Probability & Statistics

PROBABILITY AND STATISTICS TABLE OF CONTENTS

PROBABILITY AND STATISTICS INTRODUCTION

In today's world, probabilistic and statistical methods have become a part of everyday life. They are an important part of industry, business, and both the physical and social sciences. Just being an intelligent consumer requires an awareness of basic techniques of descriptive statistics, for example, the use of averages and graphs. Probability and statistics are key to the present day modeling of our world in mathematical terms. The lessons in this strand utilize both fictional stories and real data as vehicles for presenting problems and applications.

Probability stories fascinate most students and encourage their personal involvement in the situations. They often relate the probability activities, such as marble games, to games they have encountered outside the classroom. This personal involvement builds students' confidence and encourages them to rely on their intuition and logical thinking to analyze the situations. The questions and paradoxes that arise focus attention on key concepts of probability such as randomness, equally likely events, and prediction.

Some applications of mathematics are more easily investigated than others. The probability and statistics applications in this strand lean towards the easy end of the spectrum, both because of the small amount of theoretical knowledge required, and because the link between the situation and the mathematical model is easily perceived. In particular, the development of pictorial models for analyzing the problems facilitates the ease of solutions.

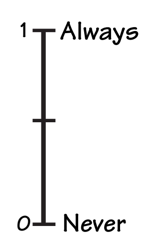
Content Overview

Probability

Bruce's games in *IG-I* led to a combinatoric analyses of probabilities based on drawing dot-and-cord pictures. In *IG-II*, students apply this technique to new stories and games. One story concerns the expected amount of money selected when two coins are drawn at random from a child's bank. A game addresses the paradox that when two coins are flipped, the outcome "one head and one tail" is more likely than either "two heads" or "two tails."

Many probability situations involve random devices such as coins, spinners, or marbles. Several lessons in this strand investigate the concepts of randomness by focusing interest on the random devices involved. One story explores the problem of using a random device with only two equally likely outcomes (e.g., a coin) to fairly select one child from any size group of children. Another lesson contrasts randomly selecting a letter of the alphabet versus randomly selecting a letter from a book.

After describing a game or story situation involving a probability event, students enjoy predicting the answer. The students' predictions indicate their current understanding of the situation and also commit them to a precise opinion of the expected outcome. Rather than expecting students to state their prediction in terms of fractions, decimals, or percents, with which many students still struggle, these lessons offer a pictorial method for making predictions. Given a segment drawn on the board, a student draws a dot to indicate a prediction that an event will, for example, always occur, never occur, have a 50-50 chance, almost never occur, and so on. The probability segment offers an appealing and intuitively visual means for students to clearly express their predictions. Later, labeling the segment as a 0–1 interval of the number line allows interpretation of the students' dots as numerical probabilities.



Lessons: P1, 3, 4, 5, 6, 7, 8, and 9

Statistics

Simulation of a probabilistic event leads to a better understanding of a situation, provides a basis for predicting probabilities, and aids in confirming analysis of the problem. Playing the two games described in the storybook *The Square Trap* leads students to insights concerning winning sequences and similarities of the games. In other lessons, personal involvement in simulating a game or story motivates students and provides a comparison to the formal analysis of the situation. Students learn to use graphs and averages to summarize the data collected in the simulations. Through these experiences, students realize that randomness causes all predictions to be only approximations. On the other hand, randomness is not entirely chaotic; the simulations and the analyses are usually in close agreement.

Several other lessons in this strand focus on descriptive statistics—the use of numerical and graphical techniques to summarize and compare sets of data. The activities continue to develop students' abilities to read, draw, and interpret bar graphs, and introduce scatter plot graphs and concepts of averages. The goal is to increase students' familiarity with these topics through rich experiences rather than to drill the techniques of computing an average or drawing a graph. Therefore, students learn to compute the mean of a set of numbers by "balancing" the numbers. This process emphasizes the role of the mean as a "central number" and later the process will lead to an efficient way to compute the mean: add and divide. Also, students interpret two fairly complex scatter plot graphs in order to answer questions about a football team's results, and about the areas and populations of states of the United States. A goal of these experiences is to augment students' confidence in using statistics as a tool to understand their world.

Lessons: P1, 2, 3, 4, 6, 7, 8, and 9

P1 PROBABILITY APPLICATIONS

Capsule Lesson Summary

Present two story situations in which probabilities are used to answer questions about what event is most or least likely to occur, what we expect to happen in a longer period, what we expect as an average, and how to make a good choice of procedures.

Materials

Teacher
- Colored chalk
- Container with coins: a quarter, four dimes, and a nickel

Student
- Paper

Description of Lesson

Exercise 1

Tell this story to the class. Use coin props to illustrate the story.

T: ***Each morning Marcy's parents put four coins in her bank: one quarter, two dimes, and one nickel. Marcy shakes the bank and takes the first two coins that come out to buy a lunch drink. She must leave the other two coins for her brother. How much money might she get?***

The class should find four possibilities:

- 35¢; a quarter and a dime
- 30¢; a quarter and a nickel
- 20¢; two dimes
- 15¢; a dime and a nickel

T: ***At school Marcy has four choices for a lunch drink.***

Write these selections on the board.

White Milk	15¢
Chocolate Milk	20¢
Fruit Punch	30¢
Soft Drink	35¢

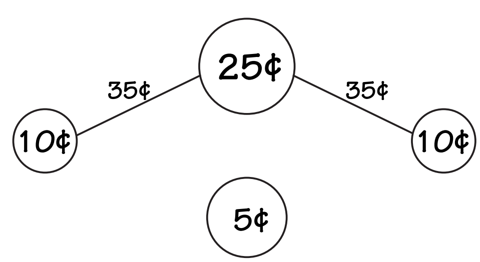
T: ***Marcy always spends her money on one drink; she buys the most expensive drink she can with the money she has.***

On any given day, what is the probability that Marcy will be able to buy a soft drink?

Let students think about and discuss this question. Listen to their ideas for a few minutes before suggesting they look at a cord picture with the four coins. Represent the four coins on the board.

T: ***How can Marcy get 35¢? How many ways are there?***

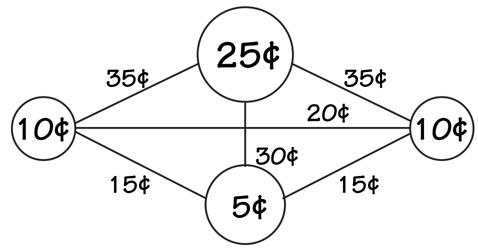
S: ***She must get the quarter and one of the two dimes, so there are two ways.***



Draw cords in your picture to show this.

T: ***What other ways are there to take two coins?***

Students may respond that there are four other ways (looking at the other possible cords in the picture), or they may go back to the other amounts of money and look at the number of ways for each. Follow the suggestions of the class, but be sure they find four, not three, other ways. Eventually you will want to look at both the total number of ways to take two coins and the number of ways to get each amount of money.

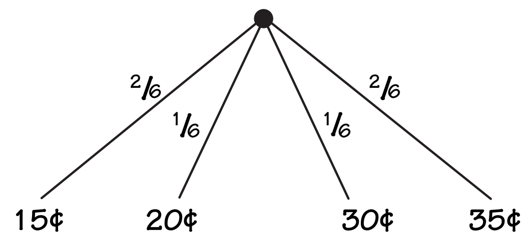


S: ***There are two ways to get 15¢, one way to get 20¢, and one way to get 30¢.***

T: ***So what is the probability that Marcy will be able to buy a soft drink for 35¢?***

S: ***There are two chances out of six to get 35¢, so the probability is 2/6.***

Also ask for the probabilities of getting 30¢, 20¢, and 15¢. Make a tree to record the probabilities.



T: ***What kind of drink will Marcy buy most often; least often?***

S: ***She will buy white milk or a soft drink most often and chocolate milk or fruit punch least often.***

T: ***Suppose we keep a record of what drink Marcy gets each day for a 30-day period. About how often would we expect her to get each kind of drink?***

Let the class discuss this question and make some estimates of how many times out of 30 Marcy would buy each kind of drink. Then lead the discussion to consider six days at a time.

T: ***Since there are two out of six chances that Marcy will get 15¢, we expect that on about two out of six days she will buy white milk. How many days out of six would we expect Marcy to buy the other kinds of drinks?***

S: ***One day out of six for chocolate milk and for fruit punch; two days out of six for a soft drink.***

Make a tally on the board to indicate these expectations.

Consider the 30 days in six-day increments to complete an expected tally for the 30 days.

White Milk	Chocolate Milk	Fruit Punch	Soft Drink
II	I	I	II
II	I	I	II
II	I	I	II
II	I	I	II
II	I	I	II

T: ***So, how often in 30 days would we expect Marcy to buy each kind of drink?***

S: ***White milk, ten times; chocolate milk, five times; fruit punch, five times; and a soft drink, ten times.***

T: ***How much money would we expect Marcy to spend on lunch drinks during this 30-day period?***

Let students work on this problem with a partner for a few minutes before asking for a response. Ask several students to explain what calculations they did to get an answer.

S: ***\$7.50. 10 x \$0.15 = \$1.50; 5 x \$0.20 = \$1.00; 5 x \$0.30 = \$1.50; and 10 x \$0.35 = \$3.50; which all adds up to \$7.50.***

T: ***What do we expect Marcy's average daily expense for a lunch drink to be?***

Let the class discuss this question, reminding them, as necessary, about what is meant by *average* (here, *mean* average). Lead the discussion to consider a six-day period with expenditures for two white milks, one chocolate milk, one fruit punch, and two soft drinks.

15¢ 15¢ 20¢ 30¢ 35¢ 35¢

Go through the balancing method for finding the (mean) average; or if your students already use a method of adding and dividing, use that method.

S: ***Her average daily expense is 25¢.***

Your students might suggest other methods for finding the (mean) average, for example, dividing \$7.50 by 30. Accept correct suggestions. Observe that the average daily expense is 25¢, and in 30 days we expect Marcy to spend about 30 x \$0.25, or \$7.50.

Exercise 2

Present a new story, this time about Marcy's brother, Mark. Again, use your coin props to illustrate the story.

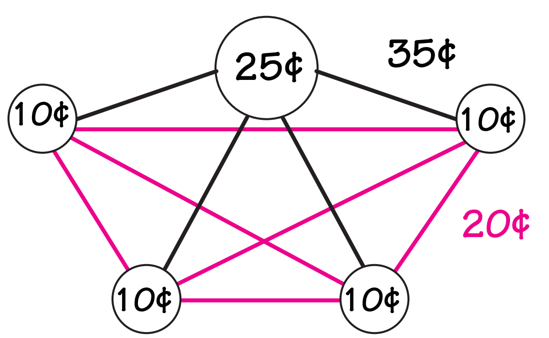
T: ***Marcy's brother, Mark, sells newspapers for 25¢ apiece. One day a customer says, "I have one quarter and four dimes in my pocket. I'll either give you the quarter or I'll randomly choose two coins from my pocket to give you."***

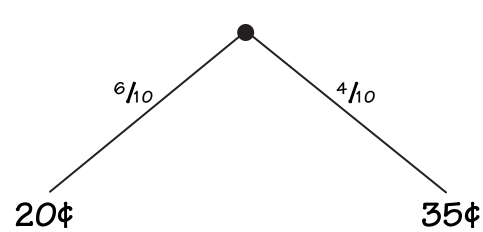
What should Mark take?

Let the class discuss the two choices. In the first case, Mark gets exactly the price of the newspaper; in the second case, he could get 10¢ extra if the quarter and a dime are selected, or 5¢ less if two dimes are selected. Lead a discussion to look at the problem in two ways.

A. One-time sale

Find that the probability of getting 35¢ is 4/10 and the probability of getting 20¢ is 6/10. Use a cord picture to determine the probabilities. Students can do this individually or with a partner before you put it on the board.





The probability of losing 5¢ (6/10) is more than that of gaining 10¢ (4/10), so Mark will more likely lose than gain on a one-time sale.

B. Extended period of sales

Proceed as in Exercise 1 to look at sales over 100 days, for example. In this case, Mark should expect that about 40 times out of 100 he would sell the customer a paper for 35¢ and about 60 times out of 100 he would sell a paper for 20¢. The total sales for 100 days would be (40 x \$0.35) + (60 x \$0.20) = \$14.00 + \$12.00 = \$26.00. If, on the other hand, Mark always sells a paper for 25¢, the total sales for 100 days would be 100 x \$0.25 = \$25.00.

Writing Activity

You may like students to take lesson notes on some, most, or even all their math lessons. The "Lesson Notes" section in Notes to the Teacher gives some suggestions and refers to forms in the Blacklines you may provide to students for this purpose. In this lesson, for example, students may note how cord pictures are used to determine probabilities and how to record probabilities in a tree. They may also consider what happens if the coins in the bank or in the customer's pocket were different than in the story.

