Geometry & Measurement

GEOMETRY & MEASUREMENT TABLE OF CONTENTS

GEOMETRY & MEASUREMENT INTRODUCTION

Geometry is useful and interesting, an integral part of mathematics. Geometric thinking permeates mathematics and relates to most of the applications of mathematics. Geometric concepts provide a vehicle for exploring the rich connections between arithmetic concepts (such as numbers and calculations) and physical concepts (such as length, area, symmetry, and shape). Even as children experience arithmetic notions in their daily lives, they encounter geometric notions. Common opinion recognizes that each child should begin developing a number sense early. There is no sound argument to postpone the development of geometric sense. These are compelling reasons why the systematic development of geometric thinking should be a substantial component of an elementary school curriculum.

A benefit of early development of geometric thinking is that it provides opportunities for all learners. Children develop concepts at individual rates. We are careful to allow students a great deal of time to explore quantitative relationships among physical objects before studying arithmetic. In the same spirit, students should be allowed to explore geometric relationships through free play and guided experiences. Before studying geometric properties of lines and shapes, they should have free play with models and with drawing instruments, using both to create new figures and patterns. Before considering parallelism and perpendicularity as relations on geometric lines, they should have free play in creating patterns with parallel and perpendicular lines. Before learning length and area formulas, they should have informal experiences to establish and sharpen intuitive notions of length and area. These early experiences are aesthetically rewarding and just as stimulating and suggestive of underlying concepts as early experiences leading to arithmetic. Such experiences are a crucial first step in the development of a geometric sense.

As with most educational development occurring through informal experiences, the payoffs come unexpectedly and spasmodically. In the development of geometric thinking, the suggested sequence of activities is so varied, reaches into so many other facets of the program, and continues for so long a span that it would be nonsense to set *a priori* milestones of accomplishments.

Thus, it is important to allow students to move through the free play activities that precede the more substantive lessons without rushing them. These preliminary activities have implicit significance for geometric thinking and also provide aesthetic opportunities and explicit practice— they are not mere play. On the other hand, some activities appear to have important consequences or clearly relate geometry to other parts of the program. Here, your inclination might be to spend more time on these activities than is suggested, but try to remember that similar activities will continue to be experienced later in a slightly different context.

This strand also includes measurement activities, many that lead to the use of metric units, as well as topics not usually found in traditional programs at this level. Measurement activities eventually lead to notions of distance. Traditionally, teachers wait for a good deal of maturity in their students before introducing properties of figures, such as circles, that are defined in terms of distance. The reason is that Euclidean distance (distance "as the crow flies") requires sophisticated notions of "exact" measurement in terms of the real number system in order to be able to measure every line segment. But in the *CSMP* curriculum very young children (even in kindergarten) begin thinking about distance in the simplified setting of "taxi-geometry." Taxi-distance (as a taxicab travels) between points is measured in terms of whole numbers by easy counting processes. Experiences in taxi-geometry lead to some creative situations, yet do not require special technical skills or vocabulary.

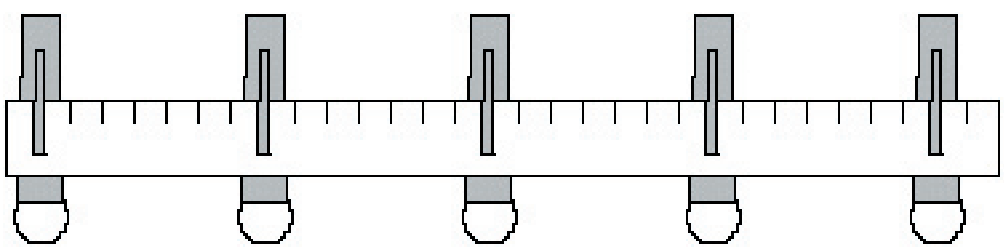
GEOMETRY & MEASUREMENT INTRODUCTION

Putting the emphasis on experience rather than on mastery frees you to engage in imaginative activities with children that you ordinarily might not attempt. For example, even when considering such commonplace topics as area and length, feel free to devise activities that help students recognize patterns, develop cognitive strategies, and relate the topic to others in the program, while suppressing the urgency to get the basic formulas. What is basic here, at this level, is that children are thinking, rather than doing tasks on command. The skills and mastery will come eventually, in their own time and place in the curriculum.

Note on Grids

A demonstration grid board is needed for several lessons, especially those on taxi-geometry. This should be a large square grid board at least 12 squares by 12 squares (with grid lines spaced 4 to 8 centimeters apart) on which you can draw with colored chalk or colored pens. A portable, commercially made, permanent grid chalkboard is ideal. However, if such a grid board is not available, try one of these alternatives.

- Printed grids on large sheets of paper are provided in classroom sets of materials for *CSMP* for the upper primary grades. Tape one of these sheets to the board, use thick felt-tipped pens for drawings, and discard the sheet after use. You may prefer to laminate or mount one of the sheets on cardboard and cover it with clear contact paper to provide a protective surface. Then use erasable markers or wax crayons for drawings. These grid sheets also are useful as graphing mats.
- For each lesson, carefully draw a grid on the chalkboard using an easy-to-make tool. Either (1) use a music staff drawer with chalk in the first, third, and fifth positions; or (2) clip five pen caps, each having the diameter to hold a piece of chalk tightly, to a ruler at intervals of 4–8 centimeters and secure them with tape.



- Make a grid transparency for use on an overhead projector and use erasable overhead markers. The Blackline section includes several size grids to use in making a grid transparency.

Capsule Lesson Summary

Find two objects in the room that are about the same length. Measure the objects in both decimeters (orange units) and centimeters to compare the lengths. Find an object that is about 100 cm long, a second object that is about 10 cm long, and a third object that is about 1 cm long. Use a centimeter tape measure to find the lengths of a selection of other objects in the room.

Materials

Teacher
- Centimeter tape measure
- Orange Cuisenaire rods (optional)

Student
- Paper
- Pencil
- Centimeter tape measure

Description of Lesson

Exercise 1

T: ***I would like to find two objects in this classroom that are about the same length.***

Allow several students to make suggestions; then select two that the class agrees upon and that are accessible. Suppose, for example, students suggest the side edge of the teacher's desk and the bottom edge of the bulletin board.

T: ***How can we check that the side of my desk and the bulletin board have about the same lengths?***

S: ***Measure them with a tape measure.***

Display a tape measure, and review the fact that on one side of the tape an orange rod fits between two orange marks. If you have not used Cuisenaire rods, simply refer to this as "the orange unit."

T: ***Let's measure my desk and the bulletin board in orange rods.***

Invite students to make these measurements and record the results on the board. For example:

Teacher's Desk (side):	between 7 orange and 8 orange
Bulletin Board:	between 7 orange and 8 orange

T: ***It appears that the side of my desk and the bulletin board are about the same length. How could we get a more accurate measurement and perhaps be able to say which one is longer?***

Accept suggestions that indicate measuring with smaller units. Another natural suggestion is to put a mark on the tape measure indicating the length of one of the objects and then comparing it with the length of the other. If this suggestion is made, invite students to do it.

Suppose, for example, that the class determines that the bulletin board is longer than the desk.

T: ***Let's use the other side of the tape measure and measure in centimeters.***

If appropriate, review that one centimeter is the length of a white (Cuisenaire) rod.

Write *centimeter* and its abbreviation *cm* on the board. Ask several students to read the word aloud. Invite students to measure the lengths of your desk and the bulletin board in centimeters and to record the results on the board. For example:

Teacher's Desk (side):	73 cm
Bulletin Board:	76 cm

Compare the measures in centimeters with the measures in orange rods.

Exercise 2

T: ***Now I would like to find something in this classroom that is about 10 orange rods long. How many centimeters is 10 orange rods?***

S: ***100 centimeters.***

T: ***This tape measure is 10 orange rods or 100 centimeters long. What do you see in the room that is about the same length?***

Accept several suggestions and choose one that is accessible. Suppose, for example, a student suggests the front edge of the teacher's desk.

T: ***Do you think the front edge of my desk is longer or shorter than 100 cm?***

Allow several students to comment and then invite one student to make the measurement. If it is longer than 100 cm, you will need to discuss how to mark 100 cm and then measure the excess. Record the result on the board.

Teacher's Desk (front):	114 cm

T: ***If we were to measure the front edge of my desk in orange rods, what would we get?***

S: ***Between 11 orange rods and 12 orange rods.***

If necessary, ask a student to make the measurement in orange rods.

T: ***Now, I would like to find something in this classroom that is about 10 centimeters long. How many orange rods is 10 centimeters?***

S: ***1 orange rod.***

Show the class 10 centimeters on the tape measure or one orange rod; then accept suggestions of objects in the room that are about the same length. Suppose a student suggests his pencil.

T: ***Hold up your pencil, Virgil. Do you think Virgil's pencil is longer or shorter than 10 centimeters?***

Allow several students to comment, and then ask a student to measure the pencil. If necessary, suggest that the student measure to the nearest centimeter. For example, if the pencil is between 10 cm and 11 cm, ask which length is closer. Record the result on the board.

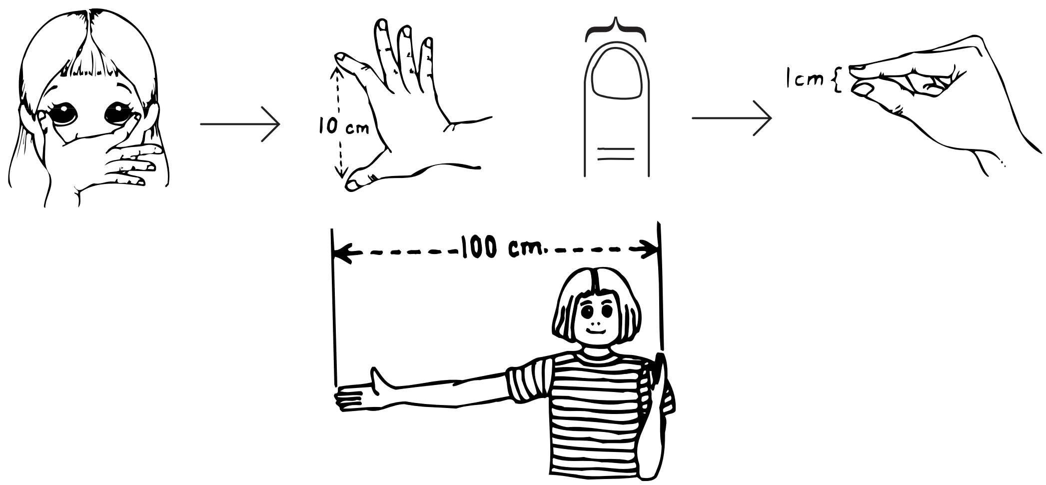
Virgil's pencil: 11 cm

T: ***Now I would like to find something in the room that is about 1 cm long.***

Accept several suggestions and let the students measure the suggested objects. This problem may be more difficult because of the relative difference in length of two very short objects. Also, sometimes it may be hard for students to describe what it is they wish to measure; for example, a student may suggest the diameter of a button.

Erase the board. Ask the entire class to follow these instructions. Observe their gestures and then give a suggestion yourself, as illustrated in the next picture.

T: ***Hold up two fingers so that they are about 10 centimeters apart.***
Hold up two fingers so that they are about 1 centimeter apart.
Hold up your two hands so that they are about 100 centimeters apart.



Distribute tape measures and paper. Ask students to do the following activity with a partner, taking turns, one doing the measuring, and the other doing the checking and writing. On the board, list four or five objects that are easily accessible (for example, an eraser, a paper clip, the width of doorway, the height of desk, the height of chair, a crayon, a longest finger). Choose objects that were not measured in a previous lesson. Then ask the partners to copy the list of objects on their papers, measure the objects to the nearest centimeter, and record their results on the paper.

Home Activity

Help students make a 100 cm string, a 10 cm string, and a copy of the recording chart (Blackline G1) to take home. Working with family members, they can measure different items they think are 10 cm or 100 cm long, and list them in the chart.

Capsule Lesson Summary

Find the lengths (taxi-measure) of shortest paths between two points of a grid. Find all the points that are 5 blocks from a given point along a shortest path and observe that there are twenty such points. Locate a point knowing that it is 3 blocks east and 2 blocks north of a given point.

Materials

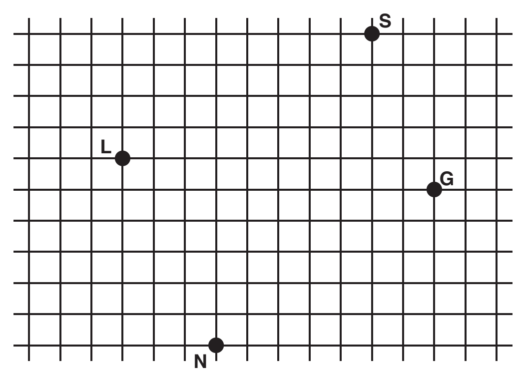
Teacher
- Grid board†
- Colored chalk
- Overhead projector (optional)

Student
- Worksheets G2(a) and (b)
- Colored pencils

Description of Lesson

Exercise 1

Display your grid board. Draw and label four dots as shown here and on Worksheet G2(a). Students should have a copy of the worksheet to follow along during this exercise.



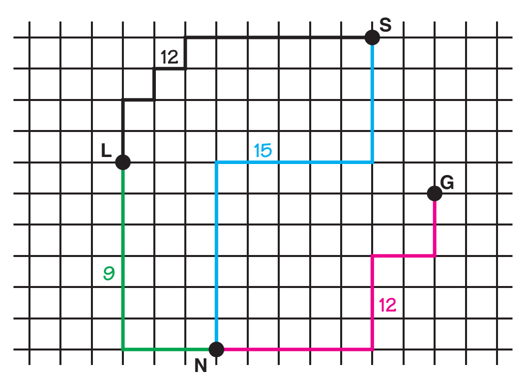
T: ***This is the map of the city where Nora lives. Here is Nora's house*** (point to **N**)***, here is her grandmother's house*** (point to **G**)***, here is Nora's school*** (point to **S**)***, and here is the library*** (point to **L**)***. Who can tell us something about our friend Nora?***

Let students briefly recall some of the previous activities involving Nora. Encourage them to discuss long and short walks and counting the number of blocks in a path. If possible during this discussion, ask students to guess which place, **L**, **S** or **G**, is closest to Nora's house. (**N**)

T: ***Suppose it is raining and Nora wants to take a shortest path from her house*** (**N**) ***to school*** (**S**). ***How many blocks is a shortest path from* N *to* S*? Who can show us a way for Nora to walk?***

Call on a volunteer to trace a path from **N** to **S**. Ask the class if anyone can find a shorter path. When the students agree on a shortest path, draw it on the grid board. Ask a student to count the blocks in this shortest path. (15 blocks)

Repeat the question to get a shortest path from **N** to **G**, and a shortest path from **N** to **L**, and a shortest path from **S** to **L**. Possible responses are illustrated.



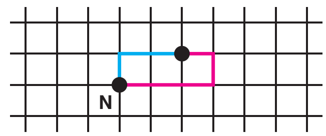
†See the "Note on Grids" section in the introduction to this strand.

Exercise 2

Erase the grid board. Draw a dot near the center of the grid and label it **N** as on Worksheet G2(b). Students should have a copy of the worksheet and colored pencils to follow along during this exercise.

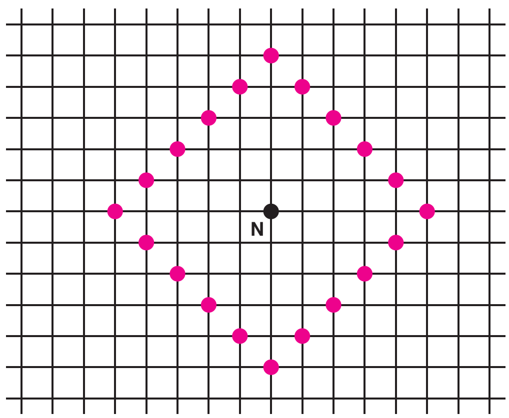
T: ***Nora has a friend, Paula. I'm not going to show you where Paula lives, but I will tell you that a shortest path from Nora's house to Paula's house is 5 blocks. Where could Paula's house be?***

There are twenty points where Paula's house could be (see the illustration below). Call on students to locate some of these points, each time tracing a shortest 5-block path from **N** to a point where Paula's house could be. Draw red dots at correctly located points. You may have to remind students that a shortest walk from **N** to Paula's house is 5 blocks. A student may suggest, for example, the following 5-block path (the red path in the next illustration).



