Workbooks

WORKBOOKS TABLE OF CONTENTS

WORKBOOKS INTRODUCTION

There are many opportunities for the student to work individually during the course of the lessons described in the other content strands. In the Workbooks strand, however, it is this individualized work which becomes the chief end of the majority of lessons. The goal in this strand is to provide students with opportunities

- to review many of the ideas they have met in other content strands;
- to apply their acquired knowledge to new situations requiring various kinds of strategic thinking; and
- to learn how to read and use mathematics workbooks.

The following workbooks are provided:

- *Set of Problems #1*
- *Set of Problems #2*
- *Set of Problems #3*
- *Set of Problems #4*
- *Set of Problems #5*

One story-workbook is provided...

- *Clinton Street*

... and one storybook:

- *Nabu Wins an Award*

There are also two lesson titled *Not Too Close #1* and *#2* in this strand.

Each workbook contains problems of varying levels of difficulty. Approximately the first ten pages of each workbook are easy problems, the next ten to twelve pages are average level difficulty, and the last ten pages are more challenging problems. For each workbook, we suggest that all students start work at the easiest level (i.e., on page 2) and then work through as many pages as they can handle during the two lessons scheduled for that workbook. We estimate that, in a typical class, about two-thirds of the students will correctly finish the first ten pages, about one-third will finish the first twenty pages, and a few will finish all or most of the workbook. These proportions will vary from class to class.

This guide contains an answer key for each workbook. The key follows an introduction to the workbook and a suggested collective lesson. The lesson either presents the workbook to the whole class or provides a warm-up activity on a problem similar to one found in the workbook.

The story-workbook *Clinton Street* and the storybook *Nabu Wins an Award* combine the motivation of a storybook and the problem-solving opportunities of a workbook. These two booklets allow students to become deeply involved in an appealing fantasy as they struggle with difficult mathematics problems. The situations support topics and strategies developed in other strands.

Use of the Workbooks Strand for Evaluation Purposes

The workbooks provide an excellent instrument to assess the progress of your students on a regular basis. You may not feel it is necessary to check every page and problem for each student, but you should develop a procedure for checking students' work with which you are comfortable. This may include checking one or more specific pages, discussing some particular mistakes with individual students and letting them correct their work, or just looking carefully at a few pages to be sure the students have understood the general idea of the problems in that particular book.

In the Blacklines, you will find a record-keeping tool for each workbook to help you assess student progress in the various strands. This tool may also assist you in parent conferences and in filling out periodic progress reports.

Here are some important points to bear in mind for workbooks.

- Always read the introductory material for each workbook and give the short introductory collective lesson(s).
- All students should start at the beginning of each workbook and progress as far as they can.
- All students should begin a new title on the same day, even if some students have not finished work on the previous title.
- Not all students should be expected to complete a given workbook. Only some students will reach the most challenging problems. Other students may succeed only in doing the easiest problems, although you should not assume this automatically—surprises are not at

Content Overview

Workbooks

The five *Set of Problems* workbooks both review and extend many of the ideas introduced in the content strands. The extensions occur through problems which require students to apply the mathematics to new situations or to synthesize their knowledge in new ways.

Lessons: W1, 2, 5, 6, 7, 8, 12, 13, 15, and 16

Not Too Close

In these lessons, the problems presented involve the distance between pairs of integers on the number line. Several pictures, including one with cords, are used to pose and solve inequalities. The cord pictures allow clear statements of inequalities and encourage students to experiment as they search for solutions. The use of a number line greatly aids in analysis of many of the problems.

Lessons: W3 and 4

Clinton Street

The story-workbook *Clinton Street* presents an intriguing mystery story about Spike, a detective who receives cryptic messages from the unknown "**.**" Messages in three distinct secret codes all lead Spike
to number 88 Clinton Street. There he meets "J," a young girl and self-proclaimed "Joker."

The story motivates students to explore revealing interrelationships among three codes which involve the Minicomputer, an arrow road with 2x and +1 arrows, and a binary abacus. Once the primary problem is solved, additional problems challenge students to consider other arrow roads and related abaci (Base 3, Base 4, and Base 8).

Lessons: W9, 10, and 11

Nabu Wins an Award

In honor of Nabu's tenth birthday, the numbers organize a special show that involves numbers in spectacular dances. The rivals 0 and 1 cooperate in the choreography of several ballets with distinct numerical themes. The story highlights many important aspects of the structure of a number system.

Some dances reveal the inverse relationship between ½x and 2x and between +2 and –2. Other performances focus on the central roles that 0 and 1, as identity elements, play in our number system.

This storybook illustrates how an imaginative story can enliven a mathematical topic. Also, the arrow pictures suggest relationships, relying on student discussion to discover and clarify the ideas being presented.

Lesson: W14

Capsule Lesson Summary

Do mental arithmetic exercises involving related multiplication and division problems. Generate a sequence of division problems by increasing the dividend by 1 each time but keeping the same divisor. Begin the workbook *Set of Problems #1*. (This is the first of two lessons using this workbook.)

Materials

Teacher
- None

Student
- *Set of Problems #1* Workbook
- Colored pencils, pens, or crayons
- Metric ruler

Description of Lesson

Exercise 1: Mental Arithmetic

Pose several sequences of related multiplication and division facts, one at a time. In the following examples, student responses are in boxes.

$6 \times 8 = \boxed{48}$	$9 \times 7 = \boxed{63}$	$8 \times 4 = \boxed{32}$	$6 \times 12 = \boxed{72}$	$4 \times 125 = \boxed{500}$
$48 \div 6 = \boxed{8}$	$63 \div 9 = \boxed{7}$	$32 \div 8 = \boxed{4}$	$72 \div 6 = \boxed{12}$	$500 \div 4 = \boxed{125}$
$48 \div 8 = \boxed{6}$	$63 \div 7 = \boxed{9}$	$32 \div 4 = \boxed{8}$	$72 \div 12 = \boxed{6}$	$500 \div 125 = \boxed{4}$

Write the division expression on the board as you pose this problem.

T: ***Suppose 7 friends share 49 pieces of bubble gum.***
How many pieces will each person get?
What number is 49 ÷ 7? (7)

$$7\overline{)49}$$

Record the answer and add a second division expression as you pose a similar problem.

T: ***Suppose 7 friends share 50 pieces of bubble gum.***
How many pieces will each person get?
What number is 50 ÷ 7?

$$\begin{array}{r} 7 \\ 7\overline{)49} \end{array} \qquad 7\overline{)50}$$

S: ***7, but there will be 1 left over.***

T: ***We can write the answer as 7 R = 1*** (read as "seven with a remainder of one")***.***

Continue in this manner with the following expressions and related problems.

$$\begin{array}{r} 7 \\ 7\overline{)49} \end{array} \qquad \begin{array}{r} 7\ R{=}1 \\ 7\overline{)50} \end{array} \qquad \begin{array}{r} 7\ R{=}2 \\ 7\overline{)51} \end{array} \qquad \begin{array}{r} 7\ R{=}3 \\ 7\overline{)52} \end{array} \qquad \begin{array}{r} 7\ R{=}4 \\ 7\overline{)53} \end{array}$$

T: ***In all of these problems the number of friends is the same.***
What did I change?

S: ***You changed the number of pieces of gum. So each time there is one more piece of gum, there is one more in the remainder.***

T: ***What is the next problem with no remainder?***

Check students' answers by continuing the sequence.

$$\begin{array}{r} 7\ R{=}5 \\ 7\overline{)54} \end{array} \qquad \begin{array}{r} 7\ R{=}6 \\ 7\overline{)55} \end{array} \qquad \begin{array}{r} 8 \\ 7\overline{)56} \end{array}$$

If someone says that 56 divided by 7 is 7 with a remainder of 7, show that 8 is a better answer because each of the seven friends can take another piece (from the remainder).

Continue with the following problems, and ask what the next problem with no remainder will be.

$$\begin{array}{r} 8\ R{=}1 \\ 7\overline{)57} \end{array} \qquad \begin{array}{r} 8\ R{=}2 \\ 7\overline{)58} \end{array} \qquad \cdots \qquad \begin{array}{r} 9 \\ 7\overline{)63} \end{array}$$

Exercise 2

Distribute copies of the workbook *Set of Problems #1* and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of the lesson, collect the workbooks for your review. They will be used again in Lesson W2.

Capsule Lesson Summary

Finish labeling evenly spaced marks on part of a number line. Indicate an interval on the number line and ask for some numbers belonging to it. Then ask for an interval in which to place a given number. Continue individual work in the workbook *Set of Problems #1*. (This is the second of two lessons using this workbook.)

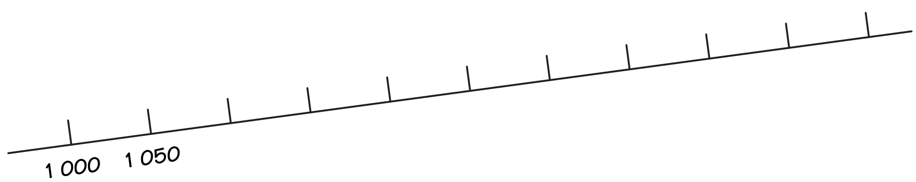
Materials

Teacher
- Meter stick

Student
- *Set of Problems #1* Workbook
- Colored pencils, pens, or crayons
- Metric ruler

Description of Lesson

Draw this number line on the board.



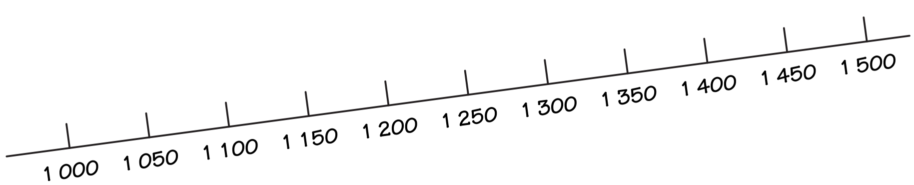
T: ***I've drawn part of a number line on the board. The marks are for numbers 50 apart.***

Point to the third mark from the left.

T: ***What number is here?***

S: ***1 100.***

Continue until all of the marks are labeled.

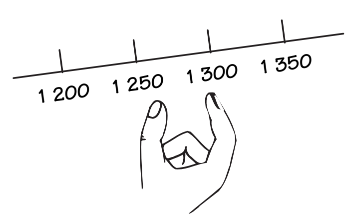


Point to various intervals, one at a time, and ask for numbers that fall within them. For example:

T: ***What are some numbers between 1 250 and 1 300?***

S: ***1 251.***

S: ***1 289.***



Repeat this activity with a few other intervals. Then give some numbers and ask where they are located. For example:

T: ***Where is 1 391 on this number line? Between what two marks?***

S: ***Between 1 350 and 1 400.***

Continue with other numbers such as 1 001, 1 010, and 1 449.

Distribute students' copies of the workbook *Set of Problems #1*. Ask students first to correct or complete pages from the previous week's work, and then to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

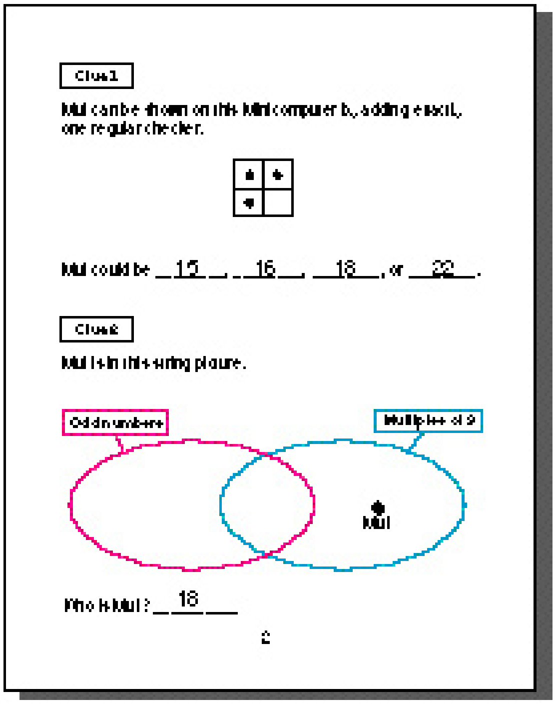
At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in the workbook during a study time or to take it home as an assignment.

Assessment Activity

An individual student progress record for the workbook is available on Blackline W2(a). You many like to use this form to monitor student work.

Home Activity

If you choose to send workbooks home with students, you may want to include a letter (reminder) to parents/guardians with this workbook. Blackline W2(b) has a sample letter.



Clue 1

Mul can be shown on this Minicomputer by adding exactly one regular checker.

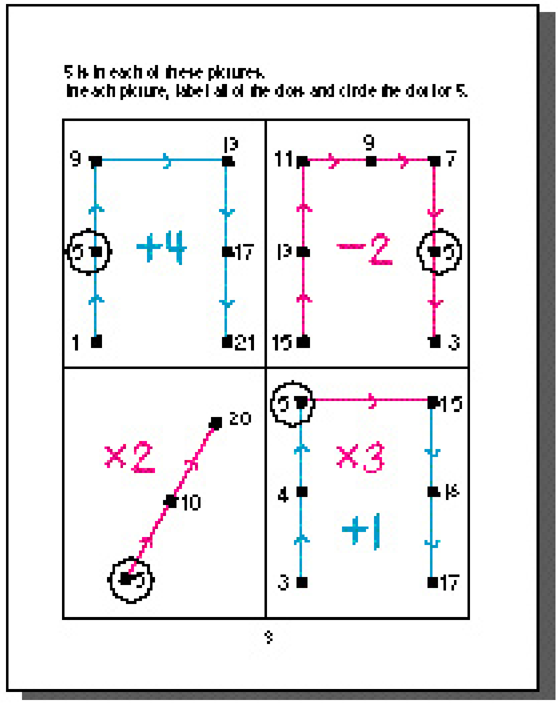
Mul could be 15, 16, 18, or 22.

Clue 2

Mul is in this string picture.

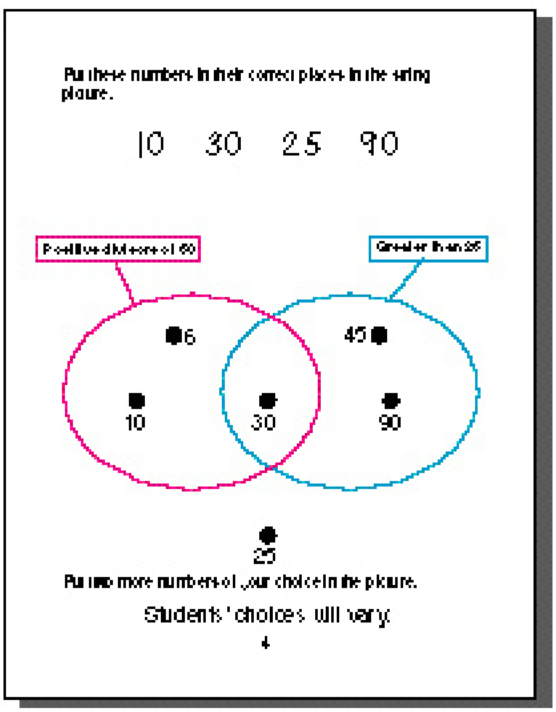
Who is Mul? 18

2



5 is in each of these pictures.
In each picture, label all of the dots and circle the dot for 5.

3



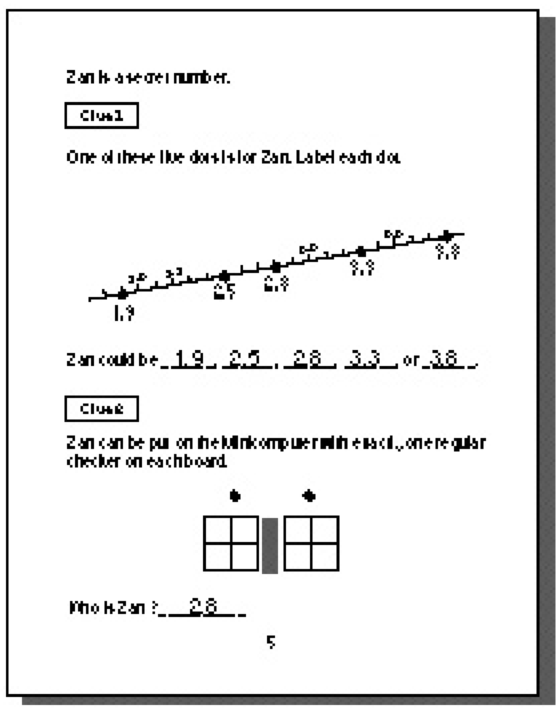
Put these numbers in their correct places in the string picture.

10 30 25 90

Put two more numbers of your choice in the picture.

Students' choices will vary.

4



Zan is a secret number.

Clue 1

One of these five dots is for Zan. Label each dot.

Zan could be 1.9, 2.5, 2.8, 3.3, or 3.8.

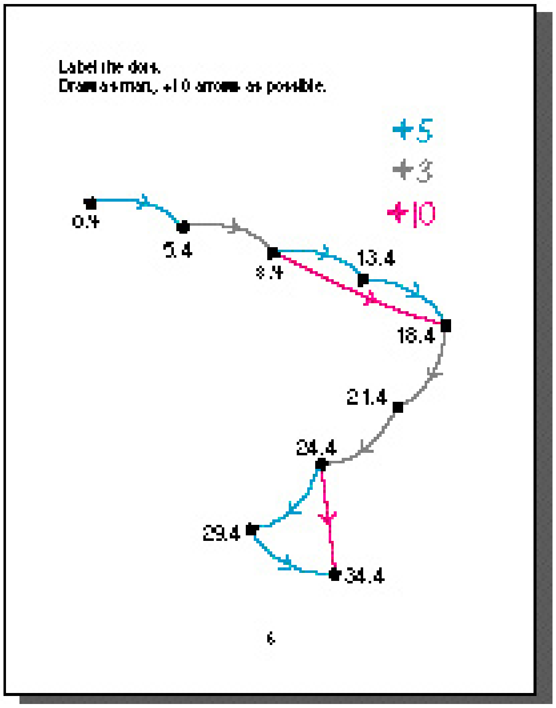
Clue 2

Zan can be put on the Minicomputer with exactly one regular checker on each board.

Who is Zan? 2.8

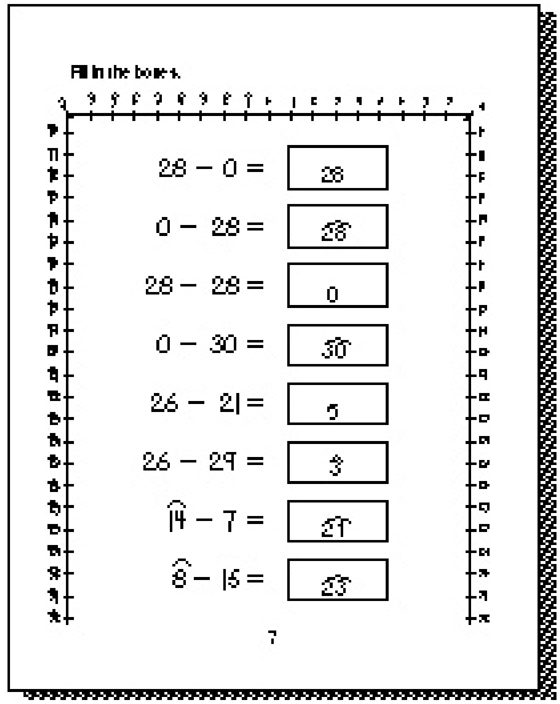
5

W2



Label the dots.
Draw as many +10 arrows as possible.

6



Fill in the boxes.

$28 - 0 = \boxed{28}$

$0 - 28 = \boxed{\hat{28}}$

$28 - 28 = \boxed{0}$

$0 - 30 = \boxed{\hat{30}}$

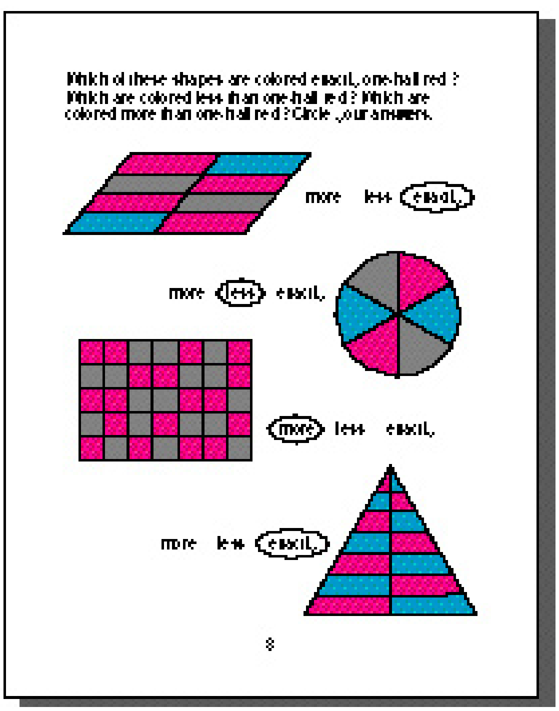
$26 - 21 = \boxed{5}$

$26 - 29 = \boxed{\hat{3}}$

$\hat{14} - 7 = \boxed{\hat{21}}$

$\hat{8} - 15 = \boxed{\hat{23}}$

7



Which of these shapes are colored exactly one-half red? Which are colored less than one-half red? Which are colored more than one-half red? Circle your answers.

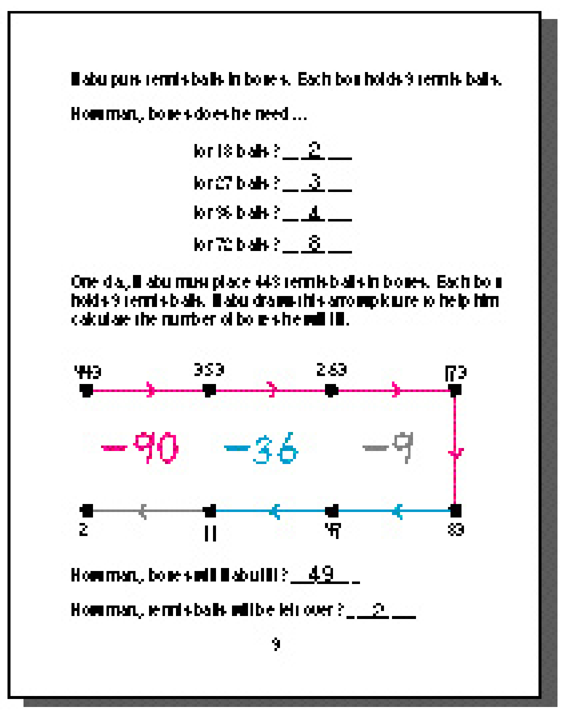
more less exactly

more less exactly

more less exactly

more less exactly

8



Babu puts tennis balls in boxes. Each box holds 9 tennis balls.

How many boxes does he need ...

for 18 balls? __2__

for 27 balls? __3__

for 36 balls? __4__

for 72 balls? __8__

One day, Babu must place 443 tennis balls in boxes. Each box holds 9 tennis balls. Babu draws this arrow picture to help him calculate the number of boxes he will fill.

How many boxes will Babu fill? __49__

How many tennis balls will be left over? __2__

9

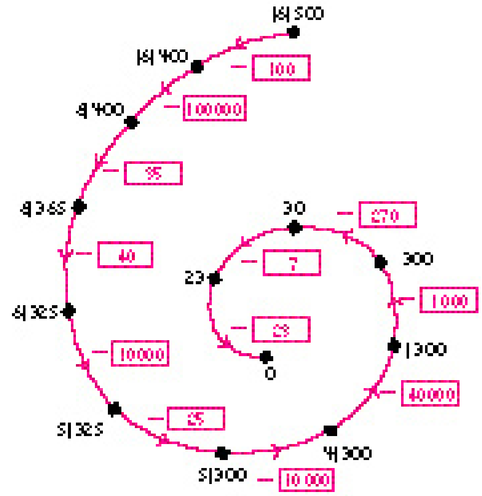
Do these calculations.

```
  158        385
   49        439
+2036       +127
 2243        951

  431        560
 -280       -352
  151        208

   26        193
  × 4       ×  8
  104       1544
```

10

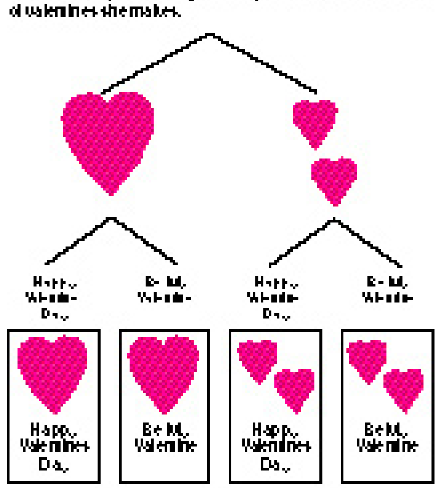
Wipe-Out

Label the arrows.



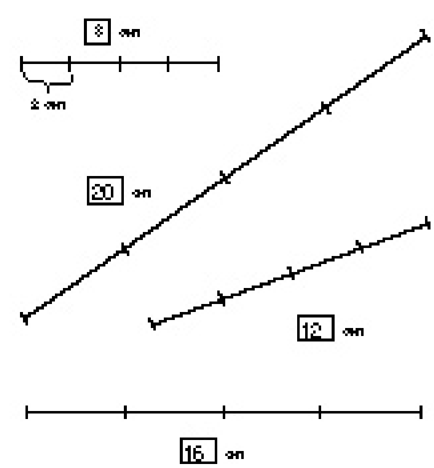
11

Lori designs her own valentines on pieces of white paper. On each, she draws either one big red heart or two small red hearts and writes either "Be My Valentine" or "Happy Valentines Day." Following this tree, show the different kinds of valentines she makes.



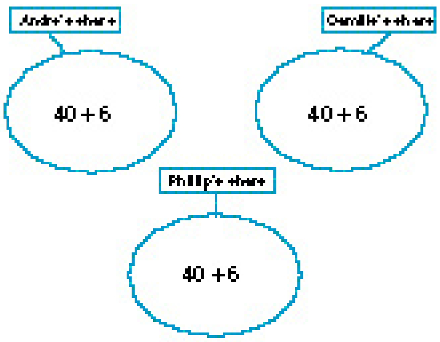
12

Measure each segment to the nearest centimeter and record its length. Then divide each segment into four pieces of the same length. One is done for you.



13

Andre, Philip, and Camille share 138 sheets of colored paper evenly. How many sheets does each person get? 46



What fraction of the total number of sheets does each person get? 1/3

Andre gives his share to Camille. What fraction of the colored paper does Camille have now? 2/3

How many sheets of paper does Camille have now? 92

14

Other ways to do the sharing are possible.

Do these calculations.

$6 \times 8 = \boxed{48}$ $7 \times 9 = \boxed{63}$

$48 \div 6 = \boxed{8}$ $63 \div 7 = \boxed{9}$

$48 \div 8 = \boxed{6}$ $63 \div 9 = \boxed{7}$

$24 \div 8 = 3$ $25 \div 8 = 3$ R 1 $26 \div 8 = 3$ R 2

$27 \div 8 = 3$ R 3 $28 \div 8 = 3$ R 4 $29 \div 8 = 3$ R 5

$30 \div 8 = 3$ R 6 $31 \div 8 = 3$ R 7 $32 \div 8 = 4$

$3 \times 12 = \boxed{36}$ $4 \times 25 = \boxed{100}$

$36 \div 3 = \boxed{12}$ $100 \div 4 = \boxed{25}$

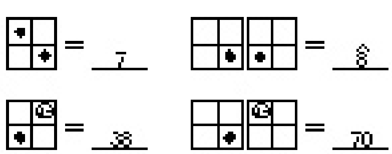
$36 \div 12 = \boxed{3}$ $100 \div 25 = \boxed{4}$

15

Ling is a secret number.

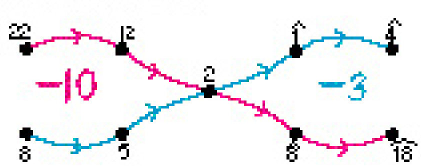
Clue 1

Ling is one of these numbers.



Clue 2

Ling is in this arrow picture.



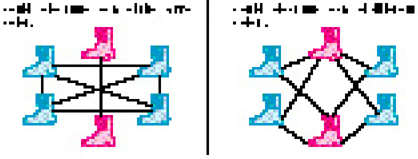
Who is Ling? 8

16

One morning, Olga has four blue socks and two red socks in her sock drawer. In the dark, Olga selects two socks and puts them on. Find the probability that she is wearing two socks of the same color.

Same

Different

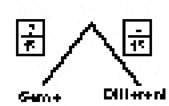


How many ways can Olga select two socks of the same color? 7

How many ways can Olga select two socks of different colors? 8

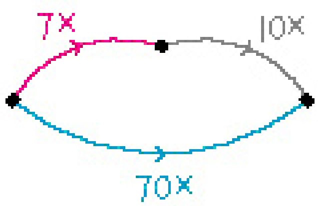
Altogether, how many ways can Olga select two socks? 15

Write the probabilities in the boxes.



Is Olga more likely to select socks of the same or of different colors? different

17



Do these calculations. This arrow picture can help you.

70 × 13 = 910

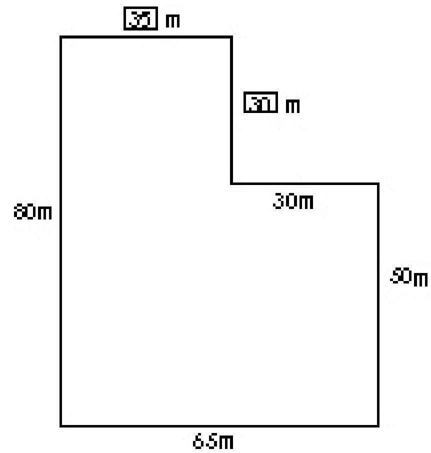
70 × 24 = 1680

70 × 50 = 3500

70 × 100 = 7000

18

This is a map of a playground. Find the length of the two sides whose lengths are not given. The shape has only square corners. Do not use a ruler.



19

Fill in the boxes. One is done for you. Hint: You do not need to add these long columns of numbers.

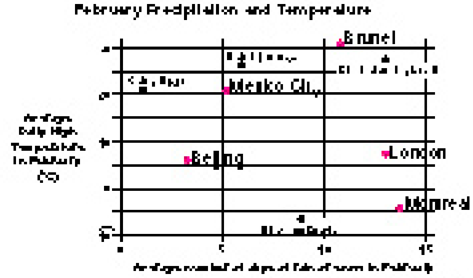
239	239	239
239	239	239
239	239	239
239	239	239
239	239	239
239	239	239
239	239	239
239	239	239
239	239	239
239	239	239
239	239	239
239	239	239
+ 239	239	239
$\boxed{3107}$	+ 239	239
	$\boxed{3346}$	+ 239
		$\boxed{3585}$
13 × 239 = $\boxed{3107}$	14 × 239 = $\boxed{3346}$	15 × 239 = $\boxed{3585}$

26 × 239 = 6 214

27 × 239 = $\boxed{6453}$

20

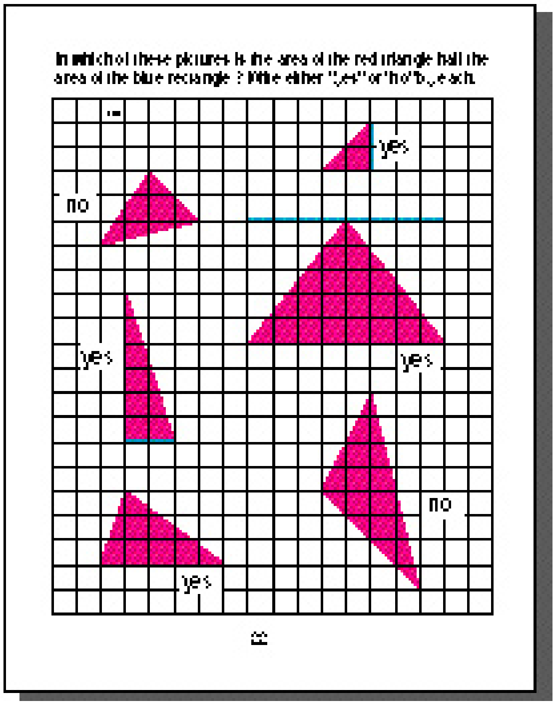
February Precipitation and Temperature



How many days of snowfall does Moscow average in February? 9
What is Moscow's average high temperature in February? -9
Which city is driest in February? Cairo
How many more days of rain does Rio de Janeiro have than Nairobi? 7
How much hotter is Nairobi than Moscow in February? 33° C
In February, London's temperature is 7°C and they receive 13 days of rain. Draw a dot for London.

