The Languages of Strings and Arrows

LANGUAGES OF STRINGS & ARROWS TABLE OF CONTENTS

THE LANGUAGES OF STRINGS & ARROWS INTRODUCTION

The language of strings is the nonverbal language of sets, that is, collections of objects put together in some way. The language of arrows is the nonverbal language of relations among objects. These languages permeate the *CSMP* strands and are used, separately or together, to present an unending variety of numerical and nonnumerical situations.

Classification: The Language of Strings

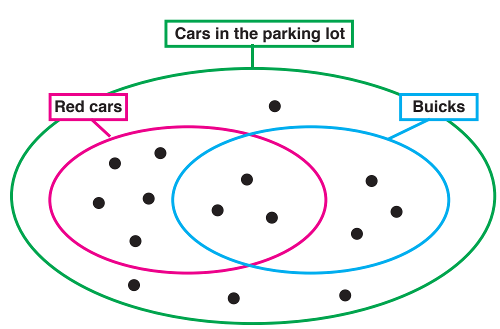
As the word implies, classifying means putting things into classes, or as the mathematician says, sets. The mathematics of sets can help students to understand and use the ideas of classification. The basic idea is simple: Given a set S and any object x, either x belongs to S (x is in S) or x does not belong to S (x is not in S). We represent this simple act of sorting—in or out—by using pictures to illustrate in and out in a dramatic way. Objects to be sorted are represented by dots, and the sets into which they are sorted are represented by drawing strings around dots. A dot inside the region delineated by a set's string is for an object in the set and a dot outside a set's string is for an object not in the set.

This language of strings and dots provides a precise (and nonverbal) way of recording and communicating thoughts about classification. The ability to classify, to reason about classification, and to extract information from a classification are important skills for everyday life, for intellectual activity in general, and for the pursuit and understanding of mathematics in particular. The unique quality of the language of strings is that it provides a nonverbal language that is particularly suited to the mode of thinking involved in classification. It frees young minds to think logically and creatively about classes and to report their thinking long before they have extensive verbal skills.

In this strand we present situations and ask carefully phrased questions to continue to advance skills in classification, always remembering that the skills grow out of such experiences. To be able to draw strings and dots is not an objective in itself; to develop the mode of thinking involved is the objective. Thus it is important for us to construct the situations carefully. The sets into which we ask students to classify objects must be determined by well-defined attributes; otherwise, there is the added problem of deciding whether a certain object does or does not have a certain attribute. For this reason we make extensive use of A-blocks (attribute blocks) and sets of numbers in designing classification situations. Students can immediately say whether or not a block is red, whether or not it is a square, and so on. They know whether or not a given number is less than 15, whether or not it is odd, and so on.

One reason for classifying objects is to count the objects that have a certain attribute. Suppose, for example, there are eight red cars and six Buicks in a parking lot. If four cars in the lot are neither red nor Buicks, must there be 18 cars in the lot? Suppose we count the cars in the parking lot and find there are exactly 15. Is this possible? A string picture immediately settles this apparent paradox.

There are three red Buicks.



Relations: The Language of Arrows

Relations are interesting and important to us in our everyday lives, in our careers, in school, and in scientific pursuits. We are always trying to establish, explore, and understand relations. In mathematics it is the same; to study mathematics means to study the relations among mathematical objects like numbers or geometric shapes. The tools we use to understand the everyday world are useful to understand the world of mathematics. Conversely, the tools we develop to help us think about mathematical things often serve us in nonmathematical situations.

A serious study of anything requires a language for representing the things under investigation. The language of arrows provides an apt language for studying and talking about relations. Arrow diagrams are a handy graphic representation of a relation, somewhat the same way that a blueprint is a handy graphic representation of a house. By means of arrow diagrams, we can represent important facts about a given relation in a simple, suggestive, pictorial way—usually more conveniently than the same information could be presented in words. The convenience of arrow diagrams has important pedagogical consequences for introducing children to the study of relations in the early grades. A child can read—and also draw—an arrow diagram of a relation long before he or she can read or present the same information in words. The difficulty of presenting certain ideas to children lies not in their intellectual inability to grasp the ideas; rather, the limitations are often mechanical. Arrow diagrams have all the virtues of a good notation: they present information in a clear, natural way; they are attractive, colorful things to look at; they are easy and fun for children to draw. Another educational bonus occurs when arrow diagrams are used imaginatively. A story or problem may be captured by an arrow diagram, and at the same time, the resulting arrow diagram may suggest other, similar situations. This allows students to call upon previous experience or to expand their creativity.

One of the purposes of this strand is to use the power of the language of arrows to help children think logically about relations. Again it must be remembered that our goals concern the thinking process and not the mechanism. The ability to draw prescribed arrows is not the objective in itself, nor is viewing an arrow diagram just another format for drill problems in arithmetic.

The general aim of The Languages of Strings and Arrows strand is to suggest situations that are inherently interesting and thought provoking, and to give children modes of thinking and appropriate languages with which they can organize, classify, and analyze. In addition to a varied assortment of lessons concerning sets and relation, this strand includes lessons involving systematic methods for solving combinatorial (counting) problems; methods for collecting, recording and interpreting data in real life situations; probabilistic thinking; and networks.

L1 STRING GAMES #1

Capsule Lesson Summary

Describe each of the A-blocks by its shape, color, and size. Introduce and play The String Game with two strings and nine possible string labels. Discuss not-cards as string labels and place the A-blocks in a two-string picture where one string is labeled with a not-card. Play The String Game allowing that the strings could have any of sixteen possible labels.

Materials

Teacher	• A-Block String Game kit • Box • Colored chalk	**Student**	• None

Advance Preparation: Before this lesson, you will need to prepare some materials from the A-Block String Game kit. These materials will be used again in all future lessons on The String Game, so keep them together in the envelope with the kit.
1. Punch out one set of shapes (24). If you have a magnetic board, magnetize each A-block by sticking a small piece of the magnetic material to the back. If there is no magnetic board available, you can still use a regular chalkboard. In this case, have loops of masking tape ready to stick to the back of each A-block so that they can be stuck to the board. Masking tape loses its stickiness quickly, so be prepared to reinforce the A-blocks regularly with new loops of tape.
2. Prepare the string cards in the same manner as the A-blocks.
3. Prepare a team board as pictured on the next page. This board should be metallic if you have magnetized A-blocks. Post a list of attributes above the team board (posters found in the A-Block String Game). You may want to laminate the A-Block pieces and cards to make them more durable.

Description of Lesson

Exercise 1

Put the 24 A-block pieces in a box (a greeting card box is a good size).

T: ***In this box I have some pieces called A-blocks with different shapes, colors, and sizes. There are three different shapes. What shapes do you think the pieces are?***

S: ***Squares.***

S: ***Triangles.***

S: ***Circles.***

Many students have seen A-blocks and are familiar with them. In any case, these responses usually come quickly.

T: ***There are four different colors. What colors do you think the pieces have?***

S: ***Red.***

S: ***Blue.***

S: ***Green.***

S: ***Yellow.***

T: ***There are two different sizes. What should we call them?***

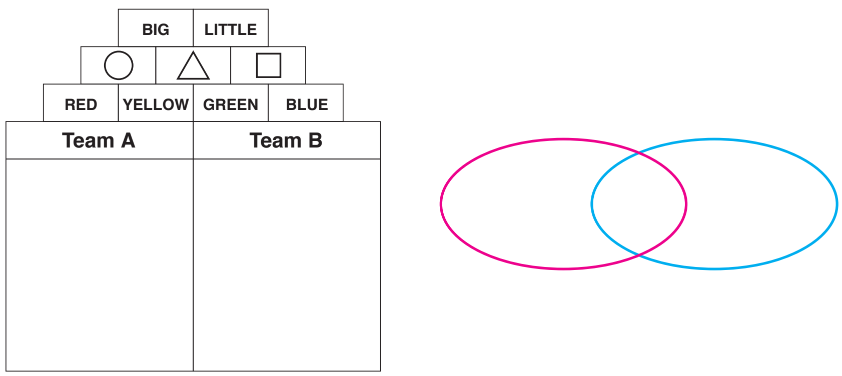
S: ***Big and little.***

T: ***Each A-block piece is described by its shape, color, and size. I have one and only one piece in the box for every combination of a shape, color, and size. How many A-block pieces are there in the box?***

Let students make guesses, perhaps listing a few guesses on the board. Then count the pieces by looking at just one shape (or one color). There are 8 of each shape (or 6 of each color) so there are 3 x 8 (or 4 x 6) or 24 pieces altogether.

Tape a copy of the poster of nine possible string labels (A-Blocks String Game Poster, Version A) to the board and prepare for The String Game as shown below.

Note: If you prefer, the game can be played with more than two teams and the team members can choose names for their teams (rather than A or B). For example, you might use three or four cooperative groups as teams. Prepare the team board accordingly.



T: ***Your first task will be to get all the pieces out of the box and onto this board*** (point to the team board)***. Teams will take turns; someone on Team A will tell me a piece to put on Team A's side of the board, and then someone from Team B will tell me a piece to put on Team B's side of the board. Remember you must describe a piece that is still in the box and I need to know exactly which piece you want me to put on your side of the board.***

Alternating teams, call on students to describe A-block pieces. Insist on complete descriptions as in the following discussion.

S: ***A big circle.***

T (looking in the box)**:** ***I have several big circles. Which one do you want?***

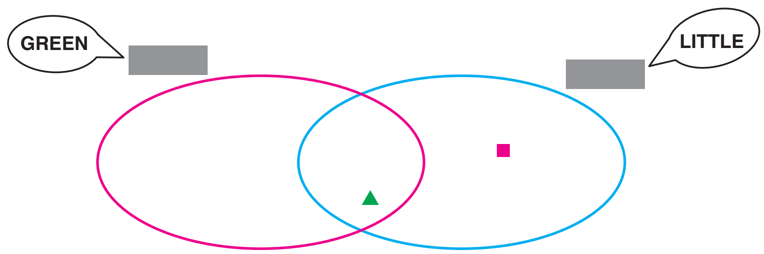
S: ***The red one.***

T: ***I have a big red circle*** (holding up this piece)***. Is this the piece you want on your side of the team board?***

If a student describes a piece already on the board, point to it and ask for a different piece, one that is still in the box. You may want to select students to call on early in the exercise when there are many choices open to them. Encourage all students to be thinking about a piece they will ask for when you call on them.
Continue until all the pieces have been divided evenly among the teams. Leave them on the board for the next exercise.

Exercise 2

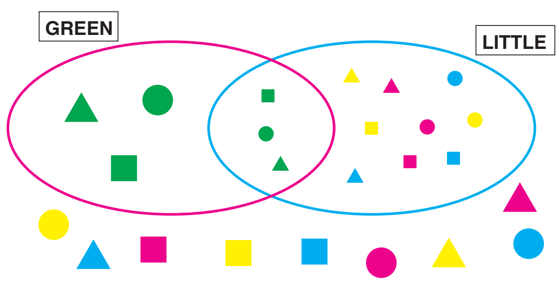
Prepare to play The String Game in the usual way, placing the string card **GREEN** face down near one string and the string card **LITTLE** face down near the other. Select one piece from each team's choices—allow the teams to each select one piece for you—to place correctly in the picture. These then serve as starting clues.



Remember that the rules of the game call for you to be the judge. If a piece is correctly placed, say yes, and immediately invite the player to try to place a second piece (bonus turn). No player should have more than two consecutive turns. If a piece is incorrectly placed, say no, and ask the player to return the piece to his or her team's side of the team board.

For this game, the player who correctly places the last piece from a team's choices may then attempt to identify both string cards. If the player gives an incorrect label for one of the strings, the next player on the opposing team has a chance to either place pieces from that team's unplayed pieces or to identify the strings if all their pieces have been correctly placed. A team wins by being first to get all of its pieces in the string picture and to identify the strings.

The following picture shows correct placement for all the A-block pieces. You may use this picture as a crib sheet during play of the game.



Exercise 3

Clear the board from the last exercise and draw one string on the board. Have the not-cards ready for discussion.

T: ***I am going to show you some other possibilities for string labels.***

Show the class the string card **NOT BLUE** and place it next to the string on the board.

T: ***If this string were for* NOT BLUE*, what pieces would go inside the string?***

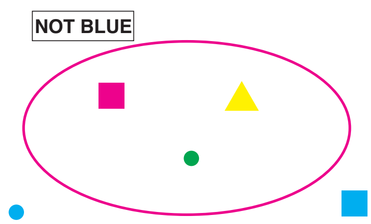
S: ***The little green circle.***

S: ***The big red square.***

Let students place several pieces that would be inside the string and discuss their placement.

T: ***Which pieces would go outside the string?***

S: ***The big blue square.***



Take the A-blocks off the board and erase the string.

T: ***What other new string labels do you think we have?***

Show the class each not-card when it is mentioned.

S: **NOT RED.**

S: **NOT GREEN.**

S: **NOT YELLOW.**

T: ***I also have a card* NOT ○.** (Show the card to the class.) ***Can you describe a piece that is not a circle?***

S: ***A small blue triangle.***

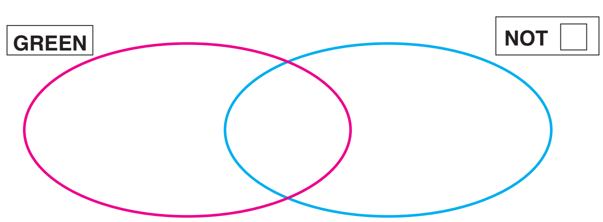
Continue in this manner considering **NOT △** and **NOT □**.

T: ***Do I need a* NOT LITTLE *or* NOT BIG *card?***

S: ***No,* BIG *is the same as* NOT LITTLE.**

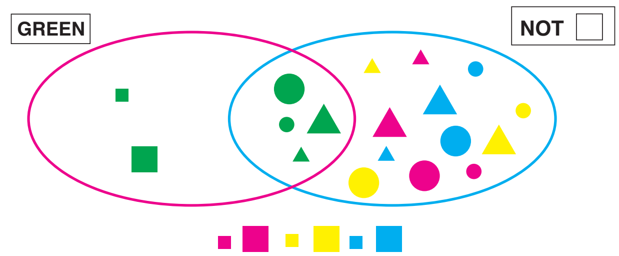
S: ***No,* LITTLE *is the same as* NOT BIG.**

Draw two overlapping strings on the board using two different colors. Label them as shown below (with face-up cards).



Let students place various pieces of their choice (or yours) in the picture. After about eight to ten pieces have been put on the board, the class should be better acquainted with the new labels.

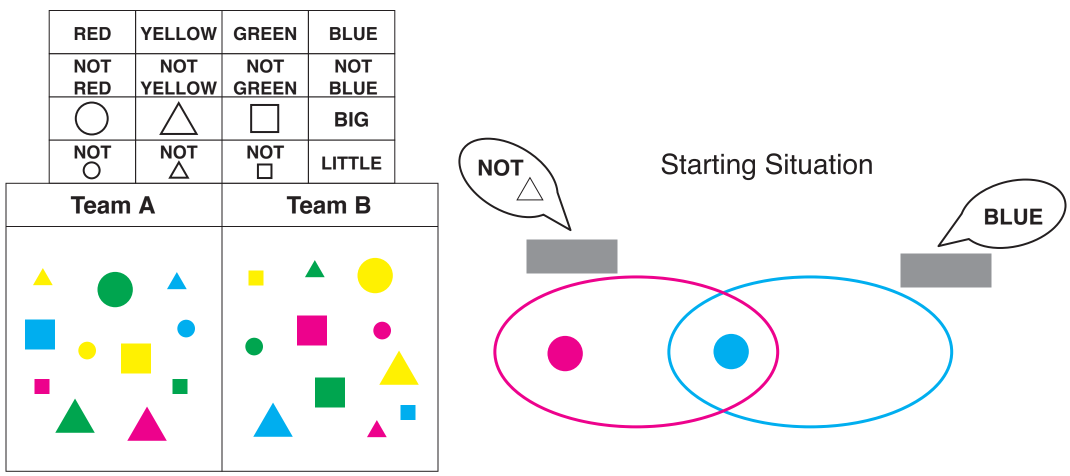
The next picture shows the correct placement of all the A-block pieces.

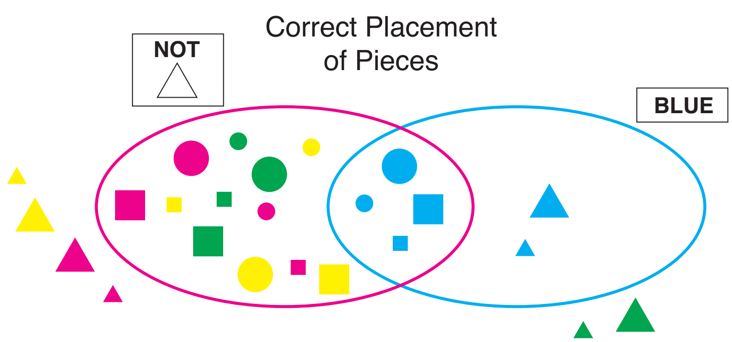


Exercise 4 (optional)

Use this exercise if the~~re is time. Otherwise,~~ there will be other lessons (such as L6) where the game will be played. You may also use the game during unscheduled time.

Prepare your board to play a two string version of The String Game in the usual way with face-down string cards. Tape the poster list of sixteen possible string labels above the team board. Distribute A-block pieces for Team A and Team B on the team board. The next illustration shows the starting situation for a game followed by the correct placement of all the A-blocks.





Center Activity

Place materials fromThe String Game kit in a center for students to conduct their own games.

Capsule Lesson Summary

Play a composition game with colored arrows where the only rule for the game is this: yellow followed by blue is red. Composition is an important relation concept, as often relations must be combined to get new relations. Exercise 1 provides an opportunity to experiment with composition paying attention to direction and order without worrying about specific relations. Exercise 2 provides a numerical example.

Materials

Teacher
- Colored chalk

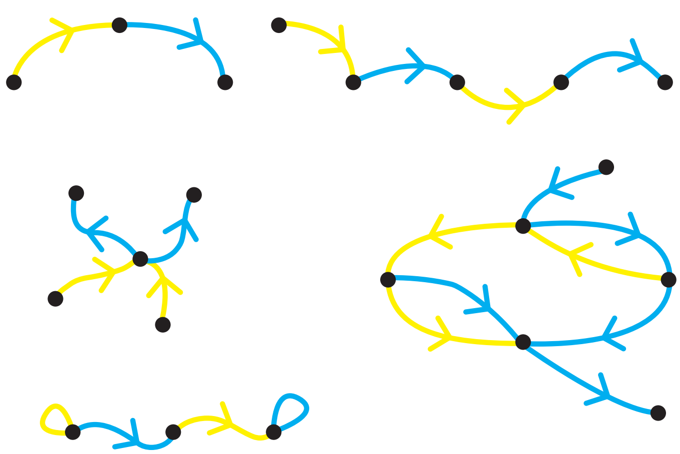
Student
- Worksheets L2* and **; L2(a) and (b)
- Colored pencils, pens, or crayons

Advance Preparation: Before the lesson begins, draw the arrow picture from Exercise 1 on the board or prepare it on a transparency. Give the picture plenty of space, and draw arrows in colors that can be easily distinguished.

Description of Lesson

Exercise 1

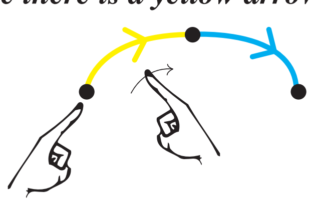
Invite students to comment on this arrow picture.



T: ***We are going to play a game with yellow and blue arrows. Some of you will remember this game from last year. The object of the game is to draw red arrows, but there is a rule for drawing red arrows.***

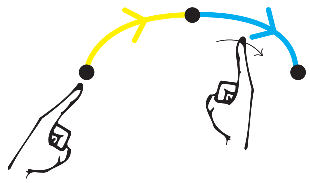
Use the upper left corner of the arrow picture for your explanation. As you explain the rule stated in the left column below, make the motions described in the right column.

T: ***Each time there is a yellow arrow…***



(Point and hold your left forefinger on a dot at which a yellow arrow starts. Follow the yellow arrow with your right forefinger in the direction of the arrowhead.)

T: ***…followed by a blue arrow…***



(Stop the motion of your right forefinger at the middle dot; tap the dot, and then follow the blue arrow. Hold your right forefinger at the ending dot of the blue arrow.)

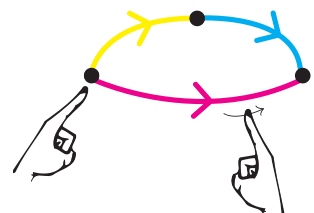
T: ***…then we can draw a red arrow from the dot where the yellow arrow starts…***

(Tap this dot several times with your left forefinger.)

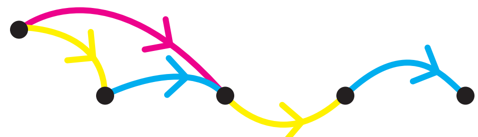
…to the dot where the blue arrow ends.

(Tap this dot several times with your right forefinger.)

First trace and then draw the red arrow. Emphasize that your left forefinger marks the start of the red arrows and that your right forefinger is at the end of the red arrow.



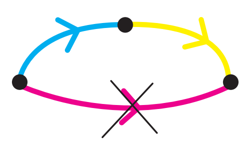
Repeat the statement of the rule using the same words and motions, this time demonstrating with the upper right portion of your picture. Trace and then draw this red arrow.



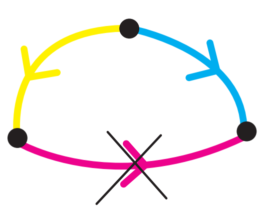
T: ***Where can we draw other red arrows?***

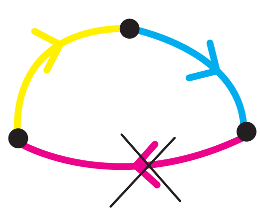
Invite students to the board, one at a time, to show where other red arrows can be drawn. Ask students to first trace a yellow arrow and a blue arrow following it, and then to trace how a red arrow can be drawn. Stop a student who starts to trace against the direction of an arrow, and emphasize that the direction of an arrow must be followed. Encourage the class to help you check for mistakes. Let a student draw a red arrow if it has been traced correctly.

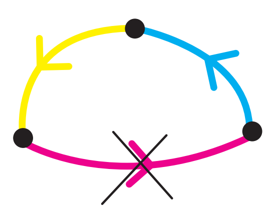
Common mistakes and difficulties: The most common mistake at first will occur when students confuse the rule "yellow followed by blue" with "blue followed by yellow."



Another mistake might occur when the directions of the arrows are not correctly followed.

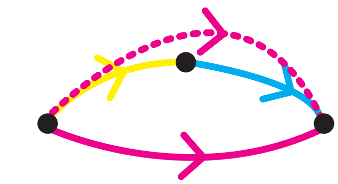






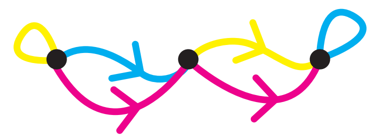
Remind students making such mistakes that they must follow the direction of the arrows. The arrows are like one-way streets and they are not allowed to go the wrong way.

If it happens that a student proposes to draw a red arrow such as the one indicated by the dotted arrow in this illustration…



…another student will probably remark that the red arrow has already been drawn. There is nothing wrong with the dotted arrow; it is merely redundant. That portion of the picture is complete without it.

