Workbooks

WORKBOOKS TABLE OF CONTENTS

WORKBOOKS INTRODUCTION

There are many opportunities for the student to work individually during the course of the lessons described in the other content strands. In the Workbooks strand, however, it is this individualized work which becomes the chief end of the majority of lessons. The goal in this strand is to provide students with opportunities

- to review many of the ideas they have met in other content strands;
- to apply their acquired knowledge to new situations requiring various kinds of strategic thinking; and
- to learn how to read and use mathematics workbooks.

The following five workbooks are provided:

- *Variety of Problems #1*
- *Variety of Problems #2*
- *Variety of Problems #3*
- *Variety of Problems #4*
- *Variety of Problems #5*

...and three story-workbooks.

- *Summer Camp*
- *Number 1000's Dream*
- *Seven Secret Numbers*

Each workbook contains problems of varying levels of difficulty. Approximately the first ten pages of each workbook are easy problems, the next ten to twelve pages are average level difficulty, and the last ten pages are more challenging problems. For each workbook, we suggest that all students start work at the easiest level (i.e., on page 2) and then work through as many pages as they can handle during the two lessons scheduled for that workbook. We estimate that, in a typical class, about two-thirds of the students will correctly finish the first ten pages, about one-third will finish the first twenty pages, and a few will finish all or most of the workbook. These proportions will vary from class to class.

This guide contains an answer key for each workbook. The key follows an introduction to the workbook and a suggested collective lesson. The lesson either presents the workbook to the whole class or provides a warm-up activity on a problem similar to one found in the workbook.

The story-workbooks *Summer Camp*, *Number 1000's Dream*, and *Seven Secret Numbers* combine the motivation of a storybook and the problem-solving opportunities of a workbook. These three booklets allow students to become deeply involved in an appealing fantasy as they struggle with difficult mathematics problems. The situations support topics and strategies developed in other strands.

Use of the Workbook Strand for Evaluation Purposes

The workbooks provide an excellent instrument to assess the progress of your students on a regular basis. You may not feel it is necessary to check every page and problem for each student, but you should develop a procedure for checking students' work with which you are comfortable. This may include checking one or more specific pages, discussing some particular mistakes with individual students and letting them correct their work, or just looking carefully at a few pages to be sure the students have understood the general idea of the problems in that particular workbook.

In the Blacklines, you will find a record-keeping tool for each workbook to help you assess student progress in the various strands. This tool may also assist you in parent conferences and in filling out periodic progress reports.

Here are some important points to bear in mind for workbooks.

- Always read the introductory material for each workbook and give the short introductory collective lesson(s).
- All students should start at the beginning of each workbook and progress as far as they can.
- All students should begin a new title on the same day, even if some students have not finished work on the previous title.
- Not all students should be expected to complete a given workbook. Only some students will reach the most challenging problems. Other students may succeed only in doing the easiest problems, although you should not assume this automatically—surprises are not at all uncommon.
- Students should be allowed to use calculators except on pages you specify beforehand.

Note: Some pages in a workbook have problems that are related to calculators or are best attacked with the support of calculators; however, there also are pages for which the use of calculators may not be appropriate. We encourage you to review a workbook before distributing copies of it and to inform the class beforehand on which pages your are not allowing the use of calculators. Certain pages you may wish to use for special evaluation purposes; other pages would not be effective if calculators were allowed, for example, pages that primarily focus on paper-and-pencil computation, "wipe-out" pages that involve subtraction of decimal numbers, pages where using patterns permits students to solve problems from previously solved problems, and pages where the problem is to insert missing decimal points in answers to calculations.

Content Overview

Workbooks

The five *Variety of Problems* workbooks both review and extend many of the ideas introduced in the content strands. The extensions occur through problems which require students to apply the mathematics to new situations or to synthesize their knowledge in new ways.

Lessons: W3, 4, 7, 8, 9, 10, 12, 13, 14, and 15

Summer Camp

In the story-workbook *Summer Camp*, the number 100 recounts the summer adventures of a group of whole numbers. 100 invites the reader to locate 100 and friend 40 in colorful arrow pictures showing boat outings, games of hide-and-seek, mountain climbing, scuba diving, and so on. Students must label the dots; they may use trial-and-error but are encouraged to use composition of arrows, always looking for possible locations of two numbers whose difference is 60.

Lessons: W1 and 2

Number 1000's Dream

In the story-workbook *Number 1000's Dream*, the number 1000 enjoys jumping on four Minicomputer boards for morning exercise. During the exercise, 1000 is always displayed on the Minicomputer but the configurations change. In a dream, 1000 imagines continuing to jump on an infinite number of boards and awakens sad about this seemingly impossible freedom. Upon hearing about the dream, the clever number 0 shows 1000, through a series of jumps, how to make the dream come true. The number 0 introduces 1000 to a decimal bar for the Minicomputer and boards to the right of it. Continuing a sequence of jumps past the decimal bar, the number 1000 discovers a new signature, 999.999.... In this delightful story the student is introduced to a name for a number with infinitely many decimal places.

Lessons: W5 and 6

Seven Secret Numbers

The story-workbook *Seven Secret Numbers* presents a detective story in which the reader discovers the identities of seven secret numbers and four spy numbers through a series of arrow pictures. After the initial clue which limits the range of all 11 stars, each clue involves only some of the secret numbers and the spy numbers, setting up various numerical relationships between them. Throughout the story, the reader must decide how the possibilities from several clues overlap to arrive eventually at a set of numbers that satisfy all of them.

Capsule Lesson Summary

Locate 40 and 100 in arrow pictures in the story-workbook *Summer Camp*. In each picture, arrow labels are given but no dot is labeled. (This is the first of two lessons using this story-workbook.)

Materials

Teacher	• *Summer Camp* Story-Workbook • Colored chalk	Student	• *Summer Camp* Story-Workbook • Paper • Colored pencils, pens, or crayons

Description of Lesson

Students should have scrap paper available during the lesson. Begin with a short discussion about going to summer camp. Let students share their camp experiences for a few minutes.

T: ***We are going to read a story about a summer camp for whole numbers.***

Distribute copies of the story-workbook *Summer Camp*. You may like to let students work with a partner during the discussion of the story-workbook.

Pages 2 to 5

Invite volunteers to read pages 2 and 3 aloud.

T: ***40 and 100 are good friends. How much greater is 100 than 40?***

S: ***60 more.***

While a volunteer reads pages 4 and 5, record this sequence of numbers on the board.

0 1 1 2 3 5 8 13 21 34 55 89

T: ***Let's look for patterns in this sequence of numbers. What do you notice about 5*** (point to it on the list) ***compared to the numbers before it.***

S: ***5 = 2 + 3; 2 and 3 are the two numbers right before 5.***

Look at other numbers in the list in a similar way.

T: ***If this pattern continues, what is the next number after 89?***

S: ***144; 55 + 89 = 144.***

Record 144 as the next number in the list.

T: ***Try to find at least the next five numbers. Record them on your paper.***

Observe and check students' work. Then collectively find the next five or six numbers and record them in sequence on the board.

Answers:

89	144	233	377	610	987
+144	+233	+377	+610	+987	+1 597
233	377	610	987	1 597	2 584

0 1 1 2 3 5 8 13 21 34 55 89

144 233 377 610 987 1597 2584 ⋯

T: ***We could go on forever*** (draw three dots at the right end of the list). ***What if we go in the other direction?***

Put a box to the left of 0 in the list.

T: ***What number goes in the box if the numbers follow the same pattern?***

Accept suggestions without comment, and then lead the class to check using the pattern suggested above.

T: ***We are looking for some number that when added to 0 equals 1.*** $\square + 0 = 1$

S: ***The number in the box must be 1.***

Record 1 to the left of 0 in the list. Draw a box to the left of 1.

T: ***Let's go another step to the left. We want some number that when added to 1 equals 0.*** $\square + 1 = 0$

S: $\hat{1}$.

Continue until there are several numbers recorded to the left of 0.

⋯ $\hat{8}$ 5 $\hat{3}$ 2 $\hat{1}$ 1 0 1 1 2 3 5 8 13 ⋯

T: ***And again we could go on forever*** (draw three dots at the left end of the list). ***What patterns do you notice?***

S: ***The numbers to the left of 0 look like the numbers to the right of 0, except they alternate positive and negative.***

Pages 6 and 7

Allow a few minutes for students to read these pages silently and to try to find 40 and 100 in the picture. Then ask,

T: ***What clue are we given?***

S: *40 and 100 are in the same boat.*

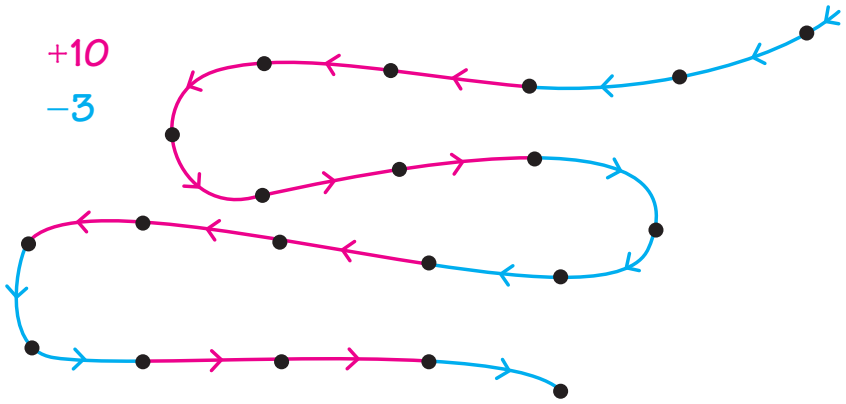
T: *So they could be in either a yellow boat or the green boat.*

S: *They are in the green boat because 40 + 60 = 100.*

T: *Put 40 and 100 in the picture and then label the other dots. When you finish, start reading the next two pages.*

Pages 8 and 9

While students are working, copy the arrow picture on pages 8 and 9 onto the board.



T: *What clue are we given?*

S: *40 is getting logs and 100 is helping cook.*

T: *Where should we look for 40 in the picture?*

Ask a volunteer to indicate the general area in the picture on the board. Likewise, ask a volunteer to indicate the general area in which 100 can be found.

T: *Try to find 40 and 100. Remember that 100 is 60 more than 40.*

After a minute or two ask,

T: *Can someone give us a hint as to how to find 40 and 100 in the picture?*

S: *Six +10 arrows is the same as +60.*

S: *We don't have a +60 arrow, but we do have six +10 arrows.*

Invite a student to label the dots for 40 and 100 in the picture on the board. Direct students to label the dots on pages 8 and 9 in their workbooks.

Pages 10 and 11

When many students are ready, go on collectively to pages 10 and 11.

T: *Look at the picture on pages 10 and 11. Where is the least number in this picture?*

S: *In the first sleeping bag (on the right). At the starting dot of the arrow road.*

T: *And where is the greatest number?*

S: *In the last sleeping bag (on the left). At the ending dot of the arrow road.*

T: ***How many blue +5 arrows are there between these two numbers?*** (Ten)
How many red +1 arrows are there? (Ten)

Hold up your copy of the workbook open to pages 10 and 11.

T: ***How much greater is the number here*** (point to the ending dot of the arrow road) ***than the number here*** (point to the starting dot of the arrow road)***?***

S: ***60 more.***

T: ***Why is that important?***

S: ***Because 100 is 60 more than 40.***

The class should conclude that 40 is the starting number of the road and that 100 is the ending number.

Let students work at their own pace for the rest of the period, completing pages 2 through 11 and possibly continuing to other pages in the book. Collect the story-workbooks at the end of the class period for your review and for use in Lesson W2.

Writing Activity

You may like students to take lesson notes on some, most, or even all their math lessons. The "Lesson Notes" section in Notes to the Teacher gives suggestions and refers to forms in the Blacklines you may provide to students for this purpose. In this lesson, for example, students can write about how they try to locate 40 and 100 in one of the arrow pictures.

Capsule Lesson Summary

Continue working in the story-workbook *Summer Camp* to locate 40 and 100 in arrow pictures. In each picture, arrow labels are given but no dot is labeled. (This is the second of two lessons using this story-workbook.)

Materials

Teacher	• *Summer Camp* Story-Workbook • Colored chalk	Student	• *Summer Camp* Story-Workbook • Colored pencils, pens, or crayons • Worksheets W2*, **, and ***

Description of Lesson

Return students' copies of the story-workbook *Summer Camp*.

T: ***Who is telling this story?***

S: ***The number 100.***

T: ***Who is 100's best friend?***

S: ***The number 40.***

T: ***How much greater is 100 than 40?***

S: ***60 more.***

Pages 12 and 13

Allow a few minutes for students to read these pages silently and to try to find 100 and 40 in the picture.

T: ***What game are 100 and 40 playing on these pages?***

S: ***Hide and Seek.***

T: ***Did you discover where they are hiding? How did you find them?***

S: ***The blue arrows are for +30, and adding 30 twice is the same as adding 60 once. So I looked for two blue arrows together.***

S: ***One of them is in the box, and one of them is in the gray trash can.***

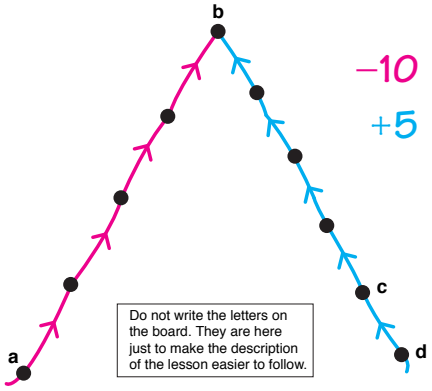
T: ***Which goes where?***

S: ***100 is greater, so 100 must be at the ending dot of the two blue arrows. 100 must go in the box.***

Direct students to label all of the dots in the picture if they have not already done so. Then they should continue working at their own pace on other problems in the book.

Pages 14 and 15

If many students are having difficulty with the problem on pages 14 and 15, lead a collective discussion of it. Draw the arrow picture on the board.



Guide students in comparing the numbers at **a**, **b**, and **d**. Conclude that the number at **a** is 40 more than the number at **b**, and that the number at **b** is 25 more than the number at **d**. Therefore, the number at **a** is 65 more than the number at **d**, and 100 must be at **a** and 40 at **c**.

Direct students to label all of the dots in the picture if they have not already done so. Then they should continue working at their own pace on other problems in the book.

Pages 18 and 19

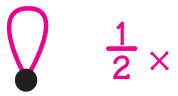
If many students are having difficulty with the problem on pages 18 and 19, lead a collective discussion of it.

T: ***Do you know what the big hint is on these pages?***

S: ***To find who the guide is.***

S: ***The guide is at the dot with the loop because it has a hat above it.***

Draw the loop on the board and write the key for red arrows.



T: ***What does the loop tell us?***

S: ***Half of the number is the same number.***

T: ***What number is the guide?***

S: ***0, ½ x 0 = 0, and 0 is the only number that could be at the dot with a loop.***

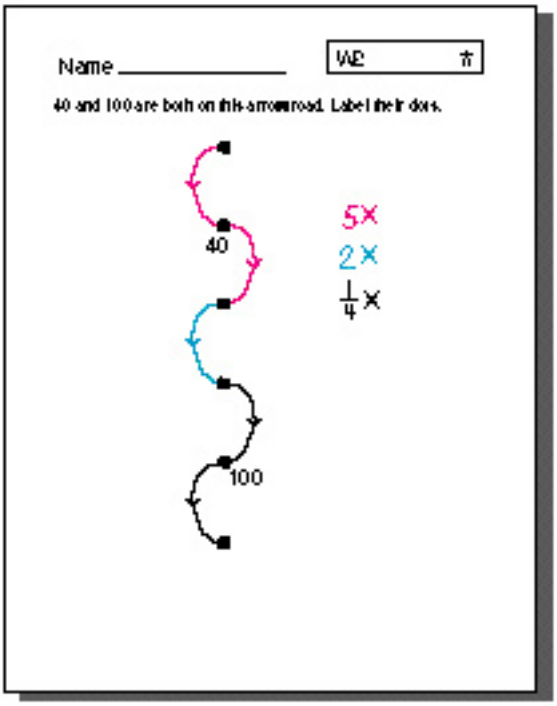
Students should conclude that 0 is the guide for the numbers visiting the Caverns of Ma Kro.

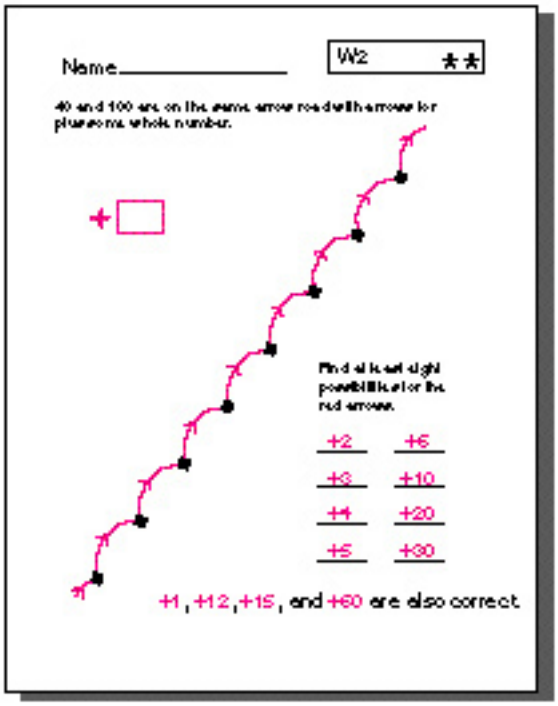
Direct students again to work at their own pace on the problems in the book. If many students are having difficulty with a particular problem, do not hesitate to discuss the page with the class or a small group. At the end of the class period, collect the story-workbooks for your review.

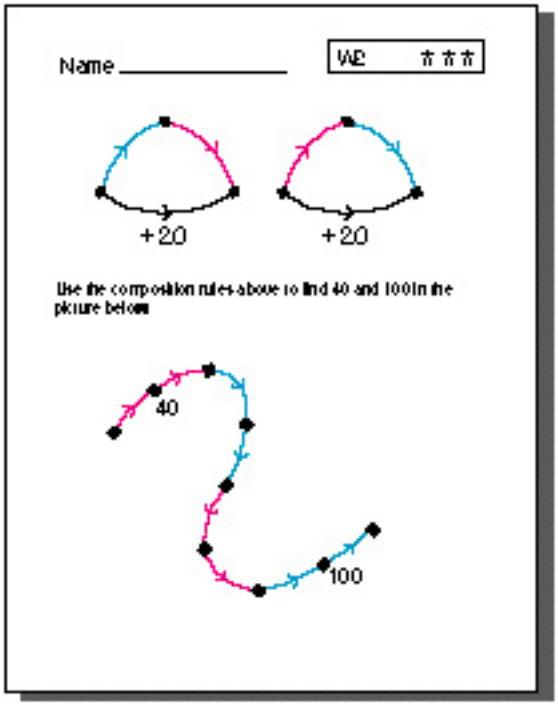
Worksheets W2*, **, and *** are available for students who finish the story-workbook and have time to solve some other problems with 40 and 100.

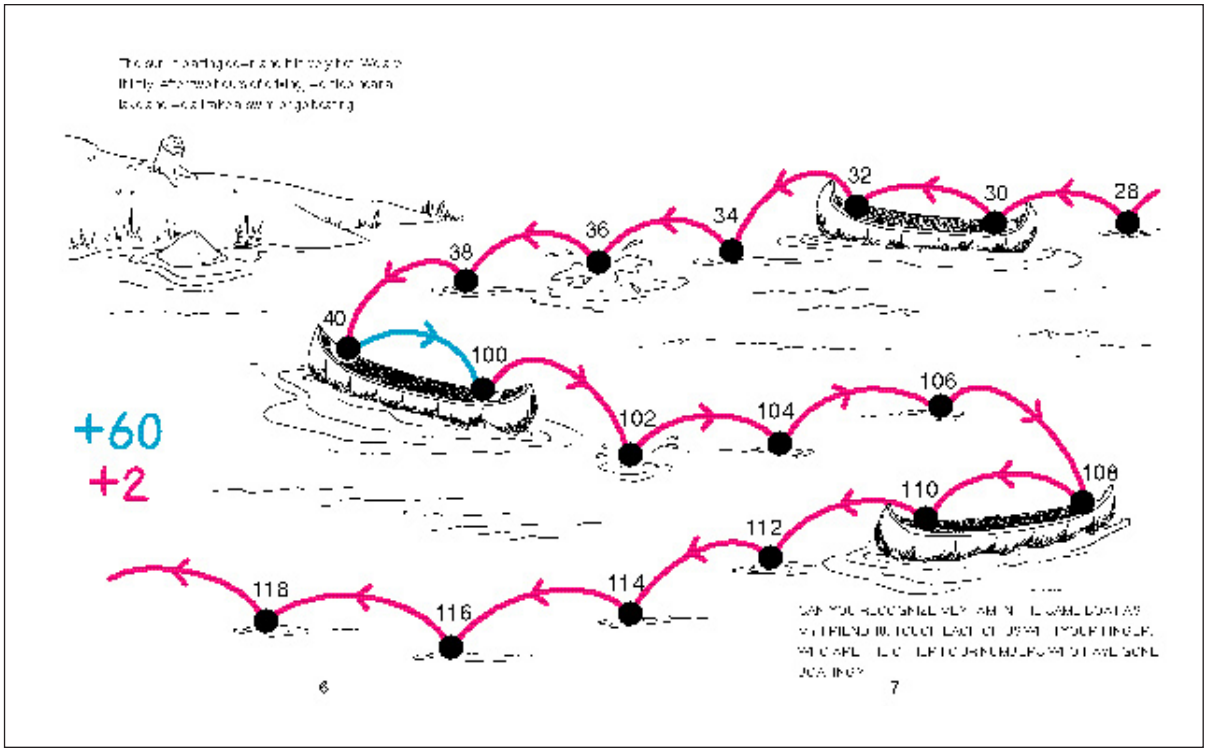
Writing Activity

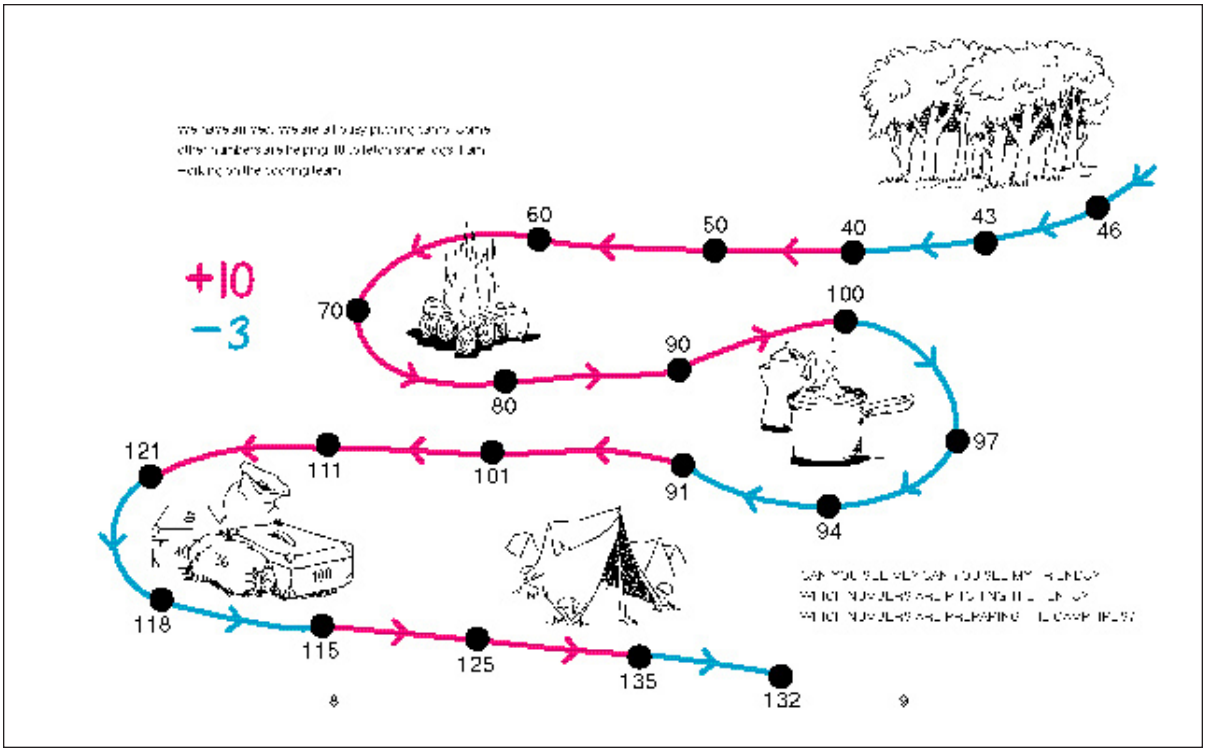
Some students may enjoy writing another adventure for the friends 40 and 100 at summer camp. Instruct them to include an arrow picture in which to locate 40 and 100.


Name _____
W2 ★
40 and 100 are both on this arrow road. Label their dots.
40
100
5×
2×
1/4×


Name _____
W2 ★★
40 and 100 are on the same arrow road with arrows for
plus some whole number.
+ □
Find at least eight
possibilities for the
red arrows.
+2 +6
+3 +10
+4 +20
+5 +30
+1, +12, +15, and +60 are also correct.


Name _____
W2 ★★★
+20
+20
Use the composition rules above to find 40 and 100 in the
picture below
40
100


+60
+2
28
30
32
34
36
38
40
100
102
104
106
108
110
112
114
116
118


+10
−3
40
43
46
50
60
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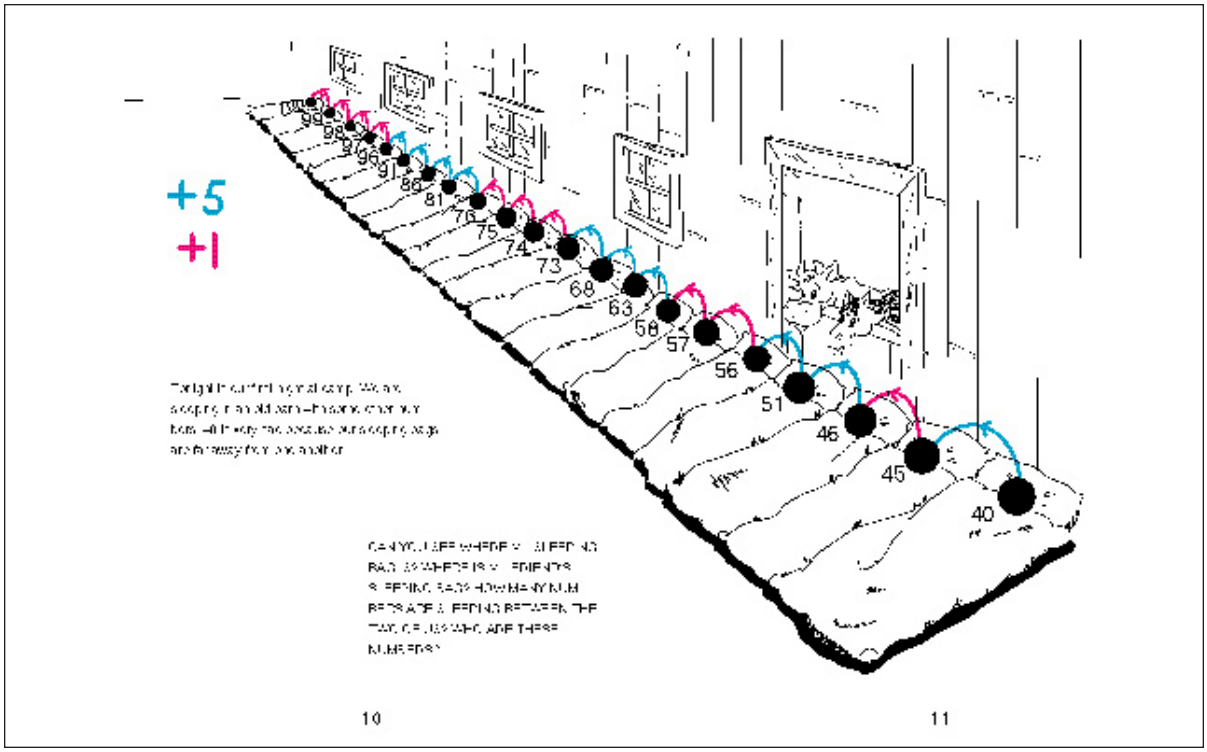
90
100
91
94
97
101
111
121
118
115
125
135
132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+1
99
96
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86
81
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74
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68
63
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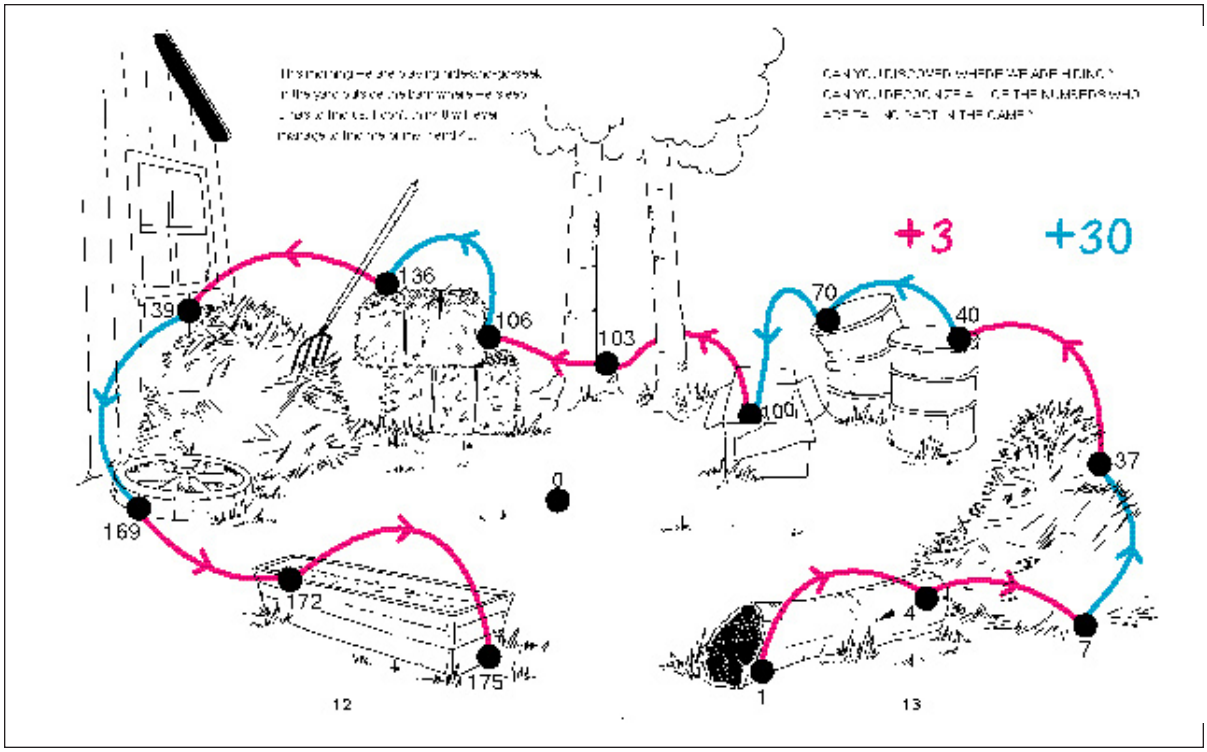
51
46
45
40
10
11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139
136
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37
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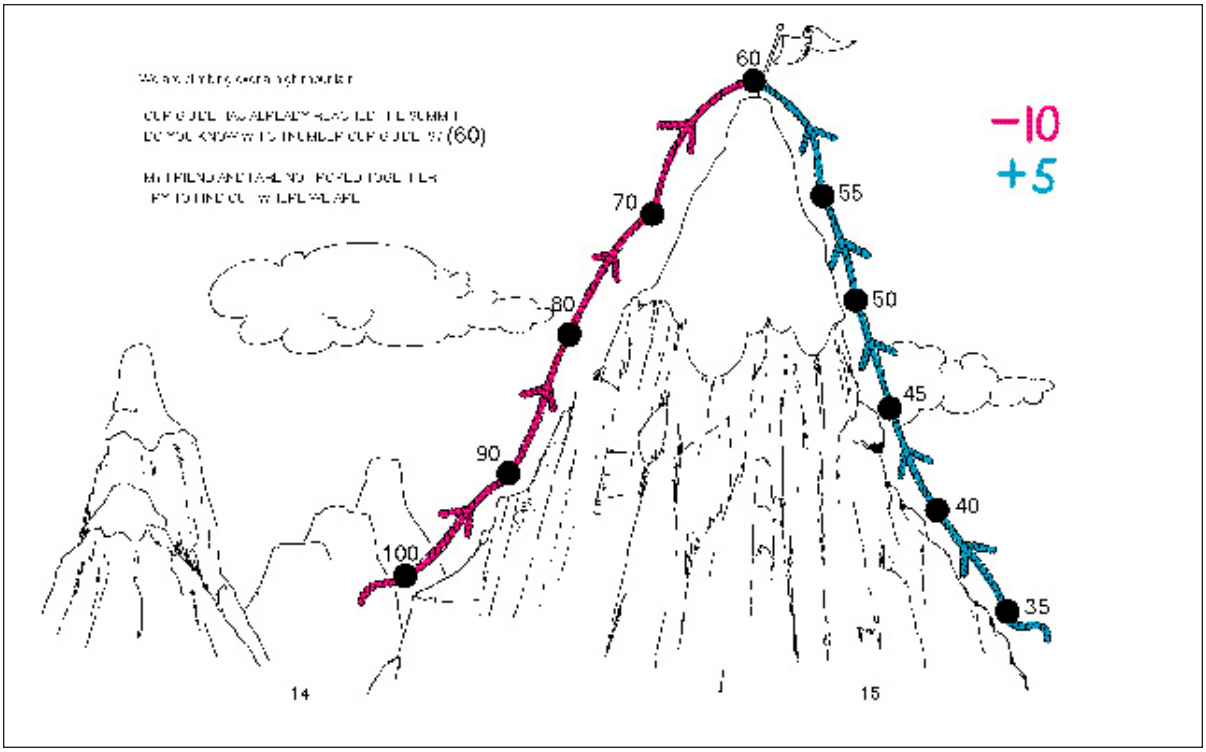
172
175
4
7
1
12
13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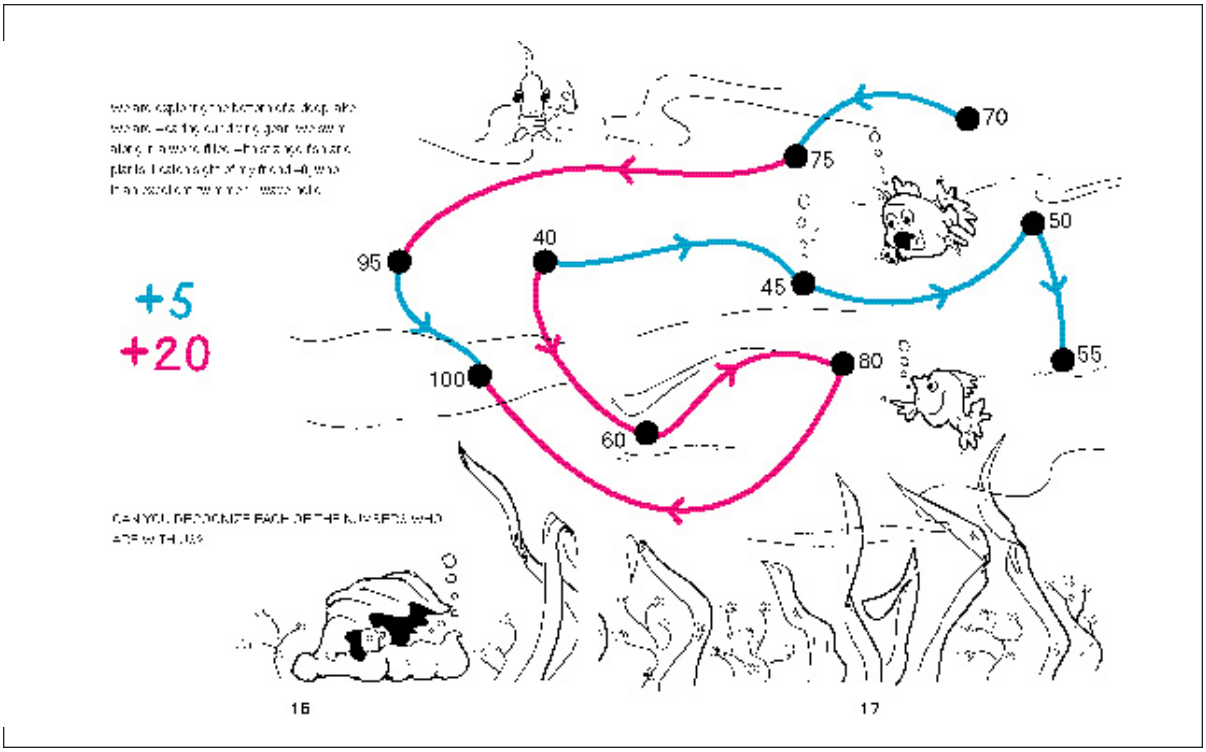
35
−10
+5
14
15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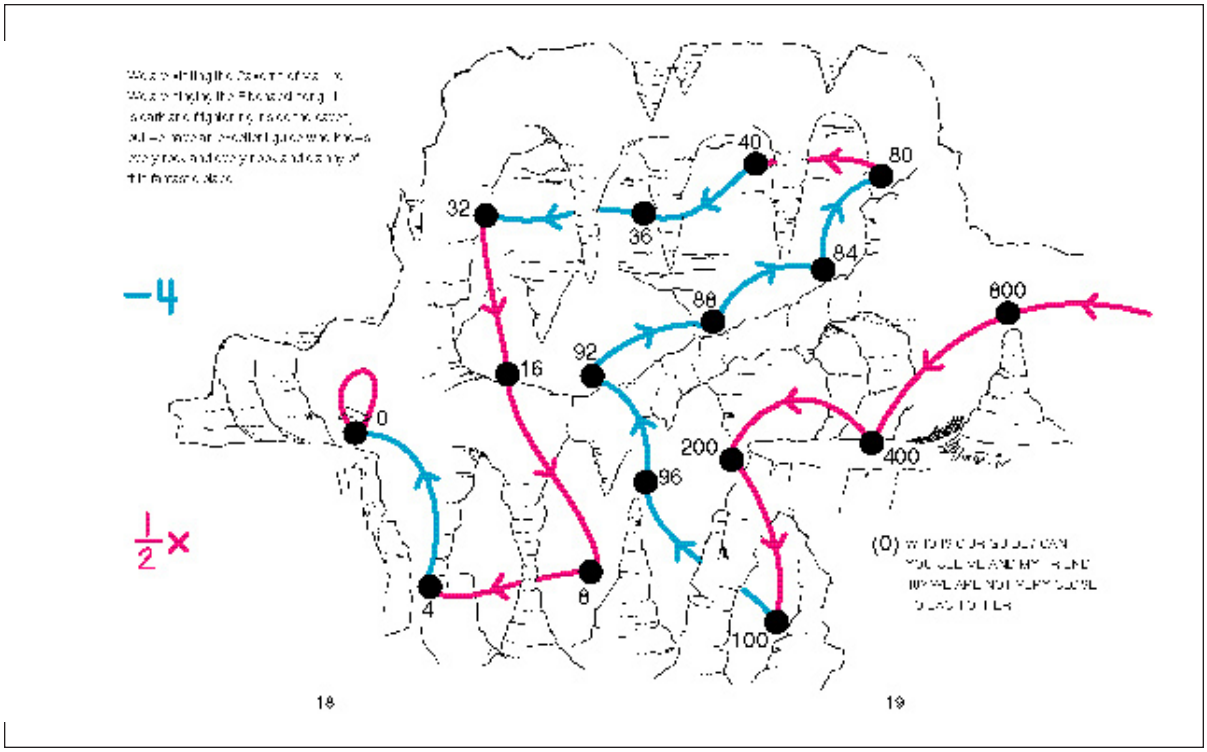
60
+5
+20
16
17


