The World of Numbers

WORLD OF NUMBERS TABLE OF CONTENTS

Numbers are among the most important things that mathematics (at all levels) is about. Mathematicians are interested in numbers just as astronomers are interested in stars, botanists in plants, and sociologists in the organization and dynamics of human societies. Surely everyone agrees that one of the primary goals of any elementary school mathematics curriculum should be to introduce students to the world of numbers—to give them the opportunity to become familiar with numbers, their properties, and the relations between them. The ability to calculate with numbers is an important part—but not the only part—of being familiar and comfortable with numbers. The World of Numbers strand of *CSMP Mathematics for the Upper Primary Grades* is designed to provide students with a wide variety of challenging experiences with numbers so that gradually they will become not only familiar, but comfortable with numbers; they will, so to speak, get to know numbers on a "first name basis" and develop number sense.

The Minicomputer and the World of Numbers

CSMP Mathematics for the Upper Primary Grades uses the Papy Minicomputer as a support for the positional system of numeration; for calculations and estimation; for number patterns and mental arithmetic; and for modeling the basic operations involving whole numbers, integers, and decimal numbers. Although the Minicomputer can be seen as a tool for calculation and as a device to help students learn routine methods for calculations, its more exciting use is as a vehicle for posing interesting problems that challenge a child's intellectual curiosity about numbers, and for presenting situations that both encourage strategic thinking and reinforce numerical skills. The lessons that make use of the Minicomputer are often intended to be explorations into the world of numbers.

Standard Algorithms of Arithmetic

CSMP seeks to develop basic numerical skills as well as an understanding of the underlying mathematical ideas. We are fully in agreement with the thesis that, along with the growth of understanding of the world of numbers, there must be a concommitant growth of familiarity and facility with numbers and operations on them. But facility should not be confused with understanding; they are partners in the growth of mathematical maturity. A balanced growth of each must be maintained, neither being sacrificed for the other.

Students must eventually learn mechanical algorithms for the basic operations (addition, subtraction, multiplication, division). However, premature presentation of these algorithms may actually stunt a student's ability to develop alternative algorithms, to do mental arithmetic, or to estimate.

Consider the problem of calculating 385 + 99. A second grader may have difficulty performing the standard (paper and pencil) addition algorithm. An easier and more efficient way to proceed is to add 100 to 385 and then to subtract 1 (385 + 100 = 485 and 485 – 1 = 484). To insist on a mechanistic response to such a problem would be to encourage inefficiency and might also inhibit the development of the flexibility necessary for problem solving. On the other hand, a rich array of situations in which students interact with numbers provides them with opportunities to gain the necessary facility with standard algorithmic procedures while retaining the openness required to respond creatively to new situations in the world of numbers.

Numerical Relations

One of the main aims of the World of Numbers strand is to familiarize students with numbers by studying relations between numbers, both explicitly and in a variety of contexts. (For more general comments about relations, see the introduction to the Languages of Strings and Arrows strand.) Arrow diagrams represent relations in a simple, suggestive, and pictorial way—usually more conveniently than the same information could be given in words or other symbols.

Students are brought into contact with an assortment of challenging situations, many of which would be totally inaccessible to them were it not for the arrow diagrams. The problems and activities of this strand include solving linear equations presented in terms of arrows; studying iterated processes and patterns in sequences of numbers; tackling problems that may have many solutions or no solution; estimating or testing that a solution is reasonable; and exploring properties of operations on numbers.

In summary, what is most important in the study of numbers is to confront the students with a variety of problems and situations that capture their interests, challenge their abilities to reason, and stimulate their curiosities about numbers—and, at the same time, provide them with tools for coping with these situations and problems.

N1 MINICOMPUTER INTRODUCTION #1

Capsule Lesson Summary

Review (or introduce) the values of the squares on the Minicomputer. Use the Minicomputer to represent numbers less than 20. Sometimes represent the same number in different ways. Model some trades and represent numbers less than 100.

Materials

Teacher	• Minicomputer set[†] • Base-10 blocks or other place-value manipulative	Student	• None

Note: Two different initial exercises have been included in this lesson. If all or most of the students in your class have had no previous experience with the Minicomputer, you should start with Exercise 1A and proceed to Exercise 2. If most of the students in your class have been introduced to the Minicomputer, begin with Exercise 1B and proceed to Exercise 2.

Description of Lesson

Exercise 1A (for entry students)

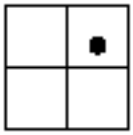
Note: If students are familiar with Cuisenaire Rods (C-rods), you may like to display a staircase of the C-rods. Then, in addition to other representations of numbers, you can relate the square values on the Minicomputer to corresponding color C-rods.

Display a Minicomputer board.

T: ***Show me four fingers. Trace a 4 on your desk. That's one way to write 4. I'm going to show you how to put 4 on the Minicomputer.***

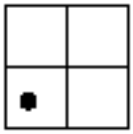
Point to the Minicomputer and put one checker on the purple square.

T: ***This is the Minicomputer. We put numbers on the Minicomputer using checkers. This is the number 4 on the Minicomputer.***



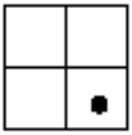
Move the checker to the red square.

T: ***This is the number 2 on the Minicomputer. Show me two fingers. Trace a 2 on your desk.***



Move the checker to the white square.

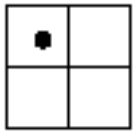
T: ***This is the number 1 on the Minicomputer. Show me one finger. Trace a 1 on your desk.***



[†]A teacher's Minicomputer set consists of four demonstration Minicomputer boards and a sufficient number of demonstration Minicomputer checkers.

Review the configurations for 4, 2, 1 and then again for 1, 2, 4 on the Minicomputer. Do this a couple times letting students tell you the numbers.

Move the checker to the brown square.



T: ***What number do you think this is?***

Some students may guess 3 or 5.

T: ***No, it is not 3 (or 5).***

Pause. Review again 1, 2, 4 and then pause as you put the checker on the brown square. After a moment a student may suggest 8. If not, simply say,

T: ***This is 8.***

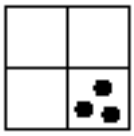
Move the checker quickly from one square to another and ask the class to call out each number. In doing this, follow the doubling pattern: 1, 2, 4, 8. Visually suggest the doubling pattern by putting two checkers on the white square and saying, "1 plus 1 is 2." Then take off the two checkers and put one checker on the red square. Repeat for "2 plus 2 is 4" and "4 plus 4 is 8." You may also ask the students to show 1 + 1 = 2 with fingers, and so on.

Remove the checkers from the Minicomputer.

T: ***Who can put 3 on the Minicomputer?***

Ask volunteers to tell you first how many checkers they will need. (Two and three are both correct answers.) Let a student put 3 on the Minicomputer.

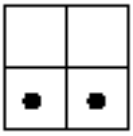
A student might put three checkers on the white square.



If a student places the checkers this way, lift the checkers one by one and say, "1 + 1 + 1 = 3." Remove the checkers from the Minicomputer. Ask if someone can put 3 on the Minicomputer in another way.

If your students need a hint, alternately wave two fingers on one hand and one finger on the other hand saying, "2 + 1 = 3."

By now someone should be able to put this configuration for 3 on the Minicomputer. After a student places the checkers in this way, lift each checker as you mention its value and then replace it quickly on the Minicomputer.



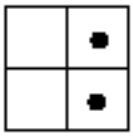
T: ***What number is on the red square?*** (2) ***What number is on the white square?*** (1)
2 + 1 = 3. 1 + 2 = 3.

Remove the checkers from the Minicomputer.

T: ***Who can put 5 on the Minicomputer?***

Ask volunteers to tell you how many checkers they will need. (Two, three, four, and five are all correct answers.) Ask a volunteer to place the checkers on the Minicomputer. Add the numbers on the various squares out loud as you did for 3. Whenever more than two checkers are used, ask for another way until you get the standard configuration.

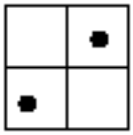
When the checkers are in this position, lift each checker as you mention its value and replace it quickly on the Minicomputer.



T: ***What number is on the purple square?*** (4)
… on the white square? (1)
4 + 1 = 5. 1 + 4 = 5.

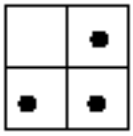
Remove the checkers from the Minicomputer.

T: ***Can anyone put 6 on the Minicomputer?***



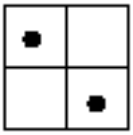
Accept all correct answers. If more than two checkers are used, ask for another way until you get the standard configuration.

T: ***Can anyone put 7 on the Minicomputer?***



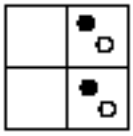
Accept all correct answers. If more than three checkers are used, ask for another way until you get the standard configuration.

T: ***Can anyone put 9 on the Minicomputer?***



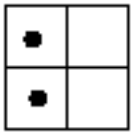
Accept all correct answers. If more than two checkers are used, ask for another way until you get the standard configuration.

Put on 5 and ask what number it is. Then put on 5 again and ask what number is 5 + 5. (10)



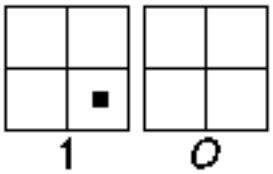
T: ***Can anyone put 10 on the Minicomputer in another way?***

At some moment someone will probably suggest this configuration.



T: ***There is a way to put 10 on the Minicomputer using only one checker, but we will need another board.***

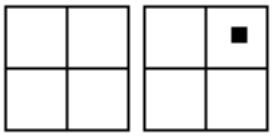
Display a second board (the tens board) to the left of the first board and place one checker on the white square of the tens board. As you do this say, "8 + 2 = 10." Write 1 below (or above) the tens board and 0 below the ones board.



At this point you may like to use base-10 blocks or some other place-value manipulative to model a trade of 10 ones for 1 ten and 0 ones.

Exercise 1B

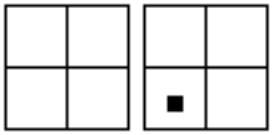
Display two Minicomputer boards.



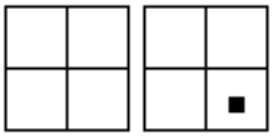
T: ***Show me four fingers.***
This is the number 4 on the Minicomputer.

To prevent confusion, always remove the checkers from the Minicomputer before asking the next question.

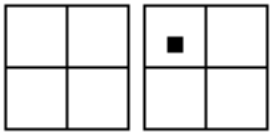
T: ***Who can put 2 on the Minicomputer?***



Who can put 1 on the Minicomputer?



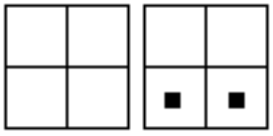
Put a checker on the brown square.



T: ***What number is this?***

S: ***8.***

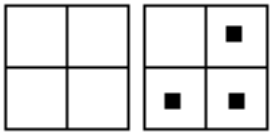
Very quickly review the placement of checkers for 1, 2, 4, and 8.



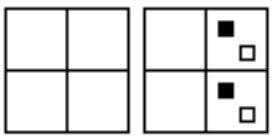
T: ***I'll give you two checkers.***
Can you put 3 on the Minicomputer?

Can you put 5 on the Minicomputer with two checkers?
… 6 with two checkers?
… 9 with two checkers?
How many checkers do you need to put 7 on the Minicomputer? (At least three)

Ask someone to put 7 on the Minicomputer with three checkers.

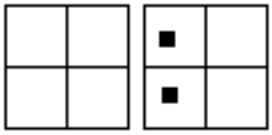


Put 10 on the Minicomputer as 5 plus 5.

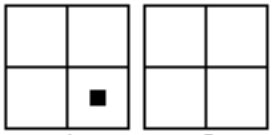


T: ***Can anyone put 10 on the Minicomputer in another way?***

Different configurations for 10 include the following:

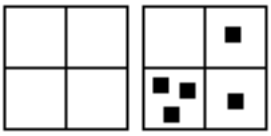


and



In the second case write the numerals below the boards.

Exercise 2



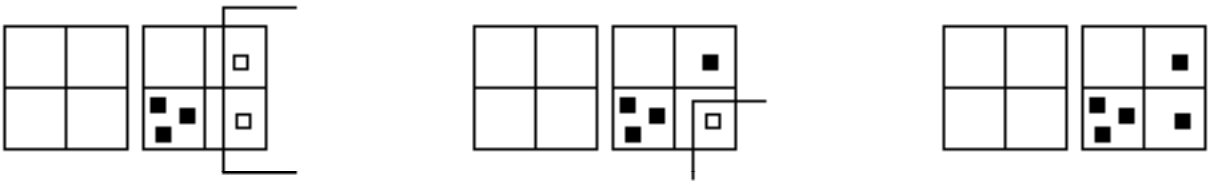
Put some checkers on the ones board very gradually. For example:

- place three checkers on the red square, pause;
- then one checker on the purple square, pause again;
- then one checker on the white square.

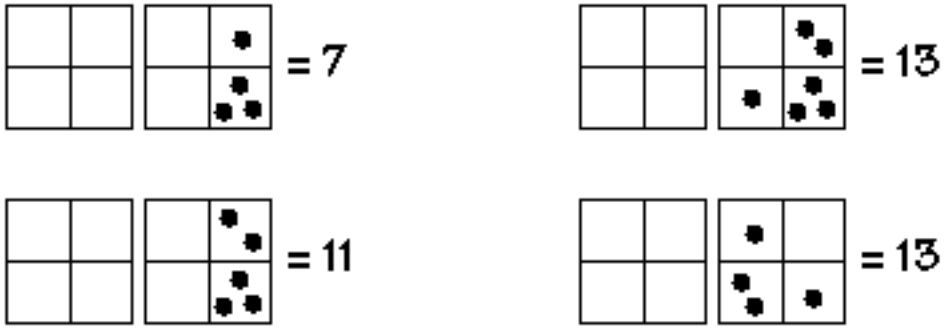
This gives your students a chance to calculate mentally.

T: ***What number is on the Minicomputer?*** (11)

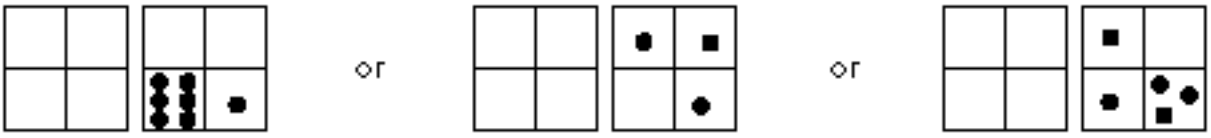
No explanation is necessary if everybody gets the right number. If someone gives the wrong number, you might use this procedure: Cover part of the board with a piece of paper to focus the class's attention on certain checkers, and gradually uncover the full set of checkers.



Repeat this activity with other examples; some possibilities are suggested below.



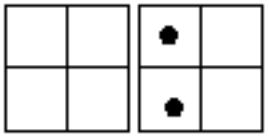
Note: It is interesting to present the same number several times, each time with a different number of checkers. For instance, 13 can be represented as



and so on.

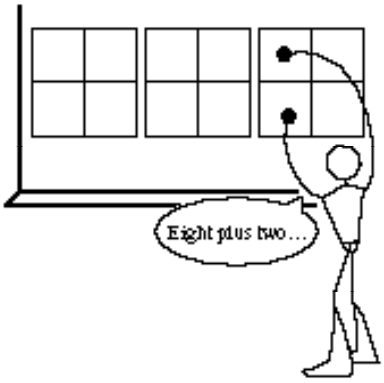
Exercise 3

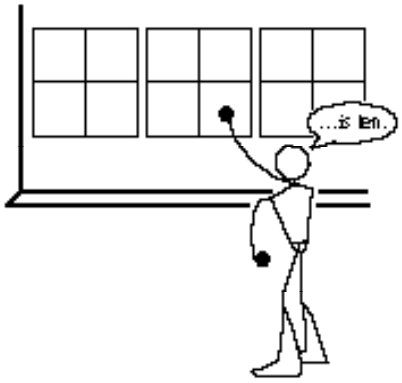
Put one checker on the 8-square and one on the 2-square.



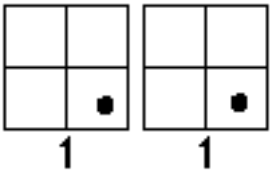
T: ***What number is on the Minicomputer?*** (10)

Pick up the checker on the brown square with one hand and the checker on the red square with the other. Then put one of these checkers on the 10-square and take the other checker away (put it in the chalk tray). As you are making the trade, say, "8 + 2 = 10."





T: ***This is a way to put 10 on the Minicomputer with just one checker. With one more checker, can you put 11 on the Minicomputer?***



Ask students to remove the checker from the ones board.

T: ***With one more checker, can you put on 12?*** (Yes)
Can you put on 14? (Yes)
Can you put on 17? (No)
How many more checkers do you need? (At least three more)
Can you put on 18? (Yes)
19? (No)

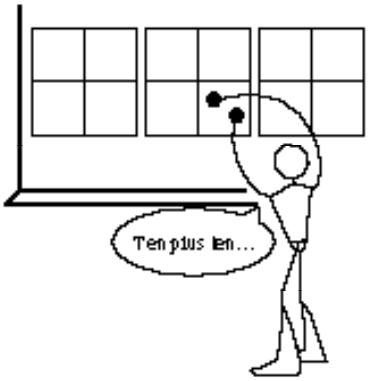
Note: The question, How many checkers do you need for 17? has many answers. For the standard configuration you need four checkers, but it is possible to put 17 on the Minicomputer with three checkers: 8 + 8 + 1 = 17. In this case, however, one checker is given on the 10-square.

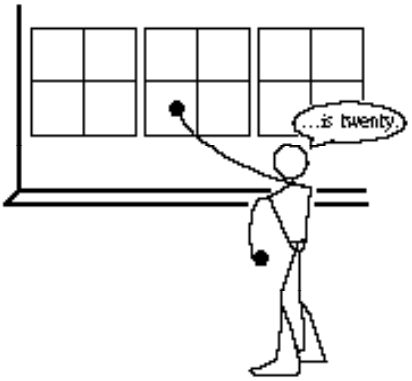
In some cases, you may request a student to model a number as well with base-10 blocks or other place-value manipulatives; for example, model 14 as 1 ten and 4 ones.

Ask someone to put 20 on the Minicomputer using two checkers. If no one volunteers, put two checkers on the 10-square.

T: ***There is a way to put 20 on the Minicomputer with one checker.***

Make the trade and say, "10 + 10 = 20."



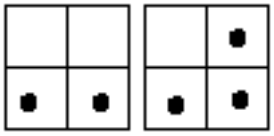


Remove the checkers and continue with other numbers.

T: ***Who can put 25 on the Minicomputer? 27? 30?***

Again you may invite students to model 25 as 2 tens and 5 ones with base-10 blocks or other place-value manipulatives.

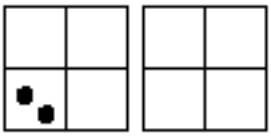
Put this configuration on the Minicomputer.



T: ***What number is this?***

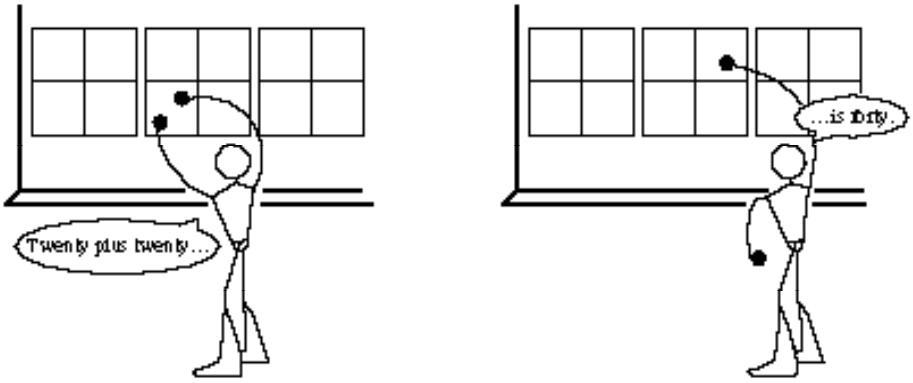
When you receive the correct answer, write 3 below the tens boards and 7 below the ones board.

Put two checkers on the 20-square.



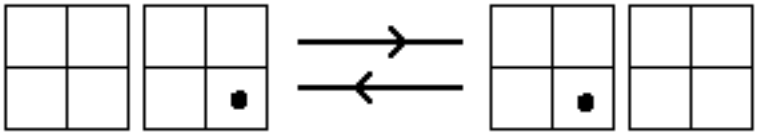
T: ***What number is this?*** (40)

Make the trade yourself and say, "20 + 20 = 40."



Remove the checker from the 40-square.

Move a checker back and forth very quickly from the 1-square to the 10-square. Each time you move the checker ask the class which number is on the Minicomputer: one—ten; one—ten; and so on.



Repeat this with 2 and 20; 4 and 40; 8 and 80.[†]

T: ***Who can put 47 on the Minicomputer? 53? 96? 32?***

Center Activity

Put two Minicomputer boards and checkers in a center. Let pairs of students practice putting on numbers and reading each other's numbers on the Minicomputer. Task cards made with a number on one side and its standard Minicomputer configuration on the other can be used by individual students.

[†]Entry classes might need help with the introduction of the 80-square.

N2 THE FUNCTIONS +3 AND –3 #1

Capsule Lesson Summary

Explore ways to put a number on the calculator when the keys for that number cannot be used. Teach a calculator to count by ones and by threes. Label the dots and draw the return arrows in a +3 arrow picture. Relate number sentences such as 7 + 3 = 10 and 10 – 3 = 7.

Materials

Teacher	• Overhead calculator • Colored chalk • Number line	Student	• Calculator • Paper

Note: Paper for students can be scratchpads, notebooks, slates, or whatever serves your classroom management and record-keeping purposes.

Description of Lesson

Display an overhead calculator (if available) and distribute calculators to individual students. Depending on your students' experience with calculators, review the parts and some features of the calculator. In particular, students should be able to turn on the calculator, enter numbers, read the display, clear the calculator, use the calculator for simple addition and subtraction problems, and use the [=] key to display the result.

Exercise 1

Tell the students you want them to pretend that a particular number key on the calculator is broken. Then ask them to display that number without using the broken key. For example,

T: ***Suppose the [8] key is broken. Try to display 8 without using the [8] key.***

S: ***Press*** [5] [+] [3] [=]**.**

S: ***Press*** [9] [–] [1] [=]**.**

S: ***Press*** [2] [×] [4] [=]**.**

Allow students to find several solutions and demonstrate their solutions on the overhead calculator.

Repeat this exercise a couple times with other broken keys.

T: ***Suppose both the [2] and the [5] keys are broken. Try to display 25.***

S: ***Press*** [1] [4] [+] [1] [1] [=]**.**

S: ***Press*** [9] [+] [8] [+] [7] [+] [1] [=]**.**

S: ***Press*** [3] [1] [–] [6] [=]**.**

S: ***Press*** [1] [0] [0] [÷] [4] [=]**.**

Again, allow students to find several solutions and demonstrate their solutions on the overhead calculator. You might not expect the same variety given here, but be open to many different solutions and encourage students' experimentation.

Exercise 2

Review (or introduce) with students how we can teach a calculator to count. You may want to let students describe the counting process first. Demonstrate with the overhead calculator while students use their calculators.

T: ***We teach a calculator to count by ones by***
(1) putting on the starting number;
(2) pressing [+] [1]; and then
(3) pressing [=] [=] [=] [=] and so on.

Let students spend a few minutes making their calculators count up to "big" numbers.

T: ***How do you suppose we can teach a calculator to count by threes?***

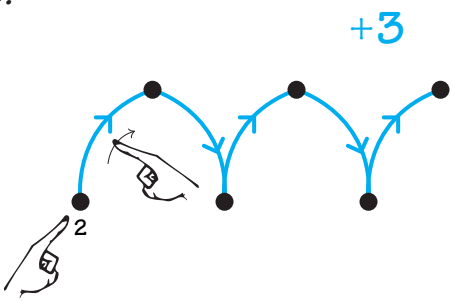
S: ***Put on the starting number.***
Press [+] [3].
Then press [=] [=] [=] [=] and so on.

Spend a few minutes exploring the counting calculator. Some students may want to make their calculators count by other numbers as well.

Exercise 3

Draw the arrow picture below on the board. Put your left forefinger on the dot for 2.

T: ***This dot is for the number 2. The blue arrow is for +3.***



Trace the blue arrow starting at 2 with your right forefinger in the direction of the arrowhead as you say,

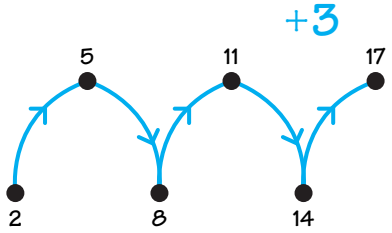
T: ***2 + 3. What number is this?*** (5)

Label the second dot 5. Put your left forefinger on the dot for 5 and trace the blue arrow starting at 5 with your right forefinger as you say,

T: ***5 + 3. What number is this?*** (8)

Label the third dot 8. Point to the unlabeled dots.

T: ***What are these numbers?***



Invite students to point to the dots as they announce the numbers. Continue until all the dots are labeled. Occasionally, you may want to write a number sentence corresponding to an arrow on the board; for example, 8 + 3 = 11.

T: ***Could we go on with more +3 arrows?*** (Yes)

Do not draw more arrows. You can trace more arrows if you like.

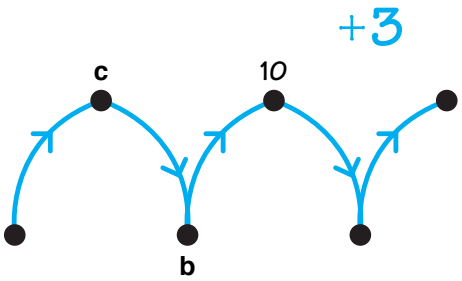
T: ***If I keep drawing arrows, do you think we will ever meet the number 20?*** (Yes)

If several students respond correctly but many are uncertain, trace an imaginary arrow and ask the students which number comes next in the picture. Ask if you will ever meet the number 25. (No) How about 30? (No) Encourage students to explain why they think yes or no to such questions.

At this time you may like to let students use their calculators, counting by threes, to follow the arrow picture and to help answer such questions. Students may also use the number line to explain answers to questions about an extended arrow picture. That is, the arrow picture starts at 2 and makes "jumps" of 3 (+3 arrows). It lands on 20 but jumps over 25 and 30.

Exercise 4

Erase the numerals but not the dots or arrows.
Label the fourth dot from the left 10.



Point to the dot labeled **b** in the illustration.

T: ***What number is here?***

Invite students to whisper their answers to you or to write on the paper for you to check†. Then ask a student to answer aloud.

Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

Label the dot.

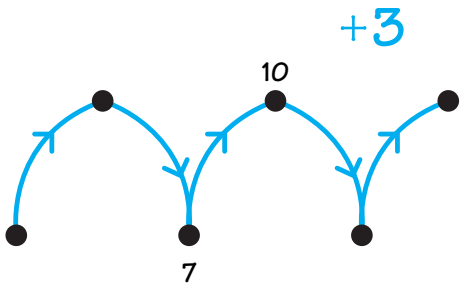
T: ***How do you know this is 7?***

S: ***7 + 3 = 10.***

Write the number sentence on the board.

$7 + 3 = 10$

$10 - 3 = 7$



T: ***Does someone have another way to see that this is 7?***

S: ***10 – 3 = 7.***

It may be necessary to give the 10 – 3 = 7 observation yourself.
Write the number sentence under 7 + 3 = 10 on the board.

T: ***We also know that this number is 7 because 10 – 3 = 7. We could draw a red arrow for –3.***

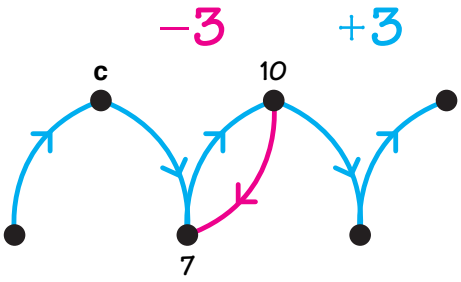
Put your left forefinger on the dot for 10. Trace an arrow (starting at 10 and ending at 7) with your right forefinger as you say, "10 – 3 = 7."

Draw the return arrow in red from 10 to 7 and write –3 in red near the arrow picture.

†Often in a lesson description it is suggested that students whisper or write an answer on paper to give many students an opportunity to respond. If appropriate, indicate that an answer is correct. In some situations, e.g., detective stories, you will not say whether or not an answer is correct because you want students to continue to think about the situation.

N2

Point to the dot labeled **c** in this illustration.



T: ***What number is here?***

S: ***4, because 7 – 3 = 4 (or 4 + 3 = 7).***

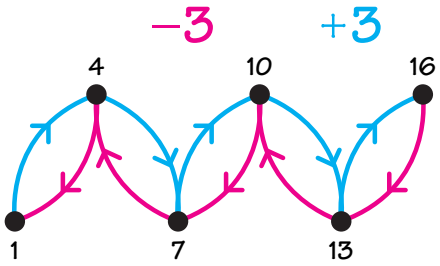
T: ***What are the other numbers in the picture?*** (1, 13, 16)

Quickly label the remaining dots.

T: ***Could we draw more –3 arrows in this picture?*** (Yes)

Do not write the letter on the board. It is here just to make the description of the lesson easier to follow.

Ask students to draw the red arrows. After each red arrow is drawn, point to its starting number and trace the arrow to its ending number while you say, for example, "13 – 3 = 10." Continue until the arrow picture is complete.



T: ***Could we use the calculator to check our work?***

S: ***Yes, we could make it count backward by threes from 16 to see if we meet each dot on the arrow road.***

T: ***Tell me how to do this.***

S: ***Put 16 on the display. Press [−] [3] [=]. It will show 13. Continue to press [=] and check each number.***

N3 MINICOMPUTER INTRODUCTION #2

Capsule Lesson Summary

Review the value of the squares on the Minicomputer. Practice reading numbers on the Minicomputer. Identify some Minicomputer trades and practice making trades.

Materials

Teacher	• Minicomputer set • Base-10 blocks or other place-value manipulatives	• Base-10 blocks or other place-value manipulatives • Calculator • Paper • Worksheets N3*, **, ***, and ****
Student	• Individual Minicomputer set†	

Description of Lesson

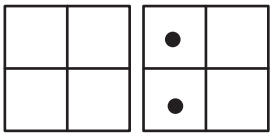
Exercise 1

Repeat Exercise 1B from lesson N1 especially for entry students. Other classes may want to start this lesson with Exercise 2.

With entry classes you may also want to review trades on the ones board; i.e., 1 + 1 = 2, 2 + 2 = 4, and 4 + 4 = 8.

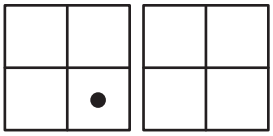
Exercise 2

Put 10 on the Minicomputer as 8 + 2.

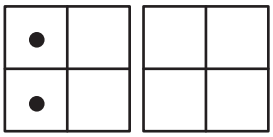


T: ***What number is this?*** (10)

Who can put 10 on the Minicomputer with one checker?



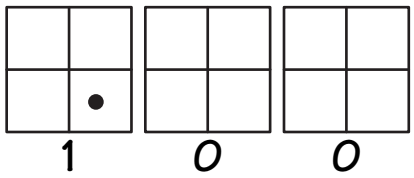
Put 100 on the Minicomputer as 80 + 20.



T: ***What number is this?*** (100)
How do you know? (80 + 20 = 100)

T: ***There is a way to put 100 on the Minicomputer with just one checker, but we need another board.***

Display the hundreds board and make the trade yourself as you say, "80 + 20 = 100." Write 100 below the Minicomputer.



You may want to model a trade of 10 tens for 1 hundred with your place-value manipulative.

†A student's Minicomputer set consists of two sheets of Minicomputer boards (two boards per sheet) and cardboard checkers.

T: ***Who can put 105 on the Minicomputer? 126? 157?***

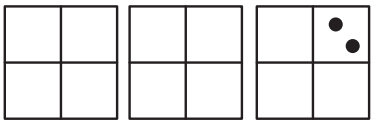
Occasionally, you may wish to ask a student to write a numeral below the Minicomputer while other students model the number with their place-value manipulatives.

Move checkers quickly from one board to another to show the following:

1, 10, 100	8, 80, 800
2, 20, 200	5, 50, 500
4, 40, 400	3, 30, 300

Exercise 3

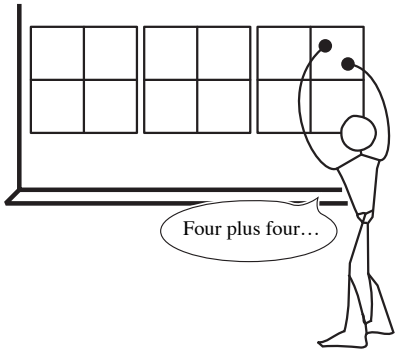
Put two checkers on the 4-square of the Minicomputer.

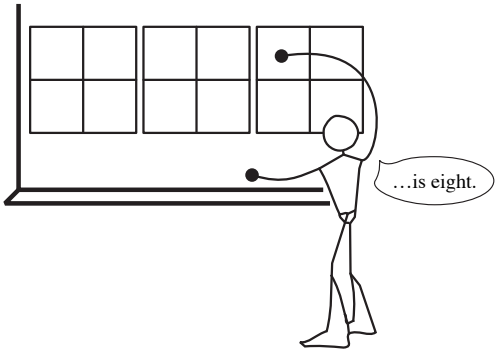


T: ***What number is this?*** (8)

T: ***Can someone put 8 on the Minicomputer with just one checker?***

Repeat the trade very clearly and say, "4 + 4 = 8 and 8 = 4 + 4."

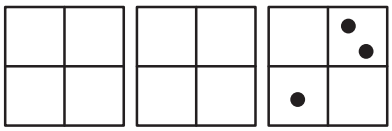




Similarly, demonstrate the following trades:

40 + 40 = 80	and	80 = 40 + 40
100 + 100 = 200	and	200 = 100 + 100
200 + 200 = 400	and	400 = 200 + 200
8 + 2 = 10	and	10 = 8 + 2

Put this configuration on the Minicomputer.

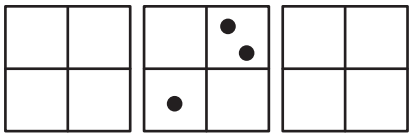


T: ***What number is this?*** (10)

Invite students to make trades until the standard configuration for 10 is on the Minicomputer. Students should name trades before and as they move checkers.

Repeat the exercise with this configuration.

T: ***What number is this?*** (100)
What do you notice?

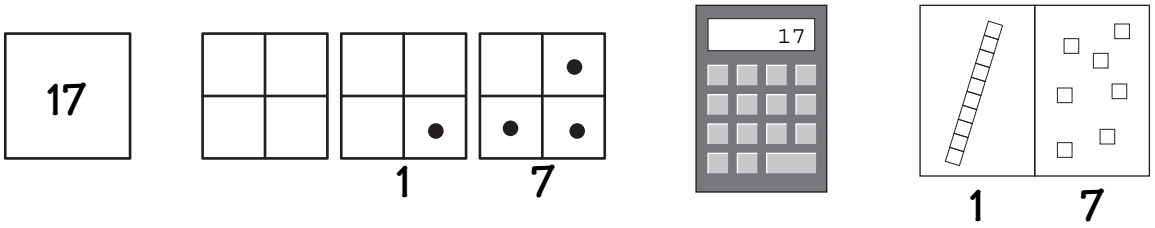


Encourage the students to comment on this new situation and to explain why the number is 100 before asking for volunteers to make trades.

Exercise 4

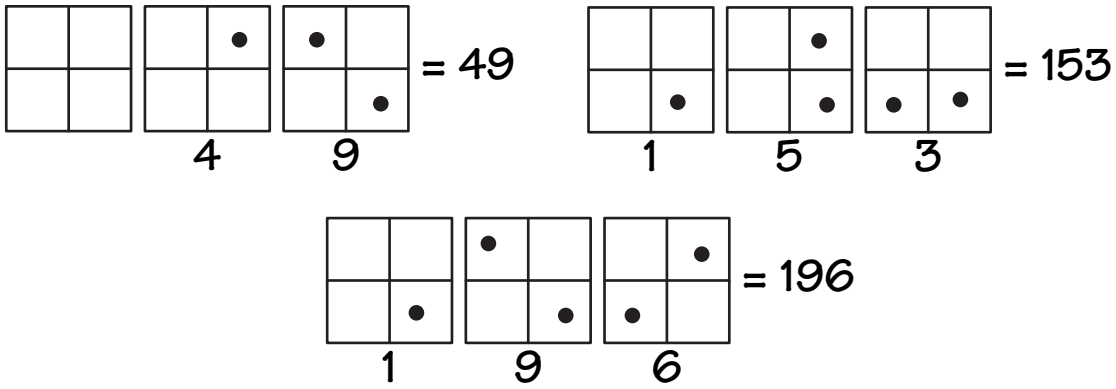
Place students in groups of four with a Minicomputer set, base-10 blocks or other place-value manipulative, a calculator, and paper and pencil. This activity should move quickly. Choose numbers appropriate for your class and encourage all students to participate.

T: ***Each group is going to show the number 17 in several ways. One person will write 17 on paper, another person will put 17 on the Minicomputer, still another person will put 17 on the calculator, and finally one person will show 17 with the base-10 blocks.***



Check to see that each group has 17 represented in all four ways. Then let some students share and explain. Continue by directing students to switch jobs within their groups.

Repeat this activity with other numbers such as those illustrated (on the Minicomputer) below.

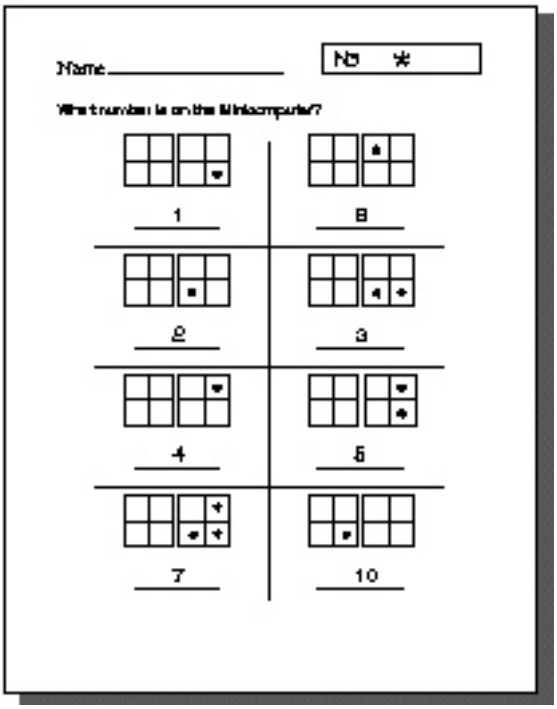


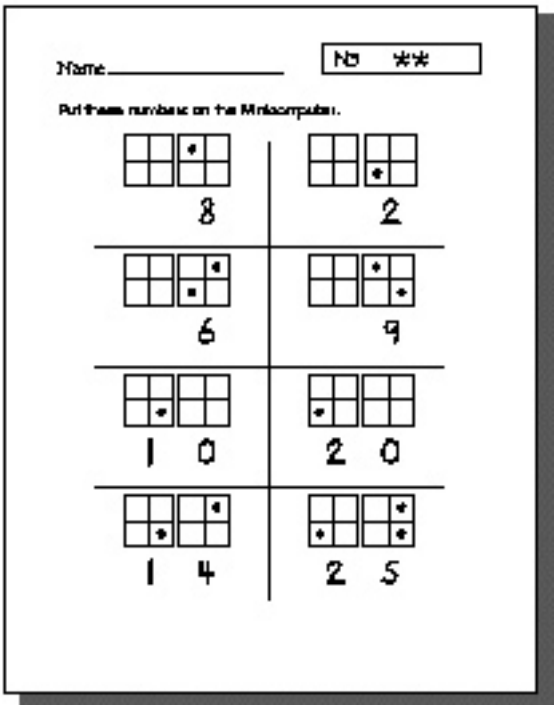
You may like to change the activity so that instead of reading a number to the groups, you put the number on the Minicomputer and the groups read the number. The person with the Minicomputer can copy the configuration you display.

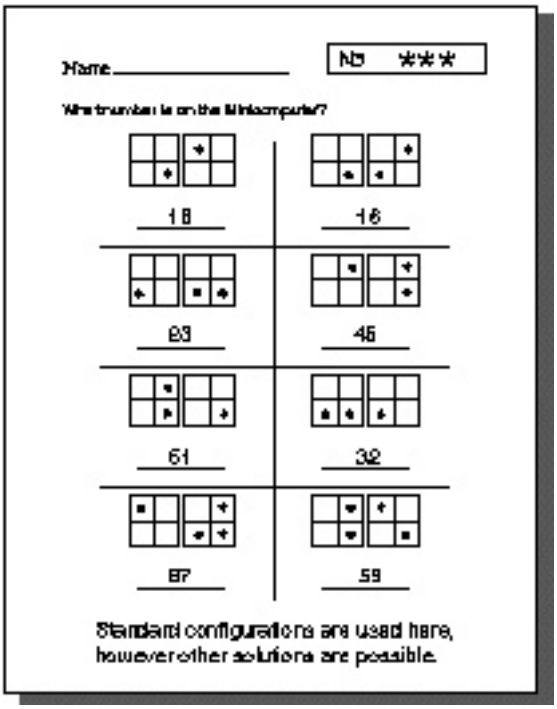
Assessment Activity

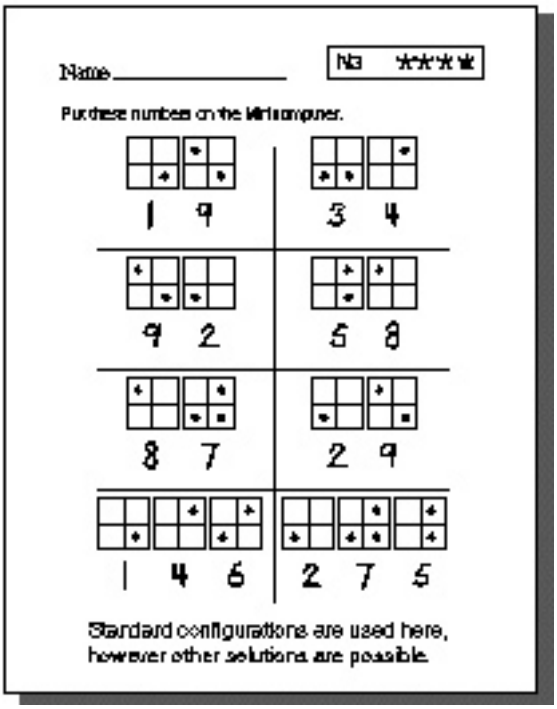
N3

Worksheets N3*, **, ***, and **** are available for independent work. You may like to use these worksheets to check which students need additional help putting on and reading numbers on the Minicomputer. Some students may like to use individual Minicomputers and checkers while doing the worksheets.









