Appendix

Number Concepts

Abacus: A Pocket Computer, The by Jesse Dilson
Base Five by David A. Adler
Binary Numbers by Clyde Watson
Counting by Henry Pluckrose
Count Your Way Through … (series highlighting different countries) by Jim Haskins
Day the Numbers Disappeared, The by Leonard Simon and Jeanne Bendick
First Number Book, A by Shari Robinson
How Did Numbers Begin? by Mindel and Harry Sitomer
How Much Is a Million? by David M. Schwartz
How to Count like a Martian by Glory St. John
How to Count Sheep without Falling Asleep by Ralph Leighton and Carl Feynman
I Can Count the Petals of a Flower by John and Stacey Wahl
I've Got Your Number, John by Olive S. Berg
King's Commissioners, The by Aileen Friedman
Knots on a Counting Rope by Bill Martin Jr. and John Archambault
Less Than Nothing Is Really Something by Robert Froman
Lucy and Tom's 1, 2, 3 by Shirley Hughes
Magic House of Numbers by Irving Adler
Math Puzzles by Irving and Peggy Adler
Math for Smarty Pants by Marilyn Burns
Millionth Egg, The by Bernice Myers
Mouse Count! by Felicia Law and Suzanne Chandler
Number Art: Thirteen 1 2 3's from Around the World by Leonard Everett Fisher
Number Families by Jane Jonas Srivastava
Numbers by Philip Carona
Numbers by Kate Petty
One Is Unique by Marnie Luce
Roman Numerals by David A. Adler
Sesame Street One, Two, Three Storybook by Emily Kingsley et al.
Solomon Grundy, Born on Oneday: A Finite Arithmetic Puzzle by Malcolm E. Weiss
Twelve Days of Christmas, The by Jan Brett
Twelve Days of Christmas, The by Jack Kent
Twelve Days of Christmas, The by June Williams
Wacky Wednesday by Theo Le Sieg
Zero: Is It Something? Is It Nothing? by Claudia Zaslavsky
Zero Is Not Nothing by Mindel and Harry Sitomer
Zero Is Something by Marnie Luce

Fractions

Black and White by David Macauley
Dad's Diet by Barbara Comber
Earrings! by Judith Viorst
Ed Emberley's Picture Pie by Ed Emberley
Fractions Are Parts of Things by Richard J. Dennis
Gator Pie by Louise Mathews
Half-Birthday Party, The by Charlotte Pomerantz
Magic School Bus at the Waterworks, The by Joanna Cole
Nobody Stole the Pie by Sonia Levitin

Pezzettino by Leo Lionni
Phantom Tollbooth, The by Norton Juster
Piemakers, The by Helen Cresswell
Really Eager and the Glorious Watermelon Contest by Richard Cheney
Tom Fox and the Apple Pie by Clyde Watson

Operations

Anno's Mysterious Multiplying Jar by Masaichiro and Mitsumasa Anno
Building Tables on Tables: A Book about Multiplication by John V. Trivett
Bunches and Bunches of Bunnies by Louise Mathews
Calculator Fun by David A. Adler
Calculators by Jan P. Haney
Doorbell Rang, The by Pat Hutchins
Grain of Rice, A by Helena C. Pittman
Greatest Guessing Game: A Book about Dividing, The by Robert Froman
Great Take-Away, The by Louise Mathews
Harriet's Halloween Candy by Nancy Carlson
Hilton Hen House, The by Jo Hinchcliffe
How Big Is the Moon? by D. Baker, C. Semple, and T. Snead
King Kaid of India published by The Victorian Readers
King's Chessboard, The by David Birch
Melisande by E. Nesbit
Moira's Birthday by Robert Munsch
Phoebe and the Hot Water Bottles by Terry Furchgott and Linda Dawson
Rajah's Rice, The by David Barry
17 Kings and 42 Elephants by Margaret Mahy
666 Jellybeans! All That? An Introduction to Algebra by Malcolm C. Weiss
Where Did My Little Fox Go? by Nancy Robinson

Patterns and Relationships

Anno's Math Games by Mitsumasa Anno
Anno's Math Games II by Mitsumasa Anno
Clue in Code, A by Marilyn Singer
Game of Functions, A by Robert Froman
Graph Games by Frédérique and Papy
Hailstones and Halibut Bones by Mary O'Neill
I Hate Mathematics! Book, The by Marilyn Burns
If I Were in Charge of the World and Other Worries by Judith Viorst
MatheMagic: Magic, Puzzles, and Games with Numbers by Royal Vale Heath
Math for Smarty Pants by Marilyn Burns
Mirror, Mirror by Rosemary and Calvin Irons
Number Families by Jane Jonas Srivastava
Number Ideas through Pictures by Mannis Charosh
Number Mysteries by Cyril and Dympna Hayes
Number Patterns Make Sense. A Wise Owl Book by Howard Fehr
Pattern by Henry Pluckrose
Polka Dots, Checks, and Stripes by Carol Cornelius
Puzzlooney by Russell Ginns
Sea Witches by Joanne Robertson and Laszlo Gal

Sneaky Square and 113 Other Math Activities for Kids, The by Richard M. Sharp and Seymour Metzner
Too Hot to Hoot by Marvin Terban
Visual Magic by David Thomson

Geometry

ABC's of Origami by Claude Sarasas
*Angles Are Easy as Pie b*y Robert Froman
Anno's Math Games III by Mitsumasa Anno
Another, Another, Another, and More by Marion Walter
Boy with Square Eyes, The by Juliet and Charles Snape
Circles by Mindel and Harry Sitomer
Cloak for the Dreamer, A by Aileen Friedman
Ellipse by Mannis Charosh
Exploring Triangles: Paper-Folding Geometry by Jo Phillips
Great Optical Illusions, The by Gyles Brandreth
Greedy Triangle, The by Marilyn Burns
Hexagrams by Kenneth Saunders
In Shadowland by Mitsumasa Anno
Keeping Quilt, The by Patricia Polacco
Lines by Philip Yenawine
Lines, Segments, Polygons by Mindel and Harry Sitomer
Listen to a Shape by Marcia Brown
Logo Fun by Pat Parker and Teresa Kennedy
Look at Annette by Marion Walter
Make a Bigger Puddle, Make a Smaller Worm by Marion Walter
Maps, Tracks, and the Bridges of Königsberg: A Book about Networks by Michael Holt
Mirror Puzzle Book, The by Marion Walter
Mirrorstone, The by Palin
Optical Illusions by Bill Martin, Jr.
Optical Illusions by Laurence B. White and Ray Broekel
Opt: An Illusionary Tale by Arlene and Joseph Baum
Origami in the Classroom by Chiyo Araki
Origami, Japanese Paper Folding by Florence Sakade
Paper John by David Small
Purpose of Forms, The by Eric Laithwaite
Puzzle Maps U.S.A. by Nancy Clouse
Quilt, The by Ann Jonas
Reflections by Ann Jonas
Right Angles: Paper-Folding Geometry by Jo Phillips
Round Trip by Ann Jonas
Rubber Bands, Baseballs, and Doughnuts. A Book about Topology by Robert Froman
Shadow Geometry by Daphne H. Trivett
Shape by Henry Pluckrose
Shapes by Philip Yenawine
Shape: The Purpose of Forms by Eric Laithwaite
Simple Science Experiments with Circles by Eiji and Masako Orii
Spirals by Mindel and Harry Sitomer
Stop and Look! Illusions by Robyn Supraner

Straight Lines, Parallel Lines, Perpendicular Lines by Mannis Charosh
String, Straightedge, and Shadow: The Story of Geometry by Julia E. Diggins
Take Another Look by Edward Carini
Tangrams: Picture-Making Puzzle Game by Peter Van Note
Thirteen by Renny Charlip and Jerry Joyner
Three-D, Two-D, One-D by David A. Adler
Topsy-Turvies: Pictures to Stretch the Imagination by Mitsumasa Anno
Turn About, Think About, Look About Book, The by Beau Gardner
Upside-Downers by Mitsumasa Anno
Village of Round and Square Houses, The by Ann Grifalconi
What Can She Be? An Architect by Gloria and Esther Goldreich
What Is Symmetry by Mindel and Harry Sitomer
Where Is It? A Hide-and-Seek Puzzle Book by Demi
Wing on a Flea: A Book about Shapes, The by Ed Emberley
World's Best Optical Illusions by Charles H. Paraquin

Measurement

Area by Jane Jonas Srivastava
Bigger and Smaller by Robert Froman
Borrowers, The by Mary Norton
Capacity by Henry Pluckrose
Diary of a Church Mouse by Graham Oakley
8,000 Stones by Diane Wolkstein
How Big Is a Foot? by Rolf Myllar
How Big Is Big? by Herman and Nina Schneider
How Little and How Much: A Book about Scales by Franklyn M. Branley
How Much Is a Million? by David M. Schwartz
How Tall Was Milton? by Lowery
King's Flower, The by Mitsumasa Anno
Length by Henry Pluckrose
Let's Find Out about What's Big and What's Small by Martha and Charles Shapp
Let's Find Out about What's Light and What's Heavy by Martha and Charles Shapp
Let's Talk about the Metric System by Joyce Lamm
Liter, The by William J. Shimek
Long, Short, High, Low, Thin, Wide by James Fey
Magic School Bus Inside the Earth, The by Joanna Cole
Magic School Bus Inside the Human Body, The by Joanna Cole
Making Metric Measurements by Neil Ardley
Measurements: Fun, Facts, and Activities by Caroline Arnold
Measure with Metric by Franklyn M. Branley
Melisande by E. Nesbit
Metric Puzzles by Peggy and Irving Adler
Popcorn Book, The by Tomie de Paola
Rules of Thumb by Tom Parker
Shrinking of Treehorn, The by Florence Heide
Size: The Measure of Things by Eric Laithwaite
Sizes by Gillian Youldon
Something Absolutely Enormous by Margaret Wild
Spaces, Shapes, and Sizes by Jane Jonas Srivastava

Temperature and You by Betsy Maestro
Think Metric! by Franklyn M. Branley
Weighing and Balancing by Jane Jonas Srivastava
Your Amazing Body by Jeanne K. Hanson

Money

Alexander Who Used to Be Rich Last Sunday by Judith Viorst
Alice and the Boa Constrictor by Laurie Adams and Allison Coudert
All Kinds of Money by David A. Adler
Chair for My Mother, A by Vera Williams
Dollars and Cents for Harriet by Betsy and Giulio Maestro
Hundred Penny Box, The by Sharon Bell Mathis
If You Made a Million by David M. Schwartz
Jason and the Money Tree by Sonia Levitin
Kids' Complete Guide to Money, The by Kathy S. Kyte
Managing Your Money by Elizabeth James and Carol Barkin
Money by Audrey Briers
Money by Benjamin Elkin
Money and Banking by Lois Cantwell
$1.00 Word Riddle Book, The by Marilyn Burns and Martha Weston
Toby's Millions by Morris Lurie
Toothpaste Millionaire, The by Jean Merrill
Turtle Street Trading Co., The by Jill Ross Klevin
What's Cooking, Jenny Archer? by Ellen Conford

