The World of Numbers

WORLD OF NUMBERS TABLE OF CONTENTS

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WORLD OF NUMBERS INTRODUCTION

Numbers are among the most important things that mathematics (at all levels) is about. Mathematicians are interested in numbers just as astronomers are interested in stars, botanists in plants, and sociologists in the organization and dynamics of human societies. Surely everyone agrees that one of the primary goals of any elementary school mathematics curriculum should be to introduce students to the world of numbers—to give them the opportunity to become familiar with numbers, their properties, and the relations between them. The ability to calculate with numbers is an important part—but not the only part—of being familiar and comfortable with numbers. The World of Numbers strand of *CSMP Mathematics for the Upper Primary Grades* is designed to provide students with a wide variety of challenging experiences with numbers so that gradually they will become not only familiar, but comfortable with numbers; they will, so to speak, get to know numbers on a "first name basis" and develop number sense.

The Minicomputer and the World of Numbers

CSMP Mathematics for the Upper Primary Grades uses the Papy Minicomputer as a support for the positional system of numeration; for calculations and estimation; for number patterns and mental arithmetic; and for modeling the basic operations involving whole numbers, integers, and decimal numbers. Although the Minicomputer can be seen as a tool for calculation and as a device to help students learn routine methods for calculations, its more exciting use is as a vehicle for posing interesting problems that challenge a child's intellectual curiosity about numbers, and for presenting situations that both encourage strategic thinking and reinforce numerical skills. The lessons that make use of the Minicomputer are often intended to be explorations into the world of numbers.

Standard Algorithms of Arithmetic

CSMP seeks to develop basic numerical skills as well as an understanding of the underlying mathematical ideas. We are fully in agreement with the thesis that, along with the growth of understanding of the world of numbers, there must be a concommitant growth of familiarity and facility with numbers and operations on them. But facility should not be confused with understanding; they are partners in the growth of mathematical maturity. A balanced growth of each must be maintained, neither being sacrificed for the other.

Students must eventually learn mechanical algorithms for the basic operations (addition, subtraction, multiplication, division). However, premature presentation of these algorithms may actually stunt a student's ability to develop alternative algorithms, to do mental arithmetic, or to estimate.

Consider the problem of calculating 294 – 89. A third grader may have difficulty performing a standard (paper and pencil) subtraction algorithm. An easier and more efficient way to proceed is to subtract 90 from 294 and then to add 1 (294 – 90 = 204 and 204 + 1 = 205). To insist on a mechanistic response to such a problem would be to encourage inefficiency and might also inhibit the development of the flexibility necessary for problem solving. On the other hand, a rich array of situations in which students interact with numbers provides them with opportunities to gain the necessary facility with standard algorithmic procedures while retaining the openness required to respond creatively to new situations in the world of numbers.

Numerical Relations

One of the main aims of the World of Numbers strand is to familiarize students with numbers by studying relations between numbers, both explicitly and in a variety of contexts. (For more general comments about relations, see the introduction to the Languages of Strings and Arrows strand.) Arrow diagrams represent relations in a simple, suggestive, and pictorial way—usually more conveniently than the same information could be given in words or other symbols.

Students are brought into contact with an assortment of challenging situations, many of which would be totally inaccessible to them were it not for the arrow diagrams. The problems and activities of this strand include solving linear equations presented in terms of arrows; studying iterated processes and patterns in sequences of numbers; tackling problems that may have many solutions or no solution; estimating or testing that a solution is reasonable; and exploring properties of operations on numbers.

Capsule Lesson Summary

Subtract one number from another by doing a sequence of subtractions. Record the sequence using an arrow picture.

Materials

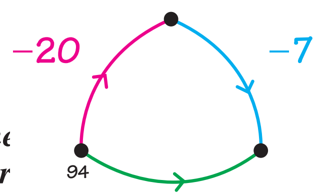
Teacher
- Colored chalk
- Minicomputer set

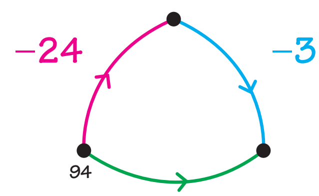
Student
- Paper
- Colored pencils, pens, or crayons
- Minicomputer set
- Worksheets N1*, **, ***, and ****

Description of Lesson

Exercise 1: Composition of Functions

Draw these arrow pictures on the board.





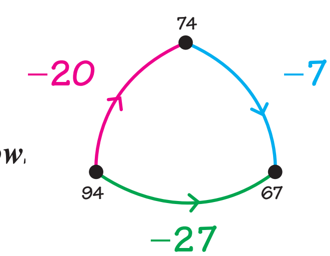
T: ***What can you tell me about these two arrow pictures? In each of the what could the green arrow be for***

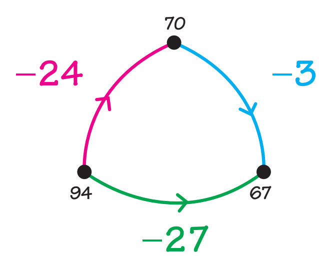
Encourage students to make observations.

S: ***In both pictures, the green arrow could be for –27.***

S: ***The ending dot is for 67 in both of these pictures.***

Call on students to label the dots and the green arrows.





T: ***Each of these arrow pictures show a way to calculate 94 – 27.***

Point to the arrow picture on the left and trace the appropriate arrows as you say,

T: ***This arrow picture shows that subtracting 20 and then subtracting 7 is the same is subtracting 27.***

Complete the calculation following the first arrow picture.

T: ***What number is 94 – 20?***

S: ***74.***

T: ***What number is 74 – 7?***

S: ***67.***

T: ***What number is 94 – 27?***

S: ***67.***

T: ***This is one way to calculate 94 – 27.***

$$\begin{array}{r} 94 \\ -\ 20 \\ \hline 74 \\ -\ 7 \\ \hline 67 \end{array} \qquad \begin{array}{r} 94 \\ -\ 27 \\ \hline 67 \end{array}$$

Point to the arrow picture on the right and trace the appropriate arrows as you say,

T: ***This arrow picture shows that subtracting 24 and then subtracting 3 is the same as subtracting 27.***

Complete the calculation again following the second arrow picture.

T: ***What number 94 – 24 ?***

S: ***70.***

T: ***What number is 70 – 3?***

S: ***67.***

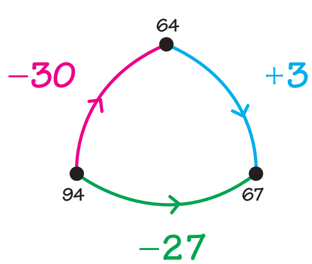
T: ***What number is 94 – 27?***

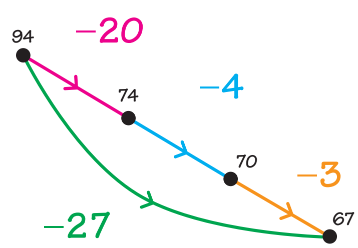
S: ***67.***

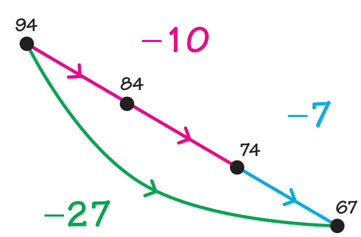
T: ***Is there another way to calculate 94 – 27?***
Can you show another way to calculate 94 – 27 in an arrow picture?

$$\begin{array}{r} 94 \\ -\ 24 \\ \hline 70 \\ -\ 3 \\ \hline 67 \end{array} \qquad \begin{array}{r} 94 \\ -\ 27 \\ \hline 67 \end{array}$$

Record students' suggestions on the board. A few possible arrow pictures are shown below.







Conclude that there are many ways to calculate 94 – 27.