N4 THE FUNCTIONS +5 AND –5

Capsule Lesson Summary

Practice counting by fives and relate counting by fives to money and telling time. Collectively, complete an arrow picture with +5 arrows and another with –5 arrows making use of return arrows. Individually, draw another arrow picture using +5 and –5 arrows. Observe a digits pattern when adding fives to a number.

Materials

Teacher	• Colored chalk • Calculator • Number line	Student	• Colored pencils, pens, or crayons • Unlined paper

Description of Lesson

Exercise 1

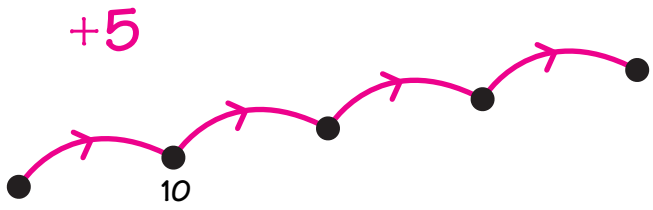
Start the lesson with some practice in counting by fives.

- Ask five children to stand and, one at a time, to hold up each hand, counting by fives to 50.
- Count how much money is in a collection of nickels.
- Look at a clock and count the minutes in an hour by fives.
- Ask students to tell the class how many cars or bikes there are in their families while you keep a running total on the board with tally marks ( ). Count by fives to find out how many cars or bikes the class's families have.

Note: Blackline N4 is a clockface to use for counting the minutes in an hour (or telling time in 5-minute intervals).

Exercise 2

Draw this arrow picture on the board.



T: ***Where is the least number in this arrow picture?***
Who can come to the board and point to the dot for the least number?
What is the least number in this picture? Show me with your fingers.

S (showing five fingers)**:** ***5.***

T: ***Who can point to the dot for the greatest number. What number is it?*** (25)

Instruct students to write the number on paper for you to check or allow them to whisper the number to you. Then ask someone to give the answer aloud.

Point to the dot to the right of 10.

T: ***What number is here? How do you know?***

S: ***15, because 10 + 5 = 15.***

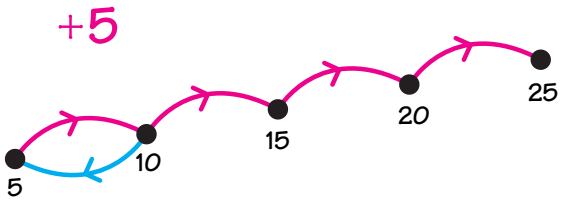
Invite a student to label this dot 15. Continue to label the other two dots to the right in this way. Then point to the first dot (on the left).

T: ***What number is here? How do you know?***

S: ***5, because 10 – 5 = 5 (or 5 + 5 = 10).***

Solicit both explanations and write the two number sentences on the board.

Draw a return arrow in blue from 10 to 5.



T: ***What could the blue arrow be for?***

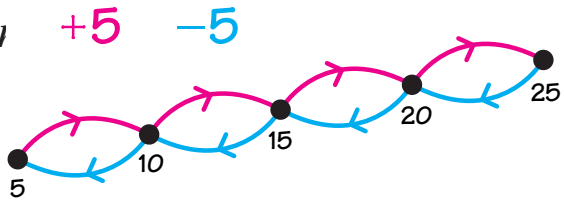
S: ***–5.***

Write –5 in blue near the arrow picture. As you trace the arrow from 10 to 5, say,

T: ***10 – 5 = 5. Could we draw more –5 arrows in this picture?***

Complete the picture quickly with your class.

T: ***If we go on drawing +5 arrows (to the right), will we ever meet 31?*** (No)
Will we meet 50? (Yes) ***63?*** (No)
How do you know?



Accept any response that is reasonably correct. It may be difficult for students to verbalize their ideas. The number line may help to explain answers to questions about an extended arrow picture. That is, the arrow picture starts at 5 and makes "jumps" of 5 (+5 arrows). It lands on 50 but jumps over 31 and 63.

You may like to remind students of the counting calculator. First, teach the calculator to count by fives by

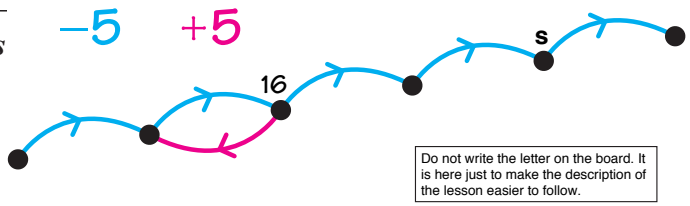
(1) putting on the starting number;
(2) pressing [+] [5] ; and then
(3) pressing [=] [=] [=] and so on.

Then use the counting calculator to follow the +5 arrows and to check which numbers an extended arrow picture would meet.

Exercise 3

Erase the board and draw this arrow picture.

T: ***What are the blue arrows for?*** (–5 +5) ***Where is the least number in this arrow picture?***
Where is the greatest number in this arrow picture?



Point to the dot labeled **s** in the illustration.

T: ***What number is here?*** (6)

Instruct students to write the number on paper or take whispers. Then ask someone to answer aloud.

T: ***Who would like to label a dot?***

Occasionally, ask students to explain how they know which numbers correspond to particular dots. Encourage students to use return arrows when they are useful and to draw any missing return arrows to complete the picture.

T: ***If we go on drawing more +5 arrows (to the left), do you think we will ever meet 50?*** (No) ***How do you know?***
What do you notice about the numbers on the arrow road?

S: ***They all have the digit 1 or 6 in the ones place.***

Encourage students to observe this pattern in the numbers the arrow road would meet.

Again, you may like to use the counting calculator (count by fives starting at 1) to check which numbers an extended arrow picture would meet.

Erase the board and distribute unlined paper. Ask the students to draw their own arrow pictures using +5 arrows. Allow them to begin with a number of their own choosing and encourage them to continue as long as possible. Students who quickly finish a rather extensive picture can be asked to draw all the –5 arrows in their pictures.

Home Activity

Suggest that parents/guardians find opportunities to count by fives with their child.

N5 MINICOMPUTER INTRODUCTION #3

Capsule Lesson Summary

Represent a sum on the Minicomputer by the use of checkers of two different colors. Explore the effect of adding a board to the right. Present a situation where a backward trade is needed to get the standard configuration of a number. Find many ways to represent a given number on the Minicomputer.

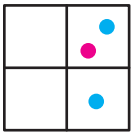
Materials

Teacher	• Minicomputer set	Student	• Individual Minicomputer set

Description of Lesson

Exercise 1

Display one Minicomputer board with this configuration of checkers. (Use two different colors for the checkers.)



T: ***What number is on the Minicomputer?***

S: ***9.***

T: ***How do you see 9?***

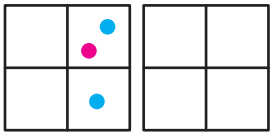
Let students make suggestions. If no one suggests that 9 can be seen as 4 (in red) plus 5 (in blue), ask,

T: ***What number do you see in red?*** (4)
What number do you see in blue? (5)

Write 4 + 5 = 9 on the chalkboard.

T: ***Suppose we counted some things, red cars and blue cars, for example. We can show this on the Minicomputer. Then we can put words in the number sentence and read, "Four cars + five cars = nine cars."***

Accept a couple more such examples and write corresponding sentences on the board. Before continuing erase these sentences leaving only 4 + 5 = 9 on the board. Without removing the checkers, add a second Minicomputer board to the right.



T: ***Now, what number is on the Minicomputer?***

S: ***90.***

T: ***How do you see 90?***

S: ***40 in red plus 50 in blue.***

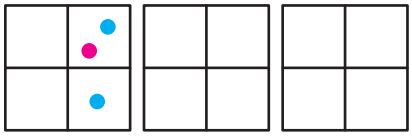
Write 40 + 50 = 90 on the chalkboard below 4 + 5 = 9.

T: ***What things could the 40 and 50 be for?***

S: ***40 red marbles and 50 blue marbles.,***

Write 40 marbles + 50 marbles = 90 marbles on the board. Accept a couple of other suggestions, and write the corresponding sentences on the board. Erase these sentences before continuing.

Add a third board to the right.



T: ***What number is on the Minicomputer now?***

S: ***900.***

T: ***How do you see 900?***

S: ***400 in red plus 500 in blue.***

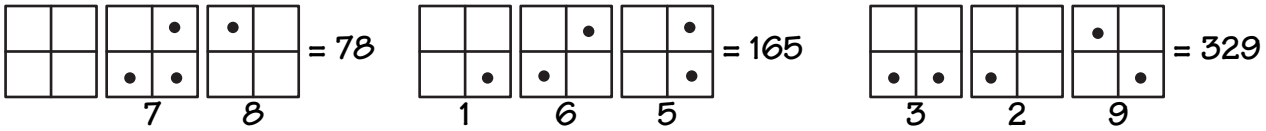
Write 400 + 500 = 900 on the chalkboard below 40 + 50 = 90. Again, allow the class to suggest what things the 400 and 500 could be for and write the corresponding sentences.

If your class seems to enjoy this exercise, you may go on adding boards and asking students to write the calculations:

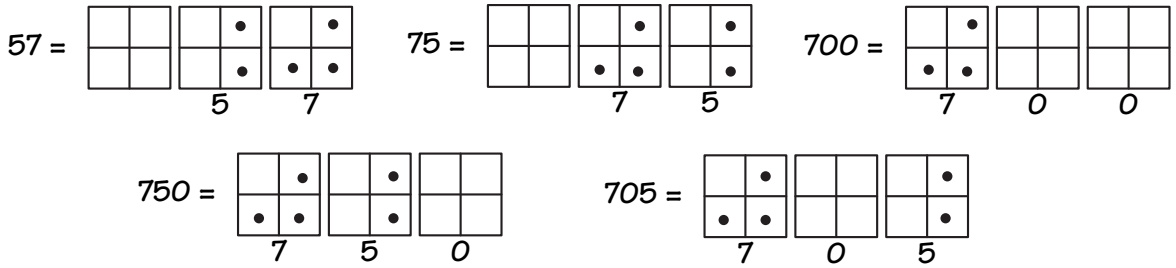
$$4 + 5 = 9$$
$$40 + 50 = 90$$
$$400 + 500 = 900$$
$$4{,}000 + 5{,}000 = 9{,}000$$

Exercise 2

Display three Minicomputer boards. Ask the students to read various numbers as you put them on the boards and to write the numerals, centering each digit below the appropriate board. Occasionally, write the numeral to the right of the boards as well.

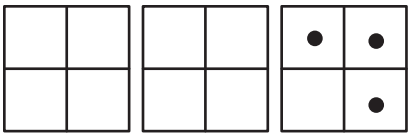


Vary the activity by asking the students to put various numbers on the Minicomputer and, again, to write the numerals below the boards. A good sequence of numbers to ask for is suggested here. Make your request by writing the numeral to the left of the boards.



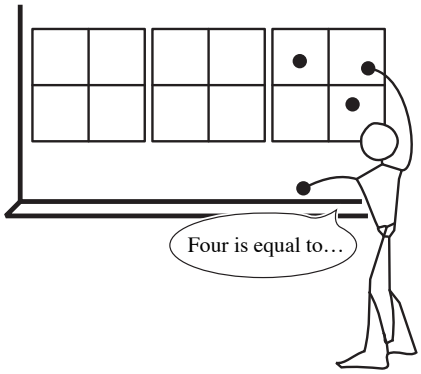
Exercise 3

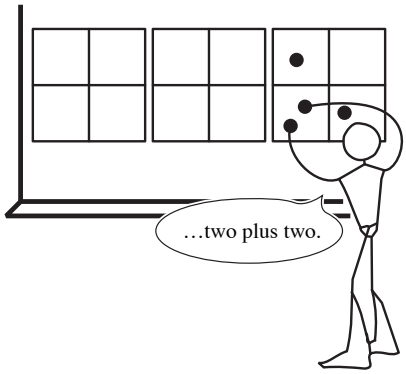
Put this configuration on the Minicomputer.



T: ***What number is this?*** (13)
Without changing the number, I want to get a checker on the red square so we can make the jump to the tens board. What trade should we make?†

If no one suggests a 4 = 2 + 2 trade, mention it yourself and make the trade.

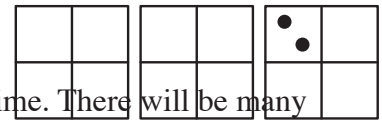




T: ***Who can make a trade so we will have a checker on the tens board?***

Ask the students to name the trade they intend to make before and as they move checkers. Do not accept the 2 + 2 = 4 trade since you have asked for a trade which results in a checker on the tens board.

Repeat the exercise with this configuration.



Note: Backward trades may be difficult for your students at this time. There will be many opportunities to work on these in the future.

Exercise 4

Distribute individual Minicomputers and checkers to pairs of students. Each pair will need two Minicomputer boards (one sheet) and at least twenty positive checkers.

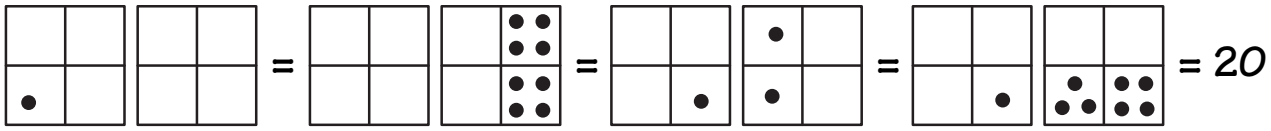
T: ***There are many ways to put 20 on the Minicomputer.***
Try to show some of them.

To be enjoyable, this exercise must move quickly. As students discover different configurations for 20, let them put their solutions on the demonstration Minicomputer. Do not attempt to get all the possible solutions; there are too many of them.

Possible configurations include the following:

†It is possible that a student will suggest an 8 + 4 = 10 + 2 trade. Although such trades are not emphasized, they should not be discouraged. In this case, you are interested in demonstrating the 4 = 2 + 2 trade. If another valid trade is suggested, you should indicate that it is correct, but that you would like to see how to use the 4 = 2 + 2 trade.

N5



If there is time remaining, you can repeat this exercise with other numbers (for example, 50, 100, 24, 35, 80, and so on).

Home Activity

This is a good time to send home a letter about the Minicomputer. Blacklines N5(a), (b), and (c) provide samples. Instruct students to color the Minicomputer boards on the full page to show their parents/guardians how the boards look. They should explain the Minicomputer and practice putting numbers on the Minicomputer and reading them. Objects such as paper clips, pennies, or dried beans can be used as checkers.

Capsule Lesson Summary

Introduce the idea of fact families and use sets of three numbers to write four facts, two addition and two subtraction. Consider an example of three numbers that cannot be used in a fact family. Give students numbers from 1 to 20 to sort into fact families and then ask them to write about what they did.

Materials

Teacher	• Chalk • Index cards	Student	• Worksheet N6 • Pencil

Advance Preparation: You will need one index card for each student. Write the numbers 1 through 20 on the first twenty cards and leave the remaining cards blank.

Description of Lesson

Exercise 1

Write the numbers 7, 5, and 2 on the board, or hold up the cards for these numbers.

T: ***What can you do with these three numbers?***

S: ***Make 752.***

S: ***Change them around to make 527.***

S: ***Write a number sentence: 5 + 2 = 7.***

T: ***I am going to record that number sentence on the board. That is one of the sentences I thought of when I looked at these numbers.***

S: ***How about 2 + 5 = 7.***

T: ***Yes, I'll write that down too.***

S: ***7 – 2 = 5.***

T: ***I'm writing that down.***

S: ***7 – 5 = 2.***

$$5 + 2 = 7$$
$$2 + 5 = 7$$
$$7 - 2 = 5$$
$$7 - 5 = 2$$

T: ***I'll put that down. Are there other number sentences that use these three numbers?***

S: ***25 + 2 = 27.***

T: ***Yes, but let's just use each of the numbers 7, 5, and 2 only once. These numbers make a fact family. There are two addition and two subtraction facts that use these three numbers.***

Exercise 2

Leave the fact family for 7, 5, and 2 on the board.

T: ***Let's look at another fact family.***

Write 8, 3, and 11 on the board, or hold up the cards for these

numbers.

T: ***Can you tell me four facts that use these three numbers?***

Write these facts as they are suggested

S: ***8 + 3 = 11.***

S: ***3 + 8 = 11.***

S: ***11 – 8 = 3.***

S: ***11 – 3 = 8.***

8 + 3 = 11
3 + 8 = 11
11 – 8 = 3
11 – 3 = 8

T: ***You found the two addition and two subtraction facts for that family quickly. Try this one.***

Write 5, 12, and 4 on the board or hold up the cards for these numbers.

S: ***5 + 4 = 9, but 9 is not one of the numbers.***

S: ***12 – 4 = 8 but 8 is not one of the numbers.***

S: ***We can't write facts with just these three numbers. You need to change some of the numbers.***

T: ***Can you suggest a change that would work?***

S: ***Change the 12 to 9; 5, 4, and 9.***

S: ***Change the 5 to 8; 12, 4, and 8.***

S: ***Change the 4 to 17; 12, 5, and 17.***

T: ***Those are good suggestions. I see that not just any three numbers make a number fact family. Here are some numbers; can you suggest which three make a fact family?***

S: ***13, 3, 10: 13 – 3 = 10; 13 – 10 = 3; 10 + 3 = 13; 3 + 10 = 13.***

S: ***13, 3, 16: 13 + 3 = 16; 3 + 13 = 16; 16 – 3 = 13; 16 – 13 = 3.***

S: ***14, 13, 1: 14 – 1 = 13; 14 – 13 = 1; 13 + 1 = 14; 1 + 13 = 14.***

S: ***6, 10, 16: 6 + 10 = 16; 10 + 6 = 16; 16 – 10 = 6; 16 – 6 = 10.***

Note: If a student suggests 3 and 6 with the facts 3 + 3 = 6 and 6 – 3 = 3, agree that this is a special family with just two members and two facts because 3 is used twice. You might say 3 is a twin in this family.

T: ***Now I am going to give some of you cards for the numbers 1–20. The rest of you will have a job shortly. Those with number cards are to look around and find two other numbers with which you make a fact family.***

Pass out the number cards and let the students move around and talk until most have found groups of three. Instruct those groups to stand together. Offer some assistance if help is needed to get started.

Once most are in groups of three continue as suggested in this dialogue.

T: ***I see 1, 19, and 20 are in a group. So are 2, 8, and 10; 3, 11, and 14; 4, 5, and 9; 6, 12, and 18.***

Use this time to check the groups. Ask the class to help with any groups that seem to be incorrect.

T: ***But some numbers are not in groups. Who are they?***

S: ***7, 13, 15, 16, and 17.***

T: ***I am going to give blank cards to those of you at your seats. Now, those with blank cards can join groups by writing numbers from 1 to 20 on your cards.***

Encourage the students already with numbers to pair up and decide on a number from 1 to 20 to join them to complete a fact family. For example, 7 and 13 might ask another student to write 6 or 20 on a blank card and join them.

Any students left with blank cards can form groups of three and make up their own number fact family.

Once all the students are in groups, give each group a copy of Worksheet N6 to complete.

Center Activity

Place the number cards 1–20 in a center, along with several blank cards. Direct students to put the number cards into fact families, writing additional numbers on the blank cards if necessary. Challenge them to use the least number of blank cards possible.

Home Activity

Introduce parents/guardians to fact families and suggest they work with their child to select three numbers from 1–20 that will make a fact family. Suggest they check the three chosen numbers by writing the addition and subtraction facts.

N7 MINICOMPUTER INTRODUCTION #4

Capsule Lesson Summary

Practice making trades on the Minicomputer and use trades in addition and subtraction calculations. Recall the symbols > and <.

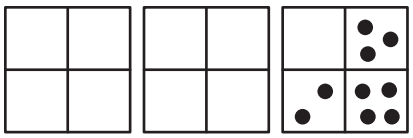
Materials

Teacher	• Minicomputer set	Student	• Paper • Individual Minicomputer set

Description of Lesson

Exercise 1

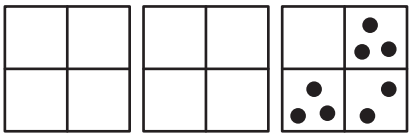
Display three Minicomputer boards. Put checkers on the Minicomputer gradually, allowing the students to calculate the number mentally.



T: ***What number is on the Minicomputer?*** (20)
Can you put 20 on the Minicomputer with one less checker (that is, with eight checkers)?

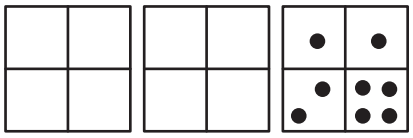
If the students do not respond, suggest that they make a trade. Any forward trade will solve the problem. For example:

Trade A: 1 + 1 = 2



= 20 or

Trade B: 4 + 4 = 8

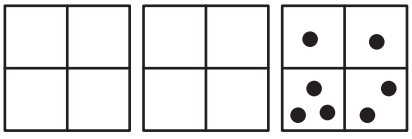


= 20

Other solutions are also possible.

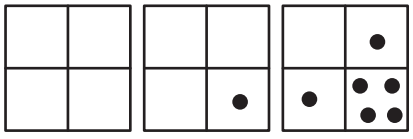
T: ***This is 20 with eight checkers. Can you put 20 on the Minicomputer with seven checkers?***

After Trade A (above)
trade: 4 + 4 = 8



= 20 or

After Trade B (above)
trade: 8 + 2 = 10



= 20

Other solutions are also possible.

T: ***Can you put 20 on the Minicomputer with one less checker (that is, with six checkers)?***

Do not overextend this activity, but return to similar exercises from time to time to review trades on the Minicomputer.

Exercise 2: Addition Problems

Write this problem on the board.

28 + 16 =

T: ***Can you think of a story problem for which we need to do this calculation?***

Allow students to compose (write) a story problem with a math partner. Accept a couple example story problems, but don't allow anyone to give an answer to the calculation at this moment. You may like to give students manipulatives to help them compose problems.

When students respond to the following estimation questions, ask them to explain their answers.

Note: Some students may know 28 + 16 = 44. Accept this knowledge but discuss why you may want an estimate rather than an exact answer. Estimates are sometimes what we use to compare numbers or to check that our results are reasonable.

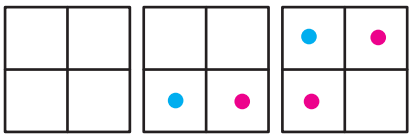
T: ***Do you think 28 + 16 is more or less than 20?*** (More)
More or less than 30? (More) ***More or less than 50?*** (Less)
We know that 28 + 16 is between 30 and 50. Would it be more or less than 40?

If the students disagree, say, "You don't know if it is more or less than 40, but you do know it is between 30 and 50." If the class is certain that 28 + 16 is more than 40, say, "Now you know it is between 40 and 50."

T: ***What number is 28 + 16?***

Accept several guesses and list them on the board. You can insist that guesses be between 30 and 50, (or between 40 and 50 if this has been determined) and so remind any student who gives an inappropriate guess.

T: ***Who would like to put 28 on the Minicomputer using blue checkers?***
Who would like to put 16 on the Minicomputer using red checkers?



Invite students to make trades so that the number will be easier to read. When all the trades have been made, call on a student to write 44 below the Minicomputer and conclude that 28 + 16 = 44.

Point to the list of estimates on the board. Acknowledge anyone who correctly predicted the answer. If no one guessed correctly, determine which estimate was the best. Indicate whether certain estimates are more or less than the sum. For example, if you had 42 and 45 as estimates, you would indicate that 42 is less than 44 and that 45 is more than 44. However, 45 is the best estimate because it is only 1 more than 44.

40
42 < 44
45 > 44
46

Note: Use the symbols > and < throughout the year whenever you have made a list of estimates and wish to decide which is the best estimate. These symbols are introduced in the *CSMP* first grade program. Most entry students will have been introduced to these symbols in other math programs. There is a short supplemental exercise following this lesson for classes with little or no experience with these symbols.

Repeat this exercise with 23 + 48 = 71.

Exercise 3: Subtraction Problems

37 – 14 =

First pose a story problem for subtraction. For example,

T: ***One day I opened a package from a friend and found 37 bright orange rocks. I gave 14 of them away. How many did I have to keep? What do you think I should do with them?***

Write this problem on the board.

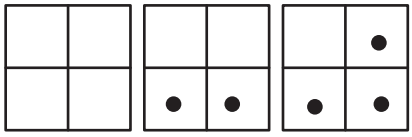
When students respond to these estimation questions, ask them to explain their answers.

T: ***Do you think 37 – 14 is more or less than 30?*** (Less)
More or less than 20? (More) ***We know that 37 – 14 is between 20 and 30. What number do you think it is?***

Accept several guesses and list them on the board. If you get a guess that is not between 20 and 30, remind the class that they already know 37 – 14 is between 20 and 30 and do not record that guess.

T: ***Who can put 37 on the Minicomputer? What should we do next?***

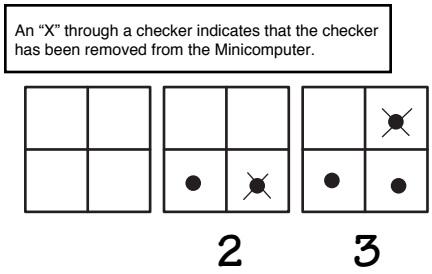
S: ***Take away 14.***



Note: When students are asked to put numbers on the Minicomputer, they may or may not use standard configurations for the numbers. The lesson description assumes that standard configurations of numbers are put on the Minicomputer. Adjust the lesson depending upon which configurations your students display.

A student should remove a checker from the 10-square and a checker from the 4-square.

Ask a student to write 23 below the Minicomputer and conclude that 37 – 14 = 23.



Repeat this exercise with 53 – 42 = 11 and 76 – 34 = 42. As in Exercise 1, give math partners a few minutes to compose (write) a story problem for which you need to do this calculation. Let students share some of their stories.

Writing Activity

Ask students to write an addition or a subtraction story problem of their choice and to also write how to solve their problem. Allow them to use manipulatives, pictures, or the Minicomputer.

N7

INTRODUCTION TO < and > (for entry classes)

Tell the following story making it as imaginative as possible.

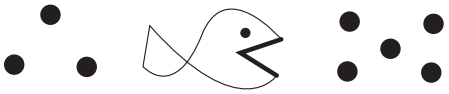
T: ***Goldy is a very large fish who lives in a large lake with smaller fish. The smaller fish swim together in "schools." Being such a large fish, Goldy gets very hungry and eats the small fish.***

Draw this picture of dots on the chalkboard.

T: ***The dots are for fish. How many fish are in this school*** (point to the dots on the left)***?*** (Three) ***How many are in this school*** (point to the dots on the right)***?*** (Five) ***If Goldy swims toward one school of fish, the other school swims away. So, if you were Goldy and very hungry, which school would you go after?***

Direct the responses so that the school with more fish is always chosen. Students should compare the numbers of fish (dots) in the two schools.

T: ***We'll show that Goldy goes after the school with five fish.***

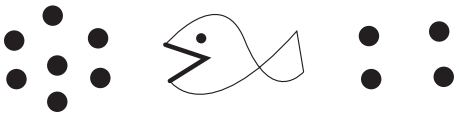


Draw Goldy between the schools of fish. Goldy's mouth should stand out.

Now draw this picture.

T: ***How many fish are in this school*** (point to the dots on the left)***?*** (Seven)
How many are in this school (point to the dots on the right)***?*** (Four)
Who would like to show us which school of fish Goldy will go after?

Draw Goldy, making sure the open mouth suggests that Goldy is ready to eat the larger school.



After a few examples, tell the students that from now on you are just going to draw Goldy's mouth.

Do several examples like the following, each time inviting a student to draw in Goldy's mouth.

Announce that, from now on, instead of drawing dots for fish, you will write a numeral to show how many fish there are in each school. For example, write 7 and 5 with space between them.

T: ***How many fish are in this school*** (point to 7)***?*** (Seven)
How many fish are in this school (point to 5)***?*** (Five)
Who can draw Goldy's mouth to show the school of fish Goldy would go after?
Yes, Goldy goes after the school with more fish. 7 is more than 5.

7 > 5

Do several examples asking students to read the number sentence each time. Then introduce = in an example comparing 4 and 3 + 1.

T: ***Goldy can't decide which school to go after, so we write this:***

4 = 3 + 1

N8 ADDITION PROBLEMS #1

Capsule Lesson Summary

Calculate 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 by pairing numbers whose sum is 10. Play a Tic-Tac-Toe number game using the numbers from 1 to 10 and involving sums to 15.

Materials

Teacher
- Minicomputer set
- Numeral cards (optional)
- Unifix® cubes (optional)

Student
- None

Advance Preparation: If you choose to use numeral cards in Exercise 1, use index cards to make cards for 1 to 10 and four extra 10-cards.

Description of Lesson

Exercise 1: Addition Problem

Choose one of your students to be the star of this story.

T: ***Last summer Steve went to the beach. Sometimes he swam and sometimes he played in the sand. One day he found a stick and started writing in the sand. He wrote the numbers 1, 2, 3, 4, and so on.***

Make a list of the whole numbers from 1 to 10 on the board as you say them, leaving space between them.

1 2 3 4 5 6 7 8 9 10

T: ***Steve's big brother came along just then and said, "This will keep you busy for awhile." He wrote plus signs between all the numbers. At the very end, he drew an equals sign and a big question mark.***

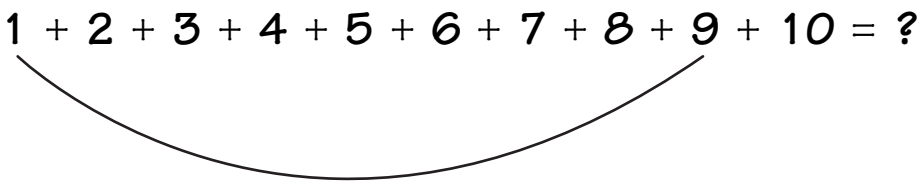
1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = ?

T: ***Steve wanted to show his brother how smart he was, so he looked for an easy way to add these numbers. Can you see a good way to pair these numbers so that they will be easy to add?***

Accept suggestions that pair two numbers whose sum is 10. If a student suggests pairing numbers whose sum is not 10, say, "Do you think that will make the addition problem easier?" If no one suggests pairing numbers whose sum is 10, continue with the story.

T: ***Soon Steve saw that he could pair numbers to get 10. "Ten is an easy number to add to other numbers," he thought.***

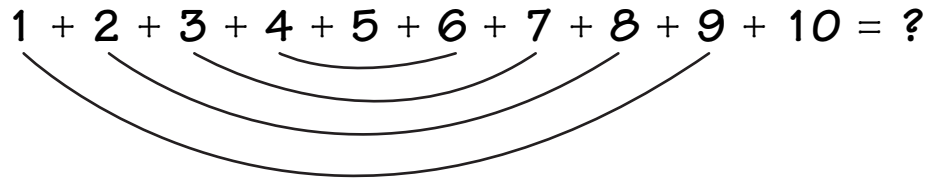
Show the pairing of 9 and 1 in your list.



You may like to model this problem with Unifix® cubes or numeral cards. Make "towers" of cubes from one to ten corresponding to the ten numbers being added. When you pair the numbers 9 and 1, snap the 1-tower together with the 9-tower to get another 10-tower. Students can refer to the towers in pairing other numbers. With numeral cards, let students bring the 9 and 1 cards together and trade them for another 10 card.

T: ***Which other numbers could Steve pair to make 10?***

Encourage students to continue pairing numbers whose sum is 10 until you have all the numbers except 5 and 10 in pairs. (Using different colors to show the pairings gives a nice rainbow effect.)



T: ***How do these pairs help Steve solve the problem?*** (10's are easy to add.)

You may need to suggest this idea yourself. If using Unifix® cubes you will now have five 10-towers and one 5-tower. With numeral cards you will now have five 10-cards and a 5-card.

T: ***There are five 10's and one 5.***

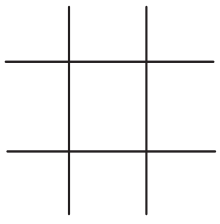
Write this calculation on the board:

$$10 + 10 + 10 + 10 + 10 + 5 = ?$$

This should be enough to convince most of the class that the sum is 55. Afterward, let the students check the calculation on the Minicomputer or with a calculator.

Exercise 2

Draw a Tic-Tac-Toe grid on the board.



T: ***Has anyone played Tic-Tac-Toe?***

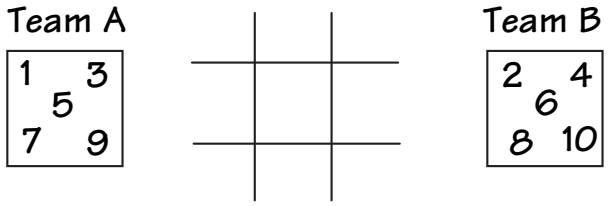
Call on a student who has played the game to describe the game to the class and to play a game with you or a classmate on the chalkboard so everyone can see.

T: ***I will teach you a new game called Tic-Tac-Toe 15. Just like the regular game, we need two players or two teams.***

Divide the class into two teams (Team A and Team B).

T: ***We will play the game with a regular Tic-Tac-Toe grid. However, instead of X's and 0's, your teams will use numbers. Team A will use the numbers 1, 3, 5, 7, 9. Team B will use the numbers 2, 4, 6, 8, 10.***

Write the teams numbers in boxes on the board near the Tic-Tac-Toe grid.



T: ***What do you notice about each team's numbers?***

S: ***Team B has numbers you get when you count by twos starting at 2.***

T: ***Do you remember what we call these numbers?***

S: ***Even numbers.***

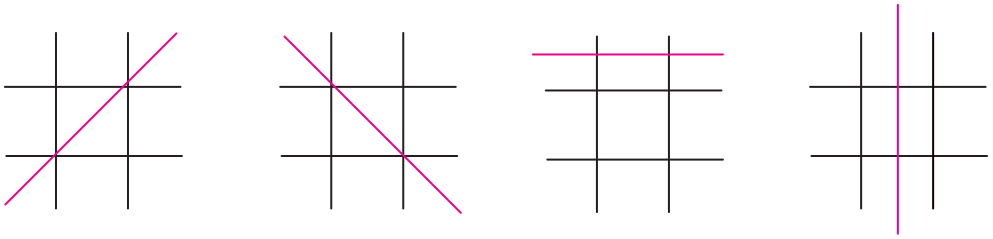
T: ***And what are Team A's numbers called?***

S: ***Odd numbers.***

If your students do not remember the terms *even* and *odd*, mention them casually in describing Team A's and Team B's numbers.

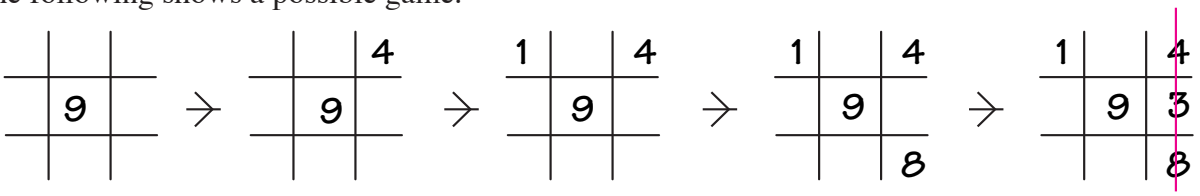
T: ***When I call on you, select one number from your team's numbers and write it in the grid in any empty space. Then erase the number you use from your team's numbers.***

The object of the game is to get three numbers in a row that add up to 15. Three numbers in a row is just like X's and 0's in a row for Tic-Tac-Toe.



T: ***Let's play a game. Remember, three numbers in a row in any direction that add to 15 wins.***

The following shows a possible game.



Team A wins

Play as many team games as time allows or put students in groups (even number per group) to play the game.

Center Activity

Put laminated Tic-Tac-Toe grids in a center for students to use playing Tic-Tac-Toe 15. For variation students can change the game to Tic-Tac-Toe 14 (or 16).

Home Activity

Describe the Tic-Tac-Toe 15 game for parents/guardians to play with their child at home.

Capsule Lesson Summary

Repeatedly double a set of objects, going from 1 to 64. Relate this doubling activity to a 2x arrow picture and label the dots.

Materials

Teacher	• Colored chalk • Angel fish pictures (optional) • Dot sheets • Masking tape	Student	• Colored pencils, pens, or crayons • Unlined paper • Minicomputer set or calculator (optional)

Advance Preparation: Use Blacklines N9 (a) and (b) to prepare angel fish pictures and dot sheets.

Description of Materials

Choose one of your students to be the star of this story. You may like to draw a picture—a simple open rectangle for the aquarium—and use cut-out pictures of angel fish as you tell the beginning of the story.

T: ***Cheryl is eight years old today. Her aunt has given her a beautiful angel fish. Cheryl watched the fish all morning. She liked to watch it swim around the aquarium. It swam around and around. The angel fish seemed very lonely. Cheryl wished she had more fish .***

Cheryl's aunt saw that she was sad and asked, "Don't you like your angel fish?"

"Yes," said Cheryl, "but I wish I had at least one more fish. It looks very lonely swimming around and around by itself."

Cheryl's aunt smiled and said, "Your angel fish is lucky. I have a magic wand. When I touch the aquarium with this wand, there will be twice as many fish." Then Cheryl's aunt touched the aquarium with the magic wand. Suddenly there were twice as many fish in the aquarium.

Ask the students how many fish are in the aquarium now and put a second angel fish in your picture.

T: ***Did Cheryl get her wish ?*** (Yes)
Cheryl was very happy to see two fish in the aquarium. She asked her aunt to touch the aquarium again with the magic wand. So, Cheryl's aunt touched the aquarium again. Immediately there were twice as many fish.
How many fish are in the aquarium now?

S: ***Four.***

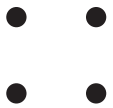
Put two more angel fish in your picture.

T: ***How many fish did Cheryl get for her birthday?***

S: ***One.***

Draw one large dot on the board for the one angel fish Cheryl got for her birthday. Ask what happened when the aquarium was touched once with the magic wand. (Then there were twice as many fish.) Draw another large dot next to the first as you say, "2 x 1 = 2."

Ask what happened when the aquarium was touched the second time. (Then there were four fish, twice as many fish.) Draw two more large dots as you say, "2 x 2 = 4."



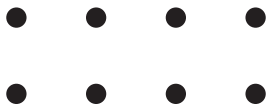
T: ***Cheryl jumped up and down and sang, "Now I have four beautiful angel fish." It made Cheryl's aunt happy to see Cheryl so excited, so she touched the aquarium again with the magic wand.***

Now, how many fish are there in the aquarium?

S: ***Twice as many as four.***

S: ***Eight.***

If students have difficulty, point to each of the four dots or the four angel fish pictures and count by twos to determine the number of fish after doubling. On the board, draw four more dots.



T: ***Cheryl was very excited and cried, "Now my angel fish has seven friends. It will never be lonely again." Cheryl watched her fish swim around and around in the aquarium. Sometimes they seemed to be talking to each other. "These fish are nice," thought Cheryl, "but the magic wand is more fun." She asked her aunt to give her the magic wand.***

Ask the students what Cheryl could do with the magic wand, and let them discuss the possibilities for a minute.

T: ***"Why do you want the magic wand?" asked Cheryl's aunt.***

"I want to touch the aquarium 100 times," answered Cheryl.

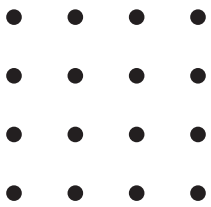
"Oh my! Then there would certainly be too many fish for your aquarium," said Cheryl's aunt.

"I'll only touch it ten times," promised Cheryl.

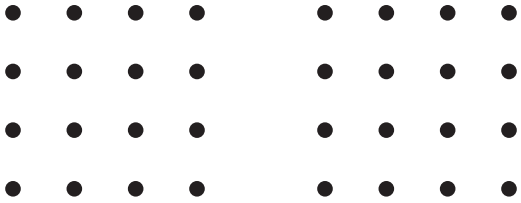
"I'm afraid that even ten times would give you too many fish to fit in one aquarium," answered Cheryl's aunt. "I will let you touch the aquarium just two more times with the magic wand." She gave Cheryl the wand and Cheryl ran to the aquarium.

Ask the students to write on their paper or whisper to you how many fish there will be after two more touches. (32)

On the board, draw eight more dots as you say, "2 x 8 = 16."



Draw 16 more dots as you say, "2 x 16 = 32." At this point you may prefer to replace the dots drawn on the board with a paper having 16 dots on it. Then, in the next couple of doubling steps, you will use the paper sheets of dots rather than draw them on the board.

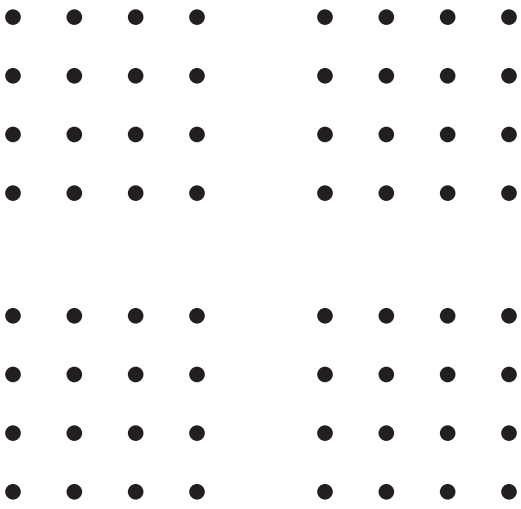


T: ***Cheryl will have 32 fish if she touches the aquarium two more times.***

But Cheryl was a naughty girl. She touched the aquarium still another time (a third time). Cheryl was going to touch the aquarium again but she stopped just in time. There were so many fish in the aquarium that they could not swim around anymore.