Time

All in a Day by Mitsumasa Anno
Anno's Sundial by Mitsumasa Anno
Calendar Art: Thirteen Days, Weeks, Months, Years from Around the World by Leonard E. Fisher
Calendars by Necia H. Apfel
Chicken Soup with Rice: A Book of Months by Maurice Sendak
How Did We Get Clocks and Calendars? by Susan Perry
One Hand at a Time by Patricia E. Smith
Story of Our Calendar by Ruth Brindze
Ten Cuckoo-Clock Cuckoo by Annegert Füchshuber
This Book Is about Time by Marilyn Burns
Time! by Jane Edmonds and Mark Sachner
Time by Werner Kirst
Time by Henry Pluckrose
Time by Elizabeth Thompson and Feenie Ziner
Time and Clocks by Herta S. Breiter
Time Book, The by John Cassidy
Time for Clocks by Daphne and John Trivett
Time for Horatio by Penelope Colville Paine
Time in Your Life by Irving Adler
What Time Is It Around the World? by Hans Baumann
Why Rat Comes First: A Story of the Chinese Zodiac by Clara Yen
Wise Owl's Time Book by Jane Belk Moncure

Classification and Logic

Alice in Puzzleland by R. Smulleyman
Anno's Hat Tricks by Akikiro Nozaki and Mitsumasa Anno
Beach Ball by Peter Sis
Board Games Round the World by Robbie Bell and Michael Cornelius
Book of Classic Board Games, The by Sid Sackson
Bread, Bread, Bread by Ann Morris
Bruno Brontosaurus by Nicole Rubel
Domino Games by John Belton and Joella Cramblit
Frog and Toad are Friends by Arnold Lobel
Hats, Hats, Hats by Ann Morris
Jumanji by Chris VanAllsburg
Logic for Space Age Kids by Lyn McClure Butrick
Mathematical Games for One and Two by Mannis Charosh
Mathnet™ Casebooks by David D. Connell and Jim Thurman
More Sideways Arithmetic from Wayside School by Louis Sacher
Odds and Evens by Thomas O'Brien
Perplexing Puzzles and Tantalizing Teasers by Martin Gardner
Pet for Mrs. Arbuckle, A by Gwenda Smyth
Puzzles from Other Worlds by Martin Gardner
Sideways Arithmetic from Wayside School by Louis Sachar
Tic Tac Toe by Claudia Zaslavsky
263 Brain Busters: Just How Smart Are You, Anyway? by Louis Phillips
Venn Diagrams by Robert Froman
Yes-No; Stop-Go: Some Patterns in Mathematical Logic by Judith L. Gersting and Joseph E. Kuczkowski

Statistics and Probability

Averages by Jane Jonas Srivastava
BASIC Fun: Computer Games, Puzzles, and Problems Children Can Write by Susan Drake Lipscomb and Margaret Ann Zuanich
Bunches and Bunches of Bunnies by Louise Mathews
Charts and Graphs: Fun, Facts, and Activities by Caroline Arnold
Do You Wanna Bet? Your Chance to Find Out about Probability by Jean Cushman
Fleet-Footed Florence by Marilyn Sacks
In One Day by Tom Parker
Make It Graphic! Drawing Graphs for Science and Social Studies Projects by Eve and Albert Stwertka
Math Menagerie by Robert R. Kadesch
Million Dollar Jeans by Ron Roy
Millions of People by John Dunworth and Thomas Drysdale
Miss Pickerell and the Weather Satellite by Ellen MacGregor and Dora Pantell
Search for Delicious, The by Natalie Babbitt
Secret Winners, The by E.W. Hildick
Socrates and the Three Pigs by Tuyosi Mori
Statistics by Jane Jonas Srivastava
What Do You Mean by "Average"? Means, Medians, and Modes by Elizabeth James and Carol Barkin
Winning with Numbers: A Kid's Guide to Statistics by Manfred G. Riedel

The Papy Minicomputer is used throughout the *Comprehensive School Mathematics Program*, but its use changes over time. In the intermediate grades, the Minicomputer is used more as a context in which to pose problems and develop strategic thinking. Two mathematically rich games, *Minicomputer Tug-of-War* and *Minicomputer Golf* illustrate how the Minicomputer can play this role. Together they give rise to an intriguing array of mathematical considerations as each play presents, in turn, a new problem to be solved. Playing these games, students sharpen their skills in mental arithmetic and employ techniques of estimation. Equally important, students have the opportunity to develop strategic thinking—discovering advantageous plays and anticipating their opponents' responses.

Introduction

Both of the games are designed to be used collectively by one teacher and an entire class. The variety of problem-solving situations that the games provide makes them appropriate for the spectrum of ability levels in a usual heterogeneous classroom. However, both games can be adapted easily for small or cooperative group use.

As students play the games, you may find it easy to evaluate their strategies. But one advantage of using a game as a teaching device is that it allows the teacher for a period of time to be a facilitator and not an evaluator. Without constant evaluation of their actions, students have a better chance to explore a situation freely and to discover the effects of the options available. Obviously a simple comment on the progress of a game may be helpful at times.

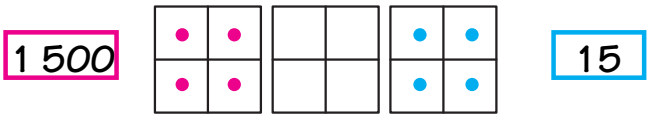
Equipment

For either game you need a Teacher Minicomputer Kit. When playing the game in groups, the students should have desk Minicomputers and checkers. Student Minicomputer sheets and checkers are available from McREL or can be made using the home Minicomputer Blackline.

MINICOMPUTER TUG-OF-WAR

Preparation for the Game

Display three Minicomputer boards and checkers as in the illustration below.



Divide the class into two teams—the **RED** team and the **BLUE** team. Each team plays with the colored checkers corresponding to the team's name. The starting number for each team is the number represented on the Minicomputer by the appropriately colored checkers.

Object of the Game

The first team to meet or pass the other team's number loses the game. In other words, the **RED** team loses by making the red number **LESS THAN** or **EQUAL TO** the blue number. Similarly, the **BLUE** team loses by making the blue number **GREATER THAN** or **EQUAL TO** the red number.

B

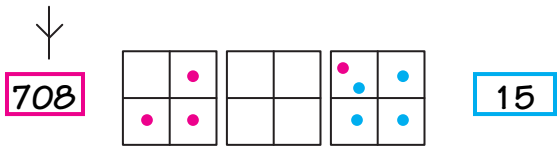
Rules of the Game

1) Teams alternate play, and the members take turns within each team.
2) The **RED** team may move red checkers only on their turn. A player for the **RED** team moves one red checker to a square of lower value, so the **RED** team's number decreases.
3) The **BLUE** team may move blue checkers only on their turn. A player for the **BLUE** team moves one blue checker to a square of higher value, so the **BLUE** team's number increases.

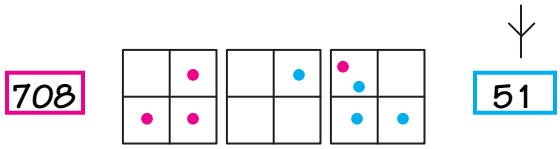
We suggest letting students volunteer to make moves during the first game rather than asking them to play in some order. This will speed up the game and allow students who are unsure of the rules to become more familiar with the game before playing themselves. You may want to impose a no talking rule to give students an opportunity to analyze the game individually. With small teams, it may be logistically possible to let a team work as a cooperative group and discuss their strategy.

Description of a Minicomputer Tug-of-War Game

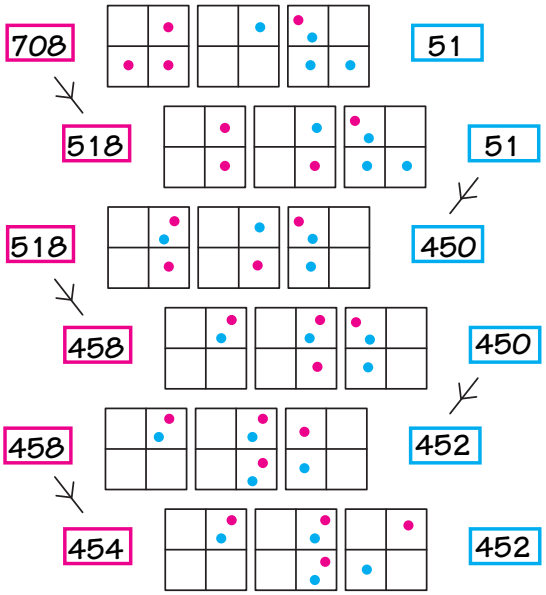
The following is a description of a possible game. In this game the **RED** team plays first, moving one red checker from the 800-square to the 8-square. The player determines that the red number is now 708 and changes the number in the red box.



Next, the **BLUE** team moves a blue checker from the 4-square to the 40-square. The player determines that the blue number is now 51 and changes the number in the blue box.



The game continues as shown below.

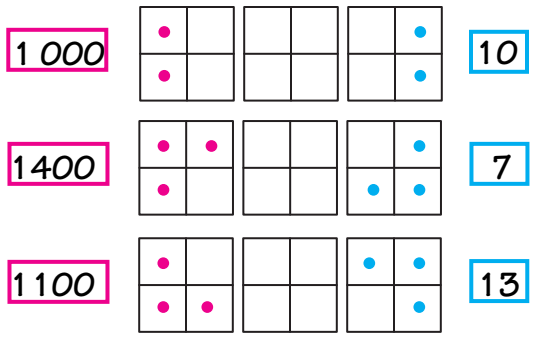


It is the **BLUE** team's turn. The smallest move they can make (it must be an increase) is from the 2-square to the 4-square. However, this move puts 454 on the Minicomputer in **BLUE** and meets the **RED** team. The **BLUE** team concedes their loss.

By studying the preceding game, you will see that the **BLUE** team could have won by going from 450 to 456 (moving a blue checker from the 2-square to the 8-square). However, students will not always play at such a sophisticated level, nor are they expected to. Such a situation occurs often during the games. You may note it yourself without comment and allow the students to enjoy the games.

