Geometry & Measurement

GEOMETRY & MEASUREMENT INTRODUCTION

Geometry is useful and interesting, an integral part of mathematics. Geometric thinking permeates mathematics and relates to most of the applications of mathematics. Geometric concepts provide a vehicle for exploring the rich connections between arithmetic concepts (such as numbers and calculations) and physical concepts (such as length, area, symmetry, and shape). Even as children experience arithmetic notions in their daily lives, they encounter geometric notions. Common opinion recognizes that each child should begin developing a number sense early. There is no sound argument to postpone the development of geometric sense. These are compelling reasons why the systematic development of geometric thinking should be a substantial component of an elementary school curriculum.

A benefit of early development of geometric thinking is that it provides opportunities for all learners. Children develop concepts at individual rates. We are careful to allow students a great deal of time to explore quantitative relationships among physical objects before studying arithmetic. In the same spirit, students should be allowed to explore geometric relationships through free play and guided experiences. Before studying geometric properties of lines and shapes, they should have free play with models and with drawing instruments, using both to create new figures and patterns. Before considering parallelism and perpendicularity as relations on geometric lines, they should have free play in creating patterns with parallel and perpendicular lines. Before learning length and area formulas, they should have informal experiences to establish and sharpen intuitive notions of length and area. These early experiences are aesthetically rewarding and just as stimulating and suggestive of underlying concepts as early experiences leading to arithmetic. Such experiences are a crucial first step in the development of a geometric sense.

As with most educational development occurring through informal experiences, the payoffs come unexpectedly and spasmodically. In the development of geometric thinking, the suggested sequence of activities is so varied, reaches into so many other facets of the program, and continues for so long a span that it would be nonsense to set *a priori* milestones of accomplishments.

Thus, it is important to allow students to move through the free play activities that precede the more substantive lessons without rushing them. These preliminary activities have implicit significance for geometric thinking and also provide aesthetic opportunities and explicit practice— they are not mere play. On the other hand, some activities appear to have important consequences or clearly relate geometry to other parts of the program. Here, your inclination might be to spend more time on these activities than is suggested, but try to remember that similar activities will continue to be experienced later in a slightly different context.

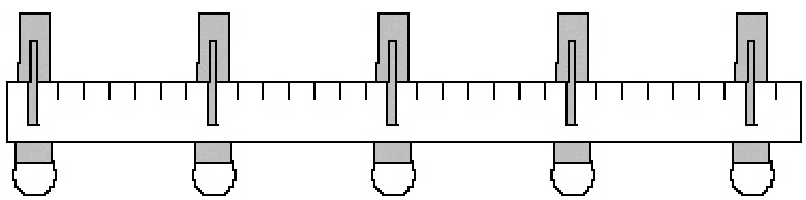
This strand also includes measurement activities, many that lead to the use of metric units as well as topics not usually found in traditional programs at this level. Measurement activities eventually lead to notions of distance. Traditionally, teachers wait for a good deal of maturity in their students before introducing properties of figures, such as circles, that are defined in terms of distance. The reason is that Euclidean distance (distance "as the crow flies") requires sophisticated notions of "exact" measurement in terms of the real number system in order to be able to measure every line segment. But in the *CSMP* curriculum very young children (even in kindergarten) begin thinking about distance in the simplified setting of "taxi-geometry." Taxi-distance (as a taxicab travels) between points is measured in terms of whole numbers by easy counting processes. Experiences in taxi-geometry lead to some creative situations, yet do not require special technical skills or vocabulary.

Putting the emphasis on experience rather than on mastery frees you to engage in imaginative activities with children that you ordinarily might not attempt. For example, even when considering such commonplace topics as area and length, feel free to devise activities that help students recognize patterns, develop cognitive strategies, and relate the topic to others in the program, while suppressing the urgency to get the basic formulas. What is basic here, at this level, is that children are thinking rather than doing tasks on command. The skills and mastery will come eventually, in their own time and place in the curriculum.

Note on Grids

A demonstration grid board is needed for several lessons, especially those on taxi-geometry. This should be a large square grid board at least 12 squares by 12 squares (with grid lines spaced 4 to 8 centimeters apart) on which you can draw with colored chalk or colored pens. A portable, commercially made, permanent grid chalkboard is ideal. However, if such a grid board is not available, try one of these alternatives.

- Printed grids on large sheets of paper are provided in classroom sets of materials for *CSMP* for the upper primary grades. Tape one of these sheets to the board, use thick felt-tipped pens for drawings, and discard the sheet after use. You may prefer to laminate or mount one of the sheets on cardboard and cover it with clear contact paper to provide a protective surface. Then use erasable markers or wax crayons for drawings. These grid sheets also are useful as graphing mats.
- For each lesson, carefully draw a grid on the chalkboard using an easy-to-make tool. Either (1) use a music staff drawer with chalk in the first, third, and fifth positions; or (2) clip five pen caps, each having the diameter to hold a piece of chalk tightly, to a ruler at intervals of 4–8 centimeters and secure them with tape.



- Make a grid transparency for use on an overhead projector and use erasable overhead markers. The Blackline section includes several size grids to use in making a grid transparency.

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G1 AREA #1

Capsule Lesson Summary

Use different size units to cover an irregular shape. Estimate and check results. Observe that the number of units needed to cover a shape will vary with the size of the unit.

Materials

Teacher
- Shoeprint
- Collection of pennies
- Blackline G1

Student
- Paper
- Covering materials
- Chart

Advance Preparation: Use Blackline G1 to make copies of the chart for student pairs. Prepare multiple collections of several different covering materials such as same size buttons, Unifix® and/or centimeter cubes, fish crackers, miniature and/or regular marshmallows, small shape counters, and so on.

Description of Lesson

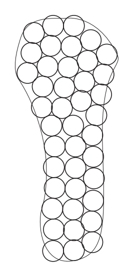
Note: A shoeprint is the object covered by pennies in this lesson description; however, you may choose to use other shapes of interest to your students such as leaves, handprints, hearts, teddybears, and so on.

Show the class your shoeprint and a collection of pennies (same size buttons, small cubes, or other covering materials).

T: ***How many pennies do you think it would take to just cover my shoeprint?***

Let students make guesses and then ask students to help you place pennies. It will be impossible to cover the shoeprint completely with pennies unless you allow the pennies to overlap. As the class works on this project you may like to discuss the difficulties and ask for suggestions of materials to use that would do a better job of covering.

When the class feels they have done a pretty good job of just covering the shoeprint with pennies, remove the pennies and ask a student to count how many were used in the covering. Compare the results to the students' guesses.



41 Pennies

Prepare students to conduct a similar activity with partners. Give each pair of students sheets of plain white paper on which to draw around each other's shoe. Then make available several collections of covering materials. For example:

- same size buttons
- cubes (Unifix or centimeter)
- miniature marshmallows
- regular size marshmallows
- fish crackers
- small shape counters

Direct the student pairs to repeat the activity several times with both shoeprints and to use a different covering material each time. Make a chart (Blackline G1) for students to record the covering unit used, their guess, and a check of the actual number of units used in a covering.

As students are working, observe how they make guesses and check. You may like to discuss methods of guessing and checking with some student pairs.

What I Used (unit)	Guess	Check (How many units to cover)

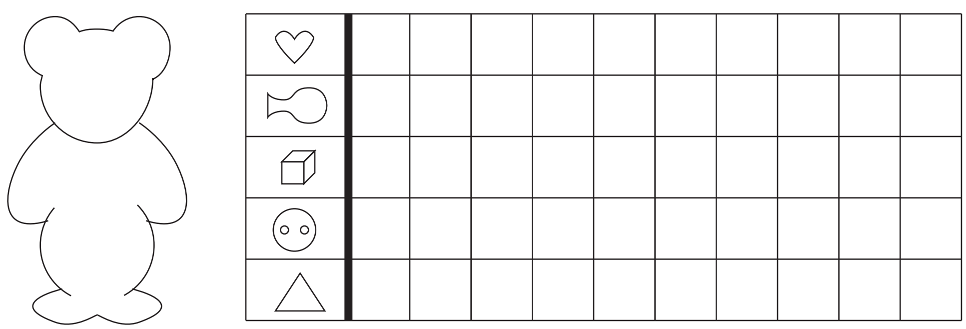
You may also ask students to note which unit had the least number in a covering and which unit had the greatest number. Ask why they needed more of one unit than another.

Writing Activity

Suggest student pairs write a report about what they found and then, if appropriate, ask some pairs to share their findings with the class.

Extension Activity

Select an interesting irregular shape (outline of an animal or a cartoon character). Organize the class in groups and let each group use a different unit to cover a copy of the shape. Then make a graph of the results. For example:



You may let students place the actual objects used in a covering in the graph or make x's (or color in boxes) to indicate the number of units used.

Capsule Lesson Summary

Find many different-looking four-sided shapes and observe similarities and differences. Organize four-sided shapes in a graph or table. Put similar four-sided shapes on a geoboard.

Materials

Teacher	• Blackline G2	Student	• Loop of yarn • Unlined paper • Geoboard and geobands • Geopaper

Advance Preparation: Make loops from several 150-200 cm lengths of yarn. You will need one loop for each group of five students. Use Blackline G2 to make geopaper.

Description of Lesson

Begin the lesson by asking the class if they know about any shapes that have four corners.

S: ***A square has four corners.***

S: ***A rectangle.***

S: ***A kite (diamond).***

T: ***How many sides does the shape you are thinking about have?***

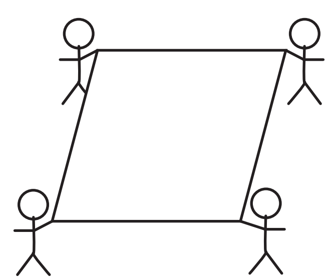
S: ***Four.***

T: ***We're going to see how many different-looking four-sided (four corner) shapes we can find.***

Exercise 1

Organize the class into groups of five students. Provide each group with a loop of yarn and unlined paper. Direct the groups to choose one student to be the recorder. The other four students will be the corners of a shape and they will use the loop of yarn to make the sides.

Direct the groups to make shapes with four corners by having four students stand and each hold a point on the loop of yarn so it is taut between adjacent students. The four students then sit down while the recorder draws a similar four-sided shape on unlined paper. (A recorder may need help getting started.) Direct groups to find several different-looking shapes and record them on separate pieces of paper.



As you observe the groups, encourage students to try to find as many different-looking shapes as they can.

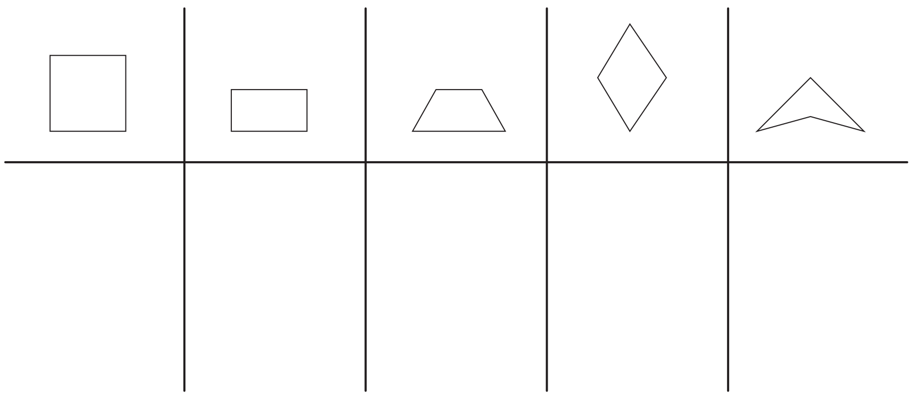
T: ***How are your shapes alike? How are they different?***

S: ***All the shapes have four sides (and four corners).***

S: ***One of our shapes looks like a kite; another look like an arrow.***

S: ***Some of our shapes are squares (or rectangles).***

After a while, let the groups share their findings with the class and display different-looking four-sided shapes. If students are able to identify similarities and differences, you may like to organize the recorded shapes in a graph or table, or you may like to make a class collage.



Exercise 2

Provide students with geoboards, geobands, and geopaper. Refer the class to some of the four-sided shapes they found in Exercise 1 and ask students to put many different four-sided shapes on a geoboard and record them on geopaper. Challenge students to find other different-looking four-sided shapes using the geoboard.

Extension Activity

As an additional challenge project, ask a group of students to repeat this lesson looking for different-looking three-sided or five-sided shapes.

Capsule Lesson Summary

Examine many shapes each of which has area ten little squares. Find the area of these shapes by measuring with little triangles that are exactly half of the little squares. Look for some shapes with area less than ten little squares and more than ten little squares.

Materials

Teacher
- Square tiles
- Grid board† or transparency
- Colored chalk
- Blackline G3

Student
- Square tiles
- Grid paper
- Colored pencils, pens, or crayons
- Worksheets G3*, **, ***, and ****

Advance Preparation: Use Blackline G3 to make a two-sided, one-inch grid paper for students and possibly an overhead transparency grid for demonstration. The grid board in this lesson should model the students' grid paper. Square tiles should match the size of the grid and can be made from tagboard, or use commercial square tiles.

Description of Lesson

Provide each student with ten one-inch square tiles and one-inch grid paper. Display similar materials on an overhead projector or another grid board. Students can work with a partner, but each should have grid paper.

Model with the demonstration materials as you give instructions to the students.

T: ***Today we are going to use these tiles. What do you think we could do with them?***

S: ***Build (or make) shapes.***

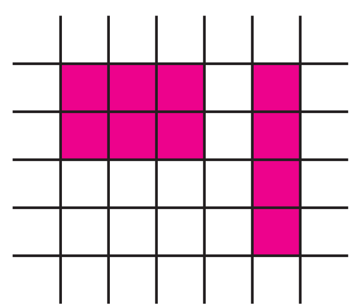
S: ***Make patterns.***

T: ***Square tiles are often used to cover things—a bathroom floor, for example, might be covered with square tiles. We are going to use these square tiles to cover shapes.***

On the grid paper you are to make two different shapes. Each shape is to have exactly 10 little squares in it. First cover a shape you want to make with 10 square tiles; then color it as you take the tiles away.

While the students work, provide individual help as necessary to correct mistakes. Most mistakes will be in counting and, in these cases, ask the student to count the number of squares in the shape. Working with a partner, students can check each other.

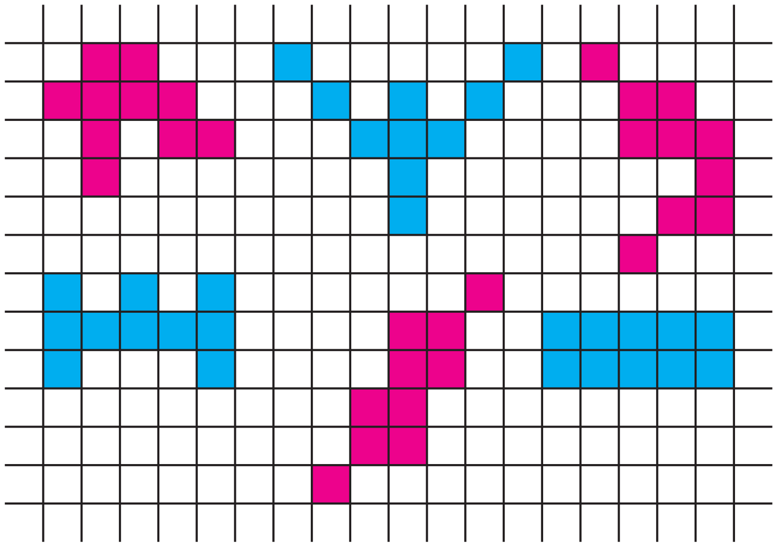
You may find some students have drawn disconnected shapes.



In a case like this, point out that although in these two shapes together there are 10 little squares, you would like one (connected) shape with 10 squares and then a second (connected) shape with 10 little squares.

†See the "Note on Grids" section in the introduction to this strand.

Ask several students who finish quickly and have drawn interesting shapes to copy one of their shapes on the demonstration grid board. The following illustration shows some examples of shapes, each of which has area 10 little squares.



Let the class react to the shapes on the demonstration grid board.

T: ***These shapes are really beautiful. Do they all look the same? Do your shapes look like these?***

S: ***No (some do, some don't).***

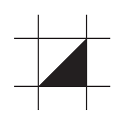
T: ***But 10 little squares cover each of them. The area of each of these shapes is 10 little squares.***

Be sure to use the words "little squares." If a unit of area—here the little square—has not been specified, then the expression "the area is 10" has no meaning.

Put this information on the grid board next to each shape.

Area = 10 ■

T: ***Suppose we cut each little square into two little triangles. How many little triangles would cover each of your shapes?***



S: ***20.***

T: ***Can you explain your answer?***

S: ***Each little square has two little triangles and 2 x 10 = 20 (or 10 + 10 = 20).***

If this or a similar answer is not forthcoming, suggest that the students draw cutting lines (lines dividing each little square into two little triangles) in one of their shapes and then count the number of little triangles. You may want to do this with one of the shapes on the demonstration grid board.

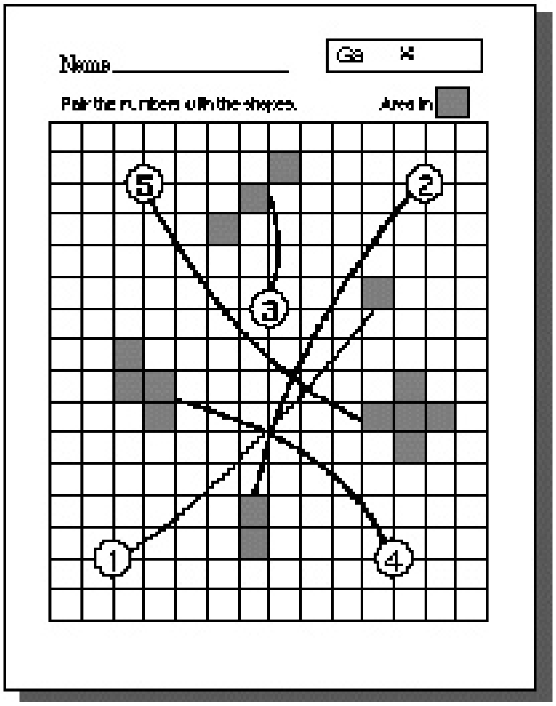
Next to each shape on the grid board, write the area in little triangles.

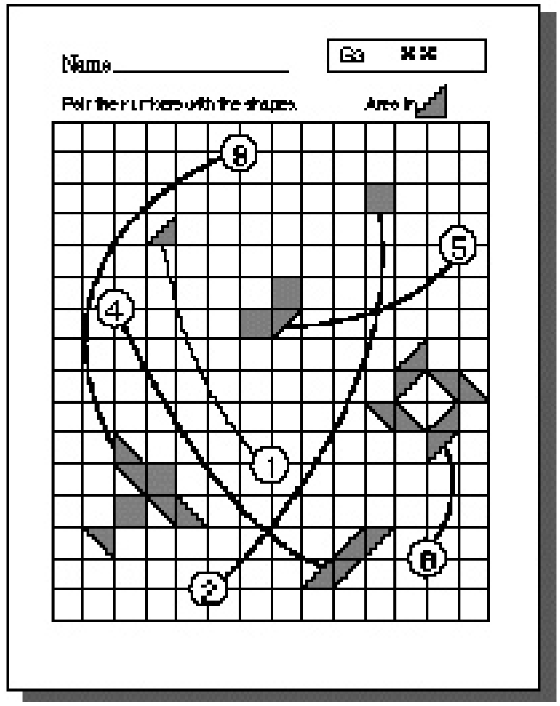
Area = 10 ■ or 20 ◢

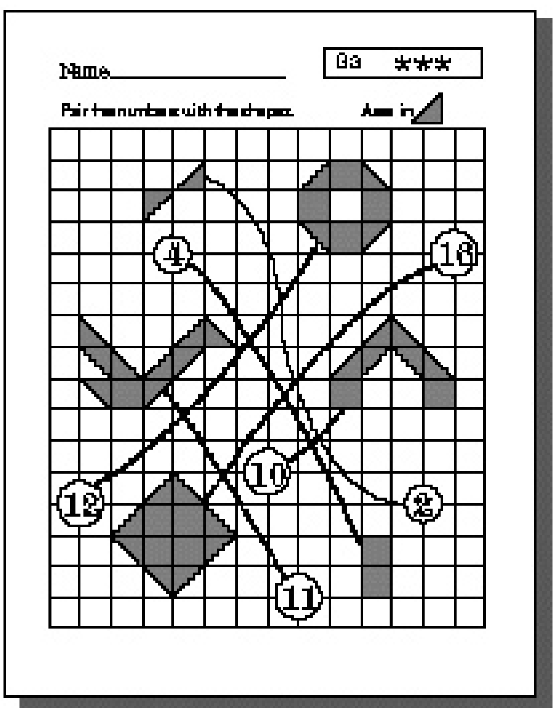
T: ***On another grid paper (or the backside) you will again make two shapes. This time one shape should have an area smaller than 10 little squares, and the other one should have an area larger than 10 little squares.***

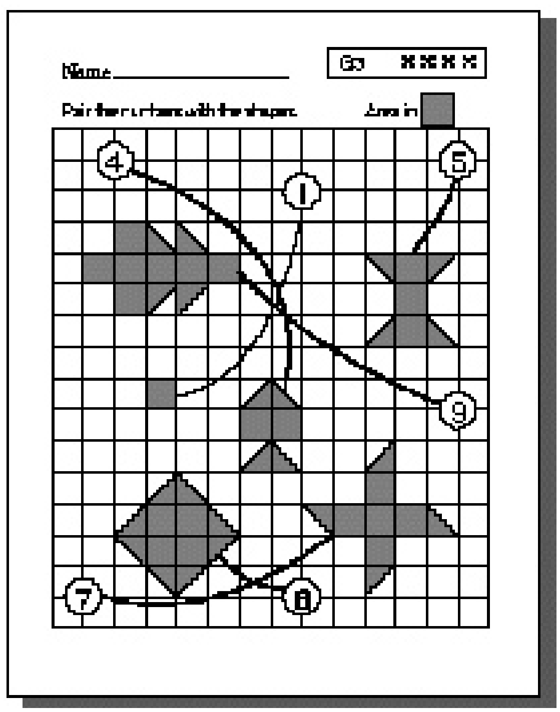
As you observe your students' drawings, you might suggest that they write the area either next to each shape in little squares or in both little squares and little triangles. Working in pairs, students can count and check each other's drawings.