Ask the students to write on their papers or whisper to you how many fish are in the aquarium now. (64)

Draw 32 more dots (arranged as two groups of 16) on the board as your say, "2 x 32 = 64."



T: ***We can tell this story with an arrow picture.***

Near the array of dots you have been developing, draw an arrow picture with five dots as in the next illustration. Leave room to the right of the arrow picture for more dots to be added later.

T: ***How many fish did Cheryl get for her birthday?***

S: ***One.***

Label the first dot on the left 1.

T: ***How many fish were there after touching the aquarium with the magic wand once?***

S: ***Two.***

Label the second dot 2. Trace the first arrow from 1 to 2 as you ask,

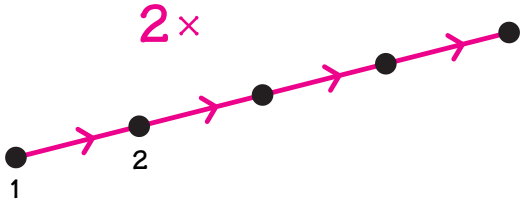
T: ***What do you think this arrow could be for?***

S: ***2x.***

If someone says +1, say, "This first arrow could be for +1, but we are thinking of the story. The magic wand always made twice as many. What else could it be for?" When someone suggests 2x write the key near the arrow picture in the same color as the arrow. Point to the third dot.

T: ***What number is this?***

S: ***4, because 2 x 2 = 4 .***



Encourage an explanation, perhaps reminding the class about how many fish were in the aquarium after two touches of the magic wand.

Label this dot. Point to the fourth dot.

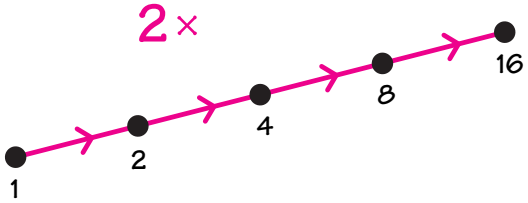
T: ***What number is this?***

S: ***8, because 2 x 4 = 8 (or 4 + 4 = 8).***

Label this dot. Point to the last dot.

T: ***What number is this?***

S: ***16.***



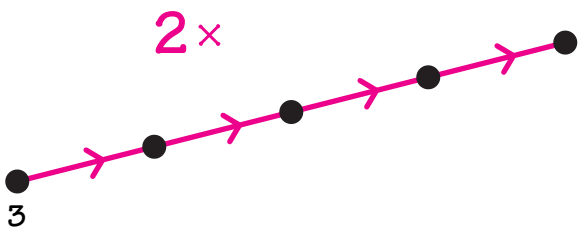
Label this dot.

T: ***How many fish did Cheryl have at the end of our story?*** (64)
How many more 2x arrows must we draw to reach 64?

S: ***Two.***

With the class, draw two more arrows and label their ending dots.

Erase the board and draw a 2x arrow picture starting at 3.



Ask the students what the second dot is for.

Distribute unlined paper. Ask students to copy the picture, to label the other dots, and then to extend the arrow picture using more 2x arrows. Some students may wish to use the Minicomputer or a calculator for the more difficult calculations.

Writing Activity

Invite students to write a doubling story (like the magic wand story) to go with their arrow picture.

Home Activity

Suggest parents/guardians discuss a simple exercise program with their child in which they will exercise every day for one week. Each day after the first, they will exercise twice as long as the day before.

Suggest parents/guardians ask their child to determine how long they must exercise each day of the week if they exercise five minutes on the first day… one minute? How many minutes of exercise would they do (total) in the week?

N10 THE FUNCTIONS +2, –2, +5, AND –5

Capsule Lesson Summary

Do some mental arithmetic with adding and subtracting 2 and 5. Label the dots and draw return arrows in a +2 arrow picture and in a -5 arrow picture.

Materials

Teacher
- Colored chalk
- Calculator
- Number line

Student
- Paper
- Worksheets N10*, **, ***, and ****

Description of Lesson

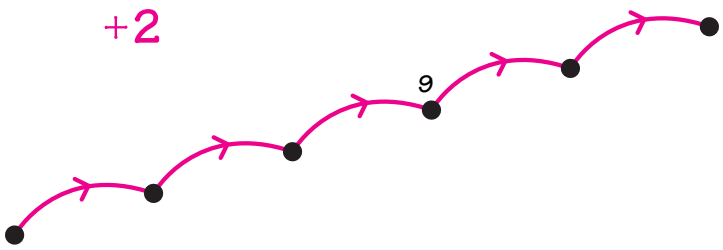
Exercise 1

Conduct a mental arithmetic practice in adding and subtracting 2 and 5. For example:

4 + 2 = ?	5 + 2 = ?	3 + 5 = ?	12 – 2 = ?	10 – 5 = ?	1 + 5 = ?
6 + 2 = ?	15 + 2 = ?	8 + 5 = ?	10 – 2 = ?	9 – 5 = ?	6 – 5 = ?
8 + 2 = ?	25 + 2 + ?	18 + 5 = ?	8 – 2 = ?	8 – 5 = ?	11 + 5 = ?
10 + 2 = ?	65 + 2 = ?	28 + 5 = ?	6 – 2 = ?	7 – 5 = ?	16 – 5 = ?
20 + 2 = ?	105 + 2 = ?	128 + 5 = ?	4 – 2 = ?	17 – 5 = ?	25 + 5 = ?
40 + 2 = ?	165 + 2 = ?	228 + 5 = ?	2 – 2 = ?	27 – 5 = ?	30 – 5 = ?

Exercise 2

Draw this arrow picture on the board.



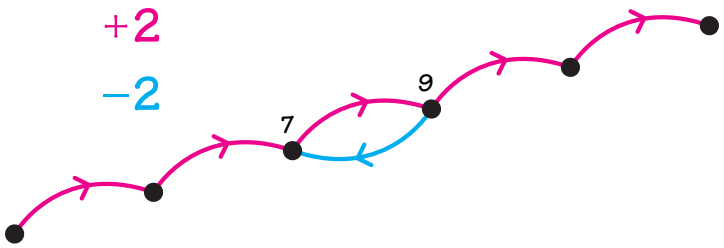
T: ***Point to the greatest number in this arrow picture. Write on your paper (or whisper to me), what is the greatest number.*** (13)
Point to the least number. What is the least number? (3)
What are the other numbers? (5, 7, 11)

With the help of several students, label all the dots. Each time a dot is labeled, ask how the student knows that dot is for the number suggested. For the dots to the left of 9, encourage students to think of subtraction calculations. For example, when someone suggests that the first dot to the left of 9 is for 7, direct the explanation to mention "9 – 2 = 7." Then ask for a volunteer to first trace and then draw (in blue) the –2 return arrow from 9 to 7. Occasionally write related number sentences on the board, for example, 7 + 2 = 9 and 9 – 2 = 7.

T: ***What could the blue arrow be for?***

S: ***–2.***

Write –2 in blue near the arrow picture.



T: ***Do you see where we could draw other –2 arrows?***

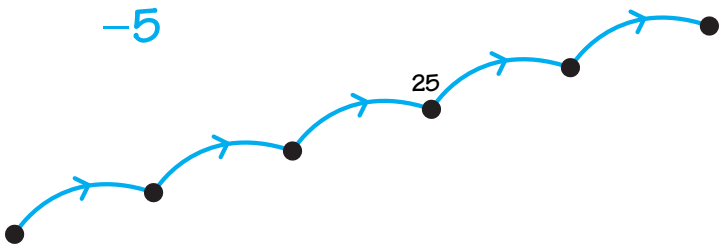
Let students suggest as many other –2 arrows as they can. You need not insist that they find them all immediately. They can continue to label other dots, and then you can return to the request for other blue arrows. Each time a student wants to show where a blue arrow can be drawn, ask the student to trace the arrow and then, if correct, to draw it in blue.

T: ***If I were go on drawing more +2 arrows (to the right), would we ever meet 20?*** (No) ***Would we meet 21?*** (Yes) ***Would we meet 30?*** (No) ***…40?*** (No) ***…45?*** (Yes)

Ask students to explain their answers, but be prepared to help if they have trouble verbalizing at this stage. You may let students check some of these questions with a counting calculator (counting by twos starting at 3) or the number line.

Exercise 3

Erase the board and draw this arrow picture.



T: ***Point to the greatest number in this arrow picture. Write the greatest number on your paper.*** (40)

Point to the least number. Write the least number on your paper. (15)

Point to the first dot to the left of 25.

T: ***What number is here? How do you know?***

S: ***30, because 30 – 5 = 25.***

T: ***Do you know another way to see that it is 30?***

S: ***25 + 5 = 30.***

T: ***How can we see 25 + 5 in our picture?***

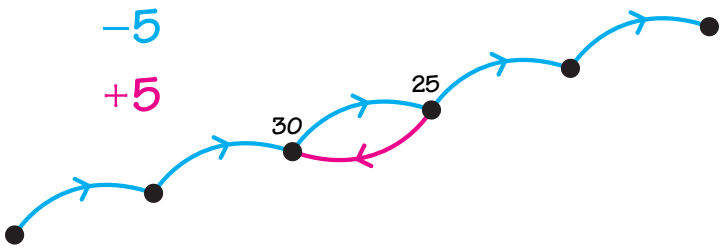
S: ***Draw a return arrow in another color.***

Ask a volunteer to draw the return arrow in red.

T: ***What could the red arrow be for?***

S: ***+5.***

Write +5 in red near the arrow picture.



T: ***Can we draw other +5 arrows in this picture?***

If the students do not immediately suggest other +5 arrows, ask them first to label some of the other dots first. Then ask them again if they see where more red arrows could be drawn. Complete the arrow picture quickly with the students.

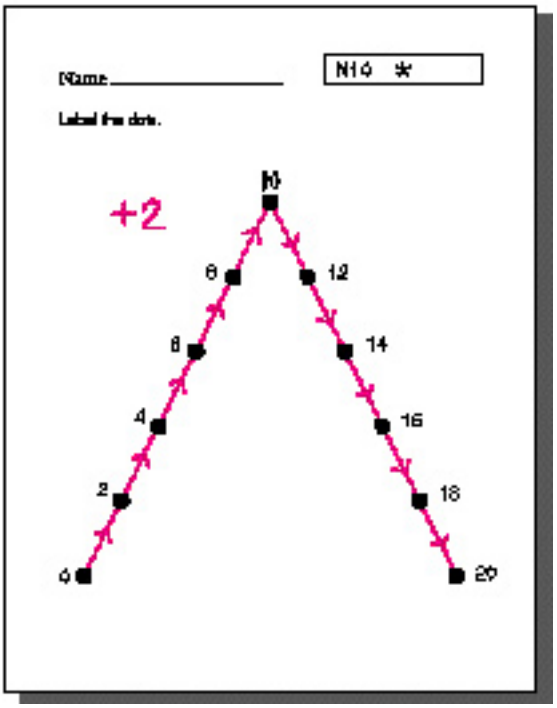
T: ***If we drew still more +5 arrows (to the left), would we meet the number 100?*** (Yes) ***200?*** (Yes) ***500?*** (Yes) ***101?*** (No)

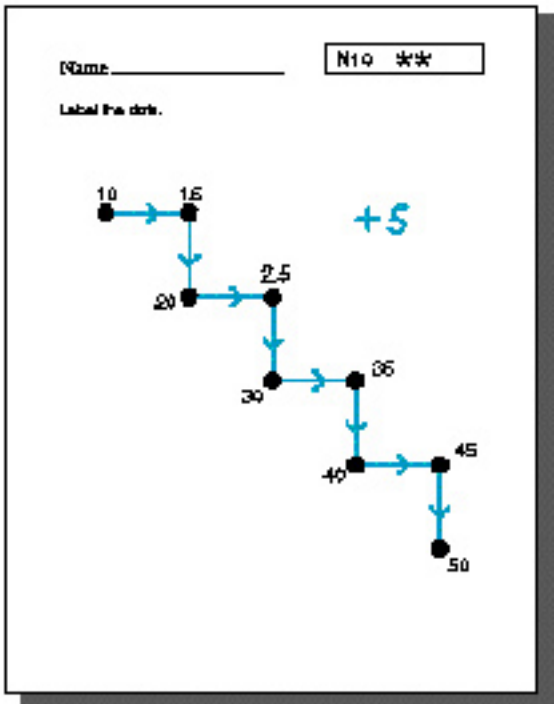
Ask students to explain, but be prepared to help if they have trouble verbalizing at this stage. Again, you may let students check some of these questions with a counting calculator (counting by fives starting at 15) or the number line.

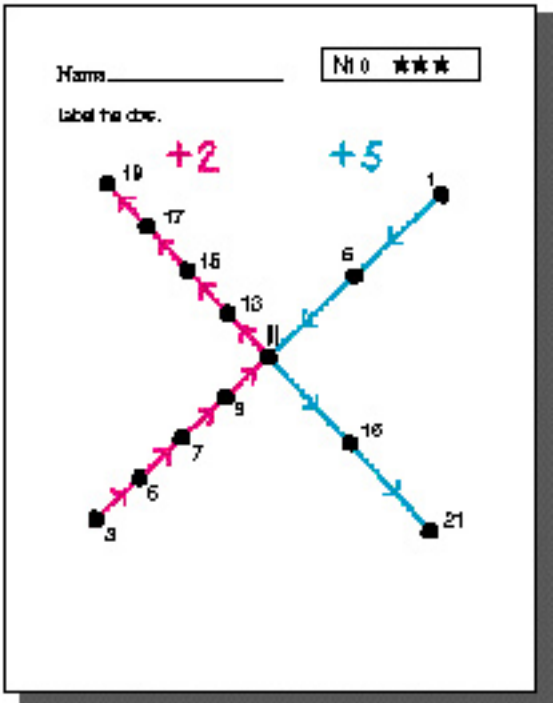
Worksheets N10*, **, ***, and **** are available for independent work.

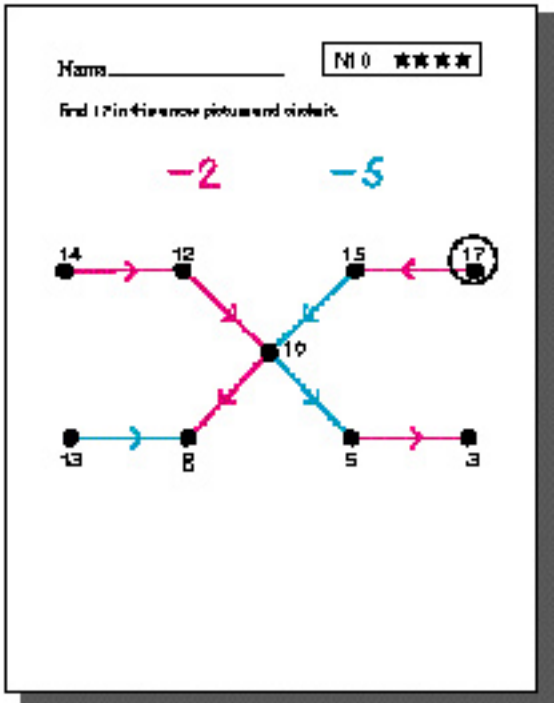
Home Activity

Suggest that parents/guardians find opportunities to count by twos and by fives with their child. This would be a good time to send a letter to parents/guardians about working on basic facts. Blackline N10 has a sample letter.


Name
N10 ★
Label the dots.
+2
0
2
4
6
8
10
12
14
16
18
20


Name
N10 ★★
Label the dots.
+5
10
15
20
25
30
35
40
45
50


Name
N10 ★★★
Label the dots.
+2
+5
19
17
15
13
11
9
7
5
3
1
6
16
21


Name
N10 ★★★★
-2
-5
14
12
15
17
10
13
8
5
3

N11 ADDITION PROBLEMS #2

Capsule Lesson Summary

Decide which coins to use to pay given prices. Pair numbers whose sum is 20 to simplify an addition problem. Practice putting numbers on the Minicomputer. Investigate the ways of putting 10 on the Minicomputer using from one to four checkers.

Materials

Teacher
- Colored chalk
- Minicomputer set
- Calculator

Student
- Coins
- Minicomputer set

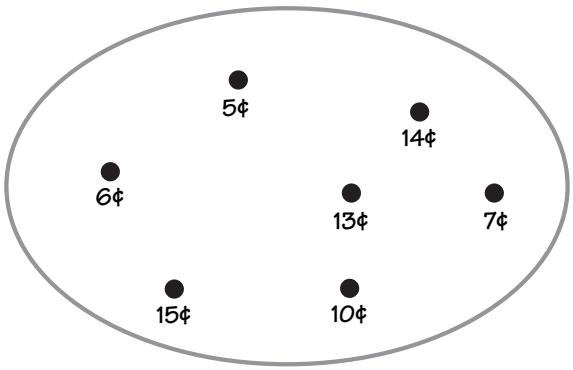
Description of Lesson

Exercise 1: Addition Problems

Choose one of your students to be the star of this story.

T: ***Nancy went to the candy store to buy some candy. She bought seven pieces of candy. Inside this blue string are the prices of the candies.***

Draw this string picture on the board.



Provide pairs of students (math partners) with a small collection of coins (play or real money) including at least five pennies, three nickels, and one dime. Invite students to show the class which coins Nancy could use to pay each of the prices in the blue string. For example, 15¢ could be paid with three nickels, or one dime and five pennies.

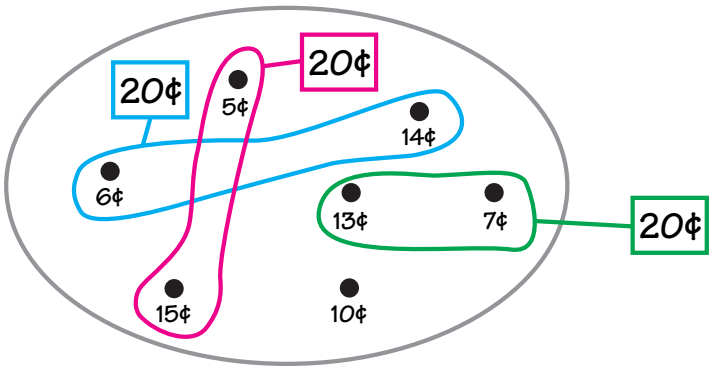
Instruct students to put away the coins before continuing.

T: ***How much did Nancy spend?***

Let the students react to this problem and let them suggest ways of solving it. Undoubtedly, some will suggest using the Minicomputer or a calculator. Others may want to use coins.

T: ***We could use the Minicomputer (or calculator), but first let's try to solve the problem by pairing the numbers so they will be easier to add.***

When a student observes, for example, that 13 + 7 = 20, circle these two numbers. If no one suggests pairing two numbers whose sum is 20, comment that 20 is an easy number to add to other numbers and ask the class to find two numbers whose sum is 20. Continue pairing numbers whose sum is 20 until your picture looks like the one below.



T: ***How can we use this string picture to find what Nancy spent?***

S: ***Add 20 + 20 + 20 + 10.***

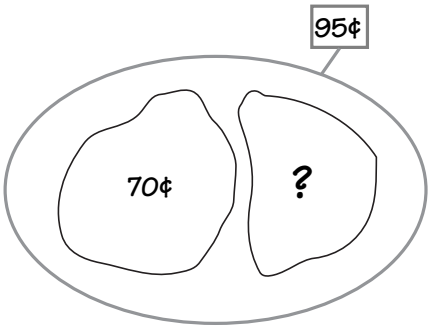
Trace an appropriate string as you write each number in the addition calculation on the board: 20 + 20 + 20 + 10.

T: ***How much did Nancy spend? How do you know?***

S: ***70¢, because 20 + 20 + 20 + 10 = 70.***

Accept any correct methods, complete the calculation on the board, and then check the calculation on the Minicomputer or with a calculator.

Draw a picture as you pose the following subtraction problem.



T: ***Suppose Nancy had 95¢ in all.***
She spent 70¢ for the candy.
Does she have money left? (Yes)
How much?

Let students explain how to find how much money is left.
Encourage them to describe both 95 – 70 = $\boxed{25}$ and 70 + $\boxed{25}$ = 95

T: ***What coin(s) might she have left in her pocket?***

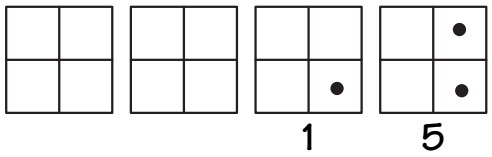
S: ***A quarter.***

S: ***Two dimes and a nickel***

Exercise 2

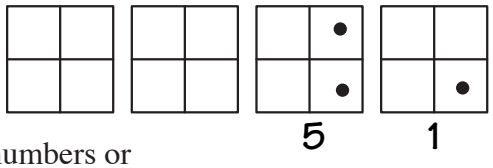
Display four Minicomputer boards. Try to include all the students, especially new students, in this activity.

T: ***Who can put 15 on the Minicomputer?***
Who can write 15 below the Minicomputer?



Always remove the checkers and erase the board before continuing to another number.

T: ***Who can put 51 on the Minicomputer?***
Who can write 51 below the Minicomputer?



Continue this activity with the following sequences of numbers or any similar ones appropriate to the numerical abilities of your students.

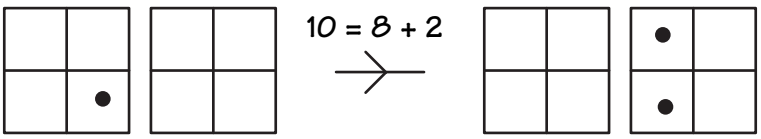
150	28
105	82
510	280
501	802
1,050	2,800
5,001	8,020

Exercise 3

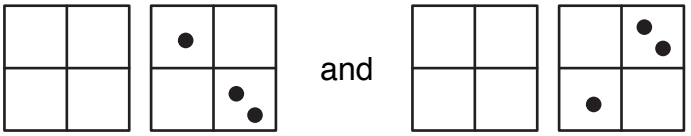
Pair students and provide each group with one individual Minicomputer sheet (two boards) and checkers. Display two Minicomputer boards.

Ask the students to put 10 on their (desk) Minicomputers using exactly one checker. Invite a student to put 10 on the demonstration Minicomputer in the same way. (There is only one way to show 10 with exactly one positive checker.)

Then ask the students to put 10 on their Minicomputers with exactly two checkers. Invite a student to show his or her solution on the demonstration Minicomputer. (There is only one way to show 10 with exactly two positive checkers.) If a student does not mention the backward trade 10 = 8 + 2, do this yourself.

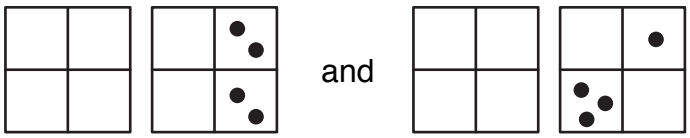


Continue this activity with three checkers . . .



(There are two ways to show 10 with exactly three positive checkers. The first you can get using the backward trade 2 = 1 + 1; and the second using the backward trade 8 = 4 + 4).

…and with four checkers.



(There are two ways to show 10 with exactly four positive checkers. Use either an 8 = 4 + 4, or a 4 = 2 + 2, or a 2 = 1 + 1 backward trade.)

Do not insist that your class find all the ways to show 10 with three or with four positive checkers. If some students put on numbers other than 10, tell them what numbers they have and remind them that they are trying to show 10.

Exercise 4 (optional)

If your class does well ~~with the previous exer~~cise, you can ask the students (in pairs) to find many ways to put 40 on their Minicomputers. As students find solutions, invite them to show their solutions on the demonstration Minicomputer.

Writing Activity

Instruct students to write about a trip to the store and two or more items they purchased. Ask them to tell how they found the total price and how they paid for them.

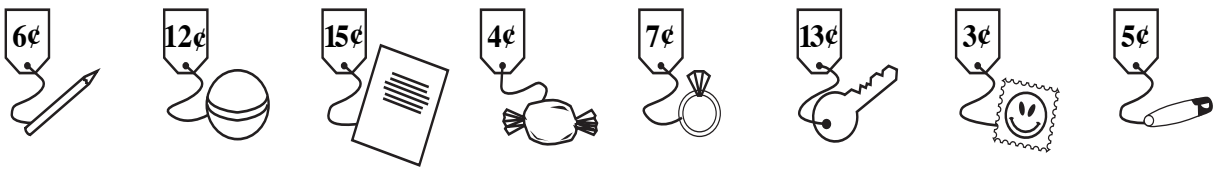
Center Activity

Cut pictures of toys, games, clothing, and so on out of Sunday circulars and mount on tag board. Put prices (within the abilities of your students) on each item. Let students choose two or three of their favorite items and "purchase" them with play money.

Home Activity

Suggest that parents/guardians give prices to some small items around the house and then give their child some coins with which to "purchase" items.

For example:



- What coins would you use to buy the key?
- Choose two or three items. What would they cost? What coins would you use to buy them?
- What could you buy and spend exactly 20¢? … between 25¢ and 30¢?

N12 COMPOSITION OF FUNCTIONS #1

Capsule Lesson Summary

Solve a story problem involving adding 2 and adding 3. Using arrow pictures, investigate the composition of the addition functions +2 followed by +3, and +3 followed by +7.

Materials

Teacher
- Colored chalk
- Counters or cubes (optional)
- Paper
- Worksheets N12*, **, and ***

Student
- Colored pencils, pens, or crayons

Description of Lesson

Exercise 1

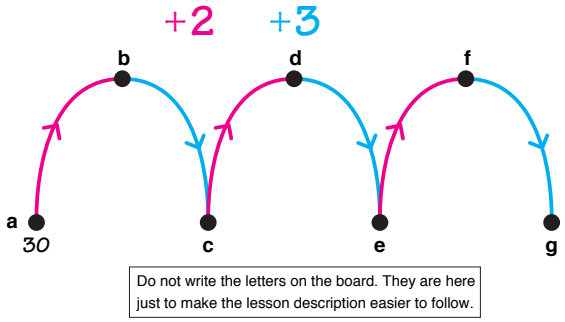
Pose this or a similar problem to the class.

T: ***Joanne collects animal figurines. Her collection has 30 figurines in it. For her birthday, Mother gives her 2 rabbits and 3 frogs. Then, Father also gives her 2 rabbits and 3 frogs. And Brother gives her 2 rabbits and 3 frogs, too.***
How many animal figurines does Joanne have in her collection now?

Let students suggest how to solve this problem and follow their suggestions. They may want to use manipulatives, draw pictures, use the Minicomputer, or write calculations. Exercise 2 will introduce an arrow picture that could be used for this problem, but do not impose this picture on their thinking unless students suggest it.

Exercise 2

Draw this arrow picture on the board.



T: ***Where is the least number in this arrow picture?*** (At **a**)
Where is the greatest number in this arrow picture? (At **g**)
Write on your paper (or whisper to me), the greatest number in this picture. (45)

This last question may be a challenge. If no one gives the correct answer, proceed to labeling dots. Wait until **c** has been labeled to ask again for the greatest number in the picture.

T: ***Does anyone see where we could draw a +5 arrow?*** (Write +5 in green near the arrow picture.)

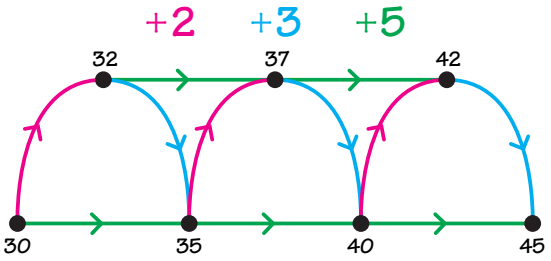
Allow the students to react, but do not allow any new dots to be added to the picture. As students suggest +5 arrows, let them trace the arrows on the board. Occasionally ask,

T: ***How do you know we can draw a +5 arrow there?***

S: ***Here is 30*** (pointing to **a**) ***and here is 35*** (pointing to **c**)***, and 30 + 5 = 35.***

S: ***A +2 arrow followed by a +3 arrow is a +5 arrow.***

Allow students to draw +5 arrows which they have traced correctly and label dots which they have referred to numerically. If no one suggests a correct +5 arrow, ask students to label the dots and then return to the request for +5 arrows. You may wish to model the composition of +2 followed by +3 with counters or cubes. In this case, put 30 counters of one color in a cup. Hold the cup near **a**. Invite a student to add two counters of another color to the cup as you move it from **a** to **b**. How many counters are in the cup at **b**? (32) Invite a student to add three counters of the second color to the cup as you move the cup from **b** to **c**. How many counters are in the cup at **c**? (35) Together we added (look at the number of counters of the second color) five counters. Trace and draw a +5 arrow from **a** to **c**. All possible +5 arrows are shown in the picture below.

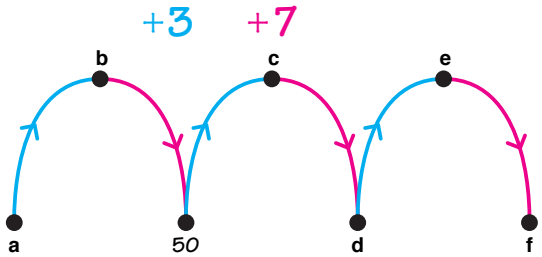


Some students may suggest that this arrow picture is like the problem in Exercise 1. In this case, make the connections (a +2 arrow for getting 2 rabbits and a +3 arrow for getting 3 frogs) and observe that Joanne has 45 figurines. If no student comments on this, leave it for future lessons.

Erase the board before going on to Exercise 3.

Exercise 3

Draw this arrow picture on the board.



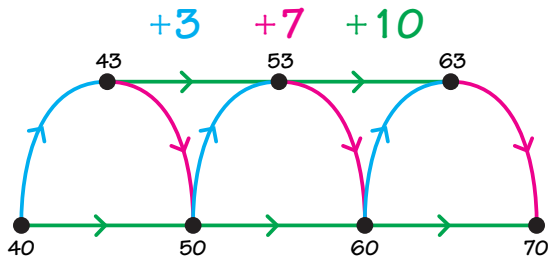
Do not write the letters on the board. They are here just to make the lesson description easier to follow.

T: ***Where is the least number in this arrow picture?*** (At **a**)
Write this number on your paper. (40)
Where is the greatest number in this arrow picture? (At **f**)
Write this number on your paper. (70)

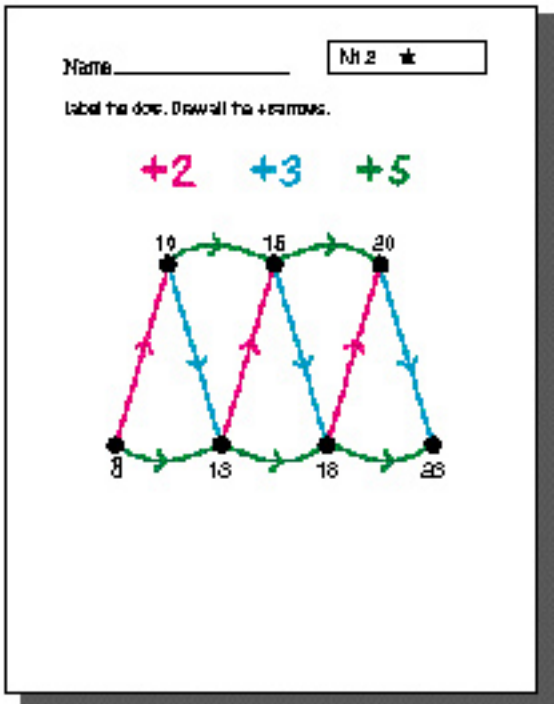
If you observe correct answers to your requests, ask students to answer aloud and explain their answers. Otherwise, wait until the dots are labeled to check the answers.

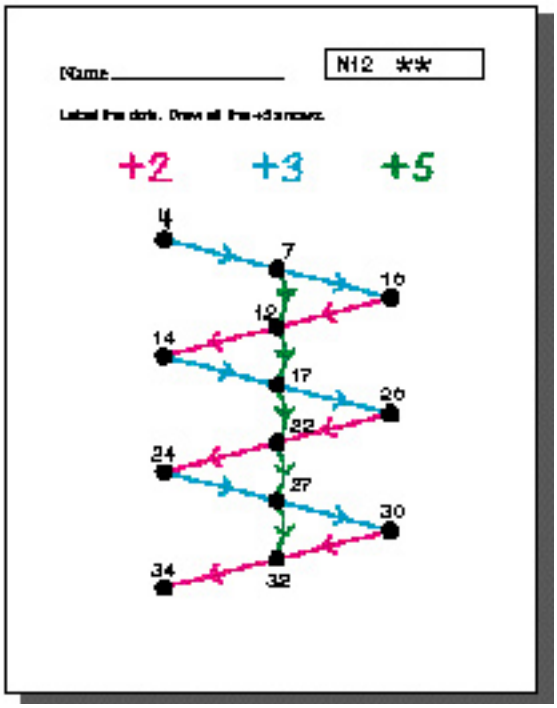
T: ***Does anyone see where we could draw a +10 arrow?*** (Write +10 in green on the board.)

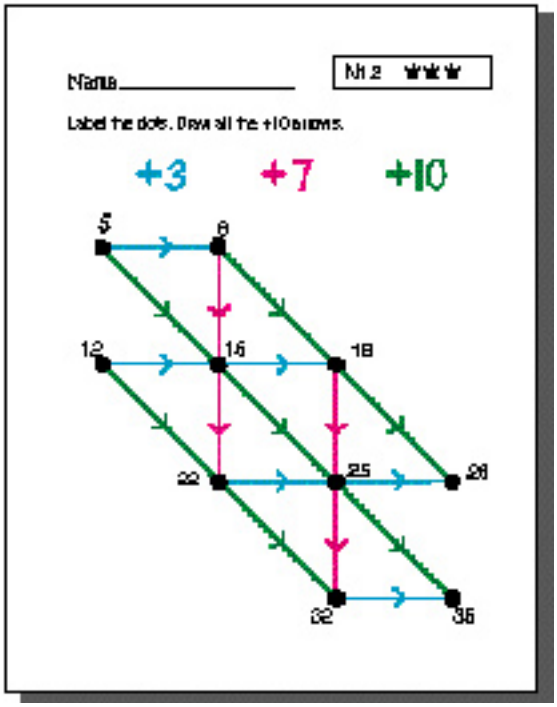
Continue until all the +10 arrows are drawn in the picture, and then label the dots with the class. Make use of the +10 arrows to label dots whenever possible. If students have trouble finding some of the +10 arrows, suggest labeling some dots first. A completed picture is shown below.



Worksheets N12*, **, and *** are available for individual work.


Name
N12 ★
Label the dots. Draw all the +2 arrows.
+2 +3 +5
10
15
20
8
13
18
23


Name
N12 ★★
+2 +3 +5
4
7
10
12
14
17
20
22
24
27
30
34
32


Name
N12 ★★★
Label the dots. Draw all the +10 arrows.
+3 +7 +10
5
8
12
15
18
22
25
28
32
35

Capsule Lesson Summary

Tell number stories about a rectangular array of dots. In particular, note that 4 x 3 = 3 x 4 as indicated by the array. Subtract numbers less than 100 with the help of the Minicomputer.

Materials

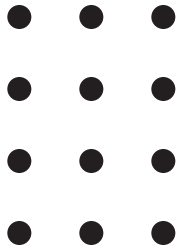
Teacher
- Straight edge
- Paper

Student
- Collection of objects (counters, cubes, odds and ends)
- Paper
- Straight edge
- Worksheet N13

Description of Lesson

Exercise 1: Multiplication Facts

Organize the class in cooperative groups (two to four students per group) and give each group a collection of objects (counters, cubes, odds and ends). Draw this pattern of dots on the board and ask the groups to reproduce your picture with their objects. Dots should be large and well-spaced. Check that the groups have arranged their objects neatly with clearly defined rows and columns.



T: ***Look at this pattern of dots or your pattern of objects. What do you see?***

Let students express themselves. Many reactions are possible. If a student says, for example, that there are 4 rows with 3 dots in each row, express this as 3 + 3 + 3 + 3 or 4 x 3.

Write expressions (for 12) on the board as they are given and invite groups to write them on paper. Other examples might include the following:

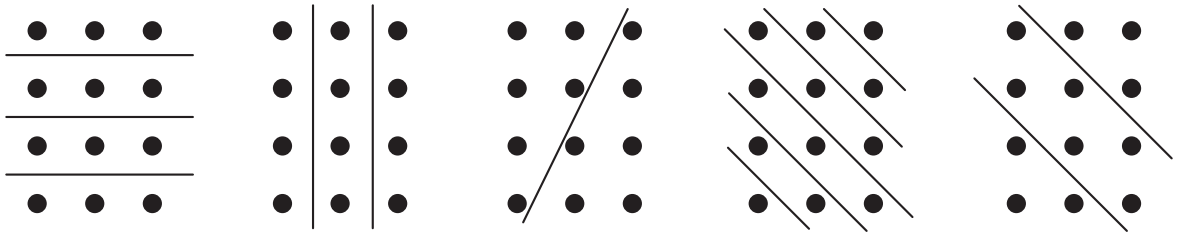
S: ***I see 4 + 4 + 4 (or 3 x 4).***

S: ***There are 12 dots.***

S: ***I count 6 + 6.***

S: ***I count 1 + 2 + 3 + 3 + 2 + 1.***

If your students do not give a wide variety of suggestions, you can prompt them by holding a straight edge in different positions to partition the array of dots (but drawing no lines on the board). Encourage groups to experiment with their array of objects and straight edges. For example:



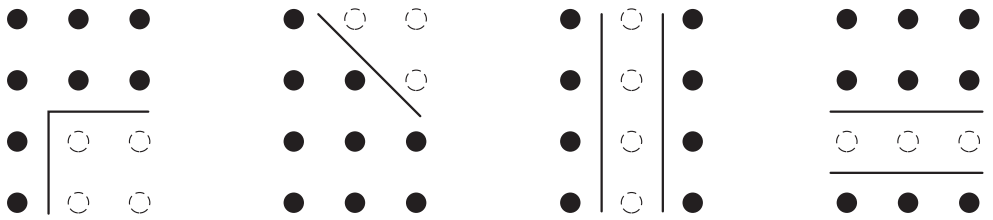
N 13

Emphasize the fact that there are two very special number facts related to this picture. Write the number sentences on the board and conclude that 3 x 4 = 4 x 3.

3 x 4 = 12 4 x 3 = 12 3 x 4 = 4 x 3

Exercise 2: Subtraction Facts

With a sheet of paper, hide some of the dots and ask the class how many of the dots are hidden. Fold the paper as necessary.



Write subtraction facts to show how many dots were hidden (from 12) and how many you still see.

12 – 4 = 8 12 – 3 = 9

Groups can work together to do a few other hidden dots (objects) examples.

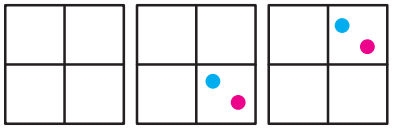
Exercise 3: Multiplication Problems

Write this problem on the board. 2 x 14 =

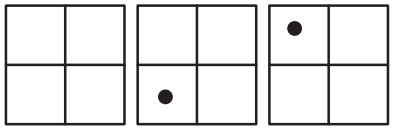
Ask each group to compose (write) a story problem for which you need to do this calculation. You may need to remind the students that 2 x 14 means 14 + 14. Let several groups share their story problems with the class.

Display three Minicomputer boards and ask,

T: ***Who would like to put 14 on the Minicomputer with blue checkers?***
Who would like to put 14 on the Minicomputer again with red checkers?



What number is on the Minicomputer? (2 x 14, or 28)
Who can make some trades so it will be easier to read?



2 8

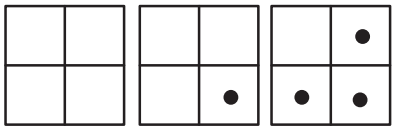
Conclude that 2 x 14 = 28.

Repeat this exercise with one or two more multiplication problems such as 2 x 23 and 3 x 12.

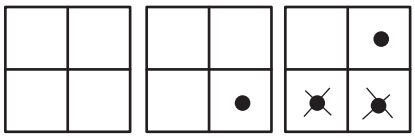
Exercise 4: Subtraction Problems

As you read this problem, call on a student to put 17 on the Minicomputer.

T: ***Marty has 17 grapes. He eats 3 of them.***
How many are left?



Some students will be able to give an answer immediately. The Minicomputer will help others find the solution almost as quickly.



T: ***Who can take 3 off the Minicomputer?***

Let a student remove the appropriate checkers from the Minicomputer.

T: ***What number is on the Minicomputer?***
How many grapes did Marty have left?

S: ***14.***

Read the number sentence as you write it on the board.

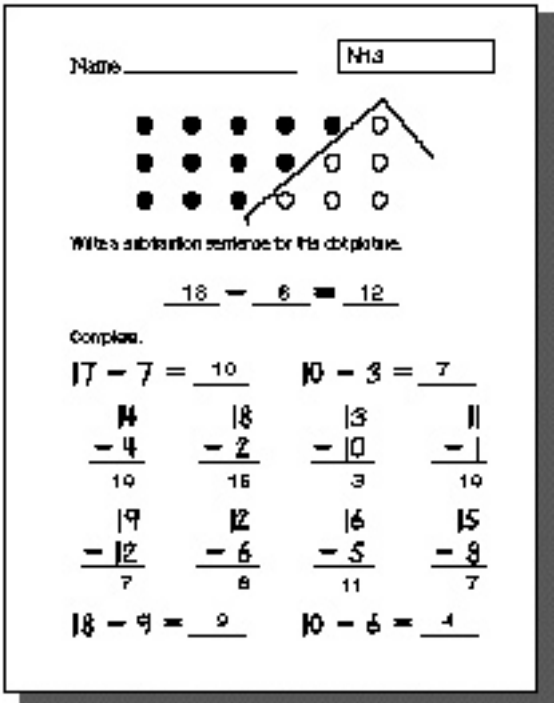
$$17 - 3 = 14$$

In a similar way, the class can do some of the following calculations on the Minicomputer, starting with 17, 75, and 367, respectively. For each calculation, no trades are needed. Write each calculation on the board as it is considered. Occasionally, invite a student to tell a story problem for a calculation.

