

# Appendix



**Number Concepts**

*Abacus: A Pocket Computer, The* by Jesse Delson  
*Base Five* by David A. Adler  
*Binary Numbers* by Clyde Watson  
*Counting* by Henry Pluckrose  
*Count Your Way Through ...* (series highlighting different countries) by Jim Haskins  
*First Number Book, A* by Shari Robinson  
*How Much Is a Million?* by David Schwartz  
*How to Count like a Martian* by Glory St. John  
*How to Count Sheep without Falling Asleep* by Ralph Leighton and Carl Feynman  
*I Can Count the Petals of a Flower* by John and Stacey Wahl  
*I've Got Your Number, John* by Olive S. Berg  
*King's Commissioners, The* by Aileen Friedman  
*Less Than Nothing Is Really Something* by Robert Froman  
*Lucy and Tom's 1, 2, 3* by Shirley Hughes  
*Math for Smarty Pants* by Marilyn Burns  
*Millionth Egg, The* by Bernice Myers  
*Mouse Count!* by Felicia Law and Suzanne Chandler  
*Number Art: Thirteen 1 2 3's from Around the World* by Leonard Everett Fisher  
*Numbers* by Philip Carona  
*One Is Unique* by Marnie Luce  
*Roman Numerals* by David A. Adler  
*Sea Squares* by Joy N. Hulme  
*Sesame Street One, Two, Three Storybook* by Emily Kingsley et al.  
*Solomon Grundy, Born on One Day: A Finite Arithmetic Puzzle* by Malcolm E. Weiss  
*Twelve Days of Christmas, The* by Jan Brett  
*Twelve Days of Christmas, The* by Jack Kent  
*Twelve Days of Christmas, The* by Brian Wildsmith  
*Twelve Days of Christmas, The* by June Williams  
*Wacky Wednesday* by Theo Le Sieg  
*Zero: Is It Something? Is It Nothing?* by Claudia Zaslavsky  
*Zero Is Not Nothing* by Mindel and Harry Sitomer  
*Zero Is Something* by Marnie Luce

**Fractions**

*Dad's Diet* by Barbara Comber  
*Eating Fractions* by Bruce McMillan  
*Ed Emberley's Picture Pie* by Ed Emberley  
*Fractions Are Parts of Things* by Richard J. Dennis  
*Gator Pie* by Louise Mathews  
*Half-Birthday Party, The* by Charlotte Pomerantz  
*How Many Ways Can You Cut a Pie?* by Jane Belk Monaire  
*Pezzettino* by Leo Lionni  
*Phantom Tollbooth, The* by Norton Juster  
*Really Eager and the Glorious Watermelon Contest* by Richard Cheney  
*Tom Fox and the Apple Pie* by Clyde Watson

# A

---

## Operations

*Anno's Mysterious Multiplying Jar* by Masaichiro and Mitsumasa Anno  
*Building Tables on Tables: A Book about Multiplication* by John V. Trivett  
*Bunches and Bunches of Bunnies* by Louise Mathews  
*Calculator Fun* by David A. Adler  
*Doorbell Rang, The* by Pat Hutchins  
*Grain of Rice, A* by Helena Pittman  
*Greatest Guessing Game: A Book about Dividing, The* by Robert Froman  
*Great Take-Away, The* by Louise Mathews  
*How Big Is the Moon?* by D. Baker, C. Semple, and T. Snead  
*King Kaid of India* published by The Victorian Readers  
*King's Chessboard, The* by David Birch  
*Melisande* by E. Nesbit  
*Moria's Birthday* by R. Munsch  
*Phoebe and the Hot Water Bottles* by Terry Furchgott and Linda Dawson  
*Rajah's Rice, The* by David Barry  
*17 Kings and 42 Elephants* by Margaret Mahy  
*666 Jellybeans! All That? An Introduction to Algebra* by Malcolm C. Weiss  
*Where Did My Little Fox Go?* by Nancy Robinson

## Patterns and Relationships

*Anno's Math Games* by Mitsumasa Anno  
*Anno's Math Games II* by Mitsumasa Anno  
*Game of Functions, A* by Robert Froman  
*Graph Games* by Frédérique and Papy  
*I Hate Mathematics! Book, The* by Marilyn Burns  
*MatheMagic: Magic, Puzzles, and Games with Numbers* by Royal Vale Heath  
*Math for Smarty Pants* by Marilyn Burns  
*Mirror, Mirror* by Rosemary and Calvin Irons  
*Number Families* by Jane Jonas Srivastava  
*Number Ideas through Pictures* by Mannis Charosh  
*Number Mysteries* by Cyril and Dympna Hayes  
*Number Patterns Make Sense* a Wise Owl Book by Howard Fehr  
*Pattern* by Henry Pluckrose  
*Polka Dots, Checks, and Stripes* by Carol Cornelius  
*Puzzlooney* by Russell Ginns

## Geometry

*Angles Are Easy as Pie* by Robert Froman  
*Anno's Math Games III* by Mitsumasa Anno  
*Another, Another, Another, and More* by Marion Walter  
*Circles* by Mindel and Harry Sitomer  
*Cloak for the Dreamer, A* by Aileen Friedman  
*Ellipse* by Mannis Charosh  
*Exploring Triangles: Paper-Folding Geometry* by Jo Phillips  
*Greedy Triangle, The* by Marilyn Burns  
*In Shadowland* by Mitsumasa Anno  
*Lines* by Phillip Yenawine

---

*Lines, Segments, Polygons* by Mindel and Harry Sitomer  
*Listen to a Shape* by Marcia Brown  
*Look at Annette* by Marion Walter  
*Make a Bigger Puddle, Make a Smaller Worm* by Marion Walter  
*Maps, Tracks, and the Bridges of Königsberg: A Book about Networks* by Michael Holt  
*Mirror Puzzle Book, The* by Marion Walter  
*Mirrorstone, The* by Palin  
*Opt: An Illusionary Tale* by Arlene and Joseph Baum  
*Puzzle Maps U.S.A.* by Nancy Clorise  
*Reflections* by Ann Jonas  
*Right Angles: Paper-Folding Geometry* by Jo Phillips  
*Round Trip* by Ann Jonas  
*Rubber Bands, Baseballs, and Doughnuts: A Book about Topology* by Robert Froman  
*Shadow Geometry* by Daphne H. Trivett  
*Shape* by Henry Pluckrose  
*Shapes* by Philip Yenawine  
*Shape: The Purpose of Forms* by Eric Laithwaite  
*Simple Science Experiments with Circles* by Eiji and Masako Orii  
*Spirals* by Mindel and Harry Sitomer  
*Stop and Look! Illusions* by Robyn Supraner  
*Straight Lines, Parallel Lines, Perpendicular Lines* by Mannis Charosh  
*String, Straightedge, and Shadow: The Story of Geometry* by Julia E. Diggins  
*Take Another Look* by Edward Carini  
*Tangrams: Picture-Making Puzzle Game* by Peter Van Note  
*Thirteen* by Renny Charlep and Jerry Joyner  
*Three-D, Two-D, One-D* by David A. Adler  
*Topsy-Turvies: Pictures to Stretch the Imagination* by Mitsumasa Anno  
*Turn About, Think About, Look About Book, The* by Beau Gardner  
*Upside-Downers* by Mitsumasa Anno  
*What Is Symmetry* by Mindel and Harry Sitomer  
*Where Is It? A Hide-and-Seek Puzzle Book* by Demi  
*Wing on a Flea: A Book about Shapes, The* by Ed Emberley

## Measurement

*Area* by Jane Jonas Srivastava  
*Bigger and Smaller* by Robert Froman  
*Capacity* by Henry Pluckrose  
*How Big Is a Foot?* by Rolf Myllar  
*How Big Is Big?* by Herman and Nina Schneider  
*How Little and How Much: A Book about Scales* by Franklyn M. Branley  
*How Much Is a Million?* by David M. Schwartz  
*How Tall Was Milton?* by Lowery  
*Inch by Inch* by Leo Lionni  
*King's Flower, The* by Mitsumasa Anno  
*Length* by Henry Pluckrose  
*Let's Find Out about What's Big and What's Small* by Martha and Charles Shapp  
*Let's Find Out about What's Light and What's Heavy* by Martha and Charles Shapp  
*Let's Talk about the Metric System* by Joyce Lamm  
*Liter, The* by William J. Shimek

# A

---

*Long, Short, High, Low, Thin, Wide* by James Fey  
*Making Metric Measurements* by Neil Ardley  
*Measurements: Fun, Facts, and Activities* by Caroline Arnold  
*Measure with Metric* by Franklyn M. Branley  
*Popcorn Book, The* by Tomie de Paola  
*Rules of Thumb* by Tom Parker  
*Size: The Measure of Things* by Eric Laithwaite  
*Sizes* by Jan Pienkowski  
*Sizes* by Gillian Youldon  
*Something Absolutely Enormous* by Margaret Wild  
*Spaces, Shapes, and Sizes* by Jane Jonas Srivastava  
*Temperature and You* by Betsy Maestro  
*Think Metric!* by Franklyn M. Branley  
*Weighing and Balancing* by Jane Jonas Srivastava  
*Your Amazing Body* by Jeanne K. Hanson

## Money

*Alexander Who Used to Be Rich Last Sunday* by Judith Viorst  
*All Kinds of Money* by David A. Adler  
*Chair for My Mother, A* by Vera Williams  
*Dollars and Cents for Harriet* by Betsy and Giulio Maestro  
*Hundred Penny Box, The* by Sharon Bell Mathis  
*If You Made a Million* by David M. Schwartz  
*Managing Your Money* by Elizabeth James and Carol Backin  
*Money* by Audrey Briers  
*Money* by Benjamin Elkin  
*Money and Banking* by Lois Cantwell  
*\$1.00 Word Riddle Book, The* by Marilyn Burns and Martha Weston  
*Toothpaste Millionaire, The* by Jean Merrill

## Time

*All in a Day* by Mitsumasa Anno  
*Anno's Sundial* by Mitsumasa Anno  
*Calendar Art: Thirteen Days, Weeks, Months, Years from Around the World* by Leonard E. Fisher  
*Calendars* by Necia H. Apfel  
*Chicken Soup with Rice: A Book of Months* by Maurice Sendak  
*How Did We Get Clocks and Calendars?* by Susan Perry  
*One Hand at a Time* by Patricia E. Smith  
*Story of Our Calendar* by Ruth Brindze  
*Ten Cuckoo-Clock Cuckoo* by Annegert Fuchshuber  
*Time!* by Jane Edmonds and Mark Sachner  
*Time* by Werner Kirst  
*Time* by Henry Pluckrose  
*Time* by Elizabeth Thompson and Feenie Ziner  
*Time and Clocks* by Herta S. Breiter  
*Time Book, The* by John Cassidy  
*Time for Clocks* by Daphne and John Trivett  
*What Time Is It Around the World?* by Hans Baumann  
*Wise Owl's Time Book* by Jane Belk Moncure