When students finish these drawings, Worksheets G3*, **, ***, **** are available for individual work.


Name
G3
Area in


Name
G3
Area in


Name
G3
Area in


Name
G3
Area in

Capsule Lesson Summary

Use a mirror to extend, expand, or otherwise change a picture. Use a mirror and geometric shapes to create many other shapes and designs.

Materials

Teacher
- Mirror
- Three bears picture
- Blacklines G4(a) and (b)

Student
- Mirror
- Three bears picture
- A-blocks
- Worksheets G4(a), (b), and (c)

Advance Preparation: Use Blackline G4(a) to make a three bears picture for demonstration and Blackline G4(b) to make copies of the picture for students.

Description of Lesson

Pair the students and provide each pair with a mirror and some attribute blocks. Allow several minutes for free play.

Note: Exercises 2 and 3 below are described with attribute blocks, but you may choose to use other shape manipulatives such as pattern blocks.

Exercise 1

Display a copy of Blackline G4(a). Also provide each pair of students with a smaller copy of the picture as in Blackline G4(b).

T: ***What do you notice about this picture?***

S: ***There are three bears in a row.***

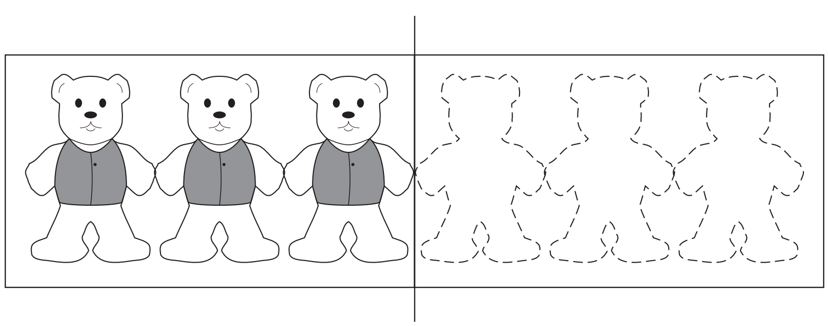
S: ***The bears are all the same.***

S: ***The bears touch hands.***

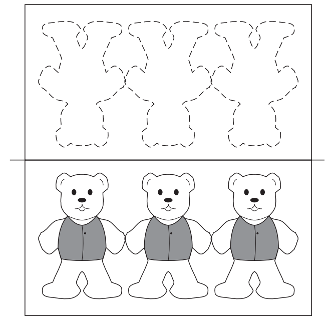
Show the class a mirror and ask,

T: ***Do you think we could place this mirror so we could see six bears in a row?***

Call on a student to do so at the board. If student pairs have a copy of the picture, ask them to place their mirrors to see six bears in a row.

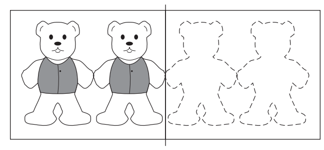


T: ***Could we place the mirror to see six bears in another way?***

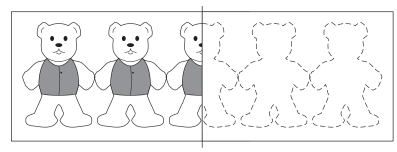


Continue in this way, asking for the following:

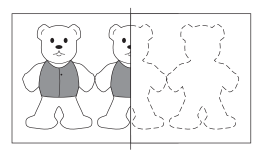
- five bears in a row



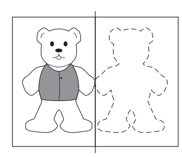
- four bears in a row



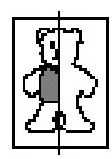
- three bears in a row



- two bears



- one bear



Let students experiment for a few more minutes with their mirrors and the three bears picture.

Exercise 2

Provide each student pair with a mirror, different-colored small A-block squares and triangles, and copies of Worksheets G4(a), (b), and (c). Direct the partners as follows:

- Use the mirror and a square to see the shapes on Worksheet G4(a).
- Use the mirror and a triangle to see the shapes on Worksheet G4(b).
- Use the mirror and a square and triangle combination to see the shapes on Worksheet G4(c).

As you observe the students working, you may occasionally want to ask a group to show you how they used the mirror and a square, triangle, or combination to see a particular shape.

Exercise 3

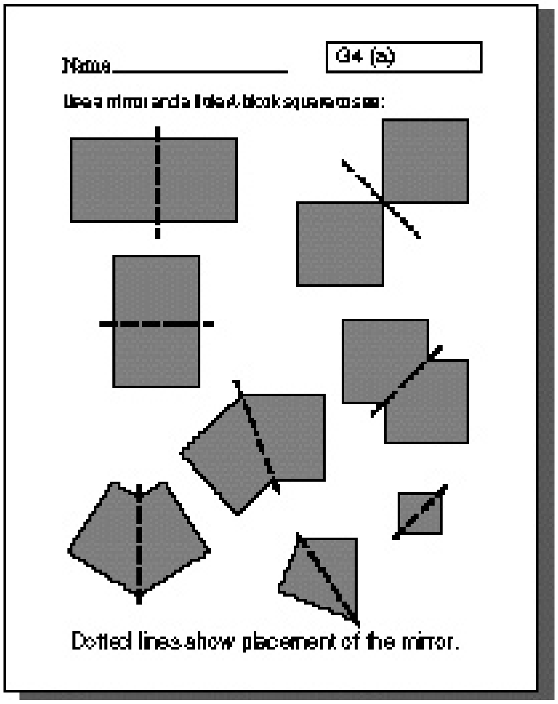
Let student pairs create designs (shapes) of their own that they can see with the mirror and the square, triangle, or combination. Students may like to draw the designs or shapes they make and challenge another pair to see their design.

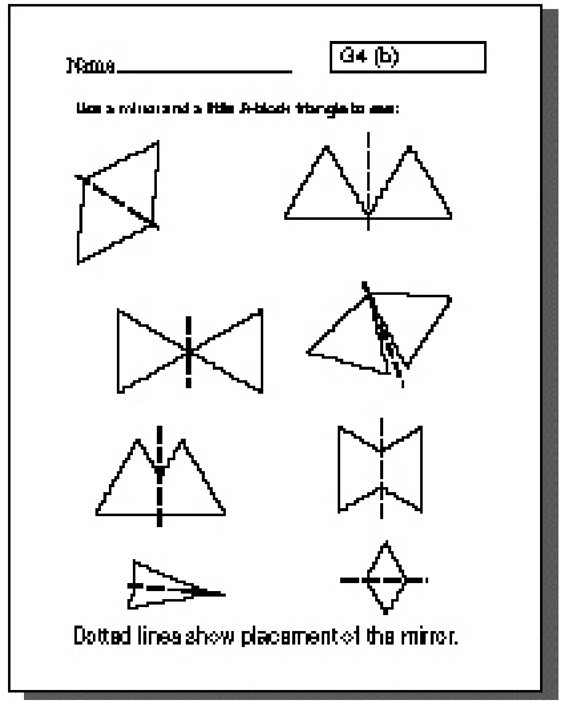
Center Activity

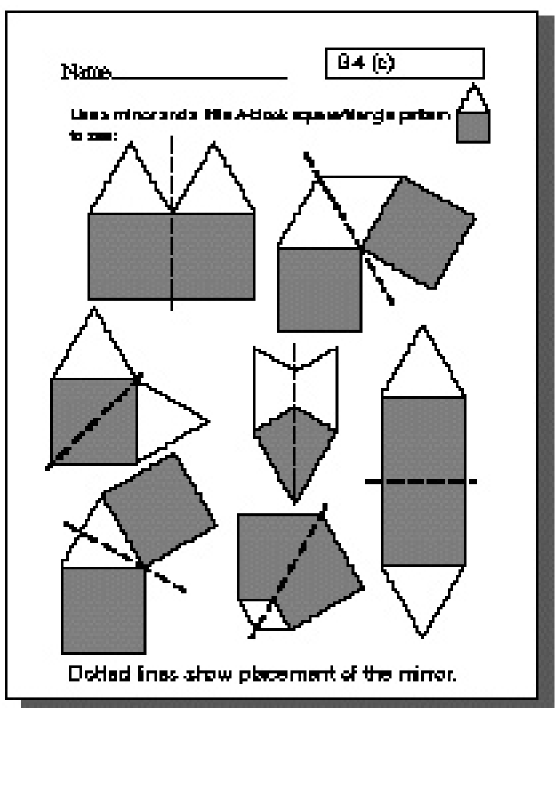
Place A-blocks or other shape manipulatives and mirrors in a center for further exploration. There are also some commercial games available which can be put in a center for exploring mirror symmetry; for example, Jeu du Miroir (Mirror Game).

Reading Activity

Several books by Marion Walter (*Look at Annette, Make a Bigger Puddle, Make a Smaller Worm)* and Ann Jonas (*Round Trip, Reflections*) use patterns and reflections in the story illustrations.


Name
G4 (a)
Dotted lines show placement of the mirror.


Name
G4 (b)
Dotted lines show placement of the mirror.


Name
G4 (c)
Dotted lines show placement of the mirror.

Capsule Lesson Summary

Measure the length of the classroom with shoes and discuss how the measurements will differ depending on whose shoes are used. Estimate the same length measured with other objects in the room such as a piece of chalk or a pointer. Measure other objects in the room with erasers or smaller things. Measure the length of a desk with different color C-rods.

Materials

Teacher		Student	
	• None		• C-rods • Pencil

Advance Preparation: Cuisenaire rods (C-rods) are used in Exercise 2. If these materials are not available, arrange to have some other uniform length items, such as popsicle sticks, toothpicks, new pencils, straws, paper clips, and so on to use for measuring.

Description of Lesson

Exercise 1

Choose a wall that you can walk along from corner to corner. Try to arrange that all of the students are able to see your feet as you step off the distance. Alternatively, you can choose to measure the length of a center aisle between two prearranged marks on the floor.

T: ***Today let's measure the length of the wall. How long do you think it is?***

S: ***What are we using to measure?***

T: ***That's a good question. What could we measure the wall with? What could we use to find out how long it is?***

Accept any plausible suggestions. If you wish, allow students to perform the measurements with some of the objects they suggest using. For example:

S: ***The wall is between nine and ten (meter) sticks long.***

S: ***The wall is between thirty and thirty-one (foot or 30 cm) rulers long.***

After a while, suggest the following:

T: ***Suppose we measure the wall with shoes. How can we do that?***

S: ***Walk along the wall.***

T: ***Can you estimate how many shoes long this wall is?***

Perhaps some students will ask about whose shoes to use. The class may want to agree on what size shoe they use. Be sure to observe that the person measuring should put one foot directly in front of the other as they walk.

Measure the wall with someone's (perhaps your own) shoes so that the students can watch and count each step taken. Announce the result. For example:

T: ***35 is too short and 36 is too long. The wall is between 35 and 36 shoes long.***

Write this information on the board.

Length of wall: between 35 shoes and 36 shoes

Choose a student whose shoes are clearly shorter than the ones used in the previous measurement. Ask the class to predict whether the number of shoes using this student's shoes will be more or less or the same as before. Then let the student do the measuring and record the result on the board. For example:

Length of wall: between 35 shoes and 36 shoes (teacher's)
between 44 shoes and 45 shoes (Jason's)

T: ***Why didn't Jason get the same number of shoes as I did?***

S: ***Because his shoes are shorter than yours (or your shoes are longer than his).***

Repeat this measuring activity with a student whose shoes are clearly longer than Jason's.

Hold up a new piece of chalk, an eraser, or something obviously shorter than anyone's shoes.

T: ***If we were to measure the wall with this piece of chalk (or this eraser), would we get a number more or less than we did with shoes?***

S: ***More, because the piece of chalk is shorter than a shoe.***

T: ***Suppose I had a stick and I measured the wall with it and found that the wall was between 12 and 13 sticks long. What could you tell us about the stick? Is it longer than our shoes or shorter?***

S: ***Longer, because it took fewer sticks to measure the wall.***

Find some other things in the room to measure and to measure with. For example, measure the chalkboard with an eraser, a piece of chalk, and a book. Students working with a partner can perform some of the measurements. For example, one pair of students can measure with one thing while other pairs measure with other things. Continue to ask questions as suggested in the following dialogue.

T: ***The chalkboard is between 19 and 20 erasers long. Suppose we measure it with a piece of chalk*** (hold up a new piece of chalk)***. Would it be more or fewer pieces of chalk long than the number of erasers long?***

S: ***More, because the piece of chalk is shorter than the eraser.***

T: ***How many pieces of chalk long do you think the measurement will be?***

Record several guesses and then do or ask for the measurement. Compare the results with the guesses.

T: ***Suppose we measure the chalkboard with this book*** (hold up a book at least twice as long as your eraser)***. Would it be more or fewer books long than erasers?***

S: ***Fewer.***

Let the students guess how many books long the chalkboard would be and then do or ask for the measurement.

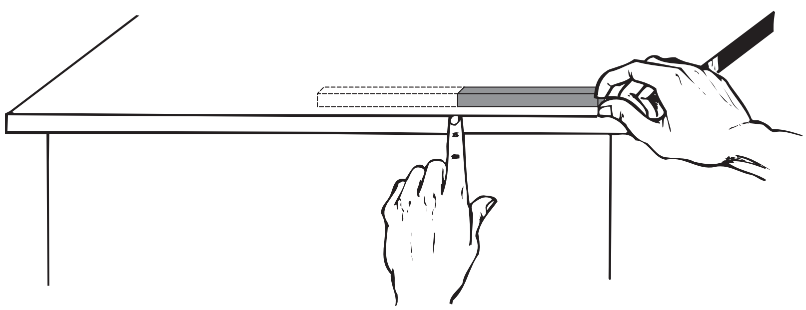
Exercise 2

This exercise can be done by cooperative groups of students at tables or pairs of students at a desk.

Provide each group of students with several C-rods of only one color. Different groups may use different colored C-rods, but give just one color C-rod to any one group. Arrange that all groups will measure the length of the same size table or desk.

T: ***I want each of you to measure the length of this table (or desk) with your C-rods. Work with your group (or partner) to do the measuring.***

Pick a table or desk in a central spot that everybody can see and demonstrate how to measure the length with a C-rod. Use your finger to mark the end point of the C-rod and count each time you set the rod down. If a group has sufficient quantity of one color C-rod, they may prefer to lay down the rods end to end.



After all students have finished measuring, record the results from the groups (or pairs) on the board.

T: ***Why are there several different numbers?***

S: ***We are using different lengths of C-rods.***

Point to the greatest number of the set of measures.

T: ***Whose measure is this? Why did this group get a greater number when they measured the length of the table (desk) than the rest of us?***

S: ***They must have a short rod.***

Ask the group to hold up their C-rod for the class to see and to compare it with the C-rods used by other groups.

Point to the least number in the set of measures.

T: ***Is the C-rod that was used to get this length shorter or longer than the others?*** (Longer) ***How do you know?***

Students may have difficulty verbalizing an explanation. Do not insist that their explanations be precise. Then ask the group that found the least measure to compare its C-rod with others.

Choose two numbers in the set of measures that are the same. Point to these numbers.

T: ***What can we say about the C-rods used to get these two measures?***

S: ***They must be the same length.***

Ask the students who had these measures to compare their C-rods for the class.

If appropriate, compare the measures or the C-rods when one is double the other. You may like to order the measures and then observe the order of the C-rods used.

Writing Activity

Ask students to write about this measurement activity. Suggest that they explain how the measurements were done and why different numbers were obtained when different things were used to measure.

Reading Activity

Read *How Big is a Foot?* by Rolf Myller to the class, or recommend students read it.

Home Activity

Suggest that parents/guardians work with their child to measure some things around the house (such as the kitchen floor or a table or a porch) using different unit objects such as shoes, pencils, paper clips, and so on. Compare the measurements made from different units.

Capsule Lesson Summary

Use a story about sharing cakes and cookies to discuss dividing a shape exactly in half. Use the symmetry of a shape to find different ways of dividing it in half. Examine shapes such as the letters of the alphabet to determine which can be easily cut in half.

Materials

Teacher
- Shape cut-outs
- Meter stick
- Colored pens
- Blacklines G6(a)–(d)

Student
- Ruler or straight edge
- Worksheets G6* and **

Advance Preparation: For Exercise 1, prepare a circle cut-out with a radius of about 20 cm and one or two additional cut-outs, for example, an equilateral triangle, a regular hexagon, or a rectangle. For Exercise 2, make copies of Blacklines G6 (a)–(d). Make multiple copies of the letters for group work.

Description of Lesson

Exercise 1

T: ***Do any of you know about Booker's Bakery? What kinds of things do you get at a bakery?***

After a brief discussion, continue:

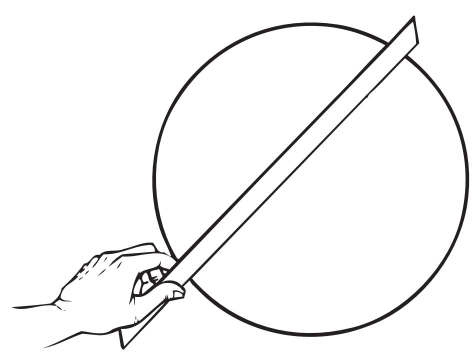
T: ***Mr. Booker is very famous in his town, because he is a very good baker and makes delicious cakes and cookies. Next door to the bakery live twins, Jack and Jill, and they especially like the cakes and cookies that Mr. Booker makes. One day they had saved enough money between them to buy a chocolate fudge cake.***

Display a circle cut-out on the board.

T: ***Jack and Jill want to share the cake fairly between the two of them. How can they do this?***

S: ***Cut the cake in two pieces.***

T: ***Like this?***



Don't draw a cut line, but demonstrate where it could be with a meter stick.

S: ***No, in half.***

Ask a volunteer to show how to cut the cake. Allow students to suggest folding the circle in half or using a mirror. If it is a fairly accurate job and the class is in agreement, draw the cut line.

T: ***Yes, I think that this is a fair cut. Both pieces look the same.***

G6

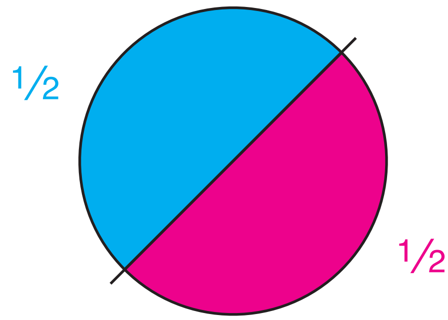
Color one side of the circle blue as you say,

T: ***This is Jack's share. How much of the cake does he get?***

S: ***One-half.***

T: ***I'll color Jill's part in red. How much of the cake does she get?***

S: ***One-half.***



Write this information on the board.

T: ***When both Jack and Jill have eaten their halves of the cake, what is left?***

S: ***Nothing; the cake is eaten.***

Write this equation on the board.

$$\frac{1}{2} + \frac{1}{2} = 1$$

T: ***One-half of the cake plus one-half of the cake equals the whole cake.***

Repeat Exercise 1 with one or two other shaped cakes; for example, you may choose an equilateral triangle, a regular hexagon, or a rectangle.

Exercise 2

For this exercise, organize the class into cooperative groups of three to five students each. Before giving a group task, present this situation to the class.

T: ***On their birthday, Jack and Jill went to visit Mr. Booker at the bakery. Mr. Booker had a nice surprise for them; he had baked 26 special cookies just for the twins. Do you have any idea what shapes these 26 cookies were?***

Let students make suggestions. Perhaps a student will think of the letters of the alphabet; otherwise, you should tell them that the cookies were shaped like the 26 letters of the alphabet.