In this case, demonstrate that the red path is not a *shortest* path. There is a 3-block path (in blue) from **N** to this point.

After several points have been located, some students should recognize a pattern. When this happens, encourage them to describe the pattern and perhaps to locate several more points following the pattern. A completed picture should have twenty red dots as shown here.

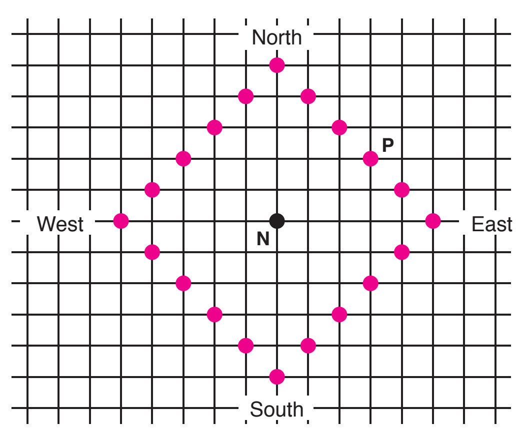


T: ***How many places did we find where Paula's house could be?***

S: ***Twenty.***

T: ***Just one of them is where Paula's house actually is. Do you know which directions are north, south, east, and west on a map?***

Ask students to indicate each of these directions or, if necessary, explain them yourself and write them on the grid board.



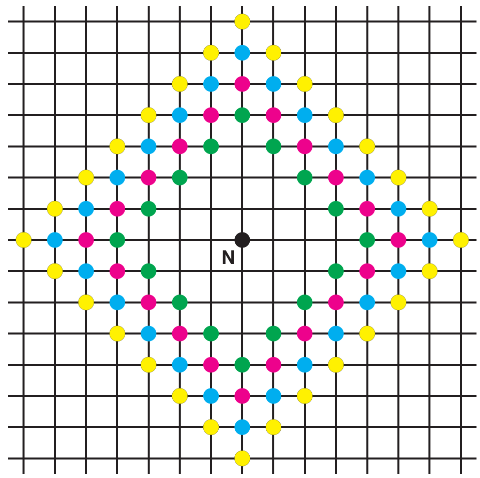
T: ***Now I can tell you how to find Paula's house. A shortest path from Nora's house to Paula's house is 3 blocks east and 2 blocks north.***

Write **3E**, **2N** on the board. Let a student locate Paula's house and label the dot **P**.

T: ***Here is Nora's house, the dot labeled* N*. Nora's friend Maria lives 6 blocks away from Nora's house. A shortest walk from Nora's house to Maria's house is 6 blocks. On your paper draw blue dots at all the places where Maria's house could be.***

Students who have difficulty with this activity will benefit from working with a partner. Both partners can count the blocks before drawing a dot. You may want to have extra copies of the worksheet available for students who make several mistakes.

Students who finish this problem quickly can be asked to find all the places that are 4 blocks or 7 blocks (along a shortest path) from Nora's house. Use different colors for these places. The picture below has blue dots for places 6 blocks from **N**, green dots for places 4 blocks from **N**, and yellow dots for places 7 blocks from **N**.



Extension Activity

Recall with the class that there were 20 red dots—places 5 blocks from **N**. Count to find how many blue dots there are—places 6 blocks from **N**. (24) How many green dots are there—places 4 blocks from **N**? (16) How many yellow dots are there—places 7 blocks from **N**? (28) Put this information in a chart, and ask students to predict how many places they would find 10 blocks from **N**.

4 blocks from **N** — 16
5 blocks from **N** — 20
6 blocks from **N** — 24
7 blocks from **N** — 28
⋮
10 blocks from **N** — ?

Center Activity

Put copies of Worksheet G2(b) in a center, along with task cards that ask students to draw dots at various places. For example:

3N, **1W** from **N**	**4E**, **2S** from **N**
3S, **3W** from **N**	**2W**, **5N** from **N**

Writing Activity

Instruct students to write about a trip Nora might take to school, then to the library, then to Grandmother's house, and then back home. Ask them to describe the route Nora takes using directions north, south, east, and west.

Capsule Lesson Summary

Observe the similarity between two pictures where one is simply a reduction of the other. Note the number of dots in a given picture and then look at other pictures that are reductions of the original picture. Find out how many dots there would be in all the copies of a reduced picture needed to cover the given picture.

Materials

Teacher
- *UPG-II Geometry Poster* #1
- Calculator

Student
- Paper
- Calculator
- Pencil

Advance Preparation: Before this lesson begins, cut out the pictures on *UPG-II Geometry Poster* #1. You may want to label the backs of the pictures as they are referenced in the lesson.

Description of Lesson

Exercise 1

Tape Picture A and the reduced copy of Picture A to the board.

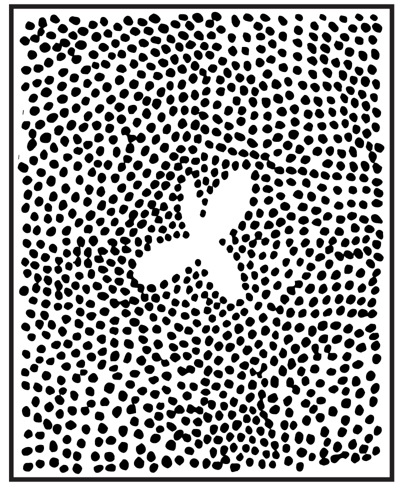
T: ***What do you think about these two pictures?***

Let students react. Try to bring out of the discussion the fact that the two pictures are the same except that one is a reduced copy of the other. You might observe, for example, that there are nine animals in the big picture and also nine animals in the small one.

Picture A (1)

Picture A (2)

Exercise 2



Picture B (1)

Hold up Picture B(1).

T: ***How many dots do you think there are in this picture?***

Accept several estimates from the students. In fact, there are exactly 1,000 dots in the picture. If the guesses are wild, you might suggest counting the dots in some small part of the picture and then asking for an estimate for the entire picture.

T: ***There are exactly 1,000 dots in this picture.***

Tape Picture B(1) to the board and write "1,000 dots" under it. Then hold up Picture B(2) and let the class comment. They should notice that it is just a reduced copy of Picture B(1).

T: ***How many dots are there in this picture?***

S: ***1,000.***

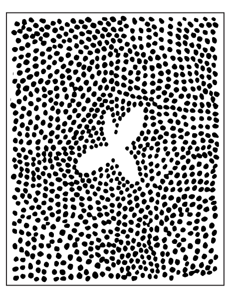
Perhaps this answer will not be obvious to everyone in the class. If necessary, refer back to Picture A and its reduced copy to illustrate that there are the same number of animals in both the big and small picture.

Hold Picture B(2) in front of Picture B(1).

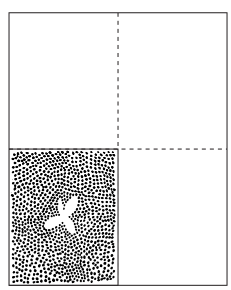
T: ***How many of these smaller pictures would we need to cover the big picture?***

S: ***Four.***

Ask a student to demonstrate how four copies of Picture B(2) would cover Picture B(1). Then tape Picture B(2) to the board and draw the dotted lines illustrated below to make it clear.



1,000 dots



T: ***Suppose we had four copies of this picture here. Together they would make a new big picture. How many dots would be in the new big picture?***

S: ***4,000.***

Write "4,000 dots" under the outline for the new big picture. Then hold up Picture B(3) and let the class comment. They should notice that it is an even smaller copy of the same picture.

T: ***How many dots in this picture?***

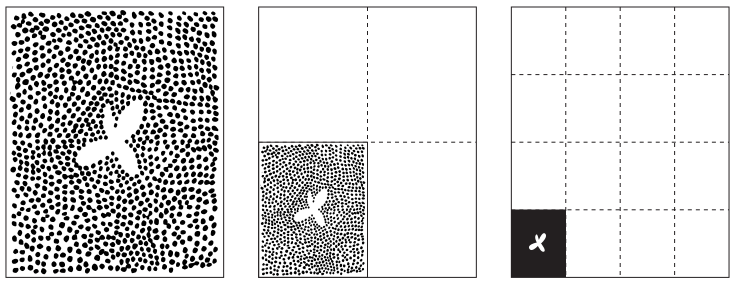
S: ***1,000; just like the big one.***

Hold Picture B(3) in front of Picture B(1).

T: ***How many of these small pictures would we need to cover the big picture?***

S: ***Sixteen.***

Again, ask a student to demonstrate how sixteen copies of Picture B(3) would cover Picture B(1). Then tape Picture B(3) to the board and draw dotted lines as in the next illustration.



1,000 dots 4,000 dots 16,000 dots

T: ***Suppose we had sixteen copies of this picture here. Together they would make another new big picture. How many dots would be in this new big picture.***

S: ***16,000.***

Write “16,000 dots” under the outline for this new big picture. Then hold up Picture B(4).

T: ***This is a very small copy of our big picture. How many dots are there in this little picture?***

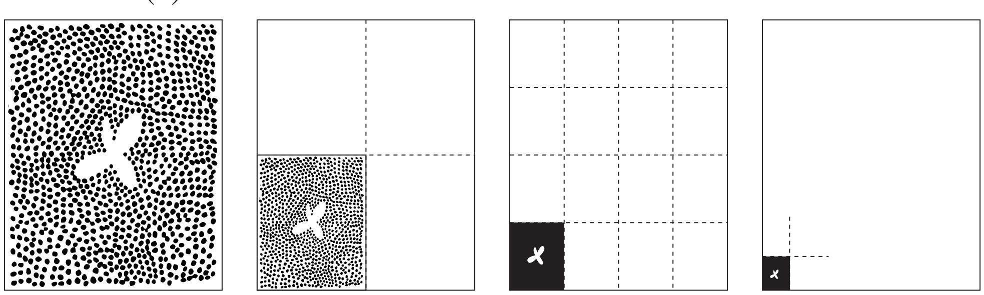
S: ***1,000.***

Hold up Picture B(4) in front of Picture B(1).

T: ***How many of these small pictures would we need to cover the big picture?***

Let the class think about this problem for a few minutes. Suggest that students who think they know the answer (64) write their answers on a piece of paper or whisper it to you. Ask them also to write the number of dots in that many copies of the little picture. When several members of the class have found the answer, invite a student to answer aloud.

Tape Picture B(4) to the board and prepare to draw dotted lines to indicate all the copies you would need to cover Picture B(1).



1,000 dots 4,000 dots 16,000 dots

Encourage students to discuss how they found the answer.

S: ***Four copies of the little picture cover the next size picture (B[3]), so I counted by fours sixteen times.***

S: ***I used the calculator and found 16 x 4 = 64.***

S: ***Sixteen copies of the little picture cover this size picture (B[2]), and four copies of this picture (B(2)) cover the big picture, so I added: 16 + 16 + 16 + 16 = 64.***

As the students discuss their methods of solution, draw in the rest of the dotted lines and observe that you need eight rows of eight copies. This picture might suggest other methods of calculating 8 x 8. For example:

8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 = 64;
4 x 4 = 16, 2 x 16 = 32, and 2 x 32 = 64.

T: ***These dots are very small, but we could make them even smaller. Try to imagine how many dots we could have on a piece of paper this size*** (indicate the size of picture B[1]) ***if we made them so tiny you would need a microscope to see them. Perhaps there would be so many no one could ever count them.***

Extension Activity

Read part or all of the book *How Much is a Million?* by David Schwartz to the class.

T: ***Do you think we could put 1,000,000 dots on the wall?***
Which of these pictures—B(1), B(2), B(3), or B(4)—should we use?

Let the class discuss how much space you would need on the wall if you used B(1), B(2), B(3), or B(4). In the discussion, include how many copies of each picture you would need. Use a calculator to help with the calculations. At the end of the discussion you might comment that the office might not let you make 1,000 copies of Picture B(1), but if you had a single page like B(4) containing 64 reduced copies of B(1), the office might let you make 16 copies to put on the wall. And 16 x 64,000 = 1,024,000 which is more than a million.

The class may enjoy making a collection of a million of some kind of object. This would be a good project. Usually, students are not aware how difficult it is to get a million of something.

Center Activity

Place newspaper photos or the colored comic strips along with magnifying glasses out for students. Let them observe how dots are used to produce the photos and to mix colors.

Reading Activity

Some other books about a million that you might like to introduce to your students are *If You Made a Million* by David Schwartz, *One Million* by Hendrik Hertzberg, and *Millions of Cats* by Wanda Guy. Also, section 3, “Dots, Dots, and More Dots” of *Anno’s Math Games II* by Mitsumaso Anno has several examples of things made of tiny parts (dots).

Capsule Lesson Summary

Find the lengths of zigzag paths between two points by measuring, in centimeters, the lengths of the parts and adding them together. Find shorter paths between the points and compare the lengths of these zigzag paths to the lengths of the shorter paths.

Materials

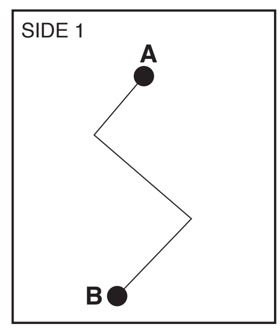
Teacher	• Centimeter tape measure • *UPG-II Geometry Poster #2*	Student	• Centimeter tape measure • Straightedge • Calculator • Worksheets G4*, **, ***, and ****

Advance Preparation: Cut *UPG-II Geometry Poster #2* in half along the center line. One side is used in each exercise of the lesson.

Description of Lesson

Exercise 1

Tape side 1 of *UPG-II Geometry Poster #2* to the board.



T: ***Here*** (point to **A**) ***there is a bug, and here*** (point to **B**) ***is some food. The bug walks along this zigzag path*** (trace the path from **A** to **B**) ***to get the food. I would like to know the length of this path. How can we find its length?***

S: ***Measure it.***

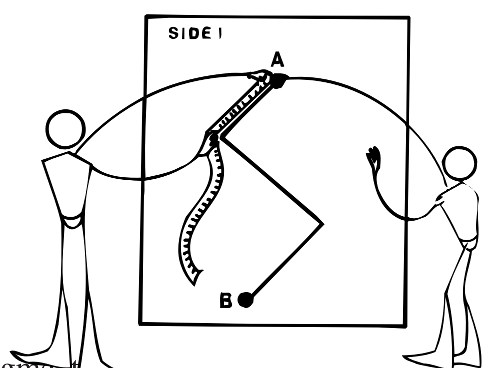
T: ***Yes, and how should we measure it?***

S: ***With a tape measure or a ruler.***

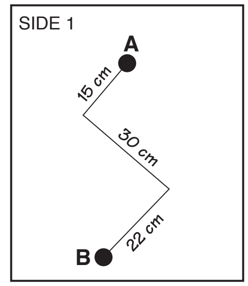
Display a tape measure and let the class discuss how to do the measuring. Direct the discussion so that the class realizes the three pieces of the zigzag path may be measured separately, and then their three lengths added to find the total length of the path.

T: ***We will measure the path in centimeters*** (review that the marks on one side of the tape measure are for centimeters)***, but before we do it, can you predict how long this path is?***

Record several students' predictions on the board. Then call on three different students to assist you in measuring the three pieces of the zigzag path. With each student, emphasize that they hold the end mark on the tape at one end of the path segment while you read the number of centimeters at the other end.



After each piece is measured, write the length next to the segment.



T (tracing the appropriate segments)**:** ***We found that the bug walked 15 cm along this part of the path, then 30 cm along this part, and then 22 cm along this part. How can we find how long the entire path is?***

S: ***Add 15, 30, and 22.***

Write the addition problem on the board and collectively do the calculation.

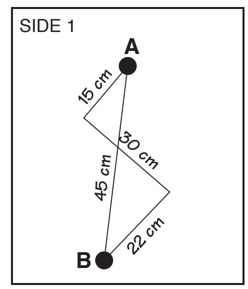
$$\begin{array}{r} 15\text{ cm} \\ 30\text{ cm} \\ +\ 22\text{ cm} \\ \hline 67\text{ cm} \end{array}$$

T: ***So, this path is 67 cm long.***

Compare the length of the path with the students' predictions, and determine which prediction was closest.

T: ***Is there a shorter path the bug could take to get the food?***

Allow students to make suggestions until the shortest path, a straight line from **A** to **B**, is mentioned. Then draw this path on the poster and ask a student to measure it.



