Workbooks

WORKBOOKS TABLE OF CONTENTS

WORKBOOKS INTRODUCTION

There are many opportunities for the student to work individually during the course of the lessons described in the other content strands. In the Workbooks strand, however, it is this individualized work which becomes the chief end of the majority of lessons. The goal in this strand is to provide the students with opportunities

- to review many of the ideas they have met in other content strands;
- to apply their acquired knowledge to new situations requiring various kinds of strategic thinking; and
- to learn how to read and use mathematics workbooks.

The following workbooks are provided:

- *Which Road?*
- *Festival of Problems #1*
- *Fishing for Numbers, Part III*
- *Road Map*
- *Festival of Problems #2*
- *20?-100?*

One story-workbook is provided:

- *Rollerskating 37*

Each workbook contains problems of varying levels of difficulty. Approximately the first ten pages of each workbook are easy problems, the next ten to twelve pages are average level difficulty, and the last ten pages are more challenging problems. For each workbook, we suggest that all students start work at the easiest level (i.e., on page 2) and then work through as many pages as they can handle during the two lessons scheduled for that workbook. We estimate that, in a typical class, about two-thirds of the students will correctly finish the first ten pages, about one-third will finish the first twenty pages, and a few will finish all or most of the workbook. These proportions will vary from class to class.

This guide contains an answer key for each workbook. The key follows an introduction to the workbook and a suggested collective lesson. The lesson either presents the workbook to the whole class or provides a warm-up activity on a problem similar to one found in the workbook.

Ten of the 18 lessons in the strand have the individual work in the workbooks as their main activity. The story-workbook *Rollerskaing 37*, used in two lessons, introduces the student to congruence classes of whole numbers and provides early notions of division. Two lessons (W3 and W8) support the addition, subtraction, and ordering of integers. The other four lessons present detective stories where there are one or two secret numbers and several clues that lead to their discovery. The clues review many ideas from the other content strands as well as involve students in new situations involving patterns and counting techniques.

Use of the Workbooks Strand for Evaluation Purposes

The workbooks provide an excellent instrument to assess the progress of your students on a regular basis. You may not feel it is necessary to check every page and problem for each student, but you should develop a procedure for checking students' work with which you are comfortable. This may include checking one or more specific pages, discussing some particular mistakes with individual students and letting them correct their work, or just looking carefully at a few pages to be sure the students have understood the general idea of the problems in that particular book.

In the Blacklines, you will find a record-keeping tool for each workbook to help you assess student progress in the various strands. This tool may also assist you in parent conferences and in filling out periodic progress reports.

Here are some important points to bear in mind for workbooks.

- Always read the introductory material for each workbook and give the short introductory collective lesson(s).
- All students should start at the beginning of each workbook and progress as far as they can.
- All students should begin a new title on the same day, even if some students have not finished work on the previous title.
- Not all students should be expected to complete a given workbook. Only some students will reach the most challenging problems. Other students may succeed only in doing the easiest problems, although you should not assume this automatically—surprises are not at all uncommon.

Capsule Lesson Summary

Solve a detective story with clues involving the Minicomputer, an arrow road, and a string picture.

Materials

Teacher
- Minicomputer set
- Colored chalk

Student
- Minicomputer set
- Paper
- Colored pencils, pens, or crayons
- Worksheets W1* and **

Description of Lesson

Before the lesson begins, write a large 17 on a slip of paper and fold the paper so that 17 is hidden.

Ask students if they have ever read a detective story, and ask students what they know about detectives. Guide the discussion to include the idea that a detective tries to solve mysteries or uncover secrets by following clues. Tell the students that they are going to be detectives today, and they will be able to discover a secret number from your clues.

Note: You may like to make the secret number something real for students to figure out such as "How old is my brother?" or "How many miles do I drive to school?" or "How many spots on my dog?"

T: ***I wrote a secret number on this paper. Listen carefully and follow my clues, so you can discover my secret number.***

Clue 1

Display one Minicomputer board. Provide student pairs with one Minicomputer sheet and three checkers.

T: ***The secret number can be put on the ones board of the Minicomputer using three regular checkers.***

Direct student partners to cover one of the Minicomputer boards on their sheet with paper. Then allow them to work together to find possibilities for the secret number and to compile a list. After a few minutes, begin a class list. Each time a number is suggested, ask a student to show how it can be put on the Minicomputer with three regular checkers. Record each number on the chalkboard as it is displayed, leaving space so that the possibilities can be recorded in order. Accept three or four suggestions from students before asking,

T: ***What is the least number it could be?*** (3)
What is the greatest number it could be? (24)

W1

Continue finding possibilities for the secret number until your list has all seventeen. Ask student partners to check that they, too, have this complete list of possible secret numbers.

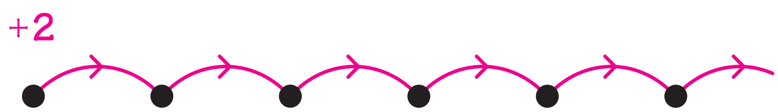
3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 20, 24

T: ***Now we know that the secret number is one of these numbers, but we do not know which one. What do we need?***

S: ***Another clue.***

Clue 2

Draw a +2 arrow road on the board, and ask students to copy it on their papers.



T: ***The secret number is on a +2 arrow road that starts at 1.***

Suggest student pairs label the dots in their arrow roads. Invite a few students to label dots in the road on the board.

Point to the last partial arrow.

T: ***This arrow road can go on and on. What other numbers would be on the road?***

S: ***13, 15, 17, and so on.***

S: ***Odd numbers.***

Point to the list of numbers on the board from the first clue.

T: ***Which of these numbers can be the secret number and which cannot?***

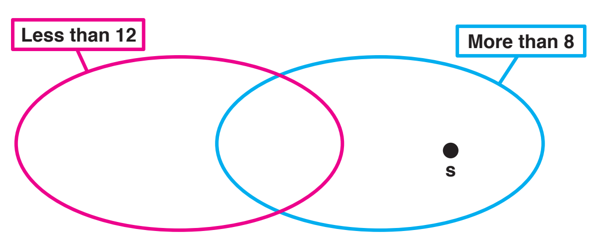
Invite students to circle numbers that can be the secret number and to erase numbers that cannot be the secret number.

(3,) (5,) (7,) (9,) (11,) (13,) (17,)

T: ***Now we know the secret numbers is one of these. Here is another clue.***

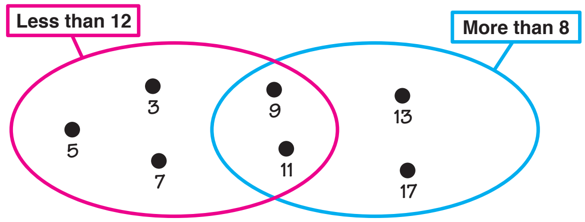
Clue 3

Draw this string picture on the board and ask student pairs to copy it on their papers. Then point to **S** as you announce,



T: ***The secret number is here. Put all the numbers left in our list of possibilities for the secret number in this picture and decide which ones can be the secret number.***

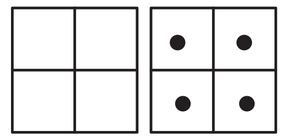
When most students have finished, invite students to put the numbers in the picture on the board. The class should find that there are still two possibilities for the secret number, 13 or 17.



Clue 4

T: ***I have just one more clue. When you know the secret number, write it on your paper.***

Give each pair of students another Minicomputer checker and direct them to put this configuration on their desk Minicomputers.



T: ***What number do we have on the Minicomputer?***

S: ***15.***

T: ***Could this be the secret number?***

S: ***No, the secret number is either 13 or 17.***

T: ***If you move one checker from the ones board to the tens board, you can get the secret number on the Minicomputer.***

Direct student pairs to show the secret number on their desk Minicomputers and to write it on a piece of paper.

T: ***What is the secret number?***

S: ***17.***

Invite a student to show how to move one checker from the ones board to the tens board on the demonstration Minicomputer to get 17. Reveal that 17 is written on your paper.

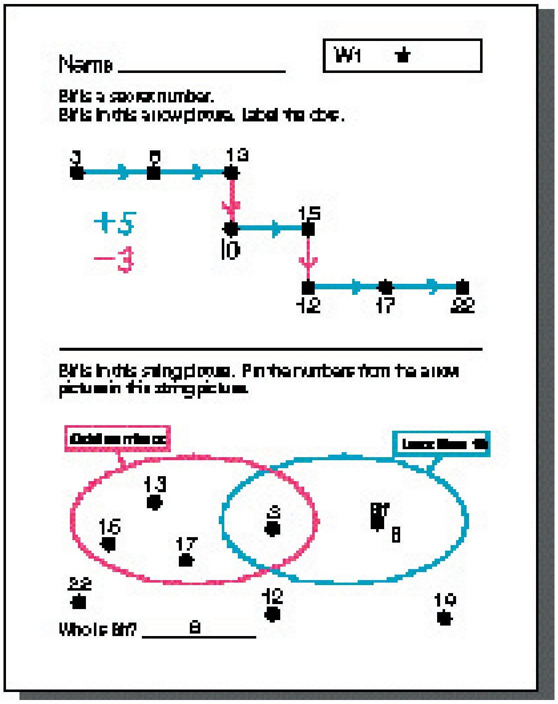
Worksheets W1* and ** are available for individual or group work.

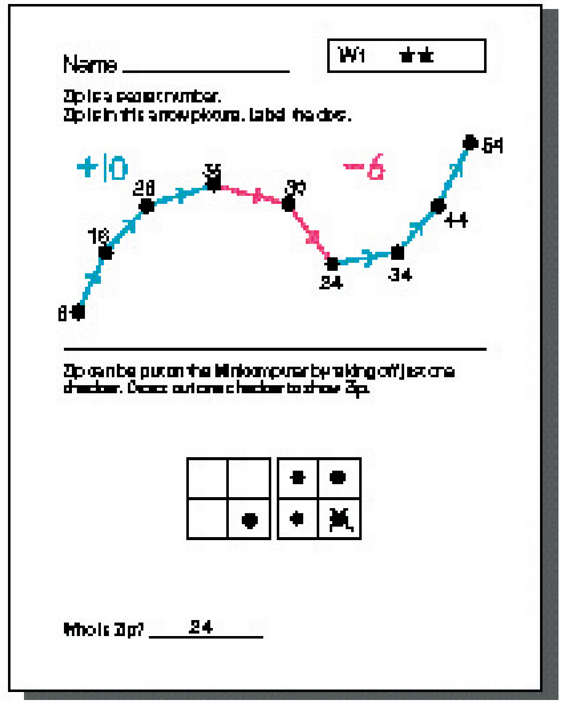
Reading Activity

You may like to read a mystery or detective story to the class.

Home Activity

You may like to send home one of the worksheets for students to share with a family member. The first time you send an activity home, send along a letter about home activities in general and/or the *CSMP* introduction letter. See the *UPG-III* Blacklines.


Name
W1
+5
−3
13
15
10
12
17
22
16
17
22
12
19
Who is Bif? 8


Name
W1
+10
−6
28
16
24
34
44
54
Who is Zip? 24

Capsule Lesson Summary

Investigate which of five functions could be used to build a road between two given numbers when all arrows in the road are the same kind. Look for shortest and longest roads. Solve similar problems in the *Which Road?* Workbook.

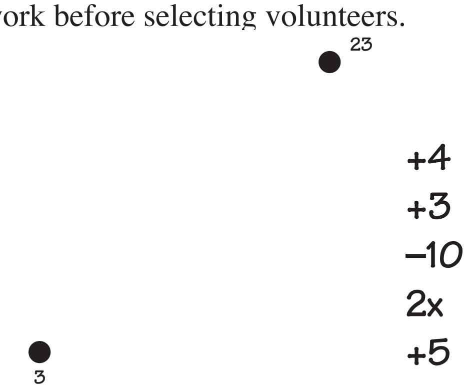
Materials

Teacher		Student	
	• Colored chalk		• Paper • *Which Road?* Workbook • Colored pencils, pens, or crayons

Description of Lesson

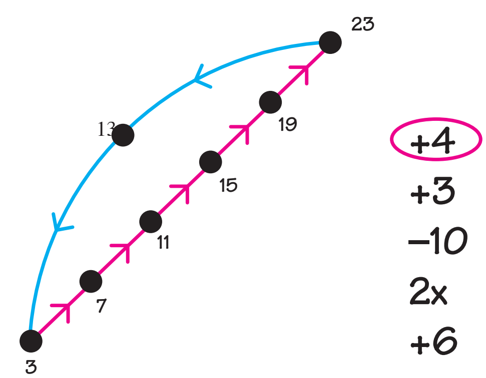
During this warm-up activity, encourage students to experiment on their papers before volunteering to put solutions on the board. You may even want to check work before selecting volunteers.

Write this information on the board.



T: ***Suppose that we want to build an arrow road between 3 and 23 using all the same kind of arrows. These are our choices for arrows*** (point to the list of functions)***. Can we build such a road? The road can start at 3 and go to 23, or it can start at 23 and go to 3.***

Call on volunteers to circle in color their choices of functions and then to build roads in the corresponding colors. In this case there are three possibilities. You may like the students to put these three roads in three separate pictures on the board.



T: ***Which of the roads is the shortest*** (has the least number of arrows)***?***

S: ***The –10 arrow road has only two arrows.***

T: ***Which is the longest?***

S: ***The +4 arrow road has five arrows.***

Your class should discover that a road cannot be built with either +3 or 2x arrows alone.

W2

Distribute copies of the *Which Road?* Workbook. Emphasize that on each page of the workbook, the students are to build a road between the given numbers. They may only use the functions listed on that page, and each road must have only one kind of arrow.

Explain that as soon as they have built a road between the two labeled dots, they have solved the problem, but that you would like them to try to build the shortest possible road. Make finding the shortest road a challenge by occasionally suggesting to students that you think there may be a shorter road than the one they have drawn.

Note: On each page of the workbook there are several functions that could be used successfully to build the road and at least one function that will not work. Of those functions that do work, some will give shorter roads than others.

At least one choice on most pages is a subtraction function, although the students are not expected to know a subtraction algorithm at this time in the school year. As necessary, help students to build a road with subtraction arrows by suggesting they use a number line, count backwards, or first draw a return road with addition arrows. Usually, only subtraction facts are needed.

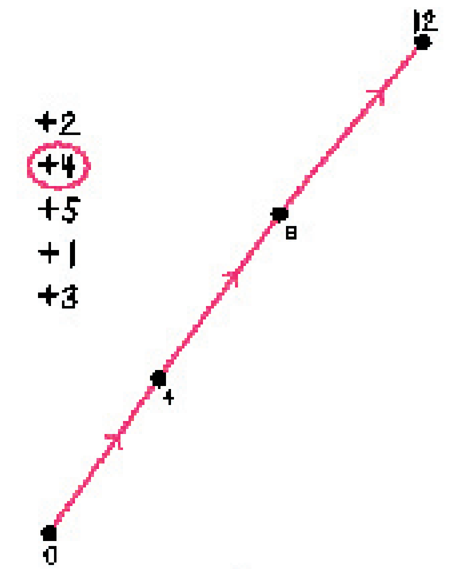
Allow approximately 30–45 minutes for the students to work individually on the *Which Road?* Workbook. Students who are able to complete the workbook quickly finding shortest roads on each page can be challenged to find all the possible roads on each page. Alternatively, invite such students to make up similar problems to challenge you or a comparable classmate. Collect the workbooks for your review.

Home Activity

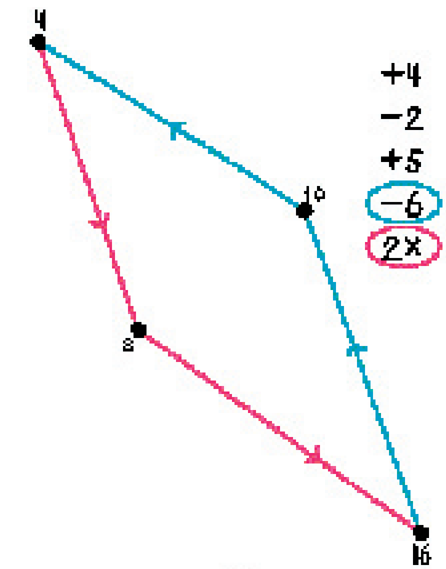
Prepare two or three problems similar to those in the workbook for students to take home and solve with a family member. You may like to put such problems in a story context. For example:

A store sells pencils in packages of 4, 6, 10, or 12. Can a person buy exactly 30 pencils buying all the same size packages?

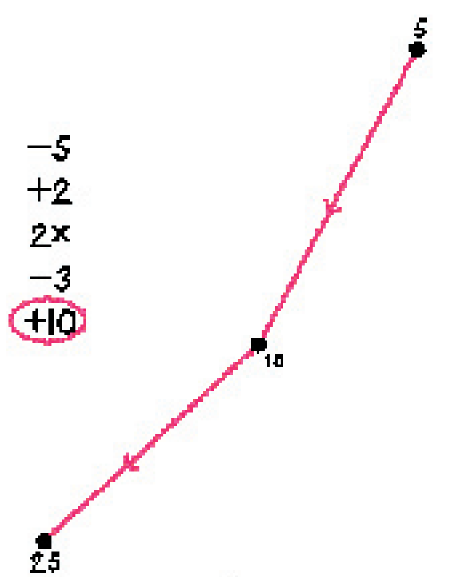
+4
+6
+10
+12



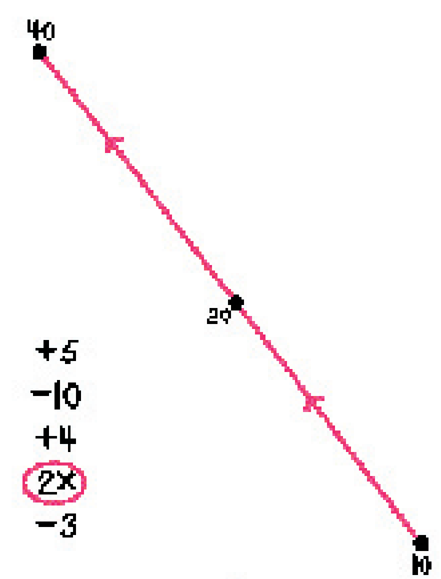
Other solutions are possible using +2, +1, +3.



Other solutions are possible using +4 or −2.

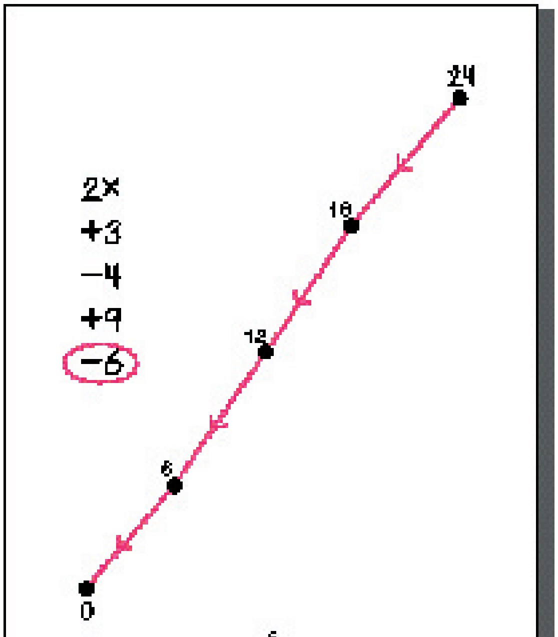


Other solutions are possible using −5 or +2.

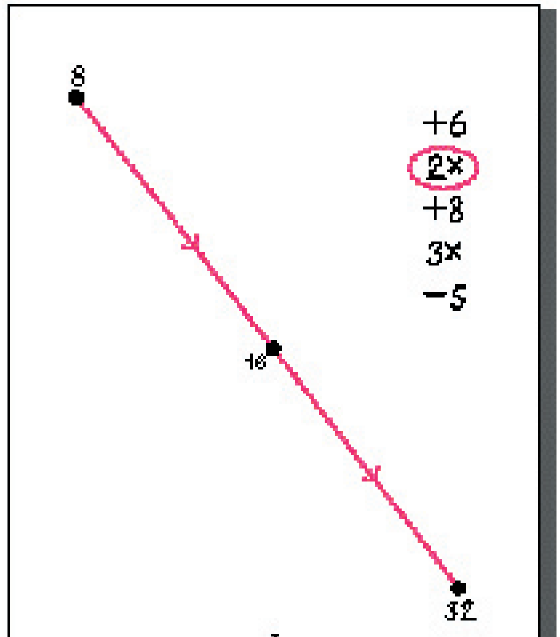


Other solutions are possible using +5, −10, or −3.

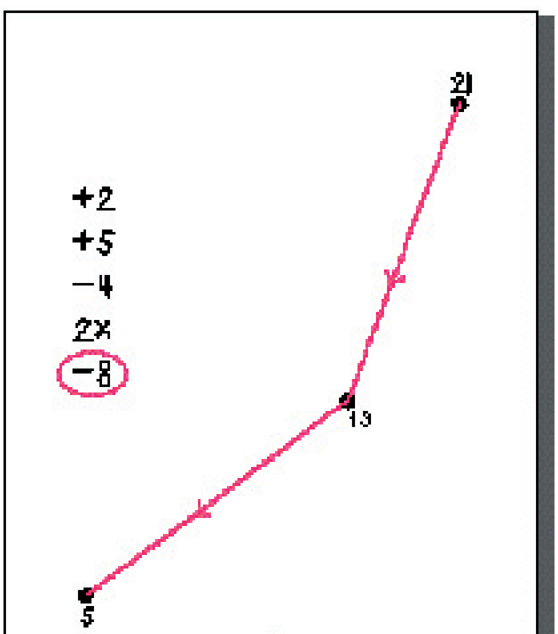
W2



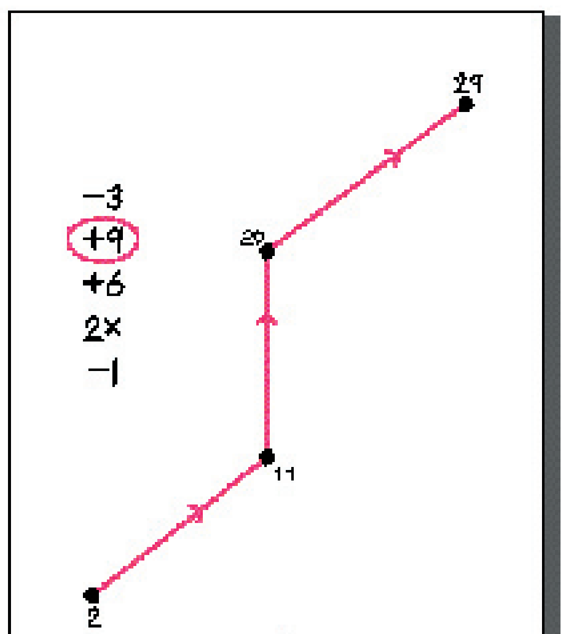
Other solutions are possible using −4 or +3.



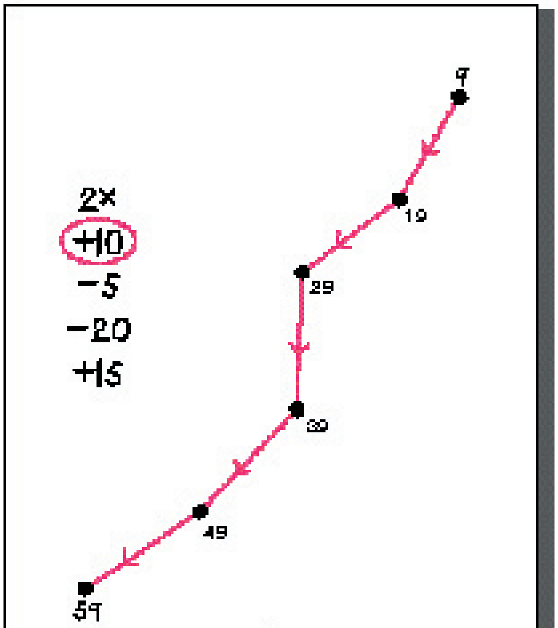
Other solutions are possible using +6 or +8.



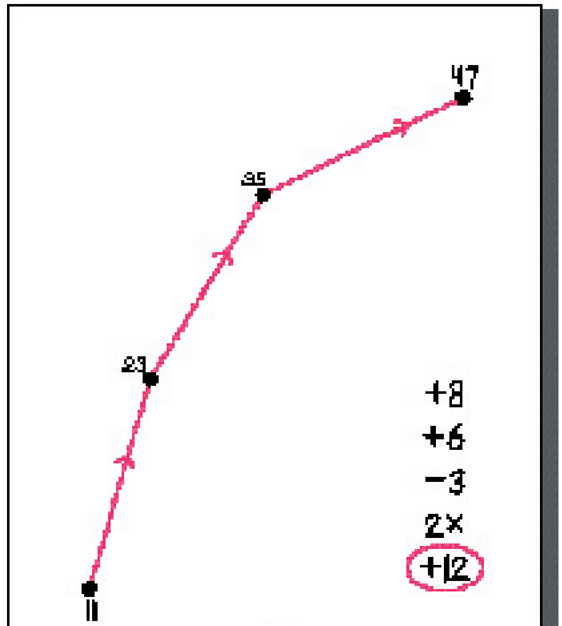
Other solutions are possible using −4 or +2.



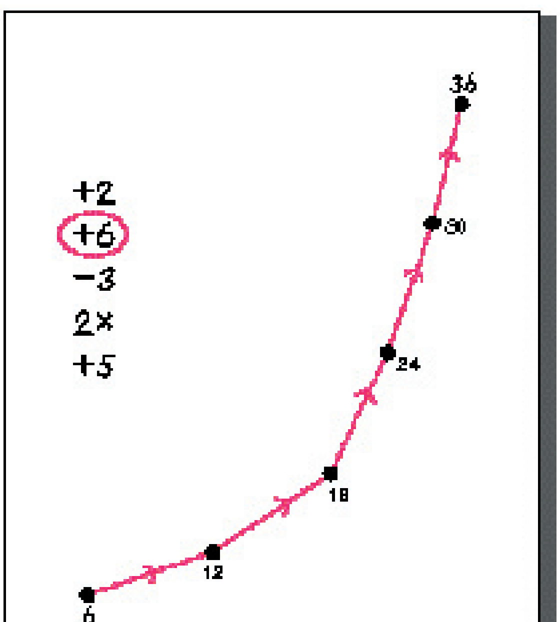
Other solutions are possible using −3 or −1.



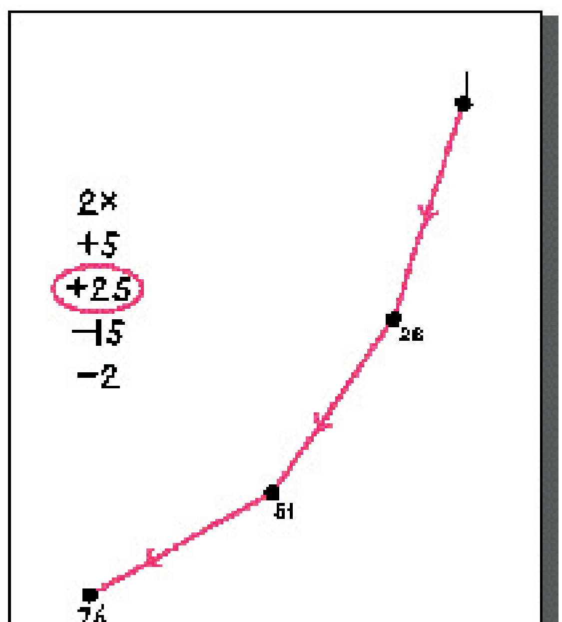
Another solution is possible using −5.



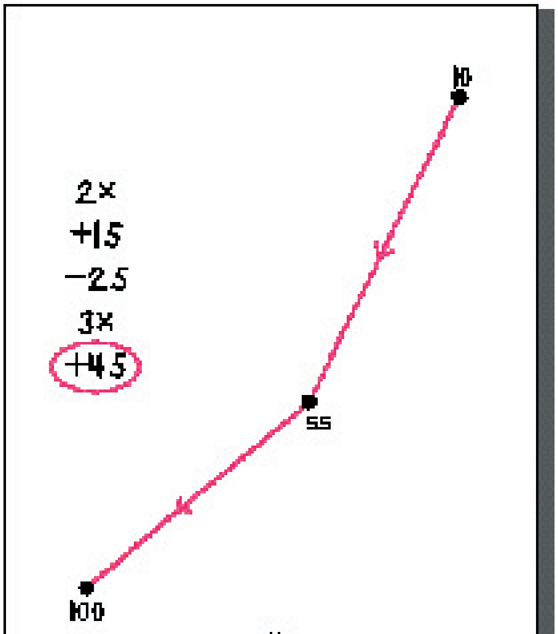
Other solutions are possible using +6 or −3.



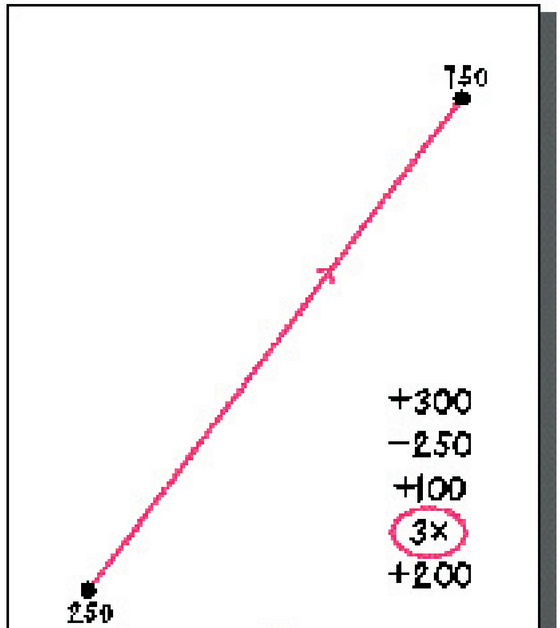
Other solutions are possible using +2, −3, or +5.



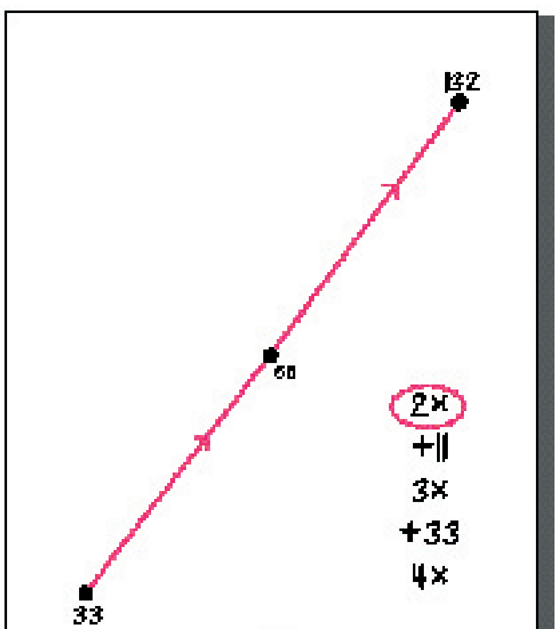
Other solutions are possible using +5 or −15.



Other solutions are possible using +5 or +15.



Other solutions are possible using −250 or +100.



Other solutions are possible using +11 or +33.

Capsule Lesson Summary

Label the floors in a partial picture of the Empire State Building. Find how many floors up or down you travel to go from one floor to another. In a story situation, consider how to go from floor to floor when the elevator will only allow +10, –10, +5, –5, +1, and –1. Label the dots in a –5 arrow road that contains some negative numbers.

Materials

Teacher
- Colored chalk

Student
- Paper
- Colored pencils, pens, or crayons

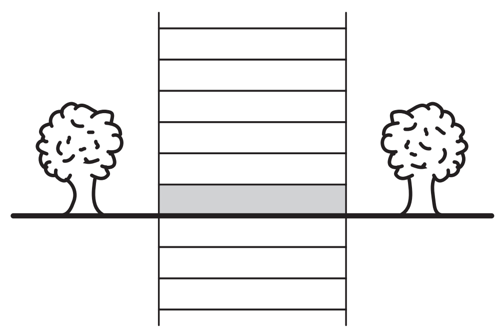
Advance Preparation: You may want to prepare the picture in Exercise 1 before teaching the lesson.

Description of Lesson

Exercise 1

Ask the students if they know of any tall buildings. Encourage them to tell you the names of some tall buildings and where they are located. Some students may have visited some tall buildings and can tell the class about their experiences. If no one mentions the Empire State Building in New York City, do so yourself, and tell the class that the Empire State Building was the tallest building in the world for many years. Some students may remember learning about the Empire State Building in second grade and know some facts, such as it has 102 floors above the ground.

Draw this picture on the board.



Explain that the picture on the board shows only the floors of the Empire State Building that are near ground level because it has too many floors to picture all of them.

T: ***Where is the ground floor in the picture?***

The student who volunteers should point to the rectangle that is shaded in this illustration.

Note: Some students may be somewhat confused by the use of the word *floor* in this lesson. In this case, floor refers to a story or level of the building rather than just the supporting surface at each level.

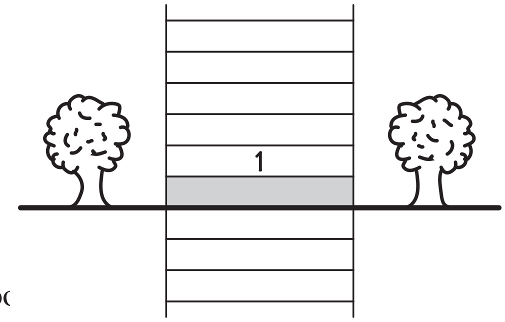
Discuss briefly the need to have a name for each floor. Indicate that the ground floor is mostly open area and has no offices. Often the ground floor has only a lobby, elevators, and stairways.

T: ***Who can show us where the first floor is?***

A student should point to the rectangle directly above the rectangle for the ground floor.

T: ***Where is the second floor? … the fifth floor?***
Let's assign a number to each of these floors.
What number should we give the first floor?

S: ***1.***



Write 1 inside the rectangle for the first floor.

With students assisting, label all the floors above the first floo

T: ***In many buildings, the ground floor has a letter name instead of a number name. Sometimes the ground floor is called G and sometimes it is called L. Why?***

S: ***G indicates ground level and L indicates lobby.***

T: ***Let's give our ground floor a number name instead of a letter name. What number should we assign to the ground floor?***

S: ***0.***

Label the ground floor 0 in your picture on the board.

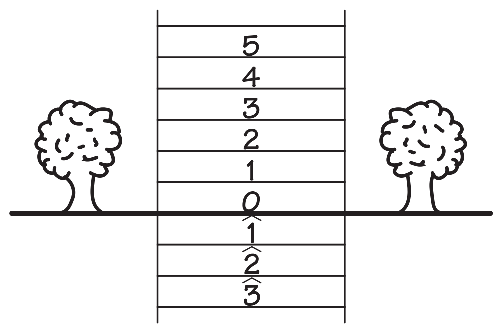
T: ***In many buildings the floors below the ground are called B1, B2, B3, B4, and so on. Why?***

S: ***B is for basement.***

T: ***What numbers should we assign to the floors below ground level?***

S: ***Negative numbers.***

Ask the class to assist you in labeling all the floors shown in the picture that are below ground level. Read the $\widehat{3}$ as "negative three."



T: ***If you are on floor 2 and want to go to floor*** $\widehat{3}$***, should you go up or down?***

S: ***Down.***

T: ***How many floors down?***

S: ***Five.***

Using the picture on the board, show that floor $\widehat{3}$ is five floors below floor 2.

T: ***If you are on floor*** $\widehat{3}$ ***and want to go to floor 4, should you go up or down?***

S: ***Up.***

T: ***How many floors up?***

S: ***Seven.***

Using the picture on the board, show that floor 4 is seven floors above floor $\widehat{3}$.

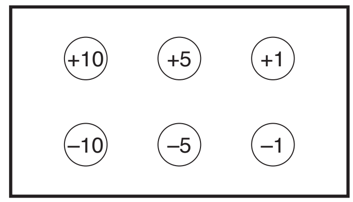
Exercise 2

Throughout this exercise, encourage students to experiment on their papers before responding.

Choose one of your students to be the star of this story, if you like. You can also call the customers in this story by names of students in the class.

T: ***There is a boy named Barry who sells sandwiches at lunchtime in the Empire State Building. Barry has regular customers on several floors and he goes to their offices everyday on the elevator. One day when Barry gets in the elevator, he discovers that there is a new control panel. The panel looks like this.***

Draw this picture on the board.



T: ***If Barry is on floor 0 and his first customer is on floor 26, which buttons should Barry press?***

The e many possibilities. The following dialogue assumes a student suggests pressing twice and six times.

T: ***Let's se hat will get Barry from floor 0 to floor 26. If he starts at floor 0 and presses , which floor will Barry arrive on?***

S: ***Floor 10.***

T: ***If he presses again, which floor will he arrive on?***

S: ***Floor 20.***

T: ***If Barry is on floor 20 and presses six times, where will he be when the elevator stops?***

S: ***Floor 26.***

Write this number sentence on the board and instruct students to write it on their papers.

$$0 + 10 + 10 + 1 + 1 + 1 + 1 + 1 + 1 = 26$$

T: ***This is one way Barry could get from floor 0 to floor 26. Is there another way?***

Consider each suggestion and decide collectively if it will get Barry to floor 26. Take advantage of any opportunity to use multiplication facts in a natural way. For example:

S: ***Barry can press five times and then one time.***

T: ***Let's see if that will get him to the floor 26. What number is 5 x 5?***

S: ***25.***

T: ***If Barry presses five times, he will go up 25 floors. 25 + 1 = …?***

S: ***26.***

W3

Write this number sentence on the board.

$$0 + 5 + 5 + 5 + 5 + 5 + 1 = 26$$

T: ***Barr ustomer on floor 26 is getting hungry; Barry decides to press , , , and .***

Record the appropriate number sentence on the board if it is not already there.

$$0 + 10 + 10 + 5 + 1 = 26$$

T: ***Now that Barry is on floor 26 he needs to deliver a sandwich on floor 36. Do you have any suggestions for Barry?***

Consider each suggestion and decide collectively if it will get Barry to floor 36. You may need to remind students that Barry is already on floor 26, so he does not need to go up 36 floors to get to floor 36.

If no one suggests pressing once, give the hint that Bar nly needs to press one button to go from floor 26 to floor 36. When someone suggests pressing once, write this number sentence on the board.

$$26 + 10 = 36$$

T: ***The next customer is on floor 45. What could Barry do now?***

S: ***Press four times and then once.***

T: ***Why does that work?***

S: ***Because 36 + 4 = 40 and 40 + 5 = 45.***

S: ***Press and then .***

T: ***Why does that work?***

S: ***36 + 10 = 46 and 46 – 1 = 45.***

If no one suggests pressing and , ask how Barry cou et to f 45 by pressing only two buttons (same or different). When someone suggests pressing and , record the appropriate number sentence on the board.

$$36 + 10 - 1 = 45$$

or

$$36 - 1 + 10 = 45$$

T: ***Now that Barry is on floor 45, how can he get to floor 60?***

S: ***He could press three times.***

S: ***He could press and .***

S: ***He could press fifteen times.***

There are many possibilities; check any that are offered. Record one student's solution on the board. For example,

$$45 + 10 + 5 = 60$$

T: ***Barry's last customer is on floor 79. How can Barry go from floor 60 to floor 79?***

S: ***By pressing , , and .***

…n the …e many possibilities; check any that are offered. If no one suggests pressing , , and , ask how Barry could get to floor 79 by pressing only three buttons (not necessarily different).

When a student suggests pressing , , and , record the appropriate number story on the board.

$$60 + 10 + 10 - 1 = 79^{\dagger}$$

T: ***Barry has delivered sandwiches to all of his customers, so he is ready to leave. How can he get back to floor 0?***

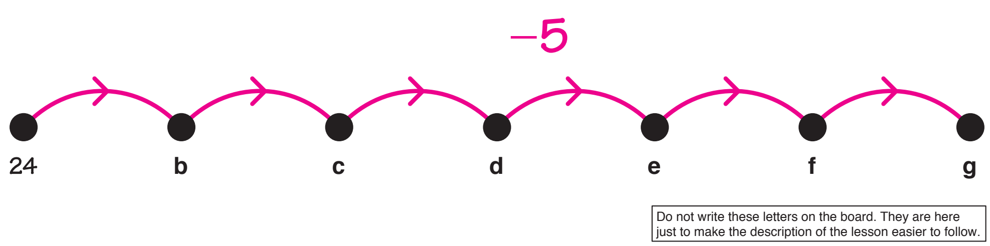
Allow the students to make many …gestions. To pres… fewest buttons in order to go from floor 79 to floor 0, Barry should press eight times and once.

$$79 - 10 - 10 - 10 - 10 - 10 - 10 - 10 - 10 + 1 = 0$$

Erase the board except for the drawing of the Empire State Building.