---

## Classification and Logic

*Anno's Hat Tricks* by Akikiro Nozaki and Mitsumasa Anno  
*Beach Ball* by Peter Sis  
*Bread, Bread, Bread* by Ann Morris  
*Bruno Brontosaurus* by Nicole Rubel  
*Clive Eats Alligators* by Alison Lester  
*Domino Games* by John Belton and Joella Cramblit  
*Hats, Hats, Hats* by Ann Morris  
*Logic for Space Age Kids* by Lynn McClure Butrick  
*Mathematical Games for One and Two* by Mannis Charosh  
*Mathnet™ Casebooks* by David D. Connell and Jim Thurman  
*Odds and Evens* by Thomas O'Brien  
*Tic Tac Toe* by Claudia Zaslavsky  
*263 Brain Busters: Just How Smart Are You, Anyway?* by Louis Phillips  
*Venn Diagrams* by Robert Froman  
*Yes-No; Stop-Go: Some Patterns in Mathematical Logic* by Judith L. Gersting  
and Joseph E. Kuczkowski

## Statistics and Probability

*Averages* by Jane Jonas Srivastava  
*Bunches and Bunches of Bunnies* by Louise Mathews  
*Charts and Graphs: Fun, Facts, and Activities* by Caroline Arnold  
*Do You Wanna Bet? Your Chance to Find Out about Probability* by Jean Cushman  
*In One Day* by Tom Parker  
*Make It Graphic! Drawing Graphs for Science and Social Studies Projects* by Eve and  
Albert St. Wertka  
*Millions of People* by John Dunworth and Thomas Drysdale  
*Socrates and the Three Pigs* by Tuyosi Mori  
*Statistics* by Jane Jonas Srivastava  
*What Do You Mean by "Average"? Means, Medians, and Modes* by Elizabeth James and  
Carol Barkin  
*Winning with Number: A Kid's Guide to Statistics* by Manfred G. Riedel





The Papy Minicomputer is used throughout the *Comprehensive School Mathematics Program*, but its use changes over time. In the intermediate grades, the Minicomputer is used more as a context in which to pose problems and develop strategic thinking. Two mathematically rich games, *Minicomputer Tug-of-War* and *Minicomputer Golf* illustrate how the Minicomputer can play this role. Together they give rise to an intriguing array of mathematical considerations as each play presents, in turn, a new problem to be solved. Playing these games, students sharpen their skills in mental arithmetic and employ techniques of estimation. Equally important, students have the opportunity to develop strategic thinking—discovering advantageous plays and anticipating their opponents’ responses.

## Introduction

Both of the games are designed to be used collectively by one teacher and an entire class. The variety of problem-solving situations that the games provide makes them appropriate for the spectrum of ability levels in a usual heterogeneous classroom. However, both games can be adapted easily for small or cooperative group use.

As students play the games, you may find it easy to evaluate their strategies. But one advantage of using a game as a teaching device is that it allows the teacher for a period of time to be a facilitator and not an evaluator. Without constant evaluation of their actions, students have a better chance to explore a situation freely and to discover the effects of the options available. Obviously a simple comment on the progress of a game may be helpful at times.

## Equipment

For either game you need a Teacher Minicomputer Kit. When playing the game in groups, the students should have desk Minicomputers and checkers. Student Minicomputer sheets and checkers are available from McREL or can be made using the home Minicomputer Blackline.

### MINICOMPUTER TUG-OF-WAR

#### Preparation for the Game

Display three Minicomputer boards and checkers as in the illustration below.



Divide the class into two teams—the **RED** team and the **BLUE** team. Each team plays with the colored checkers corresponding to the team’s name. The starting number for each team is the number represented on the Minicomputer by the appropriately colored checkers.

#### Object of the Game

The first team to meet or pass the other team’s number loses the game. In other words, the **RED** team loses by making the red number **LESS THAN** or **EQUAL TO** the blue number. Similarly, the **BLUE** team loses by making the blue number **GREATER THAN** or **EQUAL TO** the red number.

# B

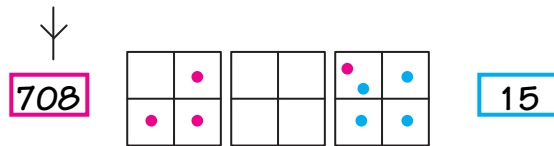
## Rules of the Game

- 1) Teams alternate play, and the members take turns within each team.
- 2) The **RED** team may move red checkers only on their turn. A player for the **RED** team moves one red checker to a square of lower value, so the **RED** team's number decreases.
- 3) The **BLUE** team may move blue checkers only on their turn. A player for the **BLUE** team moves one blue checker to a square of higher value, so the **BLUE** team's number increases.

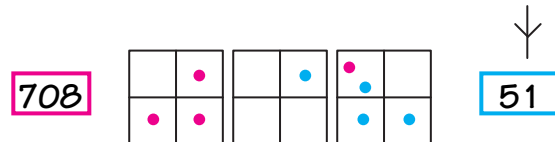
We suggest letting students volunteer to make moves during the first game rather than asking them to play in some order. This will speed up the game and allow students who are unsure of the rules to become more familiar with the game before playing themselves. You may want to impose a no talking rule to give students an opportunity to analyze the game individually. With small teams, it may be logistically possible to let a team work as a cooperative group and discuss their strategy.

## Description of a Minicomputer Tug-of-War Game

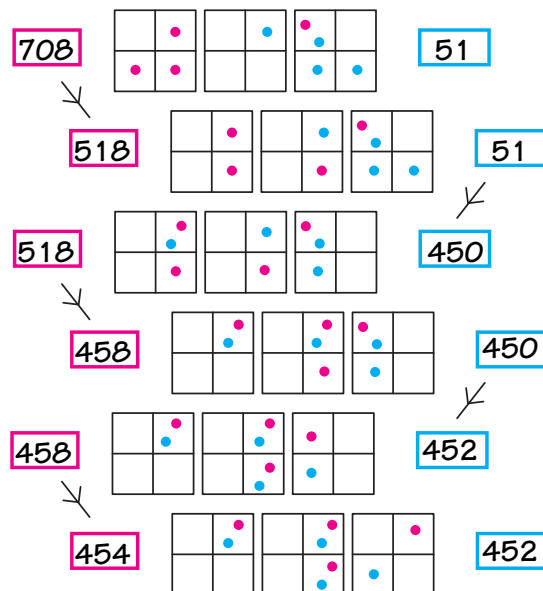
The following is a description of a possible game. In this game the **RED** team plays first, moving one red checker from the 800-square to the 8-square. The player determines that the red number is now 708 and changes the number in the red box.



Next, the **BLUE** team moves a blue checker from the 4-square to the 40-square. The player determines that the blue number is now 51 and changes the number in the blue box.



The game continues as shown below.

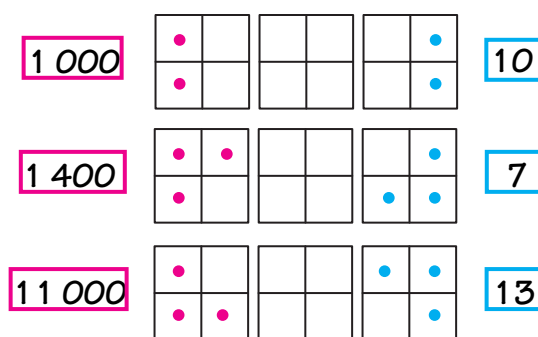


It is the **BLUE** team's turn. The smallest move they can make (it must be an increase) is from the 2-square to the 4-square. However, this move puts 454 on the Minicomputer in **BLUE** and meets the **RED** team. The **BLUE** team concedes their loss.

By studying the preceding game, you will see that the **BLUE** team could have won by going from 450 to 456 (moving a blue checker from the 2-square to the 8-square). However, students will not always play at such a sophisticated level, nor are they expected to. Such a situation occurs often during the games. You may note it yourself without comment and allow the students to enjoy the games.

### Variations of the Game

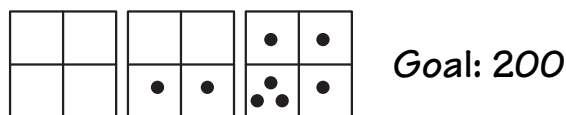
There are several different starting configurations, some of which you may like to use to help students become more proficient at playing *Minicomputer Tug-of-War*.



### MINICOMPUTER GOLF

#### Preparation for the Game

Display three Minicomputer boards and a starting configuration of checkers. Choose a goal or target number. See the illustration below.



Divide the class into two teams—the **RED** team and the **BLUE** team. In this case, the color names for teams correspond to a color-coded record of the game. See the game description.

#### Object of the Game

Start with a number on the Minicomputer and, by moving checkers one at a time, try to put the target number on the Minicomputer.

#### Rules of the Game

- 1) Teams alternate play and the members take turns within each team.
- 2) A player takes a turn by moving exactly one checker from the square it is on to another square. As long as the number on the Minicomputer is below the goal, the play must increase the number. If the number on the Minicomputer is above the goal, the play must decrease the number.

# B

You may want to impose a no talking rule to give students an opportunity to analyze the game individually. With small teams it may be logistically possible to let a team work as a cooperative group and discuss their moves. A red-blue arrow road records the progress of the game. Such a record is illustrated in the next section.

## Description of a Minicomputer Golf Game

The following is a description of a sample *Minicomputer Golf* game that begins with 49 on the Minicomputer. The dialogue gives an idea of how you might conduct the game.

In this game, the **RED** team goes first and the player moves a checker from the 20-square to the 40-square.

**T:** *Did you increase or decrease the number on the Minicomputer?*

**S:** *Increase.*

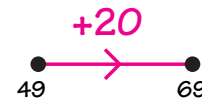
**T:** *From 20 to 40 ... how much greater?*

**S:** *20 greater.*

**T:** *What number is on the Minicomputer now?*

**S:** *69;  $49 + 20 = 69$ .*<sup>†</sup>

On the board, a red arrow records the result of the **RED** team's move.



It is the **BLUE** team's turn and the player moves a checker from the 2-square to the 20-square.

