The World of Numbers

WORLD OF NUMBERS INTRODUCTION

Numbers are among the most important things that mathematics (at all levels) is about. Mathematicians are interested in numbers just as astronomers are interested in stars, botanists in plants, and sociologists in the organization and dynamics of human societies. Surely everyone agrees that one of the primary goals of any elementary school mathematics curriculum should be to introduce students to the world of numbers—to give them the opportunity to become familiar with numbers, their properties, and the relations between them. The ability to calculate with numbers is an important part—but not the only part—of being familiar and comfortable with numbers. The World of Numbers strand of *CSMP Mathematics for the Upper Primary Grades* is designed to provide students with a wide variety of challenging experiences with numbers so that gradually they will become not only familiar, but comfortable with numbers; they will, so to speak, get to know numbers on a "first name basis" and develop number sense.

The Minicomputer and the World of Numbers

CSMP Mathematics for the Upper Primary Grades uses the Papy Minicomputer as a support for the positional system of numeration; for calculations and estimation; for number patterns and mental arithmetic; and for modeling the basic operations involving whole numbers, integers, and decimal numbers. Although the Minicomputer can be seen as a tool for calculation and as a device to help students learn routine methods for calculations, its more exciting use is as a vehicle for posing interesting problems that challenge a child's intellectual curiosity about numbers, and for presenting situations that both encourage strategic thinking and reinforce numerical skills. The lessons that make use of the Minicomputer are often intended to be explorations into the world of numbers.

Standard Algorithms of Arithmetic

CSMP seeks to develop basic numerical skills as well as an understanding of the underlying mathematical ideas. We are fully in agreement with the thesis that, along with the growth of understanding of the world of numbers, there must be a concommitant growth of familiarity and facility with numbers and operations on them. But facility should not be confused with understanding; they are partners in the growth of mathematical maturity. A balanced growth of each must be maintained, neither being sacrificed for the other.

Students must eventually learn mechanical algorithms for the basic operations (addition, subtraction, multiplication, division). However, premature presentation of these algorithms may actually stunt a student's ability to develop alternative algorithms, to do mental arithmetic, or to estimate.

Consider the problem of calculating 385 + 99. A second grader may have difficulty performing the standard (paper and pencil) addition algorithm. An easier and more efficient way to proceed is to add 100 to 385 and then to subtract 1 (385 + 100 = 485 and 485 – 1 = 484). To insist on a mechanistic response to such a problem would be to encourage inefficiency and might also inhibit the development of the flexibility necessary for problem solving. On the other hand, a rich array of situations in which students interact with numbers provides them with opportunities to gain the necessary facility with standard algorithmic procedures while retaining the openness required to respond creatively to new situations in the world of numbers.

Numerical Relations

One of the main aims of the World of Numbers strand is to familiarize students with numbers by studying relations between numbers, both explicitly and in a variety of contexts. (For more general comments about relations, see the introduction to the Languages of Strings and Arrows strand.) Arrow diagrams represent relations in a simple, suggestive, and pictorial way—usually more conveniently than the same information could be given in words or other symbols.

Students are brought into contact with an assortment of challenging situations, many of which would be totally inaccessible to them were it not for the arrow diagrams. The problems and activities of this strand include solving linear equations presented in terms of arrows; studying iterated processes and patterns in sequences of numbers; tackling problems that may have many solutions or no solution; estimating or testing that a solution is reasonable; and exploring properties of operations on numbers.

In summary, what is most important in the study of numbers is to confront students with a variety of problems and situations that capture their interests, challenge their abilities to reason, and stimulate their curiosities about numbers—and, at the same time, provide them with tools for coping with these situations and problems.

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N1 SUBTRACTION PROBLEMS #1

Capsule Lesson Summary

Explore the effect of moving various checkers in a configuration on the Minicomputer—in each case, does the move increase, decrease, or leave the same number on the Minicomputer? Put a number on the Minicomputer with many checkers; then alternate estimating the number and making some trades until standard configuration of the number is displayed. Estimate the result of a subtraction calculation involving three-digit numbers and then do the calculation on the Minicomputer.

Materials

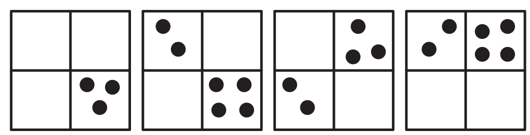
Teacher
- Minicomputer set

Student
- Index card
- Paper
- Worksheets N1 *, **, ***, and ****
- Minicomputer set (optional)

Description of Lesson

Exercise 1: Transforming a Number

Put this configuration on the Minicomputer.



T: ***I put a number on the Minicomputer. It's not easy to read, but let's try to estimate. Is this number more than 1,000? How do you know?***

S: ***Yes. The number is more than 3,000, because there are three checkers on the 1,000-square and more checkers on other squares.***

T: ***We know this number is more than 3,000. Is it more than 4,000? How do you know?***

S: ***Yes, because there are three checkers on the 1,000-square and there are two checkers on the 800-square. 800 + 800 is more than 1,000.***

T: ***Is this number more than 5,000?***

It is possible that a student will be able to explain and to convince the class why this number is more than 5,000. In any case, your class should estimate the number to be more than 4,000 and possibly more than 5,000.

T: ***We know that the number on the Minicomputer is more than 4,000 (5,000). We do not need to know exactly what number it is to compare it to other numbers.***

Write these words on the board close to the Minicomputer.

T: ***I am going to move, remove, or add some checkers. Each time tell me if the number on the Minicomputer is more than, less than, or the same number as before.***

Move a checker from the 1,000-square to the 2,000-square.

Decide on a method for students to show whether they believe the new configuration is for a number more, same, or less than before. For example, they can write the three words on an index card and then hold up the card pinching it on their choice.

Repeat the move very obviously if many students do not know that the number now on the Minicomputer is more than the previous one.

T: ***How much more is this number than the number we had before? How do you know?***

S: ***1,000 more, because you moved the checker from the 1,000-square to the 2,000-square.***

Return the checker to its original position.

Continue this activity with the following moves or similar ones. After each move return the checkers to their original positions.

- Move a checker from the 4-square to the 10-square. (6 more)
- Replace any checker by a checker of another color. (Same)
- Move a checker from the 20-square to the 1-square. (19 less)
- Make a 40 + 40 = 80 trade. (Same)
- Move two checkers from the 100-square to the 200-square. (200 more)
- Make an 800 = 400 + 400 trade. (Same)
- Make an 8 + 4 = 10 + 2 trade. (Same)

When appropriate, ask the students exactly how much more or less a number is than the previous one. After you make four or five moves yourself, invite students to transform the number on the Minicomputer.

At the end of this activity, leave the checkers on the Minicomputer for Exercise 2.

Exercise 2: Estimation

Remind the class that they previously estimated this number to be more than 4,000 (5,000). Invite students to make several trades, and then guide the class to make a closer estimate. You may allow students to guess what the number is and record some guesses on the board.

Continue making trades until standard configuration is obtained. Invite a student to write the number below (above) the Minicomputer and to determine which guess was the closest. Erase the board and remove the checkers from the Minicomputer.

Exercise 3: Subtraction Problems

T: ***Marta is making a bead necklace with 535 beads in all. She has strung 284 so far; how many more will she need to string to complete the necklace?***

S: ***We need to know what 535 take away 284 is.***

T: ***What would that number sentence look like?***

535 – 284 =

Call on a student to write the subtraction problem on the board.

T: ***The problem is to find what number 535 – 284 is.***
Is it more than 100? How do you know?
Is it more than 200? How do you know?
Is it more than 300? How do you know?

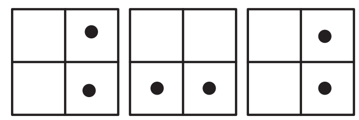
Let the students react freely to these questions. Accept their explanations, even if they are not well-formed. Suggest students estimate the number and write their estimates on their papers.

T: ***How can we find out what number 535 – 284 is?***

S: ***Put 535 on the Minicomputer and then take off 284.***

Note: Some of your students might suggest calculating 535 + $\widehat{284}$. In this case, let the class decide which calculation they prefer to make. The lesson description does not include a discussion of the calculation 535 + $\widehat{284}$, but you can modify it by making minor changes.

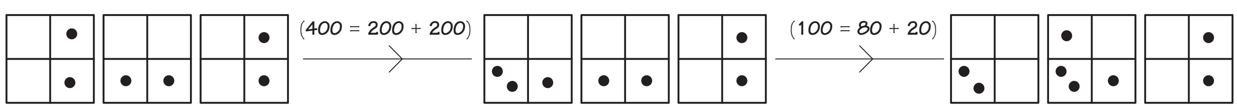
Invite someone to put 535 on the Minicomputer.



Ask for a volunteer to point to the squares where checkers are needed for 284. There are many possibilities, but very likely the student will indicate the standard configuration for 284 by pointing to the 200-square, to the 80-square, and to the 4-square.

T: ***Who can make a trade that will help us get checkers on these squares?***

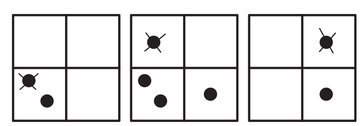
Only two trades are needed, although students may make other unnecessary trades.



T: ***Who can take 284 off the Minicomputer?***

T: ***What number is 535 – 284?***

S: ***251.***



Complete the number sentence on the board.

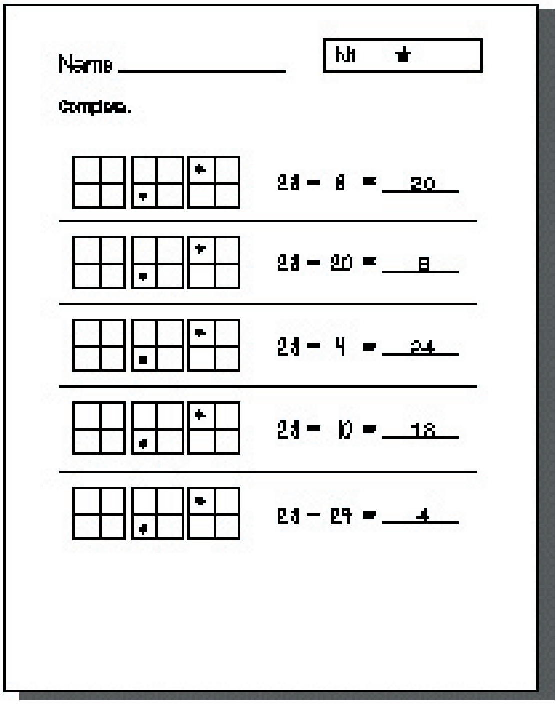
535 – 284 = 251

Suggest students compare their estimates to 251.

Worksheets N1*, **, ***, and **** are available for individual work. Allow students to use individual Minicomputers. You may want to model crossing out checkers that are subtracted (taken away) by completing one or two problems from the * worksheet on the board.

Home Activity

Send home several subtraction problems for students to do on their home Minicomputers with family members.


Name ____________ N1 ★

Complete.

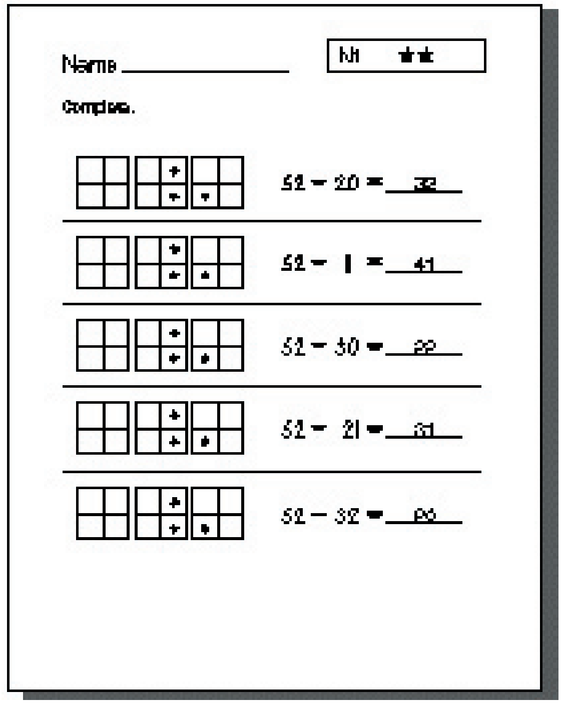
28 − 8 = 20

28 − 20 = 8

28 − 4 = 24

28 − 10 = 18

28 − 24 = 4


Name ____________ N1 ★★

Complete.

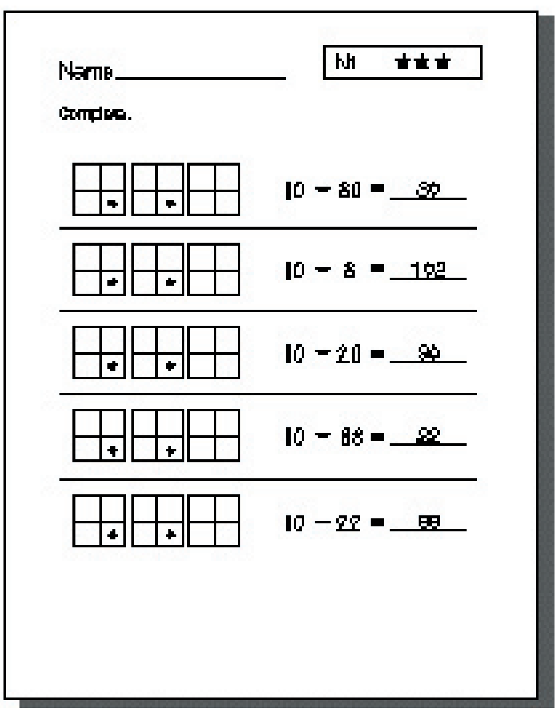
52 − 20 = 32

52 − 11 = 41

52 − 30 = 22

52 − 21 = 31

52 − 32 = 20


Name ____________ N1 ★★★

Complete.

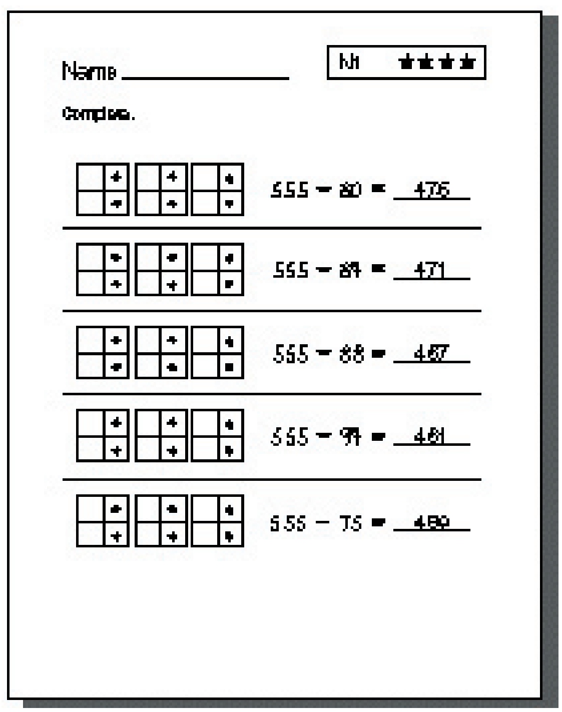
110 − 80 = 30

110 − 8 = 102

110 − 20 = 90

110 − 88 = 22

110 − 22 = 88


Name ____________ N1 ★★★★

Complete.

555 − 80 = 475

555 − 84 = 471

555 − 88 = 467

555 − 94 = 461

555 − 75 = 480

Capsule Lesson Summary

Assign numbers to each letter of a name from the alphabet chart. Use a calculator to add these numbers and to find the value of the name. Make a graph showing the value of each name in the class. Find out if some names have the same value even though they are not spelled alike. Make up names or words with a given number value.

Materials

Teacher
- Overhead or class calculator
- Post-it™ notes

Student
- Calculator
- Worksheet N2

Description of Lesson

Distribute copies of Worksheet N2

T: ***Today we are going to find the "values" of our names . Then we will look for names with the greatest value and least value, and we will try to make up names that have given values. Look at Worksheet N2. On it you will see that each letter of the alphabet has been assigned a number. What number goes with the letter D?***

S: ***4.***

T: ***What letter has the number 25?***

S: ***Y.***

T: ***Now write the letters of your first name on the blanks. Then below each letter write the number for that letter.***

When students finish writing their names and associating number values with the letters, direct them to use calculators to add the numbers for the letters in their names. The total is the name value.

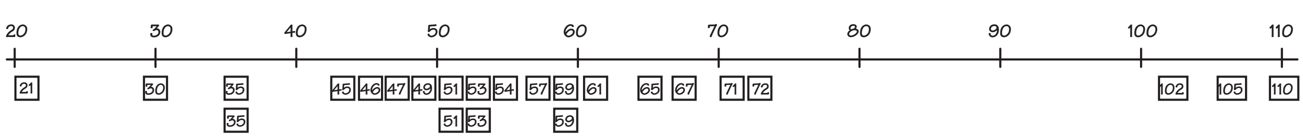
Note: You may like to show an example for the class before students find their own name values. Use a popular character's name or your own, and the overhead or class calculator.

T: ***What value did you find for your name?***

S: ***My name is Anna and the value is 30.***

S: ***My name is Zoey and the value is 71.***

Discuss with students what some of their name values are. Ask for the greatest and least values. Instruct students to write their name values on Post-it™ notes, and construct a number line graph of the numbers. For example:



Some possible questions or observations to make include the following:

- What is the difference between the greatest and least name value?
- How can two people have the same name value? Do they have the same name?
- Most of our name values are in the 40's or 50's.
- Does the person with the greatest (least) name value have the most (fewest) letters in their name?
- There was no one with a name value between 80 and 100 (or with the name value 75). Can you make up a name that has such a name value?

For this last question, let students work with a partner to find a name that fits. You might start by asking partners to find a name with a value between 80 and 100; then ask those who are able to do that to find one with value 75.

When many students have solved one of these problems, ask student pairs to share the names they found and the strategies they used to find names.

Center Activities

- Place lists of spelling words, or other words of interest to your class, in a center with a letter value chart and a calculator. Direct students to find the value of each word.
- Give students opportunities to use calculators to play games such as "20" (see *UPG-I* Lesson N26, Exercise 5) or "Target" (see *UPG-I* Lesson W4). Both games can be modified to use different numbers.

Writing Activity

Suggest that students write a letter to a friend about how they find the value of their names.

Capsule Lesson Summary

Locate numbers on the number line. Make some consecutive +2 and –3 jumps on the number line and label the ending points of the corresponding arrows. Play The Number Line Game.

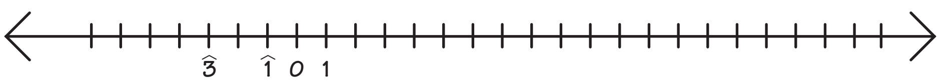
Materials

Teacher	• Colored chalk	**Student**	• Paper

Description of Lesson

Exercise 1

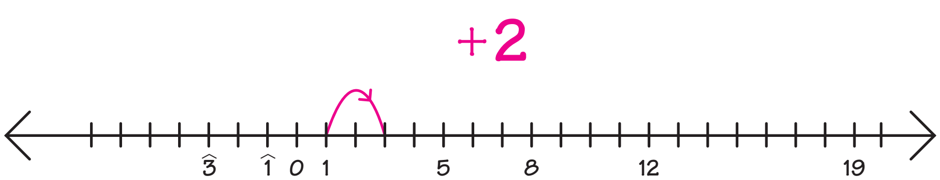
Draw a line on the board with equally spaced marks at least six centimeters apart. Label some of the marks as shown below.



Point to some of the marks to the right of 1, each time moving further to the right. When you point to a mark, ask, "What number is here?" and then label the mark when the correct number is given. Continue until students have helped label four or five marks.

T: ***Let's start at 1 on the number line and make jumps to the right, moving two spaces each time.*** (Write +2 in red above the number line.) ***I'll make the first +2 jump.***

Draw a +2 arrow starting at 1.

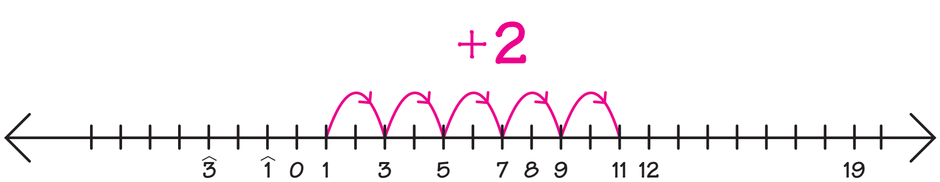


T (pointing to the ending point of the arrow)**:** ***1 + 2 = …?***

S: ***3.***

Ask a student to label the appropriate mark 3.

Invite students to make some consecutive +2 jumps, to draw the appropriate arrows, and to label the ending points of the arrows.



N3

When you have about five +2 jumps, ask some of the following questions or similar ones. The questions and answers given below are based on the preceding illustration.

- How many jumps have we made so far? (Five jumps)
- How many spaces to the right of 1 have we moved altogether? (Ten spaces)
- Which numbers have we landed on? (1,3,5,7,9,11)
- Which numbers have we jumped over? (2,4,6,8,10)
- Do you see a pattern? (We land on odd numbers and jump over even numbers)
- If we were to continue making +2 jumps from 11, we would land on many more numbers. Name some of them. (Any odd integer more than 11 is correct)
- Would we ever land on 100? (No, 100 is an even number)
- What is a number greater than 100 that we would land on?
- What is a number greater than 100 that we would jump over?
- What is the least number greater than 100 that we would land on? (101)

Note: Numbers such as 102.5 and 117 $^1/_4$ are jumped over as well as all the even numbers. Also, the +2 jumps will not land on 0 or any negative number, because we started at 1.

You may like to ask students to list the odd numbers between 0 and 20 on their papers.

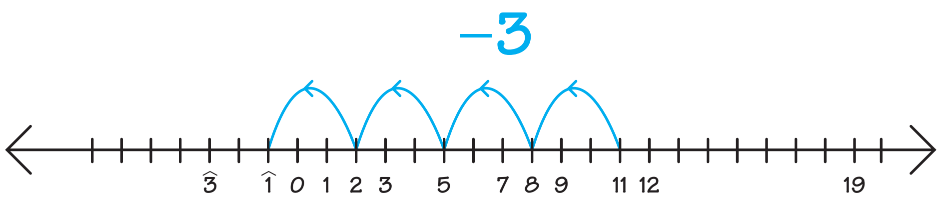
Erase only the arrows and the key, leaving the labeled number line.

Exercise 2

Write –3 near the number line.

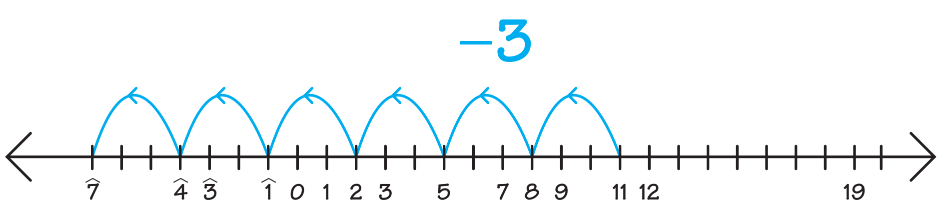
T: ***This time we will make –3 jumps*** (read as "minus three jumps") ***starting at 11.***

Ask for volunteers to trace some consecutive –3 jumps and then to draw the appropriate arrows. Your picture should look similar to this one.



Students should have little trouble making the –3 jump from 2 to $\hat{1}$. Those who seem puzzled should be reminded that the –3 jump from 2 goes three spaces to the left, just as the other –3 jumps.

Continue to ask students to draw –3 jumps from $\hat{1}$ until you have six or seven jumps altogether.



You may want to ask questions similar to those asked about the +2 jumps. Other suggestions follow.

- How is this picture different from the one showing +2 jumps?
  (Possible answers: The jumps go to the left instead of to the right. These jumps are longer. Each of these jumps took us to a smaller number; each +2 jump took us to a greater number.)
- The last –3 jump drawn on the board ends at $\hat{7}$. If we made another –3 jump, where would we land? ($\widehat{10}$)

If your class is interested and enjoying the lesson, vary the activity by making +5 (or –14, +10, –5) jumps or by changing the starting points so that you occasionally begin at a relatively large number, such as 59, and occasionally at a relatively small positive number, such as 6. You may even wish to start with a negative number. Otherwise, proceed to Exercise 3.

Note: The problems in these exercises are directly related, of course, to simple addition and subtraction calculations. For example:

Labeling the landing point of a –3 jump starting at 2	corresponds to	doing the calculation $2 - 3 = \hat{1}$.
Labeling the landing point of a –3 jump starting at $\hat{5}$	corresponds to	doing the calculation $\hat{5} - 3 = \hat{8}$.
Labeling the landing point of a +2 jump starting at 11	corresponds to	doing the calculation $11 + 2 = 13$.
Labeling the landing point of a +2 jump starting at $\hat{9}$	corresponds to	doing the calculation $\hat{9} + 2 = \hat{7}$.

This information is for you; do not make any formal statement to your students about rules for addition or subtraction of positive and negative integers.

Exercise 3: The Number Line Game

The Number Line Game can be played frequently throughout the year whenever you finish a lesson early or have some extra time. Usually, a game will take about five minutes. Several variations of the game are given at the end of this lesson. Use these variations when your class is ready for them. The following is a description of a possible game. Suppose the secret number is 114.

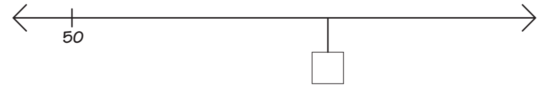
T: ***Today we are going to play The Number Line Game. I am thinking of a number between 0 and 200. I'll show you where it is on the number line; my number is hidden in this box.***

Draw a line on the board and position a box for the secret number near the center of your picture.

T: ***Guess what my secret number is.***

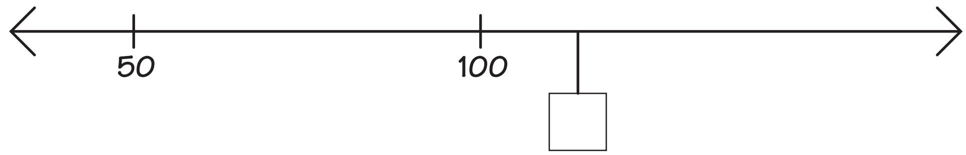
S: ***50.***

T: ***I'll show you where 50 is on this number line. It might not be in exactly the right place, but I'll try to make it as close as possible.***



50 is less than my secret number. Who has another guess?

S: ***100.***



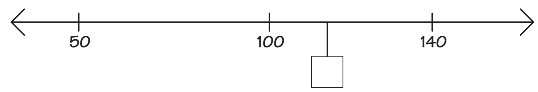
T: ***100 is less than my number.***

S: ***210.***

T: ***My number is between 0 and 200. 210 cannot be my number.***

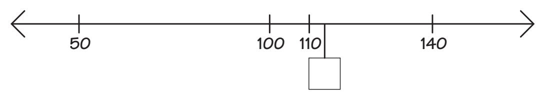
Anytime the guess is a number that is not between 0 and 200, do not write anything. Just remind students that your number is between 0 and 200, and ask someone else to guess.

S: ***140.***



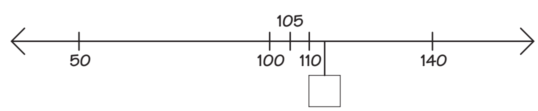
T: ***140 is more than my secret number.***

S: ***110.***



T: ***110 is less than my secret number.***

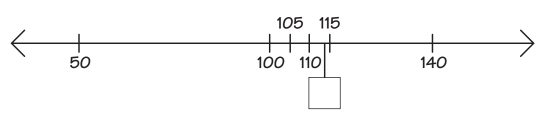
S: ***105.***



T: ***105 is less than my secret number.***

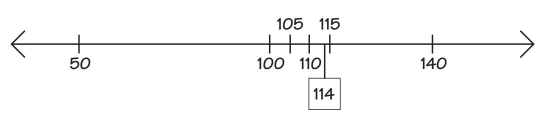
Note: Although a guess may give no new information, such as 105 in this sample game, do not criticize or comment except to say that it is more or less than your number.

S: ***115.***



T: ***115 is more than my number.***

S: ***114.***



T: ***Yes, you discovered my secret number.***

Play the game a few more times. Let the student who guesses your number choose the secret number for the next game and whisper it to you. Then ask the student to keep the number a secret and not to play in that game.

VARIATIONS OF THE NUMBER GAME

Variation 1

When your class is comfortable with the game using numbers between 0 and 200, you can expand the range of possible numbers. Some possibilities include the following:

- numbers between 0 and 500
- numbers between 500 and 1,000
- numbers between $\widehat{50}$ and 50
- numbers between $\widehat{100}$ and 100

Variation 2

As the students become familiar with non-integers such as $2\frac{1}{2}$, occasionally choose a non-integer as your secret number. It is best to limit the range of possible numbers to those between 0 and 10 when you use this variation.

Variation 3

When you are using only whole number for your range of possible numbers, ask a student to choose a secret number and to judge the guesses of the other students. The student who chooses the secret number then tells the class whether a guess is more or less than the secret number. You draw and label a mark on the number line for each guess and, if necessary, help the student to judge. Remind the student who is judging not to give additional clues.

Variation 4

Divide the class into two teams. The teams alternate turns and the members of each team take turns guessing the secret number.

N4 ROADS #1

Capsule Lesson Summary

Consider four arrow roads that start at the same number and have only +10 and +1 arrows. Determine which road has the greatest ending number and which has the least. Label the dots in all the roads. Draw roads that have three +10 and two +1 arrows, and discover that the order in which the arrows are drawn does not change the ending number. Build a road from 0 to 24 using +10 and +1 arrows, and decide which is the shortest possible road and which is the longest. Do some mental arithmetic asking for the ending number of a road described in terms of the number of +10 and +1 arrows.

Materials

Teacher
- Colored chalk

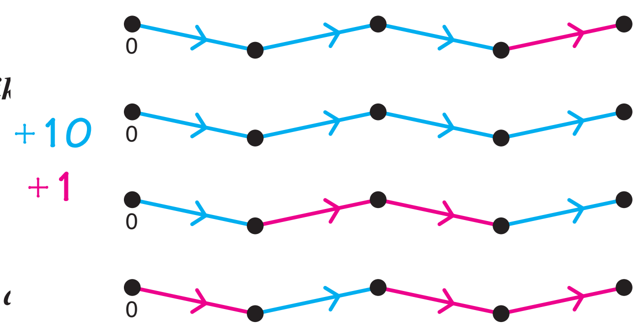
Student
- Colored pencils, pens, or crayons
- Unlined paper
- Worksheets N4* and **

Advance Preparation: Prepare the arrow picture for Exercise 1 on the board before starting the lesson.

Description of Lesson

Exercise 1

Refer to this arrow picture on the board.



T: *How are these four arrow roads alik*
How are they different?

S: *They all have four arrows.*

S: *They all start at 0.*

S: *Three of the roads start with a +10 a*
and one road starts with a +1 arrow.

S: *There is a different number of red (or blue) arrows in each road.*

T: *Which of these roads has the greatest ending number? How do you know?*

S: *The second one, because it has the most +10 arrows.*

T: *What is the ending number of this road? How do you know?*

S: *40. I counted by tens: 10, 20, 30, 40.*

S: *40, because 4 x 10 = 40.*

Ask a volunteer to label the dots in the second road.

T: *Which of these roads has the least ending number? How do you know?*

S: *The last one, because it has the fewest +10 arrows.*

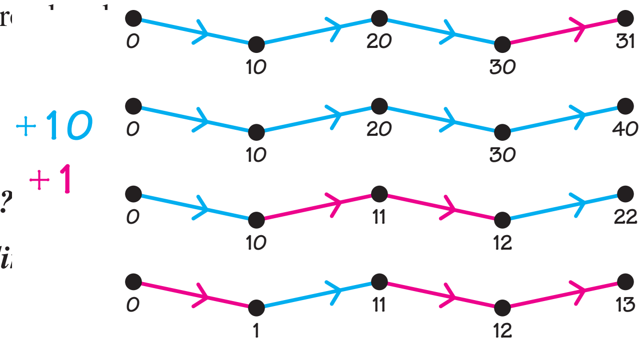
T: *What is the ending number of this road?*

S: *13.*

Call on students to label the dots in this r
then in the other roads.

T: ***Can you think of a way to get a number less than 13 by starting at 0 and using only four arrows?***

S: ***Use four +1 arrows and the endi number will be 4.***



Exercise 2

Instruct students to draw arrow roads on their papers that have exactly three +10 arrows and two +1 arrows, in any order they wish, starting at 0. Summarize this information on the board.

Start at 0
three +10 arrows
two +1 arrows

As students are working, invite several of them to each draw a road on the board. Continue until there are several different solutions on the board. It is likely students will notice that all of the roads end at 32; the order in which you draw +10 and +1 arrows does not matter.

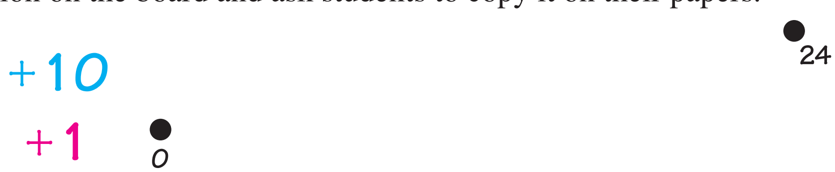
T: ***These roads always end at 32. What does this tell us?***

S: ***Three 10's and two 1's is 32.***

Erase the board.

Exercise 3

Put this information on the board and ask students to copy it on their papers.



T: ***Draw an arrow road from 0 to 24 using +10 arrows and +1 arrows.***

Let students work individually for a few minutes. Invite students who finish quickly to draw their roads on the board.

T: ***Which of the roads on the board has the most arrows?***
Did any of you build a longer road?
What is the longest road we can build?

S: ***The longest road is the one with twenty-four +1 arrows.***

T: ***Which of the roads on the board has the fewest arrows?***
Did any of you build a shorter road?
What is the shortest road we can build?

S: ***The shortest road is the one with two +10 arrows and four +1 arrows.***

Exercise 4: Mental Arithmetic

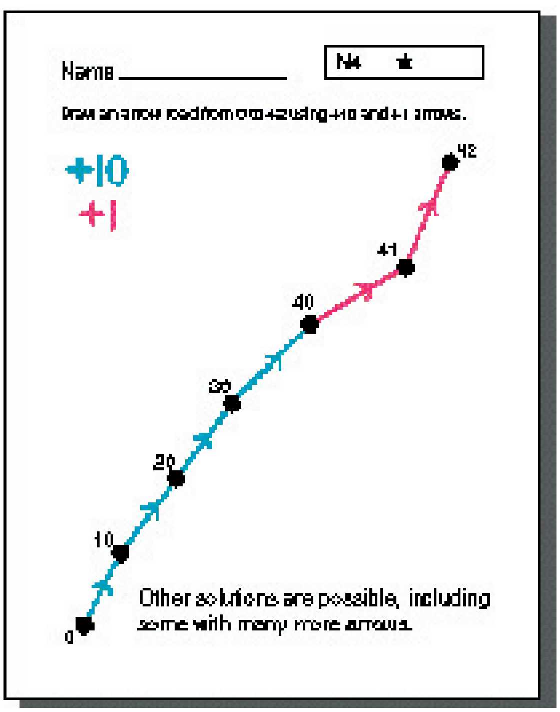
Ask students for the ending number of each of the roads described below. They should think about the arrow road, not draw it.

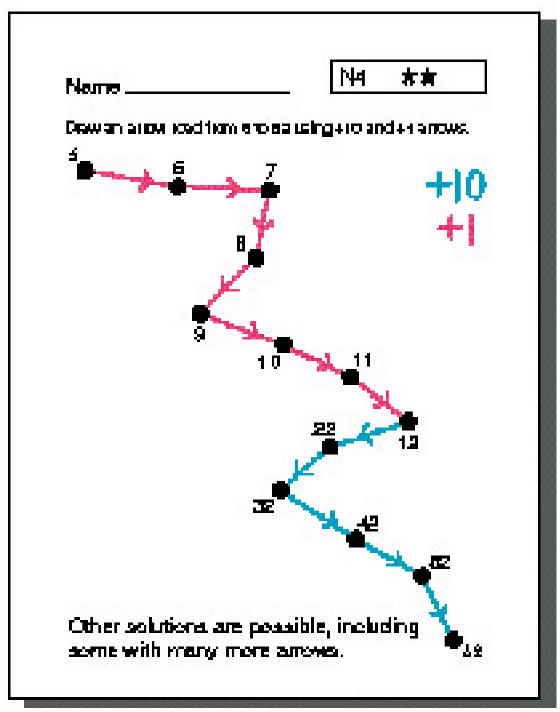
- Start at 0; use five +10 arrows and three +1 arrows. (Ending number is 53)
- Start at 0; use seven +10 arrows and nine +1 arrows. (Ending number is 79)
- Start at 0; use nine +10 arrows. (Ending number is 90)
- Start at 0; use nine +10 arrows and ten +1 arrows. (Ending number is 100)
- Start at 0; use one hundred thirty-three +1 arrows. (Ending number is 133)
- Start at 0; use one hundred thirty-three +1 arrows and one +10 arrow. (Ending number is 143)

Distribute Worksheets N4* and ** and let the students work independently. Direct students who finish quickly to find a friend or partner to check their work. You may like to give another problem to these students. For example: draw an arrow road from 0 to 452 using only +1, +10, and +100 arrows.

Assessment Activity

Ask students to imagine an arrow road that starts at 0 and has exactly seven arrows. Again, the road has +10 and +1 arrows. Direct students to record as many roads as they can think of and to make a list of the ending numbers. (If any student finds all such roads, the list of ending numbers will be 70, 61, 52, 43, 34, 25, 16, 7.) You may like to ask students to describe patterns they see in the ending numbers.





N5 ADDITION ALGORITHM #1

Capsule Lesson Summary

Introduce a trade that replaces ten checkers on a square with one checker on the square of the same color on the next board to the left. Observe the effect of multiplying by 10. Practice the addition algorithm to do addition calculations such as 453 + 276.

Materials

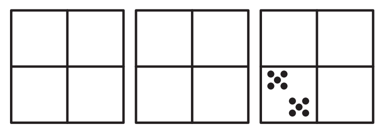
Teacher
- Minicomputer set
- Base-10 blocks or other place-value manipulatives

Student
- Paper
- *Addition Problems* Booklet (optional)

Description of Lesson

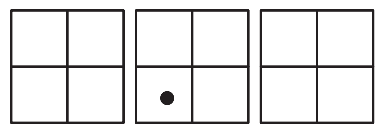
Exercise 1: Multiplying by 10

Put this configuration on the Minicomputer.



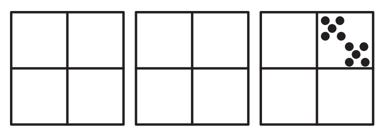
T: ***Ten checkers on the same square. What is this number?***

Many students will recognize immediately that this number is 20 because 10 x 2 = 20. Call on volunteers to make trades until the standard configuration for 20 is obtained. Emphasize that there were ten checkers on the 2-square and now there is one checker on the 20-square.



= 20

Put ten checkers on the 4-square of the Minicomputer.



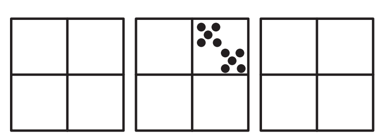
T: ***What number is this?***
What multiplication fact could we write about this number?
Do we need to make some trades to know what number this is?

Let students react freely to these questions. Many of them already know that 10 x 4 = 40 and no longer need to make trades on the Minicomputer. Nevertheless, invite students to make trades until you obtain the standard configuration for 40. Try to keep this part of the lesson moving quickly and involve many students in making trades.

T: ***Ten checkers on this purple square (4-square) is the same as one checker on this purple square (40-square).***

Record 10 x 4 = 40 on the chalkboard.

Put ten checkers on the 40-square of the Minicomputer.



T: ***What multiplication fact could we write about this number?***

S: ***10 x 40 = ?***

T: ***Is this very different from the previous multiplication fact?***

S: ***No, because we still have ten checkers on a purple square. The only difference is they are on the tens board instead of the ones board.***

T: ***After we make trades on the Minicomputer, we could get just one checker on the Minicomputer. Where would this checker be?***

S: ***On the 400-square.***

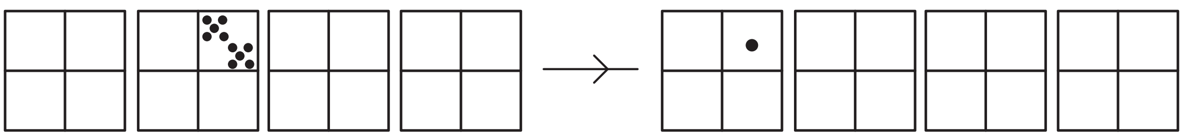
Invite students to make the appropriate trades until they obtain the standard configuration for 400.

T: ***Ten checkers on this purple square (40-square) is the same as one checker on this purple square (400-square).***

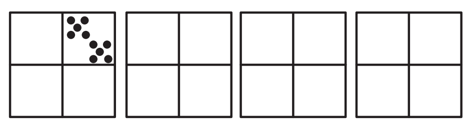
Record 10 x 40 = 400 on the chalkboard.

10 x 4 = 40
10 x 40 = 400

Display the thousands board and let the class predict and discover that 10 x 400 = 4,000.



Put ten checkers on the 4,000-square of the Minicomputer.



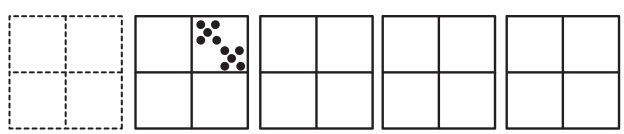
T: ***What multiplication fact could we write about this number?***

S: ***10 x 4,000 = ?***

Instruct students to write and complete the number sentence on their papers.

T: ***Could we find out what this number is without making any trades? If we were to make trades until there was only one checker remaining on the Minicomputer, where would this checker be?***

Let students react freely to these questions. Students will probably tell you that they do not need to make trades but that they need another Minicomputer board. When students ask for a fifth Minicomputer board, draw it on the chalkboard.



T: ***Ten checkers on this square (4,000-square) is the same as one checker on which square?***

Call on a student to point to the appropriate square (40,000-square).

Take all the checkers off the 4,000-square and put (or draw) one checker on the 40,000-square.

T: ***What number is this?***

S: ***40,000.***

It is possible that no one will know how to read this number, but some students may know how to write it.

T: ***Who can write this number below (above) the Minicomputer?***

Students may be able to write the numeral correctly but misplace the comma. If necessary, insert the comma yourself. Explain that the comma helps us to read large numbers and that it goes between the hundreds place (board) and the thousands place (board).

T: ***Who can read this number?***
What number fact should we write on the board?

Help students to read 40,000, if necessary.
Record the number fact in your list.

$$10 \times 4 = 40$$
$$10 \times 40 = 400$$
$$10 \times 400 = 4{,}000$$
$$10 \times 4{,}000 = 40{,}000$$

T: ***Do you notice a pattern?***

S: ***Each time there is another 0.***

S: ***Each time the 4 moves over a place.***

Exercise 2: Addition Algorithm

Write this problem on the board.

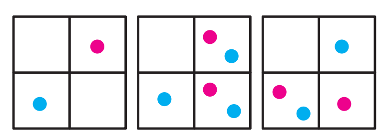
$$\begin{array}{r} 453 \\ +\ 276 \\ \hline \end{array}$$

T: ***What number is 453 + 276?***
Can you give an estimate?

Write some estimates on the board.

T: ***Let's do the calculation on the Minicomputer.***

Invite someone to put 453 on the Minicomputer using red checkers and someone else to put 276 on using blue checkers.



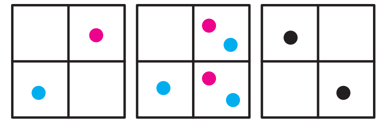
Note: You may wish to model this calculation with base-10 blocks also. As checkers are counted or moved do the same with the base-10 blocks.