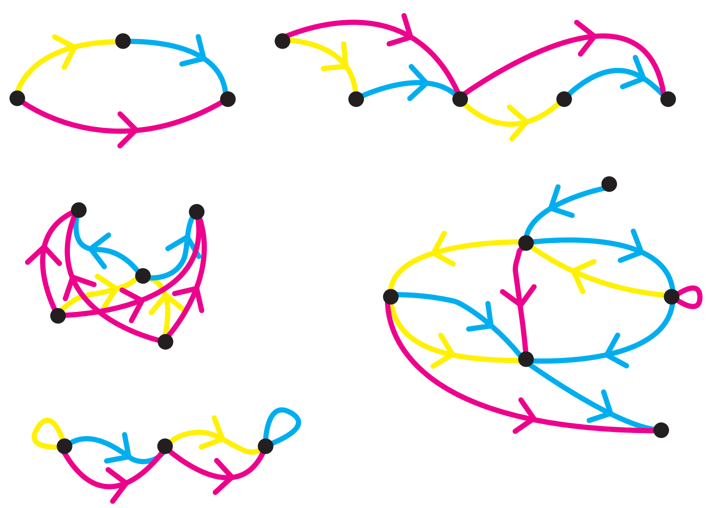
The yellow and blue loops may cause difficulty. If no one discovers one of these red arrows, suggest one of them yourself.



T: ***Yellow*** (trace around the yellow loop) ***followed by blue*** (trace the blue arrow)***, then red. Yellow*** (trace the yellow arrow) ***followed by blue*** (trace around the blue loop)***, then red.***

It is important to keep your left finger on the dot where a yellow arrow (loop) starts and to trace with your right forefinger, ending at the dot where the blue arrow (loop) ends. Then you can demonstrate that a red arrow starts at your left forefinger and ends at your right forefinger.

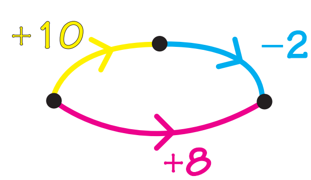
When all the red arrows have been found, your picture should look like this one.



Worksheets L2* and ** are available for students to work on individually, using the yellow followed by blue composition to draw red arrows. You may like to allow about five minutes now for this individual work. Then continue with Exercise 2.

Exercise 2

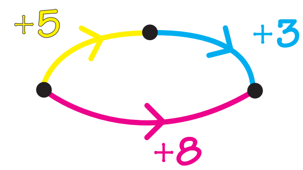
Erase the board except for the upper left portion of your picture.



T: ***Suppose the yellow arrow is for +10 and the blue arrow is for –2. What could the red arrow be for?***

S: *+8.*

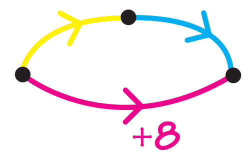
Let students check +8 for the red arrow with several examples of assigning numbers to the dots. Then erase all the labels but not the picture.



T: ***Suppose the yellow arrow is for +5 and the blue arrow is for +3. What could the red arrow be for?***

S: ***+8***

Again, check several examples of assigning numbers to the dots. Then erase the labels on the yellow and blue arrows.



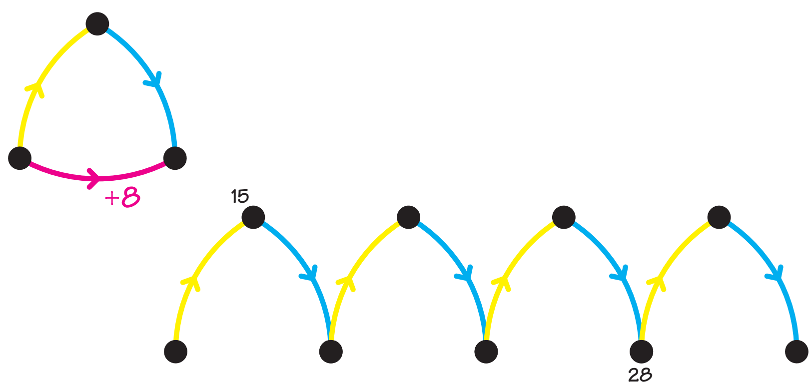
T: ***Suppose you know the red arrow is for +8. Are there other possibilities for the yellow and blue arrows? Let's only consider + some number or – some number for the arrows.***

S: ***+2 for yellow and +6 for blue.***

S: ***+12 for yellow and –4 for blue.***

Consider several possibilities and observe that the possibilities are endless.

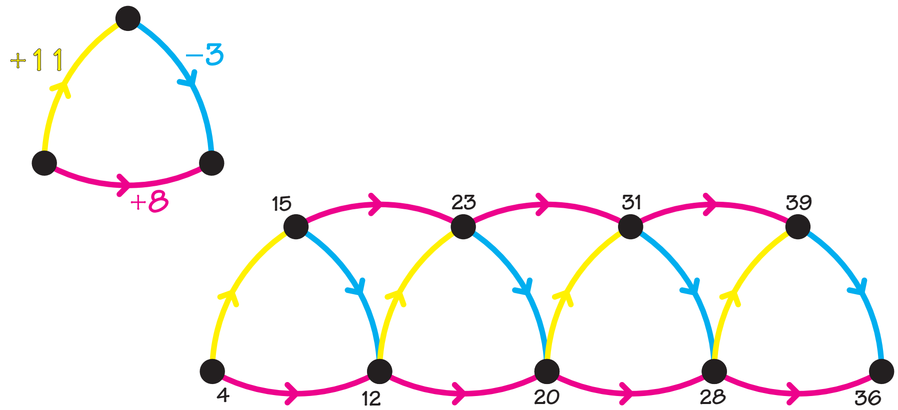
Distribute copies of worksheet L2(a) and draw the picture from the worksheet on the board.



T: ***In this picture, you are not told what the yellow and blue arrows are for, but you do know that the red arrow (yellow followed by blue) is for +8. I will tell you that the yellow and blue arrows are for + some number or – some number. Notice that there are a couple dots already labeled. Can you label the rest of the dots?***

You may let students work with partners on this problem. As you observe students working, discourage too much trial and error, and remind students to use the fact that yellow followed by blue is red (+8).

When most students have found a solution, discuss the problem collectively. You may want to ask students to draw as many red (+8) arrows as they can in the picture. A completed picture showing the additional red arrows and labeled yellow and blue arrows is shown below. You may observe that, in this case, blue followed by yellow is also red.

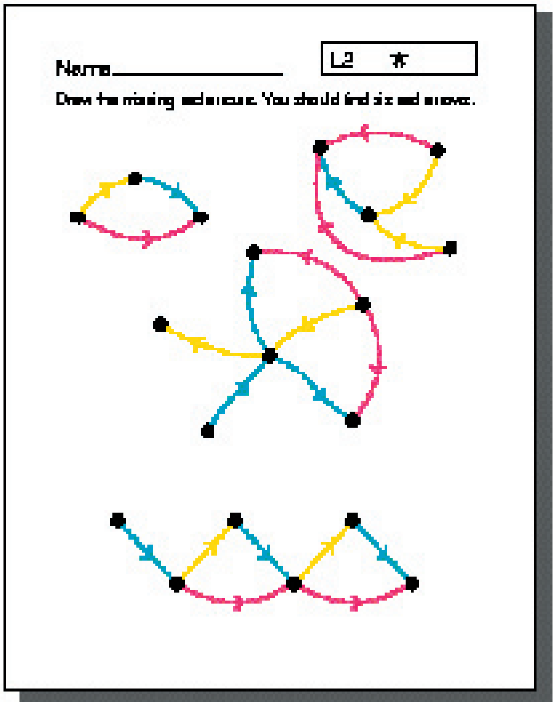


Worksheet L2(b) has another problem similar to L2(a) for individual work. Students who finish quickly may be encouraged to make up their own problems of this kind.

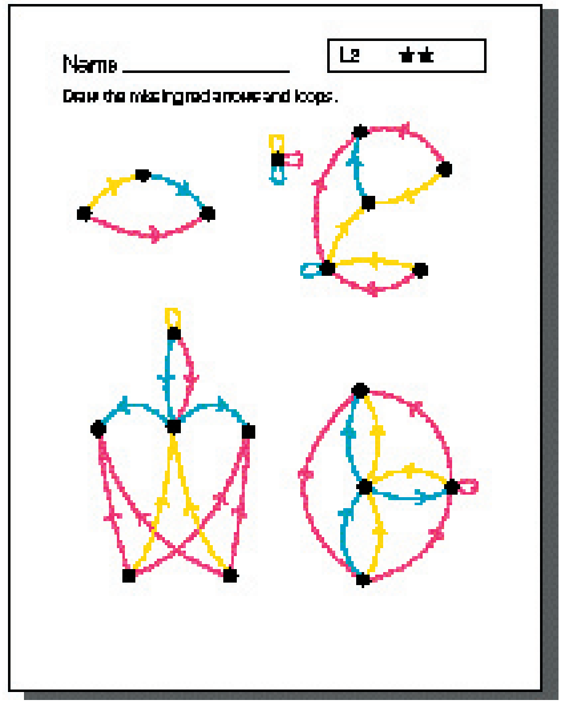
Home Activity

Suggest that parents/guardians practice some mental arithmetic with their child using composition. For example, practice adding 7 by adding 5 and then adding 2, or practice adding 8 by adding 10 and then subtracting 2.

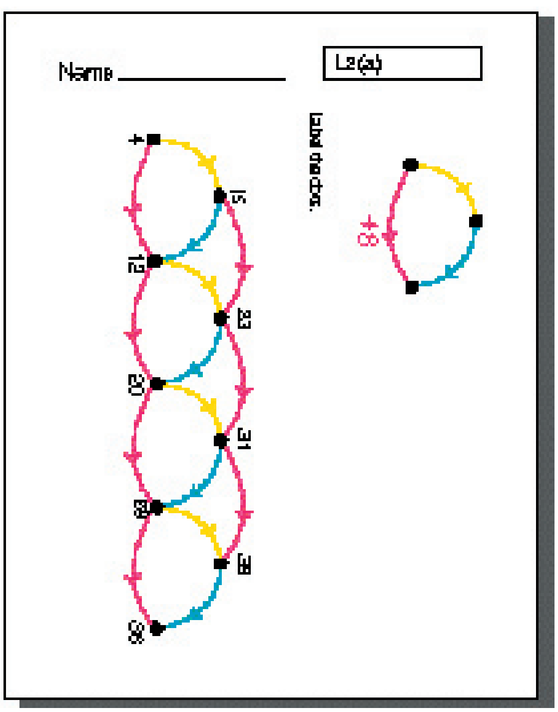
L2



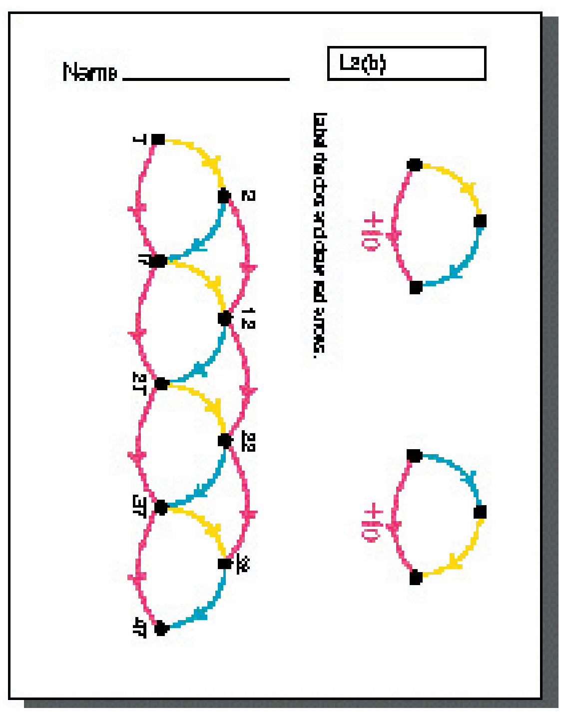
Name
L2 ★



Name
L2 ★★
Draw the missing red arrows and loops.



Name
L2(a)
+8



Name
L2(b)
+10
+10

L3 TWO LISTS

Capsule Lesson Summary

Discuss birthday parties and make class graphs from ideas about birthday parties. Determine how many people are invited to the birthday party of twins when it is known that one twin invites six friends and the other invites eight friends. Use string pictures to examine the situation. Make up and solve some similar problems.

Materials

Teacher
- Colored chalk
- Lists of names

Student
- Unlined paper
- Counters
- Colored pencils, pens, or crayons

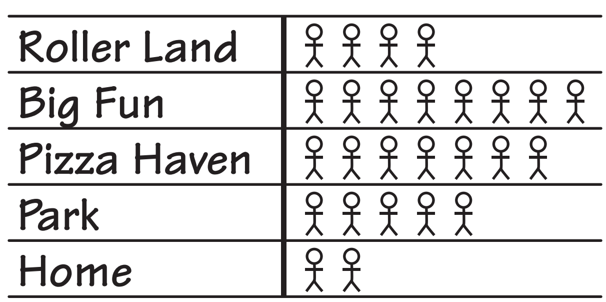
Description of Lesson

Begin this lesson with a discussion about birthday parties (or some other kinds of parties). You may like to let students talk about the following:

- what places they like to have parties
- what games they like to play at parties
- what food they like to eat at parties
- how they invite friends to a party
- how much time to plan for a party

Exercise 1

Choose one or several ideas about parties to use in a graphing activity. For example, you could select "favorite place to have a birthday party" and then collect data from the students in a class graph.



Be sure to ask some questions and make some observations about the class graph.

Note: If you feel your class has enough experience creating such graphs, you may like to divide the class into four or five groups and give each group a different topic (question or favorite thing) about parties. Each group can then collect data from the rest of the class and decide how to display the data in a class graph. Each group can present their findings to the class.

Exercise 2

Copy these lists of names using two separate slips of paper. You may want to substitute names of your students, but arrange that exactly three names are common to both lists. Do not let your students see the names on these lists.

Jill's List

George
Angie
Christy
Melissa
David
Ellen

Jack's List

Sam
George
Alice
Russell
Christy
Mark
Laura
Ellen

T: ***Do you know about the twins Jack and Jill?***

Let the students comment. (Veteran *CSMP* students may recall that Jack and Jill are the twins who live next to Booker's Bakery in a story from UPG-I geometry lessons.)

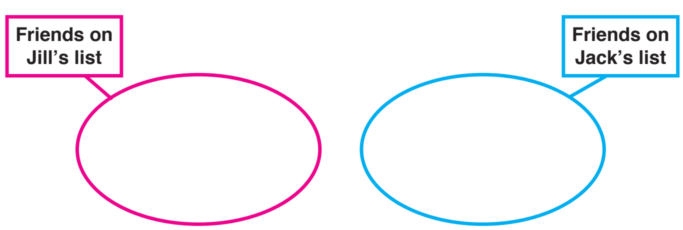
T: ***Jack and Jill are going to have a birthday party. Each of them has made a list of children to invite. Jill put*** (look at Jill's list and count) ***1, 2, 3, 4, 5, 6 children on her list, and Jack put*** (look at Jack's list and count) ***1, 2, 3, 4 , 5, 6, 7, 8 children on his list. How many children will they invite altogether?***

Predictions are likely to include 14 because students may add the number of names on each list. Perhaps some will notice that a name could be on both lists. On the board, record the students' predictions without comment.

T: ***Let's draw a picture to see if any of you are correct. I will draw a red string for the children on Jill's list.***

Who can draw a blue string for the children on Jack's list?

Let a student draw a blue string on the board. There are several possible pictures that could result and you will have to adjust your next questions to the situation. You might get this picture; if so, initiate the following discussion.

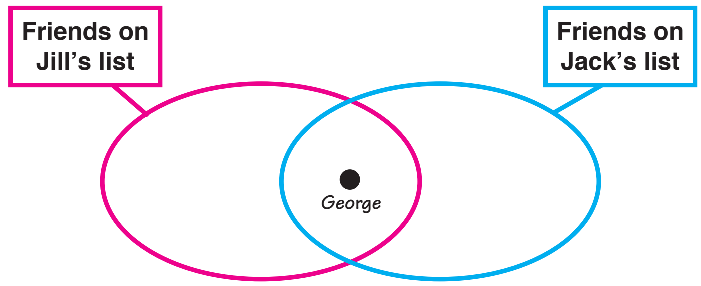


T: ***I have a problem. George is on both Jill's list and Jack's list. Where should the dot for George be?***

A student might suggest that two dots be drawn, one in each string.

T: ***But George is just one person. There should only be one dot for George. How can we solve the problem?***

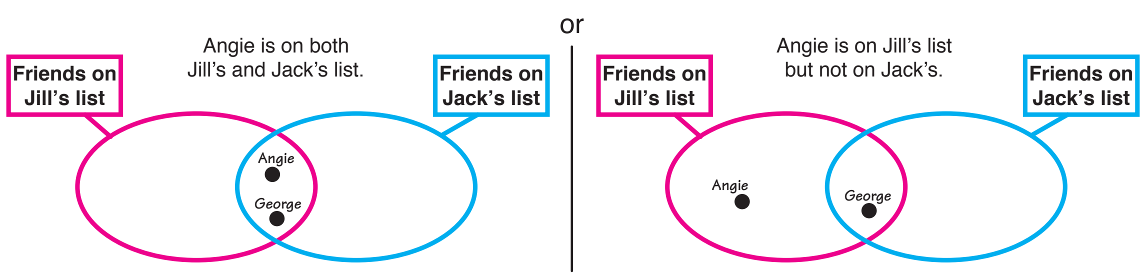
Perhaps after a couple of attempts, a student will suggest overlapping the strings and putting a dot in the middle. Invite a student to redraw the strings and put the dot in the intersection.



Note: If overlapping strings are drawn immediately in your class, go on with the lesson without mention of the preceding problem. In this case, adjust the dialogue that follows to consider George first.

T: ***Now I will read the names on Jill's list. The first one is George and we have a dot for George in our picture. The next name is Angie. Where should we draw a dot for Angie?***

A student may respond by guessing where to put the dot or by hesitating since at this point Angie could be in one of two places in the string picture.



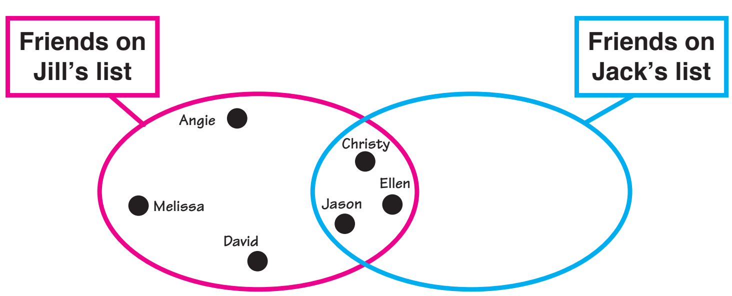
If a guess is made, ask, "How do you know where to put the dot?" Guide the discussion until it is clear that the class cannot be sure at this time where to put the dot.

T: ***You are right to think about the situation. Perhaps you can ask a question so you will be sure where to put the dot for Angie.***

S: ***Is Angie on Jack's list?***

T: ***That's a very good question. No, Angie is not on Jack's list.***

Let the student draw and then label a dot and ask the class to check that it is correct. Continue in this way until all of the names on Jill's list have been considered.



L3

At this point you can ask students whether they still think their predictions of the number of children who will be invited to the party are correct. If some students wish to change their predictions, ask them to explain why. Again, do not comment on the correctness of their responses.

T: ***Now I'll look at Jack's list. How many names are on his list?***

S: ***Eight.***

T: ***How many children from Jack's list are already in our picture?***

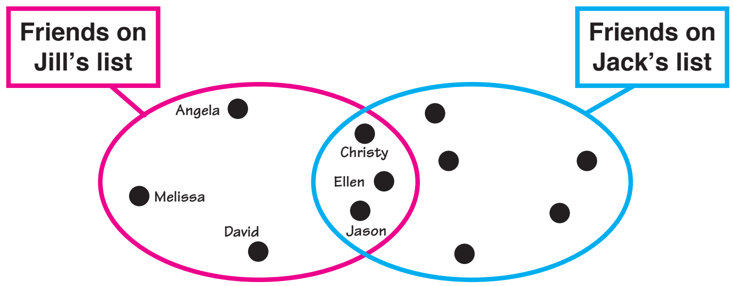
S: ***Three: George, Christy, and Ellen.***

T: ***So how many other children did Jack put on his list?***

S: ***Five.***

T: ***Without knowing their names, who can put dots for these five children in our picture?***

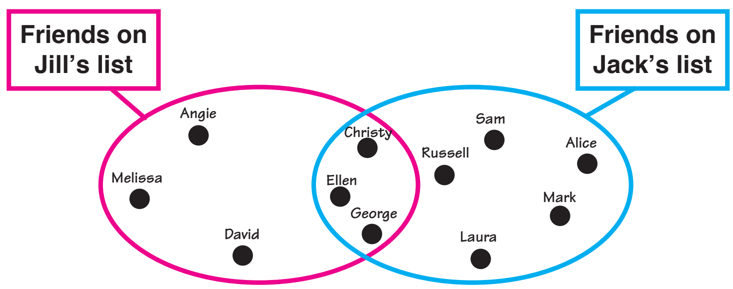
Let a student draw the dots.



T: ***How do you know they are there?***

S: ***We found all the children whose names are on both lists when we went through Jill's list.***

Read the names from Jack's list one at a time, and let students label the dots.



T: ***How many children will be invited to Jack and Jill's party? Count with me as I point to each d***

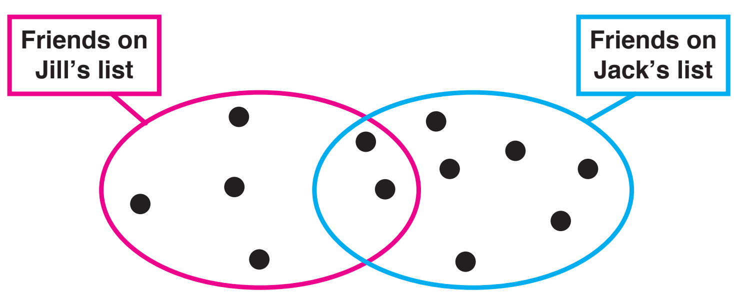
S: ***1, 2, 3, …, 11.***

T: ***Many of you predicted fourteen. Can anyone explain why the number of children is eleven instead of fourteen?***

S: ***George, Christy, and Ellen are on both lists.***

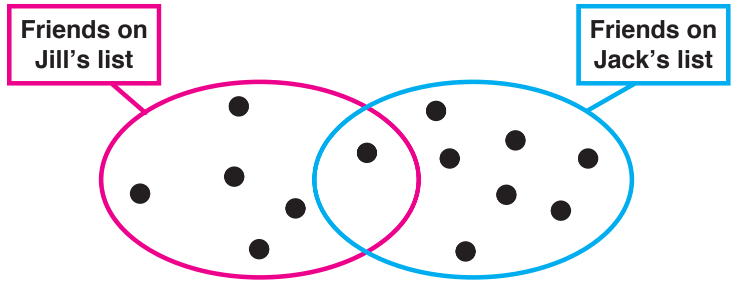
Erase the names from the picture. Erase one of the dots in the middle region, and put an additional dot in each string (as illustrated) saying,

T: ***Suppose there were two names that occurred on both lists, but there were still six names on Jill's list and eight names on Jack's list. Then, how many children would be invited to the birthday party?***



S: ***Twelve.***

Erase another dot from the middle region and replace it with an additional dot in each string (as illustrated) saying,



T: ***What if only one name occurred on both lists, but there were still six names on Jill's list and eight names on Jack's list. Then how many children would be invited?***

S: ***Thirteen.***

Erase the dots from the picture.

T: ***Think about this problem and try to draw a picture. Suppose there were seven names on Jill's list and seven names on Jack's list, and altogether there were ten names.*** (Write 7 by each string for the students' reference.) ***Can you show where the dots would be in the strings?***

Let students work independently or in pairs for a few minutes. Suggest they draw the strings on their papers and use counters for the dots. As you observe student's work, trace appropriate strings and ask,

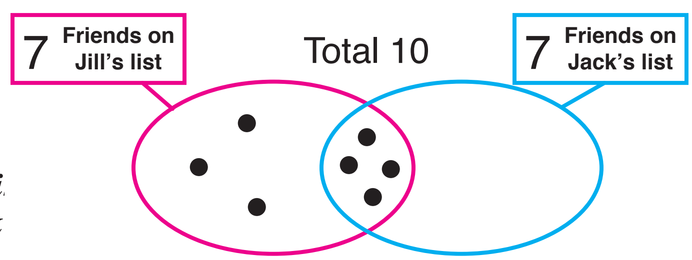
T: ***Are there seven dots in Jill's string?***
Are there seven dots in Jack's string?
Are there ten dots altogether?

