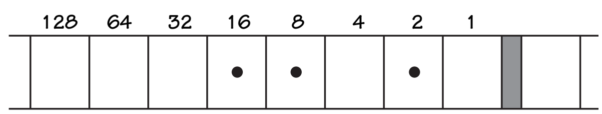



**S:** ***26.***

**T:** ***Why?***

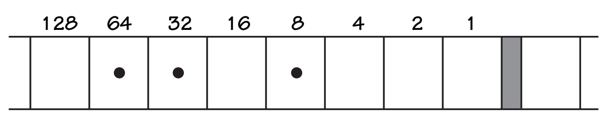
**S:** ***By moving all of the checkers one board to the left, you double the number.***

**T:** ***Suppose I move them one board to the left again?***




**S:** ***52.***

**T:** ***Again?***



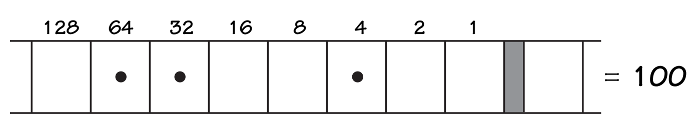

**S:** ***104.***

**T:** ***Check it to be sure.***

**S:** ***64 + 32 + 8 = 104.***

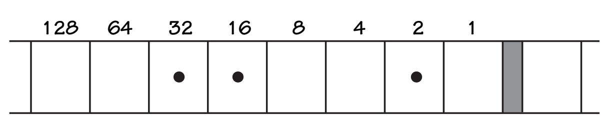
Clear the abacus.

**T:** ***Who can put on 100?***



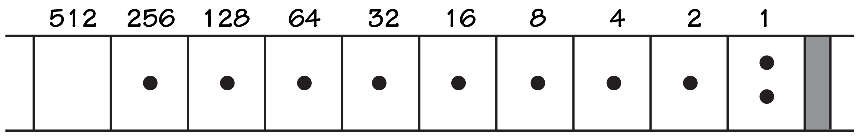

***I'll move each checker one board to the right. What number is on the abacus now?***

**S:** ***50, because moving the checkers one board to the right is the same as taking one-half of the number.***



## Exercise 2

Extend the places on your binary abacus, if necessary, and display this configuration.




**T:** ***What number is on the abacus?***

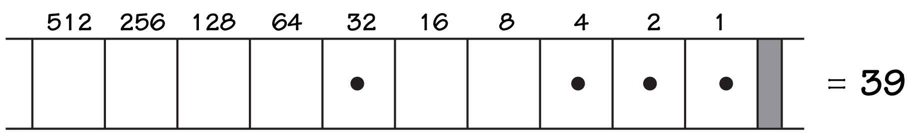
**S:** ***512.***

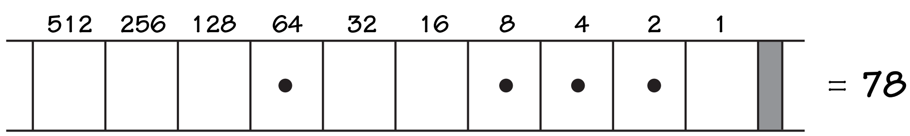
**T:** ***How did you get 512?***

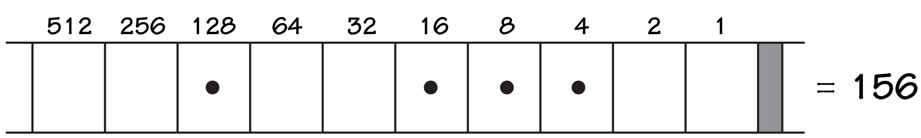
**S:** ***Trade two checkers on the 1-board for one checker on the 2-board. But then there are two checkers on the 2-board, which trade for one checker on the 4-board. But then there are two checkers on the 4-board, which trade for one on the 8-board. We can keep doing this until there are two checkers on the 256-board, which trade for one checker on the 512-board.***

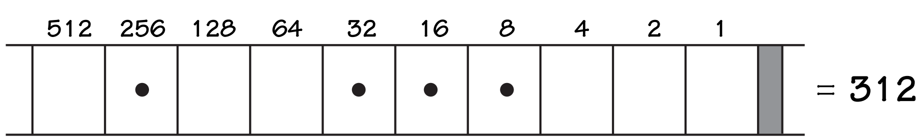
Distribute copies of a binary abacus, or ask students to draw a binary abacus on their papers. Suggest they label the boards.

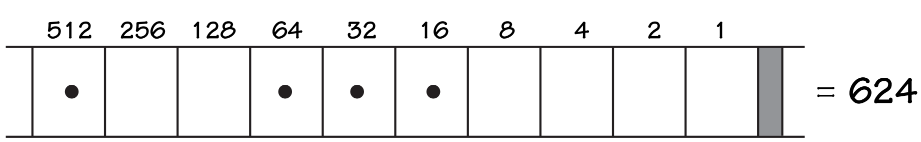
Then ask students to put numbers on the binary abacus; each time call on one student to do it at the board. For example:



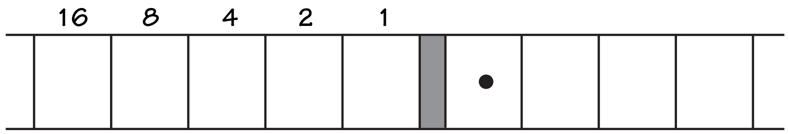



# N31

### Exercise 3

Extend the places on your binary abacus to the right. Then put on this configuration. With students, observe that the checker is to the right of the bar. Direct students to write the number on their papers.

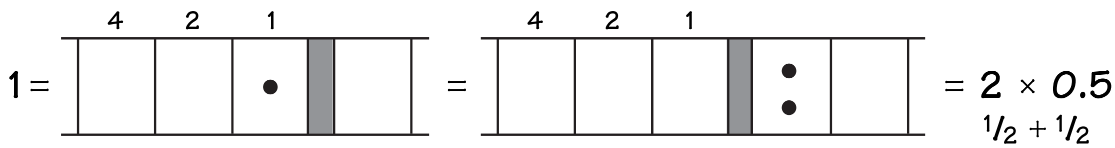



**T:** ***What number is this?***

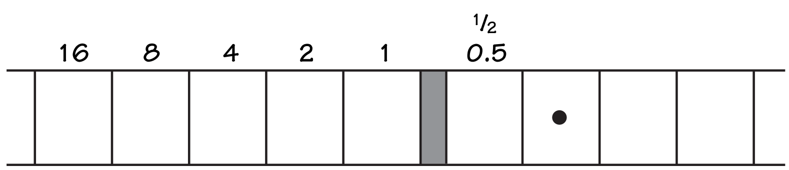
**S:** ***0.5.***

**S:** ***½.***

If some students suggest 0.8, 0.1, or 0.2, remind them that we are working with a binary abacus and not with the Minicomputer or a decimal system. Make the following trade to convince the class that the value of a checker on that board is 0.5 or ½.



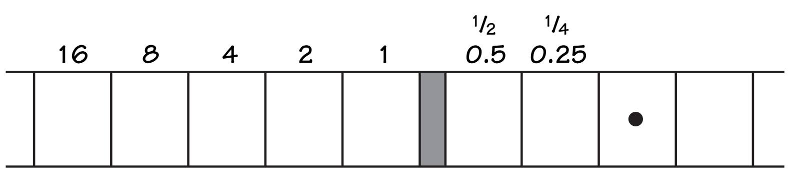

Label the board and move the checker one board to the right.




**T:** ***What number is this?***

**S:** ***0.25 or ¼.***

Label the board and again move the checker one board to the right.



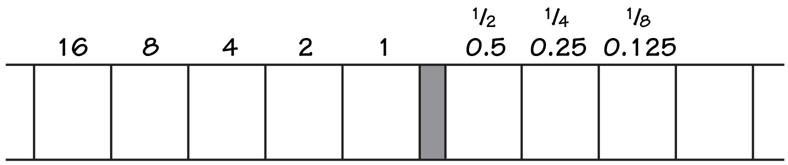

**T:** ***What number is this?***

**S:** ***One half of 0.25.***

**S:** ***0.25 = 0.250 and ½* x *0.250 = 0.125. It is 0.125.***

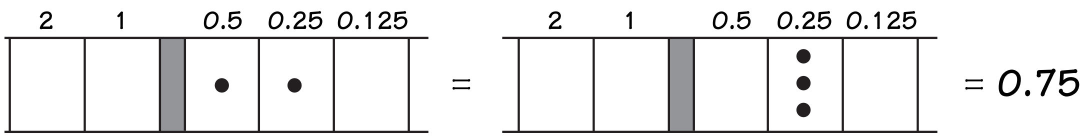
**S:** ***⅛; ½* x *¼ = ⅛.***

Add these labels to your drawing.




Distribute copies of a binary abacus with places mostly to the right of the bar, or ask students to draw one on their papers. Suggest they label several boards right and left of the bar. Then ask students to put 0.75 on the binary abacus.

