Probability and Statistics

PROBABILITY AND STATISTICS TABLE OF CONTENTS

PROBABILITY AND STATISTICS INTRODUCTION

In today's world, probabilistic and statistical methods have become a part of everyday life. They are an important part of industry, business, and both the physical and social sciences. Just being an intelligent consumer requires an awareness of basic techniques of descriptive statistics, for example, the use of averages and graphs. Probability and statistics are key to the present day modeling of our world in mathematical terms. The lessons in this strand utilize both fictional stories and real data as vehicles for presenting problems and applications. The problems and questions that arise focus attention on key concepts of probability and statistics such as randomness, equally likely events, and prediction.

Probability stories fascinate most students and encourage their personal involvement in the situations. They often relate the probability activities to games they have encountered outside the classroom. This personal involvement builds students' confidence and encourages them to rely on their intuition and logical thinking to analyze the situations. In *IG-V*, students use number cubes and other devices to simulate a situation or to play a game. These activities help students understand the story and also form a basis for predicting the likelihood of particular outcomes. Yet simulations produce only estimates of the probabilities, leaving open the question of a true probability. Pictorial techniques make the analysis of theoretical probabilities accessible. This combination of simulation and analysis of situations demonstrates the strong interdependence between probability and statistics.

Some applications of mathematics are more easily investigated than others. The probability and statistics applications in this strand lean towards the easy end of the spectrum, both because of the small amount of theoretical knowledge required, and because the link between the situation and the mathematical model is easily perceived. In particular, the development of pictorial models for s the ease of solutions.

Content Overview

Probability

There are many methods available for determining probabilities. The simplest techniques, though usually tedious, require listing all possible outcomes. Most powerful techniques rely on formulas involving the multiplication of probabilities. The lessons in this strand review and introduce several efficient pictorial techniques that elementary students can readily apply.

Three lessons this semester make use of the familiar area model for finding probabilities. In the first lesson, a story about making random selections of paths through a swamp and still getting to the other side within a specified amount of time uses the division of a square to reflect the choices and their consequences. In the second lesson, students find the probabilities involved in a fortune teller's method of deciding whether or not the time is right for a couple to marry. The method involves tieing the ends of three ropes (in different ways) together and hoping the result will be one long piece of rope. In the third lesson, students find that the analysis of a two-person game with four cubes yields a paradoxical result.

By studying the possible results of an extrasensory perception (ESP) test, students construct Pascal's triangle again (originally generated in the story-workbook *The Hidden Treasur*e in *IG-IV*). In Pascals's triangle, students discover patterns and predict the equivalence of certain tests.

PROBABILITY AND STATISTICS INTRODUCTION

While the major goal in these activities is the development of efficient and accessible pictorial techniques for determining probabilities, the lessons also reflect the continual development of other probabilistic themes; randomness, equally likely events, simulation, fair games, and predictions.

Lessons: P2, 3, 5, 6, and 7.

Statistics

Several lessons in this strand include descriptive statistics—the use of numerical and graphical techniques to summarize and compare sets of data. The activities continue to develop students' abilities to use averages, and to read, draw, and interpret bar graphs. The goal is to increase students' familiarity with these topics through rich experiences rather than to drill the techniques of computing an average or drawing a graph.

In Lesson P1 *Braille*, students use patterns and counting techniques to predict characters missing from a list of braille characters.

Another lesson this semester asks students to consider how three kinds of averages (mean, mode, and median) can be used to decide the person with the fastest reaction time. They discuss elements of a good class experiment on reaction time, conduct the experiment, and study the results. Advantages and disadvantages of the different methods for deciding the fastest reaction time evolve as part of the study.

In a third lesson on misleading advertisements, students investigate the meaning and visual effect of using different ways to graph data.

Lessons: P1, 4, and 8.

Capsule Lesson Summary

Use patterns to determine characters missing from a list of braille characters. Determine the number of possible characters in the revised and the original braille systems.

Materials

Teacher
- Braille characters transparency

Student
- Worksheets P1(a) and (b)

Advance Preparation: You may want to use Blackline P1(a) to make a transparency of braille characters for use on an overhead during the lesson.

Description of Lesson

Exercise 1

Distribute Worksheets P1(a) and (b).

Ask if anyone recognizes the symbols on Worksheet P1(a). Perhaps some students will realize that the symbols are braille characters. Ask a student to explain how the braille characters actually appear (raised dots on paper) and when they are used. Lead a short discussion on the history of braille. Here are a couple points you may wish to mention:

- Braille is one of several systems devised to allow the blind to read. Early attempts included letters carved in wood, pins inserted in cushions, and large wooden raised letters. For a long time the standard was to use raised letters on paper, however, this system was not effective with persons who were born blind.
- Louis Braille (1809–1852) invented a system of reading that proved effective with most blind people. At first the braille system met a great deal of resistance, but it was widely adopted after Louis Braille's death.

T: ***Look at Worksheet P1(a) and try to find some patterns that will help us to discover the braille characters for "Q," "the," and "W." Write these characters on the worksheet.***

Allow a few minutes for students to investigate patterns in the characters.

T: ***What do you notice about the arrangement of the dots for each character?***

S: ***There are at most three rows of dots for each character.***

S: ***There are at most two columns of dots for each character.***

T: ***A braille character is made up of six cells arranged in two columns and three rows. Let's label the cells to make it easier to explain patterns.***

Draw a diagram on the board.

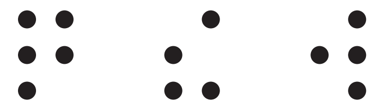
1	4
2	5
3	6

Let students describe patterns they discover. Perhaps they will mention the following:

- Each character in the second row is formed by adding a dot in cell 3 to the character directly above it in the first row. For example, the character for **L** ( ) comes from the character for **B** ( ) by adding a dot in cell 3.
- Each character in the third row is formed by adding a dot in cell 6 to the character directly above it in the second row. For example, **V** ( ) comes from **L** ( ).
- Each character in the fourth row is formed by removing a dot from cell 3 in the character directly above it in the third row. For example, **gh** ( ) comes from **V** ( ).

The class should find these missing characters.

Q **the** **W**



Exercise 2

Pose a counting problem.

T: ***How many braille characters are shown on Worksheet P1(a)?*** (40)
Are more characters possible using this six-cell dot system? (Yes)
How many different characters are possible?

Encourage students to make predictions, and record some estimates on the board.

T: ***Let's see if we can determine how many characters are possible. What could be in each cell?***

S: ***A blank or a dot.***

T: ***Suppose there were only one cell. How many characters would be possible?*** (2)

Begin a list of possibilities on the board.

T: ***Suppose there were two cells. How many different characters would be possible?***

Instruct students to use the first row of Worksheet P1(b) to draw all of the possible two-cell characters. After a few minutes, record the results on the board.

Number of Cells	Number of Characters	Characters
1	2	□, ⊡
2	4	□□, □⊡, ⊡□, ⊡⊡

T: ***What happens when we add another cell?***

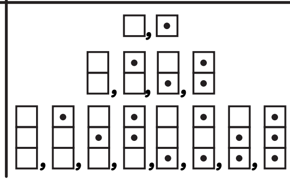
S: ***The number of characters doubles.***

Cover the bottom cell in each of the newly formed characters with a piece of paper.

T: ***What do you notice about the top cell of these characters and the characters with one cell?***

S: ***Each character with one cell is repeated twice.***

Extend the list on the board by asking the class to predict the number of possible characters using three cells. Then instruct students to use the second row on Worksheet P1(b) to draw all of the possible characters. Record the results on the board.

Number of Cells	Number of Characters	Characters
1	2	
2	4	
3	8	

T: ***What happens to the number of possible characters when we add another cell?***

S: ***The number of characters doubles again.***

Cover the bottom cell in each of the newly formed characters and ask the class to notice similarities between the new characters and the characters formed by using only two cells. Students should notice that each character with two cells is repeated twice. If necessary, ask a student to point to a previous character and the two new characters that are similar.

T: ***Why are each of the previous characters repeated twice?***

S: ***Because there are two possibilities for the third cell: a blank or a dot.***

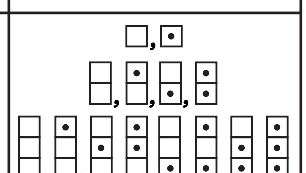
T: ***How many characters would be possible using four cells? Why?***

S: ***16, because the number of characters doubles each time we add another cell.***

S: ***16, because the characters will be the previous eight characters with a blank in the fourth cell and the previous eight characters with a dot in the fourth cell.***

Note: This last comment assumes that the arrangement of the four cells is like the arrangement for three cells with an additional cell attached.

Record this result in the list on the board, and continue in the same manner until you find the number of possible characters using six cells.

Number of Cells	Number of Characters	Characters
1	2	
2	4	
3	8	
4	16	
5	32	
6	64	

A class project could be to determine which of the 64 characters are missing on Worksheet P1(a) and then to research what they represent in the braille system.

Exercise 3

T: ***The original system that Louis Braille invented was inspired by Morse code. What kind of symbols are used in Morse code?***

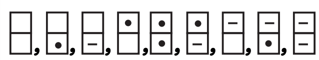
S: ***Dots and dashes.***

T: ***Can you guess how the characters of Braille's original system differed from the characters on Worksheet P1(a)?***

S: ***He used dashes as well as dots.***

T: ***How many characters do you suppose were possible in the original system?***

Accept estimates and then proceed in the same manner as Exercise 2, considering the number of characters using only one cell and then two cells. The bottom half of Worksheet P1(b) is available for students to draw all of the possible characters using two cells. Make another list on the board.

Number of Cells	Number of Characters	Characters
1	3	
2	9	

T: ***What happens to the number of characters when we add another cell?***

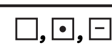
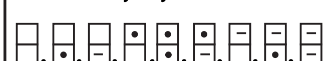
S: ***It multiplies by 3.***

Cover the bottom cell in each of the two-cell characters and compare with the one-cell characters. The class should notice that each one-cell character is repeated three times.

T: ***How many characters would there be if we used three cells?***

S: ***27; because each two-cell character would be repeated three times, once with a blank in the third cell, once with a dot in the third cell, and once with a dash in the third cell. 3 x 9 = 27.***

Record the result in the list, and continue in the same manner until the number of possible characters using six cells is determined.

Number of Cells	Number of Characters	Characters
1	3	
2	9	
3	27	
4	81	
5	243	
6	729	

T: ***Why does the number of characters triple with this system when a cell is added and only double with the standard braille system?***

S: ***In the standard system, there are two possibilities for each cell. Therefore, each time a cell is added the number of characters doubles. In this system there are three possibilities for each cell, so each time a cell is added the number of characters triples.***

Extension Activity

Present students with Morse code and suggest they look for patterns. In Morse code, letters are given signaling elements of from one to four dots or dashes. The dot is a short duration electric current and the dash is a longer duration signal. How many possible signaling elements are possible with from one to four signals? Which signaling elements are not used in Morse code? Are there enough to include Morse code signaling elements for the ten digits 0 to 9? Why does Morse code use five signals for the digits? Blackline P1(b) has the elements for Morse code.

Writing Activity

You may like students to take less on notes on some, most, or even all their math lessons. The "Lesson Notes" section in Notes to the Teacher gives some suggestions and refers to forms in the Blacklines you may provide to students for this purpose. In this lesson, for example, students can note how to determine the number of characters possible in a code with a given number of cells and one or two symbols.

Capsule Lesson Summary

Tell a story about Sylvia who is invited to a beach party on an island. A problem arises since the last ferry leaves in 60 minutes which is not enough time for Sylvia to take her usual route. Find the probability that Sylvia will get to the ferry in time if she follows paths through an unfamiliar swamp.

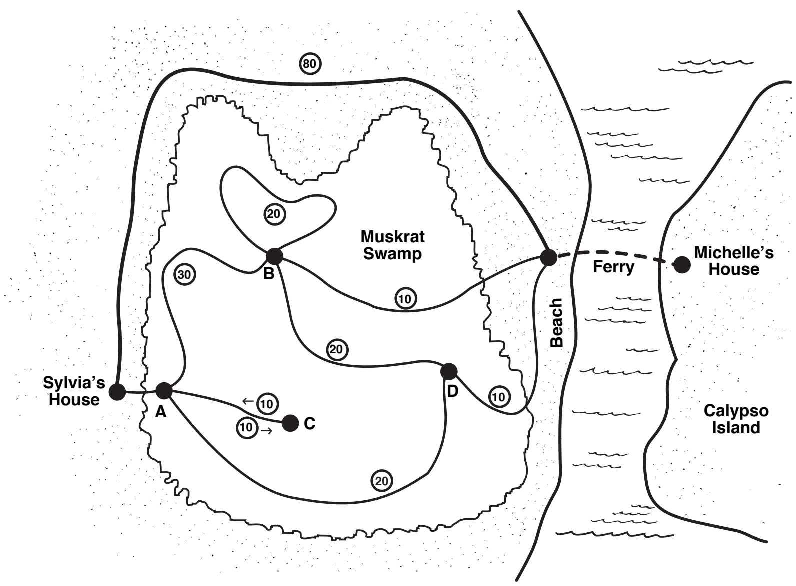
Materials

Teacher		Student	
	• *IG-V Probability Poster* #1		• Colored pencils
	• Colored chalk and pens		• Muskrat Swamp map
	• Meter stick		• Ruler
	• Blackline P2		• Worksheets P2* and **

Advance Preparation: Use Blackline P2 to make copies of the Muskrat Swamp map for students. You may also want to make an overhead transparency.

Description of Lesson

Display the Muskrat Swamp map on *IG-V Probability Poster* #1 or with a transparency.



Refer to the map as you tell the following story about Sylvia's vacation near the coast. Make the story as interesting as possible, stopping occasionally to let students discuss the situation.

T: ***Sylvia enjoys the time she spends every summer with her aunt and uncle near the ocean. She often rides her bicycle to the beach, and she always rides on the main road around Muskrat Swamp.*** (Trace the heavy black path from **Sylvia's House** to the **Beach**.) ***It takes her 80 minutes if she does not stop. There are other paths through Muskrat Swamp on which Sylvia could ride, but she doesn't know which paths to take and is a little afraid of getting lost.***

Sylvia's friend, Michelle, lives on Calypso Island. To visit Michelle, Sylvia must take a ferry which runs only three times a day: morning, noon, and evening.

Let students talk about the story; for example, they may have comments on the swamp, the ferry, the island, and so on.