Capsule Lesson Summary

Present a graph of the scores of a football team's games for one season. Interpret the graph to answer questions about the team's wins, losses, and scores. As a second activity, present a map of the United States and discuss and compare the areas and populations of certain states. Given dots for states on a population/area graph, use the map and other facts to correctly identify the states marked.

Materials

Teacher
- Colored chalk
- Blacklines P2(a), (b), (c), and (d)
- Football scores graph
- State population/area graph

Student
- Colored pencils, pens, or crayons
- Football scores graph
- U.S. map
- State population/area graph

Advance Preparation: Use Blackline P2(a) to make copies of the graph of football scores for use in Exercise 1. Students will need individual copies and you will need a display copy. Use Blacklines P2(c) and (d) to make copies of the U.S. map and a state population/area graph for use in Exercise 2. Students will need individual copies and you will need a display copy of the graph.

Note: For Exercise 1 your class may prefer to use data from a locally popular football team. In this case obtain the last season's record from the newspaper, a TV station, or the school (team) athletic department. Use Blackline P2(b) to prepare a graph for the data you obtain.

Description of Lesson

Exercise 1

Let students briefly discuss their favorite football teams and tell how well the teams played this (or last) season.

Provide students with a graph of football scores for a team of your choice. The dialogue here uses data for the 1993 season of the Colorado State University Rams on Blackline P2(a). Display the data so you can discuss it as a class.

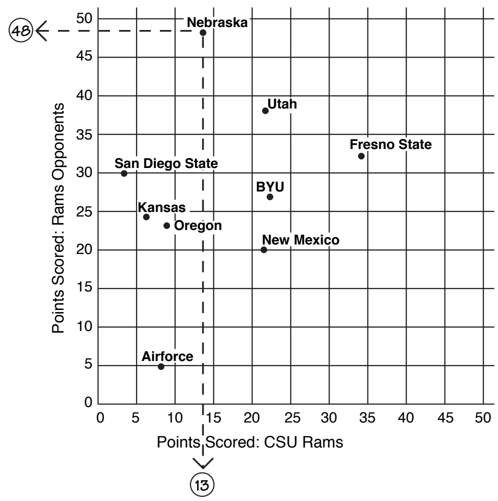
T: ***Today let's look at the 1993 football season of the Colorado State Rams. Each dot on the graph represents a game the Rams played in 1993. We can read how many points the Rams scored in a game*** (trace the horizontal axis) ***and how many points their opponent scored in that game*** (trace the vertical axis)***.***

Who can tell me the score of the Colorado State-Nebraska game?

S: ***Colorado 13, Nebraska 48. Colorado lost.***

T: ***How do you know that?***

Invite a student to show on the graph how to determine the score of a game. Emphasize that in this case, to locate the dot labeled **Nebraska**, you trace straight down for Colorado State's score (13), and trace straight across to the left for Nebraska's score (48).



T: ***What was the score in the Colorado State-Fresno State game?***

S: ***Colorado 34; Fresno State 32.***

T: ***Who won the Colorado State-Oregon game? What was the score?***

S: ***Oregon beat Colorado State 23-9.***

T: ***By how many points did Colorado State lose to Oregon?***

S: ***14 points; 23 – 9 = 14 (or 9 + 14 = 23).***

Continue with questions similar to these until it is clear that most students can correctly read the graph.

T: ***In which game did Colorado State score exactly 8 points?***

S: ***In the game against Airforce.***

T: ***Which Rams opponent scored exactly 24 points?***

S: ***Kansas.***

T: ***The graph records the results of only the first nine games the Rams played in 1993. Let's draw dots for their other games.***

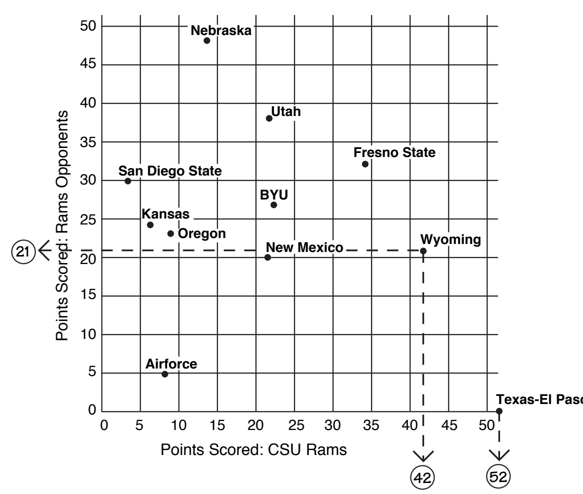
Put this information on the board.

CSU RAMS

52 ——— 0 Texas-El Paso

42 ——— 21 Wyoming

Invite students to draw dots for these two games, both on their sheets and on the class graph. At the board, emphasize locating the Colorado State score horizontally and the opponent's score vertically.



T: ***There are now dots on our graph for all 11 games that the Colorado State Rams played in 1993.***

In which game did the Rams score their greatest number of points? How many points?

S: ***They scored 52 points in the game against Texas-El Paso.***

T: ***Where do you look on the graph to find out in which game the Rams scored the greatest number of points?***

S: ***The dot farthest to the right.***

Present the following questions in a similar manner.

- In which game did the Rams' opponent score the greatest number of points? (48 points by Nebraska; the dot highest on the graph is for that game)
- In which game did the Rams score the least number of points? (3 points against San Diego State; the dot farthest to the left is for that game)
- In which game did the Rams' opponent score the least number of points? (Texas-El Paso scored 0 points)

T: ***Did the Rams play any tie games this season?*** (No)
Which game was closest to a tie?

S: ***The 21-20 game against New Mexico. Colorado State won by only one point.***

T: ***What could the score of a tie game be?***

S: ***10-10.***

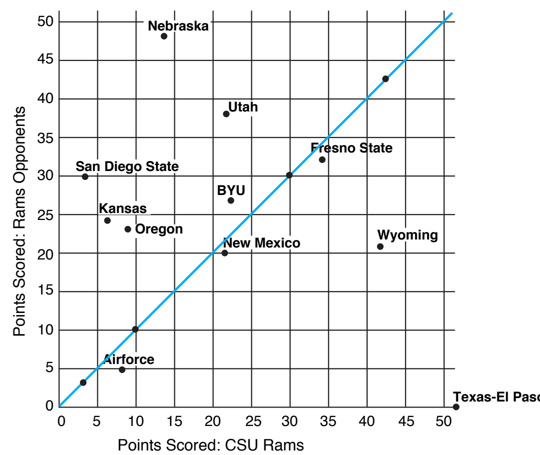
T: ***Show on our graph where a dot for a 10-10 tie game would be.***

Invite several other students to suggest possible scores of tie games, and draw blue dots on the graph to represent those scores.

T: ***What do you notice about these blue dots?***

S: ***They all are on a line.***

S: ***All tie games would be on that same line.***



With the class, observe that dots above the line are for games the Rams lost and dots below the line are for games the Rams won. You may notice that the Rams had a losing season with a record of 5 wins to 6 losses. Dots close to the line are for close games, and in this season the Rams had several close games.

At this time you may like to put students in groups and give them data for another team. The groups can then make a graph for this team and compare the team to the Colorado State Rams. Blackline P2(b) has a blank graph, and data for the University of Colorado Buffaloes is provided here. Your class may be more interested in a local team.

University of Colorado Buffaloes
1993 Football Season

CU Buffaloes	Opponents
36	14 – Texas
45	21 – Baylor
37	41 – Stanford
29	35 – Miami
30	18 – Missouri
27	10 – Oklahoma
16	16 – Kansas State
17	21 – Nebraska
31	14 – Oklahoma State
38	14 – Kansas
21	16 – Iowa State
41	30 – Fresno State

P2

Exercise 2

Distribute copies of the U.S. map and graph on Blacklines P2(c) and (d), and display the graph for class discussion.

T: ***On this map of the United States, some states are shaded and some states are striped. Who can explain what the shading means?***

S: ***The shaded states have a population of over five million, and the striped states have a population between two million and five million. The unshaded states have a population of less than two million.***

T: ***What are some states that have more than five million people?***

S: ***California, Texas, Ohio, Georgia, … .***

Similarly, ask students to name states with a population between two million and five million, and states with fewer than two million people.

T: ***We're also interested in the area of the states. What are some of the largest states?***

S: ***Alaska, Texas, California, Montana, … .***

T: ***What are some of the smallest states?***

S: ***Rhode Island, Delaware, Hawaii, … .***

Refer to the graph (Blackline P2(d)).

T: ***Each dot on the graph tells us both the state's approximate area*** (trace the vertical axis) ***and the state's approximate population in 1990*** (trace the horizontal axis)***.***

What is this state's area and population (point to the dot farthest right)***?***

S: ***Its area is about 150 000 square kilometers and its population is about 12.9 million.***

Select one or two other dots and ask students to determine each state's approximate area and population.

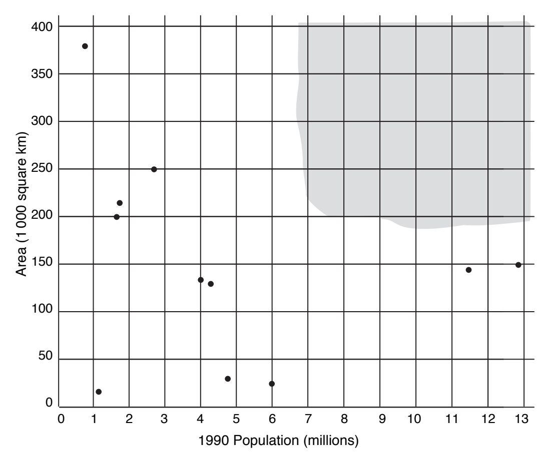
T: ***Which states are large in both area and population? Your map may help you.***

S: ***Texas and California. Both states are shaded, so they have large populations. We can see that both are large in area.***

T: ***About where in the graph would I put a dot for California?***

Accept any estimate in the upper right region of the graph.

T: ***All states with dots in the upper right region of the graph have both a large area and a large population. In fact, California is so large in area and population that its dot is off the graph.***



T: ***Which states are large in area but have a small population?***

S: ***Alaska and Montana. Both states appear large on the map, and are not shaded or striped.***

T: ***About where on the graph could I put a dot for Montana?***

S: ***In the upper left region of the graph.***

In a similar manner, determine that dots for states small in both area and population (for example, Rhode Island, Delaware, Hawaii, and Vermont) are located in the lower left region of the graph. Also, dots for states small in area and large in population (for example, New Jersey and Massachusetts) are located in the lower right region of the graph.

Refer students to the map and to the facts at the bottom of the page. Then refer to the states listed at the top of the graph.

T: ***The 11 dots on the graph are for the 11 states listed at the top of the page. Let's use the map and the facts at the bottom of the page to match the states with the dots. Can you locate the dot for any state immediately?***

Accept correct matches with reasonable explanations. Perhaps the first state students will locate is Florida.

S: ***Florida has more than 12 million people, so it must be this dot (the farthest right). That is the only one with a population of more than 12 million.***

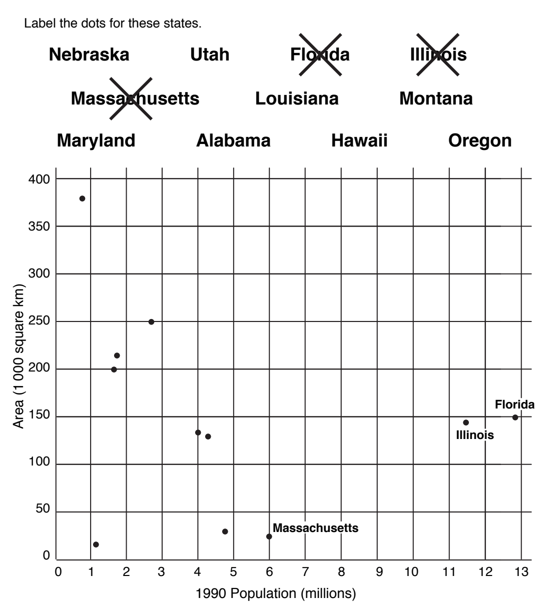
T: ***Which other states in the list have large populations?***

S: ***Illinois and Massachusetts are both shaded on our map.***

T: ***Which dots on the graph are for states with larger populations?***

S: ***Dots farther to the right (or dots to the right of the line for a population of five million).***

Invite students to use the relative size (area) of these two states to find their dots on the graph. Label dots as they are matched with states, and cross out the states.



