CSMP Mathematics for the Intermediate Grades Part II

Teacher's Guide

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CSMP Mathematics for the Intermediate Grades (IG) owes its existence to the vision and leadership of *CSMP*'s first director, Burt Kaufman, and a special group of people concerned not only about the kind of mathematics being taught to children but also the manner in which it is taught. This distinction is crucial to a child's early exposure to and future relationship with mathematics.

The *CSMP* elementary school curriculum is in large part the outgrowth of a great deal of active involvement with students in ordinary classroom settings. Its current form has been strongly influenced by information gathered from classes in extended pilot test sites nationally. But its underlying manner of presentation and many of the pedagogical ideas and tools, which make *CSMP* unique among other curricula, are based on the pioneering works of Georges and Frédérique Papy. We acknowledge them here with deep gratitude. We also acknowledge the work of mathematician and psychologist Caleb Gattegno for the fundamental idea of presenting mathematics to students via pedagogical situations.

Earlier versions of *CSMP Mathematics for the Intermediate Grades* dating back to 1976 were due to the efforts of former *CSMP* staff members Richard Armstrong, Jim Harpel, Burt Kaufman, Frédérique Papy, Pamela Pedersen, and Joel Schneider.

This revised edition of the program—*CSMP/21*—is a result of new information gleaned thanks to the interest and concern of *CSMP* teachers nationwide.

In addition, we acknowledge the following individuals for their work in producing the *CSMP/21* edition of the Intermediate Grades program.

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Notes to the Teacher

IG-II WELCOME

Welcome! You are about to teach the second semester (Part II) of *CSMP Mathematics for the Intermediate Grades,* perhaps for the first time. We wish you success and hope both you and your students enjoy the activities suggested in this teacher's guide. The purpose of this section is to help you use this book effectively, to describe the *CSMP* materials, and to give suggestions for organizing the program. In Section Two, you'll find answers to some frequently asked questions; in Section Three, a suggested schedule of lessons; and in Section Four, a suggested teaching guide organized in strands.

As you and your students take part in these activities during the year, you will discover a definite *CSMP* viewpoint about elementary school mathematics, one we hope you will come to understand and share. Part of that viewpoint is that learning takes place when students react to interesting real-life or fantasy situations (sometimes called a *pedagogy of situations),* such as those in stories or games. You'll find that *CSMP* presents situations that involve students personally and allow arithmetic to take the form of adventures in the world of numbers.

We believe that every child can and does learn something from each situation encountered in the program, sometimes suddenly and dramatically, sometimes latently. For this reason, *CSMP* teachers do not stick to one topic until it is mastered. Indeed, our experience indicates that learning often stops when the purpose of a lesson is merely the mastery of a skill. Instead, we view learning as more of a spiral rather than a linear process in which intuitive leaps play as big a role as the acquisition of small successive pieces of information, and that a variety of situations can provide the interrelated experiences through which we learn.

Further explanation of what we mean by a pedagogy of situations and spiral development can be found in the question and answer section, but you will learn their meanings primarily by experiencing them for yourself as you teach the curriculum. Meanwhile, we assure you that a large number of *CSMP* teachers across the country have come to understand and embrace the *CSMP* style of teaching and learning and are very pleased with the results.

CSMP Mathematics for the Intermediate Grades (IG) is designed as a program for students in grades four through six. There are six parts to the curriculum, each corresponding to one semester's work. This is *Part II* of the six.

Since the learning process occurs through many interrelated experiences, we have developed this curriculum so that no single lesson is an end in itself. It is neither intended nor expected that every student will meet the full challenge of a particular situation in any given lesson. Furthermore, we have found that it is most effective to vary the situations from day to day rather than to continue one type of situation or topic until so-called mastery has occurred. This spiral development is reflected in the materials and scheduling of various topics in the curriculum. It is consistent with *CSMP*'s pedagogy of situations since with such an approach, a child experiences each of several ideas a little at a time and then proceeds through increasing levels of sophistication as the situations become more challenging. We hope you will keep the spiral approach in mind as you teach the *CSMP* curriculum.

Description of Materials

Teacher's Guide

This 3-ring binder is known as the *CSMP Mathematics for the Intermediate Grades, Part II* Teacher's Guide. It contains lessons grouped by topic areas or *strands*. The five strands (with abbreviations given in parentheses) are:

- World of Numbers - (N)
- Languages of Strings and Arrows - (L)
- Geometry and Measurement - (G)
- Probability and Statistics - (P)
- Workbooks - (W)

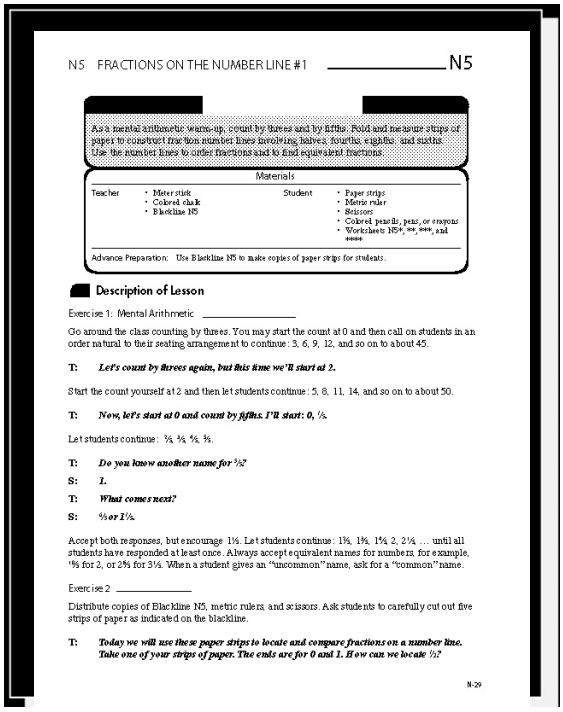
Each strand is separated and identified by a colored tab. Each strand has its own table of contents and is paginated individually.

Within a strand you will find the following arrangement of material:

- **A table of contents.** This enables you to locate whatever you need in a particular strand.
- **An introduction to the strand.** This provides you with information explaining why certain material finds its place in the strand, the overall goals of study in the strand, and how this material fits in with that which is described in other strands.
- **A series of lesson plans.** These plans include *capsule lesson summaries*, lists of needed materials for both teacher and students, and lesson descriptions which suggest possible classroom dialogue, and indicate the use of worksheets and manipulative materials. The lesson descriptions also provide illustrations of diagrams the teacher puts on the board or otherwise displays for the class, and answer keys for worksheets or other independent work during the lessons.

In each of the strands, N, L, G, P, and W, the lessons are numbered consecutively. Each lesson is designed to be used during one full class period. For example, Lesson N8 is the eighth lesson in the N (World of Numbers) strand. Since the N lessons are interspersed among lessons from other strands, N8 might be taught on, say, Wednesday of the fourth week of the term and the next N lesson (N9) might be taught on Monday of the fifth week.

Each lesson is presented in a form that includes its strand and number, title, summary, materials, and description. For example, here is the first page from the fifth lesson in the N (World of Numbers) strand.



N5 FRACTIONS ON THE NUMBER LINE #1 N5

As a mental arithmetic warm-up, count by threes and by fifths. Fold and measure strips of paper to construct fraction number lines involving halves, fourths, eighths, and sixths. Use the number lines to order fractions and to find equivalent fractions.

Materials

Teacher
- Meter stick
- Colored chalk
- Blackline N5

Student
- Paper strips
- Metric ruler
- Scissors
- Colored pencils, pens, or crayons
- Worksheets N5*, N5**, N5***, and N5****

Advance Preparation: Use Blackline N5 to make copies of paper strips for students.

Description of Lesson

Exercise 1: Mental Arithmetic

Go around the class counting by threes. You may start the count at 0 and then call on students in an order natural to their seating arrangement to continue: 3, 6, 9, 12, and so on to about 45.

T: *Let's count by threes again, but this time we'll start at 2.*

Start the count yourself at 2 and then let students continue: 5, 8, 11, 14, and so on to about 50.

T: *Now, let's start at 0 and count by fifths. I'll start: 0,* ⅕.

Let students continue: ⅗, ⅘, ⅝, ⅗.

T: *Do you know another name for* ⅖?

S: *1.*

T: *What comes next?*

S: ⅔ *or* 1⅕.

Accept both responses, but encourage 1⅕. Let students continue: 1⅗, 1⅘, 1⅝, 2, 2⅕ ... until all students have responded at least once. Always accept equivalent names for numbers, for example, ⅝ for 2, or 2⅝ for 3⅛. When a student gives an "uncommon" name, ask for a "common" name.