Considering only the checkers on the ones board, point to the red checkers and then to the blue checkers as you say,

T: ***3 (red) + 6 (blue) = …?***

S: ***9.***

Call on students to make trades on the ones board until this configuration of checkers is on the Minicomputer.†



T: ***What numbers are in the ones column?***

S: ***3 and 6.***

T: ***3 + 6 = …?***

S: ***9.***

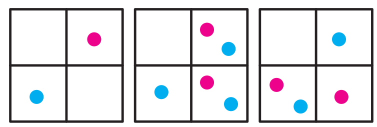
$$\begin{array}{r} 453 \\ +\ 276 \\ \hline 9 \end{array}$$

Considering only the checkers on the tens board, point to the red checkers and then to the blue checkers as you say:

T: ***50 (red) + 70 (blue) = …?***

S: ***120.***

Ask a student to make the trades involving the checkers on the tens board until this configuration of checkers is on the Minicomputer. Emphasize that the 80 + 20 = 100 trade results in another checker on the hundreds board.



T: ***What numbers are in the tens column?***

S: ***5 and 7.***

T: ***Five tens plus seven tens is …?***

S: ***Twelve tens; 120.***

T: ***120; one hundred and two tens. We record the one hundred in the hundreds column and the two tens in the tens column.***

$$\begin{array}{r} {\scriptstyle 1} \\ 453 \\ +\ 276 \\ \hline 29 \end{array}$$

Point to the appropriate checkers on the hundreds board as you say,

T: ***One hundred + four hundreds + two hundreds = …?***

S: ***Seven hundreds.***

T: ***Do we need to make a trade?***

S: ***No.***

T: ***What numbers are in the hundreds column?***

S: ***1, 4, and 2.***

T: ***One hundred + four hundreds + two hundreds = …?***

S: ***Seven hundreds.***

$$\begin{array}{r} {\scriptstyle 1} \\ 453 \\ +\ 276 \\ \hline 729 \end{array}$$

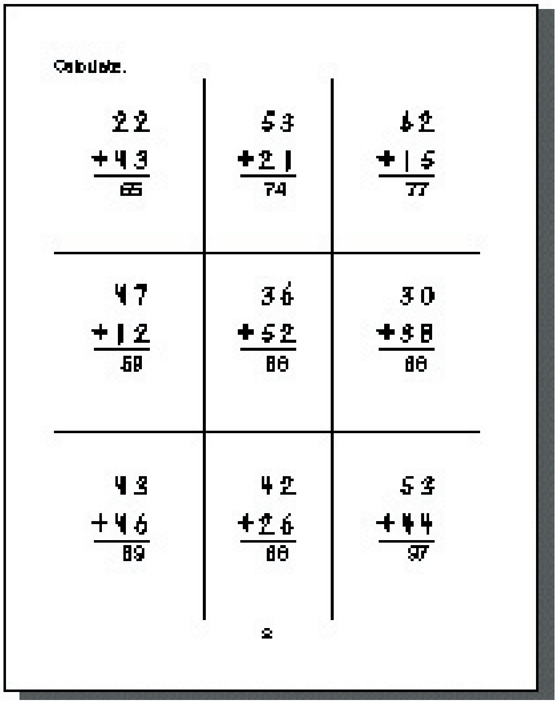
Let the students determine which estimate was closest to 729.

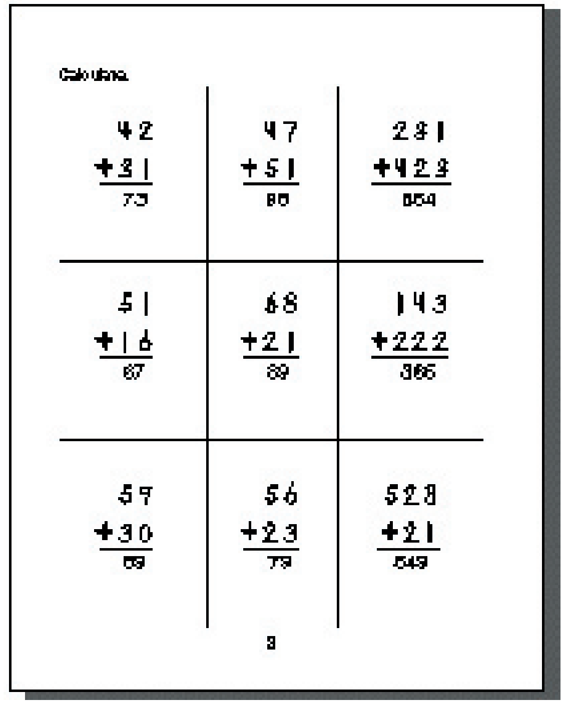
Write the following problems on the board and ask students to copy them. Emphasize the importance of lining up the digits in the ones, tens, and hundreds places. Allow the students a few minutes to do the calculations individually; then ask volunteers to solve the problems at the board and explain what they are doing.

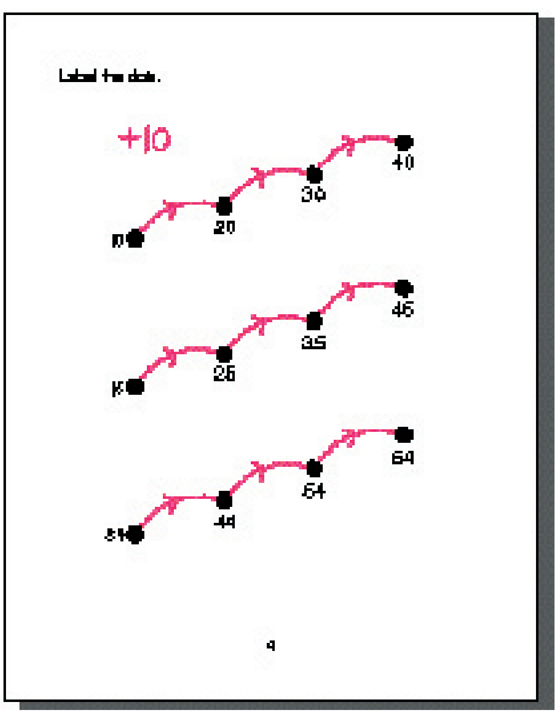
† In this picture, the checkers shown in black are those on the Minicomputer after certain trades are made. In actuality these checkers will be red or blue.

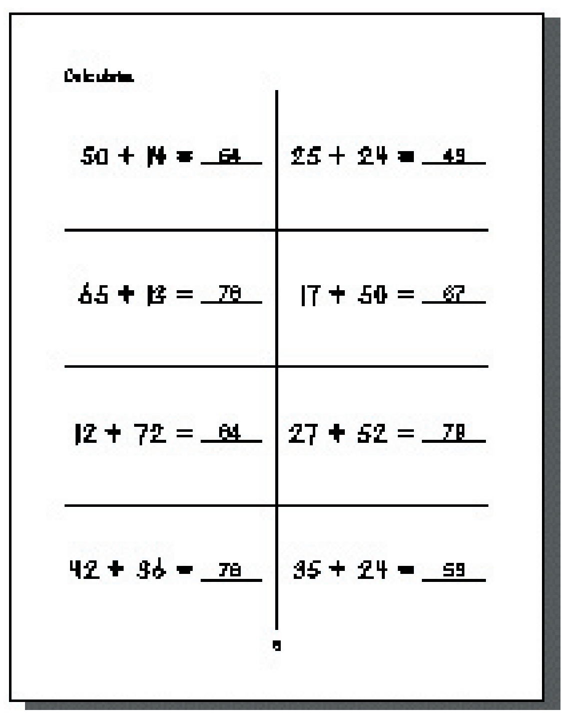
$$\begin{array}{r} 745 \\ +\ 193 \\ \hline \end{array} \qquad \begin{array}{r} 288 \\ +\ 371 \\ \hline \end{array} \qquad \begin{array}{r} 4{,}376 \\ +\ 2{,}492 \\ \hline \end{array}$$

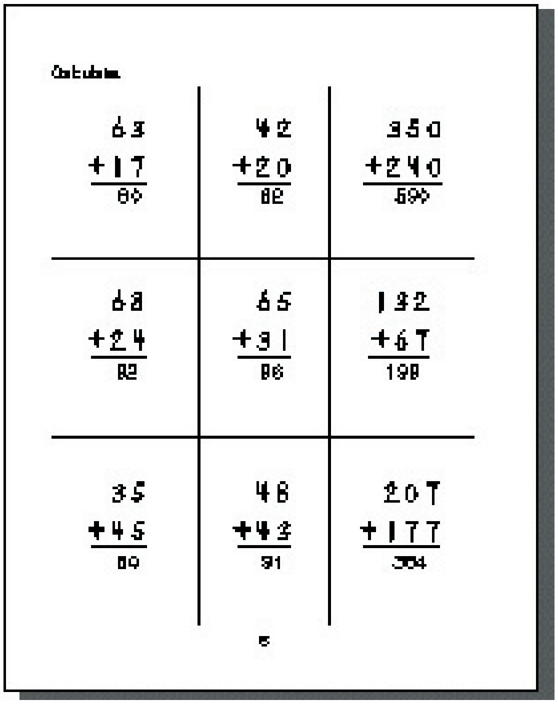
You may wish to let students work in the *Addition Problems* Booklet for about ten minutes. (The students began this booklet in *UPG-I*.) Encourage students to correct mistakes or complete pages started earlier before going on to new pages. This booklet will be used again in Lesson N8.


Calculate.
22 + 43 = 65
53 + 21 = 74
62 + 15 = 77
47 + 12 = 59
36 + 52 = 88
30 + 38 = 68
43 + 46 = 89
42 + 26 = 68
53 + 44 = 97
2


Calculate.
42 + 31 = 73
47 + 51 = 98
231 + 423 = 654
51 + 16 = 67
68 + 21 = 89
143 + 222 = 365
59 + 30 = 89
56 + 23 = 79
528 + 21 = 549
3


Label the dots.
+10
10
20
30
40
15
25
35
45
34
44
54
64
4

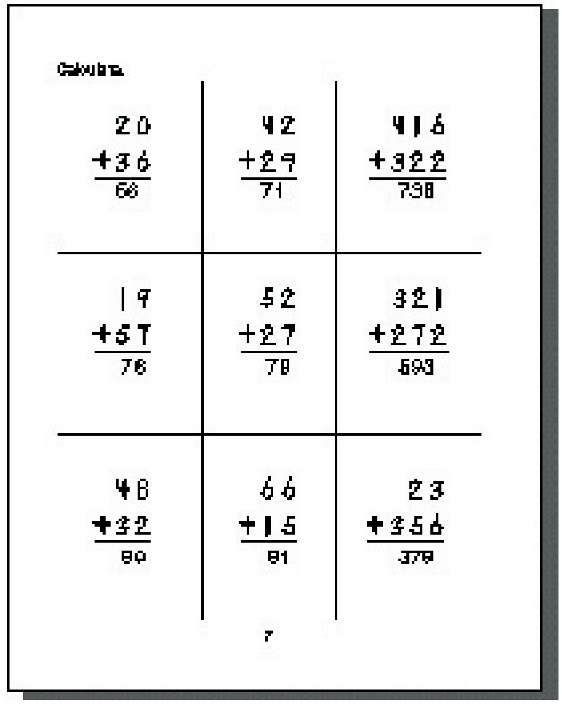

Calculate.
50 + 14 = 64
25 + 24 = 49
65 + 13 = 78
17 + 50 = 67
12 + 72 = 84
27 + 52 = 79
42 + 36 = 78
35 + 24 = 59
5



Calculate.

63 +17 80	42 +20 62	350 +240 590
68 +24 92	65 +31 96	132 +67 199
35 +45 80	48 +43 91	207 +177 384

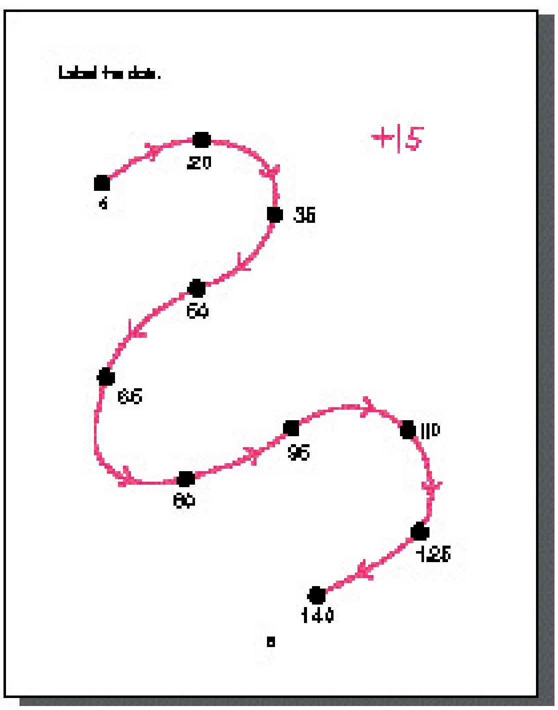
6



Calculate.

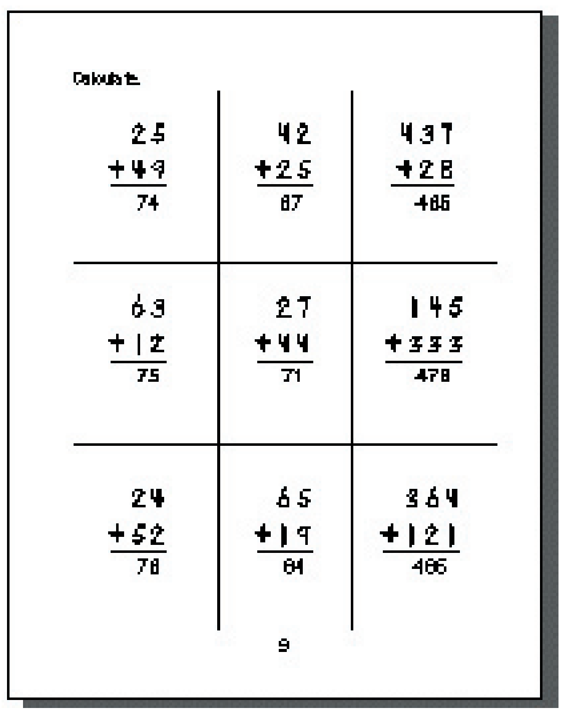
20 +36 56	42 +29 71	416 +322 738
19 +57 76	52 +27 79	321 +272 593
48 +32 80	66 +15 81	23 +356 379

7



Label the dots.

8



Calculate.

25 +49 74	42 +25 67	437 +28 465
63 +12 75	27 +44 71	145 +333 478
24 +52 76	65 +19 84	364 +121 485

9

Calculate.

35 + 45 = 80

263 + 121 = 384

74 + 26 = 100

343 + 434 = 777

26 + 46 = 72

34 + 27 = 61

10

Calculate.

25 +46 71	62 +25 87	37 +28 65
63 +43 106	45 +53 98	47 +44 91
32 +56 88	55 +17 72	20 +76 96
48 +12 60	84 +50 134	59 +23 82

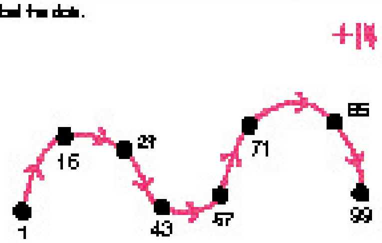
11

Calculate.

37 +17 64	58 +35 93	424 +355 779	746 + 45 791
37 +36 73	58 +60 116	129 +717 844	328 +533 861

Label the dots.

+14



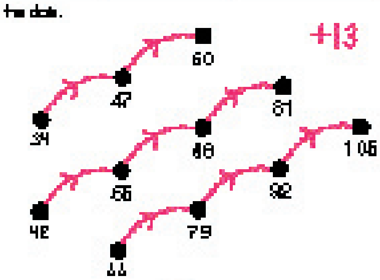
12

Calculate.

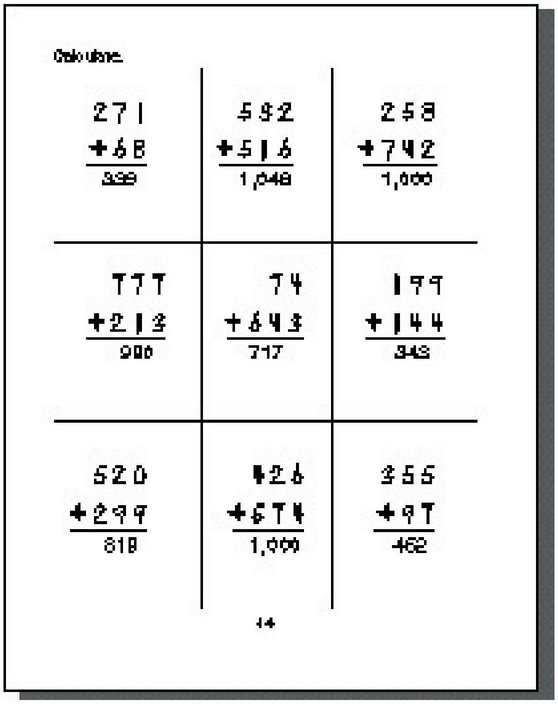
450 +139 589	263 +165 428	88 +25 113
356 +244 844	751 + 59 814	89 +88 177

Label the dots.

+13



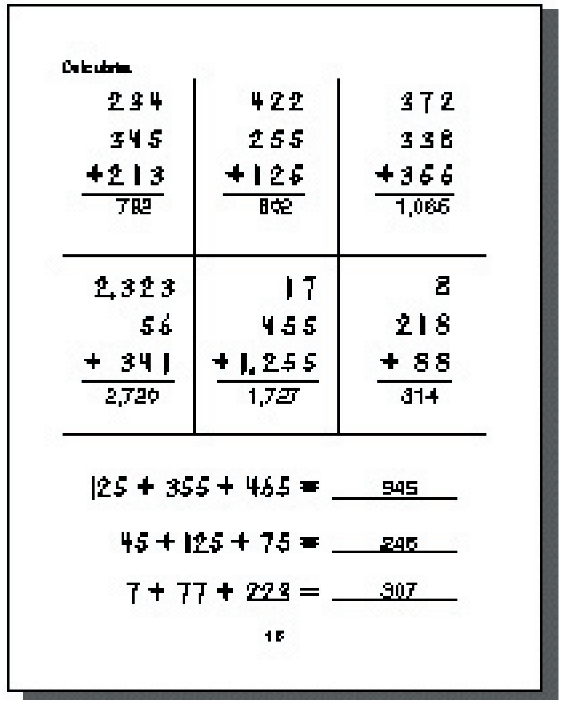
13



Calculate.

271 + 68 = 339	532 + 516 = 1,048	258 + 742 = 1,000
777 + 213 = 990	74 + 643 = 717	199 + 144 = 343
520 + 299 = 819	426 + 574 = 1,000	355 + 97 = 452

14



Calculate.

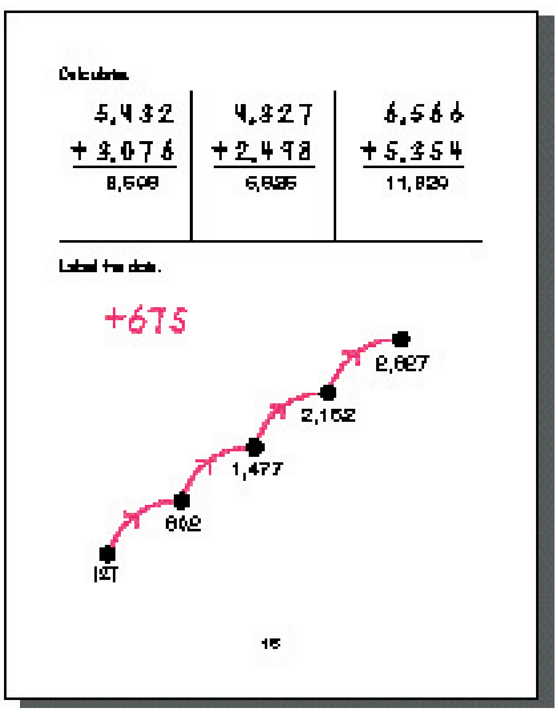
234 + 345 + 213 = 792	422 + 255 + 126 = 803	372 + 338 + 366 = 1,076
2,323 + 56 + 341 = 2,720	17 + 455 + 1,255 = 1,727	8 + 218 + 88 = 314

125 + 355 + 465 = 945

45 + 125 + 75 = 245

7 + 77 + 223 = 307

15



Calculate.

5,432 + 3,076 = 8,508	4,327 + 2,498 = 6,825	6,566 + 5,354 = 11,920

Label the dots.

16

N6 DOUBLING AND HALVING #1

Capsule Lesson Summary

Ask the class how many ears there are in the classroom today. Relate the functions 2x and ½x through the use of a simple arrow diagram; the arrow for one is the return arrow for the other. Draw 2x and ½x arrows whenever possible between the dots for three numbers. In the same picture, ask if any 4x arrows or ¼x arrows can be drawn.

Materials

Teacher
- Colored chalk
- Minicomputer set (optional)
- Calculator

Student
- Unlined paper
- Worksheets N6* and **
- Minicomputer set (optional)

Description of Lesson

Begin this lesson with a few minutes of mental arithmetic emphasizing doubling and halving.

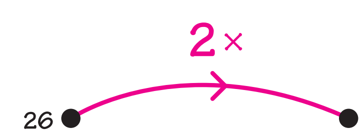
Exercise 1

T: ***How many students are here today?***

(The lesson description assumes there are 26 students present. Adjust the lesson to the number of students present in your class.)

T: ***How many ears do 26*** (use the number of students in your class) ***students have?*** (52) ***How do you know?***

Commend any clear explanation and repeat it for the benefit of the class. If no one answers 52, ask how many ears 20 students have and how many ears the other six students have and then add 40 + 12. Some students may suggest using the Minicomputer to calculate 26 + 26. When your class is aware that 26 students have 52 ears, draw this arrow picture on the board.



T: ***What number is here*** (point to the dot on the right)***?***

S: ***52.***

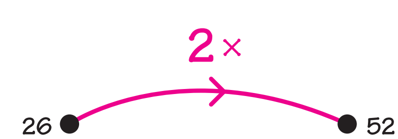
Label the dot on the right 52.

2 x 26 = 52

T: ***What number sentence is told by this arrow picture?***

S: ***2 x 26 = 52.***

Trace an arrow from 52 to 26, but do not draw it.



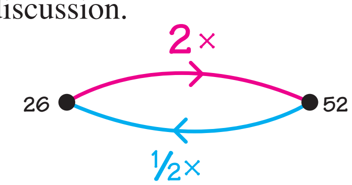
T: ***If we draw an arrow from 52 to 26, what could it be for?***

S: ***½x*** (read as "one-half of")***.***

S: ***÷2*** (read as "divided by 2")***.***

Note: The arrow also could be for –26, "is more than," or other relations, but in this case we are interested in ½x. You may like to use ÷2 as equivalent to ½x in the discussion.

Draw this arrow in blue and label it ½x.



$\frac{1}{2} \times 26 = 52$

T: ***What number sentence is told by this blue arrow?***

S: ***½ x 52 = 26*** (read as "one-half of fifty-two equals twenty-six").

Erase 26 and 52, and point to the dot on the left. Do not label the dots during this next activity. Use these or similar problems more appropriate for your students. Occasionally ask students to explain how they found an answer.

T: ***If the red arrow starts at 3, what number would be at the end of the 2x arrow?*** (6) ***Write it on your paper (or whisper it to your partner).***

Continue with several other starting numbers at the left dot such as the following. Corresponding numbers at the right are given in parentheses.

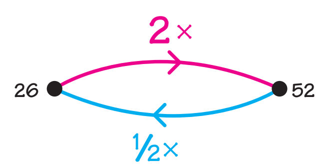
5	(10)	20	(40)	100	(200)
6	(12)	25	(50)	200	(400)
9	(18)	30	(60)	500	(1,000)

When a student correctly doubles 500, ask someone to write 1,000 on the board.

T (pointing to the dot on the right)**:** ***If the blue arrow starts at 1,000, what number would the other dot be for?*** (500) ***If it starts at 100?*** (50) ***If it starts at 60?*** (30) ***If it starts at 2,000?*** (1,000)

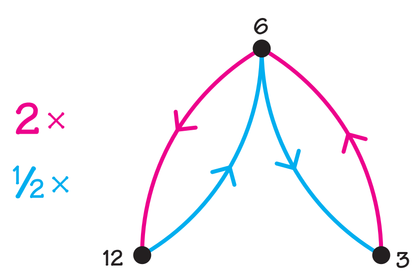
Exercise 2

Erase the board and then draw this picture. Instruct students to copy this picture on their papers and then to draw as many 2x and ½x arrows as they can.



T: ***How many 2x arrows can we draw in this arrow picture?*** (Two)
How many ½x arrows can we draw in this arrow picture? (Two)

Ask for volunteers to draw the missing arrows and to tell the appropriate number facts for the arrows. Continue until the arrow picture is complete.



$2 \times 3 = 6$ $\quad$ $2 \times 6 = 12$

$\frac{1}{2} \times 6 = 3$ $\quad$ $\frac{1}{2} \times 12 = 6$

T (tracing an arrow from 3 to 12)**:** ***What could this arrow be for?***

S: ***4x.***

Invite a student to draw an arrow from 3 to 12 and label it 4x. If no one suggests this, proceed to Exercise 3.

T (tracing an arrow from 12 to 3)**:** ***What could this arrow be for?***

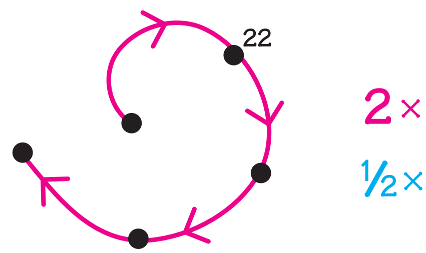
S: ***¼x.***

S: ***÷ 4.***

Invite a student to draw an arrow from 12 to 3 and label it ¼x. If no one suggests this, do not include the arrow in your picture.

Exercise 3

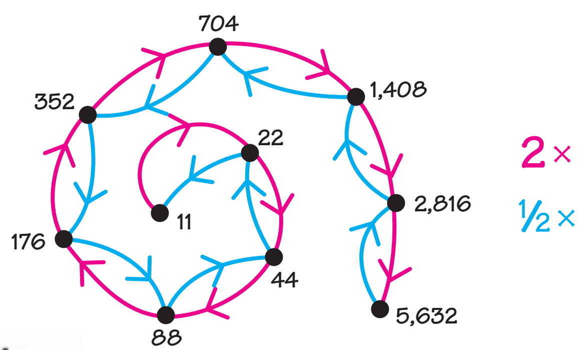
Erase the board and then draw this arrow picture.



T: ***Do you remember the Double Up game? This picture shows several days of the game. Do you remember that each day the winner can double her money? Our contestant has $22.00 at the end of the first day.*** (Trace the first arrow ending at 22.) ***How much did she start the day with?*** (Point to the first dot). ***How much could the contestant win if she stays on 3 more days?*** (Trace the next three arrows, starting at 22.)

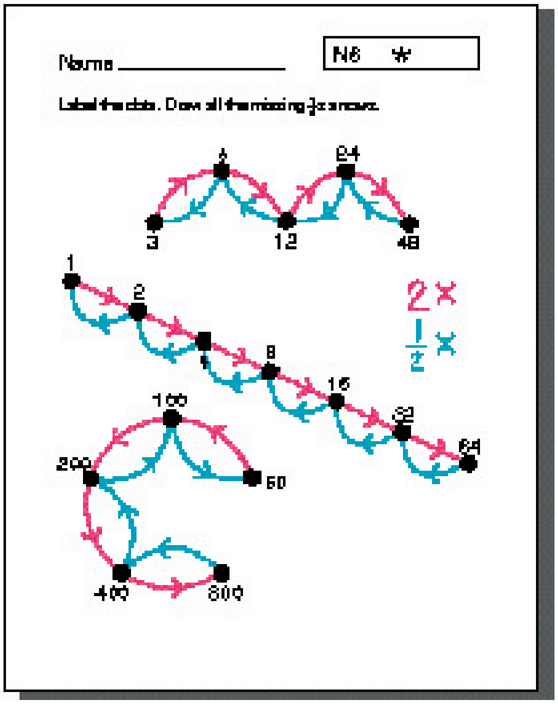
Ask volunteers to either label a dot or draw a missing ½x arrow. Use the Minicomputer or a calculator to calculate 2 x 88 or to check a student's mental calculation of 2 x 88.

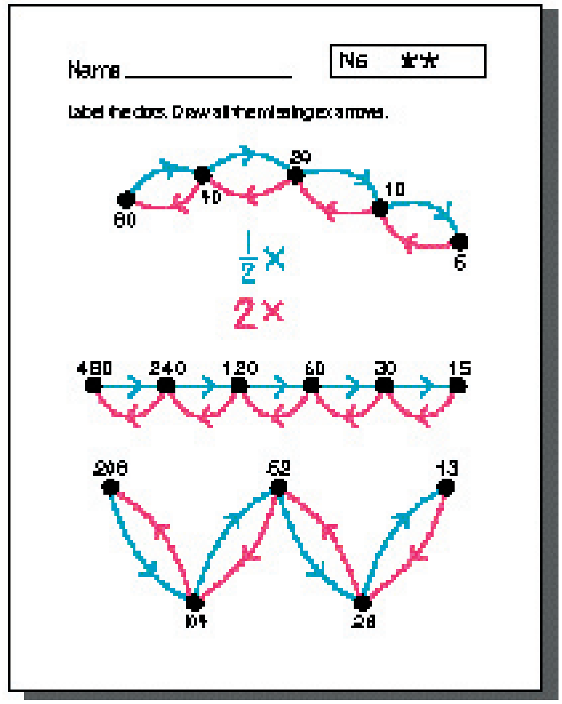
Using the Minicomputer or a calculator, continue the spiral (picturing more days in the Double Up game) as long as there is student interest. If you have several students who are good calculators, ask the class to predict the next number before it is figured on the Minicomputer or a calculator. Save approximately ten minutes for individual work on Worksheets N6* and **.



Reading Activity

Suggest students read and illustrate the Grimm's fairy tale *The Elves and the Shoemaker*, showing that each night the leather for shoes doubled.


Name
N6 *
Label the dots. Draw all the missing ½x arrows.
6
24
3
12
48
1
2
8
16
32
64
2x
½x
100
200
50
400
800


Name
N6 **
Label the dots. Draw all the missing ex arrows.
20
10
40
80
5
½x
2x
480
240
120
60
30
15
208
52
13
104
26

Capsule Lesson Summary

Consider three arrow roads that start at the same number and each have +10 and –1 arrows. Determine which road has the greatest ending number and which has the least. Label the dots in all the roads. Do a similar activity with another three roads, but discover that the roads all have the same ending numbers. Introduce other arrows into the picture.

Materials

Teacher
- Colored chalk

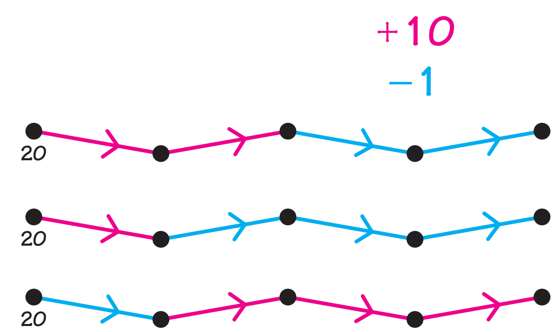
Student
- Colored pencils, pens, or crayons
- Unlined paper
- Worksheets N7* and **

Advance Preparation: Draw the arrow pictures for Exercises 1 and 2 on the board before starting the lesson.

Description of Lesson

Exercise 1

Refer to this arrow picture on the board.



T: ***The red arrows are for +10. The blue arrows are for –1. What do you notice about these arrow roads?***

S: ***All the roads start at 20.***

S: ***Each road has four arrows.***

T: ***Which road has the greatest ending number? How do you know?***

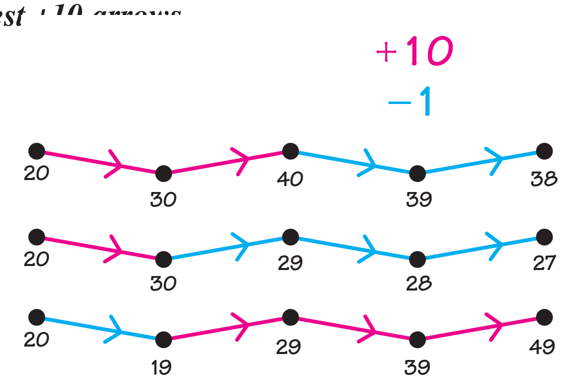
S: ***The last road, because that road has the most +10 arrows.***

Ask a volunteer to label the dots in the last arrow road.

T: ***Which road has the least ending number? How do you know?***

S: ***The one in the middle, because it has the fewest +10 arrows.***

Invite students to label the dots in the middle arrow road and then in the first one.



T: ***If we draw another arrow road that starts at 20 and has exactly four of these arrows, could it end at a number greater than 49? How?***

S: ***Yes; use all +10 arrows.***

T: ***What would the ending number be if we used four +10 arrows and started at 20?***

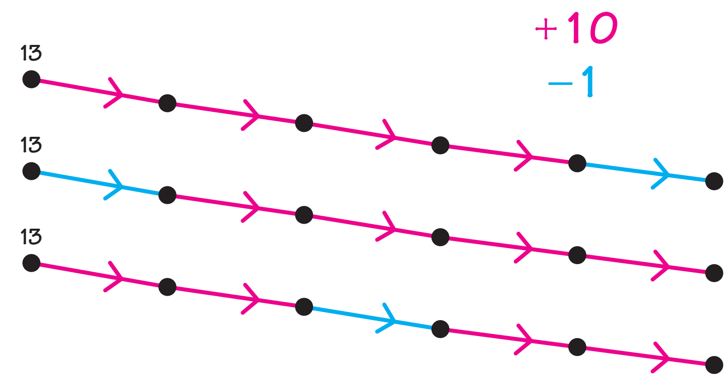
S: ***60.***

T: ***What is the least ending number we could have with exactly four of these arrows?*** (16) ***How do we get 16?***

S: ***Draw four –1 arrows. 20 – 4 = 16.***

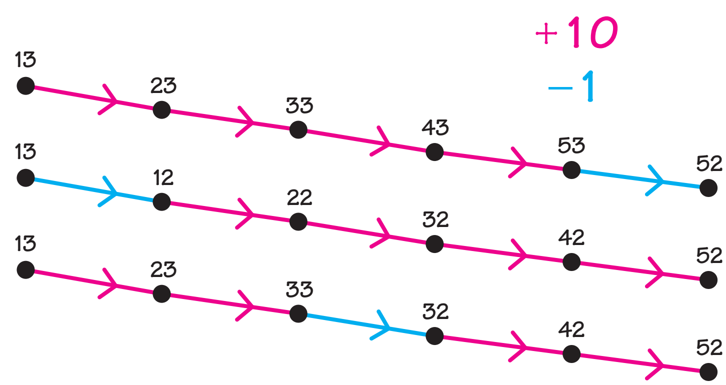
Exercise 2

Refer to this arrow picture on the board.



T: ***Which of these roads has the greatest ending number? …the least ending number?***

Allow students to predict which road has the greatest and which has the least ending number; then ask them to copy the roads and label all the dots. When many students have finished, call on volunteers to label the dots in the roads on the board.



T: ***What do you notice about these three roads?***

S: ***The ending numbers are all the same.***

S: ***They each have four red arrows and one blue arrow.***

You may like to compare these three arrow roads to those in Exercise 1. Why, in this case, do they all end at the same number? (All three arrow roads have the same number of +10 arrows and the same number of –1 arrows.)

Erase the top two roads.

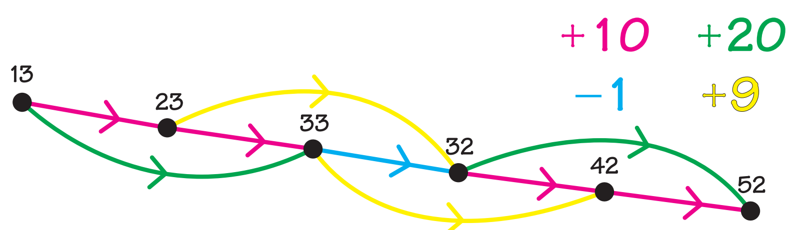
T: ***Who can draw a different kind of arrow in our picture without drawing any more dots? Tell us what kind of arrow you could draw.***

Allow volunteers to trace possible arrows and tell the class what the arrows could be for. There are many possibilities; some are listed below. Do not expect your class to find more than a few of them.

from 13 to 33	+20	from 33 to 13	–20
from 13 to 32	+19	from 32 to 13	–19
from 13 to 42	+29	from 42 to 13	–29
from 13 to 52	+39	from 52 to 13	–39
from 23 to 32	+ 9	from 32 to 23	– 9
from 23 to 42	+19	from 42 to 23	–19
from 23 to 52	+29	from 52 to 23	–29
from 33 to 42	+ 9	from 42 to 33	– 9
from 33 to 52	+19	from 52 to 33	–19
from 32 to 52	+20	from 52 to 32	–20

Afterwards, write +20 in green and +9 in yellow on the board. Instruct students to draw +20 and +9 arrows in their pictures and call on students to do the same on the board.

With all possible +20 and +9 arrows added, your picture should look like this one.



Worksheets N7* and ** are available for individual work.

Center Activities

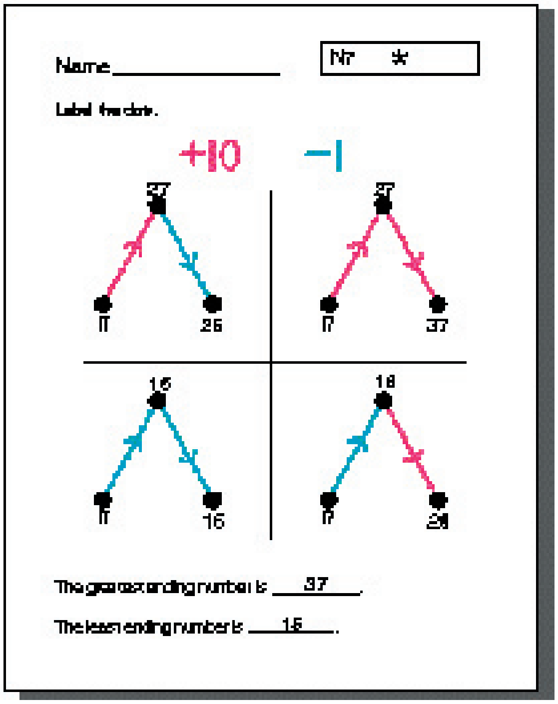
- Make laminated task cards or worksheets with arrow road problems.
- Give students opportunities to play the arrow game “20” (see *UPG-I* Lesson N24, Exercise 3). This game can easily be modified to use different kinds of arrows and different ending numbers.

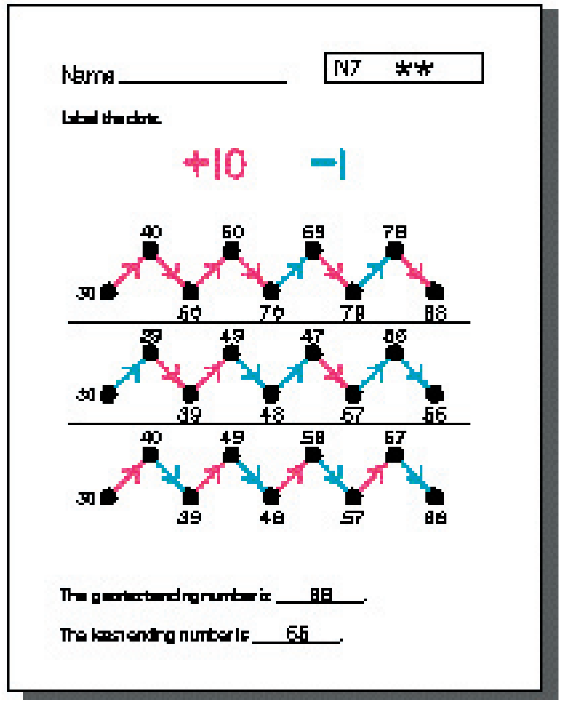
Home Activity

Prepare a few arrow road problems for students to work on at home with a family member. For example:

1) Build an arrow road starting at 9, using three +10 and two –1 arrows.
   What is the ending number?

2) Build an arrow road starting at 17, using five arrows (+10 or –1).
   What is the ending number?


Name
N7 *
+10
−1
27
11
26
27
17
37
16
11
18
17
37
15


Name
N7 **
+10
−1
40
60
69
78
30
30
39
48
57
30
40
49
58
67
39
48
57
88
88
66

Capsule Lesson Summary

Given a configuration of ten checkers on a square of the Minicomputer, make a trade replacing them with one checker on the square of the same color on the next board to the left. Practice using the addition algorithm to do addition calculations such as 576 + 287.

Materials

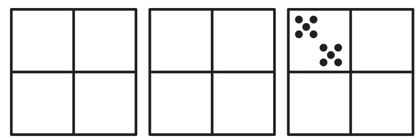
Teacher
- Minicomputer set

Student
- *Addition Problems* Booklet†
- Paper

Description of Lesson

Exercise 1: Multiplying by 10

Put this configuration on the Minicomputer.



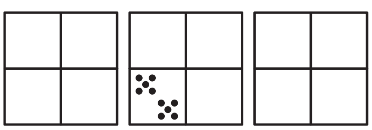
T: ***Ten checkers on the same square. What number is this?***

Many students may recognize immediately that this number is 80 because 10 x 8 = 80. Ask for a volunteer to make trades until the standard configuration for 80 is obtained.

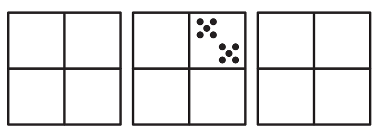
Emphasize that there were ten checkers on the 8-square and now there is one checker on the 80-square. Record this number fact on the board.

$$10 \times 8 = 80$$

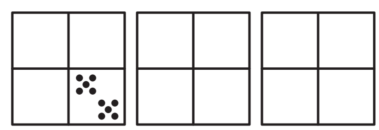
Repeat this activity with these configurations.



$10 \times 20 = 200$



$10 \times 40 = 400$



$10 \times 100 = 1{,}000$

When considering the third configuration above, emphasize that ten hundreds is another name for 1,000. Ask a student to write 1,000 below the Minicomputer after the standard configuration for 1,000 is obtained.

Exercise 2: Addition Algorithm

T: ***Fontina was taking tickets at the Winter Carnival. In the first hour she collected 576 tickets and in the second hour she collected 287 tickets. How many tickets did she collect in two hours? How can we solve this problem?***

S: ***Add 576 and 287.***

Write this problem on the board.

†The *Addition Problems* Booklet was used earlier in Lesson N5 *Addition Algorithm #1*. The answer key for this booklet follows that lesson.

N8

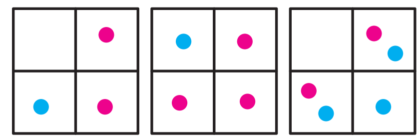
T: ***What number is 576 + 287?***
Can you give an estimate?

$$\begin{array}{r} 576 \\ +\ 287 \\ \hline \end{array}$$

Write some estimates on the board.

T: ***Let's do the calculation on the Minicomputer.***

Invite someone to put 576 on the Minicomputer using red checkers and someone else to put 287 on the Minicomputer using blue checkers.



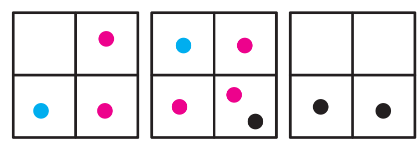
Considering only the checkers on the ones board, point to the red checkers and then to the blue checkers as you say,

T: ***6 (red) + 7 (blue) = …?***

S: ***13.***

S: ***We can make some trades.***

Call on students to make the trades involving the checkers on the ones board until this configuration of checkers is on the Minicomputer.†



S: ***Now we have another checker on the tens board.***

T: ***What numbers are in the ones column of this addition problem?***

S: ***6 and 7.***

T: ***6 + 7 = …?***

S: ***13.***

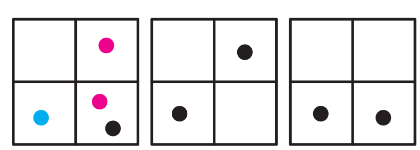
$$\begin{array}{r} {\scriptstyle 1} \\ 576 \\ +\ 287 \\ \hline 3 \end{array}$$

T: ***13; one ten and three ones. We record the one ten in the tens column and record the three ones in the ones column.***

Point to the appropriate checkers on the tens board as you ask,

T: ***What number is 80 + 70?*** (150) ***…and 10 more is what number?*** (160)
So 80 + 70 + 10 = 160.

Invite students to make the trades involving the checkers on the tens board until this configuration of checkers is on the Minicomputer.



