Geometry& Measurement

GEOMETRY & MEASUREMENT TABLE OF CONTENTS

GEOMETRY & MEASUREMENT INTRODUCTION

Geometry is useful and interesting, an integral part of mathematics. Geometric thinking permeates mathematics and relates to most of the applications of mathematics. Geometric concepts provide a vehicle for exploring the rich connections between arithmetic concepts (such as numbers and calculations) and physical concepts (such as length, area, symmetry, and shape). Even as children experience arithmetic notions in their daily lives, they encounter geometric notions. Common opinion recognizes that each child should begin developing a number sense early. There is no sound argument to postpone the development of geometric sense. These are compelling reasons why the systematic development of geometric thinking should be a substantial component of an elementary school curriculum.

A benefit of early development of geometric thinking is that it provides opportunities for all learners. Children develop concepts at individual rates. We are careful to allow students a great deal of time to explore quantitative relationships among physical objects before studying arithmetic. In the same spirit, students should be allowed to explore geometric relationships through free play and guided experiences. Before studying geometric properties of lines and shapes, they should have free play with models and with drawing instruments, using both to create new figures and patterns. Before considering parallelism and perpendicularity as relations on geometric lines, they should have free play in creating patterns with parallel and perpendicular lines. Before learning length and area formulas, they should have informal experiences to establish and sharpen intuitive notions of length and area. These early experiences are aesthetically rewarding and just as stimulating and suggestive of underlying concepts as early experiences leading to arithmetic. Such experiences are a crucial first step in the development of a geometric sense.

As with most educational development occurring through informal experiences, the payoffs come unexpectedly and spasmodically. In the development of geometric thinking, the suggested sequence of activities is so varied, reaches into so many other facets of the program, and continues for so long a span that it would be nonsense to set *a priori* milestones of accomplishments.

Thus, it is important to allow students to move through the free exploration activities that precede the more substantive lessons without rushing them. These preliminary activities have implicit significance for geometric thinking and also provide aesthetic opportunities and explicit practice— they are not mere play. On the other hand, some activities appear to have important consequences or clearly relate geometry to other parts of the program. Here, your inclination might be to spend more time on these activities than is suggested, but try to remember that similar activities will continue to be experienced later in a slightly different context.

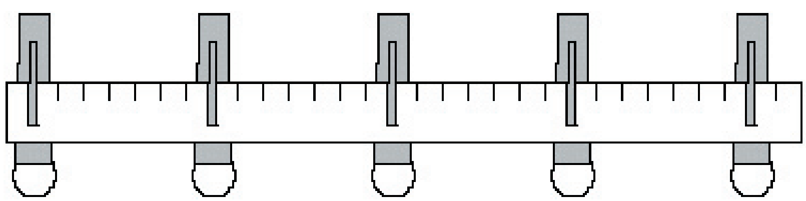
This strand also includes measurement activities, many that lead to the use of metric units, as well as topics not usually found in traditional programs at this level. Measurement activities eventually lead to notions of distance. Traditionally, teachers wait for a good deal of maturity in their students before introducing properties of figures, such as circles, that are defined in terms of distance. The reason is that Euclidean distance (distance "as the crow flies") requires sophisticated notions of "exact" measurement in terms of the real number system in order to be able to measure every line segment. But in the *CSMP* curriculum very young children (even in kindergarten) begin thinking about distance in the simplified setting of "taxi-geometry." Taxi-distance (as a taxicab travels) between points is measured in terms of whole numbers by easy counting processes. Experiences in taxi-geometry lead to some creative situations, yet do not require special technical skills or vocabulary.

Putting the emphasis on experience rather than on mastery frees you to engage in imaginative activities with children that you ordinarily might not attempt. For example, even when considering such commonplace topics as area and length, feel free to devise activities that help students recognize patterns, develop cognitive strategies, and relate the topic to others in the program, while suppressing the urgency to get the basic formulas. What is basic here, at this level, is that children are thinking, rather than doing tasks on command. The skills and mastery will come eventually, in their own time and place in the curriculum.

Note on Grids

A demonstration grid board is needed for several lessons, especially those on taxi-geometry. This should be a large square grid board at least 12 squares by 12 squares (with grid lines spaced 4 to 8 centimeters apart) on which you can draw with colored chalk or colored pens. A portable, commercially made, permanent grid chalkboard is ideal. However, if such a grid board is not available, try one of these alternatives.

- Printed grids on large sheets of paper are provided in classroom sets of materials for *CSMP* for the upper primary grades. Tape one of these sheets to the board, use thick felt-tipped pens for drawings, and discard the sheet after use. You may prefer to laminate or mount one of the sheets on cardboard and cover it with clear contact paper to provide a protective surface. Then use erasable markers or wax crayons for drawings. These grid sheets also are useful as graphing mats.
- For each lesson, carefully draw a grid on the chalkboard using an easy-to-make tool. Either (1) use a music staff drawer with chalk in the first, third, and fifth positions; or (2) clip five pen caps, each having the diameter to hold a piece of chalk tightly, to a ruler at intervals of 4–8 centimeters and secure them with tape.



- Make a grid transparency for use on an overhead projector and use erasable overhead markers. The Blackline section includes several size grids to use in making a grid transparency.

Capsule Lesson Summary

Review the story of Nora's neighborhood and the notion of taxi-distance between two points. From several clues about the location of the new house belonging to Nora's cousin Linda, positively determine where Linda will live.

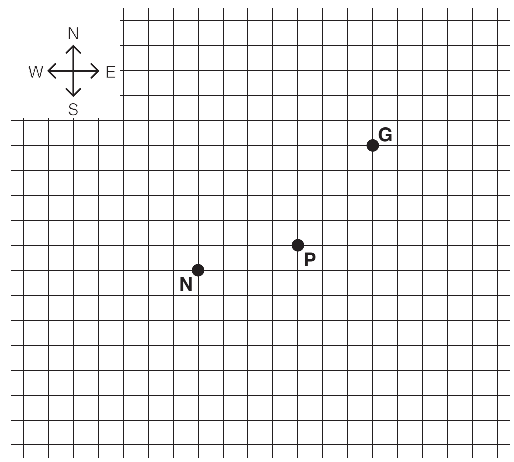
Materials

Teacher
- Grid board†
- Colored chalk or markers

Student
- Worksheet G1
- Colored pencils, pens, or crayons

Description of Lesson

Display a grid board. Mark and label three points **N**, **P**, and **G** as in this illustration. Indicate directions in the upper left corner of the board.



Briefly review the story of Nora's neighborhood. Ask someone to trace on the grid a long walk from Nora's house (**N**) to her grandmother's house (**G**), and then ask someone to trace a shortest walk.

T: ***How long is a shortest walk from Nora's house to her grandmother's house?***

S: ***Twelve blocks.***

T: ***How do you know?***

S: ***Nora has to walk five blocks north and seven blocks east.***

Students may need help wording their responses in terms of directions.

T: ***The taxi-distance from Nora's house to her grandmother's house is 12 blocks.***

Record this information on the board. This will be the first line in a list of such information.

taxi-distance from N to G = 12 blocks

T (pointing to **P**)**:** ***This is where the park is on the map. What is the taxi-distance from Nora's house to the park?***

Invite a student to count the blocks in a shortest path.

S: ***Five blocks.***

†See the Note on Grids in the introduction to this strand.

G1

Record this information on the board.

taxi-distance from N to G = 12 blocks
taxi-distance from N to P = 5 blocks

T: ***Nora has a cousin Linda. Linda is moving to Nora's neighborhood and Nora wants to know where Linda will live. Nora's mother gives her a clue.***

Clue 1

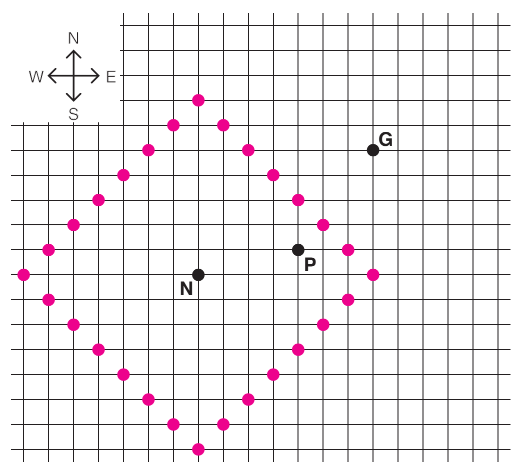
T: ***Her mother says, "Linda will live seven blocks from our house." Can you find some places where Linda's new house might be?***

Let students, one at a time, point to possible places on the grid. Draw a dot for each correct suggestion. After four or five places have been found, hold up a copy of Worksheet G1 as you say,

T: ***I would like you to find all the places where Linda's new house might be. On your paper draw red dots to record the possibilities.***

Distribute copies of Worksheet G1 and colored pencils. Let students work independently or in pairs for five to ten minutes on the worksheet.

Invite students who finish quickly to draw the complete taxi-circle on the grid board or as much of it as the size of your grid board allows.



Note: Taxi-circles are explored again in Lesson G3. At this time, just emphasize that all the dots are seven blocks from Nora's house.

T: ***What do you think of this picture?***

S: ***It makes a diamond (or a square).***

If your grid board is not large enough for the complete taxi-circle, ask about missing dots and invite students to point to where they would be.

T (pointing to the grid)**:** ***If you do not have all of these dots on your paper, draw them, and then count how many places are seven blocks from Nora's house.***

When many students have marked the 28 points on their worksheets, ask a student to count the red dots on the grid board. You may need to help the student count if there are corners that cannot be marked because of the limitations of your grid board.

Add this information to the list on the board.

taxi-distance from N to G = 12 blocks
taxi-distance from N to P = 5 blocks
taxi-distance from N to L = 7 blocks

28 red dots

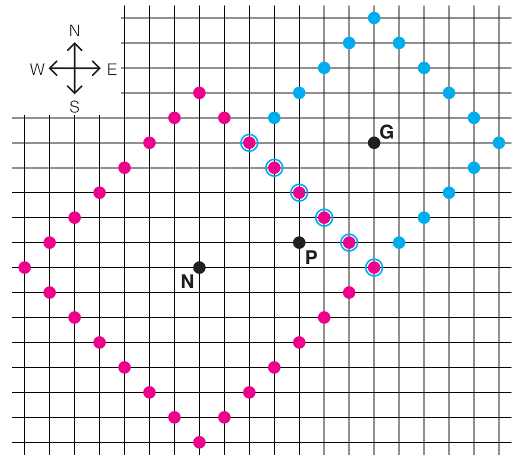
T: ***Nora is not very happy. Linda will be moving to her neighborhood, but there are 28 places where her new house might be. Which place is it? Nora's grandmother gives her another clue.***

Clue 2

T: ***Nora's grandmother says, "Linda will live five blocks from my house." On your worksheet, draw blue dots for all the places that are five blocks from Nora's grandmother's house. If a place is already marked on the map with a red dot, draw a blue circle around it.***

Let students work independently or in pairs for about five minutes. Then invite someone to draw the complete taxi-circle on the demonstration grid board or as much of it as the size of your grid board allows.

Ask a student to count the blue dots on the grid. You may need to help the student count if there are corners that cannot be marked because of the limitations of your grid board.



Add this information to the list on the board.

taxi-distance from N to G = 12 blocks
taxi-distance from N to P = 5 blocks
taxi-distance from N to L = 7 blocks
taxi-distance from G to L = 5 blocks

28 red dots

20 blue dots

T: ***Now do you know where Linda will live?*** (No) ***Where might Linda's new house be?***

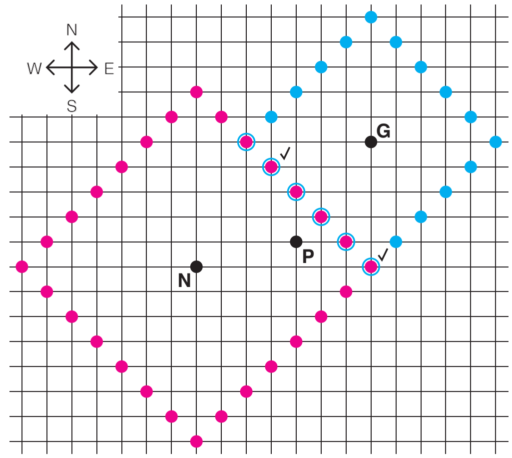
Let a student point to the new set of possibilities (red dots circled in blue).

T: ***Nora decides to call Linda and ask her which of the six places is the correct one.***

Clue 3

T: ***Linda gives Nora another clue. She says, "When I move to your neighborhood, I'll be one block closer to the park than you are."***
(Point to the red-blue dots.) ***Do any of these six places fit Linda's information?Where could Linda's new house be?***

When a place is suggested by a student, check the taxi-distance from that place to the park and from Nora's house to the park. Since the taxi-distance from **N** to **P** is five blocks, the places you are looking for are a taxi-distance of four blocks from the park. The students should find that two of the six dots are one block closer to the park than Nora's house. Put a check mark (√) by each of them.



S: ***But we still don't know for sure which of these two places is for Linda's new house.***

Clue 4

T: ***This is the last clue. Nora finds out that there is only one shortest path to walk from her house to Linda's new house. Which dot is for Linda's new house?***

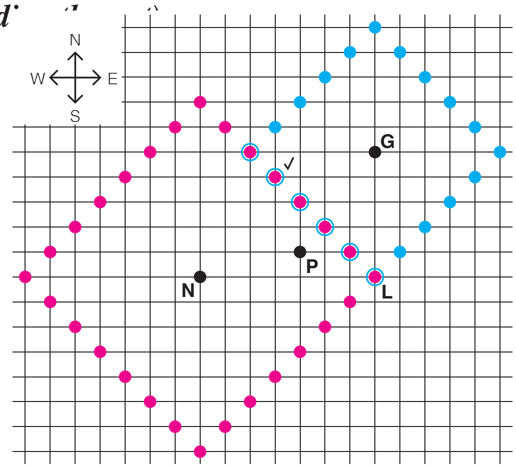
Let a student point to the dot.

T: ***How do you know that dot is for Linda's new house?***

S: ***Because the only shortest path is straight across (d***

Erase the check mark and label the appropriate dot **L**.

T (pointing to the dot that still has a check by it)**:** ***Can you show me several different shortest ways to walk from* N *to here?***



Let three or four students each trace a different shortest path.

T: ***How many shortest paths from* N *to here do you think there are?*** (35)

Accept several responses without comment. Do not expect to receive the exact answer.

S: ***At least 10 or 12.***

S: ***Less than 50.***

Writing Activity

- Invite students to write clues to locate a secret dot on the grid board.
- Suggest that students write about how Nora discovered where her cousin Linda will live.

G2 FRACTIONAL PARTS OF SHAPES AND ________ G2

Capsule Lesson Summary

Divide a rectangle with area 240 cm^2 into halves, fourths, eighths, and sixteenths and find the areas of those fractional parts. Determine many fractional names for $\frac{1}{2}$. Locate 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{16}$ on the number line and draw $\frac{1}{2}$x arrows between points on the number line. Divide a circle with area 270 cm^2 into thirds, sixths, and twelfths and find the areas of those fractional parts. Determine many fractional names for $\frac{1}{3}$.

Materials

Teacher
- Colored chalk or markers
- Blacklines G2(a) and (b)

Student
- Straightedge
- Rectangle and circle cut-outs
- Worksheets G2* and **
- Colored pencils, pens, or crayons

Advance Preparation: Use Blacklines G2(a) and G2(b) to make rectangle and circle cut-outs.

Description of Lesson

Allow students to work in pairs during this lesson. Provide each pair with five rectangle cut-outs and four circle cut-outs.

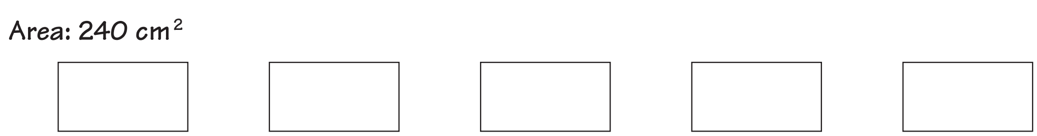
Exercise 1

Put five rectangles of the same size on the board and refer the students to their rectangle cut-outs.

T: ***What shapes are these on the board?*** (Rectangles)
Are they about the same size? (Yes)

They are supposed to be the same size. Each of them has an area of 240 square centimeters.

Record the area above the first rectangle from the left, and suggest students do the same for one of their rectangles.



T (pointing to the second rectangle from the left)**:** ***How can we divide this shape into two pieces of the same size?***

Give a straightedge to a volunteer and ask the student to use it to indicate where the shape could be divided. Draw the line segment yourself while the student holds the straightedge in position. One possibility is shown in the next illustration. Student pairs should do the same on one of their rectangles.

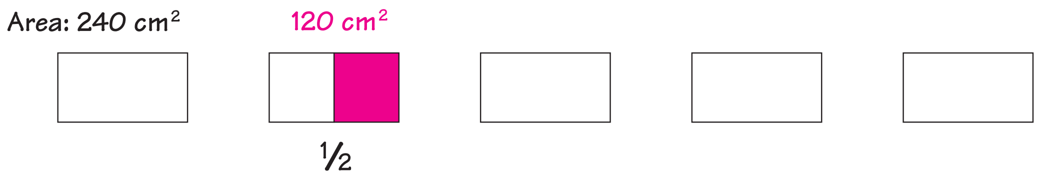
Note: You may like to let students fold a rectangle in half to find an accurate cutting line.

T: ***Let's color one-half of this shape red.***

T: ***What is the area of the red part?***

S: ***120 square centimeters.***

Record 120 cm^2 in red above the second rectangle.



T: ***Are there other ways to divide this shape into two pieces of the same size?***

Allow students to identify as many solutions as possible, but do not draw them on the board.

T (pointing to the third rectangle from the left)**:** ***How can we divide this shape into four pieces of the same size?***

Let a student hold a straightedge in positions that indicate a way to divide the shape while you draw the appropriate line segments. If the student does not divide the shape in half first, strongly urge that this be done because it is easier to see one-half of a shape than to see one-fourth of a shape. Point out to the class that the shape is first being divided in half, and then each piece is being divided in half. One possibility is shown below. Again, student pairs can do the same on one of their rectangles.

T: ***Let's color one-fourth of this shape blue.***

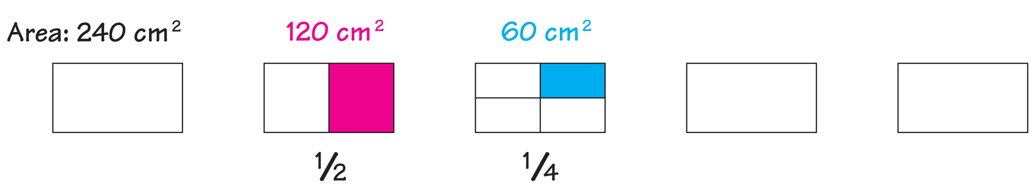
T: ***What is the area of the blue part?***

S: ***60 square centimeters.***

T: ***How do you know?***

S: ***$^1/_2$ of 120 is 60.***

Record 60 cm^2 in blue above the third rectangle.



T: ***Are there other ways to divide the rectangle into four pieces of the same size?***

Allow students to point out as many solutions as possible, but do not draw them on the board

Refer to the rectangle you colored one-half red and then to the one-fourth section as you ask,

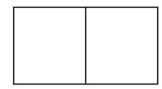
T: ***How many fourths make $^1/_2$?*** (Two)
Another name for $^1/_2$ is $^2/_4$.

Write $^1/_2 = {}^2/_4$ near the pictures on the board.

