

W Strand

# Workbooks

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# WORKBOOKS INTRODUCTION

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There are many opportunities for student to work individually during the course of the lessons described in the other content strands. In the Workbooks strand, however, it is this individualized work which becomes the chief end of the majority of lessons. The goal in this strand is to provide students with opportunities

- to review many of the ideas they have met in other content strands;
- to apply their acquired knowledge to new situations requiring various kinds of strategic thinking; and
- to learn how to read and use mathematics workbooks.

The following six workbooks are provided:

- *Collage of Problems #1*
- *Collage of Problems #2*
- *Collage of Problems #3*
- *Collage of Problems #4*
- *Collage of Problems #5*
- *Collage of Problems #6*

...and one storybook.

- *A Very Strange Neighborhood*

Each workbook contains problems of varying levels of difficulty. Approximately the first ten pages of each workbook are easy problems, the next ten to twelve pages are average level difficulty, and the last ten pages are more challenging problems. For each workbook, we suggest that all students start work at the easiest level (i.e., on page 2) and then work through as many pages as they can handle during the two lessons scheduled for that workbook. We estimate that, in a typical class, about two-thirds of the students will correctly finish the first ten pages, about one-third will finish the first twenty pages, and a few will finish all or most of the workbook. These proportions will vary from class to class.

This guide contains an answer key for each workbook. The key follows an introduction to the workbook and a suggested collective lesson. The lesson either presents the workbook to the whole class or provides a warm-up activity, usually on a problem similar to one found in the workbook.

The storybook *A Very Strange Neighborhood* touches some important areas of mathematics with the motivation of a story context. This booklet allow students to become deeply involved in an appealing fantasy as they struggle with difficult mathematics problems. The situations support topics and strategies developed in other strands.

## Use of the Workbook Strand for Evaluation Purposes

The workbooks provide an excellent instrument to assess the progress of your students on a regular basis. You may not feel it is necessary to check every page and problem for each student, but you should develop a procedure for checking students' work with which you are comfortable. This may include checking one or more specific pages, discussing some particular mistakes with individual students and letting them correct their work, or just looking carefully at a few pages to be sure the students have understood the general idea of the problems in that particular workbook.

In the Blacklines, you will find a record-keeping tool for each workbook to help you assess student progress in the various strands. This tool may also assist you in parent conferences and in filling out periodic progress reports.

Here are some important points to bear in mind for workbooks.

- Always read the introductory material for each workbook and give the short introductory collective lesson(s).
- All students should start at the beginning of each workbook and progress as far as they can.
- All students should begin a new title on the same day, even if some students have not finished work on the previous title.
- Not all students should be expected to complete a given workbook. Only some students will reach the most challenging problems. Other students may succeed only in doing the easiest problems, although you should not assume this automatically—surprises are not at all uncommon.
- Students should be allowed to use calculators except on pages you specify beforehand.

**Note:** Some pages in a workbook have problems that are related to calculators or are best attacked with the support of calculators; however, there also are pages for which the use of calculators may not be appropriate. We encourage you to review a workbook before distributing copies of it and to inform the class beforehand on which pages you are not allowing the use of calculators. Certain pages you may wish to use for special evaluation purposes; other pages would not be effective if calculators were allowed, for example, pages that primarily focus on paper-and-pencil computation, “wipe-out” pages that involve subtraction of decimal numbers, pages where using patterns permits students to solve problems from previously solved problems, and pages where the problem is to insert missing decimal points in answers to calculations.

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## Workbooks

The six *Collage of Problems* workbooks both review and extend many of the ideas introduced in the content strands. The extensions occur through problems which require students to apply the mathematics to new situations or to synthesize their knowledge in new ways.

Lessons: W1, 2, 4, 5, 6, 7, 8, 9, 10, ~~11, 12,~~ and 13

## A Very Strange Neighborhood

Your students are familiar with many different types of numbers and ways to name them; for example, 73,  $\widehat{24}$ ,  $\frac{3}{4}$ , 8.517, and  $0.\overline{3}$ . The storybook *A Very Strange Neighborhood* introduces a new kind of number, namely the irrational numbers<sup>†</sup>. During the story, students use a number line to explore the ordering and density of decimal numbers.

Lesson: W3

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<sup>†</sup> Rational numbers are numbers which have fractional names; for example,  $\frac{1}{2}$ ,  $-2\frac{3}{8}$ , and 5.02 ( $5\frac{2}{100}$ ) are rational numbers. Irrational numbers do not have fractional names although they are on the (real) number line.



### Capsule Lesson Summary

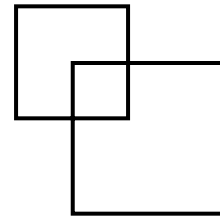
Find or draw sets of line segments of the same length in a picture of intersecting squares without using a ruler to measure. Begin the workbook *Collage of Problems #1*. (This is the first of two lessons using this workbook.)

#### Materials

Teacher	Student
<ul style="list-style-type: none"> <li>• Colored chalk</li> <li>• Meter stick</li> </ul>	<ul style="list-style-type: none"> <li>• <i>Collage of Problems #1</i> Workbook</li> <li>• Colored pencils, pens, or crayons</li> <li>• Compass</li> <li>• Metric ruler</li> <li>• Calculator</li> </ul>

### Description of Lesson

Carefully draw this picture on the board. The intersection of the two squares should also be a square.



**T:** *There are three squares in this picture.*

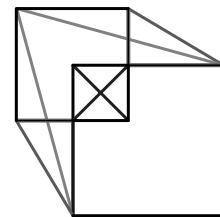
Invite someone to the board to trace the three squares.

**T:** *Can someone trace some line segments that have the same length without using a ruler to measure?*

**S:** *The four sides of a square are the same length.*

**T:** *Yes. Are there any other segments already in the picture or that you could draw in the picture that have the same length?*

As students suggest possibilities, use different colors for different sets of segments. Illustrate several different sets, but do not attempt to find or draw all of the possibilities in the picture. For example, your picture might look similar to this one.



Explain to the class that page 12 of the workbook for today's lesson has a similar activity.

Distribute copies of the workbook *Collage of Problems #1*, and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of the class period, collect the workbooks for your review. They will be used again in Lesson W2.

## Writing Activity

You may like students to take lesson notes on some, most, or even all their math lessons. The “Lesson Notes” section in Notes to the Teacher gives some suggestions and refers to forms in the Blacklines you may provide to students for this purpose. In this lesson, for example, students may note problems in the workbook they found especially interesting. They may also like to create other problems, similar to ones in the workbook, for their classmates or a family member to solve.



### Capsule Lesson Summary

Review divisibility tests by asking which of the whole numbers 2 through 10 are divisors of 58 212. Continue individual work in the workbook *Collage of Problems #1*. (This is the second of two lessons using this workbook.)

#### Materials

Teacher	Student
<ul style="list-style-type: none"> <li>• None</li> </ul>	<ul style="list-style-type: none"> <li>• <i>Collage of Problems #1</i> Workbook</li> <li>• Colored pencils, pens, or crayons</li> <li>• Compass</li> <li>• Metric ruler</li> <li>• Calculator</li> </ul>

### Description of Lesson

Write the following information on the board.

$$\begin{array}{cccccccccc}
 & & & & 5 & 8 & 2 & 1 & 2 & & \\
 & & & & \hline
 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & & 
 \end{array}$$

**T:** *Which of the whole numbers 2 through 10 are divisors of 58 212? Why?*

As students explain which of the numbers are divisors, circle them in the list. Cross out numbers that are not divisors.

**S:** *2, because 58 212 is even.*

**S:** *4, because the last two digits of 58 212 are for a multiple of 4. 4 is a divisor of 12.*

**S:** *58 212 is not divisible by 5 because its ones digit is not 0 or 5. It also is not divisible by 10 because its ones digit is not 0.*

**S:** *3 and 9, because  $5 + 8 + 2 + 1 + 2 = 18$  and 18 is divisible by 3 and by 9.*

**S:** *6, because 58 212 is divisible by 2 and by 3.*

**S:** *58 212 is not divisible by 8 because when we divide 58 212 by 4 we get an odd number. Another test is to look at the last three digits, 212, which is not divisible by 8. 200 is a multiple of 8, so 208 and 216 are multiples of 8, but 212 is not a multiple of 8.*

**S:** *We do not have an easy test for 7.*

Invite a student to do the division at the board. For example:

$$\begin{array}{r|l}
 8316 & \\
 7 \overline{)58212} & \\
 \underline{-49000} & 7000 \\
 9212 & \\
 \underline{-7000} & 1000 \\
 2212 & \\
 \underline{-2100} & 300 \\
 112 & \\
 \underline{-70} & 10 \\
 42 & \\
 \underline{-42} & 6
 \end{array}$$

Conclude that 58 212 is divisible by 7.

$$\begin{array}{c}
 58212 \\
 \hline
 \end{array}$$

2
3
4
5
6
7
8
9
10

Distribute students' copies of the workbook *Collage of Problems #1*. Ask students first to correct or complete pages from the previous week's work, and then to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in their workbooks during a study time or to take them home as an assignment.

## Assessment Activity

An individual student progress record for this workbook is available on Blackline W2(a). You may like to use this form to monitor student work.

## Home Activity

If you choose to send workbooks home with students, you may want to include a letter to parents/guardians about the workbook. Blackline W2(b) has a sample letter.

Label the roads among try to label some arrows in two ways.

Other arrow labels are possible.

2

Build an arrow road from 9 to 119. Each arrow must be for  $\times$ ,  $\div$ ,  $+$  or  $-$  a one-digit whole number. Use as few arrows as you can.

Other solutions are possible. A shortest solution has two arrows.

3

Yabu likes even numbers.

**Clue 1**

Yabu likes the number 13.

**Clue 2**

Yabu could be  $\frac{1}{2}$  or  $\frac{3}{4}$ .

**Clue 3**

Who is Yabu?  $\frac{1}{2}$

4

Draw a quadrilateral that has the black background areas of dots and that has lines of the length of the red segment. You will need a compass.

Other solutions are possible.

5

**Wipe-out**

**Fill in the boxes or the arrows.**

6

**Put a one-digit number in each box to make the calculation correct.**

$$\begin{array}{r} | \boxed{3} 9 5 \\ 3 3 \boxed{4} \\ + \boxed{4} 4 8 8 \\ \hline 6 2 \boxed{1} 7 \end{array}$$

$$\begin{array}{r} 7 \boxed{9} 8 6 \\ - \boxed{3} 2 \boxed{2} 7 \\ \hline 4 7 5 \boxed{9} \end{array}$$


---

Add	Subtract.
$340 + 97 + 16\,823$ $\begin{array}{r} 340 \\ 97 \\ +16\,823 \\ \hline 17\,260 \\ 17\,260 \end{array}$	$76\,092 - 1\,459$ $\begin{array}{r} 76\,092 \\ -1\,459 \\ \hline 74\,633 \end{array}$

7

**All City School they need 1 teacher for every 25 students. How many teachers do they need for 275 students? 11**

$$\begin{array}{r} 11 \\ 25 \overline{) 275} \end{array}$$

10 teachers for 250 students  
1 teacher for 25 students  
11 teachers for 275 students

**During lunch time at school, they need help from 2 mothers for every 10 kindergarten students. How many mothers do they need for 40 kindergarten students? 8**

$$\begin{array}{r} 4 \\ 10 \overline{) 40} \end{array} \text{ and } 2 \times 4 = 8$$

8

**Complete.**

 $\begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array} \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \end{array} = 0.9$ 
 $\begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \end{array} = 0.09$ 
 $\begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \end{array} = 2.4$ 
 $\begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \end{array} = 0.24$ 

**Label these dots with the four numbers above.**

Label them  $\rightarrow$

9

Using a ruler, draw a dot for each of these numbers on the number line.

$\frac{1}{3}$     $\frac{2}{3}$     $\frac{3}{3}$     $\frac{4}{3}$

$\frac{1}{2}$     $\frac{2}{2}$     $\frac{3}{2}$

10

Put a one-digit number in each box to make the calculation correct.

$$\begin{array}{r} \boxed{7} \boxed{6} \boxed{8} \\ \times 3 \\ \hline 2 \ 3 \ \boxed{0} \ 4 \end{array}$$

$$\begin{array}{r} \boxed{2} \boxed{2} \text{ R}=\boxed{1} \boxed{3} \\ 4 \overline{) 1137} \\ \underline{-860} \phantom{0} \\ 277 \\ \underline{-258} \phantom{0} \\ 19 \end{array}$$

---

Multiply.

$$\begin{array}{r} 84 \\ \times 47 \\ \hline 588 \\ 3360 \\ \hline 3948 \end{array}$$

Divide.

$$\begin{array}{r} 350 \text{ R}=27 \\ 76 \overline{) 26627} \\ \underline{22800} \phantom{00} \\ 3827 \\ \underline{3800} \phantom{00} \\ 27 \phantom{00} \end{array}$$

11

Do not use a ruler to measure. In doing these problems.

- 1) The blue segments all have the same length. Draw more blue segments in the picture so that all of the blue segments have the same length.
- 2) Use red to draw another set of line segments in the picture so that all of the red segments have the same length.
- 3) Use green to draw a third set of line segments in the picture so that all of the green segments have the same length.

There are other possibilities for the sets of red and green segments. 12

Complete.

$3 \times 18 = \boxed{54}$

$3 \times 1.8 = \boxed{5.4}$

$3 \times 0.18 = \boxed{0.54}$

$8 \times 910 = \boxed{7280}$

$8 \times 91.0 = \boxed{728.0}$

$8 \times 9.10 = \boxed{72.80}$

$8 \times 0.910 = \boxed{7.280}$

$7 \times 6 = \boxed{42}$

$0.7 \times 6 = \boxed{4.2}$

$0.7 \times 0.6 = \boxed{0.42}$

13

Label the blue arrows.

Complete these calculations.

$$\frac{3}{2} \times 18 = 27 \qquad \frac{2}{3} \times 30 = 20$$

$$\frac{2}{3} \times 18 = 12 \qquad \frac{5}{2} \times 30 = 75$$

$$\frac{5}{4} \times 40 = 50 \qquad \frac{4}{3} \times 40 = 53$$

14

Timely Questions

Show your work for each problem in the space provided.

- Mr. Ghoshan works  $7\frac{1}{2}$  hours each day and also has a 40-minute lunch break. If he begins work at 7:45 AM, what time does he finish? 3:55 PM  
(Remember to indicate AM or PM)

$$7 \text{ hr. } 30 \text{ min.} + 40 \text{ min.} = 8 \text{ hr. } 10 \text{ min.}$$

$$7:45 \text{ AM} + 8 \text{ hr. } 10 \text{ min.} \rightarrow 3:55 \text{ PM}$$

- Emile is 11 years and 11 months old and his mother is 56 years and 2 months old. How old would Emile's mother have been when he was born? 24 years 3 months

$$\begin{array}{r} 56 \text{ years } 2 \text{ months} \\ -11 \text{ years } 11 \text{ months} \\ \hline 24 \text{ years } 3 \text{ months} \end{array}$$

There are many good ways to solve these problems.

15

Color in the indicated fractional part of each square. Use a ruler to divide the squares equally.

Draw all of the possible red arrows between these numbers.

1 is less than  $\frac{1}{2}$

Other divisions of the square and other colorings are possible.

16

Energy Temperature (Degrees Celsius)

Month	Minneapolis	Washington, D.C.	Los Angeles
Jan	10	15	20
Feb	12	18	22
Mar	15	22	25
Apr	20	28	28
May	25	30	28
Jun	28	30	25
Jul	25	28	22
Aug	20	22	18
Sep	15	18	15
Oct	12	15	12
Nov	10	12	10
Dec	8	10	8

Minneapolis      Washington, D.C.      Los Angeles

What is the average February temperature in Minneapolis? 8°C

How much colder is Minneapolis than Los Angeles in April but 7°C in August? 24°C

In which month is Minneapolis's energy temperature closest to Los Angeles? June How close? 1°C

In which months is Washington's energy temperature below 10°C? January, February, March, November, December

In which months is Los Angeles all day > 10°C or more than Washington, D.C.? January, February, and December

17

Fill in the boxes for the arrows. Label the dots.

---

How Old?

Lillian is 3 years older than Olive. The sum of their ages is 30 years. How old is Lillian? 19 Olive? 11

$$19 - 11 = 8$$

$$19 + 11 = 30$$

How Much?

Andre has exactly ten coins in his pocket. He has only dimes and quarters. If he has \$2.05, how many quarters does he have? 7 dimes? 3

$$7 \times \$0.25 = \$1.75$$

$$3 \times \$0.10 = \$0.30$$

\$1.75
<u>\$0.30</u>
\$2.05

The work here shows that the given solution is correct, not how to get it. Most students will use guess-and-check techniques.

