The World of Numbers

WORLD OF NUMBERS TABLE OF CONTENTS

WORLD OF NUMBERS INTRODUCTION

Numbers are among the most important things that mathematics (at all levels) is about. Mathematicians are interested in numbers just as astronomers are interested in stars, botanists in plants, and sociologists in the organization and dynamics of human societies. Surely everyone agrees that one of the primary goals of any elementary school mathematics curriculum should be to introduce students to the world of numbers—to give them the opportunity to become familiar with numbers, their properties, and the relations between them. The ability to calculate with numbers is an important part—but not the only part—of being familiar and comfortable with numbers. The World of Numbers strand of *CSMP Mathematics for the Upper Primary Grades* is designed to provide students with a wide variety of challenging experiences with numbers so that gradually they will become not only familiar, but comfortable with numbers; they will, so to speak, get to know numbers on a "first name basis" and develop number sense.

The Minicomputer and the World of Numbers

CSMP Mathematics for the Upper Primary Grades uses the Papy Minicomputer as a support for the positional system of numeration; for calculations and estimation; for number patterns and mental arithmetic; and for modeling the basic operations involving whole numbers, integers, and decimal numbers. Although the Minicomputer can be seen as a tool for calculation and as a device to help students learn routine methods for calculations, its more exciting use is as a vehicle for posing interesting problems that challenge a child's intellectual curiosity about numbers, and for presenting situations that both encourage strategic thinking and reinforce numerical skills. The lessons that make use of the Minicomputer are often intended to be explorations into the world of numbers.

Standard Algorithms of Arithmetic

CSMP seeks to develop basic numerical skills as well as an understanding of the underlying mathematical ideas. We are fully in agreement with the thesis that, along with the growth of understanding of the world of numbers, there must be a concommitant growth of familiarity and facility with numbers and operations on them. But facility should not be confused with understanding; they are partners in the growth of mathematical maturity. A balanced growth of each must be maintained, neither being sacrificed for the other.

Students must eventually learn mechanical algorithms for the basic operations (addition, subtraction, multiplication, division). However, premature presentation of these algorithms may actually stunt a student's ability to develop alternative algorithms, to do mental arithmetic, or to estimate.

Consider the problem of calculating 294 – 89. A third grader may have difficulty performing a standard (paper and pencil) subtraction algorithm. An easier and more efficient way to proceed is to subtract 90 from 294 and then to add 1 (294 – 90 = 204 and 204 + 1 = 205). To insist on a mechanistic response to such a problem would be to encourage inefficiency and might also inhibit the development of the flexibility necessary for problem solving. On the other hand, a rich array of situations in which students interact with numbers provides them with opportunities to gain the necessary facility with standard algorithmic procedures while retaining the openness required to respond creatively to new situations in the world of numbers.

Numerical Relations

One of the main aims of the World of Numbers strand is to familiarize students with numbers by studying relations between numbers, both explicitly and in a variety of contexts. (For more general comments about relations, see the introduction to the Languages of Strings and Arrows strand.) Arrow diagrams represent relations in a simple, suggestive, and pictorial way—usually more conveniently than the same information could be given in words or other symbols.

Students are brought into contact with an assortment of challenging situations, many of which would be totally inaccessible to them were it not for the arrow diagrams. The problems and activities of this strand include solving linear equations presented in terms of arrows; studying iterated processes and patterns in sequences of numbers; tackling problems that may have many solutions or no solution; estimating or testing that a solution is reasonable; and exploring properties of operations on numbers.

In summary, what is most important in the study of numbers is to confront students with a variety of problems and situations that capture their interests, challenge their abilities to reason, and stimulate their curiosities about numbers—and, at the same time, provide them with tools for coping with these situations and problems.

N1 MINICOMPUTER INTRODUCTION #1

Capsule Lesson Summary

Review the value of the squares on the Minicomputer. Use the Minicomputer to represent a variety of numbers from 1 to 8,000.

Materials

Teacher
- Minicomputer set†

Students
- Paper
- Minicomputer set†
- Base-10 blocks
- Calculator
- Worksheets N1*, **, ***, and ****

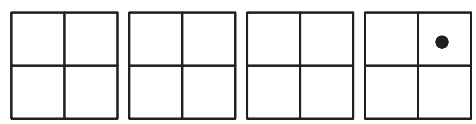
Note: Paper for students can be scratchpads, notebooks, slates, or whatever serves your classroom management and record-keeping purposes.

Description of Lesson

This lesson is for review and should move at a quick pace. Target students who need extra help. You may want to let students work with a partner; team those who are less proficient with the Minicomputer with "experts."

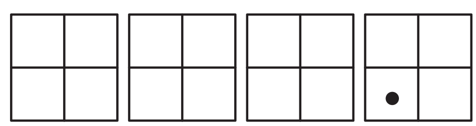
Exercise 1

Display four Minicomputer boards.

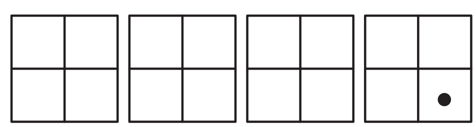


T: ***This is the number 4 on the Minicomputer.***

To prevent confusion, always remove the checkers from the Minicomputer before asking the next question.

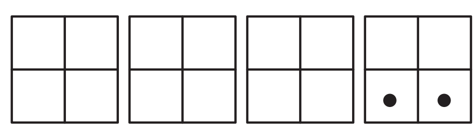


T: ***Who can put 2 on the Minicomputer?***



Who can put 1 on the Minicomputer?

Very quickly review placement of the checker for 1, 2, 4, and 8.



T: ***I'll give you two checkers.***
Can you put 3 on the Minicomputer?

T: ***Can you put 5 on the Minicomputer using exactly two checkers?***
… 6 using exactly two checkers?
… 9 using exactly two checkers?
How many regular checkers do you need to put 7 on the Minicomputer? (At least three)

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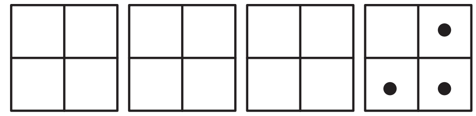
†A teacher's Minicomputer set consists of four demonstration Minicomputer boards and a sufficient number of demonstration Minicomputer checkers. A student's Minicomputer set consists of two sheets of Minicomputer boards (two boards per sheet) and cardboard checkers.

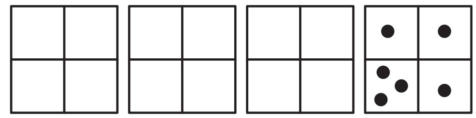
Ask someone to put 7 on the Minicomputer using exactly three checkers.



Exercise 2

Gradually make the following moves on the Minicomputer, pausing after each.

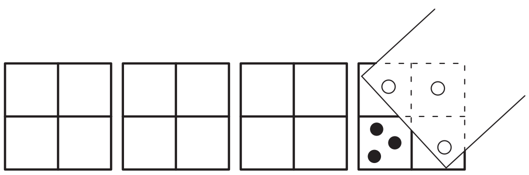
- Put one checker on the 8-square.
- Put three checkers on the 2-square.
- Put one checker on the 4-square.
- Put one checker on the 1-square.

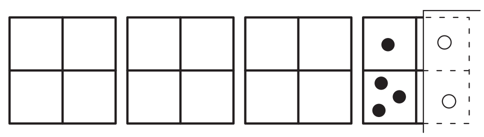


This gives your students a chance to do a mental calculation of the number.

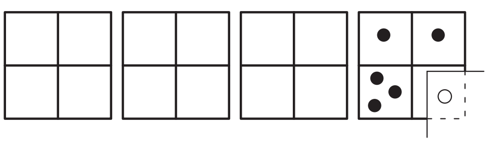
T: ***What number is on the Minicomputer?*** (19)

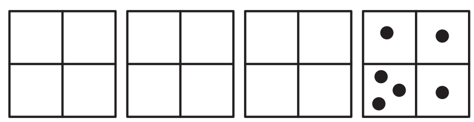
No explanation is necessary if everybody gets the right number. If someone is wrong, you might use this procedure: cover part of the board with a piece of paper to focus the class's attention on certain checkers, and gradually uncover the full set of checkers.





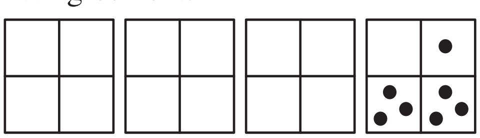
T: ***What number is this?*** (6) ***And 8 more?*** (14)



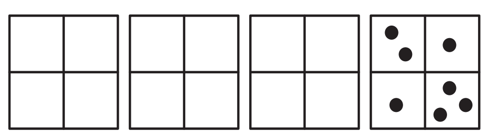


T: ***And 4 more?*** (18) ***And 1 more?*** (19)

Repeat this activity with other configurations; some possibilities are suggested below. Ask the students to write their answers on scratch paper or whisper them to their partners; then quickly get class agreement.



= 13

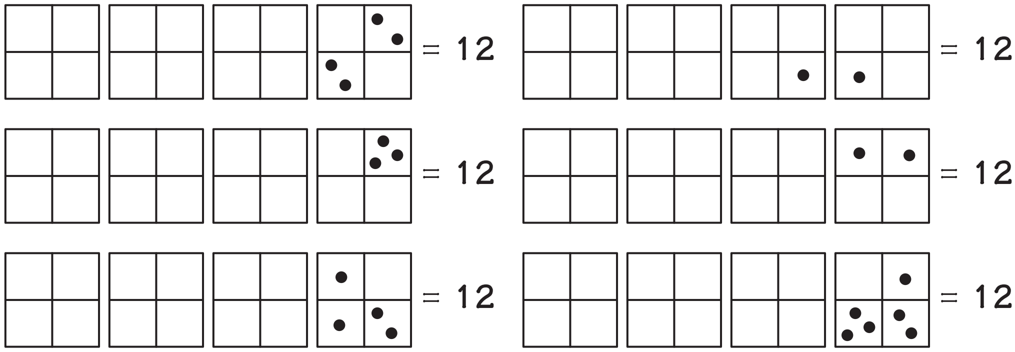


= 25

Exercise 3

T: ***Who can put 12 on the Minicomputer?***

Let students suggest a variety of ways of putting 12 on the Minicomputer. Some of the many possibilities are shown below.



Repeat this exercise for 15.

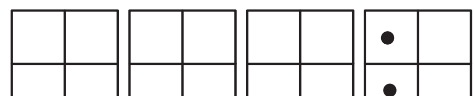
Exercise 4

Quickly review trades on the ones board:

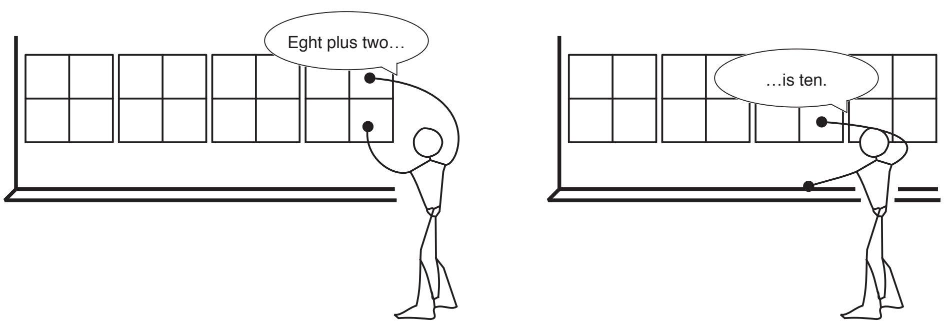
1 + 1 = 2 and 2 = 1 + 1
2 + 2 = 4 and 4 = 2 + 2
4 + 4 = 8 and 8 = 4 + 4

Put one checker on the 8-square and one on the 2-square.

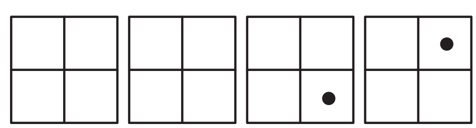
T: ***What number is on the Minicomputer?*** (10)



Pick up the checker on the brown square with one hand and the checker on the red square with the other. Then put one of these checkers on the 10-square and take the other checker away (put it in the chalk tray). As you are making the trade, say, "8 + 2 = 10."

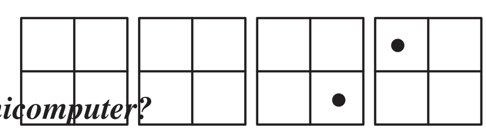


T: ***This is a way to put 10 on the Minicomputer using just one checker. I'll give you one more checker. Can you put 14 on the Minicomputer?***



Remove the checker from the ones board (4-square).

T: ***With one more checker, can you put 18 on the Minicomputer?***



T: ***Who would like to put a secret number on the Minicomputer?***

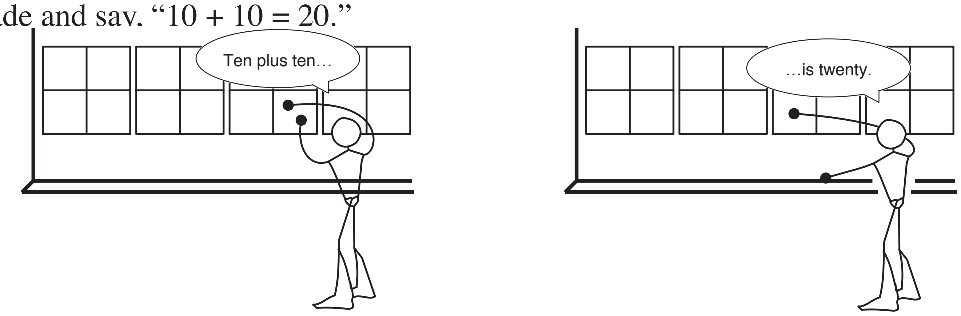
N1

Whisper a whole number less than 20 to a volunteer. When that number has been put on the Minicomputer, call on another student to say which number it is. Repeat this activity several times.

Ask someone to put 20 on the Minicomputer using two checkers. If no one volunteers, put two checkers on the 10-square.

T: ***There is a way to put 20 on the Minicomputer using just one checker.***

Make the trade and say. “10 + 10 = 20.”

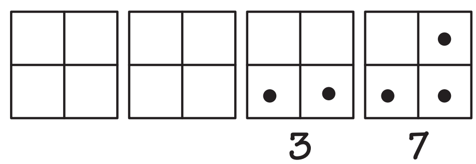


Remove the checkers and continue with other numbers.

T: ***Who can put 23 on the Minicomputer? 25? 30?***

Put this configuration on the Minicomputer.

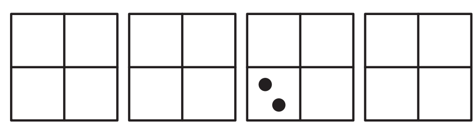
T: ***What number is this?*** (37)



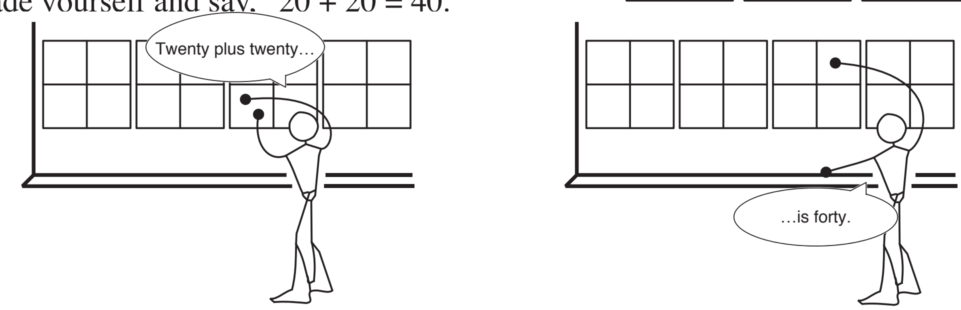
Write 3 below the tens board and 7 below the ones board.

Put two checkers on the 20-square.

T: ***What number is this?*** (40)



Make the trade yourself and say. “20 + 20 = 40.”



Remove the checker from the 40-square.

Move a checker back and forth very quickly from the 1-square to the 10-square. Each time you move the checker, ask the class which number is on the Minicomputer, one—ten, one—ten, and so on.

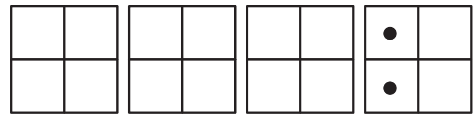
Repeat this activity with 2 and 20; 4 and 40; and 8 and 80.

T: ***Who would like to put a secret number on the Minicomputer?***

Whisper a whole number less than 100 to a volunteer. When that number has been put on the Minicomputer call on another student to say what the number is. Repeat this activity several times.

Exercise 5

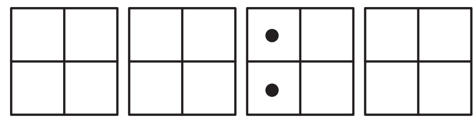
Put this configuration on the Minicomputer.



T: ***What number is this?*** (10)
Who can put 10 on the Minicomputer with only one checker?

Encourage the volunteer to say, "8 + 2 = 10," as the trade is made.

Put this configuration on the Minicomputer.



T: ***What number is this?*** (100)
Who can put 100 on the Minicomputer with only one checker?

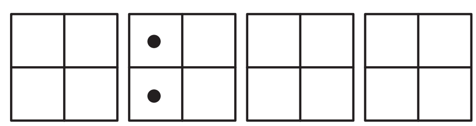
Encourage the volunteer to say, "80 + 20 = 100," as the trade is made.

Move a checker from the 1-square to the 10-square to the 100-square. Each time you move the checker, ask the class what number is on the Minicomputer: 1, 10, and 100. Repeat this activity with 2, 20, and 200; 4, 40, and 400; and 8, 80, and 800.

T: ***Who would like to put a secret number on the Minicomputer?***

Whisper a whole number between 100 and 200 to a volunteer. When that number has been put on the Minicomputer, call on another student to say which number it is. Repeat this activity several times.

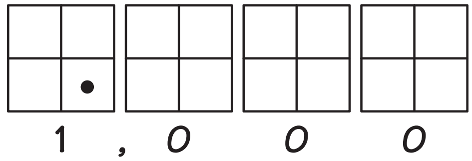
Put this configuration on the Minicomputer.



T: ***What number is this?*** (1000)
Who can put 1,000 on the Minicomputer with only one checker?

Encourage the volunteer to say, "800 + 200 = 1,000," as the trade is made.

T: ***Who can write 1,000 below the Minicomputer?***



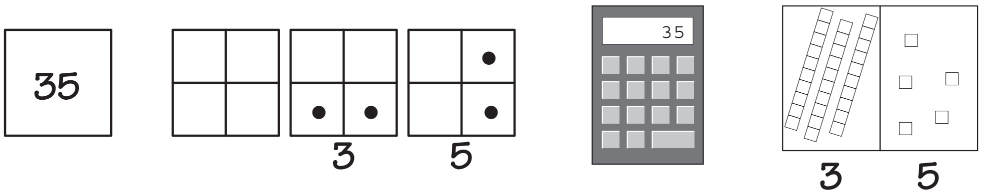
Move a checker from the 1-square to the 10-square to the 100-square to the 1000-square. Each time you move the checker, ask the class what number is on the Minicomputer: 1, 10, 100, and 1000. Repeat this activity with the 2, 20, 200, and 2000; 4, 40, 400 and 4000; 8, 80, 800, and 8000.

Exercise 6

N1

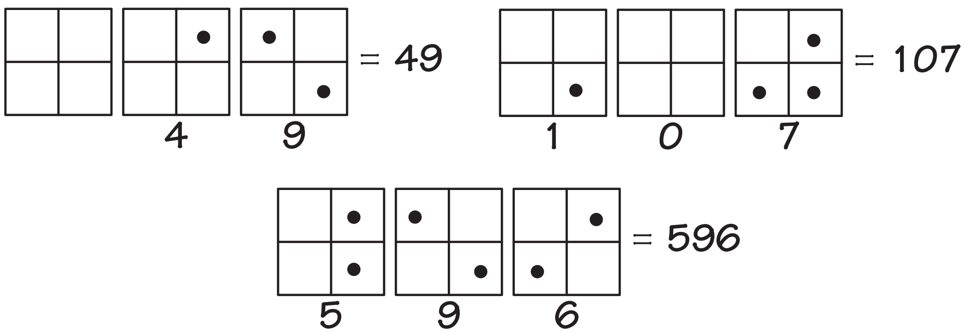
Place students in groups of four with a Minicomputer set, base-10 blocks or other place-value manipulative, a calculator, and paper and pencil. This activity should move quickly. Choose numbers appropriate for your class, and encourage all students to participate.

T: ***Each group is going to show the number 35 in several ways. One person will write 35 on paper, another person will put 35 on the Minicomputer, still another person will put 35 on the calculator, and finally one person will show 35 with the base-10 blocks.***



Check to see that each group has 35 represented in all four ways. Then let some students share and explain. Continue by directing students to switch jobs within their groups.

Repeat this activity with other numbers such as those illustrated (on the Minicomputer) below.

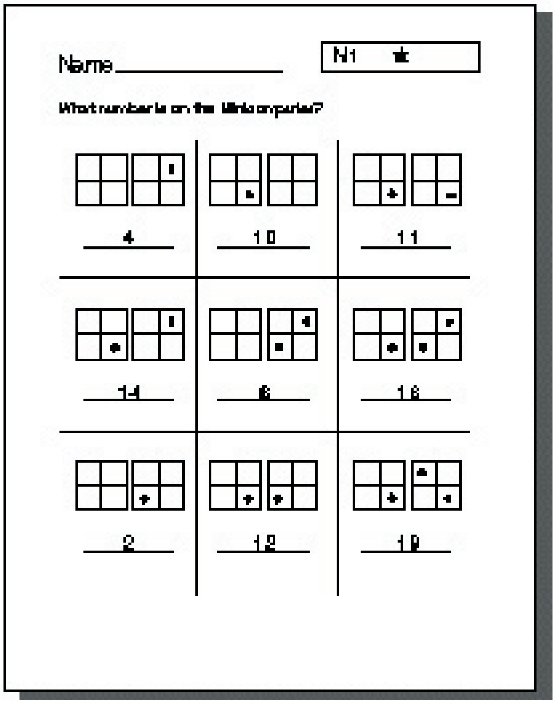


You may like to change the activity so that instead of reading a number to the groups, you put the number on the Minicomputer and the groups read the number. The person with the Minicomputer can copy the configuration you display.

Worksheets N1*, **, ***, and **** are available for individual work.

Center Activity

Place individual Minicomputer sets in a center for further exploration and practice.


Name

N1 ★

What number is on the Minicomputer?

4

10

11

14

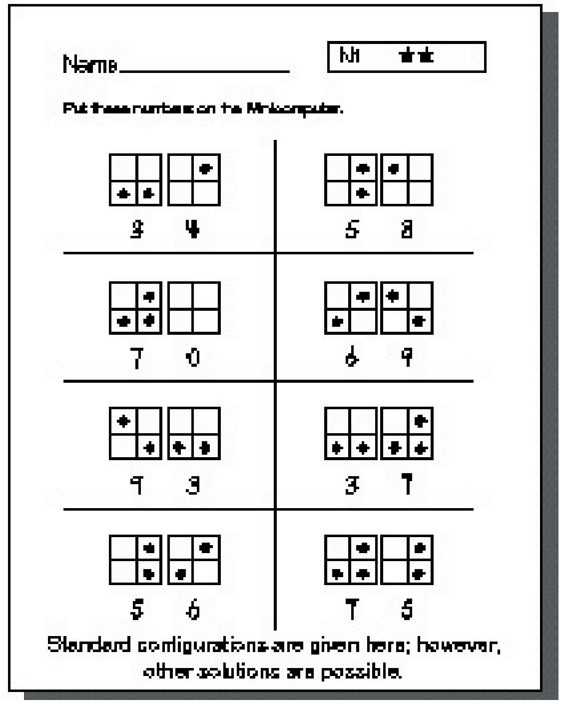
6

16

2

12

19


Name

N1 ★★

Put these numbers on the Minicomputer.

3 4

5 8

7 0

6 9

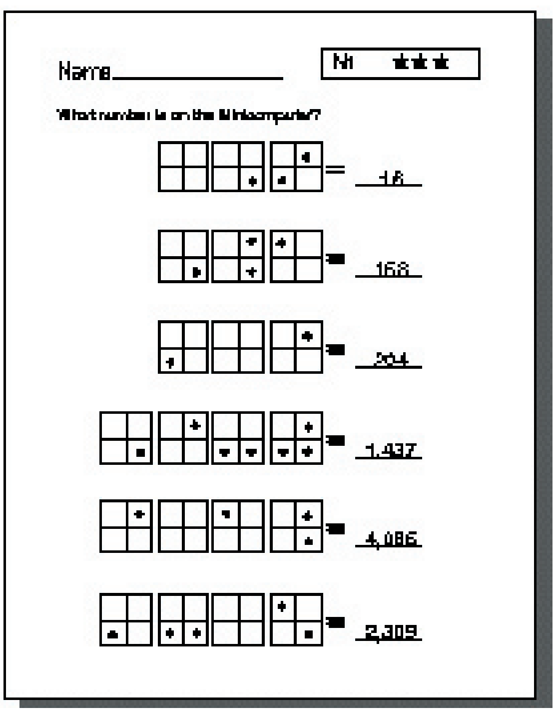
9 3

3 7

5 6

7 5

Standard configurations are given here; however, other solutions are possible.


Name

N1 ★★★

What number is on the Minicomputer?

= 16

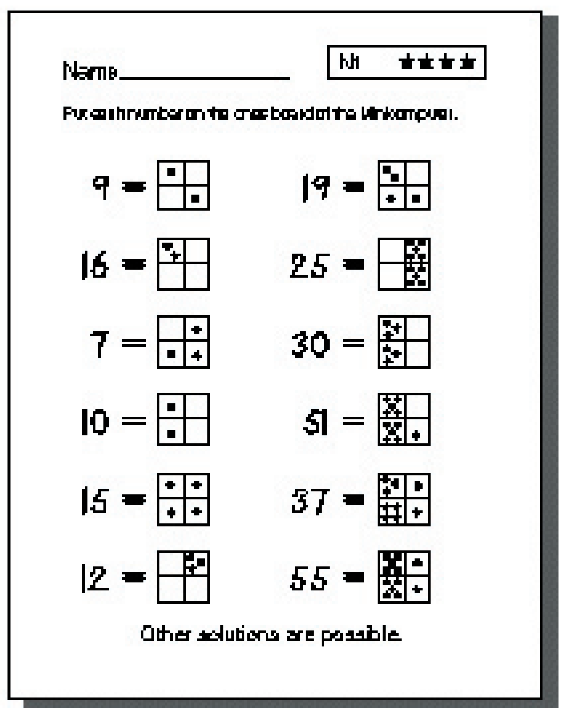
= 168

= 204

= 1,437

= 4,086

= 2,309


Name

N1 ★★★★

Put each number on the checkerboard of the Minicomputer.

9 =

19 =

16 =

25 =

7 =

30 =

10 =

51 =

15 =

37 =

12 =

55 =

Other solutions are possible.

N2 INTRODUCTORY ARROW PROBLEMS #1

Capsule Lesson Summary

Discover that
- five +2 arrows have the same effect as one +10 arrow;
- ten +2 arrows have the same effect as one +20 arrow;
- five +4 arrows have the same effect as one +20 arrow; and
- ten +4 arrows have the same effect as one +40 arrow.

Optional: Given the middle number of a road consisting of eight +5 arrows, decide what its starting and ending numbers are. Then locate and draw +20 arrows in the same picture.

Materials

Teacher
- Colored chalk
- 0–109 numeral chart

Student
- Paper
- Colored pencils, pens, or crayons

Description of Lesson

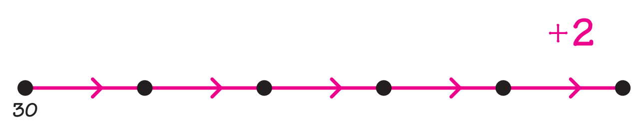
Exercise 1

Conduct a brisk mental arithmetic activity with facts similar to the following:

1 + 3	2 + 5	4 + 3	8 + 6
10 + 30	20 + 50	40 + 30	80 + 60
100 + 300	2,000 + 5,000	4 million + 3 million	800 + 600
2 x 3	2 x 6	3 x 3	4 x 2
2 x 30	2 x 60	3 x 30	4 x 20
2 x 300	2 x 600	3 x 300	4 x 200

Exercise 2

Draw this arrow picture on the board.



T: ***This arrow road has five +2 arrows. Its starting number is 30. What should its ending number be? Write it on a piece of paper (or whisper it to your partner).***

Refer students who are having trouble to the number line or the 0–109 numeral chart.

Look at (or listen to) many of the students' answers.

T: ***Many of you have 40 for the ending number. Can you explain why?***

S: ***30 + 2 + 2 + 2 + 2 + 2 = 40.***

S: ***I counted on my fingers.***

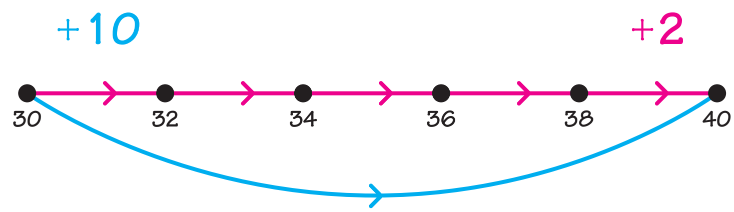
S: ***I started at 30 and counted by twos.***

S: ***5 x 2 = 10 and 30 + 10 = 40.***

Call on volunteers to label all the dots in the arrow picture.

T: ***How many +2 arrows?*** (Five)
What number is 5 x 2? (10)
Five +2 arrows is the same as one +10 arrow.
Where can we draw a +10 arrow in this arrow picture without drawing any more dots?

Ask a volunteer to draw the +10 arrow in blue. Write +10 in blue near the arrow picture.



Put your left forefinger on the dot for 30. Trace the +2 arrow road, and then trace the +10 arrow as you say,

T: ***Five +2 arrows is the same as one +10 arrow.***

Erase the labels for the dots. Point to the dot on the left.

T: ***If this dot were for 20, what would the ending dot be for? How do you know?***

S: 30, because 20 + 10 = 30.

Continue this activity with the following or similar starting numbers. Corresponding ending numbers are show in parentheses. Try to include all the students by alternating difficult and simple calculations. Occasionally, ask students to explain how they calculated an answer. Do not label the dots in the arrow picture.

50	(60)	1,000	(1,010)
51	(61)	43	(53)
10	(20)	59	(69)
12	(22)	0	(10)
100	(110)	95	(105)

You may find that some students would benefit from another visual support for adding 10. With the Minicomputer, put on a number (such as 43) and then add 10 (follow the +10 arrow); you will quickly see 53 on the Minicomputer. With the 0–109 numeral chart, note that you add 10 when you look at the number directly below a given number.

Label the starting dot 32.

T: ***Suppose an arrow road consists of ten +2 arrows. If its starting number were 32, what would its ending number be? Write it on a piece of paper (or whisper it to your partner).***

Look at (or listen to) many of the students' answers. You may need to repeat the question.

T: ***Many of you think the ending number would be close to 50. How can we find the exact answer?***

S: ***Add 2 ten times (count by twos).***

S: ***Add 10 two times, because five +2 arrows is +10.***

Invite students to demonstrate their methods. If necessary, demonstrate yourself. When counting by twos, keep track of how many twos are added by holding up a finger each time you add two.

T: ***The ending number would be 52. Is there another way to find this ending number?***

Perhaps one of your students will suggest that there are ten +2 arrows. 10 x 2 = 20 and 32 + 20 = 52, so the ending number would be 52.

T: ***We are thinking about a +2 arrow road with ten arrows. What number is 10 x 2?*** (20)
If we draw one arrow from the starting number to the ending number, what could that arrow be for? How do you know?

S: ***+20, because 10 x 2 = 20.***

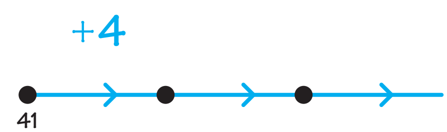
T: ***Ten +2 arrows is the same as one +20 arrow, because 10 x 2 = 20.***
If the starting number were 67, what would the ending number be?
Remember, the road has ten +2 arrows.

S: ***87.***

Erase the arrow picture.

Exercise 3

Draw this arrow picture on the board.



T: ***Suppose an arrow road has five +4 arrows. This arrow road goes until there are five arrows. If the starting number of this road were 41, what would its ending number be? Write your answer on a piece of paper (or whisper it to your partner).***

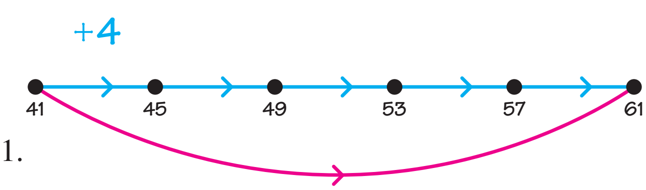
Look at (or listen to) many of the students' answers. Make sure they understand that there are five arrows in the road.

T: ***Some of you think the ending number would be 61. Can you explain why?***

S: ***I counted by fours from 41.***

S: ***5 x 4 = 20 and 41 + 20 = 61.***

Invite students to extend the arrow picture and to label the dots. Draw a red arrow from 41 to 61.

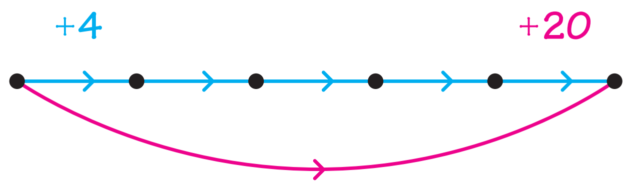


T: ***What could this red arrow be for? How do you know?***

S: ***+20; 41 + 20 = 61.***

S: ***+20, because 5 x 4 = 20.***

Write +20 in red near the arrow picture. Erase the labels for the dots.



Point to the dot on the left.

T: ***If this dot were for 10, what would the ending number be? How do you know?***

S: ***30, because 10 + 20 = 30.***

Continue this activity with the following or similar starting numbers appropriate for the abilities of your class. Corresponding ending numbers are given in parentheses. Try to include all of the students in explaining how they calculated an answer. Do not label the dots in the arrow picture.

15	(35)	100	(120)
60	(80)	69	(89)
27	(47)	1,000	(1,020)
51	(71)	83	(103)
48	(68)	7	(27)
24	(44)	98	(118)

Again, you may find that some students would benefit from the visual support of the Minicomputer or the 0–109 numeral chart.

Point to the dot on the right.

T: ***If this dot were for 25, what would the starting dot be for?***

S: ***5.***

T: ***How do you know?***

S: ***5 + 20 = 25.***

S: ***25 – 20 = 5.***

Label the dots for 25 and 5. Draw a green arrow from 25 to 5.

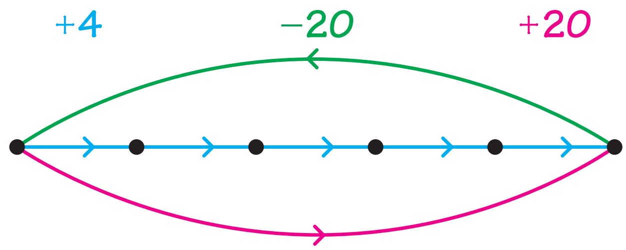
T: ***What could this green arrow be for? How do you know?***

S: ***–20. The red arrow is for +20, so the green arrow could be for –20.***

Trace the +20 arrow and then the –20 arrow.

T: ***This arrow is for +20, so the opposite (return) arrow is for –20.***

Write –20 in green near the arrow picture, and erase the labels for the dots.



Point to the dot on the right.

T: ***If this dot were for 100, what would the starting dot be for?***

S: ***80, because 100 – 20 = 80.***

Continue this activity with the following or similar ending numbers appropriate for the abilities of your class. Corresponding starting numbers are given in parentheses. Occasionally ask students to explain how they calculated an answer. Do not label the dots in the arrow picture.

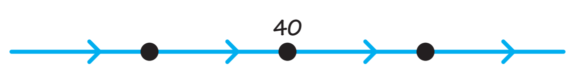
89	(69)	271	(251)
40	(20)	871	(851)
51	(31)	20	(0)
70	(50)	1,020	(1,000)
170	(150)	104	(84)
270	(250)	204	(184)

Vary this activity by asking for the ending number of a road that starts at 57 and consists of ten +4 arrows. (97)

Erase the board before going on to Exercise 4.

Exercise 4 (optional)

Draw this arrow picture on the board.



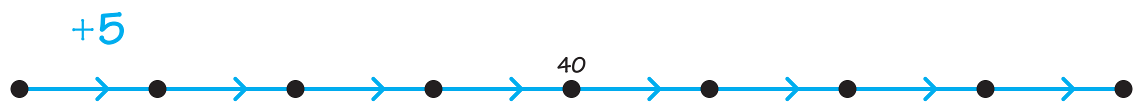
T: ***40 is the middle number of a +5 arrow road that has exactly eight arrows. Before completing the arrow picture, can you predict the starting number and the ending number of this road? Write them on a sheet of paper.***

Instruct students to talk with a partner, draw the arrow picture, and come to agreement on the starting and ending numbers.

Look at many of the students' answers. Point to the dot for 40.

T: ***If 40 is the middle number of a road with exactly eight arrows, how many arrows are there to the right of 40? How many arrows are there to the left of 40?***

Invite students to extend the arrow picture on the board and instruct others to do so on their papers.



T: ***What is the ending number of this arrow road?*** (60)
How do you know?

S: ***I started at 40 and counted by fives.***

S: ***4 x 5 = 20 and 40 + 20 = 60.***

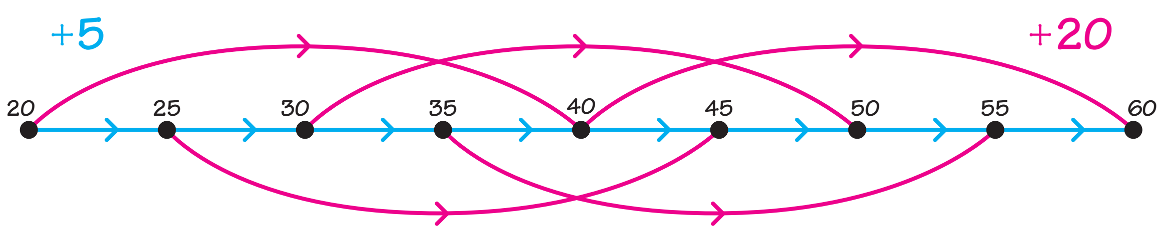
T: ***What is the starting number of this arrow road?*** (20) ***How do you know?***

S: ***4 x 5 = 20 and 40 – 20 = 20.***

S: ***I counted backward by fives from 40.***

Invite students to label the dots in the arrow picture on the board as well as on their papers. Write +20 in blue near the arrow picture.

Instruct students to work with their partners to find and draw as many +20 arrows as they can in their arrow pictures without drawing any more dots. Then call on some students to draw the +20 arrows in the picture on the board. Emphasize that four +5 arrows is the same as one +20 arrow.



N3 MINICOMPUTER INTRODUCTION #2

Capsule Lesson Summary

Review trades on the Minicomputer and the value of the squares. Explore the effect of moving various checkers in a configuration on the Minicomputer. In each case, does the move increase, decrease, or leave the same number on the Minicomputer? Put numbers on the Minicomputer with many checkers, estimate, and make trades to make them easier to read. Find many ways to represent a given number on the Minicomputer.

Materials

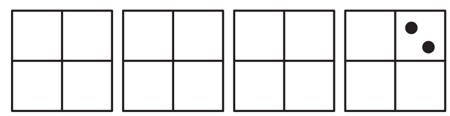
Teacher
- Minicomputer set

Student
- Paper
- Minicomputer set

Description of Lesson

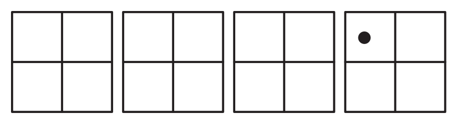
Exercise 1: Minicomputer Review

Put this configuration on the Minicomputer.



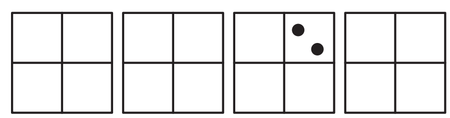
T: ***What number is on the Minicomputer?*** (8)
Could we put on 8 with one checker?

Invite a student make the 4 + 4 = 8 trade.



Demonstrate the 8 = 4 + 4 and 4 + 4 = 8 trades again yourself.

Put this configuration on the Minicomputer.



T: ***What number is on the Minicomputer?*** (80)

Ask a volunteer to make a trade so that 80 will be on the Minicomputer with only one checker. As the trade is made, the student should say, "40 + 40 = 80."

Repeat this activity with the following trades:

400 + 400 = 800	20 + 20 = 40	8 + 2 = 10
4,000 + 4,000 = 8,000	200 + 200 = 400	80 + 20 = 100
	2,000 + 2,000 = 4,000	800 + 200 = 1,000

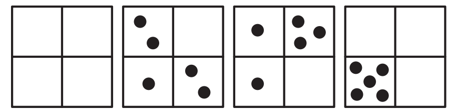
Move a checker from one square to another very quickly to review the value of the squares. Each time you move the checker, ask the class which number is on the Minicomputer.

Review the standard configurations of the following numbers. Move the checkers very quickly from one board to another, each time asking which number is on the Minicomputer.

5; 50; 500; 5,000	9; 90; 900; 9,000
3; 30; 300; 3,000	7; 70; 700; 7,000
6; 60; 600; 6,000	12; 120; 1200; 12,000

Exercise 2: Transforming a Number

Put this configuration on the Minicomputer.



T: ***I put a number on the Minicomputer.***
It's not easy to read, but let's try to estimate.
Is this number more than 100? How do you know?

S: ***Yes. There are several checkers on the hundreds board.***

T: ***Is this number more than 1,000?*** (Yes)

S: ***The two checkers on the 800-square make 800 + 800 = 1600 and that is more than 1,000.***

T: ***We know this number is more than 1,000. Is it more than 2,000?*** (Yes)
How do you know?

S: ***Yes, this pair of checkers (800 + 200) is 1,000 and these three checkers (800 + 100 + 100) equal 1,000, so the number is more than 2,000.***

T: ***We know this number is more than 2,000. Is it more than 3,000?***

It is possible that a student will be able to explain and to convince the class why this number is less than 3,000. In any case, your class should estimate the number to be more than 2,000 and less than 3,000. Some classes may estimate the number between 2,000 and 2,500.

T: ***We don't know exactly what number is on the Minicomputer, but we can still compare it to other numbers.***

Write these words on the board.

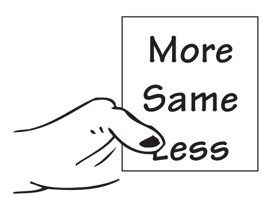
More
Less
Same

T: ***I am going to move, remove, or add some checkers.***
Each time, tell me if the number on the Minicomputer is more than, less than, or the same as this number.

Move a checker from the 800-square to the 400-square. Point to each of the three words in turn as you say,

T: ***If you think this number is more than the number we had, hold up your hand.***
If you think this number is less than the number we had, hold up your hand.
If you think this number is the same, hold up your hand.

Decide on a method for students to show whether they believe the new configuration is for a number more, same, or less than before. For example, they can write three words on an index card and then hold up the card, pinching it on their choice.



Repeat the move very obviously if many students do not know that the number now on the Minicomputer is less than the previous one.

Return the checker to its original position.

Continue this activity with the following moves or similar ones. After each move, return the checkers to their original positions.

When appropriate, ask the students exactly how much more or less a number is than the previous one.

- Move a checker from the 80-square to the 200-square. (120 more)
- Replace any checker with a checker of another color. (Same)
- Make a 2 + 2 = 4 trade. (Same)
- Move a checker from the 20-square to the 1-square. (19 less)
- Make an 80 + 20 = 100 trade. (Same)
- Move two checkers from the 40-square to the 10-square. (60 less)
- Make an 800 = 400 + 400 trade. (Same)
- Move a checker from the 2-square to the 1,000 square. (998 more)

After you make several moves yourself, invite students to transform the number on the Minicomputer.

At the end of this activity, leave the checkers on the Minicomputer for Exercise 3.

Exercise 3: Estimation

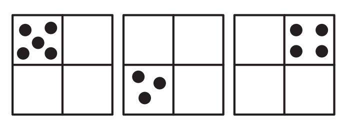
Remind the class that they previously estimated this number to be between 2,000 and 3,000. Invite students to make several trades, and then guide the class to make a closer estimate. You may allow students to guess what the number is and to record some guesses on the board.

Continue making trades until standard configuration is obtained. Invite a student to write the number below (above) the Minicomputer and to determine which guess was the closest. Erase the board and remove the checkers from the Minicomputer.

Display three Minicomputer boards. Give a student twelve checkers as you say,

T: ***Put these checkers on the Minicomputer wherever you want.***

Suppose the student chooses this configuration.



T: ***Is this number more than 100? How do you know?***
Is it more than 1,000? How do you know?
Is it more than 2,000? How do you know?
Is it more than 3,000? How do you know?

Allow the students to guess what number is on the Minicomputer, and record their guesses on the board.

T: ***How can we make the number easier to read?***

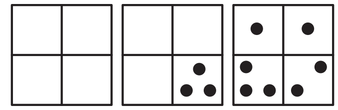
S: ***Make some trades.***

Invite students to make trades and encourage them to announce the trades they make. You may need to initiate backward trades the first few times they are needed. Display a fourth Minicomputer board if the configuration is for a number greater than 999.

When standard configuration for the number is obtained, ask a student to record the number below the Minicomputer. Determine which guess was the closest.

Exercise 4

Put the following configuration of checkers on the Minicomputer gradually, allowing students to calculate the number mentally.



T: ***What number is on the Minicomputer?*** (50)
How many checkers did I use to put 50 on the Minicomputer? (Ten)
Can you put 50 on the Minicomputer using one less checker (that is, nine checkers)?