−4
$\frac{1}{2}\times$
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16
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36
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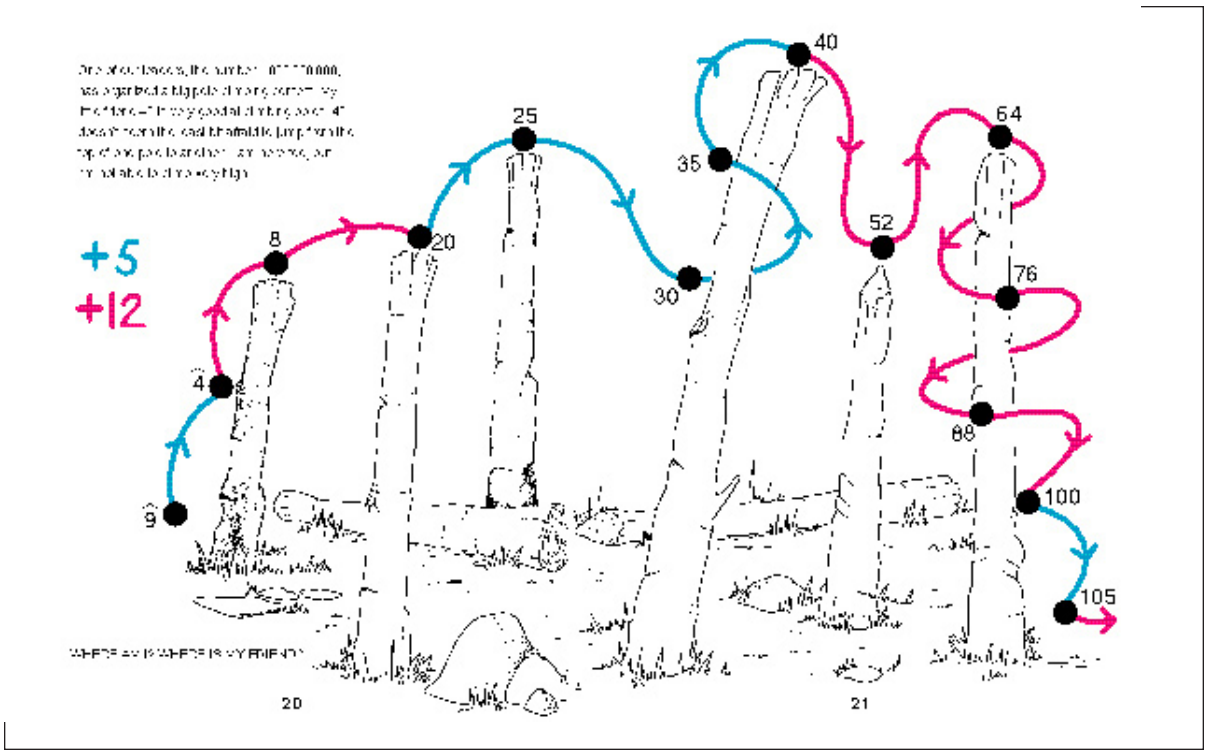
80
84
88
92
96
100
200
400
800
18
19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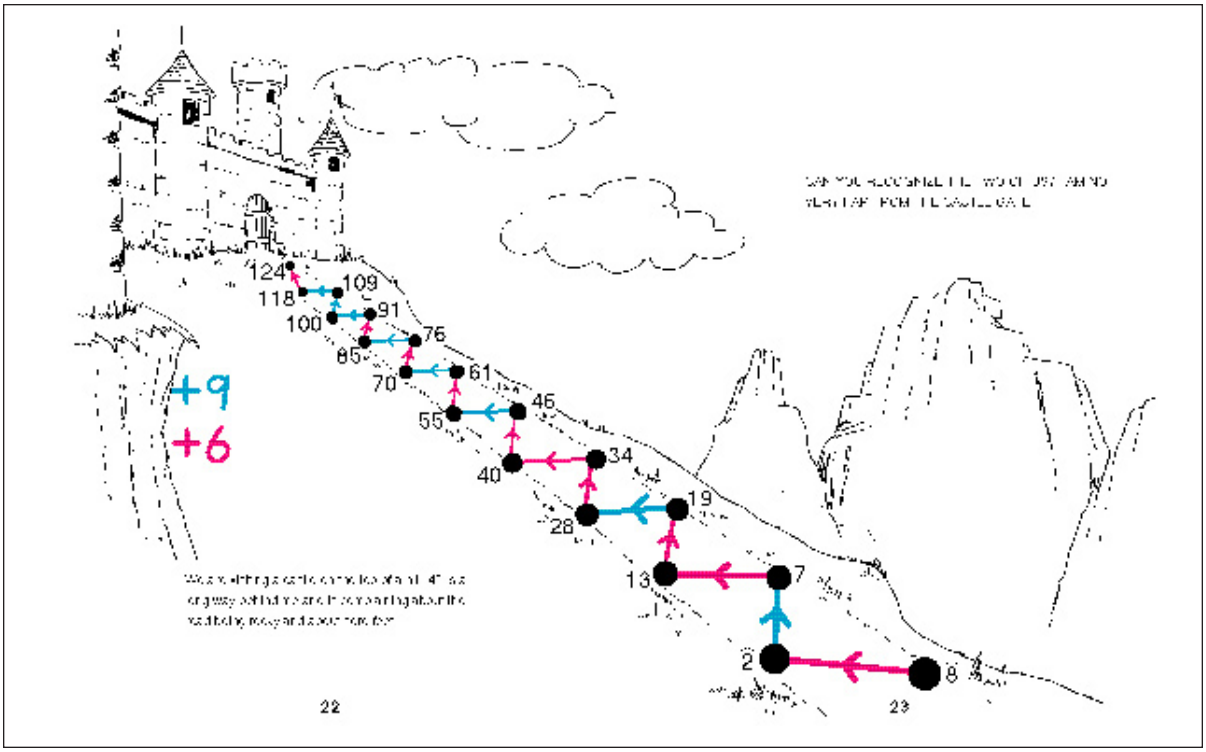
52
64
76
88
100
105
20
21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118
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100
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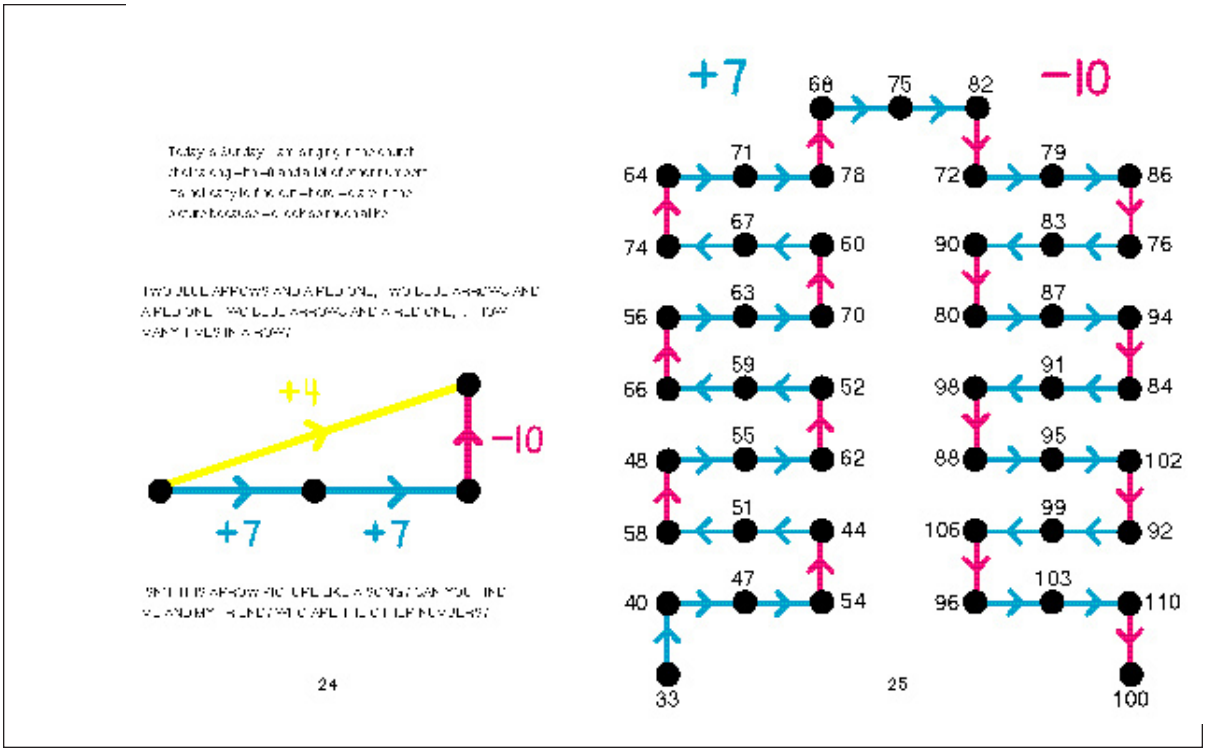
61
55
46
40
34
28
19
13
7
2
8
+9
+6
22
23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−10
+7
+7
24
+7
−10
68
75
82
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90
83
76
56
63
70
80
87
94
66
59
52
98
91
84
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100
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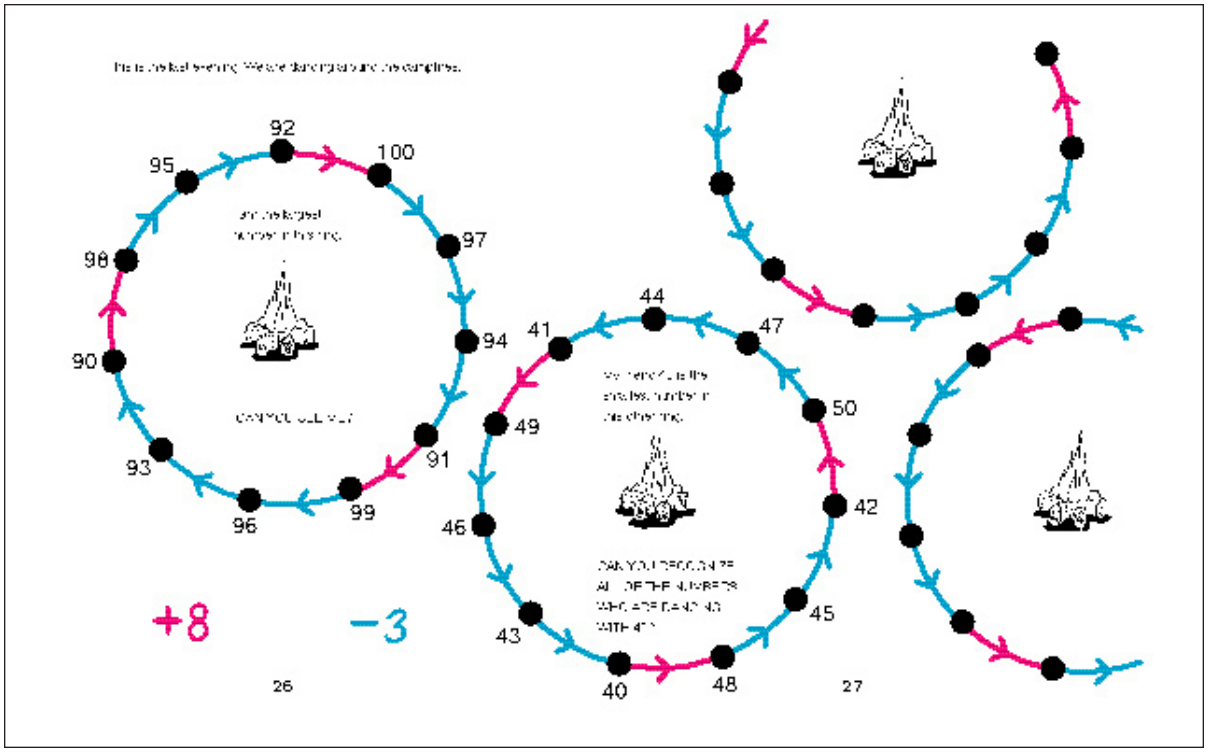
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+8
-3
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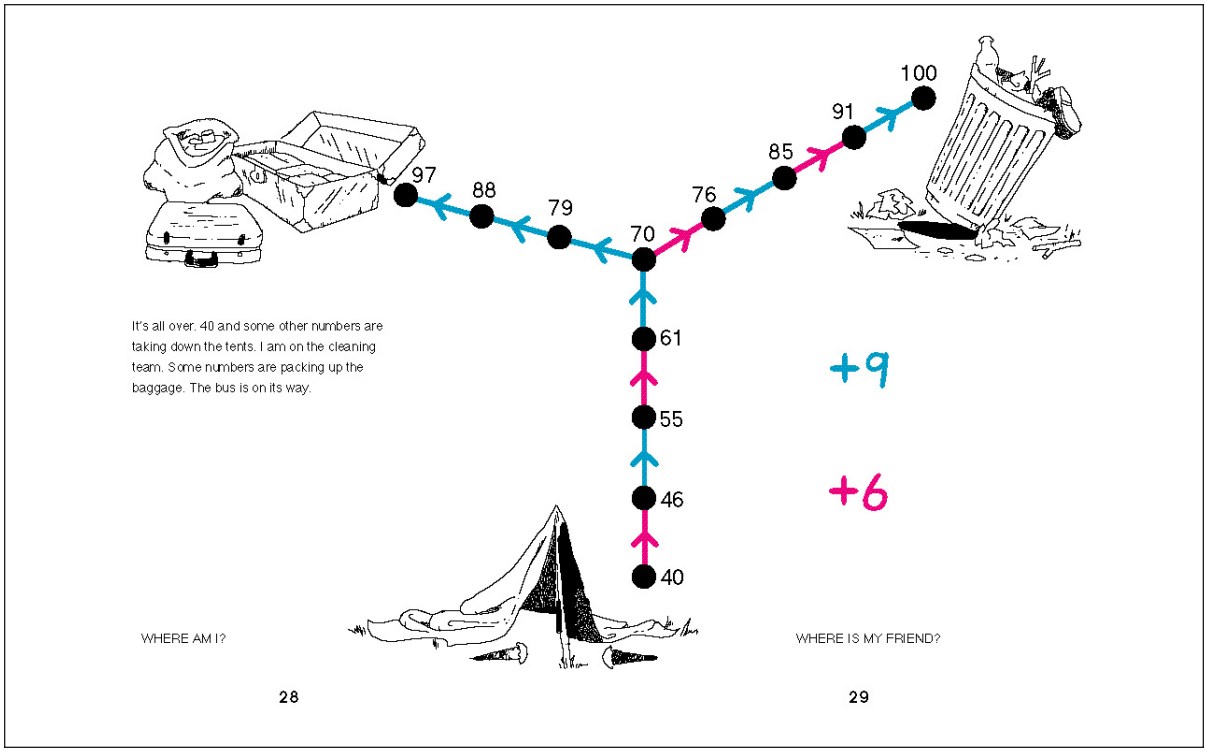

100
91
85
97
88
79
76
70
61
55
46
40
+9
+6
It's all over. 40 and some other numbers are taking down the tents. I am on the cleaning team. Some numbers are packing up the baggage. The bus is on its way.
WHERE AM I?
WHERE IS MY FRIEND?
28
29

Capsule Lesson Summary

Review encoding and decoding numbers on the binary abacus. Begin the workbook *Variety of Problems #1*. (This is the first of two lessons using this workbook.)

Materials

Teacher	Student
• Colored chalk	• *Variety of Problems #1* Workbook • Metric ruler • Colored pencils, pens, or crayons • Calculator

Description of Lesson

Draw a binary abacus on the board and label the ones place.

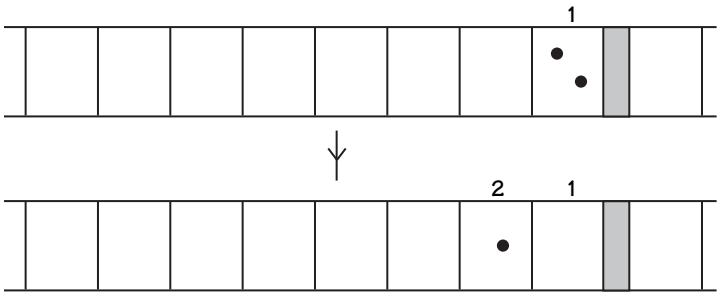
T: ***This is a binary abacus. You may remember this abacus from the story about Clinton Street.***

If students recall the binary abacus, let them make comments and describe how it works. In particular, review the rule for trading checkers on the binary abacus.

> Two checkers on a board
> trade for
> one checker on the next board to the left.

Put two checkers on the ones board.

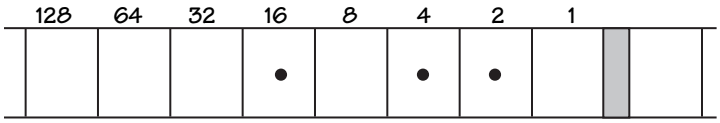
T: ***What number is this?*** (2)
What trade can we make?



Let a student trade the two checkers on the ones board for one checker on the next board to the left. Again ask what number, and give the board its binary place value.

Continue in this way, labeling eight or nine binary places. Each time, observe that two checkers on a board trade for one checker on the next board to the left. Students should recognize the doubling pattern that results.

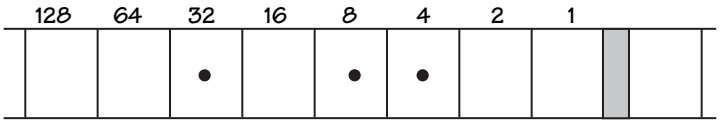
Put this configuration on the abacus.



T: ***What number is this?***

S: ***22. 16 + 4 + 2 = 22.***

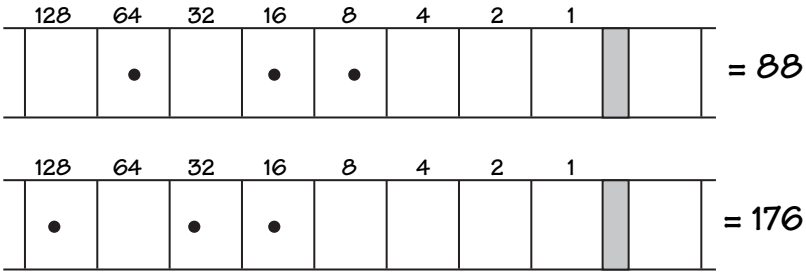
Move each checker one board to the left.



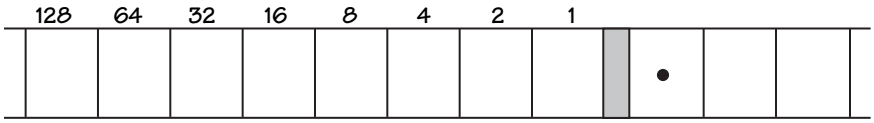
S: ***44. 32 + 8 + 4 = 44.***

S: ***44. 2 x 22 = 44. Moving the checkers one board to the left doubles the number.***

Ask students to identify these numbers on the binary abacus (answers are on the right). Emphasize the doubling pattern.



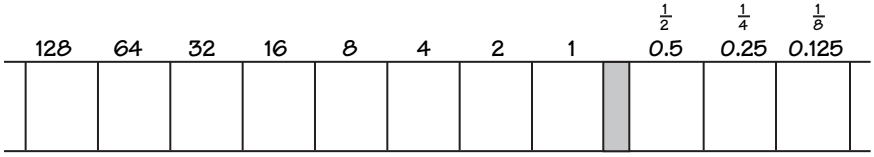
Extend the abacus a few places to the right and put on the following configuration.



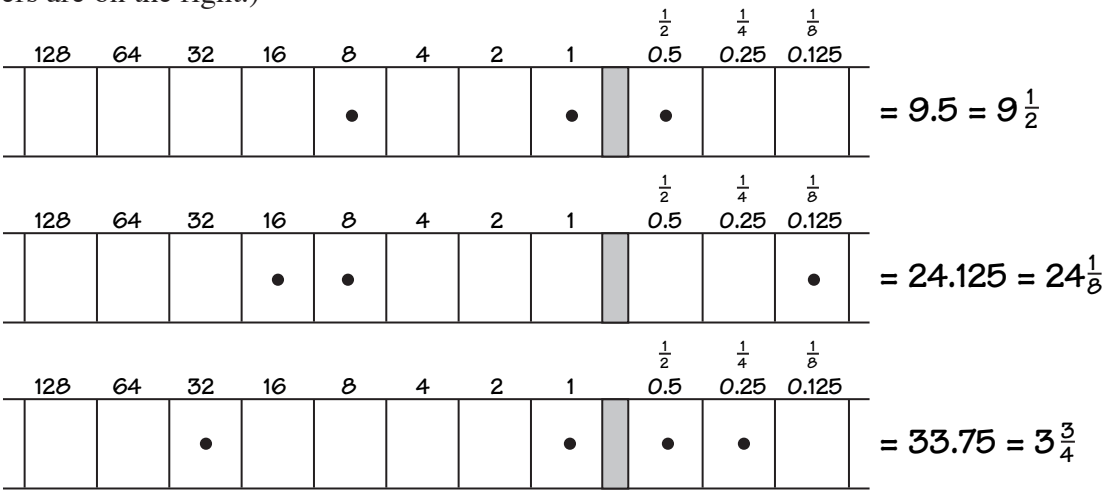
T: ***What number is this?***

S: ***½ or 0.5. Two checkers on that board shows 1, so one checker shows ½ or 0.5.***

Ask students to label several boards to the right of the bar.



Put the following configurations on the abacus and ask students to identify the numbers. (Answers are on the right.)



Distribute copies of the workbook *Variety of Problems #1* and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of the class period, collect the workbooks for your review. They will be used again in Lesson W4.

Capsule Lesson Summary

Solve a calculator puzzle in which 100 is put on the calculator using only a limited set of keys. Continue individual work in the workbook *Variety of Problems #1*. (This is the second of two lessons using this workbook.)

Materials

Teacher	• Calculator	Student	• Calculator • *Variety of Problems #1* Workbook • Colored pencils, pens, or crayons • Metric ruler

Description of Lesson

Arrange for every student to have access to a calculator. You may like to let students work with a partner.

T: ***Today I have a calculator puzzle for you. The puzzle requires you only use a few of the keys on the calculator.***

Write these key symbols on the board: [4], [6], [+], [−], [×], [÷], [=].

T: ***You may use just these keys:*** [4], [6], [+], [−], [×], [÷], [=]. ***But you may use them in any way you like. Start with 0 on your calculator display, and then try to put 200 on the display.***

You may need to remind students that they may use only the keys in the list on the board. To help them remember, suggest students record the sequence of keys they press to get 200.

When many students have found at least one solution, begin to record some of their suggestions on the board. For example:

200: [6][6] [×] [4] [−] [6][4] [=]
[4][4] [×] [6] [−] [6][4] [=]
[6][4] [+] [4][4] [=] [=] [=] [+] [4] [=]
[+] [6][4] [=] [=] [=] [+] [4] [=] [=]
[+] [6][6] [=] [=] [=] [+] [6] [−] [4] [=]
[6] [×] [=] [=] [−] [4] [=] [=] [=] [=]
[6][4][4] [−] [4][4][4] [=]
[4][6] [+] [4] [×] [4] [=]

Note: Some of these solutions depend on special features of the calculator. You may want to read "Role and Use of Calculators" in Section One: Notes to the Teacher to learn more about the features used here.

Try to get a variety of solutions. Sometimes one student's solution will result in several similar solutions from other students. One way to put a little additional challenge in this activity is to announce that it costs a dollar (or penny) for each key pressed. Challenge students to make their solutions as cheap as possible.

Distribute students' copies of the workbook *Variety of Problems #1*. Ask students first to correct or complete pages from their previous work in this workbook. Your review of the workbooks may indicate that a short collective discussion about a particular page is needed.

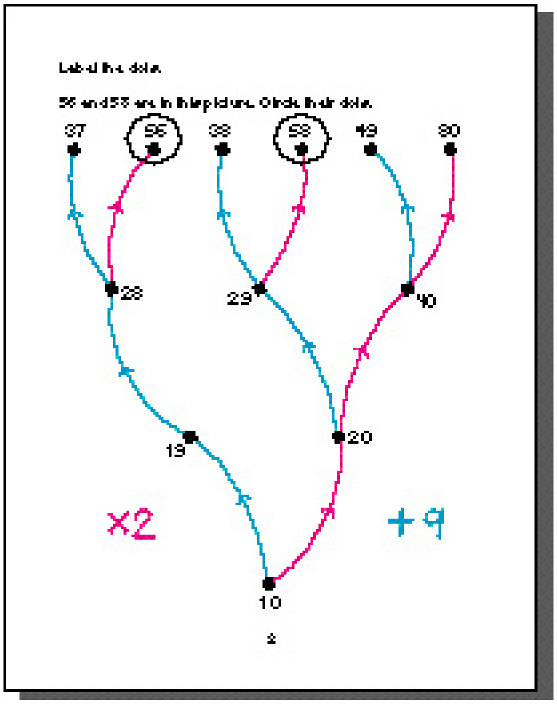
At the end of the class period, collect the workbooks for your review. After checking them, you may wish to ask some students to work further in their workbooks at school during a study time or at home as an assignment.

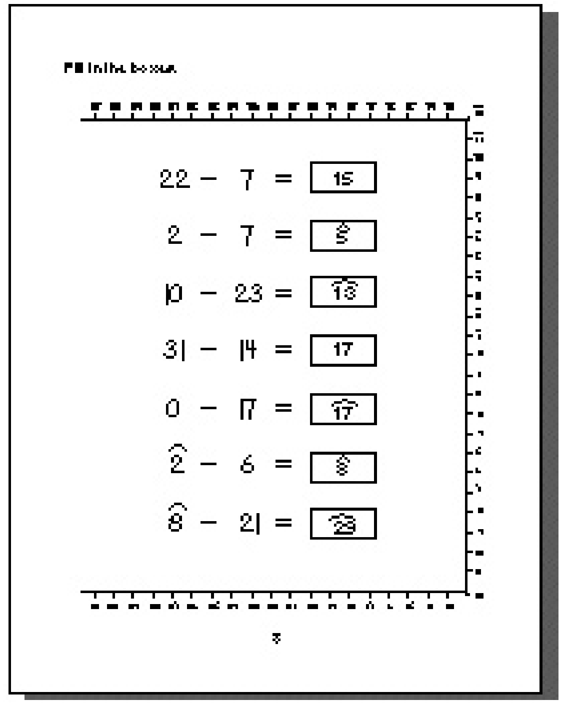
Assessment Activity

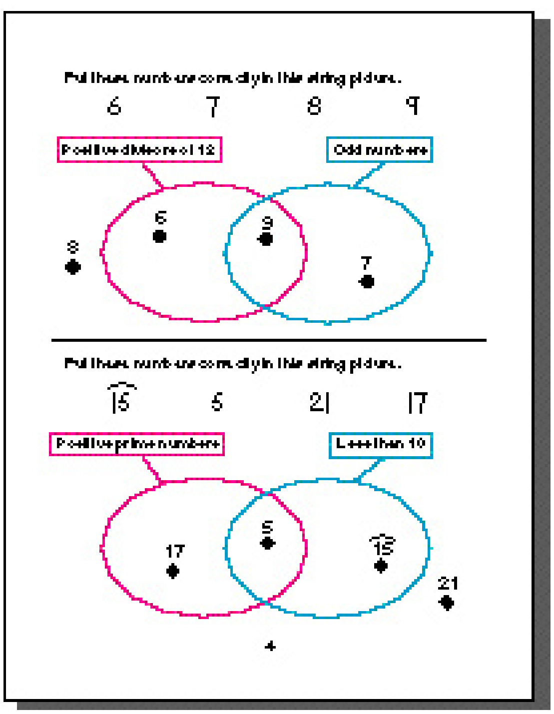
An individual student progress record for the workbook is available on Blackline W4(a). You may like to use this form to monitor student work.

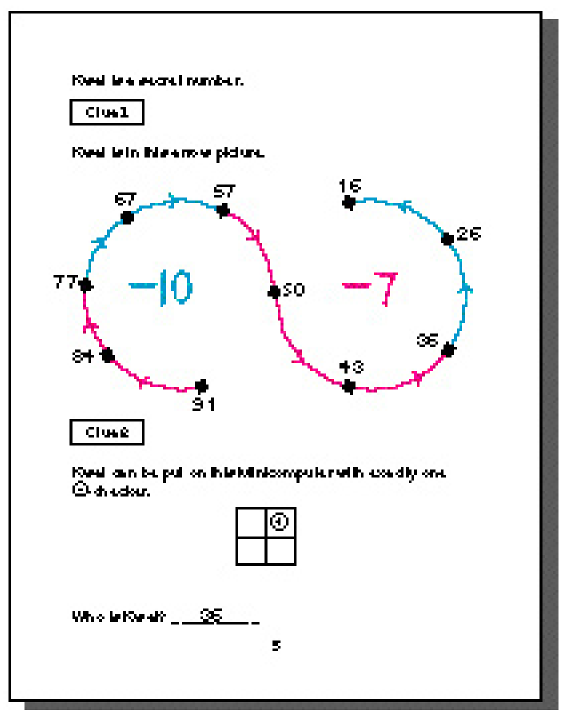
Home Activity

If you choose to send this workbook home with students, you may want to include a letter to parents/guardians. Blackline W4(b) has a sample letter.


37
56
38
58
49
80
28
29
40
19
20
×2
+9
10
2


22 − 7 = 15
31 − 14 = 17
3


6 7 8 9
Positive divisors of 12
Odd numbers
6
9
8
7
5
17
15
21
4


Clue 1
67
57
16
26
77
−10
50
−7
84
36
43
91
Clue 2
5

For each picture, circle the statement you think is true about the length of the two blackline segments. Do not measure them.

A is longer than B. | A is the same length as B. | A is shorter than B.

C is longer than D.
C is the same length as D.
C is shorter than D.

E is longer than F.
E is the same length as F.
E is shorter than F.

Measure the blackline segments to check if your guesses were correct.

A 5.5 cm B 5.5 cm Were your guesses correct? _____

C 2.7 cm D 2.7 cm Were your guesses correct? _____

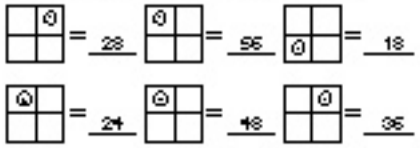
E 4.4 cm F 3.4 cm Were your guesses correct? _____

6

Kiki is a secret number.

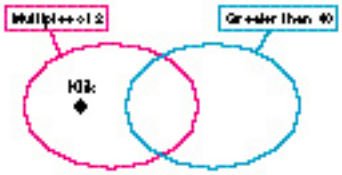
Clue 1

Kiki is one of these numbers on the Mini-computer.



Clue 2

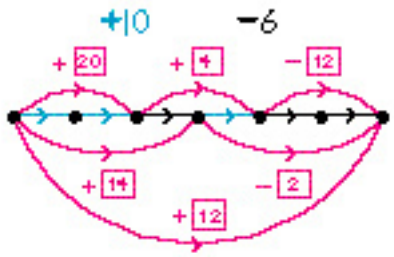
Kiki is in the starting picture.



Who is Kiki? 24

7

Label the red arrows.

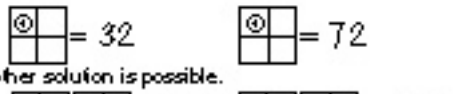


Draw three +18 arrows in red in this picture.

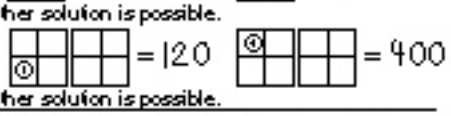
8

Put each number on the Mini-computer using exactly one of these clockwise:

② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨



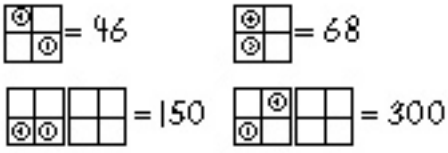
Another solution is possible.



Another solution is possible.

Put each number on the Mini-computer using exactly two of these clockwise:

② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨



9

Other solutions are possible.

Zoe is a secret number.

Clue 1

Zoe can be put on this Mini-computer by adding one negative checker.

Zoe could be 15, 15, 13, or 9.

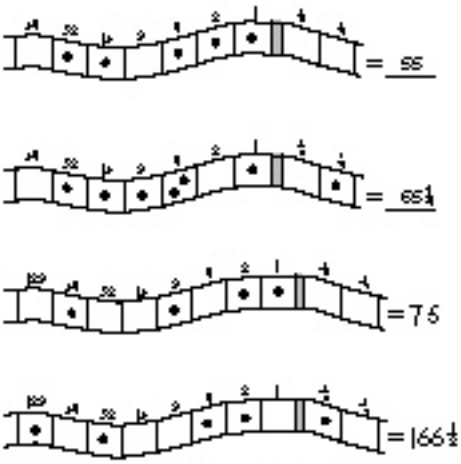
Clue 2

Who is Zoe? 15

10

BINARY ABACUS

Complete.



Other solutions are possible, but not with fewer checkers.

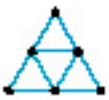
11

For each picture, count the dots and the small triangles.

1 row

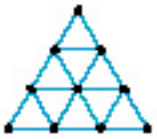
Number of dots 3
Number of small triangles 1

2 rows



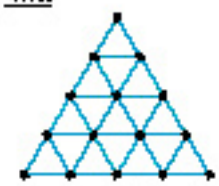
Number of dots 6
Number of small triangles 4

3 rows



Number of dots 10
Number of small triangles 9

4 rows



Number of dots 15
Number of small triangles 16

12

Use your answers on page 12 to complete this table. Ask your teacher to check the number in your table below you finish the page.

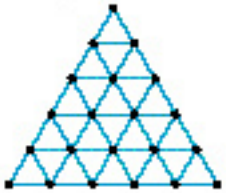
Number of rows	1	2	3	4	5
Number of dots	3	6	10	15	?
Number of small triangles	1	4	9	16	?

Look for number patterns in the table.

Use a pattern to predict the number of dots in a figure with 5 rows. 21

Use a pattern to predict the number of small triangles in a figure with 5 rows. 25

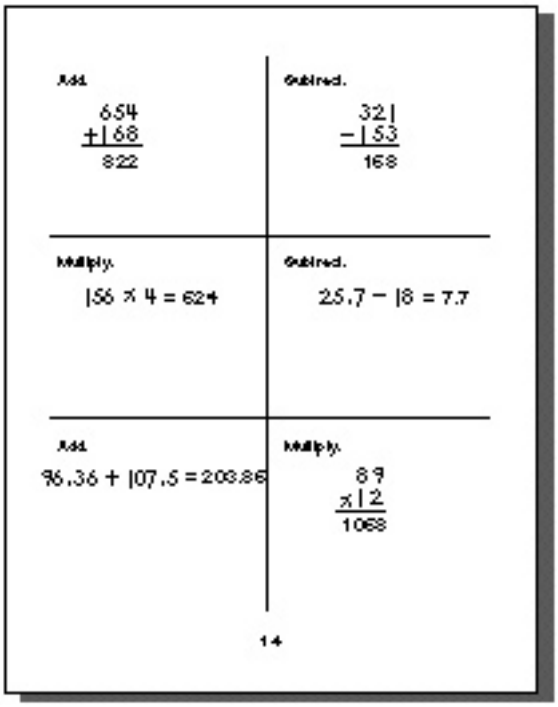
Use this picture to check your predictions.