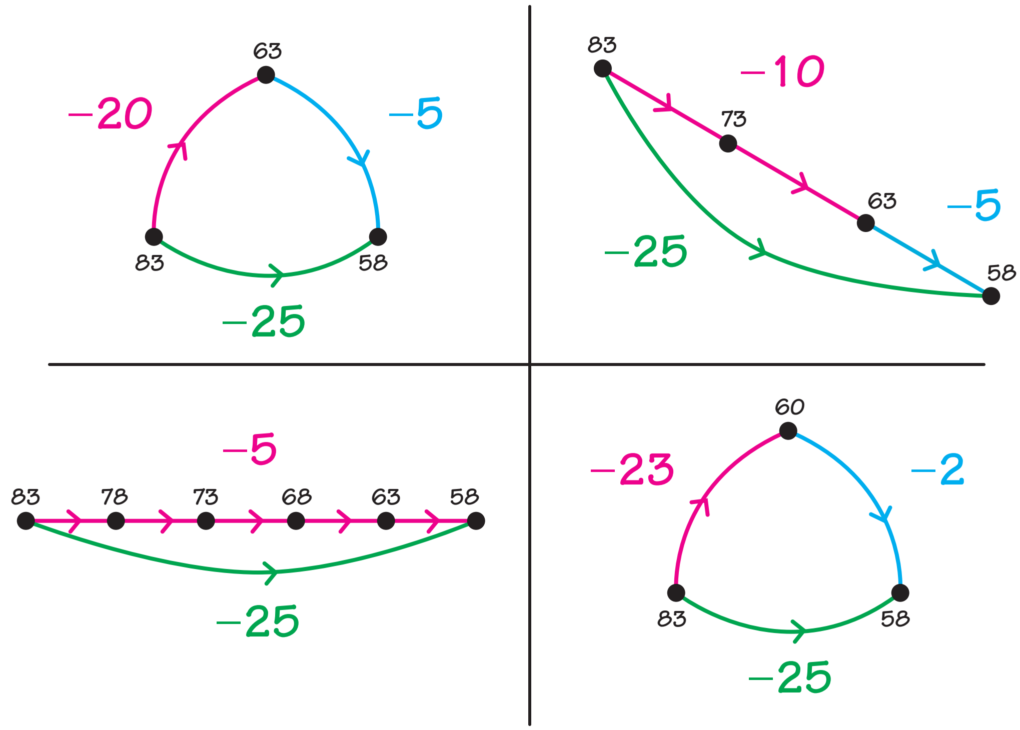
Erase the board, and then write this problem on the board.

$$83 - 25 = ?$$

T: ***Solve this problem in your own way, and then draw an arrow picture to show how you did the calculation.***

Some students may need help drawing an arrow picture. If these students are able to explain what calculation they did, you can describe the corresponding arrow picture for them.

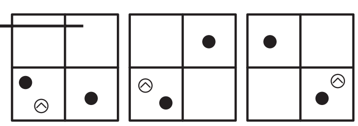
Ask students who finish quickly to draw their arrow pictures on the board. Try to get at least three different arrow pictures on the board. A few possibilities are shown below. Discuss each arrow picture briefly and conclude that 83 – 25 = 58.



Erase the board and repeat this activity with 105 – 49. If some of the students finish quickly, give them a subtraction problem such as 354 – 297 to solve with an arrow picture.

Exercise 2

Invite students to put 369 and $\widehat{221}$ on the Minicomputer, and write the problem to one side of the Minicomputer.

 $= 369 + \widehat{221}$

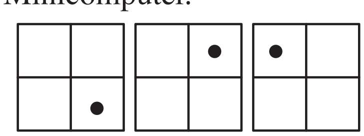
T: ***What number is 369 + $\widehat{221}$? Write it on your paper (or whisper it to a neighbor).***

Check many answers before asking a student to answer aloud.

S: ***369 + $\widehat{221}$ = 148.***

T: ***How did you do the calculation?***

Encourage several students to explain their methods. If no one suggests removing the pairs with one negative checker and one regular checker from the Minicomputer, suggest this yourself. Emphasize that when a negative checker is on the same square as a regular checker, both checkers can be removed from the Minicomputer.

 $= 369 + \widehat{221} = 148$

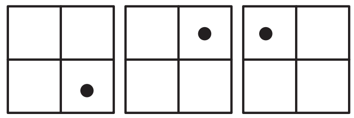
T: ***What subtraction sentence could we write for this*** (point to 369 + $\widehat{221}$ = 148)***?***

S: ***369 – 221 = 148.***

Write the subtraction sentence under the addition sentence.

Repeat this activity to calculate 824 + $\widehat{400}$. After students have had an opportunity to work on the problem, complete the number sentence on the board and ask several students to explain how they did the calculation.

For example:



$= 369 + \widehat{221} = 148$

S: ***I made an 800 = 400 + 400 trade in my head.***

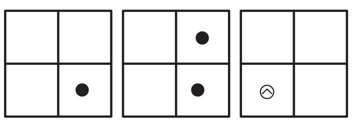
Make this trade yourself, pair the negative checker with a regular checker on the 400-square, and then remove those two checkers.

S: ***I subtracted 800 – 400 and then added 24 to that answer; 400 + 24 = 424.***

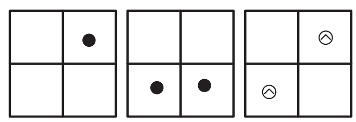
T: ***What subtraction sentence could we write?***

S: ***824 – 400 = 424.***

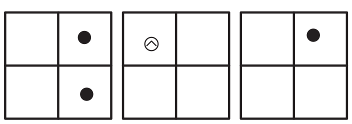
Continue this activity with these calculations.



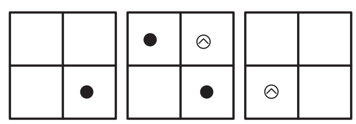
$= 150 + \widehat{2}$



$= 430 + \widehat{6}$



$= 504 + \widehat{80}$



$= 190 + \widehat{42}$

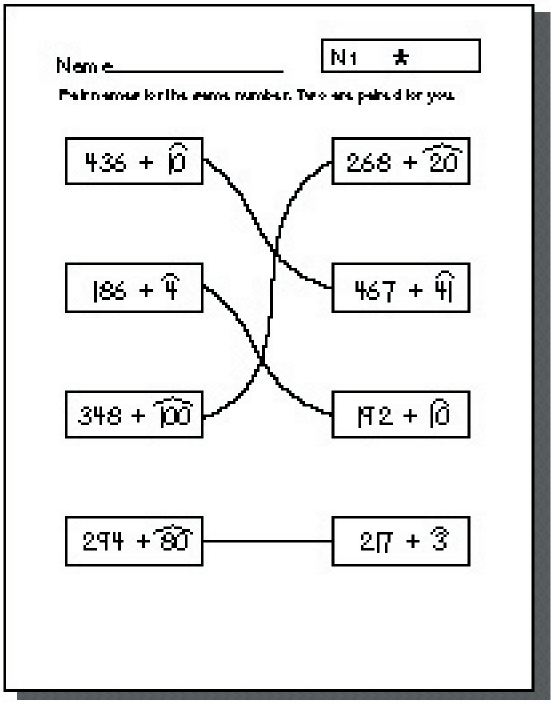
Encourage students to notice that different calculations can have the same result.

Worksheets N1*, **, ***, and **** are available for independent work. Make individual Minicomputers available, but allow students to use whatever methods they prefer to solve the problems. You may want to suggest that many of the calculations can be done mentally, but students should use other calculation methods as necessary.

Home Activity

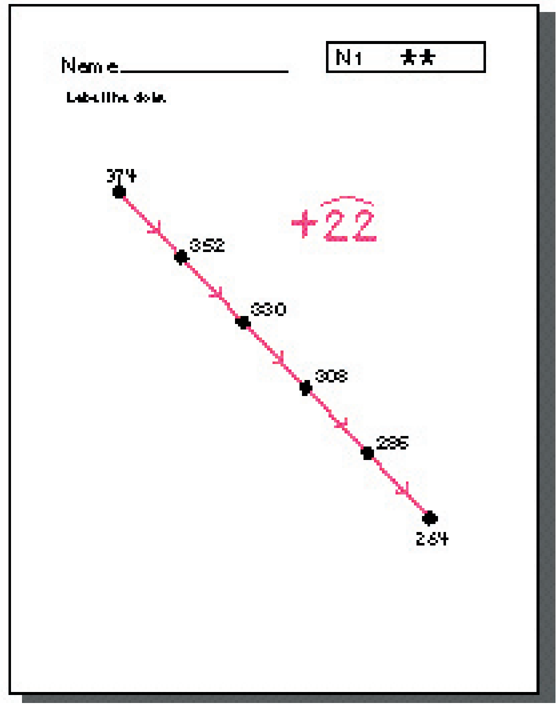
Suggest that parents/guardians practice subtraction facts with their children at home.