T: ***Late one afternoon, Michelle calls Sylvia on the telephone and invites her to a party that evening on Calypso Island. They are planning to swim, build a bonfire on the beach, and Sylvia is invited to stay overnight. Sylvia is excited, but she realizes suddenly that it is almost 5:00 and the last ferry to Calypso Island leaves at 6:00. She tells Michelle, "It takes me 80 minutes to bicycle to the ferry so I will be too late. My aunt and uncle are not here now so I can't get a ride with them." Michelle tells Sylvia about the shortcuts through Muskrat Swamp. Sylvia is afraid of the swamp and of getting lost, but Michelle finally convinces her to try it.***

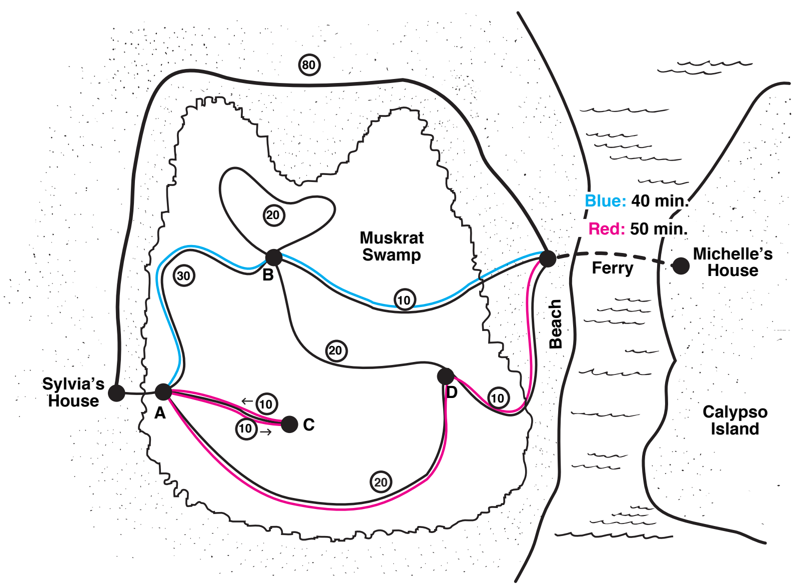
So Sylvia jumps on her bicycle and begins her ride through the swamp. Michelle could not give her good directions and there are no signs, so she has to guess which paths to follow. The number on each path tells how many minutes it takes to ride a bicycle along it. Sylvia has exactly one hour (60 minutes). Do you think she will get to the ferry on time?

Allow students to study the map and express their opinions. Be sure that both the dead-end path from **A** to **C** and the loop at **B** are noticed.

Distribute copies of the map (Blackline P2).

T: ***Who can trace a path that Sylvia could take and get to the ferry in 60 minutes or less?***

Invite several students to trace paths starting at Sylvia's house, ending at the ferry, and taking at most 60 minutes. Two such paths are shown below.

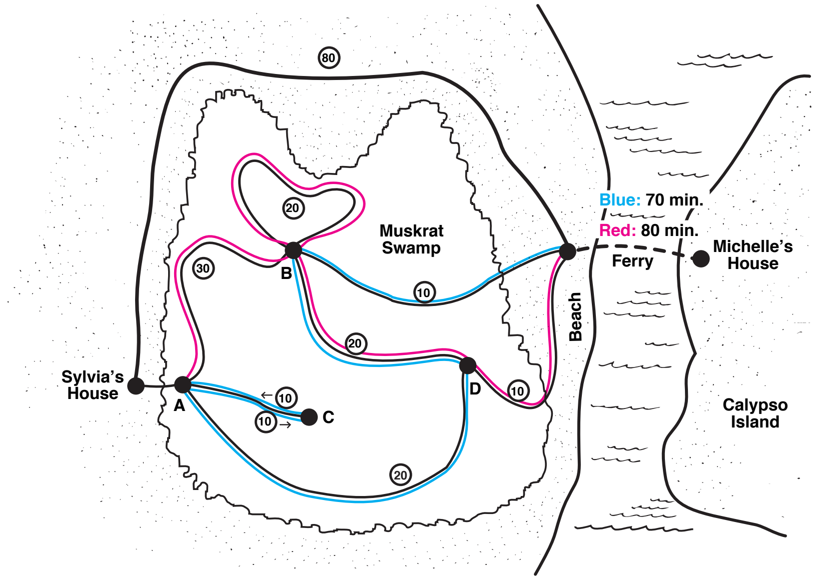


T: ***Do you think Sylvia would take the same path twice?***

Lead to the idea that if Sylvia is clever and marks each path she takes, she can avoid taking the same path twice.

T: ***Who can trace a path that Sylvia could take and not get to the ferry in 60 minutes or less?***

Invite students to trace paths starting at Sylvia's house, ending at the ferry, and taking more than 60 minutes. Two such paths are shown below.



T: ***Sylvia does get to the ferry on time and joins Michelle's party on Calypso Island. When Sylvia arrives, she tells Michelle, "I was really lucky to get to the ferry on time because there are a lot of paths through the swamp." Michelle answers, "There's not much luck involved. It's easy to ride through Muskrat Swamp in 60 minutes or less."***

What do you think? How lucky would Sylvia be if she always had to guess which path to take?

Encourage students to estimate Sylvia's chances or probability of getting through Muskrat Swamp in 60 minutes or less and to list their estimates on the board. Ask students if they feel Sylvia's probability of arriving on time is low, say less than ½, or high, say more than ¾.

Draw a large square on the board.

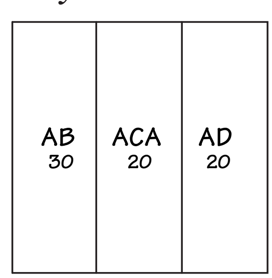
T: ***Let's use a square to calculate Sylvia's probability of riding through the swamp in 60 minutes or less. We must study all the paths Sylvia could take and all the choices she could make. Let's represent these paths and choices on the square.***

Sylvia enters the swamp here (point to **A** on the map)***. How many choices of paths does she have at* A*?***

S: ***Three; she could take the path to* B*, to* D*, or to the dead-end at* C *and back to* A*.***

T: ***How could I show on the square that Sylvia has three choices?***

S: ***Divide the square into thirds because each of the three paths is equally likely to be chosen by Sylvia.***



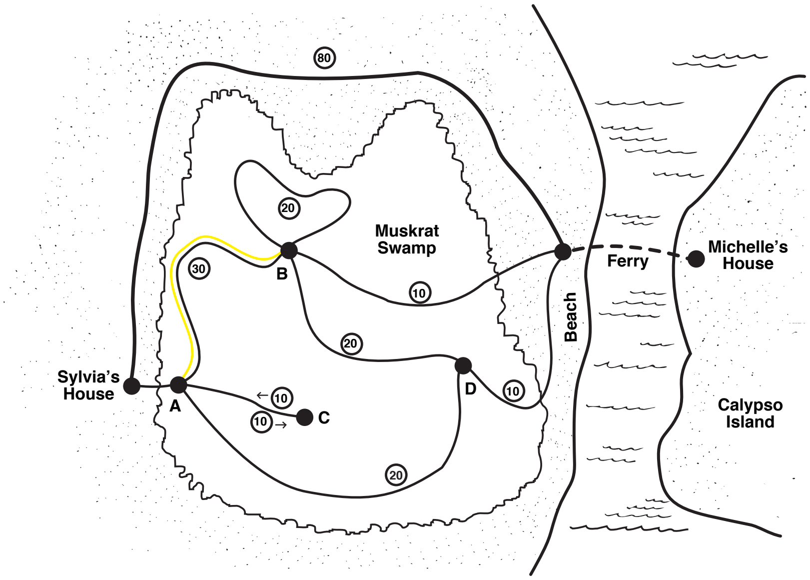
Invite a student to divide the square into thirds, using a meter stick for accuracy. For example, if the square is about 60 cm wide, each section should be about 20 cm wide.

Label each region for one of the paths and the time it would take to ride along that path.

P2

T: ***In each region, indicate a path Sylvia could choose and the number of minutes to ride that path. Suppose Sylvia chooses the path from A to B*** (point to this region)***.***

Highlight the path from **A** to **B** on the map.



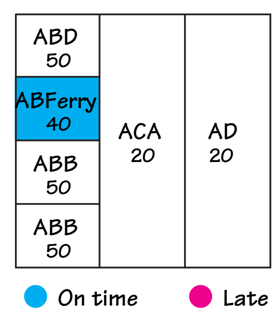
T: ***Now Sylvia is at B. How many paths could she take from B?***

S: ***Four; she could take the path from B to D or from B to the ferry. Also, there are two ways to go around the loop back to B. She won't go back to A.***

T: ***How do I show this on the square?***

S: ***Divide the region into four equal parts.***

Ask students to help you divide the region and then to label its four parts.



Note: The labels should indicate the entire path taken thus far and the total time.

Indicate that the path **ABFerry** gets Sylvia to the ferry on time. Choose a code for marking the regions, as illustrated here.

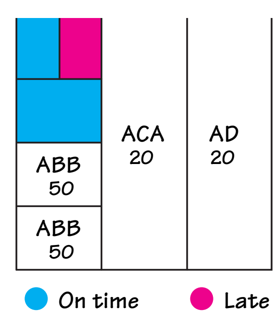
Point to the region for **ABD** and trace a path from **A** to **B** to **D**.

T: ***If Sylvia rides from A to B to D, she uses up 50 minutes. What are her chances of getting to the ferry on time?***

S: ***One out of two. She could take the path to the ferry and arrive just in time (60 minutes), or she could ride to A and use up too much time (70 minutes).***

T: ***How could I show that on the square?***

S: ***Divide the region in half; color one-half red and one-half blue.***



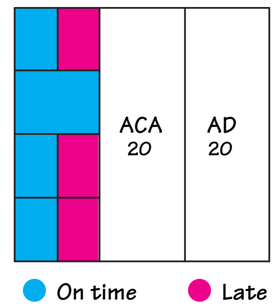
Point to the **ABB** regions and trace the two paths from **A** to **B** and around the loop (once in each direction) to **B**.

T: ***If Sylvia takes one of these paths and uses up 50 minutes, what are her chances of getting to the ferry on time?***

S: ***1 out of 2. She could either go from B directly to the ferry and arrive in time (60 minutes) or go from B to D and use up too much time (70 minutes). She wouldn't go back to A and wouldn't go around the loop again.***

T: ***How do I show this on the square?***

S: ***Divide each region into two pieces of the same size; color one piece red and one piece blue.***



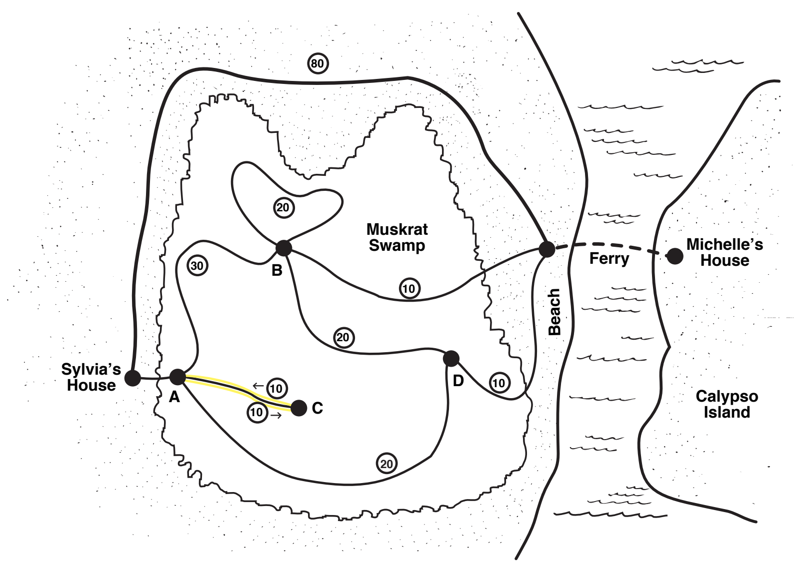
Refer to the third of the square that is colored.

T: ***If Sylvia first chooses the path from A to B, is she more likely to be on time or late for the ferry?***

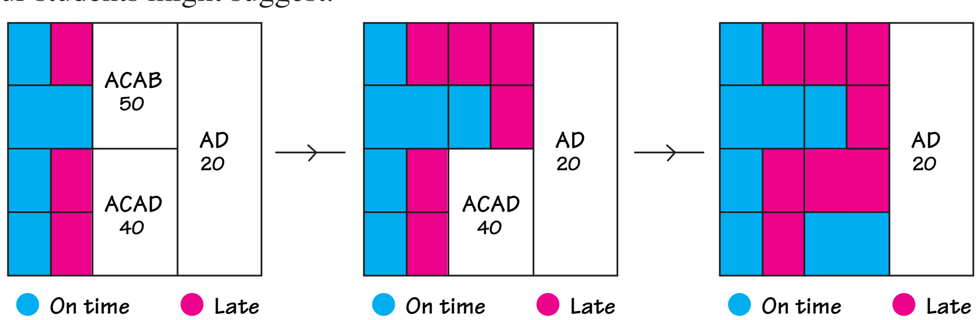
S: ***On time; more than half the region is colored blue.***

T: ***Let's calculate Sylvia's chances of being on time if she first chooses the dead-end path to C and then comes back to A*** (point to the region for **ACA**).

Highlight this path on the map.



Analyze this possibility in a similar manner as above. The following sequence of squares indicates the steps your students might suggest.

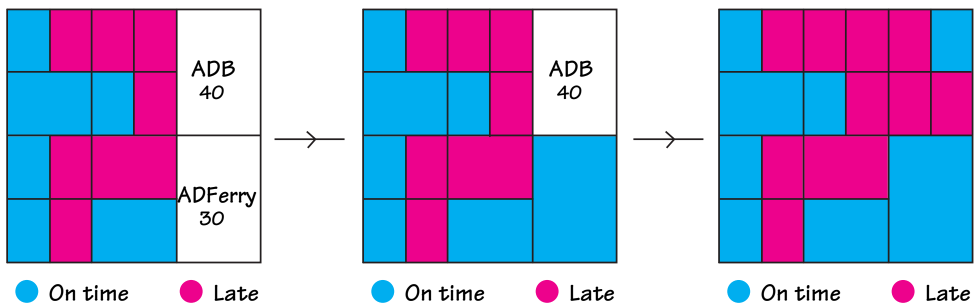


T: ***We still have to calculate Sylvia's chances of being on time if she first chooses the path from* A *to* D** (point to the region for **AD**)**.**

Highlight a path from **A** to **D** on the map.

T: ***On your own paper, try to decide how we should color the last third of the square.***

Let students work (with a partner) for a few minutes, helping those who are having difficulty. Then complete the problem collectively at the board. The following sequence of squares indicates the steps your students might suggest.



T: ***We examined all the different paths Sylvia could take and determined for each if she would get to the ferry on time. Is she more likely to be late or on time?***

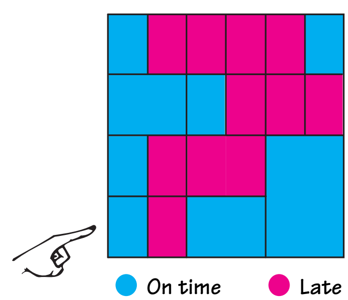
S: ***The chances are about the same because about half the square is red and about half is blue.***

T: ***Can we calculate exactly what Sylvia's chances are to be on time?***

Let students discuss this question. Encourage any suggestion to compare the number of small pieces which fit into the entire red region to the number of small pieces which fit into the entire blue region. If necessary, ask the following questions.

