Workbooks

WORKBOOKS INTRODUCTION

There are many opportunities for the student to work individually during the course of the lessons described in the other content strands. In the Workbooks strand, however, it is this individualized work which becomes the chief end of the majority of lessons. The goal in this strand to provide the students with opportunities

- to review many of the ideas they have met in other content strands;
- to apply their acquired knowledge to new situations requiring various kinds of strategic thinking; and
- to learn how to read and use mathematics workbooks.

The following workbooks are provided:

- *Caravan of Problems #1*
- *Length and Area*
- *Caravan of Problems #2*
- *Nora's Neighborhood*
- *Fishing for Numbers, Part II*

Each workbook contains problems of varying levels of difficulty. Approximately the first ten pages of each workbook are easy problems, the next ten to twelve pages are average level difficulty, and the last ten pages are more challenging problems. For each workbook, we suggest that all students start work at the easiest level (i.e., on page 2) and then work through as many pages as they can handle during the two lessons scheduled for that workbook. We estimate that, in a typical class, about two-thirds of the students will correctly finish the first ten pages, about one-third will finish the first twenty pages, and a few will finish all or most of the workbook. These proportions will vary from class to class.

This guide contains an answer key for each workbook. The key follows an introduction to the workbook and a suggested collective lesson. The lesson either presents the workbook to the whole class or provides a warm-up activity on a problem similar to one found in the workbook.

The storybook *Summer School in the Old Days* and its accompanying story-workbook *Summer School: 0's Discovery* provide the context for two lessons. Representation of numbers is the main focus of these richly illustrated books.

Several lessons present detective stories in which there is a secret number and several clues that lead to it. The clues provide review of many ideas from the other content strands and also present students with new situations involving patterns and counting techniques.

Use of the Workbooks Strand for Evaluation Purposes

The workbooks provide an excellent instrument to assess the progress of your students on a regular basis. You may not feel it is necessary to check every page and problem for each student, but you should develop a procedure for checking students' work with which you are comfortable. This may include checking one or more specific pages, discussing some particular mistakes with individual students and letting them correct their work, or just looking carefully at a few pages to be sure the students have understood the general idea of the problems in that particular book.

In the Blacklines, you will find a record-keeping tool for each workbook to help you assess student progress in the various strands. This tool may also assist you in parent conferences and in filling out periodic progress reports.

Here are some important points to bear in mind for workbooks.

- Always read the introductory material for each workbook and give the short introductory collective lesson(s).
- All students should start at the beginning of each workbook and progress as far as they can.
- All students should begin a new title on the same day, even if some students have not finished work on the previous title.
- Not all students should be expected to complete a given workbook. Only some students will reach the most challenging problems. Other students may succeed only in doing the easiest problems, although you should not assume this automatically—surprises are not at all uncommon.

WORKBOOKS TABLE OF CONTENTS

Capsule Lesson Summary

Label the dots in an arrow picture with +3 and –2 arrows. Draw all the possible +6 arrows in the same picture. Review and extend experiences from various content strands in the workbook *Caravan of Problems #1*.

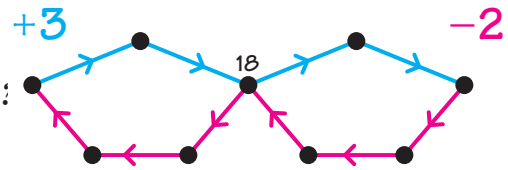
Materials

Teacher
- Colored chalk

Student
- *Caravan of Problems #1* Workbook
- Colored pencils, pens, or crayons

Description of Lesson

Draw this arrow picture on the board.



T: ***What are the blue arrows for in this picture?***

S: ***+ 3***

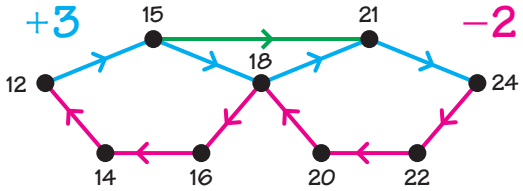
T: ***…and the red arrows?***

S: ***–2.***

T (pointing to 18)**:** ***Here is the number 18. What other numbers are in this picture?***

Call on students, one at a time, to label the other dots. Each time a dot is labeled, check the appropriate calculation and ask students to write the corresponding number fact.

After all the dots are labeled, draw this green arrow.



T: ***What could this green arrow be for?***

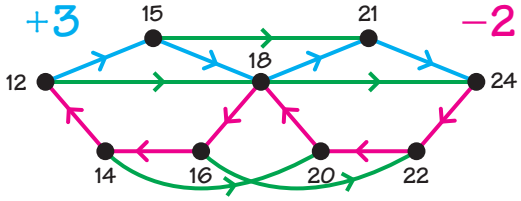
S: ***+6, because 15 + 6 = 21.***

S: ***+6, because adding 3 two times is the same as adding 6.***

Encourage the class also to observe that +3 followed by +3 is +6, and label the green arrow +6.

T: ***Are there more places where we can draw +6 arrows in our picture? Try to find them.***

Invite students first to trace +6 arrows and then, if correct, to draw them in green. Check the calculations each time. After a few minutes, and perhaps with a little prompting, your students should find four more +6 arrows.



Distribute copies of the workbook *Caravan of Problems #1* and let students work independently for the rest of the class period. If many students have difficulty with a particular page, you may wish to have a collective discussion about that page.

At the end of the lesson, collect the workbooks for your review. They will be used again in Lesson W2.

Capsule Lesson Summary

Build an arrow road from 7 to 25 using +5 and –1 arrows. Examine a road from 25 to 7 using the return arrows. Continue work in the *Caravan of Problems #1* Workbook. (This is the second of two lessons using this workbook.)

Materials

Teacher	• Colored chalk	• Colored pencils, pens, or crayons
Student	• *Caravan of Problems #1* Workbook	

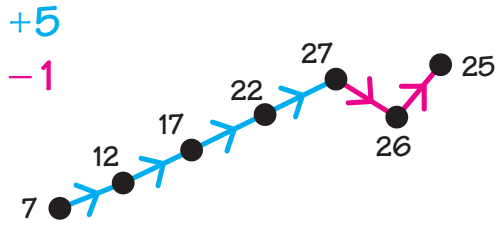
Description of Lesson

Put this information on the board.

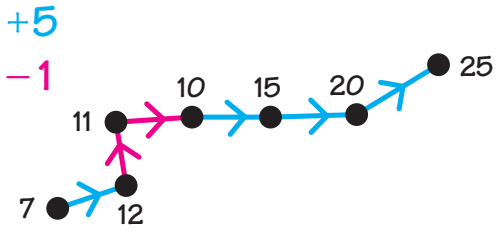
T: ***Let's build a road from 7 to 25 using blue + 5 arrows and red –1 arrows. How could we start?***

Let students tell you which arrows to draw and follow their suggestions. There are no wrong answers in this situation as long as +5 or –1 arrows are suggested. Several examples of arrow roads from 7 to 25 are given below. The first two are shortest possible roads. The third affords the opportunity of discussing how one can sometimes eliminate arrows, one blue and five red, to get a shorter road from 7 to 25. In the first two examples, you might note that at least four blue and two red arrows are needed. Let the class determine the direction of the discussion.

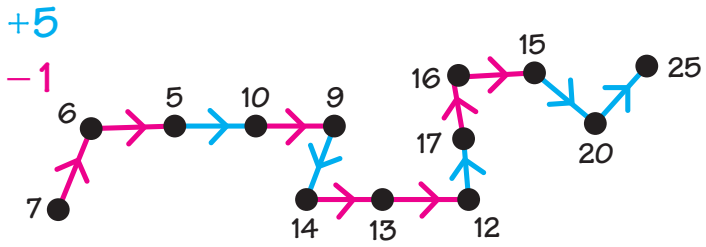
Example 1



Example 2



Example 3

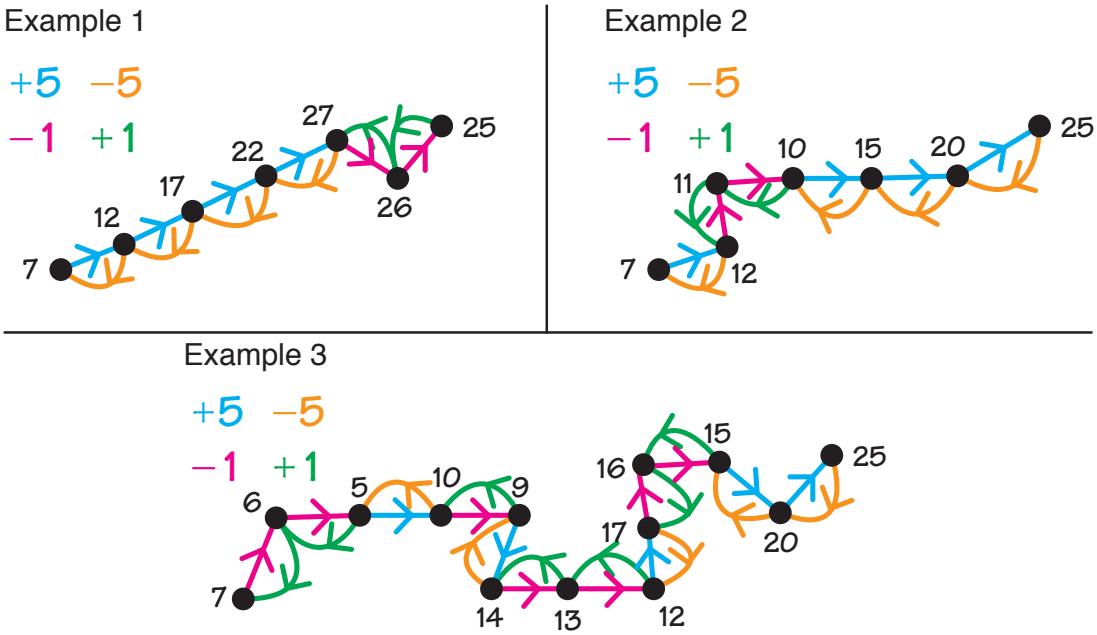


After your students build a road from 7 to 25 using +5 and –1 arrows, direct them to consider a return road from 25 to 7.

T: ***Now, let's try to find a road returning from 25 to 7 and landing on the same numbers as we did going from 7 to 25. How could we do this?***

S: ***Use –5 arrows and +1 arrows.***

Lead the class to observe that the opposite (return) of a +5 arrow is a –5 arrow and the opposite (return) of a –1 arrow is a +1 arrow. Then draw an appropriate return road from 25 to 7. The return roads, for our three examples, are shown below.



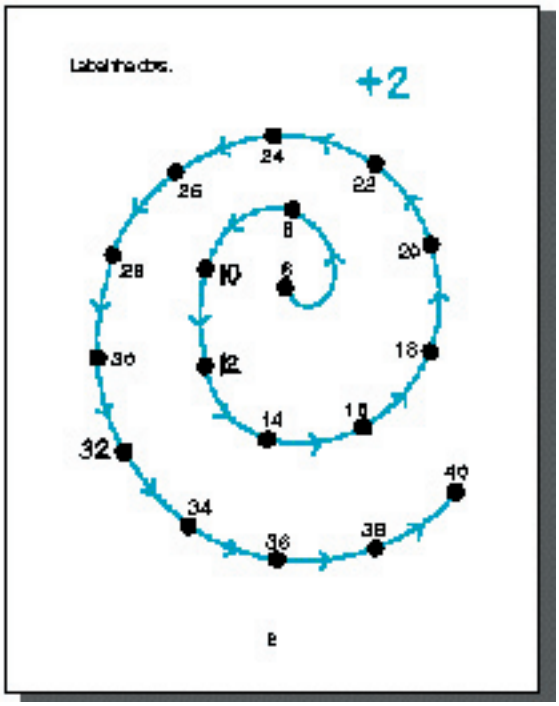
Distribute students' copies of the workbook *Caravan of Problems #1*. Ask students first to correct or complete pages from the first week's work and then to continue. You may wish to discuss collectively some problems that were difficult for many students the first week. At the end of the class period, collect the workbooks for your review.

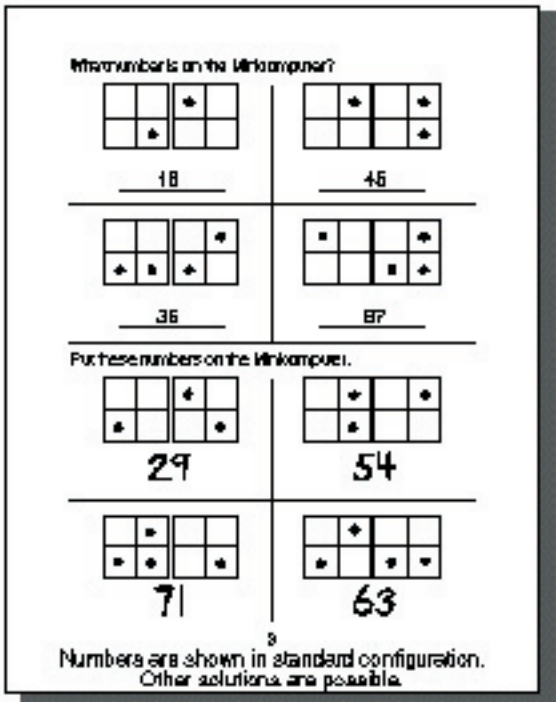
Assessment Activity

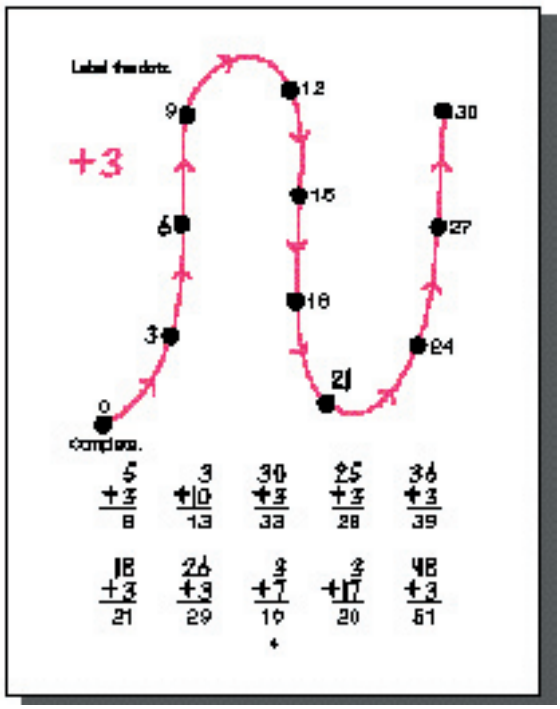
An individual student progress record for the workbook is available on Blackline W2(a). You may like to use this form to monitor student work.

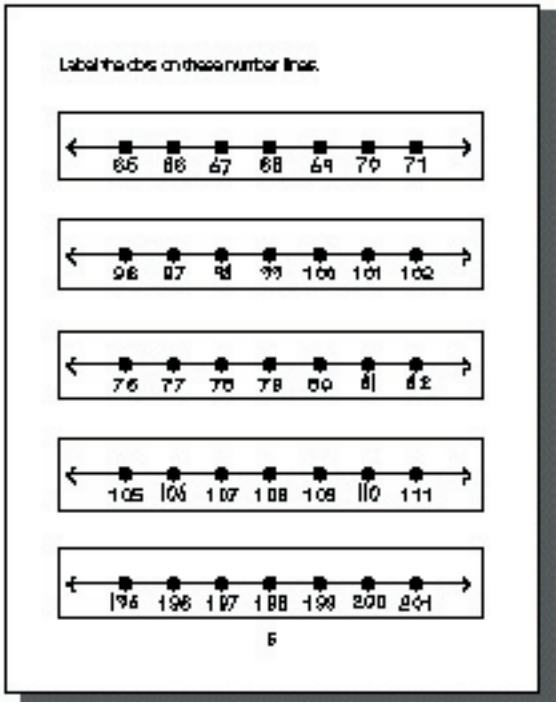
Home Activity

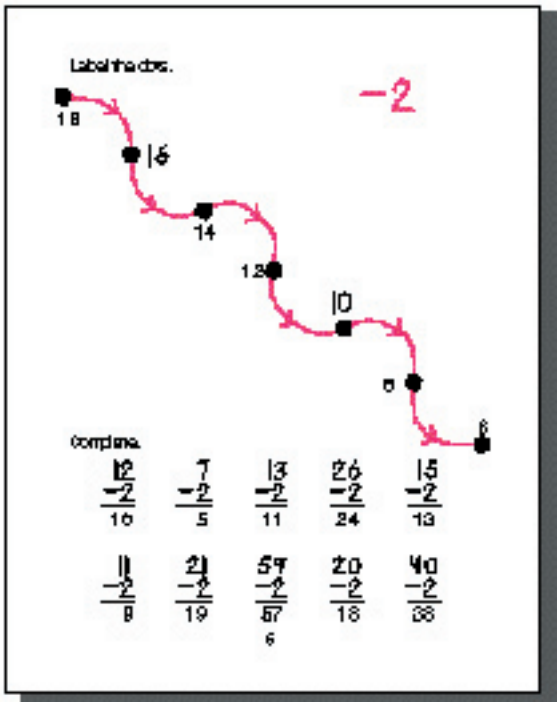
If you choose to send workbooks home with students, you may want to include a letter (reminder) to parents/guardians with this workbook. Blackline W2(b) has a sample letter.

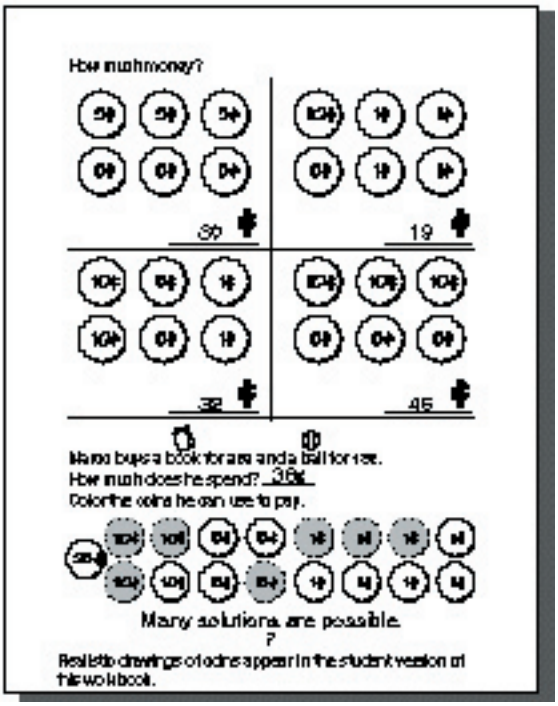

Label the dots.
+2
24
22
26
8
20
28
10
6
30
12
18
14
16
32
40
34
36
38
8

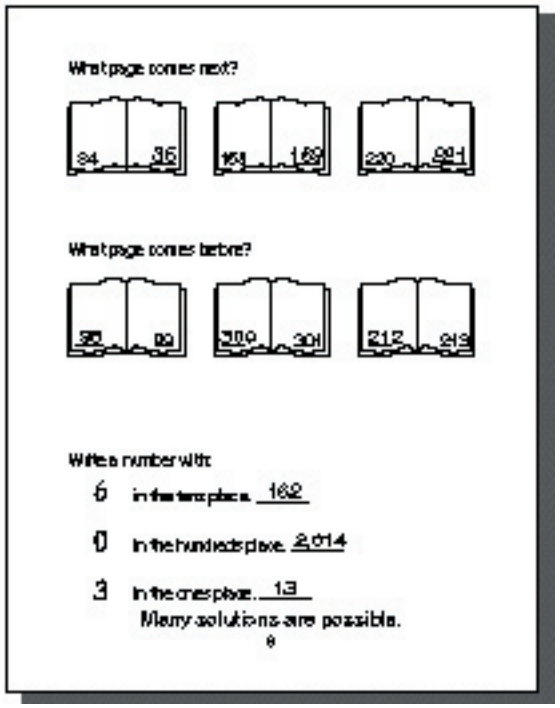

Write numbers on the Minkomputer.
18
46
36
87
Put these numbers on the Minkomputer.
29
54
71
63
9
Numbers are shown in standard configuration.
Other solutions are possible.

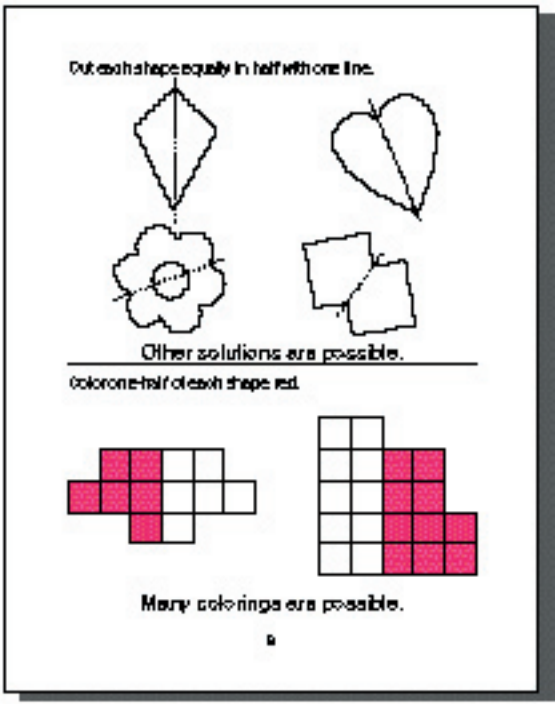

Label the dots.
+3
12
9
30
15
6
27
18
3
24
21
0
Complete.
5 +3 = 8
3 +10 = 13
30 +3 = 33
25 +3 = 28
36 +3 = 39
18 +3 = 21
26 +3 = 29
9 +7 = 16
9 +17 = 26
48 +3 = 51
4

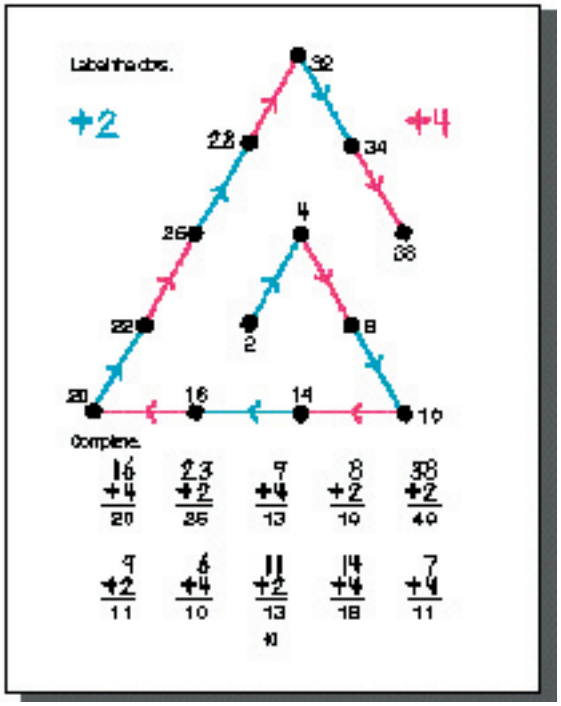

Label the dots on these number lines.
65 66 67 68 69 70 71
98 97 96 95 100 101 102
76 77 78 79 80 81 82
105 106 107 108 109 110 111
196 196 197 198 199 200 201
5

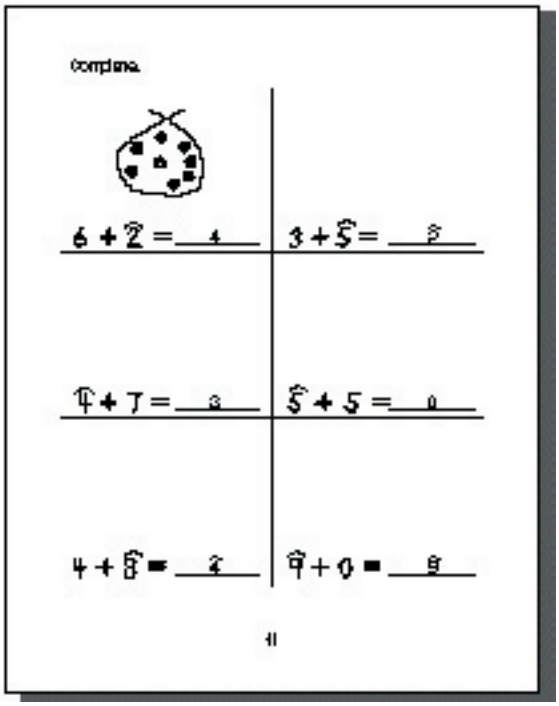

Label the dots.
−2
18
16
14
12
10
8
6
Complete.
12 −2 10
7 −2 5
13 −2 11
26 −2 24
15 −2 13
11 −2 9
21 −2 19
59 −2 57
20 −2 18
40 −2 38
6

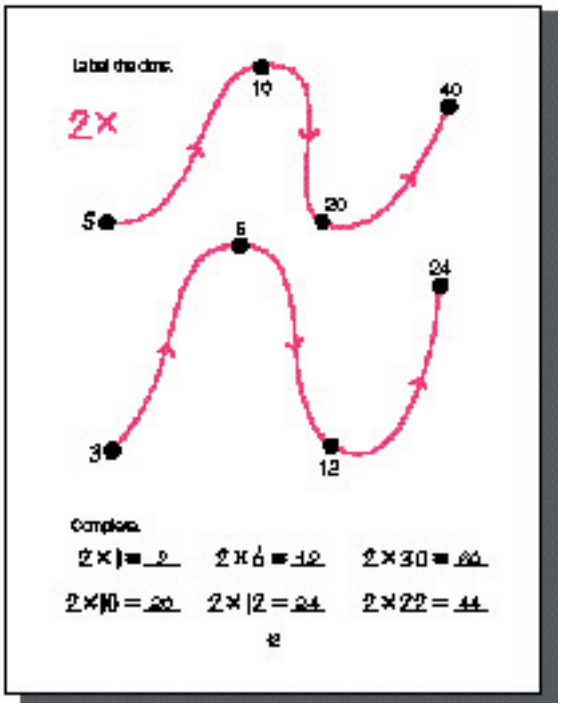

How much money?
89¢
19¢
32¢
45¢
Mario buys a book for 25¢ and a ball for 10¢.
How much does he spend? 35¢
Color the coins he can use to pay.
Many solutions are possible.
7
Realistic drawings of coins appear in the student version of this workbook.