T: ***Jack and Jill were now faced with a problem; they had to share the cookies fairly between them. How could they do this?***

S: ***Each of them gets 13 cookies.***

T: ***Why 13?***

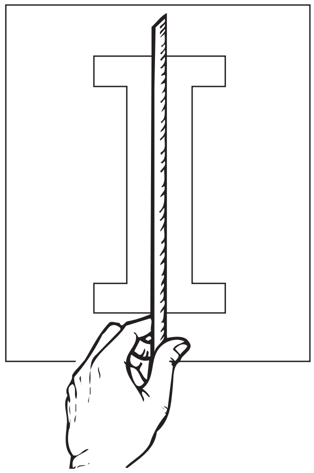
S: ***One-half of 26 is 13.***

S: ***13 + 13 = 26.***

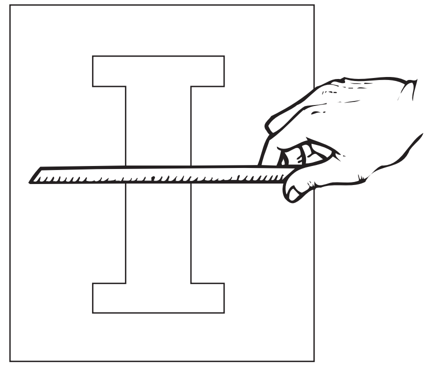
T: ***Jack thought hard about which 13 cookies he wanted, and Jill thought about which 13 cookies she wanted. It was a hard decision because some of the cookies looked bigger than others. Can you suggest another way for them to share the cookies that would be fair to both Jack and Jill?***

S: ***Cut each cookie in half.***

If necessary, suggest this idea yourself. Then tape a large copy of the letter **I** from the Blackline G6(a) on the board and give each group copies of the letter. Discuss how to cut the **I** cookie and call on a volunteer to demonstrate a cutting. Let the student use a ruler or other straight edge held on edge against the letter to show where it should be cut.



or



T: ***Would this cut allow Jack and Jill to share the cookie fairly?***

S: ***Yes.***

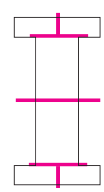
T: ***How can you tell?***

Stress that the cut should be made so that both pieces look the same. If the class and you agree that the cut is fair, draw a colored line to show it.

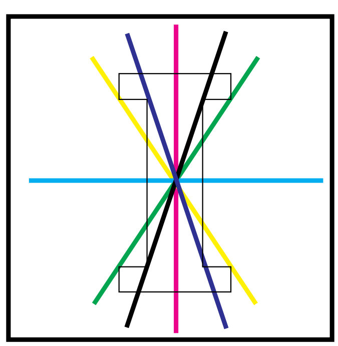
T: ***Are there other ways we could cut this cookie?***

Instruct students to find as many ways as they can to cut the **I** cookie in half fairly. Allow groups to draw cuts or make real cuts.

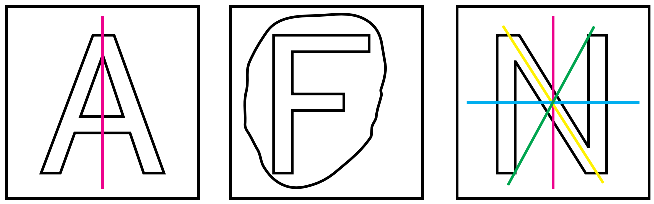
You may need to restrict ways to cut the **I** in half to those that use a single cutting line—do not allow students to cut the **I** in half using multiple cutting lines as pictured here.



For your information, several possible single cutting lines are shown below.



Repeat this cutting activity with several other letters; for example, **A**, **N**, and **F**. Use at least one letter that has only one cutting line (for example, **A**) and at least one that has no straight cutting line (for example, **F**). For your information, some cutting lines for the letters mentioned above are illustrated below. **F** is circled to indicate that it is hard to cut; that is, there is no one straight cutting line.



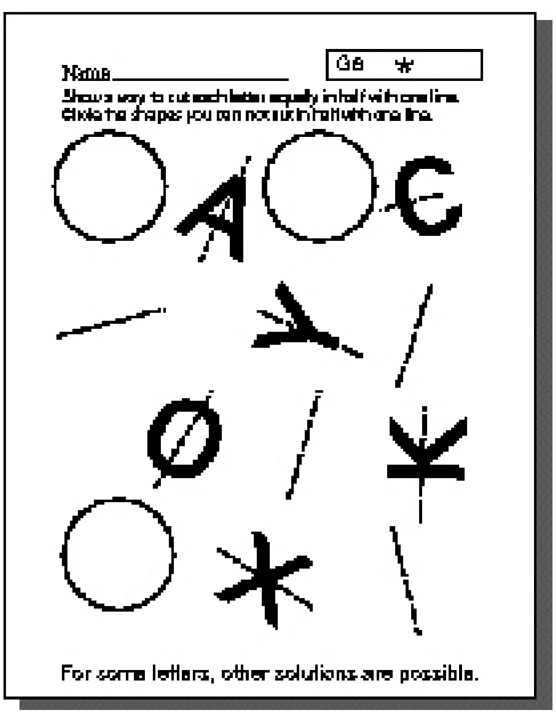
Worksheets G6* and ** are available for the rest of the lesson. Explain the instructions to the class and be sure the students understand that if they cannot find a cutting line, then they should circle the letter.

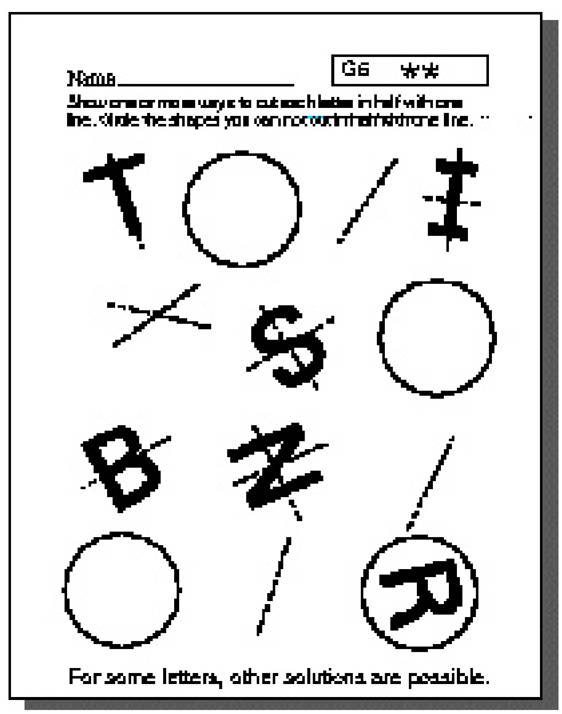
Center Activity

Place alphabet or other shaped cookie cutters in a center along with clay or paper. Students can mold, cut out, or trace the letters and shapes, and then find cutting lines or lines of symmetry—ways to cut the "cookies" in half.

Home Activity

Send several alphabet letters home with students (Blacklines G6 [e] and [f] contain alphabet letters). Instruct them to work with family members to find cutting lines.





Capsule Lesson Summary

Imagine that a grid is a map of Nora's neighborhood and investigate paths from one point to another (paths must follow the lines [streets] of the grid). Discuss the lengths of these paths (taxi-measure) and determine which is the longest and which is the shortest.

Materials

Teacher
- Grid board†
- Colored chalk

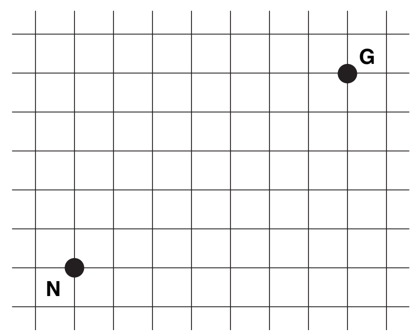
Student
- Worksheet G7
- Colored pencils, pens, or crayons

Description of Lesson

Exercise 1

Display your grid board. Draw and label two dots as shown.

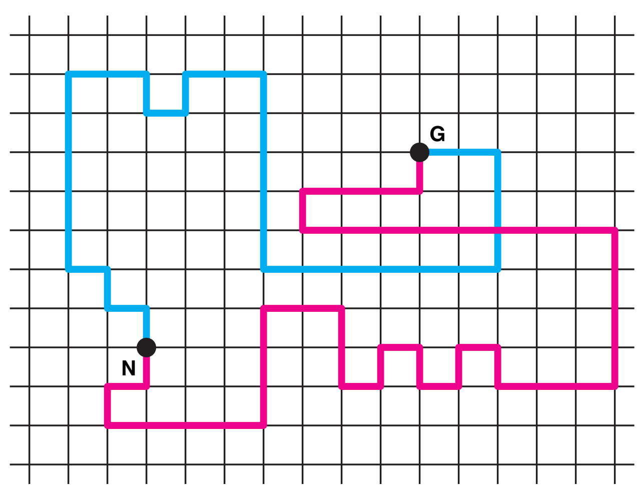
T: ***This is a map of a city. The lines are for streets. A girl named Nora lives in this city. Her house is here*** (point to **N**)***. Nora's grandmother lives here*** (point to **G**)***. Nora likes to visit her grandmother so she often walks from her house to grandmother's house. Nora knows it is not polite to walk across other people's yards, so she always walks on the sidewalks (along the streets). Who can show us a way for Nora to walk from her house*** (point to **N**) ***to her grandmother's house*** (point to **G**)***?***



By tracing with a pointer or finger, invite students to show paths from **N** to **G**. Accept any path that follows the grid lines.

T: ***Sometimes Nora likes to walk around the city and sightsee on the way to her grandmother's. Can someone show us a long path that Nora could take? Remember, Nora has to follow the streets.***

Invite two or three students to show different paths from **N** to **G**. Do not draw the paths; just let the students trace the paths with their fingers or a pointer. For your information, two different paths are shown here.



Distribute Worksheet G7 and colored pencils or crayons. Students can work with a partner, but each should have a worksheet.

†See the "Note on Grids" section in the introduction to this strand.

T: ***On this worksheet you are to draw some paths from* N *to* G*. Use different colors for your different paths. Remember, follow the streets!***

As the students work on this problem, give individual help to those who have not understood what a path from **N** to **G** is. Working with a partner, students can check each other. Allow sufficient time for the students to be free and creative in finding interesting paths.

You may want to ask student partners to describe paths to each other. For example, one student chooses a path he or she has drawn. Then, without showing it to the partner, the student gives directions to the partner to draw (copy) this path on the partner's worksheet. The students should then compare the paths to check that they are the same.

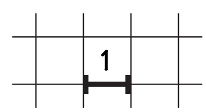
When several students have drawn many interesting paths on their papers, ask some of them to copy one of their paths on the demonstration grid board. Alternatively, ask a student to describe a path as you or another student draw it on the board. Try to get paths of various lengths and paths going through many different parts of the grid. Give students different colors of chalk (or crayon or marker) to use.

Exercise 2

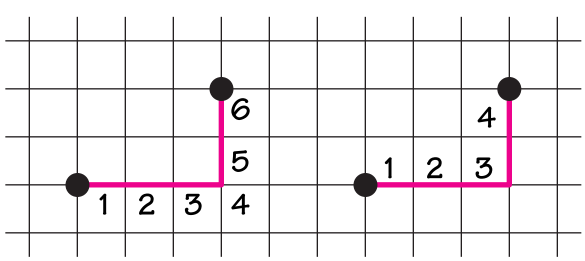
After a while (perhaps about ten minutes), call the class's attention to the paths that have been drawn on the demonstration grid board.

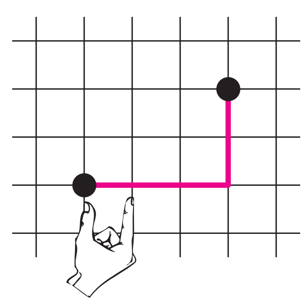
T: ***Here we have several different paths that Nora could take when she goes to her grandmother's house. Which one do you think is the longest? Which one is the shortest?***

Let the students discuss how to measure the paths and look for the longest and the shortest paths. If no one suggests that to measure the paths you can count the number of blocks, suggest it yourself. Indicate one block in an upper corner of your grid board.



Note: Some very common mistakes can occur when students count the blocks in a path, especially at the corner turns. The next illustration shows two common counting errors. Both paths in the illustration are, in fact, five blocks long. To avoid counting mistakes, it may be helpful to instruct the students to show each block with their thumb and index finger as they count. This technique is illustrated below.





Ask the students to measure the paths on the grid board and to determine which path is the longest and which is the shortest. Pay close attention to each student's counting technique and correct if necessary.

Then ask students to measure (count the blocks in) the paths they have drawn. Instruct them to find which of their paths is the longest and which is the shortest. Partners can check each other.

T: ***Some days Nora is in a hurry and wants to take a short way to her grandmother's house. Who can show us a short way for Nora to walk from her house to her grandmother's house.***

Call on students to trace paths. Check the length of the path each time, and then ask if someone can find a shorter way. At some point someone may argue that there is no path shorter than 12 blocks. If the class agrees, you can stop trying to find shorter paths.

End the lesson by making a list on the board of the different lengths students found for paths from **N** to **G**. If a student suggests a path could have an odd number length, ask the student to recount the blocks in that path. Perhaps the class will observe that all paths from **N** to **G** have an even number length, that a shortest path is 12 blocks (even though it may not be the length of their shortest path), and that paths can always be made longer.

Extension Activity

Let the class discuss trick-or-treating on Halloween. Then describe how Nora goes trick-or-treating: she starts at her house and walks along the streets stopping at houses along the way. Finally she returns to her house.

Invite students to trace paths starting at **N** and eventually returning to **N**. For each such path count the number of blocks. Make a list of possible lengths of trick-or-treating paths and observe that all such paths have even lengths.

Writing Activity

Ask students to write about paths from **N** to **G** and/or trick-or-treating paths. Suggest they look for patterns and describe both short and long paths.

Capsule Lesson Summary

Introduce a tape measure for measuring the lengths of certain objects. If appropriate, compare the separation of marks on the tape measure to centimeter rods. Measure many different objects in centimeters (white rods) and decimeters (orange rods).

Materials

Teacher
- C-rods (optional)
- Centimeter tape measure
- Blackline G8

Student
- Centimeter tape measure
- Recording chart

Advance Preparation: Use Blackline G8 to make student copies of a recording chart.

Description of Lesson

Note: If you have a set of Cuisenaire rods or other centimeter rods available, begin with Exercise 1A. Otherwise start with Exercise 1B.

Exercise 1A

Exhibit one white rod (or a one centimeter rod).

T: ***I would like to measure this side*** (point out the width) ***of my desk*[†] *with white rods. About how many white rods long do you think this side is?***

Let many students give estimates and list them on the board for later discussion and comparison with the actual measurement.

T: ***How shall we make this measurement?***

Discuss with the class the merits of the different ways suggested by students.

If the students are quite familiar with C-rods, it is likely that someone will suggest using the long orange rods. For example:

S: ***Use orange rods and count ten white rods for each orange rod.***

Call on some students to actually make the measurement with orange rods and white rods, or use any other method the class agrees upon. Record the result on the board. For example:

7 orange rods + 6 white rods = 76 white rods

Compare the actual measurement with the students' estimates.

T: ***Now I would like to measure the side of my desk with red rods. Do you know an easy way to do this?***

[†]You can substitute any other suitable object in your classroom. Try to choose something between 50 and 100 white rods (centimeters) long.

S: ***A red rod is the same as two white rods; we need half as many red rods.***

Accept any reasonable suggestions. You can direct the discussion so as to use one of the following methods. The first method suggests counting by fives because five red rods make one orange rod.

7 orange rods + 6 white rods = 35 red rods + 3 red rods = 38 red rods

76 white rods = 38 red rods

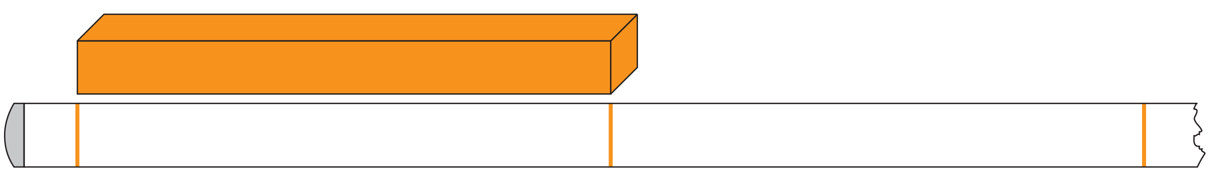
If the class is enjoying this activity and is quite familiar with C-rods, continue by asking for the same measurement with yellow rods and with red rods. For example:

7 orange rods & 6 white rods = 76 white rods
or
between 15 and 16 yellow rods
or
38 red rods

Display one of the tape measures as you say,

T: ***Measuring my desk by putting rods end to end wasn't so easy. But I have something here that may make the job of measuring a little easier. It's called a "tape measure." On one side there are orange marks, and between the two orange marks we can fit an orange rod.***

Demonstrate this fact.

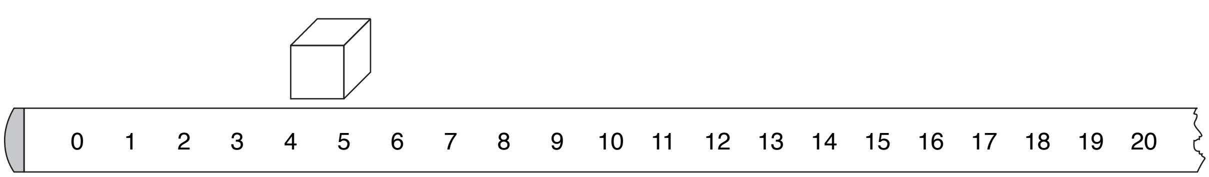


T: ***So we could measure the desk with this tape measure.***

Demonstrate measuring the desk with the tape measure and ask a student to count the spaces between orange marks. Be careful to put the end with the "0" mark at one edge of the desk. For this example, a student would count seven full spaces, and find that the other edge of the desk lands within the eighth space between orange marks.

T: ***On the other side of the tape measure a white rod will fit between the marks.***

Demonstrate this and then measure the desk with this side of the tape measure.



Exercise 1B

Display one of the tape measures.

T: ***I would like to measure this side*** (point out the width) ***of my desk with this tape measure. On one side of the tape measure are the marks for centimeters*** (write the word on the board and show the class that side of the tape measure)***. About how many centimeters long do you think this side of my desk is?***

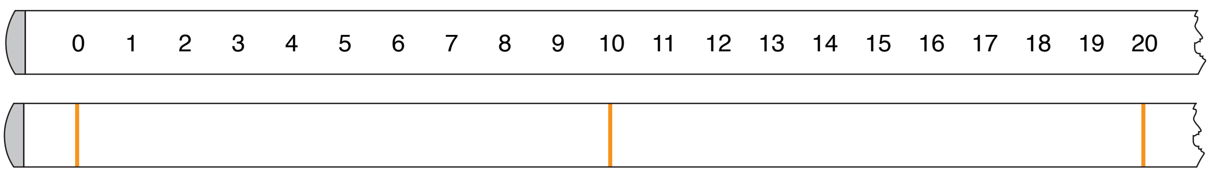
Let many students give estimates and list them on the board. Then, with a couple of students assisting you, measure the width of your desk. Record the result on the board; for example, it might be about 76 centimeters. As you record the result, point out that "cm" is an abbreviation for centimeters.

76 cm

Compare the actual measurement with the students' estimates.

T: ***Now I would like to measure the side of my desk with the other side of the tape measure. There are orange marks on this side of the tape measure. Between two orange marks there are ten centimeters; we will call this an orange length.***

Demonstrate this to the class.



T: ***How long is this side of my desk if I measure it with the orange lengths?***

S: ***More than seven orange lengths.***

S: ***Between seven and eight orange lengths.***

Do not be concerned if your students do not give these responses; let them make guesses and then measure the width of your desk. Ask a couple of students to assist you and to count the orange lengths. Record the result on the board.

76 cm

T: ***We could be more exact if we counted as many orange lengths as possible and then measured the remaining part with centimeters.***

Ask a couple of students to measure the desk again in this way; then write the measurement on the board.

76 cm

between 7 orange and 8 orange

Exercise 2

Allow students to work in pairs for this exercise. Distribute tape measures and a recording chart.

Refer students to the recording chart.

T: ***We just finished measuring my desk. Let's record the result in this chart.***

Direct the students to record whatever you measured in Exercise 1. Then give them several other objects in the room to measure in a similar way. Leave space in the chart for students to add other items they wish to measure.

What we measured †	Orange rods	White rods (centimeters)
Teacher's desk	Between 7 orange and 8 orange	76 cm
Pencil		
Paper (long side)		
Paper (short side)		
My shoe		
Student desk		
My arm		
My height		

Instruct student partners to find the measurements and record them in their charts. Individual students should record measurements for their own shoes, arms, and so on. Partners should check to see that the tape measure is used correctly. They may like to compare personal measurements.

Attach a tape measure to the wall somewhere in the classroom so that you can measure the height of each student. Call them one at a time to be measured. Student partners who finish measuring everything on their lists can be asked to measure bigger objects, such as the width or length of the chalkboard or a bulletin board.

When most of the class has finished measuring everything on their lists, collectively discuss some of the measurements. It may be interesting for the class to know who is the tallest and who is the shortest, for example. If some students measured the dimensions of the chalkboard, you might ask the rest of the class to predict how long it is before the result is announced.