Exercise 2

Distribute copies of Blackline N5, metric rulers, and scissors. Ask students to carefully cut out five strips of paper as indicated on the blackline.

T: *Today we will use these paper strips to locate and compare fractions on a number line. Take one of your strips of paper. The ends are for 0 and 1. How can we locate* ½?

N-29

The organization of lessons in this teacher's guide is a strand-to-strand presentation. This means that on one day of the week a lesson is taught from a certain strand; and then on the next day of the week a lesson is taught from another strand; and so on. In this way we can achieve a spiraling effect and at the same time gain the benefits of organization of lessons according to strands. For example, suppose you are interested in the way a topic in geometry is developed in the fourth grade. With a day-by-day format you would need to search through the entire guide for geometry lessons, but with a strand-to-strand format all these lessons are found, in sequence, in one part of the guide, the G strand. You can then read the lessons in their natural order to gain an overview of the Geometry strand for this semester. At the same time, the strand-to-strand arrangement allows you to anticipate what is coming on the next topic within the spiral development simply by reading ahead in the strand in which that topic is developed.

Teacher Materials

In addition to the teacher's guide, there are some demonstration manipulative materials you will need. The following materials are contained in the *CSMP* Classroom Set.

- Demonstration Minicomputer Set
  (4 magnetic Minicomputer boards; 1 set of magnetic checkers)
- Weighted Checker Set
- Number Line
- IG-II Poster Set
  (posters to accompany specified lessons)
- Numerical String Game Kit
  (2 sets of numerical string game pieces, 10 numerical string game posters, 2 sets of numerical string game cards, magnetic material)
- Numerical String Game Analysis Pad

- Marble Set
- Blacklines

Other materials needed for individual lessons are listed at the beginning of each lesson. Materials not in your classroom set are generally readily available or can be easily prepared from usual classroom supplies.

Student Materials

There are some individual materials that each student (or pair of students) will need. The following materials are contained in the *CSMP* Classroom Set.

- IG-II Worksheet Booklet
- *Multiplication Problems* Booklet
- *Set of Problems #1, #2, #3, #4,* and *#5* Workbooks
- *Clinton Street* Story-Workbook
- Individual Minicomputer Set
  (2 sheets of Minicomputer boards; 1 sheet of punch-out checkers)
- Translator
- *The Square Trap* and *Nabu Wins an Award* Storybooks

These and/or other materials needed are listed at the beginning of each lesson. Often, commercially available manipulatives are suggested (such as Unifix® cubes, geoboards, counters, base-10 blocks, and so on) along with several alternatives. Many lessons call for the students to have calculators (see page 1-8 on the role and use of calculators).

Some teachers find it convenient to provide each student with a sturdy cardboard box for storing manipulative materials. Others find it more convenient to store all the materials of one kind together in a container of appropriate size and to distribute these materials to the students as they are needed. In any case, you should probably keep the worksheet booklets, workbooks, and storybooks in your storage area and distribute them as called for in the lessons.

Two copies of the storybook *Two by Two* from the *Stories By Frédérique* collection are provided in your classroom set of materials although they are not used in any *IG-II* lessons. This storybook is a fun introduction to the Papy Minicomputer for new students or students who need a reminder. We suggest that these copies be available in a reading corner for students to read whenever they wish.

Lesson materials frequently call for the student to have paper available for writing problems and answers or for copying from the board. You may want students to keep such papers in a folder or spiral notebook. It may also be appropriate for students to have a “think pad” or other scratch pad handy for these responses, or to use a small slate chalkboard which can be displayed and erased for continued use. Devise whatever method for written responses that seems effective and efficient in your classroom.

Role and Use of Worksheets, Workbooks, and Storybooks

Worksheets

Some lessons include worksheets for class, group, or independent work. Worksheets are designated with numbers corresponding to the lessons with which they are used, and with stars or letters indicating the order in which they are to be completed. The stars also suggest an increasing level of difficulty which provides for various abilities of students. It is important that you do not insist that every student finish every worksheet before going on to the next lesson. Though some students will finish more worksheets than others during the allotted time, don't worry. The same ideas will emerge in new settings in later lessons so that students will have other opportunities to work independently on the same topics.

Workbooks and Story-Workbooks

CSMP Mathematics for the Intermediate Grades, Part II (IG-II) includes five workbooks titled *Set of Problems #1, Set of Problems #2, Set of Problems #3, Set of Problems #4, Set of Problems #5*, and one story-workbook titled *Clinton Street*. Each is a 32-page booklet that reviews problems which have been encountered up to that point in the curriculum. Two lessons are scheduled for each workbook, and we suggest that all students be directed to start at the beginning of the workbook and work independently through as many pages as they can during the two scheduled periods.

Ordinarily the first 10 pages of a workbook are easy enough to guarantee some success for all the students; the next 10 to 12 pages are average level difficulty; and the remaining pages are more difficult, often challenging even to your best students. We estimate that about two-thirds of your students should correctly finish the first 10 pages during the two scheduled lessons, one-third should finish the first 20 pages, and a few might finish all or most of the pages. Of course, these proportions will vary from class to class.

Two full class periods are devoted to each workbook. At the end of the first period, you should collect the workbooks and check some or all the pages completed. Then when you return the workbooks at the beginning of the next workbook period, the students can correct their mistakes before going on with new problems.

Notice that the cover page of each workbook includes spaces for marking your evaluation of the work on each page. One way to do this is to use these symbols:

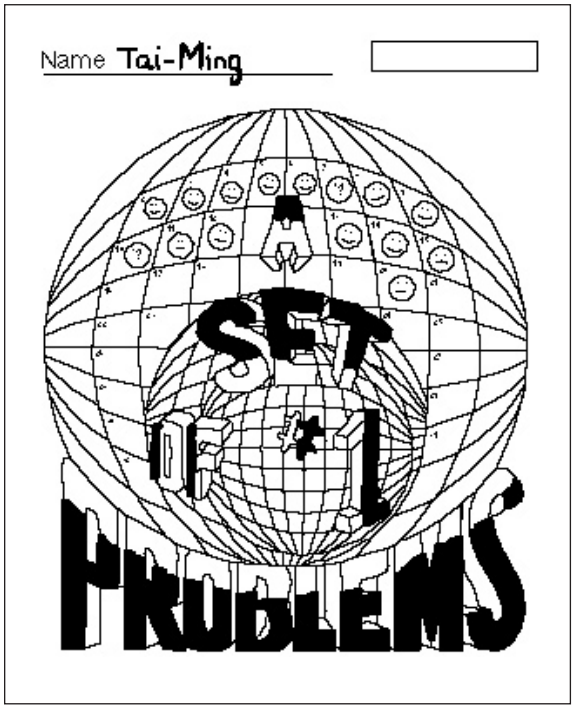
well done
or
correct

something wrong
try again

did you forget this problem?
or
do you need a hint?

The next page shows the front cover of one student's *Set of Problems #1* workbook after checking.

Note: This is the way Tai-Ming's workbook was returned to him at the beginning of the second *Set of Problems #1* lesson. The empty bubbles numbered 16–19, and 21–32 indicate that he has not yet worked on these pages. Before Tai-Ming begins on these pages during the second lesson, he should correct his mistakes on pages 11, 15, and 20, and finish his works on page 7 and 10.



Important: Again we ask you not to require or even encourage every student to finish every problem in a workbook. If at the end of the two allotted lessons, some students have not completed as much of the workbook as you expect they are capable of, you might suggest they spend some extra time on it. It is not useful to demand completion or mastery before permitting work to begin on the next workbook in the series.

After the second workbook period, and after you have checked the results, you might want to send the workbooks home with students. This can be one way to keep parents informed of their child's progress. A letter to parents/guardians concerning workbooks might accompany the first workbook sent home (see Lesson W2 and Blackline W2[b]). You may want to record student performance on workbooks using the recording tools provided in the Blackline section and keep them for use at parent conferences and/or in student progress reports.

One story-workbook from the *CSMP* Library, *Clinton Street*, is used as the basis for three lessons in the Workbooks strand. A story-workbook has both the flavor of a storybook and the advantages of a workbook. In each lesson based on a story-workbook, students discuss the story in a collective activity and stop periodically to do some individual work.