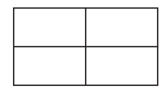
T (pointing to the fourth rectangle from the left)**:** ***How can we divide this shape into eight pieces of the same size?***

If a student starts by estimating about how large one-eighth is, ask if someone has a more accurate way of dividing it. Draw the appropriate line segments when someone suggests the following:

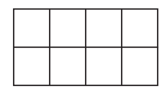
First divide the shape in half…



then divide each of those two pieces in half…



and then divide each of those four pieces in half.



Note: The preceding illustration only shows one of many ways of doing this halving process to divide the shape into eighths. You may wish to let the students show other ways.

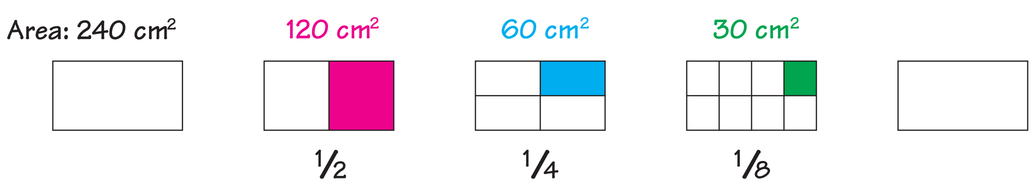
T: ***Let's color one-eighth of this shape green?***
What is the area of the green part?

S: ***30 square centimeters.***

T: ***How do you know?***

S: ***$\frac{1}{2}$ of 60 is 30.***

Record 30 cm^2 in green above the fourth rectangle.



Refer to the rectangles colored one-half red and then to the one-eighth section as you ask,

T: ***How many eighths make $\frac{1}{2}$?*** (Four)

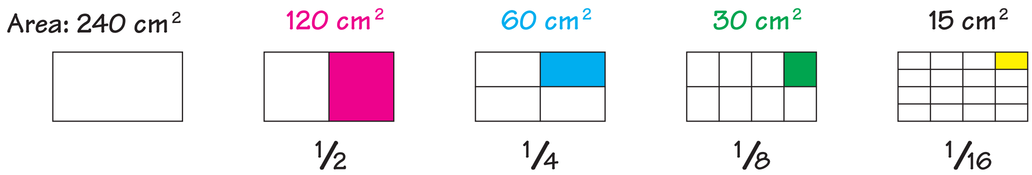
Extend the equalities for $\frac{1}{2}$ to include $\frac{4}{8}$.

T (pointing to the fifth rectangle from the left)**:** ***How can we divide this shape into 16 pieces of the same size?***

S: ***We could divide it into eighths as we did with that one*** (pointing to the fourth rectangle) ***and then divide each of the eight pieces in half.***

When someone suggests this idea, ask that student to hold the straightedge in the positions needed to divide the shape while you draw the appropriate line segments.

Proceed as before and conclude with the class that one-sixteenth of the shape has area 15 cm^2 and that another name for $\frac{1}{2}$ is $\frac{8}{16}$.



T: ***We found several names for $\frac{1}{2}$. Are there still other names for $\frac{1}{2}$?***

S: *$\frac{16}{32}$.*

S: *$\frac{32}{64}$.*

S: *$\frac{64}{128}$.*

On the board write the names for $\frac{1}{2}$ that are suggested. Students may or may not continue the pattern as suggested here; for example, they may include $\frac{3}{6}$ or $\frac{5}{10}$.

$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16} = \frac{16}{32} = \frac{32}{64} = \frac{64}{128}$$

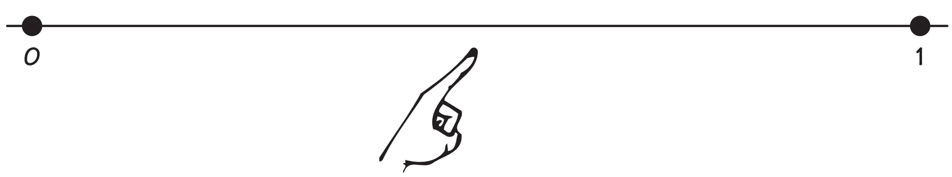
Keep the pictures and the names for $\frac{1}{2}$ on the board for reference in Exercise 2.

Exercise 2

Note: This exercise shows the relationship between the geometry of the previous exercise and the location of ~~various fractions o~~n the number line. If your students do not see this relationship easily, do not belabor the topic but move on to Exercise 3.

On the board draw a line segment about 160 cm long. Draw a dot for 0 near the left end of the segment; draw a dot for 1 near the right end of the segment.

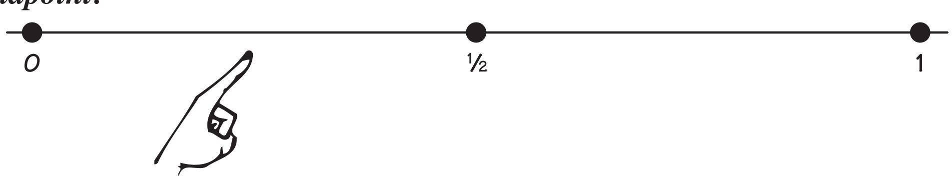
T (tracing the segment between 0 and 1)**:** ***If we divide this segment in half, what would we label the midpoint?***



S: *$\frac{1}{2}$.*

Draw a dot halfway between 0 and 1; label it $\frac{1}{2}$.

T (tracing the segment between 0 and $\frac{1}{2}$)**:** ***If we divide this segment in half, what would we label the midpoint?***



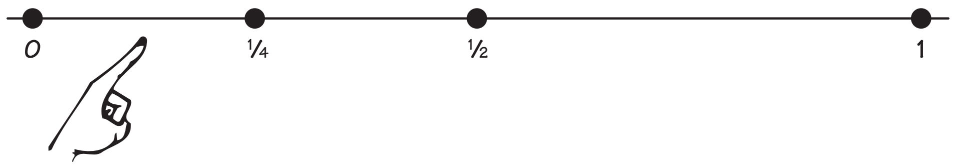
S: *$\frac{1}{4}$.*

Draw a dot halfway between 0 and $\frac{1}{2}$; label it $\frac{1}{4}$. If no one suggests $\frac{1}{4}$, refer to the third rectangle on the board from the preceding exercise as you say,

T: ***We divided this shape in half and then we divided each piece in half. Here we divided this line segment in half and now we are dividing this piece*** (trace from 0 to $\frac{1}{2}$) ***in half.***

Repeat your original question. If students still are having difficulty, simply proceed to Exercise 3.

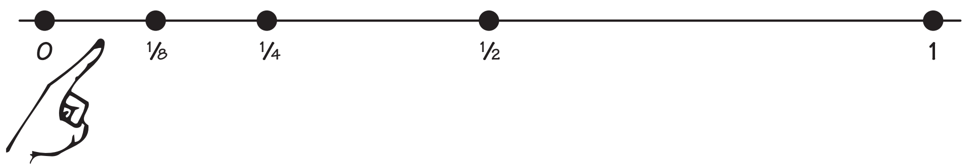
T (tracing the segment between 0 and $\frac{1}{4}$)**:** ***If we divide this segment in half, what would we label the midpoint?***



S: *$\frac{1}{8}$.*

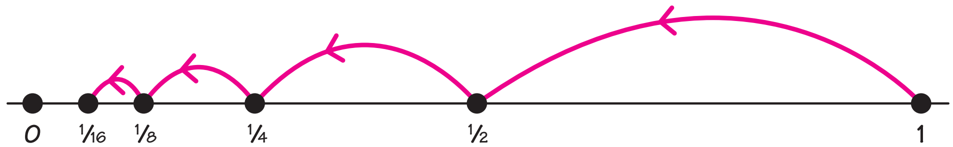
Draw a dot halfway between 0 and $\frac{1}{4}$; label it $\frac{1}{8}$.

T (tracing the segment between 0 and $\frac{1}{8}$)**:** ***If we divide this segment in half, what would we label the midpoint?***



S: *$\frac{1}{16}$.*

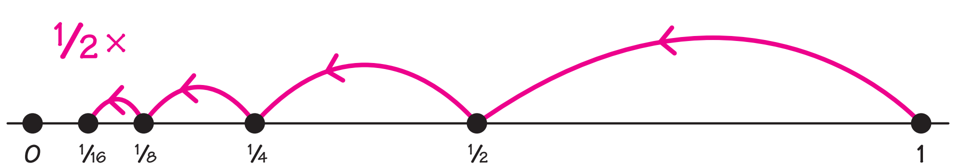
Draw a dot halfway between 0 and $\frac{1}{8}$; label it $\frac{1}{16}$. Then draw these red arrows.



T: ***What could the red arrows be for?***

S: *$\frac{1}{2}$x.*

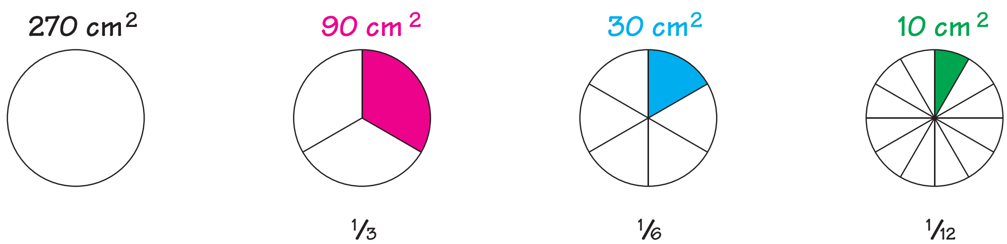
If a student suggests that the red arrows could be for "is more than," agree but indicate in this case they are for $\frac{1}{2}$x. If someone suggests 2x, trace the red arrows in the opposite direction (left to right) and say that if the red arrows went in that direction they could be for 2x. When someone suggests $\frac{1}{2}$x, write the key on the board in red.



Exercise 3

Put four circles all the same size on the board.

In the same manner as Exercise 1, divide the second disk into thirds, the third disk into sixths, and the fourth disk into twelfths. Use the pictures to suggest other names for $\frac{1}{3}$.



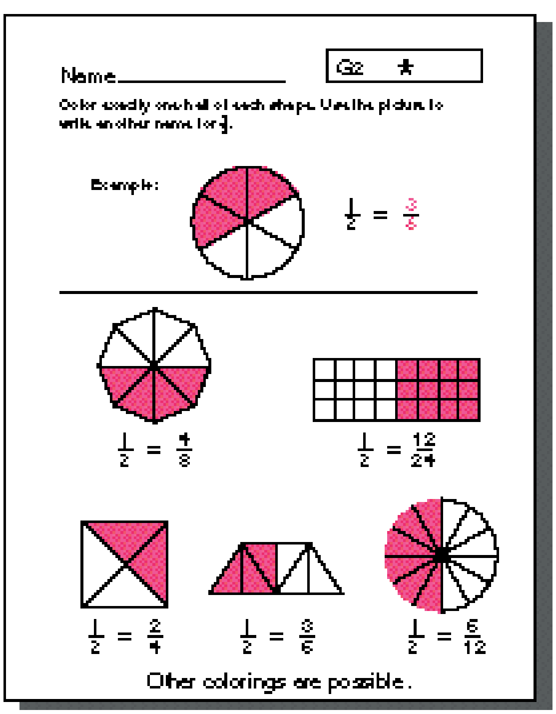
Ask the students to extend the list of names for $\frac{1}{3}$; for example,

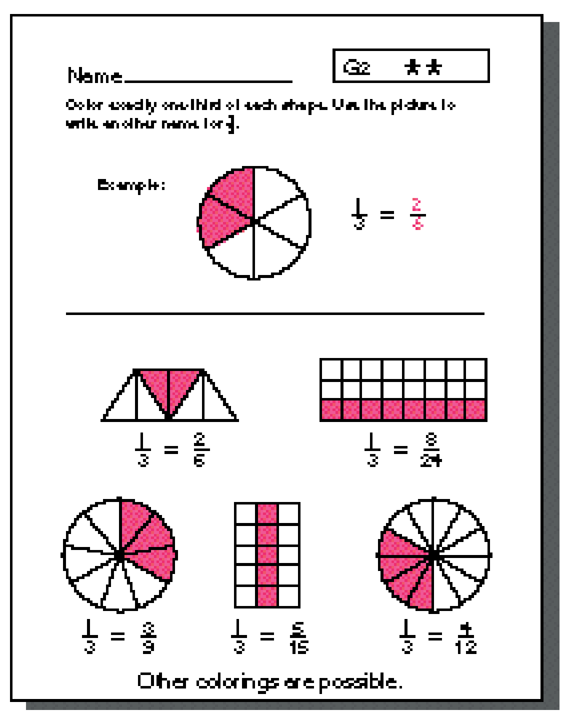
$$\frac{1}{3} = \frac{2}{6} = \frac{4}{12} = \frac{8}{24} = \qquad \frac{3}{9} = \frac{6}{18} = \frac{12}{36} =$$

Worksheets G2* and ** are available for individual work. Students having difficulty finding one–third of a shape may like to use the method of coloring one out of every three pieces.

Center Activity

Place fraction models and manipulatives in a center for further exploration.





Capsule Lesson Summary

Review the idea of a taxi-circle from Lesson G1. Find the number of places at a specific taxi-distance from a given point. Repeat the activity for several taxi-distances and find a pattern for determining this number from the taxi-distance. Relate the notion of a taxi-circle to that of a usual (Euclidean) circle.

Materials

Teacher
- Grid board†
- Colored chalk or markers
- String

Student
- Worksheet G3
- Colored pencils, pens, or crayons

Description of Lesson

Display a grid board. Draw a dot at a cross-point close to the center of the grid. Label it **N**.

T: ***Do you remember how Nora used clues to find the place where her cousin Linda's new home would be?***

Let students comment on the previous taxi-circles lesson (G1).

T: ***Nora's mother gave her the clue that the taxi-distance from Nora's house to Linda's new house is seven blocks.***

Write this information on the board.

Taxi-distance from N to L is 7 blocks.

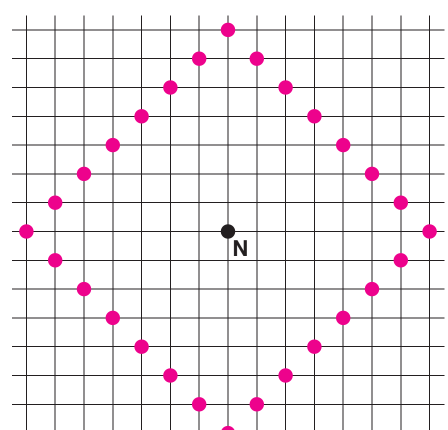
T: ***Who can show us some places that are seven blocks from Nora's house?***

Let several students trace seven-block shortest paths from Nora's house. Mark each ending point with a red dot. Some students may recall the pattern of red dots and may remember that there were 28 such places.

T: ***How many red dots like these can we draw?***
What kind of pattern are the dots forming?

S: ***A diamond (or a square).***

Encourage students to use the diamond or square shape pattern to find the remaining points in the taxi-circle. If someone, as a result of this suggestion, points to a place that is not seven blocks from Nora's house, check the taxi-distance for that point. Continue until the taxi-circle is complete or as much of it is drawn as the size of your grid board allows.



T: ***Count the red dots while I draw a chart on the board.***

†See the Note On Grids in the introduction to this strand.

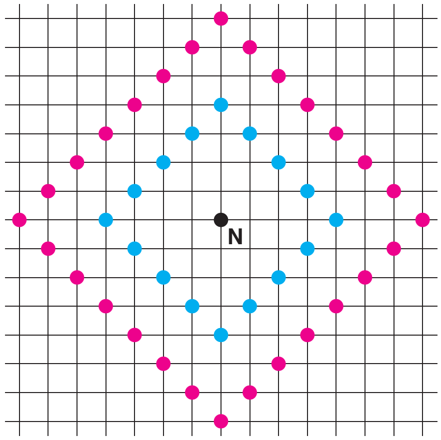
Invite a student to go to the board and count the red dots out loud. You may need to help the student count if there are places that cannot be marked because of the limitations of your board.

S: ***1, 2, 3, …, 28.***

Record 28 in the chart to the right of 7 blocks.

Taxi-distance from N	How many places?
7 blocks	28

Repeat this activity to find the places that are a taxi-distance of four blocks from Nora's house. See the blue dots in the next illustration.



Taxi-distance from N	How many places?
4 blocks	16
7 blocks	28

T: ***How many places are a taxi-distance of zero blocks from Nora's house?***

S: ***Just one—Nora's house.***

Record this information in the chart.

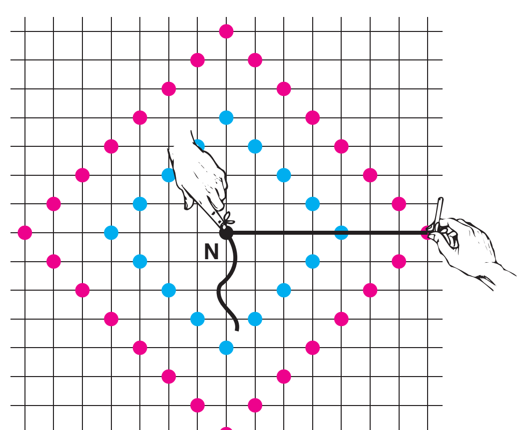
Taxi-distance from N	How many places?
0 blocks	1

Trace each taxi-circle in turn as you say,

T: ***These places (red dots) are a taxi-distance of seven blocks from Nora's house. These places (blue dots) are a taxi-distance of four blocks from Nora's house. These diamonds (squares) are called taxi-circles. In a few minutes we'll see why they are called this.***

Take out a piece of string with chalk attached to one end. Hold the piece of chalk at the red dot seven blocks east of **N** and pull the string taut to **N**.

T: ***I have set this piece of string to the length of a seven-block path. Now, suppose Nora were not restricted to staying on the sidewalks; perhaps she's in the country where there are big fields. She wants to find all the places that are the same distance from her house as this seven-block path. Where are those places? How can we find them?***

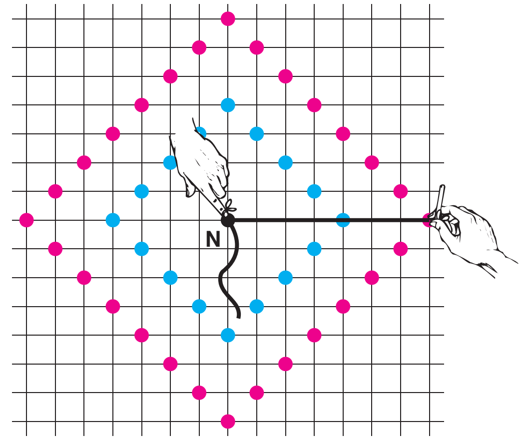


Let the students discuss the problem and experiment with finding possible points the same distance from **N** as the red dot seven blocks directly east of **N**. Use the piece of string to check that a suggested point is the correct distance from **N**. If necessary, lead the class to observe that other points can be found by holding one end of the string at **N** and moving the chalk end (keeping the string taut). Observe with the class that, in fact, all possible points can be found this way.

S: ***All the places are on a circle with the center at N.***

If no one mentions this, suggest it yourself. Carefully swing the chalk to complete the circle.

T: ***Who can show us how to draw a regular circle for a four-block path centered at N?***

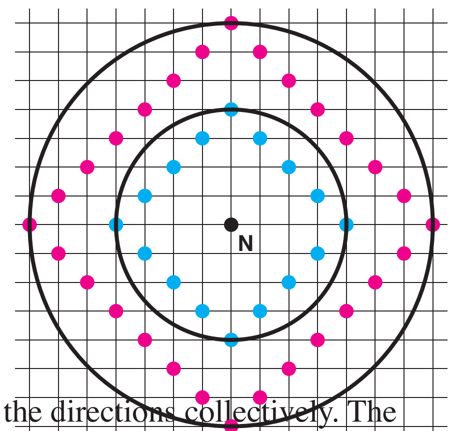


Invite a student to tell you how to hold the string and the chalk. Then draw the circle.