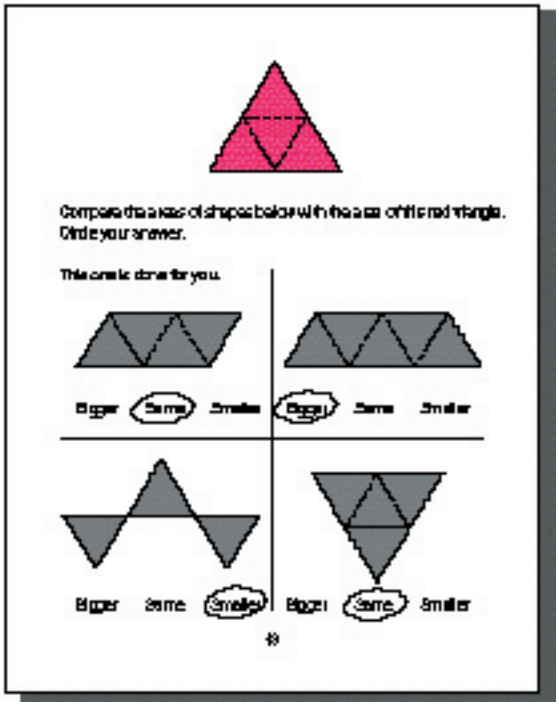

What page comes next?
84 85
150 151
220 221
What page comes before?
98 99
300 301
212 213
Write a number with:
6 in the tens place. 162
0 in the hundreds place. 2,014
3 in the ones place. 13
Many solutions are possible.
8

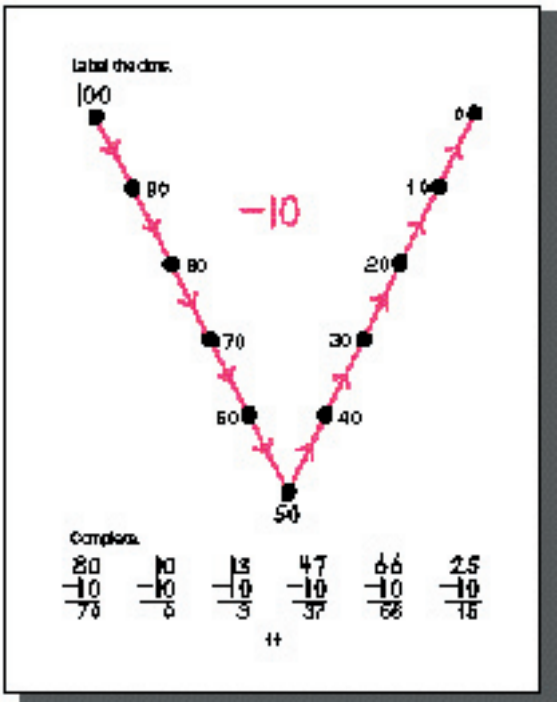

Cut each shape equally in half with one line.
Other solutions are possible.
Color one-half of each shape red.
Many colorings are possible.
9

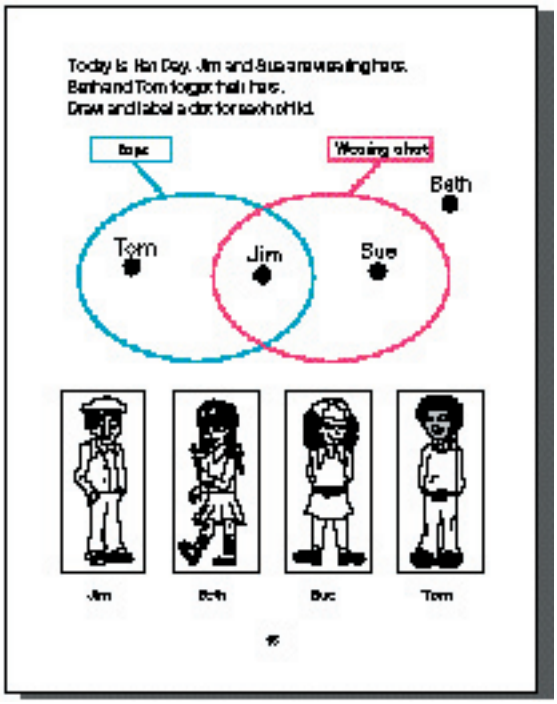

Label the dots.
+2
+4
32
28
34
26
4
38
22
2
8
20
16
14
10
Complete.
16 +4 = 20
23 +2 = 25
9 +4 = 13
8 +2 = 10
38 +2 = 40
9 +2 = 11
6 +4 = 10
11 +2 = 13
14 +4 = 18
7 +4 = 11
10

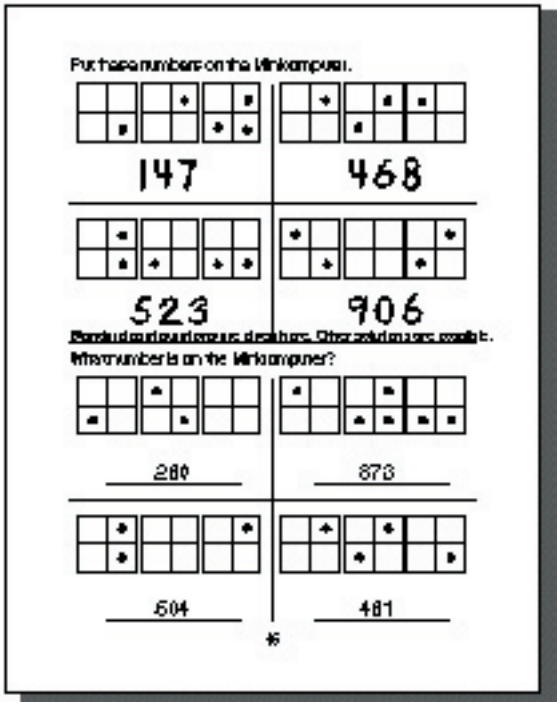

Complete.
6 + 2 = 4
3 + 5 = 8
4 + 3 = 7
5 + 5 = 10
4 + 8 = 12
9 + 0 = 9
11

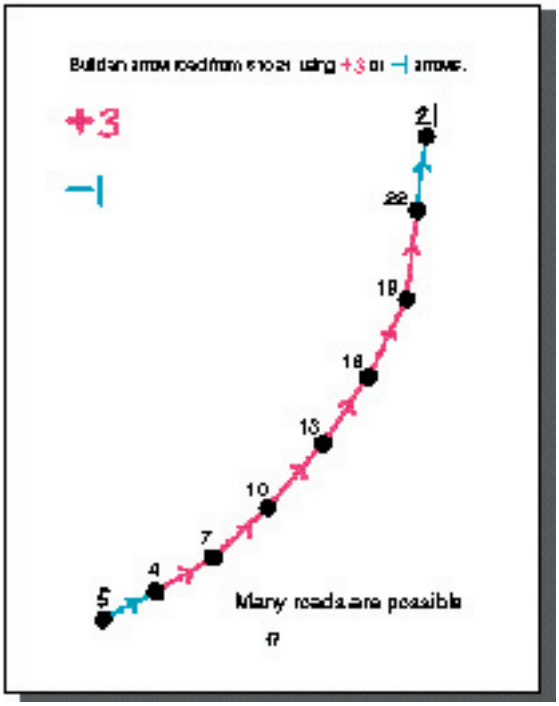

Label the dots.
2×
10
40
5
20
6
24
3
12
Complete.
2 × 1 = 2
2 × 6 = 12
2 × 30 = 60
2 × 10 = 20
2 × 12 = 24
2 × 22 = 44
12


Compare the area of shapes below with the area of this red triangle.
Circle your answer.
This one is done for you.
Bigger Same Smaller
Bigger Same Smaller
Bigger Same Smaller
Bigger Same Smaller
13


Label the dots.
100
90
80
70
60
50
40
30
20
10
0
−10
Complete.
80 − 10 = 70
10 − 10 = 0
13 − 10 = 3
47 − 10 = 37
66 − 10 = 56
25 − 10 = 15


Today is Hat Day. Jim and Sue are wearing hats.
Beth and Tom forgot their hats.
Draw and label a dot for each child.
Hat
Wearing a hat
Beth
Tom
Jim
Sue
Jim
Beth
Sue
Tom


Put these numbers on the Mirkomputer.
147
468
523
906
Which number is on the Mirkomputer?
280
873
504
481


Build an arrow road from 5 to 21 using +3 or −1 arrows.
+3
−1
5
4
7
10
13
16
19
22
21
Many roads are possible

Mr. Jumpet's class voted for which day to have show and tell.

Monday	11
Tuesday	7
Wednesday	6
Thursday	13
Friday	15

Which day had the most votes? Friday

Which day had the least votes? Tuesday

How many votes for Wednesday? 6

How many votes for the beginning of the week (Monday or Tuesday)? 7

How many votes for the end of the week (Thursday or Friday)? 13

How many votes altogether? 26

Which day would you vote for? Why? _____

Answers will vary.

8

Label the dots.

+10

18, 28, 38, 48, 58, 68, 78

Complete.

73 +10 = 83	10 +53 = 63	44 +10 = 54	10 +16 = 26	27 +10 = 37
19 +10 = 29	19 +20 = 39	65 +10 = 75	65 +30 = 95	36 +20 = 56

18

12

Compare the lengths of the paths below with this path.
Circle your answers.

Shorter Same Longer

Shorter Same Longer

Shorter Same Longer

Shorter Same Longer

Shorter Same Longer

20

Label the dots.

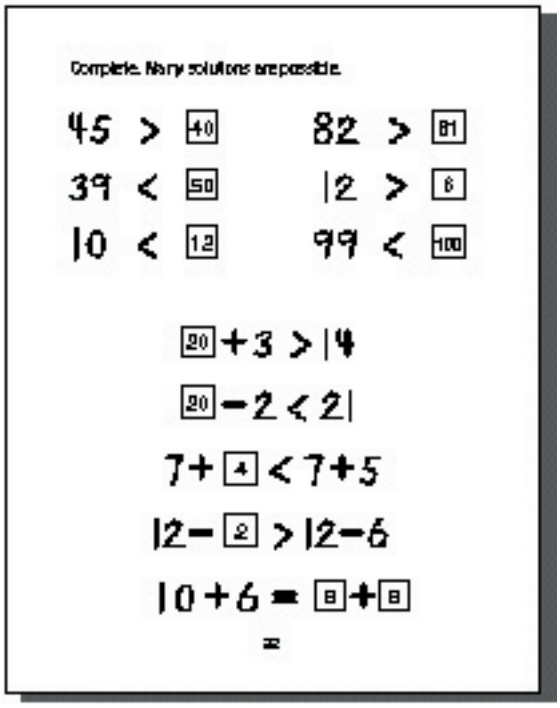
+5

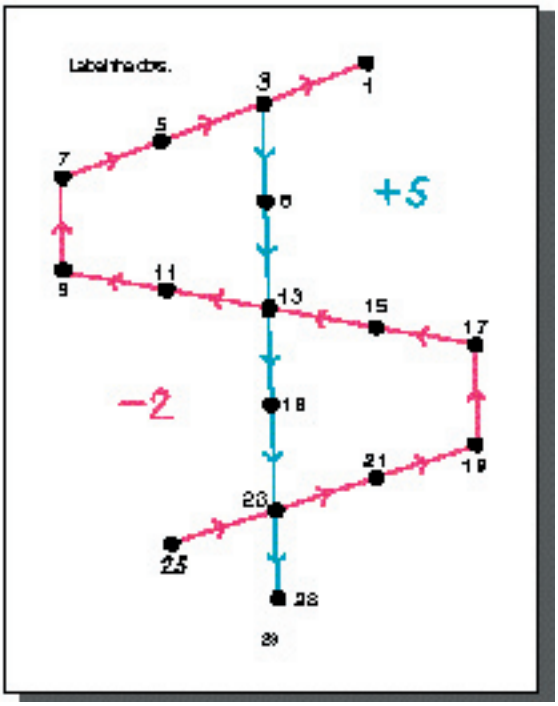
7, 12, 17, 22, 27, 32, 37, 42, 47, 52, 57

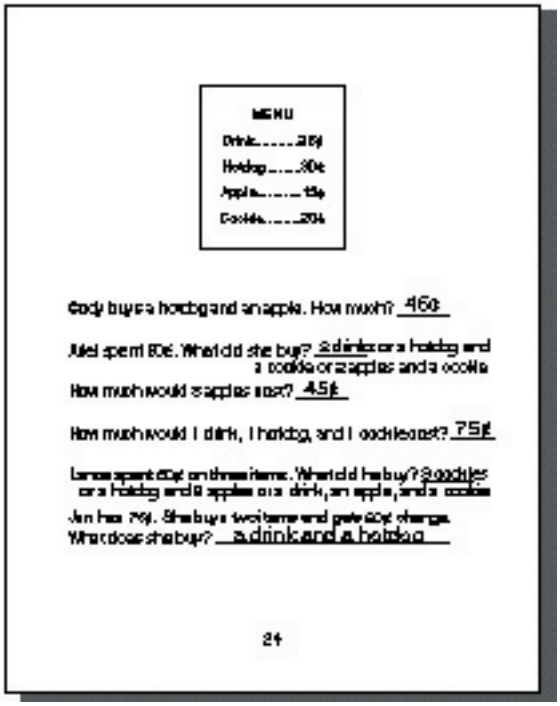
Complete.

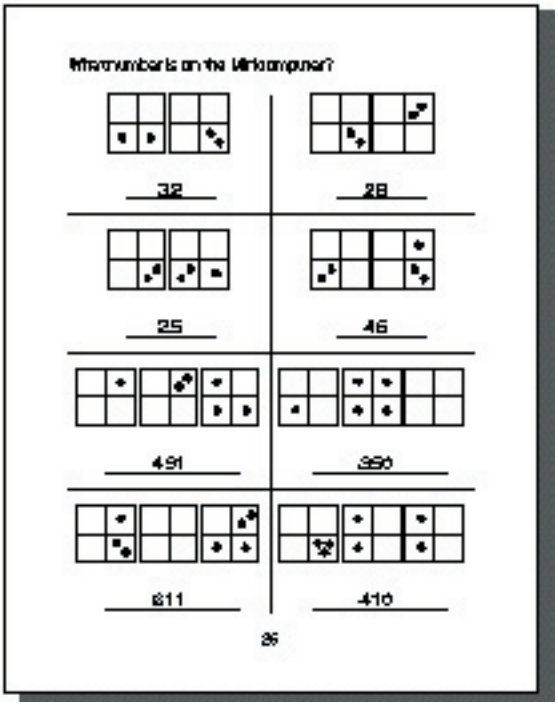
22 +5 = 27	13 +5 = 18	5 +15 = 20	35 +5 = 40	36 +5 = 41
57 +5 = 62	67 +5 = 72	67 +10 = 77	67 +15 = 82	107 +5 = 112

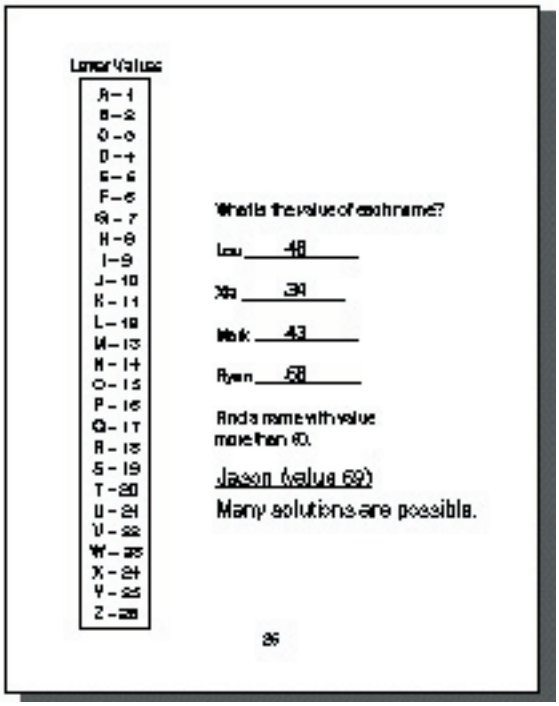
81


Complete. Many solutions are possible.
45 > 40
82 > 81
39 < 50
12 > 6
10 < 12
99 < 100
20 + 3 > 14
20 − 2 < 21
7 + 4 < 7 + 5
12 − 2 > 12 − 6
10 + 6 = 8 + 8


Label the dots.
+5
−2


MENU
Cody buys a hotdog and an apple. How much? 46¢
Abel spent 80¢. What did she buy? 2 drinks or a hotdog and a cookie or 2 apples and a cookie
How much would 3 apples cost? 45¢
How much would 1 drink, 1 hotdog, and 1 cookie cost? 75¢
Jan has 70¢. She buys two items and gets 10¢ change. What does she buy? a drink and a hotdog


What number is on the Wii computer?
32
28
25
46
491
950
811
410


Letter Values

A = 1
B = 2
C = 3
D = 4
E = 5
F = 6
G = 7
H = 8
I = 9
J = 10
K = 11
L = 12
M = 13
N = 14
O = 15
P = 16
Q = 17
R = 18
S = 19
T = 20
U = 21
V = 22
W = 23
X = 24
Y = 25
Z = 26

What is the value of each name?

Lou 48

Mia 34

Mark 43

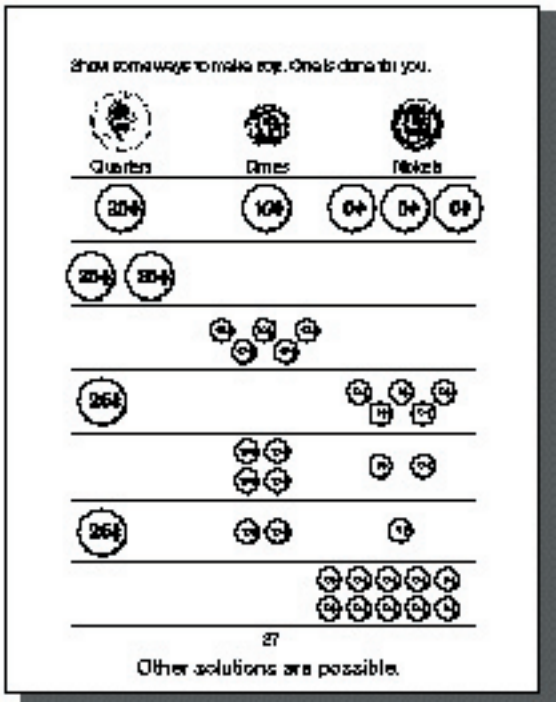
Ryan 58

Find a name with value more than 60.

Jason (value 69)

Many solutions are possible.

25


Show some ways to make 25¢. One is done for you.

Quarters Dimes Nickels

25¢ 10¢ 5¢ 1¢ 1¢

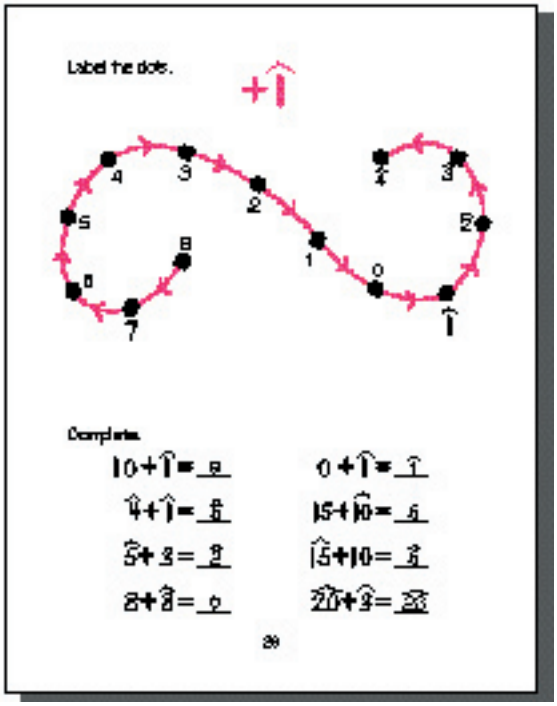
25¢ 25¢

25¢

25¢

27

Other solutions are possible.


Label the dots.

+1

Complete.

10 + 1 = 11

0 + 1 = 1

4 + 1 = 5

15 + 10 = 25

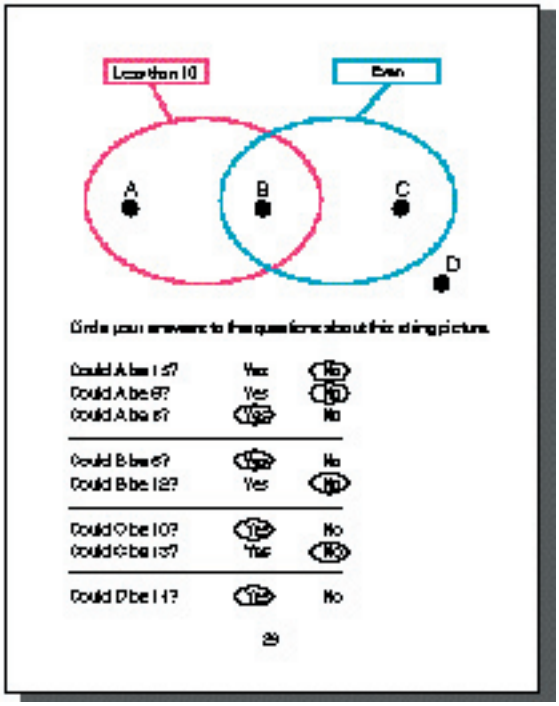
5 + 3 = 8

15 + 10 = 25

8 + 8 = 16

20 + 3 = 23

29


Less than 10

Even

A B C D

Circle your answers to the questions about this string picture.

Could A be 1? Yes No

Could A be 6? Yes No

Could A be 8? Yes No

Could B be 6? Yes No

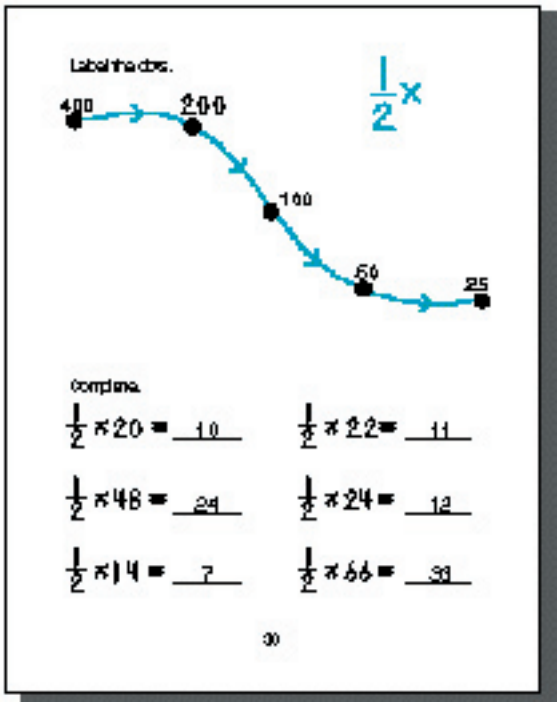
Could B be 12? Yes No

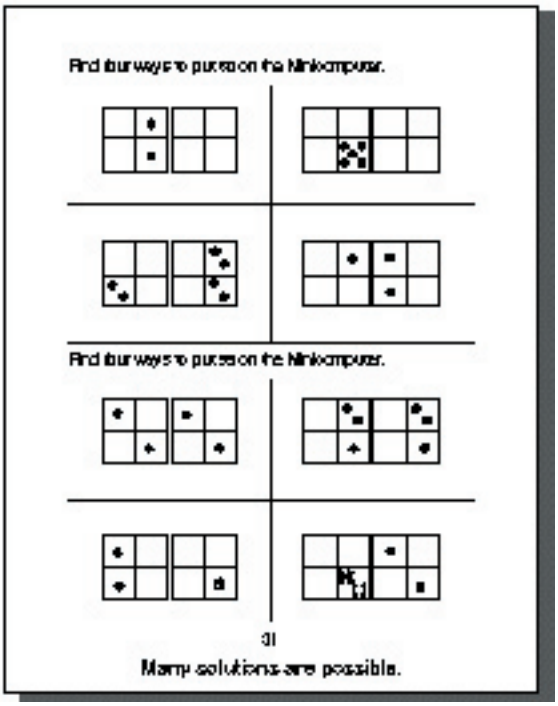
Could C be 10? Yes No

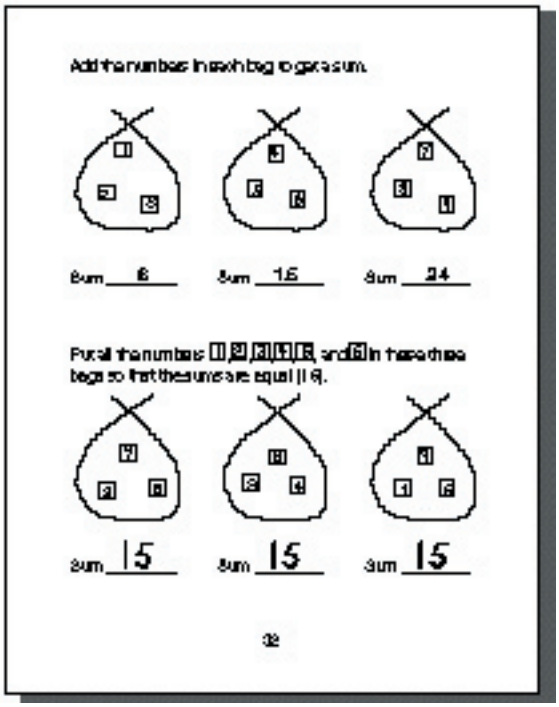
Could C be 13? Yes No

Could D be 14? Yes No

29


Label the dots.
400
200
100
50
25
$\frac{1}{2}\times$
Complete.
$\frac{1}{2}\times 20 = 10$
$\frac{1}{2}\times 22 = 11$
$\frac{1}{2}\times 48 = 24$
$\frac{1}{2}\times 24 = 12$
$\frac{1}{2}\times 14 = 7$
$\frac{1}{2}\times 66 = 33$
30


Find four ways to place on the Ninkompoop.
Find four ways to place on the Ninkompoop.
31
Many solutions are possible.


Add the numbers in each bag to get a sum.
Sum 6
Sum 15
Sum 24
Put all the numbers 1, 2, 3, 4, 5, and 6 in these three bags so that the sums are equal to 9.
Sum 15
Sum 15
Sum 15
32

Capsule Lesson Summary

Solve a detective story in which the clues involve the Minicomputer with positive and negative checkers, an arrow picture, and a string picture.

Materials

Teacher	• Minicomputer set • Colored chalk	• Paper • Colored pencils, pens, or crayons • Worksheet W3
Student	• Minicomputer set	

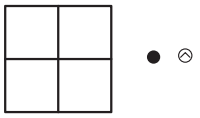
Description of Lesson

You may like to let students work with a partner during this lesson. Arrange that each pair of students has one individual Minicomputer sheet (two boards), one regular checker, and one negative checker. Write a large $\hat{1}$ on a slip of paper and fold the paper so that $\hat{1}$ is hidden.

T: ***I wrote the name of a secret number on this paper. Listen carefully to the clues and you can discover what number it is.***

Clue 1

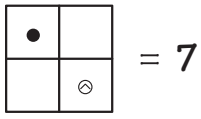
Display one demonstration Minicomputer board and place one regular checker and one negative checker near it.



T: ***The secret number can be put on the ones board of the Minicomputer using exactly one regular checker and one negative checker. What could the secret number be? Use your Minicomputer to help you decide.***

Allow several minutes for students to find many possible secret numbers. After a few minutes, check the numbers students have found, asking them to put the numbers on the demonstration Minicomputer using one regular and one negative checker. Record correct possibilities on the board, spacing the numbers appropriately so that they can be recorded in order. After three or four possibilities are recorded, ask the following questions.

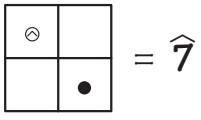
T: ***What is the greatest number that can be put on the ones board using exactly one regular and one negative checker?***



S: *7.*

T: ***What is the least?***

S: $\hat{7}$ (read as negative seven).



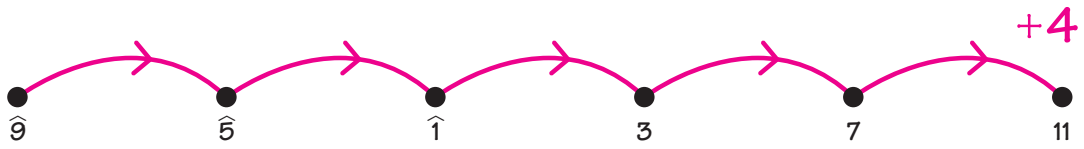
Continue until all the possible numbers are recorded. Be prepared to give suggestions if students have difficulty finding some of the numbers. Refer to Eli the Elephant and magic peanuts whenever it is helpful. Eventually, you should have this list on the board.

$\hat{7},\ \hat{6},\ \hat{4},\ \hat{3},\ \hat{2},\ \hat{1},\ 0,\ 1,\ 2,\ 3,\ 4,\ 6,\ 7$

T: ***Now we know that the secret number is one of these numbers, but we don't know which one. We need another clue.***

Clue 2

Draw the following arrow picture on the board, labeling only the dot for $\widehat{9}$. Ask students to copy the picture on their papers. Point to the dot for $\widehat{9}$ and trace the first arrow as you say, "Negative nine plus four is …?" If your class is not certain that the second dot from the left is for $\widehat{5}$, draw the appropriate picture of magic and regular peanuts and calculate $\widehat{9} + 4$. Instruct students to label the dots in their pictures and then continue to label the dots on the board.



T: ***The secret number*** (point to the list from Clue 1) ***is also in this arrow picture. Which numbers could be the secret number?***

S: ***$\widehat{1}$ is in the arrow picture and on the list.***

S: ***3 and 7 are in the arrow picture and on the list.***

T: ***Are any other numbers both on the list and in the arrow picture?***

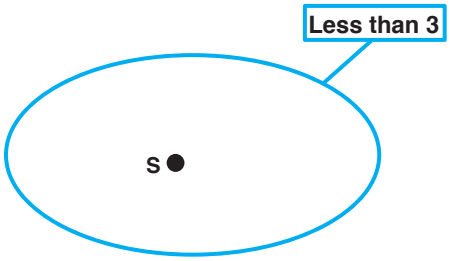
S: ***No.***

Erase the board except for these three numbers. $\widehat{1},\ 3,\ 7$

T: ***Now we know that the secret number is either $\widehat{1}$ or 3 or 7. There is one more clue.***

Clue 3

Draw this string picture on the board and instruct students to copy it.



T (pointing to **S**)**:** ***The secret number is inside this string. When you know the secret number, label the dot in your picture.***

Check the answers of most of the students (pairs) before asking someone to give the answer aloud.

T: ***What is the secret number?***

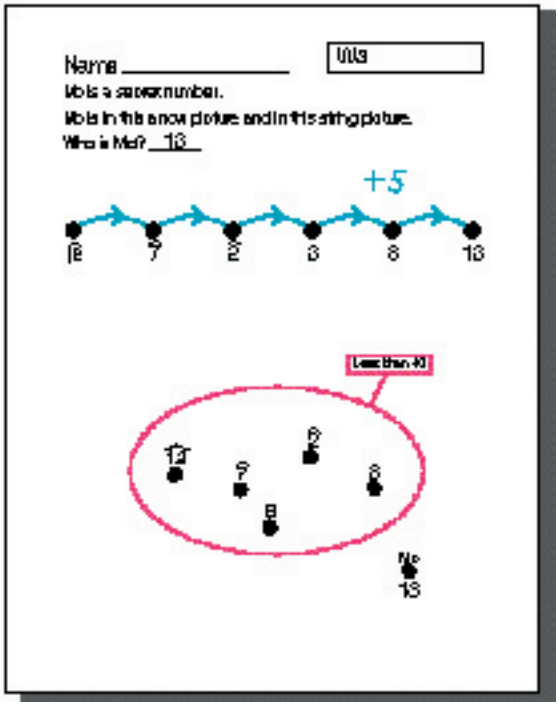
S: ***$\widehat{1}$.***

Reveal that $\widehat{1}$ is written on your paper.

Worksheet W3 is available for students (partners) to solve another detective story.

Writing Activity

Some students may enjoy trying to write a detective story.


Name
+5
13

Capsule Lesson Summary

Draw shapes on a grid all with area 9 ■ (grid squares). Compare the lengths of the borders of these shapes and determine which shape has the longest border and which shape has the shortest border. Begin work in the workbook *Length and Area*.