If the students do not respond, suggest that they make a trade. Any forward trade will solve the problem. For example:

Other solutions involve making a 1 + 1 = 2 trade or a 8 + 2 = 10 trade.

T: ***This is 50 with nine checkers.***
Can you put 50 on the Minicomputer with one less checker (that is, eight checkers)?

Continue by representing the number 50 on the Minicomputer with seven and then six checkers.

Provide student pairs with one Minicomputer sheet and ten checkers. Instruct them to put a number on with the ten checkers on the ones boards, calculate the number, and then proceed to put the same number on with nine, eight, seven, six checkers, and so on (whatever is possible).

Home Activity

This is a good time to send home a letter about the Minicomputer. Blacklines N3(a), (b), and (c) provide samples. Instruct students to color the Minicomputer boards on the full page to show their parents/guardians how the boards look. They should explain the Minicomputer, and practice putting numbers on the Minicomputer and reading them. Objects such as paper clips, pennies, or dried beans can be used as checkers.

Capsule Lesson Summary

Find 2x a number and possibly use the answer to find 3x the same number. Find ways to put 29¢ postage on a letter using only 2¢ stamps and 3¢ stamps. Show the various solutions in arrow pictures.

Materials

Teacher
- Colored chalk
- Unifix® cubes or links of two colors, grouped in twos and threes

Student
- Paper
- Colored pencils, pens, or crayons

Description of Lesson

Exercise 1: Mental Arithmetic

Begin this lesson with a few minutes of mental arithmetic involving the functions 2x and 3x. One example of a possible sequence of calculations is given here.

T: ***What number is 2 x 7?*** (14)
2 x 7 = 14, so what number is 3 x 7? (21)
How do you know?

S: ***I added 7 to 14.***

In the same manner, continue with these calculations or similar ones. Answers are given in parentheses.

2 x 5 (10)	2 x 6 (12)	2 x 9 (18)	2 x 10 (20)	2 x 12 (24)
3 x 5 (15)	3 x 6 (18)	3 x 9 (27)	3 x 10 (30)	3 x 12 (36)

Exercise 2

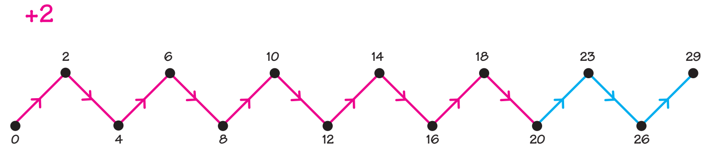
Tell the following or a similar story to your class. You may like to choose one of your students to be the star of the story.

T: ***Lori has a letter that needs 29¢ postage. She couldn't find a 29¢ stamp, but she did find several 2¢ stamps and 3¢ stamps. Can Lori put 29¢ postage on her letter by using 2¢ stamps and 3¢ stamps?***

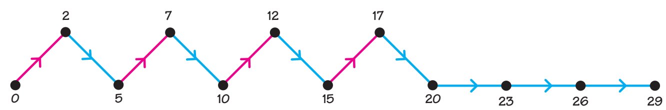
You may need to repeat the story and the question to be sure students understand the problem. Allow students to work independently or with a partner until some of them discover ways for Lori to put 29¢ postage on her letter. Suggest that students picture their solutions with an arrow picture; that is, an arrow road from 0 to 29 using +2 arrows (2¢ stamps) and +3 arrows (3¢ stamps).

Note: To support this activity with a manipulative, give student partners Unifix® cubes or links grouped in twos (red cubes or links) and threes (blue cubes or links). Then they can build towers or chains of the cubes or links until they have 29 cubes. Each addition to the tower must be a group of two (2¢ stamp) or a group of three (3¢ stamp).

After a short while, ask students to share their solutions with the class. A couple possible solutions are shown here in arrow roads. If students have not drawn arrow pictures for their solutions, ask how you could picture their solutions on the board in an arrow road.



Ten 2¢ stamps
Three 3¢ stamps



Four 2¢ stamps
Seven 3¢ stamps

T: ***Is there another way Lori could put 29¢ postage on her letter? Draw arrow roads to show other solutions.***

Encourage students to use the same colors for their +2 and +3 arrows as in the arrow pictures on the board. Ask each student (or student pair) who finishes an arrow picture either to put it on the board or to look for a different solution. Remind students they can count the +2 arrows and the +3 arrows to find how many of each kind of stamp. You may like to do this by asking the class about one of the arrow pictures on the board.

Point to one of the arrow pictures on the board.

T: ***If Lori solves her problem this way, how many 2¢ stamps does she put on her letter?***

The answer will vary according to which arrow road you are discussing. For the purpose of this dialogue we will assume the arrow road has four +2 arrows.

S: ***Four.***

T: ***How can you tell from this picture?***

S: ***There are four +2 arrows in the arrow picture.***

T: ***How many 3¢ stamps does she use?***

S: ***Seven.***

T: ***How can you tell from this arrow picture?***

S: ***There are seven +3 arrows.***

T: ***Would this give Lori exactly 29¢ postage?*** (Yes)
How many stamps would she use altogether? (Eleven)

Record this information in a table.

2¢ stamps	3¢ stamps	total number of stamps
4	7	11

T: ***What number is 4 x 2?*** (8)
What number is 3 x 7? (21)

Write 8 + 21 on the board.

T: ***What number is 8 + 21?***

8 + 21

S: ***29.***

Complete the number sentence on the board.

8 + 21 = 29

T: ***This is one way Lori can put 29¢ postage on her letter.***

Continue this activity until you have discussed all the arrow pictures on the board. If several arrow pictures have the same number of each type of arrow, do not record the information in the table more than once. It is very likely your students will find all five solutions.

2¢ stamps	3¢ stamps	total number of stamps
1	9	10
4	7	11
7	5	12
10	3	13
13	1	14

T: ***Suppose Lori loves to lick stamps and she has a big envelope. What stamps would she use? Why?***

S: ***Thirteen 2¢ stamps and one 3¢ stamp, because that is the most stamps.*** (Fourteen)

T: ***If Lori has a small envelope, what stamps would she use? Why?***

S: ***One 2¢ stamp and nine 3¢ stamps, because that is the least number of stamps.*** (Ten)

T: ***If Lori had only five 2¢ stamps and many 3¢ stamps, what could she use?***

S: ***She could use one 2¢ and nine 3¢ stamps.***

S: ***She could use four 2¢ and seven 3¢ stamps.***

You may also want to ask the class if they notice any patterns in the table of solutions.

If your class did well with this activity, pose the following problem for students to solve independently or with a partner.

T: ***Imagine you have a package that needs 52¢ postage. You have 3¢ stamps, 5¢ stamps, and 7¢ stamps. How many of each type stamp do you need to put on the package?***

Encourage students who finish quickly to find more than one solution, but insist that everyone find at least one solution to share with the class.

Note: This problem has many solutions (seventeen) so you may not expect any student to find all possible solutions.

Home Activity

Pose a stamp problem for students to solve with a family member. For example:

> Imagine a package that needs 75¢ postage. You have 3¢, 5¢, and 7¢ stamps. How many of each type stamp do you need to put on the package? Try to find several different solutions.

Capsule Lesson Summary

With Minicomputer support, write expanded notation of numbers and model these same numbers with other manipulatives. Practice reading numbers on the Minicomputer. Use the Minicomputer to model subtraction calculations. Relate what is done on the Minicomputer to the written record.

Materials

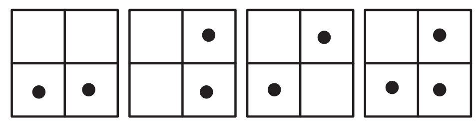
Teacher
- Minicomputer set
- Base-10 blocks (optional)

Student
- Minicomputer set
- Lined paper

Description of Lesson

Exercise 1: Place Value

Put this configuration on the Minicomputer.

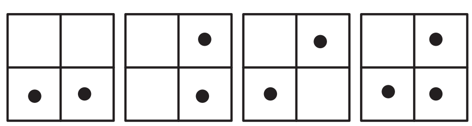


T: ***What number is on the Minicomputer? Write it on your paper.***

Look at many of the students' answers. Then ask a student to write the number below the Minicomputer and to read the number aloud.

Call attention to each board of the Minicomputer and ask for the number on that board. Record the information on the board as an addition calculation with addends corresponding to each Minicomputer board.

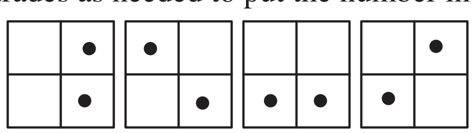
3,000 + 500 + 60 + 7 = 3,567



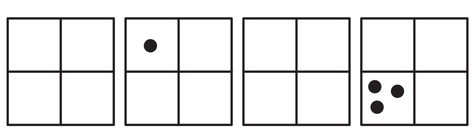
3 5 6 7

```
3,000
  500
   60
    7
-----
3,567
```

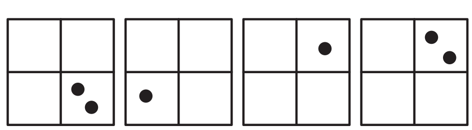
You may also like to model the number with base-10 blocks or some other place value manipulative. Repeat this activity with these configurations. After a number is written below the Minicomputer, invite students to make trades as needed to put the number in standard configuration.



= 5,936



= 806



= 2,248

Exercise 2: Subtraction Problems

Pose a problem such as the following in a story context.

T: ***A Lego® set has 337 pieces. It takes 214 pieces to build a spaceship. How many pieces would not be used in the spaceship?***

What calculation should we do to answer this question?

S: ***337 – 214.***

337 – 214

T: ***What number is 337 – 214?***

Accept several estimates and list them on the board.

T: ***How could we use the Minicomputer to calculate 337 – 214?***

S: ***Put 337 on the Minicomputer and then take off 214.***

Ask a student to put 337 on the Minicomputer. (The following dialogue assumes that the standard configuration for 337 is put on the Minicomputer.)

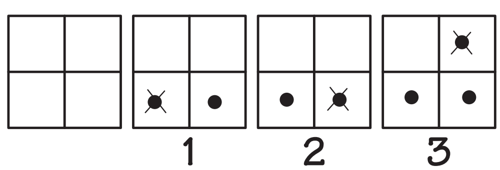
T: ***Who can take 214 off the Minicomputer?***

A student should remove the checkers on the 200-square, 10-square, and 4-square.

T: ***What number is on the Minicomputer now?***

S: ***123.***

Complete the number sentence on the board.



337 – 214 = 123

Let the students decide which estimate was the closest. Erase the board and remove the checkers from the Minicomputer.

Assign students partners and let them solve several other subtraction problems on their desk Minicomputers. For example:

625 – 214
589 – 145
267 – 136

In each case, students may need to make one or two (backward) trades to easily remove the appropriate checkers. This practice activity should only take a short time. Ask the students to put their desk Minicomputers aside before starting Exercise 3.

Exercise 3: Addition Problems

Tell this or a similar story to your students.

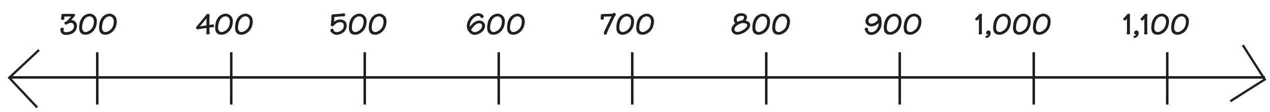
T: ***The students at an elementary school voted on where the school picnic would be held. Each student voted for one place. 278 students voted to have the picnic at the zoo, 436 students voted to have the picnic at the lake, and 129 students voted to have the picnic at an amusement park.***

Write this information on the board.

Zoo	Lake	Amusement Park
278	436	129

T: ***Which picnic site was the most popular with these students?*** (The lake)
Which picnic site was the least popular? (An amusement park)

Draw this number line on the board.



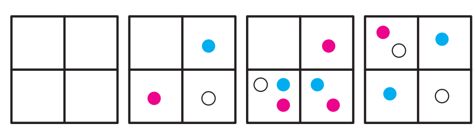
T: ***About how many students voted in the election?***

Ask several volunteers to show approximately where the answer would be on the number line and to explain why they think it would be close to that location. Record several estimates on the board.

T: ***On your paper calculate the number of students who voted in this election.***

While students are working, write the problem on the board. You may also want to ask students to put the numbers on the Minicomputer.

$$\begin{array}{r} 278 \\ 436 \\ +\ 129 \\ \hline \end{array}$$



Look at many of the students' papers, and then ask someone with the correct answer to solve the problem at the board and to explain each step. Make trades on the Minicomputer as you go over the algorithm again. A sample dialogue follows:

T: ***Point to the ones board and the ones column. First we add the numbers in the ones column. What number is 8 + 6?*** (14) ***… and 9 more?*** (23) ***How shall we record 23?***

S: ***Carry 2 tens and write 3 below the ones.***

T: ***Why do we write 2 in the tens column and 3 below the ones column?***

S: ***Because 23 is two tens and three ones.***

T: ***Next we add the numbers in the tens column.***
What calculation do we need to do?

$$\begin{array}{r} {}^{2} \\ 278 \\ 436 \\ +\ 129 \\ \hline 3 \end{array}$$

N5

S: *2 + 7 + 3 + 2.*

T: *2 tens + 7 tens + 3 tens + 2 tens.*
What number is 2 tens + 7 tens? (9 tens or 90) … *+ 3 tens?* (12 tens or 120) … *+ 2 tens?* (14 tens or 140)
How shall we record 14 tens or 140? Why?

S: *Write 1 at the top of the hundreds column and 4 below the tens column. 140 is one hundred and four tens.*

$$\begin{array}{r} {\scriptstyle 1\,2\ } \\ 278 \\ 436 \\ +129 \\ \hline 843 \end{array}$$

T: *Next we add the numbers in the hundreds' column. What calculation do we need to do?*

S: *1 + 2 + 4 + 1.*

S: *100 + 200 + 400 +100.*

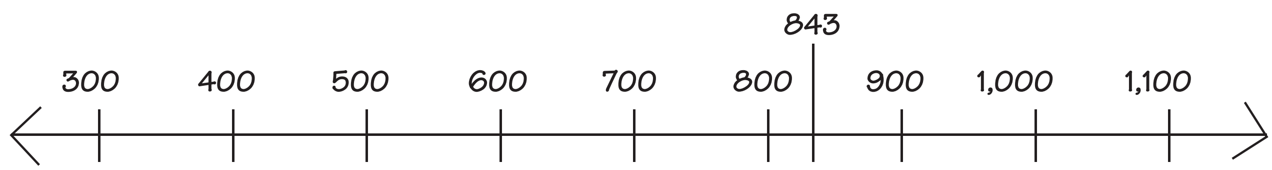
T: *What number is 100 + 200?* (300) … *+ 400?* (700) … *+100?* (800)
How shall we record 800?

S: *Write 8 below the hundreds column.*

$$\begin{array}{r} {\scriptstyle 1\,2\ } \\ 278 \\ 436 \\ +129 \\ \hline 43 \end{array}$$

T: *How many students voted in the election?* (843)
Approximately where is 843 on the number line?

Ask a volunteer to draw and label a mark for 843.



Write these problems on the board and ask the students to solve them individually. Show students how to carefully copy problems—neatly spaced, columns lined up, and legible numerals. Answers are in the boxes.

$$\begin{array}{r} 235 \\ +\ 423 \\ \hline \boxed{658} \end{array} \qquad \begin{array}{r} 378 \\ +\ 414 \\ \hline \boxed{792} \end{array} \qquad \begin{array}{r} 546 \\ +\ 66 \\ \hline \boxed{612} \end{array} \qquad \begin{array}{r} 2{,}349 \\ +\ 3{,}723 \\ \hline \boxed{6{,}072} \end{array}$$

Help students who are having difficulty. If some students finish quickly, write a second set of problems on the board. Again, answers are in the boxes.

$$\begin{array}{r} 528 \\ 140 \\ +\ 264 \\ \hline \boxed{932} \end{array} \qquad \begin{array}{r} 2{,}307 \\ 645 \\ +\ 4{,}089 \\ \hline \boxed{7{,}041} \end{array} \qquad \begin{array}{r} 5{,}328 \\ 1{,}659 \\ +\ 3{,}184 \\ \hline \boxed{10{,}171} \end{array}$$

Writing Activity

Ask students to write a story for a subtraction and/or an addition problem.

N6 THE FUNCTION +10 AND COMPOSITION

Capsule Lesson Summary

Using the calculator, start with a number other than 0 and count by tens. Discover that three +10 arrows has the same effect as one +30 arrow. Extend the picture until it consists of six +10 arrows, and locate all possible +30 arrows.

Materials

Teacher
- Colored chalk
- Calculator
- 0–109 numeral chart

Student
- Unlined paper
- Colored pencils, pens, or crayons
- Calculator
- Worksheets N6* and **

Description of Materials

Exercise 1

Provide one calculator for each student or pair of students. All students should be involved in the predicting and verifying tasks of this exercise.

T: ***Listen carefully to my directions. Turn on your calculators. What number is on the display?***

S: ***0.***

Be sure students know how to clear the display if they have something other than 0.

T: ***Start with 0 on the display. Press [7]. What number is on the display now?***

S: ***7.***

Record 7 near the top of the board.

T: ***Now press [+] [1] [0] [=]. What number is on the display?***

S: ***17.***

Record 17 below 7 on the board.

T: ***Press [=] again. What number is on the display?***

S: ***27.***

T: ***Press [=] again. What number is on the display?***

S: ***37.***

Record the results in a list on the board.

7
17
27
37

T: ***What does the calculator do each time you press [=]?***

S: ***Adds 10.***

T: ***How do you know?***

S: ***7 + 10 = 17 and 17 + 10 = 27.***

S: ***It is counting by tens starting at 7.***

T: ***Predict what number we will get if we press ⊟ again.*** (47)
Press ⊟ and check.

Record the result in your list on the board.

T: ***Predict what number we will get next?*** (57)
Press ⊟ and check.

Record this result in the list on the board.

T: ***Does anyone see a pattern?***

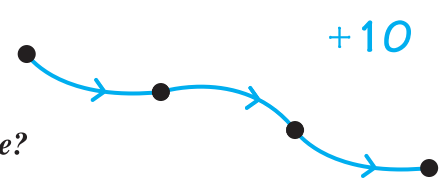
Encourage students to comment on the list of numbers on the board. You may like to locate these numbers in a 0–109 numeral chart. Continue this activity until the list includes 127. Ask again for a pattern.

Repeat this activity starting with a number that does not end in 7, such as 23. Occasionally ask the class to predict the next two or three numbers that will appear on the display.

Erase the board and put the calculators aside.

Exercise 2

Draw this arrow picture on the board.



T: ***Where is the greatest number in this arrow picture?***

The students should point to the dot on the far right. Point to that dot yourself as you ask,

T: ***How do you know that this number is the greatest one in this arrow picture?***

S: ***The blue arrows are for +10. When you add 10 to a number, you get a greater number.***

T: ***Where is the least number in this arrow picture?***

The students should point to the dot on the far left.

Label the left-most (starting) dot 5.

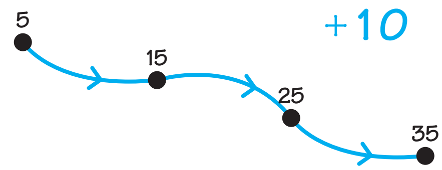
T: ***If this number is 5, what is the ending number? Write it on a piece of paper (or whisper it to your partner).***

Look at (or listen to) many answers.

T: ***Let's label the dots and find out what the ending number is.***

Call on volunteers to label the dots in the arrow picture.

Invite a student who correctly predicted that the ending number is 35 to explain how to calculate the ending number. Erase the dot labels and then label the starting dot 18.



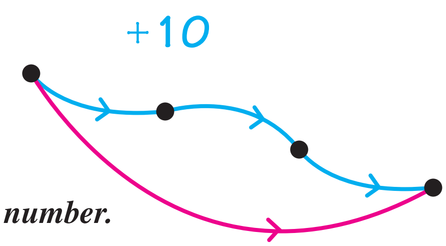
T: ***If this number is 18, what is the ending number? Write it on a piece of paper (or whisper it to your partner).***

Observe (or listen to) many answers. If there are several students who do not know the correct answer (48), ask someone to label the dots in the arrow picture on the board. Invite a student who predicted that the ending number is 48 to explain how to calculate the ending number. Erase the dot labels before continuing.

At a faster pace, give a starting number and ask the class for the corresponding ending number. Choose starting numbers appropriate for the abilities of your students, and try to alternate easy and difficult calculations in order to involve most of your students. Frequently ask students how they calculated an answer. Perhaps some students will observe that they can add 30 to the starting number. Suggestions for starting numbers are given below; corresponding ending numbers are given in parentheses.

0	(30)	51	(81)	1,000	(1,030)
9	(39)	100	(130)	29	(59)
38	(68)	105	(135)	92	(122)
80	(110)	74	(104)	1,000,000	(1,000,030)

Draw an arrow in red from the starting dot to the ending dot.



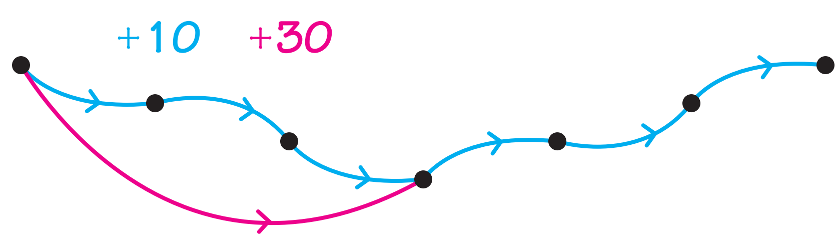
T: ***What could this red arrow be for?***

S: ***+30, because 3 x 10 = 30.***

S: ***+30, because there are three +10 arrows.***

S: ***The ending number is always 30 more than the starting number.***

Write +30 in red near the arrow picture and extend the picture to include three more blue (+10) arrows.

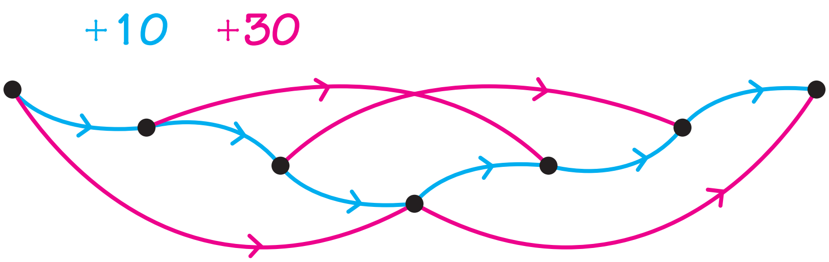


T: ***Where can we draw other +30 arrows?***

Invite volunteers to trace +30 arrows and if they are correct, draw the arrows. If many students have difficulty locating +30 arrows, label the starting dot 61 and call on students to label the other dots. Then ask again where other +30 arrows can be drawn.

Continue until all the possible +30 arrows are drawn.

N6



Distribute colored pencils and unlined paper. Ask the students to build their own +10 arrow road starting with any whole number between 0 and 10. Encourage all students to continue their +10 arrow road until the ending number is more than 200. You may want to suggest to some students that they follow their arrow road with the calculator counting by tens.

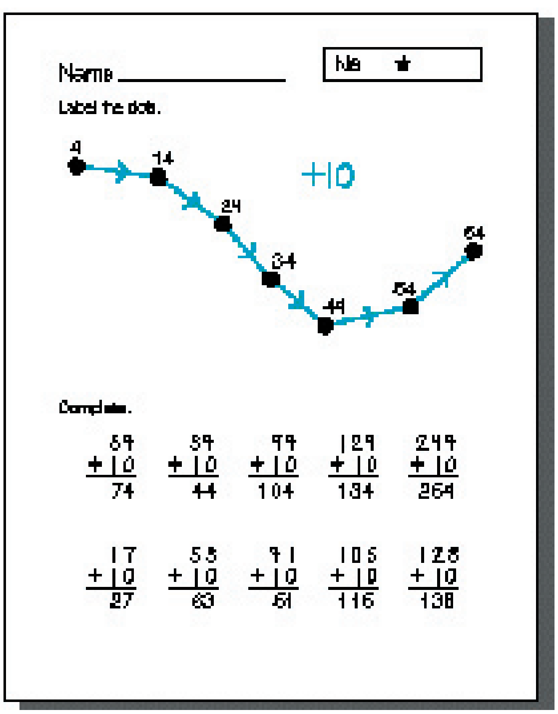
Worksheets N6* and ** are available for additional practice.

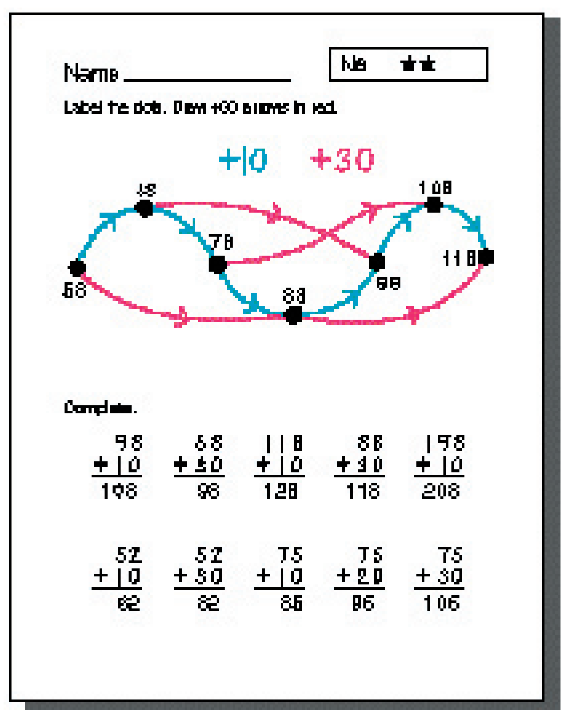
Home Activity

Suggest to parents/guardians that they practice counting by tens with their child, sometimes starting at numbers other than 0. One way to do this is to teach a calculator to count by tens:

1) Put on the starting number.
2) Press [+] [1] [0].
3) Then press [=] [=] [=] and so on.

When counting by tens, observe and listen for patterns.





N7 MULTIPLICATION PROBLEMS #1

Capsule Lesson Summary

Present a context for the concept of multiplication. Find a variety of real-world things that come in groups of twos, threes, fours, and so on up to twelves. Use these examples to generate multiplication problems.

Materials

Teacher	• 4 tennis ball cans • Chart paper	**Student**	• Chart paper or prepared chart

Advance Preparation: This lesson calls for four empty tennis ball cans. You may choose other things that come in packages of threes (or twos, four, fives …), if that is more convenient. Adjust the lesson description accordingly.

Before Exercise 2, you may want to prepare a class chart. Also, read Exercise 2 in advance to decide whether or not you wish to use Blackline N7 to make a prepared chart for students.

Description of Lesson

Exercise 1

Display four empty tennis ball cans and discuss these containers with the class.

T: ***Do you recognize these containers?***

S: ***They are cans for tennis balls.***

T: ***How many balls come in one can?***

S: ***Three.***

T: ***I have four cans. How many tennis balls would I need to fill all four cans? Explain your answer.***

S: ***Twelve. I counted by threes: 3, 6, 9, 12.***

S: ***I added 3 + 3 + 3 + 3 = 12.***

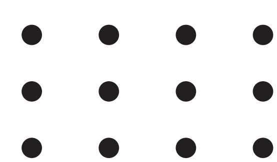
S: ***One can has 3, two cans has 6, so four cans has 12.***

S: ***12; 4 x 3 = 12.***

Try to include the multiplication fact in an explanation, and write it on the board.

$4 \times 3 = 12$ *or* $3 \times 4 = 12^{\dagger}$

You may also like to picture the balls in an array of dots for a multiplication fact.



T: ***Multiplication is a way to find out how many there are altogether when things come in groups of one size.***

[†]Usually 4 x 3 is used for four groups of threes, as in this case four cans of three balls. However, your students may use 3 x 4 and 4 x 3 interchangeably because, of course, 3 x 4 = 4 x 3.

T: ***How many tennis balls would fill ten cans? What multiplication fact would answer this question?***

S: ***30; 10 x 3 = 30.***

T: ***What are some other things that come in threes?***

Let students make suggestions of objects that are often grouped in threes. For each example, pose a counting (multiplication) problem for those objects.

S: ***Tricycles have three wheels.***

T: ***Suppose there are five tricycles on the playground.***

S: ***15; 5 x 3 = 15.***

You may like to begin a list of things that come in threes.

Exercise 2

Observe that although you found several things that come in groups of threes, there are many other sizes for groups. Ask for the group size of some other things, or take suggestions from students for things that come in other size groups.

T: ***Could I put shoes in our list of things that come in threes?***

S: ***No, shoes come in twos.***

T: ***What kind of list would I put days of the week on?***

S: ***Sevens.***

Organize the class into cooperative groups and explain that each group is going to brainstorm about things that come in different size groups from twos to twelves, and make lists of such things. Provide each group with a large sheet of paper on which to make their list, or use Blackline N7 (front and back) to make a chart. The blackline may save some organization time, but you may prefer the size of a large sheet of paper and the responsibility required for the groups to organize their own lists.

Note: It will be harder to find items that come in some group sizes than others. Do not insist that students find entries for the entire list, but do insist that they try to find real objects for many of the group sizes.

While the groups are working, prepare a class chart on the board or on chart paper. When most groups have entries in many of the columns, ask the groups to contribute to the class chart. Solicit two or three ideas from each group. Ask a group to report an entry from their chart but not to tell which list it came from. The others in the class should then decide where it belongs.

You need not get everything in the class chart, but try to get two or three suggestions from each group.

2's Twos	3's Threes	4's Fours	5's Fives	6's Sixes	7's Sevens	8's Eights	9's Nines	10's Tens	11's Elevens	12's Twelves
legs on people	tricycle wheels	car wheels	fingers on a hand	insect legs	days of week	octopus legs	players on a baseball team	fingers on a person		dozen eggs
eyes	clover leaves	legs on horses	tally marks	soda 6-packs	red stripes on flag	8-packs soda		¢ in a dime		inches on a ruler
shoes	triangle sides	square sides	¢ in a nickel	sides on stop sign		ounces in a cup		years in a decade		hours on clock
twins	tennis balls		gum in a pack	faces of a cube				numbers in row of 0–109 chart		
	triplets									

Exercise 3

Invite students to use the class chart or their groups' chart to generate multiplication problems in story contexts. You may begin with an example, and then let two or three students suggest problems. For example:

> There are seven days in a week.
> How many days are there in four weeks?

Ask students to write some of their problems on a paper.

Capsule Lesson Summary

When playing checkers with his grandfather, Pedro wins 3¢ or loses 2¢ depending upon the outcome of a game. Decide what Pedro's money situation could be after playing four games if he starts with no money. Describe the possibilities in arrow pictures. Pose related problems.

Materials

Teacher		Student	
	• Colored chalk • Pennies		• Paper • Colored pencils, pens, or crayons

Description of Lesson

You may like to let students work in small groups or with a partner for this lesson.

Tell the following or a similar story to the class.

T: ***Pedro likes to play checkers with his grandfather. Each time Pedro wins, his grandfather gives him 3¢. Each time his grandfather wins, Pedro gives his grandfather 2¢.***

Record this information on the board.

Pedro wins: Pedro gets 3¢
Grandfather wins: Pedro pays 2¢

Choose two students to be Pedro and Grandfather and to act out a couple scenarios such as the following:

- Pedro wins two games, Grandfather wins one
- Pedro wins one game, Grandfather wins two.

Give each student about ten pennies to start and after acting out the scenario, check Pedro's money. Is he ahead (gain) or behind (loss)?

Exercise 1

Today Pedro and his grandfather are going to play checkers.

T: ***Pedro starts with no money. Suppose they play four games What could happen? Draw an arrow picture to show how much money Pedro might gain or lose at the end of four games. What kind of arrows will you use?***

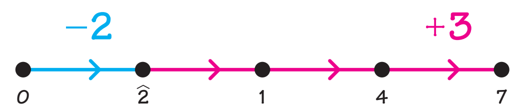
S: ***+3 arrows and –2 arrows.***

Write a key for the arrows on the board and ask the students to copy it on their papers.

Remind students that they will need to draw four arrows (one for each game); +3 arrows will show what happens when Pedro wins a game, and –2 arrows will show what happens when Pedro loses a game (Grandfather wins).

Encourage students to work with their groups (or partners) to find several possibilities of what could happen to Pedro. As you observe students' work, find students with roads having different ending numbers, and ask these students to copy their pictures onto the board.

After a short while, discuss several of the arrow pictures on the board. For example, consider this road and ask questions such as the following:



- How many games did Pedro win? (Three)
- How many games did Pedro lose? (One)
- How much money would Pedro have after the four games? (7¢)
- What does $\hat{2}$ tell us here? (Pedro owes his grandfather 2¢ after the first game.)

After looking at several pictures, suggest organizing the information in a table. (See the next illustration.)

T: ***What is the best Pedro could do after four games of checkers?***

S: ***Pedro could win all four games and gain 12¢.***

T: ***What is the worst Pedro could do after four games of checkers?***

S: ***Pedro could lose all four games and end up owing his grandfather 8¢.***

Record this information in the table, leaving several lines of space between the entries.

Pedro Wins	Grandfather Wins	Pedro's Gain
0	4	$\hat{8}$¢
4	0	12¢

Complete as much of this table as possible by referring to the arrow pictures the students drew on the board. Record the information so that patterns become obvious. If your students have not found one of the solutions, the order in which the information is recorded should be sufficient to prompt their discovery of it.

Pedro Wins	Grandfather Wins	Pedro's Gain
0	4	$\hat{8}$¢
1	3	$\hat{3}$¢
2	2	2¢
3	1	7¢
4	0	12¢

Note: A student might suggest the possibility of a tie game. You should point out to the class that in checkers it is not possible to have a tie; someone always wins if the game is completed.

T: ***Do you see any patterns?***

It is likely that students will comment on patterns in the first two columns. You may need to ask specifically for a pattern in the last column.

T: ***What happens to Pedro's gain each time he wins one more game? How much more is $\widehat{3}$ than $\widehat{8}$?… 2 than $\widehat{3}$?… 7 than 2?…12 than 7?***

Refer to the number line if the students have difficulty answering these questions.

You may wish to ask why Pedro will have 5¢ more if he wins one game more. Commend any student who suggests that it is 5¢ more because 3 + 2 = 5.

Note: When Pedro wins a game rather than losing it, he gets 3¢ from his grandfather, but he also keeps 2¢ he would otherwise have to pay his grandfather if he lost. Do not insist on this explanation.

Erase the arrow pictures and the table, but leave the arrow keys on the board.

Exercise 2

Give the following problem to the groups or partners to solve cooperatively.

T: ***One afternoon Pedro and his grandfather played several games of checkers. When they finished, Pedro had gained exactly 8¢. Is this possible? Draw an arrow picture to show how Pedro could gain 8¢?***

Write this information on the board.

Pedro's gain: 8¢

You may need to restate this problem several times before everyone understands the task.

T: ***Draw an arrow picture to find out how many games Pedro might have won and how many games Pedro might have lost to end up with a gain of 8¢. Remember, Pedro starts with no money.***

Allow five to ten minutes for groups or partners to work on the problem. When a group finds a solution, ask them to put it on the board.

If after several minutes, no one decided to build an arrow road from 0 to 8 using +3 and –2 arrows, suggest this method.

T: ***Before Grandfather and Pedro started playing checkers, Pedro had no money.***

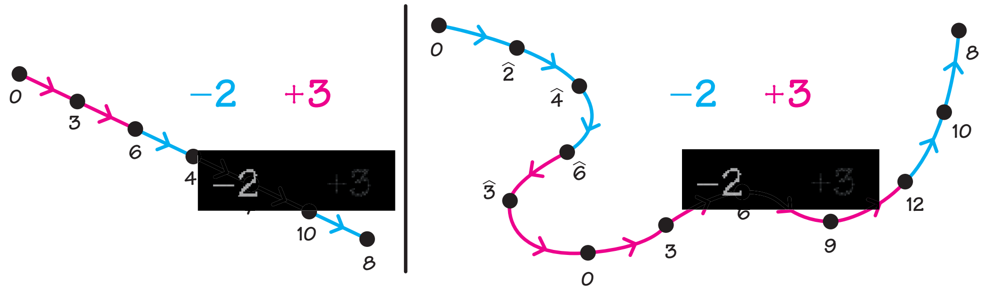
Draw a dot on the board and label it 0.

T: ***When they stopped playing checkers. Pedro had gained exactly 8¢. He started with 0¢ and gradually won 8¢.***

Draw another dot and label it 8.

T: ***Each time Pedro wins, we draw one red (+3) arrow*** (point to +3)***.***
Each time Pedro loses, we draw one blue (–2) arrow (point to –2)***.***
Using just these two kinds of arrows, build a road from 0 to 8 to show how Pedro could gain exactly 8¢.

Observe the students' work. Find groups with different roads and ask them to put their arrow pictures on the board. Two of the many solutions are given here.



Note: To show the sequence of events, you may allow more than one dot to have the same label.

Discuss a few of the arrow pictures on the board, in each case asking how many wins and losses for Pedro and for the total number of games played.

T: ***Have we found all the possibilities?***

Encourage discussion. There are infinitely many solutions to this problem, but it is not necessary that your students realize this.

Exercise 3

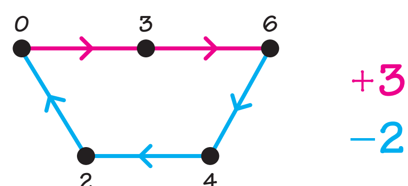
T: ***One day Pedro and his grandfather were playing checkers again. After several games, they had ~~both won as much~~ money as they had lost; so Pedro was back to 0¢. How could this have happened?***

Allow several minutes for students' to explore this new problem; then continue with a collective discussion.

T: ***How could Pedro and his grandfather break even?***

S: ***It could happen after five games if Pedro wins twice and his grandfather wins three times.***

Ask students who find solutions to describe them in arrow pictures. Copy one arrow picture on the board; for example:



Note: There are many other arrow roads from 0 back to 0 with +3 and −2 arrows. Perhaps one of your students will build a road with ten or even fifteen arrows before returning to 0.

T: ***If Pedro and his grandfather are even after five games and they continue playing, could they be even again?***

S: ***After five more games they could be even again.***

Perhaps a student will observe that they could break even after every fifth game and this could continue forever; i.e., they could be even after five games, ten games, fifteen games, and so on.

Writing Activity

Give students another problem along the lines of Exercise 2 to solve and write about. For example:

At the end of the game, Pedro owes his grandfather 3¢. Explain how this could happen.

Home Activity

Suggest that students describe the Pedro-Grandfather arrangement to their parents/guardians and then solve a problem such as the following:

Pedro and Grandfather play exactly ten games. Pedro gains 10¢.
Explain (show) how this could happen.

N9 ELI THE ELEPHANT

Capsule Lesson Summary

When Eli the Elephant awakes, five of his thirteen peanuts are missing. Clarence the Crafty Crocodile has played a trick on him! Decide that Clarence could have added five magic peanuts to Eli's bag, or he could have taken five regular peanuts out. Conclude that $13 - 5 = 13 + \widehat{5}$. Using the Minicomputer, extend this experience to show that $154 - 89 = 154 + \widehat{89}$. Practice adding a negative number to a positive number on individual Minicomputers.

Materials

Teacher
- Minicomputer set

Student
- Paper
- Minicomputer set
- Worksheets N9*, **, ***, and ****

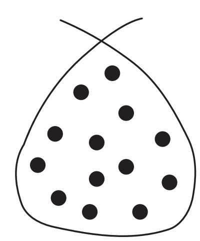
Description of Lesson

For the benefit of new students, ask the class what they remember about Eli the Elephant.[†] Be sure a student explains what happens when a regular peanut meets a magic peanut (both peanuts disappear). Ask the class if they remember Clarence the Crafty Crocodile. Students should tell you that Clarence likes to trick Eli and sometimes he changes the number of peanuts in Eli's bag while Eli is asleep.

Exercise 1

T: ***One day Eli became very tired after a long walk in the jungle and he decided to take a nap. Before he went to sleep, Eli counted his peanuts very carefully and found that there were thirteen regular peanuts in his bag.***

Draw this bag of peanuts on the board.



T: ***While Eli slept, Clarence the Crafty Crocodile came by. Clarence saw that Eli was asleep so he opened Eli's bag of peanuts and did something. Then Clarence closed Eli's bag and quietly went away. When Eli awoke, he opened his bag and counted the peanuts. Eli discovered that now there were only eight peanuts in his bag. What do you think Clarence did?***

Allow the students to discuss this problem. Two possibilities should emerge: Clarence could have removed five peanuts from Eli's bag, or Clarence could have put five magic peanuts into the bag.

Note: There are other more complex possibilities which might be suggested. For example, Clarence could have removed four regular peanuts and added one magic peanut. Accept such correct suggestions, but for the purpose of this lesson focus on these two possibilities: Clarence removed five regular peanuts, or Clarence added five magic peanuts.

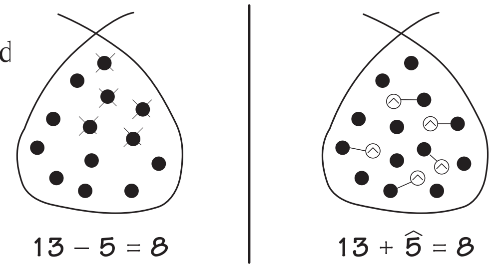
[†]Eli is a very hungry elephant whose favorite food is peanuts. Eli always keeps a bag for peanuts with him, and whenever he goes walking he gathers peanuts. Sometimes Eli finds magic peanuts. They are as good as regular peanuts but whenever a magic peanut meets a regular peanut, they both disappear.

N9

Illustrate each of these situations when it is suggested. Write the appropriate number sentence under each bag of peanuts.

Arrange the board so that the two pictures are side by sid

Emphasize that you do not know what Clarence did when he opened Eli's bag because removing five regular peanuts or adding five magic peanuts have exactly the same effect. Write this as a number sentence on the board.



$$13 + \widehat{5} = 13 - 5 = 8$$

T: ***Another day Eli was walking in the jungle and he gathered 154 regular peanuts. Eli became very tired and decided to take a nap. What do you think happened while Eli was sleeping?***

S: ***Clarence the Crafty Crocodile came by and played another trick on Eli.***

T: ***Eli felt hungry when he awoke, so he opened his peanut bag. Eli counted his peanuts and found that 89 peanuts were missing. What do you think Clarence the Crafty Crocodile had done?***

S: ***Clarence could have removed 89 peanuts from Eli's bag.***

$$154 - 89 = ?$$

S: ***Clarence could have put 89 magic peanuts into Eli's bag.***

$$154 + \widehat{89} = ?$$

As each possibility is suggested, write the appropriate number sentence on the board.

Ask students to estimate how many peanuts were left in Eli's bag and write their estimates on paper.

T: ***How are we going to find out how many peanuts Eli had left in his bag?***

If someone suggests drawing a picture of Eli's bag, accept this as a good but not very practical method, because you would have to draw so many peanuts. If no one suggests using the Minicomputer, suggest it yourself. Point to the number sentences on the board.

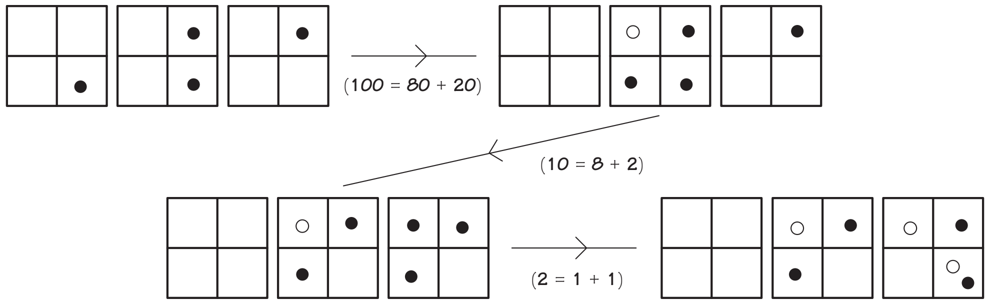
T: ***Which of these problems do you want to do first on the Minicomputer?***

Suppose the students want to calculate 154 – 89 first.
Invite someone to put 154 on the Minicomputer.

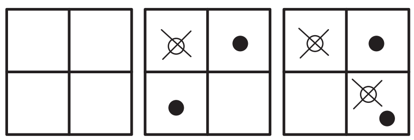
T: ***Who can make a backward trade that will help us to calculate 154 – 89?***

Ask a student to identify a trade before moving checkers. This will help to discourage trades that are not useful. Whenever a trade is made that puts a checker in position for subtraction, mention this to the class and indicate that they are getting closer to the goal. You may like to highlight checkers for 89 by changing them to a different color.

Let students make trades until 89 can be taken off the Minicomputer. A possible sequence of trades is shown below.



Now it is possible to remove checkers for 89. Invite a student to do so and complete the corresponding calculation.

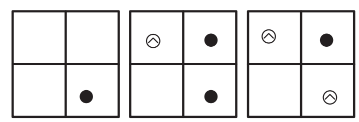


= 154 − 89 = 65

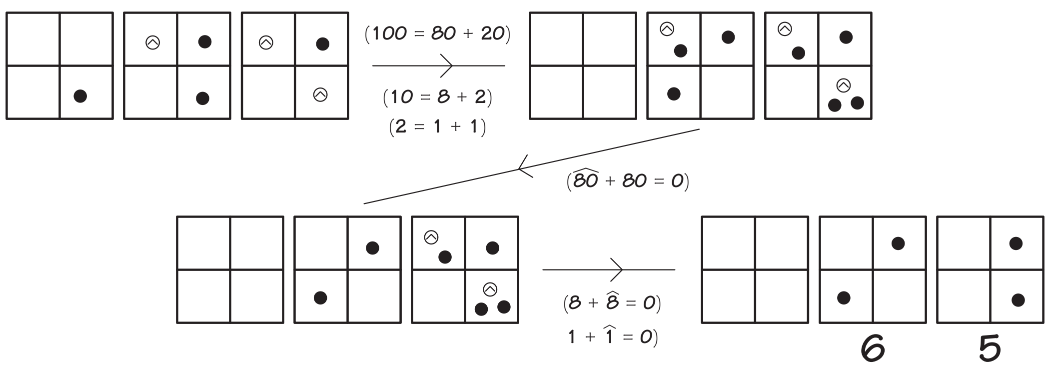
T: ***Now let's suppose Clarence put 89 magic peanuts into Eli's bag.***

Invite a student to put 154 on the Minicomputer and another to put $\widehat{89}$ on the Minicomputer.

