Workbooks

WORKBOOKS TABLE OF CONTENTS

WORKBOOKS INTRODUCTION

There are many opportunities for student to work individually during the course of the lessons described in the other content strands. In the Workbooks strand, however, it is this individualized work which becomes the chief end of the majority of lessons. The goal in this strand is to provide students with opportunities

- to review many of the ideas they have met in other content strands;
- to apply their acquired knowledge to new situations requiring various kinds of strategic thinking; and
- to learn how to read and use mathematics workbooks.

The following six workbooks are provided:

- *Selection of Problems #1*
- *Selection of Problems #2*
- *Selection of Problems #3*
- *Selection of Problems #4*
- *Selection of Problems #5*
- *Selection of Problems #6*

...and two storybooks.

- *A Valentine Mystery*
- *The Hidden Treasure*

Each workbook contains problems of varying levels of difficulty. Approximately the first ten pages of each workbook are easy problems, the next ten to twelve pages are average level difficulty, and the last ten pages are more challenging problems. For each workbook, we suggest that all students start work at the easiest level (i.e., on page 2) and then work through as many pages as they can handle during the two lessons scheduled for that workbook. We estimate that, in a typical class, about two-thirds of the students will correctly finish the first ten pages, about one-third will finish the first twenty pages, and a few will finish all or most of the workbook. These proportions will vary from class to class.

This guide contains an answer key for each workbook. The key follows an introduction to the workbook and a suggested collective lesson. The lesson either presents the workbook to the whole class or provides a warm-up activity on a problem similar to one found in the workbook.

The storybooks *A Valentine Mystery* and *The Hidden Treasure* provide problem-solving opportunities in a story context. These two booklets allow students to become deeply involved in an appealing fantasy as they struggle with difficult mathematics problems. The situations support topics and strategies developed in other strands.

Capsule Lesson Summary

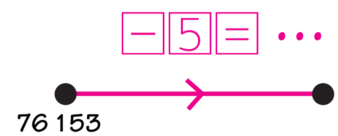
Use a calculator relation to review patterns in both positive and negative integers through repeated subtraction of 5's. Begin the workbook *Selection of Problems #1*. (This is the first of two lessons using this workbook.)

Materials

Teacher	• Colored chalk	Student	• *Selection of Problems #1* Workbook • Calculator • Metric ruler • Colored pencils, pens, or crayons

Description of Lesson

Draw this arrow picture on the board.



T: ***Put 76 153 on your calculator. Press the keys $\boxed{-}$, $\boxed{5}$, $\boxed{=}$ and then slowly press $\boxed{=}$ many times. Watch the numbers that appear on the display. What pattern do you notice?***

S: ***The ones digit alternates between 3 and 8.***

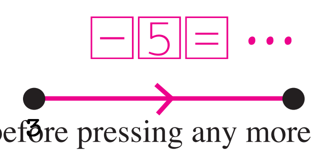
T: ***If you keep pressing $\boxed{=}$, what are some numbers less than 100 that would eventually appear?***

S: ***98, 93, 88, 78, 73, and so on.***

T: ***What is the least positive number that would appear?***

S: ***3.***

Relabel the starting dot of the arrow on the board.



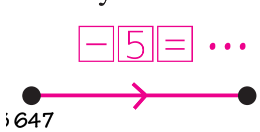
T: ***What negative numbers would appear?***

Encourage students to predict some negative numbers that would appear before pressing any more keys.

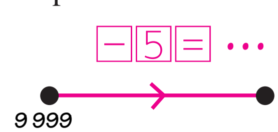
T: ***Put 3 on your calculator and again press $\boxed{-}$ $\boxed{5}$ $\boxed{=}$ ... . What numbers appear?***

S: ***$\widehat{2}$, $\widehat{7}$, $\widehat{12}$, $\widehat{17}$, and so on. All of the negative numbers end in 2 or 7.***

Repeat the above activity with one or both of the following arrow pictures.



Pattern: All positive numbers end in 7 or 2.
All negative numbers end in 3 or 8.



Pattern: All positive numbers end in 9 or 4.
All negative numbers end in 1 or 6.

W1

Distribute copies of the workbook *Selection of Problems #1* and let the students work independently for the rest of the class period. You may need to work more closely with students new to *CSMP* or let those students work with veteran *CSMP* students for awhile. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of the lesson, collect the workbooks for your review. They will be used again in Lesson W2.

Writing Activity

You may like students to take lesson notes on some, most, or even all their math lessons. The "Lesson Notes" section in Notes to the Teacher gives some suggestions and refers to forms in the Blacklines you may provide to students for this purpose. In this lesson, for example, students may note problems in the workbook they found especially difficult or especially interesting. They may also like to create other problems similar to ones in the workbook for their classmates to solve or to give to a family member to solve.

Capsule Lesson Summary

Review using a circle model to add and subtract fractions. Continue individual work in the workbook *Selection of Problems #1*. (This is the second of two lessons using this workbook.)

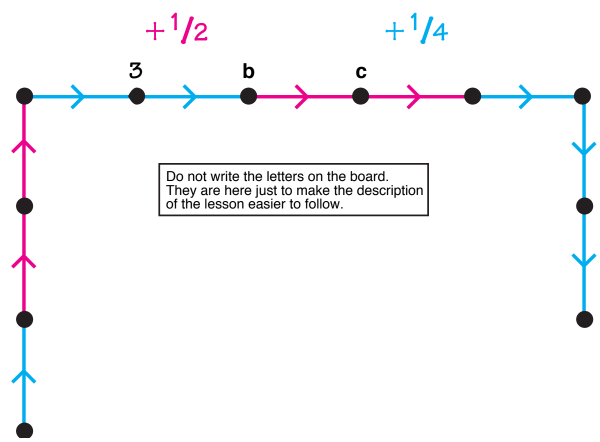
Materials

Teacher
- Colored chalk

Student
- *Selection of Problems #1* Workbook
- Metric ruler
- Colored pencils, pens, or crayons
- Calculator

Description of Lesson

Draw six equal size circles and the following arrow road on the board.



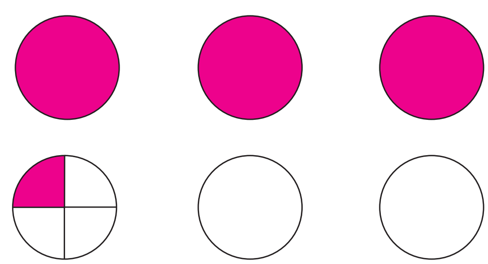
Point to 3 and invite a student to color in three of the circles. Trace the blue arrow from 3 to **b** as you ask,

T: ***What is 3 + $\frac{1}{4}$?***

S: ***3$\frac{1}{4}$.***

T: ***How could we show this using the circles?***

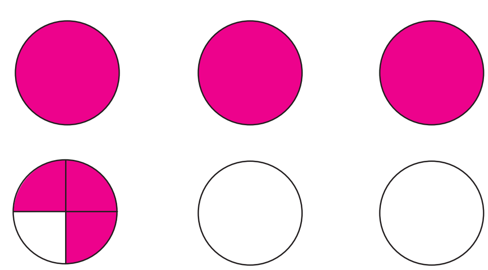
S: ***Color in one-fourth of another circle.***



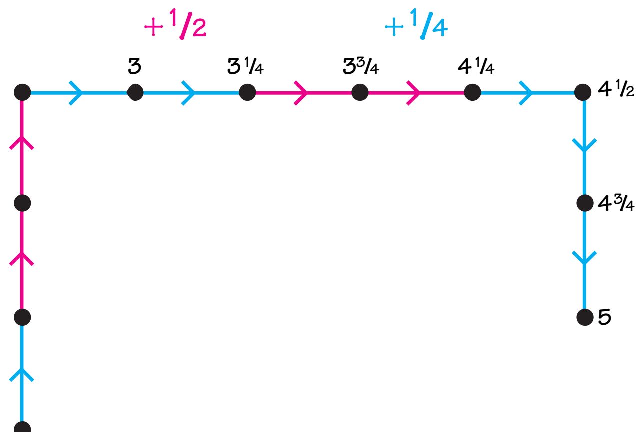
Put 3$\frac{1}{4}$ at **b**. Then trace the red arrow to **c** as you ask,

T: ***What is 3$\frac{1}{4}$ + $\frac{1}{2}$?***

S: ***3$\frac{3}{4}$. Color another half of the partially-colored circle.***



Put 3¾ at **c**. Continue to invite students to label the dots to the right of 3 and to use the circles to support the calculations.



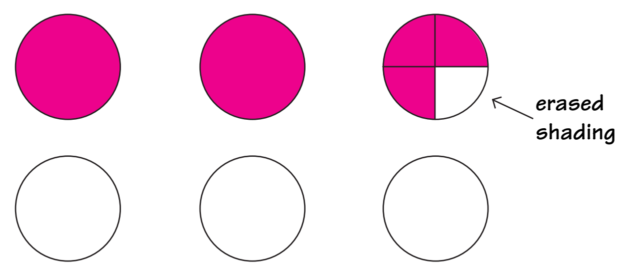
Point to the dot to the left of 3.

T: ***What number is here?***

S: ***2¾.***

T: ***Use the circles to convince us.***

S: ***Go back to three colored circles. Then erase the coloring from one-fourth of the inside of one colored circle.***



Label the dot for 2¾. Invite students to finish labeling the dots in the arrow road, using the circles to support their calculations.

Distribute students' copies of the workbook *Selection of Problems #1*. Ask students first to correct or complete pages from the previous weeks' work, and then to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

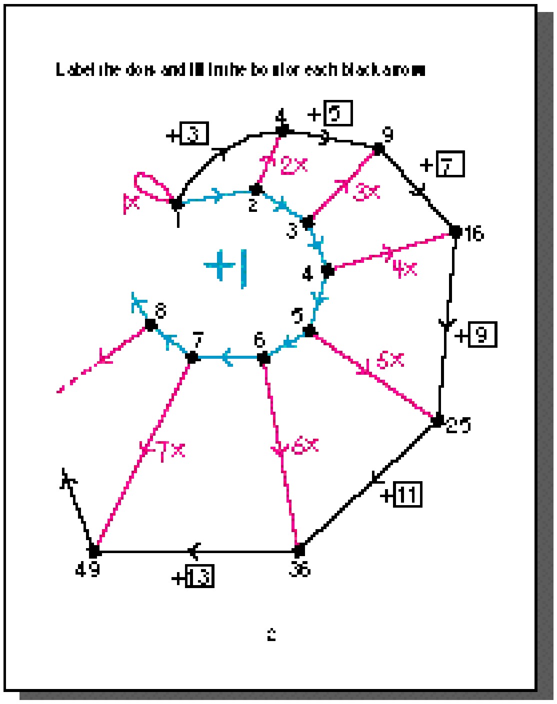
At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in the workbook during a study time or to take it home as an assignment.

Assessment Activity

An individual student progress record for this workbook is available on Blackline W2(a). You may like to use this form to monitor student work.

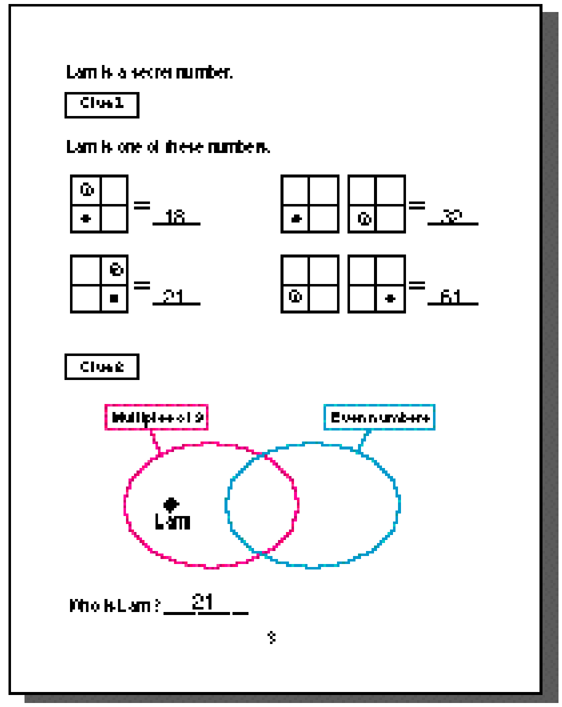
Home Activity

If you choose to send workbooks home with students, you may want to include a letter (reminder) to parents/guardians. Blackline W2(b) has a sample letter.



Label the dots and fill in the box for each black arrow.

2



Lam is a secret number.

Clue 1

Lam is one of these numbers.

= 18

= 32

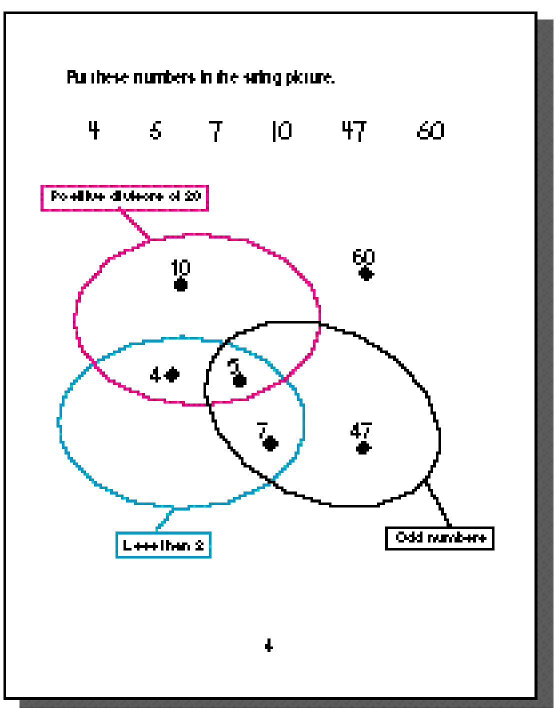
= 21

= 61

Clue 2

Who is Lam? 21

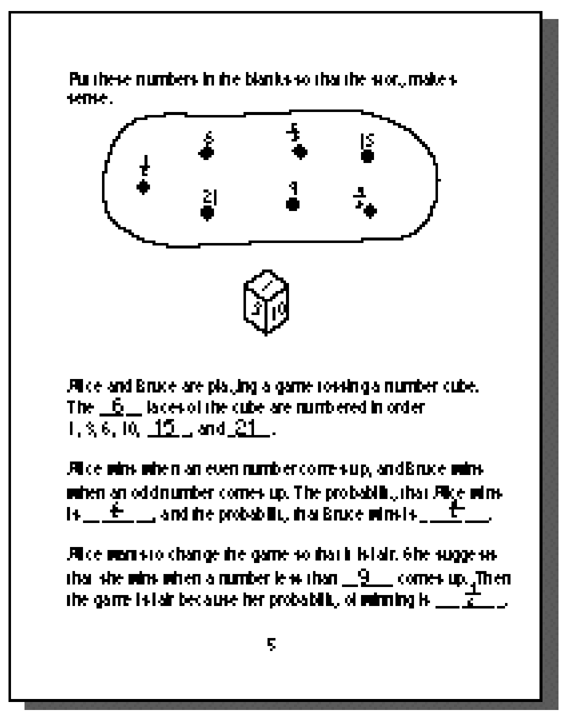
3



Put these numbers in the string picture.

4 6 7 10 47 60

4



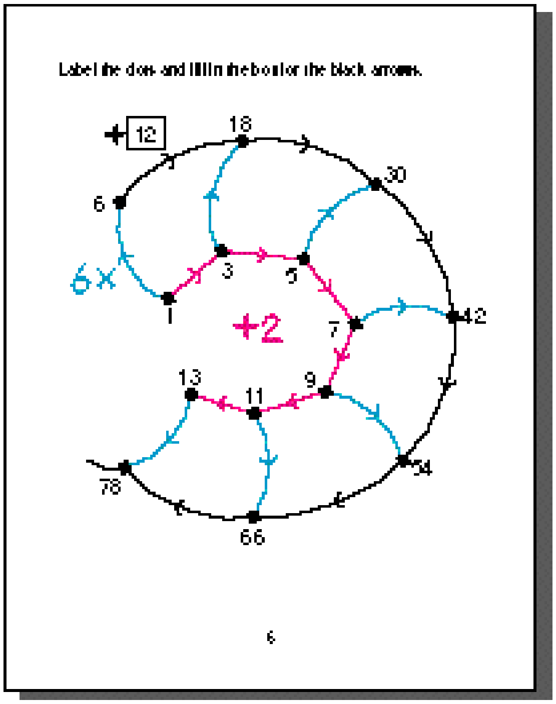
Put these numbers in the blanks so that the story makes sense.

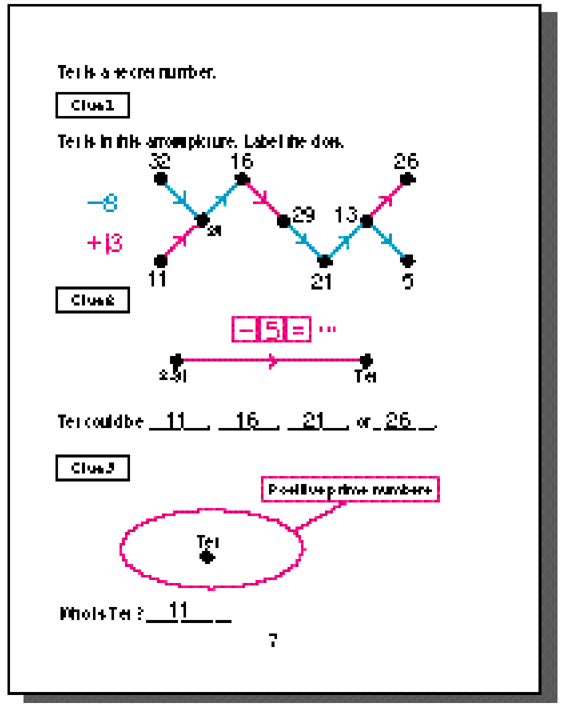
Alice and Bruce are playing a game tossing a number cube. The 6 faces of the cube are numbered in order 1, 3, 6, 10, 15, and 21.

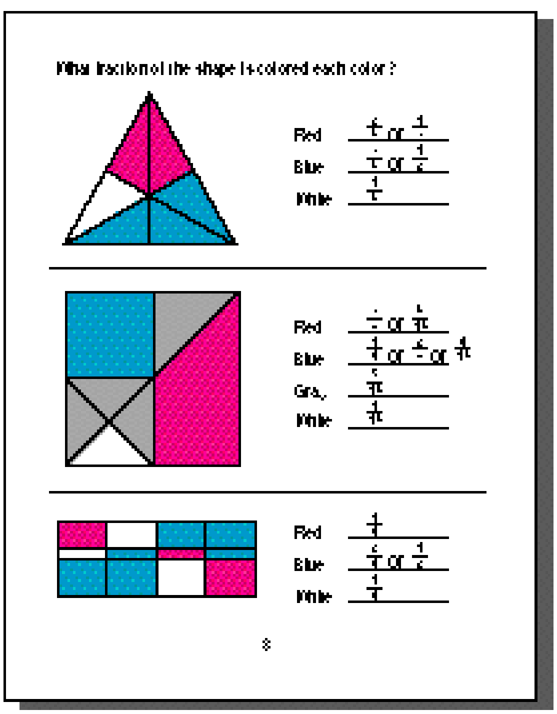
Alice wins when an even number comes up, and Bruce wins when an odd number comes up. The probability that Alice wins is [illegible], and the probability that Bruce wins is [illegible].

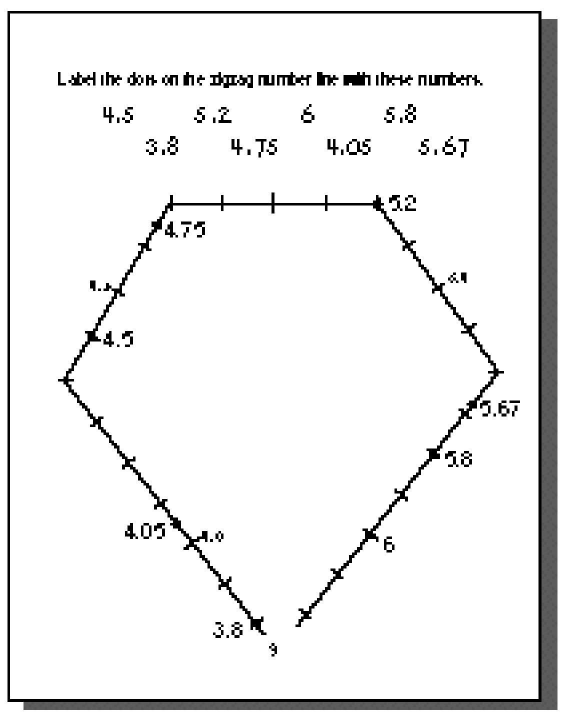
Alice wants to change the game so that it is fair. She suggests that she wins when a number less than 9 comes up. Then the game is fair because her probability of winning is $\frac{1}{2}$.

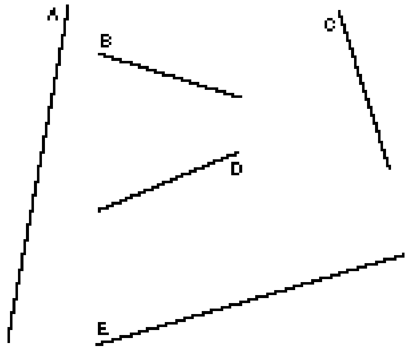
5


Label the dots and fill in the box for the black arrows.
+ 12
18
30
6
6x
3
5
1
+2
7
42
13
11
9
78
54
66
6


Ten is a secret number.
Clue 1
Ten is in this arrow picture. Label the dots.
32
16
26
−8
+13
29
13
11
21
5
Clue 2
Ten
Ten could be 11, 16, 21, or 26.
Clue 3
Positive prime numbers
Ten
Who is Ten? 11
7


What fraction of the shape is colored each color?
Red
Blue
White
Red
Blue
Gray
White
Red
Blue
White
8


Label the dots on the zigzag number line with these numbers.
4.5
5.2
6
5.8
3.8
4.75
4.05
5.67
5.2
4.75
4.5
5.67
5.8
4.05
6
3.8



Which line segment is closest to 6.4 cm long? __D__

Which line segment is closest to 13.9 cm long? __A__

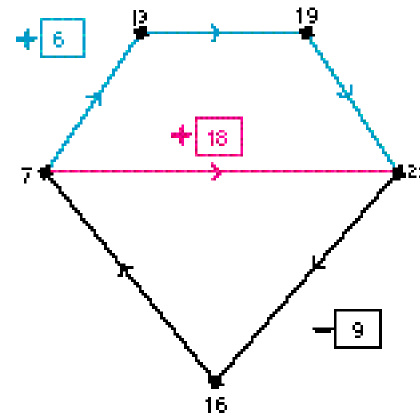
What is the sum of the lengths of segments B and D? 12.8 cm

Which segment is longer, A or E? __A__

How much longer is it? 0.9 cm

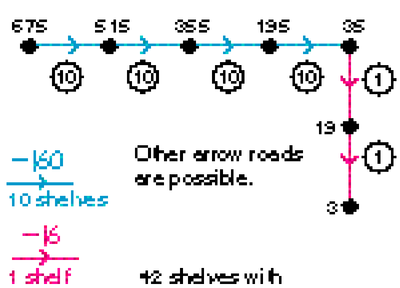
10

Label the dots and fill in the boxes for the arrows.



11

Jerome has 675 books to put on shelves. A full shelf holds 16 books. Use an arrow picture to find how many shelves Jerome can fill and how many books will be left over.



42 shelves with 3 left over.

Complete.

42 R 3

$$16\overline{)675}$$

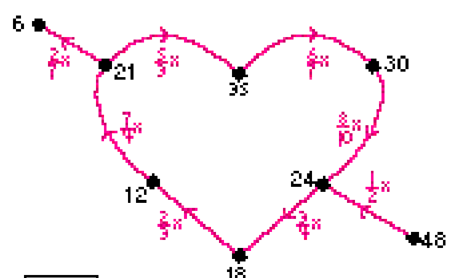
640	40
35	
32	2
3	

12

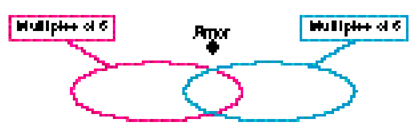
Amor is a secret number.

Clue 1

Amor is in this arrow picture. Label the dots.

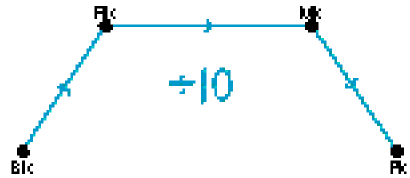


Clue 2



Who is Amor? __21__

13

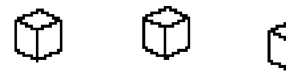


Complete the table.

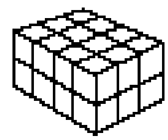
Bic	Flic	Mic	Pic
7 200	720	72	7.2
3	0.3	0.03	0.003
60 300	6 030	603	60.3
84	8.4	0.84	0.084

14

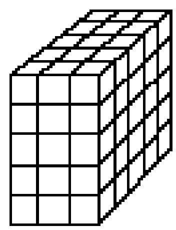
The boxes shown below are made from small cubes like these.



How many small cubes does it take to make this box? 24

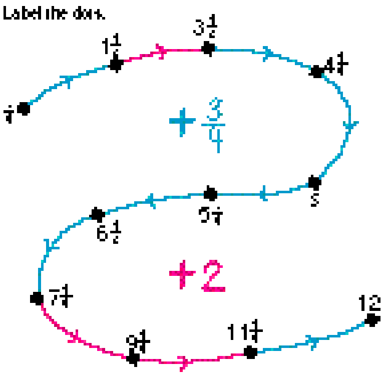


How many small cubes does it take to make this box? 75



15

Label the dots.



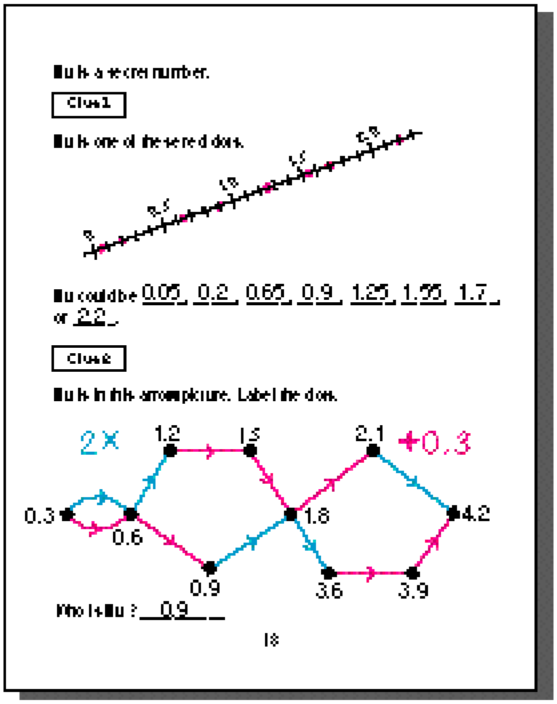
16

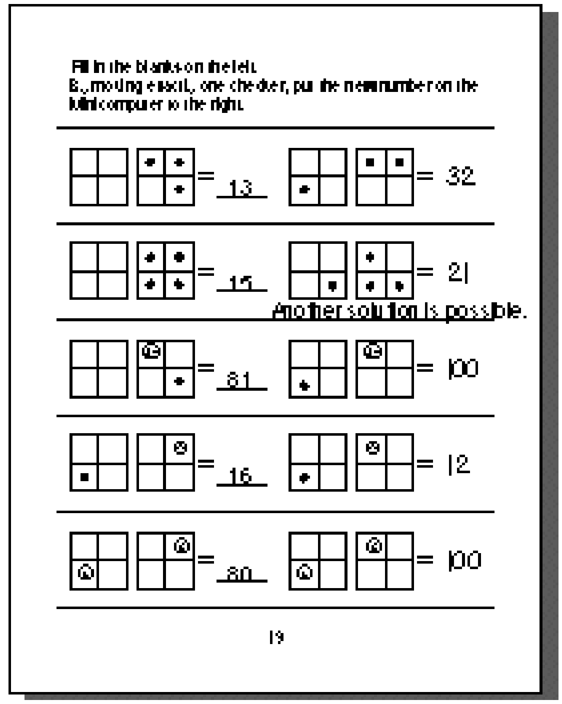
Fill in the boxes to make the calculations correct.

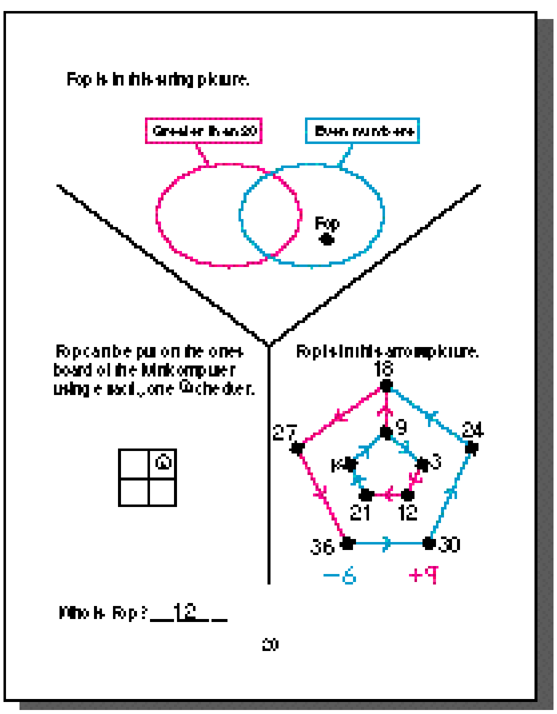
$$\begin{array}{r} \boxed{4}\,5\,9\,7 \\ 8\,3\,\boxed{2} \\ +\ 1\,6\,\boxed{5}\,6 \\ \hline 7\,\boxed{0}\,8\,5 \end{array} \qquad \begin{array}{r} \boxed{7}\,3\,\boxed{3}\,8 \\ -\ 3\,2\,9\,\boxed{6} \\ \hline 4\,\boxed{0}\,4\,2 \end{array}$$

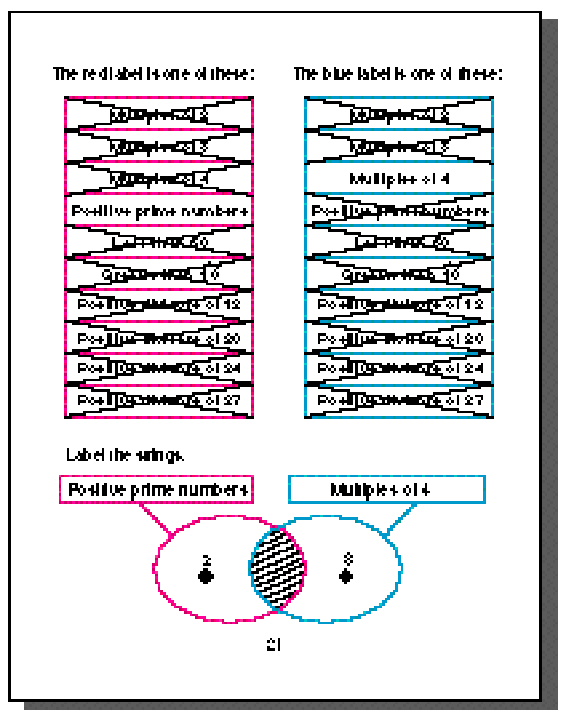
$$\begin{array}{r} 7\,4 \\ \times\ \boxed{8} \\ \hline 5\,\boxed{9}\,2 \end{array} \qquad 5\boxed{4} \div 2 = \boxed{2}7$$

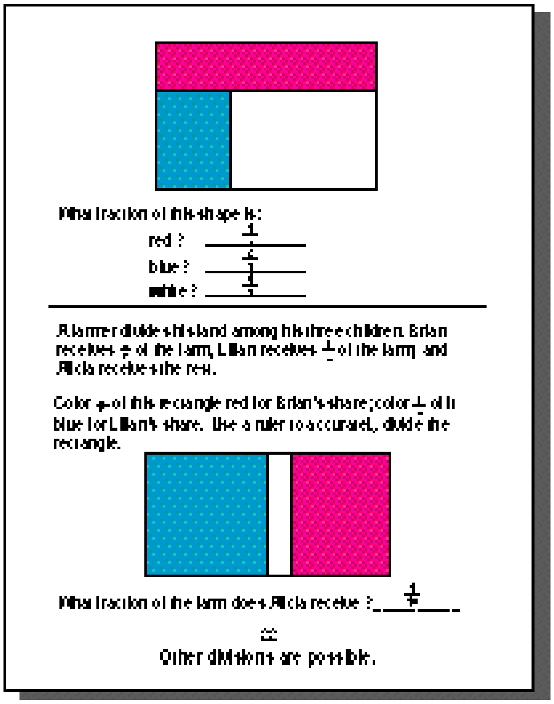
17

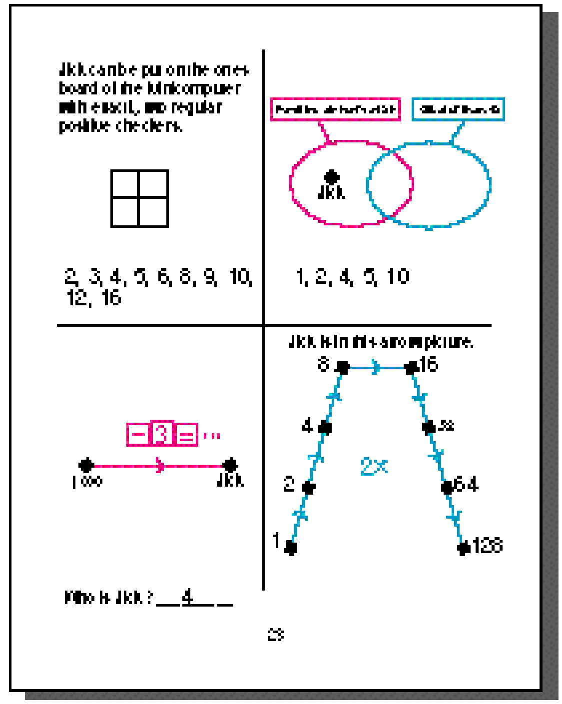

is a secret number.
Clue 1
is one of these red dots.
could be 0.05, 0.2, 0.65, 0.9, 1.25, 1.55, 1.7, or 2.2.
Clue 2
is in this arrow picture. Label the dots.
2×
+0.3
1.2
1.5
2.1
0.3
0.6
1.8
4.2
0.9
3.6
3.9
0.9
18

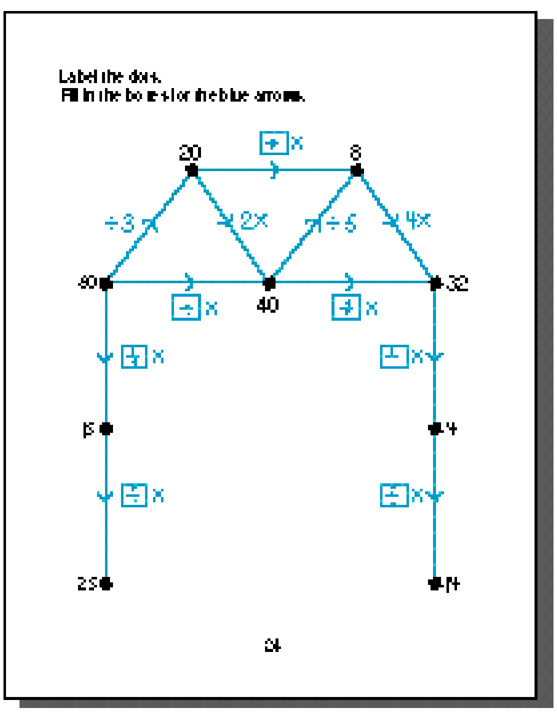

Fill in the blanks on the left.
By moving exactly one checker, put the new number on the Minicomputer to the right.
= 13
= 32
= 15
= 21
Another solution is possible.
= 81
= 100
= 16
= 12
= 80
= 100
19

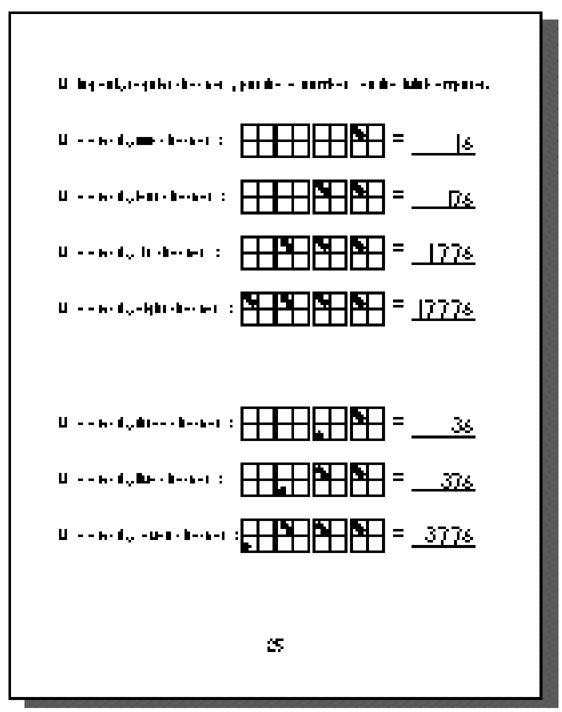

Fop is in this string picture.
Greater than 20
Even numbers
Fop
Fop can be put on the ones board of the Minicomputer using exactly one ⓪ checker.
Fop is in this arrow picture.
18
27
9
24
3
21
12
36
30
−6
+9
Who is Fop? 12
20

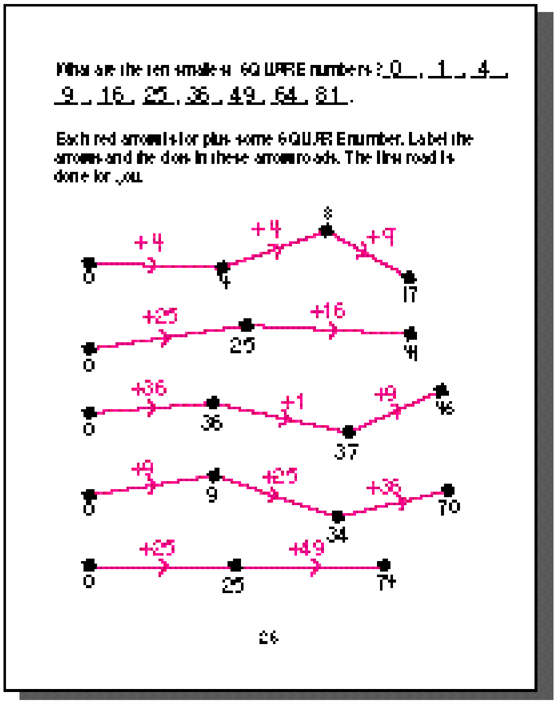

The red label is one of these:
The blue label is one of these:
Positive prime numbers
Multiples of 4
Label the strings.
Positive prime numbers
Multiples of 4
21


red ?
blue ?
white ?
rectangle.
Other divisions are possible.


2, 3, 4, 5, 6, 8, 9, 10,
12, 16
1, 2, 4, 5, 10
8
16
4
2x
2
64
1
128
4
23

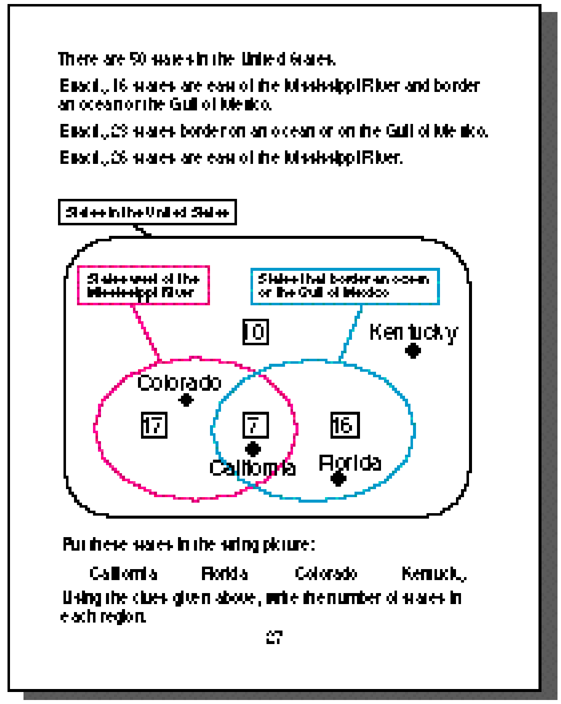

Label the dots.
Fill in the boxes for the blue arrows.
20
8
÷3
2x
÷5
4x
40
32
4
24


25


What are the ten smallest SQUARE numbers? 0, 1, 4, 9, 16, 25, 36, 49, 64, 81.