**T:** *Did you increase or decrease the number on the Minicomputer?*

**S:** *Increase.*

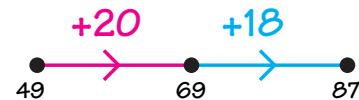
**T:** *From 2 to 20 ... how much greater?*

**S:** *18 greater.*

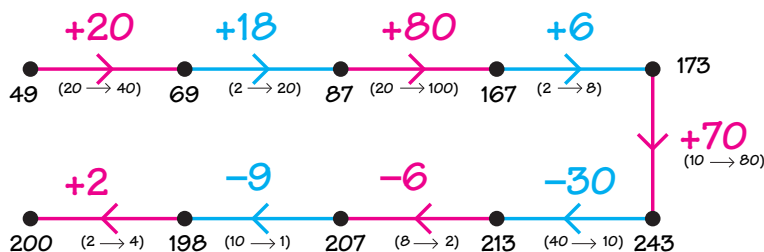
**T:** *What number is on the Minicomputer now?*

**S:** *87;  $69 + 18 = 87$ .*

A blue arrow records the result of the **BLUE** team's move.



The game continues in this manner until the goal (200) is reached—the **RED** team wins. The red-blue arrow road below describes the game from start to finish. In your classroom, draw the arrow picture on the board large enough to be clearly visible by all players.



<sup>†</sup>You should require the player to be able to announce the effect of a move and the team to say what the new number is on the Minicomputer. If they cannot, the player must choose a different move.

Refrain from making comments on the quality of the moves, letting students enjoy the game as they gradually improve their strategies.

It's a good idea to let your class play collectively several times so that students are comfortable with the rules. Then, you might arrange for some games to be played in small groups; for example, groups of four (with two students per team) work well. The games are most effective when the teams are evenly matched in numerical abilities. Most students can play the game with little supervision.

### An Optional Rule

Experience playing the game yourself may lead you to discover an inadequacy of the rules of *Minicomputer Golf* as they have been presented. The problem is that the rules allow the possibility of never-ending games.

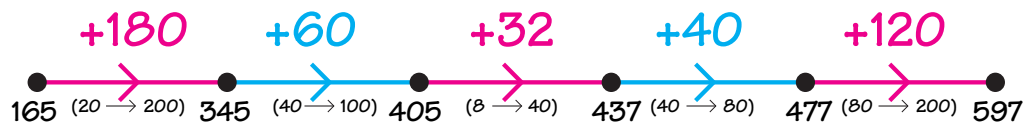
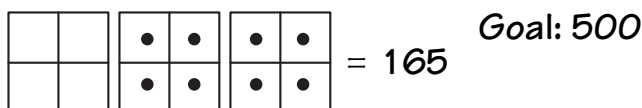
Students soon learn that even if they do not know a winning move, they can prevent opponents from winning by making the number as far from the goal as possible. The result is a continuing game whose conclusion remains out of reach for both teams. For this reason, introduce this extra rule only after students have played many times and have discovered how to make the game unending.

If a player increases the number and passes the goal, the resulting number must be less than any previously played number higher than the goal.

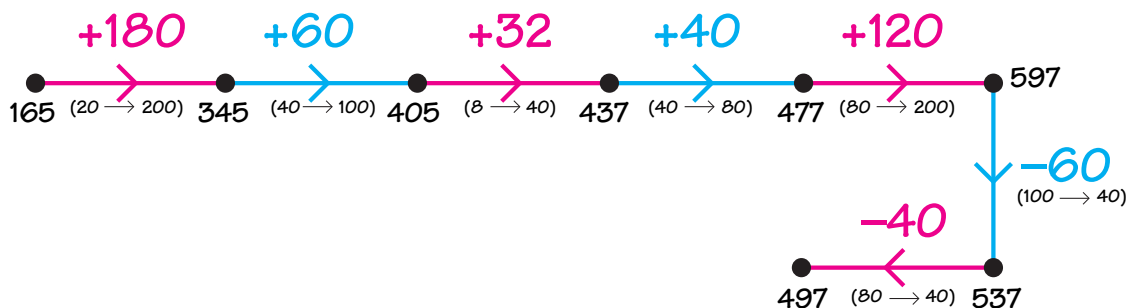
If a player decreases the number and passes the goal, the resulting number must be greater than any previously played number lower than the goal.

Using the extra rule, the teams must put the number closer and closer to the goal as the game progresses.

The following is a description of a game using this extra rule.

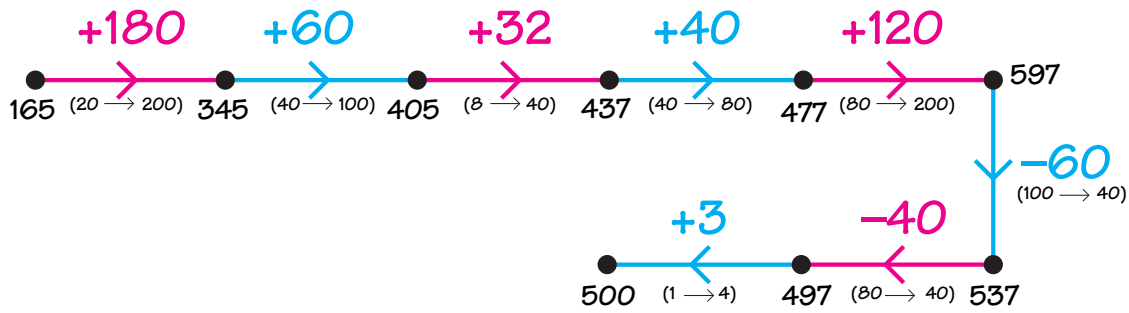


At this point the goal is exceeded. The next numbers must be less than 597 but greater than 477.

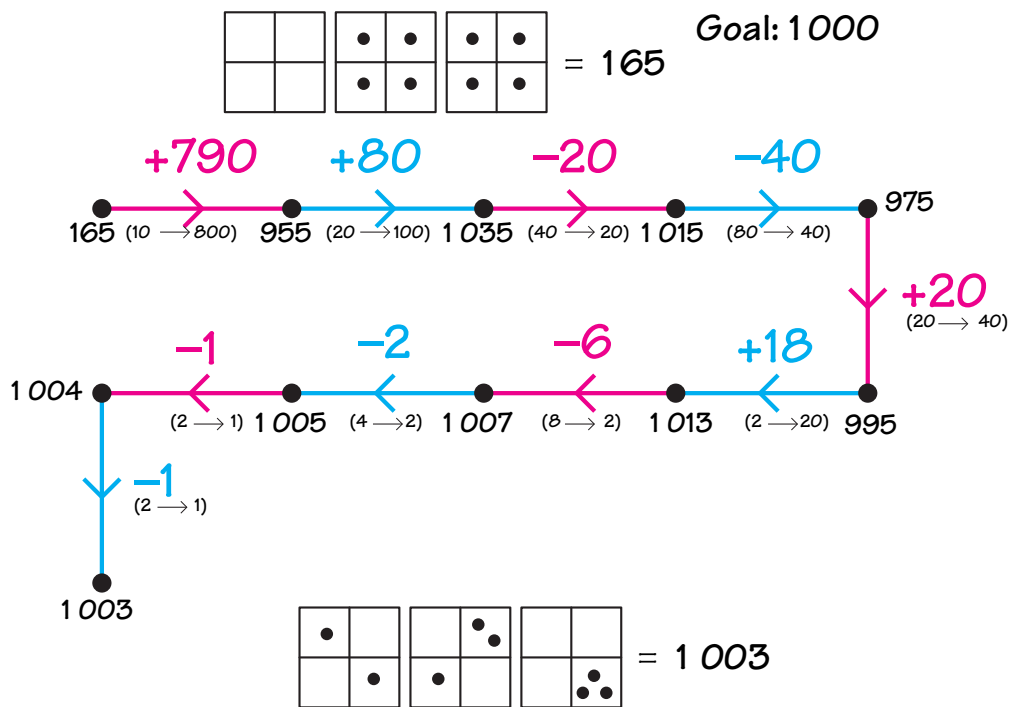


# B

Since the number is now less than 500, subsequent moves must increase the number, but the number must remain less than 537. In fact, there is a winning move.



**Note:** During a game using the extra rule, a situation can arise where there is no possible legal move. For example,

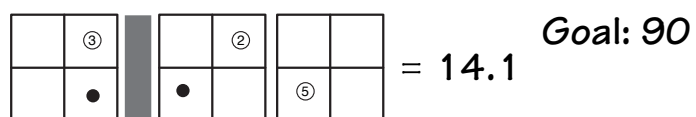
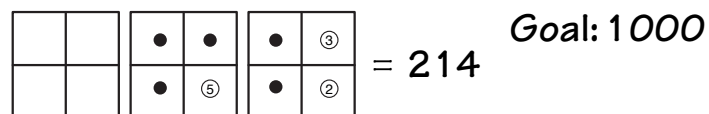
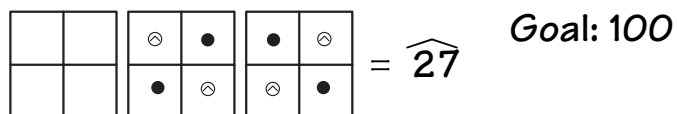
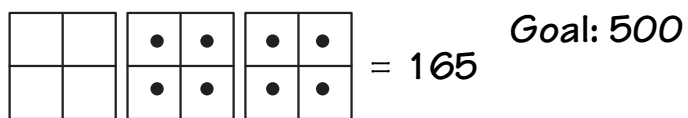


According to the extra rule, the next move must make the number less than 1003, but must keep the number greater than 995. No such move is possible with the present configuration.

If such a situation occurs during a game you and your students are playing, you have two options for play. The game can be ended and declared a draw, or the extra rule may be suspended for one move (e.g., in the game described above, allow a student to decrease the number to one less than 995). However, subsequent moves must again follow the extra rule, whenever possible.

## Variations of the Game

As your class becomes familiar with *Minicomputer Golf*, you can alter the starting number, the goal, and the type of checkers used. Several starting configurations are suggested below.







Mental arithmetic activities are short, fast-moving question and answer sessions in which students are asked to calculate mentally, preferably without the aid of paper and pencil. Often a mental arithmetic session consists of several sequences in which the answers to one or more questions lead to the answer of another, more difficult, question. For example:

$$\begin{aligned}3 \times 6 &= ? & (18) \\3 \times 10 &= ? & (30) \\3 \times 16 &= ? & (48)\end{aligned}$$

There are specific lessons that call for mental arithmetic; however, try to include such activities for five to ten minutes several times a week. Use mental arithmetic as a warm-up activity to a lesson, as a transition between two exercises in a lesson, as a conclusion to a lesson, or as a quick change of pace at any time during the day.

## Why Is Mental Arithmetic Important?

- 1) It develops mental computational skills.
- 2) It provides an opportunity to involve and help students at all ability levels.
- 3) It creates an awareness of patterns and mathematical relationships.
- 4) It helps students to be able to recall arithmetic facts easily.
- 5) It provides an opportunity to keep alive and to reinforce concepts previously introduced.
- 6) It helps students to see that there is often more than one way to solve a problem.
- 7) It uses and develops number sense.
- 8) It helps students construct their understanding of the arithmetic operations.