Home Activity

Measure some items at home (such as the length and width of a bed, parents' shoes, the height of a lamp, and so on) in centimeters and decimeters (orange rods). Send home a tape measure to be returned to school.

†This is only a suggested list; you may think of other objects in your classroom that would be more interesting for the students to measure. You may also let students suggest some things they would like to measure.

Capsule Lesson Summary

Construct a Tangram using a story to help describe the pieces.

Materials

Teacher
- Paper square
- Scissors
- Blackline G9 (optional)

Student
- Paper square
- Scissors
- Geoboard (optional)

Advance Preparation: Use colored paper to make paper squares approximately 15 cm or 6 inches on a side. Blackline G9 has a good size square to copy.

Description of Lesson

Note: In this lesson, the students will construct their own Tangram. It will be difficult for many children to do an accurate construction (folding and cutting); however, it is not very important that the final product be perfect. The more important aspects of the lesson are in following directions, describing shapes, and reassembling the pieces.

Begin the lesson with a brief mention of a Tangram as a Chinese puzzle. You may like to read the book *Grandfather Tang's Story* by Ann Tolbert.

T: ***Now we are all going to make our own Tangram.***

Distribute paper squares and scissors to individual students or to student partners. Hold up a paper square.

T: ***What shape is this? How many sides? How many corners?***

S: ***Square. A square has four sides and four corners.***

T: ***Trace the four sides of your square. Point to the corners. Show me opposite corners.***

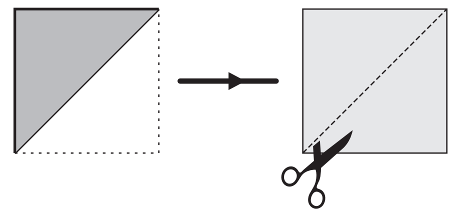
After a brief discussion of a square, continue with directions on construction of a Tangram. Model each direction as you give it.

T: ***I'm going to tell a story that will help us give names to the Tangram pieces.***

The following is a story outline with action steps. Give the story as much play or adornment as is comfortable. At each step, be sure to solicit discussion of the shapes—what they look like, number of sides, number of corners, and so on. Also, after each step, direct the students to reassemble the pieces until they again have the original square.

T: ***This is a giant's house.*** (Hold up the square.)

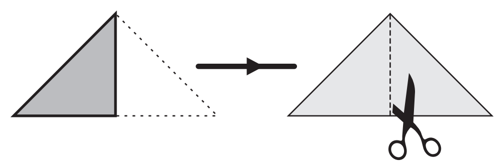
T: ***Let's go inside.*** (Fold the square along the diagonal—bring two opposite corners together and fold.)



(Open the square and cut along the fold to get two big triangles.)

The students should describe the two pieces at this point and reassemble them to get the original square.

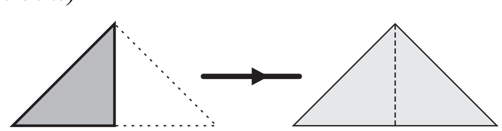
T: ***The giant has two triangle chairs.*** (Fold one big triangle in half—bring the corners [endpoints] of the longest side [base] together and fold.)



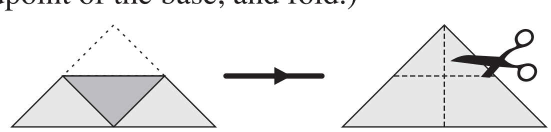
(Open the big triangle and cut along the fold to get two smaller triangles.)

The students should describe the three pieces at this point and reassemble them to get the original square.

T: ***The other triangle is the giant's tent.*** (Fold the other big triangle in half [as above] but do not cut.)



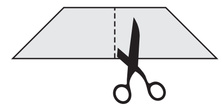
T: ***Now this big triangle is the giant's boat and motor.*** (Open up the big triangle and fold the top down; that is, bring the top corner down to meet the midpoint of the base, and fold.)



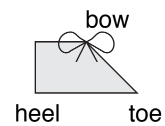
(Open the big triangle and cut along this fold to get a small triangle and a trapezoid.)

The students should describe the four pieces at this point and reassemble them to get the original square.

T: ***When we cut the boat in half, we get the giant's two boots.*** (Cut the trapezoid along the fold line.)

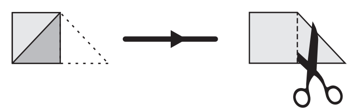


T: ***Here is the heel, the toe, and the bow.*** (Orient boots to illustrate.)



The students should describe the five pieces at this point and reassemble them to get the original square.

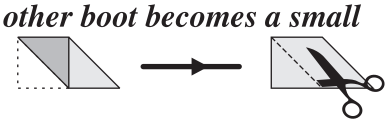
T: ***One boot becomes a small square table and triangle chair.*** (Fold one boot—bring toe to heel and fold).



(Open the boot and cut along the fold line.)

T: ***The other boot becomes a small triangle chair and bed.***

(Fold other boot—bring heel to bow and fold.)



(Open the boot and cut along the fold line.)

Now you have the seven pieces of a Tangram. Describe the pieces (two large triangles, one medium triangle, two small triangles, one square, and one parallelogram). Reassemble them to make the original square.

If you like, tell the students about the legend of the Tangram puzzle or read the book *The Tale of Tangram* by Allan Hirsch. The Tangram is a Chinese puzzle. A favorite square tile was dropped and broke into seven pieces. Scholars spent years trying to put it back together and in the meantime created many designs out of the seven pieces.

Allow some time for students to play with their Tangram pieces. Suggest they try to make different shapes (designs) with the seven pieces. You may also like to give students geoboards and ask them to make the Tangram piece shapes on their geoboards.

Home Activity

Let students take home their Tangram and try to reassemble it with family members.

Capsule Lesson Summary

Determine which of two shapes is bigger by comparing their respective areas in Tangram triangles or Tangram squares. Find the areas of some shapes in both Tangram triangles and Tangram squares.

Materials

Teacher
- Worksheet G10

Student
- Worksheets G10 (no star), *, **, ***, and ****
- Tangram triangles and squares

Advance Preparation: Prepare a set of ten Tangram triangles and ten Tangram squares for each student or pair of students. These triangles and squares can be punched out of the Tangram cards (right side) either by the students or by you, and stored in envelopes.

Description of Lesson

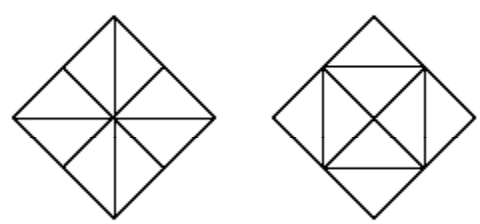
Distribute a set of Tangram triangles and Tangram squares to each student or pair of students. This is a good lesson for students to work on with partners. Give the students a few minutes to make their own designs and play with these shapes before proceeding.

Distribute individual copies of Worksheet G10. Hold up your copy of the worksheet so that the class knows which side to look at first.

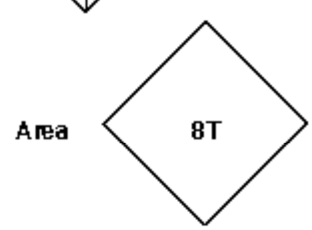
T: ***Look at the shapes on this page. Can you guess which shape is bigger?***

Let students comment on this situation. Instruct students to explain to their partners why they think that one shape is bigger than another. There may be different ideas of what it means to be bigger, and you may like to discuss different aspects of size. Eventually, direct attention to the figures. Hold up the worksheet and point to the large square.

T: ***Look at this shape at the top of the page. How many of these little (Tangram) triangles do you think it will take to cover this shape?***



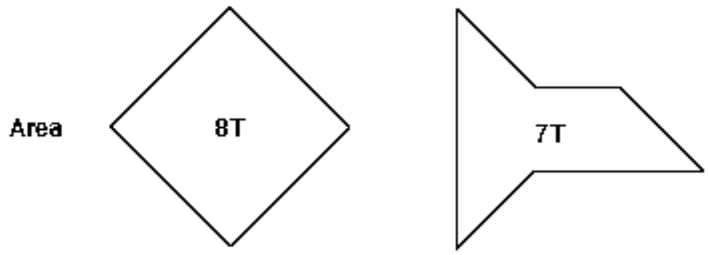
Accept several guesses from the students, and then ask them to find exactly how many little triangles it takes to cover the shape. Circulate and give help as needed. This illustration shows two ways to cover the big square with small Tangram triangles.



When students find that the correct number is eight, instruct them to write **8T** on the shape. To illustrate, sketch the shape on the board and write **8T** on it.

T: ***The area of this shape is eight little triangles. Now look at the shape at the bottom of the page. How many little triangles do you think it will take to cover this shape?***

Again, accept some guesses from the students, and then ask them to find exactly how many little triangles it takes to cover the shape. When they have found that the correct number is seven, instruct them to write **7T** on the shape. Sketch the shape on the board and write **7T** on it.



T: ***The area of this shape is seven little triangles. Which of these two shapes has a bigger area?***

S: ***The square (or diamond).***

Direct the students to put aside the small Tangram triangles and to take out the Tangram squares. Hold up the worksheet so the class knows that now you will look at the other side.

T: ***Turn over your worksheet. This time we will use our little squares to find the areas of these shapes. Which of these shapes do you think is bigger?***

Instruct students to explain to their partners why they think (without measuring) that one shape is bigger than the other.

T: ***How many little squares do you think it will take to cover this shape?*** (Point to the shape on the left.) ***How many little squares do you think it will take to cover this shape?*** (Point to the shape on the right.)

Let the students make some guesses, and then let them cover each shape with little squares. When they find that both shapes have area six little squares, instruct them to write **6S** on both shapes.

Your students may be surprised that the areas are the same while the shapes look very different. Let them comment on this situation. They may suggest or you may want to suggest that one shape is *long and narrow* and the other shape is *short and wide*.

T: ***How many little triangles will make one little square?***

S: ***Two.***

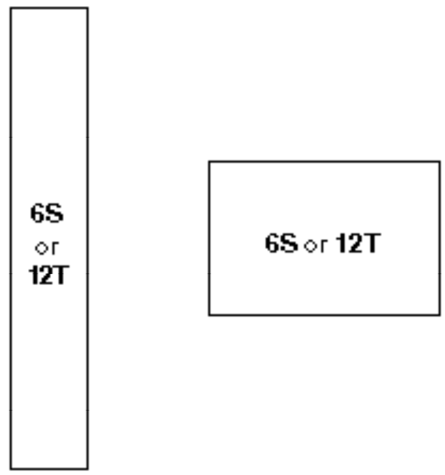
T: ***Suppose we want to cover this shape*** (point to the shape on the left) ***with little triangles. How many little triangles will it take?***

S: ***12.***

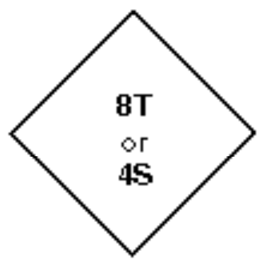
T: ***How do you know that?***

S: ***Because 6 x 2 = 12 (or 2 + 2 + 2 + 2 + 2 + 2 = 12).***

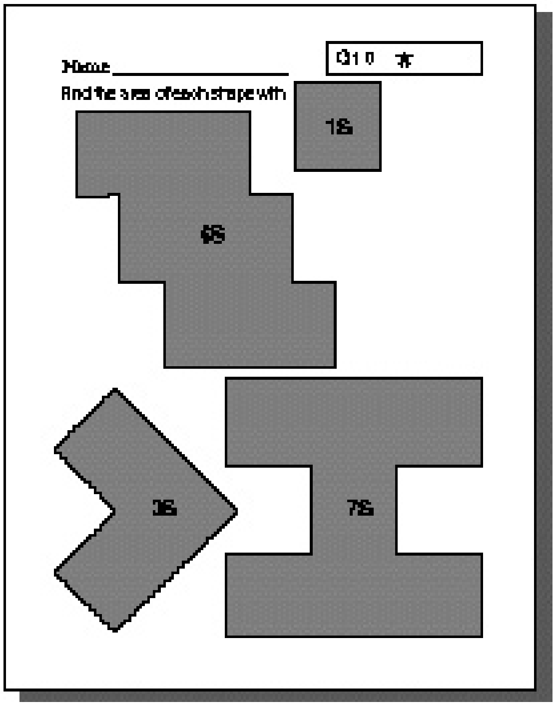
If your students are not certain about this answer, ask them to cover the shape with little triangles and count how many it takes. Repeat the same questions about the other rectangle; then record the information on the board.

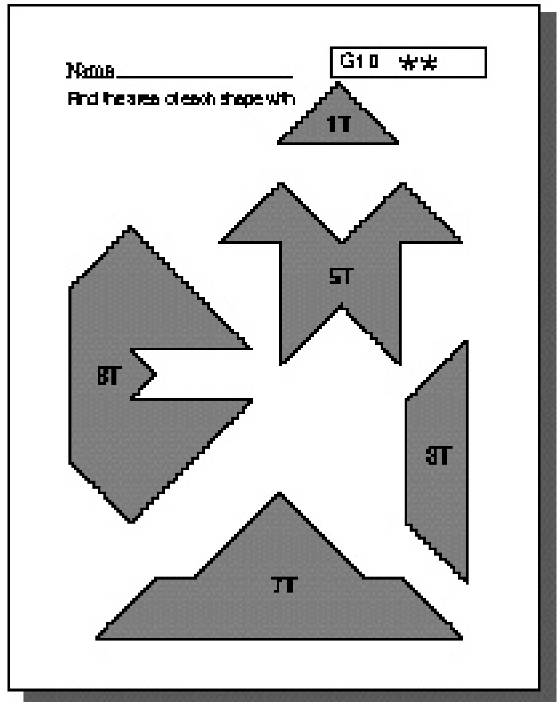


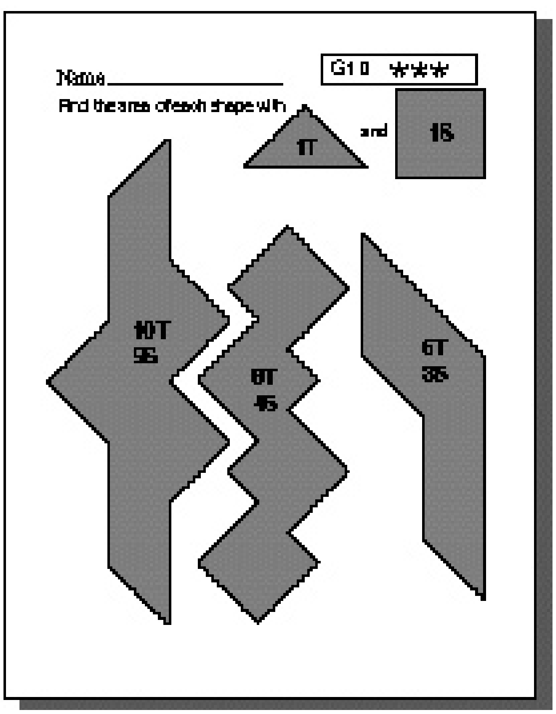
Ask similar questions about the large square on side 1 of the worksheet.

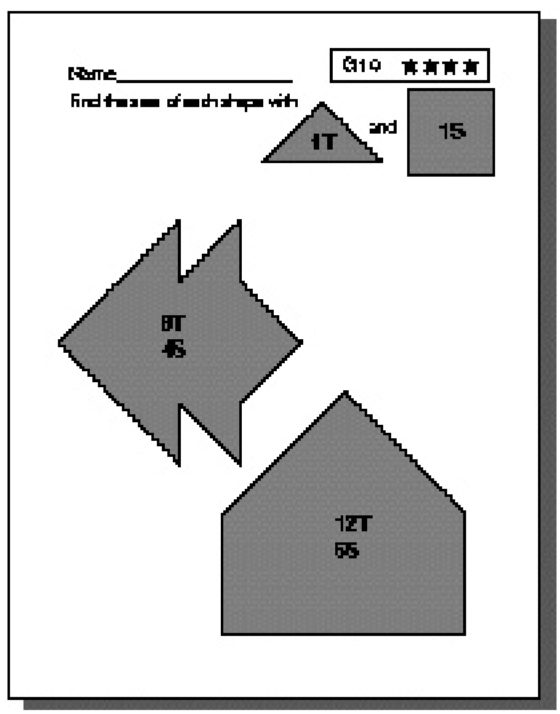


Worksheets G10*, **, ***, and **** are available for individual or partner work. Explain to the students that, on G10*, they are to find the area of each shape with little squares. Instruct them to write the area on the shape. On G10** they are to find the area of each shape with little triangles and to write the area on each shape. Students who correctly complete Worksheets G10* and ** should continue with Worksheets G10*** and ****. On these worksheets, they are to measure the area of each shape with both little squares and little triangles. Be sure that when they write the areas they indicate the units (**S** or **T**).


Name
G10 ★
Find the area of each shape with
1S
6S
3S
7S


Name
G10 ★★
Find the area of each shape with
1T
5T
8T
3T
7T


Name
G10 ★★★
Find the area of each shape with
1T
and
1S
10T
5S
8T
4S
6T
3S


Name
G10 ★★★★
Find the area of each shape with
1T
and
1S
8T
4S
12T
6S

G11 TAXI GEOMETRY #2

Capsule Lesson Summary

Review the idea of a path that follows the lines on a grid and find the lengths of some paths between two given points. Look for shortest paths between two points and observe a pattern in the lengths of paths. Introduce a hide-and-seek game to find all the points that can be reached with a four- or five-block path.

Materials

Teacher	• Grid board • Colored chalk or pens • Overhead projector (optional)	**Student**	• Worksheets G11(a) and (b)

Advance Preparation: You may want to make some extra copies of the worksheets for students who need a "new" copy.

Description of Lesson

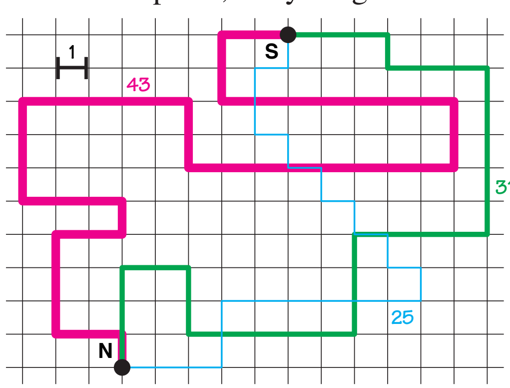
Exercise 1

Display your grid board. Draw and label two dots **N** and **S** as in the next illustration. In an upper corner of the board, indicate a one-block measure.

T: ***Do you remember our friend Nora? This is a map of the city where Nora lives and here is Nora's house*** (point to **N**). ***Today we are going to find paths that Nora could take to school. Here is Nora's school*** (point to **S**). ***Some days Nora likes to take a long walk going to school. Who can show us one way for Nora to go? Remember, Nora follows the streets.***

Let two or three volunteers trace paths from **N** to **S** with their fingers or a pointer. Accept any path that follows the grid lines. Then draw the suggested paths on the grid board.

Ask the students which of the paths is longest. They should suggest counting the blocks in each path; otherwise, make the suggestion yourself. Point to ├1┤ to remind the class what one block is. Invite students to count the blocks in each path and watch that they count correctly. Then label each path as illustrated below. You will have different paths, but your grid board should look similar to this.



T: ***Some days Nora is in a hurry and wants to take a short way to school. Would someone like to show us a way for Nora to walk to school that is shorter than any of these?***

Call on students to trace paths. Draw paths and ask the students to count the number of blocks in their paths. It is likely that one student will have chosen a 15-block path. Whether or not this is so, continue to ask for a shorter path until the class agrees that there is no path shorter than 15 blocks.

Distribute Worksheet G11(a) and colored pencils.

T: ***On this worksheet, you are to draw several paths from Nora's house to school. Try to draw some long paths and some very short paths. Each time you draw a path, count the number of blocks and label the path like we did on the board.***

While the students are working, observe individual work. You can easily catch some counting errors because all paths from **N** to **S** should have an odd number of blocks.

After about five to ten minutes, make a list in numerical order of the lengths of some of the students' paths. Your list should suggest that 15 is the shortest length path and that all paths have odd lengths. Many students will observe that an even number cannot be the length of one of these paths though they may not verbalize the pattern.

Exercise 2

Clear your grid board except for the dot labeled **N**. You may like to move **N** so it is more centrally located on your grid.