17 – 4	75 – 4	367 – 7
17 – 5	75 – 5	367 – 45
17 – 6	75 – 40	367 – 60
17 – 10	75 – 50	367 – 100
17 – 12	75 – 70	367 – 240
17 – 13	75 – 21	367 – 205
17 – 15	75 – 34	367 – 246

Assessment Activity

Worksheet N13 is available to assess student progress in subtraction.


Name _____ N13

Write a subtraction sentence for this dot picture.

18 − 6 = 12

Complete.

17 − 7 = 10 10 − 3 = 7

14 − 4 = 10 18 − 2 = 16 13 − 10 = 3 11 − 1 = 10

19 − 12 = 7 12 − 6 = 6 16 − 5 = 11 15 − 8 = 7

18 − 9 = 9 10 − 6 = 4

Capsule Lesson Summary

Fill in a box (frame) with a number to make an addition sentence true.

Materials

Teacher	Student
• Balance or counters (optional)	• Paper • Worksheets N14*, **, ***, and ****

Description of Lesson

Note: Encourage students to write and solve the problems on their papers along with the class activity.

Write this open sentence on the board as you tell a story problem.

T: ***Chad has four tickets. He wants to know how many more tickets he needs to have nine tickets.***

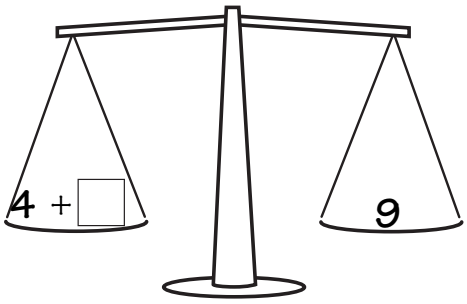
$4 + \square = 9$

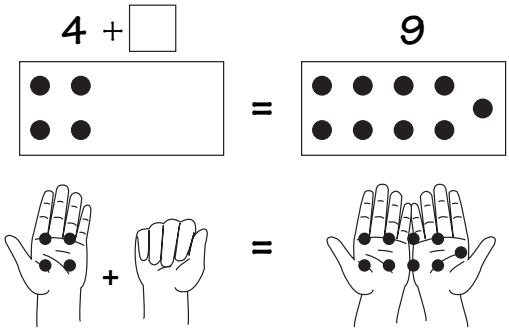
Read: Four plus box equals nine.
or
Four plus what number equals nine?

What number goes in the box to make the sentence true?

S: *5.* $4 + \boxed{5} = 9$

Note: You may like to use some manipulative to model the concept of an open sentence. For example, use a balance or counters:





Keep the first equation on the board and write this open sentence directly under it. $4 + \square = 11$

T: ***What number goes in the box to make this sentence true?***

S: *7.*

Continue giving problems, each time writing the equations under each other until there is no more space on the board.

$$4 + \boxed{5} = 9$$
$$4 + \boxed{7} = 11$$
$$4 + \boxed{9} = 13$$
$$4 + \boxed{11} = 15$$
$$4 + \boxed{13} = 17$$

T: ***Do you see a pattern?***

S: ***4 is always at the beginning.***

S: ***Each time the number over to the right of the equal sign is two more.***

S: ***The number in the box is also two more each time.***

If no one volunteers any information, direct the students' attention to what part of the equation stays the same each time, what numbers increase, and how the numbers in the boxes are affected.

Repeat this activity using these sequences of problems.

$10 + \square = 15$	$10 = 3 + \square$	$50 + 50 = \triangle$
$10 + \square = 20$	$10 = 4 + \square$	$50 + \triangle = 101$
$10 + \square = 25$	$10 = 5 + \square$	$50 + \triangle = 105$
$10 + \square = 30$	$10 = 6 + \square$	$50 + 57 = \triangle$

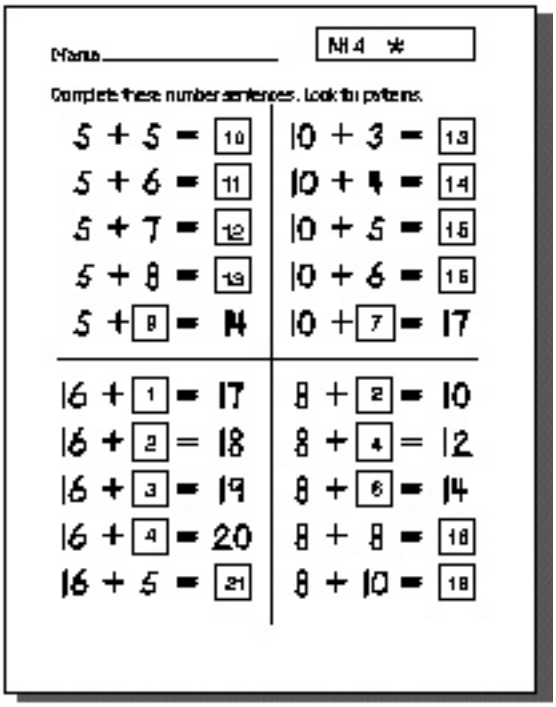
Note: You should vary the difficulty of the problems to fit the abilities of your students. But whatever sequences you make up, encourage students to begin to seek answers to a problem by using information from the previous problems.

For example, if students know that 25 + 25 = ~~50~~ $\square$, they can use this fact to figure out that 25 + 26 = ~~51~~ $\square$ and 25 + ~~27~~ $\square$ = 52, and so on.

Worksheets N14*, **, ***, and **** are available for independent work. You may prefer to allow students to work in pairs on the worksheets. As a follow-up activity, the students can make up problems of their own using just one frame. With a partner, one student can make up a problem for the other student to solve.

Writing Activity

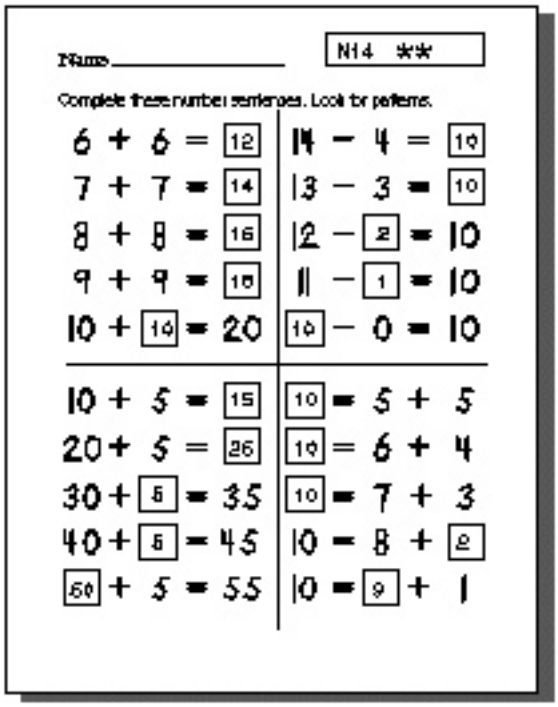
Ask students to write a story problem to go with an open sentence.



Name _____ N14 *

Complete these number sentences. Look for patterns.

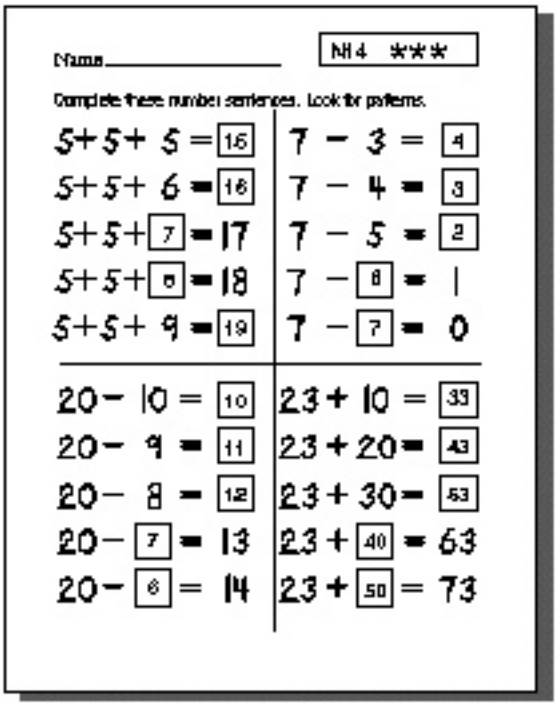
5 + 5 = 10	10 + 3 = 13
5 + 6 = 11	10 + 4 = 14
5 + 7 = 12	10 + 5 = 15
5 + 8 = 13	10 + 6 = 16
5 + 9 = 14	10 + 7 = 17
16 + 1 = 17	8 + 2 = 10
16 + 2 = 18	8 + 4 = 12
16 + 3 = 19	8 + 6 = 14
16 + 4 = 20	8 + 8 = 16
16 + 5 = 21	8 + 10 = 18



Name _____ N14 **

Complete these number sentences. Look for patterns.

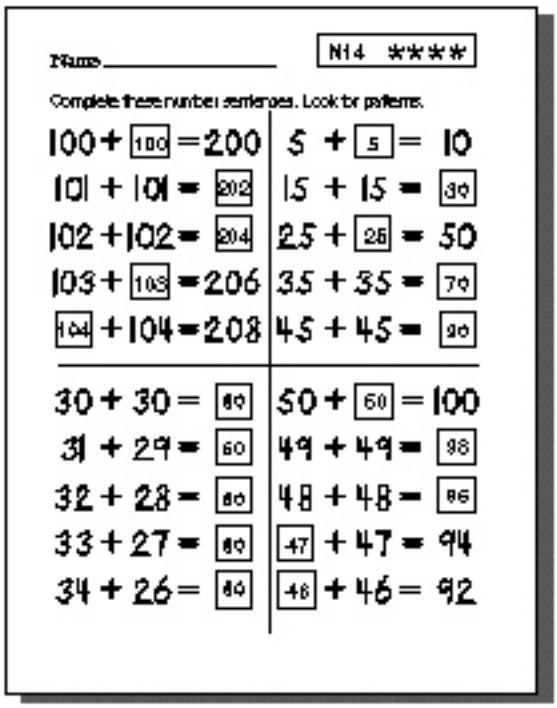
6 + 6 = 12	14 − 4 = 10
7 + 7 = 14	13 − 3 = 10
8 + 8 = 16	12 − 2 = 10
9 + 9 = 18	11 − 1 = 10
10 + 10 = 20	10 − 0 = 10
10 + 5 = 15	10 = 5 + 5
20 + 5 = 25	10 = 6 + 4
30 + 5 = 35	10 = 7 + 3
40 + 5 = 45	10 = 8 + 2
50 + 5 = 55	10 = 9 + 1



Name _____ N14 ***

Complete these number sentences. Look for patterns.

5+5+ 5 = 15	7 − 3 = 4
5+5+ 6 = 16	7 − 4 = 3
5+5+ 7 = 17	7 − 5 = 2
5+5+ 8 = 18	7 − 6 = 1
5+5+ 9 = 19	7 − 7 = 0
20− 10 = 10	23 + 10 = 33
20− 9 = 11	23 + 20 = 43
20− 8 = 12	23 + 30 = 53
20− 7 = 13	23 + 40 = 63
20− 6 = 14	23 + 50 = 73



Name _____ N14 ****

Complete these number sentences. Look for patterns.

100+ 100 =200	5 + 5 = 10
101 + 101 = 202	15 + 15 = 30
102 +102= 204	25 + 25 = 50
103+ 103 =206	35 + 35 = 70
104 +104=208	45 + 45 = 90
30 + 30 = 60	50 + 50 = 100
31 + 29 = 60	49 + 49 = 98
32 + 28 = 60	48 + 48 = 96
33 + 27 = 60	47 + 47 = 94
34 + 26 = 60	46 + 46 = 92

N15 MULTIPLICATION AND SUBTRACTION #2

Capsule Lesson Summary

Draw a picture with 5 x 3 dots and one with 3 x 5 dots. Conclude that 5 x 3 = 3 x 5. Do some subtraction problems using backward trades on the Minicomputer.

Materials

Teacher
- Minicomputer set

Student
- Paper
- Minicomputer set
- Objects such as counters (optional)

Description of Lesson

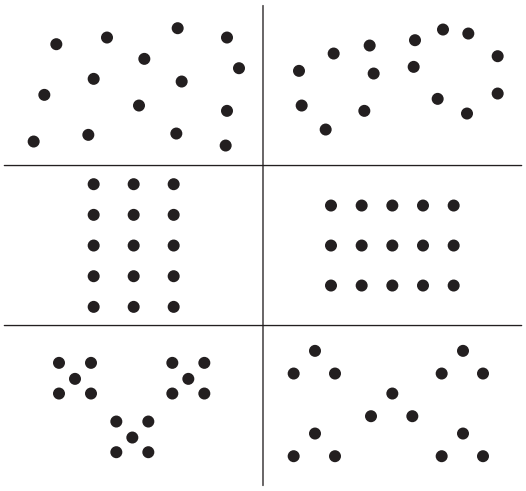
Exercise 1: Multiplication Facts

Ask the students to draw a picture with 5 x 3 dots and a picture with 3 x 5 dots. For some students, you may prefer to give them objects (counters) to show 3 x 5 and 5 x 3. Let them solve this problem in their own ways, watch how the pictures are drawn, and ask some students to draw or tape one of their pictures on the board. Students using objects can draw dots for their objects on the board, or you can draw the picture with dots arranged like the objects are arranged. There should be at least one picture organized in rows and columns (5 rows and 3 columns, or 3 rows and 5 columns) put on the board.

If some of your students do not understand what 3 x 5 means, ask someone to explain. You may want to write 3 x 5 = 5 + 5 + 5 and 5 x 3 = 3 + 3 + 3 + 3 + 3 on the board. Do not expect all of your students to have this understanding of multiplication. There will be more lessons on multiplication later.

Optional: Discuss with the class how to show 3 x 5 (or 5 x 3) using children in the classroom. Accept many suggestions and choose one that puts children in three groups of five children each (for example, three rows of five children or three circles of five children). Call up five children at a time, three times.

These are some examples of what students might draw.



With your class, count the dots in each of the pictures drawn on the board. Wherever possible, count by fives or by threes.

Ask the students which pictures they like best and why.

Do not criticize the pictures in which the dots are not drawn in rows. They might be interesting from an aesthetic point of view. Be happy if there are 15 dots!

T: ***How many dots are in your picture with 5 x 3 dots?*** (15)
How many dots are in your picture with 3 x 5 dots? (15)
Are there any pictures on the board where we can easily see that 5 x 3 = 3 x 5 without counting all the dots?

Any picture organized in rows and columns (5 rows and 3 columns, or 3 rows and 5 columns) illustrates nicely that 5 x 3 = 3 x 5, but actually any picture with 15 dots can be used to show this.

Repeat Exercise 1 for 4 x 6 and 6 x 4.

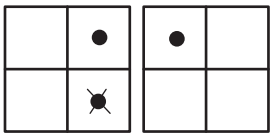
Exercise 2: Subtraction Problems

Display two Minicomputer boards.

T: ***Duane has 58 marbles. He gave 10 of them to his friend Max. How many marbles are left?***

Some students will be able to answer immediately. The Minicomputer will help others find the solution almost as quickly.

T: ***Can someone put 58 on the Minicomputer?***
Who can take 10 off the Minicomputer?



Let a student remove the appropriate checker from the Minicomputer.

T: ***How many marbles are left?*** (48)

Read the number sentence as you write it on the board.

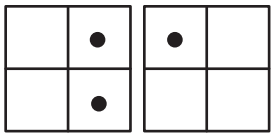
58 – 10 = 48

T: ***But what if Duane had given Max 20 marbles . . .?***

Write this number sentence on the board directly under 58 – 10 = 48.

58 – 20 = ?

Invite someone to put 58 on the Minicomputer.

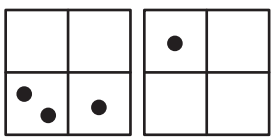


T: ***Can we take 20 off the Minicomputer?***

S: ***We do not have a checker on the 20-square.***

T: ***Can we make a trade so that we will have a checker on the 20-square?***

S: ***Yes, make a backward trade. 40 = 20 + 20.***

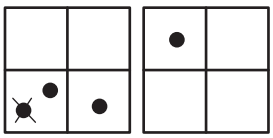


T: ***Now it is easy to take 20 off the Minicomputer.***

Let a student take a checker off the 20-square.

T: ***What number is left on the Minicomputer?***

S: ***38.***



Complete the number sentence on the board.

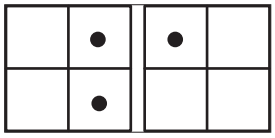
58 – 20 = 38

T: ***Let's suppose Max needed 24 marbles and that Duane agreed to give him that many. How many marbles would Duane have left?***

Write this number sentence on the board under 58 – 20 = 38.

58 – 24 = ?

Invite someone to put 58 on the Minicomputer.

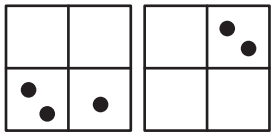


T: ***Can someone take 24 off the Minicomputer?***

S: ***We do not have a checker on the 20-square nor on the 4-square.***

S: ***We could make some backward trades.***

Invite students to make the required trades.

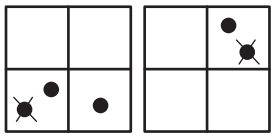


S: ***40 = 20 + 20.***

S: ***8 = 4 + 4.***

T: ***Now it is easy to take 24 off the Minicomputer.***

Invite a student to take the appropriate checkers off the Minicomputer.



T: ***What number did we get?***

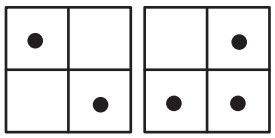
S: ***34.***

Complete the number sentence on the board.
You should have this list.

58 – 10 = 48
58 – 20 = 38
58 – 24 = 34

For the rest of this exercise, each student (or pair of students) will need one individual Minicomputer sheet (two boards) and several positive checkers.

T: ***Put 97 on your (desk) Minicomputer.***

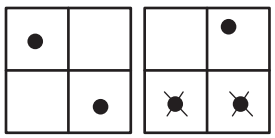


Invite someone to put 97 on the demonstration Minicomputer.

Write this number sentence on the board near the Minicomputer.

97 – 3 = ?

T: ***Calculate 97 – 3 on your Minicomputer.***



Ask a student to do the calculation on the demonstration Minicomputer. Conclude that 97 – 3 = 94 and complete the number sentence on the board.

97 – 3 = 94

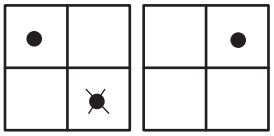
T: ***You now have 94 on your Minicomputer.***

Occasionally check to make sure that the students have the correct number on their Minicomputers.

N15

Write this number sentence on the board directly under 97 – 3 = 94.

94 – 10 = ?



T: ***Calculate 94 – 10 on your Minicomputer.***

Ask a volunteer to do the subtraction problem on the demonstration Minicomputer. Conclude that 94 – 10 = 84 and complete the number sentence on the board.

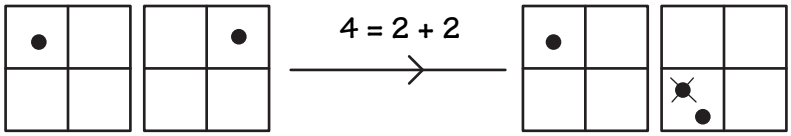
T: ***You now have 84 on your Minicomputer.***

94 – 10 = 84

Write this number sentence on the board under 94 – 10 = 84.

84 – 2 = ?

Ask the students to calculate 84 – 2. A backward trade is required before a checker can be removed from the 2-square.



Conclude that 84 – 2 = 82. Complete the number sentence on the board. You should have this list.

97 – 3 = 94
94 – 10 = 84
84 – 2 = 82

Continue in the same manner with one or more of the following sequences of subtraction calculations.

22 – 10 = 12
12 – 1 = 11
11 – 11 = 0

63 – 22 = 41
41 – 20 = 21
21 – 11 = 10

82 – 40 = 42
42 – 20 = 22
22 – 10 = 12

59 – 48 = 11
11 – 8 = 3

Home Activity

Send home a page of subtraction problems (approximately ten to fifteen). Students can work with their parents/guardians and show them how to use the Minicomputer to solve the problems.

Capsule Lesson Summary

Build an arrow road from one number to another using two types of arrows.

Materials

Teacher	• Colored chalk	Student	• Colored pencils, pens, or crayons • Unlined paper • Unifix® cubes

Description of Lesson

Exercise 1

Draw two dots well spaced on the board. Label one of the dots 6 and the other 11.

T: ***Today we are going to build arrow roads. Let's begin by building a road from 6 to 11 using blue arrows for +2 and red arrows for –1.***

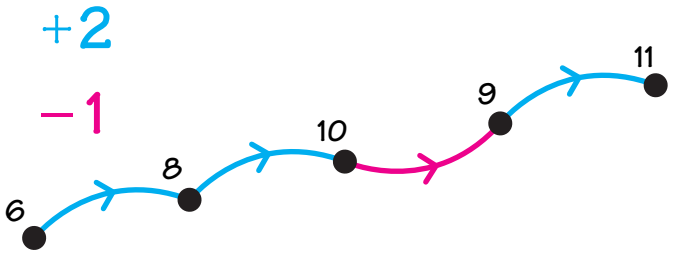
Write +2 in blue and –1 in red on the board. You might refer to these as the "key" or "color code" for this road.

T: ***Which kind of arrow would you like to start with?***

Take whatever suggestion is made (for example, +2) and draw the first arrow yourself. Point to the ending dot of this arrow.

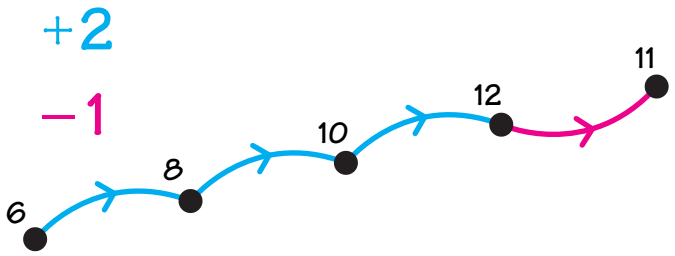
T: ***Which number is here?*** (8, if the arrow is blue; 5, if the arrow is red)

Ask students to complete the road one arrow at a time. One of the many possible roads is shown below.



Note: If a student is concentrating on a color pattern for the arrows, you might encourage them to keep in mind what the target number is. In this particular example, alternating +2 and –1 arrows will build a road from 6 to 11 using seven arrows (four +2 and three –1) or ten arrows (five –1 and five +2). There is nothing wrong with such a road, but you might ask if it is possible to build a road using fewer arrows. In this case, there is a shorter road as illustrated.

There is no rule against "overshooting" the target number. For example, here is a perfectly acceptable road.



Construct two or three different roads from 6 to 11 with suggestions from the class. The purpose of this collective exercise is to show what an arrow road is and that many different solutions are possible.

Exercise 2

Distribute paper and colored pencils and put the following information on the board.

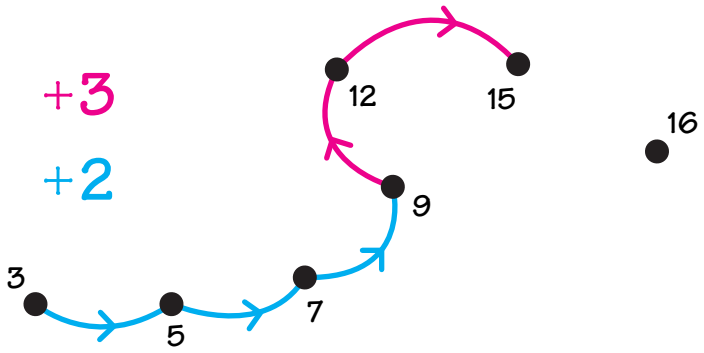
Ask the students to copy what is on the board and to build a road from 6 to 21 using +2 and +1 arrows. Students who draw cramped pictures should be encouraged to use the whole sheet of paper.

This might be a good problem for students to work with a partner. To support the activity with a manipulative, give the student partners Unifix® cubes grouped in ones (red cubes) and twos (blue cubes). Start with a tower of six cubes. One student then either adds one red cube or two blue cubes to the tower while the partner records the addition with a corresponding arrow. The end product should be a tower of 21 cubes and an arrow road from 6 to 21.

As the rest of the class continues to work, ask a couple of students with different numbers of arrows in their roads to copy their pictures on the board. When most of the students have finished, direct the class's attention to the examples on the board. Check each picture on the board with the class to see that it is correct. Point out that there are many correct solutions to this problem. Afterward, ask which road on the board has the fewest arrows and determine if anyone has a road with even fewer arrows.

Repeat this activity building a road from 3 to 16 using +3 and +2 arrows.

Note: A student might draw this road and discover that one can not reach 16 from 15 using only +3 and +2 arrows.



In such a case, you may need to suggest changing the color for the last arrow. In the preceding picture, changing the arrow starting at 12 from red (+3) to blue (+2) would enable the student to complete a road with one additional blue arrow.

If there is time remaining, the students can build one or more of the following:

1) Build an arrow road from 17 to 30 using +1 (red) and +2 (blue) arrows.
2) Build an arrow road from 8 to 19 using +3 (red) and –2 (blue) arrows.
3) Build an arrow road from 10 back to 10 using +3 (red) and –2 (blue) arrows.
4) Build an arrow road from 27 back to 27 using +3 (red) and –2 (blue) arrows.

Building these arrow roads can be made into a game for partners. For example, in Road 1 the students start by drawing and labeling a dot for 17. Then they take turns choosing a +1 or +2 arrow to put into the arrow road picture. Each time a student draws an arrow he or she also labels the ending dot. The next arrow, of course, must start at that ending dot. The student who puts in the arrow that ends at 30 (the target number) wins.

Home Activity

This would be a good time to send a letter home about the language of arrows. Blackline N16 has a sample letter. Students may like to teach their parents/guardians the road building game to solve problems such as the following:

1) Build an arrow road from 7 to 26 using +2 (red) and +3 (blue) arrows.
2) Build an arrow road from 13 to 30 using +3 (red) and –1 (blue) arrows.
3) Build an arrow road from 14 back to 14 using +3 (red) and –4 (blue) arrows.

N17 ADDITION PROBLEMS #3

Capsule Lesson Summary

Review facts for 10, 20, and 30 in a mental arithmetic activity. Pair numbers whose sum is 20 to solve an addition problem. Put 20 on the Minicomputer in a variety of configurations looking at number sentences for some representations.

Materials

Teacher
- Colored chalk
- Coins
- Minicomputer set
- Calculator
- Blackline N17 (a)

Student
- Paper
- Minicomputer set
- Minicomputer paper
- Worksheets N17*, **, ***, and ****

Advance Preparation: Use Blackline N17 (a) to make Minicomputer paper for students.

Description of Lesson

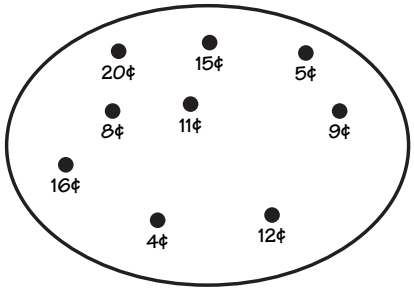
Exercise 1

Conduct a mental arithmetic activity involving facts for 10. Continue with some of the following sequences of problems. Write the calculations on the board as you say them and ask for patterns.

8 + 2	4 + 6	7 + 3
18 + 2	14 + 6	17 + 3
8 + 12	4 + 16	7 + 13
28 + 2	24 + 6	27 + 13
8 + 22	4 + 26	37 + 13

Exercise 2: Addition Problem

Tell this story or a similar one. Choose one of your students to be the star of the story. Draw this string picture on the board, leaving some space above it to write number sentences.



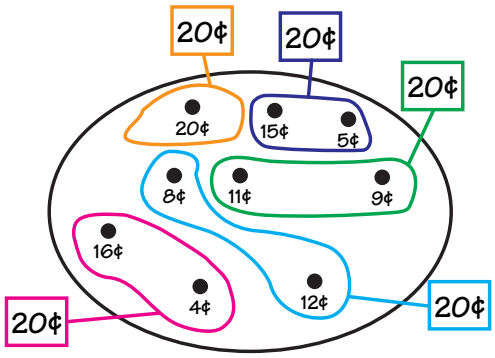
T: ***Wanda had a party, and she bought each of her guests a balloon. She went to a store which had balloons in many shapes: cat balloons, dog balloons, rabbit balloons, and many others. The dots in this string are for the prices of the balloons that Wanda bought. How many balloons did Wanda buy?*** (9)
How much did Wanda spend on balloons?

Let the students think about the problem and suggest ways of solving it. Do not expect anyone to give an exact solution now. Encourage students to estimate how much Wanda spent. Commend any student who suggests pairing two numbers whose sum is 20 to make the addition easier.

T: ***Wanda looked at these numbers a long time, and then she found an easy way to add them. She paired numbers whose sum is 20. Does anyone see two numbers whose sum is 20?***

S: ***I see 5 and 15; 5 + 15 = 20.***

Allow the students to make suggestions. Whenever a pair of numbers whose sum is 20 is offered, draw a string around those two prices. Occasionally, you can use coins to display two prices being paired. Then put the coins together and check that the paired amount is 20¢. You may want to ask students to record the facts for 20 on their papers. Ask if the number 20 needs to be paired with another number.



T: ***How can we use this string picture to find what Wanda spent?***

S: ***Add 20 + 20 + 20 + 20 + 20.***

Trace the appropriate string as you write each number in the addition calculation on the board.

T: ***Does anyone know another calculation we could write?***

S: ***5 x 20.***

You may need to mention the multiplication calculation yourself. Record 5 x 20 below 20 + 20 + 20 + 20 + 20 on the board.

T: ***How many cents did Wanda spend? How do you know?***

S: ***100¢, because 20 + 20 + 20 + 20 + 20 = 100.***

Accept any reasonable explanation and then check the answer by counting by twenties. Afterward, calculate 5 x 2 on the Minicomputer or with a calculator. Complete the number sentences on the board.

T: ***Wanda spent 100 cents. How many dollars is 100 cents?***

S: ***One dollar.***

T: ***Who can write $1.00 on the board?***

Your class may need help with the last two questions.

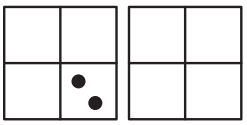
Exercise 3

Pair the students and provide each pair with two individual Minicomputer boards (one sheet) and some checkers. Display two Minicomputer boards.

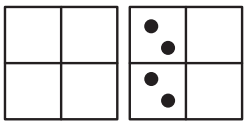
Ask the student pairs to put 20 on their desk Minicomputers in a variety of configurations. Observe the students working and help those having difficulty. You may like to require students to record the configurations for 20 they find on the Minicomputer paper.

Choose several students to put their configurations for 20 on the demonstration Minicomputer. Each time a configuration is displayed, call the class's attention to it and ask for an appropriate number sentence.

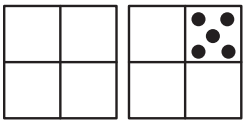
These are some of the possibilities and several of the number sentences corresponding to each.



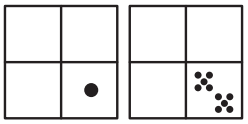
10 + 10 = 20
or
2 x 10 = 20



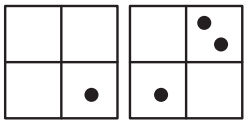
(8 + 2) + (8 + 2) = 20
or
(8 + 8) + (2 + 2) = 20
or
(2 x 8) + (2 x 2) = 20



5 x 4 = 20
or
4 + 4 + 4 + 4 + 4 = 20



10 + 10 = 20
or
10 + (10 x 1) = 20



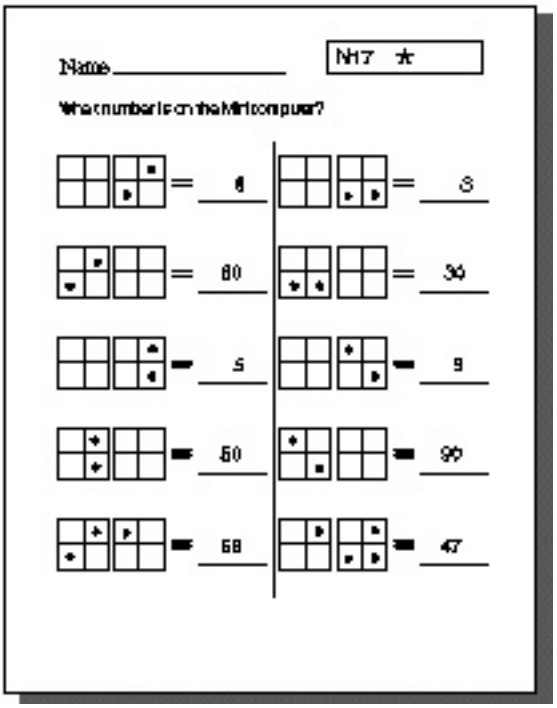
10 + 4 + 4 + 2 = 20
or
10 + (2 x 4) + 2 = 20
or
10 + 6 + 4 = 20

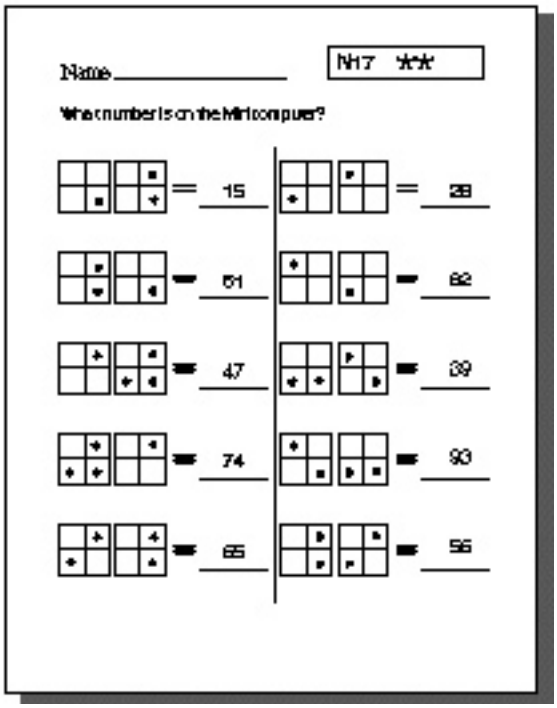
Assessment Activity

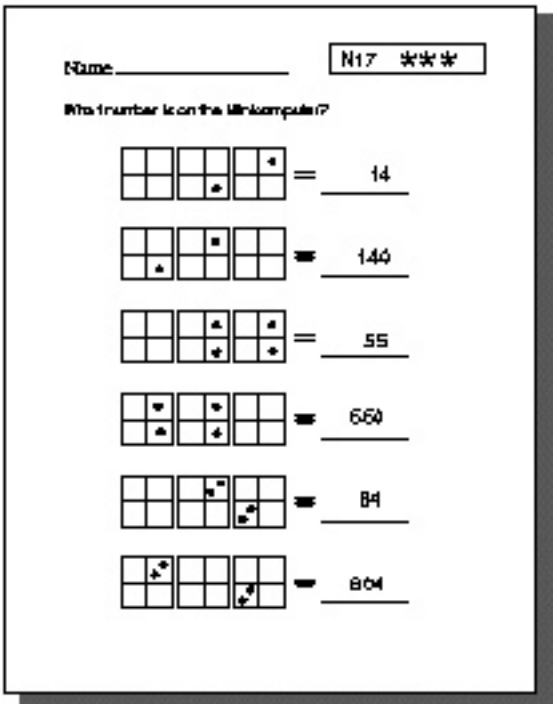
Worksheets N17*, **, ***, and **** are available for assessing students' ability to read two- and three-digit numbers on the Minicomputer. Students may want to use individual Minicomputers.

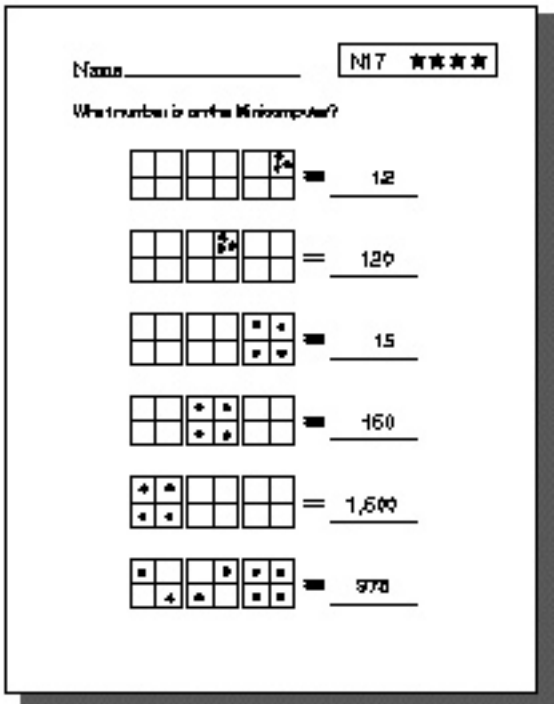
Home Activity

Suggest that students use their home Minicomputers to try to find several ways to put on numbers such as 40, 50, or 100.


Name
N17 ★
Who number is on the Mini computer?
= 6
= 8
= 80
= 36
= 5
= 9
= 50
= 99
= 68
= 47


Name
N17 ★★★
Who number is on the Mini computer?
= 15
= 28
= 61
= 82
= 47
= 39
= 74
= 93
= 65
= 56


Name
N17 ★★★
Who number is on the Mini computer?
= 14
= 140
= 55
= 660
= 84
= 804


Name
N17 ★★★★
Who number is on the Mini computer?
= 12
= 120
= 15
= 160
= 1,600
= 978

Capsule Lesson Summary

Discuss 2x as adding a number to itself. Use 2x in mental arithmetic and arrow pictures. Note that the only dot with a 2x loop is the number 0.

Materials

Teacher	• Colored chalk	Student	• Paper • Colored pencils, pens, or crayons • Worksheets N18*, **, and ***

Description of Lesson

Exercise 1

The following dialogue assumes there are 23 students present. For your lesson, use the number of students present in your class.

T: ***Everyone raise one hand. How many hands are raised?***

S: ***23.***

T: ***Now, everyone raise both hands. How many hands are up now?***

S: ***46, because 23 + 23 = 46.***

T: ***23 + 23 = 46. That is the same as saying that 2 x 23 = 46.***

Continue with a mental arithmetic activity involving 2x. Below are some suggested sequences of problems. Write the calculations on the board as you say them and ask for patterns.

2 x 5	2 x 3	2 x 2	2 x 6
2 x 10	2 x 30	2 x 22	2 x 10
2 x 20	2 x 300	2 x 202	2 x 16
2 x 40	2 x 3,000	2 x 220	2 x 26
2 x 80	2 x 3,000,000	2 x 222	2 x 36

Comments:

- You may wish to show the class a couple more concrete examples of adding a number to itself and 2x the number. For example:

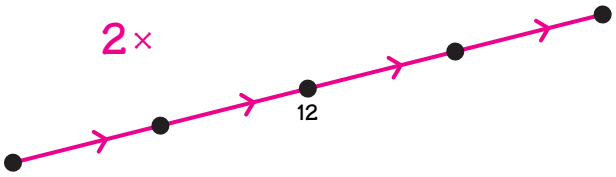
  How many fingers on one hand? (5)
  How many fingers on two hands? (10)
  5 + 5 = 10 and 2 x 5 = 10.

- If students have difficulty with 2 x 20, ask for 2 x 2 and then return to 2 x 20. Repeat this strategy as often as necessary.
- If students have difficulty with 2 x 16, ask again for 2 x 10 and 2 x 6, and then return to 2 x 16. Repeat this strategy as often as necessary.

N18

Exercise 2

Draw this arrow picture on the board and ask the students to copy it on their papers.



T: ***What is a red arrow for?***

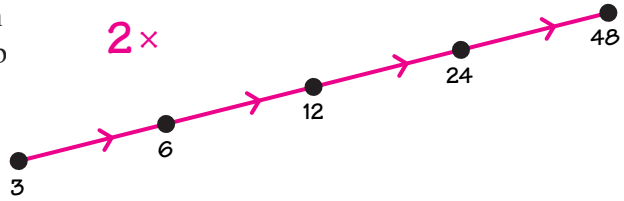
S: ***2x.***

T: ***Where is the least number? Guess what the least number is and write it on your paper. Where is the greatest number? Estimate the greatest number and write it on your paper.***

T: ***How can we find out what the least number is and the greatest number is?***

S: ***Label the dots.***

Invite students to label the dots. Occasionally ask students how they know a dot is for a certain number. Continue until all the dots have been lab



T: ***Suppose we extend this picture with another 2x arrow starting at 48. Where would it end? Can you estimate 2 x 48?***

Let students discuss how to find 2 x 48 and encourage estimation. For example, 2 x 50 = 100 so 2 x 48 is less than 100. If appropriate, include the use of the Minicomputer or a calculator in the discussion. Some students may suggest that 2 x 40 = 80, and 2 x 8 = 16, so 2 x 48 = 96. Leave the arrow picture from Exercise 2 on the board.

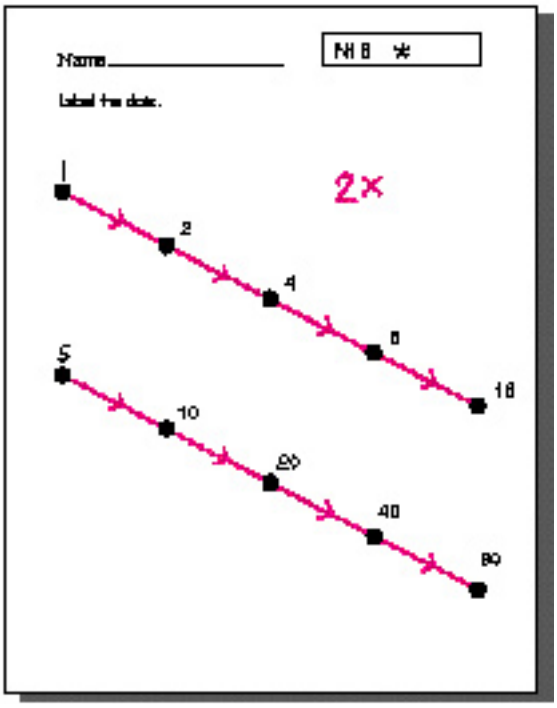
Exercise 3

Draw this loop on the board.

