

L Strand

# The Languages of Strings and Arrows



# THE LANGUAGES OF STRINGS & ARROWS INTRODUCTION

The language of strings is the nonverbal language of sets, that is, collections of objects put together in some way. The language of arrows is the nonverbal language of relations among objects. These languages permeate the *CSMP* strands and are used, separately or together, to present an unending variety of numerical and nonnumerical situations.

## Classification: The Language of Strings

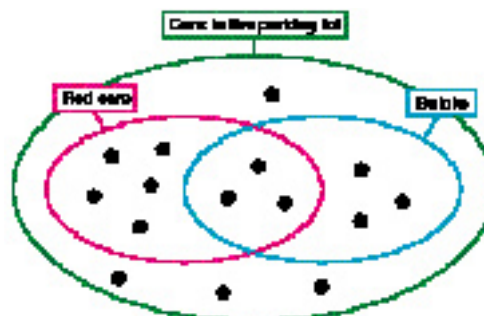
As the word implies, classifying means putting things into classes, or as the mathematician says, sets. The mathematics of sets can help students to understand and use the ideas of classification. The basic idea is simple: Given a set  $S$  and any object  $x$ , either  $x$  belongs to  $S$  ( $x$  is in  $S$ ) or  $x$  does not belong to  $S$  ( $x$  is not in  $S$ ). We represent this simple act of sorting—in or out—by using pictures to illustrate in and out in a dramatic way. Objects to be sorted are represented by dots, and the sets into which they are sorted are represented by drawing strings around dots. A dot inside the region delineated by a set's string is for an object in the set and a dot outside a set's string is for an object not in the set.

This language of strings and dots provides a precise (and nonverbal) way of recording and communicating thoughts about classification. The abilities to classify, to reason about classification, and to extract information from a classification are important skills for everyday life, for intellectual activity, in general, and for the pursuit and understanding of mathematics, in particular. The unique quality of the language of strings is that it provides a nonverbal language that is particularly suited to the mode of thinking involved in classification. It frees young minds to think logically and creatively about classes and to report their thinking long before they have extensive verbal skills.

In this strand we present situations and ask carefully phrased questions to continue to advance skills in classification, always remembering that the skills grow out of such experiences. To be able to draw strings and dots is not an objective in itself; to develop the mode of thinking involved is the objective. Thus it is important for us to construct the situations carefully. The sets into which we ask students to classify objects must be determined by well-defined attributes; otherwise, there is the added problem of deciding whether a certain object does or does not have a certain attribute. For this reason we make extensive use of A-blocks (attribute blocks) and sets of numbers in designing classification situations. Students can immediately say whether or not a block is red, whether or not it is a square, and so on. They know whether or not a given number is less than 15, whether or not it is odd, and so on.

One reason for classifying objects is to count the objects that have a certain attribute. Suppose, for example, there are eight red cars and six Buicks in a parking lot. If four cars in the lot are neither red nor Buicks, must there be 18 cars in the lot? Suppose we count the cars in the parking lot and find there are exactly 15. Is this possible? A string picture immediately settles this apparent paradox.

There are three red Buicks.



# THE LANGUAGES OF STRINGS & ARROWS INTRODUCTION

## Relations: The Language of Arrows

Relations are interesting and important to us in our everyday lives, in our careers, in school, and in scientific pursuits. We are always trying to establish, explore, and understand relations. In mathematics it is the same; to study mathematics means to study the relations among mathematical objects like numbers or geometric shapes. The tools we use to understand the everyday world are useful to understand the world of mathematics. Conversely, the tools we develop to help us think about mathematical things often serve us in nonmathematical situations.

A serious study of anything requires a language for representing the things under investigation. The language of arrows provides an apt language for studying and talking about relations. Arrow diagrams are a handy graphic representation of a relation, somewhat the same way that a blueprint is a handy graphic representation of a house. By means of arrow diagrams, we can represent important facts about a given relation in a simple, suggestive, pictorial way—usually more conveniently than the same information could be presented in words. The convenience of arrow diagrams has important pedagogical consequences for introducing children to the study of relations in the early grades. A child can read—and also draw—an arrow diagram of a relation long before he or she can read or present the same information in words. The difficulty of presenting certain ideas to children lies not in their intellectual inability to grasp the ideas; rather, the limitations are often mechanical. For example, a second grader simply may not, as yet, have developed the motor skills of writing or reading necessary to communicate about the ideas in conventional systems. Arrow diagrams have all the virtues of a good notation: they present information in a clear, natural way; they are attractive, colorful things to look at; they are easy and fun for children to draw. Another educational bonus occurs when arrow diagrams are used imaginatively. A story or problem may be captured by an arrow diagram, and at the same time, the resulting arrow diagram may suggest other, similar situations. This allows students to call upon previous experience or to expand their creativity.

One of the purposes of this strand is to use the power of the language of arrows to help children think logically about relations. Again it must be remembered that our goals concern the thinking process and not the mechanism. The ability to draw prescribed arrows is not the objective in itself, nor is viewing an arrow diagram just another format for drill problems in arithmetic.

The general aim of The Languages of Strings and Arrows strand is to suggest situations that are inherently interesting and thought provoking, and to give children modes of thinking and appropriate languages with which they can organize, classify, and analyze. In addition to a varied assortment of lessons concerning sets and relation, this strand includes lessons involving systematic methods for solving combinatorial (counting) problems; methods for collecting, recording and interpreting data in real life situations; probabilistic thinking; and networks.

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### Capsule Lesson Summary

Present an arrow picture where the dots are for children and the arrows are for the relation “sent a letter to.” Collectively, discuss some of the many observations that can be made about this picture. Individually, draw similar arrow pictures.

#### Materials

**Teacher** • Colored chalk

**Student** • Unlined paper  
• Colored pencils, pens, or crayons

### Description of Lesson

Begin the lesson with a brief exchange about writing letters to friends.

**T:** *I have a friend, Edie, who just started second grade at another school. She gave me an arrow picture that she had drawn herself. Edie’s picture is very interesting and I would like to show it to you. I will draw it on the board so everyone can see.*

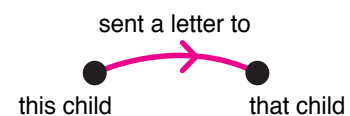
Draw eighteen well-spaced dots on the board. Suggest that the class count along as you draw the dots.

**T:** *These are the children in Edie’s second grade class. How many children are there in Edie’s class?*

Call on one or two students to check the count, pointing to each dot as they count. The class should agree that there are eighteen children in Edie’s class.

**T:** *Edie told me that during the summer vacation some of the children in her class wrote letters to other children in the class. She drew red arrows to show this.*

On another part of the board, draw this picture. Ask a student to read the message of the red arrow as you point to the starting dot, trace the arrow, and then point to the ending dot.

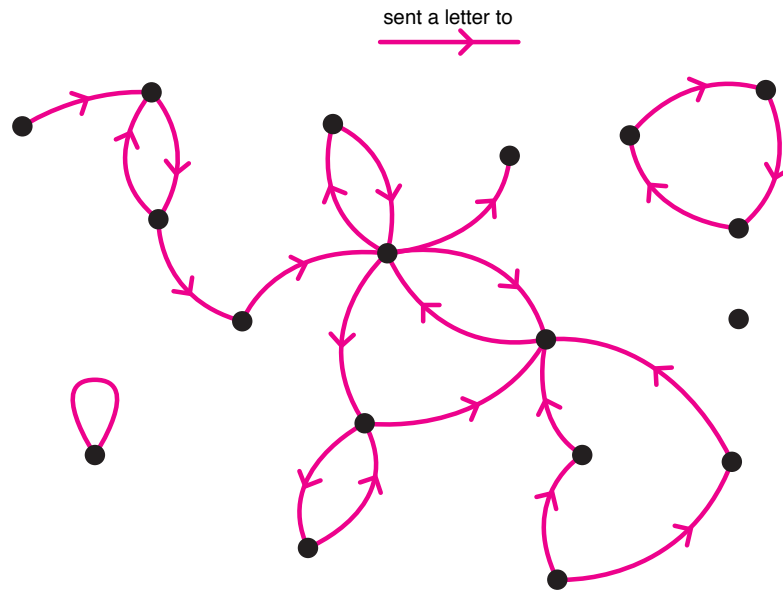


**S:** *This child sent a letter to that child.*

Erase this picture and draw a key arrow near the eighteen dots. (See the next illustration.)

**T:** *Edie’s picture has many red arrows so it will take me a couple minutes to finish the drawing on the board. I would like to surprise you, so everyone cover your eyes while I complete the picture.*

**T:** *You may open your eyes now. Look at the picture on the board. Think carefully about this picture for a moment without saying anything.*



After about a minute, ask the class what they think about Edie's picture. Let students comment spontaneously, even if their remarks are not directly related to the meaning of the arrows. For example, the following comments might be made.

- S:** *It's a very complicated picture.*
- S:** *There are lots of arrows.*
- S:** *It looks like a dancer.*
- S:** *I see a child who sent a lot of letters.*
- S:** *I see a child who received a lot of letters.*
- S:** *I see someone who didn't receive any letters.*

When comments like the last three are made, ask students to explain how they know. Be sure that arrows starting at a dot are counted as letters sent and that arrows ending at a dot are counted as letters received.

Let the discussion continue as long as new observations are being made. When the students stop making comments, ask some specific questions. For example:

**T:** *Edie told me that her friend Andrew sent the most letters. Where is Andrew? How many letters did he send? (Four)*

Ask a student to point to the dot for Andrew and to trace the four arrows for the letters Andrew sent. Label the dot.

- T:** *Did Andrew receive any letters?*
- S:** *Yes, three.*

Invite a student to trace the three arrows for letters Andrew received.



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**T:** *Can you find a child who sent just one letter and received just one letter?*

There are many, so allow several students to point to different dots at which just one red arrow starts and just one red arrow ends.

**T:** *Edie told me that her friend Jenna received the most letters. Where is Jenna? How many letters did she receive?* (Four)

Ask a student to point to the dot for Jenna and to trace the four arrows for the letters Jenna received. Label the dot.

**T** (pointing to the dot with a loop): *What do you think about this child?*

**S:** *That child sent a letter to herself (himself).*

**T:** *Find some children who exchanged letters, that is, sent letters to each other.*

There are several such pairs of children so allow students to find as many as they can.

**T:** *Edie told me that there is a new student in her class. This child sent no letters and received no letters. Where is the new student in this picture?*

Ask a student to point to the appropriate dot.

**T:** *In the story told by Edie's picture, how many letters were sent in all? (22) How many letters were received? (22)*

*If you could be one of the children in Edie's class, who would you like to be? Explain why.*

Students will react differently to such a question; encourage them to express themselves.

Distribute unlined paper and colored pencils. Ask the students to draw a picture for their own story about sending letters. One way to get students started is to ask them to draw a dot for themselves and label it, and then to draw dots for some of their friends. Now ask them to draw some red arrows as they tell a story about sending letters. Encourage students to write something about their pictures, such as how many letters were sent and received or how many letters a particular person sent and received.

You will receive many different pictures. Show some of them to the whole class and let other students comment briefly. You may wish to display some pictures on a bulletin board.



### Capsule Lesson Summary

Progressively present a detective story about finding where certain things are in a picture of fourteen dots. Use colored strings (classifications) for clues.

### Materials

**Teacher** • *Where's My Nose?* Storybook

**Note:** A classroom set of materials for *CSMP Mathematics for the Upper Primary Grades, Part*

**Student** • *Where's My Nose?* Storybook

*I* contains 15 copies of the *Where's My Nose?*

Storybook. This number should allow one copy of the storybook for every two students. If you desire, additional copies can be ordered so that students can have their own storybook to read or take home.

### Description of Lesson

We suggest that you tell your class the story of *Where's My Nose?* holding up your copy of the storybook at crucial moments so that your students can see the pictures. If you prefer, distribute copies of the storybook to pairs of students and ask that they stay on the same page as you while you read the story.

Before you start the story, draw fourteen dots well spread across your chalkboard.

**T:** *Today I'm going to tell you a story about my friend Nick and his grandmother. Nick loved his grandmother very much and used to visit her often. She lived in the country with her charming black poodle and her three lovely cats. Whenever Nick came to see her, she made up very interesting games for them to play together. She knew that Nick wanted to be a detective when he grew up so she often invented detective games for him.*

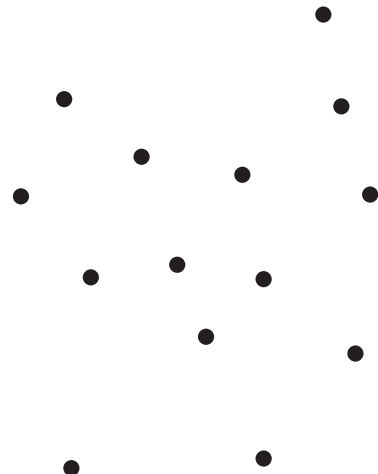
*One day Nick went to visit his grandmother. She was very happy to see him and he was happy to be with her again.*

*He had been wondering for several days what new game she would have for him when he arrived. He was very excited when she told him that she had prepared a detective game for him. She gave him a piece of paper with lots of dots on it just like the picture you see on the board.*

*Nick was puzzled by this picture because he had been expecting to receive a detective game from his grandmother. But she told him to be patient and that he would soon be surprised. First she said he should count the dots. How many dots are there?*

**S:** *Fourteen.*

Ask a student to point to the dots and count aloud with the class watching to be sure all of the dots are counted just once.



**T:** *She told him the dots were for three rabbits, her charming black poodle, her three lovely cats, three bugs, a squirrel, herself, Nick, and Nick's nose. Then she asked him if he could tell her which dots were for which things.*

Make a list on the board of what the dots are for. Allow students to guess which dot is for which object, for example, which dot is for Nick's nose.

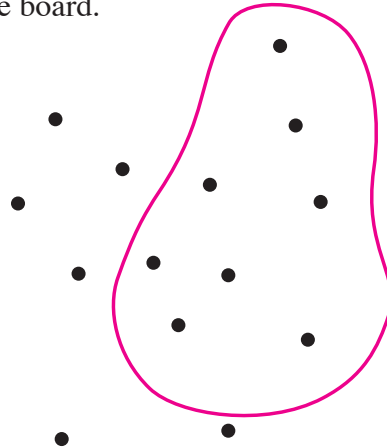
**T:** *Of course it was not possible for Nick to know for sure which dot was for which thing. His grandmother told him he was going to have a chance to be a detective. She would give him some clues so that he could identify each dot. First she drew this red string.*

Draw a red string around eight of the dots you have drawn on the board.