At this point you may like to ask students to read the graph, giving the approximate area and population of each of these three states.

T: ***Let's try to label some other dots. Which states in the list have a population of less than two million people? Your map should help you.***

S: ***Hawaii, Montana, Nebraska, and Utah. Each of those states is unshaded on the map.***

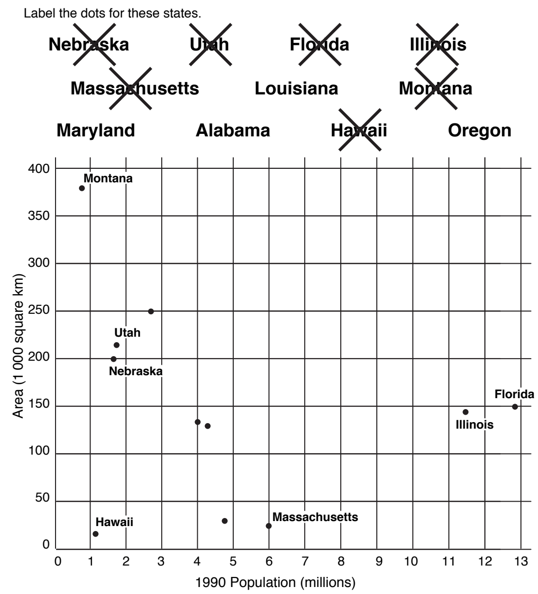
T: ***Which four dots on the graph are for these four states with small populations?***

Students should identify four dots to the left of the line for a population of two million.

P2

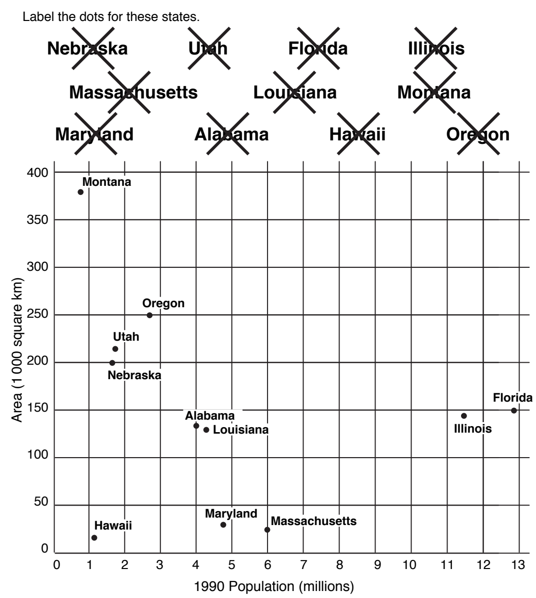
T: ***These four dots are for Hawaii, Montana, Nebraska, and Utah. Which dot is for which state?***

Students should use the relative size (area) of the four states to locate their dots. Montana has the largest area, so it must be the top-most dot. Hawaii has the smallest area, so it must be the lowest dot. The fact (#2 below the map) that Utah has a larger area than Nebraska will distinguish the remaining two dots.



T: ***We have four dots to label and four states left: Alabama, Louisiana, Maryland, and Oregon. Which dot is for which state?***

Again, students should use the relative size (area) of these four states to locate their dots. Maryland has the smallest area of the four, so it is the lowest of the remaining four dots. Oregon has the greatest area of the four, so it is the highest of the remaining four dots. Louisiana and Alabama are close in area, but the third fact says Louisiana has the greater population, so its dot is to the right of Alabama's dot.



Writing Activity

Instruct students to write a few statements about the states in the graph on P2(d).

Extension Activity

Invite students to find the population and area of a few more states to put in the graph on P2(d).

Capsule Lesson Summary

Introduce the storybook *The Square Trap* and play one of the two games presented in this story. Compile and examine the results. Calculate an average score for the results of many games.

Materials

Teacher
- *The Square Trap* Storybook
- A checker
- Colored chalk
- Blackline P3

Student
- *The Square Trap* Storybook
- Random device
- A checker or some other kind of marker
- Game record

Advance Preparation: Use Blackline P3 to make game records for students. The random device for student use may be any that has two equally likely outcomes; for example, a coin (marked 0 and 1 for heads and tails), a spinner with two equal regions for 0 and 1, or one red and one blue marble in a cup.

Description of Lesson

Pair students and distribute copies of the storybook *The Square Trap*.

Pages 1–6

Read these pages collectively, stopping for students' comments or questions. For example, you may ask students to comment on how and why spinners are used in games. Provide direction in reading the chart on page 4 as a record of the results pictured in the spiral on page 3.

Draw a square as shown on page 5 on the board, and illustrate moving a checker on the square as suggested at the bottom of page 6.

Pages 7 and 8

Read page 7 and discuss what random devices could be used to simulate the spinner in playing the game. Choose or let students choose a random device to use, and then instruct student pairs to play the game several times.

With a pair of students, one student can use the random device while the other student moves a marker on the square (page 7), counting how many moves before the checker is trapped at **E**. Suggest that students create a chart like, the one on page 8 (Blackline P3), to record how many moves in a game. Then they can switch roles and play another game.

Monitor students as they play the game, checking that each pair understands the game and is keeping accurate records. If you notice that a game is recorded with an odd number of moves, question how this could happen. Students should begin to notice that games do not end after an odd number of moves and that most games end rather quickly (after two or four moves).

When most pairs of students have completed about ten games, ask everyone to stop playing and to participate in discussing the results. Collect the results from all pairs of students in a class record on the board. For each column (2 moves, 4 moves, and so on), total the number of games from all pairs of students in which the checker was trapped after that number of moves. For example, a typical distribution of results for a class playing a total of 150 games might be as follows.

CHECKER TRAPPED AFTER										
1 move	2 moves	3 moves	4 moves	5 moves	6 moves	7 moves	8 moves	9 moves	10 moves	more than 10 moves
0	72	0	41	0	19	0	8	0	4	6

Encourage comments on the results.

S: ***Most games end after only two or four moves.***

S: ***The checker can only be trapped after an even number of moves.***

Ask students to explain why this is so, but do not expect a well-formulated reply.

S: ***The checker starts at S. After one move, it can be at the upper left or lower right corner, but not trapped. After two moves, it can be back at S or trapped. After three moves, it can be at the upper left or lower right corner, but not at S or trapped. After four moves, it can be at S or trapped. This can go on and on. After an even number of moves, the checker can be at S or trapped; after an odd number of moves, it can be at the upper left or lower right corners.***

Pages 9 and 10

Read and discuss these pages collectively. This discussion will be similar to the discussion on the class's results.

Pages 11 and 12

Read page 11 collectively. The concept of *average* was presented in earlier lessons along with a balancing method for finding an average. Use balancing or another method (for example, sum and divide by 10) to find the average number of moves for the ten games described on page 11. A class might use these steps in balancing the number of moves to find an average:

2	4	6	2	6	2	4	2	4	12
(+2)			(+2)		(+2)		(+2)		(−8)
4	4	6	4	6	4	4	4	4	4
		(−2)		(−2)		(+1)	(+1)	(+1)	(+1)
4	4	4	4	4	4	5	5	5	5

The average is between 4 and 5.

Note: To explain the storybook's statement that the average is 4.4, suggest that to share four moves among ten games one gives 0.4 to each game. Of course, a game never has 4.4 moves and this can only be understood as a theoretical average.

Read page 12 and again calculate the average number of moves (four) for the ten games described there. For example:

2	2	6	8	2	2	4	8	2	4
(+2)	(+2)		(−4)	(+2)	(+2)		(−4)		
4	4	6	4	4	4	4	4	2	4
		(−2)						(−2)	
4	4	4	4	4	4	4	4	4	4

Instruct each pair of students to use the scores from the games that they played to calculate the average number of moves for their games.

Collect the class data on the board. Fifteen pairs of students might generate these average scores:

4.2	3	3.6	3.8	4.2
4	4.4	2.8	5.2	3.8
4.6	3.6	5	4	3.4

Note: If you prefer, let students report averages as between 3 and 4 rather than 3.6. Students who have played more than ten games may have difficulty calculating their average number of moves as a decimal.

With the class data on the board, discuss the questions at the bottom of page 12.

T: ***Could an average score be 2?***

S: ***It's possible, but in that case all games would have to end after two moves.***

T: ***Could an average score be 1.5?***

S: ***No; the least number of moves is two, so the average could not be less than 2.***

Your students may observe that the average score is usually between 3 and 5, or close to 4.

Pages 13 and 14

Suggest students read these pages on their own or with their partners.

Collect the storybooks for use again in Lesson P4.

Capsule Lesson Summary

Review the story and the first game introduced in the storybook *The Square Trap*. Play the second game introduced in the storybook and compile results. Find that this second game is very much like the first game in the results and, in fact, the two games end at the same time when played simultaneously.

Materials

Teacher
- Colored chalk
- Checkers (one red and one blue)
- *The Square Trap* Storybook
- Blackline P4

Student
- *The Square Trap* Storybook
- Random device
- Paper
- Checkers (one red and one blue)
- Game record

Advance Preparation: Use Blackline P4 to make game records for students. See Lesson P3 for a description of a random device.

Description of Lesson

Pair students and distribute copies of the storybook *The Square Trap*. Review the story and the game introduced in the first half of this storybook. Let students recall some of the results and observations that were made in the first lesson.

Pages 15 and 16

Read page 15 collectively and stop to ask the class if they understand the new game. You may need to again have a discussion on simulating the spinner with another random device such as a coin, a die, or red and blue marbles. Let students comment on how long they think a typical game might take to play; that is, would most games end after only a few moves or after many moves?

Read page 16 collectively and act out this game at the board. Let students select the random device they wish to use.

Page 17

Prepare pairs of students to play the game. Each pair of students needs a piece of paper marked with a separation line down the center (a playing mat), a red checker marked 0 and a blue checker marked 1, a random device to simulate the spinner, and a chart like that shown on page 17 (Blackline P4).

Instruct each pair of students to play the game ten times and to record their results in the chart. Monitor students as they play the game, checking that each pair understands the game and is keeping accurate records. If you notice that a game is recorded as ending in an odd number of moves, question how this could happen.

As pairs of students complete playing the game ten times, ask them to recall how to find the average score (number of moves) for their games. Instruct them to calculate their average score and to save it for use in a collective discussion of results.

P4

Collect results from all pairs of students in a class record on the board. For example, a typical distribution of results and average scores for a class playing 150 games might be as follows.

GAME ENDED AFTER

1 move	2 moves	3 moves	4 moves	5 moves	6 moves	7 moves	8 moves	9 moves	10 moves	more than 10 moves
0	77	0	38	0	17	0	10	0	3	5

Average Scores

3.2	4.8	4.4	3.8	5
5.2	3.6	4	4.2	4.2
2.8	3	3.6	3.4	4

Note: If you prefer, let students report averages as between two whole numbers.

Direct a discussion of the class results. Students should comment that this game is similar to the game from Lesson P3 (the first half of the storybook). Examples of such similarities are the following:

- A game ends after an even number of moves.
- Most games end after only two or four moves.
- The average score is usually between 3 and 5.
- The average score cannot be less than 2.

Some students may wish to compare the results of the two games. Avoid drawing any conclusions at this time.

Page 18

Ask students to work with their partners to answer the questions at the top of the page. Perhaps you can suggest that they cover the answers at the bottom of the page with a piece of paper and then uncover the answers to check their own responses.

Pages 19–22

Read and discuss these pages. Students should see that the two games are much the same. The results from one game cannot be distinguished from the results of the other. The games end at the same time when played simultaneously.

Extension Activity

Ask students to look at the back page of the storybook. Invite them to make up games, similar to the ones in the storybook, that can be played with this spinner (or with three marbles: red, blue, and yellow). They might enjoy predicting the kind of results that such games would produce.

Capsule Lesson Summary

Discuss the use of marbles, coins, and spinners to make a random choice among several alternatives. Devise a method using one red marble and one blue marble to choose one child fairly from a given number of children.

Materials

Teacher
- Colored chalk
- Two marbles (one red and one blue)
- A coin
- Blackline P5
- Compass or string

Student
- Code word

Advance Preparation: Make a copy of Blackline P5 and cut out the 32 code words for use in Exercise 3.

Description of Lesson

Exercise 1

Feel free to adapt the following story to reflect the interests of your students. You may wish to choose one of your students to star in the story. Show the random devices as they are mentioned.

T: ***Stephanie likes both canoeing and bicycle riding. One group of friends has invited her to go on a canoe trip. Another group of friends has asked her to take a long bicycle ride on the same day. Both are all-day activities, so Stephanie must make a difficult choice. She likes both groups of friends and both activities very much. How could she decide what to do?***

Let students discuss briefly what Stephanie might consider in making her decision. Then, write the word *random* on the board.