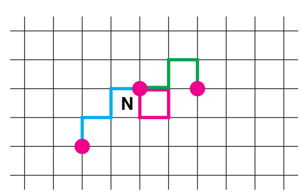
T: ***This is a map of the city where our friend Nora lives and here is Nora's house*** (point to **N**)***. Today Nora is playing a kind of hide-and-seek game with her brother, Nick. She tells Nick that she is going to take a four-block walk and then hide. Nick has to try to find her. Where should Nick look?***

Let the students come to the board and point to places where Nora could be hiding. Each time a place is correctly located, draw a red dot there. After two or three correct places have been located, ask the students to find as many places as they can on worksheet G11(b) where Nora could be hiding and to draw a red dot at each one. Remember, Nora can take any four-block walk. Observe and help students who are having difficulty. When you notice a new place on a student's paper, ask the student to draw a red dot at that place on the demonstration grid board. Occasionally, ask a student to explain how he or she found a place Nora could be hiding. For example:

S (tracing the blue path)**:** ***She could take this path.***

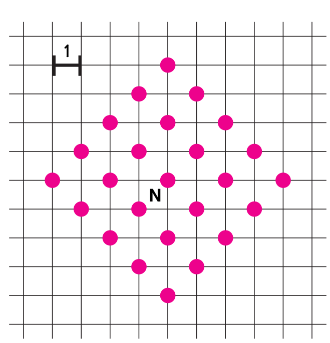
S (tracing the green path)**:** ***She could take this path.***

S (tracing the red path)**:** ***She could be hiding at her house.***

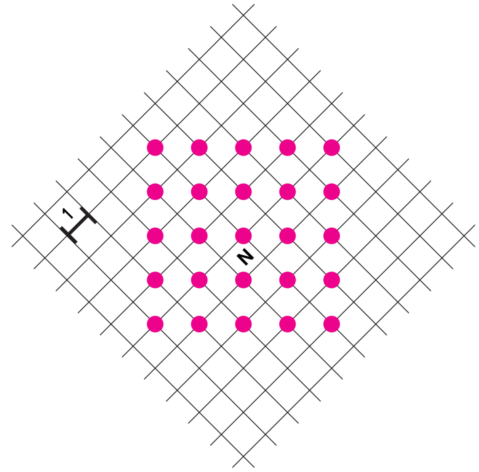


A completed picture should have 25 red dots as shown here.

This problem will not be easy at first. Do not be discouraged if students make many counting errors in the beginning; just encourage them to be careful. You may have to remind the class several times that Nora can take any four-block path. Soon some students should observe a pattern. Encourage them to describe any patterns they see in the picture.



After all the red dots have been drawn, ask a student to find the number of places where Nora could be hiding. (25)

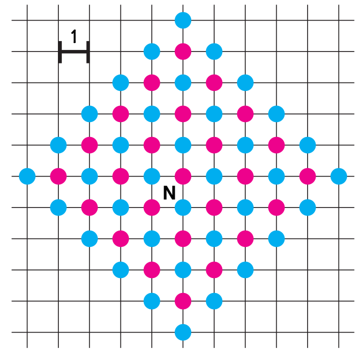


The students may notice that if you tilt the picture, you can easily see five rows of five dots each.

This observation makes counting the dots much easier: either count by fives or calculate 5 x 5. You may suggest looking at the picture in this way if it escapes the notice of your students.

T: ***Now it is Nick's turn to hide. He tells Nora he will take a five-block walk and then hide. Where are the places that Nick could hide?***

First, let several students locate some points on the demonstration grid board and, when a correct place is located, draw a blue dot there. Then ask the students to try to find on their worksheets all the places where Nick could hide and draw a blue dot at each one. As new places are found, invite students to draw the blue dots on the demonstration grid board. Occasionally ask students to explain how they found places where Nick could hide. Once a pattern is observed, one student can draw several blue dots at a time.



A completed picture should have 36 blue dots as shown here.

Encourage the students to comment on patterns in the picture and to notice that Nick's hiding places are all different from Nora's. Call on a student to count the dots. (36)

Again, if you tilt the picture, you will notice six rows of six blue dots. This observation should not be too difficult if you previously observed five rows of five dots this way. To count the blue dots, calculate 6 x 6 or count by sixes.

T: ***How many more places does Nick have to hide than Nora?*** (11)

Writing Activity

Instruct students to write a story about Nora and Nick's hide-and-seek game and to illustrate their stories.

Capsule Lesson Summary

Individually work on the Tangram booklet to make designs with all the Tangram pieces.

Materials

Teacher	• None	**Student**	• Tangram • Tangram booklet

Advance Preparation: For this lesson involving the Tangram, each student will need a set of Tangram pieces. The set consists of seven pieces (two large triangles, two small triangles, one medium-size triangle, one square, and one parallelogram) and is found on the left side of the Tangram card. Punch out a set for each student and keep each individual Tangram set in an envelope.

Description of Lesson

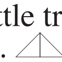
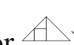
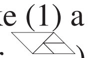
Exercise 1

You may like to use an overhead projector to demonstrate this exercise.

Distribute individual Tangram sets and spend a few minutes discussing the Tangram. For example:

T: ***How many pieces are there?*** (Seven)
What shapes are the pieces? (Triangles, squares, parallelograms)
What sizes are the pieces?

Place the Tangram pieces on the projector and demonstrate putting two or three pieces together to form another shape. Allow a few minutes of free exploration putting pieces together to form shapes. Then give some explicit assembly instructions such as the following:

- Use two pieces to make another square. (Two little triangles to make a square like the Tangram square or two big triangles to make a big square: )
- Use two pieces to make another triangle. (Two little triangles to make a medium triangle, or two big triangles to make an even bigger triangle: )
- Use two pieces to make another parallelogram. ()
- Use three pieces to make (1) a triangle ( or ); (2) a square (); (3) a rectangle ( or ).
- Use four pieces to make (1) a triangle ( or ); (2) a square (or); (3) a parallelogram ( or )

Exercise 2: Tangram Puzzles

Distribute Tangram booklets a~~nd tell the students that~~ the Tangram is thought to be an ancient Chinese puzzle. Legend says that a favorite square tile was dropped and broke into seven pieces. Scholars spent years trying to put it back together and in the meantime created many interesting designs with the seven pieces. Show the students the square on the cover of the booklet. Instruct them to use their Tangram pieces to cover the square. Then ask them to make the same square next to the booklet.

Tell the students that in the Tangram booklet there are many different designs they can make using all the Tangram pieces. The instructions for each page of the booklet are the same: use all seven Tangram pieces to cover the shaded picture on that page. When they have done so, direct students to raise their hands and you will check. Each time you check a page indicate that the student (or pair of students) has successfully made the design with Tangram pieces.

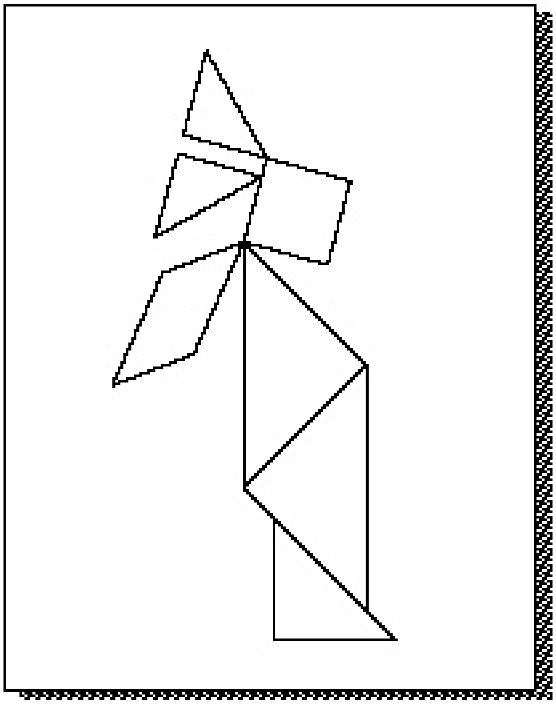
The puzzles vary in difficulty; encourage students to be patient but persistent. Be prepared to give occasional clues (for example, place one or two key pieces or draw some lines to show how some of the pieces fit in a puzzle). If students become frustrated, instruct them to go on to another page and come back to that page later. If some students think Tangram pieces are missing from their sets, suggest they check by putting all their pieces on the cover square. Students who do not want to wait for you to check their pages can draw dividing lines (as on the cover) to show how pieces are put together to make a shape.

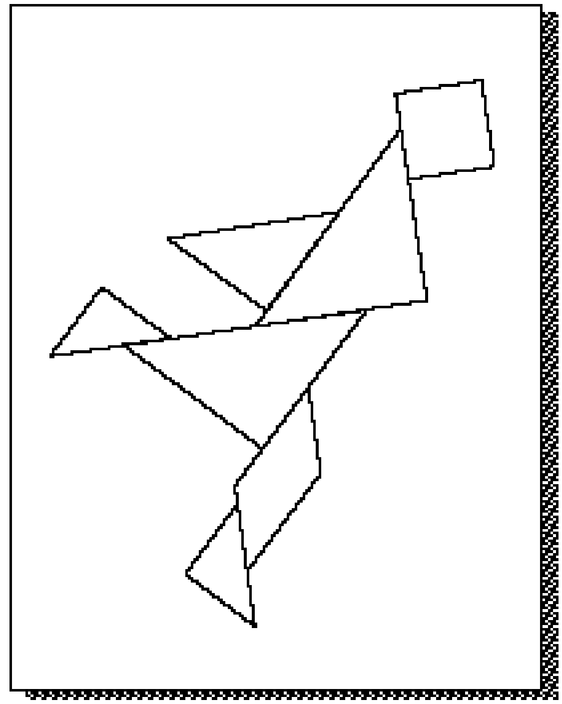
Center Activity

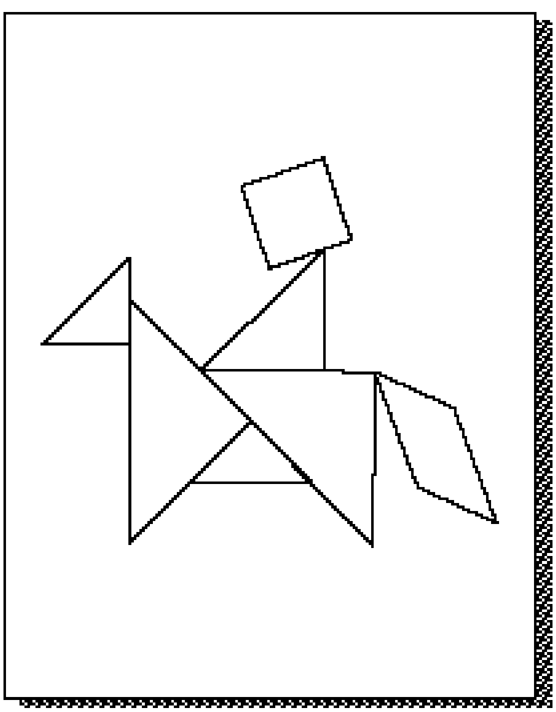
Tangram booklets and Tangram sets can be put in a center for free exploration. Later you can put other designs on task cards. There are many books available with designs to make with the seven Tangram pieces.

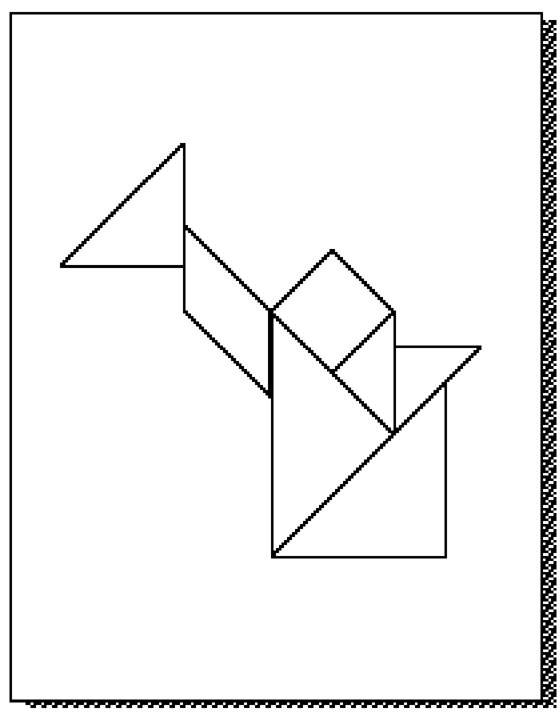
Home Activity

Copy the Tangram blackline master (Blackline G12) on card stock and send it home with a puzzle for students to complete with a family member.