**T:** *She told him that the dots inside the red string were for animals with four legs. Nick thought for a moment and then shouted that the dot for him must be outside the red string. Was he right?*

**S:** *Yes, because he has only two legs.*

Discuss placement of the fourteen things. Decide that, indeed, eight things (three rabbits, the poodle, three cats, and the squirrel) are inside the red string and the other six (Nick, Nick's nose, grandmother, and three bugs) are outside the red string. Do not label the dots yet.



**T:** *Nick's grandmother then asked if it was possible to know for sure which dot was for which thing. Was it now possible to locate Nick's nose? Nick thought and thought but could not think of any way to know which dot was the nose, so he asked his grandmother to give him a second clue.*

Draw a blue string as in this illustration. Draw carefully, so that there are eight dots in the blue string, and four of them are also in the red string.

**T:** *His grandmother then drew this blue string and told him that the blue string was for all things that were able to climb trees. She also told Nick that as a young girl she could climb trees but no longer.*

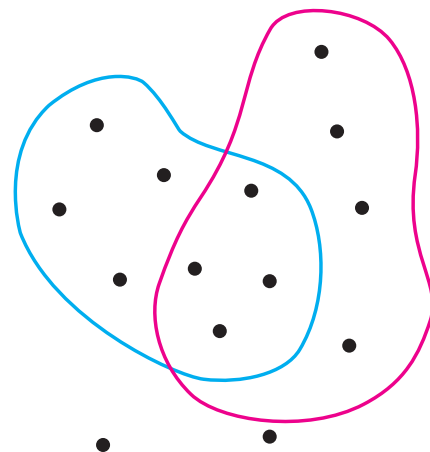
*Nick was a great tree climber so surely he was in the blue string. What do you think about those two dots that are outside of both strings? Nick was sure he knew something about them.*

**S:** *One is for Nick's nose and one is for Nick's grandmother.*

**T:** *How are you sure?*

**S:** *Well, Nick's nose doesn't climb a tree (not by itself) and Nick's grandmother said she cannot climb trees. Also, they do not have four legs.*

**T:** *But which is which?*



Again, discuss placement of the fourteen things. Consider particular objects and decide where they must be—the cats, for example, must be in the middle because they have four legs and climb trees. Or point to a dot and ask what it could be for. In doing so, the class should note both placement of the fourteen things and the four distinct regions of the picture. Further, they should realize that they still do not know exactly which dot is for which thing.

**T:** *Now Nick's grandmother was ready to give the final clue. After this clue all the dots will be known. Let's see how good a detective you are.*

Draw a green string as in the next illustration. Be careful that you have the correct number of dots in each region of your picture.

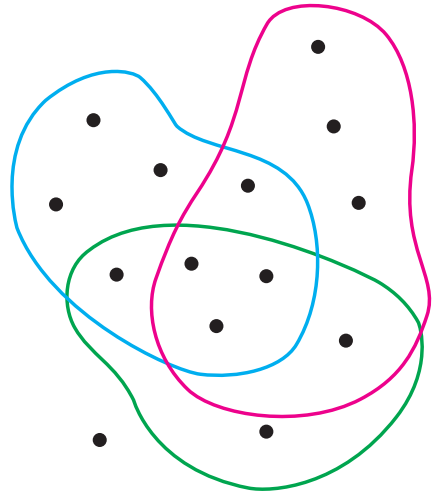
**T:** *She drew this green string and told Nick it was for all things that like to eat meat.*

*Nick became very excited. He loved hamburgers, so now he could find the dot for himself. Where is Nick?*

**S:** *There* (pointing to the only dot inside both the green and blue string but outside the red one).

**T:** *Explain your answer.*

**S:** *Nick eats meat and can climb trees but does not have four legs. So Nick is in the green string and also in the blue string, but he is outside the red string. This has to be Nick because it's the only dot there.*



Label the dot Nick.

**T:** *Very good. Where is Nick's grandmother?*

**S:** *She must be there* (pointing to the dot inside the green string but outside both the red and blue strings).

**T:** *Why?*

**S:** *Because she eats meat but cannot climb trees and does not have four legs. And there is only one dot inside the green string and outside both the red and blue strings.*

**T:** *Good thinking. I will label this dot so we will remember it's Nick's grandmother.*

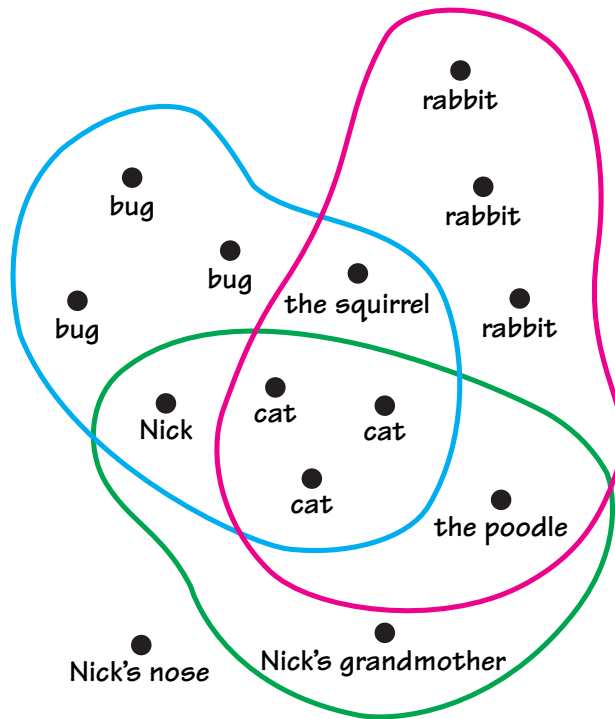
You should now have two of the fourteen dots labeled, one for Nick and one for his grandmother.

**T:** *The three lovely cats also love meat as most cats do. Who can locate the three cats?*

**S:** *There* (pointing to the three dots in the middle), *because we already know cats have four legs and can climb trees.*

**T:** *And the poodle likes meat too. So now you should be able to finish the problem completely. Where are the rabbits, the squirrel, the poodle, the bugs, and Nick's nose?*

Choose different students to locate the nose, the poodle, the bugs, the rabbits, and the squirrel. Ask students to explain their answers. After a correct answer, label the appropriate dot(s). The final picture will have all the dots labeled.



**T:** *Nick was very proud of his successful detective work and you should be proud of yourselves also. This was a problem that could only be solved by careful thinking. Did you enjoy the story?*

Distribute copies of the storybook. Let the students look at the pictures and read the story again on their own.

## Writing Activity

Students or student partners can write their own classification stories. This is a good activity for a “Writers Workshop” or a special project.

### Capsule Lesson Summary

Locate dots for students in a three-string picture and identify which students a certain dot could be for. Determine where a person belongs in a string picture by asking appropriate questions about the person.

#### Materials

**Teacher** • Colored chalk

**Student** • Sheet of paper  
• Worksheets L3\* and \*\*

### Description of Lesson

#### Exercise 1 \_\_\_\_\_

Begin with a discussion of clothes with long sleeves, short sleeves, or no sleeves (sleeveless). Observe that some days many people wear long sleeves, and on other days many people wear short sleeve or sleeveless clothes. What about our class today?

**T:** *To help us get a better picture of our class today, let's make a people graph.*

Make three labels—"long sleeves," "short sleeves," and "sleeveless." Place these on the floor. Next, give each student a piece of paper. Instruct them to line up by the label that best describes their clothes today. Tell them to place their paper on the floor and stand on it in line. The papers should help organize the lines.

**T:** *Look around. What kind of sleeves do most of us have?*

Direct the students to walk away from the graph, leaving their papers on the floor. Ask some comparison and counting questions about the graph.

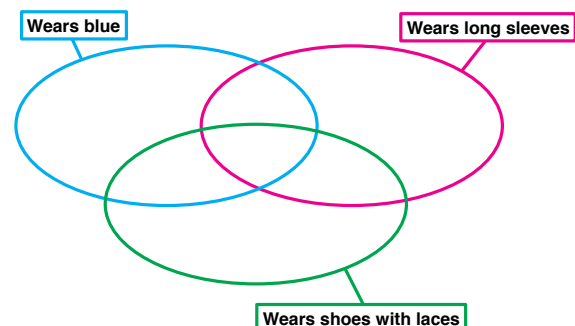
**T:** *What kind of sleeves do the greatest number of us have? ... the least number?  
How many of us have short sleeves?  
How many more of us have short sleeves than have long sleeves?  
Do you think this graph would look different if we made it again in January? Why?*

#### Exercise 2 \_\_\_\_\_

Before starting this exercise, look over the attire of the students in your class today. Try to determine labels for three overlapping strings so that there is at least one student represented in each of the eight regions (see the illustration).

Draw and label three different-colored strings. You may want to discuss the string labels so students understand what it means for a person to be inside a string.

**T:** *Who would like to show us where they are in this picture?*



# L3

**T:** *Laura, stand on this chair so everyone can see you. Is Laura wearing some blue?*

**S:** *Yes, blue stripes.*

**T:** *Is Laura wearing long sleeves?*

**S:** *No.*

**T:** *Is Laura wearing shoes with laces?*

**S:** *No.*

**T:** *Laura, draw a dot for yourself in our picture.*

If the student needs help locating a dot for herself, use large sweeping motions with your open hand to indicate the interior of the strings.

**T:** *Laura is wearing blue, so is she inside or outside the blue string? (Inside) Laura is not wearing long sleeves, so is she inside or outside of the red string? (Outside) Laura is not wearing shoes with laces, so is she inside or outside of the green string? (Outside)*

Help the student draw a dot for herself in the correct place. (See the illustration in Exercise 3.)

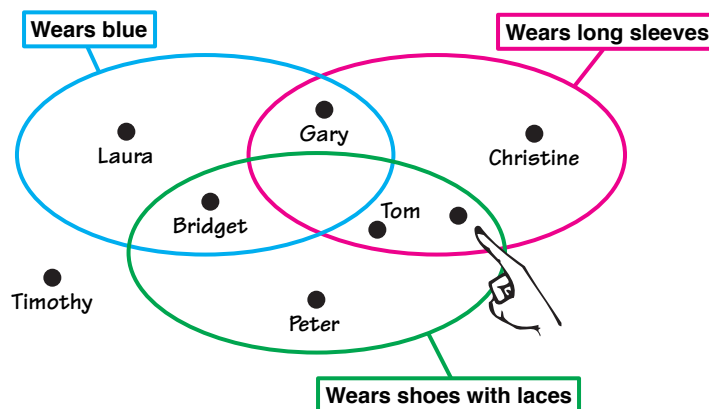
**T:** *Let's write Laura's name by her dot so we can remember that this dot is for Laura.*

Call on several other students to locate themselves in the picture. If necessary, direct the student to stand on a chair while you ask the three important questions, and then ask the student to draw his or her dot. Try to select students whose dots go in different regions, including the region outside of all three strings.

## Exercise 3 \_\_\_\_\_

Leave all the dots drawn thus far in the picture. Draw a new dot, making sure the class knows which dot you have just drawn.

**T:** *I am thinking of somebody in this class and the dot for that person is here.*



**T:** *Who could this be?*

**S:** *Kenneth.*



(If students answer with a name already in the picture, for example, Tom, remind them that you have a dot for Tom and this dot should be for someone else.)

**T:** *Stand up Kenneth. Is Kenneth wearing some blue?*

**S:** *No.*

**T:** *So a dot for Kenneth should be outside the blue string, and this dot is. Is Kenneth wearing long sleeves?*

**S:** *Yes.*

**T:** *Then a dot for Kenneth should be inside the red string, and this dot is. Is Kenneth wearing shoes with laces?*

**S:** *No.*

**T:** *Then a dot for Kenneth should be outside the green string. This dot is inside the green string.*

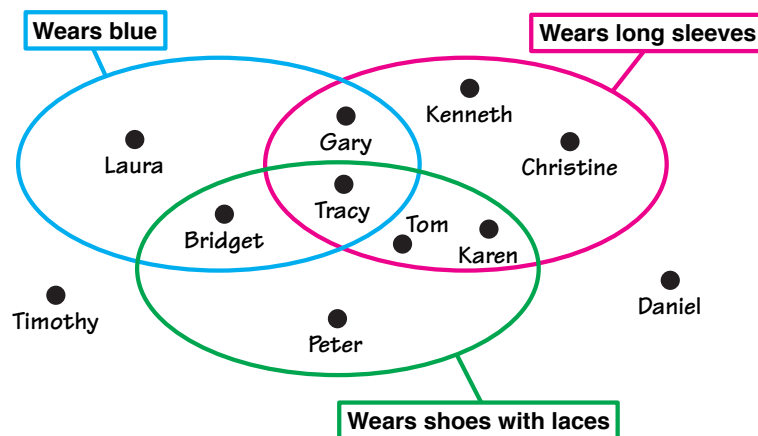
**S:** *It can't be for Kenneth.*

**T:** *Does someone else have an idea?*

**S:** *Karen.*

Ask the students to check with you to see if Karen is not wearing blue, is wearing long sleeves, and is wearing shoes with laces.

Repeat this exercise, drawing a dot inside all three strings (Tracy's dot in the picture below) and a dot outside all three strings (Daniel's dot).



Students may suggest that a dot outside all three strings could be for things other than people or students in your classroom.

**S:** *My nose.*

**S:** *An elephant.*

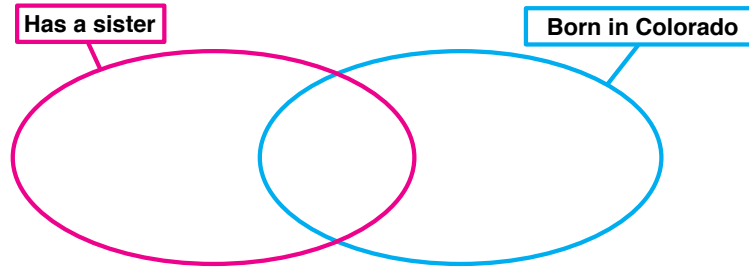
**T:** *Yes, those things are not people and they do not wear clothes or shoes.*

# L3

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## Exercise 4

Erase the board and draw two new strings. Explain that the red string is for people who have a sister and the blue string is for people who were born in Colorado.



**T:** *I am thinking about my friend Nick. Who can draw a dot for Nick in this picture?*

When you ask this question, the students cannot correctly respond without first asking you some questions. Do not allow the students to just guess where a dot should be drawn. Discourage guessing by asking if they are sure that is where the dot belongs.

**S:** *Does Nick have a sister?*

**T:** *That's a very good question. Yes, he does have a sister.*

**S:** *Was he born in Colorado?*

**T:** *No, he was not.*

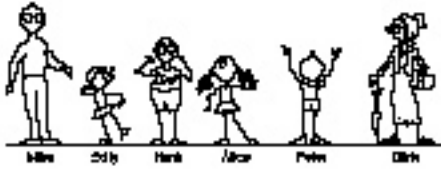
Invite a student to draw and label a dot for Nick.