Exercise 3

Draw this arrow picture on the board and ask students to copy it on their papers.



T: ***The ne… …y, after visiting a friend on floor 24, Barry gets on the elevator and presses the button . On which floor should the elevator stop?***

S: ***On floor 19.***

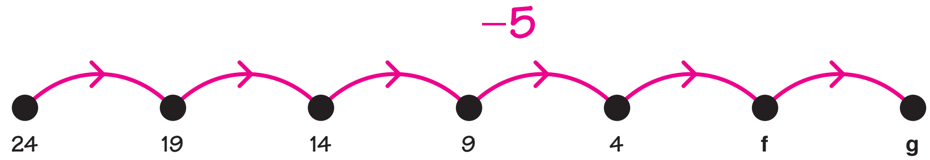
Label **b** 19.

†The order in which +10, +10, and –1 occur in this number sentence may vary depending upon how one of your students gives the solution.

T: ***The elevator does stop b ¯ie door does not open. After a moment, it continues to go down as if Barry had pressed again. This keeps happening*** (trace the arrow road with your finger) ***and Barry wonders where he will be able to get off! What is the next floor*** (point to **c**) ***that the elevator will stop at momentarily?***

S: ***Floor 14.***

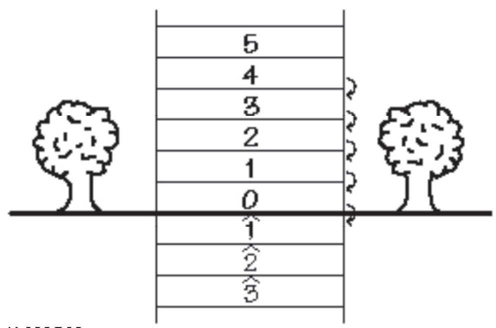
Label **c** 14, and continue in the same manner until **b**, **c**, **d**, and **e** are labeled.



T: ***The elevator moves down another five floors. Which floor does it come to*** (point to **f**)***?***

S: ***Floor*** $\hat{1}$.

Refer to the picture of the Empire State Building from Exercise 1 and count down the floors from floor 4.



T (tracing the arrow from 4 to **f**): $4 - 5 = \hat{1}$

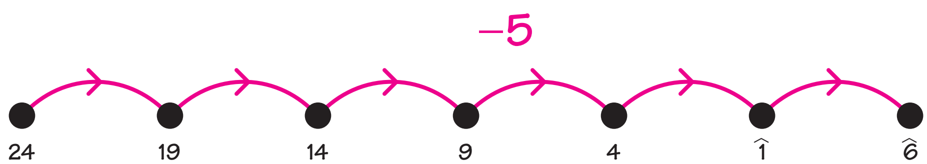
Label **f** $\hat{1}$.

T: ***The elevator continues going down again*** (point t ***before it completely stops and the door opens. On floor is Barry able to get off?***

S: ***On floor*** $\hat{6}$.

T (tracing the arrow from $\hat{1}$ to **g**): $\hat{1} - 5 = \hat{6}$.[†]

Label **g** $\hat{6}$.



Instruct students to draw two or three more –5 arrows in their pictures (from $\hat{6}$) and label two or three more dots.

Writing Activity

Write about Barry's trip in the elevator from the 5th floor to the 36th floor and then to the 2nd floor.

[†]Read as "negative one minus five equals negative six."

Capsule Lesson Summary

Introduce a Minicomputer game called Minicomputer Tug-of-War. Extend experiences from various content strands in the workbook *Festival of Problems #1*.

Materials

Teacher
- Minicomputer set
- Colored chalk

Student
- *Festival of Problems #1* Workbook
- Colored pencils, pens, or crayons

Description of Lesson

Display three Minicomputer boards. Position four red and four blue checkers on the Minicomputer as in the illustration below.[†] Divide the class into two teams (the **RED** team and the **BLUE** team).

T: ***The* BLUE *team starts with the number on the Minicomputer shown with blue checkers. What is the starting number of the* BLUE *team?***

S: ***15.***

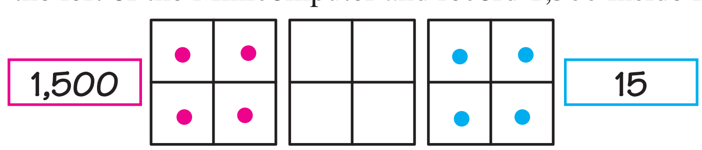
Draw a blue rectangle to the right of the Minicomputer and record 15 inside it.

T: ***We are going to play a game called Minicomputer Tug-of-War. The* BLUE *team will play with the blue checkers and the* RED *team will play with the red checkers.***

The* RED *team starts with the number on the Minicomputer shown with red checkers. What is the starting number of the* RED *team?

S: ***1,500.***

Draw a red rectangle to the left of the Minicomputer and record 1,500 inside it.



T: ***Teams alternate turns. A player moves one checker on a turn from the square it is in to another square. Players on the* BLUE *team can move blue checkers only and must increase their number. Players on the* RED *team can move red checkers only and must decrease their number. If the* RED *team always decreases its number and the* BLUE *team always increases its number, eventually the red and the blue numbers will tie or pass each other. If the* RED *team makes the red number the same as or less than the blue number, the* RED *team loses. If the* BLUE *team makes the blue number the same as or more than the red number, the* BLUE *team loses.***

Begin playing the game.

[†]You may prefer to play this first game with only three red and three blue checkers. For example, start with 7 in blue and 700 in red. However, four checkers gives more options in the game.

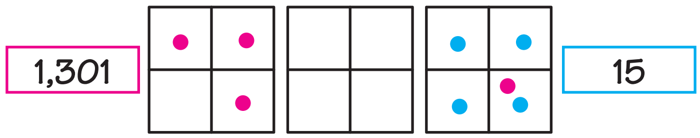
W4

Note: We suggest that during this first time playing the Minicomputer Tug-of-War game you call on volunteers from each team rather than asking students to play in some order. This will speed up the game and allow students who are unsure of the rules to become more familiar with the game before they take a turn.

A description of a possible game is given here.

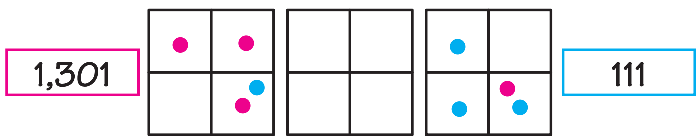
T: ***The* RED *team plays first. You must decrease the red number without tying or passing the blue number.***

A student from the **RED** team moves a checker from the 200-square to the 1-square. The student determines that the red number is now 1301 and changes the number inside the red box to 1301.



T: ***The* BLUE *team plays next. You must increase the blue number without tying or passing the red number.***

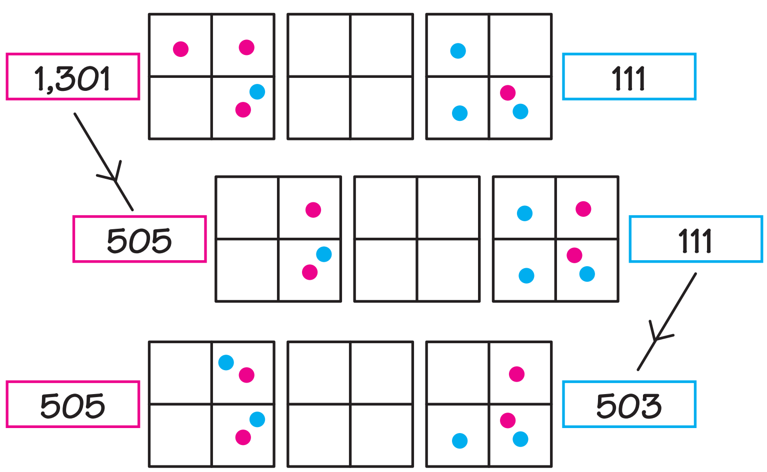
A student from the BLUE team moves a checker from the 4-square to the 100-square. The student determines that the blue number is now 111 and changes the number inside the blue box to 111.



T: ***111 is less than 1301, so the blue number is still less than the red number.***

Note: Do not expect your students to make such big moves on the first few turns. Often students are very cautious at first as they are learning how the game is played.

The game continues as shown below.



It is the **RED** team's turn to play, but no one volunteers and some of students claim it is impossible for the **RED** team to play without tying or passing the blue number.

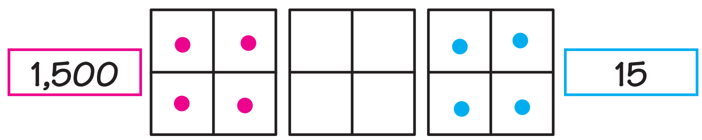
T: ***The* RED *team has to move one of its checkers and decrease the red number without tying or passing the blue number. Is this possible?***

S: ***We can't move the red checker on the 1-square, because there is no square less than 1.***

S: ***We can't move the checker on the 100-square or the 400-square, because we have to keep the 500 (any move with one of these checkers would put us below 500).***

S: ***We can't move the checker on the 4-square either, because the least backward move we can make would be from 4 to 2, and then the red number would be 503.***

Conclude that the **RED** team cannot move without losing, and congratulate the **BLUE** team. Return the checkers to their original positions, change the numbers in the rectangles, and play the game again.



Note: When you play the game with students playing in order, avoid forcing a student to make a losing play. Whenever a student declines to play because he or she cannot find a move that will prevent his or her team from losing, ask if any other member of the team is able to play. If none of the team's members is able to play, conclude that the team has lost the game.

Distribute copies of the workbook *Festival of Problems #1* and let students begin working independently. If the questions you receive indicate that many students are having difficulty with a particular page, hold a collective discussion about that page.

At the end of the class period collect the workbooks for your review. They will be used again in Lesson W5.

Center Activity

Put desk Minicomputer sets in a center so that students can play Minicomputer Tug-of-War in pairs, or groups of four or six.

Capsule Lesson Summary

Use patterns in the 0–109 numeral chart to extend the chart. Complete a puzzle piece of an extended numeral chart. Continue working in *Festival of Problems #1* Workbook. (This is the second of two lessons using this workbook.)

Materials

Teacher
- 0–109 numeral chart

Student
- *Festival of Problems #1* Workbook
- Colored pencils, pens, or crayons

Description of Lesson

Tape a copy of the 0–109 numeral chart on the board leaving space under it to extend it with several more rows.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	54	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99
100	101	102	103	104	105	106	107	108	109
110	111	112	113	114	115	116	117	118	119
120	121	122	123	124	125	126	127	128	129
130	131	132	133	134	135	136	137	138	139

T: ***We often use the 0–109 numeral chart. Do you suppose we could extend this numeral chart beyond 109?***

Invite students to help you extend the chart a few rows on the board. Ask about row and column patterns, and check that these patterns extend along with the chart.

On another section of the board, draw three or four rows of a ten square wide grid. Put a number such as 192 in an appropriate place in the grid.

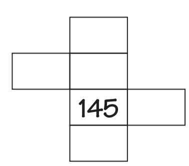
		192							

T: ***Can we extend the numeral chart to include 192? What other numbers would be in this part of the chart?***

Invite students to help you complete this part of a numeral chart. Again, check that row and column patterns can be found in this part of an extended numeral chart.

170	171	172	173	174	175	176	177	178	179
180	181	182	183	184	185	186	187	188	189
190	191	192	193	194	195	196	197	198	199
200	201	202	203	204	205	206	207	208	209

Draw this configuration of squares on the board.



T: ***This is a piece cut out of a numeral chart. What numbers should be in the other boxes?***

Let students suggest numbers to complete this piece of a numeral chart. Insist that they explain their choices, using patterns from the original 0–109 numeral chart and the extended numeral chart.

W5

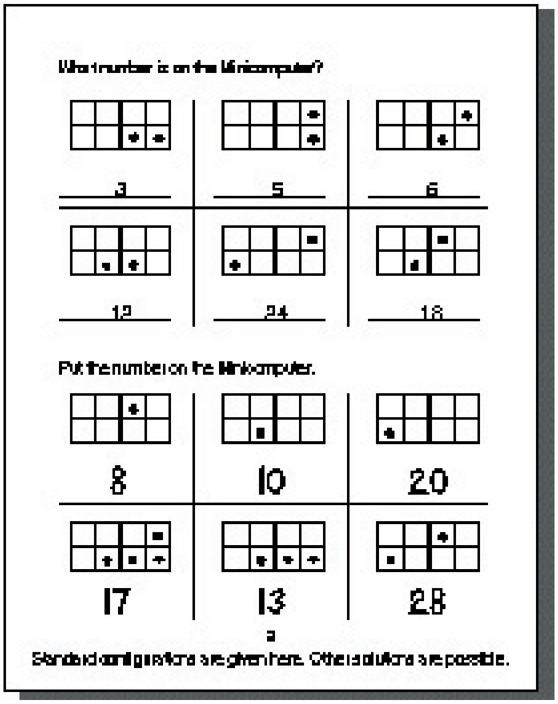
Distribute students' copies of the workbook *Festival of Problems #1*. Ask students first to correct or complete pages from the first weeks' work and then to continue. You may wish to discuss collectively some problems that were difficult for many students the first week. At the end of the class period, collect the workbooks for your review.

Assessment Activity

An individual student progress record for the workbook is available on Blackline W5(a). You may like to use this form to monitor student work.

Home Activity

If you choose to send workbooks home with students, you may want to include a letter to parents/ guardians with this workbook. Blackline W5(b) has a sample letter.


What number is on the Minicomputer?

3 | 5 | 6

12 | 24 | 18

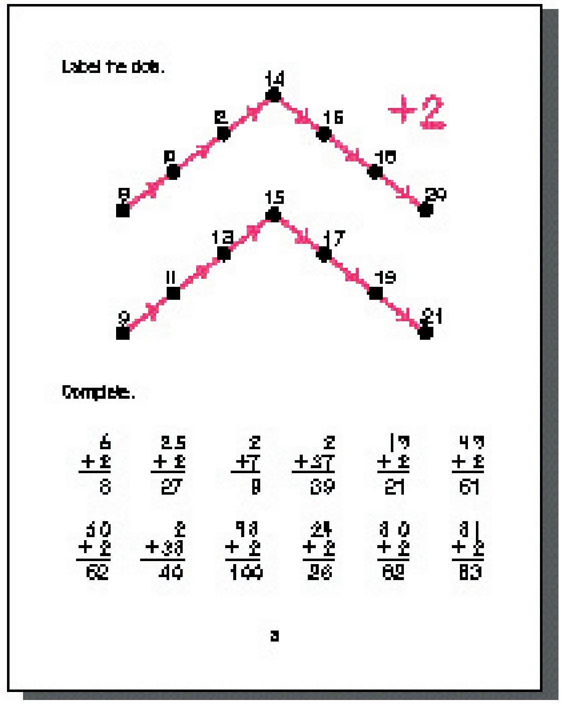
Put the number on the Minicomputer.

8 | 10 | 20

17 | 13 | 28

2

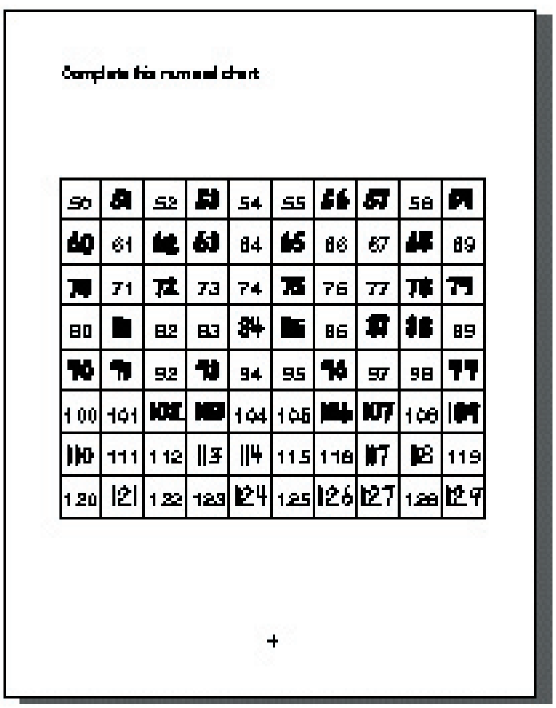
Standard configurations are given here. Other solutions are possible.



Complete.

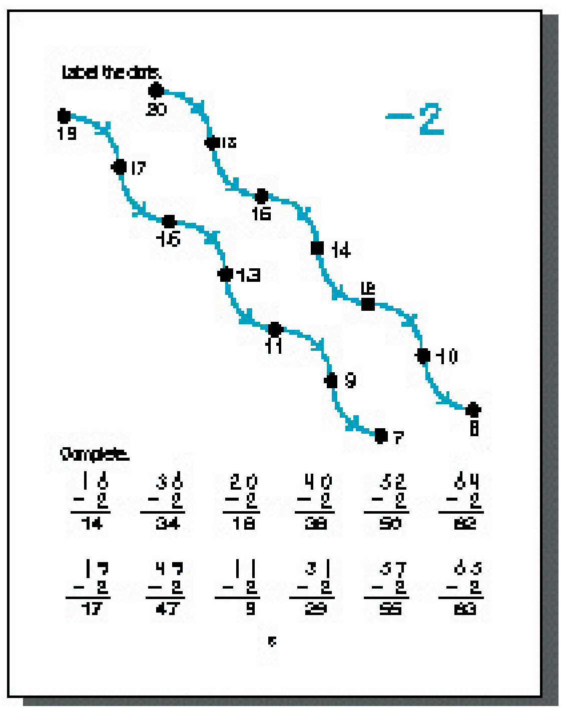
6 + 2 = 8	25 + 2 = 27	2 + 7 = 9	2 + 37 = 39	19 + 2 = 21	49 + 2 = 51
60 + 2 = 62	2 + 38 = 40	98 + 2 = 100	24 + 2 = 26	80 + 2 = 82	81 + 2 = 83

3


Complete this numeral chart.

50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99
100	101	102	103	104	105	106	107	108	109
110	111	112	113	114	115	116	117	118	119
120	121	122	123	124	125	126	127	128	129

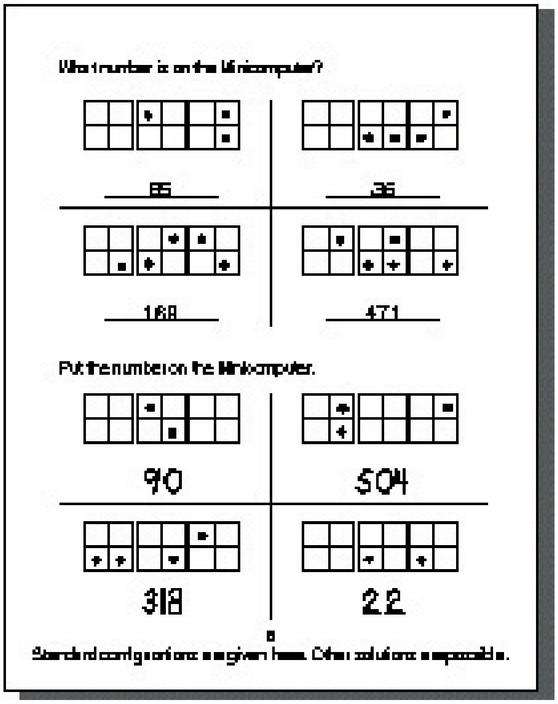
4

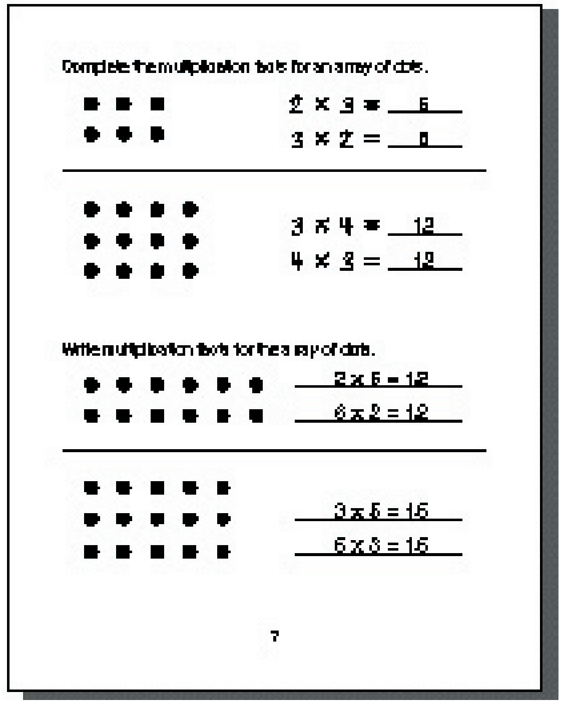


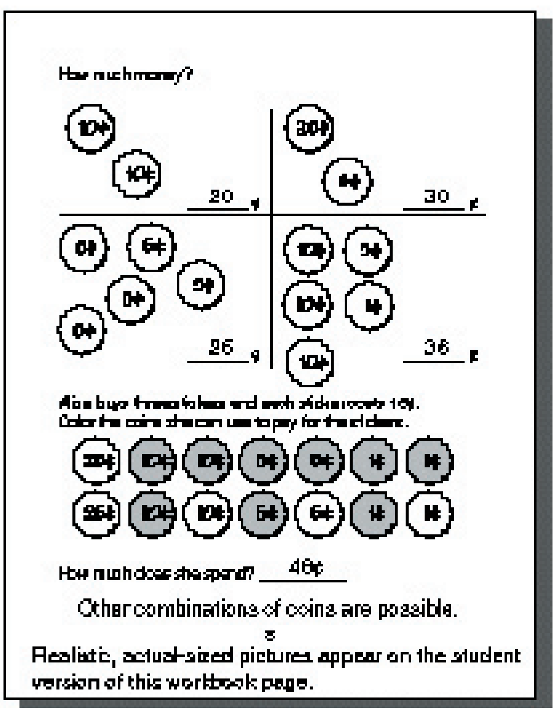
Complete.

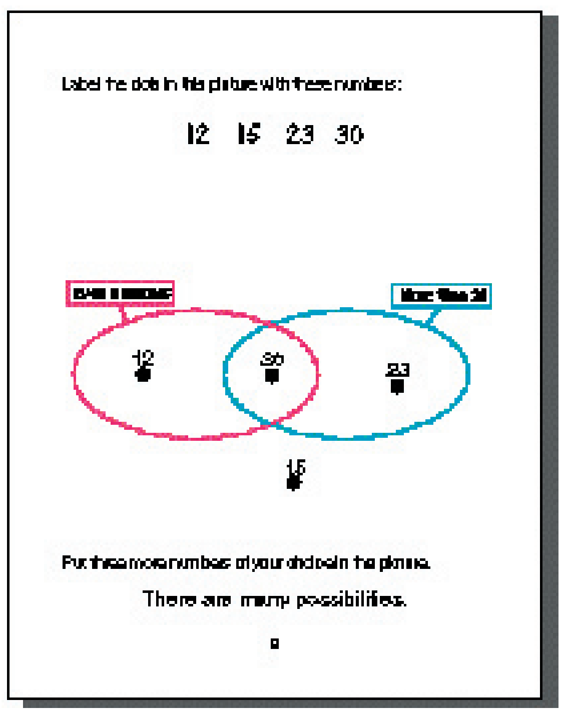
16 − 2 = 14	36 − 2 = 34	20 − 2 = 18	40 − 2 = 38	52 − 2 = 50	64 − 2 = 62
19 − 2 = 17	49 − 2 = 47	11 − 2 = 9	31 − 2 = 29	57 − 2 = 55	65 − 2 = 63

5

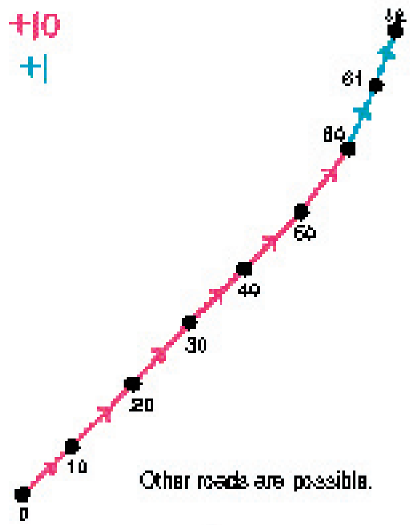

What number is on the Minicomputer?
85
36
168
471
Put the number on the Minicomputer.
90
504
318
22
6
Standard configurations are given here. Other solutions are possible.


Complete the multiplication facts for an array of dots.
2 × 3 = 6
3 × 2 = 6
3 × 4 = 12
4 × 3 = 12
Write multiplication facts for the array of dots.
2 x 6 = 12
6 x 2 = 12
3 x 5 = 15
5 x 3 = 15
7


How much money?
20 ¢
30 ¢
26 ¢
36 ¢
Color the coins she can use to pay for the stickers.
How much does she spend? 48¢
Other combinations of coins are possible.
8
Realistic, actual-sized pictures appear on the student version of this workbook page.


Label the dots in this picture with these numbers:
12 15 23 30
12
30
23
15
Put three more numbers of your choice in the picture.
There are many possibilities.
9

Build an arrow road from 0 to 62 using +10 and +1 arrows.



Other roads are possible.

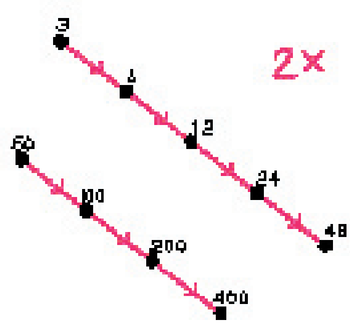
Write number facts for each number. One is done for you.

20	100
2 × 10 = 20	200 ÷ 2 = 100
10 + 10 = 20	50 + 50 = 100
21 − 1 = 20	2 x 50 = 100
$\frac{1}{2}$ x 40 = 20	99 + 1 = 100

36	75
24 + 12 = 36	80 − 5 = 75
30 + 6 = 36	3 x 25 = 75
40 − 4 = 36	100 − 25 = 75
3 x 12 = 36	50 + 25 = 75

Many facts are possible.

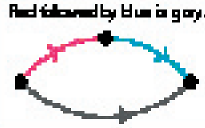
Label the dots.



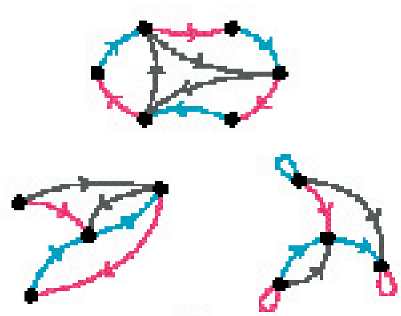
Complete.

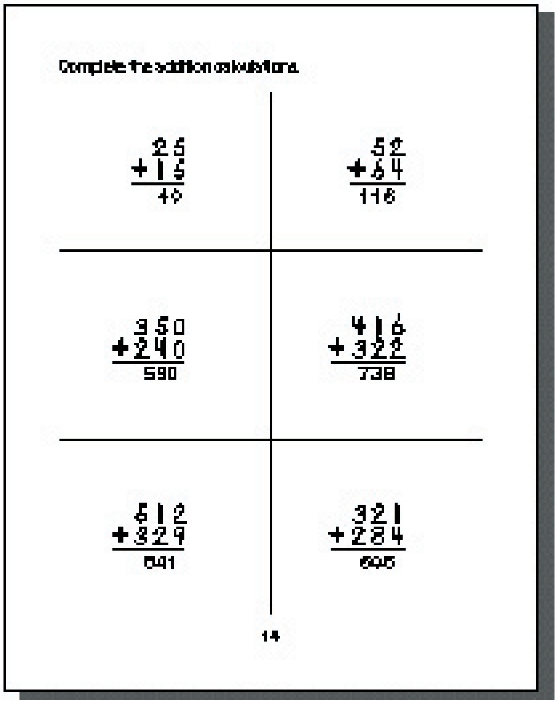
2 × 5 = 10
2 × 25 = 50
2 × 24 = 48
2 × 200 = 400
2 × 7 = 14
2 × 4 = 8
2 × 11 = 22
2 × 204 = 408

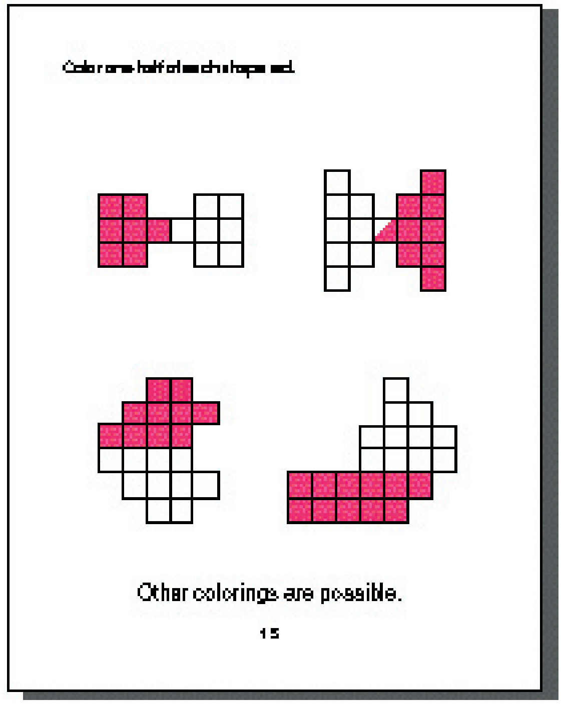
Red followed by blue is gray.

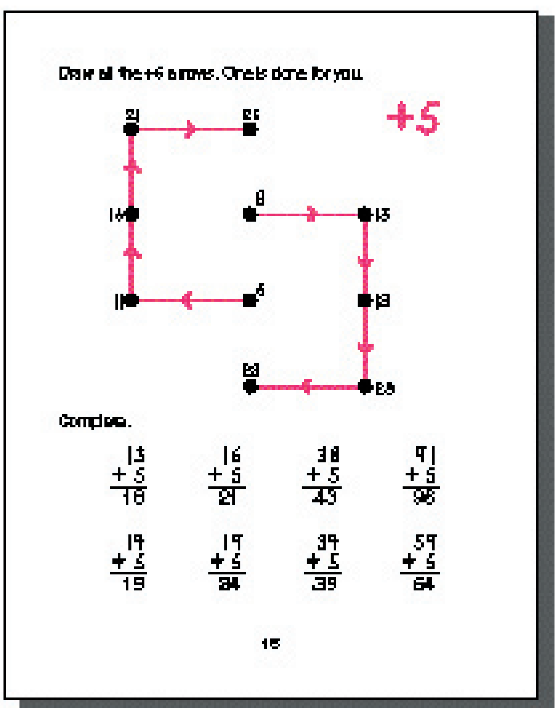


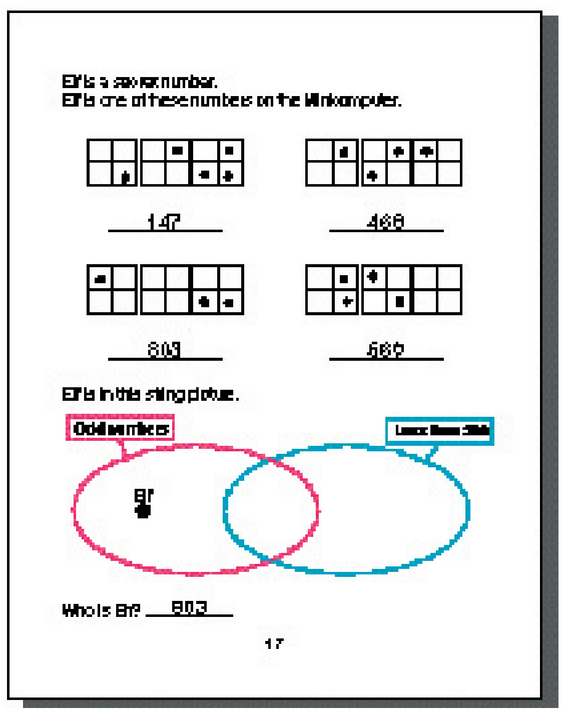
Draw the missing gray arrows (use a pencil).

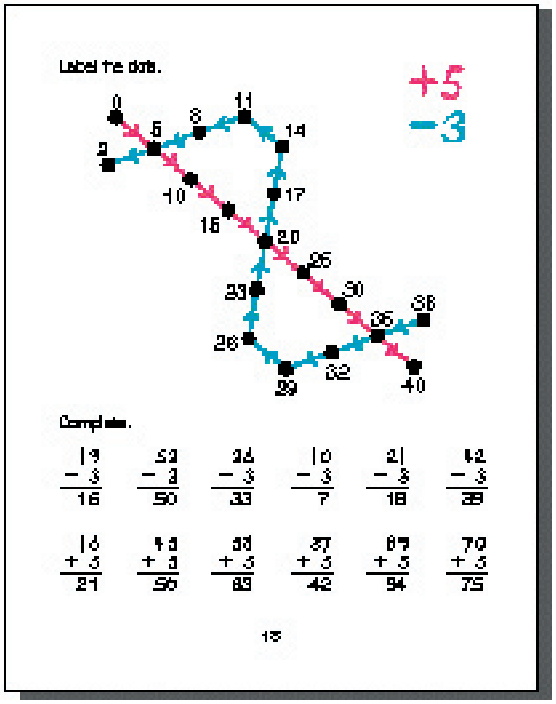



Complete the addition calculations.
25 + 15 = 40
52 + 64 = 116
350 + 240 = 590
416 + 322 = 738
612 + 329 = 941
321 + 284 = 605
14


Other colorings are possible.
15


Draw all the +5 arrows. One is done for you.
+5
Complete.
13 + 5 = 18
16 + 5 = 21
38 + 5 = 43
91 + 5 = 96
14 + 5 = 19
19 + 5 = 24
34 + 5 = 39
59 + 5 = 64
16


147
468
803
609
Odd numbers
Who is Elf? 803
17

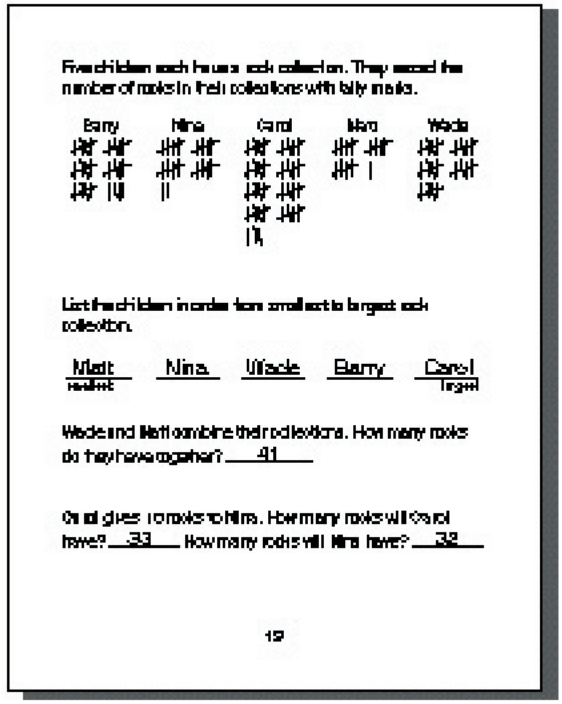


Label the dots.

Complete.

19 − 3 = 16	53 − 3 = 50	36 − 3 = 33	10 − 3 = 7	21 − 3 = 18	42 − 3 = 39
16 + 5 = 21	51 + 5 = 56	78 + 5 = 83	37 + 5 = 42	89 + 5 = 94	70 + 5 = 75

18



Five children each have a rock collection. They record the number of rocks in their collections with tally marks.

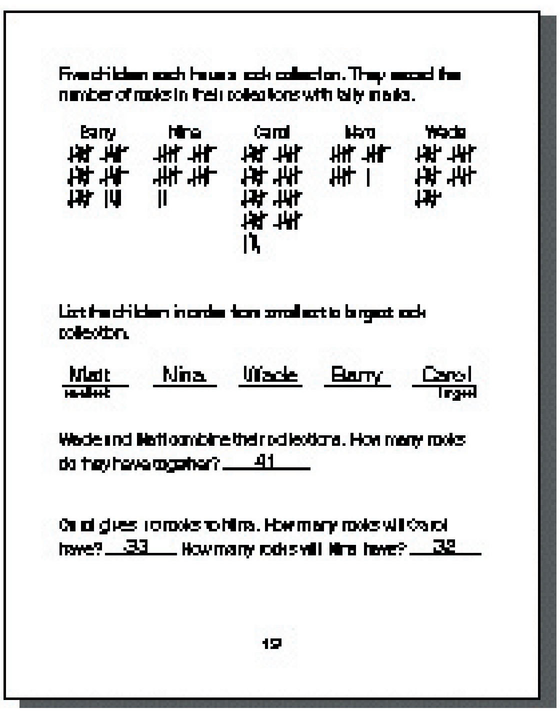
List the children in order from smallest to largest rock collection.

Matt (smallest), Nina, Wade, Barry, Carol (largest)

Wade and Matt combine their collections. How many rocks do they have together? 41

Carol gives 10 rocks to Nina. How many rocks will Carol have? 33 How many rocks will Nina have? 32

19



Five children each have a rock collection. They record the number of rocks in their collections with tally marks.

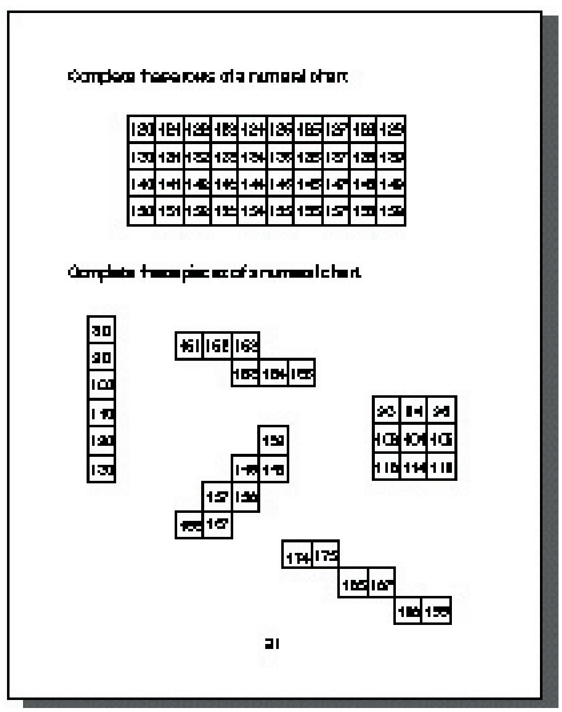
List the children in order from smallest to largest rock collection.