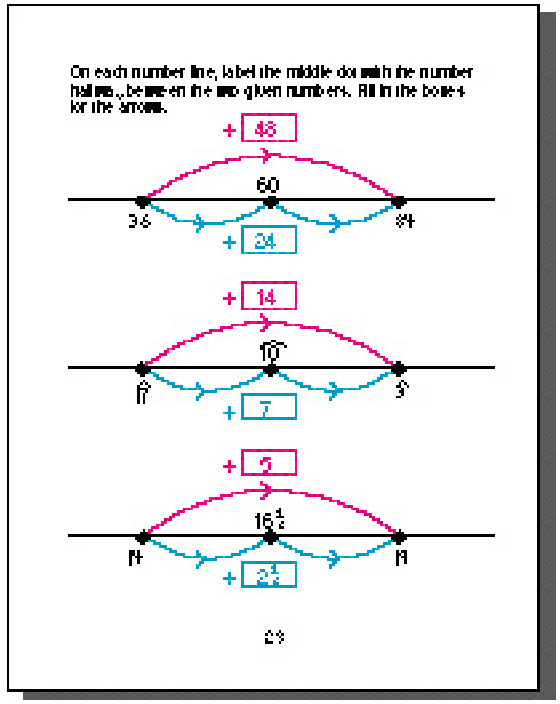
The red dots are for these cities. Label the red dots.
Montreal, Canada is cold and receives much snow in February.
Brunei, Borneo is very hot and quite rainy in February.
Mexico City, Mexico has the same temperature as Cairo in February. Beijing, China is cool and dry in February.

21



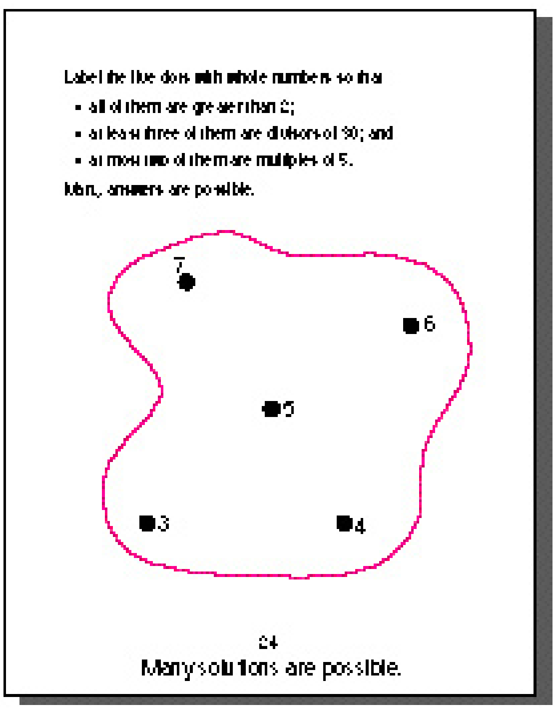
In which of these pictures is the area of the red triangle half the area of the blue rectangle? Write either "yes" or "no" by each.

22



On each number line, label the middle dot with the number halfway between the two given numbers. Fill in the boxes for the arrows.

23



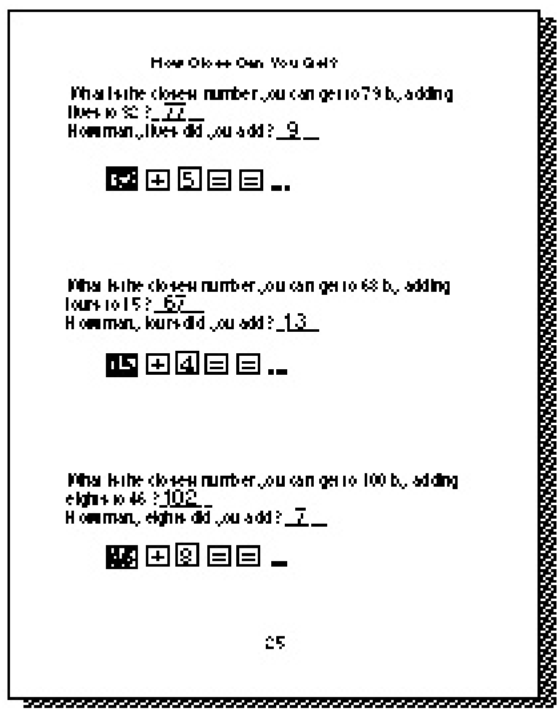
Label the five dots with whole numbers so that

- all of them are greater than 2;
- at least three of them are divisors of 30; and
- at most two of them are multiples of 5.

Many answers are possible.

24

Many solutions are possible.



How Close Can You Get?

What is the closest number you can get to 79 by adding fives to 32? 77
How many fives did you add? 9

32 + 5 = = ...

What is the closest number you can get to 68 by adding fours to 15? 67
How many fours did you add? 13

15 + 4 = = ...

What is the closest number you can get to 100 by adding eights to 46? 102
How many eights did you add? 7

46 + 8 = = ...

25

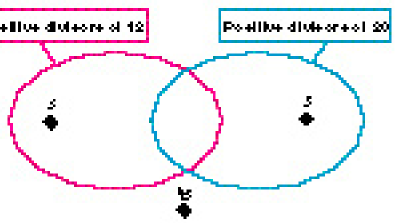
The red label is one of these:

Positive prime numbers
Multiples of 3
Multiples of 6
Positive divisors of 12
Positive divisors of 20
Greater than 10
Less than 100
Odd numbers

The blue label is one of these:

Positive prime numbers
Multiples of 3
Multiples of 6
Positive divisors of 12
Positive divisors of 20
Greater than 10
Less than 100
Odd numbers

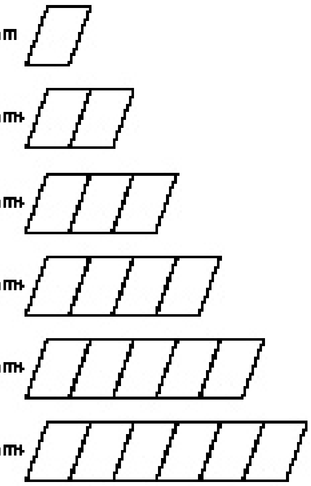
Label the strings.



26

How Many Parallelograms?

How many parallelograms can you find in each picture?



1 parallelogram

3 parallelograms

6 parallelograms

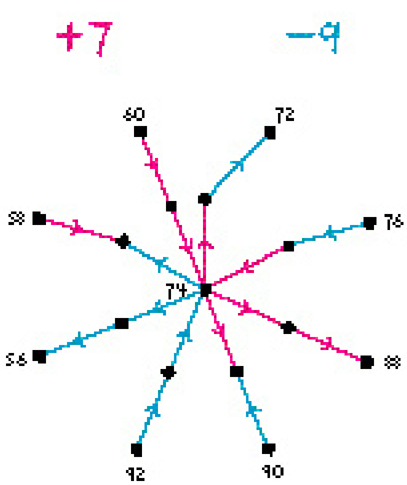
10 parallelograms

15 parallelograms

21 parallelograms

27

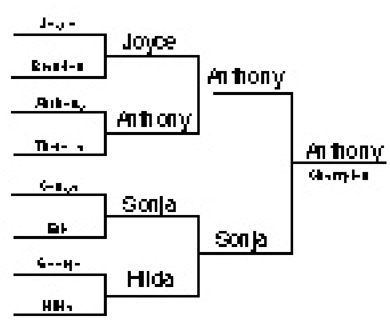
Connect 74 to each of the other numbers using exactly two arrows. One is done for you.



28

Other solutions are possible.

Eight players compete in a tennis tournament. The chart below shows how a winner is determined after three rounds.

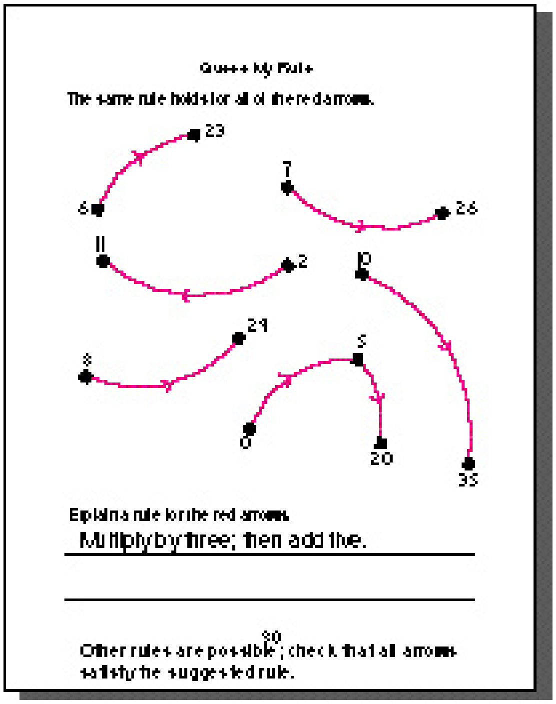


- Anthony wins the second round against Joyce.
- Sonja wins two matches and loses one.
- George loses to Hilda in the first round.

Complete the chart to show the tournament information.

Who is the champion? Anthony

29



Guess My Rule

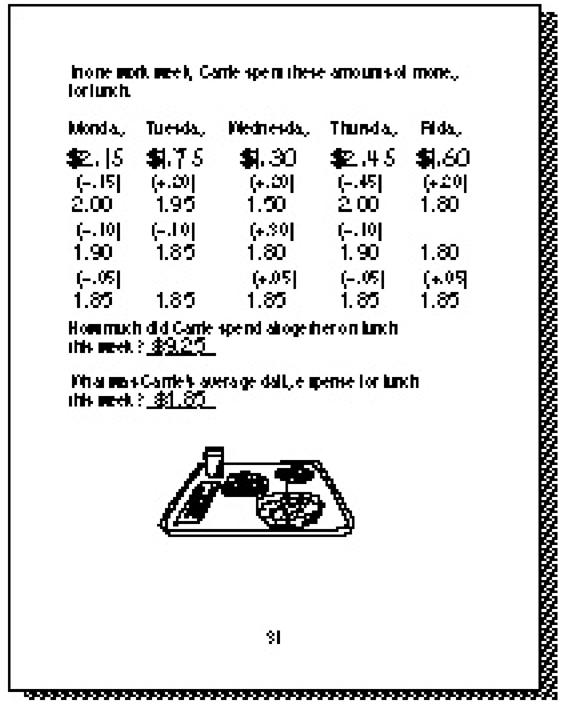
The same rule holds for all of these red arrows.

Explain a rule for the red arrows.

Multiply by three; then add five.

30

Other rules are possible; check that all arrows satisfy the suggested rule.



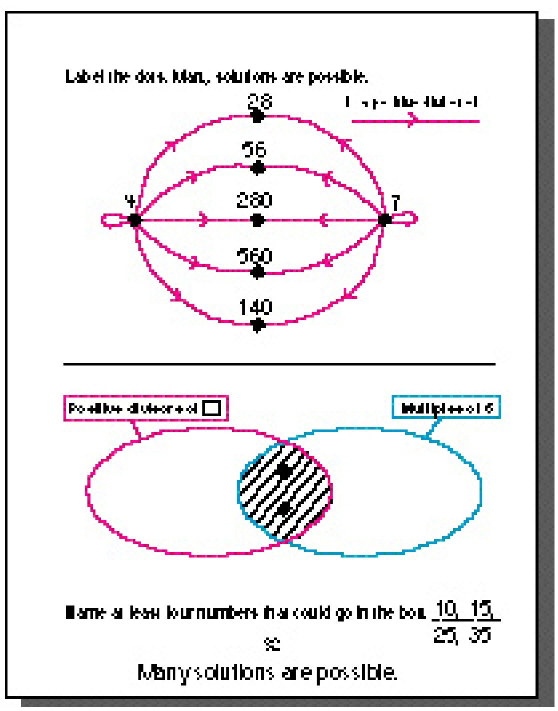
In one work week, Carrie spent these amounts of money for lunch.

Monday	Tuesday	Wednesday	Thursday	Friday
\$2.15	\$1.75	\$1.30	\$2.45	\$1.60
(−.15)	(+.20)	(+.20)	(−.45)	(+.20)
2.00	1.95	1.50	2.00	1.80
(−.10)	(−.10)	(+.30)	(−.10)	
1.90	1.85	1.80	1.90	1.80
(−.05)		(+.05)	(−.05)	(+.05)
1.85	1.85	1.85	1.85	1.85

How much did Carrie spend altogether on lunch this week? \$9.25

What was Carrie's average daily expense for lunch this week? \$1.85

31



Label the dots. Many solutions are possible.

Name at least four numbers that could go in the box. 10, 15, 25, 35

32

Many solutions are possible.

Capsule Lesson Summary

Find the distance between two numbers. Solve inequalities involving integers and distance on the number line. Draw either a red cord or a blue cord between two numbers in a picture depending upon whether their distance is less than 10, or more than or equal to 10.

Materials

Teacher
- Colored chalk
- Blackline W3
- Straightedge

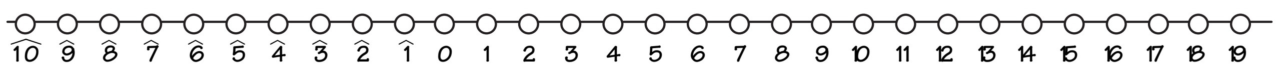
Student
- Colored pencils
- Paper
- Number line picture

Advance Preparation: Before the lesson, draw the number line picture for Exercise 1 on the board, or make a transparency of Blackline W3 to use on an overhead projector. Use Blackline W3 to make copies of this number line picture for students.

Description of Lesson

Exercise 1

Display a number line with open circles, and announce to the class that today they will work with integers only. Provide students with copies of the number line.



T: ***What is the distance between 8 and 12?*** (4)

Check by starting at 8 on the number line and counting as you point to 9, 10, 11, and 12 in succession.

T: ***What is the distance between 12 and 8?*** (4)

Perhaps students will say that the distance is $\widehat{4}$. Observe that the distance between home and school is the same as the distance between school and home; here the distance between 8 and 12 is the same as between 12 and 8. Mention that distance is not negative.

T: ***What is the distance between 40 and 64?*** (24)
… between 81 and 75? (6)
… between 990 and 1 001? (11)
… between 57 and 57? (0)

Point to the circles for $\widehat{3}$ and 5 on the number line.

T: ***What is the distance between $\widehat{3}$ and 5?***

S: *8.*

T: ***How do you know?***

S: *I counted.*

S: ***The distance between $\widehat{3}$ and 0 is 3, and the distance between 0 and 5 is 5. 3 + 5 = 8.***

T: ***What is the distance between 2 and $\widehat{2}$?*** (4)
… between $\widehat{1}$ and 7? (8)
… between $\widehat{10}$ and $\widehat{4}$? (6)
… between $\widehat{1}$ and $\widehat{8}$? (7)

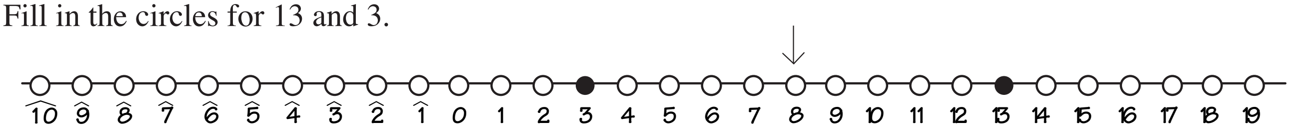
Draw an arrow pointing to 8 on the number line as you ask,

T: ***Can you tell me a number whose distance from 8 is exactly 5?***

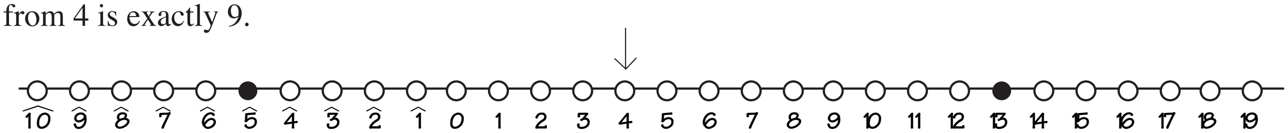
S: ***13.***

S: ***3.***

Fill in the circles for 13 and 3.



Erase the arrow and the shading in the circles for 13 and 3. Then ask for numbers whose distance from 4 is exactly 9.



Erase the arrow and the shading in the circles for $\widehat{5}$ and 13.

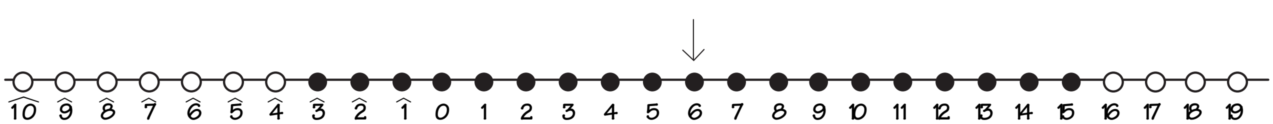
Draw an arrow at 6 as you say,

T: ***Tell me some numbers whose distance from 6 is less than 10.***

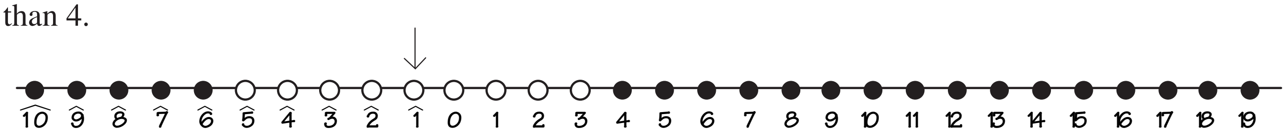
Fill in circles for correct numbers as they are offered. After two or three numbers have been given, ask,

T: ***What is the greatest integer whose distance from 6 is less than 10?*** (15) ***… the least?*** ($\widehat{3}$)

Students should now be able to quickly identify all the integers whose distance from 6 is less than 10.



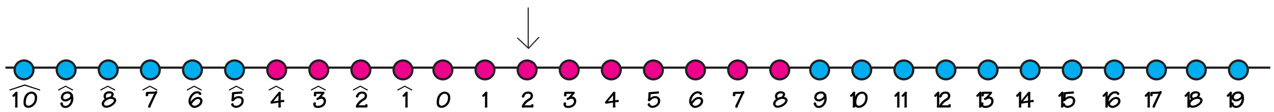
Erase the arrow and the shading in the circles. Then ask for integers whose distance from $\widehat{1}$ is more than 4.



T: ***Are there any integers other than those we've shown on this number line that are further than 4 away from $\hat{1}$?***

S: ***Lots more! 20, 21, 22, 23, and so on; $\widehat{11}$, $\widehat{12}$, $\widehat{13}$, $\widehat{14}$, and so on.***

Repeat this activity, asking first for numbers whose distance from 2 is less than 7 (show in red below) and then for numbers whose distance from 2 is 7 or more than 7 (shown in blue below). You may ask students to color in circles accordingly on their number lines.



Exercise 2

T: ***Name two numbers whose distance from each other is 6.***

List some possible pairs of numbers as students respond.

Distance of 6

3 and 9
$\hat{2}$ and 4
16 and 10
$\widehat{20}$ and $\widehat{14}$
1 and $\hat{5}$
138 and 132

S: ***3 and 9.***

T: ***How do you know?***

S: ***$3 + 6 = 9$ (or $9 - 6 = 3$).***

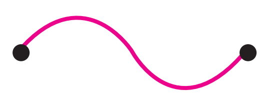
S: ***$\hat{2}$ and 4. $\hat{2}$ and 0 are 2 apart, and 0 and 4 are 4 apart; $2 + 4 = 6$.***

Prompt students, if necessary, to include pairs with one or two negative numbers. There are infinitely many solutions.

Erase the board before going on to Exercise 3.

Exercise 3

On the board, draw a red cord connecting two dots and record this rule.



A red cord joins two numbers
if and only if
the distance between them is less than 10.

T: ***Could the dots be for 32 and 40?***

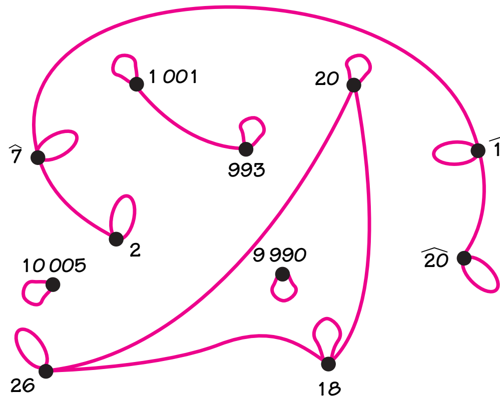
S: ***Yes, the distance between 32 and 40 is 8, and 8 is less than 10.***

Continue by asking if the dots could be for other pairs of numbers. For example:

T: ***Could the dots be for 17 and 7?*** (No)
… for 95 and 104? (Yes)
… for 4 and $\hat{4}$? (Yes)
… for $\hat{5}$ and 5? (No)
… for 4 and $\widehat{14}$? (No)
… for $\hat{5}$ and $\widehat{14}$? (Yes)

W3

Draw dots for ten to fifteen numbers on the board, and ask students to draw as many red cords as they can between the numbers. For example:



When a student takes a turn drawing a red cord, you may occasionally want to ask what the actual distance between the two numbers is. It might take a suggestion from you that the distance between a number and itself is 0, certainly less than 10. Once students see one red loop in the picture, they should observe that a red loop belongs at every dot.

Erase the picture but not the rule for red cords.

Exercise 4

Near the rule for red cords, draw a blue cord connecting two dots and record another rule—a rule for blue cords.

A blue cord joins two numbers
if and only if
the distance between them is 10 or more than 10.

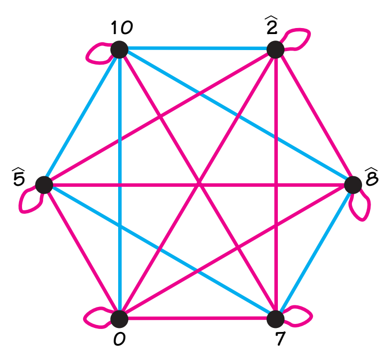
T: ***Could these dots be for 5 and 21?***

S: ***Yes; the distance from 5 to 21 is 16 and 16 is more than 10.***

Continue by asking if the dots could be for other pairs of numbers. For example:

T: ***Could these dots be for 8 and $\widehat{8}$?*** (Yes)
… for 595 and 603? (No)
… for $\widehat{5}$ and $\widehat{15}$? (Yes)
… for $\widehat{8}$ and $\widehat{1}$? (No)
… for $\widehat{13}$ and $\widehat{26}$? (Yes)
… for $\widehat{18}$ and $\widehat{11}$? (No)
… for 9 995 and 10 005? (Yes)

On the board, draw dots for six numbers as in the next illustration.



Ask the class first to draw all of the possible blue cords and then to draw all of the possible red cords. Frequently ask for actual distances between numbers in the picture.

T: ***What do you notice about the picture?***

S: ***Every dot has a red loop.***

S: ***Every dot has five other cords connected to it.***

S: ***There are more red cords than blue cords.***

T: ***How many red cords are there?*** (15, counting loops as well)
Blue cords? (6)
Altogether? (25)

S: ***Any two numbers are joined by either a red or a blue cord.***

Check this observation with several pairs of numbers.

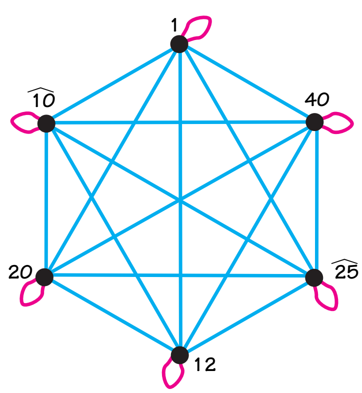
You may need to prompt your students to notice that any two numbers in the picture are joined by either a red cord or a blue cord.

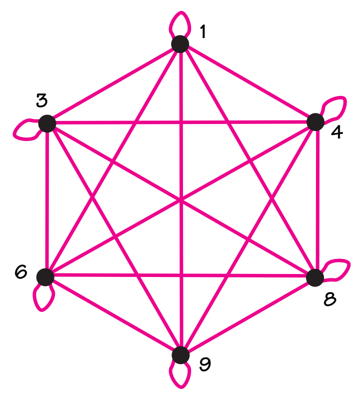
T: ***We looked at just these six numbers, but do you think any two numbers are joined by either a red cord or a blue cord?***

S: ***Yes!***

Direct students to draw six dots in a circle on a piece of paper. Suggest they label the dots with numbers of their choice, and then draw red and blue cords between them.

As students finish, challenge them to draw a second picture with six numbers so that only blue cords can be drawn between them (except, of course, for red loops at each number). For some students, you may prefer that they first draw a picture with six numbers between which only red cords can be drawn. Examples of both situations are given below.





Capsule Lesson Summary

Solve a detective story with clues involving the distance between numbers. Find labels for the dots in the four possible red-blue cord pictures with exactly three dots.

Materials

Teacher
- Colored chalk
- Straightedge

Student
- Worksheets W4(a), (b), and (c)

Description of Lesson

Exercise 1

Begin by announcing that again today the class will work only with integers.

Ask for the distance between the following numbers:

8 and 20	(12)	$\widehat{13}$ and $\widehat{21}$	(8)
20 and 8	(12)	100 and $\widehat{100}$	(200)
$\hat{4}$ and 5	(9)	$\hat{6}$ and $\hat{6}$	(0)

On the board, draw a red cord and a blue cord, and record these rules.

T: ***Give us two integers that are joined by a red cord.***

S: ***16 and 20; the distance between 16 and 20 is 4, and 4 is less than 10.***

S: ***$\hat{1}$ and 1; they are only 2 apart.***

T: ***Give us two integers that can be joined by a blue cord.***

S: ***25 and 35; the distance between 25 and 35 is 10.***

S: ***0 and 1 000; they are very far apart!***

Draw a number line on the board but only put a mark for 29 on it.

Draw two dots joined by a red cord and put 29 at one dot.

T: ***29 is here. What integers could the other dot be for?***

As students give correct responses, list them on the board. After four or five numbers have been given, ask,

T: ***What is the greatest integer that could be at the other dot?*** (38)
… and the least? (20)

W4

Locate 20 and 38 equidistant from the mark for 29 on the number line and color red dots for them. (See the illustration below.)

T: ***Is 29 connected to itself by a red cord?***

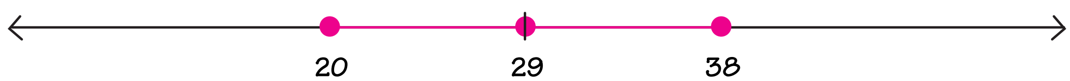
S: ***Yes, we can draw a red loop at the dot for 29.***

Draw a red loop at 29, and color a red dot at its mark on the number line.

T: ***On the number line, where are the other integers whose distance from 29 is less than 10?***

S: ***Between 20 and 38.***

Connect the dots for 20 and 38 with a red segment.



Emphasize that all the integers between 20 and 38 (inclusive) can be connected to 29 with a red cord.

Draw two dots connecting a blue cord, and put 22 at one dot.

T: ***22 is here. What integers could the other dot be for?***

List several correct responses on the board; then ask for the least integer to the right of 22 (greater than 22) that the other dot could be for (32) and the greatest integer to the left of 22 (less than 22) that the other dot could be for (12). Your students should then be able to tell you where all of the possibilities lie. There are infinitely many. On the board, illustrate this information in a second number line picture drawn below the first.

Emphasize that the other dot could be for 32 or any integer greater than 32, or for 12 or any integer less than 12.

T: ***I'm thinking of an integer whose distance from 29 is less than 10*** (trace the red segment in the first number line) ***and whose distance from 22 is 10 or more than 10*** (trace the blue rays in the second number line)***. What number could I be thinking of?***

List the possibilities as the students identify them.

32 33 34 35 36 37 38

T: ***I'll give you another clue: The number I'm thinking of is closer to 25 than it is to 40.***

Invite students to check several of the possibilities in the list and find that your number is 32. Perhaps someone will notice that once one number is ruled out as a possibility, the numbers greater than it are also ruled out.

32 ~~33~~ ~~34~~ ~~35~~ ~~36~~ ~~37~~ ~~38~~

Erase whatever is on the board except for the rules for red and blue cords.

Exercise 2

Draw three dots on the board. In this exercise we will ignore red loops. Agree with the class that since they know there are always red loops, they do not need to consider loops for the time being.

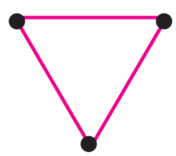
T: ***How many cords can we draw between three dots?***

S: ***Three cords.***

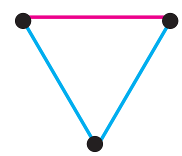
T: ***We have red and blue cords. How many cords of each color could a picture with exactly three dots have?***

Draw the possible pictures as they are described.

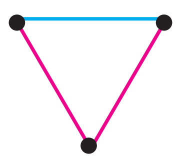
S: ***Three red cords and no blue cord.***



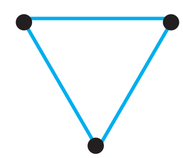
S: ***Two blue cords and one red cord.***



S: ***Two red cords and one blue cord.***



S: ***Three blue cords and no red cord***



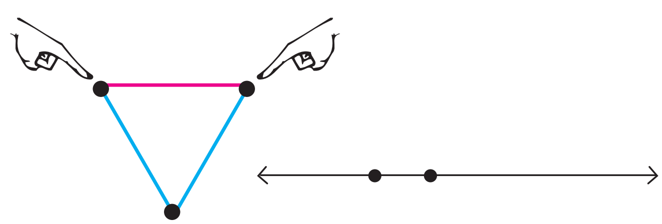
Note: Consider a rotation of a coloring as the same coloring.

T: ***Let's label the dots in these pictures. What could they be for?***

Use number lines to help your students find and check possible labelings.

The following is a possible discussion of the picture with one red and two blue cords.

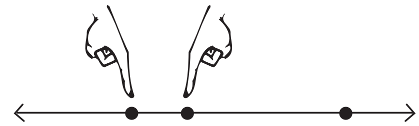
T: ***These two numbers must be close together. By "close together," we mean at a distance less than 10. I'll draw two dots close together on this number line for them.***



This number (point to the bottom dot) ***is far away from both of the other numbers*** (trace the blue cords)***. By "far away," we mean at a distance of 10 or more than 10. Where could we put a dot for this number on the number line?***

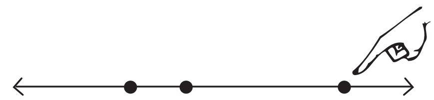
W4

Let a student draw a dot on the number line. It can be on either side of the pair of dots already there, but it should be quite a bit farther away from them than they are from each other.



T: ***What integers could these dots be for? Remember, these two numbers are close together; the distance between them is less than 10.***

S: ***1 and 5.***

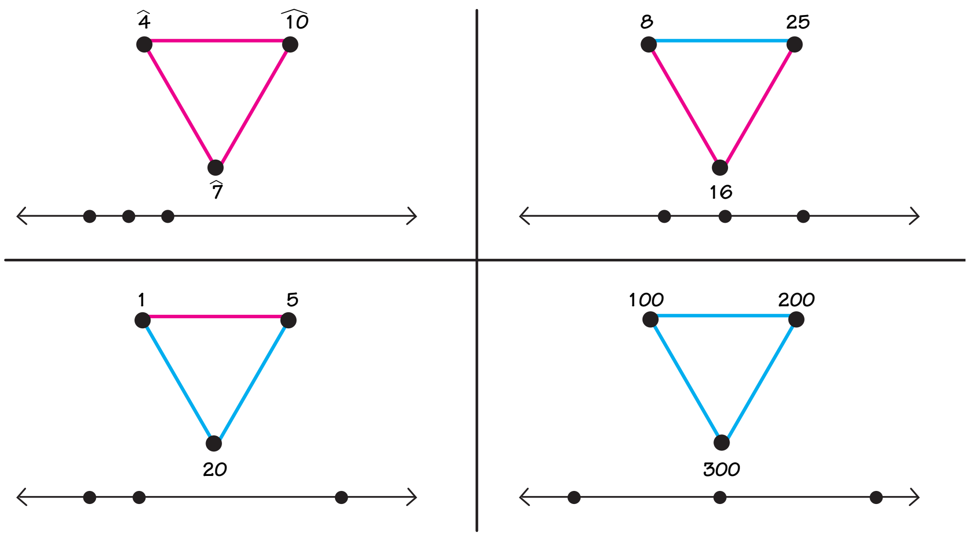


T: ***This number must be a distance of 10 or more than 10 from each of the other numbers.***

S: ***20.***

Label the dots in the cord picture with the numbers suggested. Check the solution with the class by finding the three distances involved.

Likewise, consider the other three pictures. One solution for each is shown below.



Exercise 3

T: ***Let's look at the possible cord pictures with four dots. How many possibilities do you think there are?***

S: ***Eight.***

S: ***More than four.***

T: ***Actually there are ten cord pictures† with four dots. They are all drawn for you on these worksheets. Your job will be to label the dots in each picture. I'll warn you that for two pictures it is impossible to find labels for the dots. When you find them, × them out.***

Distribute copies of Worksheets W4(a), (b), and (c). Let your students work individually or with a partner for the rest of the class period.

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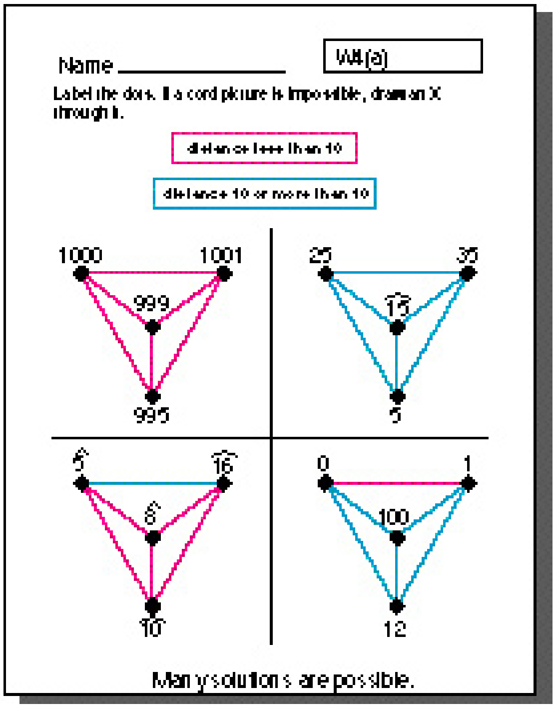
†Flips and rotations are not part of this count.