Variations of the Game

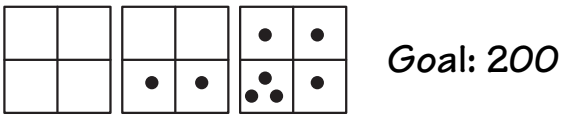
There are several different starting configurations, some of which you may like to use to help students become more proficient at playing *Minicomputer Tug-of-War*.



MINICOMPUTER GOLF

Preparation for the Game

Display three Minicomputer boards and a starting configuration of checkers. Choose a goal or target number. See the illustration below.



Divide the class into two teams—the **RED** team and the **BLUE** team. In this case, the color names for teams correspond to a color-coded record of the game. See the game description.

Object of the Game

Start with a number on the Minicomputer and, by moving checkers one at a time, try to put the target number on the Minicomputer.

Rules of the Game

1) Teams alternate play and the members take turns within each team.

2) A player takes a turn by moving exactly one checker from the square it is on to another square. As long as the number on the Minicomputer is below the goal, the play must increase the number. If the number on the Minicomputer is above the goal, the play must decrease the number.

B

You may want to impose a no talking rule to give students an opportunity to analyze the game individually. With small teams it may be logistically possible to let a team work as a cooperative group and discuss their moves. A red-blue arrow road records the progress of the game. Such a record is illustrated in the next section.

Description of a Minicomputer Golf Game

The following is a description of a sample *Minicomputer Golf* game that begins with 49 on the Minicomputer. The dialogue gives an idea of how you might conduct the game.

In this game, the **RED** team goes first and the player moves a checker from the 20-square to the 40-square.

T: ***Did you increase or decrease the number on the Minicomputer?***

S: ***Increase.***

T: ***From 20 to 40 … how much greater?***

S: ***20 greater.***

T: ***What number is on the Minicomputer now?***

S: ***69; 49 + 20 = 69.***[†]

On the board, a red arrow records the result of the **RED** team's move.

It is the **BLUE** team's turn and the player moves a checker from the 2-square to the 20-square.

T: ***Did you increase or decrease the number on the Minicomputer?***

S: ***Increase.***

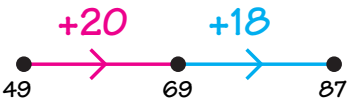
T: ***From 2 to 20 … how much greater?***

S: ***18 greater.***

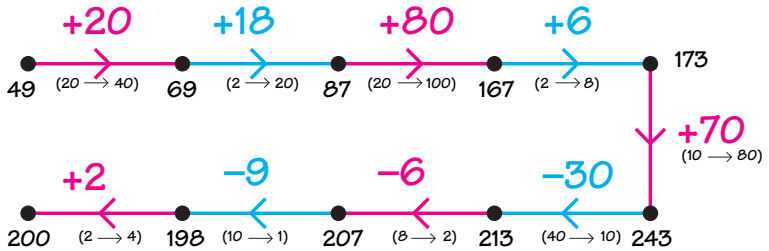
T: ***What number is on the Minicomputer now?***

S: ***87; 69 + 18 = 87.***

A blue arrow records the result of the **BLUE** team's move.



The game continues in this manner until the goal (200) is reached—the **RED** team wins. The red-blue arrow road below describes the game from start to finish. In your classroom, draw the arrow picture on the board large enough to be clearly visible by all players.



[†]You should require the player to be able to announce the effect of a move and the team to say what the new number is on the Minicomputer. If they cannot, the player must choose a different move.

Refrain from making comments on the quality of the moves, letting students enjoy the game as they gradually improve their strategies.

It's a good idea to let your class play collectively several times so that students are comfortable with the rules. Then, you might arrange for some games to be played in small groups; for example, groups of four (with two students per team) work well. The games are most effective when the teams are evenly matched in numerical abilities. Most students can play the game with little supervision.

An Optional Rule

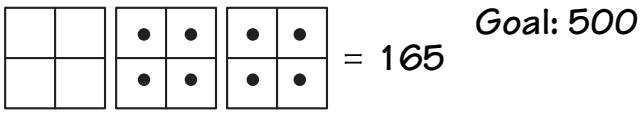
Experience playing the game yourself may lead you to discover an inadequacy of the rules of *Minicomputer Golf* as they have been presented. The problem is that the rules allow the possibility of never-ending games.

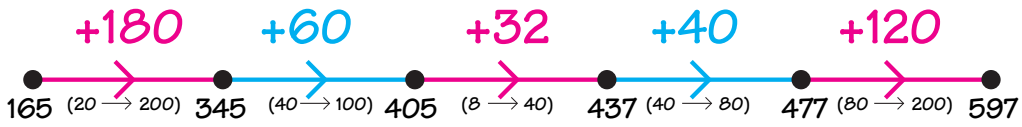
Students soon learn that even if they do not know a winning move, they can prevent opponents from winning by making the number as far from the goal as possible. The result is a continuing game whose conclusion remains out of reach for both teams. For this reason, introduce this extra rule only after students have played many times and have discovered how to make the game unending.

> If a player increases the number and passes the goal, the resulting number must be less than any previously played number higher than the goal.
>
> If a player decreases the number and passes the goal, the resulting number must be greater than any previously played number lower than the goal.

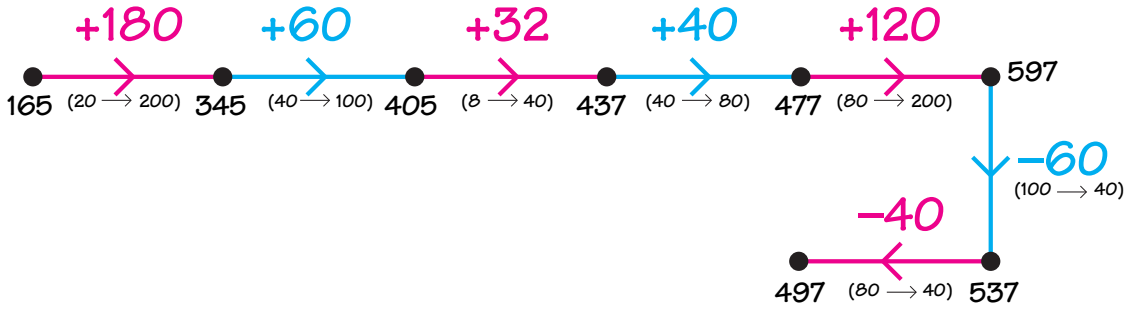
Using the extra rule, the teams must put the number closer and closer to the goal as the game progresses.

The following is a description of a game using this extra rule.

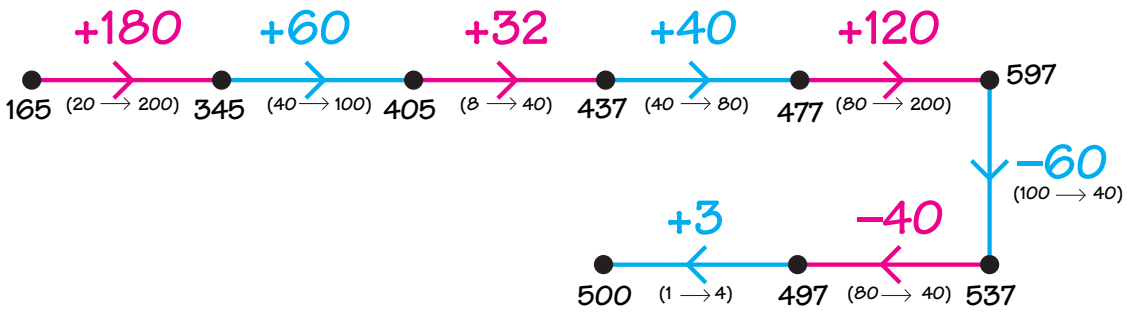




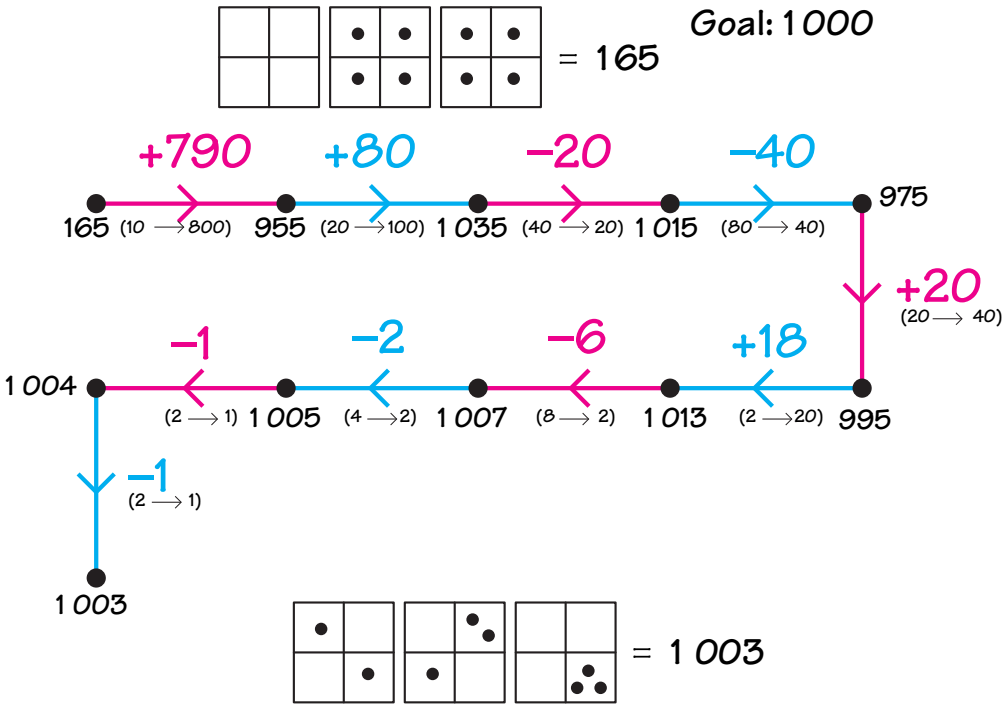
At this point the goal is exceeded. The next numbers must be less than 597 but greater than 477.



Since the number is now less than 500, subsequent moves must increase the number, but the number must remain less than 537. In fact, there is a winning move.



Note: During a game using the extra rule, a situation can arise where there is no possible legal move. For example,

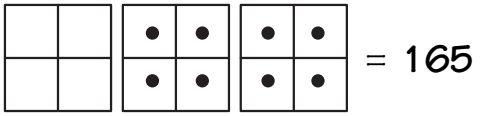


According to the extra rule, the next move must make the number less than 1 003, but must keep the number greater than 995. No such move is possible with the present configuration.