## Hints for Mental Arithmetic Activities

- 1) Vary the level of difficulty of questions throughout a session.
- 2) Involve many students.
- 3) Follow a pattern for a while and then start a new pattern.
- 4) Keep a brisk pace, although you should allow students time for thought.
- 5) Give extra attention to a student who shows signs of improvement.
- 6) Illustrate, whenever possible, the value of estimation in making calculations.
- 7) Occasionally ask a student to explain an answer.
- 8) Occasionally use the chalkboard to emphasize a pattern.
- 9) Occasionally ask students to whisper an answer to you so that many students will have an opportunity to answer.
- 10) Occasionally discuss the reasonableness of an answer.

## Suggested Mental Arithmetic Activities

The following section contains specific examples that illustrate patterns in arithmetic. You should elaborate and expand upon ideas depending upon the abilities and interests of your students. As you work with your class on the regularly scheduled mathematics lessons, you might make a note of computational skills you would like to improve; then plan a short mental arithmetic activity that develops those skills. Do not expect every one of your students to be good at all of these suggested activities, even at the end of the year.

## Counting Activities

- 1) Count by twos, threes, fours, fives, or tens collectively. Refer to the number line or 0–109 numeral chart as the class counts, if you wish. Sometimes count by twos, then fours; or by threes, then sixes; or by fours, then eights.
- 2) Count by twos, threes, fives, or tens as a group activity with each student saying one number. Use a natural seating order in your class (such as going up and down rows), and ask each student in turn to say the number that comes next. If you are counting by threes, for example, the first student will say, “3”; the second will say, “6”; and so on. Repeat the counting but start with a different student.
- 3) Count by twos, threes, fours, fives, or tens, starting at 1 or any number other than 0.
- 4) Count backward by twos, fives, or tens. Do this immediately following a similar forward count.
- 5) Count by halves, thirds, or fourths starting at 0. When you say an improper fraction (e.g.,  $\frac{4}{3}$ ) or a fraction that has a reduced form (e.g.,  $\frac{2}{4}$ ), ask the class for another name for that number before proceeding.
- 6) Play *Buzz*. The students count by ones starting at 0 but skip every fourth number by saying “buzz.” This should be done rapidly. This can be done as a whole class or small group activity with each student saying, in turn, either the number that comes next or “buzz.”

More challenging variations of this activity involve

- skipping every sixth number rather than every fourth number;
- starting the counting at 2 and skipping every fourth number;
- counting by twos or fives and skipping every fourth number.

## Addition and Subtraction

The first 17 examples are lists of calculations to be done mentally. Each list suggests an arithmetic pattern that students should begin to use in doing such calculations.

1) $26 + 498 + 2$	$30 + 20$	$7 + 4$	$9 + 5$
$26 + 598 + 3$	$30 + 21$	$7 + 14$	$19 + 15$
$26 + 698 + 4$	$30 + 22$	$17 + 14$	$19 + 25$
$26 + 798 + 5$	$30 + 23$	$17 + 24$	$29 + 25$

$21 + 35702 + 100$	$536 + 100$	$600 + 213$
$21 + 135702 + 200$	$536 + 200$	$600 + 313$
$21 + 145702 + 300$	$536 + 300$	$600 + 413$
$121 + 145$		

2) $10 - 440 - 1$	$100 - 10$
$10 - 540 - 2$	$100 - 11$
$10 - 640 - 3$	$100 - 12$
$10 - 740 - 4$	$100 - 13$

$44 - 10$	$52 - 3$	$186 - 10$
$44 - 20$	$52 - 13$	$186 - 20$
$44 - 30$	$52 - 23$	$186 - 30$
$44 - 40$	$52 - 33$	$186 - 40$

$307 - 100$	$520 - 105$	$1,000 - 100$
-------------	-------------	---------------

$$307 - 200520 - 205 \qquad 1,000 - 200$$

$$307 - 300520 - 305 \qquad 1,000 - 300$$

$$3) \quad \begin{array}{ll} 2 + 5 & 8 + 8 \\ 20 + 50 & 80 + 80 \\ 200 + 500 & 800 + 800 \\ 2,000 + 5,000 & 8,000 + 8,000 \\ 2,000,000 + 5,000,000 & 8,000,000 + 8,000,000 \end{array}$$

$$\begin{array}{lll} 23 - 1051 - 10 & 98 - 20 & \\ 230 - 100510 - 100 & 980 - 200 & \\ 2,300 - 1,000 & 5,100 - 1,000 & 9,800 - 2,000 \end{array}$$

$$4) \quad \begin{array}{ll} 4 + 107 + 10 & 6 + 10 \\ 14 + 1017 + 10 & 18 + 10 \\ 24 + 10117 + 10 & 33 + 10 \\ 44 + 101,017 + 10 & 71 + 10 \end{array}$$

$$\begin{array}{ll} 5 + 203 + 20 & 19 + 20 \\ 15 + 2013 + 20 & 100 + 20 \\ 25 + 20113 + 20 & 600 + 20 \\ 35 + 201,013 + 20 & 619 + 20 \end{array}$$

$$\begin{array}{ll} 16 - 10100 - 10 & 40 - 20 \\ 26 - 10200 - 10 & 50 - 20 \\ 36 - 10600 - 10 & 60 - 20 \\ 46 - 101,000 - 10 & 90 - 20 \end{array}$$

$$5) \quad \begin{array}{ll} 10 + 613 + 14 & 19 + 3 \\ 20 + 623 + 14 & 29 + 3 \\ 30 + 633 + 14 & 39 + 3 \\ 40 + 643 + 14 & 49 + 3 \end{array}$$

$$\begin{array}{ll} 7 - 510 - 3 & 100 - 8 \\ 17 - 520 - 3 & 200 - 8 \\ 27 - 530 - 3 & 300 - 8 \\ 37 - 540 - 3 & 400 - 8 \end{array}$$

$$6) \quad \begin{array}{ll} 5 + 310 + 20 & 106 + 105 \\ 8 - 330 - 20 & 211 - 105 \\ 7 + 690 + 40 & 150 + 150 \\ 13 - 6130 - 40 & 300 - 150 \end{array}$$

$$7) \quad \begin{array}{ll} 16 + 554 + 12 & 3 + 2 + 5 \\ 5 + 1612 + 54 & 3 + 5 + 2 \\ & 5 + 3 + 2 \end{array}$$

$$8) \quad 17 + 5 + 57 + 28 + 3 \qquad 7 + 13 + 6 + 4$$

# C

$$\begin{array}{ll} 38 + 7 + 38 + 35 + 2 & 19 + 1 + 9 + 1 \\ 49 + 6 + 46 + 76 + 4 & 14 + 6 + 8 + 2 \\ 112 + 9 + 15 + 27 + 5 & 25 + 5 + 3 + 7 \end{array}$$

$$\begin{array}{ll} 9) \quad 80 - 39127 - 100 & 59 - 30 \\ 79 - 38126 - 99 & 57 - 28 \\ 78 - 37125 - 98 & 55 - 26 \\ 77 - 36124 - 97 & 53 - 24 \end{array}$$

$$\begin{array}{ll} 10) \quad 6 + 1028 + 10 & 27 + 100 \\ 6 + 928 + 9 & 27 + 99 \end{array}$$

$$\begin{array}{ll} 20 - 1043 - 10 & 126 - 100 \\ 20 - 943 - 9 & 126 - 99 \end{array}$$

$$\begin{array}{ll} 11) \quad 50 + 4030 + 20 & 40 + 60 \\ 8 + 25 + 7 & 9 + 4 \\ 58 + 4235 + 27 & 49 + 64 \end{array}$$

$$\begin{array}{ll} 50 - 2090 - 30 & 60 - 10 \\ 8 - 67 - 2 & 9 - 4 \\ 58 - 2697 - 32 & 69 - 14 \end{array}$$

$$\begin{array}{ll} 12) \quad 5 + 216 + 2 & 64 + 2 \\ 5 + 116 + 1 & 64 + 1 \\ 5 + 016 + 0 & 64 + 0 \\ 5 + \widehat{1}16 + \widehat{1} & 64 + \widehat{1} \\ 5 + \widehat{2}16 + \widehat{2} & 64 + \widehat{2} \end{array}$$

$$\begin{array}{ll} 13) \quad 12 - 521 - 4 & 145 - 30 \\ 12 + \widehat{5}21 + \widehat{4} & 145 + \widehat{30} \end{array}$$

$$\begin{array}{ll} 14) \quad 2 + \widehat{3} \widehat{5} + \widehat{1} & 8 + \widehat{5} \\ 20 + \widehat{30} \widehat{50} + \widehat{10} & 80 + \widehat{50} \\ 200 + \widehat{300} \widehat{500} + \widehat{100} & 800 + \widehat{500} \end{array}$$

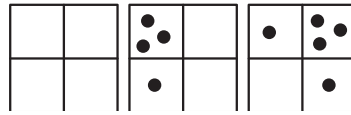
$$\begin{array}{ll} 15) \quad 10 - 611 - 4 & 27 - 13 \\ 6 - 104 - 11 & 13 - 27 \end{array}$$

$$\begin{array}{ll} 16) \quad 7 + \widehat{9} \quad 3 + \widehat{8} & 12 + \widehat{15} \\ \widehat{7} + 9 \quad \widehat{3} + 8 & \widehat{12} + 15 \end{array}$$

$$\begin{array}{ll} 17) \quad \frac{1}{2} + \frac{1}{2}\frac{1}{4} + \frac{1}{4} & 3 + 1 \\ 1\frac{1}{2} + \frac{1}{2}1\frac{1}{4} + \frac{1}{4} & 2\frac{1}{2} + 1\frac{1}{2} \\ 2\frac{1}{2} + \frac{1}{2}1\frac{1}{4} + 1\frac{1}{4} & 2 + 2 \\ 2\frac{1}{2} + 1\frac{1}{2}2\frac{1}{4} + 3\frac{1}{4} & 1\frac{1}{2} + 2\frac{1}{2} \end{array}$$

- 18) Choose some numbers and ask the students to add 1 to each number. For example, if you say, “108,” a student responds, “109.” You can vary this activity by asking the students to add 2, 3, 5, or 10 to each of your numbers, or to subtract 1, 2, 3, or 10 from each of your numbers.

- 19) Conduct an activity similar to Exercise 2 in Lesson N26 *Minicomputer Golf #2*. Put six to ten checkers on the ones and tens boards. Move one checker from some square (it makes no difference which square you choose) to another square. Ask students how much more or less the new number is. For example, suppose this configuration is on the Minicomputer.