Materials

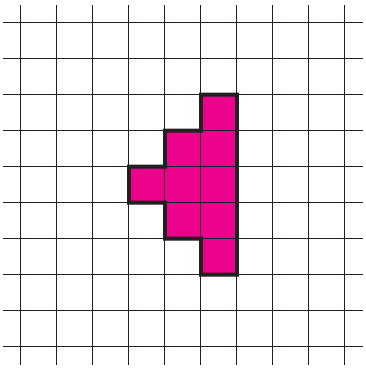
Teacher
- Grid board†
- Colored chalk
- Color tiles or paper squares
- Overhead projector (optional)
- Blackline W4

Student
- One-inch grid paper
- Colored pencils, pens, or crayons
- Nine color tiles
- *Length and Area* Workbook

Advance Preparation: Use Blackline W4 to make one-inch grid paper. If you use an overhead grid transparency, arrange that the color tiles or paper squares are the size of the grid squares.

Description of Lesson

Display a grid board and color in this shape on the board. Provide a heavy border on the shape. If your grid is on an overhead transparency, use color tiles to cover the shape.



T: ***What is the area of this shape? How many little squares?***

S: ***9 little squares.***

Write this information on the board.

Area = 9 ■

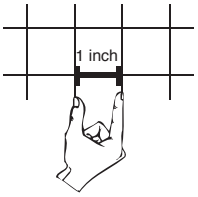
Then trace the border of the shape, removing the color tiles if necessary.

T: ***This is the border of my shape. I'd like to find out how long the border of this shape is. How can I do it?***

S: ***Count the blocks.***

T: ***Yes. Suppose that the side of one little square (or one block) is one inch.††***

Indicate the side of one square with your fingers and write "1 inch" on the board.



T: ***How long is the border of this shape in inches?***

Call on a student to do the counting.

S: ***16 inches.***

† See the "Notes on Grids" section in the introduction to the Geometry strand.

†† We are using *inch* in this lesson because most commercial color tiles are one-inch squares. You may prefer to use centimeter (cm) as the grids in the workbook have centimeter squares.

W4

Write this information on the board.

Area = 9 ■

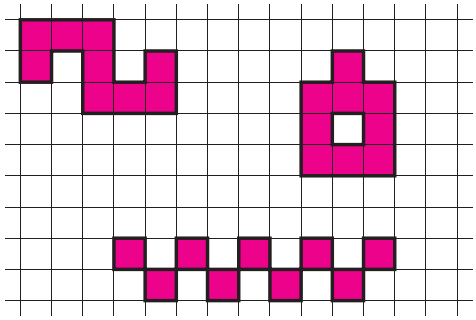
Length of border = 16 inches

Distribute one-inch grid paper, color tiles, and colored pencils.

T: ***On your paper, cover and then color in any shape you like that has area 9 ■. Then find the length of the border of your shape.***

Let the students work independently or with a partner for a few minutes. Help those students who have difficulty getting started. As necessary, tell students that the tiles covering the shape must touch at least at one point.

As you observe the work, find three students who have drawn shapes with different border lengths and ask them to copy their shapes on the demonstration grid board. For example, these three shapes might be put on the board.

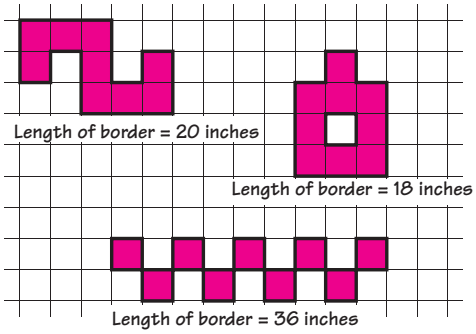


T: ***All three of these shapes on the board have area 9 little squares.***

Provide each shape with a heavy border as you say,

T: ***Let's look at the border of each shape. Which shape has the longest border? Which shape has the shortest border?***

Call on students to find the length of the border of each shape and write the information on the board. Then note the shape with the longest border and the shape with the shortest border.



Note: For a shape like the one at the upper right, be careful to count that part of the border which makes a hole in the center.

At this point you may like to extend the discussion of border length to observe that

- the border length is always an even number of inches;
- the shortest possible border length is 12 inches when the shape is a square;
- the longest possible border length is 36 inches; and
- every even number between 12 and 36 is a possible border length.

Collect the color tiles and distribute individual copies of the workbook *Length and Area*. Instruct students to work independently for the rest of the period. Observe that in this workbook the shapes are drawn on one-centimeter grids so the border length is in centimeters (cm). If many students have difficulty with a particular page, your may wish to have a full group discussion of that page.

At the end of the lesson, collect the workbooks for your review. They will be used again in Lesson W5.

Capsule Lesson Summary

Draw a shape with area 12 ■ (grid squares) on a grid board and find the length of its border. Then draw other shapes having the same area but with longer and shorter borders. Find that the shortest possible border is 14 inches and the longest possible border is 48 inches. Continue work in the *Length and Area* Workbook. (This is the second of two lessons using this workbook).

Materials

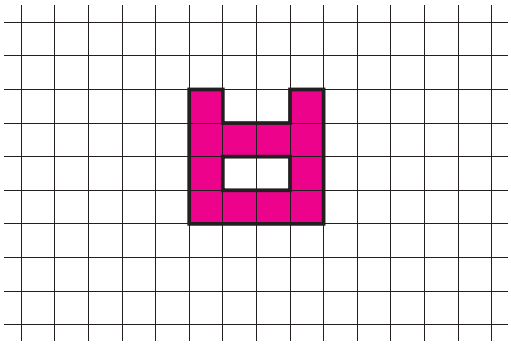
Teacher
- Grid board
- Colored chalk
- Color tiles
- Overhead projector (optional)

Student
- One-inch grid paper
- Colored pencils
- Twelve color tiles
- *Length and Area* Workbook

Advance Preparation: See Lesson W4 for notes about grids and color tiles.

Description of Lesson

Display a grid board and color in this shape on the board. Provide a heavy border on the shape. If your grid is on an overhead transparency, use color tiles to cover the shape.



T: ***What is the area of this shape? How many little squares?***

S: ***12 little squares.***

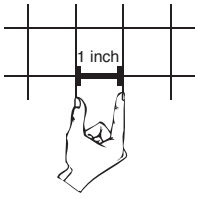
Write this information on the board.

Area = 12 ■

Then trace the border of the shape, removing the color tiles if necessary.

T (tracing the border of the shape)**:** ***I would like to find the length of the border of this shape. Suppose that the side of a little square is one inch.***

Indicate the side of one square with your fingers and write "1 inch" on the board.



T: ***How long is the border of this shape in inches?*** (24 inches)

Call on a student to do the counting. Be sure the student counts that part of the border around the hole in the center.

Write this information on the board.

Area = 12 ■

Length of border = 24 inches

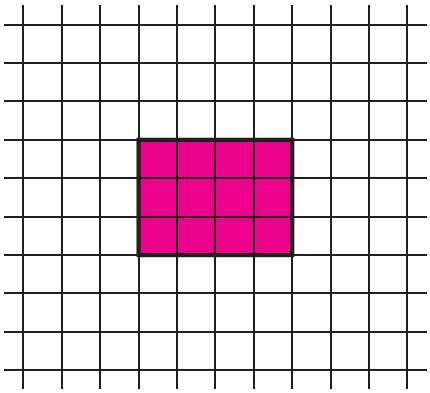
Distribute one-inch grid paper, color tiles, and colored pencils.

T: ***On your paper, color in a shape with area 12 but with a very short border. Try to find a shape that has a border shorter than 24 inches. Then color in another shape with area 12 but with a long border. Try to find a shape that has a border longer than 24 inches.***

Let the students work independently or with a partner for several minutes. Encourage students to experiment. As necessary, remind students that the tiles covering a shape must touch at least at one point. After a short while, call the class's attention back to the grid board.

T: ***Who has a shape with area 12 and with a border shorter than 24 inches?***

Let students announce their results and call on someone with the shortest possible border to put his or her shape on the board. A shape with the shortest possible border is illustrated below. Record the area and length of border information near the shape.

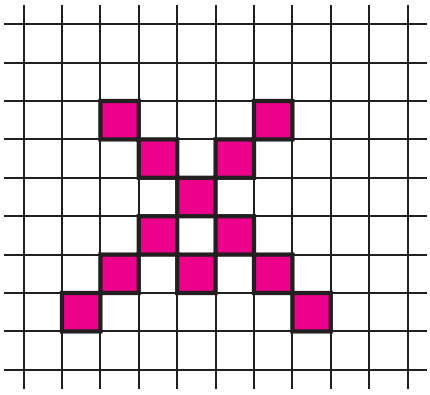


Area = 12

Length of border = 14 inches

T: ***Now, who has a shape with area 12 and with a border longer than 24 inches?***

Let students announce their results and call on someone with the longest possible border to put his or her shape on the board. A shape with longest possible border is shown below. Record the area and length of border information near the shape.

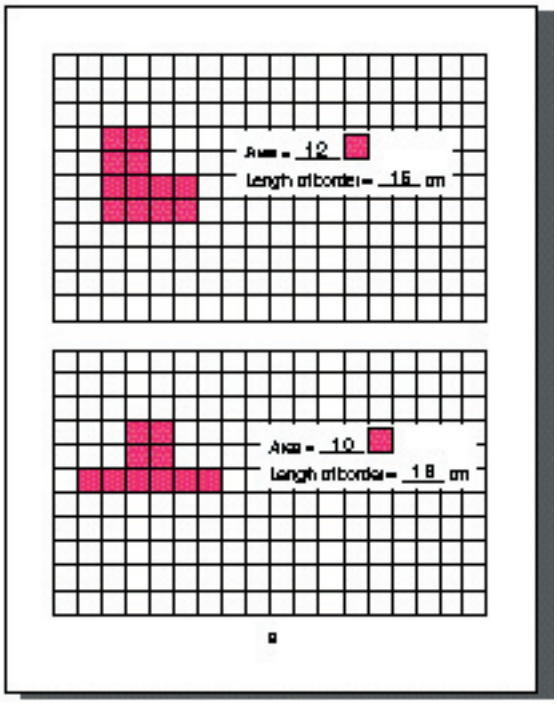


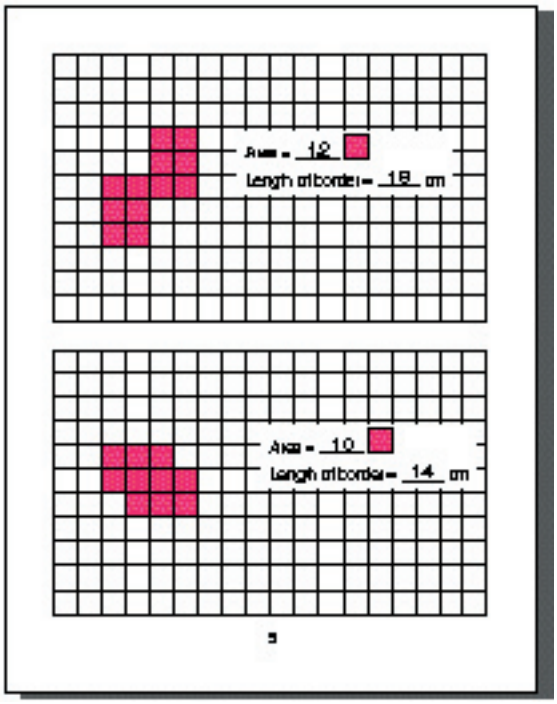
Area = 12

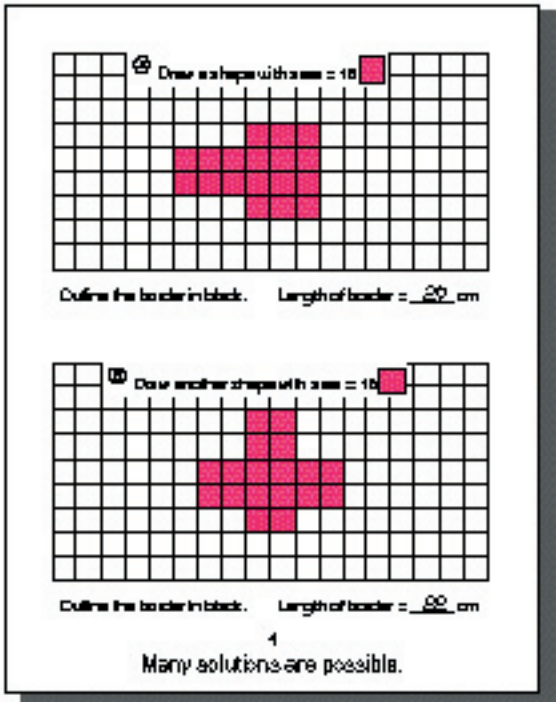
Length of border = 48 inches

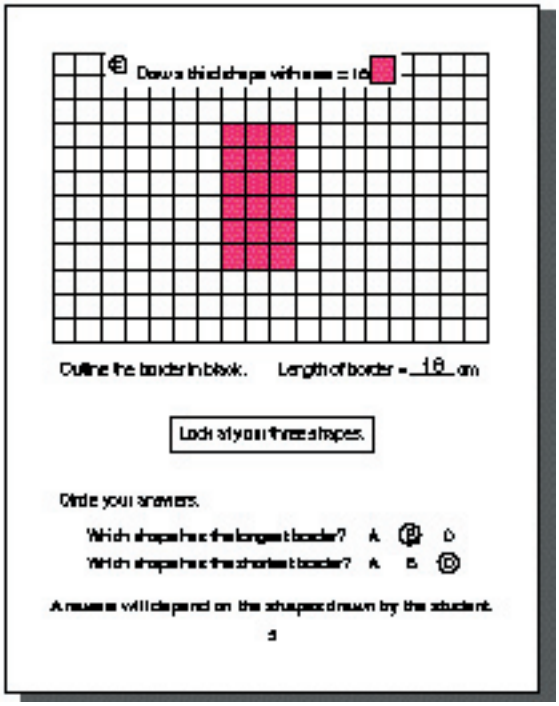
At this point you may again like to extend the discussion to observe that the border length is always an even number and that every even number between 14 and 48 is a possible border length. Return the students' copies of the workbook *Length and Area* and let them work independently for the rest of the period. If many students are having difficulty with a particular page, you may wish to have a collective discussion about that page.

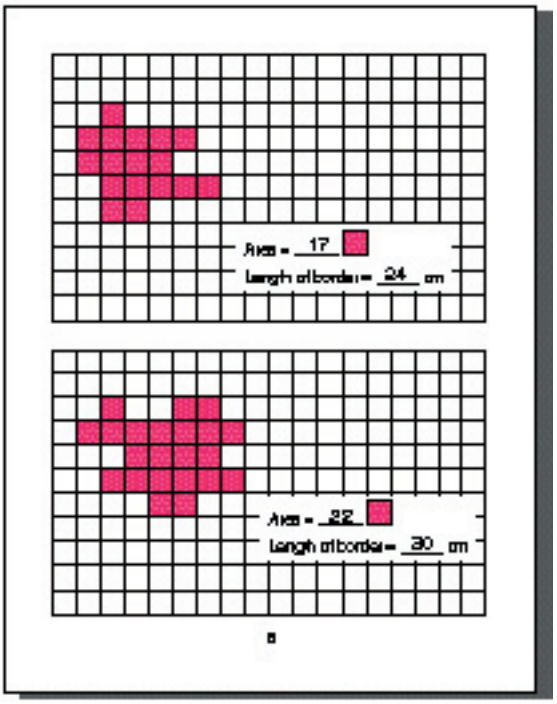
At the end of the lesson, collect the workbooks for your review.

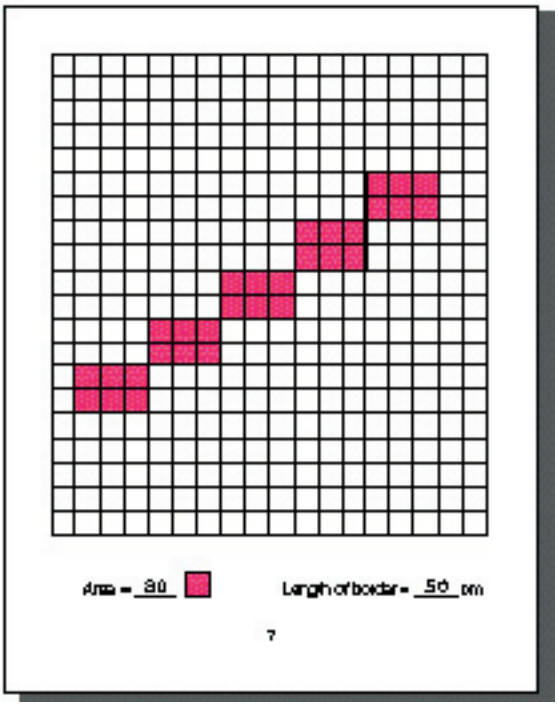

Area = 12
Length of border = 16 cm
Area = 10
Length of border = 18 cm

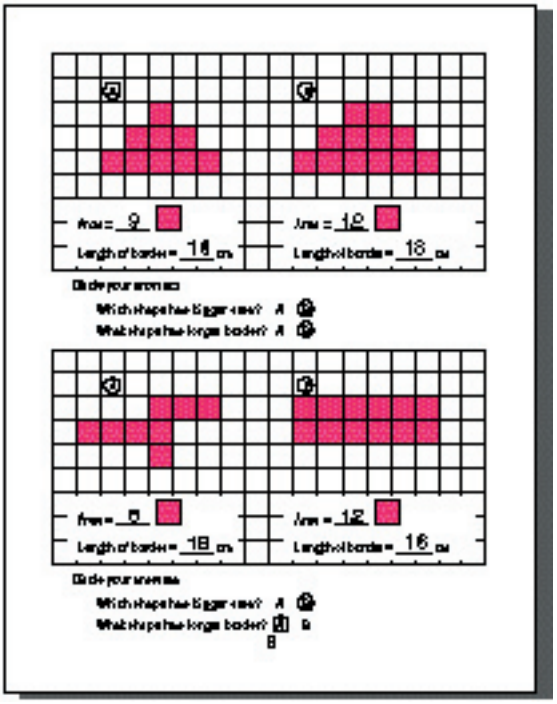

Area = 12
Length of border = 18 cm
Area = 10
Length of border = 14 cm

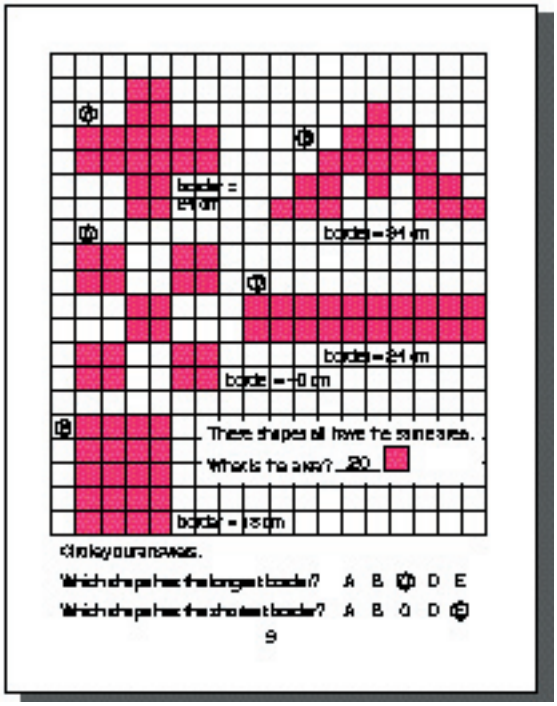

Outline the border in black. Length of border = 20 cm
Outline the border in black. Length of border = 20 cm
Many solutions are possible.

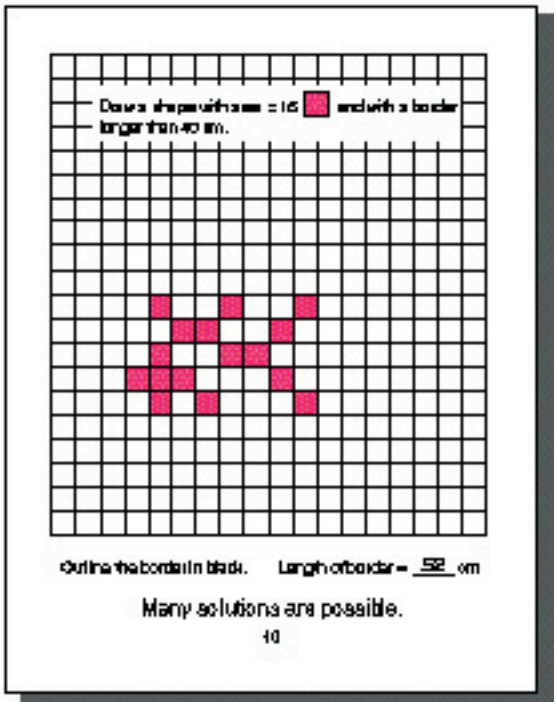

Outline the border in black. Length of border = 16 cm
Look at your three shapes.
Circle your answers.
Which shape has the longest border? A B C
Which shape has the shortest border? A B C

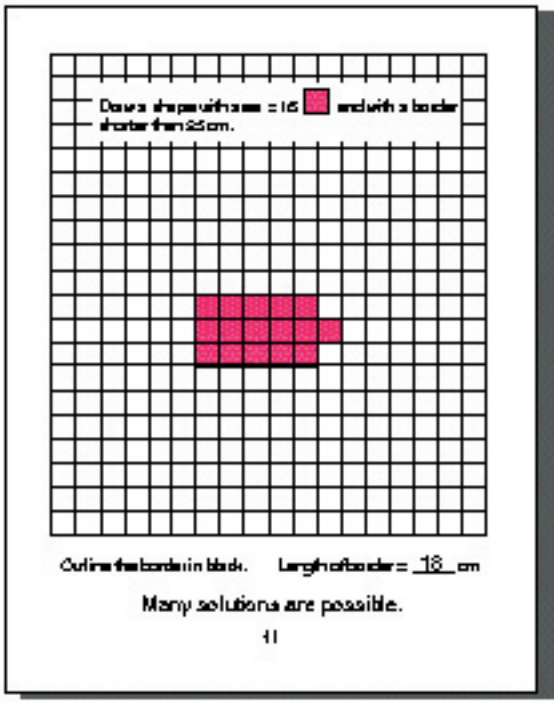

Area = 17
Length of border = 24 cm
Area = 22
Length of border = 30 cm
8

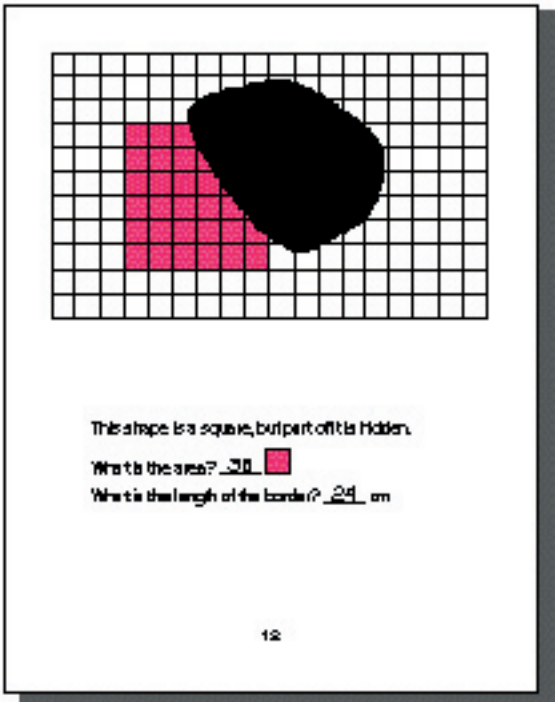

Area = 30
Length of border = 50 cm
7

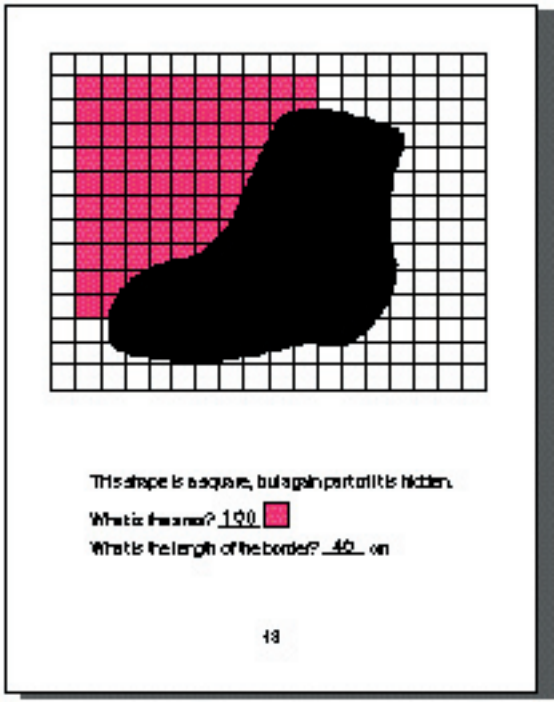

Area = 9
Length of border = 16 cm
Area = 10
Length of border = 18 cm
Circle your answers
Which shape has the bigger area? A
What shape has longer border? A
Area = 9
Length of border = 18 cm
Area = 12
Length of border = 16 cm
Circle your answers
Which shape has the bigger area? A
What shape has longer border? B
8

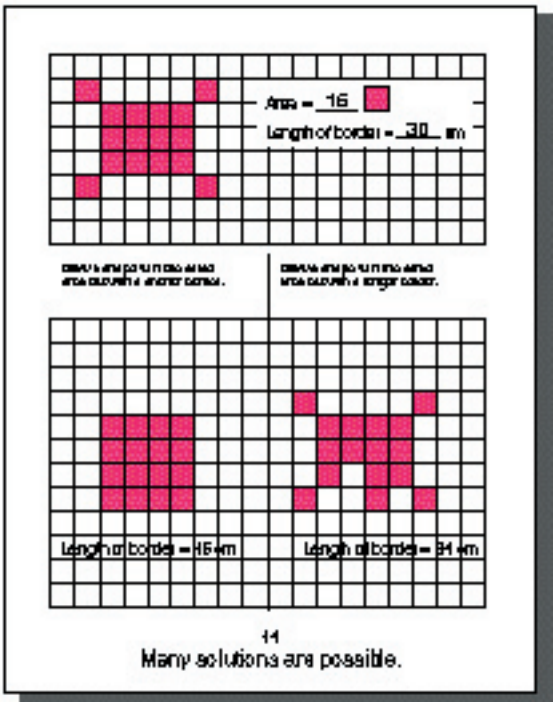

border = 24 cm
border = 24 cm
border = 24 cm
border = 18 cm
These shapes all have the same area.
What is the area? 20
border = 18 cm
Circle your answers.
Which shape has the longest border? A B C D E
Which shape has the shortest border? A B C D E
9

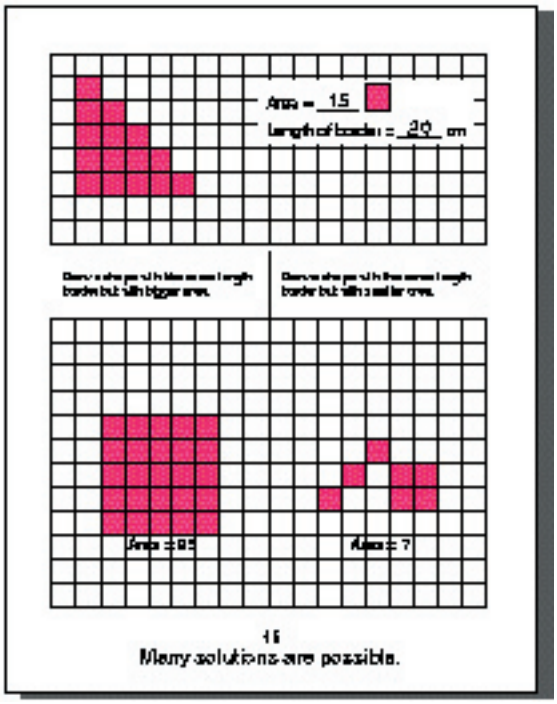

Outline the border in black. Length of border = 52 cm
Many solutions are possible.
10

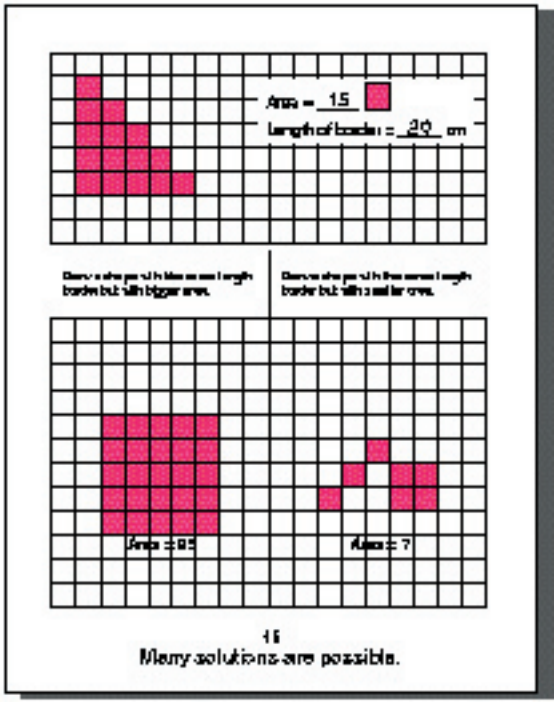

Outline the border in black. Length of border = 18 cm
Many solutions are possible.
11


This shape is a square, but part of it is hidden.
What is the area? 36
What is the length of the border? 24 cm
12


This shape is a square, but again part of it is hidden.
What is the area? 100
What is the length of the border? 40 cm
13


Area = 16
Length of border = 30 cm
Length of border = 46 cm
Length of border = 34 cm
14
Many solutions are possible.


Area = 15
Length of border = 20 cm
Area = 15
Area = 7
15
Many solutions are possible.


Area = 15
Length of border = 20 cm
Area = 15
Area = 7
15
Many solutions are possible.