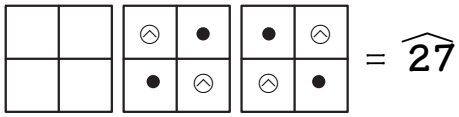
If such a situation occurs during a game you and your students are playing, you have two options for play. The game can be ended and declared a draw, or the extra rule may be suspended for one move (e.g., in the game described above, allow a student to decrease the number to one less than 995). However, subsequent moves must again follow the extra rule, whenever possible.

Variations of the Game

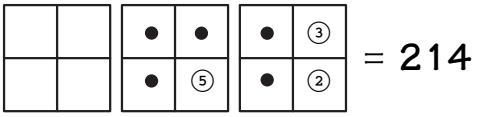
As your class becomes familiar with *Minicomputer Golf*, you can alter the starting number, the goal, and the type of checkers used. Several starting configurations are suggested below.



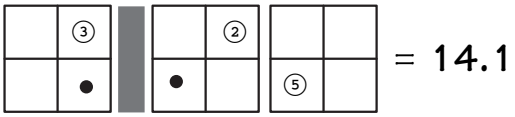
Goal: 500



Goal: 100



Goal: 1000



Goal: 90

MENTAL ARITHMETIC

Mental arithmetic activities are short, fast-moving question and answer sessions in which students are asked to calculate mentally, preferably without the aid of paper and pencil. Often a mental arithmetic session consists of several sequences in which the answers to one or more questions lead to the answer of another, more difficult, question. For example:

$3 \times 6 = ?$ (18)
$3 \times 10 = ?$ (30)
$3 \times 16 = ?$ (48)

There are specific lessons that call for mental arithmetic; however, try to include such activities for five to ten minutes several times a week. Use mental arithmetic as a warm-up activity to a lesson, as a transition between two exercises in a lesson, as a conclusion to a lesson, or as a quick change of pace at any time during the day.

Why Is Mental Arithmetic Important?

1) It develops mental computational skills.
2) It provides an opportunity to involve and help students at all ability levels.
3) It creates an awareness of patterns and mathematical relationships.
4) It helps students to be able to recall arithmetic facts easily.
5) It provides an opportunity to keep alive and to reinforce concepts previously introduced.
6) It helps students to see that there is often more than one way to solve a problem.
7) It uses and develops number sense.
8) It helps students construct their understanding of the arithmetic operations.

Hints for Mental Arithmetic Activities

1) Vary the level of difficulty of questions throughout a session.
2) Involve many students.
3) Follow a pattern for a while and then start a new pattern.
4) Keep a brisk pace, although you should allow students time for thought.
5) Give extra attention to a student who shows signs of improvement.
6) Illustrate, whenever possible, the value of estimation in making calculations.
7) Occasionally ask a student to explain an answer.
8) Occasionally use the chalkboard to emphasize a pattern.
9) Occasionally ask students to whisper an answer to you so that many students will have an opportunity to answer.
10) Occasionally discuss the reasonableness of an answer.

Suggested Mental Arithmetic Activities

The following section contains specific examples that illustrate patterns in arithmetic. You should elaborate and expand upon ideas depending upon the abilities and interests of your students. As you work with your class on the regularly scheduled mathematics lessons, you might make a note of computational skills you would like to improve; then plan a short mental arithmetic activity that develops those skills. Do not expect every one of your students to be good at all of these suggested activities, even at the end of the year.

Counting Activities

1) Count by twos, threes, fours, fives, or tens collectively. Refer to the number line or 0–109 numeral chart as the class counts, if you wish. Sometimes count by twos, then fours; or by threes, then sixes; or by fours, then eights.

2) Count by twos, threes, fives, or tens as a group activity with each student saying one number. Use a natural seating order in your class (such as going up and down rows), and ask each student in turn to say the number that comes next. If you are counting by threes, for example, the first student will say, "3"; the second will say, "6"; and so on. Repeat the counting but start with a different student.

3) Count by twos, threes, fours, fives, or tens, starting at 1 or any number other than 0.

4) Count backward by twos, fives, or tens. Do this immediately following a similar forward count.

5) Count by twos, then fours; or by threes, then sixes; or by fours, then eights.

6) Play *Buzz*. The students count by ones starting at 0 but skip every fourth number by saying "buzz." This should be done rapidly. This can be done as a whole class or small group activity with each student saying, in turn, either the number that comes next or "buzz."

   More challenging variations of this activity involve

   - skipping every sixth number rather than every fourth number;
   - starting the counting at 2 and skipping every fourth number;
   - counting by twos or fives and skipping every fourth number.

7) Count by halves, thirds, fourths, fifths, or sixths, starting at 0. When students say an improper fraction (e.g., $\frac{4}{3}$) or a fraction that has a reduced form (e.g., $\frac{2}{4}$), ask the class for another name for that number before proceeding.

8) Count backward by halves, thirds, fourths, fifths, or sixths. Do this immediately following a similar forward count.

9) Count by twos, by thirds, and then by two-thirds; or by threes, by fifths, and then by three-fifths.

Addition and Subtraction

The first 17 examples are lists of calculations to be done mentally. Each list suggests an arithmetic pattern that students should begin to use in doing such calculations.

1)

26 + 48.4 + 2	30 + 20	7 + 4	0.9 + 0.5
26 + 58.4 + 3	30 + 21	7 + 14	1.9 + 1.5
26 + 68.4 + 4	30 + 22	17 + 14	1.9 + 2.5
26 + 78.4 + 5	30 + 23	17 + 24	2.9 + 2.5

21 + 35702 + 100	536 + 100	600 + 213	
21 + 135702 + 200	536 + 200	600 + 313	
21 + 145702 + 300	536 + 300	600 + 413	
121 + 145	702 + 400	536 + 400	600 + 513

2) 144 – 103.2 – 0.1 100 – 10
144 – 203.2 – 0.2 100 – 11
144 – 303.2 – 0.3 100 – 12
144 – 403.2 – 0.4 100 – 13

52 – 31.86 – 0.6 520 – 105
52 – 131.86 – 0.7 520 – 205
52 – 231.86 – 0.8 520 – 305
52 – 331.86 – 0.9 520 – 405

3) 2 + 5 8 + 8
20 + 50 80 + 80
200 + 500 800 + 800
2 000 + 5 000 8 000 + 8 000
2,000,000 + 5,000,0008,000,000 + 8,000,000

23 – 1051 – 10 98 – 20
230 – 100510 – 100 9.8 – 2
2,300 – 1 000 5 100 – 1 000 0.98 – 0.2

4) 7 + 10 6 + 10 0.4 + 10
17 + 10 18 + 10 1.4 + 10
117 + 10 33 + 10 21.4 + 10
1 017 + 10 71 + 10 321.4 + 10

3 + 20 19 + 20 0.5 + 0.2
13 + 20 100 + 20 0.51 + 0.2
113 + 20 600 + 20 0.53 + 0.2
1 013 + 20 619 + 20 6.53 + 0.2

16 – 10100 – 10 40 – 20
26 – 10200 – 10 50 – 20
36 – 10600 – 10 60 – 20
46 – 101 000 – 10 90 – 20

5) 13 + 1419 + 3 1.9 + 0.3
23 + 1429 + 3 2.9 + 0.3
33 + 1439 + 3 3.9 + 0.3
43 + 1449 + 3 3.9 + 1.3

7 – 5100 – 8 0.10 – 0.03
17 – 5200 – 8 0.20 – 0.03
27 – 5300 – 8 0.30 – 0.03
37 – 5400 – 8 1.30 – 0.03

6) 5 + 310 + 20 106 + 105
8 – 330 – 20 211 – 105
7 + 690 + 40 150 + 150
13 – 6130 – 40 300 – 150

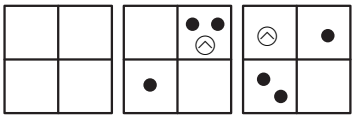
7) 16 + 554 + 12 8 + 2 + 5

	5 + 16	12 + 54 5 + 8 + 2	8 + 5 + 2
8)	17 + 5 + 57 + 28 + 3 38 + 7 + 38 + 35 + 2 49 + 6 + 46 + 76 + 4 112 + 9 + 15 + 27 + 5	7 + 13 + 6 + 4 19 + 1 + 9 + 1 14 + 6 + 8 + 2 25 + 5 + 3 + 7	
9)	80 – 39 127 – 100 79 – 38126 – 99 78 – 37125 – 98 77 – 36124 – 97	59 – 30 57 – 28 55 – 26 53 – 24	
10)	28 + 1027 + 100 28 + 927 + 99 43 – 10126 – 100 43 – 9126 – 99	1.5 +1 1.5 + 0.9 5.4 – 1 5.4 – 0.9	
11)	50 + 4030 + 20 8 + 25 + 7 58 + 4235 + 27 50 – 2090 – 30 8 – 6 7 – 2 58 – 2697 – 32	8 + 6 0.9 + 0.4 8.9 + 6.4 6 – 1 0.9 – 0.4 6.9 – 1.4	
12)	5 + 2 16 + 2 5 + 1 16 + 1 5 + 0 16 + 0 5 + $\widehat{1}$16 + $\widehat{1}$ 5 + $\widehat{2}$ 16 + $\widehat{2}$	64 + 2 64 + 1 64 + 0 64 + $\widehat{1}$ 64 + $\widehat{2}$	
13)	12 – 521 – 4 12 + $\widehat{5}$21 + $\widehat{4}$	145 – 30 145 + $\widehat{30}$	
14)	2 + $\widehat{3}\widehat{5}$ + $\widehat{1}$ 20 + $\widehat{30}$ 200 + $\widehat{300}$	8 + $\widehat{5}$ $\widehat{50}$ + $\widehat{10}$ $\widehat{500}$ + $\widehat{100}$	80 + $\widehat{50}$ 800 + $\widehat{500}$
15)	10 – 611 – 4 6 – 10	27 – 13 4 – 11	13 – 27
16)	7 + $\widehat{9}$ 3 + $\widehat{8}$ $\widehat{7}$ + 9 $\widehat{3}$ + 8	12 + $\widehat{15}$ $\widehat{12}$ + 15	
17)	$\frac{1}{2}$ + $\frac{1}{2}\frac{2}{5}$ + $\frac{1}{5}$ $1\frac{1}{2}$ + $\frac{1}{2}1\frac{2}{5}$ + $\frac{1}{5}$ $2\frac{1}{2}$ + $\frac{1}{2}$ $3\frac{1}{2}$ + $1\frac{1}{2}7\frac{2}{5}$ + $4\frac{1}{5}$	3 + 1 $2\frac{1}{2}$ + $1\frac{1}{2}$ $1\frac{2}{5}$ + $1\frac{1}{5}$ $1\frac{1}{2}$ + $2\frac{1}{2}$	$6\frac{1}{2}$ + 1 $3\frac{3}{4}$ + 1 2 + 2 $7\frac{2}{3}$ + 4
	$\frac{3}{4}$ – $\frac{1}{4}\frac{4}{5}$ – $\frac{1}{5}$	5 – 1	$4\frac{2}{7}$ – 1

$1\frac{3}{4} - \frac{1}{4}3\frac{4}{5} - 1\frac{1}{5}$ $5\frac{1}{3} - 1\frac{1}{3}$ $9\frac{3}{8} - 1$
$3\frac{3}{4} - \frac{1}{4}20\frac{4}{5} - 10\frac{1}{5}$ $5\frac{3}{4} - 1\frac{3}{4}$ $9\frac{3}{8} - 5$

18) Choose some numbers and ask the students to add 1 to each number. For example, if you say, "108," a student responds, "109." You can vary this activity by asking the students to add 2, 3, 5, or 10 to each of your numbers, or to subtract 1, 2, 3, or 10 from each of your numbers.