12

Fill in the boxes.

$$\frac{1}{2} = \frac{7}{14} = \frac{4}{8} = \frac{3}{6} = \frac{5}{10} = \frac{13}{26}$$

$$\frac{3}{5} = \frac{9}{15} = \frac{6}{10} = \frac{15}{25} = \frac{30}{50} = \frac{21}{35}$$

Complete. You may use the different names for  $\frac{1}{2}$  and  $\frac{1}{5}$  at the top of this page.

$$\frac{10}{5} + \frac{1}{2} = \frac{6}{10} + \frac{5}{10} = \frac{11}{10}$$

$$\frac{10}{5} - \frac{1}{2} = \frac{6}{10} - \frac{5}{10} = \frac{1}{10}$$

20

Scale: 1 in on the map = 100 kilometers

- On the map, what is the length of a line segment between El Paso and Houston? 8 in.  
What is the actual distance between El Paso and Houston? 800 km.
- On the map, what is the length of a line segment between Dallas and Houston? 3.5 in.  
What is the actual distance between Dallas and Houston? 350 km.
- If an airplane flies 600 kilometers in 1 hour, about how long should the flying time be from El Paso to Houston? (Circle the closest answer.)  
60 minutes  1 hour 20 minutes  1 hour 50 minutes
- If an airplane flies 400 kilometers in 1 hour, about how long should the flying time be from Dallas to Houston? (Circle the closest answer.)  
60 minutes  1 hour 10 minutes  1 hour 20 minutes

21

27810

27810 is divisible by which of these numbers? Circle your answer.

1	9	4
3	6	8

---

1010

1010 is divisible by which of these numbers? Circle your answer.

2	3	5
10	4	6

22

Deno is a secret number.


**Clue 1**

Deno can be put on the Minicomputer by adding just a 0 to each

#	#	#	#
#	#	#	#

Deno could be 5.5, 5.8, 6.2, 7.0, 7.4, 9.4, 13.4, or 21.4.

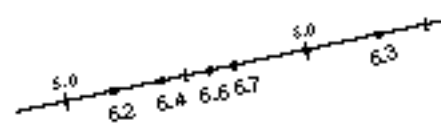
**Clue 2**



Deno could be 5.5, 6.2, 7.4, or 13.4.

**Clue 3**

Deno is one of the dots on this number line.



Who is Deno? 5.5

23

A city council must choose a committee of three people from the eligible members Arlene, Bert, Oats, Dinah, and Ed. The selection is not easy because some members jealous of others and some have close friendships.

- Arlene will serve on the committee with anyone.
- Bert won't serve on the committee with Arlene.
- Oats will serve on the committee only if Bert and Dinah also serve.
- Dinah won't serve if Arlene or Oats also serve.
- Ed won't serve unless Dinah also serves.

Can you select a committee of three people so that everyone is satisfied?

Who is on your committee? Bert, Dinah, Ed

24

Match each red flag with a blue flag.

$\frac{7}{5} + \frac{3}{5}$	$\frac{1}{2}$
$\frac{3}{2} \times \frac{3}{5}$	$\frac{4}{5}$
$\frac{2}{3} \times \frac{3}{4}$	0.9
$\frac{1}{2} - \frac{1}{5}$	2
$\frac{1}{5} - \frac{2}{5}$	$\frac{1}{3}$

25



Tim and Tam are even numbers.

**Clue 1**

(Tim, Tam) is one of the dots on the grid.

**Clue 2**  $Tim + Tam > 3$

(Tim, Tam) could be  $(1, 5), (5, 0), (5, 2), (5, 2)$  or  $(2, 4)$

**Clue 3**

(Tim, Tam) is  $(5, 2)$

Build a road from 12.5 to 38 with two arrows. Each arrow must be for  $\times, \div, +, -$  or  $\frac{1}{2}$  a one-digit whole number.

Anytho ude ths at does ths s timr or leaf or her lilik brother. If ths y deer s at does k1 6 mks n r long, whal lenth dose Amy deer? 12 m

Whal lenth dose her lilik brother deer? 4 m

$$3 \times 4 = 12$$

$$4 + 12 = 16$$

The work here shows that the given solution is correct, not how to get it. Most students will use guess-and-check techniques.

Put the legs.

The is a second whole number.

**Clue 1**

Positive divisors of 90      Multiples of The

The could be 2 3 or 5.

**Clue 2**

The can be put on this Mini-computer with exactly three two checkers.

Who is The? 3

Another way to put on 3 is with the 3 checker on the 0-4-square and the 3 checker on the 0-1-square.

Complete.

$$\begin{array}{r} 11^2 = 1 \\ 11^2 = 11 \\ 11^2 = 121 \\ 11^2 = 1331 \\ 11^2 = 14641 \\ 11^2 = 161051 \\ 11^2 = 1771561 \\ 11^2 = 19487171 \end{array}$$

Describe palindromes in digits.

The one digit is always 1.      Digits look like a row of Pascal's triangle.

The next digit goes up by 1 each time.

Complete.

Complete.	Sum of digits
$f = 1$	<u>1</u>
$11^2 = 121$	<u>4</u>
$11^2 = 12321$	<u>9</u>
$11^2 = 1234321$	<u>16</u>
$11^2 = 123454321$	<u>25</u>

Prove it.       $11111111^2 = 12345678987654321$  81

Describe palindromes in digits.

The digits in the squares increase by one up to the number of digits in the number being squared and then decrease by one back to 1. The sum of the digits is a square number.

Pin is a second number.

**Clue 1**

Pin could be 32, 32, 152, 212, 272, 332, 392, 452 and so on.

**Clue 2**

Who is Pin? 452

32

### Capsule Lesson Summary

While reading the storybook *A Very Strange Neighborhood*, explore the ordering and density of decimal numbers. Through a detective story about a lost number sending radio signals, introduce a new kind of number, an *irrational* number.<sup>†</sup>

#### Materials

**Teacher**

- Colored chalk
- Meter stick

**Student**

- *A Very Strange Neighborhood* Storybook

### Description of Lesson

Begin with a discussion of different kinds of numbers, asking students to describe or give examples. Students should mention whole numbers, negative integers, and rational numbers. Write students' examples on the board; for example:

17       $\widehat{82}$        $\frac{5}{7}$       8.2

If no suggestion uses a repeating decimal name such as  $0.\overline{3}$  or  $5.1\overline{462}$ , ask,

**T:** *Do you remember a name for  $\frac{1}{3}$  that we said was a kind of decimal “nickname”?*

**S:**  *$0.333 \dots$  or  $0.\overline{3}$ .*

Write this expression on the board.

$5.1\overline{462}$

**T:** *What does this notation mean?*

**S:** *The digits 4, 6, and 2 repeat. We could write  $5.1462462462 \dots$*

**T:** *You already know many different kinds of numbers. Today we will read a story about a new kind of number.*

Distribute copies of the storybook *A Very Strange Neighborhood*. Read the story with your class. Questions based on the mathematical ideas are suggested below. Use the questions carefully to enhance, not diminish, the spirit of the story. By keeping the discussions brief, you should finish the storybook in one lesson. At a later time, you or your students may wish to discuss further some of the ideas.

#### Pages 1–4

Read pages 1 to 4 collectively.

**T:** *Are there other numbers between 7 and 8?*

**S:** *7.1, 7.2, 7.4, 7.6, and 7.9.*

<sup>†</sup>Ask students to point to the unlabeled mark for each of these numbers on page 2 ( $\frac{1}{2}$ ), and  $5.02$  ( $\frac{502}{100}$ ) are rational numbers. Irrational numbers do not have fractional names, although they can be found on the (real) number line.

**T:** *Any others?*

**S:** *7.06, 7.29, and 7.84.*

**S:**  *$7\frac{2}{3}$ .*

**T:** *Are there numbers between 7.7 and 7.8?*

**S:** *Yes, 7.74 and 7.79.*

**S:** *7.715.*

**S:**  *$7\frac{3}{4}$ .*

## Pages 5–6

Read pages 5 and 6 together.

**T:** *On page 6 of your storybook, point to the mark on the number line for 7.73.*

Check that students are pointing to the correct mark.

## Page 7

Read page 7 together.

**T:** *Are there exactly nine numbers between 7.7 and 7.8?*

**S:** *No; there are many more, for example, 7.727 and 7.70622.*

**S:** *There are infinitely many numbers between 7.7 and 7.8.*

## Page 8

Read page 8 together. Point out that the number line has been magnified again. Now only the numbers from 7.7 to 7.8 are shown.

## Page 9

Read page 9 together.

**T:** *Marks for many of the numbers between 7.7 and 7.8 are shown on this number line, but they are not labeled. What are some of the numbers at these marks?*

**S:** *7.732.*

**T:** *Where is 7.732 on the number line?*

**S:** *Between 7.73 and 7.74; the second mark above 7.73.*

Let several students point out other numbers between 7.7 and 7.8.

## Pages 10–11

Read pages 10 and 11 together.

**T:** *Does the story ever end?*

**S:** *No, you can always find more numbers between 7 and 8.*

### Pages 12–14

Read pages 12 to 14 collectively.

**T:** *Look at the spiral and try to figure out its rule. What would be the next number if we were to continue the spiral?*

**S:** *7.888888.*

**T:** *What numbers would come after 7.888888?*

**S:** *Then 7.8888882, 7.8888884, 7.8888886, and so on.*

**T:** *Can you explain the rule?*

There are several correct explanations, but “add 2 to the last digit” is not correct.

**S:** *Start at 7. Add 0.2 four times; then add 0.02 four times; and so on.*

**S:** *Add 2 to the last digit each time, except when the last digit is 8. Then add another decimal place by putting 2 at the end.*

### Page 15

Read page 15 together.

**T:** *What is the rule in the upper part of this snake dance?*

**S:** *Put another 4 on the end of a number to get the next number.*

**T:** *What is the rule in the lower part?*

**S:** *Take off a 6 from the end of a number to get the next number.*

**T:** *Is there a smallest number in this snake dance?*

**S:** *Yes, 7.4.*

**T:** *Is there a largest number?*

**S:** *No. Although 7.666666666666 is the largest number shown on the page, the dance goes on.*

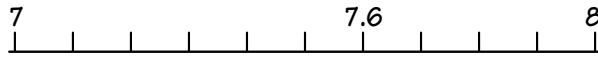
Draw this part of the number line on the board. Make the segment from 7 to 8 one meter long.



**T:** *Who can point to where 7.6 is on this number line?*

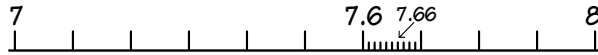
Suggest that students use a meter stick to divide the segment from 7 to 8 into ten pieces of equal length.

# W3



**T:** *Where is 7.66 on this number line?*

Again, suggest that students divide the segment from 7.6 to 7.7 into ten pieces of equal length.



**T:** *Where is 7.666?*

**S:** *Just to the right of 7.66.*

**S:** *Between 7.66 and 7.67.*

**T:** *Where is 7.6666?*

**S:** *Between 7.666 and 7.667.*

**T:** *These numbers in the snake dance, 7.6, 7.66, 7.666, and so on, are getting closer and closer to some number. That number has a fractional name. What number is it?*

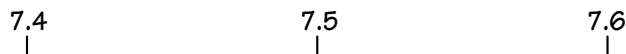
**S:**  *$7\frac{2}{3}$ , because  $0.666 \dots = 0.6\overset{*}{=} \frac{2}{3}$ .*

If no one suggests  $7\frac{2}{3}$ , remind the class that  $0.\overset{*}{3} = 0.333 = \frac{1}{3}$ , and ask about a fraction for  $0.\overset{*}{6}$ . Lead the class to observe that  $7.666 \dots = 7.\overset{*}{6} = 7\frac{2}{3}$ .

**T:** *If we start at 7.6 on the spiral and follow the arrows backwards, we get closer and closer to  $7\frac{2}{3}$ , but we never reach  $7\frac{2}{3}$ .*

## Page 16

Read page 16 together, and draw this part of the number line on the board.



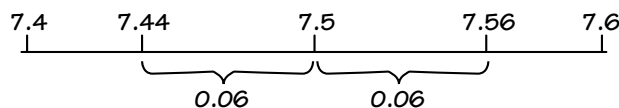
**T:** *7.5 is exactly halfway between 7.4 and 7.6. Can you suggest two other numbers that 7.5 is exactly halfway between?*

**S:** *7 and 8.*

**S:** *7.3 and 7.7.*

**S:** *7.44 and 7.56.*

Use the idea of distance from 7.5 to generate or confirm some answers; for example:



Depending on student interest, you may wish to challenge them to find pairs of numbers even closer to 7.5 which have 7.5 as their midpoint; for example, 7.49 and 7.51, 7.495 and 7.505, or 7.499 and 7.501.

## Pages 17–20

Read pages 17 to 20 collectively.

**T:** *So far in the story, you already know all of the numbers we have met.  $7.\overset{*}{6}$  and  $7.5000001$  are somewhat unusual, but they are not new to you. Now we are going to meet a new kind of number.*

### Pages 21 – 22

Read page 22.

**T:** *This sad number is singing a strange song: “Bing, bang, bing, bing, bang; bing, bing, bing, bang; and so on.” What number could be sending this message?*

Accept a few suggestions about what the sounds could mean.

### Pages 23–26

Read pages 23 to 26 collectively. Refer to the last number,  $7.5155155515555$ , on the red arrow road.

**T:** *What would be the next number on the red arrow road?*

**S:**  *$7.51551555155551$ .*

**T:** *How can we get the next number?*

**S:** *Put five 5s on the end of the new number.*

**T:** *How can we get the next numbers?*

**S:** *Put another 1 at the end.*

**S:** *Then put six 5s.*

**T:** *Who can explain the pattern?*

**S:** *You alternate between putting on a 1 and putting on a group of 5s. Each time you put on 5s, you put on one more than the last time.*

**T:** *And we could keep on going. Could we use the \*-notation to write a shorthand name for the number we are heading towards?*

After a few attempts, students should conclude that it is impossible since no one part of the number is repeated; that is, one more 5 is used each time. Students might suggest some more elaborate abbreviations, such as  $7.51\overline{55}$ . Accept these suggestions as possible notations, but mention that there is no standard notation for this type of number and that it would be impractical to invent a new notation for each non-repeating pattern we could imagine.

**T:** *Now can anyone explain the “bing-bang” song?*

**S:** *The number is trying to signal its name. Each bing is for a 5 and each bang is for a 1.*

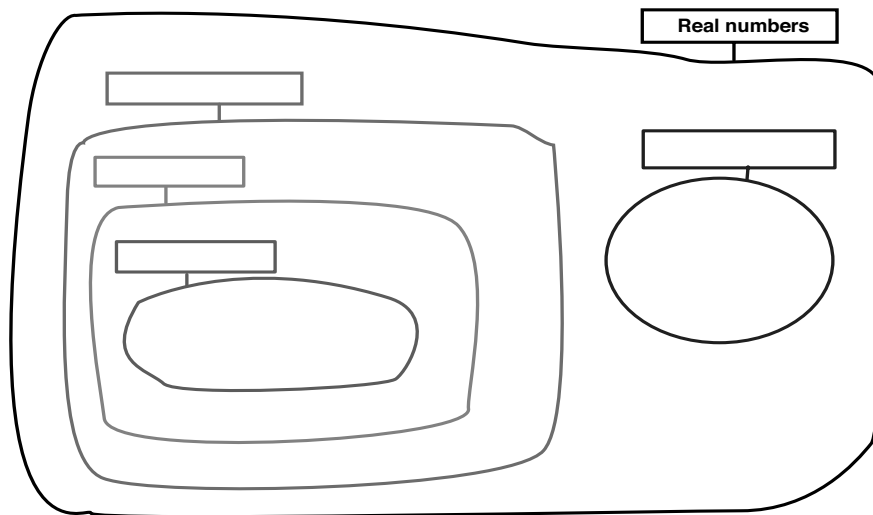
### Pages 27–30

Complete the story by reading pages 27 to 30 collectively. Let students comment on the story.



### Optional Activity

Draw the following string picture on the board.



**T** (pointing to the black string): *All of the numbers you know about are called real numbers. When you were very young, you first learned about numbers like 0, 1, 2, 10, and 100. These are called whole numbers.*

Label the red string **Whole numbers**, and put a few examples of whole numbers inside the red string.

**T:** *Then in first grade (in CSMP) you learned about negative numbers like  $\widehat{1}$ ,  $\widehat{42}$ , and  $30\ 418$ . When we combine these negative numbers and the whole numbers, we get a set of numbers called integers.*

Label the blue string **Integers**, and put some examples of negative integers inside the blue string but outside the red string.