Capsule Lesson Summary

Solve a detective story in which the secret number is not an even number and the other clues involve counting by 11 on the calculator, the order relation "is more than," and numbers on the Minicomputer with positive and negative checkers.

Materials

Teacher	Student
• Calculator • Colored chalk • Minicomputer set	• Calculator • Paper • Colored pencils, pens, or crayons • Minicomputer set • Worksheets W6*, **, ***, and ****

Description of Lesson

Provide each student or pair of students with a calculator. Allow several minutes for the students to experiment with the calculator before starting the lesson.

Write 99 on a slip of paper and fold it so the number is hidden. Announce to the class that there is a secret number on the paper, and they are going to be detectives to discover the secret number.

Clue 1

T: ***The calculator will help us with the first clue. Let's use the calculator to count by elevens starting at 0. The first clue is that the secret number will appear on the display. Turn on your calculators. What number is on the display?***

S: ***0.***

T: ***Listen carefully to my directions. Press*** [+] [1] [1] [=] (read as plus eleven equals)***. What number is on the display?***

S: ***11.***

Record 11 on the board.

T: ***Press*** [=] ***again. What number is on the display?***

S: ***22.***

Record 22 under 11 on the board.

T: ***What number will we get if we press*** [=] ***again? Press*** [=] ***. What number is on the display?***

S: ***33.***

Record 33 in the list forming on the board.

11
22
33
44

T: ***What number will we get next? Press*** ⊟ ***.***

S: ***44.***

Record 44 in the list.

T: ***Do you notice a pattern in this list of numbers?***

S: ***Here*** (pointing to the tens column) ***the numbers go 1, 2, 3, 4, and here*** (pointing to the ones column) ***they also go 1, 2, 3, 4.***

Ask students to continue predicting and verifying on the calculator multiples of 11 (in order) until the list includes 99.

Call on several students to predict what the next multiple of 11 is before letting the students press ⊟ again. Start a second column beginning with 110. Continue in the same manner at least until your list includes 143. Put three dots under the last number in your list.

11	110
22	121
33	132
44	143
55	
66	⋮
77	
88	
99	

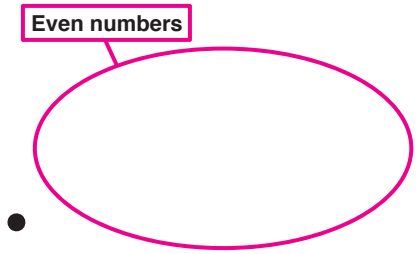
T: ***Why did I put three dots?***

S: ***To show we could go on and on with more numbers.***

Put the calculators away or ask the students to turn them off.

Clue 2

Draw this string picture on the board and point to the dot outside the string.

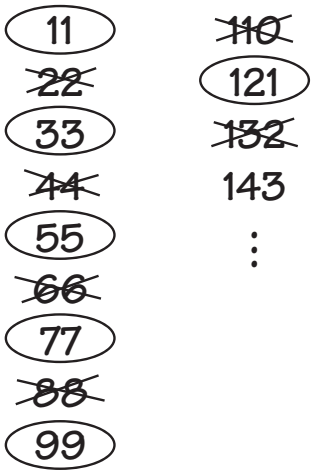


T: ***Our second clue is that the secret number is not an even number. It is here in this picture. Tell me which of the numbers in our list could be the secret number and which could not be the secret number.***

You may need to have a little discussion to remind students what even numbers are.

Note: It is possible that a student may comment that all numbers which are not even are odd. This is a natural conclusion since the students' experience is almost entirely in the realm of the integers and they are not as well-acquainted with non-integer rational numbers (such as 0.5, ⅓, 6.32, and so on) in the second grade. In such a case, rephrase the student's statement like this: "All the numbers in this list are either even or odd."

As students indicate, cross out even numbers in the list (those that could not be the secret number) and circle odd numbers (those that could be the secret number).



T: ***Does anyone see a pattern?***

It may be difficult for students to verbalize that every other number in the list is an even number; they may need prompting.

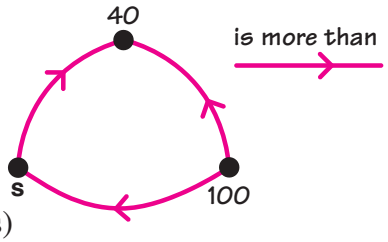
T: ***Should we cross off the three dots?***

S: ***No, because there are more odd numbers (that are multiples of 11).***

Erase the crossed-out numbers (but not the three dots) before going on to the third clue.

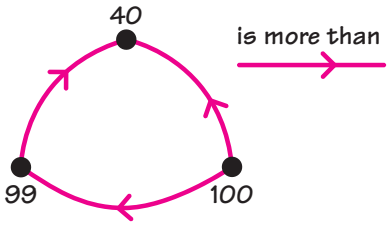
Clue 3

Draw this arrow picture on the board and ask the students to copy it on their papers.



T: ***Where is the secret number in this picture?*** (At **s**)
What are the red arrows for? (Is more than)
Is 100 more than 40 (trace the arrow from 100 to 40)***?*** (Yes)
Which of these numbers (point to the list) ***could be the secret number*** (point to **s**)***?***

Choose one number a student suggests, label **s** with that number, and then check the resulting picture. Suppose a student suggests that 99 could be the secret number. Trace the arrow from 99 to 40.



S: ***99 is more than 40.***

T: ***Is that true?*** (Yes)

Trace the arrow from 100 to 99.

S: ***100 is more than 99.***

T: ***Is that true?*** (Yes) ***99 could be the secret number.***

Circle 99 in the list.

Suppose a student incorrectly suggests that 33 could be the secret number. Trace the arrow from 33 to 40.

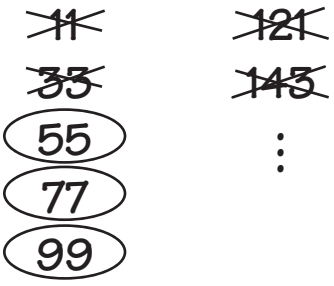
S: ***33 is more than 40.***

T: ***Is that true?*** (No)
Could the secret number be 33? (No)

Cross off 33 in the list.

W6

Allow students to work independently or with a partner to check other numbers. Then continue as a class until all the numbers in your list are either crossed out or circled.



T: ***Can we cross off the three dots?***

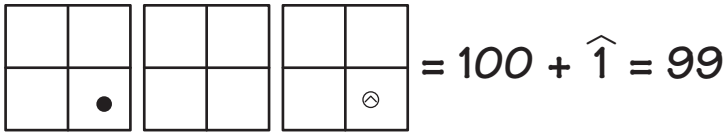
S: ***Yes, because the three dots are for numbers more than 100.***

Cross off the three dots. Erase everything that has been crossed off.

Clue 4

Display three Minicomputer boards. Hold up one regular checker and one negative checker.

T: ***We know that the secret number is 55 or 77 or 99. The last clue is this: The secret number can be put on the Minicomputer with one regular and one negative checker.***

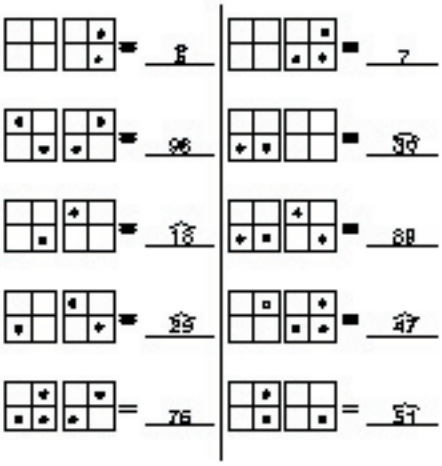


Allow student pairs to work with an individual Minicomputer to try to find the secret number. Ask the students to write the secret number on a piece of paper for you to check. Then ask someone to give the answer aloud and to put it on the Minicomputer using one regular and one negative checker. Decide with the class that neither 55 nor 77 can be put on the Minicomputer using one regular and one negative checker.

Worksheets W6*, **, ***, and **** are available for individual work. You may like to allow students to work with a partner especially on the *** and **** worksheets.

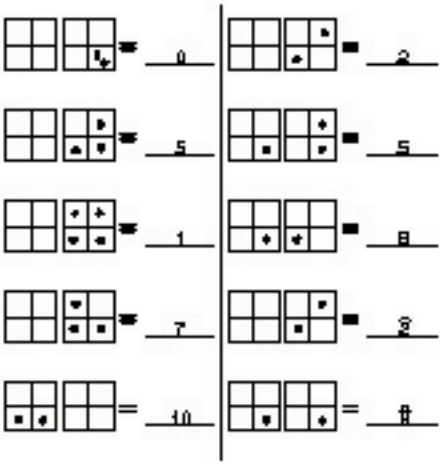
Name _____ We ★

Write number is on the Minicomputer?



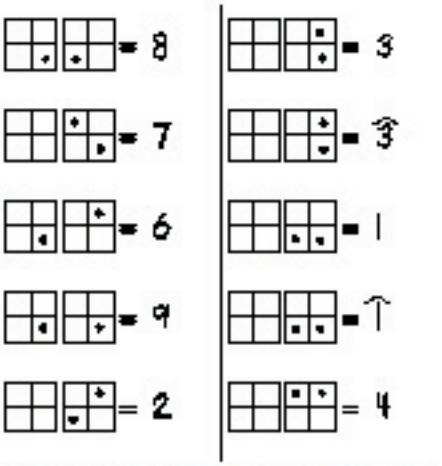
Name _____ We ★★

Write number is on the Minicomputer?



Name _____ We ★★★

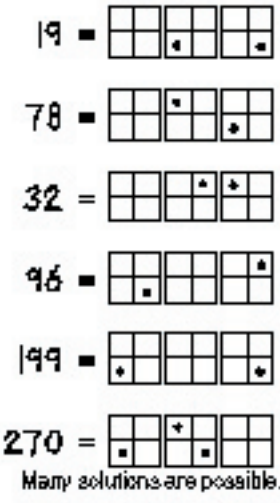
Put these numbers on the Minicomputer using one positive and one negative checker. One is done for you.



Other solutions are possible for some numbers.

Name _____ We ★★★★

Put these numbers on the Minicomputer. Use at least one negative checker for each number.



Many solutions are possible.

Capsule Lesson Summary

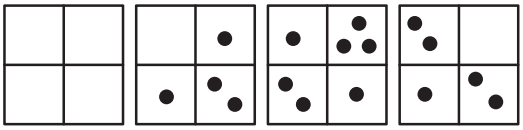
Explore the effect of moving, removing, or adding checkers to a configuration on the Minicomputer—in each case, does the numerical value increase, decrease, or stay the same? Estimate a number on the Minicomputer and then make trades, stopping periodically to get better estimates, until standard configuration is obtained. Review and extend experiences from various strands in the workbook *Caravan of Problems #2*.

Materials

Teacher	Student
• Minicomputer set	• Index card • *Caravan of Problems #2* Workbook • Colored pencils, pens, or crayons

Description of Lesson

Put this configuration on the Minicomputer.



Lead the class to estimate the number.
A possible dialogue is given here.

T: ***Is the number on the Minicomputer more than 500? How do you know?***

S: ***Yes, there is a checker on the 400-square and a checker on the 200-square. 400 + 200 = 600 and 600 is more than 500.***

T: ***Is this number more than 800? How do you know?***

S: ***Yes, there is 800 on the hundreds board and there are more checkers on the other boards.***

T: ***Is this number more than 900? How do you know?***

S: ***The number on the hundreds board is 800 and there is more than 100 on the tens board (80 + 20 = 100), so this number is more than 900.***

T: ***Is this number more than 1,000? How do you know?***

It is possible that a student will be able to explain and to convince the class that this number is more than 1,000. In any case, your class should estimate the number to be more than 900 and possibly more than 1,000.

T: ***We know that this number is more than 900 (1,000). We do not need to know exactly what number is on the Minicomputer to compare it to other numbers.***

Write these words on the board close to the Minicomputer and ask students to write them on their index card.

More
Same
Less

T: ***I am going to move, remove, or add some checkers. Each time, tell me if the number on the Minicomputer is more than, the same as, or less than before.***

Move a checker from the 1-square to the 10-square. Decide on a method for students to show whether they believe the new configuration is for a number more than, the same as, or less than before. For example, they could hold up the index card pinching it on their choice.

Repeat the move very obviously if many students do not know that the number on the Minicomputer is more than the previous number.

T: ***How much more is this number than the number we had before? How do you know?***

S: ***It is 9 more, because you moved the checker from the 1-square to the 10-square.***

Return the checker to its original position. Continue this activity with the following or similar moves. After each move return the checkers to their original positions.

- Make a 100 + 100 = 200 trade. (Same)
- Move a checker from the 8-square to the 4-square. (4 less)
- Move a checker from the 2-square to the 8-square. (6 more)
- Move a checker from the 20-square to the 80-square. (60 more)
- Move a checker from the 200-square to the 800-square. (600 more)
- Remove two checkers from the 8-square and put one checker on the 4-square. (12 less)

After you make several moves yourself, invite students to transform the number on the Minicomputer. After each move, return the checkers to their original positions.

At the end of this activity you should have the same configuration you started with on the Minicomputer. Remind students that they estimated the number to be more than 900 (or 1000). Allow students to guess what the number is and record their guesses on the board. Then invite some students to make trades that will make the number easier to read. After several trades are made, guide the class to make a closer estimate. Continue making trades until the standard configuration is obtained. Invite a student to write the number below (or above) the Minicomputer and decide which guess is the closest.

Distribute copies of the workbook *Caravan of Problems #2* and let the students work independently for the rest of the class period. If many students are having difficulty with a particular page, you may wish to have a collective discussion about that page.

At the end of the lesson, collect the workbooks for your review. They will be used again in Lesson W8.

Capsule Lesson Summary

Solve the addition problem 437 + 283 = ? collectively. Continue working in the workbook *Caravan of Problems #2*. (This is the second of two lessons using this workbook.)

Materials

Teacher	• None	Student	• *Caravan of Problems #2* Workbook • Colored pencils, pens, or crayons

Description of Lesson

Write this problem on the board.

437 + 283 =

T: ***How can we calculate 437 + 283?***

Perhaps someone will suggest writing the problem vertically.

```
 437
+283
```

T: ***What do you see in the ones column?***

S: ***7 and 3.***

T: ***7 + 3 = …?***

S: ***10.***

T: ***Where do we show the 10?***

S: ***In the tens column by writing 1 above 3.***

```
  1
 437
+283
   0
```

T: ***What should I write below the ones column?***

S: ***0, because there are no extra ones.***

T: ***What do you see in the tens column?***

S: ***1, 3, and 8.***

T: ***1 ten + 3 tens + 8 tens = …?***

S: ***12 tens.***

T: ***Where do we show the 12 tens?***

S: ***10 tens is 1 hundred, so we write a 1 in the hundreds column.***

T: ***What should I write below the tens column?***

S: ***2, because there are two extra tens.***

```
  1 1
 437
+283
  20
```

T: ***What do you see in the hundreds column?***

S: ***1, 4, and 2.***

T: ***1 hundred + 4 hundreds + 2 hundreds = …?***

S: ***7 hundreds.***

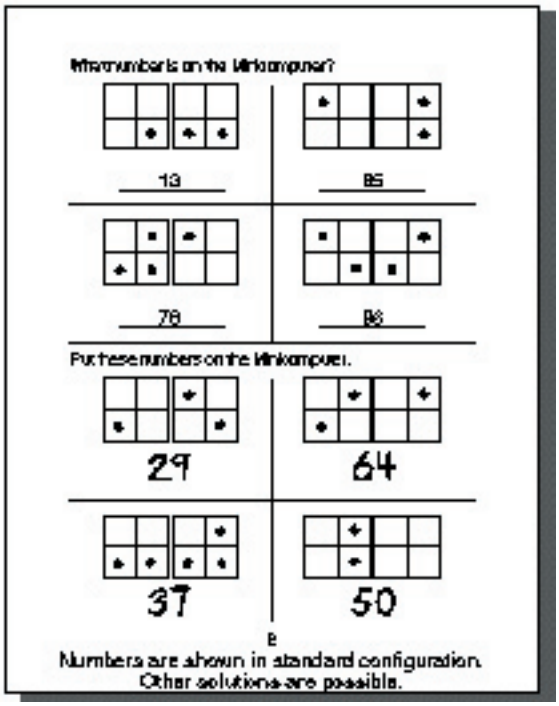
Conclude that 437 + 283 = 720.

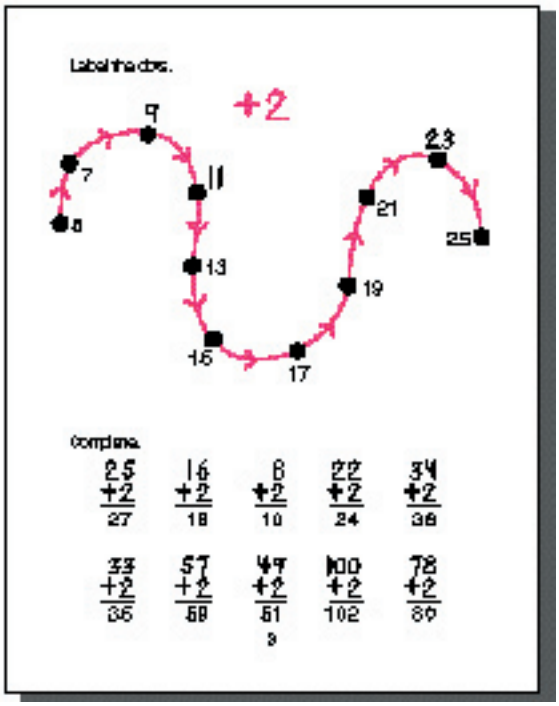
$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 437 \\ +\ 283 \\ \hline 720 \end{array}$$

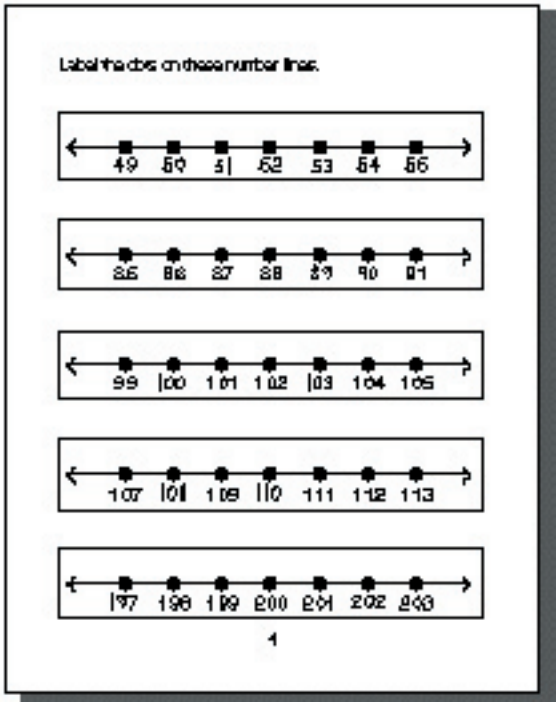
Distribute students' copies of the workbook *Caravan of Problems #2*. Ask students first to correct or complete pages from the previous week's work on this workbook and then to continue. You may wish to discuss collectively some problems that were difficult for many students the first week. At the end of the class period, collect the workbooks for your review.

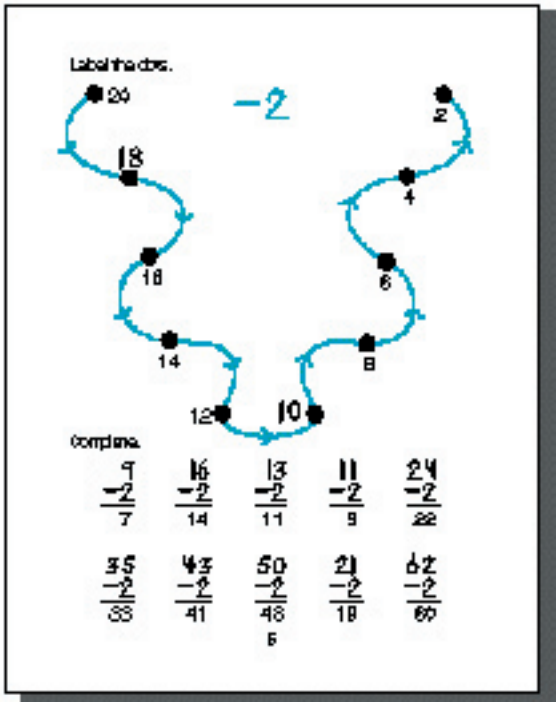
Assessment Activity

An individual student progress record for the workbook is available on Blackline W8. You may like to use this form to monitor student work.

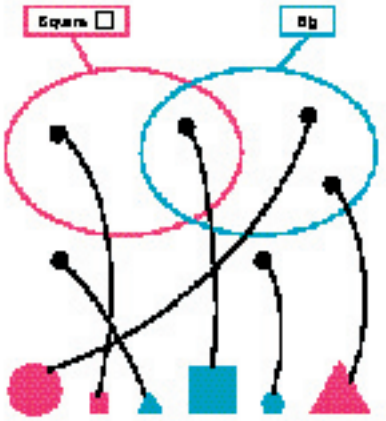

What numbers are on the Mini-computer?
13
85
78
86
Put these numbers on the Mini-computer.
29
64
37
50
8
Numbers are shown in standard configuration.
Other solutions are possible.


Label the dots.
+2
9
7
11
5
13
23
21
25
19
15
17
Complete.
25 +2 = 27
16 +2 = 18
8 +2 = 10
22 +2 = 24
34 +2 = 36
33 +2 = 35
57 +2 = 59
49 +2 = 51
100 +2 = 102
78 +2 = 80
9


Label the dots on these number lines.
49 50 51 52 53 54 55
86 88 87 88 89 90 91
99 100 101 102 103 104 105
107 101 109 110 111 112 113
197 198 199 200 201 202 203
4


Label the dots.
-2
20
2
18
4
16
6
14
8
12
10
Complete.
9 -2 = 7
16 -2 = 14
13 -2 = 11
11 -2 = 9
24 -2 = 22
35 -2 = 33
43 -2 = 41
50 -2 = 48
21 -2 = 19
62 -2 = 60
5

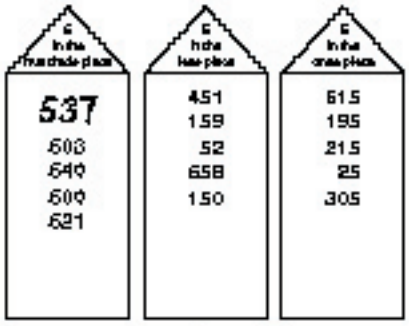
Match the dots with ABblocks. One is done for you.



5

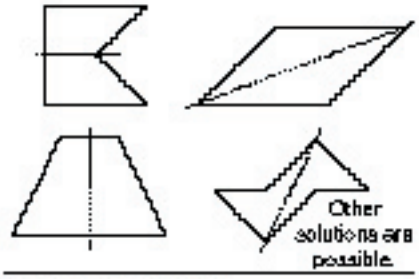
Put these numbers in the correct houses. One is done for you.

~~537~~ 52 25
615 540 215
451 658 150
503 195 305
159 500 521

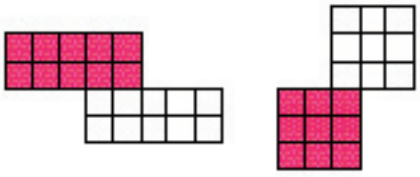


7

Put each shape equally in half with one line.



Color one-half of each shape red.



Other colorings are possible.

8

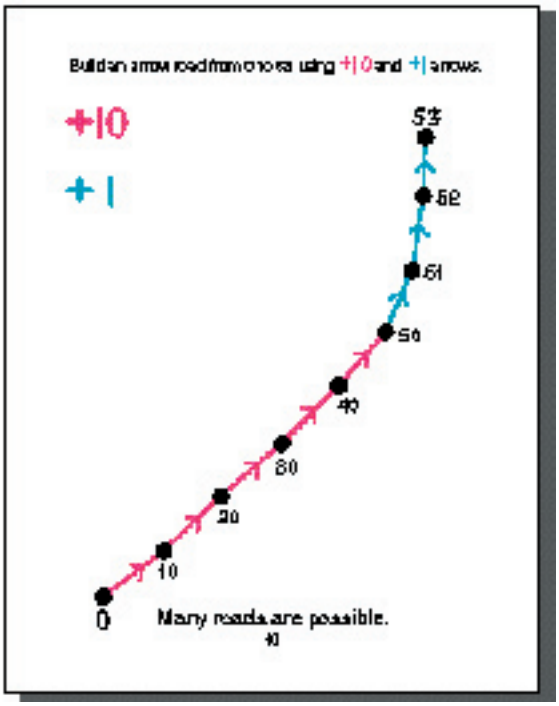
Solve these problems. You may draw pictures or use the MM computer.

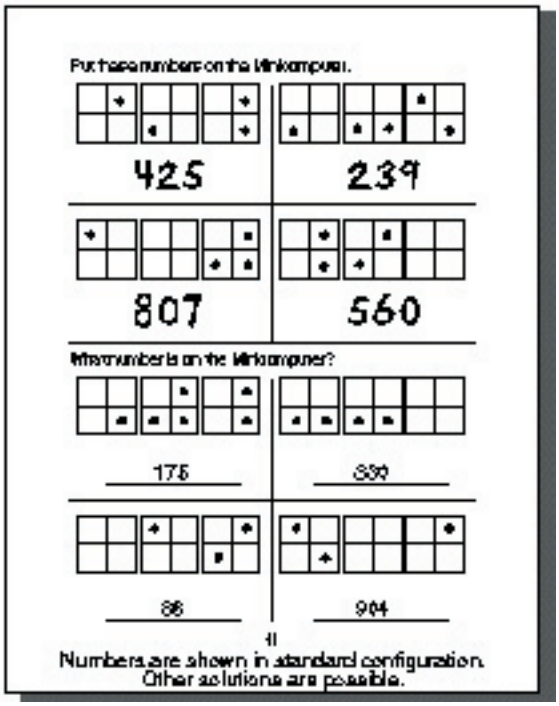
Julie has 4 packages of pencils. Each package has 8 pencils. How many pencils in all? 32

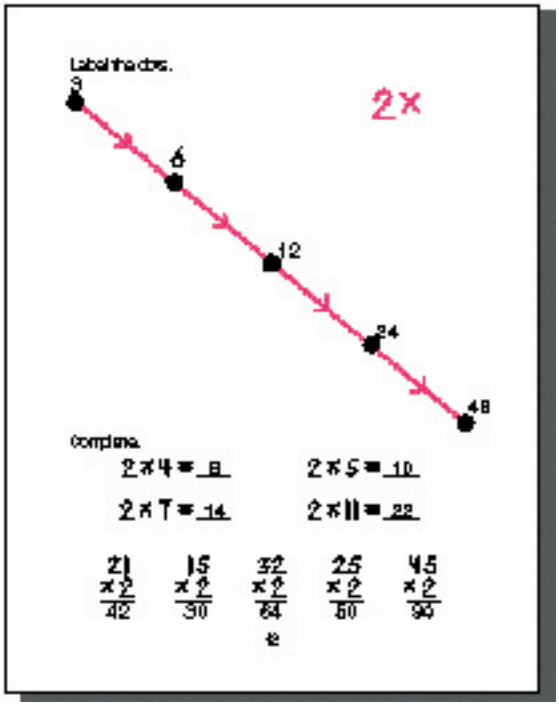
Julie took 27 flowers to the parade. He gave 15 flowers to teachers. How many flowers does he have left? 12

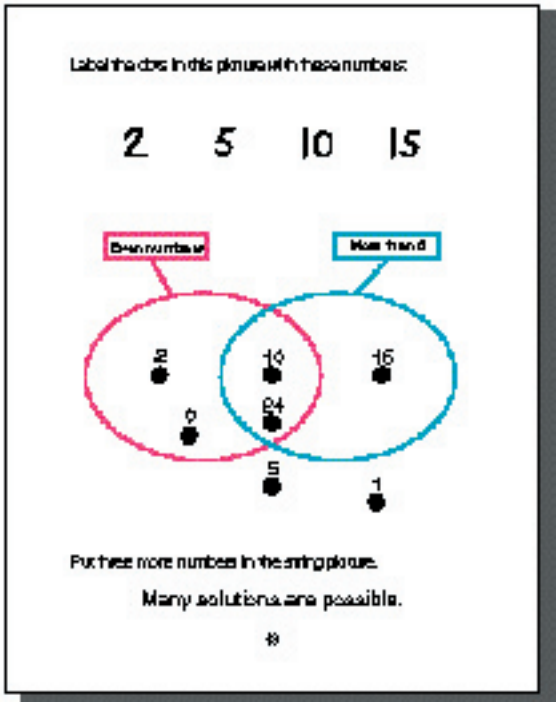
Ms. Thomas wants to share 30 bones equally among her 6 dogs. How many bones for each dog? 6

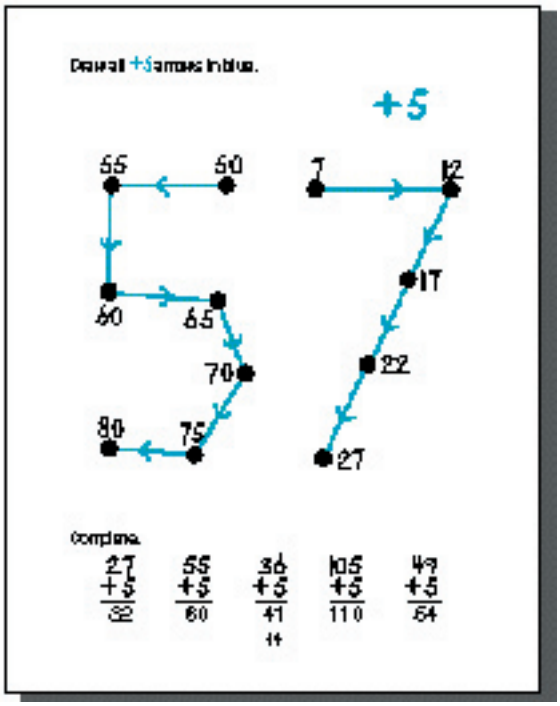
9


+10
+1
53
52
51
50
40
30
20
10
0
Many roads are possible.