Each red arrow is for plus some SQUARE number. Label the arrows and the dots in these arrow roads. The first road is done for you.

26


There are 50 states in the United States.

Exactly 16 states are east of the Mississippi River and border an ocean or the Gulf of Mexico.

Exactly 23 states border on an ocean or on the Gulf of Mexico.

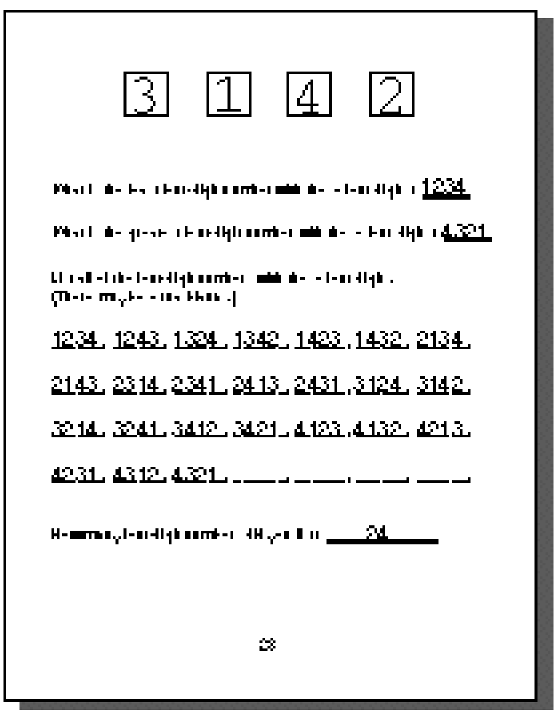
Exactly 26 states are east of the Mississippi River.

Put these states in the string picture:

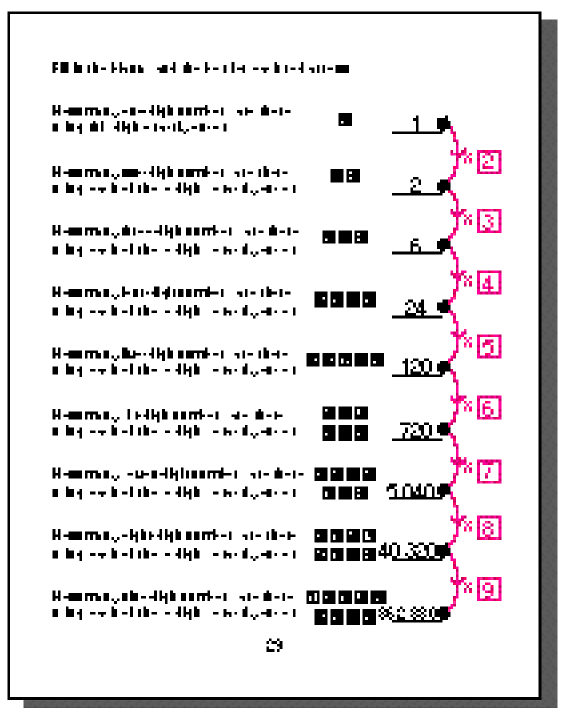
California Florida Colorado Kentucky

Using the clues given above, write the number of states in each region.

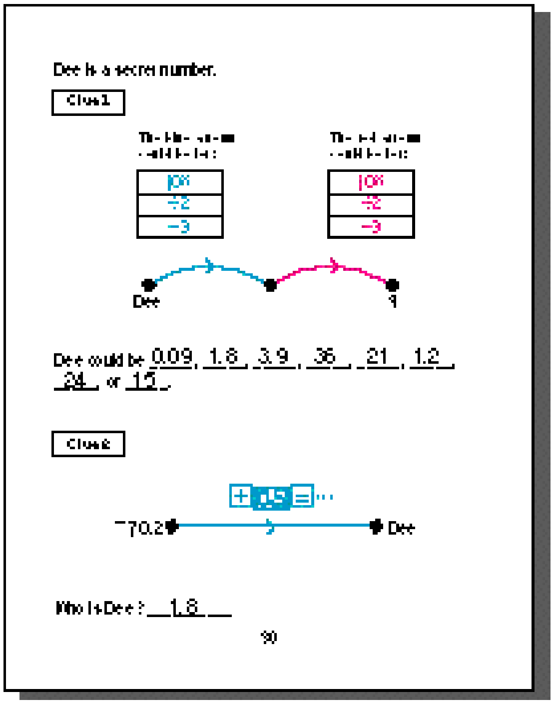
27

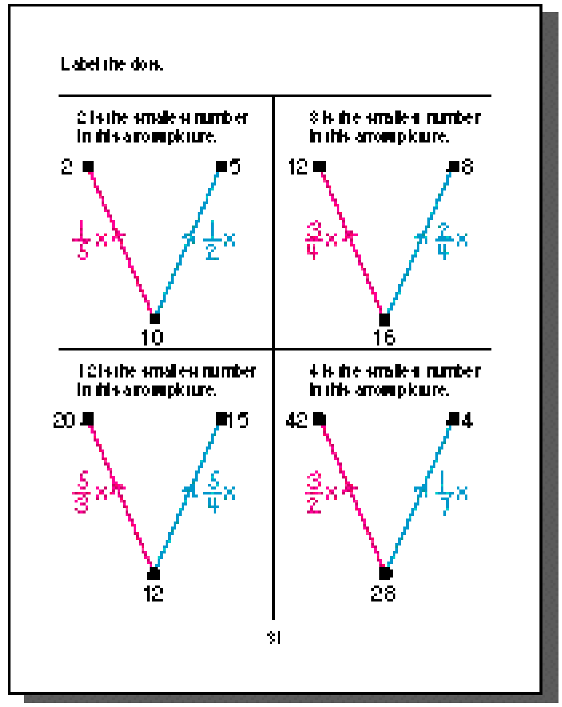


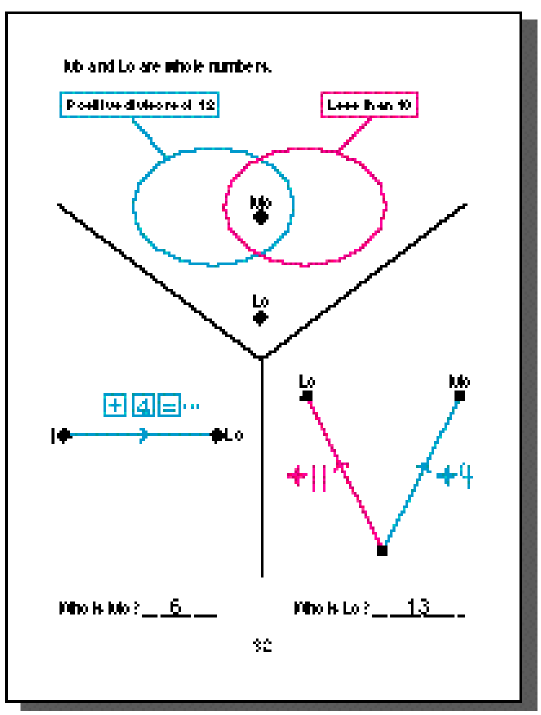
28



29


Dee is a secret number.
Clue 1
Dee
Dee could be 0.09, 1.8, 3.9, 36, 21, 1.2, 24, or 15.
Clue 2
-70.2
Dee
Who is Dee? 1.8
90


Label the dots.
2 is the smallest number in this arrow picture.
2
5
10
3 is the smallest number in this arrow picture.
12
8
16
12 is the smallest number in this arrow picture.
20
15
12
4 is the smallest number in this arrow picture.
42
4
28
91


Mo and Lo are whole numbers.
Positive divisors of 12
Less than 10
Mo
Lo
Lo
Mo
+11
+4
Who is Mo? 6
Who is Lo? 13
92

Capsule Lesson Summary

Begin the storybook *A Valentine Mystery* with a mysterious game played in the world of numbers in which each whole number sends exactly one valentine but receives ten. Discover that the rule for sending valentines involves dividing by 10. Determine which numbers send a valentine to a given number and which number receives a valentine from a given number.

Materials

Teacher
- *A Valentine Mystery* Storybook
- Colored chalk

Student
- *A Valentine Mystery* Storybook
- Unlined paper
- Colored pencils, pens, or crayons

Description of Lesson

Prepare every student with the materials listed above before starting the lesson.

Pages 1–4

Read pages 1 through 4 together; encourage class discussion of the game Zero invented for the numbers to play. Restate the rule of the game written in the red box on page 3.

Pages 5 and 6

Read pages 5 and 6 together.
Draw a red arrow starting at 79 on the board.

T: ***Study the poster on page 5. See if you can tell to whom 79 sends a valentine.***

S: ***7.***

Continue by asking to whom 709 and 4 037 send valentines. (Answers are in boxes.)

Draw a red arrow ending at 62 on the board.

T: ***From whom could 62 receive a valentine?***

S: ***623.***

T: ***Does 62 receive more than one valentine?***

S: ***Yes, 62 receives valentines from 620, 621, 622, 623, 624, 625, 626, 627, 628, and 629. 62 receives exactly ten valentines.***

T: ***Which numbers send valentines to 0?***

S: ***0, 1, 2, 3, 4, 5, 6, 7, 8, 9.***

Pages 7 and 8

Ask the class to read these pages and to study the posters.

T: ***On your paper, draw a poster for 829. When you finish the poster for 829, do one also for 6 025 also.***

While students are working, draw this much of the posters on the board.

829 →
829 =
829 =

6 025 →
6 025 =
6 025 =

When many students have finished their posters, invite students to complete the posters on the board.

829 → 829
$829 = 820 + 9$
$829 = (82 \times 10) + 9$

6 025 → 602
$6\,025 = 6\,020 + 5$
$6\,025 = (602 \times 10) + 5$

Write this division problem on the board.

$$10\overline{)829}$$

T: ***What is the result of this division problem?***

S: ***82 with a remainder of 9.***

Write the solution on the board and note that 82 is the quotient.

quotient

$$\begin{array}{r} 82 \text{ R} = 9 \\ 10\overline{)829} \end{array}$$

T: ***The result of a division calculation is the quotient. In this calculation, the quotient is 82 and the remainder is 9.***

Pages 9 and 10

Ask the class to read these pages while you write these problems on the board.

$$10\overline{)2\,386} \qquad 10\overline{)7} \qquad 10\overline{)0}$$

T: ***Can you solve these division problems?***

S: ***For the first problem, the quotient is 238 and the remainder is 6.***

S: ***For the second problem, the quotient is 0 and the remainder is 7.***

S: ***For the third problem, the quotient is 0 and the remainder is 0.***

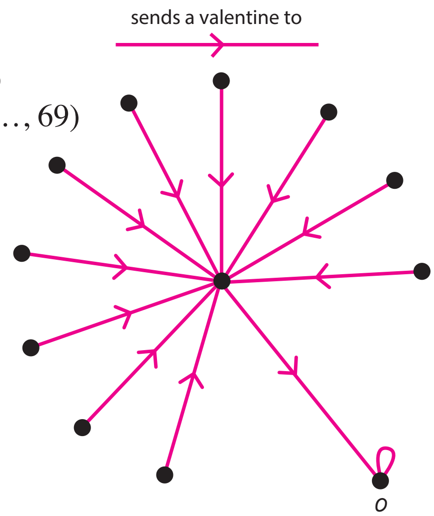
Record the solutions on the board.

Pages 11 and 12

Call on students to read these pages aloud. Note that the picture on page 12 is the completion of the poster on page 11.

T: ***On your paper, draw a flower like the one on page 12. Then put 0 at the ending dot of the longest arrow and label the other dots appropriately.***

As students work independently, draw this picture on the board.



T: ***What number could be at the center dot?*** (6, for example)
Then from whom does 6 receive valentines? (60, 61, 62, …, 69)
Does everyone have the same solution?

S: ***No, I put 8 at the center dot. The other dots are for 80 and 89.***

T: ***How many different solutions are possible?***

S: ***Nine. The center dot could be for 1, 2, …, or 9 since each of these numbers sends a valentine to 0.***

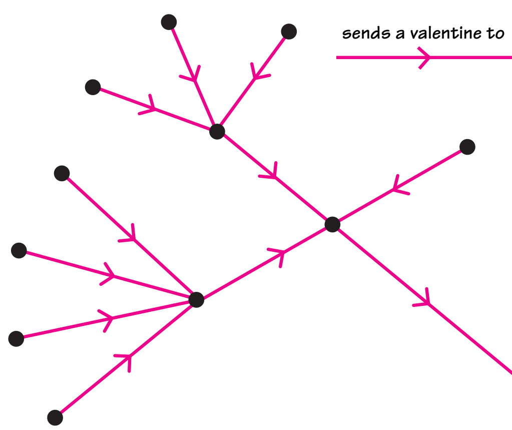
Pages 13 and 14

Call on students to read the pages aloud.

T: ***On page 14, label all of the dots that are centers of flowers and some of the other dots.***

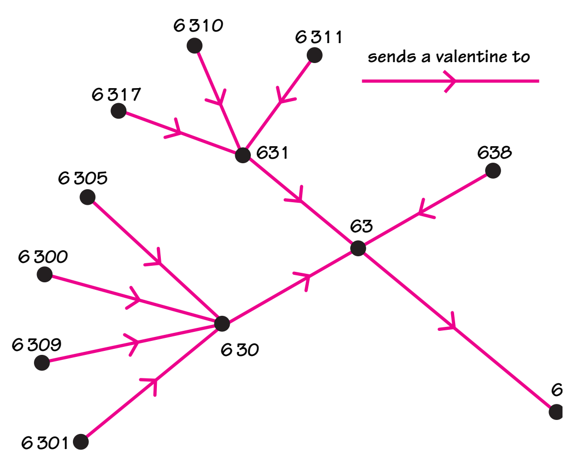
While students are working independently, draw this picture on the board.

After a few minutes, direct the class's attention to the picture on the board.



T: ***This is part of the picture on page 14. Who can label the dots?***

Invite students to label the dots. The class should observe that many solutions are possible. One possible solution in shown here.



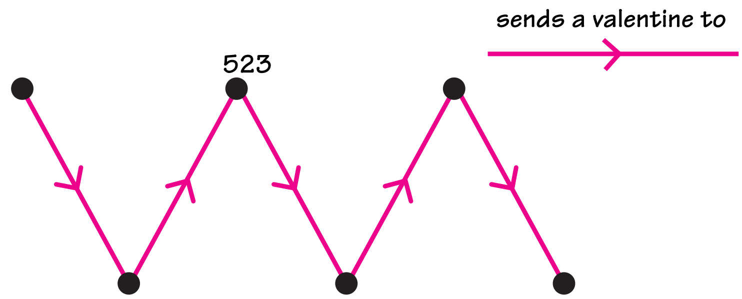
W3

Pages 15 and 16

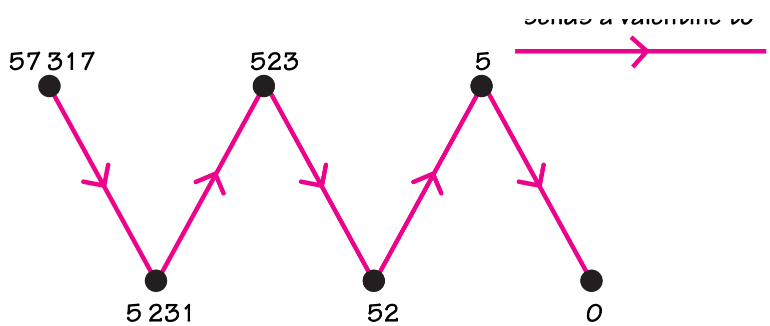
Read these pages aloud.

T: ***Now close your storybooks. I will put another problem on the board.***

Draw this arrow picture on the board, and ask students to copy it on their papers.



Invite students to label the other dots. The three dots on the right must be for 52, 5, and 0 respectively. There are many possibilities for the two dots on the left. The picture below shows one possible solution.



Collect students' copies of the storybooks for use again in Lesson W4.

W4 A VALENTINE MYSTERY (LESSON TWO)

Capsule Lesson Summary

Continue reading the storybook *A Valentine Mystery*. Find missing arrows for the relation "sends a valentine to" in an arrow picture. Discover that this game can be played with the whole numbers but not with people because there are infinitely many whole numbers.

Materials

Teacher
- *A Valentine Mystery* Storybook
- *IG-IV Workbook Poster #1* (in two parts, a and b)

Student
- *A Valentine Mystery* Storybook
- Colored pencils, pens, or crayons
- Paper

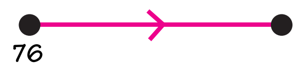
Description of Lesson

Start the lesson with students having colored pencils and unlined paper. Have the copies of the storybook *A Valentine Mystery* ready to distribute the students in Exercise 3.

Exercise 1

Ask a student to tell the story of *A Valentine Mystery* read thus far.

Draw an arrow starting at 76 on the board.



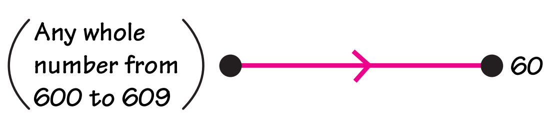
T: ***To whom does 76 send a valentine?***

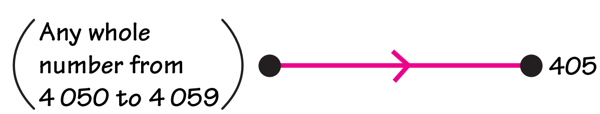
S: ***7.***

T: ***Does 7 receive other valentines?***

S: ***Yes, 7 receives exactly ten valentines—one each from 70, 71, 72, 73, 74, 75, 76, 77, 78, and 79.***

Similarly, ask who sends valentines to 60 and to 405.

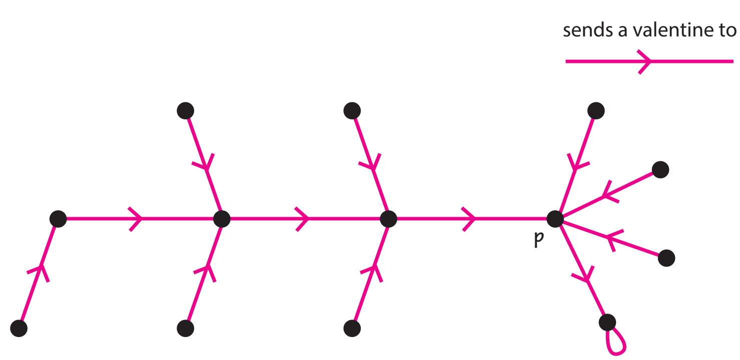




Repeat this activity using other numbers until students understand the rule for sending and receiving valentines.

Exercise 2

Draw this picture on the board and ask students to copy it on their papers.



Do not write the letter on the board. It is here just to make the description of the lesson easier to follow.

T: ***Who can label one dot?***

S: ***The dot with a loop must be 0 because 0 is the only number who sends a valentine to itself.***

W4

Label the dot for 0 and then ask students to label the other dots. Let your students work independently for a few minutes, then label the dots in the picture on the board. Suppose a student chooses to put 9 at **p**.

S: ***I put 9 here, but any number from 1 to 9 could be here because they all send valentines to 0.***

Invite students to label the other dots on the board. The class should notice that many solutions are possible. The picture below shows one possible solution.

Exercise 3

Distribute students' copies of the Storybook *A Valentine Mystery*.

Pages 17 and 18

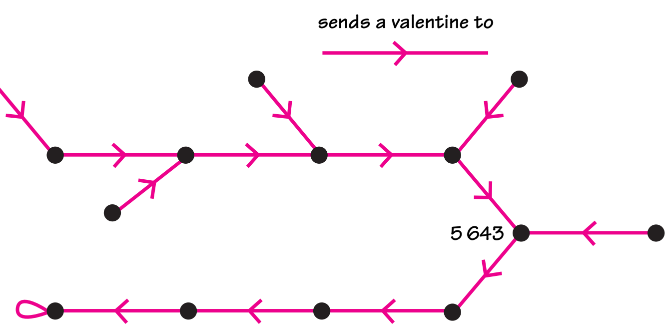
Call on students to read pages 17 and 18 aloud.

T: ***Label all of the center dots of flowers and some of the other dots on pages 17 and 18. Notice that one dot on page 18 (far right) is already labeled (5 643).***

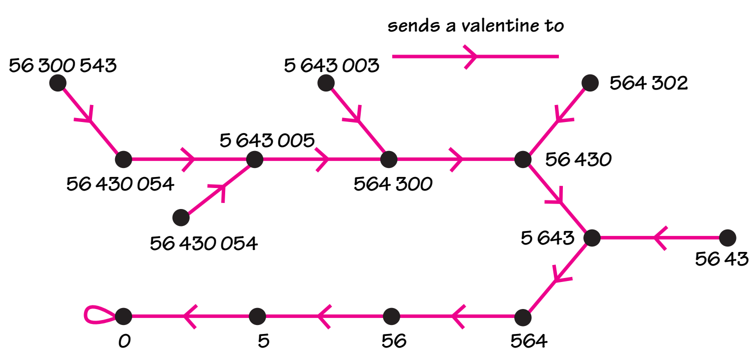
While students are working, draw this arrow picture on the board.

After several minutes, call the class's attention to the picture on the board.

T: ***This is part of the picture on pages 17 and 18. Who can label one of the dots?***



Invite students to label dots until they are all labeled. The numbers less than 5 643 must be 564, 56, 5, and 0. There are many possibilities for the other dots. Encourage students to read the numbers as they write them in the picture. One possible solution is shown here.



Pages 19–22

Read aloud and discuss these pages. Pages 19 and 20 give some hints on how to label the dots in the poster on pages 17 and 18. A full solution is on pages 21 and 22.

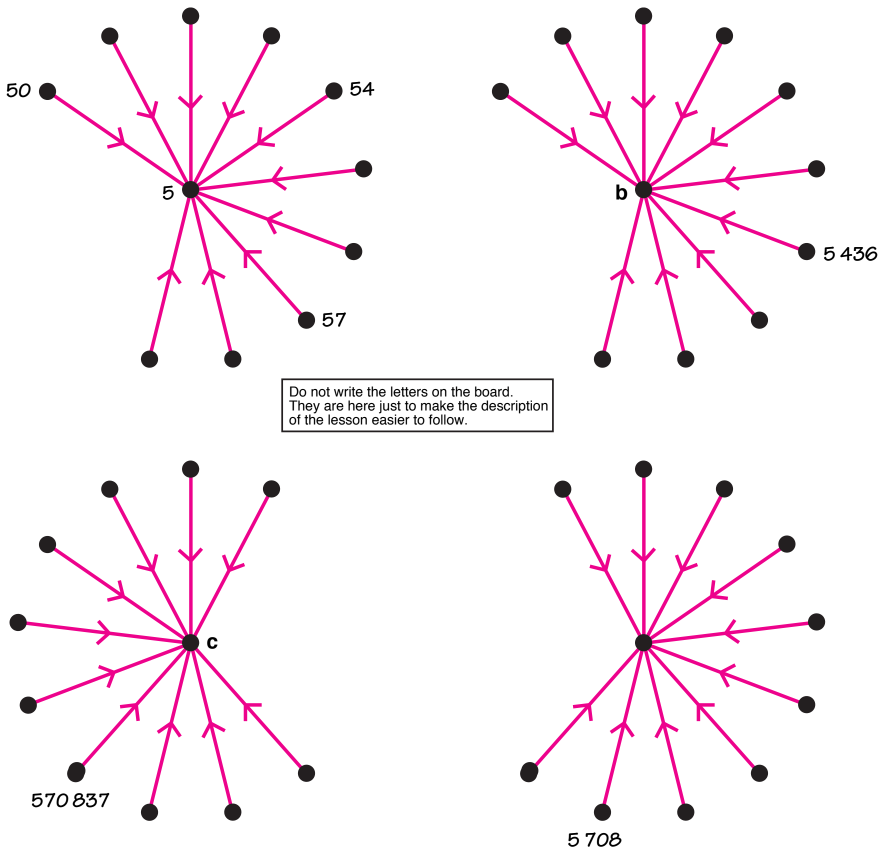
Pages 23 and 24

Call on students to read pages 23 and 24 aloud.

T: ***The flowers on pages 23 and 24 can be connected by red arrows. To help figure out where to draw the red arrows, label the center dot of each flower. I suggest that you begin on page 24 because the numbers there are smaller.***

Let students work on these pages for several minutes. Then tape *IG-IV Workbook Poster #1* on the board. This poster is in two pieces and should be displayed to look exactly like pages 23 and 24 in the storybook.

Call students' attention to the upper half of the right side of the poster.



T: ***This poster is a copy of pages 23 and 24 of your storybook. Which number is this*** (point to **b**)***?***

S: ***543, because 5 436 sends a valentine to 543.***

T: ***Where could we draw a red arrow?***

S: ***From 543 to 54.***

Label the dot for 543 and draw a red arrow from 543 to 54 on the poster.

T: ***Can someone label another center dot?***

S (pointing to **c**): ***57 083 is here.***

S: ***We can draw a red arrow from 57 083 to 5 708.***

Continue in a similar manner until all of the missing red arrows have been drawn. Use the completed picture on pages 25 and 26 of the storybook as your answer key. Labeling the center dots of the flowers aids in finding the red arrows. Encourage your students to both write and read the numbers as they are put in the picture.

Pages 25–28

After completing the poster, tell students to look at the solution on pages 25 and 26 and then to read pages 25 through 28. Discuss the picture on pages 27 and 28.

T: ***Which number is in the center of the picture?*** (0)
Is the picture finished? (No)
Could we ever finish drawing this picture?

S: ***No. Each number receives valentines from ten different numbers, and then each of those numbers receives ten more valentines and so on forever.***

Pages 29–30

Read and discuss these pages.

T: ***Why can this game be played by numbers but not by people?***

S: ***There are a limited number of people in the world, but there are always more and greater numbers.***

Extension Activity

Some students may like to try to invent another similar game that the numbers could play. For example, ask students to describe a game where every number would send one valentine and receive 12.

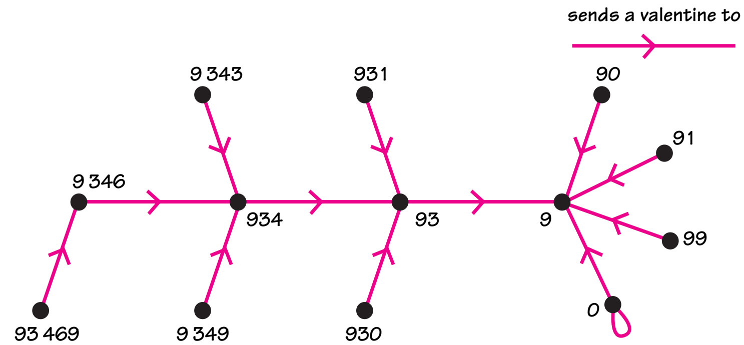
Writing Activity

Suggest that students write a description of the valentine game and then explain to a friend how this game works with numbers, but does not work with people. Some students may compare this game to the idea behind chain letters.

Label the dot for 0 and then ask students to label the other dots. Let your students work independently for a few minutes, then label the dots in the picture on the board. Suppose a student chooses to put 9 at **p**.

S: ***I put 9 here, but any number from 1 to 9 could be here because they all send valentines to 0.***

Invite students to label the other dots on the board. The class should notice that many solutions are possible. The picture below shows one possible solution.



Exercise 3

Distribute students' copies of the Storybook *A Valentine Mystery*.

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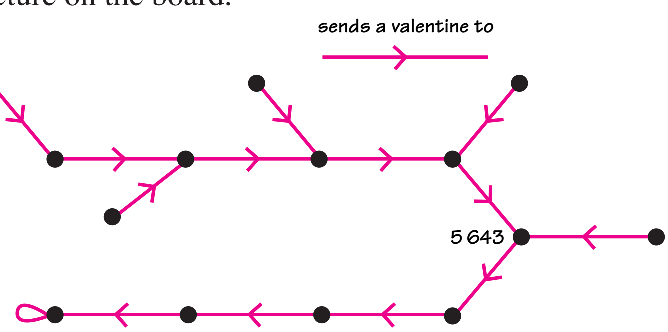

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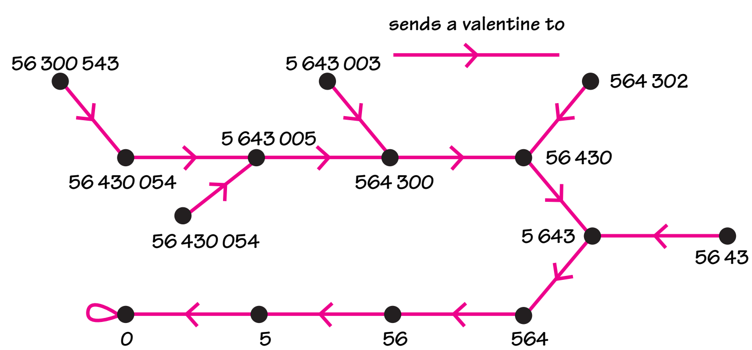

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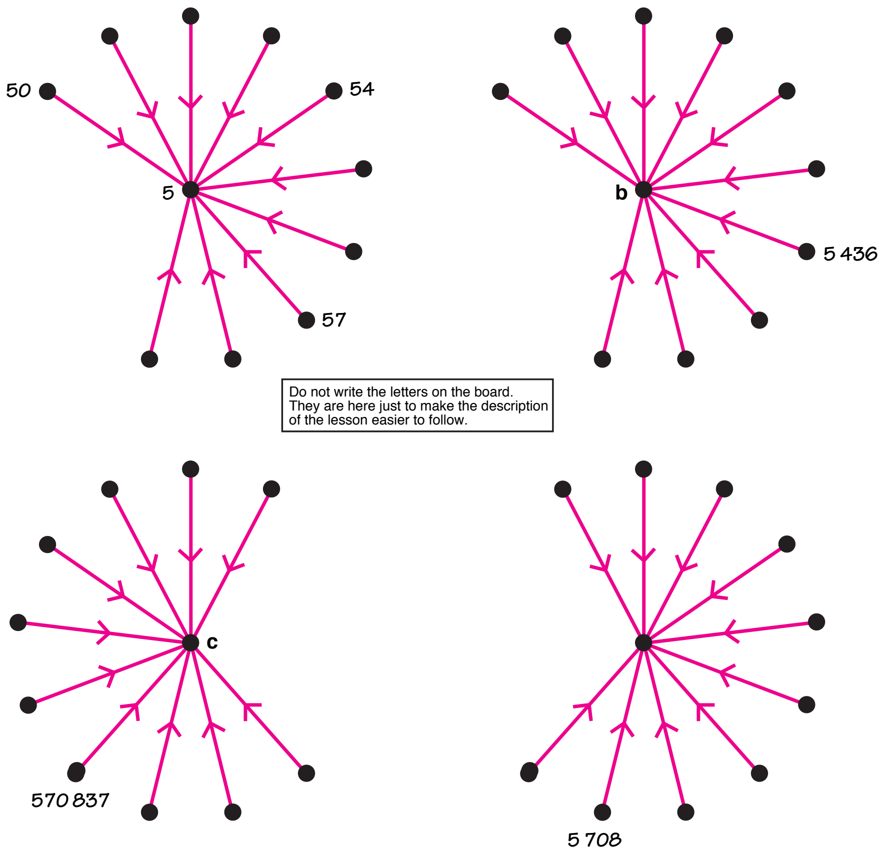

T: ***This poster is a copy of pages 23 and 24 of your storybook. Which number is this*** (point to **b**)***?***

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T: ***Where could we draw a red arrow?***

S: ***From 543 to 54.***

Label the dot for 543 and draw a red arrow from 543 to 54 on the poster.

T: ***Can someone label another center dot?***

S (pointing to **c**): ***57 083 is here.***

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Continue in a similar manner until all of the missing red arrows have been drawn. Use the completed picture on pages 25 and 26 of the storybook as your answer key. Labeling the center dots of the flowers aids in finding the red arrows. Encourage your students to both write and read the numbers as they are put in the picture.

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Is the picture finished? (No)
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S: ***No. Each number receives valentines from ten different numbers, and then each of those numbers receives ten more valentines and so on forever.***

Pages 29–30

Read and discuss these pages.

T: ***Why can this game be played by numbers but not by people?***

S: ***There are a limited number of people in the world, but there are always more and greater numbers.***

Extension Activity

Some students may like to try to invent another similar game that the numbers could play. For example, ask students to describe a game where every number would send one valentine and receive 12.

Writing Activity

Suggest that students write a description of the valentine game and then explain to a friend how this game works with numbers, but does not work with people. Some students may compare this game to the idea behind chain letters.

Capsule Lesson Summary

Find the volume of a box after viewing all or one layer of it. List the dimensions of boxes that can be made with 28 cubes of the same size. Begin the workbook *Selection of Problems #2*. (This is the first of two lessons using this workbook.)

Materials

Teacher
- 30 cubes of the same size
- Large cardboard box

Student
- *Selection of Problems #2* Workbook
- Calculator
- Pencil
- Metric ruler

Description of Lesson

Review the concepts of area and volume, mentioning that one unit for measuring area is a square centimeter and that one unit for measuring volume is a cubic centimeter.

Build a box shape (rectangular prism) with your set of cubes. Using a cube as the unit, the box should be 5 cubes by 2 cubes by 3 cubes. Display the structure where everyone in the class can see.

T: ***Pretend that each of these cubes is a cubic centimeter.***
What is the volume of this box? (30 cm^3) ***How do you know?***

S: ***There are three layers with ten cubes in each.***

S: ***There are five layers with four cubes in each.***

S: ***There are two layers with 15 cubes in each.***

Show one layer of a box 4 cubes by 3 cubes by 1 cube, laying it flat on the cardboard box.

T: ***If a box had exactly one layer like this, what would its volume be?*** (12 cm^3)
Two layers? (24 cm^3)
Ten layers? (120 cm^3)
Twenty layers? (240 cm^3)
Forty layers? (480 cm^3)

Repeat the activity, starting with a layer 8 cubes by 2 cubes by 1 cube. Turn it so that it stands tall.

Display 28 cubes. Ask what size boxes can be built with all 28 cubes. Record the dimensions as students build some of them.

28 cm^3
1 cm by 1 cm by 28 cm
2 cm by 1 cm by 14 cm
4 cm by 1 cm by 7 cm
2 cm by 2 cm by 7 cm

W5

Distribute copies of the workbook *Selection of Problems #2* and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of the lesson, collect the workbooks for your review. They will be used again in Lesson W6.

Capsule Lesson Summary

Relate a multiplication problem involving a non-whole number decimal with one that involves only whole numbers. Continue individual work in the workbook *Selection of Problems #2*. (This is the second of two lessons using this workbook.)

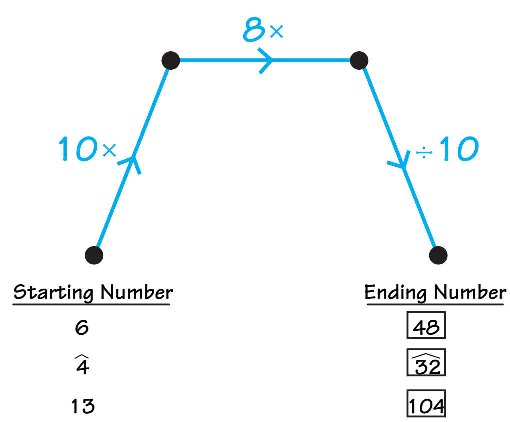
Materials

Teacher
- Colored chalk

Student
- *Selection of Problems #2* Workbook
- Calculator
- Colored pencils, pens, or crayons
- Metric ruler

Description of Lesson

Draw this arrow road on the board.



Label the starting dot with each of these numbers in turn and ask the class for the corresponding ending numbers. (Answers are in boxes.)

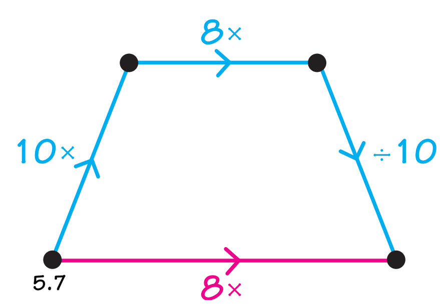
Draw a red arrow from the starting dot of the arrow road to its ending dot.

T: ***What is 10x followed by 8x followed by ÷10?***

S: ***8x.***

Label the red arrow and put 5.7 at the starting dot. Write the suggested multiplication problem near the arrow picture.

$$\begin{array}{r} 5.7 \\ \times\ 8 \\ \hline \end{array}$$



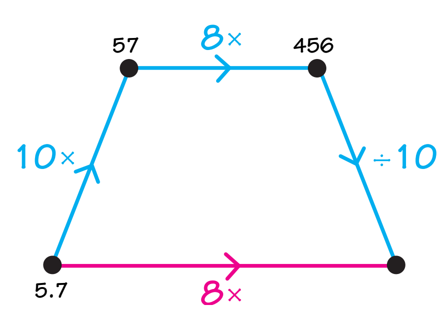
Follow blue arrows and label corresponding ending dots as you ask,

T: ***What number is 10 x 5.7?*** (57)
Now we need to multiply 8 x 57.

Write the problem next to the first multiplication problem. Invite a student to multiply 8 and 57 at the board.

T: ***What number is 456 ÷ 10?*** (45.6)

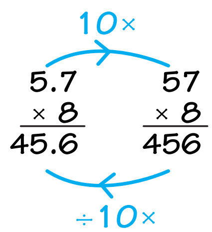
$$\begin{array}{r} 5.7 \\ \times\ 8 \\ \hline \end{array} \qquad \begin{array}{r} 57 \\ \times\ 8 \\ \hline 456 \end{array}$$

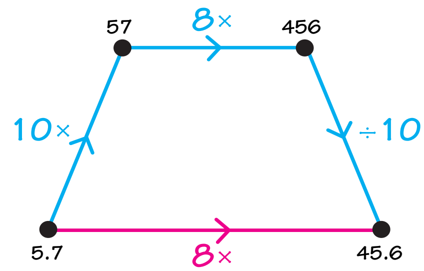


After putting 45.6 at the ending dot, trace the red arrow and announce,

T: ***So 8* x *5.7 = 45.6.***

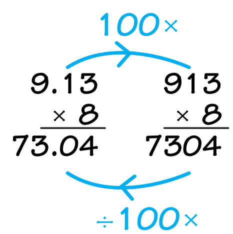
Record the answer to the first multiplication problem. Here blue arrows relate the two problems.

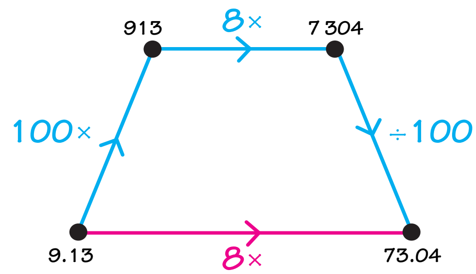




T: ***These two problems are related. To multiply 8* x *5.7, we can multiply 8* x *57 and then divide the answer by 10 since 57 is 10* x *5.7.***

Repeat the activity to calculate 8 x 9.13.





Distribute students' copies of the workbook *Selection of Problems #2*. Ask students first to correct or complete pages from the previous week's work, and then to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in the workbook during a study time or to take it home as an assignment.

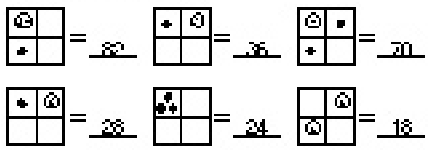
Assessment Activity

An individual student progress record for the workbook is available on Blackline W6. You may like to use this form to monitor student work.

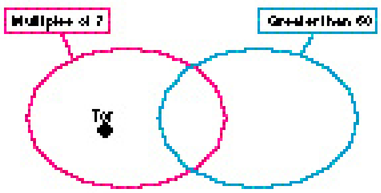
Tor is a secret number.

Clue 1

Tor is one of these numbers.