T (drawing the circle)**:** ***How many blue dots am I going to hit?***

S: ***Four.***

T: ***You get regular circles if you are in the country and do not have to follow streets. Taxi-circles are the kind of circles you get if you have to follow the streets like a taxi.***



Distribute copies of Worksheet G3 and colored pencils. Go over the directions collectively. The small circles in the chart are for students to indicate a color code for taxi-circles. You may like to let students work with a partner on this worksheet.

If some students finish the worksheet quickly, ask them to extend the table for other numbers. Do not give away the fact that the number of places is always four times the taxi-distance from **N**. When most students have completed the chart, go over the answers collectively, filling in the chart on the board. Extend the chart to include other entries that your students have found. Perhaps your chart will look like this one.

Taxi-distance from N	How many places?
0 blocks	1
1 block	4
2 blocks	8
3 blocks	12
4 blocks	16
5 blocks	20
6 blocks	24
7 blocks	28
8 blocks	32
9 blocks	36
10 blocks	40
11 blocks	44

T: ***How many places are a taxi-distance of 20 from N?***

S: ***80.***

T (pointing to the right column of the chart)**:** ***What kind of numbers are these?***

S: ***Multiples of 4.***

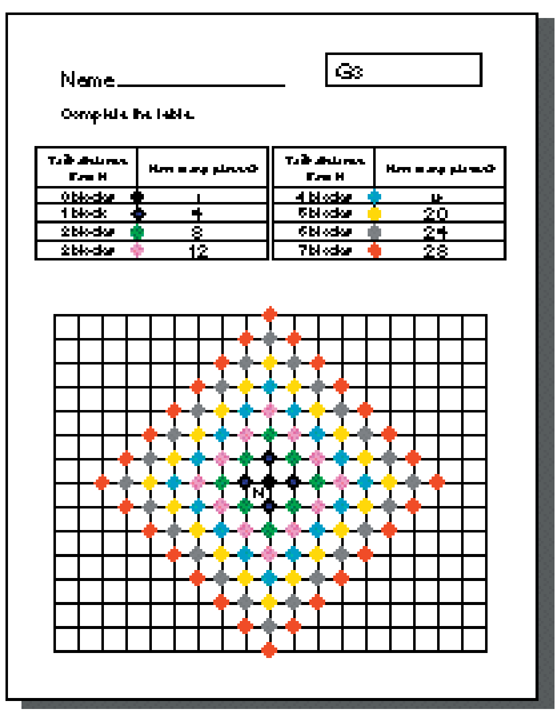
T: ***All these numbers except for 1 are multiples of 4.***

G3

Briefly indicate the 4x relation between the entries in the first and second columns (except for the 0–1 entry). Point to the appropriate places in the chart as you indicate the 4x relation.

Home Activity

Make a grid (there are several at the back of the Blackline section) with a point labeled **N**. Send copies home with students along with a problem to solve with family members. For example, ask them to find the points a taxi-distance of three (or six) blocks from Nora's house.



Capsule Lession Summary

Examine a tape measure and review how to use it to measure in centimeters. Introduce the decimeter unit of length and the equality 10 cm = 1 dm. Convert some decimeter lengths to centimeters and vice-versa. Find shortest paths between points in an irregularly shaped yard, and measure the length of these paths in centimeters and in decimeters.

Materials

Teacher
- Metric tape measure
- Colored chalk
- Straightedge

Student
- Metric tape measure
- Centimeter-rods (white Cuisenaire® rods), and decimeter-rods (orange Cuisenaire® rods)
- Worksheets G4*, **, ***, and ****

Description of Lesson

Exercise 1

Provide each pair of students with a metric tape measure, ten centimeter-rods (white Cuisenaire® rods), and one decimeter-rod (orange Cuisenaire® rod).

T: ***Place your white rods on the side of the centimeter tape that is numbered. What is the distance from one black mark to the next?***

S: ***One unit (white rod) or one centimeter.***

Say, "centimeter," as you write it and its abbreviation on the board.

centimeter (cm)

T: ***Turn the tape over to the other side. Use the rods to help answer this question. What is the distance from one orange mark to the next?***

Students should note the length of an orange rod or ten white rods. If someone suggests ten white rods, encourage them to notice that one orange rod is the same length as ten white rods.

T: ***Do you know the name for this length?***

Say, "decimeter," as you write it and its abbreviation on the board.

decimeter (dm)

T: ***Use your rods, if necessary, and look at each side of the tape. How many centimeters is one decimeter?***

S: ***Ten.***

T: ***Ten centimeters is the same length as one decimeter, just as ten white rods is the same as one orange rod.***

Begin a list of equalities on the board.

10 cm = 1 dm

G4

On the board, draw an enlargement of part of the metric tape measure as shown below.

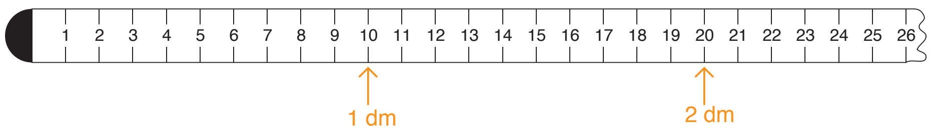
T (pointing to the mark for 10 cm)**:** ***Since 10 cm = 1 dm, this mark is also for 1 dm.***

Draw an orange arrow at the mark labeled 10. Ask students to notice that there is an orange mark on the other side of the tape measure at 10.

T (pointing to the mark for 20 centimeters)**:** ***This mark is for 20 centimeters; it is also for how many decimeters?***

S: ***Two.***

Draw an orange arrow at the mark for 20. Again, students should observe that there is an orange mark on the other side of the tape measure at 20.



Write 40 cm = ___ dm as you say,

T: ***40 centimeters is how many decimeters?***

S: ***Four.***

Complete the equality on the board.

Repeat this activity to complete these equalities.

20 dm = 2 cm
10 dm = 1 cm
40 dm = <u>4</u> cm

70 dm = <u>7</u> cm
<u>130</u> dm = 13 cm

Write 25 cm = ___ dm as you say,

T: ***25 centimeters is how many decimeters?***

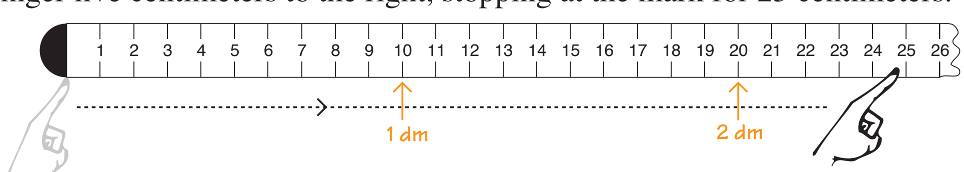
Let students make suggestions. Perhaps someone will note that 25 centimeters is between 2 decimeters and 3 decimeters.

Put your right index-finger on the mark for 0 in the picture on the board. Move your finger to the right. As you pass the first orange arrow say, "1 decimeter." Continue until you get to the second orange arrow; stop and say, "2 decimeters."

T: ***25 centimeters is more than 2 decimeters. How many more centimeters is 25 centimeters than 2 decimeters?***

S: ***Five.***

Move your finger five centimeters to the right, stopping at the mark for 25 centimeters.



T: ***25 cm = 2 dm 5 cm. What is another way to write 2 dm 5 cm?***

S: ***$2\frac{1}{2}$ dm.***

S: ***2.5 dm.***

If no one suggests 2.5 dm, compare orange rods to dollars and white rods to dimes, and ask how to write two dollars and five dimes; then return to your original question.

Complete the equality on the board. You should have this list.

10 cm = 1 dm
20 cm = 2 dm
40 cm = 4 dm
70 cm = 7 dm
130 cm = 13 dm
25 cm = 2.5 dm

Do a similar activity to complete these equalities. Students should refer to their own tape measures since the picture on the board is not sufficient to discuss these problems.

53 cm = 5.3 dm
78 cm = 7.8 dm
6 cm = 0.6 dm

Exercise 2

T (referring to the tape measure picture on the board)**:** ***Who can point to the mark for 1.5 decimeters?***

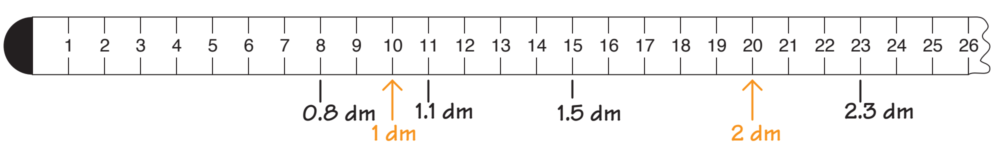
If necessary, help the volunteer by pointing to the arrow indicating 1 decimeter and by counting 5 centimeters past it. Label the appropriate mark 1.5 dm as in the next illustration.

T: ***1.5 decimeters is how many centimeters?***

S: ***15.***

Record this equality on the board.

Continue this activity asking students to locate, in turn, marks for 2.3 dm, 1.1 dm, and 0.8 dm.



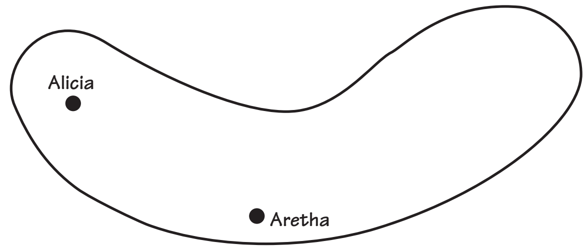
1.5 dm = 15 cm
2.3 dm = 23 cm
1.1 dm = 11 cm
0.8 dm = 8 cm

G4

Exercise 3

Erase the board and draw this picture.

T: ***This is a picture of someone's yard. The dots show you where Alicia and Aretha are standing in the yard. Aretha wants to walk to where Alicia is standing and she wants to take as short a path as possible. Who can trace the shortest path from Aretha to Alicia?***



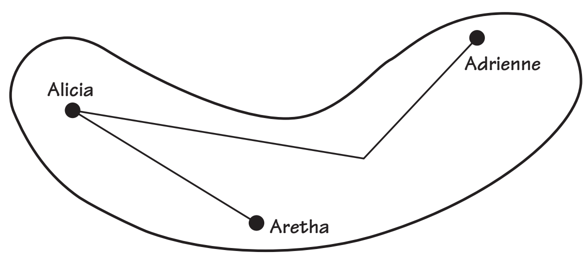
The student who volunteers should trace the line segment between the dots. Draw the segment yourself.

Help a student measure the segment in centimeters. Emphasize that you place the mark for 0 cm (the end of the tape) at one end of the segment and read the measure at the mark that lands at the other end of the segment. Then ask how long the segment is in decimeters.

Draw another dot for Adrienne as in the next illustration. Demonstrate Adrienne's manner of walking along a zigzag path as you describe it, but do not draw a path yet.

T: ***Adrienne wants to walk to where Alicia is standing without going out of the yard. She cannot take a completely straight path so she walks along a zigzag path. She walks in a straight line, turns, then walks in a straight line again, turns, and so on until she reaches Alicia. How might Adrienne walk to Alicia following a short path and always staying in the yard?***

After a student traces a zigzag path which stays inside the yard, draw the zigzag in the picture. This is one possibility; it is not the shortest such path.



T (tracing the zigzag)**:** ***How can we find out how long this path is?***

S: ***Measure the two pieces and then add their lengths.***

Help students measure the pieces of the zigzag in centimeters. Ask the class what the lengths are in decimeters and record them on the board. Then invite a student to do the addition on the board.

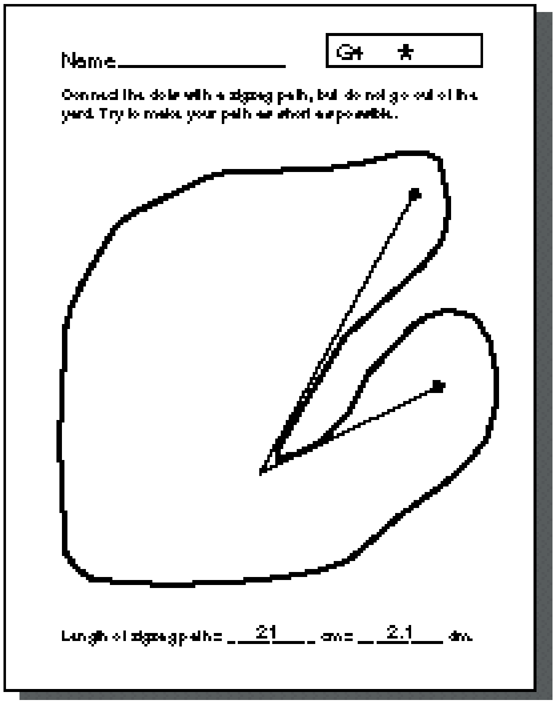
$$\begin{array}{r} 5.7 \text{ dm} \\ +\ 7.4 \text{ dm} \\ \hline 13.1 \text{ dm} \end{array}$$

Repeat this activity if there is obviously a shorter path than the zigzag drawn and compare the lengths of the two paths.

Worksheets G4*, **, ***, and **** are available for individual work.

Home Activity

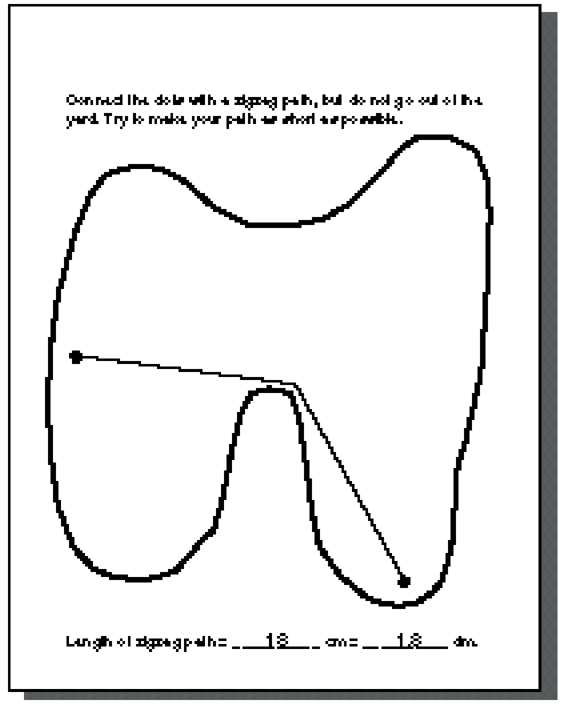
Suggest that parents/guardians practice adding decimal numbers with their child.



Name_______________ G4 ★

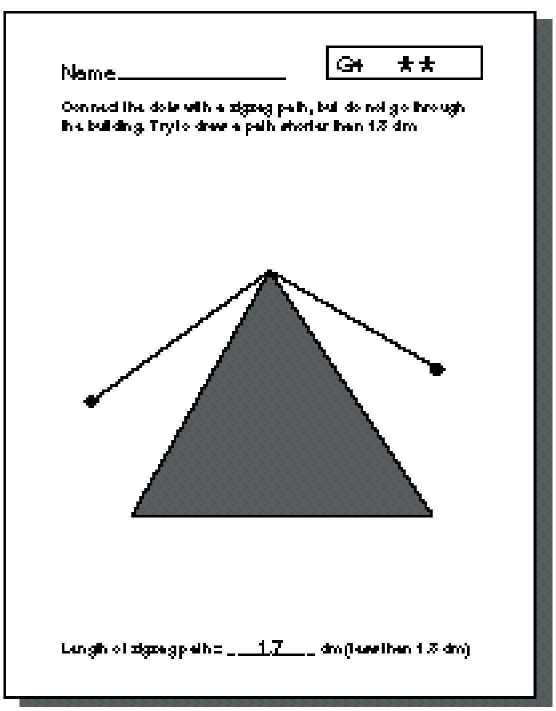
Connect the dots with a zigzag path, but do not go out of the yard. Try to make your path as short as possible.

Length of zigzag path = __21__ cm = __2.1__ dm.



Connect the dots with a zigzag path, but do not go out of the yard. Try to make your path as short as possible.

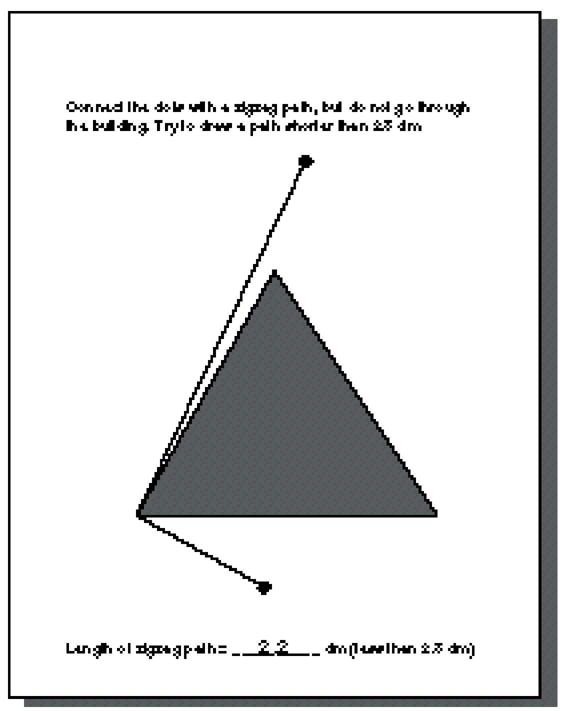
Length of zigzag path = __18__ cm = __1.8__ dm.



Name_______________ G4 ★★

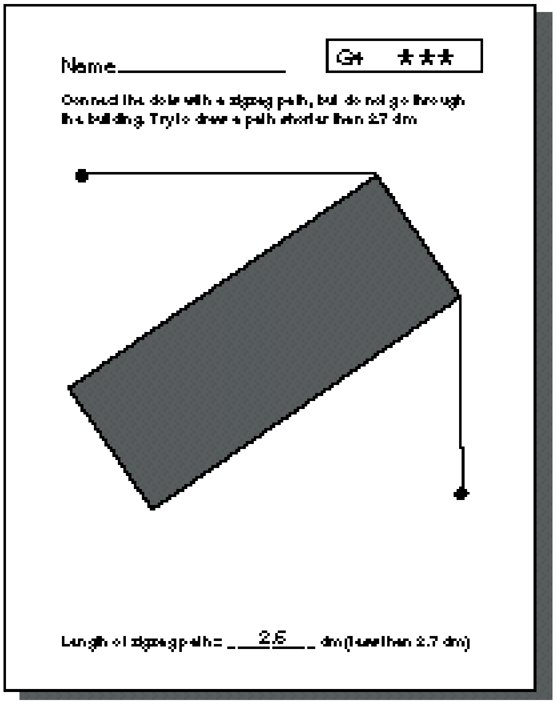
Connect the dots with a zigzag path, but do not go through the building. Try to draw a path shorter than 1.8 dm.

Length of zigzag path = __1.7__ dm (less than 1.8 dm)



Connect the dots with a zigzag path, but do not go through the building. Try to draw a path shorter than 2.3 dm.