T: ***We want to get a regular and a negative checker on the same square so that we can remove them. Who can make a trade?***



Let students make trades until the checkers on the Minicomputer are all of the same kind. In this case, they will all be regular. One possible sequence of trades is shown below.



Some of your students may prefer to remove checkers immediately as they match them (positive and negative checkers on the same square), and that is okay. Complete both calculations.

Emphasize that you don't know what Clarence did when he opened Eli's bag because removing 89 regular peanuts and adding 89 magic peanuts have exactly the same effect.

$$154 - 89 = 65$$
$$154 + \widehat{89} = 65$$
$$154 - 89 = 154 + \widehat{89}$$

T: ***Which calculation did you find easier on the Minicomputer?***

Accept either response, letting students comment on why they thought one calculation was easier than the other.

Erase the board and remove the checkers from the Minicomputer.

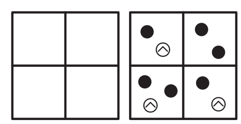
Exercise 2

You may like students to work with partners during this exercise. Each student or pair of students should have two Minicomputer boards (one sheet) and some checkers. Display two Minicomputer boards.

T: ***Put 10 on your Minicomputer.***

Invite a few students who have created different configurations for 10 to show them on the demonstration Minicomputer. Briefly check each configuration with the class. A possible dialogue is given here.

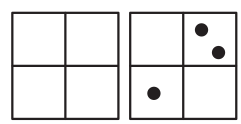
T: ***Anthony claims that this number is 10. Do you agree?***



= 10

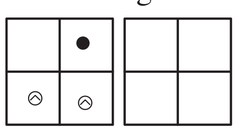
S: ***We can take off some checkers. Then it will be easier to read.***

S: ***4 + 4 + 2 = 10. The number on the Minicomputer is 10.***



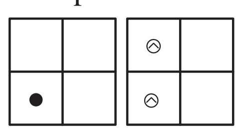
= 10

If your students enjoy this activity, ask them to put 10 on their Minicomputers using exactly one regular checker and two negative checkers. There are two possible solutions.



= 10

$40 + \widehat{30} = 10$



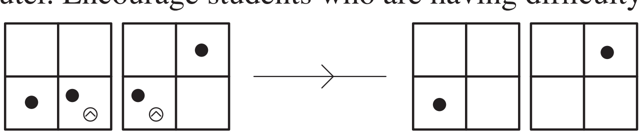
= 10

$20 + \widehat{10} = 10$

Write this problem on the board.

$$36 + \widehat{12}$$

Ask students to calculate $36 + \widehat{12}$ (read as "thirty-six plus negative twelve") on their individual Minicomputers. Invite a couple of students to do the calculation on the demonstration Minicomputer. Encourage students who are having difficulty to watch closely.



$= 36 + \widehat{12} = 24$

Continue this activity with calculations appropriate for the abilities of your students. Occasionally, after doing a calculation, write the corresponding subtraction problem on the board. For example, after the students find that $36 + \widehat{12} = 24$, write this subtraction problem on the board.

$$\begin{array}{r} 36 \\ -12 \\ \hline 24 \end{array} \qquad \text{or} \qquad 36 - 12 = 24$$

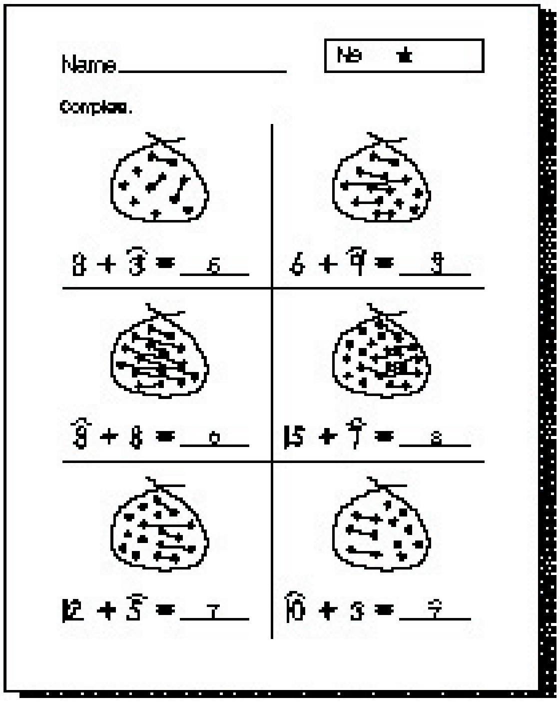
Worksheets N9*, **, ***, and **** are available for individual or partner work.

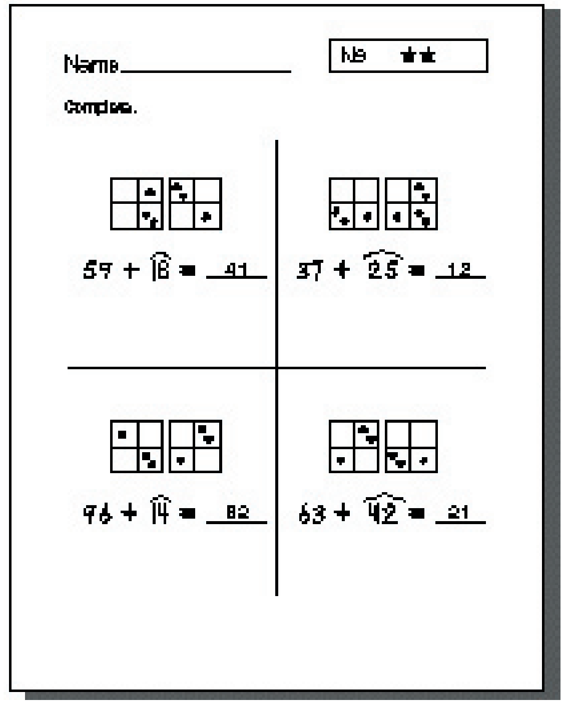
Writing Activity

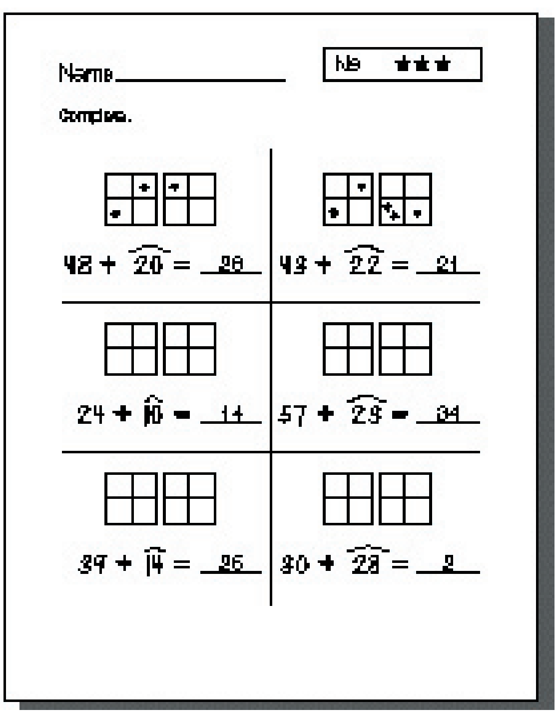
Write a story about Eli and Clarence in which Clarence plays a trick on Eli and causes Eli's bag to be empty.

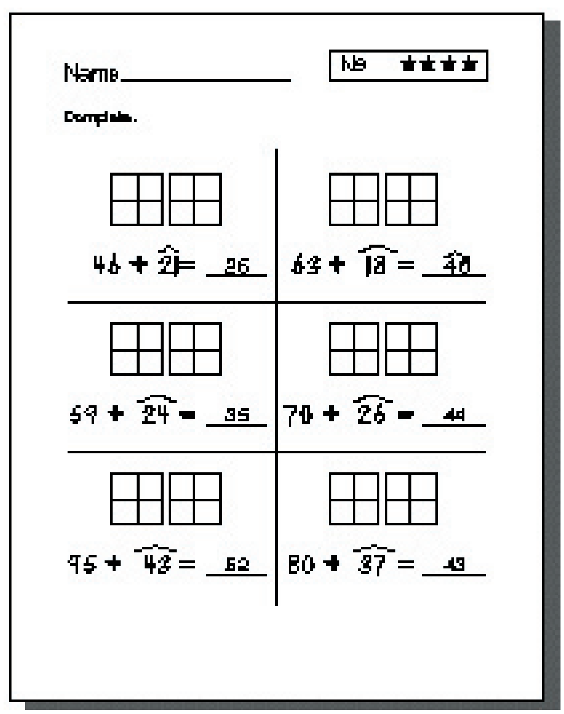
Home Activity

This is a good time to send home a letter about negative numbers. The letter can include a few calculations for students to do with a family member. Blackline N9 has a sample letter.


Name
No ★
Complete.
8 + 3 = 5
6 + 9 = 3
9 + 8 = 0
15 + 9 = 8
12 + 5 = 7
10 + 3 = 7


Name
No ★★
Complete.
59 + 18 = 41
37 + 25 = 12
96 + 14 = 82
63 + 42 = 21


Name
No ★★★
Complete.
48 + 20 = 28
43 + 22 = 21
24 + 10 = 14
57 + 23 = 34
39 + 14 = 25
30 + 28 = 2


Name
No ★★★★
Complete.
46 + 21 = 25
63 + 18 = 45
59 + 24 = 35
70 + 26 = 44
95 + 43 = 52
80 + 37 = 43

N10 INTRODUCTORY ARROW PROBLEMS #2

Capsule Lesson Summary

As a mental arithmetic activity, count by threes and then multiply various numbers by 3. Consider parts of arrow roads and ask if they could be parts of the same arrow road.

Materials

Teacher
- Colored chalk
- Calculator (optional)

Student
- Unlined paper
- Colored pencils, pens, or crayons
- Calculator (optional)

Description of Lesson

Exercise 1: The Function +3

Call on several students to count by threes, starting at 0 and continuing to about 30. Propose going around the class counting by threes. The following dialogue assumes 26 students in the class will contribute to the counting.

T: ***I'll start the count with 0 and then Elsa will say 3. We will end the count with Nathan. Can you predict which number Nathan will say?***

Accept several estimates and write them on the board. Then begin the counting.

Note: You may like to follow (check) the students' counting with a calculator, counting by threes starting at 0.

T: ***Nathan said 78. Were any of our estimates close?***

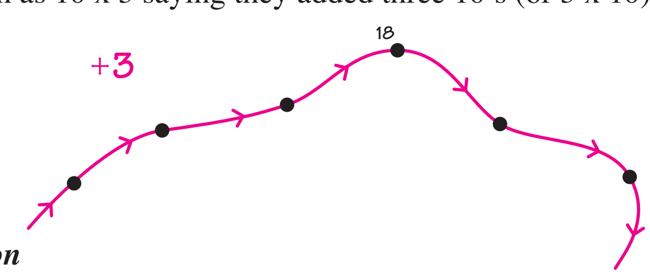
Invite students who predicted 78 or close to 78 to explain their reasoning. Encourage students to observe that counting by threes (or adding 3) twenty-six times can be an arrow road with twenty-six +3 arrows. The ending number is 3 added twenty-six times or 26 x 3.

Continue this activity with the following mental calculations; answers are shown in parentheses. Occasionally ask students to explain how they did a calculation.

5 x 3 (15)	4 x 3 (12)	12 x 3 (36)
6 x 3 (18)	10 x 3 (30)	20 x 3 (60)
8 x 3 (24)	9 x 3 (27)	30 x 3 (90)

Note: Students may explain a calculation such as 10 x 3 saying they added three 10's (or 3 x 10).

Draw this arrow picture on the board and ask students to copy it on their papers.



Point to the appropriate arrows as you say,

T: ***To show this arrow road goes on and on I have drawn an arrow without a starting dot and an arrow without an ending dot.***

Point to the dot for 18.

T: ***This dot is for 18. Which dots are for numbers greater than 18?***

Students should indicate the dots to the right of 18.

T: ***How do you know?***

S: ***The arrows are for +3; adding 3 gives a greater number.***

T: ***Which dots are for numbers less than 18?***

T: ***How do you know?***

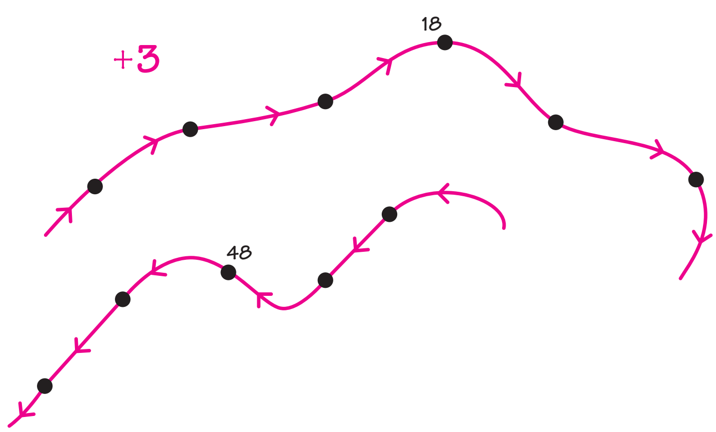
S: ***If you go backward on a +3 arrow road, the numbers decrease.***

T: ***This +3 arrow road meets 18. What are some of the other numbers this +3 arrow road would meet?***

Accept several suggestions. Any multiple of 3 is correct.

Note: You can determine easily if a number is a multiple of 3 by summing its digits. If the sum of the digits is a multiple of 3, then the number is also a multiple of 3. For example, since 7 + 2 = 9 and 9 is a multiple of 3, 72 is also a multiple of 3. Two hundred (200) is not a multiple of 3 because 2 + 0 + 0 = 2 and 2 is not a multiple of 3. This is for your information only.

Below the first picture, draw part of a +3 arrow road that meets the number 48.



T: ***This is part of a +3 arrow road that meets the number 48. Which dots are for numbers greater than 48?***

Students should indicate the dots to the left of 48.

T: ***Which dots are for numbers less than 48?***

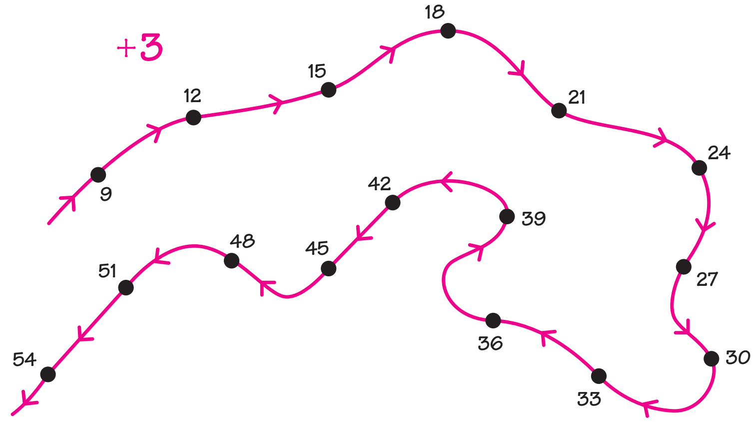
Students should indicate the dots to the right of 48 or dots in the top part of a +3 arrow road.

T: ***What are some other numbers this +3 arrow road would meet?***

Accept several suggestions. Any multiple of 3 is correct.

T: ***Could it be that these are parts of the same arrow road? Why or why not?***

Encourage students to explain their thinking. Then direct the students to extend their +3 arrow road that meets 18 until they know for sure whether or not it meets 48. After students have worked independently for several minutes, invite some students to extend the picture on the board to show that 18 and 48 are indeed on the same +3 arrow road.



Trace the arrow road from 18 to 48.

T: ***18 and 48 are on the same +3 arrow road. How many +3 arrows do we follow going from 18 to 48?***

S: ***Ten.***

T: ***If I drew an arrow from 18 to 48, what could this arrow be for?***

S: ***+30.***

T: ***How do you know?***

S: ***Ten +3 arrows is the same as one +30 arrow.***

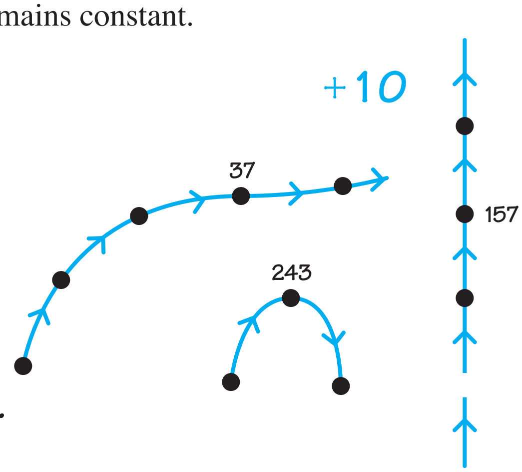
S: ***18 + 30 = 48.***

Erase the board before going on to Exercise 2.

Exercise 2: The Function +10

Call on students to count by tens starting at 0, at 1, at 5, and at 8.
Observe that in each case there is a pattern: the ones digit remains constant.

Draw this arrow picture on the board.



T: ***Do you think these could all be parts of the same +10 arrow road?***

S: ***37 and 157 are on the same +10 arrow road because both numbers end in 7.***

S: ***The +10 arrow road that meets 243 only meets positive numbers that end in 3, so 243 cannot be on the same arrow road as 37 or 157.***

If your students are not sure that 37 and 157 are on the same +10 arrow road, and that 243 is on a different +10 arrow road, ask the students to draw a +10 arrow road that meets 37 and to extend it until they know whether or not it meets 157 and 243. When a student has extended a +10 arrow road past 243, draw it to the class's attention and conclude that 157 is on the same +10 arrow road as 37, but 243 is not. As an option, support this activity with a counting calculator, counting by tens starting at 37. Students can read the display in unison to hear as well as view the pattern.

T: ***How many +10 arrows do we need to go from 37 to 157? How do you know?***

S: ***We need twelve arrows. I counted by tens from 37 to 157.***

S: ***37 + 100 = 137, so we need ten +10 arrows to go from 37 to 137, and then two more +10 arrows to take us to 157.***

T: ***If I drew an arrow from 37 to 157, what could this arrow be for? How do you know?***

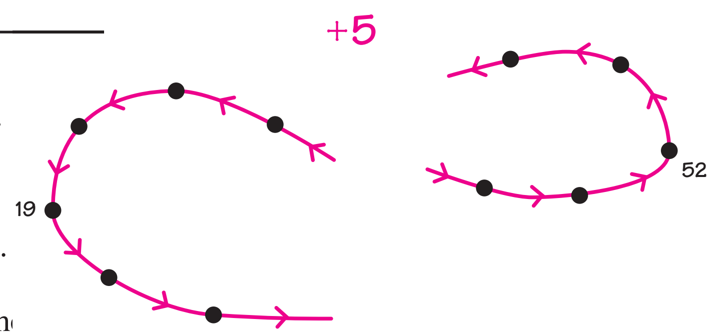
S: ***+120, because 37 + 120 = 157.***

T: ***Twelve +10 arrows is the same as one +120 arrow.***

Erase the board before going on to Exercise 3.

Exercise 3: The Function +5

Draw this arrow picture on the board.



T: ***Do you think that these are parts of the same +5 arrow road?***

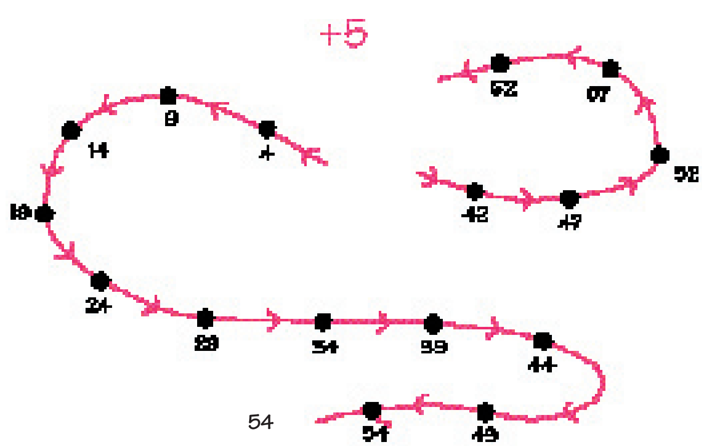
Encourage students to explain their thinking. Then ask them to draw a +5 arrow road that meets 19 and to extend it until they know wh or not it meets 52. Allow time for independent work. Then invite a student to label the dots in the picture on the board, and to extend the arrow road meeting 19 until it is clear the 52 is skipped. As an option, support this activity with a counting calculator, counting by fives starting at 19. Read the display in unison to hear as well as view the pattern of digits in the ones place.

Conclude that 19 and 52 are not on the same +5 arrow road.

T: ***Do you see any patterns in the arrow picture?***

S: ***The numbers in the arrow road that meets 19 all end in 4 or 9.***

S: ***The numbers in the arrow road that meets 42 all end in 2 or 7.***



Invite students to count by fives starting at 4. If the students have difficulty, follow the +5 arrow road starting at 4, counting by fives. Repeat this counting exercise to count by fives starting at 2, at 1, and at 3.

Additional Practice

This would be a good time to practice multiplication facts, especially facts for 3x, 5x, and 10x.

Home Activity

Suggest to parents/guardians that they find opportunities to practice multiplication facts with their child. Parents might use both oral and written exercises. The calculator is a nice tool to use in practicing multiplication facts. For example, prepare the calculator to multiply various numbers by 5 (facts for 5x) as follows:

1) Start with 0 on the display.
2) Press [5] [×] [0] [=]. 0 will be on the display again.
3) Enter any number and then press [=]. The calculator will multiply the number by 5.

N11 MULTIPLICATION TABLE

Capsule Lesson Summary

Use rectangles with areas from 1 to 30 little squares to construct a multiplication table. Observe patterns in the multiplication table.

Materials

Teacher
- Grid
- Rectangles from Lesson G3
- corner
- Blackline N11(a) and (b)

Student
- corner
- Grid paper

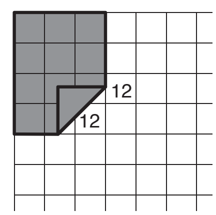
Advance Preparation: Use Blacklines N11(a) and (b) to make corners and grid paper. If you used 1-inch squares to make rectangles in Lesson G3, you can use the 1-inch square grid found on *UPG-III World of Numbers Poster* #1 for this lesson. Otherwise, prepare a grid having the square size you used for the rectangles in Lesson G3.

Description of Lesson

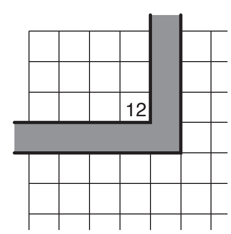
Exercise 1

Display a grid with grid squares the size of the squares used to make rectangles in Lesson G3.

T: ***Today we are going to make another table with these rectangles.***



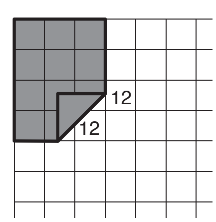
Choose a number such as 12 to demonstrate. Announce to the class what you are doing as you select the three rectangles for 12 and place it on your grid in the upper left corner. Then lift the lower right corner of the rectangle and write 12 in the grid square. Place the to highlight the rectangle formed by the upper left corner and this grid square.



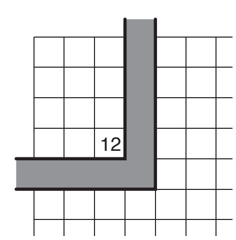
T: ***I wrote 12 here because this is a rectangle for 12. This rectangle has 3 rows of 4 squares each, or 12 squares. 3 x 4 = 12.***

Turn the rectangle to show that you can position it on the grid as 4 rows of 3 squares each.

T: ***Since I can also position the rectangle like this, I'll write 12 in this grid square.***



This rectangle has 4 rows of 3 squares each, or 12 squares. 4 x 3 = 12.



Invite students to use the other rectangles for 12 and to write 12 in other grid squares. Point out these multiplication facts:

6 x 2 = 12   12 x 1 = 12
2 x 6 = 12   1 x 12 = 12

Repeat this activity for another number such as 9. Notice that the 3 x 3 square can be positioned in only one way.

Distribute the grid paper (as on Blackline N11) and corners to pairs of students.

								9			12
					12						
		9	12								
		12									
	12										
9											
12											

T: ***Now it is your turn to complete this table with as many rectangles as will fit on your grid.***

Direct students to work with their partners, first locating the rectangles for 12 and 9 as you did in the demonstration. Point out that their grids are not big enough to include the 12 by 1 or 1 by 12 rectangle. Then, continue to locate rectangles for other numbers, and fill in the corresponding grid squares. Students should use the corner to see how a rectangle is placed.

As students are working on their own tables, call on students to help you place rectangles and fill in grid squares on the demonstration grid. A 12 by 12 table showing all the rectangles that fit for numbers from 1 to 30 is illustrated here.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30		
4	8	12	16	20	24	28					
5	10	15	20	25	30						
6	12	18	24	30							
7	14	21	28								
8	16	24									
9	18	27									
10	20	30									
11	22										
12	24										

Some students will probably go on and fill in other grid squares for numbers greater than 30. When most students have filled in at least half of their 10 by 9 table, call the class's attention to the demonstration grid. Use an corner to locate a 7 by 6 rectangle.

T: ***This rectangle has 7 rows of 6 squares each.***
How many squares in this rectangle?

S: ***42; 7 x 6 = 42.***

T: ***What number should I put in this grid square*** (point to the lower right corner)***?***

S: ***42. It is a rectangle for 42.***

1	2	3	4	5	6		8	9	10	11	12
2	4	6	8	10	12		16	18	20	22	24
3	6	9	12	15	18		24	27	30		
4	8	12	16	20	24						
5	10	15	20	25	30						
6	12	18	24	30							
7	14	21	28								
9	18	27									
10	20	30									
11	22										
12	24										

Extension Activity

Distribute copies of a 10 by 10 completed multiplication table (Blackline N11(c) in such a table) and ask students to color (shade) multiples of 4 on one page, multiples of 5 on another, and multiples of 6 on another. Look for patterns.

Multiples of 4

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Multiples of 5

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Multiples of 6

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Home Activity

Let students take home their multiplication tables to practice multiplication facts with a family member. This is a good time to send home a letter to parents/guardians about working on basic facts. Blackline N11(d) has a sample letter.

Capsule Lesson Summary

Locate numbers on a number line where the marks are for the even numbers.

In order to subtract 24 from 52, do a series of calculations. Show the corresponding jumps on the number line. Do several similar problems by subtracting tens and then the appropriate number of ones. Play The Number Line Game.

$$\begin{array}{r} 52 \\ \underline{-10} \\ 42 \\ \underline{-10} \\ 32 \\ \underline{-4} \\ 28 \end{array}$$

Materials

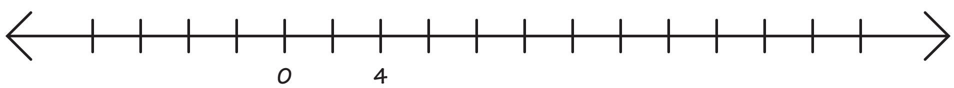
Teacher • Colored chalk

Student • Paper

Description of Lesson

Exercise 1: Jumps on the Number Line

Draw a line on the board and graduate it, using intervals about six centimeters in length. Locate 0 at one of the marks and locate the number 4 two marks to the right.

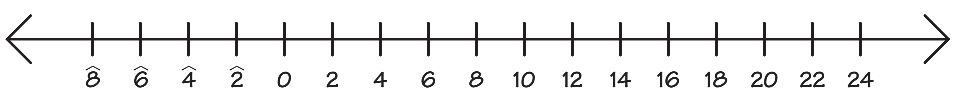


T: ***Why did I put arrows at both ends of this line?***

S: ***To show that the number line goes on and on in both directions.***

T: ***I've located 0 and 4 on this number line. Do you know where any other numbers belong on this line?***

Invite students to locate other numbers on this number line. Each time, let a student label several marks so that the activity does not drag. Continue until all the marks are labeled.



T: ***What do you notice about all the numbers on this number line?***

S: ***They are all even numbers.***

S: ***They are all multiples of 2.***

T: ***Where would 9 be on this number line?***

S: ***9 is halfway between 8 and 10.***

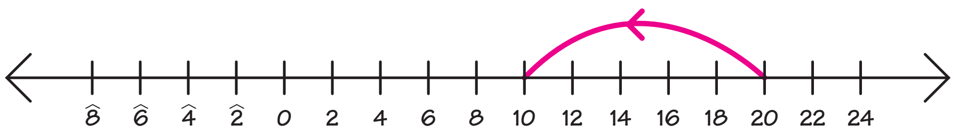
T: ***Where would 1 be on this number line?***

S: ***Between 0 and 2.***

Let students suggest other odd numbers they could locate on this part of a number line.

Draw an arrow from 20 to 10 on the number line.

N12



T: ***What could this red arrow be for? How do you know?***

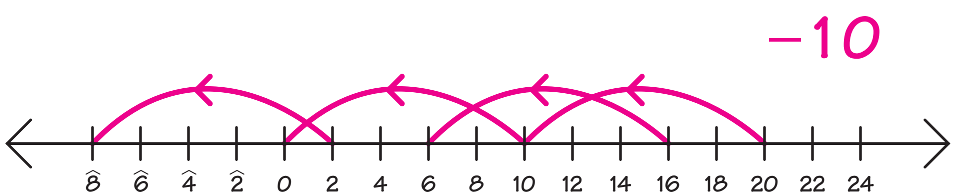
S: ***–10, because 20 – 10 = 10.***

Note: The red arrow could also be for several other relations including $\frac{1}{2}$x and “is more than.” Accept these suggestions as possibilities, but tell the class that this red arrow is for –10.

Write –10 in red on the board near the number line.

T: ***Where could we draw another –10 arrow?***

Invite students to trace other –10 arrows. Draw a few of these arrows yourself, but do not try to draw all of them or the picture will become too crowded. Perhaps your number line will look similar to this one.



T: ***Suppose we start at 40 and make a –10 jump. Where would we land?***

S: ***At 30.***

T: ***If we start at 44 and make a –10 jump …?***

S: ***At 34.***

Continue this activity with the following starting numbers or others appropriate for the abilities of your students.

62	(52)	16	(6)	0	($\widehat{10}$)
80	(70)	116	(106)	5	($\hat{5}$)
78	(68)	106	(96)	6	($\hat{4}$)

Refer to the number line if the students have difficulty calculating 5 – 10 or 6 – 10.

T: ***If we start at 30 and make two –10 jumps, where would we land?***

S: ***At 10, 30 – 10 = 20 and 20 – 10 = 10.***

T: ***And if we start at 35 and make two –10 jumps …?***

S: ***At 15.***

Continue this activity (making two –10 jumps) using the following starting numbers or others appropriate for the abilities of your students. Occasionally ask students to explain their answers.

42 (22)	89 (69)
70 (50)	66 (46)
100 (80)	166 (146)

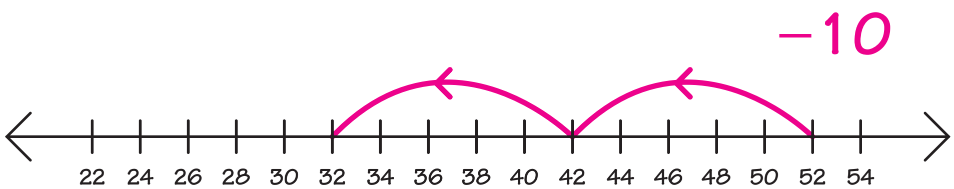
Exercise 2: Subtraction Problems

Extend your number line on the board out to 54, or relabel the marks from 22 to 54.

T: ***If I start at 52 and make two –10 jumps, what numbers will I land on?***

S: ***42 and 32.***

Start at 52 and draw two consecutive –10 arrows.



T (tracing the arrows)**:** ***We start at 52 and subtract 10; 52 – 10 = ?*** (42)
Then we subtract 10 again; 42 – 10 = ? (32) ***What number is 52 – 20?***

S: ***32.***

The illustration on the right shows what to write on the board as you reiterate.

T: ***If we subtract 10 …***

and subtract 10 again …

it is the same as if we subtract 20.

$$\begin{array}{r} 52 \\ -10 \\ \hline 42 \\ -10 \\ \hline 32 \end{array} \qquad \begin{array}{r} 52 \\ -20 \\ \hline 32 \end{array}$$

T: ***52 – 20 = 32, so what number is 52 – 24?***

S: ***28.***

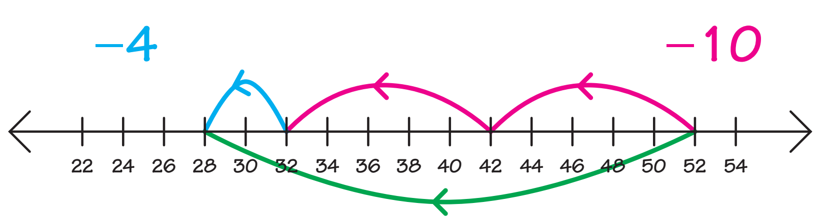
T: ***How did you do the calculation?***

S: ***I just counted backward 4 from 32.***

S: ***52 – 20 = 32 and 32 – 4 = 28.***

Draw a blue arrow from 32 to 28 on the number line. Write –4 in blue near the arrow picture.

Draw a green arrow from 52 to 28.



T: ***What could this green arrow be for?***

S: ***–24.***

Write –24 in green near the arrow picture. Trace the appropriate arrows as you say,

T: ***If we subtract 10, then subtract 10 again, and then subtract 4, it is the same as if we subtract 24.***

$$\begin{array}{r} 52 \\ -10 \\ \hline 42 \\ -10 \\ \hline 32 \\ -\ \ 4 \\ \hline 28 \end{array} \qquad \begin{array}{r} 52 \\ -24 \\ \hline 28 \end{array}$$

Erase the arrows and the subtraction problems. Write this problem on the board.

$$\begin{array}{r} 40 \\ -12 \\ \hline \end{array}$$

T: ***Can anyone suggest an easy way to solve this problem?***

Some students may know an algorithm for subtraction. Accept it as a good method, but say you are looking for an easy way they can do the problem mostly in their heads.

S: ***Subtract 10 and then subtract 2.***

T: ***What number is 40 – 10?***

S: ***30.***

T: ***Now we need to subtract 2 more. What number is 30 – 2?***

S: ***28.***

T: ***Subtracting 10 and then subtracting 2 is the same as subtracting 12. What number is 40 – 12?***

S: ***28.***

$$\begin{array}{r} 40 \\ -12 \\ \hline \end{array} \qquad \begin{array}{r} 40 \\ -10 \\ \hline 30 \\ -\ \ 2 \\ \hline 28 \end{array}$$

Complete the subtraction problem 40 – 12 = 28 on the board.

If your students suggest making jumps on the number line to find the answer, follow this suggestion and then calculate the answer as indicated in the previous dialogue. You need not insist on using the number line yourself.

Erase the board and write this problem. Ask students to write the problem on their papers.

$$\begin{array}{r} 73 \\ -26 \\ \hline \end{array}$$

T: ***When we wanted to subtract 24, we subtracted 10 twice and then subtracted 4. In this problem we want to subtract 26. What do you suggest?***

S: ***Subtract 10 twice and then subtract 6.***

Note: You might also accept "subtract 10 three times and then add 4."

Indicate the calculations on the board. Students should follow along and do the problem on their papers.

T: ***What number is 73 – 26?***

S: ***47.***

Complete the subtraction problem.

$$\begin{array}{r} 73 \\ -26 \\ \hline 47 \end{array} \qquad \begin{array}{r} 73 \\ -10 \\ \hline 63 \\ -10 \\ \hline 53 \\ -\ 6 \\ \hline 47 \end{array}$$

Suggest two or three more subtraction problems for the students to do on their papers; for example:

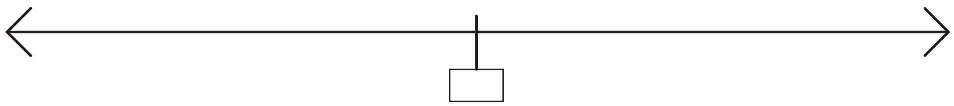
32 – 14 61 – 23 53 – 35

Exercise 3: The Number Line Game

The Number Line Game may be played frequently throughout the year whenever you finish a lesson early or have some extra time. Each game takes approximately five minutes. Several variations of this game are given at the end of the lesson. Use these variations when your class is ready for them.

T: ***I'm going to show you a number line game. Some of you may have played this game last year. I'm thinking of a number between 0 and 200. I'll show you where it is on the number line; my number is hidden in this box.***

Draw a line on the board and position a box for the secret number near the center of your picture.



Note: Begin with the box near the center of your line. For convenience in locating numbers, you may wish to extend the line during the game.

The following is a description of a possible game. Suppose the secret number is 114.

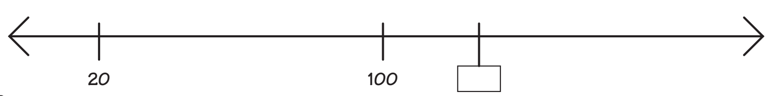
S: ***20.***

T: ***I'll show you where 20 is on this number line. It might not be in exactly the right place, but I'll try to make it as close as possible.***

T: ***20 is less than my secret number.***

S: ***100.***

T: ***100 is less than my number.***

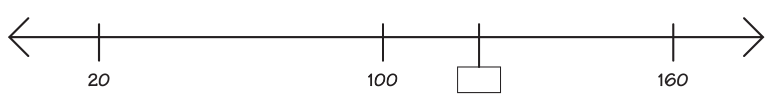


S: ***250.***

T: ***My number is between 0 and 200. 250 cannot be my number.***

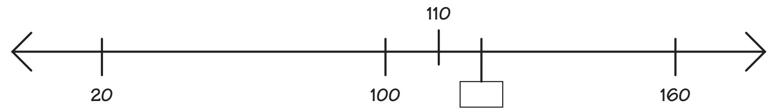
S: ***150.***

T: ***150 is more than my secret number.***



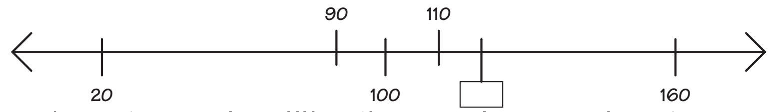
S: ***110.***

T: ***110 is less than my secret number.***



S: ***90.***

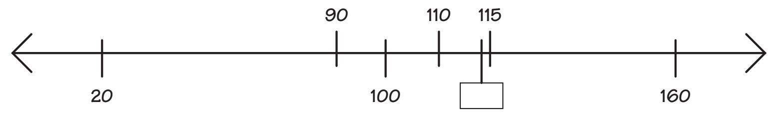
T: ***90 is less than my secret number.***



Note: Although a guess may give no new information such as 90 in this sample game, do not criticize or comment except to say that it is more or less than your number.

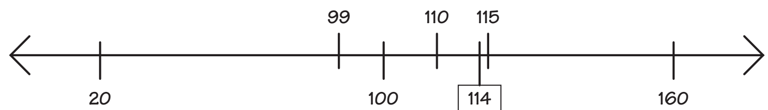
S: ***115.***

T: ***115 is more than my number.***



S: ***114.***

T: ***You have discovered my secret number!***



Play the game a few more times. Let the student who guesses the correct number choose the secret number for the next game and whisper it to you. Ask that student to keep the number a secret and not to play in that game. You may like to allow the student to help "judge" the game. (See Variation 3 of The Number Line Game).

Home Activity

Students may like to show someone at home how to play The Number Line Game and then play it with them.

VARIATIONS OF THE NUMBER LINE GAME

Variation 1

As your class becomes familiar with The Number Line Game, you can expand the range of possible numbers. Possibilities include the following:

- numbers between 0 and 500
- numbers between 500 and 1,000

Variation 2

As the students become familiar with numbers such as $2\frac{1}{2}$, occasionally you might choose a non-integer as your secret number. It is best to limit the range of possible numbers to those between 0 and 10 when you first use this variation.

Variation 3

When you are using whole numbers only for your range of possible numbers, ask a student to choose a secret number and to judge the guesses of the other students. The student who chooses the secret number tells the class whether a guess is more or less than the secret number. You label a mark on the number line for each guess and, if necessary, help the student to judge. Remind student judges not to give additional clues.

N13 MULTIPLICATION PROBLEMS #2

Capsule Lesson Summary

Do some mental arithmetic involving multiplication facts and patterns. Pose multiplication problems such as 5 x 47, 3 x 64, and 3 x 356 in story contexts and solve them with the help of the Minicomputer. Solve other similar multiplication problems.

Materials

Teacher
- Calculator (optional)
- Minicomputer set

Student
- Lined paper
- Minicomputer set
- Calculator (optional)

Description of Lesson

Exercise 1

Begin this lesson with some mental arithmetic on multiplication facts, expecially involving 2x, 3x, 4x, and 5x. One way to do this is to prepare a calculator to multiply any given number by 3 (for example) by pressing [3] [x]. Then when you enter a number and press [=], the calculator will display 3x that number. Students can enter numbers on the calculator and predict what the calculator will display when they press [=]. You may also like to refer students to the multiplication tables they made in Lesson N11.

Continue with some mental arithmetic involving multiplication patterns. For example,

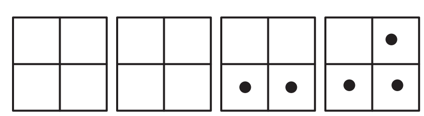
2 x 12	(24)	3 x 5	(15)	2 x 6	(12)
3 x 12	(36)	3 x 50	(150)	4 x 6	(24)
4 x 12	(48)	5 x 4	(20)	4 x 10	(40)
5 x 12	(60)	5 x 40	(200)	4 x 16	(64)

Exercise 2

Display four Minicomputer boards. Put the multiplication problems in this exercise in a story context if you like.

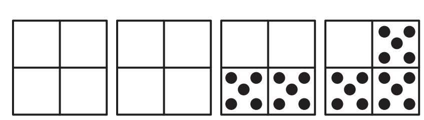
T: ***Gabe likes a certain kind of cookie that comes with 37 cookies in a package. Who can put 37 on the Minicomputer.***

If 37 is not put on in standard configuration, call on students to make some trades.



T: ***Gabe decides to buy five packages of these cookies. Can we add checkers to the Minicomputer to show five packages of 37 cookies?***

Call on students, one at a time, to put another 37 on the Minicomputer until you have this configuration for 5 x 37.



T: ***Let's calculate 5 x 37. What calculation do we have on the ones board?***

5 x 37

S: ***5 x 7.***

T: ***What number is 5 x 7?***

S: ***35.***

Record 5 x 7 = 35 above 5 x 37.

T: ***What calculation do we have on the tens board?***

S: ***5 x 30.***

T: ***What number is 5 x 30?***

S: ***150.***

You may need to ask students to calculate 5 x 3 and then 5 x 30.

Record 5 x 30 = 150 on the board above 5 x 7 = 35.

T: ***We know 5 x 30 = 150 and 5 x 7 = 35. Now calculate 5 x 37. Write the answer on a piece of paper.***

5 x 30 = 150
5 x 7 = 35

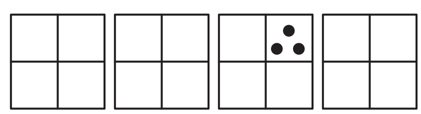
Look at students' papers before asking someone to explain how to do the calculation.

Complete the number sentence. You may also like to write the calculation in vertical format.

Some students may like to check the calculation by making trades on the Minicomputer until 185 is in standard configuration.

```
  37
 x 5
  35 (5 x 7)
 150 (5 x 30)
 185
```

Exercise 3



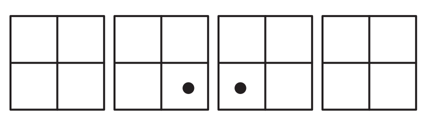
T: ***What number is this? How do you know?***

S: ***120. 40 + 40 + 40 = 120 or 3 x 40 = 120.***

T: ***Is this the way we usually put 120 on the Minicomputer?***

S: ***No, we can make some trades.***

Invite students to make trades on the Minicomputer until the standard configuration for 120 is obtained. You may need to mention the backward trade (40 = 20 + 20) yourself. Ask a student to write the number below the Minicomputer.

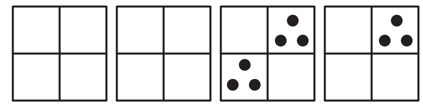


1 2 0

Remove the checkers from the Minicomputer, erase the board, and pose this or a similar problem on the board.

T: ***Julio has three bags of peanuts and each bag has 64 peanuts in it. How many peanuts does Julio have?***

Invite students to tell you what calculation to do and then to estimate the answer. Write the calculation on the board, and call on studnts to put 3 x 64 on the Minicomputer.



3 x 64

T: ***What calculation are we doing on the ones board?***

S: ***3 x 4.***

T: ***What number is 3 x 4?***

S: ***12.***

Write 3 x 4 = 12 above 3 x 64.

T: ***What calculation are we doing on the tens board?***

S: ***3 x 20 and 3 x 40.***

S: ***3 x 60.***

T: ***What number is 3 x 60?***

S: ***180.***

If necessary, prompt the students by asking what number is 3 x 6 and then 3 x 60.

Record 3 x 60 =180 on the board.

T: ***3 x 60 = 180 and 3 x 4 = 12, so what number is 3 x 64?***

S: ***192.***

```
3 x 60 = 180          64
3 x  4 =  12          x 3
3 x 64 = 192        12 (3 x 4)
                   180 (3 x 60)
                   192
```

You may like to write the calculation in vertical form as well to show students where to write 3 x 4 and 3 x 60 in this format. Call on volunteers to make trades on the Minicomputer until the standard configuration for 192 is obtained. As necessary, remind students of the need for a backward trade.