Matt (smallest), Nina, Wade, Barry, Carol (largest)

Wade and Matt combine their collections. How many rocks do they have together? 41

Carol gives 10 rocks to Nina. How many rocks will Carol have? 33 How many rocks will Nina have? 32

19

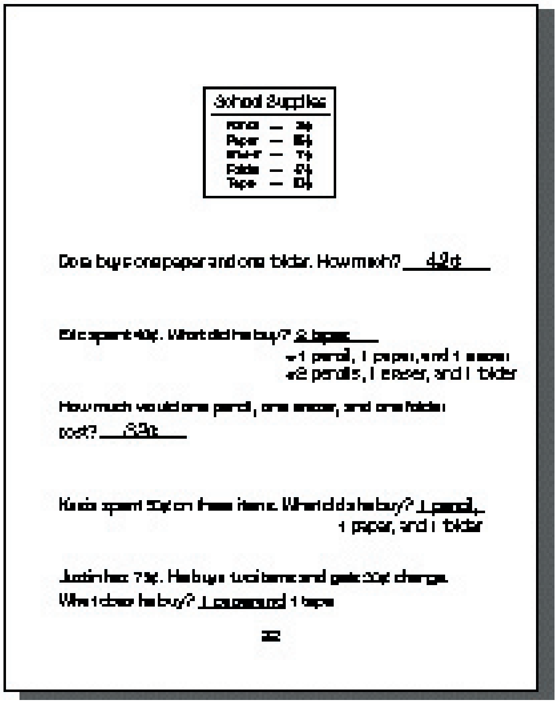


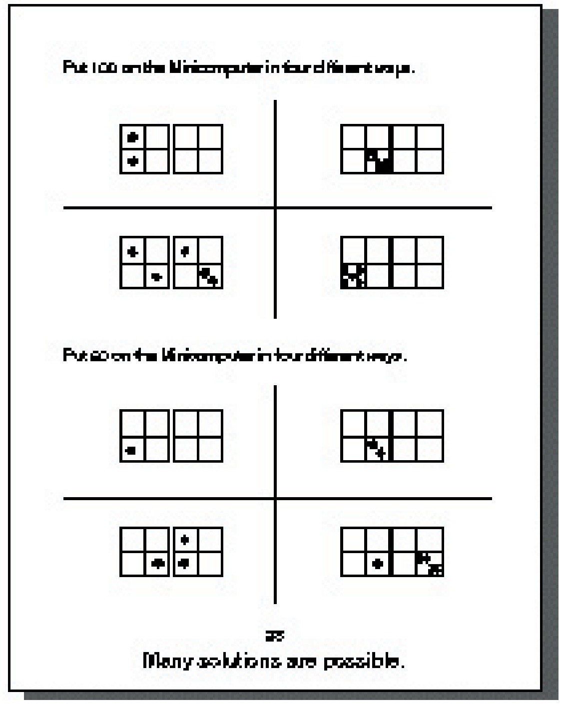
Complete these rows of a numeral chart.

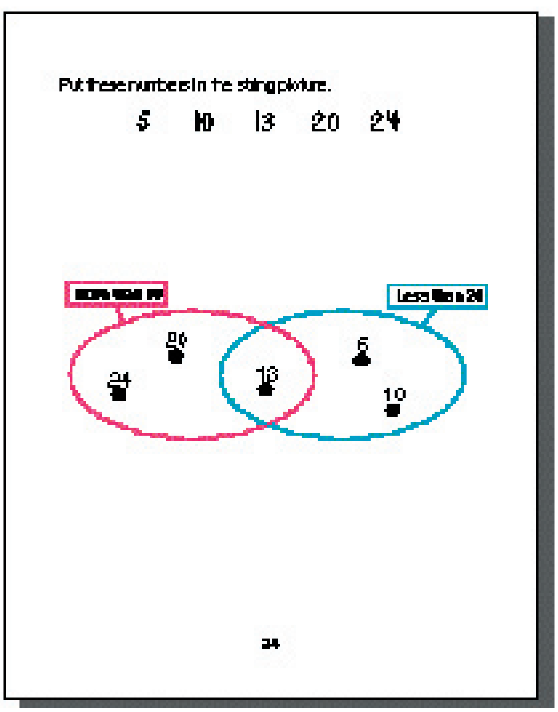
120	121	122	123	124	125	126	127	128	129
130	131	132	133	134	135	136	137	138	139
140	141	142	143	144	145	146	147	148	149
150	151	152	153	154	155	156	157	158	159

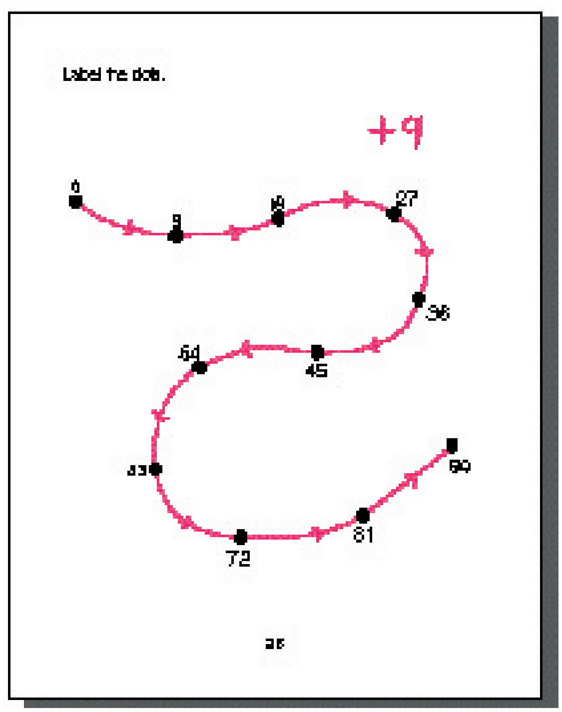
Complete these pieces of a numeral chart.

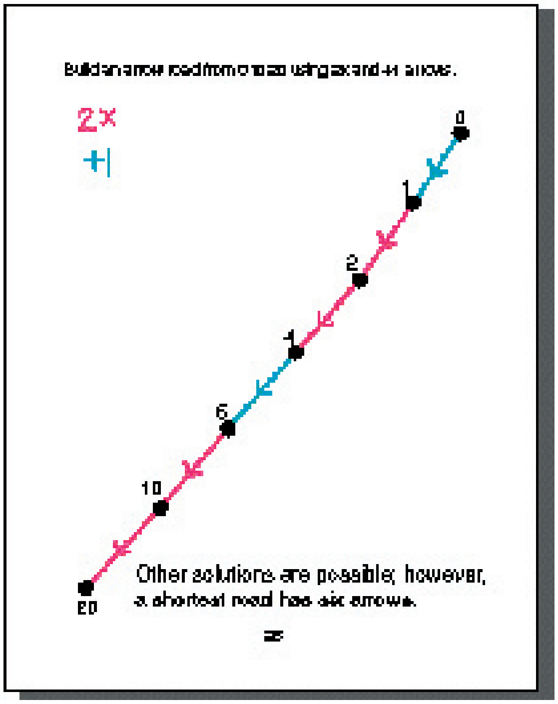
21

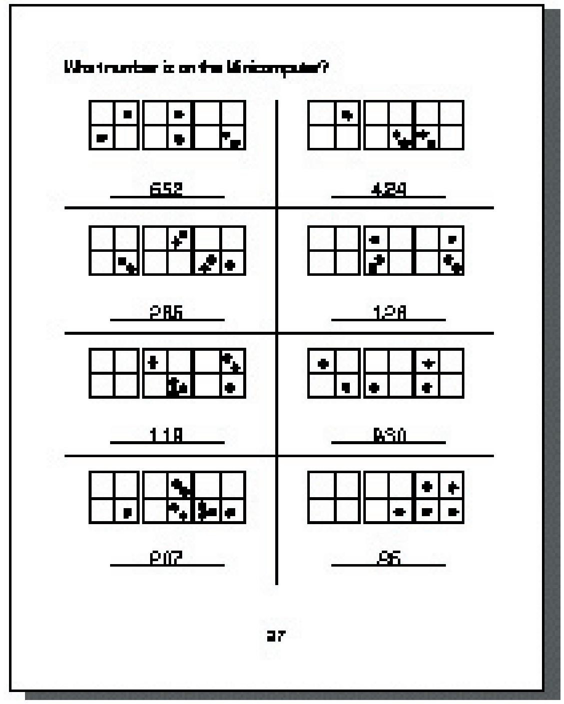

School Supplies
Dora buys one paper and one folder. How much? 42¢
2 tapes
1 pencil, 1 paper, and 1 eraser
2 pencils, 1 eraser, and 1 folder
1 paper, and 1 folder

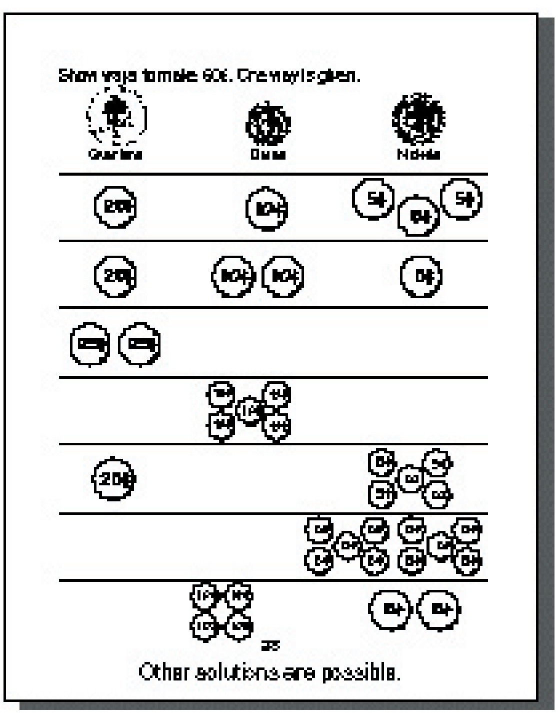

Put 100 on the Minicomputer in four different ways.
Many solutions are possible.

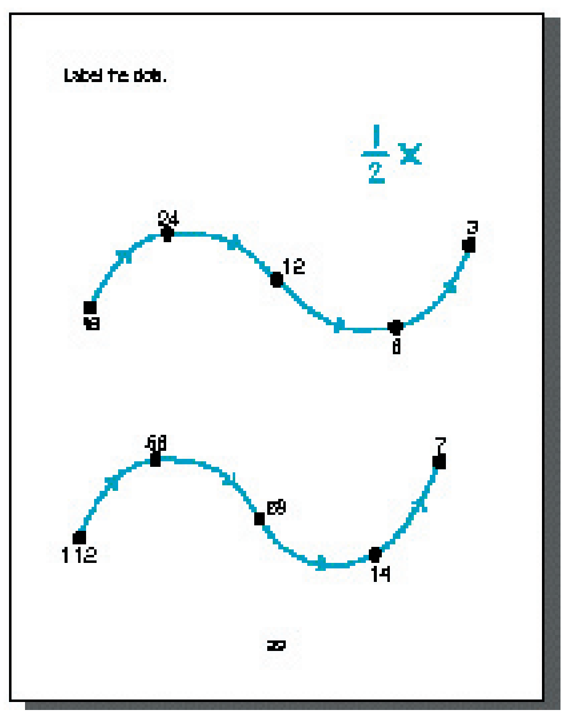

Put these numbers in the string picture.
5 10 13 20 24
Less than 20
20
24
13
5
10

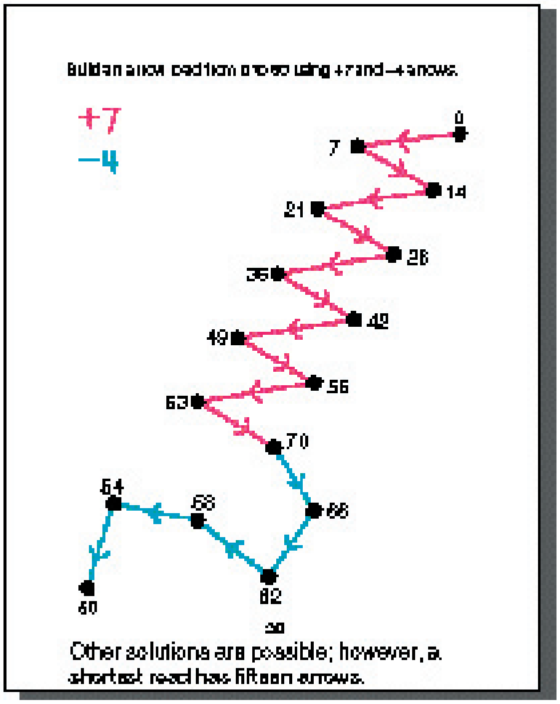

Label the dots.
+9
0
9
18
27
36
45
54
63
72
81
90


2x
+1
0
1
2
4
5
10
20
Other solutions are possible; however, a shortest road has six arrows.


What number is on the Minicomputer?
652
424
285
128
118


Other solutions are possible.

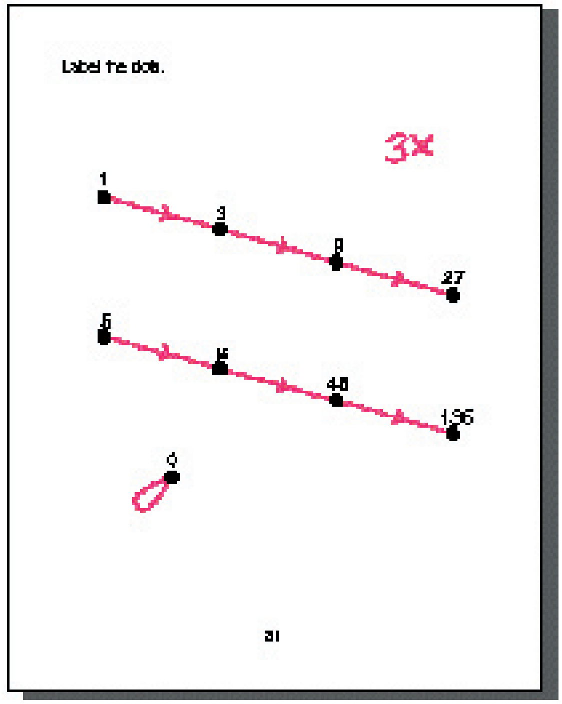

Label the dots.
24
12
3
6
7
112
14



Build an arrow road from 0 to 60 using +7 and −4 arrows.

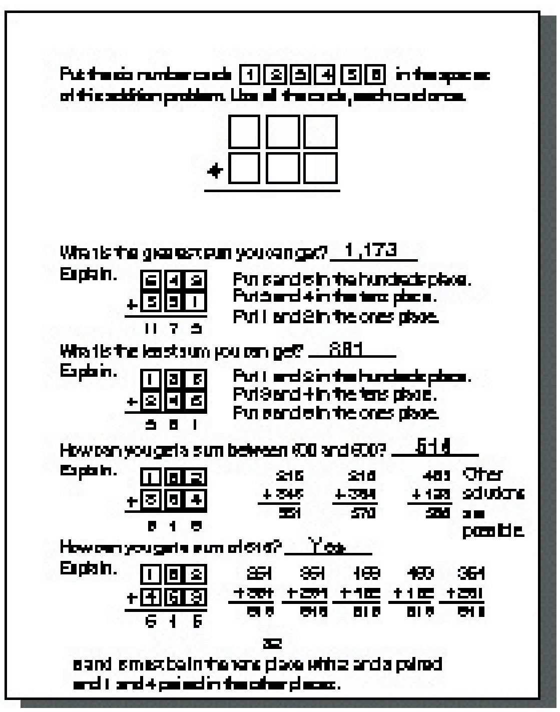
30

Other solutions are possible; however, a shortest road has fifteen arrows.



Label the dots.

31



Put the six number cards 1 2 3 4 5 6 in the spaces of this addition problem. Use all the cards, each card once.

What is the greatest sum you can get? 1,173

Explain.

Put 5 and 6 in the hundreds place.
Put 3 and 4 in the tens place.
Put 1 and 2 in the ones place.

What is the least sum you can get? 381

Explain.

Put 1 and 2 in the hundreds place.
Put 3 and 4 in the tens place.
Put 5 and 6 in the ones place.

How can you get a sum between 400 and 600? 516

Explain.

Other solutions are possible.

How can you get a sum of 615? Yes

Explain.

32

3 and 5 must be in the tens place with 2 and 6 paired and 1 and 4 paired in the other places.

Capsule Lesson Summary

Put a set of eight numbers in their proper places in an arrow picture with +10 and 3x arrows. Begin the workbook *Fishing for Numbers, Part III*.

Materials

Teacher
- Colored chalk

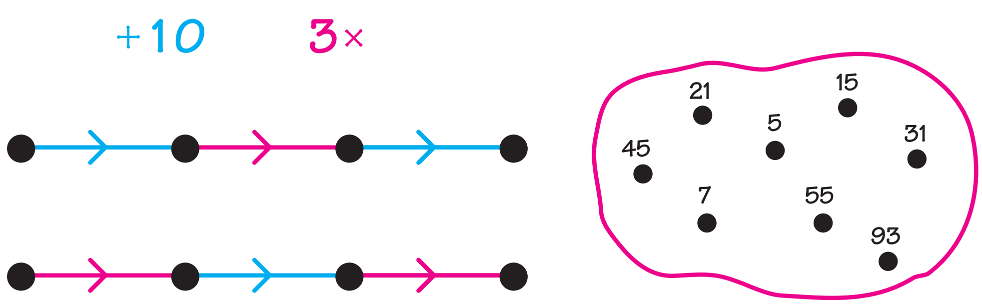
Student
- Paper
- Colored pencils, pens, or crayons
- *Fishing for Numbers, Part III* Workbook

Advance Preparation: You may like to put the picture for the warm-up activity on the board before starting the lesson.

Description of Lesson

You may like to let students work with a partner for this warm-up activity.

Draw these pictures on the board and ask students to copy the pictures on their papers. In pairs, one student can copy the arrow picture while the other copies the numbers.

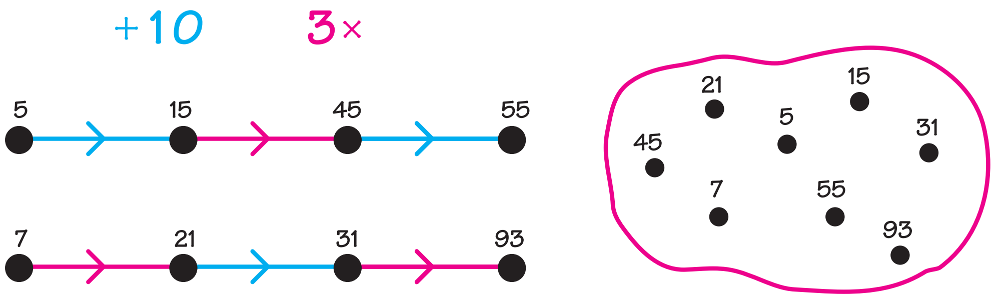


T: ***How many dots are in the arrow picture?*** (Eight)
How many numbers are inside the string? (Eight)
Each dot in this arrow picture is for one of the numbers in this string. Each number in the string has just one dot in the arrow picture. Label the dots in the arrow picture with numbers from the string.

Direct students to work with their partners on this problem for a few minutes.

When many students have finished the problem, ask someone to suggest a number for the starting dot of one of the arrow roads. Pursue this choice and see where it leads. It if leads to a number in the arrow picture that is not in the string, let the class discover this and discuss why that choice for a starting dot won't work. A student may observe that the least number (5) must be at the beginning of one of the roads; if so, ask the student to convince the rest of the class. In the course of solving this problem collectively, the opportunity to discuss various strategies should arise. Allow ample opportunity for students to discover some good strategies themselves.

The complete arrow picture is given below.



Distribute copies of the workbook *Fishing for Numbers, Part III*, and allow students to work independently for the rest of the class period. You may remind students of similar workbooks they used in second grade. Emphasize that it is likely students will need to use their erasers, and they should make several tries before asking for help.

At the end of the lesson, collect the workbooks for your review. They will be used again in Lesson W7.

Capsule Lesson Summary

Use context clues to put the numbers in a string picture in their proper places in a story. Continue working in the workbook *Fishing for Numbers, Part III* (This is the second of two lessons using this workbook.)

Materials

Teacher
- Colored chalk

Student
- Paper
- *Fishing for Numbers, Part III* Workbook

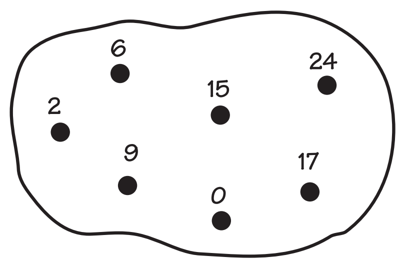
Advance Preparation: Write the story for the warm-up activity on the board or an overhead transparency before starting the lesson.

Description of Lesson

Allow students to work with a partner for this warm-up activity.

Put this story and set of numbers on the board.

> Teresa is having a Halloween party. She invited ____ friends to the party, ____ girls and____ boys. They all came. Together with Teresa and her little brother, there were ____ children at the party.
>
> Teresa's mother made cupcakes. Altogether she made ____ dozen or ____ cupcakes. Every girl at the party including Teresa ate one cupcake, and every boy including her brother ate two cupcakes. There were ____ cupcakes left over.



Tell the class that the problem is to put all the numbers inside the string into the story so that it makes sense. Invite student partners to work on this problem for a few minutes.

When many students have finished the problem, conduct a collective discussion. As students tell the class how to fill in the blanks, they should explain their reasoning. A solution is given below.

Key: Teresa is having a Halloween party. She invited __15__ friends to the party, __9__ girls and __6__ boys. They all came. Together with Teresa and her little brother, there were __17__ children at the party.

Teresa's mother made cupcakes. Altogether she made ________2__ dozen or __24__ cupcakes. Every girl at the party including Teresa ate one cupcake, and every boy including her brother ate two cupcakes. There were __0__ cupcakes left over.

Distribute the students' copies of the workbook *Fishing for Numbers, Part III*. Ask students first to correct or complete pages from the previous week's work on this workbook and then to continue. At the end of the class period, collect the workbooks for your review.

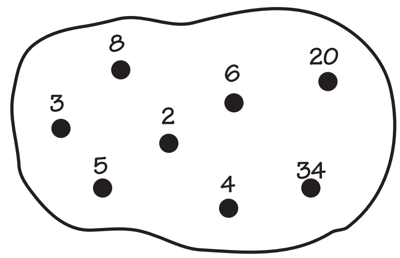
Assessment Activity

An individual student progress report for the workbook is available on Blackline W7. You may like to use this form to monitor student work.

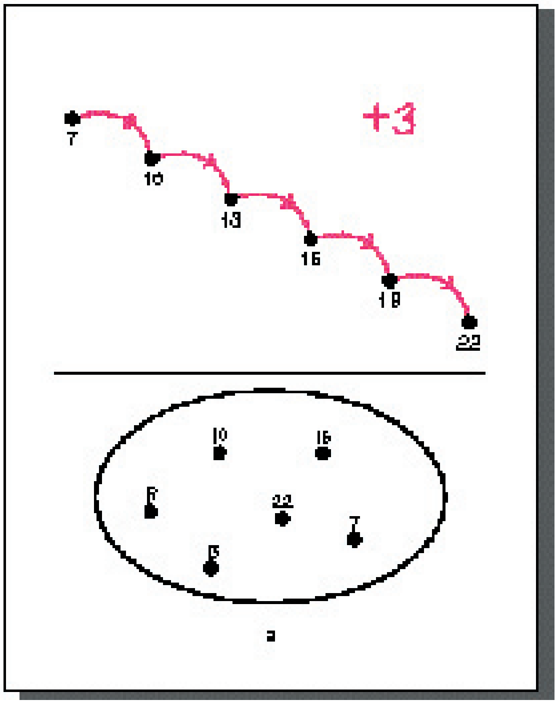
Home Activity

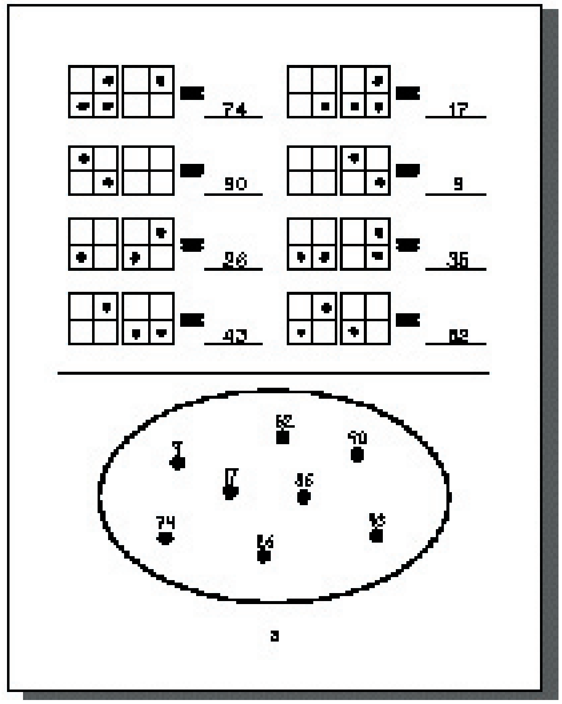
Prepare a story similar to the warm-up activity for students to do at home with family participation. For example:

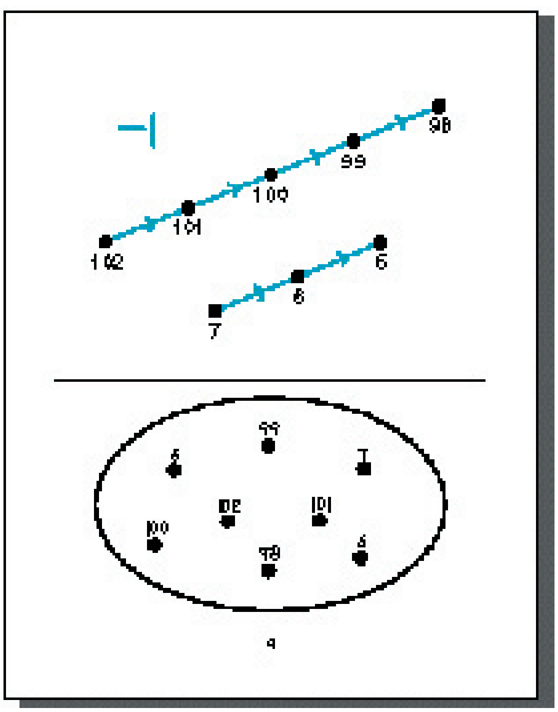
> The entire Palmer family went to an amusement park. There are ____ members of the family, ____ adults and ____ children. Children's tickets cost $____ each and adult tickets cost $____ each. The family spent a total of $____ for tickets.
>
> For lunch, the Palmer's shared a box of chicken pieces. The box contained ____ pieces. Each family member ate the same number of pieces; they each had ____ pieces of chicken.

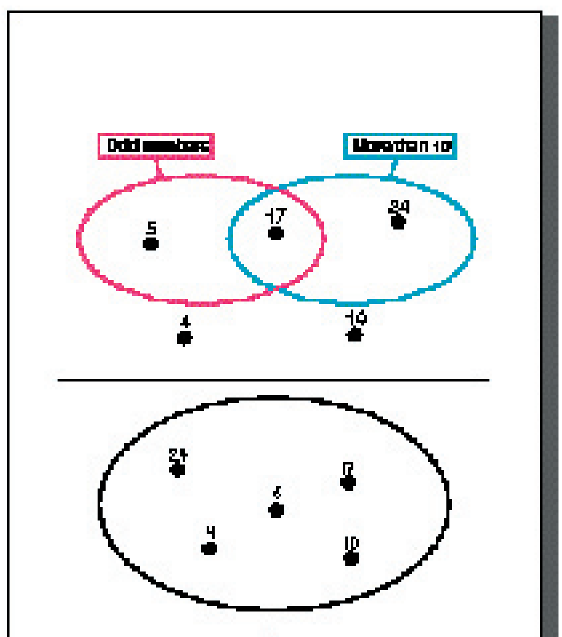


Using all the numbers in the string, fill in the blanks of the story so that it makes sense.

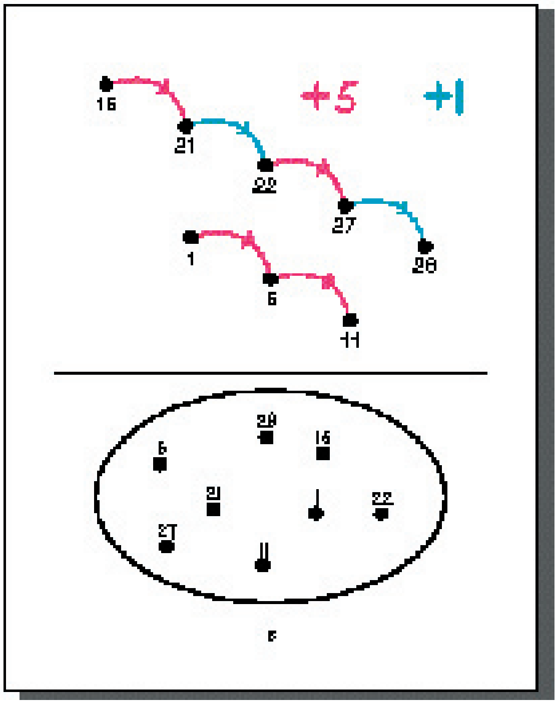


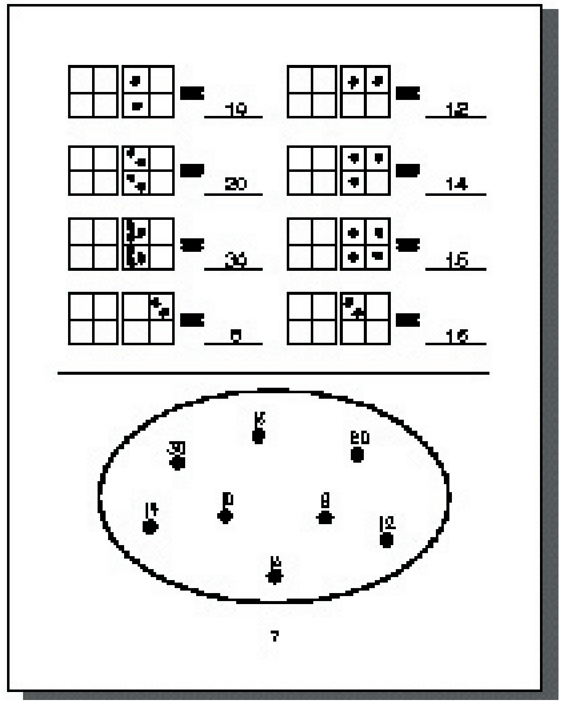


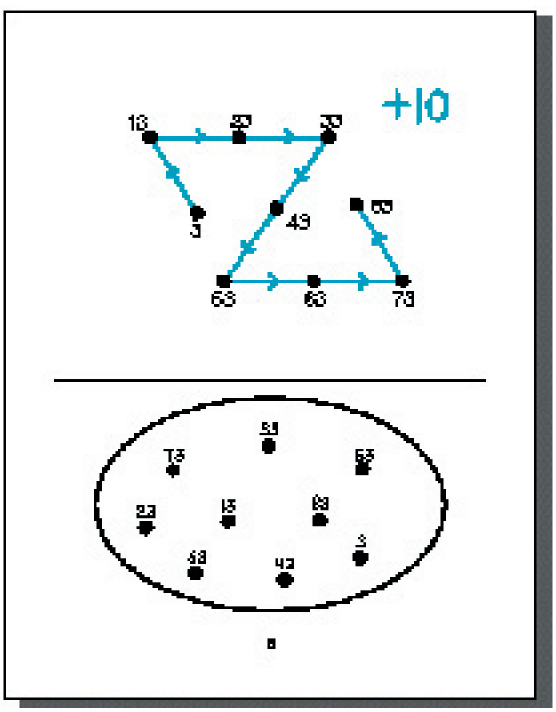


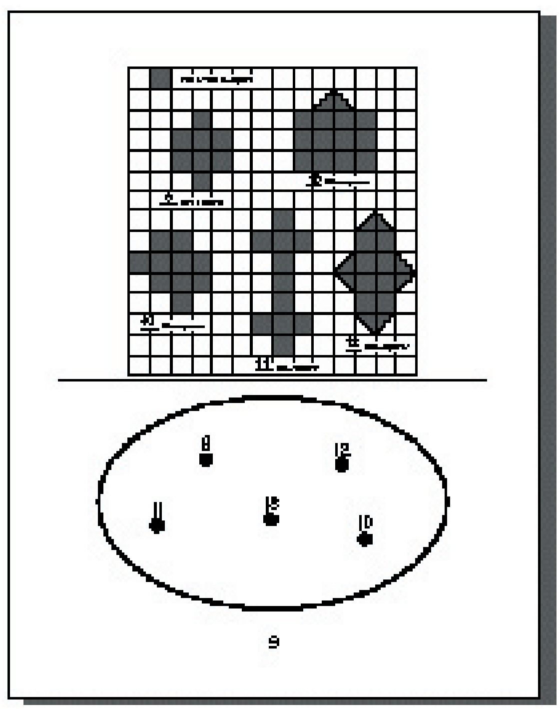


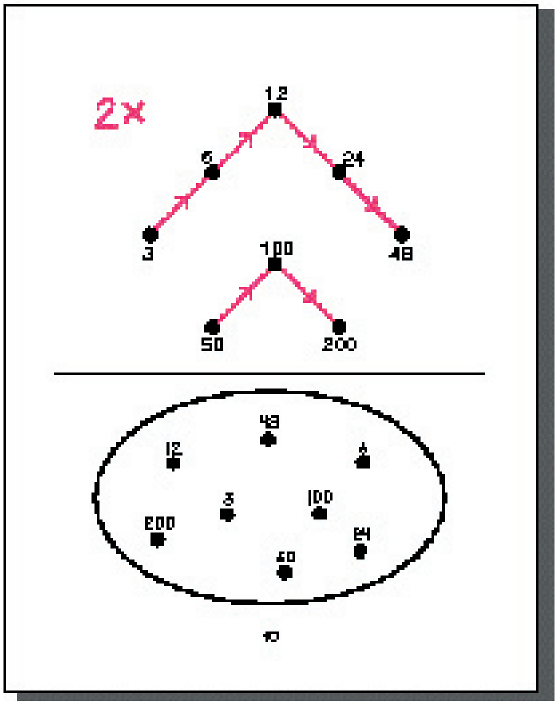
4 and 10 may be interchanged.