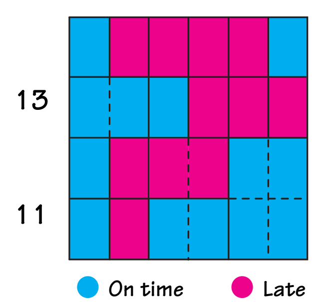
T: ***Let's compare the amount of the square colored red to the amount colored blue. Where is one of the smallest pieces in our division of the square?***

Invite a student to point to one of the smallest pieces; for example:



T: ***How many pieces of this size fit into the red region? … into the blue region?***

The class will likely find it helpful to subdivide the entire square into pieces all the same size.



S: ***13 red and 11 blue.***

T: ***What is Sylvia's probability of getting to the ferry on time?***

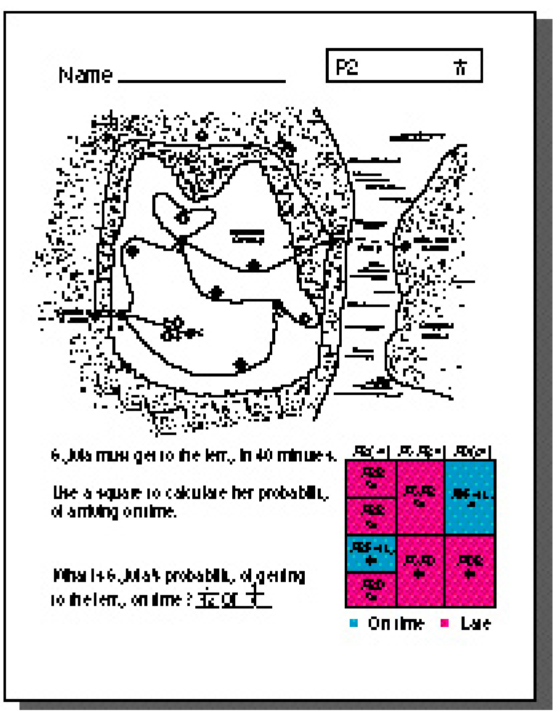
S: ***$^{13}/_{24}$ or 13 out of 24.***

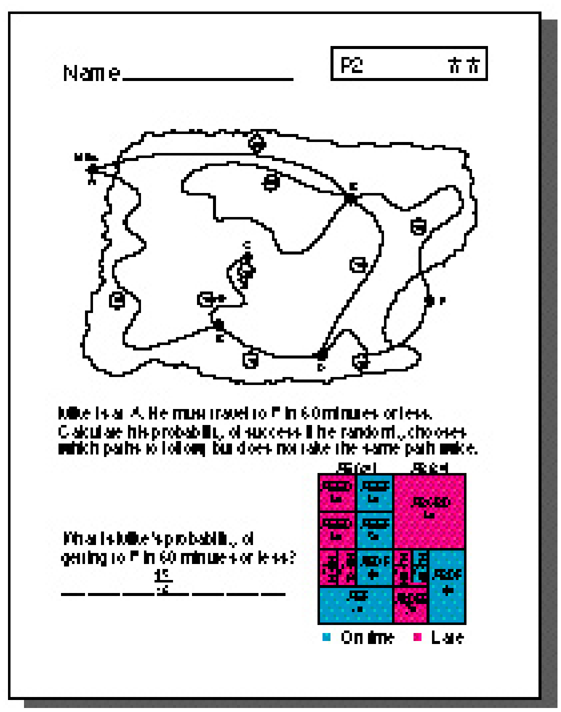
Compare the solution, $^{13}/_{24}$, with students' estimates of Sylvia's chances.

Worksheets P2* and ** are available for individual work. Point out that the map on Worksheet P2* is the same as the one used in class and that the problems are similar except that in this one Sylvia has only 40 minutes to get to the ferry.

Home Activity

Some students may like to take the map of Muskrat Swamp home to tell a family member about Sylvia's problem of getting to the ferry on time (60 minutes or less).





Capsule Lesson Summary

Calculate the probability of forming one long piece of rope when some of the ends of three ropes are joined by two knots. Vary how the ropes are held, and analyze the new situation that results.

Materials

Teacher
- Three identical pieces of light rope or cord, each about one meter in length
- Meter stick
- Colored chalk

Student
- Ruler
- Colored pencils
- Paper
- Worksheets P3(a) and (b)

Description of Lesson

Initiate a discussion of how past civilizations often put much credibility in one individual for decision making or speculation about future events. Over time, these individuals have been known as oracles, sages, prophets, chiefs, wise people, fortune tellers, astrologers, medicine men, and elders, to name but a few.

T: ***Do you know any stories about fortune tellers? What kinds of questions do people ask fortune tellers? What might fortune tellers use to predict the future?***

Exercise 1

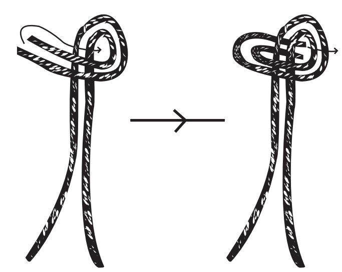
Once you have gained students' interest, tell the following story. Have the three pieces of rope on hand.

T: ***My story is about a fortune teller in a make-believe country. Many young couples planning to marry ask the fortune teller about their future together. For these romantic youths, the fortune teller has a special way to predict the future of proposed marriages using three identical pieces of rope like I have here.***

Show the class the three pieces of rope. Then fold the three ropes in half and hold them as shown here. Twist the strands lightly in your hand so that the students cannot tell which ends belong to the same piece of rope.

T: ***When young people in love ask whether or not a proposed marriage will be long and happy, this fortune teller holds three ropes like I am holding them now. The couple selects two of the six loose ends to tie together. Then they tie a second knot using two of the remaining loose ends. Let's try it.***

Invite two students to tie two knots. You may like to suggest using simple overhand knots to make it easier to untie the knots later.



P3

Note: An alternative would be to tie three knots with three pairs of rope ends. If this is done, the following description of what could happen will need minor adjustments. That is, a long piece would be a big loop with three ropes, a double piece would be a loop with two ropes, and a single piece would be a loop with one rope. In this case, there are three possibilities rather than four.

The corresponding analysis will, however, be similar.

After two knots are tied, there should still be two loose ends.

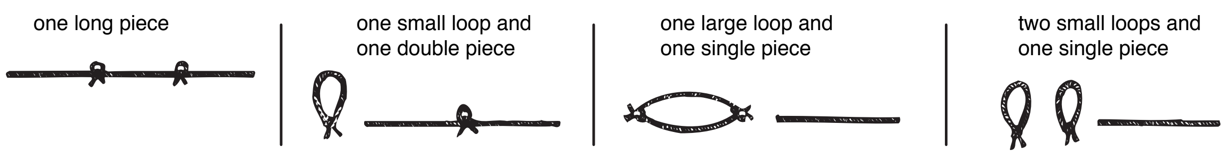
T: ***After two knots are tied, the fortune teller releases the ropes. What could happen?***

S: ***There might be one long piece of rope.***

S: ***There might be some loops.***

T: ***The fortune teller predicts the couple's future marriage will be long and happy if the result is one long piece of rope. Let's check our ropes.***

Release and untangle the ropes. Determine which of the following outcomes occurred.



Note whether or not the fortune teller would predict a long and happy marriage, that is, whether or not one long piece of rope occurs. If you wish, untie the ropes and repeat the experiment two or three more times.

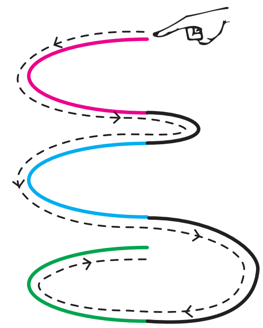
T: ***What do you think the probability is, or the chances are, that the rope will be one long piece after two knots are tied?***

List predictions of several students. Point out that the one trial (or the few trials) of the experiment made in class may not necessarily indicate the probability of success for one long piece.

Draw this picture on the board.

T: ***This is a picture of the three ropes. I've drawn each in a different color so that it is easier to talk about them. But, of course, the ropes are all the same so the couple will not know how to get a success.***

Draw two connectors, as shown here.

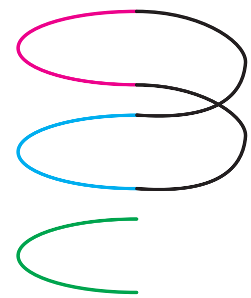


T: ***The connectors show which ends are tied in two knots. Would these two knots make one long piece of rope?***

S: ***Yes.***

Invite a student to trace the long piece of rope by starting at one loose end and following the ropes and connectors until the other loose end is reached. For example:

Change the connectors in the picture.

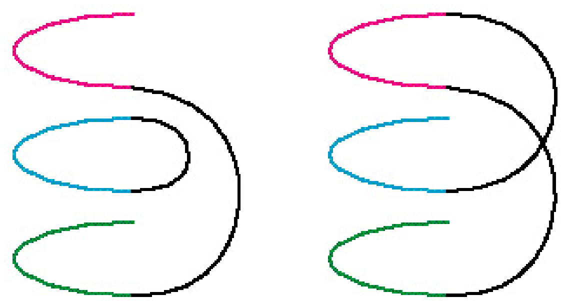


T: ***If these two knots were tied, would one long piece of rope result?***

S: ***No, the green rope is not attached.***

S: ***No, the red and blue ropes form a loop.***

Discuss one or two other possibilities in a similar manner, such as those illustrated here:



two pieces   one long piece

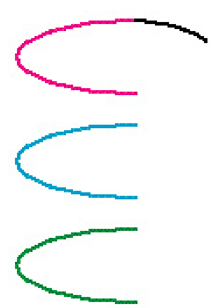
Erase any connectors from the picture and draw a square nearby.

T: ***Let's use this square to calculate the probability that one long piece of rope will result when two knots are tied randomly. We can analyze the knot-tiers' actions step-by-step. First, the couple selects one end to use in the first knot. Does it matter which end they start with?***

S: ***No.***

T: ***Since it doesn't matter which end they start with, suppose that the couple chooses an end of the red rope. Remember, the couple cannot distinguish the ropes; they are not really colored.***

Begin drawing a connector from one of the red ends.



T: ***Next, they tie this end to one of the other loose ends. In how many different ways can the first knot be completed?***

S: ***Five, since there are five other loose ends—one red, two blue, and two green.***

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Assuming the five possibilities are equally likely, divide the square into five parts of the same size. Use a meter stick for accuracy. Put colored dots above the parts to indicate which ropes are tied together (see the next illustration).

T: ***Would any of these five possible first knots be lucky or unlucky for the knot-tiers?***

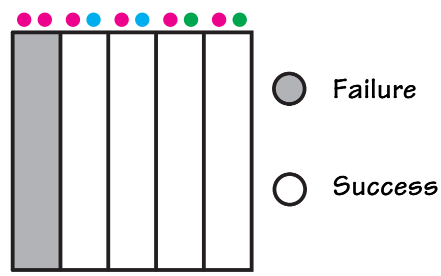
S: ***Tying the two red ends together would be unlucky since a loop would be formed.***

S: ***Tying the red end to a blue end or to a green end would be all right for the first knot since no loop is formed. But we don't know what will happen when a second knot is tied.***

T: ***What is the probability that the knot-tiers would be unlucky and would form a loop with the first knot?***

S: ***$\frac{1}{5}$; they have one chance out of five of forming a loop with the first knot.***

Indicate a code for failure or success (shaded or unshaded) next to the square, and shade the region for red-red.



T: ***We shade the first region to show that failure results if the first knot joins the two red ends. Now we must study what happens if the first knot joins a red end to one of the other four ends.***

Refer to the picture of the three ropes.

T: ***Which is luckier for the knot-tiers: to tie the red end to a blue end for the first knot or to tie the red end to a green end? Does it matter?***

S: ***It doesn't matter. There's no difference between the blue rope and the green rope.***

T: ***If the red end is tied to a blue end, does it matter which blue end it is tied to?***

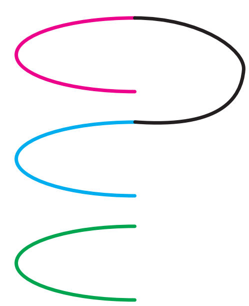
S: ***No.***

T (pointing to the four unshaded regions)**:** ***So the probability of success will be the same for each of these four knots. We need only determine how to shade one of these regions; the other three regions will be the same.***

Point to the left-most unshaded region.

T: ***Let's determine how to shade this region; that is, let's calculate the probability of success if the first knot joins the red end to one of the blue ends.***

Represent this situation by picturing a connection from a red end to a blue end.



T: ***How many different second knots could be tied?***

Invite students to the board to show all of the possibilities for the second knot. Continue until the class agrees the answer is six and students have shown the six different knots in a systematic way.

Distribute copies of Worksheet P3(a).

T: ***The six different ways the second knot could be tied are shown on this worksheet. For each picture, determine whether or not the two knots make one long piece of rope.***

Continue the collective lesson after most students have completed the worksheet.

T: ***Look at your worksheet. In how many of the six possibilities is one long piece of rope formed?***

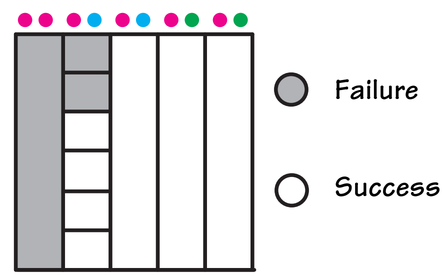
S: ***Four.***

T (pointing to the left-most unshaded region)**:** ***We are trying to shade this region. The first knot joins a red end to a blue end. What is the probability that the second knot will produce one long piece of rope?***

S: ***$\frac{4}{6}$ or $\frac{2}{3}$; there are two chances out of three.***

T: ***Explain your answer.***

S: ***We found that there were six different ways to tie the second knot, and four of those six ways resulted in one long piece of rope.***



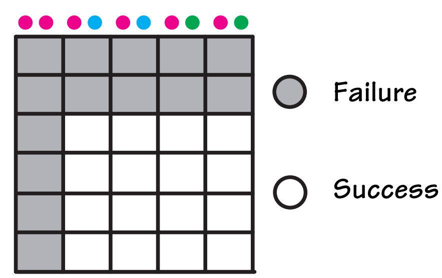
T: ***How should we shade the region?***

S: ***Divide the region into six parts of the same size; shade two parts for failure and leave four parts unshaded for success.***

S: ***It's the same if we divide that region into three parts, shade one part, and leave two parts unshaded.***

T: ***How should we shade the other three regions?***

S: ***The same as the region we just shaded, since we agreed earlier that the probability of success would be the same for each of those four regions.***



T: ***The shading for failure is complete, so now we can calculate the probability that one piece of rope will result when two knots are tied. What are the chances for success? What are the chances for failure?***

S: ***Sixteen chances for success and fourteen for failure.***

T: ***What is the probability of success?***

S: ***$\frac{16}{30}$ or $\frac{8}{15}$.***

T: ***Is success or failure more likely?***

S: ***Success, since 16 is more than 14.***

S: ***Success, since $\frac{16}{30}$ is greater than $\frac{1}{2}$.***

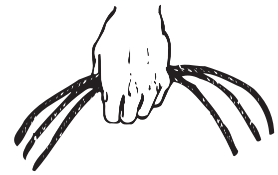
Compare earlier predictions to this result.