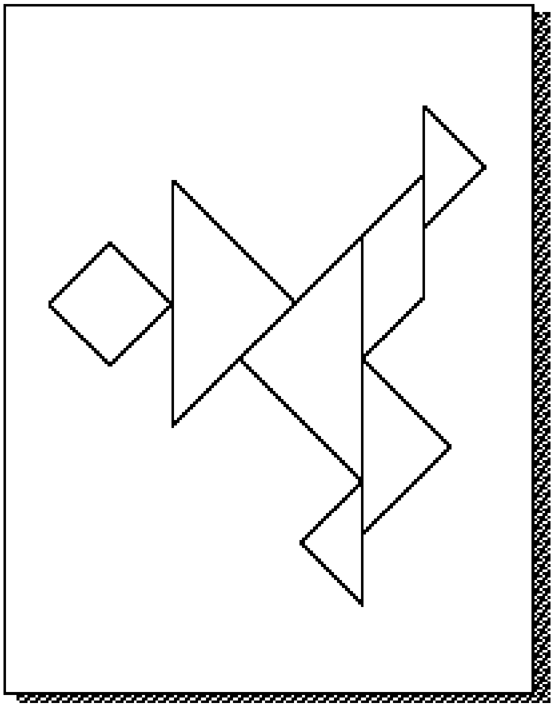


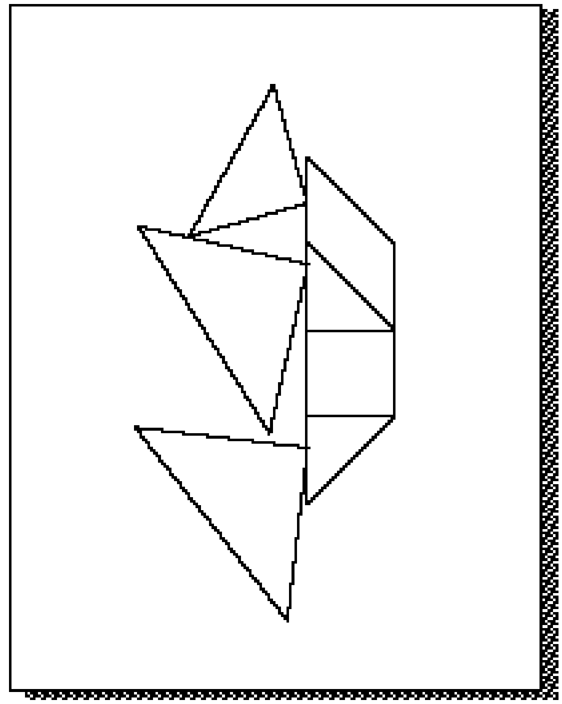


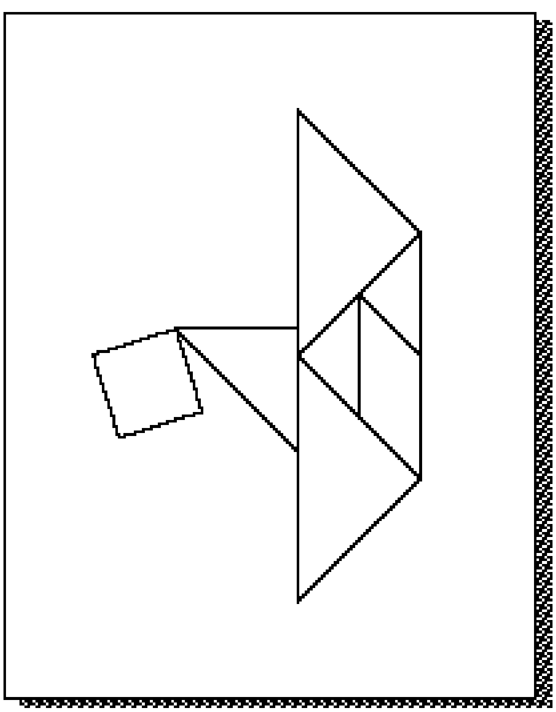


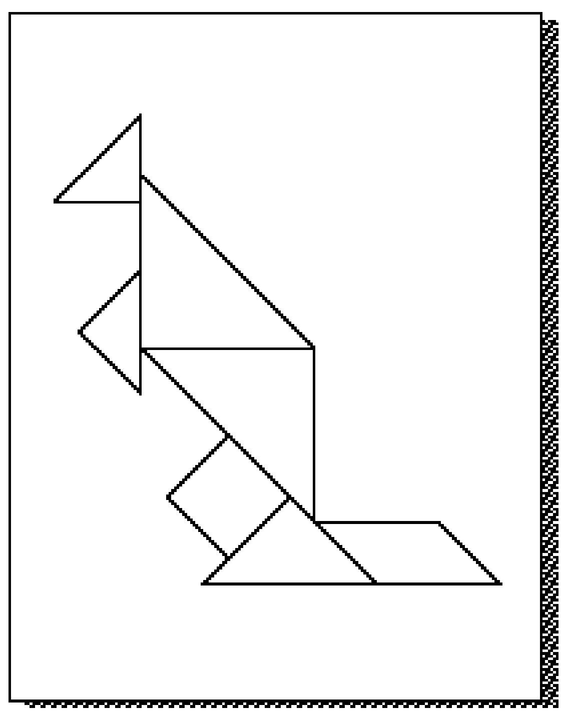


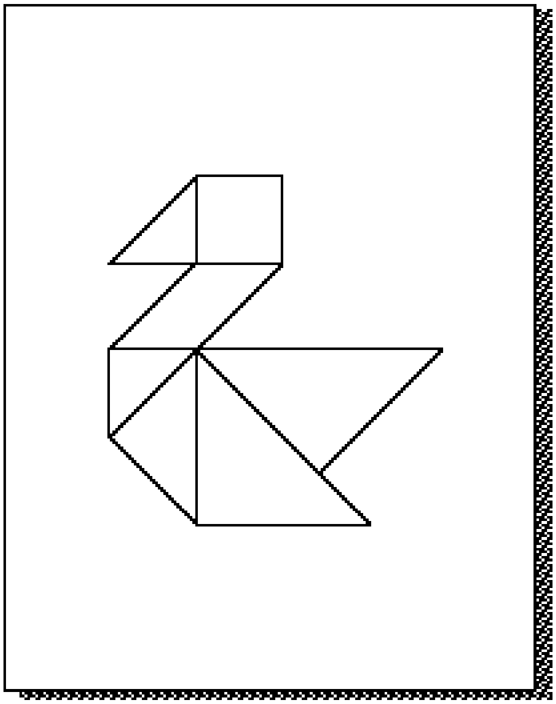
G 12

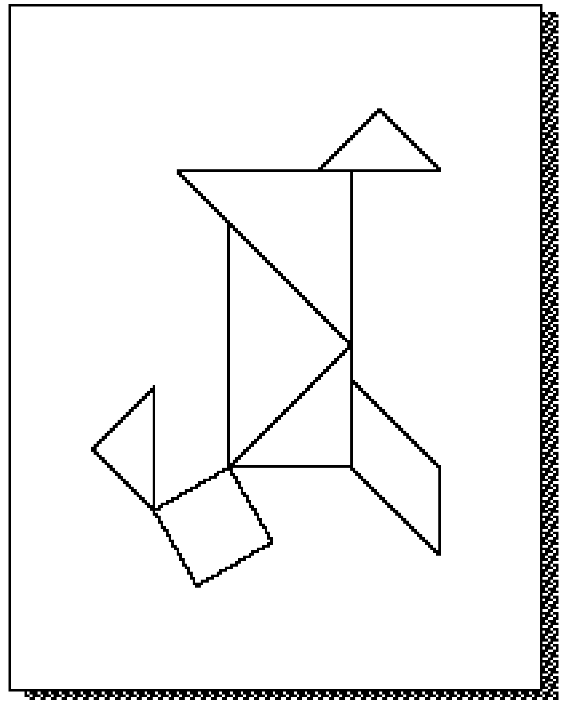


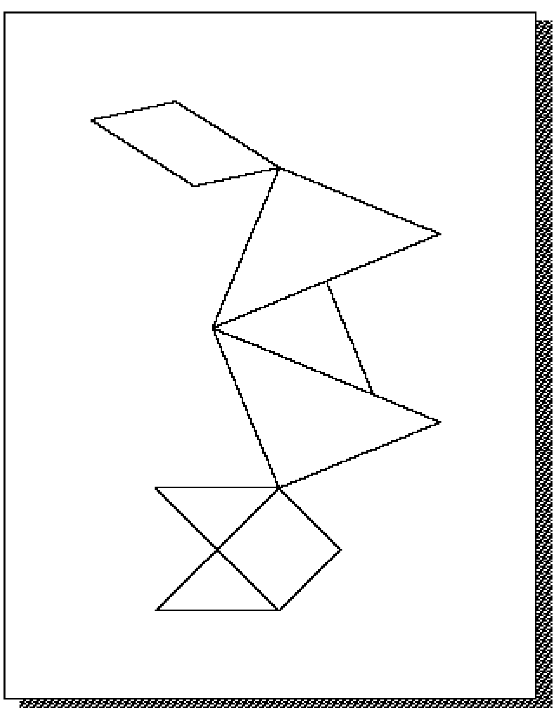


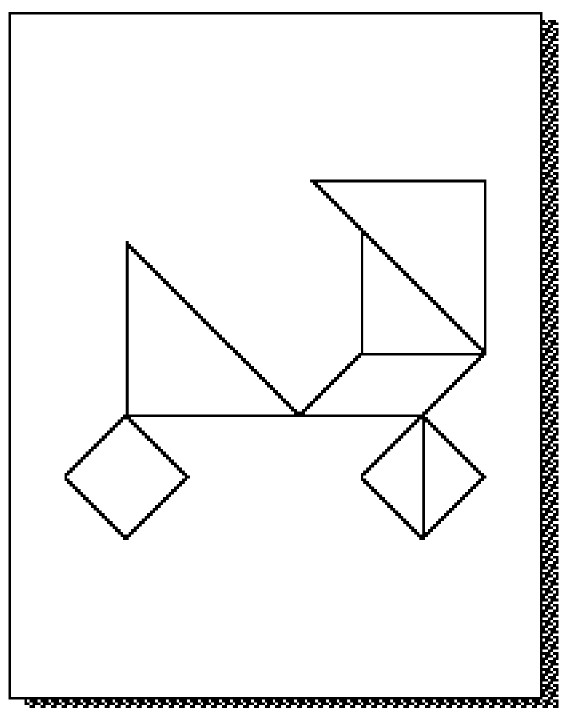




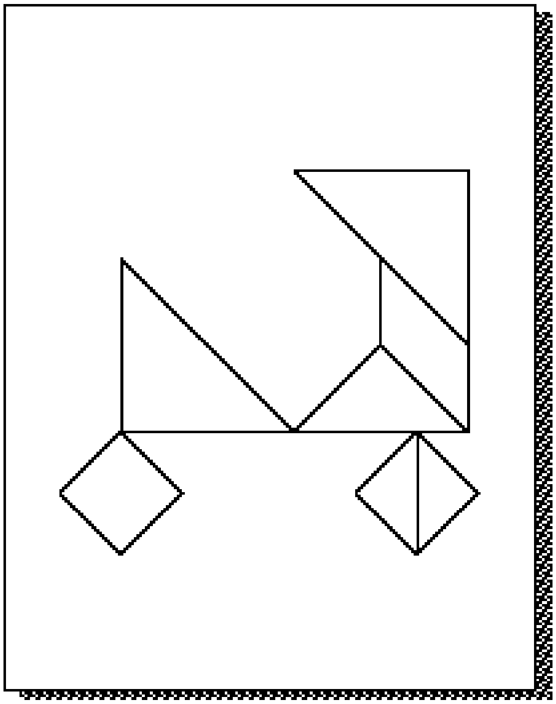


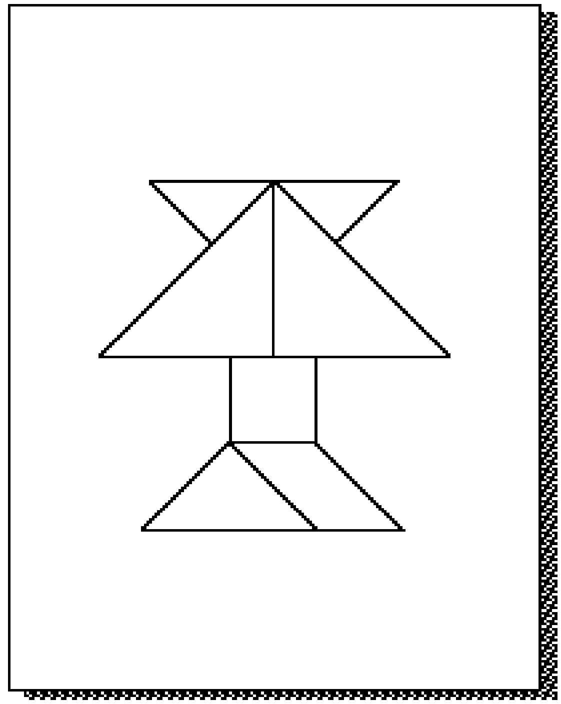


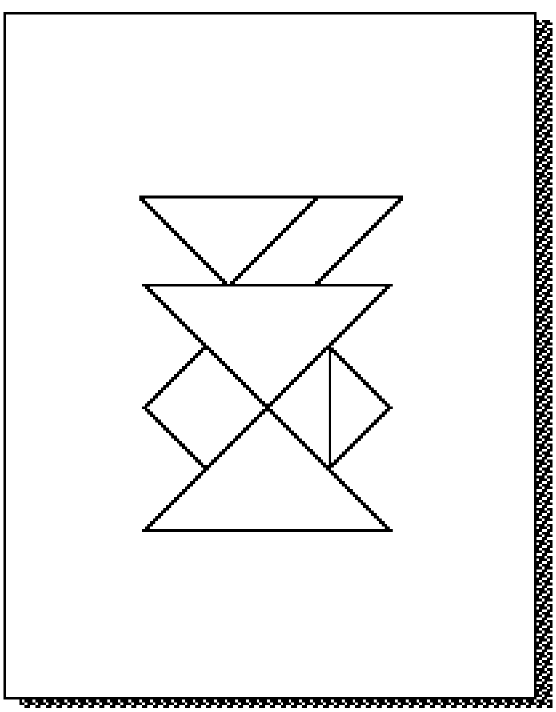


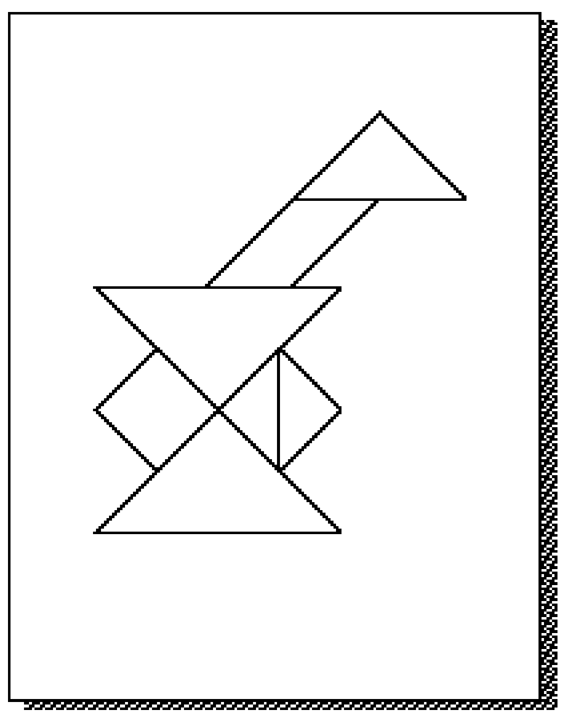


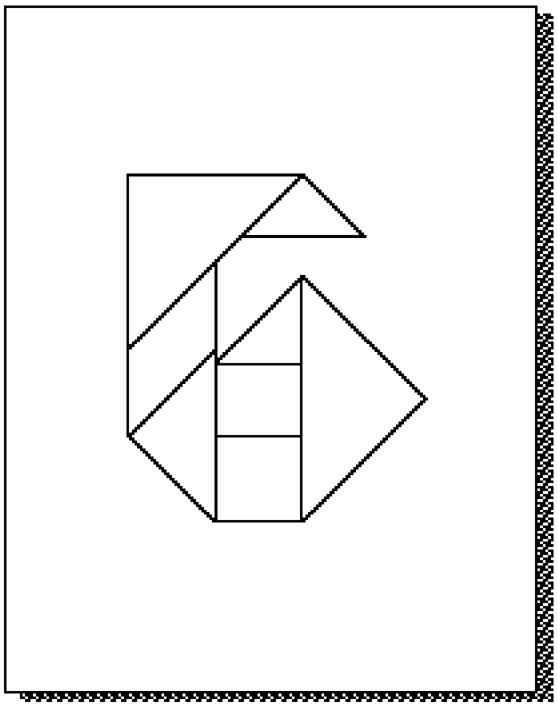
G12

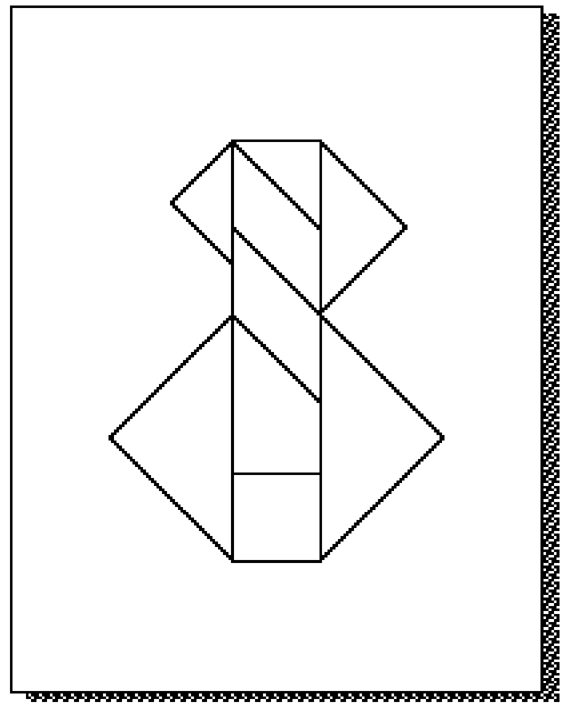


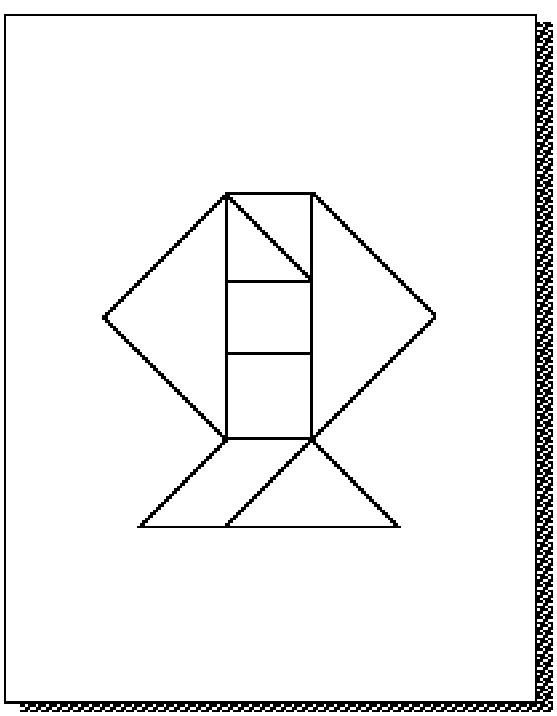


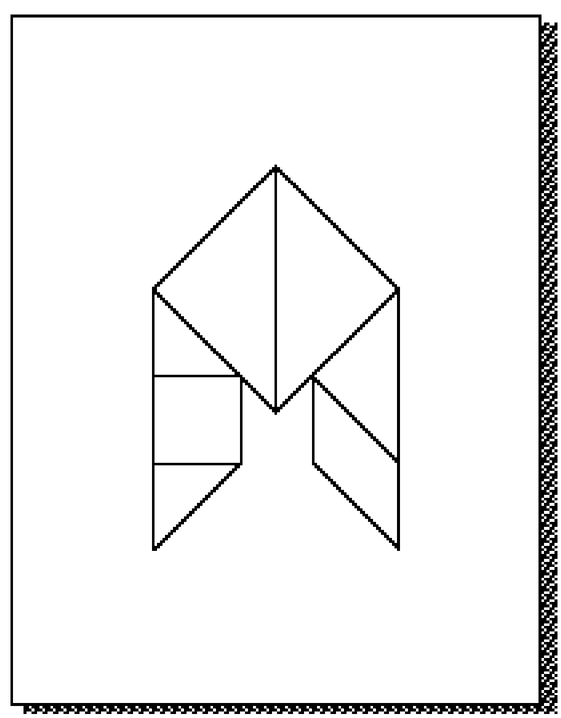


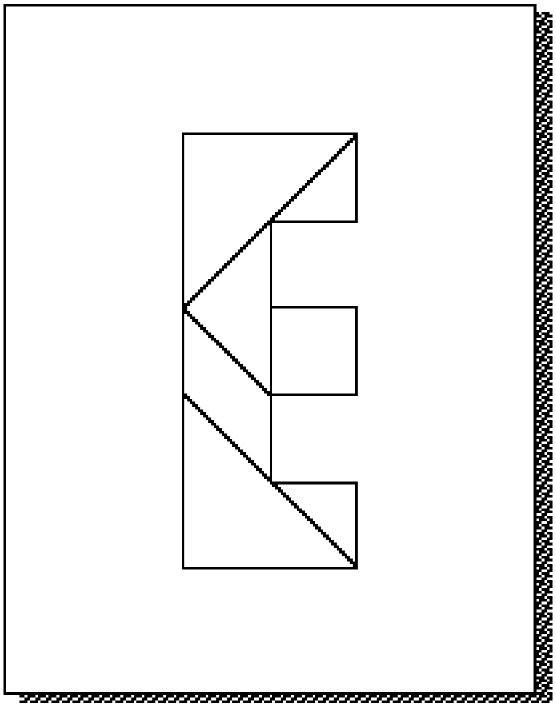


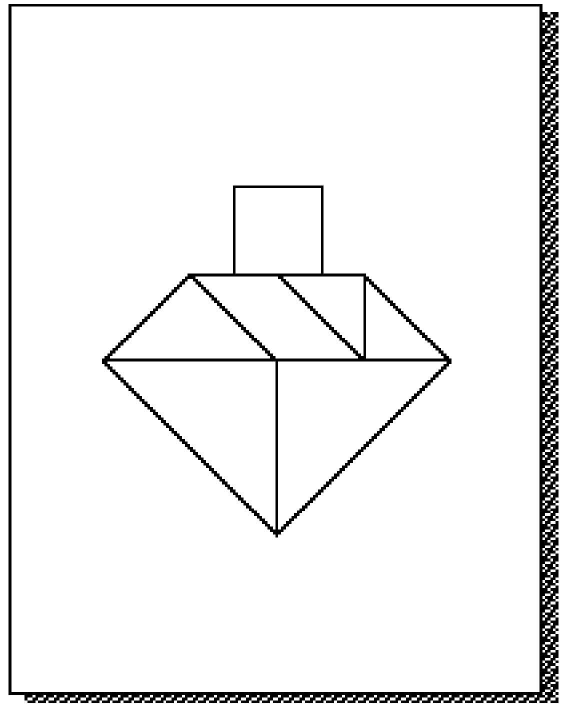


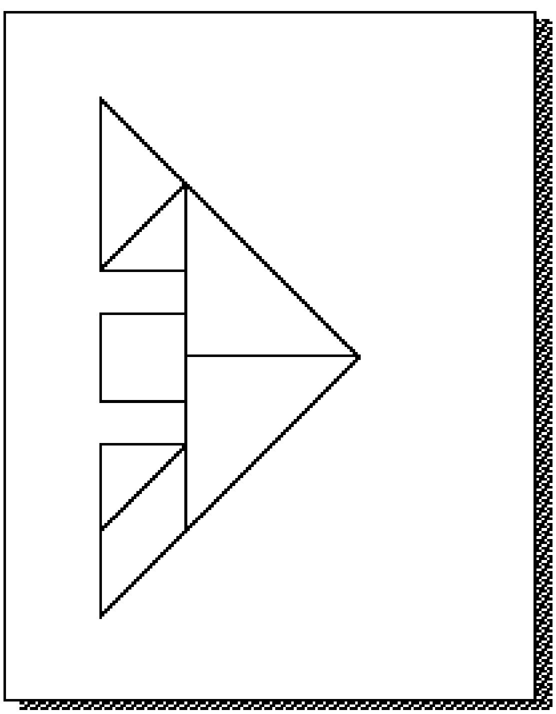


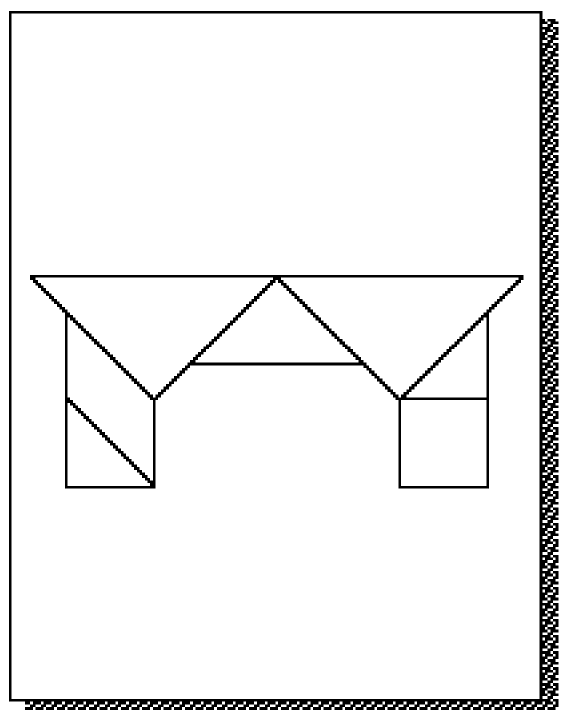


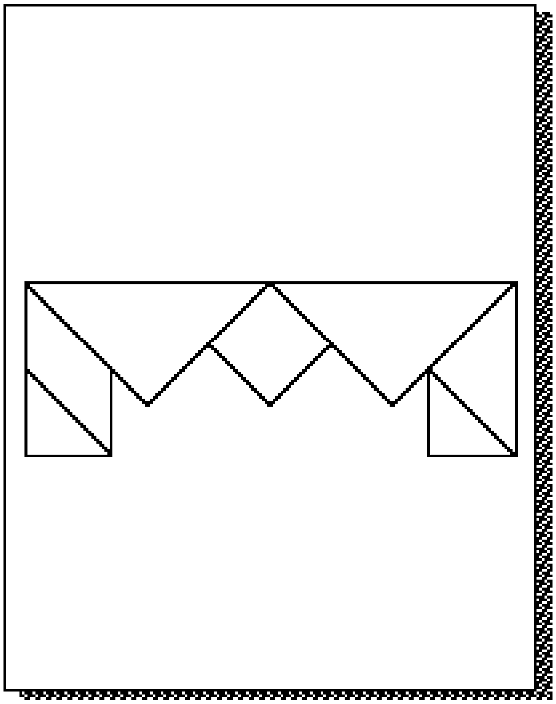


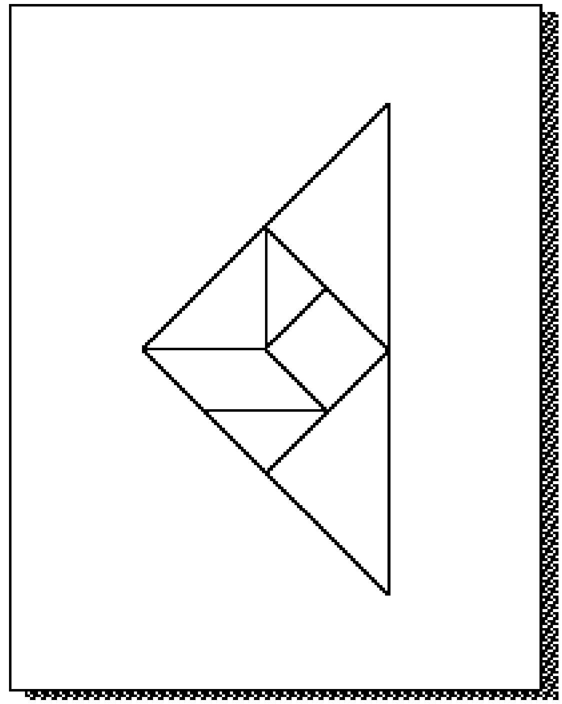


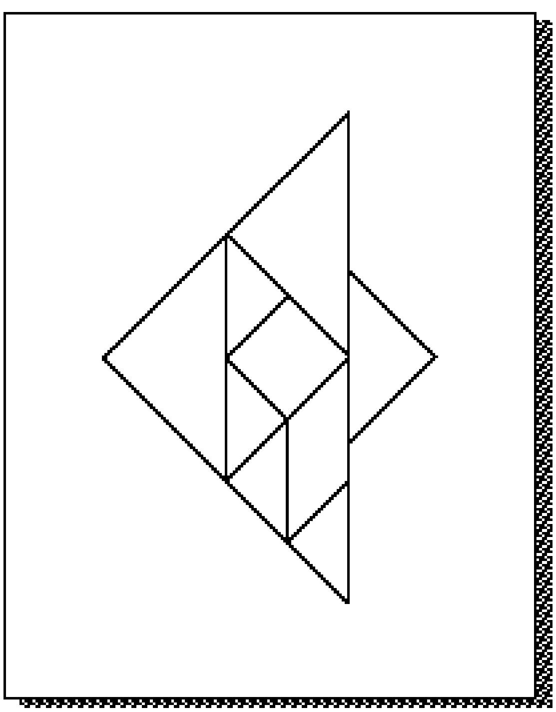


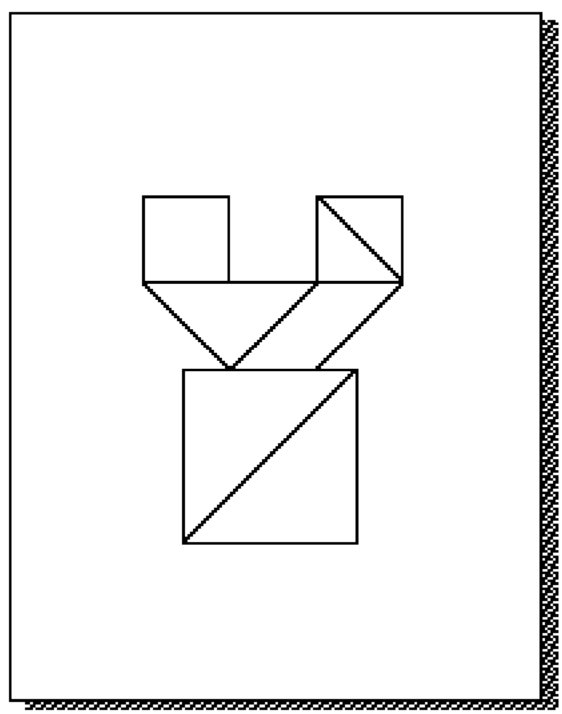




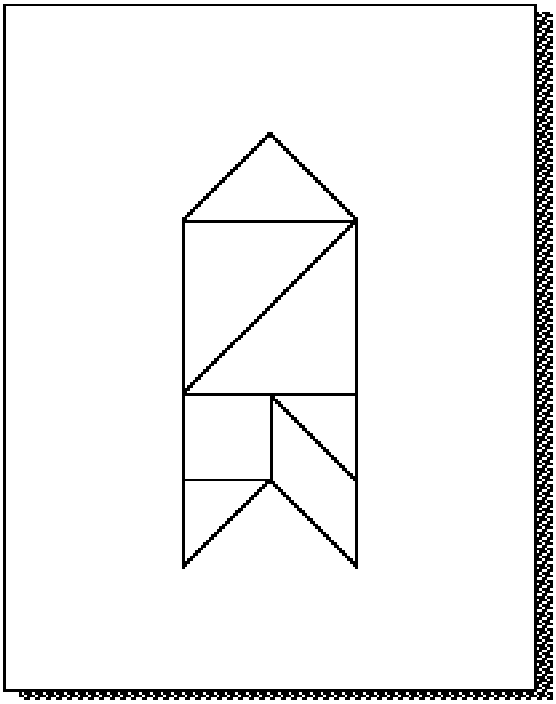


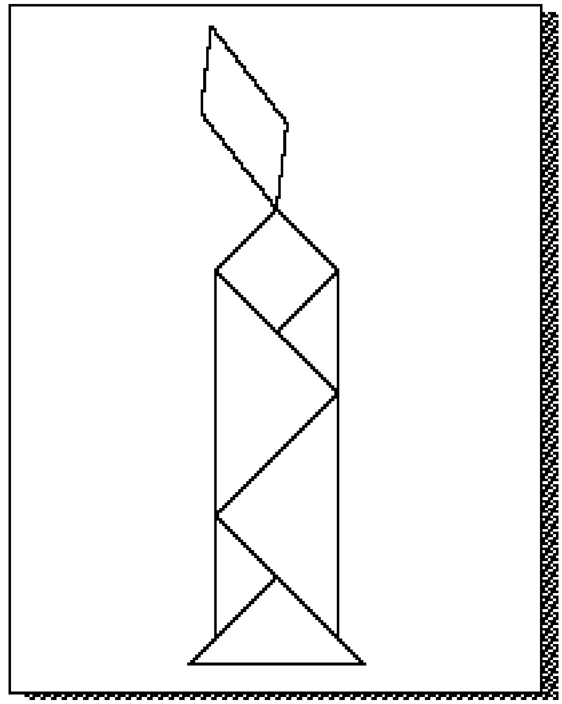






G 12





Capsule Lesson Summary

Find the area of a rectangular shape in a given size square, in triangles half the size of the square, and in other given figures as units. Without measuring, compare the areas of many other shapes to the area of the rectangle.

Materials

Teacher
- *UPG-I Geometry Poster* #1

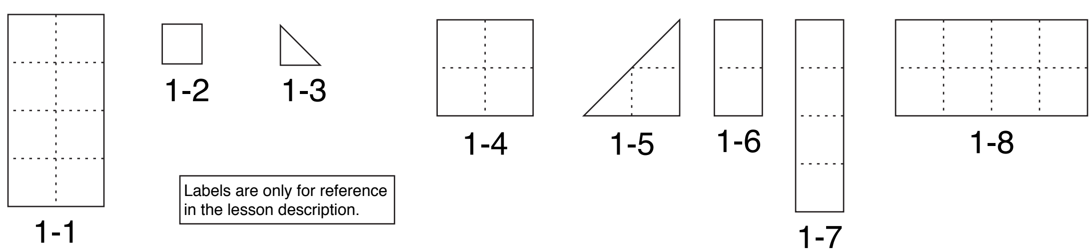
Student
- Index card
- Tangram triangles and squares
- Worksheets G13*, **, ***, and ****

Advance Preparation: The shapes on *Geometry Poster* #1 are organized according to the exercise (1 or 2) in which they are used. Cut out the shapes noting the exercise on the back of each shape. Two shapes, the 4 by 2 rectangles, are used in both exercises. Save the shapes for use again in Lesson G15.

Description of Lesson

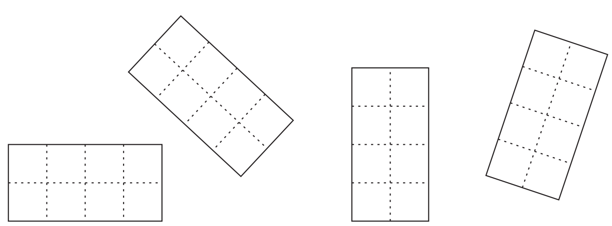
Exercise 1

Be prepared to display the following eight cut-out shapes.



Conduct this exercise at a rather brisk pace so as not to bore the class. However, if at any time many students appear to be confused, ask them for an explanation or allow class discussion of an answer.

Hold up rectangle **1-1** in different positions as you ask about its shape.

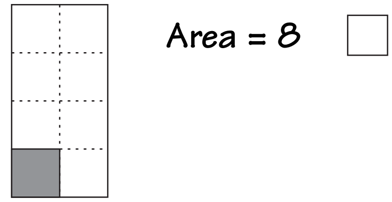


Tape rectangle **1-1** to one side of the board. Then hold up square **1-2** and ask,

T: ***How many little squares like this one do we need to cover the rectangle?***

S: *8.*

Call on a student to place square **1-2** on top of the squares of rectangle **1-1** to illustrate that eight of them would be needed to cover the rectangle. Then tape the little square **1-2** to the board and write the area of the rectangle in terms of this square.

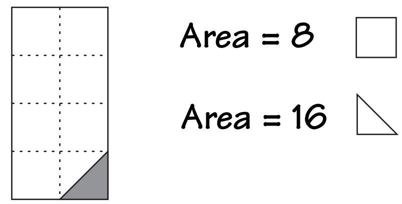


Hold up triangle **1-3** and ask:

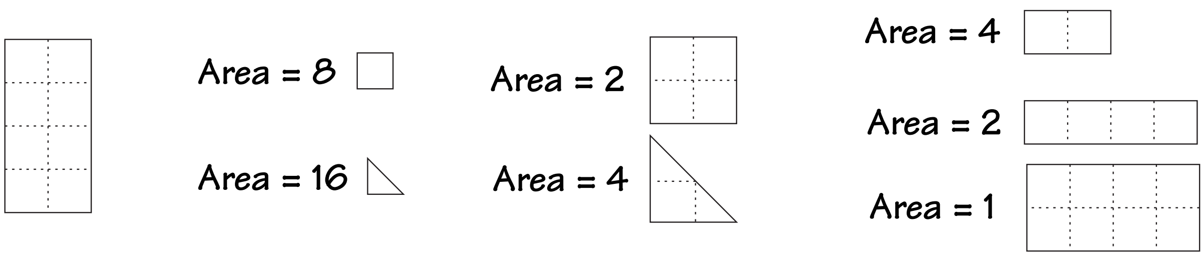
T: ***How many little triangles like this do we need to cover the rectangle?***

S: ***16.***

Call on a student to place triangle **1-3** on top of rectangle **1-1** to illustrate that two triangles are needed to cover each square of rectangle **1-1**. Conclude that sixteen triangles would be needed to completely cover the rectangle. Then tape triangle **1-3** to the board and write the area of the rectangle in terms of this little triangle.



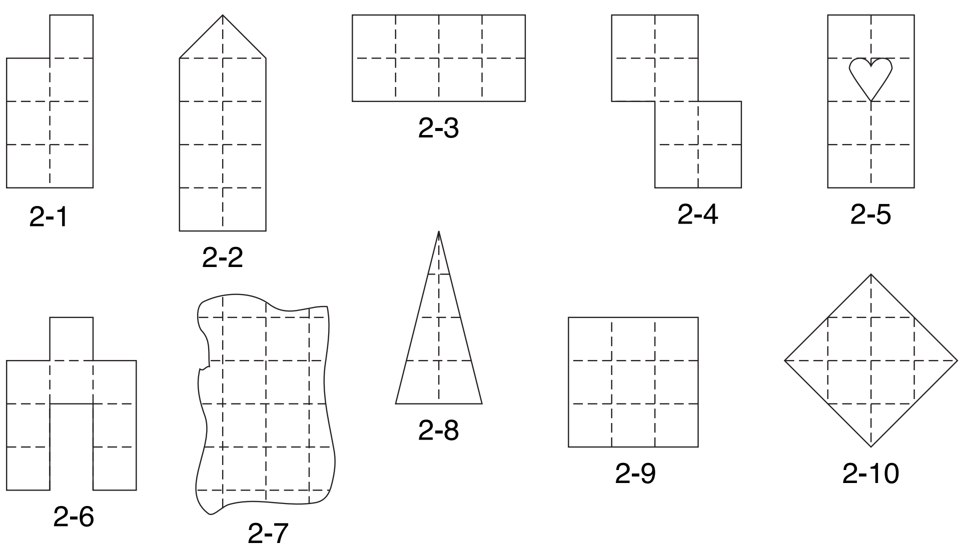
Repeat this activity for each of the remaining shapes, **1-4** through **1-8**. Record the answers on the board.



Before going on to Exercise 2, erase the board and take down all demonstration shapes except rectangle **1-1**.

Exercise 2

Next to rectangle **1-1** make a table as shown on the next page. Be sure your table is large enough to hold several shapes taped in each row. Be ready to display the shapes for Exercise 2 one at a time.



Labels are only for reference in the lesson description.

Display the shape **2-1** shown on the previous page and ask,