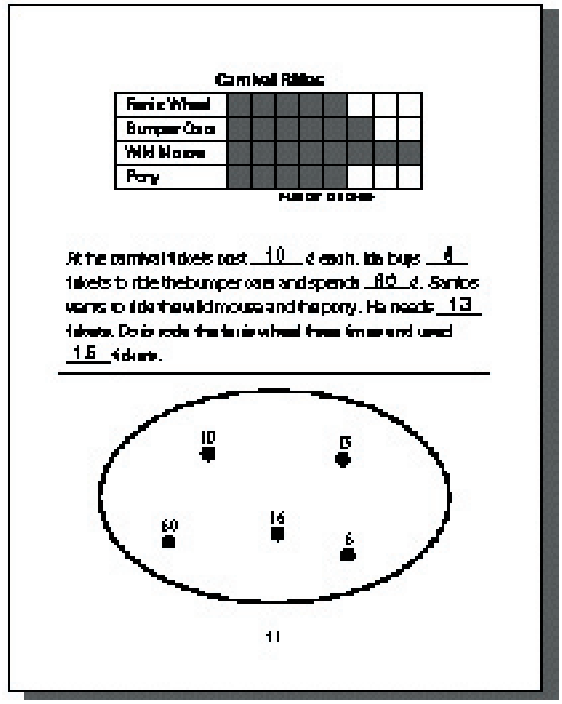

+5
+1

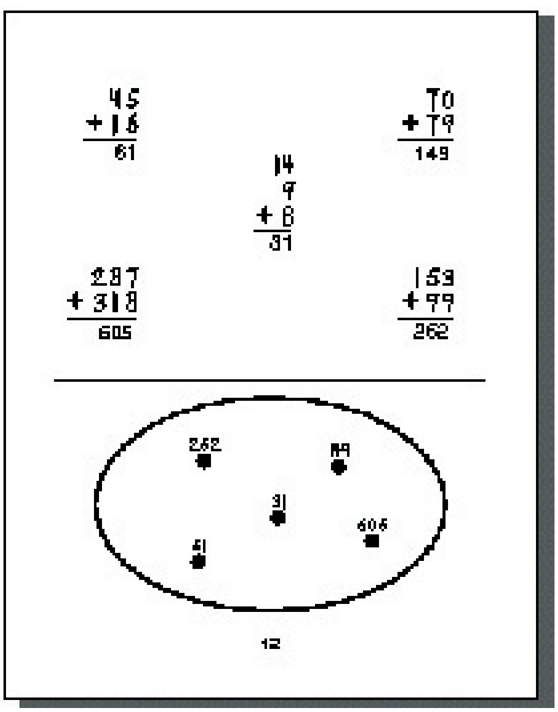


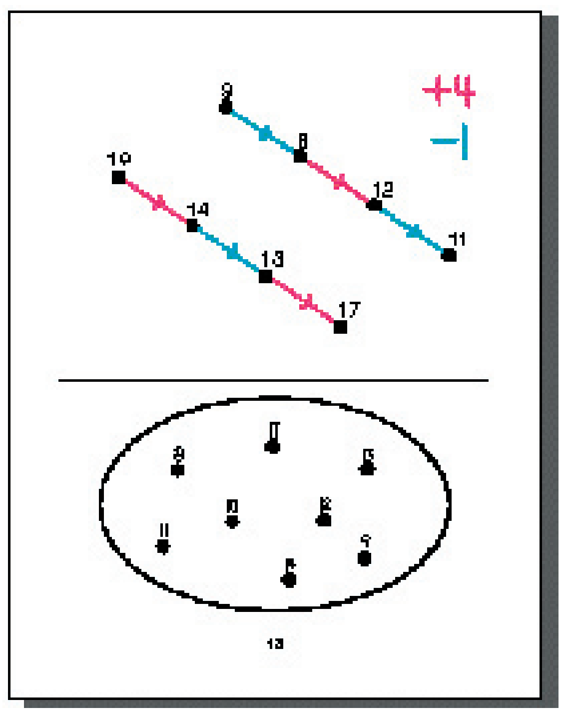

+10



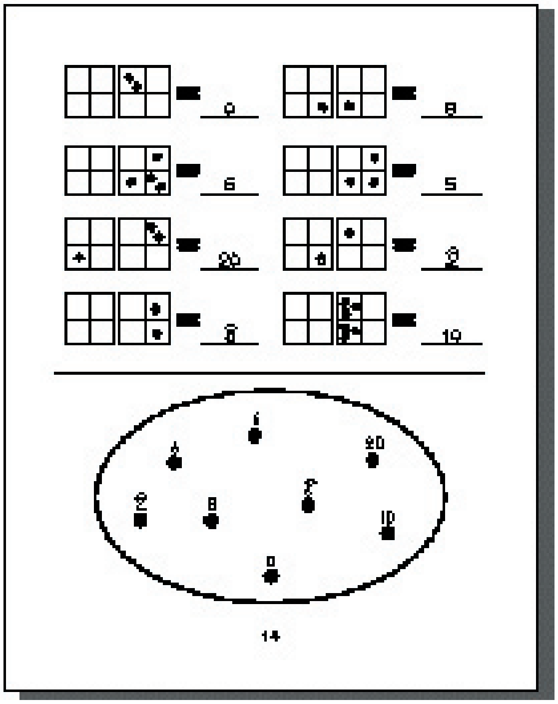

2×
12
6
24
3
48
100
50
200

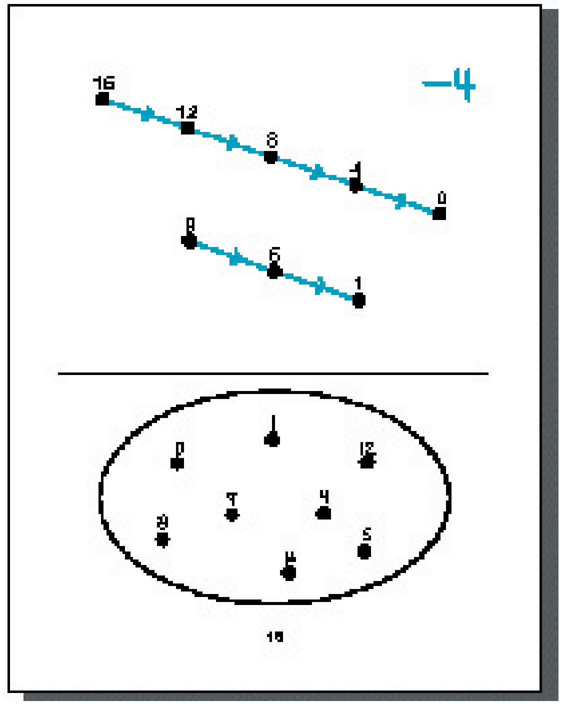

Ferris Wheel
Bumper Cars
Wild Mouse
Pony
10
8
80
13
15

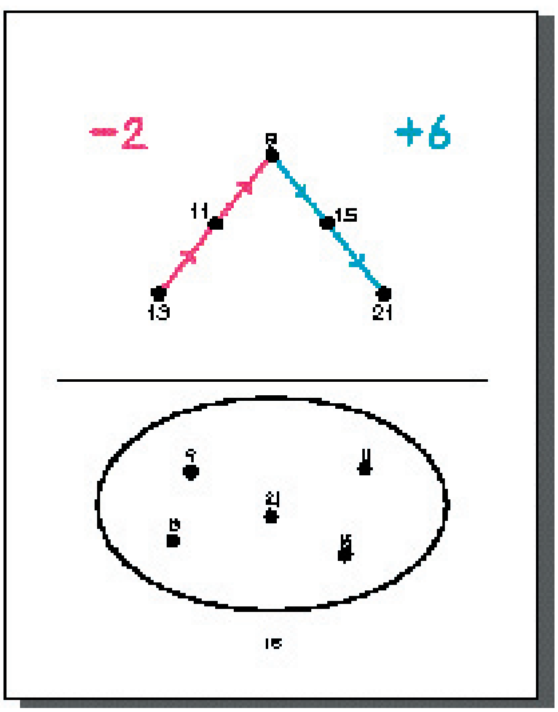

45
+ 16
61
70
+ 79
149
14
9
+ 8
31
287
+ 318
605
153
+ 99
252

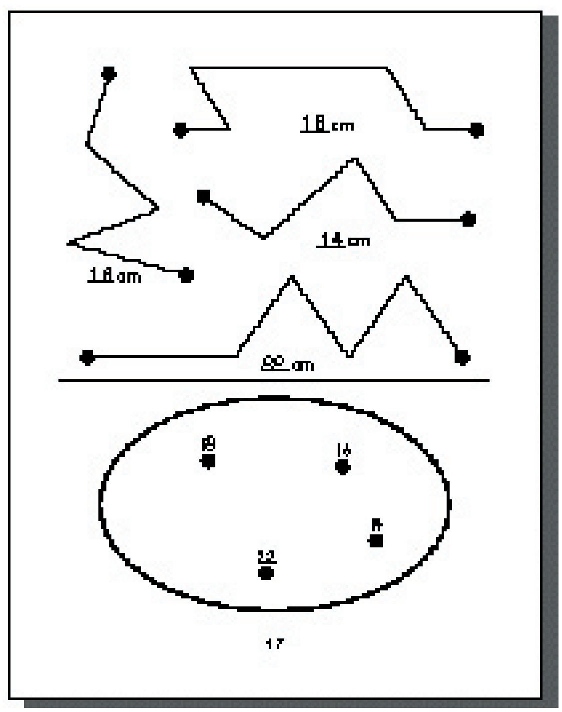

+4
−1
9
8
12
11
10
14
13
17

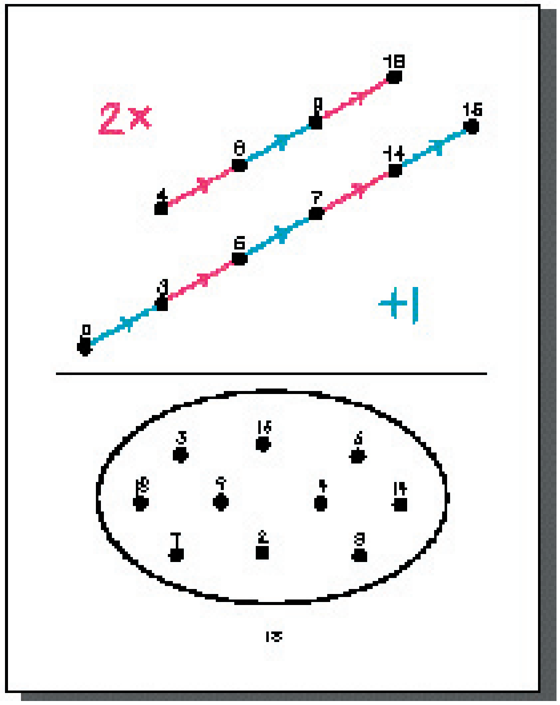
W7

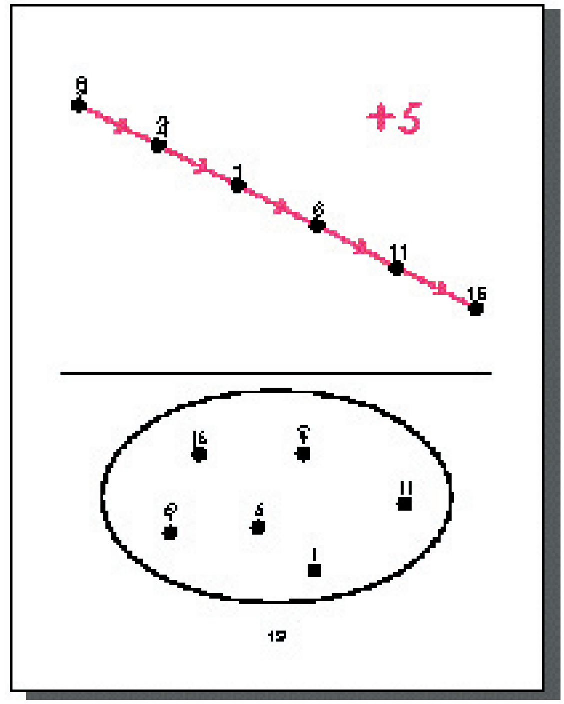


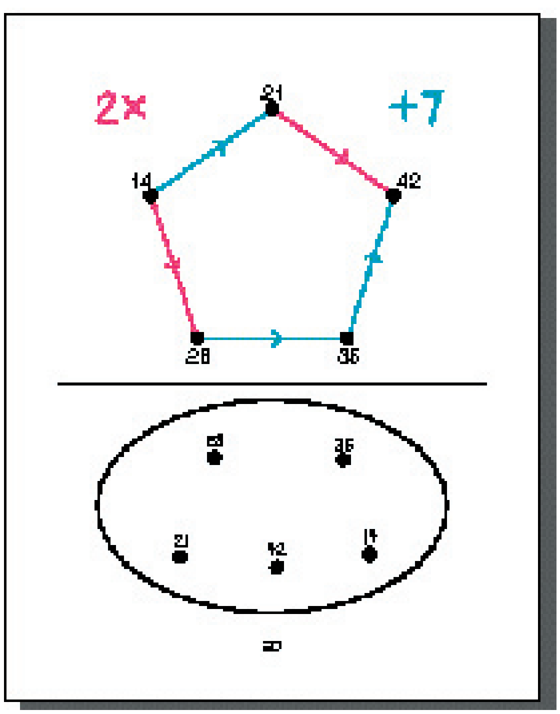


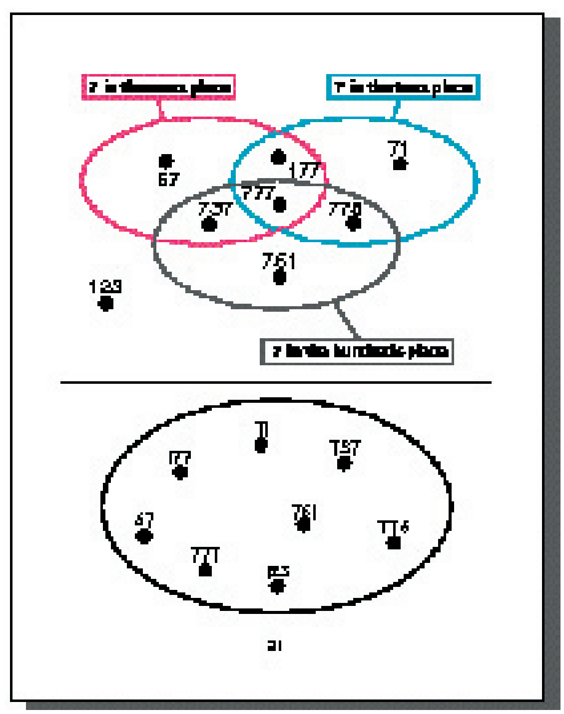


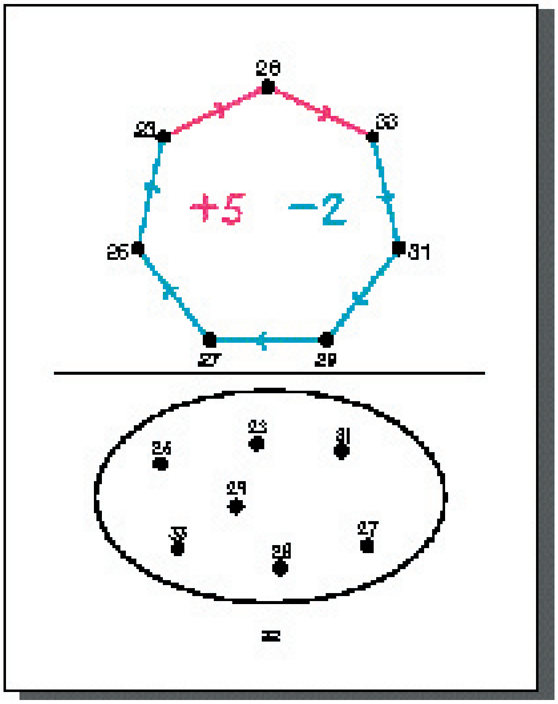


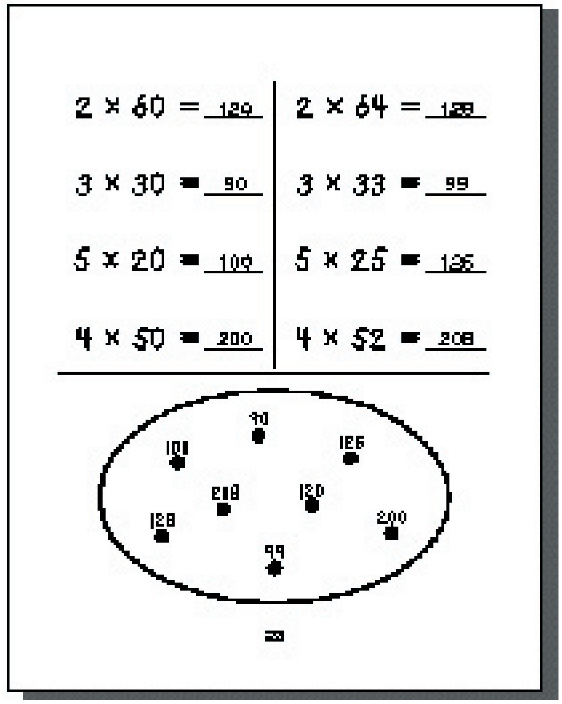

2×
+1

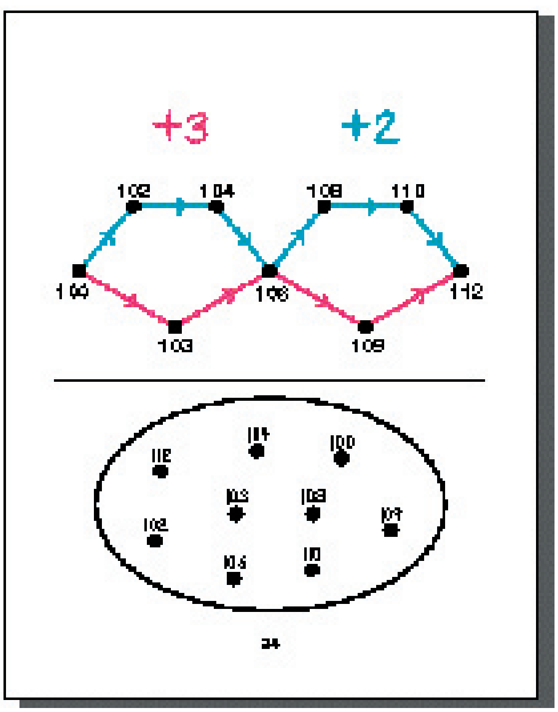

+5

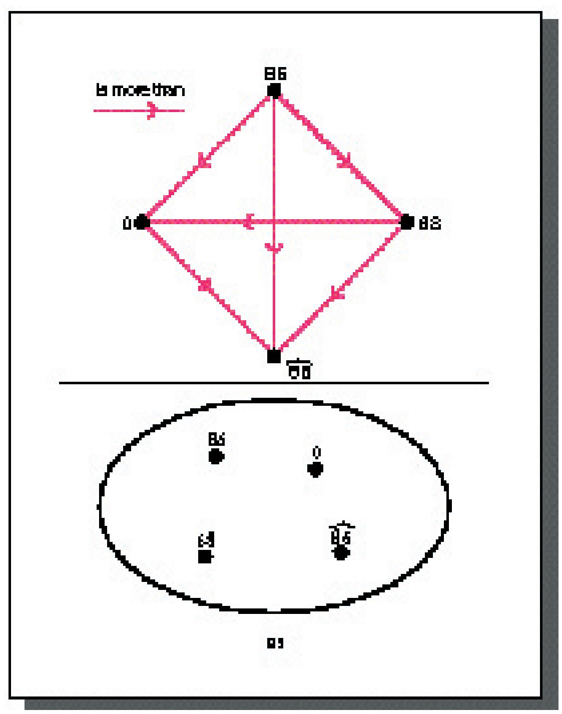

2×
+7

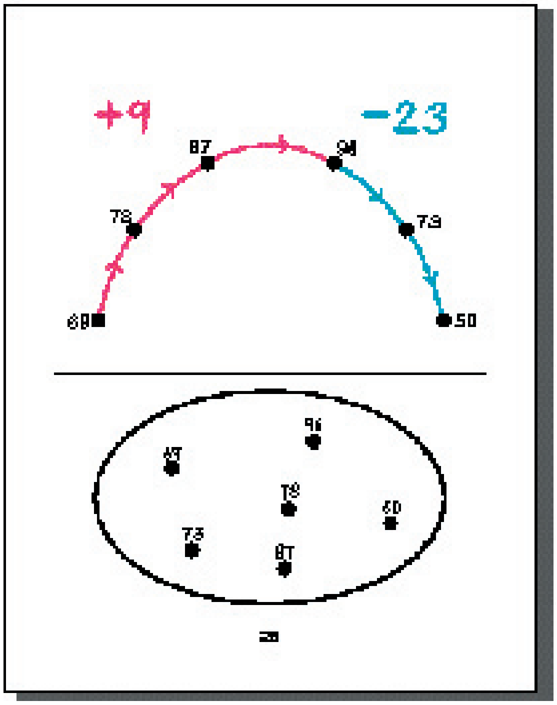


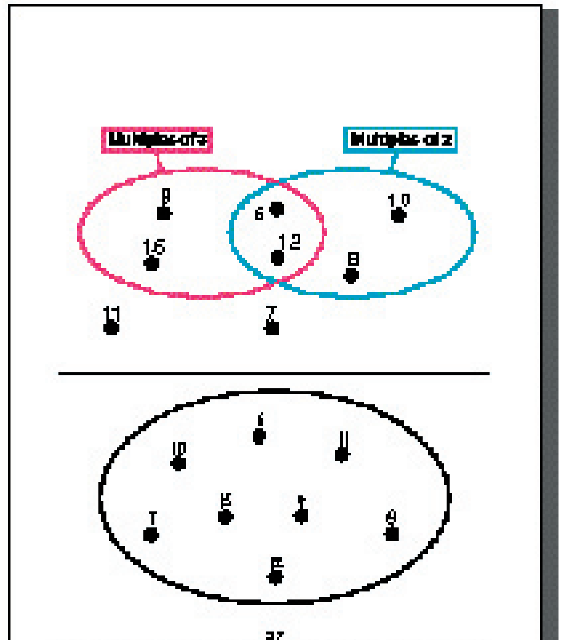

+5
−2


2 × 60 =
2 × 64 =
3 × 30 =
3 × 33 =
5 × 20 =
5 × 25 =
4 × 50 =
4 × 52 =

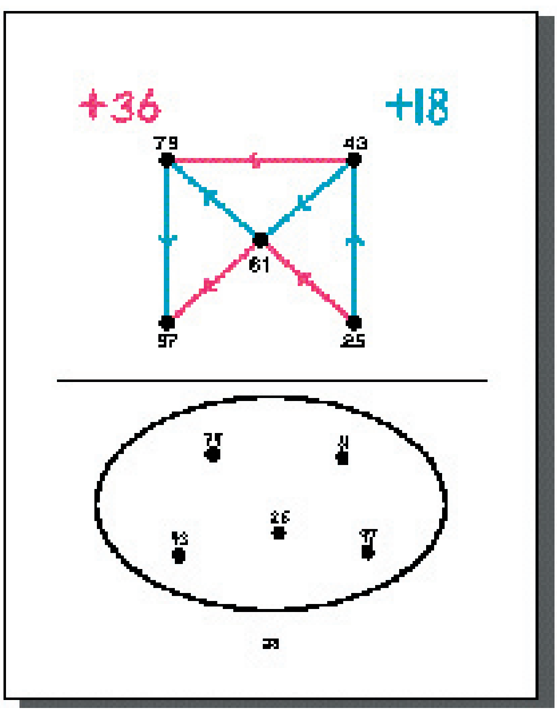

+3
+2
102
104
108
110
106
112
103
109

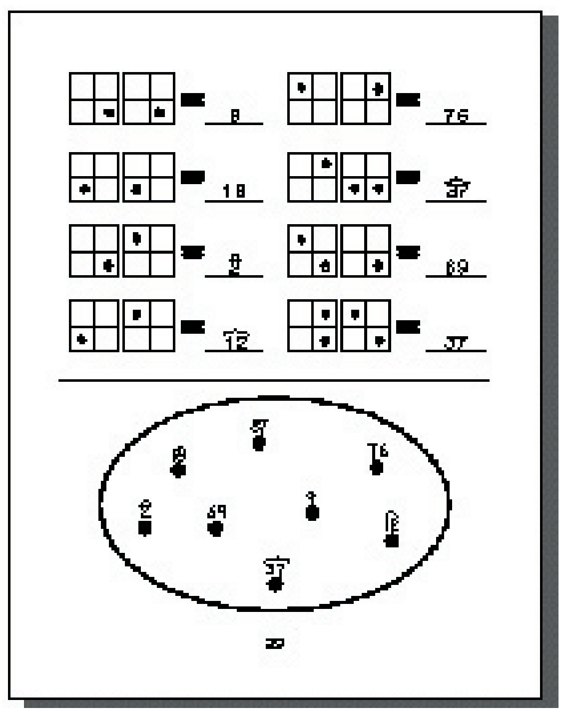

is more than

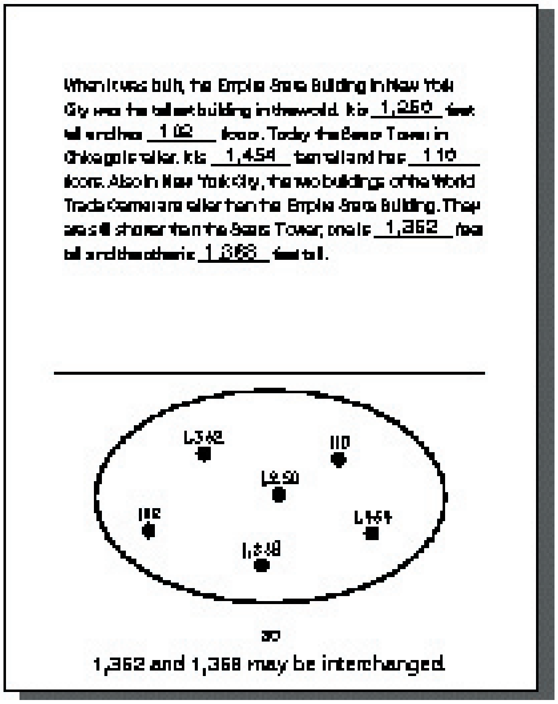




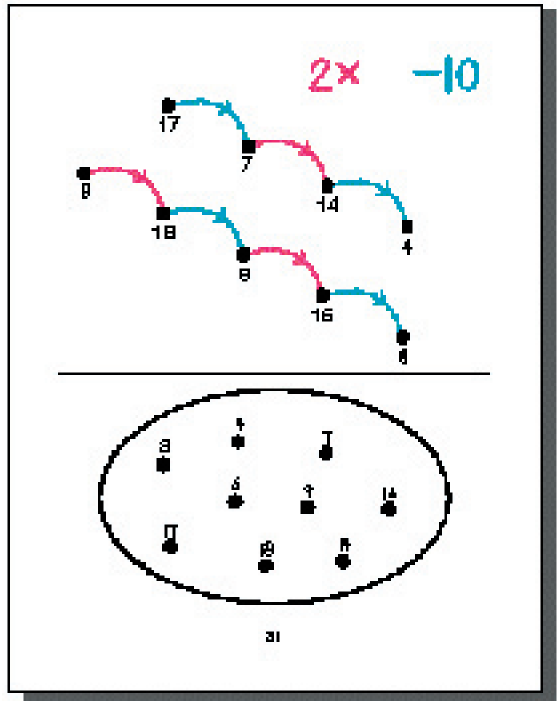
Numbers may be interchanged within a region.

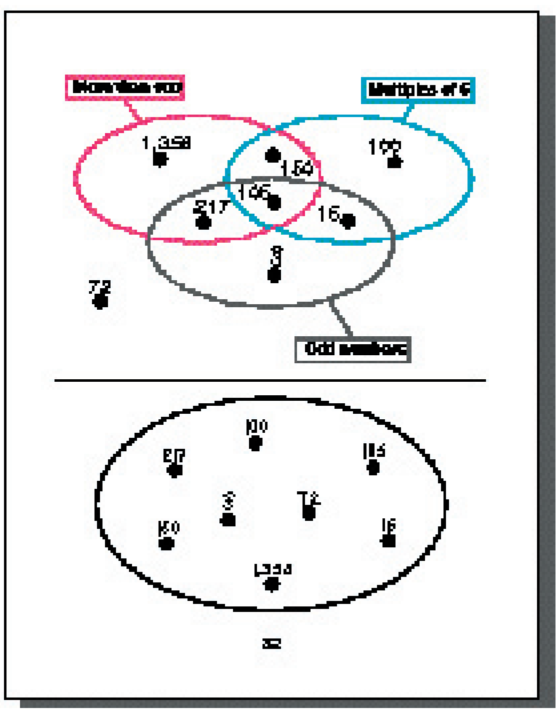






1,362 and 1,368 may be interchanged.





Capsule Lesson Summary

Using a spinner face with ten numerical relations, find some of the possible and impossible scores you can get by starting at 0, spinning the spinner four times, and following the number relations on which the spinner lands.

Materials

Teacher
- Spinner face
- Cardboard spinner arrow (optional)

Student
- Paper

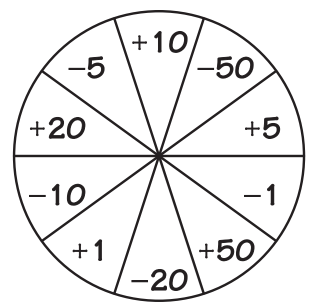
Advance Preparation: Use Blackline W8 to make a spinner face for display during Exercise 1. You may want to put the spinner face on an overhead transparency, or you may want to draw it directly on the chalkboard making it much bigger so everyone can see easily.

Description of Lesson

Allow students to work in small cooperative groups during this lesson.

Exercise 1

Display this spinner face on the board.



T: ***Have you seen anything like this before?***

Let the students tell about spinners they might have seen in games they play at home or at school. Mention that spinners usually have arrows attached at their centers. You may like to demonstrate with a cardboard spinner arrow.

T: ***I saw some children playing a game with a spinner that looks like this one. Each child would spin the spinner four times and follow the results of the four spins to get a score. That is, they would start at 0, and add or subtract according to the results of their spins. Let's pretend we are playing this game.***

Write 0 on the board as you announce that the game always starts at 0. The following dialogue assumes students choose the results of four hypothetical spins. You may prefer to make a working spinner to generate results.

T: ***Where does the arrow point on the first spin?***

S: ***+50.***

Begin forming a number sentence on the board.

$$0 + 50$$

T: ***Where does the arrow point on the second spin?***

S: ***+10.***

Record +10 in the number sentence forming on the board.

0 + 50 + 10

T: ***How many points do we have now?***

S: ***60.***

T: ***Where does the arrow point on the third spin?***

S: ***–1.***

Record –1 in the number sentence forming on the board.

0 + 50 + 10 – 1

T: ***How many points do we have now?***

S: ***59.***

T: ***Where does the arrow point on the fourth spin?***

S: ***+1***

Record +1 in the number sentence forming on the board.

0 + 50 + 10 – 1 + 1

Note: You may need to observe with the class that even though you record four spins (+50, +10, –1, +1), there are five numbers in the number sentence. Point out that you start with 0 before any spin.

T: ***We spun four times, so our turn is over. What is our score?***

S: ***60.***

0 + 50 + 10 – 1 + 1 = 60

Complete the number sentence.

T: ***What are other final scores we could get when we play this spinner game?***

Encourage student groups to find many possible final scores and to record the scores in number sentences. After several minutes take a suggestion from each group and ask the group to explain how to obtain the score. As they are suggested, record the results of four spins and the final scores on the board. When necessary, help students to subtract a greater number from a lesser number by referring to the number line.

During the discussion, ask the following questions and others that you think are appropriate.

T: ***What is the highest score we could get playing this game?*** (200)
How could we get this score? (0 + 50 + 50 + 50 + 50 = 200)
What is the lowest score we could get? ($\widehat{200}$)
How could we get this score? (0 – 50 – 50 – 50 – 50 = $\widehat{200}$)
Is it possible to get a score of 0? (Yes; 0 + 50 – 20 – 20 – 10 = 0)
Is there another way to get a score of 0? (Yes; 0 + 10 – 10 + 50 – 50 = 0)
What is the next to highest score we could get? (170)
How could we get this score? (0 + 50 + 50 + 50 + 20 = 170)
What is the next to lowest score we could get? ($\widehat{170}$)
How could we get this score? (0 – 50 – 50 – 50 – 20 = $\widehat{170}$)

Continue with this activity until you have a list of 10 to 15 scores on the board.

T: ***Suppose a person reports a final score of 115. Is this possible?***

S: ***Yes.***

Ask students to tell you at least one of the two possible ways to get a score of 115.

$$0 + 50 + 50 + 10 + 5 = 115$$
$$0 + 50 + 50 + 20 - 5 = 115$$

T: ***Another person reports a final score of 150. Is this possible?***

It is impossible to have a score of 150 after four spins. Encourage student groups to try to find a way to score 150 until some observe that it is not possible in this game. Perhaps a group will be able to explain why a score of 150 is not possible, but do not insist on an explanation.

Note: It is possible to get a score of 150 after three spins but not after four spins. It is possible to get a score of 100 after two spins, but it is not possible to increase that score by 50 in two more spins. Final scores of 151 (0 + 50 + 50 + 50 + 1) and 149 (0 + 50 + 50 + 50 – 1) are possible, but not 150.

Exercise 2

Draw a long line on the board. Point to the list of scores recorded on the board.

T: ***Let's put each of these scores on a number line in order. What is the greatest number (possible score) in this list?***

S: ***200.***

T: ***Where shall we put 200 on our number line?***

Students very likely will indicate that 200 should be on the right. Ask a student to put 200 on the number line.

Note: 200 can be assigned to any point on this number line, but students' previous experiences usually have involved number lines with the numbers increasing as you move to the right.

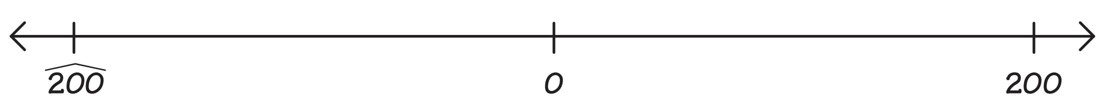
T: ***What is the least number (possible score) in our list?***

S: ***$\widehat{200}$.***

Call on a volunteer to put $\widehat{200}$ on the number line. Since $\widehat{200}$ is the least number in the discussion, it would be best if the student locates $\widehat{200}$ to the far left on the line.

T: ***Where should we locate 0?***

S: ***Halfway between $\widehat{200}$ and 200.***



Invite students to locate the other numbers in your list on the number line. Students may not locate the numbers in exactly the right place, but insist that the numbers be in the correct order. When $\widehat{170}$ is located on the number line, emphasize that $\widehat{170}$ is more than $\widehat{200}$.

T: ***There are many possible scores. We have found only some of them. On a piece of paper, write down other possibilities.***

Direct student groups to record as many possible final scores as they can find. When you observe a possible score that has not been discussed before, challenge the rest of the groups to find a way to get that score. A possible dialogue is given here.

T: ***Mark and Kasia have found a way to get a score of 20. How could we get a score of 20?***

S: ***0 + 5 + 5 + 5 + 5 = 20.***

T: ***Is that the way you got 20, Mark and Kasia?***

S: ***No, we did 0 + 50 – 10 – 10 – 10 = 20.***

S: ***I did it another way; 0 + 20 – 10 + 20 – 10 = 20.***

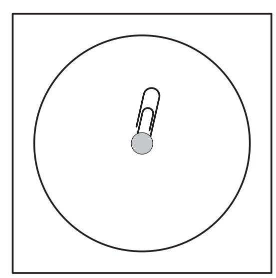
Continue this activity for a few minutes. Erase everything on the board except the spinner face.

Exercise 3

T: ***There are several numbers between 0 and 100 that are impossible to get as scores when you play this spinner game.*[†] *Try to find some of them.***

Suggest groups make lists of scores they cannot find. After a few minutes, take suggestions to check with the class. Challenge the class to try to get those numbers as scores after exactly four spins. If your students discover a way to get any of the suggested numbers as scores, cross them off your list. Continue this activity for a few minutes, but be careful not to conclude that all the remaining numbers are impossible to get as scores in the spinner game. A more accurate conclusion would be that your students have not found a way to get any of the remaining numbers as scores in the spinner game.

Extension Activity



Organize the students in groups of four and direct the groups to design and play their own spinner game. Groups can make their own spinner with a piece of cardboard, a paperclip, and a brad.

Allow groups to create their own spinner faces, but play the game with four spins, as in the lesson. Groups can then make lists of possibilities and put them on a number line.

[†]The numbers between 0 and 100 that are impossible to get are the following:

33	52	73	82	87	93
37	63	77	83	88	97
48	67	78	84	92	99

This is for your information only.

W9 A SPINNER GAME #2

Capsule Lesson Summary

Solve a detective story in which the secret number is a multiple of 11, and the other clues involve a string picture, the Minicomputer, and a spinner game.

Materials

Teacher
- Spinner face
- Colored chalk
- Minicomputer set
- Calculator

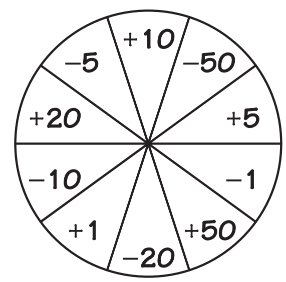
Student
- Paper
- Colored pencils, pens, or crayons
- Calculator
- Minicomputer set

Advance Preparation: See Lesson W8 for preparation of the spinner face.

Description of Lesson

Before the lesson begins, write a large 55 on a piece of paper and fold the paper so that 55 is hidden. As with other detective story lessons, you may like to let students work with partners.

Display this spinner face on the board. Ask students to recall what they can about the spinner game introduced in lesson W8.



Tell the class this story giving the children names if you wish.

T: ***Two children were playing a spinner game. Each child spun the spinner four times and from the results of the four spins arrived at a final score. One of the children told me that her score was 100. Is that possible?*** (Yes)

Ask students for at least two ways to get a score of 100. A few of the possible ways are given below.

$0 + 50 + 50 + 1 - 1 = 100$

$0 + 50 + 50 + 50 - 50 = 100$

$0 + 50 + 20 + 20 + 10 = 100$

$0 + 20 + 50 - 20 + 50 = 100$

T: ***The other child told me that his score was 101. Is that possible?***

It is impossible to have a score of 101 after four spins. Encourage students to use trial and error to find a way to score 101 until several students observe that it is not possible in this game. Perhaps someone will be able to explain why a score of 101 is not possible, but do not insist upon an explanation.

Note: It is possible to get a score of 100 after two spins but it is not possible to increase that score by 1 in two more spins. It is impossible to get a score of 100 after one spin or three spins.

Hold up the paper with 55 hidden on it.

T: ***A secret number is written on this paper. If you are careful detectives, you can discover what number it is.***

You may wish to ask a veteran student to explain what a detective story is for the benefit of new students. If students want simply to guess the number, emphasize that good detectives always look for clues until they know for sure what the secret number is.

Clue 1

Equip student pairs with calculators.

T: ***Your first clue is that the secret number is one of the numbers we see on the calculator when we make it count by elevens starting at 0.***

Invite students to teach their calculators to count by elevens starting at 0 like this:

1) Put on the starting number (0).
2) Press [+] [1] [1] (read as "plus eleven").
3) Then press [=] [=] [=] and so on.

Observe the numbers that appear, starting with 0, and begin a list on the board.

0
11
22
33
44

T: ***Do you see a pattern?***

Encourage students to predict the next number before looking at it on the calculator.

T: ***Do you know another way to describe these numbers?***

S: ***They are multiples of 11.***

More precisely, these numbers are whole number multiples of 11. You may need to mention multiples of 11 yourself. Continue listing multiples of 11 until you reach 99.

Allow several students to predict what the next multiple of 11 is before seeing it on the calculator. Record 110 in a second column. With students' help, continue listing the multiples of 11 until the list includes 143. Point to the numbers in the second column.

0	110
11	121
22	132
33	143
44	
55	
66	
77	
88	
99	

T: ***Do you see a pattern to these numbers?***

Students will likely notice the sequence in the ones place and the tens place. Continue listing the multiples of 11 with your class until the list includes 198. Allow several students to predict what the next multiple of 11 is before seeing it on the calculator. Record 209 on the board.

0	110
11	121
22	132
33	143
44	154
55	165
66	176
77	187
88	198
99	209
	⋮

T: ***Are these all the multiples of 11?***

S: ***No, we could add 11 to 209, and then add 11 to that number, and then 11 to that number, and so on.***

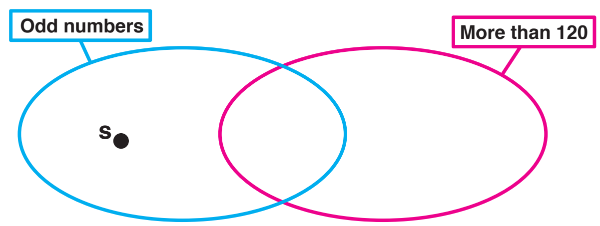
S: ***The list goes on and on.***

T: ***I will put three dots at the end of our list to indicate that the list of multiples of 11 goes on and on.***

The secret number is a multiple of 11. Here is another clue.

Clue 2

Draw this string picture on the board and instruct student partners to copy the picture on their papers.



T: ***I put a dot for the secret number in this string picture. What does this clue tell us about the secret number?***

Encourage students to observe that the secret number is an odd number and that it is not more than 120. Direct student partners to spend a few minutes deciding which numbers in the list can and cannot be the secret number. After a few minutes, discuss some possibilities with the class.

T: ***Can the secret number be more than 120?***

S: ***No. We can erase all the numbers in the list that are more than 120, because the secret number is not in the red string.***

Solicit class agreement and then erase all the numbers greater than 120.

T: ***Should I erase the three dots?***

S: ***Yes, they indicate numbers greater than 120.***

T: ***Is the secret number an odd number?***

S: ***Yes, it is in the blue string.***

S: ***We can erase all the even numbers.***

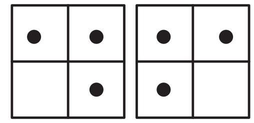
T: ***Which numbers should I erase?***

As the students name even numbers in the list, erase them. Finally, the list should be reduced to five numbers: 11, 33, 55, 77, and 99.

T: ***One of these five numbers is the secret number, but we don't know which one.***

S: ***We need another clue.***

Clue 3

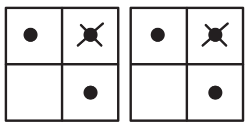


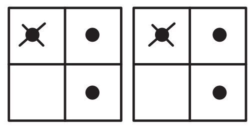
Put this configuration on the Minicomputer.

Instruct student partners to put the same configuration on their desk Minicomputers.

This is not the secret number, but you can get the secret number on the Minicomputer by removing two of these checkers. What could the secret number be?

Allow student partners to look for possibilities. After a few minutes, call on students to tell the class their findings. When a student suggests that one of the numbers in the list could be the secret number, ask the student to show which two checkers to remove. Your students should determine quickly that 99 and 55 could be the secret number.


= 99


= 55

Encourage students to explain why 11, 33, and 77 cannot be the secret number but do not expect precise explanations. Erase 11, 33, and 77 from the list.

Note: For your information only, 11 cannot be the secret number because the least possible number from the third clue results from removing the checker from the 80-square and the checker from the 40-square; that is, 23. 33 cannot be the secret number because the next to least possible number from the third clue results from removing the checker from the 80-square and the checker from the 10-square; that is, 53. 77 cannot be the secret number because after removing the checker from the 80-square, 63 is on the Minicomputer, which is less than 77, and a second checker has not yet been removed.

Clue 4

T: ***The secret number is either 55 or 99. Listen carefully to this last clue. The secret number is one of the numbers you can get as a final score playing the spinner game. If you know the secret number, write it on a piece of paper.***

Call attention to the spinner face and recall with the class that in the spinner game you need four spins to get a final score.

When many students know the secret number, ask how to get a final score of 55. The possibilities are 0 + 50 + 20 – 10 – 5 and 0 + 20 + 20 + 10 + 5. Ask why a score of 99 is impossible. Students may observe that 99 is possible after three spins but not after four spins.

Reveal that 55 is the secret number.

Writing Activity

Capsule Lesson Summary

Find the lengths of routes from one place to another on a road map. In doing so, add and subtract two-digit and three-digit numbers.

Materials

Teacher • None

Student • *Road Map* Workbook • Colored pencils, pens, or crayons

Description of Lesson

Distribute copies of the workbook *Road Map*. Ask students to look at page 2 and to tell you the names of many of the places on the map. Perhaps one of the students will recognize that the map is of Yellowstone and Grand Teton National Parks. If necessary, announce this yourself. Encourage students to share what they know about Yellowstone and Grand Teton National Parks and the places indicated on the map. Point out that the numbers on the map tell how far it is in kilometers from one place to another.

Ask the students to look at the workbook cover and to tell you about the picture.

T: ***This is a picture of a signpost somewhere in Yellowstone National Park. The signpost says that Old Faithful is 27 km to our left and that Lake is 32 km to our right. Where might we see this signpost? Look again at the map on page 2.***

S: ***In West Thumb.***

T: ***There are several routes indicated on the map on page 2. The solid blue line indicates a route between which two cities?***

S: ***Mammoth Hot Springs and Old Faithful.***

T: ***It goes through which other city?***

S: ***Madison Junction.***

T: ***How many kilometers is it from Mammoth Hot Springs to Old Faithful? How can we find the length of the solid blue route?***

S: ***Add 55 and 25.***

T: ***The distance from Mammoth Hot Springs to Madison Junction is 55 km and from Madison Junction to Old Faithful is 25 km. We can find the distance from Mammoth Hot Springs to Old Faithful by adding 55 + 25.***

Write this addition problem on the board.

T: ***Write this addition problem inside the solid blue rectangle on page 3, and then solve it.***

When most of the students have solved this problem, invite one of them to do the addition at the board.

Note: Some students will do this problem as a mental calculation, so do not insist on use of a standard addition algorithm. You may like to let students explain different methods.

$$\begin{array}{r} {}^{1} \\ 55 \\ +\ 25 \\ \hline 80 \end{array} \qquad 55 + 25 = 80$$

Conclude that the length of the solid blue route is 80 km. Then ask the students to complete the sentence inside the solid blue rectangle on page 3.

Repeat this activity for the route indicated by a solid red line. Emphasize that the columns of numbers in an addition problem should be lined up carefully so that all the ones digits are in the same column, all the tens digits are in the same column, and all the hundreds digits are in the same column.

Instruct students to continue working independently. Encourage them to read all of the directions on every page. Some students may need extra help because of the workbook's reading level. You may want to work one example from page 7 or 9 collectively to model and encourage following directions.

The solid blue route is 69 km long.

The solid red route is 152 km long.

The dashed blue route is 75 km long.

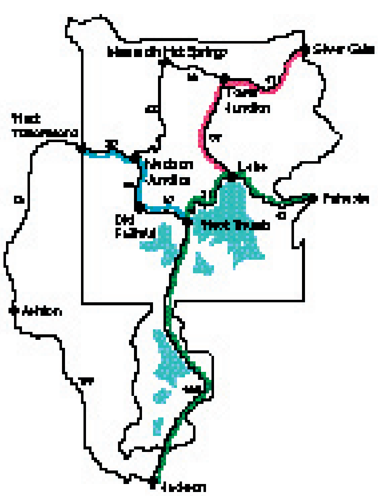
The dashed red route is 112 km long.