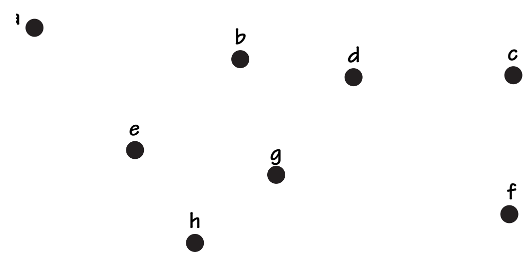
After a minute or two, let a student put 0.75 on the binary abacus on the board. There are two likely responses.




Accept any correct display. The configuration on the left is standard because it uses at most one checker on each board.

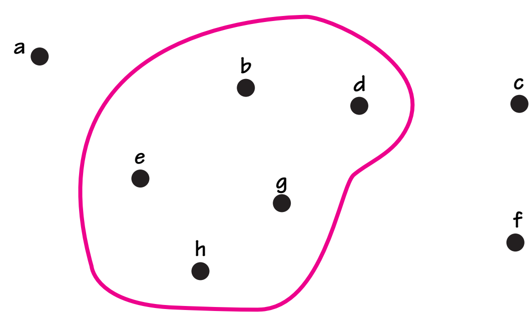
## Exercise 4

Draw and label eight dots on the chalkboard.




**T:** ***I want to tell you a story about Nabu. Nabu has eight cousins. Each cousin has a dot and a letter in my picture. Every Sunday Nabu invites some of his cousins to his house. We will draw a string to show the cousins he invites. Show me the cousins that Nabu could invite next Sunday.***

Invite a student to draw a string around any set of dots in the picture. Suppose the student makes this choice.




**T:** ***Good. The red string indicates that next Sunday Nabu will invite cousins* b, d, e, g, *and* h. *On the following Sunday, Nabu again will invite some of his cousins, but he wants to follow this rule: Each Sunday Nabu invites a different set or combination of cousins.***

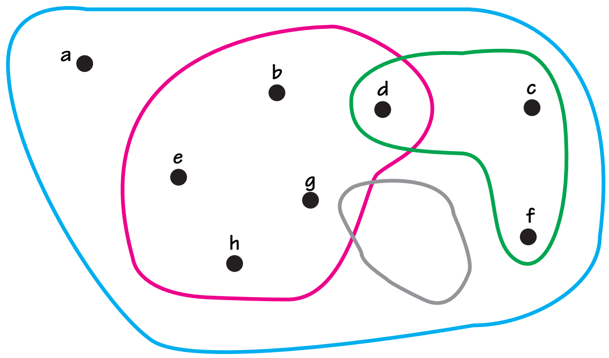
Write the rule on the board.

Each Sunday Nabu invites a different set or combination of cousins.

**T:** ***Show us a different set of cousins that Nabu could invite.***

# N31

Let students come to the board and draw different colored strings to represent different sets of cousins. There are many possibilities, but suppose your picture looks like this after several students give their choices.




**Note:** The gray string indicates that Nabu could invite no one. This set will be counted later in the lesson, but it may not be noticed by students.

**T:** ***Look at the red and green strings. On some Sunday Nabu could invite the cousins in the red string, and on another Sunday he could invite the cousins in the green string. His rule lets him invite the same cousins, but never the same set or combination of cousins.***

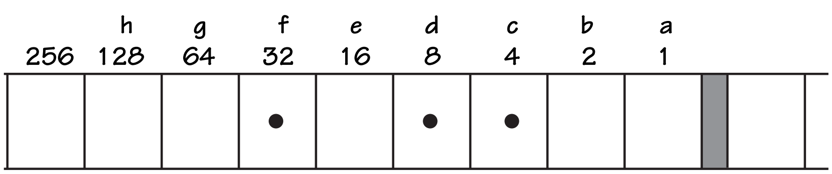
***How many Sundays will it take before Nabu invites all of the possible combinations of cousins?***

Accept estimates. You may want to list students' guesses on the board for the sake of comparison later.

**T:** ***To solve this problem, Nabu uses the binary abacus.***

Label the boards of the binary abacus as shown below. Then refer to one set (string) given above.

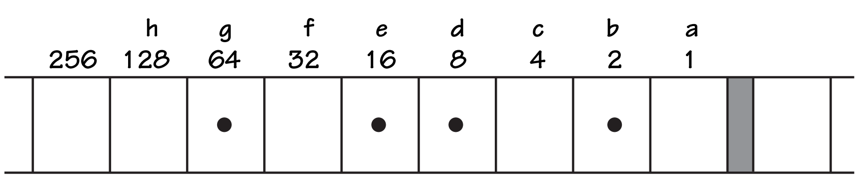
**T:** ***Cousins d, c, and f came over one Sunday. Nabu put three checkers on the abacus and wrote 44 in his notebook. Why do you suppose he wrote 44?***




**S:** ***That's the number on the abacus. 32 + 18 + 4 = 44.***

**T:** ***Another Sunday he wrote 90 in his notebook. Do you know which cousins he invited?***

Invite a student to put the number 90 on the abacus. If more than one checker is on a board, suggest that the student make a trade. You may comment that Nabu cannot invite two of the same cousin.

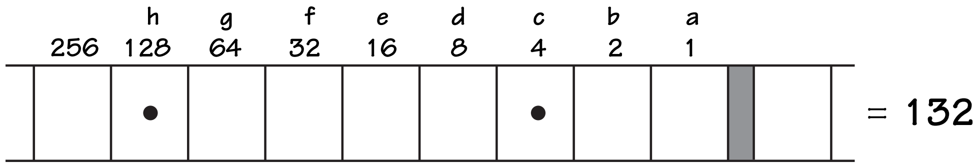



= 90

**S:** ***He invited* g*,* e*,* d*, and* b*.***

**T:** ***On still another Sunday he invited* h *and* c*. What number did he write in his notebook that day?*** (132)

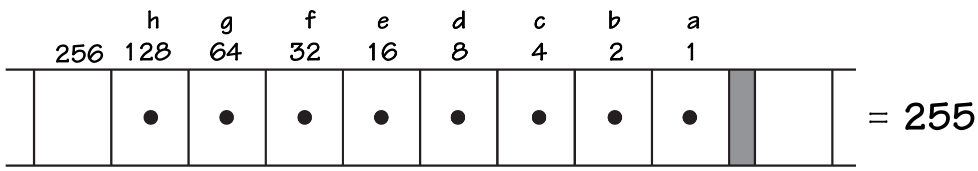
Ask a student to show the number on the abacus.



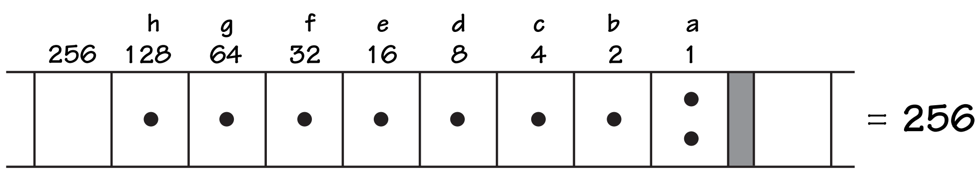

**T:** ***What is the greatest number he could write?***

**S:** ***The one for all his cousins—one checker on each board.***

**S:** ***255.***




This answer can be computed by addition. Another method, based on Exercise 2, is first to add one checker to the ones board. Then make trades to see that now the number is 256.




Since an extra checker is on the ones board, the greatest number Nabu could write is 255.