Repeat this activity to calculate 3 x 356.

```
3 x 300 =   900            356
 3 x 50 =   150            x 3
  3 x 6 =    18             18   (3 x 6)
3 x 356 = 1,068            150   (3 x 50)
                           900   (3 x 300)
                         1,068
```

Exercise 4

Allow students to work with a partner for this exercise. This would be a good time to pair a new student or a student less comfortable with the Minicomputer with an "expert."

Each pair of students should have two Minicomputer sheets (four boards) and a set of checkers. Ask the students to put 45 on their Minicomputers. Write 2 x 45 on the board, and ask the students to do the calculation on their Minicomputers. Many students will be able to do this calculation mentally, but ask them to do it on their Minicomputers also. Help students having difficulty making trades. If your class would benefit, calculate 2 x 45 on the demonstration Minicomputer, and ask the students to follow by making the same trades on their individual Minicomputers. Conclude that 2 x 45 = 90 and complete the number sentence.

Repeat this activity with 2 x 512. Conclude that 2 x 512 = 1,024.

Erase the board, and write these problems or others more appropriate for your students' abilities on the board.

2 x 107 =	4 x 59 =
3 x 73 =	5 x 216 =
2 x 134 =	4 x 260 =
3 x 625 =	3 x 187 =

Ask the students to copy these problems on their papers and to solve as many of them as they can. Encourage all students to calculate on their Minicomputers, but permit them to use other methods as well if they wish. Allow students to check results with a calculator.

Writing Activity

Suggest students write a story problem for one or two multiplication calculations.

Capsule Lesson Summary

Using as few arrows as possible in each road, build roads between pairs of numbers with +10 and +1 arrows. Describe shortest roads without actually building them. Include +100 arrows in similar road-building problems.

Materials

Teacher
- Colored chalk

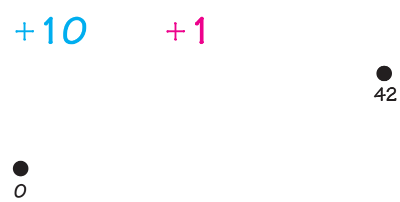
Students
- Unlined paper
- Colored pencils, pens, or crayons

Description of Lesson

Exercise 1

Ask students to count by tens beginning at 0, at 5, at 1, at 6, and at 8. Emphasize that when you count by tens the ones digit remains unchanged.

Write this information on the board.



T: *Let's build a road from 0 to 42 using only + 10 and +1 arrows. How do we build a shortest road (as few arrows as possible)?*

S: *Use as many +10 arrows as we can.*

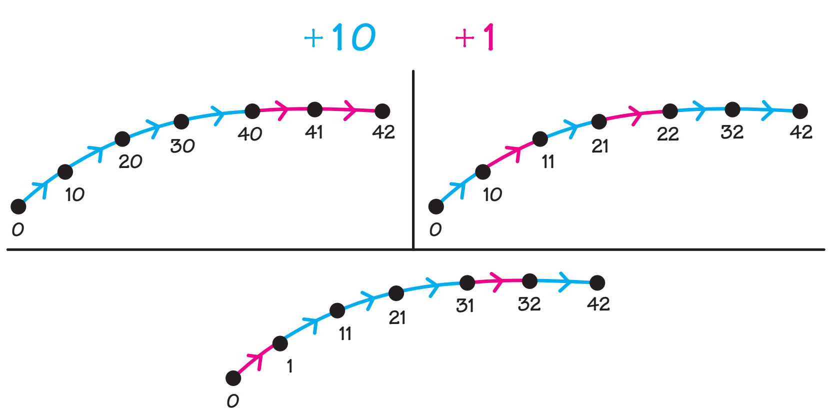
T: *How many +10 arrows and how many +1 arrows do we need for a shortest road?*

S: *Four +10 arrows and two +1 arrows.*

T: *But how do you know that would give us a road ending at 42?*

S: *Because 42 is 4 tens and 2 ones.*

Ask students to build an arrow road from 0 to 42 on their papers and to use as few arrows as possible. If necessary, remind them to copy the "key" on their papers to indicate what each color arrow represents. Students who finish quickly can build arrow roads from 0 to 42 with more arrows or put their shortest arrow roads on the board. Try to get pictures on the board in which the order of the blue and the red arrows vary. Here are three of the many (fifteen) possiblities.



N14

Count the +10 arrows and the +1 arrows in the roads on the board and confirm that each has four blue and two red arrows. Conclude that in this situation the order in which the arrows occur does not affect the ending number.

Note: If some students have drawn longer arrow roads, you may like the class to notice that they could be

- three +10 arrows and twelve +1 arrows;
- two +10 arrows and twenty-two +1 arrows;
- one +10 arrow and thirty two +1 arrows; or
- forty-two +1 arrows.

T: ***If you start at 0 and build a road using five +10 arrows and four +1 arrows, where would the road end?***

S: ***At 54.***

T: ***If you start at 0 and build a road using two +10 arrows and seven +1 arrows, where would the road end?***

S: ***27.***

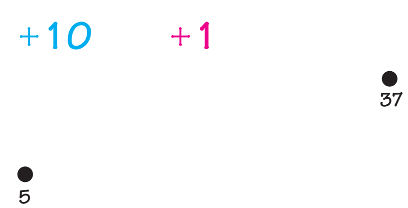
Continue this mental activity with the following or similar problems. This chart presents possible problems, but do not draw it on the board. Occasionally say and write, for example, "89 is 8 tens and 9 ones."

Starting Number	Number of +10 Arrows	Number of +1 Arrows	Ending Number
0	8	9	89
0	6	10	70
0	0	15	15
0	4	20	60
0	10	0	100
0	15	0	150
0	15	3	153

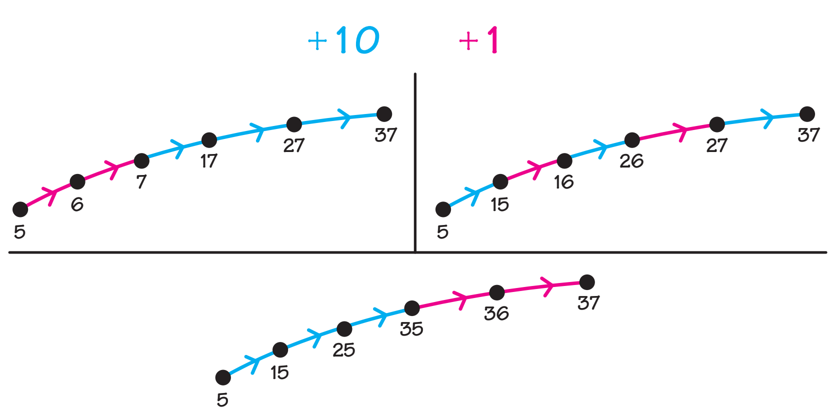
Before going on to Exercise 2, erase the board leaving only the key (+10 in blue and +1 in red).

Exercise 2

Write the following information on the board. Ask students to build an arrow road from 5 to 37 using as few arrows as possible.



After a few minutes, invite several students with different shortest roads to put them on the board. Here are three of the ten possibilities.



Ask students to count the +10 and the +1 arrows in the roads on the board and on their papers. Conclude that a shortest road has three +10 arrows and two +1 arrows.

T: ***If we start at 0 instead of 5 and build a road with three +10 arrows and two +1 arrows (and no others), where would the road end?***

S: ***At 32.***

T: ***If we start at 10 …?***

S: ***At 42.***

T: ***If we start at 12 …?***

S: ***At 44.***

T: ***If we start at 100 …?***

S: ***At 132.***

Before going on to Exercise 3, erase the board leaving only the key (+10 in blue and +1 in red).

Exercise 3

Point to the +10 and +1 on the board.

T: ***What is the least number of +10 and +1arrows we would need to build a road from 8 to 49?***

S: ***Five; four +10 arrows and one +1 arrow.***

T: ***What is the least number of +10 arrows we would need to build a road from 4 to 67?***

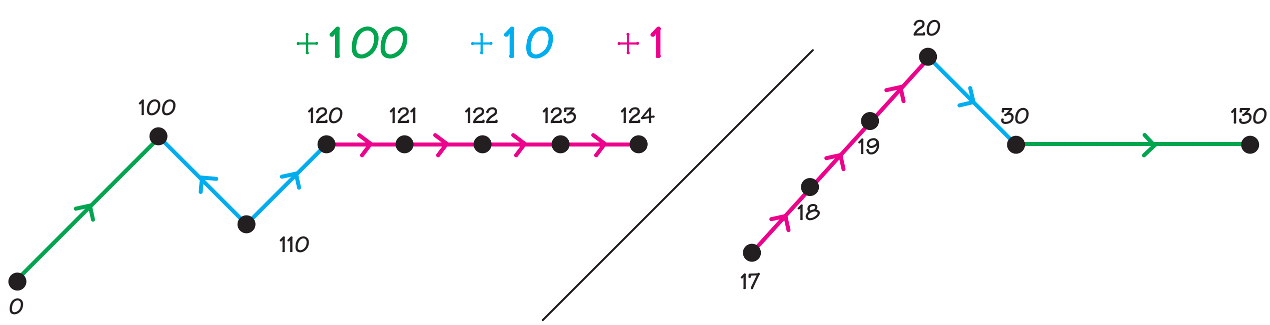
S: ***Nine; six +10 arrows and three +1 arrows.***

T: ***What is the least number of +10 and +1 arrows we would need to build a road from 9 to 21?***

S: ***Three; one +10 arrow and two +1 arrows.***

Exercise 4 (optional)

If there is time remaining and your students have done well with the exercises of this lesson, pose some road-builidng problems (as in Exercises 1 and 2) using +100, +10, and +1 arrows. For example, build an arrow road from 0 to 124 (or from 17 to 130) using only +100, +10, and +1 arrows, and as few as possible. One shortest road for each example is show below.



Center Activity

Task cards with road-building problems like those in Exercises 1 and 4 can be placed in a center to give students additional practice with expanded notation (as in an arrow picture).

Home Activity

This is a good time to send home a letter about the language of arrows. Blackline N14 has a sample letter. Students may like to show their parents/guardians how to build shortest arrow roads between two numbers using +10 and +1 arrows.

N15 DOUBLING AND HALVING ON THE MINICOMPUTER

Capsule Lesson Summary

Find a configuration of checkers on the Minicomputer. Do some mental arithmetic involving doubling and halving. Consider a configuration on the Minicomputer in which the checkers are grouped in pairs on the same square. First recognize what one-half of the number is and then find the number. Record the situation in an arrow picture. Use the Minicomputer to calculate $\frac{1}{2}$ x 1,974.

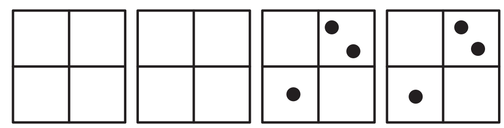
Materials

Teacher
- Minicomputer set
- Colored chalk

Student
- Paper

Description of Lesson

Exercise 1



Put this configuration on the Minicomputer.

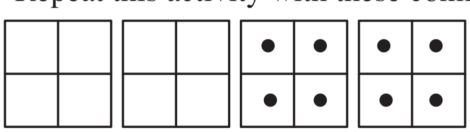
T: ***What number is on the Minicomputer? Write it on your paper (or whisper it to your neighbor).***

Look at (or listen to) many of the students' answers.

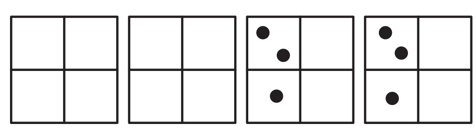
T: ***What number is on the ones board?*** (10)
What number is on the tens board? (100)
What number is this? (110) ***How do you know?***

Let students announce which calculations they did. If many are uncertain that this number is 110, ask a volunteer to make trades on the Minicomputer until you have the standard configuration for 110.

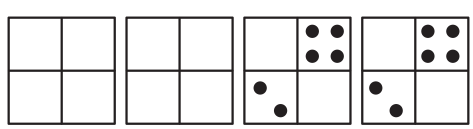
Repeat this activity with these configurations.



= 165



= 198



= 220

Exercise 2

Conduct a brief mental arithmetic exercise involving doubling and halving. A sequence of problems is suggested below; answers are in the boxes. Try to involve all the students by varying the difficulty of the calculations. Occasionally ask students to explain how they calculated an answer.

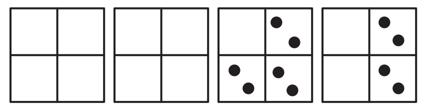
2 x 20 = 40	2 x 30 = 60	2 x 50 = 100	2 x 36 = 72
½ x 40 = 20 [†]	½ x 60 = 30	½ x 100 = 50	½ x 72 = 36
2 x 25 = 50	2 x 45 = 90	2 x 51 = 102	2 x 49 = 98
½ x 50 = 25	½ x 90 = 45	½ x 102 = 51	½ x 98 = 49

Exercise 3

T: ***I am going to put a number on the Minicomputer. As I put on checkers, try to calculate, and when you think you know the number, write it on your paper.***

Gradually put this configuration on the Minicomputer pausing after each step:

- start with two checkers on the 40-square;
- then put two checkers on the 10-square;
- then put two checkers on the 20-square;
- then put checkers on the 4-square and the 1-square (5), and then 5 again.



Look at many of the students' answers before asking someone to announce the number.

S: ***150.***

T: ***What calculations did you do to decide that the number is 150?***

Let several students tell which calculations they did; a few possibilities are given here.

S: ***40 + 40 = 80; 80 + 20 =100; and 100 + 20 + 20 = 140; and 140 + 5 + 5 = 150.***

S: ***The checkers on the 40-square and on the 10-square together make 100. 100 + 40 + 10 = 150.***

S: ***The number on the tens board is 140 because 2 x 70 = 140, and the number on the ones board is 10 because 2 x 5 = 10. 140 + 10 = 150.***

T: ***Look at the Minicomputer. What number is $^1/_2$ x 150?***

S: ***75.***

T: ***Can you tell that $^1/_2$ x 150 = 75 just by looking at these checkers on the Minicomputer?***

S: ***The number on the Minicomputer is 150. The checkers are in pairs, so you can just look at one checker in each pair.***

Record this number sentence on the board.

$$^1/_2 \times 150 = 75$$

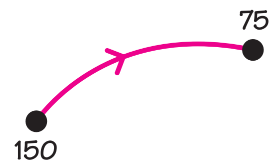
T: ***Let's show this in an arrow picture.***

Draw this arrow picture on the board.

[†]Read $^1/_2$ x 40 as "one-half of forty."

T: ***What could the red arrow be for?***

S: ***$\frac{1}{2}$x.***

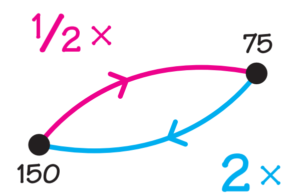


Note: The arrow could also be for –75 or "is more than" or several other relations, but in this case we are interested in $\frac{1}{2}$x.

Write $\frac{1}{2}$x in red near the arrow picture.

T (tracing the $\frac{1}{2}$x arrow): ***$\frac{1}{2}$ x 150 = 75. If I drew an arrow here*** (trace an arrow from 75 to 150)***, what could it be for?***

S: ***2x.***



Draw a blue arrow from 75 to 150 and label it 2x.

T: ***What number sentence is told by this blue arrow?***

S: ***2 x 75 = 150.***

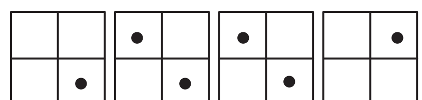
Record this number sentence on the board.

$\frac{1}{2}$ x 150 = 75

2 x 75 = 150

Erase the board and then put this configuration on the Minicomputer.

T: ***What number is on the Minicomputer?***
Write it on your paper.



Look at several students' answers. Call on a student to write this number (1,994) on the board and to read it aloud.

S: ***One thousand, nine hundred, ninety-four.***

T: ***What number is $\frac{1}{2}$ x 1,994. Is $\frac{1}{2}$ x 1,994 more than 500? How do you know?***

S: ***Yes, because $\frac{1}{2}$ x 1,000 = 500 and 1,994 is more than 1,000.***

T: ***Is $\frac{1}{2}$ x 1,994 more than 1,000? How do you know?***

S: ***No, because $\frac{1}{2}$ x 2,000 = 1,000 and 1,994 is less than 2,000.***

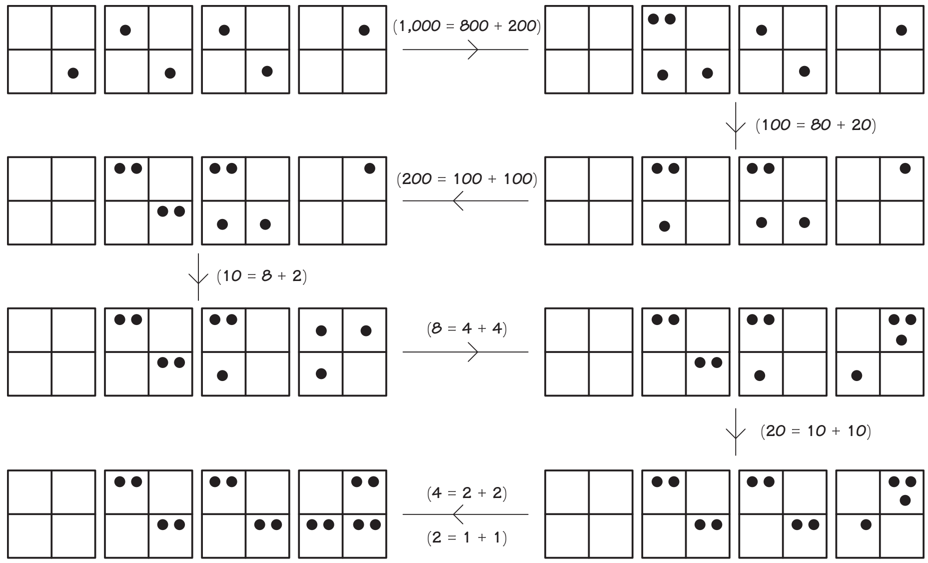
T: ***So we know that $\frac{1}{2}$ x 1,994 is between 500 and 1,000.***

If your students are able to give good explanations, ask if $\frac{1}{2}$ x 1,994 is more than 800 or even more than 900. Then solicit an estimate for $\frac{1}{2}$ x 1,994 and record some estimates on the board, but only record those between 500 and 1,000.

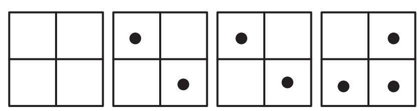
T: ***How can we find exactly what number $\frac{1}{2}$ x 1,994 is?***

S: ***We could make backward trades and get checkers in pairs (the two checkers in any pair being on the same square).***

Invite students to make trades. Any initial backward trade will result in a pair of checkers on one of the squares. Continue this activity until all the checkers are in pairs. Discourage a student who wants to make a trade with checkers that are already paired. One possible sequence of trades is shown here.



When all the checkers are paired, ask a student to remove checkers so that one-half of the number will be on the Minicomputer. The student should remove one checker from each pair of checkers. Perhaps you will have this configuration on the Minicomputer.



T: ***What number is $\frac{1}{2}$ x 1,994?***

S: ***997.***

Write $\frac{1}{2}$ x 1,994 = 997 to one side of the Minicomputer.

T: ***How could we check that 997 is $\frac{1}{2}$ x 1,994?***

S: ***We could calculate 997 + 997.***

T: ***If $\frac{1}{2}$ x 1,994 = 997, then 2 x 997 = 1,994. Calculate 997 + 997 on your paper.***

Conclude that 2 x 997 = 1,994 and $\frac{1}{2}$ x 1,994 = 997.

With the class, decide which estimate was closest to 997.

Center Activity

Write a large number on a paper near a Minicomputer set in a center. Instruct students who visit the center to estimate what one-half of that number is, write their estimate, and then use the Minicomputer to calculate one-half of the number. Change the number periodically.

Writing Activity

Instruct students to write a story in which they need to find one-half of a large number.

Capsule Lesson Summary

Find shortest routes between cities in Nevada. Find which of two cities is closer to a third. Plan a trip starting at a particular city that is approximately 800 km long.

Materials

Teacher
- *UPG-III World of Numbers Poster #2*
- Tape
- Blackline N16

Student
- Nevada map
- Worksheet N16(a) and (b)
- Calculator (optional)

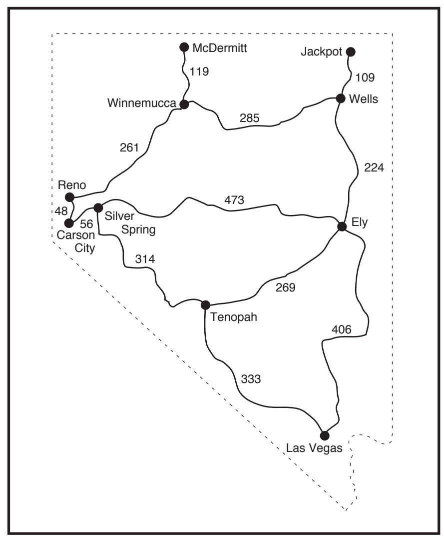
Advance Preparation: Use Blackline N16 to make copies of the Nevada map for students

Description of Lesson

Display *UPG-III World of Numbers Poster #2* and distribute copies of the Nevada map.

Note: You may prefer to hold off distributing individual copies of the map until Problem 2 so that students will give more attention to the poster map and the collective discussion of it.

If you and your students are in Nevada, adjust the following dialogue so that it is appropriate.



T: ***This is a map of one of the states of the United States of America. What is the name of this state? Do you know any of these cities?***

You may need to tell your class that this state is Nevada. Encourage students to share what they know about Nevada and the cities indicated on this map. Point out that the numbers on the map show how far it is in kilometers from one city to another. Trace the road from Las Vegas to Tenopah.

T: ***How far is it from Las Vegas to Tenopah according to this map?*** (333 kilometers)
How far is it from Reno to Winnemucca? (261 kilometers)
How many kilometers long is the shortest route from Las Vegas to Ely? (406 kilometers)
How far is it from Carson City to Silver Spring? (56 kilometers)

For the following problems, you may like to make calculators available for checking results.

Problem 1: Wells to McDermitt

Abbreviate a statement of the problem on the board.

Wells to McDermitt?

T: ***What is the shortest route from Wells to McDermitt?***

S: ***Go from Wells to Winnemucca, and then on to McDermitt.***

Ask a student to trace this route on the poster; then record it on the board.

T: ***How long is this route from Wells to McDermitt? Write your answer on a piece of paper.***

Look at many of the students' answers and help students who are having difficulty. Ask a student to answer aloud and complete the sentence on the board.

Wells to McDermitt?
Wells to Winnemucca
to McDermitt is 404 km.

T: ***How did you calculate the distance from Wells to McDermitt?***

S: ***I added 285 + 119.***

Write the addition problem on the board, and call on a volunteer to solve this problem at the board, explaining each step.

$$\begin{array}{r} 285 \\ +\ 119 \\ \hline 404 \end{array}$$

T: ***Is this the shortest route from Wells to McDermitt? How do you know?***

Let students give explanations, but do not expect well-formed answers involving calculations.

Erase the board before going on to Problem 2. Problem 2 is on Worksheet N16(a).

Problem 2: Silver Spring to Wells

Abbreviate a statement of the problem on the board.

Silver Spring to Wells?

T: ***What is the shortest route from Silver Spring to Wells?***

Some students might believe that the shortest route from Silver Spring to Wells is through Ely. Ask a student to trace this route on the poster; then record it on the board.

T: ***How long is this route from Silver Spring to Wells? Solve the problem on your worksheet.***

Silver Spring to Ely
to Wells is ____ km.

After a few minutes, invite a student to complete the problem at the board.

T: ***How did you calculate this distance?***

S: ***I added 473 + 224.***

T: ***Are you sure this is the shortest route from Silver Spring to Wells?***

$$\begin{array}{r} 473 \\ +\ 224 \\ \hline 697 \end{array}$$

If necessary, tell your class that there is a shorter route from Silver Spring to Wells on this map, and encourage them to find it. When a student suggests that the route through Carson City, Reno, and Winnemucca is shorter, ask a student to trace that route. Then record it on the board.

Silver Spring to Wells?
Silver Spring to Ely to Wells is 697 km.
Silver Spring to Carson City to Reno to Winnemucca
to Wells is _____ km.

T: ***How long is this route from Silver Spring to Wells? Solve the problem on your worksheet.***

After a few minutes, invite a student to complete the problem at the board, explaining the steps.

T: ***How did you calculate the length of this route from Silver Spring to Wells?***

S: ***I added 56 + 48 + 261 + 285.***

$$\begin{array}{r} 56 \\ 48 \\ 261 \\ +\ 285 \\ \hline 650 \end{array}$$

As you or the student writes this addition problem on the board, emphasize lining up hundreds digits in the same column, tens digits in another column, and ones digits in a third column.

Complete the sentence on the board, and conclude that the shortest distance from Silver Spring to Wells is 650 km.

Silver Spring to Wells?
Silver Spring to Ely to Wells is <u>697</u> km.
Silver Spring to Carson City to Reno to Winnemucca to Wells is <u>650</u> km.

T: ***How much shorter is the route from Silver Spring to Wells that goes through Reno than the route that goes through Ely?***

Encourage students to explain their answers by telling about the calculations they do. If necessary, suggest one or both of these calculations. Answers are in the boxes.

$$\begin{array}{r} 650 \\ +\ \boxed{47} \\ \hline 697 \end{array} \qquad \begin{array}{r} 697 \\ -\ 650 \\ \hline \boxed{47} \end{array}$$

Conclude that the route through Reno is 47 km shorter than the route that goes through Ely.

Erase the board before going on to Problem 3. Problem 3 is on Worksheet N16(b).

Problem 3: Las Vegas to Wells and Las Vegas to Silver Spring

T: ***Which city is closer to Las Vegas—Wells or Silver Spring?***

Abbreviate the problem on the board.

Las Vegas to Wells?
Las Vegas to Silver Spring?

The students are likely to tell you that Silver Spring is closer to Las Vegas than Wells, but ask them to calculate the distances so they are sure. Allow a few minutes for independent work. Then ask the students to describe the shortest routes from Las Vegas to Silver Spring and from Las Vegas to Wells, and to tell you the length of each of these routes. Record this information on the board.

Las Vegas to Wells?
Las Vegas to Ely to Wells is <u>630</u> km.
Las Vegas to Silver Spring?
Las Vegas to Tenopah to Silver Spring is <u>647</u> km.

T: ***Which distance is shorter?***

S: ***630 km is less than 647 km.***

T: ***Is Silver Spring or Wells closer to Las Vegas?***

S: ***Wells.***

T: ***How much closer is it from Las Vegas to Wells than it is from Las Vegas to Silver Spring?***

Encourage students to explain their answers by telling about the calculations they do. If necessary, suggest one or both of these calculations. Answers are in the boxes.

$$\begin{array}{r} 630 \\ +\ \boxed{17} \\ \hline 647 \end{array} \qquad \begin{array}{r} 647 \\ -\ 630 \\ \hline \boxed{17} \end{array}$$

Conclude that Wells is 17 km closer to Las Vegas than Silver Spring is.

Erase the board before going on to Problem 4.

Problem 4: Plan a Trip Starting at Ely

T: ***I want everyone to plan a trip that starts at Ely and is approximately 800 km long. Try to make the length of your trip as close to 800 km as possible. You may end your trip in any of the cities on the map. Use your map to help you plan the trip. When you get a trip that is as close as possible to 800 km, trace the route on your map with a pencil or crayon. On your paper, show the calculations you do to find the length of your trip.***

Allow several minutes for independent work. Then choose a student's trip to record on the board. Try to choose a trip that is at least 900 km long for the collective discussion. An example is given here.

T: ***Gary has a trip from Ely to Las Vegas to Tenopah and back to Ely. Did anyone else plan this trip? How long is this trip?***

S: ***1,008 km.***

Ely to Las Vegas to Tenopah to Ely is <u>1,008</u> km.

$$\begin{array}{r} 406 \\ 333 \\ +\ 269 \\ \hline 1{,}008 \end{array}$$

T: ***Is this trip more or less than 800 km long?***

S: ***More.***

T: ***How much longer than 800 km is it?***

S: ***208 km.***

$$\begin{array}{r} 1{,}000 \\ -\ 800 \\ \hline \boxed{208} \end{array} \quad \text{or} \quad \begin{array}{r} 800 \\ +\ \boxed{208} \\ \hline 1{,}008 \end{array}$$

T: ***Is there another trip we could take starting at Ely that is closer to 800 km long?***

Perhaps a student will suggest this trip.

Ely to Silver Spring to Tenopah is <u>787</u> km.

T: ***Is this trip more or less than 800 km long?***

S: ***Less.***

T: ***How much less?***

S: ***13 km.***

Conclude that 787 is closer to 800 than 1,008 is.

$$\begin{array}{r} 787 \\ +\ \boxed{13} \\ \hline 800 \end{array} \qquad \begin{array}{r} 800 \\ -\ 787 \\ \hline \boxed{13} \end{array}$$

Continue letting students suggest trips closer to 800 km long. Another trip with a length close to 800 km is from Ely to Las Vegas and back to Ely. If none of your students discover this trip, you can give them a few clues. For example:

T: ***I am thinking of a trip that has a length even closer to 800 km than 787 km. This trip begins and ends in Ely and visits only one other city. Can you discover the trip I am thinking of?***

Encourage students to find this trip on their own. When a student suggests the trip from Ely to Las Vegas and back to Ely, record this trip and its length on the board.

Ely to Las Vegas to Ely is 812 km.

T: ***How much more is 812 than 800?***

S: ***12.***

T: ***Which number is closer to 800; 787 or 812?***

S: ***812.***

T: ***Why?***

S: ***812 is 12 more than 800, and 787 is 13 less than 800.***

Your students might decide that the trip originating at Ely that is closest to 800 km in length is the one that goes from Ely to Las Vegas and back to Ely. There are other trips that have a length closer to 800 km than this one, but most of them involve traveling between Carson City and Silver Spring several times. Commend any student who finds a trip that is even closer to 800 km than 812 km, but do not insist that your students find such a trip.

Writing Activity

Direct students to write about plans for a trip. They should include a map to show where they will start, where they will travel, and the distance(s).

Center Activity

Put maps like the Nevada map in a center, and pose problems similar to ones in this lesson. Students may pose their own problems.

Home Activity

Send home the Nevada map with one or two problems like Problems 2 or 4. For example:

- How long is the shortest route from Winnemucca to Tenopah?
- Plan a trip starting at Las Vegas that is approximately 1,000 km long.

Capsule Lesson Summary

Play Minicomputer Tug of War. Use the Minicomputer to decide how much money Tracy's mother will have left out of $100 if she buys a bicycle that costs $69. Also decide how much more money she needs to buy a table that costs $253 if she presently has $186.

Materials

Teacher	• Minicomputer set	Student	• Lined paper • Minicomputer set

Description of Lesson

Exercise 1: Minicomputer Tug-of-War

Play one or two games of Minicomputer Tug-of-War as described in Lesson W4. Allow about 30 minutes for Exercise 2.

Exercise 2: Subtraction Problems

Tell the following or a similar story about one of the students in your class.

T: ***Tracy's mother is going to buy a bicycle for Tracy. She puts $100 in her purse and goes shopping for a bicycle. What bills could make exactly $100 in cash?***

Encourage students to suggest many possibilities before continuing the story.

S: ***All $10 bills.***

T: ***How many $10 bills?***

S: ***Ten.***

T: ***10 x 10 = 100, so ten $10 bills is $100.***

S: ***Two $50 bills.***

T: ***2 x 50 = 100, so two $50 bills is $100.***

S: ***Eight $10 bills and twenty $1 bills.***

T: ***8 x 10 = 80 and 80 + 20 = 100, so eight $10 bills and twenty $1 bills is $100.***

T: ***Tracy's mother finds a good bicycle for $69. Does Tracy's mother have enough money with her to buy it?***

S: ***Yes.***

Write this inequality on the board.

$$69 < 100$$

T: ***69 is less than 100. What calculation can we do to find out how much money Tracy's mother will have left after she buys the bicycle?***

S: ***100 – 69.***

Write 100 – 69 = □ on the board and point to it as you ask:

$$100 - 69 = \square$$
$$69 + \square = 100$$

T: ***Is there another way we could say this?***

S: ***69 + the number in the box = 100.***

If necessary, suggest this number sentence yourself and write it on the board.

T: ***How much money will Tracy's mother have left after she buys the bicycle?***

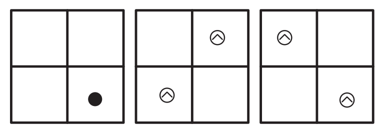
Encourage students to predict or estimate how much money Tracy's mother will have left, and ask for explanations.

T: ***How can we calculate 100 – 69 on the Minicomputer?***

S: ***Put 100 on the Minicomputer and then take off 69.***

S: ***Put 100 on the Minicomputer with regular checkers and then put $\widehat{69}$ on the Minicomputer with negative checkers.***

If no one suggests calculating 100 + $\widehat{69}$, suggest this yourself.
Ask students to put 100 and $\widehat{69}$ on the Minicomputer.



S: ***We need to make some backward trades.***

Invite students to make trades on the Minicomputer until you have the standard configuration for 31. As necessary, remind students they are trying to get pairs of checkers, one positive and one negative, on the same square. Emphasize that such pairs can be removed from the Minicomputer because a number plus its opposite equals 0; for example, 80 + $\widehat{80}$ = 0. Conclude that 100 – 69 = 31. You may like to write the vertical form of this subtraction problem on the board.

T: ***Tracy's mother buys the bicycle and has $31 left over. How could the salesperson give her $31 in change?***

S: ***Three $10 bills and one $1 bill.***

S: ***A $20 bill, a $10 bill, and a $1 bill.***

S: ***Thirty-one $1 bills.***

Erase the board and remove the checkers from the Minicomputer.

T: ***Another day Tracy's mother has $186 with her and she wants to buy a table that costs $253. Does she have enough money?***

S: ***No.***

Write this inequality on the board.

$$253 > 186$$

T: ***253 is more than 186. Tracy's mother does not have enough money to buy the table. How can we find how much more money Tracy's mother needs?***

S: ***Find a number so 186 + that number = 253.***

Write 186 + ☐ = 253 on the board and point to it as you ask,

T: ***Is there another way we could say this?***

S: ***253 – 186 = the number in the box.***

If necessary, suggest these number sentences yourself and write them on the board.

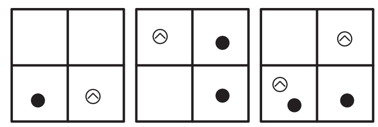
186 + ☐ = 253
253 – 186 = ☐

T: ***How much more money does Tracy's mother need?***

Record students' estimates on the board. Someone very likely will suggest calculating 253 + $\widehat{186}$ on the Minicomputer. Call on students to put 253 and $\widehat{186}$ on the Minicomputer.

Note: It is possible that a student will suggest calculating 186 + $\widehat{253}$ thinking of what is spent as negative. In this case the result is $\widehat{67}$ and that is how much money Tracy's mother lacks.

Invite students to make trades on the Minicomputer until you have the standard configuration for 67. Conclude that Tracy's mother needs \$67 more and that 243 – 186 = 67. You may like to write the vertical form of this subtraction problem on the board.



Allow the class to determine which estimate was the closest to 67.

Write these problems on the board and instruct students to solve them. The stars indicate levels of difficulty.

*	**	***	****
56 + $\widehat{10}$	75 + $\widehat{11}$	248 + $\widehat{4}$	153 + $\widehat{8}$
56 + $\widehat{4}$	75 + $\widehat{14}$	248 + $\widehat{20}$	153 + $\widehat{80}$
56 + $\widehat{40}$	75 + $\widehat{21}$	248 + $\widehat{100}$	153 + $\widehat{89}$

Make using the individual Minicomputers optional since most of these problems can be solved mentally by many students. Do not insist that students use the tool or even paper and pencil for problems they can do in their heads.

Center Activity

Put play money and task cards with whole dollar purchases in a center for students to role play making change. Students can help make the task cards by cutting pictures of toys or other items out of a catalog and writing prices next to the pictures.

N18 THE FUNCTIONS 2X AND $\frac{1}{2}$X

Capsule Lesson Summary

View the functions 2x and $\frac{1}{2}$x as opposites in a simple arrow diagram; that is, the arrow for one is the return arrow for the other. Use return arrows to label the dots in an arrow picture with 2x and –1 arrows. Notice where the greatest and least numbers are in the picture.

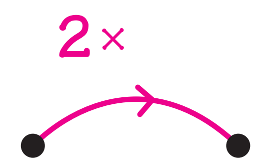
Materials

Teacher		Student	
	• Colored chalk		• Unlined paper • Colored pencils, pens, or crayons • Worksheets N18* and **

Description of Lesson

Exercise 1

Draw this arrow picture on the board and use it to do a mental arithmetic exercise.



T: ***What is this red arrow for?*** (2x)

Point to the starting dot of the red arrow.

T: ***If this dot were for 4, what would the ending dot be for?*** (8)

Repeat this activity assigning other numbers to the starting dot. Occasionally ask students to tell which calculations they did to find the ending number. Here are some possible choices with corresponding ending numbers in parentheses.

13	(26)	100	(200)	24	(48)
23	(46)	3	(6)	25	(50)
8	(16)	103	(206)	26	(52)
5	(10)	30	(60)	126	(252)
0	(0)	130	(260)	1	(2)
9	(18)	134	(268)	15	(30)

Comments on the Preceding Problems

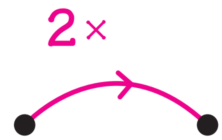
- Since 2 x 0 = 0, the arrow picture on the board would have two dots for 0. Solicit the idea of drawing a loop to show 2 x 0 = 0 and draw it on the board.

- For a problem like 2 x 134 or 2 x 126, you may like to ask students to write their answers on paper. Then, as students explain, write the corresponding addition problem as well as the vertical and horizontal forms of the multiplication number sentence on the board.

$$\begin{array}{r} 134 \\ +\,134 \\ \hline 268 \end{array} \qquad 2 \times 134 = 268 \qquad \begin{array}{r} 134 \\ \times\ \ 2 \\ \hline 268 \end{array}$$

$$\begin{array}{r} 126 \\ +\,126 \\ \hline 252 \end{array} \qquad \begin{array}{r} 2 \times 100 = 200 \\ 2 \times 26 = \ \ 52 \\ \hline 2 \times 126 = 252 \end{array} \qquad \begin{array}{r} 126 \\ \times\ \ 2 \\ \hline 252 \end{array}$$

Point to the ending dot.



T: ***If this number were 12, what would the starting dot be for?***

S: ***6.***

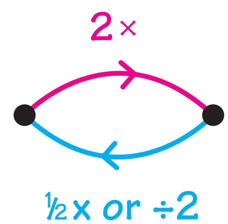
T: ***How do you know?***

Any of the following calculations are good explanations: $6 + 6 = 12$ $\quad$ $\frac{1}{2} \times 12 = 6$ $\quad$ $2 \times 6 = 12$ $\quad$ $12 \div 2 = 6$

Repeat this activity assigning other numbers to the ending dot. Frequently ask students to tell which calculations they did to find the starting number. Here are some possible choices with corresponding starting numbers in parentheses.

18 (9)	600 (300)	20 (10)
100 (50)	6,000 (3,000)	28 (14)

Draw the return arrow in blue.



T: ***The red arrow is for 2x. What could the blue arrow be for?***

S: ***½x.***

S: ***÷ 2.***

Point to the starting dot for the blue arrow.

T: ***If this dot were for 60, what number would the other dot be for?***

S: ***30.***

Repeat this activity assigning other starting numbers to the blue arrow. Occasionally ask students to tell which calculations they did to find the ending number of the blue (½x) arrow. Here are some possible choices with corresponding ending numbers of the ½x arrow in parentheses.

60 (30)	114 (57)	5 (2½)
64 (32)	40 (20)	8 (4)
200 (100)	50 (25)	9 (4½)
210 (105)	58 (29)	3 (1½)
100 (50)	4 (2)	10 (5)
106 (53)	6 (3)	11 (5½)

Comments on the Preceding Problems

- For a problem like ½ x 114 or ½ x 58, ask students to write their answers on paper. Then as students explain what they did to find the answer, record the corresponding number sentences on the board. Perhaps these calculations will be suggested.

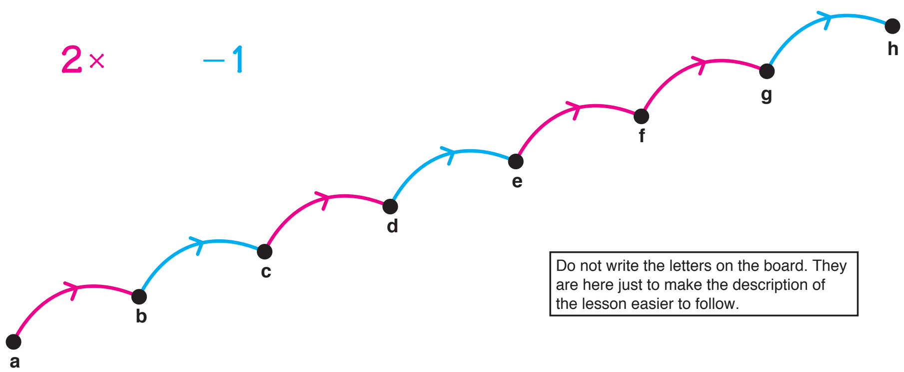
$\frac{1}{2} \times 100 = 50$	$\frac{1}{2} \times 50 = 25$
$\frac{1}{2} \times 14 = 7$	$\frac{1}{2} \times 8 = 4$
$\frac{1}{2} \times 114 = 57$	$\frac{1}{2} \times 58 = 29$

- For at least one of the calculations resulting in a non-integer, ask a student to write the corresponding number sentence on the board.

$$\frac{1}{2} \times 11 = 5\frac{1}{2}$$

Allow students to work with a partner for this exercise, but direct everyone to draw his or her own arrow picture.

Distribute unlined paper and colored pencils. Draw this arrow picture on the board and ask the students to copy it very carefully onto their papers.



Quickly check the students' drawings or ask partners to check each other's drawings to make sure the color, the order, and the direction of the arrows are correct.

T: ***Where do you think the greatest number will be in this arrow picture? How do you know?***

Let several students answer. Commend any student who gives a good explanation but do not say where the greatest number is in the arrow picture at this moment.

T: ***Where do you think the least number will be in this arrow picture? How do you know?***

Let several students answer, but do not say where the least number is in the arrow picture yet.

Note: If all the numbers in this arrow picture are more than 1, the greatest number is at **g** and the least is at **a**. This is for your information only.

Label **h** 115.

T: ***If this number*** (at **h**) ***is 115, what are these other numbers? Label all the dots in your arrow picture.***

Allow students to work with their partners for a few minutes. Look at many of their papers since several students might have difficulty getting started. If so, label **g** and **h** collectively. A possible dialogue is given here.

Perhaps several students have incorrectly labeled **g** 114.

T: ***Many of you have labeled this dot 114.***

Trace the blue arrow from **g** (114) to **h** (115).

T: ***This blue arrow is for –1. Is 114 – 1 = 115?*** (No) ***This number cannot be 114. What should it be? How do you know?***

S: ***116, because 115 + 1 = 116 (or 116 – 1 = 115).***

Trace an arrow from 115 to 116.

T: ***If we draw an arrow from 115 to 116, what could this arrow be for?*** (+1)
The blue arrow is for –1, so the return (opposite) arrow is for +1.

Draw the return arrow in another color.

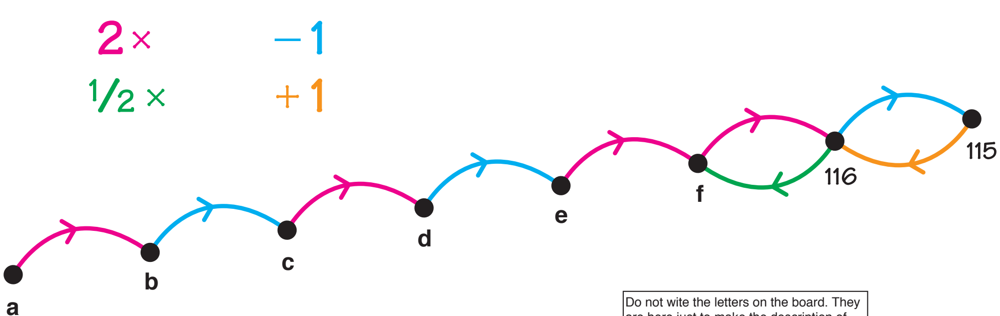
Trace the red arrow from **f** to 116.

T: ***This red arrow goes from this dot (f) to 116. But we need to go from 116 to this dot (f). We could use a return arrow.***

Draw an arrow from 116 to **f** in still another color (for example, green).

T: ***What could the green arrow be for?***

S: ***$\frac{1}{2}$x.***



Do not wite the letters on the board. They are here just to make the description of the lesson easier to follow.

T: ***Calculate $\frac{1}{2}$ x 116. How do you suggest we do this calculation?***

S: ***$\frac{1}{2}$ x 100 = 50 and $\frac{1}{2}$ x 16 = 8, so $\frac{1}{2}$ x 116 = 58.***

Record these number sentences on the board.

Label **f** 58.

$$\frac{1}{2} \times 100 = 50$$
$$\frac{1}{2} \times 16 = 8$$
$$\frac{1}{2} \times 116 = 58$$

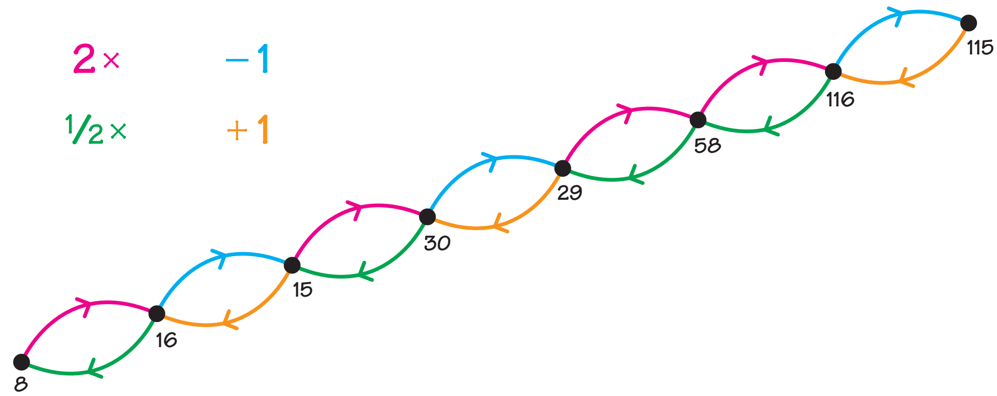
T: ***Where can we draw another $\frac{1}{2}$x arrow?***

Continue until all the $\frac{1}{2}$x arrows are drawn.