T: ***What number is here*** (point to the dot)***?***

Give the students time to think about this situation. If no one discovers that the number is 0, leave the arrow picture on the board while the students do the worksheets.

Worksheets N18*, **, and *** are available for work by individuals or math partners. Provide Minicomputers, counters, or calculators for those who would like to use them to check their work.


Name
N18 *
2×
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2
4
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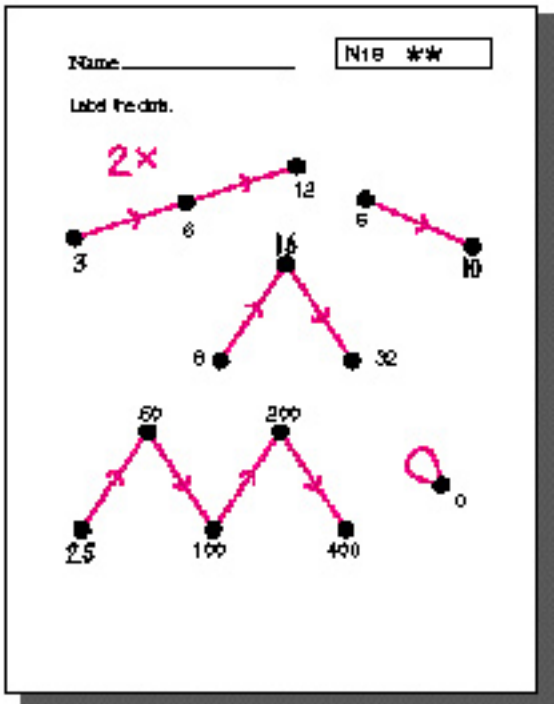
10
20
40
80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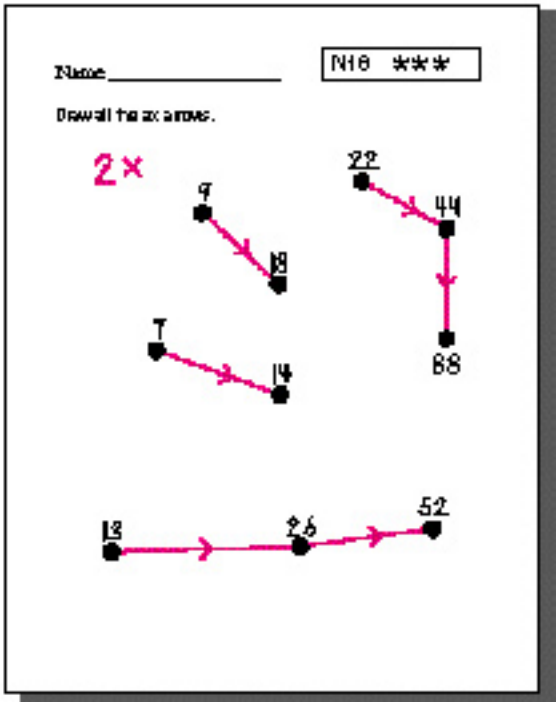
Name
N18 ***
2×
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22
44
88
13
26
52

N19 ADDITION PROBLEMS #4

Capsule Lesson Summary

Review facts for 10, 20, and 30 in a mental arithmetic activity. Pair numbers whose sum is 30 to solve an addition problem. Investigate the effect of moving checkers one board to the left on the Minicomputer.

Materials

Teacher	• Minicomputer set • Colored chalk • Coins (optional) • Base-10 blocks	• Minicomputer set • Calculator (optional)
Student	• Paper and pencil	

Description of Materials

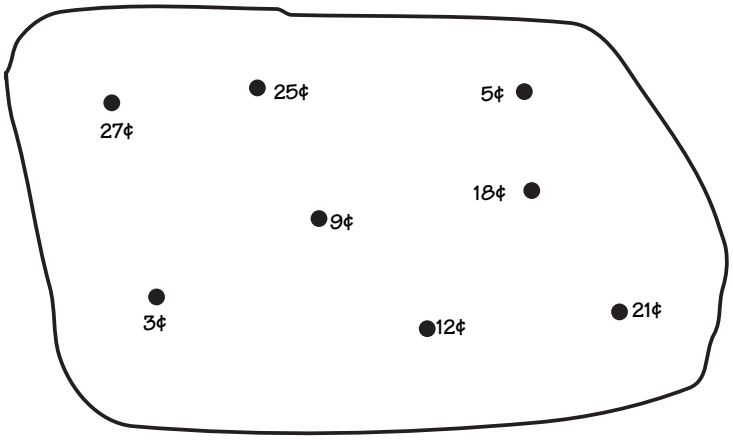
Exercise 1: Mental Arithmetic

Conduct a mental arithmetic activity involving facts for 10. Continue with some of the following sequences of problems. Write the calculations on the board as you say them and ask for patterns.

9 + 1	6 + 4	5 + 5
19 + 1	16 + 4	15 + 5
9 + 11	6 + 14	25 + 5
19 + 11	16 + 14	35 + 5
29 + 1	26 + 4	15 + 15
9 + 21	6 + 24	25 + 15

Exercise 2: Addition Problem

Draw this string picture on the board, leaving space above it to write number sentences.



T: ***Rasheed's club had a bake sale last Saturday.***

The class may want to discuss why Rasheed's club might have had a bake sale and what might have been sold.

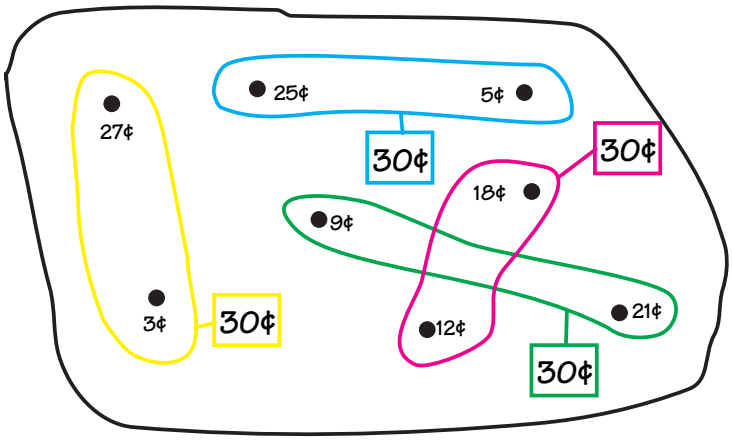
T: ***Rasheed made some chocolate chip cookies for the bake sale and his mother made some brownies. Rasheed worked at the bake sale making change and wrapping the food. Before Rasheed went home, he bought some things for his family. Each of these dots represents the price in cents of one of the baked goods Rasheed bought. How many items did Rasheed buy?*** (8) ***How much did Rasheed spend at the bake sale?***

Let the students think about the problem and suggest ways of solving it, but do not expect an exact solution now. Encourage students to estimate how much Rasheed spent. Commend a suggestion to pair two numbers whose sum is 30.

T: ***Rasheed looked at these numbers a long time before he saw an easy way to add them. He paired numbers whose sum is 30. Does anyone see two numbers whose sum is 30?***

S: ***27 + 3 = 30.***

Allow the students to make suggestions. Whenever a pair of numbers whose sum is 30 is suggested, draw a string around those two prices. Occasionally, you may like to use actual coins to display two prices being paired. Then put the coins together and check that the paired amount is 30¢. Continue until all the numbers in the string picture are paired. Ask students to record the facts for 30 on paper.



T: ***How can we use this string picture to find what Rasheed spent?***

S: ***Add 30 + 30 + 30 + 30.***

Trace the appropriate string as you write each number in the addition calculation on the board.

T: ***Does anyone know another calculation we could write?***

S: ***4 x 30.***

You may need to mention the multiplication calculation yourself. Record 4 x 30 below 30 + 30 + 30 + 30.

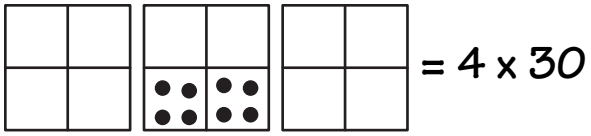
T: ***How many cents did Rasheed spend at the bake sale?***

Ask any students who respond correctly to explain how they calculated the answer.

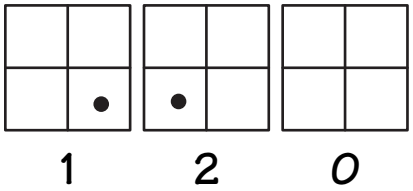
T: ***Let's calculate 4 x 30 on the Minicomputer.***
How many times should we put 30 on the Minicomputer?

S: ***4.***

Display three Minicomputer boards and ask for a volunteer to put 4 x 30 on the Minicomputer, or call on four volunteers each to put on 30.



Let students make trades until the standard configuration for 120 is on the Minicomputer.



Ask a student to write the number below the Minicomputer and ask another to complete the number sentences. Students may also like to check the calculations with a calculator.

T: ***Who can read these number sentences?***

S: ***30 + 30 + 30 + 30 = 120.***
4 x 30 = 120.

T: ***Rasheed spent 120 cents at the bake sale. Does anyone know another way to say how much money 120 cents is?***

S: ***One dollar and twenty cents.***

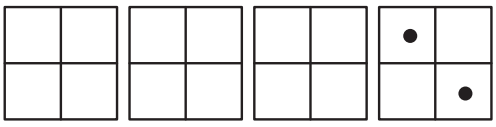
T: ***Who can write $1.20 on the board?***

The class may need help with the last two questions. You may also like to ask students to say how Rasheed could pay $1.20 (what coins and/or bills).

Exercise 3: Multiplying by 10

In this exercise, you may like to model each number on the Minicomputer with base-10 blocks (or other place-value manipulative) to emphasize the effect of multiplying by 10.

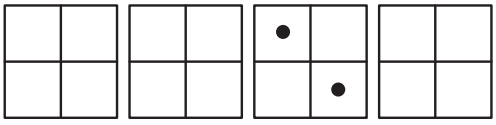
Put this configuration of checkers on the Minicomputer.



T: ***What number is this?***

S: ***9.***

Move the checkers from the ones board to the corresponding squares (same position) on the tens board.



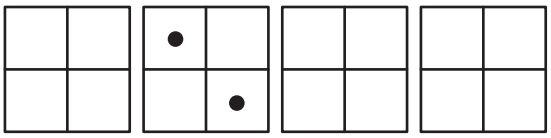
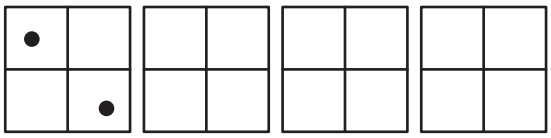
T: ***What number is this?***

S: ***90.***

T: ***Who can write 90 below the Minicomputer?***

N19

Repeat this activity with these configurations.

 and 

Record a list of the numbers you have just seen on the Minicomputer.

9
90
900
9,000

T: ***Do you see a pattern?***

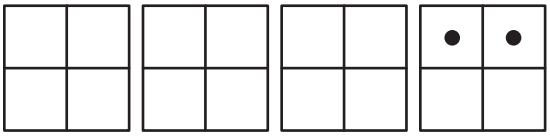
S: ***They all have a 9 in them.***

S: ***The number of zeros is changing.***

T: ***Why does each of these numbers have one more zero than the number before it?***

If a student suggests that there is another zero because all you did was move the checkers to the next board to the left, you should encourage this idea. If none of the students suggests this, continue the lesson.

Put this configuration on the Minicomputer.



T: ***What number is this?***

S: ***12.***

T: ***Do you know another way to put 12 on the Minicomputer using exactly two checkers?***

S: ***A checker on the 10-square and a checker on the 2-square.***

Let a student point to the squares where the checkers would be.

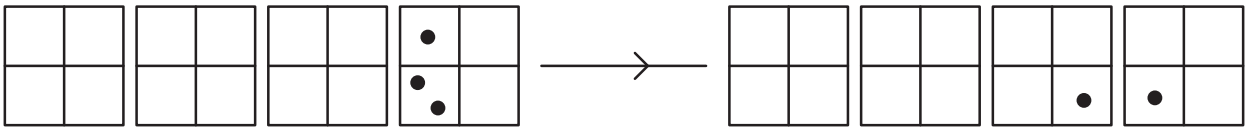
T: ***Could we make some trades which will result in a checker on the tens board.***

S: ***An 8 + 2 = 10 trade would put a checker on the tens board.***

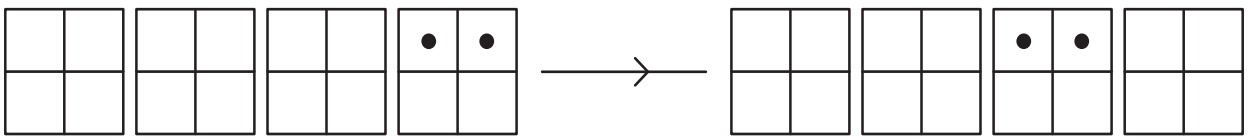
T: ***We have a checker on the 8-square. How can we get a checker on the 2-square?***

S: ***We could make a 4 = 2 + 2 trade.***

Invite students to make a 4 = 2 + 2 trade and then an 8 + 2 = 10 trade.



Return the checkers to the starting configuration. Then move the checkers from the ones board to the corresponding squares on the tens board.

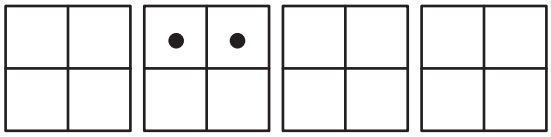


T: ***What number is this?***

S: ***120.***

T: ***Do you know another way to put 120 on the Minicomputer using exactly two checkers?***

Allow the students to make trades until 120 is in standard configuration. Ask a student to write 120 below the Minicomputer. Return the checkers to the 80-square and 40-square, and then move the checkers from the tens board to the corresponding squares on the hundreds board



T: ***What number is this?*** (1,200)

If no one knows that this number is 1,200, continue the lesson.

T: ***Who can make a trade which will make the number easier to read?***

Continue until 1,200 is in standard configuration. Ask a student to write the number below the Minicomputer. Return the checkers to the 800-square and the 400-square. Record a list of the numbers you have just seen on the Minicomputer.

T: ***Do you see a pattern?***

S: ***They all start with a 1.***

S: ***They all have a 2 in them.***

S: ***They all start with a 12.***

S: ***The number of zeros is changing.***

12
120
1,200

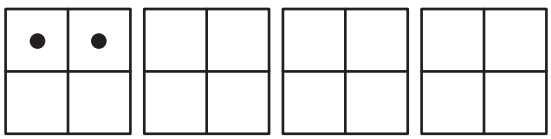
T: ***Do you know what the next number in this list would be?***

S: ***12,000.***

If no one suggests 12,000, ask if anyone knows how many zeros the next number has. After it is written on the board, say the number yourself.

T: ***Can anyone move the checkers so that 12,000 is on the Minicomputer?***

Students should suggest the following configuration or the need for a fifth board in order to put on the standard configuration.



Home Activity

This is a good time to send a letter to parents/guardians about mental arithmetic. Blackline N19 is a sample letter.

N20 BECOMING ACQUAINTED WITH 37

Capsule Lesson Summary

Get acquainted with the number 37 in a variety of situations—with base-10 blocks, with coins, on the Minicomputer, in an arrow picture, and in a string picture. Discover that 37 is not in a +2, +3, or +5 spiral that starts at 0.

Materials

Teacher
- Minicomputer set
- Colored chalk

Student
- Base-10 blocks
- Coins
- Minicomputer set
- Worksheet N20
- Calculator

Description of Lesson

The goal of this lesson is to become more familiar with the number 37. Why choose 37? you and your students may ask. The answer is that 37 is an interesting number (as are all numbers) and this is a chance to get to know it a little better. But feel free to select another number of interest to you or your students.

For most of this lesson, allow students to work with a math partner.

Exercise 1

T: ***"I am neither very small, nor very large," says the number 37. "Nevertheless, I am more than 28."***

Let students suggest other numbers 37 is more than. Occasionally locate the numbers on the number line and check the comparison.

S: ***37 is more than 17.***

S: ***37 is more than 31.***

Let students suggest numbers 37 is less than.

S: ***37 is less than 53.***

S: ***37 is less than 39.***

T: ***"I like to look at myself in lots of ways," says 37 proudly. Can we show 37 with base-10 blocks?***

Invite student partners to display 37 with base-10 blocks. With this display, read 37 as 3 tens and 7 ones or 30 + 7.

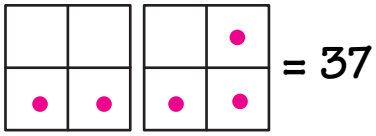
T: ***Can we show 37 with coins; that is, 37¢?***

Call on student partners to suggest several different ways to have 37¢ with coins.

N20

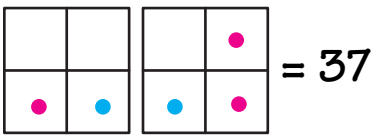
T: ***"I would like to be on the Minicomputer," says 37.***

Let a student put 37 on the Minicomputer with red checkers.



T: ***"Thank you," says 37. "But it is a little sad to be displayed in only one color."***

A student might suggest this configuration.
(There are many other possibilities.)



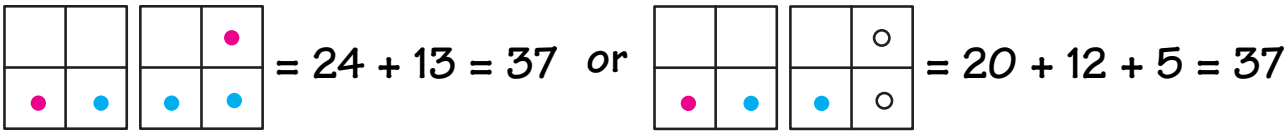
T: ***"I am happier now. Look at me and give me a new name," says 37.***

What number is in red? (25) ***What number is in blue?*** (12)
Together they make 37.

S: ***25 + 12.***

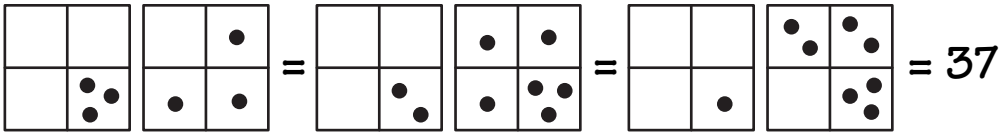
T: ***Can someone suggest another way of looking at 37 using checkers of two or more colors?***

A student might suggest this, for example:



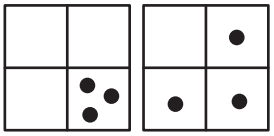
T: ***"I'm getting bored with being on the Minicomputer with only five checkers. Can you put me on the Minicomputer with more checkers?" wonders 37.***

These are some possible configurations for 37 using more than five checkers. Allow math partners to experiment with several possibilities.



T: ***"Isn't it fantastic!" observes 37. "It's always me, but each time I'm dressed differently."***

Ask the students for a number sentence corresponding to one of the configurations put on the Minicomputer. For example:



$(3 \times 10) + 7 = 37$

or

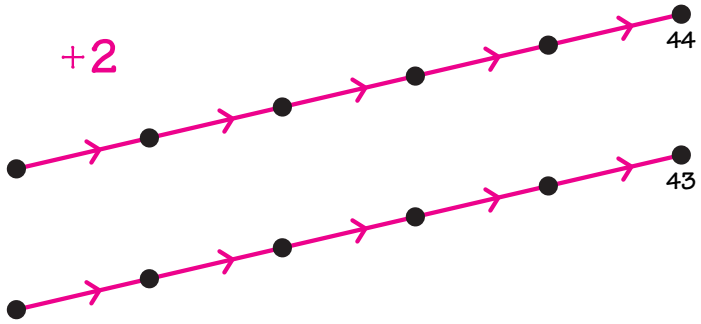
$10 + 10 + 10 + 7 = 37$

or

$(3 \times 10) + 4 + 2 + 1 = 37$

Exercise 2

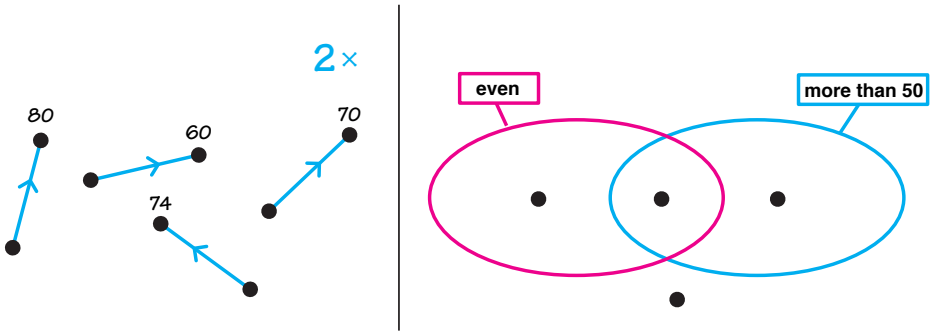
Distribute copies of Worksheet N20 and draw this arrow picture on the board.



T: ***"I am hidden in this picture," claims 37. "I am one of these dots. Can you find me?"***

Ask student partners to find and label the dot for 37 on their worksheets. Then check the responses collectively.

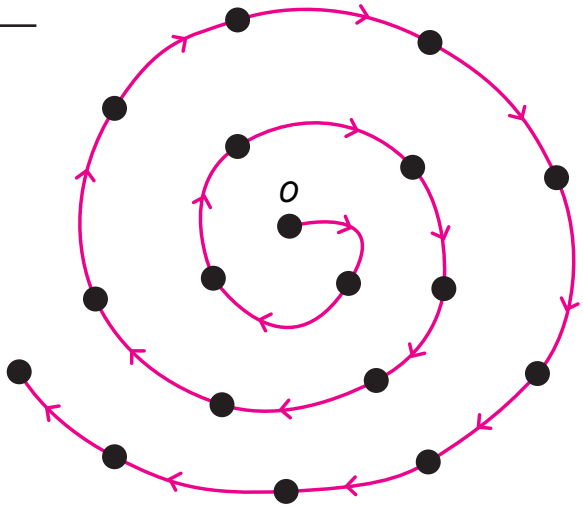
A similar activity can be done with an arrow picture using 2x arrows … or with a string picture.



Let the partners work at their own pace and with a calculator if they like. It is not necessary to solve all of these problems. Allow enough time for Exercise 3.

Exercise 3

Draw this spiral at the board.



T: ***"If the red arrows were for +2, would I be on this spiral?" asks 37.***

Let the students discuss the problem. You may suggest that they write a response to 37 on their papers. Do not allow them to label the dots in the picture.

S: ***Impossible, because 37 is not an even number (or because 37 is an odd number).***

T: ***"If the red arrows were for +3, would I be on this spiral?" asks 37.***

Count by threes starting at 0 with your class: "0, 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39…." You may suggest that students use the calculator to count by threes.

S: ***37 is not one of these dots because we skipped over it.***

T: ***"If the red arrows were for +5, would I be on this spiral?" asks 37.***

After counting by fives with the class, or using a counting calculator, conclude that 37 can not be on the spiral.

End the lesson with this open question for the students to think about by themselves.

T: ***"And if the arrows were for +4, or for +6, or for +7, or for +10, …?" continues 37.***

Writing Activity

Challenge students to use a counting calculator and see if they can find some number to count by (starting at 0) to get 37. Ask students to write about their exploration.

Home Activity

Suggest to parents/guardians that they work with their child to choose a number (perhaps a favorite number) and find lots of facts for this number. Put this number on the Minicomputer in different ways. Check whether you say this number when you start at 0 and count by twos, by threes, by fours, and so on.

N21 MULTIPLICATION AND SUBTRACTION #3

Capsule Lesson Summary

Write number sentences for a 7 x 4 array of dots and note that 7 x 4 = 4 x 7. Do a multiplication problem that is suggested by a particular configuration on the Minicomputer. Solve a sequence of subtraction problems on the Minicomputer.

Materials

Teacher	Student
• Straight edge • Minicomputer set • Blackline N17	• Minicomputer set • Minicomputer paper (optional) • Paper • Counters (optional)

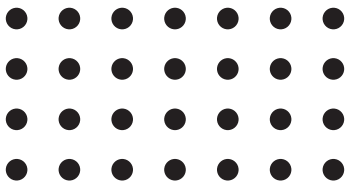
Advance Preparation: Use Blackline N17 to prepare Minicomputer paper for students.

Description of Lesson

Allow students to work in pairs or cooperative groups during this lesson.

Exercise 1

Draw this pattern of dots on the board and instruct student pairs (groups) to copy the pattern on their papers. If you prefer, students can make the pattern with objects such as counters (see Lesson N13).



T: ***Look at this pattern of dots. What do you see?***

Let the students express themselves. Many reactions are possible. If a student says there are four rows with seven dots in each row, ask how many dots there are and express this as follows:

$$7 + 7 + 7 + 7 = 28 \quad \text{or} \quad 4 \times 7 = 28$$

Invite student groups to find other number sentences for 28, as suggested by the pattern, and to write these on their papers. You may suggest other ways to look at the pattern by holding a straight edge in different positions to partition the array of dots (see Lesson N13).

Student groups may use a pencil or ruler to do the same. There are many other possible number sentences. For example:

$$4 + 4 + 4 + 4 + 4 + 4 + 4 = 28$$
$$7 \times 4 = 28$$
$$2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 28$$
$$14 + 14 = 28$$
$$12 + 16 = 28$$

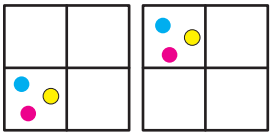
Highlight the multiplication facts related to this picture by writing them on the board.

$4 \times 7 = 28$ $7 \times 4 = 28$ $4 \times 7 = 7 \times 4$

With a sheet of paper, hide some of the dots and ask the students how many dots are hidden. Occasionally ask students how they know the number of hidden dots.

Exercise 2

Display two Minicomputer boards with this configuration of checkers.



T: ***Tell me what you see on the Minicomputer?***
Can you estimate the number on the Minicomputer?

Let the students express their observations. For the moment, do not let them make trades on the Minicomputer.

If not already suggested, ask what number is in red, what number is in blue, and what number is in yellow (28 red, 28 blue, and 28 yellow). Conclude that 3 x 28 is on the Minicomputer and write this expression on the board. Again, solicit estimates for this number.

Next, focus on the tens and ones boards separately, asking what number is in red, in blue, and in yellow on each board. Conclude that (3 x 20) + (3 x 8) is on the Minicomputer and write it under 3 x 28.

3×28
$(3 \times 20) + (3 \times 8)$

T: ***What number is 3 x 20?*** (Point to where this is written on the board.)

S: ***60.***

T: ***What number is 3 x 8?*** (Point to where this is written on the board.)

S: ***24.***

T: ***So what number is (3 x 20) + (3 x 8)?*** (Point to where this is written on the board.)

S: ***60 + 24.***

S: ***84.***

Let the class check the result by making trades on the Minicomputer. Be prepared to suggest backward trades as needed.

Exercise 3: Subtraction Problems

Each group of students will need two individual Minicomputer boards (one sheet) and several positive checkers. Choose a context for a sequence of subtraction problems; for example:

T: ***This morning there were 76 pieces of paper in this box. Put 76 on your Minicomputer.***

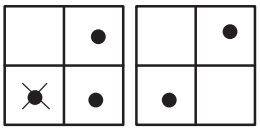
Accept, but do not encourage, non-standard configurations. Ask a student to put 76 on the demonstration Minicomputer.

T: ***Ms. Karl takes 20 pieces for her class.***

Write the subtraction calculation 76 – 20 to one side of the Minicomputer on the chalkboard.

T: ***Calculate 76 – 20 on your Minicomputer.***

Call on a student to take 20 off the demonstration Minicomputer and conclude that 76 – 20 = 56. Then complete the subtraction calculation on the board. Each group should have a recorder or take turns recording the number sentences on their papers. You may also like the groups to record the problems on Minicomputer paper and use X's to cross out the checkers they remove.



76 – 20 = 56

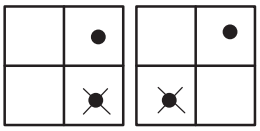
T: ***Now there are 56 pieces of paper in the box—you should have 56 on your Minicomputer.***

Check to make sure the students have the correct number.

T: ***Mr. Johnson takes 12 pieces for his class.***

On the chalkboard, write the subtraction calculation 56 – 12 under 76 – 20 = 56, and instruct the students to do that calculation on their Minicomputers.

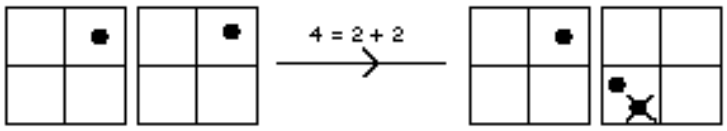
Invite a volunteer to do the subtraction on the demonstration Minicomputer and conclude that 56 – 12 = 44. Then complete the subtraction calculation on the board.



76 – 20 = 56
56 – 12 = 44

T: ***Now there are 44 pieces of paper in the box—you should have 44 on your Minicomputers. I need to take 2 pieces for a letter.***

On the board, write the subtraction calculation 44 – 2 under 56 – 12 = 44. On the Minicomputer, this calculation will require a backward trade before a checker can be taken off the 2-square. If students do not make this observation, ask how to get a checker in the 2-square to take off.



76 – 20 = 56
56 – 12 = 44
44 – 2 = 42

Continue this activity with the following sequence of subtraction problems or any similar ones appropriate to the numerical abilities of your students.

42 – 20 = 22
22 – 10 = 12
12 – 12 = 0

Invite groups of students to make up some of their own subtraction problems to solve on the Minicomputer. Encourage the groups to have a recorder keep track of the problems they solve.

N22 FRAMES OF THE SAME SHAPE

Capsule Lesson Summary

Introduce open sentences with more than one frame of the same shape. Use sharing and patterns to solve many such open sentences.

Materials

Teacher	Student
• Chalk	• Cubes or counters • Worksheets N22 (no star), *, and ** • Paper

Description of Lesson

Exercise 1

Arrange for students to work with a partner, and provide each pair of students with 20 cubes or counters. Direct the student pairs to first check that they have 20 cubes.

Write this open sentence on the board and suggest students copy it with big boxes on their papers.

$\square + \square = 20$

T: ***When frames have the same shape in a sentence, the same number goes in each frame. The boxes here are like two piles and the piles must be equal. Put your 20 cubes into two equal piles.***

If students have made sufficiently big boxes on their papers, they can put the cubes or counters into these boxes.

T: ***How many cubes in each box?***

S: ***10.***

Fill in the boxes and check with the students that the resulting number sentence is true.

$\boxed{10} + \boxed{10} = 20$

Repeat this exercise with $\boxed{5} + \boxed{5} + \boxed{5} + \boxed{5} = 20$ and $\boxed{4} + \boxed{4} + \boxed{4} + \boxed{4} + \boxed{4} = 20$

Exercise 2

Pose other problems, each time filling in the frames with students' suggestions. Encourage students to first try a solution on their papers before giving the class a solution. Again, students may use cubes or counters. You will need to reiterate that the same number goes in each frame of the same shape.

$\boxed{5} + \boxed{5} + \boxed{5} = 15$

$\triangle 6 + \triangle 6 + \triangle 6 = 18$

$(9) + (9) + 7 = 25$

Suppose a student offers an incorrect solution to the $\bigcirc + \bigcirc + 7 = 25$ problem.

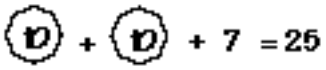
T: ***What number goes in each circle to make a true sentence?***

S: ***10?***

T: ***Let's check.***

Fill in the circles.

T: ***What number is 10 + 10 + 7?***



S: ***27.***

T: ***But we need the sum to be 25. Is 10 more or less than what we need to put in a circle?***

S: ***More.***

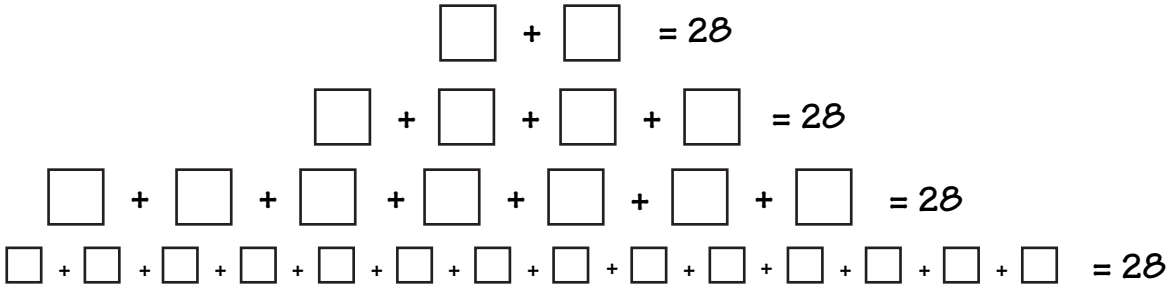
T: ***Try another number.***

Erase 10 from the circles and ask for another suggestion. Fill in the circles and check the calculations each time.

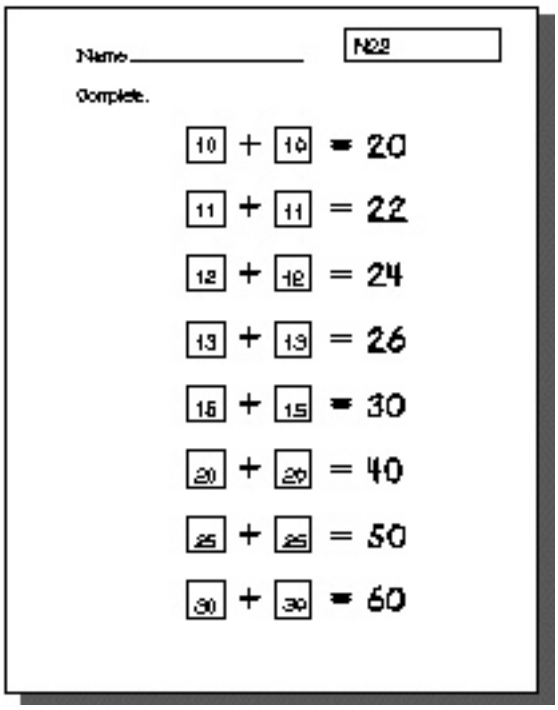
Let students work with a partner as you continue the lesson with other sequences of open sentences. Choose examples from Worksheet N22 or make up ones of your own. Encourage students to find patterns and to explain them to each other or to the class.

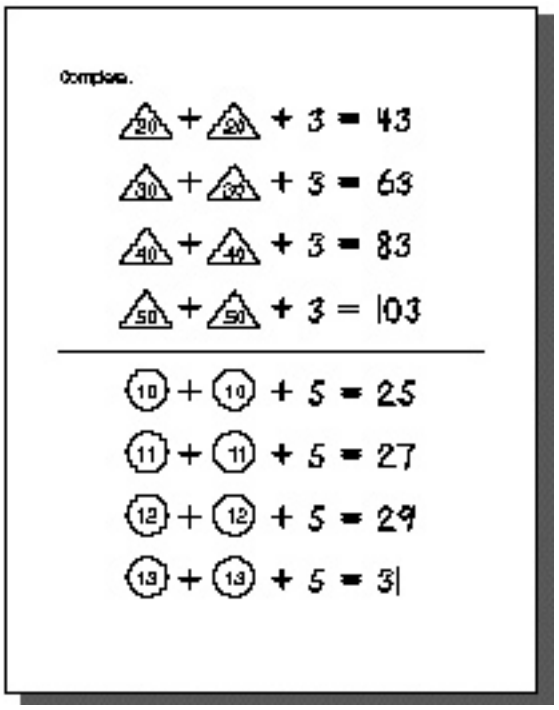
Exercise 3

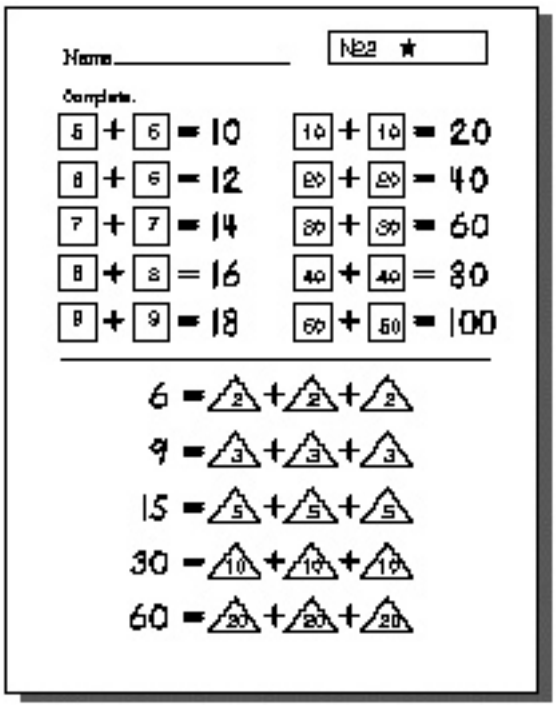
Change the number of boxes in a sequence of open sentences, but keep the sum fixed. For this exercise you may want to give students 28 cubes or counters and write the open sentences one at a time for students to copy and complete.

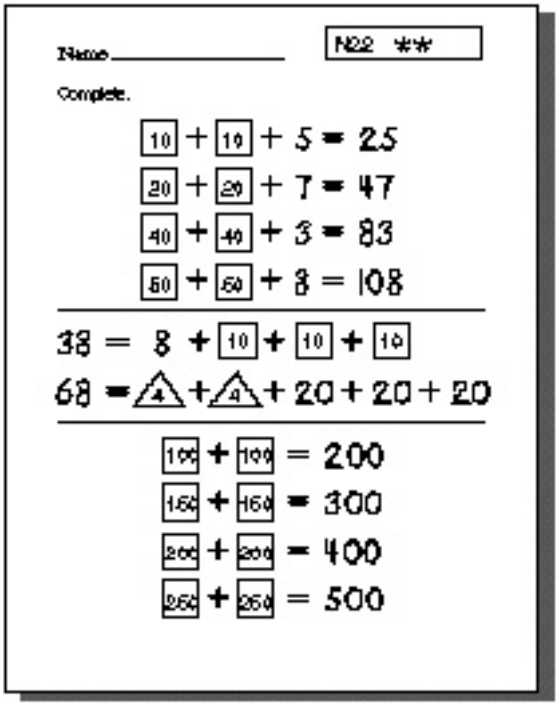


Worksheets N22* and ** are available for independent or partner work. Remind the students that in each sentence, frames of the same shape should be filled in by the same number. Students who finish quickly can make up frame sentences to challenge their partner.


Name
N22
Complete.
10 + 10 = 20
11 + 11 = 22
12 + 12 = 24
13 + 13 = 26
15 + 15 = 30
20 + 20 = 40
25 + 25 = 50
30 + 30 = 60


Complete.
20 + 20 + 3 = 43
30 + 30 + 3 = 63
40 + 40 + 3 = 83
50 + 50 + 3 = 103
10 + 10 + 5 = 25
11 + 11 + 5 = 27
12 + 12 + 5 = 29
13 + 13 + 5 = 31


Name
N22 ★
Complete.
5 + 5 = 10
6 + 6 = 12
7 + 7 = 14
8 + 8 = 16
9 + 9 = 18
10 + 10 = 20
20 + 20 = 40
30 + 30 = 60
40 + 40 = 80
50 + 50 = 100
6 = 2 + 2 + 2
9 = 3 + 3 + 3
15 = 5 + 5 + 5
30 = 10 + 10 + 10
60 = 20 + 20 + 20


Name
N22 ★★
Complete.
10 + 10 + 5 = 25
20 + 20 + 7 = 47
40 + 40 + 3 = 83
50 + 50 + 8 = 108
38 = 8 + 10 + 10 + 10
68 = 4 + 4 + 20 + 20 + 20
100 + 100 = 200
150 + 150 = 300
200 + 200 = 400
250 + 250 = 500

N23 MULTIPLICATION AND SUBTRACTION #4

Capsule Lesson Summary

Practice multiplication in some mental arithmetic activities. Extend experiences with multiplication and subtraction on the Minicomputer.

Materials

Teacher
- Minicomputer set

Student
- Worksheets N23 (no star), *, **, ***, and ****
- Minicomputer set

Description of Lesson

Exercise 1: Mental Arithmetic

Do some mental arithmetic involving multiplication facts and properties. Use the following or similar sequences of questions.

2 x 3 = ?	2 x 10 = ?	2 x 20 ?	2 x 30 = ?	3 x 10 = ?
2 x 6 = ?	2 x 4 = ?	2 x 5 = ?	2 x 2 = ?	3 x 3 = ?
2 x 12 = ?	2 x 14 = ?	2 x 25 = ?	2 x 32 = ?	3 x 13 = ?

If no one responds to 2 x 14 = ?, ask again for 2 x 10 = ? and 2 x 4 = ? writing the facts on the board. Then return to 2 x 14 = ? Ask students who answer correctly to explain to the class how they knew the answer.

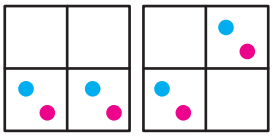
You may wish to continue with more difficult calculations, depending on the abilities of your students.

2 x 50	2 x 10	2 x 20	3 x 10	3 x 20
2 x 6	2 x 7	2 x 8	3 x 5	3 x 6
2 x 56	2 x 17	2 x 28	3 x 15	3 x 26

Exercise 2: Multiplication Problems

Distribute copies of Worksheet N23 and tell the class they are going to work together to solve these problems. For each problem, let the students read the problem and discuss what they need to do. You may solicit several ways to solve a problem but include mention of a multiplication calculation and use of the Minicomputer.