**T:** ***What is the least number Nabu could write?***

**S:** ***0, when he invites no one.***

**T:** ***How many different numbers could he write in his notebook?***

**S:** ***256 (0 through 255).***

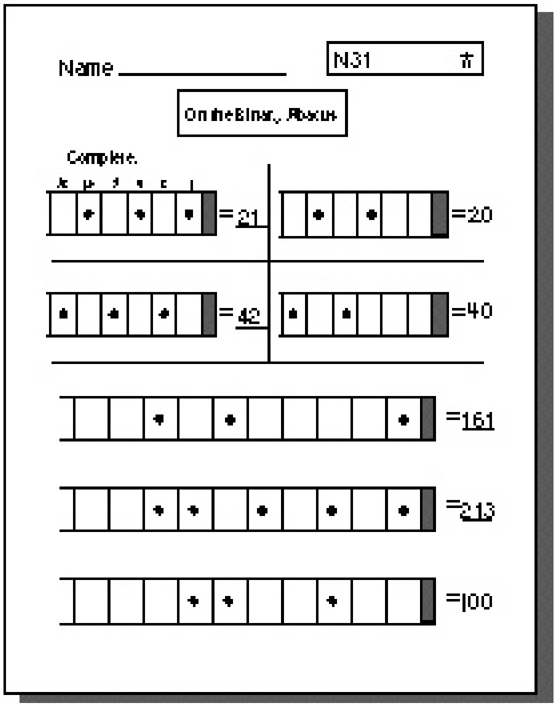
**T:** ***And how many Sundays?***

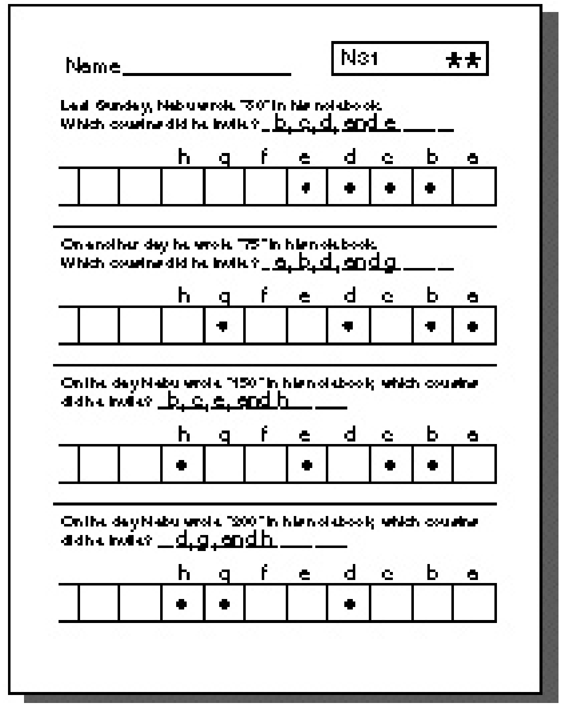
**S:** ***The same, 256.***

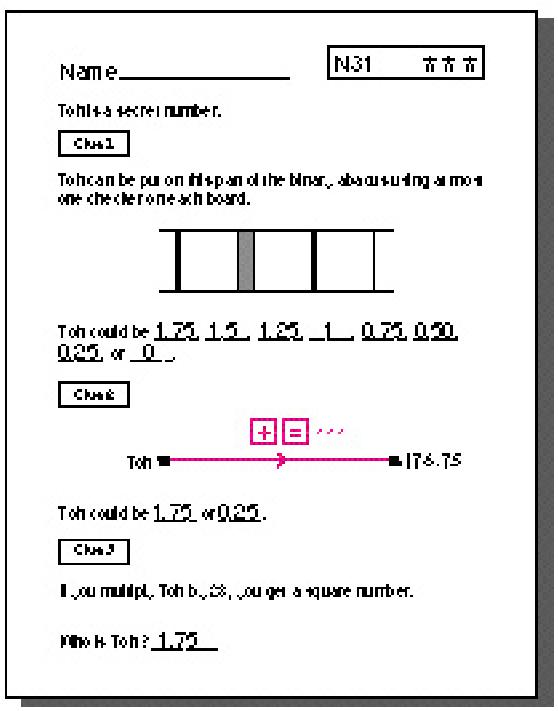
**T:** ***About how long (how many years) would it take Nabu to invite all of the possible combinations of his cousins?***

**S:** ***About five years, because there are 52 Sundays in a year.***

Worksheets N31*, **, and *** are available for individual work.


Name
N31 ☆
On the Binary Abacus
Complete.
=21
=20
=42
=40
=161
=213
=100


Name
N31 ★★
Last Sunday, Nabu wrote "30" in his notebook. Which counters did he invite? b, c, d, and e
h g f e d c b a
On another day he wrote "75" in his notebook. Which counters did he invite? a, b, d, and g
h g f e d c b a
On the day Nabu wrote "150" in his notebook, which counters did he invite? b, c, e, and h
h g f e d c b a
On the day Nabu wrote "200" in his notebook, which counters did he invite? d, g, and h
h g f e d c b a


Name
N31 ☆☆☆
Toh is a secret number.
Clue 1
Toh can be put on this part of the binary abacus using at most one checker on each board.
Toh could be 1.75, 1.5, 1.25, 1, 0.75, 0.50, 0.25, or 0.
Clue 2
+ = ...
Toh
176.75
Toh could be 1.75 or 0.25.
Clue 3
If you multiply Toh by 28, you get a square number.
Who is Toh? 1.75

# N32 TELEPHONE GAME #1

## Capsule Lesson Summary

Introduce a rule for linking numbers in a telephone network. Practice determining series of linkages and finding who can talk directly to whom, and through whom messages can be sent if necessary.

### Materials

| Teacher | | Student | |
|---|---|---|---|
| | • Colored chalk | | • Colored pencils, pens, or crayons<br>• Paper<br>• Worksheets N32*, **, ***, and **** |

## Description of Lesson

### Exercise 1

Begin the lesson with a few minutes of practice doubling, halving, adding 10, and subtracting 10.

List these four commands on the board.

$\boxed{\text{Double}}$ $\boxed{\text{Add 10}}$

$\boxed{\text{Halve}}$ $\boxed{\text{Subtract 10}}$

**T:** ***Pretend you are robots who know only integers. When I say a number (it will always be an integer because you only know integers) and point to one of the commands, you produce a new number from my number by following the command.***

**T:** *6.* (Point to $\boxed{\text{Double}}$.) **S:** ***12.***

**T:** *6.* (Point to $\boxed{\text{Halve}}$.) **S:** ***3.***

**T:** *8.* (Point to $\boxed{\text{Add 10}}$.) **S:** ***18.***

**T:** *8.* (Point to $\boxed{\text{Subtract 10}}$.) **S:** $\hat{2}$.

**T:** *7.* (Point to $\boxed{\text{Double}}$.) **S:** ***14.***

**T:** *7.* (Point to $\boxed{\text{Halve}}$.) **S:** ***3.5.***

**S:** ***Wait—we are robots who know only integers; 3.5 is not an integer.***

**T:** ***You're right. What would a robot or machine do if it could not follow instructions?***

**S:** ***It might turn off or break or just not respond.***

**T:** ***Our robots just say TILT when this occurs. When will you say TILT?***

**S:** ***We will say TILT when the result is not an integer; for example, if you say an odd number and point to*** $\boxed{\text{Halve}}$.

Continue quickly with 15 to 20 other integers. Mix your choices of odd and even numbers and your choices of positive and negative integers.

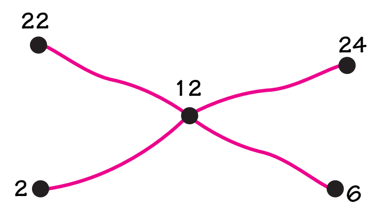
## Exercise 2

Draw a red cord on the board next to the list of commands.

| | |
|---|---|
| Double | Add 10 |
| Halve | Subtract 10 |


**T:** ***The telephone system in the World of Integers is constructed in a peculiar way. Not every number can call every other number. There is a rule for connections, and we will draw maps to show some of the connections. The rule involves these four commands: one number can call another if a robot can get one number from the other by using any one of the four commands. Who can 12 call?***

Students should find four numbers that 12 can call. Draw a picture to include all four.



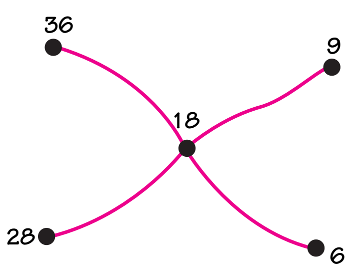

**S:** ***12 can call 24, since the double of 12 is 24.***

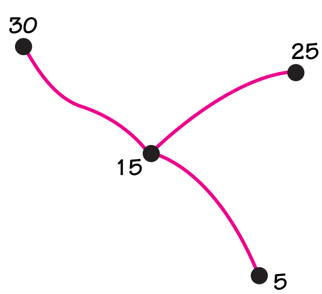
**S:** ***12 can call 6, since half of 12 is 6.***

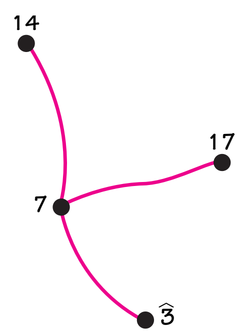
**S:** ***12 can call 22, since 12 plus 10 is 22.***

**S:** ***12 can call 2, since 12 minus 10 is 2.***

Repeat this activity to find the numbers that 18, 15, and 7 can call. If students suggest that 15 can call 7.5, remind them that in the World of Integers robots know only integers.




**T:** ***Find other numbers like 2 and 18 that can call four friends; find other numbers like 15 and 7 that can call only three friends.***

Suggest that students explore and draw pictures of the four or three calls numbers can make. Make a chart of solutions students present and verify. Perhaps your chart will look similar to the one below.

Four calls: 12, 18, 14, 24, 2, 100, ...
Only three calls: 15, 7, 17, 3, 21, 105, ...

Soon the class may observe that all odd integers can call three friends (correct) and that all even integers can call four friends (incorrect). In fact, there are four even integers (10, 20, $\widehat{10}$, and $\widehat{20}$) that can call only three friends, and one even integer (0) that can call only two friends. You might challenge the class to find these numbers. Simply say that there are some even numbers who cannot call four friends, but do not reveal their identities.