T: ***Where can we draw another +1 arrow?***

Continue until all the +1 arrows are drawn.

Allow a few minutes for students to label dots in their arrow pictures. Invite students who have completed their own arrow pictures to label the dots on the board, or to extend their arrow picture further.



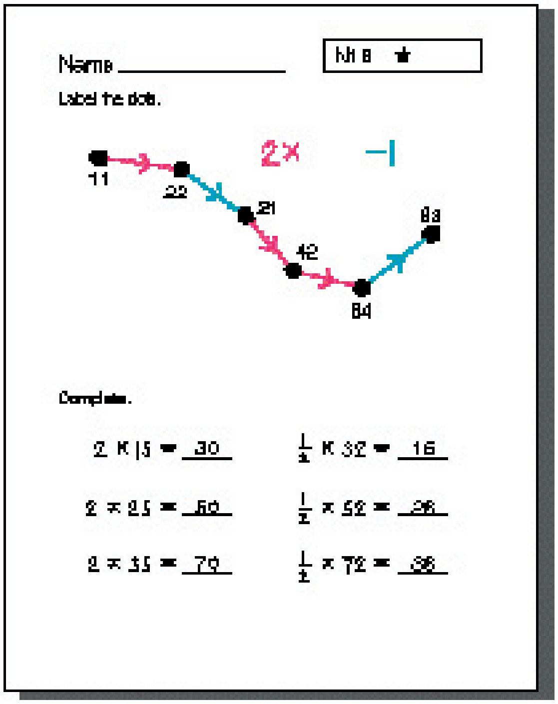
Notice where the greatest and the least numbers are in the picture.

Note: If you think the calculations in the above situation are too difficult for your class, you could start with 19 or 99 at **h** instead of 115.

Worksheets N18* and ** are available for independent work.

Home Activity

This is a good time to send a letter to parents/guardians about mental arithmetic. Blackline N18 has a sample letter.


Name ____________ N18 ★

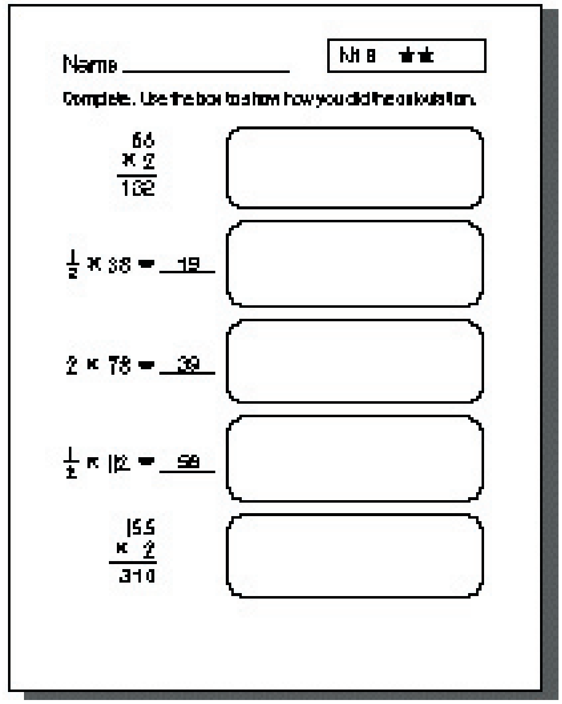
Label the dots.

Complete.

2 × 15 = 30   ½ × 32 = 16

2 × 25 = 50   ½ × 52 = 26

2 × 35 = 70   ½ × 72 = 36


Name ____________ N18 ★★

Complete. Use the box to show how you did the calculation.

66 × 2 = 132

½ × 38 = 19

2 × 78 = 39

½ × 112 = 56

155 × 2 = 310

Capsule Lesson Summary

Use the calculator to pose and solve some addition problems with a missing addend. Solve addition puzzles where sometimes one number in a calculation is missing and other times there are missing digits in the numbers.

Materials

Teacher
- Colored chalk
- Worksheets N19(a) and (b)

Student
- Calculator

Description of Lesson

Exercise 1

The following activity works best with an overhead calculator, but you can use a class calculator. Allow students to have calculators mostly for checking purposes.

T: ***I'm going to hide a number in the calculator. Let's see if you can guess my number.***

Put 68 on the display of the overhead (or class) calculator and cover the display with your hand. Be sure to do this before putting the calculator on the overhead projector so the class does not see what keys you press.

T: ***I've covered the display so you cannot see my number. Now I press [+] [2] [2] [=].***

Let students see you press [+] [2] [2] [=], and then show them that 90 is on the display.

T: ***Do you know what number I hid on the calculator?***

S: ***68, because 68 + 22 = 90.***

S: ***68, because 90 – 22 = 68.***

Let students use their calculators to find 68 and then to check the result by pressing [6] [8] [+] [2] [2] [=] to see 90 on the display.

Repeat this activity several times with some of the following examples:

Hide	Press	Show
75	[+] [3] [6] [=]	111
32	[×] [2] [+] [2] [5] [=]	89
48	[÷] [2] [+] [1] [6] [=]	40

Exercise 2

Announce to the class that today you are going to give them some addition puzzles to solve. Distribute copies of Worksheet N19(a) and refer students to the first problem. Use whatever story you like to explain the problem.

N19

T: ***I did some addition problems, but an eraser gremlin came along and erased one of my numbers each time.***

Here 235 plus a missing number is 377. Can you find the missing number?

$$\begin{array}{r} 235 \\ + \\ \hline 377 \end{array}$$

Allow a minute or two for students to work individually. Then call on a student to explain how to find the missing number while completing the problem at the board.

Accept any reasonable explanations.

S: ***I started in the ones place and thought, 5 plus what number is 7? 5 + 2 = 7, so it must be 2. Then 3 + 4 = 7, so 4 is in the tens place; and 2 + 1 = 3, so 1 is in the hundreds place.***

S: ***I subtracted 377 – 235 = 142.***

$$\begin{array}{r} 235 \\ + \boxed{142} \\ \hline 377 \end{array}$$

Let another student check the calculation, and then repeat this activity solving one or two other problems on the worksheet. Include an example where the resulting calculation involves a carry. In this example, the missing number is in the box.

S: ***In the ones place I thought, 9 plus what number ends in 1? 9 + 2 = 11, so it must be 2. But 11 is 1 ten and 1, so I put another 1 in the tens place and thought, 1 + 5 + what number is 8? Then, in the hundreds place 0 + 7 = 7, and in the thousands place 1 + 4 = 5.***

$$\begin{array}{r} 1{,}059 \\ + \boxed{4{,}722} \\ \hline 5{,}781 \end{array}$$

Let students work independently on other problems on the worksheet. You need not wait until everyone finishes the worksheet before going on to Exercise 3. Students can return to this worksheet later.

Exercise 3

Refer students to Worksheet N19(b) and write the first problem from the worksheet on the board.

T: ***These are also addition puzzles. This time the eraser gremlin erased some of the digits. I put boxes where a digit was erased. Can you fill in the boxes? One digit goes in each box.***

$$\begin{array}{r} 2\,3\,\square \\ + 1\,\square\,6 \\ \hline 3\,5\,9 \end{array}$$

Allow a minute or two for students to work individually. Then call on a student to complete the problem at the board and explain how to fill in the boxes.

S: ***3 goes in this box, because 3 + 6 = 9.***
2 goes in this box, because 3 (tens) + 2 (tens) = 5 (tens).

$$\begin{array}{r} 2\,3\,\boxed{3} \\ + 1\,\boxed{2}\,6 \\ \hline 3\,5\,9 \end{array}$$

Let another student check the calculation. Then repeat this activity with a problem such as the one illustrated here.

```
  1□4
+  6□
-----
  □08
```

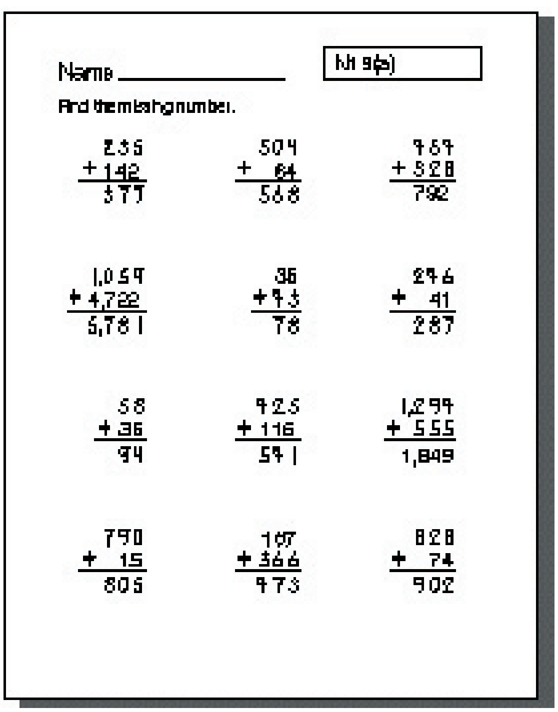
Let students continue working independently on the other problems on the worksheet. When students complete the worksheets, you may like to suggest that they trade papers with other students or use calculators to check their work.

Writing Activity

Ask students to write a story context for one or two of the problems on Worksheet N19(a).

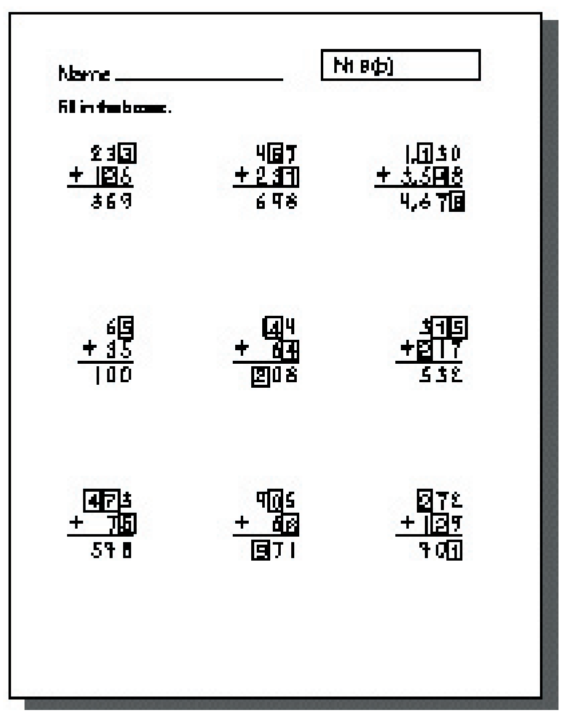
Home Activity

Send home a few addition puzzles for students to do with a family member.


Name ____________ N19(a)

Find the missing number.

235 + 142 = 377
504 + 64 = 568
264 + 528 = 792
1,059 + 4,722 = 5,781
35 + 43 = 78
246 + 41 = 287
58 + 36 = 94
425 + 116 = 541
1,294 + 555 = 1,849
790 + 15 = 805
107 + 366 = 473
828 + 74 = 902


Name ____________ N19(b)

Fill in the boxes.

N20 +4 ARROW ROADS

Capsule Lesson Summary

Use a calculator to observe multiples of 4. In a string picture, locate numbers that are multiples of 4 and numbers that are not. Display multiples of 4 in a +4 arrow road with the number 0 on it. Notice numbers that are not on this +4 arrow road and draw other +4 arrow roads to include them. Observe patterns in an arrow picture with four +4 arrow roads.

Materials

Teacher
- Colored chalk

Student
- Calculator
- Paper
- Colored pencils, pens, or crayons

Description of Lesson

Exercise 1

Organize a group of ten students to count by fours from 0 to 40. Direct one student to start the count by saying, “0, 4,” and then tell the others to continue the counting in turn.

T: ***How can we make the calculator count by fours starting at 0?***

Direct all the students to prepare their calculators to count by fours and then to count along with the calculator to about 80.

S: ***Press [0] [+] [4] [=] [=] [=] and so on.***

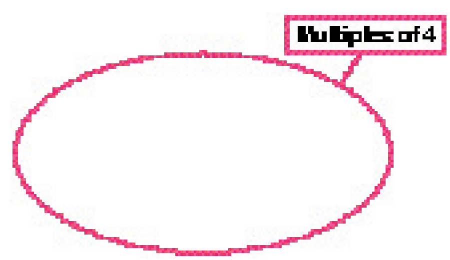
T: ***What kind of numbers do we get when we count by fours from 0?***

S: ***Even numbers.***

S: ***Multiples of 4.***

T: ***If we start at 0 and press [0] [+] [4] [=] [=] [=] and so on, all the numbers that appear on the display will be multiples of four.***

Draw this string picture on the board.



T: ***Give me some numbers to put inside this string.***

Let students suggest many multiples of 4 and record them in the string picture. If the students are not suggesting a wide range of numbers, ask for a multiple of 4 greater (or less) than a particular number such as 100 or 1,000. Explain why a number is (or is not) a multiple of 4 whenever the explanation would be fairly simple and concise. A sample dialogue follows.

S: ***100.***

T: ***100 is a multiple of 4, because 25 x 4 = 100.***

S: ***104.***

T: ***104 is a multiple of 4, because it is 4 more than 100.***

S: ***90.***

T: ***We know 80 is a multiple of 4. 80 + 4 = …?*** (84) ***84 + 4 = …?*** (88) ***88 + 4 = …?*** (92) ***We skipped 90, so 90 is not a multiple of 4.***

Record 90 outside the string.

S: ***412.***

T: ***400 is a multiple of 4, and 12 is a multiple of 4, so 412 is a multiple of 4.***

Note: Since 100 is a multiple of 4, the last two digits of a whole number indicate whether or not it is a multiple of 4. For example, 1,372 is a multiple of 4 because 72 is a multiple of 4; 494 is not a multiple of 4 because 94 is not a multiple of 4. This is for your information only.

Continue this activity until many students have participated.

T: ***Give me some numbers to put outside this string (that are not multiples of 4).***

When a number is suggested, ask students to explain why a number is not a multiple of 4. Note that the number is 1, 2, or 3 more or less than a known multiple of 4.

Exercise 2

T: ***What kind of an arrow picture would include all the multiples of 4?***

S: ***A +4 arrow road with 0 (or 4) on it.***

Draw a +4 arrow picture on the board. After drawing several arrows, ask if you've drawn enough +4 arrows. Students should tell you that the road goes on and on.

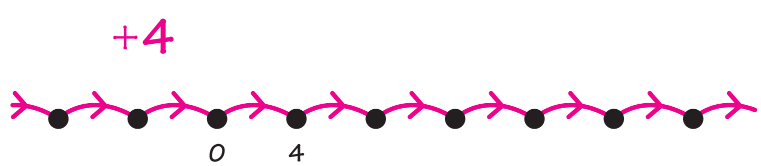
T: ***I will show that this arrow road continues by drawing an arrow without an ending dot.***

T: ***Are there any numbers less than 0 that are multiples of 4?***

S: ***Yes;*** $\widehat{4}$, $\widehat{8}$, $\widehat{12}$, $\widehat{16}$, ***… .***

Extend the arrow road a few dots to the left of 0.

T: ***I will show that there are more multiples of 4 that are less than 0 by drawing an arrow without a starting dot.***



Invite students to label the dots. Observe that even though you have only drawn a few dots on the board, all the multiples of 4 are on this arrow road.

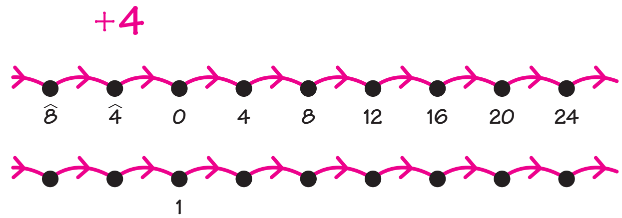
T: ***Are there any whole numbers not on this arrow road?*** (Yes)
What are some of those numbers? How do you know?

Let students suggest many numbers that are not multiples of 4. When a student indicates that a certain number is skipped, indicate where it is skipped. For example, 18 is skipped because 18 is between 16 and 20.

T: ***Is 1 on this arrow road?***

S: ***No.***

T: ***But we can put 1 on another +4 arrow road.***



Draw a second arrow road directly below the first one. Align the dots of the two arrow roads and label the dot for 1 directly below the dot for 0. Note that this arrow road also extends in both directions.

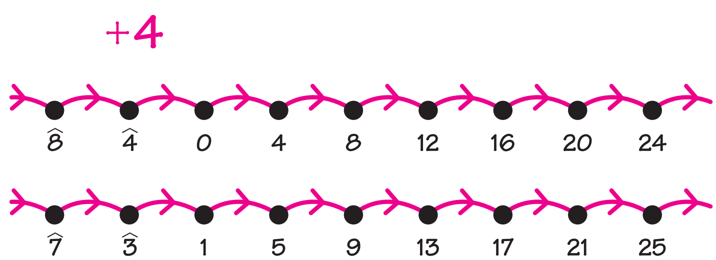
T: ***Do you know some other numbers that are on this second arrow road?***

S: ***9 is on the second arrow road because 1 + 4 = 5, and 5 + 4 = 9.***

S: ***13 is on the second arrow road because 12 is a multiple of 4, and 12 + 1 = 13.***

S: ***20 is on the first arrow road, so 20 +1 is on the second arrow road.***

Invite students to label dots in the second arrow road.



T: ***Do you notice any patterns in this arrow picture?***

There are several patterns that your students might observe. For example:

- All the numbers in the first arrow road are even.
- All the numbers in the second arrow road are odd.
- Each number in the second arrow road is 1 more than the number directly above it.
- The last digits of the whole numbers in each road form a repeating sequence. In the first road the repeating part of the sequence is 0, 4, 8, 2, 6. In the second road the repeating part of the sequence is 1, 5, 9, 3, 7.

Observe that each number in the second arrow road is 1 more than the number directly above it. If necessary, use the number line to help your students see that $\widehat{7}$ is 1 more than $\widehat{8}$, and that $\widehat{3}$ is 1 more than $\widehat{4}$.

T: ***Are there whole numbers that are not on either of these arrow roads? How do you know?***

N20

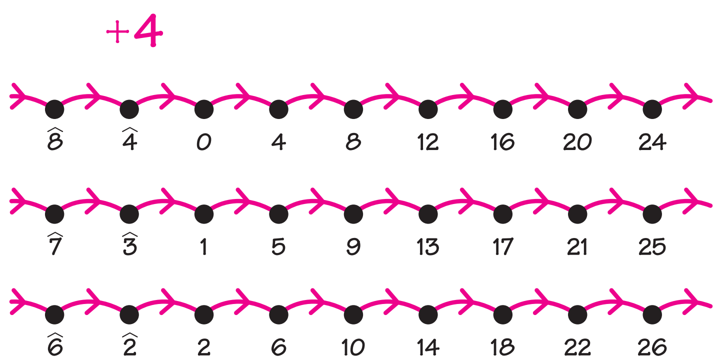
Encourage the class to find several such numbers and to explain why each number is not on either of these +4 arrow roads.

S: ***2 is not on either of these roads. Both roads skip over 2.***

T: ***Let's draw a +4 arrow road with 2 on it.***

Let students find numbers on a +4 arrow road with 2 on it. All the numbers on such an arrow road are 2 more than a multiple of 4. For example, 18 (16 + 2), 50 (48 + 2), and $\widehat{6}$ ($\widehat{8}$ + 2) are on this third arrow road.

Invite students to label the dots on the third arrow road.



T: ***What patterns do you see in this arrow picture?***

There are several patterns the students might mention, but emphasize the pattern that each number is 1 more than the number directly above it.

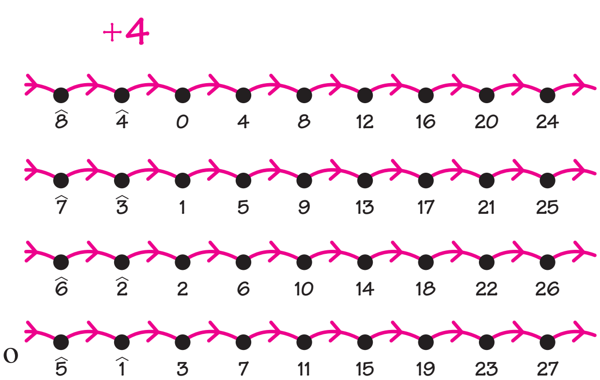
T: ***Are there whole numbers that are skipped by all three of these +4 arrow roads?*** (Yes) ***What are they? How do you know they are skipped?***

Note: All the integers 1 less (or 3 more) than a multiple of 4 are not on one of these +4 arrow roads. For example, 23 (24 – 1), $\widehat{9}$ ($\widehat{8}$ – 1), and 999 (1,000 – 1) are skipped on all three roads.

S: ***3 is not on any of these arrow roads. The first arrow road jumps from 0 to 4, the second arrow road jumps from 1 to 5, and the third arrow road jumps from 2 to 6.***

T: ***Let's draw another +4 arrow road with the number 3 on it.***

Draw a fourth arrow road below the third arrow road and align the dots. Put the dot for 3 directly below the dot for 2. Call on students to suggest many numbers that are on the fourth arrow road before asking students to label the dots.



T: ***What patterns do you see?***

There are several patterns that your students might o

- All the numbers on the first and third arrow roads are even.
- All the numbers on the second and fourth arrow roads are odd.
- Each number is 1 more than the number directly above it.
- All of the integers between $\hat{8}$ and 27 are represented by dots on the board.
- The last digits of the whole numbers in each arrow road form a repeating sequence. In the first and third arrow roads, the repeated part of the sequence is 0, 4, 8, 2, 6. In the second and fourth arrow roads, the repeated part of the sequence is 1, 5, 9, 3, 7.

T: ***Is there a whole number that would not be on one of these +4 arrow roads?*** (No)

Someone might claim that there is a number that is not on any of these +4 arrow roads. Ask the rest of the class if they agree or disagree, and why. Remind the class that each of the arrow roads continues forever in both directions. A possible dialogue is given here.

S: ***234 is not on any of these +4 arrow roads.***

T: ***How do you know? Remember that these arrow roads go on and on.***

S: ***If 234 is on one of these +4 arrow roads, it would be on either the first or the third one because 234 is an even number.***

S: ***We could find out which arrow road 234 is on by starting at 200 and counting. 200 is on the first arrow road because it is a multiple of 4; 201 is on the second road; 202 is on the third road; 203 is on the fourth road; 204 is on the first road; and so on. Eventually we would find 234 on one of these +4 arrow roads.***

T: ***It may be difficult to decide which of these arrow roads 234 would be on, but can we be sure it is on one of them? How could we use the calculator to help us decide which arrow road 234 is on?***

S: ***Test the first arrow road: start at 0 and press [+] [4] [=] [=] [=] … and see if 234 appears.***

S: ***Test the third arrow road: start at 2 and press [+] [4] [=] [=] [=] … and see if 234 appears.***

Suggest students use their calculators to find that 234 is on the third arrow road. Repeat this kind of check to find which arrow road another number, such as 357, is on.

Most of your students should be confident that all of the whole numbers are on one of these +4 arrow roads.

Extension Activity

Pose the problem of drawing +5 arrow roads to include all whole numbers. In this case, students should find they need to draw five +5 arrow roads.

N21 3X AND $\frac{1}{3}$X ON THE ________

Capsule Lesson Summary

Calculate 3 x 175 and $\frac{1}{3}$ x 378 on the Minicomputer. Do similar calculations as an individual or partner activity.

Materials

Teacher
- Minicomputer set

Student
- Minicomputer set
- Paper

Description of Lesson

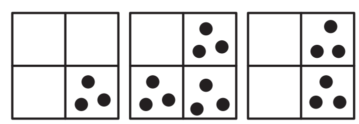
Exercise 1: Mental Arithmetic

Begin the lesson with a mental arithmetic activity involving 3x and $\frac{1}{3}$x. Use some of the following or similar calculations appropriate for the abilities of your students.

3 x 10 (30)	3 x 20 (60)	3 x 25 (75)
$\frac{1}{3}$ x 30 (10)	$\frac{1}{3}$ x 60 (20)	$\frac{1}{3}$ x 75 (25)
3 x 11 (33)	3 x 30 (90)	3 x 15 (45)
$\frac{1}{3}$ x 33 (11)	$\frac{1}{3}$ x 90 (30)	$\frac{1}{3}$ x 45 (15)
3 x 13 (39)	3 x 21 (63)	3 x 16 (48)
$\frac{1}{3}$ x 39 (13)	$\frac{1}{3}$ x 63 (21)	$\frac{1}{3}$ x 48 (16)

Exercise 2

Put this configuration on the Minicomputer.



3 x 175

T: ***What calculation are we doing on the Minicomputer?***

S: ***3 x 175. You put 175 on the Minicomputer three times.***

T: ***What calculation are we doing on the ones board?***

S: ***3 x 5.***

T: ***What number is 3 x 5?***

S: ***15.***

Write 3 x 5 = 15 on the board.

T: ***What calculation are we doing on the tens board?***

S: ***3 x 70.***

T: ***What number is 3 x 70?***

S: ***210.***

Write 3 x 70 = 210 on the board below 3 x 5 = 15.

T: ***What calculation are we doing on the hundreds board?***

S: ***3 x 100.***

T: ***What number is 3 x 100?***

S: ***300.***

$$3 \times 5 = 15$$
$$3 \times 70 = 210$$
$$3 \times 100 = 300$$

Write 3 x 100 = 300 on the board.

T: ***How could we calculate 3 x 175?***

S: ***We could do the addition problem 175 + 175 + 175.***

Record this problem on the board, but do not calculate the answer.

$$\begin{array}{r} 175 \\ 175 \\ +\ 175 \\ \hline \end{array}$$

S: ***We could make trades on the Minicomputer.***

S: ***We could add 15 + 210 + 300.***

If no one suggests adding 15 + 210 + 300, suggest this yourself by referring to the multiplication facts recorded earlier on the board.

Record the problems suggested by students that correspond to any method of calculating 3 x 175. Let the class choose a method to calculate 3 x 175, and then check the answer by calculating 3 x 175 using a second method. For example, your class might choose to calculate 3 x 175 by solving the addition problem 175 + 175 + 175. Then check the calculation by solving another addition problem, 15 + 210 + 300. You may also like to present the multiplication problem in the following way:

$$\begin{array}{r} {}^{2}\,{}^{1} \\ 175 \\ 175 \\ +175 \\ \hline 525 \end{array} \qquad \begin{array}{r} 175 \\ \times\ \ 3 \\ \hline 15 \\ 210 \\ +\ 300 \\ \hline 525 \end{array}$$

If some students insist on making trades on the Minicomputer, call on one of them to do so while the rest of the class completes the addition problems. Conclude that 3 x 175 = 525.

T: ***Let's make up a story problem for the calculation we just completed. I'll make up a story first, and then you can make up stories.***

Use this or a similar story.

T: ***I took a trip last summer to visit a friend. Each day for 3 days I drove exactly 175 miles. How far did I drive to visit my friend?***

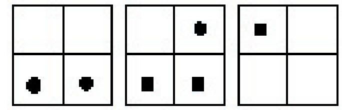
S: ***525 miles.***

T: ***Now it's your turn.***

Let students work individually or in groups. Call on several students (groups) to share their stories.

Exercise 3

Put 378 on the Minicomputer.



T: ***What number is this?***

S: ***378.***

T: ***Suppose 378 is the number of books donated to three schools. The three schools need to share these books, so each school gets the same number. Each school will get one-third of the books.***

Write $^1/_3$ x 378 = ? on the board.

T: ***Is $^1/_3$ x 378 more than 50?***

S: ***Yes, because 3 x 50 = 150.***

T: ***Is $^1/_3$ x 378 more than 100?***

S: ***Yes, because 3 x 100 = 300, and 378 is more than 300.***

T: ***Is $^1/_3$ x 378 more than 200?***

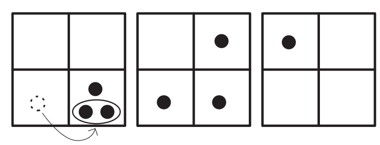
S: ***No, because 3 x 200 = 600, and 600 is more than 378.***

T: ***We know that $^1/_3$ x 378 is between 100 and 200.***

Allow students to estimate the answer. On the board, record only estimates between 100 and 200.

T: ***How can we calculate $^1/_3$ x 378?***

S: ***Make backward trades until all the checkers are in groups of three checkers.***

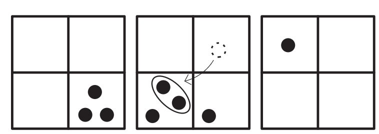


T: ***Who can make a trade that will result in three checkers on the same square?***

S: ***200 = 100 + 100.***

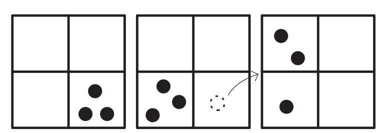
T: ***Now we have a group of three checkers on the 100-square. Each school gets 100 books. Let's push these three checkers together so we will remember not to break up this group.***

S: ***40 = 20 + 20.***



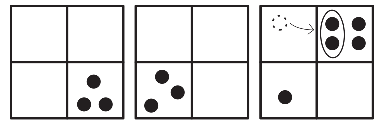
T: ***We have another group of three checkers on the 20-square. Each school gets another 20 books. Let's push them together.***

Now there are only two checkers not in a group of three. Who can make a trade to help us form another group of three checkers?

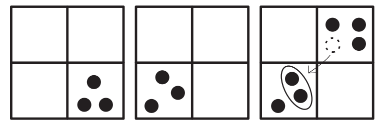


S: ***10 = 8 + 2.***

S: ***8 = 4 + 4.***



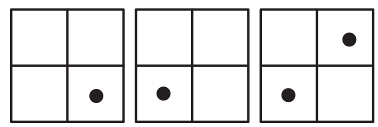
T: ***Now we have a group of three checkers on the 4-square. Let's push this group of three checkers together and make a trade with this extra checker on the 4-square.***



S: ***4 = 2 + 2.***

T: ***Now all the checkers are in groups of three. Who can remove some checkers so that one-third of this number (one school's share of the books) will be on the Minicomputer?***

The volunteer should remove two checkers from each group of three checkers on the Minicomputer.



Complete the number sentence on the board.

$$\frac{1}{3} \times 378 = 126$$
$$378 \div 3 = 126$$

Let the students determine which estimate was closest to 126.

Record some multiplication problems on the board and ask the students to do the calculations using any methods they prefer. Encourage students to use their individual Minicomputers as necessary. Problems of varying levels of difficulty are suggested below. You may like to allow students to work with partners on these problems.

*	**	***
2 x 74	3 x 164	3 x 468
3 x 36	$\frac{1}{3}$ x 159	$\frac{1}{3}$ x 528
$\frac{1}{3}$ x 36	3 x 93	4 x 484
3 x 153	$\frac{1}{3}$ x 156	$\frac{1}{4}$ x 136
$\frac{1}{3}$ x 96		

N22 SUBTRACTION AND COMPOSITION

Capsule Lesson Summary

Locate numbers on a number line where the marks are for even numbers. Make –10 jumps on the number line. Find the composite of several –10 jumps. Solve a subtraction problem by building an arrow road with arrows that compose to give the appropriate subtraction function; use as many –10 arrows as possible.

Materials

Teacher
- Colored chalk

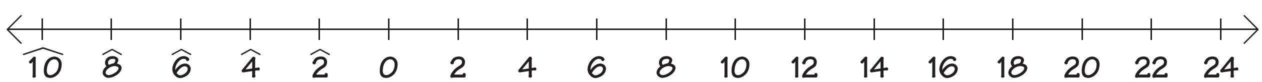
Student
- Paper
- Worksheets N22*, **, ***, and ****

Description of Lesson

Exercise 1: Mental Arithmetic

Draw a line on the board and graduate it into intervals about six centimeters in length. Locate 0 at one of the marks and locate $\hat{6}$ three marks to the left. Allow room for number sentences to be written below the number line.

Invite students, one at a time, to associate numbers with each of the marks on this number line.



Point to the number line.

T: ***What do all these numbers have in common?***

S: ***They are all multiples of 2.***

S: ***They are all even numbers***

T: ***Where would odd numbers be on this number line?***

Invite students to point to where odd numbers would be on this number line. Emphasize that between consecutive even numbers there is an odd number. Ask students to locate several specific odd numbers such as 7, $\hat{3}$, or 19 on this number line.

Write –10 in red near the number line, and invite students to trace –10 arrows or –10 jumps on the number line. As each arrow is traced, record the appropriate number sentence on the board.

When you have about ten number sentences written on the board, ask the students if they see a pattern. Emphasize that the last digit of a number does not change when you subtract 10 except for numbers between 0 and 10.

T: ***Suppose we start at 15 and make a –10 jump. Where would we land?***

S: ***At 5.***

T: ***If we start at 30 and make a –10 jump, … ?***

S: ***At 20.***

Continue this activity using some of the following or similar starting numbers. Choose problems appropriate for the abilities of your students.

31	(21)	10	(0)	19	(9)
50	(40)	100	(90)	76	(66)
52	(42)	106	(96)	43	(33)
20	(10)	200	(190)	69	(59)
25	(15)	300	(290)	5	($\hat{5}$)
35	(25)	1,000	(990)	9	($\hat{1}$)
45	(35)	1,001	(991)	2	($\hat{8}$)

Refer to the number line if the students have difficulty calculating 5 – 10, 9 – 10, or 2 – 10.

T: ***If we start at 24 and make a –10 jump, where will we land?***

S: ***At 14.***

T: ***24 – 10 = 14.***

Write this calculation on the board.

$$\begin{array}{r} 24 \\ -\ 10 \\ \hline 14 \\ -\ 10 \\ \hline 4 \end{array}$$

T: ***If we start at 24 and make two –10 jumps, where will we land?***

S: ***At 4.***

T: ***24 – 10 = 14 and 14 – 10 = 4.***

T: ***Instead of subtracting 10 twice, what number could we subtract?***

S: ***20; 24 – 20 = 4.***

$$\begin{array}{r} 24 \\ -\ 20 \\ \hline 4 \end{array}$$

T: ***Let's calculate 61 – 20. How many times do we need to subtract 10?***

S: ***Twice.***

T: ***Yes, subtracting 10 twice gives the same result as subtracting 20. What number is 61 – 20? Write it on your paper (or whisper it to your neighbor).***

Check several students' responses and then do the calculations on the board. The following illustrations on the right show what is written on the board during the dialogue.

T: ***If we subtract 10…***
… and then subtract 10 again…

…it is the same as if we subtract 20.

$$\begin{array}{r} 61 \\ -\ 10 \\ \hline 51 \\ -\ 10 \\ \hline 41 \end{array} \qquad \begin{array}{r} 61 \\ -\ 20 \\ \hline 41 \end{array}$$

T: ***Let's calculate 104 – 30. How many times do we need to subtract 10?***

S: ***Three times.***

Write the calculation on the board.

$$\begin{array}{r} 104 \\ -\ 30 \\ \hline \end{array}$$

T: ***Before we do this calculation, what can you tell me about the last digit of the answer?***

S: ***The last digit is 4, because we are subtracting tens.***

T: ***Is 104 – 30 more or less than 100?***

S: ***Less.***

T: ***More or less than 50?***

S: ***More.***

T: ***We know that 104 – 30 is between 50 and 100 and that the last digit of this number is 4. What number is 104 – 30? Write it on your paper (or whisper it to your neighbor).***

Check several students' responses and then solve the problem on the board.

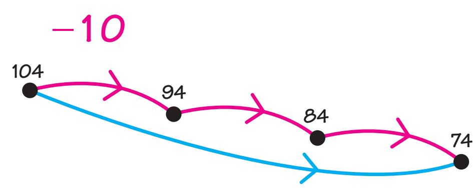
$$\begin{array}{r} 104 \\ -\ 10 \\ \hline 94 \\ -\ 10 \\ \hline 84 \\ -\ 10 \\ \hline 74 \end{array} \qquad \begin{array}{r} 104 \\ -\ 30 \\ \hline 74 \end{array}$$

T: ***We can show this calculation with a –10 arrow picture. How many –10 arrows do we need to draw?***

S: ***Three.***

Draw an arrow picture with three –10 arrows on the board.

Invite a student to label the dots. Then draw a blue arrow from 104 to 74.



T: ***What could this blue arrow be for?***

S: ***–30.***

Write –30 in blue on the board near the arrow picture.
Erase the labels for the dots and point to the starting dot.

T: ***If this number were 50, what number would the ending dot be for?***

S: ***20.***

T: ***If this number were 41, … ?***

S: ***11.***

Continue this activity with the following or similar numbers. Choose problems appropriate for the abilities of your students.

79 (49)	95 (65)
112 (82)	268 (238)

Write this calculation on the board.

$$\begin{array}{r} 120 \\ -\ 37 \\ \hline \end{array}$$

T: ***Earlier we wanted to subtract 30, so we subtracted 10 three times. In this problem we want to subtract 37. What do you suggest?***

S: ***Subtract 10 three times and then subtract 7.***

Be prepared for some student to suggest “borrowing.” Accept this as a good method, but encourage students to look for other methods that are easier to do mentally.

Extend the arrow picture on the board by adding a –7 arrow. Label the starting dot 120. Invite students to label the other dots. Draw an arrow from 120 to 83.

T: ***What could this arrow be for?***

S: ***– 37.***

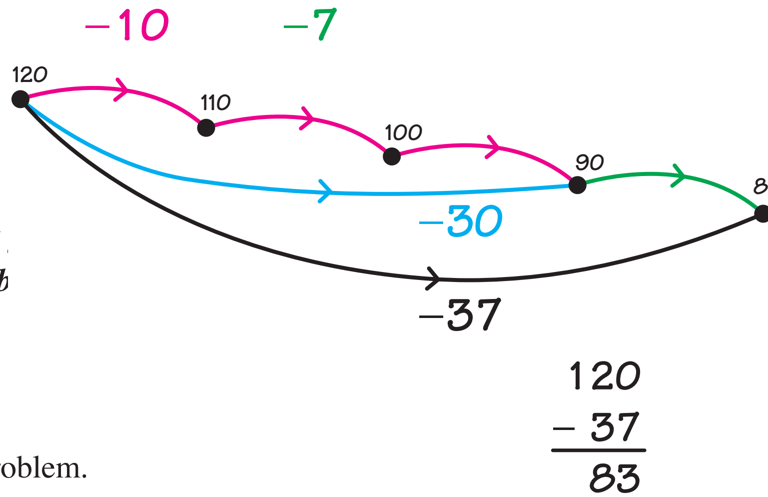
Write –37 near the arrow picture.

T: ***Subtracting 10 three times and***
subtracting 7 is the same as sub

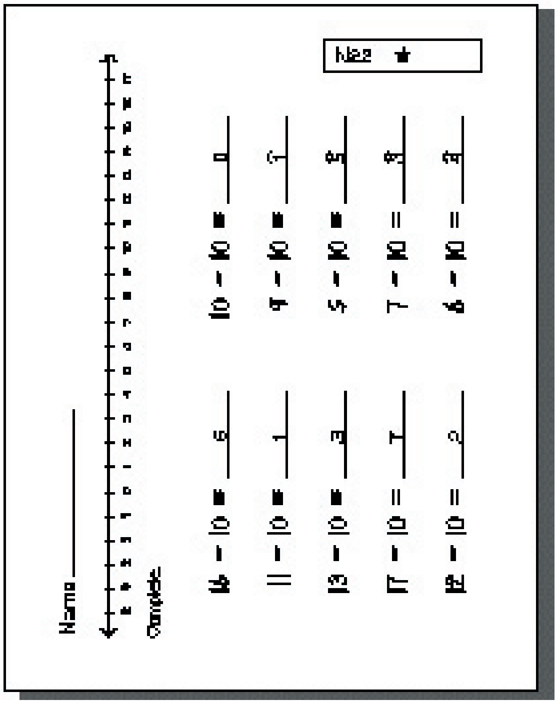
T: ***What number is 120 – 37?***

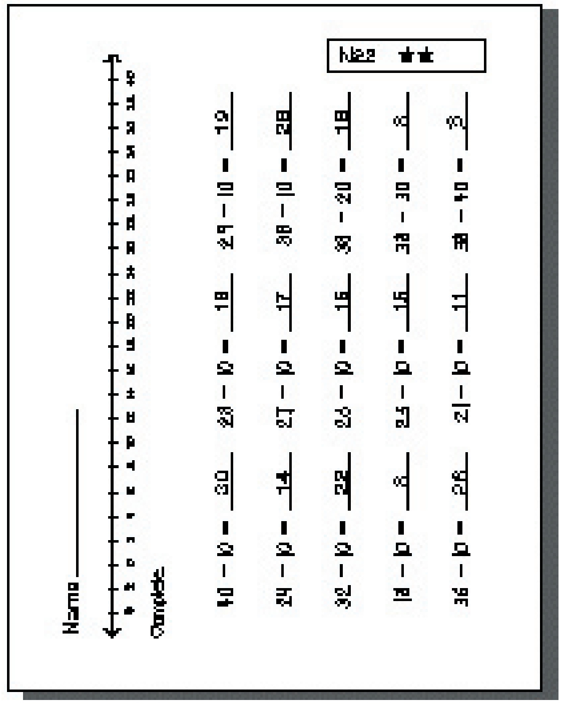
S: ***83.***

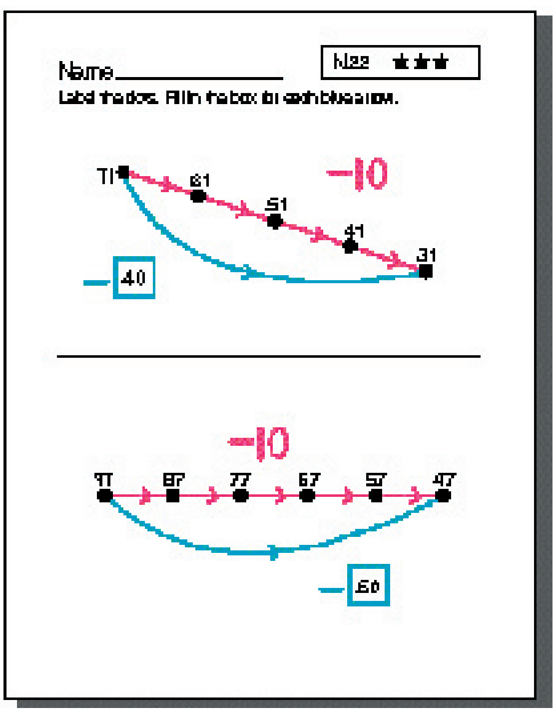
Record the answer to the subtraction problem.

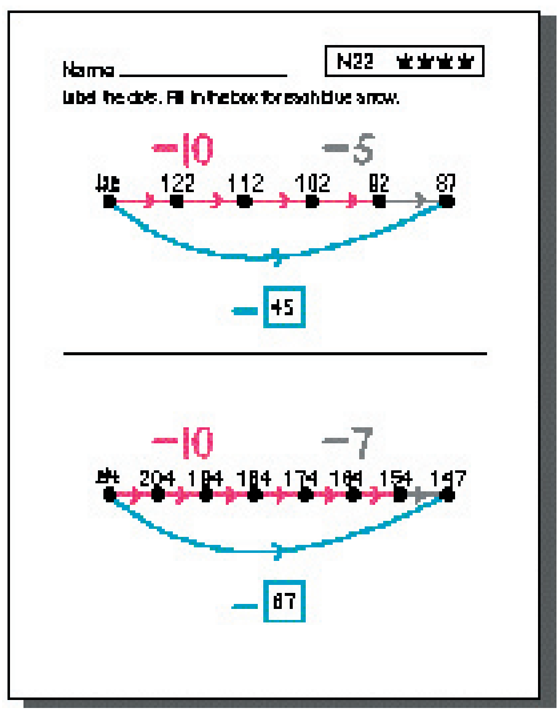


Worksheets N22*, **, ***, and **** are available for individual work.


N22 ★
Name
Complete.


N22 ★★
Name
Complete.


Name
N22 ★★★
Label the dots. Fill in the box for each blue arrow.
71
61
51
41
31
−10
− 40
97
87
77
67
57
47
−10
− 50


Name
N22 ★★★★
Label the dots. Fill in the box for each blue arrow.
−10
−5
122
112
102
92
87
− 45
−10
−7
204
154
147
− 67

Capsule Lesson Summary

Find mental methods for subtracting 8, 9, 18, or 98. Add a positive and a negative number on the Minicomputer and record corresponding subtraction sentences. Pose a subtraction problem in a story context, first estimate the result by locating it approximately on the number line, and then use the Minicomputer to do the calculation.

Materials

Teacher
- Number line or 0–109 numeral chart
- Minicomputer set
- Blackline N23

Student
- 0–109 numeral chart (optional)
- Minicomputer set
- Paper

Advance Preparation: Use Blackline N23 to make copies of the 0–109 numeral chart for students.

Description of Lesson

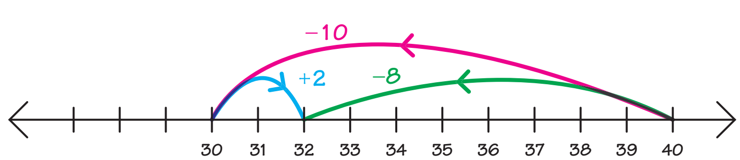
Exercise 1

This exercise should be done mostly as mental arithmetic; however, you may like to use the number line or the 0–109 numeral chart to demonstrate the subtraction methods more visually.

Note: If you use the 0–109 numeral chart, you may like students to have their own copies of the chart also.