Send home one of the arrow pictures students made in Exercise 1. Ask students to share their methods of subtraction with family members.

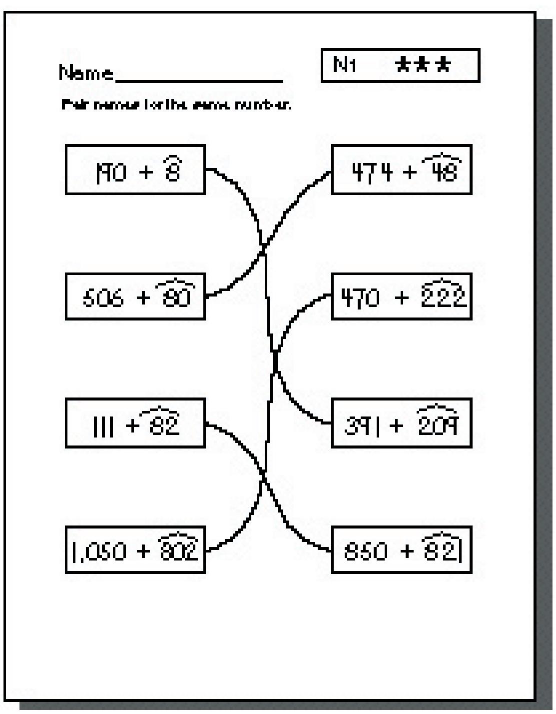

Name________________ N1 ★

Pair names for the same number. Two are paired for you.

436 + 10	268 + 20
186 + 4	467 + 41
348 + 100	192 + 10
294 + 80	217 + 3

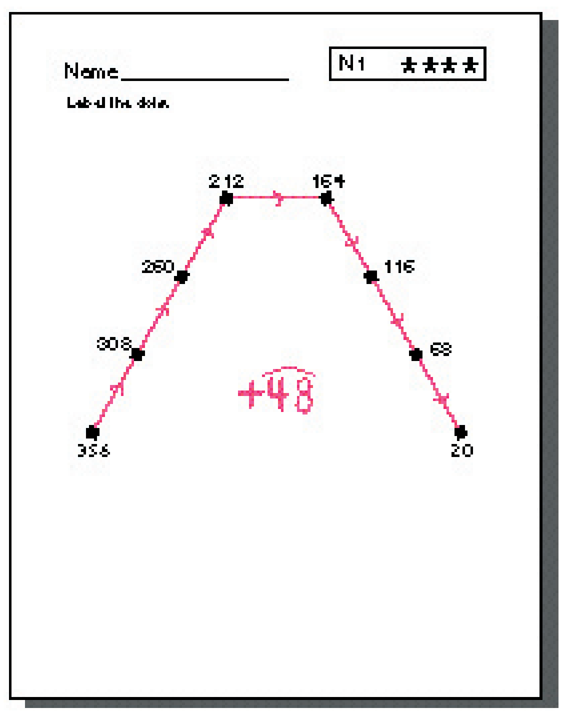

Name________________ N1 ★★

Label the dots.


Name________________ N1 ★★★

Pair names for the same number.

190 + 8	474 + 48
606 + 80	470 + 222
111 + 82	391 + 209
1,050 + 802	850 + 82


Name________________ N1 ★★★★

Label the dots.

N2 COMPOSITION OF FUNCTIONS #1

Capsule Lesson Summary

Using an array of dots and the Minicomputer, illustrate that the composition of 3x followed by 2x, or 2x followed by 3x, is 6x. Notice that to multiply a number by 6, one can first multiply it by 3 and then multiply the result by 2, or vice versa.

Materials

Teacher
- Colored chalk
- Minicomputer set
- Checkers or counters (optional)

Student
- Paper
- Colored pencils, pens, or crayons
- Worksheets N2* and **

Description of Lesson

Exercise 1

Draw a row of eight dots on the board, or use checkers or counters.

T: ***How many dots are in this row?*** (Eight)
I want to make a picture with 3 x 8 dots. How could I place the dots so we easily see 3 x 8 dots?

Consider all suggestions until someone places two more rows with eight dots.

T: ***If we place two more rows of eight dots, will there be 3 x 8 dots in this picture?***

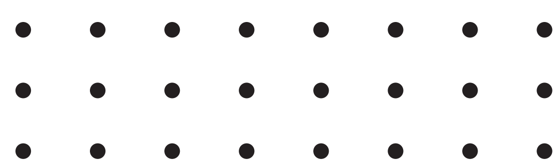
S: ***Yes, because you will have three rows of eight dots.***

T: ***How many dots will there be in my picture then?***
How do you know?

S: ***24, because 3 x 8 = 24.***

S: ***8 + 8 + 8 = 24.***

S: ***3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 24***



Complete the dot picture and ask a volunteer to count the dots by threes (or eights) to confirm that there are 24 dots in your picture.

T: ***What number sentences could we write about this dot picture?***

Record number sentences as they are mentioned, and ask the students to show how each number sentence is suggested by the dot picture. Several appropriate number sentences are given here.

$$3 \times 8 = 24 \qquad \tfrac{1}{3} \times 24 = 8$$
$$8 \times 8 \times 8 = 24 \qquad \tfrac{1}{2} \times 24 = 12$$
$$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 24$$

T: ***Let's draw an arrow picture to show how we made this array of dots. How many dots did we start with in the first row?***

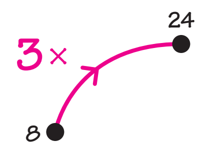
S: ***Eight.***

Draw a dot and label it 8. Instruct students to copy the arrow picture on their papers as it evolves.

T: ***How many dots are there in three rows?***

S: ***24.***

Draw another dot and label it 24. Draw an arrow from 8 to 24.



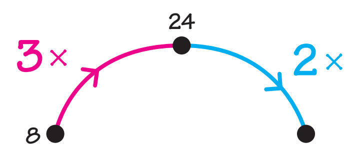
T: ***What could this arrow be for?***

S: ***3x.***

Label the arrow 3x.

Note: The arrow could also be for +16 and several other relations, but we are only interested in 3x for the purpose of this lesson. Accept other correct suggestions, but do not record them on the board.

Draw and label a 2x arrow starting at 24 as you say,

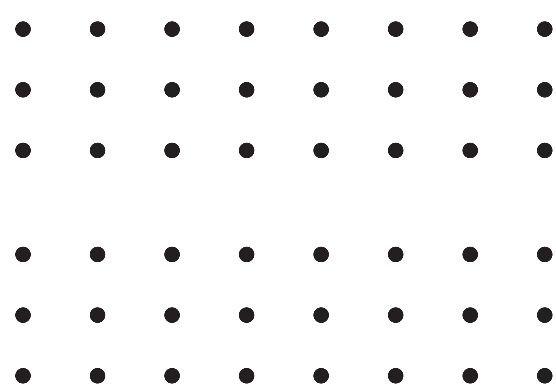


T: ***Now I would like to double the number of dots in this array. How could I do this?***

S: ***Put three more rows of dots.***

T: ***There are three rows of eight dots in the picture now. So if I want twice as many dots, I need three more rows of eight dots.***

Draw three more rows of eight dots, but leave space between the first and second group of 24 dots.



T: ***How many dots are there in each of these rows?*** (Eight)
How many rows of eight dots are there? (Six)
How many dots are there altogether in this dot picture?
How do you know?

S: ***48, because there were 24 dots in the first picture and 2 x 24 = 48.***

S: ***I knew that 6 x 8 = 48.***

S: ***I counted the dots and there are 48 of them.***

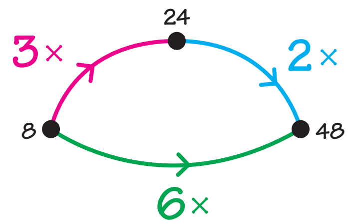
T: ***What numbers sentences could we write about this dot picture?***

Record number sentences as they are mentioned, and ask students to show how each number sentence is suggested by the dot picture.

Draw an arrow from 8 to 48.

