Workbooks

WORKBOOKS TABLE OF CONTENTS

WORKBOOKS INTRODUCTION

There are many opportunities for student to work individually during the course of the lessons described in the other content strands. In the Workbooks strand, however, it is this individualized work which becomes the chief end of the majority of lessons. The goal in this strand is to provide students with opportunities

- to review many of the ideas they have met in other content strands;
- to apply their acquired knowledge to new situations requiring various kinds of strategic thinking; and
- to learn how to read and use mathematics workbooks.

The following six workbooks are provided:

- *Arcade of Problems #1*
- *Arcade of Problems #2*
- *Arcade of Problems #3*
- *Arcade of Problems #4*
- *Arcade of Problems #5*
- *Arcade of Problems #6*

...and one storybook.

- *Election in the Number World*

Each workbook contains problems of varying levels of difficulty. Approximately the first ten pages of each workbook are easy problems, the next ten to twelve pages are average level difficulty, and the last ten pages are more challenging problems. For each workbook, we suggest that all students start work at the easiest level (i.e., on page 2) and then work through as many pages as they can handle during the two lessons scheduled for that workbook. We estimate that, in a typical class, about two-thirds of the students will correctly finish the first ten pages, about one-third will finish the first twenty pages, and a few will finish all or most of the workbook. These proportions will vary from class to class.

This guide contains an answer key for each workbook. The key follows an introduction to the workbook and a suggested collective lesson. The lesson either presents the workbook to the whole class or provides a warm-up activity, usually on a problem similar to one found in the workbook.

The storybook *Election in the Number World* touches some important areas of mathematics with the motivation of a story context. This booklet allow students to become deeply involved in an appealing fantasy as they struggle with difficult mathematics problems. The situations support topics and strategies developed in other strands.

Use of the Workbook Strand for Evaluation Purposes

The workbooks provide an excellent instrument to assess the progress of your students on a regular basis. You may not feel it is necessary to check every page and problem for each student, but you should develop a procedure for checking students' work with which you are comfortable. This may include checking one or more specific pages, discussing some particular mistakes with individual students and letting them correct their work, or just looking carefully at a few pages to be sure the students have understood the general idea of the problems in that particular workbook.

In the Blacklines, you will find a record-keeping tool for each workbook to help you assess student progress in the various strands. This tool may also assist you in parent conferences and in filling out periodic progress reports.

Here are some important points to bear in mind for workbooks.

- Always read the introductory material for each workbook and give the short introductory collective lesson(s).
- All students should start at the beginning of each workbook and progress as far as they can.
- All students should begin a new title on the same day, even if some students have not finished work on the previous title.
- Not all students should be expected to complete a given workbook. Only some students will reach the most challenging problems. Other students may succeed only in doing the easiest problems, although you should not assume this automatically—surprises are not at all uncommon.
- Students should be allowed to use calculators except on pages you specify beforehand.

Note: Some pages in a workbook have problems that are related to calculators or are best attacked with the support of calculators; however, there also are pages for which the use of calculators may not be appropriate. We encourage you to review a workbook before distributing copies of it and to inform the class beforehand on which pages your are not allowing the use of calculators. Certain pages you may wish to use for special evaluation purposes; other pages would not be effective if calculators were allowed, for example, pages that primarily focus on paper-and-pencil computation, "wipe-out" pages that involve subtraction of decimal numbers, pages where using patterns permits students to solve problems from previously solved problems, and pages where the problem is to insert missing decimal points in answers to calculations.

Content Overview

Workbooks

The six *Arcade of Problems* workbooks both review and extend many of the ideas introduced in the content strands. The extensions occur through problems which require students to apply the mathematics to new situations or to synthesize their knowledge in new ways.

Lessons: W1, 2, 3, 4, 5, 6, 8, 9, 11, 12, 13, and 14.

Positional Systems

The storybook *Election in the Number World* provides an introduction to the full decimal positional system of numeration. Special attention is given to the relationship between fractional and decimal names for numbers.

The first lesson on positional systems does not make use of the storybook, but rather discusses a variety of positional notations for numbers. For example, the lesson explores a base five and a base twelve system. Students find that in base twelve they must invent new names for ten and eleven.

W1 ARCADE OF PROBLEMS #1 (LESSON ONE)

Capsule Lesson Summary

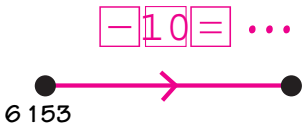
Use a calculator relation to review patterns in both positive and negative integers through repeated subtraction of tens. Begin the workbook *Arcade of Problems #1*. (This is the first of two lessons using this workbook.)

Materials

Teacher	• Colored chalk	Student	• *Arcade of Problems #1* Workbook • Calculator • Colored pencils, pens, or crayons

Description of Lesson

Draw this arrow picture on the board.



T: ***Put 6 153 on your calculator. Press − 10 =, and then slowly press = many times. Watch the numbers that appear on the display. What pattern do you notice?***

S: ***The ones digit is always 3.***

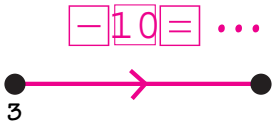
T: ***If you keep pressing =, what are some numbers less than 100 that would eventually appear?***

S: ***93, 83, 73, and so on.***

T: ***What is the least positive number that would appear?***

S: ***3.***

Relabel the starting dot of the arrow on the board.



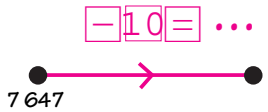
T: ***What negative numbers would appear?***

Encourage students to predict some negative numbers that would appear before pressing any more keys.

T: ***Put 3 on your calculator and again press − 10 = … . What numbers appear?***

S: ***−7, −17, −27, and so on. All of the negative numbers end in 7.***

Repeat the above activity with one or both of the following arrow pictures.



Pattern: All positive numbers end in 7.
All negative numbers end in 3.

Pattern: All positive numbers end in 9 or 4.
All negative numbers end in 1 or 6.

Distribute copies of the workbook *Arcade of Problems #1*, and let students work independently for the rest of the class period. You may need to work more closely with students new to *CSMP* or let those students work with veteran *CSMP* students for awhile. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of the class period, collect the workbooks for your review. They will be used again in Lesson W2.

Writing Activity

You may like students to take lesson notes on some, most, or even all their math lessons. The "Lesson Notes" section in Notes to the Teacher gives some suggestions and refers to forms in the Blacklines you may provide to students for this purpose. In this lesson, for example, students may note problems in the workbook they found especially difficult or especially interesting. They may also like to create other problems, similar to ones in the workbook, for their classmates or a family member to solve.

Capsule Lesson Summary

Review a division algorithm. Continue individual work in the workbook *Arcade of Problems #1*. (This is the second of two lessons using this workbook.)

Materials

Teacher	Student
• None	• *Arcade of Problems #1* Workbook • Calculator • Colored pencils, pens, or crayons

Description of Lesson

Begin the lesson with some mental arithmetic for division. Suggested problems with student responses in boxes are given below.

$$8\overline{)560} = \boxed{70} \qquad 8\overline{)16} = \boxed{2} \qquad 8\overline{)576} = \boxed{72}$$

$$8\overline{)577} = \boxed{72\text{ R} = 1} \qquad 8\overline{)580} = \boxed{72\text{ R} = 4} \qquad 8\overline{)584} = \boxed{73}$$

T: ***Nabu has various packing jobs. Suppose he is assigned to pack balls into boxes of 12. How many balls will fill 2 boxes?*** (24) ***3 boxes?*** (36) ***30 boxes?*** (360) ***31 boxes?*** (372)

One day he is given 8 717 balls to pack. How many boxes can he fill?

S: ***We need to divide 8 717 by 12.***

Write the problem on the board.

$$12\overline{)8\,717}$$

T: ***Can he fill 1000 boxes?*** (No) ***100 boxes?*** (Yes) ***200 boxes?*** (Yes) ***900 boxes?*** (No)

S: ***Let's try 500.***

T: ***How many balls can Nabu put into 500 boxes?***

S: ***6 000; 12 x 500 = 6 000.***

T: ***How many balls will he have left to pack after filling 500 boxes?***

S: ***2 717.***

Include this information in the problem on the board.

Continue letting students suggest how many boxes Nabu can fill at each step and then what is left to pack. Record the result in the division calculation. Observe that Nabu can fill a total of 726 boxes and there will be 5 balls left over.

12)8 717	
− 6 000	500
2 717	
−2 400	200
317	
−240	20
77	
−72	6
5	

Distribute students' copies of the workbook *Arcade of Problems #1*. Ask students first to correct or complete pages from the previous week's work, and then to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

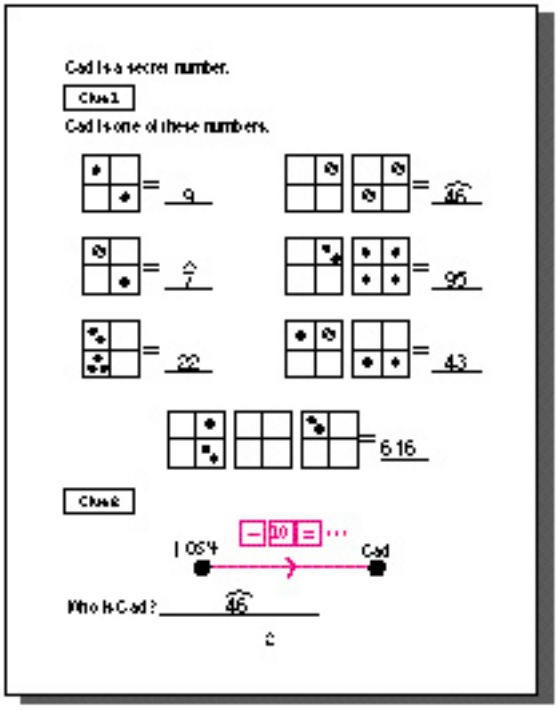
At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in the workbook during a study time or to take it home as an assignment.

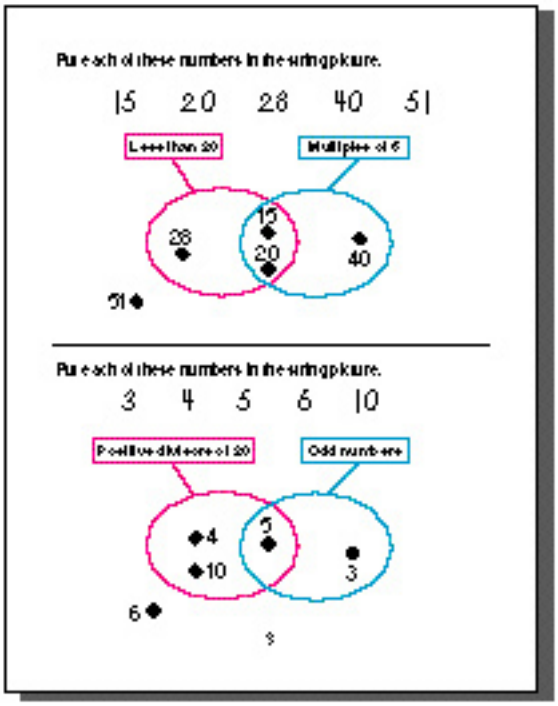
Assessment Activity

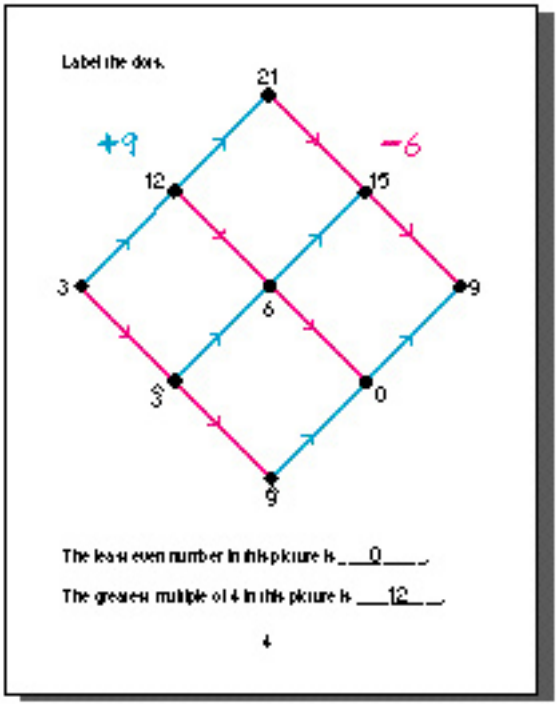
An individual student progress record for this workbook is available on Blackline W2(a). You may like to use this form to monitor student work.

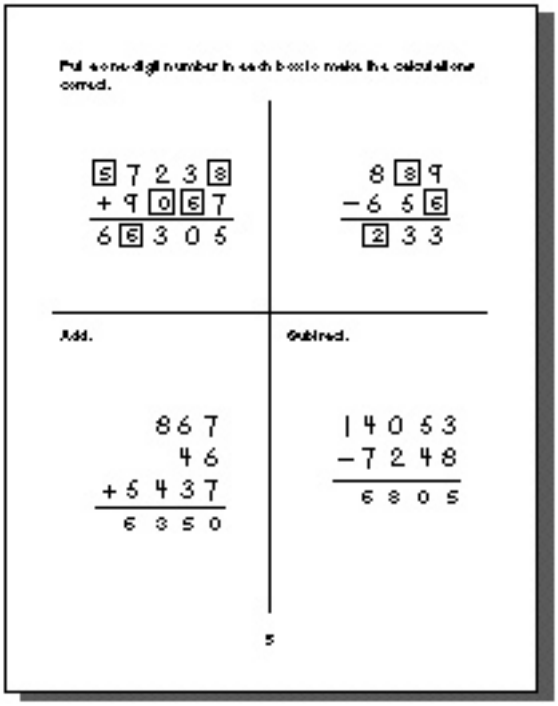
Home Activity

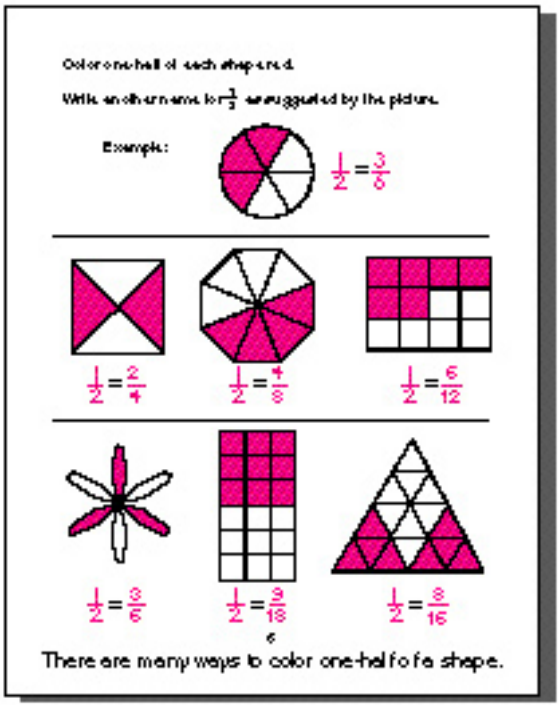
If you choose to send workbooks home with students, you may want to include a letter to parents/ guardians about the workbook. Blackline W2(b) has a sample letter.

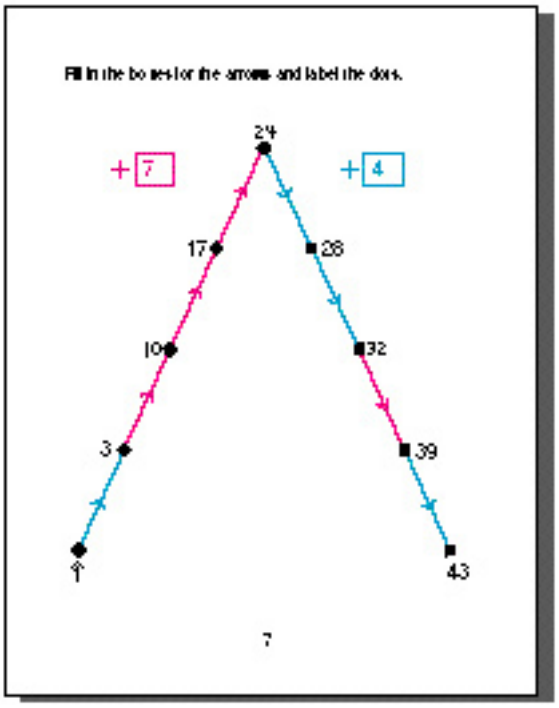

Cad is a secret number.
Clue 1
Cad is one of these numbers.
= 9
= 46
= 7
= 95
= 22
= 43
= 616
Clue 2
10 less
Cad
Who is Cad? 46
2

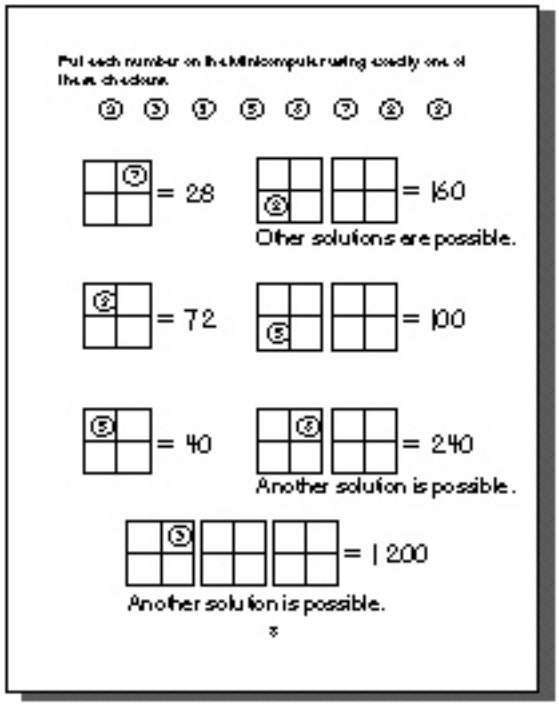

Put each of these numbers in the string picture.
15 20 28 40 51
Less than 20
Multiples of 5
28
15
20
40
51
Put each of these numbers in the string picture.
3 4 5 6 10
Positive divisors of 20
Odd numbers
4
10
5
3
6
3

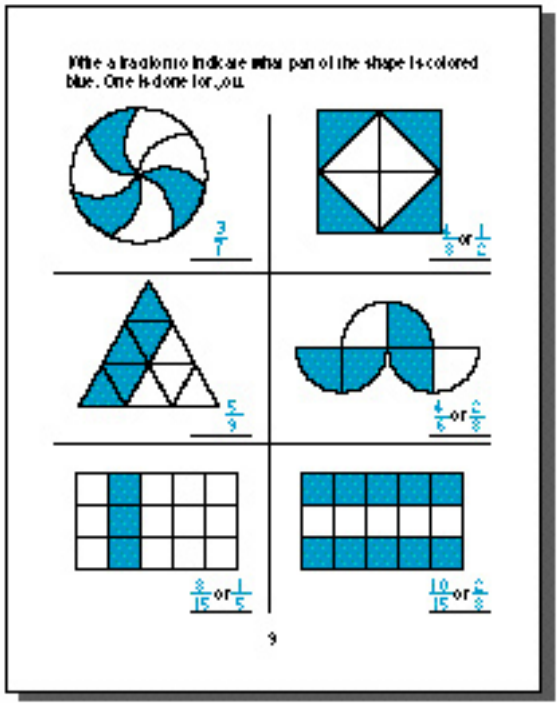

Label the dots.
21
+9
-6
12
15
3
6
9
3
0
9
The least even number in this picture is 0
The greatest multiple of 4 in this picture is 12
4

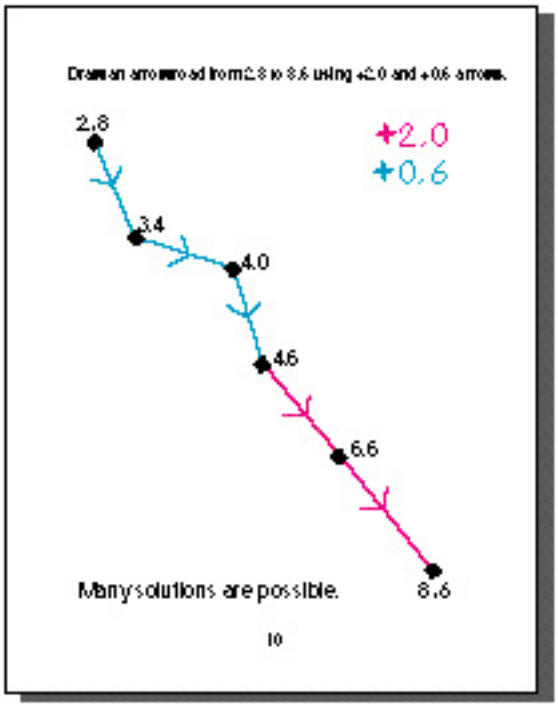

Put a one-digit number in each box to make the calculations correct.
5 7 2 3 5
+ 9 0 5 7
6 5 3 0 5
8 5 9
- 6 5 5
2 3 3
Added.
Subtracted.
867
46
+5437
6350
14053
-7248
6805
5

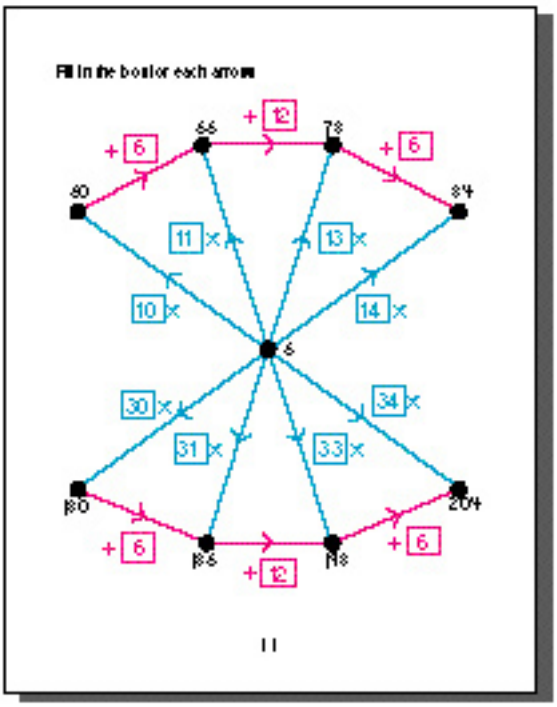

Example:
$\frac{1}{2}=\frac{3}{6}$
$\frac{1}{2}=\frac{2}{4}$
$\frac{1}{2}=\frac{4}{8}$
$\frac{1}{2}=\frac{6}{12}$
$\frac{1}{2}=\frac{3}{6}$
$\frac{1}{2}=\frac{9}{18}$
$\frac{1}{2}=\frac{8}{16}$
6
There are many ways to color one-half of a shape.

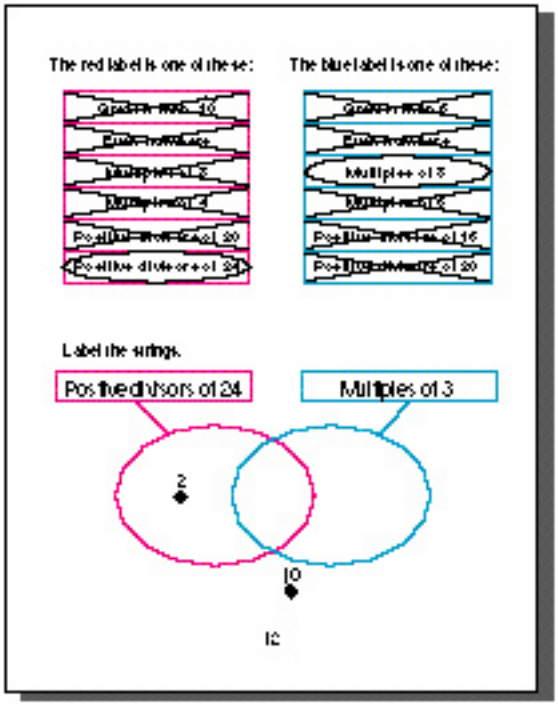

Fill in the boxes for the arrows and label the dots.
+ 7
+ 4
24
17
28
10
32
3
39
43
7

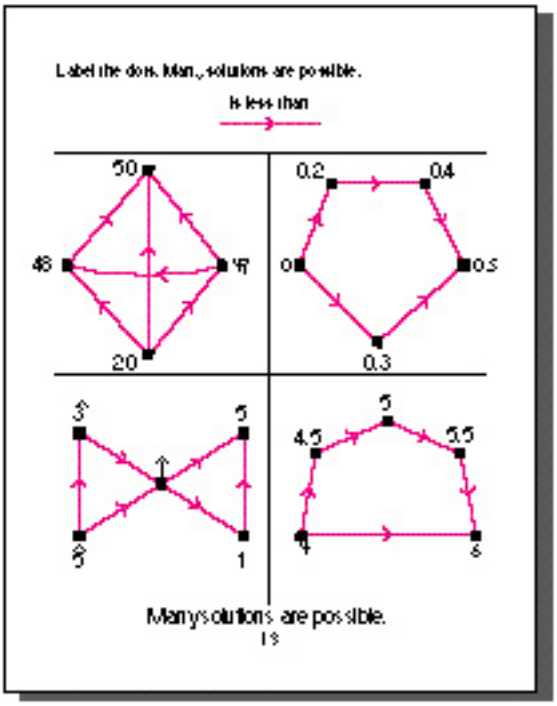

Put each number on the Minicomputer, using exactly one of these checkers.
2 5 3 5 8 7 2 2
7
= 28
2
= 160
Other solutions are possible.
2
= 72
5
= 100
5
= 40
4
= 240
Another solution is possible.
5
= 1200
Another solution is possible.
8

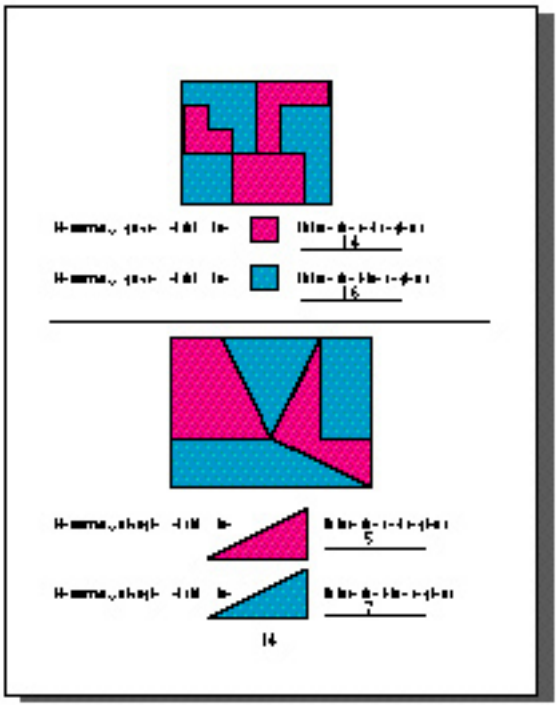

Write a fraction to indicate what part of the shape is colored blue. One is done for you.
$\frac{3}{7}$
$\frac{4}{8}$ or $\frac{1}{2}$
$\frac{5}{9}$
$\frac{4}{6}$ or $\frac{2}{3}$
$\frac{8}{15}$ or $\frac{1}{5}$
$\frac{10}{15}$ or $\frac{2}{3}$
9

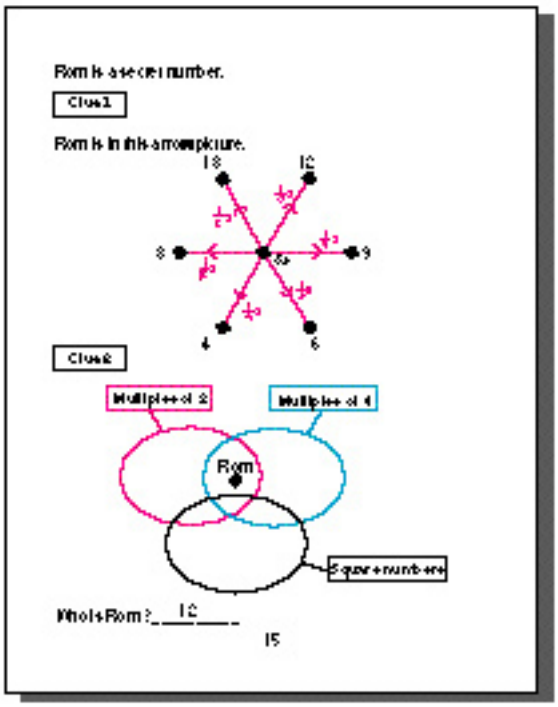

Draw an arrow-road from 2.8 to 8.6 using +2.0 and +0.6 arrows.
2.8
3.4
4.0
4.6
6.6
8.6
+2.0
+0.6
Many solutions are possible.
10

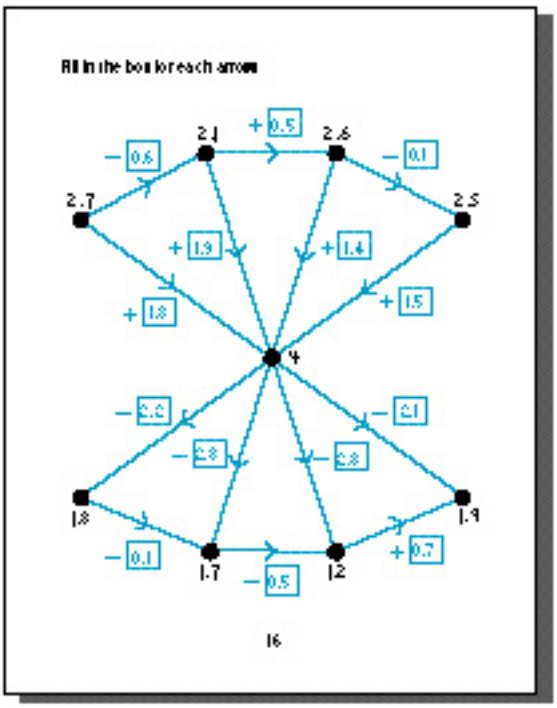

Fill in the box for each arrow.
66
78
80
84
88
86
204
6
12
11
13
10
14
30
34
31
33
11

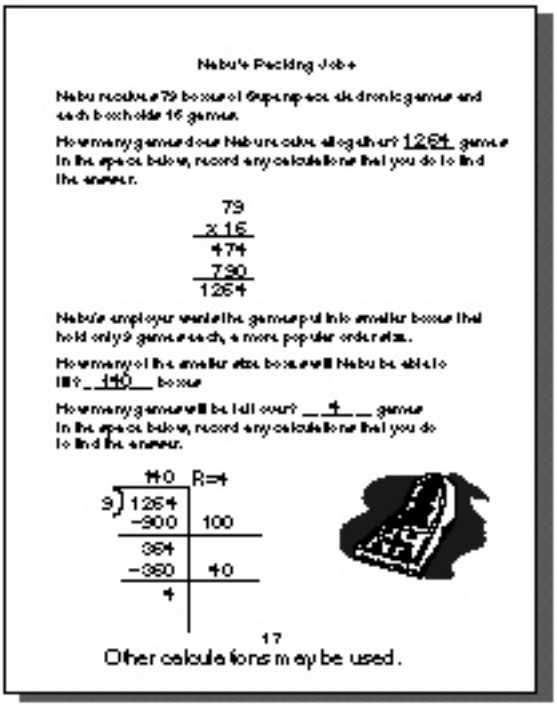

The red label is one of these:
The blue label is one of these:
Multiples of 3
Positive divisors of 24
Label the strings.
Positive divisors of 24
Multiples of 3
2
10
12