425
239
807
560
175
839
86
904
Numbers are shown in standard configuration.
Other solutions are possible.


2×
8
6
12
24
48
2 × 4 = 8
2 × 5 = 10
2 × 7 = 14
2 × 11 = 22
21 × 2 = 42
15 × 2 = 30
32 × 2 = 64
25 × 2 = 50
45 × 2 = 90

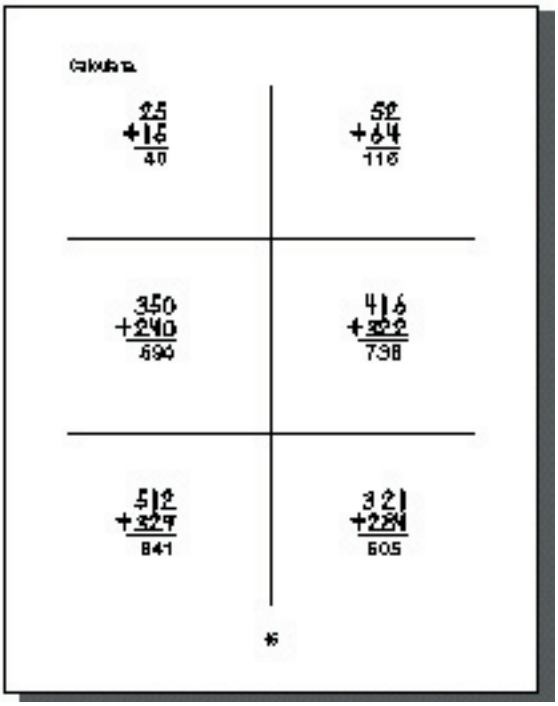

2 5 10 15
2
10
16
9
24
5
1
Many solutions are possible.



Complete.

27 +5 = 32	55 +5 = 60	36 +5 = 41	105 +5 = 110	49 +5 = 54

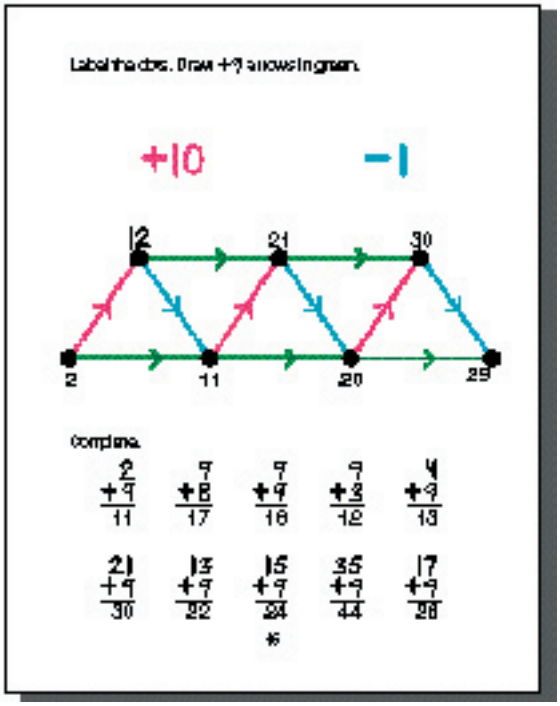
11



Calculate.

25 + 15 = 40	52 + 64 = 116
350 + 240 = 590	416 + 322 = 738
512 + 329 = 841	321 + 284 = 605

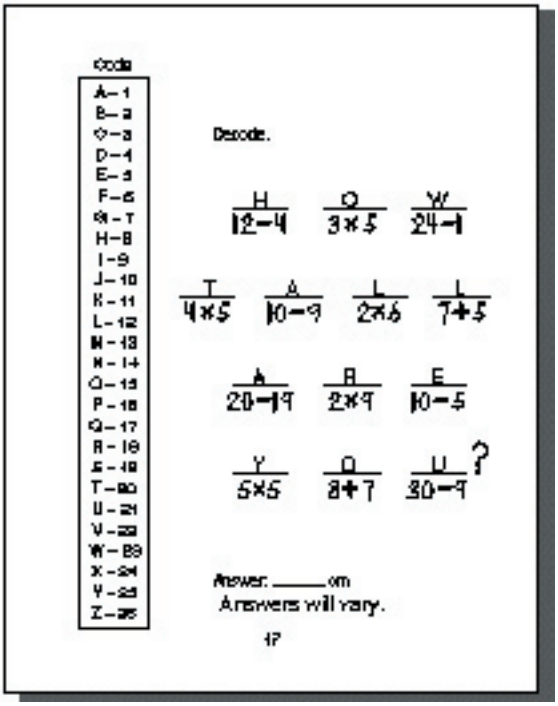
16



Complete.

2 +9 = 11	9 +8 = 17	9 +9 = 18	9 +3 = 12	4 +9 = 13
21 +9 = 30	13 +9 = 22	15 +9 = 24	35 +9 = 44	17 +9 = 26

16



Code

A – 1	N – 14
B – 2	O – 15
C – 3	P – 16
D – 4	Q – 17
E – 5	R – 18
F – 6	S – 19
G – 7	T – 20
H – 8	U – 21
I – 9	V – 22
J – 10	W – 23
K – 11	X – 24
L – 12	Y – 25
M – 13	Z – 26

Decode.

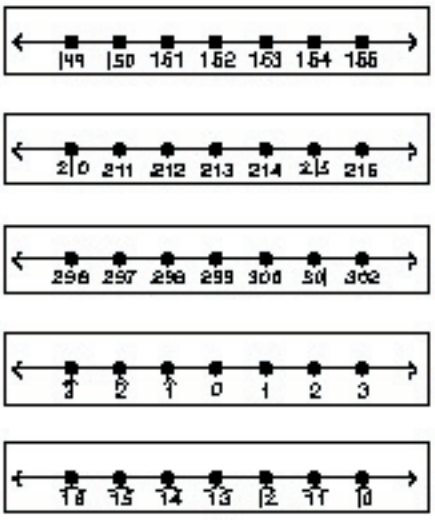
H: 12 − 4	O: 3 × 5	W: 24 − 1	
T: 4 × 5	A: 10 − 9	L: 2 × 6	L: 7 + 5
A: 20 − 19	R: 2 × 9	E: 10 − 5	
Y: 5 × 5	O: 8 + 7	U: 30 − 9	?

Answer: _____ cm

Answers will vary.

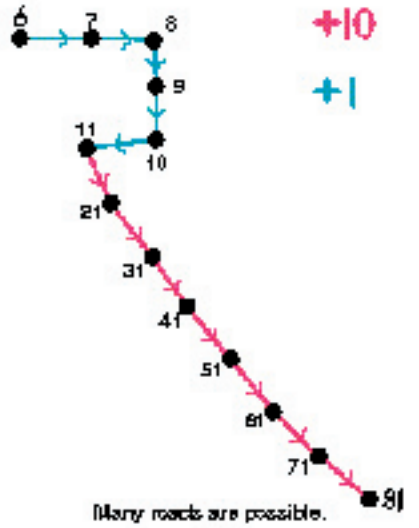
17

Label the dots on these number lines.



18

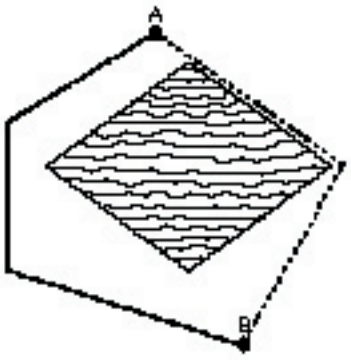
Build an arrow road from one dot using +10 and +1 arrows.



Many roads are possible.

19

How long is this zigzag path from A to B? 23 cm



Try to find a shorter zigzag path — do not go in the water. Draw it.

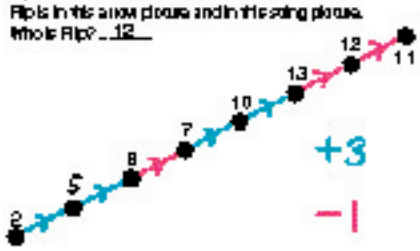
How long is your path? 18 cm

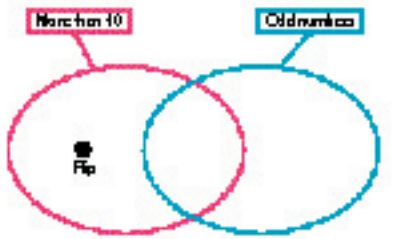
How much shorter? 5 cm

Many solutions are possible.

20

Flip is a same number.
Flip is in this arrow please and in this ring please.
Who is Flip? 12





21

Write number sentences for each number. One is done for you.

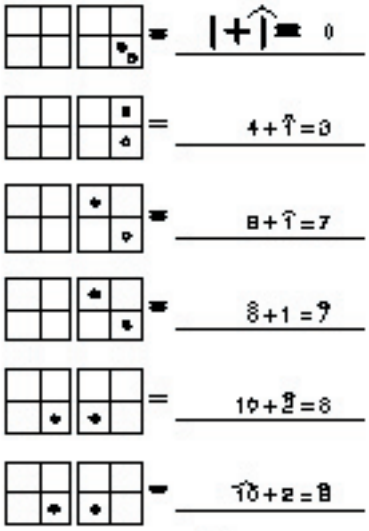
9	12
6+3	2×6
12−3	4+4+4
10+1	10+2
3×3	$\frac{1}{2}$×24

100	25
$\frac{1}{2}$×200	26−1
2×50	18+7
75+25	5×5
200+100	50÷2

Many answers are possible.

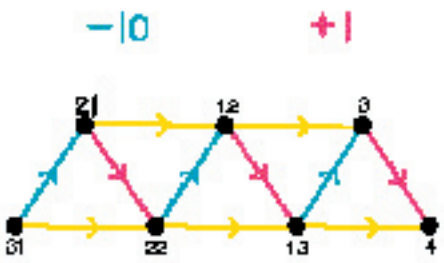
22

What number is on the Minicomputer?



23

Label the dots. Draw −9 arrows in yellow.

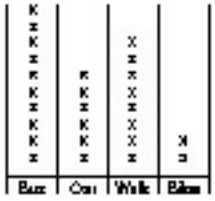


Complete.

21 −9 = 12	18 −9 = 9	10 −9 = 1	15 −9 = 6	12 −9 = 3
13 −9 = 4	16 −9 = 7	11 −9 = 2	20 −9 = 11	17 −9 = 8

24

Ms. Carlo's class made a graph of the way the students go to school. Each student put an x in the graph.



What way do the most students use to get to school? Bus

Do more students come by car or walk to school? Walk

How many students walk to school? 8

How many students do not ride the bus? 16

Where would you put an x in the graph? Why? _____
Answers will vary.

25

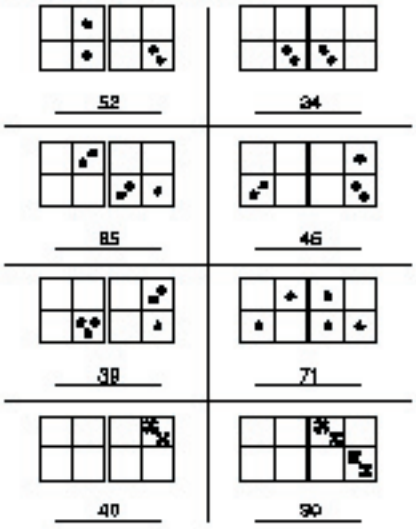
Label the dots.

Complete.

2 × 50 = 100	2 × 100 = 200
3 × 10 = 30	3 × 100 = 300
2 × 25 = 50	2 × 13 = 26
3 × 25 = 75	3 × 11 = 33

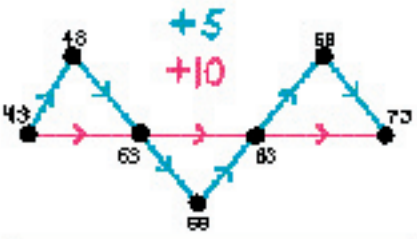
26

What numbers on the Mirkomputer?

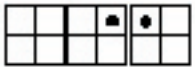


27

Mur is a green number. Mur is in this envelope.
Label the dots.



Mur can be put on the Mirkomputer with two checkers.
Put Mur on the Mirkomputer.



Who is Mur? 48

28

Letter Values

A – 1	
B – 2	
C – 3	
D – 4	
E – 5	
F – 6	
G – 7	
H – 8	
I – 9	
J – 10	
K – 11	
L – 12	
M – 13	
N – 14	
O – 15	
P – 16	
Q – 17	
R – 18	
S – 19	
T – 20	
U – 21	
V – 22	
W – 23	
X – 24	
Y – 25	
Z – 26	

What is the value of each name?

Harry 70

Zorba 62

Tammy 72

Victor 83

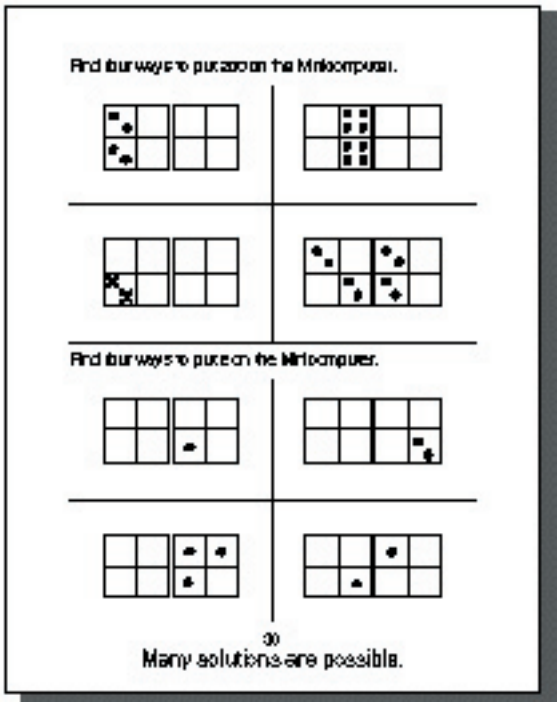
Find a name with value less than 40.

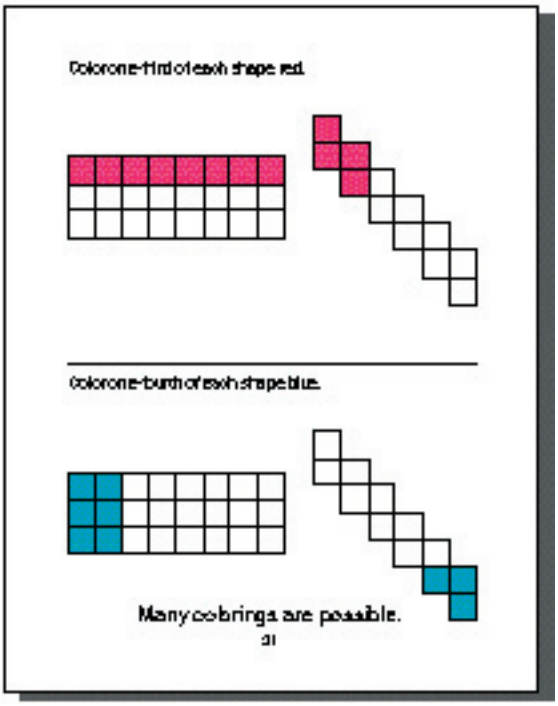
Jan (value 25)
Many solutions are possible.

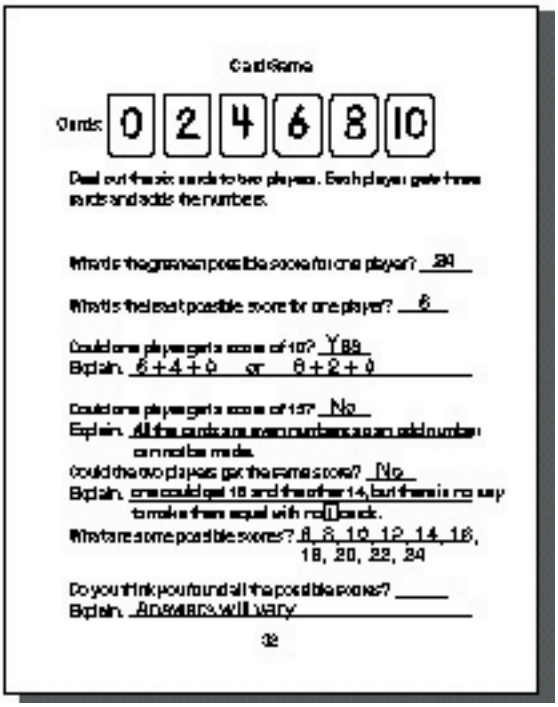
Find a name with value between 60 and 80.

Saul (value 53)
Many solutions are possible.

29


Find four ways to put zero on the Minicomputer.
Find four ways to put one on the Minicomputer.
30
Many solutions are possible.


Color one-third of each shape red.
Color one-fourth of each shape blue.
Many colorings are possible.
31


Card Game
Cards: 0 2 4 6 8 10
Deal out the six cards to two players. Each player gets three cards and adds the numbers.
What is the greatest possible score for one player? 24
What is the least possible score for one player? 6
Could one player get a score of 10? Yes
Explain. 6+4+0 or 6+2+0
Could one player get a score of 15? No
Explain. All the cards are even numbers so an odd number cannot be made.
Could the two players get the same score? No
Explain. one could get 18 and the other 14, but there is no way to make them equal with no 0 card.
What are some possible scores? 8, 8, 10, 12, 14, 16, 18, 20, 22, 24
Do you think you found all the possible scores? ____
Explain. Answers will vary
32

W9 DETECTIVE STORY #3

Capsule Lesson Summary

Solve a detective story in which the secret number is less than 100 and the other clues involve the calculator, the Minicomputer, and a string picture.

Materials

Teacher	• Calculator • Minicomputer set • Colored chalk	• Paper • Colored pencils, pens, or crayons • Minicomputer set
Student	• Calculator	

Description of Lesson

Allow students to work as partners during this lesson. Distribute calculators to pairs of students and allow a few minutes for free play. You may prefer to use an overhead calculator for Exercise 1 and wait to give students calculators until the detective story.

Exercise 1: Mental Arithmetic

Use the calculator in a mental arithmetic activity.

T: ***Make sure you have 0 on the display of your calculator. Cover the display with your hand, but be careful not to cover the light panel. Now you are going to be like the calculator. Think about what the calculator does when I tell you which keys to press.***

Slowly instruct the students to press keys. In pairs, one student can cover the display while the other presses keys.

T: ***Press*** [5] [+] [4] [−] [2] [=] ***.***
What number will be on the display? (7)

Ask students to first tell you the number on the display and then to check their calculators.

T: ***Make sure that 7 is on the display of your calculator, and then cover the display.***
Press [+] [3] [=] [=] ***. What number will be on the display*** (13)

Continue this mental arithmetic activity with a few more problems. Choose sequences that fit the abilities of your class. For example:

- 13 is on the display. Press [−] [3] [×] [2] [=]. Check.
- 20 is on the display. Press [+] [5] [=] [=] [=]. Check.
- 35 is on the display. Press [−] [5] [=] [=] [=]. Check.
- 20 is on the display. Press [÷] [2] [+] [2] [×] [2] [=]. Check.

Exercise 2: Detective Story

Write 81 on a slip of paper and fold it so the number is hidden. Announce to the class that there is a secret number on the paper, and they are going to be detectives to discover the secret number.

Clue 1

T: ***The calculator will help us with the first clue. Let's use the calculator to count by nines starting at 0. The first clue is that the secret number is one of the numbers that will appear on the display.***

Instruct students to turn on their calculators.

T: ***What number is on the display?***

S: ***0.***

T: ***Listen carefully and follow my directions. Press*** [+] [9] [=] ***. What number is on the display?***

S: ***9.***

Record 9 on the board.

T: ***Press*** [=] ***again. What number is on the display?***

S: ***18.***

Record 18 under 9. Line up the ones digits.

T: ***What number will we get if we press*** [=] ***again?***

Accept three or four predictions of what the next number will be; then record 27 in the list forming on the board.

T: ***Press*** [=] ***. What number is on the display now?***

S: ***36.***

Record 36 on the list.

9
18
27
36

T: ***Does anyone see a pattern?***

S: ***Here*** (pointing to the tens column) ***the numbers go forward: 1, 2, 3, …; and here*** (pointing to the ones column) ***they go backward: 9, 8, 7, 6….***

S: ***In each number the digits add to get 9.***

If no one see a pattern yet, press [=] a few more times and record the numbers that appear. Then ask again about a pattern.

Continue this exercise as long as your students show interest. With each number, first ask for predictions and then ask the students to press [=]. When you get to the bottom of the board begin a new column of numbers. Draw three dots at the end of your list.

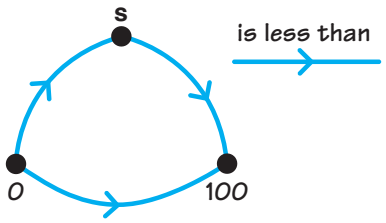
9	72	135
18	81	144
27	90	153
36	99	162
45	108	
54	117	.
63	126	.
		.

T: ***What are these three dots for?***

S: ***To show that the numbers in the list go on and on.***

Clue 2

Draw this arrow picture on the board and ask students to copy it on their papers.



T: ***Where is the secret number in this picture?*** (At **s**)
What are the blue arrows for? (Is less than)
What does the picture tell us about the secret number?
Which of these numbers (point to list) ***could be the secret number*** (point to **s**)***.***

Allow students to express their opinions. Then instruct them to write possibilities for the secret number on their papers. After a few minutes, let some students tell you which numbers to keep in the list and which to eliminate.

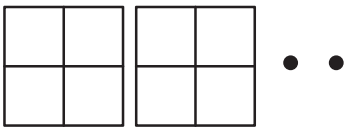
T (pointing to the three dots)**:** ***Could the secret number be any of the numbers in our list that we didn't write on the board?***

S: ***No, because those numbers are more than 100.***

Erase the three dots.

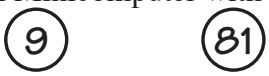
Clue 3

Display two Minicomputer boards and two regular checkers.



T: ***The secret number can be put on the Minicomputer with exactly two regular checkers.***
Which of the numbers in the list could be the secret number?

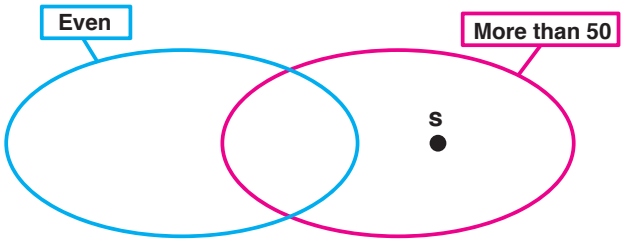
Instruct student partners to use their individual Minicomputers to find which of eleven numbers still on the list can be put on with exactly two checkers. After a few minutes, let students suggest possibilities for the secret number and put them on the demonstration Minicomputer with exactly two regular checkers.



Circle correct possibilities for the secret number and erase any number that the class decides cannot be shown with two regular checkers. Continue until all the remaining possibilities in the list have been considered.

Clue 4

Draw this string picture on the board and ask students to copy it on their papers.



T: ***We know that the secret number is 9, 18, 81 or 90. The fourth clue is that the secret number*** (point to **s**) ***is in this string picture. When you know which number is the secret number, write it in your picture.***

When most students have put a number at **s**, ask for an explanation of how they found the secret number. Conclude that 81 is the secret number.

Home Activity

Suggest to parents/guardians that they can use calculators to practice mental arithmetic with their child as in Exercise 1.

Capsule Lesson Summary

Review the idea of taxi-geometry paths between two points on a grid. Find short paths and long paths and compare the lengths (numbers of blocks) of several paths. Discuss the notion of a round trip starting at one point, going to another, and then returning to the first point. Use the workbook to review several ideas from previous taxi-geometry lessons.

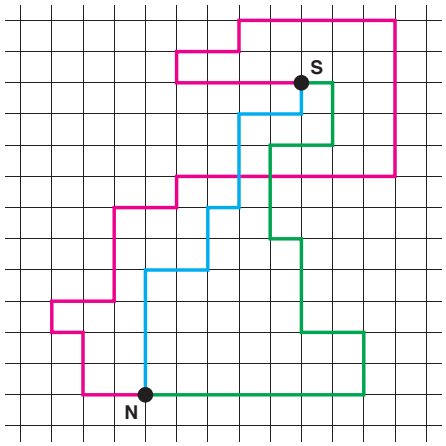
Materials

Teacher	• Grid board[†] • Colored chalk	Student	• *Nora's Neighborhood* Workbook • Colored pencils, pens, or crayons

Description of Lesson

Display your grid board with a dot labeled **N** and a dot labeled **S** as in the illustration.

Briefly recall that this is a map of a city where Nora lives, that Nora's house is at **N,** and the school is at **S**. Let the class recall that paths from **N** to **S** must follow the lines of the grid. Then ask several students to trace paths from **N** to **S**. Using three different colors, draw three of the suggested paths; choose one rather long path, one shortest path, and one in between. This illustration provides an example.



T: ***Which of these paths is the longest? Which is the shortest?***

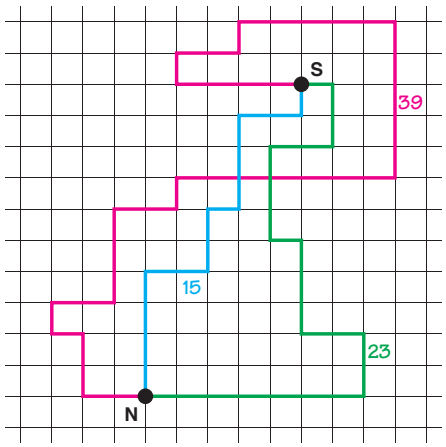
S: ***The red path is the longest.***

S: ***The blue path is the shortest.***

T: ***How could we check to be sure?***

S: ***Count the number of blocks in each path.***

Ask students to make a count for each path and to record the length on the board in the appropriate color near the path.



T: ***Each of these paths is also a path from* S *to* N.**

Trace each path starting at **S** and ending at **N** to illustrate this point.

T: ***If Nora is at school and takes the green path home, how long is her walk?***

S: ***23 blocks.***

[†]See the "Notes of Grids" section in the introduction to the Geometry strand.

T: ***Suppose Nora takes the red path going to school (from N to S) and then returns home along the blue path (from S to N). How long is her round trip?***

S: ***54 blocks, because 39 + 15 = 54.***

T: ***Very good. How long would a trip be if Nora took the green path from N to S and returned along the green path from S to N?***

S: ***46 blocks, because 23 + 23 = 46 (or 2 x 23 = 46).***

Distribute copies of the workbook *Nora's Neighborhood* and let students work independently for the rest of the class period. If many students are having difficulty with a particular page, you may wish to have a collective discussion about that page. For example, you may want to review the directions for pages 8 and 9 as students near those pages.

Here are some paths from Nora's house (N) to Grandma's house (G).

How long are these paths?

RED PATH: 14 blocks

BLUE PATH: 12 blocks

OTHER PATH: 38 blocks

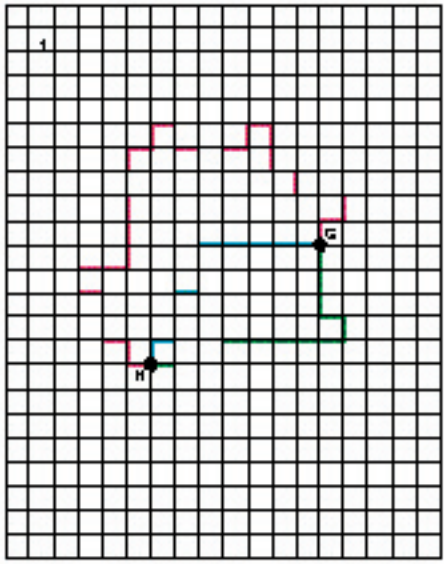
1. Which path is the shortest? (Circle your answer.)

RED BLUE OTHER

2. Which path is the longest? (Circle your answer.)

RED BLUE OTHER

8



9

Many solutions are possible.

It's raining. Nora is in a hurry.

N = Nora's house
S = School

Draw two very short paths from N to S.
Color one RED and one BLUE.

How long are your paths?

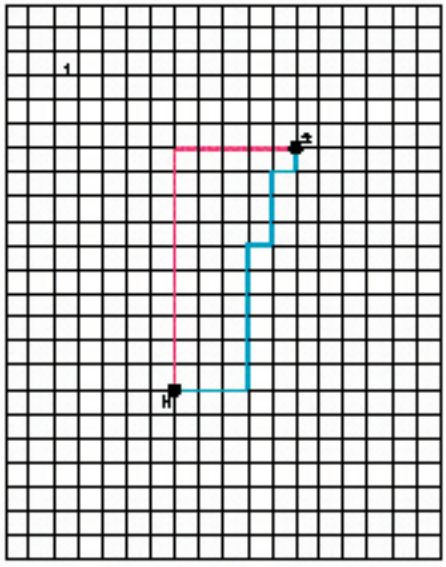
RED path: 15 blocks
BLUE path: 15 blocks

It's still raining when Nora goes home. She can run on the RED path or the BLUE path.