Clue 2



Who is Tor? 28

2

Fill in the boxes so that the calculations are correct.

$$\begin{array}{r} 8\,2\,7\,4 \\ +\ \boxed{4}\,6\,\boxed{5} \\ \hline \boxed{8}\,7\,\boxed{3}\,9 \end{array} \qquad \begin{array}{r} \boxed{4}\,\boxed{7}\,3\,9 \\ +\ 1\,2\,4\,\boxed{3} \\ \hline 5\,9\,\boxed{8}\,2 \end{array}$$

$$\begin{array}{r} \boxed{7}\,3\,\boxed{4}\,9 \\ -\ 1\,0\,3\,\boxed{3} \\ \hline 6\,\boxed{3}\,1\,6 \end{array} \qquad \begin{array}{r} 3\,8\,7\,6\,\boxed{1} \\ -\ 2\,\boxed{8}\,\boxed{1}\,4 \\ \hline \boxed{3}\,\boxed{5}\,9\,3\,7 \end{array}$$

3

Extend this line segment until it is 8 cm long.

How much longer did you make the segment? 2 cm

Extend this line segment until it is 14.3 cm long.

How much longer did you make the segment? 5.2 cm

Extend this line segment until it is 11.6 cm long.

How much longer did you make the segment? 0.8 cm

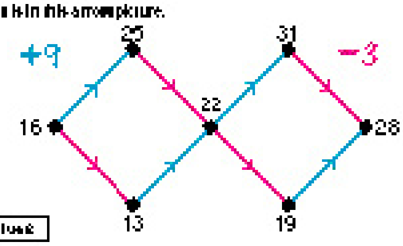
4

Answers may vary slightly.

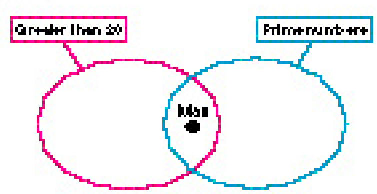
Man is a secret number.

Clue 1

Man is in this arrow picture.



Clue 2



Who is Man? 31

5

Complete these number sentences.

$37 \times 3 = \boxed{111}$

$37 \times 6 = \boxed{222}$

$37 \times 9 = \boxed{333}$

$37 \times 12 = \boxed{444}$

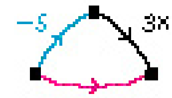
$37 \times \boxed{15} = 555$

⋮

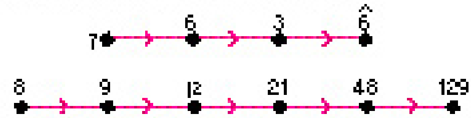
$37 \times 27 = \boxed{999}$

$37 \times 30 = \boxed{1\,110}$ SURPRISE!

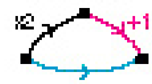
6



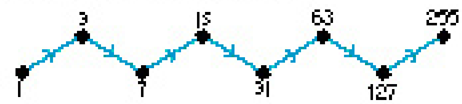
Use this two-step rule for red arrows to label the dots.



Guess My Rule



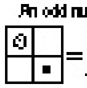
Find a two-step rule for blue arrows. Indicate the rule above and label the remaining dots using this rule.



7

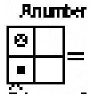
Put each number on the Minicomputer by adding exactly one regular checker. There may be more than one solution.

An odd number



= 27

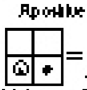
A number less than 9



= 6

7 is another solution.

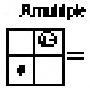
A positive divisor of 28



= 7

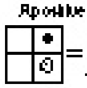
14 is another solution.

A multiple of 7



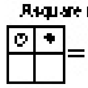
= 42

A positive prime number



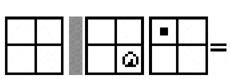
= 11

A square number



= 36

A number between 0.25 and 0.3

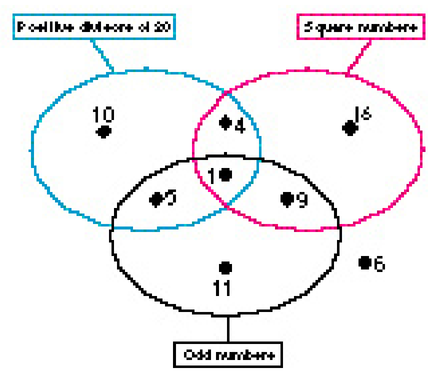


= 0.28

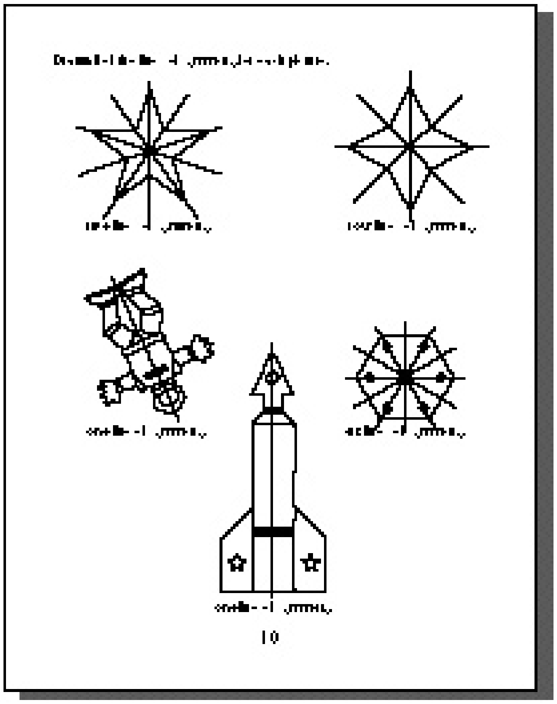
8

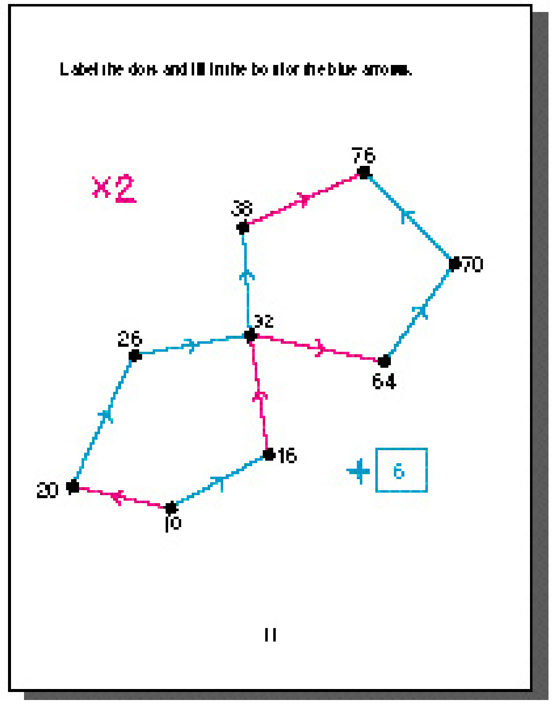
Label each dot in the string picture with one of these numbers. One dot is labeled for you.

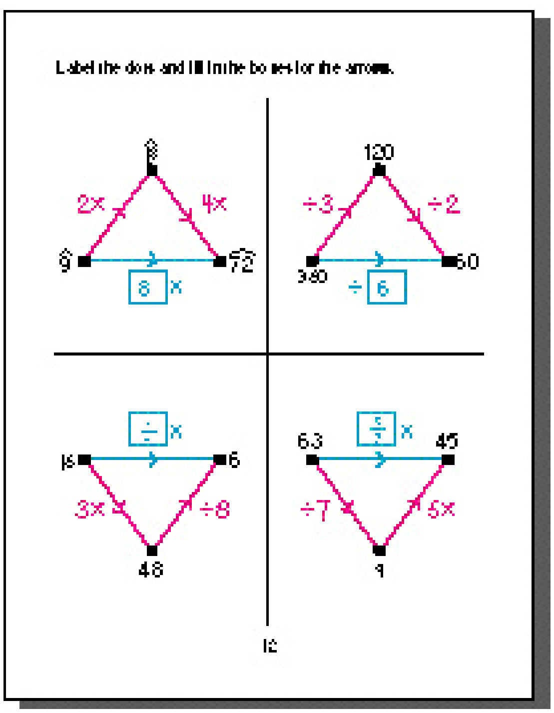
1   4   5   6   9   10   11   16

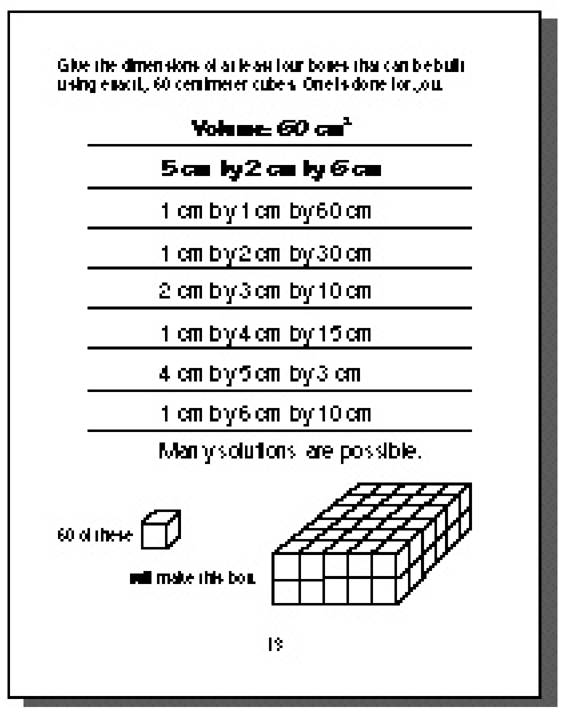


9



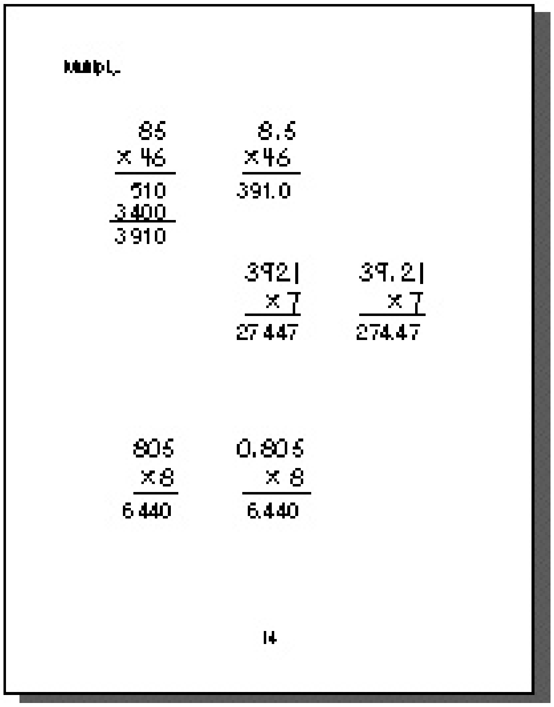


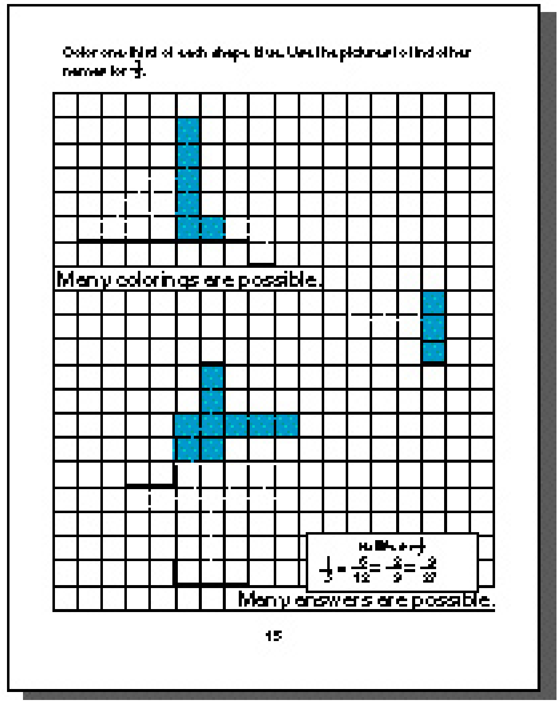


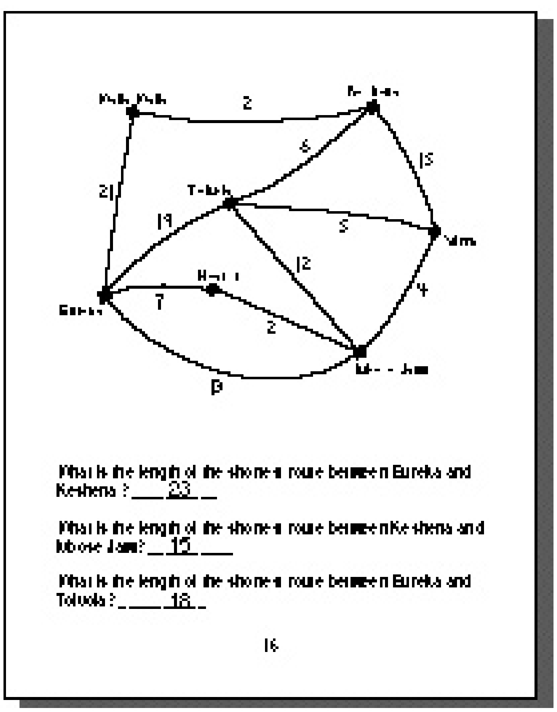

Give the dimensions of at least four boxes that can be built using exactly 60 centimeter cubes. One is done for you.

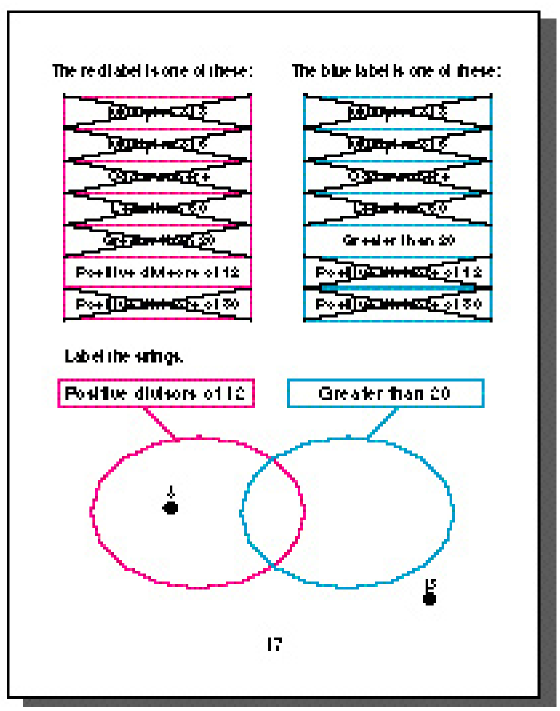
Volume: 60 cm³
5 cm by 2 cm by 6 cm
1 cm by 1 cm by 60 cm
1 cm by 2 cm by 30 cm
2 cm by 3 cm by 10 cm
1 cm by 4 cm by 15 cm
4 cm by 5 cm by 3 cm
1 cm by 6 cm by 10 cm

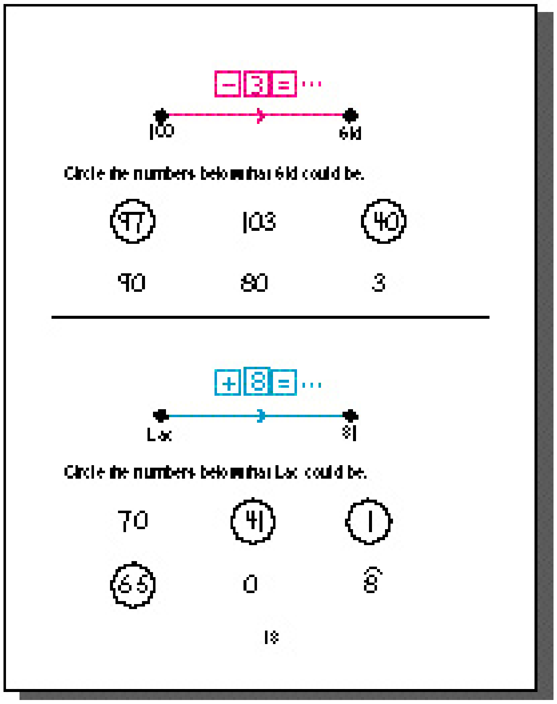
Many solutions are possible.


85
× 46
510
3 400
3 910
8.5
× 46
391.0
3921
× 7
27 447
39.21
× 7
274.47
805
× 8
6 440
0.805
× 8
6.440
14


Many colorings are possible.
Many answers are possible.
15


2
6
15
21
19
5
12
7
4
2
13
Keshena ? 23
Moose Jaw? 15
Toluola ? 18
16


The red label is one of these:
The blue label is one of these:
Positive divisors of 12
Greater than 20
Label the strings.
Positive divisors of 12
Greater than 20
17


$\boxed{-}\boxed{3}\boxed{=}\cdots$

100 → Old

Circle the numbers below that Old could be.

97 103 40

90 80 3

$\boxed{+}\boxed{8}\boxed{=}\cdots$

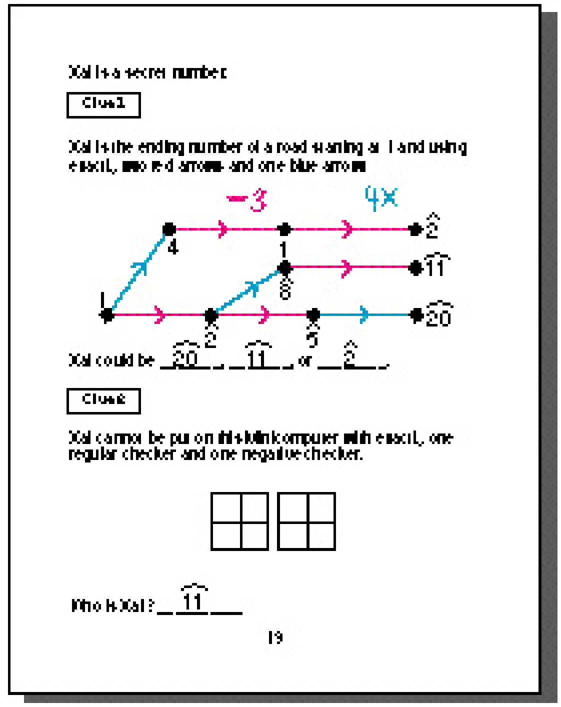
Lax → 81

Circle the numbers below that Lax could be.

70 41 1

65 0 $\hat{8}$

18


Xal is a secret number.

Clue 1

Xal is the ending number of a road starting at 1 and using exactly two red arrows and one blue arrow.

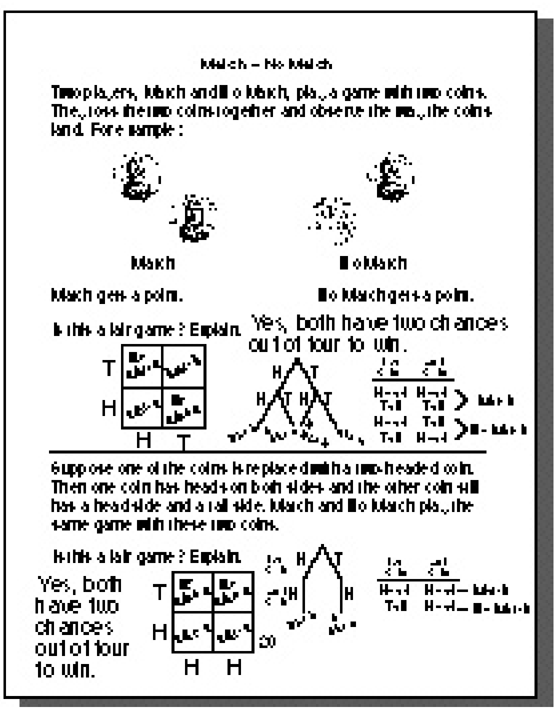
Xal could be $\hat{20}$, $\hat{11}$, or $\hat{2}$.

Clue 2

Xal cannot be put on this Minicomputer with exactly one regular checker and one negative checker.

Who is Xal? $\hat{11}$

19


Match – No Match

Two players, Match and No Match, play a game with two coins. They toss the two coins together and observe the way the coins land. For example:

Match No Match

Match gets a point. No Match gets a point.

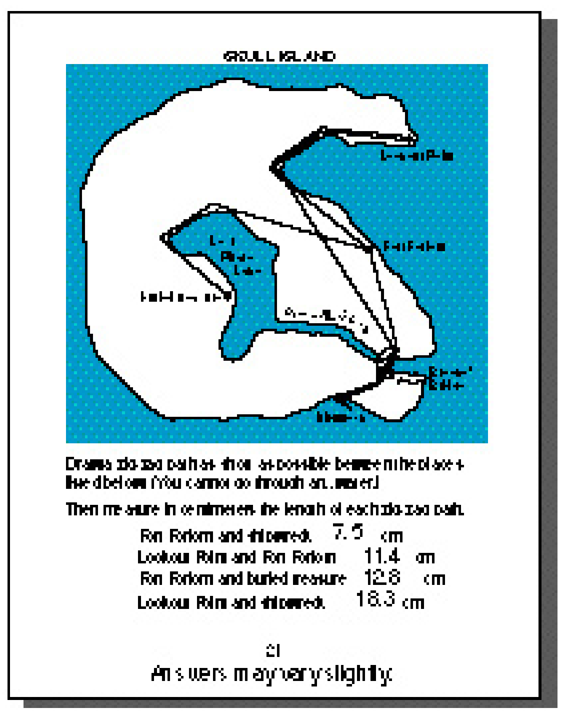
Is this a fair game? Explain. Yes, both have two chances out of four to win.

Suppose one of the coins is replaced with a two-headed coin. Then one coin has heads on both sides and the other coin still has a head side and a tail side. Match and No Match play the same game with these two coins.

Is this a fair game? Explain.

Yes, both have two chances out of four to win.

20



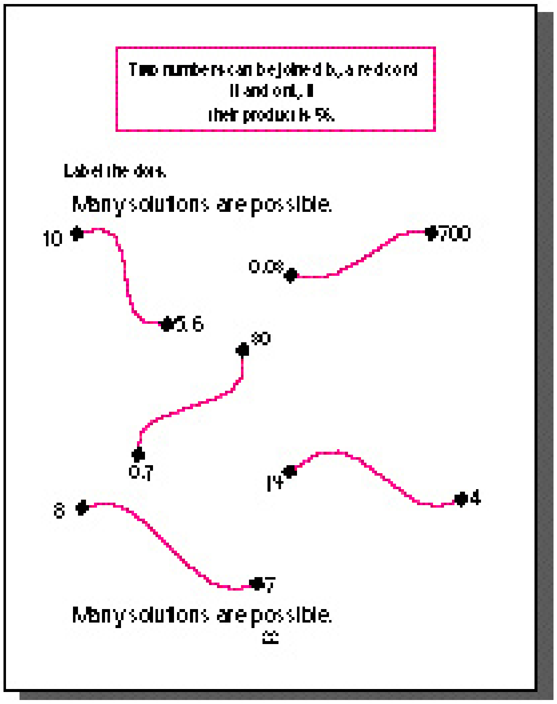
Draw zigzag paths as short as possible between the places listed below. (You cannot go through any water.)

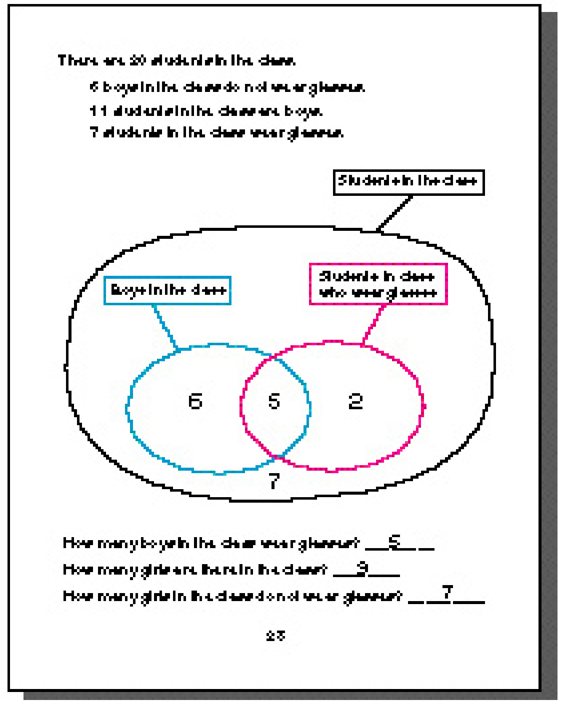
Then measure in centimeters the length of each zigzag path.

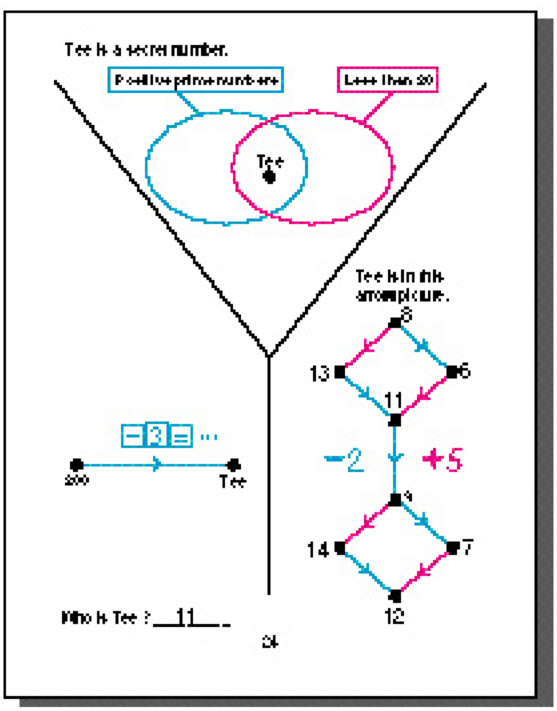
Fort Forlorn and shipwreck	7.5 cm
Lookout Point and Fort Forlorn	11.4 cm
Fort Forlorn and buried treasure	12.8 cm
Lookout Point and shipwreck	18.3 cm

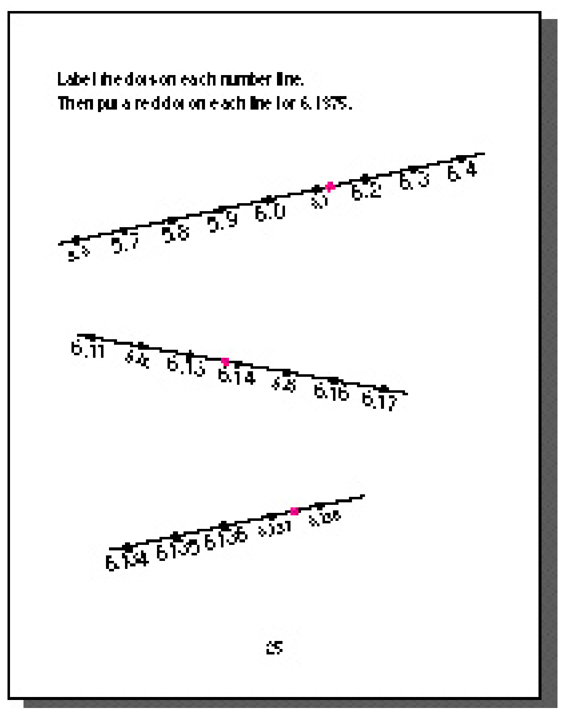
21

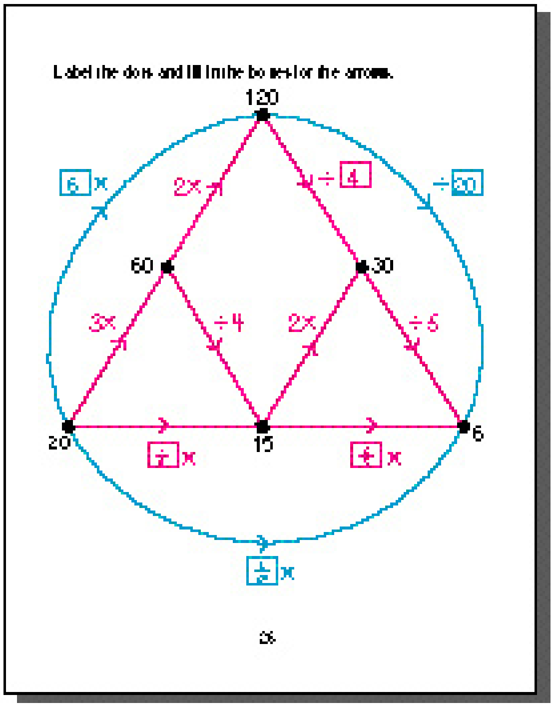
Answers may vary slightly.

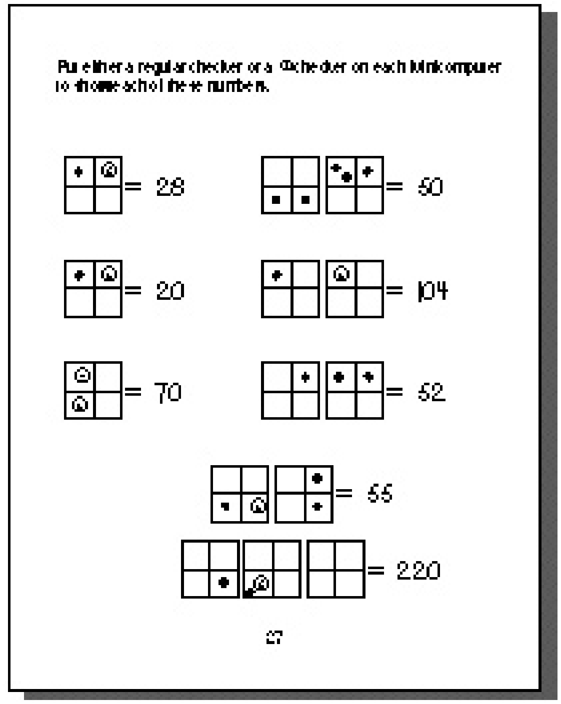

Two numbers can be joined by a red cord
if and only if
their product is 56.
Label the dots.
Many solutions are possible.
10
5.6
0.08
700
80
0.7
14
4
8
7
Many solutions are possible.
22

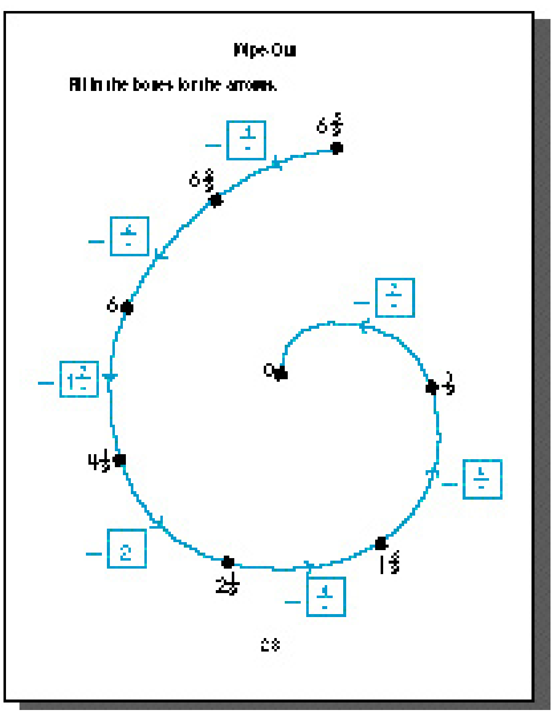

There are 20 students in the class.
6 boys in the class do not wear glasses.
11 students in the class are boys.
7 students in the class wear glasses.
Students in the class
Boys in the class
Students in class who wear glasses
6
5
2
7
How many boys in the class wear glasses? 5
How many girls are there in the class? 9
How many girls in the class do not wear glasses? 7
23

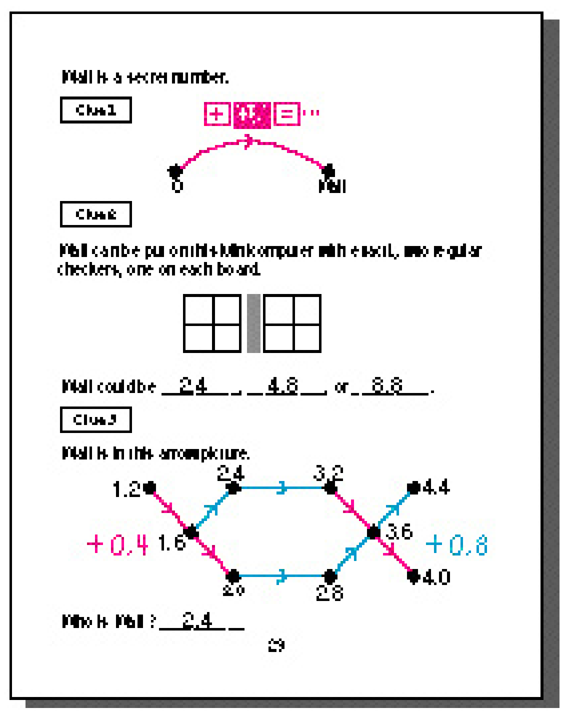

Tee is a secret number.
Positive prime numbers
Less than 20
Tee
Tee is in this arrow picture.
8
13
6
11
−2
+5
14
7
12
Tee
Who is Tee? 11
24

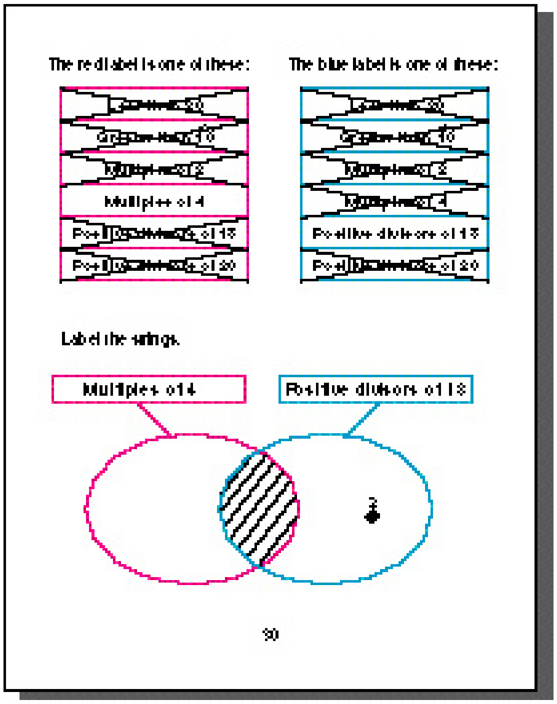

Label the dots on each number line.
Then put a red dot on each line for 6.1375.
5.7
5.8
5.9
6.0
6.2
6.3
6.4
6.11
6.13
6.14
6.16
6.17
6.134
6.135
6.136
25

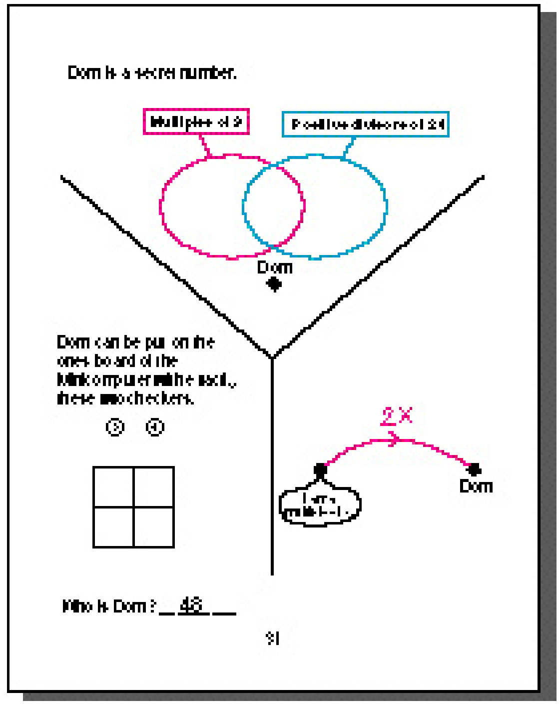

Label the dots and fill in the boxes for the arrows.
120
60
30
20
15
6
6x
2x
÷4
÷20
3x
÷4
2x
÷5
26

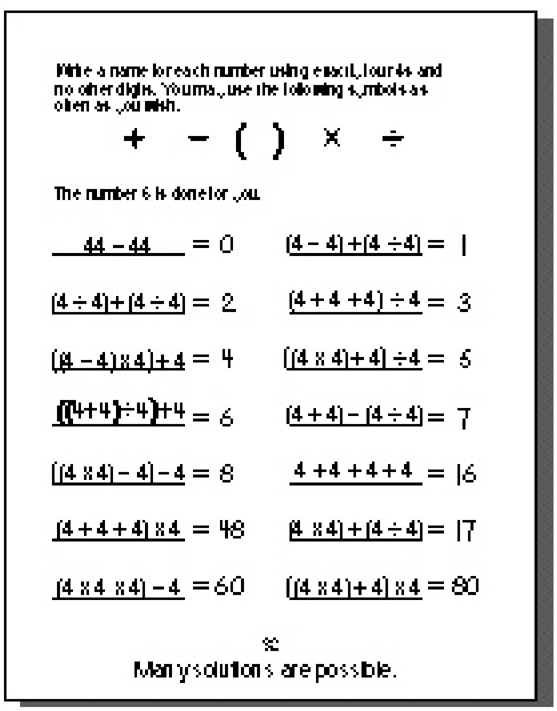

= 28
= 50
= 20
= 104
= 70
= 52
= 55
= 220
27


Wipe Out
Fill in the boxes for the arrows.
28


is a secret number.
Clue 1
Clue 2
Clue 3
2.4
4.8
8.8
1.2
2.4
3.2
4.4
+0.4
1.6
3.6
+0.8
2.0
2.8
4.0
2.4
29







Write a name for each number using exactly four 4s and no other digits. You may use the following symbols as often as you wish.

\+ − ( ) × ÷

The number 6 is done for you.

44 − 44 = 0	(4 − 4) + (4 ÷ 4) = 1
(4 ÷ 4) + (4 ÷ 4) = 2	(4 + 4 + 4) ÷ 4 = 3
((4 − 4) × 4) + 4 = 4	((4 × 4) + 4) ÷ 4 = 5
((4 + 4) ÷ 4) + 4 = 6	(4 + 4) − (4 ÷ 4) = 7
((4 × 4) − 4) − 4 = 8	4 + 4 + 4 + 4 = 16
(4 + 4 + 4) × 4 = 48	(4 × 4) + (4 ÷ 4) = 17
(4 × 4 × 4) − 4 = 60	((4 × 4) + 4) × 4 = 80

92

Many solutions are possible.

Capsule Lesson Summary

Using estimation, determine the placement of a missing decimal point in the answer to a calculation. Begin the workbook *Selection of Problems #3*. (This is the first of two lessons using this workbook.)

Materials

Teacher
- None

Student
- *Selection of Problems #3* Workbook
- Colored pencils, pens, or crayons
- Metric ruler
- Calculator
- Double mirror

Description of Lesson

Write these number sentences on the board.

$2.35 \times 7 = 9\ 3\ 5$
$1.935 \times 0.57 = 2\ 5\ 0\ 5$
$6.1 - 2.341 = 3\ 7\ 5\ 9$
$8 - 5.93 = 2\ 0\ 7$
$17.3 - 9 = 8\ 3$
$1.9 \times 43 = 8\ 1\ 7$
$8.2 \times 7.53 = 6\ 1\ 7\ 4\ 6$
$0.2 \times 81 = 1\ 6\ 2$

T: ***An eraser gremlin has erased all of the decimal points in the results of my calculations. Try to determine where a decimal point should be placed in each result to make the number sentence correct.***

As students announce where to put decimal points, ask for explanations and encourage use of estimation. For example:

S: ***8 – 5.93 = 2.07. Since 5.93 is close to 6 and 8 – 6 = 2, the result should be close to 2.***

S: ***1.9 x 43 = 81.7. Since 1.9 is close to 2, the result should be close to 2 x 43 = 86.***

Get class agreement on the placement of decimal points.