## Exercise 3

**T:** ***Tell me two numbers who cannot call each other.***

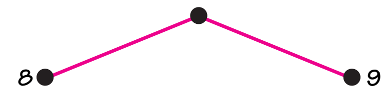
**S:** ***8 and 9 cannot. Neither is the double of the other, and their difference is not 10.***

**S:** ***5 and 30.***

**S:** ***100 and 2.***

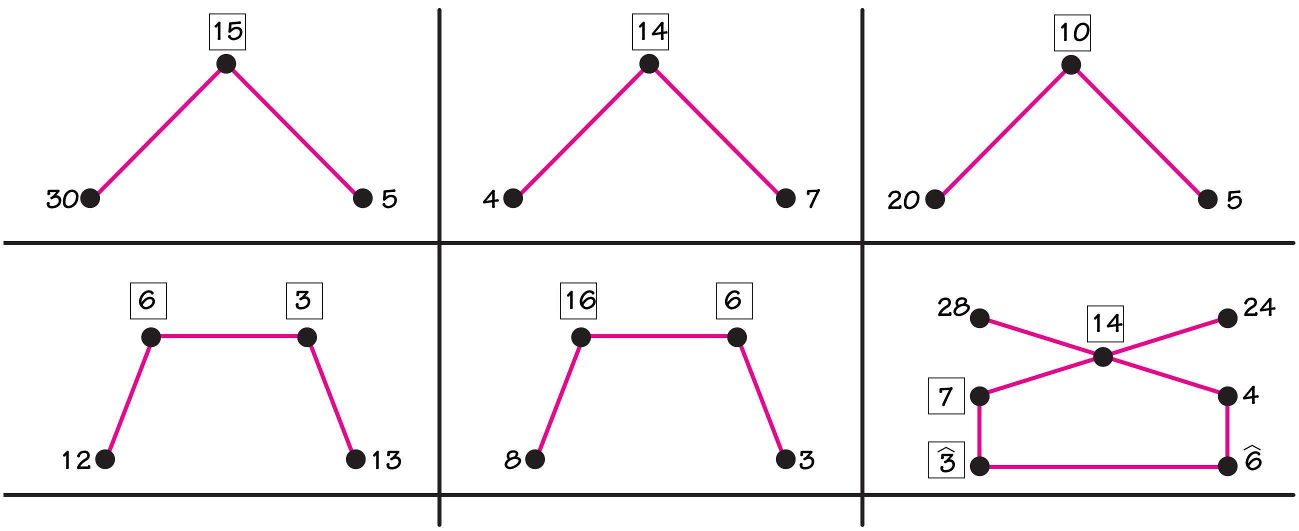
Draw this picture on the board.

**T:** ***8 must send a message to 9, but 8 cannot call 9. Who can help to carry the message?***

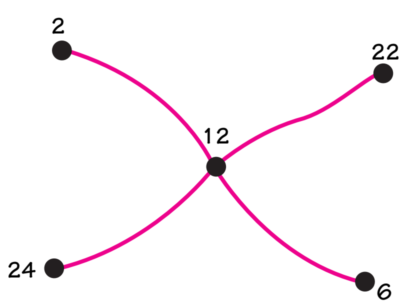


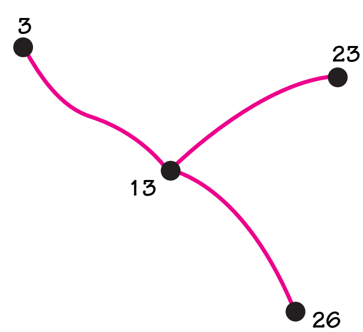

Let students solve this problem individually. Then invite a student to label the middle dot (18) on the board.

Continue in this manner with the following similar problems. (Solutions are in the boxes.)




Encourage students having difficulty to draw the maps showing all of the numbers that each of the given numbers can call and then to look for ways to make connections. For example, in the three-step connection between 12 and 13, first construct the maps showing the numbers that 12 and 13 can call.




Now the problem is to link one of the numbers 12 can call to one of the numbers 13 can call. This is accomplished by drawing a cord between 6 and 3.

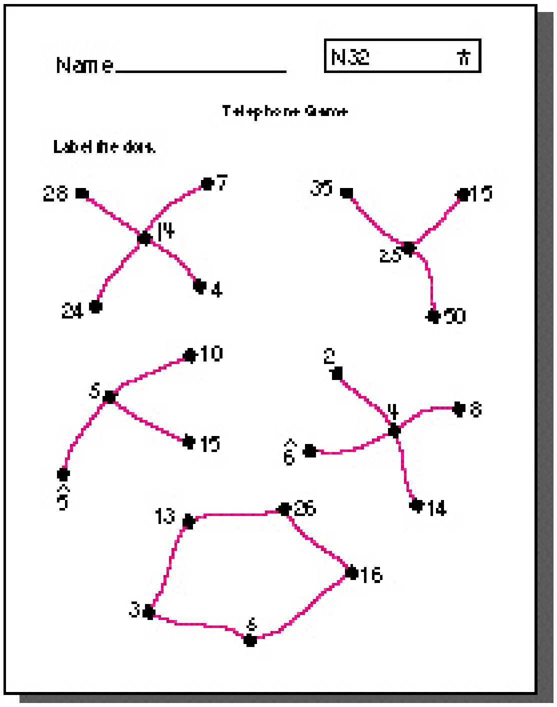
Lead a collective discussion of students' solutions to the problems. Emphasize the technique described above, if necessary.

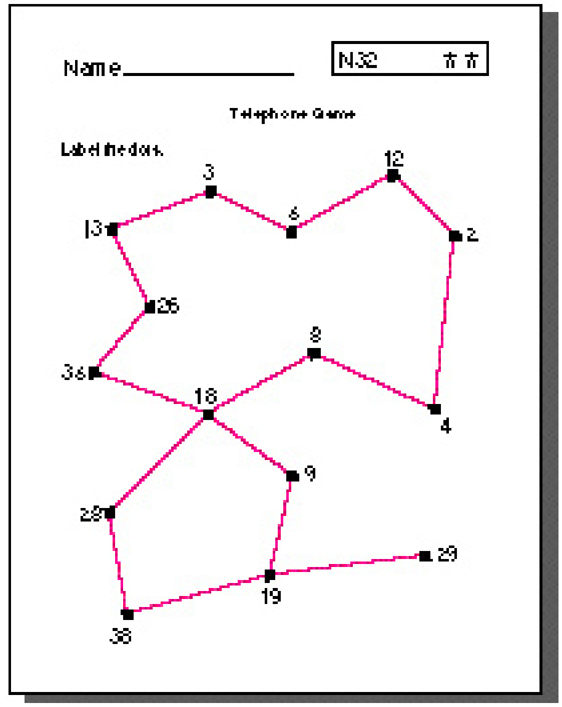
**T:** ***We can state the rule for making telephone calls in another way.***

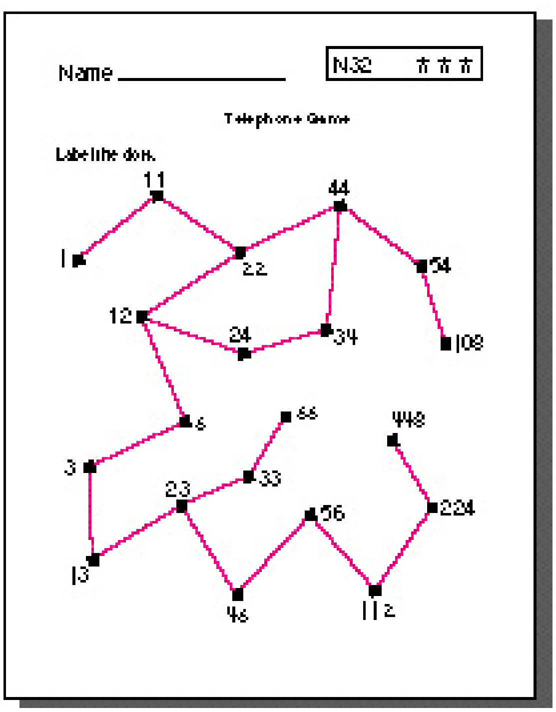
Say the following rule while writing it on the board.

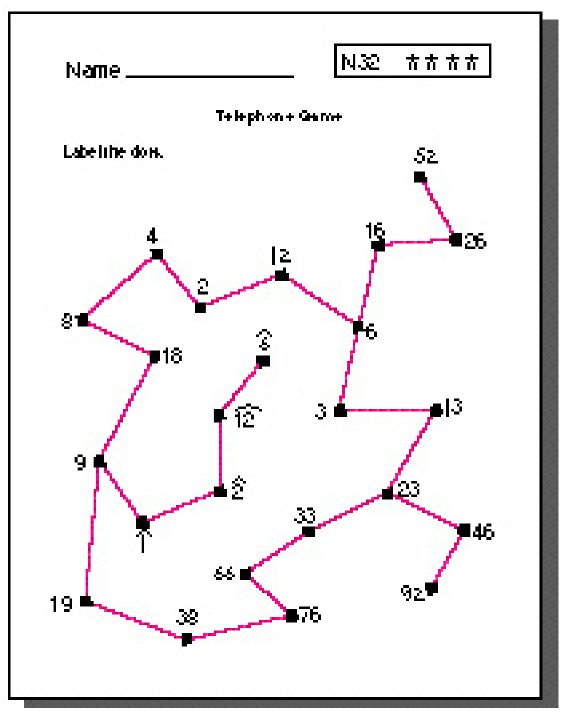
TWO INTEGERS CAN CALL EACH
OTHER BY PHONE
if and only if
ONE OF THEM IS THE DOUBLE
OF THE OTHER
or
ONE OF THEM IS 10 MORE
THAN THE OTHER

Worksheets N32*, **, ***, and **** are available for individual work.