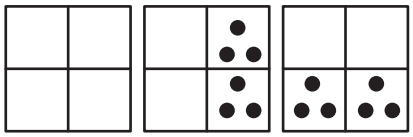
Problem 1: Put 36 on the Minicomputer two times. Then look at the boards separately to see (2 x 36) as (2 x 30) + (2 x 6).



$$2 \times 36 = (2 \times 30) + (2 \times 6)$$
$$= 60 + 12$$
$$= 72$$

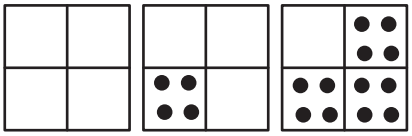
Check the results by making trades on the Minicomputer.

Problem 2:



$3 \times 53 = (3 \times 50) + (3 \times 3)$
$= 150 + 9$
$= 159$

Problem 3:



$4 \times 27 = (4 \times 20) + (4 \times 78$
$= 150 + 28$
$= 108$

Exercise 3: Subtraction Problems

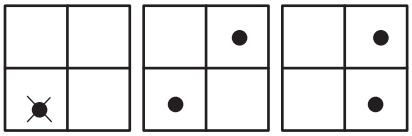
Put 265 on the Minicomputer in standard configuration. Also, draw three Minicomputer boards on the chalkboard and draw checkers for 265 in standard configuration. Tell the students you are going to show them how they will see problems on worksheets in a few minutes.

T: ***Does anyone know what number 265 – 200 is?*** (65)

Write the subtraction problem on the chalkboard and tell students they will check the answer on the Minicomputer.

T: ***Who can take 200 off the Minicomputer?***

A student should remove the checker from the 200-square. At the same time, show the class how you cross out the checker on the drawn boards.



T: ***What number is on the Minicomputer now?***

S: ***65.***

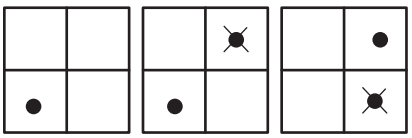
Complete the number sentence on the board.

$265 - 200 = 65$

Write 265 – 41 = ? on the board under 265 – 200 = 65. Put 265 on the Minicomputer and redraw it on the drawn boards. Let students predict the answer and then invite one student to take 41 off the Minicomputer while another crosses out the corresponding checkers on the drawn boards.

T: ***What number is 265 – 41?***

S: ***224.***



Complete the number sentence on the board.

$265 - 41 = 224$

Write 265 – 50 = ? on the board under 265 – 41 = 224. Put 265 on the Minicomputer and redraw it on the drawn boards.

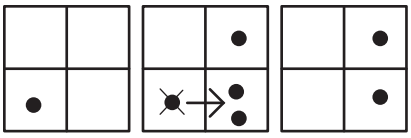
T: ***Why is this problem more difficult?***

S: ***Because we do not have checkers in position for 50.***

S: ***We can make a backward trade.***

T: ***Which trade should we make?***

S: ***20 = 10 + 10.***



Invite someone to make the backward trade while you show how to indicate it on the drawn boards. Then invite one student to take 50 off the Minicomputer and another to cross out the checkers for 50 on the drawn boards.

Complete the number sentence on the board. $265 - 50 = 215$

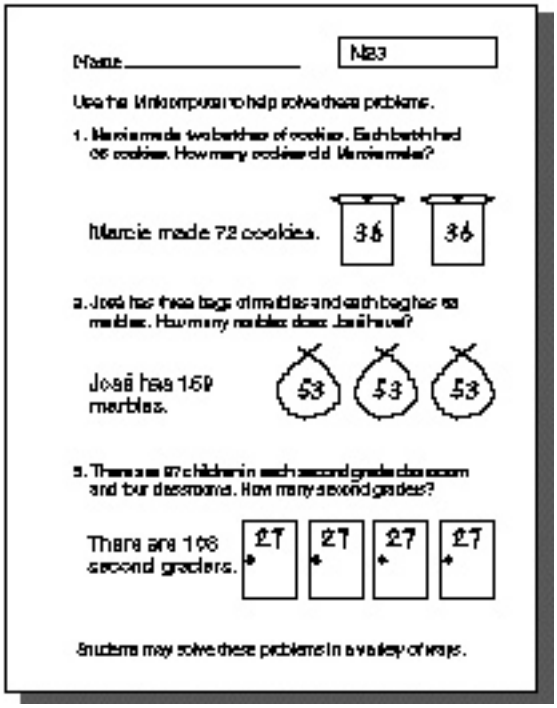
Continue this activity with a couple of the following or similar calculations appropriate for the abilities of your students.

265 – 53
265 – 120
265 – 115

Worksheets N23*, **, ***, and **** are available for independent or partner work. Allow the students to use their individual Minicomputers, if they wish, when doing subtraction calculations.

Center Activity

Place task cards with problems like those on the * and ** worksheets in a center for practice.


Name _____ N23

Use the Minicomputer to help solve these problems.

1. Marcie made two batches of cookies. Each batch had 36 cookies. How many cookies did Marcie make?

Marcie made 72 cookies. 36 36

2. Josè has three bags of marbles and each bag has 53 marbles. How many marbles does Josè have?

Josè has 159 marbles. 53 53 53

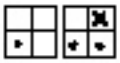
3. There are 27 children in each second grade classroom and four classrooms. How many second graders?

There are 108 second graders. 27 27 27 27

Students may solve these problems in a variety of ways.

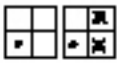
Name _____ N23 ★

Complete.


27 − 4 = 23


27 − 20 = 7


27 − 3 = 24


27 − 5 = 22


27 − 7 = 20


27 − 27 = 0

Name _____ N23 ★★

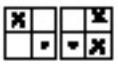
Complete.


97 − 80 = 17


97 − 5 = 92


97 − 14 = 83


97 − 82 = 15


97 − 85 = 12


97 − 17 = 80

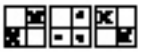
Name _____ N23 ★★★

Complete.


679 − 58 = 621


679 − 248 = 431


679 − 431 = 248


679 − 609 = 70


679 − 478 = 201


679 − 668 = 11

Name _____ N23 ★★★★

Complete.


469 − 10 = 459


469 − 200 = 269


469 − 24 = 445


469 − 248 = 221


469 − 411 = 58


469 − 255 = 214

N24 COMPOSITION OF FUNCTIONS #2

Capsule Lesson Summary

Using arrow pictures, investigate the composition of certain addition and subtraction functions; for example, –1 followed by +6, and +3 followed by +7. Introduce a game in which the players build a road using arrows for +3 and +2.

Materials

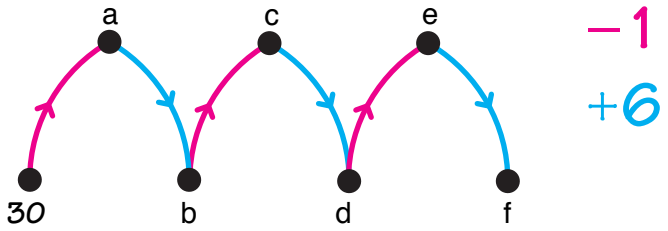
Teacher	• Colored chalk	Student	• Paper • Colored pencils, pens, or crayons

Description of Lesson

Exercise 1

Draw this arrow picture on the board.

Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.



T: ***Where is the least number in this arrow picture?*** (at **a**)
Write the least number on your paper (or whisper it to me). (29)

Where is the greatest number in this arrow picture? (at **f**)
Write the greatest number on your paper (or whisper it to me). (45)

If no one knows what the greatest number is, wait until **f** has been labeled and ask again. Write +5 in green near the arrow picture.

T: ***Where could we draw +5 arrows?***

As students make suggestions, let them trace the arrows on the board. Occasionally ask,

T: ***How do you know you can draw a +5 arrow there?***

S: ***Here is 30 and here will be 35*** (pointing to **b**) ***and 30 + 5 = 35.***

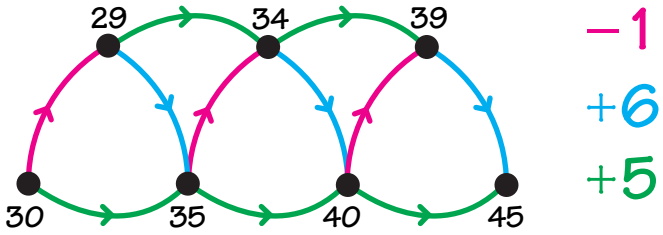
S: ***–1 followed by +6 is the same as +5.***

Let students draw any +5 arrows that they correctly trace and label any dots that they refer to numerically.

If no one suggests a correct +5 arrow, label the dots with students' help; then return to the request for +5 arrows. If necessary, draw one of the +5 arrows yourself.

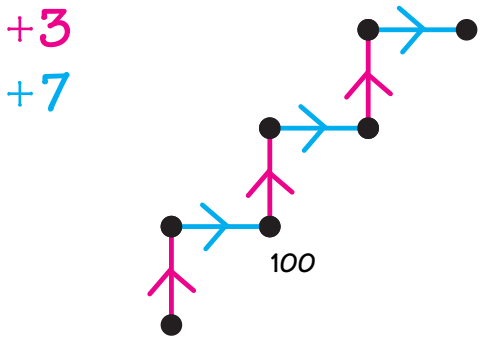
N24

Complete the arrow picture.



Exercise 2

Repeat Exercise 1 using this arrow picture. This time ask students to copy the arrow picture on their papers and follow along with the collective discussion.



Variation: If your class had no difficulty with Exercise 1, you may want to use this variation of Exercise 2. Be sure to leave time for Exercise 3.

After drawing the arrow picture on the board, ask,

T: ***What are the blue arrows for?*** (+7)
What are the red arrows for? (+3)

Draw a green arrow as illustrated.

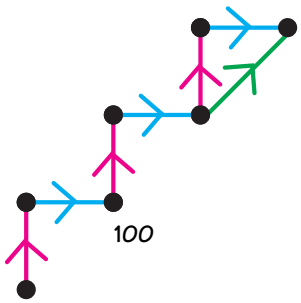
T: ***What could the green arrow be for?***

S: ***+10.***

T: ***How do you know the green arrow could be for +10?***

S: ***I figured out what numbers are at the dots.***

S: ***+3 followed by +7 is the same as +10.***

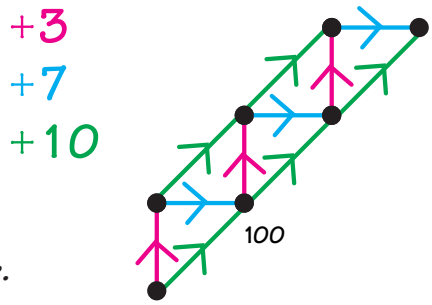


Write +10 in green near the arrow picture.

T: ***Can we draw anymore green arrows?***

Let students trace and then draw green arrows in the picture.

If not all the +10 arrows are found at this time, continue with the lesson and return to the problem of finding +10 arrows after the dots have been labeled.



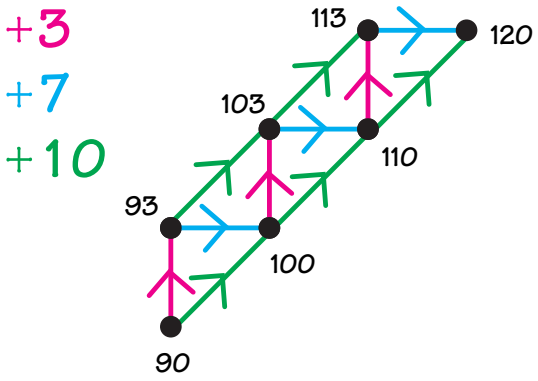
T: ***Point to the least number in this arrow picture. Write the least number on your paper.*** (90)
Point to the greatest number in this arrow picture. Write the greatest number on your paper. (120)

After looking at several responses, ask students to give the answers aloud.

T: ***How do you know the least number is 90?***
How do you know the greatest number is 120?

Encourage subtracting 10 from 100 to get 90, and counting by tens from 100 to 120.

Complete the arrow picture.



Point to 113 and trace a +10 arrow starting there.

T: ***If we drew another +10 arrow starting at 113, what number would be at the end of the arrow?*** (123)
And after that? (133, 143, 153, …)

Point to 93 and trace a +10 arrow ending at 93.

T: ***If we drew another +10 arrow here, what would be the starting number of this arrow?*** (83)
And before that? (73, 63, 53, . . .)

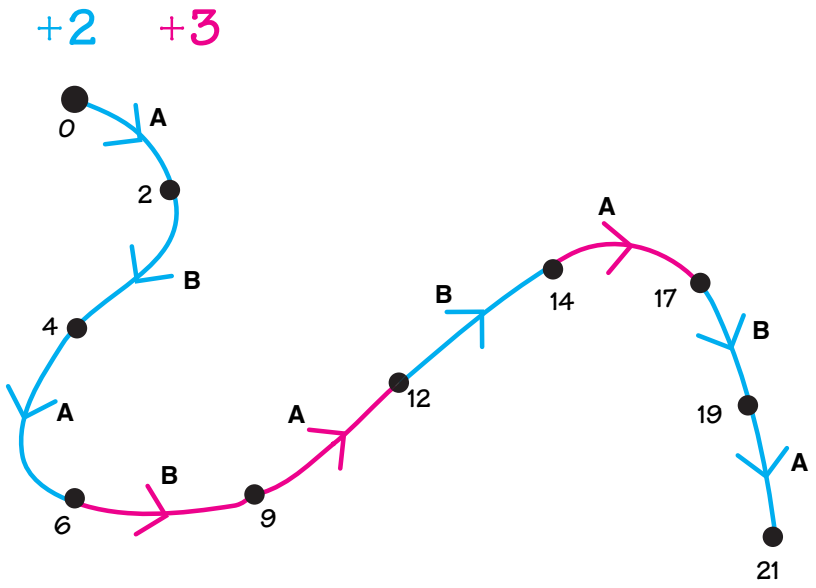
Exercise 3

Draw one ~~dot and label it 0. W~~rite +3 in red and +2 in blue.

T: ***This game is called "20" because the first person to go over 20 loses. Players take turns drawing one arrow until one of them meets a number that is more than 20. When it is your turn, you may draw either a red +3 arrow or a blue +2 arrow. Label the ending dot of each arrow you draw. We will play a game on the board to help you understand the rules.***

N24

Choose two students to play the game at the board. Give each student a piece of red and a piece of blue chalk and make sure they understand that they can use either +2 or +3 arrows. A sample game is shown below. The players are called **A** and **B**.



Player A was forced to go over 20 and therefore lost. Player B could also have won by using a +3 arrow from 17 to 20.

When the game at the board is finished, assign each student a partner. Every student will need a regular pencil, a red pencil, and a blue pencil. Each pair will need a piece of paper. If you wish, you can ask the students to change opponents after each game.

Note: This is a good small group (or partner) activity to use whenever you have some extra time.

Home Activity

Prepare the instructions to the game in Exercise 3 for students to take home. Suggest they play the game at home with family members.

N25 ADDITION PROBLEMS #5

Capsule Lesson Summary

Use the Minicomputer to explore the relationship between calculations such as 3 x 4, 3 x 40, and 3 x 400. Add the odd numbers between 0 and 40 by pairing numbers whose sum is 40.

Materials

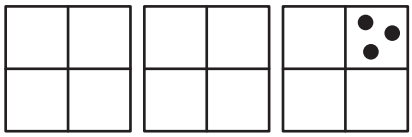
Teacher
- Minicomputer set

Student
- Paper
- Worksheets N25*, **, ***, and ****
- Calculator (optional)

Description of Materials

Exercise 1: Multiplying by 10

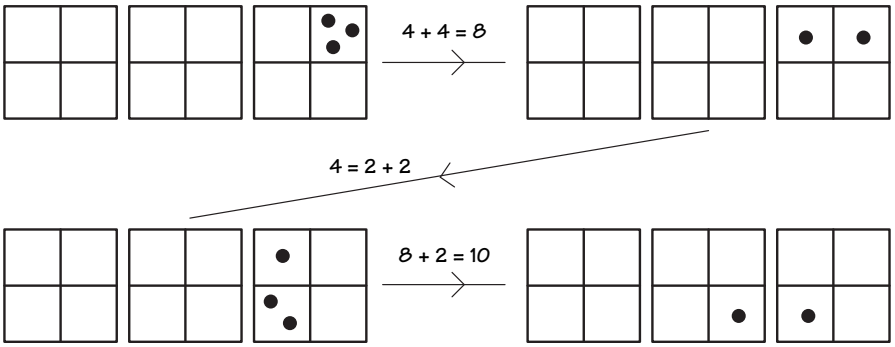
Put this configuration on the Minicomputer.



T: ***What number is this?*** (12)
How do you know?

Record number facts as given by several students. Try to include both 4 + 4 + 4 = 12 and 3 x 4 = 12. If the latter is not suggested, point to 4 + 4 + 4 = 12 and ask, "How many times did we add 4? Do you know another way to write this calculation?" If necessary, suggest 3 x 4 = 12 yourself.

T: ***We think the number on the Minicomputer is 12. Can we make some trades so that this number will be easier to read?***

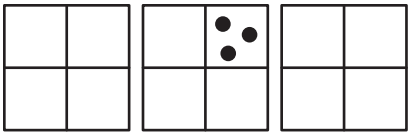


Erase the chalkboard and return the three checkers to the 4-square. Remind the class that there are three 4's on the Minicomputer and that 3 x 4 = 12. Record 3 x 4 = 12 on the chalkboard and ask students to write this fact on their papers.

T: ***Watch what I am going to do.***

Move the three checkers from the 4-square to the 40-square.

T: ***What number is this?*** (120)
How do you know?



On the board, record number sentences suggested by students. If necessary, use parentheses to record some of the sentences without calling the class's attention to them. Number sentences should include 40 + 40 + 40 = 120 and 3 x 40 = 120. If no one suggests 3 x 40 = 120, suggest it yourself.

T: ***We think the number on the Minicomputer is 120. Can we make some trades so that this number will be easier to read?***

Involve many students in making the trades. The trades will be analogous to those used to calculate 3 x 4, but this time on the tens board. Your students may observe this.

When standard configuration for 120 is obtained, ask a student to write the number below (above) the Minicomputer. Erase the chalkboard except for the number fact 3 x 4 = 12 and return the three checkers to the 40-square.

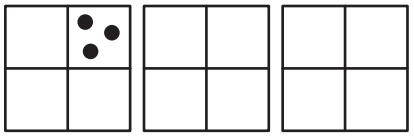
T: ***What number is 3 x 40?***

S: ***120.***

Write 3 x 40 = 120 on the board under 3 x 4 = 12, and ask students to write this fact on their papers.

T: ***Watch me again as I move the checkers.***

Move the three checkers from the 40-square to the 400-square.



T: ***What number is this?***

S: ***1,200.***

Ask students how they did the calculation. It is possible that a student will suggest putting another 0 on 120 to get the answer, but very likely many students will need to make trades before they are certain that 3 x 400 = 1,200.

If some students suggest incorrect answers, write them on the board and appeal to their sense of pattern. For example, if a student suggests 1,012, point to the calculations on the board as you read, "3 x 4 = 12 and 3 x 40 = 120." Write the suggested calculation on the board under the other two as you ask, "Does it look right to have 1,012 here?"

T: ***Can we make some trades so that this number will be easier to read?***

Help students who have trouble verbalizing the trades. For example, if a student says, "40 = 20 + 20," you should say, "400 = 200 + 200," but do not criticize the error.

Continue until standard configuration for 1,200 is obtained. Ask someone to write the number below (above) the Minicomputer.

Erase the chalkboard except for the number facts 3 x 4 = 12 and 3 x 40 = 120, and return the three checkers to the 400-square.

T: ***What number is 3 x 400?***

S: ***1,200.***

Write 3 x 400 = 1,200 under 3 x 40 = 120 and ask students to record this fact on their papers.

You should have this list on the board.

T: ***Does anyone see a pattern?***

S: ***There is a 12 in each answer.***

S: ***There is one more 0 each time.***

3 x 4 = 12
3 x 40 = 120
3 x 400 = 1,200

Do not expect answers to this question to be well formed.

If your class is doing well with the activity, put up the thousands board and ask them to calculate 3 x 4,000. Erase the board before going to Exercise 2.

Exercise 2: Addition Problems

T: ***Today we are going to add the odd numbers between 0 and 40. Does anyone know how many odd number there are between 0 and 40?***

There are twenty odd numbers between 0 and 40 but do not expect this answer; the question is intended to challenge your better students. If no one knows the answer, simply continue with the lesson.

T: ***Let's write all the odd numbers between 0 and 40 on the board. Tell me which numbers to write.***

1+ 3+ 5+ 7+ 9+ 11+ 13+ 15+ 17+ 19+ 21+ 23+ 25+ 27+ 29+ 31+ 33+ 35+ 37+ 39

T: ***Does anyone see an easy way to add these numbers?***

Allow the students to react to this problem. Perhaps someone will suggest pairing numbers whose sum is 40, but if not, make this suggestion yourself.

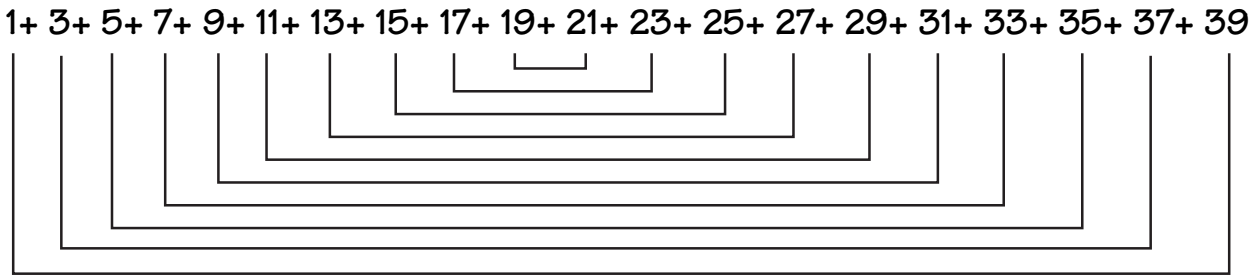
T: ***Maybe it would be easier to solve this problem if we paired numbers whose sum is 40. For example, 39 + 1 = 40.***

Indicate pairing 39 and 1 on your list.

1+ 3+ 5+ 7+ 9+ 11+ 13+ 15+ 17+ 19+ 21+ 23+ 25+ 27+ 29+ 31+ 33+ 35+ 37+ 39

T: ***Does anyone see other pairs of numbers whose sum is 40?***

On your list, indicate correct pairs as they are suggested.



T: ***How many 40's are there?*** (Point to the list of numbers on the board.)

S: ***Ten.***

T: ***What number is 10 x 40?***

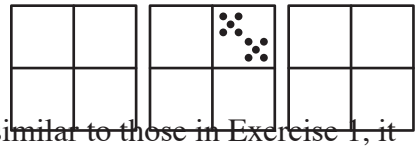
Ask students to explain their answers, but be prepared if no one knows that 10 x 40 = 400. Perhaps someone will suggest using the Minicomputer or a calculator.

T: ***Let's solve this problem on the Minicomputer. How can we show 10 x 40 on the Minicomputer?***

Guide the discussion until it is clear that 40 should be put on the Minicomputer ten times.

Invite someone to put ten checkers on the 40-square.

T: ***We put 10 x 40 on the Minicomputer.***
What can we do to make the number easier to read?



S: ***Make some trades.***

Invite students to make trades. Since this exercise requires trades similar to those in Exercise 1, it should move quickly. Continue until the standard configuration for 400 is obtained.

Assessment Activity

Worksheets N25*, **, ***, and **** are available for independent work. Read the directions with the class and emphasize that the number sentence at the bottom of each page is about the numbers in the string picture on that page. You may like to use these worksheets to assess students' progress in adding numbers to 10, 20, 30, and so on.

Writing Activity

Suggest students write a short letter to a friend that explains how to add the list of odd numbers from 1 to 39 quickly.

Name _____ N25 ★

Draw a string around each pair of numbers whose sum is 10.

10 + 10 + 10 + 10 + 5 = ___45___.

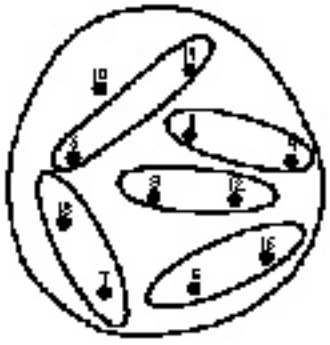
Name _____ N25 ★★

Draw a string around each pair of numbers whose sum is 20.

20 + 20 + 20 + 20 = ___80___

Name _____ N25 ★★★

Draw a string around each pair of numbers whose sum is 30.



Write a number sentence for this picture.

20 + 20 + 20 + 20 + 20 + 10 = 110

Name _____ N25 ★★★★

Draw a string around each pair of numbers whose sum is 30.

Write a number sentence for this picture.

30 + 30 + 30 + 30 + 30 + 30 = 180

6 x 30 = 180

N26 THE FUNCTIONS +3 AND –3 #2

Capsule Lesson Summary

Count forward and backward by threes. Examine the relationship between the number facts 1 + 3 = 4 and 4 – 3 = 1 by the use of a simple arrow diagram. Make use of return arrows to label some of the dots in an arrow picture using +3 arrows and in another using –3 arrows. Individually, draw arrow pictures using +3 and –3 arrows.

Materials

Teacher
- Colored chalk
- Number line or 0–109 numeral chart (optional)
- Calculator (optional)

Student
- Paper
- Colored pencils, pens, or crayons
- Calculator (optional)

Description of Lesson

Start with a short discussion of opposites such as up, down; front, back; hot, cold; and so on. You may like to choose a book using opposites to read with the class; for example, *What the Moon Saw* by Brian Wildsmith.

Exercise 1: Counting by Threes

Divide your class into three or four groups in a manner natural to the seating arrangement. Indicate an order in each group; i.e., which student is first, second, and so on.

T: ***We are going to count forward by threes starting at 0. I will start by saying, "0," then the first student in a group should say, "3," the next, "6," because 6 is three more than 3, the next, "9" because 9 is three more than 6, and so on. Try to remember the number you say.***

Let each group take its turn.

Now reverse the counting. Starting with the last student in each group, count backward by threes by asking students to recall their numbers. For example, a group of seven students would count like this:

21, 18, 15, 12, 9, 6, 3

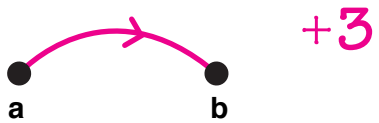
…and you would say, "0."

If students have difficulty with either activity, use the number line or the 0–109 numeral chart to help with the counting. If your 0–109 chart is laminated, highlight the numbers 0, 3, 6, 9, 12, and so on.

You may wish to have the entire class participate as a single group in counting forward and backward by threes. In this case, use a counting calculator to follow the count both forward and backward.

Exercise 2

Draw this arrow picture on the board.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: ***What is this red arrow for?***

S: *+3.*

T (pointing to the dots)**:** ***Do you know what numbers these are?***

S: *No.*

T (pointing to **a**)**:** ***If this number were 2*** (trace the +3 arrow and point to **b**)***, what number would this be?***

S: *5, because 2 + 3 = 5.*

T: ***If this number*** (**a**) ***were 5, what number would this*** (**b**) ***be?*** (8)
If this number were 8 …? (11)
If this number were 0 …? (3)
If this number were 100 …? (103)

Very obviously, change the direction of your question.

T (pointing to **b**)**:** ***If this number were 4*** (trace an imaginary return arrow and point to **a**)***, what number would this be? How do you know?***

S: *1, because 4 – 3 = 1.*

S: *1, because 1 + 3 = 4.*

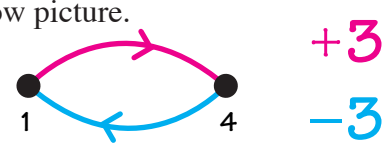
Label the dots and trace an arrow from 4 to 1.

T: ***If we drew a blue arrow from 4 to 1, what could it be for?***

S: *–3.*

T: ***Yes, –3 is the opposite of +3.***

Draw the return arrow in blue and write –3 in blue near the arrow picture.



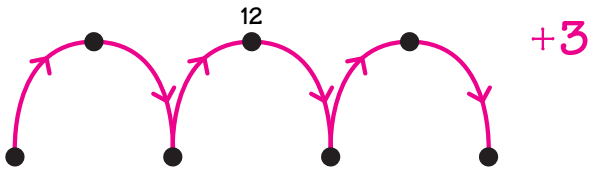
T: ***What number facts are told by this arrow picture?***

S: *1 + 3 = 4 and 4 – 3 = 1.*

Erase 1 and 4 . Alternating dots, assign a number to a dot and ask what the other number in the picture would be. Adjust the difficulty of the calculations to the abilities of your students.

Exercise 3

Draw this arrow picture on the board.



T: ***Point to the greatest number in this arrow picture. Write the greatest number in this arrow picture on your paper (or whisper it to me).***

S: *21.*

T: ***Point to the least number in this arrow picture. Write the least number in this arrow picture on your paper (or whisper it to me).***

S: *3.*

It is not necessary for you to reveal immediately the identities of the least and greatest numbers to the class. When these dots are labeled, acknowledge students who knew the correct greatest and least numbers and ask how they found them.

Call on volunteers to label the dots in this arrow picture. Occasionally ask students to explain how they know which numbers are at particular dots. Continue until the dot to the left of 12 is labeled 9. Trace the red arrow from 9 to 12.

T: ***How did you know that the starting number for this arrow is 9?***

S: ***12 – 3 = 9.***

S: ***9 + 3 = 12.***

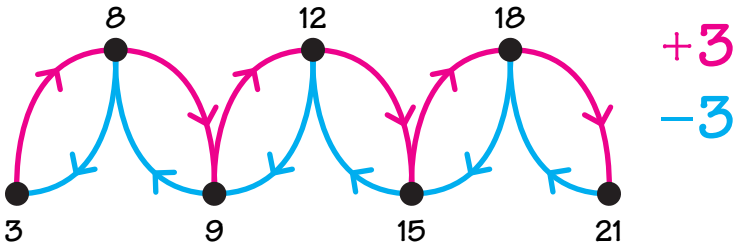
Draw a blue arrow from 12 to 9.

T: ***What could this blue arrow be for?***

S: ***–3.***

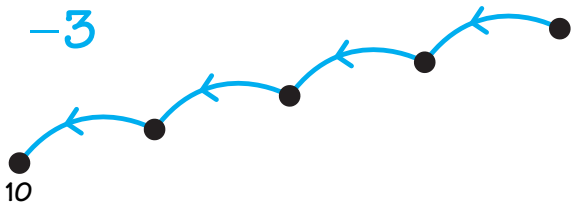
T: ***Yes, –3 is the opposite of +3.***

Write –3 in blue near the arrow picture. Invite students to draw –3 arrows and label dots until the arrow picture is complete.



Exercise 4

Draw this arrow picture on the board and ask students to copy the picture on their papers.



T: ***Point to the least number in this arrow picture. Write the least number in this arrow picture on your papers.***

S: ***10.***

T: ***Point to the greatest number in this arrow picture. Write the greatest number in this arrow picture on your papers.***

S: ***22.***

This last question is difficult. You may like to ask a student who answers correctly to explain his or her method of calculating the greatest number.

T: ***This arrow road ends at the number 10. What can we do that will help us calculate these other numbers?***

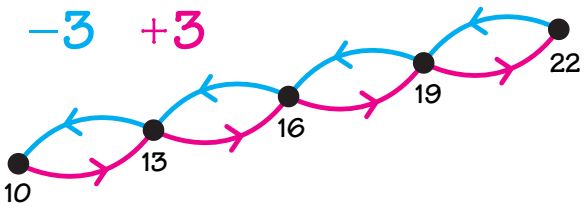
S: ***Draw return (opposite) arrows.***

You may need to suggest drawing return (opposite) arrows yourself.

T: ***What would the return (opposite) arrows be for?***

S: ***+3.***

Write +3 in red near the arrow picture and invite students to either draw +3 arrows or label dots. Complete the arrow picture while the students complete their pictures, too.



Encourage students to extend their arrow pictures, drawing more +3 and –3 arrows and labeling dots as they are added to the picture. After several minutes of individual work, you may like to let students trade papers and check each other's work.

Exercise 5 (optional)

Introduce students to a calculator game similar to the arrow game "20" in Lesson N24. Just like the arrow game, this game starts with 0 on the display of a calculator. Then, two students take turns choosing either [+] [2] [=] or [+] [3] [=]. The player to make the display of the calculator go over 20 first loses. This game can, of course, be adapted by using different functions (for example, +2 and +5) or ending numbers other than 20.

N27 BIRTHDAY PARTY PROBLEMS

Capsule Lesson Summary

Share 24 pencils several ways and write corresponding number sentences. Construct candy treat bags from three kinds of candy conforming to various given conditions.

Materials

Teacher	• 24 pencils or sticks • Three colors of counters or three kinds of wrapped candy	Student	• Three colors of counters or three kinds of wrapped candy • Colored pencils • Worksheet N27

Description of Lesson

Exercise 1

Choose one of your students to be the star of this or a similar story.

T: ***At Pablo's birthday party the guests are going to receive pencils as prizes. Pablo has a package of 24 pencils and he is wondering how many each guest should get. Can you help him?***

S: ***I need to know how many guests are coming to the party.***

S: ***Will Pablo get a pencil?***

T: ***Yes, we do need some more information. Pablo has invited six guests to his party. Each guest is to get the same number of pencils—Pablo will not get a pencil.***

Show the class your 24 pencils (or sticks) and let students count them.

Choose six students to be the guests and to act out the sharing.

T: ***Can each guest get one pencil?***

S: ***Yes; more than one.***

S: ***Each guest can get two or three pencils.***

T: ***Let's have each guest take two pencils. Now how many are left?***

S: ***12.***

S: ***So each guest gets two more.***

S: ***They each get four pencils.***

T: ***Can we write a number sentence about this?***

S: ***6 x 4 = 24.***

S: ***24 shared by six people is 4.***

Write these number sentences on the board and read the word *sharing* as *divided by.*

$6 \times 4 = 24$ $24 \div 6 = 4.$

T: ***What would happen if Pablo invited eight guests to his party?***

S: ***Each guest would get three pencils.***

Continue this exercise asking about sharing (dividing) among three guests, two guests, and twelve guests. Act out the problem as necessary and write number sentences.

Exercise 2

Change the candy types in this exercise to fit what you have, or use three colors of counters to represent the candy.

T: ***Pablo is still getting ready for his birthday. He is preparing treat bags for his guests. He wants to put some candy into each bag. He plans to put a total of 12 pieces of candy into each bag and he has two kinds of candy to use. If Pablo puts 5 candy kisses into a bag, how many lollipops would he put in the bag?***

S: ***7 lollipops.***

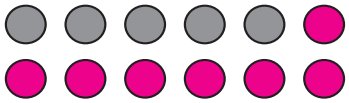
T: ***How do you know that?***

S: ***Because 5 (kisses) + 7 (lollipops) equals 12.***

S: ***There will be 12 candies in all. 12 – 5 = 7. So he would put in 7 lollipops.***

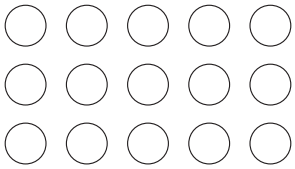
Record the solution to this problem on the board by drawing twelve circles.

Color five of them one color for candy kisses and the rest (seven) a second color for lollipops. Then write the corresponding number sentences 5 + 7 = 12 and 12 – 5 = 7.



T: ***Pablo's mother brings in a bag of candy sourballs. She tells him that now he can put 15 pieces of candy in the treat bags.***

Erase the board and draw 15 circles to represent 15 pieces of candy.



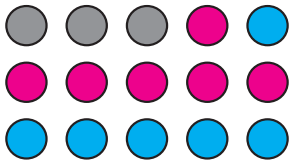
T: ***Suppose Pablo decides to put three candy kisses and equal amounts of lollipops and sour balls in each bag. How can he do this? How many of each kind of candy would he put in a bag?***

S: ***He puts in three candy kisses. Color three circles for candy kisses.***

S: ***He can put in twelve more candies, so six lollipops and six sourballs.***

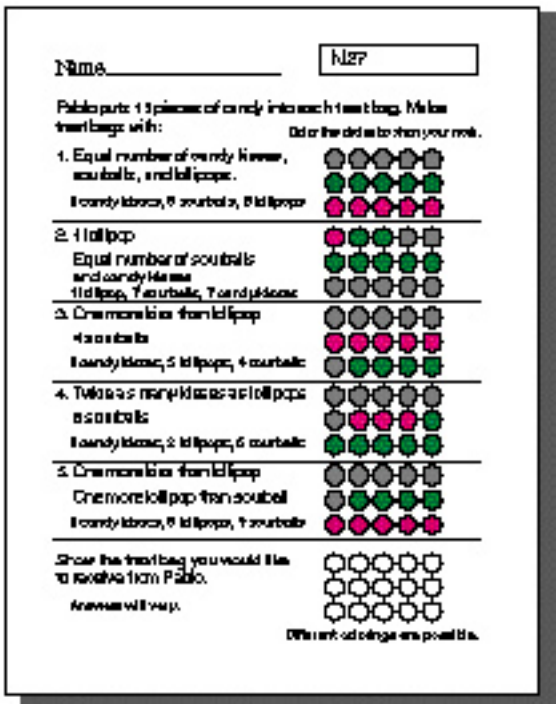
S: ***2 x 6 = 12 and 12 + 3 = 15.***

Let students act out construction of this treat bag with the three kinds of candies or the three colors of counters. Check that the total number is 15 and that there are three of one kind and equal amounts of the other two candies. Then, color in your circles to record the results. Use three colors to represent the three kinds of candy.



Put the class in groups of two to three students each, and provide each group with a supply of three kinds of wrapped candies or three colors of counters. Each student should have a copy of Worksheet N27 and colored pencils. Instruct the groups to work cooperatively to solve the problems on the worksheet.

Groups should use their manipulatives to construct the treat bags, satisfying the conditions of each problem and recording the results by coloring the circles. You may like to assign each group one of the problems to share with the rest of the class at the end. Groups that finish quickly may try to make up another problem and share this with the class.


Name_____ N27

1. Equal number of candy kisses, sourballs, and lollipops.

2. 1 lollipop
Equal number of sourballs and candy kisses
1 lollipop, 7 sourballs, 7 candy kisses

3. One more kiss than lollipop
4 sourballs

4. Twice as many kisses as lollipops
6 sourballs

5. One more kiss than lollipop
One more lollipop than sourball

Show the treat bag you would like to receive from Pablo.

N28 THE FUNCTIONS 2X AND $\frac{1}{2}$X

Capsule Lesson Summary

Conduct a mental arithmetic activity involving 2x and $\frac{1}{2}$x. Relate the functions 2x and $\frac{1}{2}$x through the use of a simple arrow diagram: the arrow for one is the return arrow for the other. Label the dots in a 2x arrow picture with the help of the Minicomputer, and then draw the return arrows.

Materials

Teacher

- Colored chalk
- Minicomputer set
- Calculator
- Base-10 blocks (optional)
- Colored pencils
- Minicomputer set (optional)

Student

- Unlined paper

Description of Lesson

Exercise 1: Mental Arithmetic

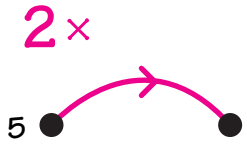
Conduct a brisk mental arithmetic activity with facts similar to the following:

10 + 10	50 + 50	30 + 30	300 + 300	25 + 25
2 x 10	2 x 50	2 x 30	2 x 300	2 x 25
$\frac{1}{2}$ x 20	$\frac{1}{2}$ x 100	$\frac{1}{2}$ x 60	$\frac{1}{2}$ x 600	$\frac{1}{2}$ x 50
20 ÷ 2	100 ÷ 2	60 ÷ 2	600 ÷ 2	50 ÷ 2

Note: $\frac{1}{2}$x should be read "one half of "; for example, $\frac{1}{2}$ x 20 is read "one half of twenty."

Exercise 2

Draw this arrow picture on the board.



Point to the dot on the right.

T: ***What number is this?***

S: ***10.***

Label the dot 10 and draw a return arrow from 10 to 5 in blue.

T: ***What could this blue arrow be for?***

S: ***$\frac{1}{2}$x.***

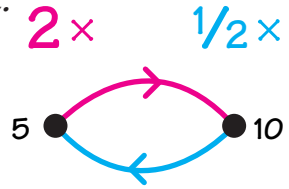
S: ***÷ 2.***

If a student suggests that the blue arrow could be for –5 (or + $\widehat{5}$), agree that 10 – 5 = 5 (or that 10 + $\widehat{5}$ = 5), but then ask what else the blue arrow could be for.

Write $\frac{1}{2}$x in blue near the arrow picture. You may want to observe that $\frac{1}{2}$x and ÷ 2 are the same.

T: ***The blue arrow is for $\frac{1}{2}$x; $\frac{1}{2}$x is the opposite of 2x. What number is $\frac{1}{2}$ x 10?***

S: ***5.***



Erase 5 and 10. Point to the dot on the left.

T: ***If this number were 3, what number would be here*** (point to the dot on the right)***?***

S: ***6***

T (tracing the 2x arrow)**:** ***Yes, 2 x 3 = 6 and*** (trace the $\frac{1}{2}$x arrow) ***$\frac{1}{2}$ x 6 = 3.***

Continue in this manner with several other starting numbers at the left dot such as 7, 20, 500, and 1,000.

Point to the dot on the right.

T: ***If this number were 8, what number would be here*** (point to the dot on the left)***?***

S: ***4.***

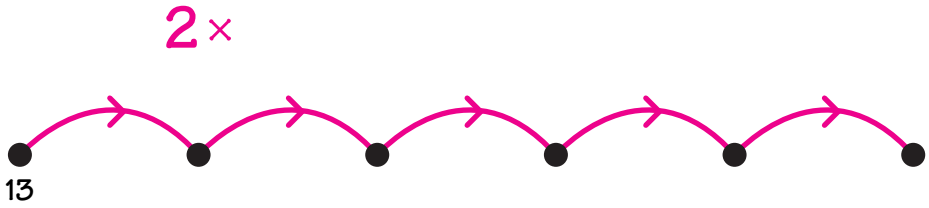
T (tracing the $\frac{1}{2}$x arrow)**:** ***Yes, $\frac{1}{2}$ x 8 = 4 and*** (trace the 2x arrow) ***2 x 4 = 8.***

Continue with several other starting numbers at the right dot such as 10, 16, 20, 80, and 100.