$2.35 \times 7 = 9.3\ 5$
$1.935 \times 0.57 = 2.5\ 0\ 5$
$6.1 - 2.341 = 3.7\ 5\ 9$
$8 - 5.93 = 2.0\ 7$
$17.3 - 9 = 8.3$
$1.9 \times 43 = 8\ 1.7$
$8.2 \times 7.53 = 6\ 1.7\ 4\ 6$
$0.2 \times 81 = 1\ 6.2$

Distribute copies of the workbook *Selection of Problems #3,* and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of the lesson, collect the workbooks for your review. They will be used again in Lesson W8.

Capsule Lesson Summary

Play a decimal number line game with intervals to focus attention on decimals with a thousandths place. Continue individual work in the workbook *Selection of Problems #3*. (This is the second of two lessons using this workbook.)

Materials

Teacher		Student	
	• Colored chalk		• *Selection of Problems #3* Workbook • Colored pencils, pens, or crayons • Metric ruler • Calculator • Double mirror

Description of Lesson

This warm up activity uses a decimal number line game called *Intervals*. The purpose of the game is to introduce the thousandths subgraduation. You need not choose the secret number until near the end of the game. As an example of how one may control the game and very quickly "home-in" on the thousandths subgraduations, dialogue of a possible game is included below.

Draw a line on the board.

T: ***I am thinking of a secret number between 0 and 10. Would someone like to guess what it is?***

S: ***6.***

Put 6 on your number line.

T: ***Each time you guess, I will respond to your guess with a number; the secret number is somewhere between your guess and my response. My response to your guess is 9. This tells you that my secret number is between 6 and 9.***

Put 9 on the number line and draw this red line segment.

T: ***The red segment is a reminder that the secret number is between 6 and 9. Another guess?***

S: ***7.***

T: ***11.***

T: ***Now what do you know about my secret number?***

S: ***It is between 7 and 11.***

S: ***Yes, but it is also between 7 and 9.***

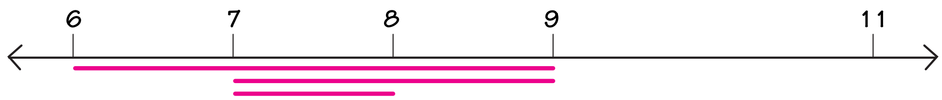
T: ***I will record this on the number line.***

T: ***Do you have another guess?***

S: ***8.***

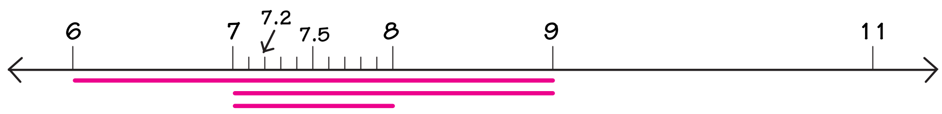
T: ***7.***

S: ***The secret number is between 7 and 8.***



S: ***7.5.***

T: ***7.2.***



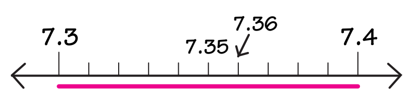
S: ***The secret number is between 7.2 and 7.5. I'll guess 7.4.***

T: ***7.3. Let's magnify the number line.***

Draw this magnified part of the number line.

S: ***7.35.***

T: ***7.36.***



Now we know the secret number is between 7.35 and 7.36. What is your next guess?

S: ***7.353.***

T: ***Can you locate 7.353 on the number line?***

S: ***Divide the segment between 7.35 and 7.36 into ten smaller segments of the same length.***

T: ***Let's magnify the number line again and locate 7.353. My response is 7.357. Do you have another guess?***

S: ***7.355.***

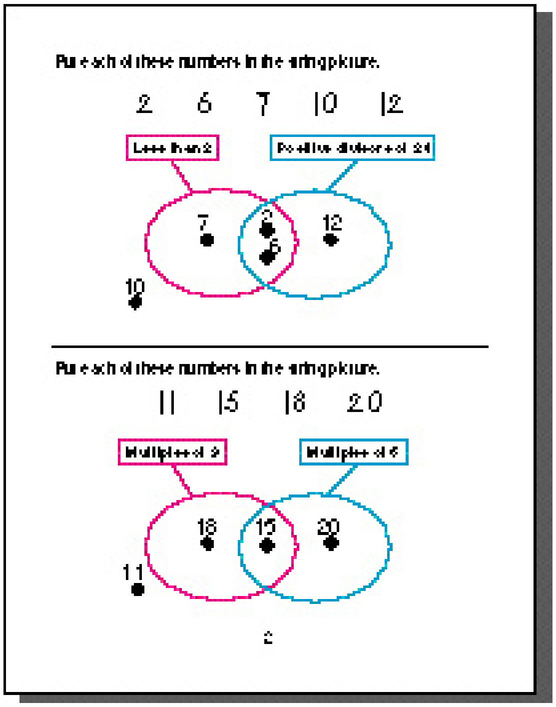
T: ***That's it!***

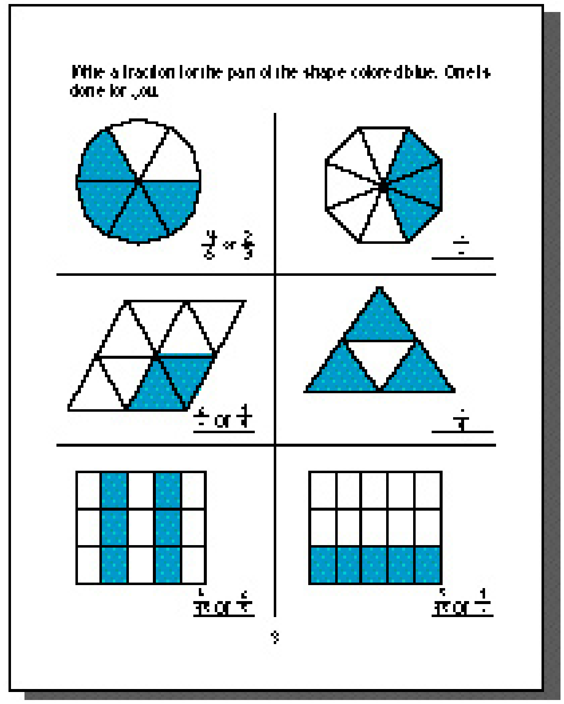
Distribute students' copies of the workbook *Selection of Problems #3*. Ask students first to correct or complete pages from the previous week's work, and then to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

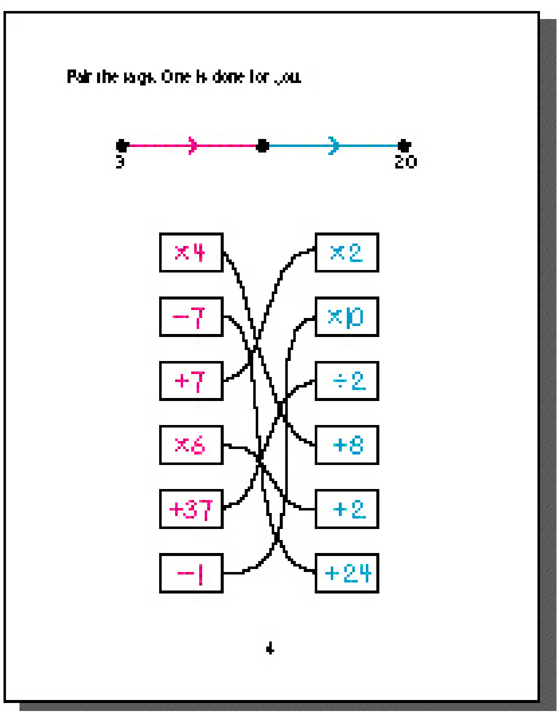
At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in the workbook during a study time or to take it home as an assignment.

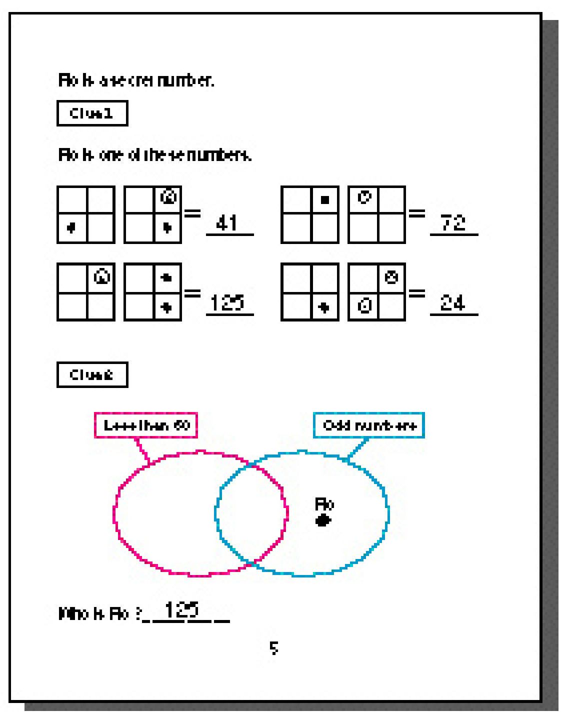
Assessment Activity

An individual student progress record for the workbook is available on Blackline W8. You may like to use this form to monitor student work.

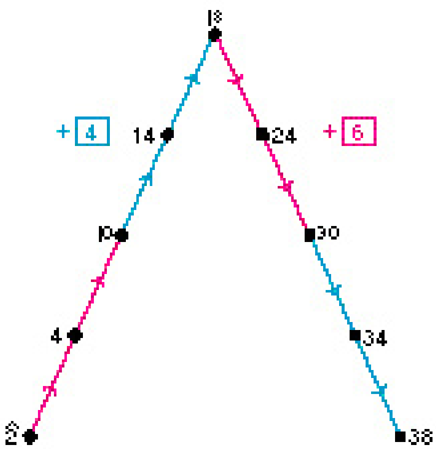

Put each of these numbers in the string picture.
2 6 7 10 12
Positive divisors of 24
Put each of these numbers in the string picture.
11 15 18 20
Multiples of 2
Multiples of 5
2


Write a fraction for the part of the shape colored blue. One is done for you.
3


Pair the tags. One is done for you.
3
20
×4
−7
+7
×6
+37
−1
×2
×10
÷2
+8
+2
+24
4


Flo is a secret number.
Clue 1
Flo is one of these numbers.
41
72
125
24
Clue 2
Odd numbers
Flo
Who is Flo? 125
5

Fill in the boxes for the arrows.
Label the dots.



6

Put a single digit in each box to make the calculations correct.

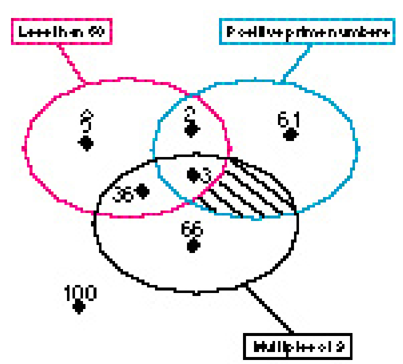
$$\begin{array}{r} 5\,\boxed{5}\,2 \\ +\,4\,8\,3\,\boxed{0} \\ \hline \boxed{5}\,\boxed{3}\,8\,2 \end{array} \qquad \begin{array}{r} \boxed{6}\,2\,6 \\ \boxed{3}\,6 \\ +\,3\,1\,6 \\ \hline 9\,7\,\boxed{8} \end{array}$$

$$\begin{array}{r} 8\,\boxed{6}\,5\,2 \\ -\,\boxed{3}\,0\,\boxed{3}\,6 \\ \hline 5\,6\,1\,\boxed{6} \end{array} \qquad \begin{array}{r} 2\,\boxed{7}\,7\,4\,4 \\ -\,4\,\boxed{2}\,8\,\boxed{0} \\ \hline \boxed{2}\,3\,4\,\boxed{6}\,4 \end{array}$$

7

Put these numbers in the string picture.

2 3 5 100 36 61 66



8

The eraser gremlin has erased the decimal point in each result.
Put in the decimal points so that the number sentences are correct.

$2.73 + 5 + 0.037 = 7.767$

$116.73 - 1.691 = 115.039$

$7 - 1.063 = 5.937$

$16.439 - 7 = 9.439$

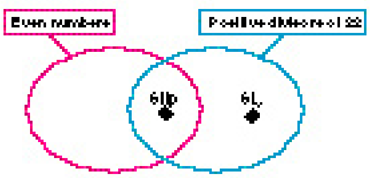
$2.83 \times 0.9 = 2.547$

$50.12 \times 2.9 = 145.348$

9

Blip and Blop are secret numbers.

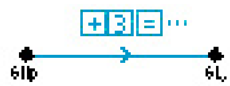
Clue 1



Blip could be __2__, __4__, __14__, or __28__.

Blop could be __1__ or __7__.

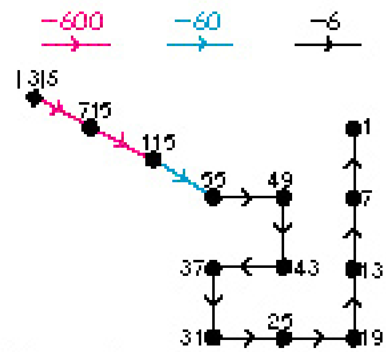
Clue 2



Who is Blip? __4__ Who is Blop? __7__

10

Build an arrow road from 1315 to the least possible positive number using these arrows.

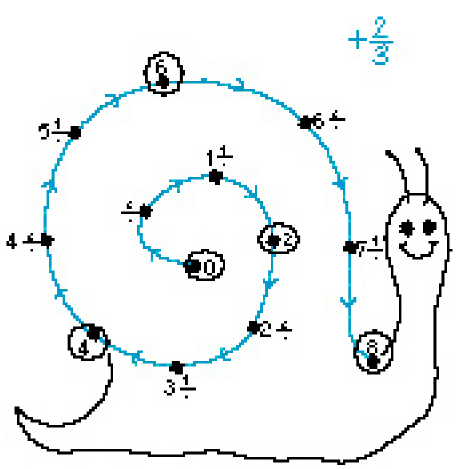


Use the arrow picture to solve this division problem.

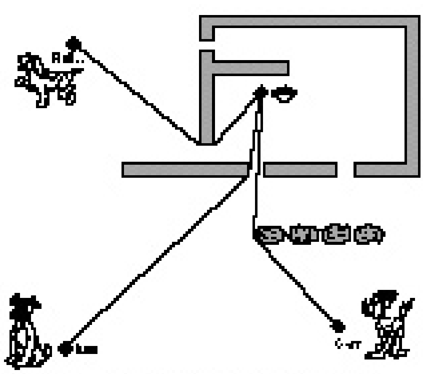
$$6\overline{)1315}\quad 219 \text{ R=1}$$

11

Label the dots. Circle all of the whole numbers in the picture.



12



Draw a zig-zag path from each dog to the dish of food. Make the paths as short as possible. Measure the length (in centimeters) of each zig-zag.

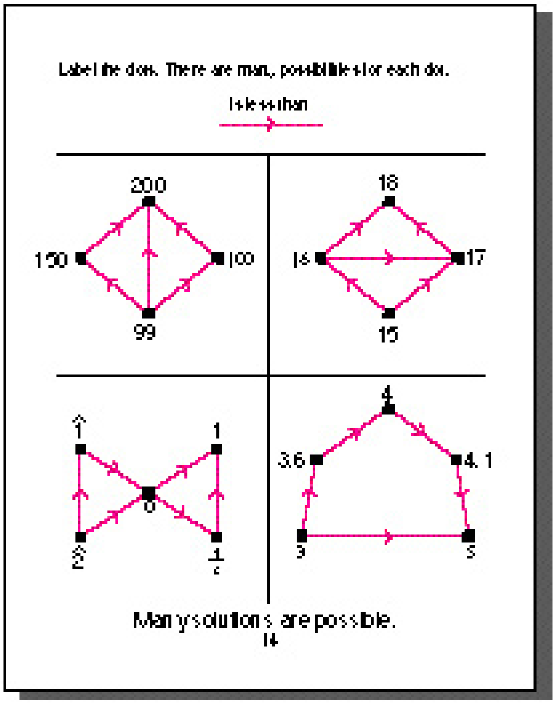
Fluffy to food __10__ cm
Mutt to food __13.6__ cm
Sam to food __10.8__ cm

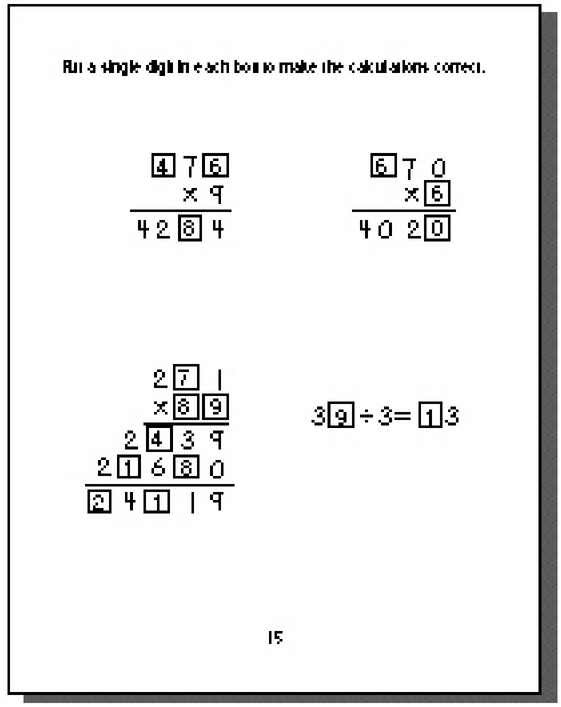
Which dog has the shortest path to the food? __Fluffy__

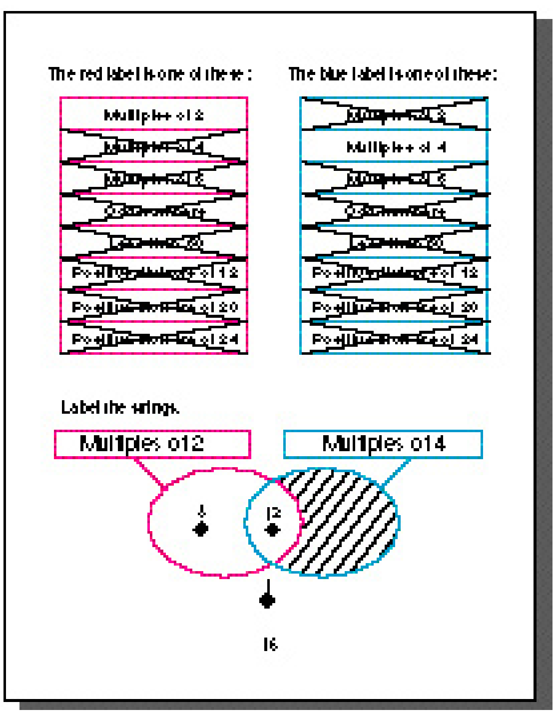
13

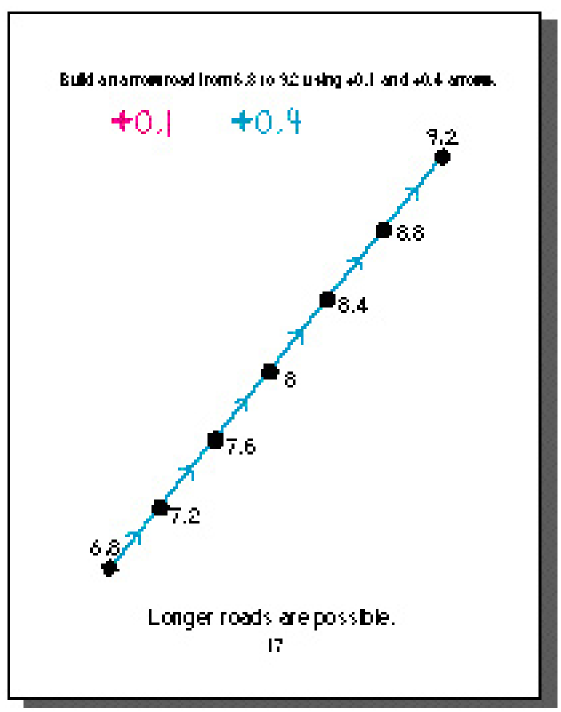
Longer paths are possible.
Answers will vary slightly

W8


Label the dots. There are many possibilities for each dot.
is less than
200
150
100
99
18
16
17
15
4
3.6
4.1
3
5
1
0
Many solutions are possible.
14


Put a single digit in each box to make the calculations correct.
15


The red label is one of these:
The blue label is one of these:
Multiples of 2
Multiples of 4
Label the strings.
Multiples of 2
Multiples of 4
8
12
16


Build an arrow road from 6.8 to 9.2 using +0.1 and +0.4 arrows.
+0.1
+0.4
9.2
8.8
8.4
8
7.6
7.2
6.8
Longer roads are possible.
17

Bill the Martian raises googels and comees. Googels have four eyes and comees have five eyes. Their eyes glow in the dark. One night Bill looks out the window and sees 26 eyes. How many of each animal does Bill see?

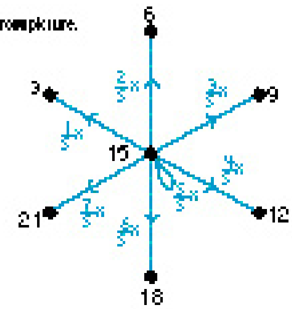
__4__ googels and __2__ comees

18

Fritz is a secret number.

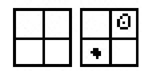
Clue 1

Fritz is in this arrow picture.



Clue 2

Fritz can be put on this Minicomputer by adding exactly one negative checker.



Who is Fritz? __18__

19

Fill in the boxes and then label the arrows.

$9 \times 3 = \boxed{27}$

$9 \times 2 = \boxed{18}$

$9 \times 1 = \boxed{9}$

$9 \times 0 = \boxed{0}$

$9 \times \hat{1} = \boxed{\hat{9}}$

$9 \times \hat{2} = \boxed{\hat{18}}$

$-\boxed{9}$

$\hat{6} \times 3 = \boxed{\hat{18}}$

$\hat{6} \times 2 = \boxed{\hat{12}}$

$\hat{6} \times 1 = \boxed{\hat{6}}$

$\hat{6} \times 0 = \boxed{0}$

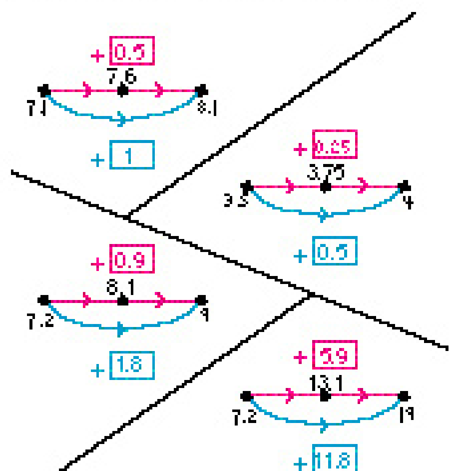
$\hat{6} \times \hat{1} = \boxed{6}$

$\hat{6} \times \hat{2} = \boxed{12}$

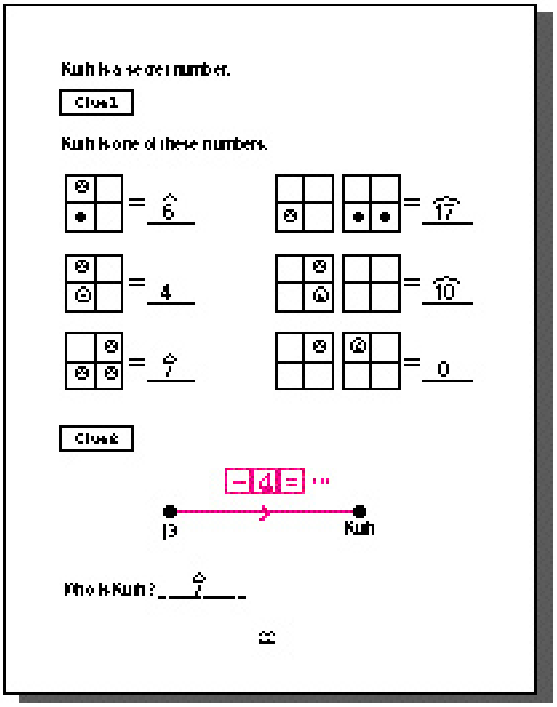
$+\boxed{6}$

20

Fill in the boxes for the arrows and then label the dots.



21



Kuh is a secret number.

Clue 1

Kuh is one of these numbers.

= $\hat{6}$

= $\hat{17}$

= 4

= $\hat{10}$

= $\hat{7}$

= 0

Clue 2

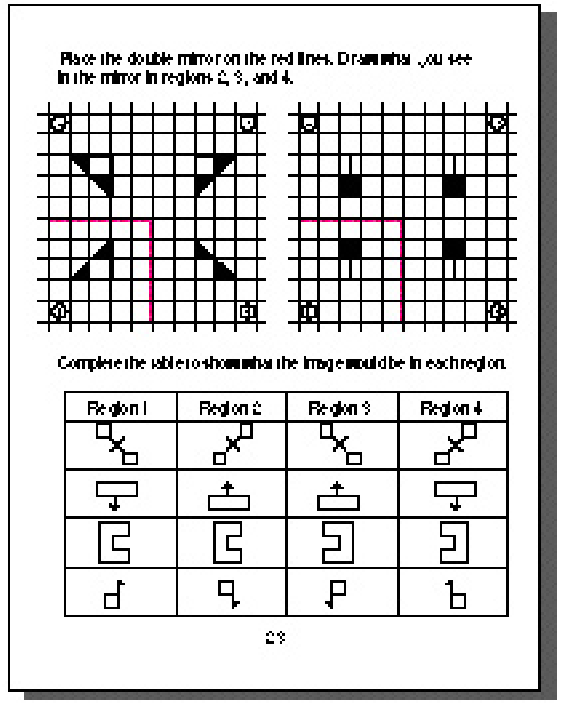
−4 = ...

13

Kuh

Who is Kuh? $\hat{7}$

22

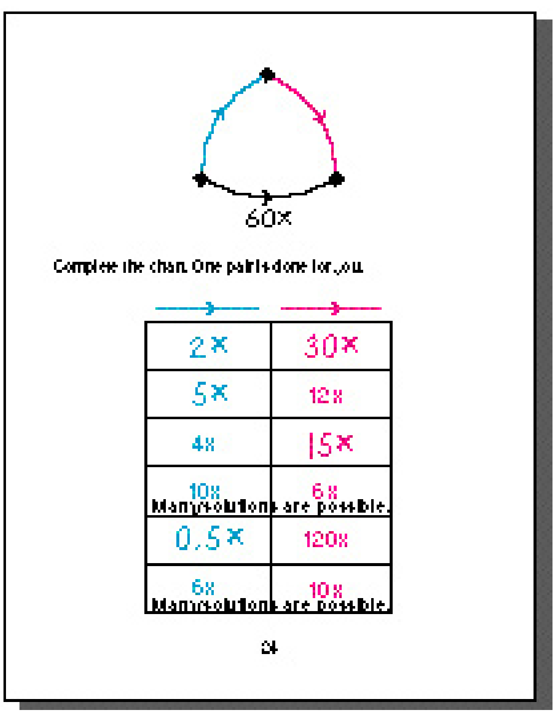


Place the double mirror on the red lines. Draw what you see in the mirror in regions 2, 3, and 4.

Complete the table to show what the image would be in each region.

Region 1	Region 2	Region 3	Region 4

23

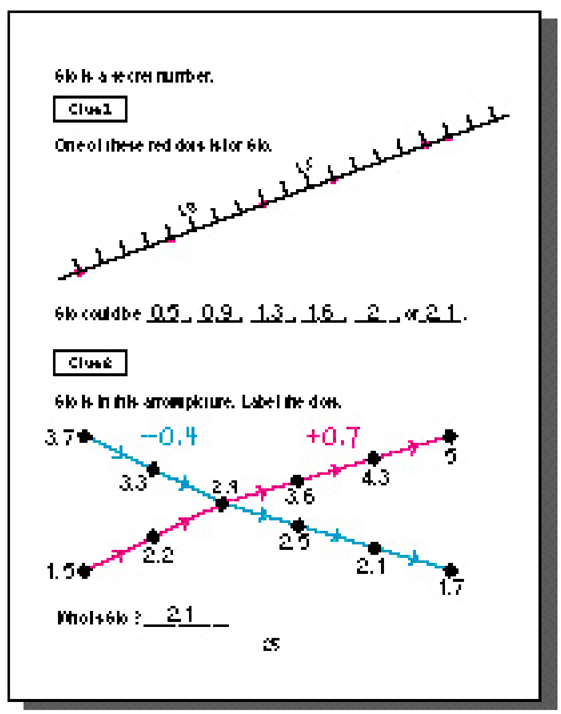


60x

Complete the chart. One pair is done for you.

2x	30x
5x	12x
4x	15x
10x Many solutions are possible.	6x
0.5x	120x
6x Many solutions are possible.	10x

24



Glo is a secret number.

Clue 1

One of these red dots is for Glo.

Glo could be 0.5, 0.9, 1.3, 1.6, 2, or 2.1.

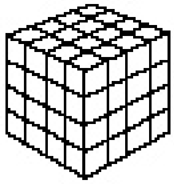
Clue 2

Glo is in this arrow picture. Label the dots.

3.7 −0.4 +0.7 5 3.3 2.9 3.6 4.3 2.2 2.5 2.1 1.5 1.7

Who is Glo? 2.1

25



Give the dimensions of the largest cube that can be made with:

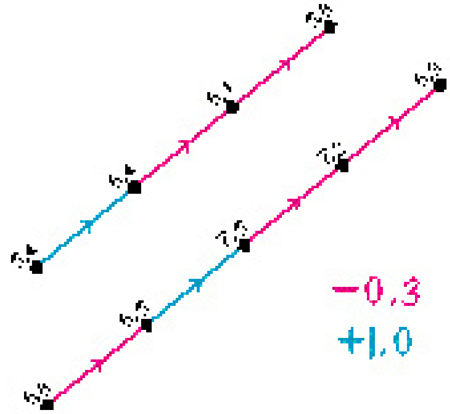
10 centimeter cubes	2 cm by 2 cm by 2 cm
64 centimeter cubes	4 cm by 4 cm by 4 cm
97 centimeter cubes	4 cm by 4 cm by 4 cm
500 centimeter cubes	7 cm by 7 cm by 7 cm

26

Label each dot in the arrow picture with one of these numbers. Use each number exactly once.

5.4 5.8 6.1 6.4

6.5 6.8 6.9 7.2 7.5

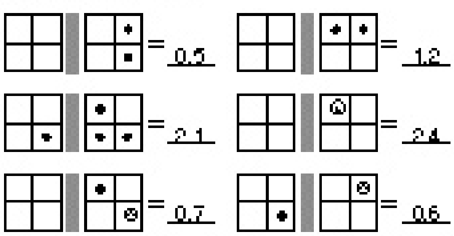


27

Klo is a secret number.

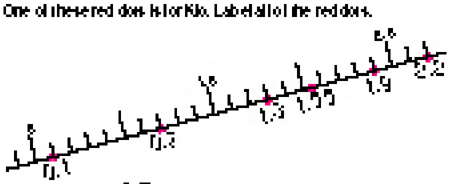
Clue 1

Klo is one of these numbers.



Clue 2

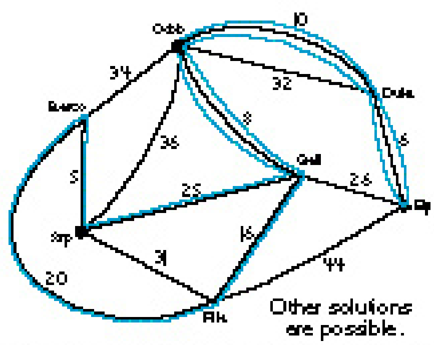
One of these red dots is for Klo. Label all of the red dots.



Who is Klo? 0.7

28

The numbers on this map indicate the length of each path.



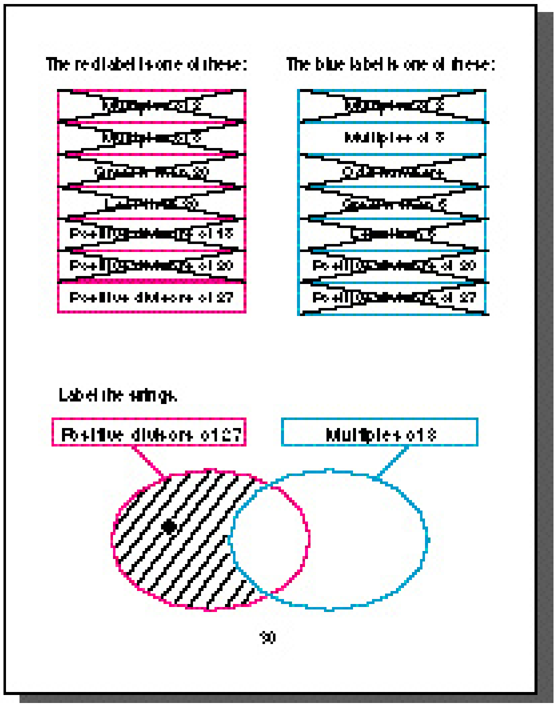
The distance between two cities is the length of a shortest route. What is the distance from Arp to Dubs? 43

List all of the cities whose distance from Fife is less than 30. Arp, Barco, Cobb, Fife, and Galt

Draw in blue a journey of length less than 120 that starts in Cobb, visits each city at least once, and then ends in Cobb.

What is the length of your journey? 114 Other solutions are possible.

29



The red label is one of these:

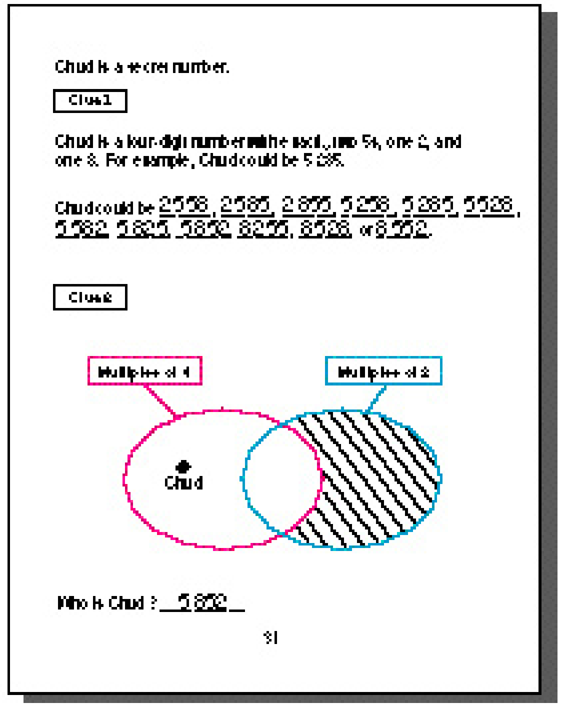
- ~~Multiples of 2~~
- ~~Multiples of 3~~
- ~~Greater than 20~~
- ~~Less than 20~~
- ~~Positive divisors of 18~~
- ~~Positive divisors of 20~~
- Positive divisors of 27

The blue label is one of these:

- ~~Multiples of 2~~
- Multiples of 3
- ~~Odd numbers~~
- ~~Greater than 5~~
- ~~Less than 5~~
- ~~Positive divisors of 20~~
- ~~Positive divisors of 27~~

Label the strings.

90



Chud is a secret number.

Clue 1

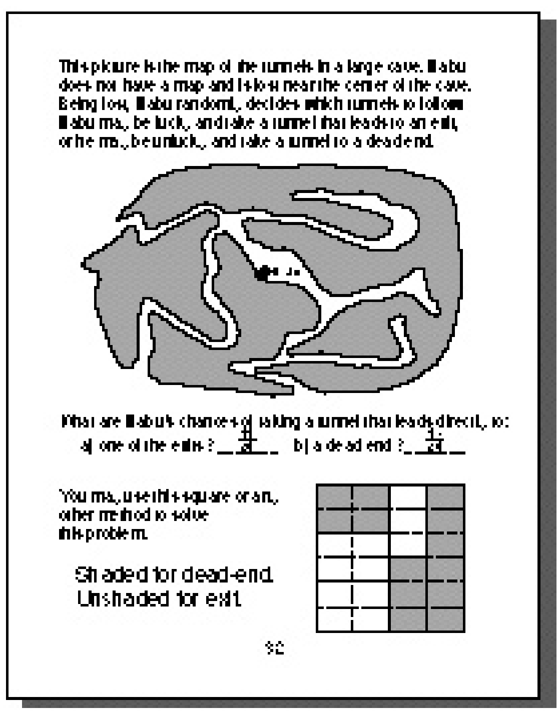
Chud is a four-digit number with exactly two 5's, one 2, and one 8. For example, Chud could be 5285.

Chud could be 2558, 2585, 2855, 5258, 5285, 5528, 5582, 5825, 5852, 8255, 8525, or 8552.

Clue 2

Who is Chud? 5852

91



This picture is the map of the tunnels in a large cave. Babu does not have a map and is lost near the center of the cave. Being lost, Babu randomly decides which tunnels to follow. Babu may be lucky and take a tunnel that leads to an exit, or he may be unlucky and take a tunnel to a dead end.

What are Babu's chances of taking a tunnel that leads directly to:

a) one of the exits? $\frac{11}{24}$ b) a dead end? $\frac{13}{24}$

You may use this square or any other method to solve this problem.

Shaded for dead-end.
Unshaded for exit.

92

Capsule Lesson Summary

Practice ordering both negative numbers and decimal numbers. Begin the workbook *Selection of Problems #4*. (This is the first of two lessons using this workbook.)

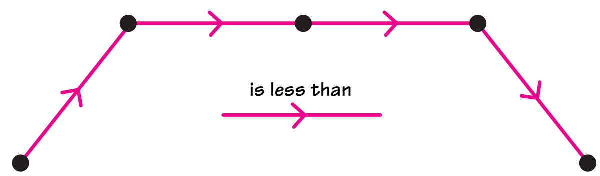
Materials

Teacher		Student	
	• Colored chalk		• *Selection of Problems #4* Workbook • Mirror • Metric ruler • Colored pencils, pens, or crayons • Calculator

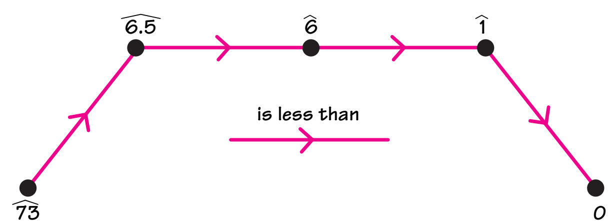
Description of Lesson

On the board, draw this arrow picture and list these numbers.

$\widehat{73}$ 0 $\hat{1}$ $\hat{6}$ $\widehat{6.5}$



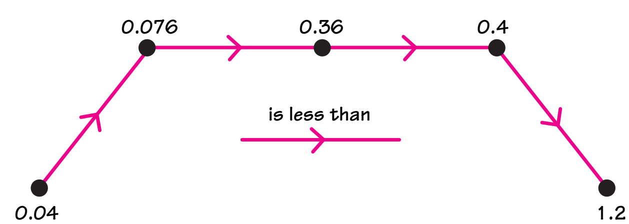
Invite students to label the dots with the given numbers. Do not specify which dot to label first. Ask students to explain their answers. They may refer to a number line. Correct locations of the numbers are shown here.



Erase the numbers from the picture and present a similar problem with these numbers.

1.2 0.4 0.04 0.36 0.076

To explain their answers, let students use any of several model; for example, money, a Minicomputer, or a number line. Correct locations of the numbers are shown here.



Distribute copies of the workbook *Selection of Problems #4*, and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

Note: You may like to make square tiles available to students when working on page 21 of the workbook.

W9

At the end of the class period, collect the workbooks for your review. They will be used again in Lesson W10.

Capsule Lesson Summary

Review coloring a specified fraction of a rectangular grid. Continue individual work in the workbook *Selection of Problems #4*. (This is the second of two lessons using this workbook.)