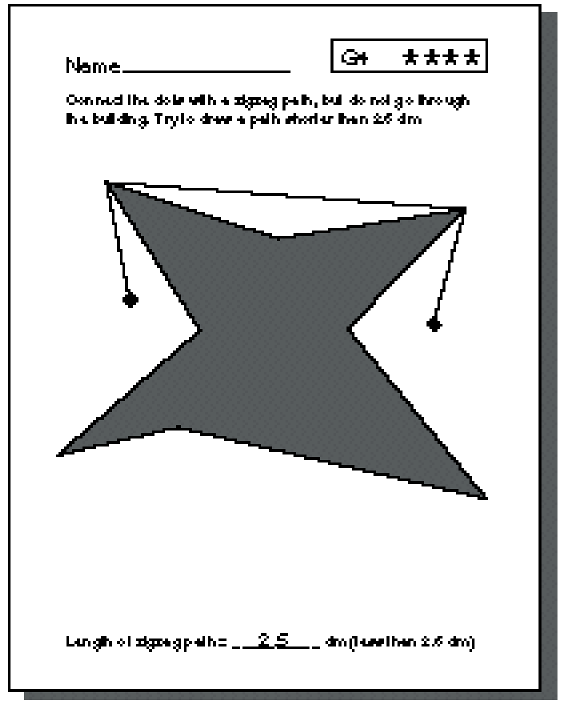
Length of zigzag path = __2.2__ dm (less than 2.3 dm)



Name_________________ G4 ★★★

Connect the dots with a zigzag path, but do not go through the building. Try to draw a path shorter than 2.7 dm.

Length of zigzag path = __2.6__ dm (less than 2.7 dm)



Name_________________ G4 ★★★★

Connect the dots with a zigzag path, but do not go through the building. Try to draw a path shorter than 2.6 dm.

Length of zigzag path = __2.5__ dm (less than 2.6 dm)

Capsule Lesson Summary

Find the taxi-distance from a given point to a given line by finding the taxi-distances from various points on the line to the given point and taking the shortest distance. Observe some patterns in the sets of numbers generated by looking at the distances from given points to the points along given lines. Notice that all the points having the shortest distance (for a given point and a given line) are along the diagonal of a square with one corner being the given point.

Materials

Teacher	• Grid board[†] • Colored chalk or markers	**Student**	• Worksheets G5 (no star), * and ** • Colored pencils, pens, or crayons

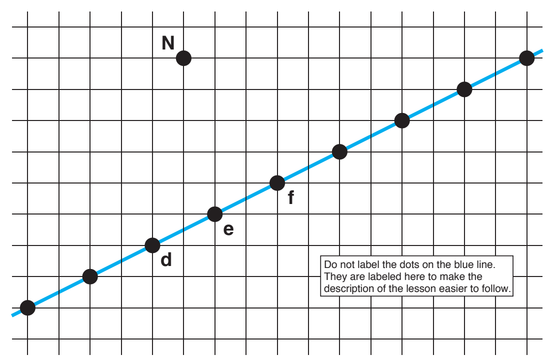
Description of Lesson

Exercise 1

On your grid board, draw the picture shown below. Be careful to give the blue line the correct slope. If you draw one dot, you can get to another dot on the blue line segment by

- moving to the right two spaces and then moving up one space; or
- moving to the left two spaces and then moving down one space.

The dot labeled **N** is positioned five spaces up and one space to the left of the dot labeled **e.** If your grid board is larger than the one pictured here, extend the blue line segment as far as possible in both directions.



Briefly review the story of Nora's neighborhood and how to find the taxi-distance between two points. Write this information on the board.

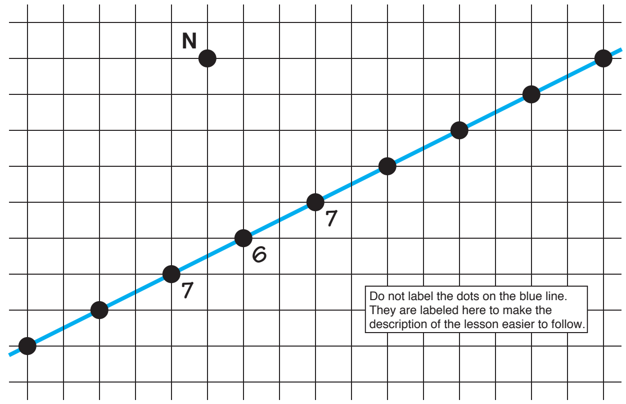
taxi-distance = length of a shortest path[††]

T: ***On this map the blue line is for part of the path of an underground railway, or a subway. The dots along that line are the subway stations—places where people can get on or off the train. Nora wants to find which subway station is closest to her house*** (point to **N**)**.**

Let students point on the map to which subway stations they think are closest to Nora's house. Most likely **d**, **e**, and **f** will be suggested.

[†]See the Note on Grids in the introduction to this strand.
[††]This is an abbreviated way of saying that the taxi-distance from one point to another is the length of a shortest path between those two points.

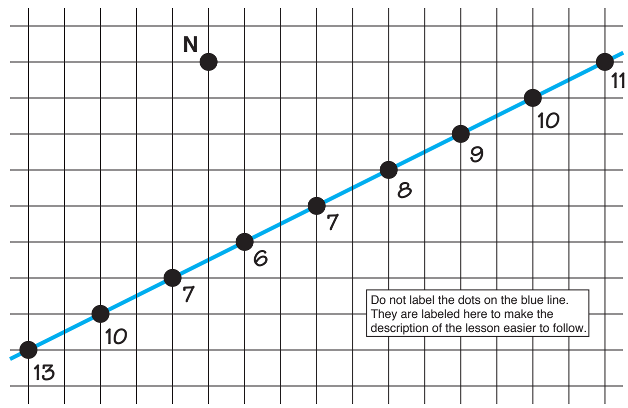
When a student suggests a possible closest station, ask the student to find the taxi-distance from Nora's house to that station. If correct, record the taxi-distance in the picture near the dot for that station.



The illustration here only shows the taxi-distance only from **N** to **d**, **N** to **e**, and **N** to **f**; your students may have considered dots other than these.

Give each student a copy of Worksheet G5 (no star). Ask students to find the taxi-distance to each of the stations on the map and to record those distances next to the dots.

Let the class work independently or in pairs for a few minutes. Then invite students to finish putting numbers (taxi-distances) next to the dots on the grid board. Note that the picture on the board may have a shorter blue line segment than the one on the worksheet.



T: ***Which station is closest to Nora's house?***

S: ***The one six blocks from Nora's house.***

On the board, erase the dot for that station and redraw it in red. Students should draw a red ring around that dot on their papers.

T: ***Do you see any number patterns in this picture?***

S: ***There is a 7 on each side of 6.***

S: ***If you move that way*** (pointing to the right) ***from 6, the numbers go up by 1 each time.***
If you move that way (pointing to the left) ***from 6, the numbers go up by 3 each time.***

Note: When you extend the blue line beyond the station 11 blocks from **N** the pattern changes. The next station will be 14 (11 + 3 = 14) blocks from **N** and the pattern is 3 more for each successive station.

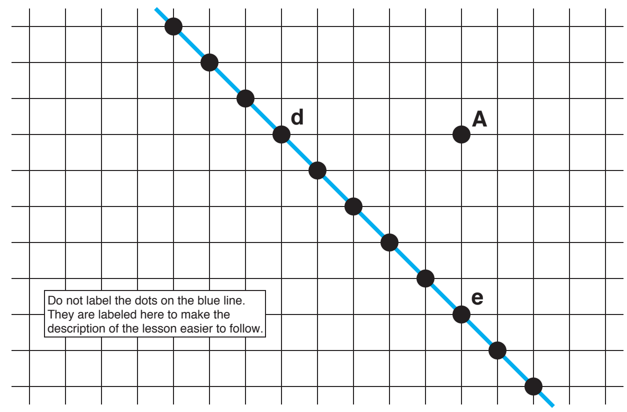
Ask students for the taxi-distances from Nora's house to stations not shown on the map on the grid board. In this picture as you move further to the right, you meet stations whose taxi-distances from Nora's house are 14, 17, 20, 23, …; as you move further to the left, you meet stations whose taxi-distances from Nora's house are 16, 19, 22, 25 … .

Exercise 2

Erase the grid board and then draw the picture shown in the next illustration. Be careful to give the line the correct slope. If you draw one dot, you can get to another on the blue line segment by

- moving one space to the right and one space down; or
- moving one space to the left and one space up.

The dot labeled **A** is positioned five spaces to the right from **d** and five spaces up from **e**. If your grid board is larger than the one pictured here, extend the blue line as far as possible in both directions.

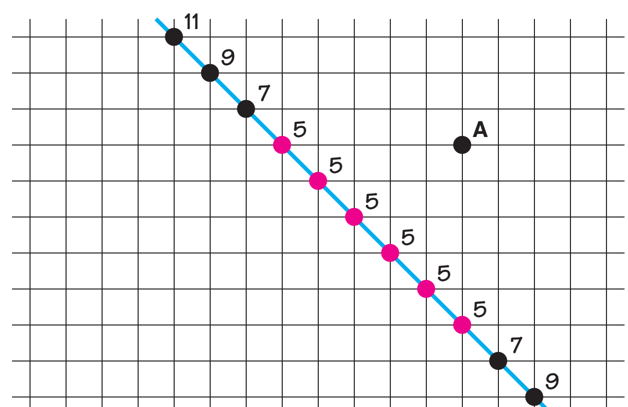


T: ***In another part of town there is a sports arena. After people go to the arena to see a basketball game or some other event, many of them ride the subway home. Most people want to board the subway at a station as close as possible to the arena.***

Turn your worksheet over to find a map like the one on the board. Is there a station close to the arena?

As students suggest possibilities, let them record the taxi-distances from the arena to those stations on the board. It is likely students will express surprise that there are several closest stations.

Continue until all six closest stations have been found. Redraw the corresponding dots in red. Collectively label the rest of the dots with the taxi-distances from the arena to the represented stations.

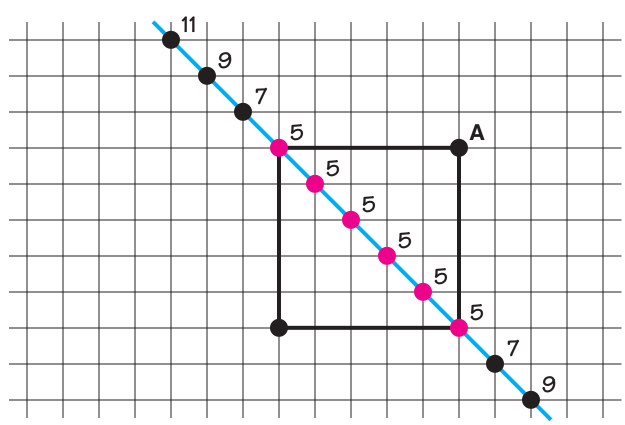


T: ***What kind of number patterns do you see in this picture?***

S: ***There are six 5's and then you count by twos moving in either direction.***

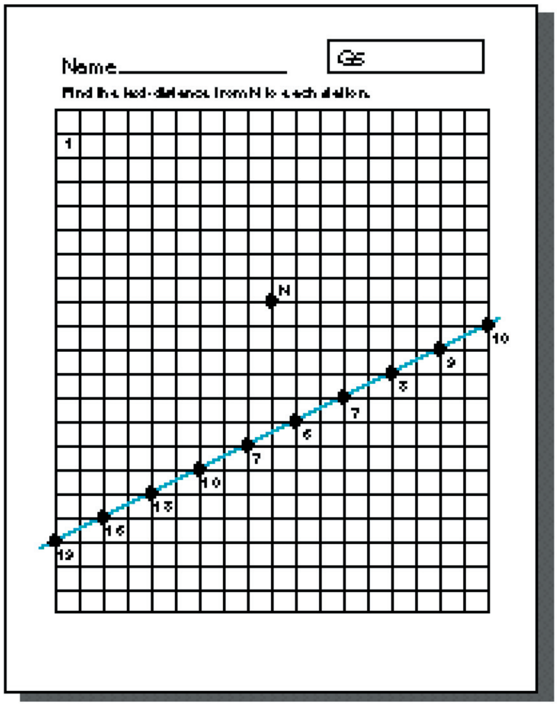
T: ***We found six different stations that are closest to the arena. Is there another place besides the arena that is a taxi-distance of five blocks from these same stations*** (point to each of the six red dots)***?***

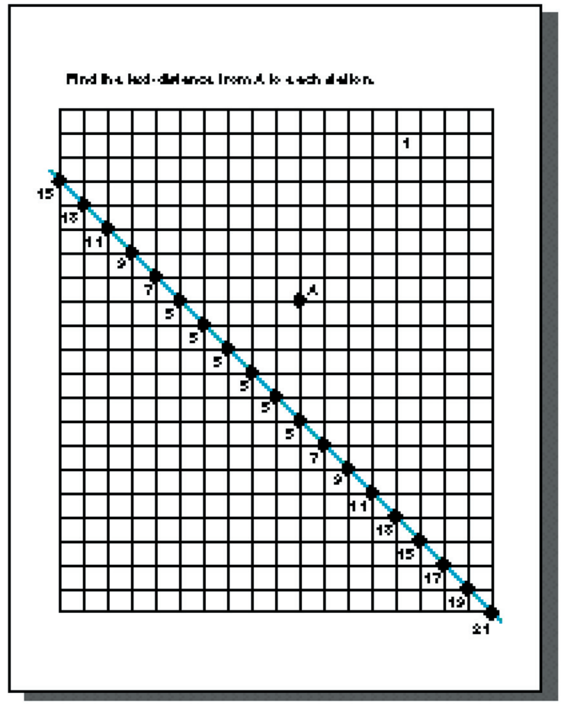
Allow time for thought. When a possible place is suggested, check the taxi-distance from the place to each of the six stations marked in red on the map. Continue until the correct place is found, and then draw a dot there and the square shown below. If no one finds this place, draw the upper right part of the square and indicate that the left side of the blue line should be checked.

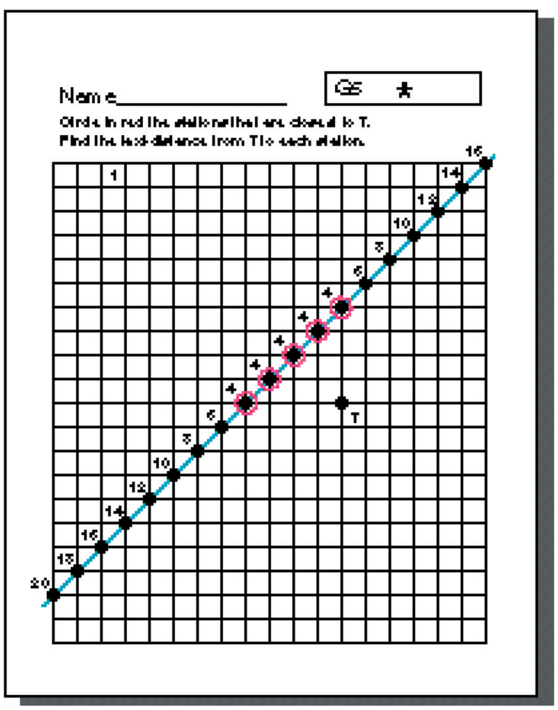


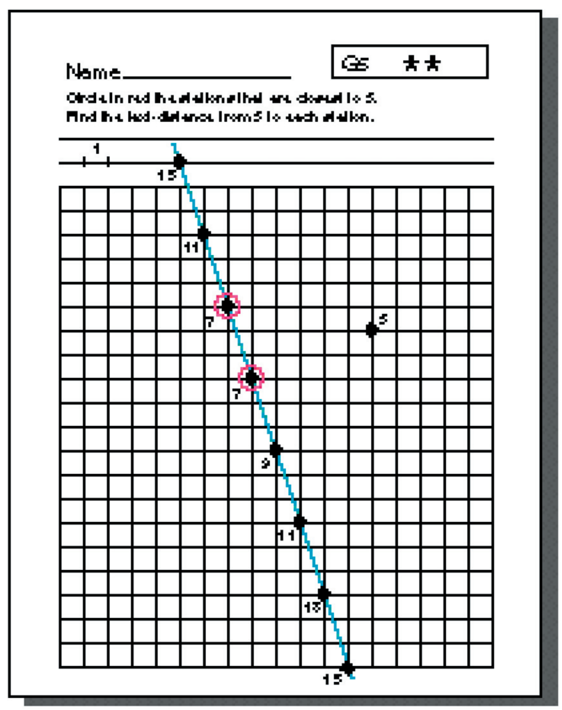
G5

Worksheets G5* and ** are available for individual or partner work. Conclude the lesson by asking for the number patterns in each picture (see the answer keys below.)









G5*: There are five 4's and then the numbers increase by twos in either direction.

G5**: There are two 7's, and then the numbers increase by fours moving upward and increase by twos moving downward. If the blue line were extended downward, there would be a station 17 blocks from **S**. After that station, successive stations would be four more blocks away.

G6 LENGTH OF SPIRALS

Capsule Lesson Summary

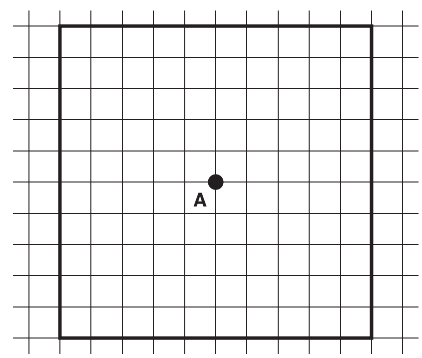
Find the perimeter of a square with sides 10 cm in length. Draw a specified spiral starting at the center of the square and ending at one corner of the square. Find the length of this spiral using an organized counting procedure. Compare the length of the spiral to that of a meter stick. Repeat the activity with a square having 20 cm sides.

Materials

Teacher
- Grid board†
- Colored chalk or markers
- Meter stick

Student
- Worksheets G6(a) and (b)
- Colored pencils, pens, or crayons

Description of Lesson



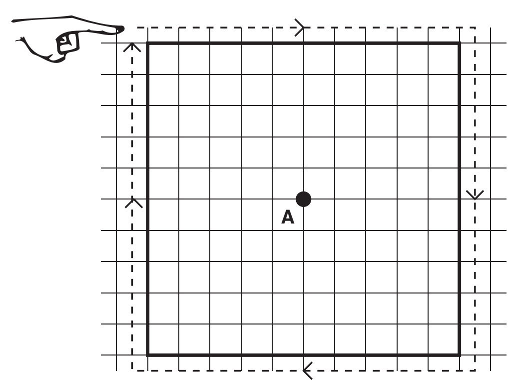
Distribute copies of Worksheet G6(a) and red pencils. Display a grid board and draw a square on it, 10 units on each side. Label the center point of the interior of the square **A**. Your grid board should look like the picture on Worksheet G6(a).

T (tracing the large square)**:** ***What shape is this?***

S: ***A square.***

T: ***On your worksheet the grid squares are 1 cm on a side. What is the length of a side of the large square?***

S: ***10 cm.***



T: ***If a bug starts here*** (point to one of the corners of the large square) ***and walks completely around the big square*** (trace around the square)***, how far will the bug have walked?***

S: ***40 cm.***

T: ***How do you know?***

S: ***I counted 10, 20, 30, 40.***

T: ***40 cm is the perimeter of this square.***

The following dialogue describes a special spiral starting at the center of this square. Direct students to draw red spirals on their worksheets just as you do so on the grid board.