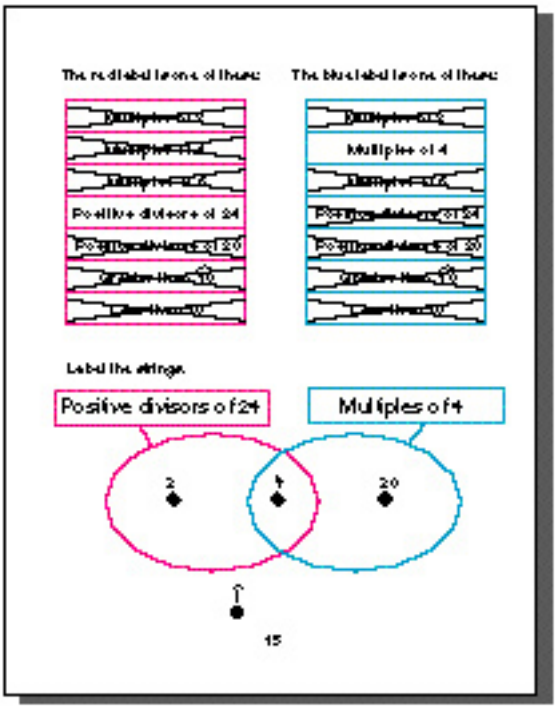


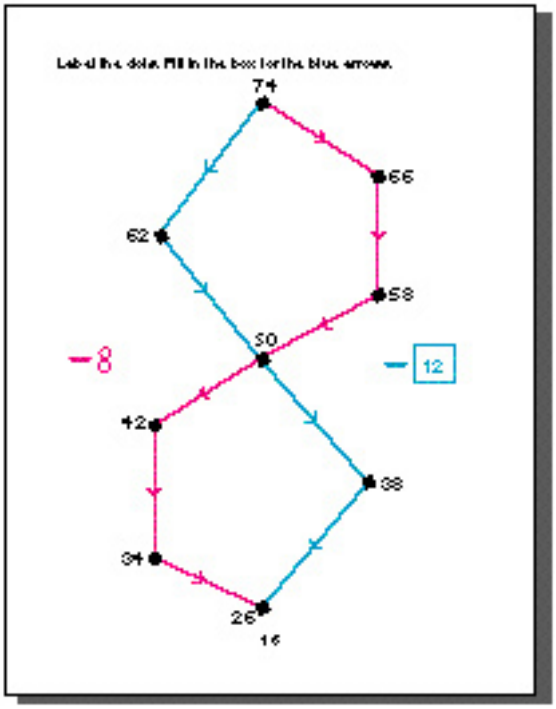
Number of dots 21 Was your prediction correct? ____

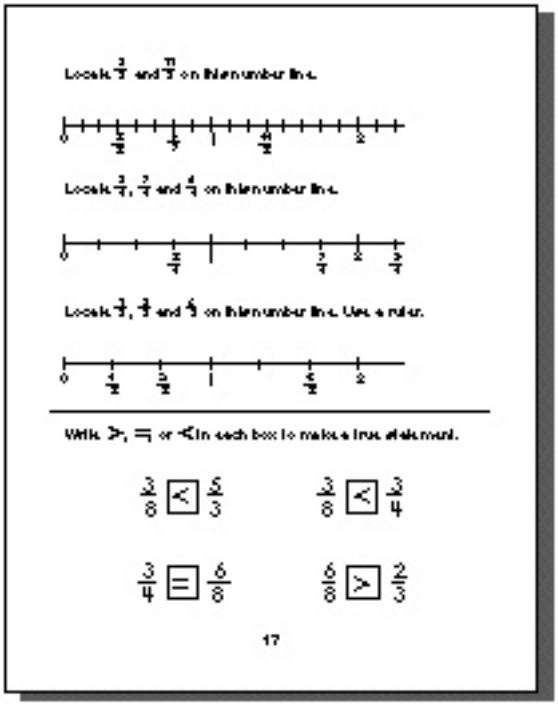
Number of small triangles 25 Was your prediction correct? ___

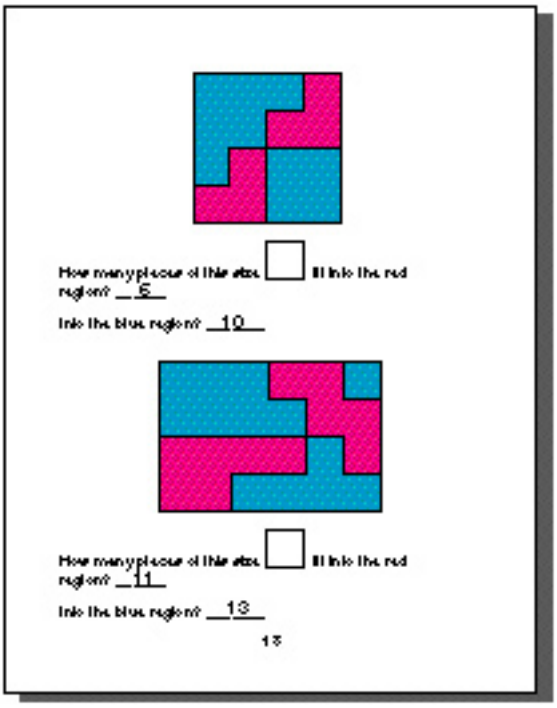
13

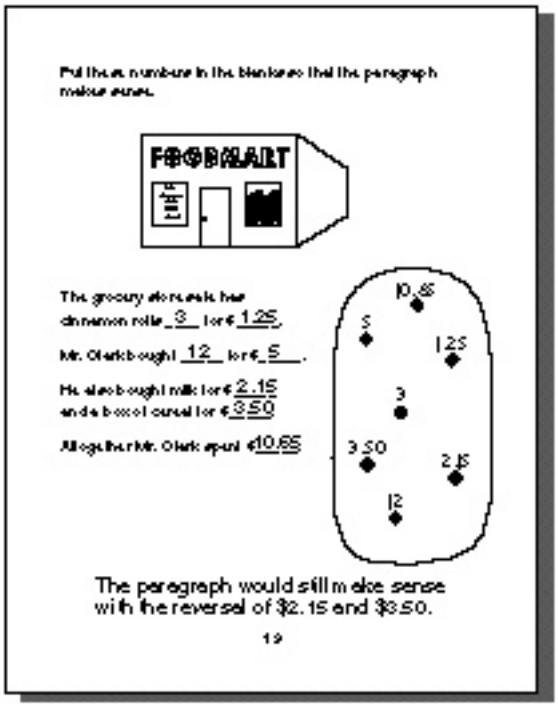

Add.
654
+168
822
Subtract.
321
−153
168
Multiply.
156 × 4 = 624
Subtract.
25.7 − 18 = 7.7
Add.
96.36 + 107.5 = 203.86
Multiply.
89
×12
1068
14

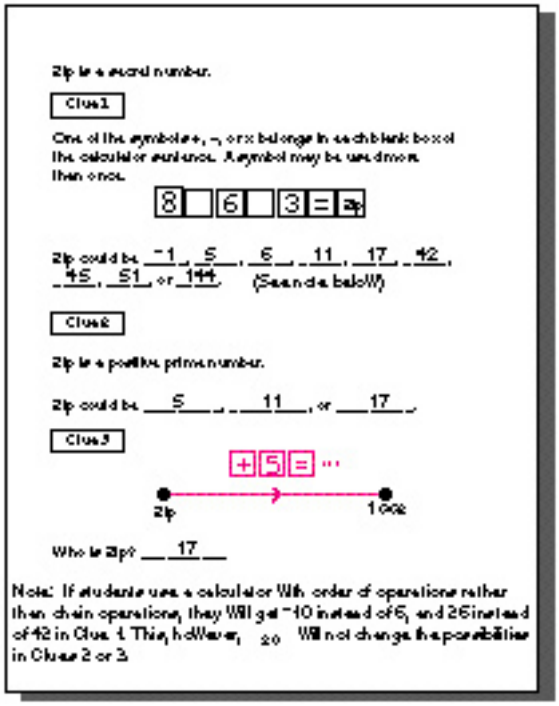

Label the strings.
Positive divisors of 24
Multiples of 4
2
4
20
1
15

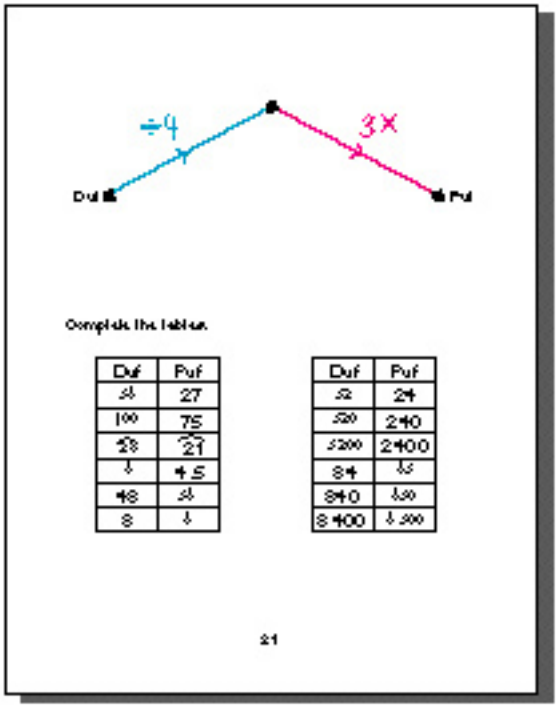

74
66
62
58
50
−8
−12
42
38
34
26
16

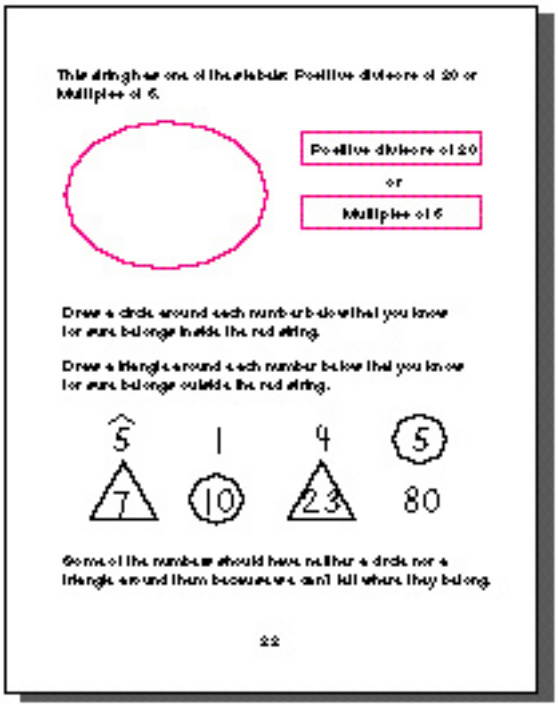

Write >, =, or < in each box to make a true statement.
17

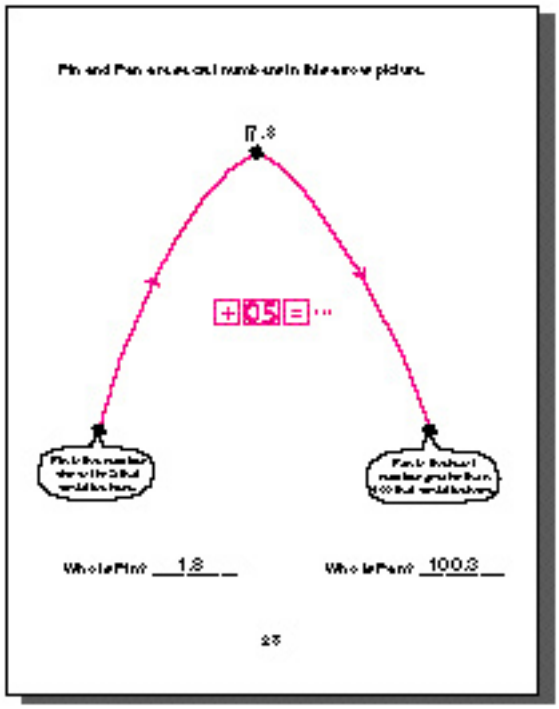

How many pieces of this size ☐ fit into the red region? 5
Into the blue region? 10
How many pieces of this size ☐ fit into the red region? 11
Into the blue region? 13
13

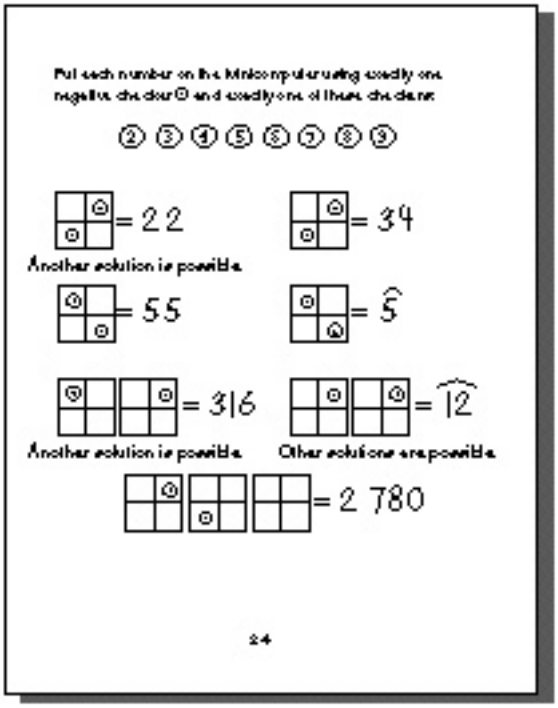

Put the six numbers in the blanks so that the paragraph makes sense.
FOOD MART
The grocery store sells 3 cinnamon rolls for € 1.25.
Mr. Clark bought 12 for € 5.
He also bought milk for € 2.15 and a box of cereal for € 3.50
Altogether Mr. Clark spent € 10.65
10.65
5
1.25
3
3.50
2.15
12
The paragraph would still make sense with the reversal of $2.15 and $3.50.
19

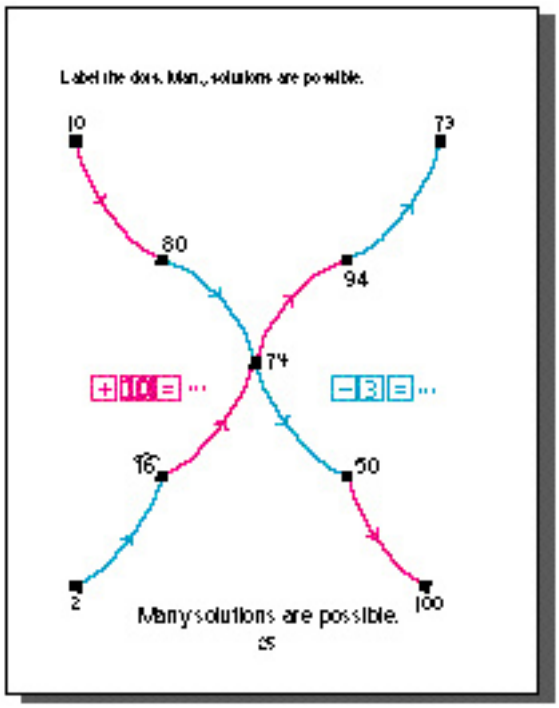

Zip is a secret number.
Clue 1
One of the symbols +, −, or × belongs in each blank box of the calculator sentence. A symbol may be used more than once.
8 ☐ 6 ☐ 3 = Zip
Zip could be −1, 5, 6, 11, 17, 42, 45, 51, or 144. (See clue 3 below)
Clue 2
Zip is a positive prime number.
Zip could be 5, 11, or 17.
Clue 3
+ 5 = ...
Zip
1 more
Who is Zip? 17
Note: If students use a calculator with order of operations rather than chain operations, they will get −10 instead of 6, and 26 instead of 42 in Clue 1. This, however, 20 will not change the possibilities in Clue 2 or 3.


÷4
3×
Duf
Puf
Complete the tables.
Duf | Puf
4 | 27
100 | 75
28 | 21
4 | 45
48 | 4
8 | 4
Duf | Puf
$2 | 24
$20 | 240
$200 | 2400
$4 | $3
$40 | $30
$400 | $300
21


This string has one of the labels: Positive divisors of 20 or Multiples of 6.
Positive divisors of 20
or
Multiples of 6
Draw a circle around each number below that you know for sure belongs inside the red string.
Draw a triangle around each number below that you know for sure belongs outside the red string.
5
1
4
5
7
10
23
80
Some of the numbers should have neither a circle nor a triangle around them because we can't tell where they belong.
22


Pin and Pan are at consecutive numbers in this arrow picture.
17.8
+ 0.5 =
Who is Pin? 18
Who is Pan? 100.3
23


Put each number on the Mini-computer using exactly one negative checker ⊖ and exactly one of these checkers:
2 3 4 5 6 7 8 9
= 22
= 34
Another solution is possible.
= 55
= 5
= 316
= 12
Another solution is possible.
Other solutions are possible.
= 2 780
24


Label the dots. Many solutions are possible.
10
73
80
94
74
+ 10 =
− 3 =
16
50
2
100
Many solutions are possible.
25

Raul has 60 tickets. Each ticket is worth either one point or two points. He has a total of 83 points. Explain.

Raul has 37 one point tickets
and 23 two point tickets.

Then, he has 37 + 23 = 60 tickets
with a total of 37 + 46 = 83 points.

Lori has $4.10 in dimes and quarters. She has one more quarter than dimes. Explain.

Lori has 12 quarters and 11 dimes.

12 quarters is $3.00
11 dimes is $1.10
Total $4.10

26

Tim is a secret number.

Clue 1

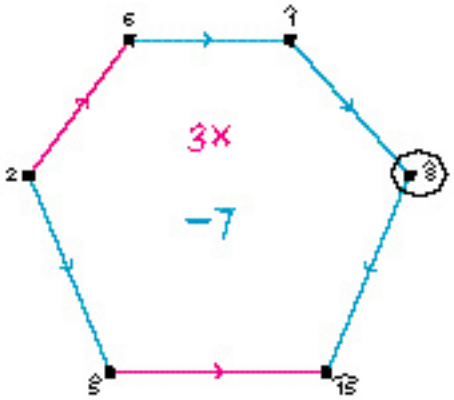
Tim could be 3 or 4.

Clue 2

Who is Tim? 4

27

8 is the only even negative number in this picture. Circle the dot for 8. Label all of the dots.



28

Fraggle has two four-sided pyramids. The faces of each pyramid are the same size and shape, and are labeled:

1 2 3 4

Fraggle rolls the two blocks on a table and adds the numbers on the faces touching the table. (for example, 2 + 3 = 5).
List the sums Fraggle could get:
2, 3, 4, 5, 6, 7, or 8.

Complete this table to show all of the possible sums with the two pyramids. One is done for you.

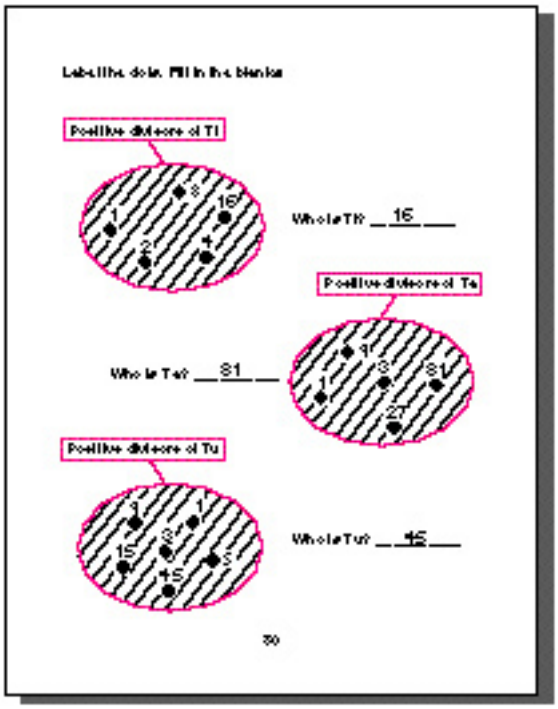
Red \ Blue	1	2	3	4
4	5	6	7	8
3	4	5	6	7
2	3	4	5	6
1	2	3	4	5

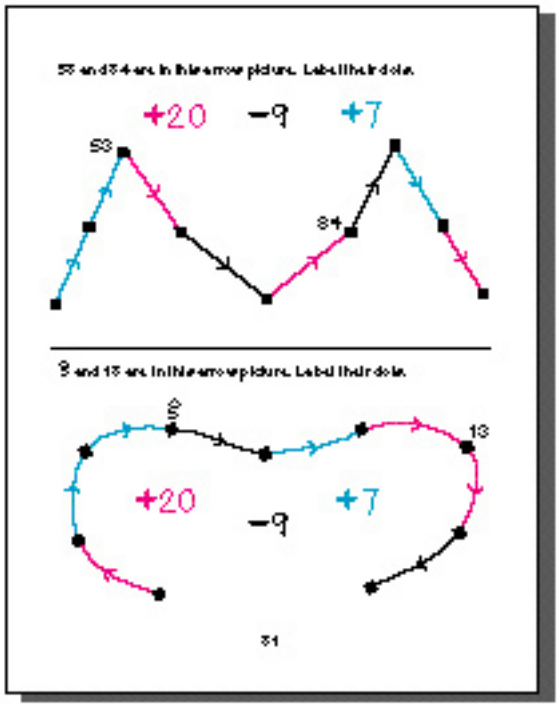
What is the probability that the sum will be 4? 3/16
Which sum is most likely? 5
What is the probability of that sum occurring? 4/16 or 1/4

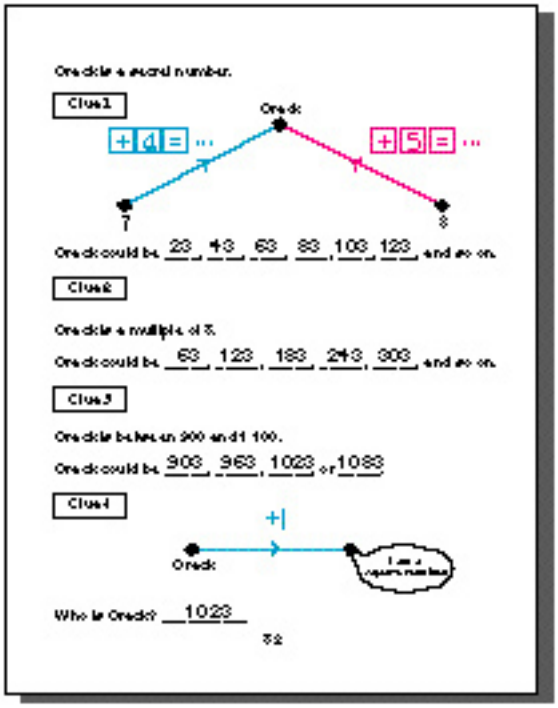
Fraggle invites Angela to play this game: Roll the two pyramids and add the results. Fraggle wins if the sum is even. Angela wins if the sum is odd.
Is this a fair game? Explain.

Yes, the probability of an even sum is 8/16 or 1/2.
and the probability of an odd sum is 8/16 or 1/2.

29


Positive difference of TI
Whole TN: 16
Positive difference of Te
Whole Te: 81
Positive difference of Tu
Whole Tu: 45
30


+20
−9
+7
83
84
9 and 13 are in this arrow picture. Label their dots.
9
13
+20
−9
+7
31


One dot is a secret number.
Clue 1
One dot
+ 4 = ...
+ 5 = ...
7
8
One dot could be 23, 43, 63, 83, 103, 123, and so on.
Clue 2
One dot is a multiple of 3.
One dot could be 63, 123, 183, 243, 303, and so on.
Clue 3
One dot is between 900 and 1100.
One dot could be 903, 963, 1023, or 1083
Clue 4
+1
One dot
Who is One dot? 1023
32

Capsule Lesson Summary

Using weighted checkers, show 200 on the Minicomputer and make trades until 200 is represented with checkers only on the 1-square. Starting with a 200-checker on the ones board of a binary abacus, make trades until 200 is represented with no more than one regular checker on a board. Notice the relationship between the weighted checkers on the 1-square in the Minicomputer configuration and the place value of the boards with checkers in the binary abacus configuration. Repeat the activity with the number 500.

Materials

Teacher	• Minicomputer set • Blackline W5	• Weighted checker set
Student	• Binary abacus	

Advance Preparation: Use Blackline W5 to make copies of a binary abacus for students.

Description of Lesson

Note: In this lesson you will not actually use the story-workbook *Number 1000's Dream*. The exercises here are in preparation for the next lesson, in which with the story-workbook is used.

Exercise 1

Display three Minicomputer boards and the following weighted checkers.

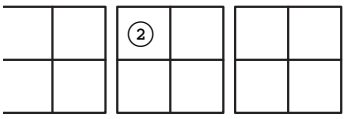
(2) (4) (8) (16) (25) (64) (128) (256) (512)

T: ***What do you notice about these checkers?***

S: ***All of the numbers are even.***

S: ***The values double: 2, 4, 8, 16 and so on.***

Put a (2)-checker on the 80-square of the Minicomputer.



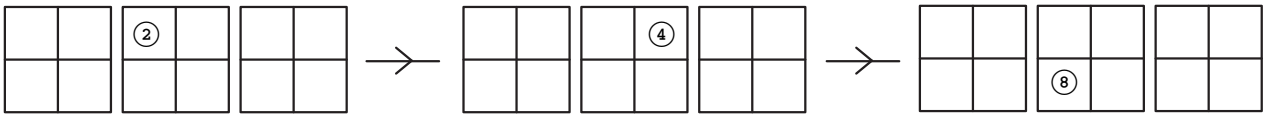
T: ***What number is on the Minicomputer? Why?***

S: ***160, because 2 x 80 = 160.***

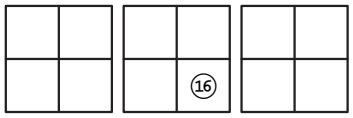
T: ***Can we put the same number on the Minicomputer using a (4)-checker?***

As a student makes the trade, emphasize that 2 x 80 = 4 x 40 = 160.

T: ***Can we get the same number by trading the (4)-checker for an (8)-checker?***
(4 x 40 = 8 x 20)



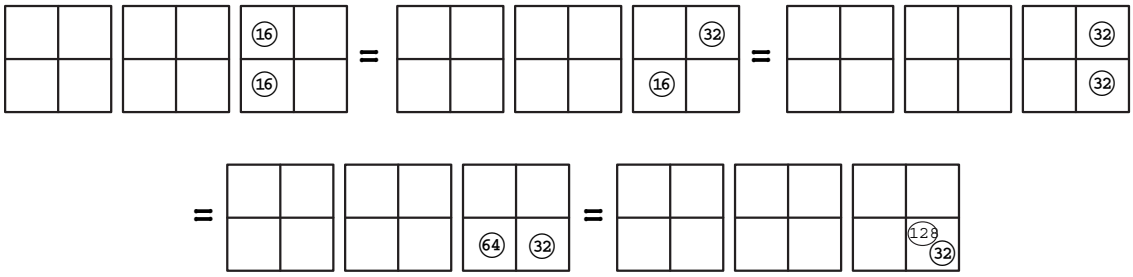
T: ***Can we get the same number by trading the ⑧-checker for a ⑯-checker?*** (8 x 20 = 16 x 10)



Now let's make a trade so that we use two ⑯-checkers instead of one ⑯-checker.

Give students a moment to consider this problem as it involves a different trade than the previous ones. Then invite a student to make the trade. If necessary, temporarily replace the ⑯-checker with a regular checker, and ask a student to make a trade so you have 10 with two regular checkers. Repeat the activity starting with three regular checkers on the 10-square; then return to the problem with a ⑯-checker on the 10-square. Remind students that a ⑯-checker is just like 16 regular checkers.

Continue asking students to make trades until there are checkers only on the 1-square. When you invite a student to make a trade, specify which checkers to trade (for example, trade a ㉜-checker for a ㊹-checker). A possible sequence of trades is shown below. Although the intermediate trades might not be exactly the same, you should finish with a ㉘-checker and a ㉜-checker on the 1-square.



T: ***What number is on the Minicomputer? Why?***

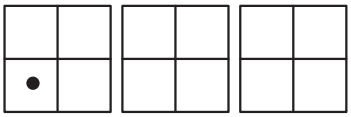
S: ***160; 128 + 32 = 160.***

T: ***Has the number changed?***

S: ***No, we started with 160.***

Exercise 2

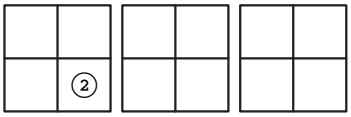
Put 200 on the Minicomputer with one regular checker.



T: ***Imagine that we are visiting the World of Numbers. When the numbers awake each day, they do exercises. Watch 200 exercise on the Minicomputer.***

200 jumps like this.

Show this jump (trade the regular checker for a ②-checker).

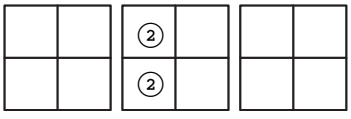


T: ***Is 200 still on the Minicomputer?***

S: ***Yes, 2 x 100 = 200.***

T: ***Then 200 jumps again. Is 200 still on the Minicomputer?***

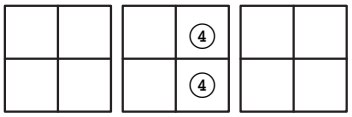
S: ***Yes. 2 x 80 = 160, 2 x 20 = 40, and 160 + 40 = 200.***



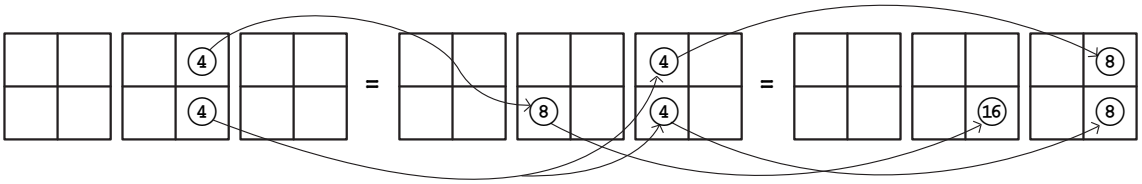
S: ***Yes. 100 = 80 + 20, so 200 = (2 x 80) + (2 x 20).***

T: ***200 jumps again. 200 trades each of these ②-checkers for a ④-checker. Who can show the jump?***

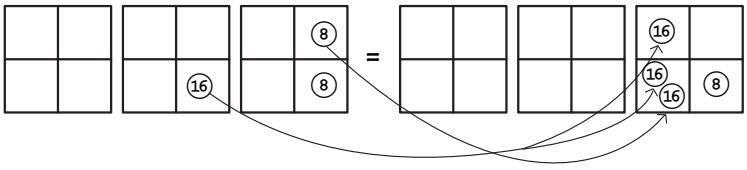
Invite a student to make the trades.



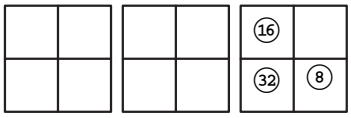
Continue inviting students to make jumps.



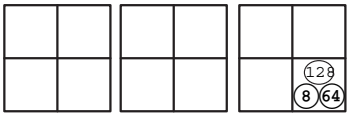
T: ***When 200 gets checkers in the 1-square, those checkers don't jump anymore. In this case, the next jump only uses the ⑯-checker and the ⑧-checker in the 4-square.***



Replace the two ⑯-checkers with a ㉜-checker.



Continue letting students make jumps until there are only checkers on the 1-square. Check that the weighted checkers add up to 200.



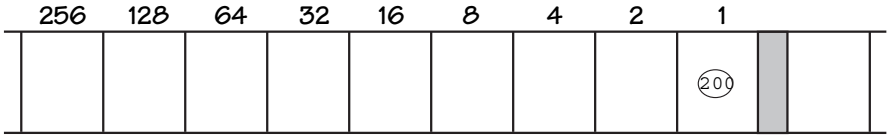
= 200

Leave this configuration on the Minicomputer for use at the end of Exercise 3.

Exercise 3

You may like to provide students with their own binary abacus to use during this exercise.

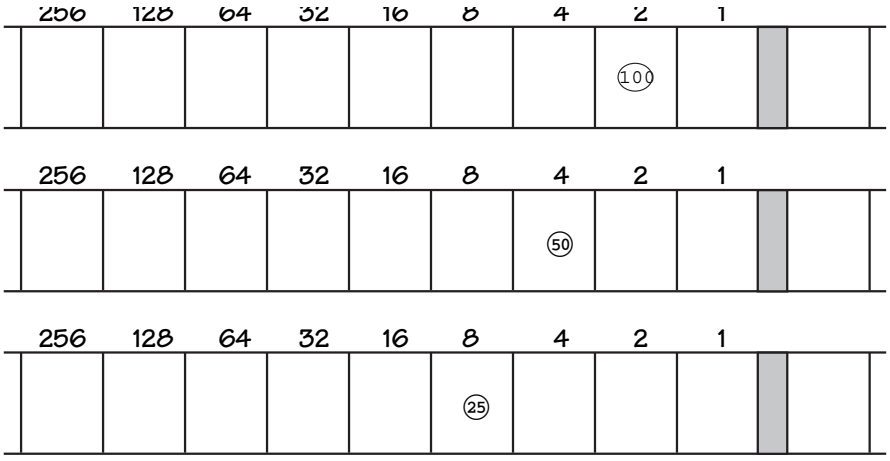
Draw a binary abacus on the chalkboard and put a ㉐⓪-checker in the ones place.



T: ***Now 200 is going to do exercises on this binary abacus. Here, though, 200 always makes forward jumps. What jump could 200 make?***

S: ***Trade the ㉐⓪-checker in the ones place for a ①⓪⓪-checker in the twos place.***

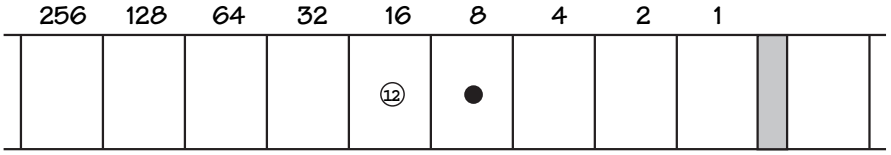
Continue in this way, making several forward jumps, until you have a ㉕-checker in the eights place.



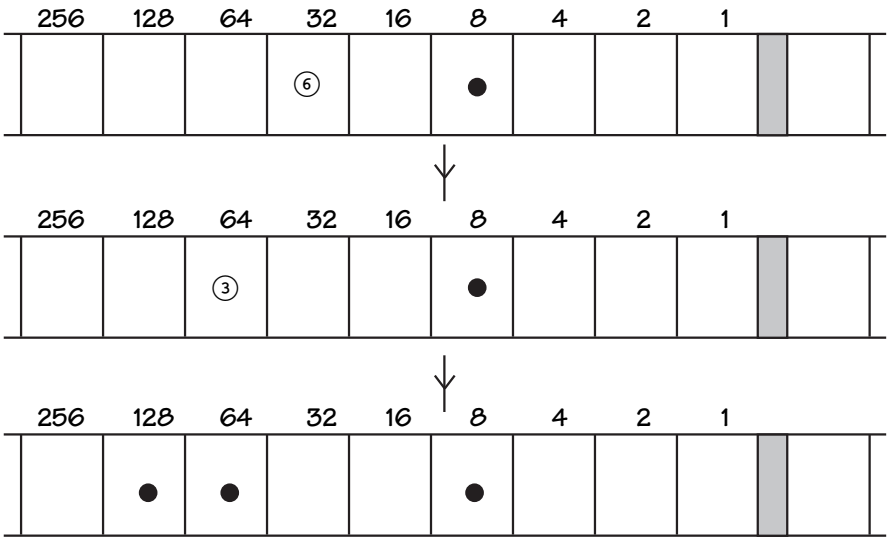
T: ***Now there are 25 checkers in the eights place. Can 200 jump again?***

You may need to give some hints or make this next jump yourself.

S: ***Leave one checker in the eights place. Then jump with the other 24 checkers. Trade 24 checkers in the eights place for 12 checkers in the sixteens place.***

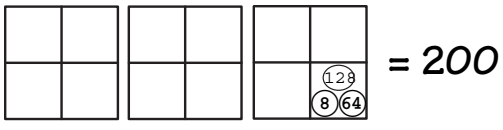


With students' help, make jumps as in the next illustration.



Ask students to check that 200 is still on the abacus. They should verify that it is by adding 128 + 64 + 8 = 200.

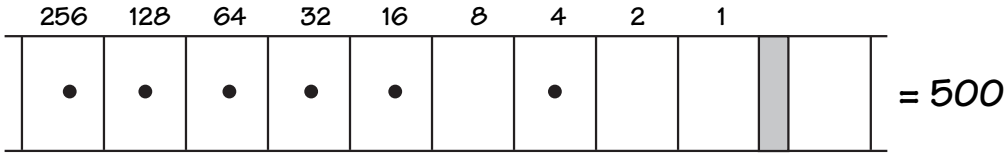
At this point, refer back to the Minicomputer with this configuration from Exercise 2.