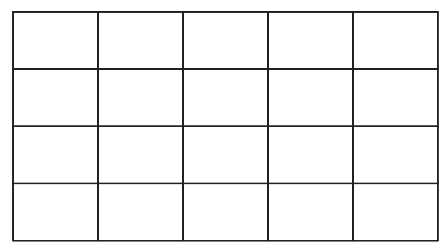
Materials

Teacher
- Colored chalk
- Meter stick

Student
- *Selection of Problems #4* Workbook
- Mirror
- Metric ruler
- Colored pencils, pens, or crayons
- Calculator

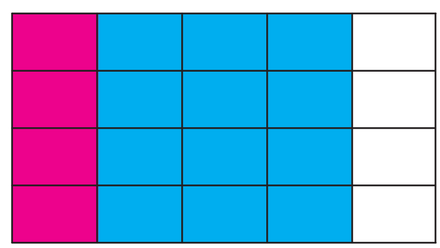
Description of Lesson

Draw this picture on the board.



T: ***How many rows of small rectangles are there?*** (Four) ***How many columns?*** (Five)

Color one column red and three columns blue.



T: ***What fraction of the whole shape is red?***

S: ***$\frac{1}{5}$, since one of the five columns is red.***

S: ***$\frac{4}{20}$, since the insides of four small rectangles are shaded red and there are 20 small rectangles altogether.***

On the board, note that $\frac{1}{5} = \frac{4}{20}$.

T: ***What fraction of the whole shape is blue?***

S: ***$\frac{3}{5}$, since three of the five columns are blue.***

S: ***$\frac{12}{20}$, since the insides of 12 small rectangles are blue and there are 20 small rectangles altogether.***

On the board, note that $\frac{3}{5} = \frac{12}{20}$.

T: ***What fraction of the whole shape is colored?***

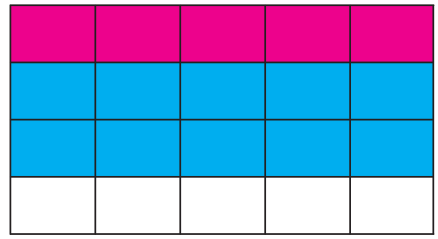
S: ***$\frac{4}{5}$ or $\frac{16}{20}$.***

On the board, indicate that $\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$ and $\frac{4}{20} + \frac{12}{20} = \frac{16}{20}$.

T: ***What fraction of the whole shape is not colored?***

S: ***$\frac{1}{5}$ or $\frac{4}{20}$.***

Erase the picture and redraw the original rectangle. Then color one row red and two rows blue.



Repeat the above activity with this coloring to reach the following results:

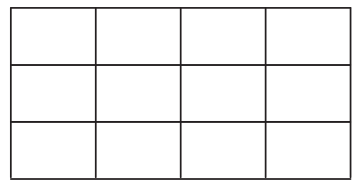
red: $\frac{1}{4} = \frac{4}{16}$

blue: $\frac{1}{2} = \frac{2}{4} = \frac{8}{16}$

colored: $\frac{1}{2} + \frac{1}{4} = \frac{3}{4} = \frac{12}{16}$

not colored: $\frac{1}{4} = \frac{4}{16}$

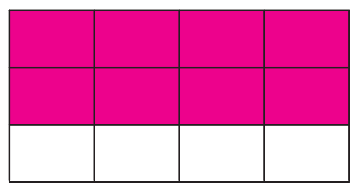
Erase the board and then draw this picture.



T: ***How many rows are there? Columns?***

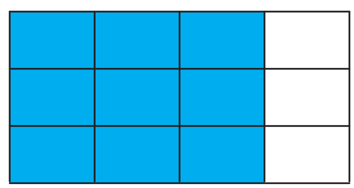
S: ***There are three rows and four columns.***

Invite a student to color two-thirds of the whole shape red, for example:



(It is correct to color the insides of any eight of the small rectangles.)

Erase the picture and redraw the rectangle; then invite a student to shade three-fourths of the whole shape blue, for example:



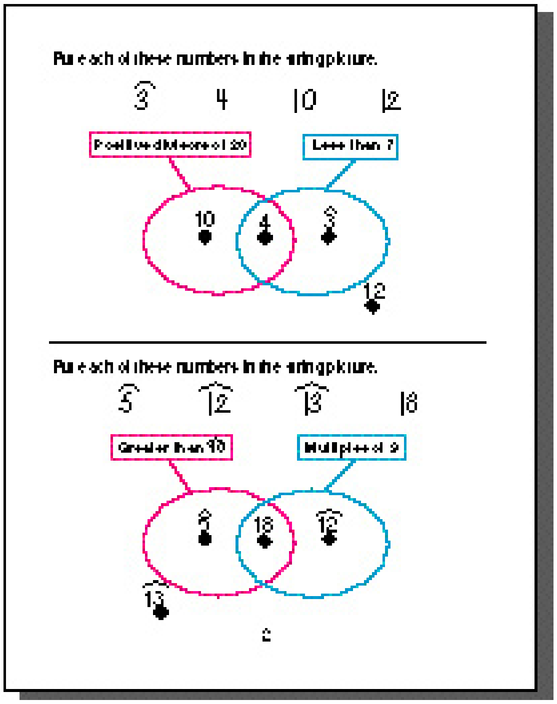
(It is correct to color the insides of any nine of the small rectangles.)

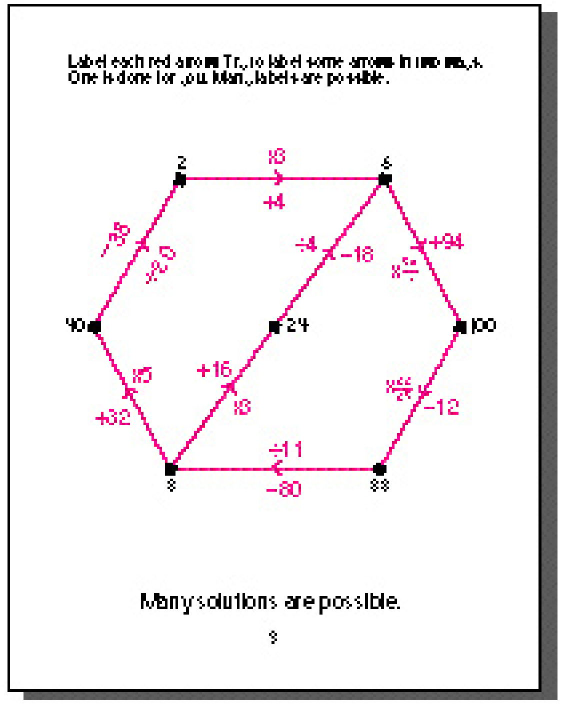
Distribute students' copies of the workbook *Selection of Problems #4*. Ask students first to correct or complete pages from the previous week's work and then to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

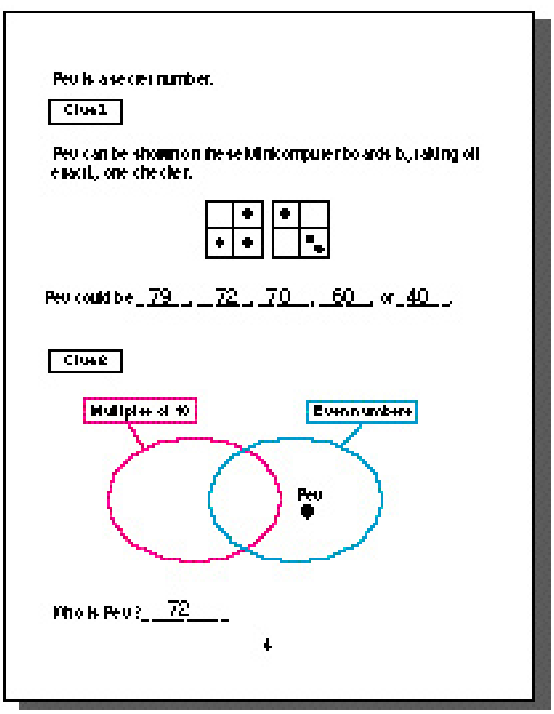
At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in the workbook during a study time or to take it home as an assignment.

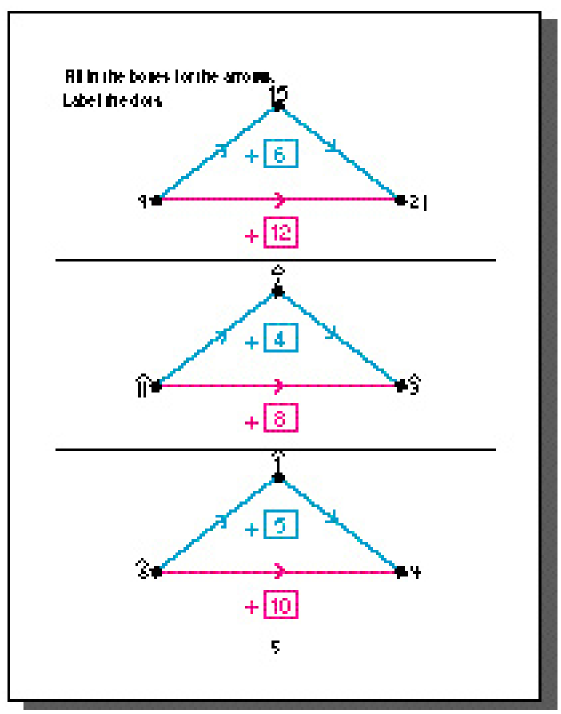
Assessment Activity

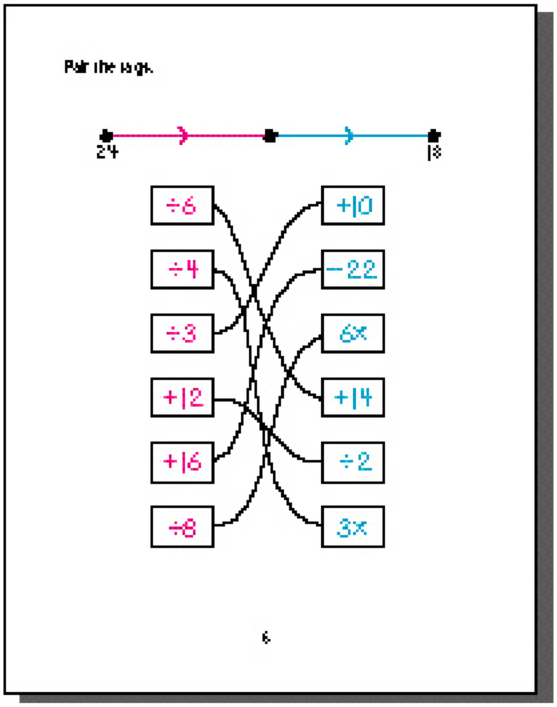
An individual student progress record for the workbook is available on Blackline W10. You may like to use this form to monitor student work.

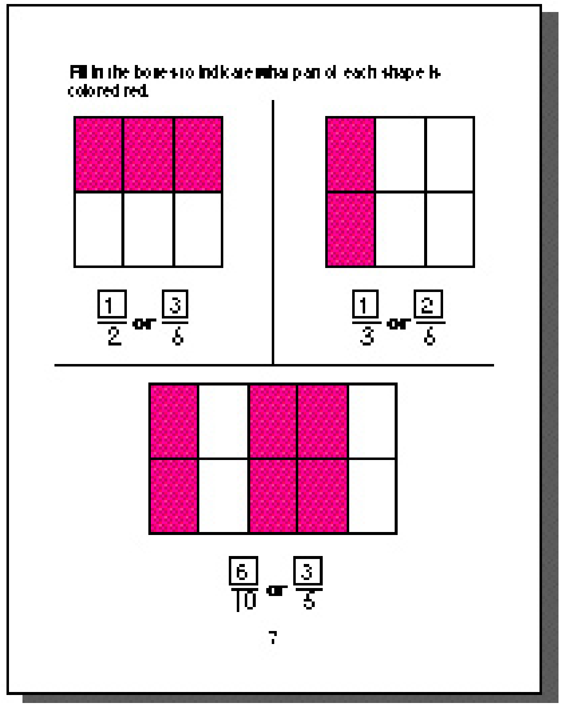

Put each of these numbers in the string picture.
3 4 10 12
Positive divisors of 20
Less than 7
10
4
3
12
Put each of these numbers in the string picture.
5 12 13 18
Greater than 10
Multiples of 3
18
12
13
2

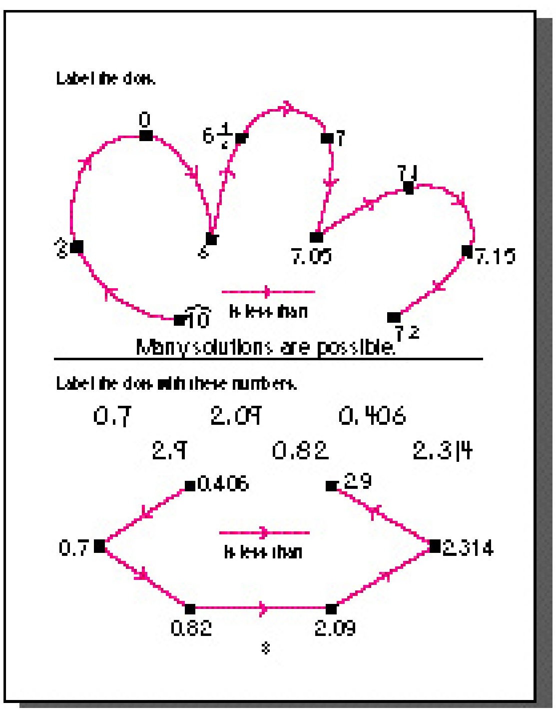

Label each red arrow. Try to label some arrows in two ways. One is done for you. Many labels are possible.
2
6
+4
24
40
100
+16
+32
−12
−18
−80
8
+94
Many solutions are possible.
3

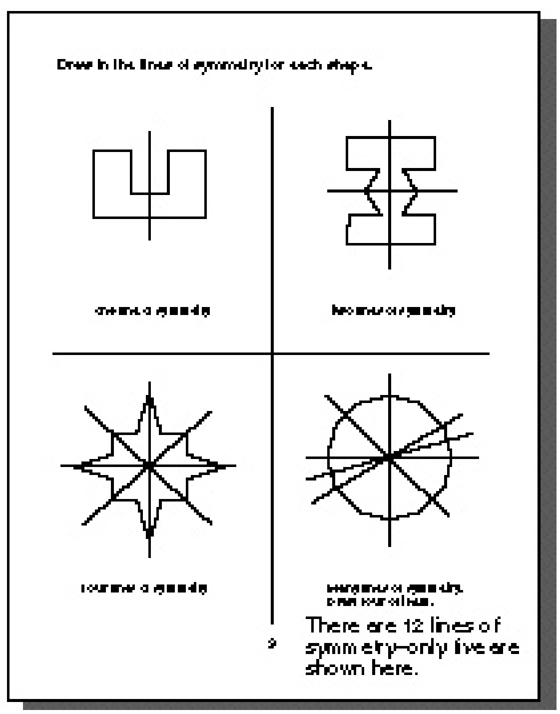

Peu is a secret number.
Clue 1
Peu can be shown on these Minicomputer boards by taking off exactly one checker.
Peu could be 79, 72, 70, 60, or 40.
Clue 2
Multiples of 10
Even numbers
Peu
Who is Peu? 72
4

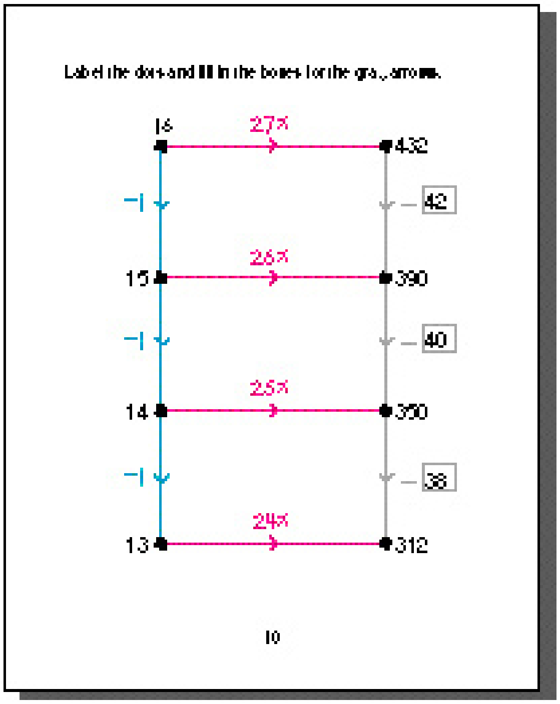

Fill in the boxes for the arrows.
Label the dots.
15
+ 6
9
21
+ 12
+ 4
+ 8
+ 5
+ 10
4
5


Pair the tags.
24
18
÷6
÷4
÷3
+12
+16
÷8
+10
−22
6x
+14
÷2
3x
6

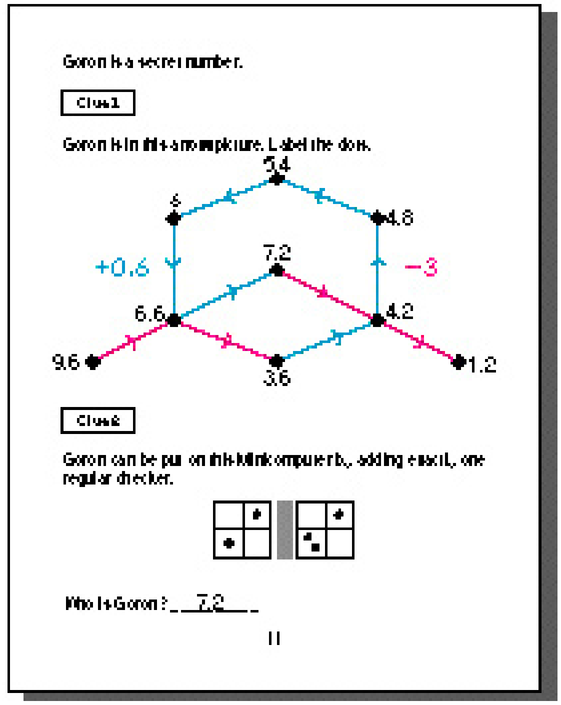

Fill in the boxes to indicate what part of each shape is colored red.
1/2 or 3/6
1/3 or 2/6
6/10 or 3/5
7


Label the dots.
0
6 1/2
7
7.1
3
6
7.05
7.15
10
is less than
7.2
Many solutions are possible.
Label the dots with these numbers.
0.7
2.09
0.406
2.9
0.82
2.314
0.406
2.9
0.7
is less than
2.314
0.82
2.09
8


Draw in the lines of symmetry for each shape.
9
There are 12 lines of symmetry—only five are shown here.


Label the dots and fill in the boxes for the gray arrows.

10


Goron is a secret number.

Clue 1

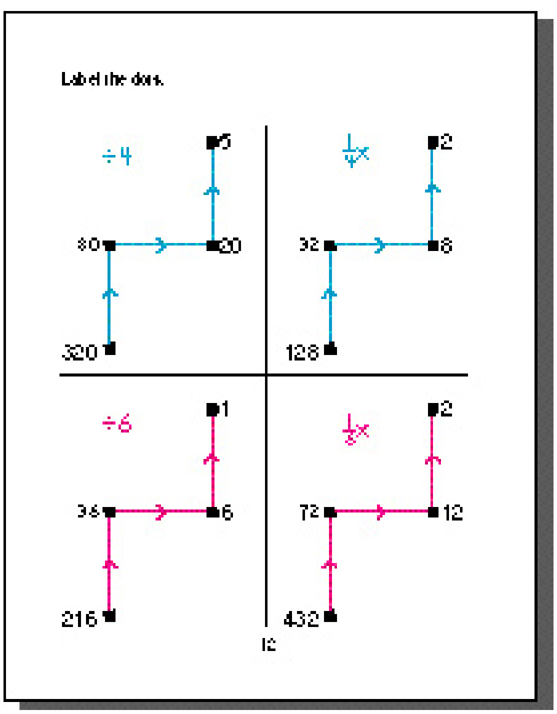
Goron is in this arrow picture. Label the dots.

Clue 2

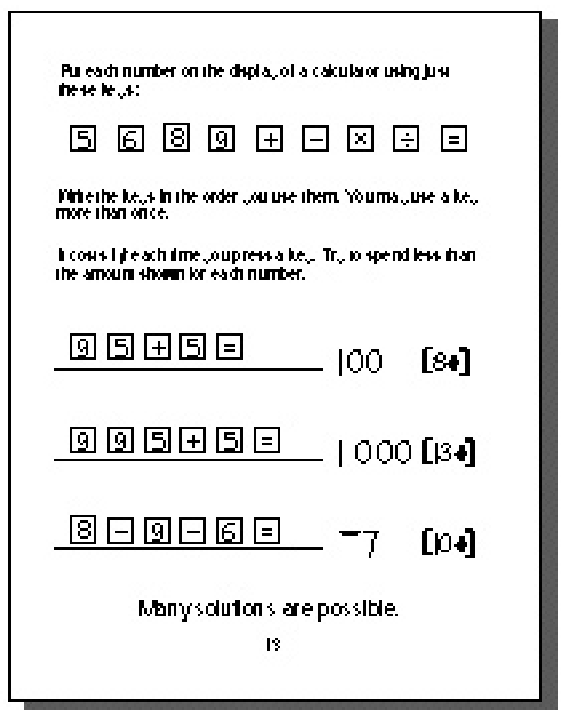
Goron can be put on this Minicomputer by adding exactly one regular checker.

Who is Goron? 7.2

11


Label the dots.

12


Put each number on the display of a calculator using just these keys:

5 6 8 9 + − × ÷ =

Write the keys in the order you use them. You may use a key more than once.

It costs 1¢ each time you press a key. Try to spend less than the amount shown for each number.

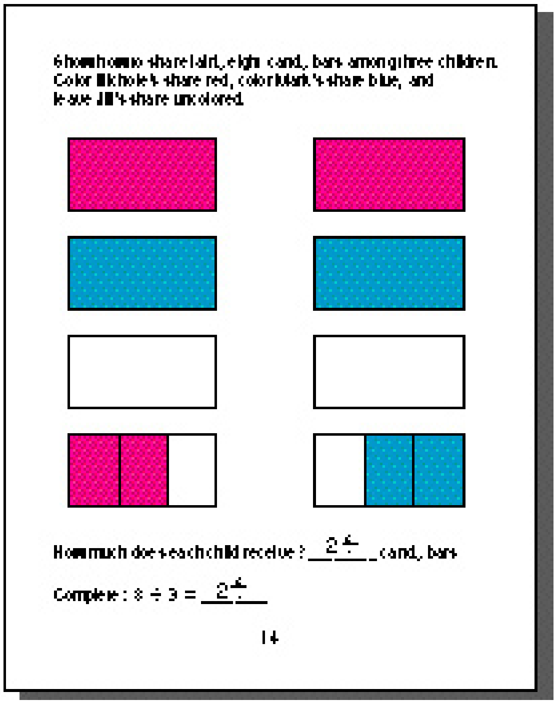
9 5 + 5 = 100 [8¢]

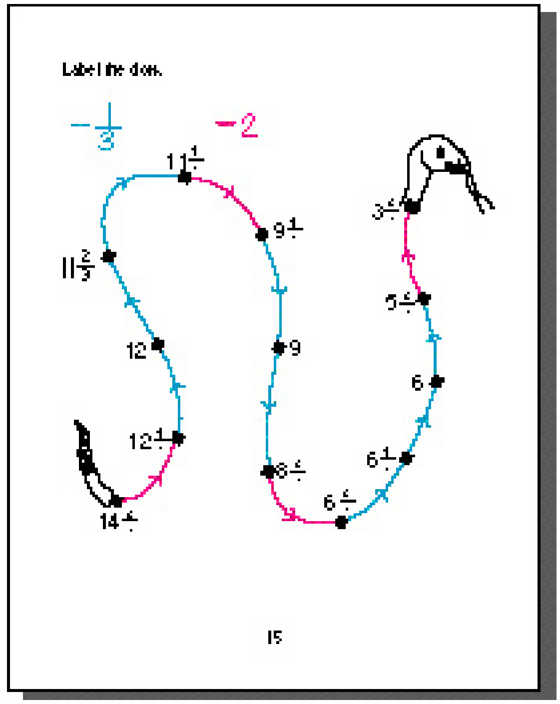
9 9 5 + 5 = 1000 [13¢]

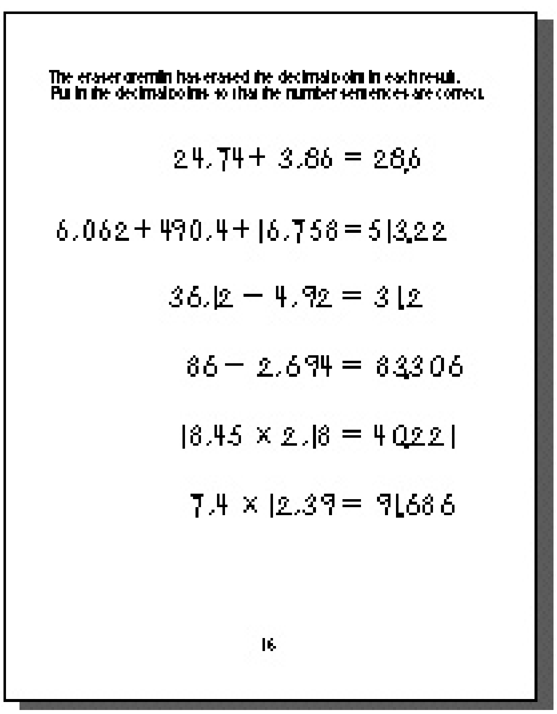
8 − 9 − 6 = ⁻7 [10¢]

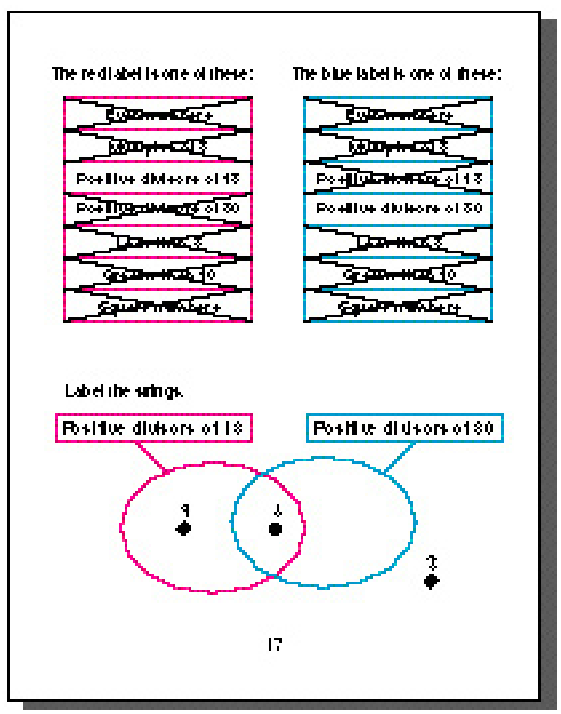
Many solutions are possible.

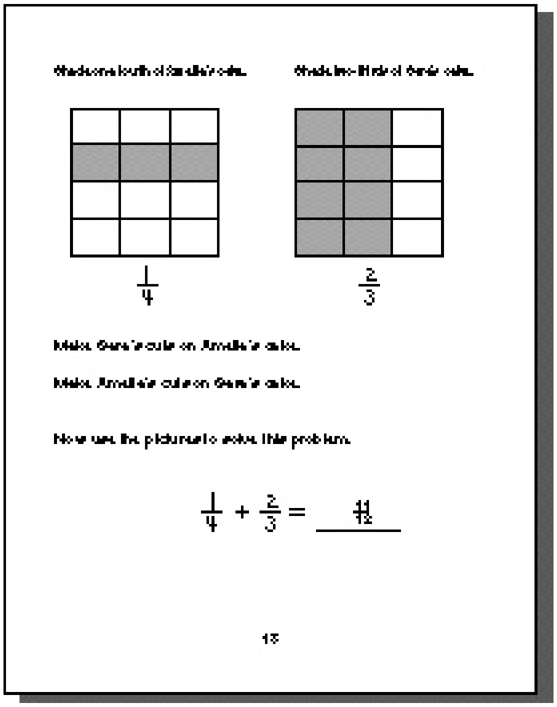
13


Show how to share fairly eight candy bars among three children.
Color Nichole's share red, color Mark's share blue, and
leave Jill's share uncolored.
How much does each child receive? $2\frac{2}{3}$ candy bars
Complete: 8 ÷ 3 = $2\frac{2}{3}$
14


Label the dots.
$-\frac{1}{3}$
-2
$11\frac{1}{3}$
$9\frac{1}{3}$
$3\frac{2}{3}$
$11\frac{2}{3}$
$5\frac{2}{3}$
12
9
6
$12\frac{1}{3}$
$8\frac{2}{3}$
$6\frac{1}{3}$
$6\frac{2}{3}$
$14\frac{2}{3}$
15


The eraser gremlin has erased the decimal point in each result.
Put in the decimal points so that the number sentences are correct.
24.74 + 3.86 = 28.6
6.062 + 490.4 + 16.758 = 513.22
36.12 − 4.92 = 31.2
86 − 2.694 = 83.306
18.45 × 2.18 = 40.221
7.4 × 12.39 = 91.686
16


The red label is one of these:
The blue label is one of these:
Positive divisors of 18
Positive divisors of 30
Label the strings.
Positive divisors of 18
Positive divisors of 30
9
6
3
17



Shade one fourth of Amelia's cake.

Shade two-thirds of Gene's cake.

$\frac{1}{4}$ $\frac{2}{3}$

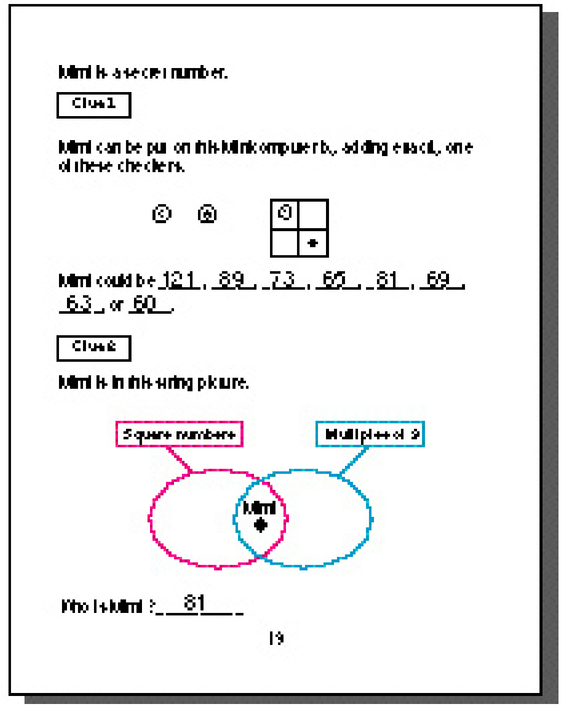
Make Gene's cut on Amelia's cake.

Make Amelia's cut on Gene's cake.

Now use the pictures to solve this problem.

$\frac{1}{4} + \frac{2}{3} = \underline{\frac{11}{12}}$

18



Mimi is a secret number.

Clue 1

Mimi can be put on this Minicomputer by adding exactly one of these checkers.

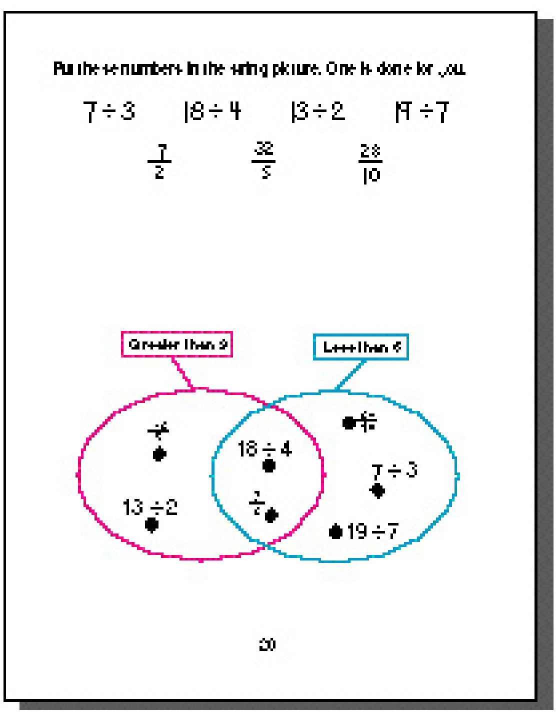
Mimi could be 121, 89, 73, 65, 81, 69, 63, or 60.

Clue 2

Mimi is in this string picture.

Who is Mimi? 81

19

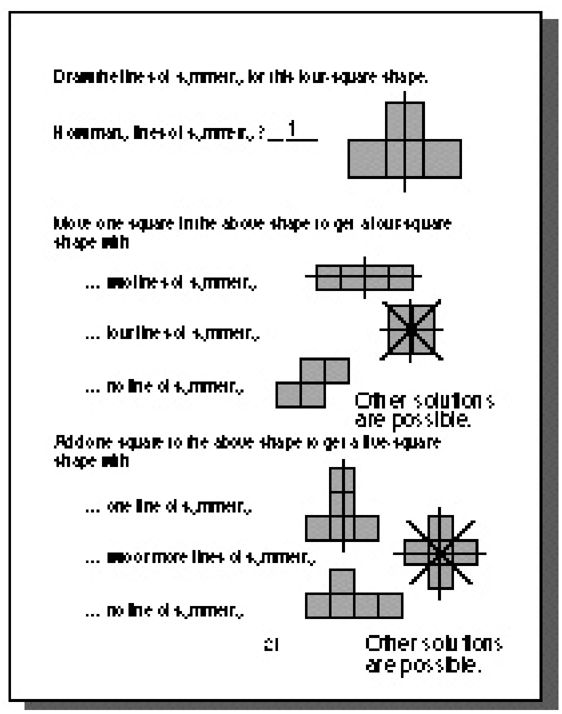


Put these numbers in the string picture. One is done for you.

$7 \div 3$ $18 \div 4$ $13 \div 2$ $19 \div 7$

$\frac{7}{2}$ $\frac{32}{5}$ $\frac{28}{10}$

20



Draw the lines of symmetry for this four-square shape.

How many lines of symmetry? 1

Move one square in the above shape to get a four-square shape with

... two lines of symmetry.

... four lines of symmetry.

... no line of symmetry.

Other solutions are possible.

Add one square to the above shape to get a five-square shape with

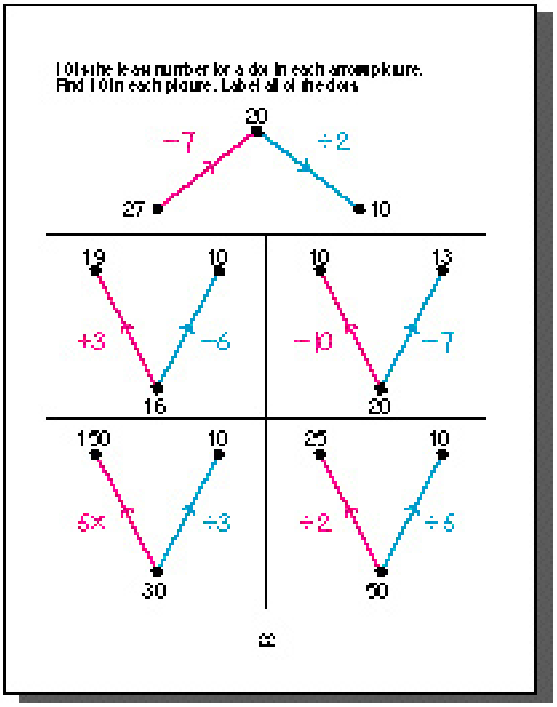
... one line of symmetry.

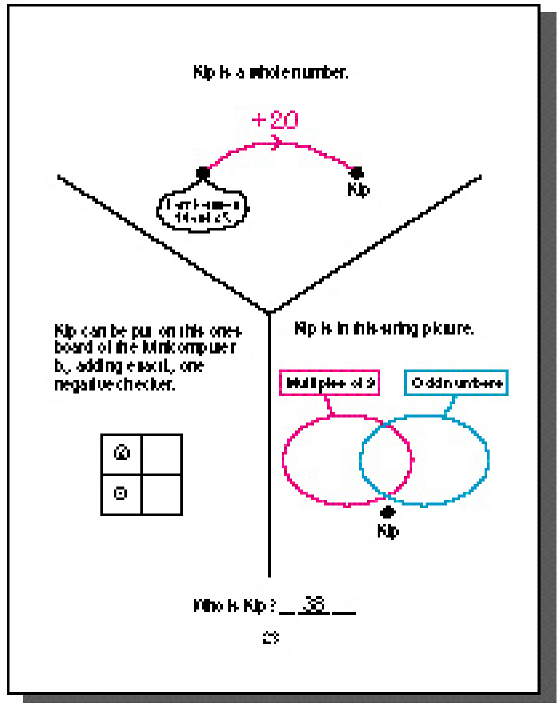
... two or more lines of symmetry.

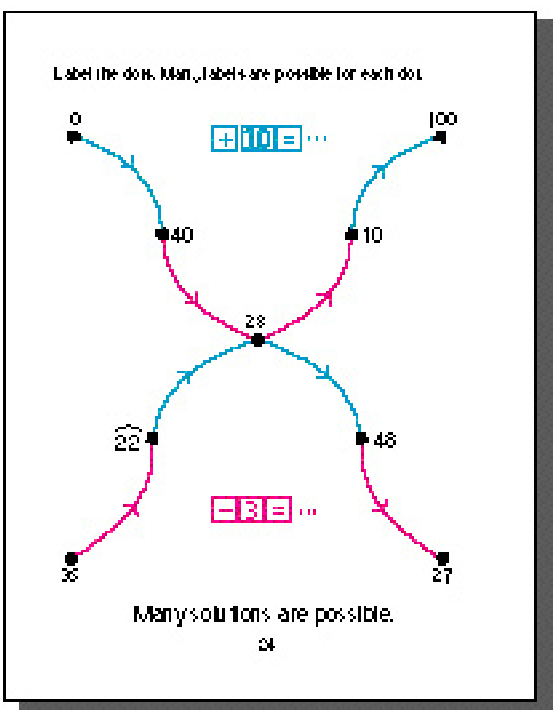
... no line of symmetry.

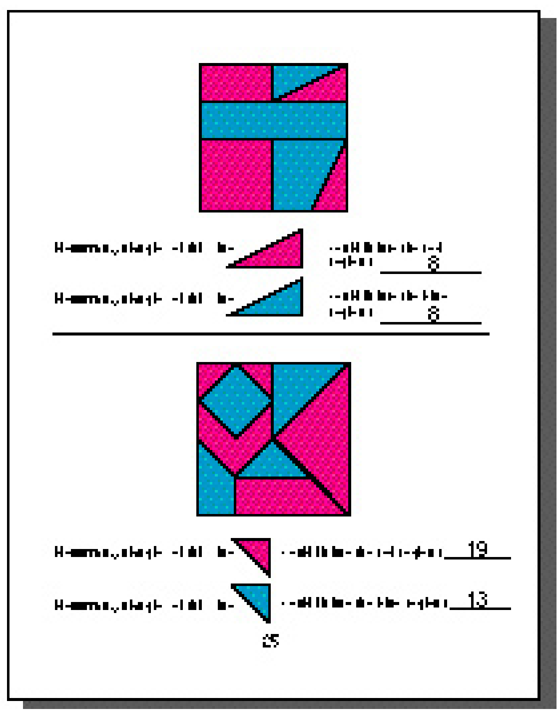
21

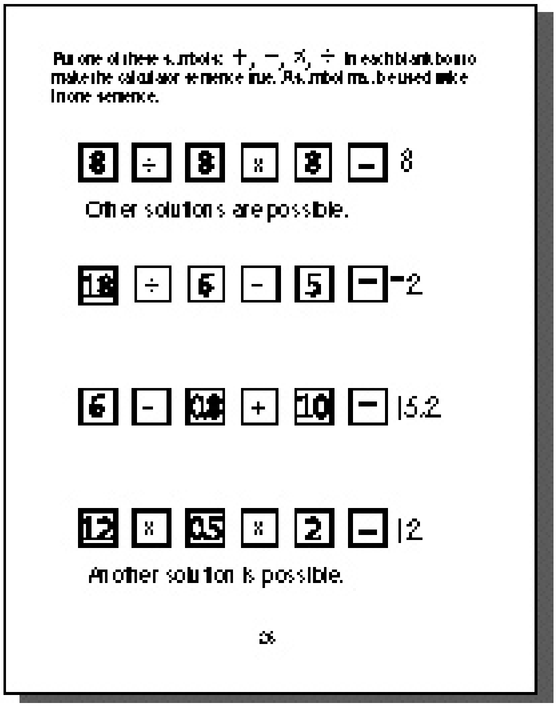
Other solutions are possible.