The solid blue route is 192 km long.

The solid red route is 84 km long.

The dashed blue route is 146 km long.

The dashed red route is 177 km long.



6

Find the shortest route between these places.

Lake to Tower Junction:	57 km
Tower Junction to Silver Gate:	47 km
Lake to Tower Junction to Silver Gate:	104 km

Color this route red.

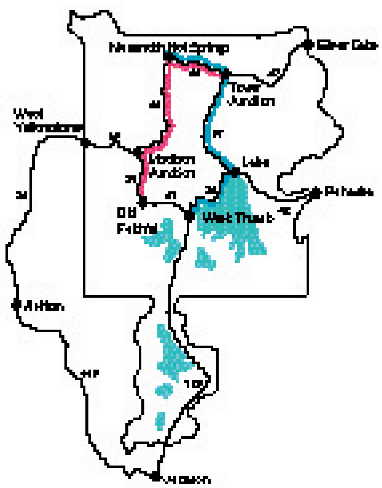
West Thumb to Old Faithful:	27 km
Old Faithful to Madison Junction:	26 km
Madison Junction to West Yellowstone:	23 km
West Thumb to Old Faithful to Madison Junction to West Yellowstone:	76 km

Color this route blue.

Pahaska to Lake:	42 km
Lake to West Thumb:	32 km
West Thumb to Jackson:	125 km
Pahaska to Lake to West Thumb to Jackson:	199 km

Color this route green.

7



8

Start at Old Faithful. Plan a trip that goes to exactly three other places. Color your route red.
How long is this trip? 109 km

Many other solutions are possible.

Plan a trip from Mammoth Hot Springs to West Thumb. Color your route blue.
How long is this trip? 118 km

Other solutions are possible.

9

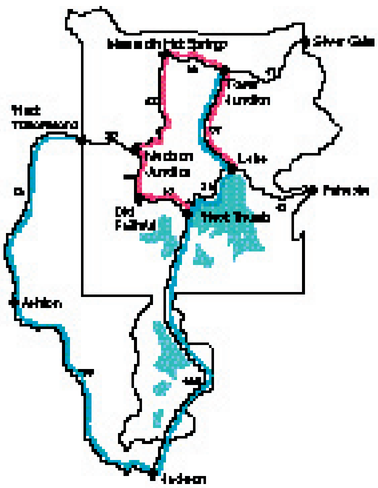
The solid blue route is 191 km long.

The solid red route is 280 km long.

The dashed blue route is 157 km long.

The dashed red route is 131 km long.

11



12

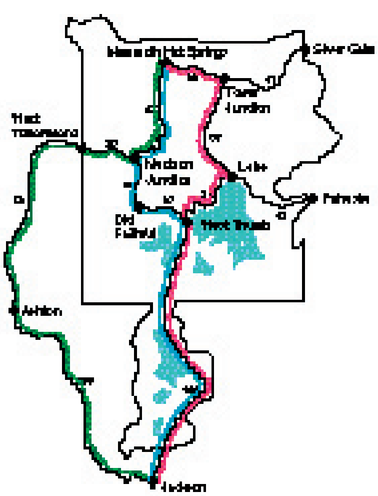
Start at Lake. Plan a trip that is less than 200 km long. Go to as many places as you can. Color your route red.
How long is this trip? __198__ km

Many solutions are possible; however, this solution goes to as many places as possible.

Start at Tower Junction. Plan a trip that is more than 400 km long.
Go to as few places as you can.
Color your route blue.
How long is this trip? __406__ km

Other solutions are possible. The only way to visit fewer places would be to go to one town more than once on the trip.

13



14

Plan three different trips from Mammoth Hot Springs to Jackson. Color each route a different color.

What color is the shortest route? __Blue__
How long is this trip? __232__ km

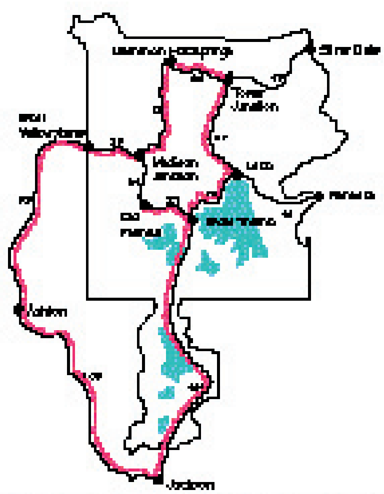
What color is the longest route? __Green__
How long is this trip? __270__ km

Your longest trip is __38__ km longer than your shortest trip.

Other solutions are possible.

15

W10



Plan a very long trip that starts at Old Faithful, goes to at least six other places, and then returns to Old Faithful.
Color your route red.
How long is this trip? 687 km

16

Other solutions are possible.

Capsule Lesson Summary

Each whole number is on one of three +3 arrow roads; locate 37. Do a similar activity to locate 37 on one of five +5 arrow roads. Given one +4 arrow road, draw three other +4 arrow roads so that every whole number belongs on one of them. Then locate 37. These activities involve whole number computation, patterns, numerical relations (order and multiples), and reasoning skills in making mathematical connections.

Materials

Teacher
- *Rollerskating 37* Story-Workbook
- *UPG-III Workbook Poster #1*
- Black marker or crayon

Student
- *Rollerskating 37* Story-Workbook
- Colored pencils, pens, or crayons

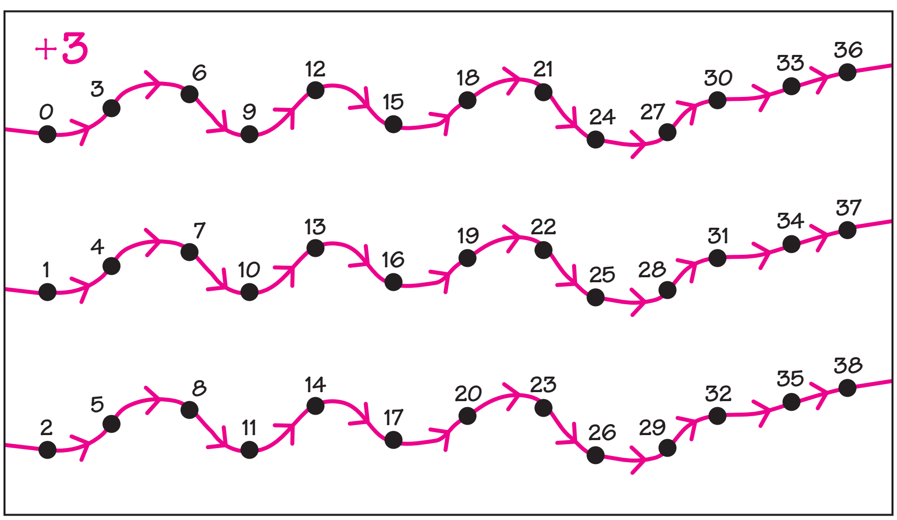
Description of Lesson

You may like to let students work with partners during this lesson. Each student should have a copy of the story-workbook *Rollerskating 37*. Briefly discuss the picture on the cover.

Pages 3–5

Read or ask students to read these pages aloud. While students are searching for 37 on pages 4 and 5, display Workbook Poster #1. When several students think they have found 37, ask one of them to label the dot for 37 on the poster and to explain why that dot is for 37. The last dot on the right on the second arrow road is for 37.

Ask students to label all the dots on pages 4 and 5. Students who finish quickly can help label the dots on the poster with a black marker or crayon so the numbers can be seen easily by all.



T: ***How many + 3 arrow roads are in this picture?*** (Three)
What patterns do you see?

There are several patterns students may observe in this arrow picture, but emphasize these two:

- Each of the whole numbers between 0 and 38 is represented by a dot on the poster.
- Each number in the second and third roads is one more than the number directly above it.

T: ***What do you notice about all the numbers in the first arrow road?***

S: ***All the numbers in the first arrow road are multiples of 3.***

T: ***Since these arrow roads go on and on, all of the multiples of 3 are in the first arrow road. What do you notice about the numbers in the second +3 arrow road?***

S: ***None of them is a multiple of 3.***

T: ***How much more are the numbers in the second arrow road than their closest multiple of 3? For example, look at 13. Which multiple of 3 is closest to 13?*** (12)
How much more is 13 than 12? (One more) ***So 13 is 1 more than a multiple of 3.***
How much more is 22 than the closest multiple of 3? (One more)

Check a few more of the numbers in the second arrow road and conclude that all the numbers in the second arrow road are one more than a multiple of 3.

T: ***Since these arrow roads go on and on, all the numbers that are 1 more than a multiple of 3 are in the second arrow road.***
What can you tell me about the numbers in the third arrow road?

S: ***They are all two more than a multiple of 3.***

T: ***Let's check 17. Is 17 two more than a multiple of 3?***

S: ***Yes, 15 is a multiple of 3 and 15 + 2 = 17.***

T: ***What is the closest multiple of 3 to 17?*** (18)
How much less is 17 than 18? (One less)

Check a few more numbers in the third arrow road and conclude that all the numbers in the third arrow road are two more (or one less) than a multiple of 3.

Pages 6 and 7

After reading page 6 aloud, let the students predict on which road 37 is skating. Direct students to label all the dots on pages 6 and 7 and to circle the dot for 37. The students who finish quickly should continue on pages 8 and 9. When many students have completed labeling the dots on pages 6 and 7, continue the collective lesson.

T: ***What patterns do you see on pages 6 and 7?***

Students may observe many patterns. Try to make mention of the following:

- Each of the whole numbers is on one of these +5 arrow roads.
- All the multiples of 5 are in the first +5 arrow road.
- All the numbers that are one more than a multiple of 5 are in the second arrow road.
- All the numbers that are two more than a multiple of 5 are in the third arrow road.

- All the numbers that are three more (or two less) than a multiple of 5 are in the fourth arrow road.
- All the numbers that are four more (or one less) than a multiple of 5 are in the fifth arrow road.

If no student mentions that the final digits of the numbers in each arrow road alternate between two numbers (0, 5; 1, 6; 2, 7; 3, 8; and 4, 9), do so yourself.

Pages 8 and 9

Read, or call on students to read, pages 8 and 9. Some students will have determined already that 37 is not on the same arrow road as 28. Ask the students to label the dots in this arrow road and to draw the three missing +4 arrow roads. Allow a few minutes for this activity and then collect the story-workbooks for use in W12.

Home Activity

Suggest that parents/guardians choose a number, such as 37, and with their child write many different number facts for the number. For example:

$$30 + 7 = 37$$
$$50 - 13 = 37$$
$$(4 \times 9) + 1 = 37$$

Capsule Lesson Summary

Explore many different representations for the number 37 on the Minicomputer and in number sentences. Remove and add checkers to a given configuration on the Minicomputer so that the number will be 37.

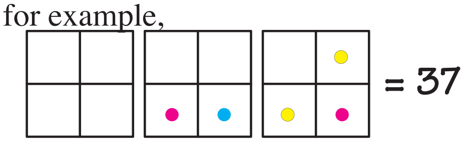
Materials

Teacher	• Minicomputer set	Student	• *Rollerskating 37* Story-Workbook

Advance Preparation: This lesson calls for more checkers than are in one classroom set. You might want to borrow a set from another teacher or magnetize some student checkers.

Description of Lesson

Display three Minicomputer boards and ask a student to put on 37. Give the student two or three colors of checkers to use. Accept any correct configuration; for example,

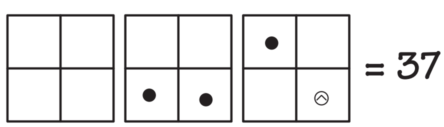


T: ***What number is on the Minicomputer with red checkers?*** (21)
…with blue checkers? (10)
…with yellow checkers? (6)

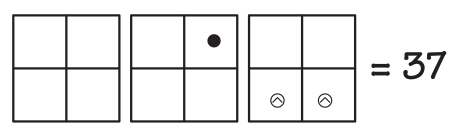
$$21 + 10 + 6 = 37$$

Write a corresponding number sentence on the board.

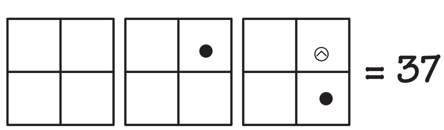
Call on volunteers to show 37 in other ways and continue recording number sentences about the configurations. Ask for a configuration using negative checkers if none has been displayed. A few of the possible configurations for 37 using negative checkers are shown below.



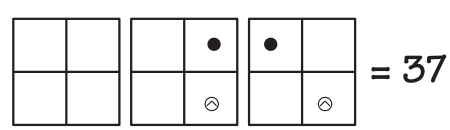
$38 + \hat{1} = 37$



$40 + \hat{3} = 37$



$41 + \hat{4} = 37$

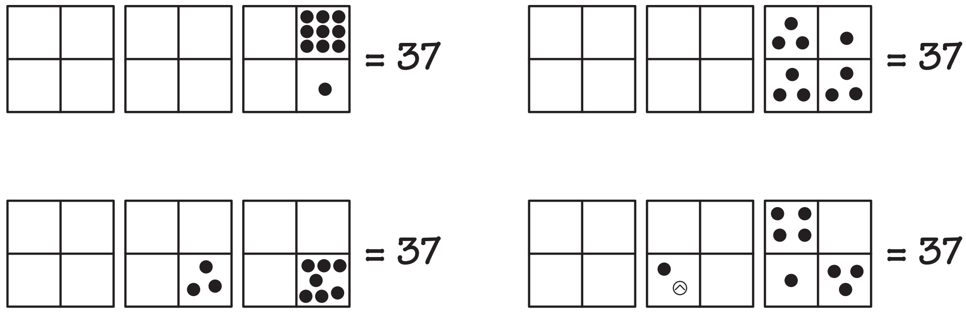


$48 + \widehat{11} = 37$

T: ***Can we put 37 on the Minicomputer using exactly ten checkers? Is this possible?*** (Yes)

W12

Invite students to put 37 on the Minicomputer using exactly ten checkers. A few of the possible configurations are shown here.



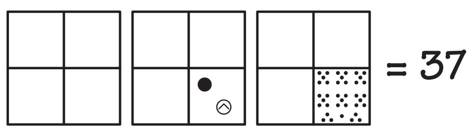
T: ***What is the greatest number of checkers we could use to put 37 on the Minicomputer?***

A student very likely will respond with the greatest number of regular checkers.

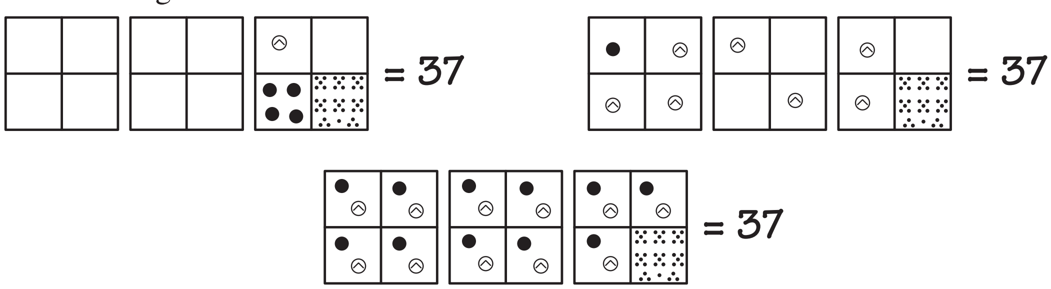
S: ***Thirty-seven, we could put thirty-seven checkers on the 1-square.***

T: ***Could we put 37 on the Minicomputer using more than thirty-seven checkers?*** (Yes)

Invite a student to put 37 on the Minicomputer using more than thirty-seven checkers. (See the next illustration.)



Ask for other volunteers to put 37 on the Minicomputer using even more checkers. Several of the many possible configurations are shown here.



T: ***Is there a greatest number of checkers we could use to put 37 on the Minicomputer?***

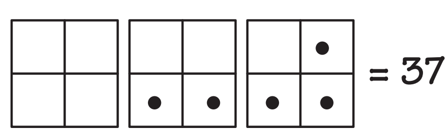
S: ***There is no greatest number; we can always put more checkers on the Minicomputer.***

Conclude with your class that if negative checkers can be used, then there is no greatest number of checkers that can be used to put 37 on the Minicomputer.

Remove the checkers from the Minicomputer.

T: ***What is the least number of checkers we could use to put 37 on the Minicomputer?***

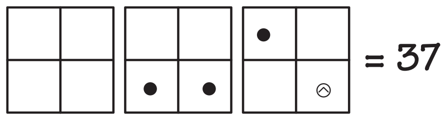
S: ***I can put 37 on the Minicomputer using five checkers.***

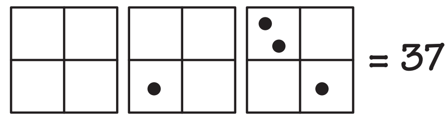


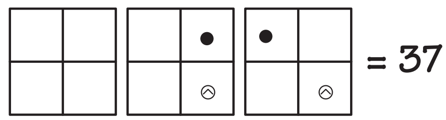
T: ***Can we put 37 on the Minicomputer using less than five checkers?***

S: ***I can put 37 on the Minicomputer using four checkers.***

Ask the student to put 37 on the Minicomputer using four checkers. A few of the possible configurations are shown below.



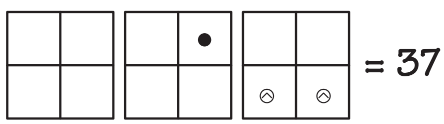


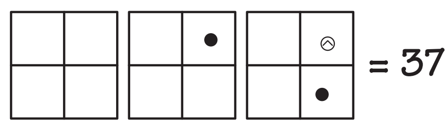


T: ***Can we put 37 on the Minicomputer using less than four checkers?***

S: ***I can put 37 on the Minicomputer using three checkers.***

Ask the student to put 37 on the Minicomputer using three checkers. There are two possible configurations.





T: ***Can we put 37 on the Minicomputer using less than three checkers?*** (No)
Why not?

S: ***We need one checker to put 40 on the Minicomputer and at least two checkers to put $\widehat{3}$ on the Minicomputer.***

Conclude that you need at least three checkers to put 37 on the Minicomputer.

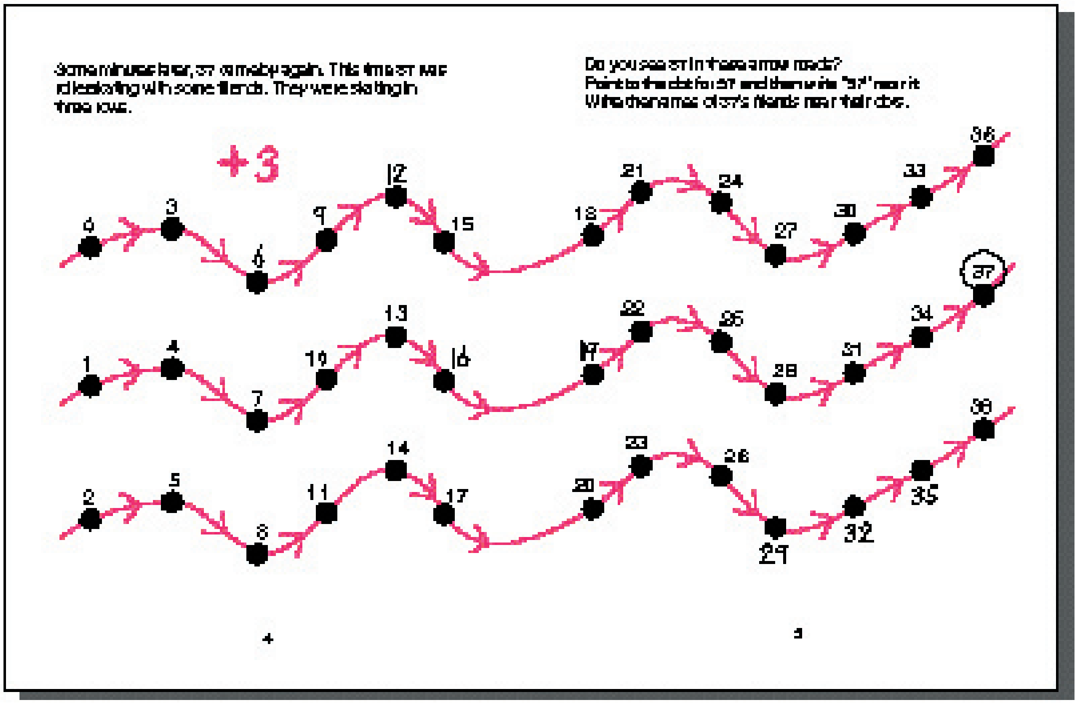
Erase the board and remove the checkers from the Minicomputer. Distribute the students' copies of the story-workbook *Rollerskating 37* and turn to page 10. Briefly read through pages 10 to 16 with the class to discuss the various problems to be solved; then ask students to complete the rest of the book independently or with a partner, beginning on page 10.

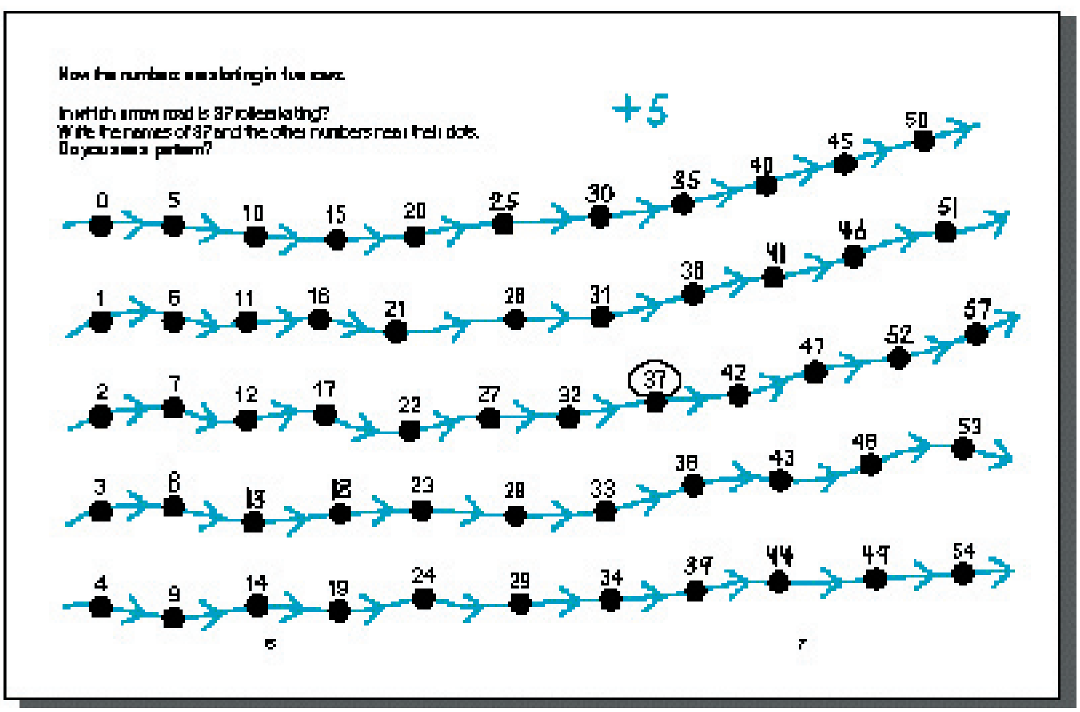
Assessment Activity

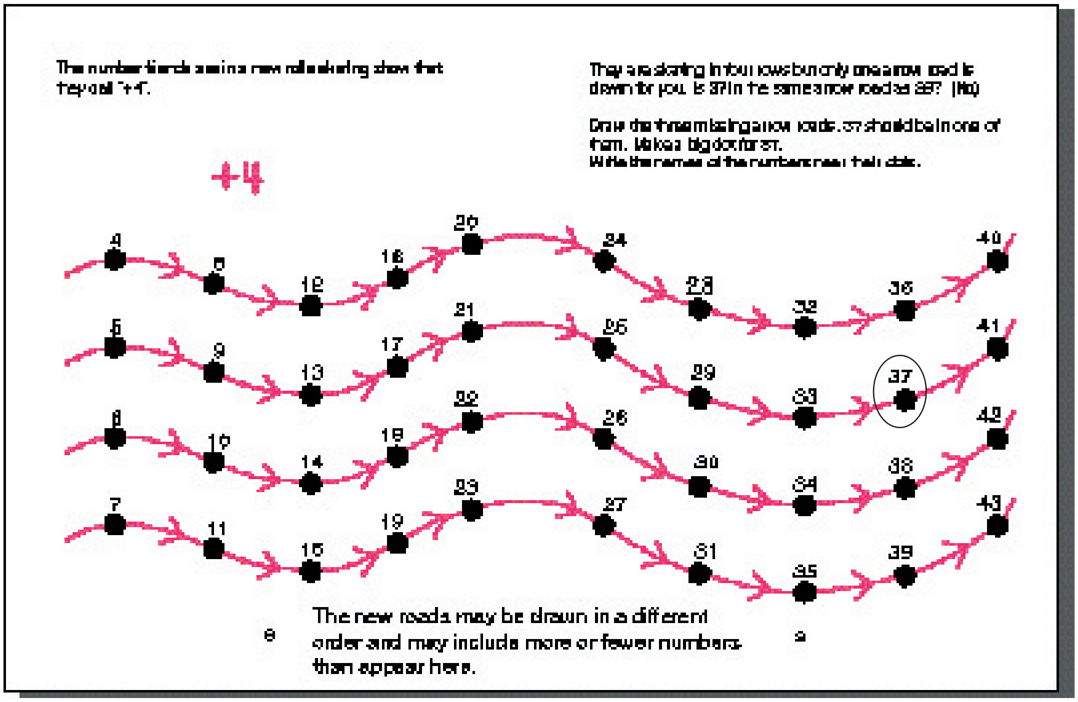
Write a specific two-digit number on a card and give it to an individual or a group of students. Challenge the students to find many different representations for the number. Allow students to use the Minicomputer, manipulatives, or paper and pencil.

Home Activity

Send home the story-workbook *Rollerskating 37* for students to share with family members.


+3


+5



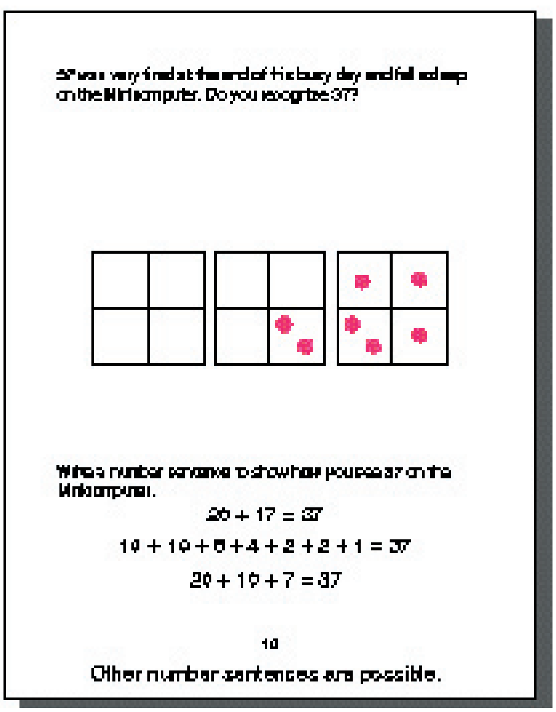
The number friends are in a new roller skating show that they call "+4".

They are skating in four rows but only one arrow road is drawn for you. Is 37 in the same arrow road as 33? (No)

Draw the three missing arrow roads. 37 should be in one of them. Make a big dot for 37.
Write the names of the numbers near their dots.

The new roads may be drawn in a different order and may include more or fewer numbers than appear here.

8 9



37 was very tired at the end of this busy day and fell asleep on the Minicomputer. Do you recognize 37?

Write a number sentence to show how you see 37 on the Minicomputer.

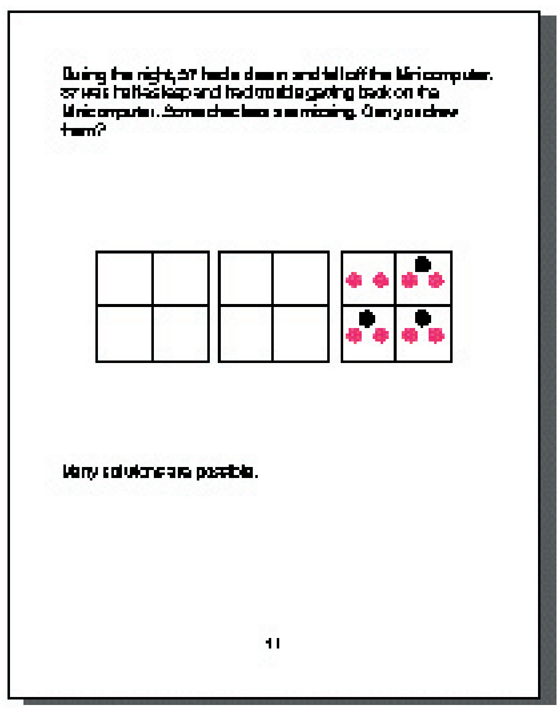
$20 + 17 = 37$

$10 + 10 + 8 + 4 + 2 + 2 + 1 = 37$

$20 + 10 + 7 = 37$

10

Other number sentences are possible.



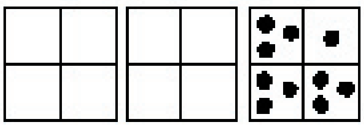
During the night, 37 had a dream and fell off the Minicomputer. 37 was half-asleep and had trouble getting back on the Minicomputer. Some checkers are missing. Can you draw them?

Many solutions are possible.

11

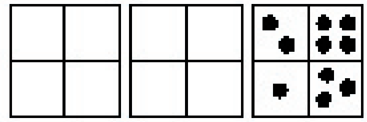
Later, 37 dreamed about sleeping on the Minicomputer with ten checkers. Do you think this is possible?

Can you show 37 with ten checkers?

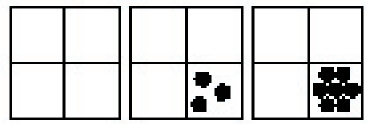


Many solutions are possible.

Can you show 37 with ten checkers in another way?

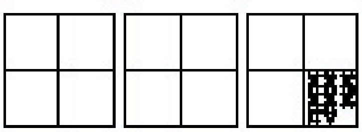


...in still another way?



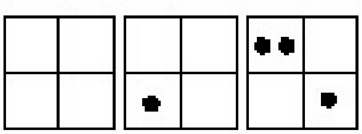
Many solutions are possible.

Put 37 on this Minicomputer using as many checkers as possible.



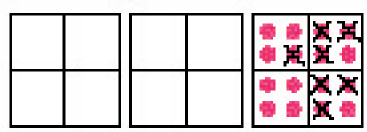
More checkers could be used if negative checkers are included.

Put 37 on this Minicomputer using as few checkers as possible.



Be careful! You can show 37 with less than four checkers. Another solution with four checkers is possible when negative checkers are included (30 + 8 + ^1). There is also a solution with three checkers when negative checkers are included (40 + ^3).

The number 37 had a nightmare. 37 dreamed of being on the Minicomputer in this way.



Other solutions are possible.

"This is not me!" cried 37. "There are too many checkers. Help me take some of them away."

Cross off the extra checkers and show 37 sleeping peacefully again. Write a number sentence to show that you have left 37 on the Minicomputer.

8 + 2 + 8 + 2 + 8 + 2 + 4 + 2 + 1 = 37

10 + 10 + 10 + 7 = 37

24 + 4 + 8 + 1 = 37

Other solutions are possible.

This is the end of the story of the number 37, who is proud to sign this book in many different ways.

40 − 3	(2 × 10) + 17
(5 × 7) + 2	50 − 13
26 + 11	2+5+6+7+8+9

Invent some other signatures for 37 and write them below.

30 + 7

(3 x 10) + 7

40 + 3

(4 x 10) − 3

18 + 19

Many solutions are possible.

16

Capsule Lesson Summary

Find the area of shapes on a grid board if each little square has area 4 cm^2. Draw shapes of a given area. Review and extend experiences from various strands in the workbook *Festival of Problems #2*.

Materials

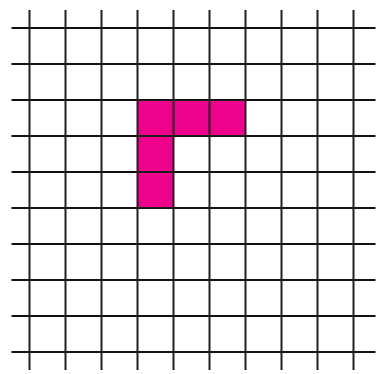
Teacher
- 0–109 numeral chart
- Grid board†
- Colored chalk

Student
- *Festival of Problems #2* Workbook
- Colored pencils, pens, or crayons

Advance Preparation: You may like to prepare colored paper squares to cover the squares on your grid, or use color tiles on a grid transparency.

Description of Lesson

Call on several students to count by fours, starting at 0. As students count, point to or highlight the numbers on a 0–109 numeral chart and observe patterns.



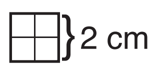
Display a grid board and color or cover this shape.

T: ***The red shape has how many little squares in it?***

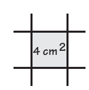
S: ***Five.***

T: ***Suppose that the area of each little square is 4 cm^2*** (read as "four square centimeters")***. Does anyone know how long each side of a little square is?*** (2 cm)

Note: A square of area 4 cm^2 measures 2 cm by 2 cm.



Indicate the area of a little square on the grid board.



T: ***What is the area of the red shape?***

S: ***20.***

T: ***20 what?***

S: ***20 cm^2*** (read as "twenty square centimeters")***.***

T: ***How do you know?***

S: ***I added 4 + 4 + 4 + 4 + 4 = 20.***

S: ***5 x 4 = 20.***

S: ***I counted by fours: 4, 8, 12, 16, 20.***

Count by fours as you point to the squares of the red shape.

---

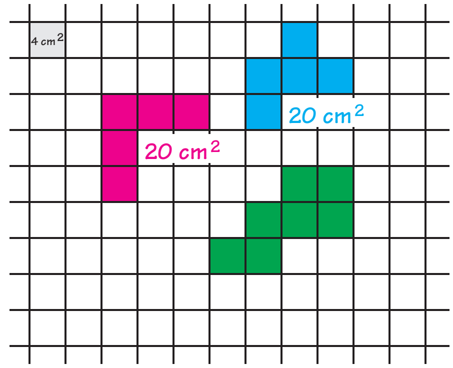
†See the "Notes on Grids" section in the introduction to the Geometry strand.

W13

T: ***Who can color (cover) another shape whose area is 20 cm^2?***

Note: It is possible that a student will color a shape whose area is difficult to determine. In such a case, reply that you are not sure exactly what the area of the shape is.

Color (cover) this green shape on the grid and ask for its area.



S: ***The area of the green shape is 28 cm^2.***

T: ***How do you know?***

S: ***It has seven little squares; 7 x 4 = 28.***

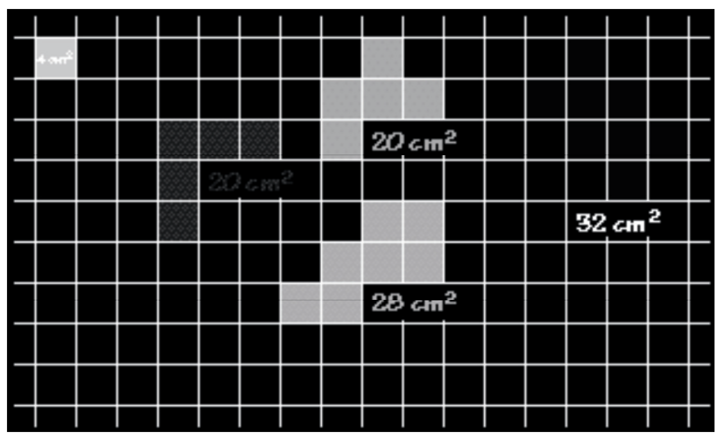
S: ***I added 4 + 4 + 4 + 4 + 4 + 4 + 4 = 28.***

S: ***I counted by fours: 4, 8, 12, 16, 20, 24, 28.***

Ask the class to count by fours as you point to the little squares in the green shape. Record 28 cm^2 on the grid.

T: ***Who can color (cover) a shape with area 32 cm^2?***

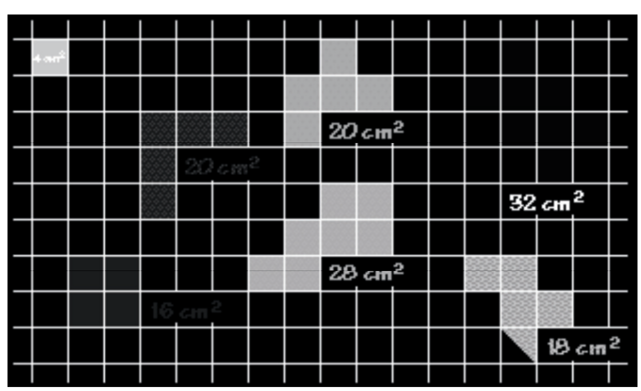
After a student has colored a shape, check the area of the shape with the class by counting by fours. If the shape does not have area 32 cm^2, erase it and ask for another volunteer. Any shape that has eight little squares is correct. One possible shape is shown in yellow here.



Repeat this activity, asking for a shape with area 16 cm^2, and then a shape with area 18 cm^2.

To draw a shape with area 18 cm^2, at least one little square must be cut in half. A student might halve a little square in one of these ways.

Your picture might look similar to this one.



If your students have difficulty finding a shape with area 18 cm^2, guide them to a solution as in the following dialogue.

T: ***Here we have a shape with area 16 cm^2*** (point to it) ***and a shape with area 20 cm^2*** (point to it). ***Now we want a shape with area 18 cm^2. What could we do?***

S: ***Cut a square in half.***

T: ***How many little squares would a shape with area 18 cm^2 have?***

S: ***Four and one half.***

T: ***How do you know?***

S: ***Because four 4's is 16 and $^1/_2$ x 4*** (read as "one-half of four") ***is 2 more; 16 + 2 = 18.***

Invite a student to color a shape with area 18 cm^2 on the board.

Distribute copies of the workbook *Festival of Problems #2,* and let students work independently for the rest of the class period. If many students are having difficulty with a particular page, you may wish to have a collective discussion about that page.

At the end of the lesson, collect the workbooks for your review. They will be used again in Lesson W14.

Capsule Lesson Summary

Put numbers on the Minicomputer. Continue working in the *Festival of Problems #2* Workbook. (This is the second of two lessons using this workbook.)

Materials

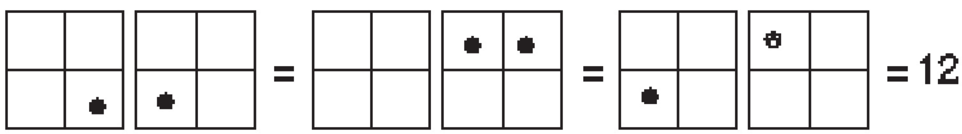
Teacher		Student	
	• Minicomputer set		• *Festival of Problems #2* Workbook • Colored pencils, pens, or crayons

Description of Lesson

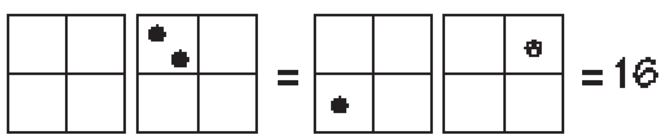
Display two Minicomputer boards.

T: ***Who can put 12 on this Minicomputer using exactly two checkers?***
Is there another way to put 12 on this Minicomputer using exactly two checkers?

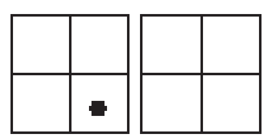
Continue until your students have found three ways to put 12 on the Minicomputer using exactly two checkers. If necessary, suggest one of these configurations yourself.



Repeat this activity with 16. There are two ways to put 16 on the Minicomputer using exactly two checkers.



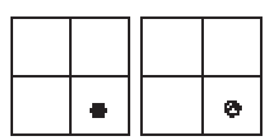
Put a regular (positive) checker on the 10-square.



T: ***What number is this?***

S: ***10.***

Add a negative checker on the 1-square.

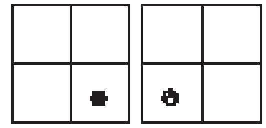


T: ***What number is this?***

S: ***9.***

T (pointing to the appropriate checkers)**:** $10 + \hat{1} = 10 - 1 = 9$.