19) Conduct an activity similar to Exercise 1 in Lesson N1 *Minicomputer Dynamics #1*. Put six to ten checkers on the ones and tens boards. Move one checker from some square (it makes no difference which square you choose) to another square. Ask students how much more or less the new number is. For example, suppose this configuration is on the Minicomputer.



Move a negative checker from the 8-square to the 20-square. When students tell you that the new number is less, ask how much less this number is than the previous number. (In this case, it is 12 less.) Sometimes include weighted checkers in the starting configuration and move a weighted checker.

20) Ask students to give various names involving addition and subtraction for a particular number. Encourage them to suggest names involving several numbers and both addition and subtraction. For example, names for 20 include 16 + 4; 30 – 10; 4 + 4 + 12; 21 + $\widehat{1}$; 15 + $\widehat{5}$ + 10; 17.3 + 2.7; 19.5 + 1 – 0.5.

Multiplication and Division

The first 25 examples here are lists of calculations to be done mentally. Each list suggests an arithmetic pattern which students should begin to use in doing such calculations.

1) 9 + 90.7 + 0.7 106 + 106
2 x 92 x 0.7 2 x 106
8 + 82.4 + 2.4 508 + 508
2 x 8 2 x 2.4 2 x 508

4 + 4 + 4$\widehat{8}$ + $\widehat{8}$ + $\widehat{8}$ 6 + 6 + 6 + 6
3 x 4 3 x $\widehat{8}$ 4 x 6
5 + 5 + 5$\widehat{12}$ + $\widehat{12}$ + $\widehat{12}$1.1 + 1.1 + 1.1 + 1.1
3 x 5 3 x $\widehat{12}$ 4 x 1.1

2) 5 x 8 1 x 5.2 7 x 60
6 x 8 2 x 5.2 7 x 61
7 x 8 3 x 5.2 7 x 62
8 x 8 4 x 5.2 7 x 63

3) 8 x 3 7 x 7 9 x 6
8 x 307 x 70 9 x 0.6
8 x 300 7 x 700 9 x 0.06
8 x 3 000 7 x 7 000 9 x 0.006
8 x 3,000,000 7 x 7,000,000 9 x 0.0006

4) 2 x 4 2 x 5 3 x 0.3
2 x 44 2 x 55 3 x 3.3

2 x 444 2 x 555 3 x 33.3
2 x 4 444 2 x 5 555 3 x 33.33

5) 4 x 7 5 x 8 2 x 1.6
7 x 4 8 x 5 1.6 x 2
4 x 70 8 x 50 3 x 1.2
70 x 450 x 8 1.2 x 3

6) 9 x 106 x 50 15 x 10
9 x 9 6 x 49 15 x 9
7 x 20 8 x 100 11 x 20
7 x 198 x 99 11 x 19

7) 5 x 8 2 x 9 7 x 10
4 x 8 10 x 9 7 x 3
9 x 8 12 x 9 7 x 13

5 x 60 4 x 200 8 x 2
5 x 4 4 x 30 8 x 0.6
5 x 644 x 1 8 x 2.6
4 x 231

8) 7 x 3 5 x 4 16 x 10
7 x 6 5 x 8 16 x 20
7 x 12 5 x 16 16 x 40

2 x 0.7 0.3 x 9 0.4 x 5
4 x 0.7 0.6 x 9 0.8 x 5
8 x 0.71.2 x 9 1.6 x 5

9) 2 x 9 3 x 9 3 x 0.9
2 x 2 x 93 x 3 x 9 2 x 3 x 0.9
4 x 9 9 x 9 6 x 0.9

4 x 7 3 x 7 4 x 0.6
2 x 4 x 72 x 3 x 7 2 x 4 x 0.6
8 x 7 6 x 7 8 x 0.6

10) 10 x 510 x 8 10 x 1.2
10 x 50 10 x 80 10 x 12
10 x 500 10 x 800 10 x 120

20 x 320 x 5 20 x 0.8
20 x 30 20 x 50 20 x 8
20 x 300 20 x 500 20 x 80

11) 15 + 1524 + 24 0.75 + 0.75
2 x 152 x 24 2 x 0.75

$\frac{1}{2}$ x 30$\frac{1}{2}$ x 48 $\frac{1}{2}$ x 1.5

11 + 11 + 11 0.2 + 0.2 + 0.2 13 + 13 + 13 + 13
3 x 113 x 0.2 4 x 13
$\frac{1}{3}$ x 33$\frac{1}{3}$ x 0.6 $\frac{1}{4}$ x 52

12) 10 x 700 10 x 900 10 x 400
10 x 70 10 x 90 10 x 40
10 x 710 x 9 10 x 4
10 x 0.7 10 x 0.9 10 x 0.4

7 x 200 3 x 500 9 x 600
7 x 20 3 x 50 9 x 60
7 x 2 3 x 5 9 x 6
7 x 0.2 3 x 0.5 9 x 0.6

13) $\frac{1}{2}$ x 24$\frac{1}{3}$ x 6 $\frac{1}{5}$ x 10
$\frac{1}{2}$ x 240 $\frac{1}{3}$ x 60 $\frac{1}{5}$ x 100
$\frac{1}{2}$ x 2 400 $\frac{1}{3}$ x 600 $\frac{1}{5}$ x 1 000
$\frac{1}{2}$ x 24 000 $\frac{1}{3}$ x 6 000 $\frac{1}{5}$ x 10 000
$\frac{1}{2}$ x 24,000,000 $\frac{1}{3}$ x 6,000, 000 $\frac{1}{5}$ x 10,000,000

14) $\frac{1}{2}$ x 20 $\frac{1}{2}$ x 6 $\frac{1}{2}$ x 10
$\frac{1}{2}$ x 30 $\frac{1}{2}$ x 16 $\frac{1}{2}$ x 100
$\frac{1}{2}$ x 40 $\frac{1}{2}$ x 26 $\frac{1}{2}$ x 108
$\frac{1}{2}$ x 50 $\frac{1}{2}$ x 36 $\frac{1}{2}$ x 148

15) 2 x 13 2 x 35 2 x 104
$\frac{1}{2}$ x 26$\frac{1}{2}$ x 70 $\frac{1}{2}$ x 208

3 x 10 3 x 8 3 x 105
$\frac{1}{3}$ x 30$\frac{1}{3}$ x 24 $\frac{1}{3}$ x 315

16) $\frac{1}{2}$ x 100 $\frac{1}{3}$ x 90 $\frac{1}{4}$ x 8
$\frac{1}{2}$ x 12$\frac{1}{3}$ x 6 $\frac{1}{4}$ x 0.4
$\frac{1}{2}$ x 112 $\frac{1}{3}$ x 96 $\frac{1}{4}$ x 8.4

17) 6 x 2 9 x 4 30 x 3
6 x $\frac{1}{2}$ 9 x $\frac{1}{3}$ 30 x $\frac{1}{2}$
6 x 2$\frac{1}{2}$ 9 x 4$\frac{1}{3}$ 30 x 3$\frac{1}{2}$

18) 7 x 8 6 x 9 3 x 8
56 ÷ 754 ÷ 9 24 ÷ 8
56 ÷ 854 ÷ 6 24 ÷ 3

2 x 432 x 17 2 x 108
86 ÷ 234 ÷ 2 216 ÷ 2

3 x 20 3 x 6 3 x 201
60 ÷ 318 ÷ 3 603 ÷ 3

19) 20 ÷ 2 400 ÷ 8 30 ÷ 3
6 ÷ 2 40 ÷ 8 1.2 ÷ 3

26 ÷ 2 440 ÷ 8 31.2 ÷ 3

20) ½ x 14½ x 7
½ x 15½ x 9
½ x 20 ½ x 11
½ x 21 ½ x 13
½ x 100
½ x 101
½ x 102
½ x 103

21) 70 ÷ 10 150 ÷ 10
7 ÷ 10 15 ÷ 10
0.7 ÷ 10 1.5 ÷ 10
0.07 ÷ 100.15 ÷ 10
125 ÷ 5
12.5 ÷ 5
1.25 ÷ 5
0.125 ÷ 5

22) 6 x 21 7 x 51
6 x 2.1 7 x 5.1
0.6 x 210.7 x 51
5 x 35
5 x 3.5
0.5 x 35

23) 17 x 3 x ⅓2 x 59 x ½
19 x 2 x ½⅕ x 27 x 5
59 x ⅕ x 5 ⅓ x 67 x 3
63 x ¼ x 44 x 100,001 x ¼

24) 2 x 3 2 x 30
2 x 2 2 x 30
2 x 1 2 x 10
2 x 0 2 x 0
2 x $\widehat{1}$ 2 x $\widehat{10}$
2 x $\widehat{2}$ 2 x $\widehat{20}$
2 x $\widehat{3}$ 2 x $\widehat{30}$
3 x 3
3 x 2
3 x 1
3 x 0
3 x $\widehat{1}$
3 x $\widehat{2}$
3 x $\widehat{3}$

25) 810 ÷ 956 ÷ 8
819 ÷ 956.8 ÷ 8
828 ÷ 957.6 ÷ 8
846 ÷ 959.2 ÷ 8
42 ÷ 6
42 ÷ 60
420 ÷ 60
420 ÷ 600

26) Ask students to give various names involving multiplication and division for a particular number. For example, names for 15 include 3 x 5; 150 ÷ 10; (5 x 6) ÷ 2; ¼ x 60; 1.5 x 10; $\widehat{3}$ x $\widehat{5}$.

This activity could be restricted to division by asking for several division problems with the same answer.

27) Ask for a multiple of 6 (or 7) between two other numbers; for example, ask for a multiple of 6 between 70 and 75.