S: ***Now we have another checker on the hundreds board.***

T: ***What numbers are in the tens column of this addition problem?***

S: ***1, 7, and 8.***

---

†In this exercise, the checkers shown in black are those on the Minicomputer after certain trades are made; in actuality, each of these checkers will be red or blue or another color which might have been used in a backward trade.

T: ***One ten + seven tens + eight tens is …?***

S: ***Sixteen tens.***

S: ***160.***

T: ***160; one hundred and six tens. We record the one hundred in the hundreds column and the six tens in the tens column.***

$$\begin{array}{r} {\scriptstyle 1} \\ 576 \\ +\ 287 \\ \hline 3 \end{array}$$

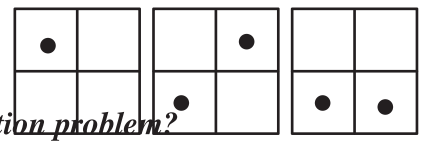
Point to the appropriate checkers on the hundreds board as you ask,

T: ***100 + 500 + 200 = …?***

S: ***800.***

S: ***We can make some trades.***

Invite students to make the trades involving the checkers on the hundreds board until this configuration of checkers is on the Minicomputer.



T: ***What numbers are in the hundreds column of this addition problem?***

S: ***1, 5, and 2.***

T: ***One hundred + five hundreds + two hundreds = …?***

S: ***Eight hundreds.***

T: ***What number is on the Minicomputer?*** (863)
What number is 576 + 287? (863)

$$\begin{array}{r} {\scriptstyle 1\,1} \\ 576 \\ +\ 287 \\ \hline 863 \end{array}$$

Let the students determine which estimate was closest to 863.

Write these problems on the board and ask students to copy them. Emphasize lining up digits in columns. Allow several minutes for students to work individually on the calculations, and then ask volunteers to solve the problems at the board and to explain what they are doing.

$$\begin{array}{r} 296 \\ +\ 357 \\ \hline \end{array} \qquad \begin{array}{r} 488 \\ +\ 376 \\ \hline \end{array}$$

Allow about 10 minutes for independent work in the *Addition Problems* Booklet. Remind students to correct mistakes or complete pages started in earlier lessons before going on to new pages. After the lesson, collect the booklets, check them, and save them for use in Lesson N23.

Writing Activity

Ask students to choose one of the addition calculations on the board and to write a story problem using that addition calculation.

Center Activity

N8

Put number ladders and dice in a center for students to use for independent addition practice or in an addition game. Draw a ladder with ten rungs and put numbers from 0 to 99 on the rungs—start with a one-digit number and increase to a number in the 90's. A student then rolls a die (or two dice) and adds the result to each number on a rung as he or she climbs the ladder. For example, suppose a student rolls a 4. The student then climbs the ladder saying "7 + 4 = 11, 16 + 4 = 20, 24 + 4 = 28, … ." In a game, other students listen for a mistake to stop the climb. Each player attempts to climb to the top with no mistakes.

97
89
71
63
58
42
35
24
16
7

Capsule Lesson Summary

Find the number halfway between two numbers on the number line. Make some consecutive +2 and +3 jumps on the number line, and label the ending points of the corresponding arrows. Decide which numbers are ending points of both the +2 and +3 arrows. Play The Number Line Game.

Materials

Teacher
- Colored chalk
- Blackline N9

Student
- Desk number line

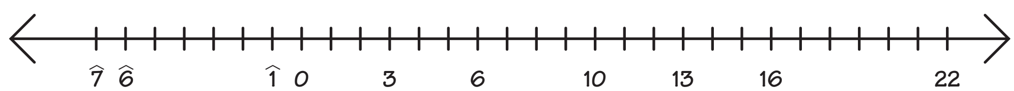
Advance Preparation: Use Blackline N9 to make copies of desk number lines for students.

Description of Lesson

Exercise 1

Draw a line on the board with marks equally spaced at least six centimeters apart. Label one of the marks 0, and label the mark three spaces to the right of it 3.

Point to some marks on the number line in random order; include some marks to the left of 0. As you point to a mark, ask, “What number is here?” Label the mark when the correct number is given. Continue until approximately ten marks are labeled.

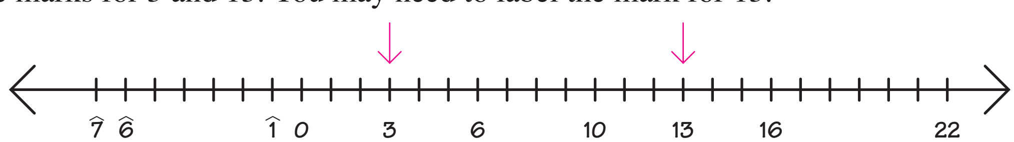


Point to the middle of the space between two consecutive marks on the number line and ask what number is there. Repeat this activity if appropriate for your students. Do not draw or label any marks for these non-integer numbers.

Distribute desk number lines for students’ reference.

T: ***Where is 3 on this number line?***
Where is 13 on this number line?

Let a student point to the marks for 3 and 13 on the number line on the board. Draw red arrows that point to the marks for 3 and 13. You may need to label the mark for 13.



Ask the students to find 3 and 13 on their number lines.

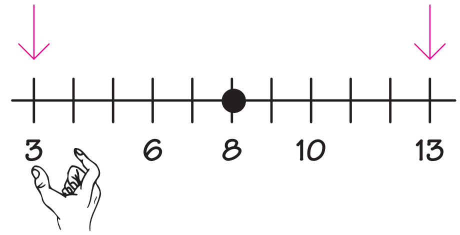
T: ***What are some numbers that are between 3 and 13 on the number line?*** (4,5,6, …12)†
Which number is exactly halfway between 3 and 13 on the number line? (8)

†There are, of course, other numbers between 3 and 13, such as 10½, but do not expect students to mention them.

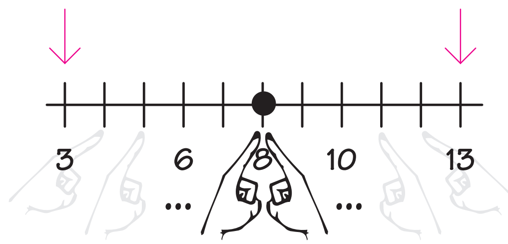
Locate and label the mark for 8 on the number line. Highlight this mark with a dot.

T: ***How can we be sure that 8 is exactly halfway between 3 and 13 on the number line?***

Encourage several students to make suggestions. If no one proposes counting the spaces from 3 to 8 and from 13 to 8, suggest this yourself. Ask the students to count the spaces using their thumb and index fingers to show each space as they count. Conclude that 8 is the same distance from 3 and from 13 (five spaces).



Another way to show that 8 is exactly halfway between 3 and 13 on the number line is to point to 3 with your left index finger and to 13 with your right index finger. Then simultaneously move your index fingers toward each other in one unit steps until both fingers point to the same mark.



When the class is convinced that 8 is halfway between 3 and 13, erase the dot and the colored arrows; then redraw the mark for 8.

Repeat this activity for other pairs of numbers where both numbers are even or where both numbers are odd.† You can use negative as well as positive numbers; you can choose numbers far apart on the number line or very close. Here are some suggestions.

Find the number halfway between

- 10 and 14 (12)
- 5 and 15 (10)
- 20 and 26 (23)
- $\hat{1}$ and 5 (2)
- $\hat{4}$ and 20 (8)
- $\widehat{20}$ and $\widehat{10}$ ($\widehat{15}$)

Note: You may extend the number line on the board, but remember that the desk number line only extends from $\hat{4}$ to 20.

Ask students to use mental calculations to find the number halfway between

- 0 and 10 (5)
- 0 and 50 (25)
- 0 and 100 (50)
- 0 and 200 (100)
- 0 and 2 million (1 million)

You may prefer to choose similar problems more appropriate for your students' abilities and interests. Occasionally ask students to explain how they found a particular number. Encourage the idea that the number halfway between 0 and any number is one-half of that number.

If your students are doing well with this exercise, continue by asking which number is halfway between 0 and 7. Or, you may choose to go to Exercise 2 at this point.

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†The number halfway between an even number and an odd number is a non-integer number. Such a problem occurs later in this exercise.

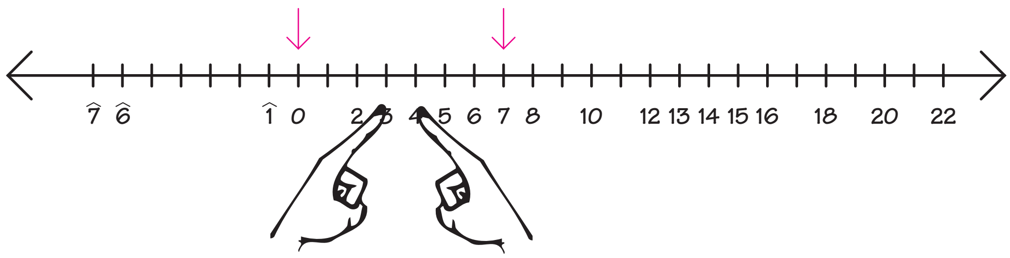
Draw red arrows pointing to the marks for 0 and 7. You may need to label the dot for 7. Ask students to find 0 and 7 on their number lines.

T: ***What number is halfway between 0 and 7?***

Students might suggest incorrectly that 3 (or 4) is halfway between 0 and 7.

T: ***How can we check to see which number is exactly halfway between 0 and 7?***

Perhaps someone will suggest pointing to 0 and 7, and moving fingers toward one another simultaneously. In this case, stop when you are pointing to the marks for 3 and 4.



T: ***What should I do now?***

S: ***Move each of your fingers in just a little bit more.***

T: ***What number is halfway between 0 and 7?***

S: ***It is the number halfway between 3 and 4.***

S: ***3½*** (read as "three and one-half")**.**

You may need to say 3½ yourself. Draw and label a mark for 3½ on the number line. If no one suggests counting the spaces from 0 to 3½ and from 7 to 3½, suggest this yourself. Conclude that 3½ is 3½ spaces from 0 and from 7.

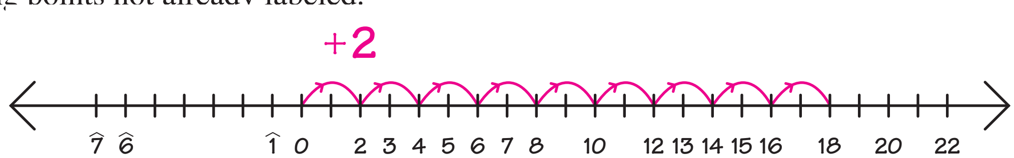
Erase the mark for 3½ before continuing with Exercise 2.

Exercise 2

Erase everything on the board except the number line. On the board, write +2 in red above the number line.

T: ***If we start at 0 and draw a +2 arrow, what number will we land on?*** (2)
If we draw two more +2 arrows, what is the last number we will land on? (6)
If we draw three more arrows …? (12)
And three more arrows …? (18)

As the students answer the above questions, draw the appropriate arrows above the number line and label ending points not already labeled.



T: ***Do you see a pattern?***

S: ***We skip every other number.***

S: ***We only land on even numbers.***

T: ***What are some numbers we would land on if we continued drawing red arrows?***

Let students make suggestions.

T: ***Would we land on 100?*** (Yes) ***…101?*** (No) ***…528?*** (Yes) ***…825?*** (No) ***…1,000?*** (Yes)

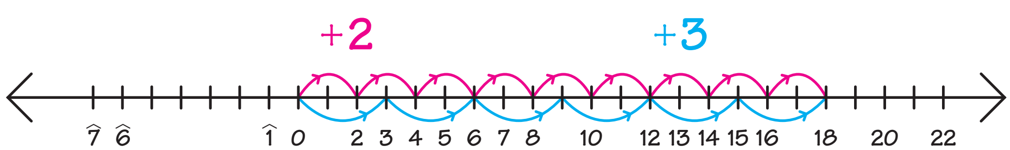
Vary these questions according to your students' abilities and interests.

T: ***How many spaces did we move in two +2 jumps?*** (Four spaces)
How many spaces did we move in five +2 jumps? (Ten spaces)
How many spaces did we move in nine +2 jumps? (Eighteen spaces)

Write +3 in blue above the number line.

T: ***If we start at 0 and draw a +3 arrow, what number do we land on?*** (3)
If we draw another +3 arrow, what number do we land on? (6)
If we draw two more +3 arrows, what is the last number we land on? (12)
And two more +3 arrows …? (18)

As the students answer the above questions, draw the appropriate arrows below the number line.



T: ***When we made +3 jumps, what numbers did we land on?*** (0, 3, 6, 9, 12, 15, 18)
What are the numbers we would land on if we drew more blue arrows? (21, 24, 27, 30, …)
Which numbers did we land on with both +2 arrows and +3 arrows? (0, 6, 12, 18)

Exercise 3: The Number Line Game

Play The Number Line Game with a secret number between 0 and 200. Then play The Number Line Game with a secret number between $\widehat{50}$ and 50. A description of The Number Line Game is given in Exercise 3 of Lesson N3.

Capsule Lesson Summary

Consider four arrow roads that each start at the same number, and have three +1 arrows and one 2x arrow. Observe that the order in which the arrows occur affects the ending number of the road. Build a road from 0 to 21 using 2x and +1 arrows.

Materials

Teacher
- Colored chalk

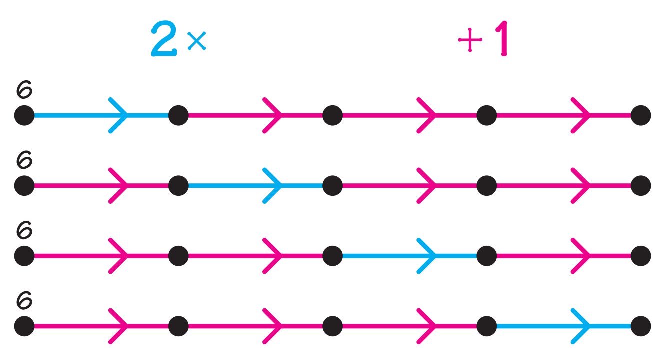
Student
- Unlined paper
- Colored pencils, pens, or crayons
- Worksheets N10* and **

Advance Preparation: On the board, draw the arrow picture for Exercise 1 before the lesson begins.

Description of Lesson

Exercise 1

Refer to this arrow picture on the board.



T: *Look at the four arrow roads on the board. How are these roads alike? How are they different?*

S: *They all have four arrows.*

S: *They all start at 6.*

S: *They all have three red arrows and one blue arrow.*

S: *The blue arrow is in a different place each time.*

T: *Which of these roads has the greatest ending number?*

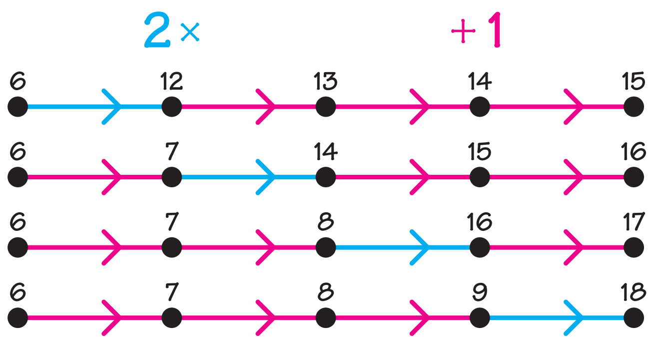
Some students may think that all the roads end at the same number. Ask students to check this by labeling the dots in two of the roads.

S: *The last one—it ends at 18.*

T: *Which of these roads has the least ending number?*

S: *The first one—it ends at 15.*

Call on volunteers to label the dots in all four roads.



T: ***Each of these roads has three +1 arrows and one 2x arrow, and they all start at 6. Why don't they all end at the same number?***

Do not expect well-formed explanations; accept students' attempts without comment. Perhaps one of your students will give this or a similar explanation.

S: ***In each arrow road the 2x arrow is in a different place, so we double a different number and that changes the ending number.***

Erase the board before going on to Exercise 2.

Exercise 2

Write this information on the board and instruct students to copy it on their papers. During this exercise, students should follow along on their papers as the class builds an arrow road on the board.

T: ***We are going to build a road from 0 to 21 with 2x and +1 arrows. Which kind of arrow do you want to use first?***

Accept any suggestion. A possible dialogue is given here.

S: ***Use a blue arrow first.***

T: ***The blue arrows are for 2x. What number is 2 x 0? How should I draw the arrow?***

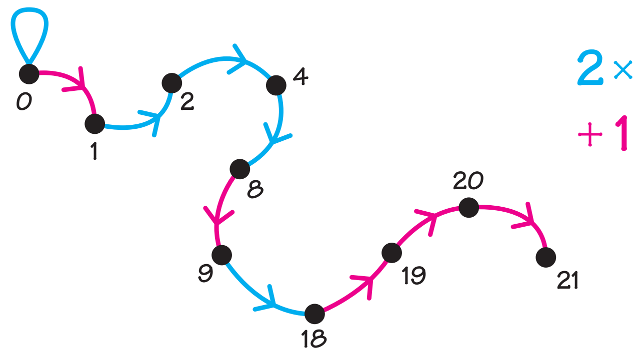
S: ***You need to draw a loop, because 2 x 0 = 0.***

T: ***Which kind of arrow do you want to use now?***

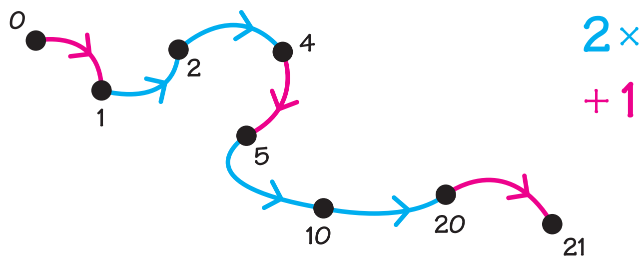
S: ***A red arrow. 0 + 1 = 1.***

Draw a red (+1) arrow starting at 0.

Continue building the arrow road from 0 to 21 until it is complete. There are many possible solutions. One solution is given here.



If this activity was easy for your class, you may wish to ask if they can build a shorter road. The shortest possible road is shown below.



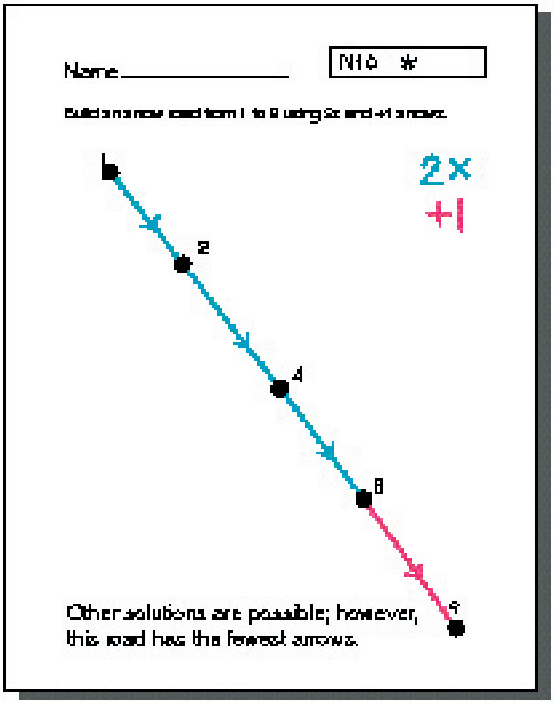
Worksheets N10* and ** are available for individual work. Students who finish quickly might be encouraged to build a road from 0 to 100 using 2x and +1 arrows.

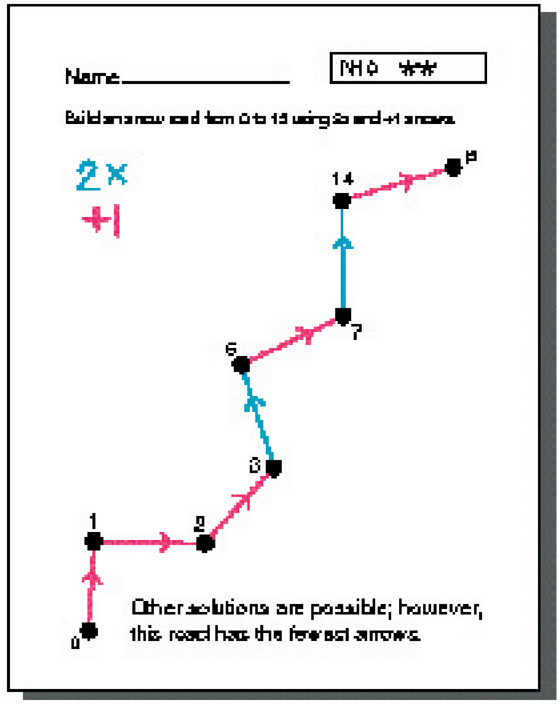
Center Activity

Make arrow road task cards to place in a center. Each task might ask the student to build an arrow road connecting two given numbers and using two different kinds of arrows.

Home Activity

Prepare a few problems like those on the worksheets for students to work on at home with a family member.


Name
N10 *
2x
+1
1
2
4
8
9
Other solutions are possible; however, this road has the fewest arrows.


Name
N10 **
2x
+1
14
7
6
3
1
2
0
Other solutions are possible; however, this road has the fewest arrows.

Capsule Lesson Summary

Use the calculator to do a mental arithmetic exercise involving open sentences and opposites. Introduce a 10-friends number relation and draw arrows for "You are my 10-friend." Draw arrow pictures for other number friends.

Materials

Teacher
- Calculator
- Colored chalk

Student
- Calculator
- Unlined paper
- Colored pencils, pens, or crayons

Description of Lesson

Exercise 1

Use the calculator to conduct a mental arithmetic activity. This activity works best with an overhead calculator, but you can use a class calculator. Allow students to have calculators for checking purposes.

T: ***I'm going to hide a number on the calculator. Let's see if you can guess my number.***

Put 17 on the display of the overhead (or class) calculator and cover the display with your hand. Be sure to do this before putting the calculator on the overhead projector.

T: ***I've covered the display so you can't see my number. Now I press $\boxed{+}$ $\boxed{3}$ $\boxed{=}$.***

Let the students see you press $\boxed{+}$ $\boxed{3}$ $\boxed{=}$ and then show them that 20 is on the display.

T: ***Do you know what number I hid on the calculator?***

S: ***17, because 17 + 3 = 20.***

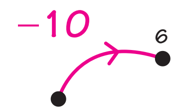
Let students use their calculators to check.

Repeat this activity several times with calculations appropriate to the abilities of your students. Try some of the following examples:

Hide	Press	Show
8	$\boxed{+}$ $\boxed{7}$ $\boxed{=}$	15
11	$\boxed{-}$ $\boxed{6}$ $\boxed{=}$	5
25	$\boxed{+}$ $\boxed{9}$ $\boxed{=}$	34
9	$\boxed{\times}$ $\boxed{2}$ $\boxed{=}$	18
16	$\boxed{-}$ $\boxed{1}\boxed{0}$ $\boxed{=}$	6

After four or five problems, show how to solve a problem with an arrow picture and frames.

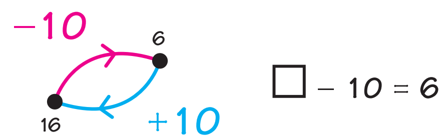
T: *I hid a number on the calculator. This dot is for the hidden number. Then I press* $\boxed{-}$ $\boxed{1}\boxed{0}$ $\boxed{=}$*. The arrow shows the calculator subtracts 10. We see 6 on the display. Here is 6. How can we find the hidden number?*



S: *Use a return (opposite) arrow.*

S: *6 + 10 = 16, so 16 was hidden.*

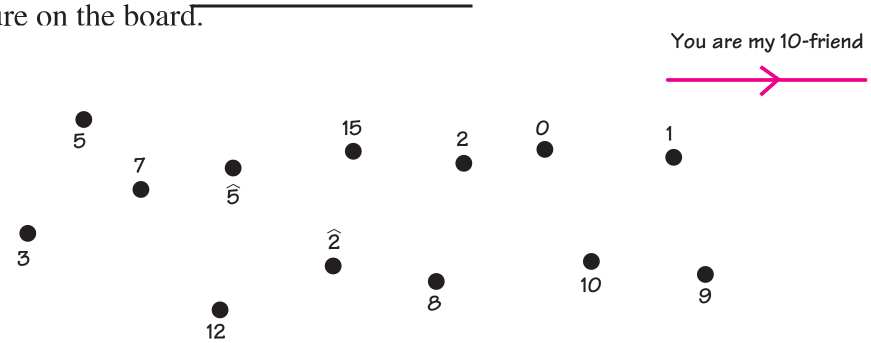
T: *Or think of the hidden number as the number in the frame* $\square - 10 = 16$*.*



If you like, pose two or three more hidden number problems and suggest to students they can think about an arrow picture to help find the hidden number.

Exercise 2: Number Friends

Draw this picture on the board.

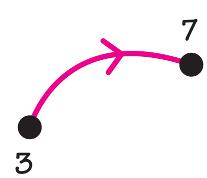


T: *These numbers are playing a game. Each number points to its 10-friend. Do you know what numbers are 10-friends?*

Perhaps a veteran student will recall something about number friends, or someone will be able to guess what it means to be 10-friends. If necessary, draw an arrow to illustrate; for example:

T: *3 says to 7, "You are my 10-friend." Can you explain?*

Allow students to offer opinions, but encourage them to observe that 3 says to 7, "You are my 10-friend," because 3 + 7 = 10.



T: *Where else can we draw arrows for "You are my 10-friend"?*

Send volunteers to trace and then draw arrows. Each time an arrow is drawn, ask the student to explain by reading a number fact. Continue until the arrow picture is complete (see next illustration). You may need to suggest an arrow going in the opposite direction such as from 7 to 3, because 7 also says to 3, "You are my 10-friend"; i.e., 7 + 3 = 10.

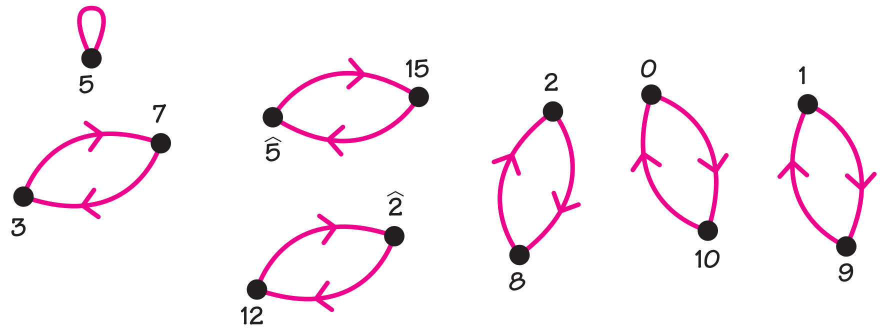
If no one suggests drawing a loop at 5, draw the students' attention to this part of the picture.

T: *5 plus what number is 10?*

S: *5.*

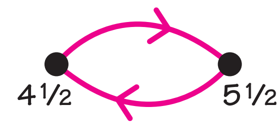
T: *How can we show that 5 is its own 10-friend?*

S: ***Draw a loop at 5.***



T: ***What are some other numbers that could play "You are my 10-friend"? Can we put some other numbers in this picture?***

Accept several suggestions, drawing appropriate dots and arrows.
You may like to include some fractions in the picture as in this example.



Exercise 3

Write this open sentence on the board.

Remind the students that since the frames have different shapes, the numbers in the square and in the triangle may be different or they may be the same.

□ + △ = 20

T: ***What number could we put in the square and what number could we put in the triangle to get a true number sentence? There are many answers.***

Allow students to work with a partner and make lists of at least ten solutions. You can remind them how to list solutions in a table by drawing a sample table on the board under the open sentence.

After awhile, record some students' suggestions in the table. When your table has four or five pairs of numbers in it accept other solutions without recording them. Ask whether a negative number could go in the square or the triangle, and record at least one solution where one of the numbers in the pair is negative.

□ + △ = 20

□	△
5	15
10	10
20	0
21	$\hat{1}$
$\widehat{80}$	100

T: ***Do you know another way to say that these pairs of numbers have a sum of 20?***

Perhaps a student will recall that these numbers are 20-friends.

T: ***This table shows us some numbers that are 20-friends.***
What does it mean to say that two numbers are 20-friends?

S: ***The sum of the two numbers is 20.***

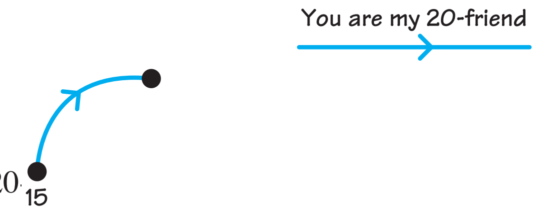
Draw this arrow picture on the board.

T: ***Which number does 15 point to when the***

numbers are playing the 20-friends game?

S: ***5.***

T: ***15 + 5 = 20, so 5 is 15's 20-friend.***



Label the dot for 5. Emphasize that two numbers are 20

T: ***What are some other numbers that are 20-friends?***

Accept students' suggestions and draw appropriate dots and arrows on the board. If no one suggests including 10 in the arrow picture, include 10 yourself.

T: ***What does 10 do when the numbers are playing the 20-friends game?***

S: ***10 points to 10. We need to draw a loop.***

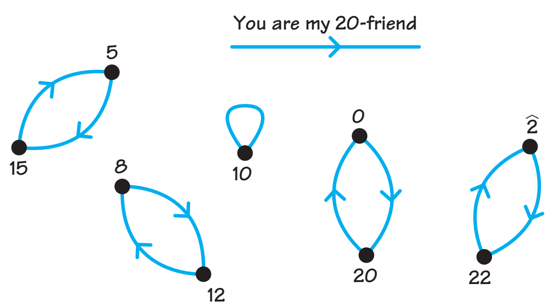
Draw and label a dot for 10 on the board.

T: ***Can negative numbers play the 20-friends game?***

S: ***$\widehat{2}$ can point to 22.***

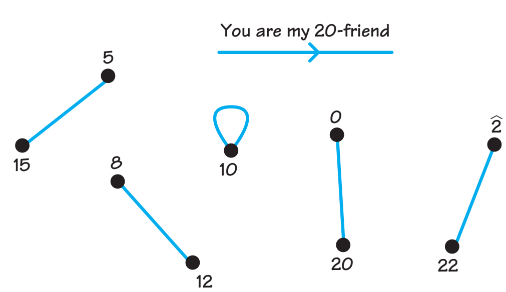
Draw an arrow from $\widehat{2}$ to 22 on the board.

Continue this activity as long as you feel it is appropriate. If no one suggests drawing return arrows, suggest it yourself, and ask for volunteers to draw the missing return arrows. Your picture will look similar to this one.



T: ***Do you notice that when we have a number friends picture, there are always two arrows between two dots? The arrows go in both directions. Instead of drawing two arrows, we can draw a cord*** (replace two arrows with a cord in the picture)***. You can go in both directions on a cord—it's a two-way street.***

Invite students to replace two arrows with a cord so that your picture looks like this.



Exercise 4

Distribute unlined paper and colored pencils. Instruct students to choose any kind of number friends they like and draw an arrow (cord) picture. Encourage them to include many numbers in their pictures including positive and negative numbers. You may like to suggest to some students that they choose "big" number friends such as 50-friends or 100-friends.

When two students complete their pictures, suggest that they trade pictures and check each other's work. Each student can add one or two more pairs of numbers and arrows (cords) to the other's picture.

Center Activity

Let students draw dot pictures with numbers appropriate for a game of "You are my 20-friend" or "You are my 50-friend." Laminate them, and place them in centers where other students can complete the pictures by drawing the arrows (cords). The student who drew the original picture can make an answer key to tape to the back.

Writing Activity

Write a story about some numbers playing together who discover that they are 12-friends.

Home Activity

Explain the number friends game to parents/guardians and invite them to play it with their child.

N12 DOUBLING AND HALVING #2

Capsule Lesson Summary

Write number sentences about an array of dots in which half of the dots are blue and half of the dots are red. Consider a configuration on the Minicomputer for some number in which the checkers are grouped in pairs. Recognize what one-half of the number is and then make trades to find the number. Find out how lunch items can be divided by two children.

Materials

Teacher
- Colored chalk
- Minicomputer set
- Base-10 blocks, tiles, or cubes
- Paper square

Student
- Paper
- Minicomputer set
- Tiles or cubes
- Paper square

Description of Lesson

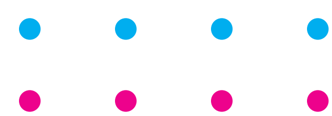
Exercise 1

Begin this lesson with about five minutes of mental arithmetic involving doubling and halving; for example, ask students to calculate the following:

2 x 10	(20)	2 x 7	(14)	2 x 23	(46)	2 x 50	(100)
½ x 20	(10)	½ x 14	(7)	½ x 46	(23)	½ x 100	(50)

Note: Read ½x as "one-half of"; and occasionally equate ½x and ÷2.

Draw this dot picture on the board or use red and blue checkers.



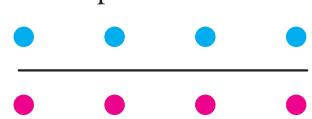
T: ***Look at this array of dots (checkers). What do you see?***

S: ***I see four blue dots and four red dots.***

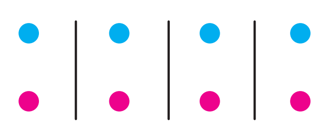
S: ***I see a blue dot and a red dot; a blue dot and a red dot; a blue dot and a red dot; and a blue dot and a red dot.***

S: ***I see eight dots—half are red and half are blue.***

Ask students to express what they see with number sentences. You may wish to use a ruler to help illustrate the different responses as illustrated here.



$4 + 4 = 8$
$2 \times 4 = 8$
$\frac{1}{2} \times 8 = 4$



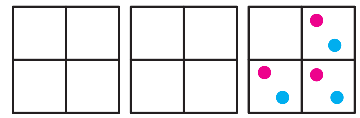
$4 \times 2 = 8$
$2 + 2 + 2 + 2 = 8$

If no one mentions ½ x 8 = 4, suggest it yourself.

Repeat this activity with another array of dots or checkers.

Exercise 2

Put this configuration on the Minicomputer.



T: ***What can you tell me about this number?***
What number is on the Minicomputer in red checkers ? (7)
What number is on the Minicomputer in blue checkers? (7)
What facts can we write about this number?

Accept and record on the chalkboard any appropriate number sentences for the configuration. These are some your students might suggest.

7 + 7 = 14	(2 x 4) + (2 x 3) = 8 + 6 = 14
2 x 7 = 14	(2 x 2) + (2 x 5) = 4 + 10 = 14
½ x 14 = 7	(2 x 6) + (2 x 1) = 12 + 2 = 14
14 – 7 = 7	(2 x 4) + (2 x 2) + (2 x 1) = 14

Ask someone to show one-half of this number on the Minicomputer. A student should remove one checker from each pair of checkers on the Minicomputer.

T: ***What number is ½ x 14?*** (7)

If ½ x 14 = 7 was not suggested previously, record it on the board at this time.

T: ***If we double 7*** (put back on the Minicomputer the three checkers that were removed), ***what number do we get?***

S: ***14.***

Make forward trades to show 14 in standard configuration, then trade back until 7 is shown twice. Erase all the number sentences except ½ x 14 = 7 and 2 x 7 = 14.

T: ***Let's show these number facts in an arrow picture. What kind of arrows should we use?***

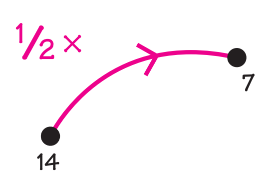
S: ***2x and ½x arrows.***

Draw a large dot on the board as you say,

T: ***This dot is for the number on the Minicomputer.***

Label the dot 14 and then draw an arrow starting from that dot.

T (drawing the ending dot)**:** ***This dot is for one-half of that number.***

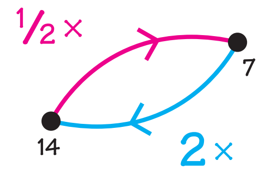


Label the ending dot 7 and then label the arrow ½x.

T (tracing an imaginary arrow from 7 to 14)**:** ***What would the return (opposite) arrow be for?***

S: ***2x.***

Invite someone to draw and label an arrow for 2x.



T: ***What number facts are told by this arrow picture?***

S: ***½ x 14 = 7 and 2 x 7 = 14.***

Exercise 3

Pair students and provide each pair with a desk Minicomputer, approximately 20 tiles or cubes, and a paper square.

T: ***Today we are going to help Kalina and Paco pack a lunch. They will share carrot sticks, raisins, cheese squares, cookies, and a sandwich equally. How many children are packing lunch?***

S: ***Two.***

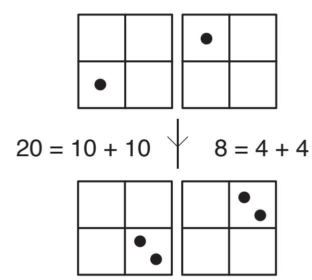
T: ***What should the two children do with each kind of food?***

S: ***Share equally two ways.***

S: ***Divide in half.***

T: ***Kalina and Paco have 28 carrot sticks. Put 28 on the Minicomputer. How can we show how the two children share the 28 carrot sticks equally?***

S: ***Make backward trades until we have groups of two.***



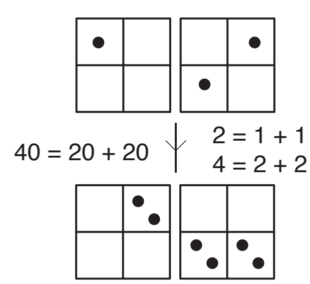
Direct the student pairs to do the problem on their desk Minicomputers while you invite a few students to do it on the demonstration Minicomputer.

When the checkers show 2 x 14 on the Minicomputer, suggest one student in a pair can pretend to be Kalina and take 14 carrot sticks, while the other student can pretend to be Paco and take 14 carrot sticks.

Write number sentences on the chalk board to describe the sharing.

$$\tfrac{1}{2} \times 28 = 14$$
$$28 \div 2 = 14$$
$$2 \times 14 = 28$$

Repeat this activity, this time suggesting that Kalina and Paco are to share a box of raisins with 86 raisins. Invite one student to be Kalina and to come to the demonstration Mincomputer to take her half of the raisins.



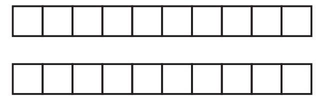
T: ***What is left for Paco?*** (43)

$$\tfrac{1}{2} \times 86 = 43$$
$$86 \div 2 = 43$$
$$2 \times 43 = 86$$

Instruct the student pairs to count out 18 tiles or cubes.

T: ***These 18 tiles (cubes) are the cheese squares Kalina and Paco have to put into their lunches. Show how they can share the 18 cheese squares equally.***

Direct the student pairs to share 18 tiles (cubes) equally between the two of them. Then record the results in number sentences on the board.



$\frac{1}{2} \times 18 = 9$
$18 \div 2 = 9$
$2 \times 9 = 18$

Repeat this activity using some of the tiles (cubes) to represent cookies for the two children to share. You may let students select a number of cookies, or you may prefer to suggest the number.

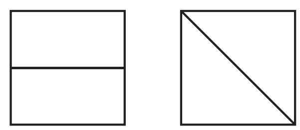
T: ***Kalina and Paco want a sandwich in their lunches but they have only one sandwich. What can they do?***

S: ***Cut it in half.***

S: ***Each child can take ½ sandwich.***

Tell the student pairs that the paper square represents the sandwich, and they should decide on one cutting line to divide the sandwich into two equal parts.

Show these two different ways to divide the sandwich into halves, and write number sentences on the board to describe the sharing.



$\frac{1}{2} \times 1 = \frac{1}{2}$
$1 \div 2 = \frac{1}{2}$

Center Activity

Put out collections of like objects (beans, counters, pennies, crayons, and so on); direct students to count them and to show how to share them equally, two ways.

Home Activity

Suggest that parents/guardians find things around the house to divide equally two ways. You may also suggest they involve their child in halving or doubling a recipe.

Capsule Lesson Summary

Calculate how much money Chuck will receive on each day of a one-week stay at his Uncle Albert's if his uncle gives him 4¢ on the first day, and each day after triples the amount of the day before.

Materials

Teacher	• Colored chalk • Minicomputer set • Calculator (optional)	Student	• Paper • Colored pencils, pens, or crayons • Calculator • Minicomputer set (optional)

Description of Lesson

Exercise 1

Begin this lesson with five minutes of mental arithmetic activities involving 2x and 3x calculations. One way to do this is to use an overhead or classroom calculator "programmed" to double or triple a number. To program the calculator to double, press $\boxed{2}$ $\boxed{\times}$ $\boxed{0}$ $\boxed{=}$. The calculator will display 0, but now is ready to multiply by 2 any number entered. For example, press $\boxed{7}$ $\boxed{=}$ and the calculator with display 14. Using the calculator, enter the number to be multiplied by 2, invite students to give the result, and then press $\boxed{=}$ to check.

Exercise 2

Tell the following story to your class. Choose one of your students to be the star of this story.

T: ***Chuck often visits his Uncle Albert whom he likes very much. Uncle Albert knows lots of tricks and sometimes plays games with Chuck. Chuck is going to stay with his Uncle Albert for one week. How many days are there in a week?***

S: ***Seven.***

T: ***Uncle Albert has a big surprise for Chuck when he arrives. He gives Chuck four pennies and promises to give him three times that much money on the second day of his visit. On the third day, he promises to give him three times as much as on the second day. Uncle Albert says he will continue doing this each day of Chuck's visit. Looking at the four pennies, Chuck isn't sure that this is such a big surprise. Let's figure out how much money Chuck will receive on each day, and how much he will receive altogether during his visit.***

Let the class discuss what they think about this problem and what they might guess Chuck will receive. Tell the class that you will record how much Chuck will receive each day in an arrow picture.

T: ***How much does Chuck receive the first day?*** (4¢)

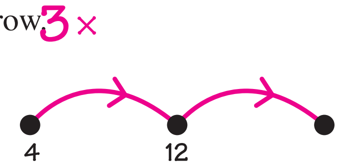
Draw a dot labeled 4. Then draw an arrow for 3x starting at 4 as you say,

T: ***On the second day Uncle Albert will give him three times as much money. How much is 3 x 4? How do you know?***

S: ***12, because 4 + 4 = 8 and 8 + 4 = 12.***

Label the ending dot 12, and extend the picture with another 3x arrow.

T: ***How much money will Uncle Albert give Chuck on the third day? What number is 3 x 12? How do you know?***



Record several of the students' calculations on the board. Perhaps they will suggest some of these number sentences.

$$(10 + 10 + 10) + (2 + 2 + 2) = 36$$
$$12 + 12 = 24 \text{ and } 24 + 12 = 36$$
$$2 \times 12 = 24 \text{ and } 24 + 10 = 34 \text{ and } 34 + 2 = 36$$

Label the third dot 36, and extend the picture another arrow.

T: ***How much will Uncle Albert give Chuck on the fourth day? What number is 3 x 36?***

Record any reasonable estimate on the board. If a suggested number is not a good estimate, discuss why not and do not record it on the board. For example:

S: ***18.***

T: ***18 is less than 36. Could 3 x 36 be less than 36?***

S: ***No.***

S: ***72, because 36 + 36 = 72.***

T: ***But Uncle Albert will give Chuck 3 x 36¢. You calculated 2 x 36.***

T: ***How can we find out for sure what number 3 x 36 is?***

S: ***Use a calculator or the Minicomputer.***

Note: If most of your students are convinced that 3 x 36 = 108, do not insist that the class find (or check) the answer with a calculator or the Minicomputer.

If the class decides to use the Minicomputer, ask,

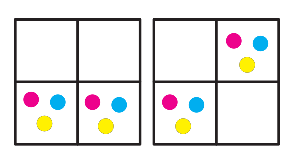
T: ***How many Minicomputer boards do we need?***

S: ***At least two.***

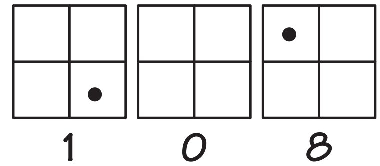
T: ***What should we do to calculate 3 x 36 on the Minicomputer?***

S: ***Put 36 on the Minicomputer three times.***

Invite a student to put 36 on the Minicomputer in red and another student to put 36 on the Minicomputer in blue. Remind the class that the number now on the Minicomputer is 2 x 36 and that you want to calculate 3 x 36. Invite a third student to put another 36 on the Minicomputer in yellow.