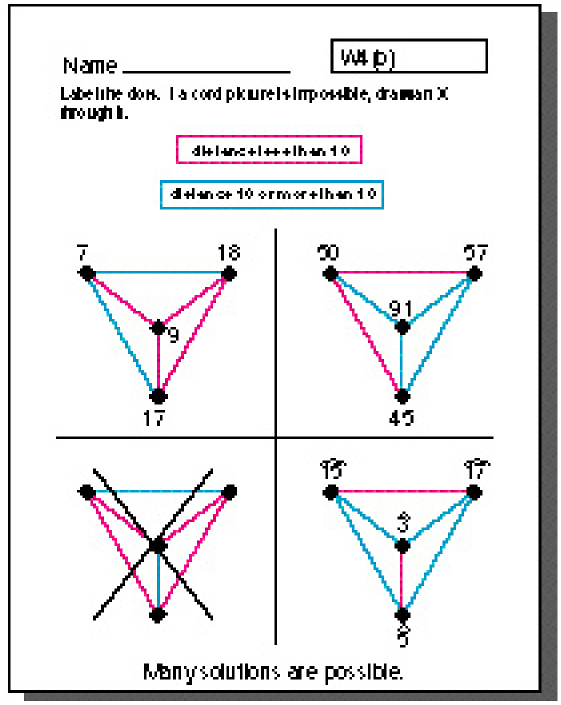

Name ____________ W4(a)

Label the dots. If a cord picture is impossible, draw an X through it.

distance less than 10

distance 10 or more than 10

Many solutions are possible.

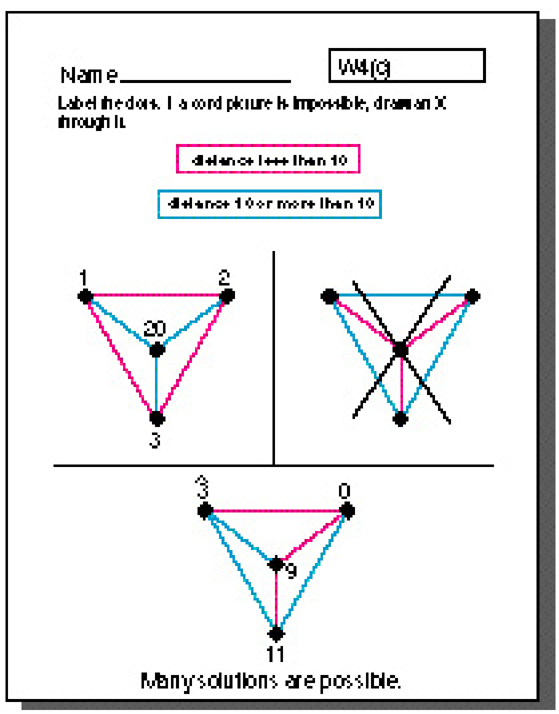

Name ____________ W4(b)

Label the dots. If a cord picture is impossible, draw an X through it.

distance less than 10

distance 10 or more than 10

Many solutions are possible.


Name ____________ W4(c)

Label the dots. If a cord picture is impossible, draw an X through it.

distance less than 10

distance 10 or more than 10

Many solutions are possible.

Capsule Lesson Summary

Solve a calculator puzzle in which 75 is put on the calculator using only a limited set of keys. Begin the workbook *Set of Problems #2*. (This is the first of two lessons using this workbook.)

Materials

Teacher	• Calculator	• *Set of Problems #2* Workbook
Student	• Calculator	• Colored pencils, pens, or crayons
		• Metric ruler

Description of Lesson

Arrange for every student to have access to a calculator. You may like to let students work with a partner.

T: ***Today I have a calculator puzzle for you. The puzzle requires that you use only a few of the keys on the calculator.***

Write these key symbols on the board: $\boxed{4}$, $\boxed{6}$, $\boxed{+}$, $\boxed{-}$, $\boxed{\times}$, $\boxed{\div}$, $\boxed{=}$.

T: ***You may use just these keys (*** $\boxed{4}$, $\boxed{6}$, $\boxed{+}$, $\boxed{-}$, $\boxed{\times}$, $\boxed{\div}$, $\boxed{=}$ ***), but you may use them in any way you like. Start with 0 on your calculator display, and then try to put 75 on the display.***

You may need to remind students that they may use only the keys in the list on the board. To help them remember, suggest students record the sequence of keys they press to get 75.

When many students have found at least one solution, begin to record some of their suggestions on the board. For example:

75: $\boxed{6}\boxed{4}\ \boxed{\times}\ \boxed{4}\ \boxed{+}\ \boxed{4}\boxed{4}\ \boxed{\div}\ \boxed{4}\ \boxed{=}$

$\boxed{4}\boxed{4}\boxed{4}\ \boxed{+}\ \boxed{6}\ \boxed{\div}\ \boxed{6}\ \boxed{=}$

$\boxed{6}\ \boxed{\div}\ \boxed{6}\ \boxed{+}\ \boxed{6}\boxed{4}\ \boxed{+}\ \boxed{6}\ \boxed{+}\ \boxed{4}\ \boxed{=}$

$\boxed{4}\boxed{4}\ \boxed{\div}\ \boxed{4}\ \boxed{+}\ \boxed{6}\boxed{4}\ \boxed{=}$

$\boxed{4}\boxed{4}\boxed{4}\ \boxed{\div}\ \boxed{4}\ \boxed{-}\ \boxed{4}\boxed{4}\ \boxed{+}\ \boxed{4}\ \boxed{=}\boxed{=}$

$\boxed{6}\boxed{6}\boxed{6}\ \boxed{\div}\ \boxed{6}\ \boxed{-}\ \boxed{6}\ \boxed{=}\boxed{=}\boxed{=}\boxed{=}\boxed{=}\boxed{=}$

§¢ å ¢¢ ß ¢ ≠≠ ƒ ¢ å ¢¢ å § ≠

Note: Some of these solutions depend on special features of the calculator. You may want to read "Role and Use of Calculators" in Section One, Notes to the Teacher to learn more about the features used here.

You may notice that some students have difficulty finding a solution because they do not use the $\boxed{\div}$ key. In this case, ask the class if they see anything common to all solutions discovered so far, and observe that every solution uses the $\boxed{\div}$ key at least once. Some students may be able to explain this fact. If you only add, subtract, and multiply (but do not divide) with even numbers, you can only get even numbers. With the $\boxed{4}$ and $\boxed{6}$ keys alone, you will only be using even numbers.

Distribute copies of the workbook *Set of Problems #2* and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of the lesson, collect the workbooks for your review. They will be used again in Lesson W6.

Home Activity

Create calculator puzzles for students to work on at home with family members. For example:

- The only keys you may use are $\boxed{4}$, $\boxed{6}$, $\boxed{+}$, $\boxed{-}$, $\boxed{\times}$, $\boxed{\div}$, and $\boxed{=}$ but you may use them in any way you like. Start at 0 and try to put 99 (or 200) on the display.
- Do a similar problem with only the $\boxed{5}$, $\boxed{7}$, $\boxed{+}$, $\boxed{-}$, $\boxed{\times}$, $\boxed{\div}$, and $\boxed{=}$ keys.

Capsule Lesson Summary

Introduce and play a game called *Minicomputer Nim*. Continue individual work in the workbook *Set of Problems #2*. (This is the second of two lessons using this workbook.)

Materials

Teacher	• Minicomputer set	• *Set of Problems #2* Workbook • Colored pencils, pens, or crayons • Metric ruler
Student	• Minicomputer set	

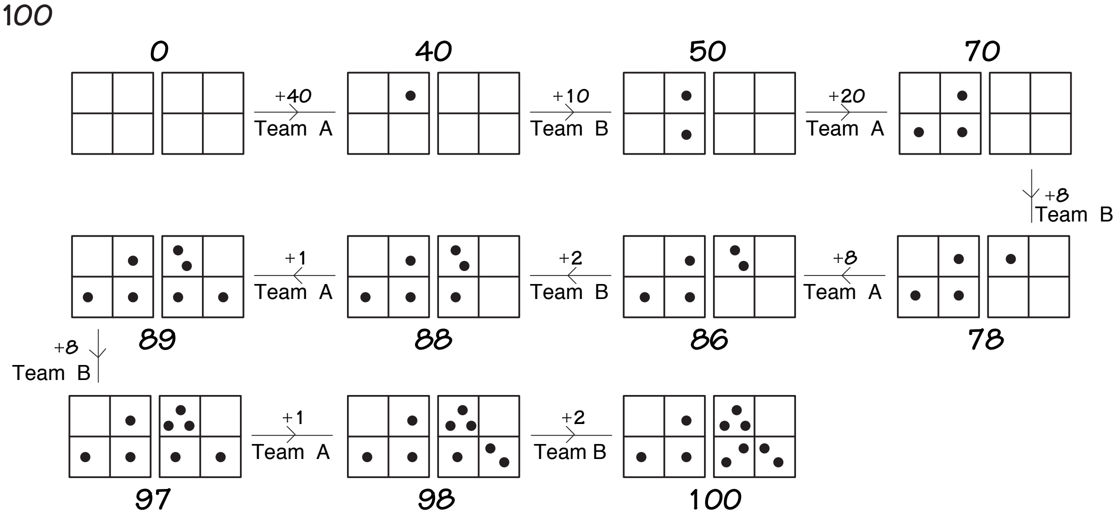
Description of Lesson

Display two Minicomputer boards for the class. Then explain how to play a two-person or two-team game called *Minicomputer Nim*.

- Start with 0 on the Minicomputer and set a goal (for example, 100).
- Players (teams) take turns putting one regular checker anywhere on the Minicomputer.
- The player (team) to reach the goal first on the Minicomputer wins the game. If a play takes the number on the Minicomputer over the goal the player (team) automatically loses.

Play a practice game with you versus the class. On your turn, include plays on the tens board so the number gets close to the goal quickly. (The game becomes boring if play is too cautious; that is, if players add checkers only to the 1- or 2-square.) By playing against the class first, you can suggest that the game gets more interesting as you draw closer to the goal. Here is a sample game:

Goal: 100



Divide the class into groups of four. In each group, pair students into two teams to play *Minicomputer Nim*. You may suggest they play the first game with 100 as the goal, and then let students set their own goals.

Distribute students' copies of the workbook *Set of Problems #2*. Ask students first to correct or complete pages from the previous week's work, and then to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in the workbook during a study time or to take it home as an assignment.

Assessment Activity

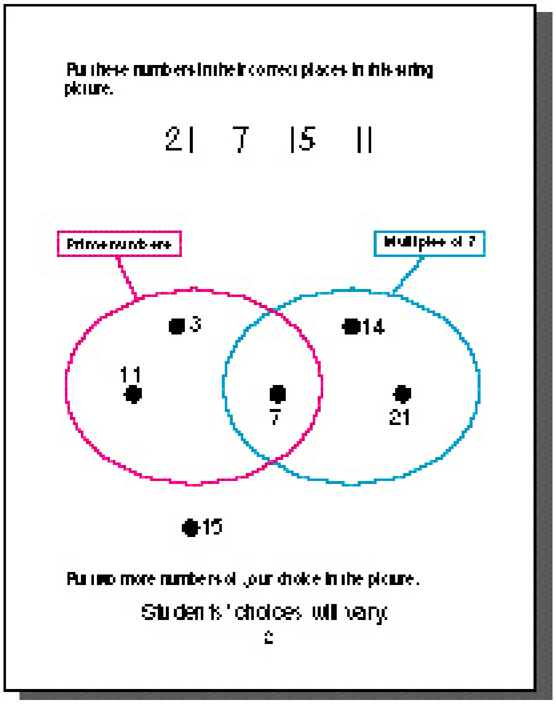
An individual student progress record for the workbook is available on Blackline W6. You may like to use this form to monitor student work.

Extension Activity

Introduce variations of the *Minicomputer Nim* game using some weighted checkers. For example, play with regular and 2-checkers so a player can put on a regular or a 2-checker on a turn. In such variations you will probably want to restrict the choice of checkers to regular checkers and one or two different weighted checkers.

Center Activity

Put Minicomputers in a center where students can pair up to play *Minicomputer Nim*. Send home the rules for *Minicomputer Nim* for students to play with family members.

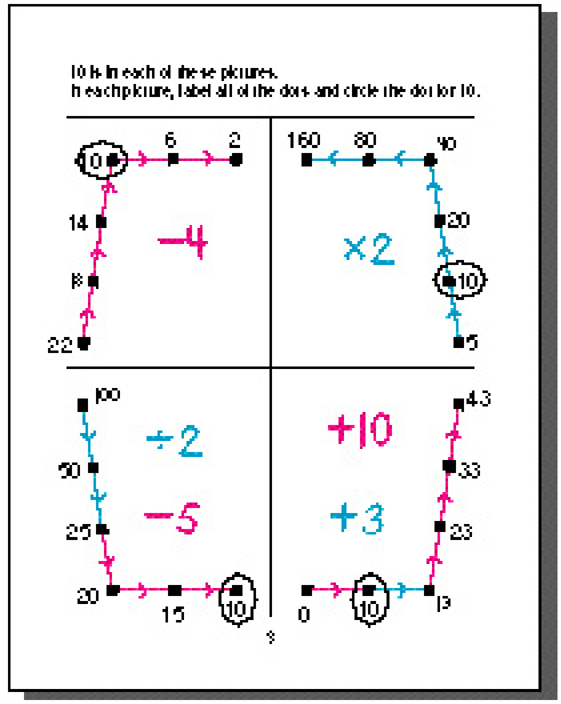

Put these numbers in their correct places in this string picture.

21 7 15 11

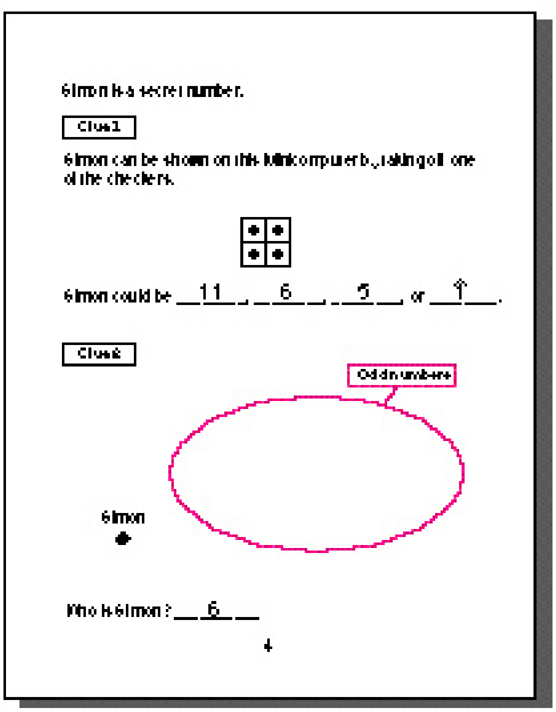
Put two more numbers of your choice in the picture.

Students' choices will vary.

2


10 is in each of these pictures.
In each picture, label all of the dots and circle the dot for 10.

3


Simon is a secret number.

Clue 1

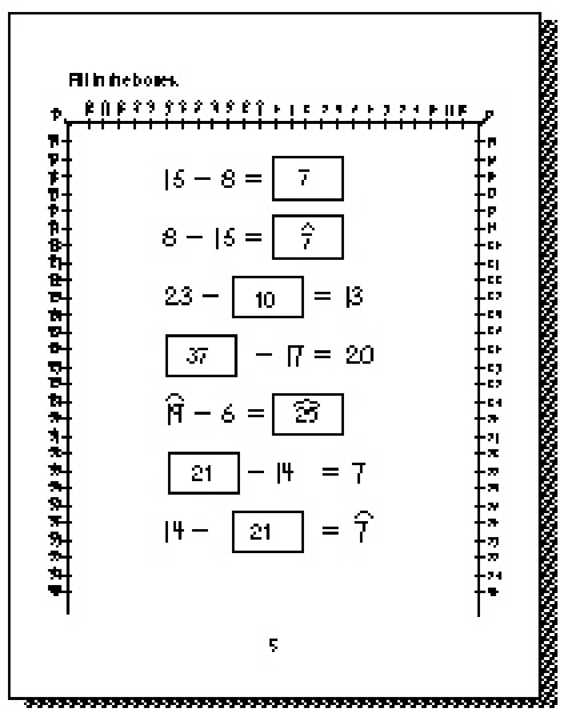
Simon can be shown on this Minicomputer by taking off one of the checkers.

Simon could be 11, 6, 5, or 7.

Clue 2

Who is Simon? 6

4


Fill in the boxes.

$15 - 8 = \boxed{7}$

$8 - 15 = \boxed{\hat{7}}$

$23 - \boxed{10} = 13$

$\boxed{37} - 17 = 20$

$\hat{19} - 6 = \boxed{\hat{25}}$

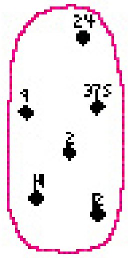
$\boxed{21} - 14 = 7$

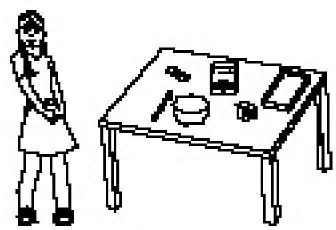
$14 - \boxed{21} = \hat{7}$

5

Put these numbers in the blanks so that the story makes sense.

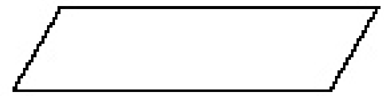
Ollie is making brownies from a box of mix. The directions call for _2_ eggs and _1⅓_ cups water to be added to the mix. The oven temperature is to be set at _375_°F. Ollie plans to bake the brownies in a _9_ inch by _12_ inch pan. When they are done, she will cut the baked brownies into _24_ pieces.



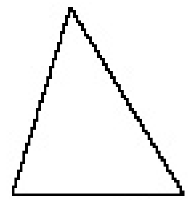


6

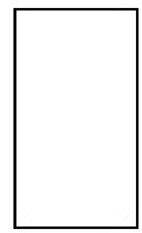
Use a ruler to find the perimeter of each shape.



perimeter = _34_ cm



perimeter = _24_ cm



perimeter = _28_ cm

7

In one week, these were Aaron's scores on the daily spelling quizzes.

~~18~~	~~14~~	~~15~~	~~20~~	~~13~~
16	~~15~~	16	~~17~~	16
16	16	16	16	16

What was Aaron's total score for the week? _80_

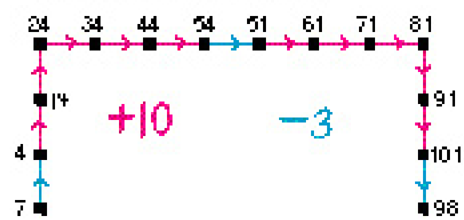
What was Aaron's average daily score that week? _16_

8

Roy is a secret number.

Clue 1

Roy is in this arrow picture. Label the dots.



Clue 2

The double of Roy is more than 200.

Who is Roy? _101_

9

Complete these calculations.

$$\begin{array}{r} 1057 \\ 281 \\ +963 \\ \hline 2301 \end{array}$$

$221 - 55 =$ 166

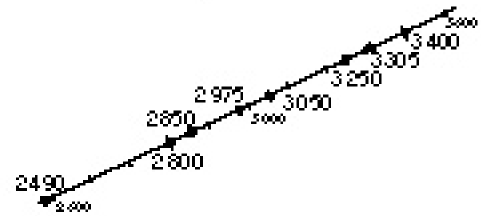
$$\begin{array}{r} 3026 \\ -818 \\ \hline 2208 \end{array}$$

$1730 \div 10 =$ 173

$$\begin{array}{r} 275 \\ \times\ 4 \\ \hline 1100 \end{array} \qquad \begin{array}{r} 394 \\ \times 10 \\ \hline 3940 \end{array} \qquad 6\overline{)426}\ \ 71$$

10

Label the dots on this part of a number line.



Draw and label a dot for 2850.

Draw and label a dot for 2975.

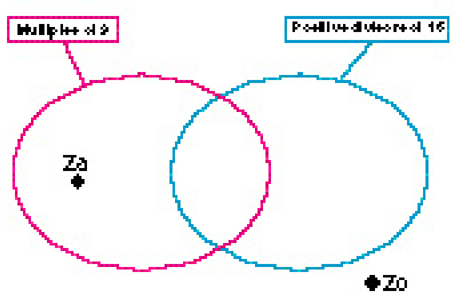
Draw and label a dot for 3305.

Draw and label a dot for 2490.

11

Za and Zo are two of these numbers.

1 3 5 6 7 15

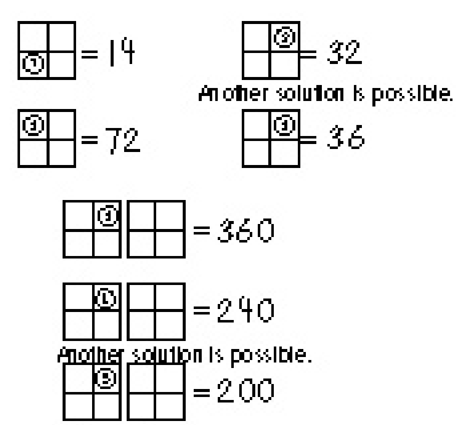


Who is Za? 6 Who is Zo? 7

12

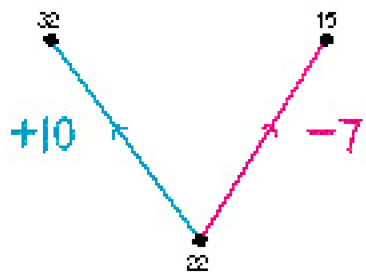
Put each number on the Minicomputer using exactly one of these checkers.

② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨



13

15 is the least number in this picture and Rick is the greatest number. Label the dots.



Who is Rick? 32

14

Complete.

$9 \times 8 = \underline{72}$
$72 \div 9 = \underline{8}$
$72 \div 8 = \underline{9}$

$7 \times 6 = \underline{42}$
$42 \div 7 = \underline{6}$
$42 \div 6 = \underline{7}$

$5 \times 24 = \underline{120}$
$120 \div 5 = \underline{24}$
$120 \div 24 = \underline{5}$

$10 \times 13 = \underline{130}$
$130 \div 10 = \underline{13}$
$130 \div 13 = \underline{10}$

$\begin{array}{r} 23 \\ 3\overline{)69} \end{array}$

$\begin{array}{r} 24 \\ 3\overline{)72} \end{array}$

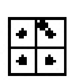
$\begin{array}{r} 25 \\ 3\overline{)75} \end{array}$

$\begin{array}{r} 26 \\ 3\overline{)78} \end{array}$

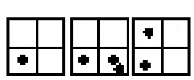
$11 \times 13 = \underline{143}$
$11 \times 14 = \underline{154}$
$11 \times 15 = \underline{165}$
$11 \times 16 = \underline{176}$
$11 \times 17 = \underline{187}$

15

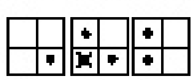
Add one checker to this Minicomputer to get 19.



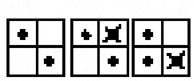
Add two checkers to this Minicomputer to get 250.



Remove one checker from this Minicomputer to get 200.

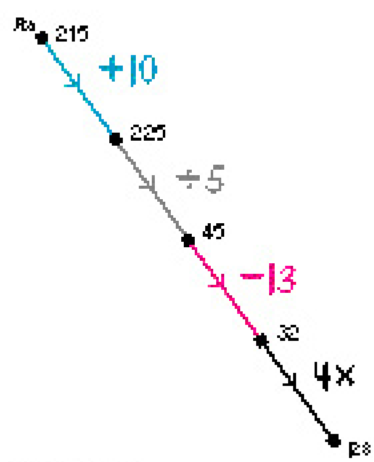


Remove two checkers from this Minicomputer to get 1 000.



16

Ra is the starting number of this arrow road.



Who is Ra? 215

17

Complete these calculations. Hint: You should not need to add the long columns of numbers.

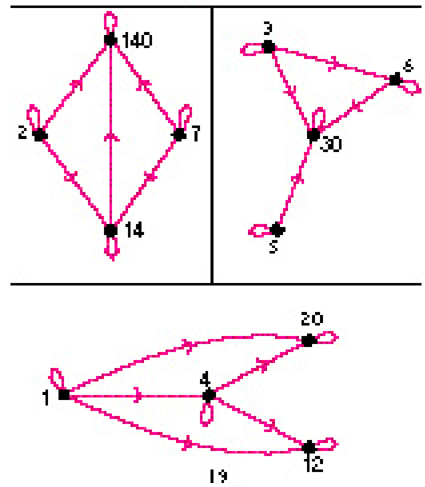
158	158	158
158	158	158
158	158	158
158	158	158
158	158	158
158	158	158
158	158	158
158	158	158
158	158	158
+ 158	158	158
1580	+ 158	158
	1738	+ 158
		1896
10 × 158 = 1580	11 × 158 = 1738	12 × 158 = 1896

22 × 158 = 3476 24 × 158 = 3792

23 × 158 = 3634 25 × 158 = 3950

18

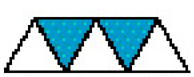
Label the dots. Many solutions are possible.
Then draw the missing red arrows and loops.



19

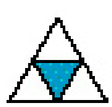
Answers will vary.

How many triangles can you find in each figure?
Write the number in the blank.



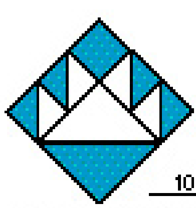
5

There are more than three here.



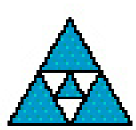
5

There are more than four here.



10

There are more than five here.

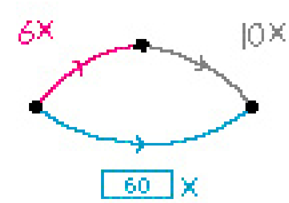


9

There are more than seven here.

20

Fill in the box for the blue arrow.



Complete these calculations. This arrow picture can help you.

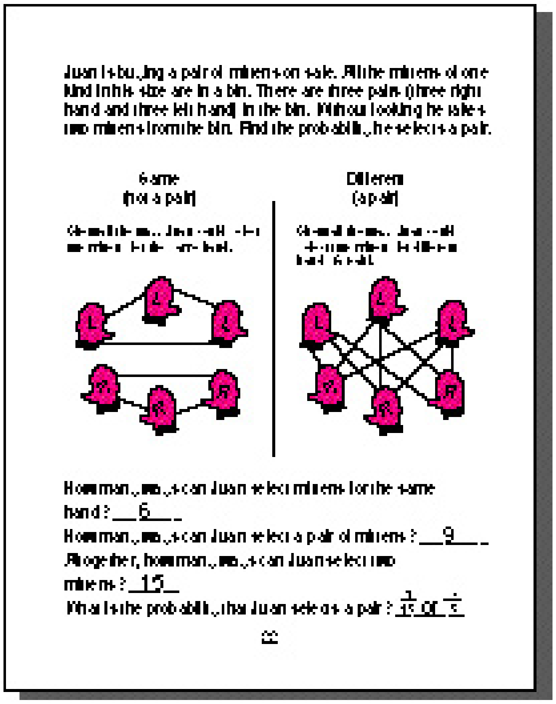
60 × 23 = 1380

60 × 52 = 3120

60 × 100 = 6000

60 × 152 = 9120

21



Juan is buying a pair of mittens on sale. All the mittens of one kind in his size are in a bin. There are three pairs (three right hand and three left hand) in the bin. Without looking he takes two mittens from the bin. Find the probability he selects a pair.

Same (not a pair)

Different (a pair)

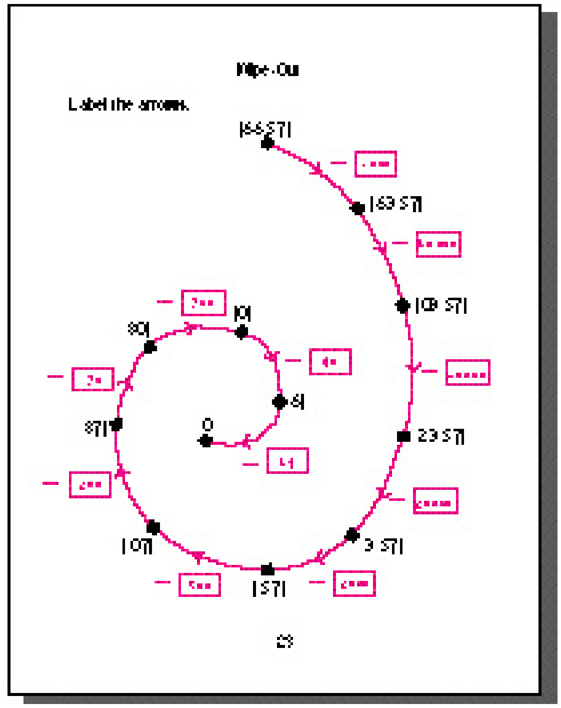
How many ways can Juan select mittens for the same hand? 6

How many ways can Juan select a pair of mittens? 9

Altogether, how many ways can Juan select two mittens? 15

What is the probability that Juan selects a pair? $\frac{9}{15}$ or $\frac{3}{5}$

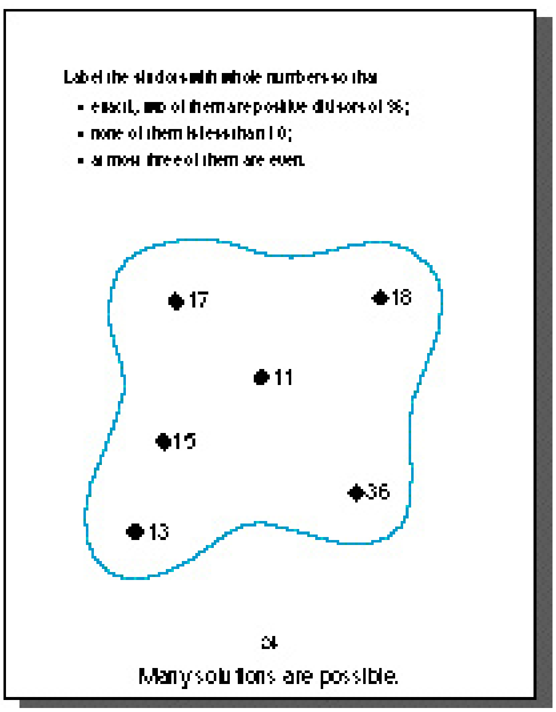
22



Wipe-Out

Label the arrows.

23

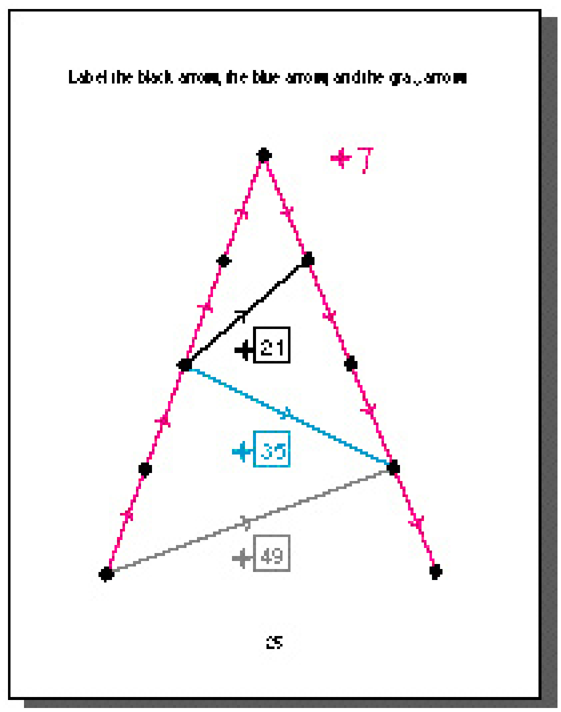


Label the dots with whole numbers so that

- exactly two of them are positive divisors of 36;
- none of them is less than 10;
- at most three of them are even.

24

Many solutions are possible.



Label the black arrow, the blue arrow and the gray arrow.

25

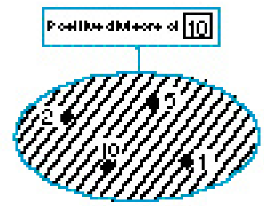
Complete these calculations.

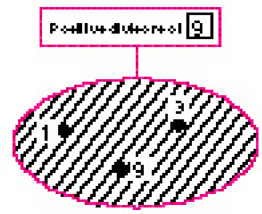
23 − 15 = 8	3.8 + 6 = 9.8
23 − 1.5 = 21.5	3.8 + 0.6 = 4.4
2.3 − 1.5 = 0.8	0.38 + 0.6 = 0.98

$314 \times 5 = 1570$	$31.4 \times 5 = 157.0$	$3.14 \times 5 = 15.70$
$256 + 387 = 643$	$25.6 + 38.7 = 64.3$	$2.56 + 3.87 = 6.43$

26

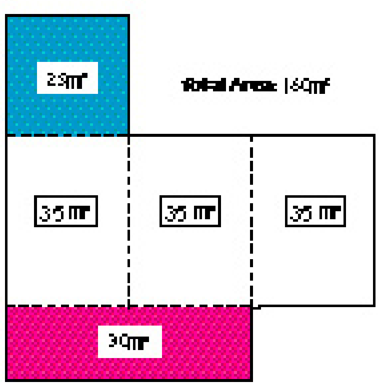
Fill in the boxes and then label the dots.