If your students do not ask these questions immediately, or if they are confused or begin to draw dots randomly, suggest that they ask you some questions about Nick. Continue this exercise with other children unknown to your students so that you get dots in different regions.

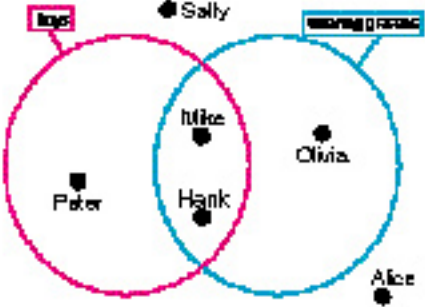
Worksheets L3\* and \*\* are available for independent work. Be sure your students understand that they are to draw and label a dot for each person pictured.

Name \_\_\_\_\_ L3 ★

Draw and label a dot for each person.




Mike Sally Hank Alice Peter Olivia




Name \_\_\_\_\_ L3 \*\*

Draw and label a dot for each person.



Amy Ben Laura Greg Dana Chris





### Capsule Lesson Summary

Introduce the A-blocks and describe each piece with three attributes: color, shape, and size. Find pieces that differ in one way, two ways, and three ways. Place pieces in a string picture and find pieces that could be in a certain region of the string picture.

#### Materials

<b>Teacher</b>	<ul style="list-style-type: none"> <li>• A-Block String Game kit</li> <li>• Colored chalk</li> <li>• Box</li> </ul>	<b>Student</b>	<ul style="list-style-type: none"> <li>• Paper</li> <li>• Colored pens, pencils, or crayons</li> </ul>
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**Advance Preparation:** Before this lesson begins, you will need to prepare some of the materials from the A-Block String Game kit. These materials will be used again in all future lessons on A-blocks and The String Game, so keep them together in the envelope with the kit.

Punch out one set of shapes (24 pieces). If you have a magnetic board, magnetize each piece by sticking a small piece of the magnetic material to the back. If you do not have a magnetic board, have loops of masking tape ready to stick to the back of each piece. Masking tape loses its stickiness quickly, so be prepared to reinforce the pieces with new loops of tape during the lesson.

### Description of Lesson

Put the 24 A-block pieces in a box the size of a shoe box. Sort them so that you can locate any given piece quickly.

#### Exercise 1 \_\_\_\_\_

**T:** *In this box I have some cardboard pieces in different shapes, colors, and sizes. They are called A-blocks. There are three different shapes. What shapes do you think the pieces have?*

**S:** *Squares.*

**S:** *Triangles.*

**S:** *Circles.*

Many students have seen the A-blocks, perhaps last year, and are familiar with them. In any case, these responses usually come immediately. If not, tell them the three shapes involved. As each shape is mentioned, write the word on the chalkboard.

**T:** *There are four different colors. What colors do you think the pieces have?*

**S:** *Red.*

**S:** *Blue.*

**S:** *Green.*

**S:** *Yellow.*

As each color is mentioned, write the word on the chalkboard.

**T:** *There are two different sizes. What should we call them?*

**S:** *Big and little.*

Write these words on the chalkboard.

Tell the class it will be their job to get the pieces out of the box. Call on students, one at a time, to describe to you which piece they want to see. As the pieces are described and taken out of the box, put them on the board for all to see.

**T:** *You tell me which piece you want, and I will take it out of the box for you.*

**S:** *A square.*

**T:** *I have many squares. Which one do you want?*

**S:** *Any of them.*

Hold up a square, for example, a big red square.

**T:** *Here's a square. I will put this piece back in the box. Now who can tell me which piece it was?*

**S:** *A square.*

**T** (holding up a green square): *This one?*

**S:** *No, a red square.*

**T** (holding up the little red square): *This one?*

**S:** *No. It was a big red square.*

**T** (holding up the big red square): *Was it this piece?*

**S:** *Yes.*

Put the piece on the board.

**T:** *Someone else ask me for a piece.*

**S:** *A big triangle.*

**T:** *Yes, I have several big triangles. Which one do you want?*

**S:** *A big blue triangle.*

Take out the big blue triangle and put it on the board.

**T:** *Very good. Sam told me the size, the color, and the shape of the piece he wanted to see.*

**S:** *A big yellow triangle.*

Put the big yellow triangle on the board and continue asking for pieces. If someone describes a piece already on the board, point to it and ask for one you have not yet seen. If the activity begins to drag, ask a student to describe all the rest of the triangles and put them up as they are described. In this way, you can get all the triangular pieces on the chalkboard.

**T:** *How many triangles do we have?*

**S:** 8.

**T:** *How many circles do you think there are?*

**S:** 8.

Take out all (the rest of) the circles and put them on the board.

**T:** *What else could be in the box?*

**S:** *Just some squares.*

**T:** *Which ones?*

Take them out as they are described and put them on the board.

**T:** *How many pieces are there altogether?*

**S:** 24.

**T:** *Can you explain?*

**S:** *I counted them.*

**S:** *8 triangles, 8 circles, and 8 squares make 24 pieces in all.*

Write this calculation on the board.

$$8 + 8 + 8 = 24$$

**T:** *Can anyone tell us another number sentence for this?*

Accept any number sentence for 24, but write only  $3 \times 8 = 24$  on the board. If necessary, suggest this multiplication sentence yourself.

$$3 \times 8 = 24$$

## Exercise 2 \_\_\_\_\_

Take the big yellow circle and put it on a separate section of the board.

**T:** *Who can show me a piece that is different from this piece in just one way?*

Suppose a student, Jason, chooses a small green circle.

**T:** *Well, Jason, your piece is a different color but it is also a different size, so it is different from my piece in two ways.*

Ask the student to make another choice. Suppose Jason selects a small yellow circle.

**T:** *Is Jason's piece a different color? (No)*  
*Is it a different shape? (No)*  
*Is it a different size? (Yes)*  
*So it is different in just one way.*

Organize a table or graph for “How many differences?” on the board. (See the illustration below.) Invite students to place pieces in the graph. Occasionally ask for a piece that goes in one column of the graph. Your graph need not be organized within the columns; the picture here is for your information.



How many differences?

One Way	Two Ways	Three Ways
 (different size)  (different color)  (different shape)	 (different size and shape)  (different size and color)  (different shape and color)	  (different size, shape, and color)

When the graph is complete (all the pieces have been placed) make some observations comparing the number of pieces in each column.

### Exercise 3

Put all the pieces back in the box. Draw two overlapping strings in two colors and label them as in the next illustration.

**T:** *The blue string is for triangles. The red string is for red pieces.*

Hold up the little red square.

**T:** *Who can put this piece in the picture?*

Call on a student to put the piece in the picture and ask the class for agreement or disagreement. If the piece is incorrectly placed, ask for an explanation and call on another student to replace the piece. If the piece is correctly placed, ask,

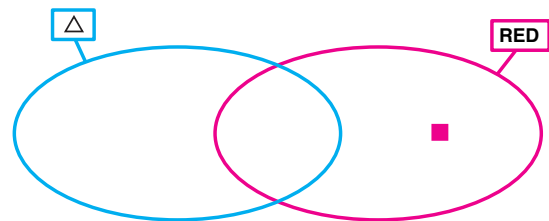
**T:** *Why do you think Kristy put it inside the red string?*

**S:** *Because the red string is for red pieces.*

**T:** *Why did Kristy put it outside the blue string?*

**S:** *Because the blue string is for triangles, not squares.*

**T:** *Yes, you must think about both strings.*



Continue in this way, asking students to place pieces in the picture. Not every piece needs to be discussed, but try to get at least one piece in every region. Be sure the students understand the two questions that help decide where a piece goes: Is it red? Is it a triangle?

Sometimes vary the activity by asking,

**T:** *Can you find a piece that belongs here?*  
 (or)  
*I'll put this piece here. Is it correct?*

You can also vary the activity by asking a student to describe a piece still in the box and then

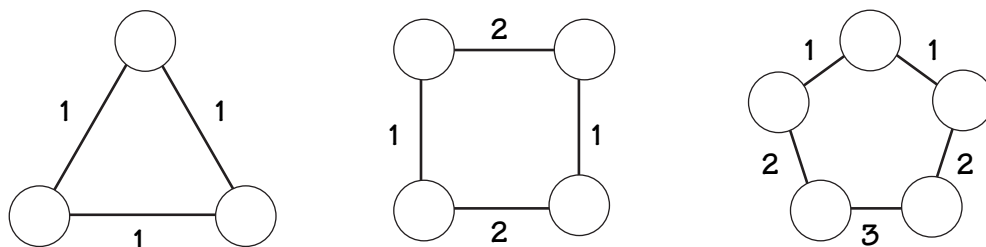


inviting another student to place it correctly.

Continue until all students have had an opportunity to participate.

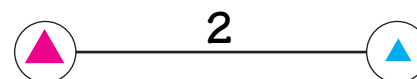
### Center Activity

Prepare some task cards with A-block puzzles. For example:



**Directions:** Use one set of A-blocks. Place pieces in the circles of a puzzle so that the number on each connection shows how many differences the pieces being connected can have.

Here are two differences (color and size).





### Capsule Lesson Summary

Use cubes with from one to five red faces and five to one white faces for cube tossing. Collect results from 20 tosses and use the results to predict which cube was used. Make a display to compare the results obtained using different cubes.

#### Materials

<b>Teacher</b> <ul style="list-style-type: none"> <li>• Ten white cubes</li> <li>• Red circle stickers</li> <li>• Blacklines L5(a) and (b) (optional)</li> </ul>	<b>Student</b> <ul style="list-style-type: none"> <li>• Counting materials</li> <li>• Cube cut-outs (optional)</li> </ul>
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**Advance Preparation:** Put red circle stickers on the faces of ten white cubes so that there are two each with one red, two red, three red, four red, and five red faces. (See the cube cut-out pictures below.) Student counting materials may be Unifix® cubes, pictures of Unifix® cubes made from Blackline L5(a), construction paper squares or strips, or chips. They should be red and white to match the color faces of the cubes. Use Blackline L5(b) to make cube cut-outs for students.

### Description of Lesson

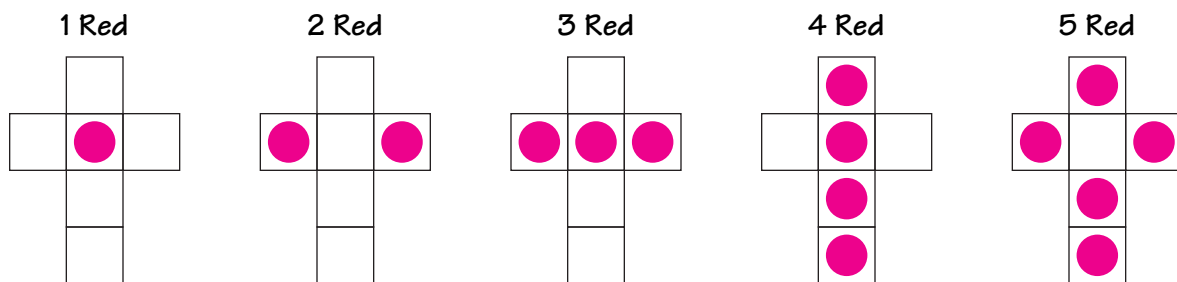
Show the class your collection of cubes and let students examine them. You may want to solicit information about a cube; for example, a cube has six faces (sides), each face is a square, a cube has eight corners, and so on.

**T:** *How are these cubes alike? How are they different?*

**S:** *All the cubes have red on a face.*

**S:** *This cube has just one red face and that cube has three red faces.*

Encourage the class to sort the cubes into five groups according to the number of red faces. You may like to record these observations on cube cut-outs. Blackline L5(b) has cube cut-outs to do this. Show the class how to fold a cut-out to make a cube.



Toss a cube, and show the class how to look at the top face and say if the result was red or white.

**T:** *Today we are going to do some cube tossing and record the results. Which cube would you choose if you wanted to get a lot of red? (The one with five red faces) ... a lot of white? (The one with only one red face)*

# L5

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Divide the class into groups of two or three students and give each group one cube. Tell the groups to check which kind of cube they have but not to tell any other group about their cube. Then provide each group with some type of counting materials with which to record the results when they toss the cube. Possibilities include the following:

- Unifix<sup>®</sup> cubes (20 red and 20 white)
- a paper with pictures of Unifix<sup>®</sup> cubes to color and cut out
- small construction paper squares
- strips of construction paper for making chains
- counters or chips to stack

Direct the groups to perform 20 cube-tossing trials and to record the result of each toss. For example, suppose a group of three students is recording with construction paper chains. One student tosses the cube. If the result is red, the second student makes a red link for a red chain. If the result is white, the third student makes a white link for a white chain. After 20 trials the group should have a red chain and a white chain. The chains may be different lengths, but the total number of links in the two chains should be 20.

After the groups finish tossing the cube 20 times and recording the results, tell them to write about their results and prepare to share with the rest of the class.

Invite each group to display its results and discuss them with the class, but not to tell the class which kind of cube they were using. Let the class predict which cube the group was using. Ask students who make predictions to explain their predictions from the group's display of results. When the class believes they know which kind of cube the group was using, the group can show the class their cube.

You may like to make a class display of the results from all the groups. To extend your display, ask the students what would happen if you used a cube with no (zero) red faces or with six red faces. Do they need to actually perform trials to show what the results would be in your display?

### Capsule Lesson Summary

Review coin recognition, coin values, and coin trades (equal amounts). Present a detective story about a secret amount of money in a piggy bank. Use clues that give information about the particular coins in the bank.

#### Materials

<b>Teacher</b>	<ul style="list-style-type: none"> <li>• One dime, three nickels and one penny in a small container</li> <li>• Colored chalk</li> <li>• Other coins (quarters, dimes, nickels, and pennies)</li> </ul>	<b>Student</b>	<ul style="list-style-type: none"> <li>• Paper</li> <li>• Colored pencils, pens, or crayons</li> </ul>
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### Description of Lesson

#### Exercise 1 \_\_\_\_\_

Begin the lesson with a discussion of the coins. Let students name the various coins and tell you their values. You may want to let them examine the coins and observe how they are different or the same. During the discussion write the names and values on the chalkboard.

Penny (1¢)

Nickel (5¢)

Dime (10¢)

Continue with a discussion of trades. For example, bring two students to the front of the room; give one a nickel and the other some pennies.