Exercise 2

Continue with the story about the fortune teller.

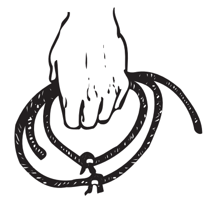
T: ***Sometimes a couple is very disappointed if the fortune teller predicts that their marriage will not be long and happy. They may ask to try again.***

Occasionally, the fortune teller agrees to give them another chance, using a different method. The three ropes are held like this.



Hold the three ropes as shown above. Twist the ropes slightly in your hand so that it is not easy to tell if two ends belong to the same rope.

T: ***The goal is the same: to form one long piece of rope by tying two knots. This time the ropes are held differently and each knot must join an end on one side with an end on the other. Who would like to try this new method?***



Select students to tie two knots according to your instructions.

Release and untangle the ropes. Determine if one long piece of rope is formed. If you wish, untie the ropes and repeat the experiment two or three more times.

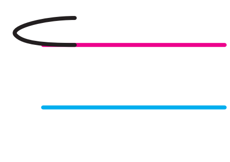
T: ***What do you think the probability of success is this time? Which of the two methods do you think has the higher probability of success?***

List some predictions on the board. Direct students to turn to Worksheet P3(b). Draw the picture from the worksheet on the board.

T: ***Let's calculate the probability of success using this second method of holding the ropes. I've drawn the three ropes straight this time to show how they are held. For the first knot, does it matter which end we start with?***

S: ***No.***

T: ***Let's start with a red end. Remember that the new method requires that each knot must join one end from the left side to one end from the right side. Now, on your worksheet, calculate the probability of success.***

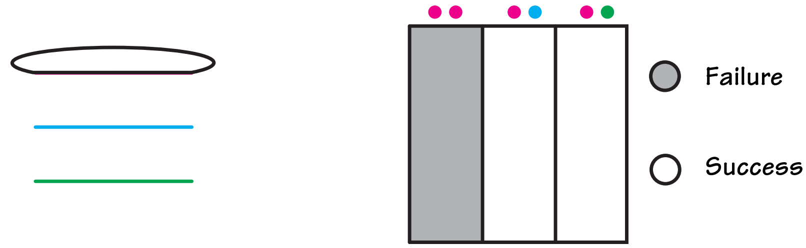


Let students work independently or with partners for a few minutes. Then do the first steps of the problem collectively.

T: ***What should we do first?***

S: ***Divide the square into three equal parts since the red end on the left can be joined to one of the three ends on the right.***

S: ***The region for red joined to red can be shaded since then the first knot forms a loop, which means failure.***



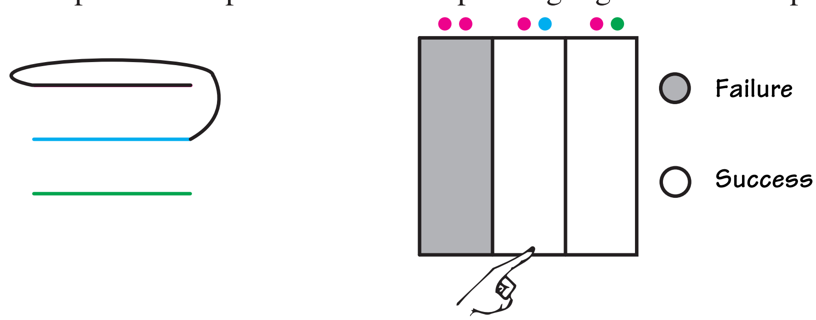
T: ***Complete the problem by determining what happens when the first knot joins a red end to either a blue end or a green end.***

After many students have finished, complete the solution collectively.

T: ***Does it matter whether the first knot joins the red end to a blue end or to a green end?***

S: ***No, the ropes are identical. The probability of success will be the same for both cases.***

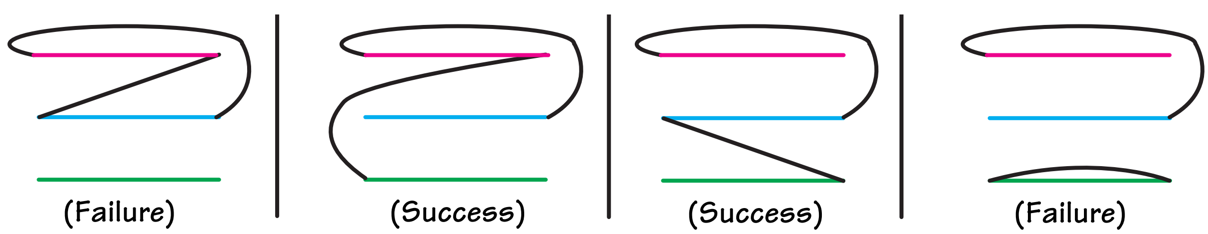
Add a connector to the picture and point to the corresponding region. For example:



T: ***Suppose that the first knot joins a red end to a blue end. How many different second knots could be tied?***

S: ***Four.***

Invite a student to trace the four possible second knots, as shown below. You may wish to sketch the pictures on the board.



T: ***Which of these four possibilities result in success? … in failure?***

The class should determine that two possibilities result in success and two result in failure, as indicated in the preceding illustration. Invite a student to divide the middle region of the square, and shade it appropriately.

T: ***How should we shade this last region?***

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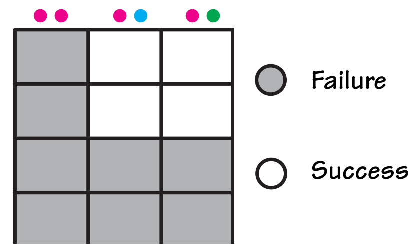
S: *The same as the middle region, since it doesn't matter whether the first knot joins the red end to a blue end or to a green end.*



T: *What are the chances for success? … for failure?*

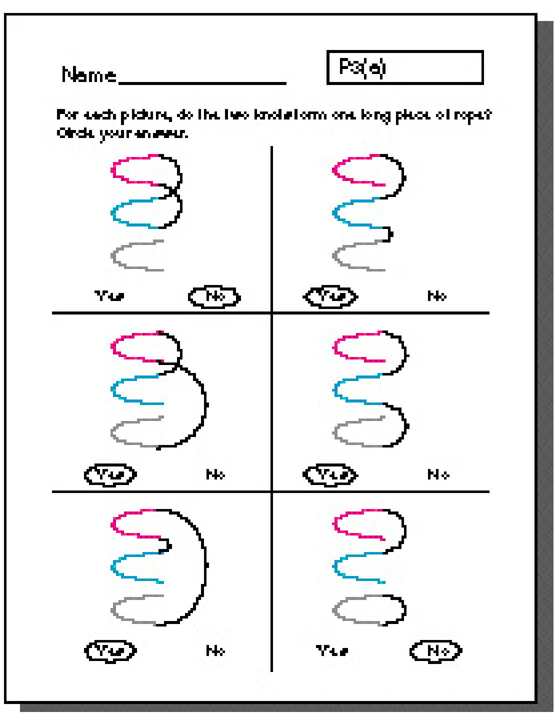
S: *Eight chances for failure and four chances for success.*

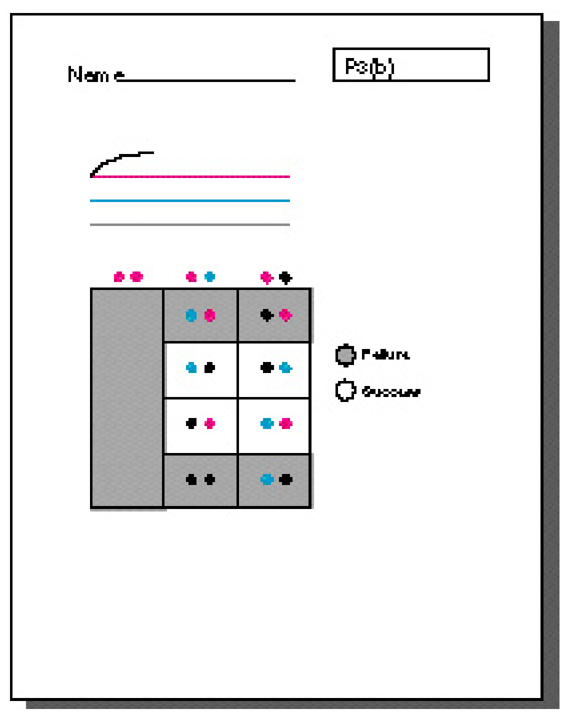
T: *What is the probability of success?*

S: *$\frac{4}{12}$ or $\frac{1}{3}$.*

T: *Which method of holding the ropes is better for a couple?*

S: *The first method; $\frac{16}{30}$ (about $\frac{1}{2}$) is greater than $\frac{1}{3}$.*





Capsule Lesson Summary

Conduct an experiment to measure students' reaction time when attempting to catch a falling ruler. Calculate the mean, median, and mode of each student's data in order to determine the student(s) with the fastest reaction time.

Materials

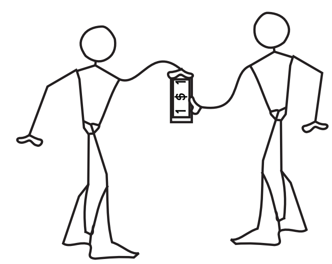
Teacher
- Crisp dollar bill

Student
- 30 cm ruler
- Calculator
- Worksheets P4(a), (b) and (c)

Description of Lesson

Note: In this lesson we make use of three statistical averages: mean, mode, and median. The lesson explains how to compute each of these numbers for a given set of data. The term *average* in everyday life often refers to the mean; however, in this lesson, use the specific terms *mean*, *mode*, and *median*. You may wish to discuss these distinctions with your students during the lesson.

Select a student and conduct the following experiment.



Hold the end of a crisp, new dollar bill so that it is hanging vertically. Tell the student to hold a thumb and forefinger apart on either side of the dollar bill near the center.

T: ***When I drop the bill, try to catch it by closing your thumb and forefinger. Do not move your hand down.***

Drop the bill and let the student try to catch it. Repeat the experiment with several other students. Most likely all, or nearly all, of the students will fail to catch the bill.

T: ***Why is the dollar bill so hard to catch?***

S: ***It falls too fast.***

S: ***We don't know when you will release the bill.***

T: ***Yes. It takes a certain amount of time for you to react and catch the bill. This is called*** **reaction time.**

Write this term on the board.

T: ***What must your brain and body do in order for you to catch the bill?***

Lead a discussion about the steps the brain and body perform:

1. The eyes see the bill start falling and send a message to the brain.
2. The brain receives that information and decides what to do.
3. The brain sends a message to the fingers telling them to close.

T: ***Each of these steps takes time, and the total amount of time is your reaction time.***

You are going to work with a partner and drop rulers to measure your reaction time. Why do you think we will use rulers instead of dollar bills?

S: ***You don't have enough dollar bills.***

S: ***Rulers fall more smoothly than dollar bills.***

S: ***Rulers are longer than dollar bills.***

S: ***With a ruler, we can measure how far it falls.***

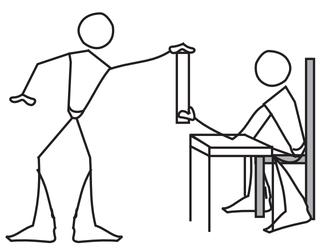
Pair students and equip each pair with a 30 cm metric ruler. Distribute Worksheet P4(a). Let students try the experiment a few times (without recording the results) before continuing.

T: ***We want to measure and compare your reaction times. In order to compare, we must all do the same thing. Who can suggest some rules for everyone to follow?***

Lead the class to establish rules similar to the following:

- The catcher's elbow must be on the table with the forearm held horizontally.
- The catcher's thumb and forefinger must be apart, away from the ruler about 2 cm on each side.
- The dropper holds the ruler with 0 at the bottom, even with the catcher's fingers.
- After a drop, measure how far the ruler drops to the nearest centimeter.
- If the catcher misses the ruler entirely and it falls to the floor, record that try as 30 cm.
- Repeat the experiment ten times and record the data on Worksheet P4(a).

Select one pair of students to demonstrate the technique three or four times at the front of the room. Draw a chart on the board to record the results, for example:



Trial	1	2	3	4	5	6	…
Drop (cm)	17	30	21	15			

The student pairs should each take a turn at being the catcher, repeating the experiment ten times. On Worksheet P4(a), a student should record only the data from his/her ten tries at being the catcher.

T: ***When you finish ten trials each, graph your own results. Connect the dots on your graph with line segments to form a line graph.***

Observe the activity to confirm that students perform the experiment correctly. As necessary, assist students who need help drawing a line graph of their data.

When most students have finished, continue the collective discussion.

T: ***We want to know who in this class has the fastest reaction time. How can we determine who is fastest?***

Encourage students to suggest several methods, for example, finding the lowest total or the lowest mean or the single smallest measurement.

Call attention to Worksheet P4(b).

T: ***Before determining who in our class has the fastest reaction time, let's look at the data on Worksheet P4(b) for four imaginary students. Each of these students thinks he or she has the fastest reaction time, but each for a different reason. Try to find the reason why each student believes he or she has the fastest reaction time.***

Let students work at least five minutes on their own or with their partners. Be sure students know that they may use calculators. While students are working, draw the table from the next illustration on the board, leaving room to draw another column on the left.

T: ***Select one of the four people and tell me why you think that person believes his or her reaction time is the fastest.***

Your students might suggest two or three of the following reasons. Explain other reasons yourself. As you consider each reason, compute the appropriate number for the four people and add one line of data to the table on the board.

S: ***Arnold might believe that he has the fastest reaction time because his drop of 5 cm is the single best measurement amongst all of the data.***

Direct students to find the single best result for each of the four students, and enter that result in the table. (See the first line in the table below.)

S: ***Lucy might believe that she has the fastest reaction time because her total of 156 cm and her mean of 15.6 cm (156 ÷ 10) are the lowest.***

Review how to compute the mean (add the ten numbers and divide by 10). Ask students to compute the mean for all four students.

Reason	Arnold	Lucy	Pierre	Michelle
Best result (cm)	5	7	14	11
Mean (cm)	16.9	15.6	16.3	16.5

S: ***Pierre might believe that he has the fastest reaction time because his most common result, 15 cm, is lower than anyone else's most common result.***

T: ***This number is called the mode. Pierre feels that he is most consistent.***

Ask students to find modes for all four students (see the third line in the following table). Note that Lucy has three modes.