T: ***Do you see any relationship between the checkers on the Minicomputer and the checkers on the binary abacus?***

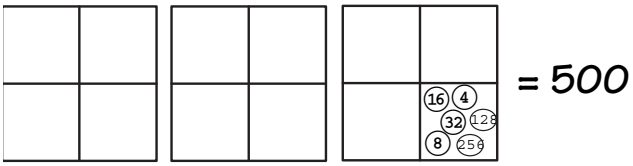
Let students make observations. Encourage students who notice that the weights of the checkers on the Minicomputer correspond to the place values of the boards containing checkers on the binary abacus.

Repeat this activity on the binary abacus with 500; then ask students to verify the resulting configuration.



256 + 128 + 64 + 32 + 16 + 4 = 500

Then put 500 on the Minicomputer and ask the students to predict which checkers will be on the 1-square after 500 does its exercises. Let students make trades as you did in Exercise 2 until there are checkers only on the 1-square. Check the class's prediction.



Capsule Lesson Summary

Read the story-workbook *Number 1 000's Dream*. Discover a new name for 1 000 (999.999 …) by making backward trades on the Minicomputer.

Materials

Teacher
- *Number 1 000's Dream* Story-Workbook
- Weighted checker set
- Minicomputer set
- Colored chalk

Student
- *Number 1000's Dream* Story-Workbook
- Paper

Advance Preparation: Borrow six additional Minicomputer boards from other teachers. If this is not possible, draw additional Minicomputer boards on the chalkboard.

Description of Lesson

Distribute copies of the story-workbook *Number 1 000's Dream*. Display four Minicomputer boards and the following weighted checkers:

T: ***What do you notice about these checkers?***

S: ***All of the numbers are multiples of 2.***

S: ***The values double: 2, 4, 8, 16, and so on.***

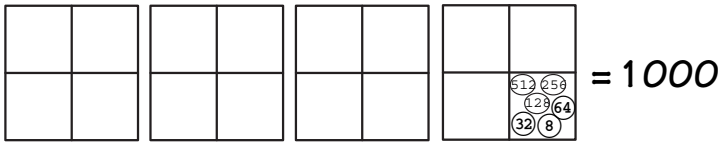
Pages 2 to 5

Ask students to read pages 2 to 5 either aloud or silently. Put the standard configuration for 1 000 on the Minicomputer and invite students to make the jumps pictured on pages 4 and 5.

Note: 1000 is making jumps involving several backward trades. A backward trade is made with every checker except any that are already on the 1-square.

Pages 6 to 11

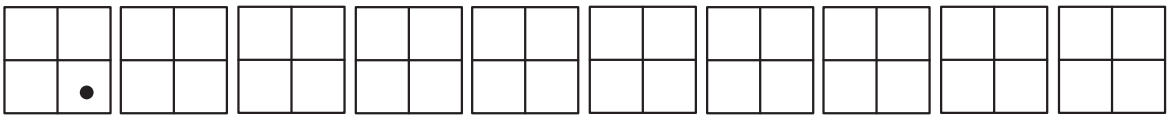
Continue by reading pages 6 and 7 and inviting students to illustrate each of the jumps on these pages at the board. Then invite students to continue making jumps until all of the checkers are on the 1-square. Ask students to add the values of the checkers to verify that 1 000 is still on the Minicomputer.



Ask students to read pages 8 to 11 aloud.

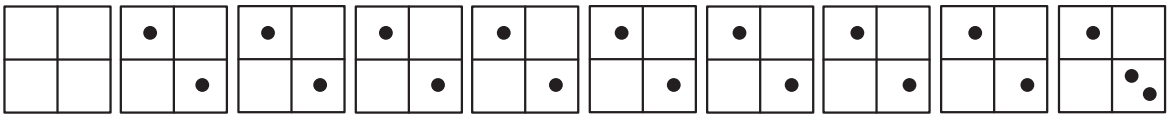
Pages 12 to 16.

Display ten Minicomputer boards with this configuration.



T: ***How many boards do we have displayed?*** (10)
What number is on the Minicomputer? (1 billion)

A different kind of exercise is suggested on page 12. Ask a student to read page 12 aloud. Then invite students to demonstrate the trades pictured on page 12. When you obtain the configuration at the bottom of the page, ask students to show the next six trades on page 13. After a few minutes, invite students to make backward trades until the following configuration is on the demonstration Minicomputer.



T: ***Can we describe what is on the Minicomputer now in a number sentence for 1,000,000,000?***

S: ***999,999,999 + 1 = 1,000,000,000.***

Record this number sentence on the board.

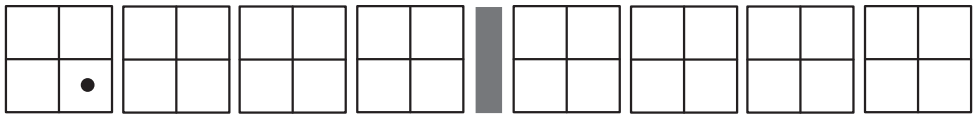
Ask students to read pages 14 and 15 aloud. Then write these problems on the board. Students should copy them and fill in the boxes. Check answers collectively.

Ask a student to read page 16 aloud.

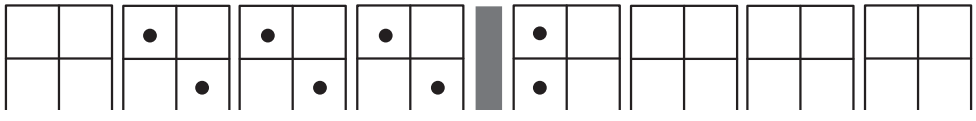
600 = 599 + 1
7 000 = 6 999 + 1
9 500 = 9 499 + 1
19 000 = 18 999 + 1
99 000 = 98 999 + 1
478 000 = 477 999 + 1

Pages 17 to 31

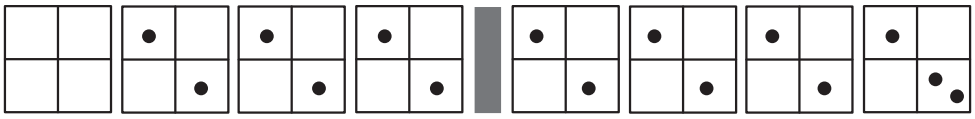
Continue reading pages 17 to 25. Display at least eight Minicomputer boards with this configuration.



As students read aloud pages 26 and 27, ask others to make the backward trades on the demonstration Minicomputer. Then invite a student to make a backward trade over the bar.



As students read aloud pages 28 and 29, ask others to continue making backward trades until this configuration is obtained.

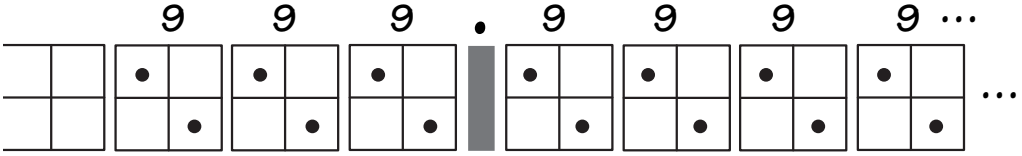


T: ***What would happen if we had more Minicomputer boards?***

S: ***We could continue making backward trades.***

Finish the lesson by reading pages 30 and 31.

Write 1 000's new signature above (or below) the Minicomputer boards for emphasis.

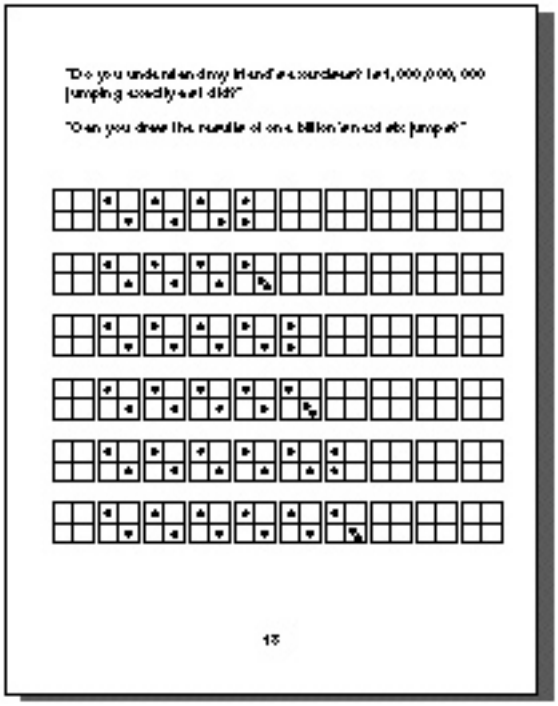


Writing Activity

Some students may like to invent other exercises for 1 000 on the Minicomputer. Suggest they write a description of their exercises and any repeating patterns that result.

Home Activity

Send home a copy of the story-workbook *Number 1000's Dream* for students to read with a family member.



Capsule Lesson Summary

Recall that the ones digit of a number determines whether it is even or odd. Add a negative checker to a configuration on the Minicomputer and decide whether the new number is even or odd. Begin the workbook *Variety of Problems #2*. (This is the first of two lessons using this workbook.)

Materials

Teacher	• Minicomputer set • Weighted checker set	Student	• *Variety of Problems #2* Workbook • Colored pencils, pens, or crayons • Metric ruler • Calculator

Description of Lesson

Begin the lesson with the following short activity: recognizing odd and even numbers.

T: ***Which of the following integers are even and which are odd?***
74. (Even) ***809.*** (Odd) ***221.*** (Odd) ***3 986*** (Even)
How do you recognize an even integer or an odd integer?

S: ***Just look at the ones digit and see if it's even or odd.***

S: ***An even integer has 0, 2, 4, 6, or 8 in the ones place. An odd integer has 1, 3, 5, 7, or 9.***

T: ***Are decimals even or odd? What about 2.31?***

S: ***Only integers are said to be even or odd.***

T: ***What about $\widehat{74}$?***

S: ***Even; 4 is the ones digit.***

S: ***74 is even; so $\widehat{74}$ is even.***

T: ***Is 3 x 7 even or odd?***

S: ***Odd; 3 x 7 = 21 and 21 is odd.***

T: ***What about 7 x 543?***

S: ***Odd.***

Invite students to complete the calculation.

$$\begin{array}{r} {\scriptstyle 3\ 2}\\ 543\\ \times\ \ 7\\ \hline 3801 \end{array}$$

T: ***Did we need to do the entire calculation to know 7 x 543 is odd?***

S: ***No, just 7 x 3, because that tells us the ones digit of the answer.***

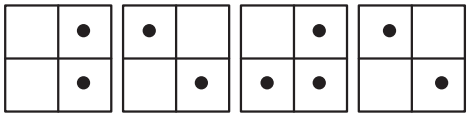
T: ***Is 86 + 32 even or odd?***

S: ***Even; the answer ends in 8.***

T: ***Is 65 – 19 even or odd?***

S: ***Even; the answer ends in 6.***

Put this configuration on the Minicomputer.

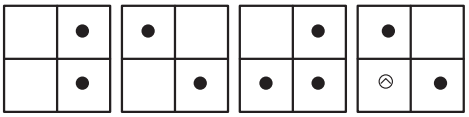


T: ***Is this number even or odd?***

S: ***Odd; the ones digit is 9.***

S: ***5 979 is odd.***

Put a negative checker on the 2-square.



T: ***Is this number even or odd?***

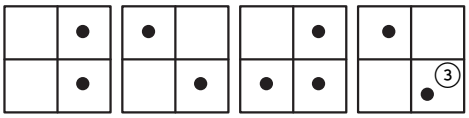
S: ***It's still odd.***

S: ***First 9 was on the ones board; now 7 is. 9 + $\widehat{2}$ = 7.***

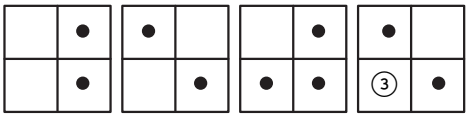
Move the negative checker to other squares and ask whether the resulting number is even or odd. For example:

- Move the negative checker to the 400-square. (Odd; 9 is still on the ones board)
- Move the negative checker to the 10-square. (Odd; 9 is still on the ones board)
- Move the negative checker to the 1-square. (Even; 8 is now on the ones board)

If this activity is going well, extend it by adding a ③-checker to the original configuration.



Even; 12 is now on the ones board, so the number on the Minicomputer ends in 2.



Odd; 15 is now on the ones board, so the number on the Minicomputer ends in 5.

Distribute copies of the workbook *Variety of Problems #2* and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of the lesson, collect the workbooks for your review. They will be used again in Lesson W8.

Capsule Lesson Summary

Given a true multiplication (division) statement, solve some related multiplication (division) problems and observe patterns. Continue individual work in the workbook *Variety of Problems #2*. (This is the second of two lessons using this workbook.)

Materials

Teacher	• None	Student	• *Variety of Problems #2* Workbook • Colored pencils, pens, or crayons • Metric ruler • Calculator

Description of Lesson

Exercise 1: Mental Arithmetic

Write this statement on the board and draw a box around it. $\boxed{12 \times 53 = 636}$

Ask students to check the calculation, preferably without pencil and paper, and invite students to explain their checking techniques.

S: *10 x 53 = 530, and 2 x 53 = 106. 530 + 106 = 636.*

S: *2 x 53 = 106, and 6 x 106 = 636.*

S: *4 x 53 = 212, and 3 x 212 = 636.*

Write this problem on the board below the boxed multiplication statement. 13×53

T: ***Try to do this calculation without too much work. The multiplication statement 12 x 53 = 636 can help you.***

S: ***13 x 53 = 689. Here there are thirteen 53s instead of twelve; one more 53 is 636 + 53 = 689.***

Continue with the following calculations. (Answers are in boxes.) Let students explain how they use an earlier problem to solve a new problem. Keep up a rather brisk pace.

$14 \times 53 = \boxed{742}$ $\quad$ $12 \times 54 = \boxed{648}$

$16 \times 53 = \boxed{848}$ $\quad$ $12 \times 55 = \boxed{660}$

$24 \times 53 = \boxed{1272}$ $\quad$ $12 \times 106 = \boxed{1272}$

Invite students to comment on patterns they observe.

Repeat this exercise by giving a division calculation and several related division problems. For example:

$168 \div 7 = 24$

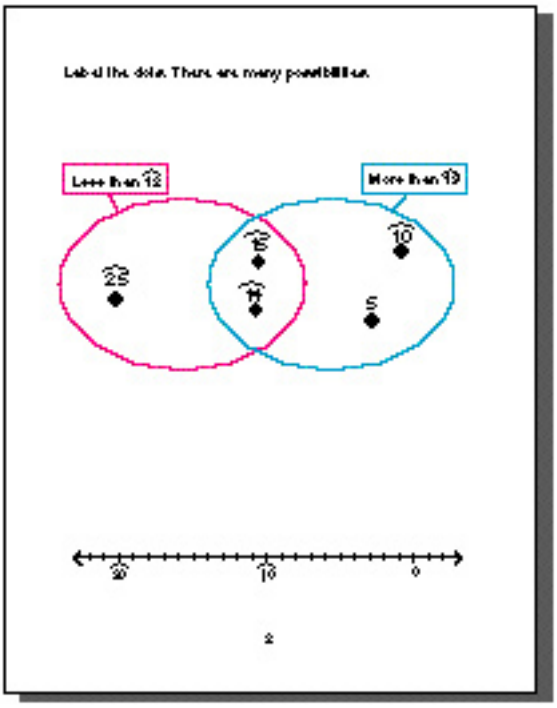
$161 \div 7 = 23$	$336 \div 7 = 48$
$175 \div 7 = 25$	$238 \div 7 = 34$
$189 \div 7 = 27$	$231 \div 7 = 33$

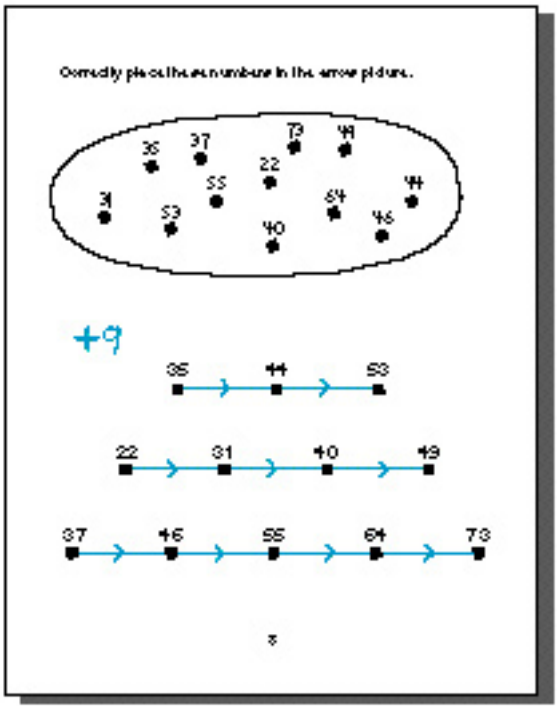
Exercise 2

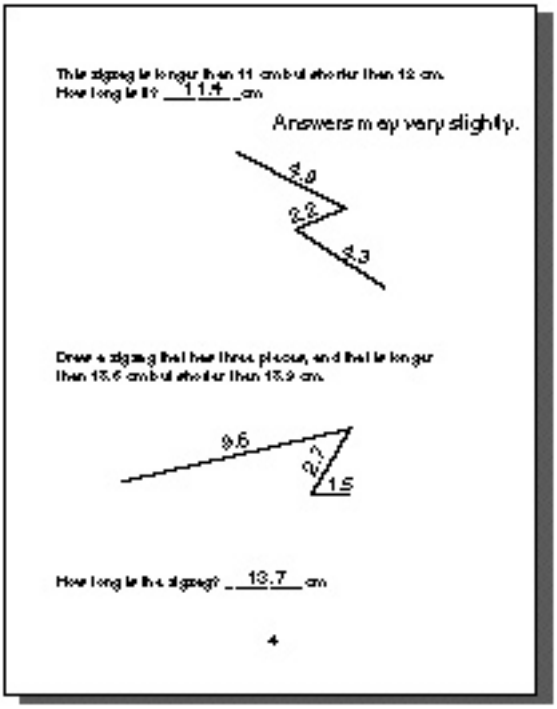
Return students' copies of the workbook *Variety of Problems #2*. Ask students first to correct or complete pages they worked on the previous week, and then to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

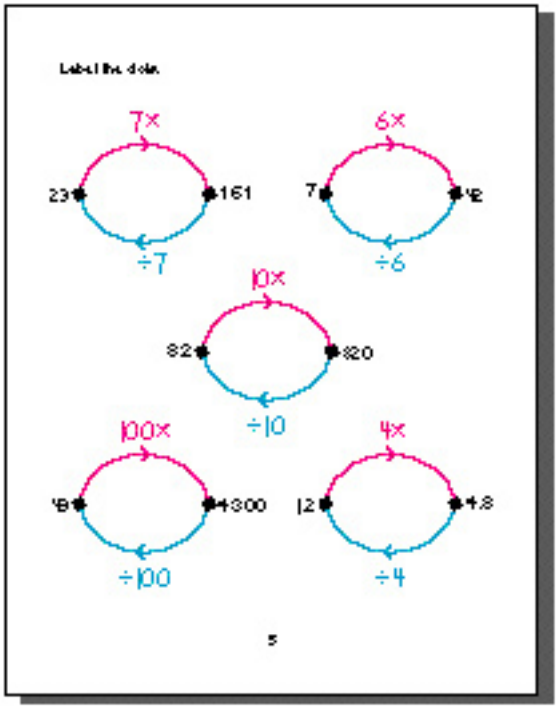
Assessment Activity

An individual student progress record for the workbook is available on Blackline W8. You may like to use this form to monitor student work.


Label the dots. There are many possibilities.
Less than 12
More than 19
25
11
5
10
20
10
0
2


Correctly place the numbers in the error picture.
35
37
73
41
22
55
44
3
53
64
46
40
+9
35
44
53
22
31
40
49
37
46
55
64
73
3

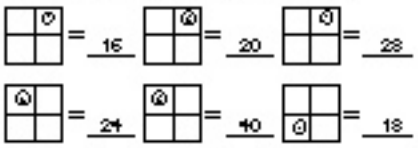

This zigzag is longer than 11 cm but shorter than 12 cm.
How long is it? 11.4 cm
Answers may vary slightly.
4.9
2.8
4.3
Draw a zigzag that has three pieces, and that is longer than 13.6 cm but shorter than 13.9 cm.
9.6
2.1
1.5
How long is the zigzag? 13.7 cm
4


Label the dots.
7×
23
161
÷7
6×
7
42
÷6
10×
82
820
÷10
100×
8
4300
÷100
4×
12
48
÷4
5

Freckles a secret number.

Clue 1

Freckles one of these numbers on the Minicomputer.



Clue 2

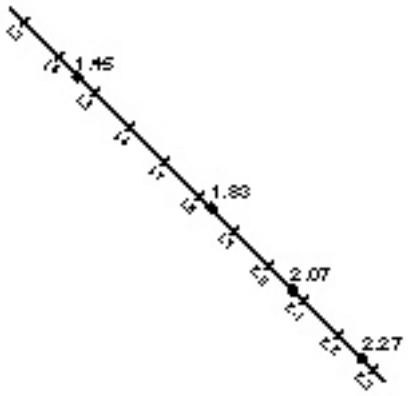
Freckles in this Venn diagram.

Who is Freck? 24

6

Draw and label dots for these numbers.

1.45 2.27 1.83 2.07



7

Janine multiplied by 4 when she meant to subtract 4.

Then she added 7 when she meant to multiply by 7.

The answer she got was 59.

What answer should she have gotten? 28

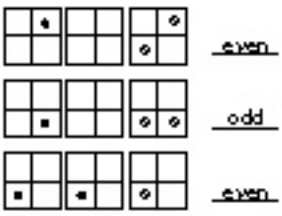
Hint: Draw an arrow picture to show what Janine did and what she meant to do.

8

Tell whether each number is even or odd.

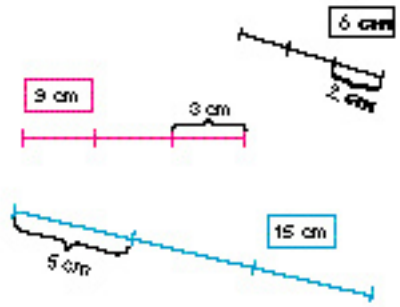
100 + 102 + 103	odd
52 + 53 + 54	odd
2 × 175	even
3 × 123	odd
4 × 51	even
10 × 99	even

Tell whether each number is even or odd.



9

Measure each line segment in centimeters and record its length in the box of the same color. Then divide each segment into three pieces of the same length. One is done for you.



Draw a line segment 12.6 cm long and divide it into three pieces of the same length.

How long is each piece? 4.2 cm

10

Calculate.

36 × 1 = 36	36 × 11 = 36 + 360 = 396	29 + 86 + 104 = 219
69 × 7 = 483	69 × 37 = 483 + 2070 = 2553	706 − 513 = 193
107 × 8 = 856	107 × 28 = 856 + 2140 = 2996	107 × 428 = 856 + 2140 + 42800 = 45796

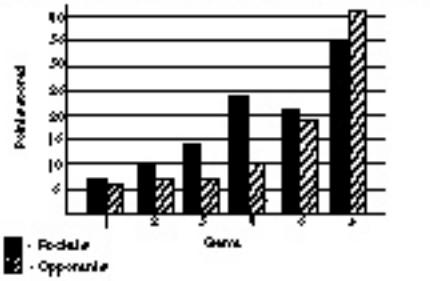
$\hat{12} + \hat{8} + 10 = \hat{10}$ | $20 + \hat{30} + \hat{14} + 1 = \hat{23}$

11

Here are the results of the Rockets' Rockets' first six football games.

Rockets	7	10	14	24	21	35
Opponents	6	7	7	10	19	41

Graph each game's results below. The first game is done for you.



Which game did the Rockets win by the most points? 4th

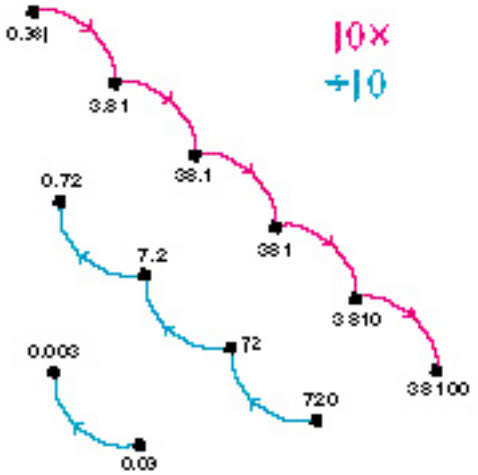
Which game did the opponents win by the fewest points? 1st

Did the Rockets improve during the season? Yes

Did the defense improve during the season? No

12

Label the data.



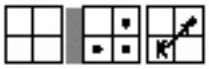
13

Move exactly one checker and make the number 20 greater.

Move exactly one checker and make the number 2 greater.

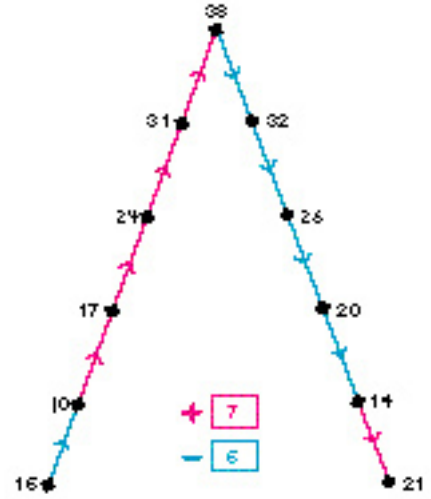
Move exactly one checker and make the number 0.2 greater.

Move exactly one checker and make the number 0.02 greater.



14

Fill in the boxes for the arrows and label the dots.



15

Share 1568 oranges among 4 grocery stores.

1568
−1200
368
−200
168
−160
8
−8

There are other ways to share the oranges.

Complete the number sentence.

$\frac{1}{4} \times 1568 =$ 392

16

Calculate.

7 × 4 = 28
28 ÷ 4 = 7
280 ÷ 4 = 70
2 800 ÷ 4 = 700
2 800 ÷ 7 = 400

8 × 6 = 48
48 ÷ 8 = 6
480 ÷ 8 = 60
4 800 ÷ 8 = 600
4 800 ÷ 6 = 800

17

A printer can print 10 pages in 3 minutes. If the printer works nonstop, how many pages can it print in 4 hours? 800 pages

10 pages in 3 minutes
100 pages in 30 minutes
200 pages in 60 minutes (1 hour)
800 pages in 4 hours

A machine can fill 80 boxes of paper clips in 15 minutes. Each box contains 100 paper clips. How many boxes can be filled in 6 hours? 1920 boxes

80 boxes in 15 minutes
320 boxes in 1 hour
1920 boxes in 6 hours

(100 paper clips is unnecessary information)

18

Put each number on the binary abacus.

20 =

39 =

33 $\frac{1}{2}$ =

Put each number on the binary abacus.

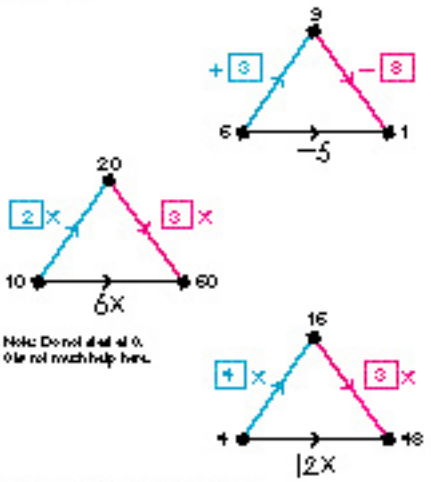
68 =

264 $\frac{1}{4}$ =

19

Other solutions are possible, but no with fewer checkers.

Fill in the boxes for the arrows. Then label the dots to check the arrows.



Note: Do not start at 0. It is not much help here.

Other solutions are possible.
In particular, there are many ways to label the dots.

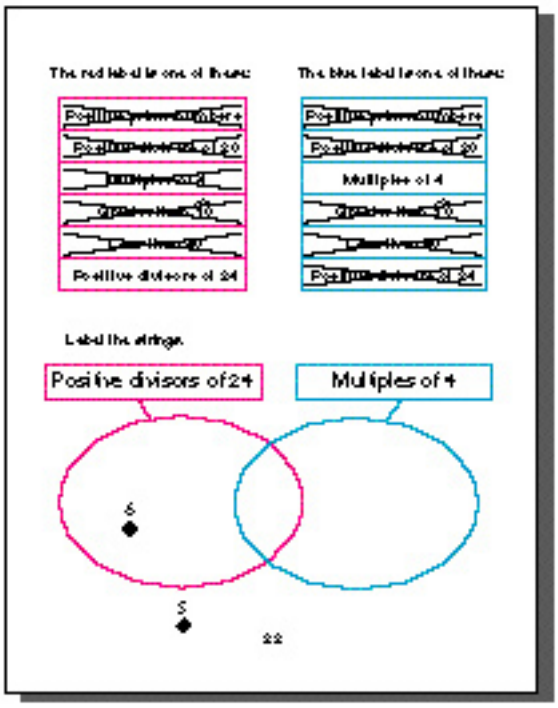
20

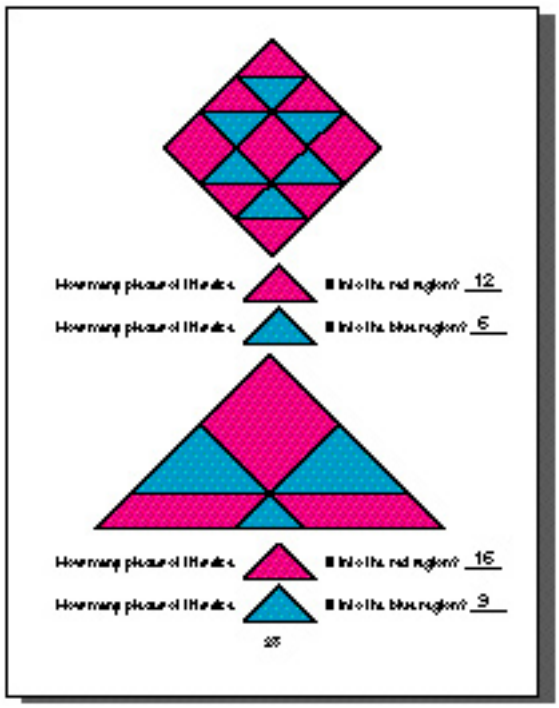
Zoe has a lemonade stand where she sells lemonade in quantities of 30 milliliters, which she finds to be the most popular size. One day she can only get cups that hold 120 milliliters and cups that hold 200 milliliters, so she buys some of both sizes. How can Zoe use the pitcher of lemonade, a 120 ml cup, and a 200 ml cup to measure 80 ml of lemonade? Explain.

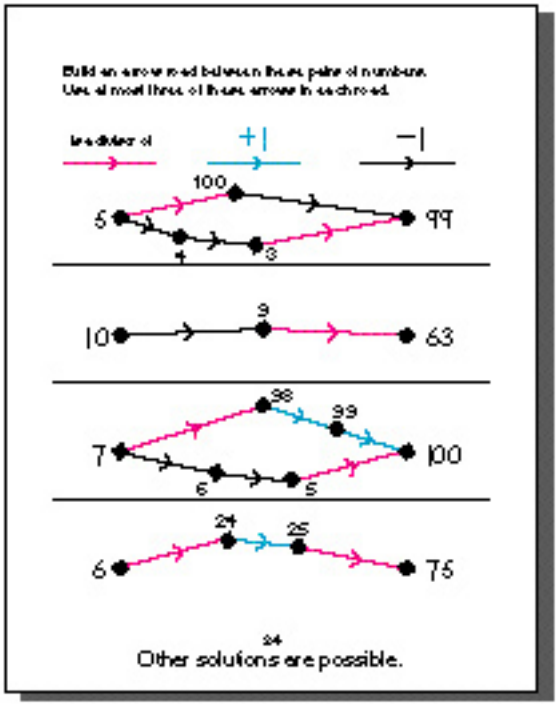
Fill the 200 ml cup with lemonade. Then use the lemonade in the 200 ml cup to fill the 120 ml cup. There will be 80 ml of lemonade left in the 200 ml cup.

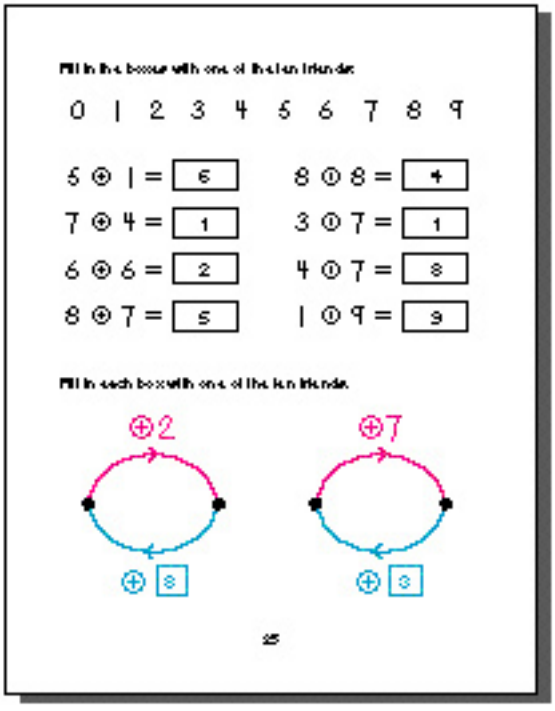
21

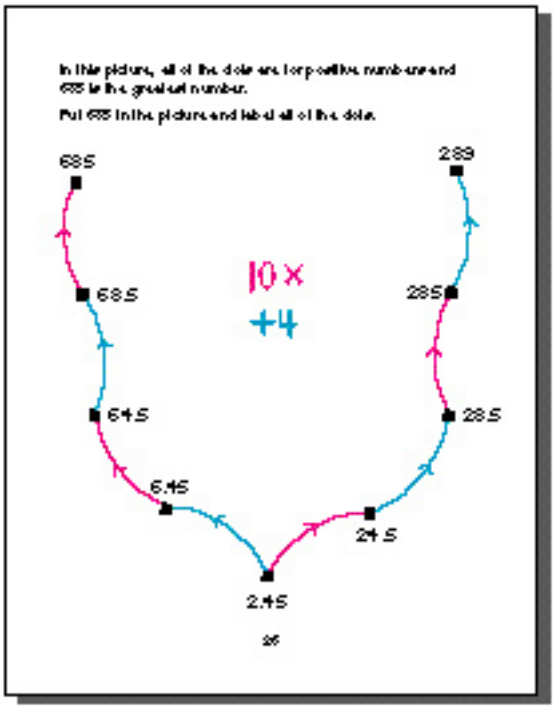
Other, more complex, solutions are possible.

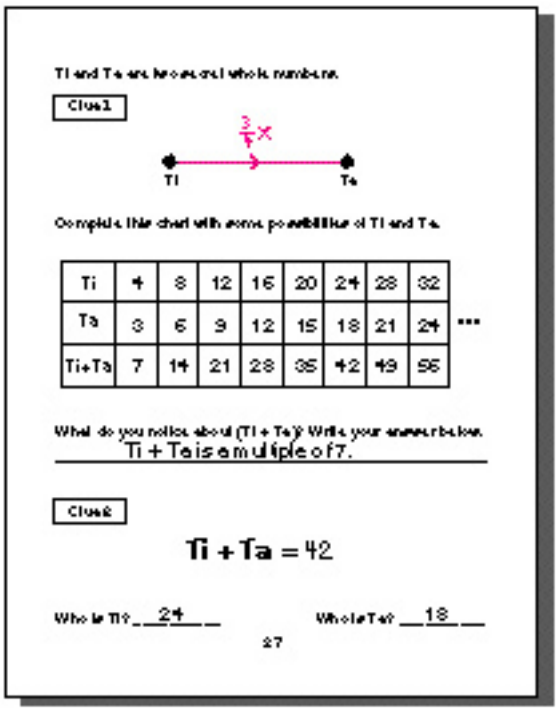

Positive divisors of 24
Multiples of 4
6
5
22

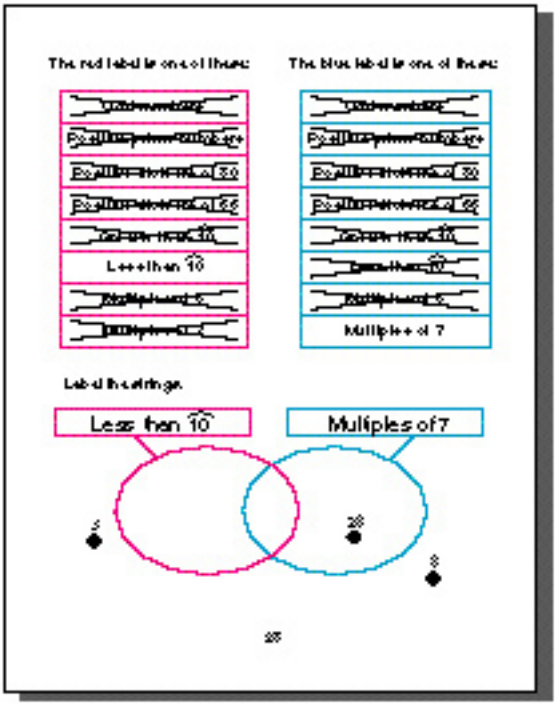

How many pieces of the shape
is in the red region? 12
How many pieces of the shape
is in the blue region? 6
How many pieces of the shape
is in the red region? 16
How many pieces of the shape
is in the blue region? 9
23

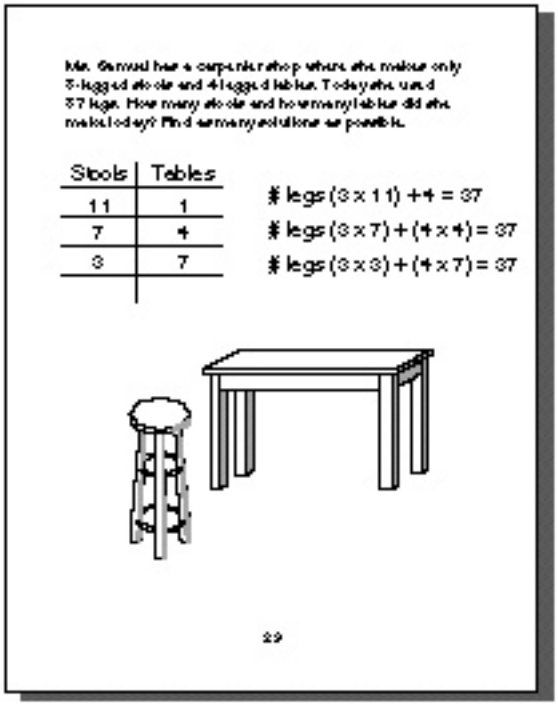

+1
−1
100
5
99
4
3
10
9
63
98
99
7
100
6
5
24
25
6
75
24
Other solutions are possible.