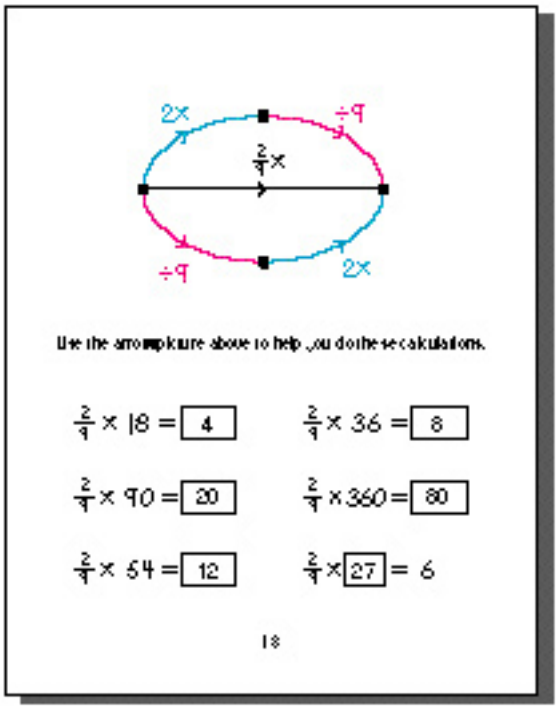

Label the dots. Many solutions are possible.
is less than
30
48
20
0.2
0.4
0
0.3
4.5
3.5
4
Many solutions are possible.
13

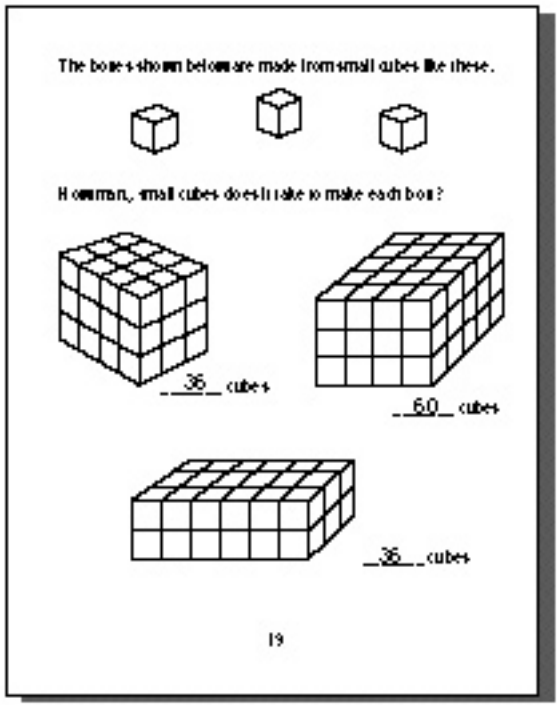

14

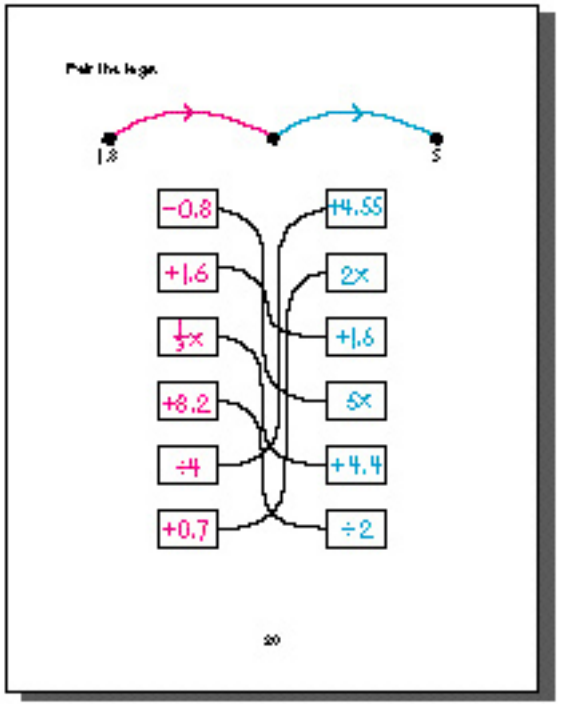

Rem is a secret number.
Clue 1
Rem is in this arrow picture.
Clue 2
Multiple of 2
Multiple of 4
Rem
Square numbers
Who is Rem? 12
15

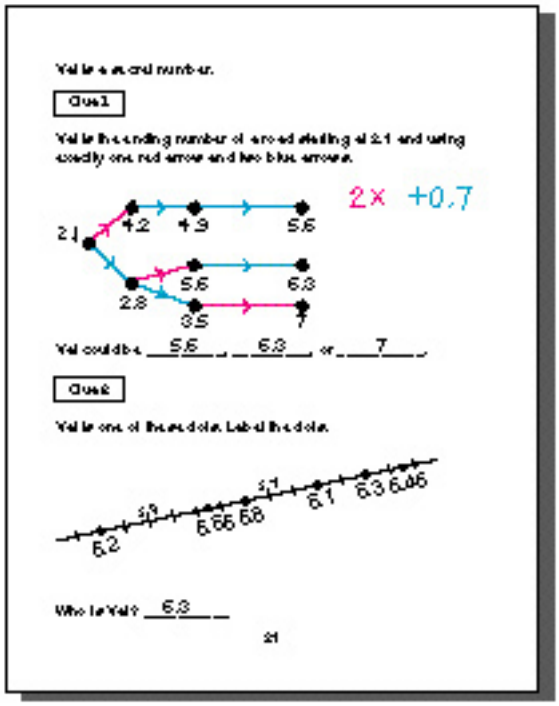

Fill in the box for each arrow
16

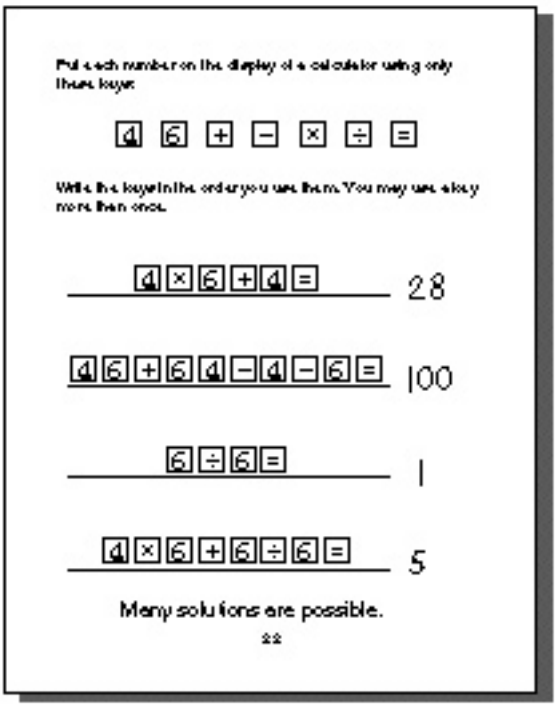

Nabu's Packing Jobs
79
x 16
474
790
1264
140 R=4
1264
−900
100
364
−360
40
4
17
Other calculations may be used.

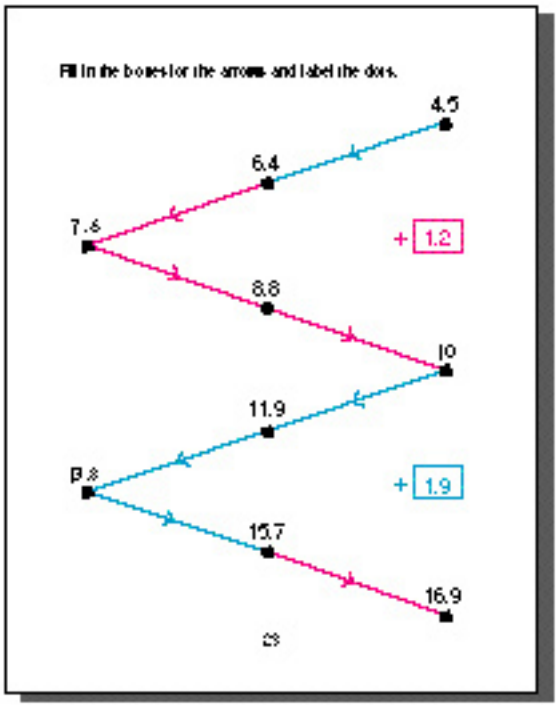

2×
÷9
$\frac{2}{9}$×
÷9
2×
Use the arrow picture above to help you do these calculations.
$\frac{2}{9}$ × 18 = 4
$\frac{2}{9}$ × 36 = 8
$\frac{2}{9}$ × 90 = 20
$\frac{2}{9}$ × 360 = 80
$\frac{2}{9}$ × 54 = 12
$\frac{2}{9}$ × 27 = 6
18

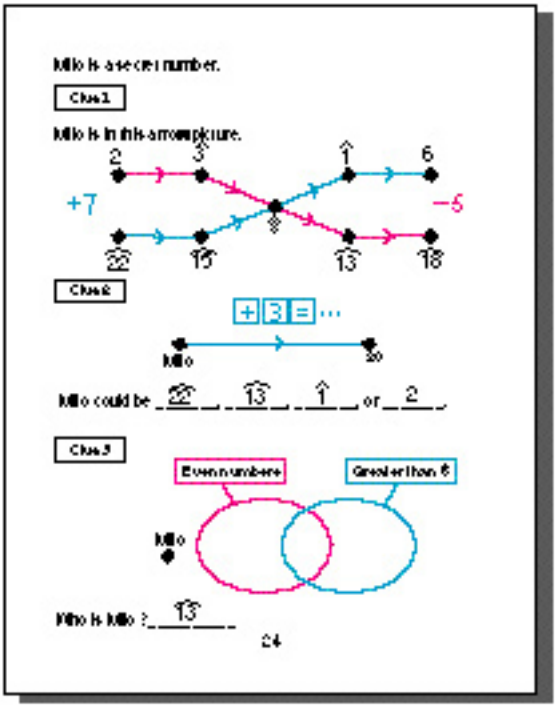

The boxes shown below are made from small cubes like these.
How many small cubes does it take to make each box?
36 cubes
60 cubes
36 cubes
19

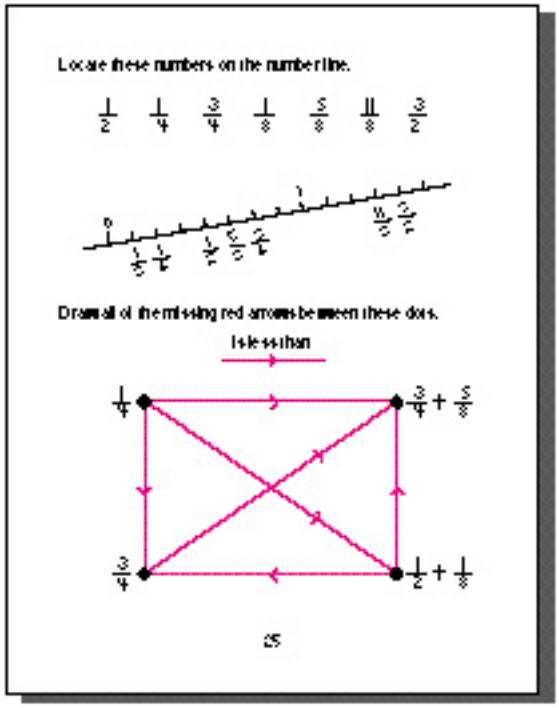

Pair the tags.
18
5
−0.8
+1.6
$\frac{1}{3}$×
+8.2
÷4
+0.7
+4.55
2×
+1.6
8×
+4.4
÷2
20


Clue 1
2×
+0.7
2.1
4.2
4.9
5.6
2.8
5.6
6.3
3.5
7
Clue 2
6.2
6.66
6.8
6.1
6.3
6.46
Who is Val? 6.3
21


Put each number on the display of a calculator using only these keys:
4 6 + − × ÷ =
Write the keys in the order you use them. You may use a key more than once.
4 × 6 + 4 = 28
4 6 + 6 4 − 4 − 6 = 100
6 ÷ 6 = 1
4 × 6 + 6 ÷ 6 = 5
Many solutions are possible.
22


Fill in the boxes for the arrows and label the dots.
4.5
6.4
7.6
+ 1.2
8.8
10
11.9
9.8
+ 1.9
13.7
16.9
23


Milo is a secret number.
Clue 1
Milo is in this arrow picture.
2
3
1
6
+7
−5
22
15
13
18
Clue 2
+ 3 = ⋯
Milo
20
Milo could be 22 , 13 , 1 , or 2
Clue 3
Even numbers
Greater than 6
Milo
Who is Milo ? 13
24


Locate these numbers on the number line.
1/2 1/4 3/4 1/8 5/8 11/8 3/2
0
1
Draw all of the missing red arrows between these dots.
1 is less than
1/4
3/4 + 5/8
3/4
1/2 + 1/8
25

In each row, show how many coins are needed to make exactly $1.00. Use exactly the number of coins at the left. The first row is done for you.

Number of coins	Half-dollar .50	Quarter .25	Dime .10	Nickel .05	Penny .01
3	1	2	0	0	0
4	0	4	0	0	0
6	1	0	5	0	0
	1	1	1	3	0
	0	3	2	1	0
12	0	2	0	10	0
	0	3	0	4	5
	1	0	3	3	5
20	0	0	0	20	0
	0	0	8	2	10
	0	3	0	2	15
43	0	2	1	0	40
	0	0	1	12	30
	0	1	1	6	35

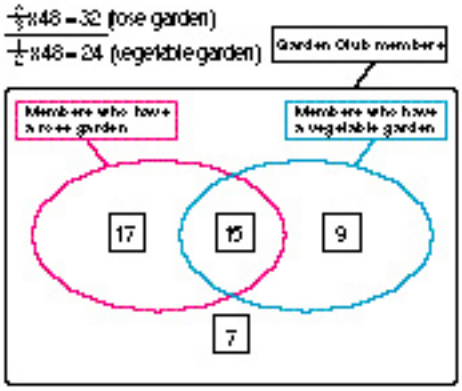
26

Find the number of Garden Club members that are in each region of the sorting picture and record these numbers in the boxes.

There are 48 members in the Garden Club.
Two-thirds of the club members have a rose garden.
One-half of the club members have a vegetable garden.
6 even club members have neither a rose garden nor a vegetable garden.

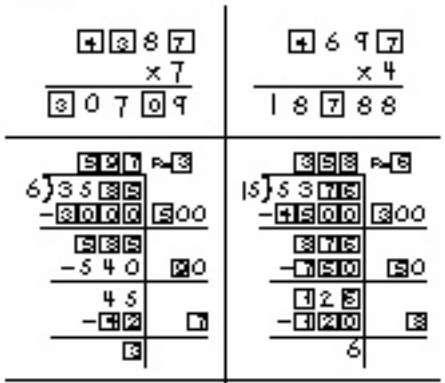
$\frac{2}{3} \times 48 = 32$ (rose garden)

$\frac{1}{2} \times 48 = 24$ (vegetable garden)



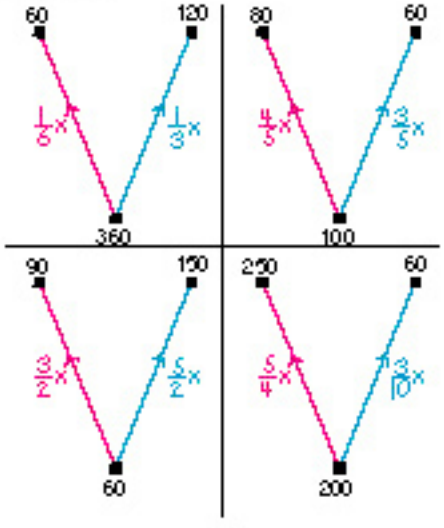
27

Put a one-digit number in each box to make the calculations correct.



28

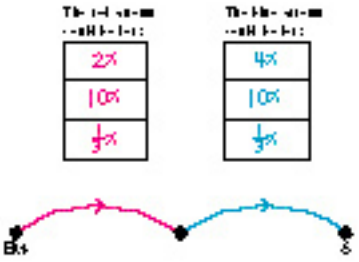
60 is the smallest number in each arrow picture. Label the dots.



29

Bix is a secret number.

Clue 1

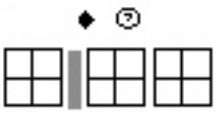


Bix could be 0.06, 0.12, 0.3, 0.72, 1.8, 4.5, 9, or 24.

30

Clue 2

Bix can be put on this mini computer with exactly these two checkers.



Bix could be 0.12, 0.3, 1.8, or 9.

Clue 3

Bix can be written by adding parentheses to this expression.

$$4 \times (0.2 + (0.5 \div 2))$$

Who is Bix? 1.8

31

Write a name for each number using exactly four 7s and no other digits. You may use the following symbols as often as you wish.

$$+ \quad - \quad (\quad) \quad \times \quad \div$$

The number 9 is done for you.

$(7-7)+(7\div7)$ = 1	$(7\div7)+(7\div7)$ = 2
$(7+7+7)\div7$ = 3	$(77\div7)-7$ = 4
$((7-7)\times7)+7$ = 7	$((7\times7)+7)\div7$ = 8
$((7+7)\div7)+7$ = 9	$(77-7)\div7$ = 10
$(7+7)+(7\div7)$ = 15	$(77\div7)+7$ = 18
$7+7+7+7$ = 28	$(7-(7\div7))\times7$ = 42
$(7\times7)-7-7$ = 35	$(7\times7)+(7\div7)$ = 50

32

Other solutions are possible.

W3 ARCADE OF PROBLEMS #2 (LESSON ONE)

Capsule Lesson Summary

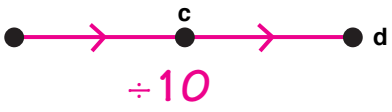
Use an arrow picture to present mental arithmetic problems involving decimal numbers and the relation ÷10. Begin the workbook *Arcade of Problems #2*. (This is the first of two lessons using this workbook.)

Materials

Teacher	Student
• Colored chalk	• *Arcade of Problems #2* Workbook • Calculator • Colored pencils, pens, or crayons

Description of Lesson

Draw this arrow picture on the board.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

Put 65 000 at **b**. Invite students to find **c** (6 500) and **d** (650).

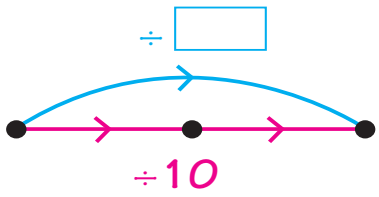
Present some of the following or similar problems. Label the indicated dot and let students label the other two dots. Discuss techniques for calculating 10x and ÷10.

dot	number
b	20 800
b	78
b	590

dot	number
c	40 300
c	8
c	0.64

dot	number
d	9.4
d	0.872
d	0.0013

Draw a blue arrow from **b** to **d**.



T: ***What could the blue arrow be for?***

S: ***÷100.***

If students suggest ÷20, check that this does not work when you label the dots.

T: ***The arrow picture suggests that one way to divide a number by 100 is to divide it by 10 twice. What is 82 ÷ 100?***

S: ***0.82; 82 ÷ 10 = 8.2 and 8.2 ÷ 10 = 0.82.***

Present some of the following problems to the class. (Student responses are in boxes.)

960 000 ÷ 100 = 9 600
470 ÷ 100 = 4.7
492 ÷ 100 = 4.92

56 ÷ 100 = 0.56
7 ÷ 100 = 0.07
3.4 ÷ 100 = 0.034

Distribute copies of the workbook *Arcade of Problems #2*, and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of the class period, collect the workbooks for your review. They will be used again in Lesson W4.

Capsule Lesson Summary

Introduce calculator sentences, and look for numbers obtained as a result of putting operation keys in this expression: 9 – 3 – 5 – 2 =. Choose a possible resulting number, and find which operation keys need to be used to get that resulting number. Continue individual work in the workbook *Arcade of Problems #2*. (This is the second of two lessons using this workbook.)

Materials

Teacher	Student
• Calculator	• Calculator • *Arcade of Problems #2* Workbook • Colored pencils, pens, or crayons • Compass

Description of Lesson

9 × 3 − 5 ÷ 2 =

Write this expression on the board.

T: ***On your calculator, press these keys in the order given here. What number is on the display?***

S: ***11.***

T: ***This is a* calculator sentence *for 11. We call it a calculator sentence because it is how a calculator does the operations in the order we press the keys.***

Note: This description of a calculator sentence assumes that the calculator does the operations in the order of entry, that is, chain operations. See "Role and Use of Calculators" in Section One, Notes to the Teacher to learn more about such features of a calculator. If your calculators do not do chain operations, you will need to adjust the lesson description accordingly.

9 – 3 – 5 – 2 =

Erase the operation symbols in the expression and ask,

T: ***If we put different operation keys in the sentence, what are some other numbers we could get?***

There are many possibilities (more than 50), so accept several. Each time, ask the student to announce which operation keys he or she used. For example:

S: ***19. I used all three* + *keys* 9 + 3 + 5 + 2 = ***19.***

S: ***6;* 9 ÷ 3 + 5 − 2 = ***6.***

T: ***What is the greatest number we could get?*** (270)
What is the least number we could get? (⁻4)

As students work on finding the greatest and least possible numbers, you are likely to get some discussion about comparing decimals or comparing decimals to negative numbers. For example, 0.5 is more than ⁻1, and ⁻1 is more than ⁻4.

W4

The greatest and least possible numbers result from the following choices of operations.

9 × 3 × 5 × 2 = 270

9 ÷ 3 − 5 − 2 = −4

or

9 ÷ 3 − 5 × 2 = −4

Choose a possible whole number that has not been mentioned, such as 15, and ask,

T: ***What operation keys would we use to get a calculator sentence for 15?***

9 − 3 − 5 − 2 = 15

In this case there are two possible solutions.

9 + 3 + 5 − 2 = 15

9 − 3 × 5 ÷ 2 = 15

Distribute copies of the workbook *Arcade of Problems #2*. Ask students first to correct or complete pages from the previous week's work, and then to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in the workbook during a study time or to take it home as an assignment.

Assessment Activity

An individual student progress record for the workbook is available on Blackline W4. You may like to use this form to monitor student work.

Home Activity

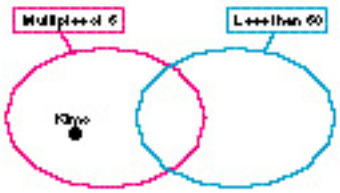
Kinvo is a secret number.

Clue 1

Kinvo can be shown on this Mini-computer by hiding all exactly one checker.

Kinvo could be 40, 70, 76, or 78.

Clue 2

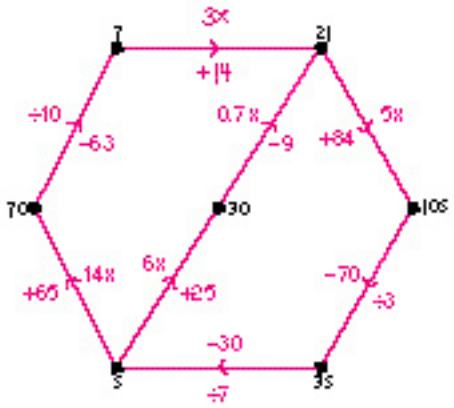


Who is Kinvo? 70

2

Label the arrows. Try to label some arrows in two ways. One is done for you.

Many solutions are possible.

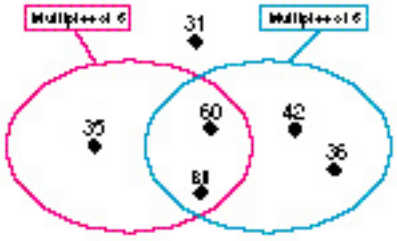


Other solutions are possible.

3

Put these numbers in the string picture. Do not label Bill.

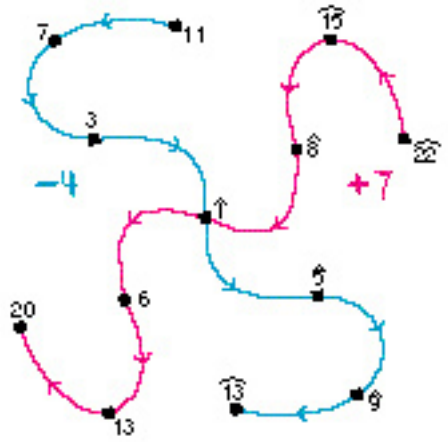
31 35 36 42 60



Bill is not one of the numbers in the list at the top of the page.
What is the smallest whole number Bill could be? 30

4

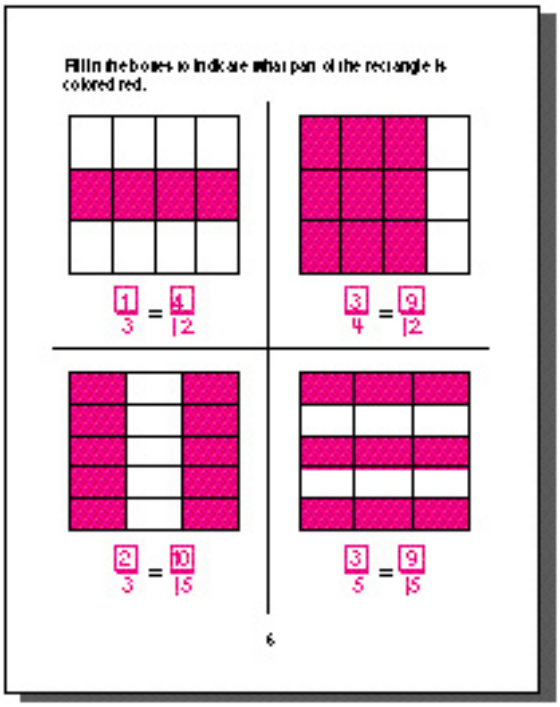
Label the dots.

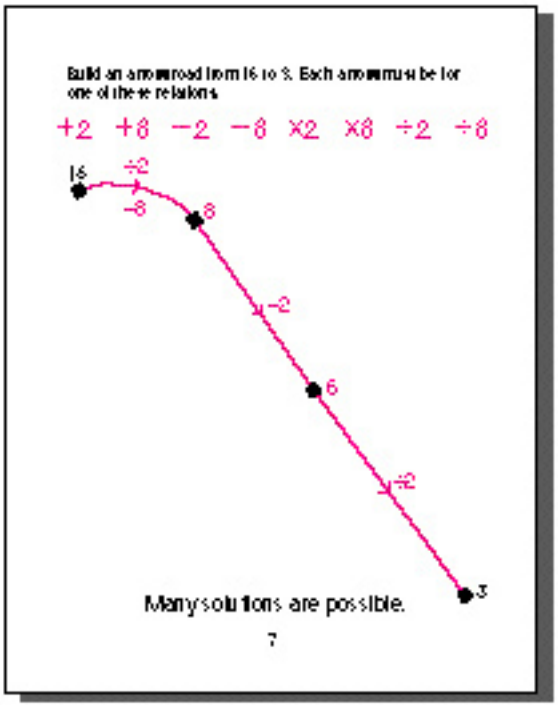


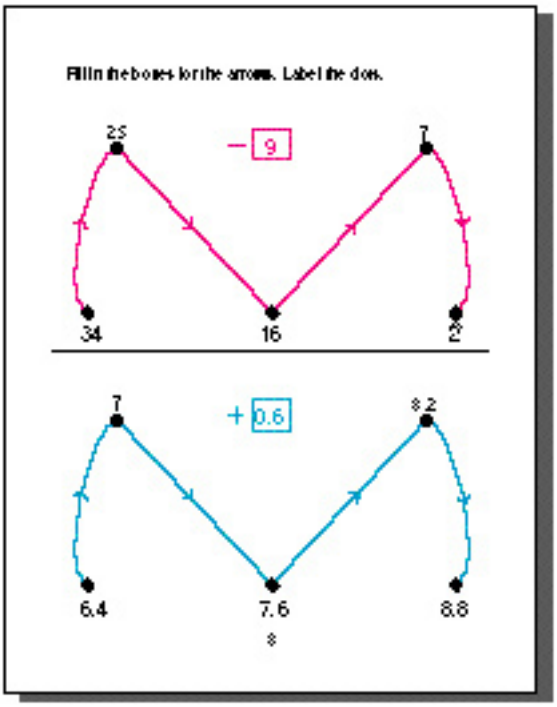
What is the least number in this picture? 22

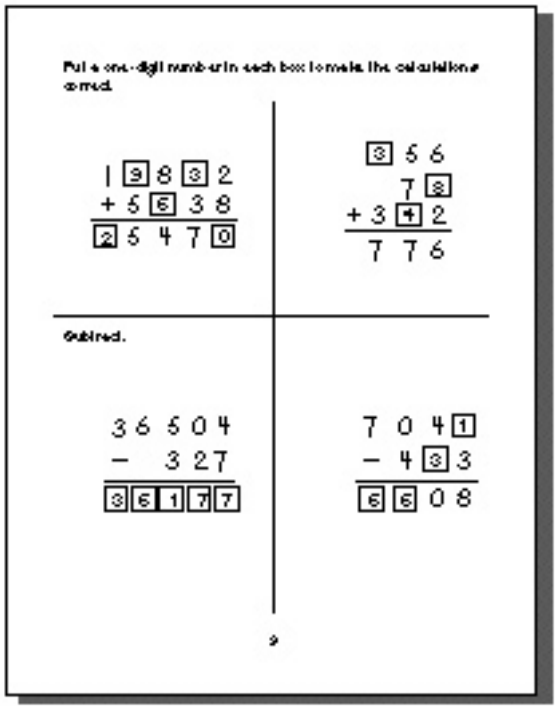
The only multiples of 4 in this picture are 20 and 8.

5

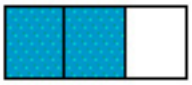

Fill in the boxes to indicate what part of the rectangle is colored red.
1/3 = 4/12
3/4 = 9/12
2/3 = 10/15
3/5 = 9/15
6


Build an arrowroad from 16 to 3. Each arrow must be for one of these relations.
+2 +8 −2 −8 ×2 ×8 ÷2 ÷8
16
÷2
−8
8
−2
6
÷2
3
Many solutions are possible.
7


Fill in the boxes for the arrows. Label the dots.
25
− 9
7
34
16
22
7
+ 0.6
8.2
6.4
7.6
8.8
8


Put a one-digit number in each box to make the calculations correct.
1 9 8 3 2
+ 5 6 3 8
2 5 4 7 0
3 5 6
7 8
+ 3 4 2
7 7 6
Subtract.
3 6 5 0 4
− 3 2 7
3 6 1 7 7
7 0 4 1
− 4 3 3
6 6 0 8
9

Complete.



$\frac{2}{3} = \frac{6}{9}$ $\frac{2}{3} = \frac{4}{6}$

$\frac{2}{3} = \frac{12}{18}$ $\frac{2}{3} = \frac{10}{15}$

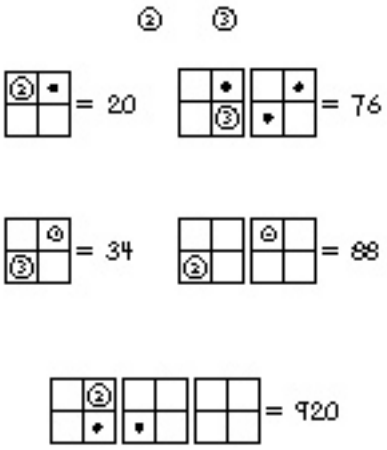
Complete. The pictures above may help you.

$3\frac{2}{3} - 2 = 1\frac{2}{3}$ $2\frac{1}{3} - \frac{1}{3} = 2$

$2\frac{1}{3} - \frac{2}{3} = 1\frac{2}{3}$ $4 - 1\frac{2}{3} = 2\frac{1}{3}$

10

Put each number on the Mini-computer by adding exactly one of these checkers.



11

Multiply.

11 × 11 = 121
11 × 22 = 242
11 × 33 = 363
11 × 44 = 484
11 × 55 = 605
11 × 66 = 726
11 × 77 = 847
11 × 88 = 968

Divide.

$9\overline{)540}$ 60 R = 0 $9\overline{)27}$ 3 R = 0 $9\overline{)567}$ 63 R = 0

$9\overline{)568}$ 63 R = 1 $9\overline{)570}$ 63 R = 3 $9\overline{)575}$ 63 R = 8

$9\overline{)578}$ 64 R = 2 $9\overline{)580}$ 64 R = 4 $9\overline{)585}$ 65 R = 0

12

Rolo's a secret number.