Invite students to make trades until the standard configuration for 108 is obtained. The class will note that a third Minicomputer board is needed. Ask a student to write the number below the Minicomputer and another to read the number aloud.



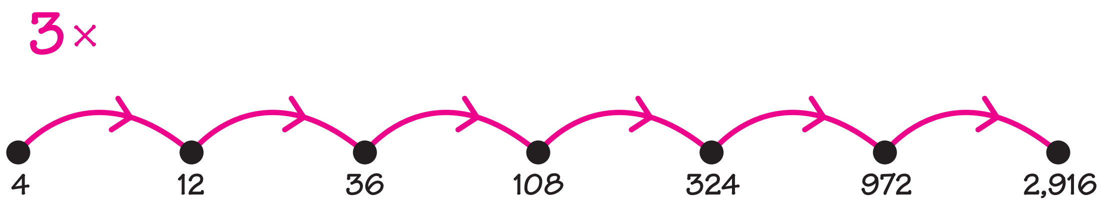
Label the fourth dot 108 in the arrow picture.

T: ***On the fourth day Chuck's uncle will give him 108 cents. Do you know another name for 108 cents?***

S: ***One dollar and eight cents.***

Extend the arrow picture to show fifth, sixth, and seventh days.

In the same manner, continue labeling dots until all seven days are accounted for. You should have this picture on the board.



Ask the students to work with a partner to calculate how much Chuck will receive altogether in seven days. They may like to do this with calculators or Minicomputers.

$$4 + 12 + 36 + 108 + 324 + 972 + 2{,}916 = 4{,}372$$

So Chuck would receive \$43.72. (Translate 4,372¢ into dollars and cents for the class.) Compare this amount to any guesses made early in the lesson.

Ask students to draw their own 3x arrow picture starting at 1, 2, or 5. Encourage students to keep extending their pictures until the numbers are too large for them to do the calculations. When this happens they should draw another 3x arrow picture starting with another number. Some students may wish to use individual Minicomputers or calculators during this activity.

Writing Activity

Ask students to write another story where they might have to multiply by 3.

Home Activity

Use Blackline N13 to make a 3x arrow road with six dots, starting at 2, 3, or 5, for students to take home and complete with a family member. Tell students to label the dots, and to use home Minicomputers or calculators to do the multiplication calculations, if they like.

N14 SUBTRACTION PROBLEMS #2

Capsule Lesson Summary

Put a number on the Minicomputer ten times. Make usual trades, and note a trade that replaces ten checkers on a square with one checker on a square of the same color on the next board to the left. Draw string pictures for two similar story situations and conclude that 13 – 5 = 13 + $\widehat{5}$. Find the solution to a subtraction calculation on the Minicomputer by adding the negative number of the number to be subtracted.

Materials

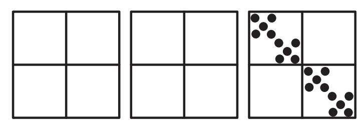
Teacher
- Minicomputer set
- Colored chalk
- Cup and marbles (optional)

Student
- Paper
- Minicomputer set

Description of Lesson

Exercise 1: Multiplying by 10

Put 10 x 9 on the Minicomputer in the following configuration.



T: ***What number is this?***
What multiplication fact can we write about this number?
Do we need to make some trades before we know what number this is?

Let the students react freely to these questions. Many of them already know that 10 x 9 = 90 and do not need to make trades on the Minicomputer in order to find the number. Nevertheless, call on volunteers to make trades until you obtain the standard configuration for 90. Let each volunteer make several trades in succession.

Record this number fact on the board.

10 x 9 = 90

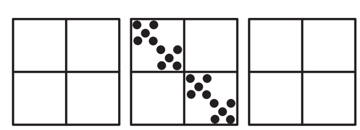
Point to the 8-square and then to the 80-square.

T: ***Ten checkers on this brown square (8-square) is the same as one checker on this brown square (80-square).***

Point to the 1-square and then to the 10-square.

T: ***Ten checkers on this white square (1-square) is the same as one checker on this white square (10-square).***

Put 10 x 90 on the Minicomputer in the following configuration.

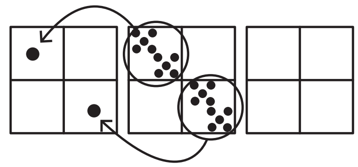


T: ***What multiplication fact can we write about this number?***
Is the calculation very different from the one we just did (point to 10 x 9 = 90)***?***

The students should notice that this problem is very similar to the first one. Many students will predict that 10 x 90 = 900.

T: ***Do we need to make lots of trades on the Minicomputer in order to find the number? How could we make it easier?***

Students may suggest these two trades; otherwise, suggest them yourself.



Allow the students to discuss these trades. If many of your students are not sure these trades are correct, let the class make usual trades until they obtain the standard configuration for 900. Then put the original configuration for 10 x 90 on the Minicomputer again and continue with this dialogue.

T: ***Ten checkers on this brown square*** (remove the ten checkers from the 80-square) ***is the same as one checker on this brown square*** (put one checker on the 800-square)***.*** ***10 x 80 = 800. We can make a trade with all the checkers on the 10-square.***

Ten checkers on this white square (remove the ten checkers from the 10-square) ***is the same as one checker on this white square*** (put one checker on the 100-square)***. 10 x 10 = 100.***

Conclude that 10 x 90 = 900, and record the number fact on the board under 10 x 9 = 90. In the same manner, ask students to calculate 10 x 900.

10 x 9 = 90
10 x 90 = 900
10 x 900 = 9,000

When they conclude that 10 x 900 = 9,000, ask if anyone sees a pattern in the number sentences you have written on the board. Instruct students to write these number sentences on their papers.

Do these calculations in the same manner. You may want to let students do the calculations first with a partner and then check the work collectively.

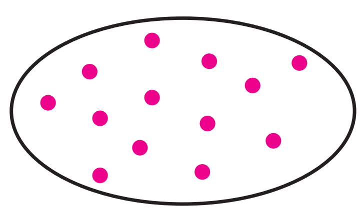
10 x 12
10 x 120
10 x 1,200

Note: Calculating 10 x 1,200 will require a fifth Minicomputer board which you can draw on the chalkboard or borrow from another class.

Exercise 2

T: ***Nick put 13 red marbles in his pocket.***

Draw this string picture on the board. You may also want to let a student pretend to be Nick and put red marbles in a cup.



T: ***Suppose Nick takes out five of these marbles. How many marbles will be left in his pocket?***

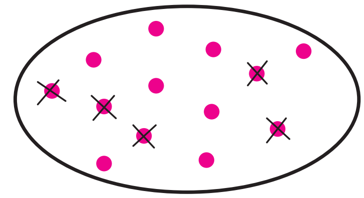
S: ***Eight.***

If appropriate, instruct the student to take out five marbles from the cup.

T: ***How can we show this in our picture?***

S: ***Erase five of the red marbles.***

S: ***Cross out five of the red marbles.***



$13 - 5 = 8$

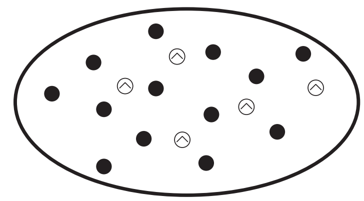
Let a student cross out or remove five of the red marbles.

T: ***What number sentence can we write about this picture?***

S: ***13 – 5 = 8.***

Write 13 – 5 = 8 below the string picture.

T: ***Eli the Elephant put 13 regular peanuts in his bag. Suppose he then put 5 magic peanuts in his bag.***



Draw this string picture on the board next to the first string picture.

T: ***How many peanuts will Eli find when he opens his bag?***

S: ***Eight, because five magic peanuts and five regular peanuts disappear.***

T: ***Who can show this in our picture?***

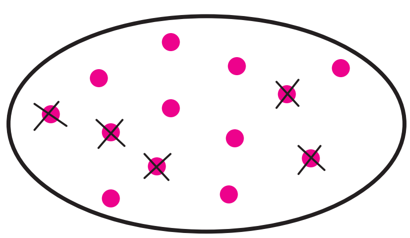
Invite a student to pair the regular and magic peanuts in the usual way.

T: ***What number sentence can we write about this picture?***

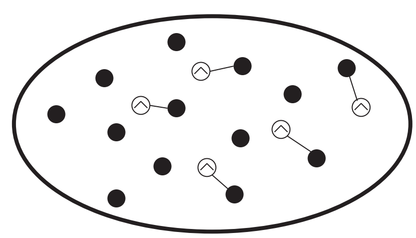
S: ***13 +*** $\widehat{5}$ ***= 8.***

Write 13 + $\widehat{5}$ = 8 below the second string picture.

T: ***What do you notice about these two pictures?***



$13 - 5 = 8$



$13 + \widehat{5} = 8$

The students should observe that subtracting 5 is the same as adding $\widehat{5}$. Erase the board.

Exercise 3

T: ***If Nick puts 100 marbles in a bag and later gives 46 of these marbles to Janet, how many marbles will be left in Nick's bag?***

Some students may be able to calculate the number mentally; others will guess.

T: ***How can we be sure how many marbles are left?***

S: ***Draw one hundred dots and then cross out forty-six of them.***

Accept this as a good idea but not a very practical one because it would take a long time.

S: ***Put 100 on the Minicomputer and then take off 46.***

Invite someone to put 100 on the Minicomputer.

T: ***Can we now subtract 46?***

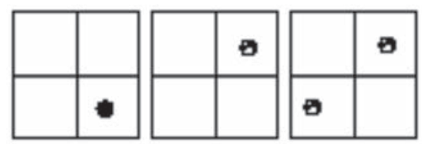
S: ***We need to make some trades first.***

S: ***We could add $\widehat{46}$.***

If no one suggests adding $\widehat{46}$, suggest it yourself. Then indicate this equality on the board.

$$100 - 46 = 100 + \widehat{46}$$

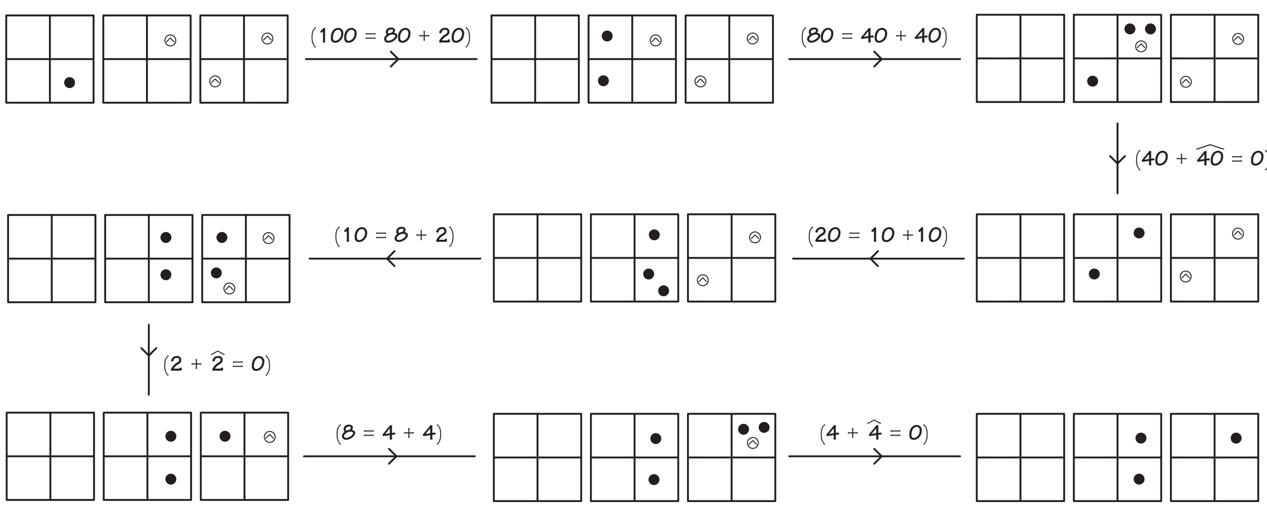
Emphasize that both calculations have the same solution.
Ask a student to put $\widehat{46}$ on the Minicomputer.



T: ***What should we do next?***

S: ***We need to make some backward trades to get negative and regular checkers on the same squares.***

Invite students to make trades on the Minicomputer. Watch closely as trades are made; a student might suggest incorrectly that $\widehat{40} = \widehat{20} + 20$ when the correct trade is $\widehat{40} = \widehat{20} + \widehat{20}$.



T: ***What number is 100 – 46?***

S: ***54.***

Erase the board and remove the checkers from the Minicomputer.

Exercise 4

Write this problem on the board. Instruct students to do the calculation on their desk Minicomputers with a partner.

296 – 165 =

Then do the problem collectively.

T: ***How did you calculate 296 – 165?***

Some students may suggest calculating $296 + \widehat{165}$ and others might suggest calculating 296 – 165. Ask the class which method they prefer, and then use that method to solve the problem on the demonstration Minicomputer. If there is sufficient time and student interest, repeat this activity to calculate 703 – 458.

Writing Activity

Ask students to write a story problem for one of the subtraction calculations in this lesson.

N15 THE FUNCTIONS –10 AND +1

Capsule Lesson Summary

Draw an arrow road alternating –10 and +1 arrows, and use composition to discover a –9 arrow. Practice taking 9 away from a number in traditional math fact practice.

Materials

Teacher	• Colored chalk	Student	• Unlined paper • Colored pencils, pens, or crayons • Worksheets N15 * and **

Description of Lesson

On the board, draw a dot for 53 and arrow keys for –10 and +1.

T: ***Today we are going to draw an arrow road using –10 and +1 arrows, starting at the number 53. We do not know where the arrow road will end, but we will alternate the two kinds of arrows. That means every time we use a –10 arrow the next arrow will be a +1, and following a +1 arrow there will be a –10 arrow.***

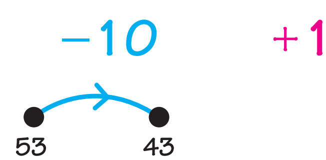
Let's begin our road and see where it takes us. Do you want to start with –10 or +1?

Let students choose how to start the road; adjust the lesson description accordingly.

S: ***Start with –10.***

As you draw the arrow road on the board, instruct students to do the same on their papers.

T: ***53 – 10 = ?*** (43) ***Now what kind of arrow comes next (starting at 43)?***

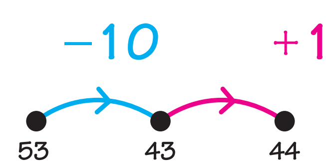


S ***A +1 arrow.***

T: ***Where will it take us?***

S ***To 44, because 43 + 1 = 44.***

T: ***What kind of arrow comes next (starting at 44)?***

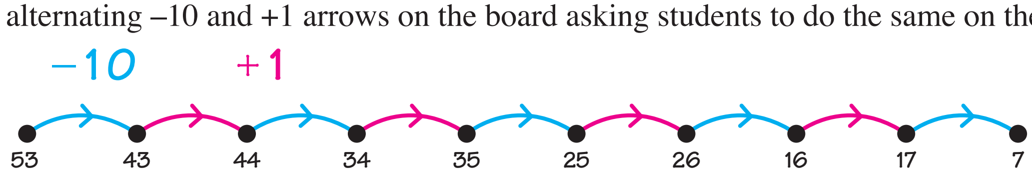


S: ***A –10 arrow.***

T: ***Where will it take us?***

S: ***To 34, because 44 – 10 = 34.***

Continue alternating –10 and +1 arrows on the board asking students to do the same on their papers.



After several more arrows, ask students what they notice about the picture.

S: ***The numbers are getting smaller.***

S: ***There are always two 3's (ones digit), two 4's, two 5's, two 6's, and so on.***

S: ***There are two 40's, two 30's, two 20's, and so on.***

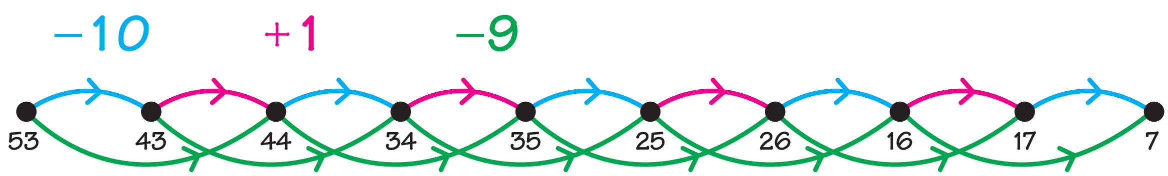
S: ***Soon we will get a negative number.***

Draw a green arrow from 53 to 44 as you say,

T: ***Let's draw a new arrow that combines –10 and +1. What could this green arrow that connects 53 to 44 be for?***

S: ***–9.***

Write –9 in green on the board and direct students to do the same on their papers. Then ask students to draw as many –9 arrows as they can find in their pictures (without adding more dots). Do not expect students to find all possibilities.



T: ***What do you notice about where we can draw –9 arrows?***

S: ***Each time there is –10 and then +1, we can draw a –9 arrow.***

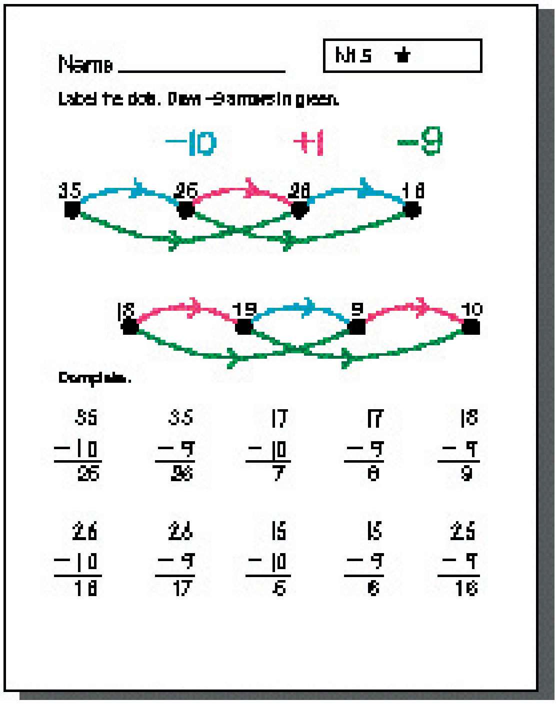
S: ***Each time there is +1 and then –10, we can draw a –9 arrow.***

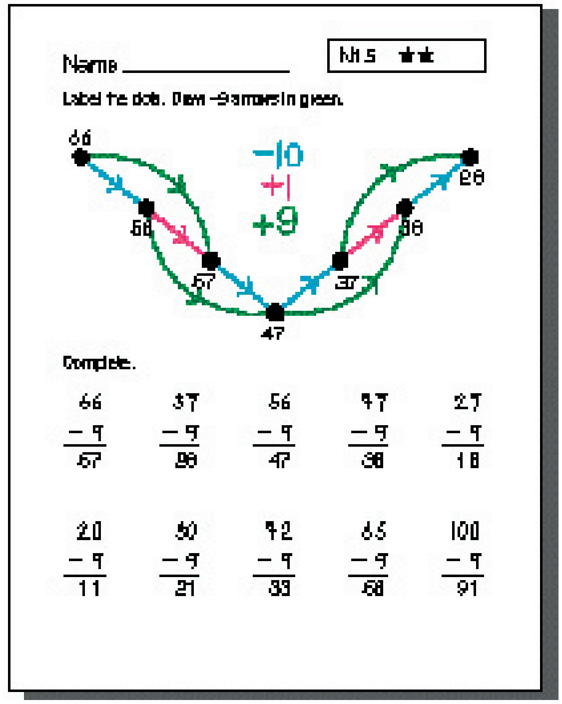
Let this discussion suggest to students that subtracting 9 is equivalent to subtracting 10 and then adding 1, or vice versa.

Worksheets N15* and ** are available for individual work.

Writing Activity

Suggest that students write an explanation of how you can subtract 9 by first subtracting 10 and then adding 1, or vice versa.


Name
N15 ★
Label the dots. Draw -9 arrows in green.
−10
+1
−9
Complete.


Name
N15 ★★
Label the dots. Draw -9 arrows in green.
−10
+1
+9
Complete.

N16 SUBTRACTION PROBLEMS #3

Capsule Lesson Summary

Put numbers on the Minicomputer ten times. Make trades replacing ten checkers on a square with one checker on the square of the same color on the next board to the left. Do a similar activity with twenty checkers on the 8-square. Use the Minicomputer to decide how much money Gina's father would have left out of $200 if he buys a television set costing $127.

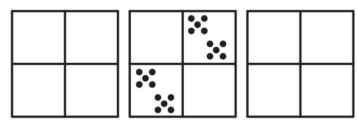
Materials

Teacher	• Minicomputer set	**Student**	• Paper • Minicomputer set • Worksheets N16*, **, ***, and ****

Description of Lesson

Exercise 1: Multiplying by 10

Put 10 x 60 on the Minicomputer in this configuration.



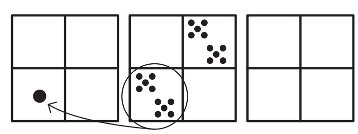
T: ***What number is this?***
What multiplication calculation can we write? (10 x 60)
Do we need to make trades before we know which number this is?

Let the students react freely to these questions. Many of them already know that 10 x 60 = 600 and no longer need to make trades on the Minicomputer.

T (pointing to the 20-square)**:** ***How many checkers are on this square?*** (Ten)
Can you make a (big) trade with all of these checkers?

S: ***Ten checkers on the 20-square is the same as one checker on the 200-square.***

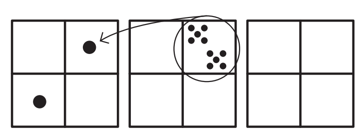
Invite a student to make this trade and say 10 x 20 = 200.



T (pointing to the 40-square)**:** ***How many checkers are on this square?*** (Ten)
Can you make a (big) trade with these ten checkers?

S: ***Ten checkers on the 40-square is the same as one checker on the 400-square.***

Invite a student to make this trade and say 10 x 40 = 400.



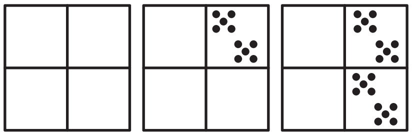
Write the corresponding 10x calculation on the board.

10 x 60 = 600

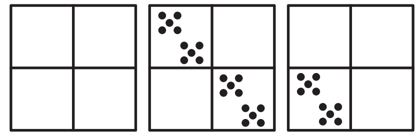
T: ***10 x 60 = ?***

S: ***600.***

Repeat this activity with these configurations. Let students whisper the number to you before making trades on the Minicomputer.



$10 \times 45 = 450$



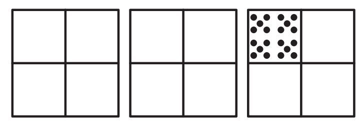
$10 \times 92 = 920$

You should have several 10x number sentences on the board.

T: ***What patterns do you see?***

Allow students to express themselves freely. Perhaps they will suggest easy ways to multiply a number (an integer) by 10; for example, simply tack on 0 thereby shifting each digit one place to the left. Do not suggest any tricks yourself; let students discover their own, if not in this lesson, in a future one.

Erase the board and then put this configuration on the Minicomputer.

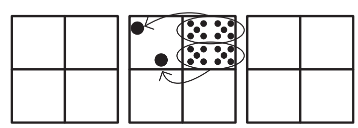


T: ***How many checkers are on the 8-square?*** (Twenty)
What multiplication calculation can we write? (20 x 8)
What number is this?

Ask students to write the answer on their papers (or whisper it to their partners).

T: ***Remember that there are twenty checkers on this square. What trades can we make with these checkers?***

Someone might suggest removing all the checkers from the 8-square and putting a checker on the 80-square. If so, ask, "Does 20 x 8 = 80?" (No) When a student suggests making a 10 x 8 = 80 trade, let the student make the trade, and call on another student to make a second 10 x 8 = 80 trade with the remaining checkers.



T: ***Now, what do we need to calculate before we know which number is on the Minicomputer?***

S: ***2 x 80.***

Invite students to make trades (some backward trades are needed) until the standard configuration for 160 is obtained. Conclude that 20 x 8 = 2 x 80 = 160.

Exercise 2: Subtraction Problem

Tell the following or a similar story about one of the students in your class.

T: ***Gina's father decides to buy a television set. He puts \$200 in his wallet and goes shopping. What bills could he have in his wallet if he has exactly \$200?***

Encourage students to suggest many possibilities before you continue with the lesson.

S: ***He could have two $100 bills.***

S: ***He could have all $20 bills?***

T: ***How many $20 bills?***

S: ***Ten.***

T: ***10 x 20 = 200, so ten $20 bills would be $200.***

When the students exhaust their ideas about what bills make $200, continue the story.

T: ***Gina's father finds a television set he likes that costs $127.***
Does Gina's father have enough money with him to buy it? (Yes)

127 is less than 200. What calculation could we do to find out how much money Gina's father will have left after he buys the television?

S: ***200 – 127.***

Write 200 – 127 = ☐ on the board, and point to it as you ask,

$$127 < 200$$

$$200 - 127 = \square$$

$$127 + \square = 200$$

T: ***Is there another way we could write a number sentence for this problem?***

S: ***127 + ☐ = 200.***

If necessary, suggest such a number sentence yourself and add this sentence to those on the board.

T: ***How much money will Gina's father have left after he buys the television set?***

Record students' estimates on the board. Perhaps someone will suggest using the Minicomputer.

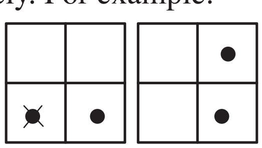
T: ***How can we calculate 200 – 127 on the Minicomputer?***

S: ***Put 200 on the Minicomputer and then take off 127.***

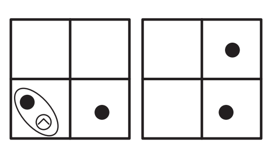
S: ***Put 200 on the Minicomputer with regular checkers and then put $\widehat{127}$ on the Minicomputer with negative checkers.***

Use whichever method most students like to solve the problem. Invite students to put the numbers on the Minicomputer, and then to make the appropriate trades. Conclude that 200 – 127 = 73. Let the students determine which guess was closest to 73.

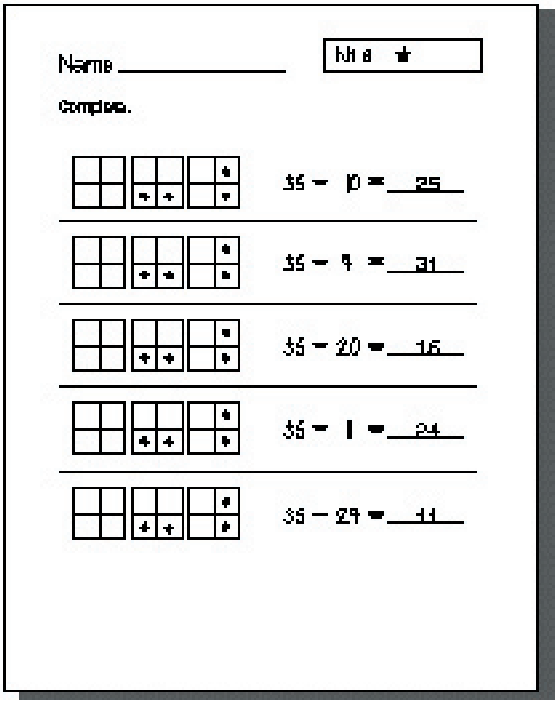
Worksheets N16*, **, ***, and **** are available for individual work. Students should be allowed to use their Minicomputers if they wish. Individual students may prefer to cross off checkers or add (by drawing) negative checkers on the worksheets. Model both methods by solving one or two problems collectively. For example:

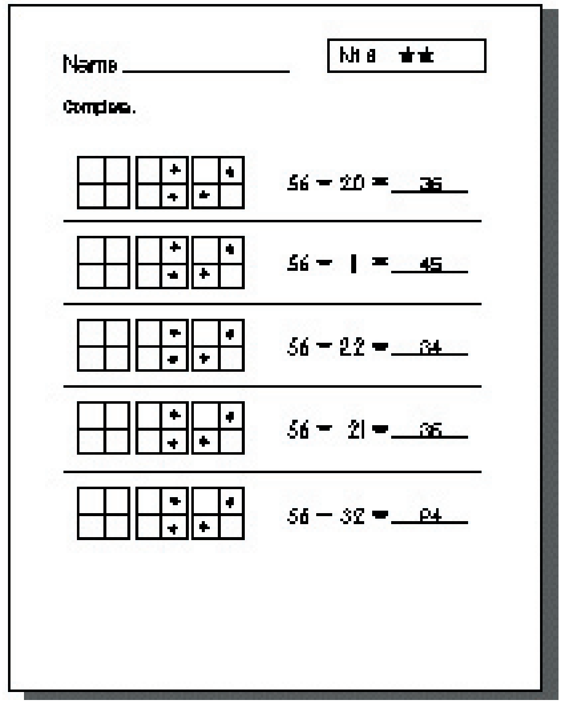


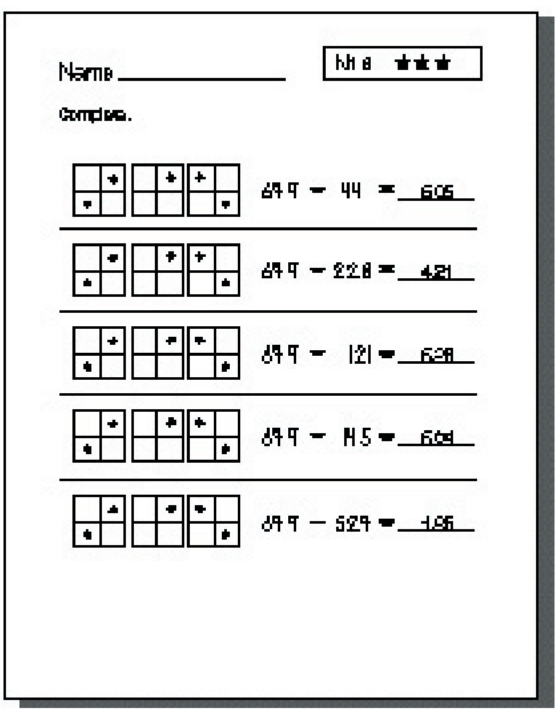
$35 - 20 = 15$

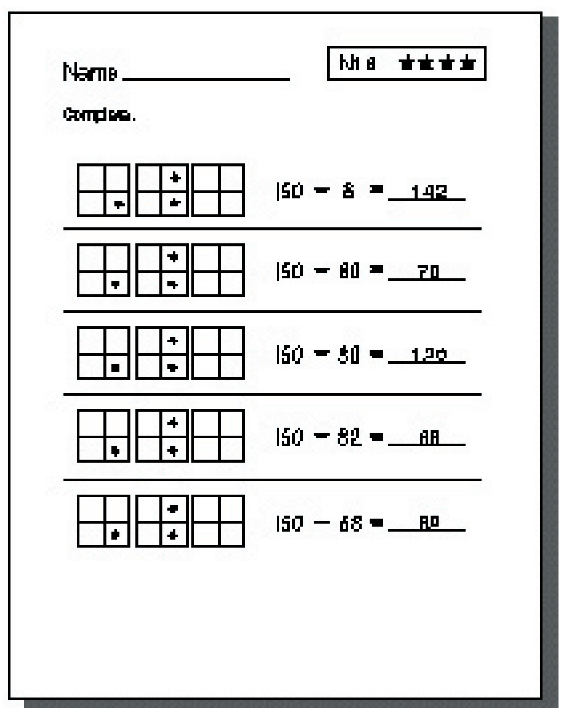


$35 + \widehat{20} = 15$


Name ____________ N16 ★
Complete.
35 − 10 = 25
35 − 4 = 31
35 − 20 = 15
35 − 11 = 24
35 − 24 = 11


Name ____________ N16 ★★
Complete.
56 − 20 = 36
56 − 11 = 45
56 − 22 = 34
56 − 21 = 35
56 − 32 = 24


Name ____________ N16 ★★★
Complete.
649 − 44 = 605
649 − 228 = 421
649 − 121 = 528
649 − 145 = 504
649 − 529 = 120


Name ____________ N16 ★★★★
Complete.
150 − 8 = 142
150 − 80 = 70
150 − 30 = 120
150 − 82 = 68
150 − 68 = 82

Capsule Lesson Summary

Determine where the greatest number is and where the least number is in an arrow road with subtraction and addition arrows. Given the least number in a road, label all the dots with the help of return arrows.

Materials

Teacher
- Colored chalk

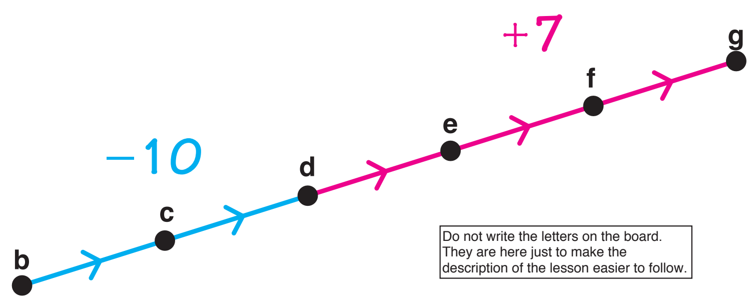
Student
- Unlined paper
- Colored pencils, pens, or crayons
- Worksheets N17* and **

Description of Lesson

You may like to let students work in pairs during this lesson.

Exercise 1

Draw this arrow picture on the board, and ask the students to copy it on their papers.



T: ***Where is the least number in this picture?*** (At **d**)
How do you know?

Students should mention these ideas in their explanations:

- The numbers from **b** to **d** in the arrow road decrease, because the arrows in that part of the road are for –10.
- The numbers from **d** to **g** in the arrow road increase, because the arrows in that part of the road are for +7.

Point to the indicated dots as you ask the following questions.

T: ***How much less is this number*** (at **d**) ***than this number*** (at **b**)***?***

S: ***20 less, because 10 + 10 = 20.***

T: ***How much less is this number*** (at **d**) ***than this number*** (at **g**)***?***

S: ***21 less, because 7 + 7 + 7 = 21.***

T: ***So which number is greater: this one*** (at **b**) ***or this one*** (at **g**)***?***

S (pointing to **g**)**:** ***This one.***

T: ***Now I will tell you that the least number in this picture is 76. In your picture, label the dot for 76 and then label the other dots.***

Invite a student to label the dot for 76 on the board. Look quickly at the individual work to ensure that each student labeled the correct dot 76. After a few minutes, check the work collectively.

T (pointing to **c**): ***What number is here? How do you know?***

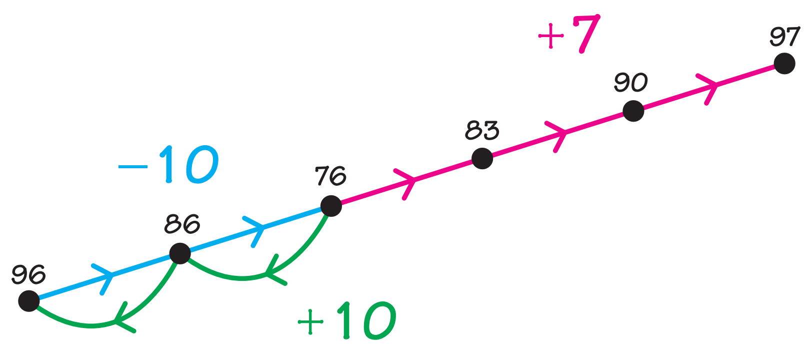
S: ***86, because 86 – 10 = 76.***

S: ***86, because 76 + 10 = 86.***

Trace return arrows from **d** to **c** and then from **c** to **b** as you ask,

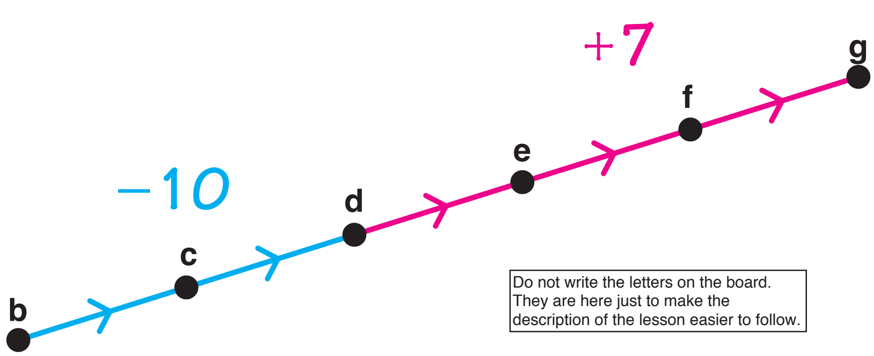
T: ***What could these arrows be for?*** (+10)

Write +10 in green near the picture on the board. Ask the students to draw green +10 arrows in their pictures, and then invite some students to draw these arrows in the picture on the board.



Exercise 2

Draw this arrow picture on the board and ask the students to copy it on their papers.



T: ***Where is the greatest number in this arrow picture?*** (At **e**)
How do you know?

Students should mention these ideas in their explanations:

- The numbers from **b** to **e** in the arrow road increase, because the arrows in that part of the road are for +6.
- The numbers from **e** to **g** in the arrow road decrease, because the arrows in that part of the road are for –11.

Point to the indicated dots as you ask the following questions.

T: ***How much more is this number*** (at **e**) ***than this number*** (at **b**)***?***

S: ***18 more, because 6 + 6 + 6 = 18.***

T: ***How much more is this number*** (at **e**) ***than this number*** (at **g**)***?***

S: ***22 more, because 11 + 11 = 22.***

T: ***So which number is less*****:** ***this one*** (at **b**) ***or this one*** (at **g**)?

S (pointing to **g**)**:** ***This one.***

T: ***How much less?***

Perhaps one of your students will know that the number at **g** is 4 less than the number at **b**. If no one suggests this, ask the question again after the dots have been labeled.

T: ***Now I will tell you that the least number in this arrow picture is 25. In your picture, label the dot for 25 and then label the other dots.***

Allow a few minutes for individual work. If several students do not know which dot to label 25, ask someone to label the dot for 25 on the board.

T (pointing to **f**)**:** ***What number is here? How do you know?***

S: ***36, because 36 – 11 = 25.***

S: ***36, because 25 + 11 = 36.***

Trace return arrows from **g** to **f** and then from **f** to **e** as you ask,

T: ***What could these arrows be for?*** (+11)

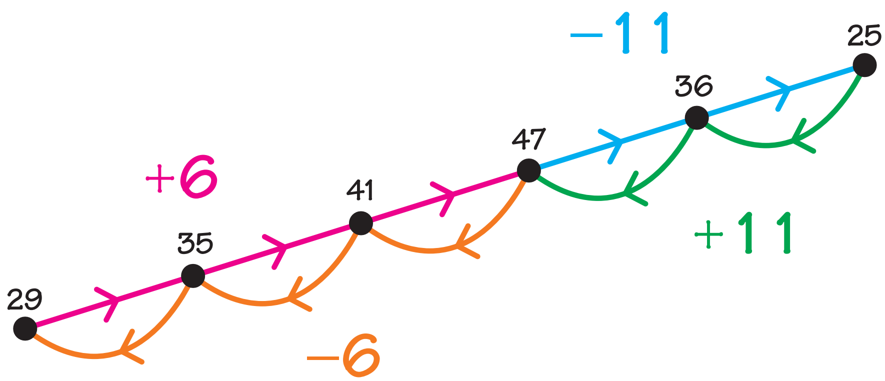
Trace return arrows from **e** to **d**, from **d** to **c**, and from **c** to **b** as you ask,

T: ***What could these arrows be for?*** (–6)

On the board, write +11 in green and –6 in orange.

T: ***Draw all the return arrows in your arrow picture.***

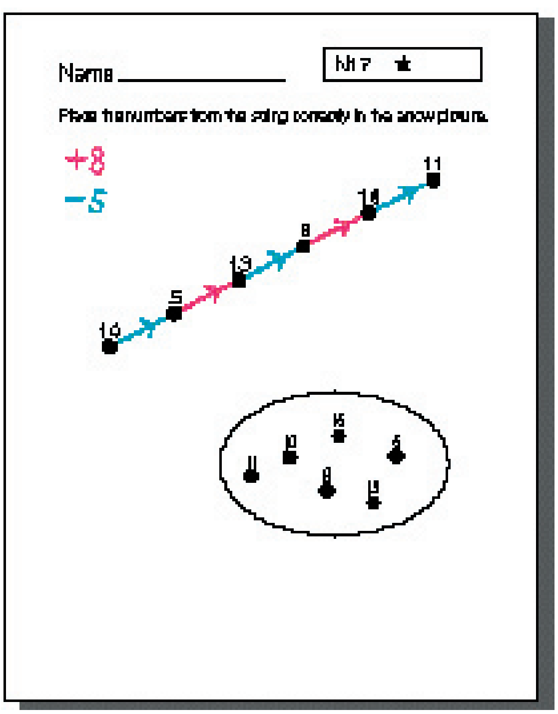
Let students complete the picture on the board.

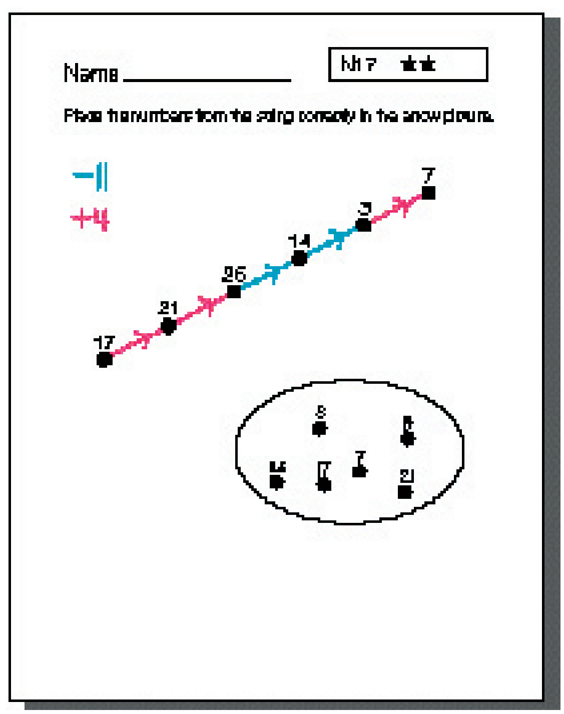


Point to the dots for 25 and 29 in turn as you ask,

T: ***How much less is this number (25) than this number (29)?*** (4)

Worksheets N17* and ** are available for individual work. Remind students of "fishing for numbers" type problems as you explain the directions.





N18 DOUBLING AND HALVING #3

Capsule Lesson Summary

Consider a configuration on the Minicomputer in which the checkers are grouped in pairs and both checkers in each pair are on the same square. Recognize what one-half of the number is, and then make trades to find the number. Record the situation in an arrow picture. Repeat this activity with variations. In one variation, the number on the Minicomputer can be recognized easily, but one-half of it cannot be because the checkers are not in pairs. In another variation, students use individual Minicomputers.

Materials

Teacher
- Minicomputer set
- Colored chalk

- Minicomputer set

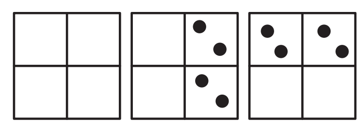
Student
- Paper

Description of Lesson

This lesson description assumes the students are paired. If you prefer, let students work individually, or in groups of three or four.

Exercise 1

Put this configuration on the Minicomputer.



Instruct the students to work with a partner and write on their papers some things they know about this number. You may want to ask a few questions to get students started.

T: ***What can you tell me about this number? Is it more than 200? How do you know? Is it more than 100? How do you know? What is one-half of this number? How can you tell?***

Encourage student pairs to find a good estimate of the number, but tell them not to worry about exactly what number it is. Allow a few minutes for students to discuss what they know about the number. If students do not suggest that one-half of the number on the tens board is 50, and one-half of the number on the ones board is 12, guide the class to these conclusions and write this number sentence on the board.

$$50 + 12 = ?$$

T: ***What is one-half of the number on the Minicomputer?***

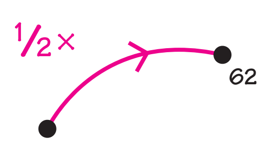
S: ***62.***

T: ***This dot*** (draw a large dot on the board) ***is for the number on the Minicomputer.***

Draw a red arrow starting at that dot, and then draw the ending dot as you say,

T: ***This dot is for one-half of that number.***

Label the arrow ½x and the ending dot 62.