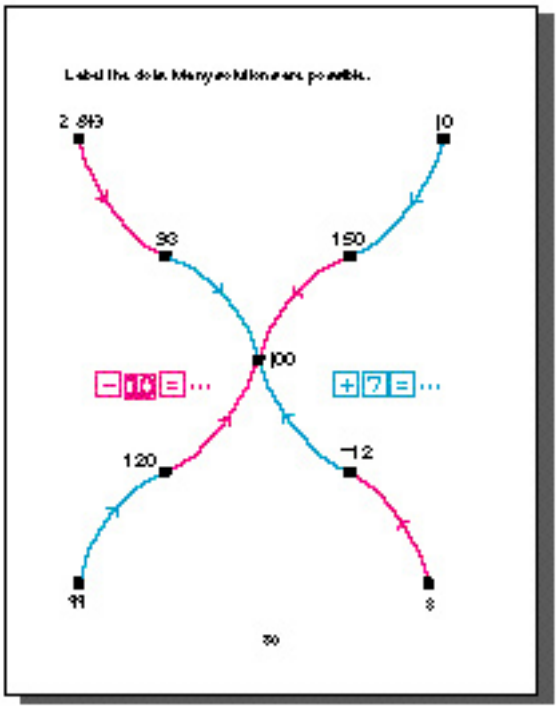

0 1 2 3 4 5 6 7 8 9
5 ⊕ 1 = 6
8 ⊕ 8 = 4
7 ⊕ 4 = 1
3 ⊕ 7 = 1
6 ⊕ 6 = 2
4 ⊕ 7 = 8
8 ⊕ 7 = 5
1 ⊕ 9 = 9
⊕2
⊕7
⊕ 8
⊕ 8
25

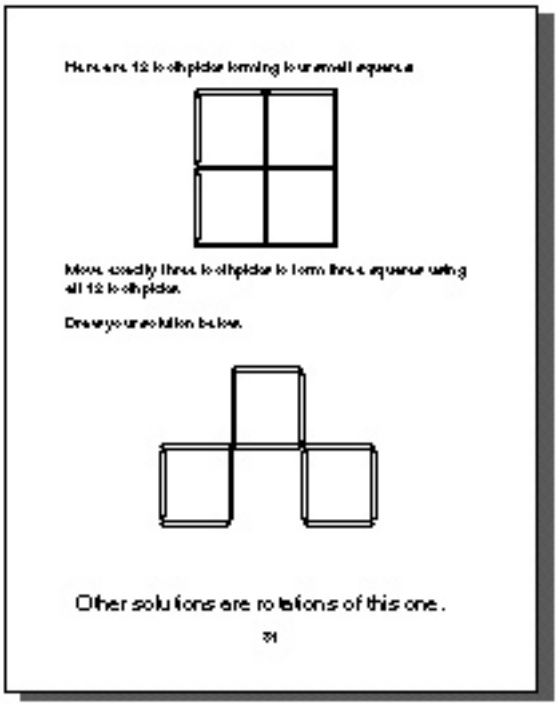

In this picture, all of the dots are for positive numbers and 68.5 is the greatest number.
Put 68.5 in the picture and label all of the dots.
68.5
28.9
10 x
+4
68.5
28.5
64.5
28.5
6.45
24.5
2.45
25

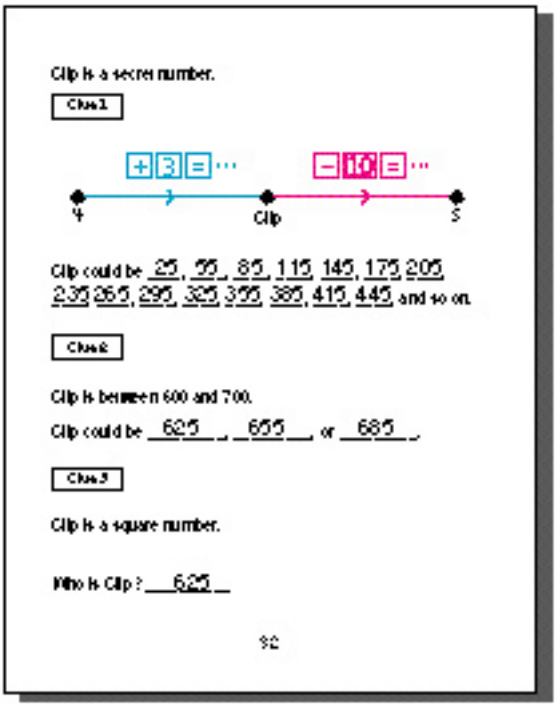

Ti and Ta are two natural whole numbers.
Clue1
3 x
Ti
Ta
Complete this chart with some possibilities of Ti and Ta.
Ti 4 8 12 16 20 24 28 32
Ta 3 6 9 12 15 18 21 24 ...
Ti+Ta 7 14 21 28 35 42 49 56
What do you notice about (Ti + Ta)? Write your answer below.
Ti + Ta is a multiple of 7.
Clue2
Ti + Ta = 42
Who is Ti? 24
Who is Ta? 18
27


The red label is one of these:
The blue label is one of these:
Less than 10
Multiples of 7
Label the strings.
Less than 10
Multiples of 7
5
28
8
25


Mr. Garrod has a carpenter shop where she makes only 3-legged stools and 4-legged tables. Today she used 37 legs. How many stools and how many tables did she make today? Find as many solutions as possible.
Stools Tables
11 1
7 4
3 7
legs (3 x 11) +4 = 37
legs (3 x 7) + (4 x 4) = 37
legs (3 x 3) + (4 x 7) = 37
29


Label the dots. Many solutions are possible.
2 849
10
93
150
100
120
12
99
8
30


Here are 12 tetrahedra forming four small squares.
Move exactly three tetrahedra to form three squares using all 12 tetrahedra.
Draw your solution below.
Other solutions are rotations of this one.
31


Clip is a secret number.
Clue 1
4
Clip
8
Clip could be 25, 55, 85, 115, 145, 175, 205, 235, 265, 295, 325, 355, 385, 415, 445 and so on.
Clue 2
Clip is between 600 and 700.
Clip could be 625, 655, or 685.
Clue 3
Clip is a square number.
Who is Clip? 625
32

Capsule Lesson Summary

Review subtracting simple decimal numbers. Begin working in the workbook *Variety of Problems #3*. (This is the first of two lessons using this workbook.)

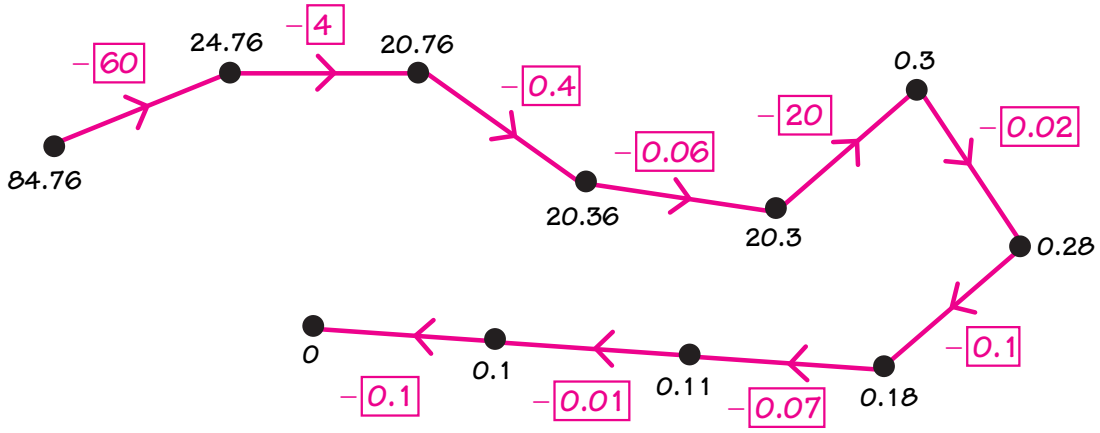
Materials

Teacher
- Colored chalk

Student
- *Variety of Problems #3* Workbook
- Metric ruler
- Colored pencils, pens, or crayons
- Calculator

Description of Lesson

Draw this arrow picture on the board, one arrow at a time. Ask students to label each arrow "minus some number." (Answers are in boxes.)



As necessary, remind students that 20.3 = 20.30 or that 0.3 = 0.30. You may also need to remind students that 0.06 ≠ 0.6. Allow students to refer either to money or to the Minicomputer in giving explanations.

Distribute copies of the workbook *Variety of Problems #3* and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

Capsule Lesson Summary

Review the use of composition (in an arrow picture) to multiply a whole number by a fraction. Continue individual work in the workbook *Variety of Problems #3*. (This is the second of two lessons using this workbook.)

Materials

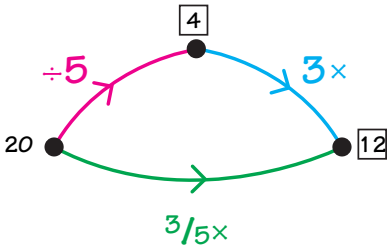
Teacher	Student
• Colored chalk	• *Variety of Problems #3* Workbook • Metric ruler • Colored pencils, pens, or crayons • Calculator

Description of Lesson

Write this problem on the board. $\frac{3}{5} \times 20$

T: ***What number is $\frac{3}{5}$ x 20?*** (12) ***Explain.***

Students most likely will explain how to do the calculation in terms of a two-step process, share (or divide) 20 five ways and then take three shares (multiply by 3). Earlier story situations, such as one about Bobo the monkey, support this idea. Illustrate the two steps in an arrow picture, or use the arrow picture as one method to do the calculation. Invite students to label the dots. (Answers are in boxes.) Conclude that $\frac{3}{5}$ x 20 = 12.



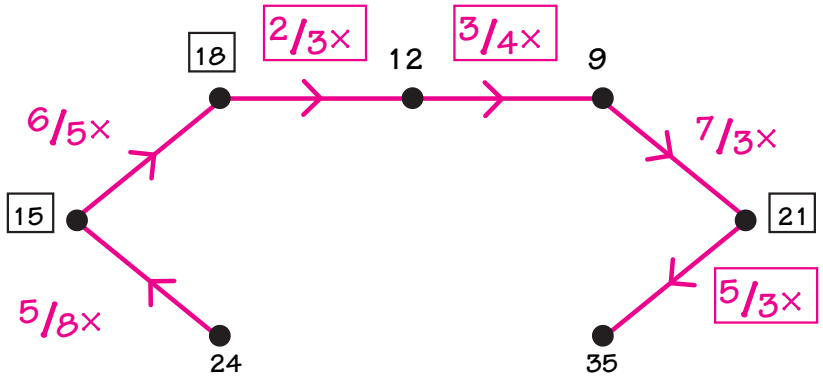
Direct students to use the arrow picture to solve a few more problems with $\frac{3}{5}$x.

$\frac{3}{5} \times 35 =$ 21

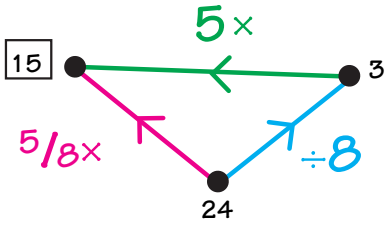
$\frac{3}{5} \times 45 =$ 27

$\frac{3}{5} \times 450 =$ 270

Draw the following arrow picture, one arrow at a time. Tell the class that each arrow is for "times some number," and invite students to label either an ending dot or an arrow. (Answers are in boxes.)



Allow students to draw "detours" to solve a problem, for example:

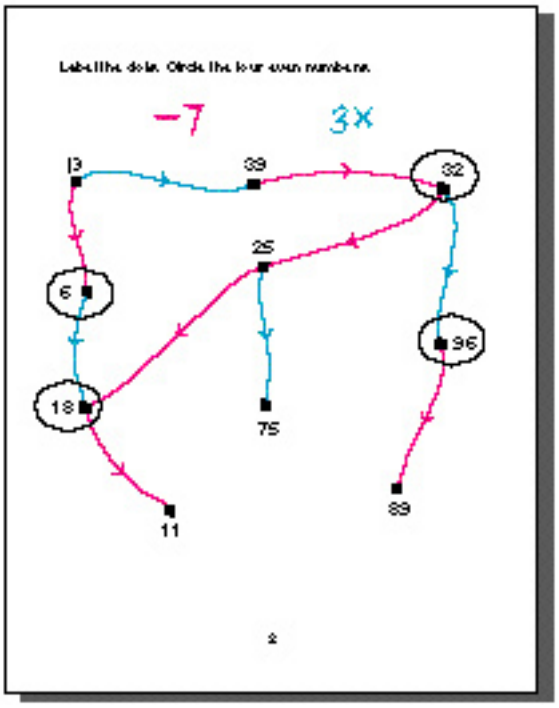


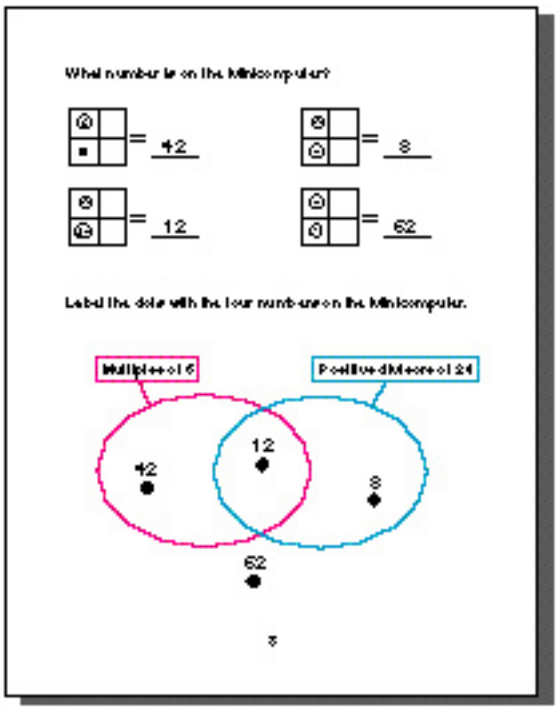
Distribute students' copies of the workbook *Variety of Problems #3*. Ask students first to correct or complete pages from the previous week's work, and then to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

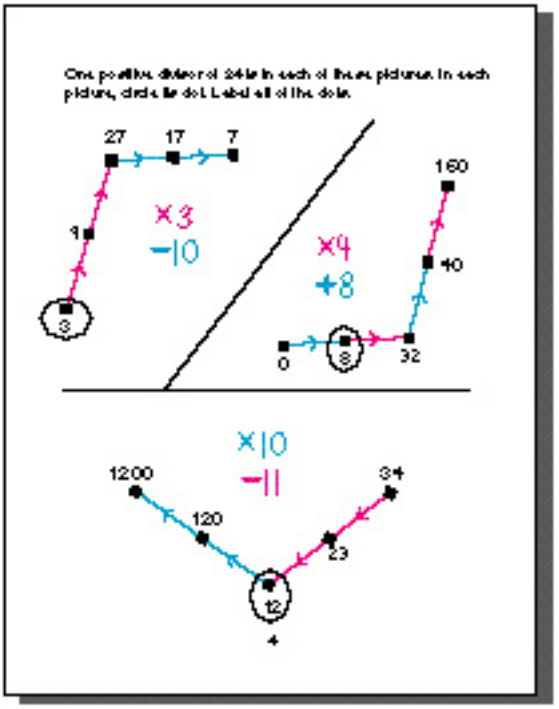
At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in their workbooks at school during a study time or at home as an assignment.

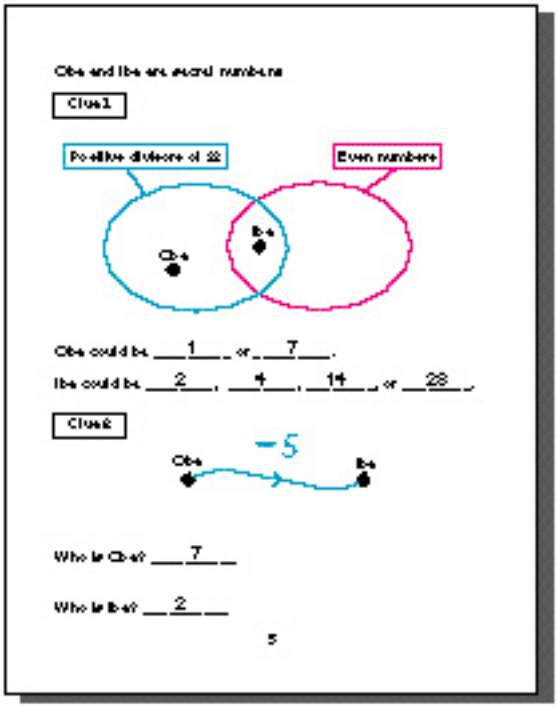
Assessment Activity

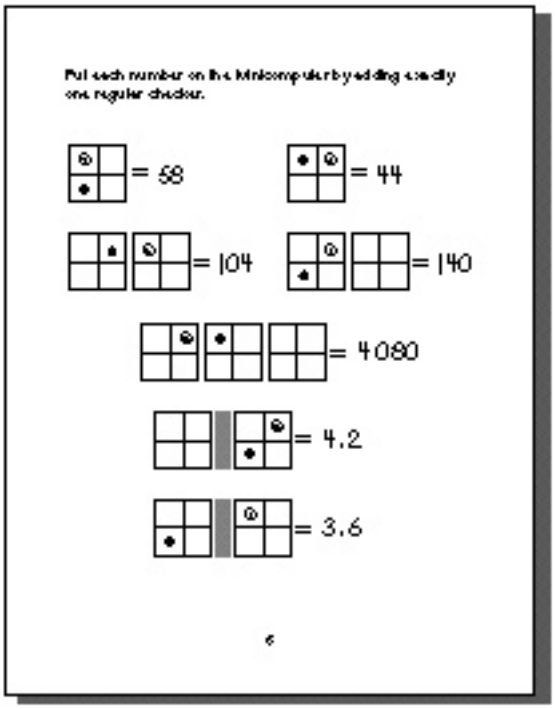
An individual student progress record for the workbook is available on Blackline W10. You may like to use this form to monitor student work.

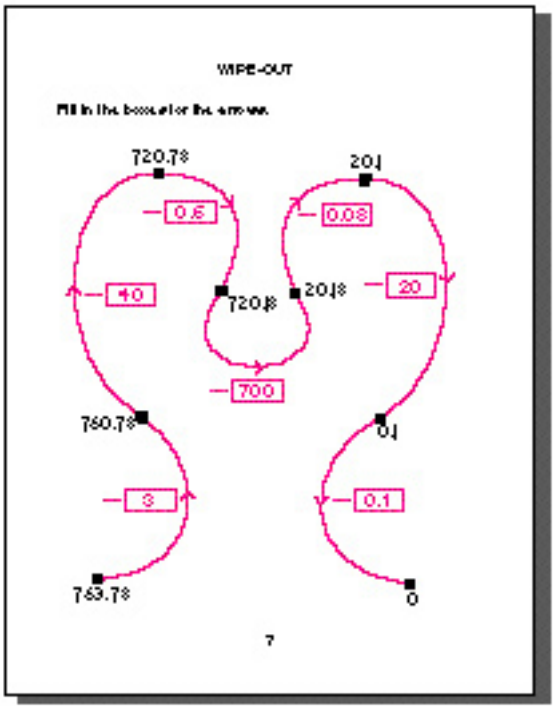

Label the dots. Circle the four even numbers.
-7
3x
13
39
32
25
6
96
18
75
89
11

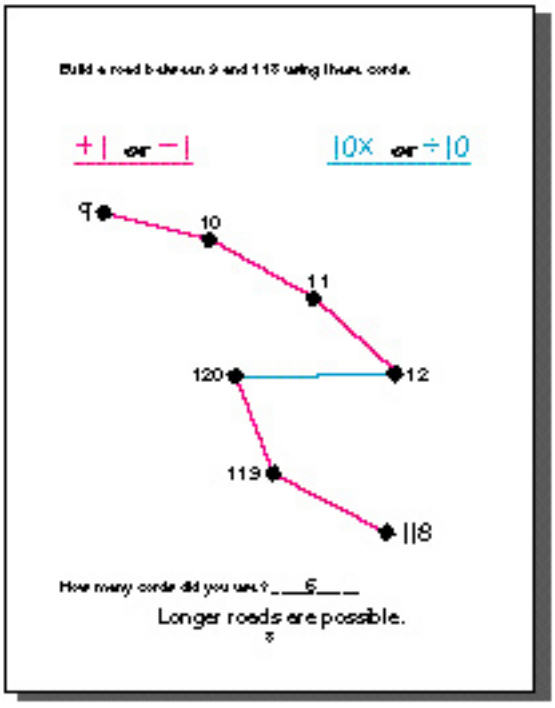

What number is on the Minicomputer?
= 42
= 8
= 12
= 62
Label the dots with the four numbers on the Minicomputer.
Multiples of 6
Positive divisors of 24
42
12
8
62

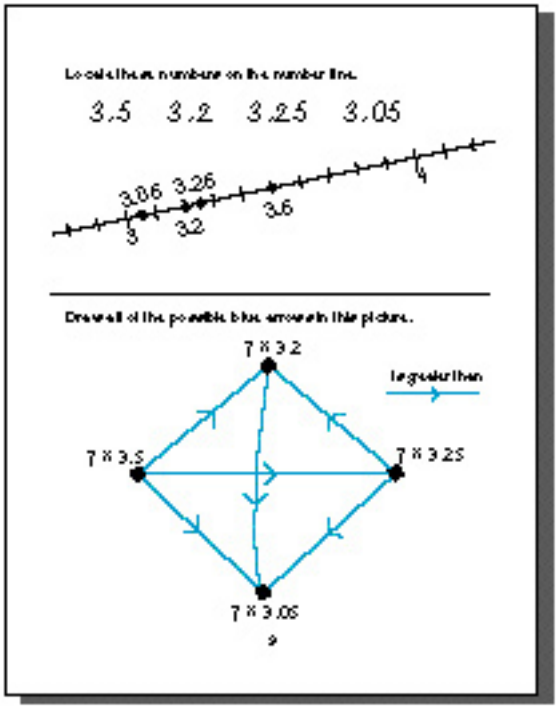

One positive divisor of 24 is in each of these pictures. In each picture, circle it. Label all of the dots.
27
17
7
×3
−10
9
3
160
×4
+8
40
0
8
32
×10
−11
1200
34
120
23
12
4


Obe and Ibe are secret numbers.
Clue 1
Positive divisors of 32
Even numbers
Obe
Ibe
Obe could be 1 or 7.
Ibe could be 2, 4, 14, or 28.
Clue 2
-5
Obe
Ibe
Who is Obe? 7
Who is Ibe? 2
5


= 58
= 44
= 104
= 140
= 4080
= 4.2
= 3.6
6


WIPE-OUT
720.7¢
20¢
0.6
0.08
40
720¢
20¢
20
700
760.7¢
0¢
3
0.1
763.7¢
0
7


+1 or −1
10× or ÷10
9
10
11
120
12
119
118
How many cards did you use? 5
Longer roads are possible.
8


3.5 3.2 3.25 3.05
3.06
3.26
3
3.2
3.6
4
7 × 3.2
is greater than
7 × 3.5
7 × 3.25
7 × 3.05
9

Add.

463 + 58.45

$$\begin{array}{r} 463 \\ +\ 58.45 \\ \hline 521.45 \end{array}$$

Subtract.

63.24 − 58.7

$$\begin{array}{r} 63.24 \\ -\ 58.7 \\ \hline 4.54 \end{array}$$

Multiply.

637 × 6 = 5 022

Multiply.

49 × 65 = 3 185

100 × 65 = 6 500
50 × 65 = 3 250
49 × 65 = 3 185

10

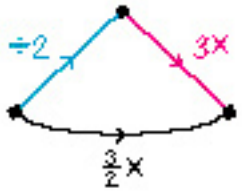
Use red to color all shapes of area 1 cm^2.

Use blue to color all shapes of area 2 cm^2.

Two shapes are colored for you.

When you finish, the picture should be completely colored red and blue.

11

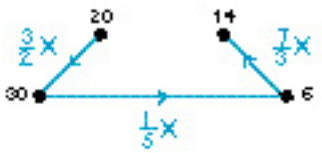


Complete.

$\frac{3}{2}$× 14 = 21 $\frac{3}{2}$× 18 = 27

$\frac{3}{2}$× 140 = 210 $\frac{3}{2}$× 5 = 7.5

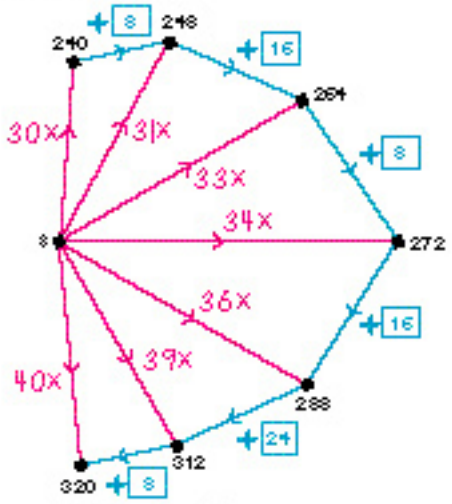
Label the dots.



12

Fill in the boxes for the blue arrows.

Label the dots.



13

What fraction of the shape is each region?

A: ½
B: ¼ or ¼
C: ¼ or ¼
D: ½

What fraction of the shape is red? ½ or ½

What fraction of the shape is blue? ½ or ½

What fraction of the shape is each region?

G: ¼ or ½
H: ½
J: ¼ or ¼
K: ¼ or ½

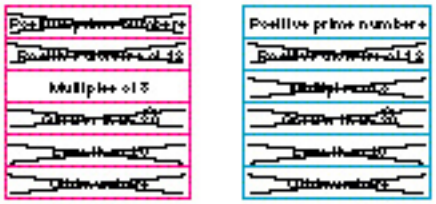
What fraction of the shape is red? ½

What fraction of the shape is blue? ½

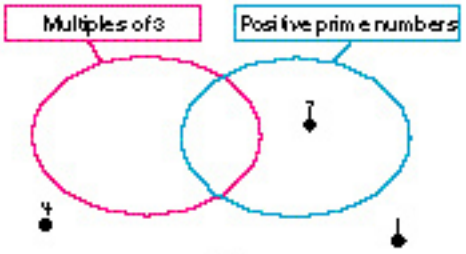
14

The red label is one of these:

The blue label is one of these:



Label the strings.



15

Share 301 newspapers fairly among seven children.

301
− 280
21
− 21

Complete.

301 ÷ 7 = 43

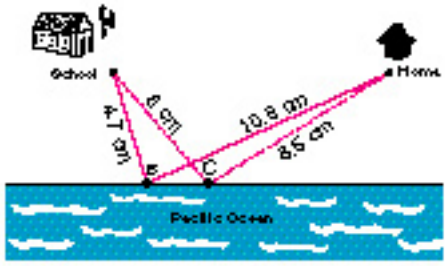
Ask your teacher to check your answer to the above division problem before you solve the problems below. Use your answer to help solve these problems.

Label the red arrows and fill in the blanks.

+ 14 ← 301 ÷ 7 = 43
+ 7 ← 315 ÷ 7 = 45
+ 21 ← 322 ÷ 7 = 46
+ 7 ← 343 ÷ 7 = 49
+ 28 ← 350 ÷ 7 = 50
378 ÷ 7 = 54

16

Alice rides to school each day. Alice walks to the Pacific Ocean for a swim. The map shows her route to go swimming at B.



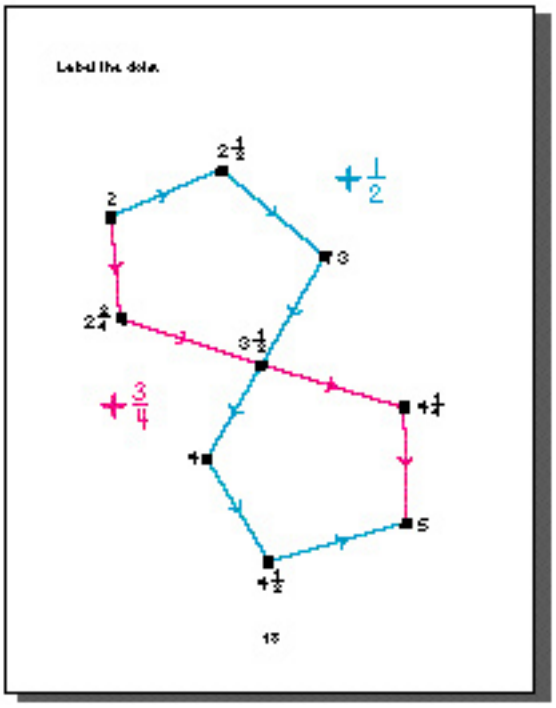
What is the total map distance of her walk from school to B to home? 15.5 cm

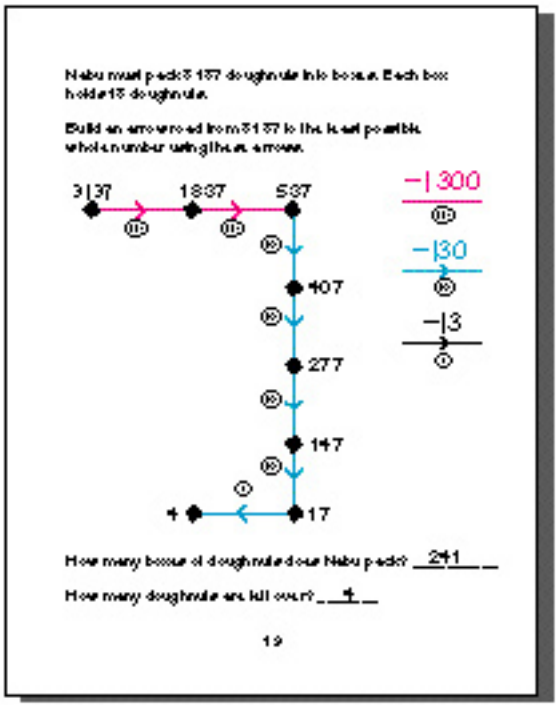
Alice thinks she can shorten her walking distance by swimming somewhere else on the beach. Draw a dot at another place Alice could swim and label it C. Measure the total map distance of her walk from school to C to home. 14.5 cm

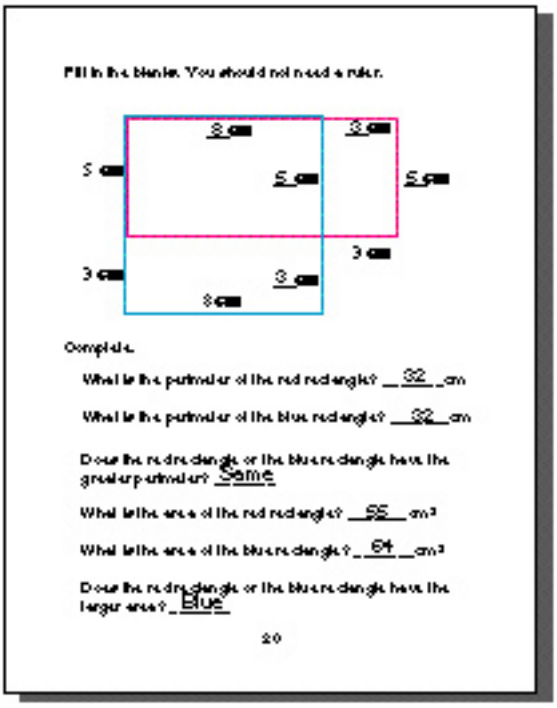
Which route is shorter? School to C to home

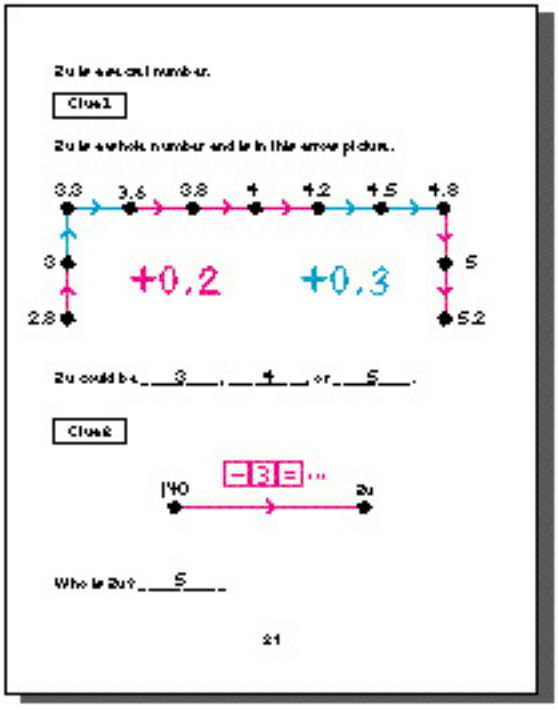
Answers will vary.