T: ***Stephanie just cannot make up her mind, so she decides to make a random choice. Does anyone know what random means?***

If no student gives a clear explanation, use an example such as the following:

T: ***At the beginning of a football game, how do they decide which team kicks off?***

S: ***The referee flips a coin.***

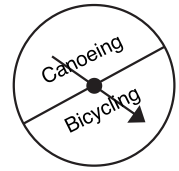
T: ***That's an example of a random choice since heads and tails are equally likely. How could Stephanie use a coin to make a random choice between bicycling and canoeing?***

S: ***Flip a coin. Go canoeing if the coin lands heads; go bicycling if it lands tails.***

T: ***Instead of a coin, how could Stephanie use marbles to make a random choice between the two activities?***

S: ***Use two marbles of different colors, for example, one red marble and one blue marble. Mix the marbles and, without looking, select a marble. Go canoeing if it's red; go bicycling if it's blue.***

Draw a picture of a spinner on the board.



T: ***Could Stephanie use a spinner to make a random choice between the two activities?***

S: ***Divide the face of the spinner in half. Mark one-half "canoeing" and one-half "bicycling." Spin the spinner to decide which activity to do.***

Accept and discuss other methods suggested for making a random choice.

T: ***If Stephanie uses this spinner*** (point to the picture of a spinner divided in half)***, what are the chances that she'll go canoeing? What is the probability that she'll go canoeing?***

S: ***50-50.***

S: ***One chance out of two, or ½.***

Write this information on the board.

Probability of canoeing = p(C) = ½
Probability of bicycling = p(B) = ½

T: ***And what is the probability that she'll go bicycling?***

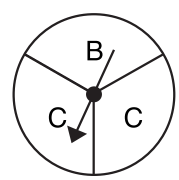
S: ***Also ½.***

T: ***Suppose Stephanie really prefers canoeing but still wants to use a spinner to choose between bicycling and canoeing? How can you change the spinner so that Stephanie is more likely to go canoeing rather than bicycling?***

S: ***Mark both halves of the spinner for canoeing.***

S: ***Mark more than half the spinner for canoeing. Then she might still go bicycling, but canoeing is more likely.***

Draw this spinner on the board.



p(B) = 1/3
p(C) = 2/3

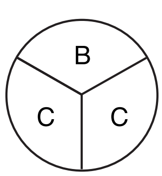
T: ***If Stephanie uses this spinner, what is the probability that she'll go canoeing?***

S: ***Two chances out of three, or ⅔.***

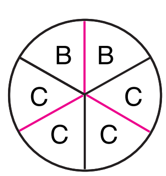
T: ***What is the probability that she'll go bicycling?***

S: ***One chance out of three, or ⅓.***

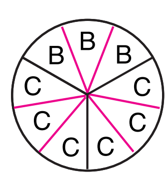
Subdivide the **B** and **C** regions of the spinner several times to generate equivalent fractions in terms of probabilities.



p(B) = 1/3
p(C) = 2/3



p(B) = 2/6
p(C) = 4/6



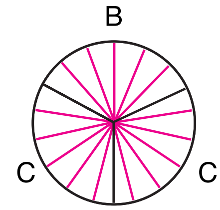
p(B) = 3/9
p(C) = 6/9

Record the equivalences on the board.

p(B) = 1/3 = 2/6 = 3/9
p(C) = 2/3 = 4/6 = 6/9

Let students name more fractions equivalent to ⅔, for example, $\frac{8}{12}$, $\frac{16}{24}$, $\frac{32}{48}$, $\frac{10}{15}$, and so on. Ask students how they generated these fractions. Encourage the use of several methods:

- **Spinner**–If you divide each region of the spinner into six parts the same size, then there are 12 regions regions for canoeing and six regions for bicycling, so the probability of canoeing is $\frac{12}{18}$.



- **Doubling**[†]–Double both the numerator and the denominator: $\frac{2}{3} = \frac{4}{6} = \frac{8}{12} = \frac{16}{24} = \frac{32}{48} = \dots$.

- **Adding on**–Since $\frac{2}{3} = \frac{6}{9}$, then $\frac{2}{3} = \frac{6+2}{9+3} = \frac{8}{12}$, because if you increase the total number of regions from 9 to 12, you increase the number of regions for canoeing from 6 to 8.

Exercise 2

In the following story, select students to play the roles of the grandchildren.

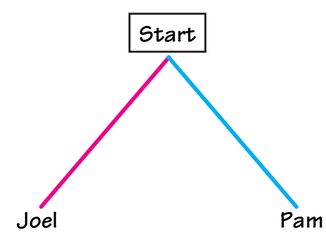
T: ***Grandmother[‡] returns from a trip to Kentucky with two small statues of horses for two of her grandchildren, Joel and Pam. But one of the statues breaks into many pieces in her luggage. What can she do?***

Discuss students' suggestions, but tell them that Grandmother decides simply to select one of the grandchildren and give that child the unbroken horse.

T: ***How could Grandmother make a fair, random choice between the two children?***

S: ***She could use coins, marbles, or spinners like Stephanie did to choose between canoeing and bicycling.***

Draw this picture near the top of the board.



Show the class one red marble and one blue marble.

T: ***Grandmother has one red marble and one blue marble handy to make a choice. I'll be the grandmother. I'll mix the marbles and select one. Who gets the gift if I select the red marble?***

S: ***Joel.***

T: ***How could Pam get the gift?***

S: ***You choose the blue marble.***

T: ***Before Grandmother has a chance to shake the marbles and select one, two more grandchildren, Amy and Brian, arrive.***

Select two more students to play the roles of the grandchildren.

T: ***How could Grandmother make a random choice of one of the four children?***

[†]Tripling, quadrupling, etc. work also.
[‡]The story could easily be changed to feature a grandfather.

S: ***Put their names in a hat and choose one name.***

S: ***Use four marbles of four different colors.***

S: ***Divide a spinner into four parts all the same size.***

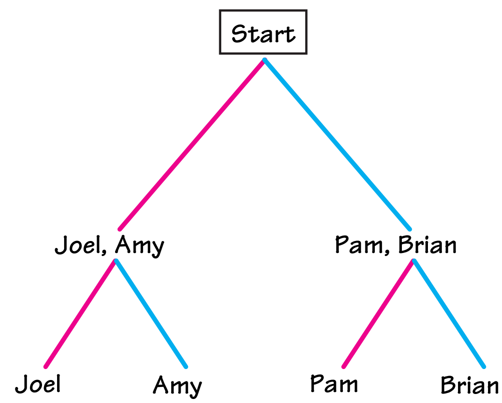
T: ***Grandmother finds a way to use just the one red marble and one blue marble she has. How could she use just these two marbles?***

S: ***Choose between Joel and Pam, and choose between Amy and Brian. Then choose between the two "winners."***

Students are likely to suggest a method similar to the one given here.

T: ***I'll show you Grandmother's method. It's like yours.***

Extend the picture on the board.



T: ***This picture shows Grandmother's method. If Grandmother mixes the marbles and selects a blue one, who could win?***

S: ***Pam or Brian.***

Trace the blue cord from Start.
Hold a finger at Pam, Brian while you ask,

T: ***Now what do you think Grandmother does?***

S: ***She mixes the two marbles and selects one again.***

T: ***Who wins if it's red?***

S: ***Pam.***

T: ***If Grandmother selects a red marble the first time and selects the red marble again the second time, who wins?***

S: ***Joel.***

Trace the path from Start to Joel along two red cords.

T: ***How could Amy win?***

S: ***Grandmother selects first a red marble and then a blue marble.***

At this point, you may want to simulate the selection by asking your students to stand appropriately near the picture. Then, after each marble is selected, ask students who do not win to sit down until you have a winner.

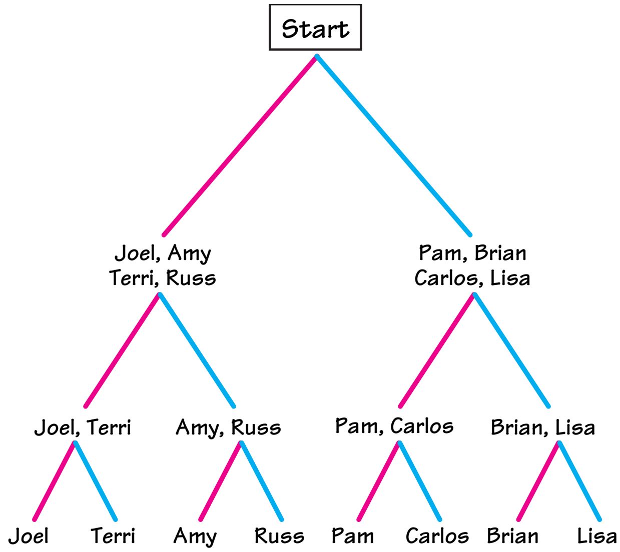
T: ***But before Grandmother has a chance to select a grandchild, four more grandchildren arrive.***

Select four more students so that you have a total of eight students representing grandchildren.

T: ***Now what can Grandmother do to select randomly one of the eight children?***

How should we change the picture on the board now that there are eight grandchildren?

Extend the picture on the board according to students' suggestions until your picture is similar to this.



T: ***How many times does Grandmother select a marble in order to choose one child from the group of eight grandchildren?***

S: ***Three times.***

T: ***If she selects "red, blue, blue," who wins?***

S: ***Russ.***

Invite a student to trace the path from Start to Russ following appropriately colored cords.

T: ***What must happen for Carlos to win?***

S: ***Grandmother must select in order a blue marble, a red marble, and then a blue marble.***

As necessary, ask more questions similar to the two preceding questions. Then model Grandmother's selection process with the students and the marbles. First ask the eight students to stand in groups appropriately near the picture. Ask the rest of the class to tell you what to do (mix and select a marble three times). After each marble selection, ask students who do not win to sit down until you have one winner.

Draw a table of information on the board as you ask,

T: ***With two children, how often did we have to select a marble?*** (Once)
With four children? (Twice)
With eight children? (Three times)
Do you notice any patterns?

S: ***The number of selections goes in order: 1, 2, 3.***
The number of children doubles: 2, 4, 8.

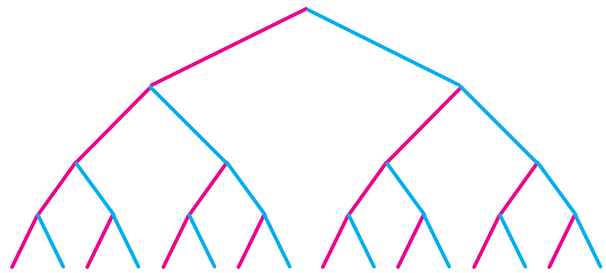
Number of Selections	Number of Children
1	2
2	4
3	8

T: ***Suppose the number of children doubles again. How many children would there be?*** (16)

How many selections of marbles do you think are needed for 16 children?

S: ***Four selections.***

Extend the picture on the board to show four selections and 16 children. Add the information to the table.



T: ***What if we extend this tree one more layer to show selecting a marble five times?***

S: ***We will have places for 32 names along the bottom row; 2 x 16 = 32.***

Extend the tree, and record five selections for 32 children in the table. Erase the names, but leave the tree picture on the board for use in Exercise 3.

Exercise 3

Indicate sections of the tree as you review Exercise 2. Select three students for this exercise.

T: ***We found a good way to use one red marble and one blue marble to select one person out of a group of 2, 4, 8, 16, or 32 people. But one day Grandmother must select one child from a group of three.***

Grandmother wants to figure out a fair way to select one child from this group of three using just her two marbles. How might she do it?

Let students suggest methods. Emphasize that Grandmother wants a fair method, and she wants to use only one red marble and one blue marble.

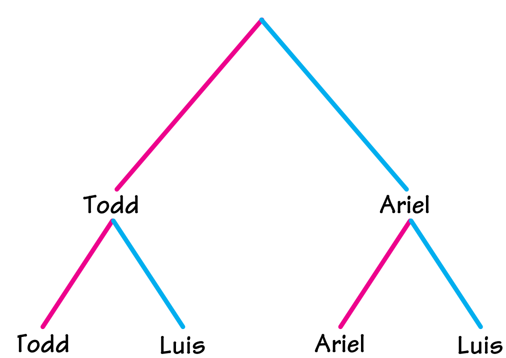
Perhaps one or both of the following incorrect methods will be suggested. The following dialogue demonstrates how to draw a tree picture to help reveal the unfairness of each method.

Method 1:

S: ***First choose between Todd and Ariel. Then let the winner go against Luis.***

T: ***Tell me how to draw a picture of this method.***

Follow students' suggestions, drawing a picture similar to this one.