How long are these paths from S to N?

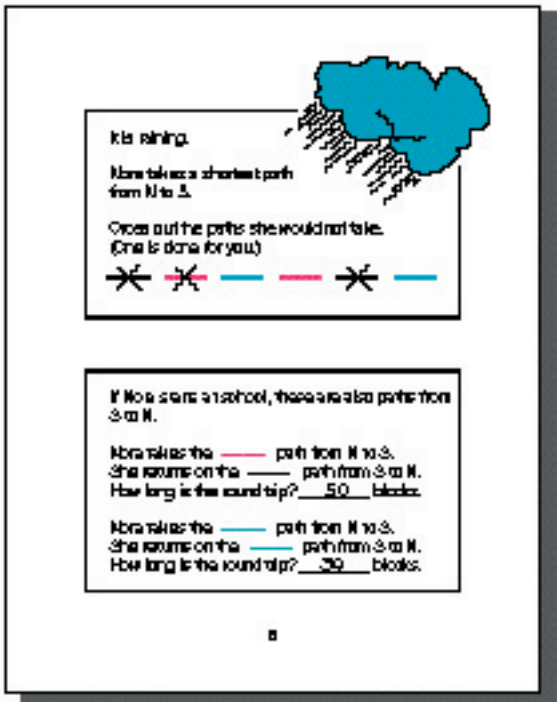
RED path: 16 blocks
BLUE path: 16 blocks

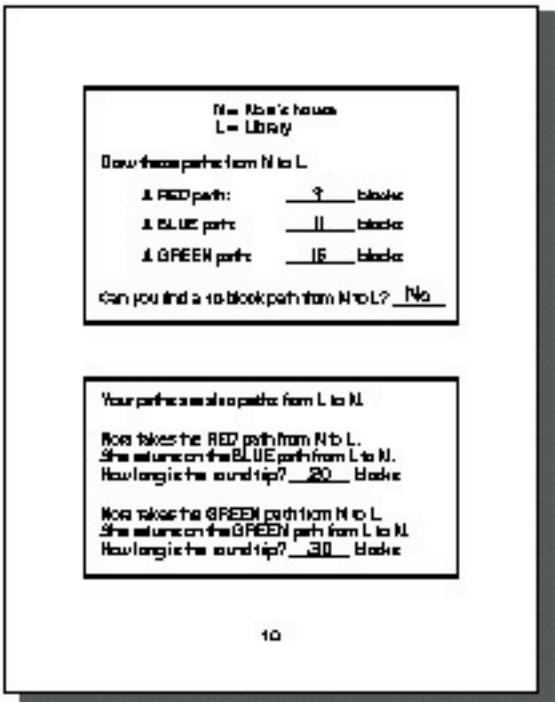
6

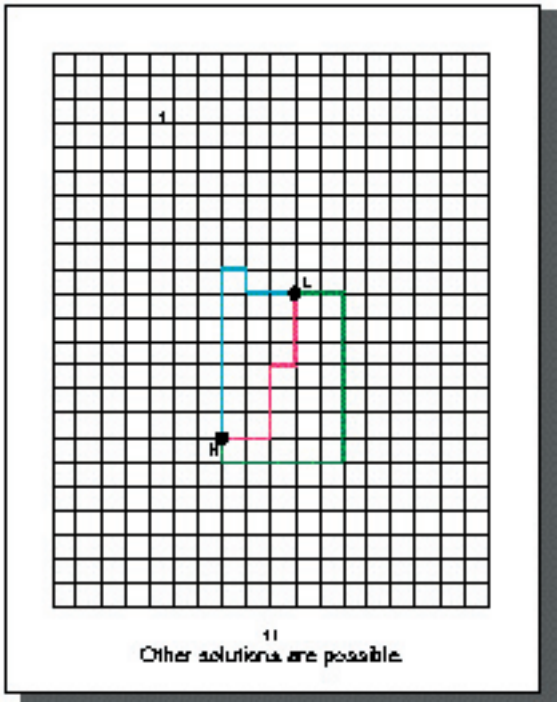


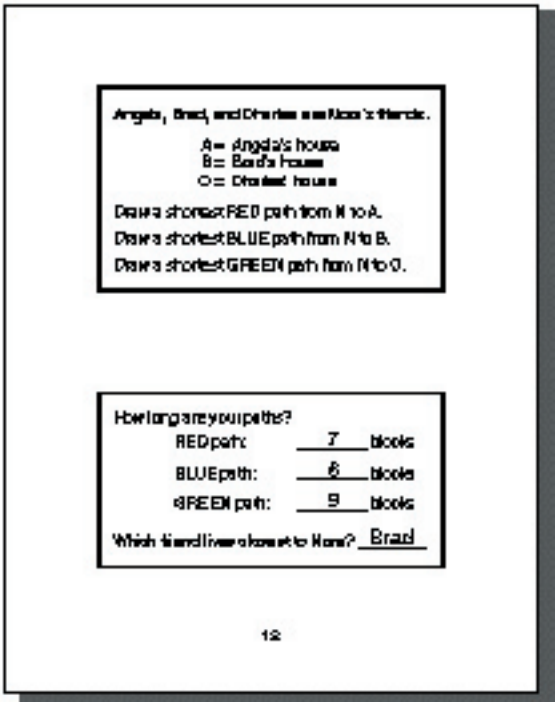
7

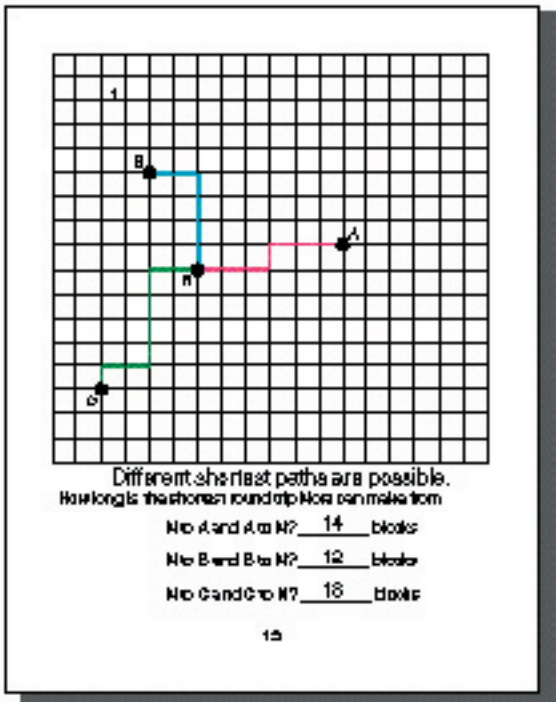
Other solutions are possible.

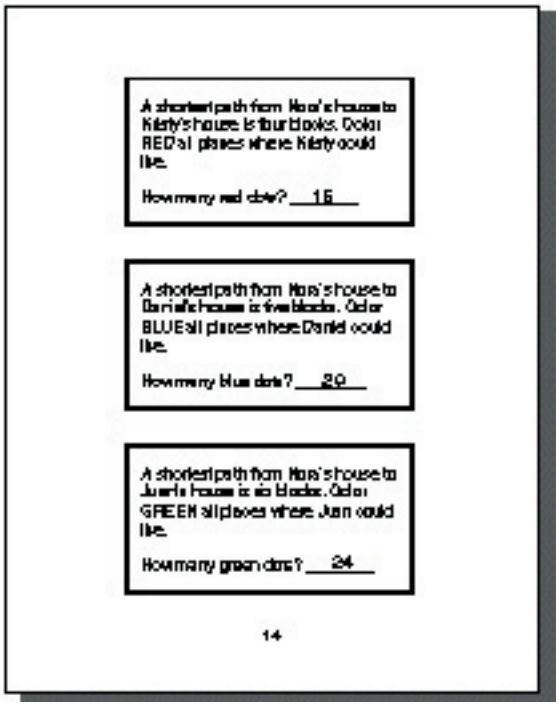

Kia skiing.
Nora takes a shortest path from N to S.
Cross out the paths she would not take. (One is done for you)
If Nora starts at school, there are also paths from S to N.
Nora takes the ____ path from N to S.
She returns on the ____ path from S to N.
How long is the round trip? 50 blocks.
Nora takes the ____ path from N to S.
She returns on the ____ path from S to N.
How long is the round trip? 30 blocks.
8

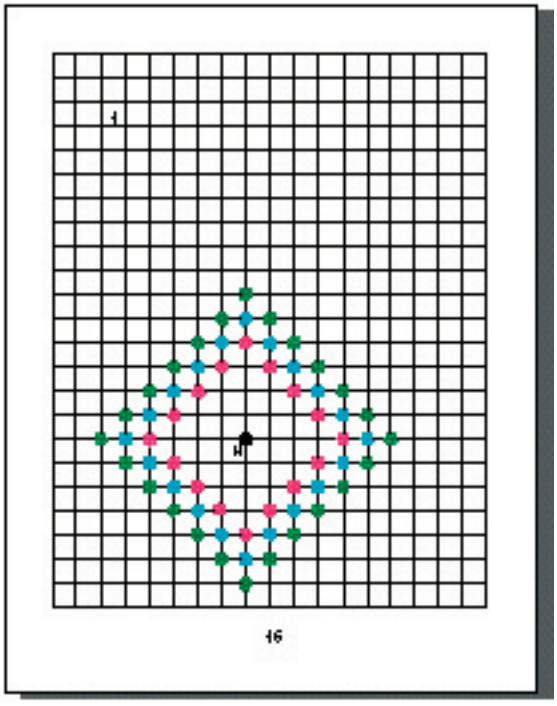

N= Nora's house
L= Library
Draw these paths from N to L.
A RED path: 9 blocks
A BLUE path: 11 blocks
A GREEN path: 15 blocks
Can you find a 10 block path from N to L? No
Your paths are also paths from L to N.
Nora takes the RED path from N to L.
She returns on the BLUE path from L to N.
How long is the round trip? 20 blocks
Nora takes the GREEN path from N to L.
She returns on the GREEN path from L to N.
How long is the round trip? 30 blocks
10

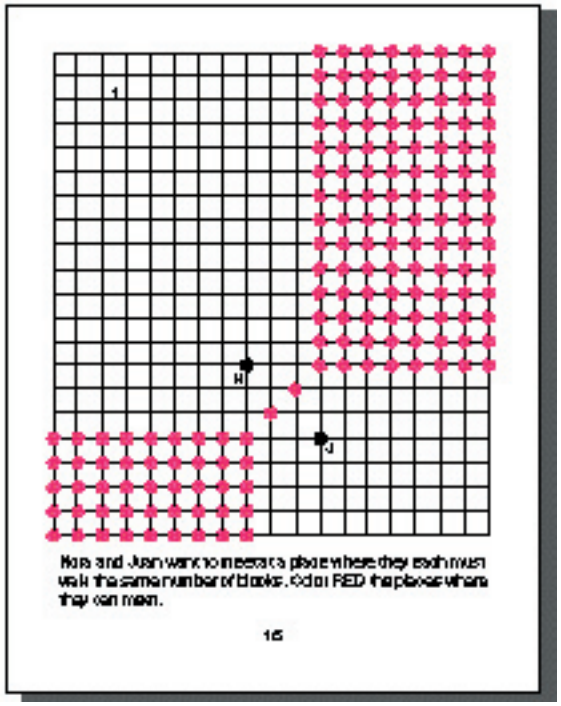

L
N
11
Other solutions are possible.


Angela, Brad, and Charles are Nora's friends.
A= Angela's house
B= Brad's house
C= Charles' house
Draw a shortest RED path from N to A.
Draw a shortest BLUE path from N to B.
Draw a shortest GREEN path from N to C.
How long are your paths?
RED path: 7 blocks
BLUE path: 6 blocks
GREEN path: 9 blocks
Which friend lives closest to Nora? Brad
12


1
B
A
H
G
Different shortest paths are possible.
How long is the shortest round trip Nora can make from
Nora A and A to N? 14 blocks
Nora B and B to N? 12 blocks
Nora C and C to N? 18 blocks
15


A shortest path from Nora's house to Kirsty's house is four blocks. Color RED all places where Kirsty could live.
How many red dots? 16
A shortest path from Nora's house to Daniel's house is five blocks. Color BLUE all places where Daniel could live.
How many blue dots? 20
A shortest path from Nora's house to Juan's house is six blocks. Color GREEN all places where Juan could live.
How many green dots? 24
14


1
H
16


1
H
J
Nora and Juan want to meet at a place where they each must walk the same number of blocks. Color RED the places where they can meet.
15

W11 DETECTIVE STORY #4

Capsule Lesson Summary

Solve a detective story in which the clues involve a string picture and two arrow pictures.

Materials

Teacher	• Colored chalk	Student	• Paper • Colored pencils, pens, or crayons

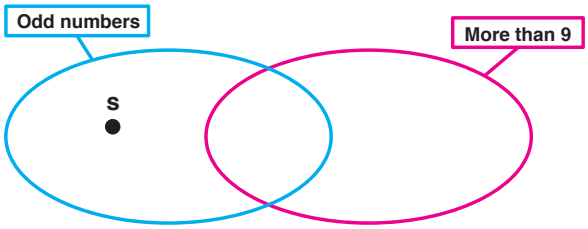
Description of Lesson

You may choose to let students work in pairs during this lesson.

Write a large 7 on a slip of paper and fold it so the number is hidden. Tell the class that there is a secret number on the paper, and they are going to be detectives to discover the secret number.

Clue 1

Draw this string picture on the board.



T: ***Your first clue is on the board. What does this string picture tell you about the secret number*** (point to **s**)***?***

First instruct students to discuss with their partner what information they get from the picture; then invite comments. The students might tell you that the secret number is odd. Ask them to name some odd numbers including some that are relatively large and some that are negative.

T: ***What else does this string picture tell you about the secret number?***

S: ***It's not more than 9.***

If someone says that the secret number is less than 9, tentatively agree, but express a little doubt. If no one suggests that the secret number could actually be 9, don't announce it at this time.

T: ***What are some odd numbers that are not more than 9?***

If necessary, ask specifically about negative numbers.

T: ***Let's make a list of numbers that could be the secret number.***

S: ***3.***

Trace the appropriate strings as you ask,

T: ***Is 3 an odd number?*** (Yes)
Is 3 more than 9? (No)
Could the secret number (point to **s**) ***be 3?*** (Yes)

Record 3 on the board but not in the string picture. Perhaps a student will suggest a number that cannot be the secret number; for example:

S: *15.*

T: ***Is 15 an odd number?*** (Yes)
Is 15 more than 9? (Yes)
Could the secret number (point to **s**) ***be 15?*** (No)
Who can show us where 15 belongs in this string picture?

A student should indicate that 15 is in the middle region—the intersection of the two strings.

As students suggest correct possibilities for the secret number, put them in the list forming on the board, spacing them appropriately so that they will be listed in order. After three or four possibilities have been given ask,

T: ***What is the greatest number the secret number could be?*** (9)

If 9 is not suggested immediately, accept another number that could be the secret number and ask if the secret number could be greater than that number. Continue until someone suggests 9.

At this point you might return to the question of whether the secret number is less than 9. Ask the class if the secret number must be less than 9. Conclude with your class that the secret number could be 9, because 9 is not more than 9.

T: ***What are some other numbers the secret number could be?***

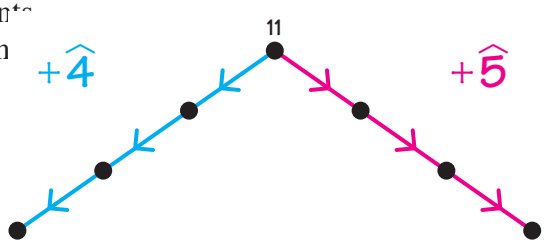
Continue recording correct possibilities until your list includes these numbers. Decide with the class that the list goes on and on, and draw three dots at the end of the list.

$$9 \quad 7 \quad 5 \quad 3 \quad 1 \quad \widehat{1} \quad \widehat{3} \quad \widehat{5} \quad \widehat{7} \quad \widehat{9} \quad \widehat{11} \quad \widehat{13} \quad \ldots$$

Note: Your list should include numbers less than $\widehat{9}$, such as $\widehat{11}$, to emphasize that $\widehat{11} < 9$.

Clue 2

Draw this arrow picture on the board and ask students to copy the picture on their papers. If students are in pairs, each pair need draw only one picture.



T: ***Your second clue is that the secret number is in this arrow picture.***
What are the blue arrows for?

Accept readings of $+\widehat{4}$ as "plus negative four" or "plus four magic (peanuts)" or "take away four," but use "plus negative four" yourself.

T: ***What are the red arrows for?*** ($+\widehat{5}$)
If we start at 11 and follow red (blue) arrows, will the numbers we meet be more or less than 11? (Less)
Who can point to where the greatest number is in this arrow picture?

A student should point to the dot labeled 11.

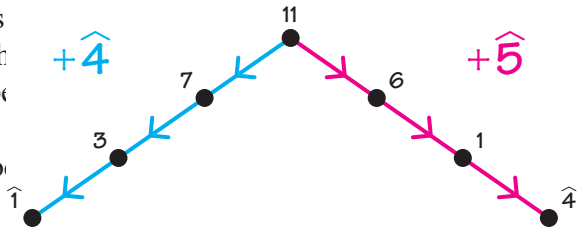
T: ***How do you know?***

S: ***All the other numbers are less than 11.***

T: ***Who can point to where the least number is in this arrow picture?*** (Lower right)

This is a difficult question; if a student answers correctly, you may wish to ask why the least number is there, but do not expect a well-formed answer. If your class is uncertain where the least number is in the arrow picture, wait until all the dots have been labeled and ask the question again.

Invite students to label dots. Label one or two dots collectively and then instruct student pairs to finish labeling the dots in their pictures. Refer to a numbe line or to magic peanuts if a student has difficulty labeling a dot. Students who finish quickly can lab dots on the board. When all the dots have been labeled, ask the class to notice that the least number is at the lower right.



T: ***The secret number is in this arrow picture and in the list we made. Which numbers could be the secret number?***

When a number is suggested, check that the number is included in the list and in the arrow picture. If the number is included in both places, circle it in the list. If the number is not in the arrow picture, cross it off the list. Continue until all the numbers on the board have been considered.

T (pointing to the three dots)**:** ***Could the secret number be any of the numbers in this list that we didn't write on the board?***

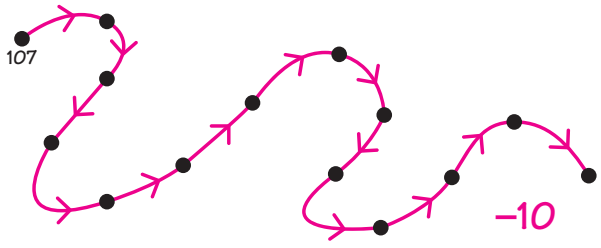
S: ***No, because the least number in the arrow picture is 4 and the other numbers in the list are less than 4.***

Cross off the three dots. Erase the board leaving only the circled numbers in the list.

T: ***Now we know that the secret number is 7, 3, 1, or 1.***

Clue 3

Draw this arrow picture on the board.



T: ***The third clue is that the secret number is in this arrow picture.***

If any student pairs think they know the secret number, ask them to write it on their papers and be ready to explain.

T: ***Let's label some of these dots. What are the red arrows for?*** (–10)
If we start at 107 and follow red arrows, will the numbers we meet be increasing or decreasing? (Decreasing)
What number is 107 – 10? (97)

If necessary, count backward with the class to find the answer. Continue labeling the dots until the dot for 67 has been labeled.

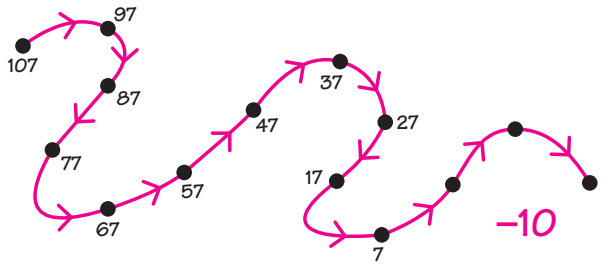
T: ***Do you see a pattern?***

S: ***All the numbers so far end in 7.***

T: ***Does this tell you anything about the secret number?***

S: ***The secret number must be 7.***

T: ***Who can label the dot for 7 in this arrow picture?***



If students have difficulty finding the correct dot, progressively label the dots from 67 to 7.

T: ***Could one of these dots be for 3?*** (No)
Why not?

Accept any reasonable explanation, but express some doubt if a student says that all the numbers in the arrow picture have 7 as their ones digit.

Point to 7 and trace the arrow that starts there.

T: ***What number is 7 – 10?*** ($\widehat{3}$)

If necessary, refer to a number line or count backward with your class.

T: ***Is one of these dots for the number 1?*** (No) ***Why not?*** (We skipped over it)
Is $\widehat{1}$ in this arrow picture? (No) ***Why not?*** (We skipped over it)

Reveal that 7 is written on the slip of paper.

Home Activity

Suggest to parents/guardians that they find opportunities to count forward and backward by tens with their child. The counting should start at different numbers. Remind them how to use a calculator to do this counting.

Capsule Lesson Summary

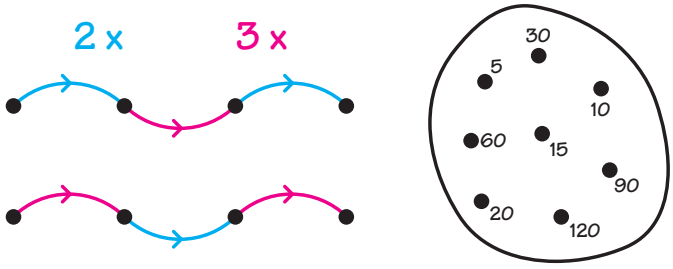
Put the numbers from a string picture in their proper places in an arrow picture with 2x and 3x arrows. Begin the *Fishing for Numbers, Part II* Workbook.

Materials

Teacher	• Colored chalk
Student	• *Fishing for Numbers, Part II* Workbook

Description of Lesson

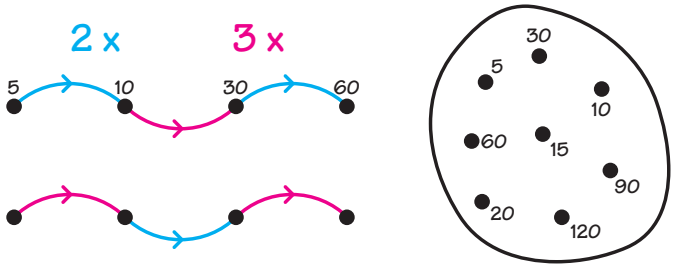
Draw these pictures on the board.



T: ***How many dots are in the arrow picture?*** (Eight)
How many numbers are inside the string? (Eight)
Each of the dots in this arrow picture is for one of the numbers in this string. Each number in the string has just one dot in the arrow picture. How can we find out where these numbers go in the arrow picture?

Who would like to label a dot in one of these arrow roads?

Do not be concerned if the first number chosen is placed incorrectly in the picture, because the subsequent discussion will be of benefit to the class. For example, suppose a student labels the starting dot of the first arrow road 5. Let the students label the other dots in that road. When a dot is labeled, ask the class to check if that number is in the string picture.



T: ***So far that worked out fine. Now, the other four numbers must go in the bottom arrow road.***

Ask for a volunteer to label one of the dots in the second arrow road. Suppose the student labels the starting dot of the second road 10. Point to the dot in the first arrow road labeled 10 as you say,

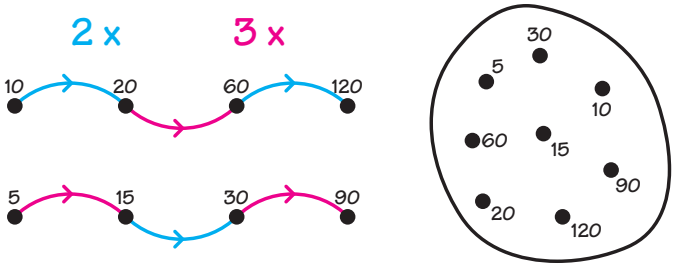
T: ***We've already used the number 10 to label a dot.***

Erase 10 from the second arrow road. Perhaps the next volunteer labels the starting dot 15. Trace the red (3x) arrow starting at 15.

T: ***What number is 3 x 15?*** (45) ***Is 45 in the string?*** (No)
Then this road cannot start at 15. (Erase 15 from the arrow road.)

Continue following students' suggestions until someone realizes it is not possible to label the dots in the bottom arrow road with 15, 20, 90, and 120. Then erase all the labels (including those from the first road) for the dots and start over again.

It is impossible to predict how your class will begin labeling the dots in the arrow picture. Pursue each suggestion and see where it leads. If it leads to an incorrect situation, let the students discover this and discuss what should be done. Perhaps someone will suggest that the least number is the starting number of one of the roads; if so, ask the student to convince the rest of the class. Continue until all the dots are labeled with the numbers from the string picture.



Distribute copies of the workbook *Fishing for Numbers, Part II* and let students work independently for the rest of the class period. You may remind the students of the similar workbook they used earlier (*Fishing for Numbers, Part I*). Notice that the first few pages have the string with numbers on the page opposite where they need to be placed. The rest of the pages have the string with numbers on the same page. Emphasize that it is very likely students will need to use their erasers, and that they should try more than one starting number before they ask for help.

At the end of the lesson, collect the workbooks for your review. They will be used again in Lesson W13.

Capsule Lesson Summary

Put the numbers in a string picture in their proper places in a story. Continue working in the *Fishing for Numbers, Part II* Workbook. (This is the second of two lessons using this workbook.)

Materials

Teacher
- Colored chalk

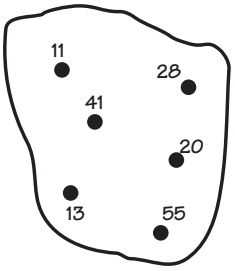
Student
- Paper
- *Fishing for Numbers, Part II* Workbook

Advance Preparation: Write the story for the warm-up activity on the board or an overhead transparency before starting the lesson.

Description of Lesson

Allow students to work with a partner for this warm-up activity.

Put this story and set of numbers on the board.



The school choir is preparing for a spring concert.
There are ____ members in the choir, ____ boys and
____ girls. The choir has more girls than boys. The date
for the concert is set for May ____. The choir will sing
____ songs, and the concert will last about ____minutes.

Tell the class that the problem is to put all the numbers inside the string into the story so that it makes sense. Invite student partners to work on this problem for a few minutes.

When many students have finished the problem, conduct a collective discussion.
As students tell the class how to fill in the blanks, they should explain their reasoning. A solution is given below.

Key: The school choir is preparing for a spring concert.
There are 41 members in the choir, 13 boys and
28 girls. The choir has more girls than boys. The date
for the concert is set for May 20. The choir will sing
11 songs, and the concert will last about 55 minutes.

Distribute the students' copies of the *Fishing for Numbers, Part II* Workbook and allow about 30–45 minutes for individual work. At the end of the lesson, collect the workbooks for your review.

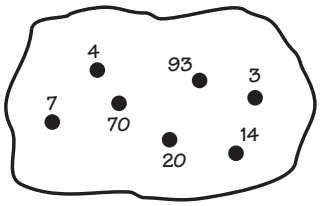
Assessment Activity

An individual student progress record for the workbook is available on Blackline W13. You may like to use this form to monitor student work.

W13

Home Activity

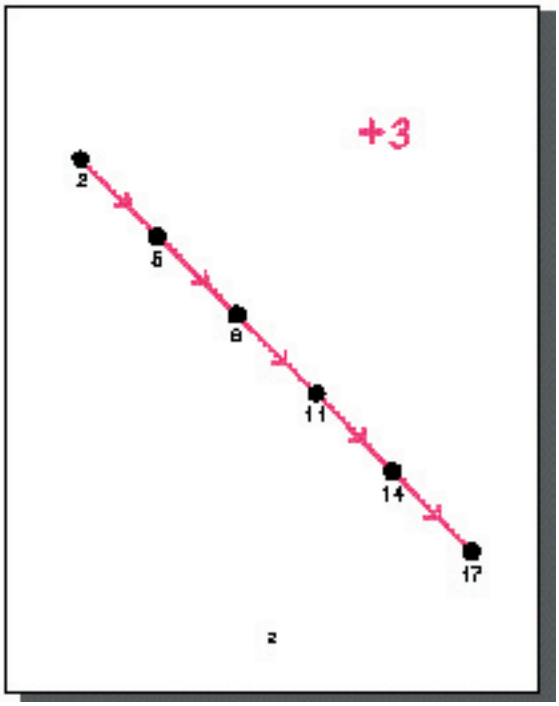
Prepare a story similar to the warm-up activity for students to do at home with family participation. The following example is a money puzzle.