Clue 1

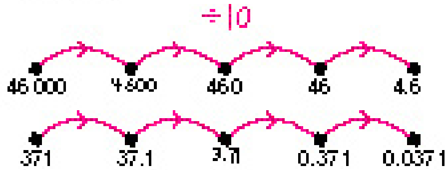
Rolo could be 12, 8, 4, or 24.

Clue 2

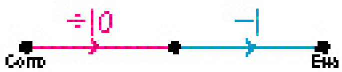
Who's Rolo? 8

13

Label the dots.



Complete.



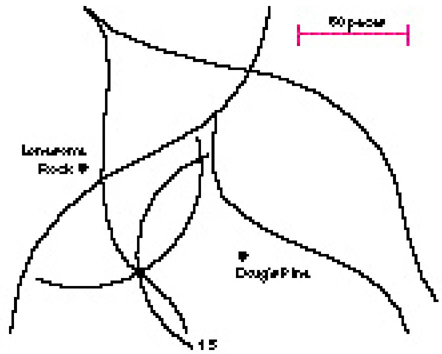
Como	Ett
1800	179
52	4.2
47.9	3.79
640	63
182	0.82

14

Laughing Lefty robbed the Longhorn Saloon. He buried the money in Brittany Woods, but was soon captured by the county sheriff. After Laughing was in prison, the sheriff caught him trying to pass the following message to one of his visitors.

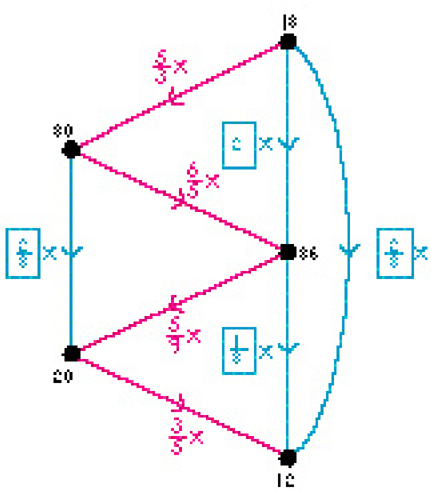
> I buried the money on a trail. The location is 50 paces from Lonesome Rock and 50 paces from Dougal's Pine.

Use this map and a compass to find the location of the buried money. Mark the location with a large dot. The red line segment shows the length of 50 paces.



15

Label the dots and fill in the boxes for the blue arrows.



16

The Gamellos and Robinsons went together to see the newest science fiction movie, *Venus and Beyond*. Tom and Marie Gamello brought their four children: Dan, Angela, Theresa, and Tony. Ms. Robinson brought her three children: Kim, Dwayne, and Paula.

Mr. Gamello paid $19.25 total for the tickets for both families. He forgot the price of the children's tickets, but paid $3.50 for each of the three adult tickets.

How much does one child's ticket cost? $1.25

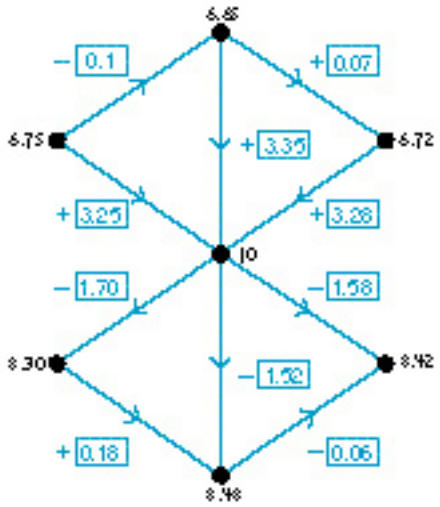
Show your computations below.

19.25 − 10.50 (3 x 3.50) = 8.75

8.75 ÷ 7 = 1.25

17

Fill in the boxes for the arrows.



18

Ra is a secret number.

Clue 1

Ra is one of these numbers.

$(7 \times (8 - 3)) + 9 =$ 44

$7 \times ((8 - 3) + 9) =$ 98

$7 \times (8 - (3 + 9)) =$ 28

$((7 \times 8) - 3) + 9 =$ 62

$(7 \times 8) - (3 + 9) =$ 44

Clue 2

Ra can be put on this Mini computer with exactly these two checkers.

Who is Ra? 62

19

Multiply.

$$\begin{array}{r} 746 \\ \times\ 68 \\ \hline 5968 \\ 44760 \\ \hline 50728 \end{array}$$

Divide.

$$\begin{array}{r|l} & 49\ \ R=8 \\ \hline 23 & 1135 \\ & -920 \quad 40 \\ \hline & 215 \\ & -207 \quad 9 \\ \hline & 8 \end{array}$$

Put a one-digit number in each box to make the calculation correct.

$$\begin{array}{r} 6\boxed{5}\boxed{3} \\ \times\ \boxed{7}\ 3 \\ \hline \boxed{1}\boxed{9}\boxed{8}9 \\ 4\boxed{5}\ 4\ 1\ 0 \\ \hline \boxed{4}\boxed{8}\boxed{3}\boxed{9}\boxed{9} \end{array}$$

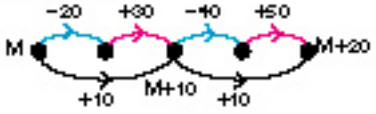
20

You buy a used stereo for $20 and later sell it to a friend for $30. Then you buy it back for $40 and sell it to your brother for $50.

Have you gained money or lost money? gained

How much money have you gained or lost? $20 gained

Hint: An arrow road recording the transactions might help to solve the problem.



Explain your answer. Each time you buy and sell the stereo, you gain $10. The picture above shows starting with an amount of money (say $M) and completing each transaction to end with $20 more ($M + 20).

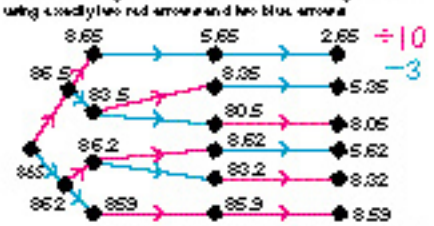
21

Accept any correct explanation.

Bill has a secret number.

Clue 1

Bill is the ending number of an arrow road starting at 365 and using exactly two red arrows and two blue arrows.



Bill could be 2.65, 5.35, 8.05, 5.62, 8.32, or 8.59.

Clue 2

Bill can be shown on the Minicomputer by hiding all exactly one checker.

Who is Bill? 8.32

22

Locate these numbers in the string picture.

$\frac{7}{2}$ $\frac{2}{3}$ $\frac{4}{3}$ $\frac{10}{3}$ $\frac{8}{5}$

$\frac{3}{10}$ 0.6 4×0.7 6×0.3

$\frac{5}{6} + \frac{1}{3}$ 0.5×1.4

23

Put one of these symbols in each box to make the calculator sentences true.

+ − × ÷

A symbol may be used twice in one sentence.

8 × 0 ÷ 3 = 0

9 − 4 ÷ 5 = 1

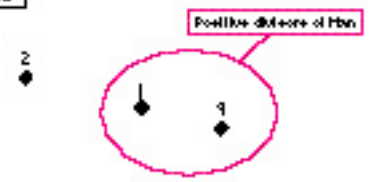
4 − 7 × 6 = −18

7 ÷ 2 + 4 = 7.5

24

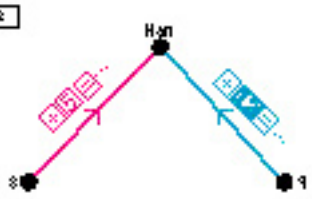
Han is a secret whole number less than 200.

Clue 1



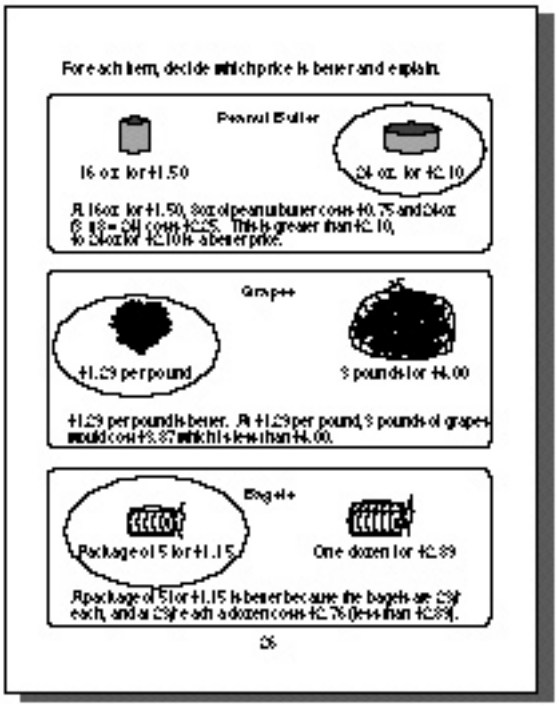
Han could be 9, 27, 45, 63, 81, 99, 117, 135, 153, 171, or 189.

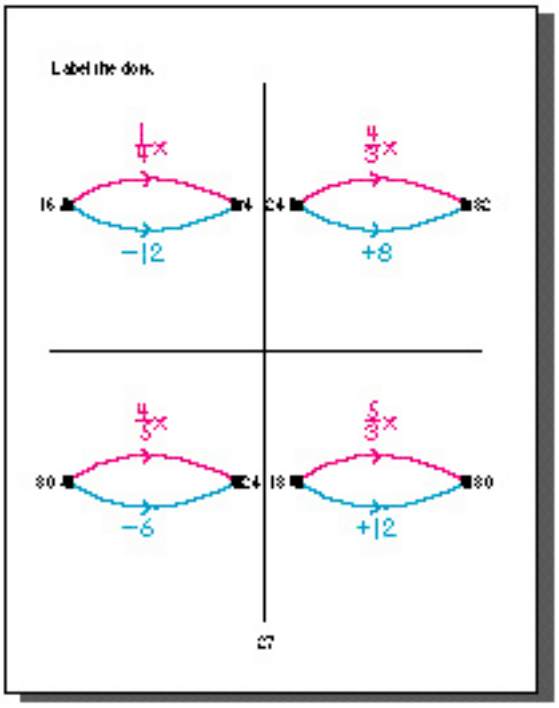
Clue 2

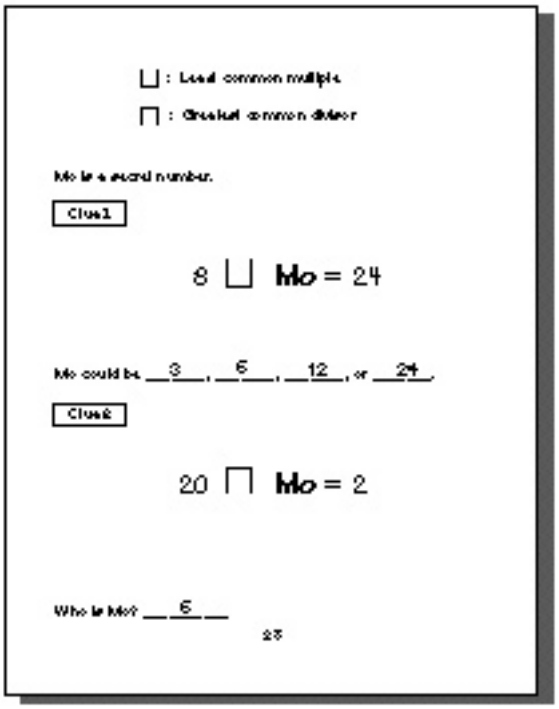


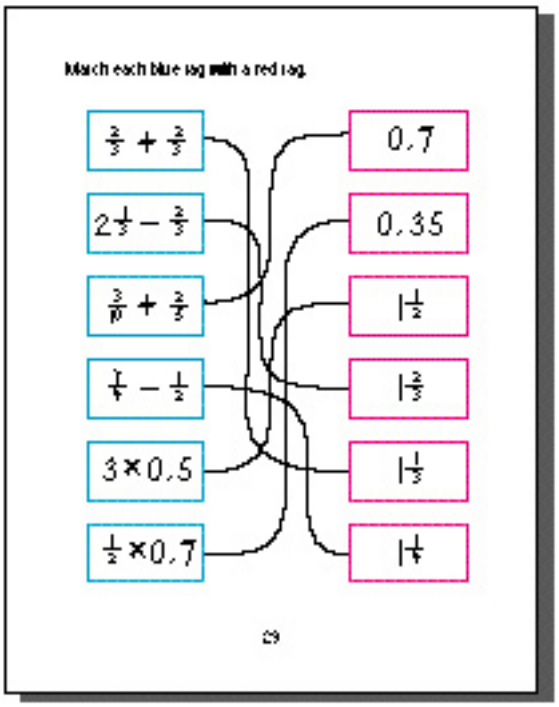
Who is Han? 153

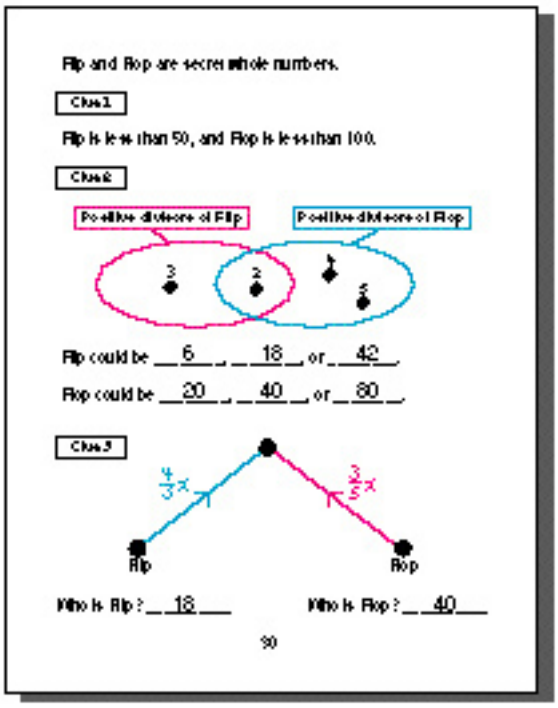
25

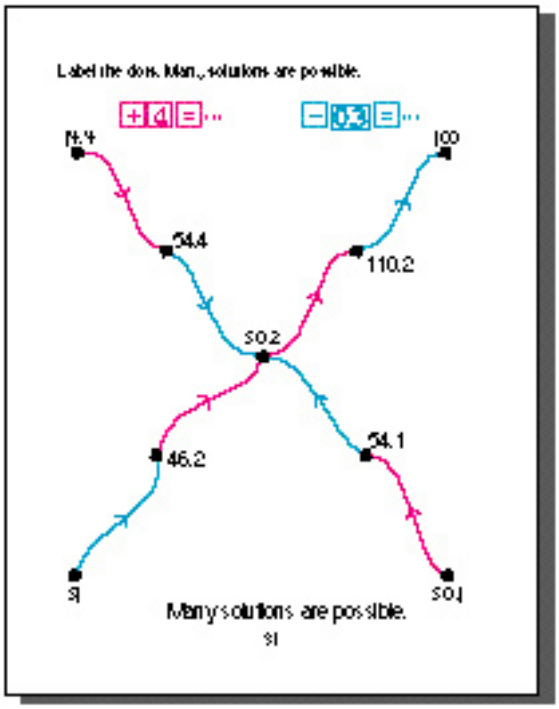

For each item, decide which price is better and explain.
Peanut Butter
16 oz. for $1.50
24 oz. for $2.10
Grapes
$1.29 per pound
5 pounds for $4.00
Bagels
Package of 5 for $1.15
One dozen for $2.89

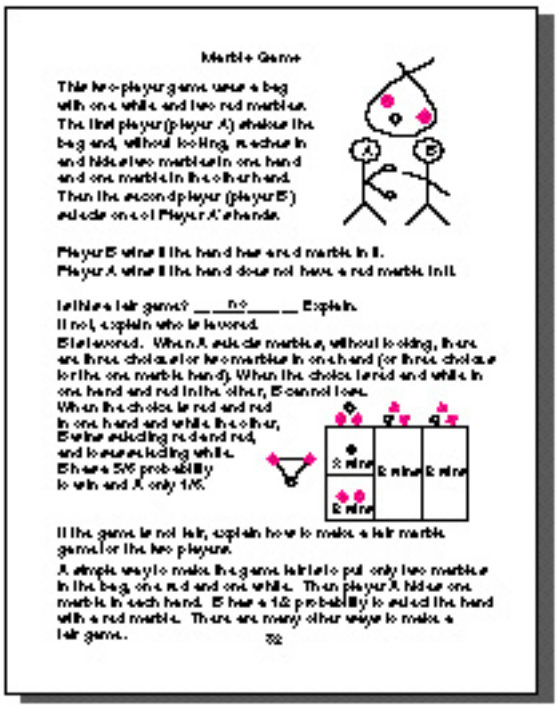

Label the dots.
16
4
24
8
80
64
80
18
−12
+8
−6
+12


□ : Least common multiple
⊓ : Greatest common divisor
Mo is a secret number.
Clue1
8 □ Mo = 24
Mo could be 3, 6, 12, or 24.
Clue2
20 ⊓ Mo = 2
Who is Mo? 6


Match each blue tag with a red tag.
0.7
0.35
3×0.5


Flip and Flop are secret whole numbers.
Clue 1
Flip is less than 50, and Flop is less than 100.
Clue 2
Positive divisors of Flip
Positive divisors of Flop
3
2
1
5
Flip could be 6 , 18 , or 42 .
Flop could be 20 , 40 , or 80 .
Clue 3
4/3 x
3/5 x
Flip
Flop
Who is Flip? 18
Who is Flop? 40
90


Label the dots. Many solutions are possible.
+ 4 = ...
− 0.3 = ...
N.4
100
34.4
110.2
50.2
46.2
34.1
91
50.1
Many solutions are possible.
91


Marble Game
This two-player game uses a bag with one white and two red marbles. The first player (player A) shakes the bag and, without looking, reaches in and hides two marbles in one hand and one marble in the other hand. Then the second player (player B) selects one of Player A's hands.
A
B
Player B wins if the hand has a red marble in it.
Player A wins if the hand does not have a red marble in it.
Is this a fair game? No Explain.
If not, explain who is favored.
B is favored. When A selects marbles, without looking, there are three choices for two marbles in one hand (or three choices for the one marble hand). When the choice is red and white in one hand and red in the other, B cannot lose.
When the choice is red and red in one hand and white the other, B wins selecting red and red, and is equal selecting white. B has a 5/6 probability to win and A only 1/6.
R
R
W
R wins
R wins
R wins
R wins
If the game is not fair, explain how to make a fair marble game for the two players.
A simple way to make the game fair is to put only two marbles in the bag, one red and one white. Then player A hides one marble in each hand. B has a 1/2 probability to select the hand with a red marble. There are many other ways to make a fair game.
92

Capsule Lesson Summary

Use estimation to determine the correct placement of missing decimal points in a list of calculations. Begin the workbook *Arcade of Problems #3*. (This is the first of two lessons using this workbook.)

Materials

Teacher • None	Student • *Arcade of Problems #3* Workbook • Compass • Metric ruler • Colored pencils, pens, or crayons • Calculator

Description of Lesson

Write the following list of problems on the board.

$$9 \times 3.73 = 1273$$
$$1.793 \times 3.02 = 4813$$
$$8 - 7.831 = 0169$$
$$73.4 - 9.9734 = 63426 6$$
$$1.7 \times 9.1 = 1547$$
$$0.93 \times 8.234 = 765762$$
$$0.3 \times 79 = 237$$

T: ***The eraser gremlin erased the decimal point in the results of all these calculations. Where should we place the decimal point in each result to make the calculation correct?***

Encourage students to use estimation in deciding where to place the decimal points. For example, in the first problem ask,

T: ***What whole number is closest to 3.73?*** (4)
What number should the result be close to?

S: ***13; 9 + 4 = 13.***

T: ***Where should we place the decimal point in the result?***

S: ***Between the 2 and the 7; 12.73 is close to 13.***

Continue in the same manner, asking students where to place the decimal point in the remaining problems. Each time, ask students to explain their responses. Encourage students to give explanations that involve estimation.

The correct placement of decimal points for all of the problems is shown below.

$$9 \times 3.73 = 12.73$$
$$1.793 \times 3.02 = 4.813$$
$$8 - 7.831 = 0.169$$
$$73.4 - 9.9734 = 63.4266$$
$$1.7 \times 9.1 = 15.47$$
$$0.93 \times 8.234 = 7.657762$$
$$0.3 \times 79 = 23.7$$

Distribute copies of the workbook *Arcade of Problems #3*, and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of the class period, collect the workbooks for your review. They will be used again in Lesson W6.

Capsule Lesson Summary

Find missing digits in a division calculation. Continue individual work in the workbook *Arcade of Problems #3*. (This is the second of two lessons using this workbook.)

Materials

Teacher	Student
• None	• *Arcade of Problems #3* Workbook • Compass • Metric ruler • Colored pencils • Calculator

Description of Lesson

Put this algorithm puzzle on the board, leaving the boxes empty.

Invite students to suggest how to fill in the boxes. Remind them that each box is for one missing digit. Encourage students to explain their responses. If many students have difficulty getting started, lead the discussion to determine that the divisor is 17. The problem is solved when all the boxes are filled in, as shown here.

```
   1 7 3  R = 9
1 7 ) 2 9 5 0
    − 1 7 0 0   1 0 0
      1 2 5 0
      − 6 8 0     4 0
        5 7 0
        5 1 0     3 0
          6 0
        − 5 1       3
            9
```

Distribute students' copies of the workbook *Arcade of Problems #3*. Ask students first to correct or complete pages from the previous week's work and then to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

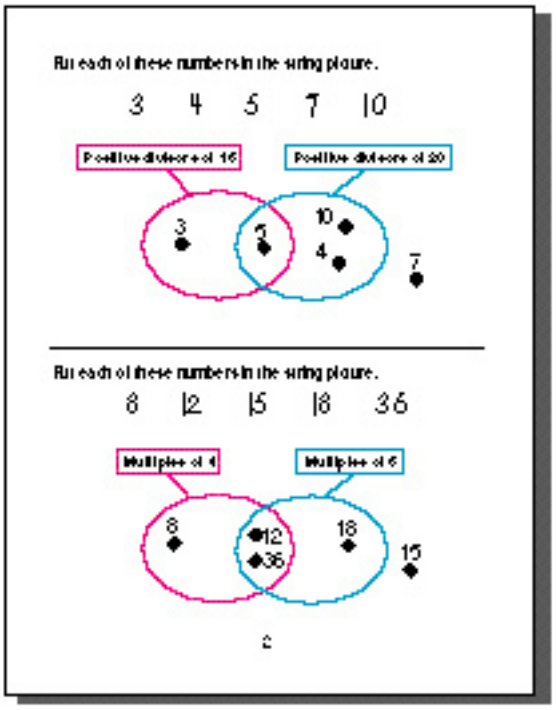
At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in the workbook during a study time or to take it home as an assignment.

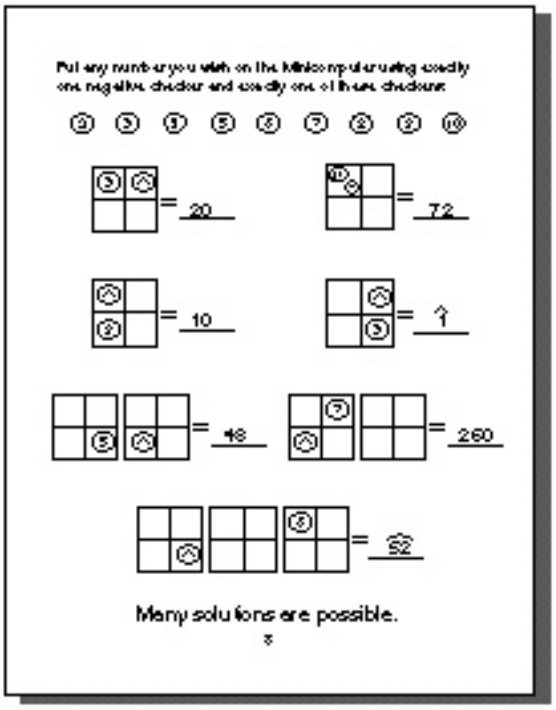
Assessment Activity

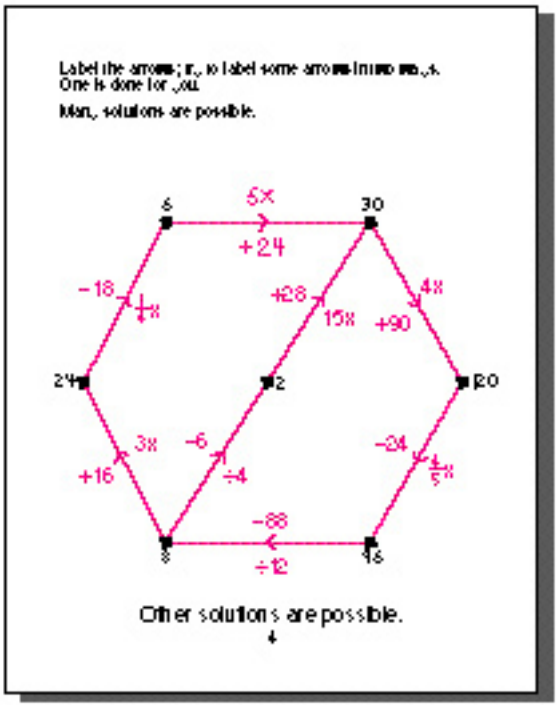
An individual student progress record for the workbook is available on Blackline W6(a). You may like to use this form to monitor student work.

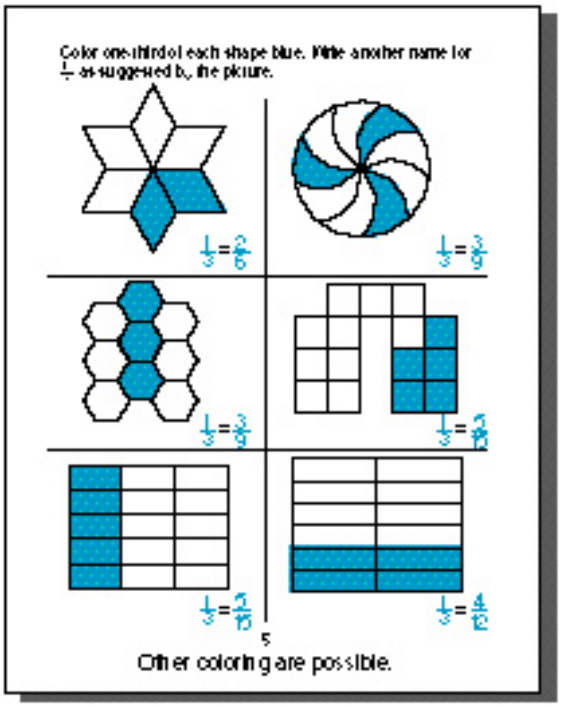
Home Activity

This would be a good time to send a letter to parents/guardians about addition, subtraction, multiplication, and division practice. Blackline W6(b) has a sample letter.

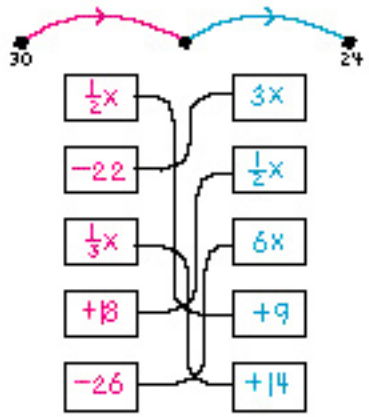

Put each of these numbers in the sorting picture.
3 4 5 7 10
Positive divisors of 16
Positive divisors of 20
3
4
10
7
Put each of these numbers in the sorting picture.
8 12 15 18 36
Multiples of 4
Multiples of 6
8
12
36
18
15
2


Put any number you wish on the blank. Use each number exactly one negative checker and exactly one of these checkers:
= 20
= 72
= 10
= 1
= 48
= 260
= 52
Many solutions are possible.
3


Label the arrows; use ,, to label some arrows in two ways.
One is done for you.
Many solutions are possible.
6
30
24
2
20
8
18
+24
−18
+28
+90
+16
−6
÷4
−24
−88
÷12
Other solutions are possible.
4


Color one-third of each shape blue. Write another name for
as suggested by the picture.
Other coloring are possible.
5

Pair the tags.



6

The eraser gremlin has erased the decimal point in each result. Put in decimal points so that the calculations are correct.

$3.73 + 9 + 0.37 = 13.1$

$297.7 - 2.977 = 294.723$

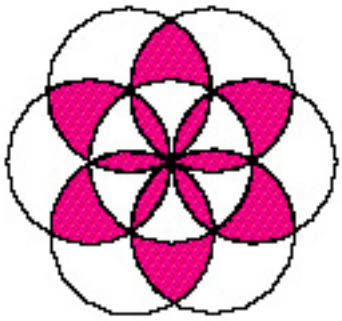
$13 - 1.673 = 11.327$

$28.3 \times 0.93 = 26.319$

$35.9 \times 60.12 = 2158.308$

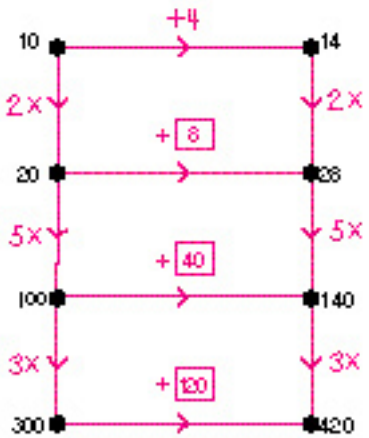
7

Complete this design with a compass and red crayon so that you get a six-petaled red flower.

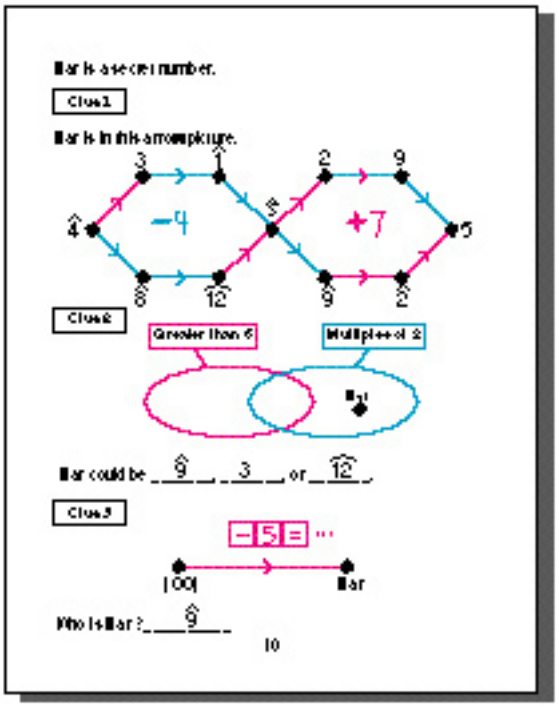


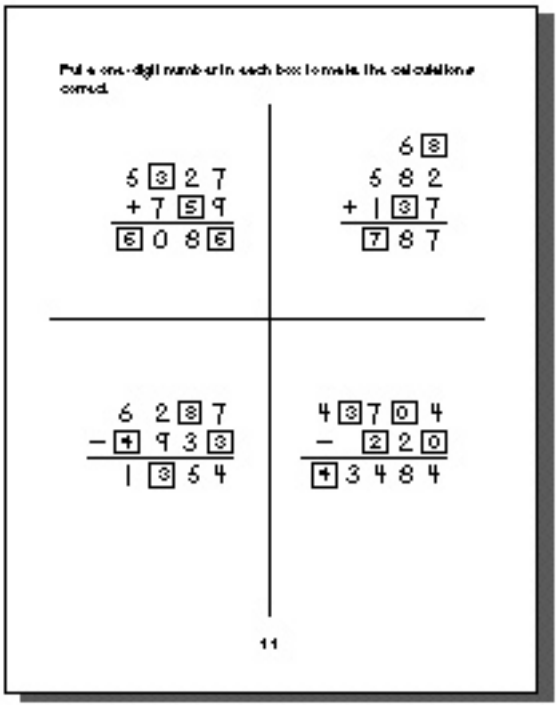
8

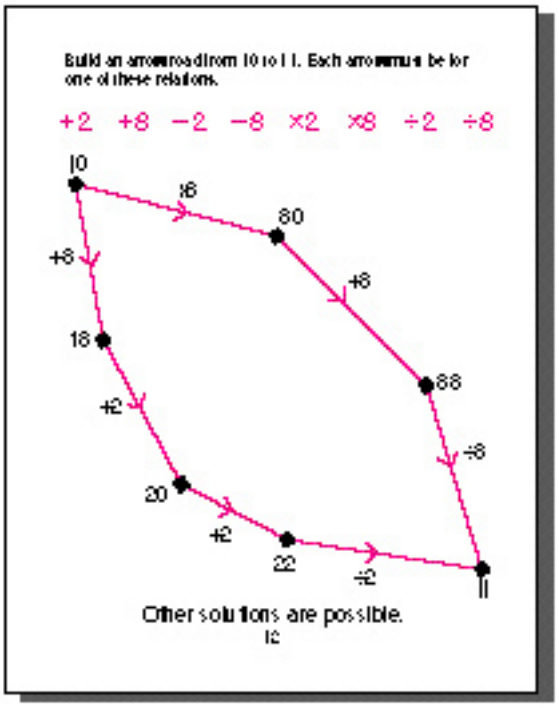
Label the dots and fill in the boxes for the arrows.