T: ***Is this method fair?***

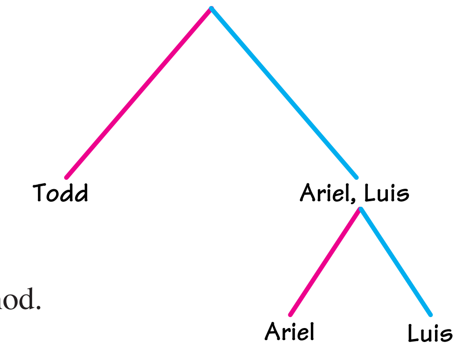
S: ***No, Luis has two ways to win, but Todd and Ariel each win in only one way.***

S: ***For Todd (or Ariel) to win, a correct marble must be selected twice. Luis needs only one correct marble selection to win.***

Method 2:

S: ***For the first selection, let Todd win if it's red and let Ariel or Luis win if it's blue. If necessary, select the marble a second time to choose between Ariel and Luis.***

Draw a picture for this method.



T: ***Is this method fair?***

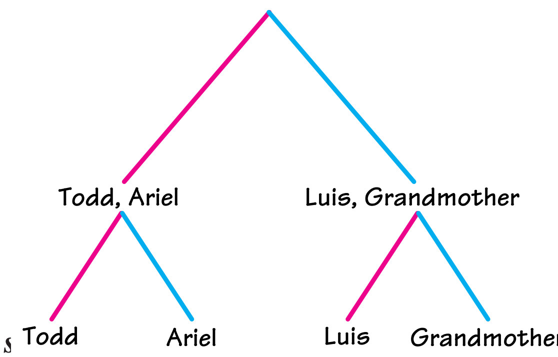
S: ***No, it favors Todd since he can win if a red marble is drawn the first time.***

S: ***Todd needs only one correct choice of a marble, whereas Ariel or Luis need two correct choices.***

You may need to lead to the following (or an equivalent) method.

T: ***If there were four people, we would know how to select one person. So let's put in Grandmother as the fourth person.***

Draw the first section of this picture on the board.



T: ***Tell me how to complete the picture.***

What do you think Grandmother does if she wins? Remember, she still wants to select one of the three children.

Accept students' suggestions or announce,

T: ***Grandmother decides that if she wins, they will s wins. Is this a fair way to select a child?***

S: ***Yes, each child has an equal chance of winning.***

Adapt the following dialogue to the number of students in your class. With 17 to 32 students, use the tree from Exercise 2. With 9 to 16 students, erase the bottom row of the tree. With over 32 students, extend the tree another row. Also, you will need to adapt the list of code words if the class has fewer than 17 or more than 32 students.

T: ***Now I'd like to select one student from our whole class. How many students are present today?***

S: ***We have 27 students here today.***

T: ***How could I randomly select one student out of 27?***

S: ***Use the picture. It works for 32 people. Put our 27 names along the bottom. You take the extra positions. If you win, we play again.***

T: ***Instead of writing your names here, I wrote each position in code on these 32 slips of paper. I'll give each of you one and keep the extras myself.***

Give each student one code word, and observe with the class how a code word represents a position in the tree.

T: ***In the code,* R *is for selecting a red marble and* B *is for selecting a blue marble. How many letters in each code?*** (Five)

I will select a marble five times. Who would win if I chose "red, red, blue, red, blue"?

S: ***Me; my code is* R R B R B.**

T: ***Suppose my first three picks are "blue, blue, red." Who could win? Raise your hands. What is true about the code of all the students raising their hands?***

S: ***The first three letters of their code should be* B B R.**

Read one of the codes you kept for yourself and ask,

T: ***Who would win if I chose "red, red, red, blue, red"?*** (No one answers.)
Why wouldn't anyone win?

S: ***It is one of your code words. We would start over; that is, make the selections again until one of us wins.***

Direct everyone to stand up. Mix the marbles and select one.

T: ***It's red. Sit down if your code word does not begin with* R.**

Mix the marbles and select one.

T: ***It's blue. Sit down if the second letter in your code word is not* B. *What do we know about the code words of everyone who is still standing?***

S: ***The first two letters of each of their code words are* R B.**

Continue the procedure until you have selected a marble five times. After each selection, ask what a winning code word should look like. After selecting a marble five times, there will be either one winner or no winner (i.e., one of your code words won). If the latter occurs, repeat the procedure

Writing/Home Activity

Suggest that students write a letter to their parents/guardians about the method used in this lesson to select fairly one person from among many (the class).

Capsule Lesson Summary

Estimate probabilities and draw representative dots on a line segment scaled from "Never" to "Always." Draw a bar graph showing the number of heads when each student flips a coin ten times. Use the experiment to consider the situation in which a particular result can be most likely and yet have a low probability of occurring. In a second experiment, each student flips a coin until it lands head up. Predict and then count the number of flips it takes until everyone has gotten a head. Simulate doing the latter experiment with a larger number of people.

Materials

Teacher
- One red and two blue marbles
- One penny
- Paper cup
- Colored chalk

Student
- One penny
- Paper cup
- Calculator (optional)

Description of Lesson

Exercise 1

The aim of this exercise is to familiarize students with using a line segment to record their rough intuitive estimates of several probabilistic events. It should proceed quite briskly.

Put one red and two blue marbles in a cup. On the board, draw a vertical line segment about 50–60 cm long.

Note: You may like to use something like socks in a drawer rather than marbles in a cup for this exercise, putting it in a "real world" context.

T: ***Suppose I blindfold Ameed and ask him to select two marbles out of this cup. Will he always, sometimes, or never get at least one blue marble?***

S: ***Always; either he'll pick one blue and one red marble, or he'll pick two blue marbles.***

Draw a dot at the top of the line segment.

Always

T: ***I'll record your answers on this line segment. I put a dot at the top of the segment for* Always.**

When Ameed selects two marbles at random, will he always, sometimes, or never get two red marbles?

S: ***Never; there is only one red marble.***

T: ***Where do you think I should put a dot for* Never*?***

S: ***At the very bottom.***

T: ***If an event is impossible, it will never occur; if it is certain, it will always occur.***

Always (Certain)

Never (Impossible)

P6

T: ***When Ameed selects two marbles, will he always, sometimes, or never select two blue marbles?***

S: ***Sometimes; he might select two blue marbles or he might select one blue and one red.***

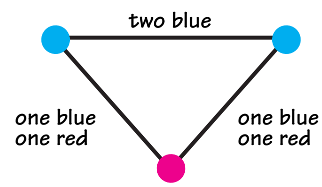
T: ***Where should I put a dot for*** **Sometimes*****?***

S: ***Somewhere between the top and bottom.***

S: ***At the midpoint of the segment.***

T: ***We could put a dot for*** **Sometimes** ***right in the middle, but*** **Sometimes** ***is a vague term.*** **Sometimes** ***could mean "almost always" and we'd put a dot here*** (point to a location just below the dot for **Always**)***, or it could mean "almost never" and we'd put a dot here*** (point to a location just above the dot for **Never**)***. A dot at the midpoint means half the time, or a 50-50 chance.***

Draw two blue dots and one red dot on the board.

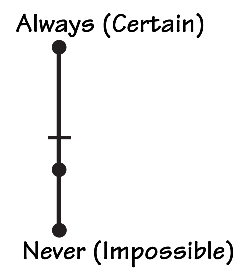


T: ***Who remembers how to use cords to calculate the probability of selecting two blue marbles?***

S: ***Connect each pair of dots with a cord. There will be three cords. Then label each cord.***

T: ***What is the probability of selecting two blue marbles?***

S: $\frac{1}{3}$***; one chance out of three since one of the three cords is for "two blue."***



T: ***Where should I draw a dot on the line segment to show the probability of this event?***

S: ***Slightly below the center, since*** $\frac{1}{3}$ ***is less than*** $\frac{1}{2}$***.***

S: $\frac{1}{3}$ ***of the way up from*** **Never.**

Erase the dots from the line segment.

Make the following or other statements of your choice. After each statement, let the class direct you as to where to draw a dot on the line segment to show the probability of it being true. Each time, erase the dot before going to the next statement.

T: ***An elephant will fly by our classroom window.*** **S:** ***Impossible.***

T: ***Someone in our class is at least 100 cm tall.*** **S:** ***Certain; I am.***

T: ***I will become the president of the United States.*** **S:** ***Almost impossible.***

T: ***Tomorrow (a school day) school will be in session.***

S: ***Almost certain.***

T: ***The next baby born at Barnes Hospital will be a boy.***

S: ***There's one chance out of two, or a 50-50 chance.***

Note: For biological reasons, the probability that a newborn is male is actually about 51%.

T: ***Jenny will get 6 when she rolls a die.***

S: ***There is one chance out of six.***

T: ***Larry will get a prime number when he rolls a die.***

S: ***There are three chances out of six or a 50-50 chance, since 2, 3, and 5 are prime, but 1, 4, and 6 are not prime.***

Exercise 2

Distribute pennies to all the students and check that they know heads and tails.

T: ***If we flip a coin, what is the probability that we'll get heads?*** (½)

Where should we draw a dot for this on the line segment? (At the center)

T: ***I'm going to ask each of you to flip a penny ten times. Before you do, predict how many times you think you'll get heads.***

Let students discuss this question. Most students will expect to get about four, five, or six heads.

T: ***If each of you flip a coin ten times, how many coin flips will there be altogether in our class?***

Adapt the following dialogue to the number of students in your class.

S: ***10 x 28 or 280, since there are 28 students here today.***

T: ***Of these 280 coin flips, how many heads and how many tails do you predict will occur?***

Record students' predictions in a table on the board. Make no comments on the predictions except to confirm that the total number of flips each time is 280.

H	T
200	80
140	140
130	150
50	230

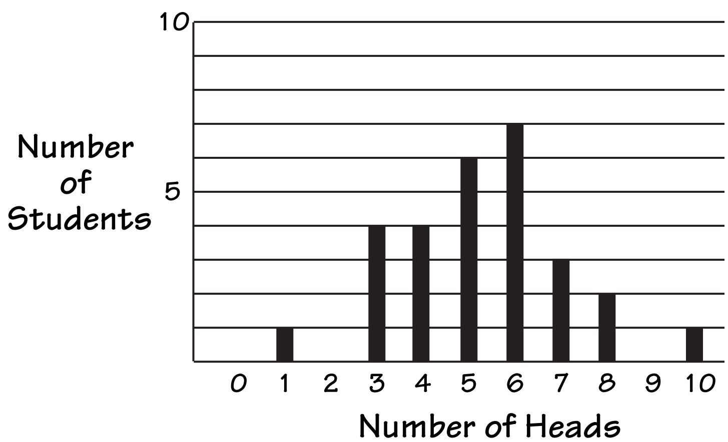
Provide each student with a cup and demonstrate how to use a cup to flip a coin. Cover the cup with your hand, shake it, and then look at the coin sitting at the bottom of the cup. Record whether it lands heads up or tails up.

T: ***Flip the coin ten times and count the number of heads. Be sure to use your first ten flips; don't skip any.***

While the students are doing the coin flipping, draw a graph on the board in which you will record results. (See the next illustration.)

T: ***Raise your hand if you had exactly five heads in ten flips.***

Count the number of students with hands raised and draw a bar above 5 to record this information. Collect the rest of the data in a similar way, and finish drawing a bar graph. The data in the sample chart below will be used in the following dialogue. You will need to adjust it according to your class's data.



T: ***What do you notice about the graph?***

Let students comment on which results were more likely and which were less likely. Conclude (if true) that most students had four, five, or six heads and that few students had zero, one, nine, or ten heads.

Note: In the long run, five heads is most likely. There is a good chance that five will not be the most frequent result in your classroom. It is not safe to determine the most likely result based on this relatively small experiment. But it is safe to use this experiment to conclude that four, five, and six heads are more likely results than zero, one, nine, and ten heads.

T: ***Which result did we get most often?***

S: ***Six heads.***

T: ***If we were to repeat the experiment, do you think that six heads would again be the most frequent result?***

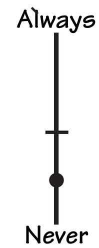
S: ***It could be, but I think it might be four heads or five heads.***

T: ***Could the most frequent result be nine heads or ten heads?***

S: ***I doubt it. Very few of us had those results.***

T: ***Six heads was our most frequent result. Let's use our experiment to estimate the probability of getting exactly six heads in ten flips. Where should I draw a dot on the line segment?***

S: ***Only seven students out of 28 had exactly six heads. According to our experiment, the probability of getting exactly six heads is about $\frac{7}{28}$ or about $\frac{1}{4}$.***



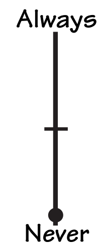
Invite a student to draw the dot for your data.