W14



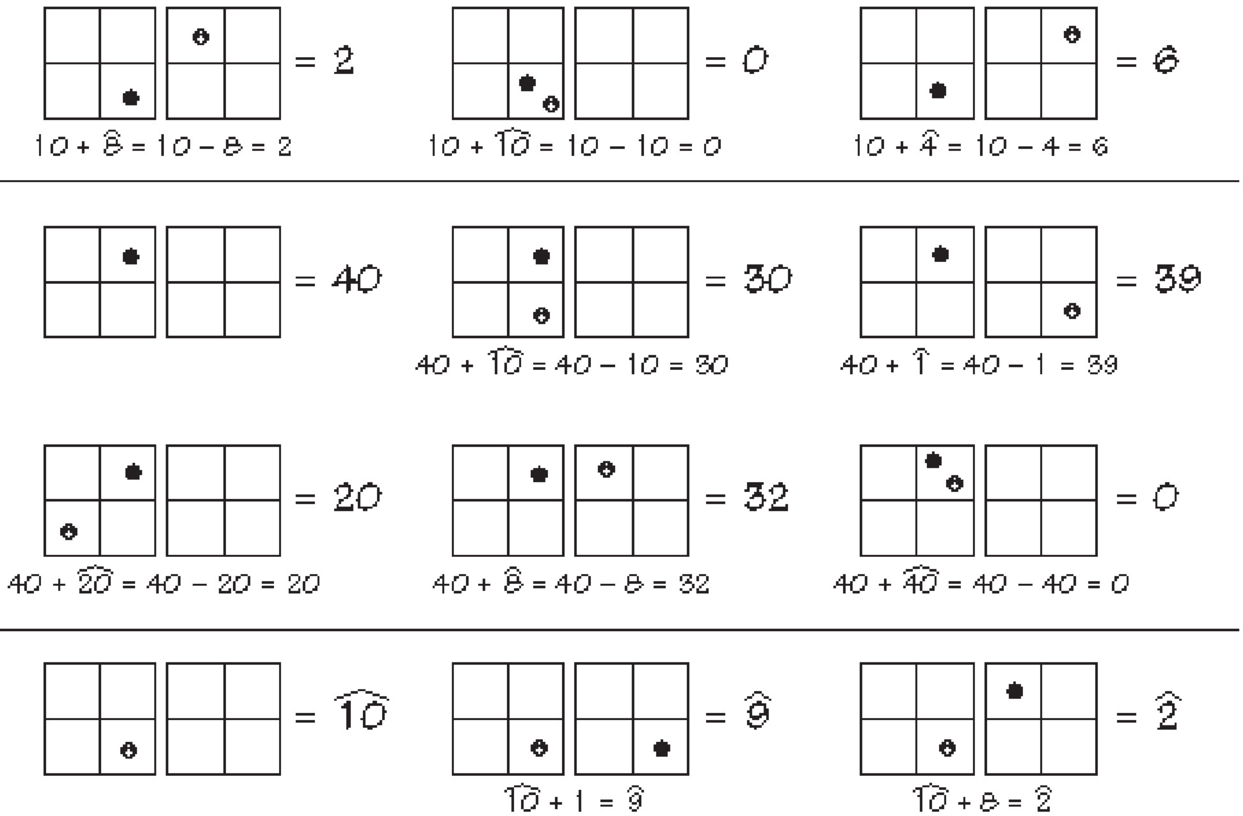
Move the negative checker to the 2-square.

T: ***What number is this?***

S: ***8.***

T (pointing to the appropriate checkers)**:** ***$10 + \hat{2} = 10 - 2 = 8$.***

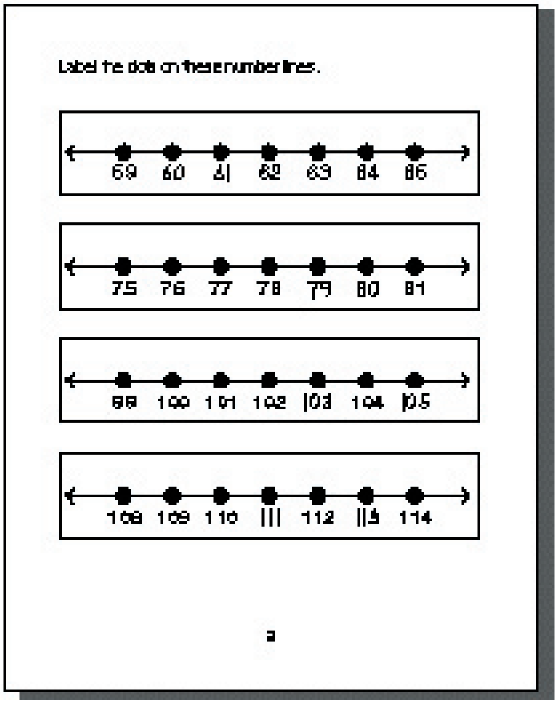
Continue this activity with these configurations.

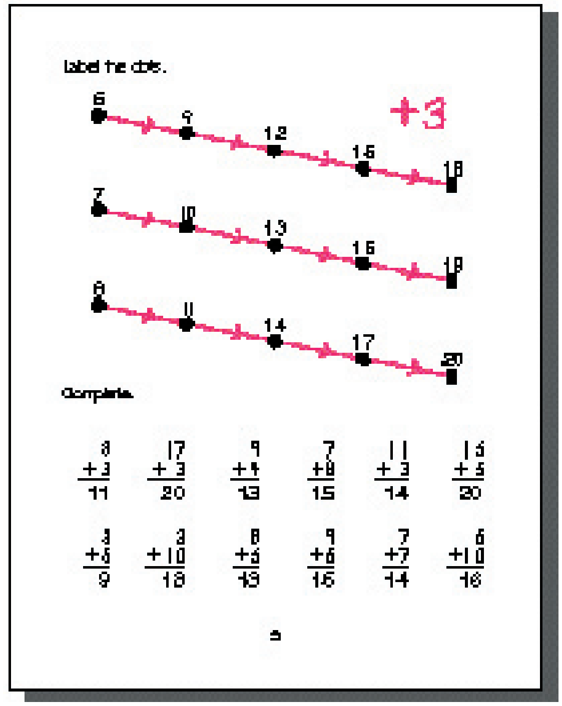


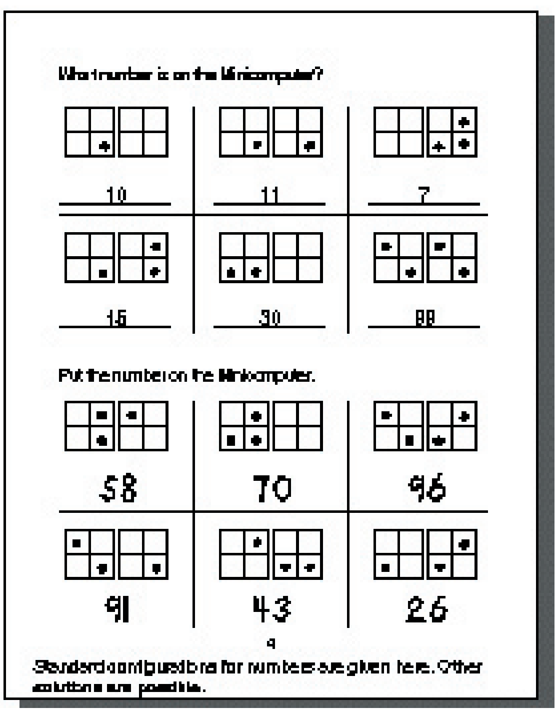
Distribute students' copies of the workbook *Festival of Problems #2*. Ask students first to correct or complete pages from the previous week's work on this workbook, and then to continue. You may wish to discuss collectively some problems that were difficult for many students the first week. At the end of the class period, collect the workbooks for your review.

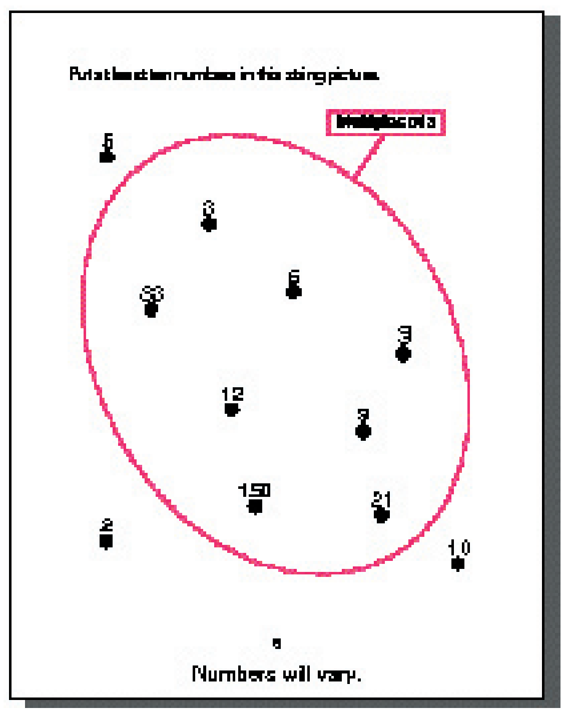
Assessment Activity

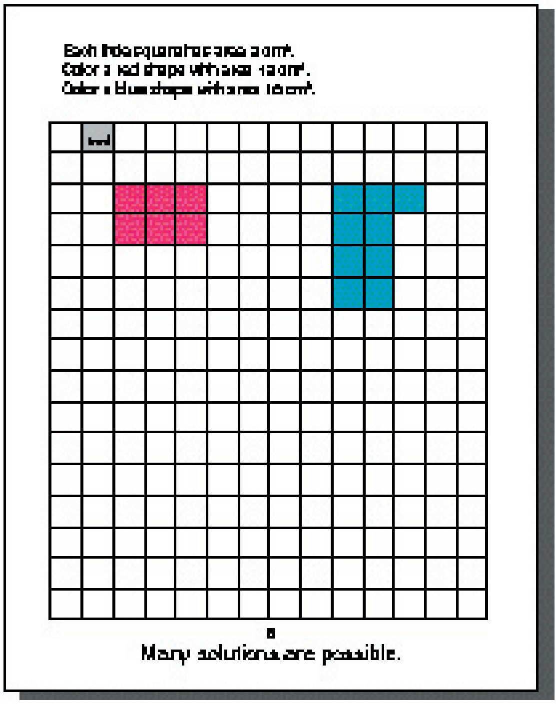
An individual student progress record for the workbook is available on Blackline W14. You may like to use this form to monitor student work.

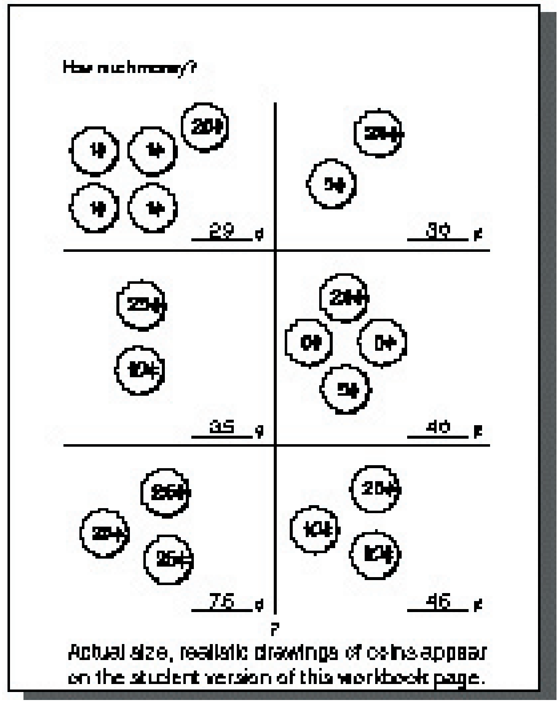

Label the dots on these number lines.
59 60 61 62 63 64 65
75 76 77 78 79 80 81
99 100 101 102 103 104 105
108 109 110 111 112 113 114
3

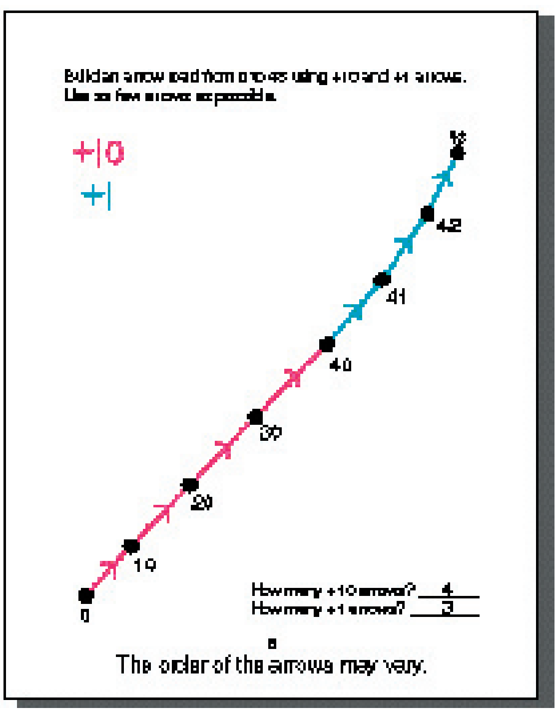

Label the dots.
+3
6 9 12 15 18
7 10 13 16 19
8 11 14 17 20
Complete.
8 + 3 = 11
17 + 3 = 20
9 + 4 = 13
7 + 8 = 15
11 + 3 = 14
15 + 5 = 20
3 + 6 = 9
3 + 10 = 13
8 + 5 = 13
9 + 6 = 15
7 + 7 = 14
6 + 10 = 16
5

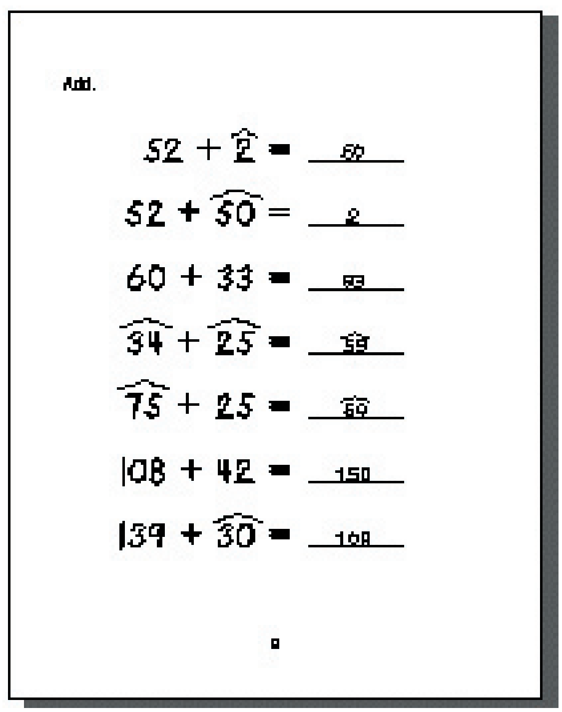

What number is on the Minicomputer?
10
11
7
15
30
88
Put the number on the Minicomputer.
58
70
96
91
43
26
4
Standard configurations for numbers are given here. Other solutions are possible.


Put at least ten numbers in this string picture.
5
3
33
6
9
12
2
150
21
2
10
6
Numbers will vary.


Many solutions are possible.


How much money?
29 ¢
30 ¢
35 ¢
40 ¢
75 ¢
45 ¢
Actual size, realistic drawings of coins appear on the student version of this workbook page.


+10
+1
How many +10 arrows? 4
How many +1 arrows? 3
The order of the arrows may vary.


Add.
60 + 33 =
108 + 42 = 150

Write number sentences for 37. Two examples are given.

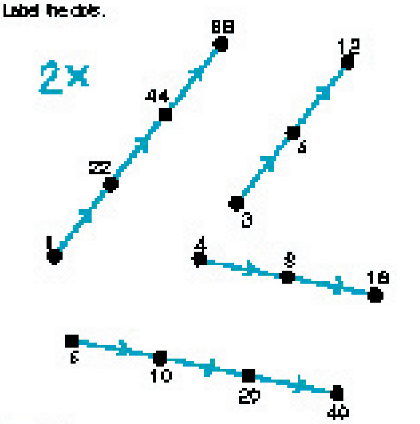
18 + 19 = 37	40 − 3 = 37
30 + 7 = 37	50 − 13 = 37
(3 x 10) + 7 = 37	74 ÷ 2 = 37
(5 x 6) + 7 = 37	20 + 17 = 37

37

Many solutions are possible.

10

Label the dots.

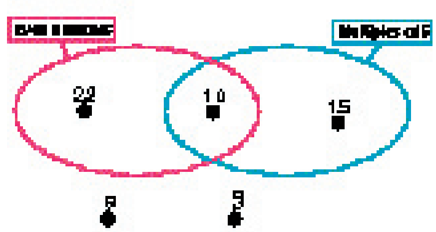


Complete.

2 × 3 = 6	2 × 7 = 14	2 × 50 = 100
2 × 13 = 26	2 × 15 = 30	2 × 51 = 102
2 × 33 = 66	2 × 12 = 24	2 × 54 = 108

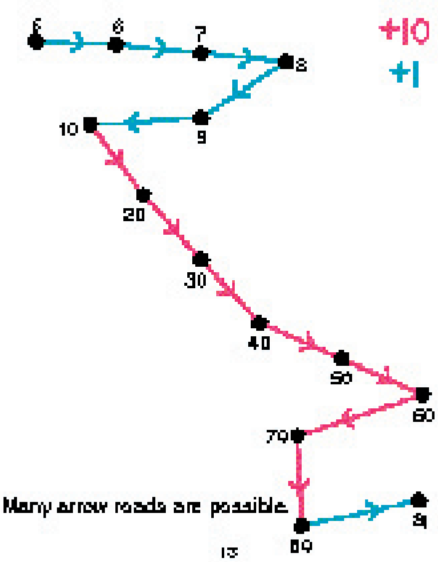
11

Label the dots in this string picture.
Many solutions are possible.



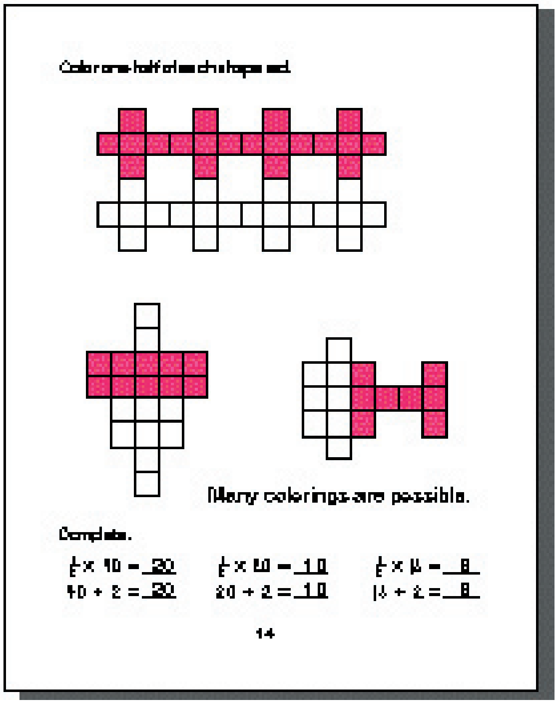
12

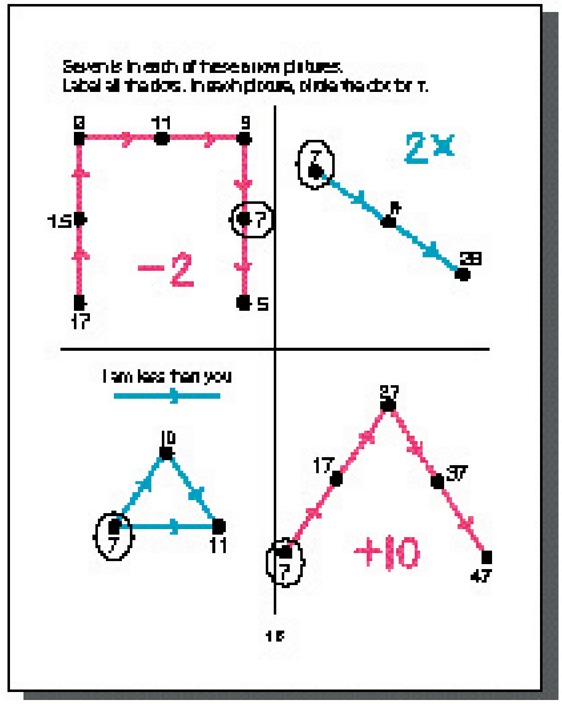
Build an arrow road from 5 to 81 using +10 and +1 arrows.

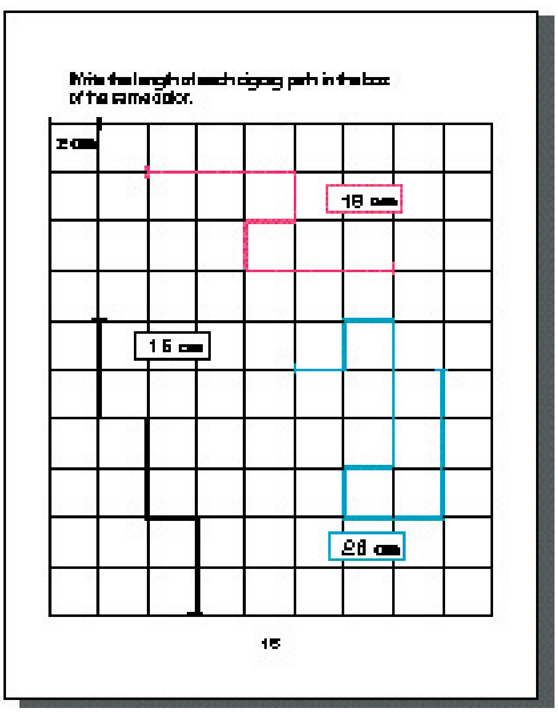


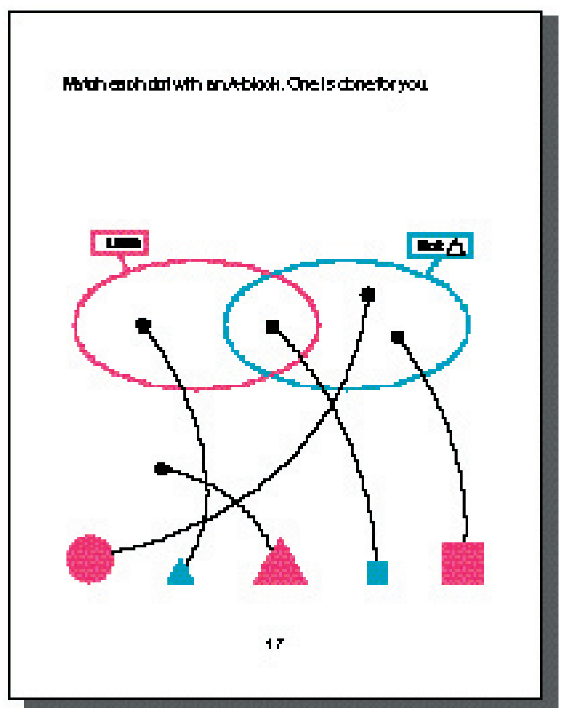
Many arrow roads are possible.

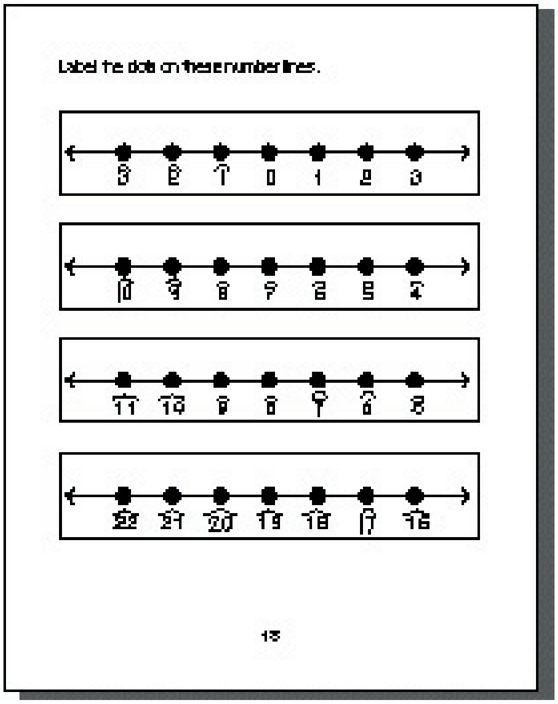
13

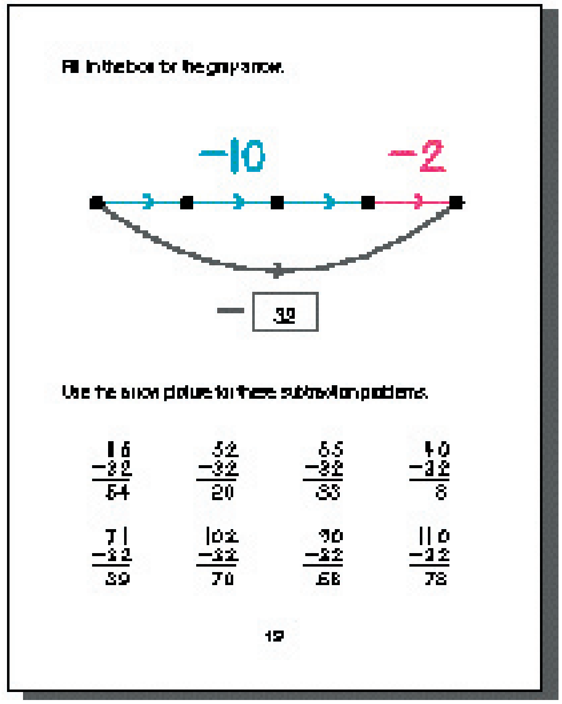

Many colorings are possible.
14

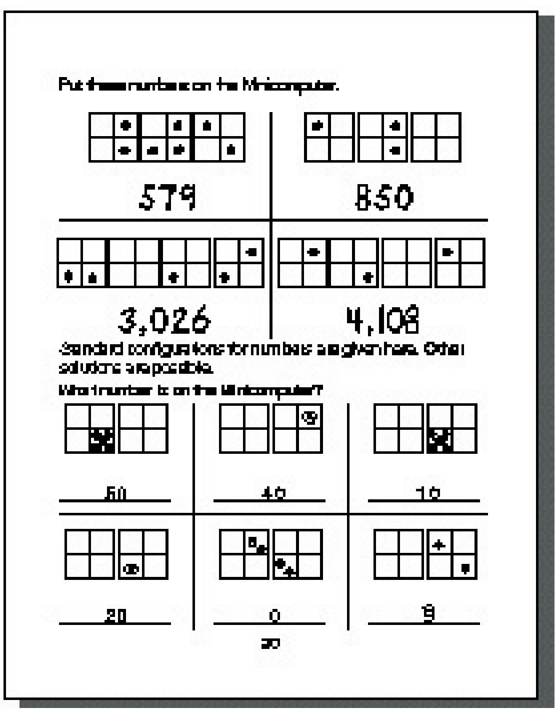

Seven is in each of these arrow pictures.
−2
2x
+10
I am less than you

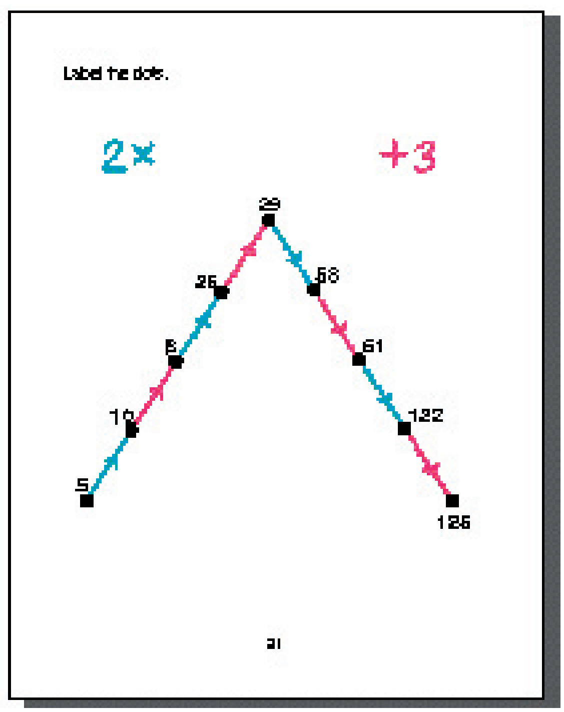

18 cm
16 cm
26 cm

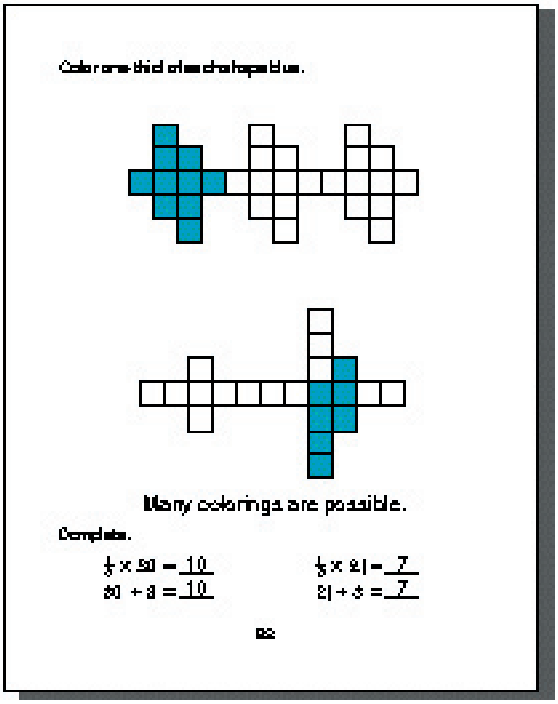

17


Label the dots on these number lines.
3 2 1 0 1 2 3
10 9 8 7 6 5 4
11 10 9 8 7 6 5
22 21 20 19 18 17 16
18


Fill in the box for the gray arrow.
−10
−2
− 32
Use the arrow picture for these subtraction problems.
86 − 32 = 54
52 − 32 = 20
65 − 32 = 33
40 − 32 = 8
71 − 32 = 39
102 − 32 = 70
90 − 32 = 58
110 − 32 = 78
19


Put these numbers on the Minicomputer.
579
850
3,026
4,108
Standard configurations for numbers are given here. Other solutions are possible.
What number is on the Minicomputer?
50
40
10
20
0
9
20


Label the dots.
2×
+3
5
10
8
26
29
58
61
122
125
21

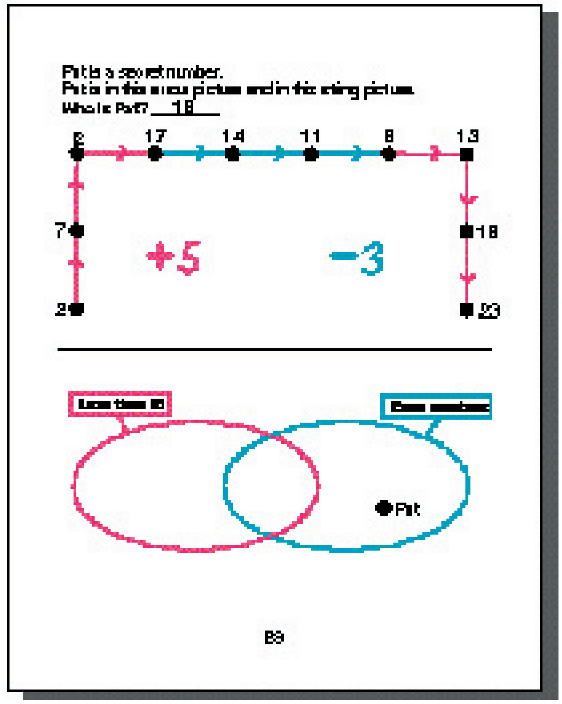

Color one-third of each shape blue.

Complete.

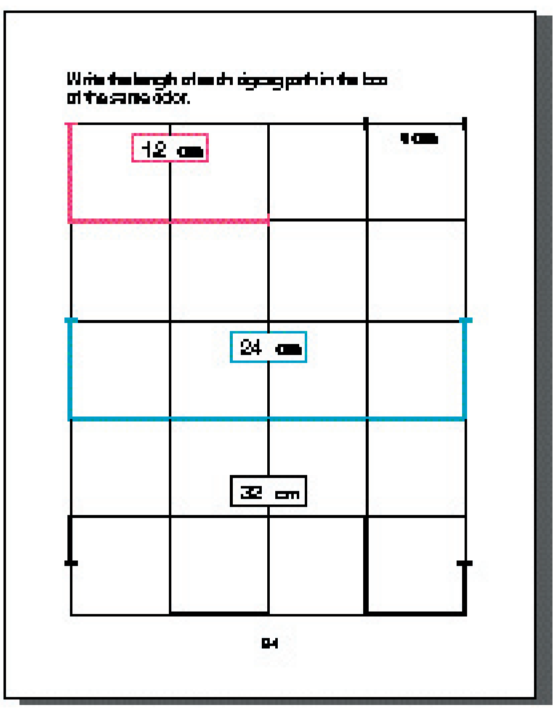
$\frac{1}{3} \times 30 =$ 10 $\qquad \frac{1}{3} \times 21 =$ 7

$30 \div 3 =$ 10 $\qquad 21 \div 3 =$ 7

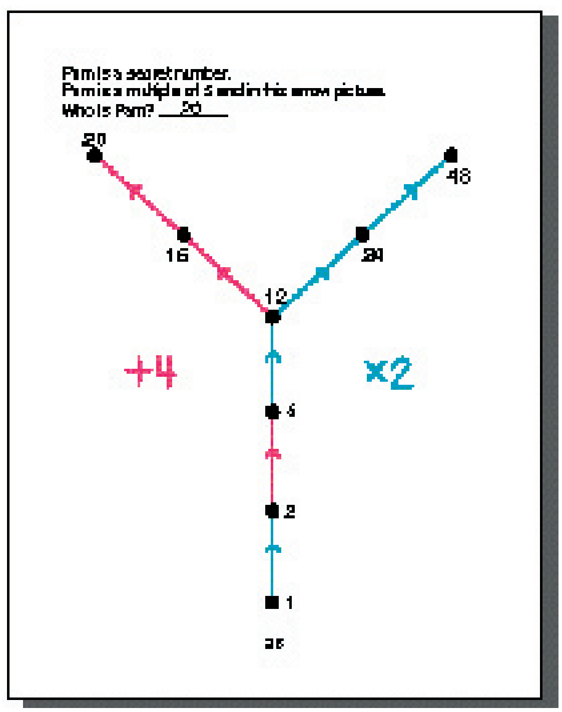
82


Pat is a secret number.
Pat is in this arrow picture and in this string picture.
Who is Pat? 18

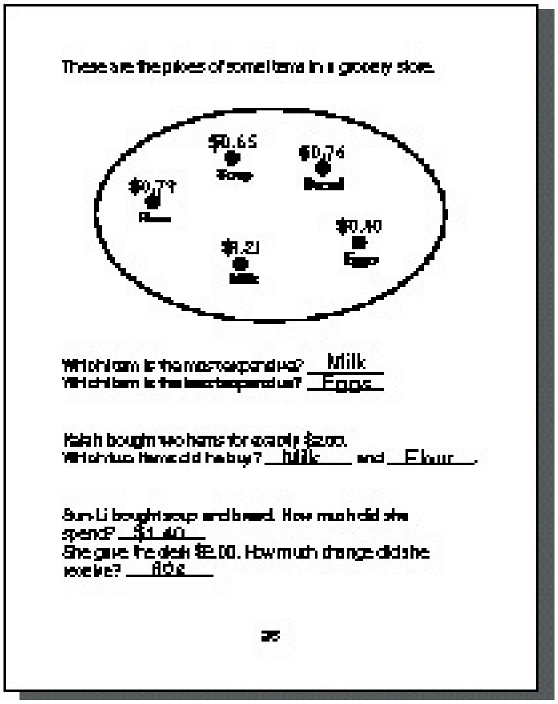
83


Write the length of each zigzag path in the box of the same color.

84


Pam is a secret number.
Pam is a multiple of 5 and in this arrow picture.
Who is Pam? 20

85

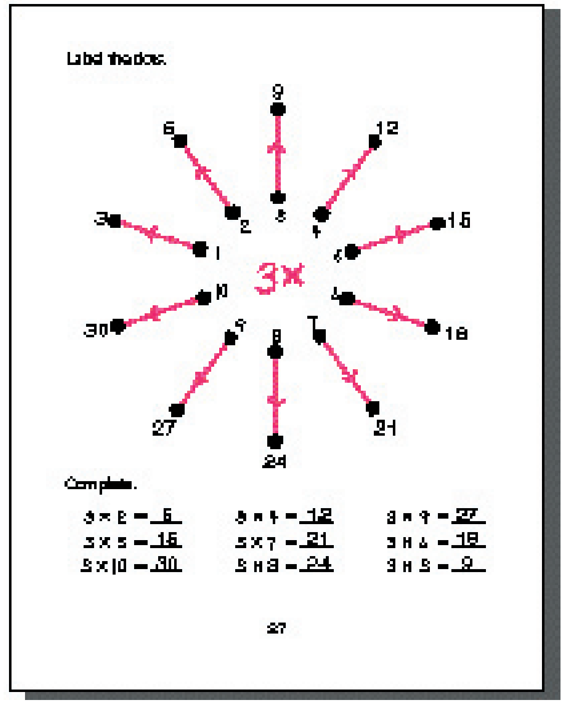


These are the prices of some items in a grocery store.

Which item is the most expensive? Milk
Which item is the least expensive? Eggs

Kaleh bought two items for exactly $2.00.
Which two items did he buy? Milk and Flour.

Sun-Li bought soup and bread. How much did she spend? $1.40
She gave the clerk $2.00. How much change did she receive? 60¢

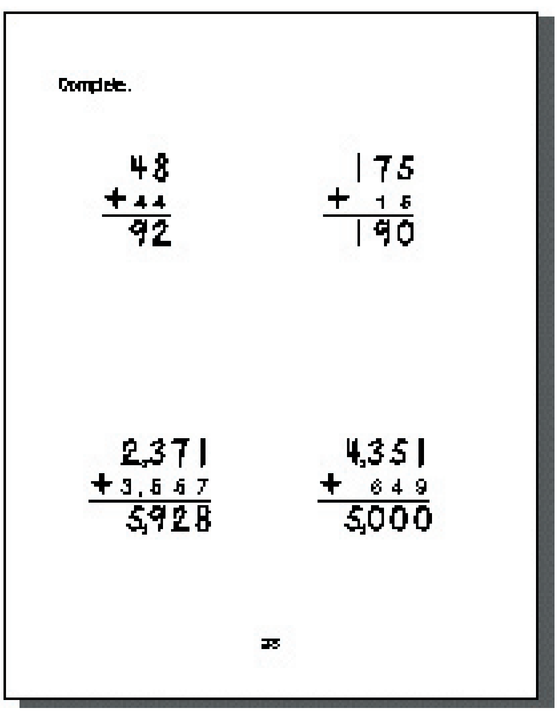


Label the dots.

Complete.