Alternating dots, assign a number to one of the dots and ask what the other number in the picture would be. Choose numbers appropriate to the abilities of your students.

Exercise 3

Draw this arrow picture on the board and ask students to copy it on their papers. Direct students to add to their pictures as is done on the board.



T: ***I drew this picture to tell you about my friend Winona. Winona was on a game show called "Double Up" for a week. The first day she started with $13 and each day she could double her money.***

Observe that the first dot in the picture is for 13 (starting amount) and the red arrows are for 2x (double). Trace the 2x arrow from 13 to the second dot.

S: ***On the first day Winona doubled her money. What number is 2 x 13?***

S: *26.*

T: *How do we calculate 2 x 13?*

S: *13 + 13 = 26.*

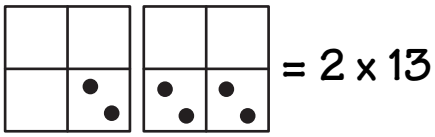
S: *2 x 10 = 20 and 2 x 3 = 6, so 2 x 13 = 26.*

S: *Use the Minicomputer.*

S: *Use a calculator.*

Use one or two suggestions made by students to calculate 2 x 13. You may like to model the calculation with base-10 blocks.

If students or you choose to use the Minicomputer, guide the discussion until it is clear that 13 should be put on the Minicomputer twice; then ask two students to each put 13 on the Minicomputer.

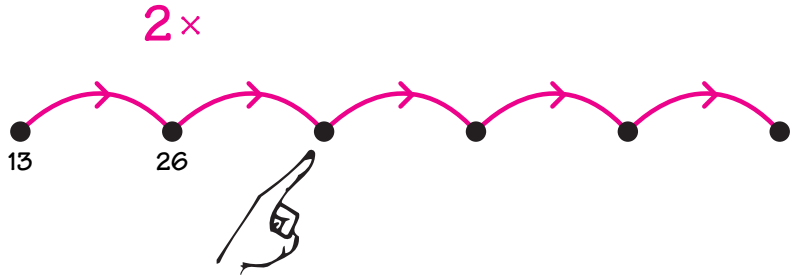


T: *What should we do to make the number easier to read?*

S: *Make some trades.*

Invite students to make trades until the standard configuration for 26 is obtained.

Label the second dot 26 and point to the third dot.



On the second day Winona doubled her money again.

T: *What number is 2 x 26? Can you predict or give an estimate?*

Allow some discussion of estimation strategies or how to find an exact solution.

T: *How can we calculate 2 x 26?*

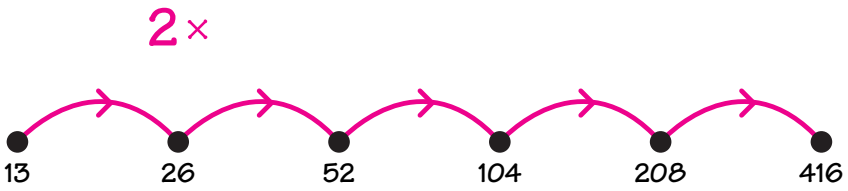
S: *2 x 20 = 40 and 2 x 6 = 12, so 2 x 26 = 40 + 12 = 52.*

S: *On the calculator.*

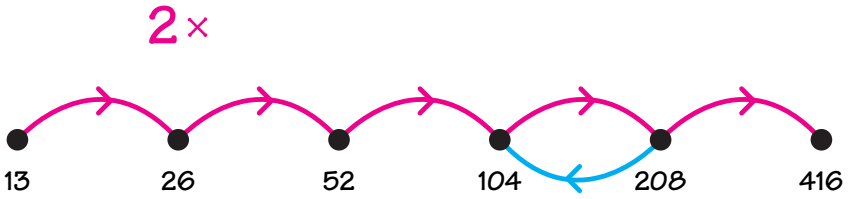
S: *On the Minicomputer; put on 26 two times.*

Again, use students' suggestions or model with base-10 blocks. If you or students choose to use the Minicomputer, ask two students to each put 26 on the Minicomputer. Then invite students to make trades until the standard configuration for 52 is obtained.

Continue in this manner until all the dots are labeled.



Draw a blue return arrow from 208 to 104.



T: ***What could this blue arrow be for?***

S: ***½x.***

S: ***÷ 2.***

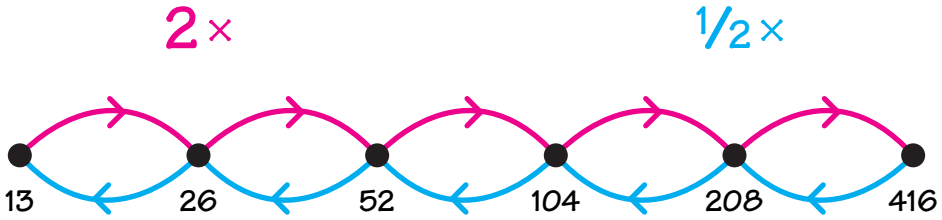
Write ½x in blue near the arrow picture.

T (tracing the blue arrow)**:** ***½ x 208 = 104.***

Write ½ x 208 = 104 on the board.

T: ***Could we draw some more ½x arrows?***

Invite students to draw the remaining ½x arrows. For each one record the appropriate number sentence on the board.



Extension Activity

Invite students to extend their arrow pictures with one or two more 2x arrows (days on the "Double Up" game show) and to label the ending dots.

Writing Activity

Suggest students write another story for their 2x arrow picture.

N29 MULTIPLICATION AND SUBTRACTION #5

Capsule Lesson Summary

Draw a picture with 4 x 9 dots and one with 9 x 4 dots. Conclude that 4 x 9 = 9 x 4. Calculate 2 x 75 in a variety of ways by looking at 75 as a sum of smaller numbers. Check this calculation on the Minicomputer. Do several sequences of subtraction problems on the Minicomputer.

Materials

Teacher	• Minicomputer set	• Counters
Student	• Paper	• Minicomputer set

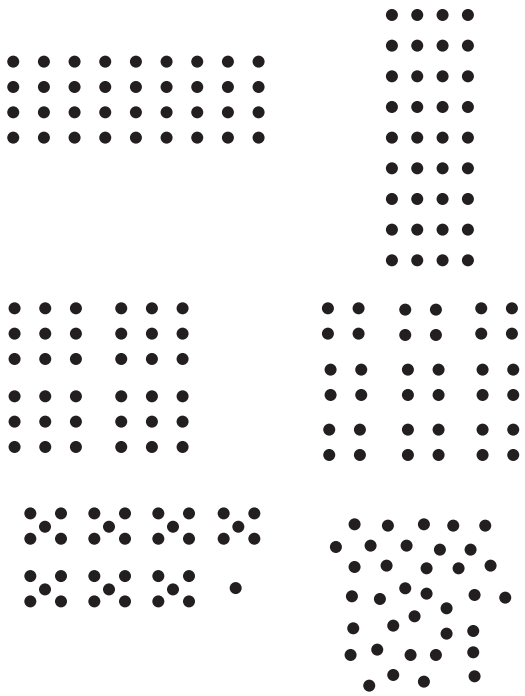
Description of Lesson

Exercise 1: Dot Pictures for Multiplication

Ask the students to draw a picture with 4 x 9 dots and a picture with 9 x 4 dots. For some students, you may prefer to give them objects (counters) to show 4 x 9 and 9 x 4. Let them solve this problem in their own ways. Watch how the pictures are drawn, and ask some students to draw one of their pictures on the board. Students using objects can draw dots for their objects on the board, or you can draw the picture with dots arranged like the objects are arranged. Put at least one picture on the board organized in rows and columns (4 rows and 9 columns, or 9 rows and 4 columns).

If some of your students do not understand what 4 x 9 means, ask someone to explain. You may want to write 4 x 9 = 9 + 9 + 9 + 9 and 9 x 4 = 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 on the board.

These are some examples of what students might draw.



It is possible that some of your students will only draw one dot picture for both 4 x 9 and 9 x 4. Do not insist that these students draw a second picture, but ask them to explain why they drew only one picture. For example, a student might say, "4 x 9 = 36 and 9 x 4 = 36. So I just drew a picture with 36 dots." Or a student might show you a picture for 4 x 9 as 4 rows of 9 dots each and then turn it and say it is also for 9 x 4 because it is 9 rows of 4 dots each.

Ask the students which pictures they like the best and why, and if there are any pictures that are clearer than others. Do not criticize pictures in which the dots are not drawn in rows. They may be interesting from an aesthetic point of view. Be happy if there are 36 dots.

T: ***How many dots are in your picture with 4 x 9 dots?*** (36)
How many dots are in your picture with 9 x 4 dots? (36)
Are there any pictures where we could see that 9 x 4 = 4 x 9 without counting all the dots?

Any picture that is organized in rows and columns (4 rows and 9 columns, or 9 rows and 4 columns) nicely shows that 4 x 9 = 9 x 4, but actually any picture with 36 dots can be organized to show this.

Allow students to work in pairs during the next two exercises.

Exercise 2: Multiplication on the Minicomputer

Display four Minicomputer boards. Distribute individual Minicomputers. Each pair of students should have four Minicomputer boards (two sheets), some checkers, and paper.

T: ***Put 2 x 75 on your (individual) Minicomputer. Remember that 2 x 75 is the same as 75 + 75. Do not make any trades yet.***

Give individual help as necessary to be sure everyone has put on 2 x 75. Ask someone to put 2 x 75 on the demonstration Minicomputer.

When students respond to these estimation questions, ask them to explain their answers.

T: ***Is this number more than 10?*** (Yes)
Is this number more than 100? (Yes)
Is this number more than 200? (No)

This last question may cause some confusion, but it is not essential that your class agree that the number is less than 200. These questions will direct the class towards a first estimation of the number.

T: ***Can we calculate 2 x 75 without making trades on the Minicomputer?***

Many suggestions are possible; for example:

2 x 75 = (2 x 70) + (2 x 5)
= 140 + 10
= 150

2 x 75 = (2 x 50) + (2 x 20) + (2 x 5)
= 100 + 40 + 10
= 150

2 x 75 = (2 x 40) + (2 x 20) + (2 x 10) + (2 x 5)
= 80 + 40 + 20 + 10
= 150

2 x 75 = (2 x 25) + (2 x 25) + 2 x 25)
= 50 + 50 + 50
= 150

Encourage students to choose their own methods of calculating, and let them share their ideas with their partner or with the class. Student pairs should write their calculation results or an estimate on paper and check it after trades are made. Indicate on the board some of their approaches to the calculations. Afterward, let the students calculate 2 x 75 by making trades on their individual Minicomputers.

Repeat this activity with a couple more calculations such as 3 x 56 and 4 x 132. When the students have put a number on their Minicomputers, ask them to try to calculate the number before they make any trades. Let students share their methods with the class.

Be patient! The goal of these exercises is not to find results quickly, but to think of many ways to solve the problems mentally.

Exercise 3: Subtraction Problems

In this exercise the students will do a sequence of subtraction calculations on the Minicomputer. If students are working in pairs, direct one student to write the problem on paper while the other puts checkers on and takes checkers off the Minicomputer. Switch roles after each problem.

You may like to put the problems in a story context. For example:

T: ***The cafeteria ordered 397 cartons of milk for today. Put 397 on your Minicomputer.***

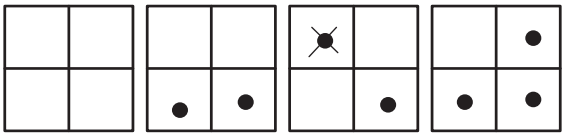
Check that all the students have 397 on their Minicomputers and invite one person to put 397 on the demonstration Minicomputer.

T: ***The first graders are first to get milk.***
They take 80 cartons.

To one side of the Minicomputer, write this subtraction problem: 397 – 80 = ?

T: ***Calculate 397 – 80 on your Minicomputer.***

Ask a student to take 80 off the demonstration Minicomputer and complete the number sentence on the board.



397 – 80 = 317

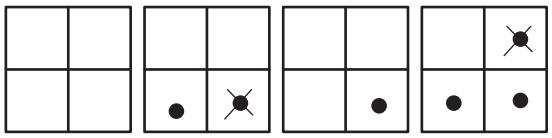
T: ***Now the cafeteria has 317 cartons of milk. Next the second graders take 104 cartons.***

Write this problem on the board under 397 – 80 = 317: 317 – 104 = ?

T: ***Calculate 317 – 104 on your Minicomputer.***

N29

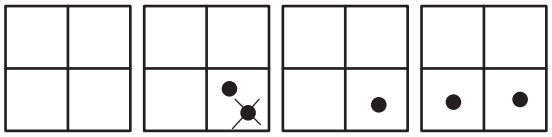
Invite someone to do the subtraction on the demonstration Minicomputer and complete the number sentence on the board.



397 – 80 = 317
317 – 104 = 213

T: ***Now the cafeteria has 213 cartons of milk. Next, the third graders take 100 cartons.***

Write 213 – 100 on the board under the first two number sentences and proceed as before. This calculation requires a backward trade: 200 = 100 + 100.



397 – 80 = 317
317 – 104 = 213
213 – 100 = 113

Continue the sequence with these calculations: 113 – 101 = 12 and 12 – 8 = 4.

If time remains repeat this activity with other sequences of subtraction problems appropriate to the numerical abilities of your class. Two possibilities are suggested below.

569 – 10 = 559	875 – 400 = 475
559 – 18 = 541	475 – 50 = 425
541 – 200 = 341	425 – 200 = 225
341 – 240 = 101	225 – 210 = 15
101 – 1 = 100	15 – 8 = 7
100 – 80 = 20	7 – 7 = 0

Writing Activity

Invite students to write subtraction problems on index cards. Then let them work with other students to solve their problems, or save the cards to put in centers.

Capsule Lesson Summary

Count by tens starting at 0 and then at 3. Relate the functions +10 and –10 through the use of a simple arrow diagram: the arrow for one is the return arrow for the other. Use return arrows to label the dots in a –10 arrow picture.

Materials

Teacher	• Overhead or class calculator • 0–109 numeral chart • Colored chalk	Student	• Unlined paper • Colored pencils • Worksheet N30 (optional)

Note: The worksheet has a Christmas holiday theme and is optional.

Description of Lesson

Exercise 1: Mental Arithmetic

Indicate to your class an order for this activity that is natural to their seating arrangement; i.e., indicate which person is first, second, and so on.

T: ***Today we are going to count by tens. I will say, "0," the first student will say, "10"; the second will say, "20" because 10 +10 = 20; the next, "30" because 20 + 10 = 30; and so on.***

How many students are here today? (Determine this number)
If everyone has exactly one turn, what number will the last person say?

Record some of the students' predictions on the board.

Start the count with "0" and walk around the room as students count by tens until each student has said a number. Make corrections as necessary. Compare the students' predictions with the last number said, commending those who correctly predicted which number would be said last. Repeat this activity again but start with a different student, and this time ask the students to remember their numbers (perhaps write them on paper). Then, start with the last student and direct the students to count backward by tens. Each student says the number they were to remember. You may like to demonstrate counting forward and backward by tens as well with a counting calculator.

Discuss with the class when they might use counting by tens; for example, counting an amount of money with dimes, counting fingers, or counting things packaged in tens.

T: ***We are going to count by tens again, but this time I will start by saying the number 3. Each person will say the number that is ten more than the last number. Remember which number you say.***

Say "3" and then point to the student who was first during the last counting. If the student hesitates, ask, "What number is 3 + 10?" Continue in exactly the same order as in the last counting. If a student has difficulty, refer to the 0–109 numeral chart and count forward (by ones) ten spaces.

T: ***Why did I ask you to remember the number you said before?***

S: ***The number I said this time was three more than the number I said last time.***

If this response is not made or is not clear, ask several students, "What was your number last time we counted? . . . this time?" until the relationship becomes clear.

T: ***Let's write some of these numbers on the board. I said 3.***

Write 3 on the board.

T: ***What number came after 3?*** (13)

Record 13 below 3. Continue in this way until 113 is written on the board.

T: ***Does anyone see a pattern?***

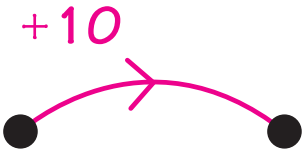
Very likely students will observe that all the numbers end in 3 and if the last digit from each numeral were removed, the new numbers would be increasing by 1. You may like to use a counting calculator (counting by tens starting at 3) to observe this pattern again.

3
13
23
33
43
53
63
73
83
93
103
113

Exercise 2

Draw this arrow picture on the board.

Ask the class to indicate which number is less, at the dot on the left or the right. Students should indicate the dot on the left. Point to this dot.



T: ***If this number were 40, what number would the other dot be? (50)***
If this number were 100, …? (110)
If this number were 5, …? (15)
If this number were 8, …? (18)
If this number were 26, …? (36)

Point to the dot on the right.

T: ***If this number were 30, what number would the other dot be?***

S: ***20, because 30 – 10 = 20.***

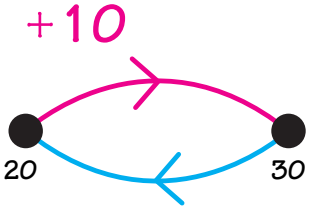
S: ***20, because 20 + 10 = 30.***

Label the dots 20 and 30 and draw a blue return arrow from 30 to 20.

T: ***What could this blue arrow be for?***

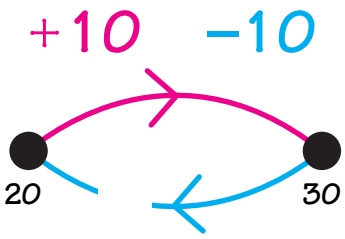
S: ***–10.***

T: ***Yes, –10 is the return (opposite) of +10.***



If a student suggests + $\widehat{10}$, record it on the board but ask if there is another name we could give this arrow. Write –10 in blue near the arrow picture. Erase 20 and 30. Point to the dot on the right.

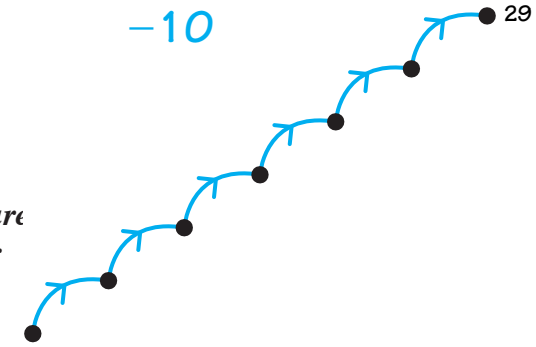
T: ***If this number were 100, what would the other number be?*** (90)
If it were 14, …? (4)
If it were 38, …? (28)
If it were 10, …? (0)
If it were 500, … ? (490)



Continue this activity with other numbers appropriate for the abilities of your students. Alternating dots, assign a number to one of the dots and ask what the other number in the picture is.

Exercise 3

Draw this arrow picture on the board and instruct students to copy the picture on their papers. Tell them to add information to their pictures just as is done on the board.



T: ***Where is the least number in this arrow picture*** (At the upper right) ***What is the least number in this arrow picture?*** (29)
Where is the greatest number in this arrow picture? (At the lower left)
Write on your paper what you predict the greatest number will be.

This last question may be especially challenging. Look at several predictions, but do not announce the correct answer now. Point to the second dot from the right.

T: ***Who can label this dot?***

Let a student label the dot 39.

T: ***How do you know this number is 39?***

S: ***29 + 10 = 39.***

S: ***39 – 10 = 29.***

Draw a red arrow from 29 to 39.

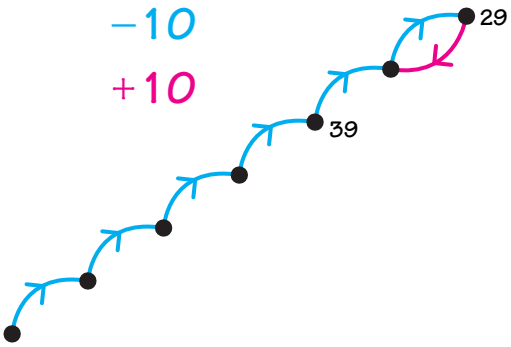
T: ***What could this red arrow be for?***

S: ***+10.***

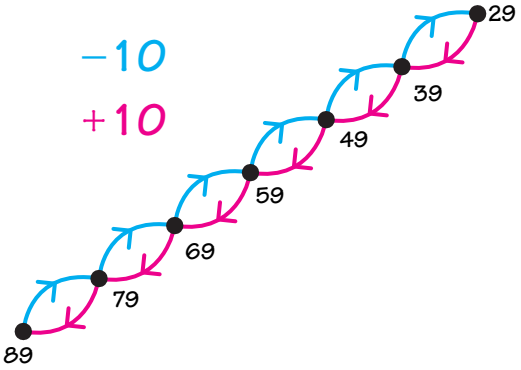
T: ***Yes, +10 is the return (opposite) of –10.***

N30

Write +10 in red near the arrow picture.



Continue by asking students to either label a dot or to draw another arrow until the arrow picture is complete. Acknowledge students who predicted that the greatest number in the picture is 89.



T: ***Does anyone see a pattern to the numbers in this picture?***

S: ***The numbers in this picture all end in 9.***

S: ***89, 79, 69, … , 29. The first number (digit) is one less each time.***

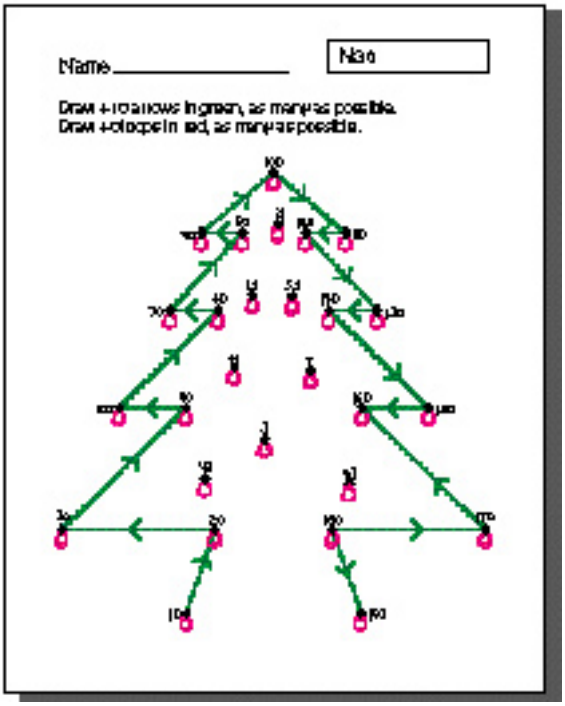
Optional: Worksheet N30 is available for independent work. Some students may need help recognizing that a +0 arrow is always a loop. Allow the students to draw the arrows as they wish; gradually curved arrows might by aesthetically more pleasing to some students.

Center Activity

Prepare a deck of cards with each card having a number on the front and the number that is 10 more on the back. Let students play in pairs with one student reading the front of a card and the other student saying what number is on the back, and vice versa.

Home Activity

Suggest parents/guardians practice counting by tens with their child. Children can show their families how to teach a calculator to count by tens and use it in their practice. Suggest that sometimes they start the counting at numbers other than 0; for example, any number from 1 to 9.


Name
N30

N31 ADDITION ALGORITHM #1

Capsule Lesson Summary

Use the Minicomputer to solve addition problems and present an addition algorithm.

Materials

Teacher	Student
• Minicomputer set	• Paper • *Addition Problems* Booklet • Individual Minicomputer sets (optional)

Description of Lesson

Exercise 1

Write this calculation on the board and invite students to suggest a story problem for which we might need to do this calculation.

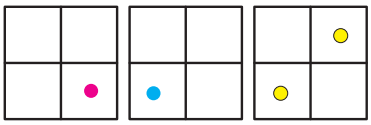
T: ***What number is 126 + 33?***

126 + 33 = ?

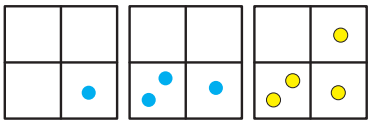
Record some students' predictions on the board.

T: ***Let's add 126 and 33 on the Minicomputer.***

Put 126 on the Minicomputer using red checkers on the hundreds board, blue checkers on the tens board, and yellow checkers on the ones board.



Add 33 to the 126 on the Minicomputer, again using blue checkers on the tens board and yellow checkers on the ones board.



Write the calculation in vertical format with colors corresponding to the checkers.

$$\begin{array}{r} 126 \\ +\ 33 \\ \hline \end{array}$$

Stand in front of the tens and hundreds boards and direct the class to look only at the ones board.

T: ***What number is on the ones board?***

S: ***9.***

T: ***Can we make a trade?***

S: ***Yes, 2 + 2 = 4 and 4 + 4 = 8.***

Let students make the trades and then ask a student to write 9 (in yellow) below the ones board.

N31

Point to the ones column of the addition problem written on the board. Cover the other two columns and ask,

T: ***What do you see in the ones column of this addition problem?***

S: ***6 and 3.***

T: ***Six ones plus three ones is …?***

S: ***9.***

T: ***So we have 9 on the ones board and below the ones column.***

$$\begin{array}{r} 126 \\ +\ 33 \\ \hline 9 \end{array}$$

Stand in front of the hundreds board and direct the class to look at the tens board.

T: ***How many tens are on this board? Can we make a trade?***

Let a student make the 20 + 20 = 40 trade on the tens board and then ask a student to write 5 (in blue) below the tens board. Point to the tens column of the addition problem written on the board.

T: ***What do you see in the tens column of this addition problem?***

S: ***2 and 3.***

T: ***Two tens plus three tens is … ?***

S: ***Five tens.***

T: ***So we have 5 on the tens board and below the tens column.***

$$\begin{array}{r} 126 \\ +\ 33 \\ \hline 59 \end{array}$$

Direct the class to look at the hundreds board.

T: ***How many hundreds are on this board?***

S: ***One.***

Ask a student to write 1 (in red) below the hundreds board. Point to the hundreds column of the addition problem written on the board.

T: ***What do you see in the hundreds column?***

S: ***1.***

T: ***What number is 126 + 33?***

S: ***159.***

$$\begin{array}{r} 126 \\ +\ 33 \\ \hline 159 \end{array}$$

Complete the number sentence on the board. Compare the result with the students' predictions, commending those who correctly predicted the answer.

Exercise 2

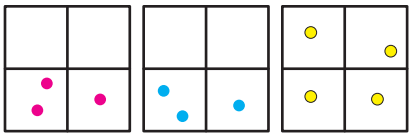
Erase the board and remove the checkers from the Minicomputer.

Write this problem on the board.

$$239 + 326 = ?$$

T: ***What number is 239 + 326?***

Record some predictions on the board.



T: ***Let's add 239 and 326 on the Minicomputer.***

As in Exercise 2, put checkers of three colors on the Minicomputer.
Write this calculation on the board with colors corresponding to the checkers.

Stand in front of the tens and hundreds board and direct the class to look only at the ones board.

$$\begin{array}{r} 239 \\ +\ 326 \\ \hline \end{array}$$

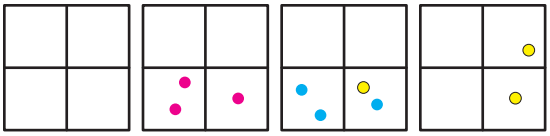
T: ***What number do you see on the ones board?***

S: ***15.***

T: ***Can we make a trade?***

S: ***Yes, 8 + 2 = 10.***

Ask a student to make the 8 + 2 = 10 trade.



T: ***What number is on the ones board now?***

S: ***5.***

Ask a student to write 5 (in yellow) below the ones board.

T: ***Before we had 15 on the ones board and now there is only 5. What happened?***

Be sure the students understand that ten ones were moved to the tens board. Point to the addition problem written on the board.

T: ***How could we show in this calculation*** (point to the problem on the board) ***that we now have another checker on the tens board?***

Allow the students to discuss this situation.

T: ***Here is the way many people show this. How many more tens do we have?*** (One)
I will write 1 above the tens column. This shows that we have another ten.

Note: The 1 is written in yellow to correspond to the yellow checker on the tens' board.

T: ***What should we write below the ones column?***

S: ***5.***

T: ***Yes, there is 5 left on the ones board.***

$$\begin{array}{r} {\scriptstyle 1} \\ 239 \\ +\ 326 \\ \hline 5 \end{array}$$

T: ***We have one more ten on the tens board*** (point to the yellow checker on the tens board) ***and in the tens column. We have 5 on the ones board and below the ones column. In the ones column of our problem there is 9 and 6. 9 + 6 = 15, and 15 is one ten*** (point to 1 above the tens column) ***and 5 ones*** (point to 5 below the ones column)***.***

Direct the class to look at the tens board of the Minicomputer.

T: ***How many tens are on this board?*** (Six) ***Can we make a trade?***

Invite students to make trades on the tens board, and then ask a student to write 6 (in blue) below the tens board. Observe that no checker moved to another board. Point to the tens column of the addition problem written on the board.

T: ***What do you see in the tens column?***

S: ***1, 3, and 2.***

T: ***One ten plus three tens plus two tens is…?***

S: ***Six tens.***

$$\begin{array}{r} {\scriptstyle 1} \\ 239 \\ +\ 326 \\ \hline 65 \end{array}$$

T: ***Did we need to write anything above the hundreds column?***

S: ***No, because we didn't make any trades that put more checkers on the hundreds board.***

T: ***What do you see in the hundreds column?***

S: ***2 and 3.***

T: ***Two hundreds plus three hundreds is…?***

S: ***Five hundreds.***

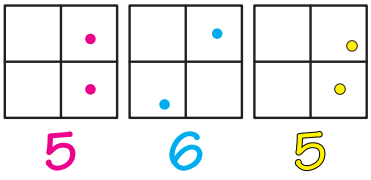
Invite a student to make the 200 + 200 = 400 trade and another student to write 5 in red below the hundreds board and below the hundreds column.

T: ***What number is 239 + 326?***

S: ***565.***

Complete the number sentence on the board.

239 + 326 = 565



$$\begin{array}{r} {\scriptstyle 1} \\ 239 \\ +\ 326 \\ \hline 565 \end{array}$$

A 16-page booklet, *Addition Problems*, contains worksheets to use with a sequence of lessons involving the addition algorithm in both Part I and Part II of *CSMP Mathematics for the Upper Primary Grades*. The students should begin on page 2 today and progress at their own pace. Today, allow about 10 minutes for independent work in this problem book. After the lesson, collect the booklets, check them, and store them for future use in the remaining lessons on the addition algorithm (*UPG-I*, Lesson N33; and *UPG-II*, Lessons N5, N8, and N23).

Home Activity

This is a good time to send a letter to parents/guardians about the addition algorithm. Blackline N31 has a sample letter.

Calculate.

22 + 43 = 65	53 + 21 = 74	62 + 15 = 77
47 + 12 = 59	36 + 52 = 88	30 + 38 = 68
43 + 46 = 89	42 + 26 = 68	53 + 44 = 97

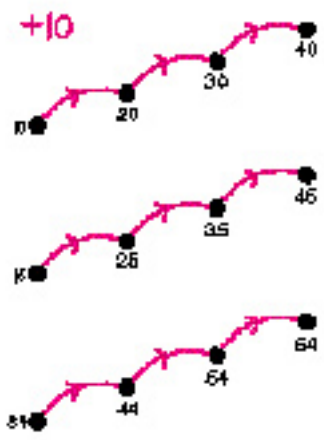
2

Calculate.

42 + 31 = 73	47 + 51 = 98	231 + 423 = 654
51 + 16 = 67	68 + 21 = 89	143 + 222 = 365
59 + 30 = 89	56 + 23 = 79	528 + 21 = 549

3

Label the dots.



4

Calculate.

50 + 14 = 64	25 + 24 = 49
65 + 13 = 78	17 + 50 = 67
12 + 72 = 84	27 + 52 = 79
42 + 36 = 78	35 + 24 = 59

5

N31

Calculate.

63 + 17 = 80	42 + 20 = 62	350 + 240 = 590
68 + 24 = 92	65 + 31 = 96	132 + 67 = 199
35 + 45 = 80	48 + 43 = 91	207 + 177 = 384

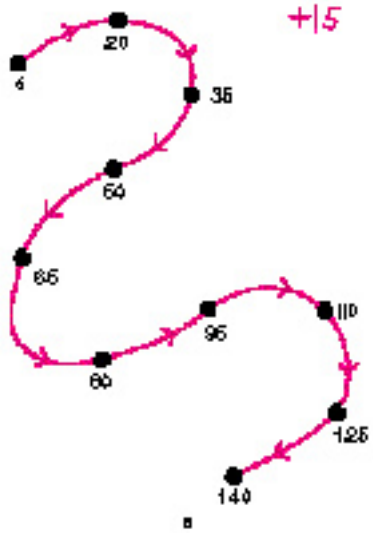
6

Calculate.

20 + 36 = 56	42 + 29 = 71	416 + 322 = 738
19 + 57 = 76	52 + 27 = 79	321 + 272 = 593
48 + 32 = 80	66 + 15 = 81	23 + 356 = 379

7

Label the dots.



8

Calculate.

25 + 49 = 74	42 + 25 = 67	437 + 28 = 465
63 + 12 = 75	27 + 44 = 71	145 + 333 = 478
24 + 52 = 76	65 + 19 = 84	364 + 121 = 485

9

Calculate.

35 + 45 = 80

263 + 121 = 384

74 + 26 = 100

343 + 434 = 777

26 + 46 = 72

34 + 27 = 61

10

Calculate.

25 + 46 = 71	62 + 25 = 87	37 + 28 = 65
63 + 43 = 106	45 + 53 = 98	47 + 44 = 91
32 + 56 = 88	55 + 17 = 72	20 + 76 = 96
48 + 12 = 60	84 + 50 = 134	59 + 23 = 82

11

Calculate.

37 + 17 = 54	58 + 35 = 93	424 + 355 = 779	746 + 45 = 791
37 + 36 = 73	58 + 60 = 118	129 + 717 = 846	328 + 533 = 861

Label the dots.

+14

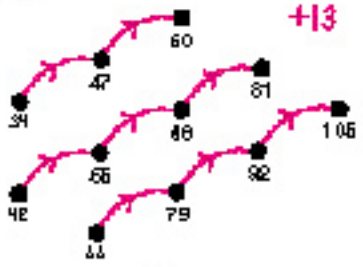
12

Calculate.

450 + 139 = 589	263 + 165 = 428	88 + 25 = 113
356 + 244 = 600	751 + 59 = 810	89 + 88 = 177

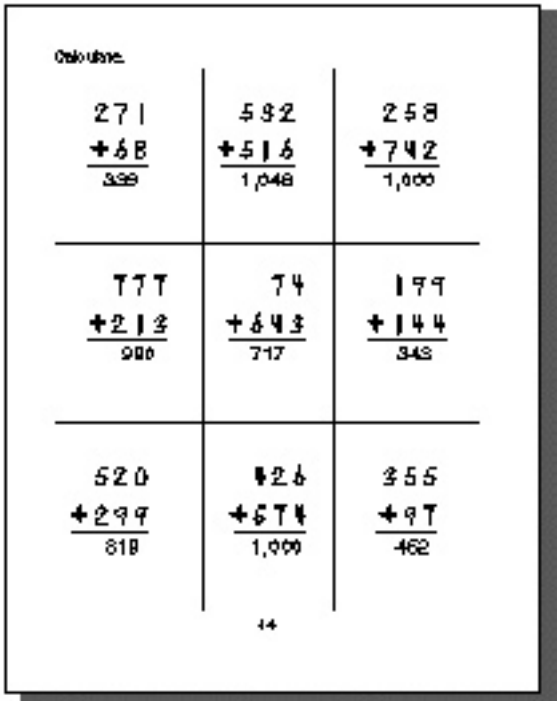
Label the dots.

+13



13

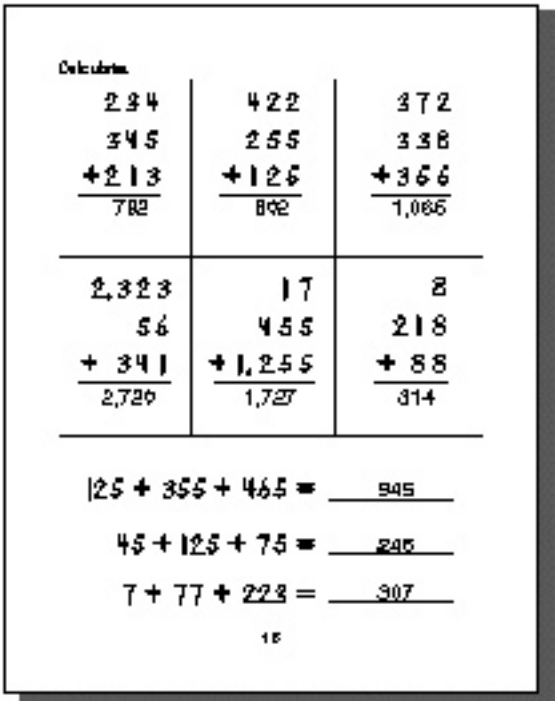
N31



Calculate.

271 + 68 = 339	532 + 516 = 1,048	258 + 742 = 1,000
777 + 213 = 990	74 + 643 = 717	199 + 144 = 343
520 + 299 = 819	926 + 574 = 1,500	355 + 97 = 452

14



Calculate.

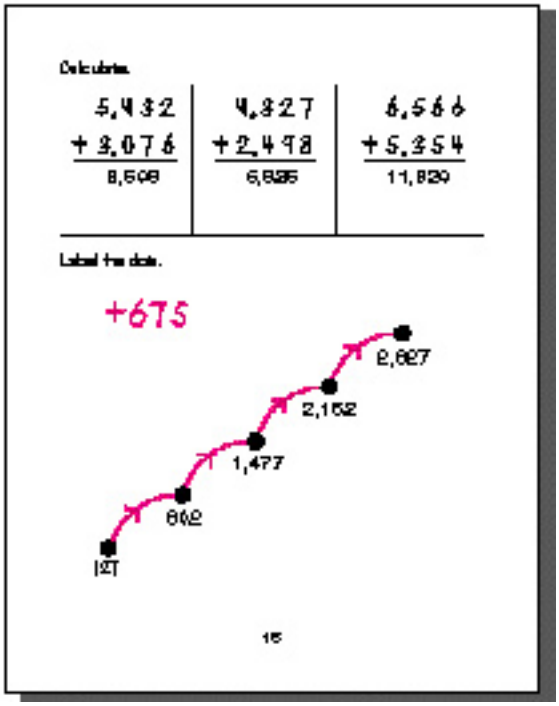
294 + 345 + 213 = 782	422 + 255 + 125 = 802	372 + 338 + 356 = 1,065
2,323 + 56 + 341 = 2,720	17 + 455 + 1,255 = 1,727	8 + 218 + 88 = 314

125 + 355 + 465 = 945

45 + 125 + 75 = 245

7 + 77 + 223 = 307

15



Calculate.

5,432 + 3,076 = 8,508	4,327 + 2,498 = 6,825	6,566 + 5,254 = 11,820

Label the dots.

+675

2,827

2,152

1,477

802

127

15

Capsule Lesson Summary

Collectively, build a road from 0 to 14, using +5 and –3 arrows. Individually, build a road from 10 to 25 using +3 and –2 arrows.

Materials

Teacher	• Colored chalk	• Unlined paper • Worksheets N32 * and **
Student	• Colored pencils, pens, or crayons	

Description of Lesson

Exercise 1

Draw and label two dots, one for 0 and one for 14.

T: ***Today we are going to tell a robot how to go from 0 to 14. Suppose the only directions the robot understands are these:***

- ***Go forward five steps (+5).***
- ***Go backward three steps (–3).***

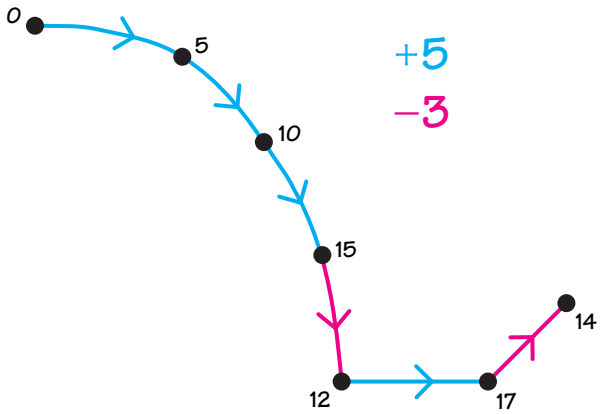
Write +5 in blue and –3 in red near the dots.

T: ***Let's show our directions to the robot in an arrow road. We can use blue +5 or red –3 arrows. What direction should we give the robot first? Which kind of arrow should we use to start the road?***

Take whatever suggestion is made and draw the arrow yourself. Point to the ending dot of this arrow.

T: ***What number is here?*** (5, if a +5 arrow was drawn; $\widehat{3}$ if a –3 arrow was drawn)

Label the dot. Call on volunteers to complete the road, one arrow at a time. This is one possible road from 0 to 14.



Exercise 2

N32

Each student should have unlined paper, a red pencil, and a blue pencil. Put this information on the board and ask students to copy it on their papers. Students should then proceed to draw an arrow road from 10 to 26 using +3 or –2 arrows.

Walk around as the students work, giving assistance to those who need it. As the rest of the class continues to work, ask two or three students, with different numbers of arrows in their roads, to copy their pictures on the board. When most of the students have finished, call the class's attention to the examples on the board. Check each picture on the board with the class to see that it is correct. Point out that there are many correct solutions to this problem.

Ask which road on the board has the fewest arrows and whether anyone has a road with even fewer arrows. Allow some discussion of how to draw the shortest (fewest number of arrows) road. Worksheets N32* and ** are available for independent work. Students who finish quickly can build all or some of the following roads on unlined paper.