Estimation

Ask students to estimate a calculation and to explain their answers. For example:

T: ***Is 218 + 157 more than 300? How do you know?***

S: ***Yes, because 200 + 100 = 300.***

T: ***Is it more than 500? How do you know?***

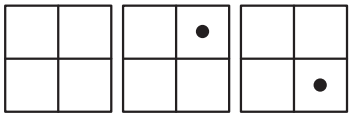
S: ***No, because 300 + 200 = 500. 218 is less than 300, and 157 is less than 200, so 218 + 157 must be less than 500.***

T: ***Is it more than 400?***

S: ***No, because 18 + 57 is less than 100.***

Other Mental Arithmetic Activities

1) Use the Minicomputer to give visual clues for addition, subtraction, and multiplication problems. For example, suppose this configuration is on the Minicomputer.



T: ***What number is on the Minicomputer?*** (41)

Hold a checker over the 8-square.

T: ***What number is 41 + 8?*** (49)

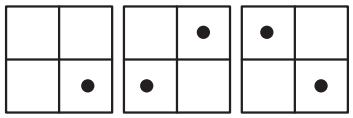
Hold a checker over the 20-square.

T: ***What number is 41 + 20?*** (61)

After a few such clues, hold a checker over the square only if the class has difficulty.

The next example involves subtraction.

T: ***What number is on the Minicomputer?*** (169)



Lift the checker temporarily from the 8-square.

T: ***What number is 169 – 8?*** (161)

Lift the checker temporarily from the 40-square.

T: ***What number is 169 – 40?*** (129)

Lift the checkers temporarily from the 40-square and the 1-square.

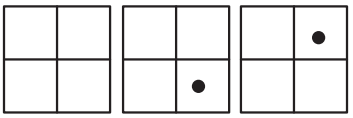
T: ***What number is 169 – 41?*** (128)

After a few such clues, lift the checkers only if the class has difficulty.

The next examples involve multiplication and division.

T: ***What number is on the Minicomputer?*** (14)

Move the checkers one board to the left as you ask,

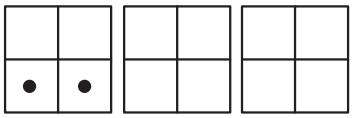


T: ***What number is 10 x 14?***

Repeat this activity starting with other starting configurations.

Put 300 on the Minicomputer.

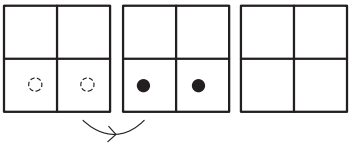
Move the checkers to the tens board as you ask,



T: ***What number is 300 ÷ 10?***

S: ***30.***

Move the checkers to the ones board as you ask,

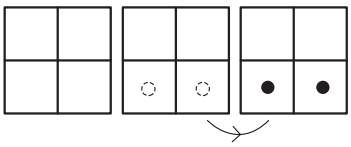


T: ***What number is 30 ÷ 10?***

S: ***3.***

T: ***Can we divide by 10 again?***

S: ***Draw a bar and put up boards to the right.***



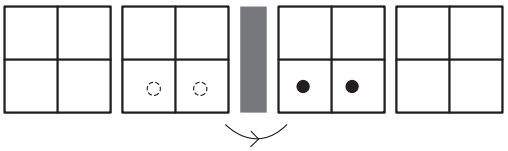
Move the checkers to the tenths (dimes) board as you ask,

T: ***What number is 3 ÷ 10?***

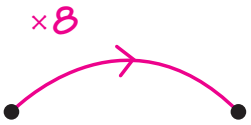
S: ***0.3.***

T: ***What is 0.3 ÷ 10?***

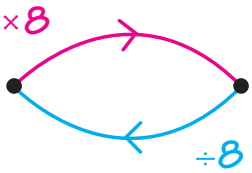
S: ***0.03.***



2) Use an arrow picture to pose mental arithmetic questions. For example, draw this arrow picture on the board.



Ask what number is at the right if the number at the left is, for example, 31. Repeat this activity several times and then draw the ÷8 return arrow.



T: ***What could this blue return arrow be for?***

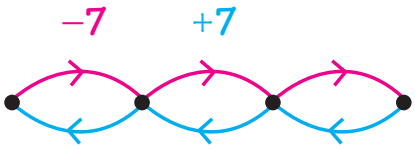
S: ***÷8 or $\frac{1}{8}$x.***

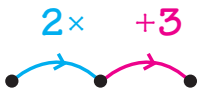
Ask students what number is at the left if the number at the right is, for example, 248. Repeat this activity several times.

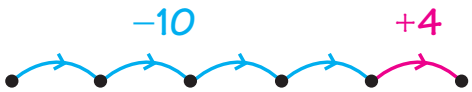
There are many variations for this activity. For example:

- Use other starting and ending numbers, including decimal numbers.
- Use functions other than x8 and ÷8.

- Use two or more arrows of the same function.
- Use a composition of two functions.







3) Use the suggestion of an arrow picture to pose place-value questions.
   - Imagine an arrow road starting at 0 and having four +10 arrows and seven +1 arrows. What is the ending number?
   - Imagine an arrow road starting at 0 and having two +100 arrows, five +10 arrows, and two +1 arrows. What is the ending number?

4) Play *Guess My Rule*. Choose a secret rule such as 10x followed by +3. Then, ask students to suggest a number to you; you respond with the number you get after applying the rule. Continue until students discover your rule. For example:

S: ***5.*** **T:** ***53.***

S: ***1.*** **T:** ***13.***

S: ***100.*** **T:** ***1 003.***

S: ***50.*** **T:** ***503.***

T: ***Does anyone know my secret rule?***

S: ***Multiply by 10 and then add 3.***

T: ***How do you know?***

S: ***Each time, you answered with a number that was three more than ten times the number we gave you.***

Calculator Activities

Use the calculator to develop and reinforce counting skills; to emphasize counting patterns; and to improve estimation skills.

1) Use one or more calculators in the class to support counting activities in which patterns are generated. For example, start with any whole number on the display of your calculator and press [+] [5] [=] [=] [=] … . On the board, record the number that appears on the display of the calculator each time you press [=]. As soon as most of your students are able to predict the sequence quickly, you can abandon the calculator. Ask students to explain the pattern.

   There are many variations of this activity, such as the following:

   - Press [+] [2] (or [3] or [4] or [1][0]) [=] [=] [=] … .
   - Start with a large number and press [−] [5] (or [2] or [1][0] or [1][0][0]) [=] [=] [=] … .

2) Play *Let's Concentrate* using one or more calculators in the class. Ask students who have the calculators to put a number you specify on the display and then to hide the display with one hand. Give a sequence of two operations and instruct students to perform them by pressing the appropriate keys without looking at the display. Then, ask what number is on the display, and let students check to see if they are correct. Continue with the new number and another sequence of two operations. Here is an example:

   Start with 14 on the display.

   - Press [+] [3] [+] [2] [=]. The resulting number is 19. (Students check that 19 is on the display.)
   - Press [+] [1] [×] [2] [=]. The resulting number is 40. (Students check that 40 is on the display.)
   - Press [−] [3] [+] [1] [=]. The resulting number is 38. (Students check that 38 is on the display.)

3) Play *Calculator Golf* with a small group of students, each one having a calculator. Ask students to put a small whole number you specify on the display of their calculators. Then let students take turns suggesting some numbers to add or subtract until a particular goal is reached. A possible game in which four students are playing and 200 is the goal is described below.

   Start with 12 on the display. (12)

Player 1:	+ 9	(21)
Player 2:	+72	(93)
Player 3:	+17	(110)
Player 4:	+7	(117)
Player 1:	+93	(210)
Player 2:	–10	(200)

   Some variations of this game include

   - changing the starting number;
   - making the goal a larger multiple of 100;
   - allowing the use of only [+], [−], or [×] some number between 0 and 10.

   For still another variation, allow only the operation [×] to be used and make the goal a number between 500 and 501. In this variation of the game, a play always begins back at the starting number. A possible game with four students is described below. The starting number is 17.

Player 1:	17 x 50 = 850
Player 2:	17 x 40 = 680
Player 3:	17 x 35 = 595
Player 4:	17 x 30 = 510
Player 1:	17 x 29 = 493
Player 2:	17 x 29.5 = 501.5
Player 3:	17 x 29.4 = 499.8
Player 4:	17 x 29.45 = 500.65

Some Short Games

1) Play a game called *High-Low* in which you choose a secret number and students try to guess it. Each time a guess is made, say if the guess is too high or too low. Depending upon the ability of your class, choose secret numbers such as 10,001; $2\frac{1}{2}$; 17.4, or 3.14.

2) Play a game called *Hit/Miss/Bull's-Eye*. The object of the game is to guess a secret number. Tell the class you have a secret three-digit number in which all three digits are different. When a student makes a guess, say the following:

   - "hit" for every digit in the guess that is correct but not in the correct place;
   - "miss" for every digit in the guess that is not in your number;
   - "bull's-eye" for every digit in the guess that is correct and in the correct place.

   The order in which you give these responses does not have to necessarily correspond to the order of the digits in the guess. In other words, if you say, "Hit, miss, miss," the students know one of the three numbers guessed is correct, but not necessarily the first one. Record the guesses and responses on the board. It is helpful to write your secret number on a piece of paper to refer to during the game. Here is a description of a sample game in which the secret number is 571.

   Secret number: 571

Guess 1:	429	miss, miss, miss
Guess 2:	389	miss, miss, miss
Guess 3:	107	hit, hit, miss (1 and 7 are correct but not in the correct place.)
Guess 4:	751	bull's-eye, hit, hit (1 is correct and in the correct place, 7 and 5 are correct but not in the correct place.
Guess 5:	715	hit, hit, hit
Guess 6:	571	bull's-eye, bull's-eye, bull's-eye

THE STRING GAME

The String Game is used in many versions throughout the *Comprehensive School Mathematics Program*. It gives students an opportunity, in a game-like atmosphere, to become familiar with the language of strings, while at the same time it involves them in the kind of reasoning that will be developed and reinforced in various contexts throughout the program.

The String Game is first played with A-blocks (shapes), but later, in the intermediate grades, variations that make use of numbers appear. This Appendix provides you with the necessary information and examples to enable you to play the game using A-blocks and using numbers, and it suggests some of the various possibilities open to you when you play the game. There are several lessons in *IG-IV* that call for *The String Game*, but we hope you will not feel restricted to playing the game only at these times. It is most beneficial and enjoyable for students if you make a regular practice of playing the game once every few weeks or whenever you have an extra 10–15 minutes during the course of the day.

Equipment

PLAYING BOARD

The equipment for this game may be most easily managed if you have a magnetic (magnet-sensitive) chalkboard available. Many permanently mounted chalkboards in classrooms are magnetic; you can test yours using a magnet. If your permanent chalkboards are not magnetic, try any portable chalkboard (dry erase board, and so on) that is available. If you do not have a magnetic chalkboard available, you can use your regular chalkboard.