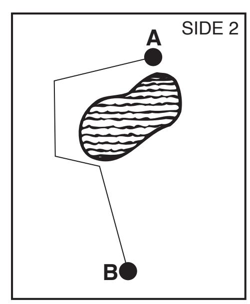
T: ***This path is only 45 cm long. How much shorter is it than the first zigzag path?***

S: ***22 cm shorter.***

The subtraction problem may be difficult for some students, but do not spend too much time on it. One way to observe that the difference is 22 cm is to note that the first two pieces of the zigzag are 45 cm together, and the third piece is the extra 22 cm.

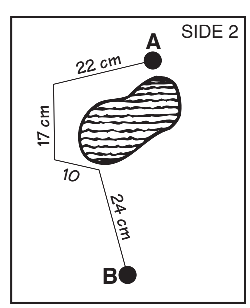
Exercise 2

Tape side 2 of *UPG-II Geometry Poster* #2 to the board.



T: ***Here*** (point to **A**) ***is the bug and here*** (point to **B**) ***is its food. There is a puddle of water between the bug and its food. The bug cannot swim, so it walks along this zigzag path*** (trace the path from **A** to **B**) ***to get to the food. About how long is this path, do you think?***

Record students' predictions on the board and then, as in Exercise 1, call on different students to assist you in actually measuring each piece of the zigzag.



T: ***How do we find the total length of this path?***

S: ***Add 22, 17, 10, and 24.***

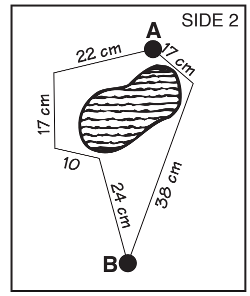
Write the problem on the board and collectively do the calculation.

$$\begin{array}{r} 15\text{ cm} \\ 30\text{ cm} \\ +\ 22\text{ cm} \\ \hline 67\text{ cm} \end{array}$$

Compare the length of the path with the students' predictions to determine which one was the closest.

T: ***Who can show us a shorter path the bug could take to get to the food? Remember, the bug cannot swim so it does not go through the puddle.***

Allow students to make suggestions and choose one that the class agrees will be shorter. Draw the path on the poster and ask students to assist you in measuring it. One possibility that comes close to a shortest path is shown here.



T: ***How long is this new path? Do the calculation on your paper.***

S: ***55 cm.***

T: ***It certainly is shorter than the first path.***

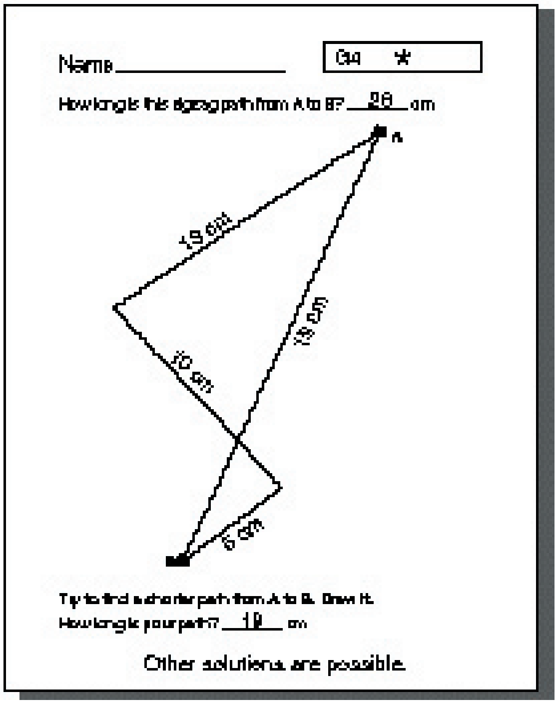
If you feel it would be appropriate for your class, ask them to determine how much shorter the new path is than the original path. (18 cm)

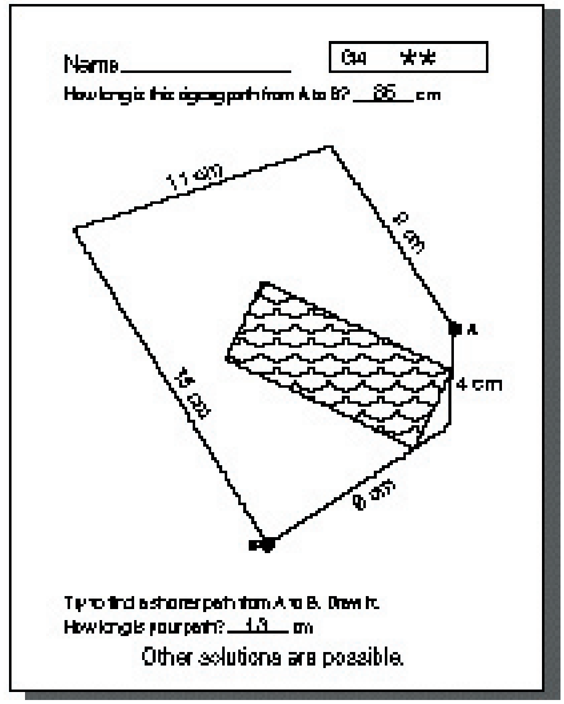
Worksheets G4*, **, ***, and **** are available for independent work. It would be helpful if students had straightedges with which to draw. Otherwise, suggest that they draw straight line segments using the edge of a crayon box or book as a guide. Then they should measure with the tape measure. Some students may need further instruction on measuring. After students have written down the measurements and added them together, they can use a calculator to check their answers.

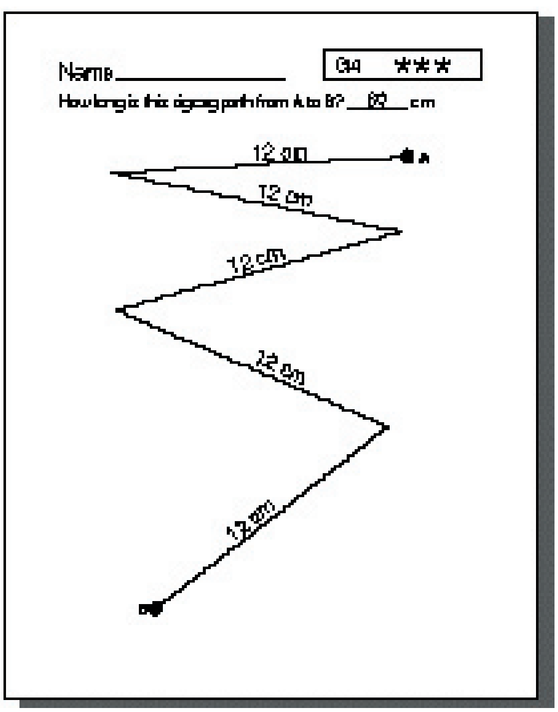
Students who finish quickly can make their own bug pictures and write directions for a partner to solve.

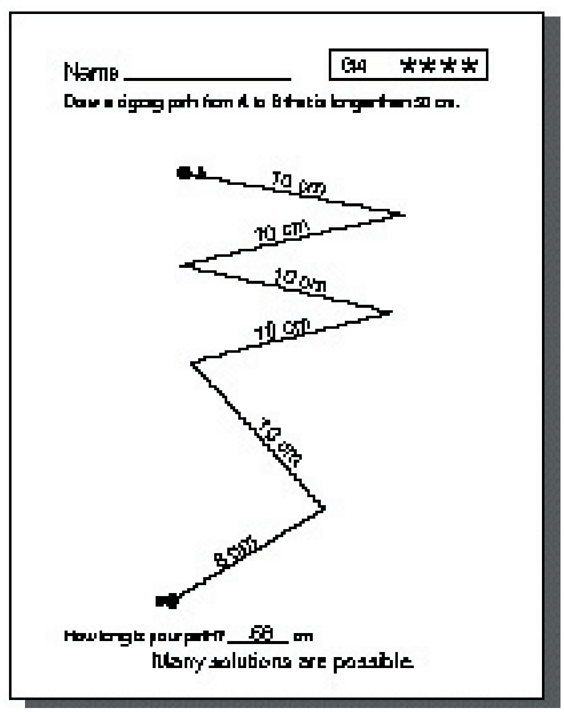
Writing Activity

Suggest students choose one worksheet and write a note to the bug explaining which path it should take and why.


Name
G4 *
Other solutions are possible.


Name
G4 **
11 cm
4 cm
Other solutions are possible.


Name
G4 ***
12 cm
12 cm


Name
G4 ****
Many solutions are possible.

Capsule Lesson Summary

Locate several points on a grid given their positions in terms of number of blocks north, south, east, or west of a specified point. Find points that are 6 blocks from one given point (along a shortest path) and 3 blocks from another given point (along a shortest path). Find many places that are the same distance (taxi-measure) from two given points.

Materials

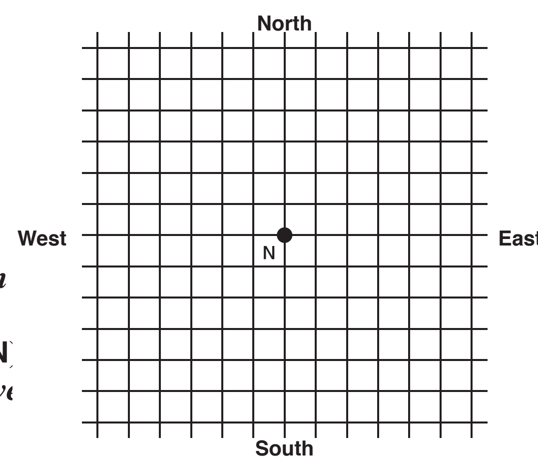
Teacher
- Grid board†
- Colored chalk or markers
- Overhead projector (optional)

Student
- Worksheets G5(a) and (b)
- Colored pencils

Description of Lesson

Exercise 1

Display your grid board. Write **North**, **South**, **East**, and **West** on the appropriate edges. Draw a dot (centrally located) and label it **N**.



T: ***This is a map of the city where our friend Nora lives. When you read a map, usually this direction is north*** (point to **North**)***, this direction is south*** (point to **South**)***, this direction is east*** (point to **East**)***, and this direction is west*** (point to **West**). ***Here is Nora's house*** (point to **N**) ***Do you remember where Nora's friend Paula live***

S: ***5 blocks from Nora's house.***

T: ***Yes, a shortest walk from Nora's house to Paula's house is 5 blocks. But we found lots of places like that. Do you remember the directions?***

S: ***3 blocks east and 2 blocks north.***

You need not insist on this recall; but if no student gives the correct directions, draw and label the dot for Paula's house (**P**) and then ask for the directions.

Call on students, and give them directions to locate several other points. For example:

- Start at **N** go 6 blocks west and 2 blocks north.
- Start at **N** go 3 blocks south and 3 blocks west.
- Start at **N** go 5 blocks north and 4 blocks east.
- Start at **N** go 4 blocks east and 5 blocks north.

Point to some intersections and ask students to give directions to those points.

†See the "Note on Grids" section in the introduction to this strand.

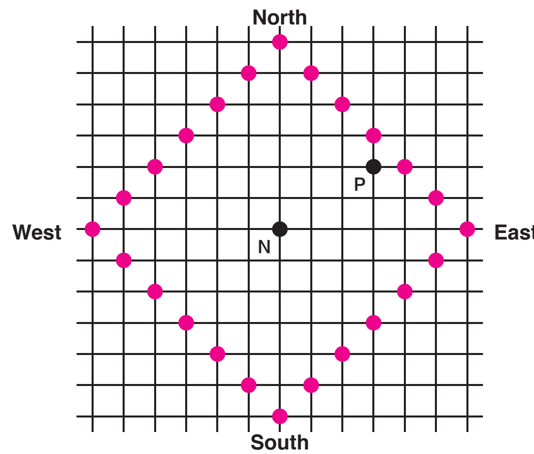
Exercise 2

Distribute Worksheet G5(a) and colored pencils. Be prepared with extra copies of the worksheet for those students who make mistakes. Ask students to look at the side where there is a dot for Nora's house (**N**) and a dot for Paula's house (**P**).

T: ***Nora and Paula have another friend, Maria. A shortest walk from Nora's house to Maria's house is 6 blocks. On your paper, draw blue dots at the places where Maria's house could be.***

Walk around and help students who have trouble getting started. As the class is working, invite some students to draw blue dots where Maria's house could be on the demonstration grid board. You may want to let one student draw several dots at a time.

A completed drawing is shown here.

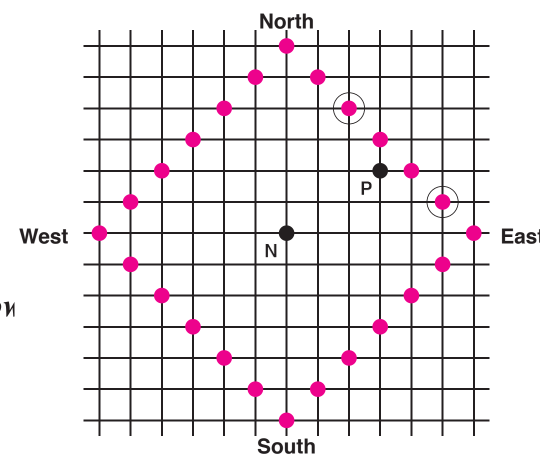


T: ***How many places are there that Maria's house could be?***

S: ***Twenty-four.***

T: ***I will give you some more information. A shortest walk from Paula's house to Maria's house is 3 blocks. Do you know where Maria's house is now?***

Let students suggest places, and check each to see that a shortest walk from **P** to that place is 3 blocks. There are two possibilities, circled in the picture below. Circle these dots on your grid board and erase the other blue dots as they are ruled out.



T: ***If you walk from Maria's house to Nora's house, you can go 2 blocks west and 4 blocks south. Do you know where Maria's house is now***

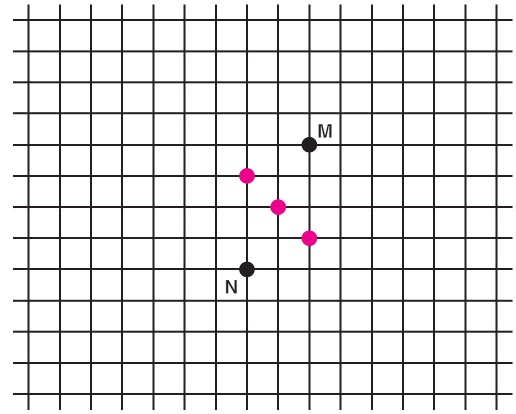
Call on a student to point to the correct dot. Label it **M**.

Exercise 3

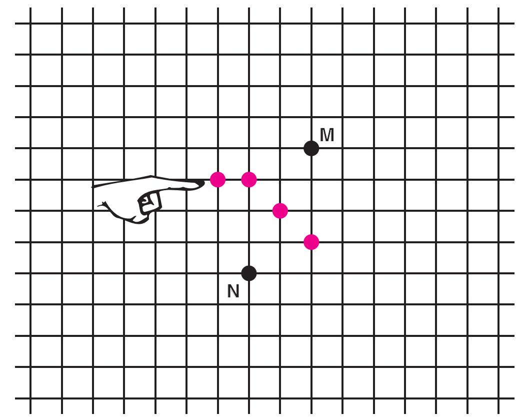
Ask the students to turn over the worksheet to G5(b) and to note that there is a dot for Nora's house (**N**) and a dot for Maria's house (**M**). Erase all but these two dots on your demonstration grid board.

T: ***Nora and Maria would like to find a place to meet to which they both have to walk the same number of blocks. On your papers try to find some places where they could meet. Remember, they both have to walk the same distance.***

Let the students work independently or with a partner on this problem. If a student finds just one place, suggest that there are more places where the girls could meet. After a while, call on students to locate dots on the demonstration grid board and to color them red, showing where the girls could meet. Most students will probably find these three red dots.



Try to encourage students to find other places the girls could meet, and ask the class to check each new place. Eventually (or perhaps you will have to make this suggestion yourself), you will consider a dot such as this:



T: ***What is a shortest walk Nora can make to this place?***

S: ***4 blocks.***

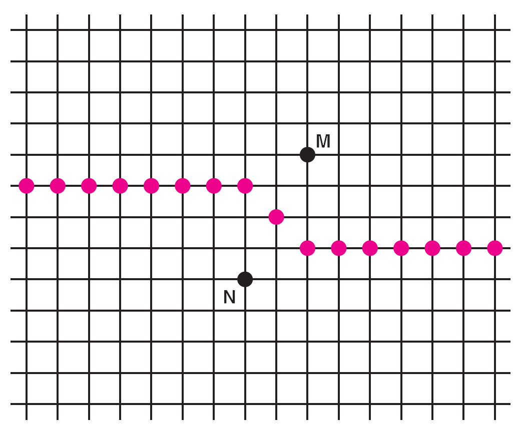
T: ***What is a shortest walk Maria can make to this place?***

S: ***4 blocks.***

T: ***Very good. They both have to walk the same number of blocks. Can you find some more places where they could meet?***

Let the students continue to work independently or with a partner. Some students may discover a pattern, but do not expect everyone to find one. Those who do not see any pattern will continue to use trial and error methods of finding new places.