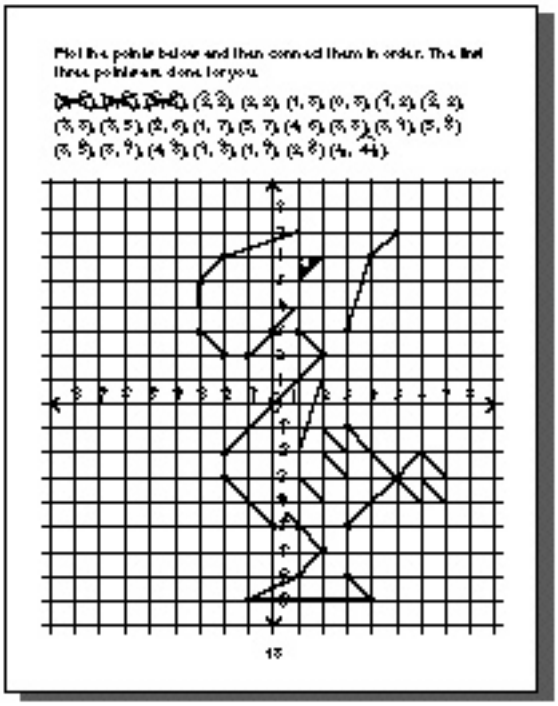


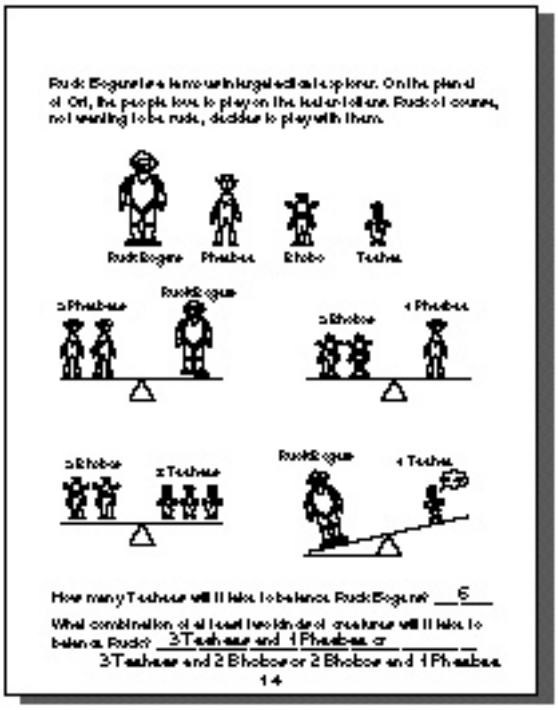
9

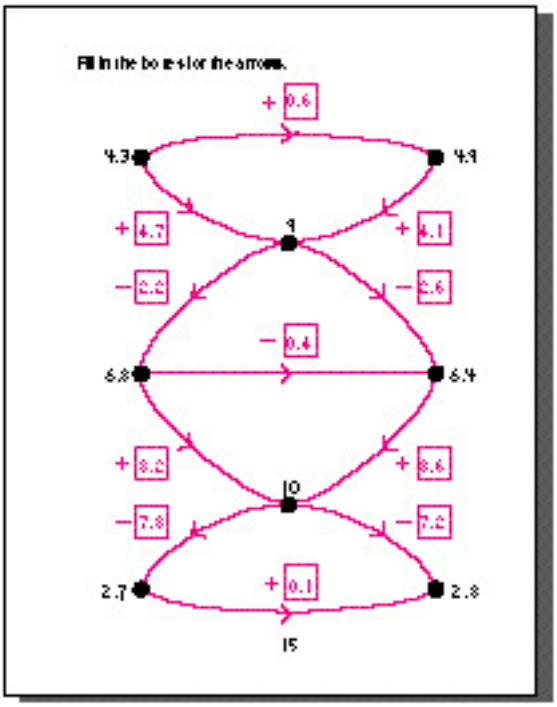

Ilar is a secret number.
Clue 1
Ilar is in this arrowpicture.
3
1
2
9
4
5
6
12
9
2
−4
+7
Clue 2
Greater than 6
Multiple of 2
Ilar
Ilar could be 9, 3, or 12
Clue 3
−5= ···
100
Ilar
Who is Ilar? 9
10

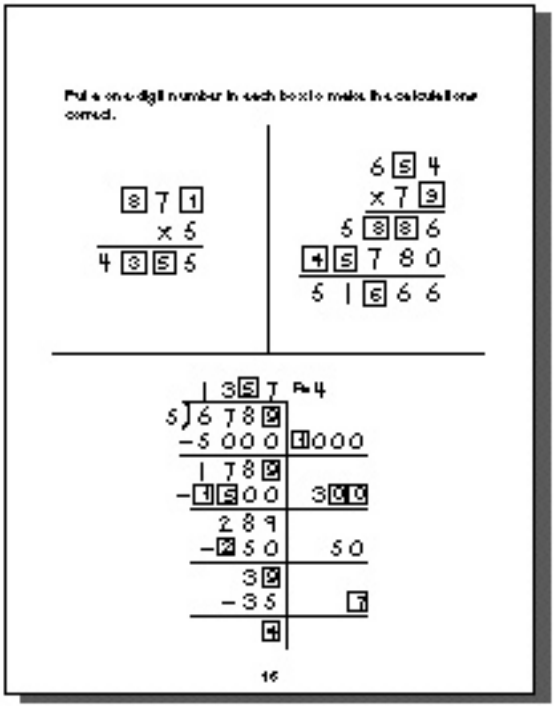

Put a one-digit number in each box to make the calculations correct.
5 3 2 7
+ 7 5 9
5 0 8 5
6 8
5 8 2
+ 1 3 7
7 8 7
6 2 8 7
− 4 9 3 3
1 3 5 4
4 3 7 0 4
− 2 2 0
4 3 4 8 4
11

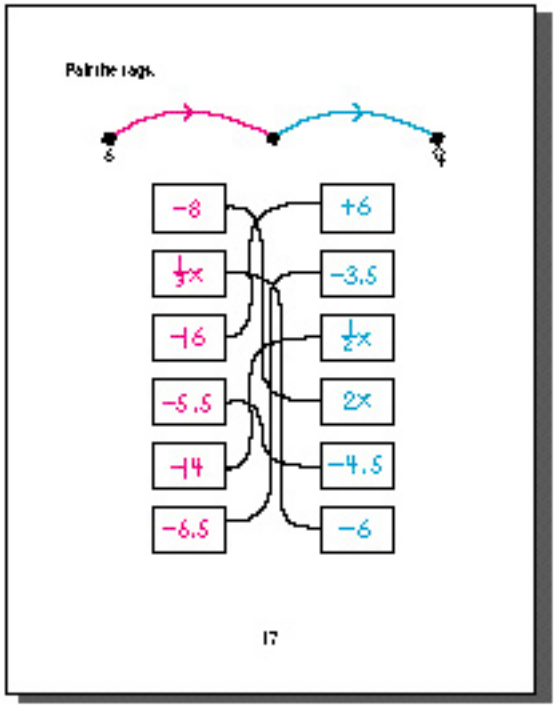

Build an arrowroad from 10 to 11. Each arrowarrow be for one of these relations.
+2 +8 −2 −8 ×2 ×8 ÷2 ÷8
10
18
80
88
20
22
11
+8
+8
+8
+2
+2
+2
÷8
Other solutions are possible.
12

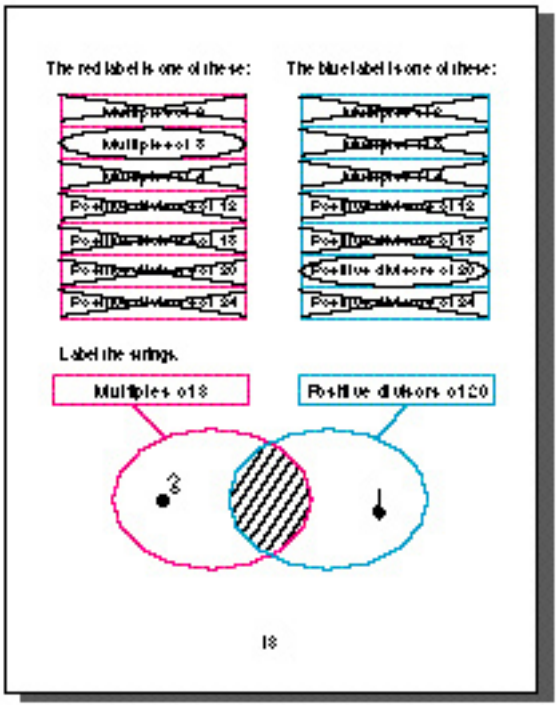

Plot the points below and then connect them in order. The first three points are done for you.
(2, 2), (3, 2), (4, 2), (1, 3), (0, 3), (1, 4), (2, 4), (3, 3), (3, 5), (2, 6), (1, 7), (3, 7), (4, 6), (3, 6), (3, 1), (2, 0) (3, 0), (3, 1), (4, 3), (1, 3), (1, 1), (2, 0) (4, 44).
13

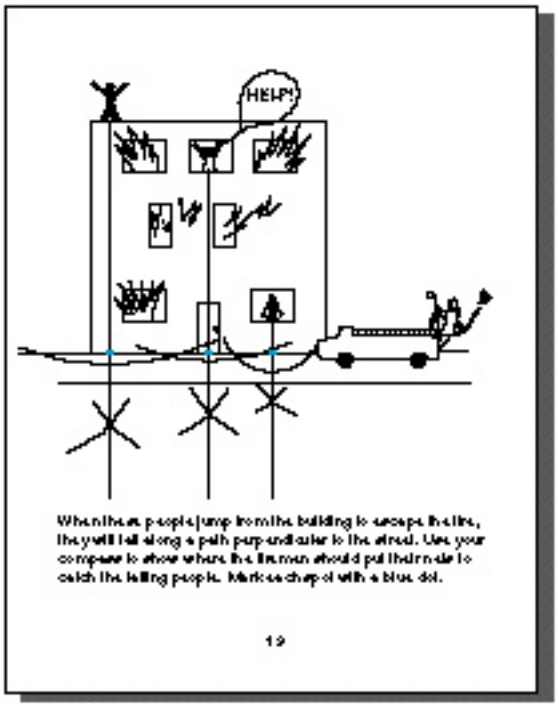

Rude Rogues have unhinged sense of humour. On the planet of Orl, the people love to play on the letter 'o'. Rude of course, not wanting to be rude, decided to play with them.
RudeRogues
Phoebes
Rhobo
Teehee
3 Phoebes
RudeRogue
3 Rhobo
1 Phoebes
3 Rhobo
2 Teehee
RudeRogue
1 Teehee
How many Teehees will it take to balance RudeRogue? 6
What combination of at least two kinds of creatures will it take to balance Rude? 3 Teehees and 1 Phoebes or
3 Teehees and 2 Rhobos or 2 Rhobos and 1 Phoebes
14

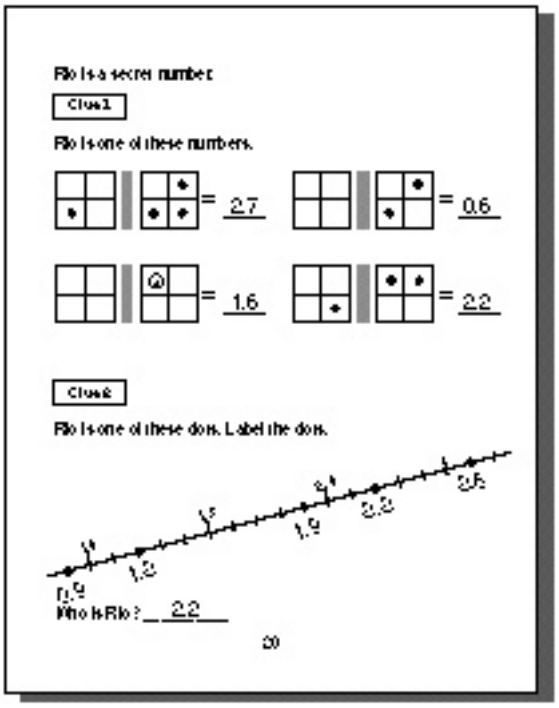

Fill in the boxes for the arrows.
+ 0.6
4.3
4.9
+ 4.7
+ 4.1
4
− 2.2
− 2.6
− 0.4
6.8
6.4
+ 8.2
+ 8.6
10
− 7.8
− 7.2
2.7
+ 0.1
2.8
15

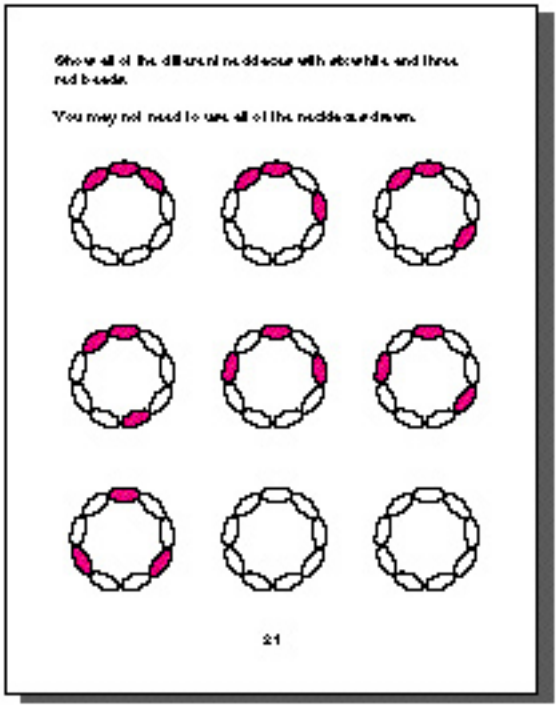

Put a one-digit number in each box to make the calculations correct.
8 7 1
× 5
4 3 5 5
6 5 4
× 7 9
5 8 8 6
4 5 7 8 0
5 1 6 6
1 3 5 7 R 4
5) 6 7 8 2
− 5 0 0 0 1000
1 7 8 2
− 1 5 0 0 300
2 8 9
− 2 5 0 50
3 2
− 3 5 5
4
16

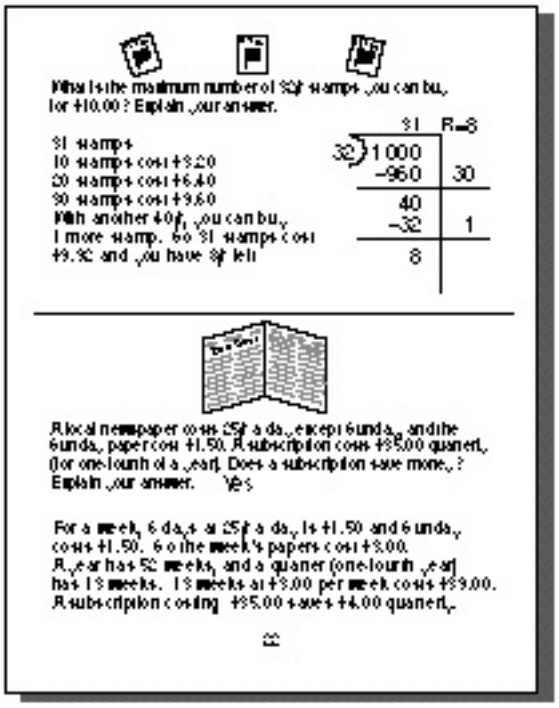

Pair the tags.
5
4
−8
+6
1/3x
−3.5
−16
1/2x
−5.5
2x
−14
−4.5
−6.5
−6
17

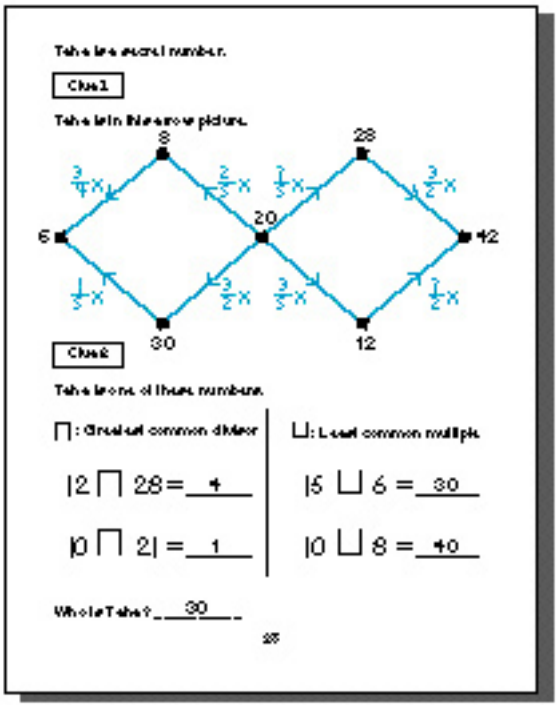

The red label is one of these:
The blue label is one of these:
Multiples of 18
Positive divisors of 120
Label the strings.
Multiples of 18
Positive divisors of 120
2
3
18

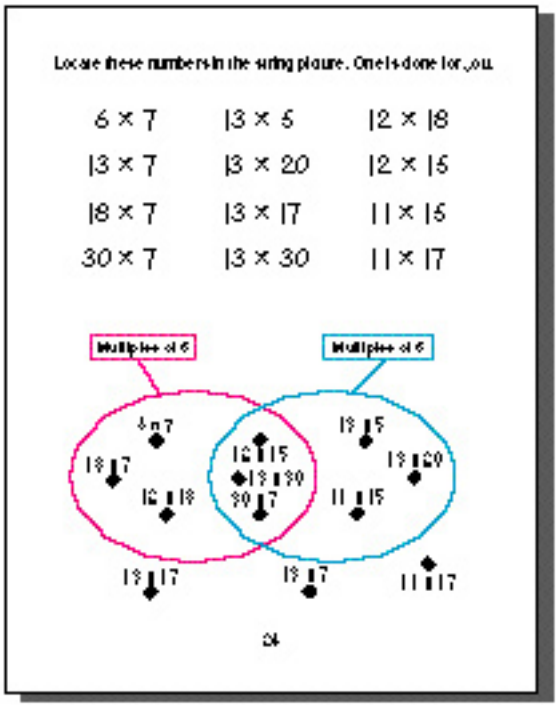

HELP!
When these people jump from the building to escape the fire, they will fall along a path perpendicular to the street. Use your compass to show where the fireman should put their ladder to catch the falling people. Mark each ladder with a blue dot.
19

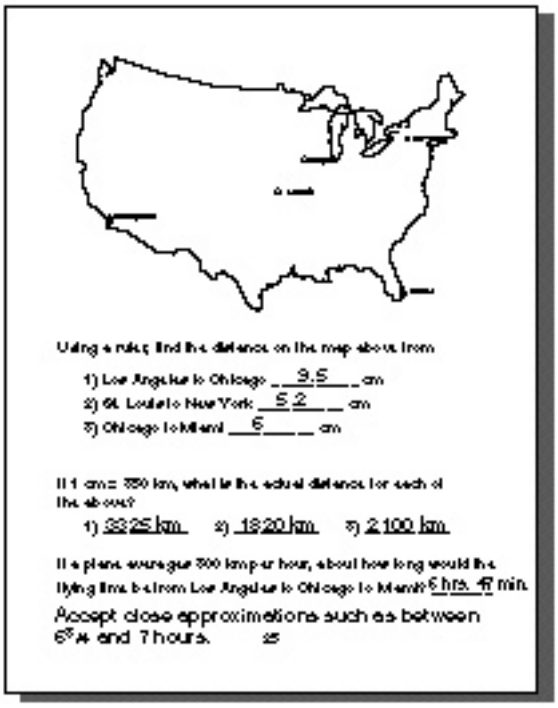

Rio is a secret number.
Clue 1
Rio is one of these numbers.
= 2.7
= 0.6
= 1.6
= 2.2
Clue 2
Rio is one of these dots. Label the dots.
0.9
1.2
1.5
1.9
2.2
2.5
Who is Rio? 2.2
20

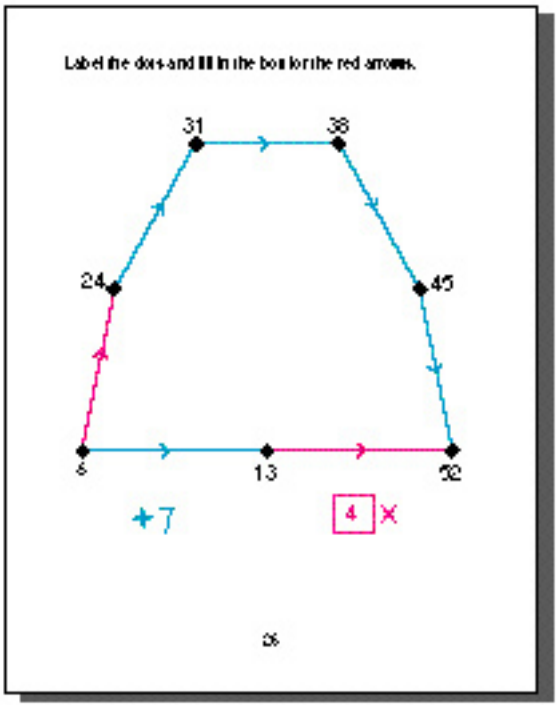

Show all of the different necklaces with six white and three red beads.
You may not need to use all of the necklaces shown.
21

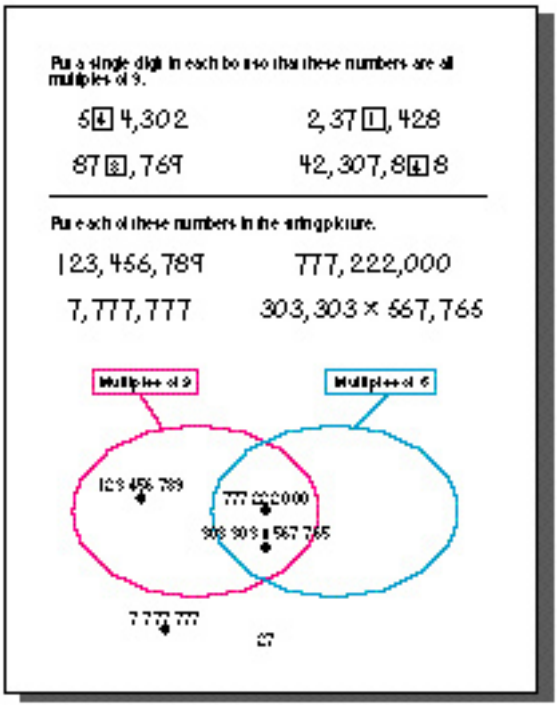

What is the maximum number of $2¢ stamps you can buy for $10.00? Explain your answer.
$1 stamps
10 stamps cost $2.20
20 stamps cost $4.40
30 stamps cost $6.60
With another 40¢, you can buy
1 more stamp. So $1 stamps cost $9.32 and you have 8¢ left
$1 R=8
32) 1000
−960 30
40
−32 1
8
A local newspaper costs 25¢ a day, except 6 undays, and the 6 undays paper cost $1.50. A subscription costs $85.00 quarterly (for one-fourth of a year). Does a subscription save money? Explain your answer. Yes
For a week, 6 days at 25¢ a day is $1.50 and 6 undays costs $1.50. So the week's papers cost $3.00.
A year has 52 weeks, and a quarter (one-fourth year) has 13 weeks. 13 weeks at $3.00 per week costs $39.00.
A subscription costing $85.00 saves $4.00 quarterly.
22

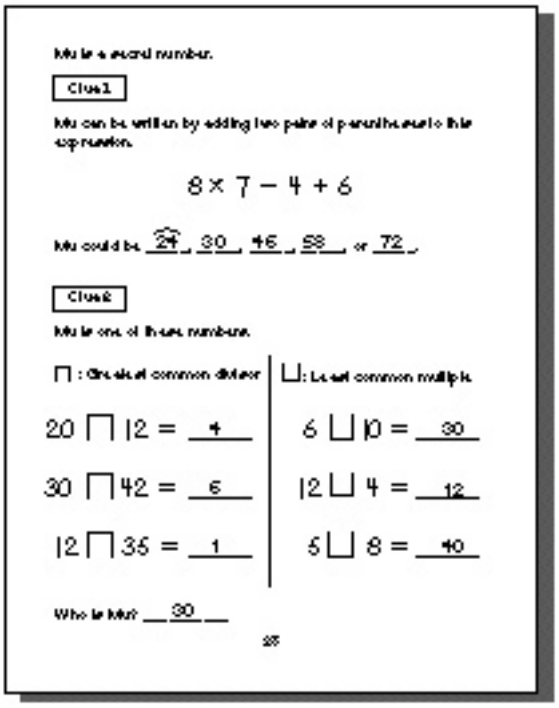

Take a whole number.
Class 1
Take it in this arrow picture.
8
28
20
6
42
30
12
Class 2
Take two of those numbers.
⊓: Greatest common divisor
⊔: Least common multiple
|2 ⊓ 28 = 4
|5 ⊔ 6 = 30
|0 ⊓ 2| = 1
|0 ⊔ 8 = 40
Whole Take? 30
23

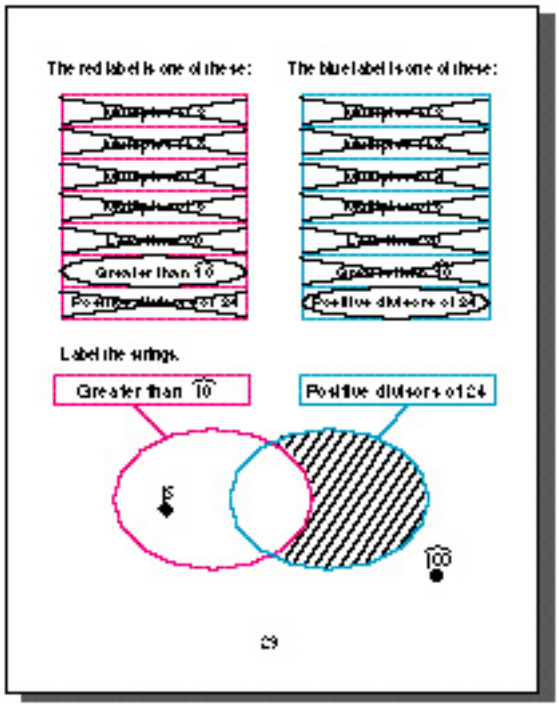

Locate these numbers in the string picture. One is done for you.
6 × 7
|3 × 5
|2 × |8
|3 × 7
|3 × 20
|2 × |5
|8 × 7
|3 × |7
|| × |5
30 × 7
|3 × 30
|| × |7
Multiples of 6
Multiples of 6
4 n 7
|8 | 7
|6 | 15
|8 | 18
|8 | 15
|8 | 20
|8 | 30
30 | 7
|| | 15
|8 | 17
|8 | 7
|| | 17
24

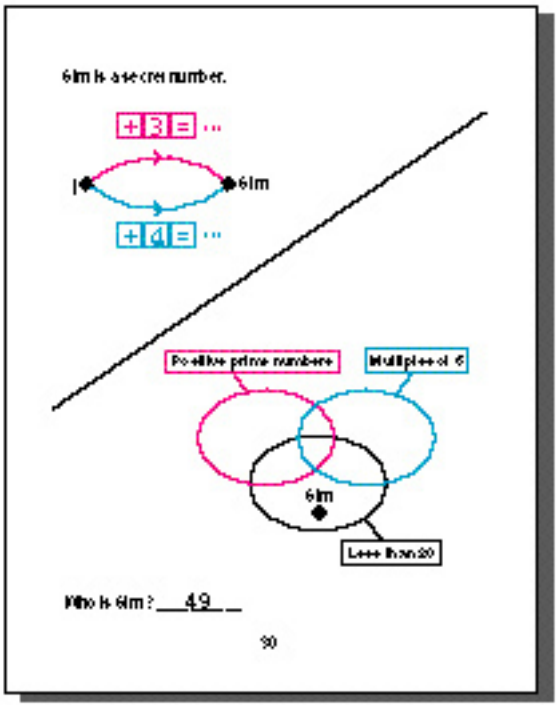

Using a ruler, find the distance on the map above from
1) Los Angeles to Chicago 3.5 cm
2) St. Louis to New York 5.2 cm
3) Chicago to Miami 6 cm
If 1 cm = 350 km, what is the actual distance for each of the above?
1) 1225 km
2) 1820 km
3) 2100 km
If a plane averages 300 km per hour, about how long would the flying time be from Los Angeles to Chicago to Miami? 6 hrs. 47 min.
Accept close approximations such as between 6¾ and 7 hours.
25

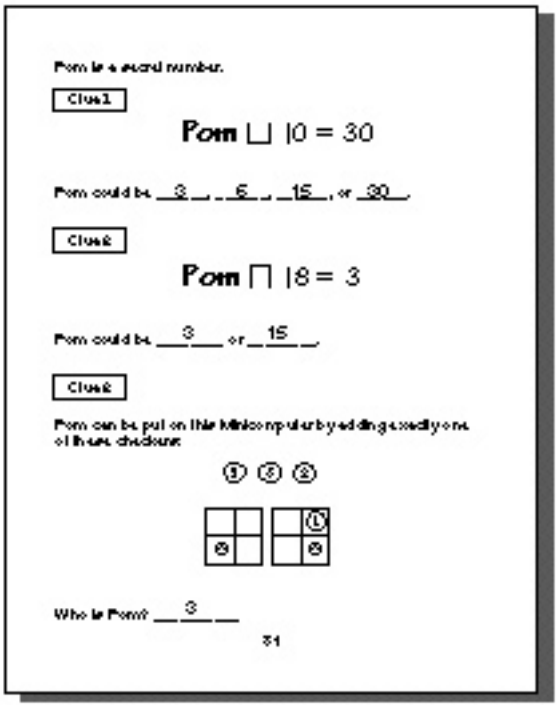

Label the dots and fill in the box for the red arrows.
31
38
24
45
6
13
32
+7
4 ×
26

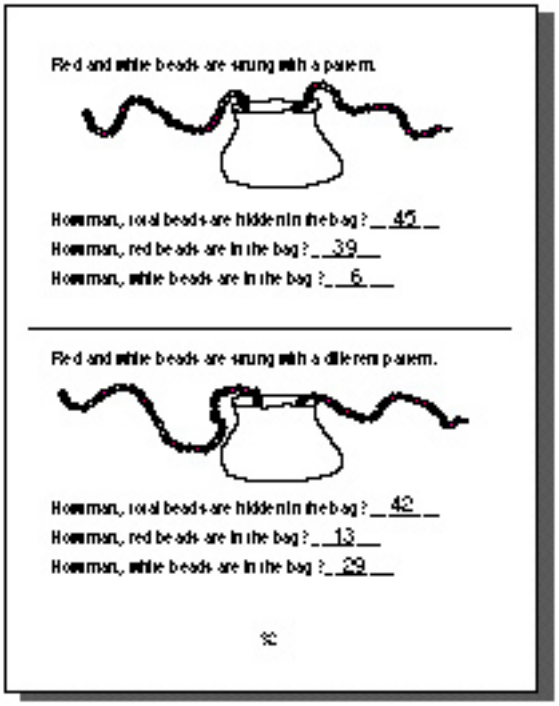

Put a single digit in each box so that these numbers are all multiples of 9.
5 4,302
2,37 1,428
87 ,769
42,307,8 8
Put each of these numbers in the right place.
123,456,789
777,222,000
7,777,777
303,303 × 567,765
Multiples of 9
Multiples of 6
123 456 789
777 222 000
303 303 × 567 765
7 777 777
27


Make a mixed number.
Clue 1
Mu can be written by adding two pairs of parentheses to this expression.
8 × 7 − 4 + 6
Mu could be 24, 30, 46, 58, or 72.
Clue 2
Make one of these numbers.
⊓ : Greatest common divisor
⊔ : Least common multiple
20 ⊓ 12 = 4
6 ⊔ 10 = 30
30 ⊓ 42 = 6
12 ⊔ 4 = 12
12 ⊓ 35 = 1
5 ⊔ 8 = 40
Who is Mu? 30
28


The red label is one of these:
The blue label is one of these:
Greater than 10
Positive divisors of 24
Label the strings.
Greater than 10
Positive divisors of 24
5
100
29


6lm is a secret number.
+3 = ...
6lm
+4 = ...
Positive prime numbers
Multiples of 6
6lm
Less than 20
Who is 6lm? 49
30


Pom is a secret number.
Clue1
Pom ☐ 10 = 30
Pom could be 3 , 6 , 15 , or 30 .
Clue2
Pom ☐ 18 = 3
Pom could be 3 or 15 .
Clue3
Pom can be put on this Mini computer by adding exactly one of these checkers:
Who is Pom? 3
31


Red and white beads are strung with a pattern.
Hmmm, total beads are hidden in the bag? 45
Hmmm, red beads are in the bag? 39
Hmmm, white beads are in the bag? 6
Red and white beads are strung with a different pattern.
Hmmm, total beads are hidden in the bag? 42
Hmmm, red beads are in the bag? 13
Hmmm, white beads are in the bag? 29
32

Capsule Lesson Summary

Using an abacus as support, discuss a base five and a base twelve system of numeration. Invent some symbols for ten and eleven to use in base twelve writing.