27

This is the floor plan of a house. The total floor space in this house is 160 m². The area of the blue room is 25 m² and the red room is 30 m². The other three rooms all have the same area. Fill in the boxes to show the areas of the three rooms.



28

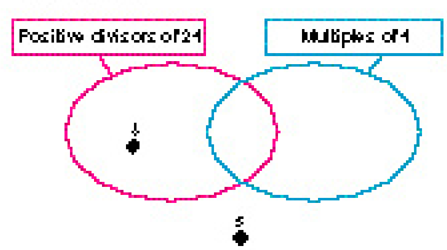
The red label is one of these:

Positive prime numbers
Positive divisors of 20
Multiples of 4
Greater than 10
Less than 60
Positive divisors of 24

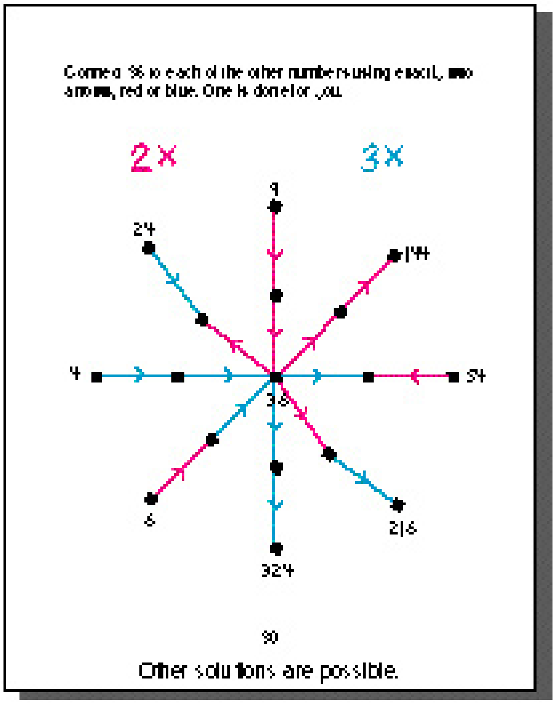
The blue label is one of these:

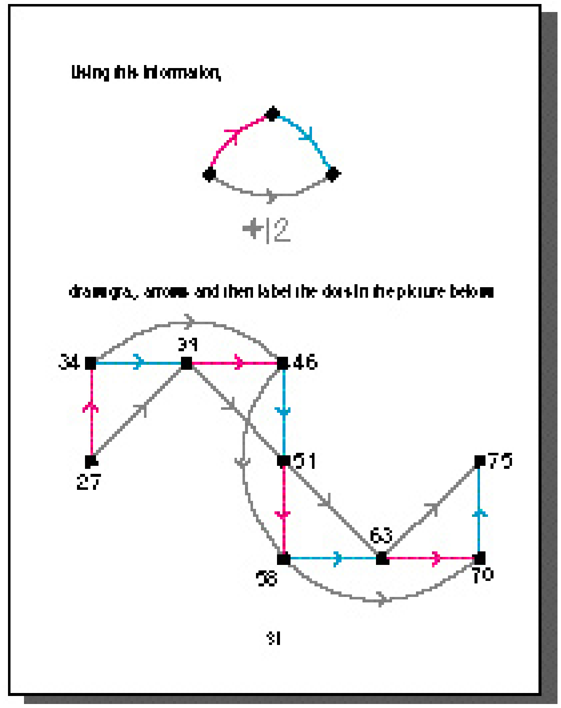
Positive prime numbers
Positive divisors of 20
Multiples of 4
Greater than 10
Less than 60
Positive divisors of 24

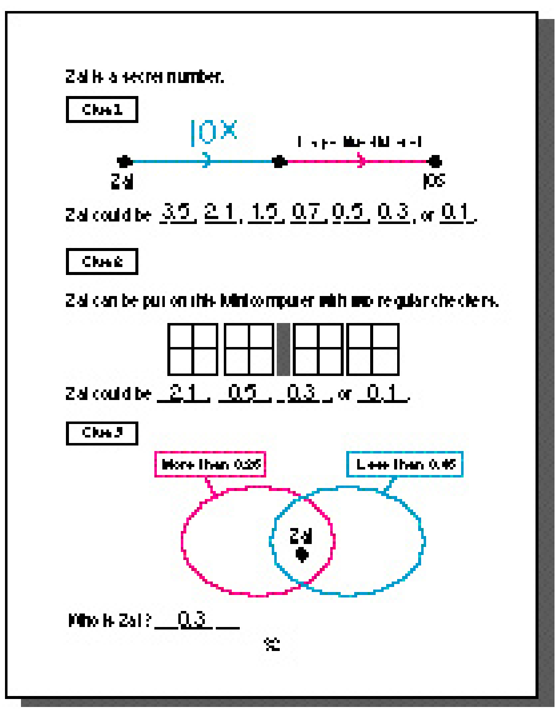
Label the strings.



29


2×
3×
9
24
144
4
36
54
6
216
324
90
Other solutions are possible.


+12
34
39
46
27
51
75
63
58
70
91


10×
Zal could be 3.5, 2.1, 1.5, 0.7, 0.5, 0.3, or 0.1.
Zal could be 2.1, 0.5, 0.3, or 0.1.
Zal
0.3
92

Capsule Lesson Summary

Using money for support, do this sequence of problems.

$$\begin{array}{r} 12 \\ \times\ 6 \\ \hline \end{array} \qquad \begin{array}{r} 1.20 \\ \times\ 6 \\ \hline \end{array} \qquad \begin{array}{r} 0.12 \\ \times\ 6 \\ \hline \end{array} \qquad \begin{array}{r} 0.012 \\ \times\ 6 \\ \hline \end{array}$$

Begin the workbook *Set of Problems #3*. (This is the first of two lessons using this workbook.)

Materials

Teacher
- Six cups or containers
- 72 paper clips or something similar

Student
- Paper
- *Set of Problems #3* Workbook
- Colored pencils, pens, or crayons
- Calculator

Description of Lesson

Ask how many dollars and cents each of the following amounts of dimes and pennies is. Occasionally ask a student to write a decimal on the board to show the amount of money being discussed.

10 dimes	($1.00)	40 pennies	($0.40)
20 dimes	($2.00)	100 pennies	($1.00)
50 dimes	($5.00)	800 pennies	($8.00)
100 dimes	($10.00)	900 pennies	($9.00)
200 dimes	($20.00)	920 pennies	($9.20)
600 dimes	($60.00)	925 pennies	($9.25)
670 dimes	($67.00)	1 000 pennies	($10.00)
673 dimes	($67.30)	1 007 pennies	($10.07)

Display six cups with 12 paper clips in each.

T: ***There are 12 paper clips in each of these six cups. How many paper clips are there altogether?***

S: ***72 paper clips.***

T: ***Who would like to count them to check?***

Let a student volunteer choose whatever method he or she wants to count—one by one, or by 12s, for example. While the volunteer is counting the paper clips, ask the following questions.

T: ***How many people are there in six vans if there are 12 people in each van? How many oranges are there in six bowls if there are 12 oranges in each bowl? How many rose buds are there on six bushes if there are 12 buds on each bush?***

Your class should quickly respond that there are 72 items or people to each question. Check back with the student who counted the paper clips.

S: ***Yes, there are 72 paper clips.***

T (pointing to the six cups)**:** ***If there were 12 one-dollar bills in each of these six cups, how many one-dollar bills would there be altogether?***

Write the multiplication fact on the board while a student answers.

$$\begin{array}{r} 12 \\ \times\ 6 \\ \hline 72 \end{array}$$

S: ***72 one-dollar bills.***

T: ***How much money is 12 dimes?***

S: ***$1.20.***

Record the corresponding multiplication calculation as you ask,

T: ***If there were 12 dimes in each of these six cups, how many dimes would there be altogether?***

S: ***72 dimes.***

T: ***72 dimes is how much money?***

S: ***$7.20.***

T: ***What decimal do we write for $7.20?***

S: ***7.20 or 7.2.***

$$\begin{array}{r} 12 \\ \times\ 6 \\ \hline 72 \end{array} \qquad \begin{array}{r} 1.20 \\ \times\ 6 \\ \hline 7.20 \end{array}$$

Likewise, ask how many pennies there would be if there were 12 pennies in each of six cups. Ask how much money is 12 pennies and how much is 72 pennies. Record the multiplication calculation next to the other two.

$$\begin{array}{r} 0.12 \\ \times\ 6 \\ \hline 0.72 \end{array}$$

T: ***We cannot think of money to help us in this next problem, but what do you think 6* x *0.012 is? Look for patterns.***

S: ***0.072.***

$$\begin{array}{r} 12 \\ \times\ 6 \\ \hline 72 \end{array} \qquad \begin{array}{r} 1.20 \\ \times\ 6 \\ \hline 7.20 \end{array} \qquad \begin{array}{r} 0.12 \\ \times\ 6 \\ \hline 0.72 \end{array} \qquad \begin{array}{r} 0.012 \\ \times\ 6 \\ \hline 0.072 \end{array}$$

Ask students to do the following four problems on their papers.

$$\begin{array}{r} 870 \\ \times\ 3 \\ \hline 2610 \end{array} \qquad \begin{array}{r} 87 \\ \times\ 3 \\ \hline 261 \end{array} \qquad \begin{array}{r} 8.70 \\ \times\ 3 \\ \hline 26.10 \end{array} \qquad \begin{array}{r} 0.87 \\ \times\ 3 \\ \hline 2.61 \end{array}$$

Distribute copies of the workbook *Set of Problems #3* and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of the lesson, collect the workbooks for your review. They will be used again in Lesson W8.

Capsule Lesson Summary

Review how Nabu uses an arrow road to calculate the number of boxes he will need to pack 1 020 kites. Each box holds 16 kites. Continue individual work in the workbook *Set of Problems #3*. (This is the second of two lessons using this workbook.)

Materials

Teacher
- Colored chalk

Student
- *Set of Problems #3* Workbook
- Colored pencils, pens, or crayons
- Calculator

Description of Lesson

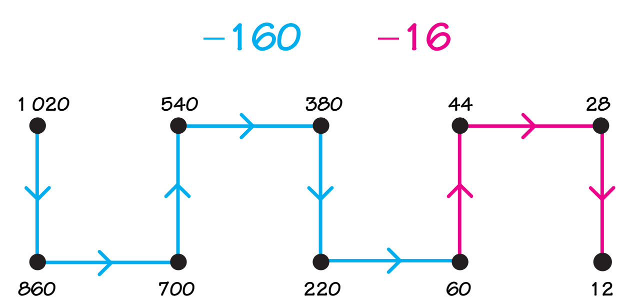
Invite students to recall the character Nabu who has so many different packing jobs.

T: ***Recently Nabu has been packing kites into boxes. Each box holds 16 kites. How many kites will fit into two boxes?***

S: ***32 kites; 2 x 16 = 32.***

Continue by asking how many kites will fit in four boxes (64 kites), ten boxes (160 kites), 20 boxes (320 kites).

Draw this arrow road on the board.

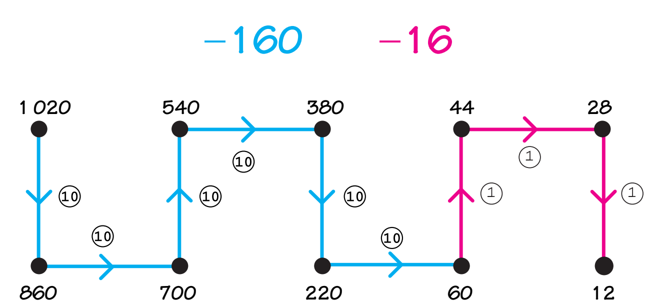


T: ***One day Nabu has 1 020 kites to pack. Nabu draws this arrow road to help him calculate the number of boxes he will fill. How does the picture help Nabu? How many boxes will he need?***

S: ***160 kites will fill ten boxes, so each blue arrow shows filling ten boxes.***

S: ***Each red arrow shows filling one box.***

Write ⑩ near each blue arrow and ① near each red arrow.



S: ***10, 20, 30, 40, 50, 60, 61, 62, 63. Nabu will need 63 boxes.***

T: ***How many kites will be left over? How can you tell?***

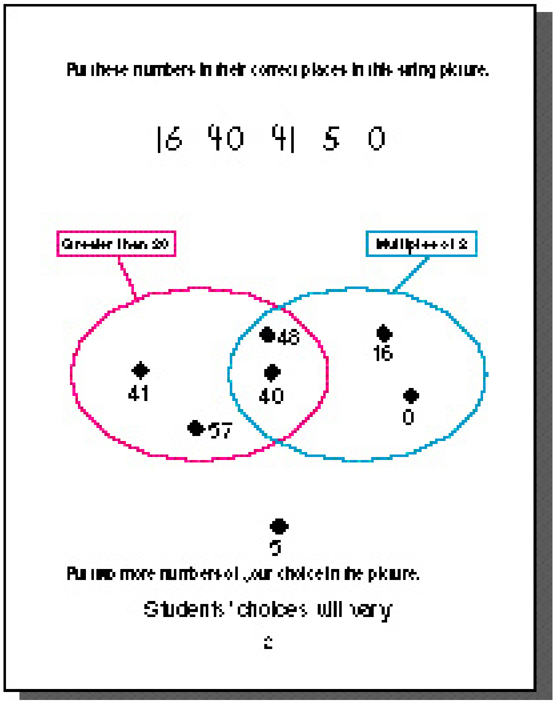
S: ***12 kites. The ending number in the road is 12, and with 12 kites Nabu cannot fill another box.***

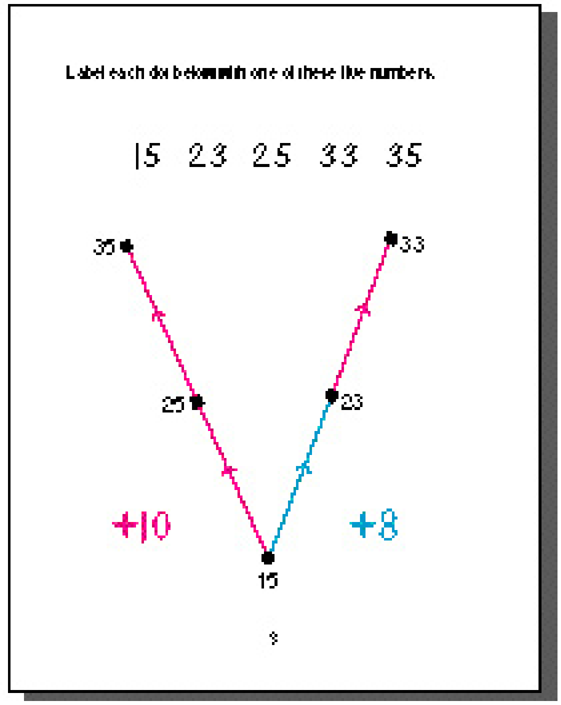
Distribute students' copies of the workbook *Set of Problems #3*. Ask students first to correct or complete pages from the previous week's work, and then to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

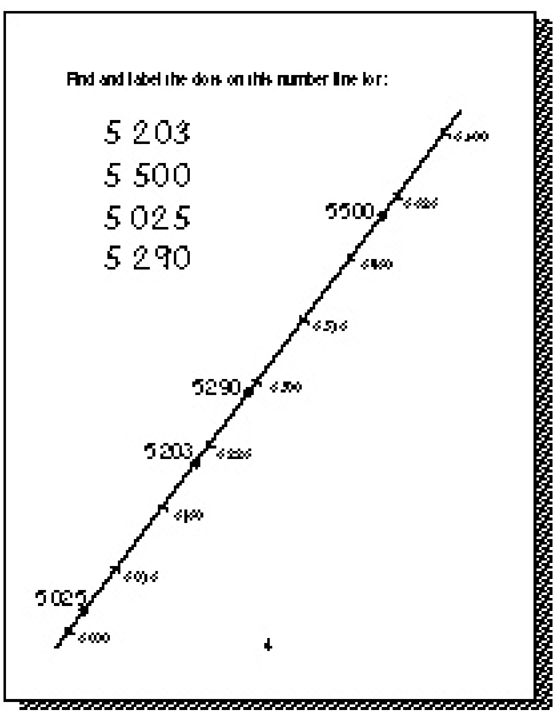
At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in the workbook during a study time or to take it home as an assignment.

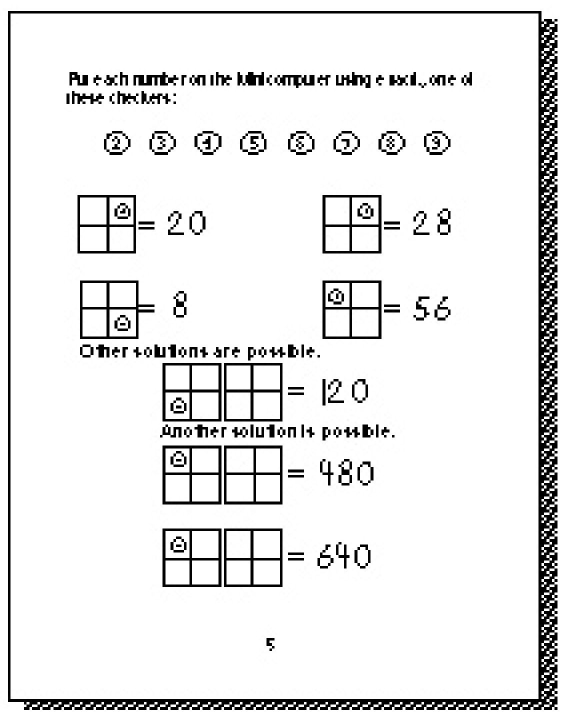
Assessment Activity

An individual student progress record for the workbook is available on Blackline W8. You may like to use this form to monitor student work.

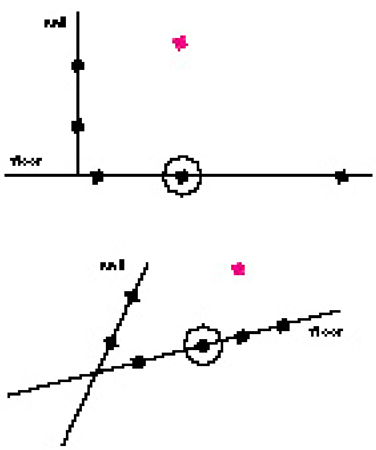

Put these numbers in their correct places in this string picture.
16 40 41 5 0
Greater than 20
Multiples of 2
48
41
40
57
16
0
5
Put two more numbers of your choice in the picture.
Students' choices will vary
2


Label each dot below with one of these five numbers.
15 23 25 33 35
35
33
25
23
+10
+8
15
3


Find and label the dots on this number line for:
5 203
5 500
5 025
5 290
5500
5290
5203
5025
4

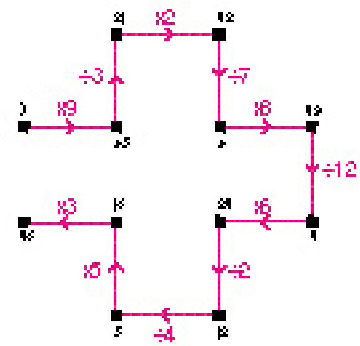

Put each number on the Minicomputer using exactly one of these checkers:
② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨
= 20
= 28
= 8
= 56
Other solutions are possible.
= 120
Another solution is possible.
= 480
= 640
5

The red ball falls to the floor parallel to the wall. In each picture, circle the black dot that shows where the ball will hit.



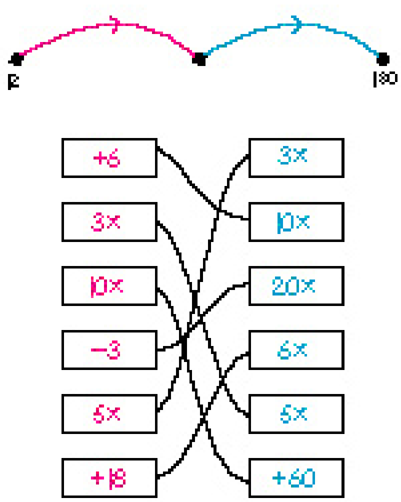
6

Label each arrow x or ÷ some whole number.



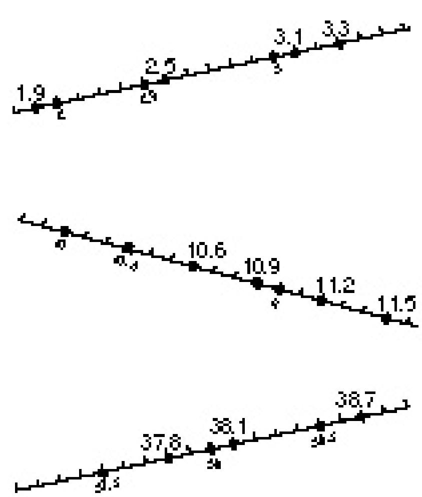
7

Pair the tags. One is done for you.



8

Label the dots.



9

Do these calculations.

$$\begin{array}{r} 709 \\ 283 \\ +564 \\ \hline 1556 \end{array} \qquad \begin{array}{r} 280 \\ -164 \\ \hline 116 \end{array} \qquad \begin{array}{r} 857 \\ \times\ 9 \\ \hline 7713 \end{array}$$

$15 - 7 = \boxed{8}$ $\quad 24 - 18 = \boxed{6}$

$7 - 15 = \boxed{\bar{8}}$ $\quad 18 - 24 = \boxed{\bar{6}}$

$4 - 0 = \boxed{4}$ $\quad 17 - 9 = \boxed{8}$

$0 - 4 = \boxed{\bar{4}}$ $\quad 9 - 17 = \boxed{\bar{8}}$

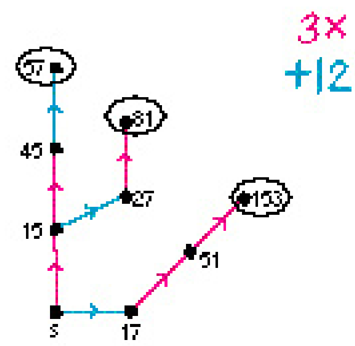
$8 \times 7 = \boxed{56}$ $\quad 6 \times 9 = \boxed{54}$

$56 \div 8 = \boxed{7}$ $\quad 54 \div 6 = \boxed{9}$

$56 \div 7 = \boxed{8}$ $\quad 54 \div 9 = \boxed{6}$

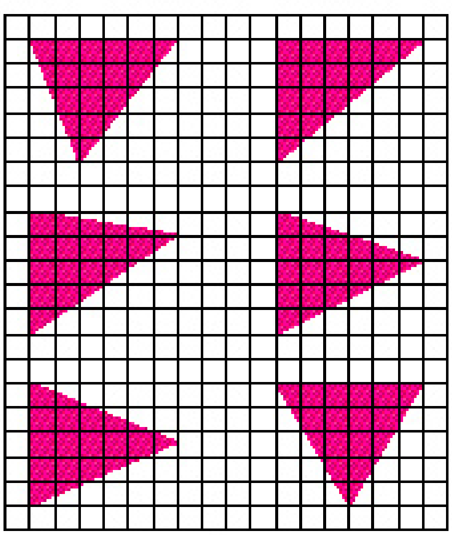
10

Find all of the possible ending numbers of an arrow road that starts at 5 and has exactly one +12 arrow and two 3x arrows.



11

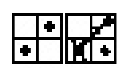
In each rectangle, draw a red triangle whose area is half that of the rectangle. One is done for you. You should have six different triangles.



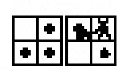
12

Many solutions are possible.

Move one checker to put 65 on the Minicomputer.

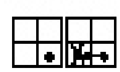


Move one checker to put 71 on the Minicomputer.

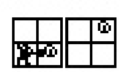


Another solution is possible.

Move one checker to put 9 on the Minicomputer.



Move one checker to put 38 on the Minicomputer.

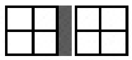


13

Joy is a secret number.

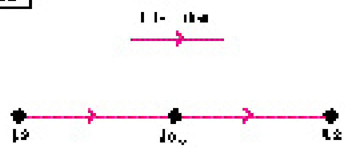
Clue 1

Joy can be put on this Minicomputer using just one checker.



Joy could be 48, 24, 12, 6, 4.8, 2.4, 1.2, or 0.6.

Clue 2



Who is Joy? 2.4

14

Divide.

$\begin{array}{r}4\\4\overline{)16}\end{array}$	$\begin{array}{r}5\\4\overline{)20}\end{array}$	$\begin{array}{r}6\\4\overline{)24}\end{array}$	$\begin{array}{r}7\\4\overline{)28}\end{array}$
$\begin{array}{r}4\ R{=}1\\4\overline{)17}\end{array}$	$\begin{array}{r}5\ R{=}1\\4\overline{)21}\end{array}$	$\begin{array}{r}6\ R{=}1\\4\overline{)25}\end{array}$	$\begin{array}{r}7\ R{=}1\\4\overline{)29}\end{array}$
$\begin{array}{r}4\ R{=}2\\4\overline{)18}\end{array}$	$\begin{array}{r}5\ R{=}2\\4\overline{)22}\end{array}$	$\begin{array}{r}6\ R{=}2\\4\overline{)26}\end{array}$	$\begin{array}{r}7\ R{=}2\\4\overline{)30}\end{array}$
$\begin{array}{r}4\ R{=}3\\4\overline{)19}\end{array}$	$\begin{array}{r}5\ R{=}3\\4\overline{)23}\end{array}$	$\begin{array}{r}6\ R{=}3\\4\overline{)27}\end{array}$	$\begin{array}{r}7\ R{=}3\\4\overline{)31}\end{array}$

15

This summer Babu packs oranges in boxes that hold a dozen (12) each.

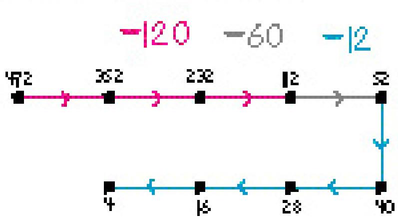
How many boxes does he need for 24 oranges? 2

... for 48 oranges? 4

... for 60 oranges? 5

... for 120 oranges? 10

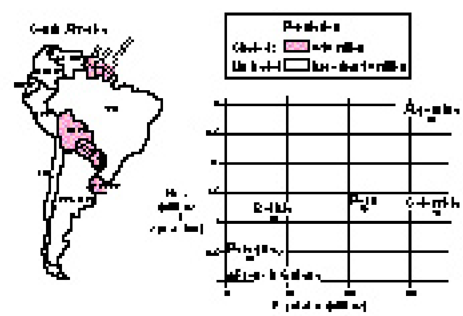
One day, Babu is given 472 oranges to pack. Babu uses this arrow road to decide how many boxes to get.



How many boxes can he fill? 39

How many oranges will not get packed? 4

16



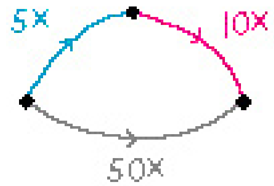
The dots on the graph are for these five South American countries. Label the dots.

Bolivia Peru Paraguay

Argentina French Guiana

Colombia has area a little less than Peru, but its population is about 12 million more than Peru's. Draw and label a dot for Colombia.

17



Use the arrow picture to help you to do these calculations.

$50 \times 61 = \boxed{3050}$

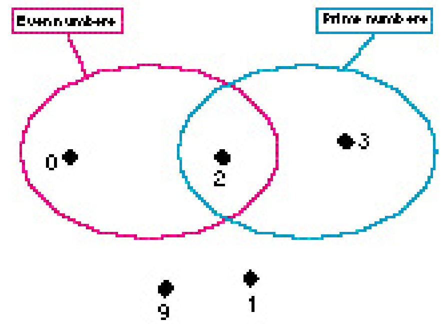
$50 \times 84 = \boxed{4200}$

$50 \times 901 = \boxed{45050}$

$50 \times 73 = \boxed{3650}$

18

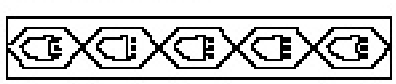
Put at most seven numbers in this picture.
Put at least one number in each of the four regions.
Put exactly two numbers in the region outside of both strings.



19

Many solutions are possible.

With this stamper Jordan can put five designs on a ribbon 2.5 cm wide and 16 cm long.



How many designs can he stamp on a ribbon 2.5 cm wide and 32 cm long? __10__

... on a ribbon 2.5 cm wide and 48 cm long? __15__

... on a ribbon 2.5 cm wide and 64 cm long? __20__

20

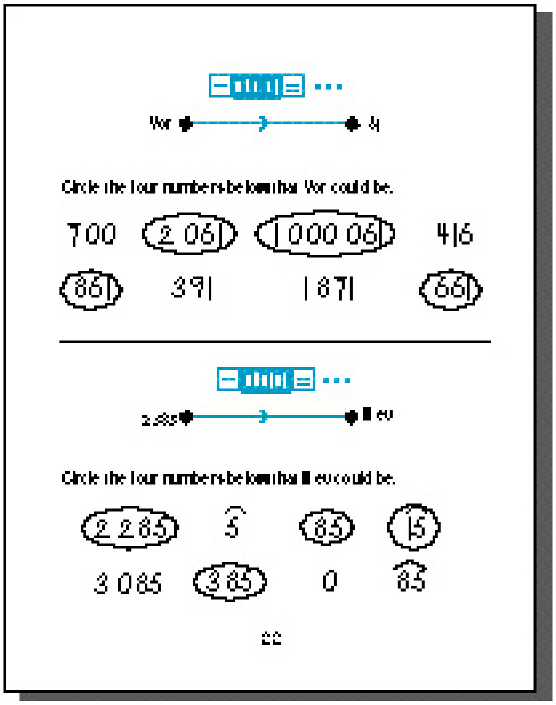
Do these computations.

$$\begin{array}{r} 56 \\ 56 \\ 56 \\ +\,56 \\ \hline 224 \end{array} \qquad \begin{array}{r} 56 \\ \times\,4 \\ \hline 224 \end{array}$$

$$\begin{array}{r} 5.6 \\ 5.6 \\ 5.6 \\ +\,5.6 \\ \hline 22.4 \end{array} \qquad \begin{array}{r} 5.6 \\ \times\,4 \\ \hline 22.4 \end{array}$$

$$\begin{array}{r} 0.56 \\ 0.56 \\ 0.56 \\ +\,0.56 \\ \hline 2.24 \end{array} \qquad \begin{array}{r} 0.56 \\ \times\,4 \\ \hline 2.24 \end{array}$$

21



Circle the four numbers below that Vor could be.

700 2 061 1 000 061 416

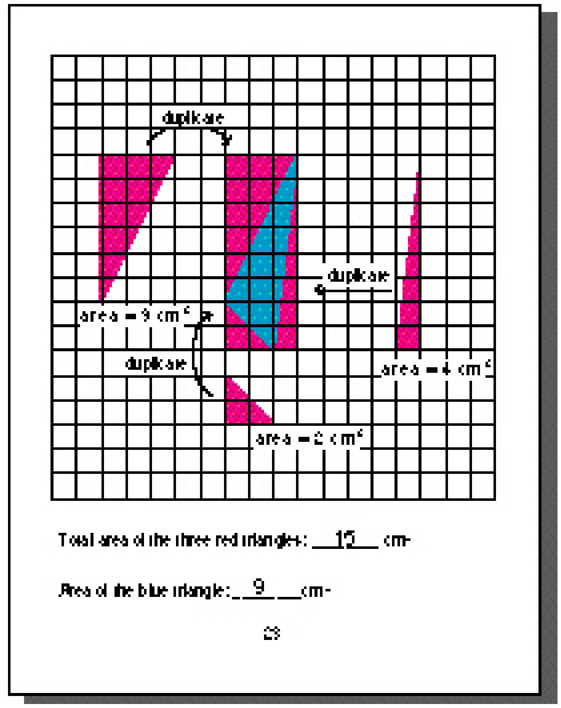
861 391 1 871 661

Circle the four numbers below that [illegible] could be.

2 285 5 85 15

3 085 385 0 85

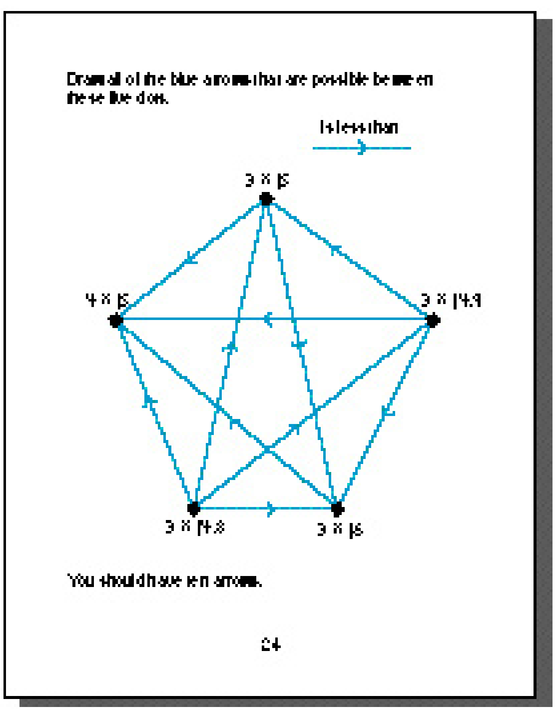
22



Total area of the three red triangles: 15 cm²

Area of the blue triangle: 9 cm²

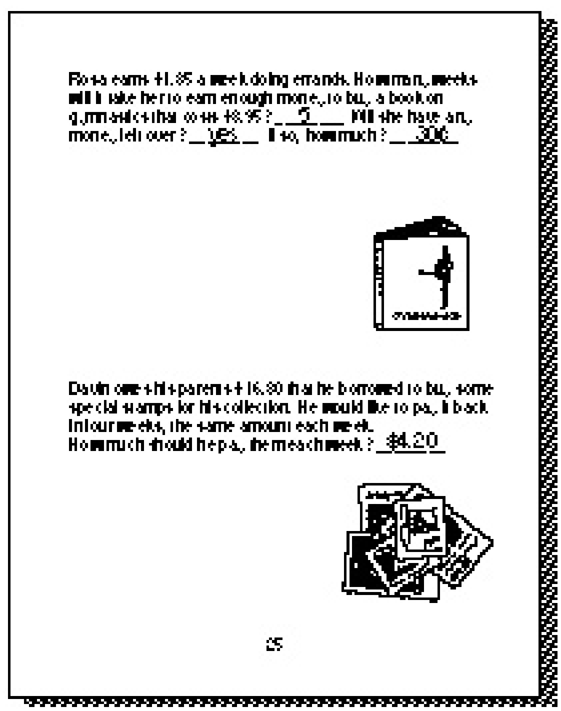
23



Draw all of the blue arrows that are possible between these five dots.