10 is the least number for a dot in each arrow picture.
Find 10 in each picture. Label all of the dots.
20
−7
÷2
27
10
19
10
+3
−6
16
10
13
−10
−7
20
150
10
5×
÷3
30
25
10
÷2
÷5
50
22


Kip is a whole number.
+20
Kip
Kip can be put on this one-board of the Minicomputer by adding exactly one negative checker.
Kip is in this string picture.
Multiples of 3
Odd numbers
Kip
Who is Kip? 38
23


Label the dots. Many labels are possible for each dot.
0
100
+10=...
40
10
28
22
48
−3=...
27
Many solutions are possible.
24


8
8
19
13
25



Put one of these symbols: +, −, ×, ÷ in each blank box to make the calculator sentence true. A symbol may be used twice in one sentence.

8 ÷ 8 × 8 = 8

Other solutions are possible.

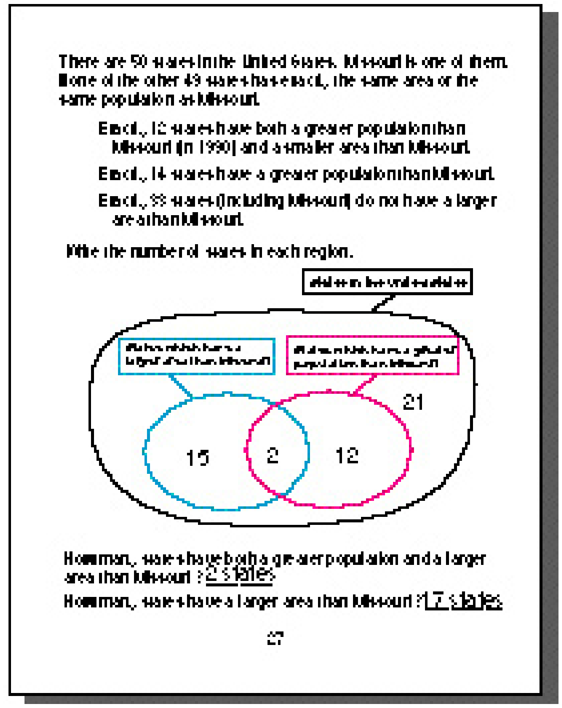
18 ÷ 6 − 5 = ⁻2

6 − 0.8 + 10 = 15.2

12 × 0.5 × 2 = 12

Another solution is possible.

26



There are 50 states in the United States. Missouri is one of them. None of the other 49 states has exactly the same area or the same population as Missouri.

Exactly 12 states have both a greater population than Missouri (in 1990) and a smaller area than Missouri.

Exactly 14 states have a greater population than Missouri.

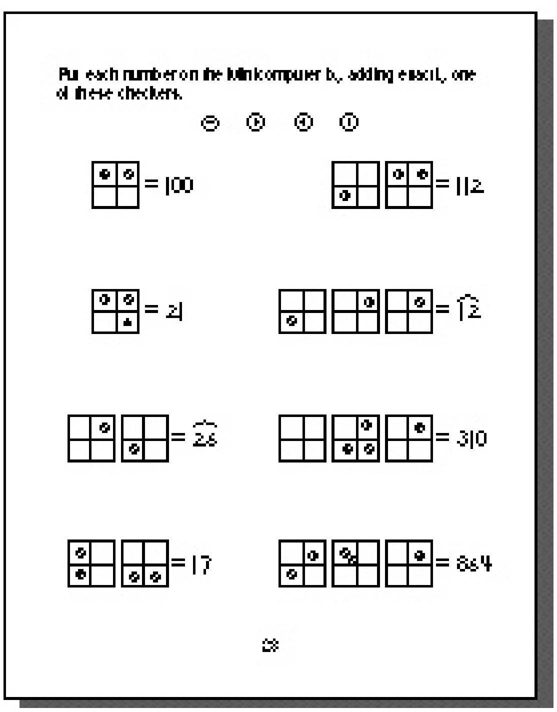
Exactly 33 states (including Missouri) do not have a larger area than Missouri.

Write the number of states in each region.

How many states have both a greater population and a larger area than Missouri? 2 states

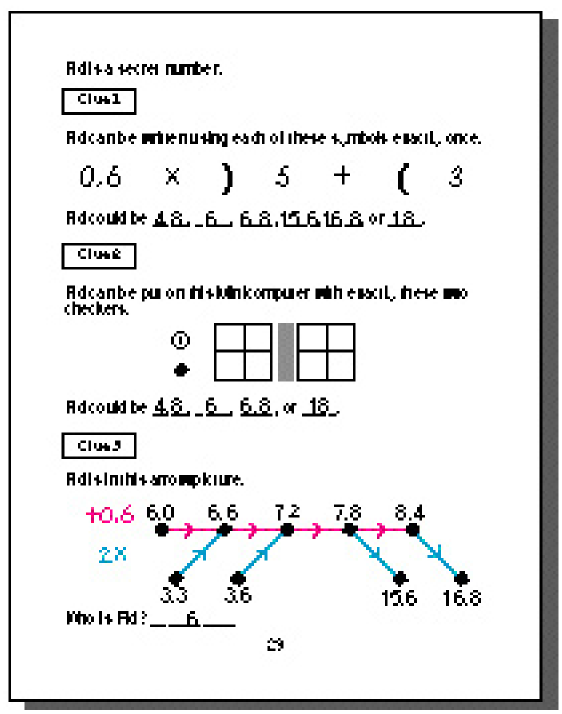
How many states have a larger area than Missouri? 17 states

27



Put each number on the Minicomputer by adding exactly one of these checkers.

28



Fid is a secret number.

Clue 1

Fid can be written using each of these symbols exactly once.

0.6 ×) 5 + (3

Fid could be 4.8, 6, 6.8, 15.6, 16.8 or 18.

Clue 2

Fid can be put on this Minicomputer with exactly these two checkers.

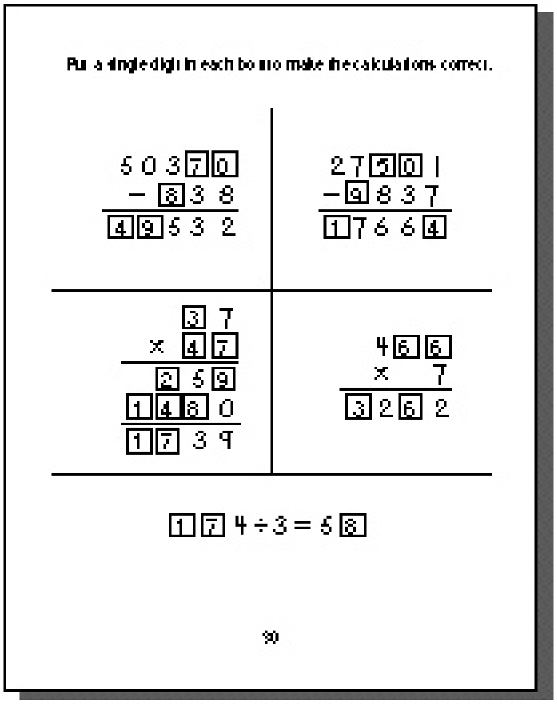
Fid could be 4.8, 6, 6.8, or 18.

Clue 3

Fid is in this arrow picture.

Who is Fid? 6

29

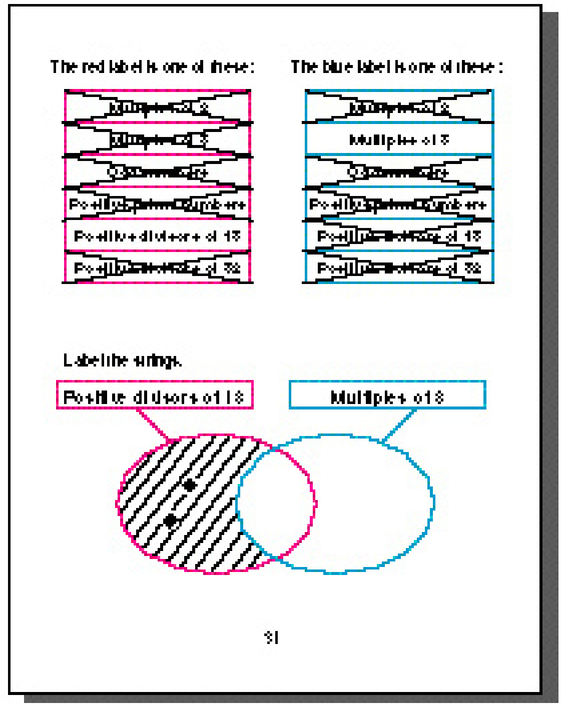

Put a single digit in each box to make the calculations correct.

$$\begin{array}{r} 503\boxed{7}\boxed{0} \\ -\ \boxed{8}38 \\ \hline \boxed{4}\boxed{9}532 \end{array} \qquad \begin{array}{r} 27\boxed{5}\boxed{0}1 \\ -\boxed{9}837 \\ \hline \boxed{1}766\boxed{4} \end{array}$$

$$\begin{array}{r} \boxed{3}7 \\ \times\ \boxed{4}\boxed{7} \\ \hline \boxed{2}5\boxed{9} \\ \boxed{1}\boxed{4}\boxed{8}0 \\ \hline \boxed{1}\boxed{7}39 \end{array} \qquad \begin{array}{r} 4\boxed{6}\boxed{6} \\ \times\ 7 \\ \hline \boxed{3}2\boxed{6}2 \end{array}$$

$$\boxed{1}\boxed{7}4 \div 3 = 6\boxed{8}$$

90


The red label is one of these:

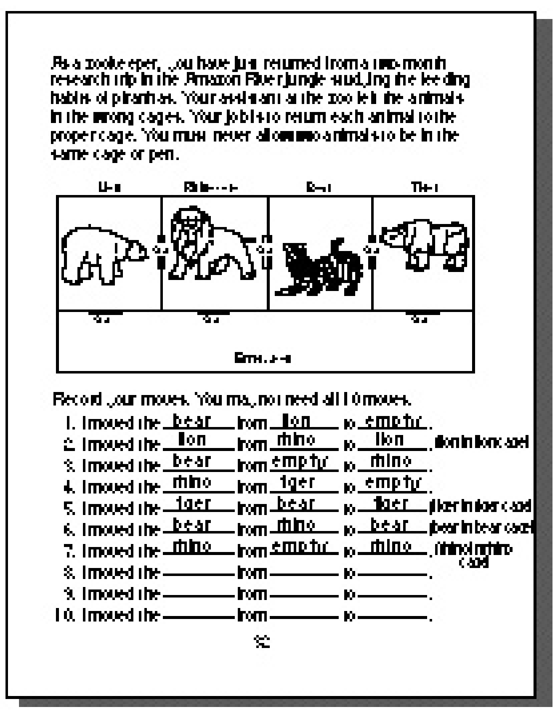
The blue label is one of these:

Multiples of 3

Positive divisors of 18

Label the strings.

91


As a zookeeper, you have just returned from a two-month research trip in the Amazon River jungle studying the feeding habits of piranhas. Your assistant at the zoo left the animals in the wrong cages. Your job is to return each animal to the proper cage. You must never allow two animals to be in the same cage or pen.

Record your moves. You may not need all 10 moves.

1. I moved the bear from lion to empty.
2. I moved the lion from rhino to lion. lion in lion cage
3. I moved the bear from empty to rhino.
4. I moved the rhino from tiger to empty.
5. I moved the tiger from bear to tiger. tiger in tiger cage
6. I moved the bear from rhino to bear. bear in bear cage
7. I moved the rhino from empty to rhino. rhino in rhino cage
8. I moved the ______ from ______ to ______.
9. I moved the ______ from ______ to ______.
10. I moved the ______ from ______ to ______.

92

Capsule Lesson Summary

As part of a detective story about a stolen diamond medallion, count the number of shortest routes along streets between two points of a city. In the process, construct part of Pascal's Triangle.

Materials

Teacher
- Overhead projector
- Grid map transparency or *IG-IV Workbook Poster #2*
- Tape
- Colored markers
- Blackline W11

Student
- Grid map
- Colored pencils, pens, or crayons
- Calculator

Advance Preparation: This lesson is described with use of an overhead projector and a transparency made from Blackline W11. *IG-IV Workbook Poster #2* may replace the transparency, but you may want to laminate a copy or to have a replacement copy for Exercise 2. Use Blackline W11 to make copies of the grid map for students

Description of Lesson

Note: Two lessons are based on the storybook *The Hidden Treasure*. In these lessons, we suggest you tell the story and present the problems. Then, after the problems have been solved, we suggest you distribute the storybooks. By reading the storybook at that point, students review the methods used in class to solve the problem. You may wish to read the storybook yourself before presenting the two lessons.

Exercise 1

Display a copy of the grid map on Blackline W11 or the *IG-IV Workbook Poster #2*. Distribute copies of the grid map for student use.

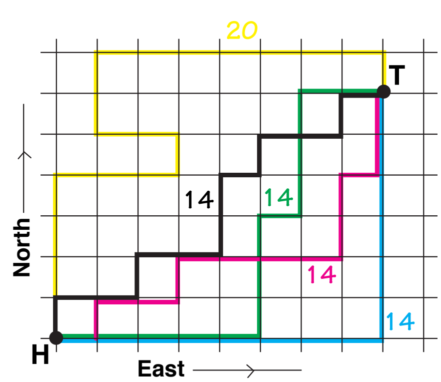
T: ***This is the street map of a town. Here at* H *is the house of a famous detective, Spike. Thieves recently stole a diamond medallion from his house. He has a lead and thinks that the thieves hid the medallion at* T*. Spike needs better evidence, so he decides to look for clues by exploring the routes the thieves might have taken from* H *to* T*. Spike knows that the thieves used a getaway car and, therefore, that they stayed on the streets. Who can trace a route that the thieves might have taken?***

Let several students trace and draw routes from **H** to **T**.

Tell students to draw three routes from **H** to **T** on their worksheets and to find the length of each route in blocks.

T: ***What is the length of a shortest route from* H *to* T*?***
(14 blocks)

Count the lengths of the routes already drawn on the board. Then invite students to draw several additional routes of length 14 from **H** to **T**. For example:



T: ***Did anyone find a route from* H *to* T *that is longer than 14 blocks?***

Invite students to draw one or two such routes on the board.

T: ***What do you notice about routes longer than 14 blocks? How can you tell, without counting, that the length of a route is more than 14 blocks?***

S: ***A route is longer than 14 blocks if it goes farther north or farther east than* T.**

S: ***A route is longer than 14 blocks if it ever goes west or south (moves away from* T.)**

Conclude that when Spike travels only north and east from **H** to **T**, his route will be exactly 14 blocks long.

T: ***Spike assumes that the thieves took a shortest route from* H *to* T*; that is, a route of length 14. He plans to explore all such routes. We found a few of these routes already. How many possible routes do you think there are for Spike to explore?***

Accept students' estimates, and record them on the board.

Let the class discuss methods for counting all the shortest routes from **H** to **T**. You may follow one or two suggestions, such as making a list or a tree diagram, until these methods become too complicated. Lead the class to consider a method involving first trying to solve a simpler but similar problem.

T: ***Often in mathematics it is a good strategy to first solve some similar but easier problems. Let's try to solve some simpler problems; then we can use their solutions to solve the original problem.***

Exercise 2

Display a clean copy of the grid map, and provide students with a clean copy as well. Refer to intersection points closer to **H**.

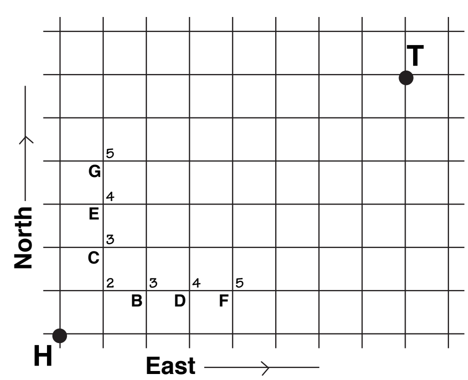
T: ***Before counting the number of routes from* H *to* T*, let's count the number of routes from* H *to some intersections closer to* H*. How many different routes are there from* H *to this point (diagonally opposite* H*).***

S: ***Two.***

Invite a student to trace the two routes, and put 2 near that corner.

In a similar manner, ask students to find the number of routes there are

- from **H** to **B** (Three)
- from **H** to **C** (Three)
- from **H** to **D** (Four)
- from **H** to **E** (Four)
- from **H** to **F** (Five)
- from **H** to **G** (Five)



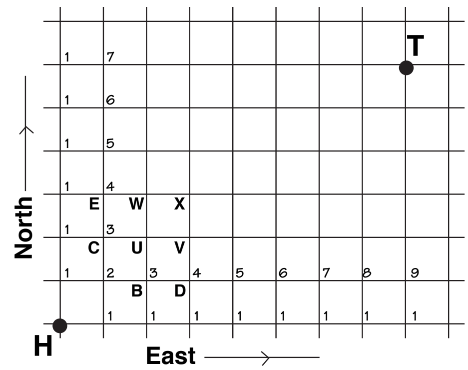
Do not write the letters **B**, **C**, **D**, **E**, **F**, and **G** on the board. They are here just to make the description of the lesson easier to follow.

Record the answers on the grid.

T: ***What patterns do you notice?***

Encourage students to identify and explain both the symmetry and the counting (2, 3, 4, 5) patterns. Extend the counting pattern by putting 6, 7, 8, and 9 by the appropriate corners.

Also note that there is only one shortest route to each intersection directly north or directly east of **H**. As you label the intersections at the board, direct students to copy the numbers on their second copy of the grid.



Do not write the letters **B**, **C**, **D**, **E**, **U**, **V**, **W**, and **X** on the board. They are here just to make the description of the lesson easier to follow.

T (pointing to **U**)**:** ***How many routes are there from* H *to this corner?***

S: ***Six.***

There may be disagreement, so invite students to defend their answers by tracing all of the routes from **H** to **U**. Encourage students to be systematic so as to find all six routes without duplication. Put 6 near the corner.

T: ***Can anyone convince me that there are six routes from* H *to this corner (*U*) without having to trace the routes?***

Perhaps some students will notice that all of the routes from **H** to **U** must pass through **B** to **C**. Since there are three routes from **H** to **U** via **B** and three routes from **H** to **U** via **C**, there are six routes in all from **H** to **U**. If this explanation is not offered by students, use the following dialogue to lead them to this observation. The dialogue uses letters to indicate where you or students point on the grid.

T: ***How many roads from* H *to* U *pass through* B*?***

S: ***Three, because there are three routes from* H *to* B.**

T: ***How many routes from* H *to* U *pass through* C*?***

S: ***Three, because there are three routes from* H *to* C.**

T: ***Are there any routes from* H *to* U *that do not pass through either* B *or* C*?***

S: ***No.***

T: ***So there are six routes from* H *to* U*: the three routes through* B *and the three routes through* C.**

Direct students to work on their own or with partners to find the number of routes from **H** to **V** and from **H** to **W**. Check that before they start they copy all of the numbers from the board onto their grids.

It is likely that some students will attempt to trace the routes and that they will have difficulty doing this. Suggest that they try to use the numbers of routes at corners **U** and **D**, or at **U** and **E**.

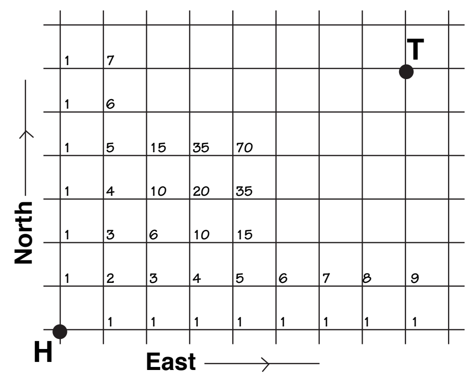
T: ***It's difficult to trace the routes and to be sure that all are counted but none are duplicated. Did anyone use a different method?***

S: ***All routes from* H *to* V *must pass through either* D *or* U*. There are four routes from* H *to* D *and six routes from* H *to* U*. Therefore, there are ten routes from* H *to* V *because 4 + 6 = 10.***

In a similar manner, lead the class to discover that there are ten routes from **H** to **W** (10 = 6 + 4) and there are 20 routes from **H** to **X** (20 = 10 + 10).

T: ***Why are there ten routes from* H *to* V *and also ten routes from* H *to* W*?***

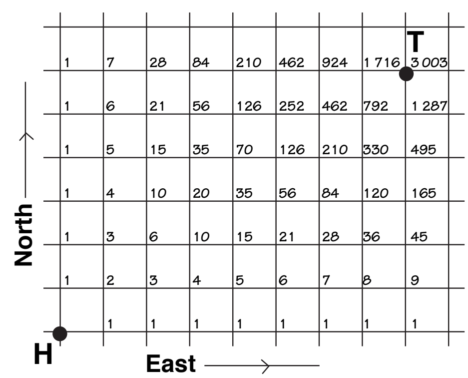
S: ***Because* V *is three blocks east and two blocks north of* H*, while* W *is three blocks north and two blocks east of* H*.***



As necessary, discuss other corners until students easily use the pattern. A solutions for part of the map is shown here.

At this time you may ask students if they wish to revise their estimates of the number of routes from **H** to **T**. Many of the original estimates may have been too low. Record new estimates.

Direct students to work individually or with partners to use the pattern discovered to find the number or routes from **H** to **T**. This will involve finding the number of routes to each intersection between **H** and **T**. Some students may want to use calculators to speed up the process.



After a few minutes, invite students to put correct numbers of routes on the grid on the board so as to gradually find the number of routes from **H** to each corner. The complete solution is shown here. The class should find that there are 3 003 routes from **H** to **T**. Compare the solution (3 003) to students' estimates.

T: ***If* T *were five blocks east and four blocks north of* H*, how many shortest routes would Spike have to explore?***

S: ***Only 126.***

Let a student find the intersection five blocks east and four blocks north of **H** to check that there are 126 shortest routes to that point.

In a similar manner, ask the number of routes to a few other intersections. Emphasize the following:

- that the class has found the number of routes not only from **H** to **T**, but also from **H** to many other intersections; and
- the number of routes from **H** to an intersection increases quickly as you get closer to **T**.

Note: You may wish to mention that the array of numbers is part of Pascal's Triangle. Often Pascal's Triangle has the numbers arranged in this format:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
⋮

Pascal lived in the sixteenth century in France. He used this array of numbers to solve many problems in algebra, probability, and combinatorics. This arrangement of numbers was discovered many centuries earlier, but Pascal popularized it by writing a treatise on its patterns and its applications. Pascal's Triangle is still widely used in the above fields of mathematics. *CSMP* students will encounter it again in sixth grade.

Capsule Lesson Summary

Review the story about a stolen diamond medallion and the problem concerning the number of routes between two locations in a city. Develop a 0–1 binary code for recording routes on a grid of streets. Use the code to determine the number of six-element subsets in a fourteen-element set.

Materials

Teacher
- Grid with numbers transparency or *IG-IV Workbook Poster #3*
- Colored chalk
- Blackline W12

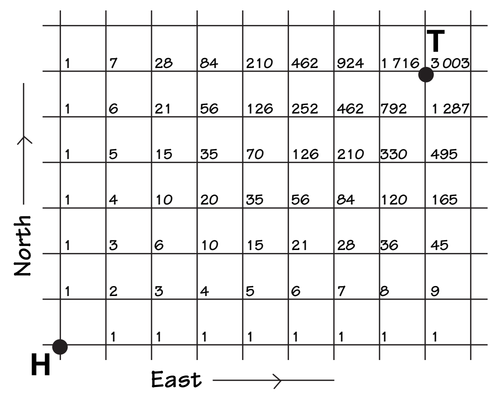
Student
- Grid with numbers
- Paper
- Colored pencils, pens, or crayons

Advance Preparation: Use Blackline W12 to make a transparency of the grid with numbers of shortest routes, or use *IG-IV Workbook Poster #3*. Make copies of the grid with numbers for students.

Description of Lesson

Exercise 1

Display the grid with numbers of shortest routes from **H** to each intersection as generated in Lesson W11.



Ask a student to recall the story from last week about Spike the detective who decided to search all of the shortest routes from his house, **H**, to the stolen treasure, **T**. Review briefly how it was determined that there were 3 003 such routes.

Point to an intersection.

T: ***What does this number tell us?***

S: ***There are that number of different shortest routes from* H *to that intersection.***

T: ***What patterns do you notice in this grid of numbers?***

Allow several minutes for students to locate and describe the patterns they observe, for example:

- 1, 1, 1, … in the first row and first column
- 1, 2, 3, … in the second row and second column
- +2 +3 +4 +5
  1, 3, 6, 10, 15 … in the third row and third column
- the basic additive pattern for labeling intersections that was discovered in the previous lesson.
- a line of symmetry along the diagonal with intersections has 2, 6, 20, 70, 252, and 924 shortest routes. That is, a point three blocks east and five blocks north has the same number of shortest routes as a point five blocks east and three blocks north of **H**.

Point to the intersection seven blocks east and four blocks north of **H**.

T: ***Where would 330 occur again if we extended the grid; i.e., where is the other corner with 330 shortest routes from H?***

S: ***Four blocks east and seven blocks north.***

Repeat this question a couple times with another point off the diagonal and with one on the diagonal to emphasize the last pattern noted above.

Exercise 2

Write this code word on the board.

1 0 1 1 0 0 0 0 1 1 1 0 0 0

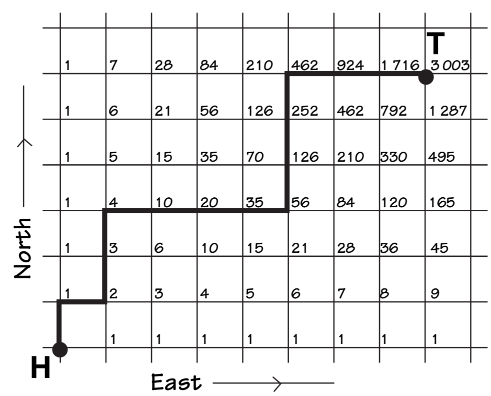
T: ***Spike travels from his house at H to the treasure at T many times. He decides to write a secret code in his notebook for each route he takes. One day Spike drives from H to T and writes 10110000111000 in his notebook.***

Can anyone guess Spike's secret rule for writing code words? If you know the secret, keep it to yourself for the moment. Which route from H to T do you think Spike travels when he writes this code word?

Ask a student to trace a route but not to explain the rule yet. As soon as the student deviates from the correct path, let another student try. Spike is using two numbers, **1** and **0**, for the two directions, north and east, that he travels to go from **H** to **T**. Some students may suspect this coding but not know which number is for which direction. Spike uses **0** to indicate that he goes one block north and **0** to indicate that he goes one block east.

The correct route is shown here.

Continue until a student traces the correct route. You may need to help the class finds this route. If necessary, trace the first few blocks of the route as you read the corresponding numbers of the code word. Then ask a student to complete the route.



Repeat this exercise by writing these code words on the board and asking a student to trace the route for each code word.

1 1 0 0 0 0 1 0 1 0 1 1 0 0 0 0
0 1 0 0 0 1 0 1 0 1 1 1

As you slowly trace a route from **H** to **T**, invite a student to write the appropriate code word. Check the student's answer. Repeat the activity one or two more times.

T: ***Who can explain the code?***

S: ***Each 1 in the code word means to go one block north. Each 0 means to go one block east.***

T: ***Who can write a code word for another route from H to T?***

Let a student write a code word on the board, for example, **11000110010001**.

T: ***Do you think that this code word represents a route from* H *to* T*? Who can check it by tracing the route?***

Let another student start at **H** and trace the route indicated by the code word. Check whether the route ends at **T**. Repeat this activity one or two more times.

T: ***How can we check whether a code word represents a route from* H *to* T*?***

S: ***Trace the route.***

T: ***Can we check it without tracing the route?***

S: ***Yes, each code word must have 14 digits.***

S: ***There must be exactly six* 1*s and eight* 0*s in each code word, because* T *is six blocks north and eight blocks east of* H*.***

Write these code words on the board.

1 0 1 0 1 0 1 0 1 0 1 0
0 0 0 0 1 1 1 1 0 0 0 0 1 1 1 1
0 1 1 0 1 1 0 1 1 0 1 0

T: ***Are each of these code words for a route from* H *to* T*?***

S: ***The first code word has only 12 digits. It needs two more* 0*s.***

T: ***The middle code word is for a route from* H *to* T *because it has six* 1*s and eight* 0*s.***

S: ***The last code word is not for a route from* H *to* T *because it has nine* 1*s and five* 0*s***

T: ***If we traced the route indicated by the second code word, where would we finish?***

S: ***Nine blocks north and five blocks east of* H*; it's off the grid on the poster.***

T: ***How many code words could we write with six* 1*s and eight* 0*s?***

If necessary, give hints to lead to the following response.

S: ***3 003; because there is one code word for each route from* H *to* T*, and we know that there are 3 003 routes from* H *to* T*.***

Students should observe that each code word with six **1**s and eight **0**s describes a different route from **H** to **T**. Also, for each route from **H** to **T**, there is exactly one such code word. There are 3 003 code words with exactly six **1**s and eight **0**s.

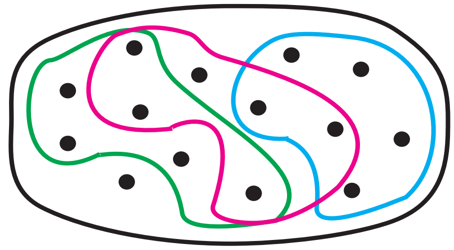
Exercise 3

On the board, draw a string with 14 dots inside it.

T: ***Spike learns that exactly six thieves stole the diamond medallion. He has 14 suspects and is sure that all six thieves are among his suspects. He feels that they will confess if he can interview all six thieves together. So he decides to interview the 14 suspects in groups of six.***

W 12

Draw a red string around six of the dots in the string picture. Then, invite students to draw strings around two different sets of six suspects. For example:

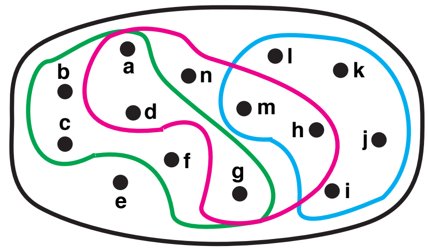


T: ***How many groups of six do you think there are for Spike to interview?***

Write students' estimates on the board. If many estimates are less than 10, let students draw a few more strings for groups of six suspects in order to suggest that there are many possible groups.

T: ***Spike decides to write a code word in his notebook for each group of six suspects that he interviews.***

First, label the dots with letters. Then write the code word for the red string on the board. Write a **1** for any dot inside the red string and a **0** for any dot outside the red string. Do not describe Spike's rule for writing code words yet; just write the word and let the class discover the rule.



T: ***Spike writes this code word in his notebook when he interviews the six suspects in the red string. Does anyone think they know his secret rule for writing code words? If you know his secret, keep it to yourself for the moment. On a piece of paper, write what you think the code word would be for the group of six suspects in the blue string.***

a	b	c	d	e	f	g	h	i	j	k	l	m	n
1	0	0	1	0	0	1	1	0	0	0	0	1	1

Check several students' papers before letting a student write the code word for the blue string on the board. Continue by asking for the code word for the green string.

a	b	c	d	e	f	g	h	i	j	k	l	m	n
1	0	0	1	0	0	1	1	0	0	0	0	1	1
0	0	0	0	0	0	0	1	1	1	1	1	1	0
1	1	1	1	0	1	1	0	0	0	0	0	0	0

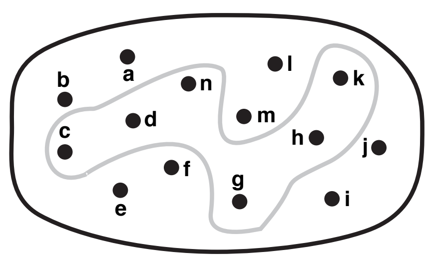
T: ***Who can write a code word for another group of six suspects?***

A student might suggest this:

a	b	c	d	e	f	g	h	i	j	k	l	m	n
0	0	1	1	0	0	1	1	0	0	1	0	0	1

T: ***Is this a correct code word for another group of six suspects?*** (Yes)

Invite a student to draw the string for the group. The string should include exactly the six dots for the six suspects marked with a **1**. This illustration shows a correct string for the preceding example.



T: ***Who can explain Spike's secret code?***

S: ***A 1 means that the suspect will be interviewed in this group of six.***
0 means that the suspect will not be interviewed with this group.

T: ***Without drawing a string, how can we tell if a code word represents a group of six suspects to be interviewed?***

S: ***The code word must have fourteen digits: six 1s and eight 0s.***

Erase all of the strings inside the original picture, leaving just the string with 14 dots inside the large white (black) string.

Invite a student to write another code word for a group of six suspects, and then ask another student to draw the corresponding string. Then reverse the problem by inviting one student to draw a string for six suspects and then asking another student to write the corresponding code word.

T: ***How many groups of six are there among the 14 suspects?***

S: ***3 003; since we know from the previous problem that there are 3 003 code words with exactly six 1s and eight 0s.***

S: ***Each road from H to T on Spike's map corresponds to a code word with six 1s and eight 0s. Each string for six suspects corresponds to a similar code word with six 1s and eight 0s. Since there are 3 003 roads from H to T, there are 3 003 groups of six suspects.***

S: ***The two problems have the same solution.***

Compare the answer (3 003) to the students' estimates.

Distribute copies of the storybook *The Hidden Treasure*. In the time remaining, let students read and discuss the story. Encourage students to read the storybook on their own if there is not time to finish it in class. The story reviews the two lessons about Spike, except that in the storybook the map of Spike's town is smaller (**T** is only six blocks east and four blocks north of **H**) and there are six thieves among only ten suspects. Thus, in the storybook there are 210 routes from **H** to **T** and there are 210 groups of six thieves among ten suspects. As the class reads the story, encourage students to comment on the similarities and the differences between the problem solved in class and the problem in the storybook.

Writing Activity

Suggest that students write about how the solution of one problem also solved a second problem in the storybook *The Hidden Treasure*.

Capsule Lesson Summary

Label the dots in an arrow road with +0.4 and –0.7 arrows. Notice the relationship between doubling sequences starting at 1, 0.1, and 0.01. Begin the workbook *Selection of Problems #5*. (This is the first of two lessons using this workbook.)

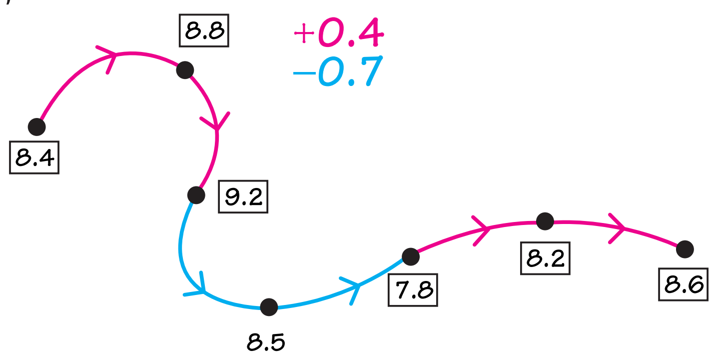
Materials

Teacher
- Colored chalk

Student
- *Selection of Problems #5* Workbook
- Calculator
- Colored pencils, pens, or crayons
- Metric ruler

Description of Lesson

Draw this arrow road on the board, labeling the dot 8.5. Ask the class to label the rest of the dots. (Answers are in boxes.)

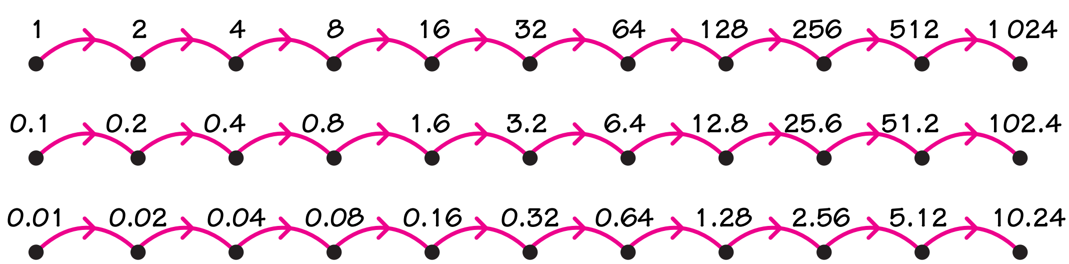


Erase the board and then draw an arrow road with ten 2x arrows. For this activity, use an order natural to the students' seating arrangement.

T: ***I'll start with 1. The first student will double 1, the second student will double that number, and so on. Let's begin. 1.***

Label the first dot in your arrow road, and then label other dots as students respond. Continue until ten students have responded.

Next begin with 0.1 and repeat the activity with the next ten students. Record the numerals on the board. Finally begin with 0.01 and repeat the activity with the next ten students.



W13

Distribute copies of the workbook *Selection of Problems #5* and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of the lesson, collect the workbooks for your review. They will be used again in Lesson W14.

W14 SELECTION OF PROBLEMS #5 (LESSON ______ W14

Capsule Lesson Summary

Discover the effect of reversing the numbers in a division problem. Review a division algorithm. Continue individual work in the workbook *Selection of Problems #5*. (This is the second of two lessons using this workbook.)

Materials

Teacher
- None

Student
- *Selection of Problems #5* Workbook
- Calculator
- Colored pencils, pens, or crayons
- Metric ruler

Description of Lesson

Pose the following problems to the class, and record number sentences about the solutions.

T: ***If we had to divide 6 candy bars among 3 people, how many would each person get?*** (2) ***If we had to divide 3 candy bars among 6 people, how much would each person get?***

$6 \div 3 = 2$
$3 \div 6 = \frac{1}{2}$

S: ***$\frac{1}{2}$; divide each candy bar into halves. There would be 6 halves; one for each of the 6 people.***

T: ***If we had to divide 8 candy bars between 2 people, how many would each person get?*** (4) ***If we had to divide 2 candy bars among 8 people, how much would each person get?***

$8 \div 2 = 4$
$2 \div 8 = \frac{1}{4}$

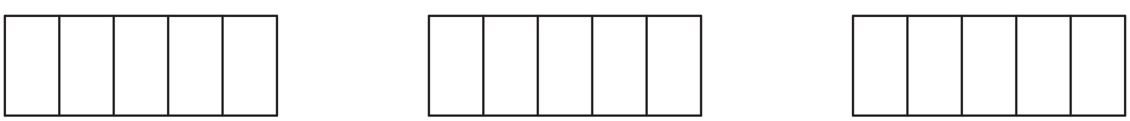
S: ***$\frac{1}{4}$; divide each candy bar into 4 pieces.***

T: ***If we had to divide 15 candy bars among 3 people, how many would each person get?*** (5) ***If we had to divide 3 candy bars among 15 people, how much would each person get?***

S: ***$\frac{1}{5}$; divide each candy bar into 5 pieces. Then there would be 15 pieces, one for each person.***

$15 \div 3 = 5$
$3 \div 15 = \frac{1}{5}$

You may wish to illustrate this last situation for the benefit of the class.



These number stories will be on the board.

Review division of whole numbers with a problem such as 1 873 ÷ 29. For example:

$$
\begin{array}{r|r}
\multicolumn{2}{r}{64\ R = 17} \\
29\overline{)1873} & \\
-870 & 30 \\
\hline
1003 & \\
-870 & 30 \\
\hline
133 & \\
-116 & 4 \\
\hline
17 &
\end{array}
$$

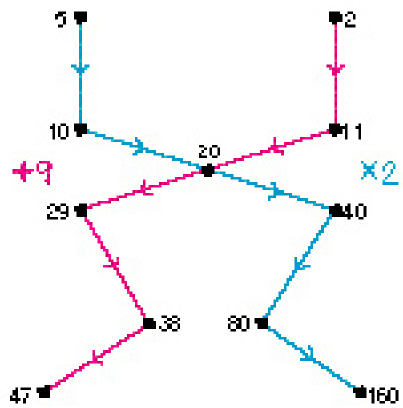
Distribute students' copies of the workbook *Selection of Problems #5*. Ask students first to correct or complete pages from the previous week's work and to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in their workbooks during a study time or to take them home as an assignment.

Assessment Activity

An individual student progress record for the workbook is available on Blackline W14. You may like to use this form to monitor student work.