Name
N32 ☆
Telephone Game
Label the dots.


Name
N32 ☆☆
Telephone Game
Label the dots.


Name
N32 ☆☆☆
Telephone Game
Label the dots.


Name
N32 ☆☆☆☆
Telephone Game
Label the dots.

## Capsule Lesson Summary

Continue examination of the telephone network in the World of Integers by considering more complex linkages among numbers.

### Materials

| Teacher | | Student | |
|---|---|---|---|
| | • Colored chalk | | • Colored pencils, pens, or crayons<br>• Worksheets N33(a) and (b) |

## Description of Lesson

### Exercise 1

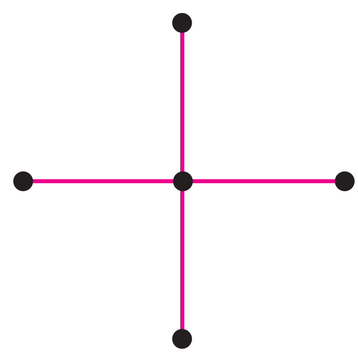
**T:** ***Do you remember the telephone game? What was the rule for making telephone calls?***

Encourage students to recall the telephone game and to explain the rule. Repeat the rule as you write it on the board.

TWO INTEGERS CAN CALL EACH OTHER BY PHONE
if and only if
ONE OF THEM IS THE DOUBLE OF THE OTHER
or
ONE OF THEM IS 10 MORE THAN THE OTHER

List the four commands from the rule and draw this cord picture on the board.

| | |
|---|---|
| Double | Add 10 |
| Halve | Subtract 10 |

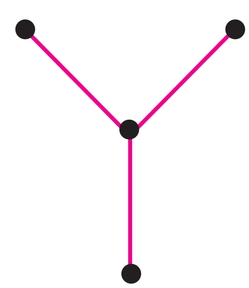


**T:** ***Find a number that can call four friends.***

Let students give many possibilities; for example, 40, 8, 2, 18, and so on.

Draw this picture on the board.

**T:** ***Find a number that can call only three friends.***



Let students give many possibilities; for example, 5, 7, 35, 20, 10, and so on.

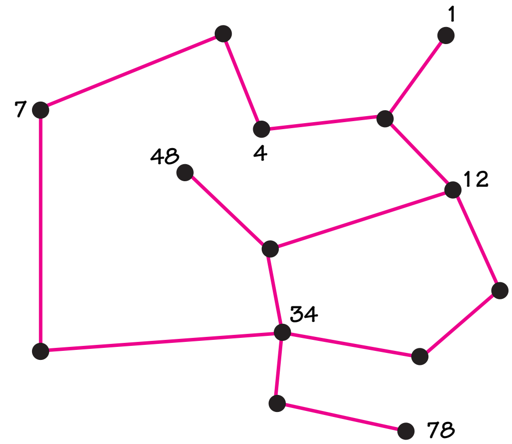
# N33

## Exercise 2

Announce to the class that you have a detective story about a secret number, Zip, and that the clues involve this telephone game. You may like to let students work in pairs on the clues.

Clue 1

Distribute copies of Worksheets N33(a) and (b). Let the class work on the first clue N33(a) for a few minutes. While students are working, copy the picture from the worksheet onto the board.




**T:** ***Zip is in this picture. Its dot is not labeled. Who can label a dot?***

Let students label the seven dots to discover that Zip could be 2, 14, 17, 22, 24, 44, or 68.

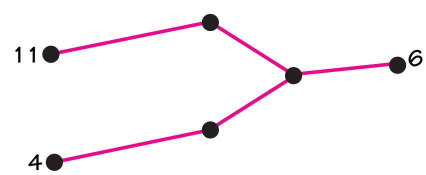
Clue 2

**T:** ***For the second clue, you need to know that a telephone call costs 10¢. Look at Worksheet N33(b) for the second clue. Try to find some of the possibilities for Zip.***

After a few minutes, bring the class together to solve the second clue collectively by drawing a map with all of the solutions. A picture showing all of the numbers that 8 can call for 20¢ is at the end of this lesson in the answer key. After comparing the numbers in the drawing with those in the list from the first clue, the class will discover that Zip could be 2 or 14.

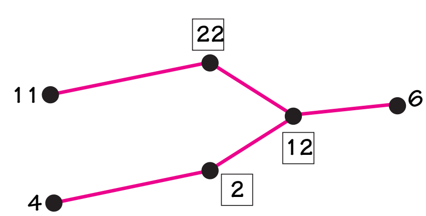
Clue 3

Draw this picture on the board.




**T:** ***Zip is in this picture but its dot is not labeled. Copy the picture on your paper, and try to label the dots.***

A completed picture is shown here with answers in boxes.



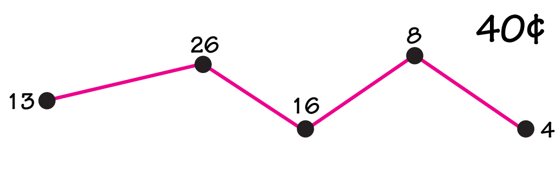

The class should discover that Zip is 2.

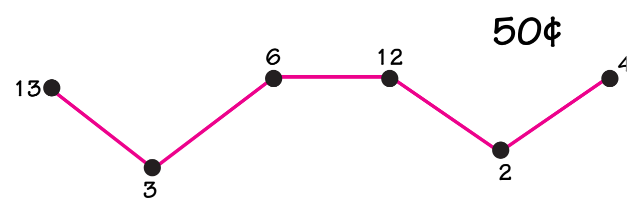
## Exercise 3

Draw dots for 13 and 4 on the board.

**T:** ***13 cannot call 4 directly. But with the help of some friends 13 can talk to 4. Who could the friends be? Try to find the least expensive way for 13 and 4 to talk to each other, remembering that it costs 10¢ to make a call.***

Let students work with partners for a short while on this problem. Since there are several solutions, ask a few students to put their solutions on the board. Two possible solutions are given below. The 40¢ solution is the least expensive.



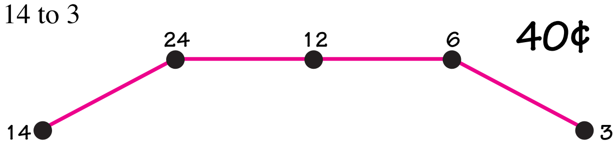




Now write these similar problems on the board for individual or partner work.

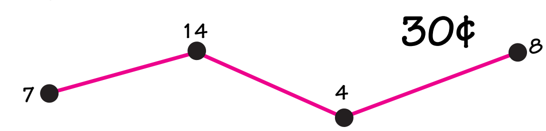
| $\boxed{*}$ | $\boxed{**}$ | $\boxed{***}$ |
|---|---|---|
| 14 to 3 | 16 to 17 | $\hat{5}$ to 50 |
| 7 to 8 | $\widehat{12}$ to 12 | $\widehat{13}$ to 13 |
| | | 15 to 16 |

As you monitor students' work, select a few solutions that they find to put on the board. Then challenge the class to find less expensive solutions, if possible. A least expensive solution to each problem is given below.