A complete picture is shown here (limited, of course, by the size of the grid board).



Home Activity

Send home a problem, similar to that in Exercise 2, for students to work on with family members. Blackline G5 has a grid for working on such a problem.

G6 FRACTIONAL PARTS OF SHAPES #1

Capsule Lesson Summary

Use a story about sharing cakes to discuss dividing a shape in half, in fourths, and in eighths. Divide a shape on a grid into halves, fourths, eighths, and sixteenths. Find the number of squares in one-half, one-fourth, one-eighth, and one-sixteenth of the shape. Do a similar activity with a shape whose symmetry lends itself to being divided into thirds, sixths, and twelfths.

Materials

Teacher
- Colored chalk
- *UPG-II Geometry Poster* #3

Student
- Colored pencils
- Worksheets G6*, **, ***, and ****

Advance Preparation: For Exercise 1, prepare a circle cut-out with a radius of about 20 cm, and a rectangle cut-out approximately 40 cm by 40 cm. Also, cut the shapes from *UPG-II Geometry Poster* #3 for Exercises 2 and 3. You may like to laminate the shapes so that you can write on them and erase.

Description of Lesson

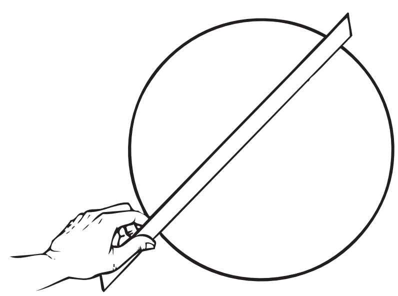
T: ***Do you remember the twins, Jack and Jill, who live next door to Booker's Bakery?***

The students might recall that Jack and Jill often buy cookies and cakes at Booker's Bakery and share them with their friends.

Exercise 1

Note: This exercise is a review of material covered in some of the *UPG-I* geometry lessons and should move quickly.

Display a circle cut-out on the board.



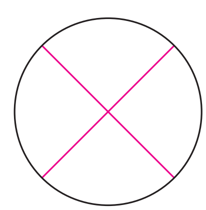
T: ***Jack and Jill want to share a circle cake*** (point to the circle) ***fairly between the two of them. How can they cut the cake?***

S: ***Cut the cake in two pieces.***

T: ***Like this?***

Don't draw a cut line, but demonstrate where it could be with a meter stick.

S: ***No, in half.***

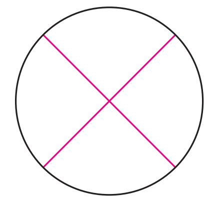


Ask a volunteer to show how to cut the cake. Allow students to suggest folding the circle in half or using a mirror. If it is a fairly accurate job and the class is in agreement, draw the cut line.

T: ***Yes, I think that is a fair cut. Both pieces look the same.***

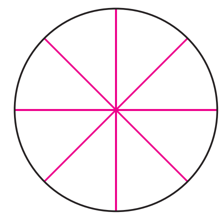
T: ***But before they eat any of the cake, two friends come over. How can they cut the cake so that the four of them can share the cake equally?***

Draw another cut line when a fairly accurate suggestion is made.



T: ***No sooner do they cut the cake, when four more friends come over. How can they cut the cake so that the eight of them can share the cake equally?***

Draw additional cut lines when fairly accurate suggestions are made.



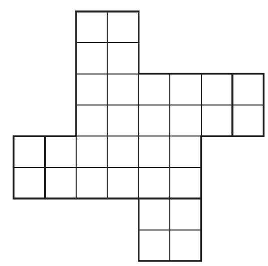
Observe how, each time, a previous piece gets cut in half.

Repeat Exercise 1 with a rectangular shaped cake.

Exercise 2

Tape this shape cut-out from *UPG-II Geometry Poster* #3 to the board.

T: ***One day Jack and Jill buy a strangely shaped cake from Booker's Bakery.*** (Point to the shape on the board.) ***The cake has vanilla icing with chocolate stripes. How many squares do the chocolate stripes divide the cake into?***

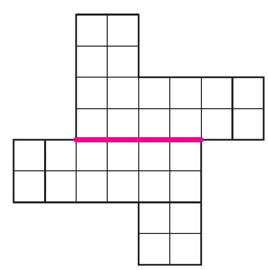


Invite a student to count them at the board.

S: ***32.***

T: ***Yes, and they are all the same size. Jack and Jill want to share the cake fairly between themselves. How could they cut the cake?***

Let students suggest several divisions and then indicate one on the shape, as illustrated here.



T: ***How many squares will Jill get? … Jack?***

S: ***They will each get 16 squares.***

$\frac{1}{2} \times 32 = 16$

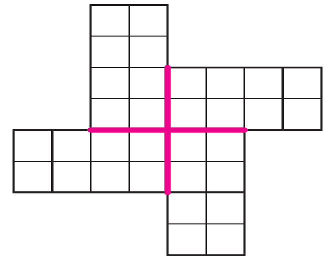
Record this number sentence on the board as you say, "One-half of thirty-two equals sixteen."

T: ***There are 32 squares in the whole cake, and there are 16 squares in one-half of the cake. Another way to write and say this is 32 ÷ 2 = 16.***

Continue the story.

T: ***But before Jack and Jill take their shares of the cake, their parents come home and want to have some cake also. How can they cut the cake so that the four of them can share the cake equally?***

Let students make suggestions. Choose a natural division and indicate it on the shape.



T: ***How many squares will each of the four people get?***

S: ***Eight.***

Record this new number sentence on the board as you say, "One-fourth of thirty-two equals eight."

$\frac{1}{2} \times 32 = 16$
$\frac{1}{4} \times 32 = 8$

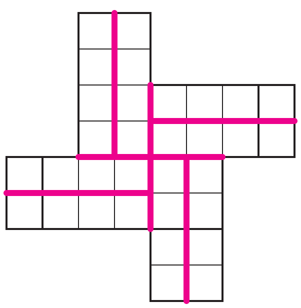
T: ***There are 32 squares in the whole cake, and there are 8 squares in one-fourth of the cake. We can also say 32 ÷ 4 = 8.***

Continue the storyline

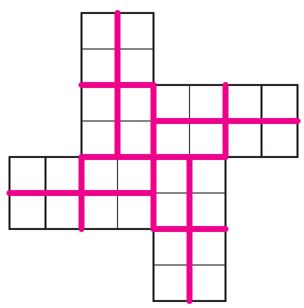
- with four neighbors coming over (eight people altogether); and then
- with their eight children coming over looking for their parents (sixteen people altogether).

The next illustration shows one way that each of the divisions could be made, along with a resulting list of number sentences.

Eight pieces



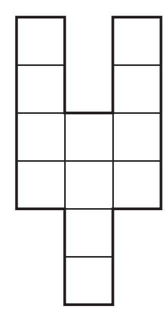
Sixteen pieces



$\frac{1}{2} \times 32 = 16$	$32 \div 2 = 16$
$\frac{1}{4} \times 32 = 8$	$32 \div 4 = 8$
$\frac{1}{8} \times 32 = 4$	$32 \div 8 = 4$
$\frac{1}{16} \times 32 = 2$	$32 \div 16 = 2$

Exercise 3

Tape this s~~hape cut-out from~~ *UPG-II Geometry Poster* #3 to the board. If you need the board space, remove the shape from Exercise 2 and erase the number sentences.

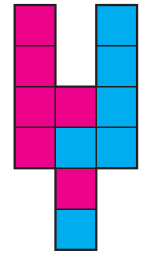


T: ***Jack and Jill and some of their friends form a Saturday club.***
Jack and Jill order a cake for the next meeting. It looks like this.
This cake has vanilla icing and chocolate stripes like the other one.
How many squares do the stripes divide this cake into?

S: ***Twelve.***

T: ***At first, only Jack and Jill are at the meeting. How can they share the cake fairly between the two of them?***

Allow students to suggest a way to share the cake. Perhaps you can let them color Jack's share one color and Jill's share another color, as illustrated here.



$\frac{1}{2} \times 12 = 6$

$12 \div 2 = 6$

T: ***How many squares will each of the twins get?***

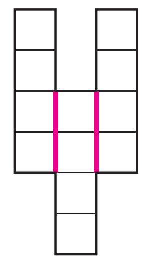
S: ***Six.***

T: ***Soon a third member of the club comes to the meeting.***

Remove the first copy of the shape and put up a clean copy.

T: ***How can they share the cake so that each of the three will get the same amount?***

Very likely your students will see the symmetry of the shape and suggest this division.



$\frac{1}{3} \times 12 = 4$

$12 \div 3 = 4$

T: ***How many squares of cake will each of the three children receive?***

S: ***Four.***

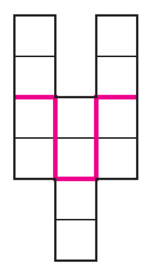
Write this number sentence close to the shape as you say, "One-third of twelve equals four."

T: ***There are twelve squares in the whole cake, and there are four squares in one-third of the cake. Also, 12 ÷ 3 = 4.***

Continue the story.

T: ***The three members decide that they really ought to wait a little longer before eating the cake. After about ten minutes, three more members arrive. How can they share the cake equally among the six of them?***

Encourage the idea of cutting each of the three pieces in half.



$\frac{1}{6} \times 12 = 2$

$12 \div 6 = 2$

T: ***How many squares of cake will each of the six children receive?***

S: ***Two.***

Record number sentences for this division on the board.

T: ***Suppose twelve members come to the meeting? How much cake will each of them receive?***

S: ***One square each.***

T: ***$\frac{1}{12} \times 12 = ?$***

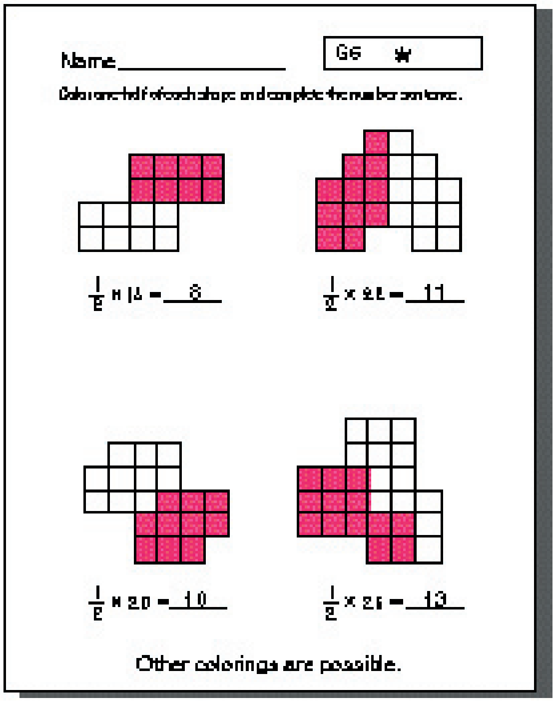
S: ***1.***

Record the number sentences in the list near the shape.

$\frac{1}{2} \times 12 = 6$	$12 \div 2 = 6$
$\frac{1}{3} \times 12 = 4$	$12 \div 3 = 4$
$\frac{1}{6} \times 12 = 2$	$12 \div 6 = 2$
$\frac{1}{12} \times 12 = 1$	$12 \div 12 = 1$

Worksheets G6*, **, ***, and **** are available for individual work. Students having trouble may benefit from some other demonstration of a method for finding one-half of a number of squares. For example, place counters of two colors on the squares of a shape. Do this, putting one of each color on the squares, then another of each color, and so on until all the squares are covered.

G6


Name____________ G6 *

Color one-half of each shape and complete the number sentence.

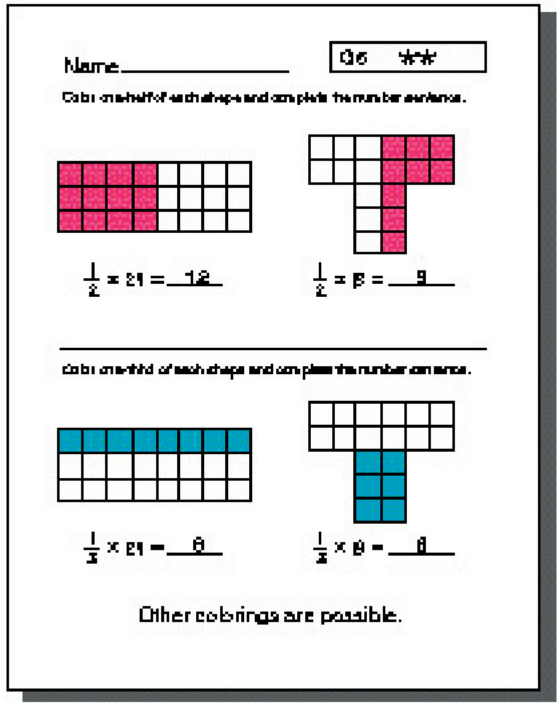
$\frac{1}{2} \times 16 =$ 8

$\frac{1}{2} \times 22 =$ 11

$\frac{1}{2} \times 20 =$ 10

$\frac{1}{2} \times 26 =$ 13

Other colorings are possible.


Name____________ G6 **

Color one-half of each shape and complete the number sentence.

$\frac{1}{2} \times 24 =$ 12

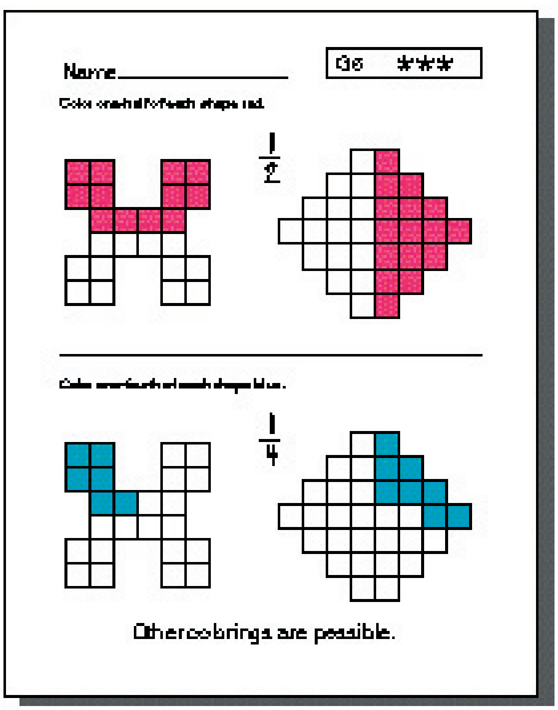
$\frac{1}{2} \times 18 =$ 9

Color one-third of each shape and complete the number sentence.

$\frac{1}{3} \times 24 =$ 8

$\frac{1}{3} \times 18 =$ 6

Other colorings are possible.


Name____________ G6 ***

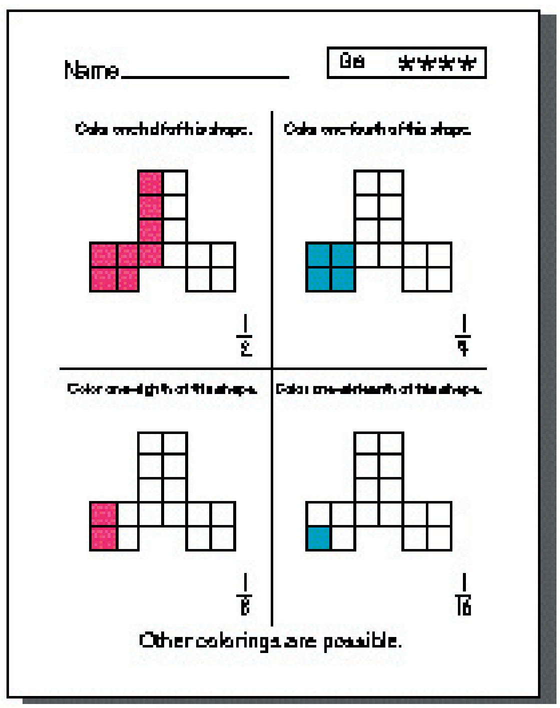
Color one-half of each shape red.