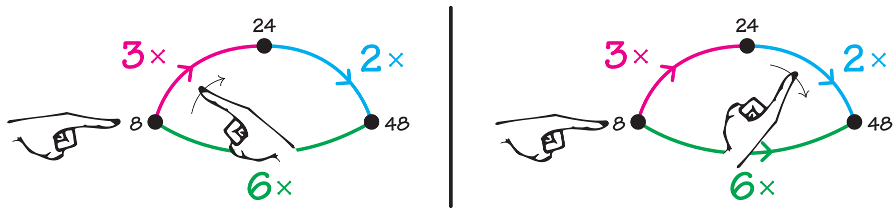
T: ***I drew an arrow from 8 to 48.***
What could this arrow be for?

S: ***6x.***



With the following dialogue, use motions as described in *Composition Games #1* and *#2* (see *UPG-III* Lessons L2 and L7).

T: ***A 3x arrow*** (trace the 3x arrow)… ***followed by a 2x arrow*** (trace the 2x arrow)**…**



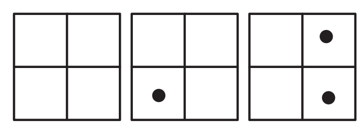
… is the same as a 6x arrow (trace the 6x arrow)**.**

Note: A common error is to say 3x followed by 2x is 5x. Using the dot pictures and the Minicomputer (see Exercise 2), emphasize that 3x followed by 2x, and 2x followed by 3x, is 6x.

Erase everything on the board except the arrow picture, and erase the labels for the dots.

Exercise 2

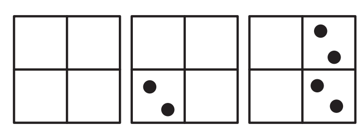
Put 25 on t~~he Minicomputer.~~



T: ***What number is on the Minicomputer?*** (25)
Let's double the number on the Minicomputer.
What is an easy way to make the number on the Minicomputer twice what it is now?

S: ***Put another checker with each checker already on the Minicomputer.***

Ask a student to double the number on the Minicomputer.



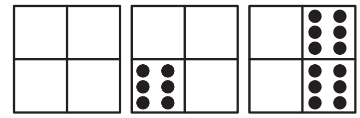
T: ***We have a new number on the Minicomputer.***
What is an easy way to make the number on the Minicomputer three times what it is now?

Encourage the students to explain their suggestions before doing anything on the Minicomputer. If necessary, ask students to focus first on the 20-square and ask how many checkers will be on that square after the number is multiplied by 3.

S: ***After we multiply by 3, there will be six checkers on each square where there are two checkers now.***

Call on volunteers to help you multiply the number on the Minicomputer by 3.

T: ***What calculation is suggested by the checkers on the Minicomputer?***



S: ***6 x 25.***

T: ***Let's draw an arrow picture about what we did on the Minicomputer. We started with 25 on the Minicomputer.***

Draw a dot and label it 25. Instruct the students to copy the arrow picture on their papers as it evolves.

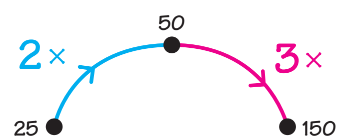
T: ***Then we doubled the number on the Minicomputer.***

Draw an arrow starting at 25 and label it 2x.

T: ***What did we do after we multiplied by 2?***

S: ***We multiplied by 3.***

Continue the arrow picture.



T (pointing to the dot on the right)**:** ***What number is here? How do you know?***

S: ***It's the number on the Minicomputer. There are six checkers on the 20-square. 6 x 20 = 120, so 120 is on the tens board. 6 x 5 = 30, so 30 is on the ones board. 120 + 30 = 150.***

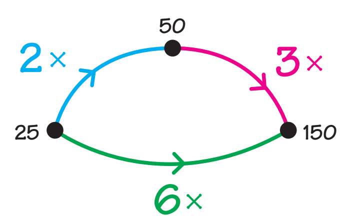
If no one multiplies 2 x 25 and then 3 x 50, suggest this as another approach.

Call on students to label the dots in the arrow picture. Then draw an arrow from 25 to 150.

T: ***What could this arrow be for?***

S: ***6x.***

Label the arrow 6x.



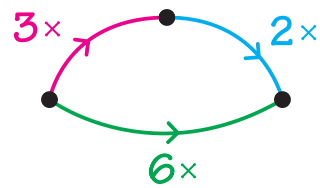
T: ***6 x 25 = 150. 6 x 25 is the calculation we did on the Minicomputer*** (point to it)***.***

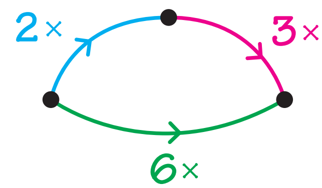
Call on students to make trades on the Minicomputer until you have the standard configuration for 150.

Point to the arrow picture and use motions as described in the *Composition Games* lessons.

T: ***A 2x arrow*** (trace the 2x arrow) ***followed by a 3x arrow*** (trace the 3x arrow) ***is the same as a 6x arrow*** (trace the 6x arrow)***.***

Erase the labels for the dots. You (and students) should have these two arrow pictures on the board (their papers).





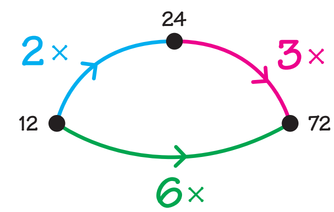
T: ***Let's see if we can use these arrow pictures to solve some multiplication problems. What number is 6 x 12? Write the answer on your paper.***

Look at many of the students' papers before asking a student to write 6 x 12 = 72. Invite several students to explain the various methods that they used to do the calculation. Whenever possible, relate a method to one or the other of the arrow pictures on the board. For example:

S: ***6 x 12 = 72. I multiplied 2 x 12 = 24; then I added 24 + 24 + 24 = 72.***

T: ***Your method is shown in this arrow picture with a 2x arrow (2 x 12) followed by a 3x arrow (24 + 24 + 24 is 3 x 24).***

Who calculated 6 x 12 another way?



S: ***I added 12 + 12 + 12 + 12 + 12 + 12 = 72.***

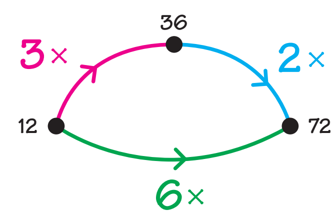
Write the addition problem on the board and invite a student to do it.

S: ***I wrote the same problem on my paper but then I saw that 3 x 12 = 36. So I put the first three twelves together and the last three twelves together and calculated 36 + 36 = 72.***

$$\begin{array}{r} {}^{1}\\ \left.\begin{array}{r}12\\12\\12\end{array}\right\} \overset{1}{36}\\ \left.\begin{array}{r}12\\12\\+\ 12\end{array}\right\} 36\\ \hline 72 \quad\ 72 \end{array}$$

T: ***Your method is shown in this arrow picture with a 3x arrow (3 x 12) followed by a 2x arrow (36 + 36 is 2 x 36).***

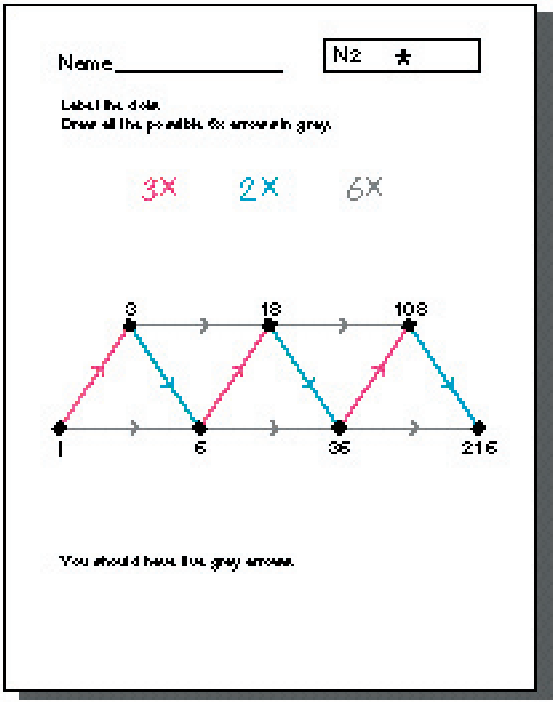
S: ***I knew that 6 x 10 = 60 and 6 x 2 = 12, so I added 60 + 12.***