You should have ten arrows.

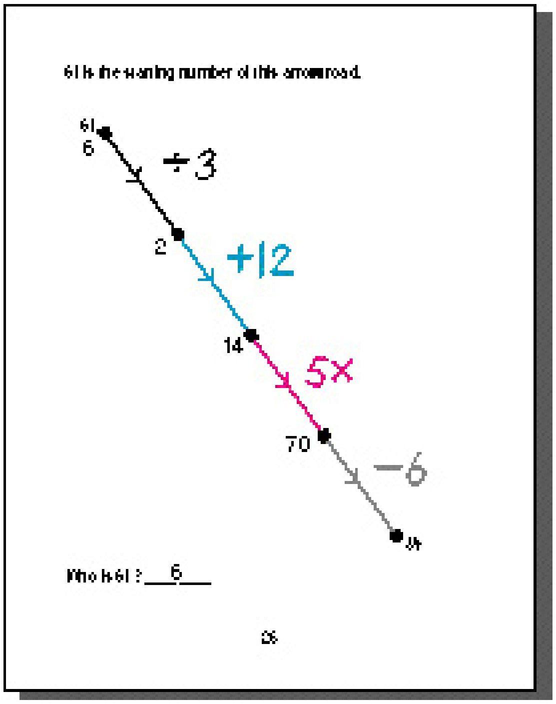
24

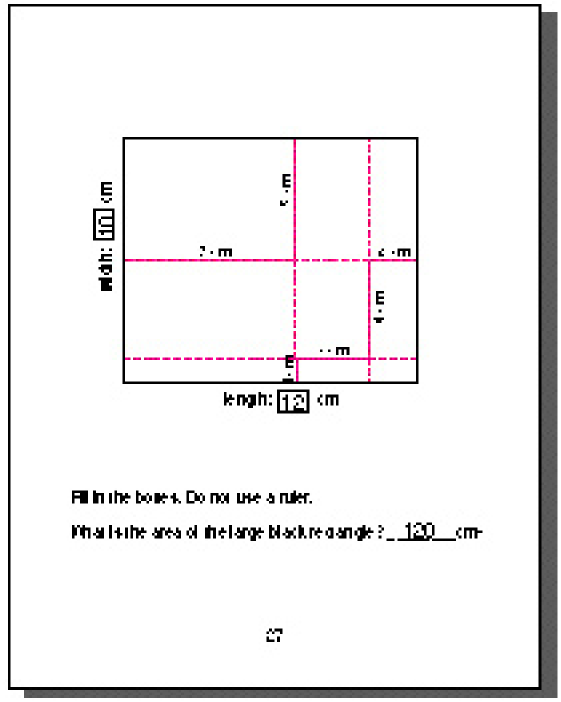


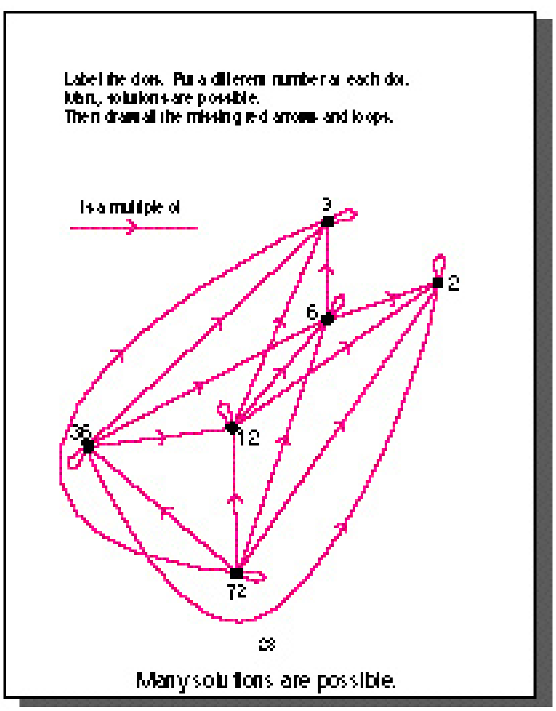
Rosa earns $1.85 a week doing errands. How many weeks will it take her to earn enough money to buy a book on gymnastics that costs $8.95? 5 Will she have any money left over? yes If so, how much? 30¢

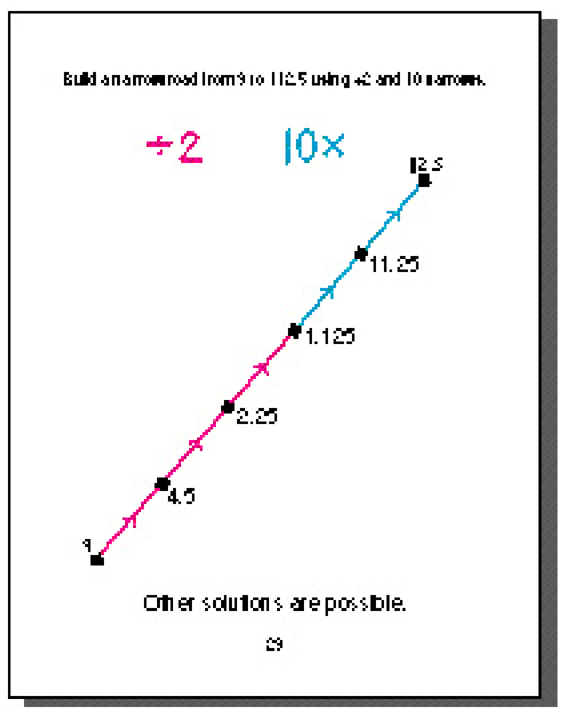
Davin owes his parents $16.80 that he borrowed to buy some special stamps for his collection. He would like to pay it back in four weeks, the same amount each week. How much should he pay them each week? $4.20

25

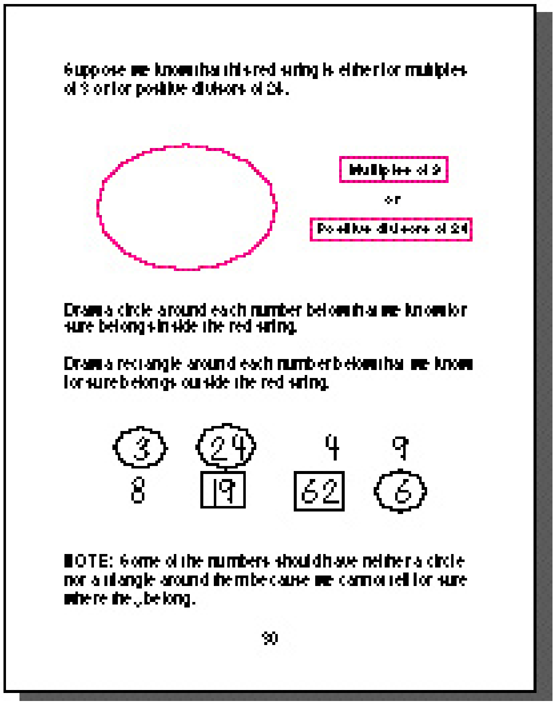

61 is the starting number of this arrowroad.
61
6
÷3
2
+12
14
5×
70
−6
What is 61 ? 6
26


width: 10 cm
length: 12 cm
Fill in the boxes. Do not use a ruler.
What is the area of the large black rectangle? 120 cm²
27


Label the dots. Put a different number at each dot.
Many solutions are possible.
Then draw all the missing red arrows and loops.
Is a multiple of
3
2
6
36
12
72
28
Many solutions are possible.


Build an arrowroad from 9 to 112.5 using ÷2 and 10× arrows.
÷2
10×
112.5
11.25
1.125
2.25
4.5
9
Other solutions are possible.
29

W8



Suppose we know that this red string is either for multiples of 3 or for positive divisors of 24.

Multiples of 3

or

Positive divisors of 24

Draw a circle around each number below that we know for sure belongs inside the red string.

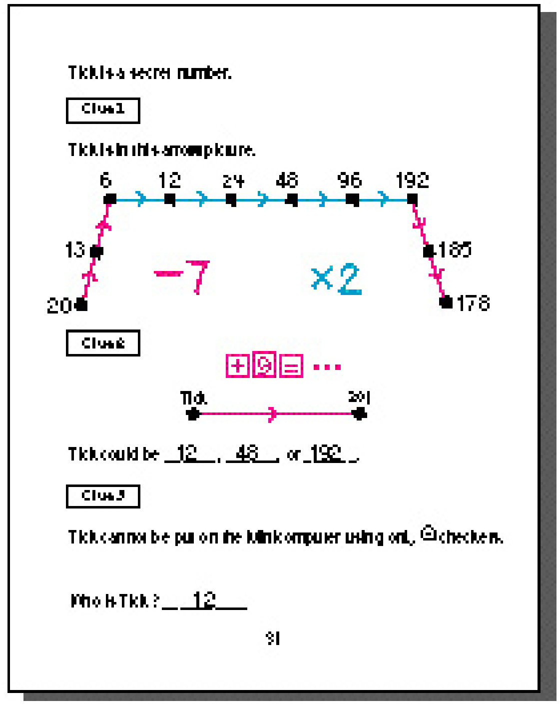
Draw a rectangle around each number below that we know for sure belongs outside the red string.

3 24 4 9

8 19 62 6

NOTE: Some of the numbers should have neither a circle nor a triangle around them because we cannot tell for sure where they belong.

90



Tikk is a secret number.

Clue 1

Tikk is in this arrow picture.

Clue 2

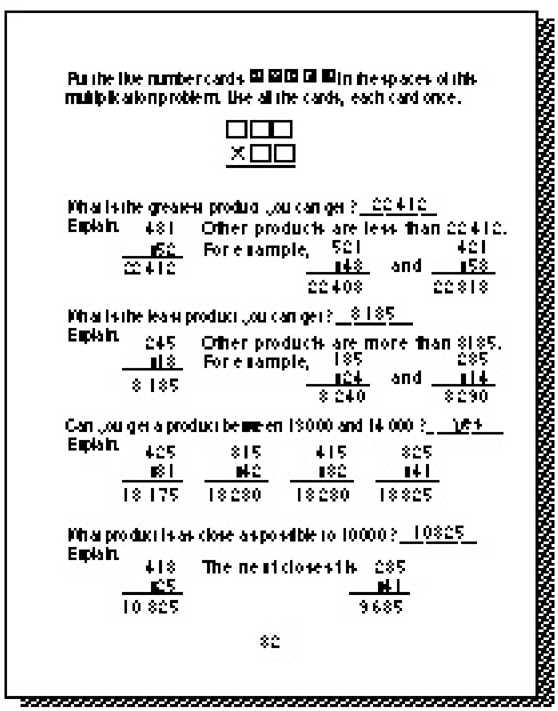
Tikk could be 12, 48, or 192.

Clue 3

Tikk cannot be put on the Minicomputer using only checkers.

Who is Tikk? 12

91



Put the five number cards 1 2 3 4 5 in the spaces of this multiplication problem. Use all the cards, each card once.

$\square\square\square$
$\times\square\square$

What is the greatest product you can get? 22412

Explain. $431 \times 52 = 22412$. Other products are less than 22412. For example, $521 \times 43 = 22403$ and $421 \times 53 = 22313$.

What is the least product you can get? 3185

Explain. $245 \times 13 = 3185$. Other products are more than 3185. For example, $135 \times 24 = 3240$ and $235 \times 14 = 3290$.

Can you get a product between 13000 and 14000? Yes

Explain. $425 \times 31 = 13175$, $315 \times 42 = 13230$, $415 \times 32 = 13280$, $325 \times 41 = 13325$

What product is as close as possible to 10000? 10325

Explain. $413 \times 25 = 10325$. The next closest is $235 \times 41 = 9635$.

92

Capsule Lesson Summary

Begin the story-workbook *Clinton Street* about a detective named Spike and his search for a mystery house. The first clues involve putting a dime and three pennies on the ones board of the Minicomputer. (This is the first of three lessons using this story-workbook.)

Materials

Teacher
- Minicomputer set
- A ⑩-checker
- *Clinton Street* Story-Workbook
- Worksheets W9*, **, and ***

Student
- *Clinton Street* Story-Workbook.

Description of Lesson

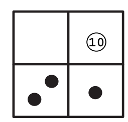
Note: You may like to let students work with a partner during this lesson.

Distribute copies of the story-workbook *Clinton Street*. Instruct students (partners) to stay with you in the book—not to go ahead on their own.

Pages 2–5

Instruct students to read pages 2 to 5 on their own and then to solve the problem on page 5.

While the students are reading, display one Minicomputer board with this configuration.



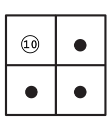
Announce that you are using regular checkers for pennies and a 10-checker for a dime.

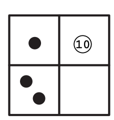
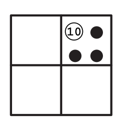
T: ***What number is on the Minicomputer? How do you figure out what number this is?***

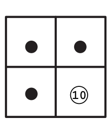
S: ***45. 4 x 10 = 40, 2 + 2 + 1 = 5, and 40 + 5 = 45.***

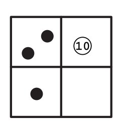
Pages 6–9

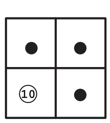
Invite a student to read page 6 aloud. Then ask the class to decode the number on page 7 and to draw some other pictures of one dime and three pennies on one Minicomputer board. Allow a few minutes for individual or partner work before asking students to check the answer to the problem on page 7 (which can be found on page 8). Collectively decode the following numbers or others in pictures drawn by students.

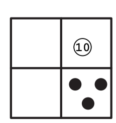
 = 87

 =  = 52

 = 24

 = 58

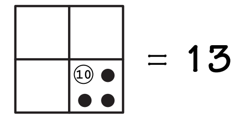
 = 33

 = 43

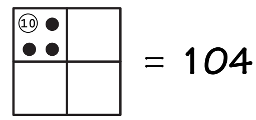
Invite a student to read pages 8 and 9 aloud. Pause briefly so students can answer the question on page 9.

T: ***What is the least number that can be put on the ones board with exactly one dime and three pennies?***

Invite a student to put the checkers on the Minicomputer.
The least such number is 13.



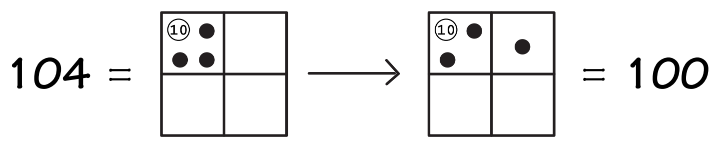
Ask for the greatest such number.



T: ***Do you think we can represent all of the whole numbers between 13 and 104 on the ones board using exactly one dime and three pennies?***

Let the class react to this question, perhaps trying more examples. It may happen that a student can show that there are at least a few of the whole numbers between 13 and 104 that cannot be represented.

S: ***104 is the greatest possible number. If we move one checker from the 8-square to the 4-square, we get the next greatest number, 100. So we can get 101, 102, or 103 with those checkers on the ones board.***



If such a suggestion is not made, do not insist on this explanation but instead ask,

T: ***Can we put 55 on the ones board using one dime and three pennies?***

Let several students try to put the checkers on the ones board. The class should conclude that it is impossible to put 55 on the ones board using exactly one dime and three pennies.

Pages 10–13

Ask the students to read and solve the problems on pages 10 to 13.

Pages 14–16

Read and discuss page 14 with the entire class. Then ask that students continue to work on pages 15 and 16. When many students (individually or with a partner) have completed these pages, continue the collective discussion.

T: ***Let's make a list of the houses Spike will have to search.***

If some students have not yet completed page 15, the class list will help them add to or check their work. In any event, the class should conclude that Spike will have to search the following 17 houses: 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 97, 98, 100, and 104.

T: ***Three of these numbers can be put on the Minicomputer in two ways. Which ones are they?***

S: ***86, 90, and 92.***

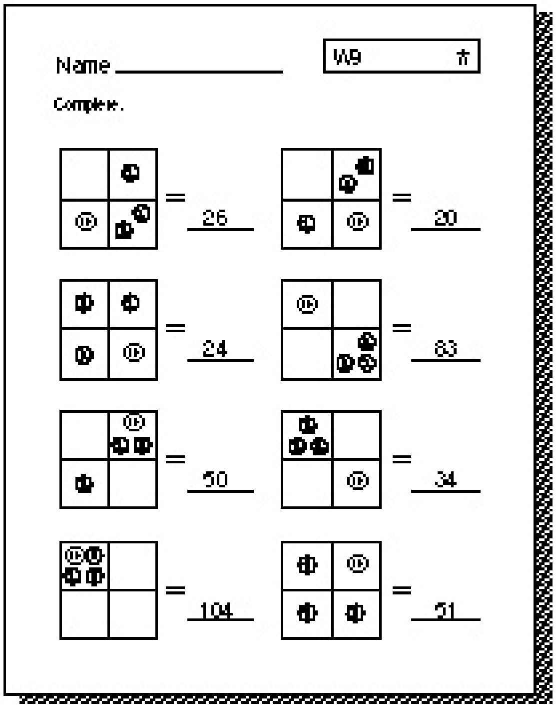
T: ***Good. Five numbers between 83 and 104 cannot be put on the Minicomputer in this way. Which ones are they?***

S: ***95, 99, 101, 102, and 103.***

Worksheets W9*, **, and *** are available for individual work. Students who finish quickly may enjoy the challenge of the problem posed on page 9. Suggest that they list all of the numbers that can be put on the ones board with one dime and three pennies. This is a difficult problem, but you may have a few students accept the challenge. A hint can make the problem easier and help to sustain interest. For example, suggest that students first consider the numbers that can be put on the Minicomputer using three pennies (there are 17 such numbers), and then using three pennies and one dime. Those that construct this list might reason that there are 68 such numbers, since there are 68 such Minicomputer configurations. But they will find that there are many duplicates.

The 60 numbers listed below can be put on the Minicomputer with exactly one dime and three pennies.

13	23	33	47	58	90
14	24	34	48	60	91
15	25	36	49	64	92
16	26	37	50	83	93
17	27	38	51	84	94
18	28	40	52	85	96
19	29	43	53	86	97
20	30	44	54	87	98
21	31	45	56	88	100
22	32	46	57	89	104



Name ________ W9 ☆

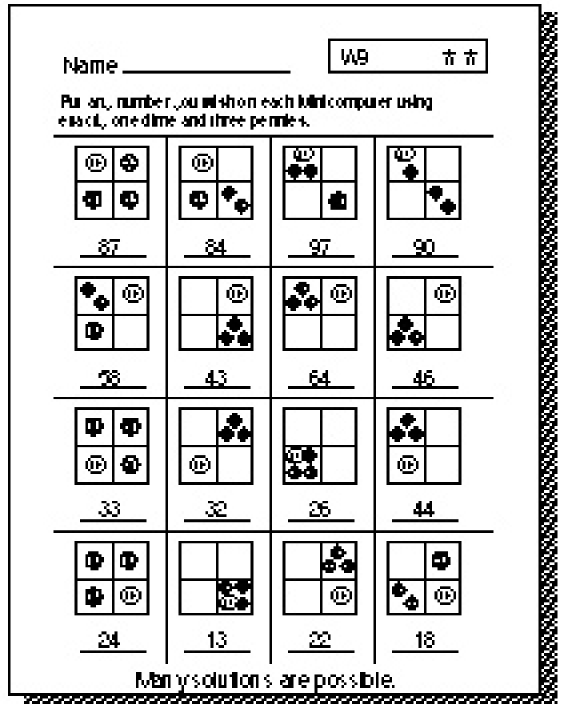
Complete.

= 26 = 20

= 24 = 83

= 50 = 34

= 104 = 51

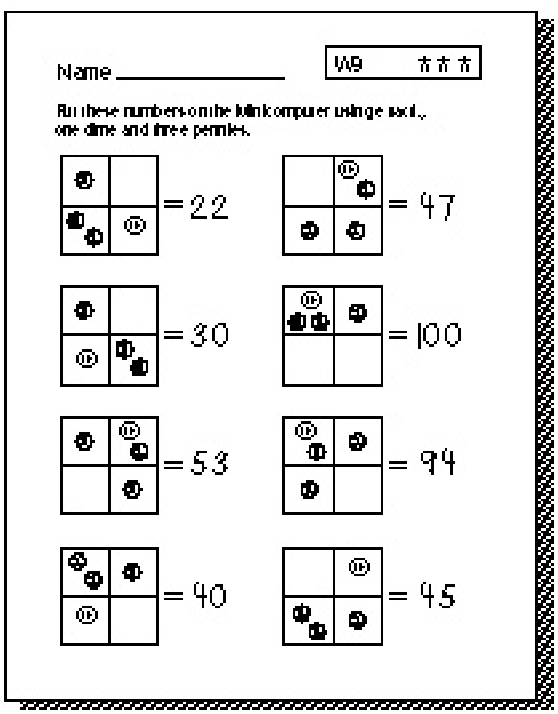


Name ________ W9 ☆☆

Put any number you wish on each Minicomputer using exactly one dime and three pennies.

87	84	97	90
58	43	64	46
33	32	26	44
24	13	22	18

Many solutions are possible.



Name ________ W9 ☆☆☆

Put these numbers on the Minicomputer using exactly one dime and three pennies.

=22 =47

=30 =100

=53 =94

=40 =45

Capsule Lesson Summary

Continue the story-workbook *Clinton Street* about a detective, Spike, and his search for a mystery house. The second clue involves an arrow road with 2x and +1 arrows. (This is the second of two lessons using this workbook.)

Materials

Teacher		Student	
	• *Clinton Street* Story-Workbook • Colored chalk		• *Clinton Street* Story-Workbook • Worksheet W10 • Paper • Colored pencils, pens, or crayons

Description of Lesson

Distribute the students' copies of the story-workbook *Clinton Street*.

T: ***Do you remember the story of Spike that we began reading some days ago? Who would like to tell the story so far?***

Let students recall as much of the story as they can. They should mention that 17 houses with house numbers between 83 and 104 inclusively will have to be searched. They may also remember that 86, 90, and 92 can be put on the Minicomputer in two ways, and that 95, 99, 101, 102, and 103 cannot be put on the Minicomputer using one dime and three pennies.

Pages 16 and 17

Read the clue at the bottom of page 16 and page 17 aloud with the class. Then instruct students to put the story-workbook aside temporarily.

T: ***Spike was not as fortunate as you. He did not know how to draw a tree diagram for this situation. Let's help Spike and draw a tree diagram.***

Draw a starting point in the center of the board.

T: ***What choices do we have for the first arrow of a road?***

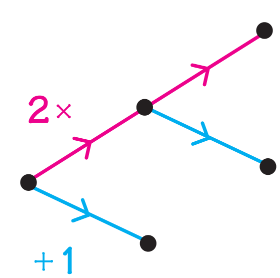
S: ***Red (2x) or blue (+1).***

Begin an arrow picture on the board by drawing one red and one blue arrow from the starting point.

T: ***Suppose we choose to start with a red arrow. What color could the second arrow of the road be?***

S: ***Red or blue.***

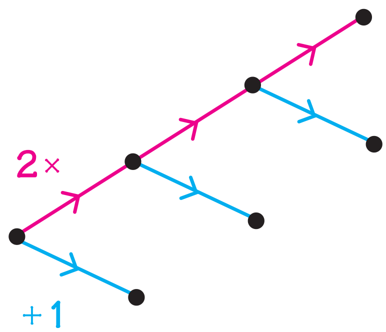
Extend the drawing.



T: ***Suppose we choose a red arrow for our second arrow. What color could the third arrow of our road be?***

S: ***Red or blue.***

Extend the drawing.

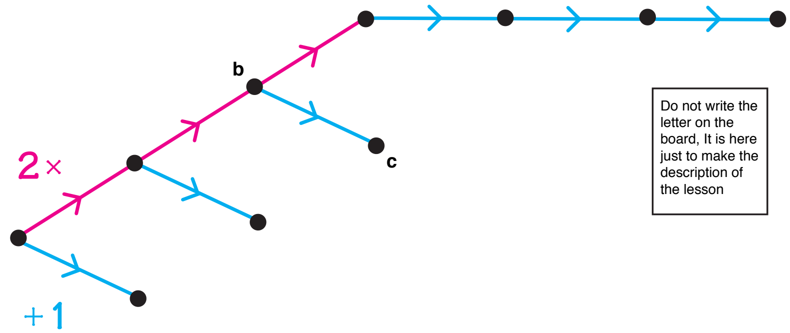


T: ***Suppose we again choose red for our third arrow. What color could the fourth arrow of our road be?***

S: ***It must be blue because we can have only three red arrows.***

S: ***Draw three blue arrows to finish the road.***

Extend the drawing.

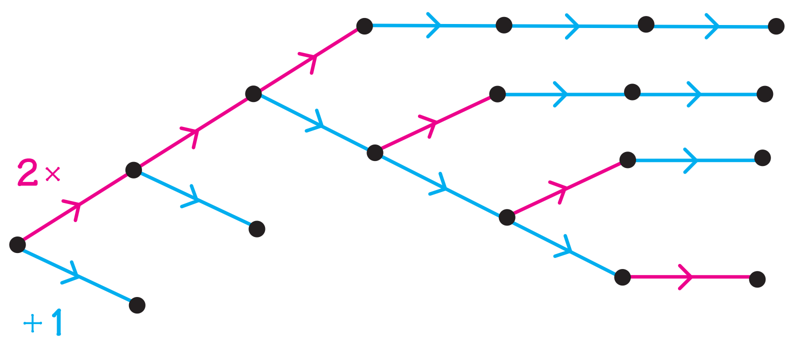


T: ***Here is one possible road.*** (Trace the red-red-red-blue-blue-blue arrow road.) ***It has three red arrows followed by three blue arrows. What if we had chosen a blue arrow for our third arrow?*** (Point to **b** and trace the red arrow starting there.)

S: ***The fourth arrow could be red or blue.***

Extend the drawing with a red and a blue arrow starting at **c**.

Continue in this way completing one or two more roads. A partially completed tree diagram is shown below.



Pages 18–23

Distribute copies of Worksheet W10.

T: ***Here is the complete tree diagram. Compare this picture to Spike's work on pages 18 and 19.***

Be sure the students understand that each arrow road pictured on pages 18 and 19 is contained within the tree picture on Worksheet W10. This can be accomplished by first asking the students to choose an arrow road on page 18 or page 19 and then to find it in the tree picture. Then reverse the situation.

T: ***Which picture do you prefer?***

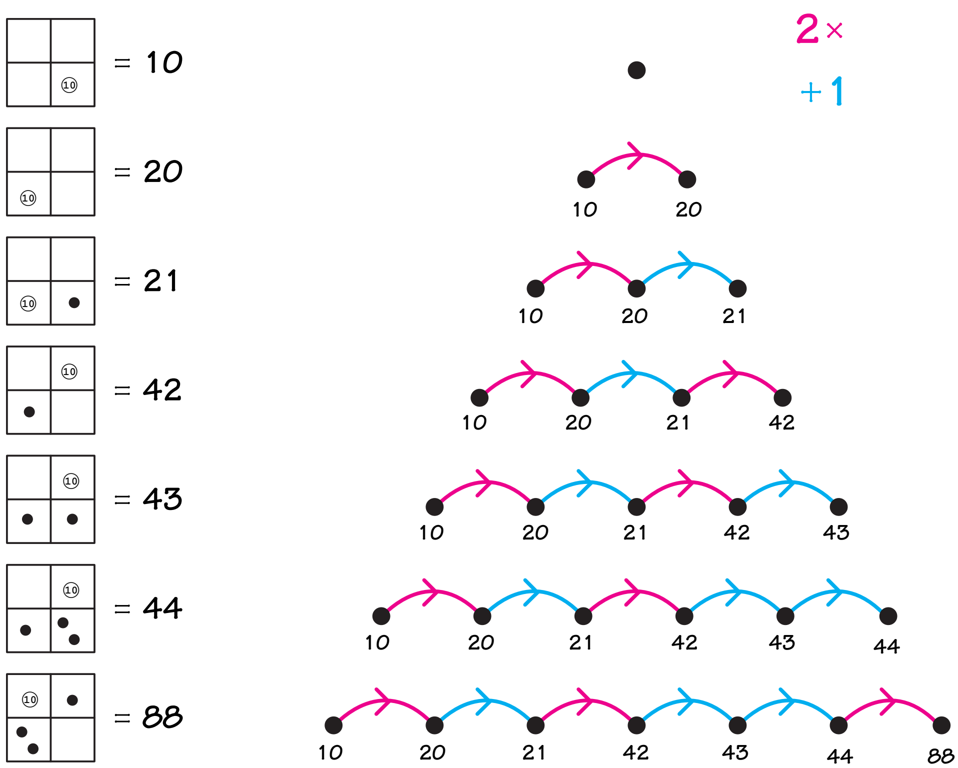
Accept students' comments. There will probably be a variety of responses.

Let the class continue by reading page 20 and labeling the dots either on Worksheet W10 or on pages 18 and 19. Students who finish should read and answer the questions on pages 21 to 23.

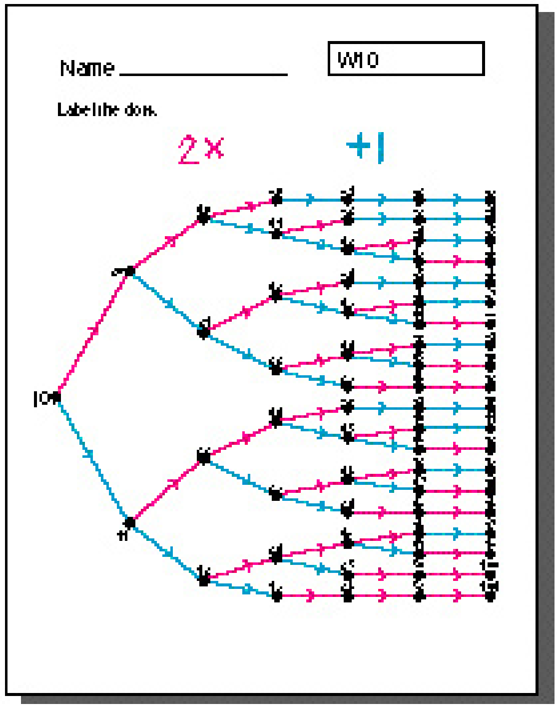
Collect the story-workbooks at the end of the lesson.

Note: On page 21 of *Clinton Street*, we state that there is a connection between the roads clue and the Minicomputer clue. One way to think of that connection is suggested below.

Compare the placing of a dime on the 8-square and three pennies anywhere on the ones board to building an arrow road starting at 10 and using exactly three 2x arrows and three +1 arrows.



For each number that can be put on the Minicomputer, there exists an arrow road and vice versa.


Name
W10
Label the dots.
2x
+1
10

Capsule Lesson Summary

Continue the story-workbook *Clinton Street* about a detective, Spike, and his search for a mystery house. The third clue involves a binary abacus. (This is the third of three lessons using this workbook.)

Materials

Teacher
- *Clinton Street* Story-Workbook
- Colored chalk

Student
- *Clinton Street* Story-Workbook
- Colored pencils, pens, or crayons
- Worksheets W11*, **, and ***

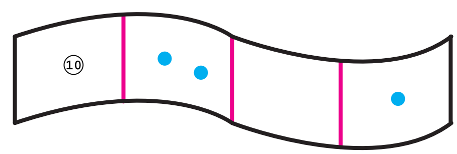
Description of Lesson

Exercise 1

Distribute students' copies of *Clinton Street* and ask the class to recall what they can of the story thus far. Include mention of the two clues (Minicomputer and arrow roads) and how they both gave the same information.

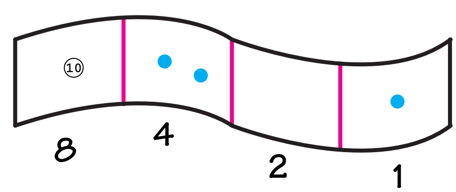
Pages 24–30

Invite a student to read pages 24 and 25 aloud. Draw the snake from page 25 on the board. Blue checkers are used for pennies here because of their relation with the blue +1 arrows in the arrow roads.



T: ***What do you think this number is? Why?***

An explanation might be like this.