$\frac{1}{2}$

Color one-fourth of each shape blue.

$\frac{1}{4}$

Other colorings are possible.


Name____________ G6 ****

Color one-half of this shape.

$\frac{1}{2}$

Color one-fourth of this shape.

$\frac{1}{4}$

Color one-eighth of this shape.

$\frac{1}{8}$

Color one-sixteenth of this shape.

$\frac{1}{16}$

Other colorings are possible.

Capsule Lesson Summary

On a geoboard, make shapes with the same area. Compare the areas of various shapes in a puzzle to the areas of two given shapes. Color shapes of the same area the same color, and find they make a design.

Materials

Teacher
- Overhead geoboard
- Tangram squares and triangles

Student
- Geoboard and bands
- Worksheet G7
- Colored pencils, pens, or crayons

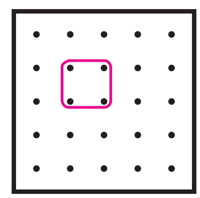
Advance Preparation: Many commercial geoboards have grid squares the size of Tangram squares. If yours does not, you may want to make paper squares the size of the geoboard square, or use Blackline G7 to make a transparency for a geoboard.

Description of Lesson

Arrange the class in groups to accommodate the number of Geoboards available. Allow a short time for students to explore with the geoboards before starting Exercise 1.

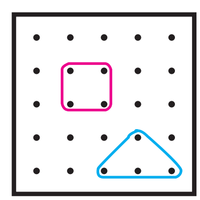
Exercise 1

Hold up one Tangram square.



T: ***Make a shape on your geoboard that has area 1 small square (1S).***

There are several possibilities, but most groups are likely to put on the square shape. If some choose to put on other shapes, encourage them to explain how they know the area is 1 small square. Check that all groups have a shape with area 1 small square on their geoboards; then invite a student to put 1 small square on the overhead geoboard or draw one on your transparency.



T: ***Leave your shape on your geoboard. With another color band, put on this shape.***

Put the shape on the overhead geoboard for student groups to copy.

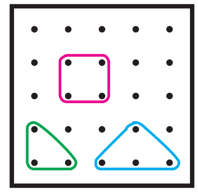
T: ***How are these shapes alike? How are they different?***

S: ***One is a square. The other is a triangle.***

S: ***They are the same size.***

Let students make many observations, but be sure to talk about size. You may use small Tangram triangles or another color of geoband to show that both shapes are covered by two small triangles; that is, both have the same area (1S).

T: ***What about this little triangle (small Tangram triangle)? How does it compare to the little square?***

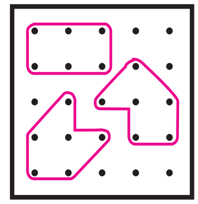


S: ***It is half of the square.***

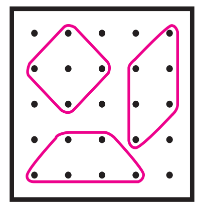
T: ***Yes, its area is ½S.***

Direct the students to clear their geoboards of shapes (geobands), and do the same on the overhead geoboard. Hold up a small Tangram square or refer to a small square on the overhead geoboard.

T: ***Now make shapes on your geoboard that have area 2S.***

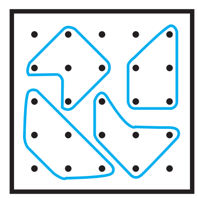


Here are several possibilities, although you might expect most groups to put on the rectangle (upper left corner) first.



As you monitor group work, encourage students to try to find more than one shape with area 2S. Invite groups to display their shapes for the class and to explain how they know the area is 2S. For example, a group might display this shape: 

S: ***Our shape has a small square and the same size triangle. Together that makes 2S.***



Repeat this activity, looking for shapes that have area 1½S.

Exercise 2

Provide each student with a copy of Worksheet G7, and red and blue pencils. Display an overhead transparency of the worksheet.

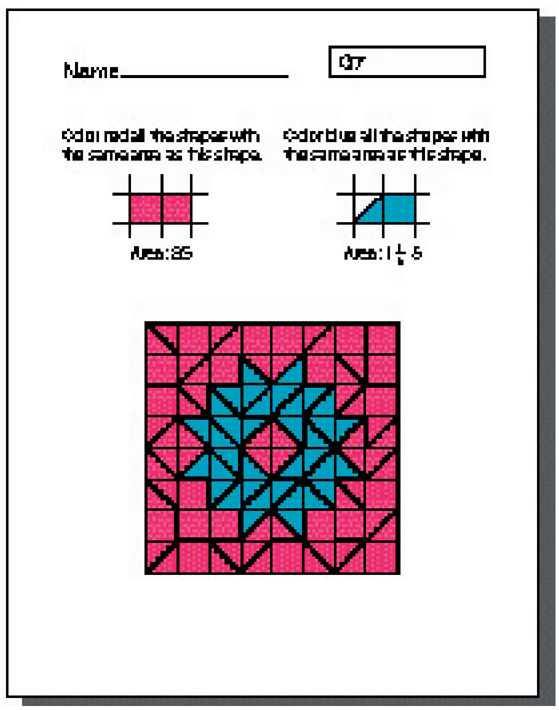
T: ***On this worksheet there is a puzzle. You are going to look for shapes having the same area as these shapes*** (point to the shapes with the directions at the top of the page)***. In the puzzle, which shapes will you color blue? Which shapes will you color red?***

S: ***Shapes with area 2S are blue.***

S: ***Shapes with area 1½S are red.***

Let the class help you locate a shape in the puzzle to color blue and another to color red. Be sure to check the areas. Remind students that they are looking for shapes with the same area, not necessarily the same shape.

Let students work in their groups to complete the puzzle. There is a second copy of the puzzle on the back of the worksheet for those who need it.



Name
G7
Color red all the shapes with the same area as this shape.
Area: 2S
Color blue all the shapes with the same area as this shape.
Area: 1 1/2 S

Capsule Lesson Summary

Find the lengths of zigzag paths between two points with an obstacle in between. Do this by measuring the lengths of the parts (in centimeters) and adding them together. Find shorter paths between the points avoiding the obstacles, and compare the lengths of the original paths to the lengths of the shorter paths.

Materials

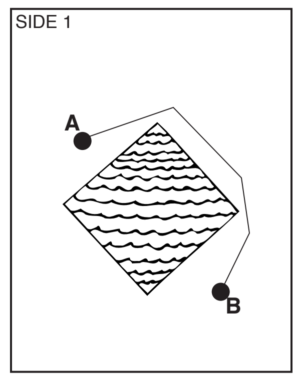
Teacher	• Centimeter tape measure • *UPG-II Geometry Poster* #4	Student	• Centimeter tape measure or 30 cm ruler • Worksheets G8*, **, ***, and ****

Advance Preparation: Cut *UPG-II Geometry Poster* #4 in half along the center line.

Description of Lesson

Exercise 1

Tape side 1 of *UPG-II Geometry Poster* #4 to the board.



T: ***Here*** (point to **A**) ***is our friendly bug and here*** (point to **B**) ***is some food. There is a pool of water between the bug and the food, so the bug walks around the pool along this zigzag path*** (trace the indicated path from **A** to **B**). ***How long is the path that the bug takes to get to the food? How can we find its length?***

S: ***Measure it.***

T: ***How should we measure it?***

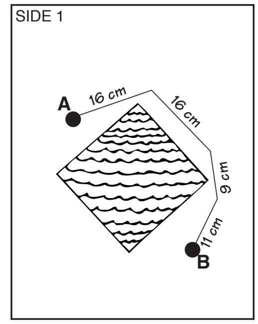
S: ***With a tape measure.***

Display a tape measure and let the class discuss how to do the measuring. Students should observe that to find the total length of the path, they can measure the four pieces of the zigzag separately and then add the four lengths.

T: ***We will measure the path in centimeters*** (review that the marks on one side of the tape measure are for centimeters), ***but before we do the measuring, can you predict how long this path is?***

Record several predictions on the board. Then, call on three students to assist you in measuring the three segments of the zigzag. With each student, emphasize that they hold the end mark of the tape measure at one end of the path segment while you read the number of centimeters at the other end.

After measuring each piece, write the length next to the segment.



T: ***Now, we know the length of the four parts of this path. How can we find the length of the entire path?***

S: ***Add 16 + 16 + 9 + 11.***

Write the addition problem on the board and collectively do the calculation.

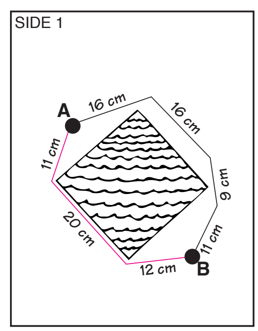
$$\begin{array}{r} 16\text{ cm} \\ 16\text{ cm} \\ 9\text{ cm} \\ +\ 11\text{ cm} \\ \hline 52\text{ cm} \end{array}$$

T: ***This path is 52 cm long.***

Compare the length of the path with the students' predictions and determine which prediction is closest.

T: ***Is there a shorter zigzag path the bug could take to get the food? Remember, the bug cannot swim.***

Allow students to make suggestions and to trace paths they think will be shorter. Let the class decide which of the suggested paths they believe is shortest, and then draw that zigzag path on the poster. It is not necessary that the class finds the shortest zigzag path from **A** to **B**; this illustration shows, however, one that is close to a shortest path.



With student assistance, determine the length of this new path by adding the lengths of its parts. For the example above, the length is 43 cm (11 cm + 20 cm + 12 cm). Compare the length of the new path with that of the original.

T: ***A zigzag path of length 43 cm is certainly shorter than the zigzag path of length 52 cm. How much shorter is it?***

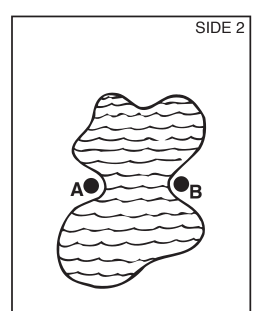
S: ***9 cm.***

With the class, observe that this last question is answered by solving a subtraction problem. Write the problem on the board and let the students use any method they wish to solve it.

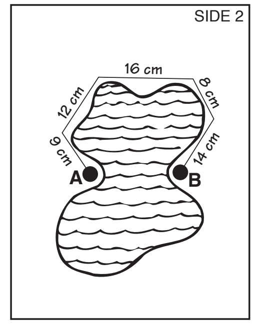
$$52 - 43 = 9$$

Exercise 2

Tape side 2 of *UPG-II Geometry Poster* #4 to the board.



T: ***Here*** (point to **A**) ***is the bug and here*** (point to **B**) ***is the food. Let's try to find a zigzag path from* A *to* B *for the bug to take to get the food. The bug would like as short a path as possible.***



Let students make suggestions and trace paths they think will be as short as possible. The class should decide which of the suggested paths is shortest and then draw that zigzag path on the poster. Again, it is not necessary that the class finds the shortest zigzag path from **A** to **B**; however, this illustration shows one that is close to a shortest path.

Direct the class to determine the length of the zigzag path. You may wish to ask the students to do the necessary addition calculation on their own before doing it collectively. In this example, the zigzag path is 59 cm long (9 cm + 12 cm + 16 cm + 8 cm + 14 cm).

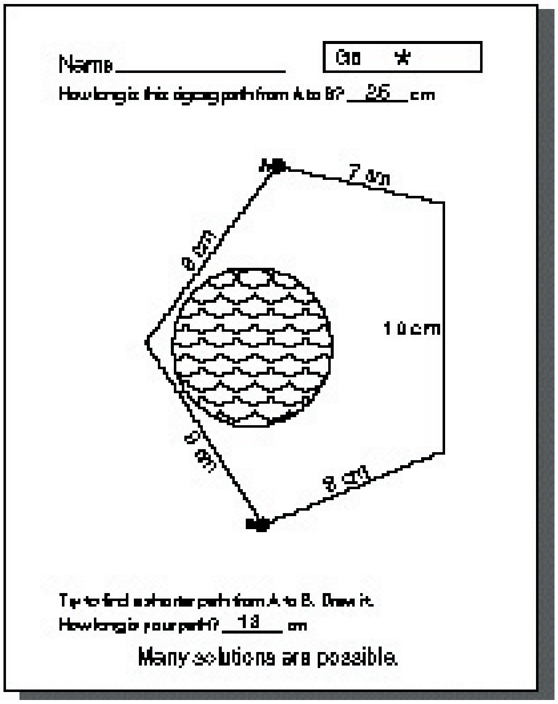
Worksheets G8*, **, ***, and **** are available for independent work. Provide students with centimeter tapes or 30 cm rulers. If necessary, suggest that they draw straight lines with the edge of a book or some other straightedge.

Writing Activity

Suggest students write directions to the bug on how to find a short path to its food.

Home Activity

You may like to send one or two of the worksheets home for students to do with a family member. Suggest they use their home Minicomputers or calculators to do calculations.

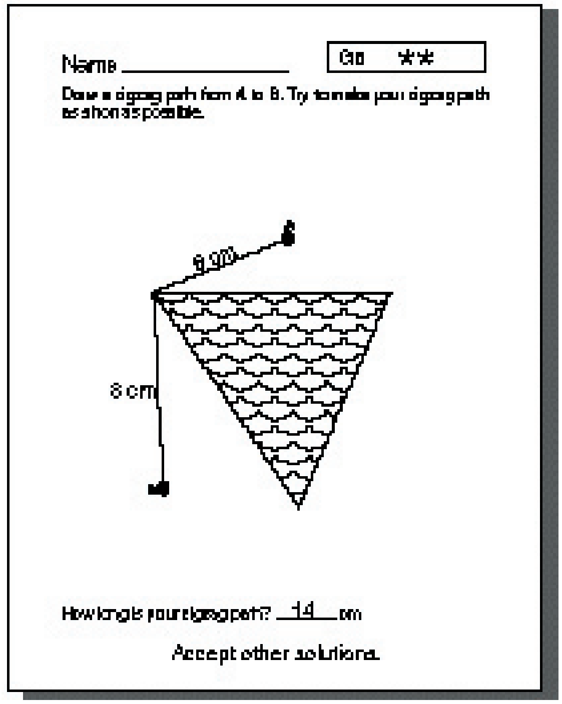

Name ____________ G8 *

How long is this zigzag path from A to B? 25 cm

Try to find a shorter path from A to B. Draw it.

How long is your path? 18 cm

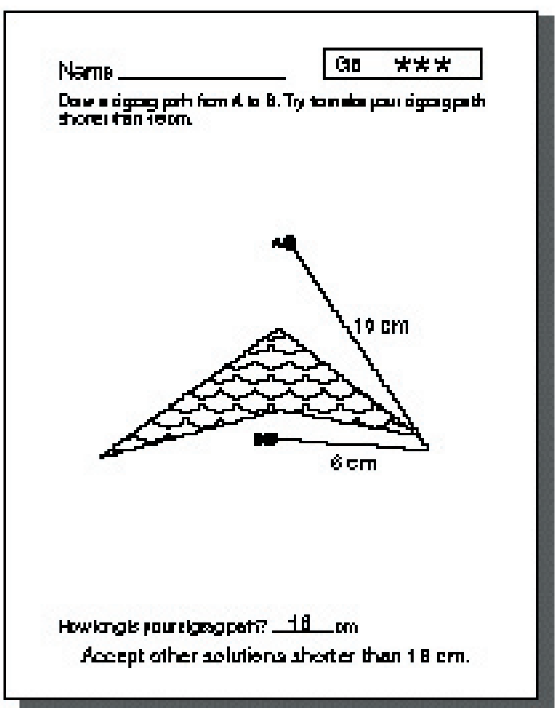
Many solutions are possible.


Name ____________ G8 **

Draw a zigzag path from A to B. Try to make your zigzag path as short as possible.

How long is your zigzag path? 14 cm

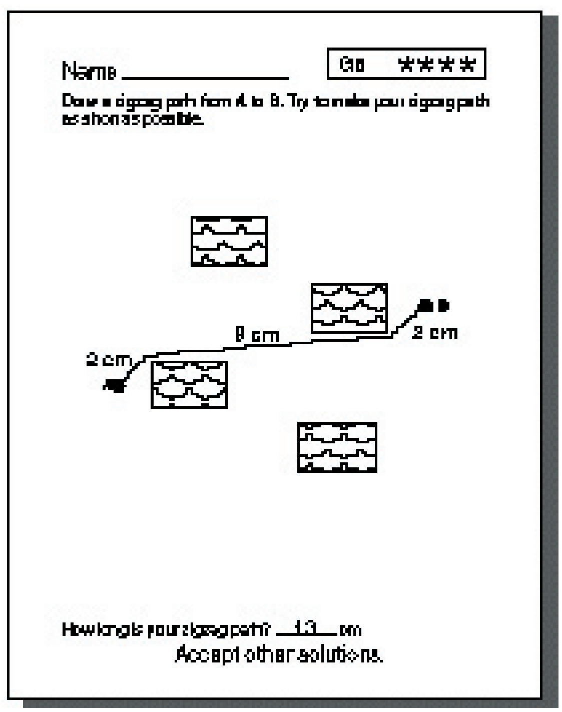
Accept other solutions.