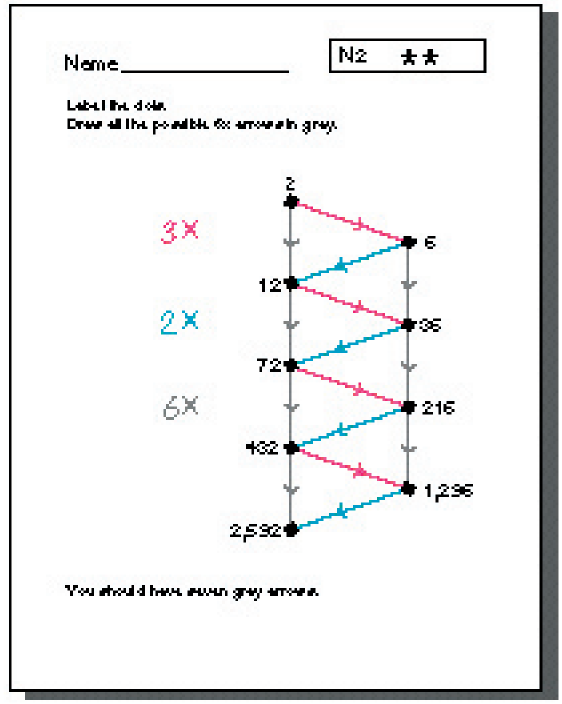


Emphasize that multiplying by 3 and then by 2 gives the same answer as multiplying by 2 and then by 3. Erase everything except the arrow pictures. Erase the labels for the dots.

Repeat this activity to calculate 6 x 17.

Worksheets N2* and ** are available for individual work. Encourage students who finish quickly to draw their own 2x and 3x arrow pictures.


Name
N2 ★
3X
2X
6X
3
18
108
1
6
36
216


Name
N2 ★★
3X
2X
6X
2
6
12
36
72
216
432
1,296
2,592

Capsule Lesson Summary

Pose sequences of subtraction calculations that have the same answer, and use properties of subtraction to generate others.

Materials

Teacher		Student	
	• Minicomputer set		• Paper • Worksheets N3*, **, ***, and ****

Description of Lesson

Note: During this lesson students who have learned a regrouping (paper and pencil) algorithm for subtraction may want to use it. Agree that it is a good method, but explain that in this lesson you would like them to use patterns, mental methods, and properties of subtraction. Encourage these students to look for patterns as another way to do the problems.

Exercise 1

Write the subtraction facts on the board as students answer.

T: ***What number is 12 – 5?***

S: *7.*

T: ***What number is 13 – 6?***

S: *7.*

$$12 - 5 = 7$$
$$13 - 6 = 7$$
$$14 - 7 = 7$$
$$15 - 8 = 7$$

Continue this activity until you have a list of four or five similar subtraction facts on the board.

T: ***What patterns do you see?***

Let students comment.

T: ***What are some other subtraction problems with 7 as the answer? Write some on your paper.***

Encourage students to contribute several problems. Record correct number sentences in your list on the board.

T: ***What number is 16 – 9?***

S: *7.*

$$16 - 9 = 7$$

Start another list on the board.

$$26 - 19 =$$

T: ***What number is 26 – 19? How do you know?***

S: ***It's still 7—you just added 10 to both numbers.***

S: ***I counted from 19 to 26, and the answer is 7.***

T: ***What do you notice about these two number sentences?***

S: ***If you add 10 to 16, you get 26. If you add 10 to 9, you get 19.***

S: ***The answer to both problems is 7.***

$$16 - 9 = 7$$
$$26 - 19 = 7$$

Record the answer and pose the next problem.

S: ***36 – 29 = 7. The answer is the same because you again added 10 to both numbers.***

$$36 - 29 = 7$$
$$46 - 39 = 7$$
$$56 - 49 = 7$$

Continue this activity until you have a list of four or five number sentences on the board.

T: ***What is another subtraction problem with 7 as the answer? Write one on your paper.***

Let students make suggestions and record them on the board. A sample dialogue is given here.

S: ***66 – 59.***

T: ***How do you know that 66 – 59 = 7?***

S: ***I added 10 to 56 and also to 49.***

Note: Students may or may not use a preceding problem to find a new problem with the same answer. Accept all valid approaches.

S: ***1,056 – 1,049 = 7.***

T: ***How do you know that problem has the same answer as the others?***

S: ***I added 1,000 to 56 and also to 49.***

Continue this activity until approximately 10 number sentences are in your list. If calculations with very large numbers are suggested (for example, 1,000,066 – 1,000,059), ask a student to write the number sentence on the board. If only large numbers are suggested, encourage students to offer number sentences with numbers less than 200.

Erase the board before going on to Exercise 2.

Exercise 2

Write this problem on the board.

T: ***What number is 28 – 15?*** (13) ***How do you know?***

S: ***20 – 10 = 10 and 8 – 5 = 3, so 28 – 15 = 13.***

S: ***28 – 10 = 18 and 18 – 5 = 13.***

$$\begin{array}{r} 28 \\ -\ 15 \\ \hline \end{array}$$

Record the answer and pose another problem.

T: ***What number is 29 – 16 ?*** (13) ***How did I change the problem?***

S: ***You added 1 to 28 and 1 to 15.***

$$\begin{array}{r} 28 \\ -\,15 \\ \hline 13 \end{array} \qquad \begin{array}{r} 29 \\ -\,16 \\ \hline \end{array}$$

Continue this activity until you have these calculations on the board.

$$\begin{array}{r} 28 \\ -\,15 \\ \hline 13 \end{array} \qquad \begin{array}{r} 29 \\ -\,16 \\ \hline 13 \end{array} \qquad \begin{array}{r} 30 \\ -\,17 \\ \hline 13 \end{array} \qquad \begin{array}{r} 32 \\ -\,19 \\ \hline 13 \end{array} \qquad \begin{array}{r} 52 \\ -\,39 \\ \hline 13 \end{array}$$

T: ***How do you know that 52 – 39 = 13?***

S: ***You added 20 to both 32 and 19, so the result is the same as 32 – 19.***

T: ***Can you suggest a different way to calculate 52 – 39 and check the result?***

Your students know many ways to solve this problem. Perhaps they will suggest one of the following methods. Be prepared to discuss their different methods.

$$\begin{array}{r} 53 \\ -\,40 \\ \hline 13 \end{array} \qquad \begin{array}{r} 52 \\ -\,30 \\ \hline 22 \\ -\;\;9 \\ \hline 13 \end{array} \qquad \begin{array}{r} 52 \\ 40 \\ \hline 12 \\ +\;\;1 \\ \hline 13 \end{array} \qquad \begin{array}{r} 52 \\ -\,39 \\ \hline 2\widehat{7} \end{array} = 20 + \widehat{7} = 13$$

Note: Some students may suggest the last method above, but do not expect students to find it. A student may suggest "borrowing" or regrouping. Accept the suggestion, but do not go into a lengthy discussion.

Encourage students to suggest other subtraction problems with 13 as the result. Record these problems on the board as they are suggested and ask students to explain how they know that the result is 13.

Erase the board before going on to Exercise 3.

Exercise 3

Write this problem on the board.

T: ***What number is 65 – 40?*** (25) ***How do you know?***

$$\begin{array}{r} 65 \\ -\,40 \\ \hline \end{array}$$

Encourage the students to explain their calculations.

Record the answer and pose another problem.