T: ***What number is 40 – 10 ?*** (30)
40 – 10 = 30; so what number is 40 – 8? (32)
How did you calculate 40 – 8?

Let students explain their methods. Highlight the idea that an easy way to subtract 8 is to subtract 10 and then add 2.



0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99
100	101	102	103	104	105	106	107	108	109

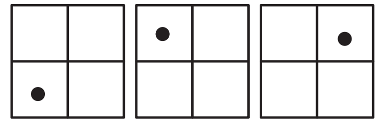
Pose several more mental subtraction calculations involving –8. For example:

30 – 8 (22)	60 – 8 (52)	100 – 8 (92)
44 – 8 (36)	72 – 8 (64)	35 – 8 (27)

Repeat this exercise, finding a mental strategy for subtracting 9, 18, or 98.

Exercise 2

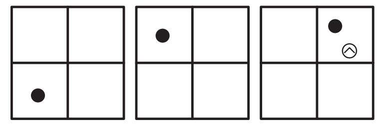
Display three Minicomputer boards and put on 284.



T: ***What number is on the Minicomputer?***

S: ***284.***

Put a negative checker on the 4-square.



T: ***What calculation am I doing?***

S: ***$284 + \widehat{4}$.***

S: ***$284 - 4$.***

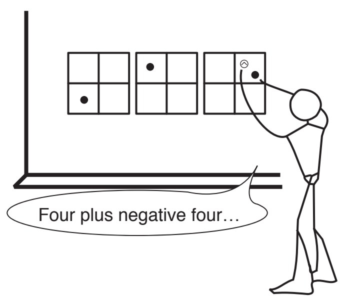
Write both calculations on the board.

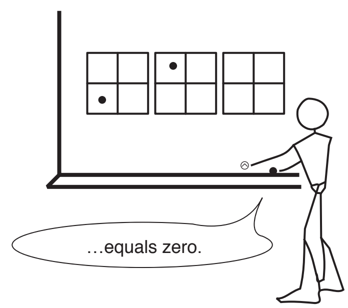
$$284 + \widehat{4} =$$
$$284 - 4 =$$

T: ***What number is this? How do you know?***

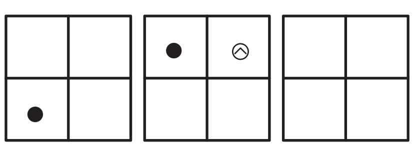
S: ***280; the checkers on the 4-square disappear.***

S: ***$284 - 4 = 280$***

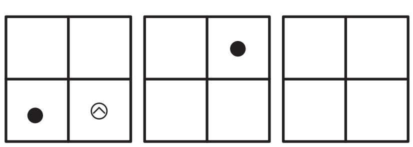




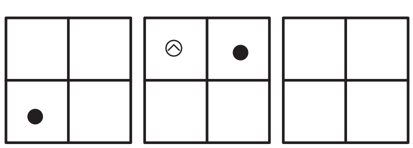
Repeat this activity with these configurations. In each case, a backward trade needs to be made to get positive and negative checkers together.



$$= 280 + \widehat{40} = 240$$
$$280 - 40 = 240$$



$$= 240 + \widehat{100} = 140$$
$$240 - 100 = 140$$



$$= 140 + \widehat{80} = 60$$
$$140 - 80 = 60$$

Exercise 3

Pose the following subtraction problem in a story context.

T: ***At a two-day carnival the balloon sellers start with 336 balloons to sell. On the first day they sell 167 balloons. How many balloons do they have left to sell the second day? What calculation do we need to do?***

S: ***336 – 167.***

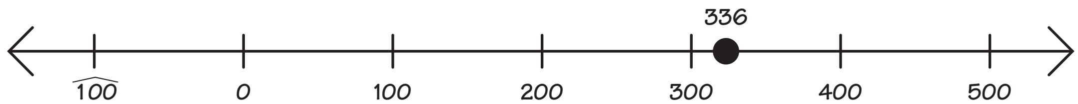
$$\begin{array}{r} 336 \\ -\ 167 \\ \hline \end{array}$$

Draw a number line graduated in hundreds on the board.

T: ***Approximately where is 336 on this number line?***

Invite a student to point to the approximate location of 336 on the number line.

T: ***Let's estimate approximately where 336 – 167 would be on this number line.***

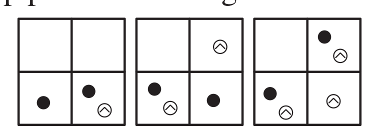


Invite students to make estimates by pointing to approximately where 336 – 167 would be. Encourage some discussion. Perhaps your class will want to locate 336 – 167 between 100 and 200, but do not insist on too close an estimate.

T: ***How should we do this calculation?***

Accept several methods, but include using the Minicomputer. On the Minicomputer use one of the following methods, whichever is more comfortable for your class.

- Put 336 on the Minicomputer and make trades until you can take off 167.
- Put 336 on the Minicomputer with positive checkers and $\widehat{167}$ on with negative checkers. Then make trades to pair up positive and negative checkers.



Note: Here the negative number represents balloons sold.

The class should conclude that 336 – 167 = 169, so the balloon sellers have 169 balloons to sell the second day.

Invite a student to point to the approximate location of 169 on the number line and to check its location with previous estimates.

Exercise 4

N23

Write the following problems on the board. Instruct the students to solve as many of them as they can and to use whatever methods they prefer. Provide Minicomputer sets to students who want to use them. Some students may prefer to draw arrow pictures, use the number line, or just write calculations.

> *
> No backward trades needed on the Minicomputer

```
  253     367     639
 - 42   - 106   - 438
```

> **
> One or two backward trades needed

```
  345     575     201
- 281   - 268   - 120
```

> ***
> More than two backward trades needed

```
  248     315     934
- 196   - 287   - 265
```

N24 THE FUNCTIONS 3X AND $\frac{1}{3}$X

Capsule Lesson Summary

Count by threes to generate multiples of 3. Locate numbers in a string picture where the string is for multiples of 3. Do 3x and $\frac{1}{3}$x (or ÷3) calculations to label the dots in a 3x arrow road. Include the calculation of $\frac{1}{3}$ x 4.

Materials

Teacher
- Colored chalk

Student
- Unlined paper
- Colored pencils, pens, or crayons
- Worksheets N24*, **, ***, and ****

Description of Lesson

Exercise 1

Organize a group of approximately 10 students to count by threes starting at 0. Direct one student to start the count at 0 and the others to follow in turn, counting by threes.

T: ***How can we make the calculator count by threes starting at 0?***

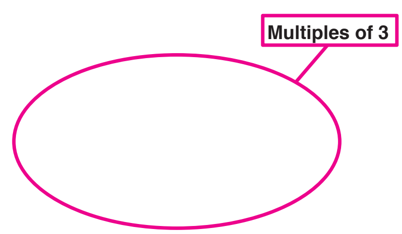
Direct all the students to prepare their calculators to count by threes and then to count along with the calculator to about 60.

S: ***Press*** [0] [+] [3] [=] [=] [=] ***and so on.***

T: ***What kind of numbers do we get when we count by threes from 0?***

S: ***Multiples of 3.***

Draw this string picture on the board.



T: ***Give me some numbers to put inside the string.***

Let students suggest many numbers and record them in the string picture. When a multiple of 3 is given, locate it inside the string. If a suggested number is not a multiple of 3, locate it outside the string. Explain why a number is or is not a multiple of 3 whenever the explanation is fairly simple and concise. Let the students' suggestions determine the direction of your discussion.

S: ***24.***

T: ***24 is how many threes?***

If necessary, count by threes from 0 to 24, and ask a student to keep track of how many times you add three. Conclude that 24 is 8 threes. Put 24 inside the string and write the number fact on the board.

8 x 3 = 24

T: ***8 x 3 = 24.***
What number is $\frac{1}{3}$ x 24 (read as "one-third of twenty-four") ***or 24 ÷ 3?***

S: *8.*

Write corresponding number sentences on the board below the first one.

$$8 \times 3 = 24$$
$$\tfrac{1}{3} \times 24 = 8$$
$$24 \div 3 = 8$$

T: ***Are there any numbers less than 24 that are multiples of 3?***

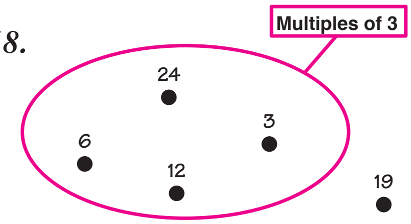
Put multiples of 3 that are less than 24 inside the string; continue to ask for multiples of 3 that are less than the least one found so far. If a student suggests a non-multiple of 3, such as 19, ask,

T: ***Is 19 a multiple of 3? How do you know?***

S: ***19 is not a multiple of 3 because 24 – 3 = 21, and 21 – 3 = 18.***

S: ***When we count by threes starting at 0, we skip 19.***

T: ***19 is not a multiple of 3, so I will put it outside the string.***



Continue this activity until a negative multiple of 3 is suggested, for example, $\widehat{6}$.

T: ***How do you know $\widehat{6}$ is a multiple of 3?***

S: ***When you count backward by threes from 0 you get $\widehat{6}$.***

T: ***Three times what number is $\widehat{6}$?***

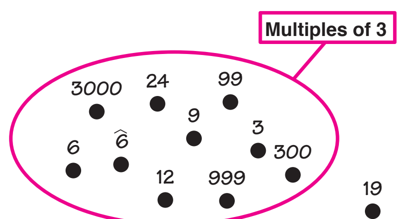
S: *$\widehat{2}$.*

$$3 \times \widehat{2} = \widehat{6}$$
$$\tfrac{1}{3} \times \widehat{6} = \widehat{2}$$

T: ***$\widehat{2} + \widehat{2} + \widehat{2} = \widehat{6}$, so $3 \times \widehat{2} = \widehat{6}$.***
What number is $\tfrac{1}{3} \times \widehat{6}$?

S: *$\widehat{2}$.*

T: ***Do you know some large numbers that are multiples of 3?***



Put any suggested multiple of 3 inside the string. If a student gives a number that is not a multiple of 3, such as 1,000, ask,

T: ***How can we decide whether or not 1,000 is a multiple of 3?***
Do you know any number close to 1,000 that is certainly a multiple of 3.

If no one suggests 999, check 9, 99, and then 999 in that order.

T: ***Three times what number is 999?***

S: ***333.***

T: ***We know that 999 is a multiple of 3, because 3 x 333 = 999. What is the next greater multiple of 3?***

S: ***1,002; so we skip 1,000.***

Continue to accept suggestions and occasionally discuss why a number is a multiple of 3.

S: ***3,021.***

T: ***Is 3,000 a multiple of 3?***

S: ***Yes, because 3 x 1,000 = 3,000.***

T: ***Is 21 a multiple of 3?***

S: ***Yes, because 3 x 7 = 21.***

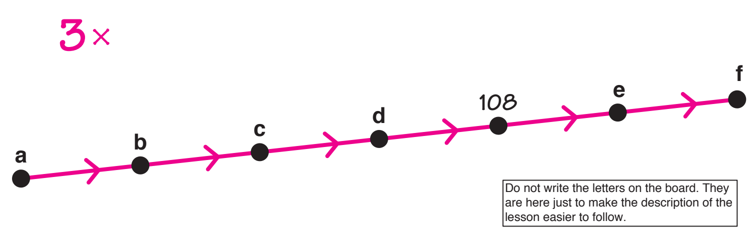
T: ***3,000 is a multiple of 3, and 21 is a multiple of 3, so 3,021 is a multiple of 3.***

Continue this activity until many students have participated, but avoid turning it into a drill.

Note: For your information, you can decide easily if a number is a multiple of 3 by summing its digits. If the sum of the digits is a multiple of 3, then the number is also a multiple of 3. For example, since 7 + 8 = 15 and 15 is a multiple of 3, then 78 is also a multiple of 3. We know that 500 is not a multiple of 3, because 5 + 0 + 0 = 5 and 5 is not a multiple of 3.

Exercise 2

Distribute ~~unlined paper and~~ colored pencils. Draw this arrow picture on the board and ask the students to copy it on their papers.



T: ***What are the red arrows for?***

S: ***3x.***

T (pointing to **e**): ***How can we find what number is here?***

S: ***Calculate 3 x 108.***

Direct students to do the calculation and to label the dot on their papers. You may want to suggest adding 108 + 108 + 108 as one method of calculating 3 x 108. After a few minutes, let students share their methods of doing the calculation and write the steps.

$$\begin{array}{r} {}^{2}\\ 108 \\ 108 \\ +\ 108 \\ \hline 324 \end{array} \qquad \begin{array}{r} 108 \\ \times\ \ 3 \\ \hline 24 \\ 300 \\ \hline 324 \end{array}$$

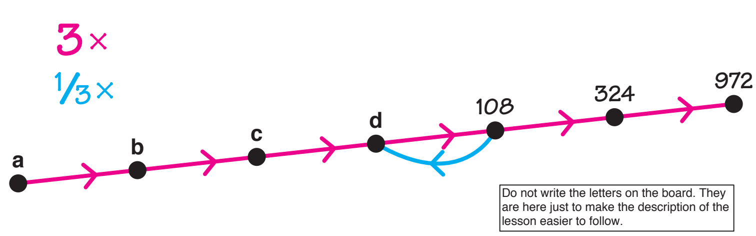
Decide which number is at **f** in a similar way. Conclude that 3 x 324 = 972, and label **f** .

T (pointing to **d**): ***How can we find what number is here?***

S: ***Draw a return (opposite) arrow.***

T: ***What is this return arrow for?***

S: ***$^1/_3$x (or ÷3).***



Ask students to estimate what number is $^1/_3$ x 108 or 108 ÷ 3.

T: ***Is $^1/_3$ x 108 more than 20?***

S: ***Yes, because 3 x 20 is only 60.***

T: ***Is $^1/_3$ x 108 more than 30?***

S: ***Yes, because 3 x 30 = 90, and 90 is less than 108.***

T: ***Is $\frac{1}{3}$ x 108 more than 40?***

S: ***No, because 3 x 40 = 120 and that's too much.***

T: ***So we know that $\frac{1}{3}$ x 108 is between 30 and 40.***

Direct students to do the calculation and label **d** on their papers. With students, observe that they can check their answer by multiplying it by 3 and seeing if the product is 108.

Conclude that $\frac{1}{3}$ x 108 = 36 and label **d**.

$$\begin{array}{r} \scriptstyle 1 \\ 36 \\ 36 \\ +\ 36 \\ \hline 108 \end{array} \qquad\qquad \begin{array}{r} 36 \\ \times\ \ 3 \\ \hline 18 \\ 90 \\ \hline 108 \end{array}$$

Repeat this activity to calculate $\frac{1}{3}$ x 36 and $\frac{1}{3}$ x 12, and label the appropriate dots. Point to 4, and trace a $\frac{1}{3}$x arrow starting at 4.

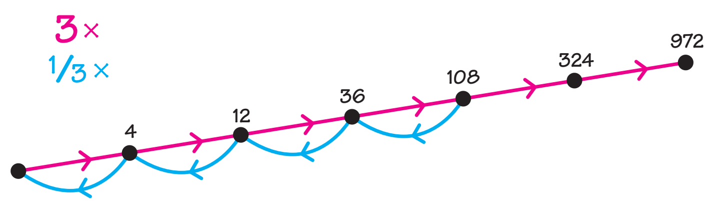
T: ***Is $\frac{1}{3}$ x 4 more than 1?***

S: ***Yes, because 3 x 1 = 3.***

T: ***Is $\frac{1}{3}$ x 4 more than 2?***

S: ***No, because 3 x 2 = 6.***

T: ***So we know that $\frac{1}{3}$ x 4 is between 1 and 2.***



Draw four circles of the same size on the board. Choose three students, and tell the class that you want to share four pies fairly among the three students.

Note: You may prefer to draw rectangles and share candy bars.

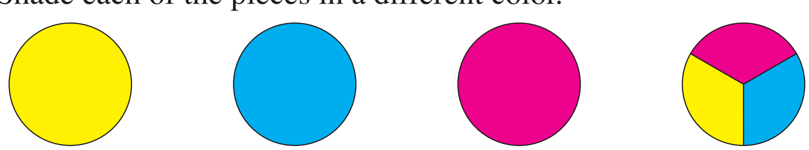
T: ***Will I be able to give each of my friends an entire pie?*** (Yes)

Shade the inside of each of the three circles in a different color.

T: ***I can give one pie to each of my friends and I will have one pie left. How can I share one pie among my three friends?***

S: ***Cut it into three pieces.***

Invite a student to show how to cut the pie. Be patient! It isn't easy to divide a circle into three pieces the same size. Be prepared to move some of the division lines if students object that the pieces are not the same size. Shade each of the pieces in a different color.

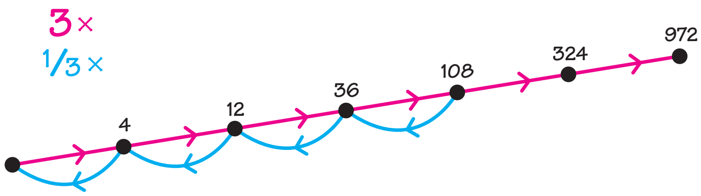


T: ***What part of the pie is each of these pieces*** (point to the fourth circle)***?***

S: $\frac{1}{3}$.

T: ***Each of my friends will receive one whole pie and one-third of a pie.*** $\frac{1}{3} \times 4 = 1\frac{1}{3}$.

Label the last dot in the arrow picture $1\frac{1}{3}$ and read the number. Trace the 3x arrow from $1\frac{1}{3}$ to 4 as you ask,



T: ***Does*** $3 \times 1\frac{1}{3} = 4$?
How can we calculate $3 \times 1\frac{1}{3}$?

S: ***Multiply 3 x 1, and*** $3 \times \frac{1}{3}$.

S: ***Add*** $1\frac{1}{3} + 1\frac{1}{3} + 1\frac{1}{3}$.

Write both the multiplication and repeated addition calculations on the board.

$$\begin{array}{r} 1\frac{1}{3} \\ 1\frac{1}{3} \\ +\ 1\frac{1}{3} \\ \hline 4 \end{array}$$

$$3 \times 1\frac{1}{3} = 4$$
$$4 \div 3 = 1\frac{1}{3}$$

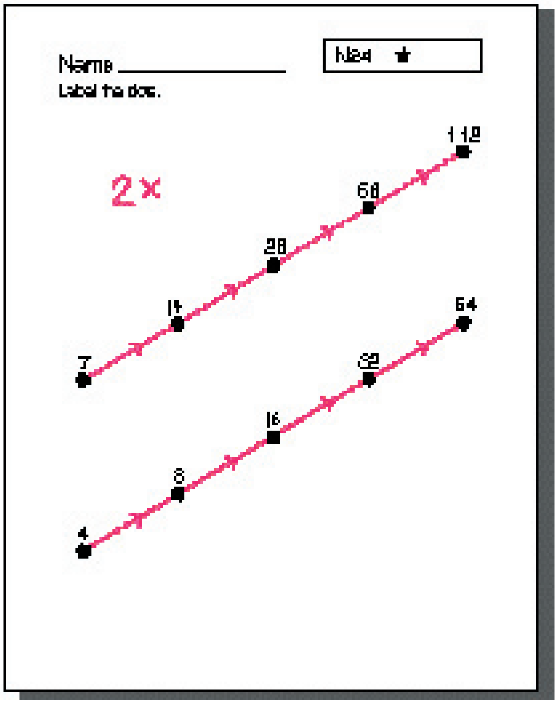
Worksheets N24*, **, ***, and **** are available for individual work.

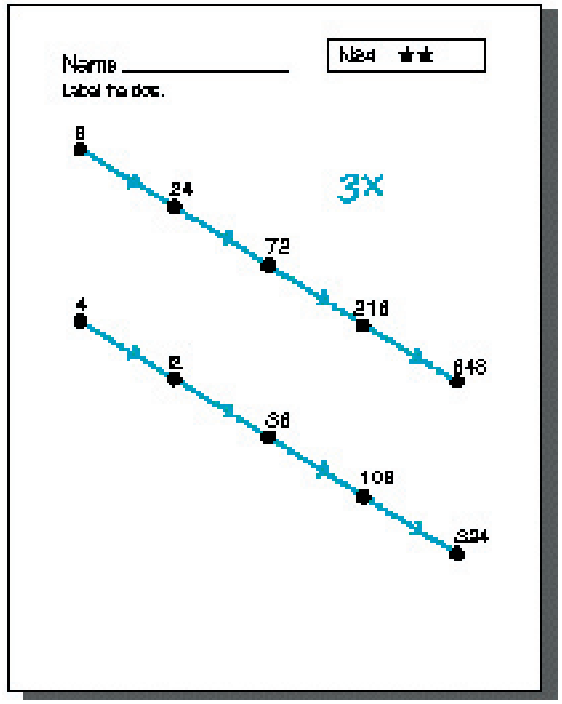
Center Activity

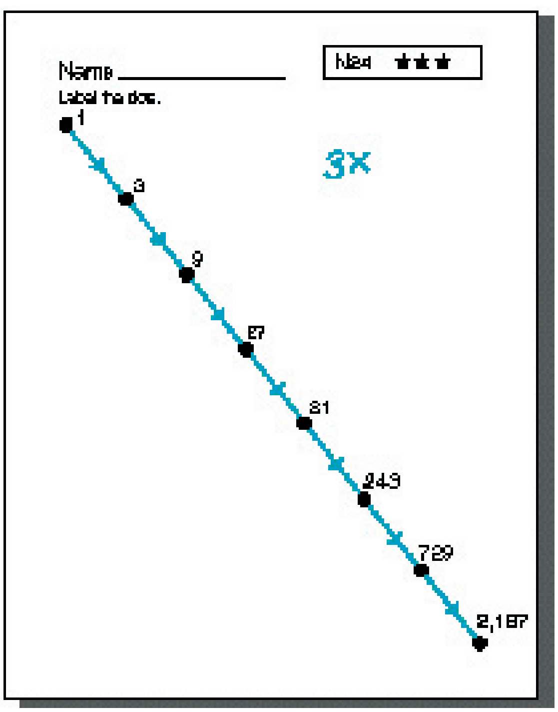
Put fraction manipulatives in a center for free exploration.

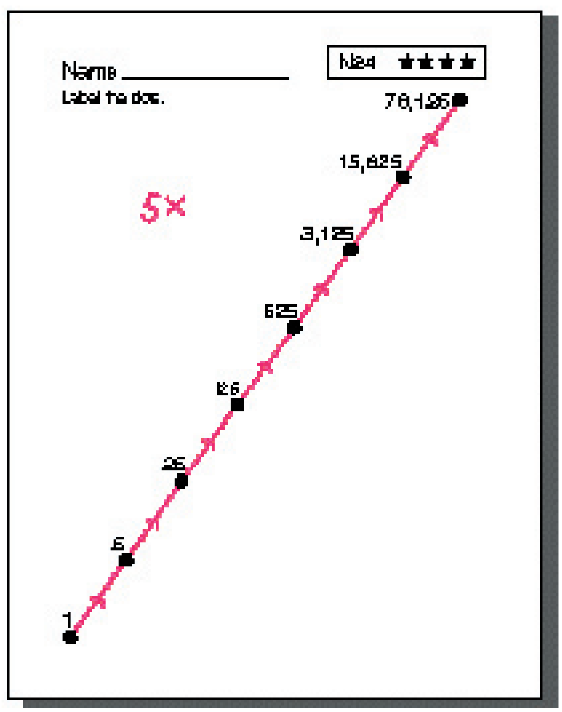
Home Activity

Suggest that parents/guardians work with their child to share five candy bars fairly among 2, 3, 4, and 5 members of the family.


Name
Label the dots.
N24 ★
2×
7
14
28
56
112
4
8
16
32
64


Name
Label the dots.
N24 ★★
3×
8
24
72
216
648
4
12
36
108
324


Name
Label the dots.
N24 ★★★
3×
1
3
9
27
81
243
729
2,187


Name
Label the dots.
N24 ★★★★
5×
1
5
25
125
625
3,125
15,625
78,125

N25 MULTIPLICATION PROBLEMS #3 N25

Capsule Lesson Summary

Use the Minicomputer to display and do multiplication calculations. Multiply by 10 on the Minicomputer and introduce ⑩-checkers together with a big trade: ten checkers on a square is the same as one checker on the same color square one board to the left.

Materials

Teacher
- Minicomputer set
- ⑩-checkers

Student
- Paper
- Worksheets N25*, ** and ***
- Minicomputer set

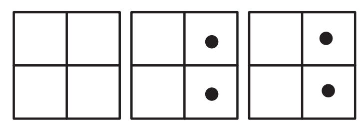
Description of Lesson

Exercise 1: Multiplication on the Minicomputer

Put the pictured configurations on the Minicomputer as you tell this or another multiplication story.

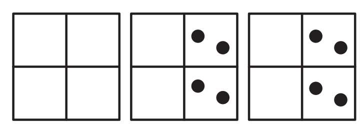
T: ***Lyle is in charge of cleaning potatoes for a big Thanksgiving dinner at the community center. He put the number of potatoes from one bag on the Minicomputer.***

How many potatoes in one bag? (55)



When he finished cleaning the potatoes from one bag, he counted the potatoes in a second bag and put the number on the Minicomputer.

How many potatoes in two bags?
What number is this?



Suggest students write their responses on paper or whisper them to a neighbor. Then call on several students to explain how to calculate the number on the Minicomputer. Include mention of 2 x 55 = 110.

T: ***What number is on the ones board?***

S: ***10; 2 x 5 = 10.***

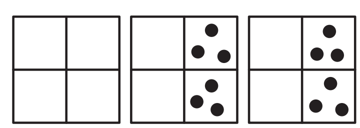
T: ***What number is on the tens board?***

S: ***100; 2 x 50 = 100.***

$$\begin{array}{l} 2 \times 5 = 10 \\ 2 \times 50 = 100 \\ \hline 2 \times 55 = 110 \end{array}$$

$$\begin{array}{r} 55 \\ \times\ 2 \\ \hline 10 \\ 100 \\ \hline 110 \end{array}$$

T: ***2 x 55 = 110.***
Now Lyle puts the number of potatoes from a third bag on the Minicomputer.
How many potatoes in three bags?
What number is this?



Again, suggest students write their responses on paper or whisper them to a neighbor. Then call on several students to explain how to calculate the number on the Minicomputer. Include mention of 3 x 55 = 165.

T (pointing to the ones board)**:** ***What number is 3 x 5?***

S: ***15.***

T (pointing to the tens board)**:** ***What number is 3 x 50?***

S: ***150.***

T: ***What number is 15 + 150?***

S: ***165.***

T: ***3 x 55 = 165***

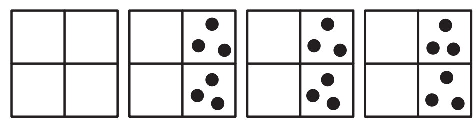
$$3 \times 5 = 15$$
$$3 \times 50 = 150$$
$$3 \times 55 = 165$$

$$\begin{array}{r} 55 \\ \times\ 3 \\ \hline 15 \\ 150 \\ \hline 165 \end{array}$$

Continue by adding three 500's to the configuration on the Minicomputer. Ask students to do this calculation on their papers and then get explanations.

$$3 \times 5 = 15$$
$$3 \times 50 = 150$$
$$3 \times 500 = 1{,}500$$
$$3 \times 555 = 1{,}665$$

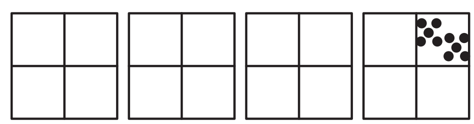
$$\begin{array}{r} 555 \\ \times\ 3 \\ \hline 15 \\ 150 \\ 1{,}500 \\ \hline 1{,}665 \end{array}$$



$3 \times 555 = 1{,}665$

Exercise 2: Multiplication by 10

Put this configuration on the Minicomputer.



T: ***What multiplication calculation is shown on the Minicomputer?***

S: ***10 x 4.***

10 x 4

Write the calculation on the board.

Note: If a student says 4 x 10, remark that although 4 x 10 = 10 x 4, four checkers on the 10-square of the Minicomputer would be a better way to show 4 x 10.

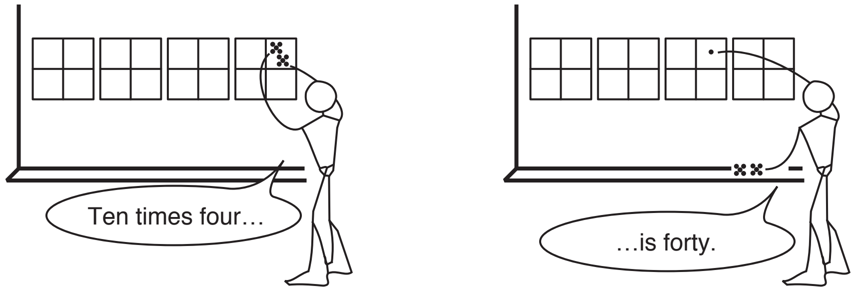
T: ***What number is this? Write it on your paper.***

Check several responses before asking a student to answer aloud.

S: ***40.***

T: ***10 x 4 = 40. Ten checkers on this purple square*** (4-square)***is the same as one checker on this purple square*** (40-square)**.**

Make a 10 x 4 = 40 trade and complete the fact on the board.



Repeat this activity for 10 x 40 and 10 x 400.
Write each number sentence below the previous one.

10 x 4 = 40
10 x 40 = 400
10 x 400 = 4,000

T: ***Do you see a pattern?***

Encourage different responses. If a student suggests that you simply write another 0 on the right when you multiply a number by 10, accept this idea but do not emphasize it yourself.

Note: Putting 0 to the right of the numeral works only when an integer is multiplied by 10. It doesn't work for non-integer decimals; for example, 10 x 0.3 is not 0.30.

Extend the list on the board another step with 10 x 4,000.

T: ***What number is 10 x 4,000? How do you know?***

S: ***40,000. 40 has one zero, 400 has two zeros, 4,000 has three zeros, and 40,000 has four zeros.***

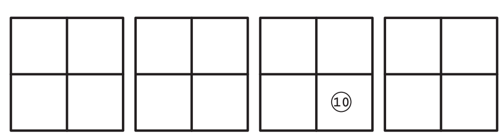
Ask a student to complete the number sentences on the board. Point out that a comma may be inserted between the thousands place and the hundreds place to help us read large numbers. The digits to the left of the comma tell us how many thousands there are.

10 x 4 = 40
10 x 40 = 400
10 x 400 = 4,000
10 x 4,000 = 40,000

Erase the board and remove the checkers from the Minicomputer.

T (holding up a ⑩-checker)**:** ***Let's use this ⑩-checker to put ten checkers on the same square.***

Put a ⑩-checker on the 10-square.

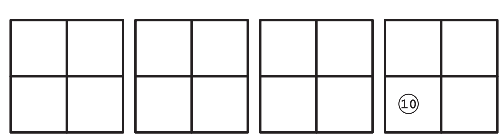


T: ***What number is this?***

S: ***100.***

T: ***10 x 10 = 100***

Move the ⑩-checker to the 2-square.



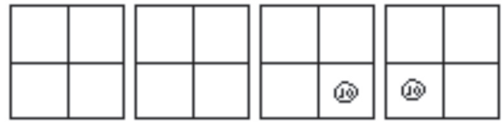
T: ***What number is this?***

S: ***20.***

T: ***10 x 2 = 20.***

N25

Put this configuration on the Minicomputer.



T: ***What calculation do you see on the Minicomputer?***

S: ***10 x 12.***

Write the calculation on the board.

10 x 12 = 120

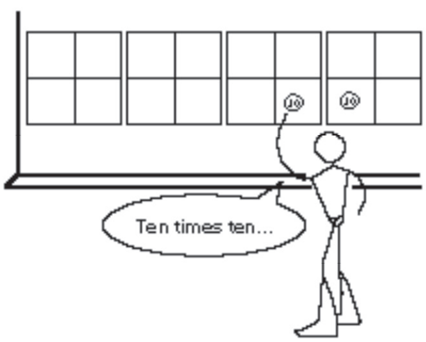
T: ***What number is this? Write it on your paper.***

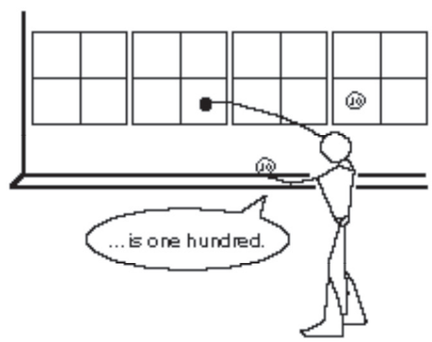
Check several responses before asking a student to answer aloud.

S: ***120.***

T: ***10 x 12 = 120. Ten checkers on this white square*** (10-square) ***is the same as one checker on this white square*** (100-square)***.***

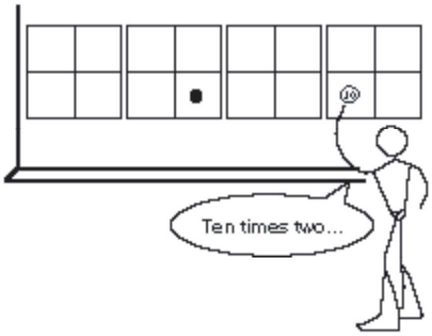
Make a 10 x 10 = 100 trade.

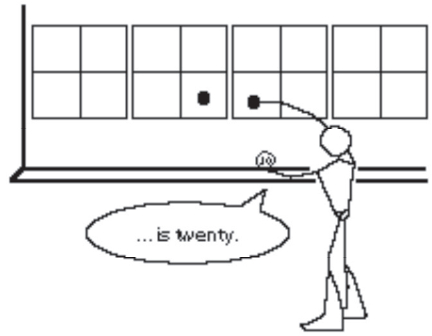




T: ***Ten checkers on this red square*** (2-square) ***is the same as one checker on this red square*** (20-square)***.***

Make a 10 x 2 = 20 trade and complete the calculation on the board.



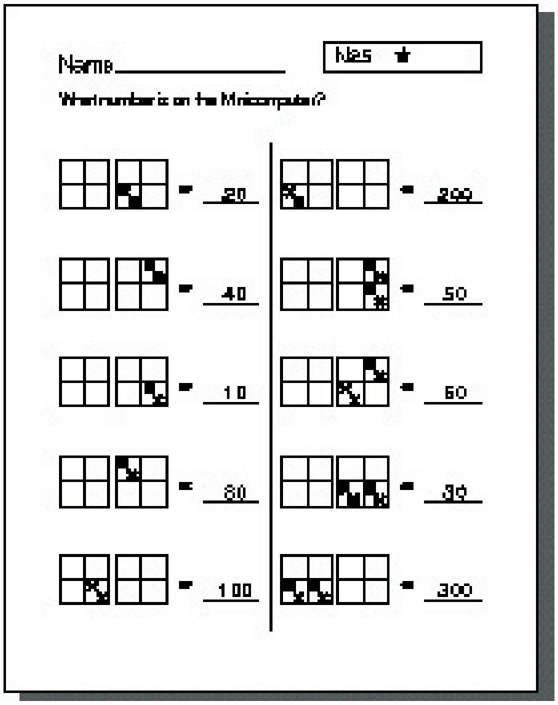


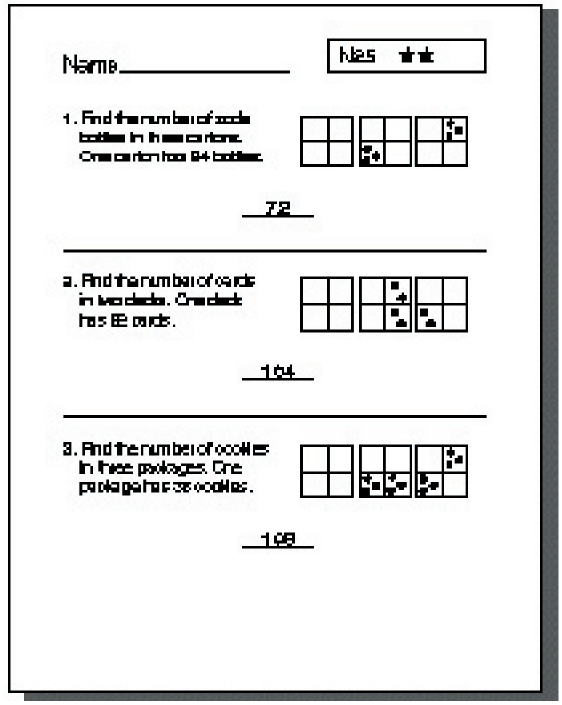
Repeat this activity with 10 x 120, 10 x 15, and 10 x 150.

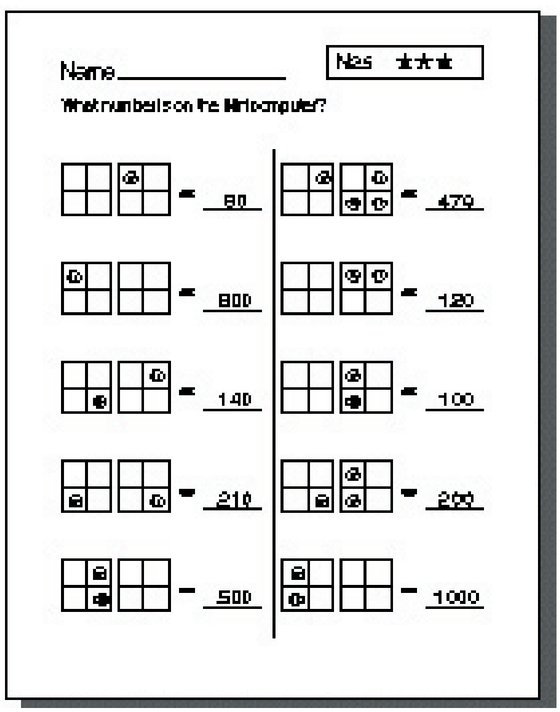
Worksheets N25*, **, and *** are available for independent work. Provide individual Minicomputers to students who wish to use them.

Home Activity

This is a good time to send home to parents/guardians a letter about multiplying by ten on the Minicomputer. Blackline N25 has a sample letter.







Capsule Lesson Summary

Solve subtraction problems by building arrow roads with arrows that compose to give the appropriate subtraction function.

Materials

Teacher
- Colored chalk
- 0–109 numeral chart (optional)

Student
- Unlined paper
- Colored pencils, pens, or crayons

Description of Lesson

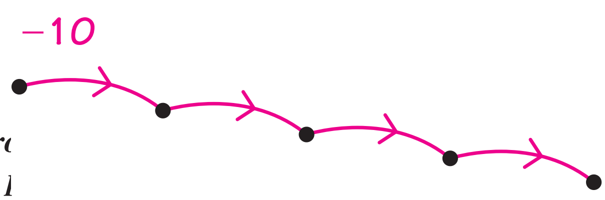
Exercise 1: Mental Arithmetic

Begin this lesson with several minutes of mental arithmetic activities involving the function –10. Use the following subtraction problems or others more appropriate for the numerical abilities of your students. You may like to use the 0–109 numeral chart as a visual support when the numbers are less than 110.

20 – 10	100 – 10	24 – 10
21 – 10	102 – 10	24 – 20
60 – 10	109 – 10	56 – 10
62 – 10	200 – 10	56 – 20
30 – 10	205 – 10	56 – 30
35 – 10	1,000 – 10	56 – 40
38 – 10	1,001 – 10	0 – 10

Exercise 2

Draw this arrow picture on the board.

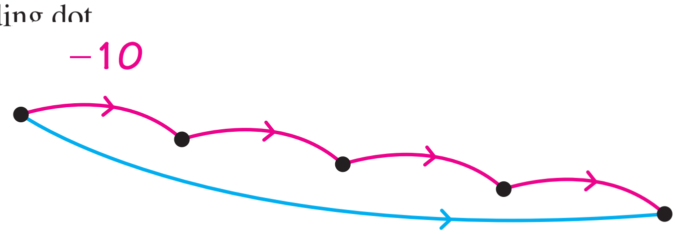


T: ***Where is the greatest number in this arr***
Where is the least number in this arrow
How do you know?

Students should indicate that the greatest number is at the far left and the least is at the far right.

S: ***If you subtract 10 from a number, you get a smaller number.***

Draw an arrow from the starting dot to the ending dot.



T: ***What could this blue arrow be for?***

S: ***–40.***

T: ***How do you know?***

S: ***There are four –10 arrows. 4 x 10 = 40.***

Label the blue arrow –40 and then label the starting dot 87.

T: ***If this number is 87, what is the ending number? Write your answer on your paper (or whisper it to your neighbor).***

Call on students to explain what calculations they did and record them on the board. For example, when a student suggests subtracting 10 four times, record the calculations as illustrated. Conclude that 87 – 40 = 47.

$$\begin{array}{r} 87 \\ -\ 10 \\ \hline 77 \\ -\ 10 \\ \hline 67 \\ -\ 10 \\ \hline 57 \\ -\ 10 \\ \hline 47 \end{array} \qquad \begin{array}{r} 87 \\ -\ 40 \\ \hline 47 \end{array}$$

Repeat this activity with the other starting numbers such as 51, 69, 113, and 338.

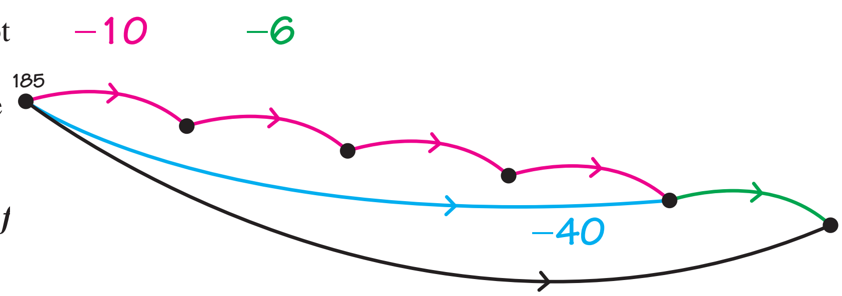
Write this problem on the board.

T: ***Subtracting 10 four times is the same as subtracting 40. Let's see if we can build an arrow road to help us subtract 46 from 185?***

$$\begin{array}{r} 185 \\ -\ 46 \\ \hline \end{array}$$

Acknowledge any correct suggestions. For example, a student might suggest using nine –5 arrows and one –1 arrow. Very likely someone will suggest starting at 185 and building an arrow road with four –10 arrows and one –6 arrow. If necessary suggest this arrow road yourself.

On the board, label the starting dot of the arrow road 185, and add a –6 arrow. Draw an arrow from the starting dot to the ending dot.



T: ***What could this arrow be f***

S: ***– 46.***

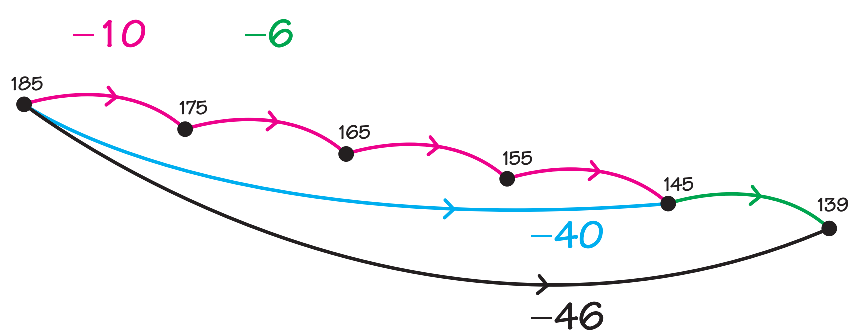
Label the arrow –46 and then ask students to calculate 185 – 46 on their papers. Check many responses before letting someone answer aloud.

S: ***139.***

T: ***Did this arrow picture help you do the calculation? How?***

S: ***The arrow picture shows that to subtract 46 we can subtract 10 four times and then subtract 6.***

Invite a student to label the dots in the arrow picture.



Conclude that 185 – 46 = 139 and record the answer on the board.

```
 185
– 46
 139
```

Erase the board, and distribute unlined paper and colored pencils. Then continue the lesson with this subtraction problem.

123 – 35

T: ***What arrow road could we build to help us solve this problem?***

Encourage the students to offer many suggestions. A few of the possible arrow roads are described below. All of the roads start at 123.

- a road with seven –5 arrows
- a road with three –10 arrows and one –5 arrow
- a road with five –7 arrows
- a road with a –20 arrow, a –10 arrow, and a –5 arrow
- a road with thirty–five –1 arrows

After discussing the problem, let the students build an arrow road to help them calculate 123 – 35. Invite students with different solutions to draw them on the board. Conclude that 123 – 35 = 88 and briefly discuss the various arrow pictures.

Erase the board and repeat this activity with 172 – 57 and 214 – 121. If necessary, suggest using a –100 arrow for the second problem.

Home Activity

Let students take home one of their papers to show parents/guardians how to build an arrow road to do a subtraction calculation.

N27 INTRODUCTION TO DECIMAL NUMBERS #1 N27

Capsule Lesson Summary

Through a sequence of situations involving money, extend the use of the Minicomputer for representing decimal numbers. Represent various amounts of money on the Minicomputer.

Materials

Teacher
- Coins
- Minicomputer set[†]
- Colored chalk

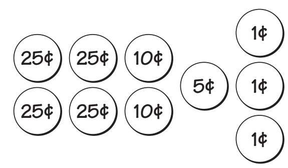
Student
- Minicomputer set
- Bar (ruler or strip of paper)

Description of Lesson

Exercise 1

Conduct a brief review of coin values and amounts of money. In particular, help students recognize an amount of money both as cents (for example, 125¢) and as dollars and cents (for example, $1.25). Pose two kinds of problems:

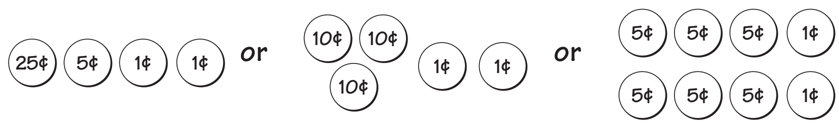
- Put out a collection of coins and ask for the amount.