**T:** *How many pennies should I give Carlos so that he can trade with Meg and still have the same amount of money?*

Emphasize trading equal amounts and invite students to try several trades. Look at trades of five pennies for a nickel, ten pennies for a dime, two nickels for a dime, five pennies and one nickel for a dime, 25 pennies for a quarter, five nickels for a quarter, and two dimes and one nickel for a quarter. You may also want to include three- or four-way trades, i.e., 10 pennies for two nickels, or one nickel and five pennies, or one dime. In your discussion, include some cases where trades cannot be made.

#### Exercise 2 \_\_\_\_\_

Before doing this exercise, put one dime, three nickels, and one penny into a small container. Choose a container in which the coins inside are not visible and you can hear them clank against the sides, for example, a small opaque plastic vial.

**T:** *How many of you have piggy banks? What do you do with a bank?*

**S:** *I save pennies in my bank.*

**S:** *My father gave me some foreign coins to put in my bank.*

**S:** *I put pennies and nickels and dimes and quarters in my bank.*

Choose one of your students to be the star of your story.

**T:** *Tracy got a new bank and it came with something already in it. She shook it and listened to the clanking sound the coins inside made. “I wish I knew how much money was in my bank,” Tracy said to herself.*

Shake the container and let the class listen.

**T:** *“Clank! Clank! Clank!” said the coins. Suddenly, Tracy realized the coins were talking to her. “Would you like to know how much money is in your new piggy bank?” they said. “Yes! Yes, I would. Can you tell me?” cried Tracy. The coins answered, “We cannot tell you our secret, but we can give you some clues. If you are a good detective, you will be able to discover our secret.”*

Clue 1

**T:** *“The first clue is this,” said the coins. “We are five coins and we are only dimes, nickels, and pennies.”*

Write this information on the board.



Dimes



Nickels



Pennies

Ask the students to suggest some possibilities for what coins are in the bank. You should get suggestions such as three dimes and two nickels; two dimes, two nickels and a penny; five dimes; four nickels and a penny; and so on. Each time ask for the amount of money. After a short while, you should have received many different suggestions. Using either real or play coins, you might like to let a student locate the coins suggested while the class counts the amount.

**T:** *Tracy was not very happy. “There are too many possibilities,” she said to the coins.*

Clue 2

**T:** *“We will give you another clue. Pay close attention. At least one of us is a dime; at least one of us is a nickel; and at least one of us is a penny.”*

*Now, what do you think could be in the bank?*

Draw a chart on the board in which to record possibilities. Let the students give you suggestions and write correct combinations in the chart. You may want to make available some other coins for students to display possibilities.

Continue this until you create the list below. After each new suggestion, check with the class that it is correct, ask how many cents it makes, and write the amount of money next to that line in your chart.

10¢ Dimes	5¢ Nickels	1¢ Pennies	Amount
○ ○ ○	○	○	<b>36¢</b>
○ ○	○ ○	○	<b>31¢</b>
○ ○	○	○ ○	<b>27¢</b>
○	○ ○ ○	○	<b>26¢</b>
○	○ ○	○ ○	<b>22¢</b>
○	○	○ ○ ○	<b>18¢</b>

For many students, you will need to model counting each coin as you put it on the chart to get the amount of money. For example, count three dimes (10¢, 20¢, 30¢) plus one nickel (35¢) plus one penny (36¢) to get the amount 36¢.

**T:** *What is the largest amount of money that could be in the bank?*

**S:** *36¢.*

**T:** *What is the smallest amount of money that could be in the bank?*

**S:** *18¢.*

**T:** *Now you know there is one of these amounts of money in Tracy's new piggy bank (point to the six possibilities). "Are you ready for another clue?" the coins asked. "Yes, yes I am," said Tracy and she listened carefully.*

Clue 3

**T:** *"If you exchange each dime for two nickels, then we would be six coins," said the coins.*

Go over the list line-by-line with your class. Cross out those lines that do not fit this clue. Again, you may let students display a possibility with actual coins, make the exchange of dimes for nickels, and observe how many coins there are. You may find it helpful to show the result of the exchange next to a line in your chart. If so, erase this before going on to the next clue.

10¢ Dimes	5¢ Nickels	1¢ Pennies	Amount	After exchange	
				Nickels	Pennies
<del>○ ○ ○</del>	<del>○</del>	<del>○</del>	<del>36¢</del>	<del>○ ○ ○ ○ ○</del>	<del>○</del>
<del>○ ○</del>	<del>○ ○</del>	<del>○</del>	<del>31¢</del>	<del>○ ○ ○ ○ ○</del>	<del>○</del>
<del>○ ○</del>	<del>○</del>	<del>○ ○</del>	<del>27¢</del>	<del>○ ○ ○ ○ ○</del>	<del>○ ○</del>
○	○ ○ ○	○	<b>26¢</b>	○ ○ ○ ○ ○	○
○	○ ○	○ ○	<b>22¢</b>	○ ○ ○ ○ ○	○ ○
○	○	○ ○ ○	<b>18¢</b>	○ ○ ○ ○ ○	○ ○ ○

**T:** *Now we know that there is either 26¢, 22¢, or 18¢ in the bank. Tracy shook her bank and listened. She heard the coins say this: “You may have one more clue. Pay close attention and think carefully about this clue.”*

Clue 4

**T:** *“If you put one more nickel in your bank, then there would be more than 30¢.”*

*When you figure it out, write on your paper (or whisper to me) how much money is in Tracy's piggy bank. (26¢)*

You may have to remind the class that there is either 26¢, 22¢, or 18¢ in the bank. After a while, choose a volunteer to say aloud how much money is in the bank. Check the three possibilities with the class so that they can eliminate 22¢ and 18¢. When you find that there is 26¢ in the bank, let a student open the container and count the money.

### Home Activity

Suggest that parents/guardians find opportunities to count the amount of money (cents) in a small collection of coins (quarters, dimes, nickels, pennies) with their child. Also, suggest they let the child choose coins to make a given amount of money up to 99¢.



### Capsule Lesson Summary

Review the different A-block pieces as to shape, color, and size by taking them out of a box, one at a time, as they are described. Place the A-blocks correctly into a two-string picture or a three-string picture. Play The String Game with hidden labels.

#### Materials

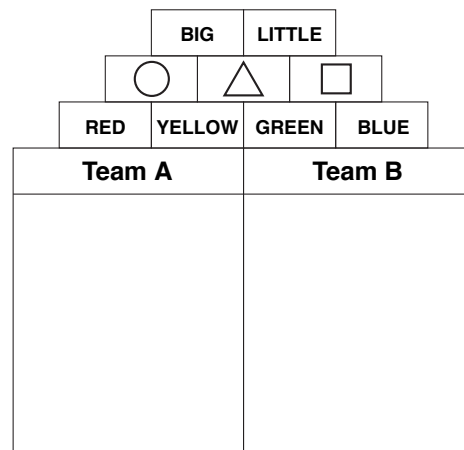
- |                |   |                |  |
|----------------|---|----------------|--|
| <b>Teacher</b> | <ul style="list-style-type: none"> <li>• A-Block String Game kit</li> <li>• Box</li> <li>• Colored chalk</li> </ul> | <b>Student</b> | <ul style="list-style-type: none"> <li>• None</li> </ul> |
|----------------|---|----------------|--|

**Advance Preparation:** Before this lesson begins, prepare a team board as pictured below either directly on your chalkboard, on the side of a file cabinet, or on another easily accessible board. Be sure this board is metallic if you have magnetized the A-block pieces. Post a list of attributes above the team board. This list can be prepared using one of the posters in the A-Block String Game kit.

### Description of Lesson

Put the 24 A-block pieces in a box about the size of a shoe box. Sort them so that you can locate any given one quickly. Divide the class into two teams. Try to arrange that the teams are roughly equal in ability. You may want to let the teams choose their own names rather than A and B.

**Note:** If you prefer, this lesson can be taught with more than two teams. For example, you might use three or four cooperative groups as teams. Prepare the team board accordingly.



#### Exercise 1 \_\_\_\_\_

Briefly discuss the different attributes of A-block pieces: size, shape, and color. Refer to the list of attributes above the team board.

**T:** *Your first task will be to get all the pieces out of my box and onto this board (point to the team board). We will take turns; someone on Team A will tell me a piece to put on Team A's side of the board, and then someone from Team B will tell me a piece to put on Team B's side. Remember, you must describe a piece that is still in the box so I know exactly which piece you want me to put on your side of the board. Team A goes first.*

**S:** *Big red circle.*

Put the big red circle on Team A's side of the board.

**T:** *Very good. Kevin told me the size, color, and shape of the piece he wanted. Now it is Team B's turn.*

**S:** *Little red circle.*

Put the little red circle on Team B's side of the board.

**T:** *Very good. Angela described a piece that is still in the box and I can put that piece on her team's side of the board.*

Continue in this way, keeping a brisk pace, until all the A-block pieces are on the board. If a student describes a piece that is already on the team board, point out that the piece has been chosen and ask the student to choose a different piece, one that is still in the box. Try to call on weaker students early in the game so that there are many choices open to them. Encourage all students to be thinking about a piece they will ask for when you call on them.

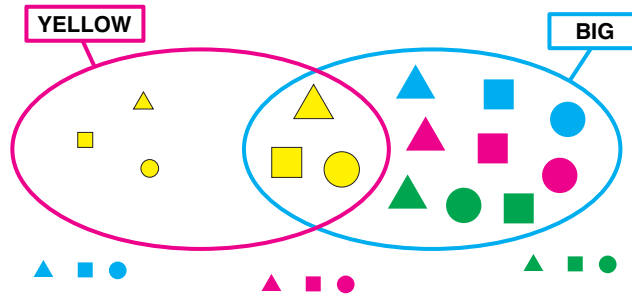
## Exercise 2 \_\_\_\_\_

**Note:** If this exercise seems too easy for your class, use the alternative Exercise 2.

Draw two different-colored, overlapping strings on the board. Label them as in the next illustration.

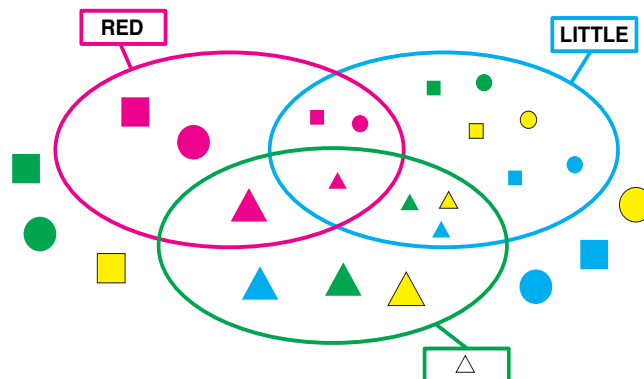
**T:** *Now the teams will take turns. Each of you will try to place a piece from your team's side of the board in this string picture. If you put it in its correct place, I will say yes and it stays there. If you do not, I will say no and you must return it to your team's side of the board. The first team to get all of its pieces in the picture is the winner.*

Play the game, alternating teams, and alternating turns among the members of each team. When all the pieces are in the picture, your board will look like this.



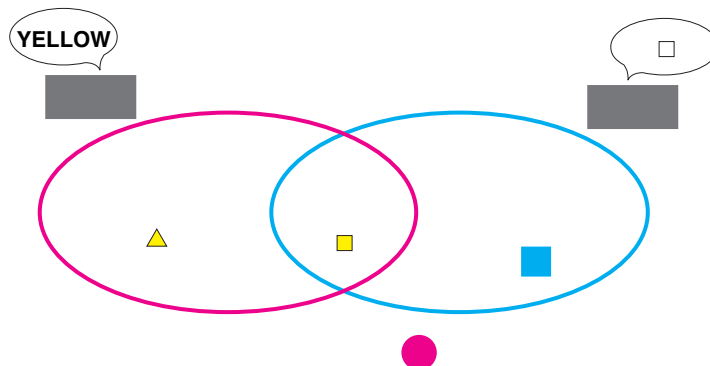
## Exercise 2 (alternative) \_\_\_\_\_

Draw three different-colored, overlapping strings on the board. Label the strings as in the next illustration. Give directions as in Exercise 2 above. When all the pieces are in the picture, the board will look like this.



### Exercise 3 \_\_\_\_\_

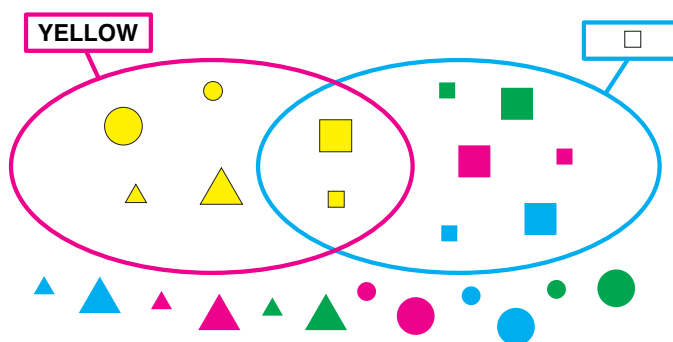
Prepare to play The String Game in the usual way.<sup>†</sup> Distribute game pieces to the two or more sides of the team board (Team A and Team B). Then, set up the playing board as shown below, but this time place the string cards face down (bubbles indicate what is hidden on the cards). Place three or four pieces correctly. Or you may want to allow the teams to each select one or two pieces for you to place correctly in the picture; these then serve as starting clues.



Remember that the rules of the game call for you to be the judge. If a piece is correctly placed, say yes, and immediately instruct the player to try to place a second piece (bonus turn). No player should have more than two consecutive turns. If a piece is incorrectly placed, say no, and ask the player to return the piece to his or her team's side of the team board.

For this game, the player who correctly places the last piece from his or her team's side of the team board may then attempt to identify each of the string cards. A team wins by being first to get all of its pieces in the string picture and to identify the strings.

The following picture shows correct placement for all the game pieces. You can use this picture as a crib sheet while the game is being played.



### Home Activity

This is a good time to send a letter to parents/guardians about the language of strings. Blackline L7 has a sample letter.

<sup>†</sup>See Appendix D on The String Game for a description of equipment and preparation for the game played in the usual way.



### Capsule Lesson Summary

Make a class graph showing the distribution of letters in the students' names. Answer questions about how many letters, most and least frequent letters, and make comparisons with other such graphs.

#### Materials

**Teacher**

- Chart paper (optional)
- Calculator

**Student**

- Paper
- Small Post-it™ notes

**Advance Preparation:** You may want to prepare for the letter distribution graph by writing the letters of the alphabet about 6 cm apart on the board or on chart paper (approximately 150 cm by 100 cm).

### Description of Lesson

Begin the lesson by asking each student to write his or her first name (the name they prefer to be called) on a piece of paper and then to count the letters.

**T:** *Who has the most letters in their name?*

**S:** *Samantha.*

**T:** *How many letters in your name, Samantha?*

**S:** *Eight.*

**T:** *Does anyone have more than eight letters in their name?*

**S:** *I have nine.*

Continue in this way until the class agrees who has the most letters. You may also want to find who has the least letters in their name.