T: ***Does this shape have a smaller area, a bigger area, or the same area as our rectangle?***

S: ***Smaller.***

Let the class discuss why the area is smaller. Then invite a student to tape the shape in the correct row of the table.

Encourage students to make different observations leading to the same conclusion. For example:

S: ***It is smaller because part of the rectangle is missing.***

S: ***The rectangle will more than cover this shape, so this shape is smaller.***

S: ***It takes seven little squares to cover this shape and eight little squares to cover the rectangle, so this shape is smaller.***

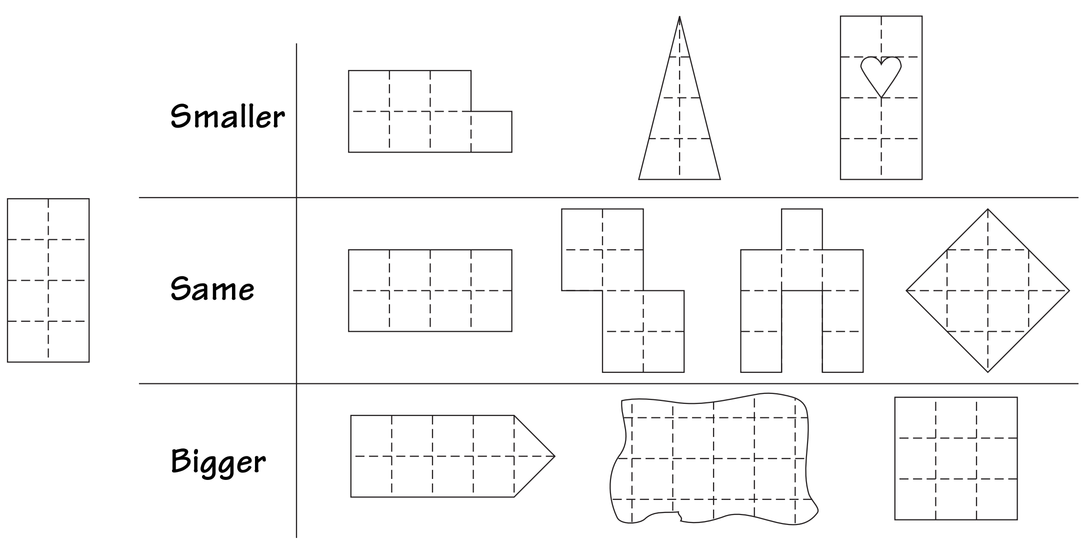
Continue this activity for the remaining shapes. Be sure to discuss where to put each shape in the table.

One way to get the entire class involved in this activity is to instruct students to write the words **Smaller, Same,** and **Bigger** on an index card.

T: ***I will show you some new shapes and you will have to compare them with our rectangle. Don't say your answer aloud. If the area of the new shape is smaller, hold up your card, pinching near* Smaller*. If the area of the new shape is the same, pinch near* Same*. If the area of the new shape is bigger, pinch near* Bigger*.***

This will allow you to see the answers of all the students.

At the end of the activity, your board should have all the shapes for this exercise taped to it.

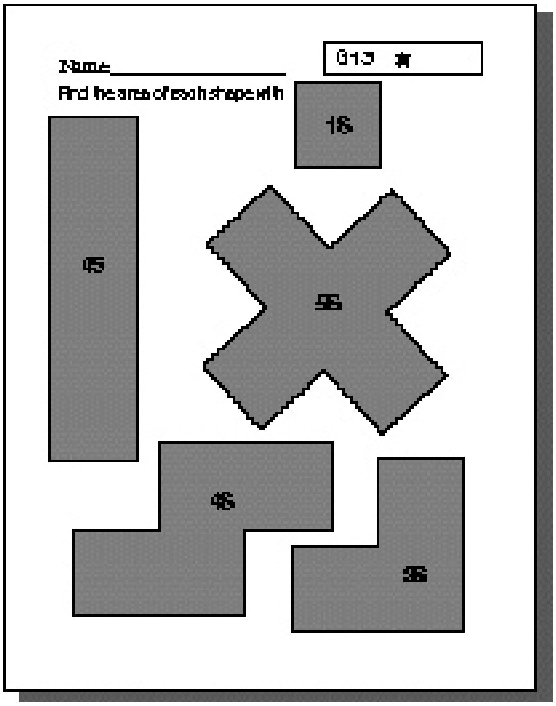


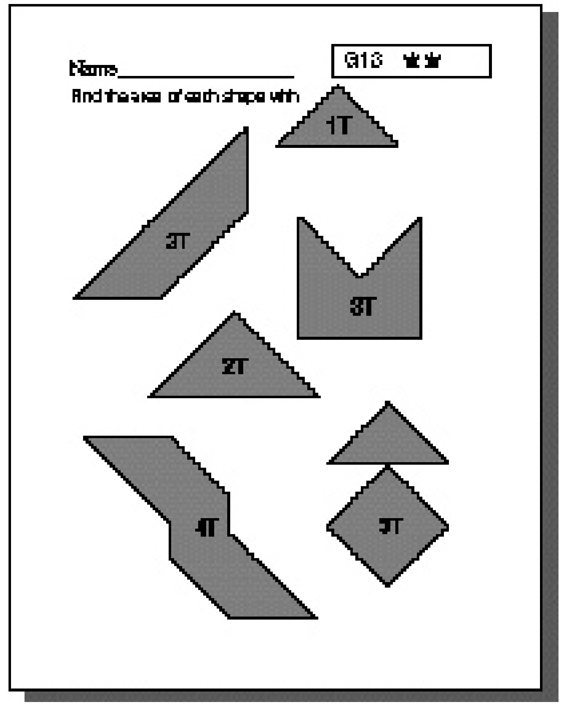
T: ***Remember, the area of our rectangle*** (point to **1-1**) ***is exactly eight little squares. Can you show me which of the shapes on the board have an area of exactly eight little squares? Which shapes have an area more than eight little squares? Which shapes have an area less than eight little squares?***

G13

Save the shapes from this lesson for use again in Lesson G15.

Distribute Tangram triangles and squares (ten little triangles and ten little squares) and Worksheets G13* and **. Students who finish these worksheets should continue with Worksheets G13*** and ****. You may like to pair students who complete the *** and **** worksheets to discuss answers. Students should give reasons for their choices, perhaps using the Tangram squares and triangles to measure the area of each shape.


Name
G13 ★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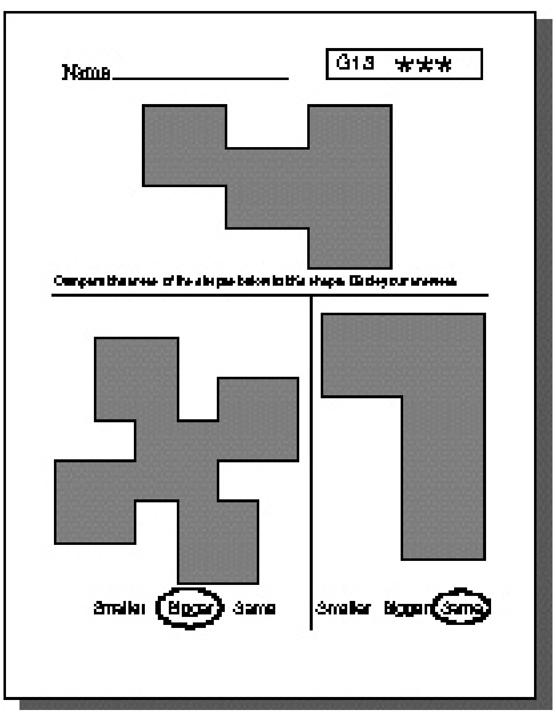
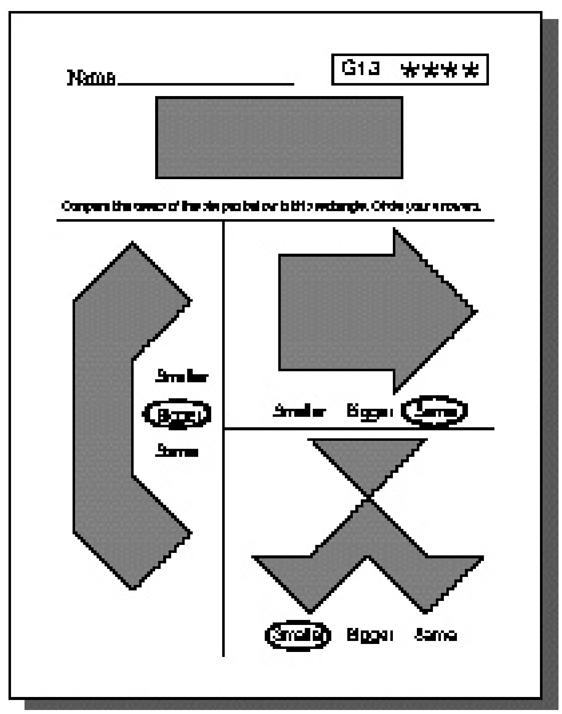
G13 ★★


Name
G13 ★★★
Smaller Bigger Same
Smaller Bigger Same


Name
G13 ★★★★
Smaller
Bigger
Same
Smaller Bigger Same
Smaller Bigger Same

G14 BOOKER'S BAKERY #2

Capsule Lesson Summary

Divide a shape into halves, thirds, and fourths. Write number stories about $^1/_2$, $^1/_3$, and $^1/_4$. Divide another shape into halves, thirds, and fourths using the symmetry of a shape or the number of squares in a shape on a grid.

Materials

Teacher
- Colored chalk
- *UPG-I Geometry Poster #2*

Student
- Colored pencils, pens, or crayons
- Worksheets G14*, **, ***, and ****
- Unlined paper

Advance Preparation: Before the lesson begins, cut each of two copies of *UPG-I Geometry Poster #2* in half. The four shapes (four copies of the same shape) will be used in Exercise 2.

Description of Lesson

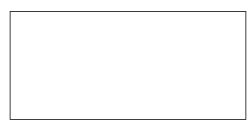
Exercise 1

T: ***Do you remember the twins Jack and Jill who live next door to Booker's Bakery?***

Allow a few minutes for the class to recall that Jack and Jill often share cakes or cookies from the bakery. Discuss that sharing equally means that each person gets the same amount.

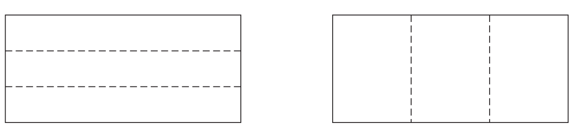
T: ***Sometimes when Jack and Jill have a friend visiting them, their mother will buy a small cake for the children to share. Today Jack and Jill have one friend visiting them, and their mother has bought a cake. I will draw a picture of the cake.***

Draw a rectangular shape on the board and give student pairs a sheet of unlined paper to represent the cake.