128¢ or $1.28

- Announce an amount of money, and ask for a collection of coins to make that amount. Accept several responses.

32¢ or $0.32



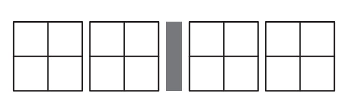
Exercise 2

Begin this exercise with about five minutes of mental arithmetic involving doubling and halving numbers.

Display three Minicomputer boards. If you like, provide student pairs with individual Minicomputers and ask them to follow along with what is done on the demonstration Minicomputer. Choose students from your class to star in these stories.

T: ***Chris and Landers are my friends. I want to share $700 between them. I want to give them both the same amount. How much money should I give Chris? How much money should I give Landers?***

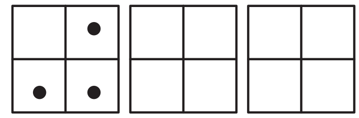
[†]This lesson requires, at times, that additional space be left between two of the demonstration Minicomputer boards and that a bar be drawn in this space.If your Minicomputers are hanging from permanently attached hooks or are displayed in some other way that fixes their position, you may need to make some adjustments before the lesson begins.



N27

Allow several students to make comments. Be sure the class understands that $700 must be shared equally. Although many students will know that ½ x 700 = 350, the following activity will prepare them for finding one-half of a number on the Minicomputer when the result is a (non-integer) decimal.

T: ***How can we use the Minicomputer to find one-half of 700?***

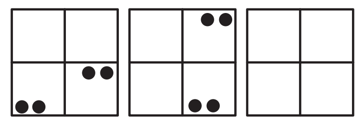


Invite a student to put 700 on the Minicomputer.

T: ***What should we do to find one-half of 700 on the Minicomputer?***

S: ***Make backward trades until all the checkers are in pairs.***

Invite students to make trades until all the checkers are in pairs. By pushing two checkers together into a corner of a square when you get them, you can discourage students from breaking up pairs.



T: ***Now we can remove checkers so that one-half of the number will be on the Minicomputer.***

Students should remove one checker from each pair. Conclude that ½ x 700 = 350, and write the number sentence on the board.

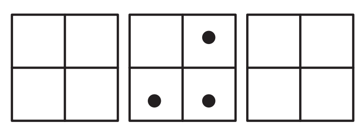
½ x $700 = $350

Clear the Minicomputer and continue with another story.

T: ***Teresa and Raoul are my friends. I want to share $70 between them. How much money should I give Teresa? How much money should I give Raoul?***

Allow several students to comment. Some will know that ½ x 70 = 35, but still proceed to use the Minicomputer.

T: ***How can we use the Minicomputer to find one-half of 70?***

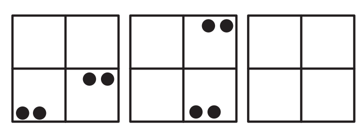


Invite a student to put 70 on the Minicomputer.

T: ***How can we find one-half of the number on the Minicomputer?***

S: ***Make backward trades until all the checkers are in pairs.***

Invite students to make trades until all the checkers are in pairs. Students should notice that the trades are like those for the first problem, but this time you started on the tens board.



Ask students to remove checkers so that one-half of the number is on the Minicomputer. Conclude that ½ x 70 = 35, and write the number sentence on the board.

½ x $700 = $350
½ x $70 = $35

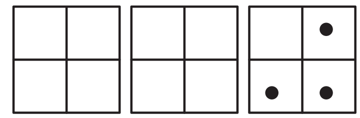
Clear the Minicomputer and continue with another story.

T: ***I have two more friends, Marvin and Erica. I want to share $7 between them. How much money should I give Marvin? How much money should I give Erica?***

Allow several students to comment. Perhaps some students will suggest immediately that each of them be given three and a half dollars.

T: ***How many cents is one-half of a dollar?*** (Fifty) ***Three and one-half dollars is three dollars and fifty cents.***

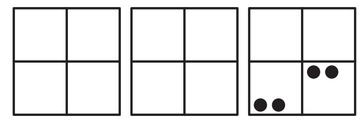
Invite a student to put 7 on the Minicomputer.



T: ***What can we do to find one-half of 7 on the Minicomputer.***

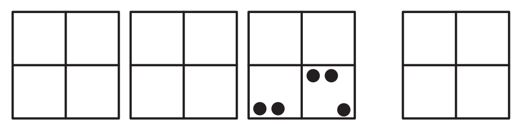
S: ***Make backward trades until all the checkers are in pairs.***

Invite students to make trades until you have this configuration on the Minicomputer.



T: ***Now we have two pairs of checkers, but we still have one checker without a partner. What should we do now?***

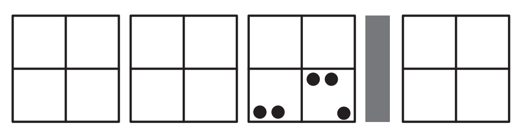
There may be some confusion. Be patient. Encourage students to express their ideas. Perhaps at least one student will suggest adding another Minicomputer board to the right. If this does not happen, suggest it yourself. Place an extra board to the right of the ones board, but leave more space than usual.



At least one of your students should object and say that now the number on the Minicomputer is not 7 but 70. Raise the question yourself, if necessary.

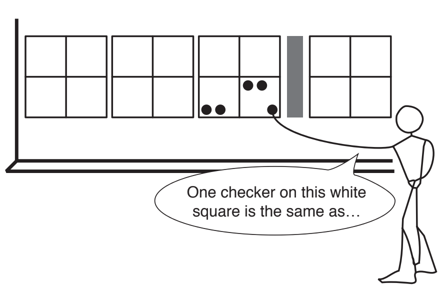
T: ***In order not to forget that this number is 7 (or that this is the ones board), let's put a bar between these two boards.***

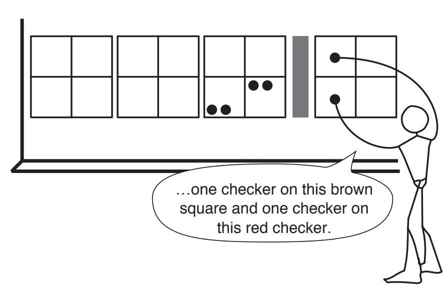
Emphasize that the bar† is there to remind us that the board to the left of the bar is still the ones board.



T: ***What should we do now?***

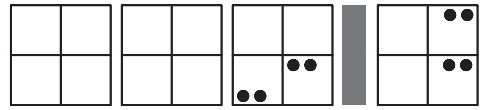
Students may still be confused and may suggest some wrong trades. It is natural for the bar to disturb them. Perhaps you will need to make one trade yourself.





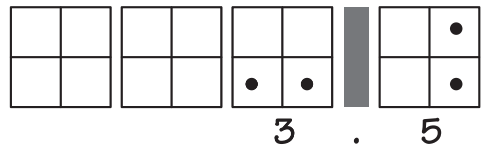
†The bar may be drawn with chalk directly on your chalkboard, or you may use a ruler (stick) or strip of paper for the bar. Students using individual Minicomputers can place a ruler or strip of paper between two sheets of Minicomputer boards.

Relate this new board (in the story context) to dimes and suggest that it is the dimes board; one dollar is eight dimes, and two dimes. Generally students will make the next two trades without any trouble.



T: ***Now we can remove some checkers so that one-half of this number will be on the Minicomputer.***

A volunteer should remove one checker from each pair.



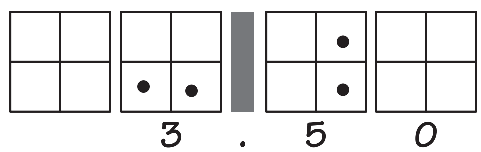
T: ***How do you suggest we write this number?***

Perhaps a student will suggest using something analogous to the bar to separate 3 and 5. If no one suggests using a point, suggest this yourself. Most students will be familiar with the decimal point from seeing prices in stores. Write 3.5 below the Minicomputer.

T: ***Is this the way people usually write three and one-half dollars?***

Without saying anything, add a new board on the far right. (You may take the board from the far left. Wait for the reactions of the class. Perhaps someone will call this new board the *pennies board*.

S: ***There are no checkers on the new board. We can write 0 below it.***



T: ***3.5*** (read as "three dollars and five dimes" or "three point five") ***is the same as 3.50*** (read as "three dollars and fifty cents" or "three point five zero").

Indicate this equality on the board.

$$3.5 = 3.50$$

T: ***How do we usually write three and one-half dollars?***

Invite a student to show how to write it on the board.

$$\tfrac{1}{2} \times \$700 = \$350.00$$
$$\tfrac{1}{2} \times \$70 = \$35.00$$
$$\tfrac{1}{2} \times \$7 = \$3.50$$

T: ***We know that one-half of $7 is $3.50, so each of my two friends will receive $3.50.***

Point to the ones board (directly to the left of the bar).

T: ***This board is for dollars.***

Point to the board directly right of the bar.

T: ***What do you think this board is for?***

S: ***Dimes.***

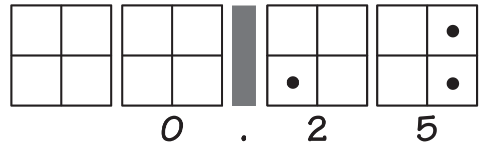
Point to the board to the right of the dimes board.

T: ***What do you think this board is for?***

S: ***Pennies.***

Remove the checkers from the Minicomputer and erase the board.

T: ***Let's put one quarter on the Minicomputer. Write this number below the Minicomputer.***

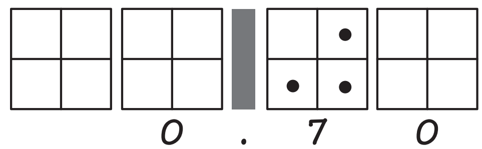


Include a 0 under the ones board, and a decimal point between the ones board and pennies board.

T: ***Yes, one quarter is 25 cents. We write 0.25*** (read as "zero point two five")***.***

Note: You may want to observe that one dollar is one hundred pennies, so 0.25 may also be read as "twenty-five hundredths."

Remove the checkers and erase the board.



T: ***Let's put seven dimes on the Minicomputer. Write this number below the Minicomputer.***

Seven dimes is seventy cents and we write 0.70 (read as "zero point seven zero")***. Is there another way to write this number?***

0.70 = 0.7

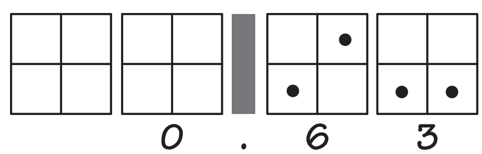
S: ***70¢.***

S: ***0.7.***

If no one suggests writing 0.7, remove the pennies board and repeat the question.

Remove the checkers and erase the board.

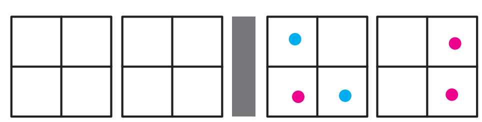
T: ***Let's put six dimes and three pennies on the Minicomputer. Write this number below the Minicomputer.***



Six dimes and three pennies is sixty-three cents and we write 0.63 (read as "zero point six three")***.***

Remove the checkers and erase the board.

T: ***Let's put nine dimes on the Minicomputer using blue checkers and a quarter on the Minicomputer using red checkers. How much is on the Minicomputer altogether ? How much is nine dimes and a quarter?***



Note: You may like to use real coins to display the problem.

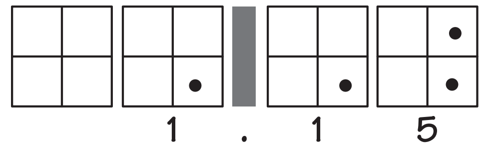
T: ***What calculation are we doing on the Minicomputer?***

S: ***0.90 + 0.25.***

T: ***Does anyone see a trade we can make?***

N27

If students have difficulty verbalizing the trade, say, "Eight dimes and two dimes is one dollar."



T: ***Who can write this number below the Minicomputer?***

How much money is nine dimes and one quarter?

S: ***One dollar and fifteen cents.***

T: ***We write 1.15*** (read as "one point one five" or "one dollar and fifteen cents")**.**

Center Activity

Put money collections and Minicomputer sets in a center. Suggest students practice putting various amounts on the Minicomputer and adding amounts of money.

Home Activity

This would be a good time to send home a review letter about the Minicomputer explaining its extension to decimal numbers. Blackline N27 has a sample letter. Suggest parents/guardians give their children opportunities to find an amount of money in a collection of coins and then to put that number on the Minicomputer.

Capsule Lesson Summary

Use ⑩-checkers to do 10x calculations on the Minicomputer. Build an arrow road from 0 to 52 using 10x and +1 arrows. Discuss where to use a 10x arrow in a shortest road. Consider how to extend the road to 523 and to 5231.

Materials

Teacher
- Minicomputer set
- ⑩-checkers
- Colored chalk
- Colored pencils, pens, or crayons

Student
- Unlined paper

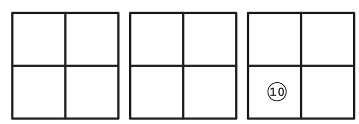
Description of Lesson

Exercise 1: 10x on the Minicomputer

Begin this exercise with a few 10x facts; for example,

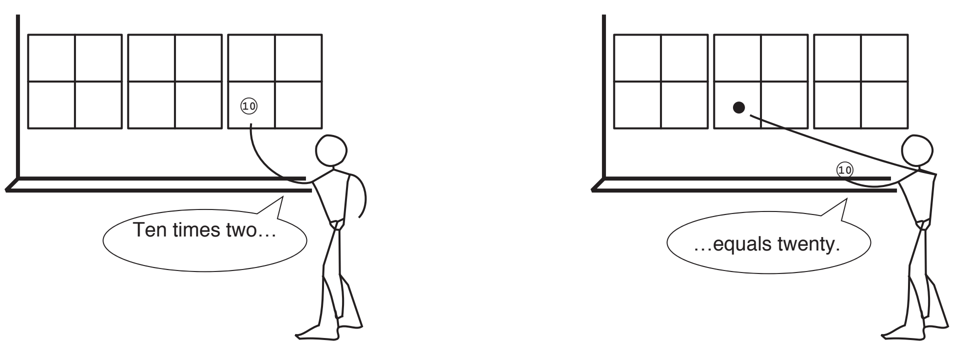
T: ***10 x 3 = …?*** (30)
10 x 5 = …? (50)
10 x 9 = …? (90)
10 x 0 = …? (0)
10 x 10 = …? (100)
10 x 2 = …? (20)

Put a ⑩-checker on the 2-square of the Minicomputer.



T: ***Ten checkers on this red square*** (2-square) ***is the same as one checker on this red square*** (20-square).

Make a 10 x 2 = 20 trade and write the fact on the board.



T: ***What number is 10 x 20?***

S: ***200.***

Write 10 x 20 = 200 under 10 x 2 = 20 on the board.

$$10 \times 2 = 20$$
$$10 \times 20 = 200$$
$$10 \times 200 = 2{,}000$$

Put a ⑩-checker on the 20–square of the Minicomputer and make a 10 x 20 = 200 trade.

T: ***What number is 10 x 200?***

S: ***2,000.***

Put a ⑩-checker on the 200-square of the Minicomputer and make a 10 x 200 = 2,000 trade.

T: ***Do you see a pattern?***

Encourage different responses. If a student suggests that when you multiply by 10 you simply write a 0 to the right of the numeral, accept this idea but do not emphasize it yourself.

Note: Writing a 0 to the right of the numeral works only when an integer is multiplied by 10. It does not work for non-integer decimals; for example, 10 x 7.9 is not 7.90.

Continue this sequence of calculations with 10 x 2,000.

T: ***What number is 10 x 2,000?***

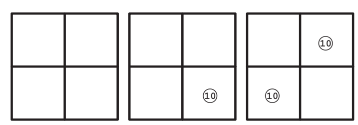
S: ***20,000.***

If no one answers correctly, ask if someone can write the answer on the board. Then let another student read the number aloud.

Continue this sequence with 10 x 20,000 = 200,000.

$$
\begin{array}{rcl}
10 \times 2 &=& 20 \\
10 \times 20 &=& 200 \\
10 \times 200 &=& 2{,}000 \\
10 \times 2{,}000 &=& 20{,}000
\end{array}
$$

Erase the board and then put this configuration on the Minicomputer.



T: ***What calculation is shown on the Minicomputer?***

S: ***10 x 16.***

Note: You may prefer to begin the following discussion with the ones board rather than the tens board.

T (pointing to the tens board)**:** ***What number is 10 x 10?***

S: ***100.***

T (pointing to the ones board)**:** ***What number is 10 x 6?***

S: ***60.***

$$
\begin{array}{rcl}
10 \times 10 &=& 100 \\
10 \times 6 &=& 60 \\
\hline
10 \times 16 &=& 160
\end{array}
$$

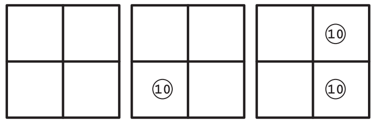
T: ***The number on the Minicomputer is 10 x 16. What number is this?***

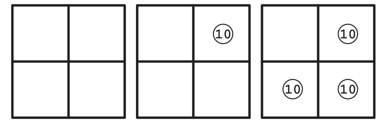
S: ***160.***

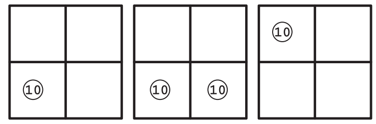
T: ***How do you know?***

Encourage students to express themselves. Some might calculate 100 + 60 while others might visually move the digits. Still others will think of writing a 0 on 16.

Repeat this activity with the following calculations.

10 x 25 = 

10 x 47 = 

10 x 238 = 

If students have difficulty with any of these calculations, ask them to make the appropriate trades to determine the number on the Minicomputer.

Erase the board and take away the Minicomputer.

Exercise 2: A 10x and +1 Arrow Road

Draw dots for 0 and 52, and write a key for 10x and +1 arrows on the board. (See the next illustration.)

T: ***Do you think we can build a road from 0 to 52 with only 10x and +1 arrows?***

S: ***Sure, we could use all +1 arrows.***

T: ***How many +1 arrows would we need to go from 0 to 52?***

S: ***Fifty-two.***

T: ***That would be a very long road. Let's try to find a shorter one using fewer arrows.***

Begin a road from 0 to 52 with the class. Follow the suggestions of your students; a sample dialogue follows.

T: ***We start at 0. Do you want the first arrow to be a 10x arrow or a +1 arrow?***

S: ***A 10x arrow.***

T: ***What number is 10 x 0?***

S: ***0.***

Draw a red loop at 0.

S: ***Start with a +1 arrow instead.***

T: ***What number is 0 + 1?***

S: ***1.***

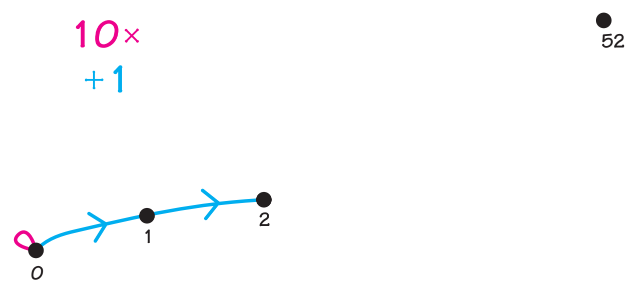
S: ***Now draw a 10x arrow.***

T: ***Will that help us get to 52?***

S: ***10 x 1 = 10, so we would be closer to 52.***

S: ***From 10 we won't be able to use anymore 10x arrows, because 10 x 10 = 100 and 100 is more than 52.***

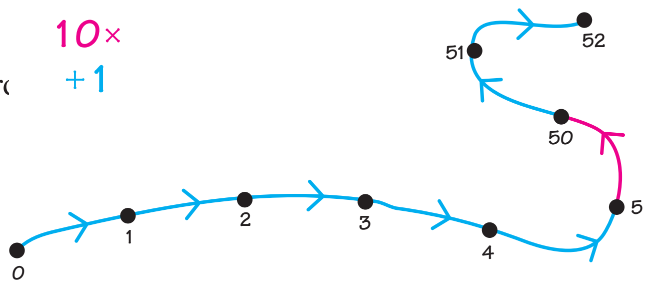
S: ***Maybe we should draw another +1 arro***



If your students want to draw a 10x arrow before they reach 5, ask them to look ahead to see what will happen if they draw a 10x arrow at that time. When your class decides that a shortest road has blue arrows until it reaches 5, ask them to draw the road on their papers.

Allow students to work independently to build a road from 0 to 52 (permitting, if they wish, roads that are longer than necessary). A student might use only +1 arrows, although the road will be quite long. Ask students who finish quickly to extend their roads to 523 and then, if there is sufficient time, to 5,231.

After a few minutes, ask a student with a short road from 0 to 52 to copy it on the boarc



Discuss this road collectively.

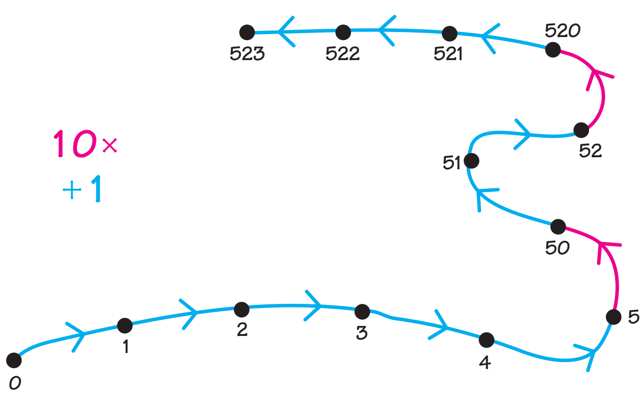
T: ***How many 10x arrows are in this road?*** (One)
How many +1 arrows…? (Seven)
How many arrows are there altogether? (Eight)
Did anyone build a road from 0 to 52 with less than eight arrows?

The shortest possible 10x and +1 arrow road from 0 to 52 has eight arrows.

Draw a dot for 523 on the board near the arrow picture.

T: ***We can extend this road until it meets 523 using all +1 arrows, but can we do it with fewer arrows?***

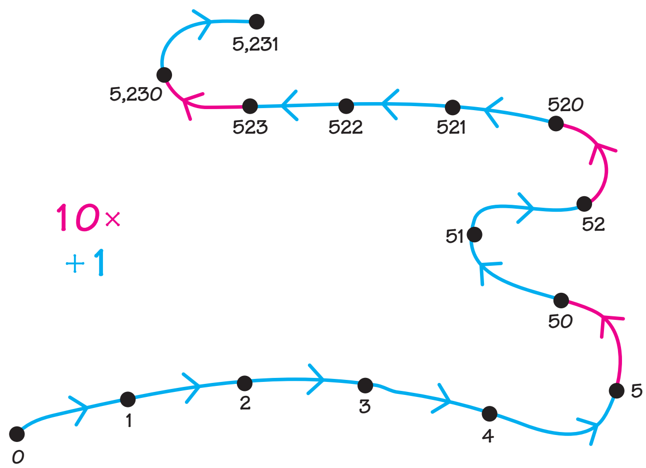
Several students may have already solved this problem. Encourage them to explain their solutions to the class. Invite a student to extend the arrow road to 523 on the board. As appropriate, help the students to see that a 10x arrow works like a ⑩-checker.



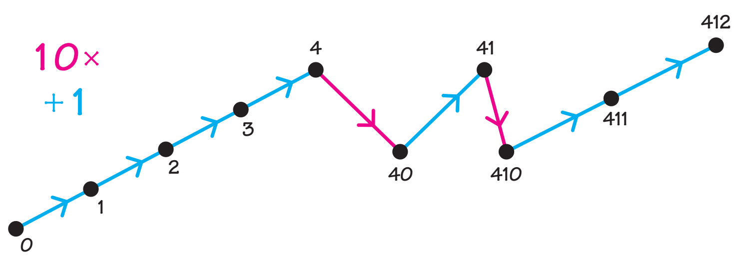
Draw a dot for 5,231 on the board near the arrow picture.

T: ***How can we extend this road to reach 5,231?***

Invite a student who has already solved this problem to explain to the class how to extend the arrow road to 5,231 on the board.



If there is time remaining, instruct students to build a 10x and +1 arrow road from 0 to 412. A shortest such road has nine arrows, but do not insist that all students find shortest roads.



N29 INTRODUCTION TO DECIMAL NUMBERS

Capsule Lesson Summary

Solve addition and subtraction problems that involve decimal numbers and arise from story situations.

Materials

Teacher	• Minicomputer set • Colored chalk	• Worksheets N29*, **, ***, and ****
Student	• Minicomputer set	

Description of Lesson

Exercise 1

Choose a student to star in this or a similar story.

T: ***Anita's grandmother gave her $2 to spend at the school fair. What coins or bills could Anita have if her total amount of spending money is $2?***

Encourage students to suggest many combinations of coins and bills. You may want to have some coins and bills for students to display $2.00. A possible dialogue follows.

S: ***Anita could have two $1 bills.***

S: ***Anita could have eight quarters.***

T: ***How many cents is one quarter?*** (25¢)
What number is 8 x 25? (200)
Eight quarters is 200¢, the same as $2.

S: ***She could have two half-dollars and ten dimes.***

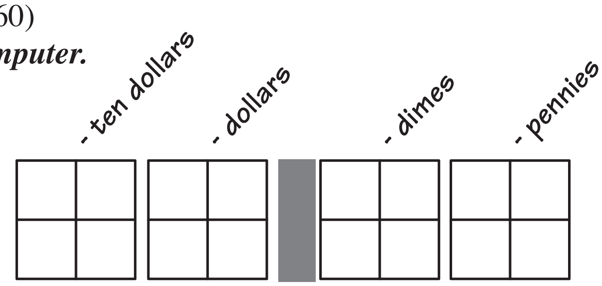
T: ***How much money is two half-dollars?*** ($1) ***2 x 50 = 100.***
How much money is ten dimes? ($1) ***10 x 10 = 100.***
So two half-dollars and ten dimes is $2.

At the school fair, Anita decides to spend 75¢ on the hoop toss and 85¢ on a surprise package.

Write $0.75 and $0.85 on the board and ask a student to read each of these prices aloud.

T: ***Does Anita have enough money?*** (Yes)
How much money would Anita spend? ($1.60)
Let's calculate $0.75 + $0.85 on the Minicomputer.

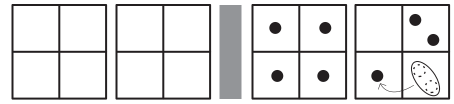
Display four Minicomputer boards and put a bar between the second and third boards. Point to each board and ask what it is for (as indicated here). You may want to let student pairs use individual Minicomputers and follow along with the class.



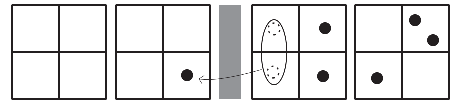
Ask a student to put $0.75 on the Minicomputer and another to put on $0.85.

Invite students to make trades, announcing the trades they make. The following is a possible sequence of trades.

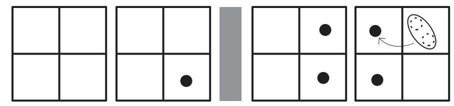
S: ***One penny plus one penny equals two pennies.***



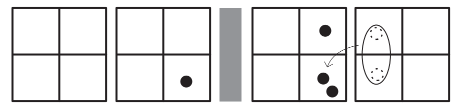
S: ***Eight dimes plus two dimes equals one dollar.***



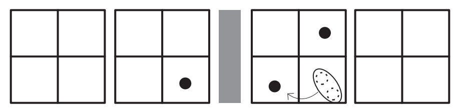
S: ***Four pennies plus four pennies equals eight pennies.***



S: ***Eight pennies plus two pennies equals on dime.***



S: ***One dime plus one dime equals two dimes.***



1 . 6 0

Call on a student to write the number below the Minicomputer.

T: ***We calculated $0.75 + $0.85 on the Minicomputer and got $1.60, as many of you predicted. We could also do this addition problem on paper.***

Write the corresponding addition problem on the board and invite a student to solve it. You may need to put in the decimal point yourself.

```
  $ 0.75
+ $ 0.85
--------
  $ 1.60
```

T: ***Remember that Anita took $2 with her to the school fair. After she spends $1.60, how much money will she have left?***

S: ***40¢.***

Write the corresponding subtraction problem on the board.

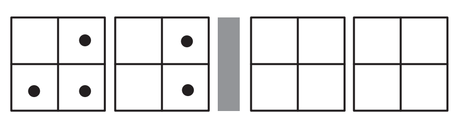
```
  $ 2.00
– $ 1.60
--------
  $ 0.40
```

T: ***$2.00 – $1.60 = $0.40.***

Erase the board and remove the checkers from the Minicomputer.

Exercise 2

Ask a student to put $75 on the Minicomputer and another to write the number below the Minicomputer.



7 5 . 0 0

T: ***Is there another way to write this number?***

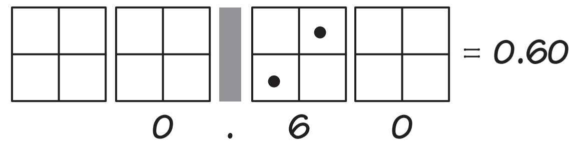
When students suggest 75 and 75.0, write those numbers on the board. If necessary, suggest them yourself.

75 = 75.0 = 75.00

Remove the checkers and erase the board. Ask students to put the following amounts of money on the Minicomputer, one at a time, and to write the appropriate number below the Minicomputer each time.

$1.06 $20.10 $8.30 $40.09

Ask a student to put six dimes on the Minicomputer, and another to write the number below the Minicomputer.



T (pointing to 0.60)**:** ***Is there another way we could write this number?***

If no one suggests writing 0.6, remove the pennies board and repeat the question. If necessary, tell the students that 0.6 is another way to write this number, and write this equality on the board.

0.60 = 0.6

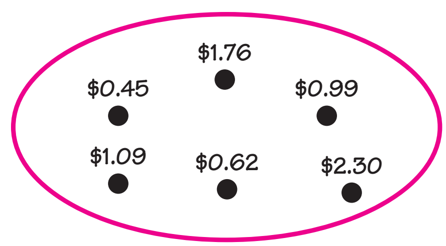
Of course someone could suggest writing .6 or .60; that is, not writing 0 to the left of the decimal point. However, do not write the number this way on the board.

Exercise 3

Choose a student to star in this or a similar story.

T: ***Tony decides to buy some new miniatures for his collection. Tony finds six miniatures he would like to buy.***

Assign six prices for the miniatures, and record these amounts inside a string on the board.



T: ***Which miniature is most expensive?***

S: ***The one that costs $2.30.***

T: ***Which miniature is least expensive?***

S: ***The one that costs $0.45.***

T: ***Do you think the bill for all six miniatures will be more or less than $5? Why?***

A student might indicate that the three (or four) most expensive miniatures cost more than $5.

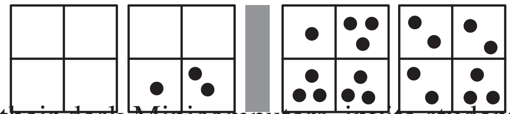
T: ***Do you think the bill will be more or less than $10? Why?***

A student might indicate that three of the miniatures cost less than $1 each and the other three miniatures cost about $5, so altogether the bill should be less than $10. Conclude that the price of all six miniatures is between $5 and $10. Invite students to estimate what the bill will be and record estimates between $5 and $10 on the board.

T: ***Use your Minicomputer to determine exactly how much Tony will have to pay.***

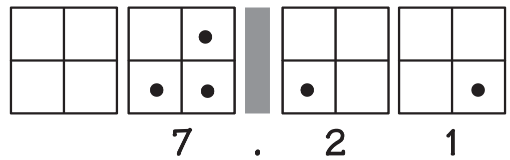
Instruct students (working with partners) to put the prices of the miniatures on their Minicomputers. You may find it convenient to move all the checkers to the corners of squares after a number is put on the Minicomputer to avoid confusion when the next number is put on.

Invite some students to help you put the numbers on the demonstration Minicomputer. After all the numbers have been put on the Minicomputer, you will have this configuration (assuming standard configurations were used for all the numbers).



While students work with their partners making trades on their desk Minicomputers, invite students to make trades on the demonstration board. Insist that students announce the trades they make.

Continue until the standard configuration for 7.21 is obtained. Ask a student to write the number below the Minicomputer.



T: ***How much will Tony pay for the miniatures?***

S: ***$7.21.***

T: ***Could we solve this problem another way?***

It's likely someone will suggest using the addition algorithm or using a calculator. Write the corresponding addition problem on the board and emphasize that all the decimal points must be lined up so you will be adding pennies together, dimes together, and dollars together. Invite a student to solve the problem.

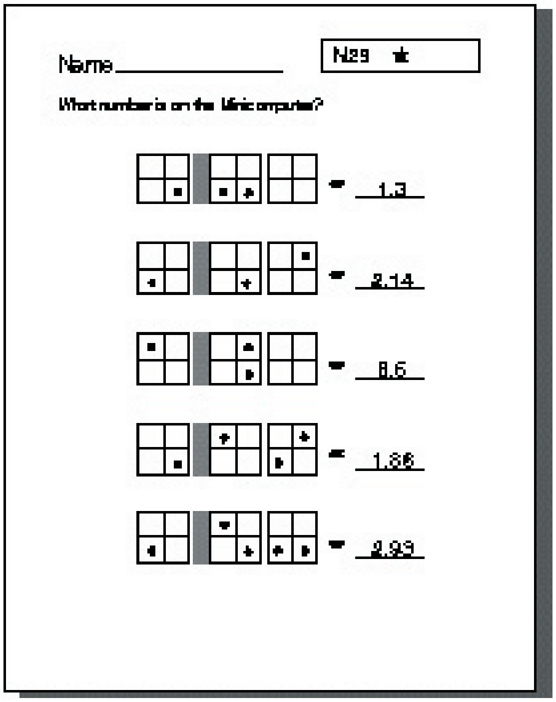
$$\begin{array}{r} {}^{3}\ \ {}^{3}\ \ \\ \$\ 0.62 \\ \$\ 0.45 \\ \$\ 1.76 \\ \$\ 1.09 \\ \$\ 0.99 \\ +\ \$\ 2.30 \\ \hline \$\ 7.21 \end{array}$$

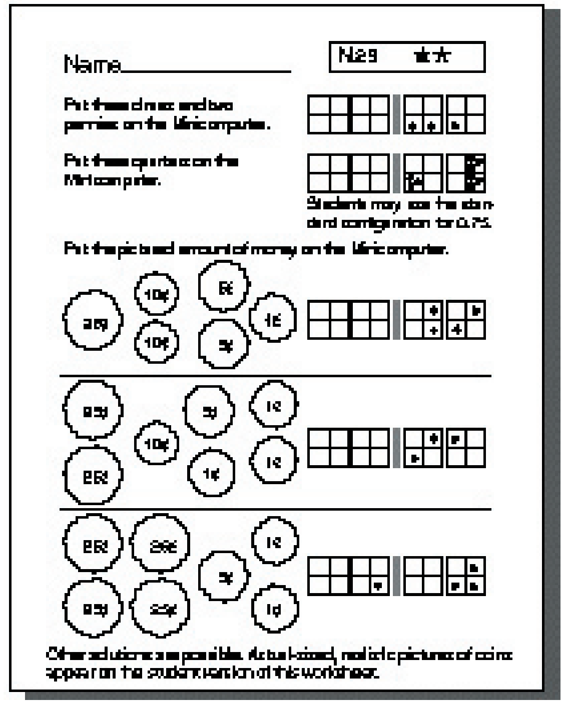
You may want to let another student check the addition with a calculator. With the class, determine which estimate was closest to $7.21.

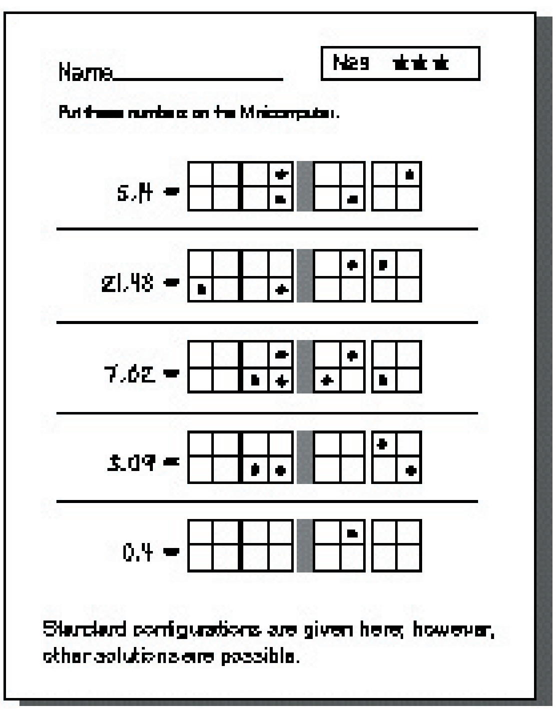
Worksheets N29*, **, ***, and **** are available for individual work.

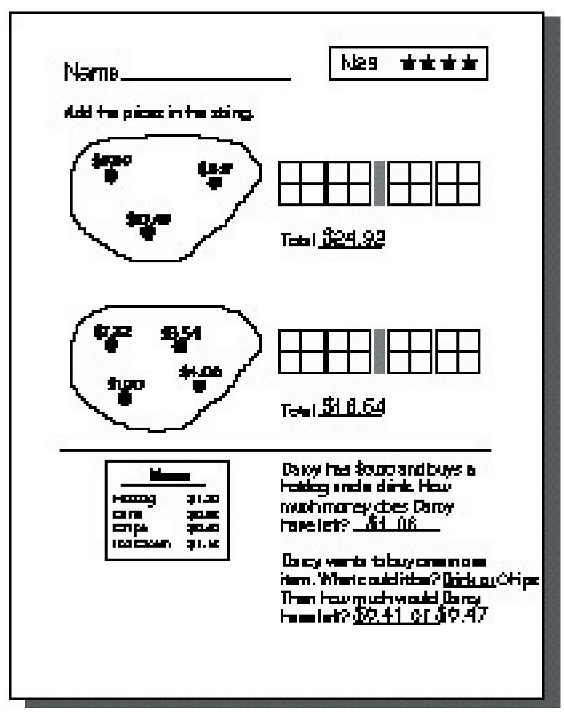
Home Activity

Suggest parents/guardians find opportunities to add several amounts of money with their child. Suggest they use the Minicomputer and a calculator to check.


Name
N29 ★
1.3
2.14
8.6
1.86
2.93


Name
N29 ★★
Students may use the standard configuration for 0.75.
10¢
10¢
5¢
1¢
1¢
1¢
10¢
5¢
1¢
1¢
1¢
5¢
1¢
Other solutions are possible. Actual-sized, realistic pictures of coins appear on the student version of this worksheet.


Name
N29 ★★★
Put these numbers on the Minicomputer.
5.14 =
21.48 =
7.62 =
3.09 =
0.4 =
Standard configurations are given here; however, other solutions are possible.


Name
N29 ★★★★
Total $24.92
$8.54
$4.06
Total $18.64
$1.08
$2.41 or $9.47

Capsule Lesson Summary

Find two numbers when you know their sum and how much greater (or less) one number is than the other. Determine many ways to share 30 (or 50) things when you require that each share is a whole number.

Materials

Teacher
- Props (including such things as play money, and counters or blocks to represent objects.)

Student
- Paper
- Colored pencils, pens, or crayons
- Props (like teacher materials)

Description of Lesson

Organize the class in pairs or small groups for this lesson. In each exercise, use props to make a story more realistic or interesting. Props may also help students to act out a story and solve the problems.

Exercise 1

Choose two of your students to be the star of this or a similar story.

T: ***Cody and Cora went shopping. Together they spent $20. Cody spent $4 more than Cora. How much did Cody spend? How much did Cora spend?***

Let students work with their partners or groups on this problem. Perhaps they will want to use props or draw pictures to find a solution. When several groups have found a solution, ask groups to share their methods.

S: ***We tried different numbers that add up to 20 until we found ones that are four apart.***

S: ***We first gave Cody $4. Then we shared the rest ($16) equally between Cody and Cora.***

You may like to ask the groups to solve a couple other similar problems, for example:

- Anita and Alex went shopping. Together they spent $50. Anita spent $8 more than Alex. How much did they each spend?
- Kyle and Libby went shopping. Together they spent $35. Kyle spent $3 less than Libby. How much did they each spend?

Exercise 2

Pose a different kind of problem to the class.

T: ***Cody bought a package with 30 stamps. He wants to share the stamps equally among his cousins. Cody wants to know how many stamps to give to each cousin. Can you help him?***

S: ***We need to know how many cousins.***

T: ***How many cousins could Cody have to share all 30 stamps equally (no leftovers and no cutting the stamps in parts)?***

Let students work with their partners or groups to find as many possible solutions as they can. Each solution should give the number of cousins and how many stamps for each. The following is a table of possible solutions:

number of cousins	number of stamps for each cousin
1	30
2	15
3	10
5	6

number of cousins	number of stamps for each cousin
30	1
15	2
10	3
6	5

You may like to pose another similar problem; for example, Cora bought a package of 50 markers to share equally with some friends.

number of friends	number of markers for each friend
1	50
2	25
5	10
10	5
25	2
50	1

Exercise 3

Pose still another kind of problem to the class.

T: ***Cody and Cora are trying to earn money to go shopping. Cody works for 2 hours and Cora works for 3 hours. Together they earn $15. How should they divide the money?***

Some students may suggest that they should share the money equally. Lead a discussion to decide that Cody should get less than Cora because he worked fewer hours. Let students work with their partners or groups to find a way to divide the money accounting for the difference in number of hours worked. Encourage students to explain their solutions.

S: ***We gave $2 to Cody and $3 to Cora, then $2 to Cody and $3 to Cora, and then $2 to Cody and $3 to Cora. Cody should get $6 and Cora $9.***

S: ***We figured they were paid $3 for each hour of work (they worked 5 hours and earned $15, 3 x 5 = 15). So Cody should get 2 x 3 or $6, and Cora should get 3 x 3 or $9.***

You may like to pose another similar problem, for example:

Cody works for 50 minutes and Cora works for 25 minutes. Together they earn $12. How much money should each get?

Capsule Lesson Summary

Do several 10x calculations on the Minicomputer using ⑩-checkers to represent 10x a number and making trades with the ⑩-checkers. Play Minicomputer Tug-of-War.

Materials

Teacher
- Minicomputer set
- ⑩-checkers
- Colored chalk

Student
- Paper
- Minicomputer set
- ⑩-checkers

Advance Preparation: Make ⑩-checkers for students by writing 10 on the back of regular student checkers.

Description of Lesson

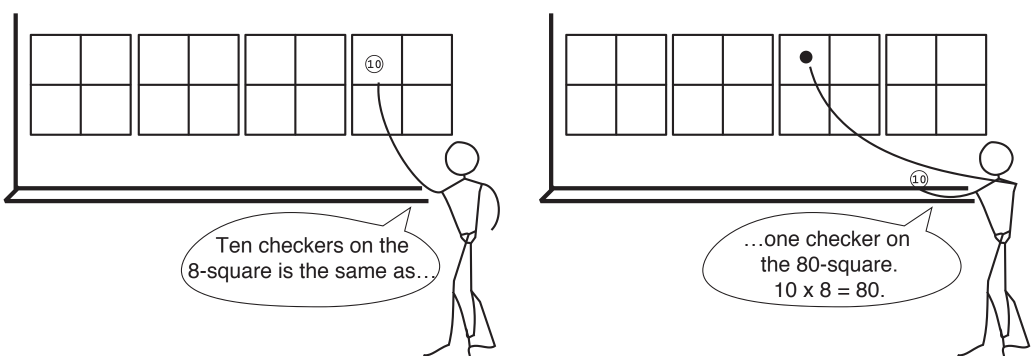
Exercise 1: 10x on the Minicomputer

Display four Minicomputer boards.

T: ***Let's put 10 x 8 on this Minicomputer using a ⑩-checker. We use a ⑩-checker to show that there are ten checkers on the same square. Ten checkers on the 8-square is 10 x 8. What trade can we make with this ⑩-checker?***

S: ***10 x 8 = 80.***

Invite a student to make the trade and then repeat the trade yourself.



Record 10 x 8 = 80 on the board and ask students to write the fact on their papers.

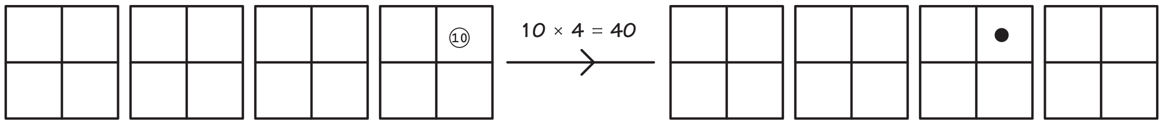
Put a ⑩-checker on the 4-square.

T: ***What calculation are we doing with this ⑩-checker?***

S: ***10 x 4.***

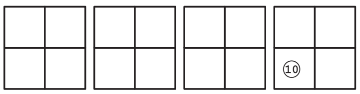
T: ***The number on the Minicomputer is 10 x 4. What trade can we make with this ⑩-checker?***

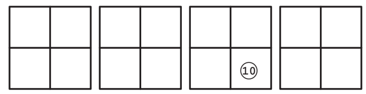
A volunteer should remove the ⑩-checker from the 4-square and put a regular checker on the 40-square. Describe the trade by saying, "10 x 4 = 40."

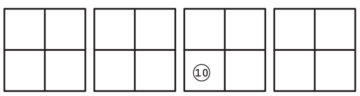


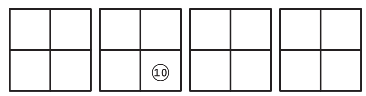
Record 10 x 4 = 40 on the board below 10 x 8 = 80. Ask students to record this fact as well on their papers.

Repeat this activity with these configurations.

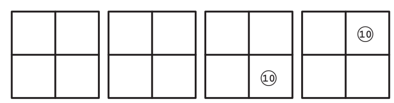
 = 10 × 2 = 20

 = 10 × 10 = 100

 = 10 × 20 = 200

 = 10 × 100 = 1,000

Put this configuration on the Minicomputer.