Then ask the class some prediction (estimation) questions.

**T:** *How many letters do you think we have altogether in our names?  
Which letter do you think will occur most often in all of our names?  
Do you think there are any letters not in any of our names?  
How could we check our answers to these questions?*

Conduct a discussion of how the class can organize all the letters in their names to check the answers to these questions.

Prepare to make a class graph by writing the letters of the alphabet about 6 cm apart on the board or large chart paper (approximately 150 cm by 100 cm). Instruct the students to write the letters of their names on small Post-it™ Notes (4 cm by 5 cm) or other self-stick notes, one letter on each note. For example:

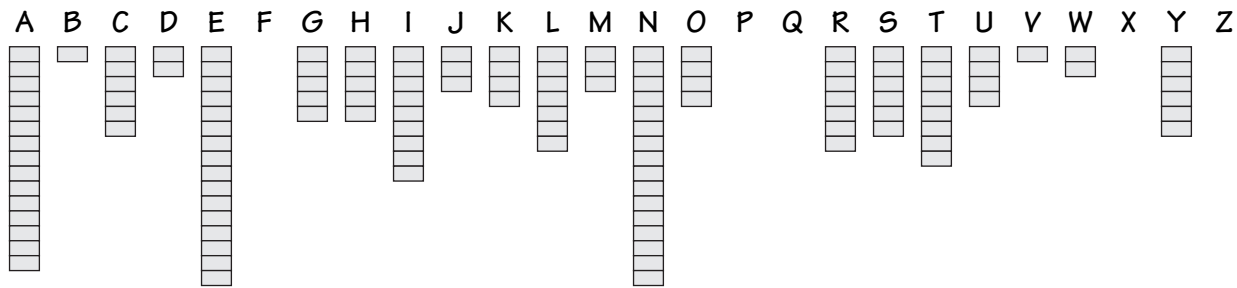
S
A
M
A
N
T
H
A

Then invite students to place the letters in their names on the graph. You may like to do this one letter at a time (everyone with A's puts them in the graph, everyone with B's ..., and so on), or one or two students at a time. If you chose to do one letter at a time, you may like to ask some questions such as the following:

- How many A's?
- Do you think we have more or less B's than A's?
- Do you think any other letter will be as plentiful as A?

If you do one or two students at a time, you may like to make observations about the emerging graph; for example, how a few letters start pulling away from the others in number.

When all the letters from students' names are in the graph, let students comment on the display and use the information in the graph to check the earlier questions. For example, the following shows a possible class graph.



- S:** *N and E are the letters of which we have the most.*
- S:** *There are no F's, P's, Q's, X's, or Z's in our names.*
- S:** *There's only one B.*
- S:** *We call my sister Pixie. If she were here, we would have a P and an X.*

Suggest that the class use a calculator to find how many letters there are altogether in their names.

- T:** *Do you think our graph of letters in our names would look the same as that of another second grade class?*
- S:** *We might have more N's than another class.*
- S:** *Another class might have some F's or P's.*
- S:** *Another class might have more or less letters altogether.*

If possible, you may like to compare your class graph with that of another class having done this lesson.

## Extension Activity

Discuss with the class what the graph would look like if they used their last names rather than their first names. Would they expect it to be the same, similar, or very different? How might it be the same? How might it be different? Then make the graph and compare.

### Capsule Lesson Summary

Draw a picture of dots for all students in the class. Locate all the students in the picture. Include arrows in the picture and change seats by following the arrows. Play the changing seats game several times, observing who is in their own seat after each round. This game involves physical experiences with the mathematical concepts of multiples and least common multiples.

### Materials

<p>Teacher</p>	<ul style="list-style-type: none"> <li>• Colored chalk</li> </ul>	<p><b>Advance Preparation:</b> Before this lesson begins, you may wish to plan an arrow picture which will be easy for your class to follow. The dialogue in this lesson is based on the arrow picture on page L-36.</p>
<p>Student</p>	<ul style="list-style-type: none"> <li>• None</li> </ul>	

### Description of Lesson

**T:** *I am going to draw some dots on the board. Watch me, and silently count them as I draw. Also, think about what my dots could be for.*

Use the largest available chalkboard and draw a dot for each student in the room. Spread out the dots as much as possible but arrange them in the same relative positions as the students are sitting. (The top of the board should correspond to the front of the room or where you are standing.) Make a point of looking at the class a few times while you are drawing.

Depending on the size and arrangement of your classroom, your board might be organized in rows and columns as in the next illustration.

**T:** *How many dots did I draw?*

**S:** *23.*

**T:** *What do you think these dots could be for?*

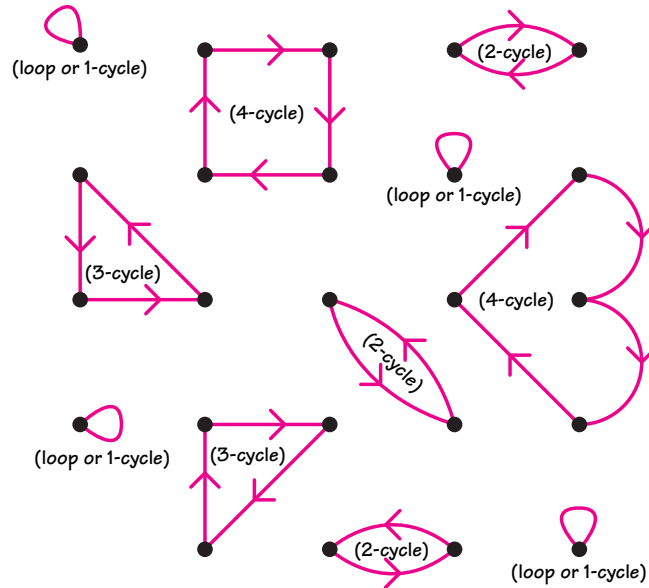
**S:** *They are for us.*

**T:** *That's close to correct. They are for your seats. Do you know where your seat is in this picture?*

Let several students come to the board and point to the dots for their seats. Point to a dot and ask the student whose seat is there to raise his or her hand. Continue this activity until you are sure all the students know where their seats are in the picture, but do not label the dots.

**T:** *We are going to play a game; you will have to follow an arrow. So, first let me put some arrows in this picture. Pay special attention to the arrow that starts at the dot for your seat.*

As quickly and carefully as possible, draw arrows in your picture. Be sure exactly one arrow starts at each dot and exactly one arrow ends at each dot. Include some loops, 2-cycles, 3-cycles, and 4-cycles as indicated in the following picture.



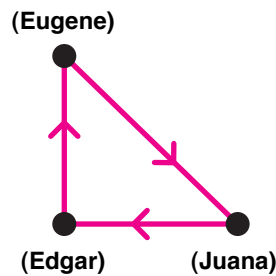
- T:** *How many arrows did I draw?*  
**S:** *23. One arrow starts at each dot.*

Encourage such observations, but do not force the class to make them.

- T:** *The game we are going to play involves changing seats. First you must look carefully at our picture, then follow the arrow that starts at the dot for your seat and find out whose seat you are going to take.*

Before you let the students play the game, be sure everyone understands where they are going to go. For example, point to a dot and ask,

- T:** *Eugene, your seat is here.  
When we change seats, where will you go?*  
**S:** *To Edgar's seat.*  
**T:** *And Edgar, where will you go?*  
**S:** *To Juana's seat.*  
**T:** *And Juana, where will you go?*  
**S:** *To Eugene's seat.*



Names in parentheses indicate who's in a seat at the start. They are not written on the board.

You need not ask all the students where they will go, but ask the class as a whole if they are ready—if they have checked the picture carefully to know whose seat they are going to take.

- T:** *Everyone quiet. Get ready. Change.*



After the students have made their moves and are quieted, point to a dot and ask,

**T:** *Now, who is in this seat?*

Do this several times. The class should discover whether anyone made a mistake when the move was made. It is likely that a student would have been corrected during the play of the game by other students.

**T:** *Are some of you still in your own seats? Raise your hands. Why are you still in your own seat?*

**S:** *There was a loop at my dot, so I had to go to my seat.*

Make a list on the board of all the students who are in their own seats. For example:

### ROUND 1

Ben

Tracy

Daniel

Kellie

**T:** *Let's play another round of the game. Look at the picture and find where you are now. Follow the arrow that starts at the dot for the seat you are in and find whose seat you are going to take this time.*

Before the students play the game again, be sure everyone knows where they are going to go.

**T:** *Everyone quiet. Get ready. Change.*

After the students move and are quieted, ask several students where they are in the picture now (after round 2).

**T:** *Who is in their own seat? Raise your hands. Sonja, you are back in your own chair. How did that happen?*

**S:** *First I went to Laura's seat. Then I had to come back to my own seat.*



Make another list on the board of all the students who are in their own seats. For example:

### ROUND 1

Ben

Tracy

Daniel

Kellie

### ROUND 2

Ben

Tracy

Daniel

Kellie

Sonja

Laura

Tammy

Kevin

Timothy

Angela

**T:** *Let's play another round of the game. But before we do, let's try to make a list of all the students we think will be in their own seats after round 3. We will check our list after we play.*

Write the list of students suggested by the class on the board and label it **ROUND 3**.

**T:** *Look at the picture and find where you are now. Follow the arrow and find whose seat you are going to take this time. Everyone quiet. Get ready. Change.*

After the students move and are quieted, check the list against those who are actually back in their own seats. Make corrections as needed. For example:

<u>ROUND 1</u>	<u>ROUND 2</u>		<u>ROUND 3</u>	
Ben	Ben	Laura	Ben	Eugene
Tracy	Tracy	Tammy	Tracy	Juana
Daniel	Daniel	Kevin	Daniel	Lynn
Kellie	Kellie	Timothy	Kellie	Gary
	Sonja	Angela	Edgar	Chuck

**T:** *Ben, Tracy, Daniel, and Kellie are always on our list. Why?*

**S:** *Because they always have to go to their own seats—they have loops at their dots.*

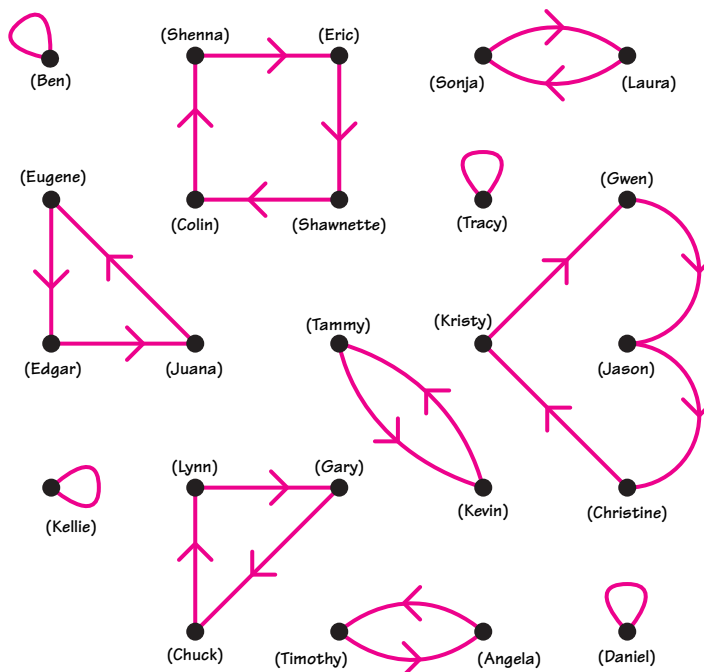
**T:** *Sonja, you are not in your own chair now. Where did you go?*

**S:** *To Laura's seat.*

Ask several such questions about the lists. Then repeat this activity for a fourth round of the game. Ask the class to first predict who will be on the list after round 4. Make a list as they suggest and check the list after the play.

<u>ROUND 1</u>	<u>ROUND 2</u>		<u>ROUND 3</u>		<u>ROUND 4</u>	
Ben	Ben	Laura	Ben	Eugene	Ben	Angela
Tracy	Tracy	Tammy	Tracy	Juana	Tracy	Shenna
Daniel	Daniel	Kevin	Daniel	Lynn	Daniel	Eric
Kellie	Kellie	Timothy	Kellie	Gary	Kellie	Shawnette
	Sonja	Angela	Edgar	Chuck	Sonja	Colin
					Laura	Kristy
					Tammy	Gwen
					Kevin	Jason
					Timothy	Christine

If the class has enjoyed commenting on this game, ask some questions about these four lists (similar to the previous ones). Then ask students to predict how many rounds of the game would have to be played before everyone would be back in his or her own seat. This is a difficult problem, and you should not expect to get an exact answer at this time. Some students may observe some patterns that enable them to predict the correct answer (which is 12 in the example below).



### A few suggestions:

(In the above example the names are in parentheses because they are not written on the board. They only indicate the starting situation, i.e., whose seat is where.)

- Don't draw a cycle larger than a 4-cycle so that the change does not become too complicated.
- Match better students in a cycle with those that might need help making the correct moves.
- Make loops at dots for students who are confident individuals and who will not be embarrassed by having to always stay in their own seats.

### Writing Activity

Ask students to write about what happens in the changing seats game. They might draw a picture to help describe how the game works in your class or in a make-believe class.



### Capsule Lesson Summary

Solve a detective story about a secret amount of money with clues about the coins, a clue about how the amount of money can be put on the Minicomputer, and an arrow picture clue. Find how much money the students would have altogether if each student had exactly one dime; draw a +10 spiral to help solve the problem.

### Materials

- |   |   |
|---|---|
| <b>Teacher</b> <ul style="list-style-type: none"> <li>• Coins (one quarter, one nickel)</li> <li>• Minicomputer set</li> <li>• Colored chalk</li> </ul> | <b>Student</b> <ul style="list-style-type: none"> <li>• Paper</li> <li>• Colored pencils, pens, or crayons</li> </ul> |
|---|---|

## Description of Lesson

### Exercise 1 \_\_\_\_\_

Before the lesson begins, put one quarter and one nickel in your pocket or a small container such as a change purse, ring box, match box, and so on.

**T:** *I have some money in my pocket (or in this small container) and you have to discover how much. I will give you some clues and, if you listen carefully, you can discover how much money I have without guessing.*

Clue 1

**T:** *My first clue is that I have two coins. They could be quarters, dimes, nickels, or pennies. How much money could be in my pocket?*

**S:** *50¢. You could have two quarters.*

**S:** *15¢. You could have one dime and one nickel.*

Draw a chart on the board in which to record the possibilities as students suggest them. Encourage the class to find all the possibilities; each time an amount of money is suggested, be sure the student says which coins make that amount of money and restate the amount as you put it in the chart. Your completed chart should have ten possibilities, although most likely they will not have been suggested in this orderly way.