Move a checker from the 4-square to the 10-square. When students tell you that the new number is more, ask how much more this number is than the previous number. (In this case, it is 6 more.) Sometimes include negative checkers in the starting configuration and move a negative checker.

- 20) Ask students to give you various names for a particular number. Encourage them to suggest names involving subtraction and also names involving addition of three numbers. For example, names for 20 include  $16 + 4$ ;  $30 - 10$ ;  $4 + 4 + 12$ ;  $21 + \hat{1}$ ;  $4 \times 5$ ;  $\frac{1}{2} \times 40$ .

This activity could be restricted to subtraction by asking for several subtraction problems with the same answer.

### Multiplication and Division

The 25 examples here are lists of calculations to be done mentally. Each list suggests an arithmetic pattern which students should begin to use in doing such calculations.

- 1)  $9 + 9$     $7 + 7$     $100 + 100$   
 $2 \times 9$     $2 \times 7$     $2 \times 100$   
 $8 + 8$     $24 + 24$     $200 + 200$   
 $2 \times 8$     $2 \times 24$     $2 \times 200$
- $4 + 4 + 4$     $8 + 8 + 8$     $6 + 6 + 6 + 6$   
 $3 \times 4$     $3 \times 8$     $4 \times 6$   
 $5 + 5 + 5$     $12 + 12 + 12$     $11 + 11 + 11 + 11$   
 $3 \times 5$     $3 \times 12$     $4 \times 11$
- 2)  $1 \times 9$     $1 \times 20$     $5 \times 8$   
 $2 \times 9$     $2 \times 20$     $6 \times 8$   
 $3 \times 9$     $3 \times 20$     $7 \times 8$   
 $4 \times 9$     $4 \times 20$     $8 \times 8$
- 3)  $8 \times 3$     $7 \times 7$     $9 \times 6$   
 $8 \times 30$     $7 \times 70$     $9 \times 60$   
 $8 \times 300$     $7 \times 700$     $9 \times 600$   
 $8 \times 3,000$     $7 \times 7,000$     $9 \times 6,000$   
 $8 \times 3,000,000$     $7 \times 7,000,000$     $9 \times 6,000,000$
- 4)  $2 \times 4$     $3 \times 3$     $2 \times 5$   
 $2 \times 44$     $3 \times 33$     $2 \times 55$   
 $2 \times 444$     $3 \times 333$     $2 \times 555$   
 $2 \times 4,444$     $3 \times 3,333$     $2 \times 5,555$

# C

$$5) \quad \begin{array}{l} 4 \times 7 \quad 5 \times 8 \\ 7 \times 4 \quad 8 \times 5 \\ 4 \times 708 \times 50 \\ 70 \times 4 \quad 50 \times 8 \end{array} \quad \begin{array}{l} 2 \times 16 \\ 16 \times 2 \\ 3 \times 12 \\ 12 \times 3 \end{array}$$

$$6) \quad \begin{array}{l} 9 \times 106 \times 50 \\ 9 \times 9 \quad 6 \times 49 \\ 7 \times 208 \times 100 \\ 7 \times 198 \times 99 \end{array} \quad \begin{array}{l} 15 \times 10 \\ 15 \times 9 \\ 11 \times 20 \\ 11 \times 19 \end{array}$$

$$7) \quad \begin{array}{l} 5 \times 7 \quad 5 \times 8 \\ 2 \times 7 \quad 4 \times 8 \\ 7 \times 7 \quad 9 \times 8 \end{array} \quad \begin{array}{l} 2 \times 9 \\ 10 \times 9 \\ 12 \times 9 \end{array}$$

$$\begin{array}{l} 7 \times 105 \times 40 \\ 7 \times 3 \quad 5 \times 2 \\ 7 \times 135 \times 42 \end{array} \quad \begin{array}{l} 8 \times 20 \\ 8 \times 6 \\ 8 \times 26 \end{array}$$

$$\begin{array}{l} 5 \times 606 \times 100 \\ 5 \times 4 \quad 6 \times 7 \\ 5 \times 646 \times 107 \end{array} \quad \begin{array}{l} 2 \times 200 \\ 2 \times 30 \\ 2 \times 1 \\ 2 \times 231 \end{array}$$

$$8) \quad \begin{array}{l} 7 \times 3 \quad 5 \times 4 \\ 7 \times 6 \quad 5 \times 8 \end{array} \quad \begin{array}{l} 6 \times 4 \\ 6 \times 8 \end{array}$$

$$\begin{array}{l} 2 \times 7 \quad 3 \times 9 \\ 4 \times 7 \quad 6 \times 9 \\ 8 \times 7 \quad 12 \times 9 \end{array} \quad \begin{array}{l} 4 \times 5 \\ 8 \times 5 \\ 16 \times 5 \end{array}$$

$$9) \quad \begin{array}{l} 2 \times 9 \quad 3 \times 9 \\ 2 \times 2 \times 93 \times 3 \times 9 \\ 4 \times 9 \quad 9 \times 9 \end{array} \quad \begin{array}{l} 3 \times 9 \\ 2 \times 3 \times 9 \\ 6 \times 9 \end{array}$$

$$\begin{array}{l} 4 \times 7 \quad 4 \times 9 \\ 2 \times 4 \times 72 \times 4 \times 9 \\ 8 \times 7 \quad 8 \times 9 \end{array} \quad \begin{array}{l} 3 \times 7 \\ 2 \times 3 \times 7 \\ 6 \times 7 \end{array}$$

$$10) \quad \begin{array}{l} 10 \times 510 \times 8 \\ 10 \times 5010 \times 80 \\ 10 \times 50010 \times 800 \end{array} \quad \begin{array}{l} 10 \times 12 \\ 10 \times 120 \\ 10 \times 1,200 \end{array}$$

$$\begin{array}{l} 20 \times 320 \times 5 \\ 20 \times 3020 \times 50 \\ 20 \times 30020 \times 500 \end{array} \quad \begin{array}{l} 20 \times 8 \\ 20 \times 80 \\ 20 \times 800 \end{array}$$

$$11) \quad \begin{array}{l} 15 + 1524 + 24 \\ 2 \times 152 \times 24 \\ \frac{1}{2} \times 30 \frac{1}{2} \times 48 \end{array} \quad \begin{array}{l} 50 + 50 \\ 2 \times 50 \\ \frac{1}{2} \times 100 \end{array}$$

$$11 + 11 + 11$$

$$20 + 20 + 20$$

$$13 + 13 + 13 + 13$$

- |     |                                                     |                                 |                                |
|-----|-----------------------------------------------------|---------------------------------|--------------------------------|
|     | $3 \times 113 \times 20$                            | $4 \times 13$                   |                                |
|     | $\frac{1}{3} \times 33\frac{1}{3} \times 60$        | $\frac{1}{4} \times 52$         |                                |
| 12) | $10 \times 70010 \times 900$                        | $10 \times 400$                 |                                |
|     | $10 \times 7010 \times 90$                          | $10 \times 40$                  |                                |
|     | $10 \times 710 \times 9$                            | $10 \times 4$                   |                                |
|     | $10 \times 0.710 \times 0.9$                        | $10 \times 0.4$                 |                                |
|     | $7 \times 2003 \times 500$                          | $9 \times 600$                  |                                |
|     | $7 \times 203 \times 50$                            | $9 \times 60$                   |                                |
|     | $7 \times 2 \quad 3 \times 5$                       | $9 \times 6$                    |                                |
|     | $7 \times 0.23 \times 0.5$                          | $9 \times 0.6$                  |                                |
| 13) | $\frac{1}{2} \times 10\frac{1}{2} \times 24$        | $\frac{1}{3} \times 6$          |                                |
|     | $\frac{1}{2} \times 100\frac{1}{2} \times 240$      | $\frac{1}{3} \times 60$         |                                |
|     | $\frac{1}{2} \times 1,000\frac{1}{2} \times 2,400$  | $\frac{1}{3} \times 600$        |                                |
|     | $\frac{1}{2} \times 1,000\frac{1}{2} \times 24,000$ | $\frac{1}{3} \times 6,000$      |                                |
|     | $\frac{1}{2} \times 10,000,000$                     | $\frac{1}{2} \times 24,000,000$ | $\frac{1}{3} \times 6,000,000$ |
| 14) | $\frac{1}{2} \times 20\frac{1}{2} \times 6$         | $\frac{1}{2} \times 10$         |                                |
|     | $\frac{1}{2} \times 30\frac{1}{2} \times 16$        | $\frac{1}{2} \times 100$        |                                |
|     | $\frac{1}{2} \times 40\frac{1}{2} \times 26$        | $\frac{1}{2} \times 108$        |                                |
|     | $\frac{1}{2} \times 50\frac{1}{2} \times 36$        | $\frac{1}{2} \times 148$        |                                |
| 15) | $2 \times 132 \times 35$                            | $2 \times 104$                  |                                |
|     | $\frac{1}{2} \times 26\frac{1}{2} \times 70$        | $\frac{1}{2} \times 208$        |                                |
|     | $3 \times 10 \quad 3 \times 8$                      | $3 \times 105$                  |                                |
|     | $\frac{1}{3} \times 30\frac{1}{3} \times 24$        | $\frac{1}{3} \times 315$        |                                |
| 16) | $\frac{1}{2} \times 40\frac{1}{2} \times 100$       | $\frac{1}{3} \times 90$         |                                |
|     | $\frac{1}{2} \times 8\frac{1}{2} \times 12$         | $\frac{1}{3} \times 6$          |                                |
|     | $\frac{1}{2} \times 48\frac{1}{2} \times 112$       | $\frac{1}{3} \times 96$         |                                |
| 17) | $6 \times 2 \quad 9 \times 4$                       | $30 \times 3$                   |                                |
|     | $6 \times \frac{1}{2} \quad 9 \times \frac{1}{3}$   | $30 \times \frac{1}{2}$         |                                |
|     | $6 \times 2\frac{1}{2} \quad 9 \times 4\frac{1}{3}$ | $30 \times 3\frac{1}{2}$        |                                |
| 18) | $7 \times 8 \quad 6 \times 9$                       | $3 \times 8$                    |                                |
|     | $56 \div 754 \div 9$                                | $24 \div 8$                     |                                |
|     | $56 \div 854 \div 6$                                | $24 \div 3$                     |                                |
|     | $2 \times 432 \times 17$                            | $2 \times 108$                  |                                |
|     | $86 \div 234 \div 2$                                | $216 \div 2$                    |                                |
|     | $3 \times 20 \quad 3 \times 6$                      | $3 \times 201$                  |                                |
|     | $60 \div 318 \div 3$                                | $603 \div 3$                    |                                |
| 19) | $20 \div 2100 \div 2$                               | $30 \div 3$                     |                                |
|     | $6 \div 2 \quad 10 \div 2$                          | $9 \div 3$                      |                                |
|     | $26 \div 2110 \div 2$                               | $39 \div 3$                     |                                |
| 20) | $\frac{1}{2} \times 14\frac{1}{2} \times 7$         | $\frac{1}{2} \times 100$        |                                |