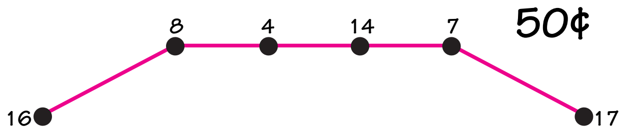
14 to 3



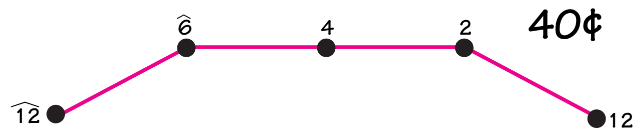

7 to 8



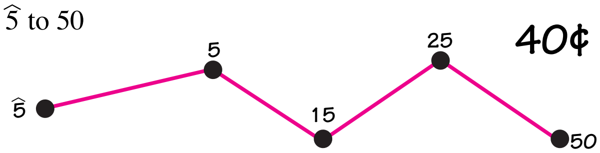

16 to 17



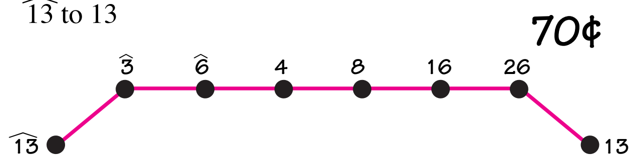

$\widehat{12}$ to 12




$\hat{5}$ to 50

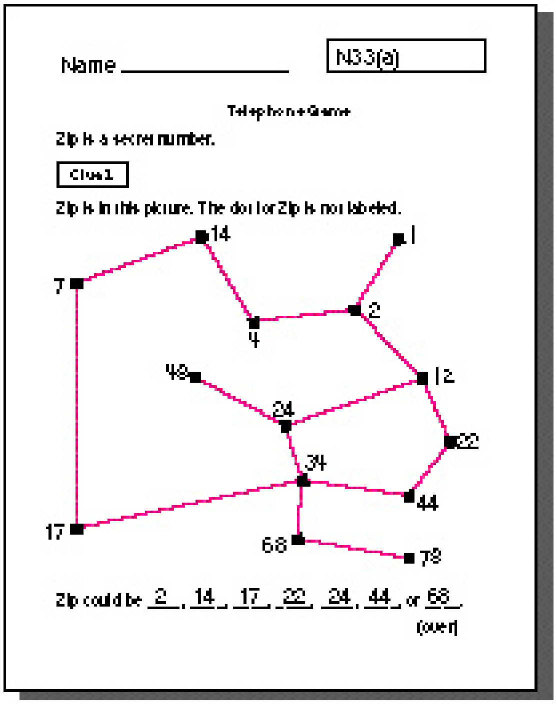


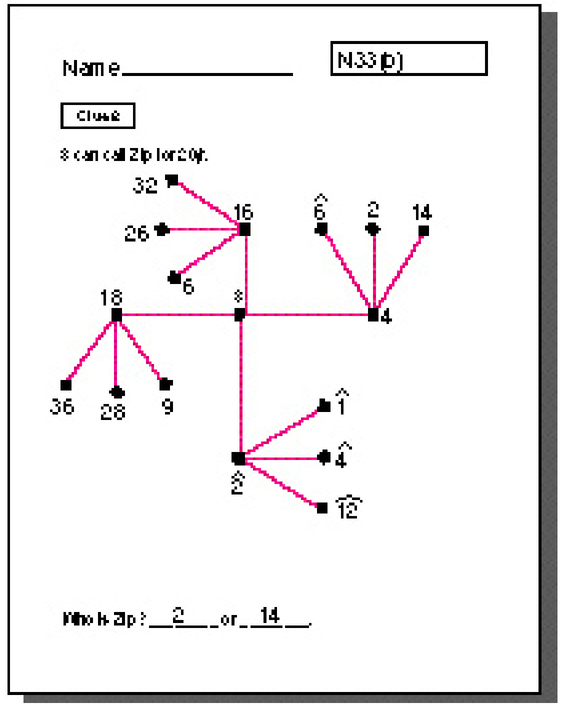

$\widehat{13}$ to 13




15 to 16

No solution. Each path from 15 leads to a number whose ones digit is 0 or 5. On the other hand, the ones digit of any number that 16 can call cannot be 0 or 5. Thus 15 cannot talk to 16.


Name
N33(a)
Telephone Game
Zip is a secret number.
Clue 1
Zip is in this picture. The dot for Zip is not labeled.
14
1
7
2
4
48
12
24
22
34
44
17
68
78
Zip could be 2, 14, 17, 22, 24, 44, or 68.
(over)


Name
N33(b)
Clue 2
8 can call Zip for 20¢.
32
16
26
6
2
14
6
18
8
4
36
28
9
1
4
2
12
Who is Zip? 2 or 14.

# N34 CALCULATOR RELATIONS #4

## Capsule Lesson Summary

Investigate how the calculator relation $\boxed{+}\ \boxed{5}\ \boxed{=}$ … partitions the set of integers into five subsets. Present a detective story in which the clues involve combining two calculator relations in an arrow picture, putting numbers on the Minicomputer with weighted checkers, and recognizing patterns in a +11 sequence.

## Materials

**Teacher**
- Calculator
- Colored chalk
- Minicomputer set
- Weighted checker set
- Paper
- Colored pencils, pens, or crayons
- Worksheets N34*, **, ***, and ****

**Student**
- Calculator

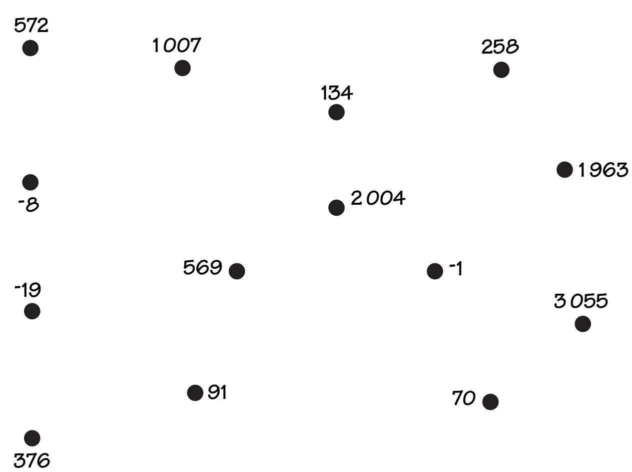
**Advance Preparation:** Prepare the picture for Exercise 1 ahead of time on the chalkboard, on an overhead transparency, or on an easel-size sheet of paper to tape onto the board. Place the dots carefully, as on Blackline N34 or as shown below, to eventually get an interesting arrow picture that suggests many patterns. Use Blackline N34 to make student copies of the picture for Exercise 1 if you wish all students to draw the arrows.

# Description of Lesson

### Exercise 1

Arrange that every student has access to a calculator for this lesson. Present this picture with carefully spaced dots.

$\boxed{+}\boxed{5}\boxed{=}$ …




**Note:** You may like every student to have a copy of the dot picture. Then you can ask students to draw arrows in their pictures as they are drawn in the class picture.

**T:** ***In this picture, we are going to draw as many red arrows for*** $\boxed{+}\ \boxed{5}\ \boxed{=}$ ***… as we can. For example, suppose we start at 572 and try to draw an arrow. Put 572 on the display of your calculator and press*** $\boxed{+}\ \boxed{5}\ \boxed{=}\ \boxed{=}$ ***and so on, as many times as you like. Watch the numbers that appear on the display. What do you notice?***

**S:** ***The ones digit of each number that appears is either 2 or 7.***

**T:** ***So we can draw a red arrow from 572 to what other number in our picture?***

**S:** ***1 007, because it has 7 in the ones place.***

Draw a red arrow from 572 to 1 007.

**T:** ***Where else can we draw red arrows?***

Encourage students to use calculators to explore the situation. Invite students to trace red arrows at the board before you draw them in the picture. Suppose a student traces an arrow from 569 to 2 004.

**T:** ***Let's see if it is correct. Put 569 on the display of your calculators. Press*** [+] [5] [=] ***. . . . Will 2 004 appear?***

**S:** ***Yes. The ones digit is always 4 or 9.***

**T:** ***So we can draw an arrow from 569 to 2 004.***

Students may be slower to suggest arrows starting at negative numbers. If necessary, ask for an arrow starting at $^{-}19$.

**S:** ***We can draw an arrow from $^{-}19$ to 91.***

**T:** ***Let's see. Put $^{-}19$†  on the display of your calculator and press*** [+] [5] [=] ***. . . . What numbers appear?***

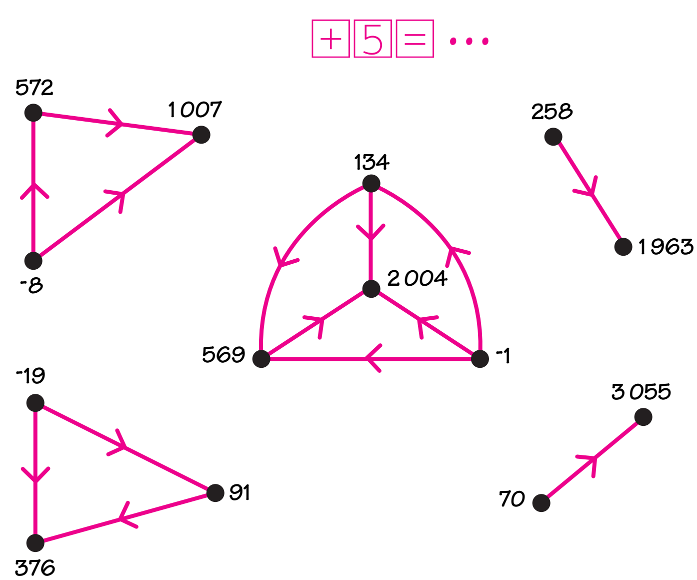
**S:** ***$^{-}14$, $^{-}9$, 1, 6, 11, 16, and so on.***

**S:** ***The pattern of the ones digit changes. The ones digit is 9 or 4 for a negative number and 1 or 6 for a positive number. So we can draw an arrow from $^{-}19$ to 91.***

**S:** ***Also, we can draw an arrow from $^{-}19$ to 376.***

**S:** ***And an arrow from 91 to 376.***

Add arrows to your picture as students suggest them. Continue this activity until the picture is complete. A careful drawing of the dots should give this arrow picture.




**T:** ***What do you notice about this arrow picture?***

**S:** ***There are five separate pieces to the picture.***

**T:** ***Why do you think there are five pieces?***

**S:** ***Because of the five pairs of ones digits: 0 and 5; 1 and 6; 2 and 7; 3 and 8; 4 and 9.***

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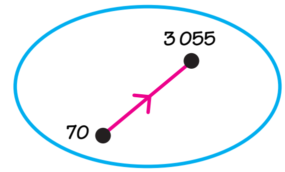
†Putting $^{-}19$ on the calculator display can be accomplished in one of at least two ways:

- If the calculator has a [+/-] (or [CS]) key, press [1] [9] [+/-].
- Press [−] [1] [9] [=] or [0] [−] [1] [9] [=].