Materials

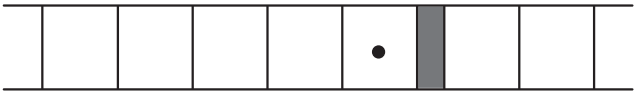
Teacher • Magnetic checkers (optional) Student • Worksheet W7

Description of Lesson

Note: Because there are several bases referred to in this lesson, it is necessary to be careful about how you read numerals. For this reason, we often indicate how to read a numeral.

Exercise 1

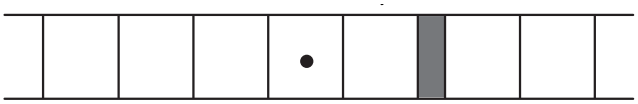
Draw this abacus on the chalkboard. Put (draw) one checker on the ones board.



T: ***What number is on this abacus?***

S: ***1.***

Label the ones board and move the checker one board to the left.

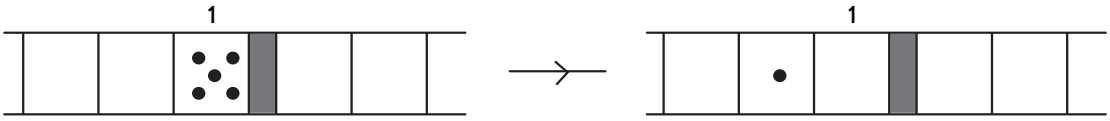


T: ***What number could this be?***

The choice is arbitrary. Let students suggest many whole numbers greater than 1. In fact, it could be a negative integer, but do not mention this if no one suggests it.

T: ***You can't know for sure what number it is until I tell you the rule for this abacus. The rule is that five checkers on a board trade for one checker on the next board to the left.***

Demonstrate the rule by putting five checkers on the ones board and then making a trade.

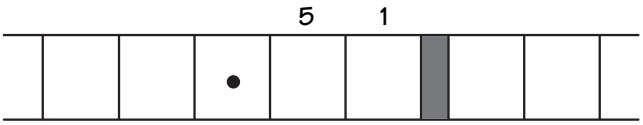


Leave the checker on the abacus.

T: ***Now, what number is this?***

S: ***5.***

Label the appropriate board and move the checker one board to the left.



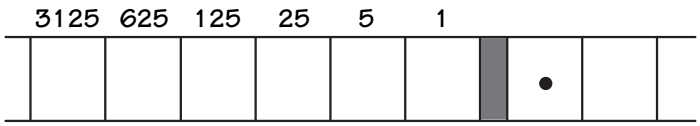
T: ***What number is this?***

S: ***25, because 5 x 5 = 25.***

S: ***25, because according to the rule we can trade this checker for five checkers on the fives board.***

Move the checker to the left one board at a time as you ask for the value of each board. When you have six or seven boards to the left of the bar labeled, put one checker on the board to the right of the bar.

T: ***What number is this?***

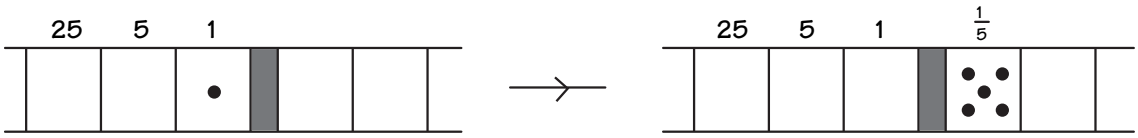


S: *⅕.*

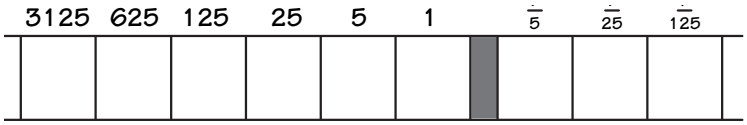
T: ***Why?***

S: ***Five checkers on that board trade for one checker on the ones board. So the value of one checker on that board is ⅕.***

Demonstrate this trade on the abacus.



Move the checker to the right one board at a time as you ask for the value of each board. Label three or four boards to the right of the bar.

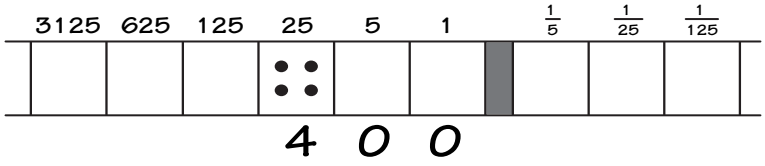


T: ***This is a* base five *abacus. Who can put 100 on this abacus?***

Accept any solution, but lead to a solution with at most four checkers on a board (see illustration below).

T: ***Who can write the base five name for 100?***

Invite a student to write the number by aligning the digits of the base five number with the boards of the abacus.



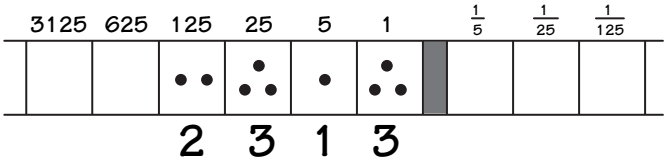
Note: Read "400" in base five as "four, zero, zero, base five" instead of as "four hundred."

T: ***Why is this 100?***

S: ***4 x 25 = 100.***

Begin this table on the board.

T: ***Who can put 333 on this abacus and write its base five name?***



Decimal writing	Base five writing
100	400
333	

T: ***In our regular decimal writing of numbers we need ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. How many digits do we need in base five?***

S: ***Five: 0, 1, 2, 3, and 4.***

T: ***Why only five? What happens if I show a number by putting more than four checkers on a board of the abacus?***

S: ***If there are five or more checkers on any board of the abacus, we can make a trade of five checkers for one checker on the next board to the left.***

Add the following information to the table.

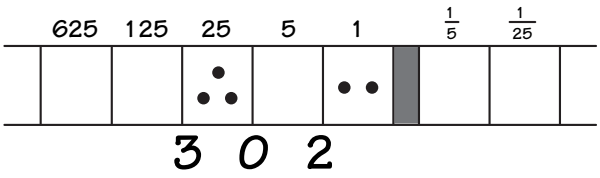
T (pointing to 302)**:** ***Which decimal number is this?***

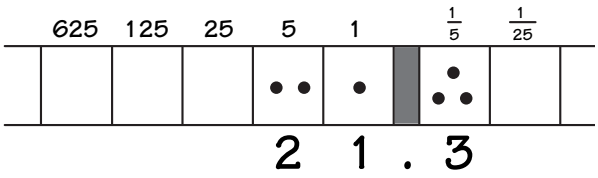
S: ***77, because (3 x 25) + 2 = 77.***

T (pointing to 21.3)**:** ***Who can put this number on the abacus? Which decimal number is this?***

S: ***11⅗ or 11.6, since ⅕ = 0.2 and ⅗ = 0.6.***

Decimal writing	Base five writing
100	400
333	2 313
	302
	21.3





Add the information to the table.

Distribute copies of Worksheet W7 and direct students to complete it. When many students are finished, invite some students to present their solutions.

Decimal writing	Base five writing
100	400
333	2 313
77	302
11.6	21.3

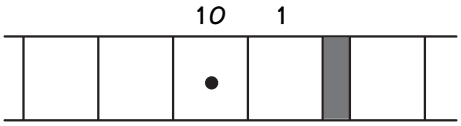
Exercise 2

Erase the labels from the abacus except for the ones board. Put one checker on the abacus on the board to the left of the ones place.

T: ***Suppose we change the rule: One checker on a board trades for ten checkers on the next board to the right. If this is the rule, what number is on the abacus?***

S: ***10.***

T: ***This is now a* base ten *abacus. How many digits do I need to write numbers in base ten?***



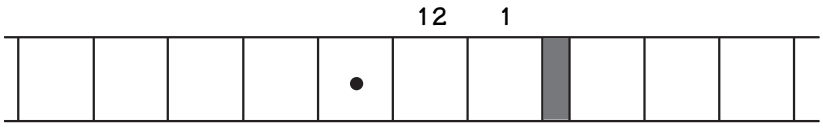
S: ***Ten: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.***

T: ***Will numbers written in base ten be familiar to you?***

S: ***Yes, it's our usual way of writing numbers.***

T: ***Base ten is another name for decimal.***

Change the rule of the abacus again and indicate it in the following way.



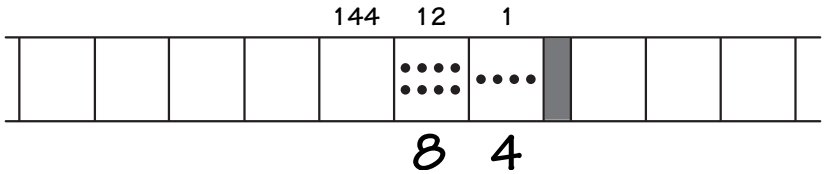
T: ***The new rule is this: One checker on a board trades for twelve checkers on the next board to the right. What number is on the abacus?***

S: ***144, because 12 x 12 = 144.***

Label that board, and begin a table on the board.

Decimal writing	Base twelve writing
144	100
100	

T: ***Who can put 100 on this abacus?***



Note: Read "84" in base twelve as "eight, four, base twelve."

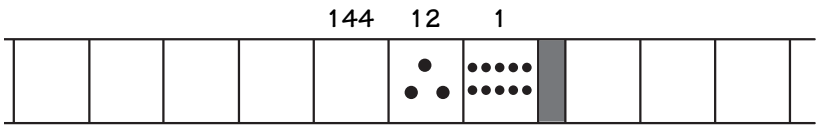
T: ***Why is this 100?***

S: ***(8 x 12) + 4 = 100.***

Extend the table with another decimal number.

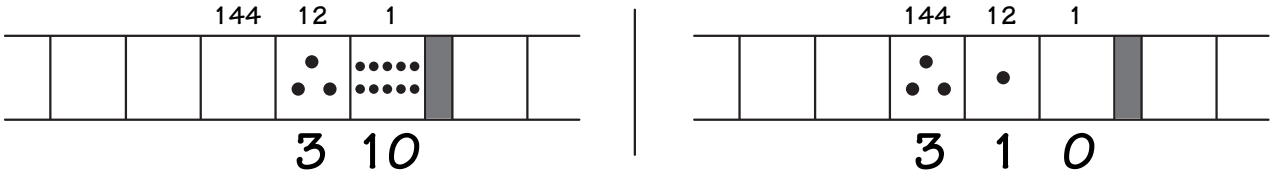
Decimal writing	Base twelve writing
144	100
100	84
46	

T: ***Who can put 46 on the abacus?***



T: ***How can we write the base twelve name for 46?***

Note: Students might suggest incorrectly 310, writing 3 beneath the twelves board and 10 beneath the ones board. If this occurs, ask the class how they would distinguish between the following two numbers written in base twelve. The first one is 46 and the second one is 444; they are different numbers.



T: ***What is the problem? Why is it difficult to write the base twelve name for 46?***

Lead the class to observe that you cannot represent ten checkers on a board until you have a base twelve name for 10.

T: ***How many digits do we need to write base twelve numbers?***

S: ***Twelve.***

T: ***Yes, but we have only ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. What numbers do we still need new symbols for?***

S: ***10 and 11.***

T: ***Let's invent a new symbol for 10 to use in base twelve writing.***

Invite several students to suggest new symbols for 10, and let other students express their opinions of the suggested symbols. The following symbols are ones like students might suggest.

After a few symbols have been drawn, present some desirable characteristics of the new symbol. For example:

T: ***The new symbol should be simple to write and not look too much like other mathematical symbols, such as the numerals 0 through 9 or the operation symbols +, –, x, or ÷. Also, the symbol need not remind us of the numeral 10.***

Write the numeral 6 on the board as you ask,

T: ***Our symbol does not have to suggest quantity; for example, does the way this symbol is written suggest six objects?***

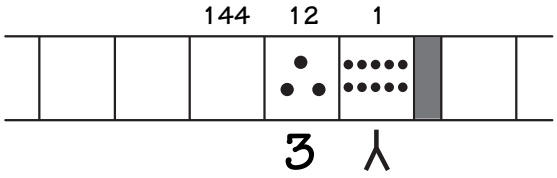
S: ***No, but we have learned to think "six" when we see that symbol.***

T: ***Our new symbol for 10 need not remind us of 10 objects or of the numeral 10. It can be entirely new.***

Using these criteria, let students discuss the symbols they have already suggested and invite them to suggest a few more symbols. Then ask the class to select the symbol they prefer. For example, suppose a class chooses ⅄.

T: ***Now who can write the base twelve name for 46?***

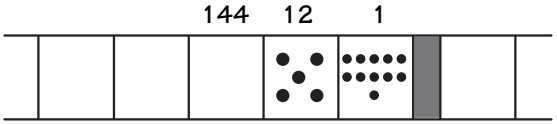
Insist that students use the new symbol chosen by your class, for example:



Decimal writing	Base twelve writing
144	100
100	84
46	3⅄

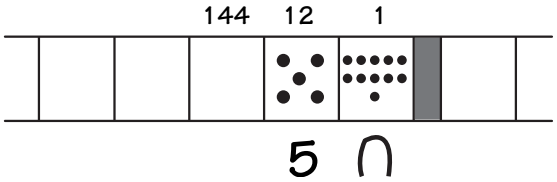
T: ***Who can put 71 on this abacus?***
Can we write the base twelve name for 71?

S: ***Not yet; we need a new symbol for eleven.***



Ask students to suggest symbols for eleven, and then let the class select one of their symbols, for example, ∩, as the base twelve name for eleven.

T: ***Who can write the base twelve name for 71?***



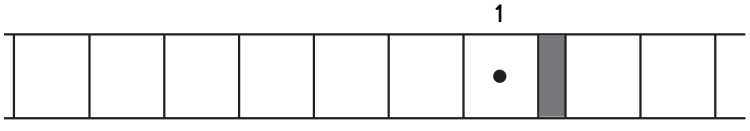
Decimal writing	Base twelve writing
144	100
100	84
46	3⅄
71	5∩

Exercise 3 (optional)

This discussion of positional notation is optional. You may wish to include it in this lesson or at some future time.

T: ***Today we looked at a base five abacus and at a base twelve abacus. In other lessons you have used a base two abacus and a base three abacus. What is similar and what is different about these different bases?***

One point you should discuss is that one checker on the first board to the left of the bar is always 1.



The rule for the abacus is arbitrary. However, once the rule is chosen (for example, five checkers on a board trades for one checker on the next board to the left), the values of the boards are determined by the multiplication rule (e.g., 5, 25, 125, 625, ⅕, 1/25, and so on).

Also, students should observe that the rule chosen for the abacus determines the number of digits needed to write names for numbers on that abacus. For example, we need five digits (0, 1, 2, 3, 4) to write base five names.

T: ***Why do you think we usually write base ten names for numbers instead of some other base?***

In the discussion you may observe that people have ten fingers and that this may have led to a base ten method of writing numerals.

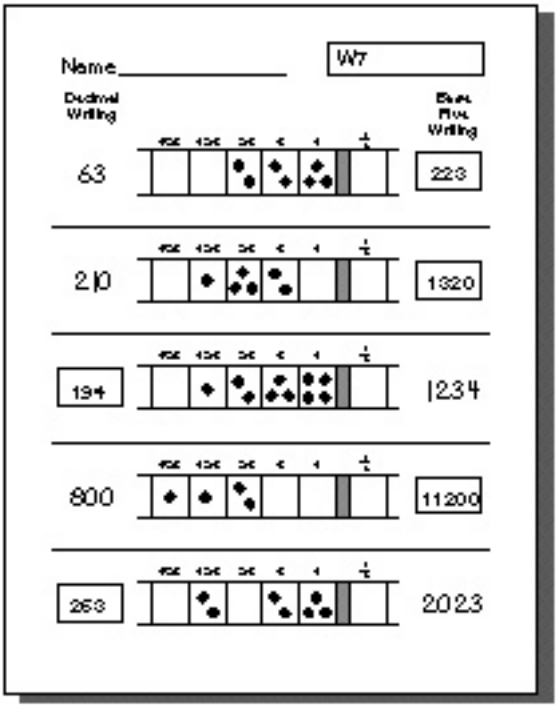
T: ***What bases do you think other earlier cultures may have used? Why?***

After the class has suggested a few possibilities, mention that the Romans used a mixture of base five and base ten; that the Mayans of Central America used a mixture of base five and base twenty; and that the Babylonians used a mixture of base ten and base sixty, from which our method of keeping time originated (60 seconds = 1 minute; 60 minutes = 1 hour).

Write this expression on the board. positional notation

T: ***Each of these ways of writing numbers is an example of*** **positional notation.** ***Why do you think we describe the notations as positional?***

S: ***The position of a checker on an abacus determines its value.***



Capsule Lesson Summary

Use rectangular regions to support addition and subtraction of fractions. Begin the workbook *Arcade of Problems #4*. (This is the first of two lessons using this workbook.)

Materials

Teacher
- Fraction manipulative (optional)
- Meter stick
- Colored chalk

Student
- *Arcade of Problems #4* Workbook
- Colored pencils, pens, or crayons
- Metric ruler
- Calculator

Description of Lesson

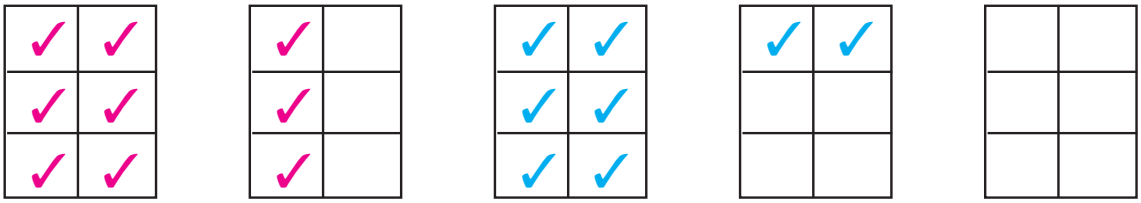
Note: If you have a fraction manipulative that would work well for this activity, you may prefer to use it rather than to draw and color rectangles.

Draw five 2-by-3 rectangles on the board, as in the next illustration.

Write this addition problem on the board. $1\frac{1}{2} + 1\frac{1}{3} =$

T: ***Let's see if we can use the rectangles to help us with the calculation.***

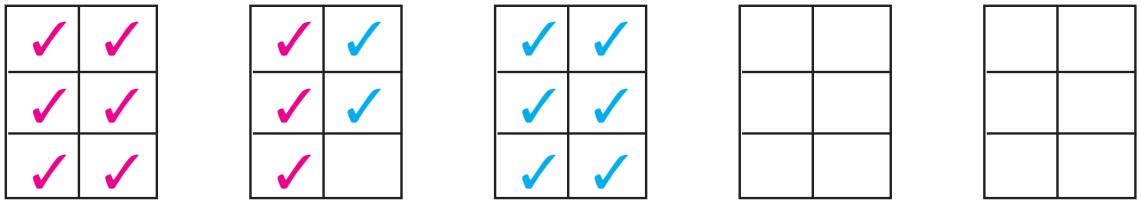
Accept student responses. Then invite one student to mark in red 1½ rectangles and another student to mark in blue 1⅓ rectangles. For example:



T: ***What is 1½ + 1⅓?***

S: ***1⅚.***

Students may suggest putting the extra three red and two blue marks in one rectangle. For example:



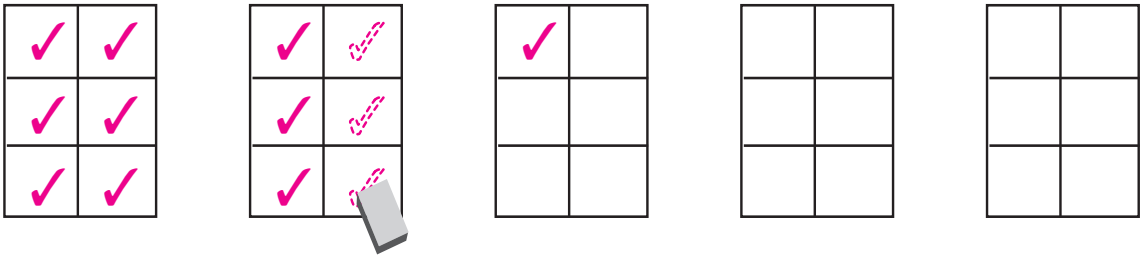
Present the following problems in a similar manner. Let students predict each sum before using the rectangles. (Answers are in boxes.)

$\frac{1}{3} + 2\frac{5}{6} = \boxed{3\frac{1}{6}}$ $\frac{1}{6} + \frac{1}{2} = \boxed{\frac{4}{6} = \frac{2}{3}}$ $2\frac{2}{3} + 1\frac{1}{2} = \boxed{4\frac{1}{6}}$

Pose this subtraction problem.

T: ***What is 2⅙ – ½?***

After accepting students' answers, let one student mark 2⅙ rectangles in red, and let another student erase marks from ½ rectangle.



T: ***What is 2⅙ – ½?***

S: ***1⅚ or 1⅔.***

Present the following problems in a similar manner. (Answers are in boxes.)

$4 - 2\frac{1}{3} = \boxed{1\frac{2}{3}}$ $2\frac{1}{2} - 1\frac{5}{6} = \boxed{\frac{4}{6} = \frac{2}{3}}$ $3\frac{1}{6} - 1\frac{1}{2} = \boxed{1\frac{4}{6} = 1\frac{2}{3}}$

Distribute copies of the workbook *Arcade of Problems #4*, and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of the class period, collect the workbooks for your review. They will be used again in Lesson W9.

Capsule Lesson Summary

Continue individual work in the workbook *Arcade of Problems #4*. (This is the second of two lessons using this workbook.)

Materials

Teacher	• None	Student	• *Arcade of Problems #4* Workbook • Colored pencils, pens, or crayons • Metric ruler • Calculator

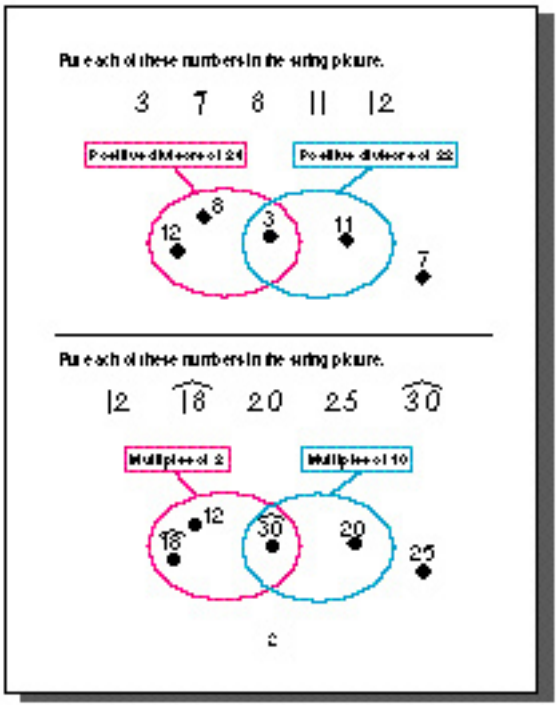
Description of Lesson

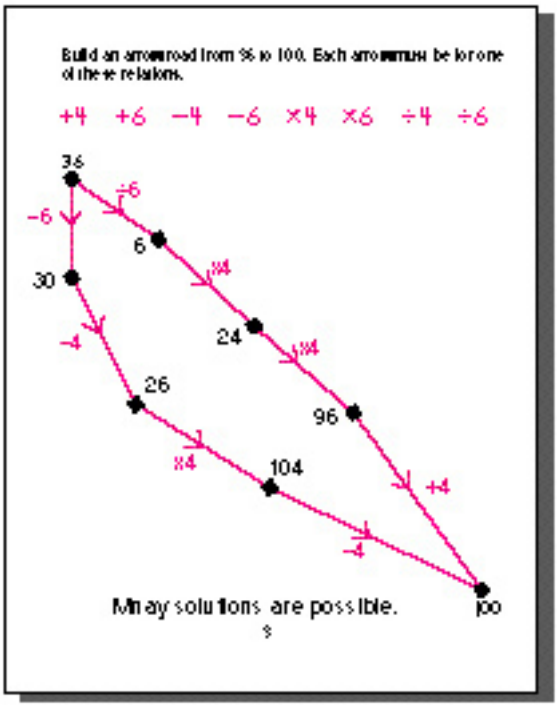
Distribute students' copies of the workbook *Arcade of Problems #4*. Ask students first to correct or complete pages from the previous week's work and then to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

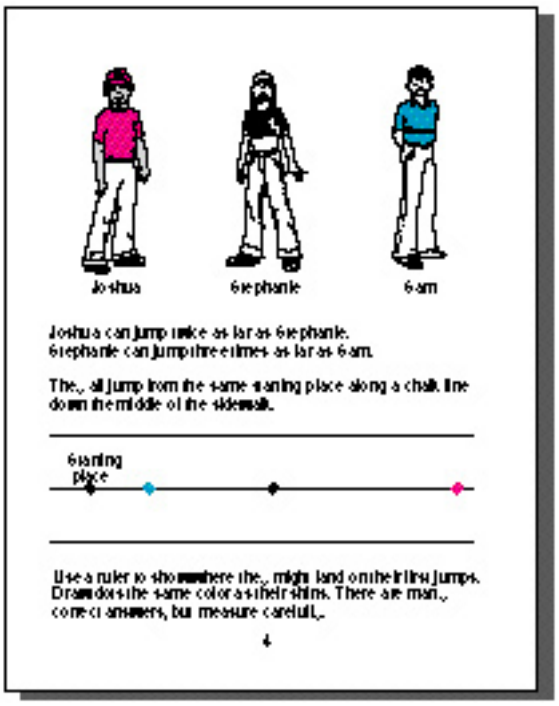
At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in the workbook during a study time or to take it home as an assignment.

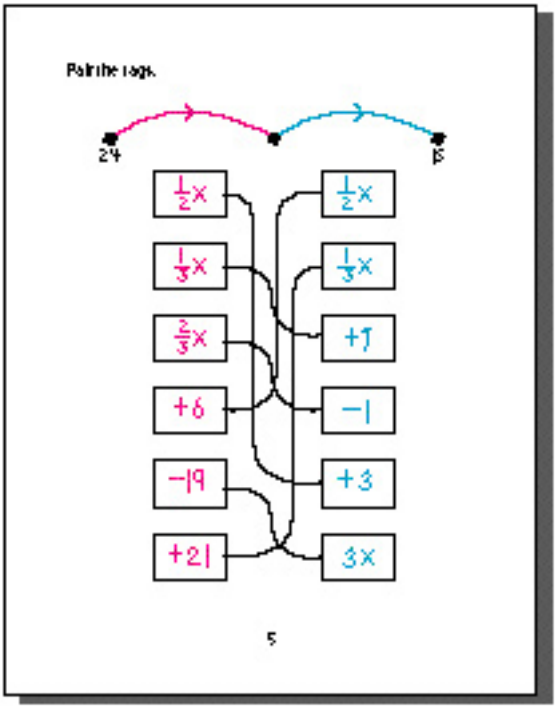
Assessment Activity

An individual student progress record for the workbook is available on Blackline W9. You may like to use this form to monitor student work.


Put each of these numbers in the sorting picture.
3 7 8 11 12
Positive divisors of 24
Positive divisors of 22
8
12
3
11
7
Put each of these numbers in the sorting picture.
12 18 20 25 30
Multiples of 2
Multiples of 10
12
18
30
20
25
2


Build an arrow road from 36 to 100. Each arrow must be for one of these relations.
+4 +6 −4 −6 ×4 ×6 ÷4 ÷6
36
÷6
−6
6
×4
30
24
×4
−4
26
96
×4
+4
104
−4
100
Many solutions are possible.
3


Joshua
Elephant
Ram
Joshua can jump twice as far as Elephant.
Elephant can jump three times as far as Ram.
They all jump from the same starting place along a chalk line down the middle of the sidewalk.
Starting place
Use a ruler to show where they might land on their first jumps. Draw dots the same color as their shirts. There are many correct answers, but measure carefully.
4


Pair the rays.
24
15
½x
⅓x
⅔x
+6
−19
+21
½x
⅓x
+7
−1
+3
3x
5

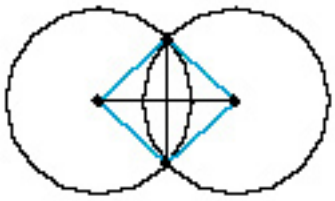
Put a one-digit number in each box to make the calculations correct.

$$\begin{array}{r} \boxed{7}09.3 \\ \boxed{5}7.4 \\ +725.\boxed{5} \\ \hline 1492.2 \end{array} \qquad \begin{array}{r} \boxed{7}8.0 \\ -2\boxed{4}.6 \\ \hline 53.\boxed{4} \end{array}$$

Add.	Subtract.
$2\frac{1}{4} + 5\frac{3}{4} + \frac{1}{2}$	$637.07 - 83.8$
$8\frac{1}{2}$	553.27

6

Draw two circles of the same radius that intersect at two places.

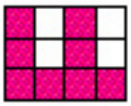


1. Draw dots at their centers.
2. Draw dots at their intersecting points.
3. Connect every dot to every other dot.
4. Can you find four line segments that have the same length? Yes
   Color them blue.