If some students finish quickly and their solutions are correct, suggest another situation. Two possibilities are given below.

a) Ten names on Jill's list, eight names on Jack's list; thirteen names altogether.

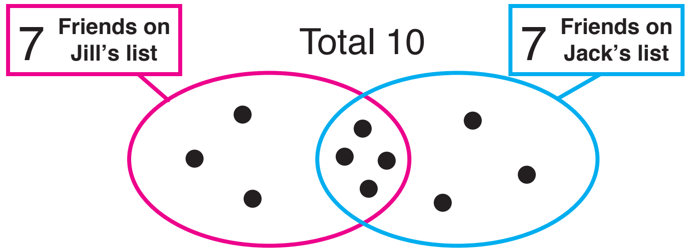
b) Twenty-five names on Jill's list, fifteen names on Jack's list; thirty names altogether.

After many students have solved the original problem, invite a volunteer to draw dots in the picture on the board. You may like to start by having the first volunteer just draw the dots in Jill's string.



T: ***Are there seven dots in Jill's string?*** (Yes)
How many names are on Jack's list? (Seven)
So we need seven dots in Jack's stri
How many dots are already in Jack string? (Four)
How many more do we need? (Three)

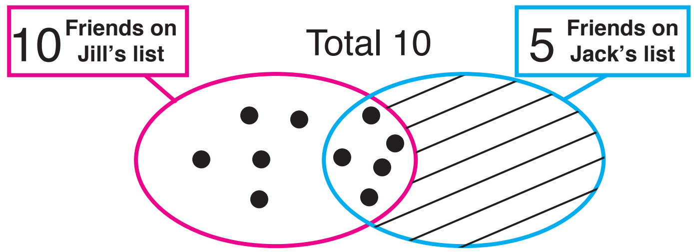
Invite a second volunteer to draw the other three dots in the picture.



T: ***We have three*** (point to the region on the left) ***and four*** (point to the middle ***dots in Jill's string—that makes seve*** ***have four*** (point to the middle region (point to the region on the right) ***dots in Jack's string—that makes seven. Are there ten dots altogether?*** (Yes)

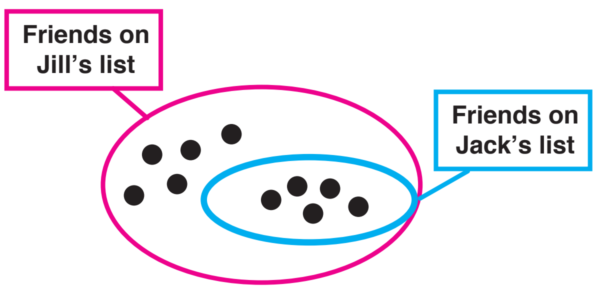
Erase the dots in the picture. Repeat this activity using the ten names on Jill's list, five on Jack's list, and ten names altogether. The string picture below corresponds to this situation.

After the dots have been drawn in the picture, consider the empty region to the right. Students may remember that we can use hatching to indicate that there is nothing in a particular part of a picture. If no one suggests the hatching, suggest it y



T: ***Do you think we could draw the strings in another way since one part of our picture is hatched?***

Someone might suggest this picture.



Home Activity

Pose this problem for the students to solve with family members:

> There are 25 students in the Chess Club and 15 students in the Drama Club. Altogether there are 30 students in the two clubs. How can this be?

This would be a good time to send home a letter to parents/guardians about the use of strings (Venn diagrams) in mathematics activities. Blackline L3 has a sample letter.

L4 MULTIPLES OF 3 AND 4

Capsule Lesson Summary

Use a calculator to generate multiples of 3 and place the multiples of 3 in a string picture. Generate multiples of 4 on the calculator and extend the string picture to include a string for multiples of 4. Discuss the various regions of the string picture using multiples of 3 and 4 for labels.

Materials

Teacher	• Calculator • Colored chalk	Student	• Calculator • Unlined paper • Colored pencils, pens, or crayons

Note: You may prefer to give students prepared string pictures to color and label for use in Exercises 3 and 4. Blackline L4(a) may be used for this purpose.

Description of Lesson

You may like to let students work in pairs during the lesson. Each pair of students should have a calculator with an automatic constant feature (see "Role and Use of Calculators" in Section One, Notes to the Teacher). Allow a few minutes for students to experiment freely with their calculators.

Exercise 1

Direct students to clear their calculators and be ready to follow your directions.

T: ***Slowly press*** $\boxed{+}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ ***and continue pressing*** $\boxed{=}$***. What numbers appear on the display?***

S: ***3, 6, 9, 12, 15, ….***

T: ***What is the calculator doing?***

S: ***It's counting by threes.***

T: ***Do you know another name for the numbers that appear?***

S: ***Multiples of 3.***

T: ***Now push the button*** $\boxed{C}$ ***to clear the display. What number is on the display now?***

S: ***0.***

Direct students to check that 0 is on the display and then to cover the display with a finger. They will need to be careful not to cover the light panel on light powered calculators.

T: ***Press*** $\boxed{+}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$***. What number do you think is on the display?***

S: ***9.***

T: ***Check the calculators.*** (Students who do not have 9 may need to start again, i.e., press $\boxed{C}$ $\boxed{+}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$)***. Hide the display again. Now press*** $\boxed{=}$ ***four more times. What number do you think is on the display?***

S: ***21.***

T: ***Check the calculators. What numbers between 9 and 21 were on the display that we did not see?***

S: ***12, 15, and 18.***

T: ***Hide the display again. Press $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$. What number do you think is on the display?***

S: ***36.***

T: ***Check the calculators.***

T: ***Hide the display on the calculators. This time press $\boxed{=}$ ten times. What number do you think is on the display?***

S: ***66.***

T: ***Why do you think it is 66?***

S: ***I counted by three's.***

S: ***I added 30 (ten 3's), 36 + 30 = 66.***

S: ***Four 3's is 12. Four more 3's is another 12; that makes 24. Two more 3's is 6—so 30 more altogether.***

T: ***Check the calculators.***

S: ***66.***

T: ***How many times would we need to press $\boxed{=}$ to go from 66 to 78?***

Suggest students whisper their answers to their partners or write answers on paper. Ask one student to respond aloud. (Four times)

T: ***Press $\boxed{=}$ four times and tell us the number that appears.***

S: ***78.***

T: ***How do you think we could go back from 78 to 0 in such a way that all the same numbers would reappear but in reverse order?***

S: ***Press $\boxed{-}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and so on.***

T: ***Hide the display. Press $\boxed{-}$ $\boxed{3}$ $\boxed{=}$. What number is on the display?***

S: ***75.***

T: ***Check your calculator. Keep pressing $\boxed{=}$ until 0 appears. Now 0 is on the display. What number would appear if we press $\boxed{=}$ once again?***

S: ***$^{-}3$.***†

T: ***…and if we press $\boxed{=}$ again?***

S: ***$^{-}6$.***

T: ***Will $^{-}13$ ever appear?***

This question may result in disagreement.

†Read $^{-}3$ just like $\widehat{3}$: negative three. This would be a good time to comment on different notations for negative numbers.

T: ***What are some numbers that appear if we kept pressing*** $\boxed{=}$***?***

S: ***$^{-}9$.***

S: ***$^{-}12$ and $^{-}15$. We missed $^{-}13$.***

Exercise 2

Instruct students to turn the calculators off and put them away for a moment.

T: ***What did the calculator do when we continued pressing*** $\boxed{+}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ ***and so on?***

S: ***Counted by threes.***

T: ***Do you know a name for the numbers that appeared on the display?***

S: ***Multiples of 3.***

If necessary, suggest this name. Then draw a string on the board and label it multiples of 3.

T: ***This string is for the multiples of 3. What are some multiples of 3?***

Let seven or eight students answer. If someone suggests a very large number, for example, 2,096, say, "I'm not sure. You'll have to convince me." (In this case 2,096 is not a multiple of 3.)

T: ***How do you think we could get the multiples of 4 to appear on the calculator?***

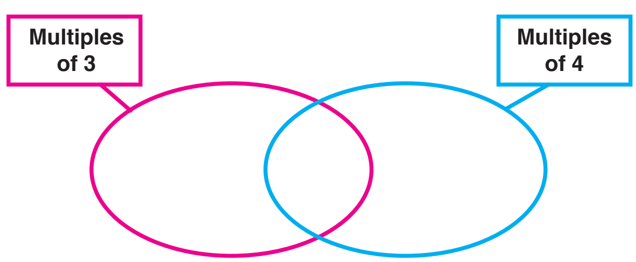
S: ***Do the same thing we did to get the multiples of 3—only press*** $\boxed{+}$ $\boxed{4}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$***....***

T: ***Let's try that. Turn the calculators on and press*** $\boxed{+}$ $\boxed{4}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ ***and so on. What numbers appear on the display?***

S: ***4, 8, 12, 16, 20, ...***

T: ***Who can draw a string on the board for the multiples of 4?***

Let a student draw the string in a different color than the first string.



If a student draws the string so that it does not overlap the multiples of 3 string, ask where 12 would be in the picture.

T: ***Give me some numbers that are multiples of 4.***

S: ***20.***

T: ***Clear your calculators. Press*** $\boxed{+}$ $\boxed{4}$***. How many times do you need to press*** $\boxed{=}$ ***to get 20?***

S: ***Five times.***

T: ***Check that on your calculator. Give me another multiple of 4.***

S: ***44.***

T: ***Does anyone know how many fours is 44?***

If this question is too difficult, approach it as follows.

T: ***How much is 10 fours?***

S: ***40.***

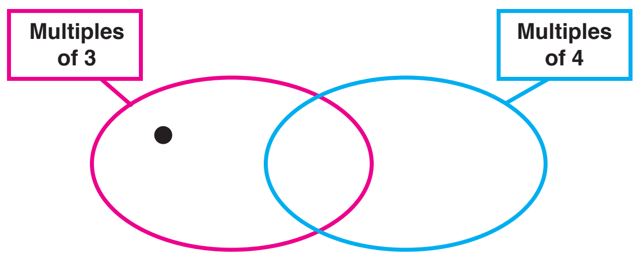
T: ***So how many fours is 44?***

S: ***11.***

Continue in this way, letting several students suggest numbers and explain why they are multiples of 4. At the end of the exercise, ask the students to turn off their calculators.

Exercise 3

Direct students to copy this picture on their papers and to follow along with the class to locate several numbers in the picture. Draw a dot in one of the regions in the string picture on the board.



T: ***What number could be here*** (point to the dot)***?***

S: ***9.***

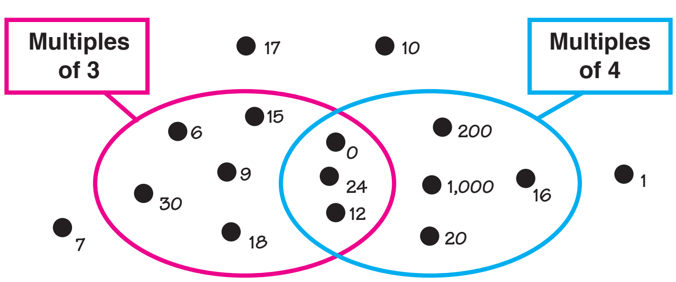
T: ***Is 9 a multiple of 3?*** (Yes)
Is 9 a multiple of 4? (No)

Label the dot in the picture. Draw a dot in another region. Continue in this manner until all regions have been considered. If someone gives a number that does not belong in a particular region, ask if the number is a multiple of 3 and then if it is a multiple of 4. Lead the student through these questions to discover where that number should be, and then let the student draw and label a dot in the correct region.

Present some numbers, one at a time, and ask students to locate them in their pictures as well as on the board. Use these or other numbers depending on your students' previous choices.

10	24	18
0	17	1
200	1,000	30

Use calculators to check the placement of a number when necessary. The next picture shows the correct location of the given numbers.

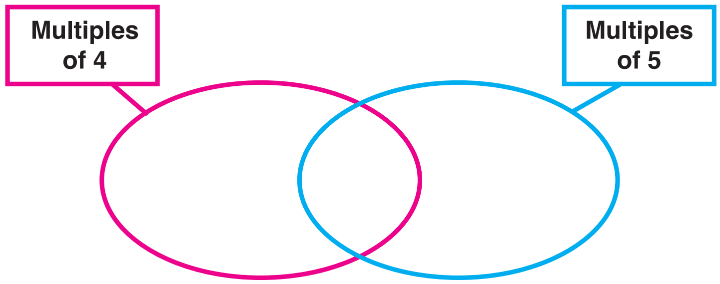


If the lesson is moving quickly, you can ask where the following numbers would be in the picture. Encourage students to locate them without first doing the calculations.

3 x 7	3 x 80
4 x 7	4 x 39

Exercise 4

Draw this picture on the board and ask students to copy it on their papers.



Instruct students to put any numbers they wish into the picture. Discuss and share ideas as time and interest permits.

Home Activity

This is a good time to send a letter to parents/guardians about the use of calculators. Blackline L4(b) has a sample letter.

L5 EXCHANGING NAMES

Capsule Lesson Summary

Play a game in which a group of students exchange names. Then, each student gives his or her right hand to the person whose name he or she receives. That person holds it with his or her left hand. Use an arrow picture to graphically record the resulting situation. Repeat the exchange of names with another group of students fewer in number. Draw pictures of what could happen if thirteen people played the game. This game involves the mathematical concept of a permutation.

Materials

Teacher		Student	
	• Colored chalk		• Unlined paper • Colored pencils, pens, or crayons

Note: Before the lesson begins, write the names of seven to ten of your students on separate slips of paper for Exercise 1, and write the names of five or six other students on separate slips of paper for Exercise 2. For each exercise, choose students whose first names begin with different letters.

Description of Lesson

You might like to precede this lesson with a literature selection involving names, such as *My Name is Andy* by Tomie DePaola.

Exercise 1

In the description that follows, we will assume that nine names are written on slips of paper: Manuel, Jenny, Barry, Lucy, Derrick, Pat, Kim, Anthony, and Scott.

T: ***Today we are going to play a game. I want these people to come to the front of the room.***

Read the names slowly. After reading a name, fold that slip of paper in half so that the name is hidden. Look at the students at the front of the room and say to them,

T: ***Hold up your right hand. I'm going to give each of you a slip of paper with one of your names on it. Don't look at the name until I tell you.***

Mix up the slips of paper and then distribute them to the students. Then call on one student (for example, Barry) to look at the name on his slip of paper.

T: ***Barry, look at the slip of paper I just gave you and see whose name is on it.*** (Suppose it is Kim). ***Give your right hand to Kim. Kim, take Barry's right hand with your left hand.***

Next, direct Kim to look at the name on her paper; then instruct Kim to give her right hand to the left hand of that person. One by one, direct the students to give their right hands to the person whose name is on their slip of paper. Several things might happen:

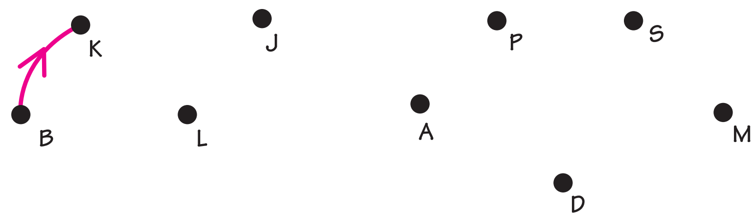
- Some students might be holding their own hands.
- Some students might be in pairs holding each other's hands.
- Some students might be in groups of three or more and have formed a circle holding hands. In this case, the students may get tangled up and need your help to form circles.

After the students are holding hands correctly, ask them to stay where they are while you and the rest of the class draw a "picture" of them on the board.

T: ***The rest of you*** (look at the students still seated) ***can help me describe what happened.***

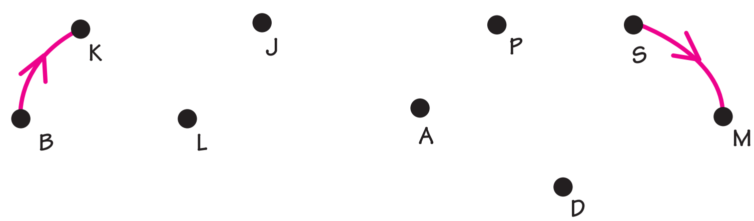
On the board, draw dots for the students in the game, labeling the dots with the initial of their first names. Draw the dots in a manner suggestive of the students' arrangement. As you label a dot, tell the class who that dot is for.

T: ***Barry has Kim's name, so Barry gives his right hand to Kim. I'm going to draw a red arrow to show this in our picture.***



T: ***Who can help me draw some more arrows?***

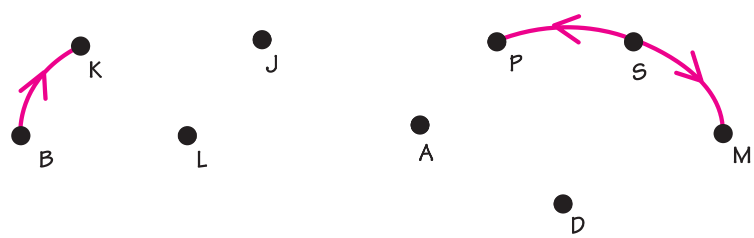
A student might draw an arrow from **S** (for Scott) to **M** (for Manuel).



T: ***Scott, did you get Manuel's name? Did you give your right hand to Manuel?***

S: ***Yes.***

Continue inviting students to draw arrows on the board. A student may draw an arrow in the wrong direction and need help. For example, consider the situation in which a student draws an arrow from **S** (for Scott) to **P** (for Pat).



T (tracing the arrow from **S** to **M**)**:** ***Scott, do you have Manuel's name?***

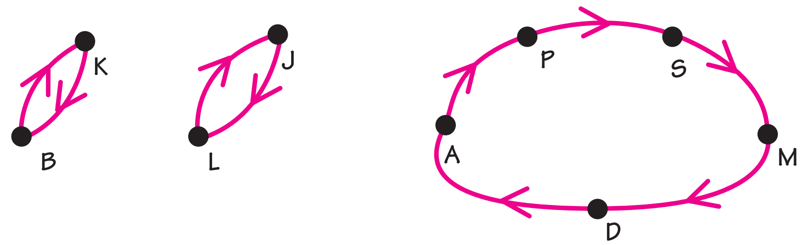
S: ***Yes.***

T (tracing the arrow from **S** to **M**): ***Scott, do you have Pat's name?***

S: ***No.***

T: ***We know that this*** (put one finger on **S** and trace each arrow from **S**) ***cannot happen because no person in the game was given two names.***

Encourage the student who drew the arrow to correct its direction.



A completed picture could look like this one.

Exercise 2

Students will need unlined paper and red pencils for this exercise. You may like to let students work in pairs on the drawing problem.

Play the game again as in Exercise 1 with a new group of five or six students. This time, ask each student still seated to draw an arrow picture describing what happens. Ask the students in the game to stay in the front of the room so the other students in the class can draw a "picture" of them.

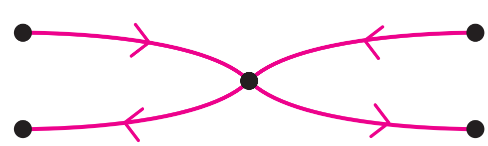
As a class, compare a few pictures and check the logic of the pictures with the students who exchanged names.

Exercise 3

Students will again need unlined paper and red pencils for this exercise.

T: ***Imagine that 13 people are playing the game. On your paper, draw an arrow picture of what might happen.***

If while observing students' work, you notice a picture with more than one arrow starting or arriving at a dot, reproduce that part of the student's picture on the board and call the class's attention to it. A possible picture and dialogue follows.



T: ***This is part of someone's picture. What do you think of it?***

S: ***It can't be.***

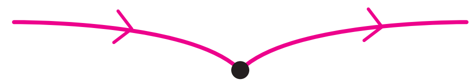
T: ***Why?***

S: ***Two people are giving their right hands to the person in the middle, but that person only has one left hand to receive them. Only one person could have gotten the name of the person in the middle.***

S: ***The person in the middle is giving his or her right hand to two people, but that person could have gotten only one name.***

T: ***In this game, exactly one arrow starts at each dot and exactly one arrow ends at each dot.***

Replace the arrow picture with this one.



On the board, draw a dot with a loop at it.
Then, point to the loop as you ask,

T: ***Does exactly one arrow start at this dot?***

S: ***Yes.***

T: ***Does exactly one arrow start at this dot?***

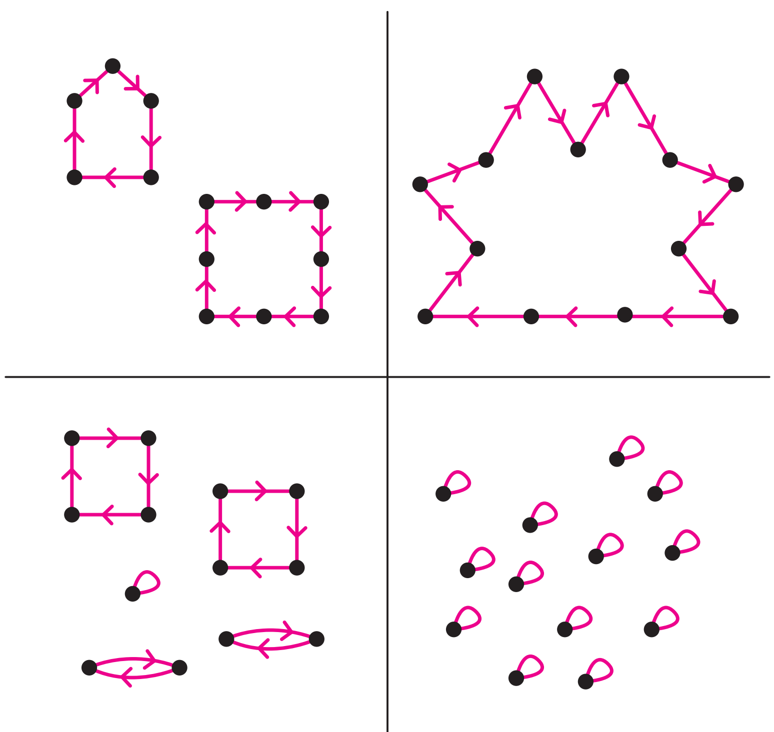
S: ***Yes.***

T: ***So a loop can be part of your picture. When would there be loops in a picture?***

S: ***When people get their own names.***

As students complete their pictures, ask several of them with significantly different pictures to put them on the board. You may like to organize the class in groups of 13 students each and to ask each group to choose a picture to act out.

These are some possibilities.



Writing Activity

Ask students to write about what happens in the exchanging names game. Let students choose a picture to help describe how the game works.

L6 STRING GAMES #2

Capsule Lesson Summary

Play The String Game with two strings and sixteen possible string labels. Discuss a two-string picture where one string is for multiples of 10 and the other is for numbers greater than 50. Introduce and play a game placing numbers in this string picture.

Materials

Teacher • A-Block String Game kit
• Colored chalk

Student • None

Advance Preparation: Before the lesson begins, put the A-blocks in a box and prepare the board for The String Game. See Appendix D on The String Game for a description of equipment and preparation for the game.

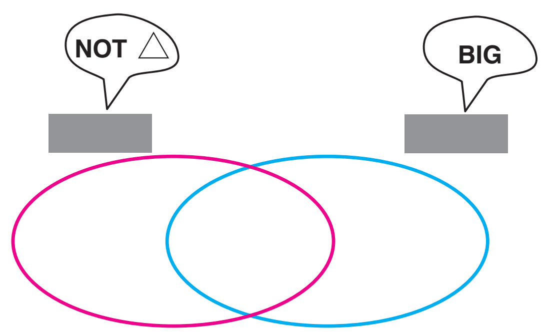
Description of Lesson

Exercise 1

Prepare your board for The String Game as shown below.