Name ____________ G8 ***

Draw a zigzag path from A to B. Try to make your zigzag path shorter than 16 cm.

How long is your zigzag path? 16 cm

Accept other solutions shorter than 16 cm.


Name ____________ G8 ****

Draw a zigzag path from A to B. Try to make your zigzag path as short as possible.

How long is your zigzag path? 13 cm

Accept other solutions.

G9 AREA PROBLEMS WITH TANGRAM PIECES

Capsule Lesson Summary

Examine the Tangram pieces and determine their relative sizes. Find the area of each piece, first using the little square as the unit, then using the little triangle as the unit, and finally using the big triangle as the unit. Build shapes with the Tangram pieces and find their areas in terms of the various units.

Materials

Teacher
- Tangram

Student
- Tangram
- Worksheets G9* and **

Description of Lesson

Exercise 1

Provide each student with a Tangram, and allow a couple of minutes for free play or discussion. Examine the pieces of a Tangram with the class and review that there are seven pieces; two small triangles, one middle-sized triangle, two big triangles, one square, and one (non-rectangular) parallelogram, which they may see as a diamond or as a "crooked" diamond.

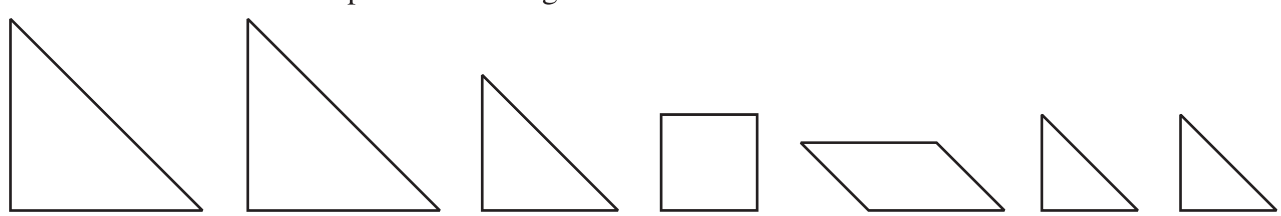
T: ***Which piece(s) is the largest?***

S: ***The big triangles.***

T: ***Which piece(s) is the smallest?***

S: ***The small triangles.***

Ask the students to line up the Tangram pieces on their desks in order by size, with the largest pieces on the left and the smallest pieces on the right.



Note: Any order of the three middle pieces is correct since they are all the same size.

T: ***Which piece did you put after the big triangles?***

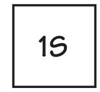
S: ***The square.***

S: ***The middle-sized triangle.***

S: ***The crooked diamond.***

Encourage students to explain how they decided which piece to put next. Eventually, the class should observe that these three pieces are all the same size and any ordering of them in the middle is correct.

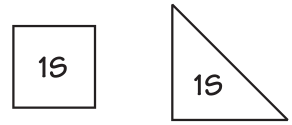
T: ***Suppose we choose the square as a unit; that is, we say the area of the square is 1.***



Draw the square on the board and write 1S inside.

T (holding up the middle-sized triangle)**:** ***What is the area of this triangle?***

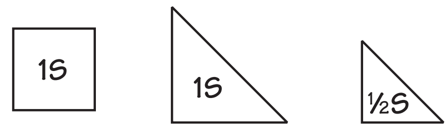
S: ***Also 1S, because it is the same size as the square.***



Draw a picture on the board to represent this information.

T (holding up a little triangle)**:** ***What is the area of this triangle?***

S: ***½S, because it takes two little triangles to cover the square.***



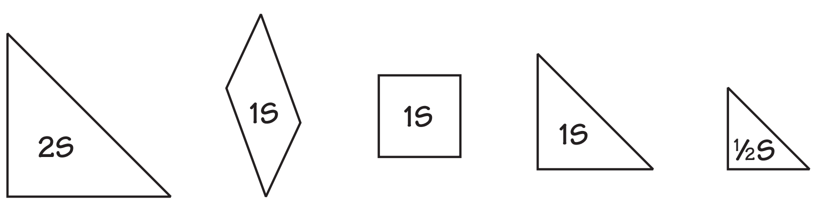
Record the information in a picture on the board.

T (holding up a big triangle)**:** ***What is the area of this triangle?***

Give the students time to think about this question as it may require experimenting with the Tangram pieces.

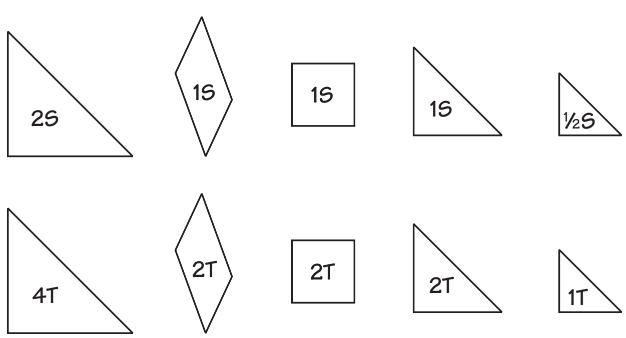
S: ***2S, because two middle-sized triangles would cover the big triangle and a middle-sized triangle has area 1S.***

Record this information on the board. Then ask about the parallelogram and record its area in a picture on the board.

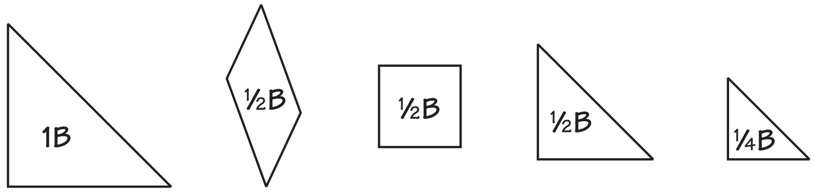


Repeat this line of questioning using the little triangle as the unit. The class should observe that in this case the unit is half the size as before, and consequently, the areas are twice as many units as before.

A good way to display this fact is to redraw each shape under the previous pictures and label them accordingly.



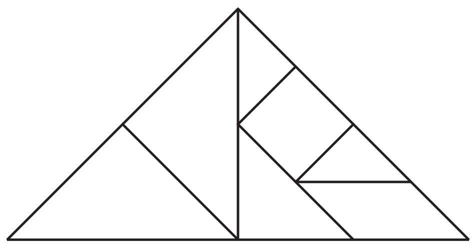
If your class is doing well with this activity, repeat the questioning using the big triangle as the unit.



Note: Here "B" is used to denote the big triangle as the unit.

Exercise 2

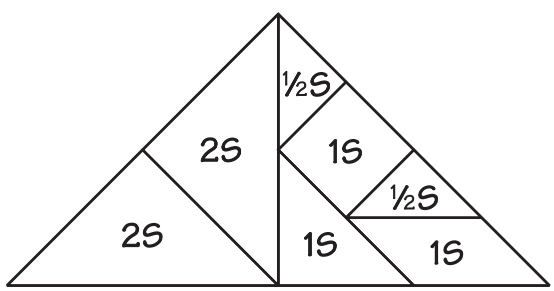
Ask the students to use all seven Tangram pieces to build a triangle on their desks. This task can be challenging so allow several minutes for this activity. If, after a few minutes, some students are having great difficulty, you might get them started by reminding them of the giant's boots (see *UPG -I* Lesson G9). Suggest they use the boots with the middle-sized triangle to make a triangle (half of the whole triangle).



When most students have completed the task, sketch this drawing on the board so all the students can build the triangle at their desks.

T: ***If we use the square as a unit (1S), what is the area of this triangle?***

S: ***8S; because each big triangle has area 2S, the middle-size triangle and the crooked diamond each have area 1S like the square, and the two small triangles together have area 1S. I added these areas and got 2 + 2 + 1 + 1 + 1 + 1 = 8.***



T: ***If we use the small triangle as a unit (1T), what is the area of this triangle?***

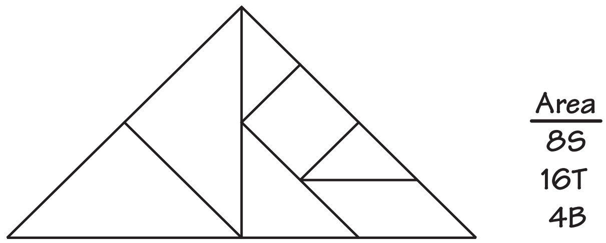
S: ***16T, because it would take twice as many small triangles as squares to cover this triangle.***

T: ***If we use the big triangle as a unit (1B), what is the area of this triangle?***

S: ***4B, because it would take half as many big triangles as squares to cover this triangle.***

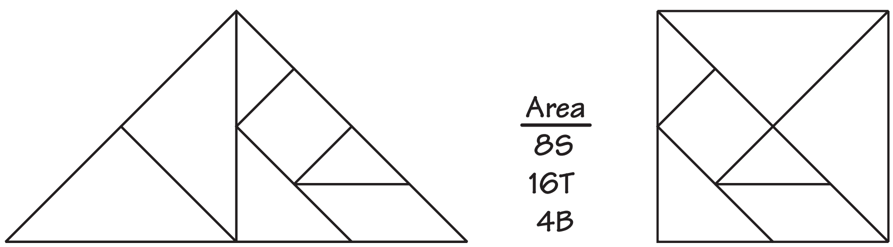
G9

Write this information on the board.

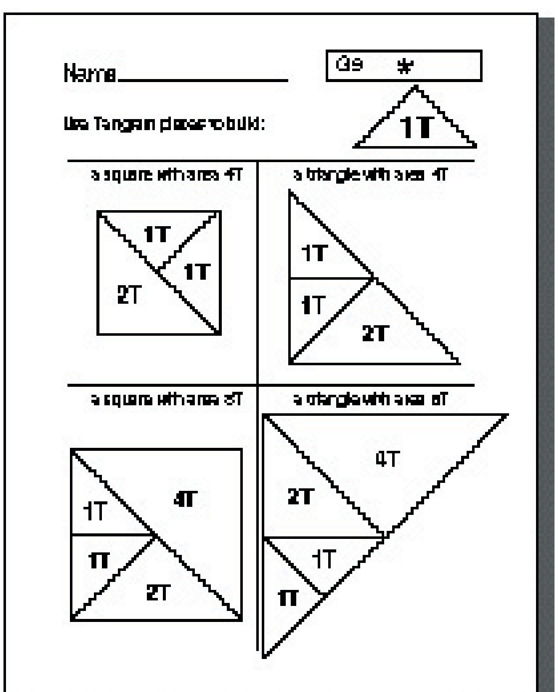


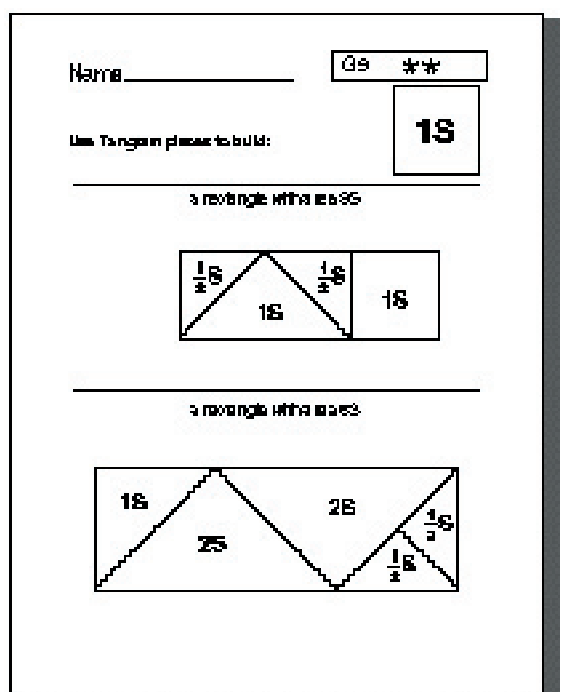
Next, ask the students to use all seven Tangram pieces to build a square on their desks. Again, allow several minutes for this activity. In this case, however, some students may observe that they only need to move the two big triangles. If students have great difficulty, lead them to consider moving just the big triangles.

When most students have completed the task, sketch a drawing on the board and ask the questions about area again. The class should quickly observe that the area of this square is the same as the area of the previous triangle.



Distribute Worksheets G9* and **. Solve the first problem collectively to give the students an idea of what they are to do. Instruct students to make a drawing of each shape they build. You may like to let students work with partners on these worksheets.





G10 FRACTIONAL PARTS OF SHAPES #2

Capsule Lesson Summary

Divide a shape on a grid into thirds in two different ways, and record appropriate number sentences. Find the number of squares in one-third, one-sixth, and one-twelfth of another shape.

Materials

Teacher
- *UPG-II Geometry Poster* #5 (two copies)
- Colored markers
- Tape

Student
- Worksheets G10*, **, ***, and ****
- Colored pencils

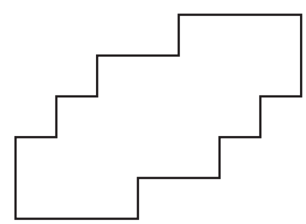
Advance Preparation: Before the lesson begins, cut out the shapes on the two copies of *UPG-II Geometry Poster* #5.

Description of Lesson

Exercise 1

Before the lesson begins, outline shape A on the board.

T: ***Jack and Jill offer to help Mr. Booker at his bakery one Saturday morning, Saturdays being Mr. Booker's busiest day. That morning, a customer tells Mr. Booker that she wants a cake like one of those on the shelf.*** (Point to the shape on the board.) ***The cake is for her son's birthday, and since he likes strawberry, blueberry and lemon icing, she doesn't know which to choose!***



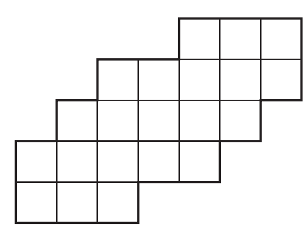
Jack and Jill excitedly ask Mr. Booker to let them ice the cake in all three flavors. Mr. Booker and the customer agree to let them.

Let the students discuss ways to ice the cake.

T: ***Jack decides to section the cake into squares.***

Tape a copy of shape A over its outline, already on the board.

T: ***How many squares are in the whole cake?***



Let a student count them at the board.

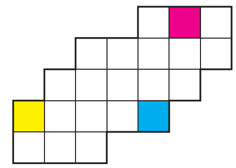
S: ***Twenty-one.***

T: ***Jill thinks that they ought to ice the same amount in each flavor. How can they ice the cake so that here is the same amount of all three flavors?***

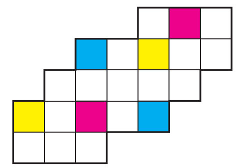
Perhaps someone will suggest icing seven squares in each flavor. Accept this suggestion and ask the class to check if this would result in the whole cake being iced. Whether or not this observation is made, continue as follows.

T: ***I would like three of you to come to the board—one for strawberry, one for blueberry, and one for lemon.***

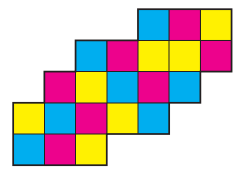
Choose three students and give each the appropriate color marker or crayon.



T: ***The three of you are going to ice the cake. When I say go, I want each of you to color one square. … Go!***



T: ***Color another square. Go!***



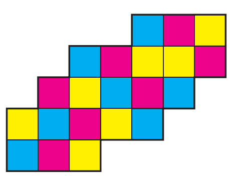
Continue signaling the students to color another square until every square in the shape has been colored. This is one possible coloring.

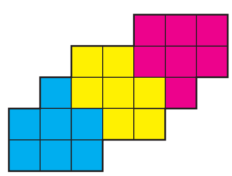
Note: To keep the lesson from slowing down because of careful coloring, you may like to suggest students simply "scribble" in the squares.

T: ***How many squares have strawberry icing?*** (Seven)

Point to each red square as the students count them. Do the same for the blue and the yellow squares. Then note that 3 x 7 = 21.