	25¢ Quarters	10¢ Dimes	5¢ Nickels	1¢ Pennies	Amount
	○ ○				50¢
	○	○			35¢
	○		○		30¢
	○			○	26¢
		○ ○			20¢
		○	○		15¢
		○		○	11¢
			○ ○		10¢
			○	○	6¢
				○ ○	2¢

Leave this chart on the board.

**Clue 2**

**T:** *My next clue is that you could have the amount of money in my pocket with just nickels.*

For this clue you might have to illustrate what is meant with an example. Be sure the students understand that this clue does not mean you have two nickels in your pocket.

**T:** *For example, you could have 15¢ with three nickels. So I could have 15¢ in my pocket. What other amounts of money could be in my pocket?*

**S:** *50¢. You have 50¢ with ten nickels.*

Go through the possibilities in the chart, crossing out or erasing those amounts that cannot be made with just nickels. This clue leaves six possibilities.

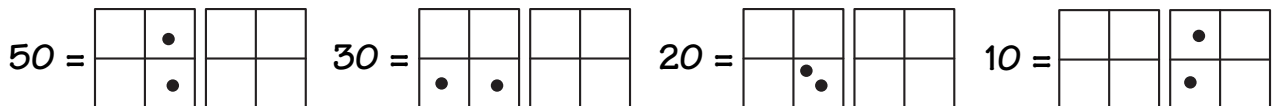
25¢ Quarters	10¢ Dimes	5¢ Nickels	1¢ Pennies	Amount	
○ ○				50¢	(ten nickels)
○	○			35¢	(seven nickels)
○		○		30¢	(six nickels)
○			○	<del>26¢</del>	
	○ ○			20¢	(four nickels)
	○	○		15¢	(three nickels)
	○		○	<del>11¢</del>	
		○ ○		10¢	(two nickels)
		○	○	<del>6¢</del>	
			○ ○	<del>2¢</del>	

**T:** *Now you know that I have one of these six amounts of money in my pocket.*

**Clue 3**

*My next clue is this: You can put this amount of money on the Minicomputer with exactly two checkers. Which of these numbers (point to the remaining six possibilities) can you put on the Minicomputer with exactly two checkers?*

Display two Minicomputer boards and let the students try each of the six numbers. They should find that four possibilities remain.



Cross out or erase the other two possibilities (35¢ and 15¢) after it has been agreed that they cannot be put on the Minicomputer with exactly two checkers.

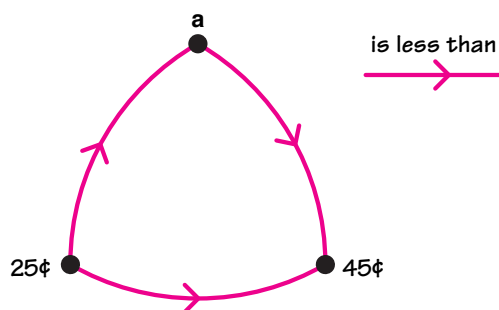
**T:** *Now you know that the amount of money in my pocket is either 50¢, 30¢, 20¢, or 10¢. I'll give you just one more clue. When you know for sure how much money is in my pocket, write it on your paper.*

Clue 4

Draw this arrow picture on the board.

**T:** *The amount of money in my pocket is here (point to **a**).*

Check some of the students' papers. After most students have the correct amount, let one student announce that you have 30¢ in your pocket, and show them the quarter and the nickel.



## Exercise 2 \_\_\_\_\_

Distribute paper and colored pencils.

**T:** *Let's pretend that every student in this class has one dime and we want to find how much money you have altogether. Think about the problem for a minute while I draw a dot for each of you on the board. How many dots should I draw?*

**S:** *26 (or the number of students in your class).*

Draw the dots spread out across the board as in the next illustration.

**T:** *We are going to draw a picture here to help us find out how much money you would have altogether. Before we do, can anyone predict how much?*

Let several students give responses and write their predictions on the board. Then start with a student in one corner of the room.

**T:** *This dot (point to the left-most dot) will be for Ming. She has one dime, so I label it 10. The person next to Ming also has one dime. Together they have how much money?*

**S:** *20¢.*

**T:** *So I draw a red arrow and label the next dot 20.*

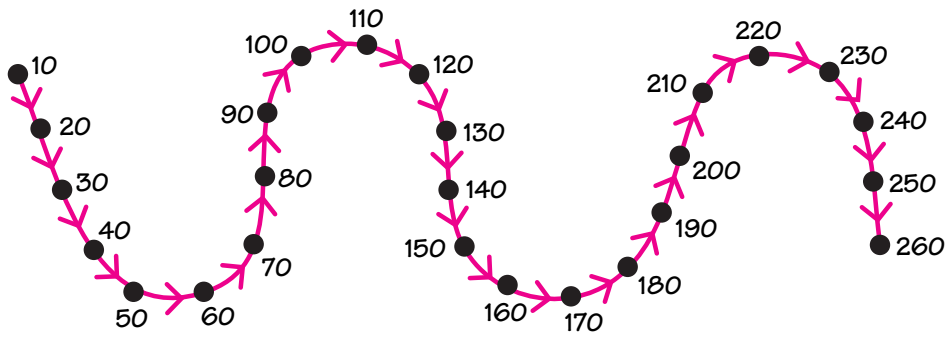
**T:** *What do you think my red arrow is for?*

**S:** *+10.*

**T:** *Good! Now let's continue going around the room adding each person's dime to the amount of money.*

# L10

Ask the student whose dot you are labeling to tell you how to label it. Try to get the students to respond quickly so that it is obvious you are just counting by tens.



**T:** *How much money altogether?*

**S:** *260¢ or \$2.60.*

Check this result with the initial predictions. Ask a student to write the amount of money on the board as \$2.60.

**T:** *Think about this problem. Suppose each of you has a nickel instead of a dime. Then how much money would you have altogether? Write what you think on your paper. Then draw a picture to show adding all the nickels together .*

You may have to help some students get started on a +5 picture. Students who finish this problem quickly can be encouraged to think about a similar problem for a quarter and to draw a +25 picture.

## Home Activity

Suggest several activities for parents/guardians to use as money practice with their child.



### Capsule Lesson Summary

Prepare to play The A-Block String Game by taking the A-block pieces out of a box, one at a time, as they are described by team members. Play a round of the game, placing the pieces in a three-string picture with visible labels. Play another round, this time with hidden labels.

### Materials

<b>Teacher</b> <ul style="list-style-type: none"> <li>• A-Block String Game kit</li> <li>• Colored chalk</li> </ul>	<b>Student</b> <ul style="list-style-type: none"> <li>• None</li> </ul>
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**Advance Preparation:** You may like to have a facts practice sheet available for students to complete while you reset the board between Exercises 2 and 3.

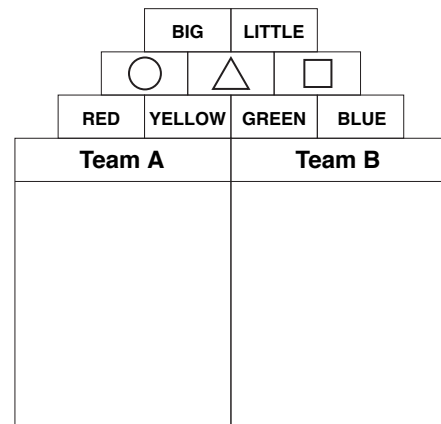
## Description of Lesson

Put the A-block pieces in a box about the size of a shoe box. Sort them so that each one is easy to locate. Divide the class into two or three teams and try to arrange that the teams are roughly equal in ability. You may like to allow the team members to choose a name for their team.

### Exercise 1 \_\_\_\_\_

Briefly review the different kinds of A-block pieces and post a list of attributes above the team board.

**T:** *We are going to play a string game with these shapes. First you have to help me take the A-block pieces out of this box. We will take turns. Someone on Team A will tell me which piece to put on Team A's side of the board, and then someone from Team B will tell me which piece to put on Team B's side of the board. Remember, you have to describe a piece that is still in the box carefully, so I know exactly which piece you want me to put on your side of the board.*

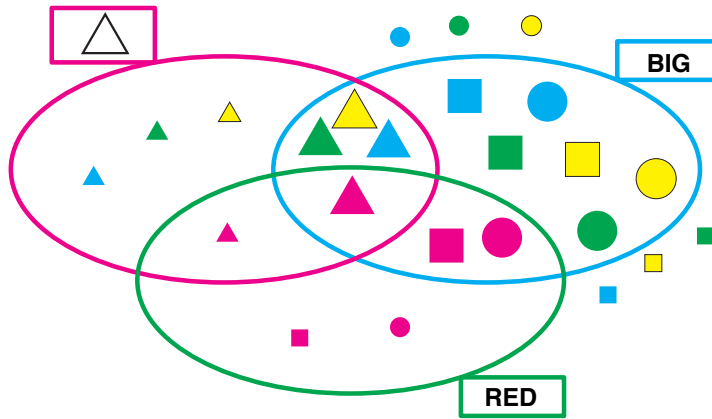


Alternating between the teams, let students describe pieces to put on the team board. This exercise serves to remind the students of what all the A-block pieces look like. Encourage the students to be decisive about which piece they want you to put out. Decide on students to call on early so that there are many choices open to them. Encourage all the students to be thinking about which pieces they will ask for when you call on them.

## Exercise 2 \_\_\_\_\_

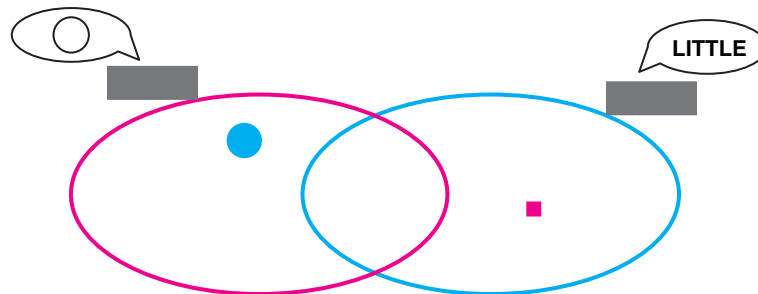
Draw three large overlapping strings on the playing board using different colors. Label the strings as shown in the next illustration.

With this situation, play a game alternating turns between the teams and among the members of a team. Because the labels are visible no starting clues are necessary. The picture below shows the correct placement of all the pieces.

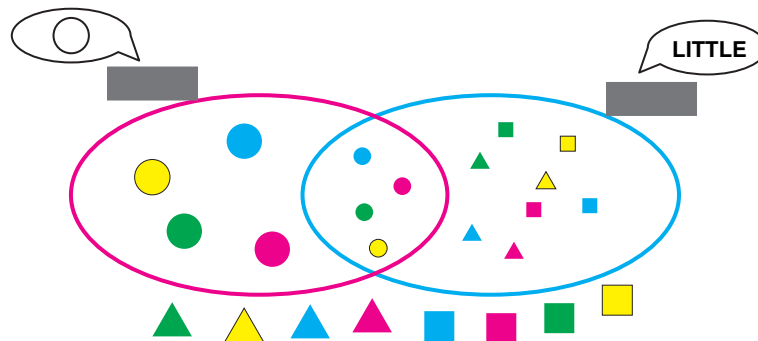


## Exercise 3 \_\_\_\_\_

Prepare to play a two-string version of The String Game<sup>†</sup> in the usual way with facedown string cards. Redistribute game pieces on the game board. Set up the playing board as shown below (the bubbles indicate what is hidden on the string cards). There are two starting clues.



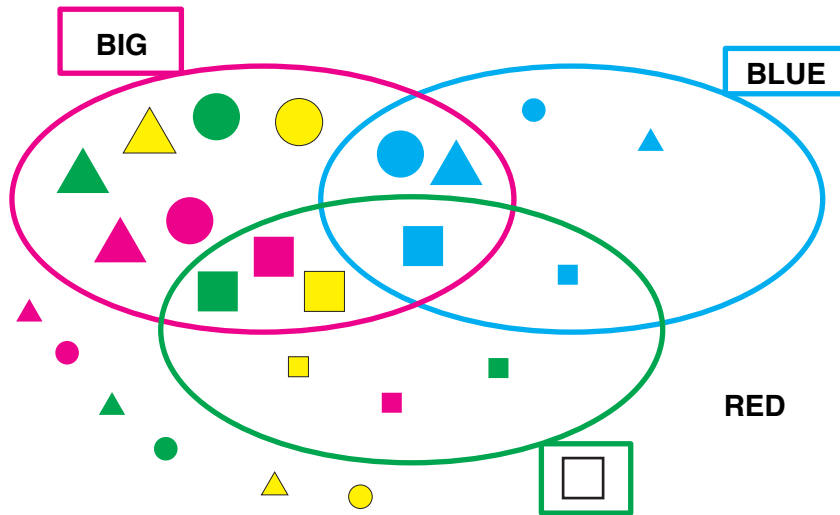
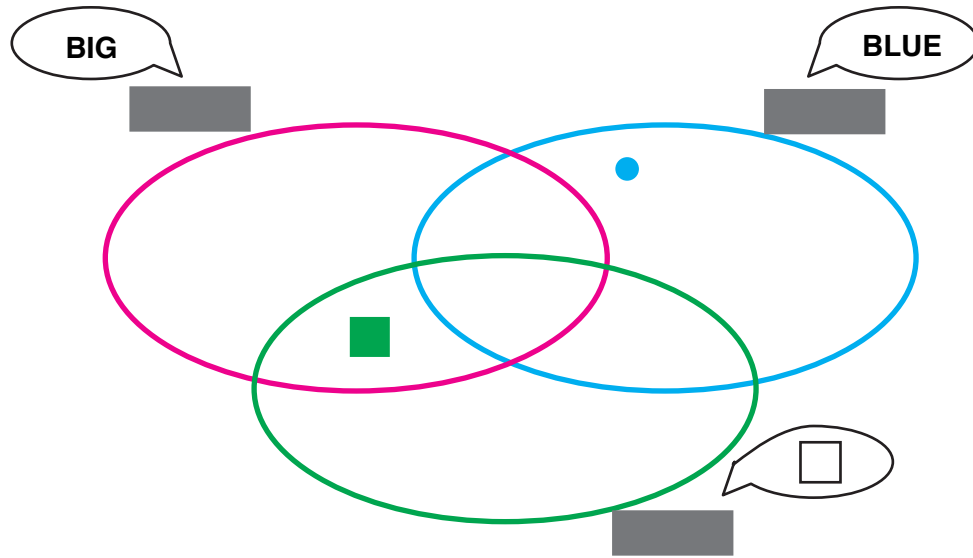
The next picture shows correct placement for all the game pieces. Use this picture as a crib sheet while the game is being played.