Storybooks

There are 24 storybooks for young readers in the *Stories by Frédérique* collection. Each story is presented in a colorful newsprint booklet and can be used for individual or collective reading at school or at home. The stories are independent of each other and for the most part need no teacher explanation. The storybook titles and their intended age range are as follows:

Ages 5 to 8

The Playful Numbers
The Baby is Born
81 Roses
One Out of Seven
The Old Shoemaker
I Am a Very Happy Boy
The Little Dreamer
Two by Two
The Weird Story of 24
Where's My Nose?
The Happy Puppet
The Magic Box
Summer School in the Old Days

Ages 8 to 12

The Little Donkey
Singing Friends
Dancing Friends
I Am Not My Name
The Living Lines
The Square Trap
Nabu Wins an Award

Ages 10 to 14

A Hidden Treasure
A Valentine Mystery
Election in the Number World
A Very Strange Neighborhood

Among the seven storybooks intended for children ages 8 to 12, two are used as the basis of lessons in *IG-II*. Young people can become personally involved in these stories; there are questions to answer, pictures to relate to the story, information to be found in the pictures, and new adventures to invent for the characters in the stories. They find that in the world of numbers there are emotions to be expressed and aesthetic senses to be developed just as in the world of people. Your students may want to reread (or just look at) the storybooks many times after they have used them for lessons. Other storybooks (from the *Stories by Frédérique* collection or from the Appendix) can be made available for individual reading during free time. Some lessons suggest reading activities using other books as extensions or enhancements. See Appendix A for a list of all the books suggested in lessons and many others you might consider.

Role and Use of Calculators

Calculators, if used creatively, can have a remarkably beneficial effect on students' interest in and understanding of arithmetic, and can contribute to their developing capabilities. The reasons for this are many and varied, but the following are certainly among them:

- Students enjoy using calculators and respond with enthusiasm to most calculator activities.
- As a pedagogical tool, the calculator contributes to the development of mental arithmetic skills, number sense, and mathematical reasoning.
- As a context in which to pose problems, the calculator stimulates inquiry and helps students construct knowledge of mathematical concepts.
- By removing the burden of mechanical computation, calculators free students to focus on the thinking that is a necessary part of the solution to any meaningful problem.
- The instant feedback provided by the calculator enables students to explore patterns and make and test conjectures, both important areas that might otherwise be overwhelming.

Calculators are recommended for many lessons in *CSMP Mathematics for the Intermediate Grades*. Ideally your class should have access to at least one calculator for every two students and an overhead calculator that matches the students' calculators. If this is not possible, you may want to encourage students to bring calculators from home. Check that your classroom calculators or any calculators brought by students have the following features:

- **Chain operations:** This means that the calculator responds to instructions given in the order in which they are entered. To check that this is the case, press 4 + 1 × 2 =. If you have 10 on the display, then the calculator does the operations in order of entry (chain).
- **Constant mode:** Some calculators have a switch so that the calculator can be set in constant mode. If your calculator lacks a switch, it is quite likely that the calculator has an *automatic constant*. For most calculators with an automatic constant, the first number entered during multiplication is the constant, while for addition, subtraction, or division, the second number entered is the constant.

To check your calculator for the constant mode operation of [+], press [0] [+] [2] [=] [=] [=] and so on. If the number on the display increases by two at each depression of [=], then [+] is operating in a constant mode. Check the constant mode operation of [−] in a similar fashion.

To check the constant mode operation of [×] (in which the first number entered will be the constant), press [2] [×] [1] [=] [=] [=] and so on. If the number on the display doubles at each depression of [=], then [×] is operating in a constant mode. A few calculator models use the second number entered during multiplication as the constant. In this case, check the constant mode operation of [×] just as [+].

Generally, we suggest that you not attempt to teach the features of the calculator. Rather, let students discover how it operates through experiences and exploration. Before teaching a lesson, check that your students' calculators operate in the way described in the lesson. If they do not, you may need to make some adjustments. For some lessons, especially those involving calculator relations, calculators without an automatic constant are unsuitable.

Schedule of Lessons

The design and schedule of the lessons in this guide should be regarded as suggestions. We cannot predict which topics will be easy for your particular class and which lessons will require more or less time. Only you will be able to judge that as you teach the lessons. The program is not meant to be rigid or followed exactly; the schedule is a suggested guide.

The master schedule in Section Three is constructed as a one semester course for a fourth grade class. It illustrates some important ideas about organizing the program. You will notice that lessons from several different strands are scheduled each week, and that even within a strand the topics are interspersed. This carries out the idea of a spiral curriculum. If you organize your own schedule, it is best to follow a similar pattern rather than to spend two or three full weeks "covering" one particular topic such as composition of functions or negative numbers. In a usual five day week, two days should be devoted to the World of Numbers strand, one day to the Geometry and Measurement strand, one day to the Workbooks strand, and one day to be shared by the Languages of Strings and Arrows strand and the Probability and Statistics strand.

In the master schedule, a block represents one full period of approximately 45–60 minutes. It may be, however, that your class will spend more than one period or, possibly, less than one period on some lessons. Exercise your own professional judgement in deciding when to stop a particular lesson and when or whether to take it up again. Just be sure you do not insist that every single child understands every single aspect of a lesson before you go on. It is better to stop a lesson at a point when student interest is high and a problem remains unsolved than to teach so methodically that everyone in the class understands everything, but is completely bored.

In each block of the master schedule, the lesson of the day is designated first by strand letter and number. For example, N5 indicates that the day's lesson is the fifth lesson in the N (World of Numbers) strand. This identifying letter/number combination also appears in the upper outside corner of each page of the lesson, so you can find a lesson simply by flipping through the pages of a strand. The master schedule also gives you the lesson title and the page number on which it begins.

Please bear in mind that the model schedule was designed for a hypothetical class with the usual range of abilities. In order to use the program successfully, you should adjust the schedule to the ability, maturity, and interests of your class. For example, you may wish to extend a lesson or a workbook for an additional period.

Adjustment Days

The *IG-II* curriculum is designed for a "typical" school semester of 18 weeks and the schedule accordingly indicates 18 weeks of lessons. However, from time to time an adjustment day appears in the schedule. These days are included to help you account for the days when no mathematics lesson is taught due to parent-teacher conferences, field trips, snow days, and so on. They give you opportunities to do many kinds of things. For example, you can use an adjustment day to review a lesson you feel needs review, to complete an unfinished lesson, or to extend some interesting ideas from completed lessons. You can also use an adjustment day to give your students time to become more familiar with the manipulative materials in the program.

If your students especially enjoy playing games such as *Minicomputer Tug-of-War*, *Minicomputer Golf*, or *The String Game*, an adjustment day can give them another opportunity to play these games.

You may wish to use an adjustment day to do some mental arithmetic exercises or to teach a topic in mathematics you think would be especially interesting or useful to your class. For suggestions on arithmetic with your class, see page 1-13 and Appendix C.

Lesson Notes

You may like to begin working with students to take notes on their mathematics lessons. Math notes can be part of a portfolio or organized into an individual math notebook. Some types of notes students might make on a lesson include the following:

- Things to remember
- Examples
- Definitions
- Descriptions of games, manipulatives, tools, and so on
- Story Synopses
- Patterns
- Student-created problems

Getting students to verbalize and write about their thinking will help them make sense of the mathematics ideas they are learning. It will also help you assess what students understand and how they think about a concept. Further, you may be able to use students' writing and math notes to communicate with parents about what their children are doing and how their thinking/understanding is progressing.

If you ask students to keep math notebooks and periodically ask them to write notes, you will probably need to give them prompts such as a question or "Tell me about … ." There are several different forms in the Blackline section that you may provide to students for this purpose.

Different lessons will suggest different kinds of notes. For example, only a few lessons will have definitions to note, and some lessons will be obvious candidates for noting a number pattern. In some lessons there will be suggestions for specific writing activities that you may like to make part of the math notes.

Home Activities/Parent Communications

Parent involvement is built into *CSMP Mathematics for the Intermediate Grades* in several ways. Included in the *CSMP* Blacklines you will find a number of parent/guardian letters designed to make parents more aware of the *CSMP* languages, tools, and methods. These letters can advise parents about some types of mathematical activities their children are involved in. They can be photocopied and sent home at various points throughout the year.

In addition, suggestions for home activities frequently follow lessons. Home activities are opportunities for parents to work with their children in doing mathematics. Some are follow-up or practice for a lesson; others are for enrichment or extension. Reproducible descriptions of the home activities, specifically written to parents, are also included in the Blacklines.

Cooperative Groups

Students can be placed in pairs or groups of three or four, depending on the situation. A cooperative group gives students a chance to talk through what they are learning with other students. They get immediate help and support, and learn to care about the progress of every other member of the class. Cooperative groups give an opportunity for peer teaching. Students enjoy having a peer to discuss their ideas with and to give them assistance when something is difficult.

Many lessons give suggestions for using partners or groups in activities. Feel free to use these suggestions in your classroom. You may also want to use cooperative or other group activities in lessons where no such suggestions are given.