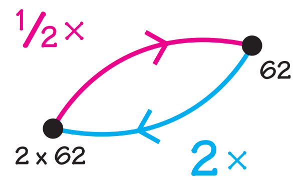
N18

Draw the return arrow in blue.

T: ***What could this blue arrow be for?***

S: ***2x.***

Label the blue arrow 2x and trace it from 62 to the dot on the left.



T: ***So the number on the Minicomputer is 2 x 62.***

Label the dot on the left 2 x 62.

T: ***Calculate 2 x 62 on your paper.***

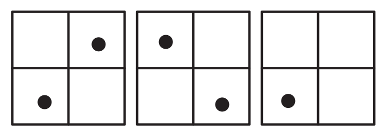
After many students have indicated their answers, invite students to make trades until 124 is in standard configuration. Conclude that 2 x 62 = 124 and label the dot on the left 124 (erasing 2 x 62).

T: ***What calculations are told by this arrow picture?***

S: ***½ x 124 = 62 and 2 x 62 = 124.***

Suggest that students write the calculations on their papers while you prepare for a new problem.

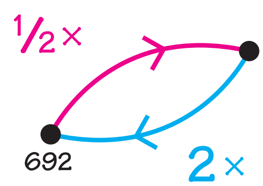
Put this configuration on the Minicomputer.



T: ***What number is this?***

S: ***692.***

In the arrow picture, label the dot on the left 692.



T (tracing the red arrow)**:** ***What is one-half of 692?***

Some students might guess the answer, but others might say they do not know because the checkers are not in pairs.

T: ***How can we put these checkers in pairs without changing the number?***

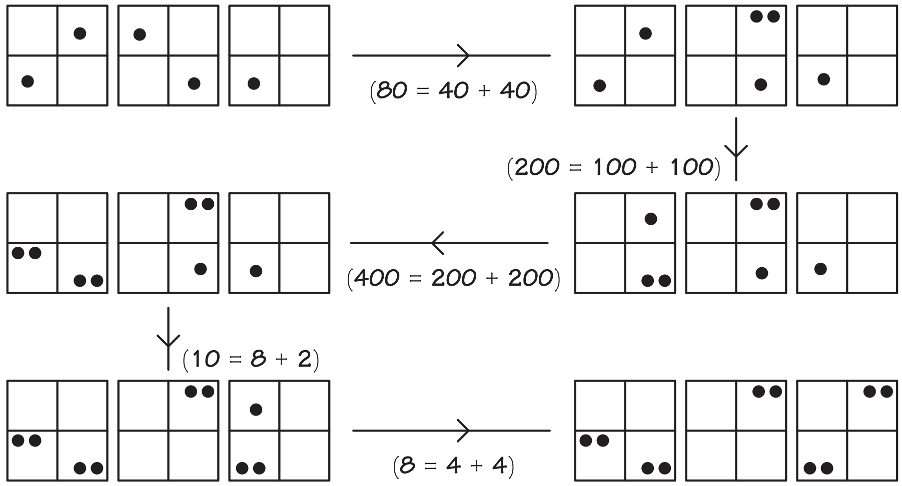
S: ***Make some backward trades.***

Invite someone to make a trade. Any backward trade will result in a pair of checkers on one of the squares. You may need to guide the discussion by pointing to a checker and asking what trade you could make that would result in a pair of checkers. Whenever you get a pair of checkers on the same square, push the two checkers into a corner of that square.

T: ***We now have a pair of checkers. Can someone make a trade that will give us another pair of checkers?***

Continue this activity until all the checkers are in pairs. Discourage any student who wants to make a trade with checkers that are already paired. Tell the students it is a correct trade but not very useful for the goal of getting pairs.

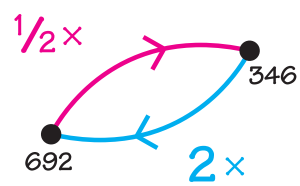
One possible sequence of trades is given below.



T: ***All the checkers are in pairs. What is one-half of this number?***

S: ***346.***

Label the dot on the right in the arrow picture 346.



T: ***What calculations are told by this arrow picture?***

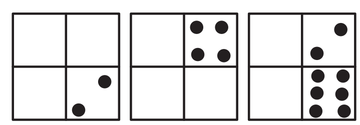
S: ***2 x 346 = 692 and ½ x 692 = 346.***

Suggest students write the calculations on their papers while you prepare for Exercise 2.

Exercise 2: Individual Minicomputers

Distribute desk Minicomputers (two Minicomputer sheets and checkers) to student pairs.

Erase the board and put this configuration on the Minicomputer. Instruct students to put the same configuration on their desk Minicomputers.

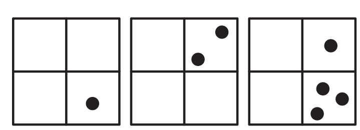


As in Exercise 1, direct students to work with a partner and write some things they know about this number on their papers. Collectively discuss some of the students' findings.

T: ***Remove some checkers so that one-half of this number is on your Minicomputer.***

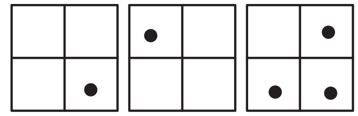
Do not change the number on the demonstration Minicomputer. Some students may not know what to do if there are more than two checkers on a square. If this is a common difficulty, ask student pairs to explain their methods. One group might pair all the checkers and take off one out of every two, another might count the checkers on a square and take off half that many. Either approach is correct.

Student pairs should have this configuration on their desk Minicomputers.

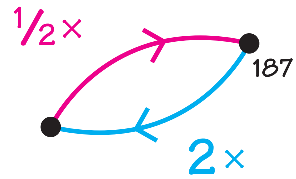


T: ***Is the number on your Minicomputer more than 100? How do you know? Is it more than 200? How do you know? What number is on your Minicomputer?***

Allow students to answer by telling their partners; then suggest they make trades on their desk Minicomputers to make the number easier to read. Conclude that 187 is one-half of the number on the demonstration Minicomputer.



In the arrow picture, label the dot on the right 187.



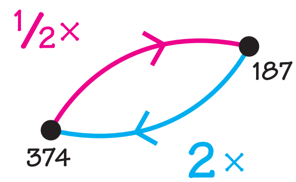
T (tracing the blue arrow)**:** ***2 x 187. How can we calculate this number without using the Minicomputer? Write it on your papers.***

Let students suggest their own methods of calculation. Perhaps some will do it this way.

100 + 100 = 200 and 80 + 80 = 160 and 7 + 7 = 14
so
2 x 187 = 200 + 160 + 14 = 374

Afterward, suggest they check this answer by calculating 2 x 187 on their Minicomputers.

Label the dot on the left 374.



T: ***What calculations are told by this picture?***

S: ***½ x 374 = 187 and 2 x 187 = 374.***

Exercise 3

Give student pairs the following problem to solve. Suggest they use their Minicomputers or any methods of their choice to do the doubling.

T: ***Corey has a puppet design that tells him how much ribbon, buttons, and yarn he needs to make one puppet. But Corey wants to make two puppets. What should he do?***

S: ***Double.***

Write these design quantities on the board, and direct students to find out how much material is needed for two puppets.

Materials for One Puppet

Ribbon — 17 inches
Buttons — 9
Yarn — 36 inches

Writing Activity

Write a story telling how to double a recipe or how two friends can each get one-half of a number of cards.

N19 INTRODUCTION TO DECIMAL NUMBERS #1

Capsule Lesson Summary

Through a sequence of situations involving money, extend the use of the Minicomputer for representing decimal numbers.

Materials

Teacher
- Minicomputer set†
- Colored chalk

Student
- None

Description of Lesson

Begin this lesson with five to ten minutes of mental arithmetic involving 2x and ½x.

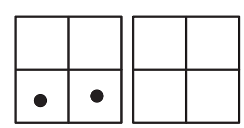
Choose students from your class to star in the following stories. Substitute their names for the names that appear here.

T: ***Jamie and Timothy are my friends. I want to share $30 between them. I want to give them both the same amount of money. How much money should I give Jamie? How much money should I give Timothy?***

Allow several students to comment. Be sure the class understands that the $30 must be shared equally between the two children. Although many students will know that ½ x 30 = 15 without the use of the Minicomputer, the following activity of calculating ½ x 30 will review the process of halving on the Minicomputer. This review is in preparation for finding one-half of a number on the Minicomputer when the result is a (non-integer) decimal. Display two Minicomputer boards.

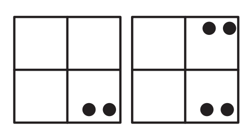
T: ***How can we use the Minicomputer to find one-half of 30?***

Invite someone to put 30 on the Minicomputer.



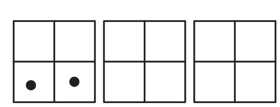
S: ***Make backward trades until all the checkers are in pairs.***

Invite students to make trades. Whenever a trade results in a pair of checkers on the same square, ask the student who made the trade to move the two checkers together into a corner of the square. Discourage students from breaking up pairs. Continue until all the checkers are in pairs.



T: ***Who can remove checkers so that one-half of this number will be on the Minicomputer?***

A student should remove one checker from each pair. Write ½ x 30 = 15 on the chalkboard.



†This lesson requires, at times, that additional space be left between two of the demonstration Minicomputer boards and that a bar be drawn in this space.

If your Minicomputers are hanging from permanently attached hooks or are displayed in some other way which fixed their position, you may need to make some adjustments before the lesson begins.

N19

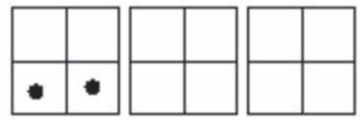
Clear the Minicomputer and continue with another story.

T: ***I have two more friends. Carla and Laura are my friends. I want to share $300 between them. How much money should I give Carla? How much money should I give Laura?***

Allow several students to comment. Some students might know that ½ x 300 = 150.

T: ***How can we use the Minicomputer to find one-half of 300?***

Display a third Minicomputer board and invite someone to put on 300.



S: ***Make backward trades until all the checkers are in pairs.***

Invite students to make trades until all the checkers are in pairs. Students may notice that the trades are like those for the first sharing problem. Ask a student to remove checkers so that one-half of the number is on the Minicomputer and write ½ x 300 = 150 on the chalkboard.

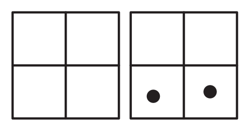
Clear the Minicomputer, take away the hundreds board, and continue with another story.

T: ***I have two more friends, Jason and Lynn. I want to share $3 between them. How much money should I give Jason? How much money should I give Lynn?***

Let students comment. Perhaps they will suggest that each child be given one and a half dollars.

T: ***How many cents is a half dollar?*** (Fifty)
So one and one-half dollars is one dollar and fifty cents.

Invite a student to put 3 on the Minicomputer.



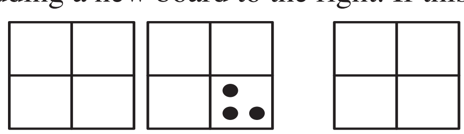
T: ***What can we do to find one-half of 3 on the Minicomputer?***

S: ***Make backward trades until all the checkers are in pairs.***

There may be some confusion, but very likely someone will make the 2 = 1 + 1 trade resulting in three checkers on the 1-square. Move two of these checkers into a corner of the square.

T: ***Now we have one pair of checkers ($1 for Jason and $1 for Lynn), but we still have a checker without a partner (another dollar). What should we do now?***

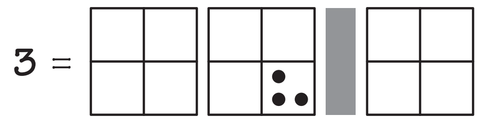
There will probably be even more confusion now. Be patient. Encourage students to express their ideas. In many cases, at least one student will suggest adding a new board to the right. If this does not happen, suggest it yourself. Place the new board to the right of the ones board, but leave more space than usual between these boards.



At least one of your students should object and say that now the number on the Minicomputer is not 3 but 30. Raise the question yourself, if necessary.

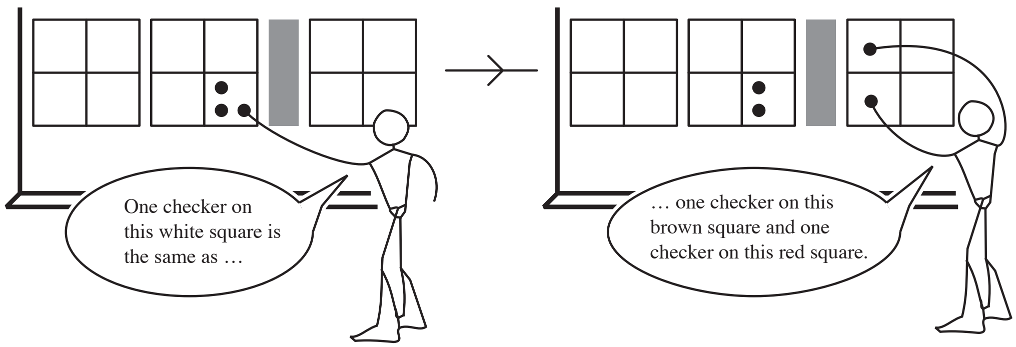
T: ***In order not to forget that this is 3 (or that this is the ones board), let's put a bar between these two boards.***

Emphasize that the bar is there to remind us that the board to the left of the bar is still the ones board.



T: ***What should we do now?***

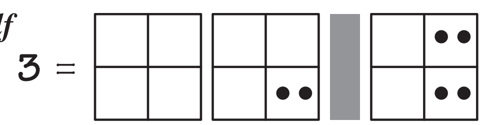
Students may still be confused and may suggest some wrong trades. It is natural for the bar to disturb them for a while. If this happens, make one trade yourself.



You may like to ask students what to call this new board, or you may suggest *dimes board* yourself.

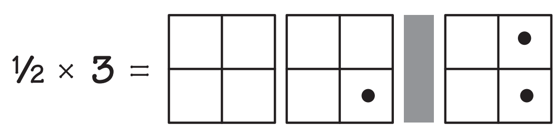
Generally students will make the next two trades without any trouble.

T: ***Now we can remove checkers so that one-half of this number will be on the Minicomputer?***

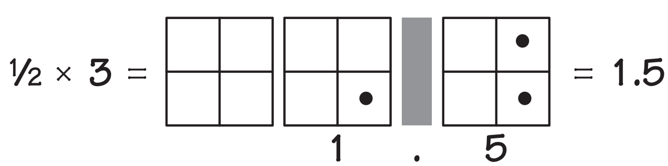


Let a volunteer remove one checker from each pair.

T: ***How do you suggest we write this number below the Minicomputer?***



Perhaps a student will suggest using something analogous to the bar to separate 1 and 5. If no one suggests using a point, suggest this yourself. Many students will be acquainted with the decimal point from seeing prices in stores. Write 1.5 below and to the side of the Minicomputer.

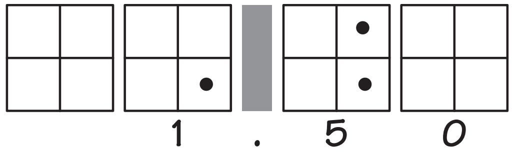


T: ***Is this the way people usually write one and one-half dollars?***

Without saying anything, add a new board to the right. Wait for the reactions of the class. Perhaps they will recognize this new board as the *pennies board*.

S: ***There are no checkers on the new board. We can write 0 below it.***

$\frac{1}{2} \times 3 =$



$= 1.50$

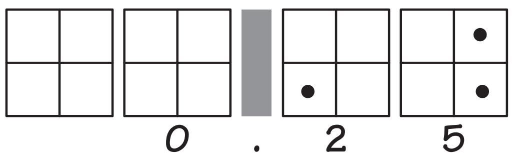
If students suggest writing 0 below the tens board, say that it is correct but not necessary.

T: ***1.5*** (read as "one dollar and five dimes" or "one point five") ***is the same as 1.50*** (read as "one dollar and fifty cents" or "one point five zero")***.***

Remove the checkers from the Minicomputer and erase the board.

T: ***Who can put one quarter on the Minicomputer?***
Who can write this number below the Minicomputer?

If necessary, write zero under the ones board yourself.

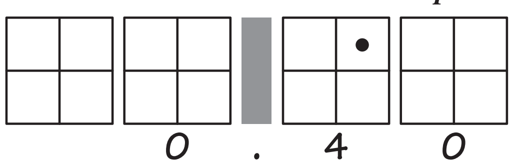


$= 0.25$

T: ***Right. One quarter is twenty-five cents which we can read as two dimes and five pennies. Zero point two five.***

Remove the checkers and erase the board.

T: ***Who can put four dimes on the Minicomputer?***
Who can write this number below the Minicomputer?



$= 0.40$

T: ***Four dimes is forty cents and we write 0.40*** (read as "zero point four zero")***.***

Point to the ones board (directly to the left of the bar).

T: ***This board is for dollars.***

Point to the board directly to the right of the bar.

T: ***What is this board for?***

S: ***Dimes.***

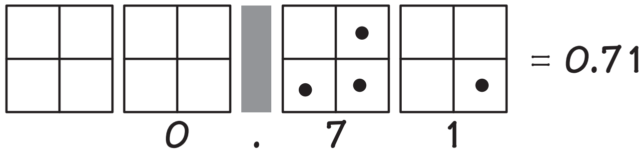
Point to the board to the right of the dimes board.

T: ***What is this board for?***

S: ***Pennies.***

Remove the checkers and erase the board.

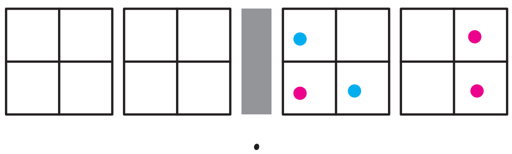
T: ***Who can put seven dimes and one penny on the Minicomputer?
Who can write this number below the Minicomputer?***



T: ***Seven dimes and one penny is seventy-one cents; we write 0.71*** (read as "zero point seven one")***.***

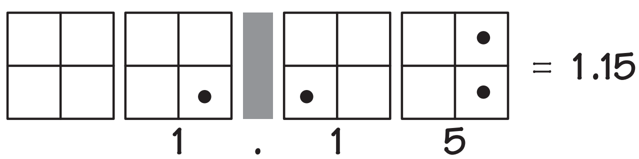
Remove the checkers and erase the board.

T: ***Who can put nine dimes on the Minicomputer with blue checkers?
Who can put a quarter on the Minicomputer with red checkers?***



If students have difficulty verbalizing the trade, say, "Eight dimes and two dimes is one dollar."

T: ***Who can write this number below the Minicomputer?***



T: ***How much money is nine dimes and one quarter?***

S: ***One dollar and fifteen cents.***

T: ***We write this as 1.15*** (read as "one point one five")***.***

Center Activity

Put money collections and Minicomputers in a center for practice.

Home Activity

Suggest parents/guardians practice counting how much money is in a small collection of pennies, nickels, dimes, and quarters (up to $2.00). This would be a good time to send home a review letter on the Minicomputer explaining its extension to decimals. Blackline N19 has such a letter.

N20 FRAMES OF DIFFERING SHAPES

Capsule Lesson Summary

Complete open sentences that have frames of differing shapes in the same sentence.

Materials

Teacher
- None

Student
- Paper
- Counters

Description of Lesson

Exercise 1

Write this open sentence on the board.

$$\boxed{} + \boxed{} = 20$$

T: ***Do you remember when we looked at a sentence like this and found what numbers to put in the frames to make the sentence true?***

Encourage the students to comment. Remind the students that when the frames in a sentence have the same shape, they are for the same number.

T: ***Today we are going to look at some sentences with frames, but this time the frames will be different shapes.***

Erase one of the squares and replace it with a triangle.

$$\boxed{} + \triangle = 20$$

T: ***When the frames have different shapes, we do not need to put the same number in both of them. What numbers could we put in these frames to make the sentence true?***

Instruct students to copy the open sentence and choose numbers to put in the frames to get a true sentence. You may like to let students work with a partner.

As students suggest pairs of numbers, write them inside the square and triangle respectively. Then ask the class to determine if the number sentence is true. The following is a sample dialogue.

S: ***Put 19 in the square and 1 in the triangle.***

$$\boxed{19} + 1 = 20$$

T: ***19 + 1 = 20. Is this true?*** (Yes)

Erase the numbers in the frames before asking for another possibility. Continue until three or four possible solutions have been found.

T: ***Let's record some of our solutions in a table.***

Draw a table below the number sentence and record solutions found by the class on the table.

If someone suggests putting 10 in both the square and the triangle, be sure your class understands that this is a correct solution. (10 + 10 = 20 is true.) The numbers do not have to be the same if the frames are different shapes, but they may be.

$\square + \triangle = 20$

$\square$	$\triangle$
19	1
5	15
10	10
20	0
25	$^{-}5$
½	19½

Continue asking for suggestions until your table has five or six pairs of numbers in it. Accept other solutions without recording them in the table. There are an infinite number of solutions to this open sentence.

Exercise 2

Write this open sentence and chart on the board. Instruct students to copy only the open sentence.

$\square + \square + \triangle = 20$

$\square$	$\triangle$

T: ***This number sentence has two squares and one triangle. The same number has to be in both squares, but a different number could be in the triangle.***

Give students a moment to choose numbers to make the sentence true. Students having difficulty can be given 20 counters to distribute among the frames. Insist they put the same number of counters in each square and possibly a different number of counters in the triangle. Suppose a student suggests putting 8 in each square and 4 in the triangle. Record the suggestion inside the frames before recording it in the table.

$\boxed{8} + \boxed{8} + 4 = 20$

$\square$	$\triangle$
8	4

T: ***8 + 8 + 4 = 20. Is this number sentence true?***

S: ***Yes.***

Record 8 and 4 in the table.

You may need to repeat the rule that the same number must be put in each square. Some students may have difficulty verbalizing their suggestions. The following is a sample dialogue.

S: ***10 + 10 = 20.***

T: ***Do you want 10 in each square and also in the triangle?***

S: ***Yes.***

Write 10 inside each frame.

$\boxed{10} + \boxed{10} + 10 = 20$

T: ***10 + 10 + 10 = 20. Is this number sentence true?*** (No)

S: ***I mean 5 in each square and 10 in the triangle.***

$\boxed{5} + \boxed{5} + 10 = 20$

Record 5 and 10 in the table.

If students have difficulty finding correct solutions, suggest a number for the square and ask them to find the number for the triangle. Encourage and help students who are trying to find solutions with negative numbers.

Continue until you have four or five pairs of numbers in the table.

Accept other solutions without recording them in the table. Again, there are an infinite number of solutions to this open sentence.

$\square + \square + \triangle = 20$

$\square$	$\triangle$
8	4
5	10
0	20
$\hat{1}$	22
½	19

Exercise 3

Write this open sentence and draw this chart on the board. Suggest students work with a partner to find many solutions and record them on their papers. You may need to remind students that the same number goes in both squares and the same number goes in both triangles.

$\square + \square + \triangle + \triangle = 20$

$\square$	$\triangle$

After several minutes, collect some of the students' solutions in the table on the board. For example:

$\square + \square + \triangle + \triangle = 20$

$\square$	$\triangle$
5	5
8	2
10	0
$\hat{1}$	11
9	1

T: ***Do you notice a pattern to these pairs of numbers: 5 and 5; 8 and 2; 10 and 0; $\hat{1}$ and 11; 9 and 1?***

S: ***The numbers in each pair have a sum of 10.***

If necessary, mention this yourself.

T: ***If 4 is in the square, what should be in the triangle?*** (6)
If $\hat{2}$ is in the triangle, what should be in the square? (12)

Continue with these or similar questions if your class is interested.

Exercise 4

Repeat Exercise 1 for this open sentence.

$\square + \triangle = 1$

After you have recorded several suggestions in the table, write ½ in a space below the square.

T: ***If ½ is in the square, what number should be in the triangle?*** (½)

Continue until the table has at least five pairs of numbers; for example:

$\square + \square + \triangle + \triangle = 20$

$\square$	$\triangle$
5	5
8	2
10	0
$\hat{1}$	11
9	1

Capsule Lesson Summary

Add the cost of several items on the Minicomputer.

Materials

Teacher
- Minicomputer set
- Colored chalk

Student
- Minicomputer set
- Paper bar
- Worksheets N21 *, **, ***, and ****

Note: Use strips of construction paper for the bar between boards of a desk Minicomputer.

Description of Lesson

Choose students in your class to star in these stories.

Exercise 1

T: ***Garth had a birthday last week and his father took him to a toy store to buy a birthday present. Garth likes models very much, so his father told him he could choose four models as his birthday present.***

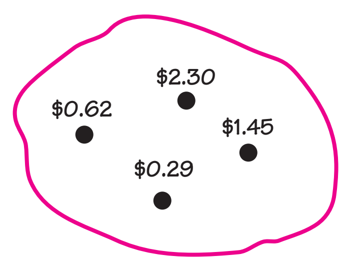
Allow the class to discuss what models are and what kinds of models they have. Ask the student who stars in this story to choose the kinds of models. In the following dialogue, Garth chooses two motorboats, a car, and an airplane. Draw a large string on the board with colored chalk. As you mention the price of each model, draw a dot for it inside the string. For example, draw a dot labeled \$0.62 and say,

T: ***Garth chose a car that costs this much.***
How much does the car cost? (62¢)

Draw another dot labeled \$2.30.

T: ***Garth chose an airplane that costs this much.***
How much does the airplane cost? (\$2.30)

Draw two more dots labeled \$0.29 and \$1.45.



T: ***Garth also chose two different motorboats.***
How much do the motorboats cost? (29¢ and \$1.45)

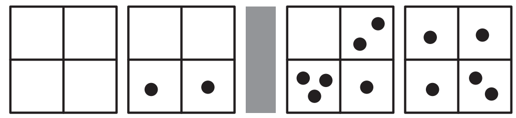
How much must Garth's father pay for the four models?
Can you estimate how much?

Allow students a minute to think; then record several estimates on the board. If the estimates are not particularly good, you may wish to ask the class to agree on a range (for example between \$4.00 and \$6.00) and only accept estimates within this range.

Display four Minicomputer boards. Leave space between the second and third boards in which to draw a bar.

T: ***Let's find the price of the four models on the Minicomputer. Who can put $2.30 on the Minicomputer?***

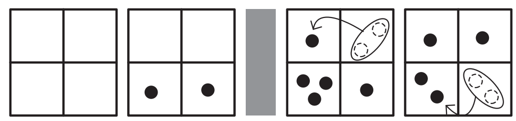
Continue until all four numbers have been put on the Minicomputer. Take advantage of any errors to explain that the dollars go on the boards to the left of the bar and the cents go on the boards to the right of the bar.



T: ***How can we make this number easier to read?***

S: ***Make trades.***

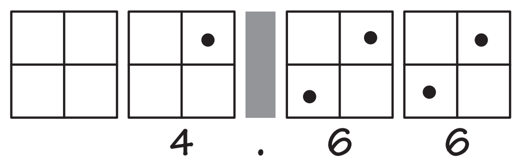
Invite students, one at a time, to make trades. Watch their trades closely because the bar may cause some confusion. You may need to help students verbalize the trades they are making. For example:



S: ***1 penny + 1 penny = 2 pennies.***

S: ***4 dimes + 4 dimes = 8 dimes.***

Continue until you have this configuration on the Minicomputer. Ask someone to write the number below the Minicomputer.



T: ***How much did the four models cost Garth's father?***

S: ***$4.66.***

Exercise 2

Equip pairs of students with two Minicomputer sheets (four boards) and a construction paper bar to put between the sheets. Tell the class another story.

T: ***Latricia's mother gave her two dollar bills to take to the supermarket. Latricia wants to buy a package of coconut and a package of chocolate chips. What do you think she is going to make with the coconut and chocolate chips?***

Let students make a few suggestions and then continue.

T: ***The coconut costs 75¢ and the chocolate chips cost 80¢. Does Latricia have enough money to buy both?***

Write this problem on the board.

$$0.75 + 0.80 = ?$$

S: ***Yes, 75¢ is less than $1 and 80¢ is less than $1.***

T: ***What coins make 75¢?***

S: ***Three quarters.***

S: ***Seven dimes and a nickel.***

S: ***Seventy-five pennies.***

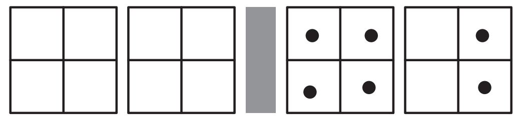
T: ***What coins make 80¢?***

S: ***Eight dimes.***

S: ***Three quarters and one nickel.***

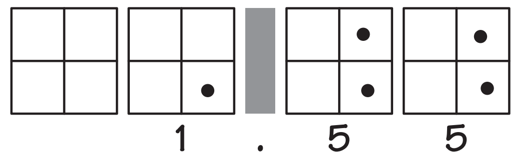
T: ***How much is 75¢ + 80¢?***

Suggest students confer with their partners and write their answer on a piece of paper. Then ask students to put 0.75 and 0.80 on the Minicomputer. Student pairs should follow along on their individual Minicomputers.



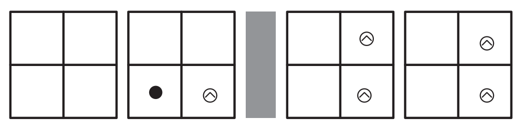
T: ***What number is on the Minicomputer?***

Invite a student to make a trade to get this configuration.
Ask a student to write the number below the Minicomputer.



T: ***How much do the coconut and the chocolate chips cost?*** ($1.55)
Does Latricia have enough money? (Yes)
How much change will Latricia get back from the two dollars?

Suggest students confer with their partners and write their answer on a piece of paper. Then ask how to use the Minicomputer to answer this question. Follow students' suggestions to calculate 2.00 – 1.55. For example, put 2 on the Minicomputer with regular checkers and put $\widehat{1.55}$ on the Minicomputer with negative checkers.



T: ***What should we do now?***

S: ***Make some backward trades.***

Make trades to get pairs of checkers, one positive with one negative on the same square. Remove such pairs when they are formed. Continue until 0.45 is on the Minicomputer with all positive checkers.

Complete a number sentence on the board.

$$2.00 - 1.55 = 0.45$$

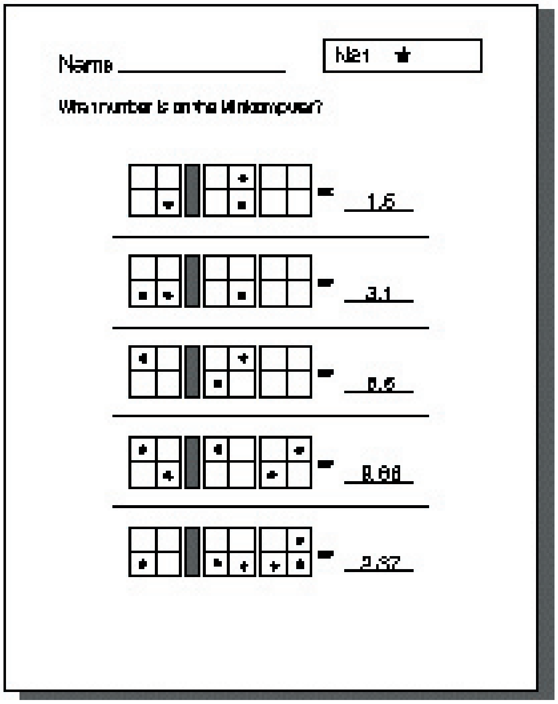
Worksheets N21 *, **, ***, and **** are available for individual work.

Center Activity

Set up a store. Let students "purchase" several items and add the prices to determine the bill. Other students can act as clerks to make change or accept payment.

Writing Activity

Write about going on a shopping trip and buying two items. Tell what each item costs, how much the two items cost together, and what coins and bills can be used to pay for them. Do you receive any change?



Name ____________ N21 ★

What number is on the Minicomputer?

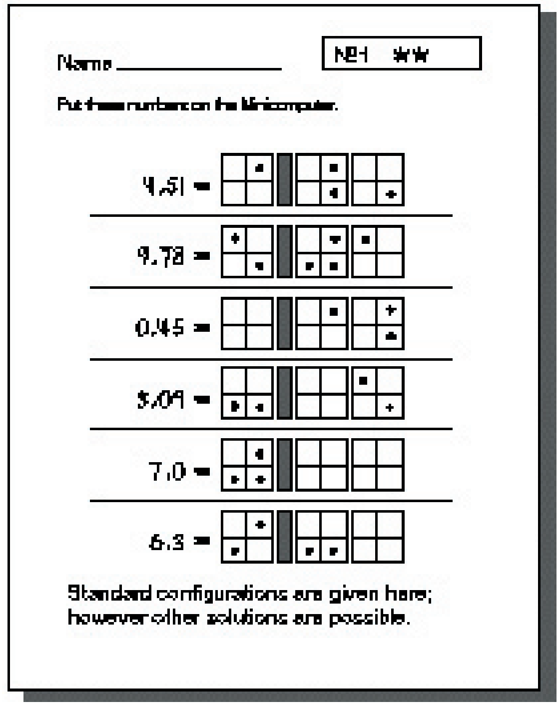
= 1.6

= 3.1

= 8.6

= 9.86

= 2.37



Name ____________ N21 ★★

Put these numbers on the Minicomputer.

4.51 =

9.78 =

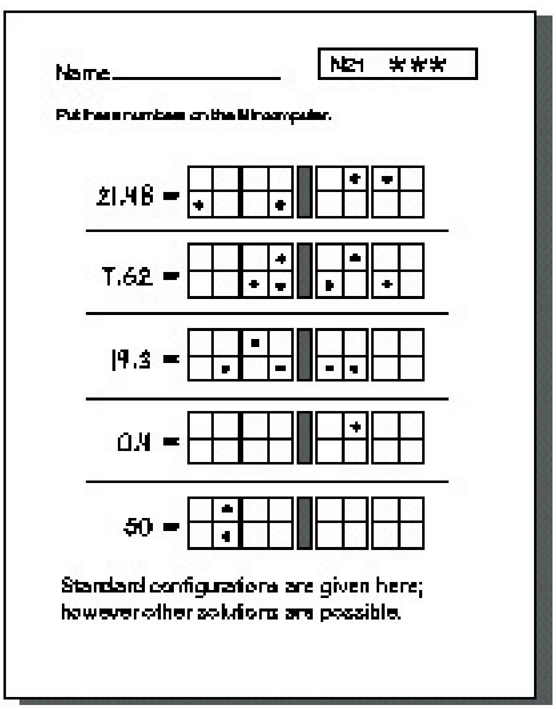
0.45 =

3.09 =

7.0 =

6.3 =

Standard configurations are given here; however other solutions are possible.



Name ____________ N21 ★★★

Put these numbers on the Minicomputer.

21.48 =

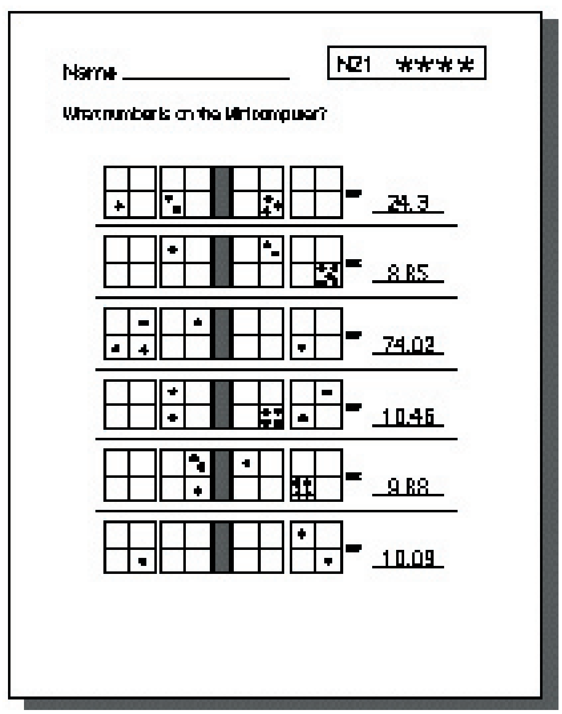
7.62 =

19.3 =

0.4 =

50 =

Standard configurations are given here; however other solutions are possible.



Name ____________ N21 ★★★★

What number is on the Minicomputer?

= 24.3

= 8.85

= 74.02

= 10.46

= 9.88

= 10.09

N22 SUBTRACTION WITH –10 ARROWS

Capsule Lesson Summary

Draw an arrow picture on the board and discover that the effect of four –10 arrows is the same as one –40 arrow. In the same arrow picture, draw –20 and –30 arrows. Build many arrow roads using only –10 arrows. In each road, draw an arrow from the first dot to the last dot and decide what the arrow could be for. Do a similar activity with a road having three –10 arrows and one –3 arrow.

Materials

Teacher
- 0–109 numeral chart
- Colored chalk

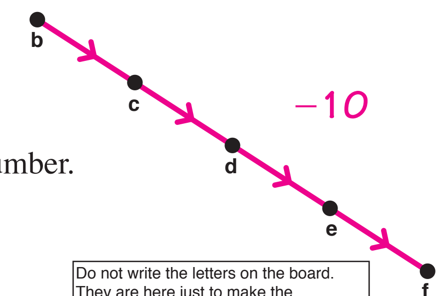
Student
- Unlined paper
- Colored pencils, pens, or crayons

Description of Lesson

Exercise 1

Do some mental arithmetic to practice counting forward by tens, starting with different numbers from 0 to 9. Use the 0–109 numeral chart to observe patterns. After counting forward, count backward by tens starting at a number between 100 and 109.

Draw this arrow picture on the board.



T: ***Point to the greatest number in this arrow picture.***

Students should point to **b** and explain why this is the greatest number.

T: ***Point to the least number in this arrow picture.***

Students should point to **f** and explain why this is the least number.

T: ***The arrows here are for –10, so when we follow the arrows, the numbers decrease.***

Label **b** 75. Point to 75 and trace the first arrow.

T: ***If this number is 75, what number is here*** (at **c**)***?***

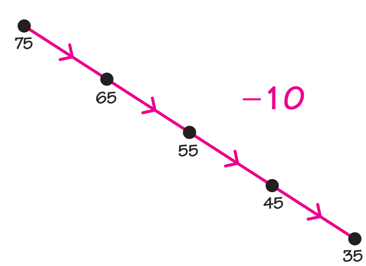
S: ***65.***

Label **c** 65. Trace the arrow starting at 65.

T: ***If this number is 65, what number is here*** (at **d**)***?***

S: ***55.***

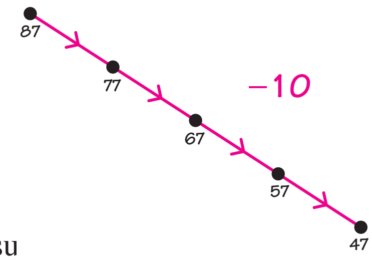
Continue this activity until all the dots are labeled.



N22

Erase all the numbers, put 87 at the beginning of the arrow road, and call on students to label the other dots.

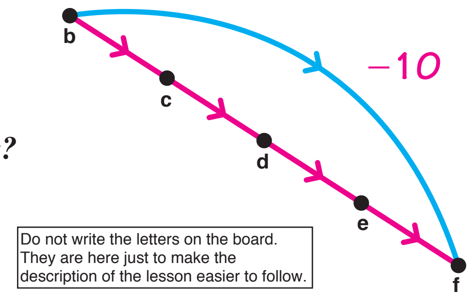
Trace (but do not draw) another arrow starting at 47.



T: ***If we draw another red arrow, which number will we meet next?*** (37)
And if we continue drawing more red arrows ...? (27, 17, 7)

Note: Accept but do not require that students find negative numbers su

Erase the labels for the dots.
Draw a blue arrow from **b** to **f**.



T: ***What could this blue arrow be for? How do you know?***

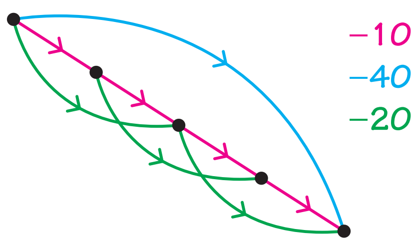
S: ***If could be for – 40, because 4 x 10 = 40.***

Note: The blue arrow could be for other relations such as "is more than." If a student suggests this, reply that the arrow could be for "is more than" but this arrow is for something else.

Write – 40 in blue and –20 in green near the arrow picture.

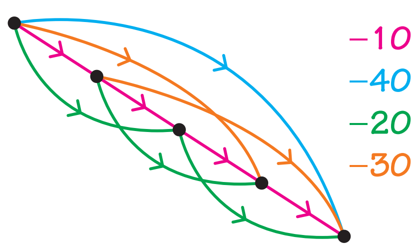
T: ***Where could we draw –20 arrows in this picture without drawing any more dots?***

Let the students first trace and then draw –20 arrows.
Continue until all three –20 arrows are in the picture.



T: ***Where could we draw –30 arrows in this picture without drawing any more dots?***

Write –30 in orange near the arrow picture and invite students to trace –30 arrows. You may wish to draw these arrows yourself if your picture is becoming complex. Continue until your class finds both –30 arrows.



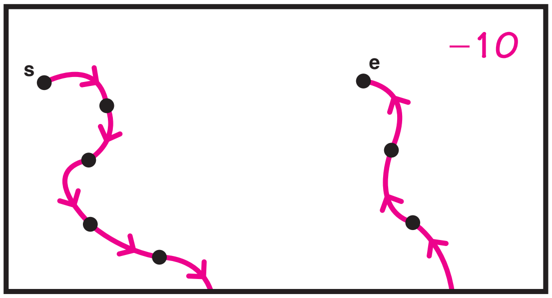
Erase the arrow picture.

Exercise 2

Distribute unlined paper and colored pencils. Write –10 on the board.

T: ***Draw a –10 arrow road with as many arrows as you like on your paper. Work quickly. Do not label the dots.***

Help individual students get started. Then, while the students are working, draw a –10 arrow road that goes partially off the board.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

After all students have drawn at least three arrows, continue the collective lesson.

T: ***On the board I have drawn parts of a –10 arrow road, but you can't see all of it. How many arrows could be in this arrow road?***

The students should conclude that the arrow road on the board has at least eight arrows.

T: ***Count the number of –10 arrows in your arrow road.***

Identify a student who has a road with at least eight arrows. (In the following description, her name is Ellen.)

T: ***Imagine that this is Ellen's arrow road. Ellen, how many arrows does your road have?***

S: ***Nine.***

T: ***How much less is this number*** (point to **e**) ***than this number*** (point to **s**)***?***

Perhaps one of your students will know that the number at **e** is 90 less than the number at **s**.

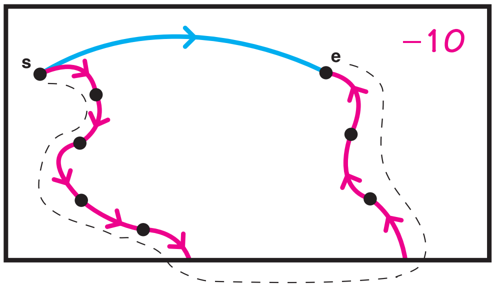
T: ***Each red arrow is for –10. How much will we subtract from the starting number of this road if it has nine red arrows? Let's count by tens.***

Hold up nine fingers for the nine arrows, and point to them as you count by tens.

Note: If the road your class is discussing has more than 10 arrows, use another method of keeping track of how many tens you count.

T: ***Each finger is for a –10 arrow. How much will we subtract with one red arrow?*** (10) ***… with two red arrows?*** (20) ***… with three red arrows?*** (30) ***… (and so on to) … with nine red arrows?*** (90) ***So the ending number of this road is 90 less than the starting number. 9 x 10 = 90.***

Put your left index finger on **s** and trace the arrow road with your right index finger until you reach **e**. The dotted line in the next illustration indicates how to trace the arrow road. Trace and then draw a blue arrow from **s** to **e**.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: ***If the –10 arrow road has nine arrows, what could this blue arrow be for?***

S: ***–90.***

T: ***Who has an arrow road that has a different number of red arrows?***

Choose another student whose road has more than eight arrows. Imagine that the arrow road on the board is for that student's road. Decide with the class what the blue arrow is for in this case.

T: ***On your paper draw a blue arrow from the first dot in your road to the last dot in your road. Decide what your blue arrow is for and label it.***

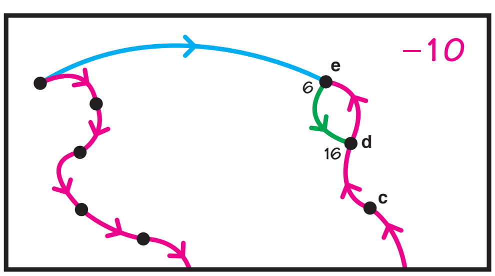
Help students who have drawn blue arrows in the wrong direction. Allow a few minutes for students to draw and label a blue arrow; then suggest they choose a partner and check each other's work.