It may surprise your students that the probability for the most frequent event, six heads for example, can be less than ½. Discuss that this often occurs when there are many possible outcomes.

T: ***Where should I draw a dot for the probability of getting exactly ten heads?***

S: ***Almost at the bottom; it could happen but it is very unlikely.***

S: ***I agree. Only one student out of 28 had all ten heads.***



Note: For ten flips, there are 1024 possible outcomes. Since each is equally likely, there is one chance out of 1 024 of getting any one of them and, in particular, of getting ten heads. This is for your information only.

T: ***Let's use the graph to count the total number of heads flipped in our class.***

Adapt the following dialogue to your graph. Point to the columns of your graph in turn as you count.

T: ***One student got only one head.*** 1

How many students got exactly two heads? (None)

How many students got exactly three heads? (Four)
Each of these four students got three heads. How many heads did they get altogether?

S: ***12; 4 x 3 = 12.***

T: ***We have a total of 13 heads so far, since 1 + 12 = 13.*** 1 × 12

Continue counting, considering one column at a time, until you have the total number of heads. For our data, there are 148 heads.

1 × 12 × 16 × 30 × 42 × 21 × 16 × 10 = 148

T: ***Altogether you got 148 heads. How many tails did you get?***

S: ***132 tails; 280 – 148 = 132.***

T: ***Which estimate of the total number of heads and tails is closest?***

If true, note that the number of heads and number of tails are roughly equal. Conclude that with 28 students making a total of 280 flips, estimates near 140-140 are more likely to be closer than estimates like 50-230.

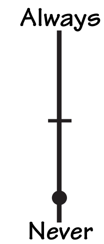
T: ***In fact, 140 heads-140 tails is the best prediction. In the long run, this prediction will be closer than other predictions.***

P6

T: ***Where would you draw a dot on the line segment for the probability of our class getting exactly 140 heads-140 tails if we repeated the experiment?***

Students may think that since 140 heads-140 tails is the best prediction, its probability of occurring is near ½. The following comment should clarify this misconception.

T: ***Since 140 heads is the best prediction, we are very likely to get close to 140 heads; for example, between 130 heads and 150 heads. But we might get 133 heads, 138 heads, 141 heads, or 148 heads. So the probability of getting exactly 140 heads is quite low.***

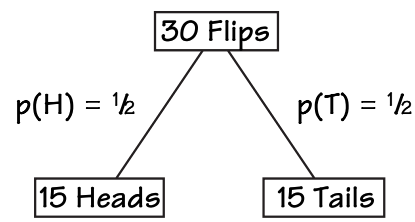


Draw a dot on the line segment to represent the probability of getting exactly 140 heads.

T: ***If we flip a coin 30 times, what is the best prediction for the number of heads? Tails?***

S: ***15 each. ½ x 30 = 15.***

Draw this diagram on the board.



In a similar manner, ask for the best guess for the number of heads and tails if you flip a coin 500 times (250 heads-250 tails), 80 times (40 heads-40 tails), and 10 000 times (5 000 heads-5 000 tails).

Exercise 3

Draw this table on the board. Record the number of students in your class across from 0.

Number of flips	Number of students standing
0	28
1	
2	
3	

T: ***For the next experiment, you will stand up and flip your coin once. Everyone who gets tails will sit down. Those who get heads will flip their coins once again. Those with tails will sit down. We'll keep on doing this until everyone is sitting down. How many rounds of coin-flips do you expect we'll need until everyone is sitting?***

Record students' estimates on the board.

T: ***Let's try the experiment. Everyone stands up. If you each flip your coin once, about how many heads do you expect there will be?***

S: ***About 14; ½ x 28 = 14.***

T: ***Let's do it. Flip your coin once. Sit down if it's tails. How many students are standing?***

Record the number of students standing, for example, 17.

Number of flips	Number of students standing
0	28
1	17
2	8
3	5
4	2
5	1
6	1
7	0

T: ***Now all standing students will flip their coins once again. About how many should get heads?***

S: ***Eight or nine of us.*** $\frac{1}{2} \times 17 = 8\frac{1}{2}$.

T: ***If you are standing, flip your coin once again. Sit down if it is tails.***

Record the results on the board. Repeat the procedure until every student is sitting. For example:

T: ***How many rounds of coin-flips were there?***

Count the number of rounds and compare it to the students' estimates. The sample data in the preceding illustration indicates how seven rounds could be needed.

Exercise 4 (optional)

Adapt this exercise according to the number of students in your school.

T: ***There are about 600 students in our school. If we did this experiment with all 600 students, how many round of flips would you estimate it would take before everyone was seated?***

Record the students' estimates on the board. Fill in a table as you get student responses to the following questions.

Number of flips	Number of students standing
0	600
1	300
2	150
3	75
4	37 or 38
5	18 or 19
6	9 or 10
7	4 or 5
8	2 or 3
9	1 or 2
10	0 or 1

T: ***After one flip, about how many students will be standing?***

S: ***300 students;*** $\frac{1}{2}$ ***of 600 = 300.***

T: ***After the second flip, about how many students will still be standing?***

S: ***150 students;*** $\frac{1}{2} \times 300 = 150$.

Continue in a similar manner until the number of students standing is zero or one.

Compare the number of rounds of coin-flips, ten, with the students' estimates.

If you wish, repeat the above activity starting with the population of your city, the United States (about 250 million), or the world (about 5.6 billion). If you choose to use a large population, let students use a calculator to repeatedly take one-half of the population. Your students are likely to be surprised at the fairly low number of rounds of coin-flips most likely needed even with a large population.

P7 GAMES OF CHANCE #2

Capsule Lesson Summary

Review using a line segment to estimate probabilities on a scale of “Never” to “Always.” Relate that scale to a 0-to-1 probability scale. Conduct two probability games involving coins and marbles. Determine the fairness of each game through use of cord diagrams and probability trees. Given the probability of an event, predict its number of occurrences when an experiment is repeated many times.

Materials

Teacher
- Two pennies and one nickel
- Cup
- Two red and four blue marbles
- Small bag
- Colored chalk

Student
- Two pennies
- Paper cup
- Paper

Description of Lesson

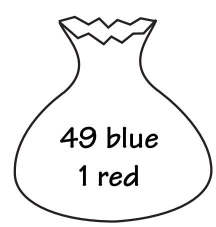
Exercise 1

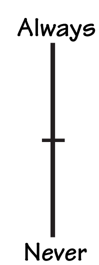
Draw a vertical line segment about 50–60 cm long on the board.

T: ***How do we show our estimates of probabilities on a line segment like this?***

S: ***A dot at the top of the line segment is for events that will always happen (certain). A dot at the bottom is for events that will never happen (impossible). For other events, we estimate where to put a dot.***

Put this information on the board and hold up a bag of marbles.





T: ***Pretend this bag has 49 blue marbles and 1 red marble in it. If we select one marble, is it likely to be blue?***

S: ***Yes, almost certainly.***

T: ***Where should we draw a dot on the line segment to show the probability of blue?***

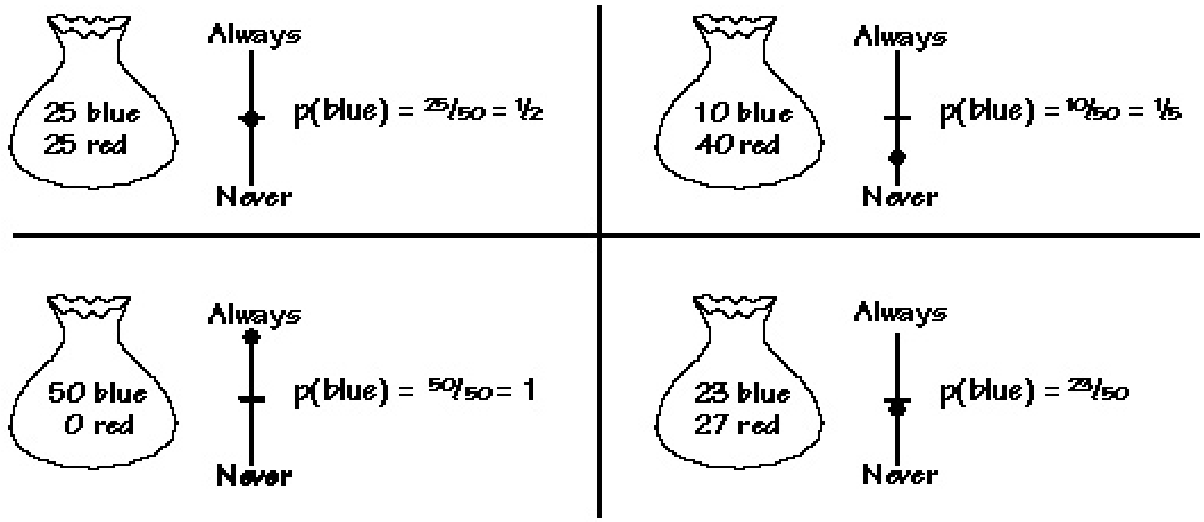
S: ***Almost at the top.***

Put a dot near the top of the line segment and write this:

$$p(\text{blue}) = {}^{49}/_{50}$$

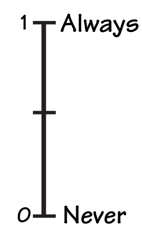
Present the following problems in a similar manner, changing the distribution of the 50 marbles in the bag for each problem. Each time, describe selecting one marble and asking for the probability of blue. Erase the dot before presenting a new problem.

P7



In this activity, students may suggest dividing the segment into 50 parts of equal length in order to locate exactly where to place a dot. If this occurs, acknowledge that this would be a good method and commend the insight, but do not insist that measurement be used. Encourage good estimates.

Locate 0 and 1 at the endpoints of the line segment.



T: ***If an event never occurs, we say its probability is 0. If an event always occurs, we say its probability is 1. The probability of other events is between 0 and 1.*** (Point to the midpoint.) ***What number is here?***

S: ***0.5 or ½.***

T: ***Yes, we can use either decimal or fraction names. So if an event has probability ½ of occurring, we would show this by putting a dot at the midpoint of the segment.***

The weather report often gives predictions for rain in percent. (Point to the top of the line segment.) ***If rain is certain, what percent do we associate with the probability of rain occurring?***

S: ***100%.***

T (pointing to the bottom of the line segment)**:** ***If rain is impossible, what percent do we associate with the probability of rain occurring?***

S: ***0%.***

T (pointing to the midpoint of the segment)**:** ***If it's just as likely that it will rain as not rain, what percent do we associate with this probability of rain occurring?***

S: ***50%.***

Note: We do not expect that students have been taught to use percents. Rather, this discussion is just to relate the probability notions to everyday statements that convey information about chance events. Rely on your students' experience or tell them the percents yourself.

Exercise 2

Organize the class in groups of three for this activity. Each group will need paper, two pennies, and a cup. Select one group (in this example, Charles, Luke, and Evie) to demonstrate how the groups will play a three-person game.

T: ***Let me show you a game three people can play using two pennies. One player shakes the two pennies in the cup and lets them land on the bottom. Charles wins if neither coin is heads, Luke wins if exactly one coin is heads, and Evie wins if both coins are heads. The winner gets a point.***

Let each student in the selected group take a couple turns shaking their two pennies in a cup. Instruct these students to cover the cup, shake the coins, and let them land on the bottom of the cup. Then tell the students to look at the coins and announce the results.

Keep a tally of the results. Be sure students understand that regardless of who shakes the coins, Charles gets a point for 0 heads, Luke for 1 head, and Evie for 2 heads.

Charles 0 Heads	Luke 1 Head	Evie 2 Heads
I	III	II

T: ***Do you think this is a fair game?***

S: ***Yes, each student has one way to win.***

S: ***No, I think Luke is favored.***

Without reaching a conclusion, let students state their opinions and explanations. Then instruct students to play the game in their groups. Ask that the groups all play the game ten times and tally the results in a table similar to the one on the board.

Check that each group is playing the game correctly. Prepare a table on the board in which you will collect results from all the groups

Record scores from all the groups in your table and ask the class to total the scores. Sample results are given here.

	0 Heads	1 Head	2 Heads
Scores	1	6	3
	4	4	2
	2	6	2
	3	5	2
	3	3	4
	2	5	3
	2	7	1
	3	6	1
Totals	20	42	18

T: ***Was this a fair game?***

S: ***No, the player getting a point for 1 head won most of the time.***

S: ***No, 0 heads and 2 heads are harder to get than 1 head.***

T: ***Why is this game not fair?***

S: ***You can get 1 head in two ways—one of the two coins lands heads up or the other one does. But you get 0 heads (or 2 heads) in just one way.***

Do not insist on a precise explanation. Illustrate the game in a tree picture to give a reason for 1 head being most likely.