Tape the other copy of shape A to the board and invite three other students to repeat the activity.





Again, count the squares in each color.

T: ***These two pictures have the squares colored differently. But what is the same about the pictures?***

S: ***They both have red, blue, and yellow squares.***

S: ***Each has seven red, seven blue, and seven yellow squares.***

T: ***How many squares altogether in the cake?***

S: ***Twenty-one.***

T: ***We had three colors. What number is*** ${}^{1}/_{3}$ ***x 21*** (read as "one-third of twenty-one")***?***

S: *7.* $\frac{1}{3} \times 21 = 7$ $3 \times 7 = 21$ $21 \div 3 = 7$

Record these number stories on the board.

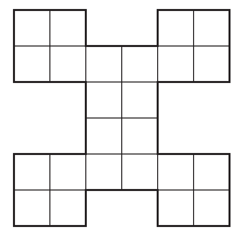
Remove the shapes and erase the board before going on to Exercise 2.

Exercise 2

Cut a copy of shape B from *UPG-II Geometry Poster* #5, and tape it to the board.

T: ***Mr. Booker is pleased with Jack and Jill's work and rewards them with a cake to bring to their Saturday club meeting.***

Shortly before the meeting is to begin, Jack divides the cake into square pieces. How many squares?

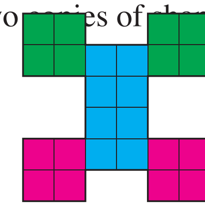


S: ***Twenty-four.***

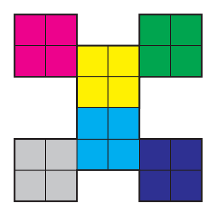
T: ***Jill protests, "But we only have twelve members!" Jack replies that it doesn't matter because whoever comes can have more than one piece.***

Ask how many pieces each person receives if the following numbers of members show up for the meeting. Be sure to mention that they must share the cake equally among the members present. Invite students to indicate how to section off the cake in each situation. Although colorings are shown below, do colorings only if you feel your students need them. (Two copies of shape B are provided.)

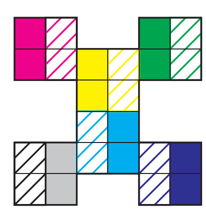
- Three members come to the meeting.



- Six members come to the meeting.



- Twelve members come to the meeting.

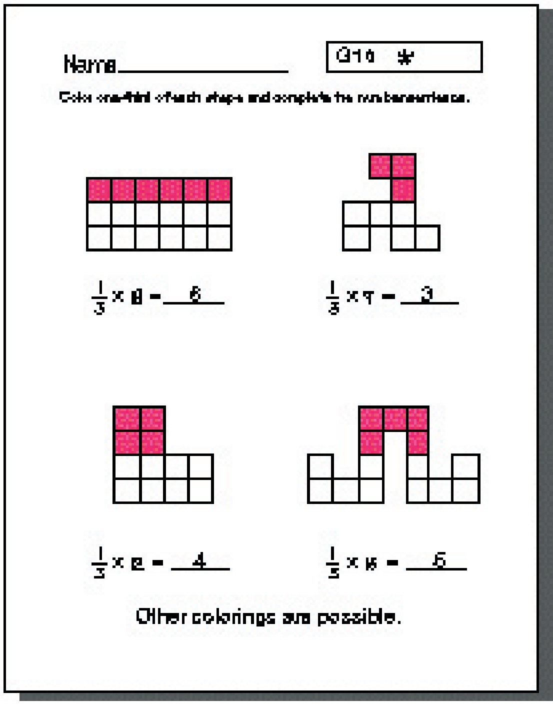


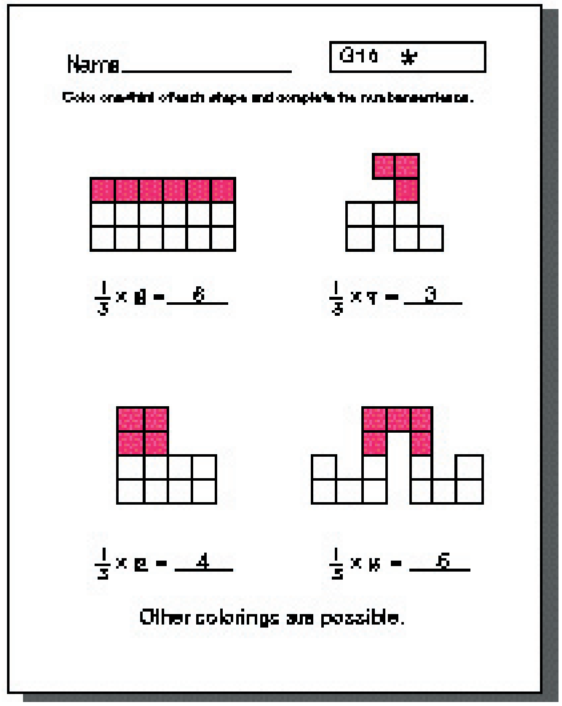
$\frac{1}{3} \times 24 = 8$ $24 \div 3 = 8$
$\frac{1}{6} \times 24 = 4$ $24 \div 6 = 4$
$\frac{1}{12} \times 24 = 2$ $24 \div 12 = 2$

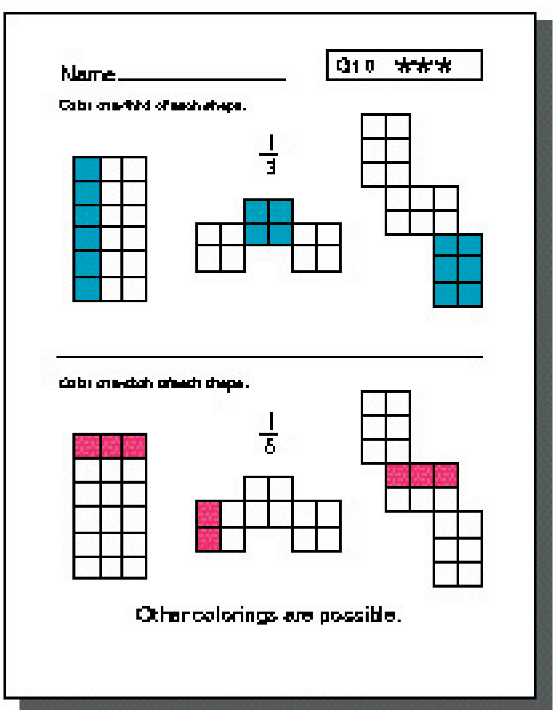
Record the corresponding number story for each situation as you consider it. You should have this list on the board.

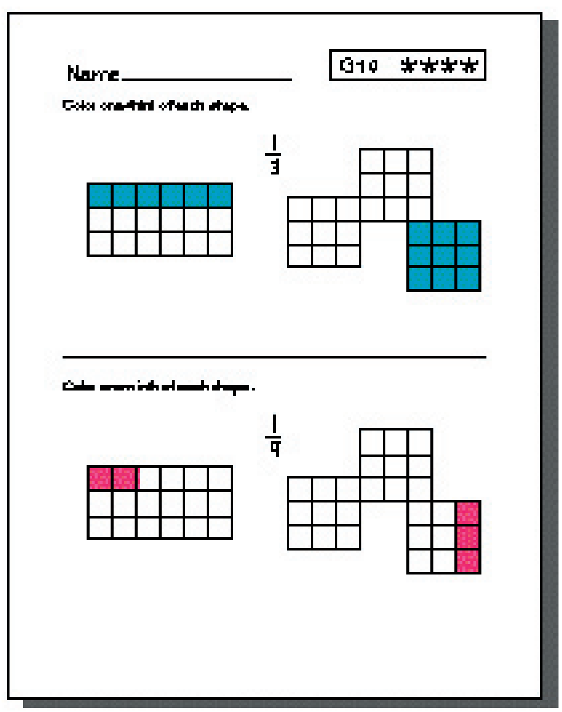
Note: If your students are concerned about what to do if five, seven, nine, or eleven members come, suggest that each member present could get the same number of pieces, and then the members could decide on a nice way to deal with any extra pieces. For example, they could give them to the members' younger brothers and sisters.

Wo ble for









Capsule Lesson Summary

Describe some geometric solids and investigate some of their properties. Using various geometric solids, make two-dimensional “maps” of their faces and record the results in a chart. Match two-dimensional face maps with geometric solids. Create a string picture in which geometric solids are classified according the shapes of faces or number of faces.

Materials

Teacher		Student	
	• Collection of geometric solids • Circle map • Chart paper • String loops • Index cards		• Geometric solid • Paper • Scissors

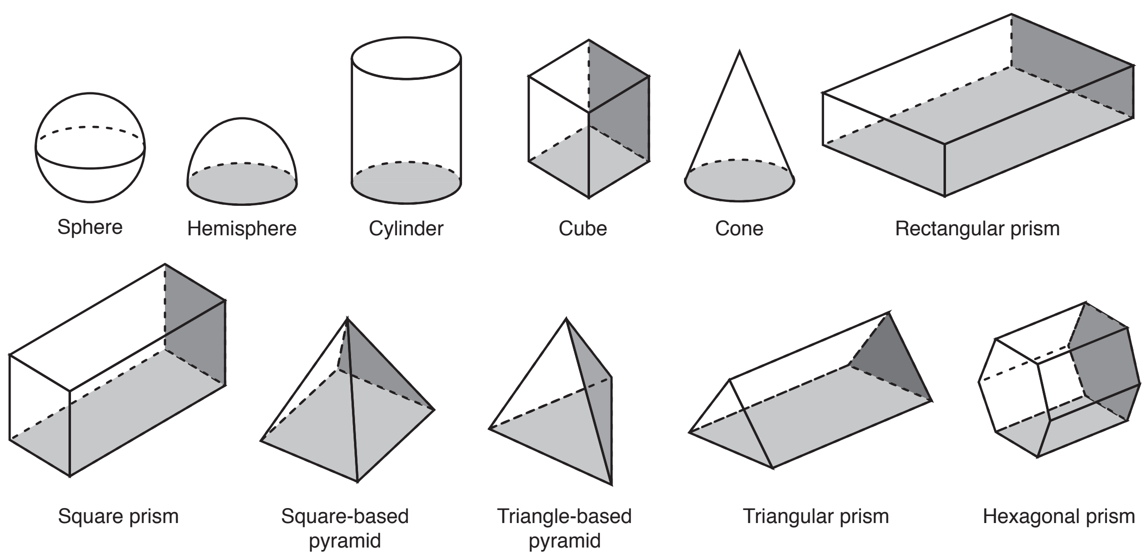
Advance Preparation: Use one of the solids to draw a circle map of a face; for example, use the cone.

Description of Lesson

Exercise 1

Display a collection of geometric solids and let students spend a few minutes examining and playing with them.

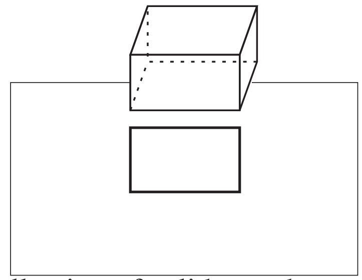
Note: This lesson description uses the following collection of geometric solids. Adjust the lesson to fit the collection you have available.



Begin a discussion by asking students to describe some of the solids and to observe similarities and differences. They may note color, material (for example, wood, plastic), size, number of corners, shape of faces, and so on. Encourage students to give names to the solids and to recall things they know that look like some of these solids. If you think your students would like to hear the mathematical terms for these solids, use the vocabulary, but do not expect students to recall it.

Exercise 2

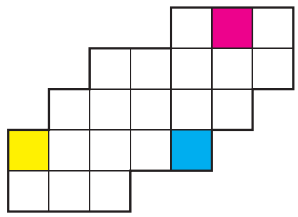
Choose one of the solids and show the class how to use it as a template to draw a map on one of its faces as illustrated here.



Show the class the circle map you made earlier with one of the solids.

T: ***Which of these solids do you think I used to make this circle map?***

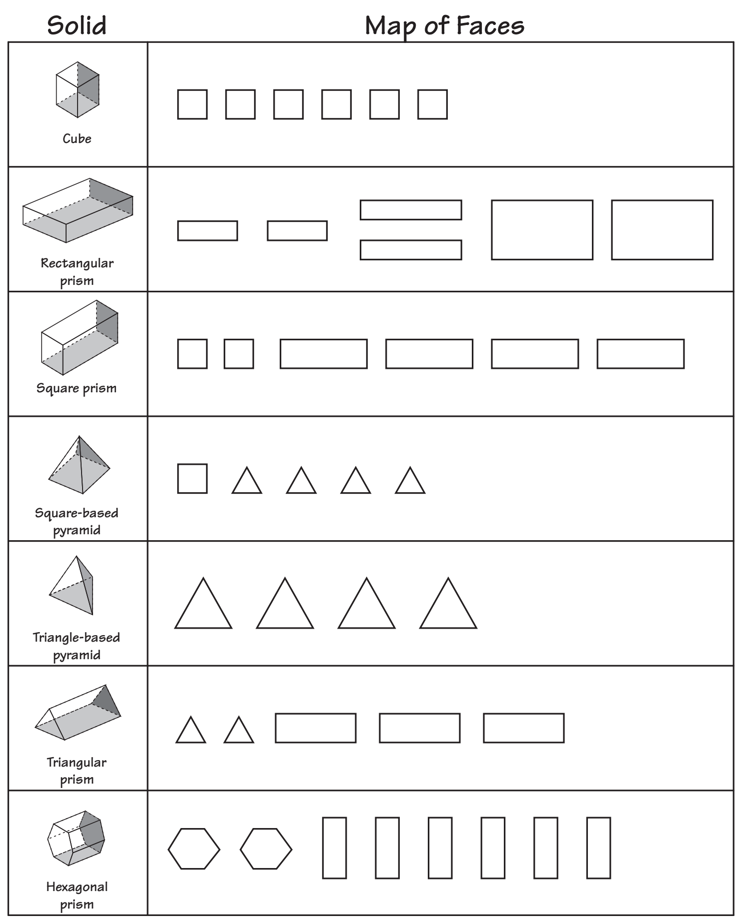
Let students find solids you might have used. Depending on your collection of solids, students might find several candidates. For example:



Exercise 3

Organize the class into cooperative groups. Provide each group with one of the solids that has corners but does not have a circular face, and a sheet of paper. Direct the groups to make a map of each face of their solid, and then to cut out the face maps. As you observe group work, check that each group has a method of making sure they make one and only one map of each face of their solid. For example, the group might use the cut-outs to cover the faces of their solid and check that every face is covered.

Prepare a chart, such as the following, on which the groups can record the results. Invite groups to place the face maps into the chart and to describe the solid they used as they do. For example:



S: ***Our solid was a box (rectangular prism) like this. It had two small rectangle faces the same size and four bigger rectangle faces.***

Occasionally invite a group to put the face maps into the chart without showing the class its solid. Then let the class guess which solid the group had.

After the chart is complete, call on students to again observe similarities and differences among the solids. For example, students may notice there are several solids that have a square face, but only one solid that has exactly four faces.

Exercise 4

Place a large string loop on the floor or a table. Write **Has a △ face** on an index card and place it near the string as a label.

T: ***This string is for all the solids having a triangular face. Which solids should we put inside the string? Which should we put outside the string?***

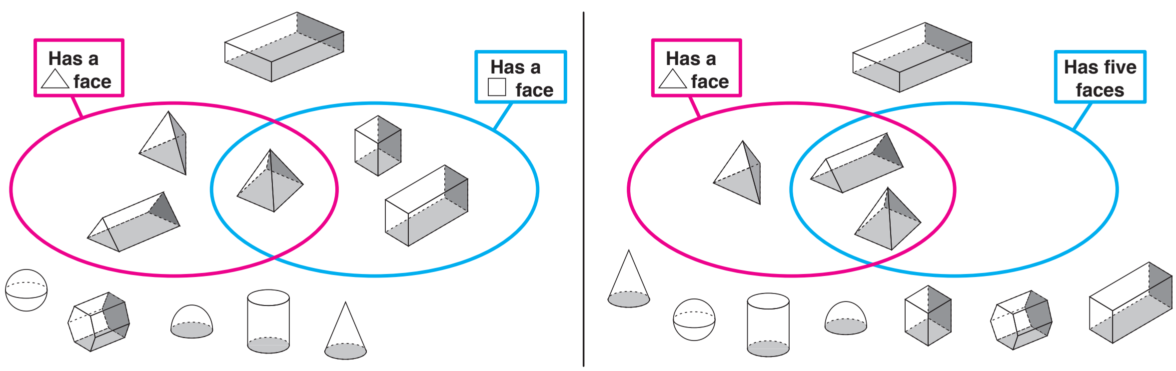
Invite students to place solids and solicit class agreement on placements.