7

Color four-sixths of each shape red.

$\frac{4}{6}$



$\frac{4}{6}$

Other colorings are possible.

Put a whole number in each box.

$\frac{4}{6} = \frac{\boxed{2}}{3}$ $\qquad$ $\frac{4}{6} = \frac{\boxed{8}}{12}$

$\frac{4}{6} = \frac{\boxed{20}}{30}$ $\qquad$ $\frac{4}{6} = \frac{\boxed{16}}{24}$

$\frac{4}{6} = \frac{12}{\boxed{9}}$ $\qquad$ $\frac{4}{6} = \frac{400}{\boxed{600}}$

8

All of the numbers had decimal points, but the eraser gremlin erased a decimal point from one number in each equation. Please put them back.

$2.3 \times 4.6 = 10.58$

$4.82 + 39.6 = 44.42$

$59.7 - 49.81 = 9.89$

$0.3 \times 98.7 = 29.61$

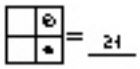
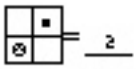
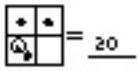
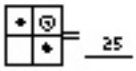
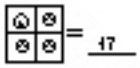
$1.2 - 0.8 = 0.4$

9

Put each type of number on the one-to-end of the Mini-computer by adding exactly one of these checkers.

● ⑤

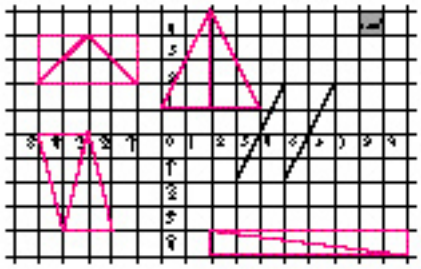
Many solutions are possible.

An odd number	A positive distance of 30
 = 21	 = 2
23 is another solution.	1, 6, and 10 are other solutions.
A multiple of 10	**A multiple of 5**
 = 20	 = 25
20 is another solution.	
A positive prime number	**A number greater than 40**
 = 17	 = 42
5 is another solution.	54 is another solution.

10

Draw a triangle that

- has corners on grid points, and
- has an area of 4 cm^2 (each small grid square is 1 cm^2).



What are the ordered pairs of the three corners of the triangle you drew? (3 , 2) (5 , 2) (5 , 2)

If you double the area of the triangle, you get 8 cm^2. Enclose your triangle in a shape that has area 8 cm^2.

What are the ordered pairs of the corners of this shape?
(3, 2) (5, 2) (5, 2) (7, 2)

Other solutions are possible, for example, as shown in red above.

11

Kelsey and Edun work on a job together, but Kelsey does less work than Edun. They agree to share their earnings as follows:

Kelsey gets $5 whenever Edun gets $5.

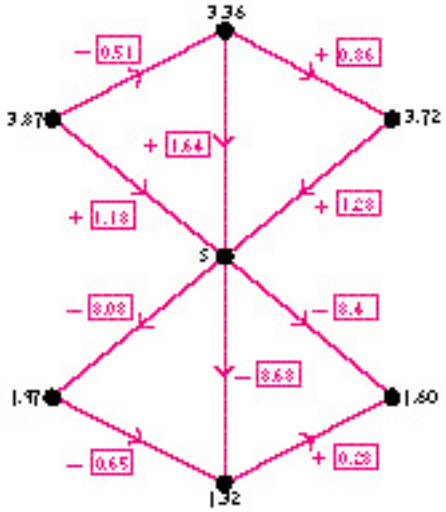
When Kelsey gets $24, how much does Edun get? $40.00

When Edun gets $25, how much does Kelsey get? $15.00

Suppose the total payment for the job is $200.
How much does Kelsey get? $75.00
How much does Edun get? $125.00

12

Fill in the boxes for the arrows.



13

Multiply.

$$\begin{array}{r} 938 \\ \times\ 6 \\ \hline 5628 \end{array} \qquad \begin{array}{r} 938 \\ \times 70 \\ \hline 65660 \end{array} \qquad \begin{array}{r} 938 \\ \times 76 \\ \hline 5628 \\ 65660 \\ \hline 71288 \end{array}$$

$$\begin{array}{r} 938 \\ \times 0.6 \\ \hline 562.8 \end{array} \qquad \begin{array}{r} 938 \\ \times 70.6 \\ \hline 5628 \\ 65660 \\ \hline 66222.8 \end{array}$$

Divide.

$$\begin{array}{r|l} 425 & \\ \hline 15)6375 & 6375 \div 15 = 425 \\ 6000 & 400 \\ \hline 375 & 637.5 \div 15 = 42.5 \\ -300 & 20 \\ \hline 75 & 63.75 \div 15 = 4.25 \\ -75 & 5 \\ \hline 0 & \end{array}$$

14

Gar is a secret number.

Clue 1

Gar can be put on this Mink computer by adding exactly one of these checkers.

Gar could be 4, 5, 7, 11, 1, 5, 13, or 29.

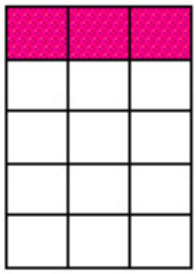
Clue 2

Gar is in this arrow picture.

Who is Gar? 3

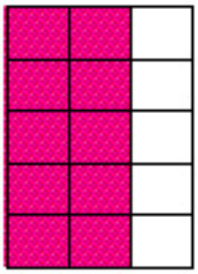
15

Color one-fifth of this region red and fill in the box.



$\frac{1}{5} = \frac{3}{15}$

Color two-thirds of this region red and fill in the box.



$\frac{2}{3} = \frac{10}{15}$

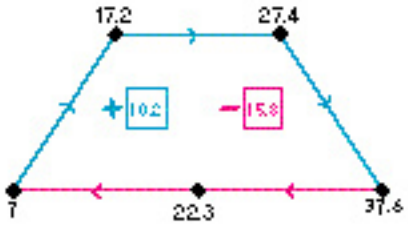
Other colorings are possible.

Complete.

$$\frac{1}{5} + \frac{2}{3} = \frac{3}{15} + \frac{10}{15} = \frac{13}{15}$$

16

Fill in the boxes for the arrows and label the dots.



17

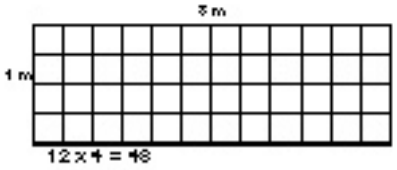
Mrs. Olsen buys eggs for 5¢ apiece and sells them for 34¢ a dozen. What is her profit on 14 dozen eggs? $3.36
Show your work.

Profit on one dozen
$84 - (12 \times 5) = 84 - 60 = 24$

Profit on 14 dozen

$$\begin{array}{r} 24 \\ \times 14 \\ \hline 96 \\ 240 \\ \hline 336 \end{array}$$

Elizabeth buys 3 meters of fabric with width 1 meter. How many 25 cm squares can she cut from it? 48
Remember 1 m = 100 cm.



$12 \times 4 = 48$

Other methods of solution are possible.

Fill in the boxes with one-digit numbers to make multiples of 9.

$$3\,8\boxed{3}4$$
$$66\,66\boxed{3}$$
$$9\boxed{7}2$$

Fill in the boxes and triangles with one-digit numbers to make different multiples of 9.

Other solutions

$$4\boxed{2}6\triangle \quad 4\boxed{6}6\boxed{2}$$
$$4\boxed{7}6\triangle \quad 4\boxed{1}6\boxed{7}$$
$$4\boxed{4}6\triangle \quad 4\boxed{3}6\boxed{5}$$
$$\qquad\qquad 4\boxed{5}6\boxed{3}$$
$$\qquad\qquad 4\boxed{8}6\boxed{0}$$
$$\qquad\qquad 4\boxed{0}6\boxed{8}$$

Del and Lal are several numbers.

Clue 1

Del and Lal are two of these numbers.

$\sqcap$: Greatest common divisor $\sqcup$: Least common multiple

$20 \sqcap 12 = 4$ $\quad 8 \sqcup 6 = 24$

$18 \sqcap 45 = 9$ $\quad 4 \sqcup 7 = 28$

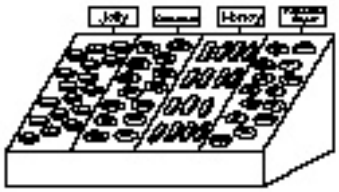
$27 \sqcap 21 = 3$ $\quad 10 \sqcup 4 = 20$

$45 \sqcap 20 = 5$ $\quad 16 \sqcup 12 = 48$

Clue 2

Who is Del? 3

Who is Lal? 48



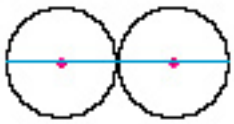
Don goes to the bakery. The baker sells only four types of donuts. He tells the baker to give him two donuts, any kind. (The two donuts do not have to be different kinds.) What are his chances of getting a jelly donut and a honey donut? Use the squares to find the probability.

	J	C	H	PS
J				
C				
H				
PS				

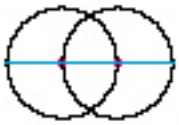
What is the probability of getting a jelly donut and a honey donut? $\frac{2}{16}$ or $\frac{1}{8}$

HINT: How many choices does the baker have when she picks the first donut? 4 Show that in the picture. After the first donut is chosen, how many choices does she have for the second donut? 4 Show that in the picture.

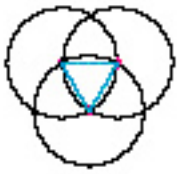
Each circle has its center marked with a red dot. The diameter of each circle is 4.5 cm.



What is the length of the blue segment? 9 cm



What is the length of the blue segment? 6.75 cm



What is the perimeter of the blue triangle? 6.75 cm

22

Label the dots. Some of the numbers have two names listed. Write both names for those numbers beside their dots.

$3 \div 4$ $1.8 + 0.4$ $2 - \frac{8}{5}$

$\frac{1}{3} \times \frac{2}{3}$ $\frac{3}{2} \times \frac{5}{3}$ $\frac{3}{2} - \frac{3}{4}$ $\frac{7}{10} + \frac{1}{2}$

$3.3 - 2.9$ 5×0.5 3×0.4

23

Fill in the boxes with whole numbers.

Example: $4^3 = 4 \times 4 \times 4 = \boxed{64}$

$7^2 = \boxed{49}$ $5^2 = \boxed{25}$

$7^3 = \boxed{343}$ $5^3 = \boxed{125}$

$20^3 = \boxed{8000}$ $5^4 = \boxed{625}$

$4^{\boxed{2}} = 16$ $\boxed{11}^2 = 121$

$3^{\boxed{4}} = 81$ $\boxed{30}^2 = 900$

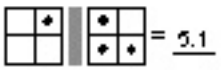
$2^{\boxed{9}} = 512$ $\boxed{6}^3 = 216$

24

Pat's a secret number.

Clue 1

Pat's one of these numbers.


= 5.1

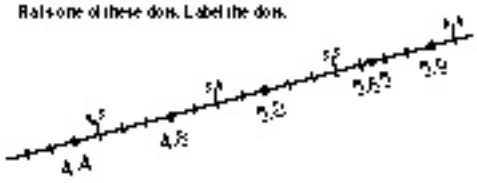

= 1.6


= 4.4


= 6.1

Clue 2

Pat's one of these dots. Label the dots.



Who is Pat? 4.4

25

Match each red tag with a blue tag.

$\frac{7}{10} - \frac{1}{5}$

$1\frac{3}{10} - \frac{7}{10}$

$\frac{1}{4} + \frac{3}{10}$

0.7×0.9

7×0.9

0.7×0.09

0.55

6.3

0.063

0.5

0.63

0.6

26

A market research survey of 200 people found that three-fourths of the people owned a VCR and only two-fifths of the people owned a computer. In the survey, 50 people owned both a VCR and a computer. How many in the survey owned neither a VCR nor a computer? 20 people

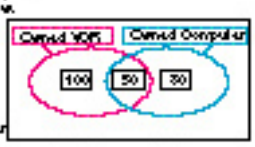
Explain your answer below.

$\frac{3}{4} \times 200 = 150$

150 people owned a VCR

$\frac{2}{5} \times 200 = 80$

80 people owned a computer



With 50 people who owned both a VCR and a computer, there are 130 people in the survey who owned either a VCR or a computer. So, 20 people owned neither.

In the same survey, the researchers found that one-half of the people used their VCR to watch movies they rent and one-half used the VCR to tape TV shows. Still, one-fifth of the people said they never used their VCR. Explain how this can be true.

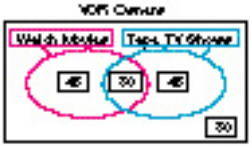
There were 150 people in the survey who owned VCRs.

$\frac{1}{2} \times 150 = 75$

75 people used their VCR to watch movies. 75 people used their VCR to tape TV shows.

$\frac{1}{5} \times 150 = 30$

30 people never used their VCR.



27

Thus, could be 30 people who used their VCR to both watch and tape TV shows

Guess My Rule

The operation * works on two numbers. Here are some clues.

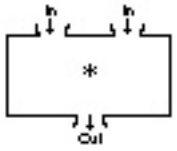
$3 * 2 = 8$

$2 * 3 = 9$

$6 * 4 = 28$

$0 * 5 = 5$

$7 * 1 = 8$



Describe the rule for *.

$a * b = (a \times b) + b$

or $a * b = (a + 1) \times b$

Use the above rule for * to fill in the boxes.

$7 * 6 = \boxed{48}$

$9 * 12 = \boxed{120}$

$3 * 7.2 = \boxed{28.8}$

$\boxed{11} * 1.5 = 18$

$4.5 * \boxed{8} = 44$

$6 * 7 = \boxed{49}$

$\boxed{3} * 8 = 32$

$9 * \boxed{4.4} = 44$

$\widehat{11} * 4 = \boxed{48}$

$\widehat{8} * 9 = \boxed{63}$

28

Other descriptions of the rule are possible.

Bea is a secret number.

Clue 1

Bea can be put on this Mini computer by adding exactly one ⊘ checker.

Bea could be 6.5, 7.4, 9.2, 12.8, 14.6, 23.6, 41.6, or 77.6.

Clue 2

Bea could be 9.2, 12.8, 23.6, 41.6, or 77.6

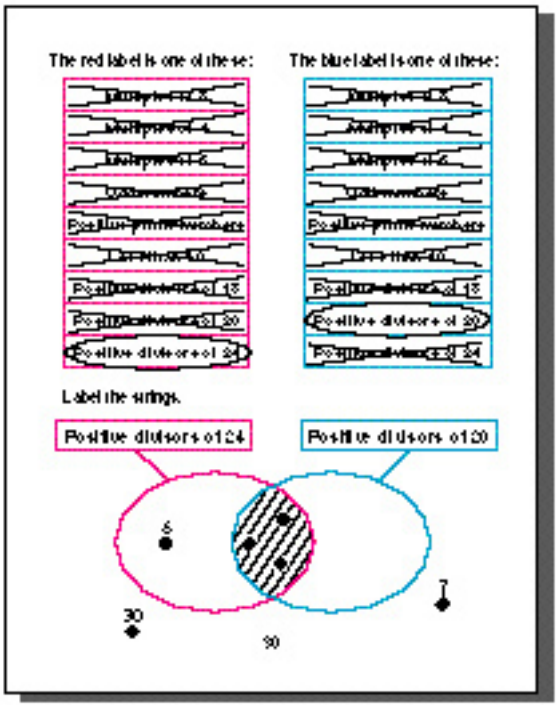
Clue 3

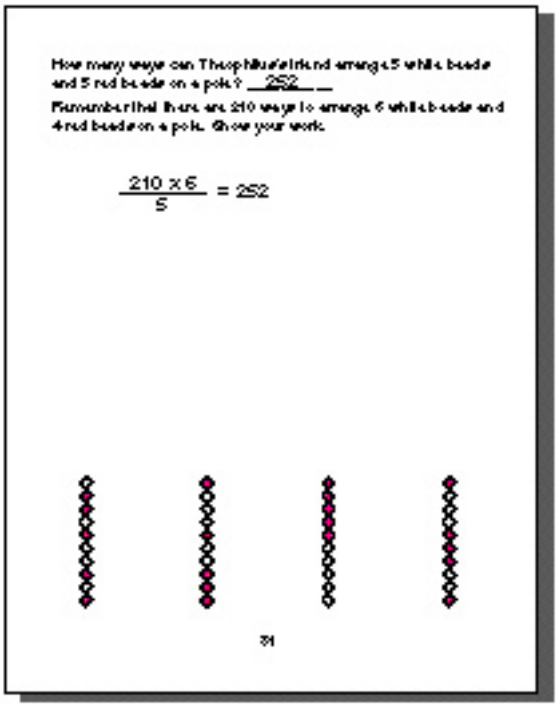
A name for Bea can be written by adding one set of parentheses to this expression.

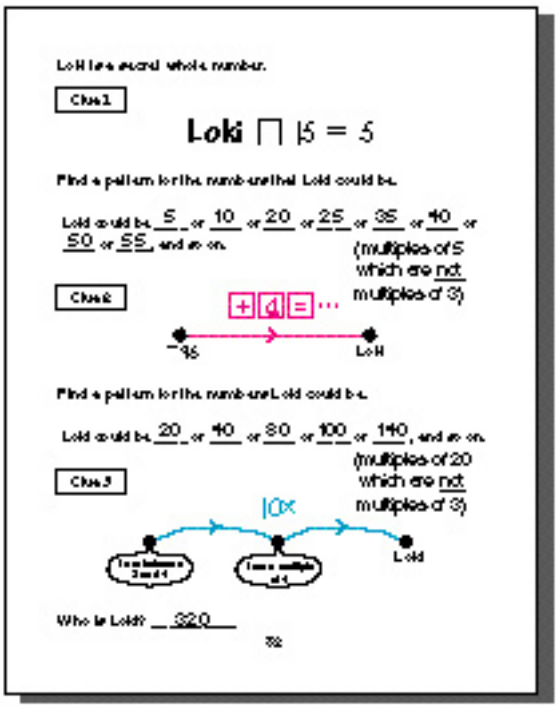
$8 \times (4 + (6 \div 5))$

Who is Bea? 41.6

29


The red label is one of these:
The blue label is one of these:
Positive divisors of 18
Positive divisors of 20
Positive divisors of 24
Label the strings.
Positive divisors of 24
Positive divisors of 20
6
30
90


How many ways can Theophilus/Erlind arrange 5 white beads and 5 red beads on a pole? 252
Remember that there are 210 ways to arrange 6 white beads and 4 red beads on a pole. Show your work.
210 × 6 / 5 = 252
91


Loki is a secret whole number.
Clue 1
Loki ⊓ 15 = 5
Find a pattern for the numbers that Loki could be.
Loki could be 5 or 10 or 20 or 25 or 35 or 40 or 50 or 55 and so on.
(multiples of 5 which are not multiples of 3)
Clue 2
+4=...
-35
Loki
Find a pattern for the numbers Loki could be.
Loki could be 20 or 40 or 80 or 100 or 140, and so on.
(multiples of 20 which are not multiples of 3)
Clue 3
10×
Loki
Who is Loki? 320
92

Capsule Lesson Summary

Review positional systems. Introduce a relationship between the fractional and the decimal names for numbers through reading the storybook *Election in the Number World*.

Materials

Teacher	• *Election in the Number World* Storybook • Magnetic checkers (optional)	Student	• *Election in the Number World* Storybook

Description of Lesson

Note: Because there are several bases referred to in this lesson, it is necessary to be careful about how you read numerals. For this reason, we often indicate how to read a numeral.

Wait to distribute copies of the storybook until after Exercise 1.

Exercise 1

T: ***I'm going to put a number on the board, but I'm not going to tell you what kind of writing I'm using.***

Write 213 on the board.

T: ***What number is this?***

S: ***Two hundred thirteen.***

T: ***It looks like two hundred thirteen, but could it be any other number?***

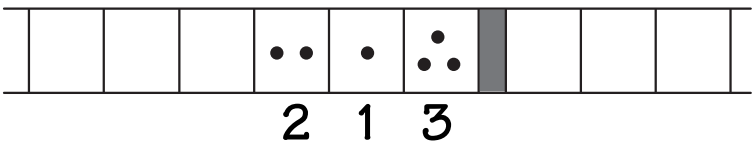
Let the class discuss the question. 213 could name a number written in any base greater than three. If no one suggests that you could be writing in a base other than base ten, proceed as follows.

T: ***I'll give you a hint. Do you remember putting numbers on different kinds of abaci (abacuses)?***

Your students should recall that they have used abaci for several bases; for example, base two, base three, base five, and base twelve. Then the class should realize that 213 could name several numbers depending upon which abacus or base writing is being used.

Draw an abacus on the board (see the next illustration).

T: ***Who can use checkers to put 213*** (read as "two, one, three") ***on the abacus regardless of which kind of abacus it is?***



Point to the board with three checkers.

T: ***What is the value of this board?***

S: ***One.***

Label the ones board and point to the board with one checker.

T: ***Could the value of this board be five?***

S: ***Yes.***

T: ***If so, what would be the rule of the abacus?***

S: ***Five checkers on a board trade for one checker on the next board to the left.***

Label that board 5 and point to the board with two checkers.

T: ***What is the value of this board for a base five abacus?***

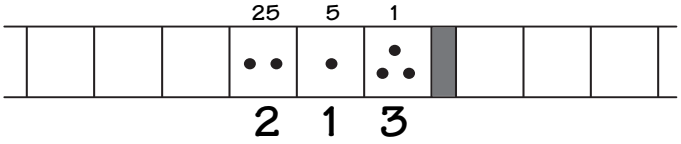
S: ***Twenty-five, since 5 x 5 = 25.***

Label the board 25.

T: ***Therefore, what decimal number is on the abacus?***

S: ***58, since (2 x 25) + (1 x 5) + (3 x 1) = 58.***

Display the equivalent base five and decimal names for the number.



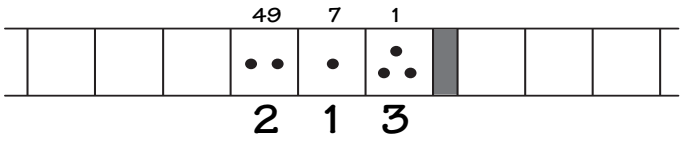
Base five writing 213_{five} = Decimal writing 58_{ten}

Erase the base five labels from the abacus. Again point to the board with one checker.

T: ***Could the value of this board be seven?***

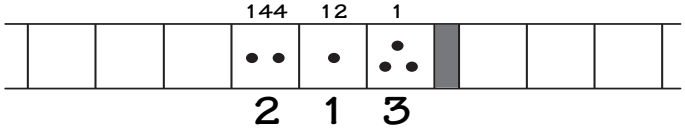
S: ***Yes.***

In a similar manner as above, ask students to determine the decimal number equivalent to 213 (read as "two, one, three") in base seven.



Base seven writing 213_{seven} = Decimal writing 108_{ten}

Repeat the activity with base twelve and obtain the following result.



Base twelve writing 213_{twelve} = Decimal writing 303_{ten}

T: ***We've looked at 213*** (read as "two, one, three") ***as a number written in base five, in base seven, and in base twelve. Could 213 be the name of a number written in any other base?***

It could be the name of a number written in any base more than three. Accept several student suggestions and then ask,

T: ***Could 213 be the base two name for a number?***

S: ***No, only 0 and 1 are used in writing base two names for numbers.***

Similarly, conclude that 213 cannot be the base three name of a number since only 0, 1, and 2 are used in base three writing.

Write this expression on the board.

positional notation

T: ***Writing numbers in base five, in base seven, and in base twelve are all examples of* positional notation.** ***We call it positional because the position of a digit in a number or of a checker on an abacus determines its value.***

Exercise 2

Distribute copies of the storybook *Election in the Number World*.

Pages 1–2

Read pages 1 and 2, and write the following information on the board.

Fractional writing	Positional writing
$1\frac{2}{5} + 3\frac{1}{2}$	1.4 + 3.5

T: ***On page 2, Nabu is considering the positional way of writing numbers and the fractional way. I have written two calculations on the board, one with fractions and one with decimal writing. Which calculation is easier to do?***

Even without doing the calculations, students should agree that adding with the decimal writing of numbers is often easier than adding with fractions.

T: ***One advantage of a positional notation is that it is usually easier to add numbers with it.***

Pages 3–7

Read pages 3 through 7 together.

T: ***Look at the arrow road on page 7. What could the blue arrows be for?***

S: ***10x, because 10 x 0.1 = 1; 10 x 1 = 10; 10 x 10 = 100; and so on.***

T: ***Find 1 000 on the arrow road. Let's start at 1000 and read the numbers in order as we follow blue arrows.***

Giving help when needed, invite students to read the numbers in order:

ten thousand, one hundred thousand;
one million (1,000,000), ten million, one hundred million;
one billion (1,000,000,000), ten billion, one hundred billion;
one trillion (1,000,000,000,000), ten trillion, one hundred trillion;
one quadrillion (1,000,000,000,000,000), and ten quadrillion.

Pages 8–9

Read page 8. Then write these expressions on the board.

decimal _____
binary _____
(or base two)
fractions _____

T: ***The numbers are going to vote on which method of writing numbers they prefer. Let's take a vote in this class. If we were planning to use only one of these ways of writing numbers, which would you vote for? Why?***

After a class discussion, take a class vote. Point out that the results from the numbers voting are on page 9. Ask students to read page 9.

T: ***Which system received the most votes?***

S: ***Decimal writing.***

T: ***Who can explain these results: 95%, 4.7%; and 0.3%?***

Encourage a discussion of the voting results. For example, the class may realize that in this situation 95% means 95 out of every 100 were for decimal writing.

Pages 10–21

As you and the class read pages 10 through 21, stop briefly to solve each problem by asking students to point to the answer in their storybooks.

Pages 22–31

Read pages 22 through 31 collectively. Briefly discuss the ideas as you wish, but try to finish the storybook in this one lesson.

Home Activity

You may like to allow students to take home copies of the storybook to read with family members.

Capsule Lesson Summary

Review addition and subtraction of decimal numbers. Begin the workbook *Arcade of Problems #5*. (This is the first of two lessons using this workbook.)

Materials

Teacher	• None	Student	• *Arcade of Problems #5* Workbook • Colored pencils, pens, or crayons • Compass • Metric ruler

Description of Lesson

Write these problems on the board.

$$428.655 \times 36.935 \times 591.07 = 1\,0\,5\,6\,6\,6$$
$$829.4 - 373.801 = 4\,5\,5\,5\,9\,9$$

T: ***These calculations are correct except that a decimal point is missing from each result. Who can place a decimal point correctly?***

Let students place the decimal points and explain their answers. Encourage the use of estimation. The correct results are 1056.66 and 455.599.

Write these two problems on the board.

$$98.037 \times 627.44 \times 85 \times 0.9 =$$
$$82.174 - 5.8 =$$

Ask students to copy these problems. First ask for estimates of each result. Students' estimates should be near 800 for the addition problem, and about 75 or 76 for the subtraction problem. Then instruct students to complete the calculations and to check their work by comparing their results to the estimates.

After a while, invite students to solve the problems at the board.

$$\begin{array}{r} 98.037 \\ 627.44 \\ 85.0 \\ +\ 0.9 \\ \hline 811.377 \end{array} \qquad \begin{array}{r} 82.174 \\ -\ 5.8 \\ \hline 76.374 \end{array}$$

Review the discussion from Lesson N20 *Decimals #4* about the need for aligning the decimal point when adding or subtracting decimal numbers.

Distribute copies of the workbook *Arcade of Problems #5*, and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of the class period, collect the workbooks for your review. They will be used again in Lesson W12.

Capsule Lesson Summary

Find equivalent names for two fractions with different denominators in order to add them. Continue individual work in the workbook *Arcade of Problems #5*. (This is the second of two lessons using this workbook.)

Materials

Teacher	• None	Student	• *Arcade of Problems #5* Workbook • Colored pencils, pens, or crayons • Compass • Calculator

Description of Lesson

Begin with a discussion about adding fractions.

T: ***When are fractions easy to add and when are they hard to add?***

S: ***They are easy to add when their denominators are the same. They are harder to add when their denominators are different.***

T: ***What could we do to add fractions with unlike denominators?***

S: ***Find other names for the fractions that have the same denominators.***

Write this problem on the board. $\frac{1}{2} + \frac{3}{5}$

T: ***Why are we sure that ½ + ⅗ is not equal to ⅘?***

S: ***½ + ⅗ is greater than 1, but ⅘ is less than 1.***

T: ***Let's find some other names for these fractions.***

On the board, list equivalent fractions for ½ and ⅗ as students name them. Continue until names are given that have the same denominator, for example, ⅤⅩ and ⅥⅩ. Using these names, complete the problem with the class.

$$\frac{1}{2} + \frac{3}{5} = \frac{5}{10} + \frac{6}{10} = \frac{11}{10} + 1\frac{1}{10}$$

Distribute students' copies of the workbook *Arcade of Problems #5*. Ask students first to correct or complete pages from the previous week's work and to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in their workbooks during a study time or to take them home as an assignment.

Assessment Activity

An individual student progress record for the workbook is available on Blackline W12. You may like to use this form to monitor student work.

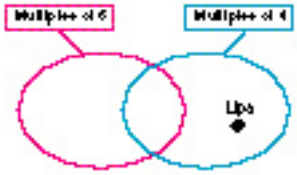
Lipa is a secret number.

Clue 1

Lipa can be put on this Mini computer by adding exactly one regular checker.

Lipa could be 29, 30, 32, or 36.

Clue 2

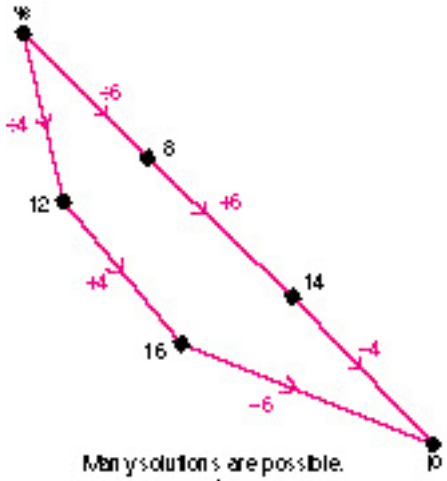


What is Lipa? 32

2

Build an arrow road from 48 to 10. Each arrow must be for one of these relations.

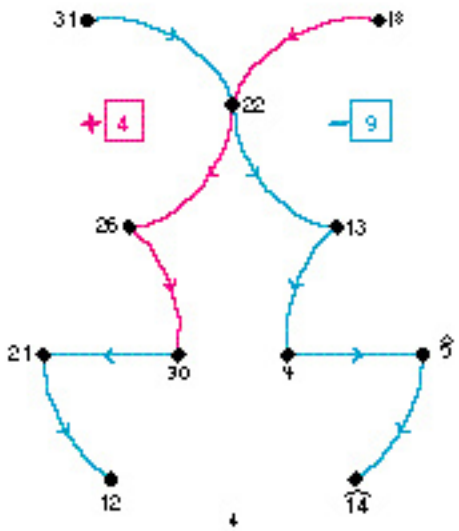
+4 +6 −4 −6 ×4 ×6 ÷4 ÷6



Many solutions are possible.

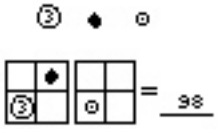
3

Fill in the boxes for the arrows and label the dots.



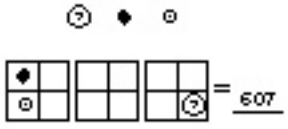
4

Use exactly these three checkers to put a number between 95 and 105 on this Mini computer.



Other solutions are possible.

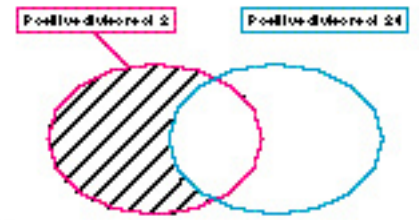
Use exactly these three checkers to put a number between 595 and 625 on this Mini computer.