TEAM BOARD

The team board is divided into regions as illustrated below.

(Attach a poster list of string cards here.)	
Team A	**Team B**

Note: The game may be played with three or four teams rather than two. In this case, create a team board with sections for more teams.

a) Magnetic: If you have a large magnetic classroom chalkboard, you can draw the team board directly on a portion of it. However, if you have a relatively small (portable) magnetic chalkboard, you may need to obtain a sheet of metal (minimum size 60 cm by 80 cm) or locate a convenient metallic surface in the classroom, such as the side of a file cabinet, on which to put the team board. In such a case, draw the team board on a large sheet of (chart) paper and tape this paper to your metallic surface.

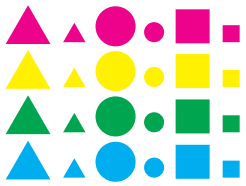
b) Non-magnetic: If you do not have a magnetic chalkboard available for the playing board, your team board can be a large piece of poster board (minimum size 60 cm by 80 cm). Or, a team board may be drawn directly on a portion of the classroom chalkboard.

GAME PIECES AND STRING CARDS

One set of game pieces and string cards is needed for each version of the game. A poster list of the string cards should be posted above the team board—it is a constant reminder during the game of the possible labels for the strings.

Game Pieces

String Game with A-Blocks



String Cards

RED	YELLOW	GREEN	BLUE
NOT RED	NOT YELLOW	NOT GREEN	NOT BLUE
○	△	□	BIG
NOT ○	NOT △	NOT □	LITTLE

String Game with Numbers

$\widehat{100}$	$\widehat{80}$	$\widehat{55}$	$\widehat{15}$	$\widehat{10}$	$\widehat{5}$
$\widehat{1}$	0	1	2	3	4
5	6	7	8	9	10
12	18	20	24	27	40
45	50	60	99	100	105

MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4	MULTIPLES OF 5
MULTIPLES OF 10	LESS THAN 50	LESS THAN $\widehat{10}$	ODD NUMBERS
POSITIVE DIVISORS OF 12	GREATER THAN 50	GREATER THAN $\widehat{10}$	POSITIVE PRIME NUMBERS
POSITIVE DIVISORS OF 18	POSITIVE DIVISORS OF 20	POSITIVE DIVISORS OF 24	POSITIVE DIVISORS OF 27

Game pieces, string cards, and the poster of the string cards can be found in the corresponding *String Game* kit, (*A-Blocks* or *Numerical*).

a) Magnetic: You can magnetize the game pieces (A-blocks or number cards) by sticking a small piece of magnetic material to the back of each one. (Magnetic material is included in *The String Game* kits, or it is available in many stores, in the hobby or notions departments.) Similarly, you can magnetize string cards by sticking a small piece of magnetic material to the front of each card, taking care not to obscure what is written on it.

b) Non-magnetic: Game pieces can be attached to the team board using loops of masking tape stuck to the backs. A string card should have a loop of masking tape stuck to the front in such a way that what is written on the card is not obscured. With this type of equipment, be prepared to make necessary repairs by having masking tape on hand so that if a loop of tape loses its stickiness it can be replaced on the spot. As an alternative, use a small wad of a plastic caulking compound (Rope Caulk or Mortite, for example) in place of the loop of masking tape.

Preparation for the Game

Draw two (or three, depending on which variation you are using) large, overlapping strings on the playing board using two (or three) different colors. Next to each of these strings attach one string card facedown. Place the team board conveniently nearby. Randomly distribute the game pieces among the sections of the team board. Divide the class into teams using whatever method is acceptable to your class, and assign each team a section of the team board.

Before any student takes a turn, correctly place an equal number (at least one) of each team's game pieces in the string picture. This eliminates the necessity of beginning the game on the basis of pure guesswork. You can influence how long the game will take by the number of pieces you place in the string picture before the game begins.

Object of the Game

Each team tries to place all of its game pieces correctly (according to the facedown string cards) in the string picture. The winning team is the one that places all of its game pieces correctly and identifies the facedown cards correctly first.

Rules of the Game

1) The teams alternate making plays, and the members take turns within each team. A player comes to the board and selects a piece from his or her team's collection to place in one of the regions of the string picture.

2) You are the judge. If the piece is correctly placed, say yes. The piece then remains in the string picture and the player immediately has a second (bonus) turn (no player may have more than two consecutive turns). If the piece is incorrectly placed, say no. The player returns the piece to the team's unplayed collection and play passes to the next team.

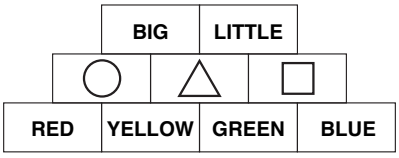
   As an aid in judging, prepare a crib sheet showing the correct position of each game piece or at least reminding you of what is on the facedown cards. If at any time you discover that you have made an error, say so immediately and rectify the mistake. Then, either move an incorrectly placed piece to its correct region or replace a correctly placed piece that has been removed.

3) When a team has correctly placed all of its pieces, the player who placed the last piece may then attempt to identify the string cards. If he or she is correct, the team wins. If a mistake is made (even if it is only in the case of one of the string cards), simply indicate that the identification is incorrect and let the game continue.

4) If a team has exhausted its stock of game pieces and the strings have not been identified, that team continues to attempt to identify the strings on its turn, while the other team(s) works to place its game pieces.

The String Game with A-Blocks (Version A)

This simplest version of the game uses 24 A-blocks as game pieces and only nine string cards.

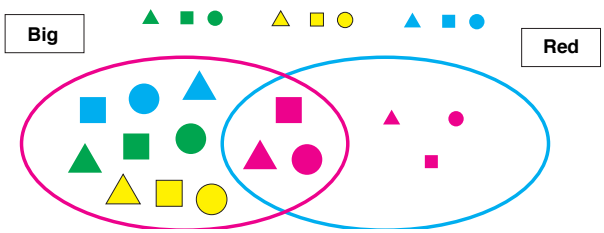


This list of string cards should be attached above the team board.

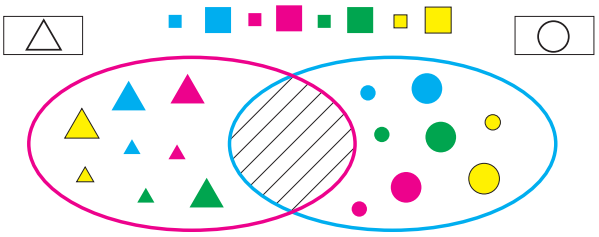
Below are several crib sheets for variations of the game with two and with three strings.

TWO STRINGS

Example 1: No empty regions



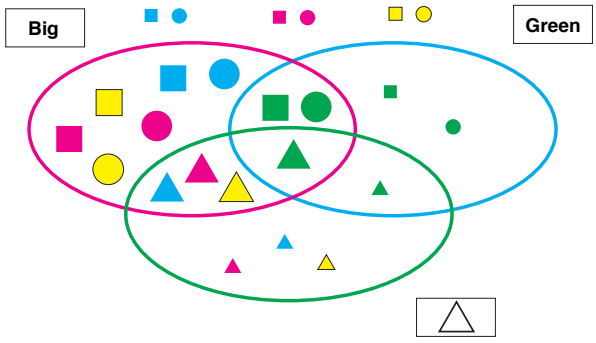
Example 2: One empty region



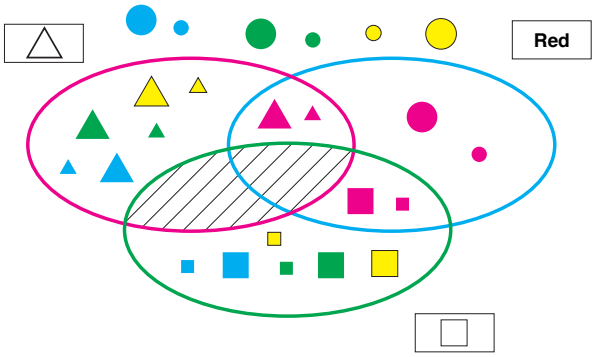
Note: We have indicated that the intersection of the strings is empty by "hatching" that region.

THREE STRINGS

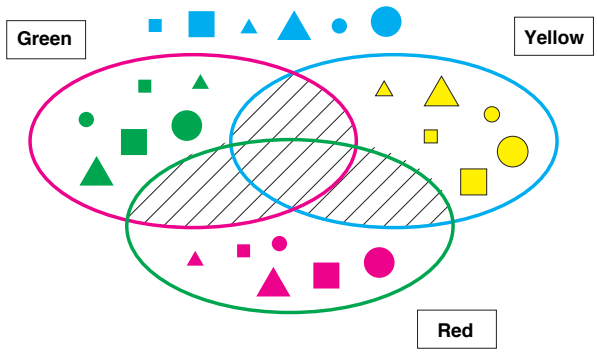
Example 1: No empty regions



Example 2: Two empty regions



Example 3: Four empty regions

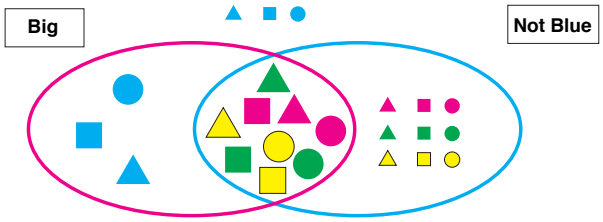


The String Game with A-Blocks, Using Not-Cards (Version B)

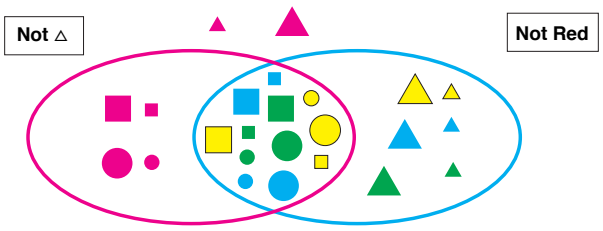
A more complicated version of the game with A-blocks uses all 16 of the string cards. A list of all 16 string cards should be posted above the team board.

Here are several crib sheets for this version of the game played with two strings.