**T:** *You next learned about fractions, decimal numbers like 6.02 and  $8.\overline{5}$ , and a few unusual numbers like  $7.\overline{6}^*$  and  $5.1\overline{462}^*$ . These numbers, along with the integers, are called rational numbers.*

Label the green string **Rational numbers**, and put some examples of non-integer rational numbers inside the green string but outside the blue string. (See the next illustration.)

**T** (pointing to the purple string): *The number we met today,  $7.515515551\dots$ , is an example of another kind of real number which is not a rational number. When we try to use decimal writing for this kind of number there is no end and no pattern that repeats. Numbers of this kind are called irrational numbers.*

Label the purple string **Irrational numbers**, and put  $7.515515551\dots$  inside the purple string.

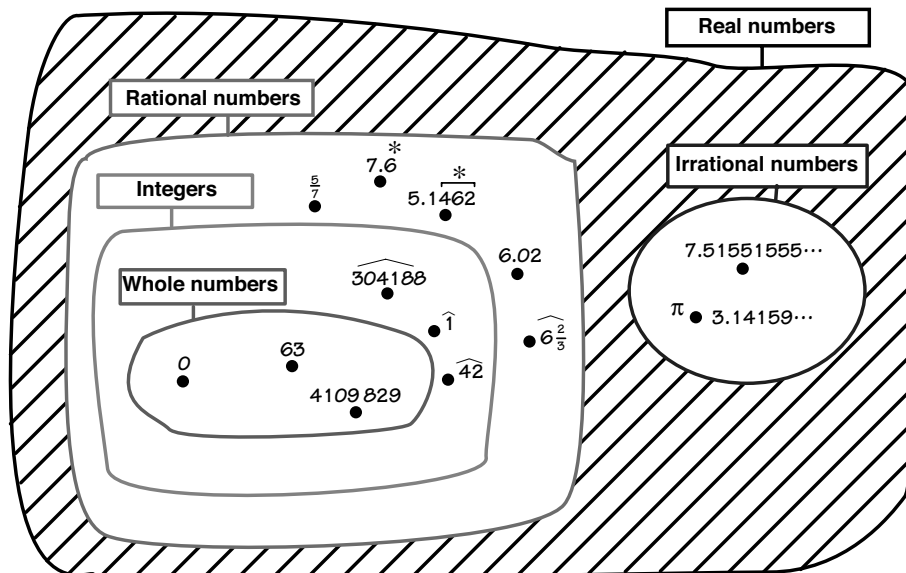
**T:** *There are infinitely many irrational numbers. You have already met one other irrational number this year. Does anyone remember another number we talked about in some geometry lessons that, when we try to use decimal writing, does not end and does not repeat?*



S:  $\pi$  (read as “pi”).

Put  $\pi$  inside the purple string. If time allows, invite students to suggest a few more examples of irrational numbers.

Hatch the indicated region of the string picture.



T (pointing to the hatching): *All real numbers are either rational numbers or irrational numbers. There are still numbers that are not real numbers,<sup>†</sup> however, you will not learn about these numbers for some time.*

<sup>†</sup>The *complex* numbers are one example of a set of numbers that contains numbers that are not real numbers.



### Capsule Lesson Summary

Compare two numbers without doing any of the indicated calculations. Begin the workbook *Collage of Problems #2*. (This is the first of two lessons using this workbook.)

#### Materials

Teacher	Student
• None	<ul style="list-style-type: none"> <li>• <i>Collage of Problems #2</i> Workbook</li> <li>• Colored pencils, pens, or crayons</li> <li>• Metric ruler</li> <li>• Compass</li> <li>• Square corner</li> <li>• Calculator</li> </ul>

### Description of Lesson

Write the following symbols on the board. Ask students to read these symbols.

<      =      >

Pose several comparison problems where you ask students to put one of these three symbols between two number expressions. (Answers are in boxes.) In each case, encourage students to make the comparisons without doing the calculations.

$1658 + 749$	<input type="text" value="•"/>	$926 + 1658$
$311 - 76$	<input type="text" value="•"/>	$311 - 47$
$\frac{1}{5}$	<input type="text" value="•"/>	$\frac{1}{20}$
$0.5$	<input "="" type="text" value="="/>	$\frac{2}{4}$
$46$	<input type="text" value="•"/>	$0.78 \times 46$

Invite students to explain their answers.

- S:**  $1658 + 749$  is less than  $926 + 1658$  because 749 is less than 926 and each number (749 and 926) is being added to 1658.
- S:**  $311 - 76$  is less than  $311 - 47$ . Each number, 76 and 47, is being subtracted from 311. Since 76 is greater than 47, we take away more if we subtract 76. So  $311 - 76$  is less than  $311 - 47$ .
- S:**  $\frac{1}{5}$  of something is more than  $\frac{1}{20}$  of the same thing, so  $\frac{1}{5}$  is greater than  $\frac{1}{20}$ .
- S:**  $0.5$  and  $\frac{2}{4}$  are both the same as  $\frac{1}{2}$ , so  $0.5$  equals  $\frac{2}{4}$ .
- S:**  $46$  is greater than  $0.78 \times 46$  because  $0.78$  is less than 1. When we multiply 46 by a number less than 1, the result is less than 46.

# W4

---

Distribute copies of the workbook *Collage of Problems #2*, and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem. Students who reach page 11 may like to skip that page temporarily as it will be discussed in Lesson W5.

At the end of the class period, collect the workbooks for your review. They will be used again in Lesson W5.

### Capsule Lesson Summary

Collectively solve a logic problem from *Collage of Problems #2*. Continue individual work in the workbook. (This is the second of two lessons using this workbook.)

#### Materials

Teacher	Student
• None	<ul style="list-style-type: none"> <li>• <i>Collage of Problems #2</i> Workbook</li> <li>• Colored pencils, pens, or crayons</li> <li>• Compass</li> <li>• Metric ruler</li> <li>• Square corner</li> <li>• Calculator</li> </ul>

### Description of Lesson

Return students' copies of the workbook *Collage of Problems #2*. Instruct everyone to turn to page 11 and to read the problem. Allow a few minutes for students to think about the problem.

**T:** *Which box contains the \$100 bill? Why?*

If during class discussion students do not arrive at a clear explanation, proceed with the following dialogue.

**T:** *Suppose that Maestro puts the \$100 bill into the Gold Box. If he does this, how many of the three statements would be true?*

**S:** *Two. The statements on the Gold Box and on the Silver Box would be true.*

**S:** *The statement of the Lead Box would be false.*

**T:** *Did Maestro put the \$100 bill in the Gold Box?*

**S:** *No, that would make two statements true, and we know that only one of the statements is true.*

In a similar manner, consider the possibilities of Maestro putting the money in the Silver Box and in the Lead Box. Conclude that Maestro put the \$100 in the Silver Box since then the only true statement is the statement on the Lead Box.

Ask students to correct or complete pages from the previous week's work, and then to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week. At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in their workbooks during a study time or to take them home as an assignment.

### Assessment Activity

An individual student progress record for the workbook is available on Blackline W5. You may like to use this form to monitor student work.



Fill in the boxes for the arrows and label the dots.

2

Find all of the pieces in this table where the entry is  $\frac{2}{2}$ . Circle the label for you.

	1	2	3	4	5	6	7	8	9
1			$\frac{2}{2}$						
2				$\frac{2}{2}$					
3					$\frac{2}{2}$				
4						$\frac{2}{2}$			
5							$\frac{2}{2}$		
6								$\frac{2}{2}$	
7									$\frac{2}{2}$
8									
9									

Did you find the other abepiece where the entry is  $\frac{2}{2}$ ? YES

3

Wipe-out  
Fill in the boxes for the arrows.

4

Fill in each box with  $<$ ,  $>$ , or  $=$ . Try to solve these problems without doing any calculation.

$782 + 149$    $782 + 188$

$621 - 17$    $621 - 84$

$567 + 138 + 495$    $316 + 567 + 138$

$71 - 25$    $73 - 27$

$0.82$    $0.135$

$0.94 \times 765$    $765$

$\frac{1}{3}$    $\frac{1}{8}$

$\frac{5}{7}$    $\frac{2}{7}$

5

Draw arrows on each number line to show the solution to each problem.

$36 \div 9 = \underline{4}$

+9

---

$4 \div \frac{1}{3} = \underline{12}$

$+\frac{1}{3}$

---

$5\frac{1}{3} \div \frac{2}{3} = \underline{8}$

$+\frac{2}{3}$

Using two lines, this circle is divided into four regions.

---

Use three lines to divide the circle into as many regions as you can.

How many regions? 7

---

Use five lines to divide the circle into as large regions as you can.

See page 20 for another style solution.

How many regions? 6

Note: To divide the circle, a line must intersect it in more than one point.

Show your work in the space provided.

Add

$4.76 + 2.8$

$$\begin{array}{r} 4.76 \\ +2.80 \\ \hline 7.56 \end{array}$$

Subtract

$4.76 - 2.8$

$$\begin{array}{r} 4.76 \\ -2.80 \\ \hline 1.96 \end{array}$$


---

Multiply

$4.76 \times 2.8$

$$\begin{array}{r} 476 \quad +100 \quad 4.76 \\ \times 28 \quad +10 \quad \times 2.8 \\ \hline 13328 \quad +1000 \quad 13328 \end{array}$$

Students' work may vary. Accept any correct method.

Circle each picture that shows a rectangle divided into at least 6 regions.

9



Complete.

$$\begin{array}{ll} 10\% \text{ of } 60 = 6 & 2\% \text{ of } 60 = 1.2 \\ 50\% \text{ of } 60 = 30 & 20\% \text{ of } 60 = 12 \\ 2.5\% \text{ of } 60 = 15 & 100\% \text{ of } 60 = 60 \end{array}$$

Use the table above to solve these problems.

$$\begin{array}{ll} 60\% \text{ of } 60 = 36 & 35\% \text{ of } 60 = 21 \\ 12.0\% \text{ of } 60 = 7.2 & 90\% \text{ of } 60 = 54 \\ 1\% \text{ of } 60 = 0.6 & 22.2\% \text{ of } 60 = 13.2 \\ 11\% \text{ of } 60 = 6.6 & 42\% \text{ of } 60 = 25.2 \end{array}$$

10

Miguelito the Magician put a \$100 bill into one of these boxes. The other two boxes are empty.



Miguelito secretly tells you that exactly one of the statements on the boxes is true.

Which box contains the \$100 bill? Silver

Which box has the true statement written on it? Lead

11

Complete these two division calculations.

$$\begin{array}{r} 133 \text{ R}19 \\ 48 \overline{) 6403} \\ \underline{- 4800} \quad 100 \\ \underline{1603} \\ \underline{- 1440} \quad 30 \\ \underline{163} \\ \underline{- 144} \quad 3 \\ 19 \end{array}$$

$$\begin{array}{r} 706 \text{ R}5 \\ 21 \overline{) 14831} \\ \underline{- 14700} \quad 700 \\ \underline{131} \\ \underline{- 126} \quad 6 \\ 5 \end{array}$$

12



SCALE: 1 cm on the map = 2 km on land

- On the map, what is the length of a line segment between Camp Kline and Moon Lake Beach? 3 cm  
What is the actual distance between Camp Kline and Moon Lake Beach? 12 km
- If a hiker walks 4 km in one hour, how long should she take to walk from Camp Kline to Moon Lake Beach? 3 hours  
From Moon Lake Beach to the Indian Mounds? 3.5 hours  
length on map = 6.5 cm, so actual distance = 13 km
- Hogback Cave is Poplar Park's km from Camp Kline and 14 km from Beaver Camp. Find the location of Hogback Cave, draw a dot, and label it.

13

Use the second number.

**Clue 1**

Use lines of the dot on this number line. Label the dots.

**Clue 2**

Use lines of these numbers.

**Clue 3**

Use lines of these numbers.

Who is Unit? 3.32

14

Timely Questions

Show your work in the space provided.

- Lloyd takes a 5 hour and 40 minute bus ride from Chicago to St. Louis to see a football game. If he leaves Chicago at 8:30 AM, what time will he arrive in St. Louis? 2:10 PM  
(Don't forget to indicate AM or PM)

$$8:30 \text{ AM} \xrightarrow{+5 \text{ hrs. } 40 \text{ mins.}} 2:10 \text{ PM}$$

- Your heart beats about 80 times per minute.  
About how many times does it beat in one day? 1,152,000  
one year? 42,048,000

$$60 \text{ min. per hr.} \times 24 \text{ hr. per day} = 1,440 \text{ min. per day}$$

$$1,440 \text{ min. per day} \times 365 \text{ days per yr.} = 525,600 \text{ min. per yr.}$$

$$1,440 \times 80 = 115,200 \quad 525,600 \times 80 = 42,048,000$$

15

Draw a quadrilateral (four-sided polygon) with at least one right angle and at least one obtuse angle.

Other solutions are possible.

16

Use and/or use second numbers.

**Clue 1**

Use and/or use in this arrow picture. Label the dots.

**Clue 2**

Who is 2a? 2 1/4      Who is 2a? 2 1/2

17

A Pair of Problems

1. There are 120 8th graders in Jackson Middle School. If there are 12 more girls than boys in the 8th grade, how many girls are there? 66 Boys? 54

$$66 + 54 = 120$$

$$66 - 54 = 12$$

2. On a summer day in Tucson, there are 9 hours more of daylight than of darkness. On that day, how many hours of daylight are there? 16½ □ darkness? 7

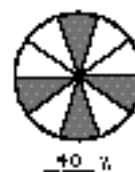
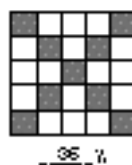
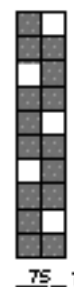
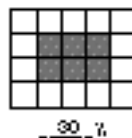
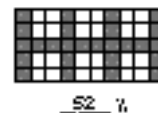
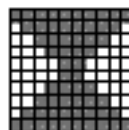
$$16\frac{1}{2} - 7\frac{1}{2} = 9$$

$$16\frac{1}{2} + 7\frac{1}{2} = 24$$

The work here shows that the given solution is correct, no how to get it. Most students will use guess-and-check techniques.

18

What percent of each shape is colored blue?



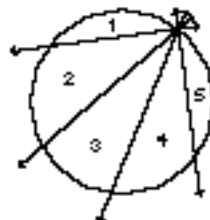
19

Use four lines to divide the circle into as many regions as you can.



How many regions? 11

Use four lines to divide the circle into as few regions as you can.

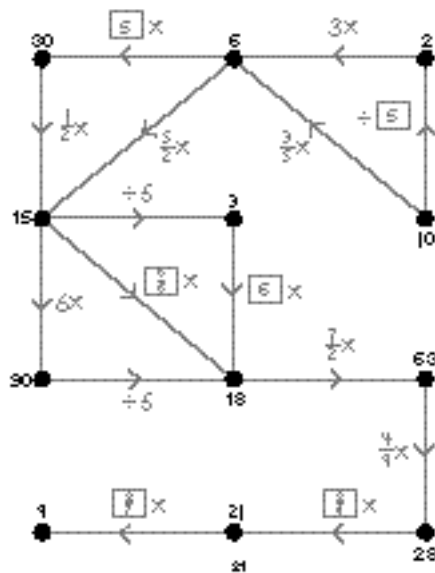


See page 7 for another style solution.

How many regions? 5

Note: To divide the circle so a line must intersect it in more than one point.


Label the dots and fill in the boxes for the arrows.



Zim has an owl number.

**Clue 1**

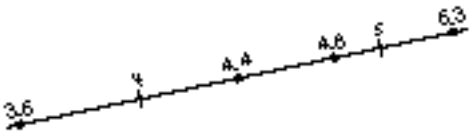
Zim can be put on the Mini computer by adding on a regular die dot.



Zim could be 3,7, 3,8, 4, 4,4, 4,6, 5,6, 7,6, or 11,6.

**Clue 2**

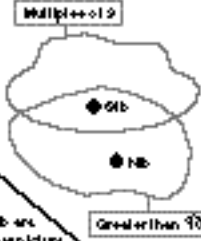
Zim lives on the dot on the number line. Label the dot. (Use a ruler)




Who is Zim? 4.4

22

OB and NB are even numbers



OB and NB are in the arrow picture.