Label **e** in the arrow road on the board 6. Point to **e** and then to **d** (see next illustration) as you ask,

T: ***If this number is 6, what number is this dot (d) for? How do you know?***

S: ***16, because 6 + 10 = 16 (or because 16 – 10 = 6).***

Label **d** 16, and then draw the return arrow from **e** to **d** in green.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: ***If the red arrows are for –10, what is this green return arrow for?***

S: ***+10.***

Write +10 in green near the arrow picture. Decide with the students that **c** is for 26 and then label it.

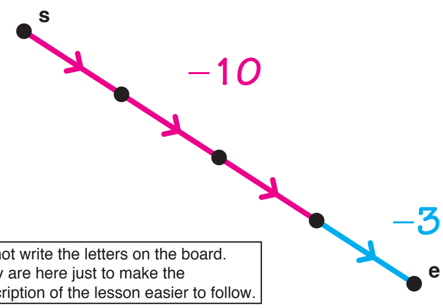
T: ***Label the ending dot in your arrow road 6, and then label all the dots.***

Encourage students who finish quickly to draw green arrows for +10 in their arrow pictures.

Erase the arrow picture on the board before going on to the next exercise.

Exercise 3

Draw this arrow picture on the board.



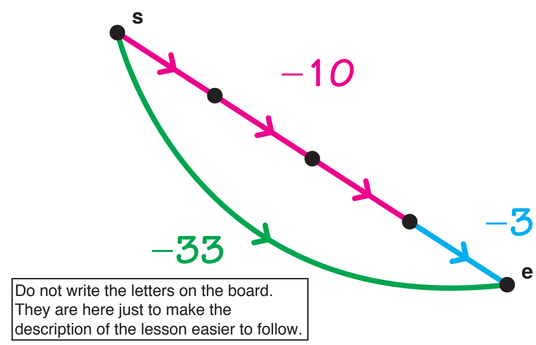
T: ***Point to where the greatest number is in this arrow picture.***

The students should point to **s**.

T: ***Point to where the least number is in this arrow picture.*** (**e**) ***How do you know this number*** (point to **e**) ***is the least?***

S: ***Because the arrows are for –10 and –3.***

Draw a green arrow from **s** to **e**.



T: ***This green arrow goes from the dot for the greatest number to the dot for the least number. What could this green arrow be for? How do you know?***

S: ***–33. Three –10's is –30 and –3 more is –33.***

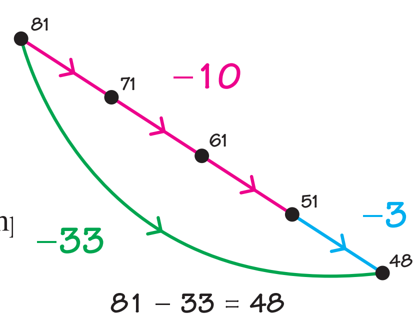
Label the green arrow –33.

T: ***Let's see if we can use this arrow picture to solve some subtraction problems.***

Write this problem on the board and label **s** 81.

81 – 33 = ?

T: ***If this number*** (point to **s**) ***is 81, what is this number*** (point to **e**)***?***



Instruct students to write their answers on their papers or whisper them to a neighbor. Label the dots collectively and com the problem. If several students have difficulty calculating 51 – 3, hold up three fingers and count backward from 51.

In the same manner, calculate 70 – 33 and 52 – 33.

Home Activity

Suggest to parents/guardians that they practice counting forward and backward by tens. They can start with different numbers from 0 to 9 and count forward, or with numbers between 80 and 90 and count backward.

Capsule Lesson Summary

Use the addition algorithm to solve addition problems with two, three, and four digit numbers.

Materials

Teacher
- None

Student
- Paper
- *Addition Problems* Booklet[†]

Description of Lesson

Tell the following story or a similar one to the class. Choose two of your students to be the stars of this story.

T: ***Oscar and Heather volunteered to bake cookies for the school picnic. Oscar baked 437 cookies and Heather baked 283 cookies. Which student prepared more cookies for the picnic?***

S: ***Oscar, 437 is more than 283.***

Write 437 > 283 on the board.

$$437 > 283$$

T: ***How many cookies did they bake altogether for the school picnic?***

S: ***We need to add 437 and 283.***

Write this problem on the board.

$$\begin{array}{r} 437 \\ +\ 283 \\ \hline \end{array}$$

T: ***First, let's add the numbers in the ones column.***

Ask a volunteer to do the addition at the board and to explain what is being done.

Note: The dialogue here assumes the student is using the usual addition algorithm; however, you should accept other addition explanations and let students use methods that work best for them.

S: ***7 + 3 = 10. Write 1 above the tens column and 0 below the ones column.***

$$\begin{array}{r} {\scriptstyle 1\ } \\ 437 \\ +\ 283 \\ \hline 0 \end{array}$$

T: ***Seven ones plus three ones is ten ones, or 10. 10 is one ten and 0 ones. Next, let's add the numbers in the tens column.***

Call on a volunteer to do the addition at the board and to explain.

$$\begin{array}{r} {\scriptstyle 1\,1\ } \\ 437 \\ +\ 283 \\ \hline 20 \end{array}$$

S: ***1 + 3 + 8 = 12. Write 1 above the hundreds column and 2 below the tens column***

T: ***One ten plus three tens plus eight tens is twelve tens, or 120.***

[†]The *Addition Problems* Booklet was used earlier in Lesson N5 *Addition Algorithm #1*. The answer key for this booklet follows that lesson.

120 is one hundred (100) and two tens (20). Next, let's add the numbers in the hundreds column.

Call on a volunteer to do the addition at the board and to explain.

S: ***1 + 4 + 2 = 7. Write 7 below the hundreds column.***

T: ***One hundred plus four hundreds plus two hundreds is seven hundreds.***

$$\begin{array}{r} {}^{1\,1} \\ 437 \\ +\ 283 \\ \hline 720 \end{array}$$

Conclude that 437 + 283 = 720. Proceed in a similar way to solve these problems.

$$\begin{array}{r} 724 \\ +\ 876 \\ \hline \end{array} \qquad \begin{array}{r} 625 \\ 236 \\ +\ 137 \\ \hline \end{array}$$

Write the following problems on the board and ask the students to solve them on their papers. Allow a few minutes for individual work, and then solve the problems collectively at the board.

$$\begin{array}{r} 217 \\ 353 \\ +\ 224 \\ \hline \end{array} \qquad \begin{array}{r} 75 \\ 142 \\ +\ 36 \\ \hline \end{array} \qquad \begin{array}{r} 3{,}156 \\ 2{,}326 \\ +\ 2{,}406 \\ \hline \end{array}$$

Distribute the *Addition Problems* Booklet and instruct students to continue working independently for 10 to 15 minutes. Suggest students correct mistakes from earlier work before going on to a new page. At the end of the class period, collect the booklets for your review. (This is the last lesson using this booklet.)

Note: If many of your student have completed the *Addition Problems* Booklet, you may like them to write their own addition problems and perhaps exchange with another student to do the calculations.

Assessment Activity

Use the *Addition Problems* Booklet or some problems of your choice to assess student progress in addition.

Capsule Lesson Summary

Multiply a number by 10 on the Minicomputer by moving each checker in the configuration from one square to the square of the same color on the next board to the left. Use this idea to help label the dots in 10x arrow roads. Build roads with exactly one 10x arrow and three +10 arrows. Notice how the order in which the arrows appear affects the ending number of the road. Decide which road has the greatest ending number and which has the least.

Materials

Teacher	• Minicomputer set • Colored chalk	**Student**	• Unlined paper • Colored pencils, pens, or crayons

Description of Lesson

Exercise 1

Begin the lesson with mental arithmetic involving 10x on the Minicomputer. Display four Minicomputer boards and put a checker on the 2-square.

T: ***What number is on the Minicomputer?*** (2)
What number is 10 x 2? (20)

Move the checker from the 2-square to the 20-square as you say, “10 x 2 = 20. ”

T: ***What number is 10 x 20?*** (200)

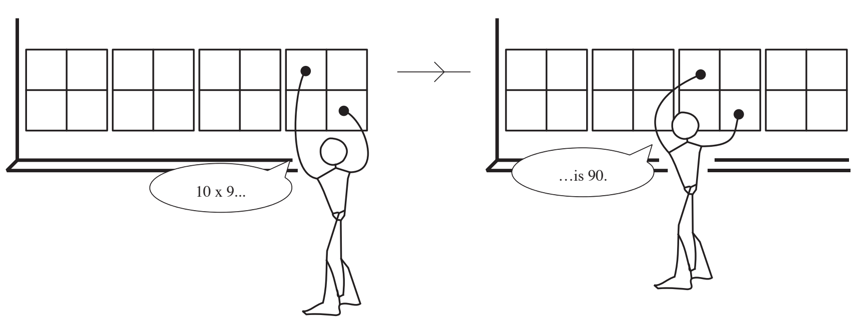
Move the checker from the 20-square to the 200-square as you say, “10 x 20 = 200.”

T: ***What number is 10 x 200?*** (2,000)

Move the checker from the 200-square to the 2,000-square as you say, “10 x 200 = 2,000.”

Repeat this activity for 9, 10 x 9 = 90, 10 x 90 = 900, and 10 x 900 = 9,000.

Move the checkers simultaneously as you announce the result of 10x. See the illustration below.



Write 10 x 7 = 70 on the board.

T: ***What number is 10 x 70?*** (700)

10 x 7 = 70
10 x 70 = 700
10 x 700 = 7,000

Write 10 x 70 = 700 on the board.

T: ***What number is 10 x 700?*** (7,000)

Write 10 x 700 = 7,000 on the board.

Ask the students if they notice any patterns in this list of number sentences, and ask for a next number sentence to put in the list.

Exercise 2

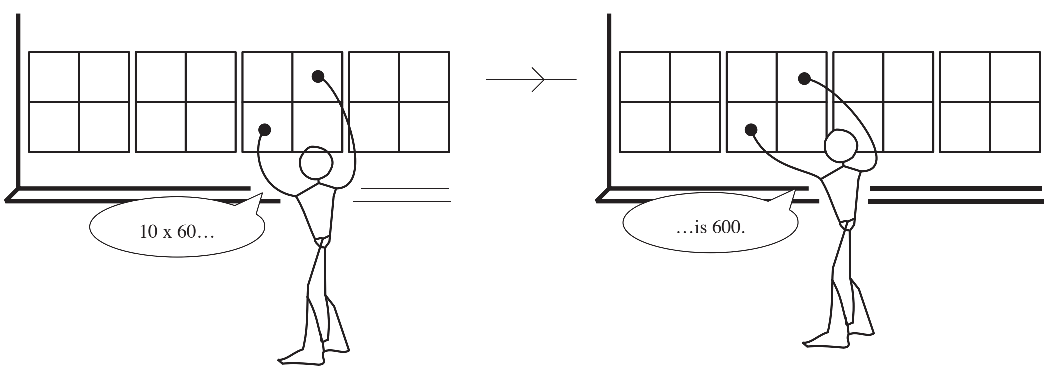
Ask a student to put 6 on the Minicomputer.

T: ***What number is 10 x 6?*** (60)
Who can move these checkers to put 10 x 6 on the Minicomputer?

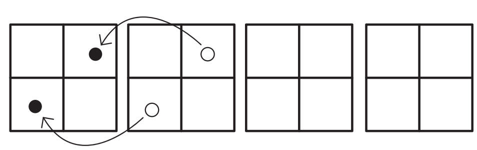
The volunteer should move the checkers from the ones board to the tens board. Repeat the move yourself, moving the checkers simultaneously.

T: ***What number is on the Minicomputer now?*** (60)
Who can move these checkers to put 10 x 60 on the Minicomputer?

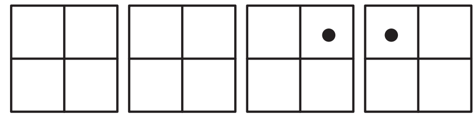
The volunteer should move the checkers from the tens board to the hundreds board. Repeat the move yourself, moving the checkers simultaneously.



T: ***What number is 10 x 600?*** (6,000)
Who can move these checkers to put 10 x 600 on the Minicomputer?



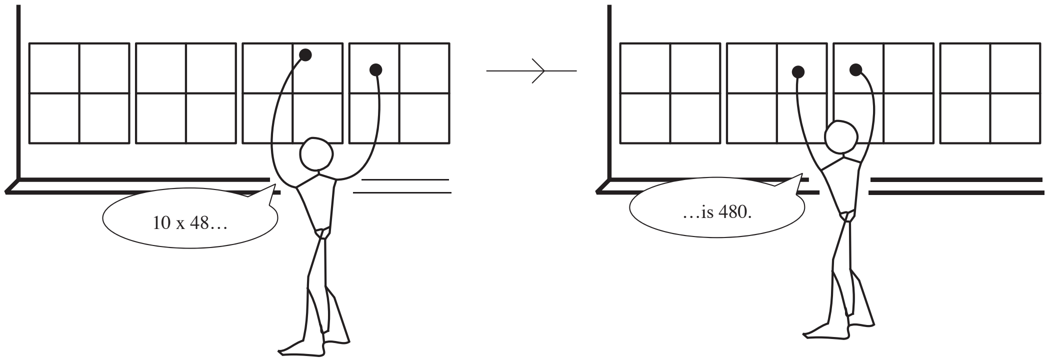
Ask a student to put 48 on the Minicomputer. If necessary, make trades until the standard configuration for 48 is obtained.



T: ***How could we move these checkers to put 10 x 48 on the Minicomputer?***

S: ***Move the checker from the 40-square to the 400-square and the checker from the 8-square to the 80-square.***

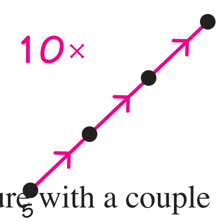
Demonstrate this move yourself.



Ask a student to move the checkers to get 10 x 480 on the Minicomputer. Conclude that 10 x 480 = 4,800.

Exercise 3

Draw this a~~rrow picture on the~~ board leaving space to extend it later. Instruct the students to copy the picture and label the other dots.

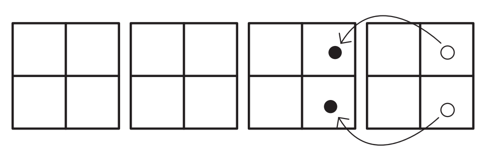


Encourage students who finish quickly to extend the arrow picture with a couple more 10x arrows. Allow enough time for all students to at least copy the arrow picture. Trace the arrow starting at 5.

T: ***What are the red arrows for?*** (10x)
What number is 10 x 5? (50)

Ask a student to put 5 on the Minicomputer.

T: ***Who can move these checkers to put 10 x 5 on the Minicomputer.***



T (tracing the arrow starting at 50)**:** ***10 x 50 = …?*** (500)
Who can move the checker so that 10 x 50 is on the Minicomputer?

Continue this activity until all the dots are labeled.

T: ***Do you see a pattern?***

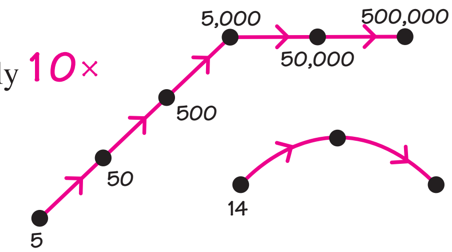
S: ***Each number has one more zero at the end of it than the number before it.***

S: ***Each time we multiply by ten, 5 moves to the next place (left).***

T: ***If we drew another red arrow, what would the next number be?*** (50,000)
And after 50,000…? (500,000)

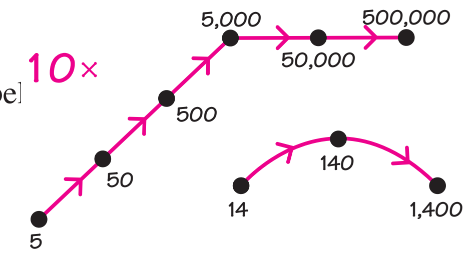
Draw another section in your 10x arrow picture.

Continue as before. Ask the students to work individually then label the dots collectively.

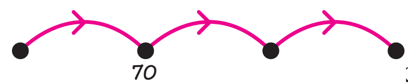


Draw another section in your 10x arrow picture.

Ask the students to work individually. When several students have labeled all the dots, ask a student to label the dots on the board. Ask again if they see a pattern.

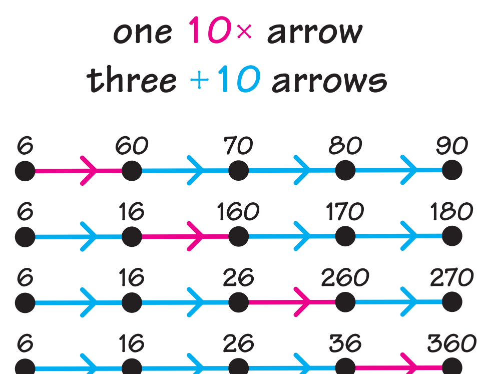


Exercise 4



Direct students to use a clean piece of paper for this ex d and +10 in blue at the top of their papers. Then instruct them to draw arrow roads with four arrows, one red (10x) arrow and three blue (+10) arrows.

Write the directions on the board. As a student finishes one such arrow road, ask the student to draw another arrow road following the same directions. When most of the students have drawn at least one arrow road, instruct students to label the starting dot of each of their arrow roads 6 and then to label the other dots.



Allow enough time so that each student can label all the dots on at least one arrow road. There are four possible arrow roads.

Suppose a student named Chang drew two different arrow roads.

T: ***Chang, were the ending numbers of your roads the same?***

S: ***No.***

T: ***Tell us one of the ending numbers.***

S: ***180.***

T: ***Does anyone else have a road that ends at 180?***

S: ***Yes.***

T: ***Chang, describe the arrow road that ends at 180.***

S: ***A +10 arrow; then a 10x arrow; then a +10; and then another +10.***

Students may comment that their arrow roads are like Chang's.

T: ***Does anyone have a road with an ending number greater than 180?***

S: ***I have a road that ends at 270.***

S: ***I have one that ends at 360.***

T: ***Describe the road ending at 360.***

S: ***+10; +10; +10; 10x.***

T: ***Can we build a road with one 10x arrow and three +10 arrows that ends at a number greater than 360?*** (No)

Let the students discuss the situation freely. Perhaps someone will comment that there are only four places in the road to put the 10x arrow and putting it at the end gives the greatest ending number.

T: ***Does anyone have a road with an ending number less than 180?***

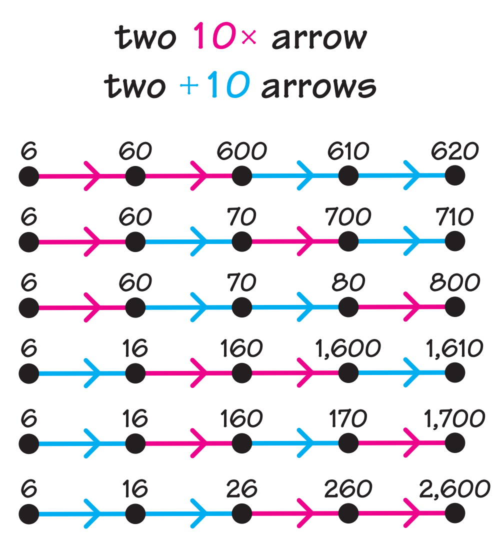
S: ***I have one that ends at 90. The arrows are 10x; +10; +10; +10.***

T: ***Can we build a road with one 10x arrow and three +10 arrows that ends at a number less than 90?*** (No)

Again let the students discuss the situation freely. Perhaps someone will comment that putting the red arrow first gives the least ending number.

Extension Activity

Repeat Exercise 4 for arrow roads with two 10x arrows and two +10 arrows. In this case, there are six possible arrow roads.



Capsule Lesson Summary

Draw +9 arrows in arrow roads with alternating +10 and –1 arrows.

Materials

Teacher
- Colored chalk
- 0–109 numeral chart

Student
- Worksheet N25
- Paper
- Colored pencils, pens, or crayons

Description of Lesson

Exercise 1

Begin the lesson with a short mental arithmetic activity involving the functions +10, –10, +20, –20. A sample dialogue is given here. Use the 0–109 numeral chart for visual reinforcement.

T: ***I will choose a number. You tell me what number is 10 more than my number.***
6. (16)
20. (30)
23. (33)
⋮

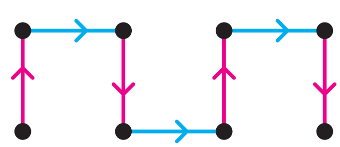
T: ***Now I am going to change the rule. You tell me what number is 10 less than my number.***
27. (17)
10. (0)
9. (1)
⋮

T: ***I am going to change the rule again. You tell me what number is 20 more than my number.***
5. (25)
10. (30)
12. (32)
⋮

T: ***Now tell me what number is 20 less than my number.***
24. (4)
50. (30)
51. (31)
⋮

Exercise 2

Copy the arrow picture from Worksheet N25 on the board, and ask students what they notice about this arrow picture.



N25

S: ***It goes red, blue, red, blue, red, blue.***

S: ***All the blue arrows are going across.***

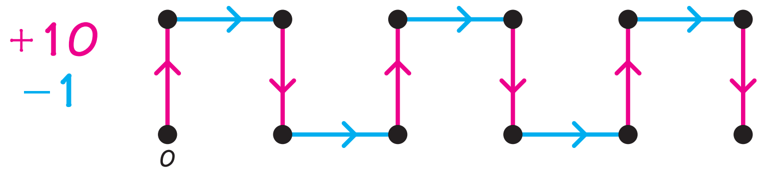
S: ***All the red arrows are going up and down.***

Direct students to extend the arrow picture on their worksheets, following the alternating color pattern. As they work, extend the arrow picture on the board. It is not necessary that students copy your layout as long as they are careful to alternate the colors. Encourage students to continue drawing arrows until everyone has a road with at least nine arrows. Some students may like to tape another piece of paper to one side of the worksheet.

Ask students to record the following keys on their papers as you do the same on the board.

T: ***The red arrows are for +10, and the blue arrows are for –1.***
The starting dot is 0.

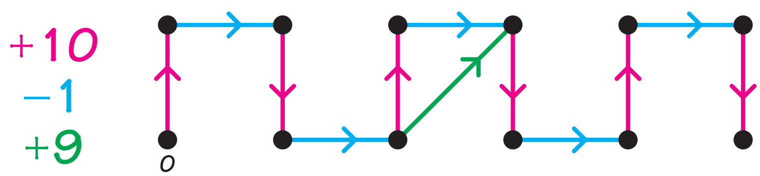
Your picture might look like this one.



Instruct students to start at 0 and follow the arrows to label the rest of the dots on their worksheets. Observe students' work and help those having difficulty. Encourage students who finish quickly to extend their arrow pictures. Continue until everyone has labeled at least the first eight dots.

T: ***We are going to draw some green arrows in our arrow picture.***
Green arrows will be for +9.

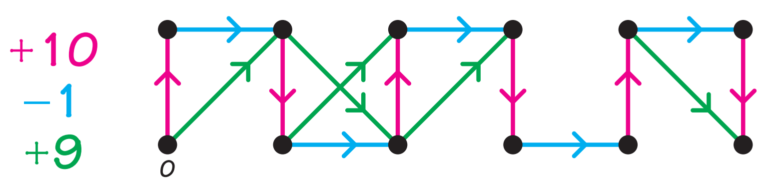
Write +9 in green near the picture. Ask someone to draw a +9 arrow on the board. Your picture might look like this one.



T: ***Look at your arrow picture and draw all the +9 arrows you can.***

As students finish, ask them to draw some green arrows in the arrow picture on the board. It is not necessary that all the possible green arrows be drawn.

For example your picture might look like this one.



T: ***How did we know where to draw green arrows in this picture when the dots were not labeled?***

Commend students who notice that a +10 arrow followed by a –1 arrow (or a –1 arrow followed by a +10 arrow) is the same as a +9 arrow.

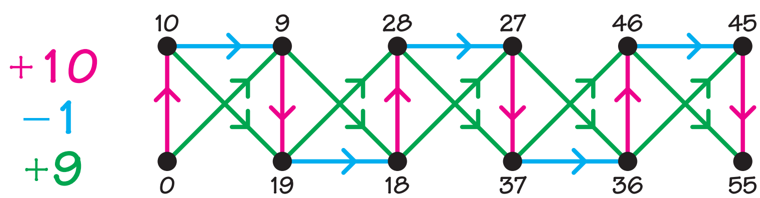
Point to the starting dot of a +10 arrow with your left index finger. Trace the +10 arrow and then the following –1 arrow with your right index finger as you say,

T: ***+10 followed by –1 is the same as +9.***

With your left index finger still on the same dot, trace the appropriate green (+9) arrow with your right index finger.

Repeat this procedure for a –1 arrow followed by a +10 arrow, noting that –1 followed by +10 is the same as +9.

Let students draw more green arrows in their pictures for a few minutes. A completed picture is shown here.



End the lesson with some mental arithmetic involving +10 and +9. Use the following or similar problems appropriate for the abilities of your students.

T: ***What number is 43 + 10?*** (53)
…43 + 9? (52)
…65 + 10? (75)
…65 + 9? (74)
…78 + 10? (88)
…78 + 9? (87)

Home Activity

Ask students to make another copy of their +10, –1 arrow road, but with unlabeled dots. Suggest students work with a family member at home to label the dots, starting with a different number (1, 3, 5, or 7), and to add green arrows for +9.

Capsule Lesson Summary

Decide how Farmer McDonald can evenly share his animals among his three children.

Materials

Teacher
- Colored chalk
- Minicomputer set

Student
- Counters
- Paper
- Minicomputer set

Description of Lesson

Group students in threes, and provide each group with about twenty counters. Tell the following or a similar story to the class. As you do, suggest groups act out the story, using counters to represent the animals and the three group members to be the three children.

T: ***Farmer McDonald has decided to retire and take a long trip around the world. He has decided to give all the animals on his farm to his three children. He wants to give the same number of horses to each child. He also wants each of his children to receive exactly the same number of dogs, cats, chickens, pigs, and so on.***

Farmer McDonald has six dogs. He wants to give one-third of the dogs to each of his children. How many dogs will each child receive? (Two) ***How do you know?***

Let groups model the sharing and then ask for an explanation. Emphasize that $3 \times 2 = 6$, and $\frac{1}{3} \times 6 = 2$ or $6 \div 3 = 2$.

T: ***Farmer McDonald has nine cats. How many cats will he give to each child?*** (Three) ***How do you know?***

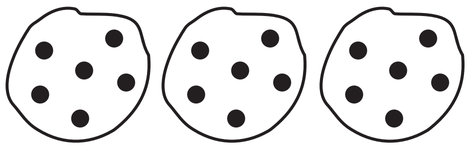
Again, let groups model the sharing and then ask for an explanation. Emphasize that $3 \times 3 = 9$, and $\frac{1}{3} \times 9 = 3$ or $9 \div 3 = 3$.

T: ***Farmer McDonald has eighteen horses. Again, he wants to share the horses equally among his three children. Show on your paper how you decide how many horses Farmer McDonald should give to each child. Either draw a picture or write a number fact to show how you solved this problem.***

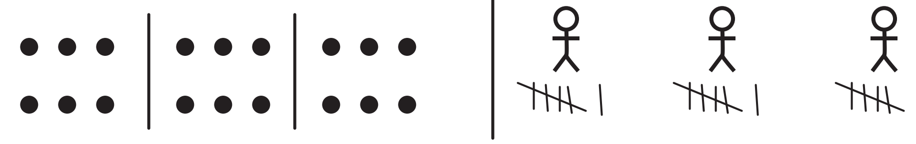
You may need to explain to groups having difficulty getting started that one method of sharing horses is to give one horse to each child, and then to give another horse to each child, and so on until all eighteen horses have been given away. Then draw three strings on the group's paper and indicate that eighteen counters can be put in the strings, one at a time, using that same method.

When all groups know how many horses each of Farmer McDonald's children will receive, discuss a few of their pictures or their number facts with the class.

Some possible pictures and facts are shown below. Conclude that each child will receive six horses.



$$6 + 6 + 6 = 18$$
$$3 \times 6 = 18$$
$$\tfrac{1}{3} \times 18 = 6$$
$$18 \div 3 = 6$$



T: ***Farmer McDonald has 36 pigs. How many pigs will he give to each of his children? Write the answer on your paper.***

Allow the groups a few minutes to work on this problem. This time they may not have sufficient counters to model, but they can draw pictures.

T: ***Will each of Farmer McDonald's children get more or less than ten pigs?*** (More) ***How do you know?***

S: ***3 x 10 = 30; so if each child got ten pigs, there would still be six pigs left to share.***

T: ***3 x 10 = 30, so ⅓ x 30 = …?*** (10)

$$\tfrac{1}{3} \text{ x } 30 = 10 \quad \text{or} \quad 30 \div 3 = 10$$

T: ***There would be six more pigs left to share. How many more pigs should each child receive?***

S: ***Two.***

T: ***3 x 2 = 6, so ⅓ x 6 = 2.***

$$\tfrac{1}{3} \text{ x } 6 = 2 \quad \text{or} \quad 6 \div 3 = 2$$

If ⅓ x 30 = 10 and ⅓ x 6 = 2, what number is ⅓ x 36?

$$\tfrac{1}{3} \text{ x } 36 = 12 \quad \text{or} \quad 36 \div 3 = 12$$

S: ***12.***

T: ***How do you know?***

S: ***I just added 10 + 2.***

T: ***⅓ x 30 = 10 and ⅓ x 6 = 2, so ⅓ x 36 = 12.***

Complete the calculations on the board and conclude that each of Farmer McDonald's children will receive twelve pigs.

T: ***Farmer McDonald also has 45 cows. How should he share 45 cows among his three children? Write your answer on your paper.***

Allow the groups a few minutes to work on this problem.

T: ***Would each of Farmer McDonald's children get more or less than ten cows?*** (More) ***Would each of them get more or less than twenty cows?*** (Less) ***How do you know?***

S: ***2 x 20 = 40, so 3 x 20 = 60. Farmer McDonald only has 45 cows.***

T: ***Each child will get between ten and twenty cows.***

Display three Minicomputer boards and ask a student to put 45 on the Minicomputer.

T: ***Do you remember how to share two ways (calculate one-half of a number) on the Minicomputer?***

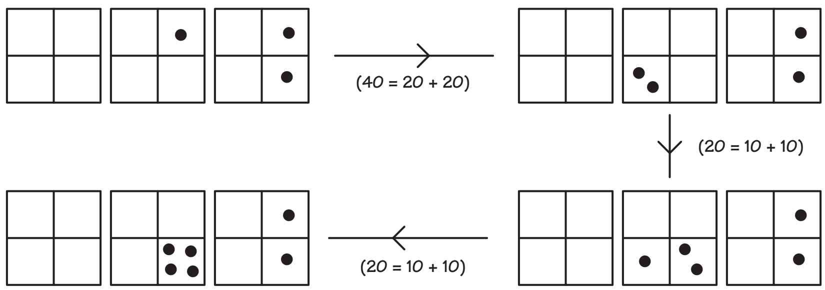
S: ***We make trades until all the checkers are in pairs.***

T: ***How could we share three ways (calculate one-third of a number) on the Minicomputer?***

S: ***We could make trades until all the checkers are in groups of three.***

T: ***Let's make trades to do this.***

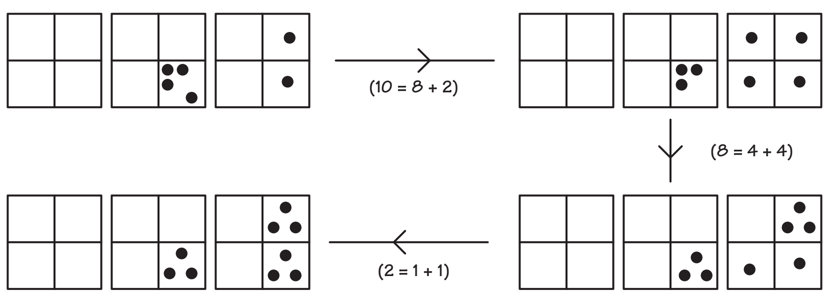
A possible sequence of trades and dialogue is given here.



T: ***Now we can form a group of three checkers on the same square. We do not need to make anymore trades with these checkers.***

Push three checkers into one corner of the 10-square.

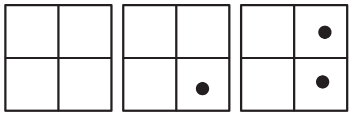
T: ***What trade can we make with the extra checker on the 10-square?***



T: ***Now all the checkers are in groups of three. Who can remove some checkers so that one-third of this number will be on the Minicomputer?***

The volunteer should leave one checker of each group of three on the Minicomputer. Depending upon which configuration is on the Minicomputer, it may or may not be necessary to make some trades to obtain the standard configuration for 15.

Write ⅓ x 45 = 15 to one side of the Minicomputer.



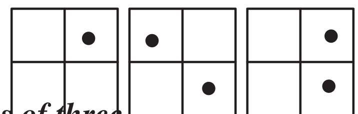
$\frac{1}{3} \times 45 = 15$ or $45 \div 3 = 15$

Conclude that each of Farmer McDonald's children will receive fifteen cows.

T: ***Farmer McDonald also has 495 turkeys. How should he share 495 turkeys among his three children? Will each child receive more or less than 100 turkeys?*** (More) ***… more or less than 200 turkeys?*** (Less) ***Each of the children will receive more than 100 turkeys but less than 200 turkeys. How many turkeys will each child receive? What number is ⅓ x 495?***

Provide desk Minicomputers to groups and allow them to work on this problem.

Ask a student to put 495 on the demonstration Minicomputer.

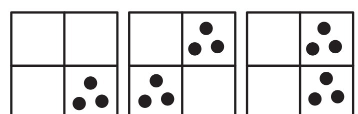


T: ***How can we calculate one-third of this number?***

S: ***Make backward trades until all the checkers are in groups of three.***

Invite students to make trades on the Minicomputer. During the activity, feel free to suggest a trade the class might be overlooking. Whenever there is a group of three checkers on the same square, push them into a corner of that square. Discourage students from making a trade with the checkers already in groups of three, but otherwise do not stop students from making inefficient trades.

Continue until all the checkers are in groups of three and the checkers in each group are on the same square. Perhaps you will have this configuration on the Minicomputer.



Ask someone to remove checkers so that one-third of this number is on the Minicomputer. Conclude that ⅓ x 495 = 165, so each child will receive 165 turkeys.

You may wish to discuss with your class some other animals Farmer McDonald might have on his farm.

At the end of the lesson, you may like to collectively recall how many the various kinds of animals each of Farmer McDonald's children will receive. Then ask the groups to calculate how many animals each child receives altogether. (203)

2 dogs
3 cats
6 horses
12 pigs
15 cows
165 turkeys

Writing Activity

Ask students to write about how Farmer McDonald would share 63 goats among his three children.

N27 COMMMON MULTIPLES #1

Capsule Lesson Summary

On the number line, find non-negative common multiples of 3 and 5 by making consecutive +3 jumps starting at 0 and consecutive +5 jumps starting at 0. Note common landing points.

Materials

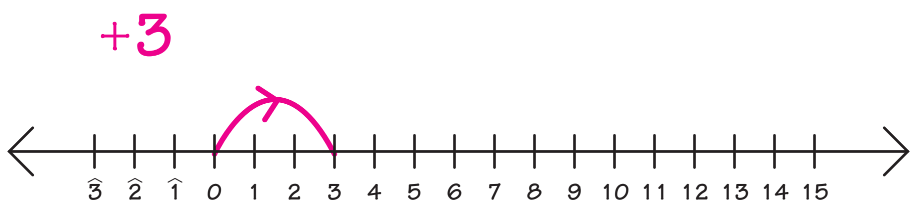
Teacher
- Colored chalk
- Overhead calculator (optional)
- 0–109 numeral chart
- Blackline N27

Student
- Number line sheet
- Colored pencils, pens, or crayons
- Calculator (optional)

Advance Preparation: Use Blackline N27 to make number line sheets for students.

Description of Lesson

Draw this picture on the board.



T: ***If we start at 0 and make +3 jumps, what are some of the numbers we land on?***

Encourage students to suggest several numbers, including numbers greater than 15, but do not draw any more arrows on the board. You may need to remind the students that the number line goes on and on in both directions.

S: ***We would land on 6.***

T: ***How many +3 jumps would we make to go from 0 to 6?***

S: ***Two.***

T: ***2 x 3 = 6. What are some other numbers we would land on?***

S: ***We would land on 15.***

T: ***How many +3 jumps would we make to go from 0 to 15?***

S: ***Five.***

T: ***5 x 3 = 15, so we would land on 15. What is a number greater than 15 that we would land on?***

S: ***36.***

T: ***If we continue making +3 jumps, would we land on 36?***

The class may be uncertain or disagree.

T: ***How many +3 jumps would we make to go from 0 to 30?***

S: ***Ten.***

T: ***10 x 3 = 30. How many +3 jumps would we make to go from 30 to 36?***

S: ***Two; 30 + 3 = 33 and 33 + 3 = 36.***

T: ***How many +3 jumps would we make to go from 0 to 36?***

S: ***Twelve.***

T: ***12 x 3 = 36. What kind of numbers do we land on if we start at 0 and make +3 jumps?***

S: ***Multiples of 3.***

Continue asking for multiples of 3. You may also like to use the overhead calculator to view multiples of 3. That is, start at 0 and press $\boxed{+}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$.... Observe that counting the number of times you press $\boxed{=}$ is the same as counting the number of +3 jumps. The 0–109 numeral chart may also reinforce a pattern of counting by threes.

Note: You can determine easily if a number is a multiple of 3 by adding its digits. If the sum of the digits is a multiple of 3, then the number is also a multiple of 3. For example, you can quickly determine that 72 is a multiple of 3 by adding 7 + 2 = 9 and 9 is a multiple of 3. By contrast, 200 is not a multiple of 3 because 2 + 0 + 0 = 2, and 2 is not a multiple of 3. This test is for your information only.

T: ***Is 100 a multiple of 3? How can we check?***

S: ***No, we skip 100. We land on 90 but then 93, 96, 99, 102.***

T: ***Are there any negative numbers that are multiples of 3?*** (Yes)

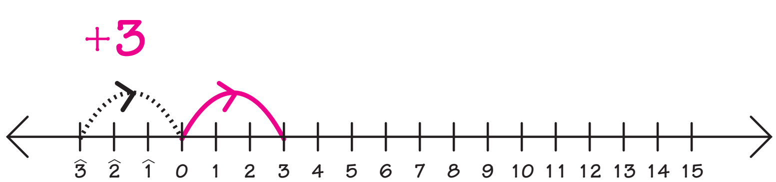
S: ***$\hat{3}$.***

T: ***How many +3 jumps would we make to go from $\hat{3}$ to 0?***

S: ***One.***

T: ***$\hat{3} + 3 = 0$.***

Trace a +3 arrow from $\hat{3}$ to 0 on the number line.

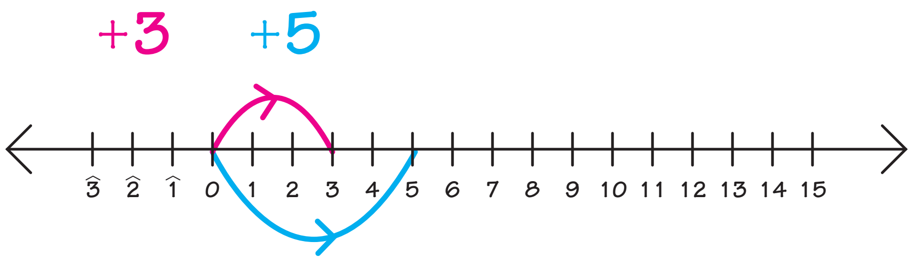


T: ***Are there other negative numbers that are multiples of 3?***

Encourage the students to suggest several negative numbers that are multiples of 3.

Note: With the calculator, you can view negative multiples of 3 by going backward; that is, by pressing $\boxed{-}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ ... .

Write +5 on the board and draw a +5 arrow from 0 to 5.



T: ***If we start at 0 and make +5 jumps, what are some numbers we land on?***

Encourage students to make many suggestions as they did for the +3 jumps. After they suggest several multiples of 5, ask students if they notice any patterns. Most likely they will observe that multiples of 5 end with 0 or 5. You need not spend as much time soliciting multiples of 5 as you did multiples of 3. You may, however, like to remind students how to use the calculator to view multiples of 5. To do so, you start at 0 and press $\boxed{+}$ $\boxed{5}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ …. Again, the 0–109 numeral chart may also reinforce a pattern of counting by fives.

Hold up a copy of the number line sheet (Blackline N27) as you say,

T: ***We can use this number line to show some of the numbers that are multiples of 5. We will start at 0 and make +5 jumps with arrows. Write +5 in blue above your number line.***

Point to 0 on the number line on the board, and trace the +5 arrow starting at 0.

T: ***If we start at 0 and make a +5 jump, where do we land?***

S: ***On 5.***

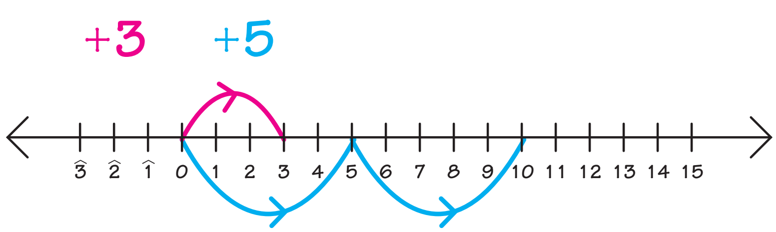
T: ***Draw a blue arrow from 0 to 5 on your number line.***
Is 5 a multiple of 5?

S: ***Yes.***

T: ***Start at 5 and draw another +5 arrow on your number line.***
5 + 5 = …?

S: ***10.***

Draw a blue arrow from 5 to 10 on the board.



T: ***Is 10 a multiple of 5?***

S: ***Yes.***

Ask students to continue drawing consecutive +5 arrows on their number lines. Indicate that when they reach 15 on the top portion of the number line, they can start another +5 arrow at 15 on the middle portion of the number line. Allow the students to work independently until several students reach 45. You may need to help some students who have difficulty drawing +5 arrows. Provide new copies of the number line sheet to students who need to start over.

T: ***What are some of the numbers that are multiples of 5?***

The class should observe that all the numbers you landed on with the +5 arrows are multiples of 5.

T: ***We are going to use this same number line to show some of the numbers that are multiples of 3. We start at 0 and make +3 jumps with arrows. Write +3 in red above your number line.***

Point to 0 on the number line on the board.

T: ***If we start at 0 and make a +3 jump, where will we land?***

S: ***On 3.***

T: ***Draw a red arrow from 0 to 3 on your number line.***

Trace this arrow on the board.

T: ***Is 3 a multiple of 3?***

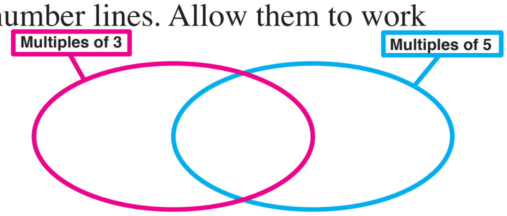
S: ***Yes.***

T: ***If we start at 3 and draw another +3 arrow, where will we land?***

S: ***On 6.***

Instruct students to continue drawing +3 arrows on their number lines. Allow them to work independently until several students reach 45.

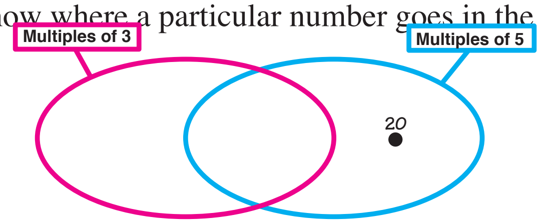
Draw this string picture on the board.



T: ***Let's put some numbers in this string picture.***

Invite students to put numbers in the string picture. Numbers should be put in all four regions of the picture. Frequently ask students to explain how they know where a particular number goes in the string picture. A possible dialogue is given here.