## Exercise 4

<sup>†</sup>See Appendix D on The String Game for a complete description of the game.

Play another game with two strings or, if you think your class is ready for the challenge, one with three strings (see Appendix D for sample games). The following pictures show starting clues and correct placement for all the game pieces in a three-string game.





### Capsule Lesson Summary

Read the storybook *The Happy Puppet* and invent new stories about the various string pictures and arrow pictures in the book. This storybook involves the reader in using the mathematical languages of strings and arrows to record a variety of real and make-believe events.

#### Materials

<b>Teacher</b>	<ul style="list-style-type: none"> <li>• Colored chalk</li> <li>• <i>The Happy Puppet</i> Storybook</li> <li>• Blackline L12</li> </ul>	<b>Student</b>	<ul style="list-style-type: none"> <li>• <i>The Happy Puppet</i> Storybook</li> <li>• Puppet picture</li> </ul>
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**Advance Preparation:** Use Blackline L12 to make copies of the puppet picture for students.

### Description of Lesson

Sometime before this lesson begins, read the *The Happy Puppet* on your own. Also read the comment by Edward Martin on the last page of the booklet. While doing this, think about the various situations and how your students might react to them. Feel free to bring your imagination, your experience, and your general familiarity with the interests and concerns of your students to bear on the lesson. This lesson description simply provides some suggestions on how students might react and on how you might prompt creative thinking.

In order to create the right climate for the fanciful adventures of *The Happy Puppet*, the story should not be read too slowly. Many second graders are not yet very good readers, so unless you have some unusually good readers, you should do most of the reading yourself. The students may follow the reading in their own books. They should have time to look at the pictures and to answer the questions by pointing to dots or to parts of the realistic drawings, or by telling their own stories. Allow spontaneous reaction and expression of feeling about *The Happy Puppet*.

If your students get very involved in telling their own stories during this lesson, do not be concerned about finishing the storybook. There will be another telling stories lesson later in the year during which you can finish this storybook.

#### Pages 1 and 2

Read both pages to the class. Encourage discussion about what a hand-puppet is and how one might use hand-puppets to tell make-believe stories. On page 2 there are many interesting pictures about which students may wish to comment. Simply ask what they see and you will get a variety of responses.

- S:** *I see lots of arrows and some loops.*
- S:** *I see +, x and ÷.*
- S:** *I see 44 + 44 and 88, and they are the same.*
- S:** *Also, 11 + 11 + 11 + 11 + 11 + 11 + 11 + 11 = 88.*
- S:** *88 is on the Minicomputer flags.*

## Pages 3 and 4

Read both pages to the class. Let the students comment on the changing interpretation of the eight dots and perhaps give some new interpretations of their own. Be sure to note how there are always eight dots; the number of dots does not change even though the interpretation may.

Students might not be familiar with the word *jubilant*. Discuss its meaning and how it fits the Happy Puppet's game.

## Pages 5 and 6

Read both pages collectively, pausing to allow students to answer the questions. Ask students to locate the different cars in the string picture. After some discussion of this situation, encourage students to invent their own stories about the string picture. Let the class discuss whether a student's story fits the picture. You may find it helpful to draw the picture on page 5 on the board during this discussion.

## Pages 7 and 8

Read both pages, pausing to allow the students to point to appropriate dots and to answer the questions. In each situation, discuss the meaning of the red arrows. You may like to draw the picture on the board and have the class make up other stories for the picture.

## Pages 9 and 10

Read and discuss pages 9 and 10. Encourage students to tell their own stories about the arrow picture. Some may recall the exchanging names activity the class played several weeks ago or the game of changing seats. Do not expect clearly stated stories that exactly fit the picture, but do encourage the class to check whether a story does fit the picture.

## Pages 11 and 12

Read these pages collectively, pausing to ask students to point to appropriate dots and answer the questions. After reading page 12, ask the students to show where the various toys are in the string picture.

## Pages 13 and 14

Allow the students to read these pages on their own or read them collectively, whichever you think is most appropriate for your class. There are no pictures to be investigated nor to be discussed on these pages.

Give students a copy of the puppet picture to color and to use to tell their own stories. They may also enjoy copying or drawing their own string and arrow pictures. The puppet and the pictures can go home with students who might like to tell their families about *The Happy Puppet*.

## Capsule Lesson Summary

Locate children and other objects in string pictures about your school. Progressively make the picture more and more detailed with strings for several different groups of children at the school. Examine different ways of picturing the same situation.

### Materials

**Teacher**

- Colored chalk

**Student**

- Unlined paper
- Colored pencils, pens, or crayons

## Description of Lesson

### Exercise 1 (optional)

If you choose to do this exercise, collect ahead of time the numbers of students in each classroom in the school or instruct students to survey all the classrooms to find this information.

Present the above information to the class and ask several questions.

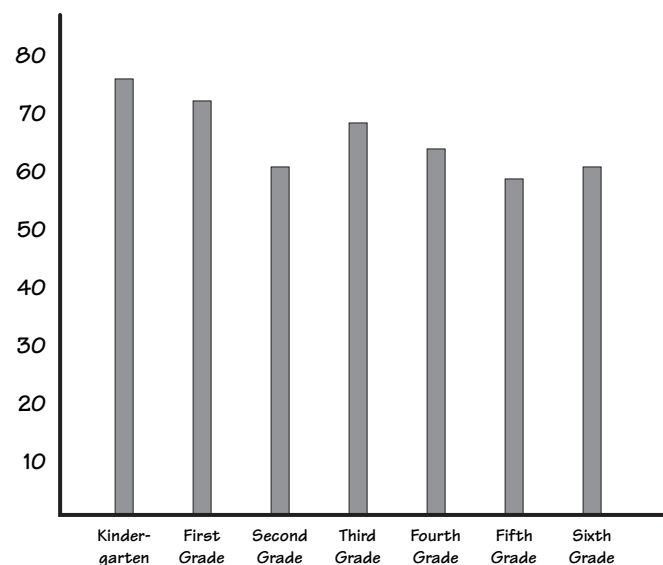
- T:**
- How many first graders do we have in the school?*
  - How many second graders?*
  - How many students are there altogether in the school?*
  - How many boys? How many girls?*

Let students decide how to use the information to answer these questions. Allow students to use calculators or other tools (Minicomputer, if they choose) to do calculations. Before doing any calculations, you may like to ask for estimates. Perhaps some students will observe that the information is not specific enough to find how many boys or how many girls.

If appropriate for the data, make a graph of the grade level sizes similar to the one shown here.

Then make some comparison observations such as these:

- The kindergarten level has the most students.
- The fifth grade level has the least students.
- There are fewer second graders than third graders.
- Second grade and sixth grade have the same number of students.



## Exercise 2

Distribute unlined paper and colored pencils or crayons. Draw a large string in red on the board and label it **Children in (your school name)**.

**T:** *This string is for all the children in our school.*

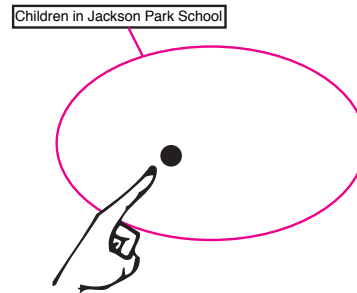
Draw a dot inside the string and ask,

**T:** *What could this dot be for?*

**S:** *It could be for me.*

**S:** *It could be for my older brother Michael.*

**S:** *It could be for anybody in our class—like Angela.*



Let the students make many suggestions, both of students in your classroom and students in other classrooms in the school. For students in other classrooms, be sure they are clearly identified so the class is sure they are children who attend your school.

Draw a dot outside of the string and ask,

**T:** *What could this dot be for?*

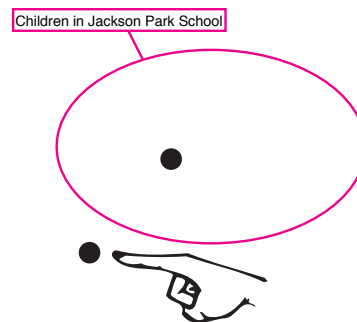
**S:** *It could be for my cousin, Joshua.  
He goes to Lincoln School.*

**S:** *It could be for my little sister.  
She doesn't go to school.*

**S:** *It could be for my grandmother.*

**S:** *It could be for you, teacher.*

**S:** *It could be for my dog.*



Let the students make many suggestions. Be sure the class agrees on the correctness of each answer. Do not impose upon their creativity in giving answers unless there is some doubt.

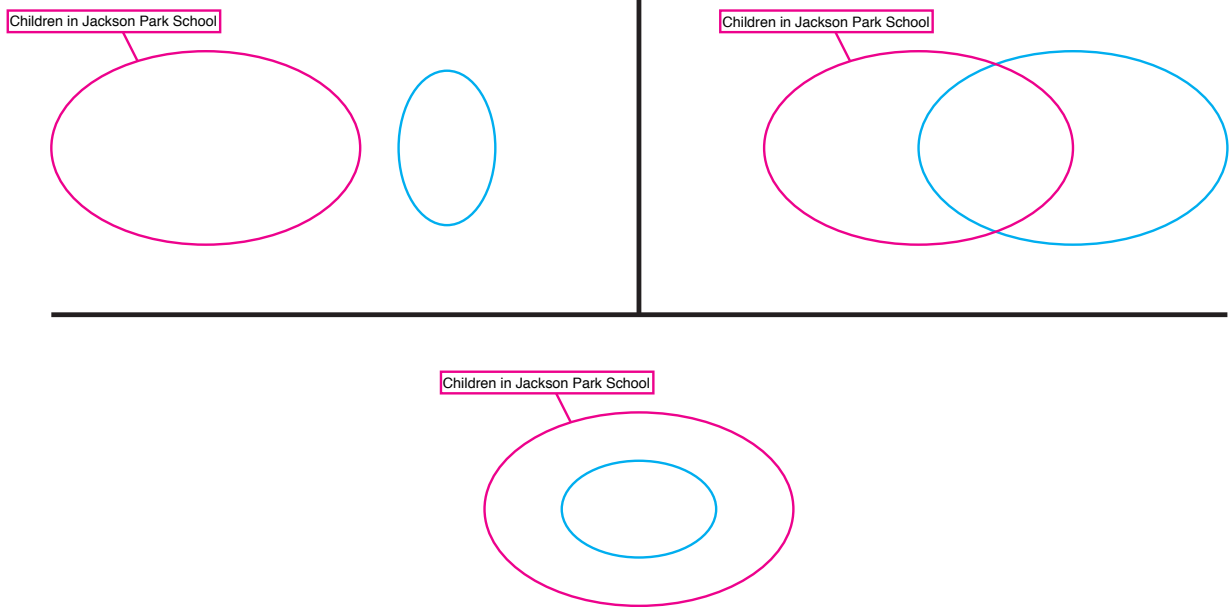
**T:** *Copy this string on your piece of paper. (Encourage large drawings.) Listen carefully. On the same piece of paper, draw another string in blue for all the children in our classroom.*

Observe the students' drawings; you are likely to see several kinds. Do not discourage anyone, but do ask some suggestive questions of students who have difficulty drawing any picture.

**T:** *I see several very interesting drawings. Some of your drawings look similar to these.*



Draw three separate pictures on the board.



For each picture, ask the class what they think about it. Discuss the pictures one at a time. If it is not suggested by a student, suggest that you think there is something wrong with the first picture. Refer back to your original drawing on the board.

**T** (pointing to the dot inside the red string): *Do you remember that, in this drawing, we said this dot inside the red string could be for Angela, who is in our class?*

**S:** *Yes.*

**T:** *Well, would Angela be inside or outside the blue string.*

**S:** *Inside.*

**T** (pointing to inside the blue string): *So Angela would be here instead?*

**S:** *No; she has to be inside both strings.*

The students should conclude that the first picture is not possible. Erase it and go on to discuss the other two pictures. Although some students might suggest that the third picture is the correct one (being the most descriptive for the time being), your students should conclude that both pictures are okay. Do not force them to make observations about which is better or about empty sets unless it happens naturally. The acceptance of a picture should be strictly on the basis that everything or person has its place in the picture.

Ask students who have drawn incorrect pictures to copy one of these two pictures on the reverse side of the paper.

**T:** *Now, we are going to add new strings to our picture, so choose two new colors (for example, green and yellow). With green, draw a string for all the boys in the school. With yellow, draw a string for all the girls in the school.*

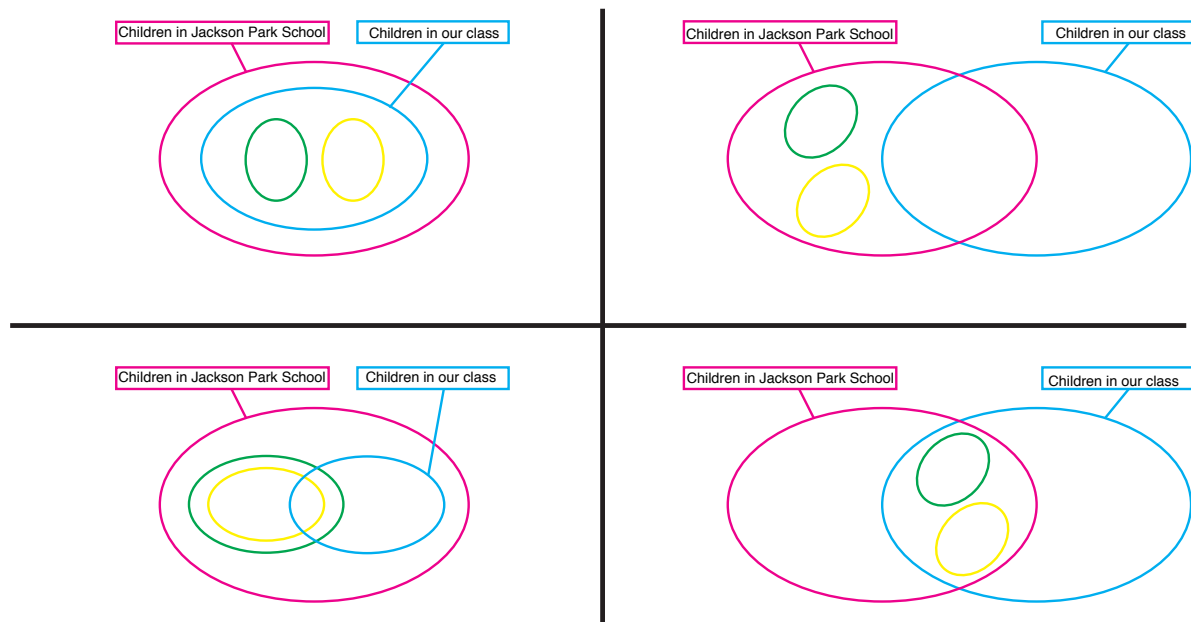
# L13

Observe students' drawings and help those who are confused by asking some leading questions. After a short while, put some of the drawings you have observed on the board. Display the drawings as you put them on the board to give students recognition. Let the class discuss each drawing. A couple possible correct pictures are shown in the next illustration; do not expect your students to find all possibilities.