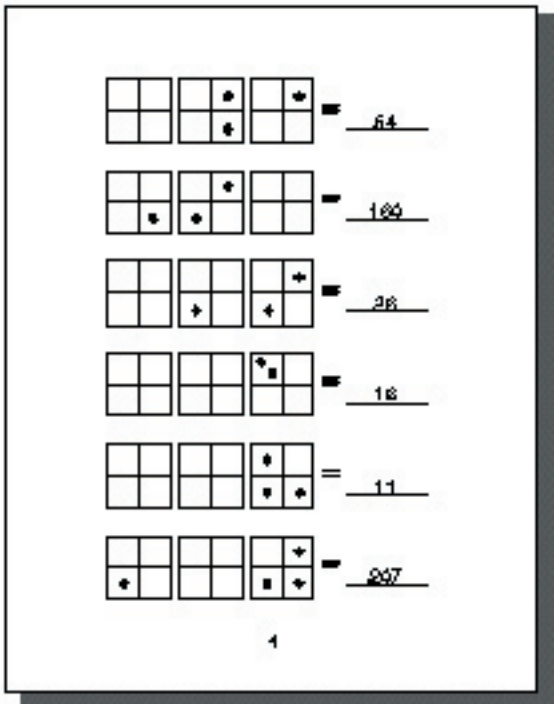


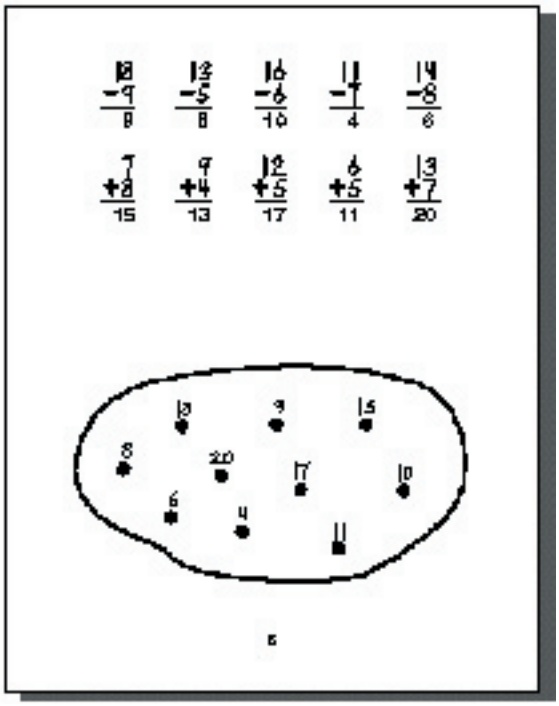
Darcy is counting her coins.
She has ___ pennies.
She has ___ nickels, or ___ ¢ in nickels.
She has ___ dimes, or ____¢ in dimes.
Altogether, Darcy has ____ coins and ____¢.

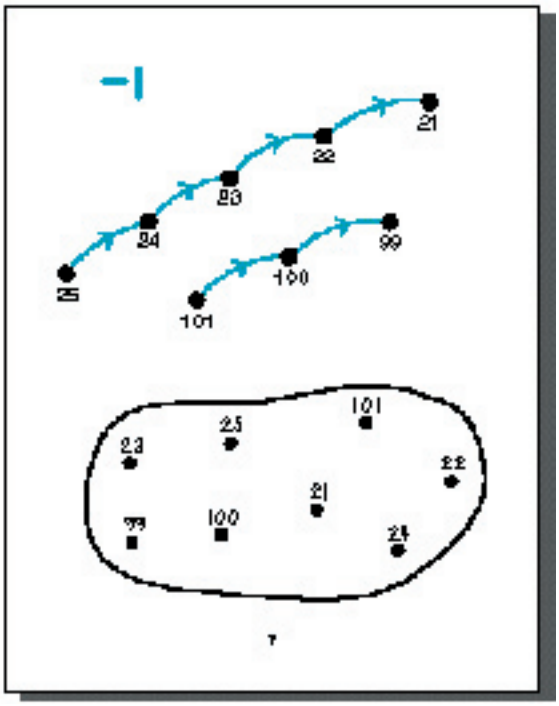
Using all the numbers in the string, fill in the blanks of the puzzle so that it makes sense.

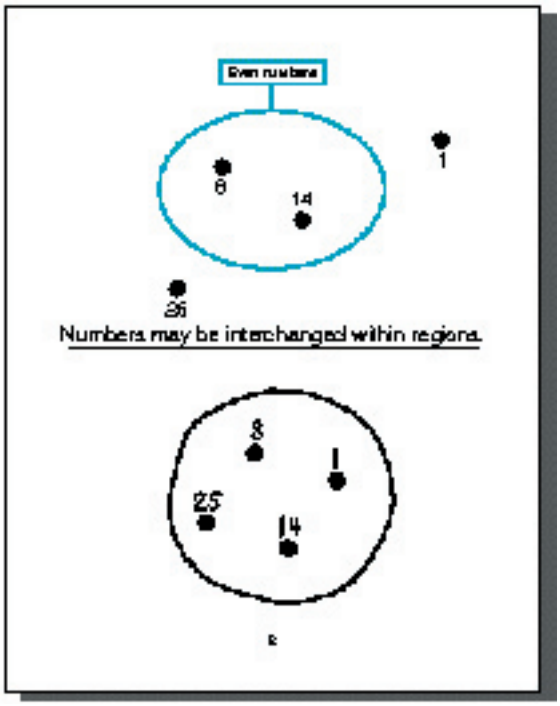
Key: Darcy is counting her coins.
She has _3_ pennies.
She has _4_ nickels, or _20_¢ in nickels.
She has _7_ dimes, or _70_¢ in dimes.
Altogether, Darcy has _14_ coins and _93_¢.

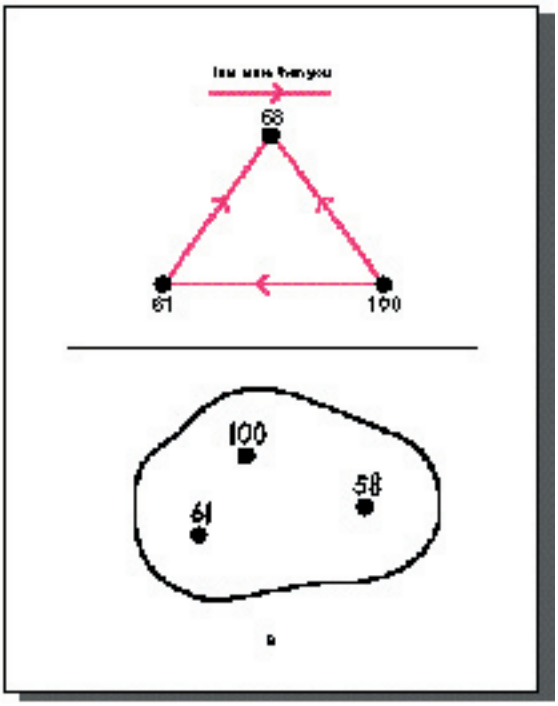

+3
2
5
8
11
14
17

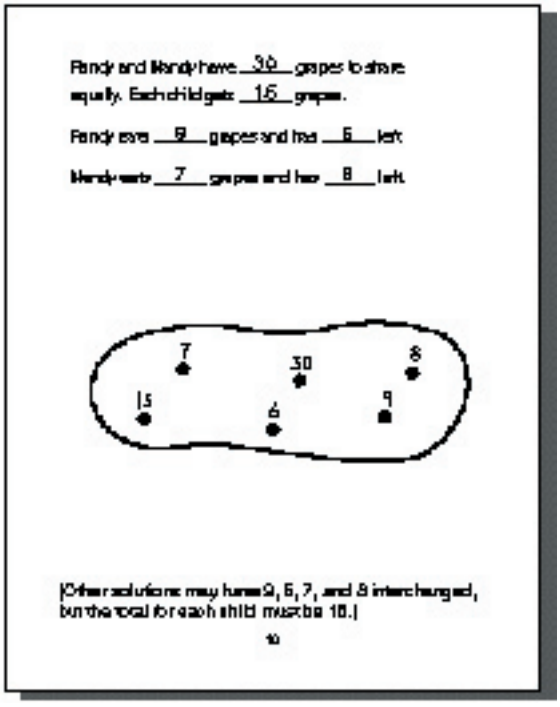

64
100
36
18
11
207

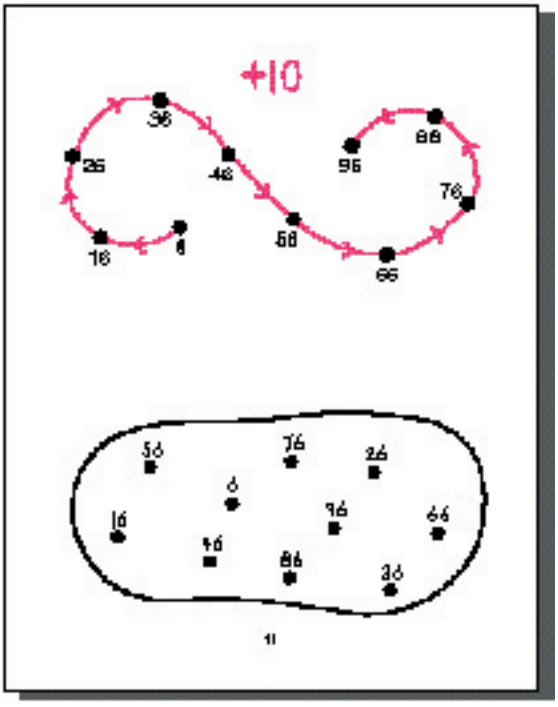

18 − 9 = 9
12 − 5 = 8
16 − 6 = 10
11 − 7 = 4
14 − 8 = 6
7 + 8 = 15
9 + 4 = 13
12 + 5 = 17
6 + 5 = 11
13 + 7 = 20
13
3
15
8
20
17
10
6
4
11

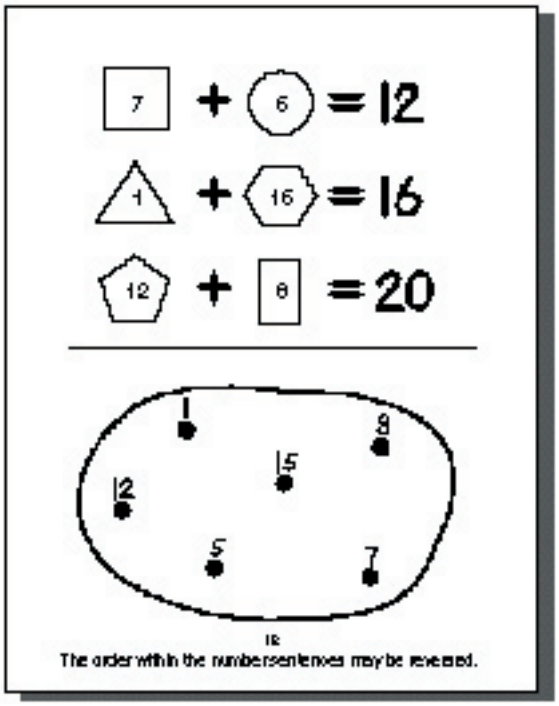

−1
21
22
23
24
25
99
100
101
101
25
23
22
21
99
100
24

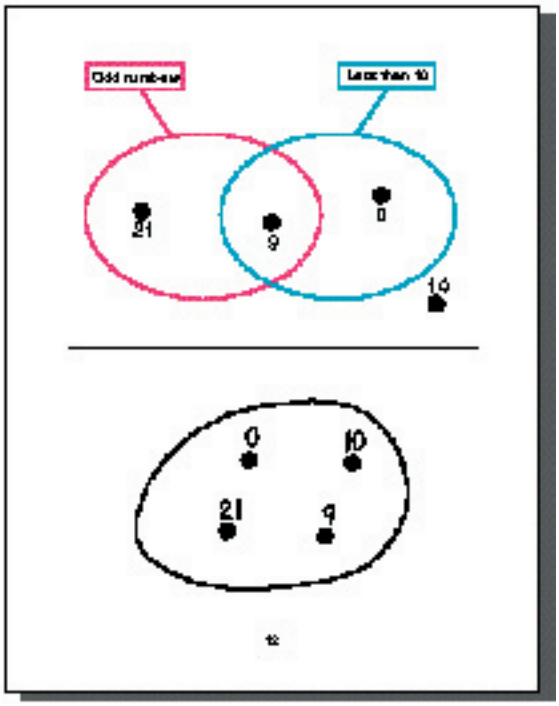

Even numbers
6
14
1
26
Numbers may be interchanged within regions.
8
1
25
14

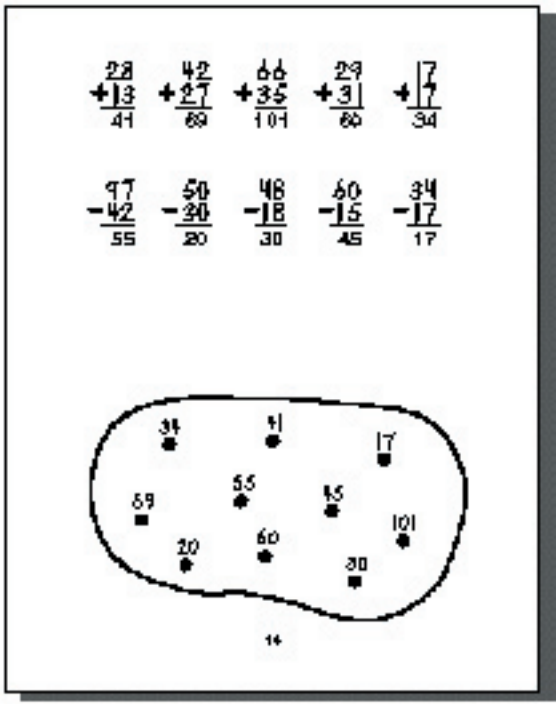

68
61
100
100
58
61

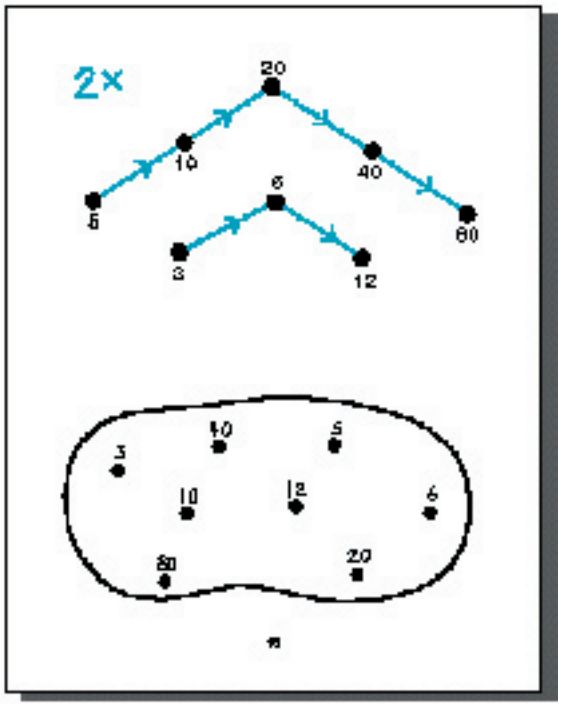

Pandy and Mandy have 30 grapes to share equally. Each child gets 15 grapes.
Pandy eats 9 grapes and has 6 left.
Mandy eats 7 grapes and has 8 left.
7
30
8
15
6
9
[Other solutions may have 0, 5, 7, and 8 interchanged, but the total for each child must be 15.]

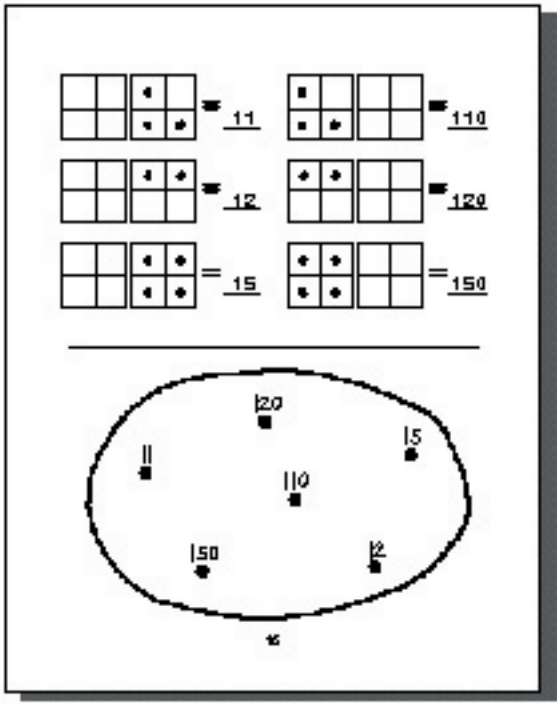

+10
36
25
46
96
88
76
16
58
66
56
76
26
6
16
96
66
46
86
36

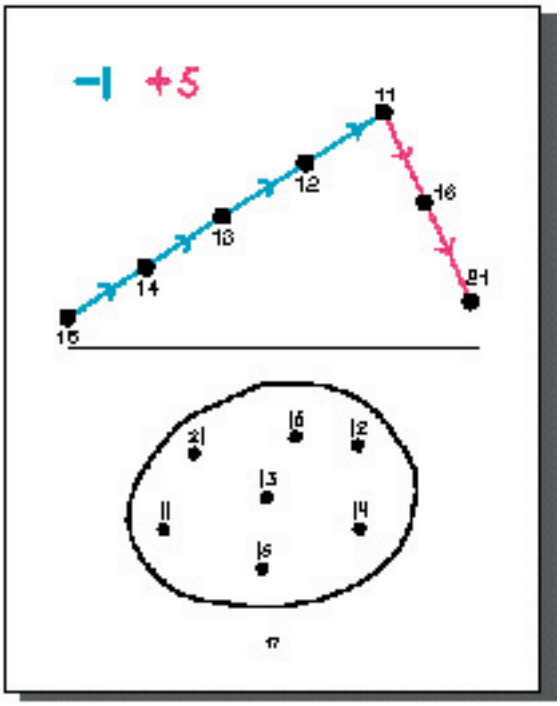

7 + 6 = 12
1 + 16 = 16
12 + 8 = 20
1
8
15
12
5
7
The order within the number sentences may be reversed.

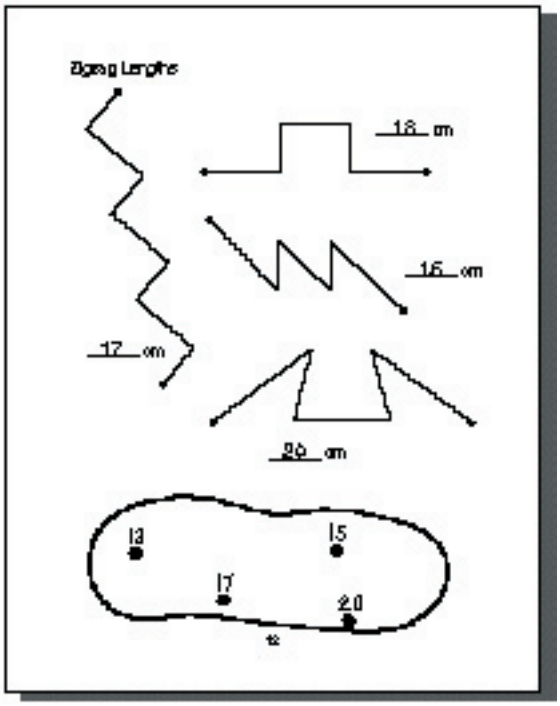

Odd numbers
Less than 10
21
9
0
10
0
10
21
9

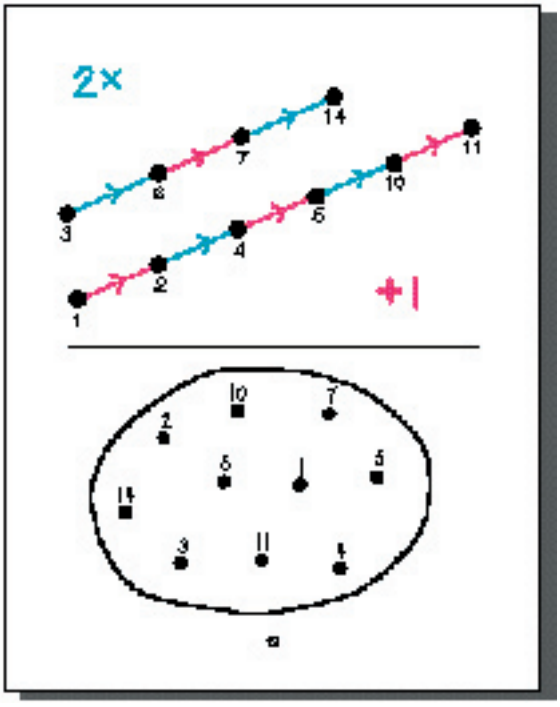

28 + 13 = 41
42 + 27 = 69
66 + 35 = 101
29 + 31 = 60
17 + 17 = 34
97 − 42 = 55
50 − 30 = 20
48 − 18 = 30
60 − 15 = 45
34 − 17 = 17
34
41
17
55
45
69
101
20
60
30

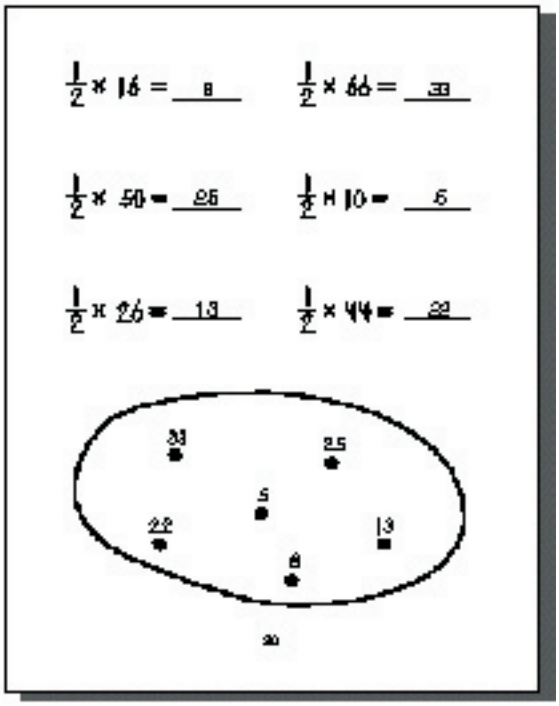

2×
20
10
40
5
6
60
3
12
40
5
3
10
12
6
30
20

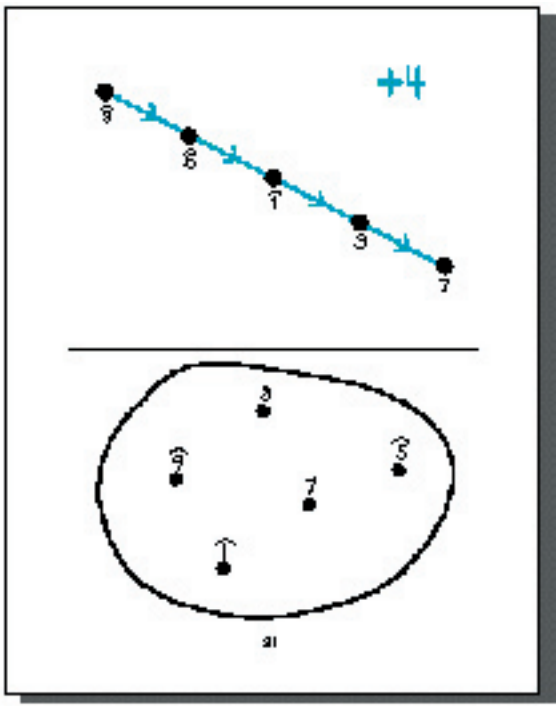

= 11
= 110
= 12
= 120
= 15
= 150
120
15
11
110
150
12

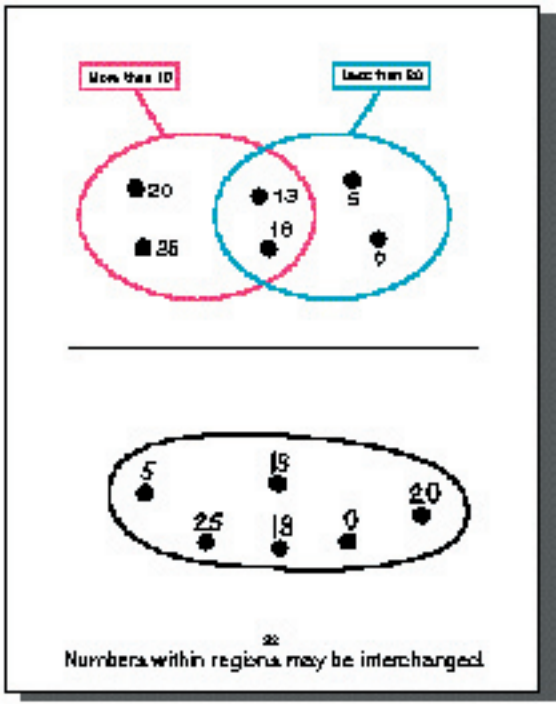

-1 +5
11
12
13
14
15
16
21
16
12
21
13
11
14
15

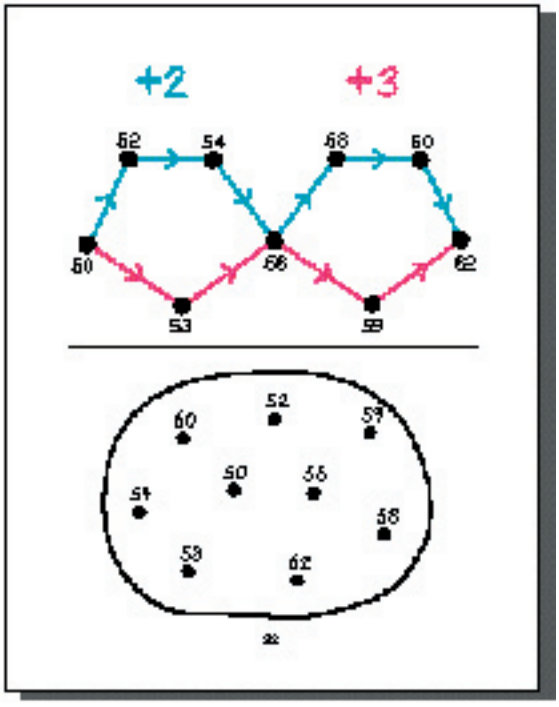

Zigzag Lengths
18 cm
16 cm
17 cm
20 cm
13
15
17
20

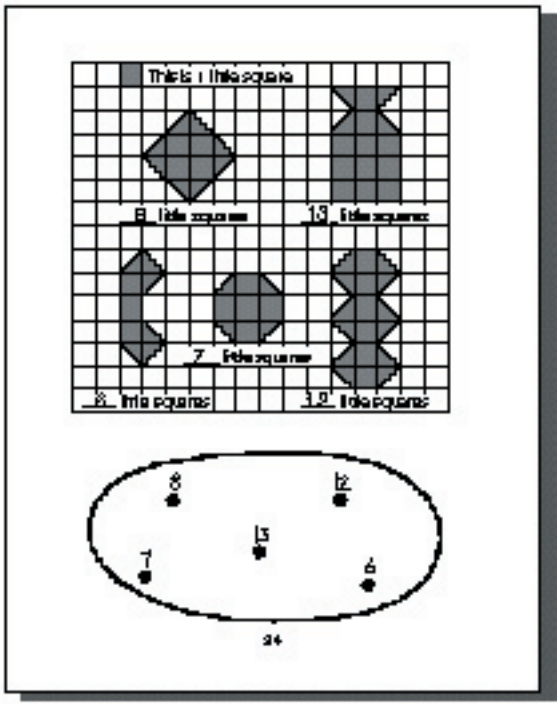

2×
+1
1
2
3
4
5
6
7
8
10
11
14
10
7
2
5
5
14
3
11
4


$\frac{1}{2} \times 16 = 8$
$\frac{1}{2} \times 66 = 33$
$\frac{1}{2} \times 50 = 25$
$\frac{1}{2} \times 10 = 5$
$\frac{1}{2} \times 26 = 13$
$\frac{1}{2} \times 44 = 22$
33
25
5
22
13
8
20


+4
9
5
1
3
7
3
9
5
7
21


More than 10
Less than 30
20
13
5
16
25
0
5
15
20
25
18
0
22
Numbers within regions may be interchanged.