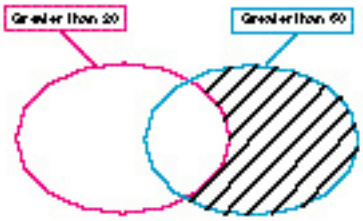
Other solutions are possible.

5

One region of this string picture can be hatched. Hatch it.



One region of this string picture can be hatched. Hatch it.



6

Add.

$13.65 + 748.9 + 8.05$

$$\begin{array}{r} 13.65 \\ 748.9 \\ +\ \ 8.05 \\ \hline 770.61 \end{array}$$

770.61

Add.

$7.6 + 0.871 + 43$

$$\begin{array}{r} 7.6 \\ 0.871 \\ +43 \\ \hline 51.471 \end{array}$$

51.471

Subtract.

$530.84 - 76.2$

$$\begin{array}{r} 530.84 \\ -\ 76.2 \\ \hline 454.64 \end{array}$$

454.64

Subtract.

$96.7 - 67.24$

$$\begin{array}{r} 96.7 \\ -\ 67.24 \\ \hline 29.46 \end{array}$$

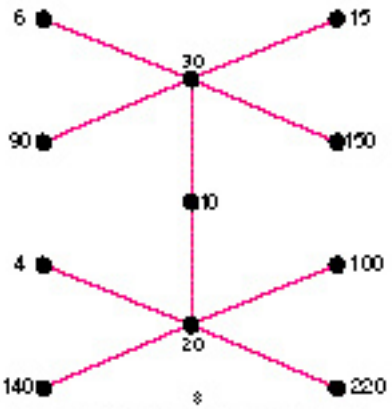
29.46

7

Label the dots so that they are all for different numbers. Many solutions are possible.

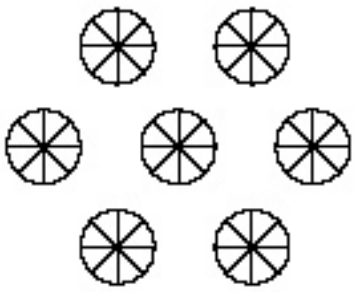
Prime Factor Relation

Two whole numbers are joined by a red cord if and only if one of the numbers equals the other number times a positive prime number.



8

Many solutions are possible, but the center dot must be for 10 or 60.



Complete. You may use the pictures above.

$3\frac{5}{8} + 2\frac{7}{8} = 6\frac{1}{2}$ $\qquad 3\frac{1}{2} - \frac{3}{8} = 3\frac{1}{8}$

$1\frac{3}{4} + \frac{7}{8} = 2\frac{5}{8}$ $\qquad 6 - 2\frac{5}{8} = 3\frac{3}{8}$

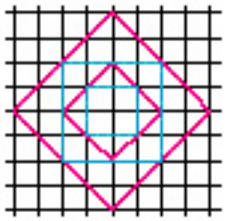
$1\frac{1}{8} + 2\frac{3}{8} = 3\frac{1}{2}$ $\qquad 3\frac{1}{4} - 2\frac{1}{2} = \frac{3}{4}$

Other fractional names are possible.

9

Find the area of each square.

Find the length of one side of each square. Use a ruler.



Square	Area (cm)	Length of one side (cm)
Small Blue	4 cm²	2 cm
Small Red	8 cm²	Between 2.8 and 3 cm
Large Blue	16 cm²	4 cm
Large Red	32 cm²	Between 5.6 and 5.7 cm

What pattern do you notice about the areas? Each square has double the area of the next smaller square.

Does the same pattern hold for the lengths? no

10

Choose a secret whole number.

Clue 1

Choose one of these numbers.

□: Greatest common divisor ⊔: Least common multiple

21 □ 6 = 3 10 ⊔ 4 = 20

20 □ 9 = 1 3 ⊔ 7 = 21

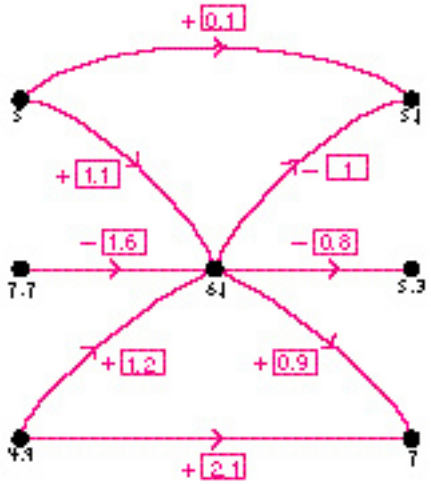
14 □ 35 = 7 9 ⊔ 6 = 18

Clue 2

Who is Clue? 20

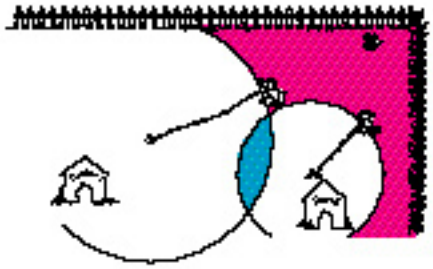
11

Fill in the box for each arrow.



12

Polly the cat enjoys hunting Bruiser and Dawg by jumping into the yard where Bruiser and Dawg can't reach him.



Scale: 1 cm = 1 m

Bruiser's chain is 5 m long and Dawg's chain is 3 m long. Color in red the safe area for Polly in the corner of the yard.

Color in blue the area where Bruiser and Dawg could fight with each other.

Use a compass.

13

Complete.

$\frac{3}{4} = \frac{12}{16}$ $\quad \frac{2}{5} = \frac{8}{20}$

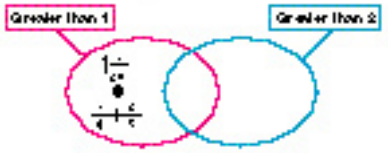
$\frac{3}{4} = \frac{21}{28}$ $\quad \frac{2}{5} = \frac{18}{45}$

$\frac{3}{4} = \frac{15}{20}$ $\quad \frac{2}{5} = \frac{40}{100}$

You may use the above results to add these fractions.

$$\frac{3}{4} + \frac{2}{5} = \frac{15}{20} + \frac{8}{20} = \frac{23}{20} = 1\frac{3}{20}$$

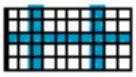
Place $\frac{1}{4}$, $\frac{1}{2}$, $\frac{4}{4}$, $\frac{5}{4}$ in this sorting picture.

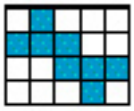


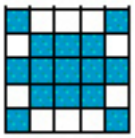
14

What percent of each rectangle is colored blue?


24 ✓


36 ✓


40 ✓


60 ✓


30 ✓


70 ✓

15

SALE!

Christmas Cards
47.50 per box
50 cards per box
Wrapping paper
4.50 per roll

Elena plans to send 150 Christmas cards this season.
The postage rate for first class mail is 60.72 per card.

How many boxes of cards must Elena buy? (She cannot buy part of a box of cards.) 3

How many extra cards will Elena have? 5

Calculate the cost of the cards and the postage:

cost of cards $67.50
postage $41.60
total $109.10

16

Divide.

$$\begin{array}{r|l} 206.3 & \\ 18\overline{)3713.4} & \\ -3600 & 200 \\ \hline 113.4 & \\ -108 & 6 \\ \hline 5.4 & \\ -5.4 & 0.3 \\ \hline \end{array}$$

Complete.

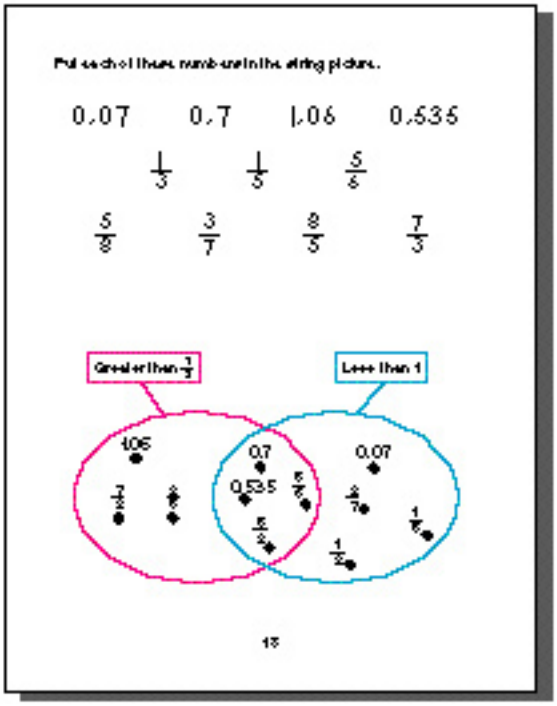
178.68 ÷ 4 = 44.67

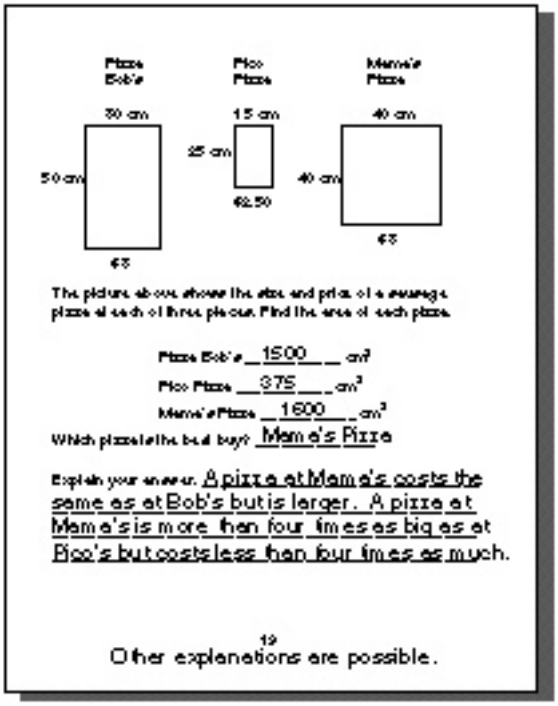
+ 8 → 186.68 ÷ 4 = 46.67 → + 2

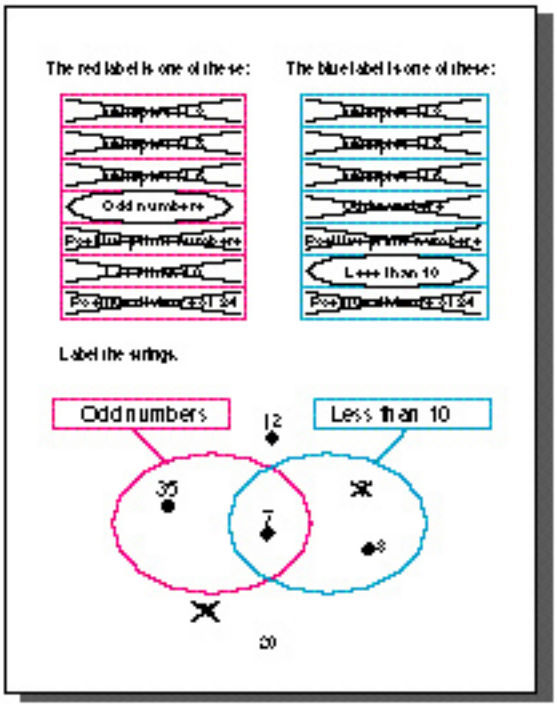
+ 0.28 → 186.96 ÷ 4 = 46.74 → + 0.07

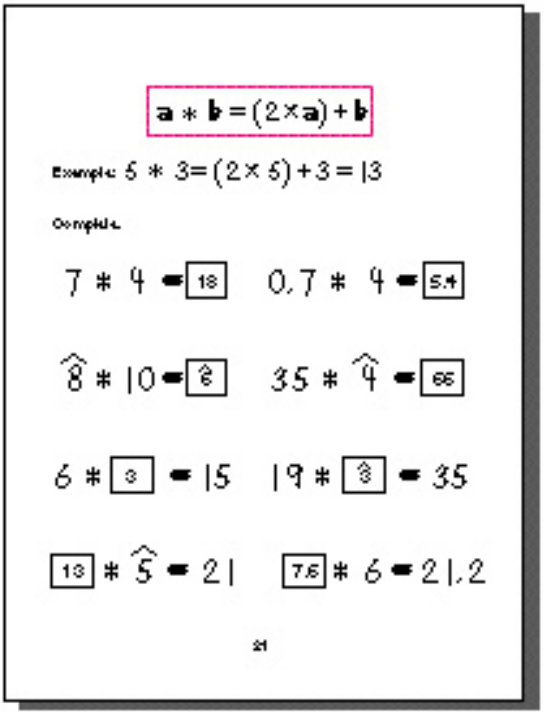
+ 1.2 → 188.16 ÷ 4 = 47.04 → + 0.3

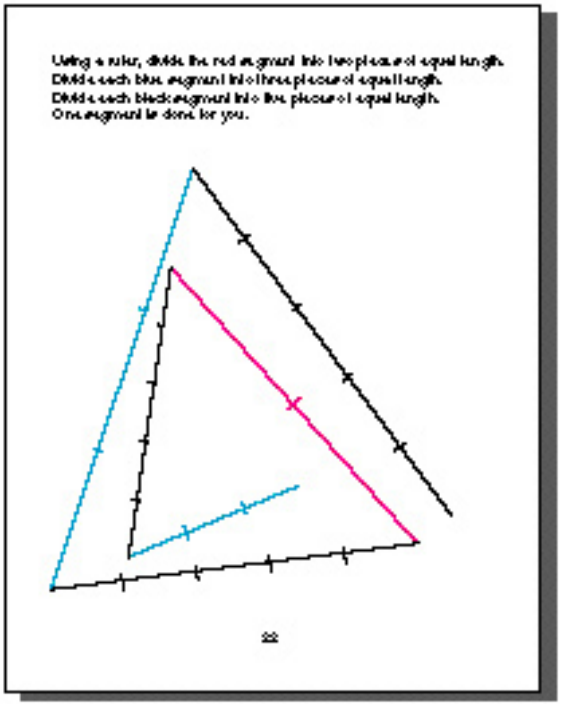
17

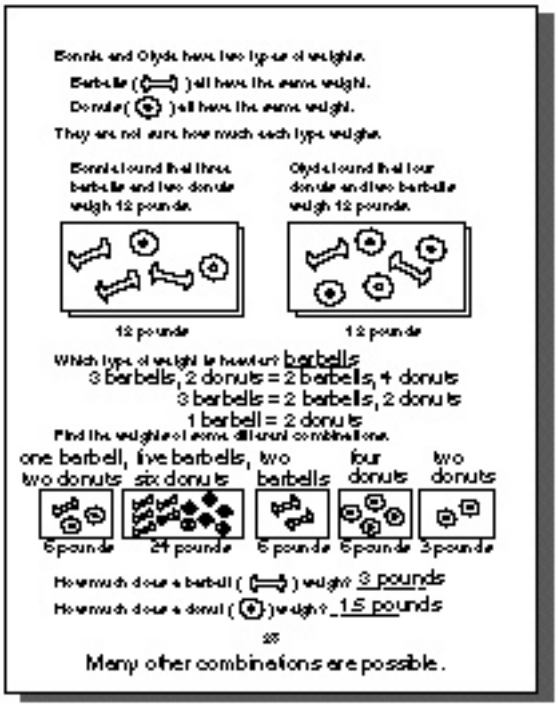

Put each of these numbers in the sorting picture.
0.07
0.7
1.06
0.535
1/3
1/5
5/6
5/8
3/7
8/5
7/3
Greater than 1/2
Less than 1
1.06
0.7
0.535
0.07
18

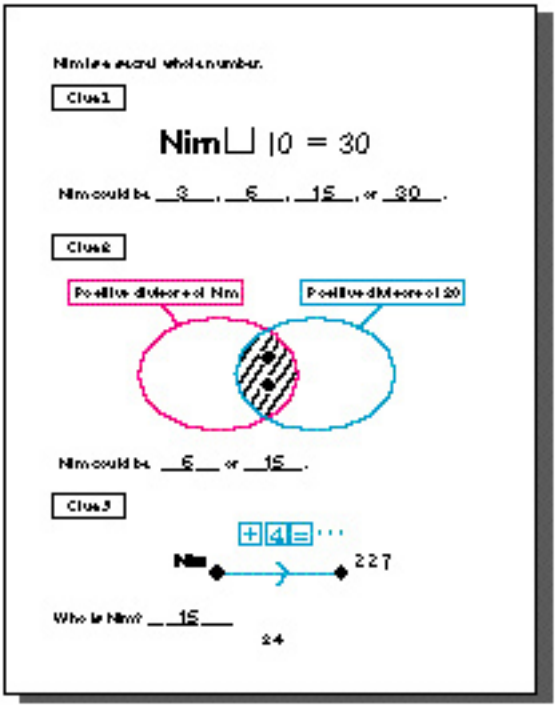

Pizza Bob's
Pico Pizza
Mama's Pizza
30 cm
50 cm
63
15 cm
25 cm
62.50
40 cm
40 cm
63
The picture above shows the size and price of a average pizza at each of three places. Find the area of each pizza.
Pizza Bob's 1500 cm²
Pico Pizza 375 cm²
Mama's Pizza 1600 cm²
Which pizza is the best buy? Mama's Pizza
Explain your answer. A pizza at Mama's costs the same as at Bob's but is larger. A pizza at Mama's is more than four times as big as at Pico's but costs less than four times as much.
19
Other explanations are possible.

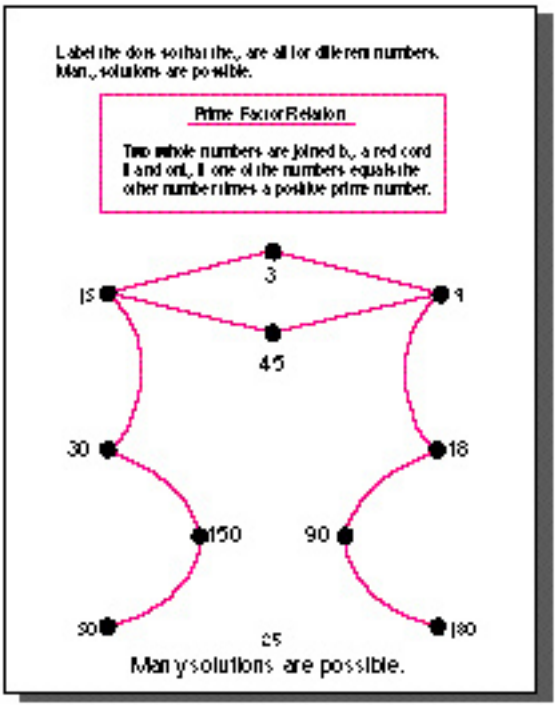

The red label is one of these:
The blue label is one of these:
Odd numbers
Less than 10
Label the strings.
Odd numbers
Less than 10
12
35
7
8
20

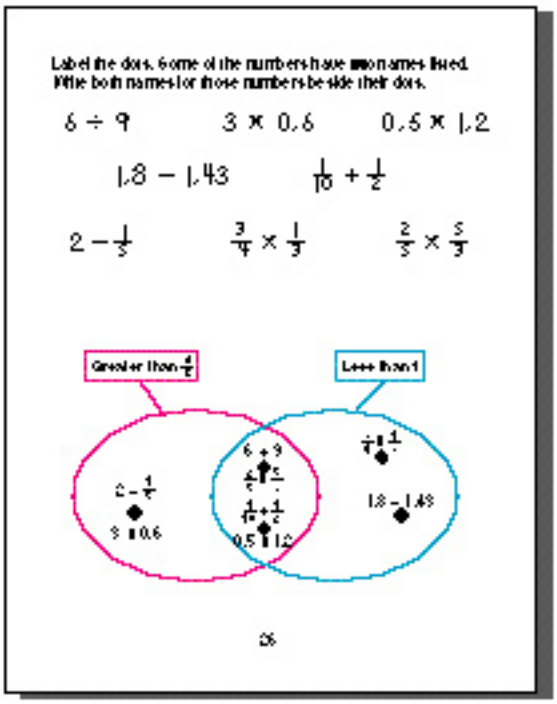

a * b = (2 × a) + b
Example: 5 * 3 = (2 × 5) + 3 = 13
Complete.
7 * 4 = 18
0.7 * 4 = 5.4
8 * 10 = 8
35 * 4 = 66
6 * 3 = 15
19 * 3 = 35
13 * 5 = 21
7.6 * 6 = 21.2
21

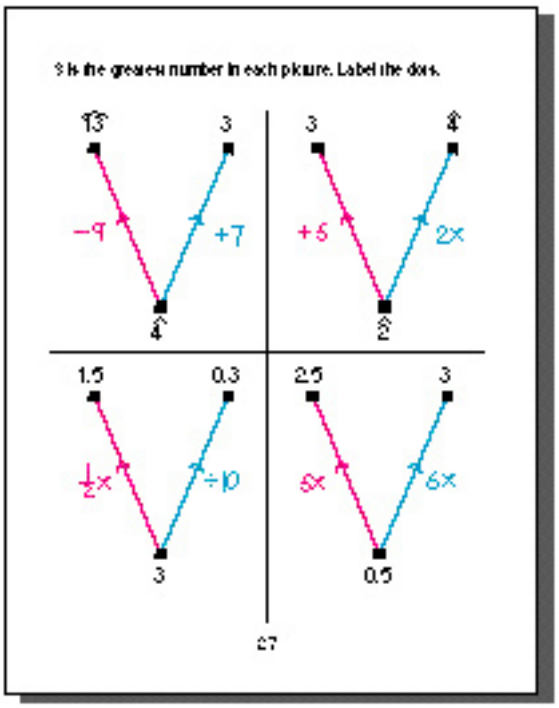

Using a ruler, divide the red segment into two pieces of equal length.
Divide each blue segment into three pieces of equal length.
Divide each black segment into five pieces of equal length.
One segment is done for you.
22

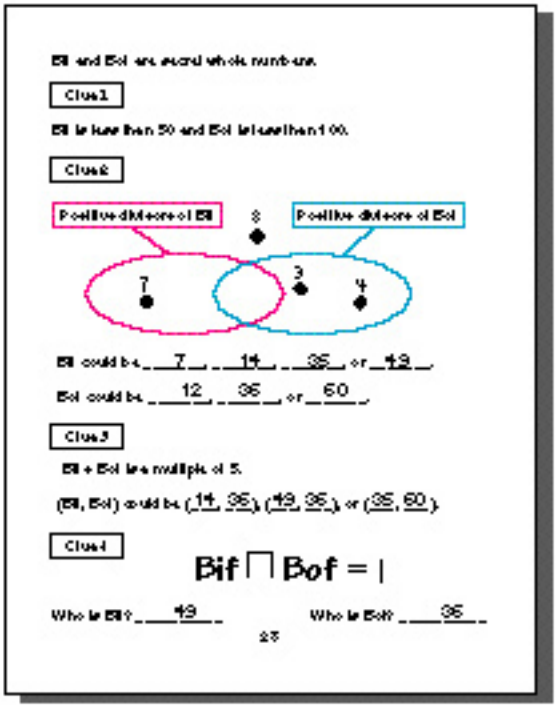

Bonnie and Clyde have two types of weights.
Barbells ( ) all have the same weight.
Donuts ( ) all have the same weight.
They are not sure how much each type weighs.
Bonnie found that three barbells and two donuts weigh 12 pounds.
Clyde found that four donuts and two barbells weigh 12 pounds.
12 pounds
12 pounds
Which type of weight is heavier? barbells
3 barbells, 2 donuts = 2 barbells, 4 donuts
3 barbells = 2 barbells, 2 donuts
1 barbell = 2 donuts
Find the weights of some different combinations.
one barbell, two donuts
five barbells, six donuts
two barbells
four donuts
two donuts
6 pounds
24 pounds
6 pounds
6 pounds
3 pounds
How much does a barbell ( ) weigh? 3 pounds
How much does a donut ( ) weigh? 1.5 pounds
23
Many other combinations are possible.

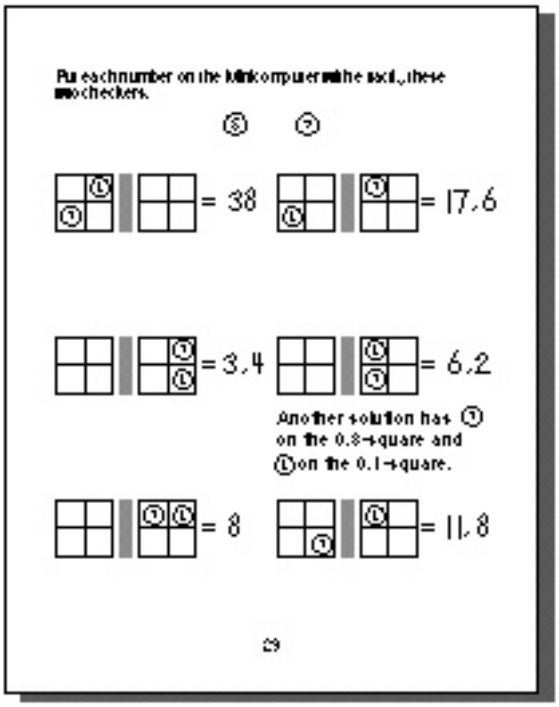

Nim is a secret whole number.
Clue 1
Nim ☐ 10 = 30
Nim could be 3, 6, 15, or 30.
Clue 2
Positive divisors of Nim
Positive divisors of 30
Nim could be 6 or 15.
Clue 3
+ 4 = ...
Nim
227
Who is Nim? 15
24

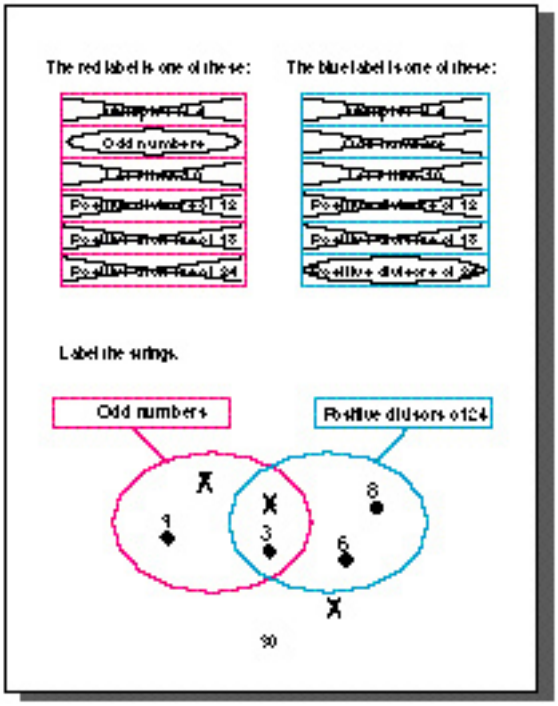

Label the dots so that they are all for different numbers.
Many solutions are possible.
Prime Factor Relation
Two whole numbers are joined by a red cord if and only if one of the numbers equals the other number times a positive prime number.
3
15
1
45
30
18
150
90
50
180
25
Many solutions are possible.

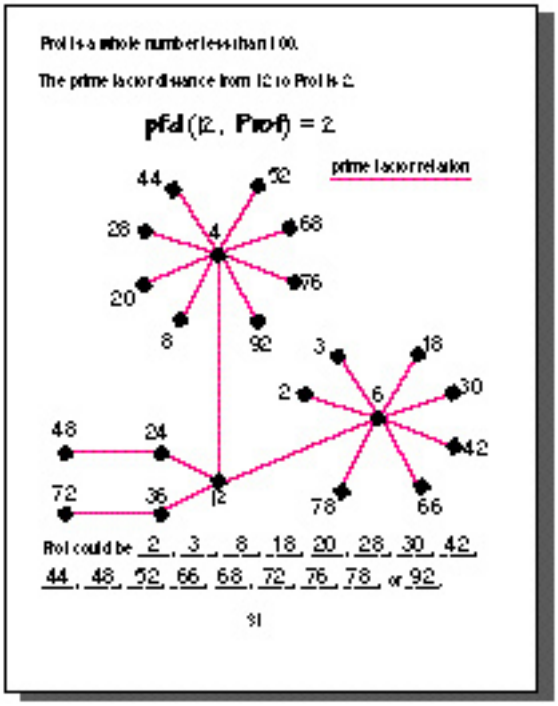

Label the dots. Some of the numbers have more names listed. Write both names for those numbers beside their dots.
6 ÷ 9
3 × 0.6
0.5 × 1.2
1.8 − 1.43
1/10 + 1/2
2 − 1/5
3/4 × 1/3
2/5 × 5/3
Greater than 4/9
Less than 1
26

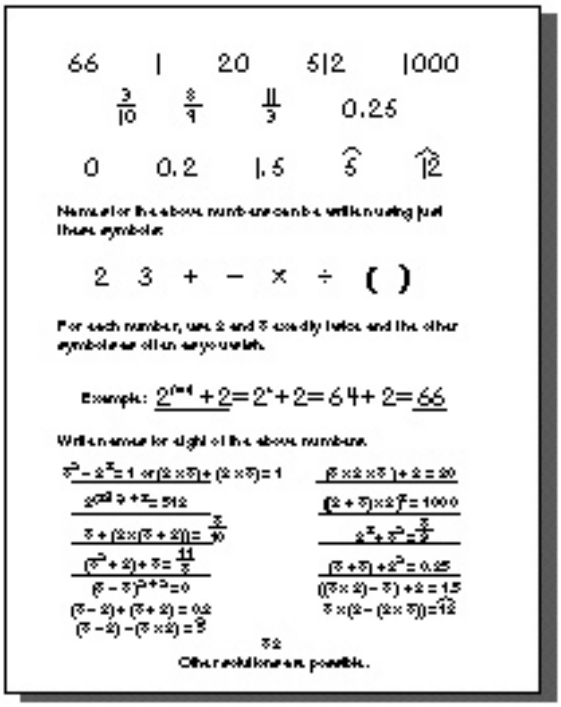

is the greatest number in each picture. Label the dots.
13
3
−9
+7
4
3
4
+5
2×
2
1.5
0.3
½×
÷10
3
2.5
3
8×
6×
0.5
27


Bif and Bof are secret whole numbers.
Clue 1
Bif is less than 50 and Bof is less than 100.
Clue 2
Positive dividers of Bif
8
Positive dividers of Bof
7
3
4
Bif could be 7, 14, 35, or 49.
Bof could be 12, 36, or 60.
Clue 3
Bif + Bof is a multiple of 5.
(Bif, Bof) could be (14, 36), (49, 36), or (35, 60).
Clue 4
Bif ⊓ Bof = 1
Who is Bif? 49
Who is Bof? 36
28


Put each number on the Mini computer with the least, these two checkers.
= 38
= 17.6
= 3.4
= 6.2
Another solution has ① on the 0.8-square and ① on the 0.1-square.
= 8
= 11.8
29


The red label is one of these:
The blue label is one of these:
Odd numbers
Positive divisors of 12
Label the strings.
Odd numbers
Positive divisors of 24
1
3
6
8
90


Prof is a whole number less than 100.
The prime factor distance from 12 to Prof is 2.
pfd (12, Prof) = 2
prime factor relation
44
32
28
4
68
20
76
8
92
3
18
2
6
30
48
24
42
72
36
12
78
66
Prof could be 2, 3, 8, 18, 20, 28, 30, 42, 44, 48, 52, 66, 68, 72, 76, 78, or 92.
91


66
1
20
512
1000
3/10
8/9
11/3
0.25
0
0.2
1.5
√5
√12
Name all of the above numbers using just these symbols:
2 3 + − × ÷ ( )
For each number, use 2 and 3 exactly twice and the other symbols as often as you wish.
Example: 2^(3+1) + 2 = 2^4 + 2 = 64 + 2 = 66
Write an answer for eight of the above numbers.
3 2
Other solutions are possible.

W13 ARCADE OF PROBLEMS #6 (LESSON ONE)

Capsule Lesson Summary

Use fractional regions of a circle to support addition and subtraction of fractions. Begin the workbook *Arcade of Problems #6*. (This is the first of two lessons using this workbook.)