RED	YELLOW	GREEN	BLUE
NOT RED	NOT YELLOW	NOT GREEN	NOT BLUE
○	△	□	BIG
NOT ○	NOT △	NOT □	LITTLE

Team A	Team B

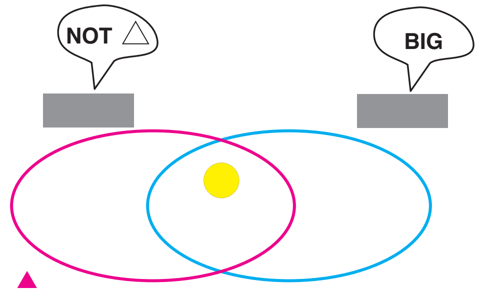


Divide the class into two or more teams (possibly cooperative groups), and allow the teams to choose names if you wish. Alternating teams, ask students to describe an A-block piece to put in their team's collection. Students should give complete descriptions of pieces—color, shape, and size. Continue in this manner until all the A-block pieces have been placed on the team board.

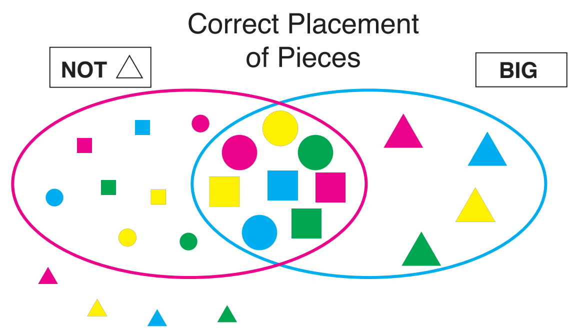
T: ***Now we are going to play The String Game. Each string is for one of these labels*** (point to the list on the board)***. I'll give you some starting clues.***

L6

Place two A-blocks (for example, the large yellow circle and the small red triangle) correctly in the picture. You may prefer to let teams each select one piece from their collections for you to place correctly in the picture.

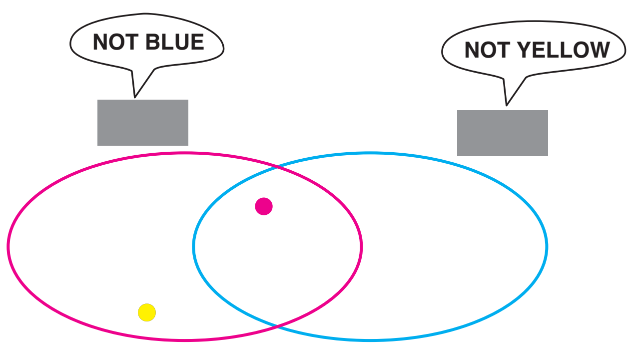


Play the game in the usual way. The next illustration shows the correct placement of all the A-block pieces. Use this picture as a crib sheet while the game is being played.

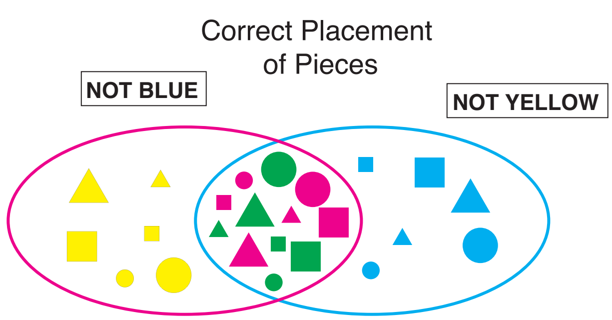


Exercise 2 (optional)

Use this exercise if time permits. Save approximately ten minutes for Exercise 3. Play another round of The String Game using not-cards for both strings. A game with starting clues is suggested below.



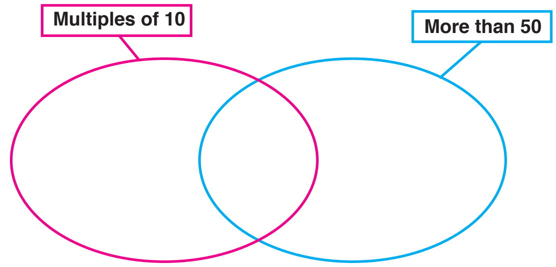
This picture shows the correct placement of all the A-block pieces.



Exercise 3

Take the list of string labels off the board and put the A-blocks away. Label the strings as shown below. Trace the appropriate string as you ask,

T: ***The red string is for multiples of 10. What are some numbers that belong in the red string?***



S: ***10 and 20.***

S: ***70.***

S: ***120 and 200.***

T: ***The blue string is for numbers more than 50. What are some numbers that belong in the blue string?***

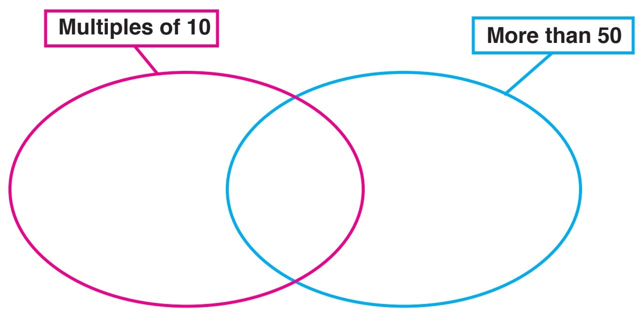
S: ***51.***

S: ***100.***

S: ***67.***

List these numbers on the team board near the strings.

TEAM A		TEAM B	
50	$\widehat{13}$	40 + 30	9 x 11
48 + 8	36 + $\widehat{6}$	83	0
5	128	60 – 15	$\widehat{50}$
6 x 10	10	20	24
72 – 24	$\frac{1}{2}$ x 110	$\widehat{14}$ + 4	$\frac{1}{2}$ x 96



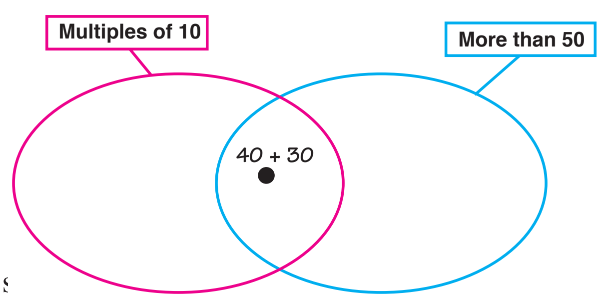
T: ***We're going to play a game with numbers. During a turn, you will have only one chance to put a number in the string picture. The first team to place all its numbers where they belong wins.***

Alternating teams, let students locate numbers in the picture. Discuss correct responses; for example, suppose a student puts 40 + 30† in the middle region.

T: ***Why does 40 + 30 belong there?***

S: ***40 + 30 = 70; 70 is a multiple of 10 and is more than 50.***

S: ***40 + 30 is more than 50, and both 40 and 30 are multiples of 10, so 40 + 30 is a multiple of 10.***

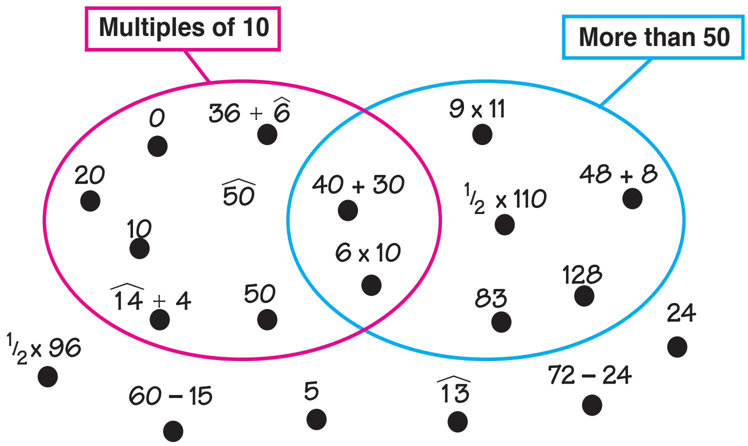


Erase 40 + 30 from the team board and leave it in the s

†Allow students to write either the given name or the standard name, whichever they prefer.

If a student locates a number incorrectly, simply erase it from the string picture and say, "No, that number does not belong there." If you were to discuss why a number does not belong in a certain region, some students might object because you would give information to the team with that number.

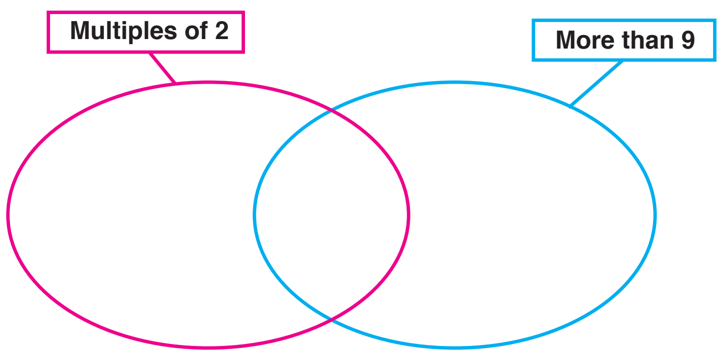
This picture shows the correct location of all the numbers in the game.



Home Activity

Suggest parents/guardians work with their child to place numbers in a string picture such as the one below.

4	2 × 10
2 × 4	15
2 × 3	1
5	3 × 7



L7 COMPOSITION GAMES #2

Capsule Lesson Summary

Play a composition game with the rule "yellow followed by blue is red." Reverse the order in which the colors of the arrows are considered to introduce a new rule: blue followed by yellow is green. Play a game using both rules.

Materials

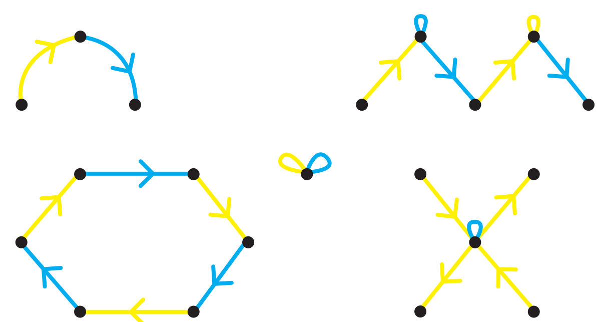
Teacher	• Colored chalk	**Student**	• Unlined paper • Worksheets L7* and ** • Colored pencils, pens, or crayons

Advance Preparation: Prepare the arrow picture for Exercise 1 before the lesson begins.

Description of Lesson

Exercise 1

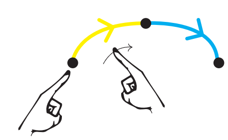
Invite students to comment on this arrow picture.



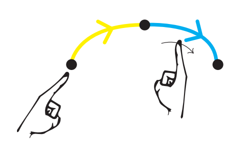
T: ***Do you remember the game with the rule "yellow followed by blue is red"?***

Use the upper left corner of the arrow picture to review the rule of the game.

T: ***Yellow*** (trace the arrow)…

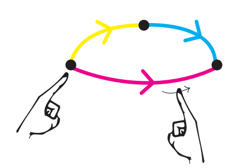


…followed by blue (trace the arrow)***…***



…is red.

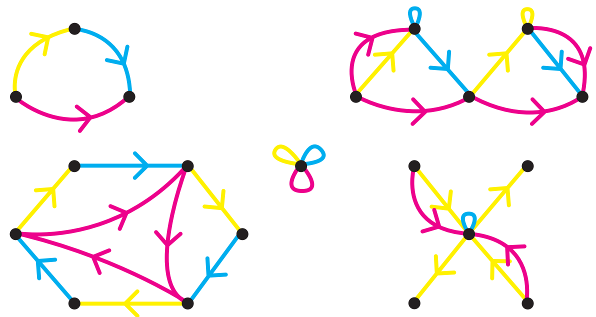
Trace and then draw the red arrow.



T: ***Where can we draw more red arrows?***

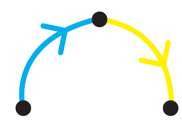
Let students, one at a time, trace red arrows in the picture. You may want to draw the red arrows yourself after they have been correctly traced. Help a student who has difficulty with this by holding the student's left forefinger at a dot where a yellow arrow starts. Then guide the student's right forefinger to trace the yellow arrow followed by a blue arrow. The student should then trace a red arrow from the start of yellow arrow (left forefinger) to the end of the blue arrow (right forefinger).

The completed picture should look like this.



T: ***Since there are no more red arrows, let's introduce a new rule.***

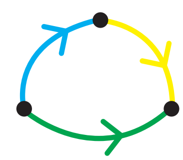
In the upper left part of your picture draw this configuration of arrows.



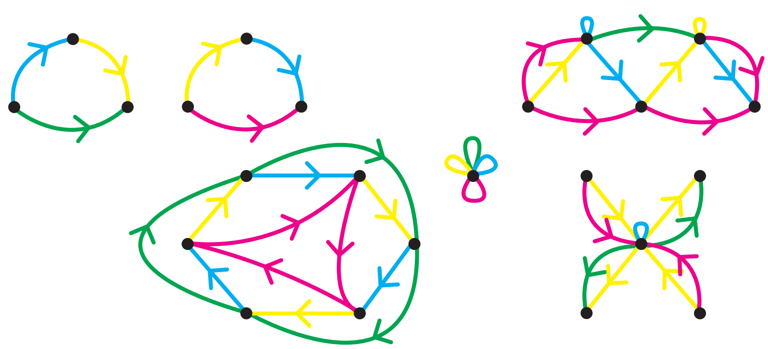
Refer to the arrows you just drew.

T: ***Blue*** (trace the blue arrow) ***followed by yellow*** (trace the yellow arrow) ***is green.***

Trace and then draw a green arrow.



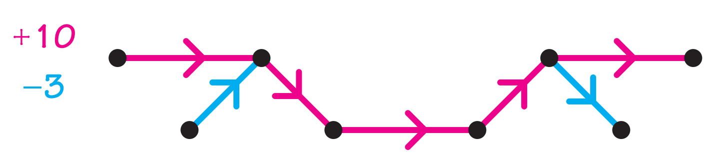
Let students, one at a time, trace where green arrows may be drawn in the picture. You may want to draw the green arrows yourself after they have been correctly traced. A completed picture should look like this.



Worksheet L7* and ** are available for students to work on individually using both composition rules to draw red and green arrows. You may like to allow five to ten minutes now for this individual work. Then continue with Exercise 2.

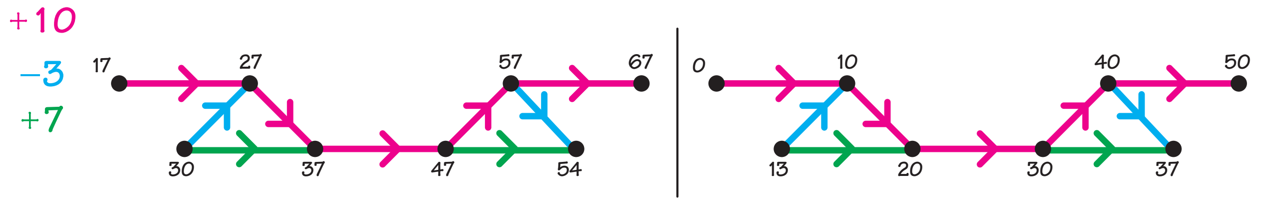
Exercise 2

Erase the board and draw this arrow picture. Ask students to copy it on their papers.



T: ***Right now there are no dots labeled in this picture. I would like to find places for the numbers 30 and 37. Try to put 30 and 37 in the picture, but remember, red arrows are for +10 and blue arrows are for –3. After you find a place for 30 and 37, label all the dots and check the arrows.***

Let students experiment. You may want to remind them to consider the composition of +10 followed by –3 or the composition of –3 followed by +10. There are two possible solutions to this problem.

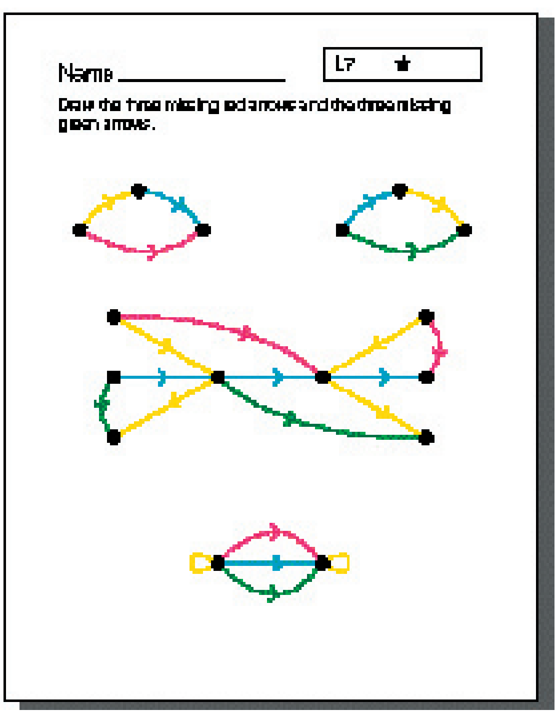


Look for both solutions and ask students to put them on the board. Encourage students to explain how they found a solution. Discuss the compositions +10 followed by –3 is +7, and –3 followed by +10 is +7. Draw green arrows for +7, and use the green arrows to help explain how to find solutions.

30 and 37 can be located at the beginning and end, respectively, of either green arrow.

Center Activity

Make task cards similar to Exercise 2 to place in a center for independent problem-solving practice.



L8 MARBLE GAME

Capsule Lesson Summary

Do some mental arithmetic involving ½x and ⅓x. Conduct an experiment by choosing one marble from a collection of marbles of two colors, and discussing the chances of getting one color marble versus that of getting the other color. Introduce and play a marble game involving prediction.

Materials

Teacher	• Marbles • Colored chalk	**Advance Preparation:** You may want to prepare the game charts for Exercise 3 before the lesson begins.
Student	• None	

Description of Lesson

Exercise 1

Begin this lesson with about five minutes of mental arithmetic involving ½x and ⅓x. For example, ask students to calculate the following.

½ x 12 (6)	½ x 6 (3)	½ x 18 (9)	½ x 16 (8)	⅓ x 9 (3)
⅓ x 12 (4)	⅓ x 6 (2)	⅓ x 18 (6)	½ x 20 (10)	⅓ x 15 (3)

Regularly ask students to explain their answers. Relate ÷2 to ½x, and ÷3 to ⅓x.

Exercise 2

For this exercise you need four red and four blue marbles. Try to keep a brisk pace.

T: ***I have two marbles in my hand, one red and one blue.*** (Show them to the students.)

Shake the marbles behind your back while you explain what you are doing to the class.

T: ***I hold the marbles with both hands behind my back and shake them; then I put one of the marbles in my right hand.***

Hold out your closed right hand.

T: ***What color marble could I have in my right hand?***

S: ***Red.***

S: ***Blue.***

Open your hand and show the marble to the class.

T: ***I have two marbles*** (hold them in an outstretched open hand) ***and one of them is red*** (hold up the red marble)***. We say the chance of getting red is 1 out of 2.***

Record the chance of getting red on the board.

●	Red	1 out of 2
●	Blue	1 out of 2

T: ***What is the chance of getting blue?***

S: ***The same—1 out of 2.***

Record this information on the board.

T: ***With one red and one blue marble, the chance of getting red is the same as the chance of getting blue. If I put another red marble and another blue marble in my hand, will I have a better chance of getting red?***

S: ***I don't think so.***

S: ***The chances for red and blue will be the same.***

Draw the marbles on the board below the information from the first situation.

T: ***I have four marbles*** (hold them in an outstretched open hand) ***and two of them are red*** (hold up the two red marbles)***. What are the chances of getting red?***

S: ***2 out of 4.***

T: ***…of getting blue?***

S: ***The same—2 out of 4.***

●	Red	1 out of 2
●	Blue	1 out of 2
● ●	Red	2 out of 4
● ●	Blue	2 out of 4

Record this information in the chart forming on the board.

T: ***Here*** (point to the record of the first situation) ***the chance of getting red and the chance of getting blue are both 1 out of 2. And here*** (point to the record of the second situation) ***the chances of getting red and the chances of getting blue are both 2 out of 4.***

Who can suggest another collection of marbles where the chances of getting red would be the same as the chances of getting blue?

Suppose a student suggests five red and five blue marbles. Draw another section to the chart on the board and draw the marbles.

T: ***How many marbles are red*** (point to the red marbles)***?***

S: ***Five.***

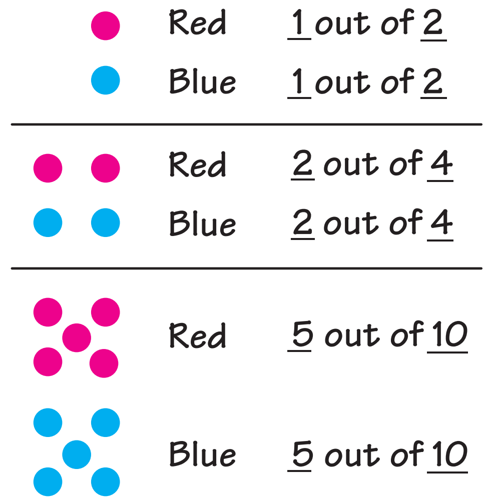
T: ***And how many marbles are there altogether?***

S: ***Ten.***

T: ***What are the chances of getting red?***

S: ***5 out of 10.***

Repeat these questions for blue, recording the chances in your chart.



T: ***What are some other collections of marbles for which the chances of getting red are the same as the chances of getting blue?***

S: ***Three red and three blue marbles.***

S: ***Four red and four blue marbles.***

S: ***Twenty red and twenty blue marbles.***

T: ***For all these collections the chances for red and for blue are the same. 1 chance out of 2 is the same as 2 chances out of 4, or 5 chances out of 10, or 3 chances out of 6, and so on.***

This time hold out two red marbles and one blue marble.

T: ***What if I put two red marbles and only one blue marble behind my back—are the chances of getting red the same as the chances of getting blue?***

S: ***No, there is a greater chance of getting red.***

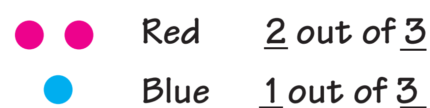
T: ***What are the chances of getting a red marble*** (show the students the three marbles)***?***

S: ***2 out of 3.***

T: ***What are the chances of getting a blue marble*** (show the students the three marbles)***?***

S: ***1 out of 3.***

Write this information on the board to the side of the first chart—not in it.



T: ***If I add two more red marbles to those in my hand, how many blue marbles should I add to keep a similar situation with the chance of getting red twice as great as the chance of getting blue?***

S: ***One red marble.***

T: ***That's right and I will have four red and two blue marbles.***

Draw the new collection on the board.

T: ***With these six marbles*** (hold out the marbles and point to the picture)***, what are the chances of getting a red marble?***

S: ***4 out of 6.***

T: ***…of getting a blue marble?***

S: ***2 out of 6.***

Write this information next to the picture of the six marbles. Your chart will look like this.

Exercise 3

Draw these charts on the board.

Red Team

Prediction	Tally count # of Reds	Points

Blue Team

Prediction	Tally count # of Blues	Points

Announce to the class that these charts will be used in a marble game that they are going to play. Divide the class into two teams and choose a scorekeeper for each team. The scorekeepers should come to the front of the room and stand near the charts.

T: ***The game we are going to play involves one red and one blue marble. For each round of the game I'll shake the marbles and choose a marble without looking, six times. We will note the color each time. Let's try a round to see how the game is played.***

The following is a sample dialogue.

T: ***Before the round starts, each team will predict how many times out of six they think I will get their color marble.***

You may find a way for teams to agree on a prediction, or select one student on each team to make a prediction.

Ask the Red scorekeeper to record the Red Team's prediction, and the Blue scorekeeper to record the Blue Team's prediction in their chart. Then perform six trials. Shake the marbles, choose one without looking, and show the class one marble each time. As you perform the trials, direct the scorekeepers to keep a tally count of the number of times you get their color marble. See the charts that follow.

T: ***This is how we keep score. Each team starts with 10 points. (Write 10 above the last column in each chart.) But a team loses points when their prediction does not match the actual count.***

In this round, the Red Team predicted four red; we had only two, so the Red Team loses two points since 4 – 2 = 2. What is their score now?