T: ***Take your red pencil. Start at* A *and go up 1 centimeter; then go 1 centimeter to the right. Now go down 2 centimeters and then 2 centimeters to the left; go up 3 centimeters and then 3 centimeters to the right; go down 4 centimeters and then 4 centimeters to the left. What are we drawing?***

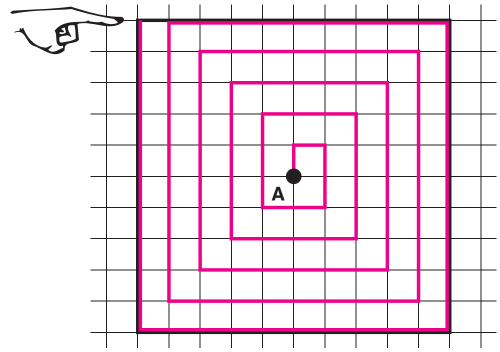
S: ***A whirl.***

†See the Notes on Grids in the introduction to this strand.

S: ***A spiral.***

T: ***Continue drawing the spiral in the same manner. You can draw along the sides of the large square but do not go outside of it.***

On the board, complete the spiral within the boundaries of the large square.



T: ***Does everyone have a spiral that ends here? How long do you think the red spiral is?***

Let students make guesses for its length; you may like to record some guesses on the board.

T: ***Let's see if we can figure out exactly how long the spiral is. How long is a shortest segment in this spiral?***

S: ***1 cm.***

S: ***There are two segments that are 1 cm long.***

Record this information on the board after tracing the two 1 cm segments.

$$\overset{2 \times 1}{2}$$

Then, trace the first (vertical) 2 cm segment in the spiral on the board.

T: ***How many 2 cm segments are there in the spiral?***

S: ***Two.***

$$\overset{2 \times 1}{2} + \overset{2 \times 2}{4}$$

Continue until you have this addition problem on the board.

$$\overset{2 \times 1}{2} + \overset{2 \times 2}{4} + \overset{2 \times 3}{6} + \overset{2 \times 4}{8} + \overset{2 \times 5}{10} + \overset{2 \times 6}{12} + \overset{2 \times 7}{14} + \overset{2 \times 8}{16} + \overset{2 \times 9}{18} + \overset{2 \times 10}{20} + 10$$

Note: There are three 10 cm segments because the boundary of the large square cuts off the last segment of the spiral at 10 cm.

T: ***How can we do this addition problem?***

Let students suggest methods, for example:

- Simply add the numbers one after the other.
- Notice that numbers can be paired that add to 22. $4 + \cdots + 18 + 20 + 10 = (5 \times 22) + 10 = 120$
- Notice that numbers can be paired that add to 20. $4 + \cdots + 10 + \cdots + 16 + 18 + 20 + 10 = 6 \times 20 = 120$

Using one of the preceding methods, conclude with your class that the spiral is 120 cm long. Hold up a meter stick as you observe the following:

T: ***The spiral you drew is 120 cm long. Is the length of the spiral more or less that 1 meter?***

S: ***More.***

T: ***How much more?***

S: ***20 cm.***

Record this information on the board.

10 cm square — 120 cm spiral
1 m 20 cm = 1.20 m

Ask students to turn their worksheets to G6(b).

T (holding up a copy of the worksheet)**:** ***Each side of this large square measures 20 cm. What is its perimeter*** (trace the large square on the worksheet)***?***

S: ***80 cm.***

T: ***How long do you think a similar spiral that starts at* A *and fits in this square would be?***

Here are some possible responses.

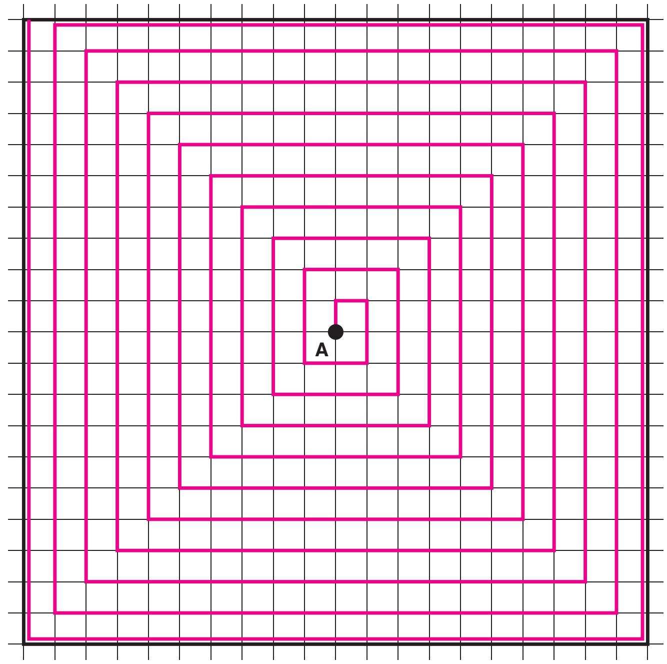
S: ***400 cm.***

S: ***This square has side length twice the other square, so maybe the spiral would be twice as long, 240 cm.***

S: ***About 420 cm.***

S: ***Four of the 10 cm squares fit inside this 20 cm square, so maybe the spiral would be four times as long, 480 cm.***

Direct students to draw a red spiral inside the 20 cm square in the same manner as they did for the 10 cm square.



This illustration is a reduction of what students should draw on their worksheets.

T: ***How long is the spiral you drew?***

Let students work on the problem individually or with a partner for a few minutes.

T: ***There are two 1 cm pieces, two 2 cm pieces, two 3 cm pieces ….***

$$2 + 4 + 6 +$$

How far do we have to go?

S: ***To 40 (or 2*** x ***20).***

S: ***And then there is still one more 20 cm piece.***

If no one responds, ask how long each side of the large square is, or write out the full addition problem as students read it off.

$$2 + 4 + 6 + \ldots + 18 + 20 + 22 + \ldots + 36 + 38 + 40 + 20$$

T: ***How shall we do this addition problem?***

Let students suggest methods, for example:

- Notice numbers can be paired that add to 40.

$$2 + 4 + \cdots + 18 + 20 + 22 + \cdots + 36 + 38 + 40 + 20 = 11 \times 40 = 440$$

- Notice numbers can be paired that add to 42.

$$2 + 4 + \cdots + 18 + 20 + (20 + 2) + (20 + 4) + \cdots + (20 + 18) + (20 + 20) + 20$$

$$(120 - 10) \quad + \quad (10 \times 20) + (120 - 10) \quad + 20$$

$$110 \quad + \quad 310 \quad + 20 = 440$$

- Use the previous problem twice.

$$2 + 4 + \cdots + 18 + 20 + (20 + 2) + (20 + 4) + \cdots + (20 + 18) + (20 + 20) + 20$$

$$(120 - 10) \quad + \quad (10 \times 20) + (120 - 10) \quad + 20$$

$$110 \quad + \quad 310 \quad + 20 = 440$$

If your students only suggest adding the numbers one after the other, show them how to pair numbers that add to 40. Conclude with your class that the length of the spiral is 440 cm.

T: ***How many meters is 440 cm?***

S: ***4 meters with 40 cm left over.***

S: ***4.40 meters.***

T: ***Do you think the ceiling in our classroom is that high?***

Perhaps your class will want to check this with a meter stick, but unless your classroom has a very high ceiling the answer most likely is no.

Record the information about the second spiral on the board.

10 cm square — 120 cm spiral
1 m 20 cm = 1.20 m

20 cm square — 440 cm spiral
1 m 20 cm = 1.20 m

T: ***440 cm is almost 4 x 120 centimeters. Why do you think that's true?***

S: ***Because four 10 cm squares fit inside one 20 cm square.***

If no one suggests this, simply leave the question open.

Capsule Lesson Summary

Find hiking trails following paths between campsites on a map of a campground. Try to use as many paths as possible in a hiking trail, but do not use a path more than once. Discuss what is meant by a round-trip hiking trail, and try to find round-trip trails using as many paths as possible.

Materials

Teacher
- Blackline G7
- Colored chalk or markers
- Overhead projector (optional)

Student
- Campground map
- Colored pencils, pens, or crayons
- Worksheets G7(a) and (b)

Advance Preparation: Use Blackline G7 to make copies of the campground map used in Exercise 1 and 2. Make copies for students and an overhead transparency for display. You may prefer to draw a large copy of this map on the board, but in this case be prepared to redraw the map as needed.

Description of Lesson

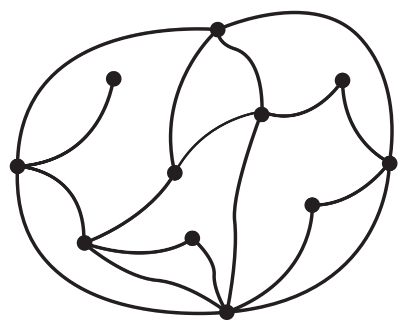
Begin the lesson with a brief discussion about camping and hiking.

T: ***Have any of you ever gone camping? Were there some hiking trails?***

Let students freely share their experiences for a few minutes.

Exercise 1

Display the campground map on Blackline G7 and refer students to their copies.



Note: You may like to make a very big copy of this campground map to put on the floor and let students actually walk the paths.

T: ***Today I want to tell you about a campground with some hiking trails. This is a map of the campground. What do you think the dots are for?***

S: ***Campsites.***

Invite a student to count the campsites aloud as you point to each dot.

S: ***1, 2, 3, …, 11. There are 11 campsites.***

Trace several cords in the map as you say,

T: ***These are paths which were made through the woods so people could hike from one campsite to another. How many paths are there?*** (18)

Invite a student to count the paths.

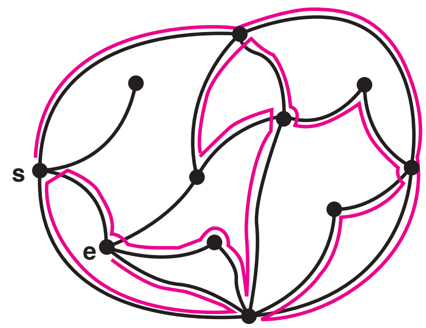
T: ***Suppose we are at this campsite*** (point to any dot) ***and we want to take a hike. Let's say that we will not take a path more than once on our hike.***

Invite several students to trace the paths in a hiking trail. Encourage them to find examples that are both long and short. Do not allow a student to use a path more than once in a trail. Sometimes note how many paths a student uses in the hiking trail he or she traces.

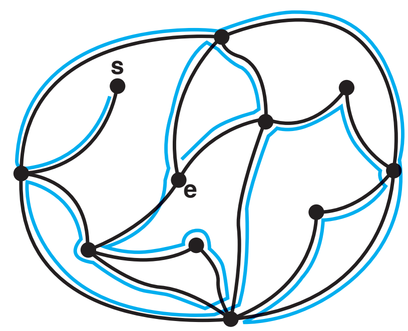
T: ***Did any of the hiking trails you suggested use most or all of the paths at this campground?***

S: ***This hiking trail uses all but two paths.***

Invite students to try to find hiking trails that use all or most of the paths. Tell students that they can start a hike any place they want; it does not have to start at a particular point. Suggest, however, that students note where they start and where they end.



Perhaps a student will find a hiking trail that uses all the paths, such as the one shown here in blue.



Do not force observations about the starting and ending point, but some students may notice the following:

1) The site that connects with only one path can be the starting or ending point of a trail, but only the starting or ending point of a trail. Otherwise, a trail will omit that path.
2) The only trails that use all the paths are ones that start and end at the same points as this blue trail, or vice versa.
3) If the starting and ending points are different, then the trail uses an odd number of paths at the starting and ending points, but an even number of paths at every other point.

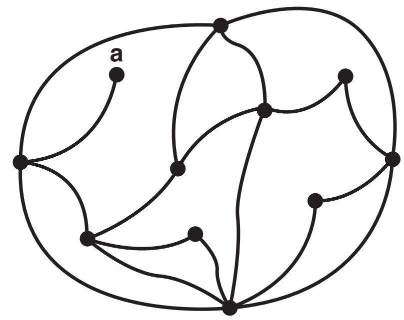
At this time, you may like to give students Worksheet G7(a) and ask them to find a hiking trail at this campground using all the paths.

Exercise 2

It is likely that in Exercise 1 (Blackline G7) many students initially found hiking trails starting and ending at the same place; such trails did not use all the paths.

T: ***Many of you found trails that started and ended at the same campsite. If you start at your campsite, it is reasonable that you might want to end your hike back at the same campsite. A trail that returns to its starting point is called a* round trip*.***

Let students comment on why such trails are called round trips while you write the words on the board. Invite several students to trace round-trip trails. Then point to the dot with only one path at it.



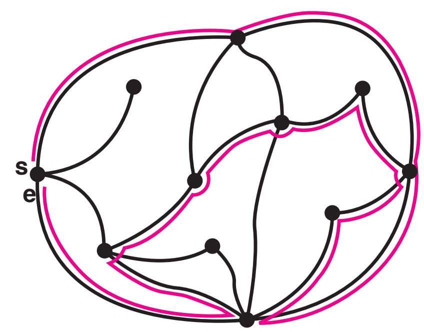
T: ***Can we find a round-trip trail that starts here* (a)*?***

S: ***No, because you would need to return to that campsite along the same path you leave it.***

T: ***Can we include this campsite* (a) *on a round-trip trail that starts somewhere else?***

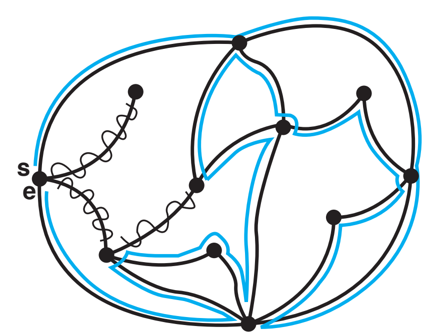
S: ***No, once you take the path to that campsite you cannot leave it without taking the same path again.***

Color a round-trip trail in red on the display map. This illustration gives an example. Count aloud how many paths you traverse in the red trail.



T: ***This trail follows 11 paths. Can you find a round-trip trail that uses more paths than this one?***

Let students experiment for awhile. When someone says they have found a round-trip trail that uses more paths, let that person demonstrate the trail and count the paths. Continue until the class concludes that a round-trip trail can use at most 15 paths, such as the one pictured here in blue.



Choose such a round-trip trail, and suggest that you close the paths you did not use. Then observe that the campground has an even number of paths at each campsite.

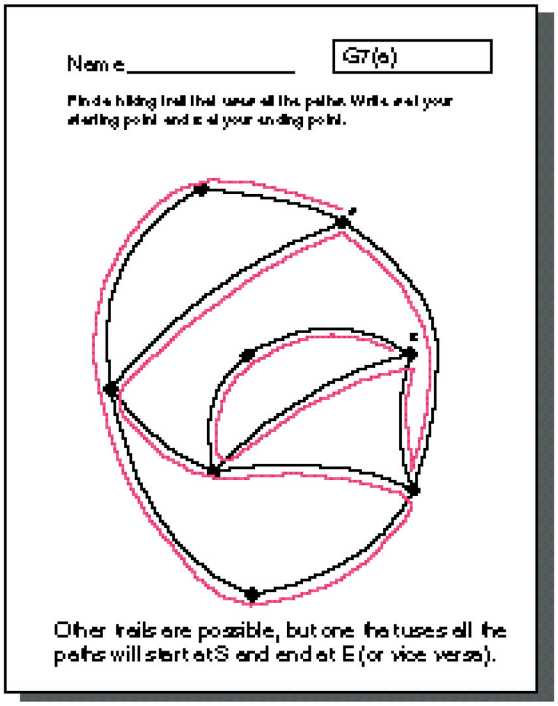
Worksheets G7(a) and (b) are available for students to find hiking trails using all the paths at other campgrounds.

Reading Activity

Read *Round Trip* by Ann Jones.

Writing Activity

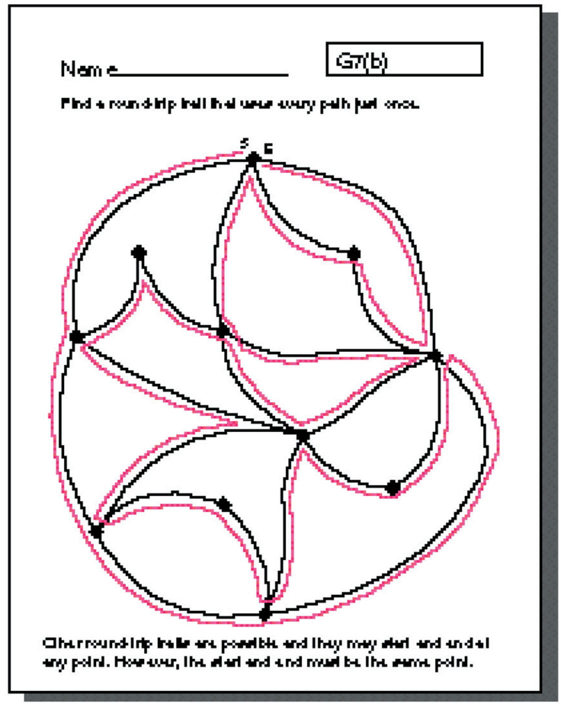
Suggest students write about how they find a hiking trail using most of the paths at a campground.



Name_______________ G7(a)

Find a hiking trail that uses all the paths. Write S at your starting point and E at your ending point.

Other trails are possible, but one that uses all the paths will start at S and end at E (or vice versa).



Name_______________ G7(b)

Find a round-trip trail that uses every path just once.

Other round-trip trails are possible and they may start and end at any point. However, the start and end must be the same point.

G8 TOURING HOUSES #1

Capsule Lesson Summary

Find walks through a house using each doorway at most once. Try to go through as many doors as possible on these walks. Define a walk that goes through each door exactly once as a *tour*. Find tours of two different houses, and observe where a tour starts and ends.

Materials

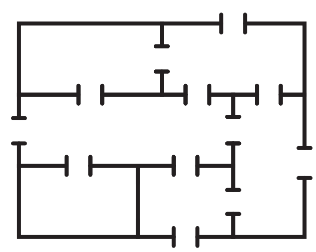
Teacher
- Blacklines G8(a) or (b)
- Colored chalk or markers
- Overhead projector (optional)

Student
- House floor plans
- Colored pencils, pens, or crayons
- Worksheets G8*, **, ***, and ****

Advance Preparation: Use Blacklines G8(a) and (b) to make copies of the house floor plans used in Exercises 1 and 2. Make copies for students and overhead transparencies for display. You may prefer to draw large copies of the floor plans on the board, but in this case be prepared to redraw them as needed.

Description of Lesson

Begin the lesson with a brief discussion about floor plans for houses. You may do this by displaying the floor plan on Blackline G8(a) and referring students to their copies.



T: ***What does this look like to you?***

S: ***A floor plan for a house.***

T: ***Who might use such a plan?***

S: ***The builder for a house.***

S: ***A person planning where to put furniture in the house.***

S: ***I saw a floor plan for a house in the newspaper.***

Exercise 1

Refer to the house floor plan on Blackline G8(a).

Note: You may like to make a very big copy of this floor plan to put on the floor and let students actually walk through the doors and rooms.

T: ***How many rooms does this house have?*** (6)
How many doors does this house have? (12)

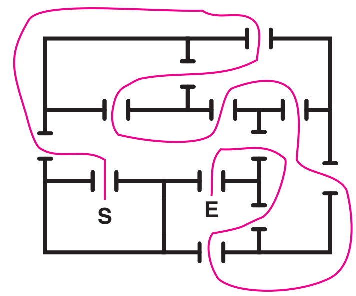
Point to any room you like in the floor plan.

T: ***Suppose we are in this room and we want to take a walk around the house. Let's say that we will not use a door more than once (we'll pretend a door closes and locks after we go through it).***

Invite several students to trace walks through the house. Do not allow a student to use a door more than once in a walk and occasionally note how many doors a student uses in the walk he or she traces.