+2
+3
62
54
68
60
60
66
62
53
59
60
52
58
50
55
54
58
59
62
22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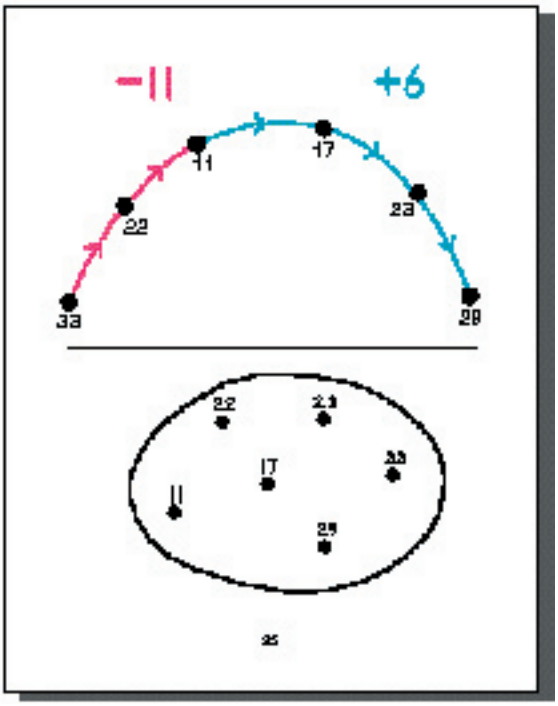
-11
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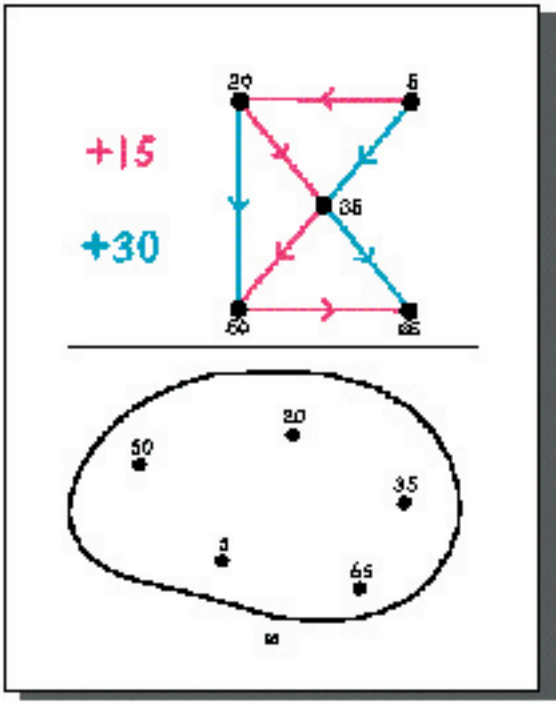
+15
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50
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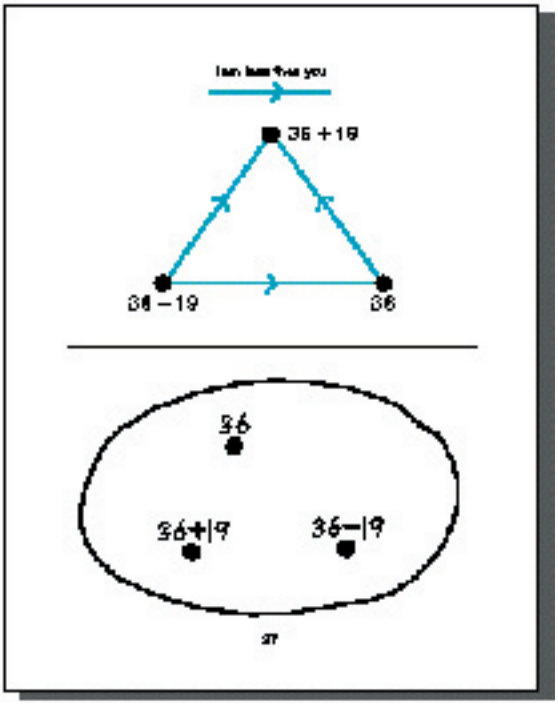
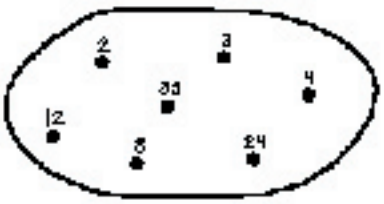
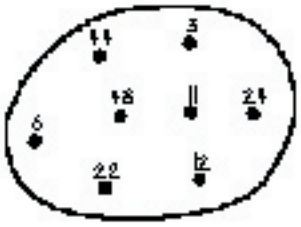
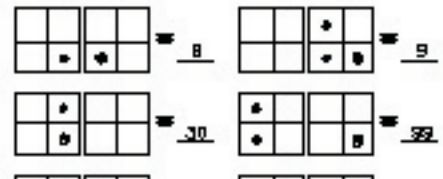
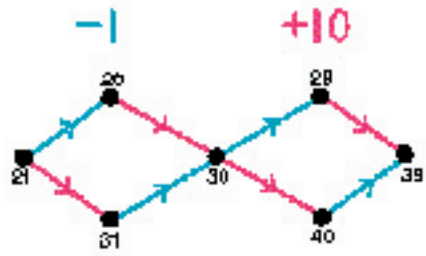

35 + 19
36 − 19
36
36
36+19
36−19

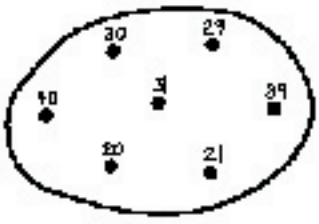
Nolah is making cupcakes. He decides to make a double recipe. The recipe calls for 2 eggs, so he needs 4 eggs. The recipe makes 12 cupcakes, so he makes 24 cupcakes. The cupcakes must bake for 35 minutes. Nolah will serve the cupcakes on 3 plates with 8 cupcakes on each plate.

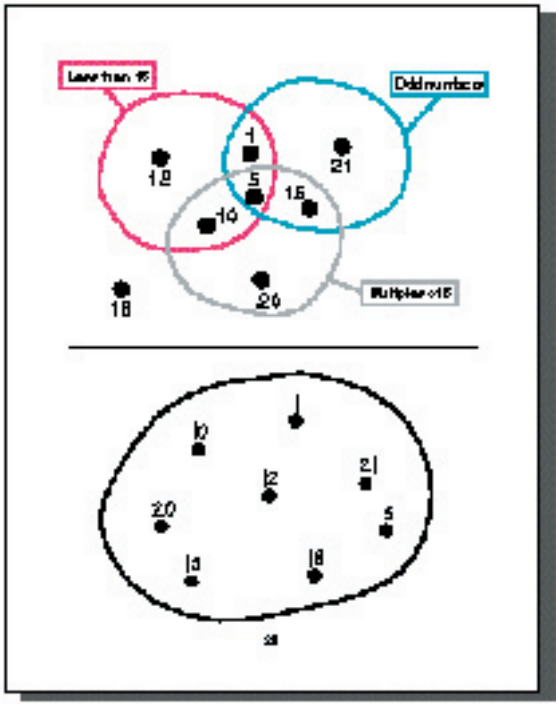











Odd number
1
21
12
5
15
10
18
20
10
21
12
20
5
13
18

Capsule Lesson Summary

Label the floors in a partial picture of the Empire State Building. Find out how many floors a person travels on the elevator to get from one floor to another. Through several clues involving order and multiples, discover on which floor of the Empire State Building an executive's office is located.

Materials

Teacher	Student
• 0–109 numeral chart • Marking pens or crayons • Minicomputer set • Colored chalk • Calculator • Blackline W14	• 0–109 numeral chart • Calculator • Minicomputer set

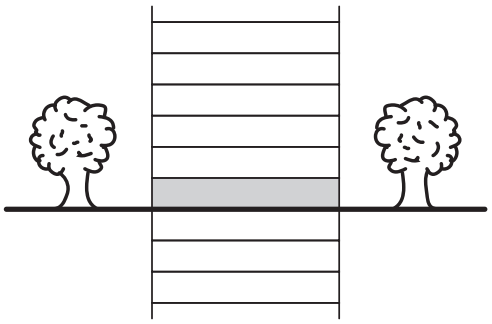
Advance Preparation: Use Blackline W14 to make student copies of the 0–109 numeral chart.

Description of Lesson

Exercise 1

Ask the students if they know of any tall buildings. Encourage them to tell you the names of some tall buildings and where they are located. Some students may have visited some tall buildings and can tell the class about their experiences. If no one mentions the Empire State Building in New York City, do so yourself and tell the class that the Empire State Building was the tallest building in the world for many years.

Draw this picture on the board.



Explain that the picture on the board shows only the floors of the Empire State Building that are near ground level because it has too many floors to picture all of them.

T: ***Where is the ground floor in the picture?***

The student who volunteers should point to the rectangle that is shaded in this illustration.

Note: Some students may be somewhat confused by the use of the word "floor" in this lesson. In this case, floor refers to a story or level of the building rather than just the supporting surface at each level.

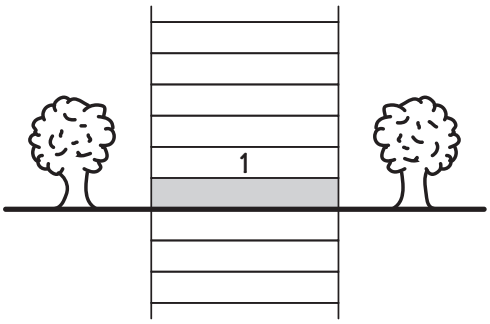
Discuss briefly the need to have a name for each floor. Indicate that the ground floor is mostly open area and has no offices. Often the ground floor has only a lobby, elevators, and stairways.

T: ***Who can show us where the first floor is?***

A student should point to the rectangle directly above the rectangle for the ground floor.

T: ***Where is the second floor? … the fifth floor? Let's assign a number to each of these floors. What number should we give the first floor?***

S: ***1.***



Write 1 inside the rectangle for the first floor.

With students assisting, label all the floors above the first floo

T: ***In many buildings, the ground floor has a letter name instead of a number name. Sometimes the ground floor is called G and sometimes it is called L. Why?***

S: ***G indicates ground level and L indicates lobby.***

T: ***Let's give our ground floor a number name instead of a letter name. What number should we assign to the ground floor?***

S: ***0.***

Label the ground floor 0 in your picture on the board.

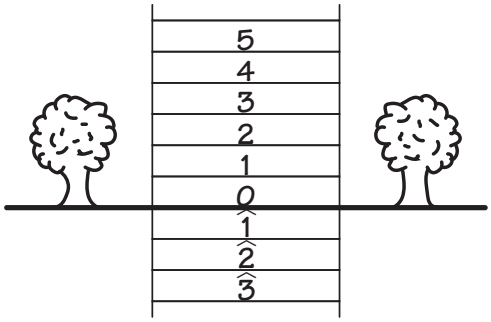
T: ***In many buildings the floors below the ground are called B1, B2, B3, B4, and so on. Why?***

S: ***B is for basement.***

T: ***What numbers should we assign to the floors below ground level?***

S: ***Negative numbers.***

Ask the class to assist you in labeling all the floors shown in the picture that are below ground level.



T: ***If an office worker walks back to the building after having lunch and rides the elevator to the fifth floor, how many floors would this person go up?***

S: ***Five.***

Demonstrate this by counting the floors in the drawing of the building.

T: ***After work the office worker rides the elevator to the second basement floor*** (point to $\hat{2}$) ***where the cars are parked. What is the number of the floor where the cars are parked?*** ($\hat{2}$) ***Should the office worker go up or down the elevator?*** (Down) ***How many floors does the office worker go down from floor 5 to floor $\hat{2}$?*** (Seven)

Demonstrate this by counting the floors in the drawing of the building. Repeat this activity with the following stories or similar ones that involve trips to floors that are included in your drawing.

T: ***The janitor has an office on floor $\hat{3}$. He is in his office when he receives an emergency call from floor 1. Should he take the elevator up or down?*** (Up) ***How many floors should the janitor go up?***

S: ***Four.***

T: ***After he takes care of the emergency on floor 1, he wants to go to floor 4 to check on the heating system. How many floors should he go up?***

S: ***Three.***

T: ***If the janitor now wants to return to his office from floor 4, should he go up or down?*** (Down) ***How many floors down?***

S: ***Seven.***

Return to your discussion about the Empire State Building.

T: ***How many floors above ground do you think the Empire State Building has?***

Tell the class the Empire State Building has 102 floors above ground level.

Exercise 2: Detective Story

T: ***Virginia Lockwood is an executive who has an office in the Empire State Building. She likes to write detective stories about secret numbers for her friends. In this story, the secret number is the number of the floor where her office is located. Let's see if you can figure out Ms. Lockwood's secret number.***

Clue 1

T: ***Her first clue is this: "The number of the floor where my office is located is on the 0–109 numeral chart."***

Display a 0–109 numeral chart and distribute copies to students.

T: ***Could Ms. Lockwood's office be in a basement?*** (No, there are no negative numbers on the chart.)
Could Ms. Lockwood's office be on floor 109? (No)
Why not?

S: ***There are only 102 floors in the Empire State Building, so the secret number can not be 103, 104, 105, 106, 107, 108, or 109.***

Indicate that the whole numbers from 103 to 109 cannot be the secret number by crossing them out on the 0–109 numeral chart. Students can do the same on their charts.

T: ***Could the secret number be 0?***

S: ***No, because there are no offices on the ground floor.***

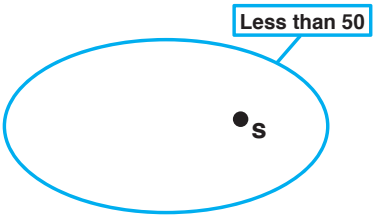
Cross out 0 on the 0–109 numeral chart.

T: ***Are there any other numbers we can cross out now?*** (No)

~~0~~	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	54	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99
100	101	102	~~103~~	~~104~~	~~105~~	~~106~~	~~107~~	~~108~~	~~109~~

Clue 2

T: ***Ms. Lockwood's second clue is a string picture.***



Begin a string picture on the board.

T: ***What does this string picture tell us about the secret number*** (point to **s**)***?***

S: ***The secret number is less than 50 so Ms. Lockwood's office is on a floor between 0 and 50.***

T: ***Which of these numbers*** (point to the 0–109 numeral chart) ***cannot be the secret number?***

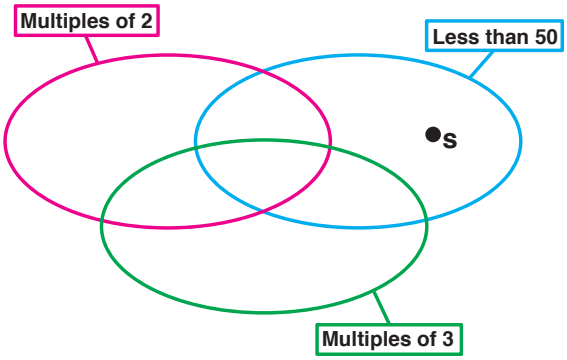
S: ***50, and all the numbers greater than 50.***

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	54	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99
100	101	102	103	104	105	106	107	108	109

Indicate that the whole numbers from 50 to 102 cannot be the secret number by crossing them out on the 0–109 numeral chart. Students can do the same on their charts.

T: ***By adding two more strings to the picture, Ms. Lockwood tells us more about the secret number.***

Extend your string picture as shown here.



T: ***What does this red string tell us about the secret number?***

S: ***The secret number is not a multiple of 2.***

T: ***What kind of numbers are not multiples of 2?***

S: ***Odd numbers.***

T: ***The secret number is outside of the red string, so we know the secret number is an odd number. Which of these numbers*** (point to the 0–109 numeral chart) ***cannot be the secret number?***

S: ***The even numbers.***

T: ***Cross out the even numbers on your charts.***

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	54	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99
100	101	102	103	104	105	106	107	108	109

As students are working, send some to the demonstration chart to cross out even numbers. At first the students may suggest even numbers in no particular order. Emphasize that all the numbers with ones digit 0, 2, 4, 6, or 8 are even numbers and lead students to see that entire columns of numbers can be ruled out at one time. Continue until your chart looks like this one.

T: ***What does the green string tell us about the secret number?***

S: ***The secret number is not a multiple of 3.***

T: ***Which numbers are multiples of 3?***

S: ***The numbers we say when we start at 0 and count by threes.***

T: ***We know that the secret number is not a multiple of 3. We have already crossed out 0. What is the next number in this chart that is a multiple of 3?***

S: ***3.***

Cross out 3. Continue asking for the next greater multiple of 3 until the students realize that every third number is a multiple of 3. Instruct students to cross them out on their charts. Then invite several students each to indicate several multiples of 3 and to cross them out on the demonstration 0–109 numeral chart. Commend any student who notices that every other multiple of 3 is an even (odd) number.

When all of the multiples of 3 between 1 and 50 have been crossed out, your chart should look like this one.

~~0~~	1	~~2~~	~~3~~	~~4~~	5	~~6~~	7	~~8~~	~~9~~
~~10~~	11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19
~~20~~	21	~~22~~	23	~~24~~	25	~~26~~	~~27~~	~~28~~	29
~~30~~	31	~~32~~	~~33~~	~~34~~	35	~~36~~	37	~~38~~	~~39~~
~~40~~	41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	49
~~50~~	~~51~~	~~52~~	~~54~~	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	~~59~~
~~60~~	~~61~~	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	~~67~~	~~68~~	~~69~~
~~70~~	~~71~~	~~72~~	~~73~~	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	~~79~~
~~80~~	~~81~~	~~82~~	~~83~~	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	~~89~~
~~90~~	~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	~~97~~	~~98~~	~~99~~
~~100~~	~~101~~	~~102~~	~~103~~	~~104~~	~~105~~	~~106~~	~~107~~	~~108~~	~~109~~

Clue 3

T: ***Ms. Lockwood's third clue is this: "Start with 4 on the calculator and press*** [+] [5] [=] [=] [=] ***. . . . You will see my secret number."***

Let students investigate this clue with a calculator for a couple minutes. A student may observe that all the whole numbers that end in 4 or 9 appear. Continue this activity until your class concludes that the secret number is 19, 29, or 49. Write these numbers separately on the board or circle them in the chart.

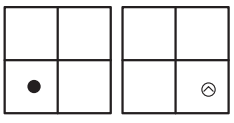
Clue 4

Display two Minicomputer boards, one positive checker, and one negative checker. Provide students with the same.

T: ***Ms. Lockwood's fourth clue is this: "The secret number can be put on the Minicomputer with exactly one regular and one negative checker." Which of these numbers can we put on the Minicomputer with one regular and one negative checker?***

Allow the students to work with their desk Minicomputers to find the secret number. Tell them to put the number on the Minicomputer and write it on a paper. After a few minutes invite a volunteer to put the secret number on the demonstration Minicomputer.

T: ***19 could be the secret number. Can we put 29 or 49 on the Minicomputer with one regular and one negative checker?***



Allow students who think they can put either of these numbers on the Minicomputer to try. Perhaps a student will comment that 29 = 30 – 1 and 49 = 50 – 1 so neither of these numbers can be put on the Minicomputer with exactly one regular and one negative checker. Conclude that Ms. Lockwood's office is on the 19th floor.

W15 SUMMER SCHOOL IN THE OLD DAYS

Capsule Lesson Summary

Read the *Summer School in the Old Days* Storybook and discuss the illustrations in it.

Materials

Teacher	• *Summer School in the Old Days* Storybook • Minicomputer set	Student	• *Summer School in the Old Days* Storybook

Note: A classroom set of materials has 15 copies of the storybook, so you may want to pair students with a storybook during the lesson.

Description of Lesson

Read the storybook *Summer School in the Old Days* with your class as you would read any other story. You may wish to read it aloud yourself or ask students to read it aloud. Most of the content of this storybook is presented in the illustrations, so it is important that the students have time to look at the pictures and to comment on them. Frequently ask students to describe what they see on a particular page of the storybook. The following suggestions as well as the text of the storybook will help you to guide the discussion of the illustrations.

Pages 3–7

Ask students how many sticks each of the numbers pictured is carrying and how the sticks are grouped. Emphasize that some sticks are loose and some sticks are bundled in tens and in hundreds. For example, 354 is carrying three bundles with 100 sticks, five bundles with 10 sticks, and four loose sticks.

Pages 8–11

Ask students to suggest number sentences for each of the presentations by 12 and 45. Record the number sentences on the board.

Pages 12 and 13

Determine the number of sticks the children stole from 74; then ask how many sticks 74 has left. Encourage students to calculate this number rather than counting the sticks in the picture.

Pages 14–17

Ask students to comment on the way 74 has arranged the sticks and to notice how the new sticks are stored. Discuss how 74 might arrange the new sticks which 0 has brought. Emphasize that 74 is 7 tens and 4 ones.

Pages 18–21

Ask students what the flag of the World of Numbers looks like. (A Minicomputer) Notice any other Minicomputers in the pictures, and ask what numbers are displayed when there are checkers on them.

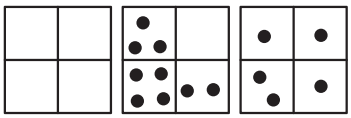
Page 22 and 23

Encourage students to explain why the numbers no longer carry any sticks and why 0 does not carry a bag of checkers.

Page 24–27

Ask students to suggest a number sentence for each of the presentations by 9.

After discussing the first two pictures for 357, put this configuration on the Minicomputer using only red and yellow checkers.



Note: It would be preferable to use all red checkers, but your Minicomputer set does not contain enough red magnetic checkers; therefore, use some yellow checkers or borrow some red checkers from another teacher. Avoid using blue checkers in this configuration.

Invite someone to explain why the number is 357.

Pages 28–32

Encourage students to think about what 0 did with the red and the blue checkers on page 32. Tell them next week there will be another lesson about summer school and in that lesson they will learn more about 0's idea.

Writing Activity

Invite students to write a story of their own about the numbers.

Capsule Lesson Summary

Extend the ideas of a previous storybook in the *Summer School: 0's Discovery* Story-Workbook.

Materials

Teacher	• *Summer School: 0's Discovery* Story-Workbook	Student	• *Summer School: 0's Discovery* Story-Workbook • Colored pencils, pens, or crayons

Description of Lesson

The story-workbook *Summer School: 0's Discovery* is a continuation of the storybook *Summer School in the Old Days*. At the end of *Summer School in the Old Days*, a boy put a bag of red checkers and blue checkers in 0's lunch box; these checkers gave 0 an idea.

After a brief discussion of *Summer School in the Old Days*, distribute copies of the story-workbook. Allow the students a few minutes to read it silently and encourage them to try to figure out 0's idea. Instruct the students not to write in the story-workbook at this time.

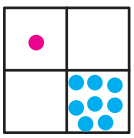
Pages 2–5

Read, or ask students to read, pages 2 and 3 aloud. Then discuss the configurations for 0 on pages 4 and 5. A sample dialogue follows:

T: ***Let's look carefully at these posters. What do you notice about the first poster 0 is showing us?***

S: ***There is a red checker and a blue checker on each square.***

T: ***Isn't it interesting that 0 did that! Let's look at the next poster. What number is on the Minicomputer with red checkers?***



S: ***8.***

T: ***…with blue checkers?***

S: ***8.***

In the same manner consider each of the other two posters.

T: ***What is 0's idea? How is 0 using the checkers?***

S: ***0 is using one color for regular checkers and the other color for negative checkers.***

T: ***That's right! In this story-workbook, red checkers are used to show positive numbers and blue checkers are used to show negative numbers.***

Invite students to show other configurations for 0 using positive and negative checkers on the demonstration Minicomputer.

Pages 6 and 7

After reading these pages aloud, discuss why 200 and 16 are angry with 0.

Pages 8–11

After reading page 8, ask students to explain why the poster shows a configuration for 1. On page 9, allow some time for students to show several ways to put 1 on the Minicomputer. Let students take turns putting their "pictures" of 1 on the demonstration Minicomputer.

Note: If students prefer, allow them to use negative checkers ⊖ instead of blue checkers to indicate negative numbers on the Minicomputer.

When a few students have completed page 9, continue reading on page 10. Discuss the posters prepared by 2 and 3; then ask students to design other posters for each of those numbers using red checkers and blue checkers. As students find solutions, call on them to show their "pictures" on the demonstration Minicomputer.

Pages 12 and 13

Read these pages aloud or ask a student to read them. Allow about five minutes for students to work on page 13. Emphasize that they are to put each of the numbers on the Minicomputer using exactly one red checker and one blue checker. Invite students to show their solutions on the demonstration Minicomputer.

Pages 14 and 15

Finish reading the story-workbook. Ask the class why the numbers formed a circle around the children and cheered. If necessary, remind them that the boy near 23 put the bag of red checkers and blue checkers in 0's lunch box.

Capsule Lesson Summary

Solve a detective story in order to find the floor on which you must change elevators in the Empire State Building. The clues involve a +3 arrow picture, a string picture, special configurations on the Minicomputer, and multiples of 4.

Materials

Teacher	• Colored chalk • Minicomputer set • Calculator	Student	• Paper • Colored pencils, pens, or crayons • Calculator

Description of Lesson

Exercise 1

Begin this lesson with about five minutes of mental arithmetic involving multiples of 4. Try to maintain an atmosphere of exploration rather than turn this activity into a drill. If the class loses interest, go on to Exercise 2.

T: ***What are some numbers that are multiples of 4?***

Encourage students to name many multiples of 4, including negative numbers and relatively large numbers such as 200 and 400. Occasionally ask what number multiplied by 4 would equal the number that has been suggested. This would be especially appropriate in the following situations:

a) when the suggested number is a multiple of 4 between 4 and 20;
b) when the number suggested is 40; 400; 4,000; 4,000,000; ….

You may also like to use a calculator counting by fours from 0 (or some other multiple of 4) to generate or to check for multiples of 4.

Whenever possible, if the suggested number is not a multiple of 4, explain how you know this number is not a multiple of 4. A sample dialogue follows:

S: ***16.***

T: ***4 times what number is 16?*** (4)
4 x 4 = 16, so 16 is a multiple of 4.

S: ***40.***

T: ***4 times what number is 40?*** (10)
4 x 10 = 40, so 40 is a multiple of 4.

S: ***44.***

T: ***How do you know that 44 is a multiple of 4?***

S: ***I just added 4 to 40.***

S: ***4 x 11 = 44.***

S: ***100.***

T: ***4 times what number is 100?*** (25)
4 x 25 = 100, so 100 is a multiple of 4.

S: ***200.***

T: ***Yes, 200 is a multiple of 4 because 200 = 100 + 100.***

S: *50.*

T: *Is 50 a multiple of 4?* (No) *Let's start at 40 and keep adding 4: 40, 44, 48, 52. We skipped 50, so 50 is not a multiple of 4.*

S: *816.*

T: *800 is a multiple of 4 and 16 is a multiple of 4, so 816 is a multiple of 4.*

S: *32.*

T: *32? Is 32 a multiple of 4?* (Yes) *32 = 16 + 16 (or 32 = 2 x 16) and 16 is a multiple of 4.*

T: *What are some large numbers that are multiples of 4?*

S: *1,000,000.*

S: *4,000,000.*

T: *4 times what number equals 4,000,000?* (1,000,000)

S: *1,000,004.*

T: *Do you know any multiples of four less than 0?*

S: *4.*

S: *44.*

Exercise 2: Detective Story

You may like to let students work with a partner during this exercise.

Ask the class what they remember about the Empire State Building. If necessary, remind them that the Empire State Building has 102 floors above the ground level.

T: *There is an observation area on floor 102. Many tourists visit this observation area because from there they have a marvelous view of New York City. If you start at ground level and ride the elevator to floor 102, how many floors will you go up?* (102) *Try to imagine going up 102 floors on an elevator.*

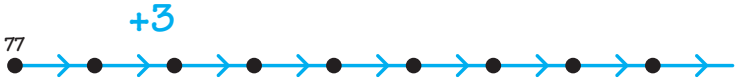
But the Empire State Building does not have an elevator that goes all the way from floor 0 to floor 102. People who wish to go all the way to the 102nd floor must change elevators.

Allow a brief discussion. Be sure the class understands that to go to the 102nd floor you need to get off one elevator and get on another one.

T: *Do you know the number of the floor in the Empire State Building where you must change elevators? You can find out which floor it is by first solving this detective story for a secret number. When you know the secret number, you will know the number of the floor where you change elevators.*

Clue 1

Draw this arrow picture on the board. The last arrow on the right should appear to go off the board.



T: *The secret number is in this arrow picture.*

Ask students to label the dots until the arrow picture is complete.

T: ***Why do you think this arrow has no ending point?***

Be sure the students understand that this arrow road keeps on going.

T: ***Let's make a list of the numbers we meet on this +3 arrow road.***

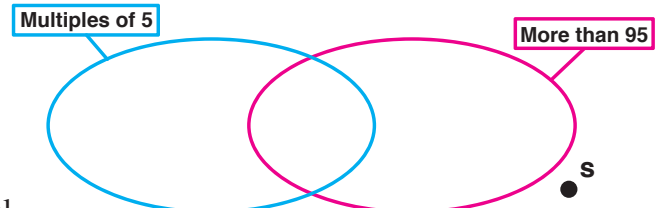
Progressively make a list on the board and ask students to make the same list on their papers. When you reach the last visible dot along this road, ask the class to imagine that the arrow road continues. Students may want to stop the list when you reach 101 because the Empire State Building has only 102 floors. If it is not suggested, stop the list when you reach 104. Ask the students if 104 could be the secret number. Someone should notice that 104 cannot be the secret number because the Empire State Building has only 102 floors. Otherwise, make this observation yourself and erase 104. You should have this list on the board.

77 80 83 86 89 92 95 98 101

Note: Some classes may suggest erasing 101 because there would be no need for an elevator which starts on floor 101. Accept this suggestion, but do not make it yourself nor insist that it be made.

Clue 2

Draw the following string picture on the board and ask students to copy it on their papers. Trace the blue string as you ask,



T: ***What are some multiples of 5?***

Accept three or four correct answers but do not place them in the picture. Trace the red string as you ask,

T: ***What are some numbers that are more than 95?***

Accept three or four answers, but again do not place them in the picture.

T: ***This dot*** (point to **s**) ***is for the secret number. What can you tell me about the secret number?***

S: ***It is not a multiple of 5.***

S: ***It is not more than 90.***

T: ***Which of these numbers in our list could be the secret number?***

Instruct students (with their partners) to put all the numbers on the list into the string picture. After

a few minutes, check with the class to either cross a number off the list if it cannot be the secret number or circle it if it could be the secret number. Continue until all the numbers in your list have been considered.

T: ***Now we know that the secret number is 78, 83, 86, 89, or 92.***

Erase everything in the list that has been crossed off.

Clue 3

Display two Minicomputer boards.

T: ***The secret number can be put on the Minicomputer with no checker in the white square on the ones board (the one-square.) What numbers left in our list could be the secret number?***

When students claim to be able to put a number on the Minicomputer, ask them to do so. Your students should conclude that it is impossible to put 77, 83, or 89 on the Minicomputer without a checker in the one-square because they are odd numbers. On the other hand, 86 and 92 can be put on the Minicomputer with no checker in the one-square.

T: ***So the secret number is either 86 or 92.***

Clue 4

T: ***The secret number is not a multiple of 4.***

Ask the students to write the secret number on a piece of paper when they know it. You may let students use calculators to check whether or not a number is a multiple of 4.

T: ***What is the secret number?***

S: ***86.***

T: ***80 is a multiple of 4 so 84 and 88 are also multiples of 4 but not 86.***
Since 80 is a multiple of 4, and 12 is a multiple of 4, 80 + 12 = 92 is a multiple of 4.
Do you know which floor the special elevator is on?

S: ***The 86th floor.***

T: ***The elevator to the observation area starts on floor 86 and goes to floor 102. How many floors does it travel?***

S: ***Sixteen.***