Who is OB? 6

Who is NB? 1

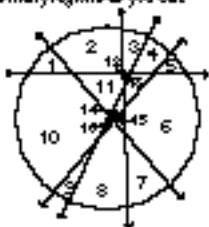
23

Ask your teacher to check your answer on page 7 and 20. Use a ruler and a protractor to fill in the boxes in the table:

Number of line segments	1	2	3	4	5	6
Maximum number of regions	2	4	7	11		

Do you see a pattern? YES. Predict the largest number of regions into which you can divide a circle using 10 line segments. 16

Check your prediction by using 10 line segments to divide the circle into as many regions as you can.

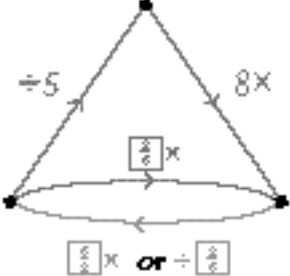


How many regions? 16. Was your prediction correct? YES

Using the same pattern, predict the largest number of regions into which you could divide a circle using 15 line segments. 22

24

Fill in the boxes or the arrows.



Complete.

$$40 \div \frac{8}{5} = 25 \quad 40 \div \frac{5}{8} = 64$$

$$56 \div \frac{8}{5} = 35 \quad 100 \div \frac{5}{8} = 160$$

$$\frac{5}{6} \div \frac{8}{5} = \frac{25}{24} \quad \frac{5}{6} \div \frac{5}{8} = \frac{40}{3} = 13\frac{1}{3}$$

25

Build an arrow road from 1.3 to 3.25. Each arrow must be for  $\times$ ,  $\div$ ,  $+$ ,  $-$ , or  $\pm$  a one-digit whole number. Use at most four arrows.

Many solutions are possible. A shortest road has three arrows. **26**

Complete the table and check the entries that belong to the blue segment.

$+$	$\frac{1}{2}$	$\frac{3}{4}$
$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
$\frac{1}{4}$	$\frac{12}{10}$	$\frac{21}{30}$
$\frac{2}{4}$	$\frac{2}{10}$	$\frac{28}{30}$

$-$	$\frac{1}{2}$	$\frac{3}{4}$
$\frac{3}{4}$	$\frac{1}{4}$	0
$\frac{1}{4}$	$\frac{2}{10}$	$\frac{1}{30}$
$\frac{2}{4}$	$\frac{1}{10}$	$\frac{7}{30}$

**27**

These are the top of two number cubes.

Figor and Ethel plays a game with these two cubes. Higher number wins. Figor rolls the red cube and Ethel rolls the blue cube.

Use this square to calculate the probability that Figor rolls the higher number.

	2	2	2	5	5	
6						
6						
3						
3						
1						
1						

What is Figor's probability of winning?  $\frac{16}{36}, \frac{4}{9}$

Who is favored in this game, Figor or Ethel? Ethel

**28**

The red label is one of these:

The blue label is one of these:

Label the strings.

**29**

**$Lm + Lon < 3$**

Shade a region on the grid to indicate all pairs that  $(Lm, Lon)$  could be.

30

It is impossible to make two of these shapes (without dust):

- A: A triangle with at least two right angles.
- B: A five-sided shape with exactly one right angle and at least two acute angles.
- C: A five-sided shape with at least four obtuse angles.
- D: A five-sided shape with the obtuse angle.

Circle the **all** one of the two shapes that are impossible to make.

(A)   B   (C)   D

Draw the other two shapes.

31

One symbol is chosen at random from each oval to form the name of a number, for example,  $\frac{2}{4} \frac{1}{2} \frac{1}{8}$ . Choose **M** so that the probability of the number being in the blue segment is exactly  $\frac{1}{2}$ .

You may use the table below to help you decide **M** could be. There are many possible choices for **M**.

+	$\frac{1}{2}$	$\frac{1}{8}$
$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{8}$
<b>M</b>	$\frac{1}{4}$	$\frac{5}{8}$

-	$\frac{1}{2}$	$\frac{1}{8}$
$\frac{2}{4}$	$\frac{2}{4}$	$\frac{12}{8}$
<b>M</b>	$\frac{1}{4}$	$\frac{1}{8}$

**M** could be  $\frac{1}{4}$ .

Choose **M** so that only one of the 32 four-digit numbers **M** is in the blue segment. For example, **M** could also be 2, or any number between  $\frac{1}{4}$  and  $\frac{1}{8}$ , or any number between  $\frac{3}{8}$  and 2.

### Capsule Lesson Summary

Count by sixths. Observe the results of successively adding  $\frac{1}{6}$ ; then use the results to add fractions with unlike denominators. Begin the workbook *Collage of Problems #3*. (This is the first of two lessons using this workbook.)

#### Materials

Teacher	Student
<ul style="list-style-type: none"> <li>• None</li> </ul>	<ul style="list-style-type: none"> <li>• <i>Collage of Problems #3</i> Workbook</li> <li>• Colored pencils, pens, or crayons</li> <li>• Compass</li> <li>• Metric ruler</li> <li>• Square corner</li> <li>• Calculator</li> </ul>

### Description of Lesson

Begin the lesson counting by sixths. Start the count at  $\frac{1}{6}$  and ask each student, in turn, to name the next fraction. Each time a student says a fraction that has a simpler name (for example,  $\frac{3}{6}$  or  $\frac{7}{6}$ ), pause, and ask for an equivalent (simpler) name (for example,  $\frac{1}{2}$  or  $1\frac{1}{6}$ ). Complete the counting after each student has participated.

Write these expressions on the board as you ask,

**T:** *What number is  $\frac{1}{6} + \frac{1}{6}$ ?*

$$\frac{1}{6} + \frac{1}{6} =$$

**S:**  $\frac{2}{6}$ .

**T:** *What is a simpler name for  $\frac{2}{6}$ ? ( $\frac{1}{3}$ )*

Complete the equation, and continue by adding another  $\frac{1}{6}$ .

**T:** *How many sixths is  $\frac{1}{3} + \frac{1}{6}$ ? Why?*

$$\frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

**S:** *Three, because  $\frac{1}{3} = \frac{2}{6}$  and  $\frac{2}{6} + \frac{1}{6} = \frac{3}{6}$ .*

$$\frac{1}{3} + \frac{1}{6} = \frac{3}{6}$$

**T:** *What is a simpler name for  $\frac{3}{6}$ ? ( $\frac{1}{2}$ )*

Continue in the same manner, following this pattern.

$$\frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

$$\frac{1}{3} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{1}{2} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

$$\frac{2}{3} + \frac{1}{6} = \frac{5}{6}$$

$$\frac{5}{6} + \frac{1}{6} = \frac{6}{6} = 1$$

# W6

---

Then pose this question.

$$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

**T:** *How many sixths is  $\frac{1}{2} + \frac{1}{3}$ ? Why?*

**S:** *Five, because  $\frac{1}{2} = \frac{3}{6}$  and  $\frac{1}{3} = \frac{2}{6}$ , so  $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$ .*

Continue with the following problems. (Answers are in boxes.)

$$\frac{5}{6} + \frac{1}{2} = \boxed{\frac{8}{6} = 1\frac{2}{6} = 1\frac{1}{3}}$$

$$\frac{2}{3} + \frac{1}{2} = \boxed{\frac{7}{6} = 1\frac{1}{6}}$$

Distribute copies of the workbook *Collage of Problems #3*, and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of the class period, collect the workbooks for your review. They will be used again in Lesson W7.



### Capsule Lesson Summary

Solve a counting problem to find the number of shortest paths from one corner to the opposite corner of a four-by-four grid square. Continue individual work in the workbook *Collage of Problems #3*. (This is the second of two lessons using this workbook.)

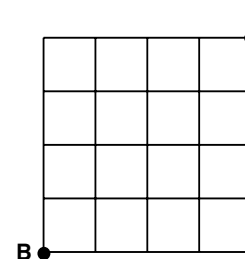
#### Materials

Teacher	Student
<ul style="list-style-type: none"> <li>• Grid board (optional)</li> </ul>	<ul style="list-style-type: none"> <li>• <i>Collage of Problems #3</i> Workbook</li> <li>• Colored pencils, pens, or crayons</li> <li>• Compass</li> <li>• Metric ruler</li> <li>• Square corner</li> <li>• Calculator</li> </ul>

### Description of Lesson

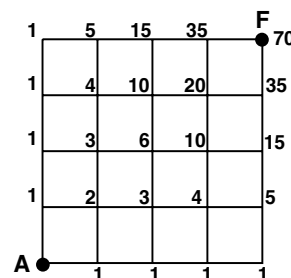
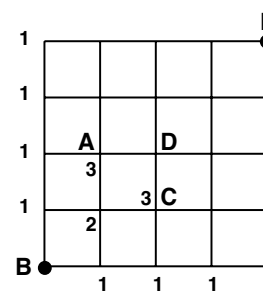
Draw a four-by-four square grid on the board, and present a counting problem to find the number of shortest paths from one corner to the opposite corner. You may put the problem in a context similar to the following.

**T:** *A bug is at B and its food is at F. How many shortest paths can it take to get the food.*



Let the class discuss how to solve this counting problem. They should recall a method used in *The Hidden Treasure* from *IG-IV* or in the “Codes” lessons in the L-strand.

- There is one way to get to any intersection directly right or up from **B**.
- There are two ways to get to the corner opposite **B** on the grid square.
- There are three ways to get to **A** (or **C**)—one through the intersection two up (or two right) from **B**, and two through the corner opposite **B** on the grid square.
- Continuing in this way, there are six ways to get to **D** and so on, as indicated in this labeled grid. There are 70 shortest paths from **B** to **F**.



Distribute students’ copies of the workbook *Collage of Problems #3*. Ask students first to correct or complete pages from the previous week’s work, and then to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

# W7

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At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in their workbooks during a study time or to take them home as an assignment.

## **Assessment Activity**

An individual student progress record for the workbook is available on Blackline W7. You may like to use this form to monitor student work.

Pair the legs.

2

Build an arrow road from 7 to 210. Each arrow must be for  $+$ ,  $-$ ,  $\times$ , or  $\div$  a one-digit whole number. Use as few arrows as you can.

Many solutions are possible.  
A shortest road has two arrows.

3

All of these calculations are correct, except that a decimal point is missing in each answer. Place a decimal point in each answer to make equations.

$$787.6 + 35.87 = 823.47$$

$$613.82 + 492.48 = 1106.3$$

$$86.15 - 12.483 = 73.667$$

$$420.36 - 381.76 = 38.6$$

$$6.2 \times 4.5 = 27.9$$

$$31.53 \times 4.97 = 156.7041$$

4

Fill in the blanks.

Many solutions are possible for the dots outside the string.

5

Fill in each box with  $<$ ,  $>$ , or  $=$ . Try to solve these problems without doing any calculation.

$736 + 587$    $587 + 426$   
 $1018 - 650$    $1018 - 200$   
 $500 + 700$    $520 + 680$   
 $82 \div 7$    $82 \div 3$   
 $70 \times 36$    $7 \times 360$   
 $58 \times 67$    $71 \times 58$   
 $\frac{2}{3}$    $\frac{2}{5}$   
 $\frac{3}{2}$    $\frac{5}{2}$

6

Fill in the boxes or the arrows.

Complete these calculations.

$\frac{4}{3} \times 12 = \underline{16}$        $\frac{2}{3} \times 20 = \underline{8}$   
 $\frac{3}{4} \times 12 = \underline{9}$        $\frac{5}{2} \times 20 = \underline{50}$

7

Use 40 arrows to build an arrow road between the two numbers in each problem. Then complete the division fact.

$31450 \xrightarrow{\div 10} 3145$   
 $31450 \div \underline{1000} = 31.45$

$570 \xrightarrow{\div 10} 57 \xrightarrow{\div 10} 5.7 \xrightarrow{\div 10} 0.57 \xrightarrow{\div 10} 0.057$   
 $570 \div \underline{10000} = 0.057$

$315 \xrightarrow{\div 10} 0.315$   
 $315 \div \underline{10} = 0.315$

$273 \xrightarrow{\div 10} 2730$   
 $27300 \div \underline{100} = 273$

8

DE is a second whole number.

**Clue 1**

is a positive divisor of  $\xrightarrow{\hspace{2cm}}$  is less than  $\xrightarrow{\hspace{2cm}}$

DE could be 6, 8, 12, or 15.

**Clue 2**

The one-digit difference between  $n$  and DE is not equal to 1.

**4 (3, 17) = 1**

Who is DE? 15

9

Fill in the boxes with one-digit numbers.

$$\frac{1}{8} + \frac{1}{8} = \frac{\boxed{2}}{8} = \frac{1}{\boxed{4}}$$

$$\frac{1}{4} + \frac{1}{8} = \frac{\boxed{3}}{8}$$

$$\frac{3}{8} + \frac{1}{8} = \frac{\boxed{4}}{8} = \frac{2}{\boxed{4}} = \frac{1}{\boxed{2}}$$

$$\frac{1}{2} + \frac{1}{8} = \frac{\boxed{5}}{8}$$

$$\frac{5}{8} + \frac{1}{8} = \frac{\boxed{6}}{8} = \frac{3}{\boxed{4}}$$

$$\frac{3}{4} + \frac{1}{8} = \frac{\boxed{7}}{8}$$

$$\frac{7}{8} + \frac{1}{8} = \frac{\boxed{8}}{8} = \boxed{1}$$

Complete:  $\frac{1}{2} + \frac{3}{8} = \frac{7}{8}$

$$\frac{1}{4} + \frac{7}{8} = \frac{9}{8} = 1\frac{1}{8}$$

$$\frac{5}{8} + \frac{1}{2} = \frac{9}{8} = 1\frac{1}{8}$$

10

The Goode are planning a 3-day camping trip for 15 people. The manual suggests they take 2 gallons of water for 4 people for 1 day. How much water should the Goode plan to take? **27 gals.**

1 day	3 days
2 gals/4 people	6 gals/4 people
8 gals/16 people	24 gals/16 people
1 gal/2 people	3 gals/2 people
9 gals/18 people	27 gals/18 people

The 15 people taking the camping trip include adults. The Goode agreed there should be 1 adult for every 5 youths. How many adults and how many youths are taking the trip? **3 adults, 15 youths**

1 adult for 5 youths
2 adults for 10 youths
3 adults for 15 youths
$3 + 15 = 18$

Suppose the Goode had not added they need 1 adult for every 3 youths. How many more adults would need to go? **2 adults**  
How much more water would they need? **3 gals.**

1 adult for 3 youths
5 adults for 15 youths

11



SCALE: 1 cm = 250 km

1. Find the length of a line segment and the actual distance between each pair of cities.

	Line Segment	Actual Distance
Los Angeles - Chicago	$\frac{11}{8}$ cm	2,750 km
Chicago - New York	$\frac{4.5}{8}$ cm	1,125 km

2. If an airplane flies 400 km in one hour, what is the flying time from Los Angeles to Chicago? (Circle the closest answer.)

3 hours    5 hours    **7 hours**    9 hours

... from Chicago to New York? (Circle the closest answer.)

$1\frac{1}{2}$  hours    **2 hours**     $3\frac{1}{2}$  hours     $4\frac{1}{2}$  hours

3. The city Truth or Consequance is 1,000 km from Los Angeles and is in the United States. Using a compass, show lines to the places where Truth or Consequance could be.

12

Fill in the boxes.

$$\widehat{5} \times 3 = \boxed{8} \times \widehat{5} = \boxed{15}$$

$$\boxed{8} \times \widehat{5} = \widehat{3} \times 5 = \boxed{15}$$

$$\widehat{3} \times 9 = \widehat{9} \times \boxed{3} = \boxed{27}$$

$$\widehat{6} \times \boxed{5} = \widehat{5} \times \boxed{6} = \widehat{30}$$

$$\widehat{6} \times \boxed{5} = \widehat{5} \times \boxed{6} = 30$$

$$4 \times \boxed{7} = \boxed{4} \times \widehat{7} = 28$$

13

Complete these two division calculations

		3 0 7 R=14
42	) 2 9 0 8	
		- 1 2 6 0 0 3 0 0
		3 0 8
		- 2 9 4
		7
		1 4

		2 3 R=12
67	) 5 5 3	
		- 1 3 4 0 2 0
		2 1 3
		- 2 0 1 3
		3
		1 2

14

Fill in the box for the arrow.

Complete

30% of 60 =

30% of 40 =

30% of 30 =

30% of  = 21

30% of  = 15

30% of  = 30

15

Put each number on the Minicomputer by adding exactly one regular die dot.

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16

1. When a printer prints the page numbers of a 100-page book, how many 7's are printed? 20

10	ones digits
+10	tens digits
20	

When a printer prints the page numbers of a 1000-page book, how many 7's are printed? 300

10 × 20 = 200	ones and tens digits
+100	hundreds digits
300	

2. Gwen and Yidd Fogar ran in the Tri-City Marathon. Gwen took 2 hours and 40 minutes to finish the race, and Yidd took 2 hours and 57 minutes.