T: ***Did any of the walks you suggested use most or all of the doors in this house? Try to find a walk that goes through as many doors as possible.***

Let students experiment. Tell students that they can start and end their walk wherever they like. When a student traces a walk, mark each doorway used with an **x**. At the completion of the walk, count how many doors the walk went through. Then ask if anyone found a walk going through more doors. Continue in this way until a walk using all the doorways is found. For example:



Students may make some observations about the places the walks start and end.

1) The room with only one door must be a starting or ending place of a walk; otherwise the walk will omit the doorway in that room.
2) The only walks that use all the doorways are ones that start and end at the same places as this red walk, or vice versa.
3) If a walk does not start and end in the same room, the number of doorways used will be odd for the starting and ending rooms but even for all the other rooms.

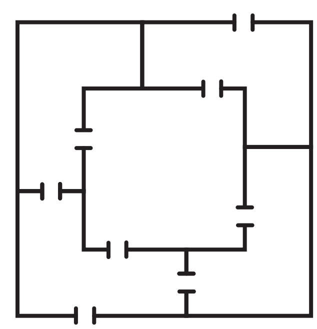
T: ***Has anyone ever been on a tour? What is a tour?***

Let students comment on what they know about tours.

T: ***We will call a walk through a house that uses every doorway exactly once a* house tour.**

Exercise 2

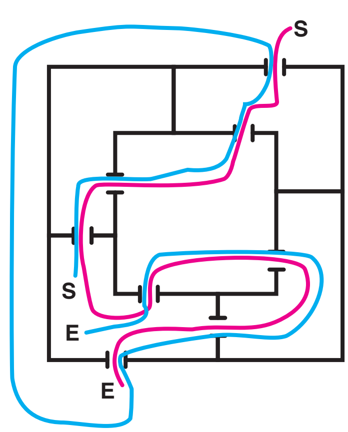
Display the floor plan on Blackline G8(b) and refer students to their copies of this floor plan.



T: ***Here is a different house's floor plan.***
How many rooms? (5)
How many doors? (8)

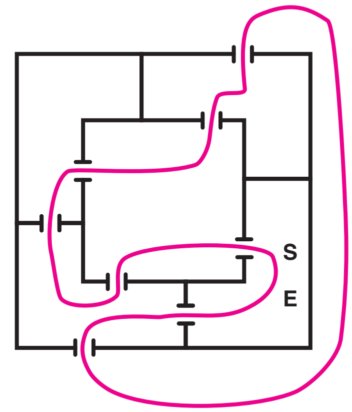
Do you think we could find a tour of this house—a walk that goes through every door just once?

Let students experiment for awhile before asking volunteers to trace tours on the display copy of the floor plan. Each time a student traces a tour, observe where the tour starts and ends. In this example, the red tour starts and ends outside the house, and the blue tour starts and ends in the room at the lower left.



Observe with your class that there are many different tours. In this case, every tour starts and ends at the same place (either in the same room or outside). After making this observation, ask if a tour could start and end in the upper right room (or a room where no one has started a tour). Students should observe that any tour they find could be considered as a tour starting and ending in that room.

For example, the red tour illustrated here can start and end in any of the rooms.



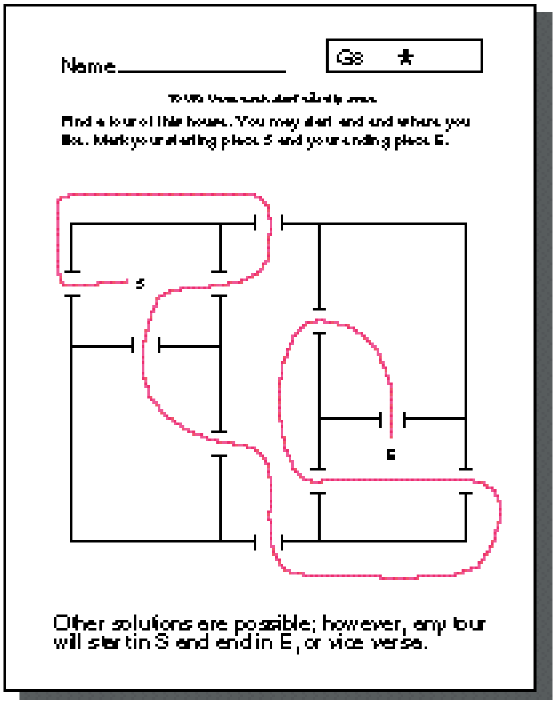
Worksheets G8*, **, ***, and **** are available for individual work. When students have completed the *** or **** worksheet, you may like to point to a floor plan that is impossible to tour and ask them how they would modify it to make touring possible. That is, what doors would they block or what doors would they build?

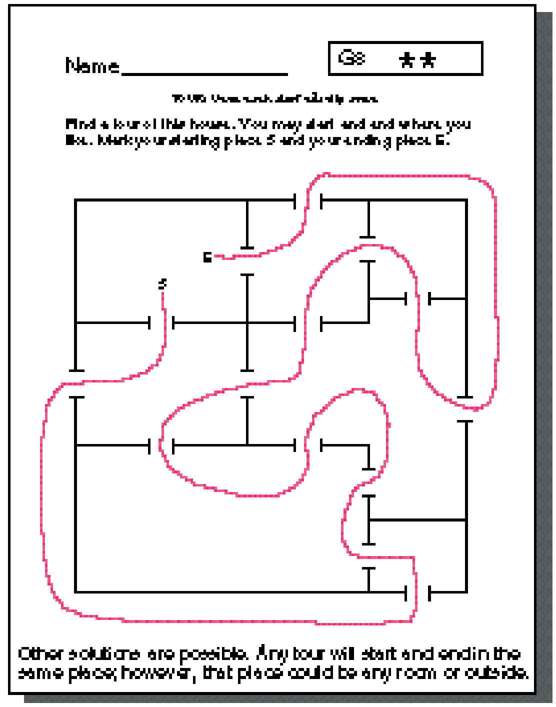
Center Activity

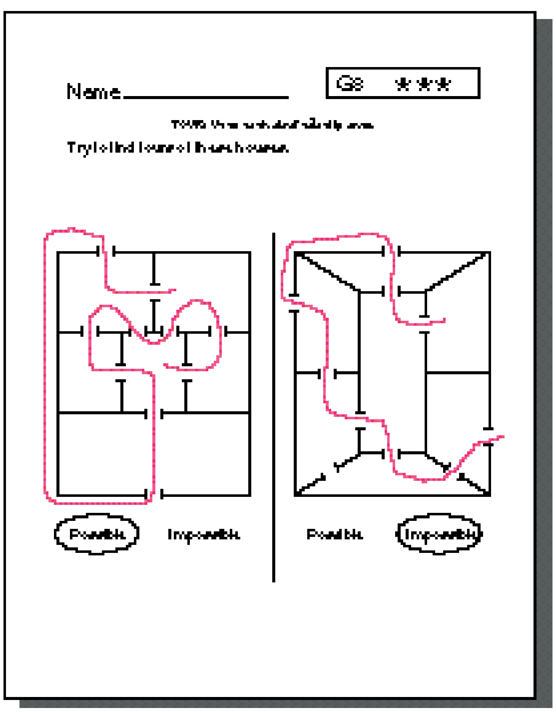
Place laminated floor plans in a center and let students try to find tours through them.

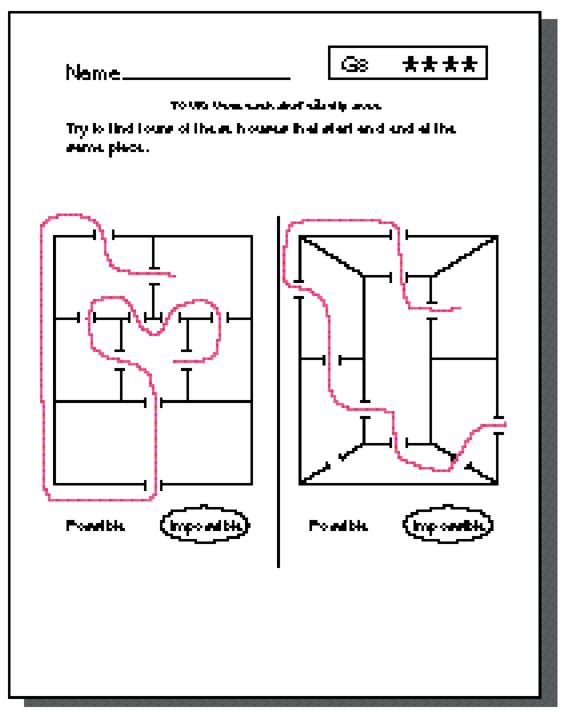
Home Activity

Suggest that students work with family members to make floor plans (for one floor) of their houses and then check to see if they can find a tour.


Name
G8 ★
Find a tour of this house. You may start and end where you like. Mark your starting place S and your ending place E.
S
E
Other solutions are possible; however, any tour will start in S and end in E, or vice versa.


Name
G8 ★★
Find a tour of this house. You may start and end where you like. Mark your starting place S and your ending place E.
E
S
Other solutions are possible. Any tour will start and end in the same place; however, that place could be any room or outside.


Name
G8 ★★★
Try to find tours of these houses.
Possible
Impossible
Possible
Impossible


Name
G8 ★★★★
Try to find tours of these houses that start and end at the same place.
Possible
Impossible
Possible
Impossible

G9 TOURING HOUSES #2

Capsule Lesson Summary

Review the idea of a tour of a house and decide whether or not certain houses have tours. Modify houses by building doors so that there are tours or so that there are special kinds of tours. Relate the idea of a tour for houses to the idea of a hiking trail by associating rooms of houses with campsites, and doors with paths between campsites.

Materials

Teacher
- Blacklines G9(a) and (b)
- Colored chalk or markers
- Overhead projector (optional)

- Colored pencils, pens, or crayons
- Worksheet G9*, **, ***, and ****

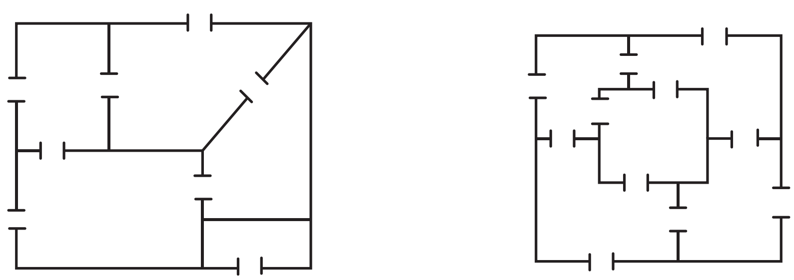
Student
- House floor plans
- Tracing paper

Advance Preparation: Use Blacklines G9(a) and (b) to make copies of the floor plans for Exercise 1. Make copies for students and an overhead transparency for display in Exercise 2. For Exercise 1, draw the floor plans from these blacklines on the board before the lesson begins.

Description of Lesson

Exercise 1

Draw the floor plans from Blacklines G9(a) and (b) on the board. Refer students to their copies.

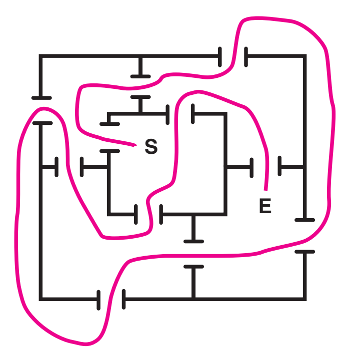


T: ***What kind of pictures are these?***

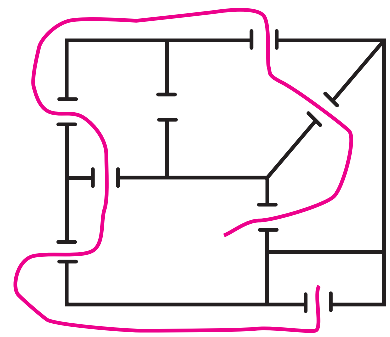
S: ***They are floor plans for houses.***

Let the class recall situations from the previous lesson on touring houses. Review the idea that a tour is a walk that goes through each door exactly once. Perhaps your students will volunteer the information that sometimes a tour starts and ends at the same place, sometimes a tour starts in one place and ends in another, and sometimes there is no tour at all. If not, mention these facts yourself and ask the class to predict (before making any trials) whether or not these houses have tours.

S: ***This house*** (on the right) ***has a tour starting in the middle room and ending here*** (lower right room).



S: ***This house*** (on the left) ***does not have a tour. For a tour we must use all the doors, and that includes the door of the room with just one door. We could start (end) there, but then we would end (start) in a room with three doors. There are several rooms with three doors and we can't end (start) in all of them. So some doors would not be used.***

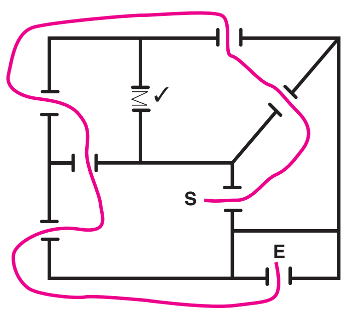


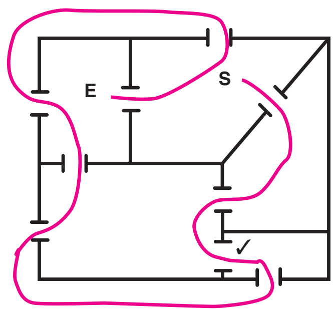
Note: Allowing the outside to be called a room, a house has a tour if and only if each room has an even number of doors (tours in this case start and end at the same place), or there are exactly two rooms with an odd number of doors (tours in this case start in one of the rooms with an odd number of doors and end in the other).

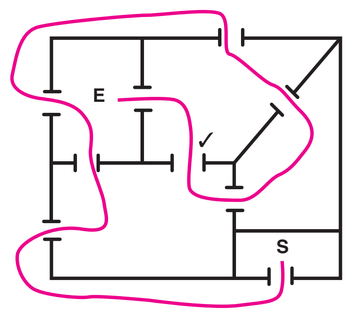
The discussion should review the roles played by rooms with an odd number of doors and by rooms with an even number of doors. Do not expect well-formed explanations; rephrase them as necessary for class understanding.

T (pointing to the house plan on the left)**:** ***Right now we cannot find a tour of this house. Could we block or build a door somewhere so that there would be a tour of the house?***

Your class might make several different suggestions. For each suggestion, check that it leads to a tour of the house. The most efficient solutions are to block a door or build a door between two rooms that each have an odd number of doors. For example:







(A check marks a blocked door or a newly built door.)

Exercise 2

If you have an overhead projector, display the floor plan on Blackline G9(b). Arrange to project on the board or a paper on which you can draw. Otherwise, refer to the house plan on the right and erase the one on the left.

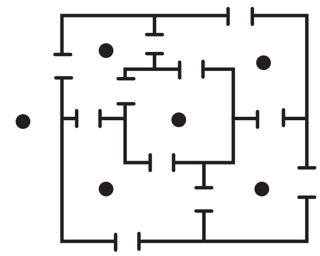
Note: This description assumes an overhead projector is used. You may need to adjust your actions if you do not have access to an overhead projector.

T: ***You predicted that this house has a tour. Who can show us a tour?***

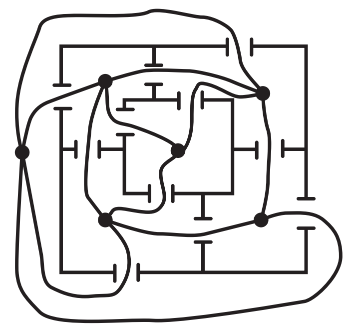
Let several students trace tours. (On an overhead transparency, you can let a student actually draw his or her tour and then erase it before another students draws a tour.) Observe that any tour either starts in the middle room and ends in the lower right room, or vice versa.

In the sequence of actions that follow, draw the dots and cords directly on the board or the paper on which you are projecting.

T: ***Now I'm going to draw a large dot in each room of the house and one dot outside.***



Then I'll connect the dots if there is a door between the rooms or between a room and the outside.



Turn off the overhead projector (or erase the outline of the house floor plan).

T: ***What does this picture remind you of?***

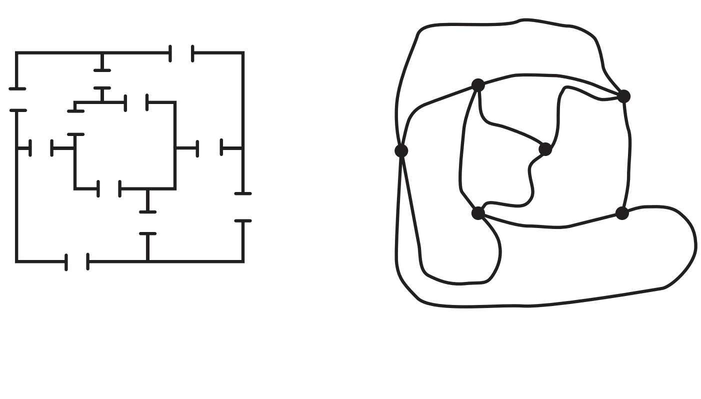
S: ***A campground with campsites and hiking trails.***

If necessary, recall the maps from Lesson G7 *Hiking Trails*. Observe that the rooms of the house correspond to campsites, and the doors correspond to trails between campsites.

T: ***If this were the map of a campground, could we find a hiking trail that uses every path exactly once?***

Let a student trace such a trail, checking (√) each path as it is used so that no path is used more than once and all the paths are used. Observe where the hiking trail starts and ends.

Redraw or display the house floor plan next to the campground map.



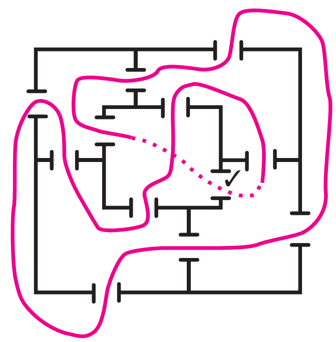
T: ***Let's see if we can take a tour of the house and trace along a hiking trail at the same time.***

Call on two students, one to draw a tour on the transparency and one to trace a hiking trail. Ask the student who is drawing a house tour to pause after going through each room so the student tracing a hiking trail can trace along the corresponding path.

Then erase the tour on the transparency. Call on two students, one to trace a hiking trail and the other to draw a tour. Ask the student at the board to trace a hiking trail very slowly, pausing after each path so that the student at the projector can draw the part of the tour going through the corresponding door.

T: ***We can follow a tour of the house with a hiking trail on the campground map, and we can follow a hiking trail on the campground map with a tour of the house.***

Look at the house plan. Each time, a tour started (ended) in the middle room and ended (started) in the upper right room. Could we block any door or build another door so that there would be a tour starting and ending at the same place?

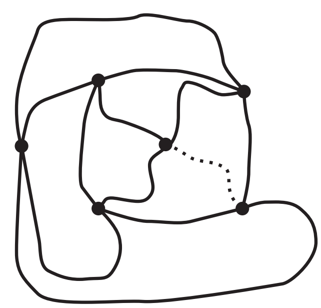


There are many solutions to this problem especially if we allow building more than one door, or both blocking and building doors. Encourage the class to look for a solution in which only one new door is built. Perhaps students will observe that they can build a door between the middle room and the lower right room since earlier they found a tour starting in one of those rooms and ending in the other.

As a result each room, including the outside, has an even number of doors.

T: ***What addition should we make to the campground map so that it corresponds to this house with an extra door?***

Draw a path between the middle campsite and the lower right campsite.



T: ***What kind of hiking trail do we get when we follow a tour of the house now?***

S: ***A round-trip trail—a trail that starts and ends at the same campsite.***

Invite a student to demonstrate this fact.