S: ***8.***

Record 8 in the Red Team's chart.

T: ***In this round, the Blue Team predicted three blue; we had four, so the Blue Team loses one point since 4 – 3 = 1. What is their score now?***

S: ***9.***

Record 9 in the Blue Team's chart.

Red Team

Prediction	Tally count # of Reds	Points
4	II	8

Blue Team

Prediction	Tally count # of Blues	Points
3	IIII	9

T: ***Now that you see how the game is played, let's begin again. We'll play several rounds each time making predictions and picking out marbles. When one team loses all of its points, the other team will be the winner.***

Charts for a possible game are shown below. In this example, the Red Team wins the game.

Red Team

Prediction	Tally count # of Reds	Points
3	III	10
4	II	8
3	~~IIII~~	6
3	III	6
3	I	4
3	III	4
3	~~IIII~~	2

Blue Team

Prediction	Tally count # of Blues	Points
4	III	9
2	IIII	7
3	I	5
2	III	4
3	~~IIII~~	2
3	III	2
4	I	1

Center Activity

Put marbles and game charts in a center for groups of two to six students to play the game.

Writing Activity

Ask students to explain how they decide on a prediction in this game.

Capsule Lesson Summary

Explore putting A-blocks in three-string pictures. Observe situations where there are empty regions and hatch such regions. Locate numbers in a three-string picture where the strings are for **Even numbers**, **Multiples of 3**, and **Less than 100**.

Materials

Teacher		Student	
	• Colored chalk • A-blocks		• Worksheets L9*, **, ***, and ****

Description of Lesson

Exercise 1

This exercise can be done with cooperative groups. If there are six groups, for example, give each group four A-block pieces.

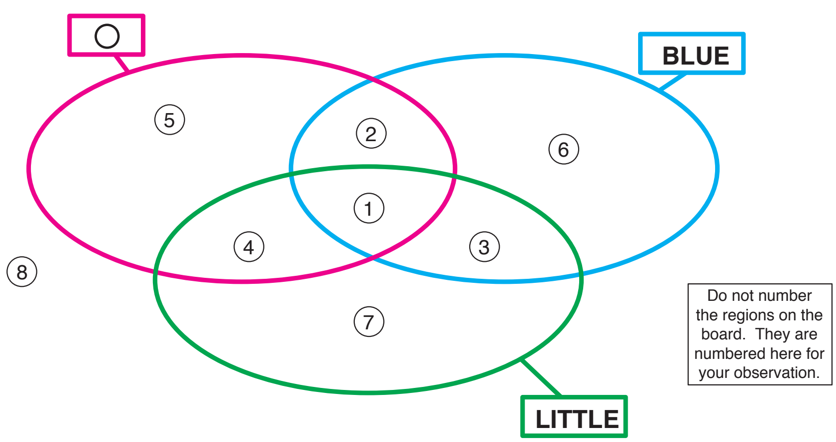
Draw a large red string on the board and label it ○.

T: ***The red string is for circles. Let's draw another string for blue pieces. Who can trace where we should draw it?***

When a student traces a string overlapping the red string, draw and label it.

T: ***Let's draw still another string for little pieces . Who can trace where we should draw it?***

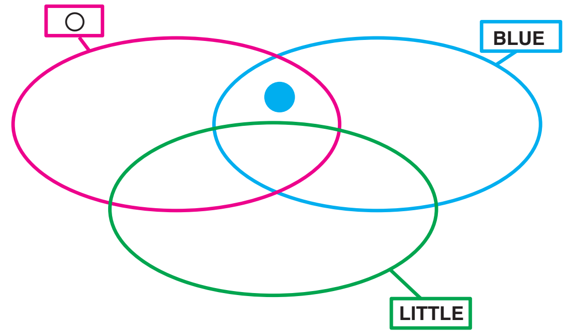
After a student traces a third string overlapping the other two strings, draw and label it. Your three-string picture should have eight regions as indicated below.



Point to one of the eight regions in the picture as you ask,

T: ***Who has an A-block that belongs here?***

Check the placement of a piece by considering each label. For example, suppose a student places the big blue circle here.

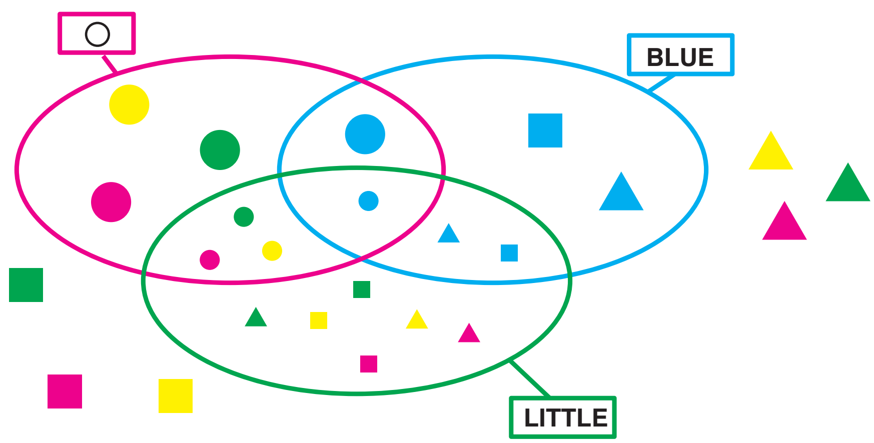


Point to the respective string labels as you ask, "Is it a circle?" (Yes) "Is it blue?" (Yes) "Is it little?" (No) "So it is in the red and the blue strings, but not in the green string."

Repeat this activity, pointing to three or four different regions. If someone places an A-block that does not belong in the region being considered at the time, the class should notice the error. Remove the piece from the picture and ask the student (or group) where the piece actually belongs in the picture.

You can change the type of question by asking one group to hold up an A-block not yet placed and directing another group to show where the piece belongs in the picture.

Although your picture might have about eight to ten pieces in it, this picture shows the correct placement of all the A-blocks.



Next, remove the A-blocks from the string picture, and relabel the strings **RED**, □ and △. Redistribute A-block pieces to the groups.

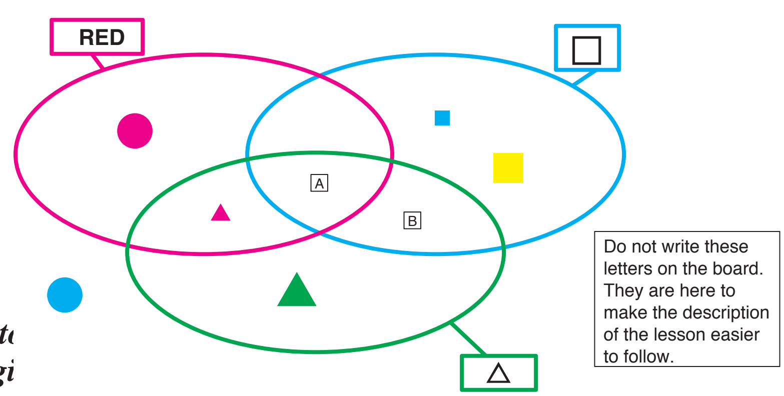
Again, point to three or four regions one at a time and ask students for pieces that belong in those regions. After several A-blocks have been placed correctly, ask groups to hold up A-blocks not already in the picture and let other groups place them in the picture.

Suppose your picture looks like this. Point to region **B**.

T: ***Does anyone have a piece that belongs here?***

S: ***No, because no piece is both a square and a triangle.***

T: ***What can we do to the picture t show that nothing is in this regi***



S: ***Hatch the region.***

Hatch region **B** yourself.

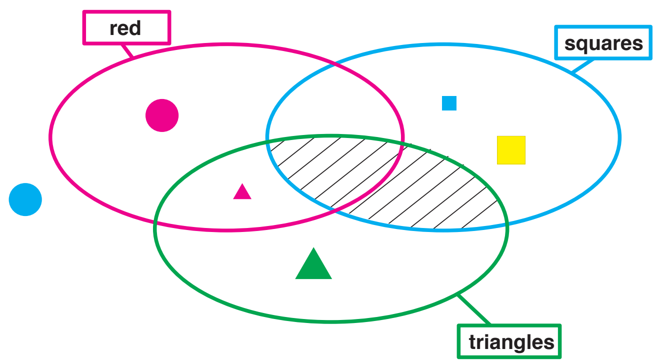
T: ***Are there other regions we could hatch?***

S: ***We can hatch the middle region (A) because no piece is red and a square and a triangle.***

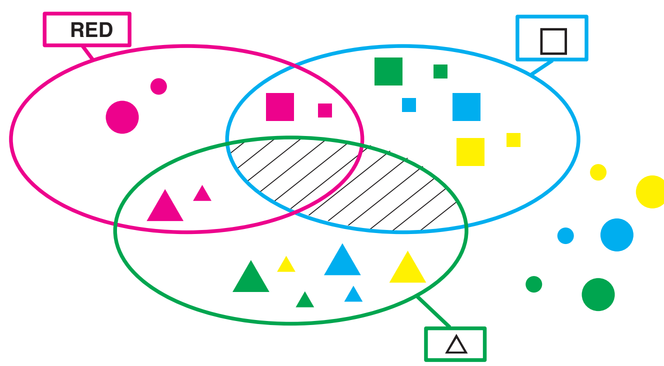
Hatch region **A**.

Consider all the regions that are not hatched and do not have pieces placed in them. Ask the class if these regions are empty, and if not, ask students to place some pieces that belong in them.

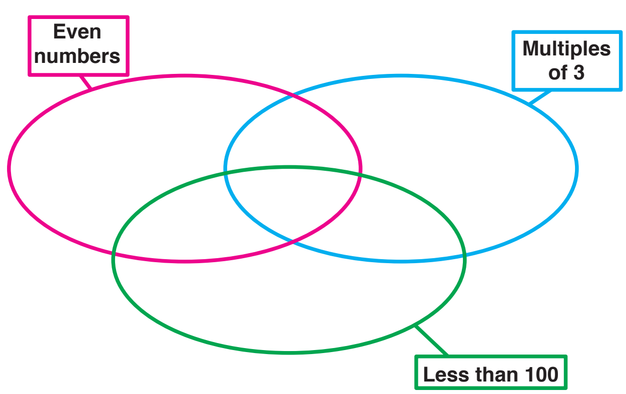
Although your picture will have about six to nine pieces in it, the next illustration shows the correct placement of all the pieces.



Optional: Repeat the exercise with different string labels.

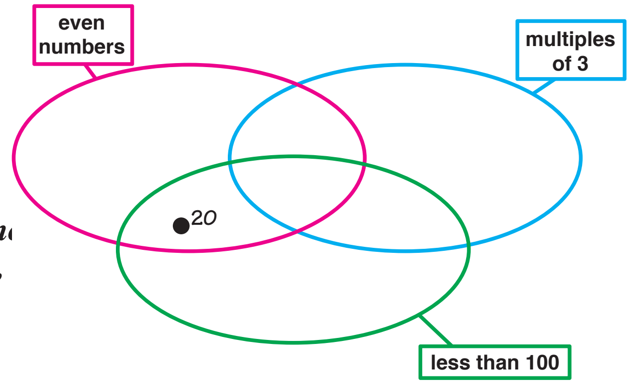


In this case, regions **C** and **D** are empty. The class should discover this fact. For your information, correct placement of all pieces is shown below.



Exercise 2

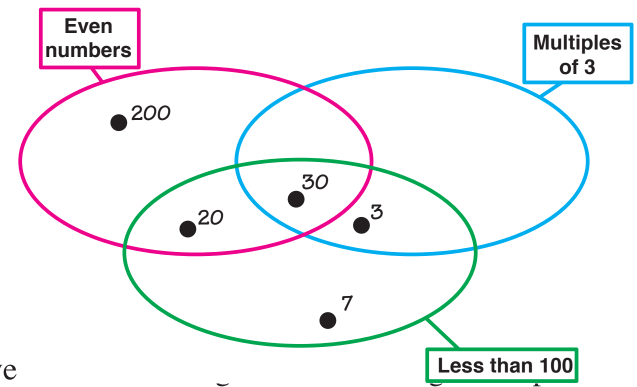
Remove the A-blocks from the string picture and relabel the strings as shown.



Conduct a brief discussion to remind students about even numbers, multiples of 3, and numbers less than 100.

T: ***This time we are going to put numbers in the picture. The red string is for even numbers, the blue string is for multiples of 3, and the green string is for numbers less than 100. Who can show us where 20 belongs?***

After a student points to the correct region, draw a dot and put 20 in that region. Similarly, ask where these numbers belong in the picture: 200, 3, 30, 7.

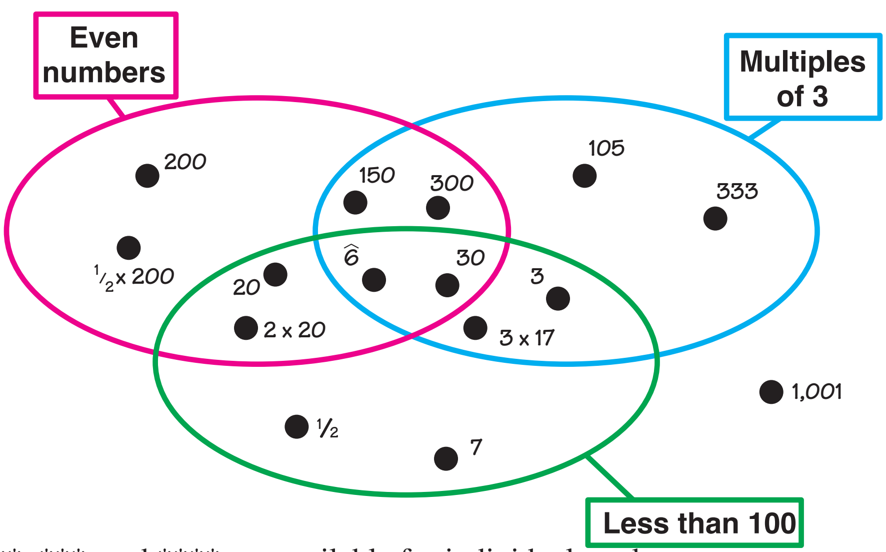


Point to each region in which a number has not yet been located. For each one ask, "Can you think of a number that belongs here?"

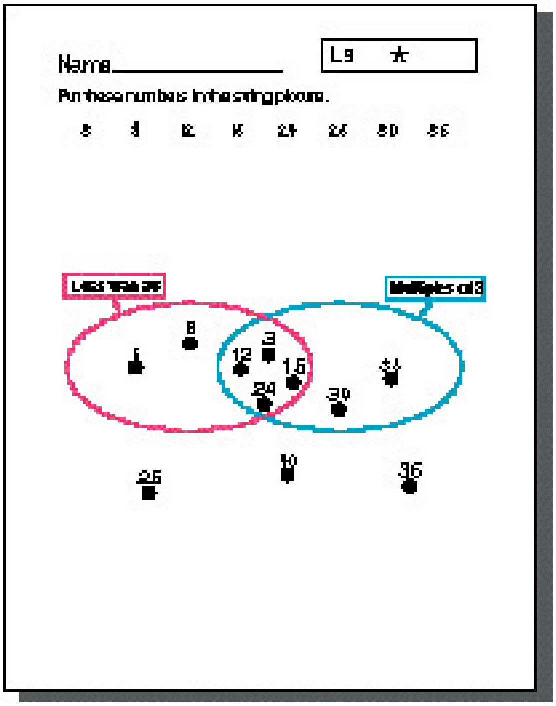
Afterward, consider with the class where four or five

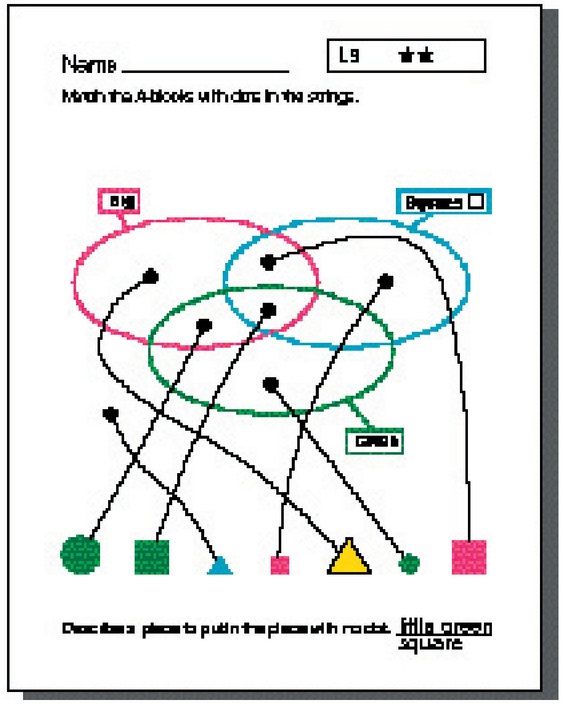
½ 333 $\widehat{6}$ 2 x 20 3 x 17 ½ x 200 300 1001

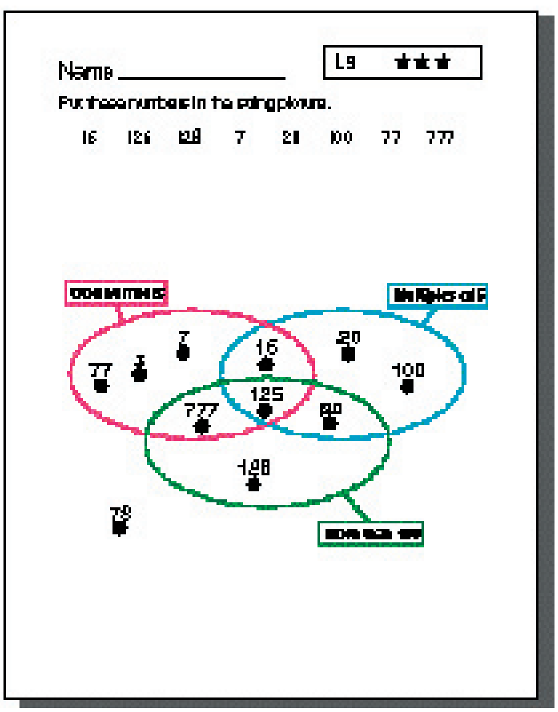
The following picture shows placement of all of the numbers in the above list as well as a few others your students might suggest.

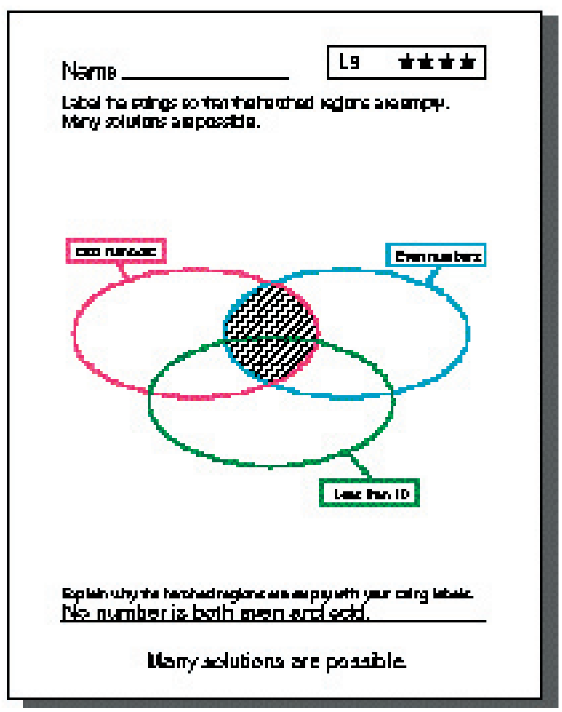


Worksheets L9*, **, ***, and **** are available for individual work.


Name
L9
8
12
3
15
24
30
25
36


Name
L9
little green square


Name
L9
7
77
16
20
100
125
777
80
128


Name
L9
Even numbers
Less than 10
No number is both even and odd.
Many solutions are possible.

Capsule Lesson Summary

Discover a secret number from three clues. The first involves putting numbers on the Minicomputer using one positive checker and one negative checker; the second involves building roads with exactly two 2x arrows and two +3 arrows; and the third gives the secret number's place in a three-string picture.

Materials

Teacher
- Minicomputer kit
- Colored chalk

Student
- Paper
- Colored pencils, pens, or crayons
- Minicomputer set

Description of Lesson

Before the lesson begins, write a large 36 on a slip of paper and fold the paper over so that 36 is hidden.

Begin the lesson with a brief discussion of detectives. Then refer to the paper.

T: ***I wrote a secret number on this paper. You are going to be detectives. I will give you some clues to help you find out the secret number. If you are good detectives, you can discover the secret number without just guessing.***

Clue 1

Display two Minicomputer boards. Provide students with one Minicomputer sheet and two checkers, one positive and one negative. You may like to let students work in pairs or cooperative groups.

T: ***The secret number can be put on these Minicomputer boards using exactly two checkers. It can be put on using one regular checker on the tens board*** (display a regular checker by the tens board) ***and one negative checker on the ones board*** (display a negative checker by the ones board)**.**

Instruct students to find all possibilities for the secret number. Encourage them to make a list on their papers. When you observe that many students (groups) have completed their work, direct the class's attention to the demonstration Minicomputer.

T: ***What are some numbers that could be my secret number?***

For each suggestion, ask the student to show the number on the Minicomputer using one regular checker on the tens board and one negative checker on the ones board. If correct, record it in a list of possibilities on the chalkboard.

Try to organize the list so that when all the possibilities are found your list delineates some patterns. In fact, the patterns here may help students find some of the solutions.

2	6	8	9
12	16	18	19
32	36	38	39
72	76	78	79

Take away the Minicomputer boards, but leave the list of numbers on the chalkboard.

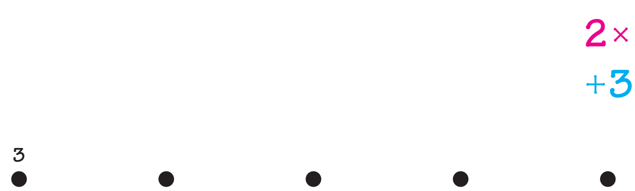
T: ***My secret number is one of these*** (point to the list)***. Do you know my secret number? What do you need?***

S: ***Another clue.***

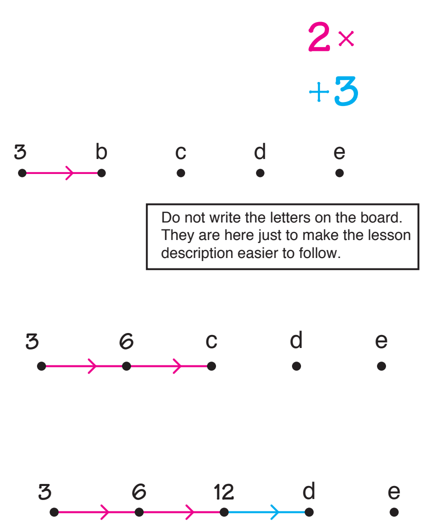
Clue 2

Draw the picture below on the board.

T: ***Here is a second clue. My secret number is the ending number of a red-blue road that starts at 3*** (point to the dot for 3), ***and has exactly two red arrows*** (point to 2x) ***and two blue arrows*** (point to +3)***.***



Invite students to suggest red-blue roads. There are six possibilities; one is described in the following sample dialogue.



T: ***What color is the first arrow?***

S: ***Red.***

T: ***What number is here (at* b*)?***

S: ***6.***

T: ***What color is the second arrow?***

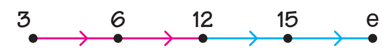
S: ***Red.***

T: ***What number is here (at* c*)?*** (12)
What color is the next arrow?

S: ***Blue.***

T: ***What number is here*** (at **d**)***?*** (15)
What color is the last arrow?