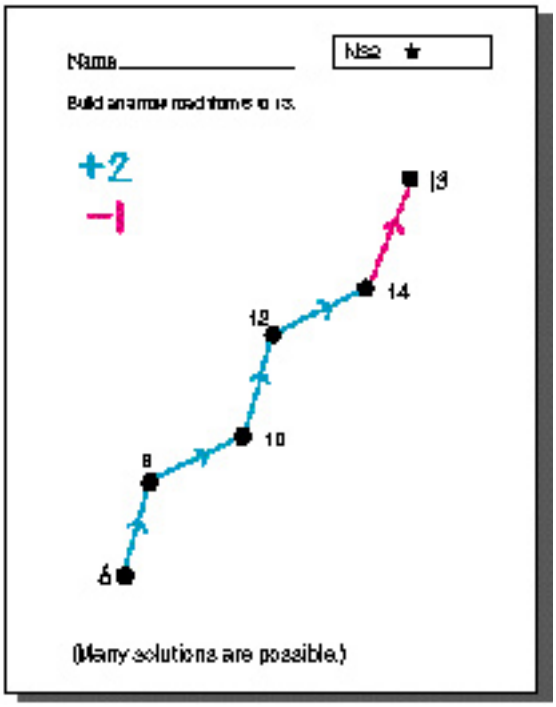
1) Build a road from 0 back to 0 using –5 or +2 arrows.
2) Build a road from 0 back to 0 using –4 or +3 arrows.
3) Build a road from 61 back to 61 using –4 or +3 arrows.

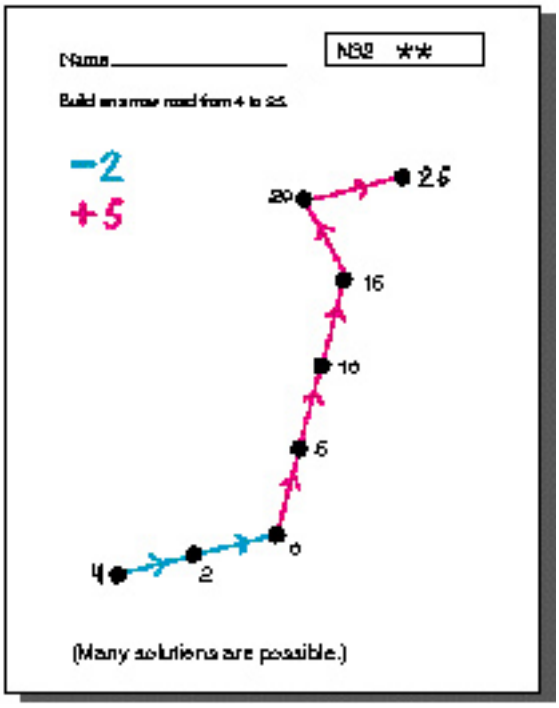
Center Activity

Describe a calculator game for students to play in pairs. Start by putting 21 on the display of the calculator. Then players take turns choosing either [+] [3] [=] or [–] [2] [=]. The first person to get 0 on the display wins.

Home Activity

You may like to repeat (or modify) the home activities from Lessons N16 and N24.


Name
N32 ★
+2
−1
6
8
10
12
14
16
(Many solutions are possible.)


Name
N32 ★★
−2
+5
4
2
6
8
10
16
20
26
(Many solutions are possible.)

N33 ADDITION ALGORITHM #2

Capsule Lesson Summary

Explore the effect of moving, removing, or adding some checkers to a configuration on the Minicomputer. In each case, does the numerical value increase, decrease, or stay the same? Estimate a number on the Minicomputer and then make trades, stopping periodically to get better estimates, until the standard configuration is obtained. Practice using the addition algorithm previously introduced to do addition calculations, such as 453 + 276.

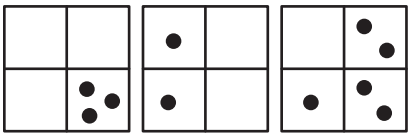
Materials

Teacher	• Minicomputer set • Colored chalk	• *Addition Problems* Booklet[†]
Student	• Index cards	

Description of Lesson

Exercise 1: Transforming a Number

Put this configuration on the Minicomputer.



T: ***I put a number on the Minicomputer. Is the number more than 100? How do you know?***

S: ***Yes, because there is 300 on the hundreds board.***

T: ***Is this number more than 300? How can you tell?***

S: ***Yes, because there is 300 on the hundreds board and more checkers on other boards.***

T: ***Is this number more than 400? How can you tell?***

S: ***Yes, because there is 100 on the tens board and 300 + 100 = 400. There are still more checkers on the ones board.***

T: ***Do you think this number is more than 500?*** (No)

To answer these questions, your students might point to some of the checkers or suggest some trades.

T: ***We do not know exactly what number is on the Minicomputer, but we do know that it is between 400 and 500. We do not need to figure out this number exactly to compare it to other numbers.***

Write these words on the board close to the Minicomputer.

More
Same
Less

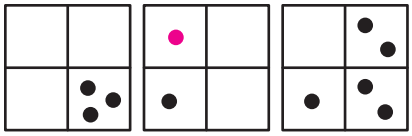
You may like to give students index cards on which to write these words for themselves.

[†]The *Addition Problems* Booklet was used previously in Lesson N31 *The Addition Algorithm #1*. The answer key for this booklet follows that lesson.

T: ***I am going to change this configuration of checkers in some way. I might move checkers, take checkers away, or add more checkers. Each time, you tell me if the number on the Minicomputer is more, less, or the same as this number.***

Decide on a method for students to respond. For example, they can hold up their index cards pinching on their choice, or you might want to invent some signs for students to show more, less, or same.

Replace one of the checkers by a checker of another color.



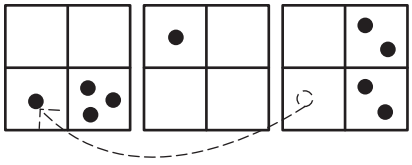
T: ***More, less, or same?***

S: ***Same.***

T: ***Why?***

S: ***Because all you did was exchange a checker for a different-colored checker.***

Move the checker from the 2-square to the 200-square.



T: ***More, less, or same?***

S: ***More.***

T: ***Why?***

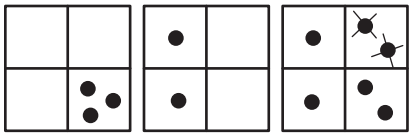
S: ***Because you took a checker off the 2-square and put it on the 200-square. 200 is more than 2.***

Return the checkers to their original positions.

T: ***Who can move one checker and make the number less?***

Ask a student to do this and discuss why the move makes the number less. Then return the checkers to their original positions.

Remove two checkers from the 4-square and put one checker on the 8-square.

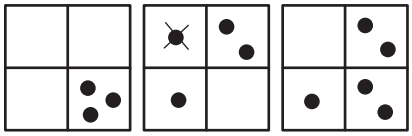


S: ***Same.***

T: ***Why?***

S: ***4 + 4 = 8.***

Return the checkers to their original positions. Then remove the checker from the 80-square and put two checkers on the 40-square.



S: ***Same.***

T: ***Why?***

S: ***80 = 40 + 40.***

Return the checkers to their original position.

T: ***How can we make the number easier to read?*** (Make some trades)

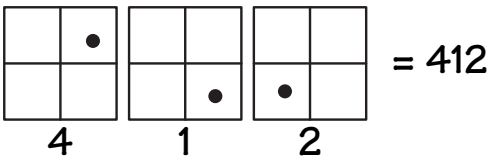
Ask students to name the trade they intend to make before they move any checkers. For example:

S: ***4 + 4 = 8.***

S: ***100 + 100 = 200.***

Be sure students understand that any trade they make must leave the number the same. Be prepared to make a backward trade yourself if one is needed and the students do not suggest one.

Continue until you obtain the standard configuration for 412. Be sure to include new students in making trades. Read the number with the students and write it below the boards and also to the right.



Exercise 3: Addition Problem

Tell the class an addition story problem. You may like to choose your own context and perhaps a student to star in the story. For example:

T: ***Manuel collects baseball cards. This year he got 453 cards with American League players and 276 cards with National League players. How many cards did he collect this year? What calculation should we do?***

S: ***Add 453 + 276.***

Write this addition calculation on the board.

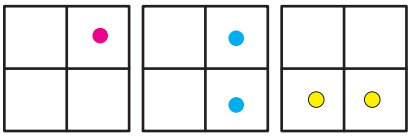
453 + 276 = ?

T: ***What number is 453 + 276?***

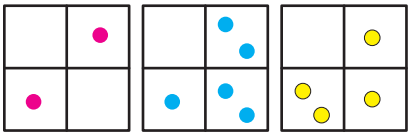
Record some students' estimates on the board.

T: ***Let's add 453 and 276 on the Minicomputer.***

Put 453 on the Minicomputer. Use red checkers on the hundreds board, blue checkers on the tens board, and yellow checkers on the ones board.



Add 276 to the 453 on the Minicomputer, using the same color scheme.



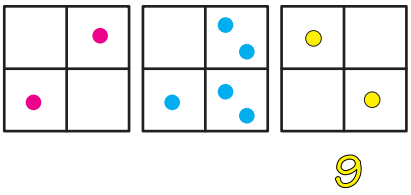
Write the problem vertically on the board with colored numerals.

453
+ 276

T: ***What number do you see on the ones board?*** (9)
Can we make a trade? (Yes)

N33

Let students make trades on the ones board only.



T: ***How many ones are on this board*** (point to the ones board)***?***

S: ***Nine.***

Ask a student to write 9 in yellow below the ones board. Point to the ones column of the addition problem.

T: ***What do you see in the ones column?***

S: ***3 and 6.***

T: ***3 + 6 = . . . ?***

S: ***9.***

T: ***What do you see in the tens column?***

S: ***5 and 7.***

T: ***Five tens plus seven tens is . . . ?***

S: ***Twelve tens.***

T: ***What should we do now?***

$$\begin{array}{r} 453 \\ +\ 276 \\ \hline 9 \end{array}$$

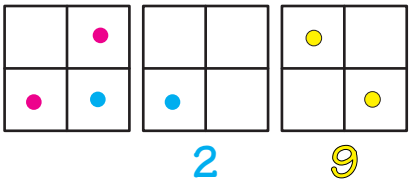
Allow the students to make suggestions, but encourage them to look at the tens board of the Minicomputer. If a student says there is another hundred, pursue the idea.

T: ***Let's see what happens on the Minicomputer.***
How many tens are on this board (point to the tens board)***?***

S: ***Twelve.***

T: ***Can we make a trade?*** (Yes)

Call on students to make the appropriate trades and then to write 2 in blue below the tens board.



T: ***How many tens are on this board now?***

S: ***Two.***

T: ***Before we had twelve tens and now there are only two tens. What happened?***

S: ***Ten tens is 100, and there is another checker on the hundreds board now.***

Be sure the students understand that the other ten tens are now on the hundreds board. Point to the addition problem written on the board.

T: ***How do we show that we have another hundred on the hundreds board.***

S: ***Write 1 above 4 in the hundreds column.***

T: ***This blue checker shows us that we have one more hundred on the hundreds board. I will write a 1 above the hundreds column. This shows that we have one more hundred. We write 2 below the tens column because there are two tens left on the tens board. Five tens plus seven tens is twelve tens and that is the same as one hundred and two tens.***

$$\begin{array}{r} {\scriptstyle 1} \\ 453 \\ +\ 276 \\ \hline 29 \end{array}$$

T: ***What do you see in the hundreds column?***

S: ***1, 4, and 2.***

T: ***100 + 400 + 200 = …?***

S: ***700.***

Point to the hundreds board of the Minicomputer.

T: ***How many hundreds are on this board?***

S: ***Seven.***

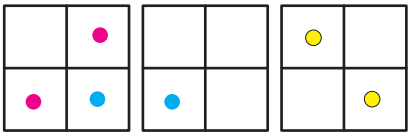
Write 7 in red below the hundreds board and below the hundreds column of the addition problem.

T: ***What number is 453 + 276.***

S: ***729.***

Complete the number sentence on the board. Compare the students' estimate with the answer and decide which guesses were the closest.

453 + 276 = 729



7 2 9

$$\begin{array}{r} {\scriptstyle 1} \\ 453 \\ +\ 276 \\ \hline 729 \end{array}$$

Distribute the *Addition Problems* Booklets and allow about 10 to 15 minutes for independent work. Encourage students to correct any errors they made the previous week. Allow students to use individual Minicomputers if they wish. After the lesson, collect the booklets, check them, and keep them for use in *UPG-II* lessons on the addition algorithm.

N34 PATTERNS GENERATED BY +5 AND –5

Capsule Lesson Summary

Use return arrows to label the dots in two arrow pictures: one with +5 arrows and one with -5 arrows. Observe a pattern to the numbers that are in these arrow pictures.

Materials

Teacher	Student
• Overhead or classroom calculator • Colored chalk • 0–109 numeral chart	• Paper • Colored pencils, pens, or crayons • Worksheets N34* and **

Description of Lesson

Exercise 1: Mental Arithmetic

Indicate to your class an order for this activity that is natural to their seating arrangement; i.e., indicate which person is first, second, and so on.

T: ***Today we are going to count by fives. I will say, "0"; the first student should say, "5," because 0 + 5 = 5; the second should say, "10," because 5 + 5 = 10; the next, "15," because 10 + 5 = 15; and so on. Try to remember the number you say.***

Determine how many students are present.

T: ***If everyone has exactly one turn, what number will the last person say?***

Record some of the students' predictions on the board.

Say zero, and then walk around the room as students count by fives until each has said a number. Make corrections as necessary. Compare the predictions to the last number said, commending those students who had correctly predicted which number would be said last.

T: ***What might we use counting by fives to count?***

S: ***Tally marks.***

S: ***Amount of money in nickels.***

T: ***Now we are going to count by fives again, but this time I am going to start with 1. Each person will say the number that is five more than the last number. What number do you think the last person will say this time?***

Record some of the students' predictions on the board.

Say one, and then go around the room (in the same order as last time) counting by fives until each has said a number. To help a student who is having difficulty, hold up five fingers and help the student count (by ones) from the previous number. Compare the predictions with the last number said, commending those students who correctly predicted which number would be said last.

T: ***Did anyone notice a pattern when we started at 1?***

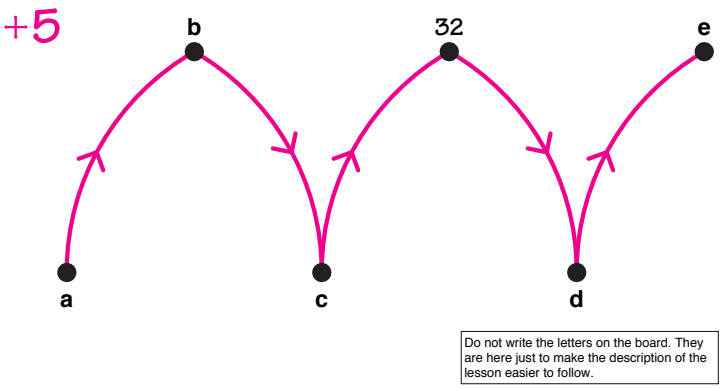
S: ***Everyone said a number 1 more than last time we counted.***

S: ***The numbers we said always ended in 1 or 6.***

Use a counting calculator (counting by fives starting at 1) to repeat the count and to observe the pattern again. You may also like to observe this pattern using the 0–109 numeral chart.

Exercise 2

Draw this arrow picture on the board



T: ***Where is the greatest numbe in this arrow picture?*** (At **e**) ***Write the greatest number or your paper (or whisper it to*** (42) ***Where is the least num this arrow picture?*** (At **a**) ***Write the least number on yo paper (or whisper it to me).***

Do not announce the least and greatest numbers now. Later, when all the dots are labeled, you can check which numbers these are.

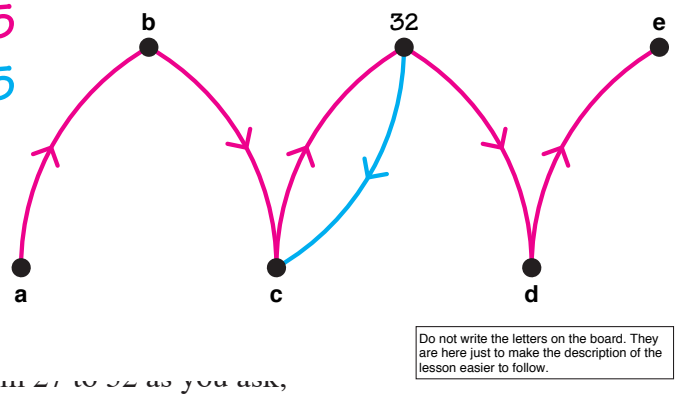
Point to **a**, **b**, and **c**.

T: ***How can we label these dots?***

Guide the discussion so that it is clear you do not follow +5 arrows from 32 to any of those three dots. Students may suggest drawing –5 arrows or, if necessary, make this suggestion yourself.

T (pointing to **c**)**:** ***Who can draw a –5 arrow which will help us label this dot?***

Let a student first trace an arrow fro +5
32 to **c** and then draw the arrow in b
Write –5 in blue near the arrow pictu –5



Point to 32 and trace the –5 arrow.

T: ***32 – 5 = … ?***

S: ***27.***

Label the dot 27. Trace the +5 arrow from 27 to 32 as you ask,

T: ***27 + 5 = …?***

S: ***32.***

T: ***Who can show us a way to go from 32 back to 32, always following arrows?***

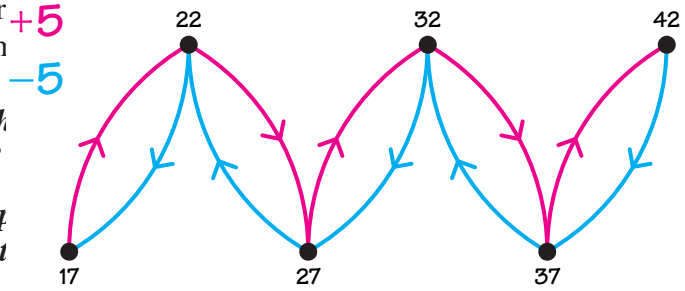
Whoever volunteers should start at 32, trace the –5 arrow to 27, and then trace the +5 arrow back to 32. Repeat the tracing yourself as you say,

T: ***32 – 5 + 5 = 32.***
Do you see a way we could follow arrows going from 27 back to 27?

Call on a student to trace the arrows and then repeat the tracing yourself.

T: ***27 + 5 – 5 = 27.***
–5 followed by +5 or +5 followed by –5 takes us back to the starting number.

Invite students to either label dots or dr
–5 arrows until the arrow picture is con



T: ***Does anyone see a pattern to th***
numbers in the arrow picture?

S: ***The numbers at the top of the p***
end with 2, and the numbers at
bottom end with 7.

T: ***If we were to draw more +5 arrows going on from 42, what are some numbers we would meet?***

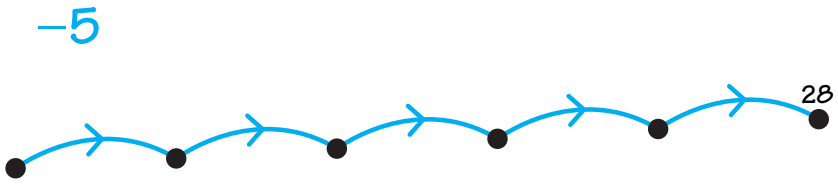
S: ***57.***

S: ***92.***

Any whole number greater then 42 with 2 or 7 in the ones place is a correct answer. Encourage students to suggest a wide range of numbers.

Exercise 3

Draw this arrow picture on the board and ask students to copy it. Direct them to add to their picture as is done on the board.



T: ***Where is the least number in this arrow picture?*** (At the right) ***What is the least number?*** (28)

Where is the greatest number in this arrow picture? (At the left) ***Write the greatest number on your paper (or whisper it to me).***

The last question may be challenging. Do not announce which number is the greatest yet.

T: ***If we follow these arrows, will we be able to label the other dots?***

S: ***No, they go in the wrong direction.***

T: ***What can we do to help us label the other dots?***

S: ***Draw return (opposite) arrows.***

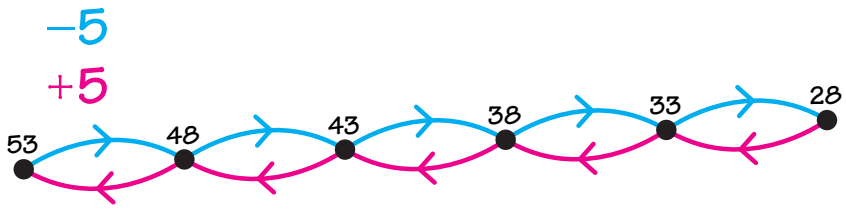
Draw a return arrow in red, starting at 28.

T: ***What could this arrow be for?***

S: ***+5.***

Write +5 in red near the arrow picture.

Call on volunteers to either label a dot or to draw another + 5 arrow until the arrow picture is completed. Observe that 53 is the greatest number in this arrow picture and ask students who predicted this how they found the answer. Accept any reasonable explanation.



T: ***Do you notice a pattern to the numbers in the picture?***

S: ***All the numbers end in 3 or in 8.***

T: ***If we were to draw more +5 arrows going on from 53, what are some other numbers we would meet?***

S: ***58.***

S: ***73.***

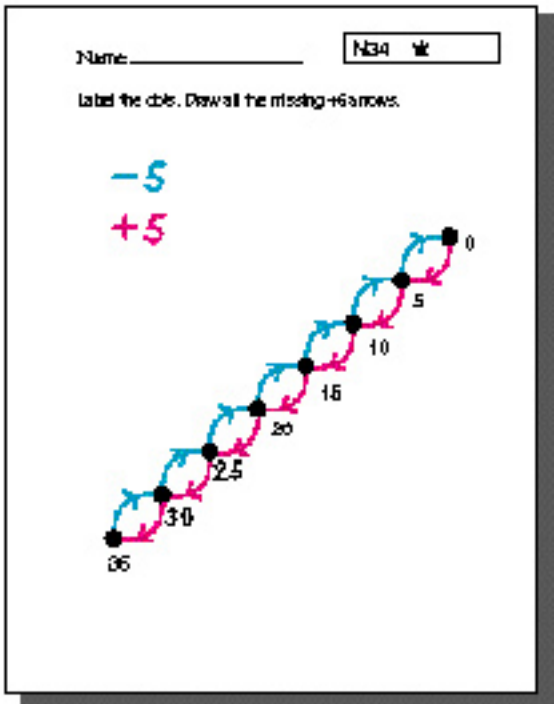
Any whole number greater than 53 with 3 or 8 in the ones place is correct. Encourage students to suggest a wide range of numbers.

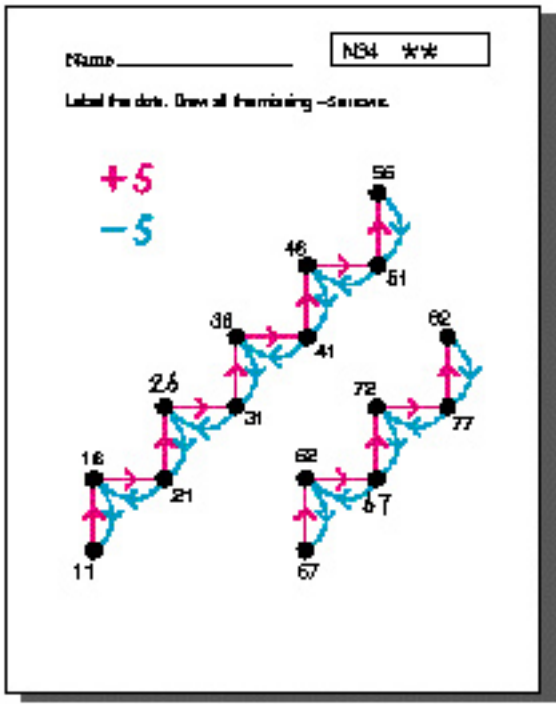
Extension: You may like to pursue with the class what happens if you extend the arrow picture in the other direction; i.e., with more –5 arrows going out from 28. This extension can take you into negative numbers where the pattern changes from positive numbers with 3 or 8 in the ones place to negative numbers with 2 or 7 in the ones place. A counting calculator (counting backward by fives) will help to look at this pattern.

Worksheets N34* and ** are available for independent work. The students should draw all the return arrows in these arrow pictures.

Home Activity

Suggest parents/guardians explore with their child the pattern of numbers generated when they count by fives starting at 0, 1, 2, 3 or 4. Occasionally list sequences of numbers (or use a counting calculator—counting by fives starting at 0, 1, 2, 3, or 4) and observe patterns.


Name:
N34 ★
Label the dots. Draw all the missing +5 arrows.
−5
+5
0
5
10
15
20
25
30
35


Name
N34 ★★
+5
−5
56
46
51
36
41
62
26
31
72
77
16
21
62
67
11
67

Capsule Lesson Summary

Review the trade which replaces ten checkers on a square with one checker on the square of the same color, on the next board to the left. Investigate ways of putting 100, 40, and 25 on the Minicomputer using exactly ten checkers.

Materials

Teacher	• Minicomputer set	Student	• Minicomputer set

Description of Lesson

Exercise 1: Multiplying by Ten

Beginning in first grade, *CSMP* students were introduced to the three binary rules (trades) of the Minicomputer: 1 + 1 = 2; 2 + 2 = 4; 4 + 4 = 8. Soon they learned to jump from the first board to the second with the decimal rule: 8 + 2 = 10.

Gradually they realized that these four rules can be extended to all of the Minicomputer boards.

1 + 1 = 2	10 + 10 = 20	100 + 100 = 200	
2 + 2 = 4	20 + 20 = 40	200 + 200 = 400	
4 + 4 = 8	40 + 40 = 80	400 + 400 = 800	
8 + 2 = 10	80 + 20 = 100	800 + 200 = 1,000	and so on.

In the language of the Minicomputer, these basic facts can be verbalized as follows. On each board:

Two checkers on the white square
is the same as
one checker on the red square.

Two checkers on the red square
is the same as
one checker on the purple square.

Two checkers on the purple square
is the same as
one checker on the brown square.

From board to board:

One checker on the brown square and one checker on the red square
is the same as
one checker on the white square of the next board to the left.

From the beginning, the students' numerical thinking has been supported by the binary structure of the Minicomputer boards.

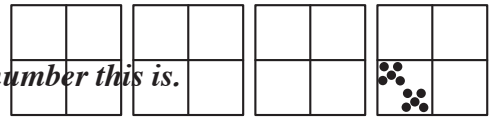
Now it is time to introduce a new trade that is a consequence of the four Minicomputer rules and is a key to the decimal positional system.

Ten checkers on a square
is the same as
one checker on the same color square of the next board to the left.

Developing this fact will be the goal of several lessons, mostly in the N-strand of *UPG-II*. Experience has shown that some students find the new idea difficult to grasp. Therefore, be patient; do not expect all students to assimilate this idea at the same moment. About 20 minutes of each of many lessons will be devoted to this topic in order to allow the student's thinking to mature in a spontaneous and natural way.

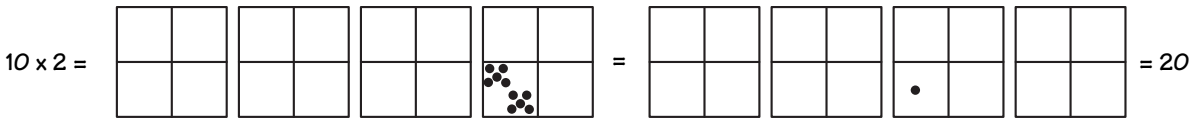
Put ten checkers on the 2-square of the Minicomputer.

T: ***What number is this?***
What multiplication fact could we write about this number?
Do we need to make some trades to know what number this is.



Let the students react freely to these questions. Many of them already know that 10 x 2 = 20 and do not need to make trades on the Minicomputer in order to find the answer. Nevertheless, invite students to make trades until you obtain the standard configuration for 20. Try to keep this part of the lesson moving quickly and involve many students in making trades.

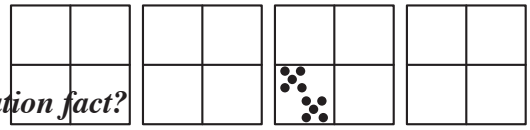
T: ***Ten checkers on this red square (2-square) is the same as one checker on this red square (20-square).***



Record 10 x 2 = 20 on the chalkboard.

Put ten checkers on the 20-square of the Minicomputer.

T: ***What multiplication fact could we write about this number?***



S: ***10 x 20 = ?***

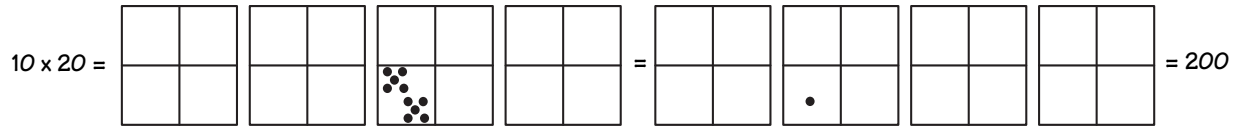
T: ***Is this very different from the previous multiplication fact?***

S: ***No, because we still have ten checkers on a red square. The only difference is they are on the tens board instead of the ones board.***

T: ***After we make trades on the Minicomputer, can we get just one checker on the Minicomputer? Where would this checker be?***

S: ***On the 200-square.***

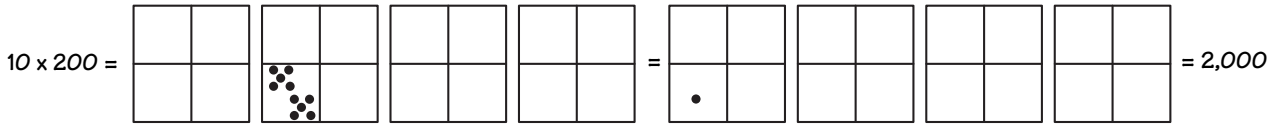
Invite students to make the appropriate trades until they obtain the standard configuration for 200.



T: ***Ten checkers on this red square (20-square) are the same as one checker on this red square (200-square).***

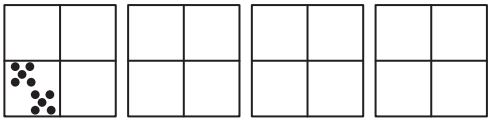
Record 10 x 20 = 200 on the chalkboard under 10 x 2 = 20.

Similarly, let the class predict and discover that 10 x 200 = 2, 000.



Put ten checkers on the 2,000-square of the Minicomputer.

T: ***What multiplication fact could we write about this number?***



S: ***10 x 2,000 = ?***

T: ***What number is this?***

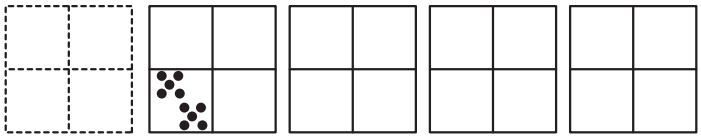
Write this problem in your list on the board.

10 x 2 = 20
10 x 20 = 200
10 x 200 = 2,000

Instruct the students to write the answer on their papers.

T: ***Could we find out what this number is without making any trades? If we make trades until there is only one checker remaining on the Minicomputer, where would this checker be?***

Let the students react freely to these questions. The students will probably tell you that they do not need to make trades but that they do need another Minicomputer board. When the students ask for a fifth Minicomputer board, draw it on the chalkboard.



T: ***Ten checkers on this red square (2,000-square) are the same as one checker on which square?***

Call on a student to point to the appropriate square (20,000-square).

Take all the checkers off the 2,000-square and put (or draw) one checker on the 20,000-square.

T: ***What number is this?***

S: ***20,000.***

It is possible that no one will know how to read this number, but some students may know how to write it.

T: ***Who can write this number below (above) the Minicomputer?***

Students may be able to write the numeral correctly but misplace the comma. Perhaps another student will be able to place the comma correctly or, if necessary, insert the comma yourself. Explain that the comma helps us read large numbers and that it goes between the hundreds place (board) and the thousands place (board).

T: ***Who can read this number?***
What number fact should we write on the board?

Help the students to read 20,000 and complete the number fact in your list.

T: ***Do you notice a pattern?***

S: ***Each time there is another 0.***

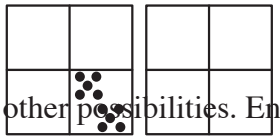
$10 \times 2 = 20$
$10 \times 20 = 200$
$10 \times 200 = 2{,}000$
$10 \times 2{,}000 = 20{,}000$

Exercise 2: Individual Minicomputers

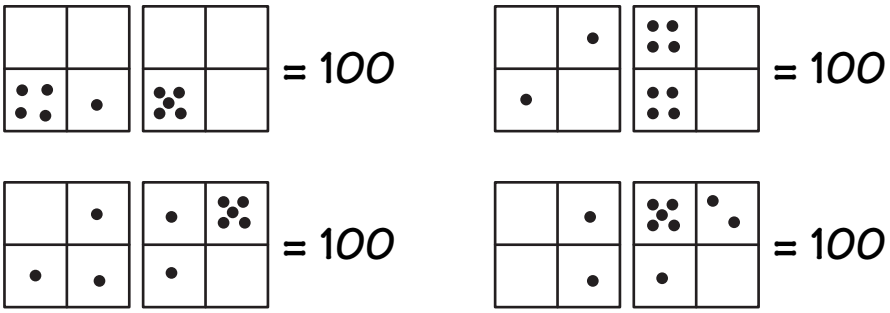
You may wish to allow students to work in pairs for this exercise. Each student (pair) will need one individual Minicomputer sheet (two boards) and at least ten positive checkers.

T: ***Can you put 100 on your Minicomputer using exactly ten checkers?***

Probably many students will put this configuration on their Minicomputers.

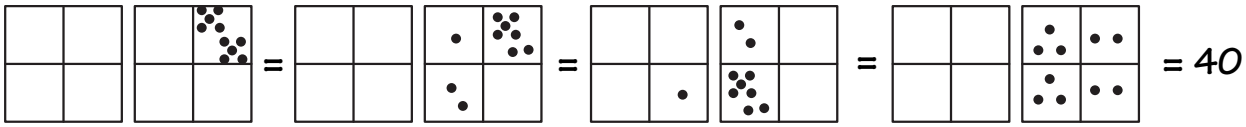


This is the most obvious and an important solution, but there are many other possibilities. Encourage students to find others. A few solutions are given below.



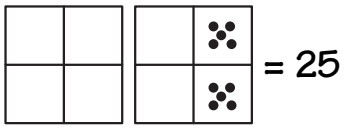
As different solutions are found, ask a student to put them on the demonstration Minicomputer. You may wish to discuss some of the solutions with the entire class.

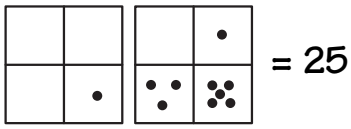
Repeat this exercise asking for 40 with ten checkers. Here are a few (of many) possible solutions:

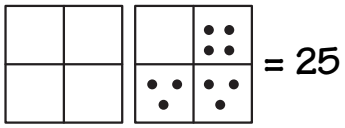


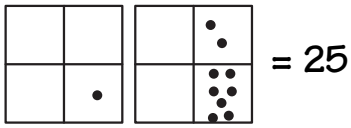
T: ***Can you put 25 on your Minicomputer with exactly ten checkers?***

Let the students work on their own for several minutes. As different solutions are found, ask a student to put them on the demonstration Minicomputer. You may wish to discuss some of the solutions with the entire class. Several solutions are given below.









Home Activity

Suggest a couple Minicomputer problems for parents/guardians to work on with their child.

- Find several different ways to put 50 on the Minicomputer with ten checkers.
- Put the number 14 (or 24 or 44) on the Minicomputer with two checkers; …three checkers;… four checkers;… five checkers;… ans so on.

Capsule Lesson Summary

Start the lesson with a mental arithmetic exercise involving +10. Explore the composition of +10 and –1 functions through the use of an arrow picture. Use knowledge of the +10 function to gain familiarity with the +9 function; for example, (7 + 10) –1 = 7 + 9.

Materials

Teacher	• Colored chalk	Student	• Paper • Colored pencils, pens, or crayons

Description of Lesson

Exercise 1: Mental Arithmetic

T: ***I will say a number and you will tell me the number that is ten more than the number I say.***

Choose numbers that fit the abilities of your students, alternating problems that are difficult with problems that are easy. When a student answers correctly, say the appropriate addition number fact. For example:

T: ***8.***

S: ***18.***

T: ***8 + 10 = 18.***
18.

S: ***28.***

T: ***18 + 10 = 28.***
28.

S: ***38.***

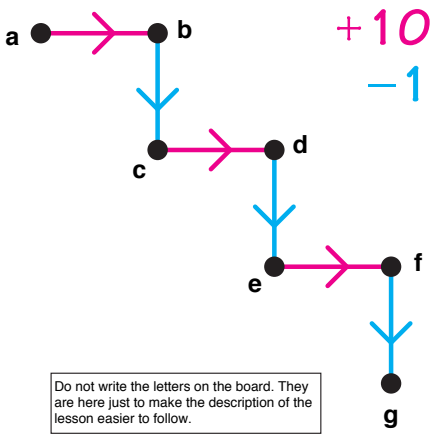
Continue in this way for a few minutes.

T:	***7.*** (17)	***25.*** (35)	***90.*** (100)	***92.*** (102)
	7 + 10 = 17.	***25 + 10 = 35.***	***90 + 10 = 100.***	***92 + 10 = 102.***
	97. (107)	***140.*** (150)	***163.*** (173)	***215.*** (225)
	97 + 10 = 107.	***140 + 10 = 150.***	***163 + 10 = 173.***	***215 + 10 = 225.***

N36

Exercise 2

Draw this arrow picture on the board.

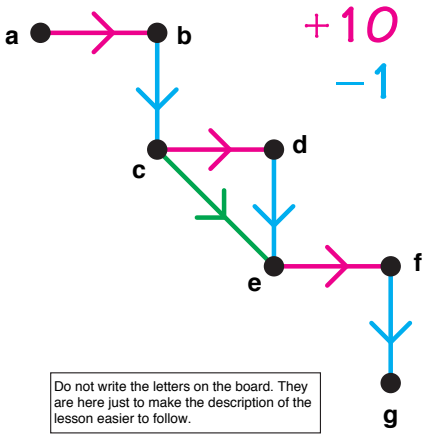


T: ***Where is the least number in this arrow picture?*** (At **a**)
How do you know where the least number is?

This may be very hard for students to verbalize, so do not expect well-formed explanations. If the students appear to be focusing only on the +10 arrows, remind them of the –1 arrows. Perhaps your class will conclude that they do not know where the least number is; others will be convinced it is at **a**.

If your class is fairly sure that the least number is at **a**, ask them to locate the greatest number in the arrow picture. (At **f**) Students might suggest that the greatest number is at **g**. Again, do not expect well-formed answers. Someone might see that the number at **g** cannot be the greatest because the number at **f** is one more.

Draw a green arrow from **c** to **e**.



T: ***What could this green arrow be for?*** (+9)
How do you know?

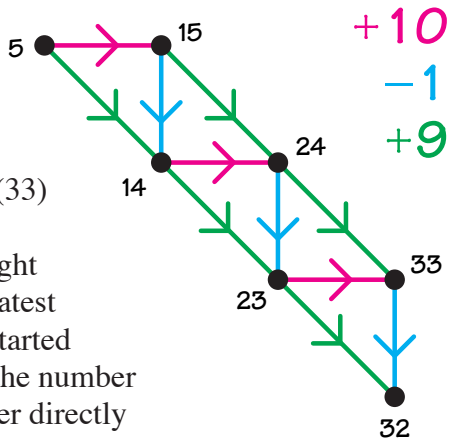
This question may be very difficult. Commend any student who says that adding ten and then taking away one is the same as adding nine. If none of the students know the green arrow could be for +9, do not reveal this yourself but continue with the lesson.

T: ***Let's label some dots. I will tell you that one of these numbers is 15.***

Put 15 at **b** and call on students to label the other dots. Each time a dot is labeled ask for an explanation. When your class is confident that the green arrow is for +9, write +9 in green near the arrow picture. Continue until all the dots are labeled.

T: ***Who can draw other green +9 arrows?***

Call on volunteers to draw +9 arrows.



T: ***What is the least number in this arrow picture?*** (5)
What is the greatest number in this arrow picture? (33)

If your class thought that the number on the extreme lower right would be the greatest, you may wish to ask why 33 is the greatest number in this arrow picture and what would happen if you started with another number instead of 15. Pursue a suggestion that the number on the extreme lower right will always be less than the number directly above it in this arrow picture.

Exercise 3

T: ***What number is 5 + 10?***

S: ***15.***

T: ***Yes, 5 + 10 = 15. So what number is 5 + 9? How do you know?***

S: ***14, because 5 + 9 is one less than 5 + 10.***

Point to 5 in the arrow picture on the board from the previous exercise and trace the appropriate arrows as you say,

T: ***5 + 9 is the same as (5 + 10) – 1.***
What number is 7 + 10?

S: ***17.***

T: ***Yes, 7 + 10 = 17. So what number is 7 + 9? How do you know?***

S: ***16, because 17 – 1 = 16.***

T: ***7 + 10 = 17, so 7 + 9 = 16.***

S: ***What number is 24 + 10?***

S: ***34.***

T: ***Yes, 24 + 10 = 34. So what number is 24 + 9? How do you know?***

S: ***33, because 34 – 1 = 33.***

T: ***What number is 38 + 10?***

S: ***48.***

T: ***If 38 + 10 = 48, what number is 38 + 9?***

S: ***47.***

Continue this activity for a few minutes, occasionally asking for an explanation. Choose problems that are appropriate for the abilities of your students.

Exercise 4 (optional)

If there is time remaining, repeat Exercises 2 and 3 with –2 in place of –1; that is, consider +10 followed by –2 is + 8.

Additional Practice

Instruct students to copy the picture in Exercise 2 without labeling the dots. Then put a number, for example 30, at **d** and let the students label the remaining dots.

Writing Activity

Ask students to write about an easy way to add 9 to a number. Help them choose some calculations to work with in their short essays; for example, 38 + 9 or 17 + 9.