If Gwen finished at 1:10 PM, at what time did he start? 3:00 AM

If Yidd started at the same time as Gwen, at what time did she finish? 1:27 PM

17

Using one line, this shape is divided into three regions.

Use two lines to divide this shape into as many regions as you can.

How many regions? 6

13

Wan is a secret number.

**Clue 1**

Wan is in this arrow picture.

**Clue 2**

In binary writing Wan is a four-digit number.

128	64	32	16	8	4	2	1
■	■	■	■	■	■	■	■
1	0	1	1	0	1	0	0

What is Wan's binary name? 1011010

Who is Wan? 30

19

**School Days**

There are 180 school days in a year. If you attend a 50-minute math class every school day, how many hours do you spend in math class?

150 hours

Show your work here.

$50 \times 180 = 9\,000$     9 000 minutes in math class

$9\,000 \div 60 = 150$     150 hours in math class

20

The red label is one of these:

The blue label is one of these:

Label the string.

21

All in a burglary the police question three men: Archie, Gus, and Paul. The police know that exactly one of these men is guilty. The suspects make the following statements:

Archie: Gus did it.  
 Gus: Paul did it.  
 Paul: Archie lied when he said Gus did it.

If exactly one of these statements is false, who is the guilty man? Paul

Whose statement is false? Archie's

22

Complete the calculations. Draw all the missing red arrows between the dots.

is less than

→

$$\frac{3}{4} + \frac{2}{3} = \frac{20}{20} = \frac{12}{20}$$

$$\frac{3}{4} - \frac{2}{3} = \frac{7}{20}$$

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{20} = \frac{3}{10}$$

$$\frac{3}{4} \div \frac{2}{3} = \frac{16}{20} = \frac{4}{5}$$

23

Use this shape to divide the shape into as many regions as you can.

How many regions? 10

24

B0 and O0 are word numbers.

**Clue 1**

(B0, O0) is one of the dots on this grid.

**Clue 2**

(B0, O0) is (7, 5)

25



Label the dots. One of the numbers has two names. Write both names for that number below the dot.

$\frac{2}{3} \div \frac{1}{2}$       $\frac{5}{2} + \frac{2}{3}$       $0.5 \times 2.2$   
 $1.56 - 0.89$       $\frac{1}{6} \div \frac{1}{5}$   
 $\frac{1}{8} \div 1$       $\frac{2}{5} - \frac{3}{2}$       $\frac{4}{7} \times \frac{7}{5}$

26

Build a road from 8.5 to 6 with exactly two arrows. Each arrow must be for +, -,  $\times$ , or  $\div$  a one-digit whole number.

27

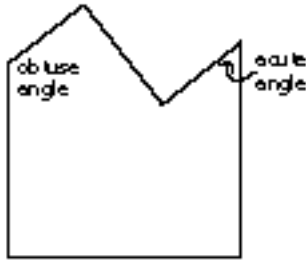
Label 11 number lines in the binary code.

28

How many shortest routes from A to B.  
Note: Routes must follow grid lines and may not go through the shaded areas.

29

Use a piece of scratch paper to make a square corner (right angle). Then construct a shape with exactly one obtuse and one acute angle.



Other shapes are possible.

30

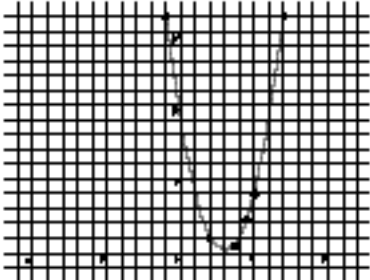
$a \rightarrow (a - 3)^2$

Fill in the blanks. Each ordered pair belongs to the red relation. One is done for you.

$(5, 4)$        $(7, 16)$        $(9, 1)$

$(1, 16)$        $(5, 4)$        $(9, 1)$

For each ordered pair, draw a dot on the grid. Sketch the Cartesian graph for this relation.




Name two more ordered pairs that belong to the red relation.

$(-2, 0)$  and  $(6, 9)$

There are many more ordered pairs in the red relation.

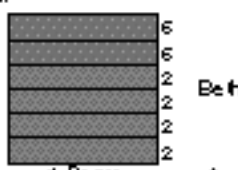
31

These are the maps of two number cubes.



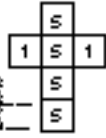
Roger rolls the red cube and Ethel rolls the blue cube. Use these maps to calculate the probability that Roger wins by rolling the high number.

Roger wins  
 Ethel wins



What is Roger's probability of winning over Ethel?  $\frac{2}{3}$

Using only the numbers 1, 3, and 5, label the cube for Ethel so that Ethel is favored over Roger, but Ethel is favored over Ethel.



What is Ethel's probability of beating Roger?  $\frac{2}{3}$

What is Ethel's probability of beating Ethel?  $\frac{2}{3}$

32

### Capsule Lesson Summary

Review the rounding operation. Begin the workbook *Collage of Problems #4*. (This is the first of two lessons using this workbook.)

#### Materials

**Teacher** • None

**Student**

- *Collage of Problems #4* Workbook
- Colored pencils, pens, or crayons
- Compass
- Metric ruler
- Calculator

### Description of Lesson

Begin the lesson by asking the class (or one student) to count by nines from  $\widehat{27}$  to 81.

**S:**  $\widehat{27}, \widehat{18}, \widehat{9}, 0, 9, 18, 27, 36, 45, 54, 63, 72, 81$ .

Write this problem on the board.

$$57 \text{ } \textcircled{R} \text{ } 9 =$$

**T:** *Do you remember the rounding operation? What is the nearest multiple of 9 to 57?* (54)

Record the answer and read the equation: 57 rounded to the nearest (multiple of) 9 is 54.

$$57 \text{ } \textcircled{R} \text{ } 9 = 54$$

Continue with these problems. (Answers are in boxes.) With the class, recall the convention that if the first number in a problem is halfway between two multiples of the second number, the result is the greater multiple.

$$30 \text{ } \textcircled{R} \text{ } 8 = \boxed{32}$$

$$18 \text{ } \textcircled{R} \text{ } 4 = \boxed{20}$$

$$1.1 \text{ } \textcircled{R} \text{ } 2 = \boxed{18}$$

$$63 \text{ } \textcircled{R} \text{ } = \boxed{63}$$

$$100 \text{ } \textcircled{R} \text{ } = \boxed{98}$$

$$100.25 \text{ } \textcircled{R} \text{ } = \boxed{98}$$

$$28 \text{ } \textcircled{R} \text{ } 3 = \boxed{2}$$

$$50 \text{ } \textcircled{R} \text{ } 6 = \boxed{48}$$

$$0.041 \text{ } \textcircled{R} \text{ } 1 = \boxed{0}$$

Ask the class (or one student) to count by tenths from 0 to 2.

**S:**  $0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2$ .

$$1.28 \text{ } \textcircled{R} \text{ } 0.1 =$$

Write this problem on the board.

**T:** *When we count by tenths starting at 0, which of the numbers is closest to 1.28?*

**S:**  $1.3$ .

$$1.28 \text{ } \textcircled{R} \text{ } 0.1 = 1.3$$

Record the answer and read the equation: 1.28 rounded to the nearest (one) tenth is 1.3.

# W8

---

Continue with these problems. (Answers are in boxes.)

$$1.15 \text{ } \textcircled{R} \text{ } 0.1 = \boxed{1.2}$$

$$2.08 \text{ } \textcircled{R} \text{ } 0.1 = \boxed{2.1}$$

$$2.02 \text{ } \textcircled{R} \text{ } 0.1 = \boxed{2}$$

$$1 .51 \text{ } \textcircled{R} \text{ } 0.1 = \boxed{1 .5}$$

$$1 .512 \text{ } \textcircled{R} \text{ } 0.1 = \boxed{1 .5}$$

$$1 .512 \text{ } \textcircled{R} \text{ } 0.01 = \boxed{1 .51}$$

Distribute copies of the workbook *Collage of Problems #4*, and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of the class period, collect the workbooks for your review. They will be used again in Lesson W9.

### Capsule Lesson Summary

Review multiplication and division with decimal numbers. Continue individual work in the workbook *Collage of Problems #4*. (This is the second of two lessons using this workbook.)

#### Materials

**Teacher** • None

**Student**

- *Collage of Problems #4* Workbook
- Colored pencils, pens, or crayons
- Compass
- Metric ruler
- Calculator

### Description of Lesson

Review multiplication and division of decimals. Suggested exercises with answers are given below.

$$\begin{array}{r} 384 \\ \times 67 \\ \hline 2688 \\ 23040 \\ \hline 25728 \end{array}$$

$$67 \times 384 = 25728$$

$$67 \times 38.4 = 2572.8$$

$$67 \times 3.84 = 257.28$$

$$67 \times 0.384 = 25.728$$

$$6.7 \times 3.84 = 25.728$$

$$0.67 \times 3.84 = 2.5728$$

$$\begin{array}{r|l} 386 & \\ 8 \overline{) 3088} & \\ - 2400 & 300 \\ \hline 688 & \\ - 640 & 80 \\ \hline 48 & \\ - 48 & 6 \\ \hline 0 & \end{array}$$

$$3088 \div 8 = 386$$

$$308.8 \div 8 = 38.6$$

$$30.88 \div 8 = 3.86$$

$$3.088 \div 8 = 0.386$$

$$0.3088 \div 8 = 0.0386$$

Distribute copies of the workbook *Collage of Problems #4*. Ask students first to correct or complete pages from the previous week's work, and then to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in their workbooks during a study time or to take them home as an assignment.

### Assessment Activity

An individual student progress record for the workbook is available on Blackline W9. You may like to use this form to monitor student work.



Dore and Fido are animal numbers.

**Clue 1**

Dore and Fido are in this arrow picture.

**Clue 2**

Dore  $\xrightarrow{+16}$  Fido

Who is Dore?  $\frac{4}{2}$       Who is Fido?  $\frac{12}{2}$

Fill in the blanks.

Many solutions are possible for the dots outside the string.

Wipe-out

Fill in the boxes for the arrows.

Fill in each box with  $<$ ,  $>$ , or  $=$ . Try to solve these problems without doing any calculation.

$55 \div 9$   $<$   $55 \div 7$

$73 \div 6$   $<$   $87 \div 6$

$23 \div 3$   $=$   $230 \div 30$

80% of 40  $>$  20

23% of 18  $<$  18

100% of 37  $=$  37

$\frac{3}{2} \times 175$   $>$  175

$\frac{2}{3} \times 175$   $<$  175

Label the gray arrows. Try labeling the dots as checks.

Hint: One has with a multiple of 5.

6

In each picture, write 1 by the vertex of the largest angle, 2 by the vertex of the next largest and so on until all of the angles are labeled. Right angles are indicated.

7

Put a one-digit number in each box to make the calculation correct.

$$\begin{array}{r} \boxed{5}.7 \\ 30.\boxed{3}8 \\ 7.09 \\ + 26.5\boxed{6} \\ \hline \boxed{6}9.73 \end{array}$$

$$\begin{array}{r} \boxed{8}9.60 \\ - 2\boxed{5}.\boxed{8}3 \\ \hline 63.7\boxed{7} \end{array}$$


---

Add

$$\begin{array}{r} 163.7 \\ 98.0 \\ + 84.53 \\ \hline 286.23 \\ \hline 286.23 \end{array}$$

Subtract

$$\begin{array}{r} 803.42 \\ - 539.6 \\ \hline 263.82 \\ \hline 263.82 \end{array}$$

8



The red string is for one of these relations.

Label the string.

$(200, 20)$

$(72, 12)$

$(72, 35)$

10

Use the rounding operation  $\textcircled{R}$  to complete these problems. The first two are done for you.

$11 \textcircled{R} 3 = \underline{12}$  The nearest multiple of 2 to 11 is 12.

$4.3 \textcircled{R} 0.5 = \underline{4.5}$  When counting by 0.5 from 0, the decimal we get to 4.2 is 4.5.

$17 \textcircled{R} 4 = \underline{16}$

$60 \textcircled{R} 7 = \underline{63}$

$38 \textcircled{R} 5 = \underline{40}$

$172 \textcircled{R} 20 = \underline{180}$

$283 \textcircled{R} 10 = \underline{280}$

$4\,527 \textcircled{R} 10 = \underline{4\,530}$

$283 \textcircled{R} 100 = \underline{300}$

$4\,527 \textcircled{R} 100 = \underline{4\,600}$

$13.58 \textcircled{R} 1 = \underline{14}$

$7.66 \textcircled{R} 0.1 = \underline{7.7}$

11

Fee and Gee are real numbers.

Clue 1

$(Fee, Gee)$  is one of the dots on this grid.

Clue 2

$Fee + Gee = 4$

On the grid, check the coordinates  $(Fee, Gee)$  could be.

Clue 3

Intersect them

$(Fee, Gee) = (\underline{2}, \underline{2})$

12

One number in each calculation is missing a decimal point. Place a decimal point in this number to make the equation true.

$376 + 14.29 + 265.8 = 656.09$

$408.27 + 255.68 + 3862.75 = 4526.7$

$3461.3 - 512.76 = 2948.54$

$7.4 \times 53.27 = 394.198$

$3135 \times 8.06 = 252.681$

$648.69 \div 2.1 = 3089$

$83.3 \div 19.6 = 4.25$

13

Using three straight lines (lines), this disk is divided so that each dog house is in a different region.

Use five straight lines (lines) to divide this disk so that each dog house is in a different region.

There are other ways to draw the three lines.

Complete the calculations. Show your work.

$\begin{array}{r} 7.36 \\ \times 1.3 \\ \hline 2.208 \\ 7.36 \\ \hline 9.568 \end{array}$	$97.2 \times 6.7$ $\begin{array}{r} 97.2 \\ \times 6.7 \\ \hline 6804 \\ 5832 \\ \hline 651.24 \end{array}$
$\begin{array}{r} 85.2 \\ 7 \overline{)596.4} \\ -560 \\ \hline 36.4 \\ -35 \\ \hline 1.4 \\ -1.4 \\ \hline 0.2 \end{array}$	$2356.2 \div 63$ $\begin{array}{r} 37.4 \\ 63 \overline{)2356.2} \\ -1890 \\ \hline 466.2 \\ -441 \\ \hline 25.2 \\ -25.2 \\ \hline 0.4 \end{array}$

Pair each red leg with a blue leg.

$\frac{5}{8} + \frac{3}{8}$	$\frac{3}{10}$
$\frac{5}{8} - \frac{3}{8}$	$\frac{1}{10}$
$\frac{3}{5} + \frac{1}{2}$	$\frac{1}{4}$
$\frac{3}{5} - \frac{1}{2}$	$\frac{6}{5}$
$\frac{3}{5} \times \frac{1}{2}$	1
$\frac{3}{5} \div \frac{1}{2}$	$\frac{1}{10}$

Alice, Brenda, and Carl are all different ages. They make the following statements:

Alice: I am older than Carl.  
 Brenda: Carl is not the oldest.  
 Carl: I am older than Brenda.

Problem 1: If all three statements are true, list the three people from youngest to oldest.

Brenda      Carl      Alice  
 youngest      oldest

---

Problem 2: If exactly one of the statements is false, who is the youngest? Carl.

Whose statement is false? Carl's.

---

Problem 3: If the only true statement is Brenda's, list the three people from youngest to oldest.

Alice      Carl      Brenda  
 youngest      oldest

Draw as many red arrows as possible in the picture.

is less than  $\rightarrow$

18

include 4 miles 60 km  
Complete.

	Length of Line Segment on Map	Actual Distance by Road
Des Moines to Des Moines	5 cm	250 km
Des Moines to Omaha	4 cm	200 km
Omaha to Iowa City	2.8 cm	140 km

Mr. Pullin drove from Omaha to Des Moines on I-80. If his average is 90 km per hour, about how long to his drive?  
2.8 hours

Mr. Pullin's car uses one liter of gasoline every 10 km. The gasoline costs 40.50 for one liter. How much gasoline does she use on her trip from Omaha to Des Moines? 2.8 liter  
How much does that much gasoline cost? \$113.50

By air, Oskaloosa is 250 km from Omaha and 136 km from Des Moines. Draw and label a dot for Oskaloosa in Iowa. You may use a compass.

19

Label the dots following the rule.

Two numbers can be joined by a red cord if and only if they are relatively prime.

With your way of labeling the dots, are there more red cords that can be drawn in a picture? 12.5. If so, draw them. Other solutions give different possibilities for additional red cords.

Label the dots in the picture so that no more red cords can be drawn.