N3

T: ***63 minus what number is 25?***

S: ***38.***

T: ***How do you know?***

S: ***The answers are the same, and since 65 – 2 = 63, I subtracted 2 from 40.***

$$\begin{array}{r} 65 \\ -\ 40 \\ \hline 25 \end{array} \qquad \begin{array}{r} 63 \\ -\ \boxed{} \\ \hline 25 \end{array}$$

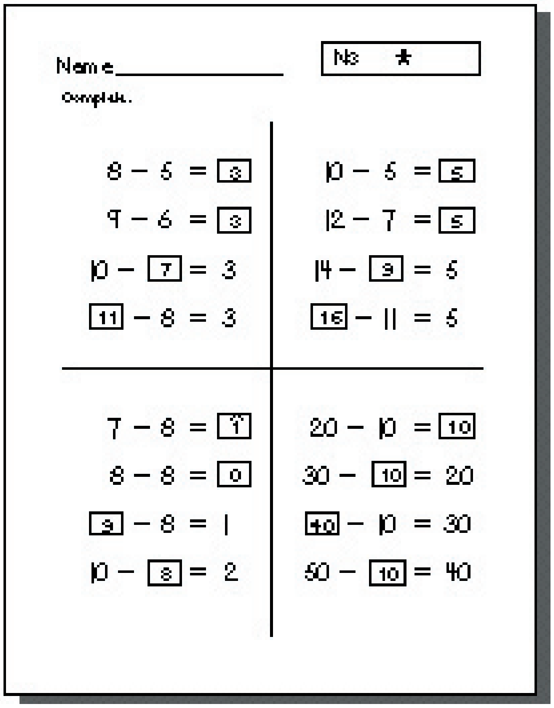
Record 38 in the box and continue in this way with several more similar problems.

$$\begin{array}{r} 65 \\ -\ 40 \\ \hline 25 \end{array} \qquad \begin{array}{r} 63 \\ -\ \boxed{38} \\ \hline 25 \end{array} \qquad \begin{array}{r} 60 \\ -\ \boxed{35} \\ \hline 25 \end{array} \qquad \begin{array}{r} 71 \\ -\ \boxed{46} \\ \hline 25 \end{array} \qquad \begin{array}{r} \boxed{} \\ -\ 45 \\ \hline 25 \end{array} \qquad \begin{array}{r} \boxed{} \\ -\ 39 \\ \hline 25 \end{array}$$

Worksheets N3*, **, ***, and **** are available for individual work.

Encourage students who do the *** and **** worksheets to try to match the numbers using what they know about subtraction without doing too much calculation. That is, do not insist that students do each calculation before matching the numbers.

Suggest that students who finish quickly write many subtraction problems with 78 as the result.



Name____________ N3 ★

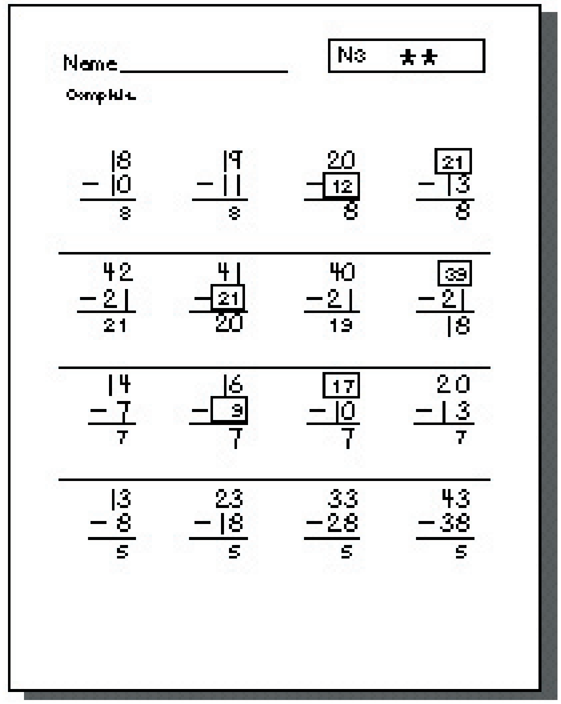
Complete.

8 − 5 = [3]
9 − 6 = [3]
10 − [7] = 3
[11] − 8 = 3

10 − 5 = [5]
12 − 7 = [5]
14 − [9] = 5
[16] − 11 = 5

7 − 8 = [1]
8 − 8 = [0]
[9] − 8 = 1
10 − [8] = 2

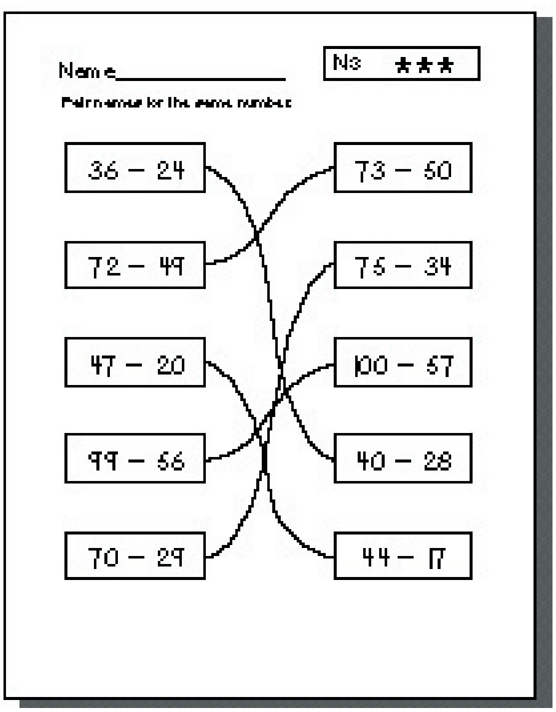
20 − 10 = [10]
30 − [10] = 20
[40] − 10 = 30
50 − [10] = 40



Name____________ N3 ★★

Complete.

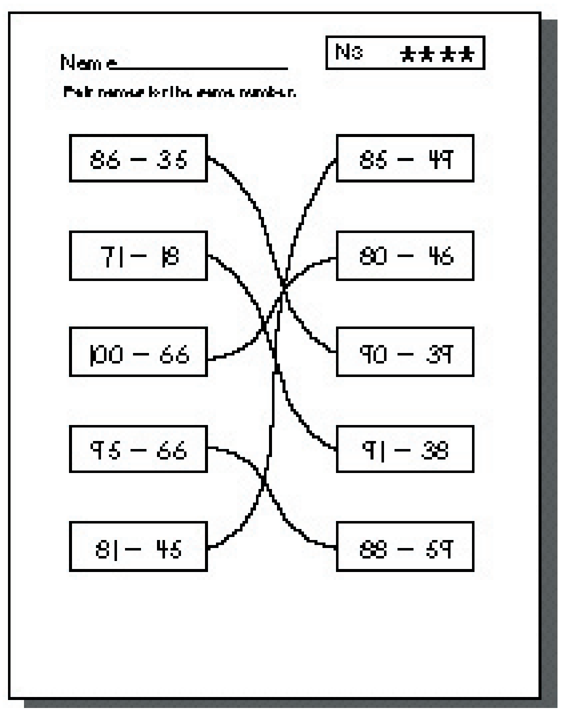
18 − 10 = 8	19 − 11 = 8	20 − [12] = 8	[21] − 13 = 8
42 − 21 = 21	41 − [21] = 20	40 − 21 = 19	[39] − 21 = 18
14 − 7 = 7	16 − [9] = 7	[17] − 10 = 7	20 − 13 = 7
13 − 8 = 5	23 − 18 = 5	33 − 28 = 5	43 − 38 = 5



Name____________ N3 ★★★

Pair names for the same number.

36 − 24	73 − 60
72 − 49	75 − 34
47 − 20	100 − 67
99 − 66	40 − 28
70 − 29	44 − 17



Name____________ N3 ★★★★

Pair names for the same number.

86 − 35	85 − 49
71 − 18	80 − 46
100 − 66	90 − 39
95 − 66	91 − 38
81 − 45	88 − 59

N4 ADDITION OF DECIMALS

Capsule Lesson Summary

Decide which of several items can be purchased within the limits of a certain amount of money. Use the Minicomputer, mental arithmetic, or paper and pencil to add costs and to calculate change.

Materials

Teacher
- Minicomputer set
- Colored chalk
- Real or play money

Student
- Paper
- Worksheets N4*, **, ***, and ****

Description of Lesson

Exercise 1

Choose a student to star in this or a similar story.

T: ***Warren is visiting a toy store. He has exactly $2 to spend. What coins or bills could Warren have to make exactly $2?***

Encourage students to suggest many ways to make $2 in cash. Combinations your class might suggest include the following:

- eight quarters
- ten dimes and four quarters
- four 50¢ pieces
- 200 pennies
- two $1 bills
- one $2 bill

For each suggestion that has a reasonable number of coins and/or bills, invite students to display the money and then count it as a class.

T: ***There are many things in the toy store that Warren would like to buy.***

Ask the student star of your story to name three small things he would like to buy in a toy store. In this example, Warren chooses a jig-saw puzzle, a model boat, and a race car track. Assign a price to each object the student chooses and record each item and its price on the board.

jigsaw puzzle	$1.25
model boat	$0.90
race track	$1.45

T: ***Which toy is the most expensive?*** (Race track)
Which toy is the least expensive? (Model boat)
Warren has exactly $2. Is there any toy he cannot afford to buy?