**S:** ***Because ⊞ ⑤ ⊟ … is like counting by fives.***

Accept any reasonable answer. Be prepared for students to suggest that there could be more pieces if additional numbers are put in the picture. Your class might want to try some other numbers. They should find that any other number can be connected to one of the five pieces.

Draw a string around this part of your picture.




**T:** ***This blue string will be for all of the numbers that can be connected to 70 and 3 055 by ⊞ ⑤ ⊟ … arrows. What other numbers could go inside the string?***

**S:** ***80.***

**S:** ***4 135.***

**T:** ***What is the least positive number that could go inside of the string?***

**S:** ***5.***

**Note:** You may get 0 as an answer. Point out that 0 would be inside the string but that 0 is not positive.

**T:** ***The greatest negative number?***

**S:** ***⁻5.***

**T:** ***Some other negative numbers?***

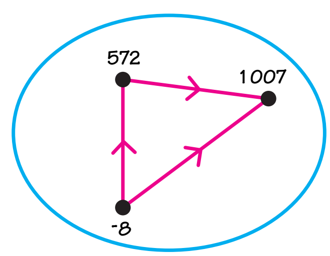
**S:** ***⁻10, ⁻25, ⁻40, and so on.***

**T:** ***What do we call the numbers that are inside this string?***

**S:** ***Multiples of 5.***

**S:** ***Numbers that end in 0 or 5.***

Draw a string around this piece of your picture.




**T:** ***What other numbers could go inside this string?***

**S:** ***577.***

**S:** ***1 012.***

**T:** ***What is the least positive number that could go inside the string?***

**S:** ***2.***

**T:** ***The greatest negative number?***

**S:** ***⁻3.***

**T:** ***Some other negative numbers?***

**S:** ***⁻13 or ⁻28.***

**T:** ***Do you see any patterns?***

**S:** ***The negative numbers end in 3 or 8.***

**S:** ***The positive numbers end in 2 or 7.***

Students might note that these numbers are all two more than a multiple of 5; for example, 572 = 570 + 2;
1 007 = 1 005 + 2; $^{-}8 = {}^{-}10 + 2$; and so on.

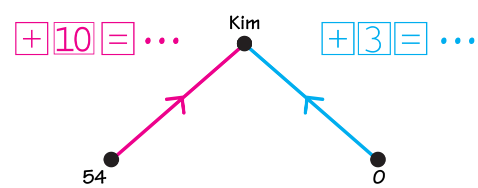
If there is interest, the other three pieces of the string picture can be investigated in a similar manner. Save time for the detective story in Exercise 2.

## Exercise 2: Detective Story

**T:** ***Kim is a secret number.***

Clue 1

Draw this arrow picture on the board.




**T:** ***Kim is in this arrow picture. If you put 54 on your calculator and press*** $\boxed{+}\ \boxed{1}\boxed{0}\ \boxed{=}$ ***..., you will get Kim. Also, if you put 0 on your calculator and press*** $\boxed{+}\ \boxed{3}\ \boxed{=}$ ***..., you will get Kim. On your paper, write some numbers that Kim could be.***

Allow a few minutes for students to explore the situation.

**T:** ***Who can tell me a number that Kim could be?***

**S:** ***84.***

**S:** ***114.***

If many students have difficulty with this clue, and if you get several incorrect answers, use one such answer to review the clue. For example, suppose 64 is given as a number Kim could be.

**T:** ***If we put 54 on the display of the calculator and press*** $\boxed{+}\ \boxed{1}\boxed{0}\ \boxed{=}$ ***..., will we get 64?*** (Yes) ***If we put 0 on the calculator and press*** $\boxed{+}\ \boxed{3}\ \boxed{=}$ ***..., will we get 64?***

**S:** ***No. We will get 60, 63, 66, but not 64.***

**T:** ***So Kim cannot be 64. What is the least number Kim could be?***

**S:** ***84.***

**T:** ***Another number Kim could be?***

As students make suggestions, list them in numerical order.

84, 114, 144, 174, 204, ...

**T:** ***Do you see a pattern? Do you need to use your calculator to find more possibilities for Kim?***

**S:** ***Just add 30 each time.***

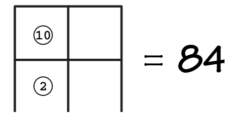
**S:** ***234.***

The class should conclude that Kim could be 84, 114, 144, 174, 204, 234, and so on.

Clue 2

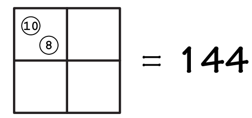
Display one Minicomputer board and the weighted checkers ②… ⑩.

**T:** ***Kim can be put on the ones board using a ⑩-checker and exactly one of these checkers*** (point to the checkers ② … ⑨)**.**



**S:** ***Kim could be 84.***

**T:** ***Show us.***



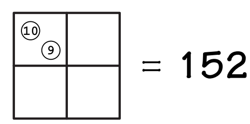
**S:** ***Kim could be 144.***

**T:** ***Could Kim be 114?*** (No) ***Why not?***

**S:** ***If we put the ⑩-checker on the 8-square, that is 80. We need 34 more because 80 + 34 = 114. But 34 cannot be put on the Minicomputer with exactly one checker.***

**S:** ***We can put the ⑩-checker on the 4-square, but then we cannot put 74 on the Minicomputer with exactly one checker.***

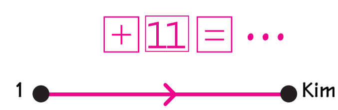
The argument can be extended to include the cases of the ⑩-checker on the 2-square and the ⑩-checker on the 1-square. In each case, a number greater than 72 is needed to reach 114, but 72 (the ⑨-checker on the 8-square) is the greatest possible number using exactly one of the given checkers on the ones board. Hence, we are sure that Kim cannot be 114.



Students may argue that Kim cannot be greater than 144, noting that the greatest number that can be put on the Minicomputer with a ⑩-checker and exactly one other checker is 152. Since 174 is the next number greater than 144 that Kim could be (according to the first clue), it is clear that 174 or any greater number cannot be Kim.

The class should conclude that Kim is 84 or 144.

Clue 3



Draw this arrow picture on the board.

Ask students to work individually or with a partner to find Kim.

**T:** ***When you think you know who Kim is, write it on your paper.***

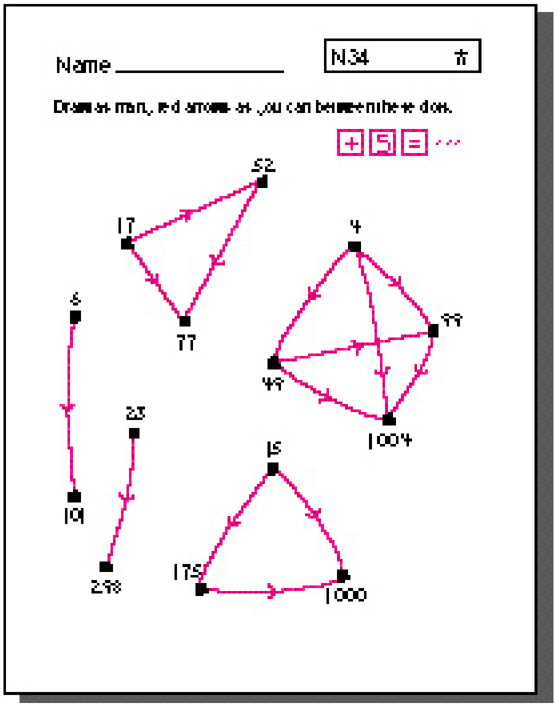
Check solutions as students indicate they know the secret number.

**S:** ***Kim is 144.***

If time permits, the class might discuss patterns in the sequence of numbers generated by ☐ + 11 = …
.

Worksheets N34*, **, ***, and **** are available for individual work.

# N34

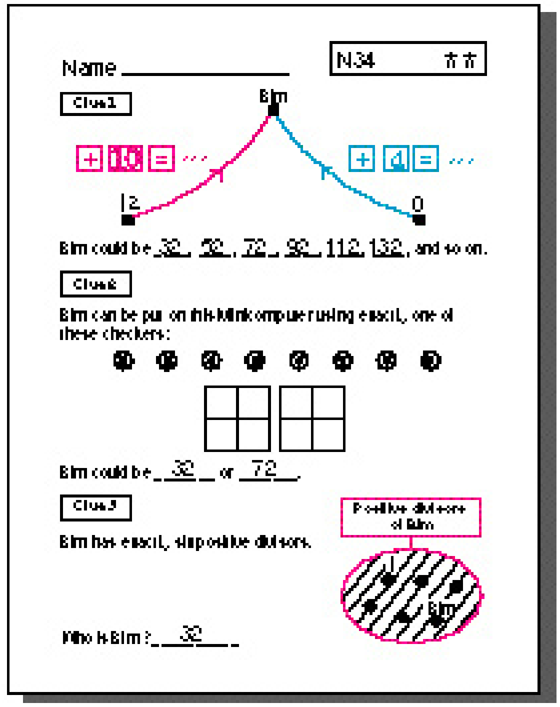

Name

N34 ☆

Draw as many red arrows as you can between these dots.