S: ***Michelle might believe that she has the fastest reaction time because she has the fastest middle result. She arranges her ten results in order (11, 12, 12, 13, 13, 16, 16, 16, 28, 28) and looks at the average of the middle two results: $\frac{13+16}{2}$ = 14.5.***

T: ***This number is her median. She has the best median of the four people.***

Ask students to compute medians for all four students. Stress that you put the ten numbers for a person in order before looking at the two middle results.

Reason	Arnold	Lucy	Pierre	Michelle
Best result (cm)	5	7	14	11
Mean (cm)	16.9	15.6	16.3	16.5
Mode (cm)	19	16 or 17 or 18	15	16
Median (cm)	18.5	16.5	15	14.5

T: ***Each person has a good reason for claiming to have the fastest reaction time. Do you understand their reasons? Who do you think has the best claim for being fastest?***

Discuss student comments and compare the four reasons.

Distribute Worksheet P4(c).

T: ***Let's use each of these four methods to determine who in this class has the fastest reaction time. To do this, use the data from your ten trials to compute these four numbers (best result, mean, mode, and median).***

After allowing time for individual students or partners to complete Worksheet P4(c), ask students for their best result and determine the best single result in the class. Also find the lowest mean, mode, and median. Record this information in a column added to the left in the table on the board. For example:

In our class		Reason
Nathan	6 cm	Best result (cm)
Carla	11.2 cm	Mean (cm)
Melinda & Ameed	14 cm	Mode (cm)
Carla	11.5 cm	Median (cm)

Let students express opinions about who they believe has the fastest reaction time. There is no one correct answer to this question; it is a matter of interpretation.

For your information, some advantages and disadvantages of using each method are listed below. Discuss some of these, especially if mentioned by students.

Method	Advantages	Disadvantages
Single Best Result	Many winners, especially in athletics (for example, the long jump or shot put), are determined by the one best effort.	Only one measurement counts for each person.
Mean	All data count equally.	One or two bad (or good) results strongly affect the mean.
Mode	To some extent, the mode reflects consistency.	The mode often reflects only two or three values and is easily changed by new data.
Median	All data affect the median, but extreme data does not affect it strongly.	Very inconsistent data may push the median one way.

Instruct students to look at the line graphs of their data.

T: ***If a person has a consistently fast reaction time, what will the line graph look like?***

S: ***The dots and lines will be near the bottom of the graph.***

T: ***What will the graph for a person with a consistently slow reaction time look like?***

S: ***The dots and lines will be near the top of the graph.***

T: ***If a person's reaction time improves during the ten trials, what will the line graph look like?***

S: ***The dots and lines will go from the upper left to the lower right.***

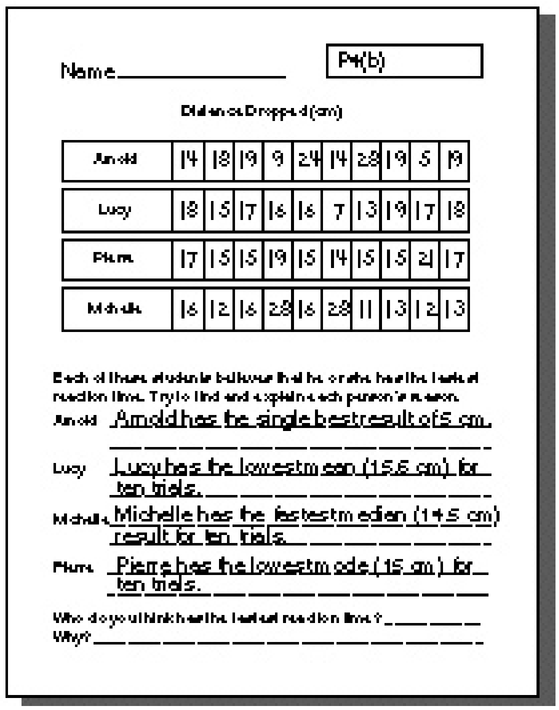
Note: The highly erratic results in this experiment indicate that this is not a very reliable technique for measuring reaction time. Still, this lesson is useful to illustrate reaction time, it is simple to perform, and it requires students to collect and analyze data.

Writing Activity

Suggest that students write about the line graphs for their reaction time.

Home Activity

Some students may like to use the experiment from this lesson to determine who in their family has the fastest reaction time.



Name________________

P4(b)

Distance Dropped (cm)

Arnold	14	18	19	9	24	14	28	19	5	19
Lucy	18	15	17	16	16	7	13	19	17	18
Pierre	17	15	15	19	15	14	15	15	21	17
Michelle	16	12	16	28	16	28	11	13	12	13

Each of these students believes that he or she has the fastest reaction time. Try to find and explain each person's reason.

Arnold: Arnold has the single best result of 5 cm.

Lucy: Lucy has the lowest mean (15.6 cm) for ten trials.

Michelle: Michelle has the fastest median (14.5 cm) result for ten trials.

Pierre: Pierre has the lowest mode (15 cm) for ten trials.

Who do you think has the fastest reaction time? ________
Why? ________________________

Capsule Lesson Summary

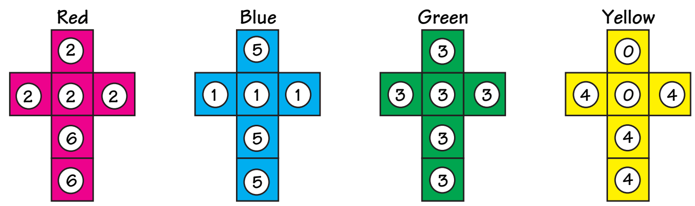
Analyze a two-person game played with four cubes to determine which player is favored and to find the best strategies. Discover some surprising results about the game, results that may be contraintuitive.

Materials

Teacher
- Colored chalk
- Four number cubes

Student
- Paper
- Ruler
- Worksheets P5(a) and (b)

Advance Preparation: Before the lesson begins, put number labels on four colored cubes, as shown below.



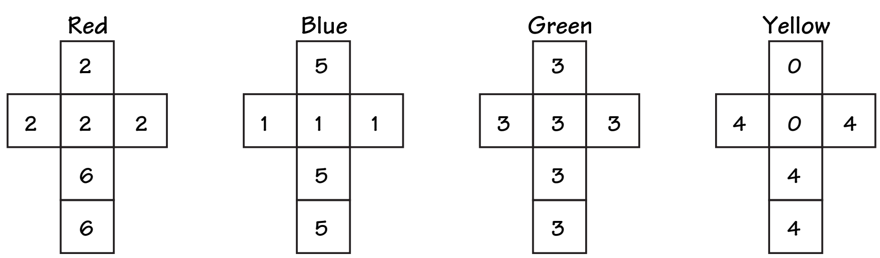
Description of Lesson

Note: Control the length of discussions and pace of the lesson so that students begin worksheets by the middle of the class period.

Draw four maps of a cube on the board. Label one for each of the four colors.

T: ***I have a game to show you. We play it with these four cubes.***

Hold up each of the four cubes for the class to observe. Let four students describe the cubes and direct other students to label each map of a cube on the board. (The face positions of the numbers on a map is not important for this lesson.)



Select a student to be your opponent in the first game.

T: ***I will choose one of the four cubes and then you*** (student opponent) ***can select any one of the other three cubes. We roll the cubes we've chosen and whoever gets the higher number wins.***

Play the game several times. Each time you, the teacher, select a cube first and then let your opponent select a cube. Encourage players to comment on why they select the cubes they do.

†These cubes were designed by Bradley Efron, a statistician at Stanford University. See Martin Gardner's *Mathematical Games*, Scientific American, December 1970.

T: ***Do you think this is a fair game?***

Encourage students to explain why they think that the game is fair, or why they think the game favors either the teacher or the student. Students may suggest ways to make the game fair. Accept such suggestions, but remind the class that your question concerns whether *this* game is fair. After the discussion, you may wish to poll the class on their opinions.

T: ***If you feel that the teacher is favored, what is the teacher's best strategy? If you feel that the student is favored, what is the student's best strategy?***

Discuss strategies students feel each player should use.

T: ***Let's use probability methods to determine who is favored in this game and to learn each player's best strategy.***

Draw a square on the board

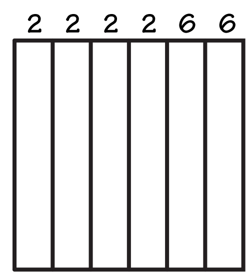
T: ***Let's use this square to analyze the game, assuming that the teacher selects the red cube and the student selects the blue cube. Does it matter who rolls first?*** (No)

Suppose the teacher rolls first. What are the possible outcomes?

S: ***The teacher could roll a 2 or a 6.***

T: ***How can we show this on the square?***

S: ***Divide the square into six equal parts for the cube's six faces. Label four parts 2 and the other two parts 6.***



Teacher: Red
Student: Blue

S: ***You could divide the square into three equal parts and label two parts 2 and one part 6.***

T: ***Yes, either way would be okay. I'll use six parts to remind us of the six faces of the cube.***

What happens if the teacher rolls a 6?

S: ***The teacher wins for sure.***

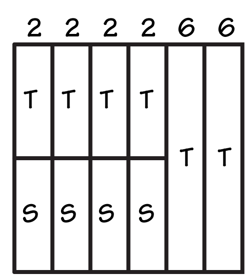
Put **T** in the two regions for 6, and point to one of the regions for 2.

T: ***What happens if the teacher rolls a 2?***

S: ***The student wins about half the time, since the blue cube has three 1s and three 5s.***

T: ***How can we show that on the square?***

S: ***Divide each region for 2 in half. Put*** **T*****, for teacher, in one half and*** **S*****, for student, in the other half.***



Teacher: Red
Student: Blue

T: ***Who is favored if the teacher selects the red cube and the student selects the blue cube?***

S: ***Teacher, since*** **T** ***has $^2/_3$ of the square and*** **S** ***has only $^1/_3$.***

T: ***What is each player's probability of winning?***

S: ***$^8/_{12}$ or $^2/_3$ for the teacher; $^4/_{12}$ or $^1/_3$ for the student.***

You may divide the regions for 6 in half to get pieces all the same size. Then eight pieces are for **T** and four pieces are **S**. Record the probabilities on the board.

T: ***The probability of red beating blue is ⅔.***

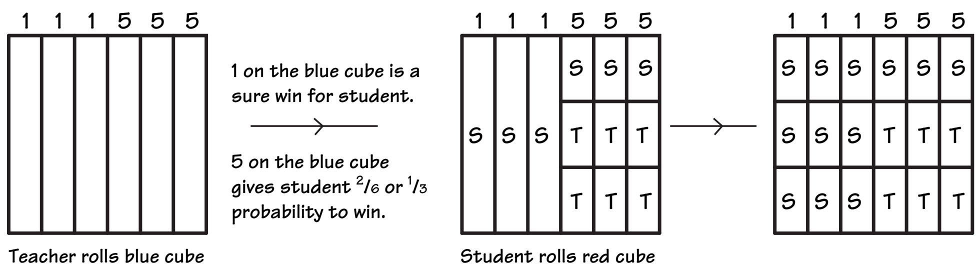
Record this information on the board as indicated here:

$p\ (R, B) = \frac{2}{3}$

T: ***If the teacher chooses the blue cube and the student chooses the red cube, what are their probabilities of winning?***

S: ***Switch the* T*s and* S*s in the square. The student would have a probability of ⅔ of winning; the teacher, ⅓.***

Note: If students suggest an analysis assuming the teacher rolls the blue cube first, the division of the square could look different, but the winning probabilities still will be: teacher ⅓, student ⅔.



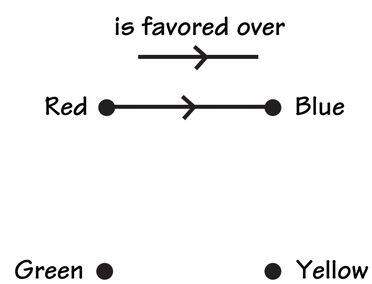
Teacher: Blue $\frac{6}{18} = \frac{1}{3}$

Student: Red $\frac{12}{18} = \frac{2}{3}$

T: ***The probability of blue beating red is ⅓.***

$p\ (B, R) = \frac{1}{3}$

Draw this picture on the board.

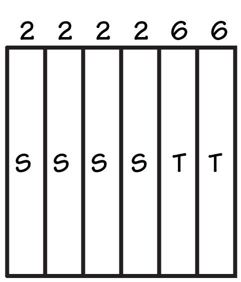


T: ***This arrow picture records the fact that the red cube is favored over the blue cube. Do you think that the green cube or the yellow cube would be a better choice for the student when the teacher selects the red cube?***

S: ***I think that the green cube is favored over the red cube.***

T: ***Let's analyze the game supposing that the teacher selects the red cube and the student selects the green cube.***

Analyze this game in a manner similar to the previous game, and record the result in the arrow picture.

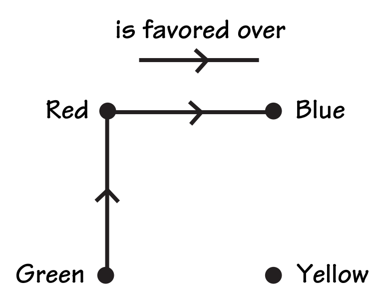


$p(G,R) = \frac{2}{3}$

Teacher: Red $\frac{2}{6} = \frac{1}{3}$

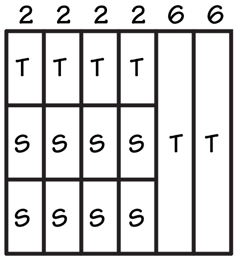
Student: Green $\frac{4}{6} = \frac{2}{3}$

$p(R,G) = \frac{1}{3}$



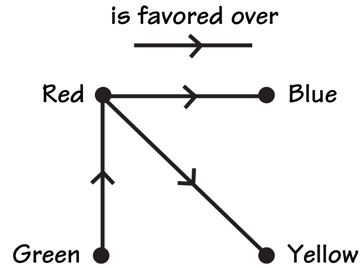
P5

In a similar manner, analyze the game in the case in which the teacher selects the red cube and the student selects the yellow cube. Draw an arrow from red to yellow to record the result.



Teacher: Red $\frac{10}{8} = \frac{5}{9}$

Student: Yellow $\frac{8}{18} = \frac{4}{9}$



$p(R,Y) = \frac{5}{9}$ $p(Y,R) = \frac{4}{9}$

T: ***In this situation, is it a good strategy for the teacher to select the red cube?***

S: ***No, because the student could select the green cube and be favored to win.***

T: ***Which cube might be better for the teacher?***

S: ***The green cube, because the green cube is favored over the red cube.***

Students might suggest that you draw an arrow from green to blue since green is favored over red and red is favored over blue. Also, they might suggest drawing an arrow from green to yellow. Indicate that these are interesting ideas, but that these suggestions need to be checked by further analysis.

Distribute copies of Worksheets P5(a) and (b). Divide the class into three groups, and write this information on the board.