S: ***Blue.***



T: ***What number is here*** (at **e**)***?***

S: ***18.***

T: ***Does this road have two red and two blue arrows?***

S: ***Yes.***

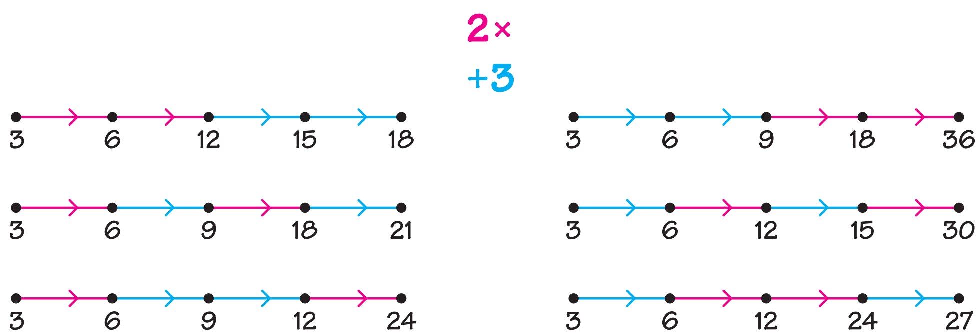
T: ***This road ends at 18. Is 18 on our list?***

S: ***Yes.***

T: ***So my secret number could be 18. I will circle it in the list.***

Instruct students to work with a partner or in groups to find all the different roads with exactly two red arrows and two blue arrows. Ask the students to draw the roads on their papers as you did on the board, and to remember all the roads start at 3.

As students are working, encourage them to be thinking about how many different roads there are. After a while, invite students to put the six different roads on the board.



With all six roads drawn on the board, draw the class's attention to the list of numbers from the first clue on the board, and consider whether the ending number of each road could be the secret number. Circle any entry in the list that is also the ending number of one of the red-blue roads. For example, suppose the road ending at 30 is being discussed.

T: ***This road is blue-red-blue-red and ends at 30. Could 30 be my secret number?***

S: ***No, 30 is not in the list.***

Note: If there is confusion that a number in the arrow pictures other than an ending number could be the secret number, erase the numbers from all but the starting and ending dots.

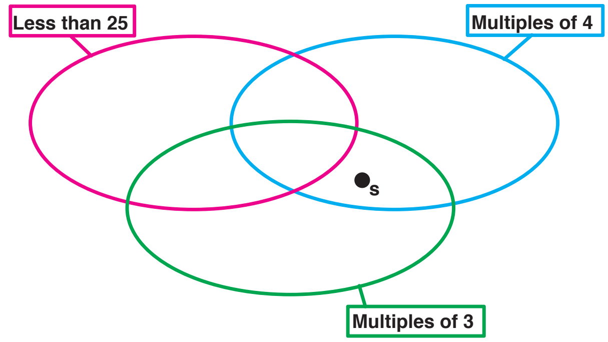
After comparing all the ending numbers of the red-blue roads with the numbers in the list from the first clue, you should have only 18 and 36 circled. Erase all others.

T: ***What do you need to help decide which of these numbers (18 or 36) is my secret number?***

S: ***Another clue.***

Clue 3

Erase the arrow roads from the board and draw this string picture. You may need to remind students about multiples of 3 and 4. Do so, counting by threes and fours, starting at 0.



Multiples of 3: 0, 3, 6, 9, 12, …
Multiples of 4: 0, 4, 8, 12, 16, …

T: ***My secret number is here*** (point to **s**). ***Decide whether 18 or 36 is my secret number and write it on your paper.***

Check papers and when most students have found that the secret number is 36, show the class the slip of paper.

T: ***Who can explain why 36 is here*** (point to **s**)***?***

S: ***36 is a multiple of 3 and a multiple of 4.***

T: ***Is 36 less than 25?***

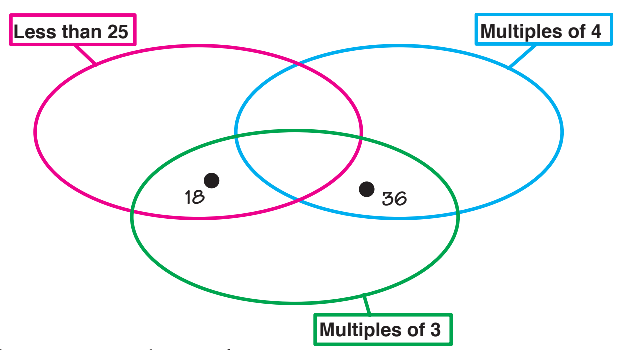
S: ***No.***

T (tracing the red string)**:** ***So 36 must be outside the red string.***

Erase the **s** and label the dot 36.

T: ***Who can show us where 18 belongs in this picture?***

When a student indicates the correct region, draw a dot in that region and label it 18.



T (tracing the appropriate strings in turn)**:**
Is 18 less than 25? (Yes)
Is 18 a multiple of 3? (Yes)
Is 18 a multiple of 4? (No)

Writing Activity

Suggest students try to write a detective story that has two or three clues.

Home Activity

This is a good time to send a letter to parents/guardians about detective stories. Blackline L10 has a sample letter.

L11 THE LITTLE DONKEY

Capsule Lesson Summary

Make a class graph by collecting some information about grandparents. Use relational thinking to discuss an arrow picture as it tells about a little girl's family in *The Little Donkey* Storybook. The arrows are for "You are my mother" and "You are my father."

Materials

Teacher
- Colored chalk
- *The Little Donkey* Storybook

Student
- *The Little Donkey* Storybook
- Colored pencils, pens, or crayons
- Worksheets L11* and **

Description of Lesson

Exercise 1

Initiate a discussion of grandparents with the class. Students may enjoy telling about

- how many grandparents they have;
- where their grandparents live;
- what they like to do with their grandparents; and
- having great-grandparents.

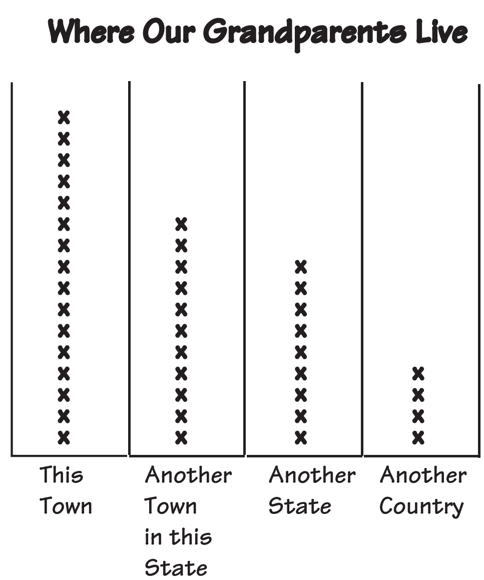
Try to mention in the discussion that many children have two sets of grandparents, one on their mother's side of the family and one on their father's side of the family.

Choose one particular characteristic about grandparent to make a class graph.

For example, if your students have grandparents living in different parts of the country or even in other countries, the class might like to make a graph of this information.

The graph may include two entries for many students, one for each set of grandparents.

Make some observations about your collection of data, such as Where do most of our grandparents live? or How many sets of grandparents live in another country?



Exercise 2

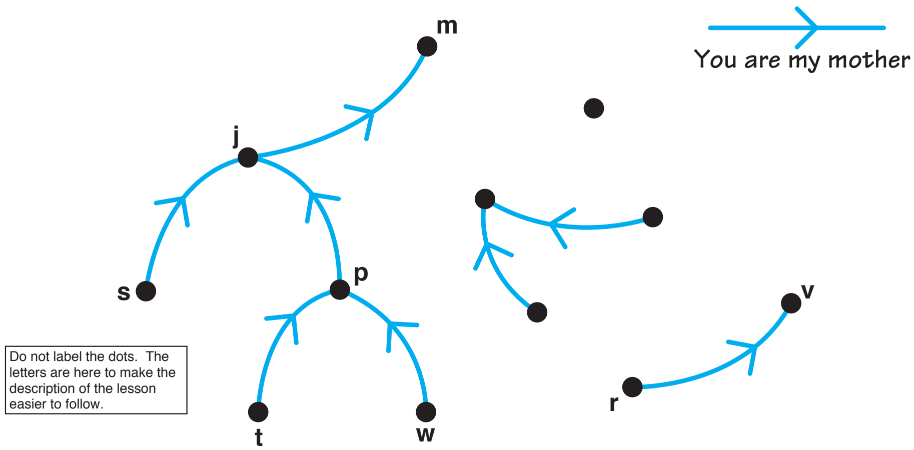
Distribute copies of *The Little Donkey* Storybook to student pairs. Ask your students not to read ahead of the class during the lesson.

Pages 1–4

Read these pages with your class.

Pages 5 and 6

After reading page 5, draw the arrow picture from that page on the board. Add the key arrow to your picture, indicating that the blue arrows are for "You are my mother."



Read page 6 with your class and label dots **r** (the donkey) and **v** (his mother) accordingly. Remind students not to turn the page.

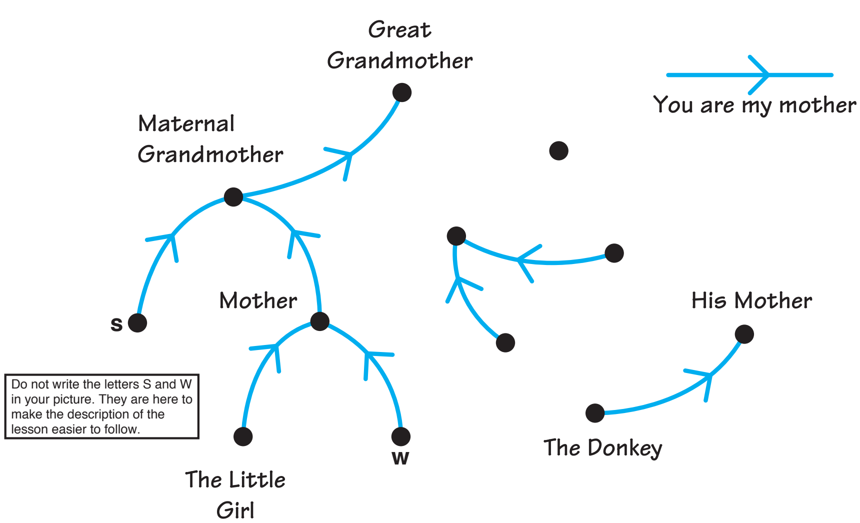
T: ***I'll show you where the little girl is in this picture.***

Label the dot (**t**) for the little girl.

T: ***Now can you tell for sure who some of the other dots are for?***

Your class should decide who the dots **p**, **j**, and **m** are for. Take the opportunity to discuss how to distinguish a person's two sets of grandparents and introduce the terms *maternal* and *paternal*. Use hints such as maternal—ma, and paternal— pa. Label **p**, **j**, and **m** appropriately. You may like to use a student as an example in the discussion about maternal grandmother and great grandmother.

Perhaps some of your students will comment that **w** must be for either a brother or a sister of the little girl, and that **s** must be for either an aunt or an uncle. If so, when page 7 is read, note which choice is made for each dot.



Pages 7 and 8

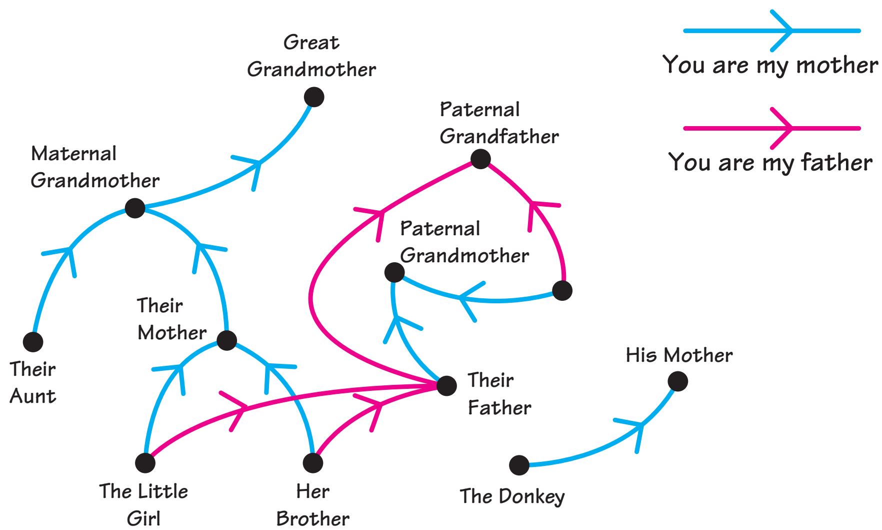
Read these pages with your class. Label the dots in the picture on the board for the girl's brother and for their aunt.

Pages 9 and 10

Read these pages with your class. Add the red arrows to your picture as shown on page 9 and add a key arrow indicating that the red arrows are for "You are my father." Let a student locate the dot for the little girl's father, and then label the dot yourself.

Pages 11 and 12

Read these pages with your class. Invite students to locate the dots for the little girl's paternal grandparents and then label them yourself.



There is still one dot that is unlabeled. Ask what it could be for. Your students should decide that it is for either a brother or a sister of the girl's father, i.e. an aunt or an uncle of the girl.

Pages 13–15

Finish reading the storybook with your class.

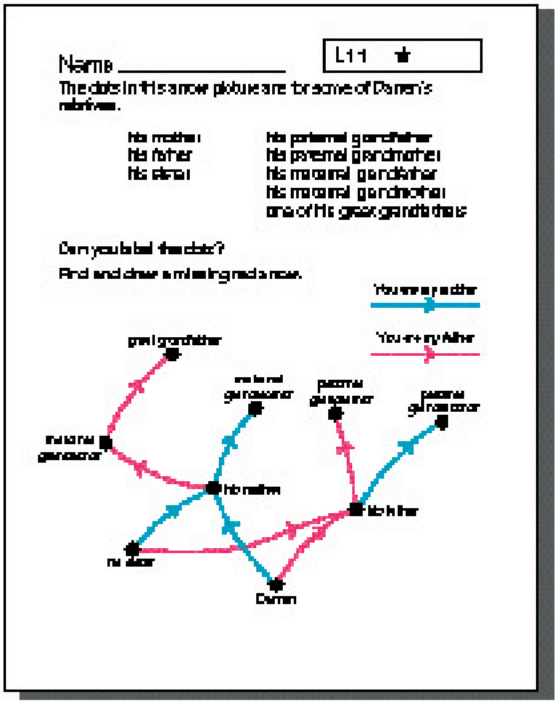
Worksheets L11* and ** are available for individual work.

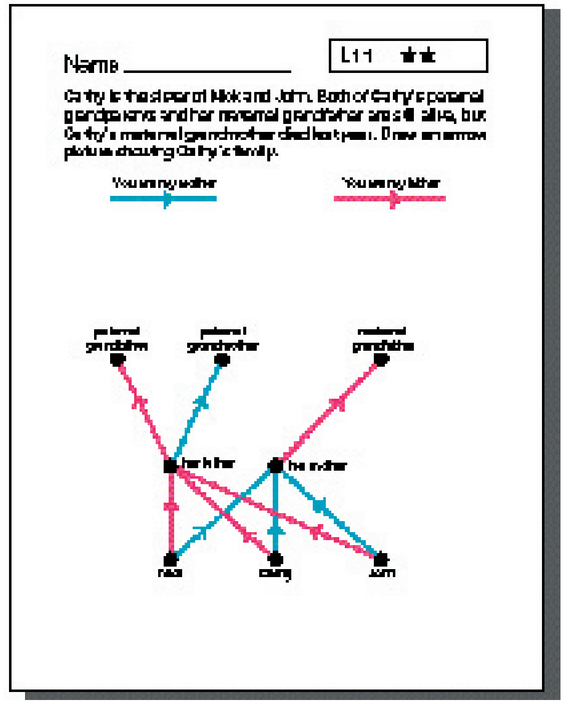
Reading Activity

Other books about relationships with grandparents may be of interest to your students; for example, *Kevin's Grandma* by Barbara Williams, *Annie and the Old One* by Miska Miles, *Song and Dance Man* by Karen Ackerman, or *Now One Foot, Now the Other* by Tomie De Paola.

Home Activity

Suggest students draw an arrow picture of some members of their families. Different relationships can be pictured, but ask that pictures include a color key for the arrows.


Name
L11


Name
L11

L12 A GAME WITH TWO CUBES

Capsule Lesson Summary

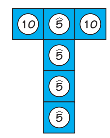
Examine two number cubes noting what numbers are on the six faces of each cube. Find all the possible scores one can get by tossing one of the cubes three times and adding the outcomes. Play a game pitting the blue cube against the red cube, and observe which cube gets the higher score most often.

Materials

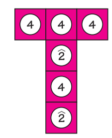
Teacher
- Two numeral cubes
- Cube cut-out

Student
- Paper
- Cube cut-out
- Colored pencils, pens, or crayons

Advance Preparation: Use Blackline L12 to make sturdy paper cube cutouts for students. Prepare approximately twenty cubes—ten blue cubes and ten red cubes—with numbers on their faces as shown here.



Blue cube



Red cube

Description of Lesson

Exercise 1

Hold up a cube that does not have numbers on its faces.

T: ***What shape is this?***

S: ***A cube.***

Perhaps students will say *square*, in which case you should point out that one side (or face) is a square, but the 3-dimensional shape is a *cube*.

If necessary, give this name to the shape yourself.

T (indicating a side of the cube)**:** ***A side of a cube is called a face. How many faces does a cube have?***

S: ***Six.***

Hold up your cube cut-out.

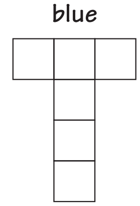
T: ***Does this look like a cube?***

S: ***It's like a cube flattened out.***

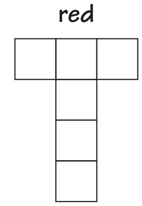
S: ***You can fold it into a cube.***

Ask students to fold their cube cut-outs to form cubes. While students do this, draw a similar map for the blue and red cubes on the board.

blue



red



Give a student the blue number cube to examine.

T: ***What numbers are on the faces of this blue cube?***

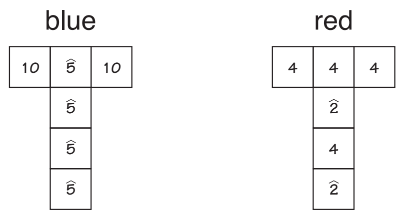
S: ***$\widehat{5}$ is on four of the faces, and 10 is on two of the faces.***

T: ***How can we show this in our picture*** (point to the map of the blue cube on the board)***?***

S: ***By writing $\widehat{5}$ in four of the squares and 10 in the other two squares.***

Invite a student to label the faces in the blue cube map; one way is shown below.

Give a student the red number cube to examine, and ask the student to announce which numbers are on the faces while another student labels the faces in the red cube map.



At this point, you may like to pair the students and instruct the pairs to label one of their cube cut-outs like the blue cube (use blue numbers), and one like the red cube (use red numbers).

Exercise 2

Tell the class you are going to describe a game with the two (a red and a blue) number cubes. In the game, one player will toss the red cube three times. The other player will do the same with the blue cube.

T: ***Suppose I toss the red cube three times and add the numbers that appear (on the top face); what score could I get?***

S: ***12.***

T: ***How could I get a score of 12 with the red cube?***

S: ***Roll 4, 4, and 4.***

Write this number sentence on the board.

$$4 + 4 + 4 = 12$$

Keep a record of the possible scores near the map of the red cube.

Continue in this manner until all four possible scores are found. Record these number sentences to show the possible scores for the red cube.

red

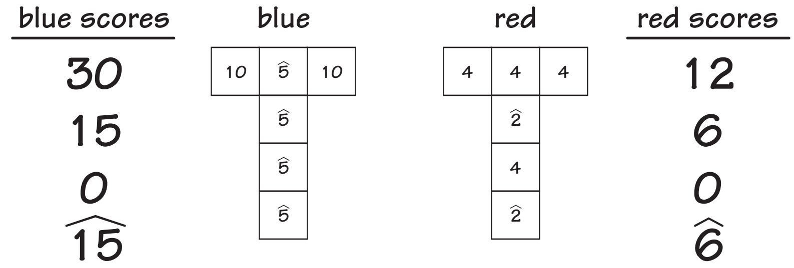
$$4 + 4 + 4 = 12$$
$$4 + 4 + \widehat{2} = 6$$
$$4 + \widehat{2} + \widehat{2} = 0$$
$$\widehat{2} + \widehat{2} + \widehat{2} = \widehat{6}$$

Similarly, ask what the possible scores could be if you toss the blue cube three times.

Erase the number sentences, leaving just the possible s

blue

$$10 + 10 + 10 = 30$$
$$10 + 10 + \widehat{5} = 15$$
$$10 + \widehat{5} + \widehat{5} = 0$$
$$\widehat{5} + \widehat{5} + \widehat{5} = \widehat{15}$$



T: ***Suppose two players play a game with these cubes, each tossing the cube three times. High score wins. Would you rather be the player with the red or the blue cube?***

Perhaps some students will have an intuitive feel for the cubes and choose red. Others might look only at the possible scores and choose blue. Be noncommittal at this time.

T: ***The red cube has more positive numbers on its faces, but the blue cube has the possibility of a higher score.***

Draw a chart on the board in which to record scores and winners from this game.

blue	red	winner

T: ***The player with the blue cube will go first.***
Who would like to toss the blue cube?
Who would like to play with the red cube?

Give the students who volunteer the respective color cubes, and ask them to sit on the floor or at a table near the front of the room to toss the cubes.

After the first student has tossed the blue cube three times, ask, “Can the second person (use actual name) win? Can the second person lose?” For example,

blue throws 10 — 10 — 10 (score 30) ⟶ red cannot win

blue throws $\hat{5}$ — $\hat{5}$ — $\hat{5}$ (score $\widehat{15}$) ⟶ red must win

blue throws $\hat{5}$ — 10 — $\hat{5}$ (score 0) ⟶ red can win or lose

As the cube tosses are made, instruct the other students to record the information as number sentences on their papers, as you do so on the board. If the student tossing the blue cube rolls $\hat{5}$, 10, and then 10, write $\hat{5} + 10 + 10 = 15$. Write the scores for each player in the chart. Repeat this activity a couple times, choosing different students to toss the cubes each time.

Occasionally you may like to consider an intermediate score. For example, suppose the student tossing the blue cube has gotten 10 and $\hat{5}$. Stop the play for a moment and ask,

T: ***After two tosses, what is the blue score?***

S: ***5, because $10 + \hat{5} = 5$.***

T: ***Do you think the blue score will be 30 after the third toss?***

S: ***No; the third toss will be 10 or $\hat{5}$, so the total blue score will be 15 ($10 + \hat{5} + 10 = 15$) or 0 ($10 + \hat{5} + \hat{5} = 0$).***

T: ***Which score, 15 or 0, do you think has a better chance of occurring?***

S: ***0, because there are more faces with $\hat{5}$ than with 10.***

Let the student make the third toss; then record the score. Proceed in this way for the red cube as well.

After a few class demonstration games, organize the class into groups of two or three to play the game.

Direct one student in each group to be the blue player and another to be the red player. If there is a third student in a group, that student can be the scorekeeper.

After each group has played the game one time, collect results from all the groups in your chart as illustrated her

blue	red	winner
0	0	tie
30	0	blue
0	12	red
$\widehat{15}$	6	red
$\widehat{15}$	6	red
15	12	blue
$\widehat{15}$	6	red
0	12	red
15	6	blue
$\widehat{15}$	$\hat{6}$	red

At this point, you might discuss which cube students prefer to use in the game. Direct the discussion toward choosing a cube with which you have the better chance of winning and avoid preferences based on favorite colors or numbers.

The results from many games will likely suggest that the player with the red cube has a better chance to win.

If there is sufficient time, you might ask the class to find thrown four times or five times.

Center Activity

Place cubes in a center for further exploration and games.

Home Activity

Allow students to take red and blue cubes home, and to play a game with another family member. They may want to keep a record from the home games and compare it with the class games.

L13 STRING GAMES #3

Capsule Lesson Summary

Play The String Game with A-blocks using sixteen attributes as possible string labels. Play a type of string game with numbers where the strings are for **Multiples of 3** and **Multiples of 5**.