20  
Many solutions are possible.

The red string is for one of these relations.

The blue string is for one of these relations.

Label the strings.

21

**Two at a Time**

1. Pamela is 14 years older than her brother Andrew. She also is three times as old as Andrew. How old is Pamela? 21  
Andrew? 7

Pamela's age must be a multiple of 3 greater than 14.

2. Only one-third of the students in Mr. Oursler's class are boys. If there are nine boys in his class, how many students are in his class? 27

22

Fill in a word number.

**Class 1**

Fill in the arrow picture.

**Class 2**

Who is **PI**? 1.25 (or 5/4)

25

Mr. and Mrs. Harle and their two children, Audrey and Glen, must cross a river. The only boat available is a small canoe which can hold at most one adult or the two children. All of the Harles know who will paddle the canoe. Describe how the family can use the canoe to cross the river. The alligators in the river prevent anyone from returning. Try to use as few trips as possible.

About each arrow, indicate who is in the boat. You may not need to use all of the arrows.

First trip across → Audrey, Glen	Fourth trip across → Mrs. Harle
First return trip ← Audrey	Fourth return trip ← Glen
Second trip across → Mr. Harle	Fifth trip across → Audrey, Glen
Second return trip ← Glen	Fifth return trip ←
Third trip across → Audrey, Glen	Sixth trip across →
Third return trip ← Audrey	Sixth return trip ←
	Seventh trip across →

24

There are other minimal solutions.

The dots are for ordered pairs of numbers. Label the dots. One dot is for you. Many solutions are possible. Hint: Some ordered pairs have negative numbers.

25

Fill in the boxes for the arrows.

26

Use the straight lines (lines) to divide this circle so that each dog house falls in a different region.

There are other ways to draw the three lines.

27

Test scores for Mr. Doctor's class are pictured below. Find the mean, median, and mode for the class. Which type of average (mean, median, or mode) best describes the class's performance on the test? Explain your answer.

Mean 78.8

Median 85

Mode 100

Answers may vary.  
In this case the median may best describe the class's performance, because the scores appear to be mostly from 70 to 100, and 85 is the middle. The low scores appear to bring the mean average down too much compared to the rest of the class.

28

Put the numbers in the Venn picture.

$\frac{1}{2} + \frac{3}{4}$        $\frac{3}{5} \times \frac{5}{4}$        $2 \times \frac{5}{6}$

25% of 10      20% of 7

$\frac{7}{8} - \frac{1}{4}$        $\frac{1}{8} \div \frac{1}{7}$        $\frac{2}{5} \div \frac{4}{5}$

$1 + \frac{2}{3}$        $2 - \frac{2}{3}$

29

Give a second number between 2 and 30.

Positive prime numbers      Multiples of 6

---

Who is Gem? 13      30

What could the red arrow be? Start With the ordered pair (a, b). Divide a by b. End With the ordered pair whose first component is the quotient and second component is the remainder.

Observe yours, and then fill in the blank in the arrow picture.

Many rule descriptions are possible.

Gem and Tam are two-dimensional numbers.

Class 1

$2 \times n \times 2 \times n$  is a possible divisor of

(Gem, Tam) could be (2, 2 x 5), (2, 2 x 5^2), (5, 2^2), (5, 2 x 5), (5, 2 x 5^2), (2 x 5, 2 x 5^2), (2 x 5, 2 x 5^2), (1, 2), (1, 2 x 5), (1, 2 x 5^2), (1, 5), (1, 5^2)

Hint: There are between 10 and 15 possibilities for (Gem, Tam)

Class 2

$$\frac{1}{5 \times \text{Gem}} + \frac{1}{7 \times \text{Tam}} < \frac{1}{4}$$

(Gem, Tam) could be (5, 5^2), (5, 2 x 5^2), (5, 2 x 5^3), (2 x 5, 2 x 5^2)

Class 3

$$5 \text{Gem}^3 > 7 \text{Tam}^2$$

(Gem, Tam) is (5^2, 2 x 5^2)

### Capsule Lesson Summary

Given a set of fractions, identify each as being between two whole numbers, and locate the fractions on a number line. Begin the workbook *Collage of Problems #5*. (This is the first of two lessons using this workbook.)

#### Materials

**Teacher** • Meter stick

**Student**

- *Collage of Problems #5* Workbook
- Colored pencils, pens, or crayons
- Protractor
- Metric ruler
- Calculator

### Description of Lesson

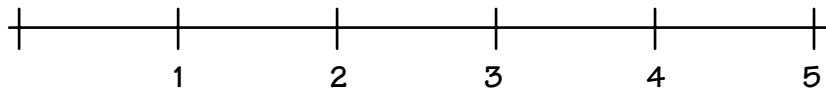
Write these five fractions on the board.

$$\frac{5}{6} \quad \frac{7}{2} \quad \frac{13}{3} \quad \frac{23}{8} \quad \frac{23}{18}$$

**T:** *Of these five numbers, which is least? Greatest?*

Let students express their opinions, but do not acknowledge which answers or explanations are correct at this time.

Write this information on the board.



$$< \square < 1 < \square < 2 < \square < 3 < \square < 4 < \square < 5$$

**T:** *There are five fractions listed on the board and there are five boxes. Exactly one of the fractions is between 0 and 1, another is between 1 and 2, and so on. Select a fraction and tell us which box it belongs in.*

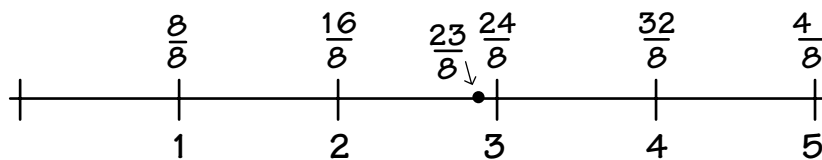
Let students consider the fractions in any order. For example:

**S:**  $\frac{23}{8}$  is between 2 and 3, since  $\frac{16}{8} = 2$  and  $\frac{24}{8} = 3$ .

**S:**  $\frac{23}{8} = 2\frac{7}{8}$ .

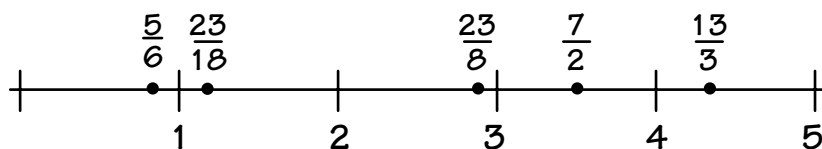
# W10

On the number line, give eighths labels to the whole numbers, as shown below. Invite a student to find the approximate location of  $\frac{23}{8}$  on the number line and to fill in the appropriate box.



$$< \boxed{\phantom{00}} < 1 < \boxed{\phantom{00}} < 2 < \boxed{\frac{23}{8}} < 3 < \boxed{\phantom{00}} < 4 < \boxed{\phantom{00}} < 5$$

Erase the eighths labels, except  $\frac{23}{8}$ , from the number line. Let students locate the other four fractions in a similar manner.



$$< \boxed{\frac{5}{6}} < 1 < \boxed{\frac{23}{18}} < 2 < \boxed{\frac{23}{8}} < 3 < \boxed{\frac{7}{2}} < 4 < \boxed{\frac{13}{3}} < 5$$

Focus attention on the least,  $\frac{5}{6}$ , and the greatest,  $\frac{13}{3}$ , of the fractional numbers to note which of the students earlier answers and explanations were correct.

Distribute copies of the workbook *Collage of Problems #5*, and let students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of the class period, collect the workbooks for your review. They will be used again in Lesson W11.



### Capsule Lesson Summary

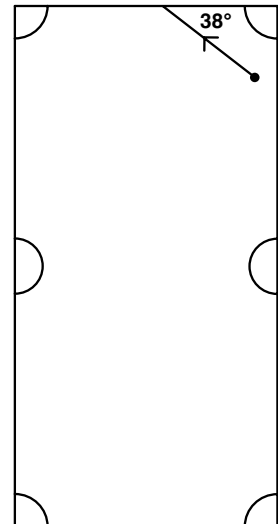
Measure angles to determine the path of a billiard ball on a pool table. Continue individual work in the workbook *Collage of Problems #5*. (This is the second of two lessons using this workbook.)

#### Materials

- |   |  |
|---|--|
| <b>Teacher</b> <ul style="list-style-type: none"> <li>• Demonstration protractor</li> <li>• Straightedge</li> </ul> | <b>Student</b> <ul style="list-style-type: none"> <li>• <i>Collage of Problems #5</i> Workbook</li> <li>• Colored pencils, pens, or crayons</li> <li>• Protractor</li> <li>• Metric ruler</li> <li>• Calculator</li> </ul> |
|---|--|

### Description of Lesson

Draw this picture on the board. A 50 cm by 100 cm rectangle is a good size. Draw the indicated ball path at an angle of  $38^\circ$  to the edge. The exact starting location for the ball is not crucial.



**T:** *This is a picture of a pool table. If I hit the pool ball (without a sideways spin) in this direction, which direction will it go after it hits the edge?*

On the board, let students trace the path they think the ball will travel.

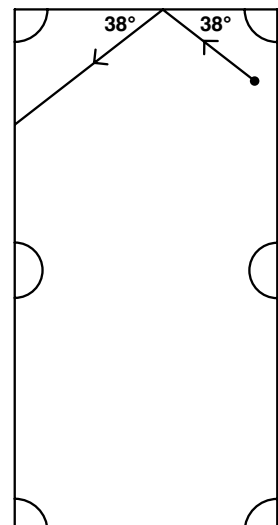
Point to the  $38^\circ$  angle you have drawn.

**T:** *To know more exactly how the ball will travel, we need to measure this angle.*

**S:** *It is a  $38^\circ$  angle.*

**T:** *If this is a  $38^\circ$  angle, the ball will also leave the edge at a  $38^\circ$  angle.*

Carefully measure a  $38^\circ$  angle, and draw the deflecting path in the picture.

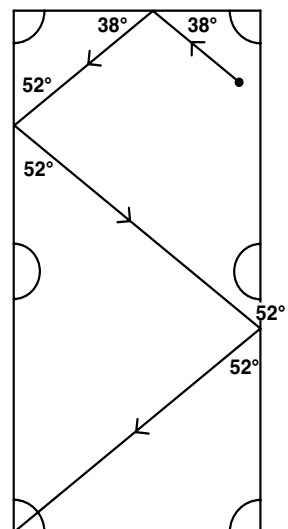


**T:** *Where will the ball travel after it hits the second edge?*

Lead students to measure the appropriate angles, and add this information to the picture.

# W11

Continue in a similar manner until the ball hits a pocket, or declare that the ball stops after two or three more rebounds. For example:



Distribute students' copies of the workbook *Collage of Problems #5*. Ask students first to correct or complete pages from the previous week's work, and then to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in their workbooks during a study time or to take them home as an assignment.

## Assessment Activity

An individual student progress record for the workbook is available on Blackline W11. You may like to use this form to monitor student work.

For each problem, list at least four whole numbers that could be in the box.

15  $\xrightarrow{+ \square - \dots}$  35  
 $\frac{10}{\quad} \frac{2}{\quad} \frac{5}{\quad} \frac{8}{\quad}$   
 also 1, 4, 20, 40

15  $\xrightarrow{- \square - \dots}$  -15  
 $\frac{4}{\quad} \frac{7}{\quad} \frac{28}{\quad} \frac{1}{\quad}$   
 also 2, 14

-10  $\xrightarrow{+ \square - \dots}$  35  
 $\frac{5}{\quad} \frac{9}{\quad} \frac{3}{\quad} \frac{15}{\quad}$   
 also 1, 15

Complete the calculations. Show your work.

$\begin{array}{r} 8.70 \\ 0.96 \\ +0.40 \\ \hline 10.06 \end{array}$	$0.08 + 7.153 + 64$ $\begin{array}{r} 0.080 \\ 7.153 \\ + 64.000 \\ \hline 71.233 \end{array}$
$\begin{array}{r} 65.70 \\ - 8.46 \\ \hline 57.24 \end{array}$	$326 - 89.7$ $\begin{array}{r} 326.0 \\ - 89.7 \\ \hline 236.3 \end{array}$

Place these ordered pairs in the right picture.

(21, 7)    (42, 12)    (40, 8)  
 (25, 8)    (100, 5)    (29, 9)

is smaller

(25, 8)    (100, 5)    (21, 7)    (40, 8)    (29, 9)    (42, 12)

is larger

Build an arrow road from 3.5 to 26. Each arrow must be for +, -,  $\times$ , or  $\div$  a one-digit whole number. Use as few arrows as you can.

3.5  $\xrightarrow{\times 8}$  28  $\xrightarrow{+ 3}$  65  $\xrightarrow{\times 4}$  26

Other solutions are possible.

Probo is a special number.

**Clue 1**

Probo is in this arrow picture. Label the dots.

**Clue 2**

It is less than

Who is Probo? 60

Fill in the boxes with these numbers.

$$\frac{8}{3} \quad \frac{9}{2} \quad \frac{5}{8}$$

$$\frac{5}{11} \quad \frac{13}{11} \quad \frac{37}{10}$$

$$0 < \boxed{\frac{5}{11}} < \frac{1}{2} < \boxed{\frac{5}{8}} < 1 < \boxed{\frac{13}{11}} < 2$$

$$2 < \boxed{\frac{8}{3}} < 3 < \boxed{\frac{37}{10}} < 4 < \boxed{\frac{9}{2}} < 5$$

7

Fill in the boxes for the arrows.

Complete.

24% of 60 = <u>14.4</u>	24% of 250 = <u>60</u>
124% of 60 = <u>74.4</u>	24% of 150 = <u>36</u>
48% of 60 = <u>28.8</u>	24% of 36 = <u>8.64</u>

8

A farmer must carry a fox, a goose, and a basket of cabbage across a river. The farmer's boat is big enough for the farmer and only one item. The farmer can't leave the goose and cabbage together because the goose will eat the cabbage. Also, the fox and goose cannot be left together (for obvious reasons). Decide how the farmer can get all this stuff across the river safely. Try to use as few trips as possible.

Above each arrow, write what the farmer carries. You may not need all the arrows.

First trip across	→	goose
First return trip	←	
Second trip across	→	fox
Second return trip	←	goose
Third trip across	→	cabbage
Third return trip	←	
Fourth trip across	→	goose
Fourth return trip	←	
Fifth trip across	→	

Another minimal solution is possible: interchange the fox and cabbage trips.

Complete.

$$60 \times \underline{7} = 420 \qquad 60 \times \underline{0.7} = 42$$

$$60 \times \underline{700} = 42\,000 \qquad 60 \times \underline{0.007} = 0.42$$


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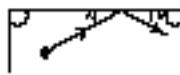
Complete the calculation and show your work.

<p>Multiply.</p> $37.6 \times 0.06$ $\begin{array}{r} 37.6 \\ \times 0.06 \\ \hline 2256 \\ \div 1000 \\ \hline 2.256 \end{array}$	<p>Multiply.</p> $63.8 \times 1.49$ $\begin{array}{r} 63.8 \\ \times 1.49 \\ \hline 5862 \\ \times 638 \\ \hline 95062 \\ \div 1000 \\ \hline 95.062 \end{array}$
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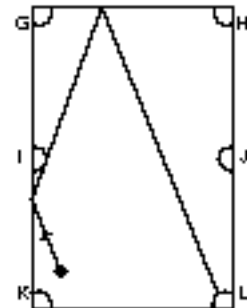
10

Students' work may vary. Accept any correct method.

Agaron Billiard Hall



If a ball is hit with no spin, angle  $B$  will always equal angle  $A$ . Use this fact to draw the path of the ball in the following picture of a pool table.



Which pocket does the ball enter? L

11

Write each number as a product of positive prime numbers. One is done for you.

$$280 = \underline{2 \times 2 \times 2 \times 5 \times 7} = 2^3 \times 5 \times 7$$

$$\textcircled{42} = \underline{2 \times 3 \times 7}$$

$$\textcircled{90} = \underline{2 \times 3^2 \times 5}$$

$$275 = \underline{5^2 \times 11}$$

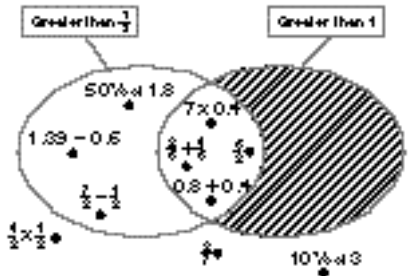
$$392 = \underline{2^3 \times 7^2}$$

Circle the two numbers above that are divisors of  $2 \times 3^2 \times 5^2 \times 7^2$ .