Centers

Many classrooms are organized with centers or stations where students work on activities without the direct assistance of the teacher. We offer these suggestions as possible ways that you might incorporate math centers into the regular classroom set-up, or might enhance and extend the math curriculum into center activities.

Exploration Center: Place materials introduced in previous lessons here and allow students to explore them without direction. Counters, pattern blocks, the Minicomputer, A-blocks, translators, geoboards, and so on might each take a turn in the center. You might also add other types of manipulatives, purchased or handmade, to support a lesson or concept.

Reading and Writing Mathematics Center: Activities here might include reading the *CSMP* Storybooks after they have been introduced, reading other books and stories about number concepts, and creative writing activities built around *CSMP* lessons.

Relating to Lessons Center: This center can include changing task cards and worksheet suggestions that follow a particular lesson and reinforce the ideas presented there. You will find suggestions for this type of center at the end of some lessons and you may wish to add others as the need arises in your particular class.

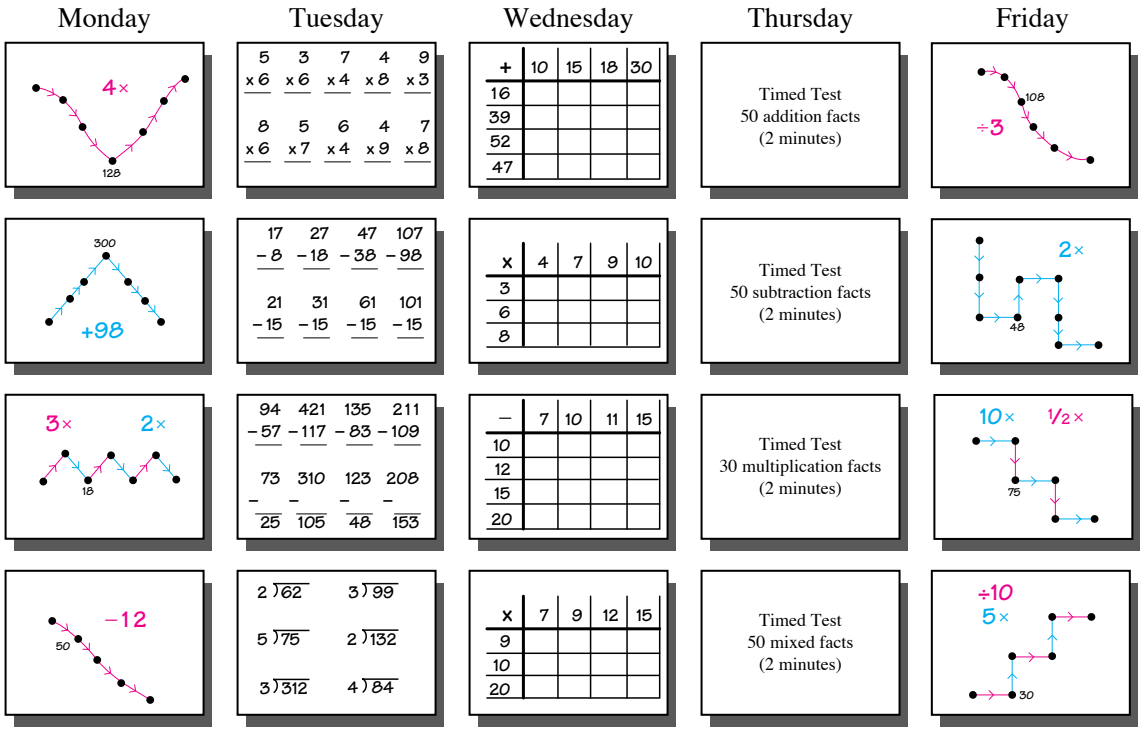
Centers may be set up so that students work alone or in groups. Many of the activities lend themselves to small groups of students working on their own but in close proximity to each other, sharing results or giving help to others as needed.

Mental Arithmetic and Supplementary Activities

Throughout the lessons there are many activities that children enjoy doing over and over in a variety of forms. For example, *The String Game* and *Minicomputer Golf* (to mention only two such activities) can be played whenever an extra time period is available. Of particular importance and enjoyment are the activities dealing with mental arithmetic; fully one-third of the lessons involve such activities. Other general suggestions about mental arithmetic are given in Appendix C. Teachers can invent clever variations of these mental arithmetic activities to be used often during a school day as adjuncts to the natural school routines. Some of the most obvious are checking class attendance (How many present? How many should be here? How many absent?); the opening exercises (What day of the month is it? How many days in this month? How many days left?); distributing materials (How many students? Three notecards for each. How many notecards?).

You might want to do some supplementary drill work on basic number facts. If so, try to keep such drill work to a minimum so as not to suggest to students that arithmetic is a chore. The following are suggestions for additional arithmetic practice that you may like to use regularly in your classroom.

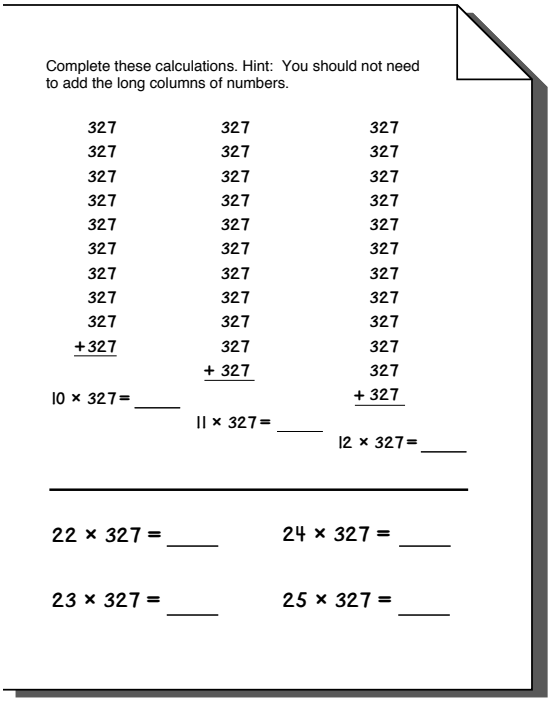
1. Use a warm-up activity every day as on this sample calendar.



2. Practice addition/subtraction facts using

   - games
   - mental arithmetic (see Appendix C)
   - calculator "concentration" or other activities (see Appendix C)
   - computer programs
   - parent help

3. After completing workbooks, create pages similar to computation pages for additional practice (possible homework). For example, the following page is similar to *Set of Problems #1* page 20 or *Set of Problems #2* page 18, and could be used after *IG-II* Lesson W2 or W6.



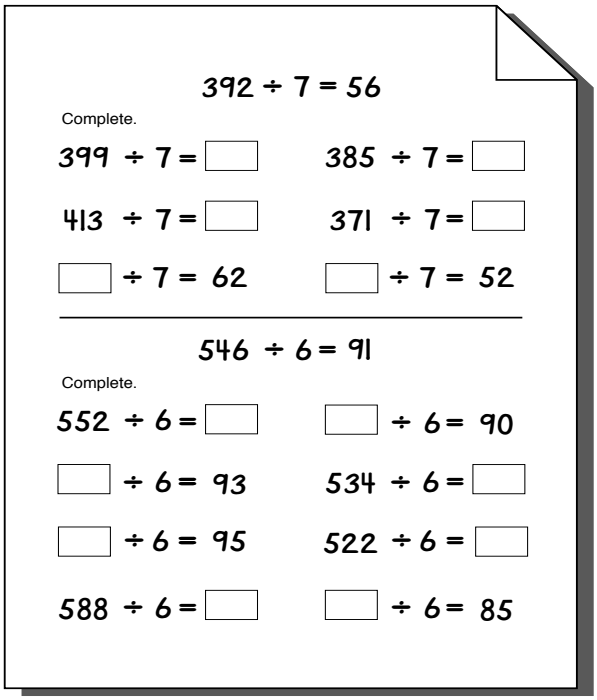
Complete these calculations. Hint: You should not need to add the long columns of numbers.

327	327	327
327	327	327
327	327	327
327	327	327
327	327	327
327	327	327
327	327	327
327	327	327
327	327	327
+327	327	327
	+ 327	+ 327
10 × 327 = ____	11 × 327 = ____	12 × 327 = ____

22 × 327 = ____ 24 × 327 = ____

23 × 327 = ____ 25 × 327 = ____

4. After lessons with computation, make additional worksheets with similar problems and include problems in traditional format. For example, the page on the right might be used after *IG-II* Lesson N24.



Assessment

Although there are no formal tests designed to be given at certain points in *CSMP Mathematics for the Intermediate Grades*, there are many suggestions for assessing student understanding and progress.

First, almost all lessons have some opportunity for class discussion or group interaction. Listen to what students say during these discussions or when they work in groups, and make mental notes. In addition, try to have individual conversations with students with whom you feel a need to get more information about what they understand.