T: ***There are three children and they would like to share this cake fairly. With your partner try to find a way for them to cut the cake. You can draw cutting lines on your paper when you agree on a good way.***

After a few minutes, invite several students to trace possible cutting lines in the rectangle on the board. There may be different suggestions; draw dotted lines each time a good suggestion is given. Then erase the cutting lines and ask for another way to cut the cake. Two divisions you are likely to receive are in the next illustration. Do not insist on other suggestions but accept them if offered. Leave one of these divisions on the board.

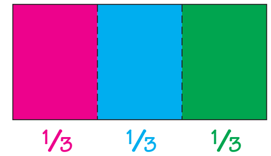


Choose three colors of chalk (for example, red, blue, and green).

T: ***I would like to color one-third of this cake red, one-third blue, and one-third green to show that the three pieces go to the three children.***

Invite students to do the coloring, but encourage them to do it quickly.

Write the fractions under each colored piece of the cake.
Write the number sentence for this situation on the board as you say,



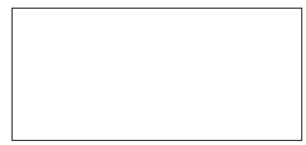
T: ***One-third of the cake plus one-third of the cake plus one-third of the cake is the whole cake.***

$$1/3 + 1/3 + 1/3 = 1$$

Erase the board and continue with a new situation.

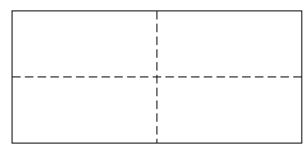
T: ***Suppose another friend comes to visit Jack and Jill today, so that now there are four children. Then the four children would have to share the cake equally.***

Draw a picture of the cake on the board again and suggest students turn over their paper cakes.

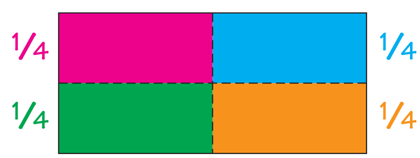


T: ***With your partner, find a way for four children to share the cake so that they all get the same amount. Draw cut lines when you agree on a good way.***

Again, invite students to trace cutting lines in the rectangle on the board. Draw dotted lines each time a correct suggestion is given. Leave one simple division on the board like the example here.



Choose four colors of chalk and ask the students to color one-fourth of the cake in each color. Write the fractions near the colored pieces.



Write a number sentence for this situation as you say,

$$1/4 + 1/4 + 1/4 + 1/4 = 1$$

T: ***One-fourth of the cake plus one-fourth of the cake plus one-fourth of the cake plus one-fourth of the cake is the whole cake.***

Suppose Jack gets this red piece and Jill gets this blue piece. What part of the cake do Jack and Jill get together?

S: ***One-half.***

You may have to rephrase the question and indicate in the picture that you want the class to look at the red piece and the blue piece together. Then write this number sentence on the board.

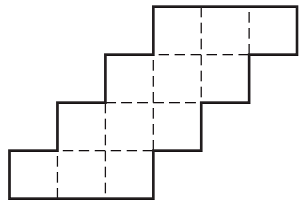
$$1/4 + 1/4 = 1/2$$

You may like to let some student pairs cut their paper cakes in thirds, some in fourths, and some in halves. Then compare the sizes of the pieces to see that $\frac{1}{4} < \frac{1}{3} < \frac{1}{2}$.

Erase the board before going on to the next exercise.

Exercise 2

Put one copy of this shape (from *UPG-I Geometry Poster #2*) on the board.



T: ***One day Jack and Jill get a strange shaped cake from Booker's Bakery. This is the shape*** (point to the board)***. Also, the cake is already cut into many pieces. How many pieces?***

S: ***12.***

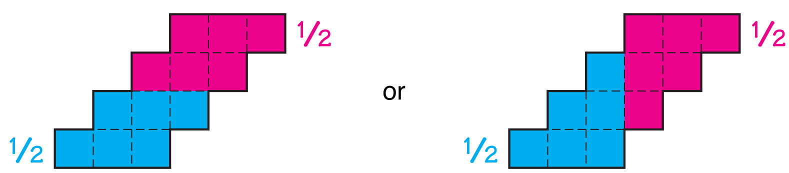
T: ***Yes, and they are all the same size. Jack and Jill would like to share the cake fairly between them. How can they do it?***

S: ***Each gets half of the cake.***

S: ***Jack takes six pieces and Jill takes six pieces.***

T: ***Which pieces should we give to Jack and which to Jill?***

Let students suggest several different divisions and then color half of the shape red and half of the shape blue. Choose a suggestion from one of your students that illustrates some symmetry in the shape. For example:



T: ***The red pieces could be Jack's and the blue pieces could be Jill's. Is it easy to see that both children get the same amount of cake?***

Encourage students to discuss this question briefly in terms of symmetry (both pieces of the cake look exactly the same) and in terms of numbers (both children get six pieces).

T: ***How many pieces are in the whole cake?***

S: ***12.***

T: ***How many pieces are in one-half of the cake?***

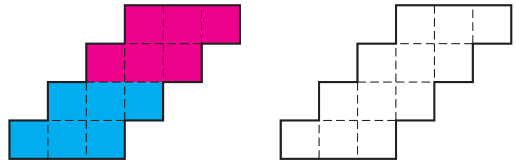
S: ***6.***

Write this equation to one side of the board as you say, "One-half of twelve is six."

$$\frac{1}{2} \times 12 = 6$$

Leave this copy of the shape and the equation on the board. Put up a new copy of the shape.

$\frac{1}{2} \times 12 = 6$



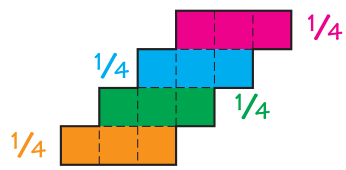
T: ***Before Jack and Jill can divide the cake, their cousins Judy and Jim come to visit. Now there are four children and they have to divide the cake fairly among them. How can they do it?***

S: ***Each one gets one-fourth of the cake.***

S: ***Each one gets three pieces.***

T: ***Which pieces should we give to Jack, which to Jill, which to Judy, and which to Jim?***

Again, let students make several suggestions. Choose one suggestion that illustrates symmetry and color the cake with four colors.



T: ***Is it easy to see that all four children get the same amount of cake?***

Again, encourage discussion which suggests both symmetry and counting.

T: ***How many pieces in the whole cake?***

S: ***12.***

T: ***How many pieces in one-fourth of the cake?***

S: ***3.***

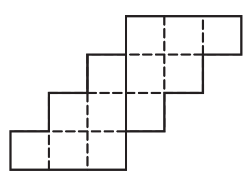
Under the first equation write a second equation as you say, "One-fourth of twelve is three."

$$\frac{1}{2} \times 12 = 6$$
$$\frac{1}{4} \times 12 = 3$$

Take down the shapes and erase the board except for these two equations. Put up another copy of the shape.

$\frac{1}{2} \times 12 = 6$

$\frac{1}{4} \times 12 = 3$



T: ***Suppose that Jack and Jill had just one friend visiting them. Then there would be three children. How could they divide this cake fairly among three children?***

S: ***Each gets one-third.***

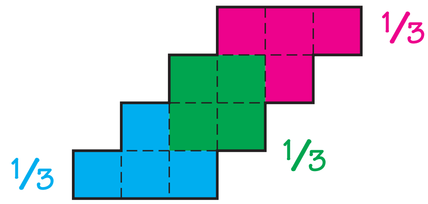
S: ***Each gets four pieces.***

As before, let students suggest which pieces would go to each of the three children and color the shape for one of the suggestions. Then, as before, observe that one-third of twelve is four and write the equation on the board.

$$\frac{1}{2} \times 12 = 6$$

$$\frac{1}{4} \times 12 = 3$$

$$\frac{1}{3} \times 12 = 4$$



Worksheets G14*, **, ***, and **** are available for independent or group work. If students are working in groups of two or three, each student should have a worksheet and the group should find more than one way to solve each problem, especially those on the * worksheet.

Reading Activity

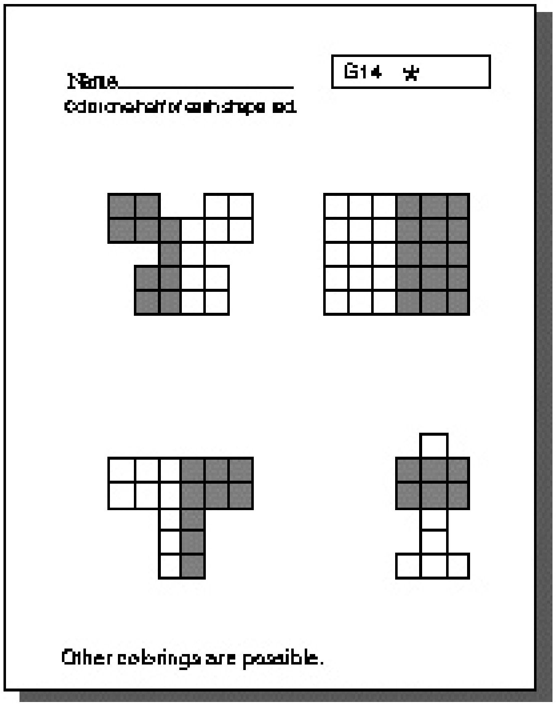
Your students might enjoy reading a book such as *The Doorbell Rang* by Pat Hutchins following this lesson.

Center Activity

Make task cards out of shapes like those on the worksheets for students to divide in halves, thirds, or fourths.

Home Activity

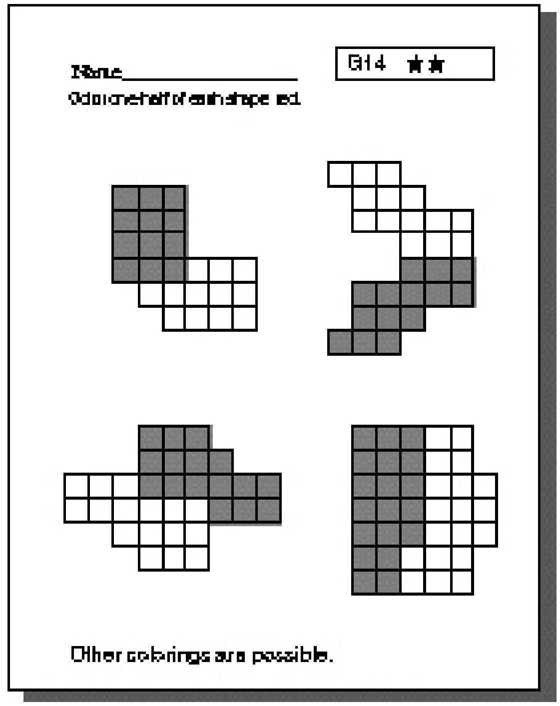
Suggest parents/guardians find opportunities for children to divide something (pizza, candy bars, and so on) in halves and fourths.



Name________________ G14 ★

Color one-half of each shape red.

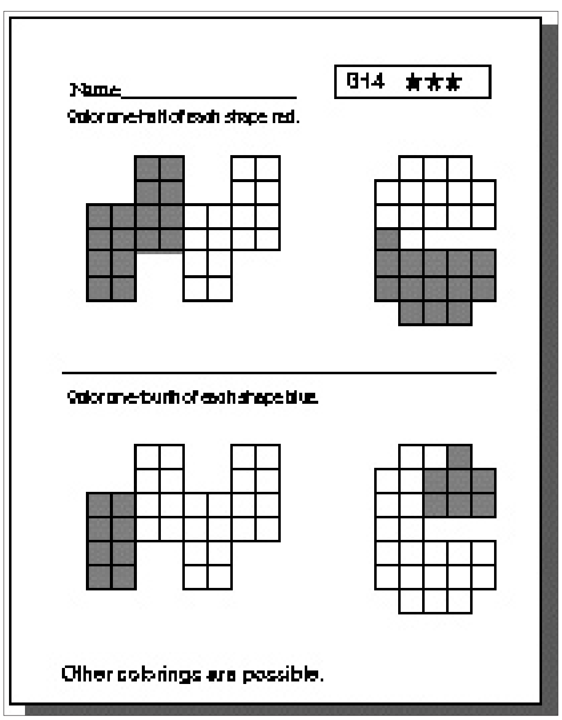
Other colorings are possible.



Name________________ G14 ★★

Color one-half of each shape red.

Other colorings are possible.

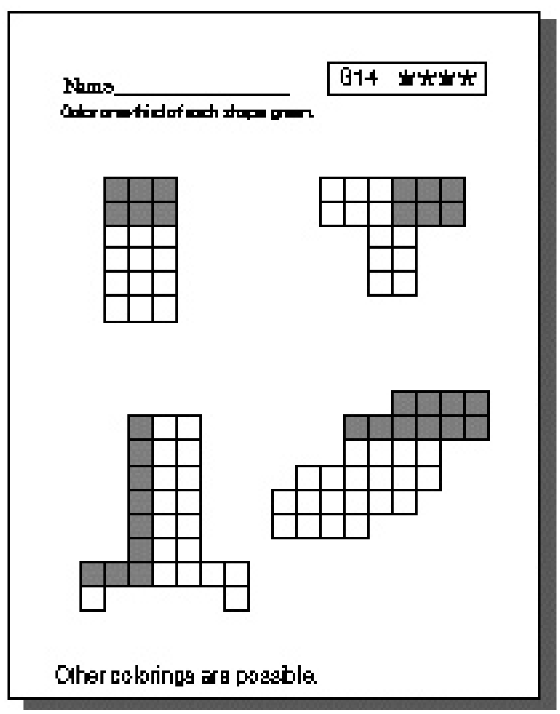


Name________________ G14 ★★★

Color one-half of each shape red.

Color one-fourth of each shape blue.

Other colorings are possible.



Name________________ G14 ★★★★

Color one-third of each shape green.

Other colorings are possible.

Capsule Lesson Summary

Compare the areas of many shapes to the area of a given rectangle. Find or estimate the areas of the rectangle and the other shapes using a small square as the unit of measure.

Materials

Teacher • Shapes from Lesson G13

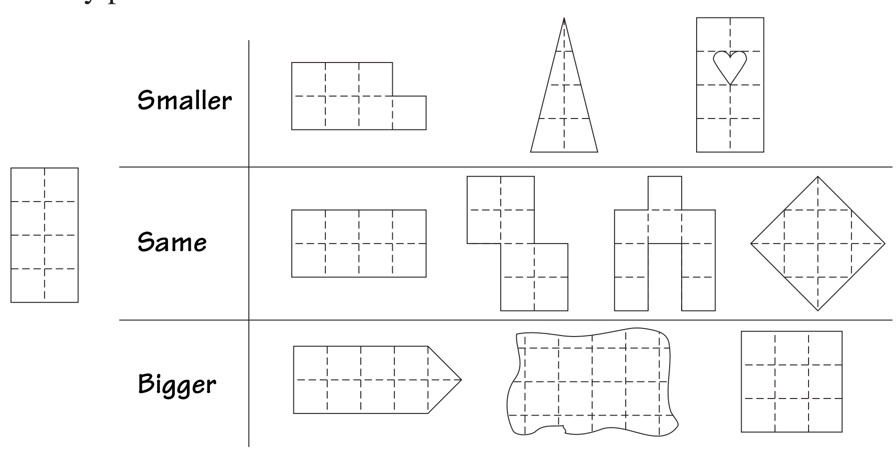
Student • Pencil • Worksheets G15* and **

Advance Preparation: Before this lesson begins, locate the shapes used in Lesson G13.

Description of Lesson

Tape one of the rectangular shapes (see illustration below) to the board and draw a table large enough to hold the other shapes from *UPG-I Geometry Poster* #1 used in Exercise 2 of Lesson G13.

Quickly compare the sizes of the other shapes to this rectangle by discussing with the class where each shape goes in the table. At the end of this activity, you should have the various shapes taped on the board and correctly placed in the table.



Hold up the small square.

T: ***We are going to measure these shapes with this little square. This little square is our unit so its area is 1. What is the area of this rectangle*** (point to the rectangle at the side of the table)***?***

S: ***8 little squares.***

Write "8 squares" under the rectangle on the board.

T: ***Are there any other shapes on the board that have area 8 little squares?***

S: ***Yes, all the shapes that have the same area as the rectangle.***

Point to all the shapes in the **Same** section of the table and write "8 squares" under each one.

Point to the first shape in the **Smaller** section of the table (see the previous illustration).

T: ***What is the area of this shape? Is it easy to find its area?***

S: ***Yes, just count the squares. The area is 7 squares.***

Write "7 squares" under the shape as you repeat the explanation.

T: ***Are there other shapes in this table whose areas you can find?***

Let the students discuss the other shapes. If a student correctly gives the area of a shape, ask for an explanation. After the class agrees on the correct area of a shape, write it underneath.

For two of the shapes, it will not be possible to say exactly what the area is. In these cases, encourage estimation. For example:

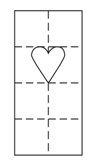
T: ***It may not be possible to find the exact area of this shape*** (point to the rectangle with a heart-shaped hole in it)***. What can we say about its area?***

S: ***It is less than 8 squares.***

T: ***Yes, and it is more than…?***

S: ***6 squares.***

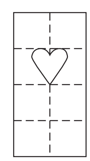
Many answers to this last question are acceptable. Ask for an explanation and, if you and the class agree, write the estimate under the shape.



Between 6 and 8 squares

Perhaps the best (whole number) estimate in this case is between 7 squares and 8 squares, because the heart cut-out is smaller than one square.

Do not insist on this close an estimate unless a student provides it. Use the best estimate that is suggested and explained.



Between 6 and 8 squares

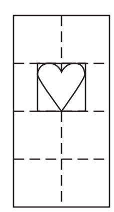
One way to discuss the irregular shape shown below is to enclose it in a larger rectangle.

T: ***How can we estimate the area of this shape now?***

S: ***It is more than 8 squares.***

T: ***Yes, and it is less than…?***

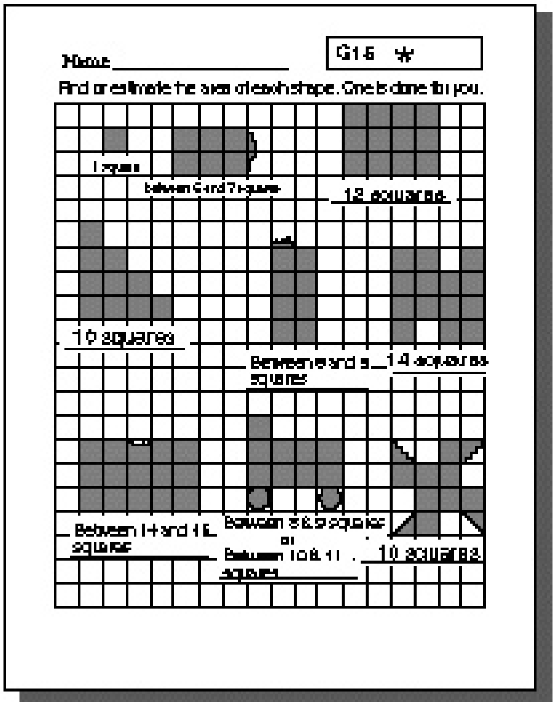
S: ***24 squares.***

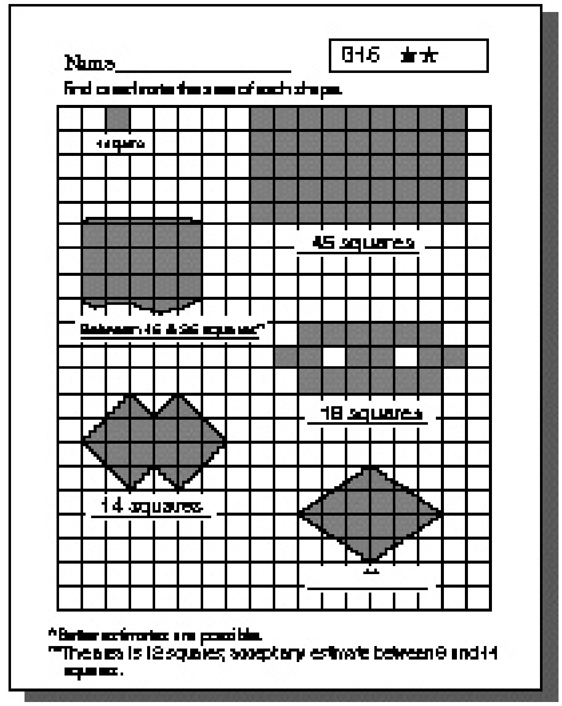


Between 7 and 8 squares

As before, better estimates may be suggested. Write the best estimate that is suggested and explained underneath the shape.

Distribute Worksheets G15* and **. Explain that for each shape the students should write its area on the blank underneath or estimate its area as best they can. As students begin to complete the worksheets, you may like to discuss different possible answers for estimates.


Name
G15 ★
Find or estimate the area of each shape. One is done for you.
1 square
Between 6 and 7 squares
12 squares
10 squares
Between 8 and 9 squares
14 squares
Between 14 and 15 squares
Between 8 & 9 squares or Between 10 & 11 squares
10 squares


Name
G15 ★★
Find or estimate the area of each shape.
1 square
45 squares
Between 16 & 26 squares*
18 squares
14 squares
**
* Better estimates are possible.
** The area is 12 squares; acceptable estimate between 8 and 14 squares.