12

Put each of these numbers in the Venn picture.

$\frac{5}{3}$	$\frac{3}{7}$	$\frac{3}{5} + \frac{4}{5}$	$\frac{7}{8} - \frac{1}{8}$
$\frac{4}{3} \times \frac{1}{3}$	50% of 1.8	10% of 3	
$0.8 + 0.4$	$1.39 - 0.6$	$7 \times 0.4$	



One region of the picture can be hatched. Hatch it.

13

The red string is for one of these relations.

The blue string is for one of these relations.

Label the strings.

Write three division problems with the same answer as the division problem.

$$18 \div 2.5$$

$$180 \div 25$$

$$1.8 \div 0.25$$

$$72 \div 10$$

Complete these calculations. You may change each problem into a problem with the same answer. Show your work.

Divide

$$18 \div 2.5$$

$$36 \div 5 = 7.2$$

or

$$72 \div 10 = 7.2$$

Answer:  ,

Divide

$$43.586 \div 0.62$$

$$4358.6 \div 62$$

Answer:   ,

In each case, there are many division problems with the same answer.

Use the arrow picture to help solve each problem. Label the dollar and amount, then answer the questions.

1. Norma buys a walk-behind mower for \$42 and then pays 6% tax. She also pays \$252 for the mower. What is her total bill?

How much tax does Norma pay? \$25.2  
 What is her total bill? \$327.2

2. Mr. Hummel buys a bicycle for her son. With a 5% tax, she pays \$450 for the bicycle. How much does the bicycle cost (without tax)? What is her total bill?

How much does the bicycle cost (without tax)? \$390  
 What is her total bill? \$394.50

Put each number on the base 2 abacus. Write the base 2 name for each number. One is done for you.

Decimal Name      Base 2 Name

1 = = 1

2 = = 10

3 = = 11

4 = = 100

5 = = 101

6 = = 110

7 = = 111

8 = = 1000

Lelida enjoys running on the hiking trail (blue) from her home to the beach. Her brother, Tony, pretends riding his bicycle around the lake along the route marked in red.

Complete.

	Length of route on map	Actual distance traveled
Lelida (blue)	8 cm	4 km
Tony (red)	24 cm	12 km

Lelida runs an average of 10 km in one hour and Tony rides 20 km in one hour. About how long does Lelida take to run on the hiking trail from her home to the beach? 24 minutes

About how long does Tony take to ride his bicycle on the road from her home to the beach? 51 minutes

18

(L1, M1) is a second ordered pair of numbers.

Clue 1

L1 could be 8 and M1 could be 10

L1 could be 9 and M1 could be 7

L1 could be 2 and M1 could be 4

L1 could be 9 and M1 could be 1

L1 could be 12 and M1 could be 2

L1 could be 17 and M1 could be 5

L1 could be 22 and M1 could be 8

Clue 2

(L1, M1) is one of the dots on the grid.

(L1, M1) = (9, 5)

19

Zun is a second number.

Clue 1

Zun is in the arrow picture.

Clue 2

Zun can be put on the Mini-computer by adding just a 0 to the dot.

Who is Zun? 6.43

20

**EARL'S**  
**Parking Lot**

---

\$0.30 for each  $\frac{1}{4}$  hour

How much does it cost to park a car for 1 hour? \$1.20

10 hours? \$3.00

About how long will a car parked if the charge is \$0.30? 1 hour

\$5.00 21 hours

Sharon parked her car from 11:20 AM to 1:20 PM. How much must she pay? \$2.40

Douglas parked his car from 5:45 PM to 7:00 PM. How much must he pay? \$3.15

Orlando parked his car at 7:50 PM. When he left the lot, he had to pay \$0.60. About what time did he leave the lot? 9:20 PM

21

The dots are for possible numbers.  
Label the dots and fill in the boxes for the blue arrows.

In the square of  $\rightarrow$

22

Place these numbers in the Venn picture.

$2 \times 3^2$	$11^2$	$3^2 \times 11$
$11^2$	$3^4 \times 11$	$3^3 \times 11$
$3^4$	$3^5$	$7 \times 11^2$

23

For each problem, fill at least three whole numbers that could be in the box.

also 1, 3, 4, 8, and 12

also 1, 2, and 6

also 1, 4, and 5

$$a * b = \left(\frac{1}{2} \times a\right) - b$$

Example:  $6 * 5 = \left(\frac{1}{2} \times 6\right) - 5 = 3 - 5 = \hat{2}$

Complete.

$12 * 3 = \boxed{3}$	$2.5 * 0.5 = \boxed{0.75}$
$23 * 8 = \boxed{35}$	$7 * 5 = \boxed{15}$
$10 * \boxed{35} = 1.5$	$\boxed{30} * 7 = 8$
$26 * \boxed{10} = 3$	$\boxed{34} * 10 = 7$
$\frac{2}{3} * \frac{1}{4} = \boxed{\frac{1}{12}}$	$\frac{4}{5} * \boxed{\frac{2}{5}} = \hat{\frac{1}{5}}$

25



Label the dots and fill in the boxes for the arrows.

$3 \times 8 = 24$   
 $2 \times 16 = 32$

$24 + 96 = 120$      $(3 \times 24) + (2 \times 96) = 72 + 192 = 264$

How many adults came to the movie? 24  
 How many children? 96

26

1. Toby is now twice as old as Leo. In eight years, Toby will be only twice as old as Leo. How old is Toby now? 24  
 Leo's age 8    Toby's Leo's age 8

$3 \times 8 = 24$   
 $2 \times 16 = 32$

2.

**State Theater**  
**How playing "The Wild and Crazy Guys!"**  
 Adults: \$3.00  
 Children: \$2.00

Theater Owner: How many customers did we have tonight?  
 Tidal: Giller: Only 80.  
 Owner: How many adults?  
 Giller: I forgot to count how many.  
 Owner: Do you know how much money we received?  
 Giller: Yes, \$134.  
 Owner: Good! Then I can calculate how many adults came.

$24 + 96 = 120$      $(3 \times 24) + (2 \times 96) = 72 + 192 = 264$

How many adults came to the movie? 24  
 How many children? 96

The Work here shows that the given solution is correct, not how to get it. Most students will use guess-and-check technique.

27

Label the dots and fill in the boxes for the arrows.

28

The red label is one of these:

- Multiple of 3
- Multiple of 4
- Multiple of 6
- Multiple of 8
- Positive prime numbers
- Negative 10
- Positive multiples of 12
- Positive divisors of 24
- Positive multiples of 40

The blue label is one of these:


- Multiple of 3
- Multiple of 4
- Multiple of 6
- Odd numbers
- Positive prime numbers
- Negative 10
- Positive multiples of 12
- Positive multiples of 24
- Positive multiples of 40

Label the strings. Label the dots.

Positive divisors of 24    Odd numbers

29

Holly and Molly are even numbers.




Holly can be shown on the Mini computer by adding one 10's dot.

Molly can be shown on the Mini computer by taking away one 10's dot.


Holly: 5, 47, 41, 29,  
23, 07, 67, 87

Molly: 43, 53, 27

is greater than



Holly + Molly is in the blue segment of this number line.



(Holly, Molly) is (0, 10, 20)

30

Find the smallest positive integer  $N$  such that the product  $N \times 135$  is a square number.

$$135 = 3^3 \times 5$$

square  $3^4 \times 5^2 = (3^2 \times 5)^2$

$$N = 3 \times 5 = 15$$

$$15 \times 135 = 2025 = 45^2$$

Find the greatest square number that has a divisor of 64800.

$$64800 = 2^4 \times 3^4 \times 5^2$$

greatest square divisor  $2^4 \times 3^4 \times 5^2 = (2^2 \times 3^2 \times 5)^2$

$$32400 = 180^2$$

How many positive integers less than 100 have an odd number of positive divisors? (Nine; 1, 4, 9, 16, 25, 36, 49, 64, and 81 have 1 divisor)

Squares of primes ( $p^2$ ) have 3 divisors (4, 9, 25, 49)

Primes to 4th power ( $p^4$ ) have 5 divisors (16, 81)

Primes to 8th power ( $p^8$ ) have 7 divisors (64)

Primes to 8th power ( $p^8$ ) or the product of two different primes squared ( $p^2 \times q^2$ ) have 9 divisors (36)

The product of two whole numbers is 1 000 000. If neither of the two whole numbers is a multiple of 10, what numbers are they?  $2^4 (64)$  and  $5^4 (625)$

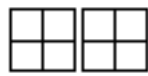
$$1\,000\,000 = 10^4 = 2^4 \times 5^4 = 64 \times 625$$

31

Gyles' even number.

Clue 1

Gyles can be put on the Mini computer using exactly one of these 4 dots: 10, 10.

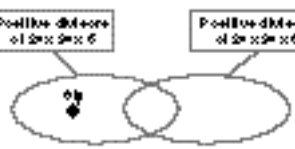


Gyles could be 3, 5, 12, 24, 30, 60, 120, 240, 3, 18, 36, 72, 90, 180, 360, 720

Clue 2

Possible divisors of  $2m \times 3m \times 6$

Possible divisors of  $2n \times 3n \times 6$



Gyles could be  $2^2 \times 3$ ,  $2 \times 3 \times 5$ ,  $2^2 \times 3^2$ , or  $2 \times 3^2 \times 5$

(24) (30) (72) (360)

Clue 3

Gyles  $\perp 14 = 2^3 \times 3^2 \times 7$

Who is Gyles?  $2^2 \times 3^2 = 72$

32

**Capsule Lesson Summary**

Use an arrow picture to first determine how much a person earns on commission when the sales are given. Then use the picture to determine the sales when the amount earned is given. Begin the workbook *Collage of Problems #6*. (This is the first of two lessons using this workbook.)

**Materials**

<b>Teacher</b>	• Colored chalk	<b>Student</b>	<ul style="list-style-type: none"> <li>• <i>Collage of Problems #6</i> Workbook</li> <li>• Colored pencils, pens, or crayons</li> <li>• Compass</li> <li>• Metric ruler</li> <li>• Protractor</li> <li>• Calculator</li> </ul>
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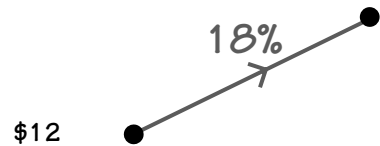
**Description of Lesson**

Draw a red arrow on the board, and label it “18%.”

**T:** *Mr. Carfinkle works on commission. He earns 18% of the sales he makes. One month his sales are \$12 000. Where should we put \$12 000 in this arrow picture?*

**S:** *At the starting dot of the arrow.*

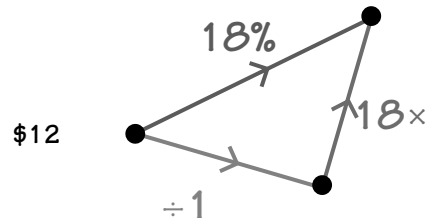
Label the appropriate dot.



**T:** *Can you suggest a detour for this arrow that will help us determine how much Mr. Carfinkle will earn this month?*

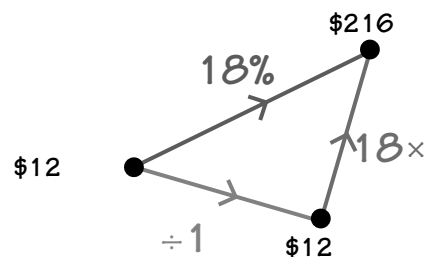
**S:** *18% is the same as  $\div 100$  followed by  $18\times$  (or  $18\times$  followed by  $\div 100$ ).*

Add arrows to the picture according to a student’s suggestion, as illustrated here.



Let students tell you how to label the dots in the arrow picture. Find that Mr. Carfinkle earns \$2 160 this month.

Erase all the dot labels.



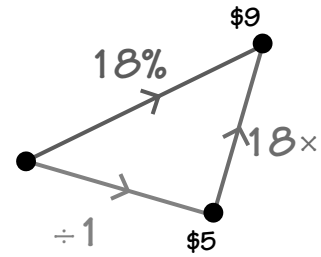
# W12

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**T:** *Mr. Carfinkle was on vacation for part of a month, but still he earned \$900. How much were his sales during this month? Where should we put \$900 in this arrow picture?*

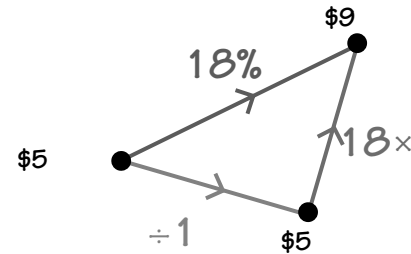
**S:** *At the ending dot of the 18% arrow.*

**T:** *Try to determine Mr. Carfinkle's sales.*



Allow a few minutes for individual work; then collectively discuss the problem.

Let students tell you how to label the dots in the arrow picture. Point out the use of return arrows to follow the detour in reverse. Find that Mr. Carfinkle's sales were \$5 000.



Distribute copies of the workbook *Collage of Problems #6*, and let the students work independently for the rest of the class period. If many students are having difficulty with a particular problem, you may wish to have a collective discussion about that problem.

At the end of the class period, collect the workbooks for your review. They will be used again in Lesson W13.

**Capsule Lesson Summary**

Continue individual work in the workbook *Collage of Problems #6*. (This is the second of two lessons using this workbook.)

**Materials**

Teacher	Student
• None	<ul style="list-style-type: none"><li>• <i>Collage of Problems #6</i> Workbook</li><li>• Colored pencils, pens, or crayons</li><li>• Protractor</li><li>• Metric ruler</li><li>• Compass</li><li>• Calculator</li></ul>

**Description of Lesson**

Distribute students' copies of the workbook *Collage of Problems #6*. Ask students first to correct or complete pages from the previous week's work, and then to continue working in their workbooks. You may wish to have a collective discussion about some problems that were difficult for many students the first week.

At the end of the class period, collect the workbooks for your review. After checking the workbooks, you may wish to ask some students to work further in their workbooks during a study time or to take them home as an assignment.

**Assessment Activity**

An individual student progress record for the workbook is available on Blackline W13. You may like to use this form to monitor student work.



Match each blue leg with a red leg.

2

Complete the calculations. Show your work.

$\begin{array}{r} 3.68 \\ 70.83 \\ +23.849 \\ \hline 98.359 \end{array}$	$6.28 + 183 + 9.764$ $\begin{array}{r} 6.280 \\ 183.000 \\ +9.764 \\ \hline 199.044 \end{array}$
$\begin{array}{r} 4 \overline{) 733} \\ -468 \\ \hline 67059 \end{array}$	$328 - 63.2$ $\begin{array}{r} 328.0 \\ -63.2 \\ \hline 264.8 \end{array}$

3

Draw all of the possible red arrows and blue arrows in this picture.

4

Write a decimal number.

Clue 1

Write one of these numbers.

$\begin{array}{ c c } \hline \square & \square \\ \hline \end{array} \begin{array}{ c } \hline \ominus \\ \hline \end{array} \begin{array}{ c c } \hline \square & \square \\ \hline \end{array} = 5.5$	$\begin{array}{ c c c } \hline \square & \square & \square \\ \hline \end{array} \begin{array}{ c } \hline \ominus \\ \hline \end{array} \begin{array}{ c c c } \hline \square & \square & \square \\ \hline \end{array} = 6.48$
$\begin{array}{ c c } \hline \square & \square \\ \hline \end{array} \begin{array}{ c } \hline \ominus \\ \hline \end{array} \begin{array}{ c c } \hline \square & \square \\ \hline \end{array} = 6.2$	$\begin{array}{ c c c } \hline \square & \square & \square \\ \hline \end{array} \begin{array}{ c } \hline \ominus \\ \hline \end{array} \begin{array}{ c c } \hline \square & \square \\ \hline \end{array} = 6.62$

Clue 2

Write one of the dots on this number line. Label the dot.

Whole Unit? 6.48

5

**Mathematic**  
(The Great Mathematician knows all...)

- Write a three-digit number whose first and last digit differ by more than 1. 725
- Write this number with the digits reversed. 527
- Find the difference between your number in step 1 and 2. (Subtract the smaller number from the larger number.) 198
- Write the number in step 3 with the digits reversed. 891
- Add the number from step 3 and 4. 1089

The Great Mathematician has written your number in step 5 on page 70.


Many choices are possible for step 1. The choice determines steps 2, 3, and 4. At step 5 the number will always be 1089.

7

Match each blue leg with a red leg.