# C

$$\begin{array}{ll} \frac{1}{2} \times 15\frac{1}{2} \times 9 & \frac{1}{2} \times 101 \\ \frac{1}{2} \times 20\frac{1}{2} \times 11 & \frac{1}{2} \times 102 \\ \frac{1}{2} \times 21\frac{1}{2} \times 13 & \frac{1}{2} \times 103 \end{array}$$

$$\begin{array}{ll} 21) \quad 70 \div 1080 \div 10 & 150 \div 10 \\ \quad 7 \div 108 \div 10 & 15 \div 10 \\ \quad 0.7 \div 100.8 \div 10 & 1.5 \div 10 \\ \quad 0.07 \div 100.08 \div 10 & 0.15 \div 10 \end{array}$$

$$\begin{array}{ll} 22) \quad 6 \times 217 \times 51 & 5 \times 35 \\ \quad 6 \times 2.17 \times 5.1 & 5 \times 3.5 \\ \quad 0.6 \times 210.7 \times 51 & 0.5 \times 35 \end{array}$$

$$\begin{array}{l} 23) \quad 17 \times 3 \times \frac{1}{3} \times 2 \times 59 \times \frac{1}{2} \\ \quad 19 \times 2 \times \frac{1}{2} \times \frac{1}{5} \times 27 \times 5 \\ \quad 59 \times \frac{1}{5} \times 5 \times \frac{1}{3} \times 67 \times 3 \\ \quad 63 \times \frac{1}{4} \times 44 \times 100,001 \times \frac{1}{4} \end{array}$$

$$\begin{array}{ll} 24) \quad 2 \times 3 \quad 2 \times 30 & 3 \times 3 \\ \quad 2 \times 2 \quad 2 \times 20 & 3 \times 2 \\ \quad 2 \times 1 \quad 2 \times 10 & 3 \times 1 \\ \quad 2 \times 0 \quad 2 \times 0 & 3 \times 0 \\ \quad 2 \times \widehat{1} \quad 2 \times \widehat{10} & 3 \times \widehat{1} \\ \quad 2 \times \widehat{2} \quad 2 \times \widehat{20} & 3 \times \widehat{2} \\ \quad 2 \times \widehat{3} \quad 2 \times \widehat{30} & 3 \times \widehat{3} \end{array}$$

25) Ask for a multiple of 3 (or 4) between two other numbers; for example, ask for a multiple of 3 between 20 and 25.

## Estimation

Ask the students to estimate a calculation and to explain their answers. For example:

**T:** *Is  $218 + 157$  more than 300? How do you know?*

**S:** *Yes, because  $200 + 100 = 300$ .*

**T:** *Is it more than 500? How do you know?*

**S:** *No, because  $300 + 200 = 500$ . 218 is less than 300, and 157 is less than 200, so  $218 + 157$  must be less than 500.*

**T:** *Is it more than 400?*

**S:** *No, because  $18 + 57$  is less than 100.*

During the lessons, students are often asked to estimate a number displayed on the Minicomputer.

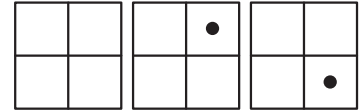
## Other Mental Arithmetic Activities

- 1) Use the Minicomputer to give visual clues for addition, subtraction, and multiplication problems. For example, suppose this configuration is on the Minicomputer.



**T:** *What number is on the Minicomputer?* (41)

Hold a checker over the 8-square.



**T:** *What number is  $41 + 8$ ?* (49)

Hold a checker over the 10-square.

**T:** *What number is  $41 + 10$ ?* (51)

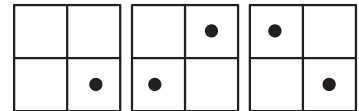
After a few such clues, hold a checker over the square only if the class has difficulty.

The next example involves subtraction.

**T:** *What number is on the Minicomputer?* (169)

Lift the checker temporarily from the 8-square.

**T:** *What number is  $169 - 8$ ?* (161)



Lift the checker temporarily from the 20-square.

**T:** *What number is  $169 - 20$ ?* (149)

Lift the checkers temporarily from the 20-square and the 1-square.

**T:** *What number is  $169 - 21$ ?* (148)

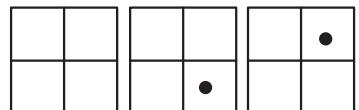
After a few such clues, lift the checkers only if the class has difficulty.

The next examples involve multiplication and division.

**T:** *What number is on the Minicomputer?* (14)

Move the checkers one board to the left as you ask,

**T:** *What number is  $10 \times 14$ ?*



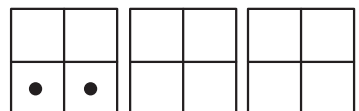
Repeat this activity starting with other starting configurations.

Put 300 on the Minicomputer.

Move the checkers to the tens board as you ask,

**T:** *What number is  $300 \div 10$ ?*

**S:** 30.



# C

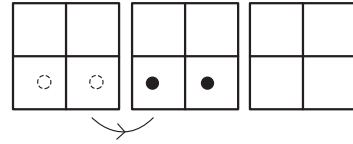
Move the checkers to the ones board as you ask,

**T:** *What number is  $30 \div 10$ ?*

**S:** 3.

**T:** *Can we divide by 10 again?*

**S:** *Draw a bar and put up boards to the right.*



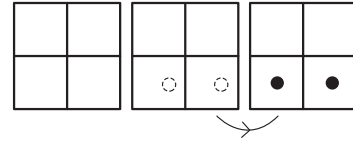
Move the checkers to the tenths (dimes) board as you ask,

**T:** *What number is  $3 \div 10$ ?*

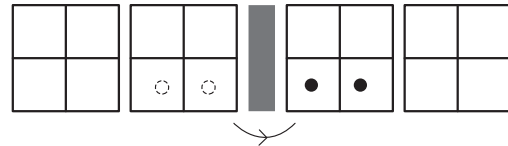
**S:** 0.3.

**T:** *What is  $0.3 \div 10$ ?*

**S:** 0.03.



- 2) Use an arrow picture to pose mental arithmetic questions. For example, draw this arrow picture on the board.



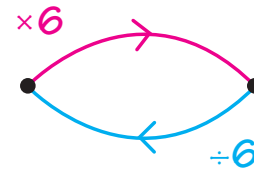
Ask what number is at the right if the number at the left is, for example, 12. Repeat this activity several times and then draw the  $\div 6$  return arrow.



**T:** *What could this blue return arrow be for?*

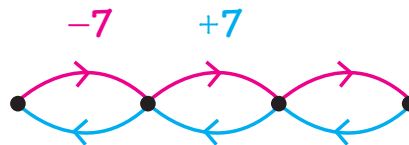
**S:**  $\div 6$ .

Ask students what number is at the left if the number at the right is, for example, 48. Repeat this activity several times.

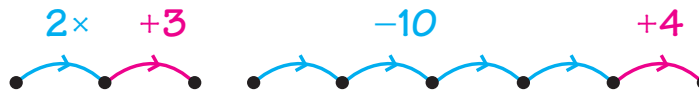


There are many variations for this activity. For example:

- Use larger starting and ending numbers.
- Use functions other than  $\times 6$  and  $\div 6$ .
- Use two or more arrows of the same function.



- Use a composition of two functions.



- 3) Use the suggestion of an arrow picture to pose place-value questions.

- Imagine an arrow road starting at 0 and having four  $+10$  arrows and seven  $+1$  arrows. What is the ending number?
- Imagine an arrow road starting at 0 and having two  $+100$  arrows, five  $+10$  arrows, and two  $+1$  arrows. What is the ending number?



- 3) Play *Calculator Golf* with a small group of students, each one having a calculator. Ask the students to put a small whole number you specify on the display of their calculators. Then let the students take turns suggesting some numbers to add or subtract until a particular goal is reached. A possible game in which four students are playing and 200 is the goal is described below.

Start with 12 on the display. (12)

Player 1:	+ 9	(21)
Player 2:	+72	(93)
Player 3:	+17	(110)
Player 4:	+7	(117)
Player 1:	+93	(210)
Player 2:	-10	(200)

Some variations of this game include

- changing the starting number;
- making the goal a larger multiple of 100;
- allowing the use of only  $\boxed{+}$ ,  $\boxed{-}$ , or  $\boxed{\times}$  some number between 0 and 10.

For still another variation, allow only the operation  $\boxed{\times}$  to be used and make the goal a number between 499 and 500. In this variation of the game, a play always begins back at the starting number. A possible game with four students is described below. The starting number is 17.

Player 1:	$17 \times 50 = 850$
Player 2:	$17 \times 40 = 680$
Player 3:	$17 \times 35 = 595$
Player 4:	$17 \times 30 = 510$
Player 1:	$17 \times 29 = 493$
Player 2:	$17 \times 29.5 = 501.5$
Player 3:	$17 \times 29.4 = 499.8$

### Some Short Games

- 1) Play a game called *High-Low* in which you choose a secret number and students try to guess it. Each time a guess is made, say if the guess is too high or too low. Depending upon the ability of your class, choose secret numbers such as 10,001;  $2\frac{1}{2}$ ; 17.4, or 3.14.
- 2) Play a game called *Hit/Miss/Bull's-Eye*. The object of the game is to guess a secret number. Tell the class you have a secret three-digit number in which all three digits are different. When a student makes a guess, say the following:
  - “hit” for every digit in the guess that is correct but not in the correct place;
  - “miss” for every digit in the guess that is not in your number;
  - “bull’s-eye” for every digit in the guess that is correct and in the correct place.

The order in which you give these responses does not have to necessarily correspond to the order of the digits in the guess. In other words, if you say, “Hit, miss, miss,” the students know one of the three numbers guessed is correct, but not necessarily the first one. Record the guesses and responses on the board. It is helpful to write your secret number on a piece of paper to refer to during the game. Here is a description of a sample game in which the secret number is 571.

---

Secret number: 571

Guess 1: 429 miss, miss, miss  
Guess 2: 389 miss, miss, miss  
Guess 3: 107 hit, hit, miss (1 and 7 are correct but not in the correct place.)  
Guess 4: 751 bull's-eye, hit, hit (1 is correct and in the correct place, 7 and 5 are correct but not in the correct place.)  
Guess 5: 715 hit, hit, hit  
Guess 6: 571 bull's-eye, bull's-eye, bull's-eye



*The String Game* is used in many versions throughout the *Comprehensive School Mathematics Program*. It gives students an opportunity, in a game-like atmosphere, to become familiar with the language of strings, while at the same time it involves them in the kind of reasoning that will be developed and reinforced in various contexts throughout the program.