Writing activities and mathematics notes done by the students will provide invaluable insights into their thinking and reasoning abilities.

Many lessons have worksheets to accompany them. You may want to select certain of these worksheets for assessment purposes.

The workbooks are perhaps the best written vehicle for monitoring student progress. They provide a variety of types of problems and presentations to review work covered in the lessons. They also give opportunities for students to extend their thinking.

See the section on "Role and Use of Worksheets, Workbooks, and Storybooks," Page 1–6, for more comments on these written materials for assessment.

Integration with Other Curriculum Areas

There are many opportunities in the daily classroom schedule to introduce and reinforce mathematics concepts with students. Some of these may be specifically planned by the teacher and some may arise incidentally in conversations and activities. Number recognition and counting will occur regularly, as will activities requiring looking at patterns, sorting, and using the vocabulary and concepts of mathematics. Literature that is read and shared in the class will suggest mathematical ideas that can be explored further. Books and stories can be found to support and reinforce mathematics topics. Looking for ways to integrate mathematics topics with other subject areas, and using concepts introduced in mathematics lessons to explore other topics will help students to see the relevance and applications of mathematics to their lives.

Mathematics activities do not just occur during the time designated for math in the daily schedule. They are a part of the total school day and occur at every turn in the curriculum. You may plan them, but they will also arise spontaneously as students work with each other. Taking attendance and lunch count, collecting money and book orders, going over the daily schedule and ordering the tasks of the day, playing games at recess, and getting to the next class on time are just a few of the math-related activities in which students participate every day. It is not necessary to always stop and point out the mathematics in a situation, but its presence provides the groundwork for concept understanding and problem solving in formal mathematics lessons.

Throughout the *CSMP* lessons we have tried to integrate other curriculum areas and to suggest ways you might do more within your own classroom. The content of mathematics is not something that students should see and use during an isolated period, but rather throughout the day. In the same way, mathematics classes should not deal only with numbers, but should incorporate elements of reading, writing, social studies, science, and so on. *CSMP* strives not only to involve all students in the learning of mathematical concepts, but to be part of an integrated school day in which students see the application of each subject to other subjects and to their own lives. Integration allows all students to participate and contribute to the total work of the class.

Section Two

Questions and Answers about CSMP

IG-II QUESTIONS AND ANSWERS ABOUT CSMP

As you teach the *CSMP* curriculum or even as you prepare to teach it, some basic questions will come to mind. They are, very likely, the same kinds of questions that parents and supervisors will ask about a curriculum that appears to be different from the traditional program. Perhaps some of the questions that concern you are answered here.

Q: What is the spiral approach? How does it work?

A: We learn through many interrelated experiences. But no experience, particularly a fourth grade math lesson, is an end in itself. With this in mind, *CSMP* introduces an idea in one lesson, then another in the next lesson, then another. Later, the first idea reappears in a slightly new setting or in a slightly expanded version, perhaps with connections to other concepts; then a second topic reappears; and so on. Each of the many times a particular idea or area of study reappears, it is treated in a different manner and at a slightly higher level than the last time. This is called *spiral development*.

Students learn at different times and at different rates. Only part of the learning of a given topic actually takes place during the classroom lesson. In between the times the topic occurs on the learning spiral, the student is mentally digesting the idea, sometimes consciously, usually unconsciously. In this way the spiral development gives each student a new chance to catch on at each stage.

We find it highly effective to follow this kind of development rather than continuing on and on with a topic until so-called mastery has occurred.

Sticking with the spiral approach requires an act of faith on your part, particularly if you are used to teaching a given topic until you are satisfied that all (or at least most) of the class knows it cold. The spiral approach implies that some lessons may be stopped before some (or even a majority of) students appear to have caught on, or before everyone has successfully completed all the worksheet or workbook pages. It involves knowing that the topic will reappear again and again, and that varying degrees of understanding will come at varying rates and times. It includes the belief that an idea planted now may not sprout until much later, and that it is best not to force its development.

Q: I understand how spiral development works, but how can I assess my students' progress using such an approach?

A: We don't believe it is desirable or reasonable to try to evaluate student mastery of skills from day to day or even from week to week. On the other hand, teachers, parents, and the students themselves benefit by some monitoring of the students' progress and involvement with the activities. Most teachers have their own methods for monitoring with which they are comfortable, and we suggest that you continue to use these methods. You will find that group interactions during the lessons, individual responses on worksheets and workbooks, and center and writing activities provide much valuable information. A spiral approach, in fact, may help rather than hinder your long-term assessment of students' progress.

Q: For what sort of student is the *CSMP* elementary curriculum designed?

A: It is designed for *all* students. Students engage with each learning situation at their own level so that they succeed in dealing with some aspects of the situation that challenges them. The pedagogy of situations (see the next question) is particularly effective with all types of learners.

Q: How does *CSMP* differ from a traditional mathematics curriculum?

A: The core content of *CSMP* is not revolutionary. The program aims to integrate ideas and facts that have always been considered important in mathematics. The major differences are in method. Traditional teaching generally focuses on the mastery of paper-and-pencil algorithms. *CSMP* students, on the other hand, become involved with real mathematical activities from which they develop mathematical understanding.

With a pedagogy of situations, they participate in the learning process which leads to the development of skills—not by rote memorization but by constructing their own understanding. Faced with challenging situations, the students are led by their reactions to important ideas in mathematics—they begin to mathematize situations. This process is fundamental whenever mathematics is applied.

CSMP also makes use of a spiral approach rather than the mastery approach found in most traditional mathematics curricula (see page 2-1).

Q: Are all the topics in the*CSMP* curriculum really mathematics?

A: Yes. Mathematics is much more than calculation with numbers, and its methods include more than paper-and-pencil algorithms. The *CSMP* curriculum is filled with experiences that directly and indirectly relate to a wide world of numbers, their interrelationships, and the kinds of logical reasoning about these relationships that make mathematics an activity of interest and concern.

Q: Why use the languages of strings and arrows?

A: It is surprising that the language of strings was not used earlier and more extensively to teach mathematics. Mathematicians have used strings for a very long time; they support classification notions so fundamental and natural that they go back to Aristotle. The scope of the string diagrams is broader than mathematics; it is a marvelous instrument for logical thinking. String diagrams permit the understanding of a situation at a glance; they generate math concepts and ideas through related discussion.

Arrow diagrams are probably the most important teaching aid in the *CSMP* curriculum. They are used everywhere in our daily lives: signals, advertisements, directions, and so on. Why not use them to teach mathematics? Arrows are a pictorial way to represent relations. Relations have always been important in mathematics, but in the past seemed to be something rather abstract, elusive, and mysterious. With arrows they become vivid, concrete, and easy to understand.

Q: Why use the Papy Minicomputer and calculators?

A: The Minicomputer allows very young students to become familiar with numbers and numerical operations (especially large numbers and more difficult calculations) before they are able to work with them on paper. Later, the Minicomputer is used as a vehicle for posing situations involving mental arithmetic and for examining the workings of arithmetic operations. It provides the necessary experience so that, rather than memorizing an algorithm and applying it to situations, the situations give rise to the algorithm.

The calculator can have a remarkably beneficial effect on students' interest in and understanding of arithmetic. In *CSMP*, the calculator is used as an instructional tool, not as a crutch. That is, with the calculator students develop mental arithmetic skills and number sense as well as the ability to construct their understandings of a variety of mathematical concepts. By removing the need to do messy calculations, the calculator frees students to think about problem solving and possibly tackle even more difficult problems.

Q: If students use the Minicomputer and a calculator, will they be able to calculate without them?

A: This question suggests that the only purpose of the Minicomputer and the calculator in the *CSMP* curriculum is to perform calculations. To be sure, the Minicomputer and the calculator do enable very young students to deal with large numbers and perform relatively difficult calculations long before they know paper-and-pencil algorithms. But very soon these experiences suggest how the paper-and-pencil algorithms come about, and from that moment the tools are no longer regarded as calculating devices. Later, usually by the second grade and for the remainder of the K–6 curriculum, the Minicomputer and the calculator assume their primary roles: sources of very interesting problems and games, effective vehicles for stimulating mental arithmetic, teaching devices for introducing more advanced topics such as the negative numbers and the decimal numbers, and, in general, alternatives to the usual paper/pencil representation of numbers.

Q: Why is there a lack of drill and little emphasis on skills training?