Group 1	Group 2	Group 3
Teacher selects the blue cube	Teacher selects the green cube	Teacher selects the yellow cube
p(B,G) =_____	p(G,B) =_____	p(Y,B) =_____
p(B,Y) =_____	p(G,Y) =_____	p(Y,G) =_____

T: ***I want you to complete the analysis of this game on the worksheets. Each group has two problems to solve, one problem for each side of the worksheet. Those in group 1 should assume that the teacher selects the blue cube. Then they should calculate the probability of the blue cube winning over the green cube and the probability of the blue cube winning over the yellow cube.***

In a similar manner, explain the responsibilities of group 2 and group 3.

T: ***On your worksheets, write your group number and use the squares to solve the two problems assigned to your group.***

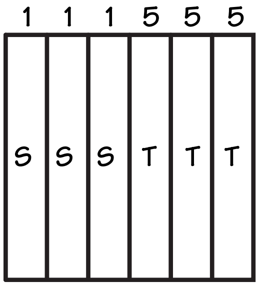
Note: You may want to form six groups and assign two groups to each pair of problems.

As you observe the group, you may need to help some get started on their problems. If some groups finish quickly, suggest they solve the problems from another group. The following pictures indicate possible solution methods for the six problems.

Group 1

Teacher selects the blue cube (1, 1, 1, 5, 5, 5).

Student selects the green cube (3, 3, 3, 3, 3, 3).



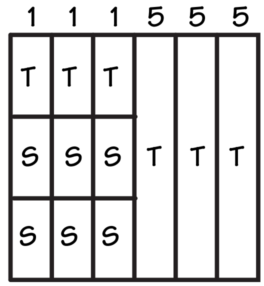
Teacher: $\frac{3}{6} = \frac{1}{2}$

Student: $\frac{3}{6} = \frac{1}{2}$

$p(B,G) = \frac{1}{2}$

$p(G,B) = \frac{1}{2}$

Student selects the yellow cube (0, 0, 4, 4, 4, 4).



Teacher: $\frac{12}{18} = \frac{2}{3}$

Student: $\frac{6}{18} = \frac{1}{3}$

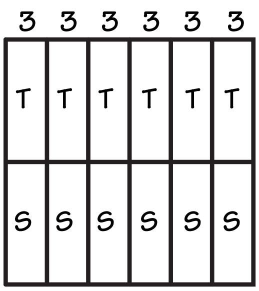
$p(B,Y) = \frac{2}{3}$

$p(Y,B) = \frac{1}{3}$

Group 2

Teacher selects the green cube (3, 3, 3, 3, 3, 3).

Student selects the blue cube (1, 1, 1, 5, 5, 5).



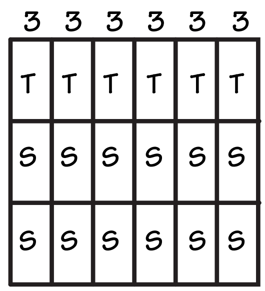
Teacher: $\frac{6}{12} = \frac{1}{2}$

Student: $\frac{6}{12} = \frac{1}{2}$

$p(G,B) = \frac{1}{2}$

$p(B,G) = \frac{1}{2}$

Student selects the yellow cube (0, 0, 4, 4, 4, 4).



Teacher: $\frac{6}{18} = \frac{1}{3}$

Student: $\frac{12}{18} = \frac{2}{3}$

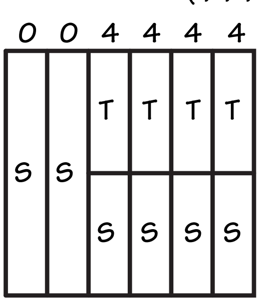
$p(G,Y) = \frac{1}{3}$

$p(Y,G) = \frac{2}{3}$

Group 3

Teacher selects the yellow cube (0, 0, 4, 4, 4, 4).

Student selects the blue cube (1, 1, 1, 5, 5, 5).



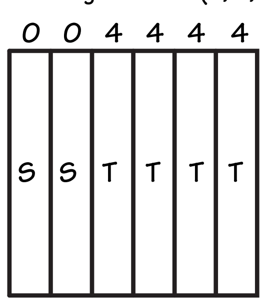
Teacher: $\frac{4}{12} = \frac{1}{3}$

Student: $\frac{8}{12} = \frac{2}{3}$

$p(Y,B) = \frac{1}{3}$

$p(B,Y) = \frac{2}{3}$

Student selects the green cube (3, 3, 3, 3, 3, 3).



Teacher: $\frac{4}{6} = \frac{2}{3}$

Student: $\frac{2}{6} = \frac{1}{3}$

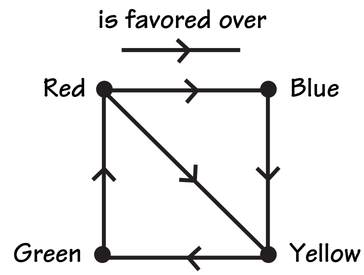
$p(Y,G) = \frac{2}{3}$

$p(G,Y) = \frac{1}{3}$

P5

During the last ten minutes of the class period, collect and discuss the results. Ask each group for the solutions to their two problems. If there is not a consensus on the correct solution to a problem, quickly solve the problem at the board. Record the results on the board, and invite students to add arrows to the arrow picture whenever possible.

Group 1	Group 2	Group 3
$p(B,G) = \frac{1}{2}$	$p(G,B) = \frac{1}{2}$	$p(Y,B) = \frac{1}{3}$
$p(B,Y) = \frac{2}{3}$	$p(G,Y) = \frac{1}{3}$	$p(Y,G) = \frac{2}{3}$



Note: Since **p (B,G) = ½**, an arrow cannot be drawn between the dots for the blue cube and the green cube.

As the results are recorded, students may make several observations:

- p (B,Y) = ⅔ and p (Y,B) = ⅓ both imply that an arrow can be drawn from blue to yellow. Similarly, p (Y,G) = ⅔ and p (G,Y) = ⅓ both imply that an arrow can be drawn from yellow to green.
- p (B, Y) + p (Y, B) = ⅔ + ⅓ = 1; p (Y, G) + p (G, Y) = ⅔ + ⅓ = 1; p (B, G) + p (G, B) = ½ + ½ = 1.
- Even though green is favored over red and red is favored over blue, green is not favored over blue. Similarly, green is favored over red and red is favored over yellow, but green is not favored over yellow.

T: ***We have analyzed all of the possible games with these four cubes. Who is favored, the teacher or the student?***

S: ***The student, because no matter which cube the teacher selects the student can choose a cube that is favored over the teacher's cube.***

T: ***Look at the arrow picture. If I select the red cube, which cube would you select?*** (Green)
If I select the green cube, which cube would you select? (Yellow)
If I select the yellow cube, which cube would you select? (Red or blue)
If I select the blue cube, which cube would you select? (Red)

Writing Activity

Suggest that students write directions to an absent student on how to select a cube in this game with four cubes.

Capsule Lesson Summary

Conduct an extrasensory perception (ESP) test with the class, and discuss the probability of getting certain results by just guessing. Assume a probability of ½ that a person guessing is correct. Construct Pascal's triangle, using the idea of an ESP test with a given number of items administered to a given number of people. Predict the number of correct responses per person.

Materials

Teacher
- Colored chalk
- Large piece of paper
- Ten index cards

Student
- Paper
- Colored pencils, pens, or crayons

Advance Preparation: Before teaching this lesson, prepare five index cards with **TRUE** written on one side and five index cards with **FALSE** written on one side.

Description of Lesson

Exercise 1

Lead a short discussion of extrasensory perception (ESP). Include the idea that ESP might be tested using a set of cards with two different words written on them. The tester could look at a card and then concentrate on the word; the person being tested could try to read the tester's mind and tell what word is on the card.

Show the class the ten index cards, five with **TRUE** and five with **FALSE** on them. Tell students that they will all take a five-item true-false test, and ask them to number 1 to 5 on their papers in preparation for the five test items. Explain that for each test item you will shuffle the cards, select a card, and then concentrate on the word that is on the card. They should try to read your mind and guess what word is on the card. Keep track yourself of the correct answers to the test.

When you finish the test, suggest students exchange papers to grade each other's tests. Then collect results from the class to see how many students had 0, 1, 2, 3, 4, or 5 correct. For example:

Correct test items	0	1	2	3	4	5
Number of students	2	4	10	9	5	0

T: ***Do you think anyone in our class has ESP?***

S: ***Certainly not the ones with no correct items.***

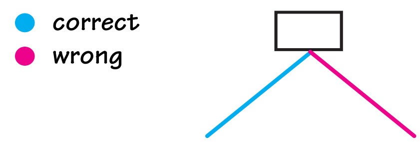
S: ***Maybe those who got four out of five correct.***

T: ***Is this a good test for ESP?***

S: ***No; a person could be just guessing and still give several correct responses.***

T: ***Let's determine the probability of getting these results if people are just guessing.***

Begin a tree diagram on the board.

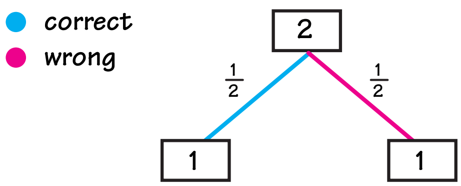


T: ***If we give the test with each person making only one guess, what might we expect the results to be?***

S: ***About one half of the people would make a correct guess and one half would make a wrong guess.***

T: ***So for every two people taking the test we might expect that one would be correct and one would be wrong.***

Indicate how the tree diagram can display this information, and begin a table for recording the results.

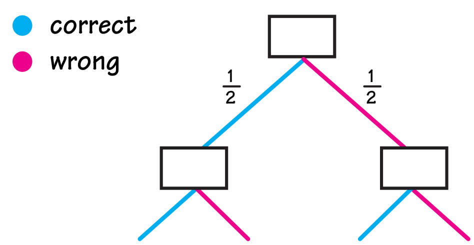


Number of guesses per person	Number of people	Number of people with 0 correct guesses	1	2	3	4	5
1	2	1	1				

Erase the numbers from the tree diagram.

T: ***Now let's look at what could happen if each person were given two guesses. What could the results be for one person?***

As a student describes a possible result, add to the tree diagram to show this possibility. In the illustration below, the student responses are given in an order that adds branches to the tree from left to right.



S: ***Both guesses are correct.***

S: ***The first guess is correct and the second is wrong.***

S: ***The first guess is wrong and the second is correct.***

S: ***Both guesses are wrong.***

T: ***For the first guess we expect about one half of the people to be correct and one half to be wrong. What about for the second guess?***

S: ***We expect the same, about one half to be correct and one half to be wrong.***

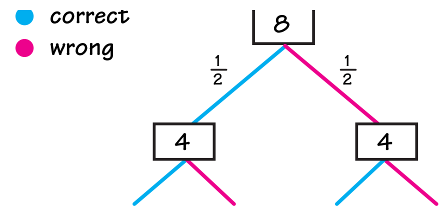
T: ***What number of people should we consider taking a two-item ESP test (with two guesses per person), if we want to see what results to expect?***

Put whatever number the students suggest in the top box of the tree diagram. If the number is not a positive multiple of 4, it will necessitate the use of fractions in the tree. When this becomes apparent, ask the class to suggest another number of people so as to avoid using fractions.

Perhaps a student will suggest giving the test to eight people.

T: ***How many of these eight people might we expect to be correct and how many to be wrong on their first guesses? Why?***

S: ***Four and four, because correct and wrong guesses are equally likely.***



Point to the four people who were correct on the first guess.

T: ***How many of these four people would we expect to be correct on their second guesses? How many wrong?***

S: ***Two correct and two wrong.***

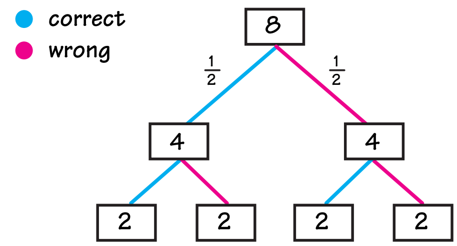
T: ***How many of the four people who were wrong on the first guess would we expect to be correct on the second guess? How many wrong?***

S: ***The same; two correct and two wrong.***

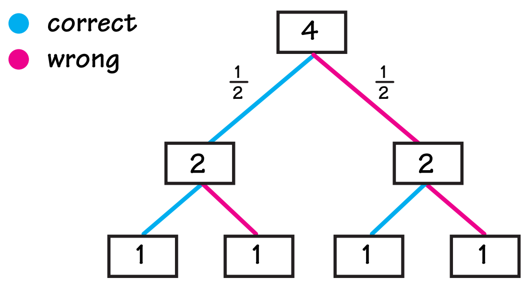
T: ***Could we use more or fewer people in this probability tree?***

S: ***Yes, we could use a large number of people.***

S: ***Yes, four would be a good number to use.***

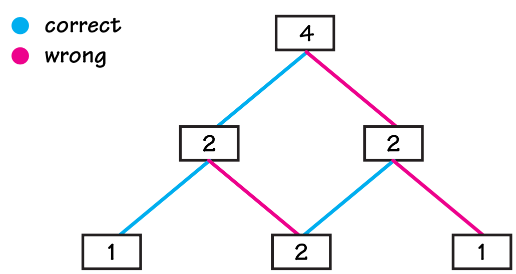


Discuss how to change the other numbers if the top number (in this case 8) is different. Erase all of the numbers in the tree diagram, and repeat the activity assuming four people are taking the test. Then let students use the information in the probability tree to tell you how to make another entry in the table.

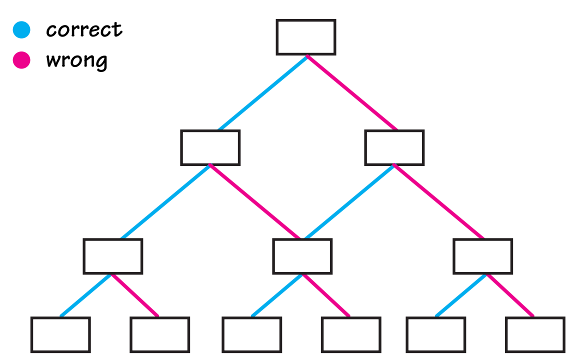


Number of guesses per person	Number of people	Number of people with 0 correct guesses	1	2	3	4	5
1	2	1	1				
2	4	1	2	1			

With the class, discuss that the probability tree shows two possible ways to get the same result (one correct guess). Since it is not important which one of the two guesses is correct, alter the probability tree to make it clear that there are really only three possible results for a two-item test.



Erase all of the numbers in the tree diagram and ask the class to help extend it to show the possible results of a three-item test, i.e., three guesses per person. Perhaps the tree the students suggest will have six boxes in the bottom row.