*The String Game* is first played with A-blocks (shapes), but later, in the intermediate grades, variations that make use of numbers appear. This Appendix provides you with the necessary information and examples to enable you to play the game using A-blocks and using numbers, and it suggests some of the various possibilities open to you when you play the game. There are several lessons in *IG-I* that call for *The String Game*, but we hope you will not feel restricted to playing the game only at these times. It is most beneficial and enjoyable for students if you make a regular practice of playing the game once every few weeks or whenever you have an extra 10–15 minutes during the course of the day.

## Equipment

### PLAYING BOARD

The equipment for this game may be most easily managed if you have a magnetic (magnet-sensitive) chalkboard available. Many permanently mounted chalkboards in classrooms are magnetic; you can test yours using a magnet. If your permanent chalkboards are not magnetic, try any portable chalkboard (dry erase board, and so on) that is available. If you do not have a magnetic chalkboard available, you can use your regular chalkboard.

### TEAM BOARD

The team board is divided into regions as illustrated below.

<b>(Attach a poster list of string cards here.)</b>	
<b>Team A</b>	<b>Team B</b>

**Note:** The game may be played with three or four teams rather than two. In this case, create a team board with sections for more teams.

- a) **Magnetic:** If you have a large magnetic classroom chalkboard, you can draw the team board directly on a portion of it. However, if you have a relatively small (portable) magnetic chalkboard, you may need to obtain a sheet of metal (minimum size 60 cm by 80 cm) or locate a convenient metallic surface in the classroom, such as the side of a file cabinet, on which to put the team board. In such a case, draw the team board on a large sheet of (chart) paper and tape this paper to your metallic surface.
- b) **Non-magnetic:** If you do not have a magnetic chalkboard available for the playing board, your team board can be a large piece of poster board (minimum size 60 cm by 80 cm). Or, a team board may be drawn directly on a portion of the classroom chalkboard.

## GAME PIECES AND STRING CARDS

One set of game pieces and string cards is needed for each version of the game. A poster list of the string cards should be posted above the team board—it is a constant reminder during the game of the possible labels for the strings.

Game Pieces

String Cards

RED	YELLOW	GREEN	BLUE
NOT RED	NOT YELLOW	NOT GREEN	NOT BLUE
○	△	□	BIG
NOT ○	NOT △	NOT □	LITTLE

String Game with A-Blocks

$\widehat{100}$	$\widehat{80}$	$\widehat{55}$	$\widehat{15}$	$\widehat{10}$	$\widehat{5}$
$\widehat{1}$	0	1	2	3	4
5	6	7	8	9	10
12	18	20	24	27	40
45	50	60	99	100	105

MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4	MULTIPLES OF 5
MULTIPLES OF 10	LESS THAN 50	LESS THAN $\widehat{10}$	ODD NUMBERS
POSITIVE DIVISORS OF 12	GREATER THAN 50	GREATER THAN $\widehat{10}$	POSITIVE PRIME NUMBERS
POSITIVE DIVISORS OF 18	POSITIVE DIVISORS OF 20	POSITIVE DIVISORS OF 24	POSITIVE DIVISORS OF 27

Game pieces, string cards, and the poster of the string cards can be found in the corresponding *String Game* kit, (*A-Blocks* or *Numerical*).

- a) **Magnetic:** You can magnetize the game pieces (A-blocks or number cards) by sticking a small piece of magnetic material to the back of each one. (Magnetic material is included in *The String Game* kits, or it is available in many stores, in the hobby or notions departments.) Similarly, you can magnetize string cards by sticking a small piece of magnetic material to the front of each card, taking care not to obscure what is written on it.
- b) **Non-magnetic:** Game pieces can be attached to the team board using loops of masking tape stuck to the backs. A string card should have a loop of masking tape stuck to the front in such a way that what is written on the card is not obscured. With this type of equipment, be prepared to make necessary repairs by having masking tape on hand so that if a loop of tape loses its stickiness it can be replaced on the spot. As an alternative, use a small wad of a plastic caulking compound (Rope Caulk or Mortite, for example) in place of the loop of masking tape.

### Preparation for the Game

Draw two (or three, depending on which variation you are using) large, overlapping strings on the playing board using two (or three) different colors. Next to each of these strings attach one string card facedown. Place the team board conveniently nearby. Randomly distribute the game pieces among the sections of the team board. Divide the class into teams using whatever method is acceptable to your class, and assign each team a section of the team board.

Before any student takes a turn, correctly place an equal number (at least one) of each team's game pieces in the string picture. This eliminates the necessity of beginning the game on the basis of pure guesswork. You can influence how long the game will take by the number of pieces you place in the string picture before the game begins.



## Object of the Game

Each team tries to place all of its game pieces correctly (according to the facedown string cards) in the string picture. The winning team is the one that places all of its game pieces correctly and identifies the facedown cards correctly first.

## Rules of the Game

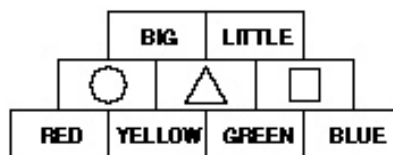
- 1) The teams alternate making plays, and the members take turns within each team. A player comes to the board and selects a piece from his or her team's collection to place in one of the regions of the string picture.
- 2) You are the judge. If the piece is correctly placed, say yes. The piece then remains in the string picture and the player immediately has a second (bonus) turn (no player may have more than two consecutive turns). If the piece is incorrectly placed, say no. The player returns the piece to the team's unplayed collection and play passes to the next team.

As an aid in judging, prepare a crib sheet showing the correct position of each game piece or at least reminding you of what is on the facedown cards. If at any time you discover that you have made an error, say so immediately and rectify the mistake. Then, either move an incorrectly placed piece to its correct region or replace a correctly placed piece that has been removed.

- 3) When a team has correctly placed all of its pieces, the player who placed the last piece may then attempt to identify the string cards. If he or she is correct, the team wins. If a mistake is made (even if it is only in the case of one of the string cards), simply indicate that the identification is incorrect and let the game continue.
- 4) If a team has exhausted its stock of game pieces and the strings have not been identified, that team continues to attempt to identify the strings on its turn, while the other team(s) works to place its game pieces.

## The String Game with A-Blocks (Version A)

This simplest version of the game uses 24 A-blocks as game pieces and only nine string cards.

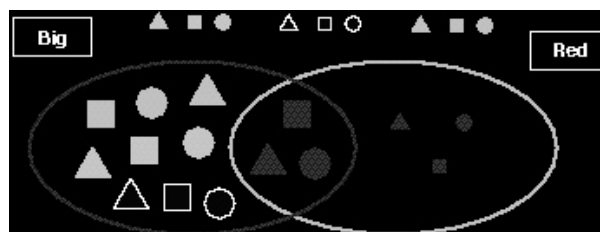


This list of string cards should be attached above the team board.

Below are several crib sheets for variations of the game with two and with three strings.

### TWO STRINGS

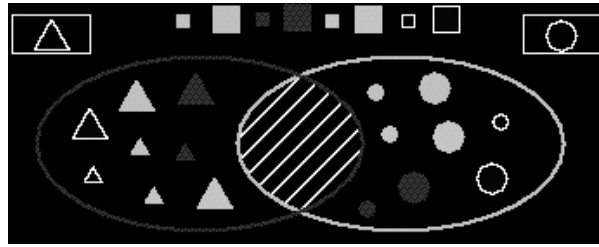
Example 1: No empty regions



Example 2: One empty region

# D

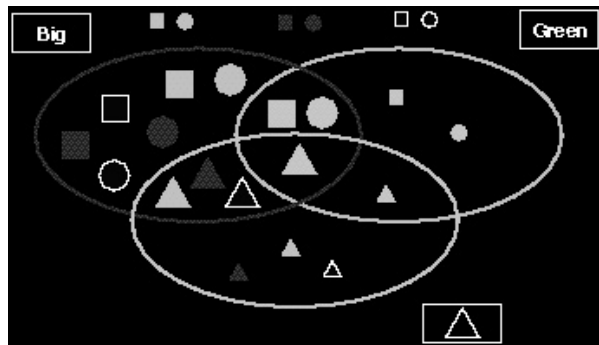
---



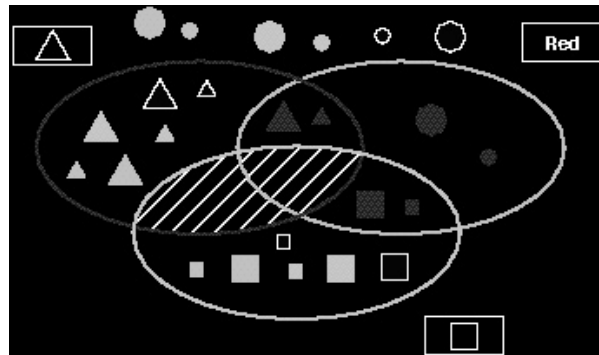
**Note:** We have indicated that the intersection of the strings is empty by “hatching” that region.

## THREE STRINGS

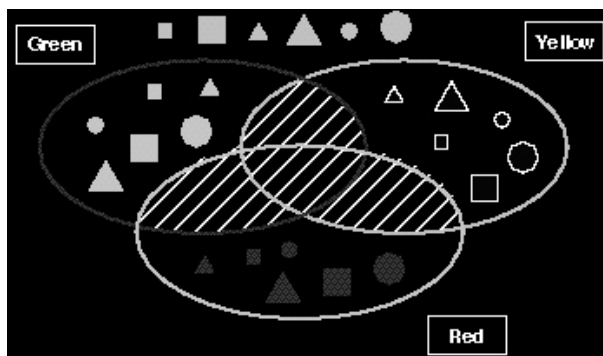
Example 1: No empty regions



Example 2: Two empty regions



Example 3: Four empty regions



The String Game with A-Blocks, Using Not-Cards (Version B)

A more complicated version of the game with A-blocks uses all 16 of the string cards. A list of all 16 string cards should be posted above the team board.

Here are several crib sheets for this version of the game played with two strings.