S: ***Each toy costs less than $2, so he can afford any one of them.***

Point to each price in turn and ask if it is less than $2.

T: ***Warren can afford to buy any one of these toys, but he would like to buy two different toys. Is that possible?***

Encourage students to comment on this problem. Suppose a student suggests that Warren can afford to buy two of the toys.

S: ***Warren could buy the jigsaw puzzle and the model boat.***

S: ***No; $1.25 + $0.90 is more than $2.***

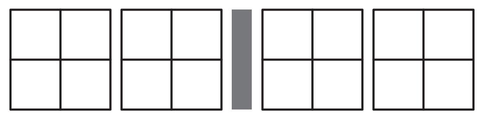
T: ***How much is $1.25 + $0.90?***

$1.25 × 0.90 = ?

Suggest students write their answers on their papers or whisper them to neighbors.

T: ***Let's calculate $1.25 + $0.90 on the Minicomputer.***

Display four Minicomputer boards with a bar between the second and third boards.

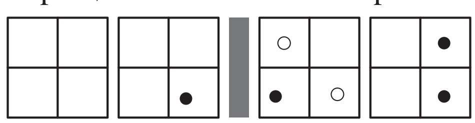


T: ***What does this bar tell us?***

S: ***The boards on the left of the bar are for dollars, and the boards on the right of the bar are for cents.***

Briefly review the value of the boards by moving a checker from one red square to the next red square of higher value and asking the students to say each amount: 2¢, 20¢, $2, $20.

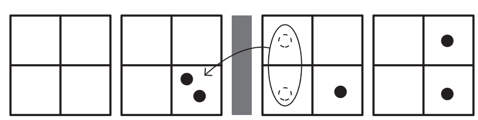
Call on a volunteer to put $1.25 on the Minicomputer and another to add $0.90.



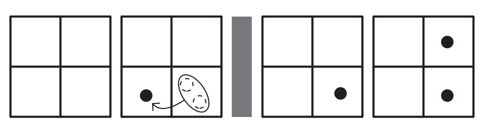
= 1.25 + 0.90

Students should then suggest making trades.

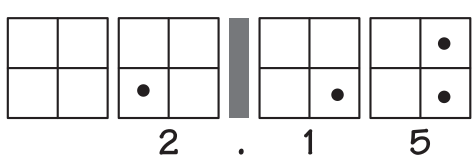
S: ***80¢ + 20¢ = $1.***



S: ***$1 + $1 = $2.***



Invite a student to write the number below (above) the Minicomputer and to complete the number sentence.



= 1.25 + 0.90 = 2.15

Some students may correctly do the calculation before the trades are made on the Minicomputer. Ask them to explain their methods.

T: ***How would we calculate $1.25 + $0.90 without using the Minicomputer?***

If necessary, suggest using the addition algorithm.

$$\begin{array}{r} {}^{1} \\ 1.25 \\ +\ 0.90 \\ \hline 2.15 \end{array}$$

Emphasize that the decimal points are lined up so that the cents will be added together and the dollars will be added together.

T: ***$1.25 + $0.90 = $2.15, so Warren cannot afford to buy these two toys. Is it possible for Warren to buy two different toys?***

Continue this activity until your students conclude that it is not possible for Warren to buy two different toys. Some students might propose alternative toys, but you should stress that Warren wants to buy two of the three toys he chose earlier.

Allow the student star to choose which of the three toys he would prefer to buy. Then ask the students to determine how much change there will be from the $2 if this toy is purchased. If several students are able to calculate the amount of change mentally, ask them to explain their calculations. You or students can suggest calculating the amount of change on the Minicomputer in one of several ways. For example, if the toy costing $1.45 is selected, you could do any of the following:

1) Calculate $2.00 – $1.45
2) Calculate $2.00 + $\widehat{1.45}$
3) Find an amount so that $1.45 + $\boxed{}$ = $2.00

Erase the board and remove the checkers from the Minicomputer.

Exercise 2

Choose a student to star in this or a similar story.

T: ***Andrea is visiting a stationery store. What could she buy at a stationery store?***

You may need to tell your class that stationery stores sell paper, envelopes, cards, pens, and other objects associated with letter writing.

T: ***Andrea is going on a trip and wants to buy some stationery so she can write to her friends. She has $5 to spend. There are four things that Andrea is interested in buying.***

List the following items and their prices on the board as they are mentioned.

T: ***Andrea would like to buy some stationery with clown faces at the top. A box of this stationery sells for $3.50. She would like to buy a package of initialed envelopes that costs $1.10. There is a box of plaid stationery she likes that costs $1.75. Andrea also would like to buy a special set of colored pens. The price of the set is $2.15.***

clown face stationery	$3.50
envelopes	$1.10
plaid paper	$1.75
pens	$2.15

T: ***Which item is most expensive?*** (Clown face stationery)
Which item is least expensive? (Envelopes)
Is there any item Andrea cannot afford to buy? (No) ***She can afford each of them, but she would like to buy as many different things as possible. What do you suggest?***

Encourage the students to comment.

S: ***If Andrea buys the stationery with clown faces, the only other thing she can afford to buy is the envelopes.***

T: ***How much will it cost to buy the stationery with clown faces and the envelopes?***

S: ***$4.60.***

$$\$3.50 \times \$1.10 = \$4.60$$

S: ***She could buy everything except the stationery with the clown faces.***

T: ***How can we be sure? What calculation should we do?***

S: ***$1.10 + $1.75 + $2.15.***

Ask a student to write the calculation on the board and to solve it.

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ \ } \\ 1.10 \\ 1.75 \\ +\ 2.15 \\ \hline 5.00 \end{array}$$

Conclude that Andrea can buy everything except the clown face stationery for exactly $5.

Worksheets N4*, **, ***, and **** are available for individual work.

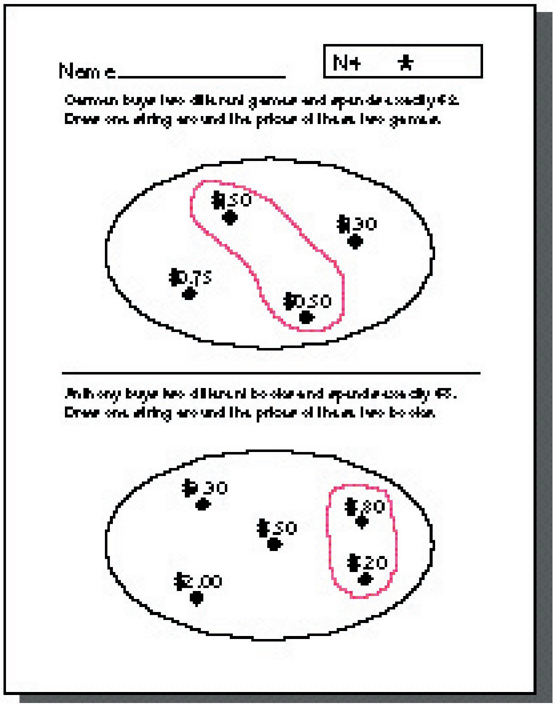
Note: On these worksheets, it will be helpful for students to recognize two amounts of money that add to $1.00. You may like to discuss several possibilities with the students before they start the worksheets. For example:

$0.50 + $0.50 = $1.00	$0.60 + $0.40 = $1.00	$0.95 + $0.05 = $1.00	$0.85 + $0.15 = $1.00
$0.80 + $0.20 = $1.00	$0.70 + $0.30 = $1.00	$0.75 + $0.25 = $1.00	$0.65 + $0.35 = $1.00

Home Activity

Suggest that parents/guardians use real money to do some of the following activities with their children.