Materials

Teacher	• None	Student	• *Arcade of Problems #6* Workbook • Colored pencils, pens, or crayons • Compass • Metric ruler • Calculator

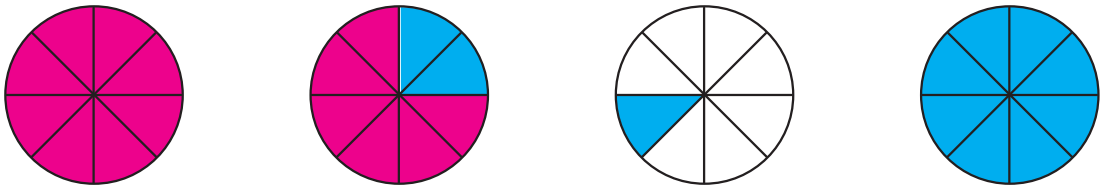
Description of Lesson

Draw four circles on the board and ask students to divide each of these circles first into halves, then into fourths, and finally into eighths. See the next illustration.

Write this addition problem on the board. $1\frac{3}{4} + 1\frac{3}{8} =$

T: ***Let's see if we can use the circles to help us with the calculation. What number is 1¾ + 1⅜?***

Invite students to color (or mark) 1¾ circles in red and 1⅜ circles in blue.



T: ***How many circles are colored?***

S: ***3⅛.***

Complete the calculation on the board: 1¾ + 1⅜ = 3⅛.

Repeat this activity with the following calculations. (Answers are in boxes.)

$1\frac{1}{2} + 1\frac{3}{8} = \boxed{2\frac{7}{8}}$ $\quad 1\frac{1}{2} - \frac{3}{8} = \boxed{1\frac{1}{8}}$ $\quad 3 - 1\frac{3}{4} = \boxed{1\frac{1}{4}}$

When doing a subtraction problem, you may wish to ask students only to mark regions with a color (rather than color entire regions) in order to make erasing easier.

Distribute copies of the workbook *Arcade of Problems #6* and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of class period, collect the workbooks for your review. They will be used again in Lesson W14.

Capsule Lesson Summary

Continue individual work in the workbook *Arcade of Problems #6*. (This is the second of two lessons using this workbook.)

Materials

Teacher	• None	Student	• *Arcade of Problems #6* Workbook • Colored pencils, pens, or crayons • Compass • Metric ruler • Calculator

Description of Lesson

Distribute students' copies of the workbook *Arcade of Problems #6*. Ask students first to correct or complete pages from the previous week's work and to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

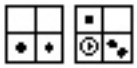
At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in their workbooks during a study time or to take them home as an assignment.

Assessment Activity

Yip is a secret number.

Clue 1

Yip can be put on this Mini computer by taking all exactly one checker.



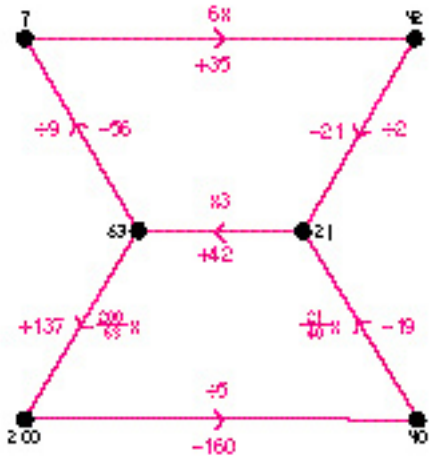
Yip could be 25, 36, 38, 40, or 45.

Clue 2

Who is Yip? 45

2

Label the red arrows; if you like, to label some arrows in two ways. Many solutions are possible.

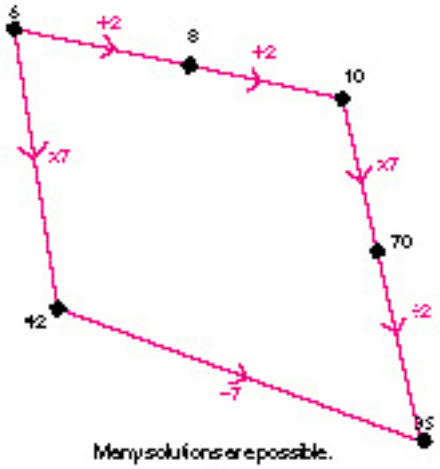


Other solutions are possible.

3

Build an arrow road from 6 to 35. Each arrow must be for one of these relations:

+2 +7 −2 −7 ×2 ×7 ÷2 ÷7



Many solutions are possible.

4

Llama is a secret number.

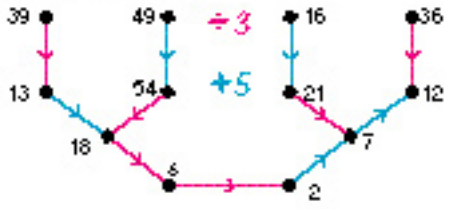
Clue 1

Llama is greater than 30 and can be put on this Mini computer with exactly these two checkers: ▼ and ⊖

Llama could be 32, 49, 30, 32, or 36.

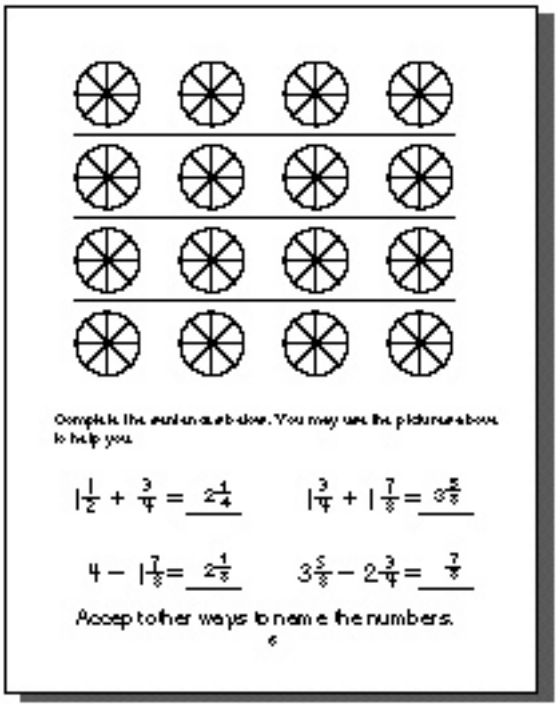
Clue 2

Llama is in this arrow picture.



Who is Llama? 49

5



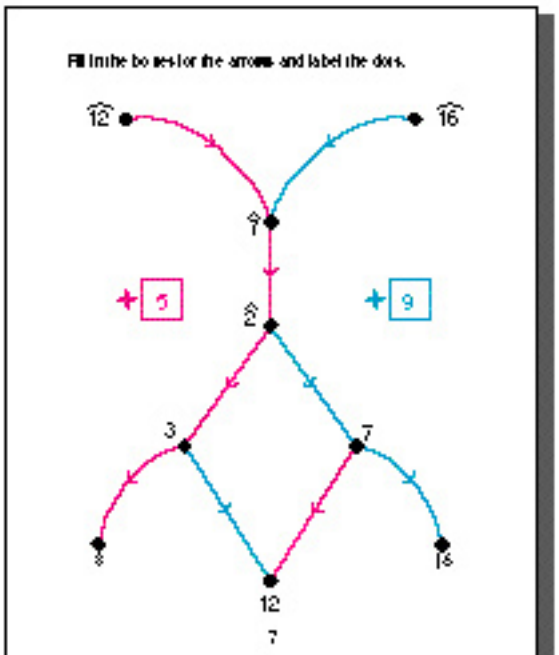
Complete the sentences below. You may use the pictures above to help you.

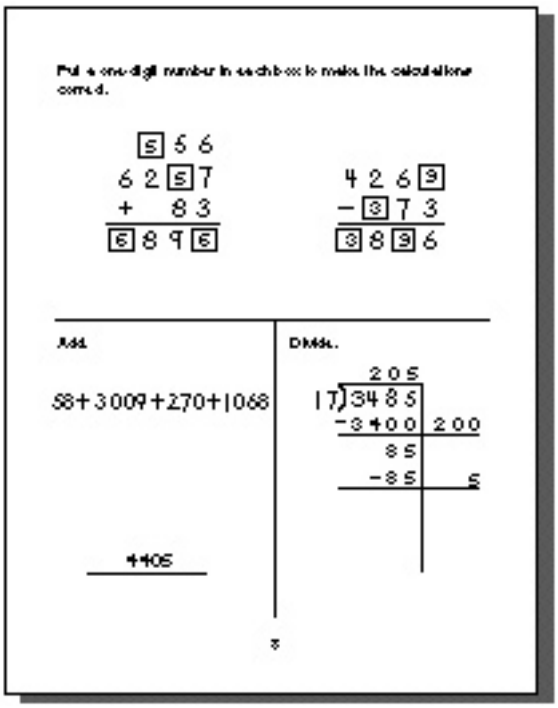
$1\frac{1}{2} + \frac{3}{4} = 2\frac{1}{4}$ $\quad 1\frac{3}{4} + 1\frac{7}{8} = 3\frac{5}{8}$

$4 - 1\frac{7}{8} = 2\frac{1}{8}$ $\quad 3\frac{5}{8} - 2\frac{3}{4} = \frac{7}{8}$

Accept other ways to name the numbers.

6





Put a one-digit number in each box to make the calculations correct.

$$\begin{array}{r} \boxed{5}56 \\ 62\boxed{5}7 \\ +\quad 83 \\ \hline \boxed{6}89\boxed{6} \end{array} \qquad \begin{array}{r} 426\boxed{9} \\ -\boxed{3}73 \\ \hline \boxed{3}8\boxed{9}6 \end{array}$$

Add.

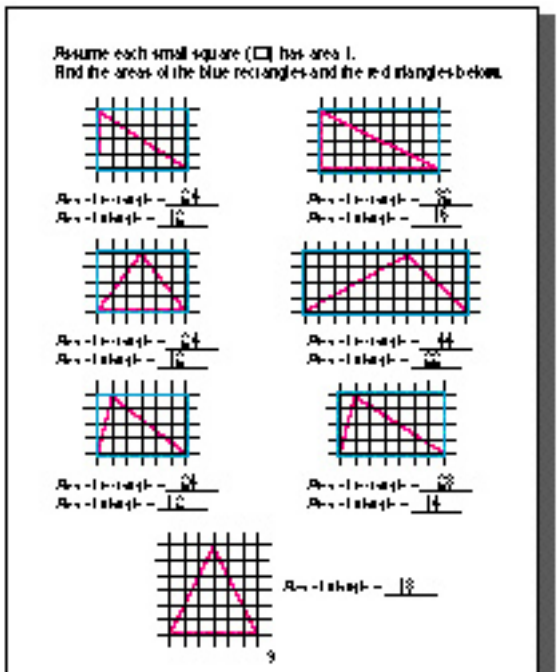
$58+3009+270+1068$

4405

Divide.

$$\begin{array}{r} 205 \\ 17\overline{)3485} \\ -3400 \\ \hline 85 \\ -85 \\ \hline 0 \end{array} \quad 200 \quad 5$$

8



Bookout's bakery sells big cookies at 4 for $1.80.

Mr. Bookout is making a chart with the prices for different numbers of cookies. Each cookie costs the same, regardless of quantity. Complete this chart for Mr. Bookout. Round prices to the nearest penny.

Number of Cookies	Price
1	$0.45
2	$0.90
3	$1.35
4	$1.80
5	$2.25
(½ dozen) 6	$2.70
7	$3.15
8	$3.60
9	$4.05
10	$4.50
11	$4.95
(dozen) 12	$5.40

10

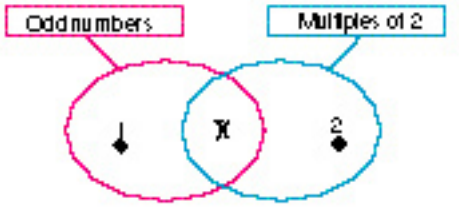
The red label is one of these:

- Multiples of 3
- Odd numbers
- Greater than 10
- Positive numbers of 12
- Positive numbers of 13
- Positive numbers of 20

The blue label is one of these:

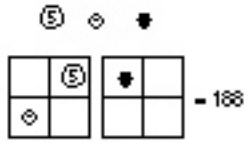
- Multiples of 2
- Odd numbers
- Greater than 10
- Positive numbers of 12
- Positive numbers of 13
- Positive numbers of 20

Label the strings.



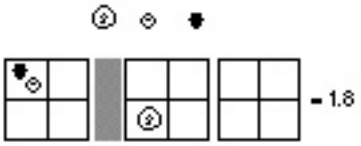
11

Use each of these three checkers to put on a number between 185 and 195.



Other solutions are possible.

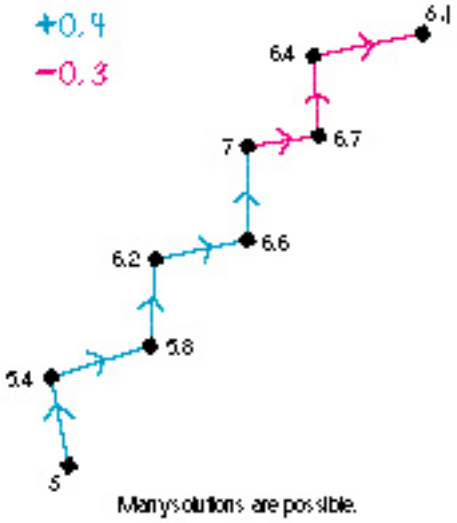
Use each of these three checkers to put on a number between 1 and 2.



Other solutions are possible.

12

Build an answer road between 5 and 6.1 using +0.4 and −0.3 arrows.



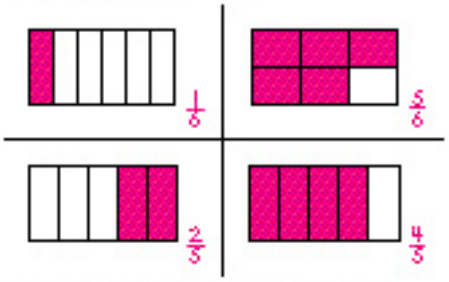
Many solutions are possible.

13

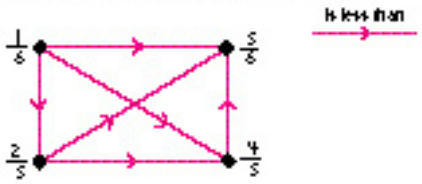
Add.	Subtract.
2.309 + 23.49 + 974.2	33 − 3.889
999999	29.111
Add.	**Subtract.**
3.147 + 45.426 + 9.2 + 20	6.003 − 2.97
777777	3.033

14

Color in red the indicated fractional part of each rectangle.
Use a ruler to divide the rectangles.



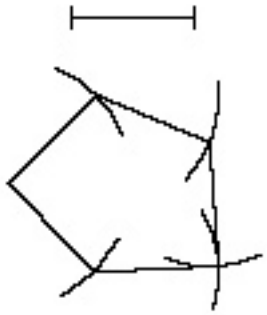
Draw all of the possible red arrows between these dots.



15

Mr. Mueller bought five sections of fence all the same length to make a pen for his dog. When he finished pulling the fence together, he had a very unusual looking dog pen.

Use your compass to show what his dog pen could look like. Make all sections of the fence the same length.



Other solutions are possible.

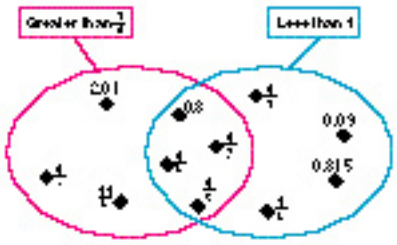
16

Put these numbers in the sorting picture.

0.09 2.01 0.87 0.315

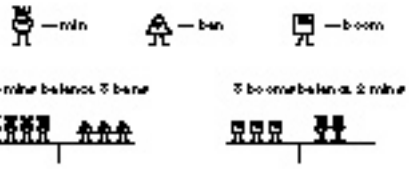
1/6 4/6 11/6

4/3 4/5 4/7 4/9



17

On the planet of Art, the inhabitants like to play on a balance.

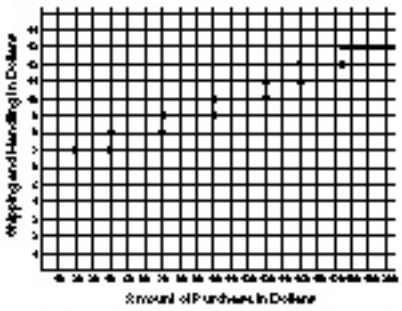


How many booms will balance 1 ban? 2

4 mins balance 3 bans.
2 mins balance 3 booms.
Therefore, 4 mins balance 6 booms.
Therefore, 6 booms balance 3 bans
and 2 booms balance 1 ban.

18

A mail order catalog has the following additional charges for shipping and handling of purchases.



How much is the shipping and handling on a purchase of:

$75? $9.00 $65? $8.00 $110? $10.00 $170? $15.00

Mr. Gunn paid $3.00 shipping and handling on his order. What is the most his total bill could have been including shipping and handling? $23.00

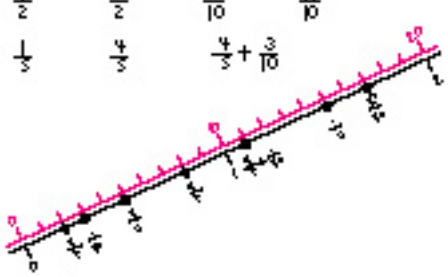
Mr. Hansen has selected two items to order from the catalog. One item costs $20 and the other item costs $95. How much can Mr. Hansen save by ordering both items together as one purchase rather than as two purchases? $4.00

19

Label the dots on the number line with these numbers.

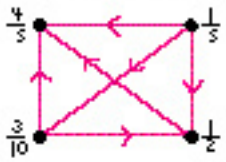
$\frac{1}{2}$ $\frac{3}{2}$ $\frac{3}{10}$ $\frac{17}{10}$

$\frac{1}{5}$ $\frac{4}{5}$ $\frac{4}{5}+\frac{3}{10}$



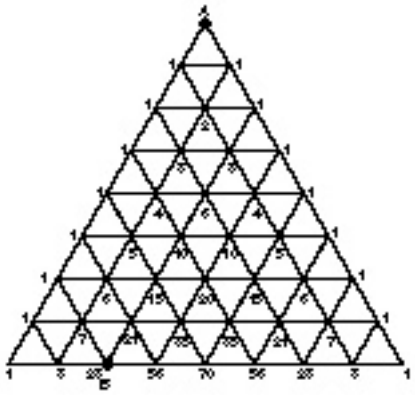
Draw all of the possible redarrows between these dots.

is less than



20

The streets of a town built on a hill are as shown below. How many shortest routes are there from point A to point B? 252



21

Zib is a secret number.

Clue 1

Zib is in this arrow picture.

Clue 2

$$\text{Zib} \square 40 = 4$$

Zib could be 12 or 36

Clue 3

$$20 \uparrow \text{Zib} = \text{Zib}$$

Who is Zib? 36

22

1 cm on the map above represents 230 kilometers.
Find the actual distance in kilometers from:

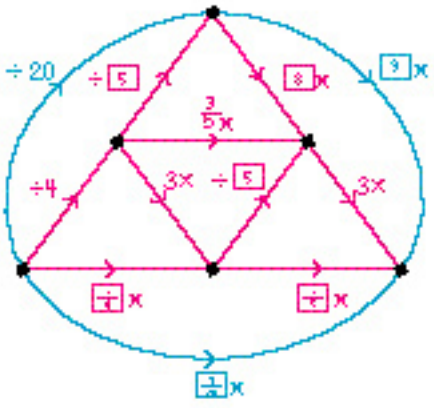
London to Rome 1380 km (6 x 230)
Madrid to Paris 1035 km ($4\frac{1}{2}$ x 230)
Warsaw to Bonn 920 km (4 x 230)
Paris to Bonn 460 km (2 x 230)

A train averages 150 km per hour. When will it arrive in Warsaw if it leaves Madrid at 7:00 a.m. and travels through Paris and Bonn? 11:06 pm

23

Accept answers between 11pm and 11:45 pm.

Fill in the boxes for the arrows.



24

Mali is a secret number.

Clue 1

Mali is divisible by 4.

$$\text{Mali} = 201\,620\,3\square6$$

Mali's name can be completed by putting a one-digit number in the box.

Which digits could go in the box? 1, 3, 5, 7, or 9

Clue 2

Mali is divisible by 8.

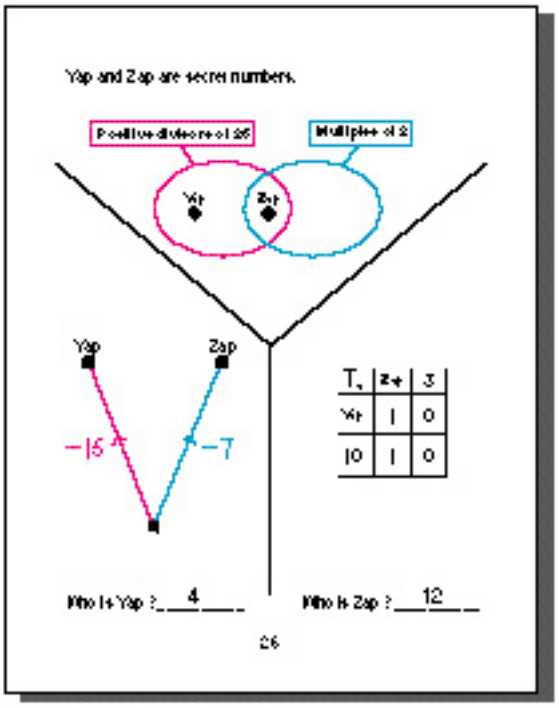
Mali could be 201,620,336 or 201,620,376

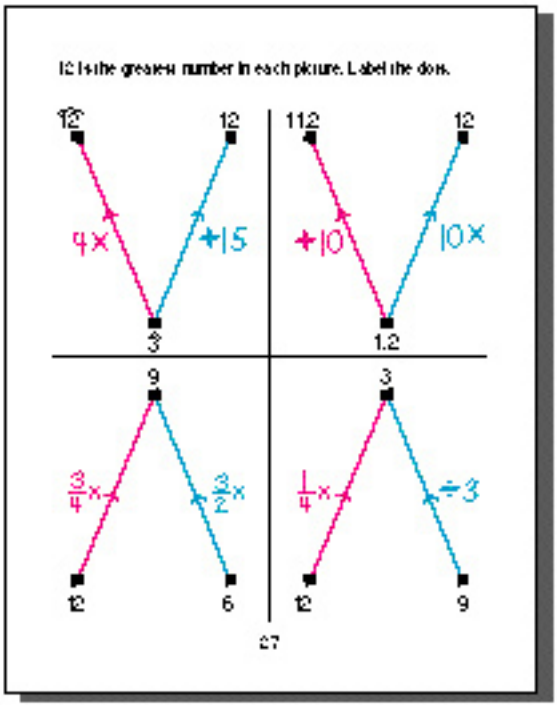
Clue 3

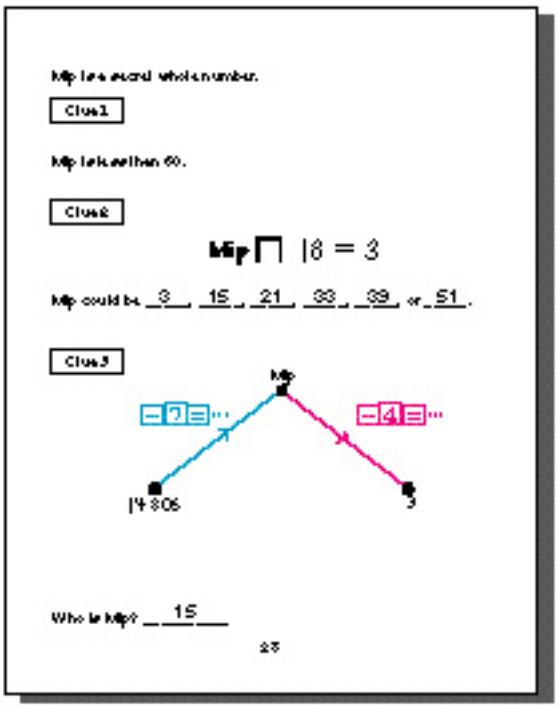
Mali is not divisible by 3.

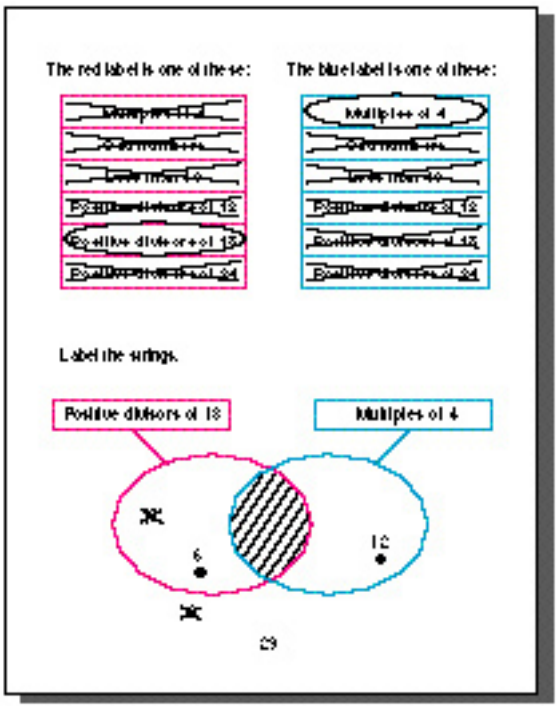
Who is Mali? 201,620,336

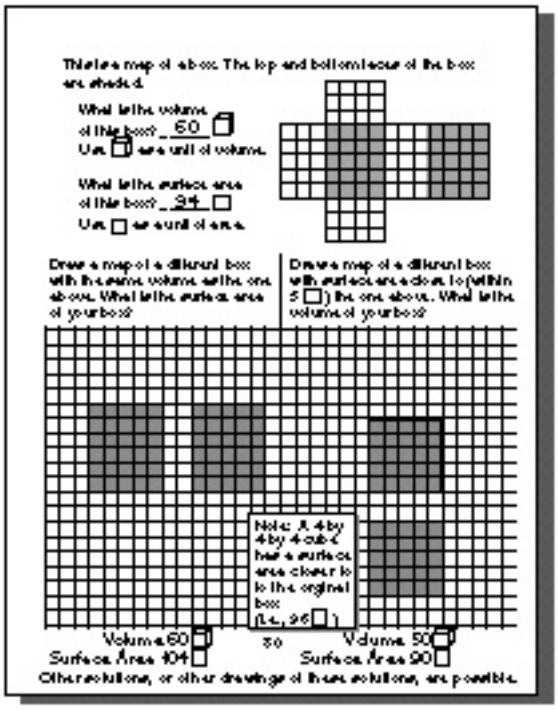
25

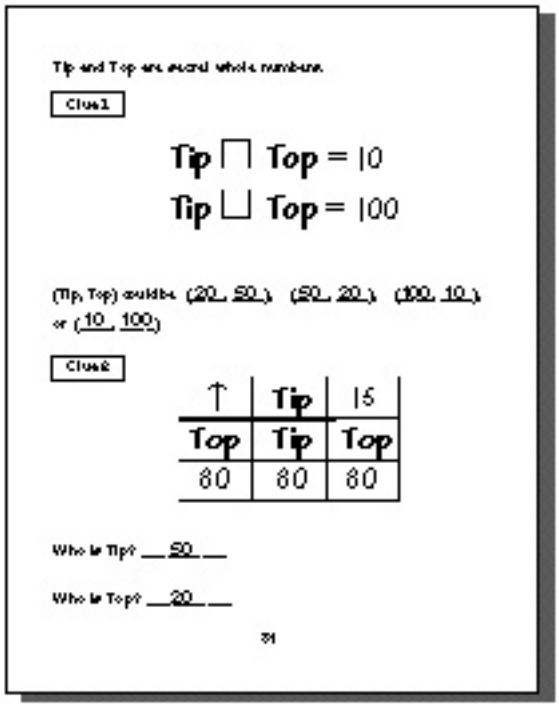

Yap and Zap are secret numbers.
Positive divisors of 28
Multiples of 2
Yap
Zap
Yap
Zap
−15
−7
T, | Z+ | 3
Yap | 1 | 0
10 | 1 | 0
Who is Yap ? 4
Who is Zap ? 12
26

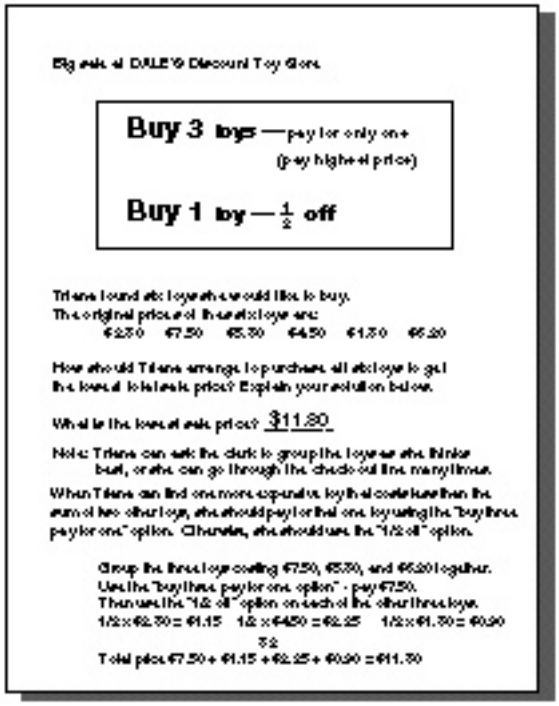

12 is the greatest number in each picture. Label the dots.
12
12
4×
+15
3
112
12
+10
10×
1.2
9
3
12
6
12
9
27


Clue 1
Clue 2
Clue 3
Who is Mip? 15
28


The red label is one of these:
The blue label is one of these:
Multiples of 4
Label the strings.
Positive divisors of 18
Multiples of 4
6
12
29


This is a map of a box. The top and bottom faces of the box are shaded.
What is the volume of this box? 60
Use ☐ as a unit of volume.
What is the surface area of this box? 94 ☐
Use ☐ as a unit of area.
Draw a map of a different box with the same volume as the one above. What is the surface area of your box?
Draw a map of a different box with surface area close to (within 5 ☐) the one above. What is the volume of your box?
Note: A 4 by 4 by 4 cube has a surface area closest to the original box (i.e., 96 ☐).
Volume 60
Surface Area 104
Volume 50
Surface Area 90
Other solutions, or other drawings of these solutions, are possible.


Tip and Top are several whole numbers.
Clue1
Tip ☐ Top = 10
Tip ☐ Top = 100
(Tip, Top) could be (20, 50), (50, 20), (100, 10), or (10, 100)
Clue2
↑ Tip 15
Top Tip Top
80 80 80
Who is Tip? 50
Who is Top? 20


Big sale at DALE'S Discount Toy Store
Buy 3 toys — pay for only one (pay highest price)
Buy 1 toy — ½ off
Triana found six toys she would like to buy.
The original prices of the six toys are:
$2.50 $7.50 $5.30 $4.50 $1.30 $6.20
How should Triana arrange to purchase all six toys to get the lowest total sale price? Explain your solution below.
What is the lowest sale price? $11.80
Note: Triana can ask the clerk to group the toys as she thinks best, or she can go through the check-out line many times.
When Triana can find one more expensive toy that costs less than the sum of two other toys, she should pay for that one toy using the "buy three, pay for one" option. Otherwise, she should use the "1/2 off" option.
Group the three toys costing $7.50, $5.30, and $6.20 together.
Use the "buy three, pay for one option" - pay $7.50.
Then use the "1/2 off" option on each of the other three toys.
1/2 × $2.50 = $1.15 1/2 × $4.50 = $2.25 1/2 × $1.30 = $0.60
1/2
Total price $7.50 + $1.15 + $2.25 + $0.60 = $11.50