A: The purpose of drill in arithmetic is to acquire the skills of calculation. And while certain numerical skills *are* essential, the usual drill techniques are, unfortunately, dull, stultifying, and sometimes even counterproductive. The *CSMP* curriculum *appears* to place little emphasis on skills because it does not resort to drill. But in fact, the essential skills of both mental and written calculation are learned in the many situations where the ability to calculate is needed in order to respond to the situation: to win a number game, to identify the secret number of the detective story, or to build a number road. In *CSMP*, calculation is always considered a tool for doing interesting things, rather than a chore done for its own sake.

Q: Should I supplement the program with more practice in number facts?

A: There is a surprising amount of such practice throughout the program. Most is implicit in the activities and is not in the form of practice for its own sake. If you still feel it necessary to give supplementary practice, we suggest you keep repetitive drill to a minimum so as not to endanger students' interest in numerical situations. Suggestions for additional fact practice can be found in Section One pages 1-13 and 1-14.

Q: How do students learn to use mathematics in practical situations?

A: For elementary school students, a situation that intrigues them, catches their imagination, or piques their curiosity will motivate them to learn whatever is necessary in order to deal with the situation. But situations that adults consider "practical" very often are outside the realm of interest of young students and do not provide motivation for learning. *CSMP* has found that students will indeed learn to use mathematics in situations they consider worthy of their attention. As they approach adulthood, they will begin to consider practical situations worthy of their attention, but in the meantime they are more interested in games, fantasy, puzzles, stories, and so on.

Q: How are money and the metric system treated?

A: The topic of money is first introduced in the *CSMP* curriculum in situations involving combinations of coins. It is treated again in the upper primary grades and the intermediate grades as a support for the teaching of the decimal numbers. Although money is not treated as a separate topic, these situations provide experience with our monetary system. Should the students need more exposure to this topic, feel free to develop supplementary materials in

order to provide extra practice.

Metric units are used whenever there is cause to measure in standard units. We recommend that you do not ask the students to convert from the English to the metric system or vice versa (inches to centimeters, yards to meters, Celsius to Fahrenheit, grams to ounces, and so on). It can be confusing for students to deal with two systems simultaneously, especially at this early stage. Since the United States has not yet "gone metric," your students will still need to be familiar with the English system. We suggest that you use whatever materials and methods you have found successful in the past for teaching this topic, but we repeat our recommendation that you not ask the students to convert from one system to the other.

Q: Why are we teaching Probability and Statistics?

A: Next to arithmetic, probability and statistics have the most direct application to life in such areas as economics, meteorology, data processing, insurance, most of the natural sciences, and most of the social sciences. Probability differs from many areas of mathematics in that it is applied directly. One does not need to combine knowledge of probability with any deep theoretical knowledge in order to apply it to "real world" problems. One encounters statistics nearly everyday, and the better developed one's intuitive sense of probabilistic situations is, the less likely one is to be "duped" by statistics.

Q: What is the role of storybooks and story-workbooks in the curriculum?

A: The storybooks are part of a 24-storybook collection called *Stories by Frédérique*. Each is presented in a colorful newsprint booklet designed for young students. Two storybooks are used with lessons in *CSMP Mathematics for the Intermediate Grades, Parts I and II*. Each story is illustrated using the mathematical languages of dots, arrows, and strings to reinforce the story idea. The stories are independent of each other and need little external explanation. They may be used for individual or group reading at home or at school. With both storybooks and story-workbooks, young readers can become personally involved in the stories; there are questions to answer, pictures to relate to the story, and new adventures to invent for the characters. They find that emotions and aesthetics are part of the world of mathematics just as in the world of people.

Among the goals of the various storybooks and story-workbooks are the following: to familiarize students with mathematical languages, and to suggest that these languages are as natural and useful as the usual language; to introduce some mathematical games or puzzles; to present detective stories that involve interesting mathematical problems; and to provide reinforcement for all students.

Other lessons will include suggestions for reading activities using children's literature as an extension or enhancement, and an appendix has still further literature suggestions.

The story-workbooks, as the name implies, are a cross between storybooks and workbooks. They present workbook-like problems for students to solve in a story setting. There are 11 story-workbooks in the collection called *The CSMP Library* and four are used with lessons in *CSMP Mathematics for Intermediate Grades, Parts I and II* .

Q: Some of the *CSMP* materials are new to me. How do I learn enough to teach a class?

A: *CSMP* training is recommended for every teacher new to the program. This training may be conducted by a local *CSMP* coordinator or a certified *CSMP* trainer. Kindergarten and first grade teachers should receive a minimum of 12 hours training; teachers of the upper primary grades, a minimum of 24 hours training; and teachers of the intermediate grades, a minimum of 30 hours training. During the training, the *CSMP* content is taught to new teachers and suggestions are given for organization of the materials.

The *CSMP* trainer will present a variety of lessons taken from the *CSMP* curriculum and use an integrated spiral approach during the training itself. In this way teachers not only learn how to teach *CSMP* mathematics, but they become familiar with the spirit and pedagogy of the program.

Q: Are there goals for each lesson?

A: At the beginning of each lesson plan there is a *capsule lesson summary* that briefly describes the activities and experiences included in the lesson. Because the goals of the curriculum spread over a whole series of lessons, specific goals for individual lessons are stated only indirectly. If you read the introduction to the strand in which a lesson appears as well as the capsule lesson summary, we expect you will easily see the direction and intent of the lessons. However, we decline to predict what will happen in terms of student learning for any one lesson.

Q: Why are the lesson plans presented as a dialogue?

A: Most of the lessons are written in considerable detail; step-by-step descriptions are frequently given, together with a possible dialogue between teacher and students. These details are given only as suggestions of how a lesson might develop. You should never feel obligated to follow a lesson word for word as it is written in the guide. By the same token, you should never insist that your students give the same responses as those given in the imaginary dialogue. Use your own creativity to draw out student responses as the situations suggest. Read the lesson plans carefully beforehand to get an idea how the lessons are expected to proceed.

Q: How often and for how long should I teach mathematics in fourth grade?

A: The *CSMP* schedule suggests that you teach mathematics five days a week for 45 to 60 minutes each day.

Q: Is it necessary to follow the suggested schedule exactly?

A: This program is not meant to be a static sequence. The schedule of lessons is only a suggestion based on the teaching experiences of many teachers and *CSMP* staff. If this is your first time teaching the program, you may like specific suggestions for organizing the lessons, so we offer you the suggested schedule as a guideline. You may wish to follow it at the beginning of the school year and then deviate from it as the needs of your students dictate.

Exercise your own professional judgment in deciding when to stop a particular lesson. It is better to stop a lesson when student interest is high than to teach so methodically that everyone in the class understands everything, but is completely bored. In order to be successful with this curriculum, you should adjust the schedule to the ability, maturity, and interests of your class.

Q: The program makes use of the students' creative freedom. How much room is there for the teacher's ingenuity?

A: *CSMP* not only allows for your ingenuity but depends on it. While the lessons may appear to be rigid by presenting a dialogue for teachers and students, remember that these are only suggested questions, answers, and comments based on our experience with development classes. Even if you follow your part of the script, your students are likely to offer responses which differ from those we have listed. It is the students' responses that will determine the content and direction of a lesson.

In a traditional program your creativity is called upon the first time you develop a particular lesson, and to a lesser extent as you revise lessons in succeeding years. There is little room for spur-of-the-moment creativity once a lesson begins. The *CSMP* curriculum, by creating a free and open situation emphasizing the students' creative freedom, continually presents you with unanticipated student responses and calls on your ingenuity at all times.

Q: What are some ways to organize materials and manage my class?

A: It is very important to organize your *CSMP* materials before school begins and to decide where you will store them. File drawers, shelves, boxes, or closets are all satisfactory as long as you can easily get to the materials. Take them all out of their shipping boxes and spread them on several tables and/or desktops in your classroom so they can be organized into the order you will be using them.

Worksheets: You should have a worksheet booklet for each student. Decide whether you will store these booklets and pass them out when a particular lesson calls for worksheets or let the students be responsible for their own worksheet booklets.

Worksheets can be removed from the booklet or left in depending on how you wish to manage them. Once a worksheet is removed, however, it cannot be replaced easily, so make sure students are careful about tearing out the correct sheet.

Manipulatives: Storing all the manipulatives of a certain type in separate marked containers is one way to make them accessible, especially for use in lessons and centers. Another way is to provide each student with a sturdy box (for example, a shoe box) in which they keep their individual manipulative materials. Properly label each box with the student's name.

Additional Materials: Be sure that you have plenty of high-quality colored chalk and large sheets of unlined white paper. Your students should have colored pencils, pens, or crayons. Plan well ahead to be sure you have any needed materials that are not supplied by *CSMP*. Capsule lesson summaries give notice about such materials. Keep all teacher materials where they are easily accessible. The demonstration Minicomputer boards, and the number line should always be on display in the room.