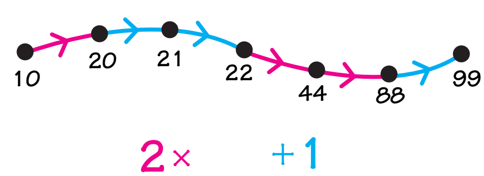
S: ***89, because it's like a Minicomputer. The 1-, 2-, 4-, and 8-squares are in a row instead of in a square.***

S: ***A dime on the 8-square, two pennies on the 4-square, and one penny on the 1-square make 89.***

Read page 26 with the students.

T: ***Notice the arrow road Spike uses to solve the snake clue.***

Draw Spike's arrow road on the board, asking students to label the dots.

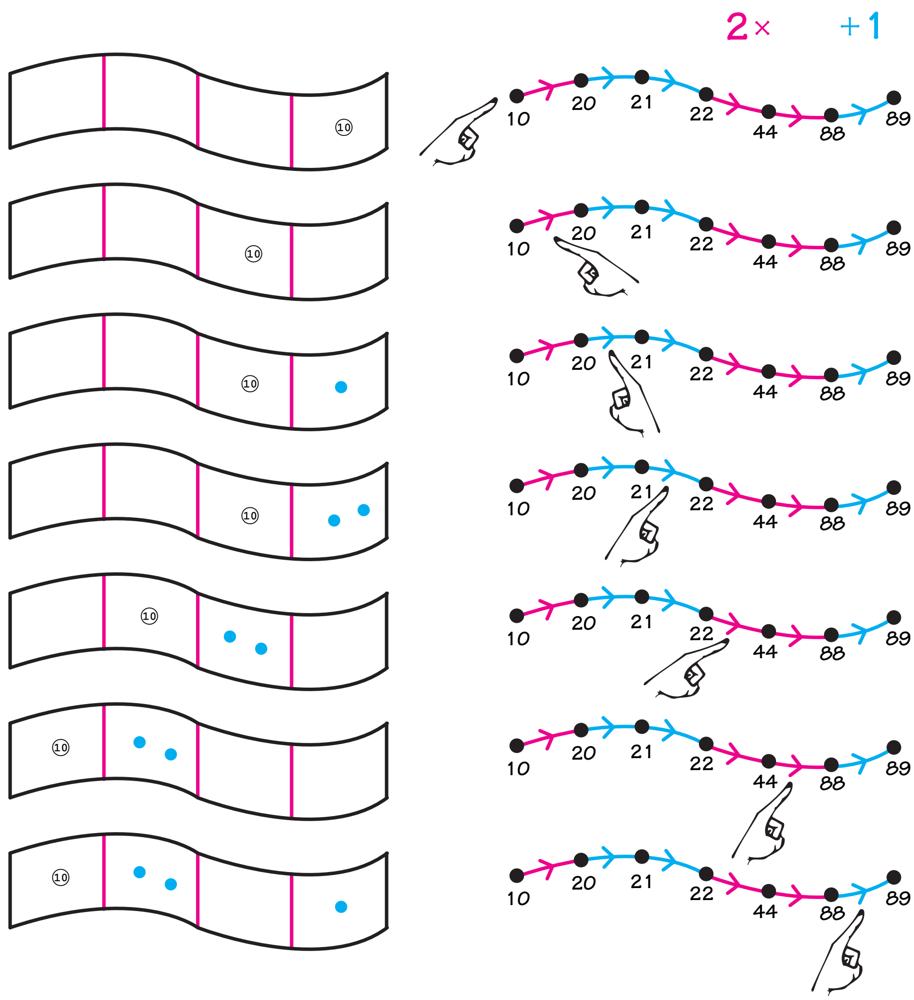


T: ***Why do you think Spike drew this arrow picture to go with this snake?***

S: ***The red and blue in the snake tell you what to do. There is a ⑩-checker, and the arrow road starts at 10. Then there is a red bar, and there is a red arrow. Next there are two blue checkers, and there are two blue arrows. There is another red bar, and there is another red arrow. And so on.***

S: ***The arrows tell us what to do. To start at 10 we put the ⑩-checker on the 1-square. Then a 2x arrow tells us to double it by moving it to the next square; for a +1 arrow we add a penny; +1, add another penny; the 2x arrow tells us to double it all by moving everything to the next squares; 2x, double again; and finally +1, add the last penny.***

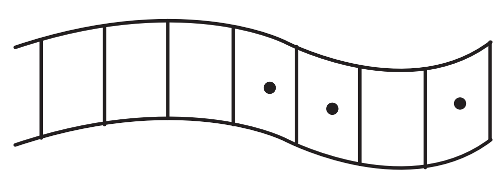
If this explanation is given, move the checkers on the snake as a student traces the arrows in the arrow road. This method is analogous to the one explained in the note at the end of Lesson W10; the only difference is that one involves the Minicomputer and the other involves the snake (base-two abacus).



At this point, let the class read and do the work on pages 27 to 30.

Exercise 2

Draw this snake on the board.



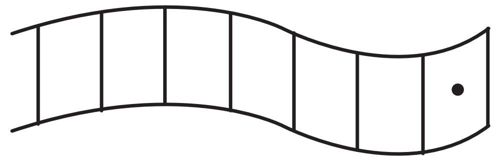
T: ***What is this number?***

S: ***13.***

T: ***Why?***

S: ***1 is on the first board, 4 is on the third board, 8 is on the fourth board. 1 + 4 + 8 = 13.***

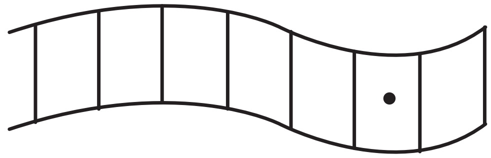
Put one checker on the first board.



T: ***What number is this?***

S: ***1.***

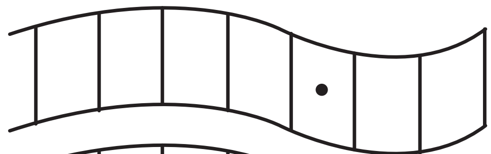
Move the checker one board to the left.



T: ***What number is this?***

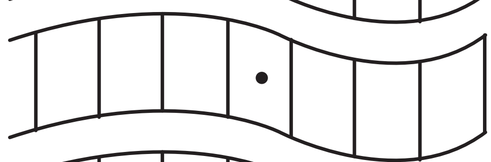
S: ***2.***

Continue moving the checker to the left and asking for the new number.

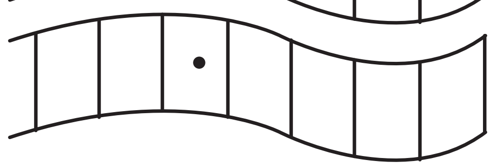


= 4

When you reach the fifth board, some students may think the number should be 10 rather than 16. If this happens, remind them that the snake is different from the Minicomputer.



= 8



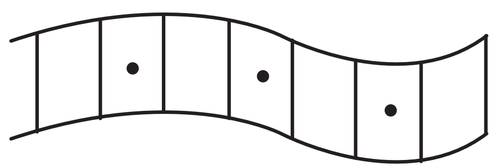
= 16

T: ***Look at the pattern of doubling; 1, 2, 4, 8, then what number?***

S: ***16.***

Continue in this manner until 1 024 is reached. You may want to label each board with its value.

Put this configuration on the snake.

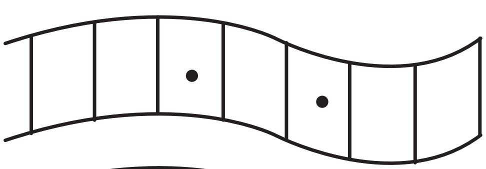


T: ***Write this number on your paper.*** (42)

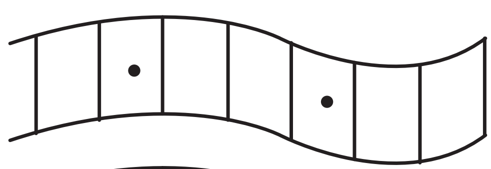
Check several students' papers.

T: ***I would like someone to put 20 on the snake, using one checker at most on each board.***

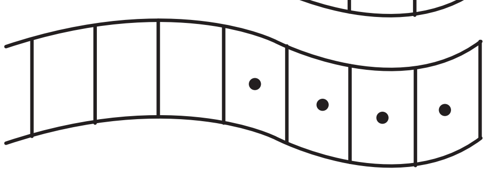
Continue in this manner with 15, 36, and 50.



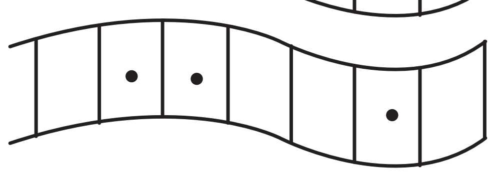
= 20



= 36



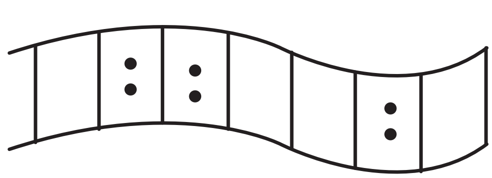
= 15



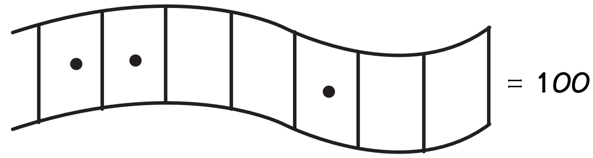
= 50

T: ***Here we have 50 on the snake. How can we get 100 immediately?***

A likely response is to add one checker to each board where there is already a checker.



= 100



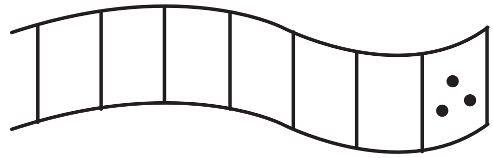
When this suggestion is made, accept it without comment but encourage students to make trades from the doubling pattern.

S: ***2 + 2 = 4; 16 + 16 = 32; 32 + 32 = 64.***

Other students may notice that moving each checker over one square to the left doubles the number.

Exercise 3

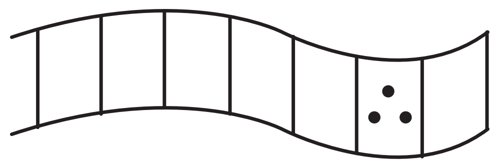
Put this configuration on the snake.



T: ***Which number is this?***

S: ***3.***

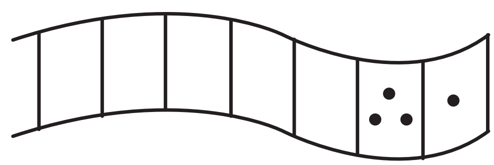
Move the checkers one board to the left.



T: ***Which number is this?***

S: ***6; you doubled 3.***

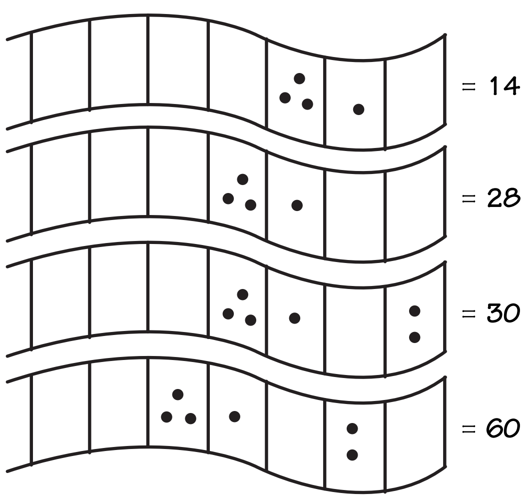
Add a checker.

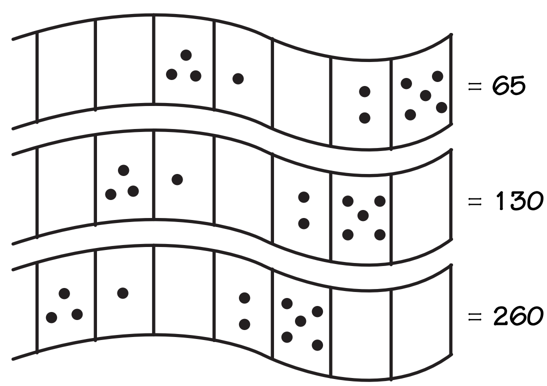


T: ***And this number?***

S: ***7, because 6 + 1 = 7.***

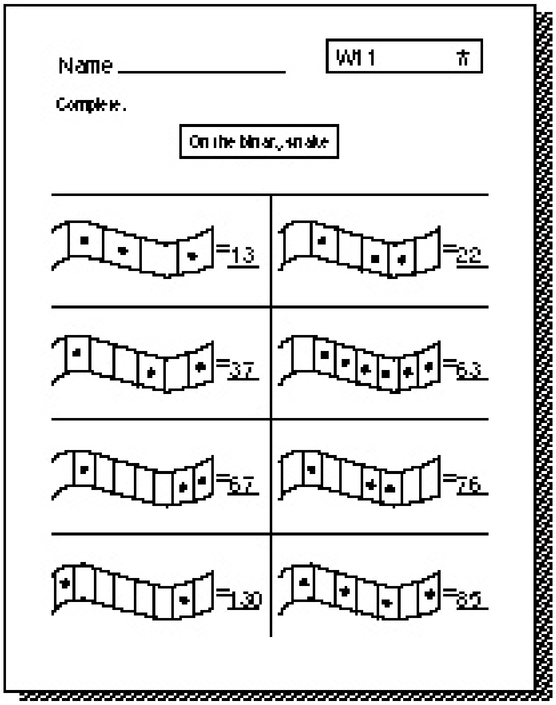
Continue in this manner, either moving all of the checkers one board to the left or adding checkers on the ones board.





Ask students to begin working on page 31 of the story-workbook. They should work as far as they can. Everyone should complete through page 35 to solve the mystery. Faster students can be encouraged to complete the story-workbook. The material beyond page 36 is an interesting extension of the abacus to other bases.

Worksheets W11*, **, and *** are available for the remainder of the lesson. If students have difficulty getting started with the * worksheet, encourage them to label each board of the snake with its value.


Name ____________ W11 ☆

Complete.

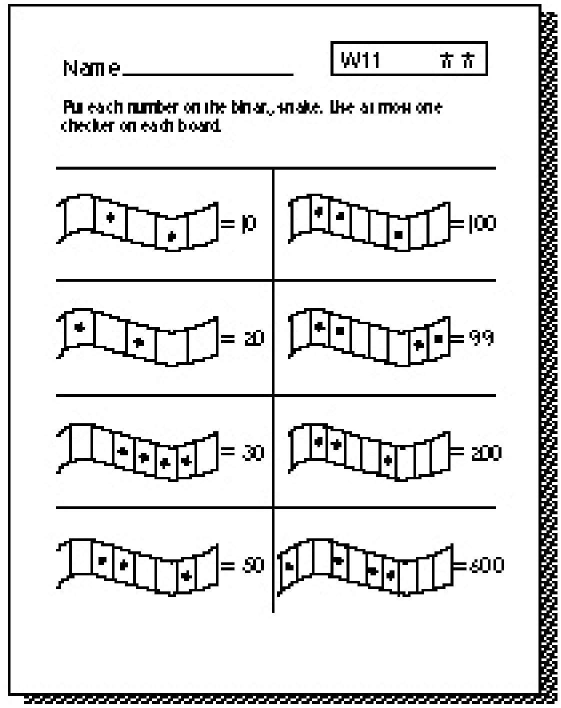
On the binary snake

=13 =22

=37 =63

=67 =76

=130 =85


Name ____________ W11 ☆☆

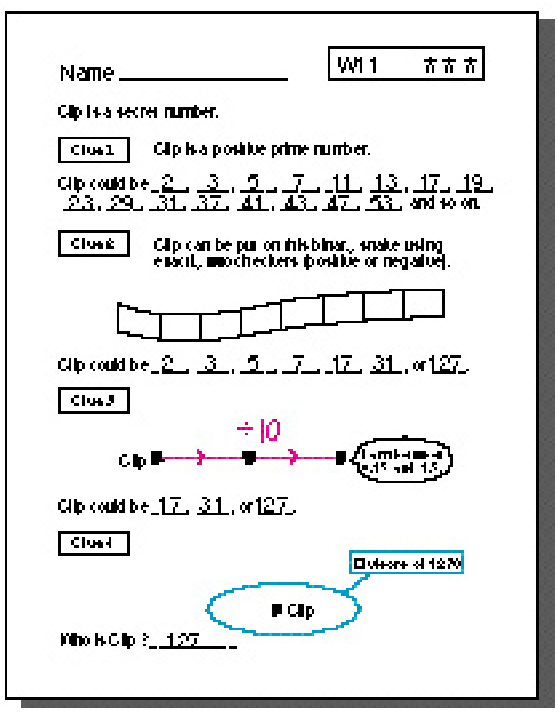
Put each number on the binary snake. Use at most one checker on each board.

=10 =100

=20 =99

=30 =200

=50 =600


Name ____________ W11 ☆☆☆

Clip is a secret number.

Clue 1 Clip is a positive prime number.

Clip could be 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, and so on.

Clue 2 Clip can be put on this binary snake using exactly two checkers (positive or negative).

Clip could be 2, 3, 5, 7, 17, 31, or 127.

Clue 3

Clip could be 17, 31, or 127.

Clue 4

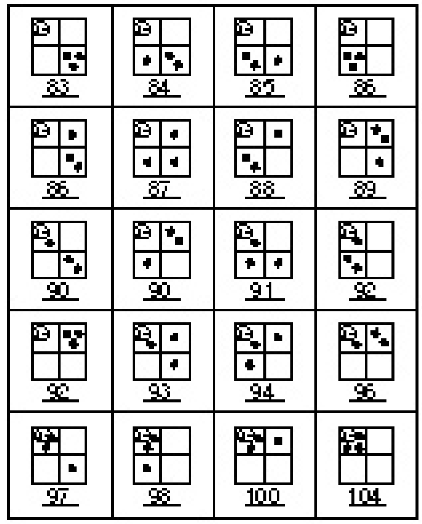
Who is Clip? 127

[illegible]

HOW MANY NUMBERS CAN YOU PUT ON A MINICOMPUTER BOARD WITH [illegible] AND THREE PEBBLES? WRITE WHAT YOU THINK BELOW.

60 numbers

(Accept any estimate for this problem. A complete solution is given at the end of Lesson 1.)



[illegible]

They are:

86, 90, and 92.

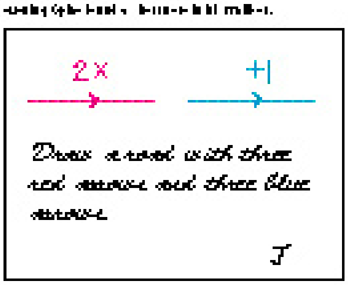
[illegible]

They are:

95, 99, 101, 102, and 103.

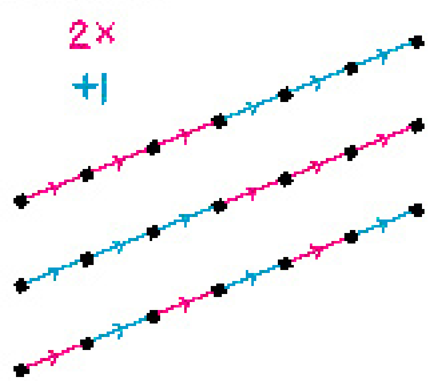
[illegible]

[illegible]



[illegible]

[illegible]



ON A SEPARATE SHEET OF PAPER, TRY TO DRAW ALL OF THE POSSIBLE ROADS. HOW MANY ARE THERE?

There are 20 roads.

YOU DID IT TOO ON THE ROADS THAT ARE DRAWN ON PAGES [illegible].
WHAT DO YOU NOTICE? The ending numbers are the same as the numbers that can be put on the Minicomputer with a dime on the 8-square and three pennies somewhere or other on the Minicomputer.

DID YOU FIND TWENTY DIFFERENT NUMBERS?

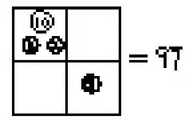
No, there are only 17 different numbers.

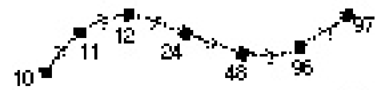
DO YOU SEE SOME CONNECTION BETWEEN THE ROAD CLUES AND THE MINICOMPUTER CLUES? IF SO, WHAT CONNECTION DO YOU SEE?

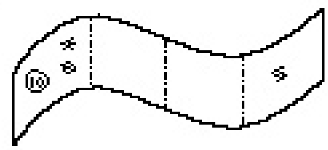
WRITE YOUR ANSWER BELOW.

Any comment relating the ending number of the arrow road, starting at 10 and consisting of three red and three blue arrows, to the numbers that can be put on the Minicomputer using exactly one dime and three pennies should be accepted. The intention is to have students be aware that there is a connection, and not to describe this connection until later in the story.

PICK ANY NUMBER (EXCEPT [illegible]) FROM [illegible] TO [illegible] AND, IN THE SPACE BELOW, DRAW A MINICOMPUTER PICTURE, AN ARROW PICTURE, AND A [illegible] PICTURE FOR IT.



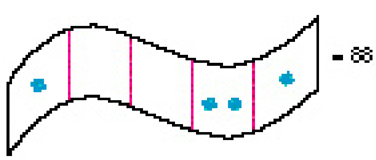


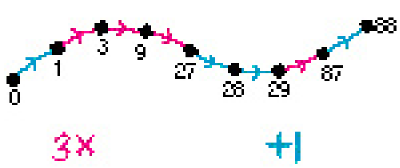


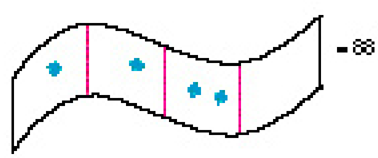
DO YOU THINK THAT THE [illegible] HELPED [illegible] TREASURE HUNT?

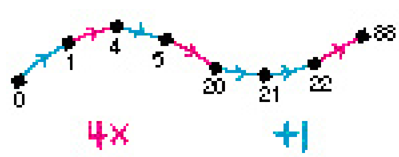
[illegible]

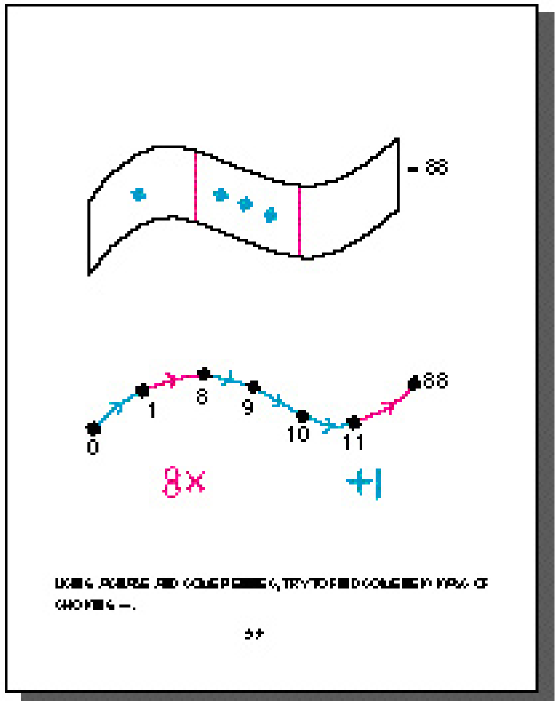
CHECK THEIR PICTURES ON PAGES [illegible] BY DECODING EACH ONE. LABEL THE DOTS IN EACH ARROW PICTURE.










88
0
1
8
9
10
11
88
8x
+1

Capsule Lesson Summary

Solve a calculator puzzle in which 352 is put on the calculator using only the keys $\boxed{0}$, $\boxed{1}$, $\boxed{+}$, and $\boxed{=}$. Begin the workbook *Set of Problems #4*. (This is the first of two lessons using this workbook.)

Materials

Teacher
- Calculator

Student
- Calculator
- *Set of Problems #4* Workbook
- Colored pencils, pens, or crayons
- Metric ruler

Description of Lesson

Arrange for every student to have access to a calculator. You may like to let students work with a partner.

Write these key symbols on the board: $\boxed{0}$, $\boxed{1}$, $\boxed{+}$, $\boxed{=}$.

T: ***Today I have a different calculator puzzle for you. For this puzzle you may use only the keys*** $\boxed{0}$***,*** $\boxed{1}$***,*** $\boxed{+}$***, and*** $\boxed{=}$***. Start with 0 on your calculator. Then try to put 352 on the display, and try to do this with as few key presses as possible.***

As necessary, remind students that they may use only the four keys in the list on the board. To help them remember and to count key presses, suggest students record the sequence of keys they press to get 352. You may like to motivate students to find solutions with fewer key presses by saying that it costs a dollar each time they press a key. Tell them to try to find as "cheap" a solution as possible.

When many students have found at least one solution, begin to record some of their suggestions on the board. Compare solutions according to how many keys are pressed or how much they cost. For example:

352:

$\boxed{+}\ \boxed{1}\boxed{0}\boxed{0}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{+}\ \boxed{1}\boxed{0}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{+}\ \boxed{1}\ \boxed{=}\ \boxed{=}$ (19 key presses; \$19)

$\boxed{+}\ \boxed{1}\boxed{1}\boxed{1}\ \boxed{=}\ \boxed{=}\ \boxed{+}\ \boxed{1}\boxed{0}\boxed{0}\ \boxed{+}\ \boxed{1}\boxed{0}\ \boxed{=}\ \boxed{=}\ \boxed{=}$ (16 key presses; \$16)

$\boxed{1}\boxed{1}\boxed{1}\ \boxed{+}\ \boxed{1}\boxed{1}\boxed{0}\ \boxed{=}\ \boxed{=}\ \boxed{+}\ \boxed{1}\boxed{1}\ \boxed{+}\ \boxed{1}\boxed{0}\ \boxed{=}$ (16 key presses; \$16)

$\boxed{+}\ \boxed{1}\boxed{1}\boxed{1}\ \boxed{=}\ \boxed{=}\ \boxed{+}\ \boxed{1}\boxed{1}\boxed{0}\ \boxed{+}\ \boxed{1}\boxed{0}\ \boxed{=}\ \boxed{=}$ (15 key presses; \$15)

$\boxed{1}\boxed{1}\boxed{0}\ \boxed{+}\ \boxed{1}\boxed{1}\boxed{1}\ \boxed{=}\ \boxed{=}\ \boxed{+}\ \boxed{1}\boxed{0}\ \boxed{=}\ \boxed{=}$ (14 key presses; \$14)

$\boxed{+}\ \boxed{1}\boxed{1}\boxed{0}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{+}\ \boxed{1}\boxed{1}\ \boxed{=}\ \boxed{=}$ (12 key presses; \$12)

$\boxed{+}\ \boxed{1}\boxed{1}\ \boxed{=}\ \boxed{=}\ \boxed{+}\ \boxed{1}\boxed{1}\boxed{0}\ \boxed{=}\ \boxed{=}\ \boxed{=}$ (12 key presses; \$12)

Note: All of these solutions make use of the constant feature of a calculator. Read "Role and Use of Calculators" in Section One, Notes to the Teacher to learn more about this feature.

Distribute copies of the workbook *Set of Problems #4* and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of the lesson, collect the workbooks for your review. They will be used again in Lesson W13.

Capsule Lesson Summary

Given a true multiplication (division) statement, solve some related multiplication (division) problems and observe patterns. Continue individual work in the workbook *Set of Problems #4*. (This is the second of two lessons using this workbook.)

Materials

Teacher
- None

Student
- *Set of Problems #4* Workbook
- Colored pencils, pens, or crayons
- Metric ruler

Description of Lesson

Exercise 1: Mental Arithmetic

Write this boxed statement on the board.

$$\boxed{12 \times 35 = 420}$$

Ask students to check the calculation, preferably without pencil and paper, and invite students to explain their checking techniques.

S: ***10 x 35 = 350. 2 x 35 = 70, and 350 + 70 = 420.***

S: ***2 x 35 = 70 and 6 x 70 = 420.***

S: ***2 x 35 = 70, 6 x 35 = 210, and 12 x 35 = 420.***

Write this problem on the board below the boxed multiplication statement.

$$13 \times 35$$

T: ***Try to do this calculation without too much work. The multiplication statement 12 x 35 = 420 can help you.***

S: ***13 x 35 = 455. Here there are thirteen 35s instead of twelve; one more 35 is 420 + 35 = 455.***

Continue with the following calculations. Answers are in the boxes. Let students explain how they use an earlier problem to solve a new problem. Keep up a rather brisk pace.

$11 \times 35 = \boxed{385}$ $\qquad$ $12 \times 36 = \boxed{432}$

$14 \times 35 = \boxed{490}$ $\qquad$ $12 \times 37 = \boxed{444}$

$24 \times 35 = \boxed{840}$ $\qquad$ $12 \times 70 = \boxed{840}$

$48 \times 35 = \boxed{1\,680}$ $\qquad$ $12 \times 140 = \boxed{1\,680}$

Invite students to comment on patterns they observe.

Repeat this exercise by giving a division calculation and several related division problems. For example:

$$\boxed{98 \div 7 = 14}$$

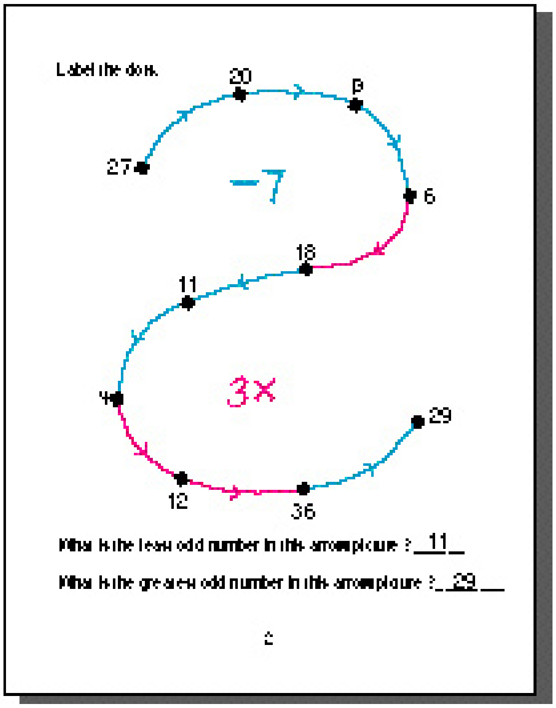
$91 \div 7 = \boxed{13}$	$196 \div 7 = \boxed{28}$
$105 \div 7 = \boxed{15}$	$168 \div 7 = \boxed{24}$
$119 \div 7 = \boxed{17}$	$175 \div 7 = \boxed{25}$

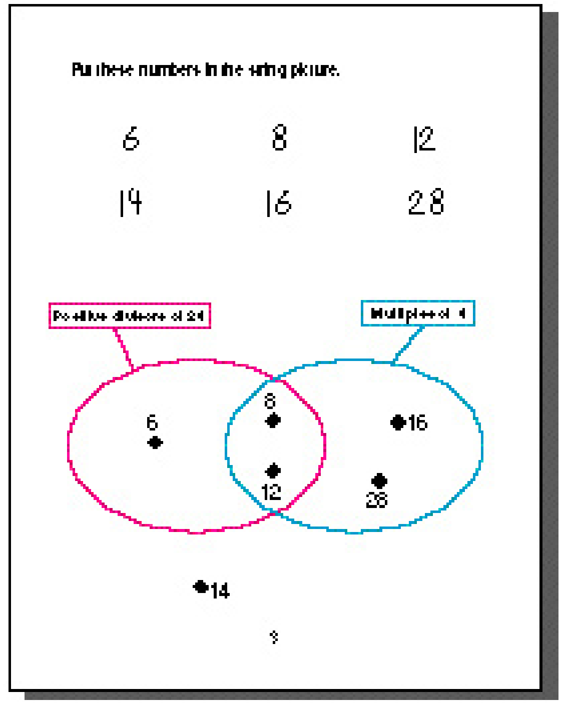
Exercise 2

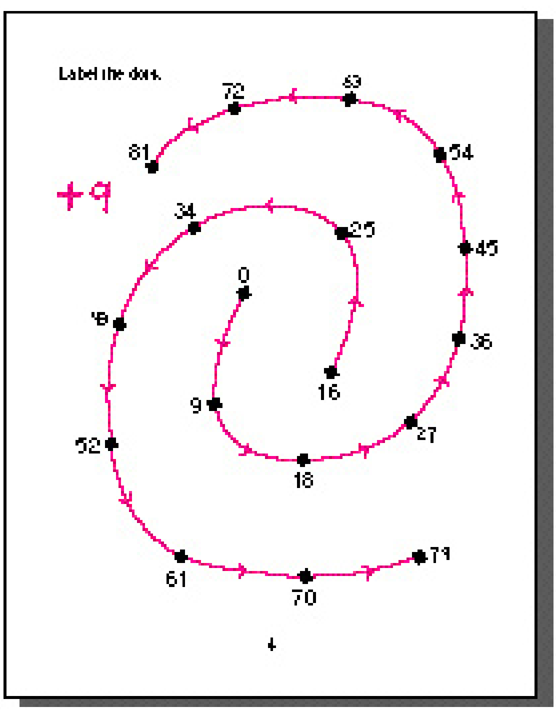
Return students' copies of the workbook *Set of Problems #4*. Ask students first to correct or complete pages they worked on the previous week and then to continue working in their workbook. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

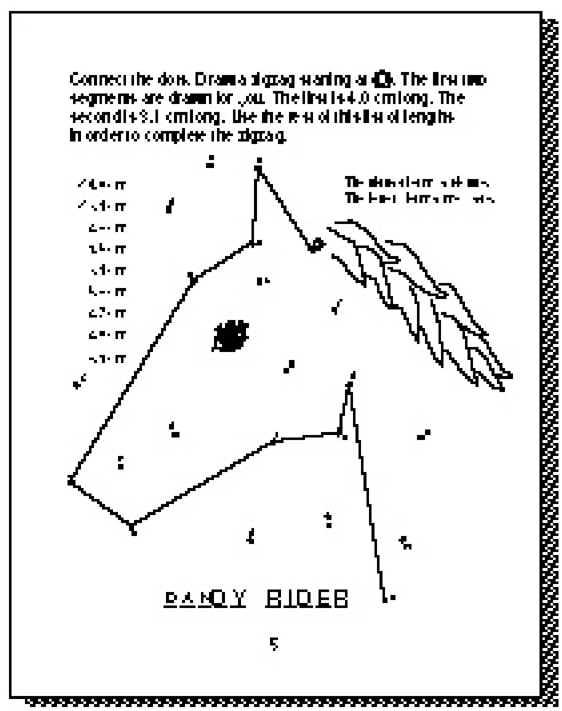
Assessment Activity

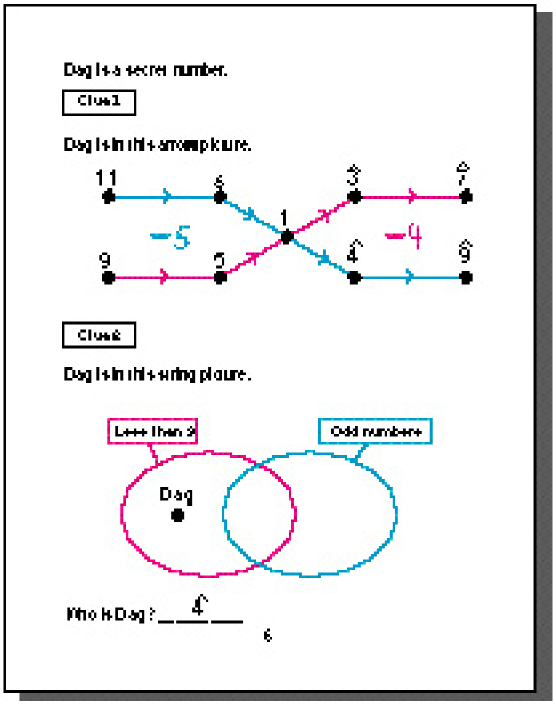
An individual student progress record for the workbook is available on Blackline W13. You may like to use this form to monitor student work.