+5= ...

52, 17, 77, 4, 99, 49, 1004, 6, 101, 23, 298, 15, 175, 1000


Name

N34 ☆☆

Clue 1

+10= ...   +4= ...

Bim, 12, 0

Bim could be 32, 52, 72, 92, 112, 132, and so on.

Clue 2

Bim can be put on this Minicomputer using exactly one of these checkers:

Bim could be 32 or 72.

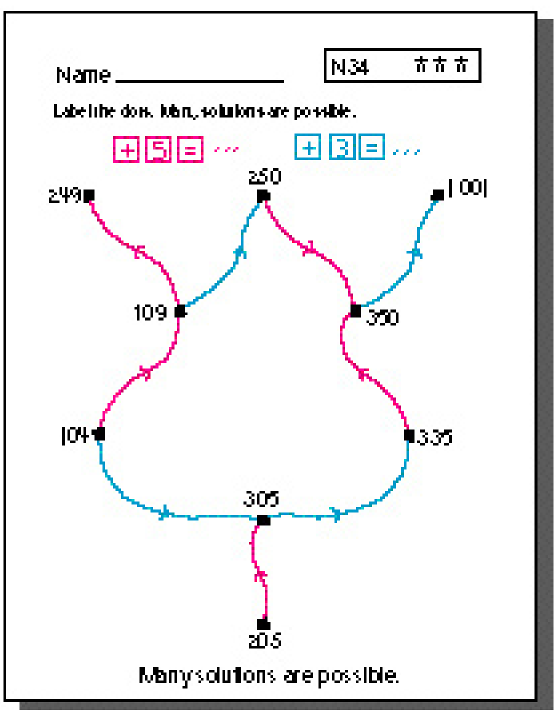
Clue 3

Bim has exactly six positive divisors.

Positive divisors of Bim

Bim

Who is Bim? 32


Name

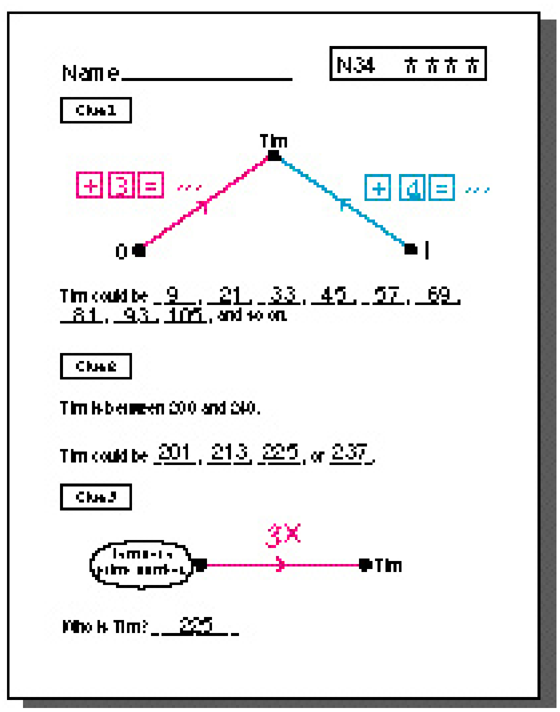
N34 ☆☆☆

Label the dots. Many solutions are possible.

+5= ...   +3= ...

249, 250, 1001, 109, 350, 104, 335, 305, 205

Many solutions are possible.


Name

N34 ☆☆☆☆

Clue 1

Tim

+3= ...   +4= ...

0, 1

Tim could be 9, 21, 33, 45, 57, 69, 81, 93, 105, and so on.

Clue 2

Tim is between 200 and 240.

Tim could be 201, 213, 225, or 237.

Clue 3

Is the square of a prime number

3x

Tim

Who is Tim? 225

## Capsule Lesson Summary

Estimate a number on the Minicomputer with positive, negative, and weighted checkers. By moving exactly one checker, change the number on the Minicomputer by a specified amount. Play *Minicomputer Golf.*

### Materials

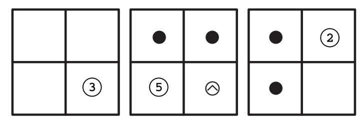
**Teacher**
- Minicomputer set
- Weighted checkers
- Colored chalk

**Student**
- Paper

## Description of Lesson

### Exercise 1

Put this configuration on the Minicomputer.



**T:** ***Let's estimate what number is on the Minicomputer. Is it greater than 100?***

**S:** ***Yes, it is more than 300 because ③ on the 100-square is 300.***

**S:** ***It is more than 400 because ⑤ on the 20-square is another 100.***

**S:** ***On the tens board there is also 80 + 40 + $\widehat{10}$ = 110, so it is more than 500.***

**T:** ***Is it closer to 500 or to 600?***

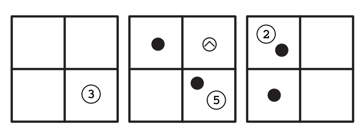
**S:** ***Closer to 500.***

**T:** ***Now I am going to change the number by moving one checker from the square it is on to another square. Each time I move a checker, tell me whether the new number is more or less than the previous number. Also, tell me how much more or less it is.***

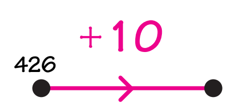
Move checkers as indicated below. Do not return checkers to their original positions. Each move starts from a new number on the Minicomputer.

- Move the ②-checker from the 4-square to the 8-square. (8 more)
- Move the negative checker from the 10-square to the 4-square. (6 more)
- Move the regular checker from the 40-square to the 10-square. (30 less)
- Move the ⑤-checker from the 20-square to the 10-square. (50 less)
- Move the negative checker from the 4-square to the 40-square. (36 less)

If all the moves suggested above were made, this configuration will be on the Minicomputer. Ask the class to decode the number on the Minicomputer at this time. (426)



Draw this arrow on the board as you say,

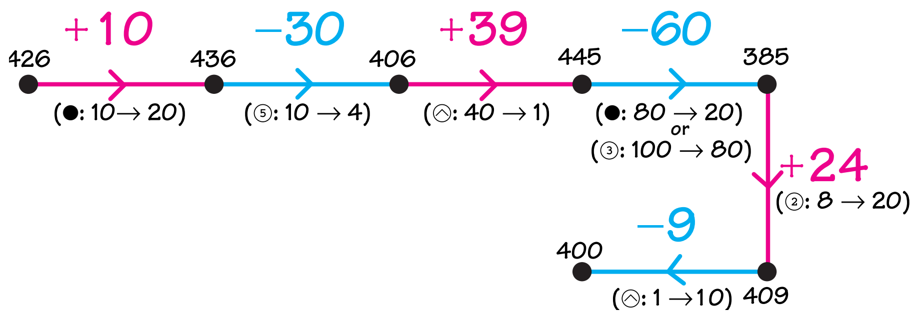



**T:** ***Yes, 426 is on the Minicomputer. Can you move just one of the checkers and make it 10 more?***

**S:** ***Move a regular checker from the 10-square to the 20-square.***

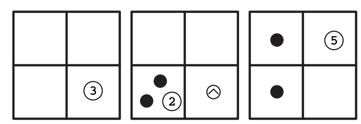
Continue in this way, drawing one arrow at a time and asking for a move of one checker to effect the change indicated by the arrow.

**Note:** You may need to adjust the changes (arrows) you request below if you have a different configuration on the Minicomputer.



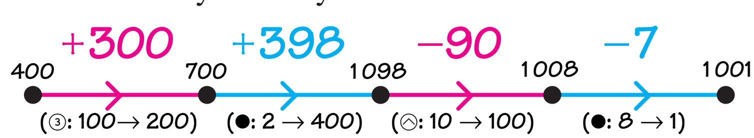

## Exercise 2

Use whatever configuration you have on the Minicomputer at the end of Exercise 1 to play a cooperative game of *Minicomputer Golf*. In a cooperative game, the entire class works at getting a target number (goal) on the Minicomputer with as few moves as possible. In this case, you may have this configuration on the Minicomputer.



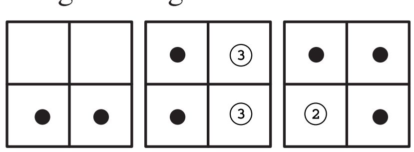
**T:** ***400 is on the Minicomputer. Suppose we are playing Minicomputer Golf and the goal is 1 001. We'll play altogether as a team and try to get 1 001 on the Minicomputer. Remember, you take a turn by moving exactly one checker from the square it is on to another square. Let's try to reach the goal with as few turns (moves) as possible.***

Keep a record of the moves with arrows as you usually do for *Minicomputer Golf*. When the class reaches the goal, count the arrows and announce how many turns it took. A possible game with four turns is shown below. Your class may certainly take more turns.




## Exercise 3

Play a usual game of *Minicomputer Golf* with teams as described in Lesson N8. A possible starting configuration and goal are given below.



= 567 Goal: 1200