$\frac{9}{10} + \frac{3}{10}$	$0.6$
$\frac{9}{10} - \frac{3}{10}$	$\frac{1}{8}$
$\frac{2}{3} + \frac{1}{2}$	$\frac{9}{8}$
$\frac{2}{3} - \frac{1}{2}$	$1\frac{2}{10}$
$\frac{3}{4} \div \frac{2}{3}$	$\frac{1}{2}$
$\frac{3}{4} \times \frac{2}{3}$	$1\frac{1}{8}$

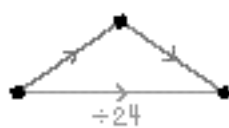
7



Alonso wants to make a ramp (like the one pictured above) for his go-cart. He has three pieces of wood of lengths 2, 3, 4, 5, and 7 inches. What combinations of the three pieces of wood could he use to build the ramp?

2	3	4
2	4	5
3	4	5
3	5	7
4	5	7
_____	_____	_____

7



Complete the chart.

$\rightarrow$	$\rightarrow$
$\div 6$	$\div 4$
$\div 12$	$\div 2$
$\div 8$	$\div 3$
$\times 2$	$\div 48$
$\div 36$	$\div \frac{1}{2}$

Many solutions are possible for the last two entries in the chart.

7



Use the given information to help solve these problems.

$20\% \text{ of } \$350 = \$70$   
 $6\% \text{ of } \$350 = \$21$

$26\% \text{ of } \$350 = \$91$      $14\% \text{ of } \$350 = \$49$   
 $60\% \text{ of } \$350 = \$210$      $18\% \text{ of } \$350 = \$63$   
 $100\% \text{ of } \$350 = \$350$      $106\% \text{ of } \$350 = \$371$   
 $40\% \text{ of } \$350 = \$140$      $10\% \text{ of } \$350 = \$35$   
 $120\% \text{ of } \$350 = \$420$      $200\% \text{ of } \$350 = \$700$

10

The red label is one of these:

- Multiples of 2
- Multiples of 3
- Greater than 10
- Less than 10
- Positive divisors of 12
- Positive divisors of 20

The blue label is one of these:

- Multiples of 2
- Multiples of 3
- Greater than 10
- Less than 10
- Positive divisors of 12
- Positive divisors of 20

Label the strings.

11

Luck is trying to figure out the money in 66 bills. This is what she knows so far.

- There are three kinds of coins
  - 
  - 
  -
- One Quarter is due for three Dimes
 
$$\begin{array}{c} \text{Q} \\ \swarrow \searrow \\ \text{D} \quad \text{D} \end{array} = 3\text{D}$$
- One Dime is due for two Nickels
 
$$\begin{array}{c} \text{D} \\ \swarrow \searrow \\ \text{N} \quad \text{N} \end{array} = 2\text{N}$$

Complete this information for Luck.

$A = \underline{6} \text{ Q}$                        $3A = \underline{9} \text{ D}$   
 $12B = \underline{4} \text{ A}$                        $40 = \underline{2} \text{ B}$   
 $12B = \underline{24} \text{ Q}$                        $A + B = \underline{8} \text{ Q}$   
 $3B + A = \underline{12} \text{ Q}$                        $2B + 20 = \underline{1} \text{ A}$

Luck has three Quarters. How many Dimes could make for Dimes and Nickels

$3A = \underline{9\text{D}}$      $3A = 9\text{D} + 2\text{C}$      $3A = 9\text{D} + 5\text{C}$

Other additions are:  $3A = 7\text{D} + 4\text{C}$      $3A = 3\text{D} + 10\text{C}$   
 $3A = 5\text{D} + 3\text{C}$      $12 \text{ B} = 2\text{D} + 14\text{C}$   
 $3A = 4\text{D} + 10\text{C}$      $3A = 5\text{D} + 16\text{C}$   
 $3A = 10\text{C}$

12

Complete the calculations. Show your work.

$4.79 \times 0.603$

$$\begin{array}{r} 4.79 \quad \times 100 \quad 479 \\ \times 0.603 \quad \times 1,000 \quad \times 603 \\ \hline 1437 \\ 28740 \\ \hline 28837 \\ + 100,000 \\ \hline 28837 \end{array}$$

Students' work may vary. Accept any correct method.

$377.4 \div 1.2$ 

$$\begin{array}{r} 314.5 \\ 12 \overline{)3774} \\ \underline{-3600} \quad 300 \\ 174 \\ \underline{-120} \quad 10 \\ 54 \\ \underline{-48} \quad 4 \\ 6 \\ \underline{-6} \quad 0.5 \end{array}$$

Answer: 314.5

$204.36 \div 1.5$ 


$$\begin{array}{r} 136.24 \\ 15 \overline{)2043.6} \\ \underline{-1500} \quad 100 \\ 543.6 \\ \underline{-450} \quad 30 \\ 93.6 \\ \underline{-90} \quad 6 \\ 3.6 \\ \underline{-3.0} \quad 0.2 \\ 0.6 \\ \underline{-0.6} \quad 0.04 \end{array}$$

Answer: 136.24

13



GERMANY



SCALE: 1 cm on the map = 100 km

On the map, what is the length of a line segment between Munich and Hamburg? 5.5 cm. What is the actual distance between Munich and Hamburg? 550 km.

If an airplane that flies 400 kilometers in 1 hour leaves Munich at 9:30 AM, what time will it arrive in Hamburg? (Circle the closest answer.)

10:40 AM    11:10 AM    11:50 PM    1:20 PM

What is the distance between Germany's northernmost point and Germany's southernmost point? 920 km.

Dresden is 350 km from Munich and 350 km from Frankfurt. Draw and label a dot for Dresden on the map.

Hannover is 240 km from Berlin. Indicate in red pixels where Hannover could be.

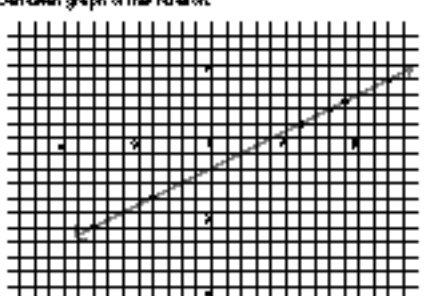
18

$$a \rightarrow \left(\frac{1}{2} \times a\right) - 2$$


Put in the blank. Each ordered pair belongs to the red relation. One is done for you.

(8, 2)    (12, 4)    (6, 1)  
 (8, 6)    (4, 4)    (9, 2.5)

For each ordered pair, draw a dot on the grid. Sketch the Cartesian graph of this relation.



19



Young Wilbur stalks the zoo on a day and later told his sister, Julie, about an area where he saw both ostriches and zebras. Julie asked, "How many ostriches and zebras did you see?" Wilbur answered, "I won't tell you that, but there were 24 eyes and 30 feet."

Julie quickly calculated the number of ostriches and zebras. Can you?

Eyes:  $2 \times 12 = 24$  (12 animals)  
 Feet:  $(2 \times 9) + (4 \times 3) = 18 + 12 = 30$   
 $9 + 3 = 12$

There were 9 ostriches and 3 zebras.

Extra problem: Wilbur saw some other common jungle animals in another part of the zoo. Altogether, these animals had 5 heads and 34 ft. Which animals and how many of each kind could Wilbur have seen? (Hint: The answer is 10k today.)

2 lions and 3 snakes, or 4 parrots and 1 snake, or 1 lion and 2 parrots and 2 snakes

20

Answers will vary.

Put each number on the base ten blocks by adding up the ones of these divisions:

①    ①    ②

$\begin{array}{|c|c|} \hline \square & \square \\ \hline \hline \end{array} \begin{array}{|c|c|} \hline \square & \square \\ \hline \hline \end{array} = 10.8$      $\begin{array}{|c|c|} \hline \square & \square \\ \hline \hline \end{array} \begin{array}{|c|c|} \hline \square & \square \\ \hline \hline \end{array} = 0.52$

$\begin{array}{|c|c|} \hline \square & \square \\ \hline \hline \end{array} \begin{array}{|c|c|} \hline \square & \square \\ \hline \hline \end{array} = 7.2$      $\begin{array}{|c|c|} \hline \square & \square \\ \hline \hline \end{array} \begin{array}{|c|c|} \hline \square & \square \\ \hline \hline \end{array} = 1.74$

$\begin{array}{|c|c|} \hline \square & \square \\ \hline \hline \end{array} \begin{array}{|c|c|} \hline \square & \square \\ \hline \hline \end{array} = 7.2$      $\begin{array}{|c|c|} \hline \square & \square \\ \hline \hline \end{array} \begin{array}{|c|c|} \hline \square & \square \\ \hline \hline \end{array} = 5.04$

21

### State Income Tax: Pile

- What is the percent of state income tax paid for a person who earns \$25,500 in one year? 17%    \$17,500? 27%
- Ms. Glavin's state income tax rate is 6%. What is the largest of her annual incomes could be? \$35,000    Smallest? \$30,000
- Mr. Doster earns \$42,000 per year. How much state income tax does he pay? \$2520 After paying his state income tax, Mr. Doster puts 5% of his remaining into a savings account this year. How much dollars put in the account? \$1932  
 $(\$42,000 - \$2520 = \$39480 \text{ and } 5\% \text{ of } \$39480 = \$1932)$
- Ms. Lepin's state income tax rate is 10%. This year she paid \$4570 in state income tax. What is her annual income? \$45,700

22

$(N, T)$  is a second ordered pair of numbers.

**Clue 1**

$(N, T)$  is a dot on the grid line.

$(N, T)$  could be  $(5, 4)$ ,  $(6, 2)$ ,  $(8, 5)$ ,  $(10, 4)$ ,  $(9, 3)$ ,  $(9, 3)$ , and so on.

**Clue 2**    Is equal to

$(N, T)$  is  $(2, 2)$

23

Pelton High School is marching band and is preparing to march in a city parade. Their director is trying to find which formation they should use.

When the band marches in rows of two, poor Waldo is alone in the last row.

When the band marches in rows of three, again poor Waldo is alone in the last row.

When the band marches in rows of four, also, poor Waldo is alone in the last row.

But when the band marches in rows of five, every row has exactly five people.

What is the least number of members the band could have? 25

If the band has between 50 and 140 members, how many students are in the band? 85

24

Pip and Pop are second numbers.

Who is Pip? 2    Who is Pop? 5

25

$a \rightarrow \frac{1}{2} \times (a + 2)^2$

Fill in the blank. Each ordered pair belongs to the blue relation. One label is for you.

(2, 8)      (0, 2)      (3, 12.5)  
 (4, 2)      (2, 0)      (7, 12.5)

For each ordered pair, draw a dot on the grid. Sketch the Cartesian graph of this relation.

Name two more ordered pairs that belong to the blue relation.  
 (1, 2.5) and (3, 2.5)

There are many possibilities for more ordered pairs belonging to the blue relation.

Put each number on the abacus and fill in the boxes.

Dotted Writing      Base Two      Base Two Writing  
 500 =  $\begin{matrix} 256 & 128 & 64 & 32 & 16 & 8 & 4 & 2 & 1 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{matrix}$  = 11111000

Base Five      Base Five Writing  
 500 =  $\begin{matrix} 625 & 125 & 25 & 5 & 1 \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{matrix}$  = 4000

Base Eight      Base Eight Writing  
 500 =  $\begin{matrix} 256 & 64 & 8 & 1 \\ \bullet & \bullet & \bullet & \bullet \end{matrix}$  = 764

Base Three      Base Three Writing  
 500 =  $\begin{matrix} 27 & 9 & 3 & 1 & \frac{1}{3} & \frac{1}{9} \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{matrix}$  = 2.02.11

Base Ten      Base Ten Writing  
 500 =  $\begin{matrix} 1000 & 200 & 10 & 1 & \frac{1}{10} & \frac{1}{100} \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{matrix}$  = 2.02.11

Base Thirteen      Base Thirteen Writing  
 500 =  $\begin{matrix} 2197 & 169 & 13 & 1 & \frac{1}{13} & \frac{1}{169} \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{matrix}$  = 2.02.11

= 340022402      27

$a * b = \frac{a}{b} + \frac{1}{2}$

Example:  $2 * 3 = \frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6} = \frac{7}{6}$

Complete.

$3 * 4 = \frac{7}{4}$        $1 * 10 = \frac{5}{10}$   
 $10 * 7 = \frac{27}{14}$        $2 * 5 = \frac{9}{10}$   
 $4 * 3 = \frac{11}{6}$        $6 * 8 = \frac{13}{4}$   
 $5 * 10 = 2$        $4 * 5 = 3.3$   
 $6 * 3 = \frac{5}{2}$        $1 * 6 = \frac{7}{6}$

27

The red label is one of these:

- ~~Composite of 2~~
- ~~Composite of 3~~
- ~~Composite of 4~~
- ~~Composite of 5~~
- ~~Composite of 6~~
- ~~Composite of 7~~
- ~~Composite of 8~~
- ~~Composite of 9~~
- ~~Composite of 10~~
- ~~Composite of 11~~
- ~~Composite of 12~~
- ~~Composite of 13~~
- ~~Composite of 14~~
- ~~Composite of 15~~
- ~~Composite of 16~~
- ~~Composite of 17~~
- ~~Composite of 18~~
- ~~Composite of 19~~
- ~~Composite of 20~~
- ~~Composite of 21~~
- ~~Composite of 22~~
- ~~Composite of 23~~
- ~~Composite of 24~~
- ~~Composite of 25~~
- ~~Composite of 26~~
- ~~Composite of 27~~

The blue label is one of these:

- ~~Composite of 2~~
- ~~Composite of 3~~
- ~~Composite of 4~~
- ~~Composite of 5~~
- ~~Composite of 6~~
- ~~Composite of 7~~
- ~~Composite of 8~~
- ~~Composite of 9~~
- ~~Composite of 10~~
- ~~Composite of 11~~
- ~~Composite of 12~~
- ~~Composite of 13~~
- ~~Composite of 14~~
- ~~Composite of 15~~
- ~~Composite of 16~~
- ~~Composite of 17~~
- ~~Composite of 18~~
- ~~Composite of 19~~
- ~~Composite of 20~~
- ~~Composite of 21~~
- ~~Composite of 22~~
- ~~Composite of 23~~
- ~~Composite of 24~~
- ~~Composite of 25~~
- ~~Composite of 26~~
- ~~Composite of 27~~

Label the string.

27

Use the space to briefly explain your answer.

The world population in 1996 is about 5.3 billion. If the present 1.6% yearly rate of net population growth continues, what will be the world population in 1997? 5,382 billion 1998? 5,562 billion 2000? 5.18 billion

In what year will the population reach 6 billion? 1999  
7 billion? 2002

Number in step 5 on page 6  
1039

30

Alice, Brenda, and Carl are three of Nabu's friends. When Nabu asked who was the oldest, they made the following statements.

Alice: I am younger than Brenda.  
Carl is not the oldest.

Brenda: I am the oldest.  
Carl is younger than Alice.

Carl: I am older than Brenda.  
Alice is the youngest.

To confuse Nabu, each person made one true statement and one false statement. Nabu was able to determine who was oldest and who was youngest.

Who is oldest, Alice, Brenda, or Carl? Alice

Who is youngest? Brenda

Circle each person's true statement.

31

Suppose you win one million dollars on your 12th birthday with the following conditions.

- Each day you must spend exactly enough to average \$15 an hour. (Use a 24-hour day and the mean average.)
- At the beginning of each year, 5% interest will be added to the amount you have left.
- You must spend all the money before your 21st birthday.

How much must you spend each day? \$360  
each year? \$13,140

Can you spend the money before your 21st birthday? No  
How long will it take to spend all the money? 83 days after  
21st birthday

Explain your answer.

12th birthday (end of year one) \$1,000,000 - \$360 x 365 = \$67,200  
+ 5% interest = \$70,560

13th birthday (end of year two) \$672,000 - \$360 x 365 = \$26,640  
+ 5% interest = \$27,972

14th birthday (end of year three) \$645,360 - \$360 x 365 = \$26,640  
+ 5% interest = \$27,972

15th birthday (end of year four) \$618,720 - \$360 x 365 = \$26,640  
+ 5% interest = \$27,972

16th birthday (end of year five) \$592,080 - \$360 x 365 = \$26,640  
+ 5% interest = \$27,972

17th birthday (end of year six) \$565,440 - \$360 x 365 = \$26,640  
+ 5% interest = \$27,972

18th birthday (end of year seven) \$538,800 - \$360 x 365 = \$26,640  
+ 5% interest = \$27,972

19th birthday (end of year eight) \$512,160 - \$360 x 365 = \$26,640  
+ 5% interest = \$27,972

20th birthday (end of year nine) \$485,520 - \$360 x 365 = \$26,640  
+ 5% interest = \$27,972

21st birthday (end of year ten) \$458,880 - \$360 x 365 = \$26,640  
+ 5% interest = \$27,972