T: ***Here is another string. Let's give it a label and make a two-string picture. What label would you like to give this second string?***

S: ***Has a square face.***

S: ***Has five faces.***

Investigate the resulting picture for one or two different choices for the second string. For example:



Center Activity

Place solids, string loops, and index card labels in a center for students to use to make other string pictures with solids.

Cut the chart in Exercise 3 into strips, and remove the solid name and picture from its map of faces. Put these pieces in a center and instruct students to match a map of faces with a solid.

Extension Activity

Tell the class riddles about the solids, and let the students try to identify a solid from a riddle. For example:

- All my faces are squares. What am I? (Cube)
- I have five faces and five corners. What am I? (Square-base prism)

Home Activity

Suggest to parents/guardians that they identify some solids such as containers or other objects around the house.

G12 FRACTIONAL PARTS OF SHAPES #3

Capsule Lesson Summary

Investigate the construction of a shape made from six half-circles. Find many ways to divide the shape into twelve pieces, and color the shape half red and half blue. Using twelve quarter-circles, create other shapes to color half red and half blue.

Materials

Teacher
- Overhead projector (optional)
- Blacklines G12(a), (b), and (c)
- Colored chalk or markers

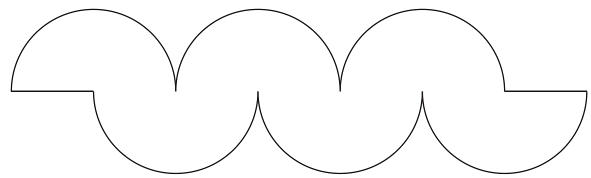
Student
- Red and blue circles
- Scissors
- Paper

Advance Preparation: Copy Blackline G12(a) on red and blue paper to make circles for students. Use Blacklines G12(b) and (c) to make overhead transparencies if you choose to use the overhead.

Description of Lesson

Exercise 1

Draw or project the following shape on the board. Blackline G12(b) can be used to make an overhead transparency.

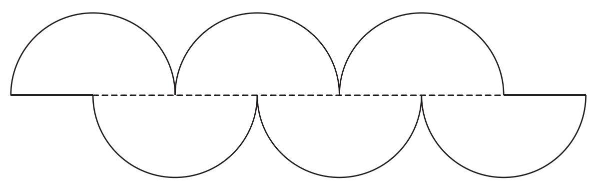


T: ***Mr. Booker is running a special at his bakery on "twist cakes." Do you have any idea about how he makes a twist cake?***

S: ***Maybe he has a special pan.***

S: ***He cuts circle cakes in half and puts the half-circles together to make a twist cake.***

Allow students to express their ideas and to demonstrate at the board or on the overhead transparency. For example, a student might make cut lines on the twist cake to show how it could be made from half-circles.

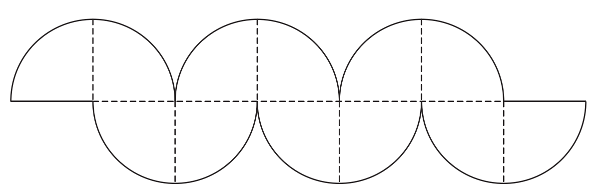


T: ***Jack and Jill are helping Mr. Booker this day. A customer orders a twist cake like this one*** (point to the shape on the board) ***and says it must serve 12 people at a party. The customer also asks for the twist cake to be frosted with strawberry and blueberry icing.***

Jack and Jill excitedly ask Mr. Booker to let them prepare the cake.

Let the class discuss how Jack and Jill might prepare the cake, reviewing that it is to have red and blue icing and it must serve 12 people. Perhaps your students will make suggestions similar to Jack and Jill's solution, as discussed here.

T: ***Jack decides to mark cut lines for half-circles first.***



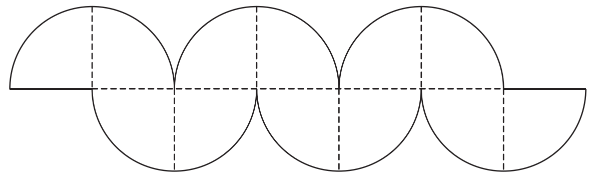
Invite a student to do this on the board or overhead; point out the six half-circles.

T: ***How many pieces does this make in the twist cake?***

S: ***Six.***

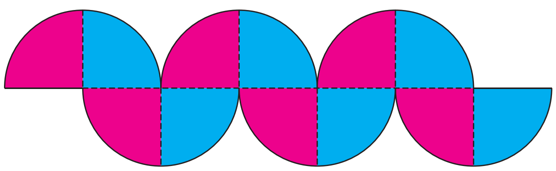
T: ***Next he thinks about the fact that the cake must serve 12 people. How could he mark cut lines to show 12 pieces?***

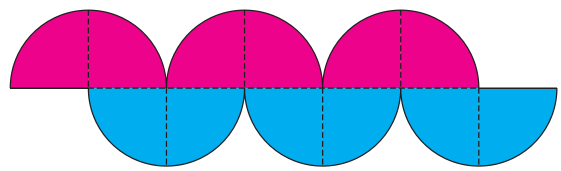
Invite students to make suggestions. It is likely someone will indicate cutting each half-circle in half. After cut lines are marked on the board drawing, call on students to count the pieces.



T: ***Jill thinks they should frost the cake so that there are the same number of pieces with each flavor icing. Remember the customer wants strawberry and blueberry icing. Can they frost the cake with an equal number of strawberry and blueberry pieces?***

Let students find different ways to do this and picture one or two of them.





T: ***How many pieces have strawberry icing?*** (Six)
How many pieces have blueberry icing? (Six)

$$2 \times 6 = 12$$
$$\tfrac{1}{2} \times 12 = 6$$

Exercise 2

Organize the class in math partners and give each pair of students three red and three blue circles.

T: ***Now you're going to find other ways for the twins to prepare the twist cake. First, cut the circles so you can make twist cakes.***

Instruct the student pairs to fold each circle carefully in half and to cut along the fold line. This may take several minutes to accomplish.

T: ***To make a twist cake, how many half-circles do you need?***

S: ***Six.***

T: ***If a twist cake is to be half red and half blue, how many red and how many blue half circles do you need?***

S: ***Three red and three blue.***

Direct student pairs to share their pieces so each one has three red and three blue half circles. Then suggest students use their pieces to make a twist cake. Check that individual students are able to do this.

T: ***Remember that the customer wanted the cake to serve twelve people. How can you make twelve pieces?***

S: ***Cut each half-circle in half.***

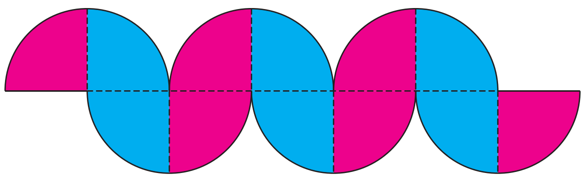
Instruct students to fold each of their six pieces and then cut along the fold line. Again, this will take several minutes. Check that each student ends up with six red and six blue quarter-circles.

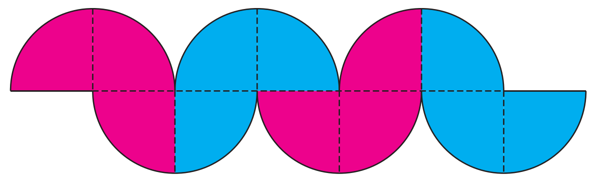
T: ***How many pieces do you have now? How many red and how many blue?***

S: ***Twelve; six red and six blue.***

T: ***Now, use your twelve pieces to design another twist cake for the customer. Remember, the cake serves twelve people and is frosted with half strawberry and half blueberry icing.***

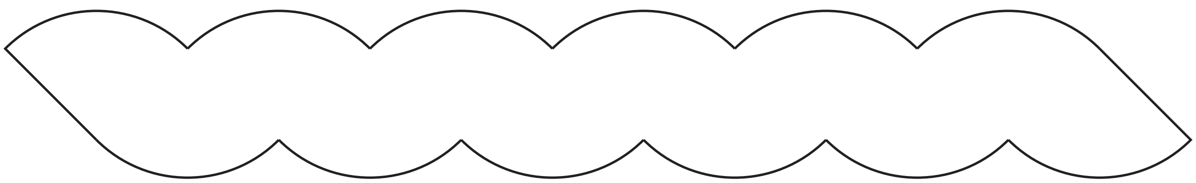
Let students find different ways of preparing a cake, and invite some students to put their designs on the board. For example:





Exercise 3

Draw or project the following shape on the board. Blackline G12(b) can be used to make an overhead transparency.



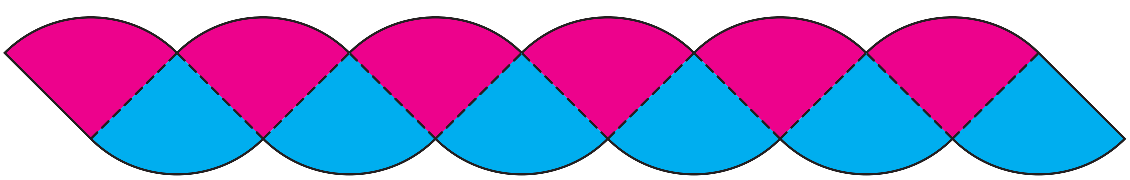
T: ***This in another twist cake Mr. Booker made. What do you notice about it?***

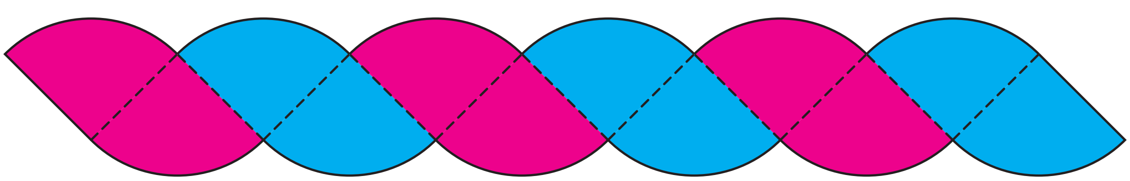
S: ***There are more twists.***

S: ***It's longer.***

T: ***See if you can use your red and blue pieces to prepare a twist cake like this one.***

It may take awhile for students to figure out how to put the twelve quarter circles together to make a twist cake with this shape. Once they do, let them investigate various ways to prepare it for a customer who wants to serve twelve people, and wants half strawberry and half blueberry icing. For example:





Encourage students to show some of their designs on the board.

Extension Activity

Repeat Exercises 2 and 3, this time with a customer who wants to serve 24 people. That is, cut each quarter-circle in half to get 24 pieces and investigate different cake designs. With 24 pieces, ask about making an even longer or twistier (tighter) cake.

Writing Activity

Suggest students record one or two different twist cakes and write directions for Mr. Booker to make these cakes.

G13 DOMINOES, TRIOMINOES, AND TETROMINOES

Capsule Lesson Summary

Investigate all the different ways of making shapes from one, two, three, and four squares put together in a specified way. Introduce *dominoes*, *triominoes*, and *tetrominoes* in this way. Use two or three tetrominoes to cover other shapes in the form of puzzles.

Materials

Teacher	• Collection of shapes • Blackline G13	**Student**	• Square tiles • Grid paper • Scissors • Worksheets G13 * and **

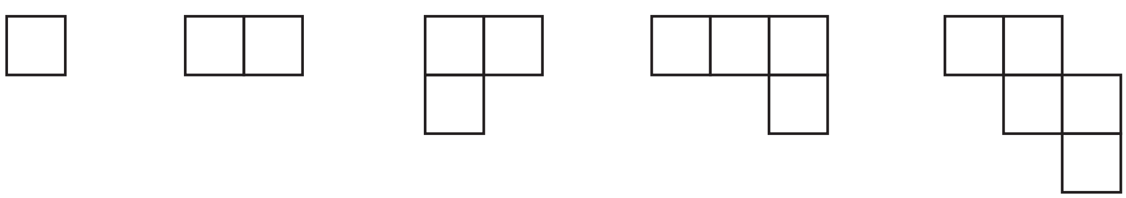
Advance Preparation: Make grid paper for students using Blackline G13. This same blackline may be used to make a collection of shapes as pictured in Exercise 1.

Description of Lesson

Arrange for students to work with a partner during this lesson. Provide each pair of students with four or five square tiles (all the same color) and grid paper with grid squares the size of the tiles. Allow a few minutes for students to play with the tiles.

Exercise 1

Display a collection of shapes you have made from one, two, three, four, or five squares. For example:



T: ***What do you notice about all these shapes?***

S: ***They are made from squares.***

S: ***They are different shapes.***

S: ***They are different sizes.***

S: ***One looks like a domino.***

Let students make many observations. Try to relate some of these shapes to familiar games or puzzles, and use names such as *domino* and *tetromino*. Some students may be familiar with the computer game called Tetris and recognize the four-square piece (tetromino) as one used in that game.

Point out that when two squares connect, the sides they touch on are touching completely; that is, they are flush with one another. This will be a rule in the construction that follows.

Direct the students to take out two square tiles and ask,

T: ***How many different shapes can we cover on this grid with two squares? Let's make our shapes like dominoes so the squares have to touch along a complete side.***

S: ***Only one.***

Check to see that all the students understand there is just one possible domino shape. You may reiterate the rule to eliminate shapes such as  and .

Begin a table on the board, putting in the information for dominoes first, and then asking how many shapes are possible with one square.

	Area	Number of Shapes
Square	1S	1
Dominoes	2S	1

Direct the students to take out three square tiles; then show them a shape made of three squares from your collection.

T: ***With three square tiles we can make shapes called triominoes. What would be the area of a triomino?***

S: ***3S.***

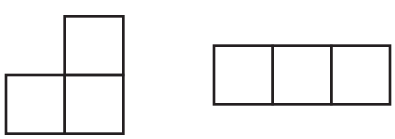
Add this information to your table.

Note: We are using the language of dominoes, triominoes, and so on to relate these shapes and provide experience with common terminology. It is not necessary that students learn to use these names at this point.

T: ***How many different shapes can we cover on this grid with three squares? Before you make the shapes, predict how many different ones there will be.***

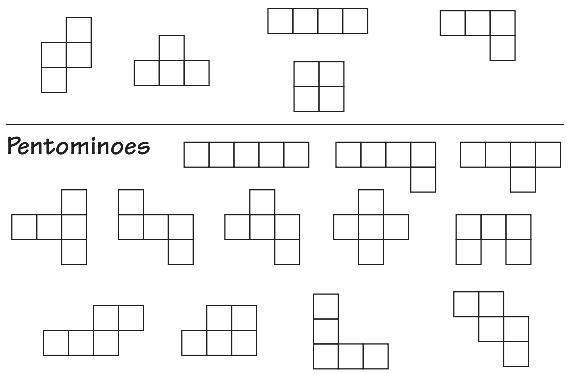
Accept predictions without comment unless someone talks about making shapes that do not follow the rule: squares must be touching along a complete side. Then instruct the students to work with their partners to find as many different three-square (tromino) shapes as possible. As they work, suggest the students use the three square tiles to cover a shape on the grid, and then color the shape. You may also direct students to cut out their triomino shapes, especially if it is difficult to see that  and  are the same (by turning or flipping).

When everyone agrees that there are only two triomino shapes, add this information to your table.



	Area	Number of Shapes
Square	1S	1
Dominoes	2S	1
Triominoes	3S	2

Continue this exercise, looking for four-square shapes (tetrominoes) and possibly five-square shapes (pentominoes). You may want to make finding pentominoes an extension activity or a challenge activity for students who find all the tetrominoes quickly. Leave some time for Exercise 2.



	Area	Number of Shapes
Square	1S	1
Dominoes	2S	1
Triominoes	3S	2
Tetrominoes	4S	5
Pentominoes	5S	12

Exercise 2

If students have not already done so, ask them to cut out the five tetrominoes so that each pair of students has a complete set. Distribute copies of Worksheets G13* and **.

T: ***The worksheets have shapes to cover with your tetrominoes. They are like puzzles. On the * side, use two different pieces to make (cover) each shape.***

You may want to work as a class to do one of the shapes on the * side of the worksheet. Then let students work with their partners to solve the rest of the puzzles. Students who finish quickly can create their own puzzles for other students to solve.