While discussing a drawing, lead the discussion to consider whether or not everything has its place in the picture. Ask about specific children in the school but outside of your class, both boys and girls; ask about other children, people, or things. This discussion can be very open, and you must decide how far it can proceed based on the interests and abilities of your students.

Some incorrect drawings you might observe are shown below. If one of these is discussed, the class should note that there is somebody they cannot place in the picture.



If your class is handling this activity very well and seems to still be interested, ask them to draw and label dots for somebody (or something) in each region of the picture. Students who do not have a correct picture should copy a correct picture off the board on a new sheet of paper. You may also like to ask students to write about their pictures.



Ask a student to do this as indicated in the chart here.

You may also choose to read and record the temperature in degrees Fahrenheit. In this case, use a similar but separate chart to record the temperature, for example, 33°F.

### Notes

1. Your class will be keeping a record of the morning temperature in degrees Celsius (and possibly also degrees Fahrenheit) for the next two months (school days in January and February). Let different students read the thermometer and record the temperature each day. Every student should have at least one or two chances to do this.
2. The first time a student reads the thermometer, check it with him or her. If the mercury is between two marks, read the temperature to the nearest degree.
3. Each day you might ask one or two questions. For example:
  - a) What do you think the temperature will be this morning?  
After taking the temperature, ask who had the closest guess.
  - b) Yesterday the temperature was 2°C and today it is 3°C. Is it warmer or colder today? How many degrees colder?
  - c) What is the coldest temperature we have recorded so far?  
I heard that this morning was the coldest morning of the year. What do you think the temperature will be this morning?
4. This lesson may be your students' first introduction to reading a thermometer so make every effort to have one available for class use. If, however, the placement of one in a secure outdoor spot or the acquisition of one is a problem, you can still do this activity by checking the daily paper for the previous day's high (low) or by arranging to have students call the local time and temperature line each day at a specified time.

For the remainder of the class time, you may choose to do some other temperature taking/comparing activity. For example, consider finding the water temperature of several containers: ice water, room temperature water, hot tap water, and boiling water. Discuss how these temperatures differ and where else you might find similar temperatures.

Otherwise, you may prefer to use the remainder of the class time to play The String Game or The Number Line Game.

### Home Activity

Suggest that students practice reading a thermometer at home.

**Capsule Lesson Summary**

Play a composition game with colored arrows where the only rule for the game is this: yellow followed by blue is red. Composition is an important relation concept as often we must combine relations to get new relations. The first exercise provides an opportunity to experiment with composition, paying attention to direction and order without worrying about specific relations. The second exercise provides a numerical example.

**Materials**

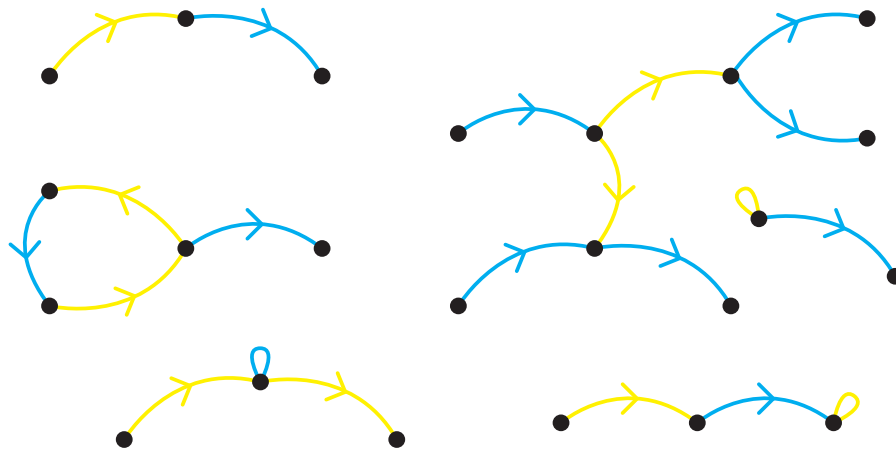
- |         |                 |                                     |
|---------|-----------------|-------------------------------------|
| Teacher | • Colored chalk | • Colored pencils, pens, or crayons |
| Student | • Unlined paper |                                     |

**Advance Preparation:** Before the lesson begins, draw the arrow picture from Exercise 1 on the board or prepare it on a transparency. Give the picture plenty of space and draw arrows in colors that can be easily distinguished.

**Description of Lesson**

**Exercise 1** \_\_\_\_\_

Invite students to comment on this arrow picture.



**T:** *We are going to play a game with yellow and blue arrows. The object of the game is to draw red arrows, but there is a rule for drawing red arrows.*

Use the upper left corner of the arrow picture for your explanation. As you explain the rule stated in the left column below, make the motions described in the right column.

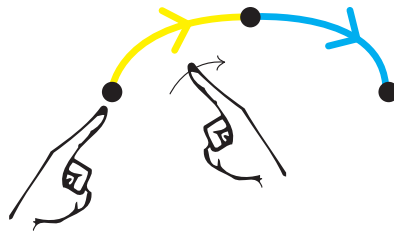
**T:** *This is the rule of the game...* (Place your left forefinger on a dot at which a yellow arrow starts.)

# L15

**T:** *Each time there is a yellow arrow...*

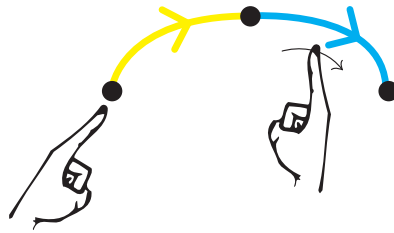
right  
arrowhead.)

(With your left forefinger in place at the dot,  
follow the yellow arrow with your  
forefinger in the direction of the



**T:** *...followed by a blue arrow...*

(Stop the motion of your right forefinger at the  
middle dot, tap the dot, and then follow the  
blue arrow.)



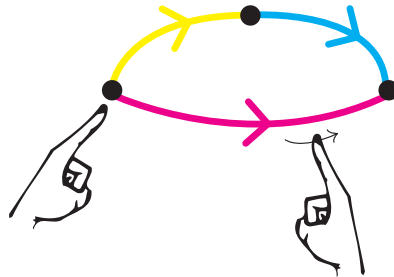
**T:** *...then we can draw a red arrow from  
the dot where the yellow arrow starts...*

(Tap this dot several times with your left  
forefinger.)

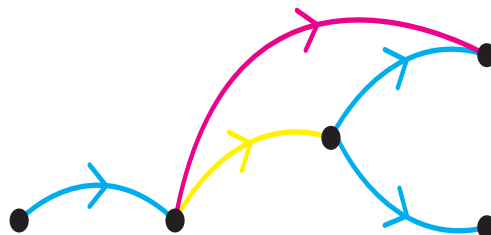
*...to the dot where the blue arrow ends.*

(Tap this dot several times with your right  
forefinger.)

First trace and then draw the red arrow.



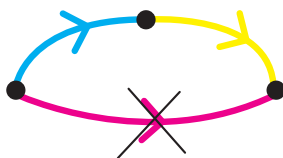
Repeat the statement of the rule using the same words and motions, this time demonstrating with the upper right portion of your picture. Trace and then draw this red arrow.



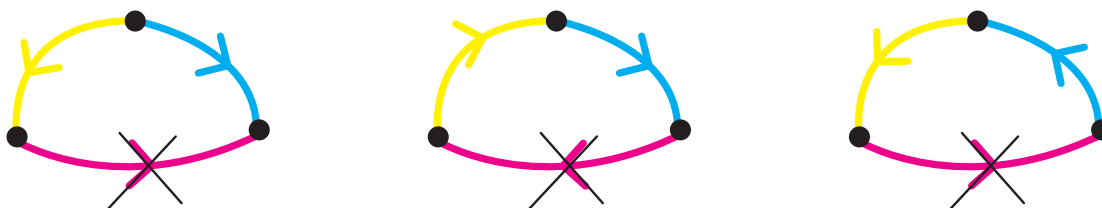
**T:** *Where can we draw another red arrow?*

Invite students to the board, one at a time, to show where other red arrows can be drawn. Ask the students to first trace a yellow arrow and a blue arrow that follows it, and then to trace how a red arrow can be drawn. Stop a student who starts to trace against the direction of an arrow and emphasize that the direction of an arrow must be followed. The rest of the class should help you check for mistakes. Let a student draw a red arrow if it has been correctly traced.

**Common mistakes and difficulties:** At first, the most common mistake will occur when students confuse the rule “yellow followed by blue” and “blue followed by yellow.”



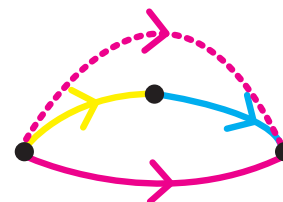
Another mistake might occur when the directions of the arrows are not correctly followed.



Remind students making such mistakes that they must follow the direction of the arrows. The arrows are like one-way streets and they are not allowed to go the wrong way.

If it happens that a student proposes to draw a red arrow as indicated by the dotted arrow in this illustration...

... another student will probably remark that the red arrow has already been drawn. There is nothing wrong with the dotted arrow; it is merely redundant. That portion of the picture is complete without it.



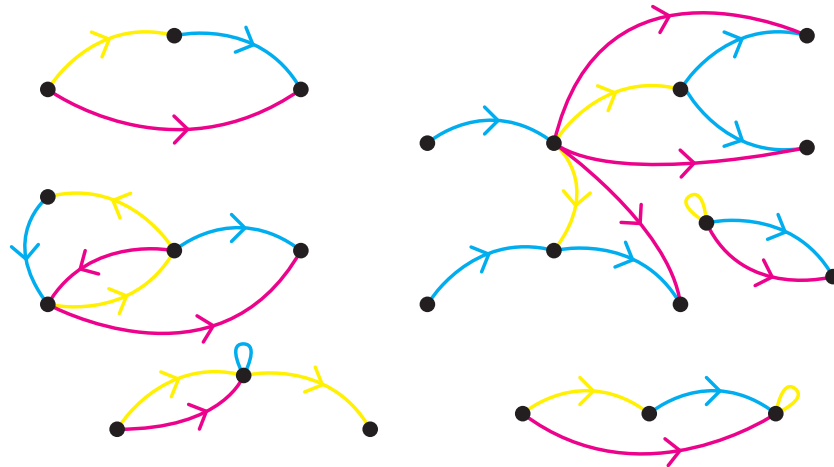
It is likely that the loops on the left side of the picture will cause difficulty. If no one suggests either of the following red arrows, suggest one of them yourself.

**T:** *Yellow* (trace around the yellow loop) *followed by blue* (trace the blue arrow), *then red*.



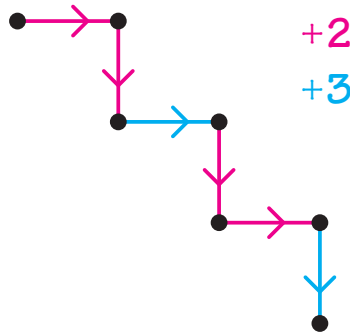
It is important to keep your left forefinger on the dot where a yellow arrow (loop) starts and then trace with your right forefinger, ending at the dot where the blue arrow (loop) ends. Then you can show a red arrow starts at your left forefinger and ends at your right forefinger.

When all the red arrows have been found, your picture should look like this one.



## Exercise 2

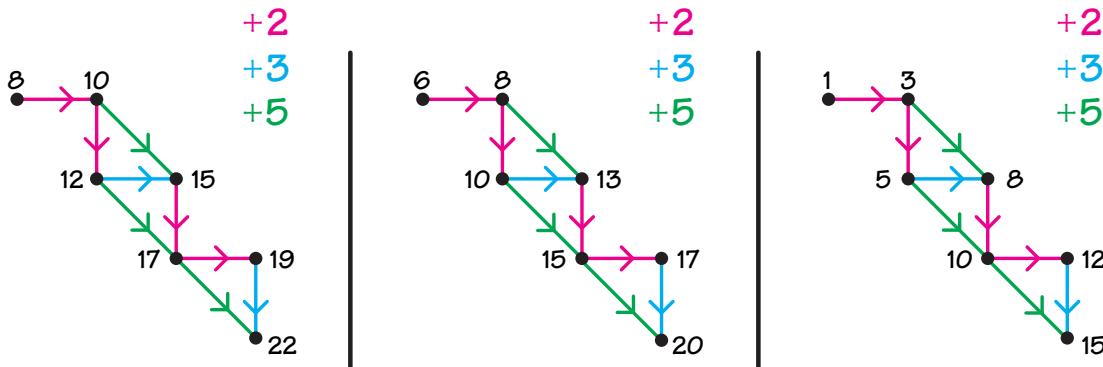
You may like to let students work with a partner for this exercise. Erase the board and draw this arrow picture. Ask students to copy it on their papers.



**T:** *Right now there are no dots labeled in this picture. I would like to find places for the numbers 10 and 15. Try to put 10 and 15 in the picture, but remember red arrows are for +2 and blue arrows are for +3. After you find a place for 10 and 15 label all the dots and check the arrows.*

Let students experiment. You may want to suggest they consider the composition of +2 followed by +3 or the composition of +3 followed by +2. Note that the +5 arrows show compositions and help to locate 10 and 15.

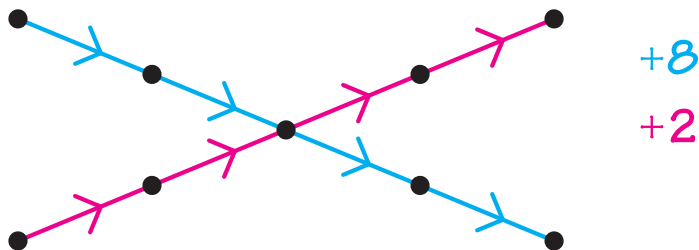
There are three possible solutions to this problem.





Look for one or more solutions and ask students to put them on the board. Encourage students to explain how they found a solution. Discuss the compositions  $+2$  followed by  $+3$  is  $+5$ , and  $+3$  followed by  $+2$  is  $+5$ . Draw arrows for  $+5$  and use the  $+5$  arrows to help explain how to find solutions.

If there is time and students enjoyed this exercise, repeat Exercise 2 for the following problem.



Locate places for 15 and 25 in this picture. After you put 15 and 25 into the picture, label all the dots and check the arrows.

There are two possible solutions to this problem.