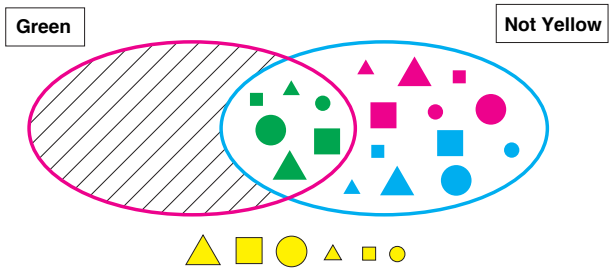
Example 1: No empty regions



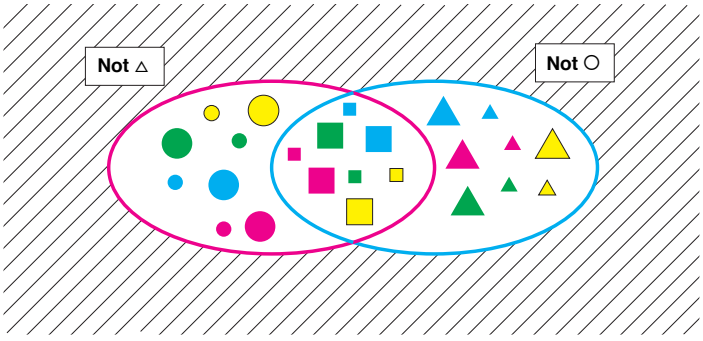
Example 2: No empty regions



Example 3: One empty region



Example 4: One empty region



Note: By hatching the "outside" region of the diagram we mean to indicate that no game pieces can be placed there correctly. Strictly speaking, that region is not empty because, for example, the number 50 is in the outside region.

You should be warned that the "not" version of the game played with three strings is very difficult to judge without a crib sheet and is equally difficult to play. Hence, use it only when you think the two-string version is no longer challenging enough for the majority of your students.

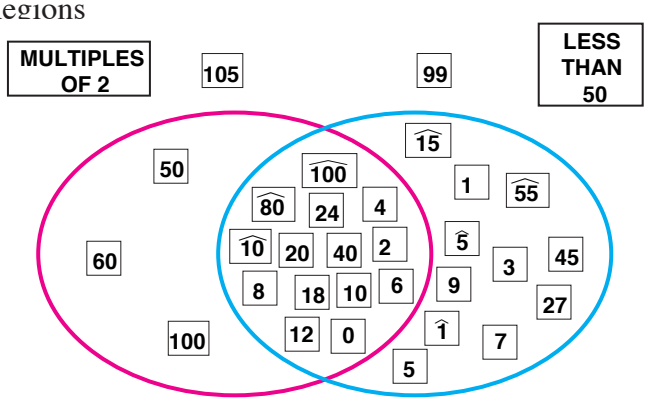
The String Game with Numbers (Version C)

This version of the game uses 30 number cards as game pieces and 16 string cards as described earlier in the section on equipment. Note that The Numerical String Game kit contains three sets of string cards. This allows you to give the same label to more than one string. Example 7 played with two strings and Example 1 played with three strings show this option; however, before using such a game players should have an understanding that this is a possibility.

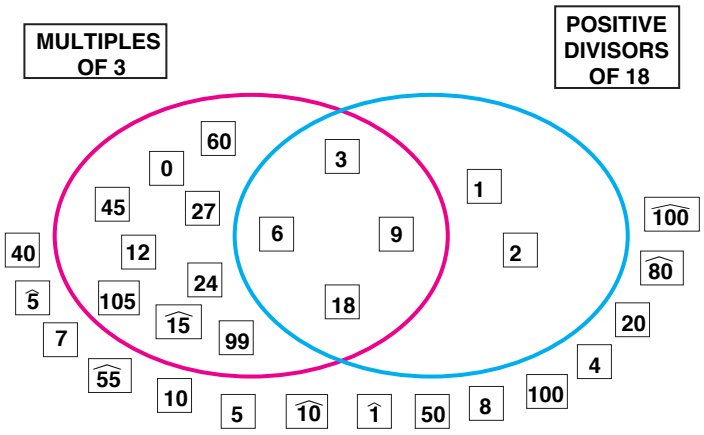
Below are several crib sheets for variations of the game with two and with three strings.

TWO STRINGS

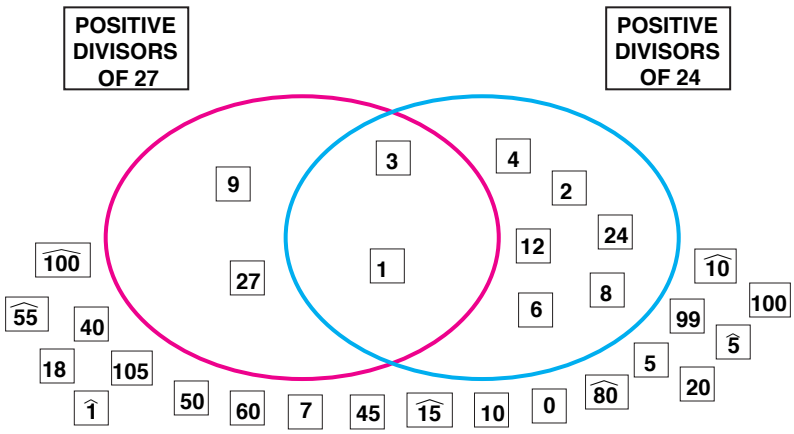
Example 1: No Empty Regions



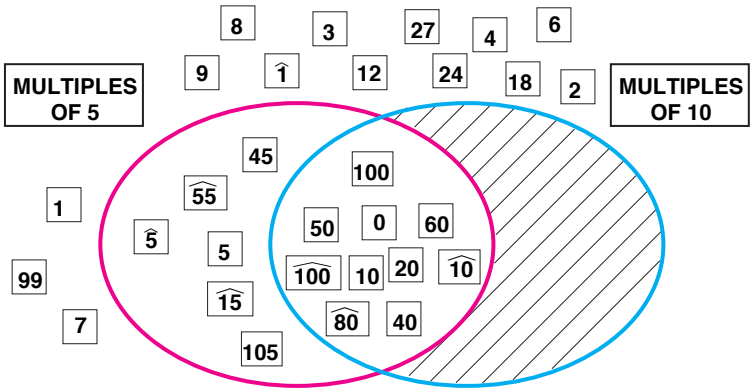
Example 2: No Empty Regions



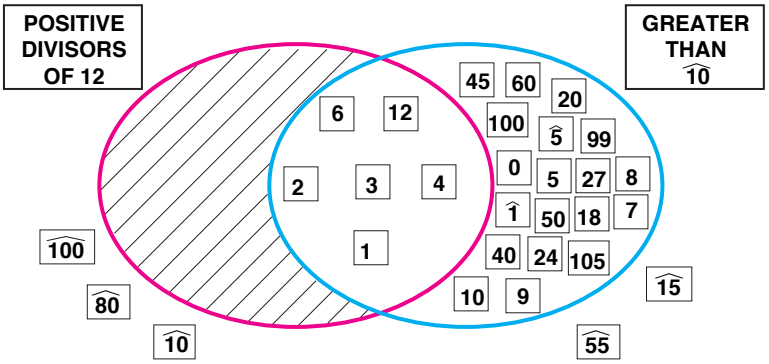
Examples 3: No Empty Regions



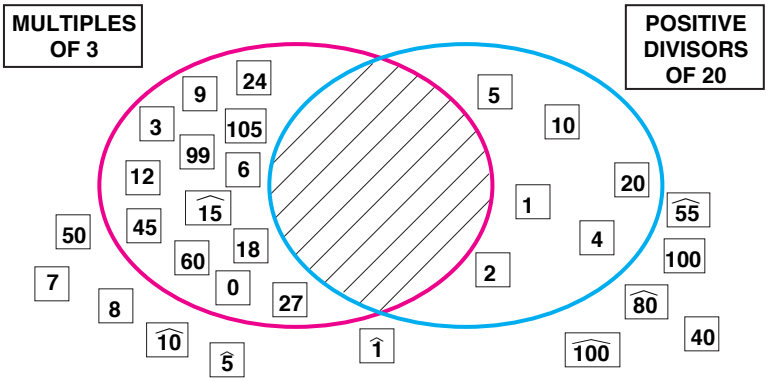
Example 4: One Empty Region



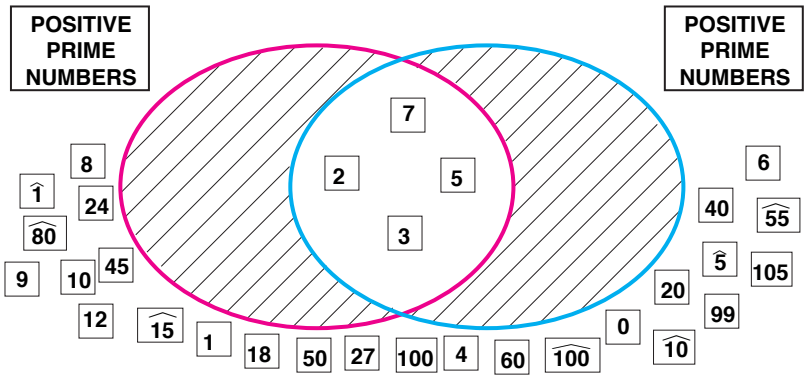
Example 5: One Empty Region



Example 6: One Empty Region



Example 7: Two Empty Regions

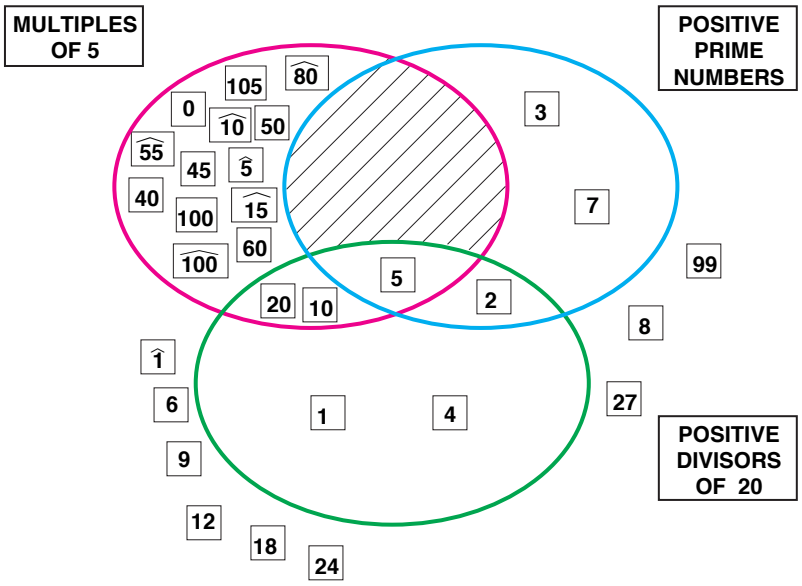


D

THREE STRINGS

Caution: The jump in difficulty from a two-string game to a three-string game is much greater in *The String Game* with numbers than it was in *The String Game* with A-blocks. Hence you should not expect to proceed to three-string games as rapidly as you might have with A-blocks.

Example 1: One Empty Region



Example 2: Five Empty Regions