Label the dots.



What is the greatest odd number in this picture? 47

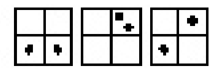
What is the greatest multiple of 5 in this picture? 160

2

Rum is a secret number.

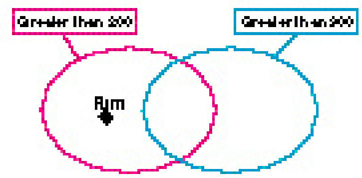
Clue 1

Rum can be shown on these Minicomputer boards by removing exactly one checker.



Rum could be 186, 286, 346, 382, or 384.

Clue 2



Who is Rum? 286

3

Put a single digit in each box to make the calculations correct.

$$\begin{array}{r} \boxed{4}\,6\,2\,8 \\ +\ \boxed{7}\,7\,\boxed{6} \\ \hline 5\,4\,\boxed{0}\,4 \end{array}$$

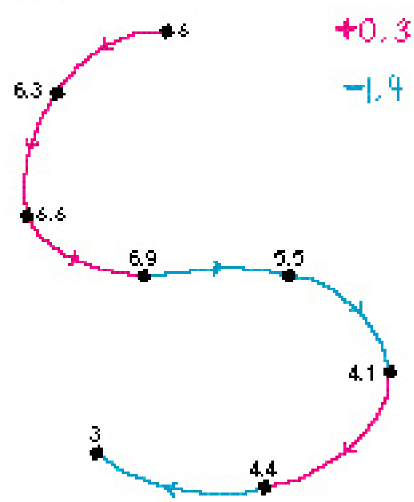
$$\begin{array}{r} \boxed{2}\,3\,6\,9 \\ 3\,9 \\ +\ \boxed{5}\,6\,\boxed{6} \\ \hline 2\,9\,\boxed{7}\,4 \end{array}$$

$$\begin{array}{r} 7\,\boxed{8}\,9 \\ -\ \boxed{3}\,6\,3 \\ \hline 4\,2\,\boxed{6} \end{array}$$

$$\begin{array}{r} \boxed{8}\,2\,\boxed{1} \\ -\ 2\,9\,5 \\ \hline 5\,\boxed{2}\,6 \end{array}$$

4

Label the dots.



5

Put each number on the display of a calculator using just these keys:

5 6 8 9 + − × ÷ =

Use the keys in the order you use them. You may use a key more than once.

It costs 1¢ each time you press a key. Try to spend less than the amount shown for each number.

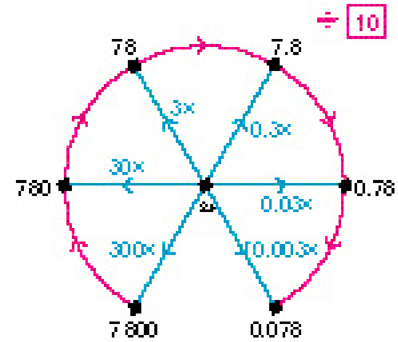
8 − 6 + 5 = 7 [9¢]

9 9 − 5 5 = 44 [10¢]

Other solutions are possible.

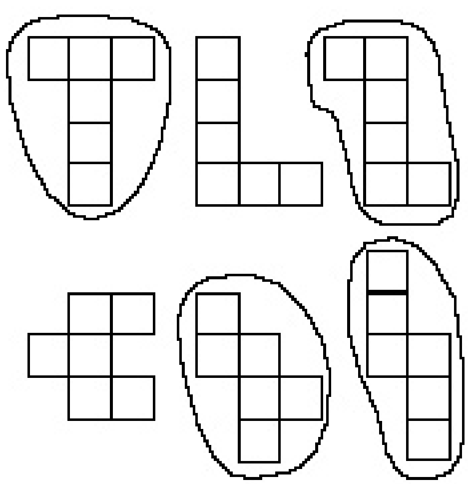
6

Label the dots and fill in the box for the red arrow.



7

Circle each shape that could be folded into a cube.



You should have circled four shapes.

8

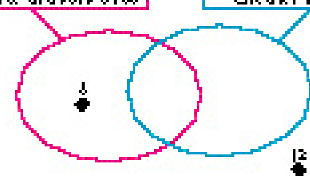
The red label is one of these:

~~Multiple of 2~~
~~Multiple of 3~~
~~Multiple of 4~~
~~Less than 20~~
~~Greater than 20~~
~~Positive divisors of 24~~
Positive divisors of 30

The blue label is one of these:

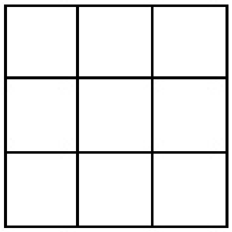
~~Multiple of 2~~
~~Multiple of 3~~
~~Multiple of 4~~
~~Less than 20~~
Greater than 20
~~Positive divisors of 24~~
~~Positive divisors of 30~~

Label the strings.



9

This square measures 9 cm on an edge. Using a ruler, carefully divide this square into nine smaller squares all the same size.



Fill in the blanks.

Each small square is what fraction of the large square? $\frac{1}{9}$

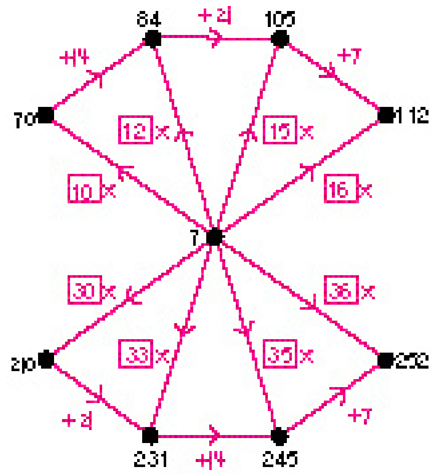
Each small square has sides of length 3 cm each.

The area of each small square is 9 cm^2.

The area of the large square is 81 cm^2.

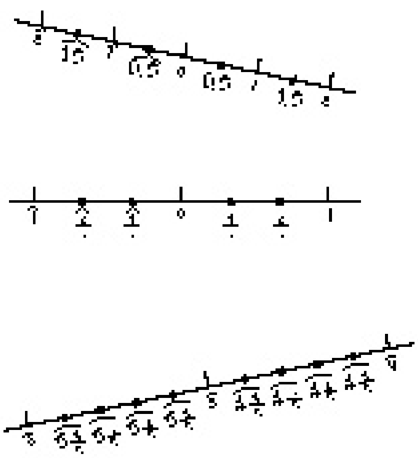
10

Label the dots and fill in the boxes for the red arrows.



11

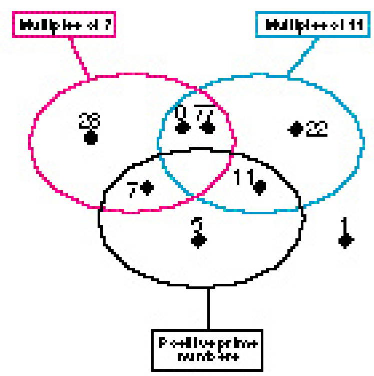
Label the dots on each number line.



12

Label each dot in the string picture with one of these numbers.

0 1 5 7 11 22 28 77



Are there any numbers that belong in the middle region? No

13

Fill in the boxes.

$21 \div 3 = \frac{21}{3} = 7$

$22 \div 3 = \frac{22}{3} = \boxed{7\frac{1}{3}}$

$23 \div 3 = \frac{23}{3} = \boxed{7\frac{2}{3}}$

$24 \div 3 = \frac{24}{3} = \boxed{8}$

$25 \div 3 = \frac{25}{3} = \boxed{8\frac{1}{3}}$

$26 \div 3 = \frac{26}{3} = 8\frac{2}{3}$

$27 \div 3 = \frac{27}{3} = \boxed{9}$

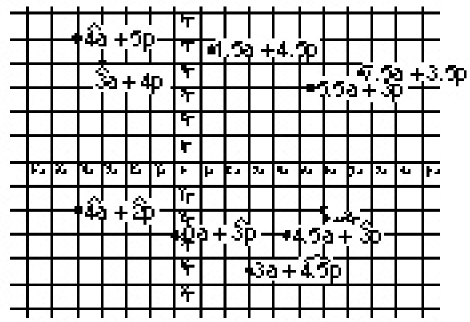
$28 \div 3 = \frac{28}{3} = \boxed{9\frac{1}{3}}$

$32 \div 3 = \frac{32}{3} = \boxed{10\frac{2}{3}}$

14

The dots on the grid represent purchases/returns of apples and peaches.

Label the dots. One is done for you.



Draw and label dots for these purchases/returns.

4a + 5p | 4.5a + 5p

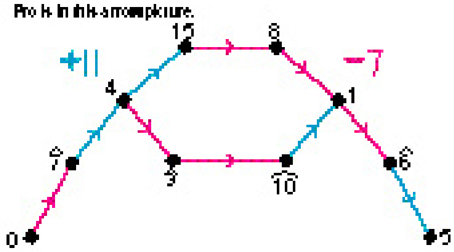
0a + 5p | 7.5a + 3.5p

15

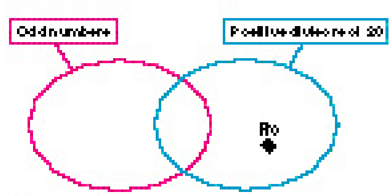
Pro is a secret number.

Clue 1

Pro is in this arrow picture.

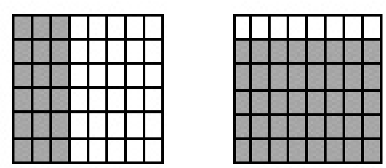


Clue 2



Who is Pro? 4

16



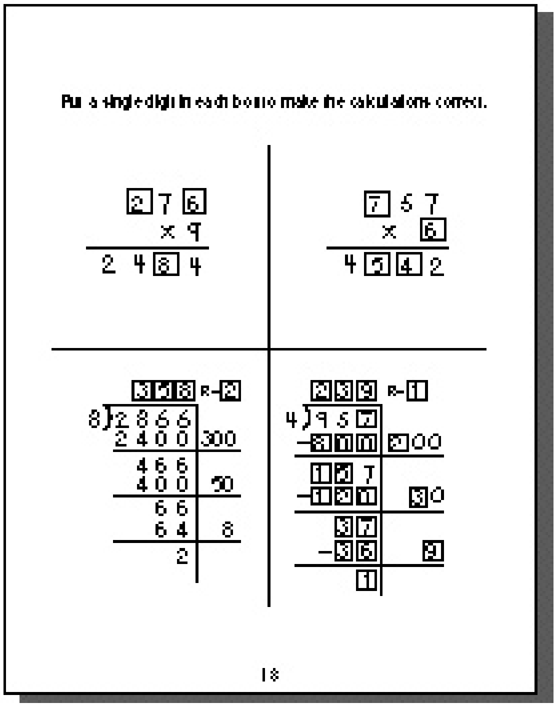
Make Sara's cuts on Amelia's cake.

Make Amelia's cuts on Sara's cake.

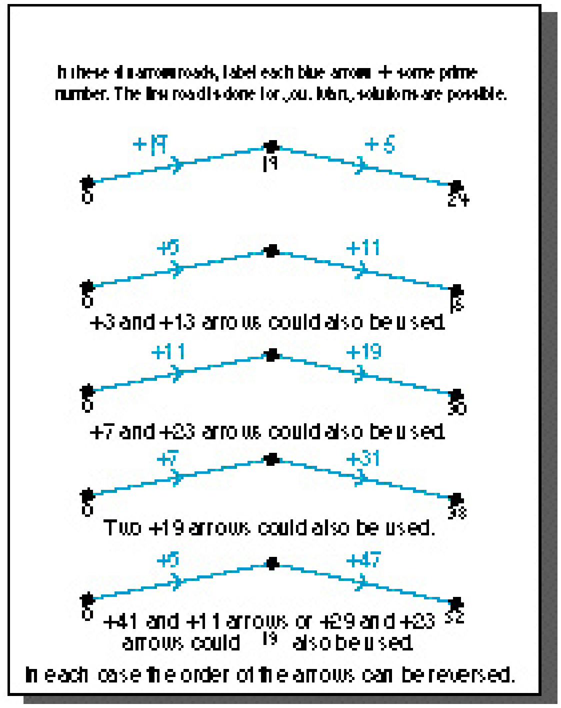
Now use the pictures to solve this problem.

$\frac{3}{8} + \frac{5}{6} = \text{[illegible]} = 1\frac{5}{24}$

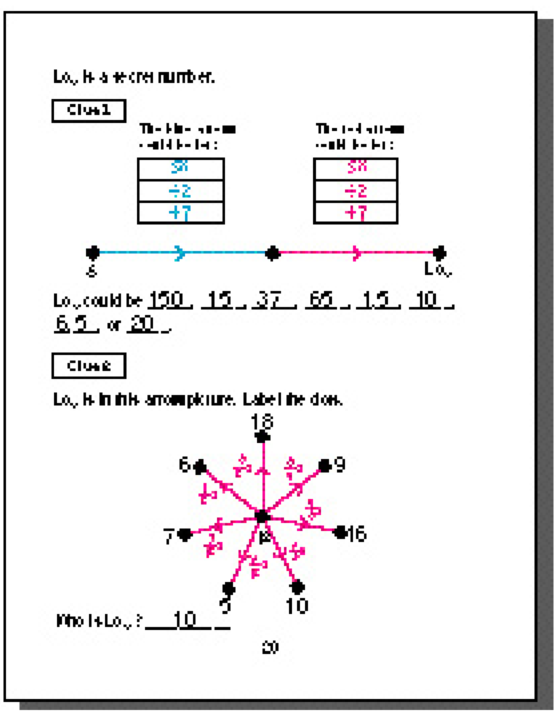
17



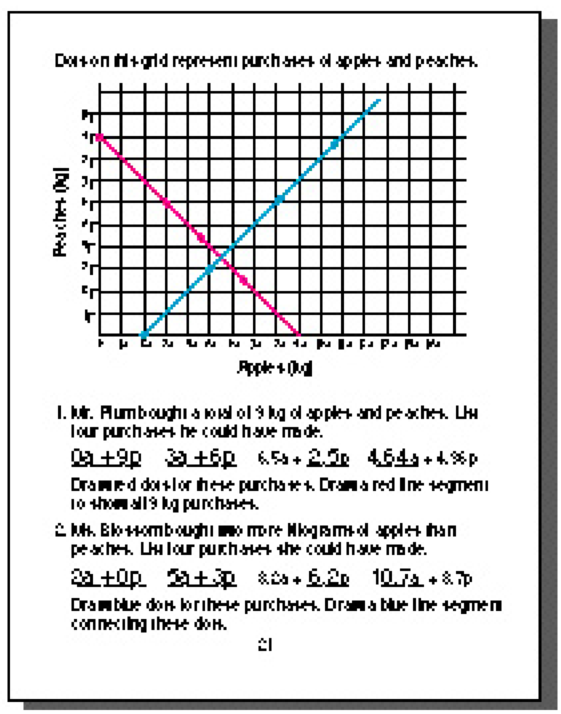
Put a single digit in each box to make the calculations correct.
276 × 9 = 2484
757 × 6 = 4542
358 R 2
8)2866
2400 300
466
400 50
66
64 8
2
239 R 1
4)957
−800 200
157
−120 30
37
−36 9
1
18



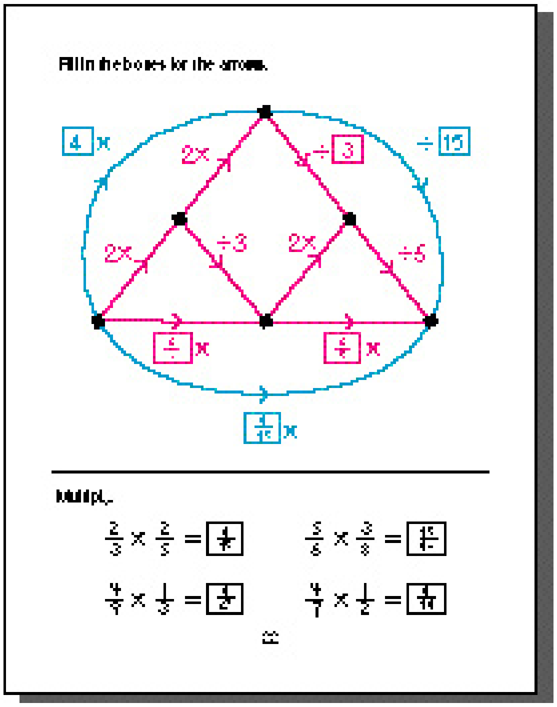
In these arrow roads, label each blue arrow + some prime number. The first road is done for you. Many solutions are possible.
+19 +5
0 19 24
+5 +11
0 16
+3 and +13 arrows could also be used.
+11 +19
0 30
+7 and +23 arrows could also be used.
+7 +31
0 38
Two +19 arrows could also be used.
+5 +47
0 52
+41 and +11 arrows or +29 and +23 arrows could also be used.
19
In each case the order of the arrows can be reversed.

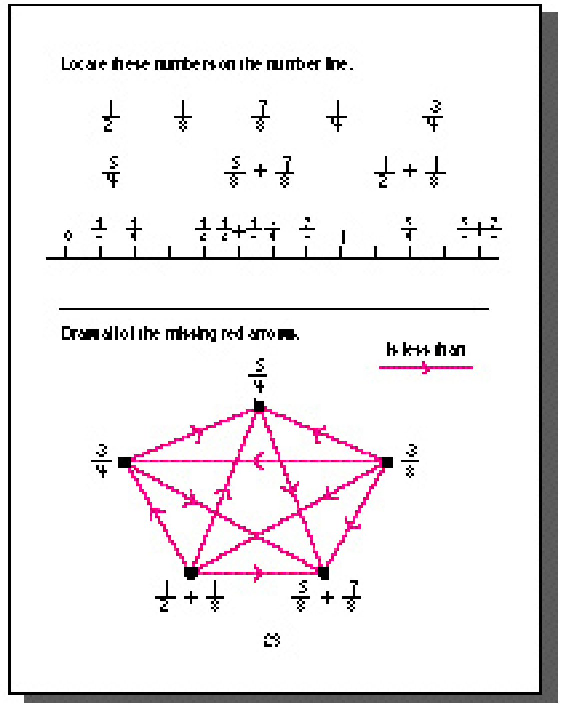


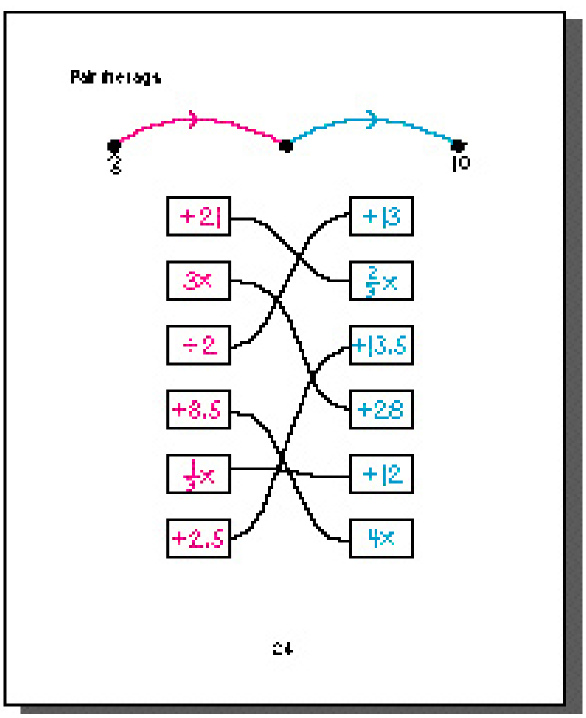
Lou is a secret number.
Clue 1
The blue arrow could be:
5x
÷2
+7
The red arrow could be:
5x
÷2
+7
6
Lou
Lou could be 150, 15, 37, 65, 1.5, 10, 6.5, or 20.
Clue 2
Lou is in this arrow picture. Label the dots.
18
6
9
7
16
5
10
Who is Lou? 10
20

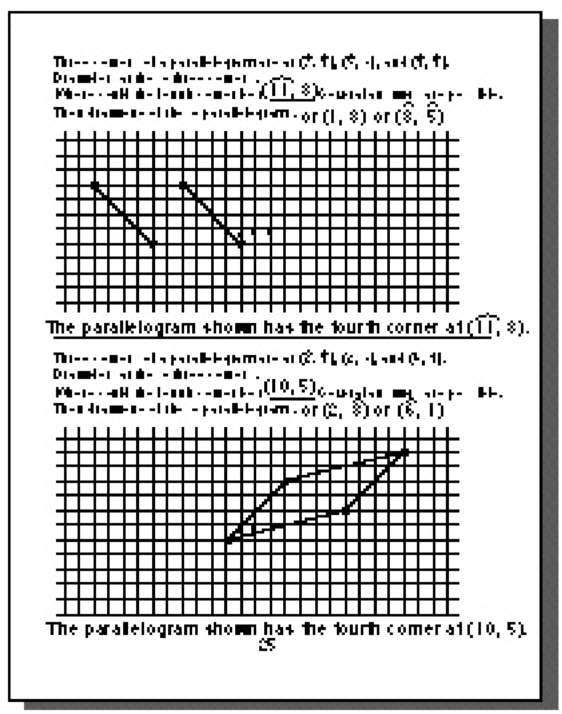


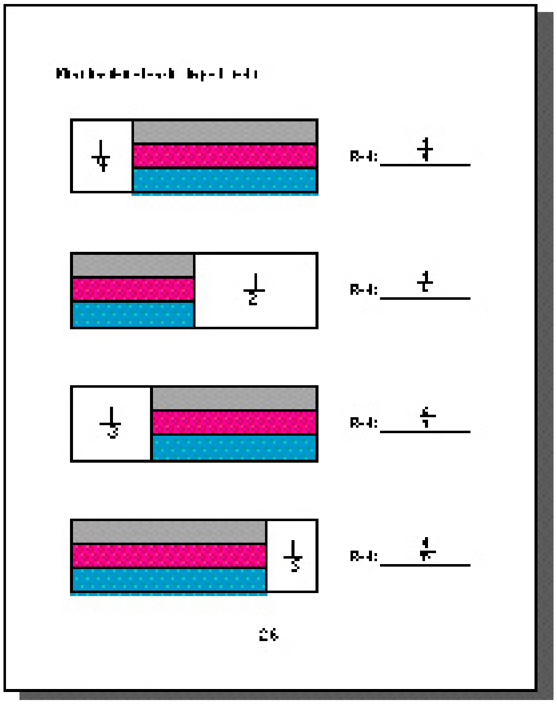
Dots on this grid represent purchases of apples and peaches.
Peaches (kg)
Apples (kg)
1. Mr. Plum bought a total of 9 kg of apples and peaches. List four purchases he could have made.
0a + 9p 3a + 6p 6.5a + 2.5p 4.64a + 4.36p
Draw red dots for these purchases. Draw a red line segment to show all 9 kg purchases.
2. Ms. Blossom bought two more kilograms of apples than peaches. List four purchases she could have made.
2a + 0p 5a + 3p 8.2a + 6.2p 10.7a + 8.7p
Draw blue dots for these purchases. Draw a blue line segment connecting these dots.
21

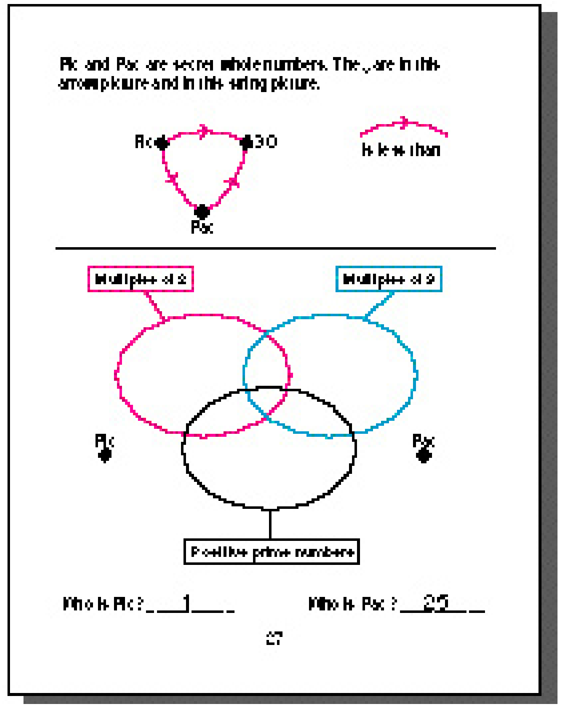

Fill in the boxes for the arrows.
4x
2x
÷3
÷15
2x
÷3
2x
÷5
Multiply.
22

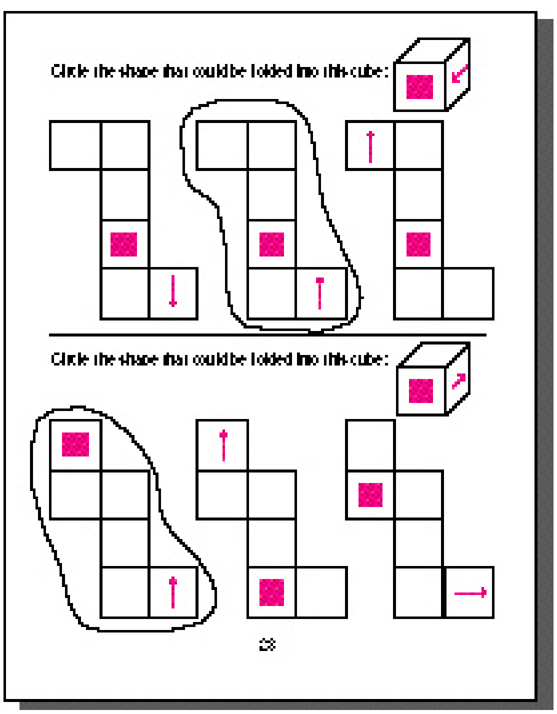

Locate these numbers on the number line.
Draw all of the missing red arrows.
is less than
23

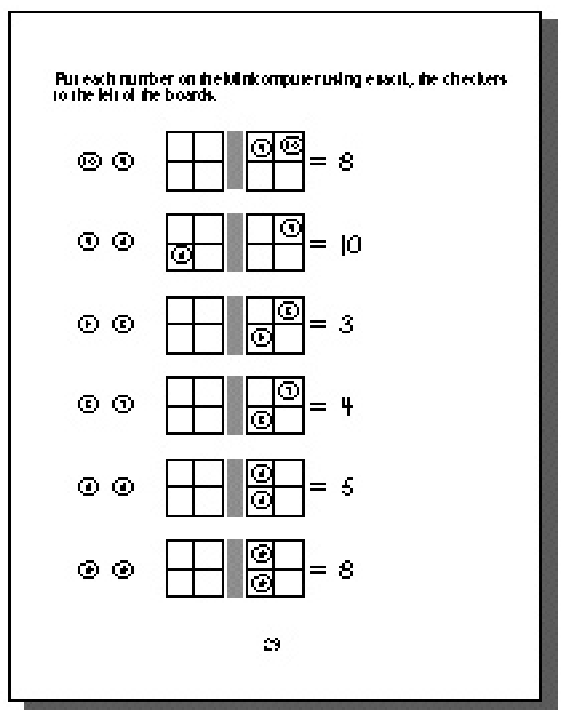

10
+21
+13
3x
÷2
+13.5
+8.5
+28
+12
4x
+2.5
24

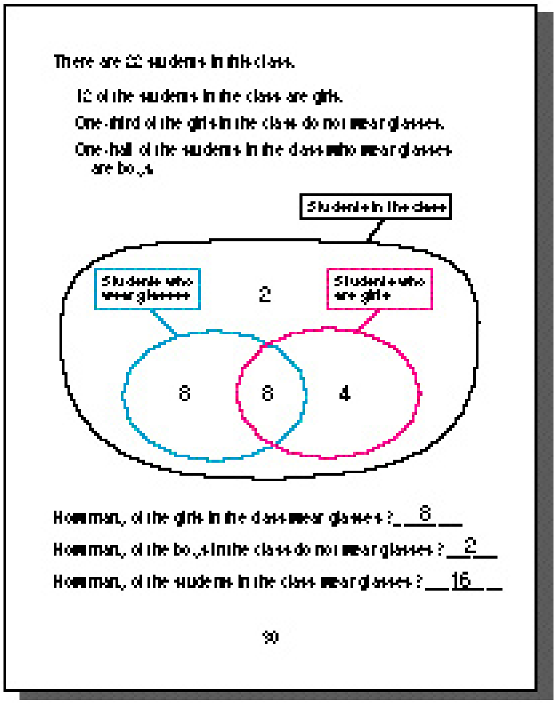

The parallelogram shown has the fourth corner at (11, 8).
The parallelogram shown has the fourth corner at (10, 5).
25


26


Pic
30
Pac
is less than
Multiples of 2
Positive prime numbers
Who is Pic? 1
Who is Pac? 25
27


Circle the shape that could be folded into this cube:
Circle the shape that could be folded into this cube:
28


= 8
= 10
= 3
= 4
= 5
= 8
29



There are 22 students in this class.

12 of the students in the class are girls.

One-third of the girls in the class do not wear glasses.

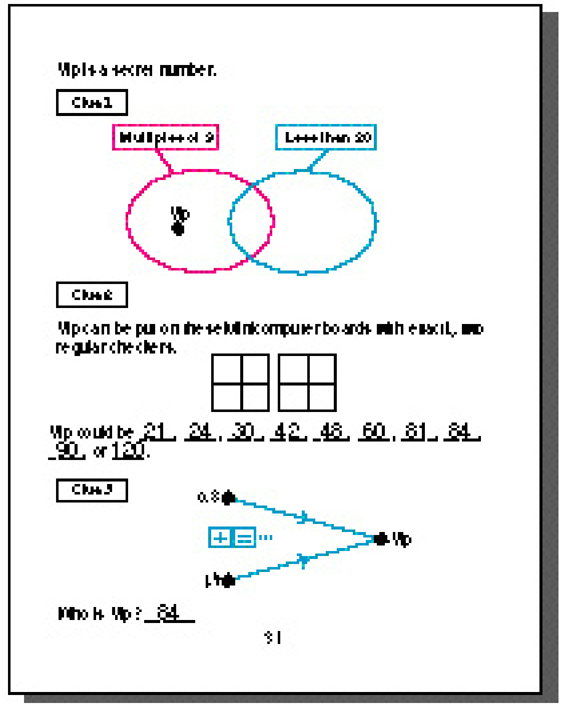
One-half of the students in the class who wear glasses are boys.

How many of the girls in the class wear glasses? 8

How many of the boys in the class do not wear glasses? 2

How many of the students in the class wear glasses? 16

90



Vip is a secret number.

Clue 1

Clue 2

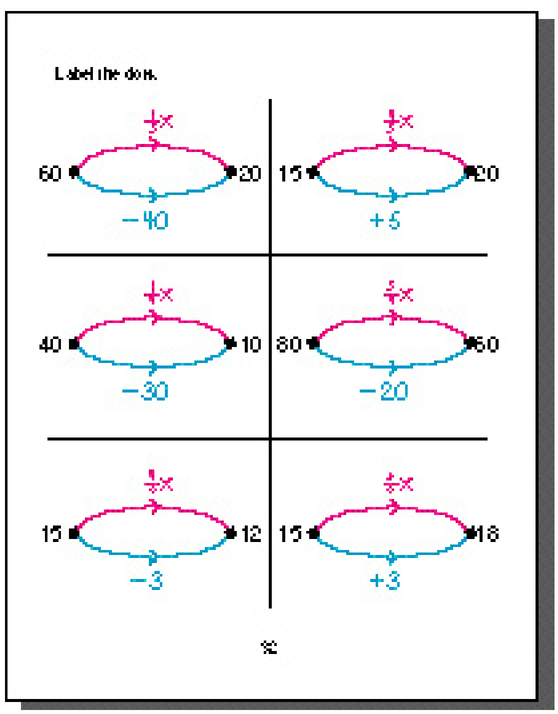
Vip can be put on these Minicomputer boards with exactly two regular checkers.

Vip could be 21, 24, 30, 42, 48, 60, 81, 84, 90, or 120.

Clue 3

Who is Vip? 84

91



Label the dots.

92

Capsule Lesson Summary

Compute the area and perimeter of several rectangles. Given the lengths of some of the sides in a shape, determine the lengths of the other sides, and then compute the shape's perimeter and area. Begin the workbook *Selection of Problems #6*. (This is the first of two lessons using this workbook.)

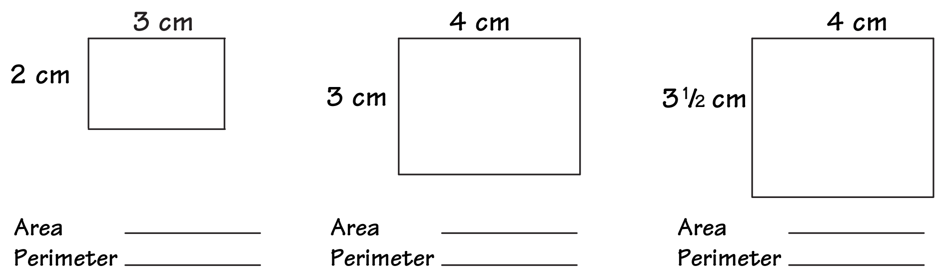
Materials

Teacher
- Colored chalk
- Meter stick

Student
- *Selection of Problems #6* Workbook
- Colored pencils, pens, or crayons
- Metric ruler
- Calculator

Description of Lesson

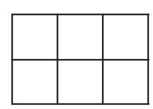
Draw three rectangles on the board with sides proportional to the following.



T: ***What is the area of a 2 cm by 3 cm rectangle? Why?***

S: ***I multiplied 2 x 3 = 6. The area is 6 cm^2.***

S: ***6 cm^2; I divided the rectangle into little squares, each a centimeter square, and counted the squares.***



T: ***What does perimeter mean?***

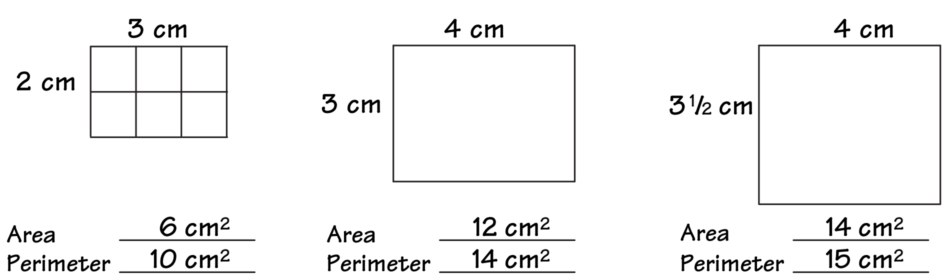
S: ***The distance around the shape, or the length of its border.***

T: ***What is the perimeter of a 2 cm by 3 cm rectangle? Why?***

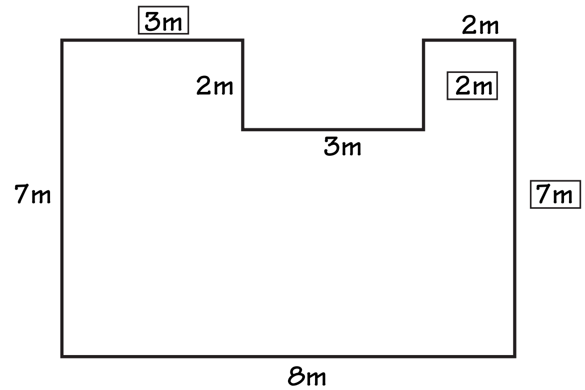
S: ***10 cm; I added the lengths of the sides.***

S: ***I added 2 and 3 and then doubled that answer.***

Record the area and perimeter of each rectangle on the board.



Draw this shape on the board and ask the students to supply the missing lengths. (Answers are in boxes.)

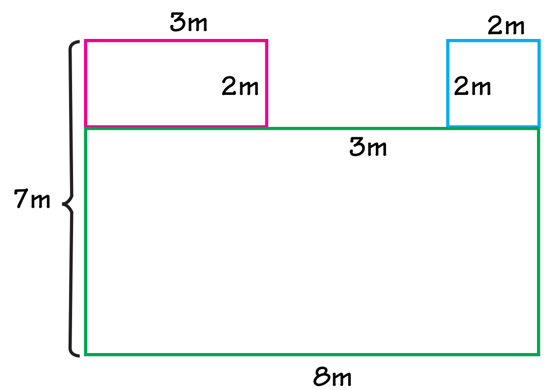


T: ***What is the perimeter of this shape?***

S: ***34 m.***

T: ***What is the area of this shape?***

If some students have difficulty with this question, partition the shape as shown here.



T: ***What is the length of a shorter side of the green rectangle?***

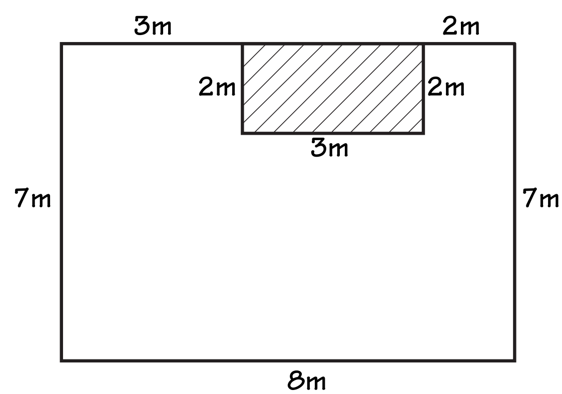
S: ***5 m, because 7 – 2 = 5.***

T: ***Using these three rectangles, find the area of the shape.***

S: ***The area of the green rectangle is 40 m²*** ***because 5 x 8 = 40, the area of the red rectangle is 6 m² because 2 x 3 = 6, and the area of the blue rectangle is 4 m² because 2 x 2 = 4. 40 + 6 + 4 = 50, so the area of the shape is 50 m².***

T: ***Can anyone see a different method we could use?***

S: ***We could look at the shape as what is left after a 2 m by 3 m rectangle is taken out of a 7 m by 8 m rectangle. 7 x 8 = 56, 2 x 3 = 6, and 56 – 6 = 50; so the area of the shape is 50 m².***



Distribute copies of the workbook *Selection of Problems #6* and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of class period collect the workbooks for your review. They will be used again in Lesson W16.

W16 SELECTION OF PROBLEMS #6 (LESSON ______ W16

Capsule Lesson Summary

Knowing only some of the digits in a record of a division computation, determine the original problem and its solution. Continue individual work in the workbook *Selection of Problems #6*. (This is the second of two lessons using this workbook.)

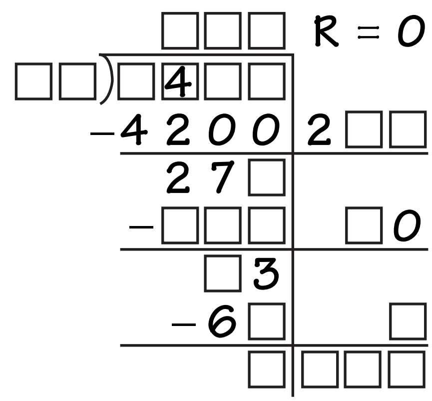
Materials

Teacher
- None

Student
- *Selection of Problems #6* Workbook
- Colored pencils, pens, or crayons
- Metric ruler
- Calculator

Description of Lesson

Write this record of a division computation on the board.



T: ***The eraser gremlin has struck again! Can you tell me any of the digits that have been erased in this problem?***

If students have difficulty getting started, assist them in determining the divisor (21) in the problem.

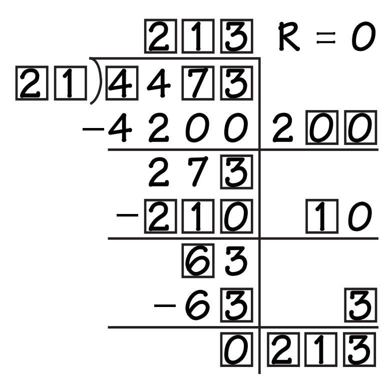
T (pointing to 2 □ □)**:** ***What digits were erased here?***

S: ***Two zeroes because 4 200 ends in two zeroes.***

T: ***Does this tell us what the divisor is?***

S: ***Yes, the divisor is 21 because 200 x 21 = 4 200.***

Continue asking students to supply the missing digits until all of the boxes are filled. Ask students to justify their suggestions.