Example 1: No empty regions



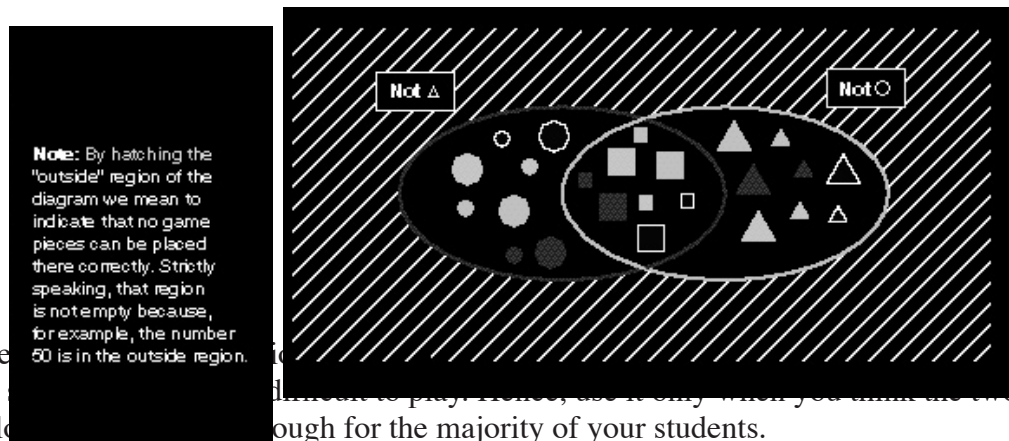
Example 2: No empty regions



Example 3: One empty region



Example 4: One empty region



**Note:** By hatching the "outside" region of the diagram we mean to indicate that no game pieces can be placed there correctly. Strictly speaking, that region is not empty because, for example, the number 50 is in the outside region.

You should be warned that this version of the game is more difficult to play. Hence, use it only when you think the string version is no longer challenging enough for the majority of your students.

### The String Game with Numbers (Version C)

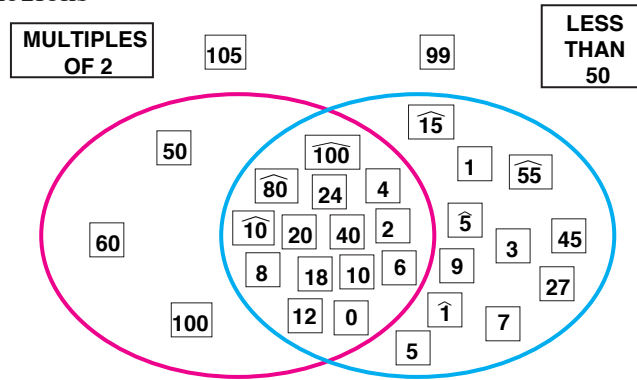
# D

This version of the game uses 30 number cards as game pieces and 16 string cards as described earlier in the section on equipment. Note that The Numerical String Game kit contains three sets of string cards. This allows you to give the same label to more than one string. Example 7 played with two strings and Example 1 played with three strings show this option; however, before using such a game players should have an understanding that this is a possibility.

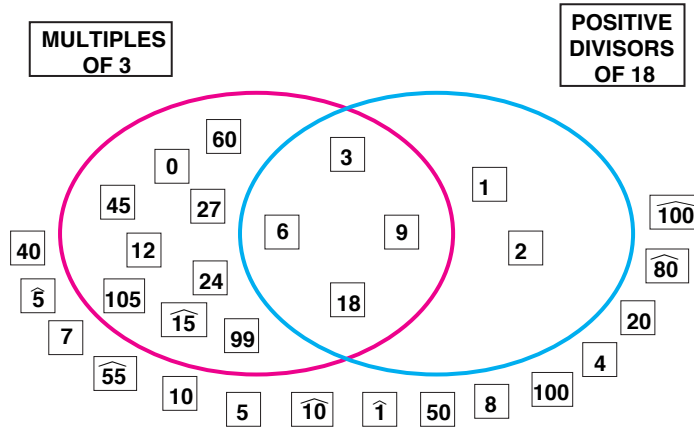
Below are several crib sheets for variations of the game with two and with three strings.

## TWO STRINGS

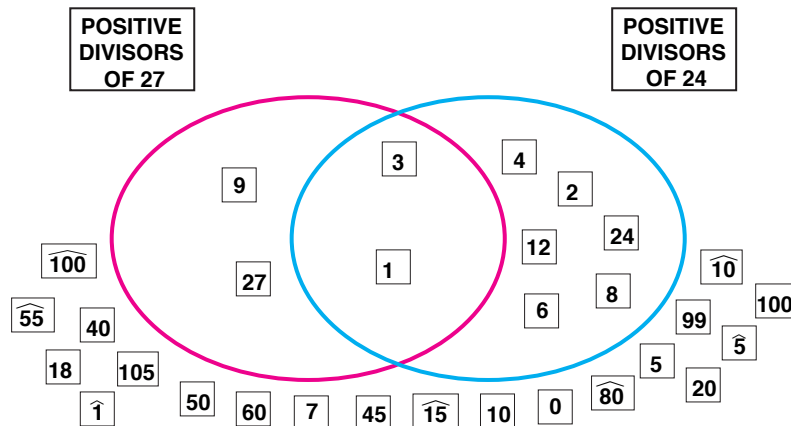
Example 1: No Empty Regions



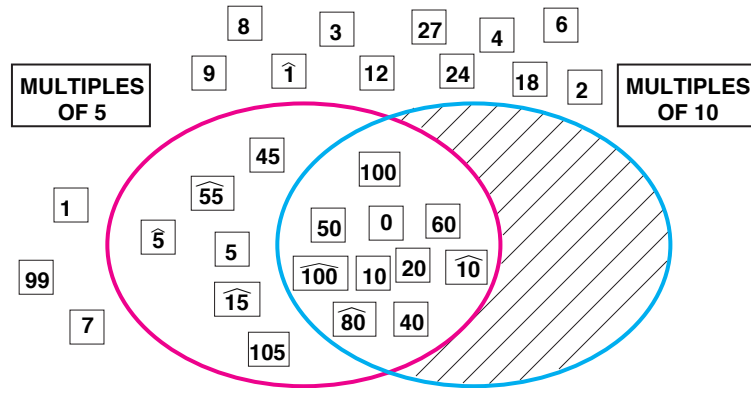
Example 2: No Empty Regions



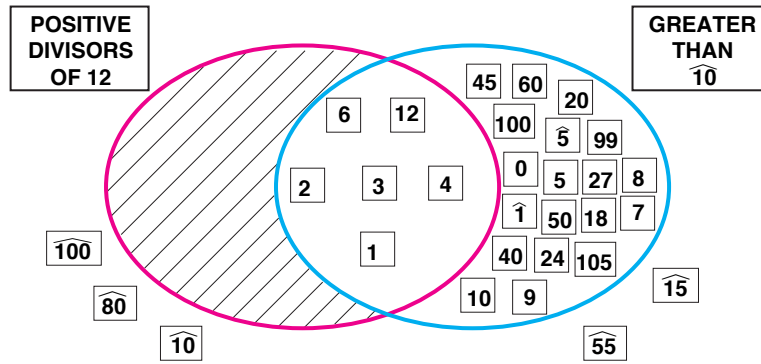
Examples 3: No Empty Regions



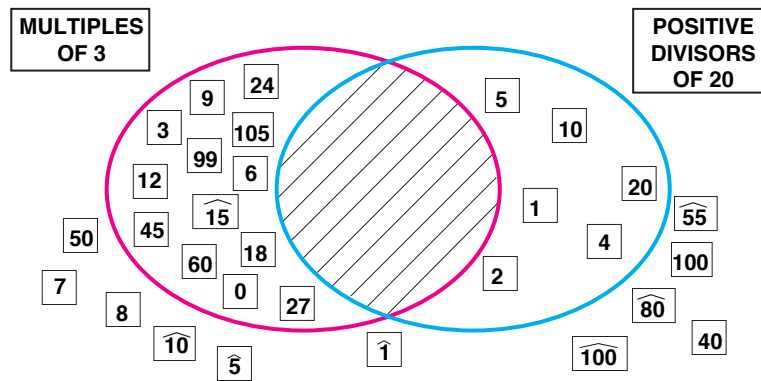
Example 4: One Empty Region



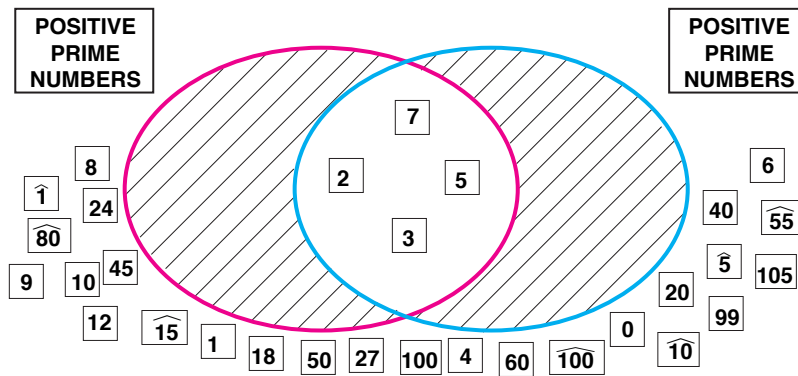
Example 5: One Empty Region



Example 6: One Empty Region



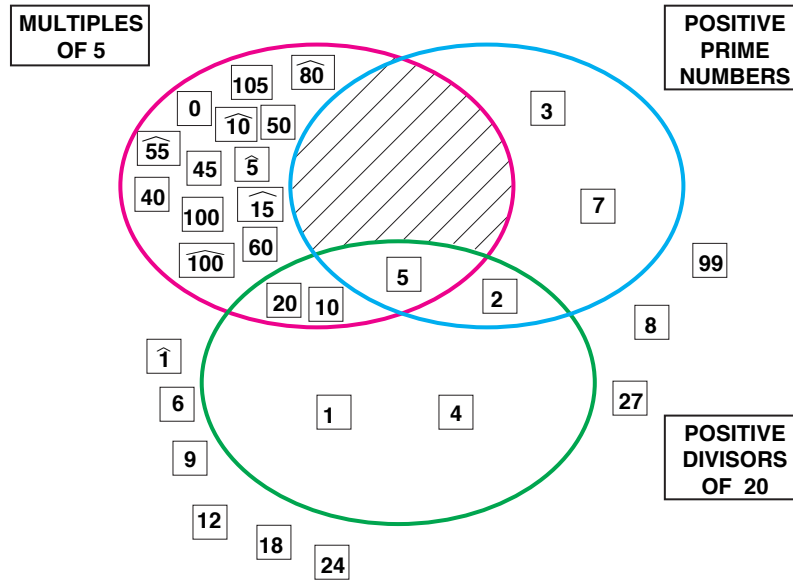
Example 7: Two Empty Regions



# D

**Caution:** The jump in difficulty from a two-string game to a three-string game is much greater in *The String Game* with numbers than it was in *The String Game* with A-blocks. Hence you should not expect to proceed to three-string games as rapidly as you might have with A-blocks.

Example 1: One Empty Region



Example 2: Five Empty Regions