A student puts 20 in the string picture …

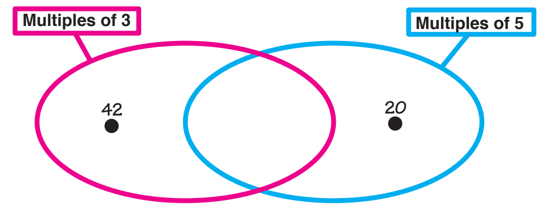


T: ***How do you know that 20 is a multiple of 5 but not a multiple of 3?***

S: ***A +5 arrow ends at 20 but none of the +3 arrows ends at 20.***

T: ***We know that 20 is a multiple of 5, because it is on the +5 arrow road that starts at 0. We also know that 20 is not a multiple of 3, because it is not on the +3 arrow road that starts at 0.***

A student puts 42 in the string picture…



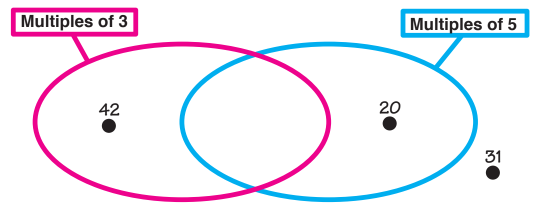
T: ***How do you know that 42 is a multiple of 3 but not a multiple of 5?***

S: ***42 is on the +3 arrow road but not on the +5 arrow road.***

S: ***42 is not a multiple of 5 because it ends in 2.***

T: ***Are there any numbers that go outside both strings?***

A student puts 31 in the string picture…



T: ***Are you sure that 31 is not a multiple of 3?***

S: ***We know 30 is a multiple of 3. 30 + 3 = 33, so 31 can not be a multiple of 3.***

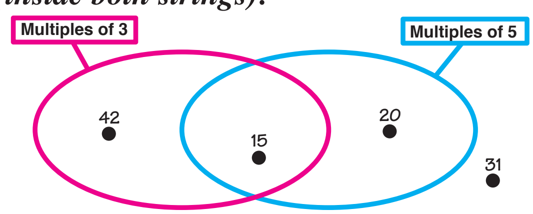
S: ***31 is not on the +3 arrow road.***

T: ***How do you know 31 is not a multiple of 5?***

S: ***It does not end in 5 or 0.***

T: ***Are there any numbers that will be in the middle (inside both strings)?***

A student puts 15 in the string picture…



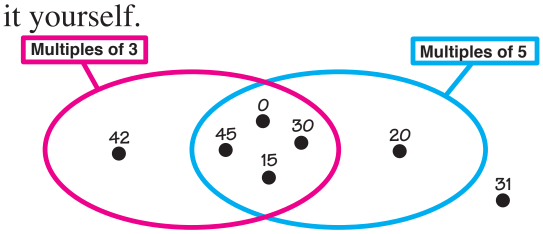
T: ***Is 15 a multiple of 3 and also a multiple of 5? How do you know?***

S: ***Yes, 15 is on both arrow roads.***

T: ***How many +3 jumps do we make to go from 0 to 15?*** (Five) ***5 x 3 = 15.***
How many +5 jumps do we make to go from 0 to 15? (Three) ***3 x 5 = 15.***
Are there any other numbers that are both multiples of 3 and multiples of 5? (Yes)

In the string picture, record the common multiples of 3 and 5 that students suggest. If no one suggests that 0 is a common multiple of 3 and 5, mention it yourself.

Perhaps your string picture will look like this one.



T: ***Look at the numbers that are multiples of 3 and multiples of 5. Do you see any patterns?***

Allow the students to express themselves. If necessary, point out that 0 + 15 = 15, 15 + 15 = 30, and 30 + 15 = 45.

S: ***They increase by 15.***

T: ***The common multiples of 3 and 5 are the numbers that we would meet if we started at zero and counted by 15's. Are there other numbers that are common multiples of 3 and 5?***

Encourage students to suggest several common multiples of 3 and 5, including some negative numbers. A possible dialogue is given here.

S: ***60 is a multiple of 5 and a multiple of 3.***

T: ***How do you know?***

S: ***60 ends in 0, so 60 is a multiple of 5. 60 is a multiple of 3, because 30 is a multiple of 3 and 30 + 30 = 60.***

S: ***45 + 15 = 60.***

T: ***45 and 15 are both common multiples of 5 and 3, so 60 is a common multiple of 5 and 3. Are there any negative numbers that are common multiples of 3 and 5?***

S: ***Yes;*** $\widehat{15}$ ***because*** $\widehat{15} + 15 = 0$.

Perhaps one of your students will suggest that the numbers in the intersection of the two strings are the multiples of 15. Commend this student, but do not expect this observation.

If time remains, play a game similar to the one described in Exercise 1 of Lesson L8 *Multiples #1*, using two strings rather than one. Or, ask students to copy the string picture and put in other numbers. Allow students to use calculators to find more multiples of 3 and 5.

Center Activity

Set up a center for students to play a game as described in Exercise 1 of Lesson L8, using two strings.

Capsule Lesson Summary

Count backward by fives and by tens starting at 163. Use an arrow road having three –10 arrows to present a way to subtract 30 from a number. Solve subtraction problems by building arrow roads with arrows composed to give the appropriate subtraction function.

Materials

Teacher
- Colored chalk
- Overhead calculator

Student
- Colored pencils, pens, or crayons
- Unlined paper

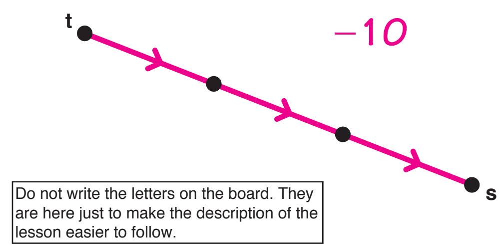
Description of Lesson

Exercise 1

Begin this lesson with some mental arithmetic involving counting backward by fives and tens, starting at various numbers. For example, teach the calculator to count backward by five or by ten starting at 163 (press 1 6 3 – 5 = = =… or 1 6 3 – 1 0 = = =…). Let students read the numbers that appear for the first few presses of =, and then ask them to predict the next entry or next two or three entries before pressing =. You may also want to ask students to predict what number they will see when you press = three (or four) times.

Exercise 2

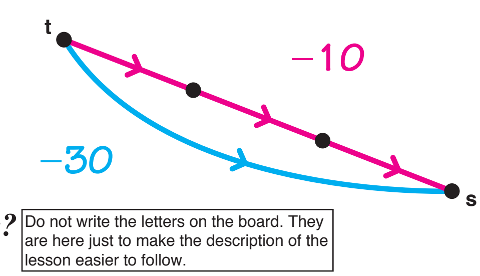
Draw this arrow picture on the board.



T: ***Where is the greatest number in this arrow picture?*** (At **t**)
Where is the least number in this arrow picture? (At **s**)
How much less is this number (point to s) than this number (point to t)? (30 less)

Draw a blue arrow from **t** to **s**.

T: ***What could this arrow be for?*** (–30)



Label the blue arrow –30.

T: ***If the greatest number*** (point to **t**) ***were 80, what would the least number*** (point to **s**) ***be?***

S: ***50.***

Record 80 – 30 = 50 on the board.

Repeat this activity several times by putting other numbers at **t**, such as 65, 153, and 29, or numbers more appropriate for the abilities of your students. Erase the board.

Exercise 3

T: ***The soccer team had a package of 124 cups. They used 35 cups at the game. How many cups remain in the package? How can we solve this problem?***

S: ***Take 35 away from 124.***

Write this subtraction problem on the board.

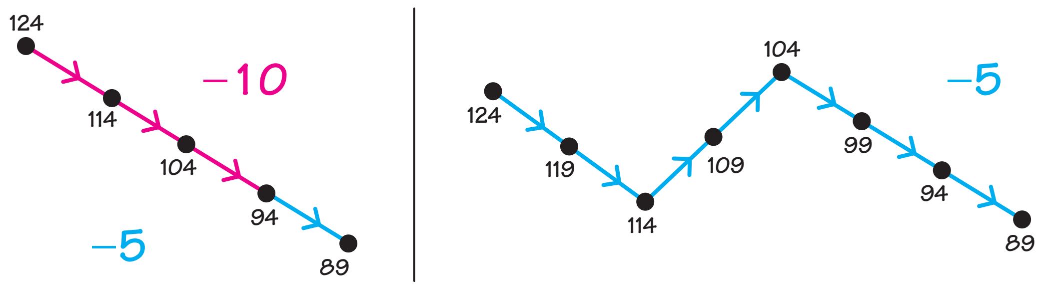
$$124 - 35$$

T: ***What kind of arrow picture could we draw to help us do this calculation?***

S: ***We could start an arrow road at 124 and draw three –10 arrows and one –5 arrow.***

S: ***We could start an arrow road at 124 and draw seven –5 arrows.***

Discuss the appropriateness of any arrow road suggested. Then, pass out unlined paper and colored pencils, and ask students to use the method they like best to calculate the answer. Encourage students to look for a pattern when they are labeling dots in their pictures. Suggest to students who finish quickly that they use a different arrow picture to calculate the answer. Invite students to draw their arrow pictures on the board and to discuss the number patterns in the arrow pictures.



Complete the subtraction calculation on the board.

$$124 - 35 = 89$$

Note: Students may, of course, use different arrow pictures and even different methods to do the subtraction calculation.

Repeat this activity with other subtraction problems such as 142 – 55, 130 – 62, and 117 – 48. Allow students to decide on their own methods/arrow pictures to use to do each calculation. Ask students who finish quickly to put their arrow pictures on the board.

Writing Activity

Ask students to choose one of the subtraction calculations and to write a story problem using that subtraction calculation.

Home Activity

Suggest parents/guardians use counting calculators to practice counting backward by fives and by tens, starting at numbers such as 127 or 154.

Capsule Lesson Summary

Solve addition problems that arise in story situations and involve decimal numbers.

Materials

Teacher	• Colored chalk • Minicomputer set	• Calculator • Worksheets N29 *, **, ***, and ****
Student	• Paper	

Description of Lesson

Exercise 1

Tell the following story or a similar one, choosing one of your students to be the star.

T: ***Katrina has a half-dollar to spend in a plant shop. She wants to buy some plants for her garden. She especially likes six of the plants at the shop. I will put some numbers inside a string to show the prices of these plants.***

On the board, draw a large string with a dot inside labeled $0.54.

T: ***This is the price of one of the plants Katrina would like to buy. How much does this plant cost?***

S: ***54¢.***

Continue in this manner until you have this string picture on the board.

T: ***After thinking awhile, Katrina finds that she can buy two of the six plants and spend exactly 50 cents for them. Which two plants are they?***

Let the students react to this problem and listen to their suggestions. Ask for some guesses and record them on the board. For example, your class may suggest the following pairs.

$0.35 and $0.14	$0.14 and $0.36
$0.26 and $0.36	$0.26 and $0.14
$0.23 and $0.26	$0.23 and $0.36

Through class discussion, try to eliminate the less accurate guesses. A sample dialogue is given here.

T: ***How can we be sure that the two plants don't cost 26¢ and 36¢?***

S: ***20¢ + 30¢ = 50¢, so 26¢ + 36¢ is more than 50¢.***

Erase this pair of prices from your list. Continue to eliminate pairs until only three or four very close guesses remain; for example:

$0.35 and $0.14 $0.14 and $0.36
$0.23 and $0.26

Ask students to write on their papers the prices of the two plants Katrina can buy and spend exactly 50¢.

T: ***Which two of these prices added together are 50¢?***

S: ***36 cents and 14 cents.***

Several of your students may be able to calculate 36 + 14 mentally, and they can explain this process to their classmates.

S: ***30 + 10 = 40; 6 + 4 = 10; 40 + 10 = 50; so 36 + 14 = 50.***

S: ***36 + 10 = 46 and 46 + 4 = 50.***

T: ***This is one way to write this addition problem. I lined up the decimal points to be sure that the dollars get added together, that the dimes get added together, and that the pennies get added together.***

$$\begin{array}{r} \$0.36 \\ +\ \$0.14 \\ \hline \$0.50 \end{array}$$

Invite someone to do the problem at the board. If necessary, write the decimal point and 0 to the left of the answer yourself.

Conclude that Katrina can buy the plants that cost 36¢ and 14¢, and spend exactly 50¢. Draw a string around $0.36 and $0.14.

Erase the board before going on to Exercise 2.

Exercise 2

Allow students to work with a partner during this exercise.

Choose one of your students to be the star of this story.

T: ***Mario likes stickers. His uncle gives him one dollar to spend in a special shop for stickers. Mario finds six stickers he would especially like to buy.***

Draw a large string on the board and ask students to read the price of each sticker as you put the corresponding number inside the string. Ask student pairs to copy the string picture on their papers.

T: ***After some hesitation, Mario chooses three stickers and spends exactly one dollar. Which three stickers are they?***

Let the student pairs work for a few minutes to try to answer this question. You may like to let students use a calculator for checking.

T: ***Which three of these prices have a sum of one dollar?***

Suppose, for example, someone suggests 14¢, 45¢, and 38¢. Write the addition calculation on the board. Remind the class that the dollars are on the left of the decimal point, and the dimes and the pennies are on the right of the decimal point. Invite a student to do the problem at the board.

$$\begin{array}{r} \$0.14 \\ \$0.45 \\ +\ \$0.38 \\ \hline \$0.97 \end{array}$$

Conclude that Mario didn't buy these three stickers, because together they cost less than one dollar. Continue until your class concludes that Mario bought the stickers with prices $0.40, $0.22, and $0.38. If the class has a great deal of difficulty solving this problem, tell them that one of the stickers Mario bought costs 40¢ and ask them for the other two prices.

When the class has found the three prices of the stickers Mario bought, draw a red string around $0.38, $0.22, and $0.40.

Worksheets N29*, **, ***, and **** are available for individual work.

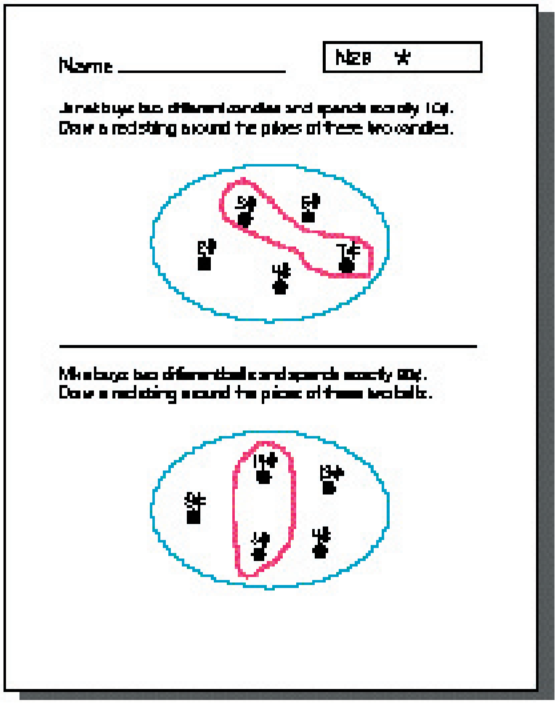
Writing Activity

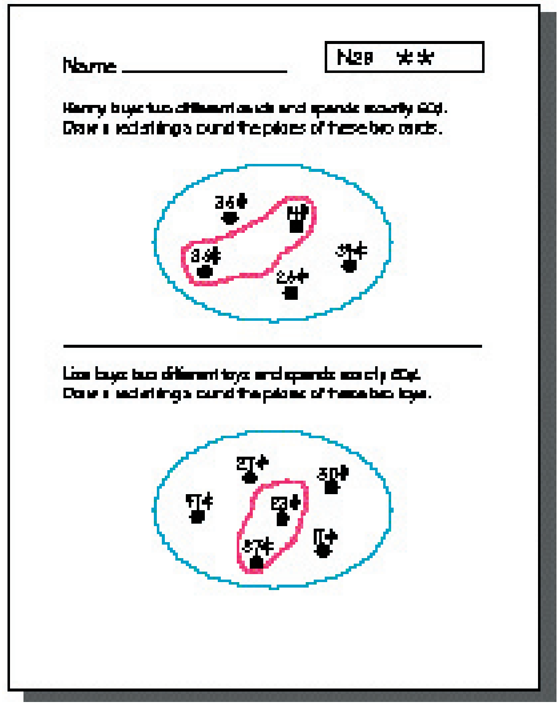
Ask students to write about a shopping trip or a meal at a restaurant where three items are purchased that cost exactly one dollar. Tell how much each item costs.

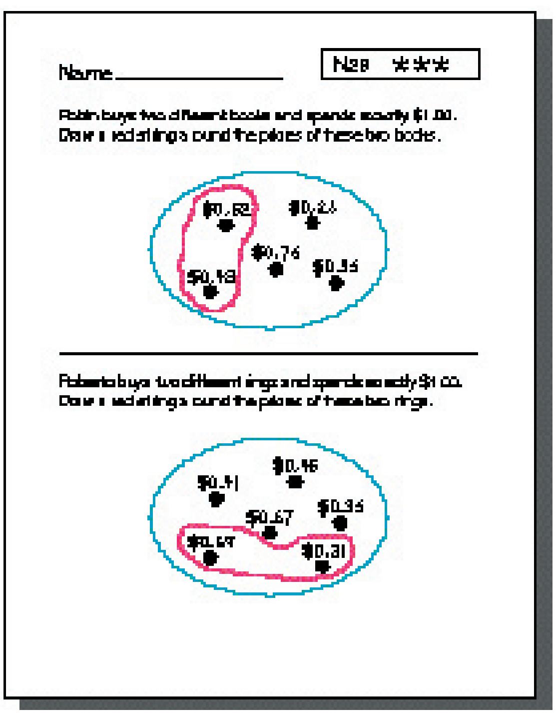
Home Activity

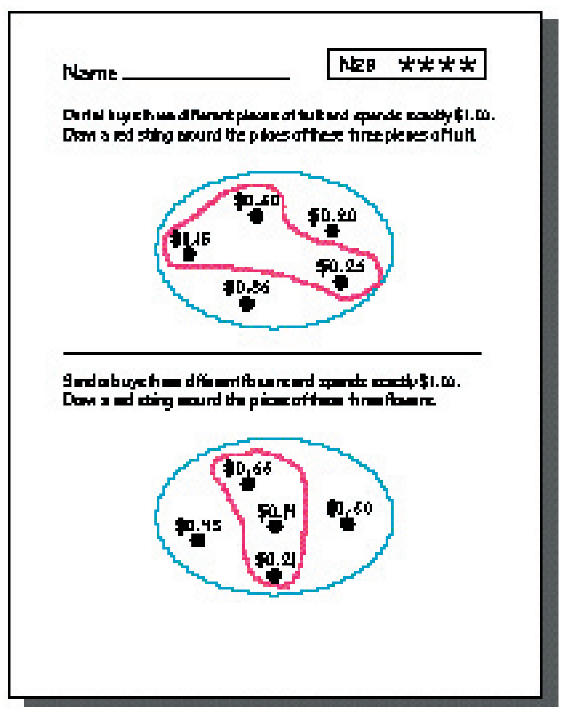
Suggest parents/guardians continue to practice counting money with their child. They can select coins to make a certain amount such as 37¢, 72¢, or 95¢. You may like to send home a problem like the one in Exercise 1 or one of the worksheets for parents to do with their child. Suggest that parents can pose other similar problems, even at a store.

N29


Name
N28 *


Name
N28 **


Name
N28 ***
$0.62
$0.74
$0.48
$0.46
$0.67
$0.31


Name
N28 ****
$0.20
$0.25
$0.66
$0.45
$0.21

N30 THE FUNCTIONS 3X AND $\frac{1}{3}$X

Capsule Lesson Summary

Write number facts about rectangular arrays of dots. Consider a configuration on the Minicomputer in which the checkers are grouped in threes on the same square. On the Minicomputer, note what one-third of a number is, and then make trades to find the number. Record the situation in an arrow picture.

Materials

Teacher
- Minicomputer set
- Colored chalk

Student
- Counters
- Paper

Description of Lesson

Exercise 1

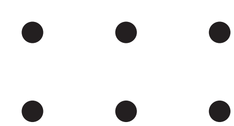
Begin the class with mental arithmetic activities involving the functions 3x and $\frac{1}{3}$ x. These are some suggestions.

3 x 10	3 x 4	3 x 5	3 x 12	3 x 14	3 x 100
$\frac{1}{3}$ x 30	$\frac{1}{3}$ x 12	$\frac{1}{3}$ x 15	$\frac{1}{3}$ x 36	$\frac{1}{3}$ x 42	$\frac{1}{3}$ x 300
30 ÷ 3	12 ÷ 3	15 ÷ 3	36 ÷ 3	42 ÷ 3	300 ÷ 3

This is also a good opportunity to review some 3x multiplication facts. Occasionally ask students to explain their methods of doing these calculations.

Exercise 2

Draw this dot picture on the board or use magnetic checkers to form this array.



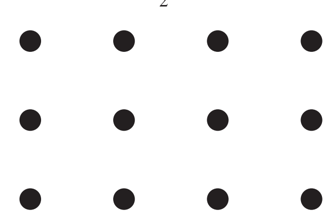
T: ***What number facts does this dot picture suggest?***

Record students' suggestions on the board.

$3 + 3 = 6$	$\frac{1}{2} \times 6 = 3$	$2 \times 3 = 6$	$\frac{1}{3} \times 6 = 2$
$2 + 2 + 2 = 6$	$6 \div 2 = 3$	$3 \times 2 = 6$	$6 \div 3 = 2$

Do not insist that students find all of these number facts. If no one mentions $\frac{1}{2}$ x 6 = 3 and $\frac{1}{3}$ x 6 = 2, suggest them yourself.

Erase the board and draw this dot picture, or use magnetic checkers.

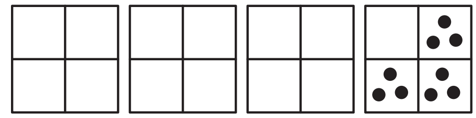


Instruct students to copy this array with counters and to write many number facts about this picture on their papers. Observe students' work and occasionally ask a student to write a number fact on the board. Try to include $\frac{1}{3}$ x 12 = 4 and $\frac{1}{4}$ x 12 = 3 in your list of facts.

Exercise 3

N30

Put this configuration on the Minicomputer.



T: ***What can you tell me about this number?***

Let students express their ideas. They probably will tell you that 3 x 7 is on the Minicomputer. Encourage students to calculate 3 x 7 noting the following:

$$\begin{aligned} 7 &= 4 + 2 + 1 \\ 3 \text{ x } 7 &= (3 \text{ x } 4) + (3 \text{ x } 2) + (3 \text{ x } 1) \\ &= 12 + 6 + 3 \\ &= 21 \end{aligned} \quad \text{or} \quad \begin{aligned} 7 &= 5 + 2 \\ 3 \text{ x } 7 &= (3 \text{ x } 5) + (3 \text{ x } 2) \\ &= 15 + 6 \\ &= 21 \end{aligned} \quad \text{or} \quad \begin{aligned} 7 &= 4 + 3 \\ 3 \text{ x } 7 &= (3 \text{ x } 4) + (3 \text{ x } 3) \\ &= 12 + 9 \\ &= 21 \end{aligned}$$

T: ***What is one-third of 21?***

S: ***7.***

Write these number facts on the board.

$$3 \text{ x } 7 = 21$$
$$\tfrac{1}{3} \text{ x } 21 = 7$$

T: ***Let's show these number facts in an arrow picture. What kind of arrows should we use?***

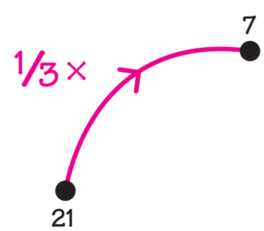
S: ***3x and $\frac{1}{3}$x*** (read as "one-third of") ***arrows.***

Draw a large dot on the board near the Minicomputer as you say,

T: ***This dot is for the number on the Minicomputer.***

Label the dot 21, and then draw a red arrow starting from that dot. Draw the ending dot of this red arrow as you say,

T: ***This dot is for one-third of that number.***



Label the ending dot 7 and label the red arrow $\frac{1}{3}$x.

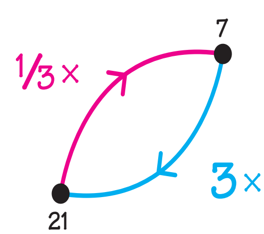
T (tracing an arrow from 7 to 21)**:** ***If I draw an opposite arrow here, what could it be for?***

S: ***3x.***

Invite someone to draw a blue arrow for 3x.

T: ***What number facts are told by this arrow picture?***

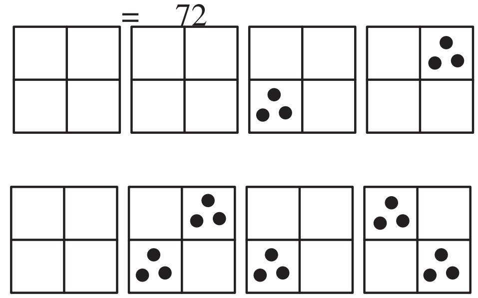
S: ***$\frac{1}{3}$ x 21 = 7 and 3 x 7 = 21.***



Repeat this activity with this configuration.

Encourage students to calculate 3 x 24 noting the following:

$$\begin{aligned} 3 \times 24 &= (3 \times 20) + (3 \times 4) \\ &= 60 + 12 \\ &= 72 \end{aligned}$$



Put this configuration on the Minicomputer and erase the dot labels in the arrow picture. Point to the dot on the left as you say,

T: ***This dot is for the number on the Minicomputer.***

Trace the $^1\!/_3$ arrow and point to the dot on the right.

T: ***What is one-third of this number?***

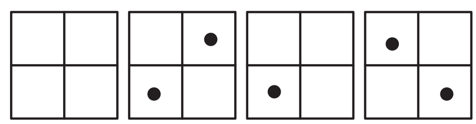
S: ***629.***

T: ***How were you able to calculate one-third of this number so quickly?***

S: ***The checkers are in groups of three.***

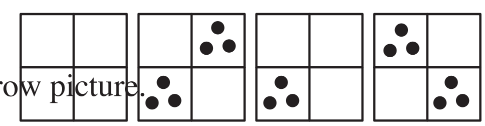
S: ***Look at one checker in each group of three.***

Invite a student to remove some checkers so that one-third of this number is on the Minicomputer. Emphasize that the checkers were in groups of three and that the student is leaving only one checker of each group of three on the Minicomputer.



T: ***Now we see that one-third of the number is 629.***

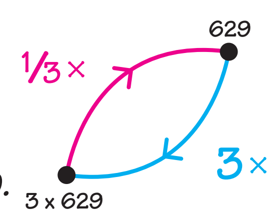
Label the dot on the right 629, and then put the original configuration back on the Minicomputer.



Trace the 3x arrow and point to the dot on the left in the arrow picture.

T: ***So the number on the Minicomputer is 3 x 629.***

Label that dot 3 x 629.



T: ***The number on the Minicomputer is 3 x 629. Look carefully at the Minicomputer. Is 3 x 629 more than 1,000?*** (Yes) ***How do you know?***

S: ***There are three checkers on the 400-square; that is more than 1,000.***

S: ***Two checkers on the 400-square and one checker on the 200-square is 1,000, and there are more checkers still.***

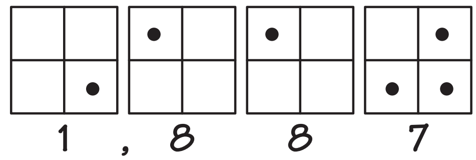
T: ***Is 3 x 629 more than 2,000?***

N30

Students probably will disagree.

Ask the students to predict what 3 x 629 is and to write their predictions on their papers.

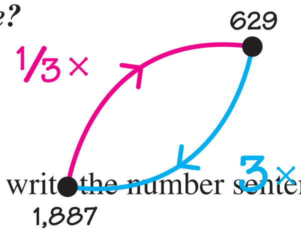
Then invite students to make trades on the Minicomputer until the standard configuration is obtained. Ask a student to write the number below (above) the Minicomputer.



Conclude that 3 x 629 = 1,887, and label the dot on the left 1,887 (erase 3 x 629).

T: ***What number sentences are told by this arrow picture?***

S: ***$\frac{1}{3}$ x 1,887 = 629 and 3 x 629 = 1,887.***



You may like to ask students to copy this arrow picture and/or write the number sentences near it.

T: ***What addition problem did we solve?***

S: ***629 + 629 + 629.***

Write this problem on the board and invite students to do the addition calculation on their papers.

$$\begin{array}{r} 629 \\ 629 \\ +\ 629 \\ \hline \end{array}$$

Capsule Lesson Summary

Find non-negative common multiples of 6 and 4 by making consecutive +6 jumps and +4 jumps on the number line (starting at 0 in both cases), and looking for common landing points.

Materials

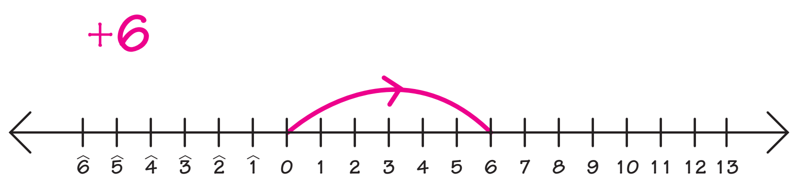
Teacher
- Colored chalk
- Overhead calculator (optional)

Student
- 0–109 numeral chart
- Blackline N31
- Number line sheet
- Colored pencils, pens, or crayons
- Calculator (optional)
- Worksheets N31 * and **

Advance Preparation: Use Blackline N31 to make copies of a number line sheet for students.

Description of Lesson

Draw this picture on the board.



T: ***If we start at 0 and make +6 jumps, what are some of the numbers we land on?***

Encourage students to make several suggestions. If necessary, solicit numbers greater than 15, but do not draw any more arrows on the board. You may need to remind the students that the number line continues in both directions.

S: ***We would meet 12.***

T: ***How many +6 jumps would we make to go from 0 to 12?***

S: ***Two.***

T: ***2 x 6 = 12. What is another number we would land on?***

S: ***18.***

T: ***How many +6 jumps would we make to go from 0 to 18?***

S: ***Three.***

T: ***3 x 6 = 18. What is another number we would land on?***

S: ***30.***

T: ***If we continue drawing +6 arrows, do we meet 30?***

S: ***5 x 6 = 30, so we would meet 30 after five jumps.***

T: ***What kind of numbers do we meet if we start at 0 and make +6 jumps?***

S: ***Multiples of 6.***

Continue asking for numbers on which you would land, that is, multiples of 6. You may also like to use the overhead calculator to view multiples of 6. To do so, start at 0 and press $\boxed{+}$ $\boxed{6}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$….

Observe that counting the number of times you press $\boxed{=}$ is the same as counting how many +6 jumps. Use the 0–109 numeral chart to reinforce a pattern of counting by sixes.

T: ***Is 100 a multiple of 6?***

The class may disagree.

T: ***Does everyone agree that 60 is a multiple of 6?*** (Yes; 10 x 6 = 60)
Let's start at 60 and count by sixes. (66, 72, 78, 84, 90)
Since 90 is a multiple of 6, can 100 be a multiple of 6?

S: ***90 + 6 = 96 and 96 + 6 = 102, so 100 is not a multiple of 6.***

T: ***What are some numbers greater than 100 that are multiples of 6?***

S: ***102 and 108.***

S: ***600.***

S: ***6,000,000.***

S: ***666.***

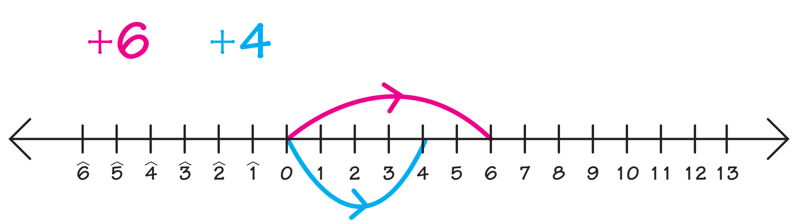
T: ***Are there any negative numbers that are multiples of 6?*** (Yes; $\widehat{6}$, $\widehat{12}$, $\widehat{18}$, and so on)
How many +6 jumps would we make to go from $\widehat{6}$ ***to 0?*** (One)

Trace an arrow from $\widehat{6}$ to 0 on the number line.

Note: For your information only, one way to check that a number is a multiple of 6 is to check that it is both a multiple of 2 (even) and a multiple of 3. For example:

- 72 is a multiple of 2 because it is an even number, and 72 is also a multiple of 3 because 7 + 2 = 9 and 9 is a multiple of 3; therefore, 72 is a multiple of 6.
- 200 is a multiple of 2 (it is even), but 200 is not a multiple of 3 because 2 + 0 + 0 = 2 and 2 is not a multiple of 3; therefore, 200 is not a multiple of 6.

Add the key +4 to your picture, and draw a blue arrow from 0 to 4 on the number line.



T: ***If we start at 0 and make +4 jumps, what are some of the numbers we meet?***

Encourage students to make several suggestions as you did for the multiples of 6. Again, you may like to use the calculator and/or the 0–109 numeral chart to view multiples of 4.

Note: The following observation is for your information only. An integer is a multiple of 4 if the number created by its last two digits is a multiple of 4. For example:

- 312 is a multiple of 4 because 12 is a multiple of 4.
- 510 is not a multiple of 4 because 10 is not a multiple of 4.

Distribute copies of the number line sheet (Blackline N31).

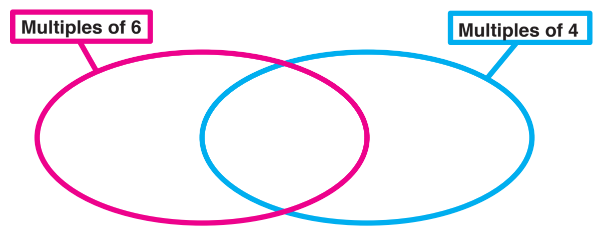
T: ***Are there numbers that are multiples of both 6 and 4?***

We can find the common multiples of 6 and 4 by drawing +6 and +4 arrow roads starting at 0 on the number line.

Ask the students to write +6 in red and +4 in blue above their number lines. Then direct them to draw +6 arrows starting at 0 and +4 arrows starting at 0.

Allow students to work independently or with partners until many students have reached 48 with both kinds of arrows. Students who have difficulty may need to begin again on a new copy of the number line sheet. If students work with a partner, suggest that one generates multiples of 6 on the calculator while the other draws +6 arrows, and vice-versa for +4 arrows.

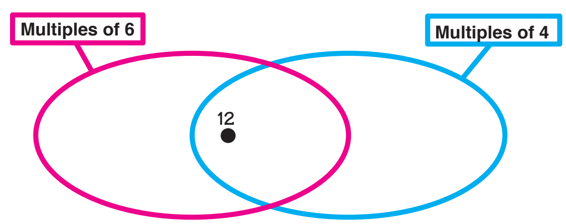
While students are working, draw this string picture on the board.



T: ***I would like to put some numbers in this string picture.***

Invite students to put numbers in the picture. Solicit numbers for all four regions. Frequently ask students to explain how they know where a particular number belongs. A possible dialogue is given here.

A student puts 12 in the string picture…

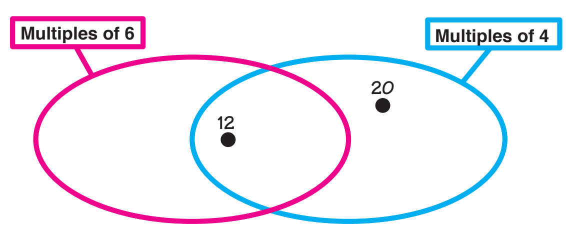


T: ***Is 12 a multiple of 4 and also a multiple of 6? How do you know?***

S: ***Yes, 12 is on both of the arrow roads.***

T: ***How many +6 jumps do we make to go from 0 to 12?*** (Two) ***2 x 6 = 12. How many +4 jumps do we make to go from 0 to 12?*** (Three) ***3 x 4 = 12.***

A student puts 20 in the string picture …



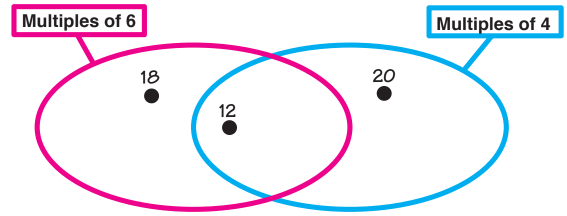
T: ***How do you know that 20 is a multiple of 4 but not a multiple of 6?***

S: ***A +4 arrow ends at 20, but none of the +6 arrows ends at 20.***

T: ***We know that 20 is a multiple of 4, because it is on the +4 arrow road starting at 0. We also know that 20 is not a multiple of 6, because it is not on the +6 arrow road starting at 0.***

N31

A student puts 18 in the string picture…

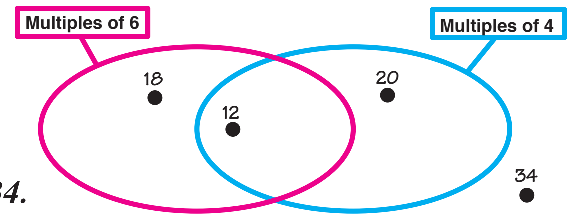


T: ***How do you know that 18 is a multiple of 6 but not a multiple of 4?***

S: ***18 is on the +6 arrow road but not on the +4 arrow road.***

T: ***Are there numbers that go outside both strings?***

A student puts 34 in the string picture …



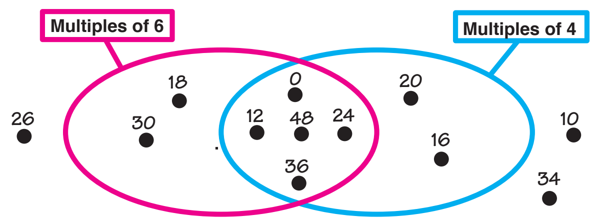
T: ***How can we be sure that 34 is neither a multiple of 6 nor a multiple of 4?***

S: ***Both the +6 and the +4 arrow roads skip over 34.***

T (pointing to the middle region)**:** ***Are there other numbers that are multiples of both 6 and 4?*** (Yes)

In the string picture, record the common multiples of 4 and 6 that students suggest. If no one mentions 0 as a common multiple of 4 and 6, do so yourself.

Perhaps your string picture will look like this one.



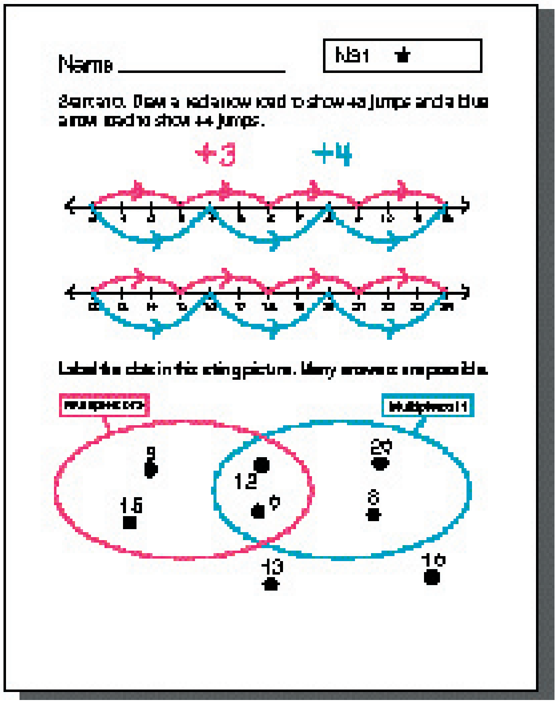
T: ***Look at the common multiples of 6 and 4. Do you see any patterns?***

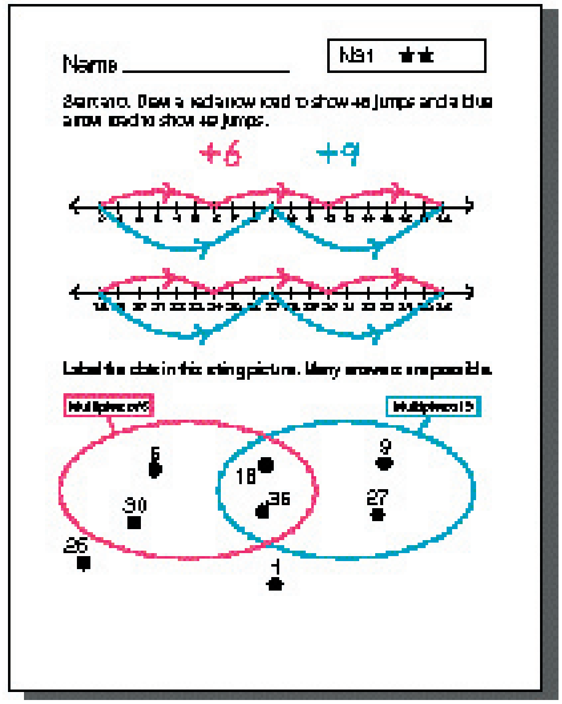
S: ***0 + 12 = 12; 12 + 12 = 24; 24 + 12 = 36; and 36 + 12 = 48.***

T: ***Are there still other numbers that are common multiples of 4 and 6?*** (Yes)

Encourage students to find many common multiples of 6 and 4, including some negative numbers. Perhaps one of your students will observe that the common multiples of 6 and 4 are the multiples of 12, but do not expect this observation.

Worksheets N31* and ** are available for individual work.


Name
N31
+3
+4
9
15
12
13
20
8
16


Name
N31
+6
+9
6
30
18
36
9
27
26
1

N32 ROADS #3

Capsule Lesson Summary

Consider several arrow roads that start at the same number and use only +100, +10, and +1 arrows. Decide which road has the greatest ending number and which has the least. Explore the question of whether or not the ending number of any of the roads would change if the order of the arrows were changed. Find the greatest and the least ending numbers of arrow roads with exactly four arrows.

Materials

Teacher
- Colored chalk
- 0–109 numeral chart

Student
- Unlined paper
- Colored pencils, pens, or crayons
- Worksheets N32* and **

Description of Lesson

Exercise 1

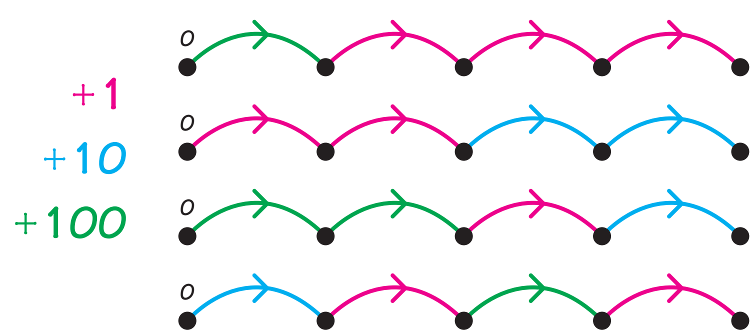
Begin the lesson with some mental arithmetic, first counting by tens and then adding 10 or 100 to a number. Ask a student to start at 0 and count by tens. Ask other students to start at different whole numbers between 0 and 10 and to count by tens. Continue by telling students you will say a number and they should add 10 (or 100) to your number. For example:

T:	***5.***	**S:**	***15 (105).***
	20.	**S:**	***30 (120).***
	22.	**S:**	***32 (122).***
	54.	**S:**	***64 (154).***
	$\hat{2}$.	**S:**	***8 (98).***

You can make the exercise a little harder by asking students to add 20. Refer students who are having difficulty to the 0–109 numeral chart.

Exercise 2

Draw these arrow roads on the board.



T: ***What do you notice about these arrow roads? How are they alike? How are they different?***

S: ***They all have four arrows.***

S: ***They all start at 0.***

T: ***Which of these arrow roads has the greatest ending number? How do you know?***

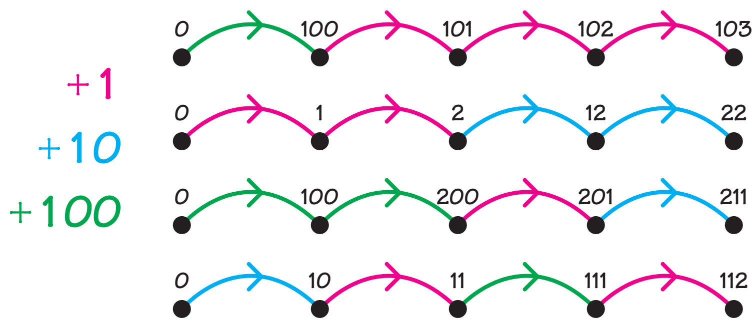
S: ***The third one because it has the most +100 arrows.***

T: ***Which of these arrow roads has the least ending number? How do you know?***

S: ***The second one, because it has no +100 arrow and the other arrow roads do.***

Emphasize that even though all the roads have four arrows and all start at 0, having different combinations of arrows gives different ending numbers in this case. Ask students to copy the arrow roads on their papers and label the dots. Encourage students who finish quickly to draw other arrow roads that start at 0, have exactly four arrows, and use only +1, +10, and +100 arrows. Ask these students to find the greatest and the least possible ending numbers.