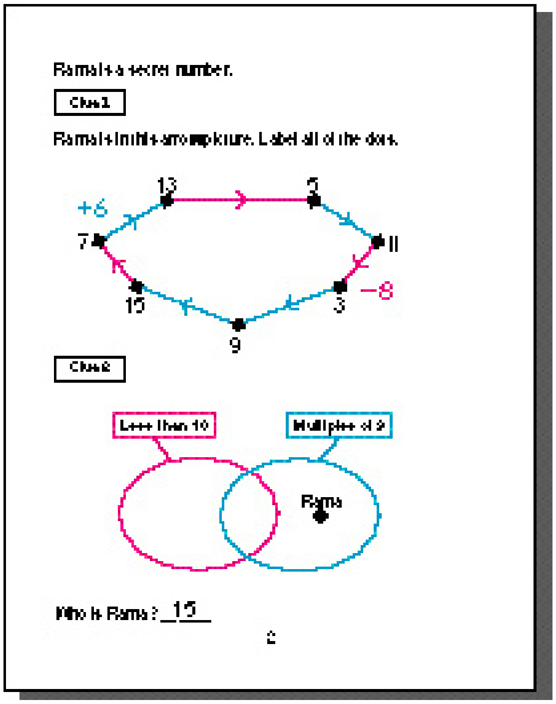
Distribute students' copies of the workbook *Selection of Problems #6*. Ask students first to correct or complete pages from the previous week's work and then to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

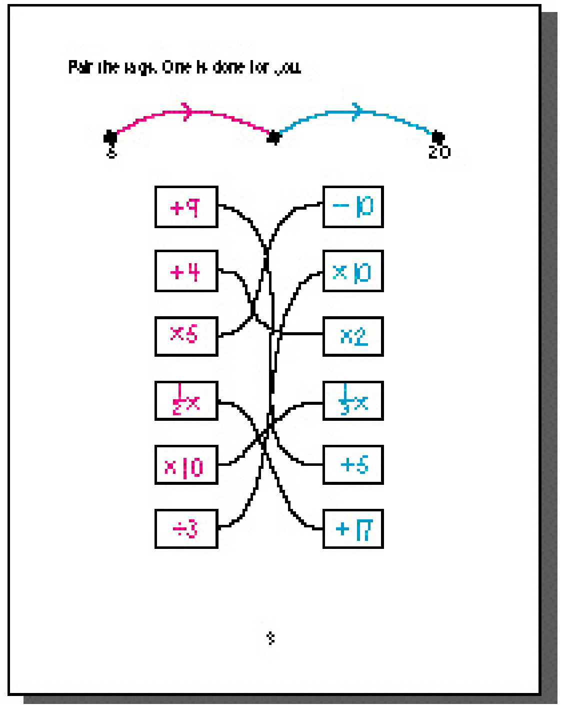
W16

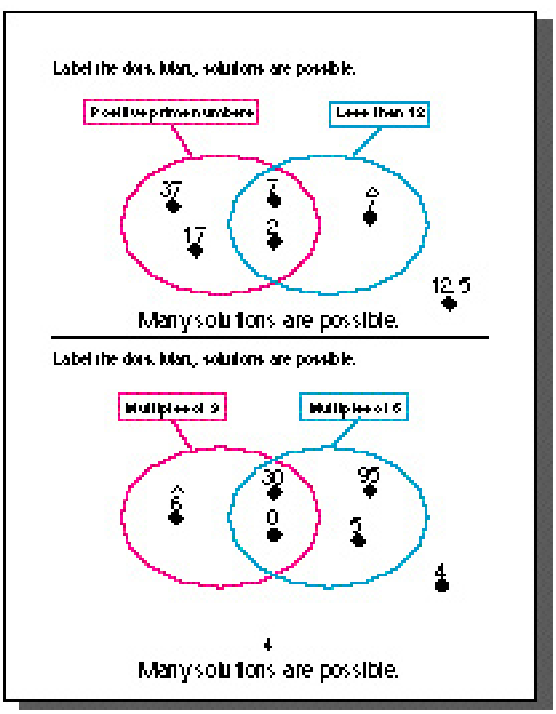
At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in their workbooks during a study time or to take them home as an assignment.

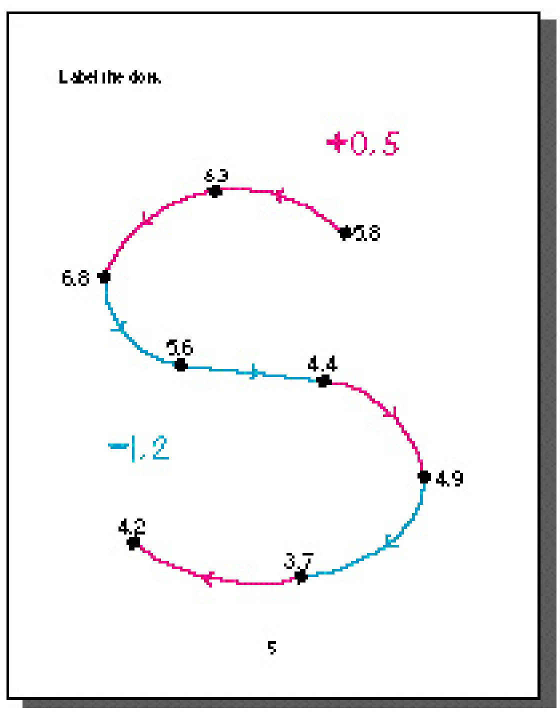
Assessment Activity

An individual student progress record for the workbook is available on Blackline W16. You may like to use this form to monitor student work.


Rama is a secret number.
Clue 1
Rama is in this arrow picture. Label all of the dots.
+6
13
5
7
11
15
3
−8
9
Clue 2
Less than 10
Multiples of 3
Rama
Who is Rama? 15
2


Pair the legs. One is done for you.
5
20
+9
+4
×5
½×
×10
÷3
−10
×10
×2
⅓×
+5
+17
3

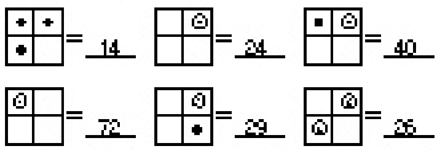

Label the dots. Many solutions are possible.
Positive prime numbers
Less than 12
37
17
7
2
6
12.5
Many solutions are possible.
Label the dots. Many solutions are possible.
Multiples of 3
Multiples of 5
6
30
0
95
5
4
4
Many solutions are possible.


Label the dots.
+0.5
6.3
5.8
6.8
5.6
4.4
−1.2
4.9
4.2
3.7
5

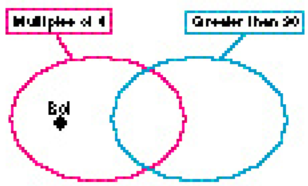
Bol is a secret number.

Clue 1

Bol is one of these numbers.



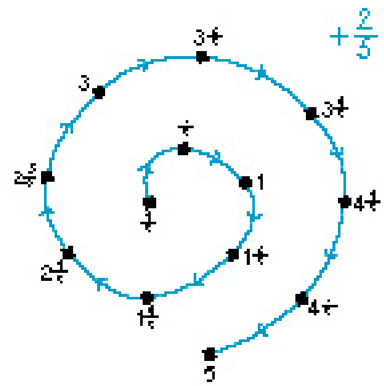
Clue 2



Who is Bol? 24

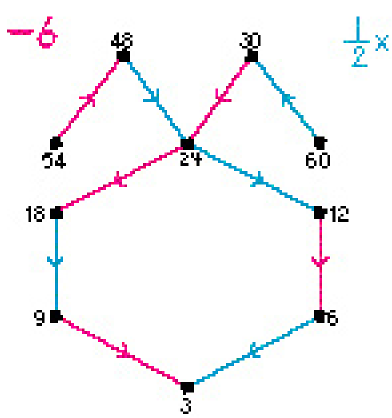
6

Label the dots.



7

Label the dots.



In this picture, exactly three numbers are multiples of 9.

Name them. 9, 18, and 54

8

Put each number on the display of a calculator using just these keys:

[4] [5] [+] [−] [×] [÷] [=]

Use the keys in the order you use them. You may use a key more than once.

It costs 1¢ each time you press a key. Try to spend less than the amount shown for each number.

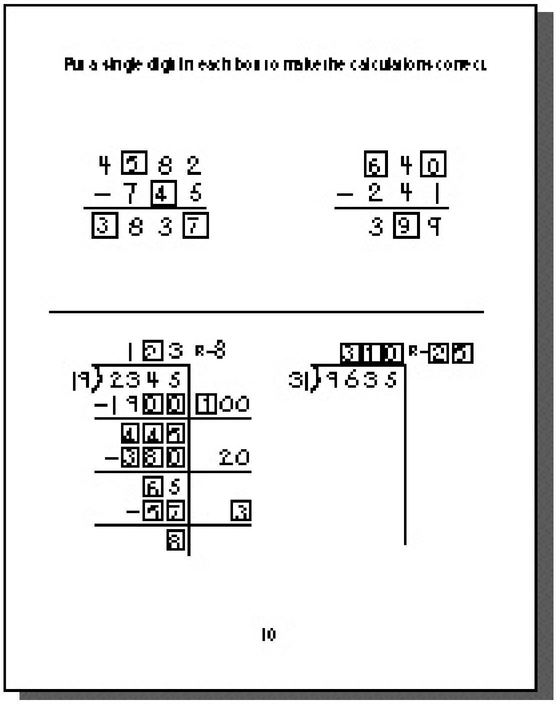
[4] [×] [=] [+] [5] [=] 21 [10¢]

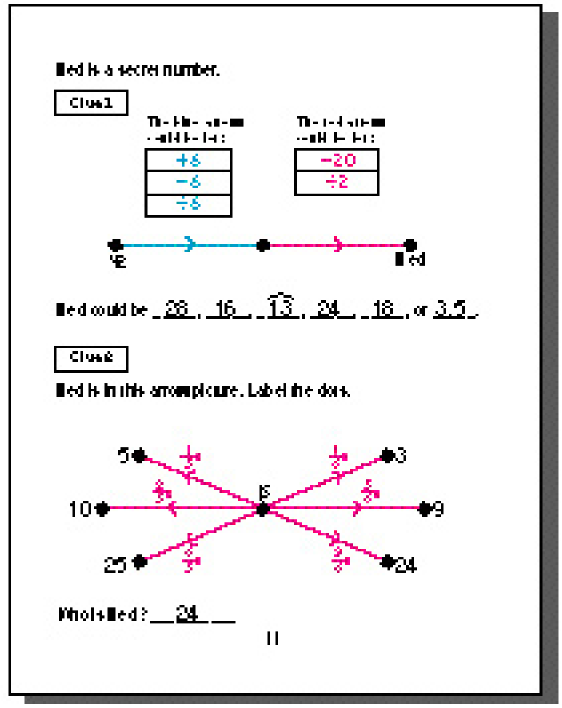
[5] [5] [+] [5] [=] 60 [8¢]

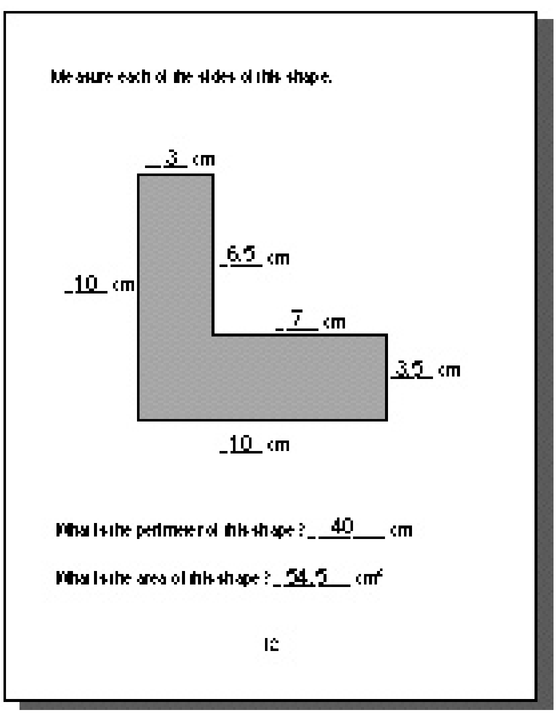
[4] [−] [5] [=] [=] ⁻6 [8¢]

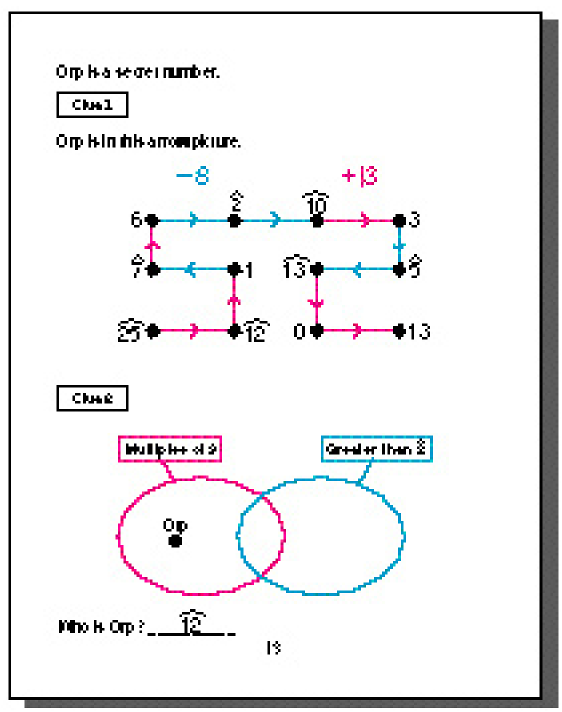
Many solutions are possible.

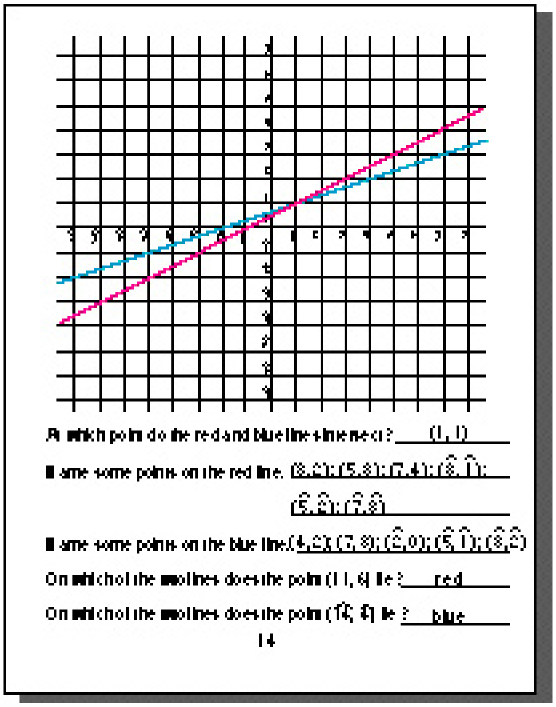
9

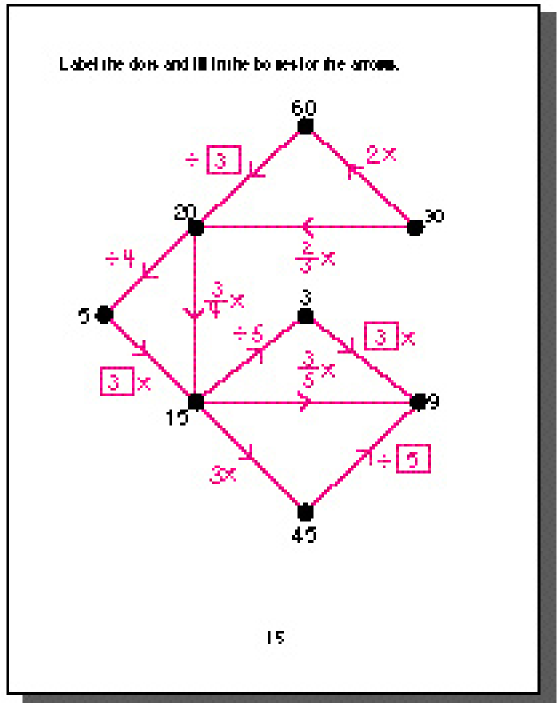

Put a single digit in each box to make the calculations correct.
10

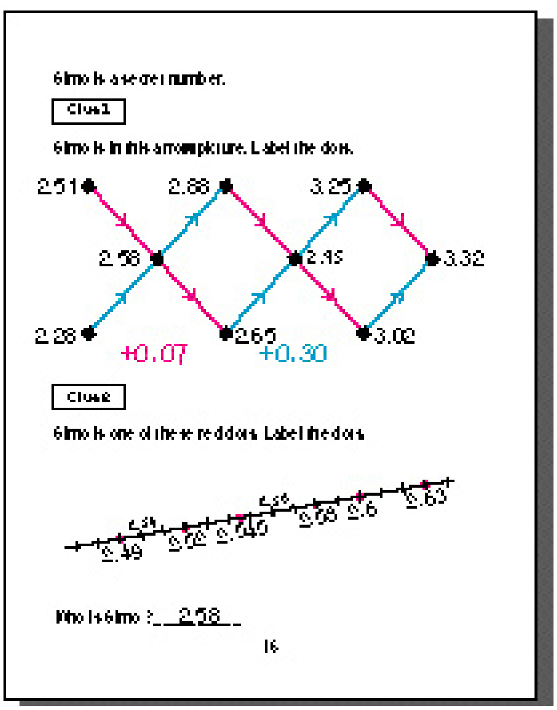

Clue 1
+6
−6
÷6
−20
÷2
Clue 2
5
3
10
9
25
24
11

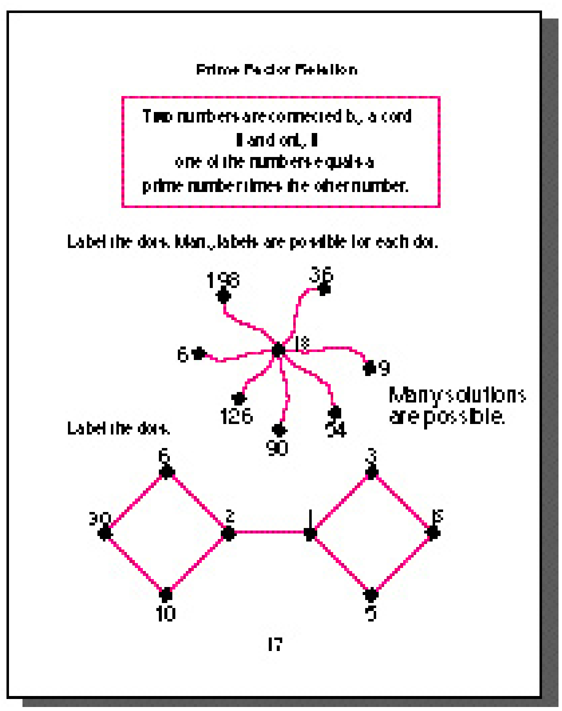

Measure each of the sides of this shape.
3 cm
6.5 cm
10 cm
7 cm
3.5 cm
10 cm
What is the perimeter of this shape? 40 cm
What is the area of this shape? 54.5 cm²
12

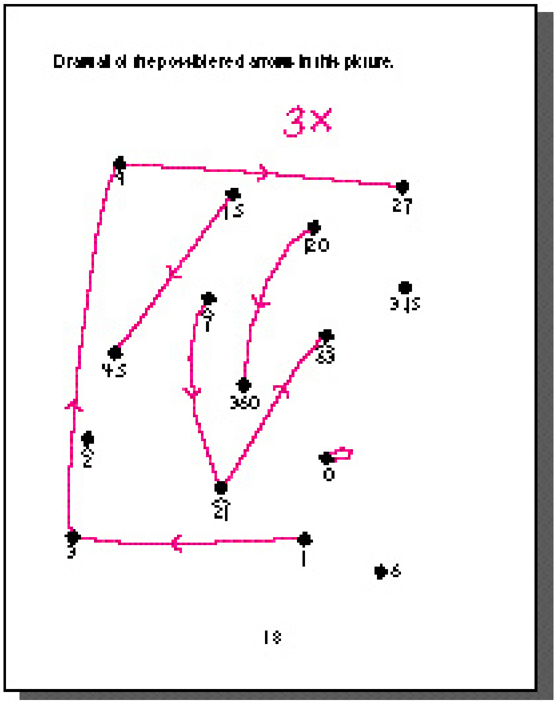

Orp is a secret number.
Clue 1
Orp is in this arrow picture.
−8
+13
Clue 2
Orp
Who is Orp? 12
13

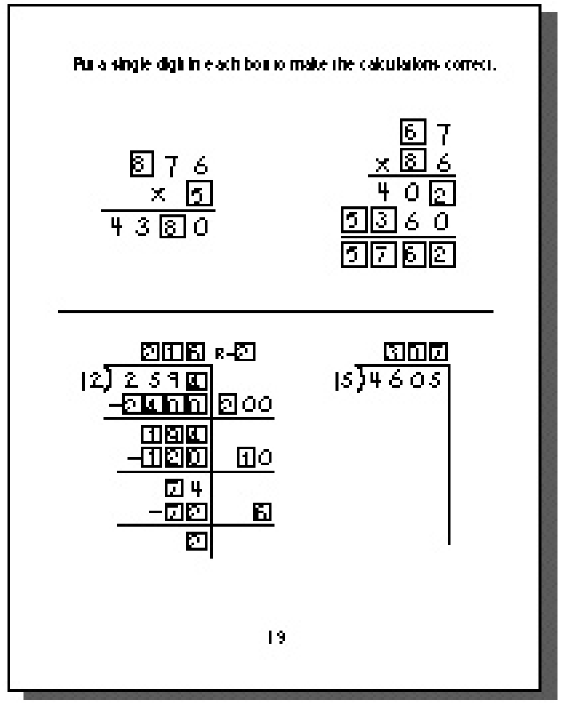

At which point do the red and blue lines intersect? (1, 1)
Name some points on the red line.
Name some points on the blue line.
On which of the two lines does the point (11, 6) lie? red
On which of the two lines does the point (14, 4) lie? blue
14

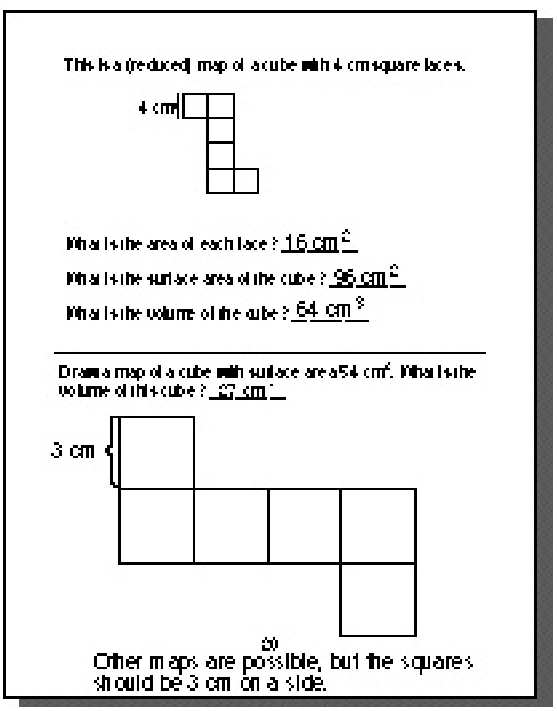

Label the dots and fill in the boxes for the arrows.
60
÷3
2×
20
30
$\frac{2}{3}$×
÷4
$\frac{3}{4}$×
3
5
÷5
3×
3×
$\frac{3}{5}$×
15
9
3×
÷5
45
15

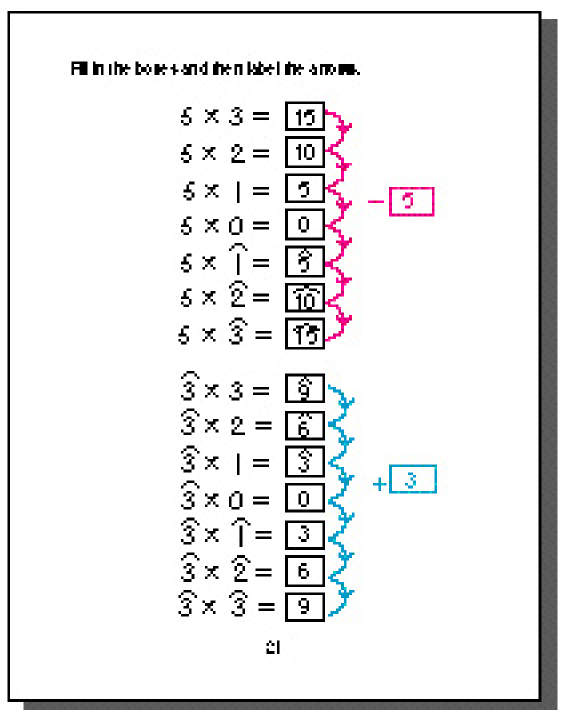

Gimo is a secret number.
Clue 1
Gimo is in this arrow picture. Label the dots.
2.51
2.88
3.25
2.58
2.95
3.32
2.28
2.65
3.02
+0.07
+0.30
Clue 2
Gimo is one of these red dots. Label the dots.
Who is Gimo? 2.58
16

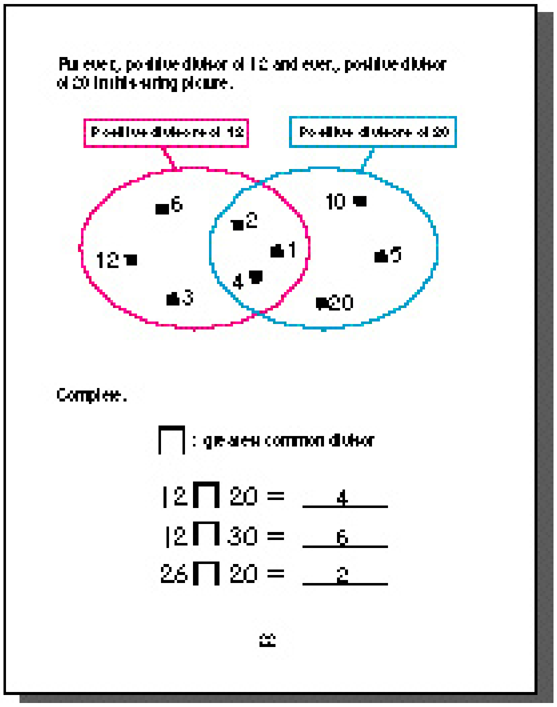

Prime Factor Relation
Two numbers are connected by a cord
if and only if
one of the numbers equals a
prime number times the other number.
Label the dots. Many labels are possible for each dot.
198
36
18
6
9
126
90
54
Many solutions are possible.
Label the dots.
6
3
30
2
1
15
10
5
17


Draw all of the possible red arrows in this picture.
3×
9
27
15
20
315
7
45
63
360
2
0
21
3
1
6
18


Put a single digit in each box to make the calculations correct.
8 7 6
× 5
4 3 8 0
6 7
× 8 6
4 0 2
5 3 6 0
5 7 6 2
12)2 5 9
15)4 6 0 5
19


This is a (reduced) map of a cube with 4 cm-square faces.
4 cm
What is the area of each face? 16 cm²
What is the surface area of the cube? 96 cm²
What is the volume of the cube? 64 cm³
Draw a map of a cube with surface area 54 cm². What is the volume of this cube? 27 cm³
3 cm
20
Other maps are possible, but the squares should be 3 cm on a side.


Fill in the boxes and then label the arrows.
5 × 3 = 15
5 × 2 = 10
5 × 1 = 5
5 × 0 = 0
5 × $\hat{1}$ = $\hat{5}$
5 × $\hat{2}$ = $\hat{10}$
5 × $\hat{3}$ = $\hat{15}$
−5
$\hat{3}$ × 3 = $\hat{9}$
$\hat{3}$ × 2 = $\hat{6}$
$\hat{3}$ × 1 = $\hat{3}$
$\hat{3}$ × 0 = 0
$\hat{3}$ × $\hat{1}$ = 3
$\hat{3}$ × $\hat{2}$ = 6
$\hat{3}$ × $\hat{3}$ = 9
+3
21


Put every positive divisor of 12 and every positive divisor of 20 in this string picture.

Positive divisors of 12

Positive divisors of 20

6
12
3
2
1
4
10
5
20

Complete.

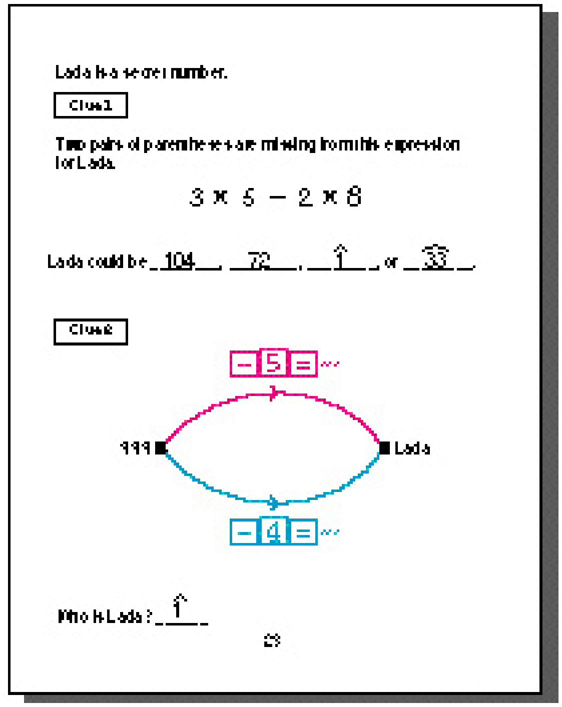
⊓ : greatest common divisor

12 ⊓ 20 = 4

12 ⊓ 30 = 6

26 ⊓ 20 = 2

22


Lada is a secret number.

Clue 1

Two pairs of parentheses are missing from this expression for Lada.

3 × 5 − 2 × 8

Lada could be 104, 72, 1, or 33.

Clue 2

−5=...

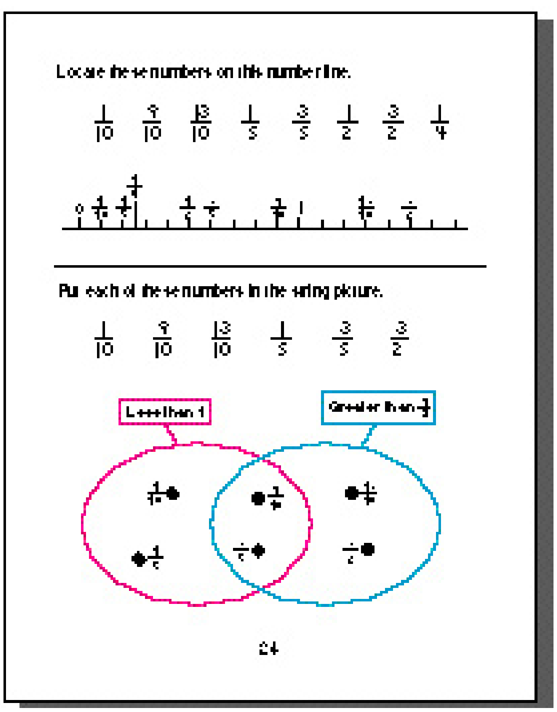
999

Lada

−4=...

Who is Lada? 1

23


Locate these numbers on this number line.

$\frac{1}{10}$ $\frac{9}{10}$ $\frac{13}{10}$ $\frac{1}{5}$ $\frac{3}{5}$ $\frac{1}{2}$ $\frac{3}{2}$ $\frac{1}{4}$

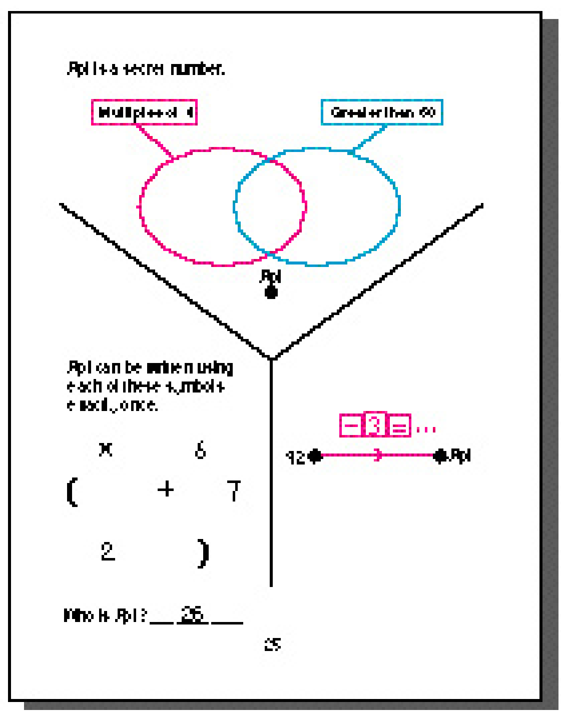
Put each of these numbers in the string picture.

$\frac{1}{10}$ $\frac{9}{10}$ $\frac{13}{10}$ $\frac{1}{5}$ $\frac{3}{5}$ $\frac{3}{2}$

Less than 1

Greater than $\frac{1}{2}$

24


Rpl is a secret number.

Multiple of 4

Greater than 60

Rpl

Rpl can be written using each of these symbols exactly once.

× 6

(+ 7

2)

−3=...

92

Rpl

Who is Rpl? 26

25

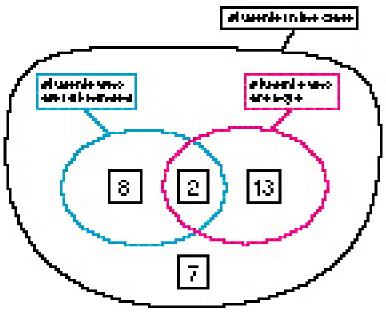
Find the number of students in each region of the string picture and record these numbers in the boxes.

There are 30 students in the class.

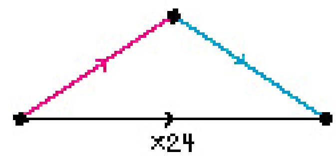
One-half of the students are boys.

13 of the boys are right-handed.

One-third of the class is left-handed.



26



Complete the charts.

→	→
×4	×6
×2	×12
×12	×2
÷2	×48
×72	÷3

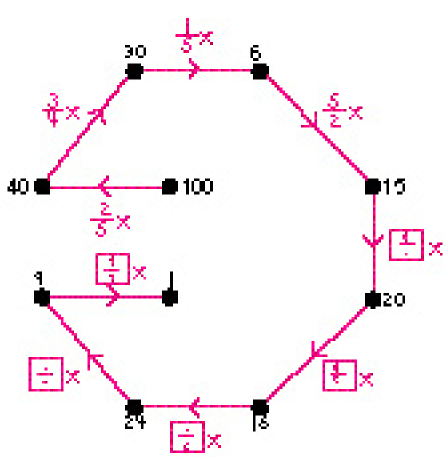
Many solutions are possible.

Many solutions are possible.

→	→
×240	÷10
÷20	×480
÷100	×2400
×4800	÷200
÷30	×720

27

Label the dots and fill in the boxes for the arrows.



28

Viva is a secret number.

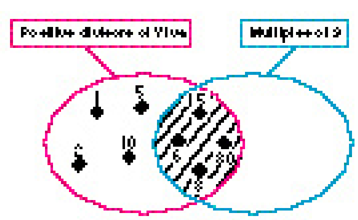
Clue 1

$\square$: least common multiple

$60 = 12 \square$ Viva

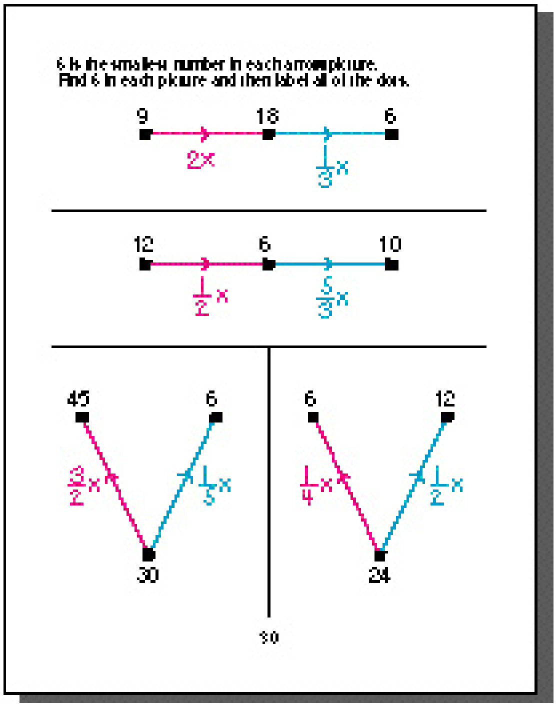
Viva could be _60_, _30_, _20_, _15_, _10_, or _5_.

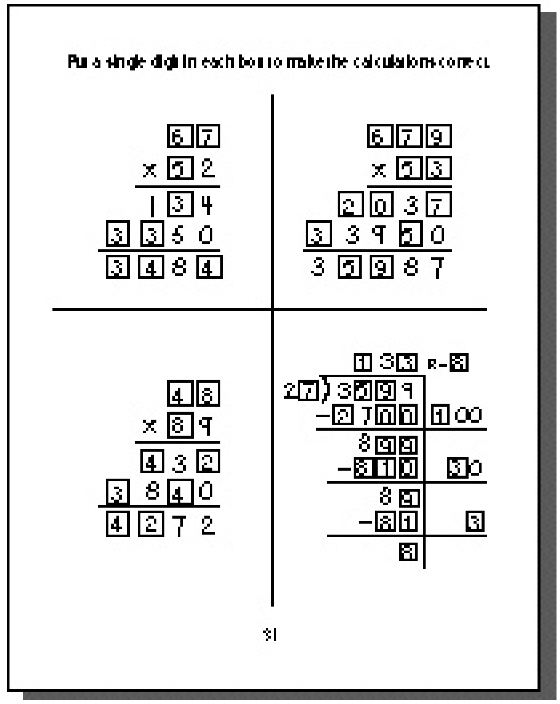
Clue 2

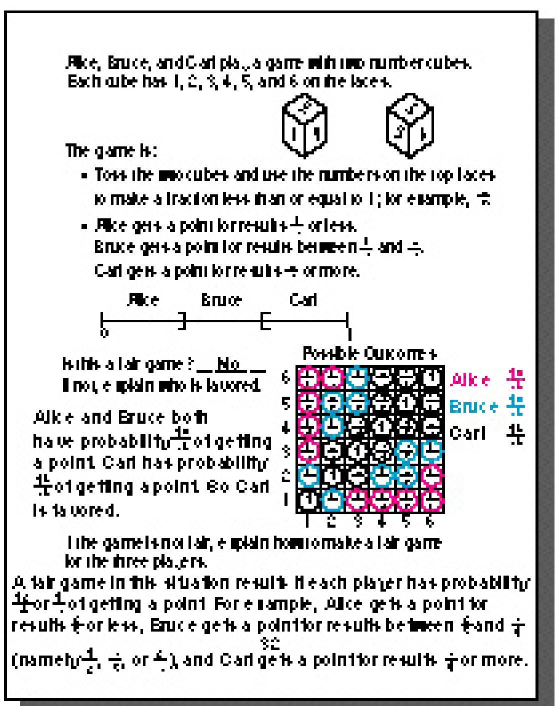


Who is Viva? _30_

29


6 is the smallest number in each arrow picture.
Find 6 in each picture and then label all of the dots.
9
18
6
2x
12
6
10
45
6
30
6
12
24
30


Put a single digit in each box to make the calculations correct.
31


Alice, Bruce, and Carl play a game with two number cubes.
Each cube has 1, 2, 3, 4, 5, and 6 on the faces.
The game is:
Toss the two cubes and use the numbers on the top faces to make a fraction less than or equal to 1; for example,
Alice gets a point for results or less.
Bruce gets a point for results between and
Carl gets a point for results or more.
Alice
Bruce
Carl
0
1
Possible Outcomes
Is this a fair game? No
If not, explain who is favored.
Alice and Bruce both have probability of getting a point. Carl has probability of getting a point. So Carl is favored.
Alice
Bruce
Carl
If the game is not fair, explain how to make a fair game for the three players.
A fair game in this situation results if each player has probability or of getting a point. For example, Alice gets a point for results or less, Bruce gets a point for results between and
32