Q: I prefer teaching small groups, but many of the lessons seem to be aimed at the whole group. What do you suggest?

A: If your school routine allows, you might try teaching a lesson twice, once to each half of your class. But if you do this, it might be wise to vary the composition of the groups each day so that you are not tempted to group students according to your perception of their ability. Always grouping by ability may guarantee that half your students have less experience than the other half, through no fault of their own. The purpose of the full group experiences is to allow interaction among all students in every situation posed in the lessons. Students learn quite a

bit from each other's reactions. You will be surprised to find that in some situations the key responses come from students you least expect.

Our answer might give the impression that *CSMP* teaching takes place only in full group settings. This is not the case. There are many opportunities to group students in a variety of ways—partners, small cooperative groups, teams, and so on. In addition, there is time allotted for individual work on worksheets and workbooks. We also encourage you to arrange small group sessions whenever you have time, particularly for students who need extra practice. Such small group activities may supplement the full group lessons, but should not replace them.

Q: What provision is made for individualization in the *CSMP* program?

A: Many of the fourth grade lessons contain worksheets to be completed individually by students. Sometimes the worksheets are done during the course of a lesson with a full or small group discussion of them. At other times a topic is first developed in a group setting, followed by individual student work on related worksheets. This allows students to work at their individual rates and levels. A series of worksheets for a given lesson offers a spectrum of problems from very simple to quite challenging. All students start with the one-star worksheet and proceed at their own pace as far as they are capable. The spiral approach accommodates individual work in the same way as full group work; that is, no single worksheet is a prerequisite for the content of any succeeding lesson.

During the school year, 20 lessons are based on the 10 fourth grade workbooks. Like the worksheets, each workbook moves through a range of levels of difficulty. We believe that the worksheets and workbooks realize the best sense of individualization.

Q: What provisions are made for the variety of abilities in a typical class?

A: Every lesson includes something that allows every student to participate in a meaningful way, regardless of differing abilities. Even your brightest students will have challenges. This means that you should not exclude any students from a full group lesson, but at the same time, you should not expect every student to fully understand everything in a lesson either. You will find that the spiral development leaves many situations open-ended, allowing students to experience them at different levels.

Evaluation has shown that students traditionally labeled "below grade level" do better in *CSMP* classes than in non-*CSMP* classes. We believe this is due to several things: the spiral development, group work on mathematical activities, and the presence of a wide variety of both numerical and non-numerical activities in the *CSMP* curriculum.

Additionally, *CSMP* provides for a range of ability levels in the student materials. Worksheets and workbooks offer problems from very easy to quite challenging. Indeed, some of the multi-starred worksheets and pages toward the end of a workbook may be accessible only to your most advanced students.

In an extreme case, you might consider the possibility of arranging for exceptionally bright students to take their math course with a *CSMP* fifth grade class.

Q: How do I handle students who have an extended absence or students, new to *CSMP*, who transfer into my class during the year?

A: The spiral approach allows a student to miss one or two weeks of school without needing much catch-up work. When the student returns, either you or classmates can help with any important material that was missed.

Use your best judgment in sending worksheets and/or workbooks home to absent students. Without participating in the collective lessons that introduce the worksheets and workbooks, it may be difficult for a student to proceed.

If several months of classes have been missed, you might need to treat the student as you would a transfer student new to *CSMP*. Give the student a little extra attention during a lesson, especially when the questions are review for most of the class.

In most cases, the differences between new and veteran students tend to fade quickly. However, if the absence or transfer occurs later in the year, lessons involving the Minicomputer might be an exception. Veteran students who are adept with the Minicomputer are often good tutors for these students.

Q: How do I handle whole classes who are new to CSMP?

A: *CSMP* does not assume that kindergarten and first grade students have any prior experience with *CSMP*. The kindergarten and first grade programs will need no adaptation for classes who are new to *CSMP*.

Second grade classes that are new to *CSMP* may initially progress more slowly than veteran classes. The lessons at the beginning of the second grade program are designed to serve either as an introduction to *CSMP* languages for new classes or as a quick review for veteran classes.

Third or fourth grade classes new to *CSMP* will need to become familiar with the *CSMP* languages and tools before beginning the lessons in *UPG-III* or *IG-I*. A special guide which consists of prerequisite lessons from earlier levels followed by a modified *UPG-III/IV* or *IG-I/II* program is available for these classes. This guide, *CSMP Third Grade Entry Supplement* or *CSMP Intermediate Grades Entry Supplement*, is available through McREL.

Q: What should I do with my class if they did not finish the previous year's schedule?

A: No special measures are required. Each level of the *CSMP* curriculum begins with introductory lessons. In addition, the spiral approach in the lessons permits students to participate at their own level of understanding, even if that level is not quite as high as it might have been if the previous year's schedule had been completed.

Of course, there may be some exceptions. For example, if the previous year's schedule completely omitted a strand or a sequence of related lessons (such as on The String Game), then you may want to start this year with introductory lessons from the Entry Supplement for that strand or lesson sequence.

Q: What should be done when there is a substitute teacher?

A: Some school systems have included individuals who frequently substitute in their *CSMP* training. When this has not been done, it may be difficult for the untrained substitute to teach from *CSMP* materials. One solution is to arrange for another *CSMP* teacher to teach your math class while the substitute teaches another subject to the *CSMP* teacher's class. If you will only miss one or two days, you may find the substitute can manage certain *CSMP* material quite well, for example, workbooks. Otherwise, let the substitute use your math period to cover material not included in the *CSMP* curriculum, such as English measurement. If your absence will be

an extended one, arrangements should be made with your coordinator to prepare a substitute, rearrange schedules to put a *CSMP* teacher in your class, or otherwise provide for your students.

Q: How do *CSMP* students do on standardized tests?

A: While the *CSMP* philosophy differs from the philosophy underlying standardized testing, test results are available to us from several sources. The results indicate that *CSMP* classes generally do as well or better on standardized tests than non-*CSMP* classes. It should be pointed out that standardized testing and traditional programs are correlated, and that *CSMP* students are exposed to a great deal of mathematics that is not taught in more traditional programs or that is not considered in the construction of the tests. There are also many immeasurable or unmeasured areas such as enjoyment of mathematics, strategic thinking, organizational ability, and depth of understanding of the methods and uses of mathematics in which we feel *CSMP* students have a definite advantage. Evaluation Report 7-A-1 in the *CSMP Evaluation Report Series* gives a more detailed answer to this question. Copies are available upon request.

Q: The non-verbal languages and instructional tools avoid the use of some standard terminology. Can I include this terminology in my teaching of the program?

A: The use of the pictorial languages and instructional tools do make some of the terms used in other mathematics programs unnecessary (*quotient*, *minuend*, *set*, *intersection*, and *factor*, to name a few). If you wish to introduce your students to this terminology, try to do so in a natural way by occasionally using it in your teaching. For example, sometimes refer to a string as a set, or to divisors as factors. Be careful not to place too much emphasis upon terminology, especially before the students have had the necessary prerequisite experience with the ideas.

Q: What problems will *CSMP* students have if they return to a more traditional program?

A: There should be no problems for *CSMP* students who return to a traditional program. Test results indicate that *CSMP* students are at or above the level of non-*CSMP* students in terms of material covered in traditional courses. In addition, *CSMP* students will have encountered many situations that are not introduced in traditional programs nor tested for on standardized tests. In particular, their reasoning ability should be much sharper than their non-*CSMP* counterparts.

There are, of course, differences in timing as to when different programs introduce the standard algorithms of arithmetic, and often *CSMP*'s introduction occurs later than in some of the other programs. The reason for this is our belief that it is essential for students to construct their own understandings of arithmetic and that this not be impaired by a premature emphasis on the standard algorithms for adding, subtracting, multiplying, and dividing. We believe that computational difficulties experienced by many elementary and junior high school students is the result of their spending an inordinate amount of time trying to master the algorithms too early in their mathematical education. This shuts students off from other experiences necessary for their mathematical growth. Students need plenty of time to explore the world of numbers. Mastering an algorithm should be the last step of this exploration, a sort of synthesis of previous experience. Furthermore, as calculators are available to nearly everyone, algorithmic mastery as a prerequisite to other numerical experiences is less important.

CSMP students usually can solve a problem before having learned a standard algorithm. If they enter a program that expects an algorithm they have not learned, they are still more than adequately prepared for the algorithm and should have no trouble quickly learning to use it.

CSMP introduces the usual addition algorithm in second grade, a standard subtraction algorithm in third grade, the multiplication algorithm in fourth grade, and a division algorithm in fifth grade.

Q: How can I explain *CSMP* to parents?