Worksheets G9*, **, ***, and **** are available for individual work. Supply tracing paper for students to draw their maps on and suggest that they show the corresponding hiking trails in color.

Center Activity

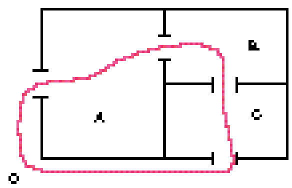
Place maze worksheets or games in a center for student exploration.

Reading Activity

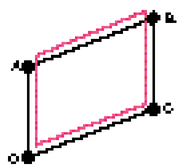
Math Games by Mitsumasa Anno contains a section on mazes and poses the Könisberg Bridge problem for students.

Name________________ G9 ★

Find a tour of this house. You may start and end where you like.

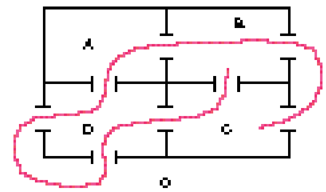


On tracing paper, draw a map of this house. Show a hiking trail corresponding to your tour.

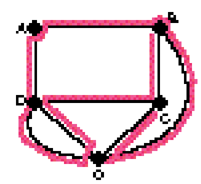


Name________________ G9 ★★

Find a tour of this house. You may start and end where you like.

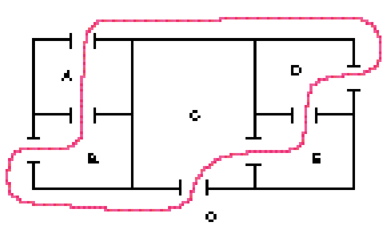


On tracing paper, draw a map of this house. Show a hiking trail corresponding to your tour.

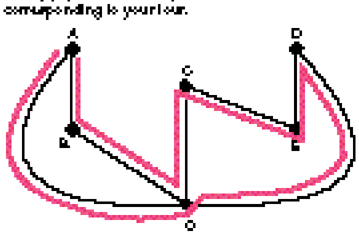


Name________________ G9 ★★★

Find a tour of this house. You may start and end where you like.

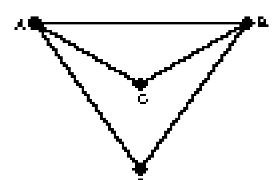


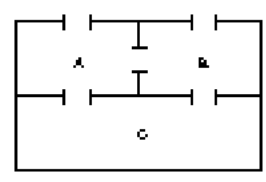
On tracing paper, draw a map of this house. Show a hiking trail corresponding to your tour.



Name________________ G9 ★★★★

Draw a house plan for this map.





Capsule Lesson Summary

Characterize rectangles as shapes with exactly four square corners and four straight sides. Find many rectangles with area 12 cm^2 and arrange them on a grid, always with one corner at a given point. Observe a pattern formed by the opposite corner points.

Materials

Teacher
- Paper
- *UPG-IV Geometry Posters* #1 and #2
- Colored markers

Student
- Paper
- Scissors
- Worksheets G10(a) and (b)
- Colored pencils, pens, or crayons

Advance Preparation: Before the lesson begins, cut out the rectangles outlined in blue on *Geometry Poster* #2. Cut along the underlying black borders even though the blue outlines may overlap them. You may prefer to use Blacklines G10(a) and (b) to prepare overhead projection materials rather than use the posters.

Description of Lesson

Exercise 1

T: ***What is a square corner? Where is there a square corner in this room?***

Students may not be able to explain precisely what a square corner is, but most likely they will be able to point them out in the room. For each item suggested, ask students to be specific about what they see as a square corner.

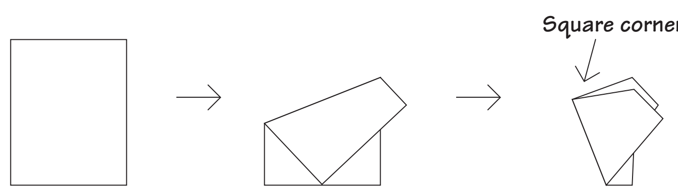
S: ***The chalkboard has four square corners.***

S: ***The corner of a window.***

S: ***A ceiling tile has square corners.***

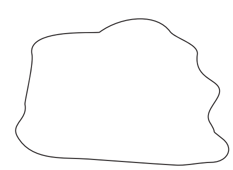
S: ***My paper has square corners.***

You and your students should each have a piece of paper. Demonstrate how to make a square corner as in the next illustration.



T: ***I'll show you how to make a square corner with a piece of paper.***

Fold the paper any way you want, … and then fold it again so that the folded edge lines up.



Note: It may impress your students if you use an irregular shaped piece of paper for your demonstration.

T: ***We can use this paper model as a template to help us check for square corners.***

Check some of the corners in the room that were suggested as square corners.

G10

T: ***I am thinking of a shape that has four square corners and four straight edges. What shape am I thinking of?***

S: ***A rectangle.***

S: ***You could be thinking of a square.***

T: ***Is a square a rectangle?*** (Yes)
A square is a special kind of rectangle in which all the sides have the same length.

Exercise 2

Distribute copies of Worksheet G10(a) and scissors. This would be a good time for students to work with a partner or in small groups. Pose a problem similar to the following:

T: ***I went to a carpet store and the store manager gave me 12 square pieces of carpet. I want to make a rectangular rug with these pieces of carpet. How many different sized rugs can I make? The store manager told me it would be easy to cut the pieces if I wanted to. Can you help me with this problem?***

Hold a brief discussion to establish that the problem is to find many different rectangles, each with area 12 squares. You may need to note that different sized rectangles does not mean different area but rather different dimensions. Refer students to Worksheet G10(a).

T: ***What size are the little squares on this worksheet?***

S: ***1 cm by 1 cm.***

T: ***What is the area of one little square?***

S: ***1 cm^2*** (read as, "one square centimeter").

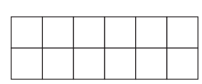
T: ***On this worksheet, color rectangles that each have an area of 12 cm^2. The rectangles will represent different rugs I can make.***

Remember, you can cut the squares, but you must make rectangles and they must have area 12 cm^2. Cut out the different rectangles you find.

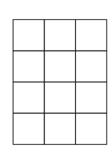
As you observe students' work, you may want to hold up some different examples to help those having trouble getting started. Allow about ten minutes before calling the class's attention back to the board.

Ask students (groups) to describe some of the rectangles they have cut out. For each description, locate the corresponding poster copy or overhead copy of that rectangle to display. Each time, announce the dimensions and check that the area is 12 cm^2. Observe that the product of the dimensions is the area.

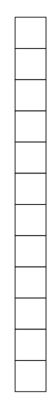
One way to encourage all students to think about the dimensions of a rectangle is to tell them about one side of the rectangle and ask for a full description.



6 cm by 2 cm
6 x 2 = 12



3 cm by 4 cm
3 x 4 = 12



1 cm by 12 cm
1 x 12 = 12

T: ***One side of this rectangle is 3 cm. How long are the other sides?***

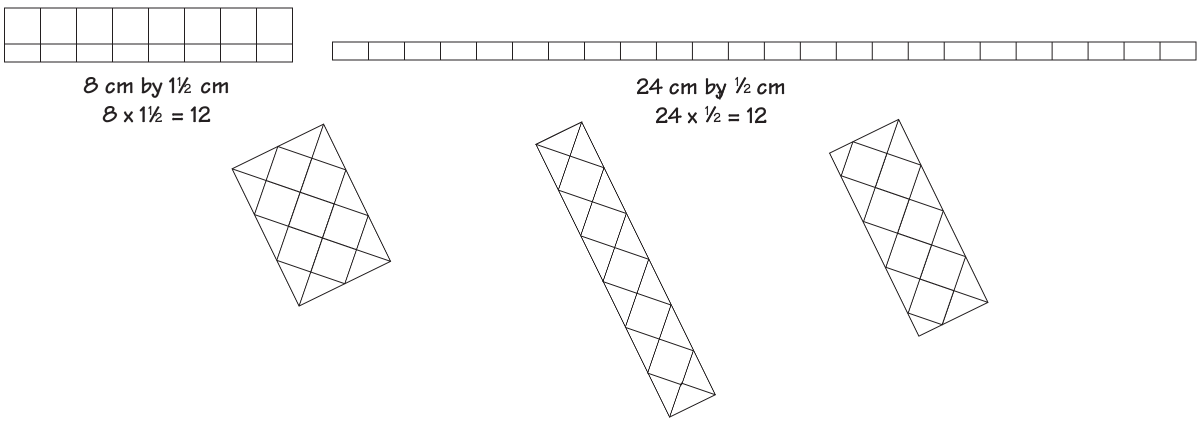
S: ***There is another 3 cm side, and the other two sides are both 4 cm.***

S: ***3 x 4 = 12.***

Each of the above rectangles may be turned to be viewed as 2 cm by 6 cm (2 x 6 = 12), 4 cm by 3 cm (4 x 3 = 12), and 12 cm by 1 cm (12 x 1 = 12), respectively.

T: ***Did you find any other rectangles with area 12 cm²?***

Perhaps students will suggest some of these rectangles.



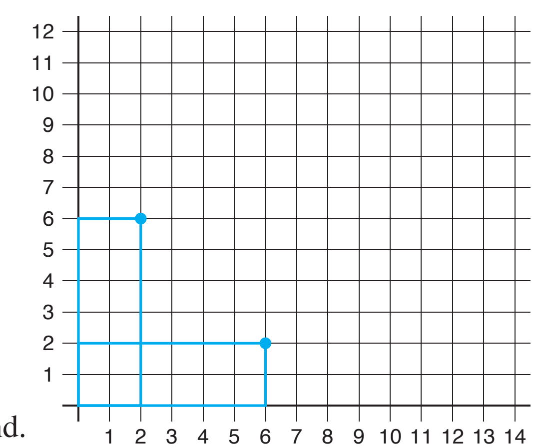
Note: The dimensions of three of the rectangles are not given because they involve irrational numbers, but it is easy to check their areas by counting squares, half-squares, and quarter-squares.

Tape *Geometry Poster* #1 to the board or put up a transparency from Blackline G10(b), and distribute copies of Worksheet G10(b).

T: ***We are going to make a picture (graph) of these rectangles. What we do on the board you should do on your worksheet as well.***

Among your rectangle cut-outs, find the 6 cm by 2 cm rectangle.

Display the cut-out of the same measurements.

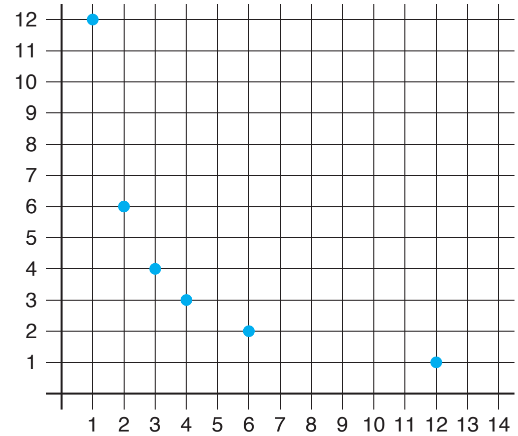


T: ***We lay the rectangle on the grid and line up the sides like this*** (see the illustration)***. Then we draw a dot at the upper right corner to picture this rectangle.***

If we turn the rectangle, we can get a dot for the 2 cm by 6 cm rectangle.

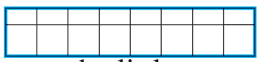
Direct students to draw blue dots on their worksheets for the other rectangles with area 12 cm² that they found. Also prompt students to draw blue dots for other rectangles with area 12 cm² even if they did not cut those rectangles out. As students are working individually or with their groups, invite some to place rectangles and to draw their corresponding blue dots on the poster. Give them the appropriate cut-outs to locate the dots. Students should check that any rectangle they put in the graph has area 12 cm².

Most students (groups) should be able to draw six dots for the rectangles with whole number dimensions.



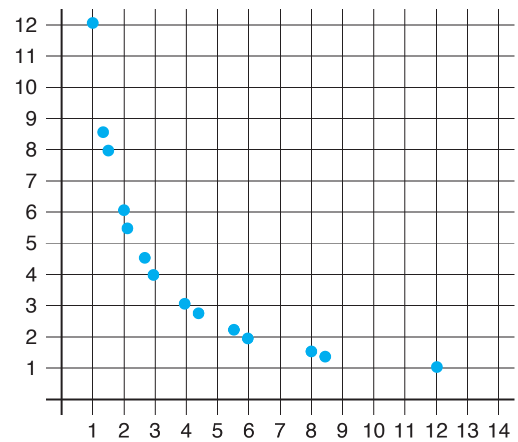
T: ***We have six blue dots in our graph. Look at this picture*** (touch each blue dot) ***and think about where other blue dots might be. See if you can find other rectangles with area 12 cm²^2.***

Show the class a rectangle (cut-out) involving half-squares. For example:



Let the class count the little squares to check that the area is 12 cm^2. On the graph plot the two corresponding blue dots for this rectangle. Such a rectangle may lead some students to discover other rectangles with area 12 cm^2, but you need not insist that your class find more.

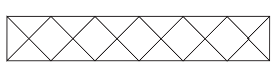
If all of the rectangles provided as cut-outs are found, the blue dots on your graph will suggest a smooth (hyperbolic) curve.

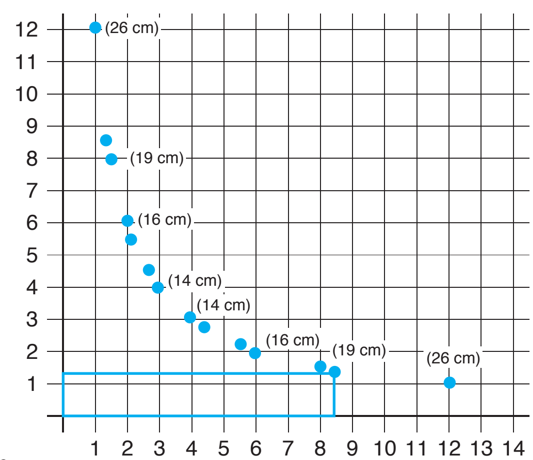


Conclude the lesson by letting the students discuss the picture.

Extension Activity

Find the perimeter for the rectangles pictured by dots in your graph (all with area 12 cm^2), if possible. Students should be able to find the perimeter of those rectangles with whole number dimensions. Make some observations about the perimeters and estimate the perimeters of other rectangles with area 12 cm^2. For example, 20 cm might be a good estimate for the perimeter of this rectangle.





Writing Activity

Suggest that students write a note to you explaining how they would answer the question about making a rug from the 12 carpet pieces. They might include in the note a suggestion of what they think would be the best idea for making a rug.

G11 RECTANGLES OF A GIVEN ____________

Capsule Lesson Summary

Discuss what is meant by the perimeter of a rectangle, and find that the perimeter is twice the sum of the lengths of two adjacent sides. Find many rectangles with perimeter 20 cm and arrange them on a grid, always with one corner at a given point. Observe a pattern formed by the opposite corner points.

Materials

Teacher
- Centimeter tape
- *UPG-IV Geometry Posters* #1, #3, and #4
- Colored markers

Student
- Centimeter tape or metric ruler
- Note pad paper
- Worksheets G11(a) and (b)
- Colored pencils, pens, or crayons

Advance Preparation: Before the lesson begins, cut out the rectangles outlined in red on *Geometry Posters* #3 and #4. Cut along the underlying black borders even though the red outlines may overlap them. You may prefer to use Blacklines G11(a) and (b) to prepare overhead projection materials rather than use the posters.

Description of Lesson

Exercise 1

Provide each student with a rectangular sheet of note pad paper and a centimeter tape or metric ruler. You may like students to have different sizes of note pad paper.

T: ***What shape is your piece of paper?*** (Rectangle)
The length of the border of a shape is called its perimeter.

Find the perimeter of your piece of paper, that is, the length of its border. Measure it in centimeters. Next to each edge write its length.

Students should measure to the nearest centimeter. After a few minutes discuss the results.

T: ***Patrick, what is the perimeter of your piece of paper?***

S: ***52 cm.***

T: ***How did you find the perimeter?***

S: ***I measured the sides and got 15 cm, 15 cm, 11 cm, and 11 cm.
Then I added 15 + 15 + 11 + 11 = 52.***

T: ***Nicole, how did you find the perimeter of your paper?***

S: ***I measured two sides together and got 26 cm. Then I multiplied; 2 x 26 = 52.***

If students have different size papers, let several share the perimeters of their papers.

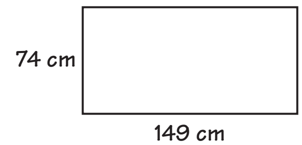
G11

Choose a fairly large rectangular object in the room to measure. The following dialogue assumes a table top is being measured.

T: ***I would like two of you to help me find the perimeter of the top of this table.***

S (measuring one side of the table)**:** ***A long side of the table is 149 cm.***

S (measuring another side of the table)**:** ***The short side of the table is 74 cm.***



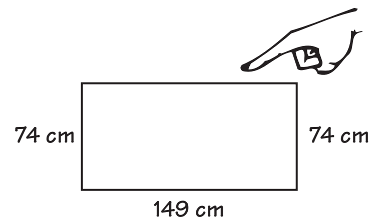
Put this information in a diagram on the board.

T: ***What is the length of this side?***

S: ***74 cm.***

T: ***… and this side?***

S: ***149 cm.***



T: ***How can we find the perimeter of the table top?***

S: ***Calculate 2 x 149 and 2 x 74 and then add those numbers.***

S: ***2 x 150 = 300, so 2 x 149 = 298.***

S: ***2 x 74 = 148.***

S: ***298 + 148 = 446.***

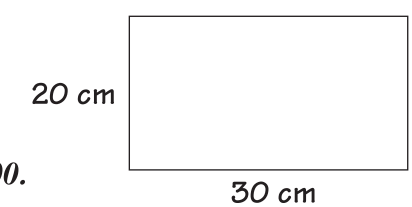
$$\begin{aligned} 2 \times 149 &= 298 \\ 2 \times 74 &= \underline{148} \\ &\ 446 \end{aligned}$$

S: ***149 + 74 = 223 and 2 x 223 = 446.***

$$\begin{aligned} 149 \times 74 &= 223 \\ 2 \times 223 &= 446 \end{aligned}$$

Erase the board and draw a rectangle.

T: ***I am thinking of a rectangle. A long side has length 30 cm; a short side has length 20 cm. What is the perimeter of the rectangle?***



S: ***100 cm, because 20 + 20 = 40, 30 + 30 = 60, and 40 + 60 = 100.***

S: ***100 cm, because 20 + 30 = 50 and 2 x 50 = 100.***

Repeat this activity for rectangles with the following or similar dimensions. With each situation let students describe their methods of calculating the perimeter.

- A long side has length 34 cm and a short side has length 15 cm. (Perimeter is 98 cm)
- A long side has length 112 cm and a short side has length 51 cm. (Perimeter is 326 cm)

Exercise 2

If you have written dimensions near the sides of the rectangle on the board, erase them.

T: ***I'm thinking of a rectangle that has perimeter 20 cm. What could be the lengths of its sides?***

Accept a few suggestions; for example:

S: ***The sides could all be 5 cm long.***

T: ***Then it would be a special kind of rectangle — it would be a square.***

Label each of the sides of the rectangle 5 cm and check that the perimeter is 20 cm.

Begin a list on the board.

5 cm by 5 cm
6 cm by 4 cm

S: ***A long side could be 6 cm long and a short side could be 4 cm long. 6 + 4 = 10 and 2*** x ***10 = 20, so the perimeter is 20 cm.***

Distribute copies of Worksheet G11(a) and scissors. This would be a good time to arrange for students to work with a partner or in small groups.