Label the dots.
20
9
27
6
−7
18
11
4
3x
29
12
36
What is the least odd number in this arrow picture? 11
What is the greatest odd number in this arrow picture? 29
2


Put these numbers in the string picture.
6 8 12
14 16 28
Positive divisors of 24
Multiples of 4
6
8
16
12
28
14
3


Label the dots.
72
63
81
54
+9
34
25
45
0
36
16
9
52
27
18
61
70
79
4


Connect the dots. Draw a zigzag starting at . The first two segments are drawn for you. The first is 4.0 cm long. The second is 3.1 cm long. Use the rest of this list of lengths in order to complete the zigzag.
DANDY RIDER
5


Dag is a secret number.

Clue 1

Dag is in this arrow picture.

Clue 2

Dag is in this string picture.

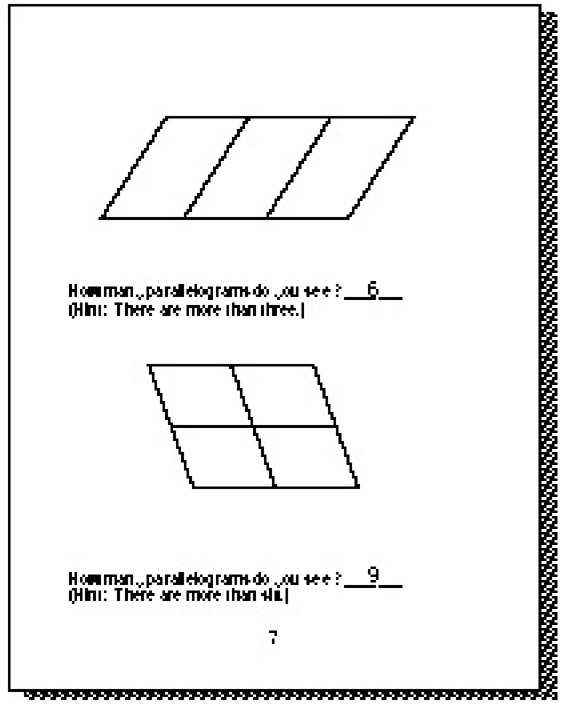
Less than 9

Odd numbers

Dag

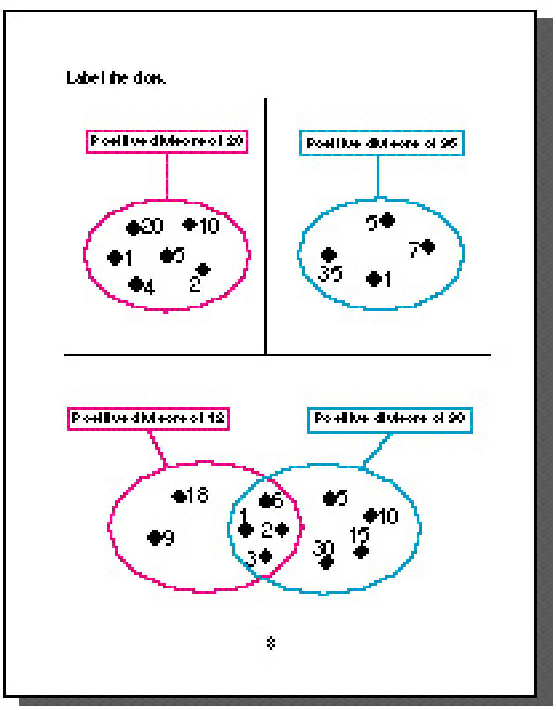
Who is Dag? 4

6


How many parallelograms do you see? 6
(Hint: There are more than three.)

How many parallelograms do you see? 9
(Hint: There are more than six.)

7


Label the dots.

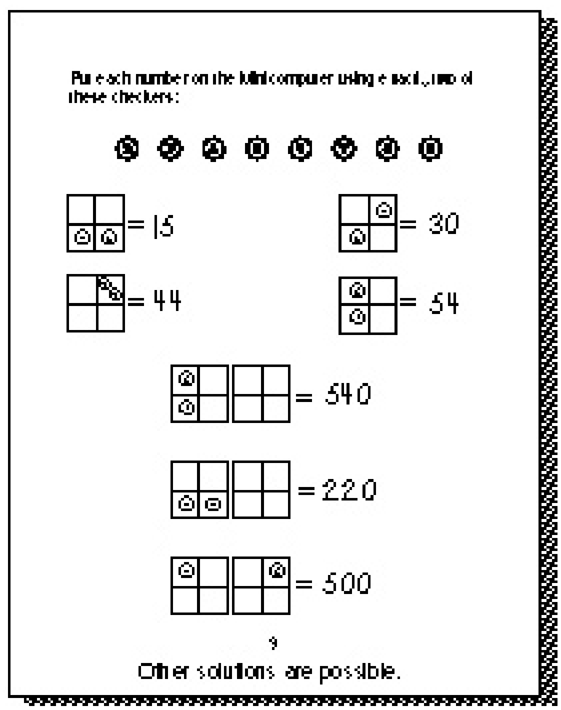
Positive divisors of 20

Positive divisors of 35

Positive divisors of 12

Positive divisors of 30

8


Put each number on the Minicomputer using exactly two of these checkers:

= 15

= 30

= 44

= 54

= 540

= 220

= 500

9

Other solutions are possible.

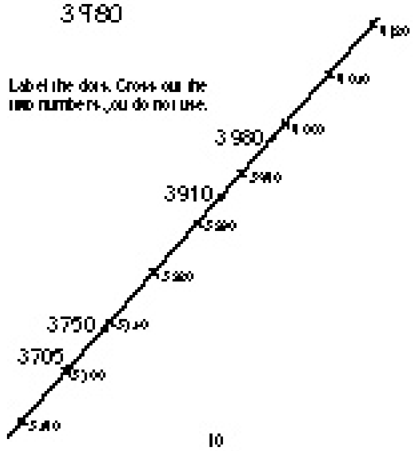
The dots on the number line are for four of these numbers.

3 750 3 910

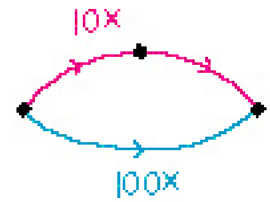
3 705

3 980

Label the dots. Cross out the two numbers you do not use.



10



Use the arrow picture to help you do these calculations.

$100 \times 7 = \boxed{700}$

$100 \times 82 = \boxed{8200}$

$100 \times 40 = \boxed{4000}$

$100 \times \boxed{68} = 6800$

$100 \times \boxed{70} = 7000$

$100 \times 0.06 = \boxed{6}$

11

Multiply.

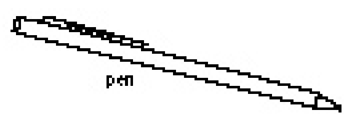
23 × 6 = 138	23 × 40 = 920	23 × 46 = 1058
19 × 5 = 95	19 × 30 = 570	19 × 35 = 665
72 × 80 = 5760	72 × 7 = 504	72 × 87 = 6264

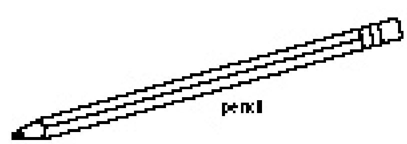
12

Find the length of the pen and the length of the pencil to the nearest millimeter.

Pen __137__ mm

Pencil __165__ mm





How much longer is the pencil? __28__ mm

13

Sherry is making boxes of art supplies. For 10 boxes she needs 60 pieces of paper; for 15 boxes she needs 45 pencils; and for 20 boxes she needs 100 paper clips. Each box has the same items in it.

Complete the list of supplies for 10, 15, and 20 boxes.

10 boxes	15 boxes	20 boxes
60 pieces of paper	90 pieces of paper	120 pieces of paper
30 pencils	45 pencils	60 pencils
50 paper clips	75 paper clips	100 paper clips

Sherry wants to make enough boxes so that 75 people can get boxes of art supplies. Write the list of supplies she needs.

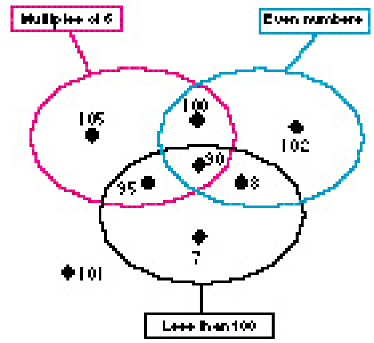
75 boxes
450 pieces of paper
225 pencils
375 paper clips

14

Put these numbers in the string picture.

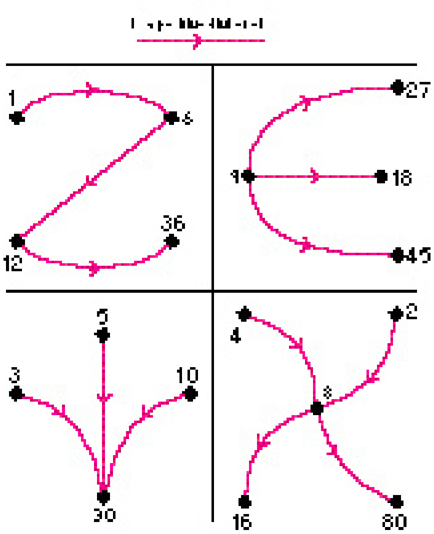
7 8 90 95

100 101 102 105



15

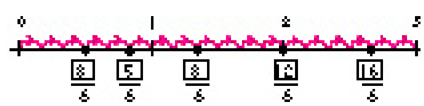
Label the dots. Many solutions are possible.



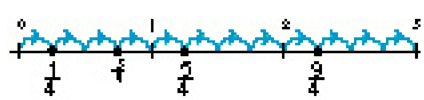
16

Many solutions are possible.

Fill in the boxes.



Label the dots. One is done for you.

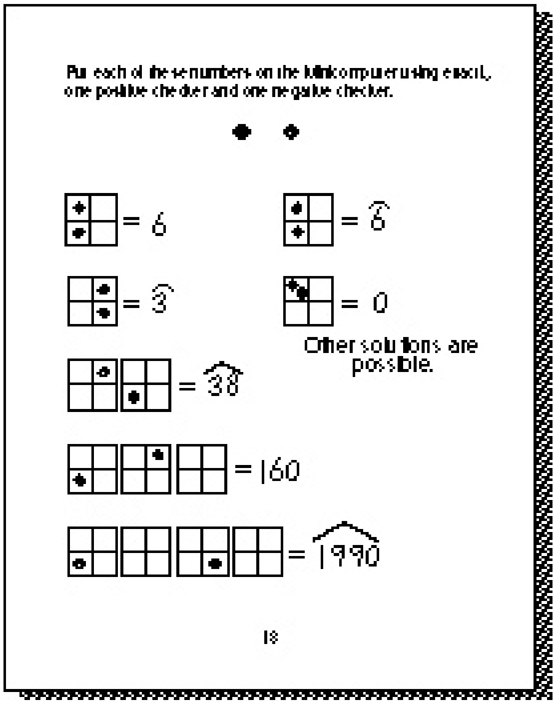


Put <, =, or > in each box. Use the number lines to help you.

$\frac{9}{4}$ $\boxed{>}$ $\frac{8}{6}$ $\qquad$ $\frac{8}{4}$ $\boxed{=}$ 2

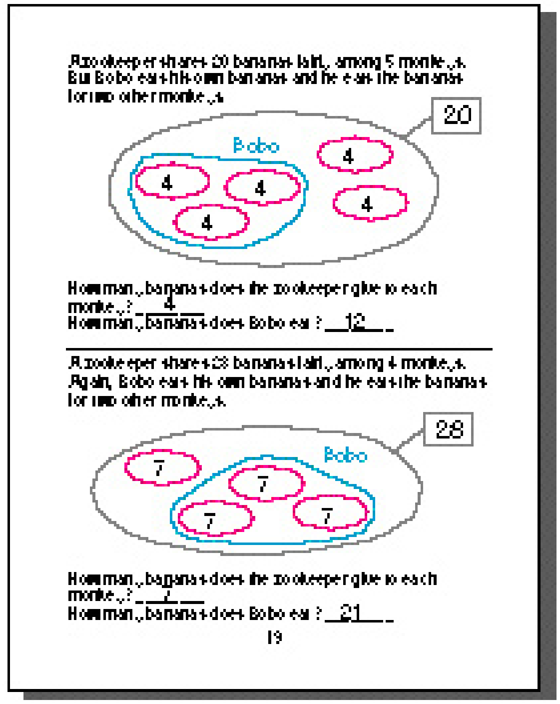
$\frac{5}{4}$ $\boxed{>}$ $\frac{5}{6}$ $\qquad$ $\frac{2}{4}$ $\boxed{=}$ $\frac{3}{6}$

17



Put each of these numbers on the Minicomputer using exactly one positive checker and one negative checker.

18



A zookeeper shares 20 bananas fairly among 5 monkeys. But Bobo eats his own bananas and he eats the bananas for two other monkeys.

How many bananas does the zookeeper give to each monkey? 4

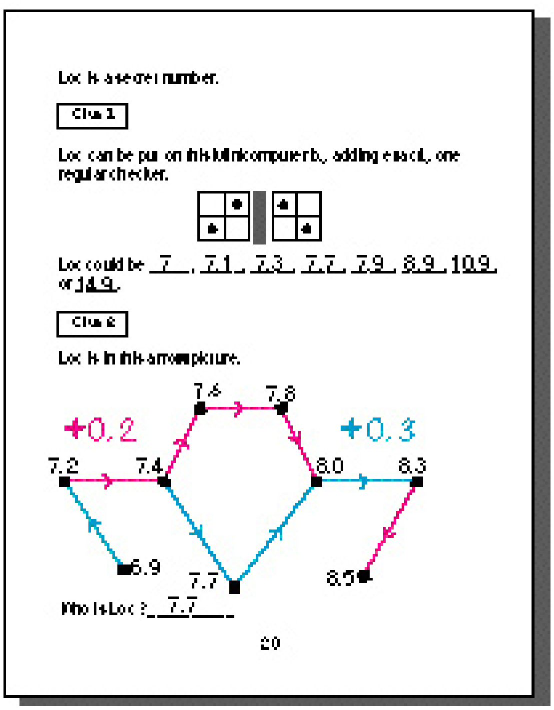
How many bananas does Bobo eat? 12

A zookeeper shares 28 bananas fairly among 4 monkeys. Again, Bobo eats his own bananas and he eats the bananas for two other monkeys.

How many bananas does the zookeeper give to each monkey? 7

How many bananas does Bobo eat? 21

19



Loc is a secret number.

Clue 1

Loc can be put on this Minicomputer by adding exactly one regular checker.

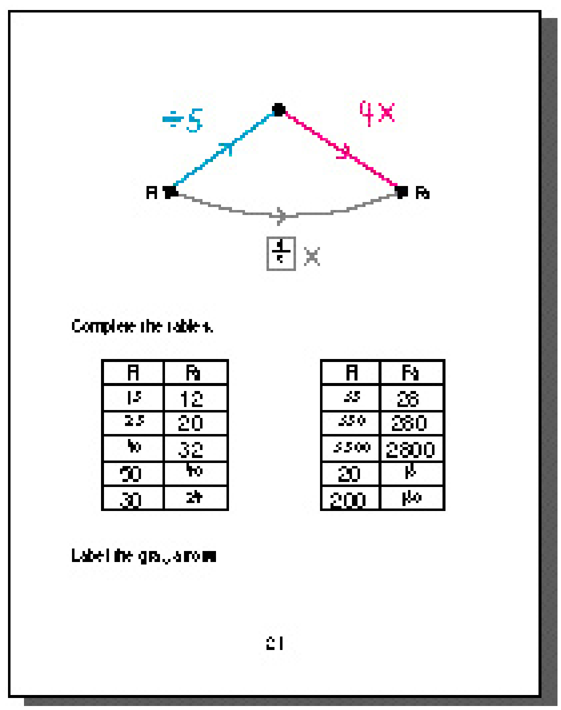
Loc could be 7, 7.1, 7.3, 7.7, 7.9, 8.9, 10.9, or 14.9.

Clue 2

Loc is in this arrow picture.

Who is Loc? 7.7

20



Complete the tables.

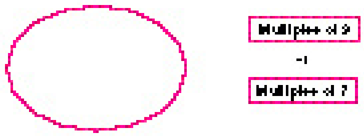
A	B
15	12
25	20
40	32
50	40
30	24

A	B
35	28
350	280
3500	2800
20	16
200	160

Label the gray arrow.

21

We know that this red string is either for multiples of 3 or for multiples of 7.



Draw a circle around each number below that we know for sure belongs inside the red string.

Draw a triangle around each number below that we know for sure belongs outside the red string.

17 21 27

56 63 103

Which two numbers have neither a circle nor a triangle around them because we cannot tell for sure where they belong? 27 and 56

22

Complete.

$680 \div 5 = 136$

$685 \div 5 = \boxed{137}$ $675 \div 5 = \boxed{135}$

$695 \div 5 = \boxed{139}$ $660 \div 5 = \boxed{132}$

Complete.

$420 \div 7 = \boxed{60}$

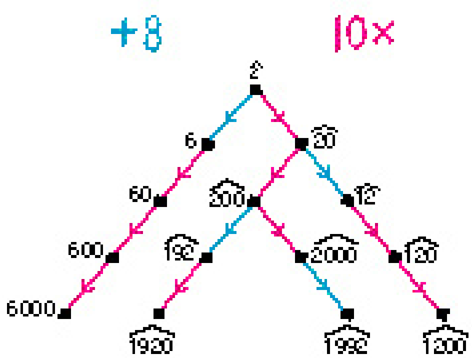
$427 \div 7 = \boxed{61}$ $413 \div 7 = \boxed{59}$

$448 \div 7 = \boxed{64}$ $399 \div 7 = \boxed{57}$

$462 \div 7 = \boxed{66}$ $378 \div 7 = \boxed{54}$

23

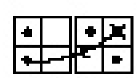
Find all of the possible ending numbers of an arrow road that starts at 2 and has exactly one +8 arrow and three 10x arrows. You may draw a tree diagram with arrows to help you.



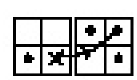
List the ending numbers. 6000, 1200, 1920, 1992

24

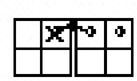
Move one checker to put 109 on the Minicomputer.



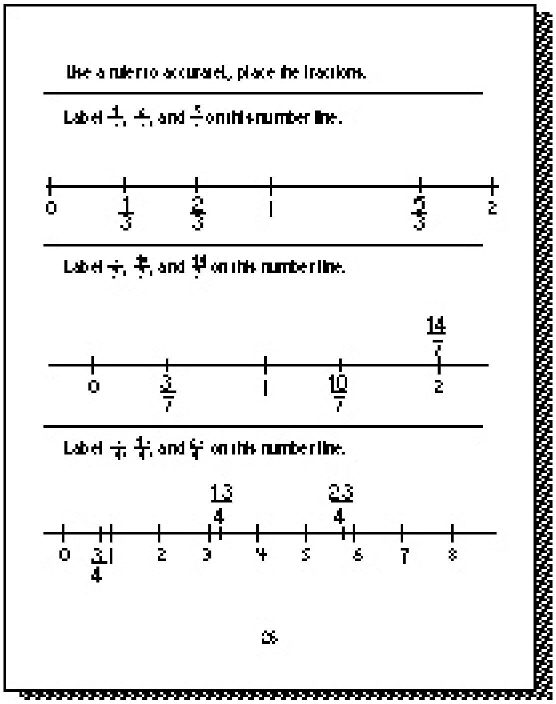
Move one checker to put 7 on the Minicomputer.



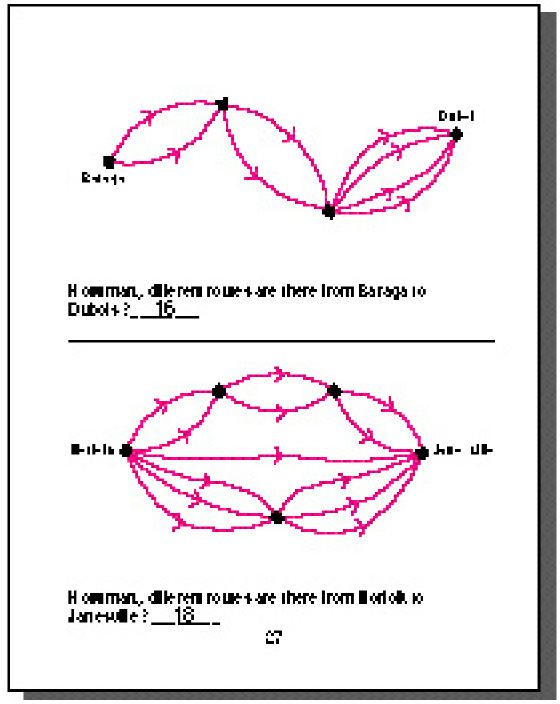
Move one checker to put 28 on the Minicomputer.



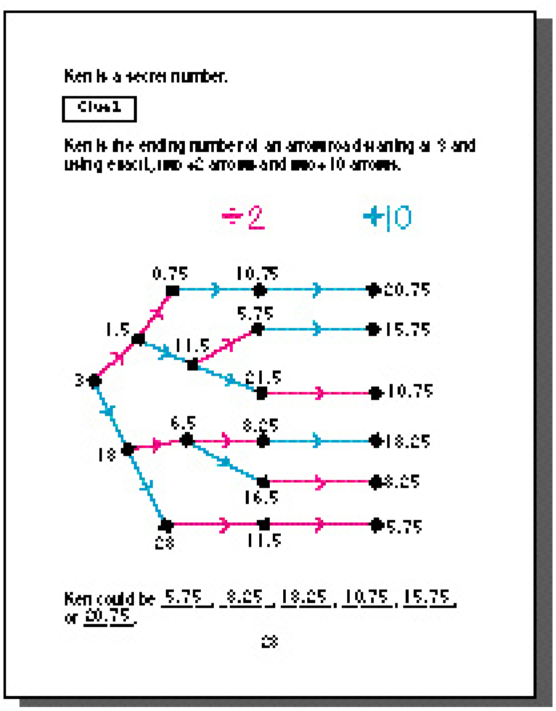
25



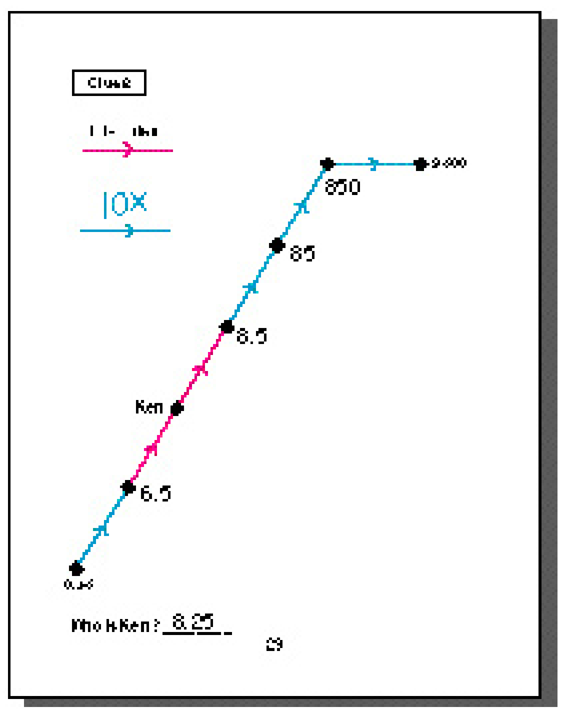
0
1/3
2/3
1
5/3
2
14/7
0
3/7
1
10/7
2
13/4
23/4
0
3/4
1
2
3
4
5
6
7
8
26



16
18
27

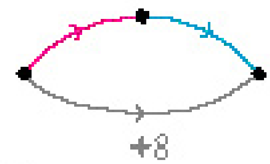


Ken is a secret number.
÷2
+10
0.75
10.75
20.75
5.75
15.75
1.5
11.5
21.5
10.75
3
6.5
8.25
18.25
13
16.5
8.25
23
11.5
5.75
Ken could be 5.75, 8.25, 18.25, 10.75, 15.75, or 20.75.
28

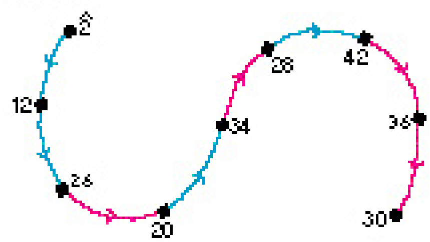


10X
850
85
8.5
Ken
6.5
Who is Ken? 8.25
29

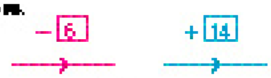
Using this arrow picture,



Label the dots.



Label the arrows.



90

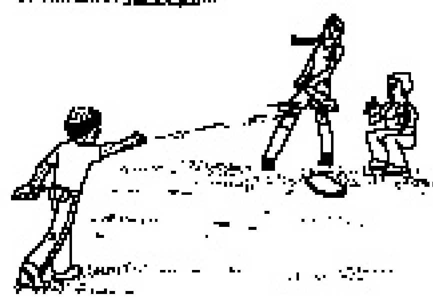
Sports Day

Halloran School is planning a sports day. The activities will begin at 8:00 a.m. and end at 3:00 p.m., with a 1 hour lunch period somewhere in between.

How many 40 minute activities can the coach schedule? (Don't forget to take time out for lunch.) ___9___

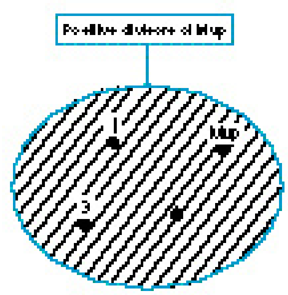
How many activities do you think the coach should schedule before lunch? ___6___ How many after lunch? ___3___

Based on your last two answers, at what time would lunch be scheduled? 12:00 p.m.



91

Many solutions are possible.



Mup must have exactly four positive divisors. List at least five numbers Mup could be.

___6___ ___15___ ___21___ ___33___ ___39___

What do you notice about numbers that Mup could be? ______

__3 x any prime number could be Mup.______

Mup could also be 27.

92

Capsule Lesson Summary

Explore certain fundamental properties of the set of integers as well as the role that 0 and 1 play in that system.

Materials

Teacher • *Nabu Wins an Award* Storybook

Student • *Nabu Wins an Award* Storybook

The purpose of this lesson is to explore certain properties of the integer number system in a very open way, a way that should provide an opportunity for students to participate at their level of understanding. It will be up to you, the teacher, to direct the discussion and to capitalize on insights that students might have.

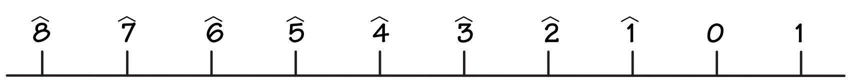
Below are some questions and students comments that might be made during the presentation of this story. Feel free to adjust your presentation of this storybook to the needs, abilities, and interests of your students. Above all, let students explore and enjoy this story of Nabu.

Description of Lesson

Distribute copies of the storybook *Nabu Wins an Award* and invite students to read the story aloud (or you can read it to the class).

Page 5

Draw this number line on the board.



Note: Much of the time *CSMP* uses the $\hat{\ }$ notation for negative numbers, although it does use the $^{-}$ notation also, for example, when calculators are being used. Your students should be familiar with both notations. The use of one notation in the storybook and the use of the other on the board should not cause difficulty.

T: ***Where is $\widehat{1/2}$ on the number line?***

S: ***$\widehat{1/2}$ is halfway between $\hat{1}$ and 0.***

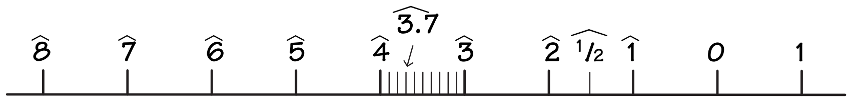
Ask a student to locate $\widehat{1/2}$ on the number line.

T: ***Where is $\widehat{3.7}$ on the number line?***

S: ***Between $\hat{3}$ and $\hat{4}$. Divide the segment between $\hat{3}$ and $\hat{4}$ into ten smaller segments of equal length.***

W14

Invite students to do the division and locate $\widehat{3.7}$.



Pages 7 and 8

T: ***What do you think of the $\frac{1}{2}$x and 2x dances?***

S: ***They are very much alike, but $\frac{1}{2}$x goes one way and 2x goes the other way.***

T: ***Yes. The $\frac{1}{2}$x dance is just the opposite of the 2x dance.***

S: ***They go on forever. There is no beginning number or ending number.***

Pages 11 and 12

T: ***What do you notice about the –2 and the +2 dances?***

S: ***There are no odd numbers in these dances.***

Note: Of course, these dances could have been made with just the odd numbers.

S: ***They are like the $\frac{1}{2}$x and 2x dances—one goes in the opposite direction from the other.***

T: ***Yes, they too are opposites of each other.***

Page 15

T: ***If 15 were in this dance, who would be its partner?***

S: ***$\frac{1}{15}$.***

T: ***Why?***

S: ***$15 \times \frac{1}{15} = 1 = \frac{1}{15} \times 15$.***

Page 16

S: ***The dances look like flowers.***

S: ***The + dance looks just like the x dance, but each positive number dances with a negative number.***

T: ***With any negative number?***

S: ***No, with its opposite: 8 and $\hat{8}$; $\widehat{26}$ and 26; and so on.***

T: ***Can 1 dance in 0's dance?***

S: ***Yes; 1 would dance with $\hat{1}$.***

T: ***Can 0 dance in 1's dance?***

S: ***No; no number times zero equals one.***

Page 19

S: ***This dance looks like teardrops.***

S: ***It's raining numbers.***

S: ***Each number dances with itself.***

T: ***Why?***

S: ***Because any number plus zero equals that number, and any number times one equals that number.***

Allow students as much time as they want to read and look at the pictures in the storybook. You may like to place several copies in the class reading corner for students to reread at their leisure.

If there is time for another activity in you math period, play *The String Game*, a Minicomputer game, or another mathematics game of your choice.

Capsule Lesson Summary

Introduce calculator sentences, and look for numbers obtained as a result of putting operation keys in this expression: $\boxed{8} - \boxed{2} - \boxed{5} - \boxed{3}\ \boxed{=}$. Choose a possible resulting number, and find which operation keys need to be used to get that resulting number. Begin the workbook *Set of Problems #5*. (This is the first of two lessons using this workbook.)

Materials

Teacher		Student	
	• Calculator		• Calculator • *Set of Problems #5* Workbook • Colored pencils, pens, or crayons • Metric ruler

Description of Lesson

Write this expression on the board.

$\boxed{8}\ \boxed{\times}\ \boxed{2}\ \boxed{+}\ \boxed{5}\ \boxed{\div}\ \boxed{3}\ \boxed{=}$

T: ***On your calculator, press these keys in the order given here. What number is on the display?***

S: ***7.***

T: ***This is a*** **calculator sentence** ***for 7. We call it a calculator sentence because it is how a calculator does the operations in the order we press the keys.***

Note: This description of a calculator sentence assumes that the calculator does chain operations. See "Role and Use of Calculators" in Section One, Notes to the Teacher to learn more about such features of a calculator. If your calculators do not do chain operations, you will need to adjust the lesson description accordingly.