Invite other students to label the dots in the original arrow roads.



T: ***Would the ending number of any of these roads change if we changed the order of the arrows in that road?***

Encourage your students to discuss this problem. Suggest students calculate the ending number of the first arrow road if one of the +1 arrows preceded the +100 arrow. Conclude that the ending number would still be 103. You may wish to repeat this process, changing the order of the arrows in another arrow road.

T: ***Some of you have drawn some more arrow roads with exactly four of these arrows. Did you get ending numbers different from the ones on the board?***

S: ***Yes, 301.***

T: ***301. What kind of arrows do you think are in Brad's arrow road?***

S: ***Three +100 arrows and one +1 arrow.***

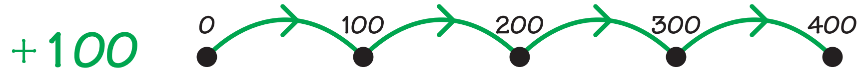
T: ***Brad, does your road have three +100 arrows and one +1 arrow?***

S: ***Yes.***

Continue this activity for several of the students' ending numbers.

T: ***What is the greatest possible ending number if we build a road starting at 0 with exactly four arrows and use only +1, +10, and +100 arrows?***

Let students announce their greatest ending number and describe how to get each ending number. Continue until someone finds that 400 is greatest and invite a student to draw the arrow road on the board. Discuss why you cannot get a greater ending number.



T: ***What is the least possible ending number if we start at 0 and draw an arrow road with exactly four of these arrows?***

Proceed as you did to find the greatest possible ending number, and conclude that 4 is the least possible ending number.

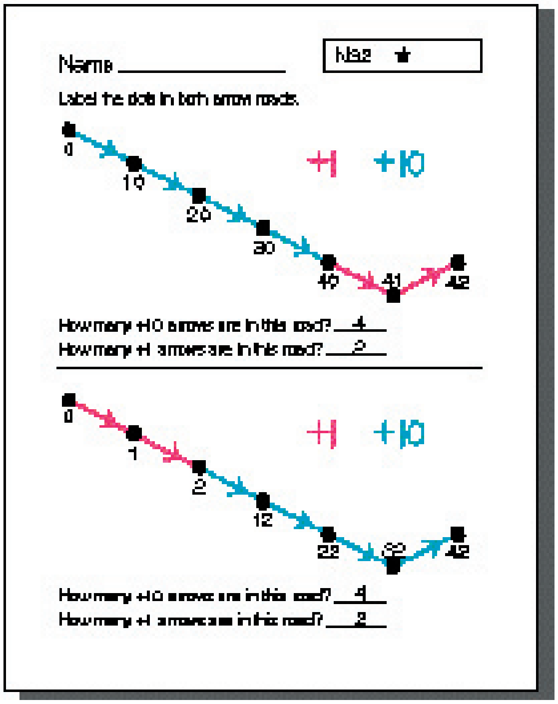
Worksheets N32* and ** are available for individual work. If some students finish quickly, challenge them to build other arrow roads such as the following:

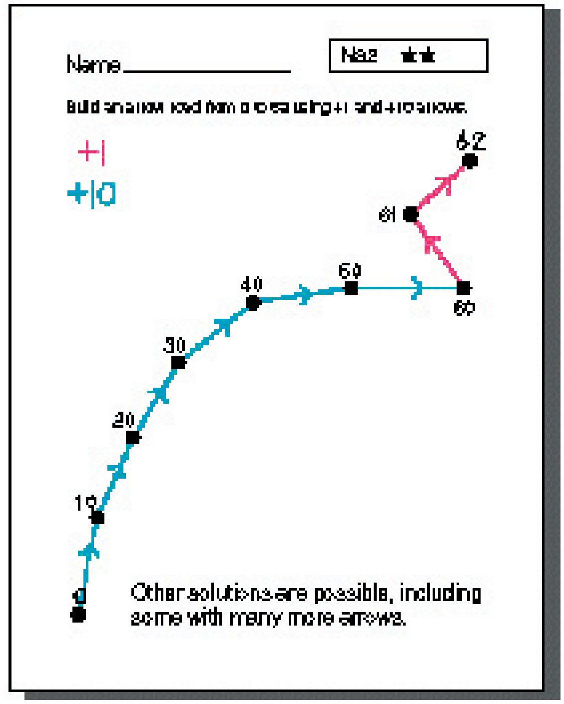
- a road from 0 to 134 using +1, +10, and +100 arrows
- a road from 98 to 452 using +1, +10, and +100 arrows

Home Activity

Suggest that parents/guardians practice mental arithmetic with their child by adding tens and hundreds, such as in Exercise 1. Send home several arrow road problems such as the following:

Using +1, +10, and +100 arrows, build an arrow road from
- 0 to 113
- 67 to 244
- 126 to 475


Name
N32 ★
Label the dots in both arrow roads.
+1 +10
0
10
20
30
40
41
42
How many +10 arrows are in this road? 4
How many +1 arrows are in this road? 2
+1 +10
0
1
2
12
22
32
42
How many +10 arrows are in this road? 4
How many +1 arrows are in this road? 2


Name
N32 ★★
Build an arrow road from 0 to 62 using +1 and +10 arrows.
+1
+10
62
61
40
50
60
30
20
10
0
Other solutions are possible, including some with many more arrows.

N33 INTRODUCTION TO DECIMAL NUMBERS #3

Capsule Lesson Summary

A stamp club is offering a prize of $78 to the winner of a contest. If there are two winners, calculate on the Minicomputer how much money each will receive. Calculate how much each winner will receive if there are three winners. Decide how to divide a prize of $100 among three winners.

Materials

Teacher
- Minicomputer set
- Colored chalk
- Play money (optional)

Student
- Minicomputer set

Description of Lesson

Exercise 1

Begin this lesson with a few minutes of mental arithmetic involving $\frac{1}{2}$x, $\frac{1}{3}$x and $\frac{1}{4}$x. Occasionally note that $\frac{1}{2}$x is the same as ÷ 2, $\frac{1}{3}$x is the same as ÷ 3, and $\frac{1}{4}$x is the same as ÷ 4.

T: ***$\frac{1}{2}$ x 10?*** (5)
$\frac{1}{2}$ x 20? (10)
$\frac{1}{2}$ x 50? (25)
$\frac{1}{2}$ x 100? (50)
$\frac{1}{2}$ x 150? (75)

$\frac{1}{3}$ x 30? (10)
$\frac{1}{3}$ x 60? (20)
$\frac{1}{3}$ x 90? (30)
$\frac{1}{3}$ x 15? (5)
$\frac{1}{3}$ x 75? (25)

$\frac{1}{4}$ x 40? (10)
$\frac{1}{4}$ x 20? (5)
$\frac{1}{4}$ x 100? (25)
$\frac{1}{4}$ x 200? (50)
$\frac{1}{4}$ x 300? (75)

Exercise 2

T: ***A stamp club decides to have a contest for the students in a local elementary school. The club announces that it will give a prize of $78 to the student with the best stamp collection. How could the club award the $78 in cash? Which bills or coins could they use?***

Encourage your class to suggest many ways to make $78 in cash. You may like to have some play money (bills) on hand for students to count out to show $78. Some possibilities will be easier to describe than to show with the play money.

- seventy-eight $1-bills
- three $20-bills, three $5-bills, and three $1-bills
- seven $10-bills and eight $1-bills
- one hundred fifty-six half-dollars
- seven hundred eighty dimes

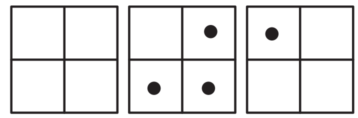
T: ***Many students are excited by this contest and begin collecting stamps from all over the world. The day for the contest judging arrives, and almost every student in the school has entered a stamp collection. There are so many stamp collections that the judges can not decide which collection is the best. Hours go by and the judges still can't decide. Then someone suggests that they choose the two best collections and give half of the $78 prize money to each of the winners.***

How much would each of the winners receive if the judges choose two best collections?

Record students' estimates on the board.

T: ***How can we calculate $^1/_2$ x 78?***

Perhaps someone will suggest using the Minicomputer; if necessary, suggest this yourself. Ask a student to put 78 on the demonstration Minicomputer and instruct the other students to put 78 on their desk Minicomputers.



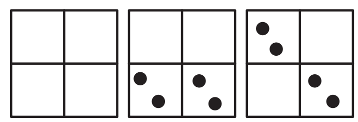
= 78

T: ***How can we calculate $^1/_2$ x 78 on the Minicomputer?***

S: ***Make some backward trades until the checkers are in pairs.***

Invite students to make trades on the demonstration Minicomputer while students follow along on their desk Minicomputers. Whenever there is a pair of checkers on the same square, push those checkers into a corner of the square and mention that you do not want to make any more trades with those checkers. Continue until all the checkers are in pairs.

Perhaps you will have this configuration on the Minicomputer.

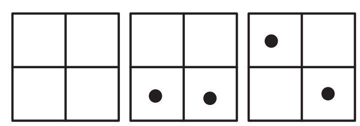


= 78

T: ***Let's remove some checkers so that one-half of this number is on the Minicomputer?***

A volunteer should remove one checker from each pair of checkers. Depending upon which configuration was on the Minicomputer, the class may or may not need to make trades to obtain the standard configuration for 39.

Write the calculation below (above) the Minicomputer.



$^1/_2 \times 78 = 39$ or $78 \div 2 = 39$

Conclude that if the judges choose two winners and give each half of the prize money, each winner will receive $39. You may like to check which estimate was closest.

Exercise 3

T: ***After a long while, the judges finally announce that they have chosen three winners to share the prize money. Everyone is glad that the contest is finally over and that the winners will be announced soon. How much money will each of the three winners receive?***

Record students' estimates on the board.

T: ***How can we determine exactly how much money each winner will receive?***

S: ***Divide 78 three ways.***

S: ***Calculate $^1/_3$ x 78.***

T: ***Is $^1/_3$ x 78 more or less than 39? Why?***

S: ***Less, because there is one more person to share in the prize money, so each winner will get less money.***

T: ***We know that $^1/_3$ x 78 is less than 39. Can we eliminate any of these estimates*** (point to the list on the board)***?***

Allow students to tell you to erase any estimate that is not less than 39. Ask a student to put 78 on the demonstration Minicomputer and instruct students to do the same on their desk Minicomputers.

T: ***How can we calculate $^1/_3$ x 78 on the Minicomputer?***

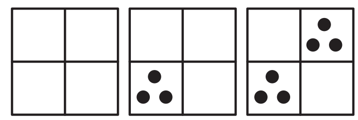
S: ***Since we are sharing three ways, we need groups of three.***

S: ***When we take one-half of a number, we need pairs of checkers. When we take one-third of a number, we need groups of three checkers.***

T: ***Let's make trades until all the checkers are in groups of three.***

Ask for volunteers to make trades on the Minicomputer and direct students to follow along on their desk Minicomputers. Whenever there is a group of three checkers on the same square, push those checkers into a corner of the square and mention that you do not want to make any more trades with those checkers. Continue until all the checkers are in groups of three.

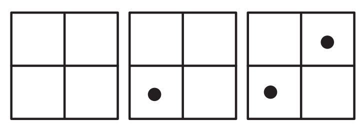
Perhaps you will have this configuration on the Minicomputer.



T: ***Let's remove some checkers so that one-third of this number is on the Minicomputer?***

A volunteer should leave one checker in each group of three on the Minicomputer. Depending upon which configuration was on the Minicomputer, the class may or may not need to make trades to obtain the standard configuration for 26.

Write the calculation below (above) the Minicomputer. Conclude that each of the winners will recieve $26.



$^1/_3 \times 78 = 26$ or $78 \div 3 = 26$

Exercise 4

T: ***At the awards ceremony, the judges announce that since they have chosen three winners instead of one, they have decided to increase the amount of prize money to $100. Each winner will receive one-third of the $100. Will each winner receive more or less than $26?***

S: ***More, because 100 is more than 78.***

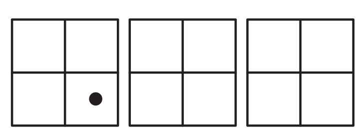
T: ***How much money will each of the three winners receive?***

Record students' estimates on the board. Do not record any guess less than 26.

T: ***How can we calculate $^1/_3$ x 100 on the Minicomputer?***

S: ***Put 100 on the Minicomputer and make trades until all the checkers are in groups of three.***

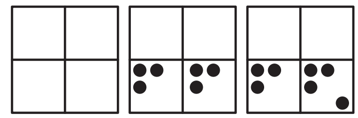
Ask a student to put 100 on the Minicomputer and instruct students to do the same on their desk Minicomputers.



Invite students to make trades on the Minicomputer, and direct students to follow along on their desk Minicomputers. Since there are many trades to be made, you may wish to allow a student to make several trades at a time. Whenever there is a group of three checkers on the same square, push these checkers into a corner of the square.

Before a jump is made from the tens board to the ones board, perhaps a student will notice that all the checkers are in groups of three except for one extra checker on the 10-square. Otherwise, mention this yourself.

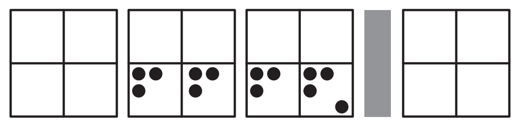
Continue this activity until all the checkers are in groups of three except for one extra checker on the 1-square. Perhaps you will have this configuration on the Minicomputer.



T: ***All the checkers are in groups of three except for this extra checker on the 1-square. This checker is for $1 which needs to be shared. What should we do now?***

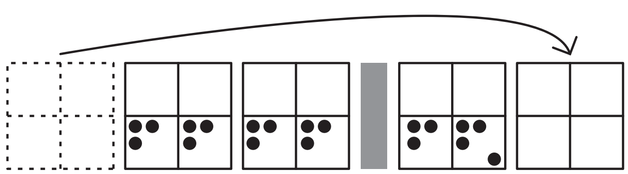
Allow the students to suggest solutions. Whether or not it is suggested, place a Minicomputer board to the right of the ones board leaving extra space between them. If no one mentions drawing a bar between the two boards on the right, ask if you have changed the number. Then suggest the bar yourself. Provide students with construction paper or other bars to use with their desk Minicomputers.

T: ***The bar reminds us that the Minicomputer boards on the left are for dollars and the boards on the right are for dimes and for pennies.***

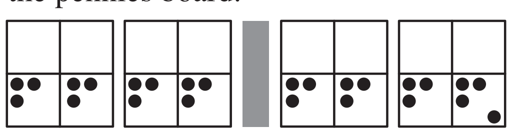


Invite students to make trades. Continue until all the checkers are in groups of three, except for one extra checker on the white square of the dimes board. Some students may foresee that to continue making trades onto the pennies board would result in the same situation—an extra checker on the white square of the pennies board.

Move a Minicomputer board to the right of the dimes board.



Continue inviting students to make trades until you have a group of three checkers and one extra checker on the white square of the pennies board.



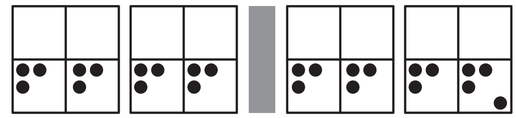
T: ***What would happen if I put another Minicomputer board on the right?***

S: ***We could make more trades, but we would still end up with an extra checker on a white square.***

T: ***No matter how many boards we put up, there will always be a checker that won't be in a group of three.***

T (pointing to the extra checker)**:** ***A checker on this white square is for what?***

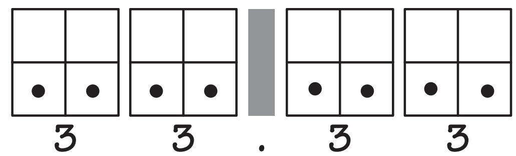
S: ***One penny.***



T: ***The judges don't know any way to share one penny*** (remove the extra checker) ***…***

… so they decide to simply keep the one penny.

Call on a volunteer to remove some checkers so that one-third of the number will be on the Minicomputer. The student should remove two checkers from each group of three. Invite another student to write the number below (above) the Minicomputer.



T: ***How much money will each of the winners receive?***

S: ***$33.33.***

T: ***If the judges give each winner $33.33, how much money will this be altogether?***

Write this problem on the board and call on a student to solve the problem. The class will discover that 3 x 33.33 = 99.99.

$$\begin{array}{r} 33.33 \\ 33.33 \\ +\ 33.33 \\ \hline 99.99 \end{array}$$

Write a number sentence on the board as you summarize that each winner receives $33.33 and that the judges have a leftover penny.

$$100 = (3 \times 33.33) + 0.01$$

Home Activity

Suggest parents/guardians practice counting an amount of money with their child. Using a small collection of bills—ones, fives, tens, and twenties—suggest they find several ways to make various amounts such as $17; $31; $45; and $63.

Capsule Lesson Summary

Decide which of two arrow roads starting at the same number has the greater ending number. Build a road from 0 to 50 using only 2x arrows and +5 arrows, and then shorten it, if possible.

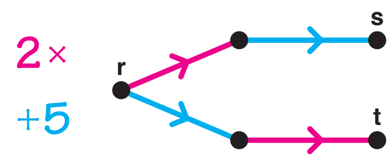
Materials

Teacher	• Colored chalk	Student	• Unlined paper • Colored pencils, pens, or crayons • Worksheets N34*, **, ***, and ****

Description of Lesson

Exercise 1

Draw this arrow picture on the board.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: ***All the dots in this arrow picture are for whole numbers. Where is the least number?*** (At **r**) ***How do you know?***

S: ***If we follow 2x or +5 arrows, the numbers increase.***

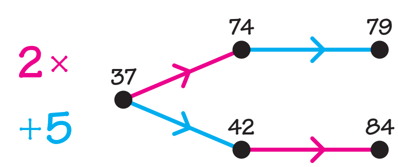
T: ***This dot*** (point to **r**) ***is the starting dot of two arrow roads. One road has a 2x arrow and then a +5 arrow. The other road has a +5 arrow and then a 2x arrow. Where is the greatest number in this arrow picture?***

Encourage students to express themselves. Some students may believe initially that **s** and **t** are for the same number. In this case, suggest that they try to find a way to check if they are correct. Very likely, students will want to choose a number for **r** and then to label the other dots accordingly. This method should lead the class to decide that the greatest number is at **t**. Insist that they do not depend, however, on only one example. Let several students suggest numbers to start with at **r**. Each time, ask where the greatest number is, and then ask how much greater **t** is than **s**.

You may like to pair students and ask each pair to find a way to label the dots. Then collectively view many examples looking for similarities.

T: ***Do you think that this number*** (point to **t**) ***is always the greatest? Let's start with another number and see if a +5 arrow followed by a 2x arrow gives us a greater ending number than a 2x arrow followed by a +5 arrow.***

Select a starting number, for example 37, and ask students to label the other dots. When all the dots are labeled, conclude that the greatest number is 84.



T: ***How much greater is 84 than 79?***

S: ***5 more.***

If appropriate, count with your class from 79 to 84 and verify that 79 + 5 = 84.

Point to the dot for 37 with your left forefinger. Trace the appropriate arrows with your right forefinger as you say,

T: ***If we start at 37 and follow a +5 arrow first and then a 2x arrow, we end at a greater number than if we follow a 2x arrow first and then a +5 arrow.***

Repeat this activity one or two more times until the students are confident that **t** will always be for the greatest number and that the number at **t** is always 5 more than the number at **s**. Choose starting numbers appropriate for the abilities of your students.

Exercise 2

Draw this picture on the board.

T: ***Can we build a road between 0 and 50 using only 2x and +5 arrows?***

S: ***We can build a road between 0 and 50 using just +5 arrows.***

T: ***If we were to build the road using all +5 arrows, how many arrows would we need?***

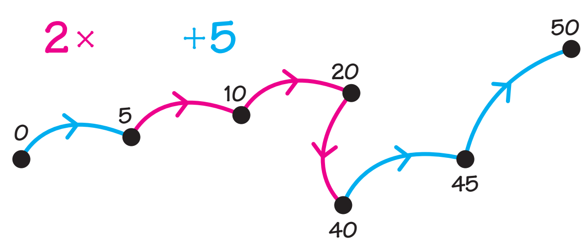
S: ***Ten.***

T: ***Could we build a shorter road?***

S: ***Yes.***

Direct student pairs to try to build such an arrow road on their papers. You may need to help some students see that they must use a + 5 arrow first. That is, direct them to observe that a 2x arrow at 0 is a loop because 2 x 0 = 0.

After several minutes, let one pair of students describe their arrow road to the class as you draw it on the board. For example:



T: ***This road starts at 0 and goes +5, 2x, 2x, 2x, +5, +5.***
How many arrows are in this road? (Six)
Could we build a shorter red-blue arrow road from 0 to 50?

Encourage other student pairs to offer suggestions. Perhaps no one will see a way to shorten this road. In such a case, start tracing the road backward; stop at each dot asking if there is a shorter road between the corresponding number and 50 until the road is shortened, if possible.

For example, point to the dots for 40 and 50.

T: ***Is there a shorter road from 40 to 50?***

S: ***No, we cannot use a 2x arrow because 2 x 40 = 80.***

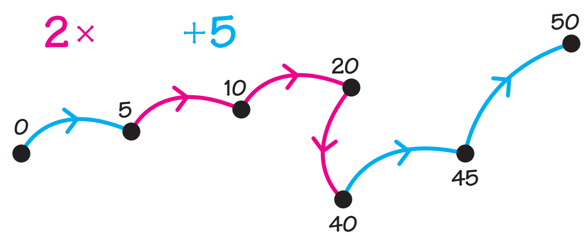
Point to the dots for 20 and 50.

T: ***Is there a shorter road between 20 and 50?***

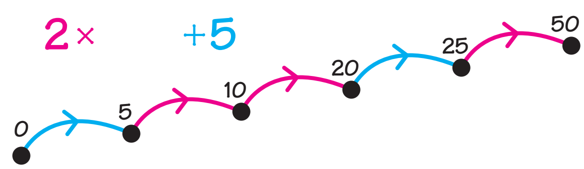
S: ***We can go from 20 to 50 with only two arrows; 20 + 5 = 25 and 2 x 25 = 50.***

T: ***We can go from 20 to 50 with a +5 arrow followed by a 2x arrow.***

Draw these arrows in the picture.



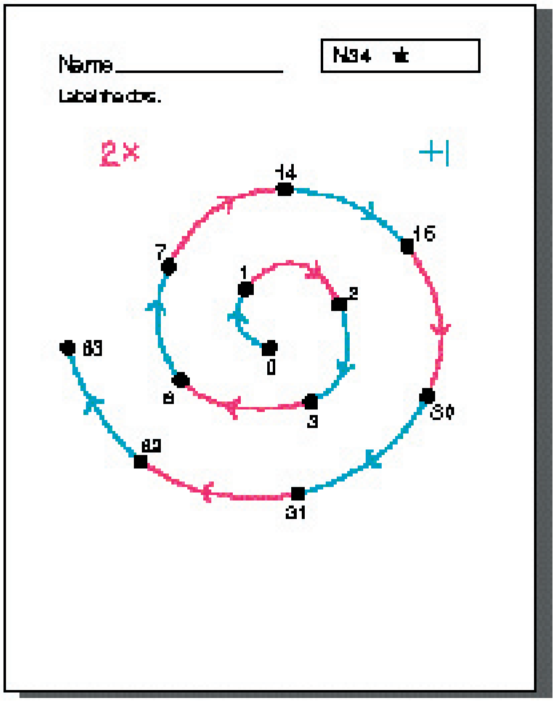
Point out the shorter road from 20 to 50 (two arrows rather than three), and erase the longer road.

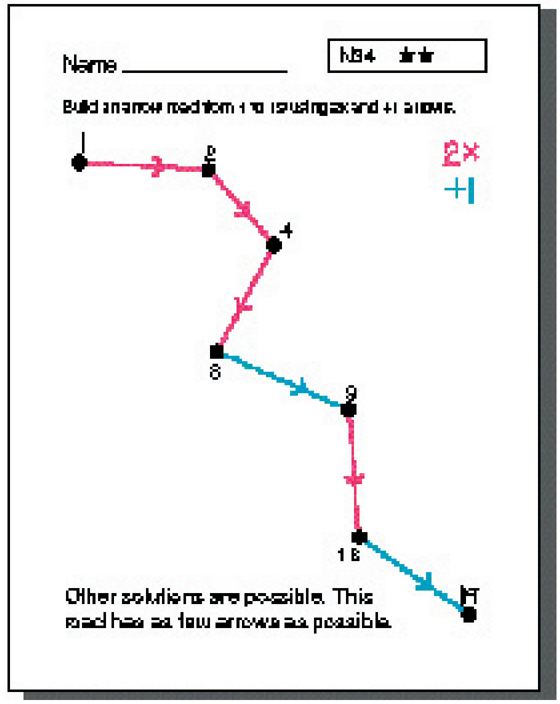


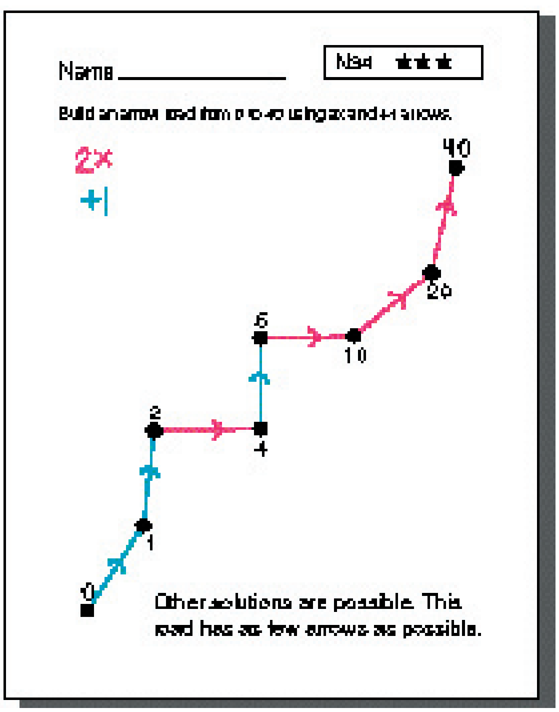
T: ***Altogether, how many arrows are in this road now?*** (Five)
Could we build a shorter road from 0 to 50?

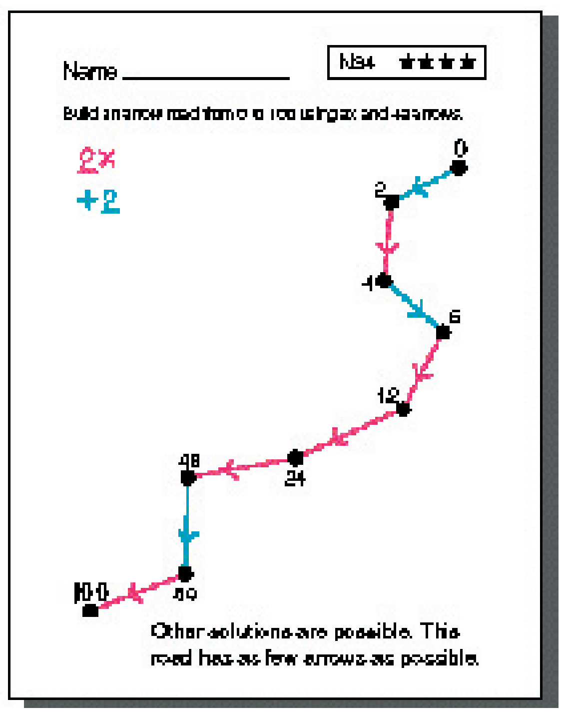
Allow the students to offer their opinions, and then tell them that you cannot build a red-blue road from 0 to 50 with less than five arrows.

Worksheets N34*, **, ***, and **** are available for individual work.


Name
N34 ★
Label the dots.
2×
+1
14
7
15
1
2
63
0
6
3
30
62
31


Name
N34 ★★
2×
+1
1
2
4
8
9
18
Other solutions are possible. This
road has as few arrows as possible.


Name
N34 ★★★
2×
+1
40
20
5
10
2
4
1
0
Other solutions are possible. This
road has as few arrows as possible.


Name
N34 ★★★★
2×
+2
0
2
4
6
12
48
24
100
50
Other solutions are possible. This
road has as few arrows as possible.

N35 ADDITION PROBLEMS WITH DECIMALS #2

Capsule Lesson Summary

Solve addition problems that arise in story situations and involve decimal numbers. Play The Number Line Game with decimal numbers.

Materials

Teacher
- Colored chalk

Student
- Paper
- Calculator (optional)

Description of Lesson

Exercise 1

Tell the following story or a similar one, appropriate for your class. Choose one of your students to be the star of the story.

T: ***Colin has one dollar to spend in a whistle shop. He especially likes six of the whistles at this shop. I will put some numbers inside a string to show the prices of these whistles.***

On the board, draw a large string with a dot inside labeled $0.09.

T: ***This is the cost of one of the whistles Colin would like to buy. How much does this whistle cost?***

S: ***9¢.***

Continue in this manner until you have this string picture on the board.

T: ***Which of these whistles is the most expensive?***

S: ***The whistle that costs $1.25.***

T: ***Which whistle is the cheapest?***

S: ***The one that costs 9¢.***

T: ***Are there any of the six whistles that Colin cannot afford to buy? Remember, Colin has $1.00.***

S: ***He can not buy the whistle that costs $1.25.***

T: ***After thinking awhile, Colin finds that he can buy two whistles and spend all of his money. Which two whistles are they?***

Ask questions to help students estimate combination costs and eliminate several possibilities. Check possible choices with the class.

S: ***Maybe he could buy the whistles that cost 25¢ and 74¢.***

T: ***Does 25¢ + 74¢ = $1.00?***

Ask students to do the addition calculation.

S: ***25 + 74 = 99.***

T: ***How did you calculate the 25¢ + 74¢?***

As appropriate, record some of the students' methods of calculation on the board.

- Some students might write an addition problem such as one of these:
- Some students might calculate mentally; for example:

$$\begin{array}{r} 25 \\ +74 \\ \hline 99 \end{array} \quad \text{or} \quad \begin{array}{r} 0.25 \\ +0.74 \\ \hline 0.99 \end{array}$$

25 + 75 = 100	(75 is 1 more than 74)	and	100 – 1 = 99
20 + 74 = 94	(25 is 5 more than 20)	and	94 + 5 = 99
20 + 70 = 90	(25 is 5 more than 20; 74 is 4 more than 70)	and	90 + 5 + 4 = 99

Decide that 25¢ and 74¢ are not the prices of the whistles. Continue this activity until your class concludes that Colin can buy the whistles that cost 64¢ and 36¢ and spend exactly $1.00 for them. Draw a red string around $0.64 and $0.36.

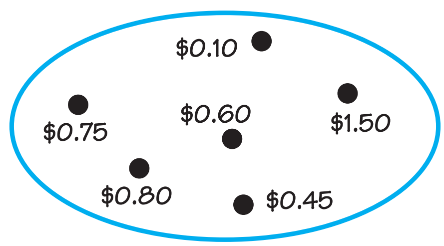
Exercise 2

Allow students to work with a partner during this exercise.

Present another story with one of your students as the star.

T: ***Laura likes to collect bicycle decals. She finds six decals that she would especially like to buy.***

Draw a large string on the board and ask students to read the price of each decal as you put the corresponding number inside the string. Ask student pairs to copy the string picture on their papers.



T: ***What is the price of the most expensive decal?*** ($1.50)
What is the price of the cheapest decal? (10¢)
After thinking very carefully, Laura selects three different decals and spends exactly $2. Which three decals does Laura buy?

Allow students to work with a partner to try to answer this question. As you observe students' work, you may want to encourage individual pairs of students to try other combinations of prices. You may like to allow students to use calculators for checking. If the class has difficulty finding a correct combinations of prices, tell them that one of the decals Laura buys costs 80¢.

After several groups find the solution ($0.80, $0.75, and $0.45), check with the class that the sum of these prices is $2.00 by writing this addition problem on the board and asking

$$\begin{array}{r} \$0.80 \\ \$0.75 \\ +\ \$0.45 \\ \hline \$2.00 \end{array}$$

someone to do the calculation.

Draw a string around the three prices.

Exercise 3: The Number Line Game

T: ***We are going to play The Number Line Game. I am thinking of a number between 0 and 10. I'll put a mark for it on the number line. The number is hidden behind the box.***

Draw a line on the board and position a box for the secret number near the center of your picture.

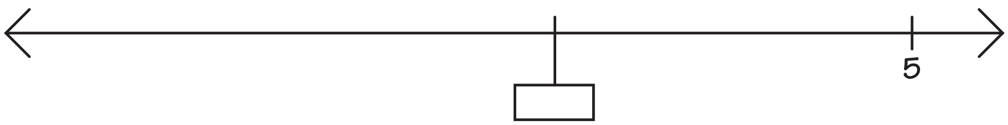
T: ***Guess what my secret number is.***

If someone suggests a number that is not between 0 and 10, do not write anything. Just remind the students that their guesses should be between 0 and 10 and ask someone else to guess.

This is a description of a possible game in which the secret number is 3.25.

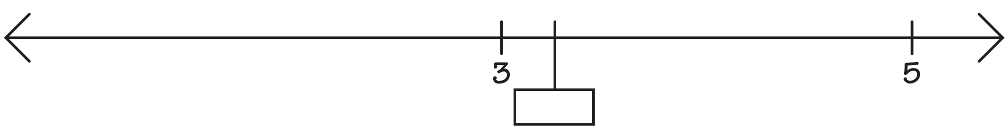
S: ***5.***

T: ***I'll show you where 5 is on this number line. It might not be in exactly the right place, but I'll try to make it as close as possible.***



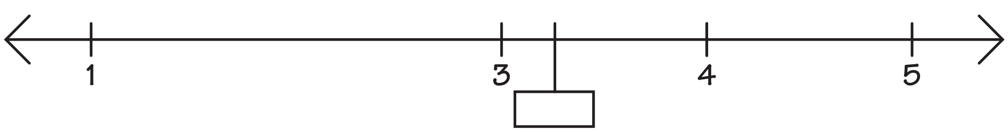
T: ***5 is more than my secret number.***

S: ***3.***



T: ***3 is less than my secret number.***

S: ***4.***



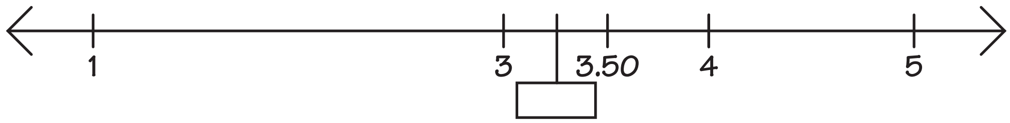
T: ***4 is more than my number.***

At this point, students may be confused and suggest several whole numbers before a number between 3 and 4 is suggested. Be patient, and record these numbers on your number line as long as they are between 0 and 10. Occasionally remind students that the secret number is between 3 and 4.

S: ***$3\frac{1}{2}$.***

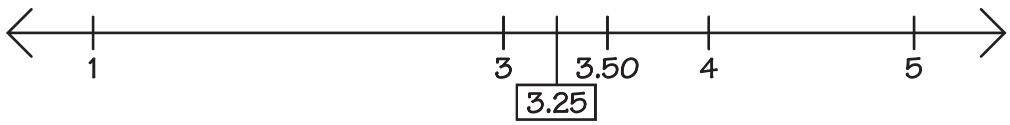
N35

T: ***Another name for $3\frac{1}{2}$ is 3.50*** (read as "three point five zero"). ***I'll show you where 3.50 is on this number line. 3.50 is more than my secret number.***



S: ***$3\frac{1}{4}$.***

T: ***How much money is one-fourth of a dollar?*** (25¢)
Another name for $3\frac{1}{4}$ is 3.25.



T: ***3.25 is my secret number.***

Play the game several times using other secret numbers such as 9.50, 7.10, and 6.80.

Capsule Lesson Summary

Calculate 2 x 65 by considering 65 as 60 + 5 or 40 + 20 + 5. Use 2 x 65 = 130 to calculate 2 x 265 and 2 x 565. Use similar methods to calculate 3 x 65, 3 x 265, and 3 x 565.

Materials

Teacher
- Minicomputer set

Student
- Paper
- Minicomputer set
- Worksheets N36 *, **, ***, and ****

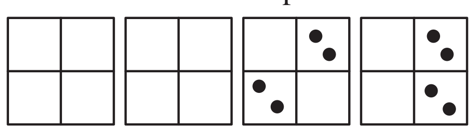
Description of Lesson

Exercise 1

Display four Minicomputer boards, and provide each student or student pair with an individual Minicomputer set. Pose a problem to the class that involves doubling a recipe or pattern. For example, two children each want to build a cube structure and the pattern for this cube structure calls for 65 cubes. How many cubes do they need altogether?

T: ***Put 2 x 65 on your (individual) Minicomputer. Remember 2 x 65 is the same number as 65 + 65. For the moment, do not make trades.***

Help students having difficulty putting this configuration on their Minicomputers. Call on a student to put 2 x 65 on the demonstration Minicomputer.



= 2 × 65

T: ***Can we calculate 2 x 65 without making trades on the Minicomputer?***

Many suggestions are possible; for example:

$$\begin{aligned} 2 \times 65 &= (2 \times 60) + (2 \times 5) \\ &= 120 + 10 \\ &= 130 \end{aligned}$$

or

$$\begin{aligned} 2 \times 65 &= (2 \times 40) + (2 \times 20) + (2 \times 5) \\ &= 80 + 40 + 10 \\ &= 130 \end{aligned}$$

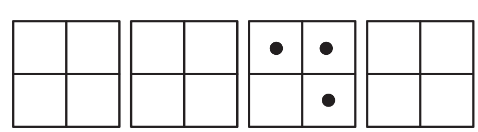
Encourage students to choose their own methods of doing the calculation, and let them share their ideas with the class. Indicate some of their methods on the chalkboard.

Write this number sentence on the board.

2 x 65 = 130

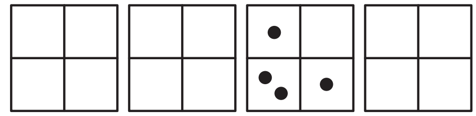
T: ***Now make trades on your Minicomputer.***

Very likely, several students will make trades resulting in this configuration.

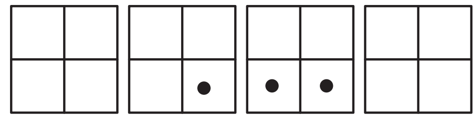


T: ***Some of you have checkers on your Minicomputers like this. Can someone suggest a useful trade? Remember, we want to make the number easier to read.***

S: ***40 = 20 + 20…***



…and then 80 + 20 = 100.



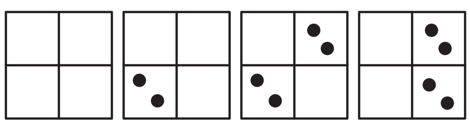
T: ***Make an 80 + 20 = 100 trade as soon as possible to avoid having to make backward trades later. What number is on the Minicomputer?***

S: ***130.***

Remove the checkers from the demonstration Minicomputer. Return to your problem of doubling a recipe or pattern and change the number needed to 265 (cubes).

T: ***Put 2 x 265 on your (individual) Minicomputer. Remember 2 x 265 is the same number as 265 + 265. Can you calculate 2 x 265 without making trades?***

Check that students put the correct configuration on the Minicomputer. Invite a student to put 2 x 265 on the demonstration Minicomputer.



= 2 × 265

Point to the checkers on the tens and ones boards as you say,

T: ***We already know that 2 x 65 = 130. Does this help us to calculate 2 x 265?***

S: ***Yes, because 2 x 65 = 130; 2 x 200 = 400; so all we need to do is add 130 + 400.***

Write the addition calculation on the board and ask a student to complete it.

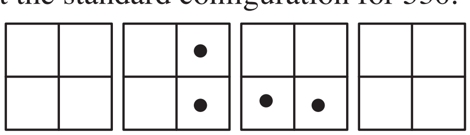
If no one suggests this calculation, refer to the Minicomputer and write number sentences on the board as you ask,

T: ***2 x 65 = …?*** (130)
2 x 200 = …? (400)
2 x 265 = …? (530)

$$\begin{array}{l} 2 \times 65 = 130 \\ \underline{2 \times 200 = 400} \\ 2 \times 265 = 530 \end{array}$$

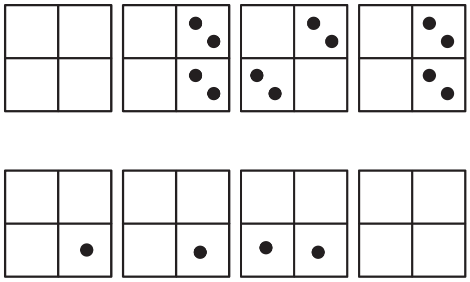
Accept other explanations, but include the method described above as an easy way to solve the problem. Instruct students to make trades on their Minicomputers. You may need to help some students if they reach a position where a 40 = 20 + 20 trade is needed.

Students should arrive at the standard configuration for 530.



= 530

Repeat this activity to calculate 2 x 565.



= 2 × 565

= 1,130

$$
\begin{array}{r}
2 \times 65 = 130 \\
2 \times 500 = 1{,}000 \\
\hline
2 \times 565 = 1{,}130
\end{array}
$$

Remove the checkers from the Minicomputer and erase the board.

Exercise 2

Return to your problem of doubling a recipe or pattern, and change the problem to one of tripling the recipe or pattern. For example, three children each want to build a cube structure and the pattern for one cube structure calls for 65 cubes. How many cubes do they need altogether?

Follow the same format as in Exercise 1 to let students calculate 3 x 65 = 195. Then use this result to help calculate 3 x 265 and 3 x 565.

$$
\begin{array}{r}
3 \times 65 = 195 \\
3 \times 200 = 600 \\
\hline
3 \times 265 = 795
\end{array}
\qquad
\begin{array}{r}
3 \times 65 = 195 \\
3 \times 500 = 1{,}500 \\
\hline
3 \times 565 = 1{,}695
\end{array}
$$

Worksheets N36*, **, ***, and **** are available for individual work. Allow students to use Minicomputers if they wish.

Writing Activity

Ask students to write directions to a friend on how to double or triple a pattern that calls for 36 of some item.

Ask students to write directions on how to do the calculation 2 x 354.

Name ______________ N36 ★

Complete.

2 × 10 = 20	2 × 20 = 40
2 × 2 = 4	2 × 1 = 2
2 × 12 = 24	2 × 21 = 42
2 × 20 = 40	2 × 10 = 20
2 × 4 = 8	2 × 3 = 6
2 × 24 = 48	2 × 13 = 26

Name ______________ N36 ★★

Complete.

2 × 20 = 40	2 × 10 = 20
2 × 5 = 10	2 × 7 = 14
2 × 25 = 50	2 × 17 = 34
2 × 30 = 60	2 × 100 = 200
2 × 4 = 8	2 × 13 = 26
2 × 34 = 68	2 × 113 = 226

Name ______________ N36 ★★★

Complete.

2 × 4 = 8	2 × 5 = 10
2 × 14 = 28	2 × 15 = 30
2 × 24 = 48	2 × 25 = 50
2 × 34 = 68	2 × 35 = 70
2 × 44 = 88	2 × 45 = 90
2 × 6 = 12	2 × 23 = 46
2 × 16 = 32	2 × 123 = 246
2 × 26 = 52	2 × 223 = 446
2 × 36 = 72	2 × 423 = 846
2 × 46 = 92	2 × 523 = 1,046

Name ______________ N36 ★★★★

Complete.

2 × 18 = 36	2 × 35 = 70
2 × 48 = 96	2 × 435 = 870
2 × 148 = 296	2 × 1,035 = 2,070
2 × 248 = 496	2 × 1,235 = 2,470
2 × 548 = 1,096	2 × 4,035 = 8,070
2 × 1,048 = 2,096	2 × 1,435 = 2,870
2 × 70 = 140	3 × 54 = 162
2 × 75 = 150	3 × 254 = 762
2 × 175 = 350	3 × 354 = 1,062
2 × 475 = 950	3 × 454 = 1,362
2 × 675 = 1,350	3 × 1,054 = 3,162
2 × 1,075 = 2,150	3 × 1,354 = 4,062