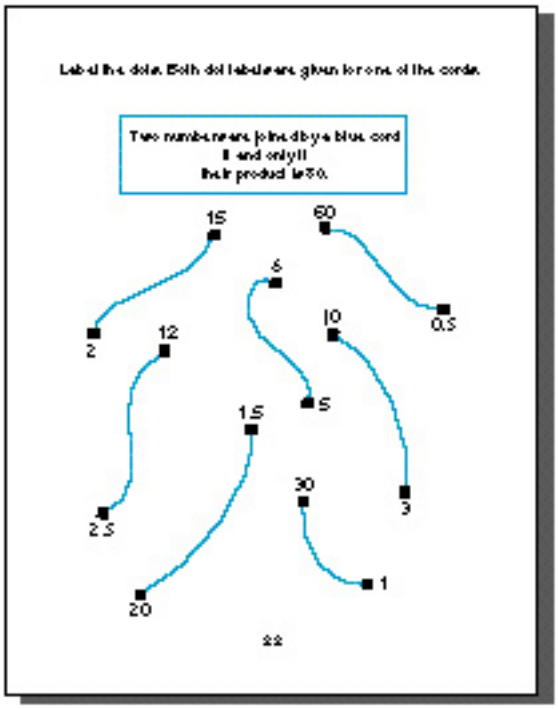
17

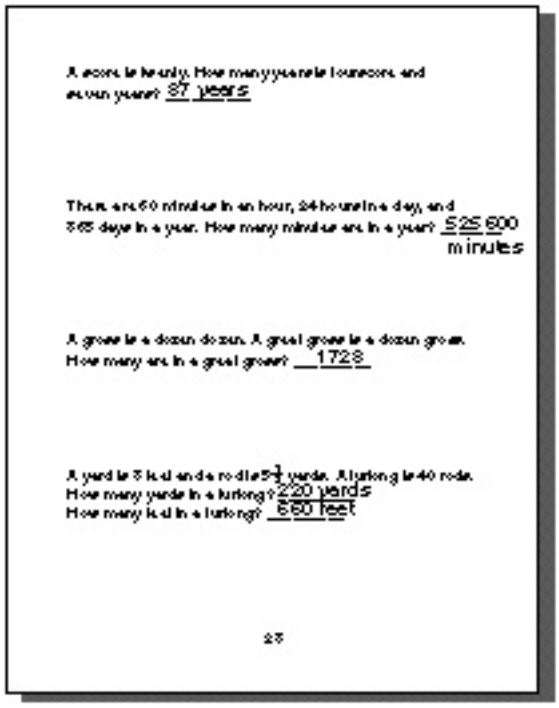

Label the dots.
$2\frac{1}{2}$
2
3
$2\frac{3}{4}$
$3\frac{1}{2}$
$4\frac{1}{2}$
4
5
$4\frac{1}{2}$
$+\frac{1}{2}$
$+\frac{3}{4}$
18

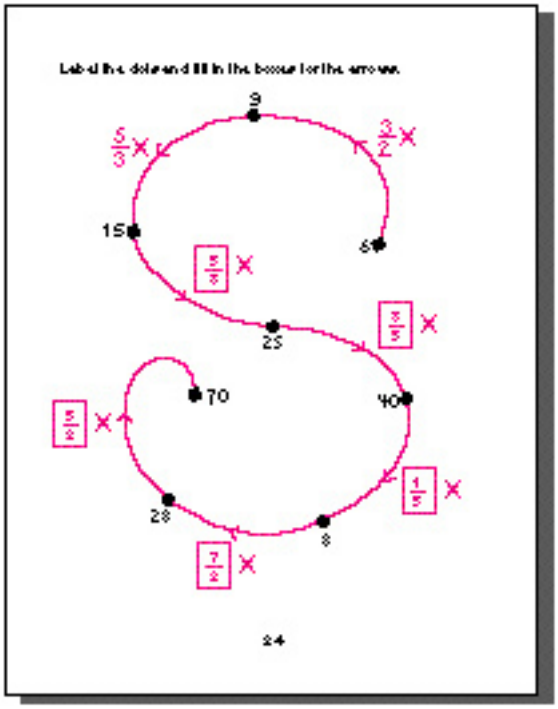

Nabu must pack 537 doughnuts into boxes. Each box holds 13 doughnuts.
Build an arrow road from 537 to the least possible whole number using these arrows.
537
1837
3137
407
277
147
17
4
−1300
−130
−13
How many boxes of doughnuts does Nabu pack? 241
How many doughnuts are left over? 4
19

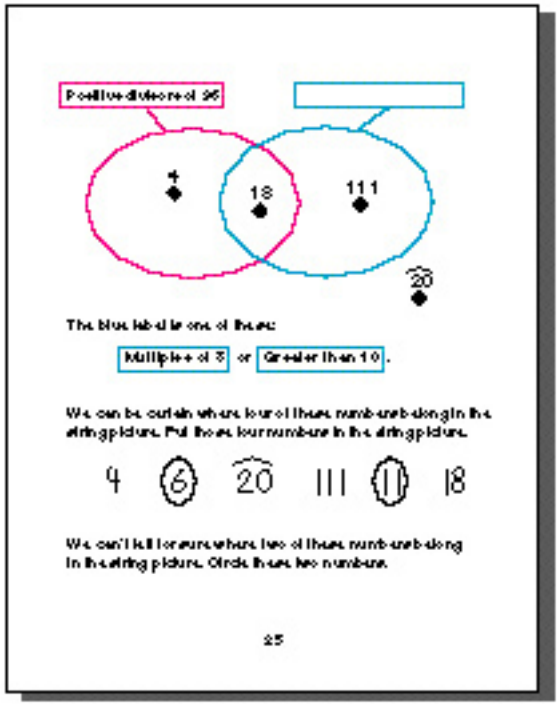

Fill in the blanks. You should not need a ruler.
8 cm
3 cm
5 cm
5 cm
5 cm
3 cm
3 cm
3 cm
8 cm
Complete.
What is the perimeter of the red rectangle? 32 cm
What is the perimeter of the blue rectangle? 32 cm
Does the red rectangle or the blue rectangle have the greater perimeter? Same
What is the area of the red rectangle? 55 cm²
What is the area of the blue rectangle? 64 cm²
Does the red rectangle or the blue rectangle have the larger area? Blue
20


Zu is a secret number.
Clue 1
Zu is a whole number and is in this arrow picture.
3.3
3.6
3.8
4
4.2
4.5
4.8
3
2.8
+0.2
+0.3
5
5.2
Zu could be 3, 4, or 5.
Clue 2
140
Zu
Who is Zu? 5
21


Label the dots. Both dot labels are given for one of the cords.
Two numbers are joined by a blue cord
if and only if
their product is 30.
15
60
6
2
12
10
0.5
5
15
3
25
30
1
20
22


A score is twenty. How many years is fourscore and
seven years? 87 years
There are 60 minutes in an hour, 24 hours in a day, and
365 days in a year. How many minutes are in a year? 525,600
minutes
A grove is a dozen dozen. A great grove is a dozen groves.
How many are in a great grove? 1728
A yard is 3 feet and a rod is 5½ yards. A furlong is 40 rods.
How many yards in a furlong? 220 yards
How many feet in a furlong? 660 feet
23


Label the dots and fill in the boxes for the arrows.
9
3/5
3/2
15
6
2/5
3/5
25
70
40
1/5
28
8
7/2
24


Positive divisors of 36
4
18
111
20
The blue label is one of these:
Multiple of 3
or
Greater than 10.
We can be certain where four of these numbers belong in the
string picture. Put these four numbers in the string picture.
4
6
20
111
11
18
We can't tell for sure where two of these numbers belong
in the string picture. Circle these two numbers.
25

Ghig is a secret number.

Clue 1

Ghig is the ending number of an arrow road starting at 0.5 and using exactly two red arrows and two blue arrows.

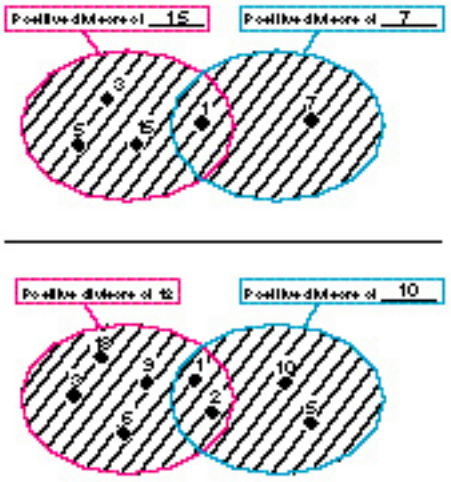
Clue 2

Ghig can be put on the Mini-computer using exactly one of these checkers:

Who is Ghig? 4.8

26

Fill in the blanks and label the dots. Many solutions are possible.

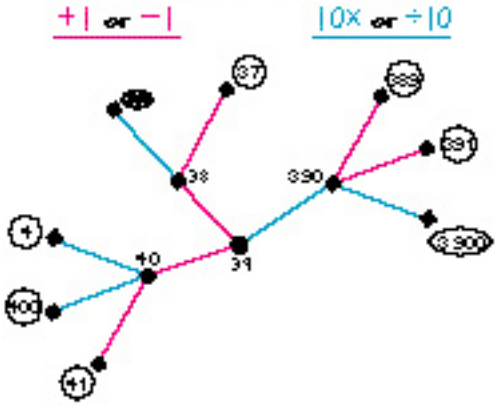


27

All of the numbers in this arrow picture are whole numbers and 153 is the greatest. Circle the dot for 153 and then label all of the dots.

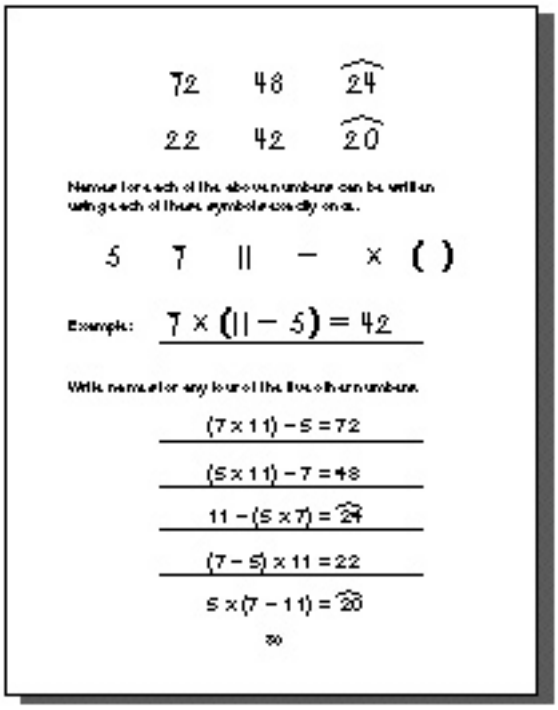
28

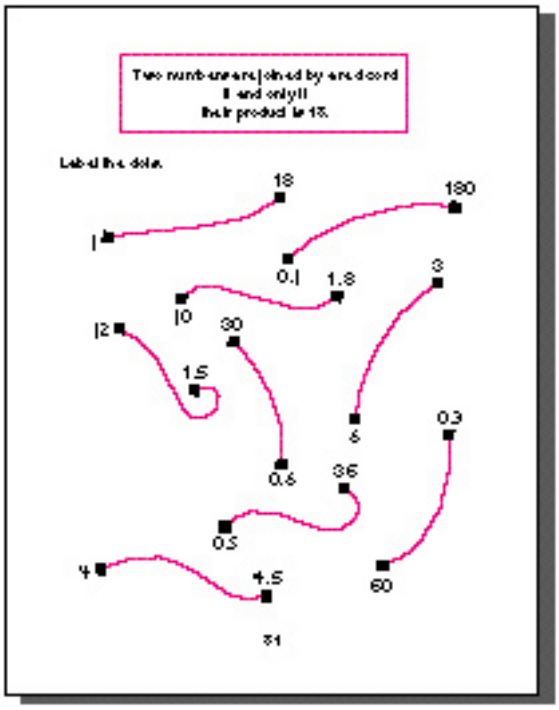
Paula is a secret whole number that is exactly two cards away from 39. Draw a card picture to show all of the numbers that Paula could be. One possibility is done for you. Find a number that Paula cannot be.

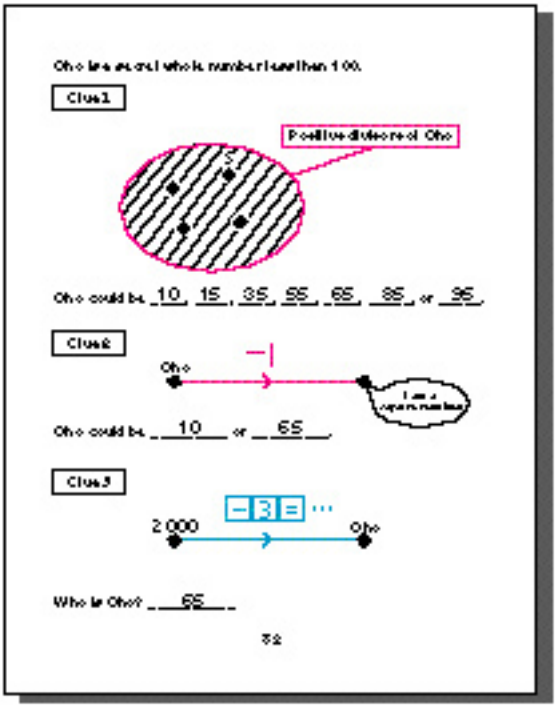


Paula could be 390, 37, 389, 391, 3900, 4, 400, or 41.

29


72 48 24
22 42 20
Names for each of the above numbers can be written using each of these symbols exactly once.
5 7 11 − × ( )
Example: 7 × (11 − 5) = 42
Write names for any four of the five other numbers.
(7 × 11) − 5 = 72
(5 × 11) − 7 = 48
11 − (5 × 7) = 24
(7 − 5) × 11 = 22
5 × (7 − 11) = 20
30


Two numbers joined by an edge
if and only if
their product is 18.
Label the dots.
18
180
1
0.1
1.8
3
10
2
30
1.5
6
0.3
0.6
36
0.5
4
4.5
60
31


Oho is a natural whole number less than 100.
Clue 1
Positive divisors of Oho
5
Oho could be 10, 15, 35, 55, 65, 85, or 95.
Clue 2
Oho
−1
Oho could be 10 or 65.
Clue 3
−3=
2000
Oho
Who is Oho? 65
32

Capsule Lesson Summary

Read the story-workbook *Seven Secret Numbers*, a detective story about seven secret numbers and four spies. Through a series of arrow pictures, determine the identities of the secret numbers and the spies.

Materials

Teacher	• *Seven Secret Numbers* Story-Workbook • Colored chalk	Student	• *Seven Secret Numbers* Story-Workbook • Colored pencils, pens, or crayons

Description of Lesson

This lesson description suggests some possible questions and gives sample student comments for a class presentation of the story-workbook *Seven Secret Numbers*. Feel free to modify the presentation to make it appropriate for your students.

Distribute copies of the story-workbook *Seven Secret Numbers*. You may like to let students work with partners.

Pages 2 to 5

Invite students to read pages 2 to 5 aloud. Students should study the arrow picture on page 4 and answer the questions on page 5. In the meantime, draw the arrow picture from page 6 on the board.

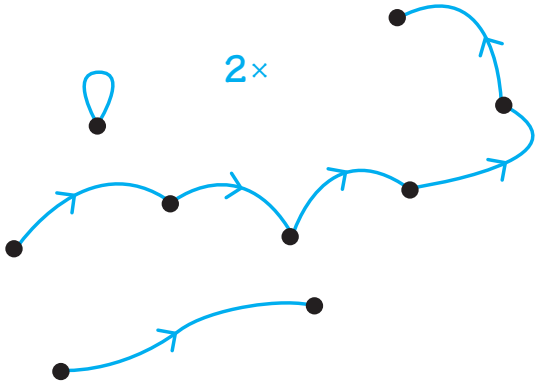
T: ***Look at the arrow picture on page 4. What is the greatest number and what is the least number in the picture?***

S: ***157 is the greatest number and $\widehat{3}$ is the least number.***

Collectively check answers to the questions on page 5. The class should conclude that the seven secret numbers and the four spies are all integers greater than $\widehat{4}$ and less than 158.

Pages 6 to 7

Invite a student to read page 6 aloud.



T: ***The arrow picture on the board is like the one on page 6. One of the secret numbers is easy to find in this picture. Can you find it?***

S: ***0 with the loop, because 2 x 0 = 0.***

T (pointing to the starting dot of the five-arrow road)**:** ***What could be the starting number for this arrow road?***

S: ***4 could be the starting number.***

T: ***If 4 were the starting number, what other numbers would be in this arrow road?***

S: ***8, 16, 32, 64, and 128.***

List the possibilities on the board.

T: ***Are there other possibilities for the secret numbers?***

As students discover possibilities, list them on the board while students record possibilities on page 7. A complete list is given here.

The seven secret numbers could be:

0, 4, 8, 16, 32, 64, 128
or
0, 2, 4, 8, 16, 32, 64
or
0, 3, 6, 12, 24, 48, 96
or
0, 1, 2, 4, 8, 16, 32

T: ***Are there other possibilities?***

S: ***If we start with 5, then the other number would be 10, 20, 40, 80, and 160. But all of the secret numbers are less than 158, so the starting number cannot be 5; it cannot be greater than 4.***

T: ***Could some of the secret numbers be negative?***

S: ***No. If $\widehat{1}$ were the starting number, then the other numbers in the picture would be $\widehat{2}$, $\widehat{4}$, $\widehat{8}$, $\widehat{16}$, and $\widehat{32}$. But all of the secret numbers must be greater than $\widehat{4}$.***

T: ***What information are we given about two of the spies on page 6?***

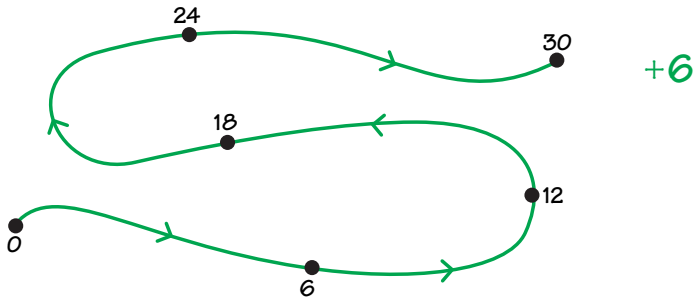
S: ***One spy is the double of another spy.***

Pages 8 and 9

Draw the arrow picture from page 8 on the board.

T: ***Let's see if we can use the clue on page 8 to identify some of the seven secret numbers. Who has an idea?***

Listen carefully to students' suggestions. Someone should notice that 0, 6, 12, and 24 will fit in the arrow picture along with 18 and 30. Invite a student to label the dots in the arrow picture.



Note: If students suggest labeling the starting dot of the road $\widehat{6}$, remind them that all of the spies must be greater than $\widehat{4}$.

T: ***Who are the four secret numbers in this picture? Who are the two spies?***

S: ***0, 6, 12, and 24 are secret numbers. The two spies are 18 and 30.***

T: ***So 0, 6, 12, and 24 are the secret numbers who run away. Who are the three other secret numbers?***

S: ***3, 48, and 96.***

The class should conclude that the seven secret numbers are 0, 3, 6, 12, 24, 48, and 96, and that two of the spies are 18 and 30.

Write this information on the board.

Secret Numbers: 0, 3, 6, 12, 24, 48, 96
Spies: 18, 30

Pages 10 and 11

Ask a student to read page 10 aloud.

T: ***Let's see if the clue on page 10 will help us identify the other two spies. Where are the secret numbers?***

S: ***Four of them are in the truck.***

S: ***0 is the driver.***

T: ***18 and 30 are two of the spies, but one of them cannot be in the helicopter. Which one?***

S: ***30. ⅓ x 30 = 10, but 10 is not a secret number.***

T: ***Which spies must be in the helicopter? Why?***

S: ***18 must be in the helicopter, because there are only four spies and 30 is not in the helicopter. So all of the other spies are in the helicopter.***

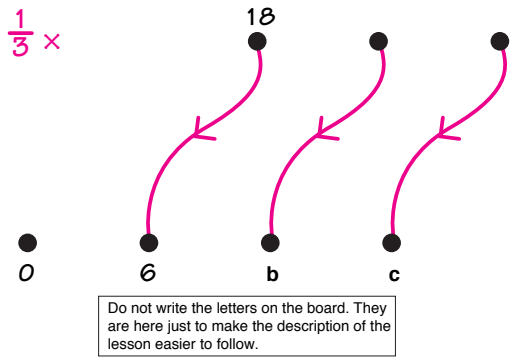
Draw this picture on the board.

T: ***We know that 0 and 6 are two of the secret numbers in the truck.*** (Point to **b** and **c**.) ***Which numbers could be here?***

S: ***3, 12, 24, 48, or 96.***

T: ***Yes, those are the other secret numbers, but one of them cannot be in the truck. Which one?***

S: ***96 cannot be in the truck, because 3 x 96 = 288 and 288 is greater than 157.***



T: ***Could 48 be in the truck? If so, then who would be the spy in the helicopter watching 48?*** (144, because 3 x 48 = 144)

If 24 were in the truck, who would be the spy in the helicopter watching 24? (72, because 3 x 24 = 72)

If 12 were in the truck, then who would be the spy in the helicopter watching 12? (36, because ⅓ x 36 = 12)

And if 3 were in the truck; then who would be the spy in the helicopter watching 3? (9, because ⅓ x 9 = 3.)

Write these numbers on the board. **9, 36, 72, 144**

T: ***This clue does not tell us exactly who the two other spies are, but we do know that they are two of these numbers.***

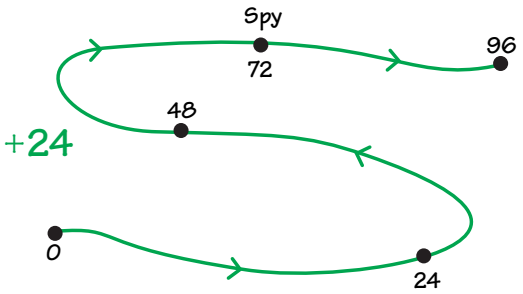
Pages 12 and 13

Ask a student to read page 12 aloud. Allow students a few minutes to label the dots on page 12 and to answer the questions on page 13. While students are working, draw the arrow picture from page 12 onto the board.

T: ***Where is the spy in this picture?*** (With binoculars)
Who are the four secret numbers in this picture?

S: ***0, 24, 48, and 96.***

Ask a student to label the dots on the board.



T: ***Who is the spy with the binoculars?***

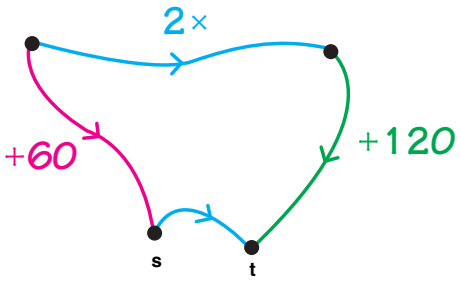
S: ***72.***

Record 72 in the list of known spies.

Pages 14 and 15

T: ***Now we know that three of the spies are 18, 30, and 72, and that the fourth spy is 9 or 36 or 144. Let's see if we can use the clue on page 14 to determine who is the fourth spy.***

Ask students to read pages 14 and 15 while you draw the arrow picture from page 14 onto the board.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: ***Where are the spies in this picture?***
(At **s** and **t**)
Who is one of the spies on the motorcycle?

S: ***72, the spy with the binoculars.***

T: ***Where does 72 go in the picture? Why?***

S: ***72 goes here (at* s*). If 72 were here (at* t*), then the numbers in the forest wouldn't be secret numbers.***

T: ***Who are the secret numbers in this picture?***

S: ***12 and 24.***

T: ***Who is the other spy?***

S: ***144.***

If time and interest permit, suggest that students write the names of the spies and secret numbers in each picture of their books.

Writing Activity

Suggest that students write their own stories about the seven secret numbers and the four spy numbers.

Could one of the secret numbers or one of the spies be:

$\frac{1}{3}$? No

75 ? Yes

325 ? No

200 ? No

2 ? Yes

10 ? No

The seven secret numbers and the four spies are all bigger than 4 and smaller than 158.

5

The seven secret numbers could be:

0, 1, 2, 4, 8, 16, 32;

or

0, 2, 4, 8, 16, 32, 64;

or

0, 3, 6, 12, 24, 48, 96;

or

0, 4, 8, 16, 32, 64, 128;

Which number is certainly one of the secret numbers? 0

7

The four secret numbers who run away are:

0, 6, 12, and 24.

The two spies who run with them are 18 and 30.

The three other secret numbers are:

3, 48, 96.

9

The truck driver is 0. Write the driver's name in the picture.

You already know that two of the spies are 18 and 30.

Can you explain why one of them (30) cannot be in the helicopter?

Can you explain why the other one (18) is in the helicopter?

$\frac{1}{3}$ x 18 = 6 and 6 is a secret number. Also, only one of the four spies is not in the helicopter and we know that spy is 30.

Write the spy's name in the picture. It will give you a clue about one of the secret numbers in the truck.

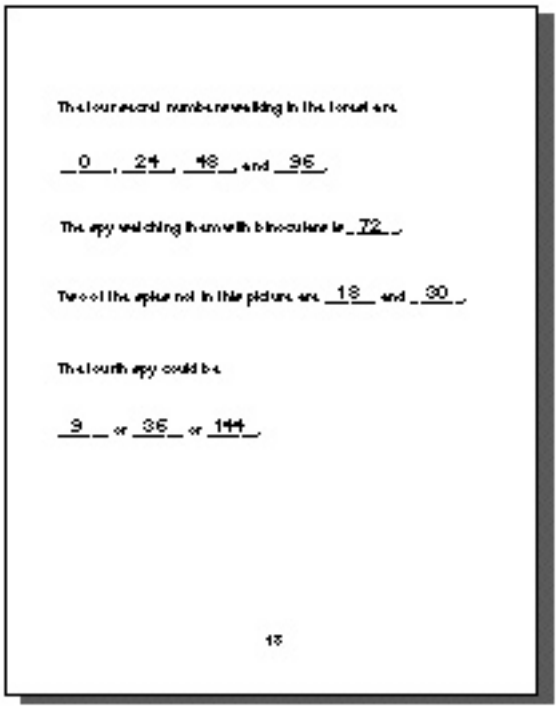
The other two spies in the helicopter are two of these four numbers:

9, 36, 72, 144.

Can you explain why these are the only possible numbers that the other two spies can be?

3 x 3 = 9, 3 x 12 = 36, 3 x 24 = 72, 3 x 48 = 144, and 3 x 96 = 288. 3 x 96 is too big—all of the spies are smaller than 158.

11



The four secret numbers hiding in the forest are

0, 24, 48, and 96.

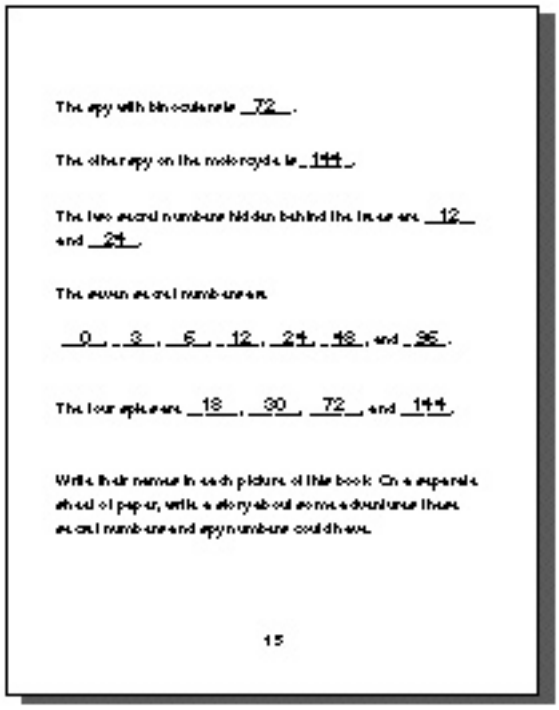
The spy watching them with binoculars is 72.

Two of the spies not in the picture are 18 and 30.

The fourth spy could be

9 or 36 or 144.

13



The spy with binoculars is 72.

The other spy on the motorcycle is 144.

The two secret numbers hidden behind the trees are 12 and 24.

The seven secret numbers are

0, 3, 6, 12, 24, 48, and 96.

The four spies are 18, 30, 72, and 144.

Write their names in each picture of this book. On a separate sheet of paper, write a story about some adventures these secret numbers and spy numbers could have.

15

Capsule Lesson Summary

Estimate the answers to three multiplication problems, and then compare the actual answers to the estimates. Begin the workbook *Variety of Problems #4*. (This is the first of two lessons using this workbook).

Materials

Teacher	• None	Student	• *Variety of Problems #4* Workbook • Colored pencils, pens, or crayons • Metric ruler • Calculator

Description of Lesson

Begin the lesson with some mental arithmetic involving multiplication and division facts.

3 x 7 = 21	56 ÷ 7 = 8	63 ÷ 7 = 9
21 ÷ 3 = 7	36 ÷ 9 = 4	32 ÷ 8 = 4
8 x 8 = 64	27 ÷ 3 = 9	18 ÷ 3 = 6
64 ÷ 8 = 8	45 ÷ 9 = 5	72 ÷ 9 = 8

Write these three problems on the board.

66 × 45	23 × 92	36 × 84

T: ***Which of these problems do you think has the greatest answer (product)?***

Let students discuss and compare their estimates. Discourage exact calculations done on paper or with a calculator at this time. A sample dialogue follows.

S: ***92 x 23, because 92 is the greatest number in any of the problems.***

S: ***84 x 36, because 36 is between 45 and 23, and 84 is between 66 and 92.***

S: ***I think 84 x 36 is greater than 92 x 23, because 84 is only 8 less than 92 whereas 36 is 13 more than 23.***

T: ***Let's estimate. About how much is 84 x 36?***

S: ***About 3 000.***

S: ***84 is less than 100, and 36 x 100 = 3 600, so the answer is definitely less than 3 600.***

S: ***80 x 30 = 2 400, so the answer is definitely more than 2 400.***

Record well-reasoned estimates on the board above the problems.

T: ***Can we get a better estimate?***

S: ***84 is close to 80, and 36 is close to 40. 80 x 40 = 3 200.***

T: ***3 200 is a good estimate for 84 x 36. What about 92 x 23?***

S: ***About 1 800. 92 is close to 90, and 23 is close to 20. 90 x 20 = 1 800.***

T: ***Is 92 x 23 more or less than 1 800?***

S: ***More, because we multiplied two numbers less than 92 and 23 to get the estimate.***

S: ***23 x 100 = 2 300, so 92 x 23 is less than 2 300.***

T: ***What about 45 x 66?***

S: ***About 3 500. 50 x 70 = 3 500.***

T: ***Is 45 x 66 more or less than 3 500?***

S: ***Less, because we multiplied two numbers more than 45 and 66 to get the estimate.***

S: ***Maybe 50 x 60 = 3 000 would be a better estimate for 45 x 66.***

S: ***Or 40 x 70 = 2 800 might be a better estimate.***

Use whatever estimates are offered by the class to again compare the three problems.

S: ***Maybe 92 x 23 is least and 84 x 36 is greatest.***

T: ***Let's do the multiplication calculations to check our predictions.***

Invite three students to do the calculations at the board. Compare exact calculations to the estimates.

66	23	36
× 45	× 92	× 84
5 × 66 = 330	2 × 23 = 46	4 × 36 = 144
40 × 66 = 2 640	90 × 23 = 2 070	80 × 36 = 2 880
2 970	2 116	3 024

Distribute copies of the workbook *Variety of Problems #4* and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of the lesson, collect the workbooks for your review. They will be used again in Lesson W13.

Capsule Lesson Summary

Play *Calculator Golf* in which players go from a starting number to a target number using the operations +, –, x, or ÷, and one-digit positive integers. Continue individual work in the workbook *Variety of Problems #4*. (This is the second of two lessons using this workbook.)

Materials

Teacher	Student
• Overhead calculator (optional) • Colored chalk	• Calculator • Paper • Colored pencils, pens, or crayons • *Variety of Problems #4* Workbook • Metric ruler

Description of Lesson

Exercise 1

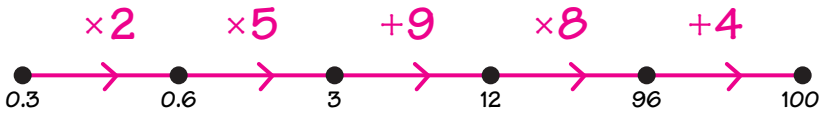
Display an overhead calculator, if available, and provide each student or pair of students with a calculator. Ask students to recall some of what they remember about golf and the *Minicomputer Golf* game.

T: ***Today we are going to play a game called Calculator Golf. We start with a number on the display of the calculator and then set a goal.***

Draw two dots on the board. Label one of them 0.3 and the other 100.

T: ***Let's start with 0.3*** (put 0.3 on the display) ***and make 100 the goal. When you play this golf game, you can press any operation key (+, –, ×, or ÷) followed by a one-digit number (1 through 9), and then =. Play continues until 100 is on the display.***

Invite students to take turns playing the game on the overhead calculator or a classroom calculator. Record the play in an arrow picture on the board. For example, the following picture is for a game with five turns (steps).



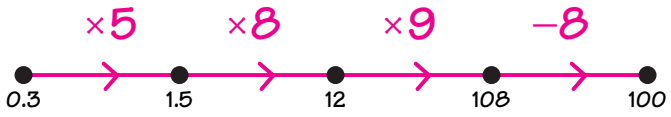
T: ***Put 0.3 on your display. You can add, subtract, multiply, or divide by any of the one-digit numbers 1 through 9. As in golf, try to reach 100 in fewer steps than we did here.***

Suggest that students press = after pressing a number key so that they can see the result before deciding which operation to use next.

Note: If your students have calculators, you need not require that they record their steps on paper. Some will want to keep track of their steps while others will find working with both pencil and calculator awkward and inhibiting.

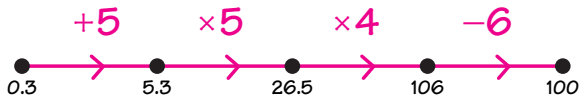
Allow a few minutes for students to work on this problem before asking them to share solutions with the class. As a student describes a solution, draw the corresponding arrow road on the board. For example:

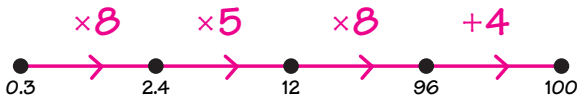
S: *I pressed* × 5 =, *then* × 8 =, *then* × 9 =, *and then* − 8 =.



T: *That took four steps (arrows). Did anyone get to 100 with fewer steps?*

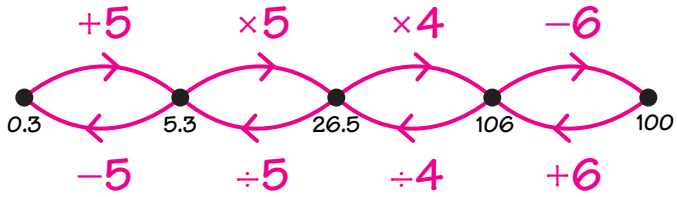
Continue this activity until several solutions are on the board. Try to include several solutions with four steps (arrows). Two of the many four-step solutions are shown here for your information.





Encourage students to try to find other solutions that use fewer steps (arrows) than those on the board. Perhaps your class will see that it is not possible to go from 0.3 to 100 using fewer than four steps.

Do not erase the arrow pictures. Ask students to go from 100 to 0.3 with the same restrictions; that is, play the game starting at 100 and make 0.3 the goal. Students should notice that to build an arrow road from 100 to 0.3, you only need to go backward on a road from 0.3 to 100. For example, if the following arrow road were suggested from 0.3 to 100, you could reverse the arrows and the return road would go from 100 to 0.3.

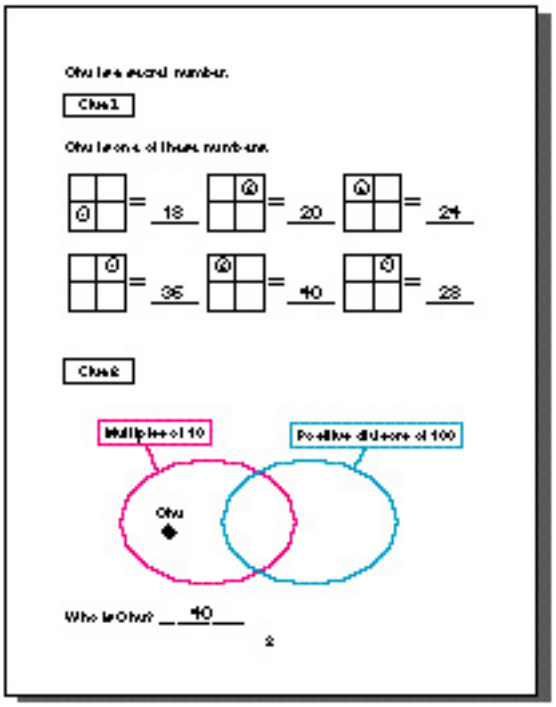


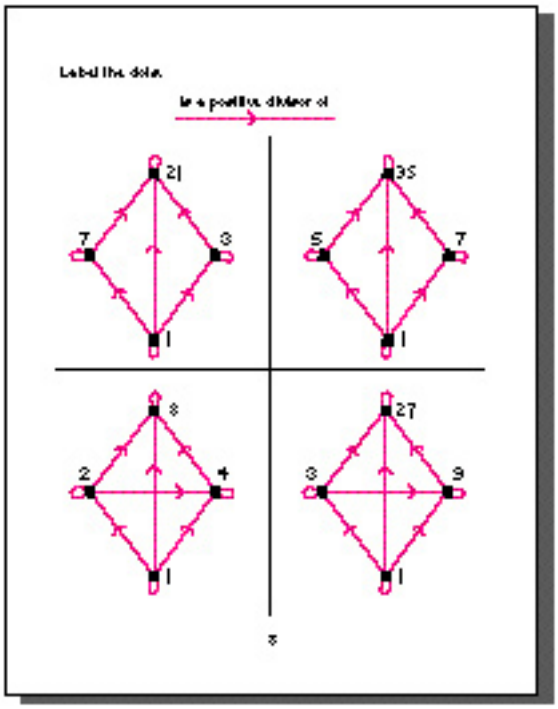
Return students' copies of the workbook *Variety of Problems #4*. Ask students first to correct or complete pages they worked on the previous week and then to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

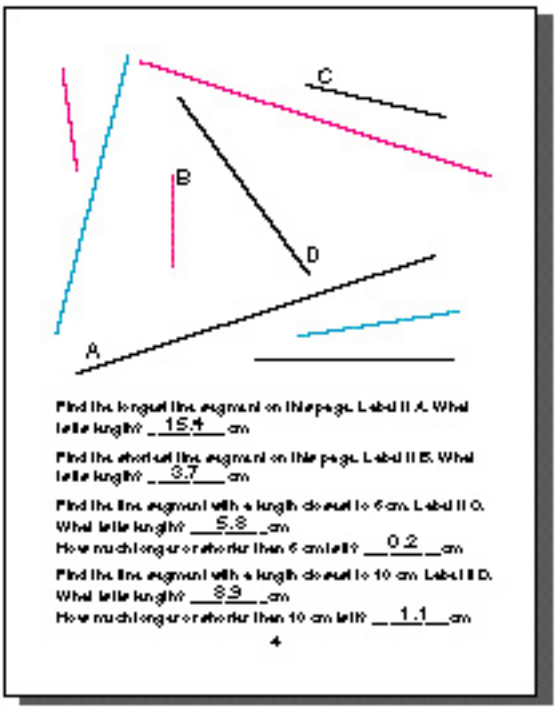
At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in their workbooks at school during a study time or at home as an assignment.

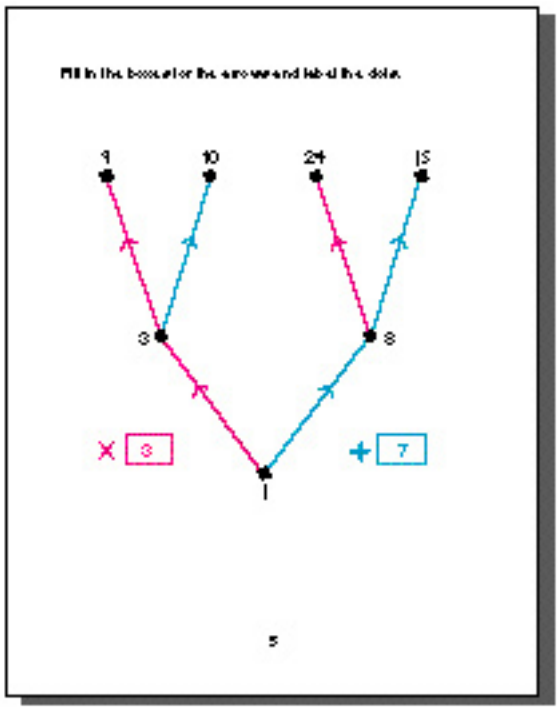
Assessment Activity

An individual student progress record for the workbook is available on Blackline W13. You may like to use this form to monitor student work.