Erase the operation symbols in the expression and ask,

$\boxed{8} - \boxed{2} - \boxed{5} - \boxed{3}\ \boxed{=}$

T: ***If we put different operation keys in the sentence, what are some other numbers we could get?***

There are many possibilities (more than 50), so accept several. Each time ask the student to announce which operation keys he or she used. For example:

S: ***18. I used all three*** $\boxed{+}$ ***keys*** $\boxed{8}\ \boxed{+}\ \boxed{2}\ \boxed{+}\ \boxed{5}\ \boxed{+}\ \boxed{3}\ \boxed{=}$ ***18.***

S: ***6;*** $\boxed{8}\ \boxed{\div}\ \boxed{2}\ \boxed{+}\ \boxed{5}\ \boxed{-}\ \boxed{3}\ \boxed{=}$ ***6.***

T: ***What is the greatest number we could get?*** (240)
What is the least number we could get? ($^-4$)

As students work on finding the greatest and least possible numbers, you are likely to get some discussion about comparing decimals or comparing decimals to negative numbers. For example, 0.3333333 is more than $^-1$.

W15

The greatest and least possible numbers result from the following choices of operations.

$\boxed{8}\ \boxed{\times}\ \boxed{2}\ \boxed{\times}\ \boxed{5}\ \boxed{\times}\ \boxed{3}\ \boxed{=}\ 240$ $\qquad$ $\boxed{8}\ \boxed{\div}\ \boxed{2}\ \boxed{-}\ \boxed{5}\ \boxed{-}\ \boxed{3}\ \boxed{=}\ {}^{-}4$

Choose a possible whole number that has not been mentioned, such as 12, and ask,

T: ***What operation keys would we use to get a calculator sentence for 12?***

$\boxed{8} - \boxed{2} - \boxed{5} - \boxed{3}\ \boxed{=}\ 12$

In this case there are two possible solutions.

$\boxed{8}\ \boxed{+}\ \boxed{2}\ \boxed{+}\ \boxed{5}\ \boxed{-}\ \boxed{3}\ \boxed{=}\ 12$ $\qquad$ $\boxed{8}\ \boxed{\div}\ \boxed{2}\ \boxed{+}\ \boxed{5}\ \boxed{+}\ \boxed{3}\ \boxed{=}\ 12$

Distribute copies of the workbook *Set of Problems #5* and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of the lesson, collect the workbooks for your review. They will be used again in Lesson W13.

Home Activity

Send home a description of a calculator sentence and some calculator sentence problems for students to work with a family member.

Capsule Lesson Summary

Do some mental arithmetic involving 1x, 10x, and 100x; 2x, 20x, and 200x; or 3x, 30x, and 300x. Continue individual work in the workbook *Set of Problems #5*. (This is the second of two lessons using this workbook.)

Materials

Teacher		Student	
	• None		• *Set of Problems #5* Workbook • Colored pencils, pens, or crayons • Metric ruler

Description of Lesson

Begin with some mental arithmetic such as in the sequences of calculations below.

$1 \times 67 = \boxed{67}$ $\quad 2 \times 81 = \boxed{162}$ $\quad 3 \times 32 = \boxed{96}$

$10 \times 67 = \boxed{670}$ $\quad 20 \times 81 = \boxed{1\,620}$ $\quad 30 \times 32 = \boxed{960}$

$100 \times 67 = \boxed{6\,700}$ $\quad 200 \times 81 = \boxed{16\,200}$ $\quad 300 \times 32 = \boxed{9\,600}$

$1 \times 2.4 = \boxed{2.4}$ $\quad 2 \times 6.3 = \boxed{12.6}$ $\quad 1 \times 0.86 = \boxed{0.86}$

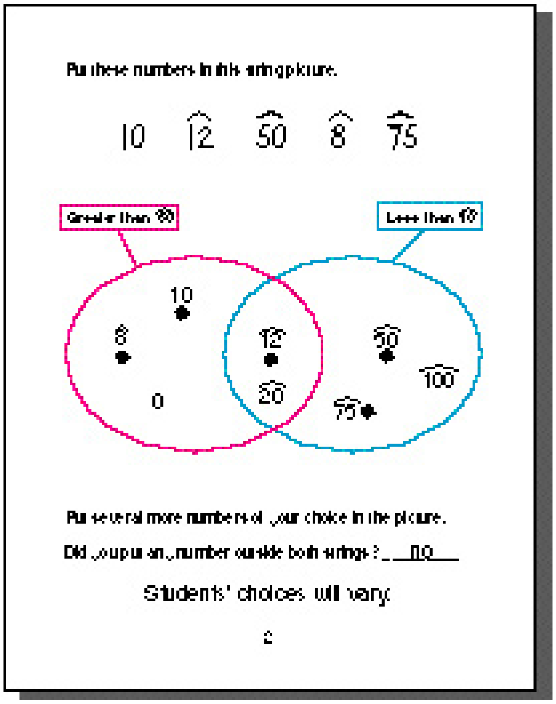
$10 \times 2.4 = \boxed{24}$ $\quad 20 \times 6.3 = \boxed{126}$ $\quad 10 \times 0.86 = \boxed{8.6}$

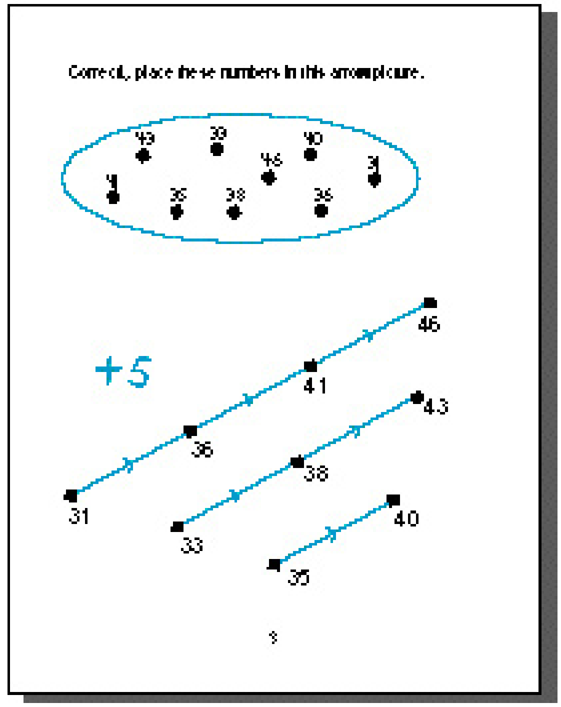
$100 \times 2.4 = \boxed{240}$ $\quad 200 \times 6.3 = \boxed{1\,260}$ $\quad 100 \times 0.86 = \boxed{86}$

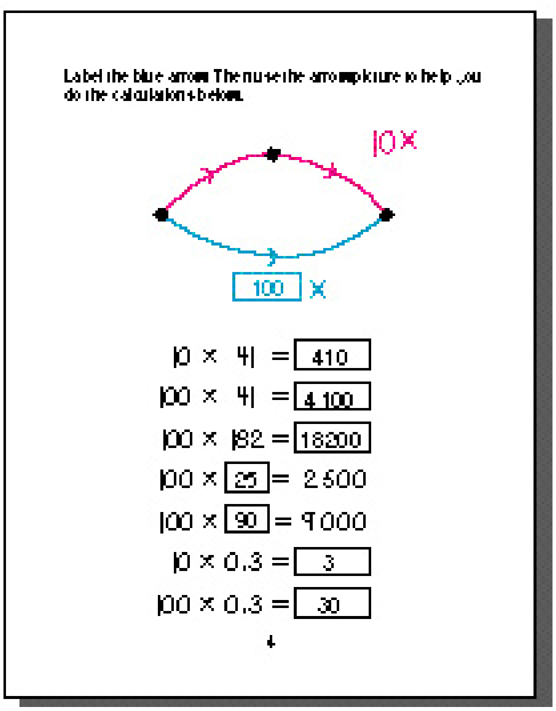
Return students' copies of the workbook *Set of Problems #5*. Ask students first to correct or complete pages they worked on the previous week and then to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

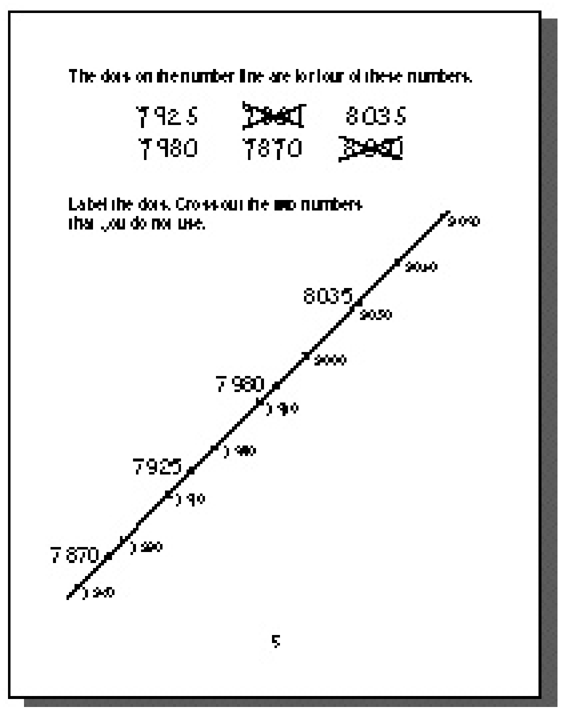
Assessment Activity

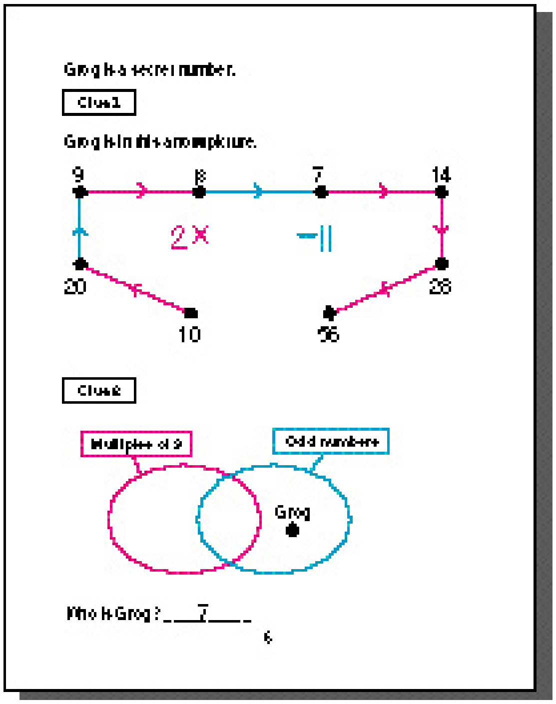
An individual student progress record for the workbook is available on Blackline W16. You may like to use this form to monitor student work.


Put these numbers in this string picture.
10 12 50 8 75
Greater than 9
Less than 10
10
8
0
12
20
50
100
75
Put several more numbers of your choice in the picture.
Did you put any number outside both strings? no
Students' choices will vary.
2


Correctly place these numbers in this arrow picture.
43 39 40 46 31 41 36 38 35 33
+5
46
41
36
31
43
38
33
40
35
3


Label the blue arrow. Then use the arrow picture to help you do the calculations below.
10×
100 ×
10 × 41 = 410
100 × 41 = 4 100
100 × 182 = 18200
100 × 25 = 2 500
100 × 90 = 9 000
10 × 0.3 = 3
100 × 0.3 = 30
4


The dots on the number line are for four of these numbers.
7 925 8 035
7 980 7 870
Label the dots. Cross out the two numbers that you do not use.
8035
7 980
7925
7 870
5


Grog is a secret number.

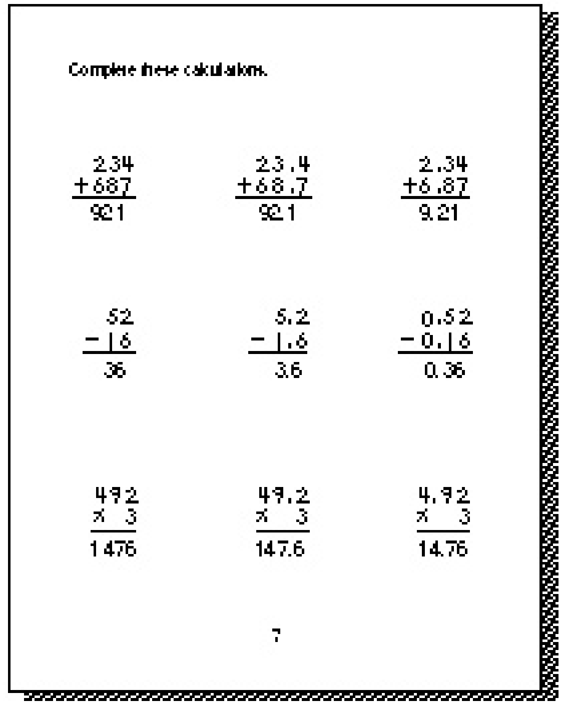
Clue 1

Grog is in this arrow picture.

Clue 2

Who is Grog? 7

6

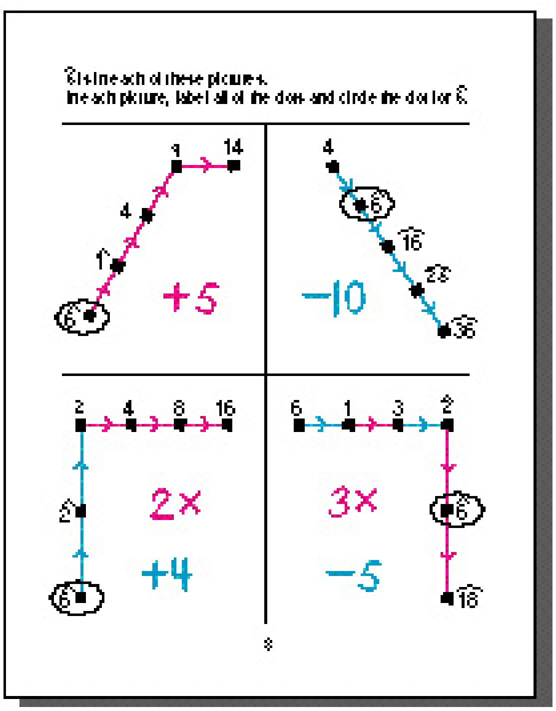

Complete these calculations.

$$\begin{array}{r}234\\+687\\\hline 921\end{array}\qquad\begin{array}{r}23.4\\+68.7\\\hline 92.1\end{array}\qquad\begin{array}{r}2.34\\+6.87\\\hline 9.21\end{array}$$

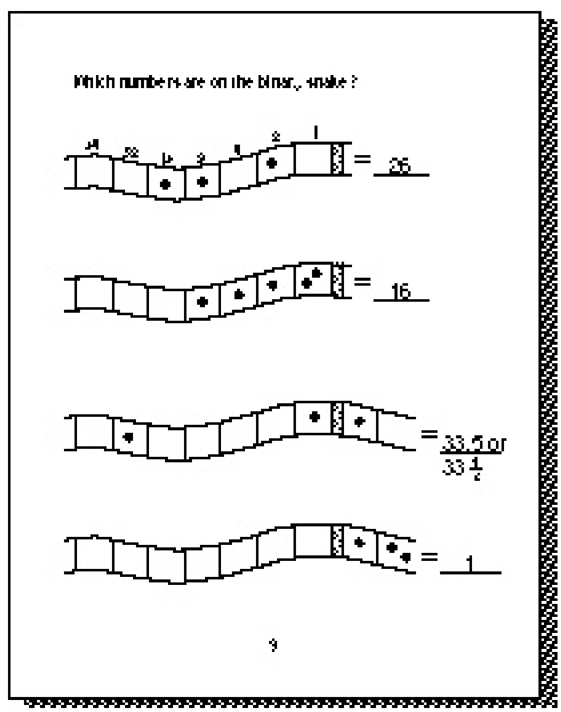
$$\begin{array}{r}52\\-16\\\hline 36\end{array}\qquad\begin{array}{r}5.2\\-1.6\\\hline 3.6\end{array}\qquad\begin{array}{r}0.52\\-0.16\\\hline 0.36\end{array}$$

$$\begin{array}{r}492\\\times\ 3\\\hline 1476\end{array}\qquad\begin{array}{r}49.2\\\times\ 3\\\hline 147.6\end{array}\qquad\begin{array}{r}4.92\\\times\ 3\\\hline 14.76\end{array}$$

7

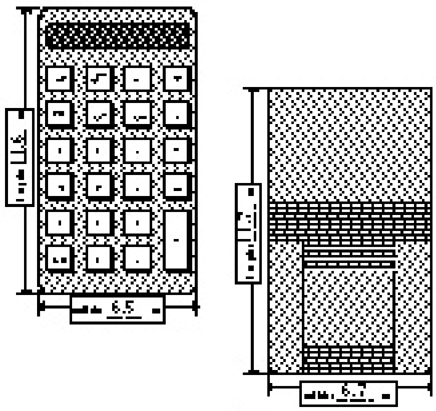

$\hat{6}$ is in each of these pictures.
In each picture, label all of the dots and circle the dot for $\hat{6}$.

8


Which numbers are on the binary snake?

9

Measure to the nearest millimeter the lengths and widths of the calculator and its case, both pictured in actual size below.



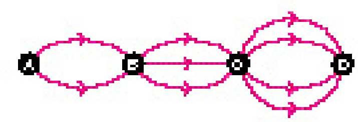
How much wider is the case? 0.2 cm

How much longer is the case? 0.1 cm

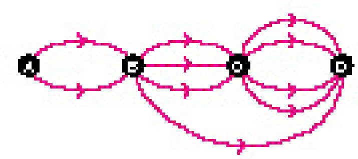
10

Answers may vary slightly.

How many ways are there to go from city A to city D following these one-way roads? 24

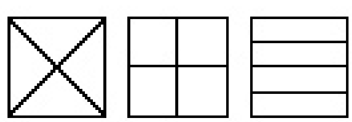


A new road is put in from city B to city D. How many ways are there to go from A to D following these one-way roads? 26

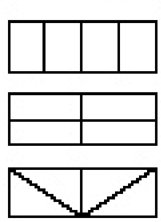


11

Show three different ways to divide a square cake into four pieces of the same size and shape. The pictures should look different from one another no matter how they are turned.



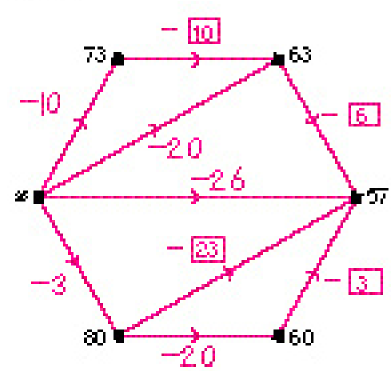
Show three different ways to divide a rectangular cake into four pieces of the same size and shape.



12

Other solutions are possible.

Fill in the boxes for the arrows.
Label the dots.



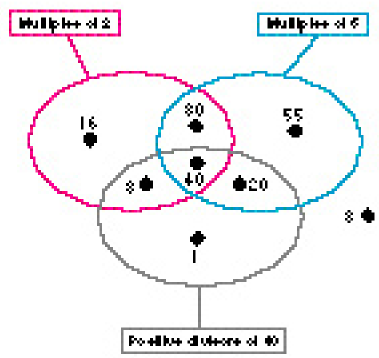
Complete this subtraction statement.

$$83 - 26 = \underline{\quad 57 \quad}$$

13

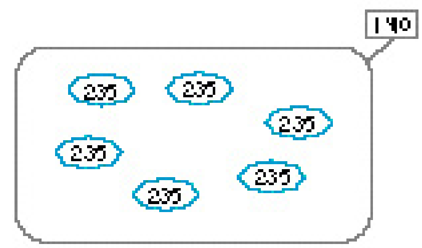
Put these numbers in the string picture.

20 1 80 16 8 55 40 3



14

Bethel School District has 1410 students and 6 elementary schools. They wish to put the same number of students in each school. You may use a string picture to determine the number of students they send to each school.



How many students are in each school? 235

The school district is buying new desks for all of the students in five of the schools. How many desks must they buy? 1175

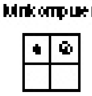
If the district had only three schools for the 1410 students, how many students would go to each school? 470

15

Zag is a secret number.

Clue 1

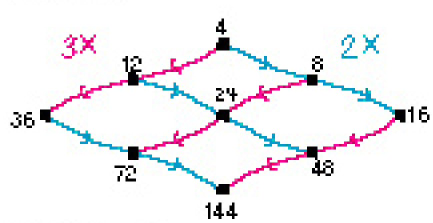
Zag can be put on this Minicomputer by adding one ⊘ checker.



Zag could be 51, 58, 72, or 100.

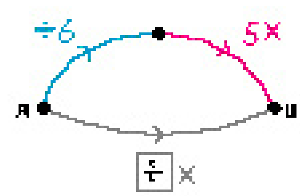
Clue 2

Zag is in this arrow picture.
Label the dots.



Who is Zag? 72

16



Complete the tables.

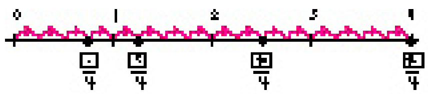
A	B
12	10
30	25
36	30
60	50
24	20

A	B
18	15
180	150
1800	1500
54	45
540	450

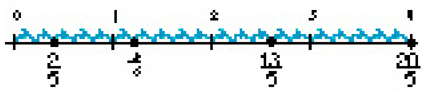
Label the gray arrow.

17

Fill in the boxes.



Label the dots. One is done for you.

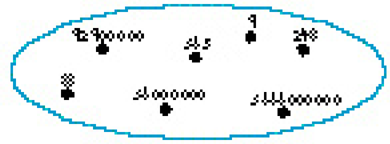


Put <, =, or > in each box. Use the number lines to help you.

$\frac{3}{4}$ $\boxed{>}$ $\frac{2}{5}$ $\frac{13}{5}$ $\boxed{>}$ $\frac{10}{4}$

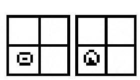
$\frac{16}{4}$ $\boxed{=}$ $\frac{20}{5}$ $\frac{6}{5}$ $\boxed{<}$ $\frac{5}{4}$

Put these numbers in the blanks so that the paragraph makes sense.

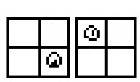


Earth is one of __9__ planets that orbit around the sun. Mercury, the planet closest to the sun, is about 36 000 000 miles from the sun. Pluto, the planet furthest from the sun, is about 3 666 000 000 miles from the sun. Earth, the third planet from the sun, is about 92 900 000 miles from the sun. Mercury travels around the sun once every __88__ days (Earth days). Earth needs a little more than four times as long as Mercury to travel around the sun, about __365__ days. Pluto takes the longest time to travel once around the sun, about 248 years (Earth years).

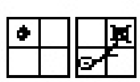
Add one checker so that 86 is on the Minicomputer.



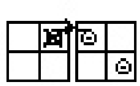
Add one checker so that 76 is on the Minicomputer.



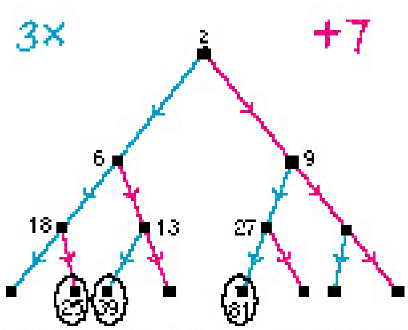
Move one checker so that 408 is on the Minicomputer.



Move one checker so that 57 is on the Minicomputer.



This tree shows all of the possible red-blue arrowroads that have exactly three arrows.



Sam is the ending number of an arrowroad that has two 3× arrows and one +7 arrow. Circle the dots in the tree that could be for Sam.

Sam is the least of the three numbers.
Who is Sam? __25__

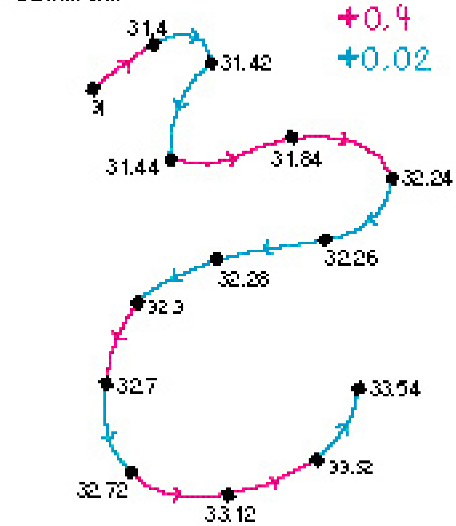
NOTE: You do not need to label all of the dots in the picture.

Profitable Yard Work

It takes Chris 2 hours and 15 minutes to do the yard work each week. Her parents pay her $2 an hour for this work. How many weeks will it take Chris to earn enough money to buy a bicycle speedometer that costs $30? ___7___
Will she have any money left over? __yes__
If so, how much? __$1.50__

22

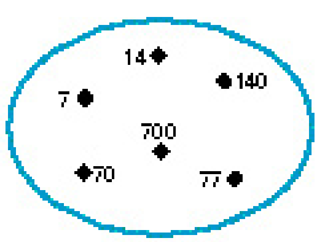
Label the dots.



23

Label the six dots with whole numbers so that

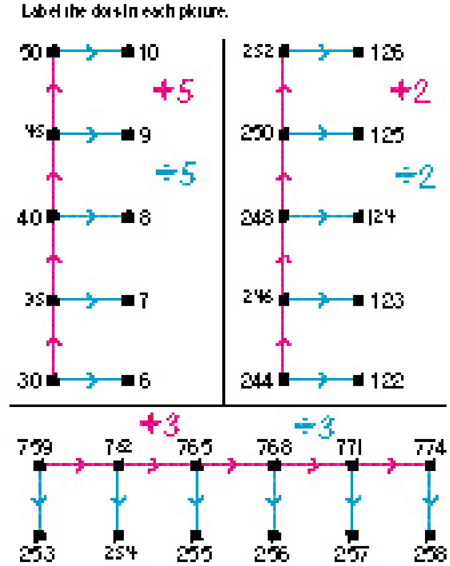
- all of them are multiples of 7;
- at least one of them is greater than 100; and
- exactly two of them are between 50 and 80.



24

Many solutions are possible.

Label the dots in each picture.



25

Ponderosa School Fair Tickets

Ponderosa School Fair Tickets	
Adults	$5.00
Students	
10–18 years old	$3.00
Under 10 years old	$2.00

Latisha sold 50 tickets to the Ponderosa School Fair and collected $150.00. How many tickets of each kind did she sell?

Adult __10__

Student (10–18) __20__

Student (under 10) __20__

NOTE: There are several solutions to this problem. Can you find another solution?

Other solutions:

Adult	11	12	13	14	15	16
Student (10–18)	17	14	11	8	5	2
Student (under 10)	22	24	26	28	30	32

26

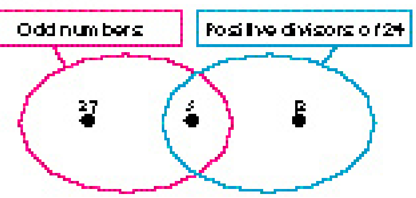
The red label is one of these:

- Multiples of 3
- Multiples of 6
- Positive divisors of 18
- Positive divisors of 24
- Greater than 10
- Less than 10
- Odd numbers

The blue label is one of these:

- Multiples of 3
- Multiples of 6
- Positive divisors of 18
- Positive divisors of 24
- Greater than 10
- Less than 10
- Odd numbers

Label the strings.



27

Period of Rotation

Fill in the blanks.

1 hour = __60__ minutes

1 hour 4 minutes = __64__ minutes

5 hours = __300__ minutes

5 hours 30 minutes = __330__ minutes

1 minute = __60__ seconds

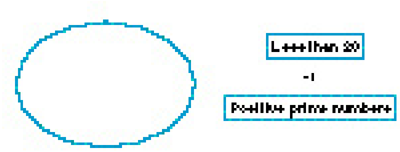
1 minute 20 seconds = __80__ seconds

It takes the Earth 23 hours 56 minutes 4 seconds (a sidereal day) to make one complete rotation, but if you were watching the Earth rotate as you were standing on the sun, it would appear to rotate every 24 hours (a solar day).

What is the difference? __3__ minutes __56__ seconds

28

This string is either for numbers less than 20 or for prime numbers.



Draw a circle around each number below that we know for sure belongs inside the blue string.

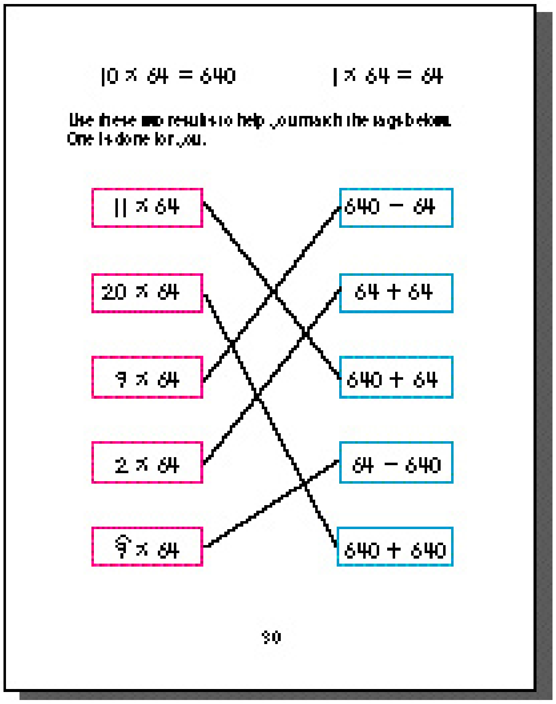
Draw a triangle around each number below that we know for sure belongs outside the blue string.

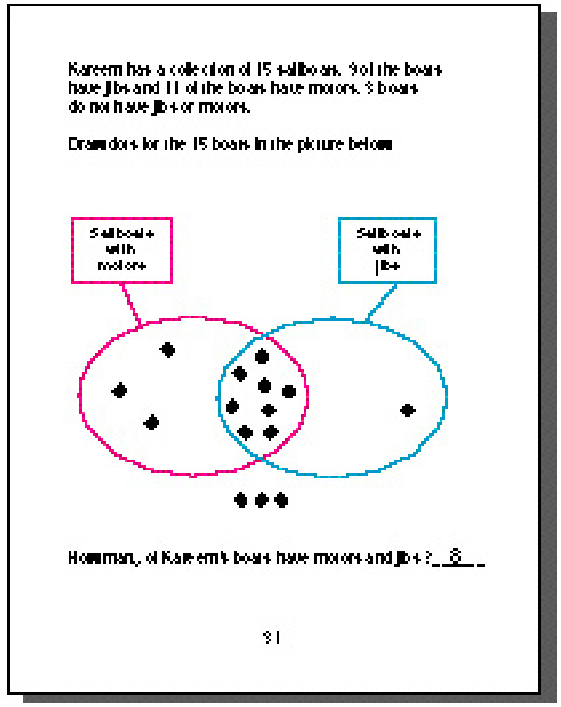
40 6 2

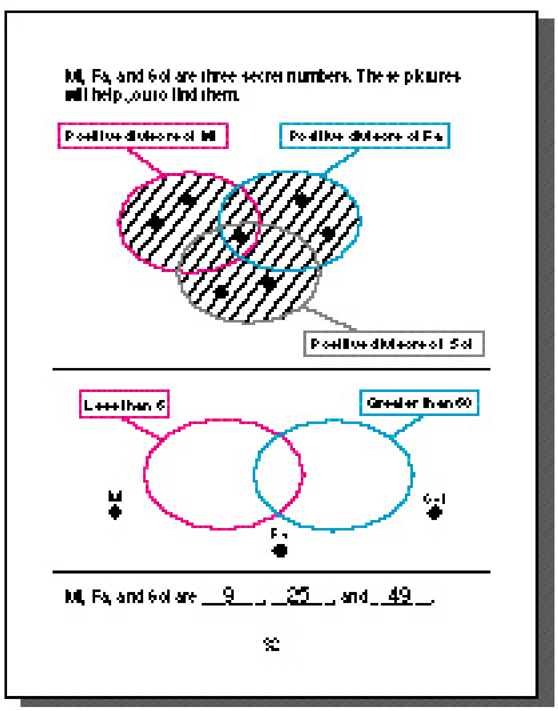
17 23 5

Note: Some of the numbers should have neither a circle nor a triangle around them because we cannot tell for sure where they belong.

29


10 × 64 = 640
1 × 64 = 64
Use these two results to help you match the tags below.
One is done for you.
11 × 64
20 × 64
9 × 64
2 × 64
9 × 64
640 − 64
64 + 64
640 + 64
64 − 640
640 + 640
30


Kareem has a collection of 15 sailboats. 9 of the boats
have jibs and 11 of the boats have motors. 3 boats
do not have jibs or motors.
Draw dots for the 15 boats in the picture below.
Sailboats with motors
Sailboats with jibs
How many of Kareem's boats have motors and jibs? 8
31


Mi, Fa, and Sol are three secret numbers. These pictures
will help you to find them.
Positive divisors of Mi
Positive divisors of Fa
Positive divisors of Sol
Less than 6
Greater than 60
Mi
Fa
Sol
Mi, Fa, and Sol are 9, 25, and 49.
32