A: The best way is for you or your *CSMP* coordinator to organize a workshop for interested parents. Parents can then see the languages of the program in use and have some of the same experiences their children are having. Workshops can be of almost any length, but anything short of two hours does not allow participants to gain much familiarity with the material.

In such a workshop, we suggest you demonstrate the Minicomputer and the calculator with particular emphasis on their uses in presenting situations that call for mental arithmetic, estimation, and number sense. A non-numerical example, such as *The String Game* with A-blocks, provides a good example of the power of the language of strings. A numerical example, such as building a road from 2 to 23 using +5 and –2 arrows, provides a nice introduction to the language of arrows. Favorite lessons of your students' are excellent choices to present to parents as well.

Throughout the year there are opportunities to communicate with and involve parents in the math program. The curriculum materials include numerous letters to parents that can provide more introduction to the *CSMP* languages, tools, and methods. Home activities accompany many lessons and are opportunities for parents to become involved with their children in learning mathematics. Sometimes they suggest follow-up or practice, and other times enrichment or extension of a class lesson.

You might also extend an invitation to parents to visit your class during the math period. Many times such an invitation leads to a closer parent-teacher relationship or to the parent volunteering as an aide.

If you believe some parents would appreciate a more detailed and sophisticated discussion of *CSMP,* the following letter may serve that purpose. It can be reproduced and sent home with students.

Dear Parent/Guardian:

Your child is participating in the *Comprehensive School Mathematics Program (CSMP)*. While the program cannot be fully explained in a brief note, we hope this letter will give you some feeling for the substance of your child's mathematics education.

The goals of the *CSMP* K–6 program include the development of the standard number systems and their operations (including an early introduction to integers, rational numbers, and decimal numbers) interwoven with the development of geometry and measurement. The *CSMP* program also includes a strong component of probability, statistics, and combinatorics. Powerful pedagogical devices are used, such as several nonverbal languages and instructional tools. These make the unifying ideas of the program accessible to very young children.

The Language of Strings: This language mimics the basic methods of collecting and classifying data. It deals with the fundamentally useful and important mathematical notion of sets.

The Language of Arrows: This language models the process of comparing and analyzing sets and performing operations on them. That is, it deals with the mathematical notions of relations and functions including, but not restricted to, ordering relations and the numerical processes of adding to, subtracting from, multiplying by, and sharing equally among.

The Papy Minicomputer: The Papy Minicomputer, an extremely effective abacus invented by Georges Papy, models the positional structure of our number system and lends itself to a multitude of numerical algorithms, including the standard algorithms for the basic numerical operations.

The Calculator: The calculator serves as an instructional tool to help students develop mental arithmetic skills and number sense as well as to focus their attention on understanding and problem solving.

Among other things, these languages and tools allow students to develop an understanding of the basic operations on numbers in a natural way based on previous experiences. They also provide a context in which situations arise that call for the repeated use of arithmetic operations without resorting to drill exercises, which most students find boring.

The content is learned in an atmosphere of constant application using a variety of situations. These vary from simple stories in the early primary grades to challenging applications and nontrivial simulations of real world problems in the upper grades.

One of the tenets of the *CSMP* philosophy is that mathematics is a unified whole and should be learned as such. The *CSMP* content is completely sequenced in spiral form so that students come into contact with each area of mathematics continuously throughout the program. This avoids atomizing the content and requiring mastery of each bit before continuing to the next. Rather, students have repeated exposures to the content, building interlocking experiences of increasing sophistication.

Another tenet of the *CSMP* philosophy, based on years of experimentation in the classroom, is that no single method of classroom management meets the needs of every student. Hence, the program is constructed to allow numerous opportunities for whole class participation, small group cooperative interactions, and independent experiences. Individualization is achieved through independent work on workbooks and worksheets of varying levels of difficulty. Due to the spiral nature of the curriculum, students can work in their workbooks without losing touch with the progress of the class.

(over)

CSMP does not prescribe a rigid system of student assessment and progress reporting. It allows the teacher to use classroom interactions, observations of individual and group work, writing activities,

and workbooks and worksheets to best advantage for evaluating and reporting student progress. While standardized tests indicate that *CSMP* students develop the basic skills as early or earlier than non-*CSMP* students, we hope you will not be obsessed with your child's development of these skills. Each child is an individual and should not be judged against norms but rather on what he or she can and does accomplish. Many extremely important areas—developing self-confidence, learning to think and reason, being able to pose and solve non-routine problems—are difficult to measure and grade. If you have any questions concerning your child's progress, the classroom teacher is in the best position to answer those questions.

THE *CSMP* STAFF

Section Three

Suggested Schedule of Lessons

Master Schedule for CSMP Mathematics for IG-II

Week	World of Numbers	Languages of Strings & Arrows / Probability & Statistics	World of Numbers	Workbooks	Geometry and Measurement
1	N1 Division #1 N-9	L1 String Game with Numbers #1 L-5	N2 New Checkers on the Minicomputer N-17	W1 Set of Problems #1 (Lesson One) W-5	G1 Constructing Zigzags G-5
2	N3 Decimals #1 N-21	L2 How Many Designs L-9	N4 The Functions 10x and ÷10 N-25	W2 Set of Problems #1 (Lesson Two) W-7	G2 Area #1 G-9
3	N5 Fractions on the Number Line #1 N-29	P1 Probability Applications P-3	N6 Guess My Rule #1 N-35	W3 Not Too Close #1 W-17	G3 Area #2 G-15
4	N7 Subtraction #1 N-39	L3 String Game with Numbers #2 L-17	N8 Minicomputer Golf #1 N-43	W4 Not Too Close #2 W-23	Adjustment Day
5	N9 Positive Divisors N-47	L4 LogicalThinking #1 L-21	N10 Composites of □x	W5 Set of Problems #2 (Lesson One) W-29	G4 ParallelProjection #1 G-21
6	N11 Who is Zip? N-57	P2 Graphsin Football and in Geography	N12 Division #2 N-61	W6 Set of Problems #2 (Lesson Two) W-31	G5 ParallelProjetion #2 G-27
7	N13 Multiplication Algorithm #1 N-67	L5 String Game with Numbers #3 L-27	N14 Calculator Relations #1 N-75	Adjustment Day	G6 ParallelProjection #3 G-31
8	N15 Multiplication with Decimals N-81	P3 The Square Trap #1 P-13	N16 Fractions on the Number Line #2 N-87	W7 Set of Problems #3 (Lesson One) W-41	G7 Area #3 G-35
9	N17 Guess My Rule #2 N-93	P4 The Square Trap #2 P-17	N18 Multiplication Algorithm #2 N-97	W8 Set of Problems #3 (Lesson Two) W-43	G8 Area #4 G-39

Week	World of Numbers	Languages of Strings & Arrows / Probability & Statistics	World of Numbers	Workbooks	Geometry and Measurement
10	N19 Fractions #1 N-101	L6 How Many Roads? L-31	N20 Decimals #2 N-107	Adjustment Day	G9 ParallelProjection #4 G-45
11	N21 Subtraction #2 N-113	P5 Distributing Gifts P-19	N22 Decimals #3 N-115	W9 Clinton Street (Lesson One) W-53	G10 ParallelProjection #5 G-49
12	N23 Fractions #2 N-121	L7 String Game Analysis #1 L-35	N24 Division #3 N-127	W10 Clinton Street (Lesson Two) W-57	Adjustment Day
13	N25 Multiplication Algorithm #3 N-135	P6 Games of Chance #1 P-27	N26 Minicomputer Golf #2 N-139	W11 Clinton Street (Lesson Three) W-61	G11 Shadows G-55
14	N27 Calculator Relations #2 N-141	P7 Games of Chance #2 P-35	N28 Which Coins? N-147	W12 Set of Problems #4 (Lesson One) W-69	Adjustment Day
15	N29 Calculator Relations #3 N-153	L8 LogicalThinking #2 L-41	N30 Fractions on the Number Line #3 N-157	W13 Set of Problems #4 (Lesson Two) W-71	G12 Estimation and Error in Measurement G-57
16	N31 Binary Abacus N-163	L9 String Game Analysis #2 L-45	N32 Telephone Game #1 N-171	W14 Nabu Wins an Award W-81	Adjustment Day
17	N33 Telephone Game #2 N-177	P8 Letter Distribution #1 P-43	N34 Calculator Relations #4 N-181	W15 Set of Problems #5 (Lesson One) W-85	P9 Letter Distribution #2 P-47
18	N35 Minicomputer Golf #3 N-187	L10 String Game Analysis #3 L-49	Adjustment Day	W16 Set of Problems #5 (Lesson Two) W-87	G13 Networks G-61

Section Four

Lesson Guide