Materials

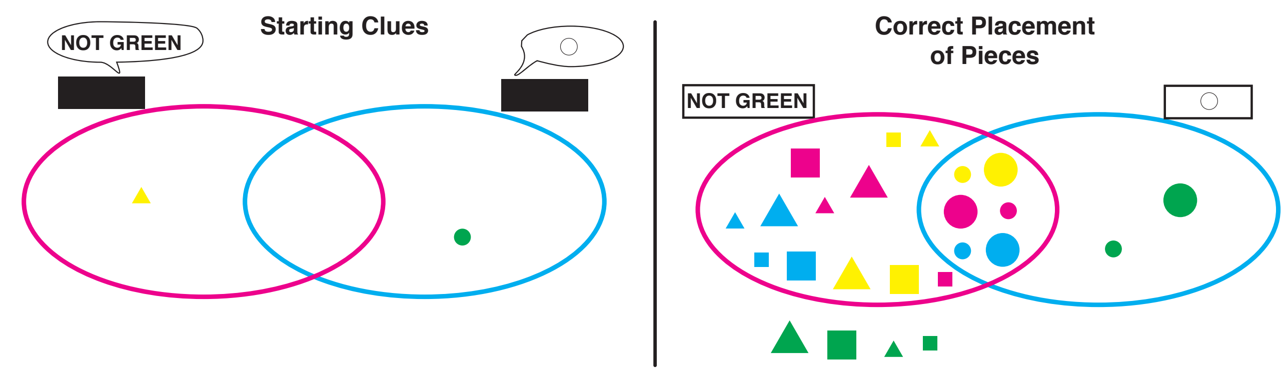
Teacher	• A-Block String Game kit • Colored chalk	**Student**	• Worksheets L13*, **, ***, and ****.

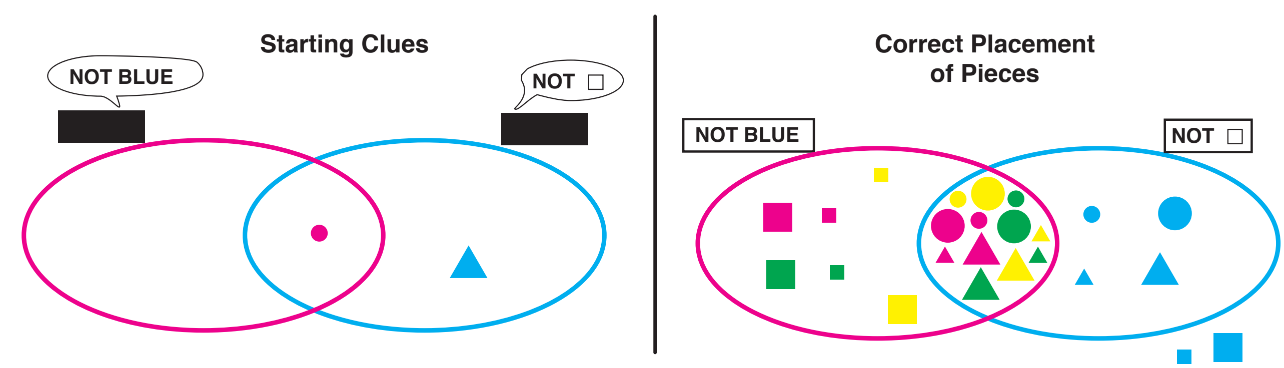
Advance Preparation: See Appendix D on The String Game with A-blocks for a description of equipment and preparation for the game.

Description of Materials

Exercise 1

Prepare to play a two-string version of The String Game with A-blocks in the usual way. Place the Version B poster list of sixteen possible string labels above the team board. Play one of the games illustrated below or another of your choice.





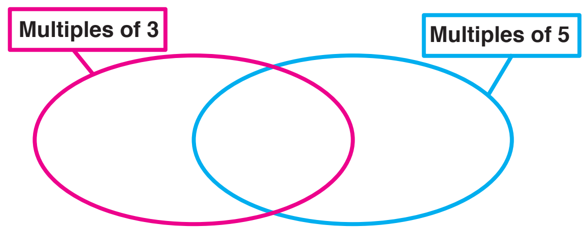
Exercise 2

Take down the list of string labels and put the A-blocks away. Label the red string **Multiples of 3** and the blue string **Multiples of 5**.

Review the classifications **Multiples of 3** and **Multiples of 5** and allow students to suggest several numbers that would fit in each region of the picture.

Assign numbers to the respective teams as shown below on the team board.

Team A		Team B	
30	5 x 30	15	3 x 13
3 x 7	17	3 x 35	60
45	3 x 40	19	$\hat{9}$
5 x 17	26	5 x 19	50
100	40	5	80
$\hat{5}$	3	5 x 12	32



T: ***This time we'll play a game with numbers. During a turn, you will have only one chance to put a number in the string picture. The first team to locate all the numbers from its list correctly, wins.***

Alternating teams, let students locate numbers in the picture. Discuss correct responses; for example, suppose a student from Team B locates 5 x 19 correctly in the string picture.

T: ***Why does 5 x 19 belong there?***

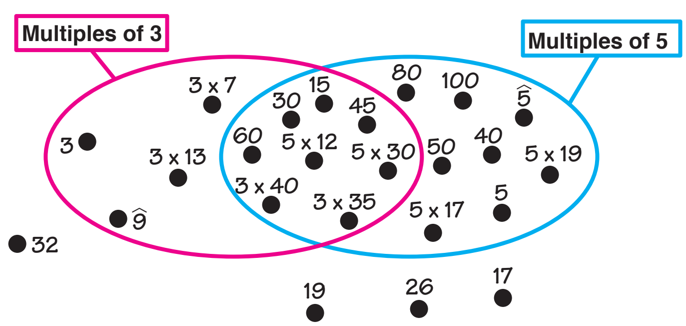
S: ***5 x 19 is a multiple of 5, but it is not a multiple of 3.***

Erase 5 x 19 from Team B's collection of numbers on the team board.

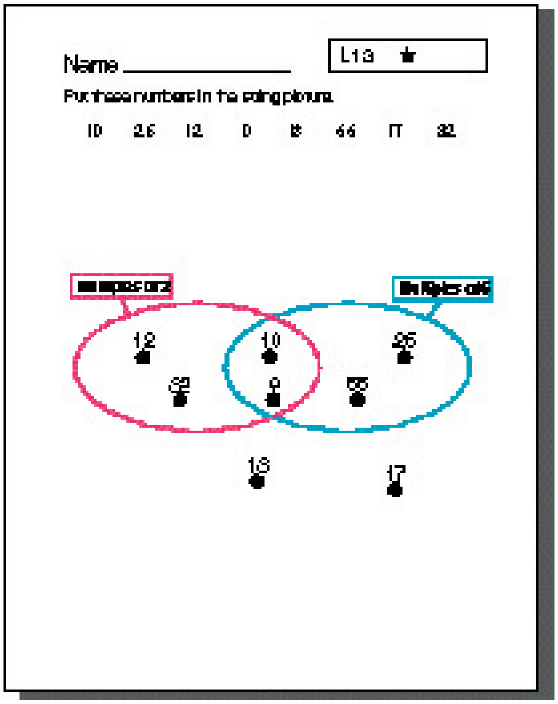
If a student locates a number incorrectly, simply erase it from the string picture and say, "No, that number does not belong there."

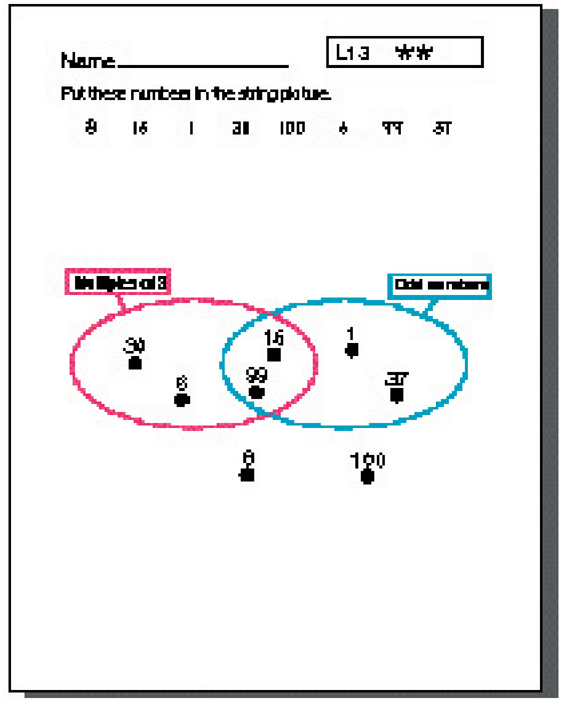
This picture shows the correct location of all the numbers in the game

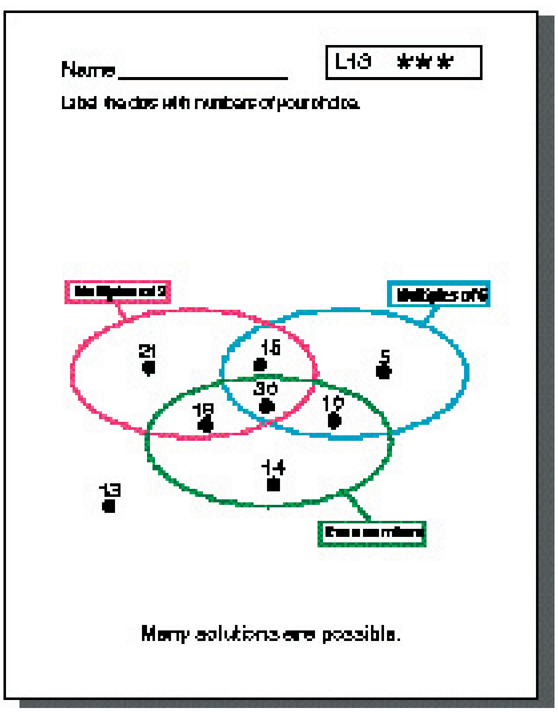
Discuss with the class how to know where to place numbers like, for example, 5 x 12. (It is a multiple of 5, because it is 5x something, and it is a multiple of 3 because 12 is a multiple of 3.)

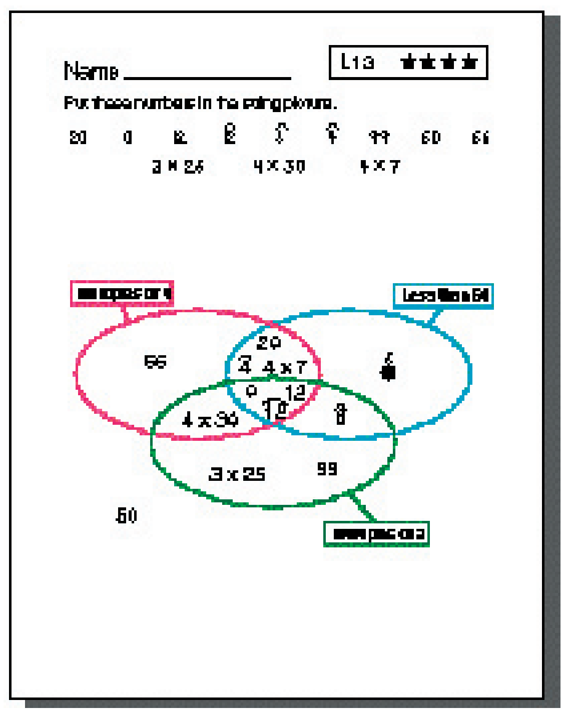


Worksheets L13*, **, ***, and **** are available for individual work.


Name
L13 ★
Put these numbers in the string picture.
10 26 12 0 18 66 17 32
12
10
26
32
0
18
17


Name
L13 ★★
Put these numbers in the string picture.
Multiples of 3
Odd numbers
30
15
1
99
37
100


Name
L13 ★★★
Label the dots with numbers of your choice.
21
15
5
30
18
10
14
13
Many solutions are possible.


Name
L13 ★★★★
Put these numbers in the string picture.
4 x 7
4 x 30
3 x 25
99
50

Capsule Lesson Summary

Collect information on the starting times of many Saturday morning cartoons. Record times on both analog and digital clockfaces. Use time to measure the length of events and to prepare an activity schedule.

Materials

Teacher	Clockface Overhead projector (optional)	**Student**	Worksheets L14(a), (b), and (c)

Advanced Preparation: Use Blackline L14 to make clock faces for every hour and half hour from 7:00 AM through 9:30 AM. Use a copy of Worksheet L14(c) to prepare the start of a class schedule for Exercise 3.

Description of Lesson

Organize the class so that students can work in pairs during this lesson.

Exercise 1

Distribute copies of Worksheet L14(a).

T: ***How many of you watch cartoons on Saturday morning? Raise your hand. Roberto, name a couple of the cartoons you watch.***

S: ***I watch* Bugs Bunny *and* Beetlejuice.**

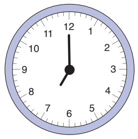
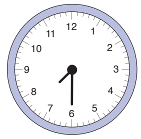
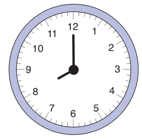
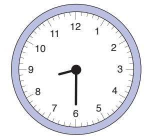
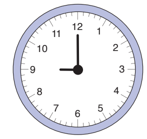
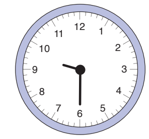
T: ***On the worksheet, there are blank lines for you to list cartoons you watch on Saturday mornings. Next to each cartoon show the time it comes on: draw hands on the clockface for the time the cartoon starts, and write the time the way it appears on a digital clock. Only include cartoons that are on between 7:00 AM and 10:00 AM (last starting time 9:30 AM).***

Allow 5–10 minutes for students to log the cartoons they watch.

T: ***On the board, I taped clock faces for every half hour between 7:00 AM and 9:30 AM. Let's list the cartoons you watch under the times on the board. Who has a cartoon starting at 7:00 AM?***

Write the names of the cartoons students give you under the clockface for 7:00 AM.

Continue listing the names of cartoons under the appropriate clockfaces. Your board may eventually look like the next illustration. Above each clockface, invite a student to write the time as it appears on the digital clock.

7:00	7:30	8:00	8:30	9:00	9:30
					
Captain N & New Mario	Pro Stars	Wishkid/ Culkin	Space Cats	Saved by the Bell	Saved by the Bell
Garfield & Friends	Garfield & Friends	Teenage Mutant Ninja Turtles	TMNT (cont.)	Back to the Future	Where's Waldo
Darkwing Duck	Beetlejuice	Hammerman	Pirates of Dark Water	Bugs Bunny & Tweety	Bugs Bunny & Tweety
Killer Tomatoes	Bobby's World	Tom & Jerry Kids	Taz-Mania	Little Shop	Bill & Ted"s Adventures

Poll the class to find how many students watch (or like) each cartoon listed. Use tally marks or let students count and record the number in parentheses next to a cartoon. In this way, you will construct a graph showing which cartoon is the favorite of the class.

Leave the chart from Exercise 1 on the board.

Exercise 2

Tape a large clockface on the board (or make an overhead transparency of the clockface to project). Refer students to Worksheet L14(b) as you pose several time problems related to the information in the chart. Each time instruct the students to write the digital time and draw in the hands on one of the clock faces on their worksheets. Encourage students to confer with their partners and agree on the time. For example,

- What time is it 15 minutes before *Tom & Jerry Kids* starts.
- Suppose you start watching *Garfield & Friends* at its starting time and quit watching T.V. one hour and ten minutes later. What time is it?
- Suppose *Bugs Bunny & Tweety* starts exactly on time. A commercial comes on after every five minutes of the cartoon and each commercial lasts three minutes. What time is it at the end of the third commercial?

After students have had time to respond to a problem on their worksheets, invite students to draw the hands on the clockface on the board (or overhead) and write the time as it appears on a digital clock.

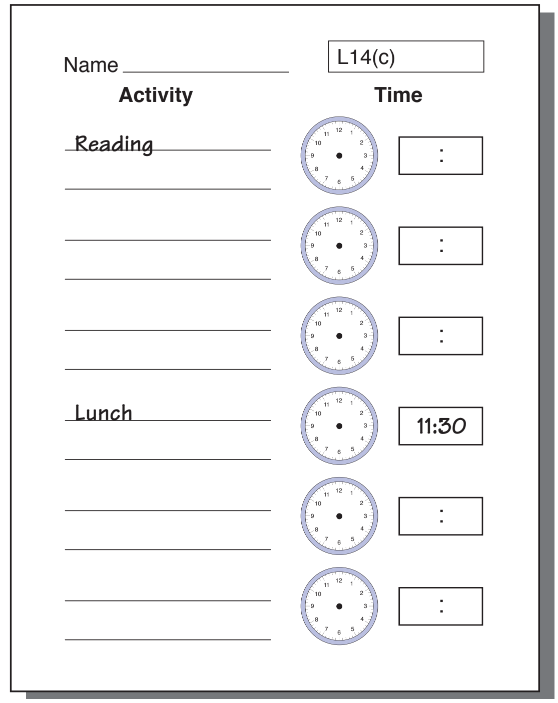
Exercise 3

Use a copy of Worksheet L14(c) to prepare the start of a class schedule. Fill in parts, but not all of the information. You may like to put this on an overhead transparency.

T: ***We are going to make a schedule for the school day.***

Refer students to Worksheet L14(c).

T: ***On this worksheet, you will fill in the class or activity and show its starting time.***



Give the students a couple activities to begin the schedule. For example, if you start the day with Reading, direct students to list Reading as the first activity and to show the time. Continue by letting students help define the class day.

Use the completed class schedule to pose some time problems. For example,

- Suppose we started Science 15 minutes late this afternoon. What time would it start?
- Lunch starts at 11:30 am and you have 35 minutes for lunch. What time is lunch over?
- Melodie spends only 15 minutes eating lunch so she has time to play outside after eating. How much time does she have for play after eating?

Home Activity

Send a copy of Worksheet L14(c) home with students to fill in their daily routine of activities before and after school. Instruct them to include activities that involve other family members as well (for example, meals, reading time, watching T.V., and so on). Also, send home copies of Worksheet L14(d). Tell students to ask a family member to help them time themselves doing activities and answer the questions.

Capsule Lesson Summary

Associate the dots in an arrow picture with the seats in a row or at a table. Change seats by following the arrows. Play the changing seats game several times in succession, observing who is in their original seat after each round. Predict how many rounds need to be played before everyone is back in his or her own seat. This game involves physical experiences with the mathematical concepts of multiples and least common multiples.

Materials

Teacher
- Colored chalk

Student
- Unlined paper
- Colored pencils, pens, or crayons

Description of Lesson

Organize your class into rows or at tables, with six or seven students each. The chart below suggests ways of doing this for class sizes between 18 and 31.

18 3 rows of 6	**19** 3 rows of 6 with one recorder	**20** 3 rows of 6 with two recorders	**21** 3 rows of 6 with three recorders	**22** 3 rows of 7 with one recorder	**23** 3 rows of 7 with two recorders	**24** 3 rows of 7 with three recorders
25 4 rows of 6 with one recorder	**26** 4 rows of 6 with two recorders	**27** 4 rows of 6 with three recorders	**28** 4 rows of 6 with four recorders	**29** 4 rows of 7 with one recorder	**30** 4 rows of 7 with two recorders	**31** 4 rows of 7 with three recorders

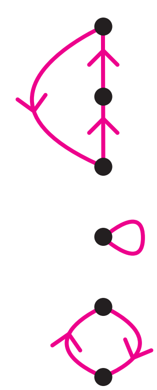
The lesson description assumes there are four rows of six students and also some recorders. The dialogue here includes names to facilitate the description; you, of course, will use the names of your students.

On the board, draw six (or seven) dots placed like the chairs in a row or at a table. As you explain the arrangement of dots to the class, indicate how the dots correspond to seats; for example, the dot at the top is for the first seat in a row and the dot at the bottom is for the last seat in a row.

If possible, assign a recorder to each row and ask recorders to take a position at the board.

T: ***These dots are for the seats in any one of the rows. I will draw some arrows to show you how we are going to change seats in each row.***

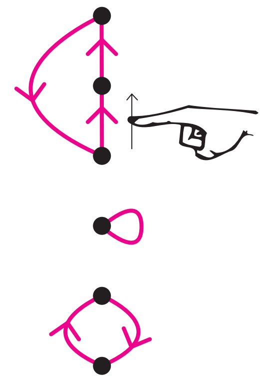
T: ***Look at the picture on the board. Mike, where***



will you go when we change seats?

S: ***To Pat's seat.***

Trace the appropriate arrow.



T: ***And Pat, where will you go?***

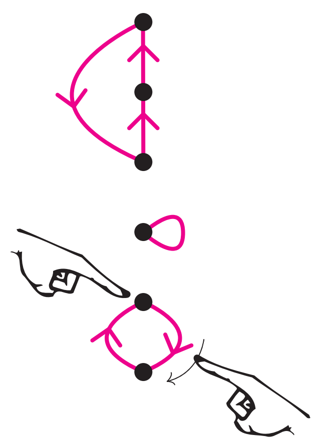
S: ***To Lucy's seat.***

T: ***Wanda, where will you go when we change seats?***

S: ***I'm not sure.***

T (pointing successively to chairs in Wanda's row)**:** ***One, two, three, four, five; you are sitting in the fifth seat in your row, Wanda.***

Then point successively to the dots on the board as you say,



T: ***One, two, three, four, five; this dot is for your chair.***
Now do you know where you will go when we change seats?

Keep one finger on the fifth dot, and trace the arrow starting at that dot.

S: ***To Cameron's seat.***

T: ***Cameron, where will you go?***

S: ***To Wanda's seat.***

You need not ask all the students where they will go, but try to check that the class as a whole has checked the picture to know which seats they are going to take.

T: ***The recorders and I are going to record on the board what happens when you change seats.***

Provide a labeled column for each row. (See the next illustration.)

T: ***Each row moves in the same way, following the arrows in this picture. Does everyone know where to go? Get ready; get set; go!***

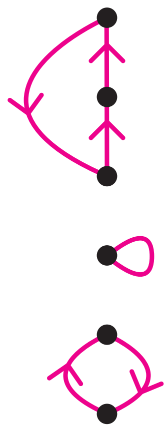
Students may get excited and noisy during this activity, so you may want to preface your introduction of this game with some remarks about moving as quietly as possible. After students have made their moves and are quieted, ask,

T: ***Are some of you still in your own seats? Raise your hands. Why are you still in your own seats?***

S: ***There is a loop at my dot, so I had to go to my own seat.***

You and the recorders should record the names of the students whose hands are raised. (See Round 1 in the next illustration.)

Repeat this activity for a second round and record the names of the students who are in their own chairs after the second round.



Seats in any row	Row 1	Row 2	Row 3	Row 4
	Round 1	Round 1	Round 1	Round 1
	Patrick	Derrick	Linda	Ki Jong
	Round 2	Round 2	Round 2	Round 2
	Patrick	Derrick	Linda	Ki Jong
	Sheila	Wanda	Terrell	Gary
	Anthony	Cameron	David	Scott

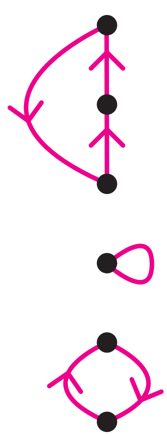
Before playing a third round of the game, ask,

T: ***If we change seats again, can you predict who will be back in their own seats?***

Let a person from each row make a prediction for that row. Repeat each response for the benefit of the class. Then let the students play the third round.

T: ***Get ready; change.***

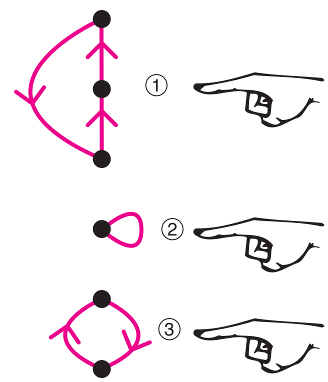
After the students move and are quieted, check the predictions of each row against those who are actually back in their own seats. You and the recorders can record the names under Round 3 on the board.