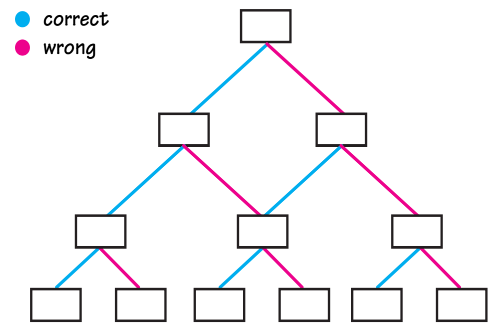


T: ***This probability tree suggests that there are six different results for a three-item test. Are all of these results really different?***

S: ***There are two boxes for the same result, one wrong and two correct guesses. We could make just one box for this result.***

S: ***There are also two boxes for one correct and two wrong guesses. We could have just one box for this result too.***

Change the probability tree to have boxes for four possible results: zero, one, two, or three wrong.

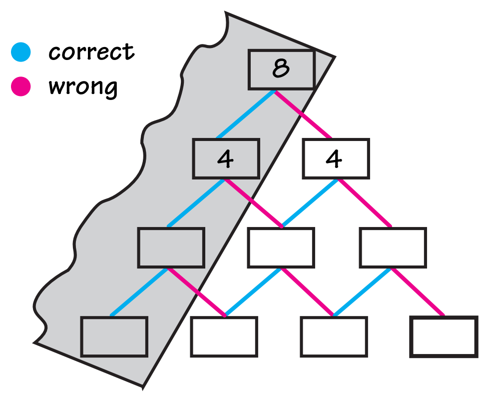


T: ***How many people could we use in this probability tree if we want to see what results to expect from a three-item test? Try to use a small number of people. Do you see a pattern in the number of people?***

S: ***Eight people. Each time we add an item to the test we double the number of people taking it.***

If necessary, mention to the class that since two was a good number of people to use for a one-item test, and four was a good number for a two-item test, we might predict that eight would be a good number for a three-item test.

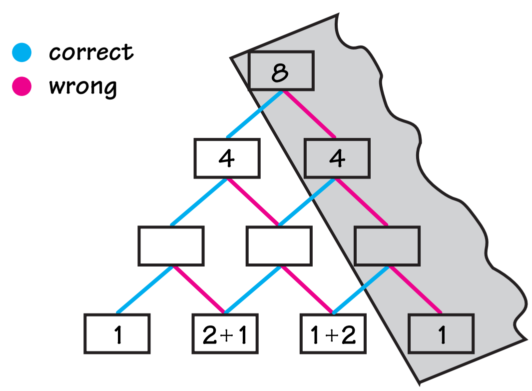
Write 8 in the top box and ask how many out of eight people we expect to be correct (four) and wrong (four) on the first guess. Add this information to the probability tree, and then cover part of the picture with a large sheet of paper, as shown here.



T: ***Look at this part of the picture. How many people are guessing?*** (Four)
How many guesses are left? (Two each)
Perhaps the table can help us fill in the bottom row of boxes.

S: ***If four people have two guesses each, the numbers in the bottom row of boxes would be 1, 2, and 1.***

Record 1, 2, and 1 in the appropriate boxes and repeat by covering the other side of the picture.



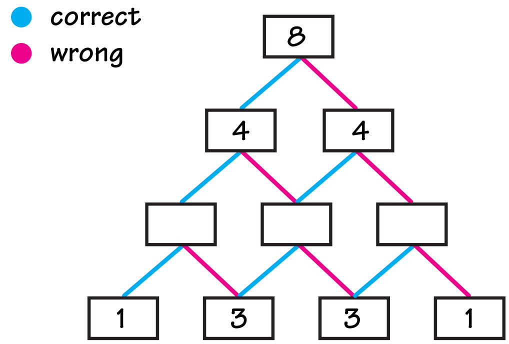
T: ***What numbers should we write in the bottom row of boxes?***

S: ***It is the same as before. If four people have two guesses each, the results could be 1, 2, and 1 respectively, for the three possibilities. We must add these numbers to those already in the boxes.***

S: ***Put 1 in the first box, 3 in the second box because 1 + 2 = 3, and 3 in the third box because 2 + 1 = 3.***

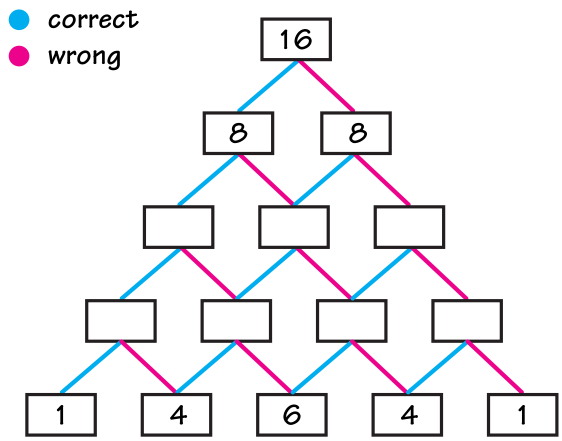
T: ***Let's use the information in this probability tree to make another entry in our table.***

Invite students to tell you what data to enter in the table on the board.



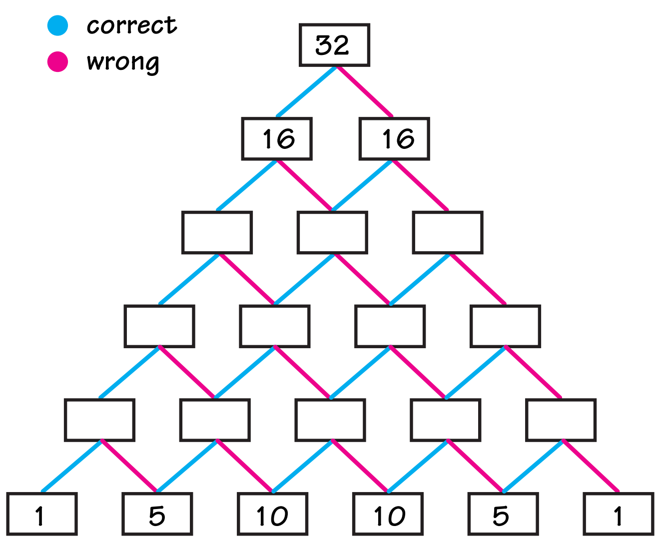
Number of guesses per person	Number of people	Number of people with correct guesses					
		0	1	2	3	4	5
1	2	1	1				
2	4	1	2	1			
3	8	1	3	3	1		

Erase all of the numbers in the tree, and repeat the activity for a four-item test. Let students tell you how to extend the tree diagram; suggest that 16 would be a good number of people to use with a four-item test. Determine that on the first guess we expect eight people to be correct and eight to be wrong. Then cover part of the tree diagram to show eight people with three guesses each, and use the previous results in the table to fill in the bottom row of boxes. Finally, enter this data in the table.



Number of guesses per person	Number of people	Number of people with correct guesses					
		0	1	2	3	4	5
1	2	1	1				
2	4	1	2	1			
3	8	1	3	3	1		
4	16	1	4	6	4	1	

Ask students to predict the entries in the table for a five-item test based upon patterns they observe; record students' predictions. If necessary, record more than one prediction for a single number. Then extend the probability tree and determine the entries in the table for a five-item test with 32 people taking the test. Decide which patterns in the table can be used to predict the next entries.



Number of guesses per person	Number of people	Number of people with correct guesses					
		0	1	2	3	4	5
1	2	1	1				
2	4	1	2	1			
3	8	1	3	3	1		
4	16	1	4	6	4	1	
5	32	1	5	10	10	5	1

Compare your class results on the five-item test to the expected results for 32 people on a five-item test.

Tell the class that you will return to the question of whether the test was a good test for ESP in another lesson. You may want to save your class test results for reference in Lesson P7.

Home Activity

Suggest that students give a five-item true-false test to family members, friends, and neighbors. They can use a method similar to the one used at the beginning of this lesson, but administer the test individually. Then, if they get results for about thirty people, they can compare the actual results to the expected results.

Capsule Lesson Summary

Use the table constructed in Lesson P6 *Pascal's Triangle #1* to determine the probability of guessing correctly a given number of times on an extrasensory perception (ESP) test. Use a tree diagram to determine the probability of always guessing correctly on two separate tests. Discover that giving two five-item tests is the same experiment as giving a ten-item test.

Materials

Teacher
- Colored chalk
- Test results from Lesson P6

Student
- Calculator
- Colored pencils
- Paper

Description of Lesson

Exercise 1

Draw the table from Lesson P6 *Pascal's Triangle #1* on the board.

Number of guesses per person	Number of people	Number of people with 0 correct guesses	1	2	3	4	5
1	2	1	1				
2	4	1	2	1			
3	8	1	3	3	1		
4	16	1	4	6	4	1	
5	32	1	5	10	10	5	1

Lead a short discussion of ESP and the test used in Lesson P6. Review how the table was constructed, and again compare your class test results to the table.

T: ***Let's use the table to determine the probabilities of some possible results from ESP tests. If we give a five-item test (each person making five guesses) to 32 people, we can expect five people to make exactly one correct guess. What is the probability that by just guessing someone will make exactly one correct guess?***

S: ***5 out of 32, or 5/32.***

Write this information on the board. Continue finding probabilities for the other possible results. For example:

p (0 correct out of 5) = $\frac{1}{32}$ p (1 correct out of 5) = $\frac{5}{32}$ p (2 correct out of 5) = $\frac{10}{32}$

p (5 correct out of 5) = $\frac{1}{32}$ p (4 correct out of 5) = $\frac{5}{32}$ p (3 correct out of 5) = $\frac{10}{32}$

T: ***Which results do we expect most often?***

S: ***Either two or three correct guesses.***

T: ***What is the probability of guessing correctly either two or three times on a five-item test?***

S: ***$^{20}/_{32}$. There are 10 chances out of 32 of guessing correctly twice and 10 chances out of 32 of guessing correctly three times; 10 + 10 = 20.***

T: ***What is the probability of guessing correctly at least three times? Why?***

S: ***$^{16}/_{32}$ or $^{1}/_{2}$. There are 10 out of 32 chances of getting three guesses correct, 5 out of 32 chances of getting four guesses correct, and 1 out of 32 chances of getting all five guesses correct. 10 + 5 + 1= 16, so there are 16 out of 32 chances of guessing correctly at least three times.***

T: ***What is the probability of guessing correctly at least four times?*** ($^{6}/_{32}$)
What result would you want before you would believe that someone has ESP?

S: ***All five guesses correct.***

T: ***What is the probability that all five guesses will be correct?*** ($^{1}/_{32}$)
So when we gave the five-item test to everyone in this class, it was likely that one of you would guess correctly all five times. Do you think this is a reliable test for ESP?

S: ***No.***

Exercise 2

Make calculators available for this exercise.

T: ***What could we do to make the test for ESP more reliable?***

S: ***Give the test several times.***

S: ***Put more items (guesses) on the test.***

S: ***Include more choices of answers on the cards; that is, make it a multiple choice test.***

If necessary, suggest some of these possibilities yourself.

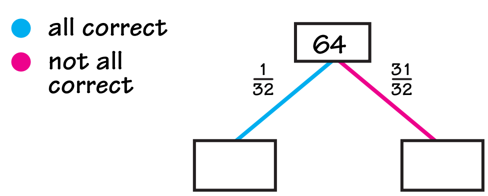
T: ***Let's determine what the probabilities would be if we gave the five-item test twice. How many correct guesses would you want a person to have each time to say that they have ESP?***

S: ***Five correct guesses on each test.***

T: ***What is the probability that a person would guess correctly five times on the first test?*** ($^{1}/_{32}$)

If a person does not guess correctly all five times, then there would be at least one wrong guess. What is the probability of having at least one wrong guess? ($^{31}/_{32}$)

Begin a probability tree on the board.



T: ***If 64 people took a five-item test, how many of them would we expect to have all their guesses correct? Why?***

S: ***2; 64 ÷ 32 = 2.***

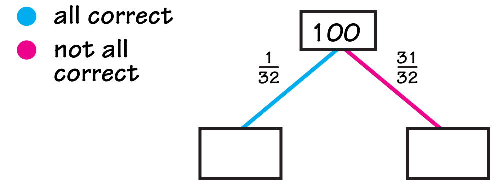
T: ***How many would we expect to have at least one wrong guess? Why?***

S: ***62; 64 – 2 = 62.***

S: ***62; 31 x 64 = 1 984 and 1984 ÷ 32 = 62.***

Put 2 and 62 in the appropriate boxes.

Erase the numbers in the boxes and start with 100 in the top box.



T: ***If 100 people took a five-item test, how many of them would we expect to get all five guesses correct? Why?***

S: ***3; because 100 ÷ 32 = 3.125 which is close to 3.***

T: ***How many of them would we expect to get at least one guess wrong? Why?***

S: ***97; 100 – 3 = 97.***

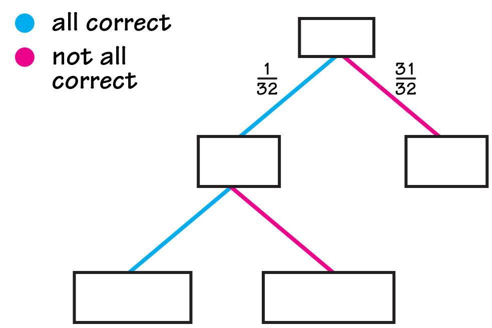
S: ***97; 31 x 100 = 3 100 and 3100 ÷ 32 = 96.875 which is close to 97.***

S: ***97; 31 x 3.125 = 96.875 which is close to 97.***

Record 3 and 97 in the appropriate boxes.

T: ***Now let's see how many people we would expect to always guess correctly if we gave them the five-item test twice.***

Erase the numbers in the boxes and extend the tree diagram. Point to each of the bottom boxes in turn and ask the class what it represents.



S: ***Blue followed by blue represents guessing correctly all five times on both tests.***

S: ***Blue followed by red represents guessing correctly all five times on the first test but not all five times on the second test.***

T: ***Do we need to extend this tree diagram on the right side in order to find the chances of guessing correctly all five times on both of two five-item ESP tests?***

S: ***No, because red represents having a wrong guess at least once.***

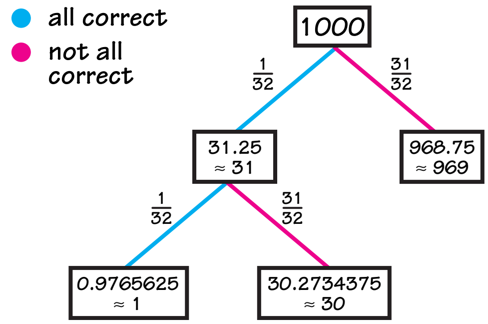
T: ***What is the probability of guessing correctly all five times on the second test?*** ($\frac{1}{32}$)
Of not guessing correctly all five times? ($\frac{31}{32}$)

Show this information on the probability tree by labeling the bottom blue and red cords, as in the next illustration.

T: ***Copy this picture on your paper. Suppose 1000 people take two five-item ESP tests. Put 1 000 in the top box, and fill in the other boxes.***

P7

Allow students to work independently or with partners for a few minutes. Then invite students to help you fill in the boxes at the board.



T: ***What is the probability of guessing correctly all five times on both tests?***

S: ***About 1/1000 because we would expect one person out of 1000 to guess correctly all five times on both tests.***

Save this tree diagram for comparison later in the lesson.