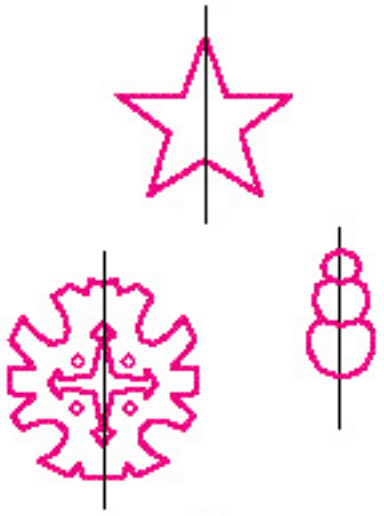

Obu is a secret number.
Clue 1
Obu is one of these numbers.
18
20
24
36
40
28
Clue 2
Multiples of 10
Positive divisors of 100
Obu
Who is Obu? 40
2


Label the dots.
is a positive divisor of
21
7
3
1
35
5
7
1
8
2
4
1
27
3
9
1
3


C
B
D
A
Find the longest line segment on this page. Label it A. What is its length? 15.4 cm
Find the shortest line segment on this page. Label it B. What is its length? 3.7 cm
Find the line segment with a length closest to 6 cm. Label it C.
What is its length? 5.8 cm
How much longer or shorter than 6 cm is it? 0.2 cm
Find the line segment with a length closest to 10 cm. Label it D.
What is its length? 8.9 cm
How much longer or shorter than 10 cm is it? 1.1 cm
4


Fill in the boxes for the arrows and label the dots.
1
10
24
15
3
8
× 3
+ 7
1
5

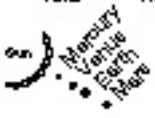
Draw what would appear in a mirror if it were placed along the black line. The drawings do not have to be exact.



6

The distance from the planet to the sun are given below in astronomical units. Using this information, label the six planets that are closest to the sun in the picture below.

Mercury	0.387	Earth	1.000
Pluto	39.44	Saturn	9.539
Mars	1.524	Neptune	30.06
Venus	0.723	Uranus	19.18
		Jupiter	5.203



Jupiter

Saturn

Three of the planets are so far away from the sun that they cannot be shown on this page in their relative positions to the other planets. Which three planets are they? Uranus, Neptune, and Pluto

Note: An astronomical unit is the average distance from the Earth to the sun.

7

Add parentheses to each number sentence to make it true.

$(7 \times 8) + 4 = 60$

$7 \times (8 + 4) = 84$

$13 - (7 + 9) = 3$

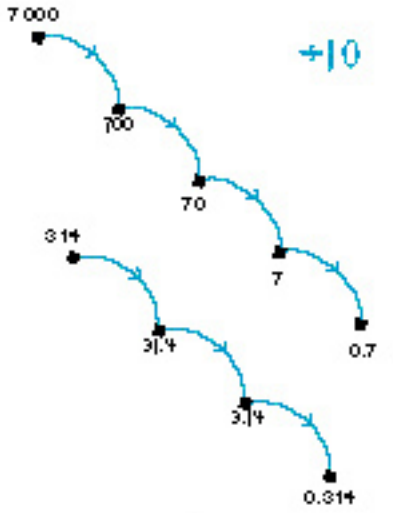
$(13 - 7) + 9 = 15$

$(2.5 - 18) \div 2 = 3.5$

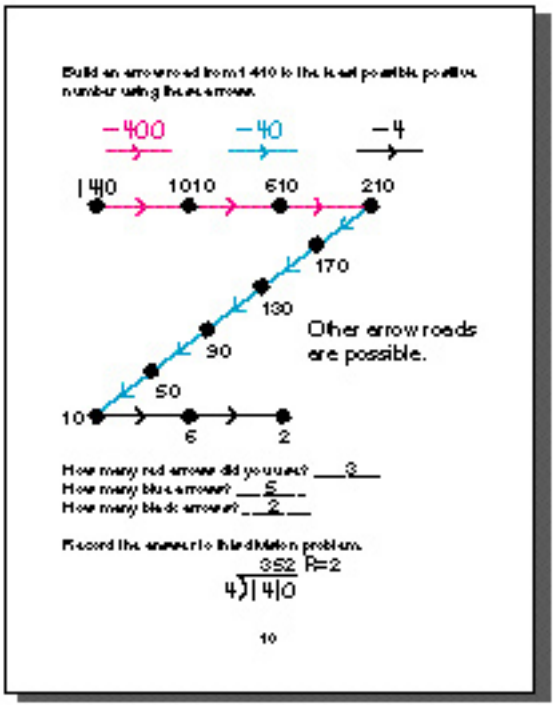
$2.5 - (18 \div 2) = 16$

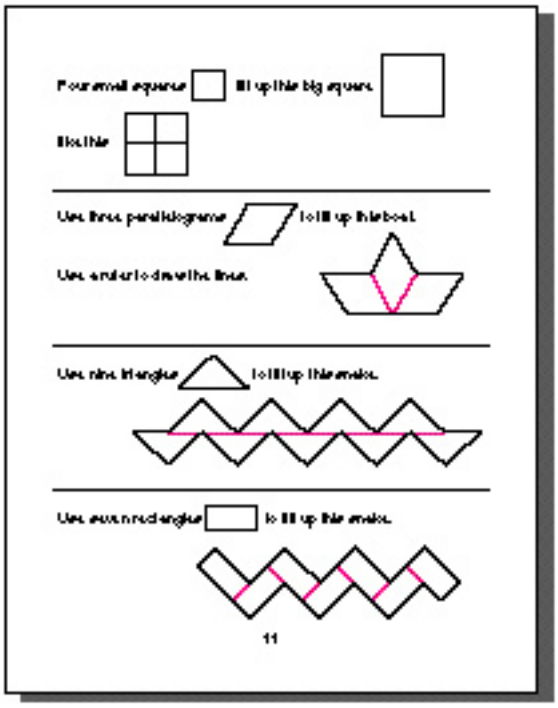
8

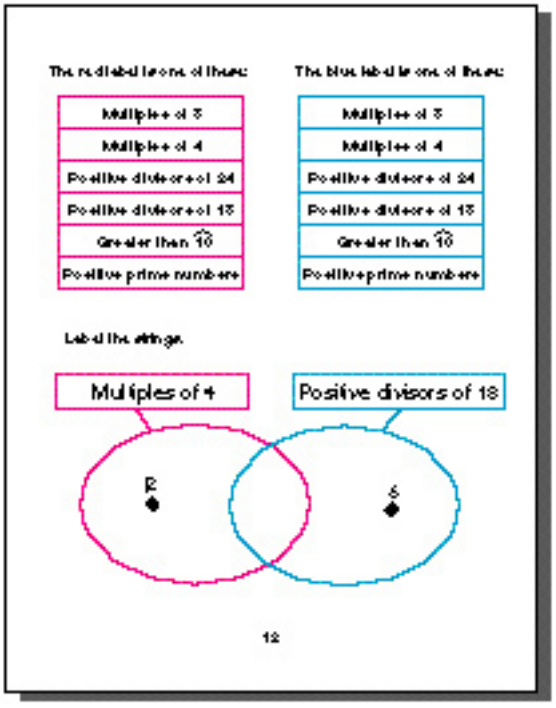
Label the dots.

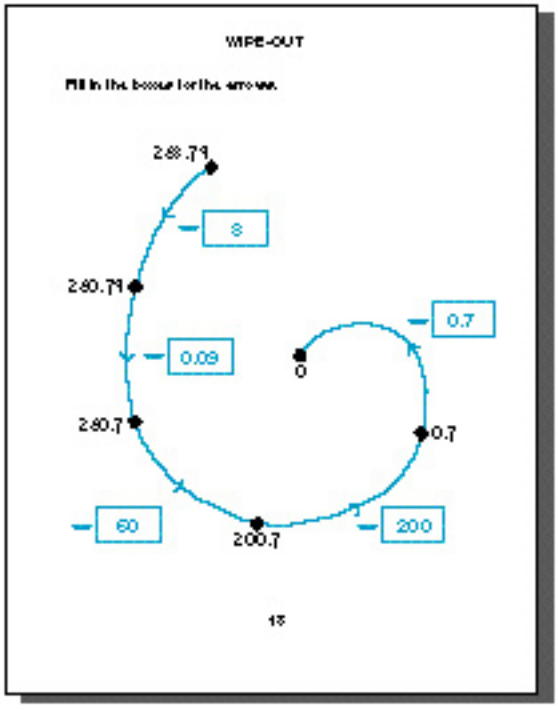


9


−400
−40
−4
1410
1010
610
210
170
130
90
50
10
6
2
Other arrow roads are possible.
How many red arrows did you use? 3
How many blue arrows? 5
How many black arrows? 2
Record the answer to the division problem.
352 R=2
4)1410
10


Four small squares fill up the big square.
Use three parallelograms to fill up the boat.
Use nine triangles to fill up the snake.
Use seven rectangles to fill up the snake.
11


The red labeled arrows all have:
The blue labeled arrows all have:
Multiples of 3
Multiples of 4
Positive divisors of 24
Positive divisors of 18
Greater than 10
Positive prime numbers
Label the strings.
Multiples of 4
Positive divisors of 18
8
6
12


WIPE-OUT
Fill in the boxes for the arrows.
268.79
8
260.79
0.7
0.09
0
260.7
0.7
60
200.7
200
13

Pizza Pie

Tom, Dara, and Zoe are going to have pizza for lunch. Tom wants $1\frac{1}{2}$ pizzas, Dara and Zoe each want $\frac{3}{4}$ of a pizza.
How many pizzas should they order? 3

Will there be any pizza left over? No

Each pizza comes cut into eight pieces.

How many pieces does Tom want? 12
How many pieces does Dara want? 6
How many pieces does Zoe want? 6

14

Eraser Gremlin Strikes Again!

Fill in the boxes.

$$\begin{array}{r} 7\boxed{8}3 \\ +45\boxed{6} \\ \hline 1239 \end{array} \qquad \begin{array}{r} 100 \\ -\boxed{7}2 \\ \hline 2\boxed{8} \end{array} \qquad \begin{array}{r} 7\boxed{4} \\ \times\ 4 \\ \hline 2\boxed{9}6 \end{array}$$

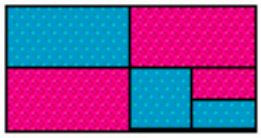
$$\begin{array}{r} 3\boxed{8} \\ \times\boxed{5}6 \\ \hline 228 \\ 1\boxed{9}00 \\ \hline 2128 \end{array} \qquad 7\overline{)5\boxed{6}}\ \boxed{8}$$

15

What fraction of each shape is colored red?
What fraction of each shape is colored blue?

Red: $\frac{3}{8}$
Blue: $\frac{5}{8}$

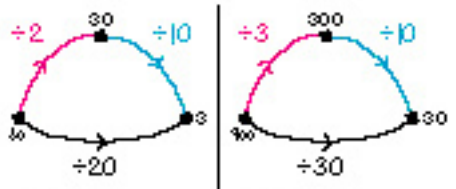
Red: $\frac{3}{12}$ or $\frac{1}{4}$
Blue: $\frac{4}{12}$ or $\frac{1}{3}$



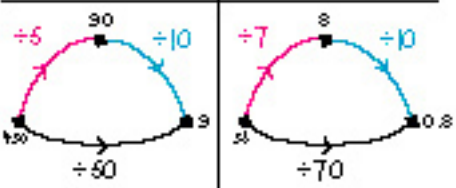
Red: $\frac{9}{16}$
Blue: $\frac{7}{16}$

16

Label the dots in the arrow pictures and then complete the number sentences.

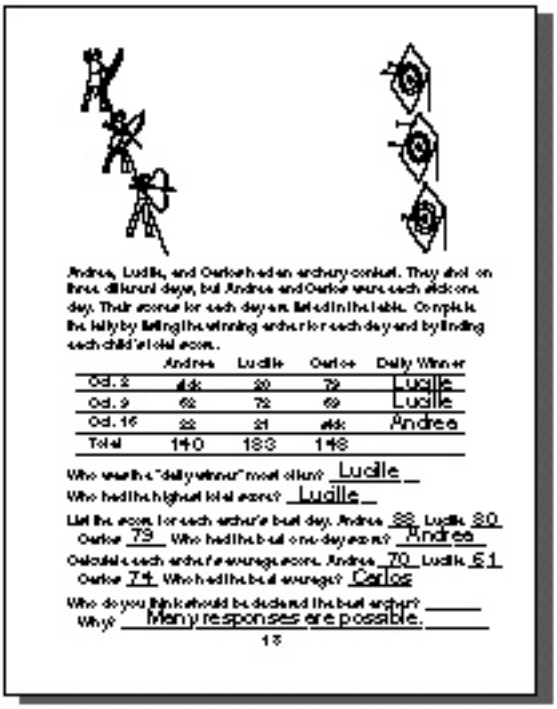


$60 \div 20 = 3$ $\qquad$ $900 \div 30 = 30$



$450 \div 50 = 9$ $\qquad$ $56 \div 70 = 0.8$

17



Andrea, Lucille, and Carlos had an archery contest. They shot on three different days, but Andrea and Carlos were each sick one day. Their scores for each day are listed in the table. Complete the tally by listing the winning archer for each day and by finding each child's total score.

	Andrea	Lucille	Carlos	Daily Winner
Oct. 2	88	80	79	Lucille
Oct. 9	62	72	69	Lucille
Oct. 16	90	91	88	Andrea
Total	140	183	148	

Who won the "daily winner" most often? Lucille

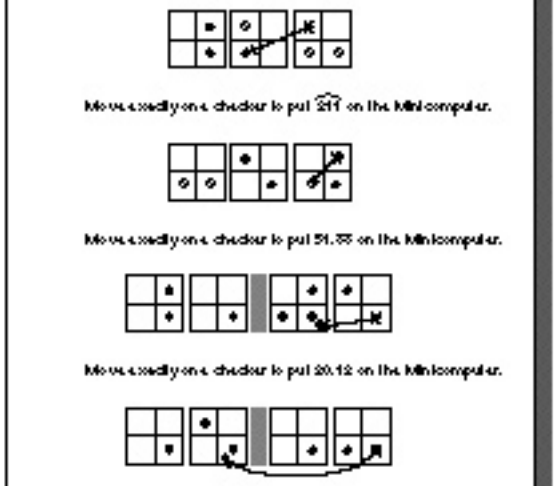
Who had the highest total score? Lucille

List the score for each archer's best day. Andrea 88 Lucille 80
Carlos 79 Who had the best one-day score? Andrea

Calculate each archer's average score. Andrea 70 Lucille 61
Carlos 74 Who had the best average? Carlos

Who do you think should be declared the best archer? _____
Why? Many responses are possible.

18



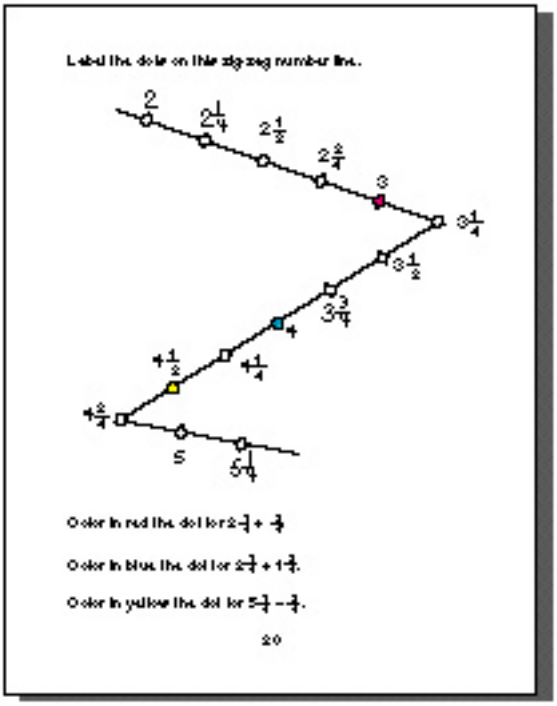
Move exactly one checker to put 457 on the Mini-computer.

Move exactly one checker to put 211 on the Mini-computer.

Move exactly one checker to put 51.35 on the Mini-computer.

Move exactly one checker to put 20.12 on the Mini-computer.

19



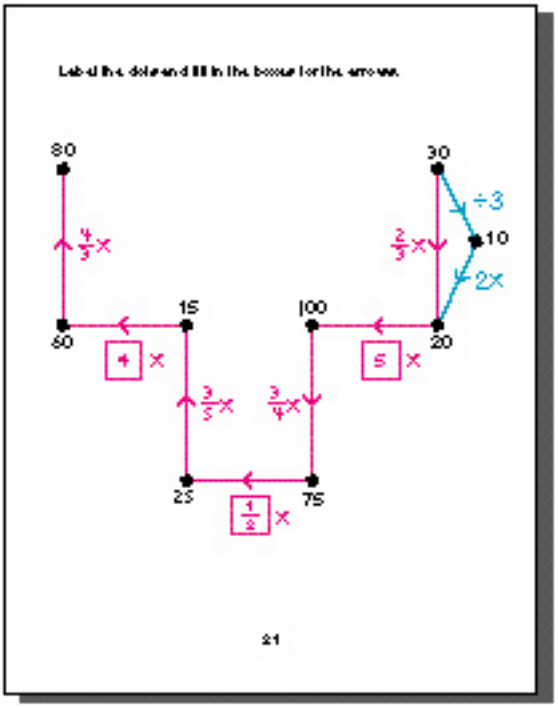
Label the dots on this zigzag number line.

Color in red the dot for $2\frac{1}{4} + \frac{3}{4}$.

Color in blue the dot for $2\frac{3}{4} + 1\frac{3}{4}$.

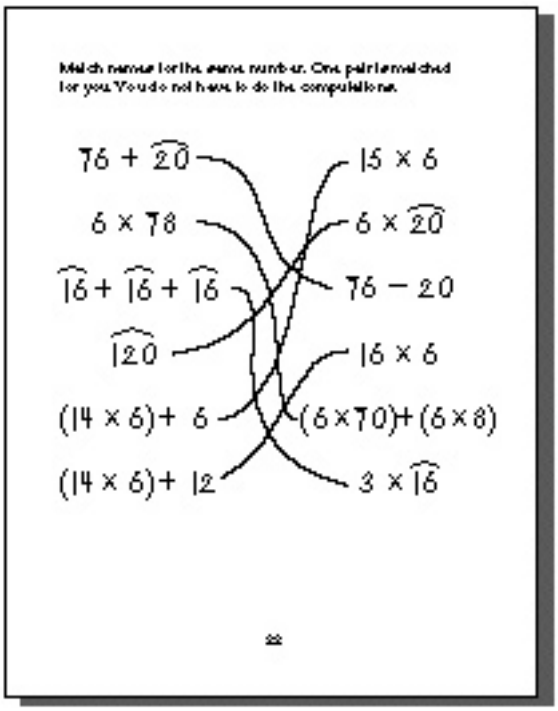
Color in yellow the dot for $5\frac{1}{4} - \frac{3}{4}$.

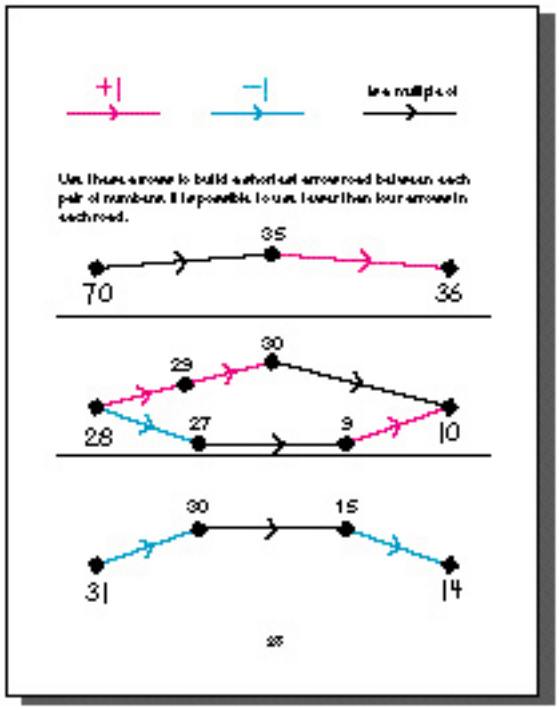
20

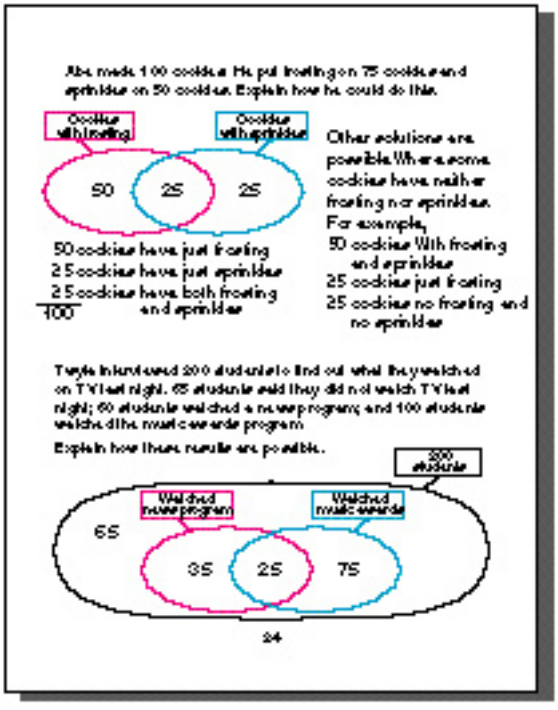


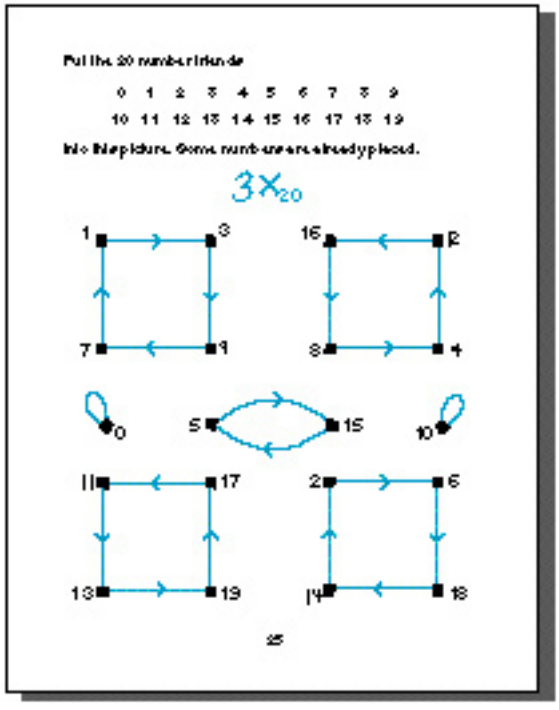
Label the dots and fill in the boxes for the arrows.

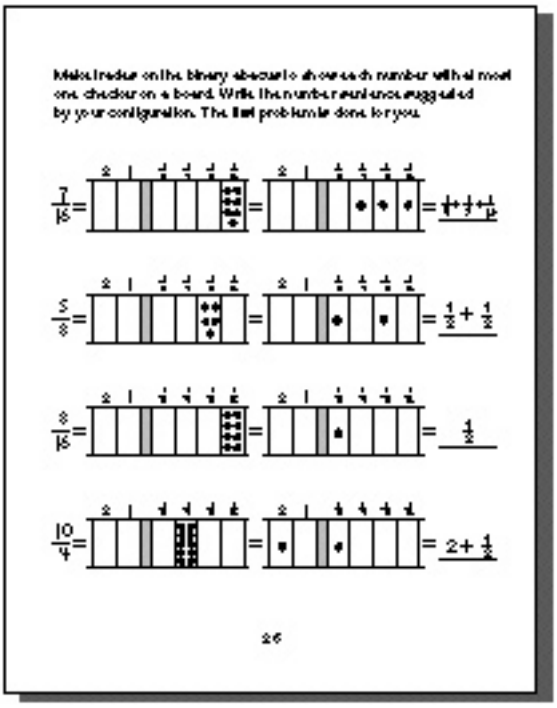
21

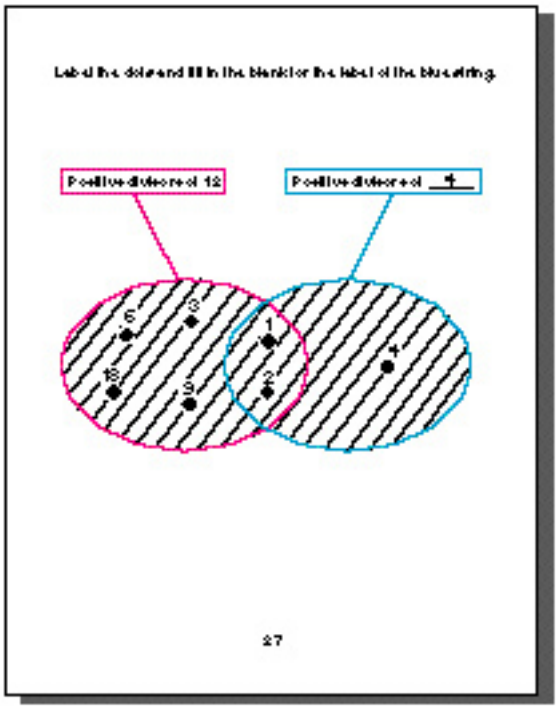

Match names for the same number. One pair is matched for you. You do not have to do the computations.
76 + 20
6 × 78
16 + 16 + 16
20
(14 × 6) + 6
(14 × 6) + 12
15 × 6
6 × 20
76 − 20
16 × 6
(6 × 70) + (6 × 8)
3 × 16
22

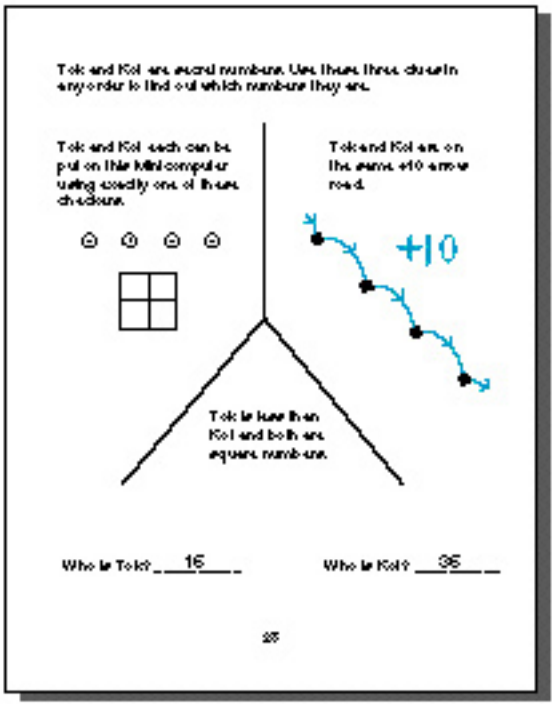

+1
−1
is a multiple of
Use these arrows to build a shortest arrowroad between each pair of numbers. It is possible to use fewer than four arrows in each road.
35
70
36
30
29
28
27
9
10
30
15
31
14
23

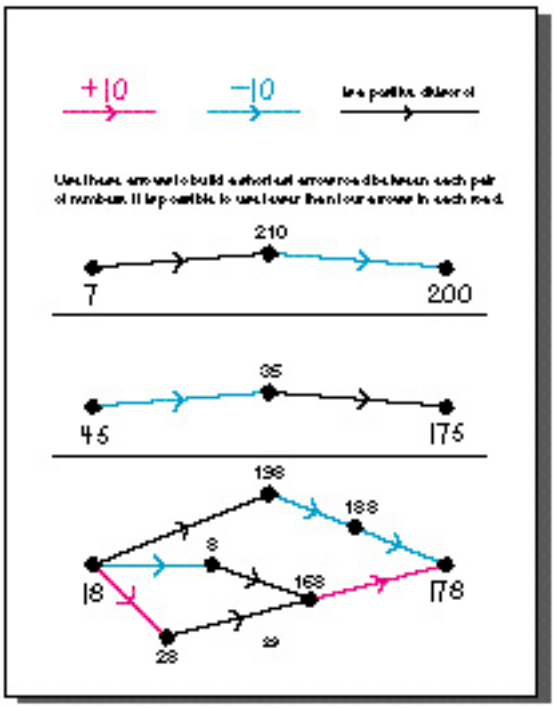

Abe made 100 cookies. He put frosting on 75 cookies and sprinkles on 50 cookies. Explain how he could do this.
Cookies with frosting
Cookies with sprinkles
50
25
25
50 cookies have just frosting
25 cookies have just sprinkles
25 cookies have both frosting and sprinkles
100
Other solutions are possible. Where some cookies have neither frosting nor sprinkles.
For example,
50 cookies With frosting and sprinkles
25 cookies just frosting
25 cookies no frosting and no sprinkles
Twyla interviewed 200 students to find out what they watched on TV last night. 65 students said they did not watch TV last night; 60 students watched a news program; and 100 students watched the music awards program.
Explain how these results are possible.
200 students
Watched news program
Watched music awards
65
35
25
75
24

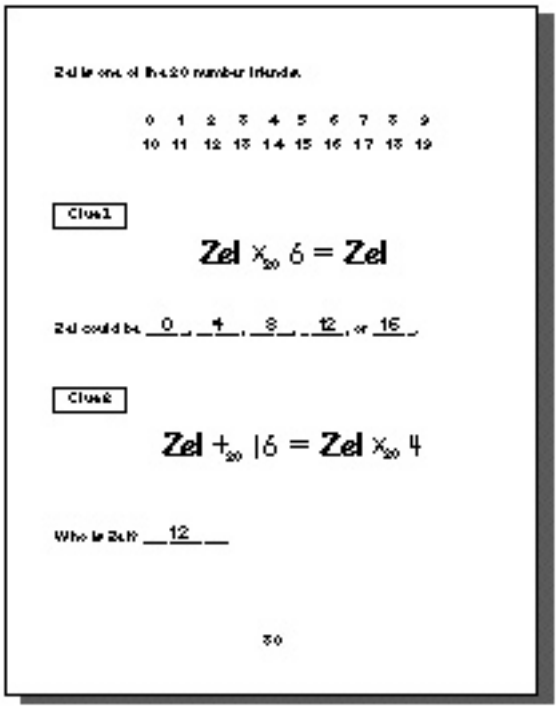

Put the 20 numbers in here:
0 1 2 3 4 5 6 7 8 9
10 11 12 13 14 15 16 17 18 19
into the picture. Some numbers are already placed.
3×20
1
3
16
12
7
9
8
4
0
5
15
10
11
17
2
6
13
19
14
18
25

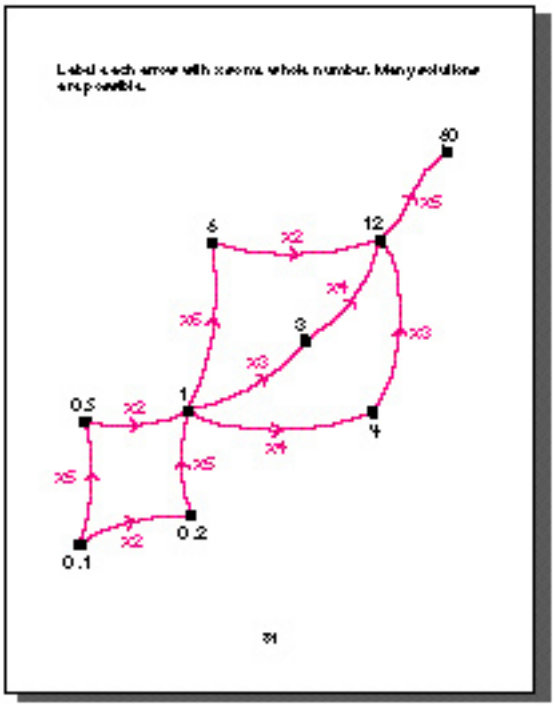

Make fraction on the binary abacus to show each number with at most one checker on a board. Write the number sentence suggested by your configuration. The first problem is done for you.
$\frac{7}{16} = \frac{1}{4}+\frac{1}{8}+\frac{1}{16}$
$\frac{5}{8} = \frac{1}{2}+\frac{1}{8}$
$\frac{3}{16} = \frac{1}{8}$
$\frac{10}{4} = 2+\frac{1}{2}$
26

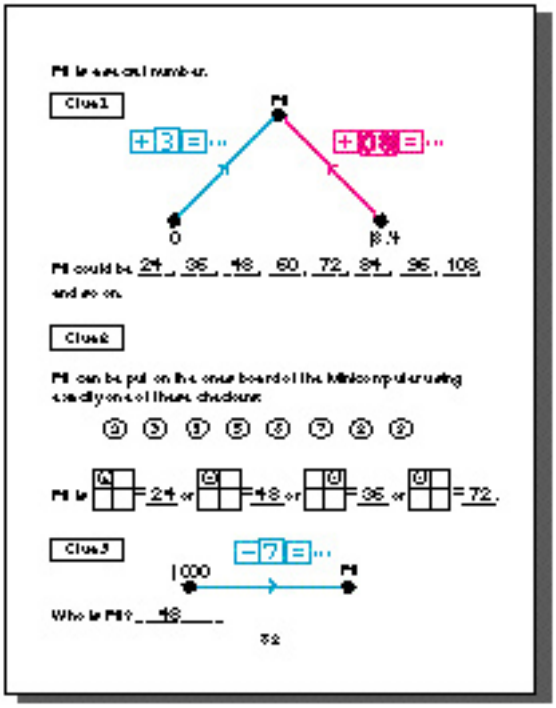

Label the sides and fill in the blanks for the label of the blue sorting.
Positive divisors of 12
Positive divisors of 4
6
3
12
9
1
2
4
27


Tok and Kol are secret numbers. Use these three clues in any order to find out which numbers they are.
Tok and Kol each can be put on the Mini-computer using exactly one of these checkers.
Tok and Kol are on the same +10 arrow road.
+10
Tok is less than Kol and both are square numbers.
Who is Tok? 16
Who is Kol? 36
28


+10
−10
is a positive divisor of
Use these arrows to build a shortcut arrow road between each pair of numbers. It is possible to use fewer than four arrows in each road.
210
7
200
35
45
175
198
188
8
168
18
178
28
29


Zel is one of the 20 number islands.
0 1 2 3 4 5 6 7 8 9
10 11 12 13 14 15 16 17 18 19
Clue 1
Zel × 6 = Zel
Zel could be 0, 1, 8, 12, or 16.
Clue 2
Zel + 16 = Zel × 4
Who is Zel? 12
30


Label each arrow with a one whole number. Many solutions are possible.
60
12
6
8
1
0.5
4
0.2
0.1
×5
×2
×4
×3
31


Pil is a secret number.
Clue 1
+ 3 =
+ 0.4 =
0
Pil could be 24, 36, 48, 60, 72, 84, 96, 108
and so on.
Clue 2
Pil can be put on the snow board of the Mini computer using exactly one of these shapes:
Pil is = 24 or = 48 or = 36 or = 72.
Clue 3
− 7 =
1000
Pil
Who is Pil? 48
32

Capsule Lesson Summary

Draw an arrow road with decimal number problems. Begin the workbook *Variety of Problems #5*. (This is the first of two lessons using this workbook.)

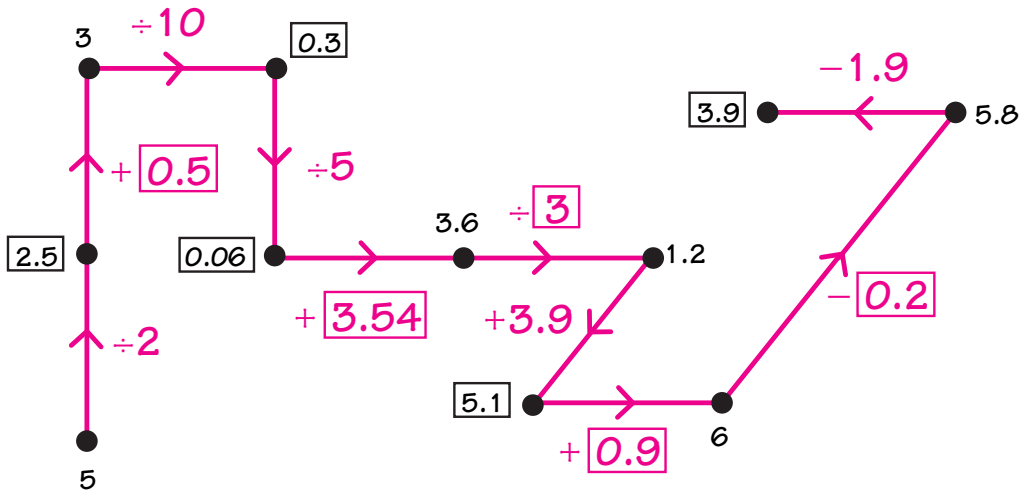
Materials

Teacher	Student
• Colored chalk	• *Variety of Problems #5* Workbook • Colored pencils, pens, or crayons • Metric ruler • Calculator

Description of Lesson

T: ***I will draw an arrow road on the board. For each arrow, you will either label the ending dot or the arrow.***

Draw one arrow at a time and call on students to label the ending dot or the arrow. (Answers are in boxes.)



Distribute copies of the workbook *Variety of Problems #5* and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of the lesson, collect the workbooks for your review. They will be used again in Lesson W15.