Seats in any row	Row 1	Row 2	Row 3	Row 4
	Round 1	Round 1	Round 1	Round 1
	Patrick	Derrick	Linda	Ki Jong
	Round 2	Round 2	Round 2	Round 2
	Patrick	Derrick	Linda	Ki Jong
	Sheila	Wanda	Terrell	Gary
	Anthony	Cameron	David	Scott
	Round 3	Round 3	Round 3	Round 3
	Patrick	Derrick	Linda	Ki Jong
	Lucy	Andy	Sharon	Mary
	Manuel	Stanley	Lisa	Patty
	Pat	Lori	Mindy	Jason

T: ***How many times do you think we will need to play this game before you are all back in your own seats?*** (Six)

Consider responses like "one more time" by asking the students who will be back in their own seats after the next round (Round 4) to raise their hands. Observe that some will and some won't. If no one gives the correct answer with a clear explanation, the following dialogue suggests how to help students arrive at the solution. The next illustration focuses only on the arrow picture.



T: ***The people who start here (1) return to their own chairs during the third round. When would the next time be?*** (During the sixth round) ***… the next time?*** (During the ninth round) ***So every third round they return to their own seats.***

In every round this person (2) stays in the same chair.

The people who start here (3) return to their own chairs in the second round. In which other rounds would they return to their own chairs? (In the fourth, the sixth, the eighth, and so on.)

When is the first time (in which round) that you all return to your own seats?

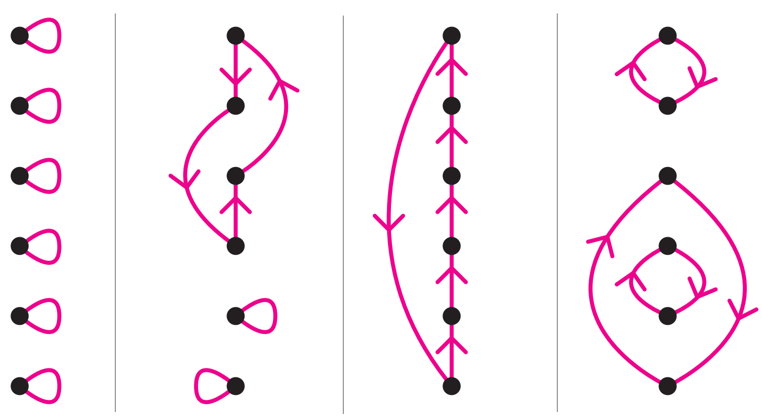
S: ***The sixth round.***

Quickly play three more rounds of the game without recording names on the board. All students should then be in their own seats.

Provide students with unlined paper and colored pencils on which to draw their own pictures of a changing seats game.

T: ***Draw six (seven) dots on your paper, and draw red arrows for another way we could play the game of changing seats.***

Check for students who have drawn different (interesting) pictures, and ask some of them to put their pictures on the board. Include a wide variety of pictures. For example:



You may like to let students, again in rows of six (seven), act out the changing seats game for one of the pictures on the board.

If time allows, consider each picture on the board and ask when is the first time (in which round) that all students return to their own seats.

The answers, starting from the left, are as follows:

- First picture: All students are always in the same seats.
- Second picture: All students return to their own seats in the fourth round, but not before.
- Third picture: All student return to their own seats in the sixth round, but not before.
- Fourth picture: All students return to their own seats in the second round, but not before.

Writing Activity

Ask students to write about what happens in the changing seats games. Let them use their pictures, or one on the board, to help describe the game.

Extension Activity

Ask students to try to find a changing seats game that does not have everyone back in their own seats until round 10 (or 12). Are there different games that have this property.

Capsule Lesson Summary

Solve several problems to show how there can be a given number of objects inside one string and a given number of objects inside a second string, but the total number of objects is less than the sum of the given numbers. Solve a problem of finding how many objects are in a second string,.given the total number of objects, the number of objects inside one string, and the number of objects common to two strings.

Materials

Teacher
- Colored chalk
- String loops
- 7 blue and 8 red pieces of paper

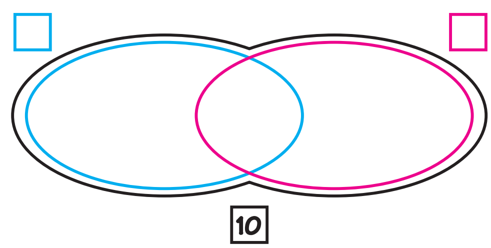
Student
- Unlined paper
- Colored pencils, pens, or crayons
- Counters (optional)
- Worksheets L16*, **, ***, and ****

Advance Preparation: To make string loops, cut long pieces of red and blue yarn, ribbon, or string approximately four yards long, and tie the ends together to form a loop.

Description of Lesson

Exercise 1

Choose ten students to help you with this exercise. Give two students one sheet of blue paper each, three students one sheet of red paper each, and five students one sheet of blue each and one sheet of red paper. Lay out large, overlapping string loops on the floor. Draw a string picture on the board similar to the string loops.



T: ***These ten students are going to stand in this string picture. All ten of them will be inside the strings.*** (Write 10 in the black box.) ***Some of them have blue pieces of paper; they will stand inside the blue string. Some of them have red pieces of paper; they will stand inside the red string.***

Instruct the ten students to find and stand in the appropriate place inside the strings. Then ask the rest of the class to check their positions, while students inside the strings hold up their papers.

T: ***How many students are inside the blue string (have blue papers)?*** (7)

Ask the students with blue papers to hold them up for counting. Write 7 in the blue box.

T: ***How many students are inside the red string (have red papers)?***

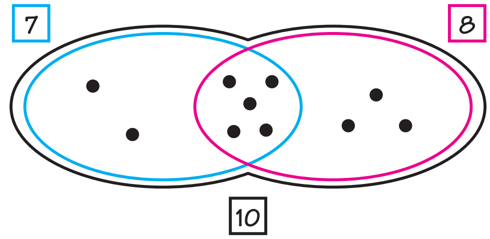
Ask the students with red papers to hold them up for counting. Write 8 in the red box.

T: ***7 students with blue papers, 8 students with red papers; how many students altogether inside the strings?*** (10)

L16

Point to 10 in the black box. Let the class discuss how there can be 7 with blue papers and 8 with red papers and 10 students with papers altogether.

Draw dots in your string picture on the board to represent the students, and then instruct the students to return to their seats.



With the class, count the numbers of dots in each region. Then erase the dots and numbers, leaving the string picture.

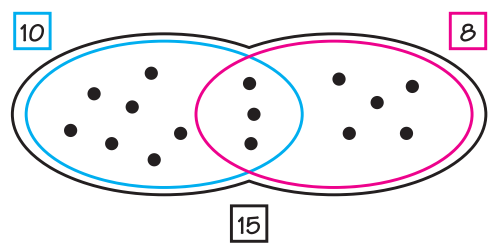
Exercise 2

You may like to allow students to work in pairs or small groups for this exercise. Distribute unlined paper and colored pencils and counters (optional) to the groups. Instruct the students to copy the string picture on their papers.

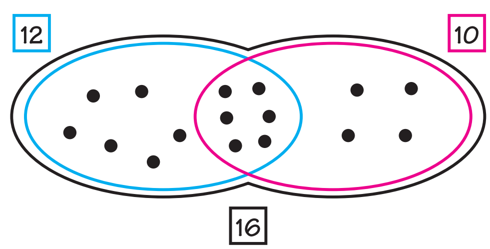
Record the appropriate numbers in the boxes as you give the following directions to the groups.

T: ***Put 10 dots (counters) inside the blue string and 8 dots (counters) inside the red string. But, there should be exactly 15 dots (counters) altogether inside the strings.***

Let the students experiment for several minutes. Students may benefit from using counters as they can be moved easily from region to region without erasing. When several groups find a solution, record it on the board and check it with the class.



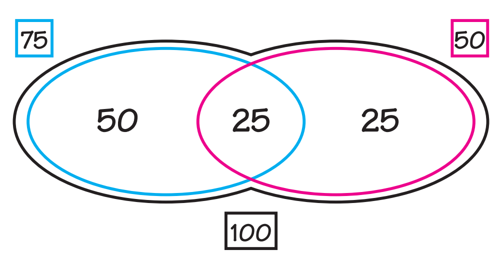
Repeat this activity with another example; such as the one illustrated here.



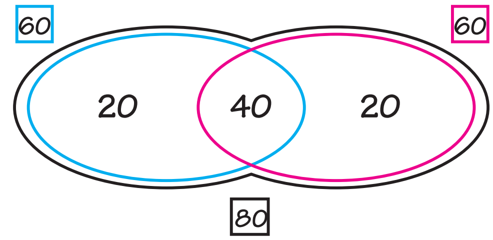
Encourage students to make observations about how to find a solution. In particular, they may talk about how many dots (counters) to put in the intersection (middle region). The number in the middle is the difference between the sum of blue and red and the string total; for example, (12 + 10) – 16 = 6. Allow students to discover the "secret." Otherwise, give clues leading to this observation in the next problems.

Vary the activity by adding story lines. Two examples are given below, with correct solutions given on the right. On their papers, suggest students write numbers in the regions rather than drawing dots or using counters.

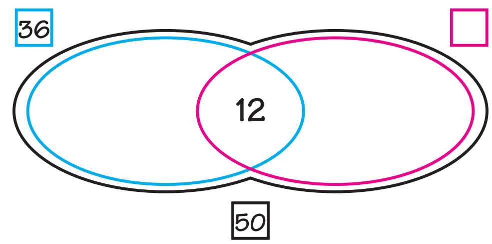
T: ***Pretend I have a large box that has one hundred blocks inside it. Seventy-five blocks are made of wood*** (blue string) ***and fifty blocks that are cubes*** (red string)***.***



T: ***Pretend I sold eighty sandwiches at a football game. Sixty were hotdogs*** (blue string) ***and sixty had mustard on them*** (red string)***.***



Vary the activity by changing what information is given. An example is presented in the remainder of the lesson description.



T: ***Pretend I made a bouquet with fifty flowers*** (record 50 in the black box)***. In this bouquet there are thirty-six roses*** (record 36 in the blue box)***. Some of the flowers are red*** (trace the red string)***, but I won't tell you how many. I will tell you this*** (write 12 in the center region)***.***

What does that tell you (point to 12)***?***

S: ***There are twelve red roses in your bouquet.***

T: ***Who can tell me the other numbers in the picture?***

Although students may suggest some numbers that do not belong in the picture and will be eventually ruled out, the discussion might proceed as follows.

S (pointing to the left region in the blue string on the left)**:** ***24 belongs here.***

T: ***How do you know?***

S: ***Because 24 + 12 = 36 (or 36 – 12 = 24).***

Let the student record 24 in that region.

S (pointing to the right region in the red string)**:** ***14 belongs here.***

T: ***How do you know?***

S: ***Because 24 + 12 + 14 = 50 (or 50 – 36 = 14).***

Let the student record 14 in that region.

Check with the class that there are a total of 50 flowers by doing the calculation 24 + 12 + 14 = 50.

T (pointing to the red box)**:** ***How many red flowers are in my bouquet? How do you know?***

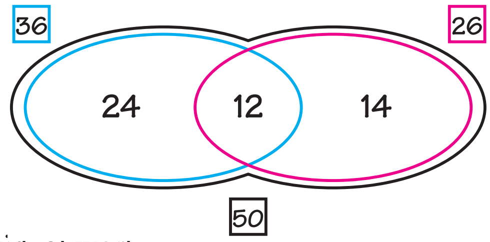
S: ***26, because 12 + 14 = 26.***

Record 26 in the red box.

You can ask other questions related to the picture. For example:

T: ***How many roses were colors other than red?*** (24)

How many flowers in the bouquet were not roses? (14)

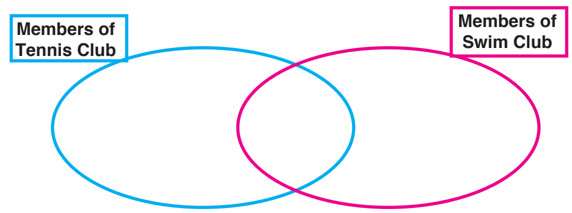


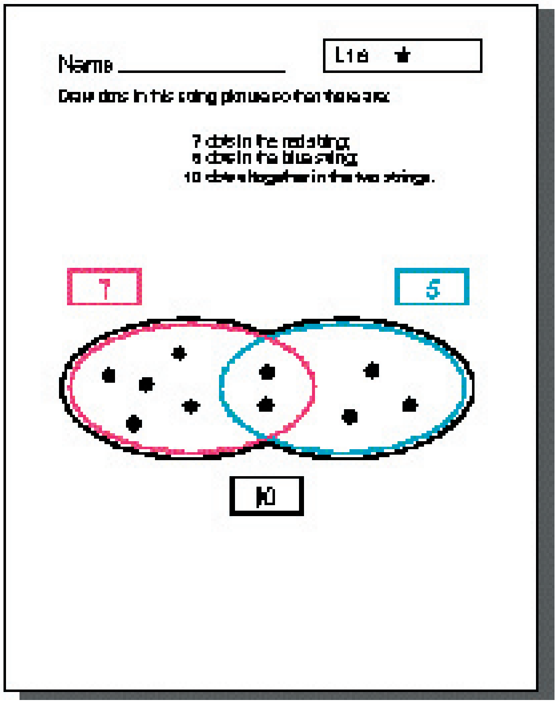
Worksheets L16*, **, ***, and **** are available for individual work.

Home Activity

Pose a story problem like those in Exercise 2 for students to solve at home with family members. For example:

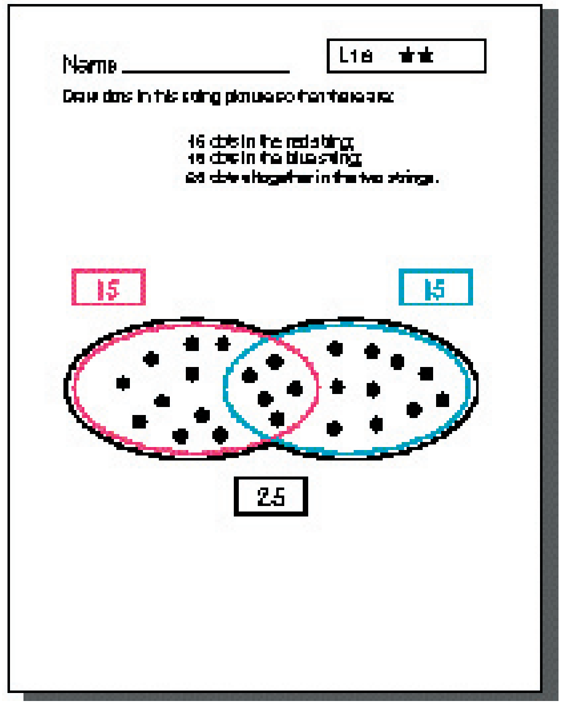
There are 15 people in the Tennis Club and 12 people in the Swim Club. Altogether there are 20 people in the two clubs. Explain how this can be.




Name ______________ L16 ★

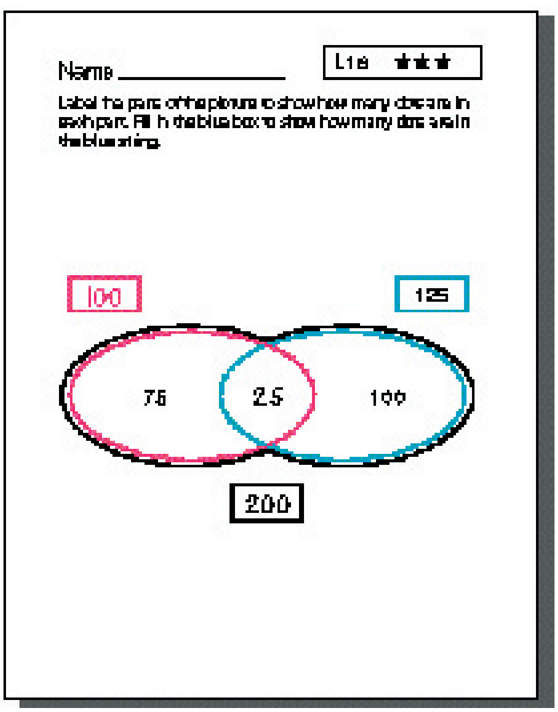
Draw dots in this string picture so that there are:

7 dots in the red string,
5 dots in the blue string,
10 dots altogether in the two strings.

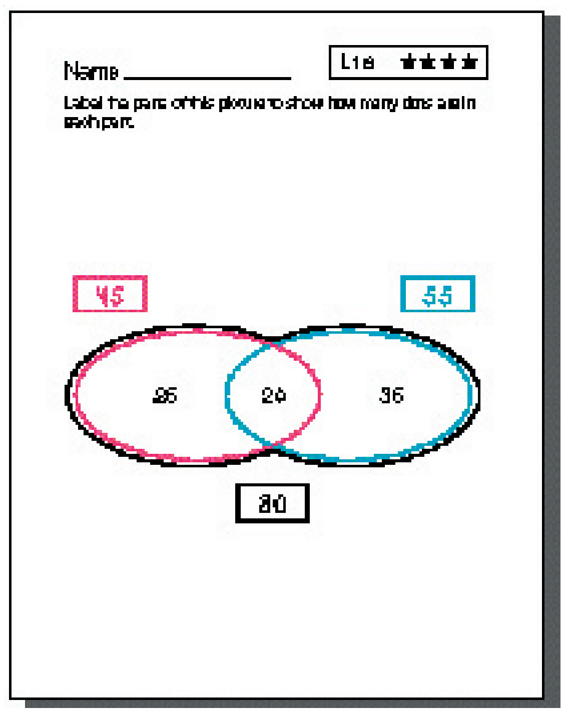

Name ______________ L16 ★★

Draw dots in this string picture so that there are:

15 dots in the red string,
15 dots in the blue string,
25 dots altogether in the two strings.


Name ______________ L16 ★★★

Label the parts of the picture to show how many dots are in each part. Fill in the blue box to show how many dots are in the blue string.


Name ______________ L16 ★★★★

Label the parts of this picture to show how many dots are in each part.

Capsule Lesson Summary

Read and discuss the storybook *Singing Friends*. Find the numbers that can be named by inserting parentheses in expressions such as 3 x 5 + 2. Explore the possibilities that result when x and + are interchanged, or when 3 and 2 are interchanged.

Materials

Teacher
- Colored chalk
- *Singing Friends* Storybook

Student
- *Singing Friends* Storybook

Description of Lesson

Distribute copies of the storybook *Singing Friends* to student pairs, and let them look at the book until you are ready to begin.

Pages 2–3

As you read page 3, stop and ask a student to read the numbers:

1 3 5 7 10 14

T: ***Do you know a number less than any of these?***

S: ***0.***

T: ***…less than 0?***

S: $\hat{5}$.

T: ***Who can read the large numbers on this page?***

After someone reads the numbers correctly, ask,

T: ***Does anyone know a number greater than 65,375?***

Let several students respond, and for each correct suggestion, ask if anyone can write that number on the board.

Pages 4–5

Read these pages aloud, or let a student do so.

T: ***What is your favorite number?***

Pages 6–9

Read these pages aloud (or let a student do so), and then ask the students to complete the +1 snake dance on page 9.

T: ***What is the greatest number in this picture?*** (38)
If we drew one more arrow starting from 38, where would the it end? (39)
If we drew ten more arrows starting from 38, what number would be at the end? (48)
If we drew twenty more arrows (from 38)…? (58)

If we drew thirty more arrows (from 38) … ? (68)

What is the least number in this picture? (0)
If we go back ten arrows, what number would we be at? ($\overset{\frown}{10}$)
If we go back twenty arrows (from 0), …? ($\overset{\frown}{20}$)
If we go back thirty-five arrows (from 0) …? ($\overset{\frown}{35}$)

Pages 10–11

Ask the students to compare their +1 snake pictures on page 9 to the one on page 10. Then read page 11 aloud or let a student do so.

T: ***Why is 38 unhappy?***

Pages 12–13

Read page 12 aloud, or let a student do so.

T: ***This +2 dance starts at 0. What do you notice about these numbers?***

S: ***They are even numbers.***

T: ***What kind of numbers would be in a +2 dance that starts at 1?***

S: ***Odd numbers.***

T: ***Turn the page and you'll see a +2 dance starting at 1.***

Pages 14–15

Read these pages aloud, or let a student do so.

T: ***What are some of the numbers in the yard who are dancing in this +2 dance?***

Let several students answer. (Any odd number greater than 203 is dancing in the yard.)

T: ***What are some of the numbers in the yard who are dancing in the +2 dance shown on pages 12 and 13?***

Students may respond that any even number greater than 240 is dancing in the yard.

Pages 16–17

As you read page 16, write this expression on the board.

3 + 5 x 2

T: ***What number is this? How do you know?***

S: ***16, because 3 + 5 = 8 and 8 x 2 = 16.***

S: ***13, because 5 x 2 = 10 and 3 + 10 = 13.***

T (after reading page 17)**: *Like you, the numbers 3 and 2 cannot decide whether this*** (point to 3 + 5 x 2) ***is 13 or 16.***

Pages 18–19

As you read page 18, write these expressions on the board.
(Be sure to emphasize the spacing.)

3 + 5   x 2
3 +   5 x 2

T (reading page 19)**:** ***I smiled and drew two parentheses. I put them like this.***

Draw parentheses around 3 + 5 in the first expression.

(3 + 5) x 2
3 +   5 x 2

T: ***How does that help?***

S: ***It shows that we calculate 3 + 5 first.***

T: ***3 + 5 = …?*** (8) ***And 8 x 2 = …?*** (16)

Complete the first number sentence (3 + 5) x 2 = 16 on the board.

T (reading)**:** ***"Now we name the number 16," agreed 3 and 2. I moved the parentheses.***

Draw parentheses around 5 x 2 in the second expression.

(3 + 5) x 2 = 16
3 +   (5 x 2)

T: ***How does that help?***

S: ***It shows that we calculate 5 x 2 first.***

T: ***5 x 2 = …?*** (10) ***3 + 10 = …?*** (13)

On the board, complete the second number sentence
3 + (5 x 2) = 13.

(3 + 5) x 2 = 16
3 +   (5 x 2) = 13

Pages 20–21

To the right of the two number sentences on the board,
write this expression.

3 + 5 x 2

T (reading page 20)**:** ***"And what happens if we switch + and x ?" asked 5.***

Switch the symbols on the board.

3 x 5 + 2

T: ***What number could this be?***

S: ***17.***

T: ***Can you put in parentheses so we can see how you are looking at this expression?***

Let the student put in parentheses, check the solution
with the class, and complete the number sentence.

(3 x 5) + 2 = 17

Write the expression again, and let a student who sees
it as 21 put in parentheses.

3 x (5 + 2) = 21

Finish reading pages 20 and 21.

Vary the activity in the storybook by asking what happens if you

- switch 3 and 2 (in the original expression);
- switch 3 and 2, and x and + (in the original expression);
- switch 5 and 2 (in the original expression); or
- switch 5 and 2, and switch x and + (in the original expression).

Your board might look similar to this.

(3 + 5) x 2 = 16	(2 + 5) x 3 = 21	(3 + 2) x 5 = 25
3 + (5 x 2) = 13	2 + (5 x 3) = 17	3 + (2 x 5) = 13
(3 x 5) + 2 = 17	(2 x 5) + 3 = 13	(3 x 2) + 5 = 11
3 x (5 + 2) = 21	2 x (5 + 3) = 16	3 x (2 + 5) = 21

Your students should notice the repetition of answers.

Pages 22–25

Read these pages aloud. Reproduce the picture on pages 24 and 25 on the board. Let students locate ½ and then draw a red arrow from 1 to ½.

Pages 26–27

Read these pages aloud.

T: ***What do you suggest we call this new dance?***

S: ***½x.***

Pages 28–31

Read the final pages aloud or let the students read them silently.

Writing Activity

Suggest students write to a friend about why they sometimes need to use parentheses in a number sentence.