T: ***What calculation are we doing with these ⑩-checkers?***

S: ***10 x 14.***

T: ***What number is 10 x 14?***

S: ***140.***

Write 10 x 14 = 140 on the board. Ask the students to explain how they know that 10 x 14 = 140, and encourage a variety of responses.

S: ***10 x 10 = 100 and 10 x 4 = 40, so 10 x 14 = 140.***

S: ***I made the trades in my head.***

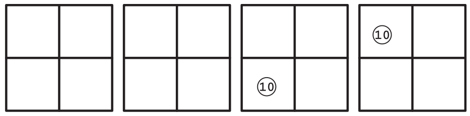
$$\begin{array}{r} 10 \times 10 = 100 \\ 10 \times 4 = 40 \\ \hline 10 \times 14 = 140 \end{array}$$

Make the 10 x 10 = 100 and 10 x 4 = 40 trades on the Minicomputer to aid a student's explanation.

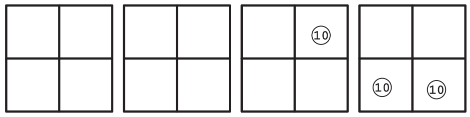
S: ***I knew it would be 140 because I put a 0 on 14.***

Note: This observation is evident in the 10x calculations on the board; however, it only works for integers. You may like to extend the list with a problem like 10 x 0.2 (⑩-checker on the 0.2 square).

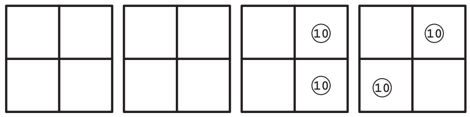
Continue this activity with these configurations. You may like students to do these calculations on their individual Minicomputers.



$= 10 \times 28 = 280$



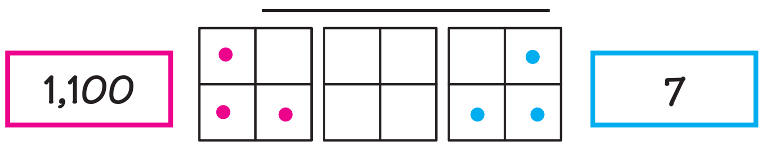
$= 10 \times 43 = 430$



$= 10 \times 56 = 560$

Exercise 2: Minicomputer Tug-of-War

Play several games of Minicomputer Tug-of-War as described in Lesson W4 *Festival of Problems #1 (Lesson 1)*. You may wish to use a variation of the game with this starting configuration.



Writing Activity

Ask students to write a note to a friend about how to multiply a number by 10.

Center Activity

Put individual Minicomputers in centers and let students play Minicomputer Tug-of-War with a partner.

Capsule Lesson Summary

Decide that the composite of four –10 arrows and a +1 arrow is a –39 arrow. Do subtraction calculations that involve the function –39. Use arrow pictures to describe various methods of doing calculations, and perform other subtraction calculations such as 74 – 28 and 111 – 43.

Materials

Teacher
- Colored chalk
- 0–109 numeral chart
- Blackline N23

Student
- 0–109 numeral chart
- Paper
- Colored pencils, pens, or crayons

Advance Preparation: Use Blackline N23 to make student copies of the 0–109 numeral chart.

Description of Lesson

Note: While this lesson intends to help students develop mental calculation strategies for subtracting, not all students should be expected to use the same method on a given problem. To insist that students view a problem from a one strategy perspective may diminish their willingness to apply their own "best fit" method to a problem or new situation.

Exercise 1

Begin this exercise with a couple minutes of mental arithmetic involving –10 and –20.

Distribute student copies of the 0–109 numeral chart and refer to the demonstration chart. Tell the class that you are going to use the numeral chart to help with some subtraction problems.

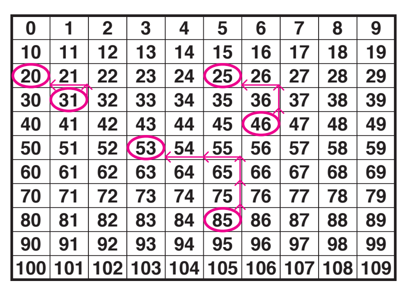
T: ***How would you find 73 – 10 on the numeral chart?***

S: ***Find 73 and then look at the number above it (go up ↑).***

Repeat the question to find 73 – 20, 73 – 30, 45 – 10, and 45 – 20.

T: ***How would you find 46 – 21 on the numeral chart?***

S: ***Find 46, go up two rows (–20), and then back one space.***



Solve a couple more subtraction problems of this kind with the 0–109 numeral chart. For example, 85 – 32 = 53, and 31 – 11 = 20.

T: ***How would you find 32 – 9 on the numeral chart?***

S: ***Find 32, go up one row (↑, –10), and then go one space to the right (→, +1).***

T: ***Count back nine spaces.***

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99
100	101	102	103	104	105	106	107	108	109

Observe with students that subtracting 9 is the same as subtracting 10 (↑) and then adding 1 (→).

T: ***How would you find 53 – 29 on the 0–109 chart?***

S: ***Find 53, go up three rows (–30), and then go one space to the right (+1).***

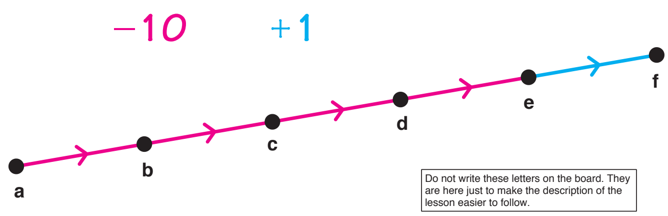
0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99
100	101	102	103	104	105	106	107	108	109

Write the calculation 53 – 29 = 24 as you observe that subtracting 29 is the same as subtracting 30 and then adding 1.

Solve a couple more subtraction problems using asimilar method with the 0–109 numeral chart. For example, 72 – 19 = 53, and 48 – 19 = 29.

Exercise 2

Draw this arrow picture on the board.



T: ***Where is the least number in this arrow picture?*** (At **e**)

Where is the greatest number in this arrow picture? (At **a**)

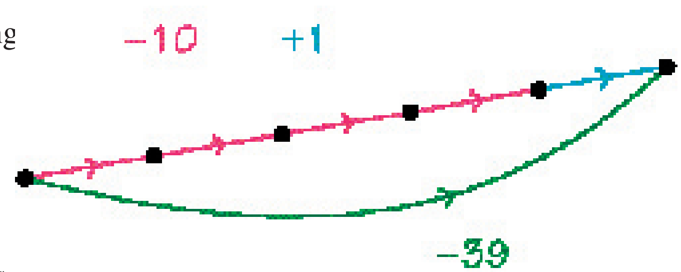
How do you know where the least and greatest numbers are in this arrow picture?

Students should notice that when you subtract 10 from a number you get a lesser number, and when you add 1 you get a greater number.

T: ***How much less is this number*** (point to **f**) ***than the starting number*** (point to **a**).

S: ***39.***

Draw an arrow from the starting dot to the ending



T: ***What could this arrow be for?***

S: ***–39.***

T: ***How do you know?***

S: ***Subtracting 10 four times is the same as subtracting 40, and then adding 1 makes it the same as subtracting 39.***

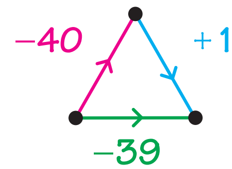
Practice several calculations with the function –39; use some of the following or similar problems.

90 – 39	40 – 39	81 – 39
93 – 39	42 – 39	67 – 39
100 – 39	105 – 39	158 – 39

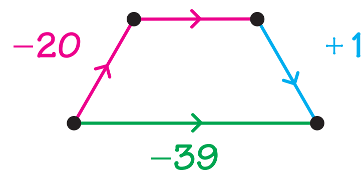
If methods other than that suggested in the above arrow picture are mentioned, draw the corresponding arrow pictures on the board.

For example, the following arrow pictures describe methods that students may suggest for subtracting 39.

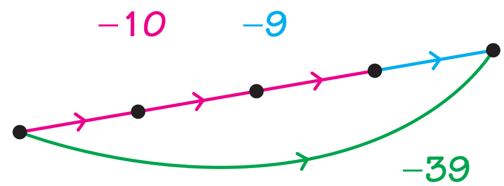
Subtract 40, and then add 1.



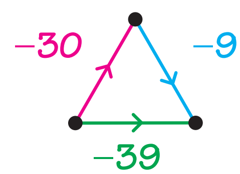
Subtract 20 two times, and then add 1.



Subtract 10 three times, and then subtract 9.



Subtract 30, and then subtract 9.



Erase the board, and then write this subtraction problem.

74 – 28

Instruct the students to solve this problem individually and to draw arrow pictures to explain their solutions. Ask students with distinct solutions to draw them on the board. Briefly discuss the various arrow pictures and conclude that 74 – 28 = 46.

Erase the board and repeat this activity to calculate 111 – 43. If some students finish quickly, ask them to calculate 242 – 137 and to show their methods in arrow pictures.

N33 MULTIPLICATION AND ADDITION PROBLEMS

Capsule Lesson Summary

Put 39 on the Minicomputer as $40 + \hat{1}$. Then double 39 by doubling the checkers in this configuration. Conclude that 2 x 39 = 80 – 2 = 78. Use a similar strategy for other multiplication calculations. Given the result of an addition calculation and one of the addends, find the other addend.

Materials

Teacher		Student	
	• Minicomputer set		• Paper • Worksheets N33(a) and (b)

Description of Lesson

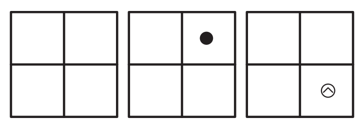
Exercise 1: Multiplication Problems

During this exercise all students should have paper and pencil for doing calculations.

T: ***What number is 2 x 40?*** (80)
2 x 40 = 80, so what number is 2 x 39? (78)
How do you know?

Let several students explain how they would calculate 2 x 39.

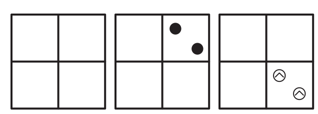
Put this configuration on the Minicomputer and ask what number it is.


= 39

Observe that $40 + \hat{1}$ or 40 – 1 is 39 and write 39 = 40 – 1 on the board.

T: ***Let's double 39 on the Minicomputer to calculate 2 x 39.***

Invite a student to put the same configuration for 39 on the Minicomputer again.


= 2 × 39

T: ***What number is on the tens board?***

S: ***80.***

T: ***What number is on the ones board?***

S: ***$\hat{2}$.***

T: ***So what number is on the Minicomputer?***

S: ***78.***

T: ***How do you know?***

S: ***$80 + \hat{2} = 78$.***

S: ***80 – 2 = 78.***

Record this information in a number sentence.

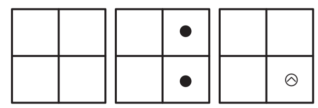
39 = 40 – 1
2 x 39 = 80 – 2 = 78

Remove the checkers from the Minicomputer and erase the board.

T: ***What number is 2 x 50?*** (100)
2 x 50 = 100; so what number is 2 x 49? (98)
How do you know?

Let students explain how they would calculate 2 x 49. Commend a student who suggests calculating 100 – 2.

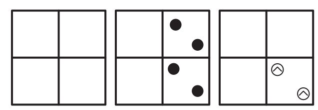
Write 49 = 50 – 1 on the board, and ask a student to put 49 on the Minicomputer as 50 + $\hat{1}$.



= 49

T: ***This number is 49. Now let's calculate 2 x 49.***

Invite a student to put the same configuration for 49 on the Minicomputer again.



= 2 × 49

T: ***What number is on the tens board?***

S: ***100.***

T: ***What number is on the ones board?***

S: ***$\hat{2}$.***

T: ***So what number is on the Minicomputer?***

S: ***98.***

T: ***How do you know?***

S: ***100 + $\hat{2}$ = 98.***

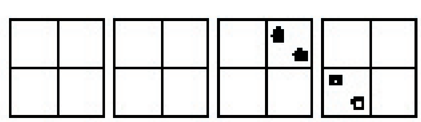
S: ***100 – 2 = 98.***

Record this information in a number sentence.

49 = 50 – 1
2 x 49 = 100 – 2 = 98

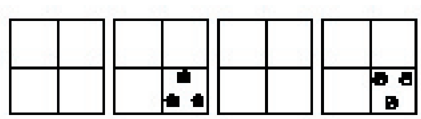
Remove the checkers and erase the board. Repeat this activity with the following multiplication problems.

2 × 38 =



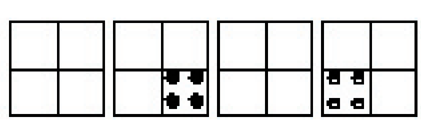
= 80 – 4 = 76

3 × 99 =



= 300 – 3 = 297

4 × 98 =



= 400 – 8 = 392

Comments: Some students may want to check the results for 3 x 99 and 4 x 98 using repeated addition. If you do this, take the opportunity to review the addition algorithm and perhaps note the relative ease of this using negative numbers or subtraction.

```
 3
 98      98
 98     × 4
 98      32
 98     360
392     392
```

Encourage the use of multiplication facts; for example, when adding 98 + 98 + 98 + 98, think of 4 x 8 = 32 and 4 x 9 = 36.

Erase the board and take away the Minicomputer.

Exercise 2: Addition Problems

Remind the students of the addition puzzles they did in Lesson N19. Distribute copies of Worksheet N33(a) and direct students to the first problem. Use a story similar to the one you used in Lesson N19 to explain the problem.

T: ***Do you remember an eraser gremlin who erases one of the numbers in addition calculations?***

Here 403 plus a missing number is 698. Can you find the missing number?

```
  403
+
  698
```

Allow a minute or two for students to work individually. Then call on a student to explain how to find the missing number while completing the problem at the board.

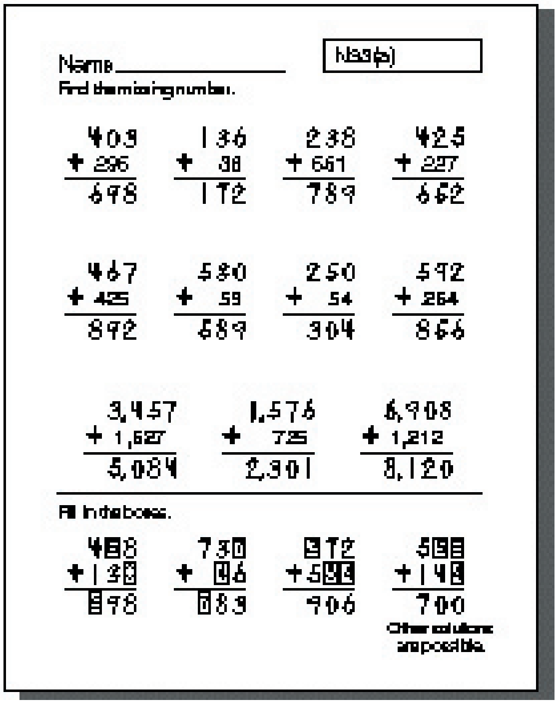
Check the addition collectively to show that 403 + 295 = 698.

```
  403
+ 295
  698
```

Continue this activity, solving one or two other problems on the worksheet. Include an example where the resulting addition calculation involves a carry.

Let students work independently to finish the worksheet. Remind them that in the bottom row of problems, the eraser gremlin just erased certain digits, and that a box is where there is a missing digit. One digit goes in each box.

Worksheet N33(b) can be used for additional practice with multiplication problems like those in Exercise 1.


Name

N33(a)

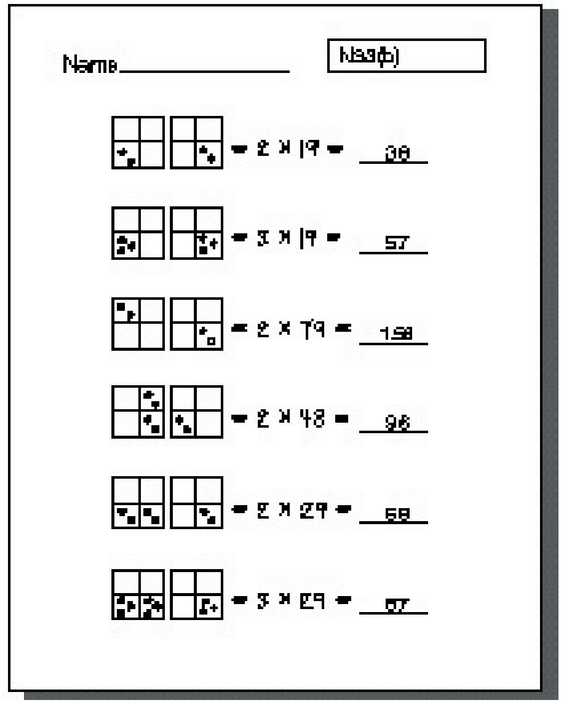
Find the missing number.

403 + 295 = 698	136 + 36 = 172	238 + 551 = 789	425 + 227 = 652
467 + 425 = 892	530 + 59 = 589	250 + 54 = 304	592 + 264 = 856
3,457 + 1,627 = 5,084	1,576 + 725 = 2,301	6,908 + 1,212 = 8,120	

Fill in the boxes.

468 + 130 = 598	730 + 106 = 836	372 + 534 = 906	552 + 148 = 700

Other solutions are possible.


Name

N33(b)

= 2 × 19 = 38

= 3 × 19 = 57

= 2 × 79 = 158

= 2 × 48 = 96

= 2 × 29 = 58

= 3 × 29 = 87

Capsule Lesson Summary

Find shortest routes between cities in Virginia. Decide which of two cities is closer to a third. Plan a trip that starts at a particular city and is approximately 500 km long.

Materials

Teacher
- *UPG-III World of Numbers Poster #3*
- Tape
- Blackline N34

Student
- Map of Virginia
- Paper
- Worksheets N34(a) and (b)
- Calculator

Advance Preparation: Use Blackline N34 to make copies of the Virginia map for students.

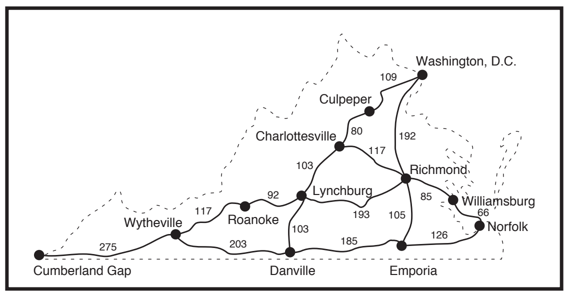
Description of Lesson

Display *UPG-III World of Numbers Poster #3*, and provide copies of the Virginia map for students.

Note: You may prefer to do one or two problems collectively, using the poster only, before giving students the Virginia map.

(If you and your students are in Virginia, adjust the following dialogue so that it is appropriate.)

T: ***This is a map of one of the states of the United States of America. What is the name of this state? Do you know any of these cities?***



You may need to tell your class that this state is Virginia. Encourage students to share what they know about Virginia and the cities indicated on this map. Point out that the numbers on the map show how far it is in kilometers from one city to another. Trace the road from Danville to Lynchburg.

T: ***How far is it from Danville to Lynchburg according to this map?*** (103 kilometers)
How far is it from Culpeper to Charlottesville? (80 kilometers)
How far is it from Wytheville to Cumberland Gap? (275 kilometers)

Problem 1: Charlottesville to Williamsburg

Abbreviate a statement of the problem on the board.

Charlottesville to Williamsburg?

T: ***What is the shortest route from Charlottesville to Williamsburg?***

S: ***Go from Charlottesville to Richmond, and then on to Williamsburg.***

Ask a student to trace this route on the poster; then record it on the board.

T: ***How long is this route from Charlottesville to Williamsburg? Write your answer on a piece of paper.***

Look at many of the students' answers and help students who are having difficulty. Ask a student to answer aloud and complete the sentence on the board.

T: ***How did you calculate the distance from Charlottesville to Williamsburg?***

S: ***I added 117 + 85.***

Charlottesville to Williamsburg?
Charlottesville to Richmond to Williamsburg is 202 km.

Write the addition problem on the board, and call on a volunteer to solve this problem at the board, explaining each step.

T: ***Is this the shortest route from Charlottesville to Williamsburg? How do you know?***

Let students give explanations, but do not expect well-formed answers involving calculations.

Erase the board before going on to Problem 2. Problem 2 is on Worksheet N34(a).

Problem 2: Williamsburg to Emporia

Abbreviate the problem on the board.

Williamsburg to Emporia?

T: ***What is the shortest route from Williamsburg to Emporia?***

Some students might believe that the shortest route from Williamsburg to Emporia is through Richmond, while other students might claim that the route through Norfolk is shorter. Ask students to trace these routes on the poster, and record them on the board. Direct students to calculate the lengths of both routes so that their lengths can be compared.

Allow a few minutes for individual work; then invite two students to solve the respective addition problems at the board.

$$\begin{array}{r} 85 \\ +\ 105 \\ \hline 190 \end{array} \qquad \begin{array}{r} 66 \\ +\ 126 \\ \hline 192 \end{array}$$

Complete the sentences on the board and conclude that the shortest distance from Williamsburg to Emporia is 190 km.

Williamsburg to Emporia?
Williamsburg to Richmond to Emporia is 190 km.
Williamsburg to Norfolk to Emporia is 192 km.

T: ***How much longer is the route from Williamsburg to Emporia through Norfolk than the route through Richmond?***

S: ***2 km.***

Erase the board before going on to Problem 3. Problem 3 is on Worksheet N34(b).

Problem 3: Wytheville to Washington, D.C. vs. Wytheville to Norfolk

T: ***Which city is closer to Wytheville — Washington, D. C. or Norfolk?***

Abbreviate the question on the board.

Wytheville to Washington, D.C.?
Wytheville to Norfolk?

The students may not be sure which city is closer to Wytheville, so ask them to calculate both distances. Allow several minutes for independent work. Then ask students to describe the shortest routes from Wytheville to Washington, D. C. and from Wytheville to Norfolk, and to tell the length of each of these routes. Record the information on the board.

Wytheville to Washington, D. C.?
Wytheville to Roanoke to Lynchburg to Charlottesville
to Culpeper to Washington, D. C. is 501 km.
(117 + 92 + 103 + 80 + 109 = 501)
Wytheville to Norfolk?
Wytheville to Danville to Emporia to Norfolk is 514 km.
(203 + 185 + 126 = 514)

T: *Which distance is shorter?*

S: *501 km is less than 514 km.*

T: *Is Washington, D. C. or Norfolk closer to Wytheville?*

S: *Washington, D. C.*

T: *How much farther is it from Wytheville to Norfolk than it is from Wytheville to Washington, D. C.?*

S: *13 km.*

Erase the board before going on to Problem 4.

Problem 4: A Trip Starting at Lynchburg

T: *I want everyone to plan a trip that ~~starts at Lynchburg an~~d is approximately 500 km long. Try to make the length of your trip as close to 500 km as possible. You may end your trip in any of the cities on the map. Keep a record of your trip by writing down the names of the cities you go through.*

Allow several minutes for independent or group work. Then choose a student's trip to record on the board. Try to choose a trip that is at least 550 km long for the collective discussion. An example is given here.

$$\begin{array}{r} 193 \\ 105 \\ 185 \\ +\ 103 \\ \hline 586 \end{array}$$

T: *Elizabeth has a trip from Lynchburg to Richmond to Emporia to Danville and back to Lynchburg.*

T: *Did anyone else plan the same trip? How long is this trip?*

S: *586 km.*

Lynchburg to Richmond
to Emporia to Danville
to Lynchburg is 586 km.

T: *Is this trip more or less than 500 km long?*

S: *More.*

T: *How much longer than 500 km is it?*

S: *86 km.*

T: ***Is there another trip we could take starting at Lynchburg that is closer to 500 km long?***

Perhaps a student will suggest this trip.

Lynchburg to Roanoke
to Wytheville
to Cumberland Gap is 484 km.

T: ***Is this trip more or less than 500 km long?***

S: ***Less.***

T: ***How much less?***

S: ***16 km.***

Conclude that 484 is closer to 500 than is 586.

T: ***Is there another trip we could take starting at Lynchburg that is still closer to 500 km long?***

Continue letting students suggest trips closer to 500 km until at least one trip with a length between 480 km and 520 km is found. There are several such trips: for example, Lynchburg to Charlottesville to Richmond to Williamsburg to Norfolk to Emporia (497 km); and Lynchburg to Danville to Emporia to Richmond to Charlottesville (510 km). Compare all the trips you have recorded on the board, and decide which of them has a length closest to 500 km.

Center Activity

Put maps like the Virginia map in a center, and pose problems similar to ones in this lessons. Students may pose their own problems.

Writing Activity

Direct students to write a letter to a friend about one of the trips they planned in this lesson. They should include the names of the cities they will go through, the distances between cities, and how they found the total length of the trip.

Home Activity

Send home the Virginia map with one or two problems like Problems 3 and 4. For example,

- Which city is closer to Washington, D. C. — Roanoke or Norfolk? How much closer?
- Plan a trip starting at Norfolk that is approximately 650 km long.

N35 MULTIPLICATION AND COMPOSITION

Capsule Lesson Summary

Create a picture with 2 x 7 objects. Then double the number of objects, and observe calculations suggested by the picture. Decide that a 2x arrow followed by a 2x arrow is the same as a 4x arrow. Put 17 on the Minicomputer, double 17, and then double the result. Notice that the number now on the Minicomputer is 4 x 17. Show this in an arrow picture.

Materials

Teacher
- Colored chalk
- Minicomputer set
- Counters or blocks
- Overhead projector (optional)

Student
- Unlined paper
- Colored pencils, pens, or crayons
- Counters or blocks

Note: Use checkers, small cubes, or other counters for this lesson. Each group of students will need approximately 30 objects.

Description of Lesson

Exercise 1

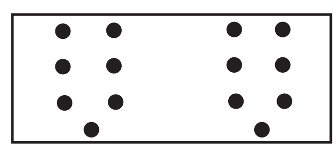
Organize the class in pairs or small groups. Each group should have a set of student materials.

T: ***On your paper, lay out 2 x 7 objects (checkers, blocks, counters). For the moment, I am not interested in calculating 2 x 7. I only want you to arrange exactly 2 x 7 objects. Try to make it easy to see that there are 2 x 7 objects.***

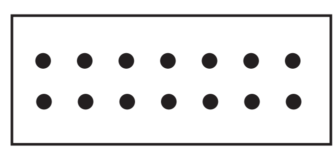
Look at the students' pictures, and select several of them in which it is easy to see that there are 2 x 7 objects. When most of the students have completed their layouts, show those you have selected to the class, and ask for volunteers to describe each of them.

Note: You may let students place their layouts on the overhead projector to project or draw a dot picture of the layout on the board.

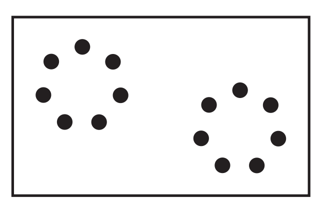
S: ***There are two groups of objects and each group has seven objects.***



S: ***There are two rows with seven in each row.***

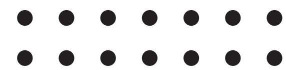


S: ***There are two circles and each circle has seven objects.***



N35

Refer to or draw the two rows of seven dots (objects).



T: ***What calculations does this picture (layout) suggest?***

S: ***2 x 7 = 14.***

S: ***7 x 2 = 14.***

S: ***7 + 7 = 14.***

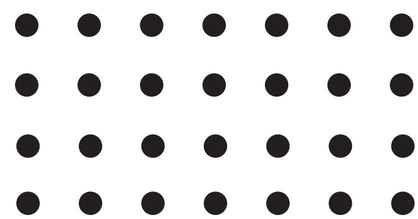
S: ***2 + 2 + 2 + 2 + 2 + 2 + 2 = 14.***

T: ***Arrange your objects in this way on your paper.***

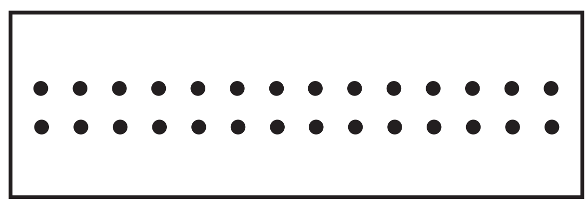
Allow a minute or two for students to complete this task.

T: ***Now continue adding objects to the layout until you have twice as many objects on your paper.***

When most students complete this task, discuss the resulting arrangement with the class as you did earlier.



S: ***There are four rows with seven in each.***



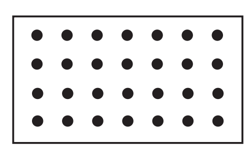
S: ***There are two rows with 14 in each.***

T: ***Now I, too, will draw dots on the board (or arrange objects on the overhead) until there are twice as many dots (objects) as there are now. I'll make two more rows of seven dots (objects) below the two rows already on the board. How many dots (objects) will I have altogether in my picture? How do you know?***

S: ***Twenty-eight, because there will be four rows of seven and 4 x 7 = 28.***

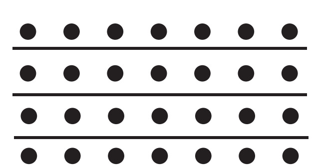
S: ***There are fourteen dots (objects) on the board now, and 2 x 14 = 28.***

Extend the dot picture on the board (or the arrangement on the overhead).



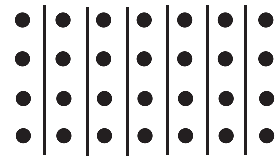
T: ***What calculations does this picture suggest?***

Record the corresponding calculations on the board as they are suggested. Relate each calculation to the picture or ask students to do so. Several examples are given below. Do not draw lines on the board; they are here to show how the collection of dots (objects) may be viewed.

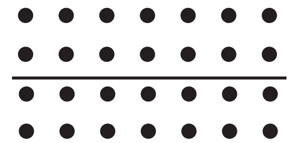


S: ***4 x 7 = 28.***

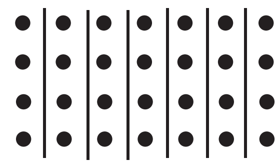
S: ***7 x 4 = 28.***



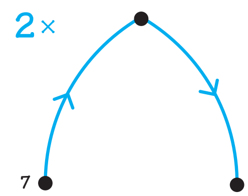
S: ***2 x 14 = 28.***



S: ***12 + 12 + 4 = 28.***

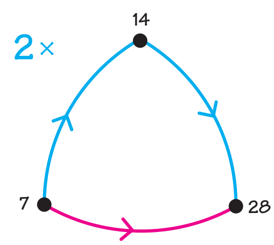


Draw this arrow picture on the board.



T: ***This arrow picture is about what we did with the dots (objects).***

Call on volunteers to label the dots, and then draw a red arrow from 7 to 28.



T: ***What could this red arrow be for?***

S: ***4x.***

T: ***Yes, 4 x 7 = 28.***

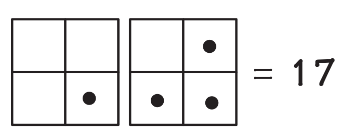
Label the red arrow 4x, and then trace the arrows using motions described in Lesson L2 *Composition Games #1* as you say,

T: ***2x followed by 2x is the same as 4x.***

Put away the materials from Exercise 1 before continuing.

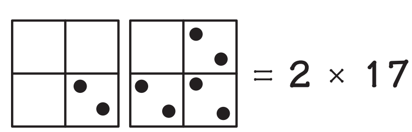
Exercise 2

Erase the board and display two Minicomputer boards. Ask a student to put 17 on the Minicomputer.



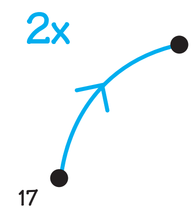
T: ***How could we get 2 x 17 on the Minicomputer?***

S: ***Put another checker with each of the checkers already on the Minicomputer.***



N35

On the board, draw a 2x arrow starting at 17.



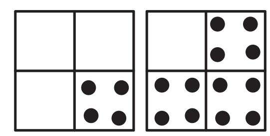
T: ***First we put 17 on the Minicomputer, and then we doubled it. Now 2 x 17 is on the Minicomputer. For the moment, I don't want to calculate 2 x 17. What could we do to get 2x this new number on the Minicomputer?***

Let the students discuss this problem and make suggestions.

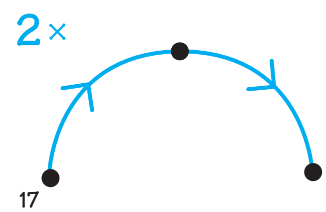
S: ***We need to put another pair of checkers with each pair of checkers on the Minicomputer.***

S: ***We should put 2 x 17 on the Minicomputer again.***

Ask a student to double the number on the Minicomputer by putting the 2 x 17 configuration on again.



Draw another blue arrow.



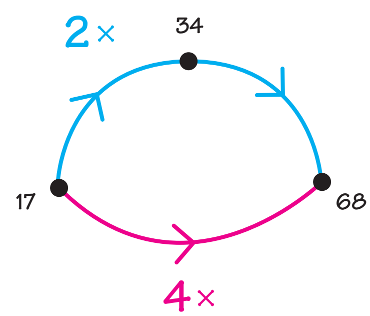
T: ***First we doubled 17 and then we doubled it again. How many times is 17 on the Minicomputer now?***

S: ***Four.***

T: ***The number on the Minicomputer is 4 x 17.***

Ask for a volunteer to draw a red 4x arrow in the picture. Call on students to label the dots.

Trace the appropriate arrows as you say,



T: ***2 x 17 = 34 and 2 x 34 = 68, so 4 x 17 = 68.***

Erase the labels for the dots and take away the Minicomputer. Use the arrow picture in a brief 4x mental arithmetic activity.

T: ***What number is 4 x 11? How do you know?***

S: ***44; I added 11 + 11 + 11 + 11.***

S: ***2 x 11 = 22 and 2 x 22 = 44.***

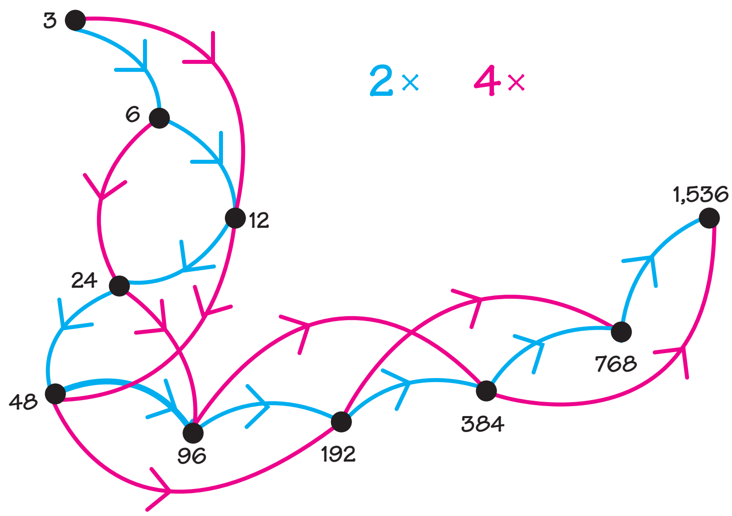
Demonstrate this calculation with the arrow picture. Point to the starting dot and say, "11"; then trace the appropriate arrows as you repeat,

T: ***2 x 11 = 22 and 2 x 22 = 44; so 4 x 11 = 44.***

Erase everything on the board except the arrow picture, and repeat this activity with these calculations: 4 x 25 (100); 4 x 6 (24); 4 x 27 (108); and 4 x 55 (220).

Exercise 3

Instruct students to start with any whole number they like between 0 and 10, and to draw a 2x arrow road on their papers. When you notice that students have several arrows in their 2x arrow roads, ask them to draw 4x arrows between numbers already in their pictures. One example of a student's picture is shown here.



Home Activity

Send home an arrow picture with 2x followed by 2x is 4x. You may use Blackline N35 to make copies of an arrow picture that students can color and label arrows. Suggest that students explain to their parents/guardians how to use the picture to do calculations such as 4 x 19 and 4 x 52.

Capsule Lesson Summary

Build roads from one number to another using only the subtraction arrows –1, –10, and –100. For each road, find the composite of the arrows in the road, and ask for the corresponding subtraction calculations.

Materials

Teacher • Colored chalk

Student • Colored pencils, pens, or crayons • Worksheets N36*, **, ***, and ****

Description of Lesson

Exercise 1

Begin this lesson with some mental arithmetic involving adding and subtracting 10.

Exercise 2

Draw two dots, one for 63 and one for 39, and indicate keys for –10 and –1 arrows on the board. (See the next illustration.)

T: ***Let's build an arrow road from 63 to 39 using –10 and –1 arrows? Can we do it?***

S: ***Yes, we can use all –1 arrows.***

S: ***Yes, we can use some –10 arrows and some –1 arrows.***

Build a road from 63 to 39 with the class. Follow the suggestions of your students.

S: ***Start with a –10 arrow.***

T: ***What number is 63 – 10?***

S: ***53.***

Each time, let students explain the calculations.

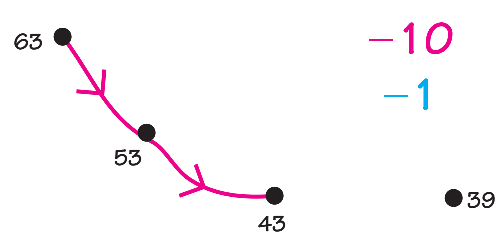
T: ***63 has 6 tens and 3 ones. If we subtract 1 ten*** (trace the –10 arrow)***, there will be 5 tens and 3 ones left, or 53.***

S: ***Draw another –10 arrow.***

T: ***What number is 53 – 10?***

S: ***43.***

T: ***53 has 5 tens and 3 ones. If we subtract 1 ten, there will be 4 tens and 3 ones left, or 43.***



Would it help to draw another –10 arrow? (No) ***Why not?***

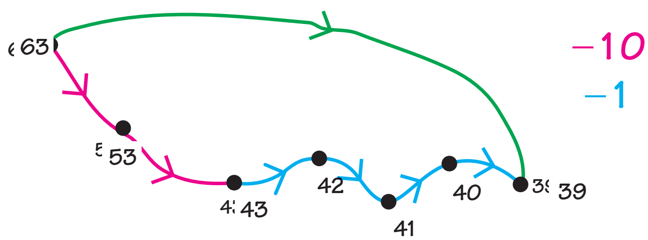
S: ***43 – 10 = 33. We cannot go from 33 to 39 because we are using subtraction arrows.***

S: ***We need to draw some –1 arrows.***

T: ***How many –1 arrows should we draw?***

S: ***Four.***

Invite students to complete the arrow road, and then draw a green arrow from 63 to 39.



T: ***What could this green arrow be for?***

S: ***–24.***

Label the green arrow –24.

T: ***What subtraction calculation does this green arrow show?***

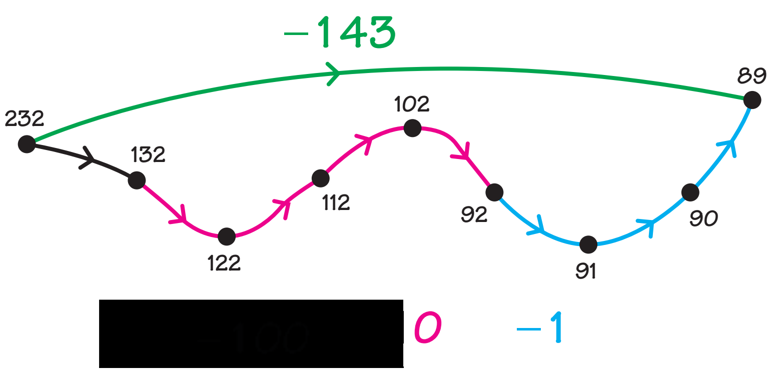
S: ***63 – 24 = 39.***

If necessary, suggest this number sentence yourself. Ask for volunteers to record it on the board in both horizontal and vertical formats.

$63 - 24 = 39$

$$\begin{array}{r} 63 \\ -\ 24 \\ \hline 39 \end{array}$$

Repeat this exercise to build an arrow road from 232 to 89 using –100, –10, and –1 arrows.

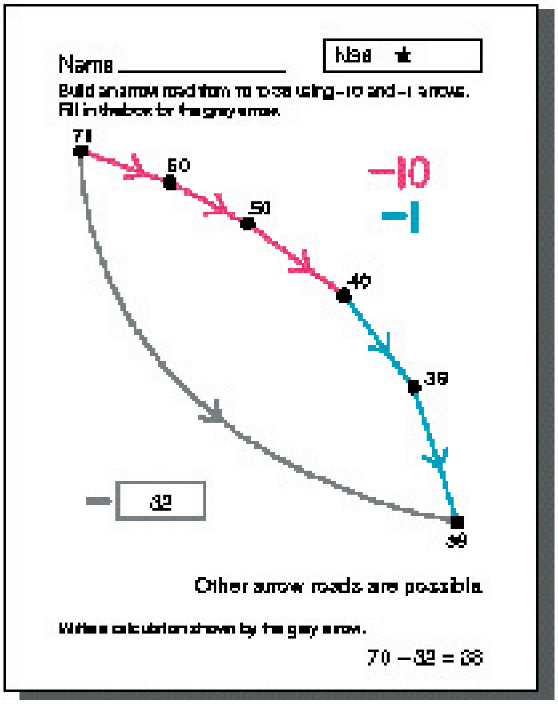


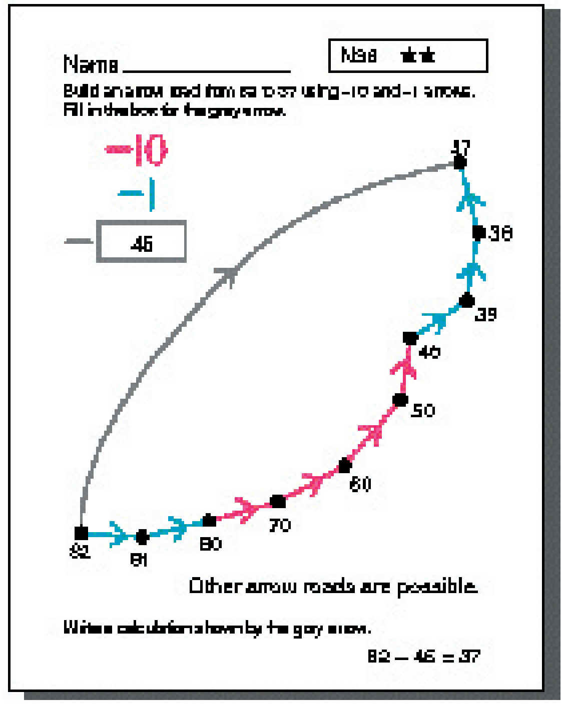
$232 - 143 = 89$

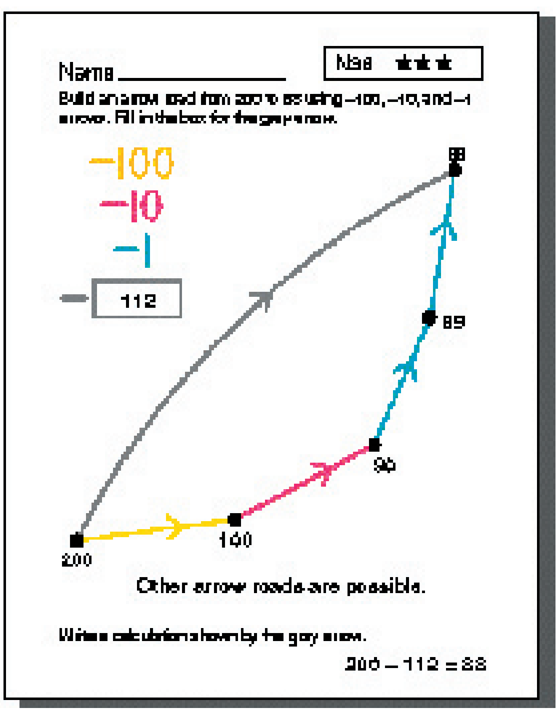
$$\begin{array}{r} 232 \\ -\ 143 \\ \hline 89 \end{array}$$

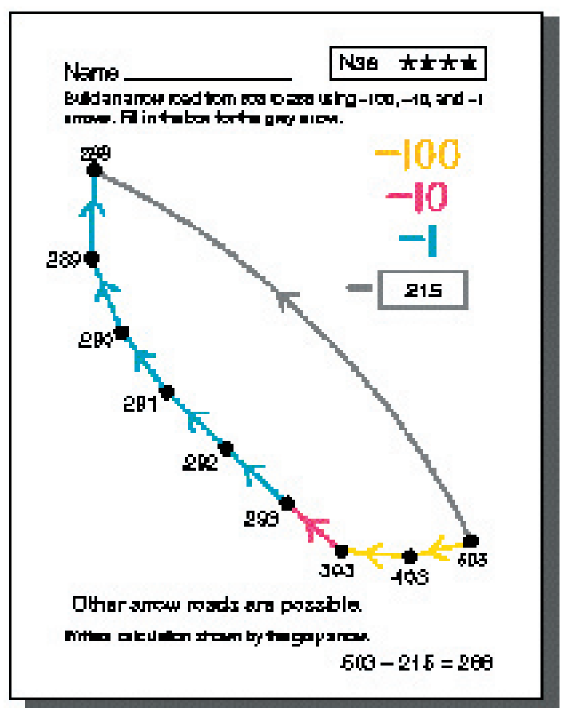
Note: Other arrow roads are possible; however, this one has the fewest number of arrows. Your students may suggest similar arrow roads with –100, –10, and –1 arrows in different orders.

Worksheets N36*, **, ***, and **** are available for individual work.


Name
N36 ★
70
60
50
40
39
38
−10
−1
32
Other arrow roads are possible.
70 − 32 = 38


Name
N36 ★★
−10
−1
45
37
38
39
40
50
60
70
80
81
82
Other arrow roads are possible.
82 − 45 = 37


Name
N36 ★★★
−100
−10
−1
112
88
89
90
100
200
Other arrow roads are possible.
200 − 112 = 88


Name
N36 ★★★★
−100
−10
−1
215
288
289
290
291
292
293
303
403
503
Other arrow roads are possible.
503 − 215 = 288