Capsule Lesson Summary

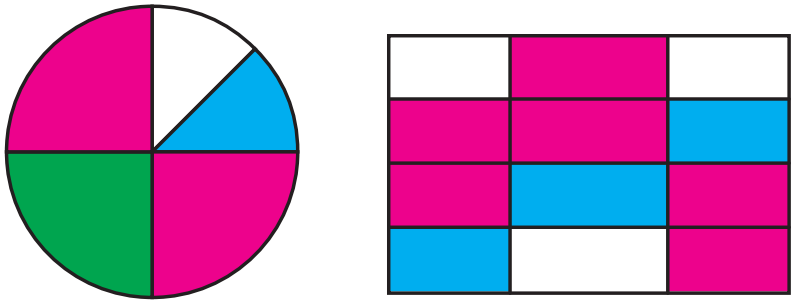
Determine what fraction of a multicolored region is shaded in each color. Continue individual work in the workbook *Variety of Problems #5*. (This is the second of two lessons using this workbook.)

Materials

Teacher	• Colored chalk	• Colored pencils, pens, or crayons • Metric ruler
Student	• *Variety of Problems #5* Workbook	• Calculator

Description of Lesson

Draw the following pictures on the board before class begins.



T: ***What fractional part of this shape*** (point to the circular shape) ***is colored green?***

S: ***¼, because four green pieces would cover the inside of the shape, so one green piece is ¼ of the shape.***

T: ***What fractional part of the shape is red?***

S: ***2/4, because there are two pieces like the green piece.***

T: ***What is another name for 2/4?***

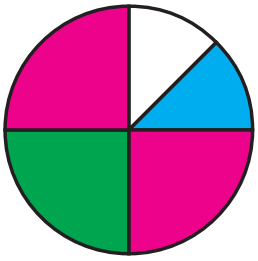
S: ***½.***

T: ***What fractional part of the shape is blue?***

S: ***⅛, because eight blue pieces would cover the shape.***

T: ***What fractional part of this shape*** (point to the rectangular shape) ***is white?***

S: ***¼, because the three white pieces would cover one horizontal strip and that is ¼ of the shape. The shape has four horizontal strips.***



Green 1/4

Red 2/4 or 1/2

Blue 1/8

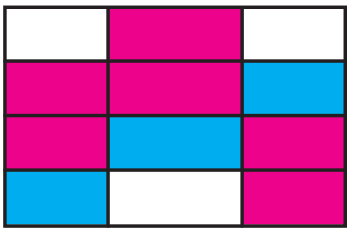
There are many ways students may explain why the rectangular shape is ¼ white.

T: ***What fractional part of the shape is red?***

S: ***¾ or ½.***

T: ***What fractional part of the shape is blue?***

S: ***¼.***



White 1/4

Red 2/4 or 1/2

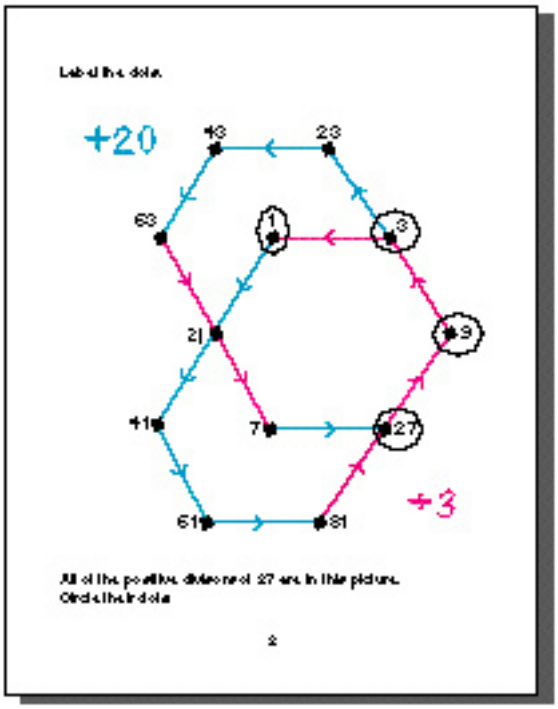
Blue 1/4

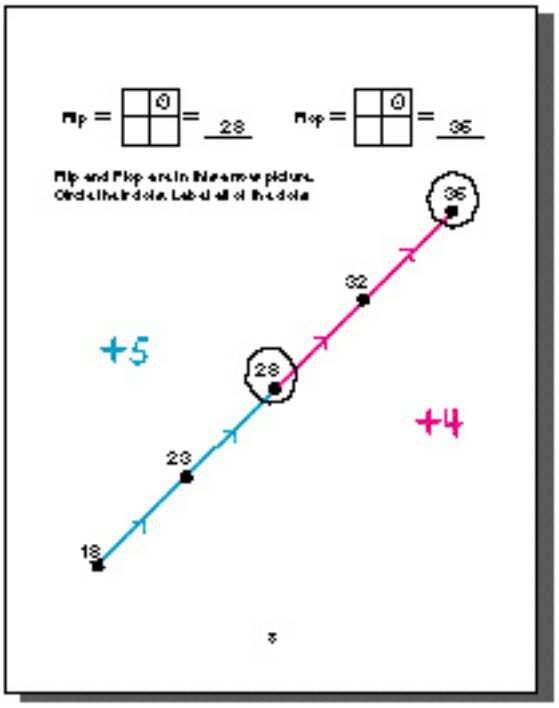
Return students' copies of the workbook *Variety of Problems #5*. Ask students first to correct or complete pages they worked on the previous week and then to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

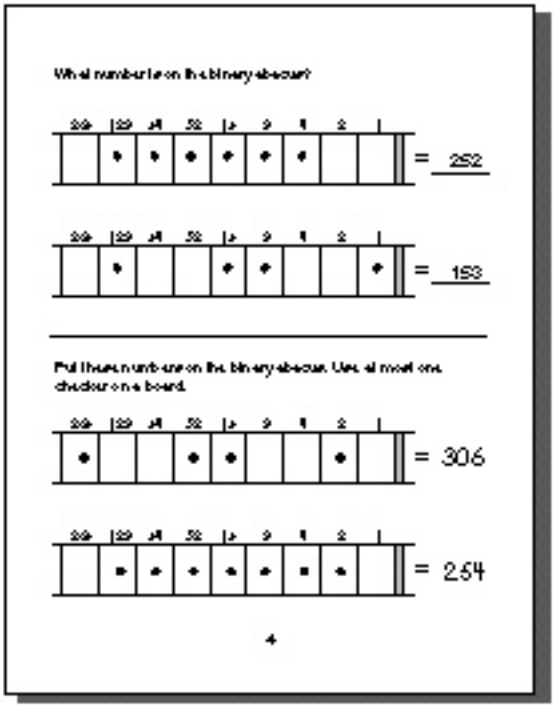
At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in their workbooks at school during a study time or at home as an assignment.

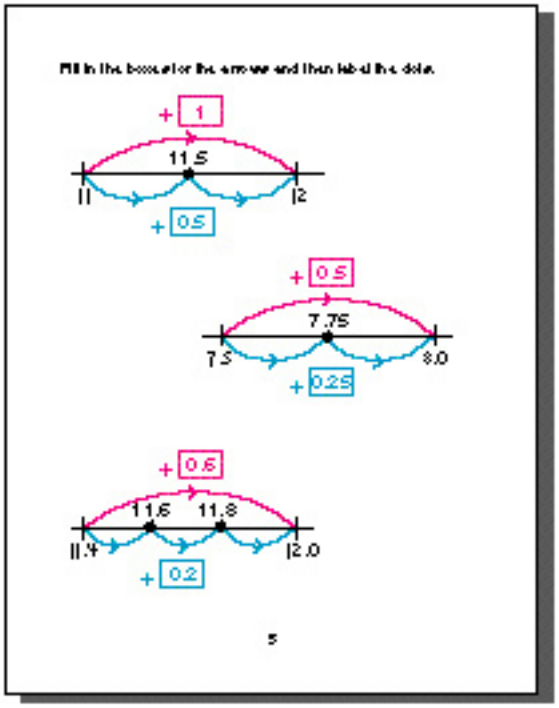
Assessment Activity

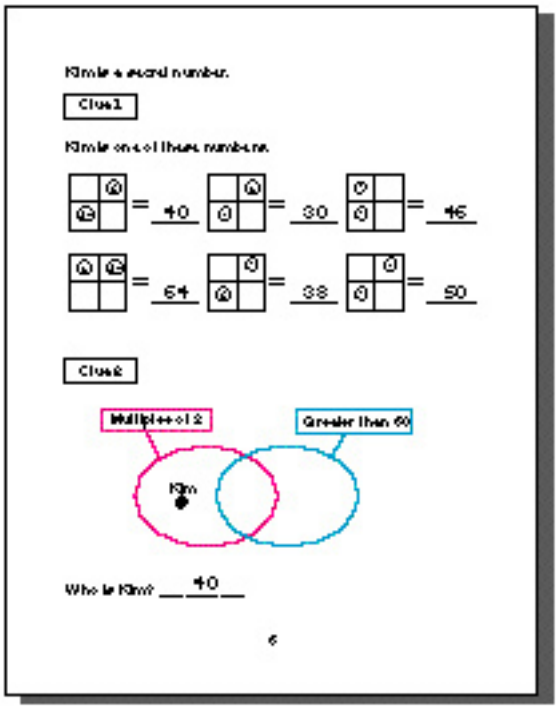
An individual student progress record for the workbook is available on Blackline W15. You may like to use this form to monitor student work.

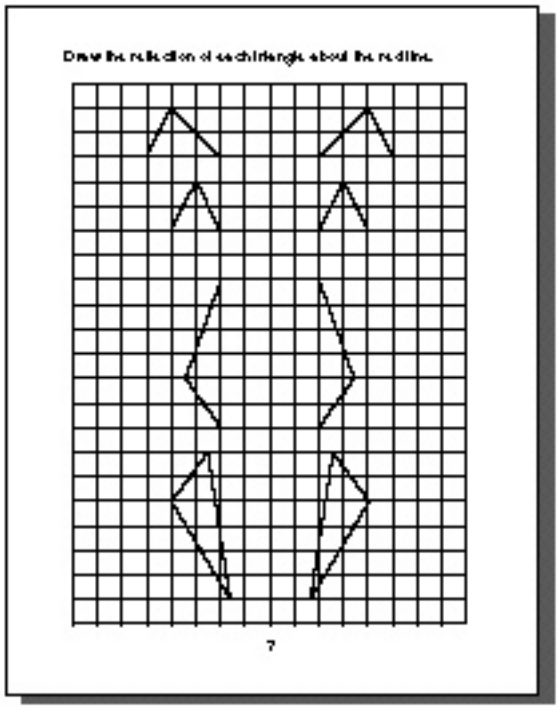

Label the dots.
+20
43
23
63
1
3
21
9
41
7
27
+3
61
81
All of the positive differences of 27 are in this picture.
Circle the dots.
2

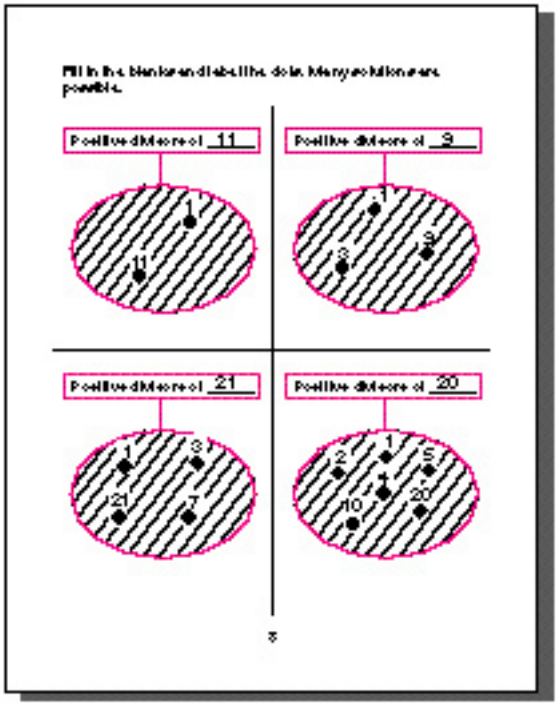

Flip =
0
= 28
Flop =
0
= 35
Flip and Flop are in this arrow picture.
Circle the dots. Label all of the dots.
35
32
+5
28
+4
23
18
3

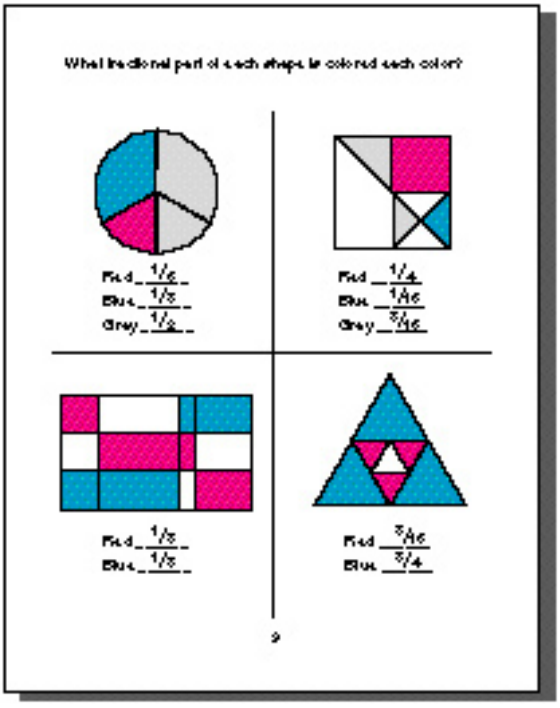

What number is on the binary abacus?
= 252
= 153
Put these numbers on the binary abacus. Use at most one checker on a board.
= 306
= 254
4


Fill in the boxes for the arrows and then label the dots.
+ 1
11.5
11
12
+ 0.5
+ 0.5
7.75
7.5
8.0
+ 0.25
+ 0.6
11.6
11.8
11.4
12.0
+ 0.2
5


Clue 1
= 40
= 30
= 46
= 64
= 38
= 50
Clue 2
Multiple of 2
Greater than 60
Kim
Who is Kim? 40
6


7


Positive difference of 11
Positive difference of 9
Positive difference of 21
Positive difference of 20
8


Red 1/3
Blue 1/3
Grey 1/3
Red 1/4
Blue 1/16
Grey 9/16
Red 1/2
Blue 1/2
Red 9/16
Blue 3/4
9

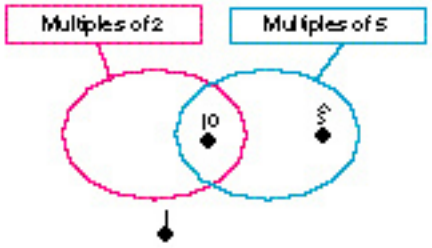
The red label is one of these:

Multiples of 2
Positive prime numbers
Positive divisors of 20
Multiples of 6
Odd numbers
Less than 60

The blue label is one of these:

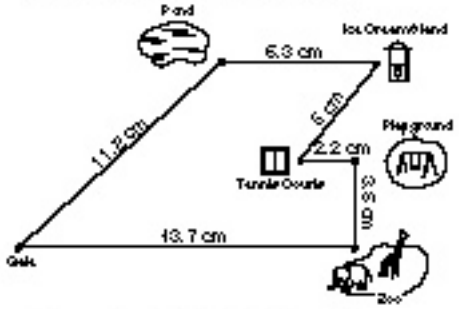
Multiples of 2
Positive prime numbers
Positive divisors of 20
Multiples of 6
Odd numbers
Less than 60

Label the strings.



10

Here is a map of the bicycle trails at Kibby Park.



On the map, how long is the shortest bike route from:
the gate to the zoo? 13.7 cm
the zoo to the ice cream island? 10.7 cm
the ice cream island to the pond? 5.3 cm
the playground to the pond? 13.5 cm

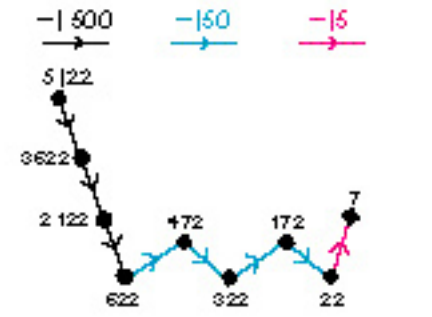
There are two routes from the gate to the ice cream island:
Route A goes by the pond.
Route B goes by the zoo, playground, and tunnel courts.

Which route, A or B, is shorter? B
On the map, how much shorter is this route? 5.9 cm

11

Answers may vary slightly.

Build an arrow road from 5122 to the least possible positive number using these arrows.



Use the arrow picture to solve this division problem.

$$5\overline{)5122} \quad 341 \text{ R } 7$$

12

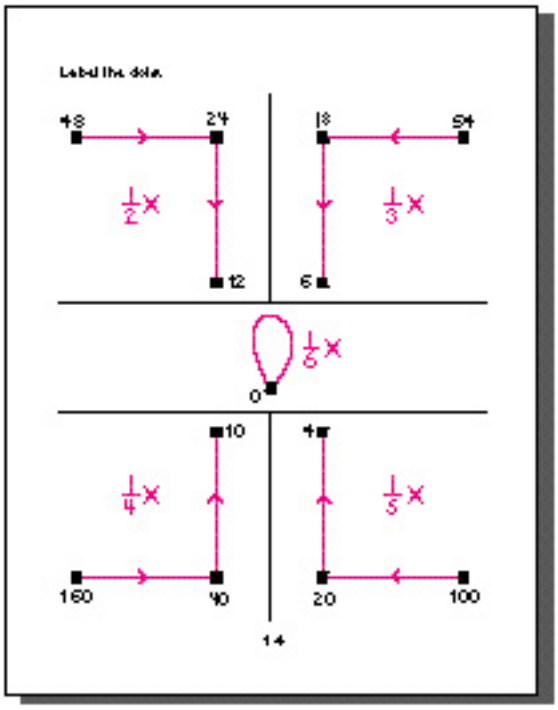
Fill in the boxes.

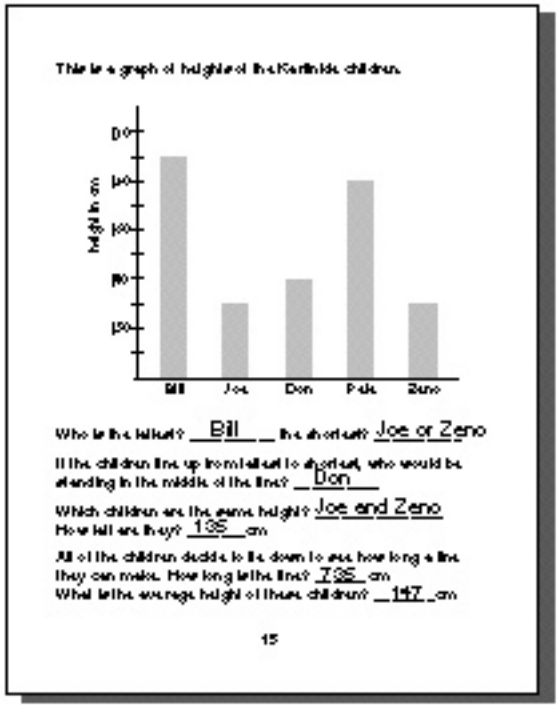
$$\begin{array}{r} 4\,5\,4\,9 \\ +\,3\,6\,7\,2 \\ \hline 8\,2\,2\,1 \end{array} \qquad \begin{array}{r} 2\,3\,1\,9 \\ -\,1\,5\,6\,0 \\ \hline 7\,5\,9 \end{array}$$

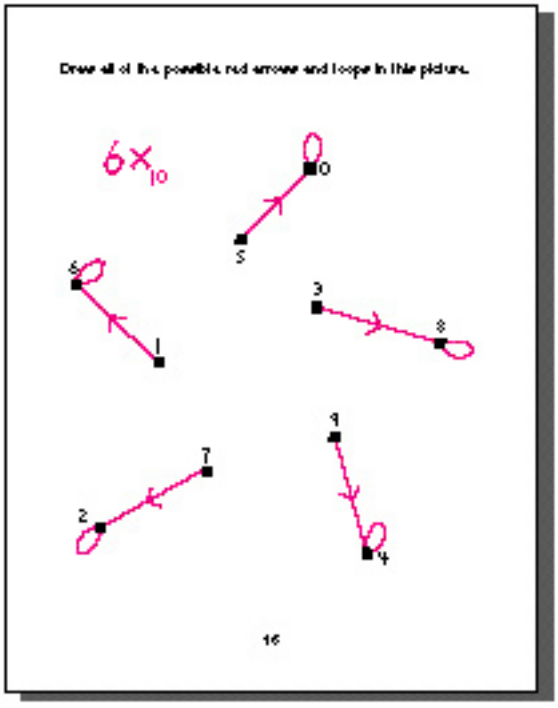
$$\begin{array}{r} 9\,3 \\ \times\ \ 9 \\ \hline 8\,3\,7 \end{array} \qquad \begin{array}{r} 3\,9 \\ \times\ 8\,2 \\ \hline 7\,8 \\ 3\,1\,2\,0 \\ \hline 3\,1\,9\,8 \end{array}$$

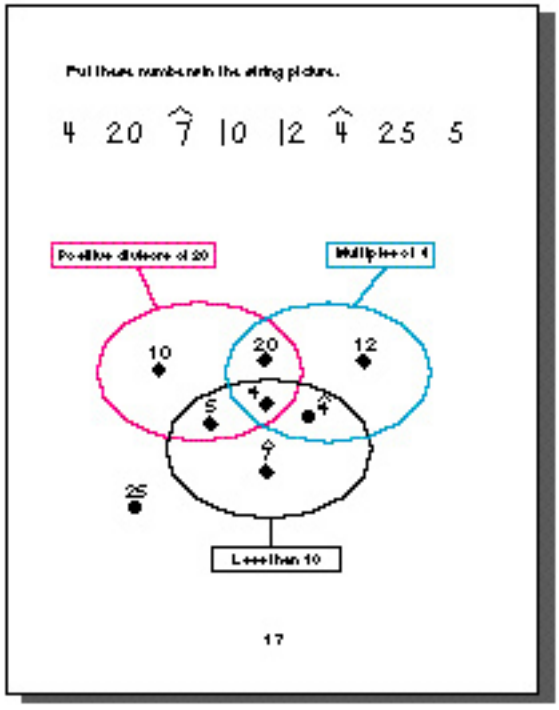
$$5\,6 \div 4 = 1\,4$$

13

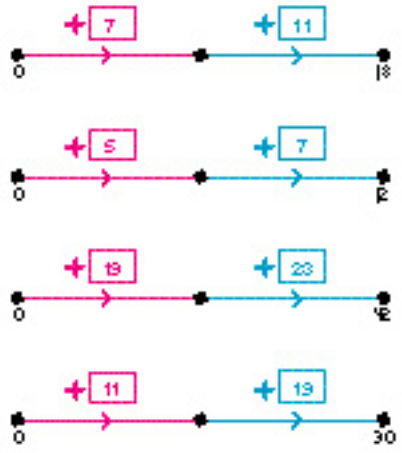

Label the dots.
48
24
12
18
54
6
0
10
40
160
4
20
100
14


This is a graph of heights of the Kerfinkle children.
Height in cm
Bill
Joe
Don
Pete
Zeno
Who is the tallest? Bill the shortest? Joe or Zeno
If the children line up from tallest to shortest, who would be standing in the middle of the line? Don
Which children are the same height? Joe and Zeno
How tall are they? 135 cm
All of the children decide to lie down to see how long a line they can make. How long is the line? 735 cm
What is the average height of these children? 147 cm
15


Draw all of the possible red arrows and loops in this picture.
6
10
0
5
6
1
3
8
9
7
2
4
16


Put these numbers in the string picture.
4 20 7 10 12 4 25 5
Positive divisors of 20
Multiples of 4
10
20
12
5
4
4
9
25
Less than 10
17

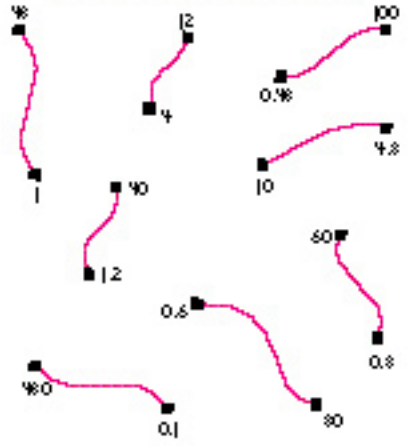
Fill in the boxes for the arrows using only prime numbers.
Many solutions are possible.



18

Draw all of the possible red cords in the picture. Two are done for you.

Two numbers are joined by a red cord
if and only if
their product is 48.



19

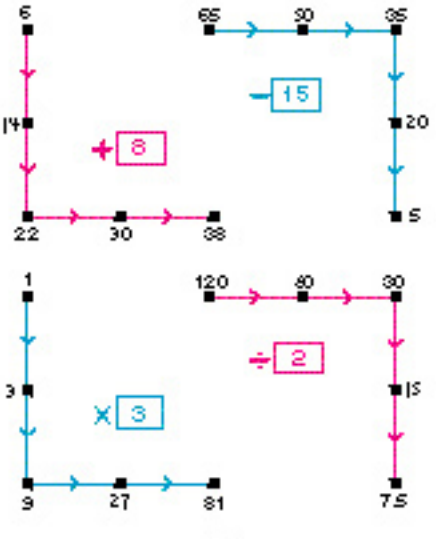
Marvin and Mark are preparing art supply kits. Each kit
has 5 pencils and 24 pieces of paper.

Marvin has 70 pencils. How many pieces of paper does he
need to prepare as many kits as possible? 336
How many kits does he prepare? 14

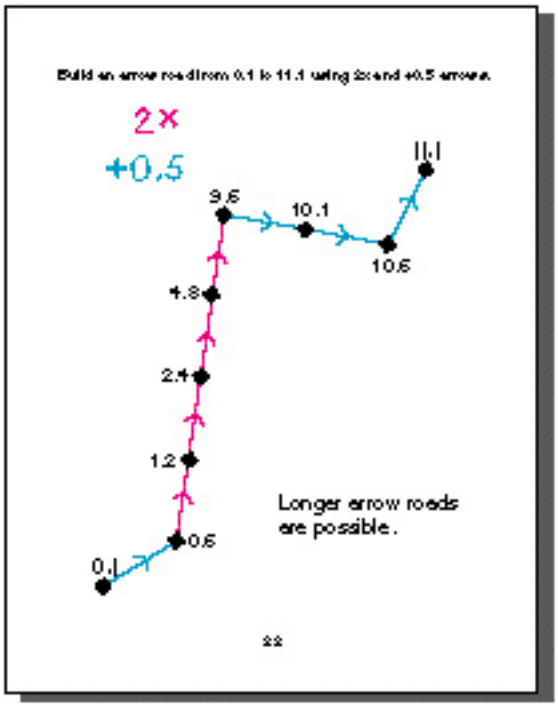
Mark has a ream of paper, 500 sheets. How many pencils does
she need to prepare as many kits as possible? 100
How many kits does she prepare? 20

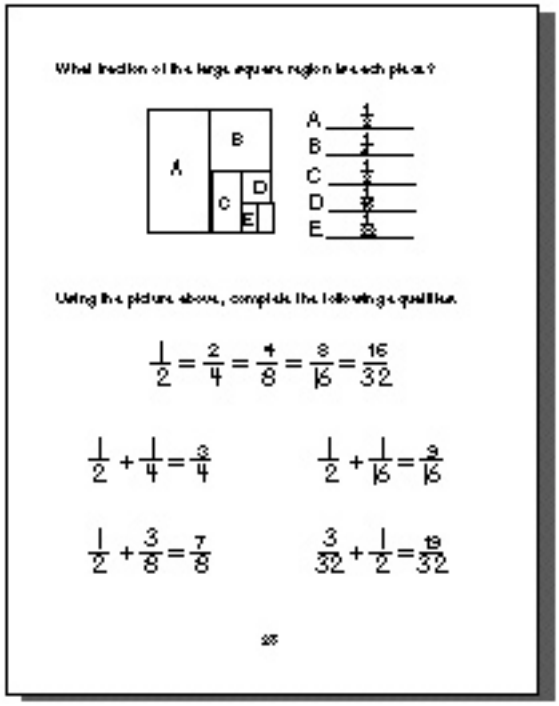
20

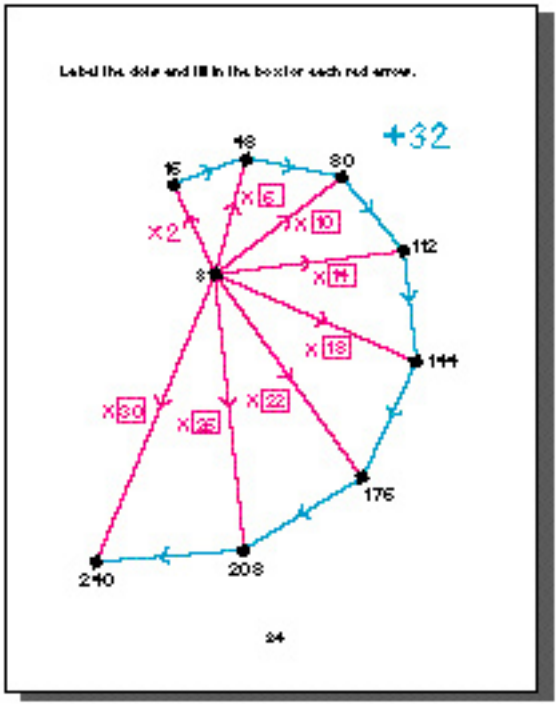
Label the dots and fill in the boxes for the arrows.

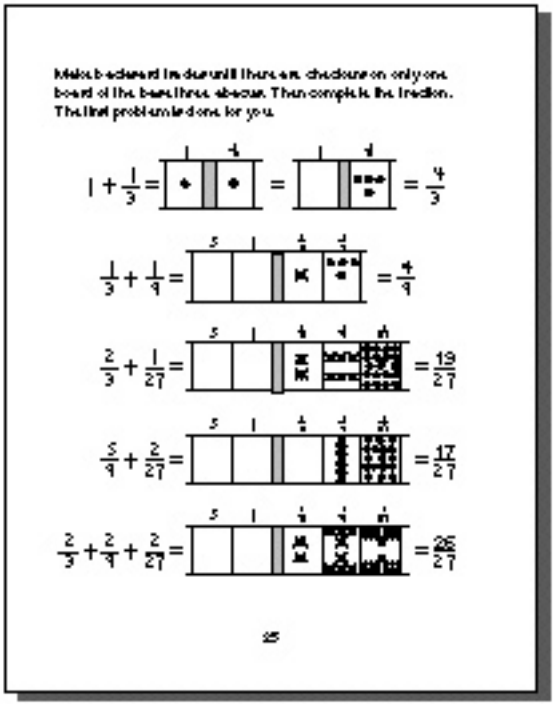


21


Build an arrow road from 0.1 to 11.1 using 2x and +0.5 arrows.
2×
+0.5
11.1
9.6
10.1
10.6
4.8
2.4
1.2
0.6
0.1
Longer arrow roads are possible.
22


What fraction of the large square region is each piece?
A
B
C
D
E
Using the picture above, complete the following equations.
$\frac{1}{2}=\frac{2}{4}=\frac{4}{8}=\frac{8}{16}=\frac{16}{32}$
$\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$
$\frac{1}{2}+\frac{1}{16}=\frac{9}{16}$
$\frac{1}{2}+\frac{3}{8}=\frac{7}{8}$
$\frac{3}{32}+\frac{1}{2}=\frac{19}{32}$
23


Label the dots and fill in the boxes for each red arrow.
+32
48
16
80
8
112
144
176
208
240
×2
×6
×10
×14
×18
×22
×26
×30
24


Make a shaded fraction until there are checkers on only one board of the bar three above. Then complete the fraction. The first problem is done for you.
$1+\frac{1}{3}=\frac{4}{3}$
$\frac{1}{3}+\frac{1}{9}=\frac{4}{9}$
$\frac{2}{3}+\frac{1}{27}=\frac{19}{27}$
$\frac{5}{9}+\frac{2}{27}=\frac{17}{27}$
$\frac{2}{3}+\frac{2}{9}+\frac{2}{27}=\frac{26}{27}$
25

Green pepper cost $0.75 per pound
Red pepper cost $2.25 per pound

Rona bought green pepper but the clerk made a mistake and charged her for red pepper. The charge was $6.75. How much was Rona overcharged? $4.50

$6.75 was the charge for 3 pounds of red peppers.
3 pounds of green peppers should cost $2.25.
$6.75 − $2.25 = $4.50

Other methods for finding the solution are possible.

Dave has $7.50 and decides to buy some pepper, both red and green. He decides to buy two pounds of red pepper. How many pounds of green pepper can he buy? 4 pounds

2 pounds of red peppers cost $4.50.
$7.50 − $4.50 = $3.00.
$3 will buy 4 pounds of green peppers
(4 × 0.75 = $3.00).

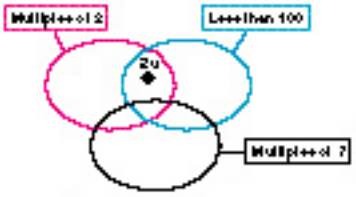
26

2u is a second number.

Clue 1

2u could be 36, 56, 76, 96, 116, 136, 156, 176, 196, 216, 236, 256, and so on.

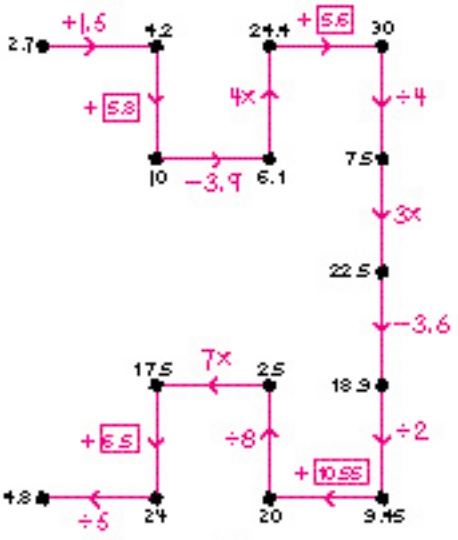
Clue 2



Who is 2u? 96

27

Label the dots and fill in the boxes for the arrows.



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Shawn, Mike, and Ling are marathon runners. They run in four different races this spring, but Shawn was unable to run in two of the races and Ling in one of the races. Their times are given in the table below.

	First Race	Second Race	Third Race	Fourth Race	Average Time
Shawn	[illegible]	[illegible]	[illegible]	———	2 hrs. 46 min.
Mike	[illegible]	[illegible]	[illegible]	[illegible]	2 hrs. 57 min.
Ling	[illegible]	[illegible]	———	[illegible]	2 hrs. 49 min.

Who came in first in the most races? Shawn
Who had the fastest time? Ling
Who had the best average time? Shawn

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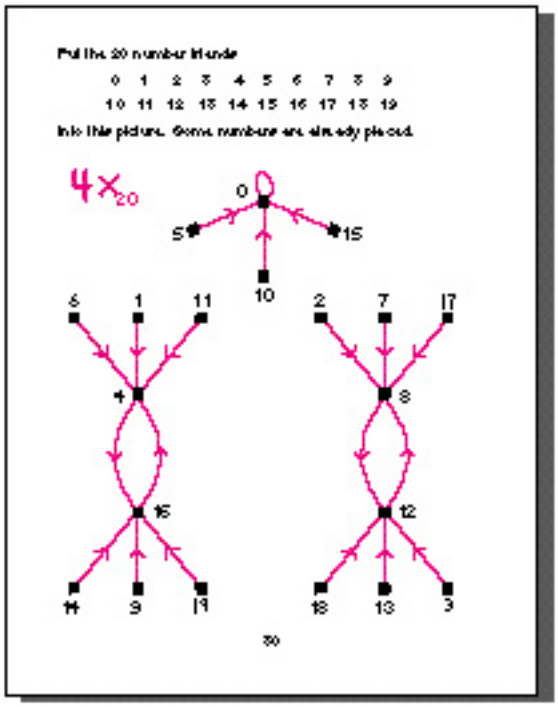

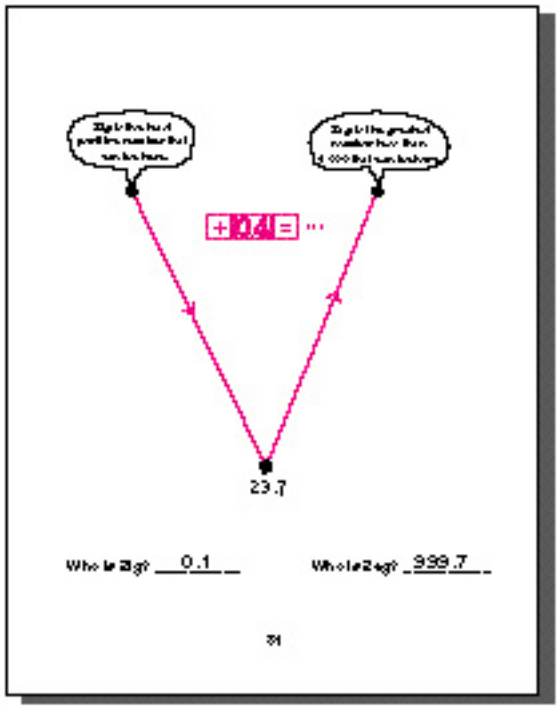

Fill in the boxes with the whole numbers 1 through 12.
Use each number once and make all the equations true.

1 + 11 = 12
2 + 7 = 9
3 + 5 = 8
4 + 6 = 10

Other solutions are possible.

Fill in the boxes with the whole numbers 1 through 12.
Use each number once and make all the equations true.

1 + 3 = 2 × 2
4 + 8 = 2 × 6
5 + 9 = 2 × 7
10 + 12 = 2 × 11

Other solutions are possible.

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