3 × 2 = 6	3 × 4 = 12	3 × 9 = 27
3 × 5 = 15	3 × 7 = 21	3 × 6 = 18
3 × 10 = 30	3 × 8 = 24	3 × 3 = 9

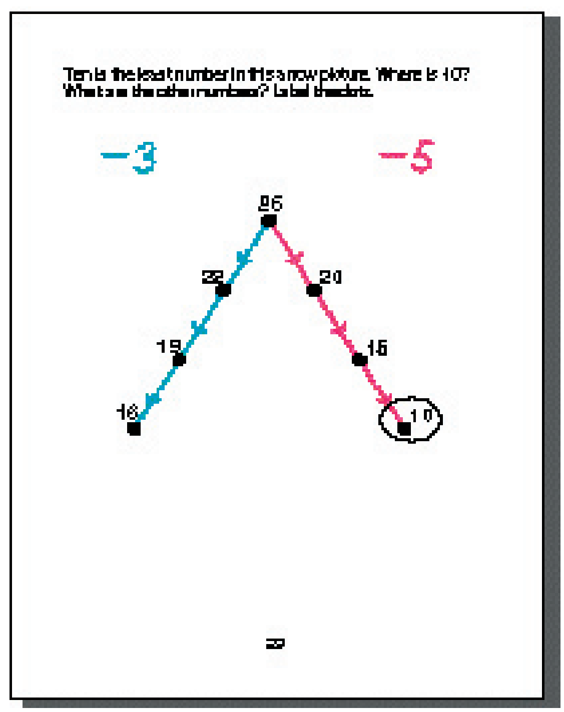
27



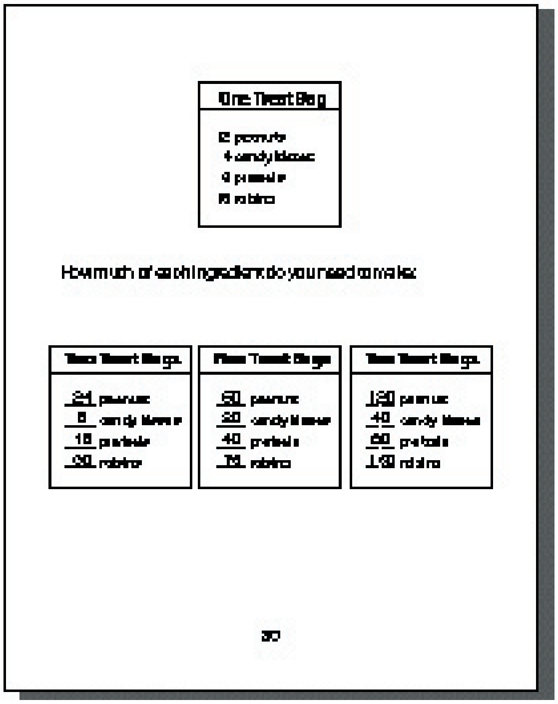
Complete.

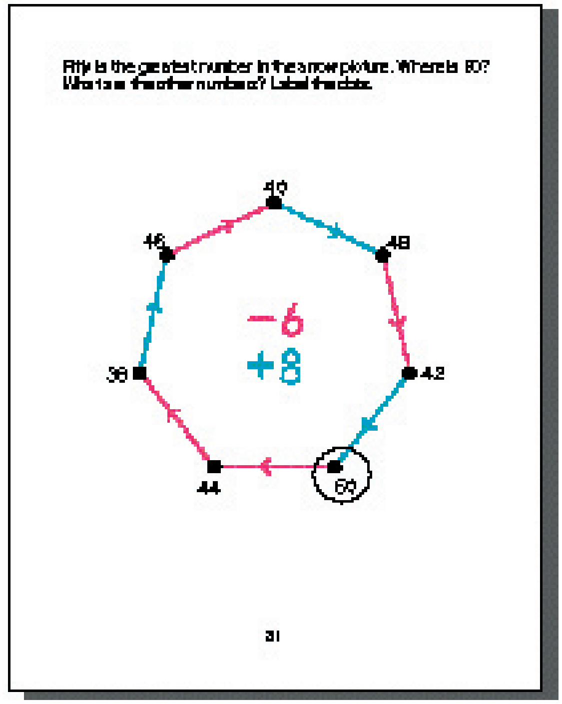
$$\begin{array}{r} 48 \\ +\ 44 \\ \hline 92 \end{array} \qquad \begin{array}{r} 175 \\ +\ 15 \\ \hline 190 \end{array}$$

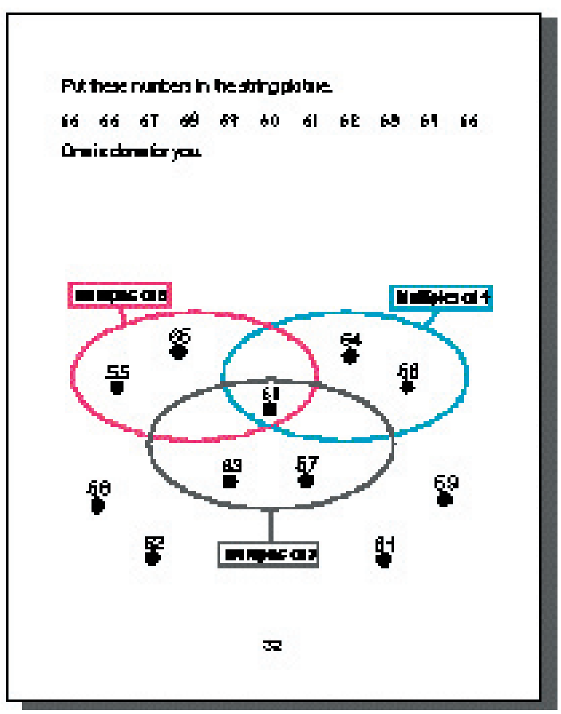
$$\begin{array}{r} 2{,}371 \\ +\ 3{,}557 \\ \hline 5{,}928 \end{array} \qquad \begin{array}{r} 4{,}351 \\ +\ 649 \\ \hline 5{,}000 \end{array}$$



Ten is the least number in this arrow picture. Where is 10? What are the other numbers? Label the dots.


One Treat Bag
How much of each ingredient do you need to make:
24
30


46
48
38
42
44
−6
+8
31


Put these numbers in the string picture.
One is done for you.
Multiples of 4
55
65
64
67
59
32

Capsule Lesson Summary

Use the calculator to do some mental arithmetic. Solve a detective story in which the clues involve the calculator, an arrow picture, and the Minicomputer.

Materials

Teacher		Student	
	• Overhead calculator (optional) • Colored chalk • Minicomputer set		• Calculator • Worksheet W15

Advance Preparation: Before the lesson begins, write a large 18 on a piece of paper and fold the paper so that 18 is hidden. You may want to prepare the arrow picture from Worksheet W15 on the board, on chart paper, or on an overhead transparency.

Description of Lesson

Exercise 1: Mental Arithmetic

Pair students and distribute calculators, one to each pair. Explain to the class that in this activity one person in each pair will use the calculator, and the other will do calculations in his or her head. The people without calculators may not look at the calculator for a few minutes.

T: ***We are going to have a little contest to see which is faster, the calculator or the "brain." Half of you have calculators, and you must do my problems on the calculator. Half of you do not have calculators, and you must do the calculations in your head. You cannot use the calculator.***

You may need to discuss the rules for a moment. If you like, suggest that students in pairs take turns using the calculators.

T: ***I will give you a calculation. As soon as you know the result, raise your hand. When I say stop, leave your hand up if it is up or down if it is down. Let's try it.***

Give the class a simple calculation such as 10 + 2; say it clearly but not too slowly. Watch hands go up, and when several hands are up say stop. You should purposely try to say stop before those with calculators have time to press all the appropriate keys.

T: ***Look around. Do more people using calculators or more using their brains have their hands up?***

S: ***Brains; but it wasn't fair. You didn't give us enough time to do the problem on the calculator. I knew the answer, but I had to press the keys on the calculator and that was slower.***

Repeat this contest several times with simple calculations and with more difficult calculations such as 17 x 56. Make some observations with the class about how some calculations are easier and faster to do in our heads, while others can be done more quickly on the calculator.

Conclude the contest, and tell the student pairs that now they will use both the calculator and their heads to do some calculations. You may want to use an overhead calculator during this activity.

T: ***Press [C] and make sure 0 is on the display. Cover the display with a hand (finger), but be careful not to cover the light panel (energy source). Now I will tell you keys to press. Try to do the calculations in your head, just the way you think the calculator is doing the calculations.***

Slowly announce which keys to press, giving students an opportunity to do the calculations mentally. Cover the display and press the same keys on the overhead calculator, if you are using one.

T: ***Press [3] [×] [4]*** (pause) ***[+] [5] [=].***
Don't look. What number do you think will be on the display?

S: ***17.***

Allow several students to answer before letting students check the display of their calculators. Also, check the display of the overhead calculator. If some students have other than 17 on the display, ask those students to press [C] and then to put 17 on the display.

T: ***Cover the display again. Now press [+] [3] [=] [=] [=].***
Don't look. What number do you think will be on the display?

S: ***26.***

Again, let several students respond before letting them check the display of their calculators. Help student pairs correct the number on the display as necessary.

Continue this activity with the following (or similar) sequence of calculations:

- Start with 26 on the display: press [−] [2] [=] [=] (22)
- Start with 22 on the display: press [×] [2] [=] (44)
- Start with 44 on the display: press [+] [1] [0] [=] [=] † (64)

Exercise 2: Detective Story

Students may continue to work in pairs during this exercise. Show the class the folded paper with 18 written on it.

T: ***Flip is my secret number today, and if you are good detectives, you can figure out who Flip is. This piece of paper hides Flip's identity.***

Clue 1

T: ***Flip is one of the numbers we get if we start at 3 and count by fives.***

Record 3 near the top of the board. Suggest to the class that the calculator can do the counting.

T: ***Press [3] [+] [5] [=]. What number is on the display?***

S: ***8.***

†Read as "plus ten, equal, equal."

Record 8 below 3 on the board.

T: ***If we continue adding fives, what is the next number we will get?*** (13)
Press [=]. What number is on the display?

S: ***13.***

Ask a volunteer to predict what the next number will be before you let students find it on their calculators. Continue this activity until you have at least eight numbers in the sequence recorded on the board.

3	23
8	28
13	33
18	38

T: ***Do you see any pattern in this list of numbers?***

Your class will probably notice that the last digits of these numbers alternate 3 and 8.

T: ***What number comes next? Why?***

S: ***43, because we are counting by fives. 38 + 5 = 43.***

S: ***The next number ends in 3.***

Record 43 in the list on the board.

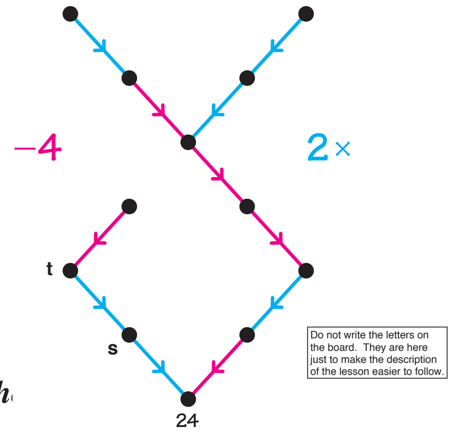
T: ***If we continued pressing [=], what are some other numbers we would get on the display?***

Encourage students to suggest many numbers that Flip could be. During this discussion, ask the following or other questions you feel are appropriate.

T: ***What is a number more than 100 that Flip could be?***
…more than 250?
…between 500 and 600?
…less than 1,000?
…the greatest possible number less than 1,000? (998)
Do you know who Flip is?

S: ***All we know is that Flip ends in 3 or 8.***

Clue 2



T: ***My next clue is that Flip is in this arrow picture.***

Put this arrow picture on the board and distribute Worksheet W15.

T: ***Where should we start if we want to know which numbers are in this arrow picture?***

S: ***The only dot that is labeled is 24.***

T: ***Can we start at 24 and follow the arrows to label th***

S: ***All the arrows are going the wrong way.***

S: ***We could draw return arrows.***

Trace the red arrow that ends at 24 as you say,

T: ***This arrow is for –4. What is its return arrow for?***

S: ***+4.***

Trace the blue arrow that ends at 24.

T: ***This blue arrow is for 2x. What is its return arrow for?***

S: ***$^1/_2$x (or ÷2).***

T: ***Label the dots in this arrow picture and be very careful about the directions of the arrows. Perhaps you can discover which number is Flip.***

This worksheet is difficult and perhaps only a few students (or pairs) will be able to complete it on their own. As you observe students' work, reiterate the need to pay attention to the direction of the arrows. After several minutes call the class's attention back to the board.

T (pointing to **s**)**:** ***How did you calculate this number?***

S: ***$^1/_2$ x 20 = 10 and $^1/_2$ x 4 = 2, so $^1/_2$ x 24 = 12.***

T: ***If this*** (point to **s**) ***is for 12, then we have the number fact 2 x 12 = 24. Is that true?***

S: ***Yes.***

Label **s** 12. Point to **t**.

T: ***What number is this? How do you know?***

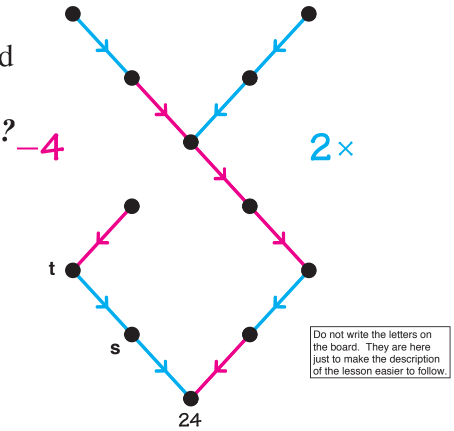
S: ***6, because 2 x 6 = 12.***

T: ***A blue arrow is for 2x, so its return arrow is for $^1/_2$x.***

Trace a return arrow from **s** to **t**.

T: ***What number is $^1/_2$ x 12?***

S: ***6.***



Label **t** 6. Continue this activity until all the dots are labeled

T: ***Does this arrow picture tell us anything about Flip?***

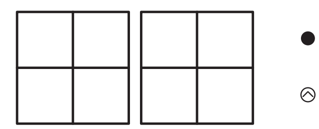
S: ***Flip could be 13, 18, or 28, because those are the only numbers in this arrow picture that end in 3 or 8.***

Circle 13, 18, and 28 in the arrow picture or record these numbers in a prominent place on the board.

T: ***Now we know that Flip is either 13, 18, or 28.***

Clue 3

Display two Minicomputer boards. Put or draw a regular checker and a negative checker to the side of the boards.



T: ***Flip can be put on the Minicomputer using exactly one regular and one negative checker. What numbers could Flip be?***

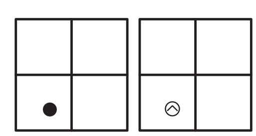
If some students think they know which number Flip is, walk over and ask them to point to Flip in their arrow pictures.

T: ***Could Flip be 13?***

Encourage students to use trial and error to find a way to put 13 on the Minicomputer using just one regular and one negative checker. Continue this activity until several students observe that 13 cannot be put on the Minicomputer using just those two checkers. Perhaps a student will explain that 13 $= 20 + \widehat{7}$ and that it is impossible to put $\widehat{7}$ on the Minicomputer using only one negative checker. Cross out 13.

T: ***Flip is either 18 or 28. Could Flip be 18?*** (Yes)

Ask a volunteer to put 18 on the Minicomputer using exactly one regular and one negative checker.



T: ***Flip could be 18. Could Flip be 28?***

Encourage students to use trial and error to find a way to put 28 on the Minicomputer using just one regular and one negative checker. Again, students should observe that 28 cannot be put on the Minicomputer using just those two checkers. Perhaps a student will explain that $28 = 30 + \widehat{2}$ or $28 = 40 + \widehat{12}$, but it is impossible to put 30 on the Minicomputer with only one regular checker and it is impossible to put $\widehat{12}$ on the Minicomputer using only one negative checker.

Cross out 28 and conclude that Flip must be 18. Reveal that 18 is written on the folded piece of paper.

Writing Activity

W15

Some students may enjoy trying to write a detective story.

Home Activity

Send a detective story home with students to challenge other family members to discover a secret number.

Capsule Lesson Summary

Look at the number 20 in a variety of ways: as a multiple of certain whole numbers; on the Minicomputer using a specified number of checkers; in a +4 arrow road; and in a three-string picture. Begin the *20?-100?* Workbook. In the first section of this workbook, find the number 20 on each page or extend the picture to include 20. In the second section, find the number 100 or extend the picture to include 100.

Materials

Teacher
- Minicomputer set
- Colored chalk

Student
- Paper
- *20?-100?* Workbook
- Colored pencils, pens, or crayons

Advance Preparation: You may like to prepare the arrow picture and the string picture for Exercises 3 and 4 before the lesson begins.

Description of Lesson

During the warm-up activities, ask students to record facts for 20 on their papers as you do so on the board. These exercises should be short and quick.

Exercise 1

T: ***Today we are going to become more acquainted with the number 20. What can you tell me about 20? Is 20 a multiple of any of the whole numbers?***

S: ***20 is a multiple of 5.***

T: ***How do you know?***

S: ***I counted by fives: 5, 10, 15, 20.***

T: ***20 is how many fives?*** (4) ***4 x 5 = 20.***

S: ***20 is a multiple of 4.***

$$4 \times 5 = 20$$
$$5 \times 4 = 20$$

T: ***Is 20 a multiple of 3?*** (No) ***How do you know?***

S: ***If we count by threes—3, 6, 9, 12, 15, 18, 21—we skip over 20.***

Continue this discussion until the class discovers that 20 is a multiple of 1, 2, 4, 5, 10, and 20. You should have the following multiplication facts on the board.

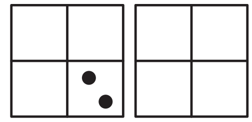
4 x 5 = 20	20 x 1 = 20	2 x 10 = 20
5 x 4 = 20	1 x 20 = 20	10 x 2 = 20

Exercise 2

Display two Minicomputer boards.

T: ***Who can put 20 on this Minicomputer using exactly two checkers?***

Perhaps a volunteer will suggest this configuration.



= 20

10 + 10 = 20
2 × 10 = 20

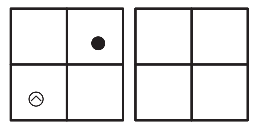
T: ***What fact about 20 do you see on the Minicomputer?***

S: ***10 + 10 = 20.***

S: ***2 x 10 = 20.***

Record these facts on the board.

Perhaps another student will suggest this configuration. If necessary, put on this configuration yourself and record the facts.

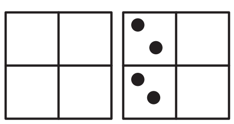


= 20

$40 + \widehat{20} = 20$
40 – 20 = 20

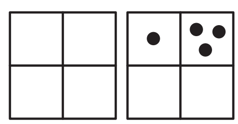
T: ***Who can put 20 on the Minicomputer using exactly four checkers?***

Encourage students to find several ways to solve this problem. As each configuration is put on the Minicomputer, record the number sentences students suggest on the chalk board, or point to the appropriate number sentences if they are already on the board. A few of the possible configurations are given here along with some of the number sentences students might suggest.



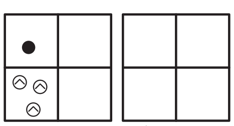
= 20

(2 × 8) + (2 × 2) = 20
16 + 4 = 20
2 × 10 = 20



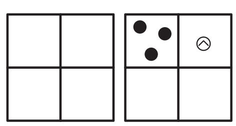
= 20

8 + (3 × 4) = 20
8 + 12 = 20



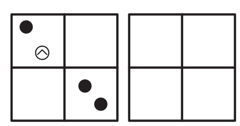
= 20

$80 + (3 \times \widehat{20}) = 20$
$80 + \widehat{60} = 20$
80 – 60 = 20



= 20

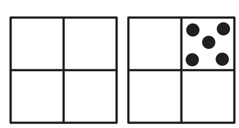
$(3 \times 8) + \widehat{4} = 20$
$24 + \widehat{4} = 20$
24 – 4 = 20



= 20

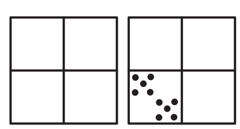
$80 + \widehat{80} + (2 \times 10) = 20$
0 + 20 = 20
100 – 80 = 20

T: ***Who can put 20 on the Minicomputer using exactly five checkers on the same square?***



= 5 × 4 = 20

T: ***Who can put 20 on the Minicomputer using exactly ten checkers on the same square?***

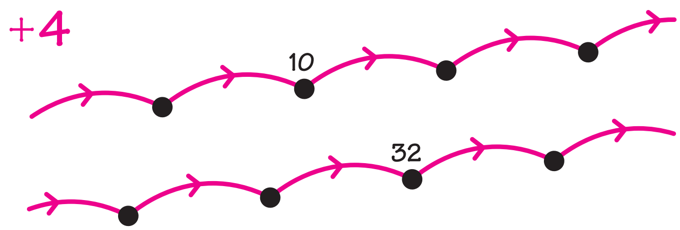


= 10 × 2 = 20

T: ***We have found many ways to put 20 on the Minicomputer and have written many number sentences about 20.***

Exercise 3

Erase the board and then draw this arrow picture.

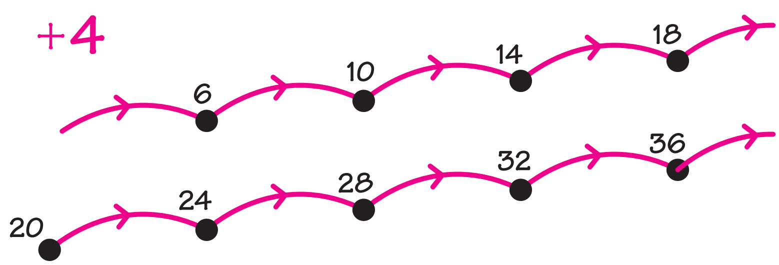


T: ***Can you find 20 in this arrow picture?***

Encourage the students to comment. Very likely a student will claim that a particular dot in the arrow picture is for 20. In this case, label the suggested dot 20 and seek your class's agreement or disagreement. Continue this activity until the class concludes that none of these dots is for 20. If necessary, collectively label all the dots to show that none of them is for 20.

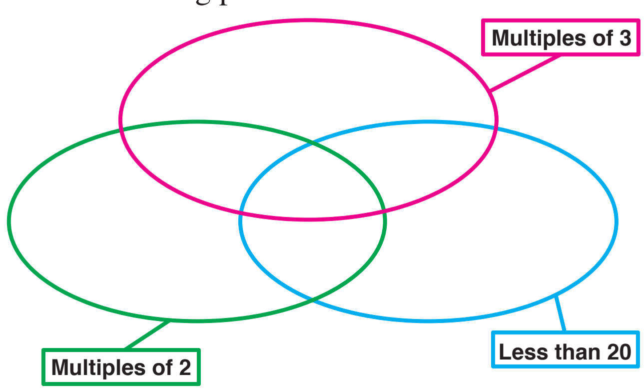
T: ***Could we extend this arrow picture to include 20?***

Let one of the students extend the picture to include 20. If a student just draws a dot and labels it 20, ask if any new arrows can be drawn. Of course, since 24 is already in the picture a +4 arrow can be drawn from 20 to 24.



Exercise 4

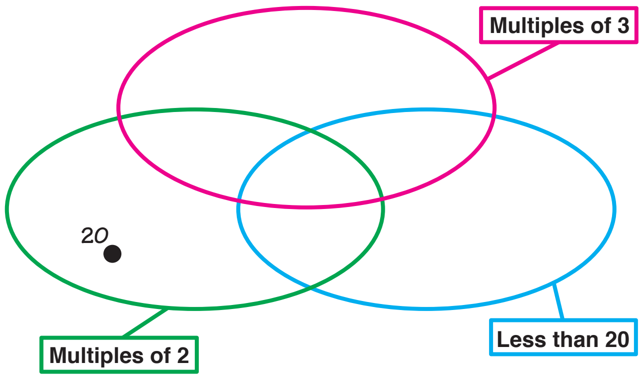
Erase the board and then draw this string picture.



T: ***Where is 20 in this string picture?***

Invite a student to draw and label a dot for 20. Ask the class if the student put 20 in the correct place. If necessary, erase an incorrect placement and ask for another volunteer to draw a dot for 20. Continue until 20 is located in the correct region.

T: ***Is 20 a multiple of 3?*** (No) ***So 20 is outside of the red string. Is 20 a multiple of 2?*** (Yes) ***10 x 2 = 20, so 20 is inside the green string. Is 20 less than 20?*** (No) ***So 20 is outside the blue string.***



Distribute the *20?-100?* workbooks. Stress that in the first section they are to find 20 in every picture or extend the picture to include 20. The second section is similar but involves the number 100 instead of 20. Tell the students to begin working on page 2. After a few minutes, ask the class if they have found 20 on page 2. Be sure all students understand that they need to extend the arrow picture on page 2 to include 20. As appropriate for your class, look at some other pages together with the students to clarify directions. You may want to require students to circle 20 when they find it on a page.

At the end of the class period, collect the workbooks for your review and to use again in Lesson W17.

Capsule Lesson Summary

Represent the number 100 1) as a multiple of certain whole numbers, and 2) on the Minicomputer, using a specified number of checkers. Continue working on the *20-100?* Workbook. (This is the second of two lessons using this workbook.)

Materials

Teacher
- Minicomputer set

Student
- Paper
- *20?-100?* Workbook
- Colored pencils, pens, or crayons

Description of Lesson

During the warm-up activities, ask students to record facts for 100 on their papers as you do so on the board.

Exercise 1

T: ***Today we are going to become more acquainted with the number 100. What can you tell me about 100? Is 100 a multiple of any of the whole numbers?***

S: ***100 is a multiple of 5.***

T: ***How do you know?***

S: ***100 is one of the numbers we say when we count by fives starting at 0.***

T: ***How many fives are there in 100?***

If the students have difficulty with such questions, guide them as follows.

T: ***How many dimes are there in a dollar?*** (10) ***10 x 10 = 100.***
How many nickels are there in a dollar? (20) ***How do you know?***

S: ***I know that two nickels is the same amount of money as one dime, so there are twice as many nickels in a dollar as there are dimes.***

T: ***How many fives are there in 100?*** (20) ***20 x 5 = 100.***

S: ***100 is a multiple of 20.***

20 x 5 = 100
5 x 20 = 100

Continue this discussion until the class discovers that 100 is a multiple of 1, 2, 4, 5, 10, 20, 25, 50, and 100.

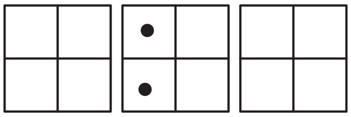
Exercise 2

Display three Minicomputers boards.

T: ***Who can put 100 on this Minicomputer using exactly two checkers?***

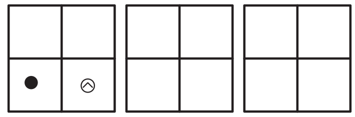
When a configuration has been put on the Minicomputer, ask the students what calculation is suggested by the checkers on the Minicomputer and record the number sentences on the board.

There are two ways to put 100 on the Minicomputer using exactly two checkers.



= 100

$80 + 20 = 100$



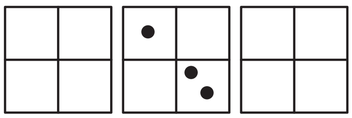
= 100

$200 + \widehat{100} = 100$

$200 - 100 = 100$

T: ***Who can put 100 on the Minicomputer using exactly three checkers?***

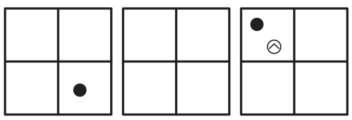
A few of the possible configurations are shown here.



= 100

$80 + (2 \times 10) = 100$

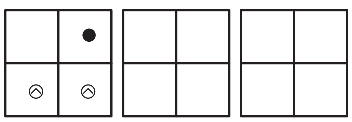
$80 + 20 = 100$



= 100

$100 + (8 + \widehat{8}) = 100$

$100 + 0 = 100$



= 100

$400 + (\widehat{200} + \widehat{100}) = 100$

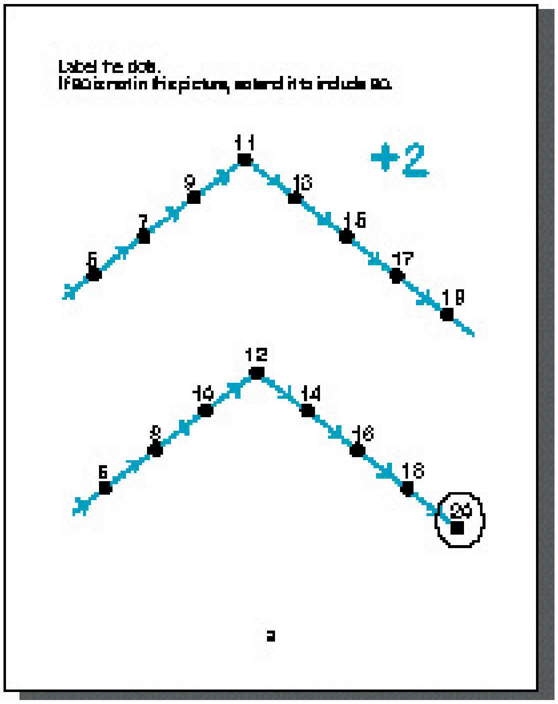
$400 + \widehat{300} = 100$

$400 - 300 = 100$

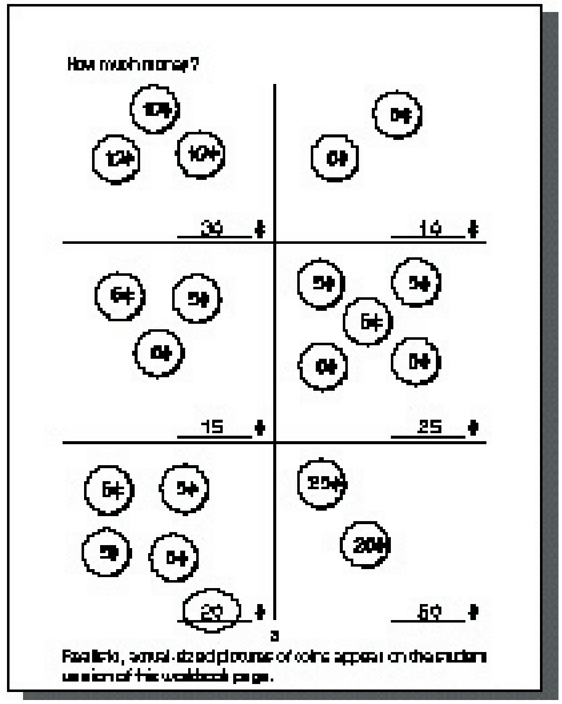
Distribute the students' copies of the *20?-100?* Workbook. Remind students that in the first section they are to find 20 in every picture or to extend the picture to include 20. The second section is similar but involves the number 100 rather than 20. As appropriate for your class, look at some pages together to clarify directions. You may want to require that students circle 20 when they find it on a page in the first section, and circle 100 when they find it on a page in the second section. If some of the students finish the workbook early, suggest they make up their own problems for a similar workbook about a number of their choice. At the end of the lesson, collect the workbooks for your review.

Assessment Activity

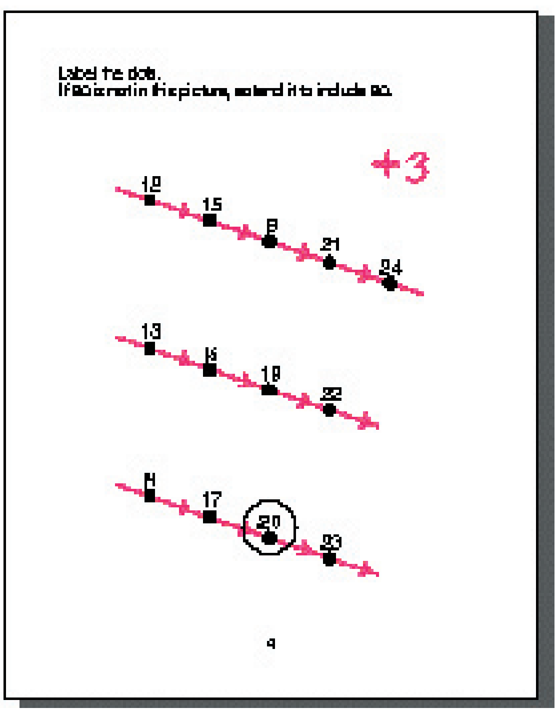
An individual student progress record for the workbook is available on Blackline W17. You may like to use this form to monitor student work.



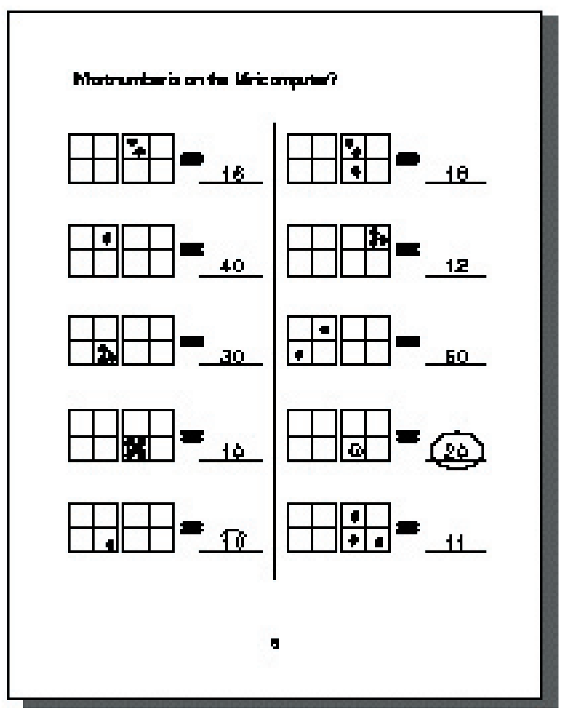
Label the dots.
If 20 is not in the picture, extend it to include 20.
+2
2



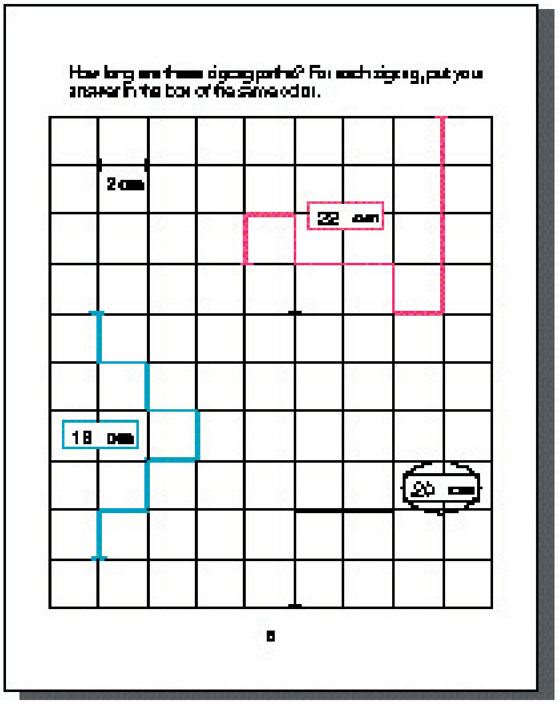
How much money?
30 ¢
14 ¢
15 ¢
25 ¢
20 ¢
50 ¢
3
Realistic, actual-sized pictures of coins appear on the student version of this workbook page.

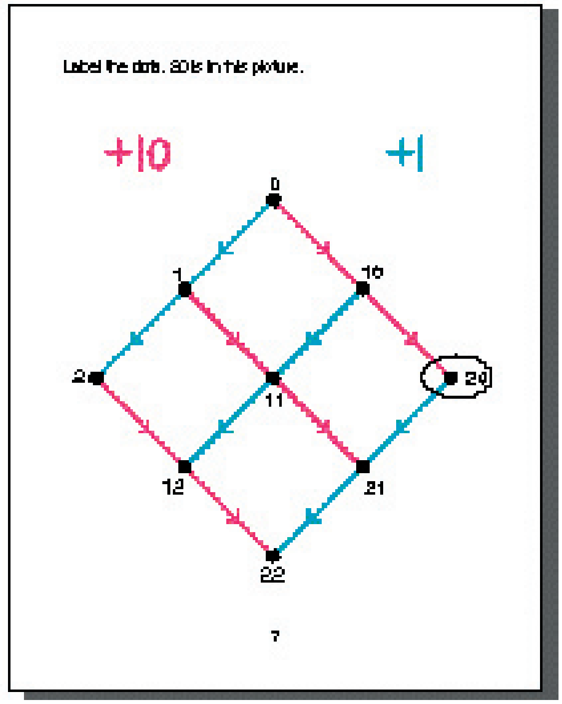


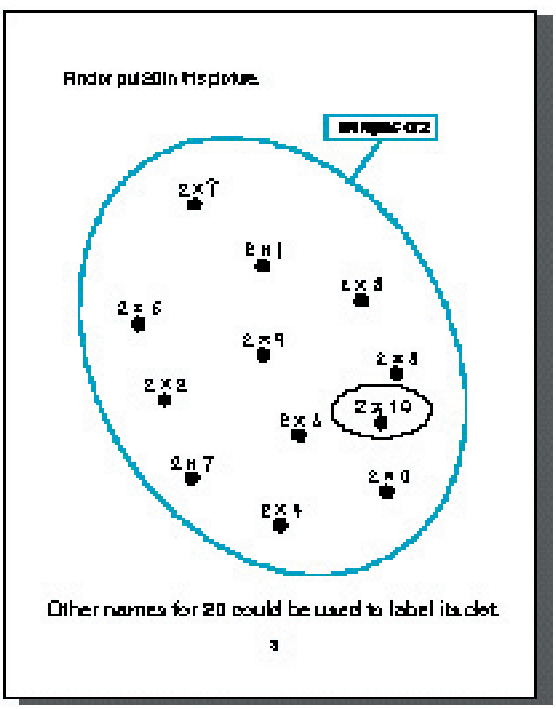
Label the dots.
If 20 is not in the picture, extend it to include 20.
+3
4

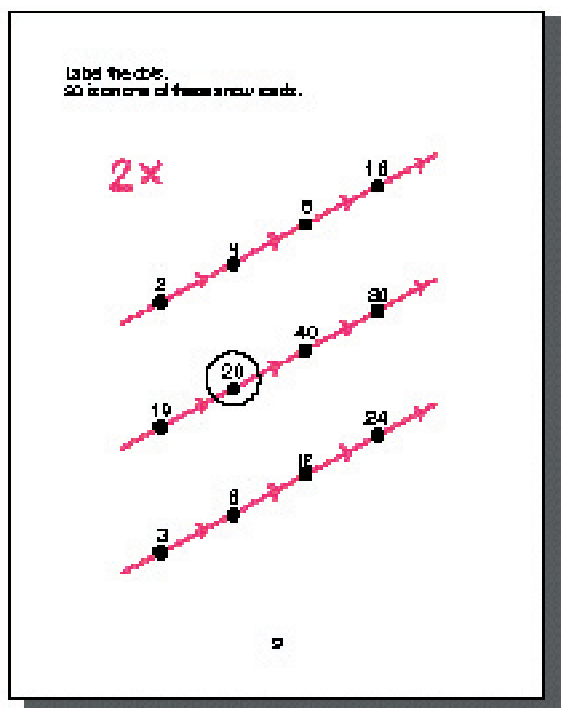


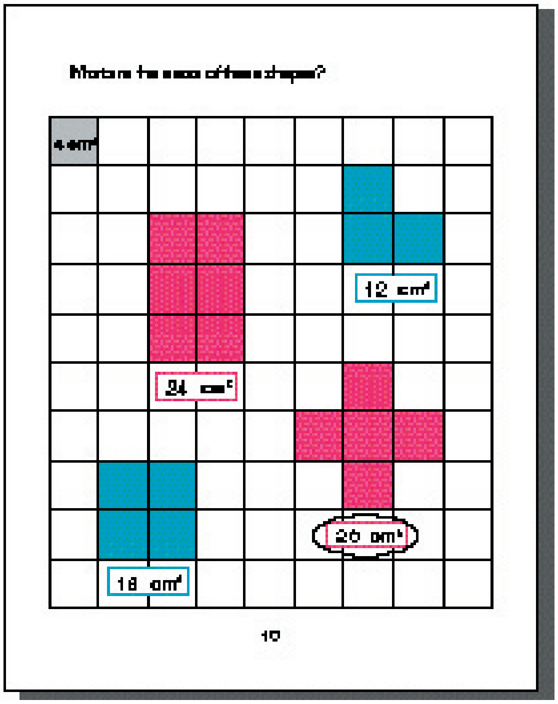
What number is on the Minicomputer?
5

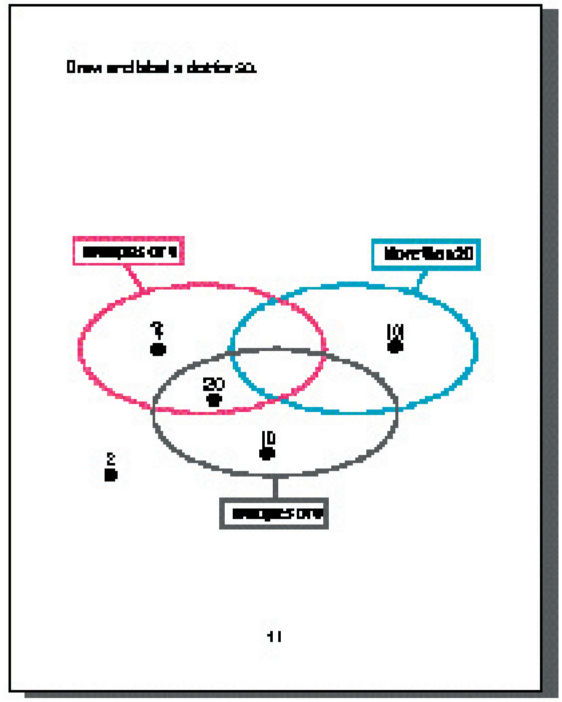

2 cm
22 cm
18 cm
20 cm

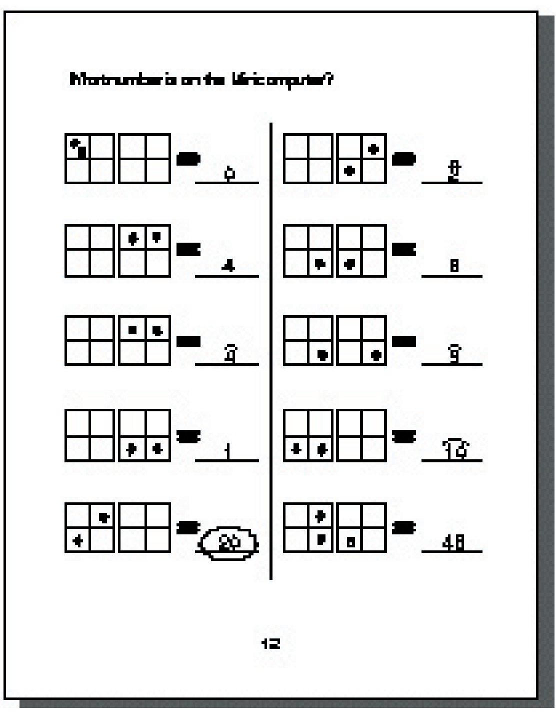

+10
+1
0
1
10
2
11
20
12
21
22

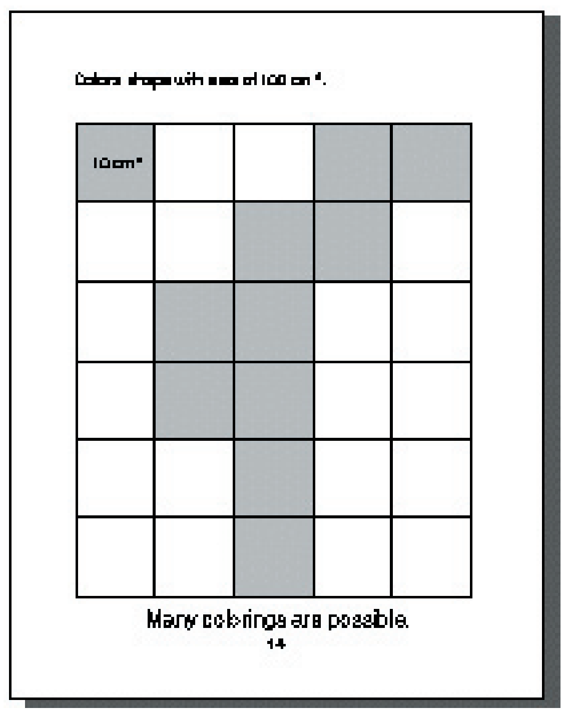

2 x 10
Other names for 20 could be used to label its dot.