P7

T: ***Suppose we were playing the game with one nickel and one penny instead of two pennies. Would that change the game?***

S: ***No.***

Shake two coins in a cup, and tell the class to look at the nickel first.

T: ***If the nickel is tails up, who could still win?***

S: ***1 head or 0 heads.***

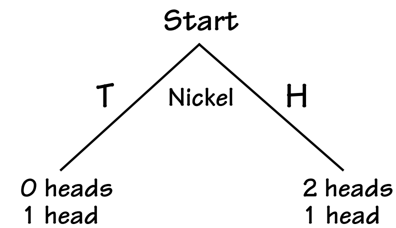
T: ***If the nickel is heads up, who could still win?***

S: ***1 head or 2 heads.***

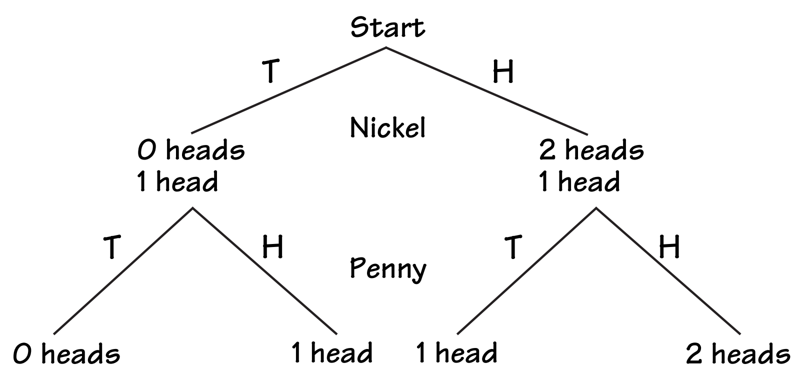
T: ***Does that seem fair?***

S: ***No; 1 head can still win regardless of whether the nickel is tails up or heads up.***

T: ***Next we look at the penny and determine the winner.***



Extend the tree diagram to show two possibilities for the penny whether the nickel was tails or heads. In each case, note the winner at the bottom of the tree.



T: ***Is this a fair game?***

S: ***No, there are two ways to get 1 head but only one way to get 2 heads or 0 heads.***

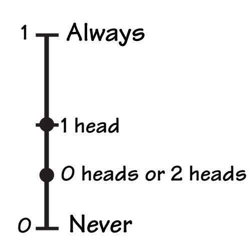
T: ***What is the probability of getting 0 head?*** ($\frac{1}{4}$)

What is the probability of getting 1 heads? ($\frac{1}{2}$)

What is the probability of getting 2 heads? ($\frac{1}{4}$)

$$p(0\text{ head}) = \tfrac{1}{4}$$
$$p(1\text{ heads}) = \tfrac{1}{2}$$
$$p(2\text{ heads}) = \tfrac{1}{4}$$

Locate these probabilities on a 0(Never)–1(Always) segment.



Note: The original three events (0 heads, 1 head, and 2 heads) are not equally likely, which is why the game was not fair. The four events indicated in the tree diagram (HH, HT, TH, TT) are equally likely and thereby we can assign the probabilities $\frac{1}{4}$, $\frac{2}{4}$, $\frac{1}{4}$ to 0 heads, 1 head, and 2 heads respectively.

Note: The preceding analysis demonstrates that the probability of 1 head in each game is ½. However in a set of ten games, the probability of the score for 1 head being greater than the score for 0 or 2 heads is much larger than ½. Mention this distinction only if it arises naturally in class. This type of situation occurs in sports: a weak team often upsets a strong team in one game, but a weak team is very unlikely to win a series of games against a strong team.

Draw the picture below on the board as you ask about the total results in the class.

T: ***How many total games were played in our class?***

S: ***80 games. There were eight groups and each group played ten games; 8 x 10 = 80.***

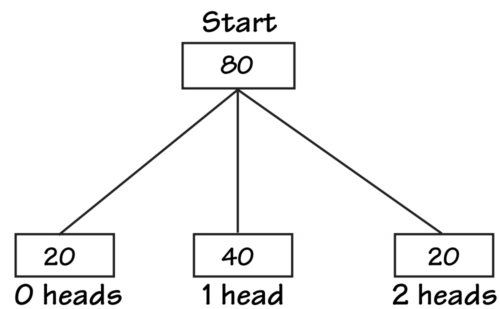
T: ***Of those 80 games, how many games should we predict will result in 1 head?***

S: ***40 games; ½ x 80 = 40.***

T: ***How many games should we predict will result in 0 heads or 2 heads?***

S: ***20 games; ¼ x 80 = 20.***

T: ***Were our results close to these predictions?***



Let students discuss the comparison of the experimental results to the predicted results. Conclude that you can expect the results in the class to be close to, but rarely identical to, the predicted results.

Exercise 3

Choose three students (in this example, Darryl, Lisa, and Kim) to help you present another game, this time with marbles.

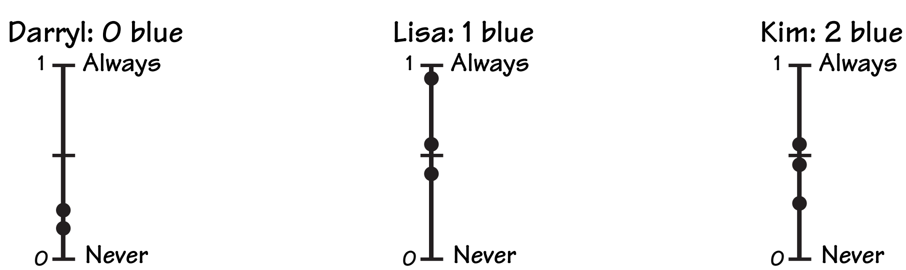
T: ***In this bag there are two red marbles and four blue marbles. I will mix the marbles and someone will select two marbles at random. Darryl wins if neither is blue, Lisa wins if exactly one is blue, and Kim wins if both are blue. Do you think this is a fair game?***

S: ***No, Darryl has little chance of winning.***

T: ***Let's record your estimates of each student's probability of winning.***

Darryl: 0 blue
Lisa: 1 blue
Kim: 2 blue

Record students' estimates on 0(Never)–1(Always) segments. You may put several dots on each of three segments because students may have different estimates.

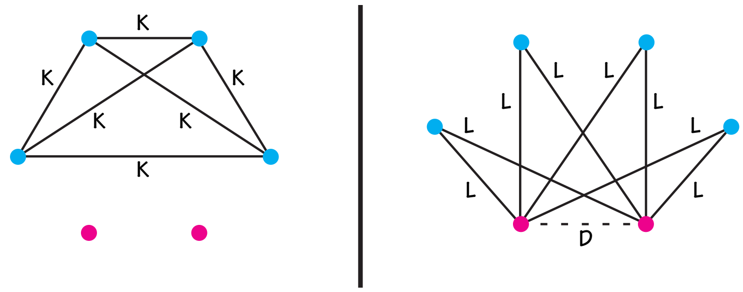


T: ***We could play the game many times, but instead let's analyze the game to determine who is favored. Who knows how to calculate each player's probability of winning?***

S: ***Draw four blue dots and two red dots on the board. Use cords to show ways two marbles can be selected.***

Follow student directions to complete a cord picture for drawing two marbles from the bag. You may ask questions such as, How can Darryl win? to motivate the drawing and labeling of cords. (Darryl can win by selecting two red marbles.)

In the following picture, cords are labeled to indicate which student wins with which draw. Also, two pictures are used to better distinguish the possibilities for Kim to win from those for Lisa to win.



Darryl: 1
Lisa: 8
Kim: 6

T: ***So Lisa is favored. What is Lisa's probability of winning? Why?***

S: ***8/15. There are 15 ways to select two marbles and Lisa wins in 8 of them.***

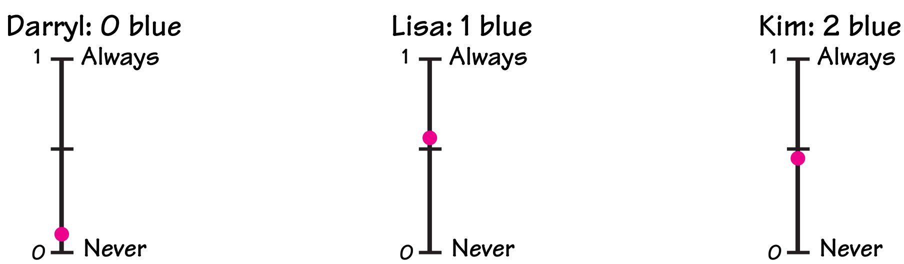
T: ***What are Darryl's and Kim's probabilities of winning?***

S: ***1/15 for Darryl and 6/15 for Kim.***

Refer to the segments on which you recorded students' estimates of Darryl's, Lisa's, and Kim's probabilities to win.

T: ***Let's draw red dots to record each player's probability of winning.***

Let students use either estimation techniques (for example, 8/15 is slightly more than ½, so place Lisa's dot just above the midpoint) or measurement techniques (divide the segment into 15 equal parts) to place the red dots.



Draw a table on the board as you review the situation.

T: ***These probabilities tell us that if we play 15 games, the most likely result is that Darryl will win 1 game, Lisa will win 8 games, and Kim will win 6 games. What is the most likely result if we play 30 games?***

Games	Darryl	Lisa	Kim
15	1	8	6
30			
45			
60			

S: ***Double each result*****:** ***Darryl should win 2 games; Lisa, 16; and Kim, 12.***

Let students complete the chart using the fact that for each additional 15 games, Darryl is expected to win 1; Lisa, 8; and Kim, 6. The addition arrows may be useful as emphasis.

Games	Darryl	Lisa	Kim
15	1	8	6
30	2	16	12
45	3	24	18
60	4	32	24

Conclude the exercise with a discussion about how this method provides a way to make a good prediction of how many games each player should win. For example, in 60 games Lisa should win about 32 games, that is, a little more than half the games. Lisa could win more or fewer games, but her number of wins should be close to 32.

Capsule Lesson Summary

Discuss methods of randomly selecting a letter, both from the alphabet and from a book, within the context of a game. Compare the distribution of vowels and consonants in the alphabet to their distribution in books.

Materials

Teacher	• 26 small square pieces of paper • Bag or box	Student	• Book • Paper

Advance Preparation: Write each letter of the alphabet on one small piece of paper, fold, and put the papers in a bag or box. Help students to select books with mostly text, such as social studies books or storybooks.

Description of Lesson

Exercise 1

Begin the lesson with a short discussion about the alphabet used in the English language (the Roman alphabet).

T: ***In the alphabet, which letters are vowels?***

S: **A**, **E**, **I**, **O** *and* **U**.

S: *Sometimes* **Y**.

T: ***Yes, depending on its use in a word,* Y *sometimes acts as a vowel. Today we'll consider only* A, E, I, O, *and* U *as vowels, because we will not care how they are used in a word.***

What do we call other letters?

S: *Consonants.*

T: ***How many consonants are there?***

S: *21 consonants; there are 26 letters in the alphabet and 5 are vowels. 26 – 5 = 21.*

Write this information on the board.

5 vowels (a, e, i, o, u)
21 consonants

T: ***I'd like to select a letter from the alphabet at random. What does* random *mean?***

Let students define or give examples to explain *random*.

T: ***If a selection is random, each object should have an equal chance of being selected. For example, when you buy tickets in a raffle, all of the tickets are mixed together and one ticket is selected as the winner. The method is random because each ticket has the same chance of being selected when the tickets are well mixed.***

How could we select a letter at random from the alphabet?

P8

Discuss students' suggestions. Note any methods that are not random because they may favor some letters over other letters. For example, letting a student pick his or her favorite letter is not random.

Show the class the bag or box with 26 pieces of paper in it.

T: ***I wrote each letter of the alphabet on a piece of paper and put all 26 pieces in this bag. To select a letter at random, I'll mix the pieces of paper and let someone select one piece.***

Select two students to help demonstrate the following game. Prepare a game recording table on the board.

Vowels (Roberto)	Consonants (Jeanette)

T: ***Someone will select a letter. If it is a vowel, Roberto receives one point; if it is a consonant, Jeanette gets one point. Is this a fair game?***

S: ***No, Jeanette should win easily since there are 21 consonants and only 5 vowels.***

T: ***How could we make the game fair?***

S: ***Take out 16 consonants; then there would be 5 consonants and 5 vowels.***

S: ***Put in 16 more pieces of paper with vowels and leave the 21 consonants in there.***

S: ***After each game, switch rules. For example, after the first game, let Roberto get points for consonants and Jeanette for vowels.***

T: ***Could we make the game fair by changing the scoring? That is, suppose Jeanette gets 1 point for each consonant chosen, but Roberto gets more than 1 point for each vowel. For example, he might get 2 points or 3 points for each vowel. To make the game fair, how many points should Roberto get for each vowel?***

One of your students may already have suggested this method.