T: ***On your worksheet color and cut out several rectangles, each with perimeter 20 cm.***

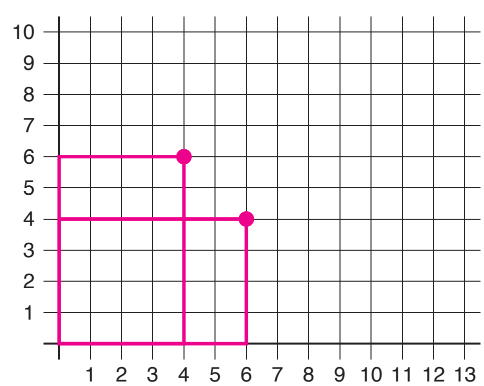
As you observe students' work, announce to the class the sizes of various rectangles being found. Each time, record the dimensions on the board and check that the perimeter is 20 cm. This should help students who are having difficulty getting started. Some sample rectangles with their dimensions are listed here.

5 cm by 5 cm
6 cm by 4 cm
3 cm by 7 cm
8 cm by 2 cm
1 cm by 9 cm
$\frac{1}{2}$ cm by $9\frac{1}{2}$ cm
$1\frac{1}{2}$ cm by $8\frac{1}{2}$ cm
$2\frac{1}{2}$ cm by $7\frac{1}{2}$ cm
$3\frac{1}{2}$ cm by $6\frac{1}{2}$ cm
$4\frac{1}{2}$ cm by $5\frac{1}{2}$ cm

Tape *Geometry Poster* #1 to the board or put up a transparency from Blackline G11(b), and distribute copies of Worksheet G11(b).

T: ***We are going to make a picture (graph) of these rectangles. What we do on the board you should do on your worksheet as well. Among your rectangle cut-outs, find the 6 cm by 4 cm rectangle.***

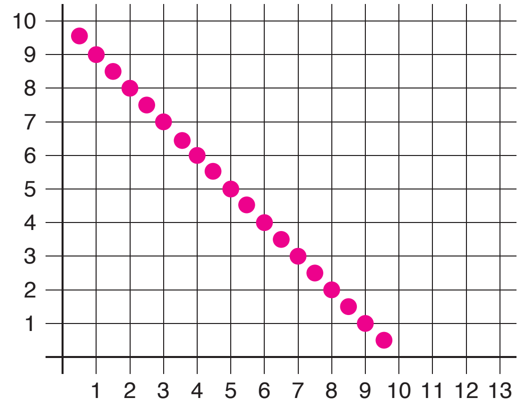
Display the cut-out of the same dimensions.



T: ***We lay the rectangle on the grid and line up the sides like this*** (see the illustration). ***Then we draw a dot at the upper right corner to picture this rectangle.***

If we turn the rectangle, we can get a dot for the 4 cm by 6 cm rectangle.

Direct students to draw red dots on their worksheets for all the rectangles they find with perimeter 20 cm. Also, encourage them to draw red dots for other rectangles in your list with perimeter 20 cm even if they did not cut out those rectangles. As students are working individually or in groups, invite some to place rectangles and draw their corresponding red dots on the poster. The cut-outs should be available for their use. This illustration shows what the picture would look like for a sample (partial) list.



T: ***Does the picture suggest we could find other rectangles with perimeter 20 cm?***

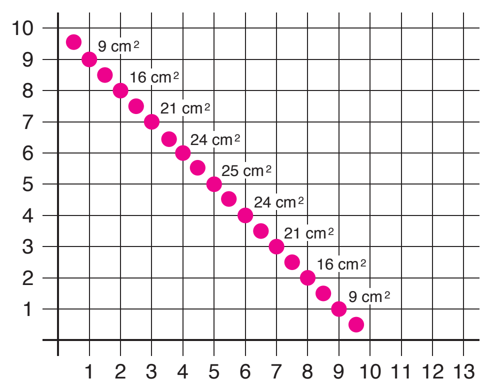
S: ***Yes, $4^1/_4$ cm by $5^3/_4$ cm.***

S: ***Any dot on this line would be for such a rectangle.***

If no one notices that the red dots lie in a straight line, point this out yourself.

Extension Activity

Find the areas of the rectangles pictured by dots in your graph (all with perimeter 20 cm), if possible. Students should be able to find the areas of those rectangles with whole number dimensions. By using the cut-outs, they may be able to find areas for most of the others. Make some observations about the areas; for example, the square (5 cm by 5 cm) has the greatest area, and the area decreases as the dimensions of the rectangle get further apart.



G12 PENCIL TRACINGS

Capsule Lesson Summary

Introduce rules for making a tracing on a grid. The rules are that you start at one corner of a little square in the grid and make a tracing without going over any edge more than once and without lifting the pencil (chalk). The score for a tracing is the number of little squares it surrounds completely. Find the highest possible scores for various arrays and observe some patterns.

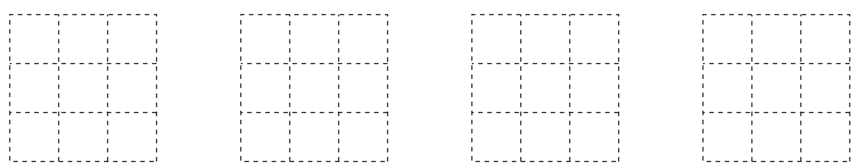
Materials

Teacher
- Colored chalk

Student
- Worksheets G12*, **, ***, and ****
- Colored pencils, pens, or crayons

Description of Lesson

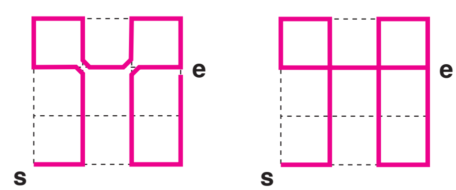
Draw several 3 by 3 grids on the board using dotted lines.



Hold a collective discussion to observe that these 3 by 3 grids are squares and each big square is made up of nine little squares. Then introduce the idea of a tracing.

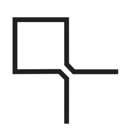
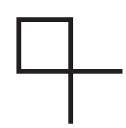
T: ***I'm going to start at a corner of one of the little squares in the grid and make a tracing. I'll try to surround as many squares as possible. The rules for a tracing are that I cannot go over any edge more than once, and I cannot lift the chalk off the board until I'm finished.***

Make a tracing such as the following. Label your starting point **s** and your ending point **e**.



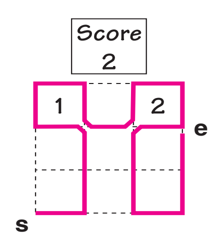
The first illustration shows how the tracing is executed. Your drawing actually would look more like the second illustration.

Note: Throughout this lesson, we have used this technique of drawing to make the illustrations easier to follow:

 instead of 

T: ***How many squares did I surround completely?***

Invite a student to count the surrounded squares. Indicate the total number as shown here.

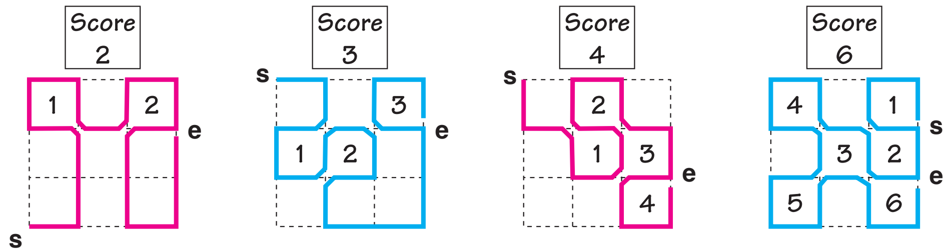


If a student wants to count the squares within the rectangle that is completely surrounded, point out that you only count a square that is itself completely surrounded.

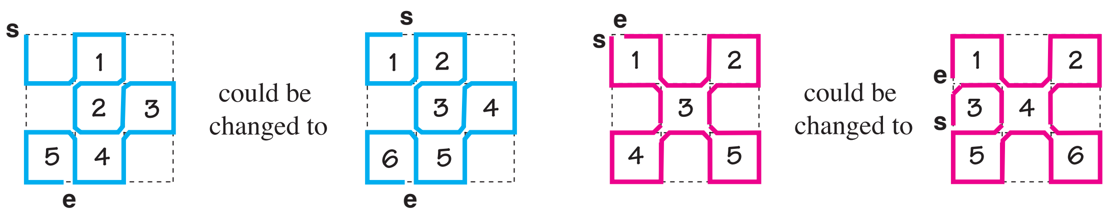
Refer students to Worksheet G12* as you say,

T: ***My score for this tracing is 2. Make you own tracings and see if you can get a higher score.***

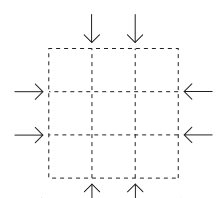
After about five minutes, call on several students to put their tracings on the board. Be sure to include a student who has surrounded six squares (maximal score).



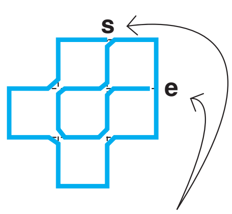
If the best score in your class is 5, it is possible that such a tracing could be slightly modified to surround six squares. For example:



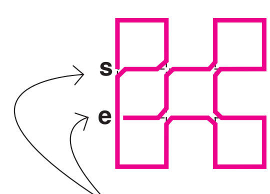
Note: One approach to this problem is to consider the number of odd vertices (an odd vertex has an odd number of edges meeting at it). A picture can be completely traced if there are exactly two odd vertices (a starting point and an ending point) or none at all. Because there are eight odd vertices in this picture, it is impossible to trace the entire grid and, therefore, impossible to surround all nine squares.



Because at least three edges must be omitted to have two odd vertices, the maximum number of squares that can be surrounded is six. The next illustration shows examples in which edges not used in the tracings have been deleted. Note the two odd vertices in each.



odd vertices

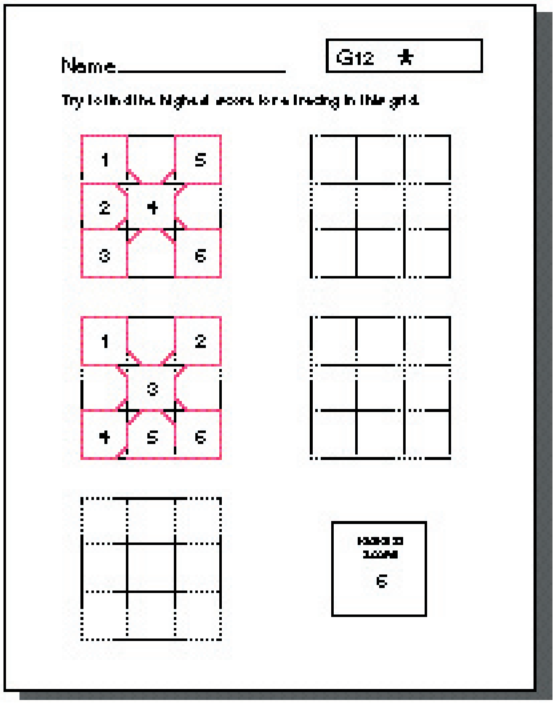


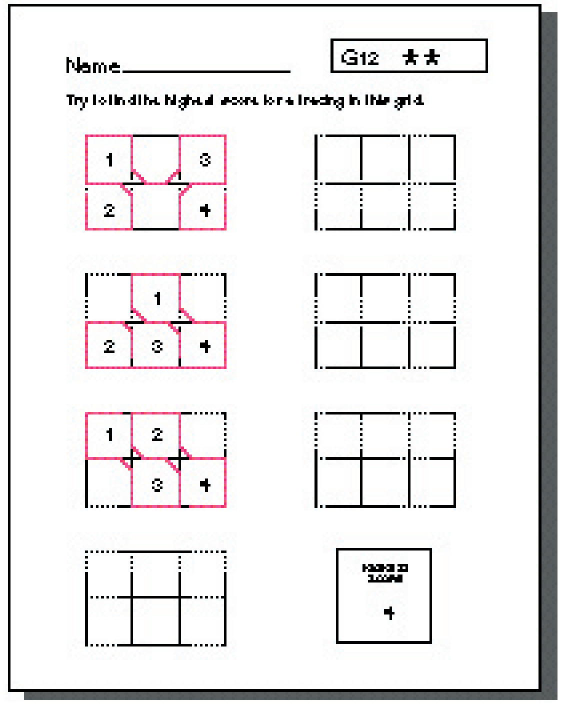
odd vertices

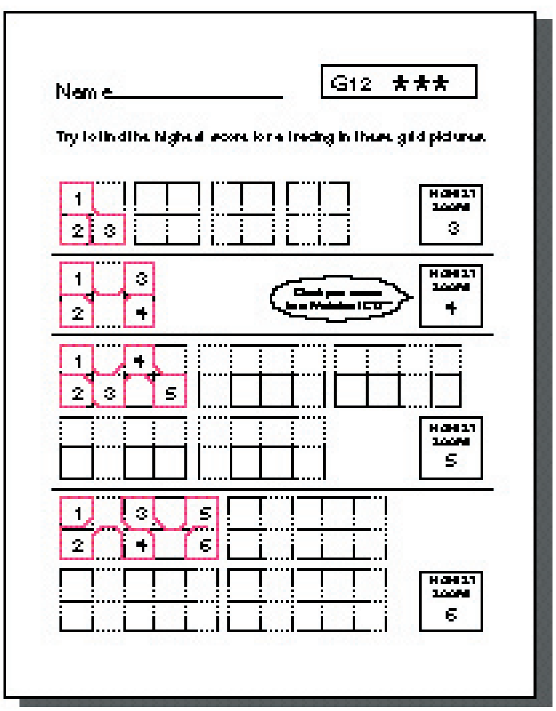
Worksheets G12**, ***, and **** are available for individual work.

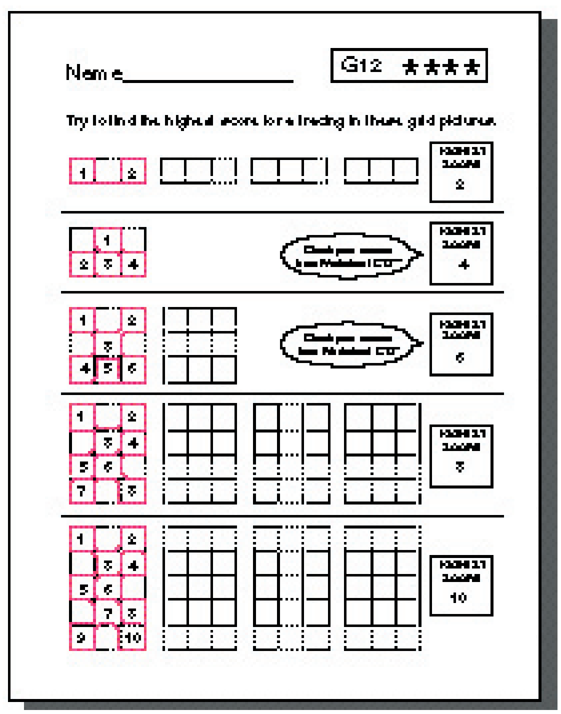
Extension Activity

Determine the length of a tracing with the unit of length being the length of a side of a small grid square. Then look for tracings that are as long as possible. You can also look for tracings that surround the most squares and are as short as possible.


Name
G12 ★


Name
G12 ★★


Name
G12 ★★★


Name
G12 ★★★★

G13 DOES YOUR CHEWING GUM LOSE ITS FLAVOR?

Capsule Lesson Summary

Discuss weighing things and what type scale to use for different size things. Conduct an experiment with chewing gum to discover how much sugar is in it. Record the mass of the gum before and after chewing, and make graphs to display the data.

Materials

Teacher
- Balance scale
- Weights
- Chewing gum
- Blackline G13

Student
- Balance scale
- Weights
- Chewing gum
- Record sheet

Advance Preparation: For this lesson you will need to obtain several different brands of chewing gum. We suggest you use bubble gum that comes in fairly good size pieces. Find two or three brands, both regular and sugarfree. You may need to check whether any student is not allowed to chew gum even for a short period of time. Use Blackline G13 to make copies of the record sheet for groups of students.

Description of Lesson

Begin this lesson with a discussion of weighing things.

T: ***How did we find a pound of potatoes?***

S: ***We weighed them on a scale.***

T: ***Right. What kind of scale did we use?***

S: ***A small diet scale.***

T: ***Could I use that scale to find out how much I weigh?***

S: ***No. You're too big!***

T: ***True. What kind of scale would I use to weigh myself?***

S: ***A bathroom scale.***

T: ***Have you ever noticed there are scales that weigh big trucks at certain spots along the highway? We need different types of scales to weigh light and heavy objects. Name some things that you might use a bathroom scale to weigh.***

S: ***My little sister.***

S: ***My dog.***

S: ***A box of books.***

T: ***What kind of scale would you use to weigh a piece of gum?***

S: ***One that can weigh very light things.***

T: ***Here is a problem we will investigate today. Which type of gum has the most sugar in it?***

Write 1g on the board and hold up a gram weight.

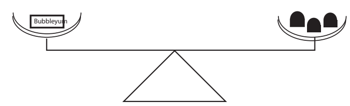
T: ***This weight is one gram. As you can tell, it is very light. Two paper clips weigh about one gram. The gram will be helpful in our investigation with the gum.***

Organize the class in groups of four or five students. If possible, provide each group with a balance scale, weights, and record sheets (Blackline G13). Give each student in a group a different brand of gum. Be sure to tell students not to open or chew their gum yet. If you have only one or two balance scales and sets of weights, locate them in a convenient place so every group has access.

T: ***Today you are going to conduct some tests to find out which kind of gum has the most sugar in it.***

Discuss with the class how they will conduct the tests. Let students talk about what happens when they chew gum, and decide that the gum loses most of its sugar. Model for the class how they will weigh the gum in its wrapper and record the beginning mass in the table on the record sheet.

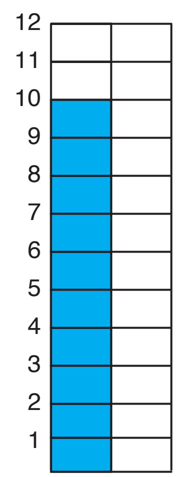
	Mass		
Brand Name (Flavor)	**Beginning**	**Chewed**	**Loss**
1. Bazooka	10 grams		
2. Bubble Yum			
3. Bubble Yum Sugarless			
4. Bubblicious			
5. Sugar Free Bubblicious			



Direct each group to choose a timekeeper. When the timekeeper says go, everyone will unwrap their gum, save the wrapper, and chew the gum for approximately ten minutes. When the timekeeper says stop, everyone will put their gum back in its wrapper and waits for a chance to weigh the chewed gum.

While students are chewing, direct them to look at the boxes at the bottom of the record sheet.

T: ***Here you will color the left side of each box to show the mass in grams of the different kinds of gum before chewing (beginning).***



Direct groups to color in these boxes for the beginning mass of their different brands of gum.

When the ten minutes of chewing time is over, direct the groups to weigh the chewed gum and record this in the chart. They also should color in the right side of the box for each brand to see the chewed gum mass compared to its beginning mass. Suggest that the groups discuss among themselves what they think happened in this experiment.

End the lesson with a class discussion to compare the groups' results, to note which brand of gum has the most sugar, to observe what part (about $\frac{1}{2}$ or $\frac{2}{3}$) of gum is sugar and flavor, to compare regular gum with sugarless gum, and so on.

Writing Activity

Suggest that students write a letter to their parents/guardians about the experiments they did with gum and to comment on the amount of sugar in gum.