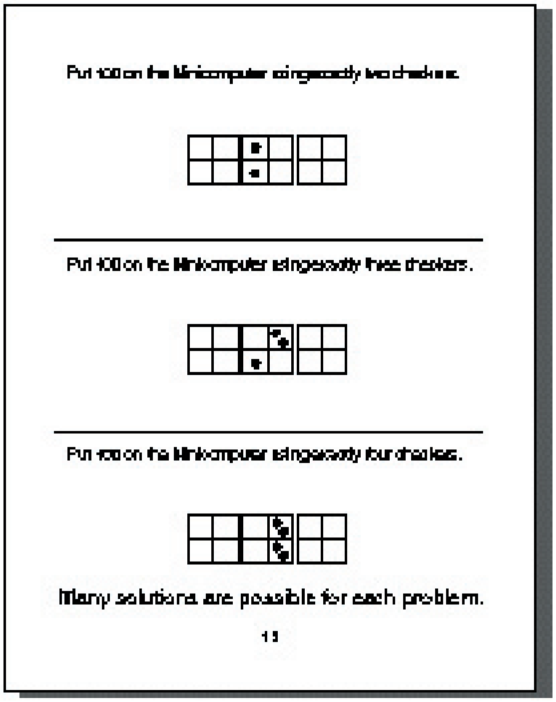

2x
2
4
16
20
40
80
10
24

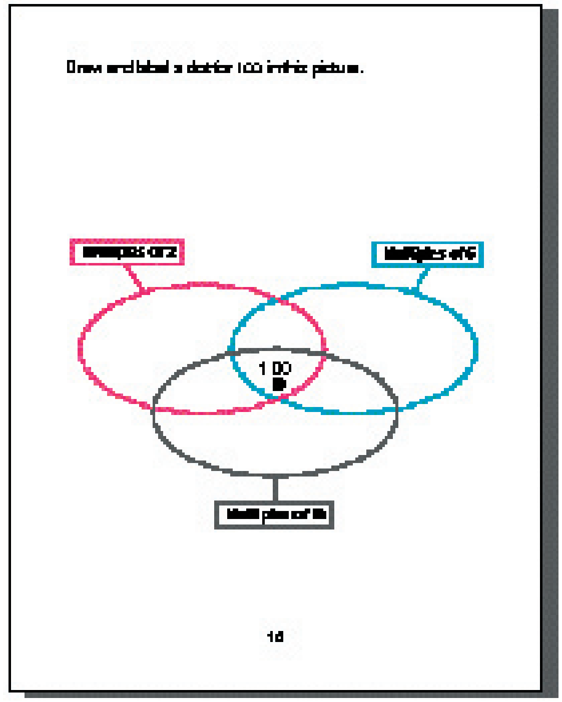

12 cm²
24 cm²
20 cm²
16 cm²
10

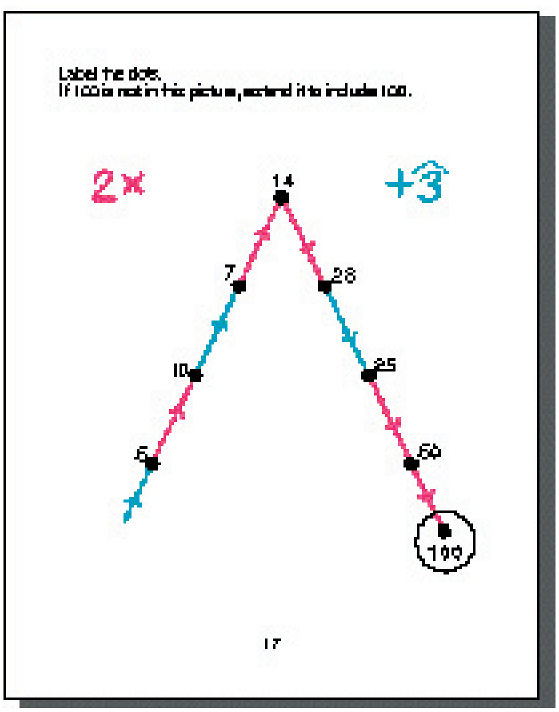

20
10
11

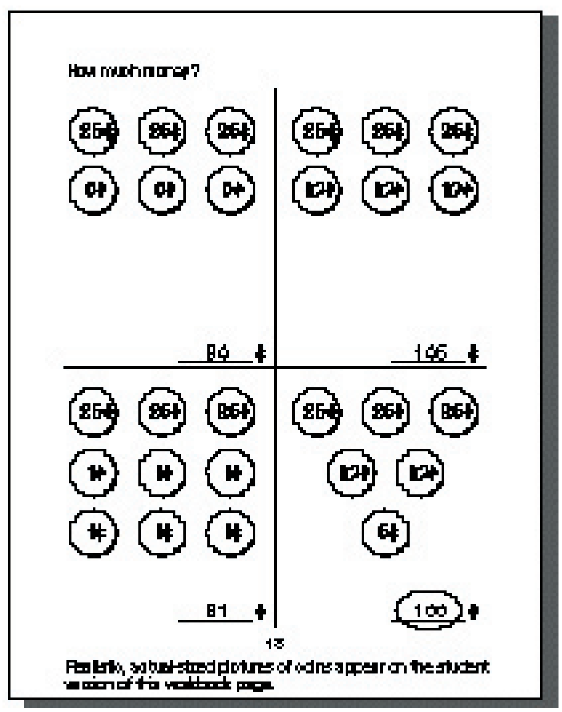

12

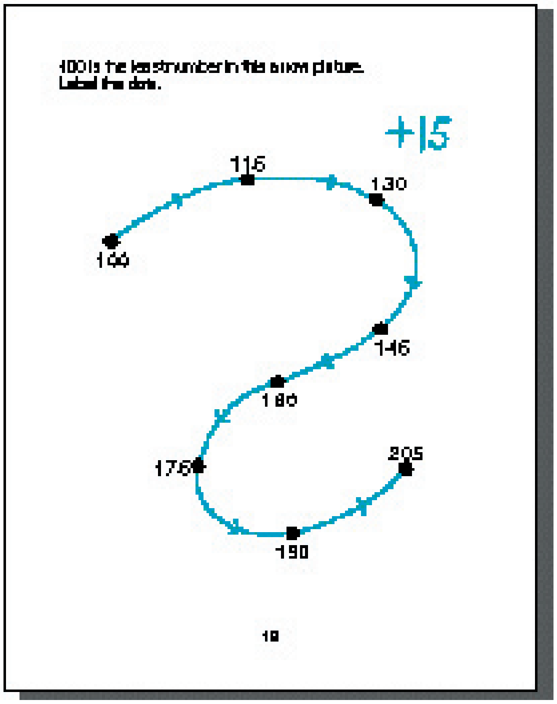

Many colorings are possible.
14

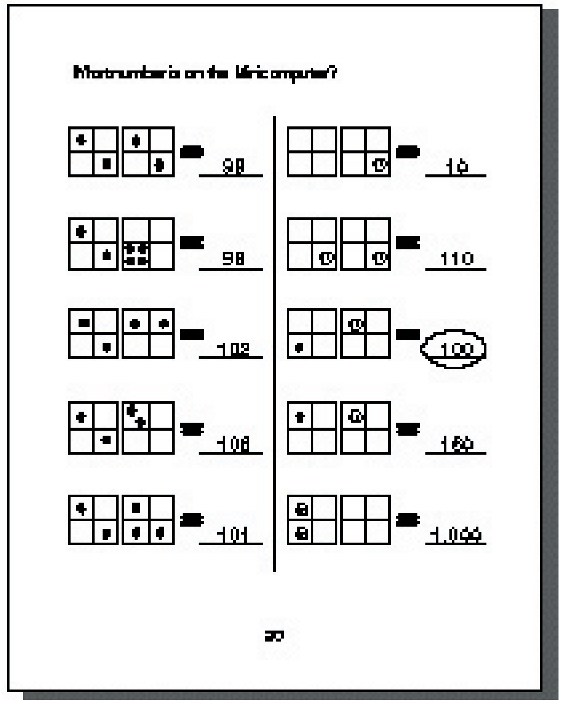

Put 100 on the Minicomputer using exactly two checkers.
Put 100 on the Minicomputer using exactly three checkers.
Put 100 on the Minicomputer using exactly four checkers.
Many solutions are possible for each problem.
15

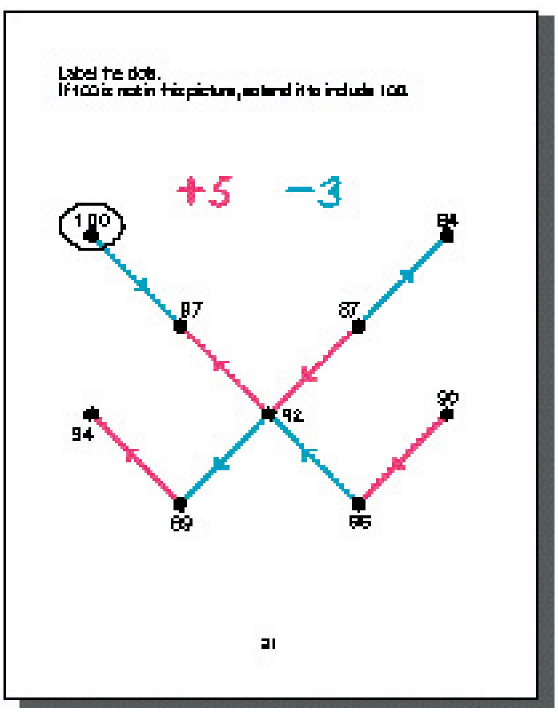

Draw and label a dot for 100 in this picture.
100
16

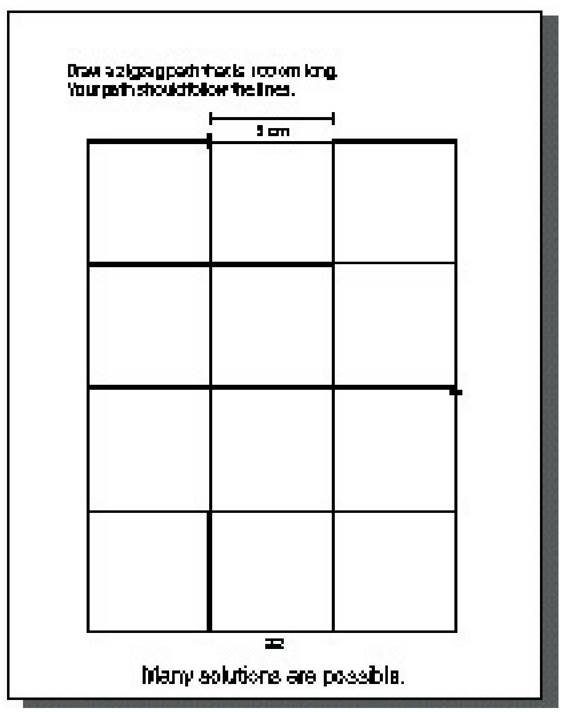

Label the dots.
If 100 is not in this picture, extend it to include 100.
2x
+3
14
7
28
10
25
5
50
100
17

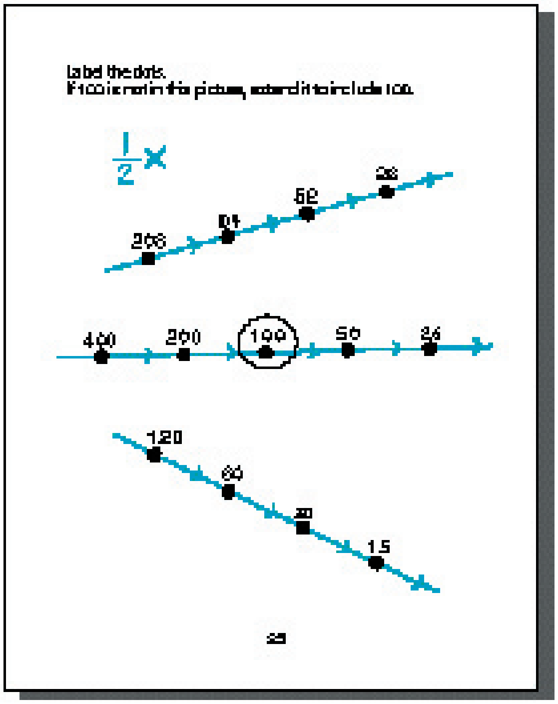

How much money?
84 ¢
145 ¢
81 ¢
100 ¢
18
Realistic, actual-sized pictures of coins appear on the student version of this workbook page.


100 is the least number in this arrow picture.
Label the dots.
+15
100
115
130
145
160
175
190
205

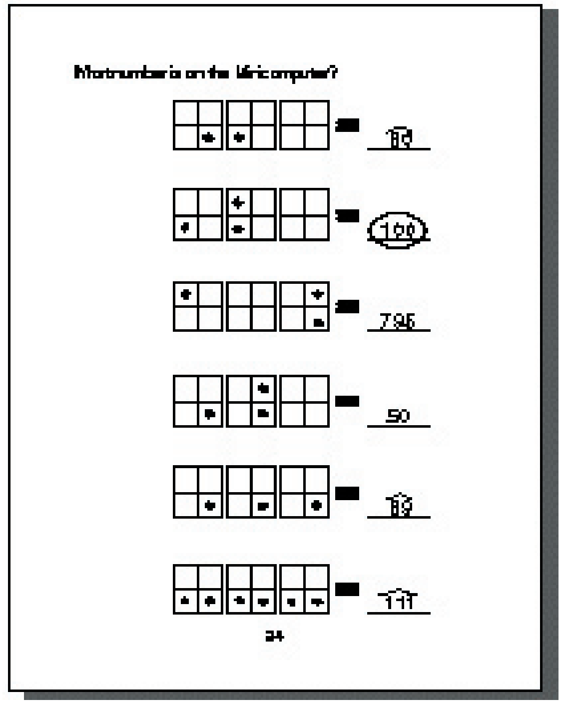

What number is on the Minicomputer?
98
98
102
108
101
10
110
100
160


Label the dots.
If 100 is not in this picture, extend it to include 100.
+5
−3
100
97
87
94
92
90
89
84

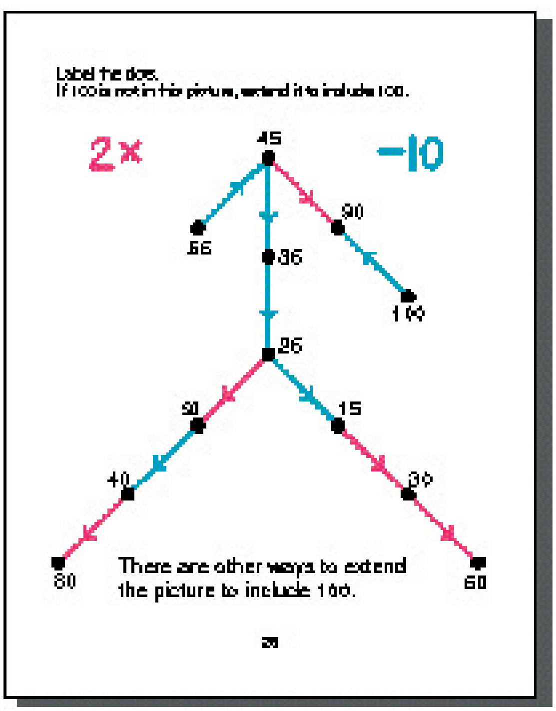

Draw a zigzag path that is 100 cm long.
Your path should follow the lines.
Many solutions are possible.



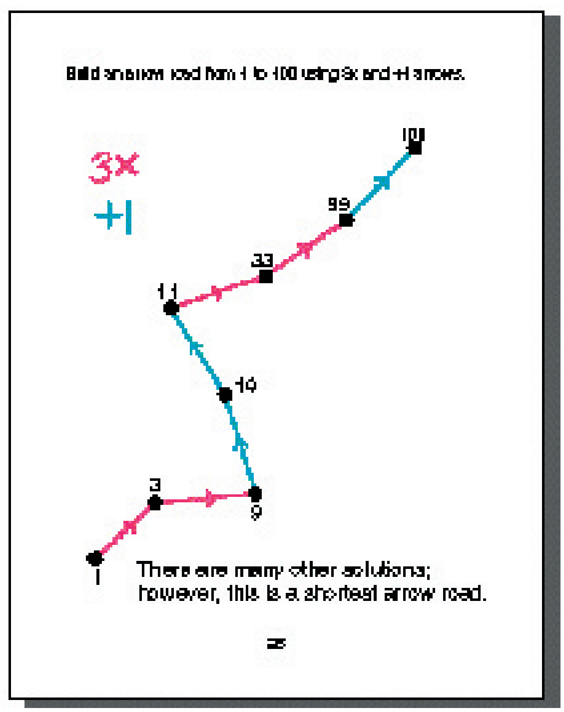
400
200
100
50
25
120
60
15



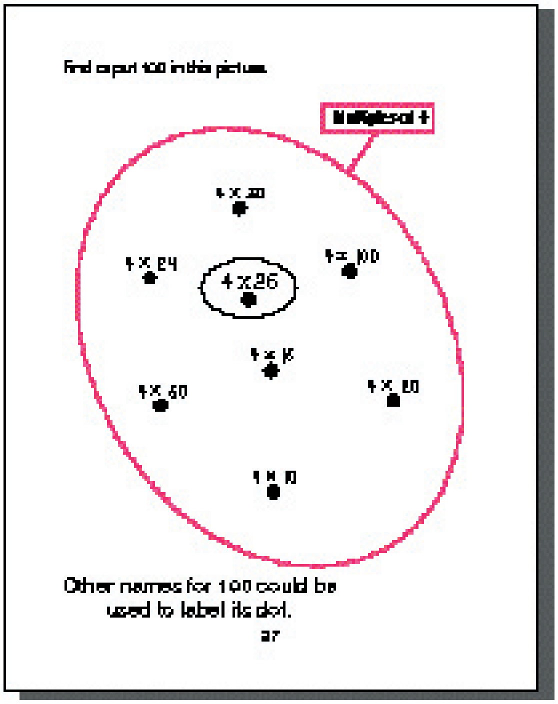
100
795
50



Label the dots.
If 100 is not in this picture, extend it to include 100.
2x
-10
45
90
36
100
26
15
40
30
80
60
There are other ways to extend the picture to include 100.

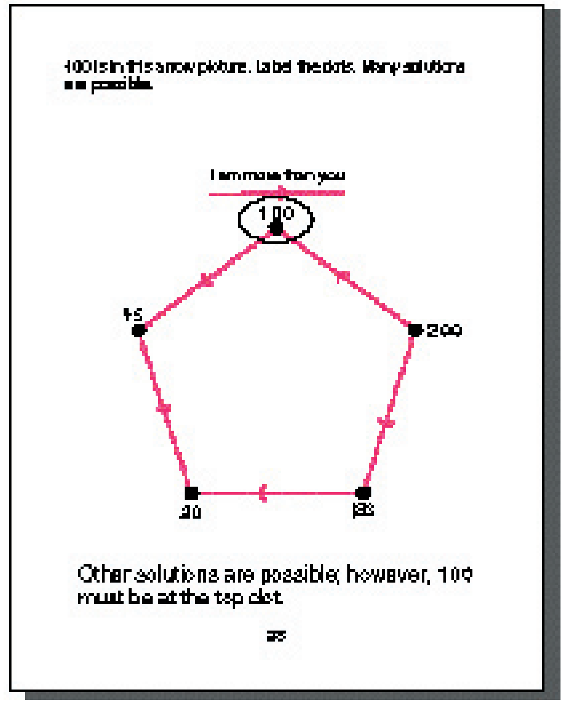


3x
+1
100
99
33
11
10
3
9
1
There are many other solutions; however, this is a shortest arrow road.


Find or put 100 in this picture.

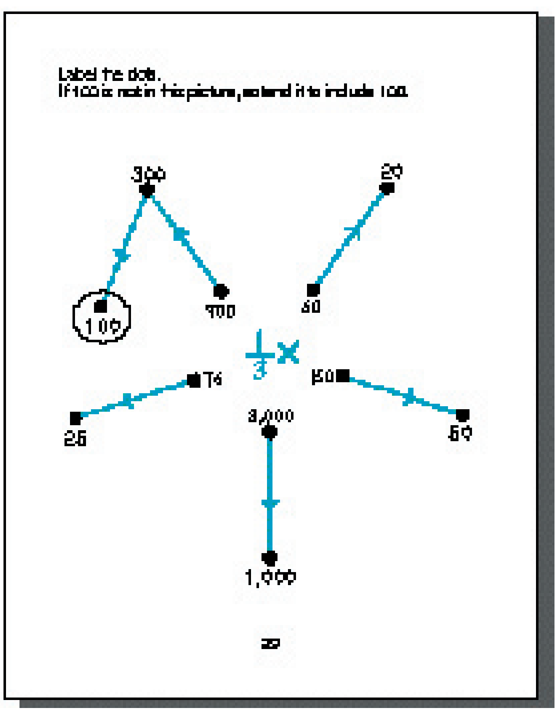
Other names for 100 could be used to label its dot.

27

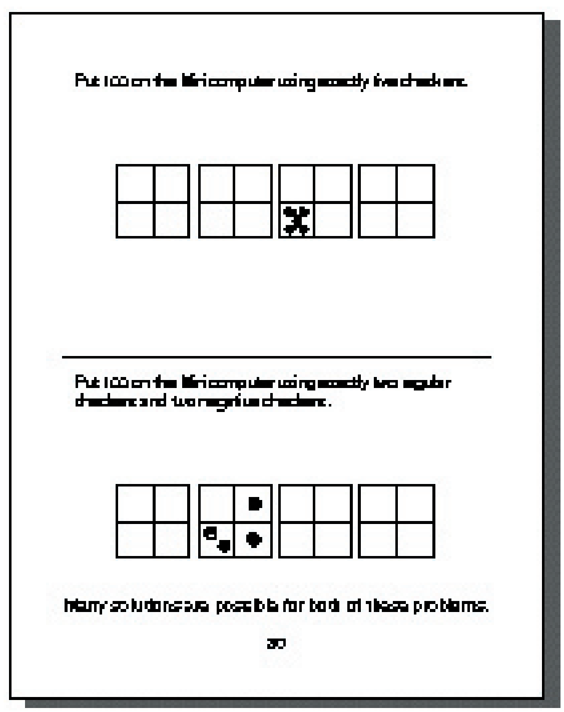

100 is in this arrow picture. Label the dots. Many solutions are possible.

Other solutions are possible; however, 100 must be at the top dot.

28


Label the dots.
If 100 is not in this picture, extend it to include 100.

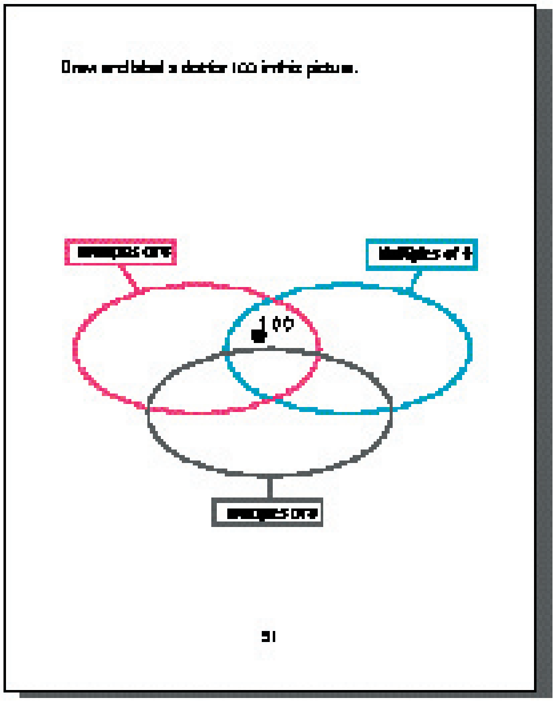
29

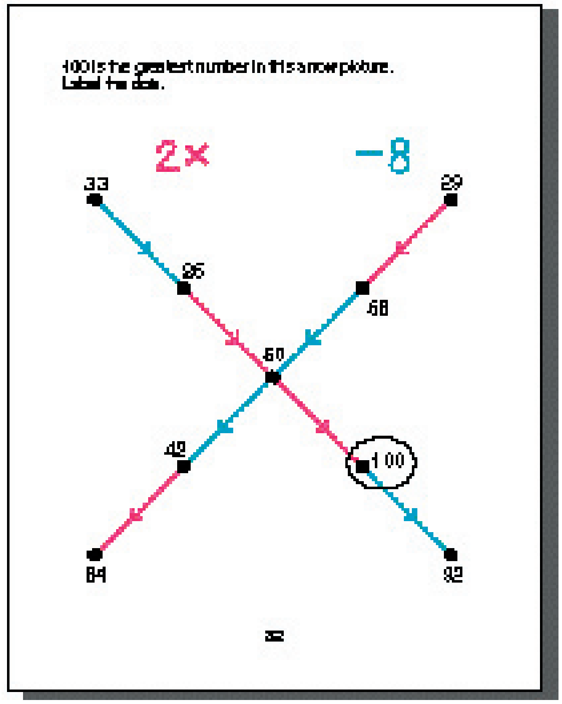

Put 100 on the Minicomputer using exactly five checkers.

Put 100 on the Minicomputer using exactly two regular checkers and two negative checkers.

Many solutions are possible for both of these problems.

30


Draw and label a dot for 100 in this picture.
100


100 is the greatest number in this arrow picture.
Label the dots.
2x
-8
33
29
25
58
50
42
100
84
92

Capsule Lesson Summary

Identify two number friends using clues that involve arrow pictures.

Materials

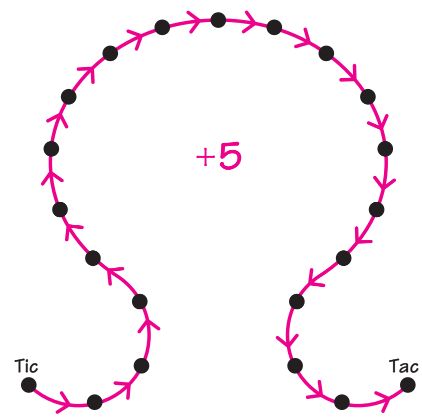
Teacher
- Colored chalk

Student
- Paper
- Worksheets W18 (no star), *, and **

Advance Preparation: Draw the arrow picture from Worksheet W18 (no star) on the board before class begins. You may prefer to prepare it on chart paper or as an overhead transparency.

Description of Lesson

This would be a good lesson for students to work on with a partner. Distribute copies of Workshee W18 (no star) and refer to this arrow picture.



T: ***I have two number friends, Tic and Tac.***
Tic and Tac are whole numbers.
Can you see them in this arrow picture?

Invite a student to point to Tic and Tac.

T: ***Look carefully at this arrow picture.***
What are the red arrows for ?

S: ***+5.***

T: ***How many red arrows are there between Tic and Tac?***

S: ***Twenty.***

T: ***Which is the greatest number in this arrow picture?*** (Tac)
Which is the least number? (Tic)

If Tic were 10, what would Tac be?

Suggest students write the answer on their papers and discuss it with their partners. Call on a student (or partner) with the correct answer to answer aloud and explain.

S: ***If Tic were 10, Tac would be 110. I started at 10 and counted by fives.***

S: ***20 x 5 = 100, so I added 10 + 100 to find Tac.***

S: ***I figured out what number would be at each dot by adding 5 each time.***

Proceed in the same way, asking students to find Tac when Tic is 100, when Tic is 20, when Tic is 50, when Tic is 27, and when Tic is 184.

Collect the results in a table drawn on the board near the arrow picture.

Tic	Tac
10	110
100	200
20	120
50	150
27	127
184	284
	361

T: ***Listen carefully. If Tac were 361, what would Tic be? Write your answer on your scratch pad*** (and discuss it with your partner)***.***

Write 361 in the column for Tac. Emphasize that Tac is given as 361 and that they are to find Tic. Again, suggest to students that they discuss their answer with their partners and write it on their papers.

S: ***If Tac were 361, Tic would be 261.***

Enter 261 in the column for Tic.

T: ***Do you notice any patterns in this table?***

S: ***Tac is always 100 more than Tic.***

Draw a blue arrow from Tic to Tac in the arrow picture.

T: ***What could this blue arrow be for?***

S: ***+100.***

T: ***How do you know?***

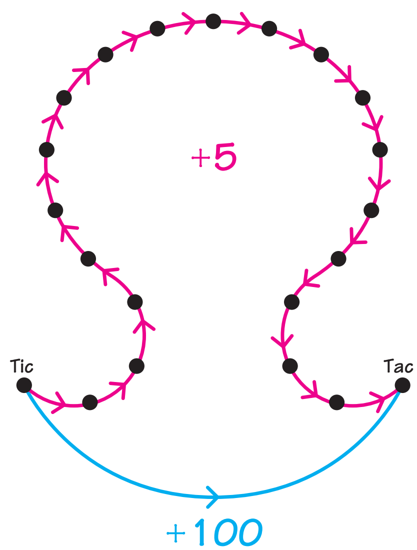
S: ***20 x 5 = 100.***

S: ***Tac is always 100 more than Tic.***

S: ***I counted by fives.***

Label the blue arrow +100 and write +100 at the top of the table.

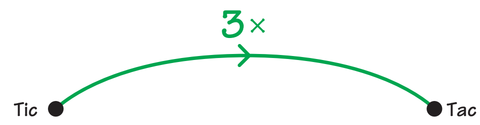
+100	
Tic	Tac
10	110
100	200
20	120
50	150
27	127
184	284
261	361



T: ***Do we know which numbers Tic and Tac are?***

S: ***No, all we know is that Tac is 100 more than Tic.***

On another part of the board draw this arrow picture.



T: ***What does this arrow picture tell you about Tic and Tac?***

S: ***Tac is three times as large as Tic.***

T: ***For the moment, let's concentrate on this arrow picture only. If Tic were 5, what would Tac be? How do you know?***

S: ***15, because 3 x 5 = 15.***

T: ***If Tic were 100, what would Tac be? Write your answer on your scratch pad (and check it with your partner).***

Ask students to write the answer on their papers and check it with their partners.

S: ***If Tic were 100, then Tac would be 300 because 100 + 100 + 100 = 300.***

S: ***300, because 3 x 100 = 300.***

Draw a new table on the board near the 3x arrow picture, and record the information for Tic and Tac when Tic is 5 or 100.

3×	
Tic	Tac
5	15
100	300
31	

T: ***If Tic were 31, what would Tac be?***

Proceed as above letting students work with their partners before asking a student to answer aloud. Each time, ask for an explanation on how to calculate Tac. Continue this activity with Tic being 120 and 251. Enter the appropriate numbers in the table.

3×	
Tic	Tac
5	15
100	300
31	93
120	360
251	753
	126

T: ***Listen carefully. If Tac were 126, what would Tic be?***

Again, suggest students work with their partners to find an answer and write it on their papers. Indicate that Tac is given as 126 and that they are to find Tic. Conclude that Tic is 42 and enter 42 in the table.

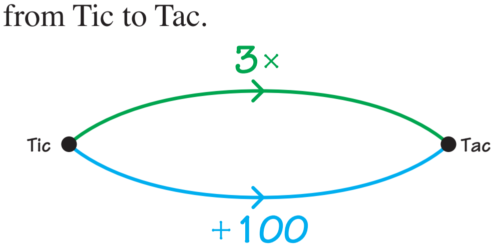
Note: This is a difficult problem. You may prefer to start with a simpler problem, such as Tac is 36. Or, if none of your students are able to find Tic, suggest that they determine what Tic would be if Tac were 120. When someone discovers that 3 x 40 = 120, return to the original problem.

Refer to the two arrow pictures on the board.

T: ***We know that Tac is 3 x Tic. We also know that Tac is 100 more than Tic. Let's put these two clues together.***

In the second arrow picture, draw a blue arrow labeled +100 from Tic to Tac.

T: ***Which numbers are Tic and Tac?***



Refer to the two tables for Tic and Tac, and encourage students to investigate this problem with their partners. Do not expect a solution to be found quickly. After a few minutes, discuss some students' suggestions for Tic and Tac collectively. A sample dialogue is given here.

S: ***I think that Tic is 100 and Tac is 300.***

Label the dot for Tic 100, and then trace the green arrow.

T: ***3 x 100 = …?***

S: ***300.***

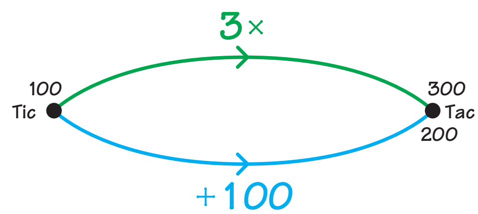
Write 300 near the dot for Tac. Trace the blue arrow.

T: ***100 + 100 = …?***

S: ***200.***

Write 200 near the dot for Tac.

T: ***Tac cannot be both 300 and 200, so Tic cannot be 100.***

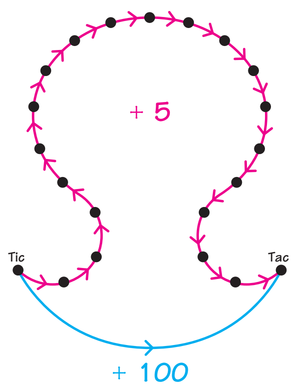


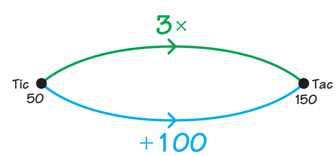
Erase the numbers for Tic and Tac from the arrow picture, and in both tables cross out the rows corresponding to 100 for Tic.

Encourage other students to suggest their ideas. Continue this activity until someone suggests that Tic is 50 and Tac is 150. If no one suggests that Tic is 50, refer to the numbers in the +100 table for Tic and Tac and ask if any of these numbers would work in the second arrow picture.

Conclude that 3 x 50 = 150 and 50 + 100 = 150, so Tic is 50 and Tac is 150. When you finish, your board might look something like this.

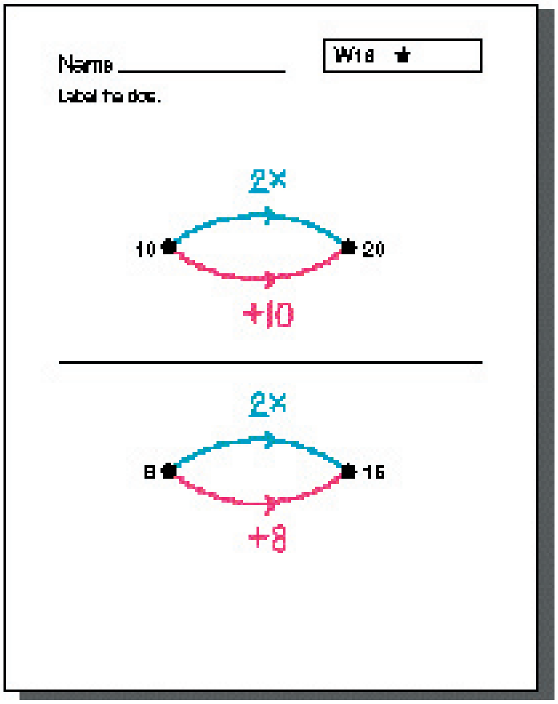
+100	
Tic	Tac
10	110
~~100~~	~~200~~
~~20~~	~~120~~
50	150
27	127
184	284
261	361

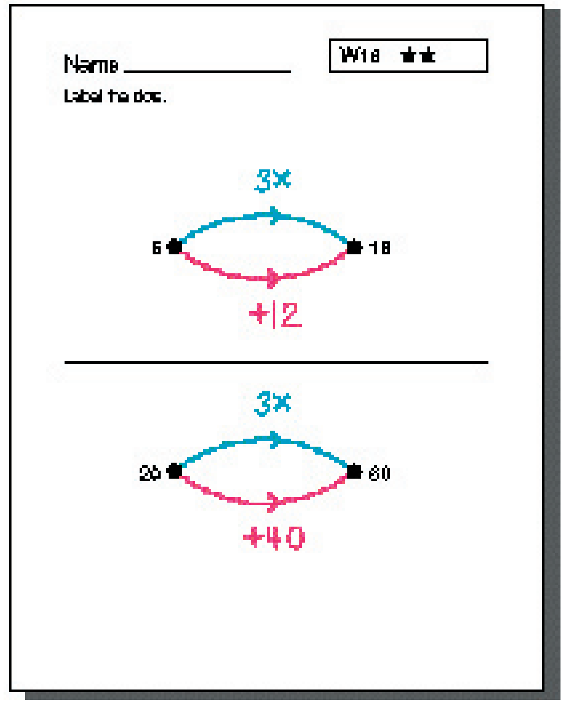




3x	
Tic	Tac
5	15
~~100~~	~~300~~
31	93
120	360
251	753
~~42~~	~~126~~
50	150

Worksheets W18* and ** are available for individual or partner work. Suggest to students that tables like you made in the lesson can be helpful in solving the problems on the worksheets.


Name
W18 ★
Label the dots.
2×
10
20
+10
2×
8
16
+8


Name
W18 ★★
Label the dots.
3×
6
18
+12
3×
20
60
+40