T: ***You also suggested that we could put more items on the ESP test to make it more reliable.***

Point to the table on the board, and ask students to consider extending it to include a six-item ESP test. Encourage them to predict the next row of entries by noticing patterns in the table. Students might observe various patterns, but, if necessary, point out the following pattern yourself.

Number of guesses per person	Number of people	Number of people with 0 correct guesses	1	2	3	4	5
1	2	1	1				
2	4	1	2	1			
3	8	1	3	3	1		
4	16	1	4	6	4	1	
5	32	1	5	10	10	5	1

The sum of two adjacent entries in a row gives an entry in the next row, as shown.

Use this pattern to extend the table until it includes information for a ten-item ESP test.

Number of guesses per person	Number of people	Number of people with 0 correct guesses	1	2	3	4	5	6	7	8	9	10
1	2	1	1									
2	4	1	2	1								
3	8	1	3	3	1							
4	16	1	4	6	4	1						
5	32	1	5	10	10	5	1					
6	64	1	6	15	20	15	6	1				
7	128	1	7	21	35	35	21	7	1			
8	256	1	8	28	56	70	56	28	8	1		
9	512	1	9	36	84	126	126	84	36	9	1	
10	1024	1	10	45	120	210	252	210	120	45	10	1

T: ***What is the probability that a person will have ten correct guesses on a ten-item test?***

S: ***1/1024.***

Point to the tree diagram on the board.

T: ***We estimated that the probability of always guessing correctly on two five-item ESP tests was about 1/1000. Let's compare the probabilities of always guessing correctly on one ten-item test and on two five-item tests.***

Erase the numbers in the boxes of the tree diagram and start with 1 024 in the top box.

T: ***How many people would we expect to always guess correctly if 1 024 people each took two five-item ESP tests?***

S: ***We expect 32 people to guess correctly all five times on the first test because 1 024 ÷ 32 = 32.***

Record 32 in the appropriate box.

S: ***We expect one of those 32 people to guess correctly all five times on the second test.***

Record 1 in the appropriate box. The students should conclude that the probability of always guessing correctly on two five-item tests or on one ten-item test is the same, $\frac{1}{1024}$.

T: ***Would either of these tests be a good indicator of ESP?***

Allow students to express their opinions freely, but note that either method would be much more reliable than the results of one five-item ESP test.

Extension Activity

Make copies of Blackline P7 (Pascal's triangle arranged differently) and suggest that students look for number patterns.

Investigate other probability situations where Pascal's triangle may be used. For example, what is the probability that a family with five children will have four boys and one girl? This problem assumes that the probability of having a boy or a girl is the same.

Capsule Lesson Summary

Use students' knowledge of percent in everyday life to relate certain fractions with percents (for example, ½ is equivalent to 50% and ¾ is equivalent to 75%). Find misleading information in a cereal advertisement. Compare the visual impact of two different bar graphs for the same data. Investigate methods for comparing the prices of two items when the given prices are for different quantities.

Materials

Teacher
- Overhead
- *Nutribest* advertisement
- Blackline P8

Student
- Calculator
- Colored pencils
- Paper
- Worksheets P8(a), (b), (c), and (d)

Advance Preparation: Use Blackline P8 to make a copy of the *Nutribest* advertisement for display in Exercise 2.

Description of Lesson

Exercise 1

Note: The goal of this exercise is to relate certain fractions with percents by using students' intuitive knowledge and concepts of percent from everyday life. Attempt to rely on students' experiences rather than presenting formal techniques yourself.

T: ***When we have 100% of the class here, what does that mean?***

S: ***Every student in the class is here.***

T: ***When 50% of the students in a class are girls, what does that mean?***

S: ***One-half of the students in the class are girls.***

Begin a table on the board.

Fraction	Percent
$\frac{1}{2}$	50%

T: ***Yes, 50% is another name for ½. Do you know the percent name for ¼?***

S: ***25%. ½ or ²⁄₄ is the same as 50%, so ¼ is the same as 25%.***

S: ***25%, since ¼*** x ***100 = 25.***

S: ***25%. 1 out of 4 is the same as 25 out of 100.***

Record this information in the table.

Similarly, lead students to find equivalent percent names for ¾, ⅕, ⅖, ⅗, and ⅘, and record the results in the table.

Fraction	Percent
$\frac{1}{5}$	20%
$\frac{1}{4}$	25%
$\frac{2}{5}$	40%
$\frac{1}{2}$	50%
$\frac{3}{5}$	60%
$\frac{3}{4}$	75%
$\frac{4}{5}$	80%

Point out that in every line of the table the fraction could be written as $\frac{\square}{100}$. For example, $\frac{3}{5} = \frac{60}{100} = 60\%$.

T: ***What is a fractional name for 100%?***

S: ***100/100.***

S: ***5/5. 1/5 = 20%, 2/5 = 40%, 3/5 = 60%, 4/5 = 80%, and so 5/5 = 100%.***

S: ***4/4. 1/4 = 25%, 2/4 = 50%, and 3/4 = 75%. So 4/4 = 100%.***

S: ***2/2. 1/2 = 50%, so 2/2 = 100%.***

S: ***100/100. 5/5, 4/4, 2/2 are all the same as 1.***

Students may be surprised that 1 is another name for 100%.

Exercise 2

Display the advertisement on Blackline P8.

T: ***Advertisers rarely outright lie to the public, but they often try to mislead you. This is an advertisement for Nutribest cereal. Let's assume that everything in this ad is true, but that the advertisers have used many "tricks" to convince us to buy Nutribest cereal. What are some of these tricks?***

Encourage students to find many, but not necessarily all, of the following misleading ideas in the advertisement.

Picture

- The pictures suggest that *Nutribest* will make you muscular.
- The *Nutribest* boy is smiling while the other boy is frowning.

Quotation

- Did they ask only five people?
- Who were the people they asked? Parents? Employees?
- In what way do people prefer *Nutribest*? Maybe Brand X tastes horrible. Is Brand X even a breakfast cereal?

Graphs

- The *Nutribest* graph has thicker bars.
- Are the scales for the two graphs the same?

Ingredients

- How much vitamin A is good for you? It may be harmful to get too much of some vitamins.
- Sodium usually comes from salt (sodium chloride), and many Americans are trying to reduce their sodium intake.
- Does Brand X have important nutrients that *Nutribest* does not have?

Price

- How much cereal does \$1.38 and \$1.60 buy? Maybe *Nutribest* comes in a small box.
- Maybe Brand X includes prizes or coupons.

Exercise 3

Distribute Worksheets P8(a), (b), and (c).

T: ***Based on tests and research, the United States Department of Agriculture has determined the recommended daily allowance of various vitamins and minerals. The graph on Worksheet P8(a) gives the percent of each allowance in Nutribest and in Brand X. Was the advertisement true for vitamin A, protein, and sodium?***

S: ***Yes, Nutribest has more vitamin A, protein, and sodium than Brand X.***

T: ***What about the vitamins and minerals not listed on the advertisement?***

S: ***Brand X has more vitamin C, vitamin D, calcium, niacin, and riboflavin than Nutribest. Nutribest has more thiamine and iron.***

T: ***Was the advertisement fair?***

S: ***No, it made Nutribest look much better than it really is.***

T: ***On the advertisement, the bars for the Nutribest graph are wider than the bars for Brand X. Let's look at another trick advertisers often use when making bar graphs.***

Assign half of the class to do the following activity on Worksheet P8(b) and the other half to do the same activity on Worksheet P8(c).

T: ***Draw a bar graph for the amount of thiamin, iron, and vitamin A in both Nutribest and Brand X.***

After a while, pair students to trade and compare Worksheets P8(b) and (c).

T: ***What do you notice about the two graphs?***

S: ***On one graph, Nutribest appears to have much more thiamine, iron, and vitamin A. On the other graph, Nutribest appears to have only a little more of each item.***

T: ***Why?***

S: ***The scales on the two graphs are different.***

S: ***One graph starts at 0%, the other starts at 20%.***

T: ***Are both graphs correct?***

S: ***Yes, although they look very different.***

T: ***Which company, Nutribest or Brand X, is more likely to use the graph on Worksheet P8(b)? On Worksheet P8(c)?***

S: ***The graph on Worksheet P8(b) makes Nutribest look better.***

S: ***Brand X might prefer the graph on Worksheet P8(c) since the two cereals appear about the same. But Brand X would probably use different vitamins or minerals on a graph.***

Exercise 4

Write the following information on the board. Make calculators available.

300 grams of Nutribest: \$138
400 grams of Brand X: \$160

T: *This information gives the weight of the boxes of cereal with the prices in the advertisement. Which is the better buy?*

S: *A box of Nutribest costs less, but you get less cereal.*

T: *Try to calculate which cereal is cheaper.*

S: *Brand X is a better buy. 400 grams of Brand X cost \$1.60 so 100 grams cost \$1.60 ÷ 4 or \$0.40. 300 grams of Nutribest cost \$1.38 so 100 grams cost \$1.38 ÷ 3 or \$0.46.*

S: *Brand X is cheaper. Three boxes of Brand X weight the same as four boxes of Nutribest. But three boxes of Brand X cost 3 x \$1.60 or \$4.80, whereas four boxes of Nutribest cost 4 x \$1.38 or \$5.52.*

S: *One gram of Brand X costs \$1.60 ÷ 400 or \$0.004, while one gram of Nutribest costs \$1.38 ÷ 300 or \$0.0046. So Brand X is cheaper.*

Worksheet P8(d) is available for individual work.

Exercise 5

Begin this table on the board.

People Interviewed	People who Preferred Nutribest
5	4
10	

T: *When Nutribest advertises "four out of five people prefer Nutribest," they might mean that they questioned many people and that four out of every five people preferred Nutribest to Brand X. If so, how many out of ten people would prefer Nutribest?*

S: *Eight. There are two groups of five and 2 x 4 = 8.*

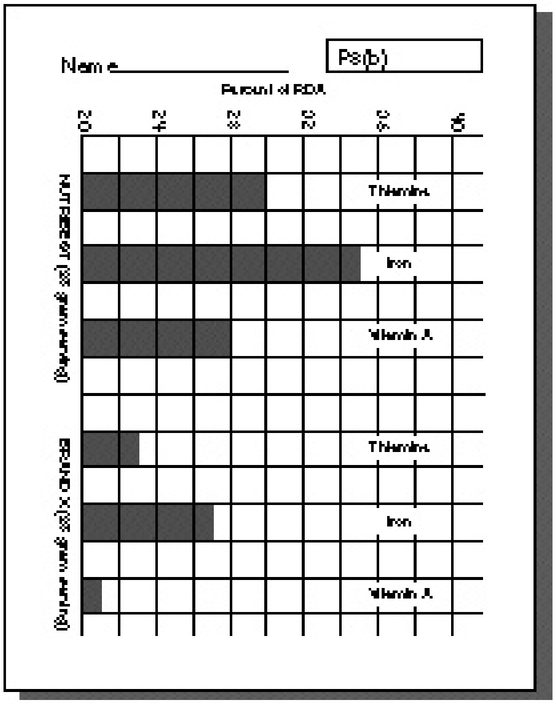
In a similar manner, continue to add several more lines to the table. (Answers are in boxes.)

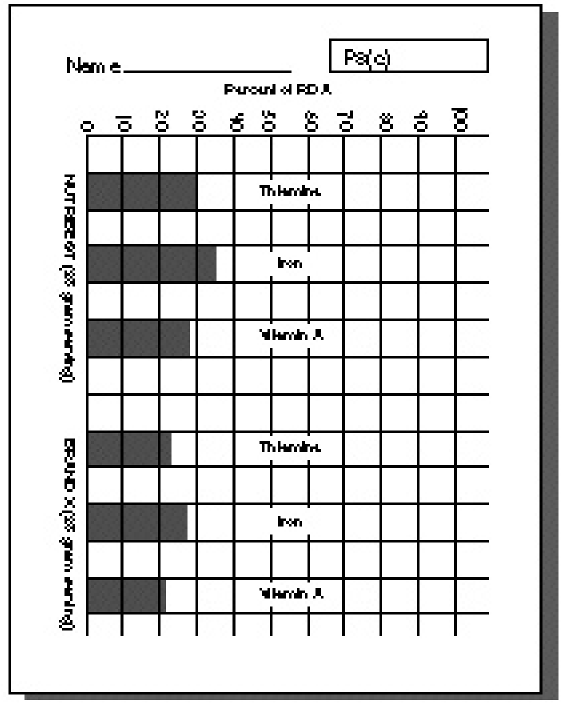
People Interviewed	People who Preferred Nutribest
5	4
10	$\boxed{8}$
15	$\boxed{12}$
20	$\boxed{16}$
50	$\boxed{40}$
100	$\boxed{80}$
1000	$\boxed{800}$

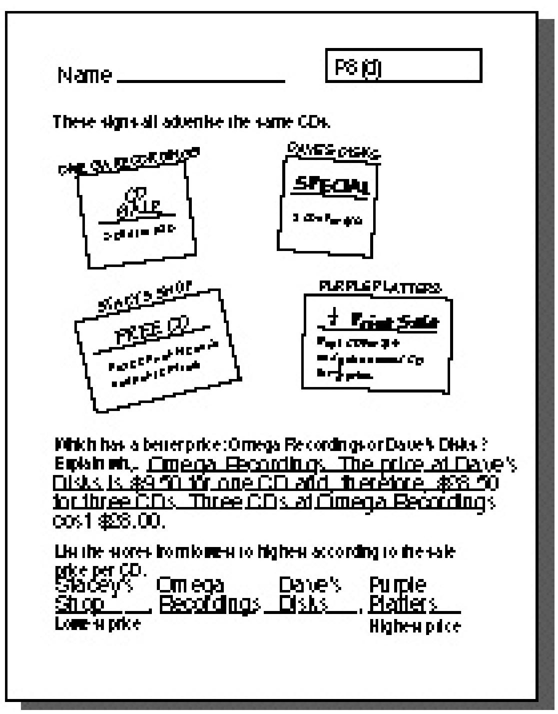
T: *If Nutribest really interviewed 1 000 people and 800 preferred Nutribest to Brand X, that is much more impressive than just four out of five people. Still, we should know who these 1000 people were.*

Extension Activity

As a class project, you may wish to ask students to look for misleading advertisements in newspapers and magazines and to share them with the class. Another project could be to collect nutrition information from cereal boxes and to compare the cereals.






Name

P8(d)

These signs all advertise the same CDs.

SPECIAL

FREE CD

PURPLE PLATTERS

Price Sale

Which has a better price: Omega Recordings or Dave's Disks?

Explain why. Omega Recordings. The price at Dave's Disks is \$9.50 for one CD and, therefore, \$28.50 for three CDs. Three CDs at Omega Recordings cost \$28.00.

List the stores from lowest to highest according to the sale price per CD.

Stacey's Shop, Omega Recordings, Dave's Disks, Purple Platters

Lowest price — Highest price