S: ***4 points. There are 5 vowels and 21 consonants, $4 \times 5 = 20$. The game is almost fair.***

A student might suggest $4\frac{1}{5}$ points since $4\frac{1}{5} \times 5 = 21$. Accept $4\frac{1}{5}$ as a correct answer, but tell the class that you will use 4 points to make it easier to keep score. Point out that this game is very close to being fair.

T: ***Let's play a game up to 10 points.***

Mix the letters in the bag and let students select letters, one at a time. For each vowel, give Roberto 4 points; for each consonant, give Jeanette 1 point. Keep score in the table on the board as illustrated here.

Vowels (Roberto)	Consonants (Jeanette)
𝍸 |||	𝍸 𝍸

Let students express their opinions of this game.

Exercise 2

T: ***Instead of selecting letters out of this box, let's randomly select letters from a book. How might we do this?***

Discuss the randomness and practicality of any methods suggested by your students; then describe the following method.

T: ***Find a book with mostly words, for example, your social studies book or a storybook. Here's a way to select a letter at random: Without looking, open your book to any page; then I'll tell you which line and which letter to choose. For example, I might tell you to pick the 6th line, 8th letter on that page.***

If Roberto and Jeanette play the same vowel-consonant game but use this method to select randomly a letter from a book, should Roberto still get 4 points per vowel to make it almost a fair game?

S: ***I think 4 points is too much. Vowels occur more frequently in words than they do in the alphabet. Every word has at least one vowel.***

T: ***Should we just give 1 point for a vowel and 1 point for a consonant?***

S: ***No, I think more consonants than vowels occur in words.***

T: ***How many points should we give Roberto for each vowel if Jeanette gets 1 point for each consonant?***

On the board, record students' answers, for example: 2, 3, 2½, and 1½.

T: ***Let's see if your answers work very well in an actual game. Open to any page in your book. I'll tell you what line and letter to look for. Then I'll ask how many of you got vowels and how many got consonants.***

Open to a page. Look for the 3rd line, 9th letter on your page. Raise your hand if it is a vowel. (Count.) ***Raise your hand if it is a consonant.*** (Count.)

Vowels (Roberto)	Consonants (Jeanette)
12	15

Record your counts in the table on the board.

Repeat the experiment about five times. For each experiment, choose new lines and letters for students to find. Record and total the results, as illustrated here.

	Vowels (Roberto)	Consonants (Jeanette)
	12	15
	8	19
	11	16
	10	17
	13	14
	9	18
Totals	63	99

T: ***To make the game nearly fair, how many points should Roberto get for each vowel?***

The answers depend on the data gathered in your class. Most likely, your answer will be about 1½. For example, with the above data, 1½ x 63 = 94½, which is close to 99.

Check the accuracy of your students' estimates. Mention that continuing the experiment would yield more accurate information. Conclude that randomly selecting a letter from the bag gives a different frequency of consonants versus vowels than randomly selecting a letter from a book.

Exercise 3

Ask students individually to predict and write on a paper what they think are the five most frequently and the five least frequently used letters in the English language. You may like to ask that they also highlight (star, box, circle) the one letter they think occurs most often and the one letter they think occurs least often.

At this time you may organize the class into groups to discuss their selections and to let groups come to a group decision about what they think are the five most and five least used letters, in order. Then write the list from the groups on the board, and solicit some discussion and explanation. Otherwise, lead a collective discussion that results in class agreement about the top five and bottom five letters. Save the predictions for use in Lesson P9.

Lead the class in a discussion about how you could check their predictions. Then announce that they will do some statistical sampling to gather information about the usage frequency of letters in English.

Note: At this time there is no need to further explain *statistical sampling* or *frequency*. These ideas will become reality as they do the following activity.

Observe that neither you nor they can examine all the writing ever done in the English language, but they can look at a small sample. Assign each student to select a sentence with at least ten words in it from his or her book. (At this point you may want to suggest that students trade books with a partner so that they can select sentences for other students.) Direct students to copy their sentences carefully on paper.

T: ***In your sentence, count to find out how many times each letter of the alphabet occurs. Make a record so that you can report to the class.***

You may allow time in class for students to do this work or you may suggest students do it as homework. In any case, let students decide themselves how to accomplish the task and make the record. You are likely to get many questions, but try to encourage students to determine their own methods. Announce that they will need the results in a couple days, and save the papers for use in Lesson P9.

P9 LETTER DISTRIBUTION #2

Capsule Lesson Summary

Use a sample to predict the frequency of use of letters in English and compare to a given frequency table. Compare this letter frequency to both the points per letter and the frequency of letters available in the game *Scrabble®*.

Materials

Teacher
- Letter frequency chart
- Overhead projector (optional)
- Letter frequency and *Scrabble* information
- Calculator

Students
- Sentence and letter frequency record from Lesson P8

Advance Preparation: Use Blackline P9(a) to make a transparency of a letter frequency chart to put on the overhead, or copy the chart on the board before the lesson. Use Blackline P9(b) to make a transparency of the information on letter frequency and *Scrabble*.

Description of Lesson

Exercise 1

Begin the lesson by letting some students report on their results in the assignment from Exercise 3 of Lesson P8. Try to select students who used different methods of organizing and recording.

Organize the ~~class into five or~~ six cooperative groups. Direct the groups to total how often each letter appeared in all the sentences for the students in the group. Then invite groups to enter their group results in the chart on the overhead or on the board.

When all the data is in the chart, assign each group to find the class totals for four or five letters, and to enter these in the chart. As a check, several group members should find the total for a letter.

T: ***In our class data, which letter occurs most often? …least often?***

Instruct the class to help you make a list of the letters, in order, from most frequent to least frequent. Announce that you have a list of letter frequency in English from another source, and display only the first column on Blackline P9(b).

Note: Many different lists for letter frequency in English exist. The frequencies may vary slightly from list to list.

Allow a little time for students to compare the class results with this list, and to check their own predictions of most and least frequent letters. Then note how letter frequency is recorded in the list on Blackline P9(b).

T: ***To get these figures, some people looked at many groups of 100 letters from many books and counted the number of occurrences of each letter. Then they calculated the (mean) average frequency for each letter. For example, on the average, they found* e *occurs 12.3 times in every 100 letters. How could the (mean) average be 12.3; you can't have 0.3 of a letter?***

S: ***In groups of 100 letters,* e *might occur 10 times, 11 times, 12 times, 13 times, and so on. When you average all these numbers, you are unlikely to get exactly a whole number like 12 or 13.***

S: ***12.3 means that, on the average,* e *occurred more than 12 times but fewer than 13 times.***

T: ***According to this list, the five most frequent letters in English are* E, T, A, O*, and* N. *Who has at least two of these letters on their list? At least three? At least four? All five?***

The five least frequent letters are* Z, J, X, Q*, and* K. *Who has at least two of these on their list? At least three? At least four? All five?

At this time students may like to compare the frequency of vowels (8.1 + 12.3 + 7.2 + 7.9 + 3.1 = 38.6)
to the frequency of consonants (100 – 38.6 = 61.4). You might like to refer back to the game played in Exercise 2 of Lesson P8.

Exercise 2

Initiate a class discussion about where the information on letter frequency could be valuable.

T: ***Who would care about letter frequency of use in English?***

You may need to mention some of the following ideas yourself.

S: ***People who send messages in secret code or who try to break the codes.***

S: ***People who play on* Wheel of Fortune.**

S: ***People who play word games or solve puzzles like cryptograms.***

S: ***People who make letters for bulletin boards or other displays.***

T: ***Can you think of any word games that use the frequencies of letters?***

S: ***Scrabble or Boggle or Hangman.***

As appropriate, let students tell about how *Scrabble* is played. Then refer to the letter frequency list.

T: ***Some of the least used letters in English are* Z, J, X*, and* Q. *In Scrabble, what is true about these letters?***

S: ***Those letters are worth the most points.***

S: ***There are not many of those letters in a Scrabble set.***

T: ***What do you expect is true in Scrabble about the most used letters, for example,* E, A, T*, and* O*?***

S: ***Those letters are each worth only one point.***

S: ***There are more of those letters in a Scrabble set.***

T: ***Let's see if these facts are really true in Scrabble.***

Display the 2nd column of Blackline P9(b).

Frequency of letters in English	Value of letters in Scrabble
E — 12.3	E — 1
T — 9.6	T — 1
A — 8.1	A — 1
O — 7.9	O — 1
N — 7.2	N — 1
I — 7.2	I — 1

Only part of the table on Blackline 9(b) is shown here.

Note: In the table on Blackline P9(b), letters with equal value in *Scrabble* and letters that occur the same number of times in *Scrabble* are placed in an order to highlight the similarity of those two columns with the first column.

T: ***Overall, are the most common letters in English worth the least in Scrabble and the rare letters worth the most?***

S: ***Yes, the letters at the top of the first column are also at the top of the second column. The same letters are also at the bottom of both columns.***

T: ***Can you find any exceptions? Can you find any letters that seem to be valued much too high or much too low?***

S: ***It seems that* H *should be worth fewer than 4 points.* H *is much higher in the first column than in the second column.***

S: **G *should be worth more than 2 points.* G *is much lower in the first column than in the second column.***

Students might notice other smaller discrepancies between the two columns. Conclude that, with only a couple of exceptions, the value of a letter in *Scrabble* is closely related to its frequency of use in English.

T: ***Let's also check if the letters that occur most frequently in English also have the most tiles in Scrabble.***

Display the third column of Blackline P9(b).

Frequency of letters in English	Value of letters in Scrabble	Number of tiles in Scrabble
E — 12.3	E — 1	E — 12
T — 9.6	T — 1	A — 9
A — 8.1	A — 1	I — 9
O — 7.9	O — 1	O — 8
N — 7.2	N — 1	T — 6
I — 7.2	I — 1	N — 6
S — 6.6	S — 1	R — 6
R — 6.0	R — 1	S — 4
H — 5.1	L — 1	L — 4
L — 4.0	U — 1	U — 4
D — 3.7	D — 2	D — 4
C — 3.2	G — 2	G — 3
U — 3.1	C — 3	H — 2
P — 2.3	P — 3	C — 2
F — 2.3	M — 3	P — 2
M — 2.3	B — 3	F — 2
W — 2.0	H — 4	M — 2
Y — 1.9	F — 4	W — 2
B — 1.6	W — 4	Y — 2
G — 1.6	Y — 4	B — 2
V — 0.9	V — 4	V — 2
K — 0.5	K — 5	K — 1
Q — 0.2	X — 8	Q — 1
X — 0.2	J — 8	X — 1
J — 0.1	Q — 10	J — 1
Z — 0.1	Z — 10	Z — 1
Total 100		

T: ***The third column tells us the number of times each letter occurs in a set of Scrabble tiles. In general, do the most frequently used letters in English occur most frequently in Scrabble?***

S: ***Yes, both the top and the bottom letters of the first and third columns are the same.***

T: ***Can you find exceptions?***

S: ***It seems that there should be only two* G*'s in Scrabble, not three.* G *is much higher in the third column than in the first column.***

S: ***There should be more than two* H*'s in Scrabble.* H *is much higher in the first column than in the third column.***

Students may notice other smaller discrepancies between the first and third columns. Conclude that the number of tiles for a letter in *Scrabble* is closely related to that letter's frequency of use in English. Observe that the correlation of the last two columns with the first column is strong evidence that the inventors of *Scrabble* referred to an English letter frequency chart when they developed the game.

If a number of your students speak or study a foreign language, you may wish to ask the following questions:

T: ***If people played Scrabble in another language, for example, French or Spanish, do you think they would use the same set of tiles?***

S: ***They could, but some letters might occur too often or too rarely.***

S: ***Also, some letters aren't used in other languages. For example, French has no* W.**

S: ***Some languages have letters we don't have in English. For example, German has ß (a "double s").***

T: ***Scrabble does make sets for other languages and does provide a different set of tiles.***

Extension Activity

As a project, suggest that students take a book in a foreign language and count how often each letter occurs in, for example, a 100-letter passage. Then compare the results of the foreign language to English.

As another project, get a *Scrabble* game for another language and compare the point value of letters and the number of tiles for each letter to that in the English version.

Home Activity

Suggest that parents/guardians try to solve a cryptogram puzzle with their child.

Suggest that parents/guardians watch *Wheel of Fortune* with their child and observe which letters are chosen by contestants.