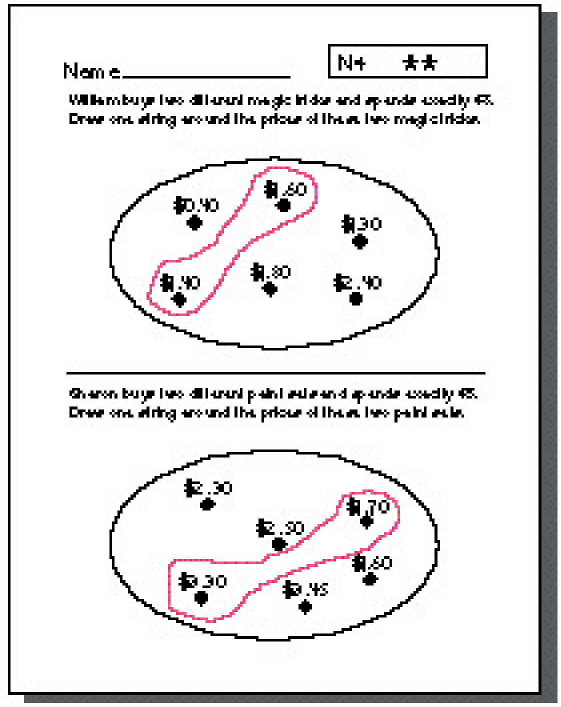
1) Review the values of various coins.
2) Make trades such as five nickels for a quarter, or five dimes for a half-dollar, or two dimes and a nickel for a quarter, and so on.
3) Count a collection of nickels, dimes, or quarters.
4) Practice making various amounts of money in several ways.
5) Calculate the amount of money in a given collection of coins.



Name ____________ N4 ★

Carmen buys two different games and spends exactly $2.
Draw one string around the prices of these two games.

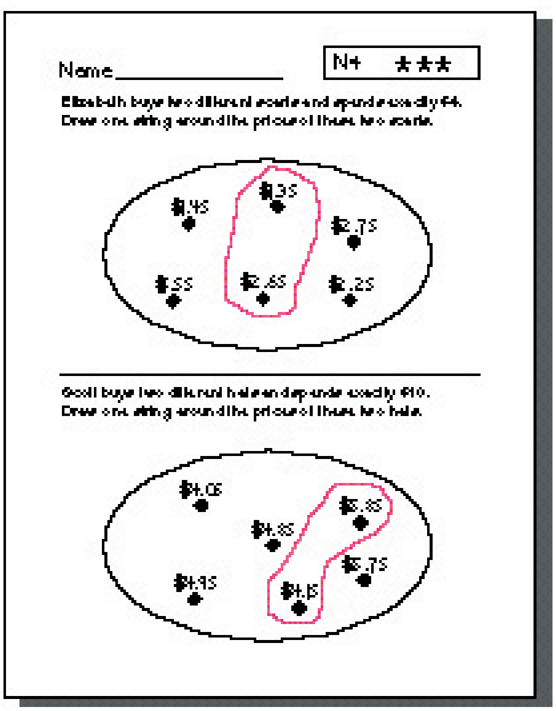
Anthony buys two different books and spends exactly $5.
Draw one string around the prices of these two books.



Name ____________ N4 ★★

William buys two different magic tricks and spends exactly $3.
Draw one string around the prices of these two magic tricks.

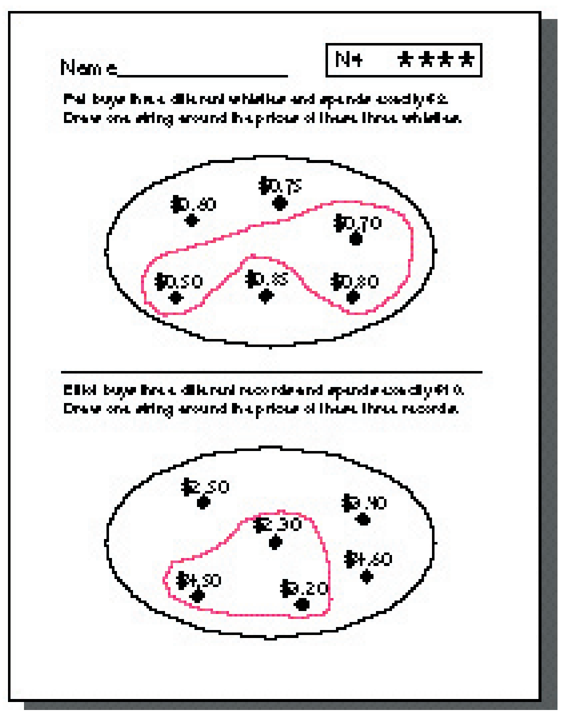
Sharon buys two different paint sets and spends exactly $5.
Draw one string around the prices of these two paint sets.



Name ____________ N4 ★★★

Elizabeth buys two different scarfs and spends exactly $4.
Draw one string around the prices of these two scarfs.

Scott buys two different hats and spends exactly $10.
Draw one string around the prices of these two hats.



Name ____________ N4 ★★★★

Pat buys three different whistles and spends exactly $2.
Draw one string around the prices of these three whistles.

Elliot buys three different records and spends exactly $10.
Draw one string around the prices of these three records.

N5 SUBTRACTION PROBLEMS #3

Capsule Lesson Summary

Do some mental arithmetic using properties of subtraction or subtraction patterns. Decide how many of Mr. Chipper's watermelons are missing if one day he has 363 and the next day only 295. Subtract one number from another by finding and doing an easier calculation with the same answer.

Materials

Teacher • Colored chalk

Student • Paper

Description of Lesson

Exercise 1

Conduct a mental arithmetic exercise with sequences of related subtraction facts such as the following.

8 – 3 = 5	13 – 5 = 8	20 – 3 = 17
9 – 4 = 5	11 – 3 = 8	21 – 4 = 17
11 – 6 = 5	15 – 7 = 8	23 – 6 = 17
14 – 9 = 5	25 – 17 = 8	26 – 9 = 17
24 – 19 = 5	45 – 37 = 8	36 – 19 = 17
124 – 119 = 5	245 – 237 = 8	66 – 49 = 17

Exercise 2

Tell the following or a similar story to your class.

T: ***There is a man named Mr. Chipper who grows watermelons. Every day Mr. Chipper counts his watermelons. Why would he do this?***

Allow students to express their opinions.

T: ***Yesterday Mr. Chipper counted 363 watermelons but today he counted only 295. What could have happened?***

Students very likely will suggest that some watermelons were stolen.

T: ***Mr. Chipper is very upset because some of his watermelons are missing. He thinks that they may have been stolen, so he decides to report the missing watermelons to the sheriff. Mr. Chipper tells the sheriff, "Yesterday I had 363 watermelons and now I have only 295. I think some of my watermelons may have been stolen." The sheriff pulls out the official forms and begins to complete them; he asks Mr. Chipper how many watermelons were stolen.***

Mr. Chipper thinks for a while and then he tells the sheriff that he can't count the missing watermelons until they are found. The sheriff says he can't report the missing watermelons until he knows how many watermelons are missing. Can you help them figure this out?

N5

Let the students work on this problem independently or with partners for a few minutes. Collectively discuss students' ideas for solving the problem.

T: ***After a long time the sheriff writes this problem on a piece of paper. "You had 363 watermelons, then someone took away some of the watermelons and now you have 295 watermelons left," explains the sheriff.***

$$\begin{array}{r} 363 \\ - \\ \hline 295 \end{array}$$

Mr. Chipper says "I think they took about 100 watermelons." The sheriff writes this problem on the paper. What number is 363 – 100?

$$\begin{array}{r} 363 \\ -\ 100 \\ \hline \end{array}$$

S: ***263.***

Record that 363 – 100 = 263.

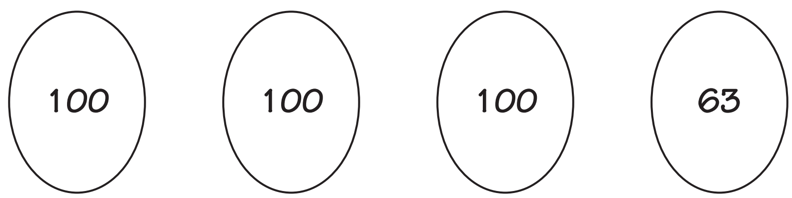
T: ***Are more or less than 100 watermelons missing?***

S: ***Less.***

T: ***If 100 watermelons were taken, Mr. Chipper would have only 263 watermelons left. So we know that less than 100 watermelons are missing.***

Draw this picture on the board.

$$\begin{array}{r} 363 \\ - \\ \hline 295 \end{array}$$

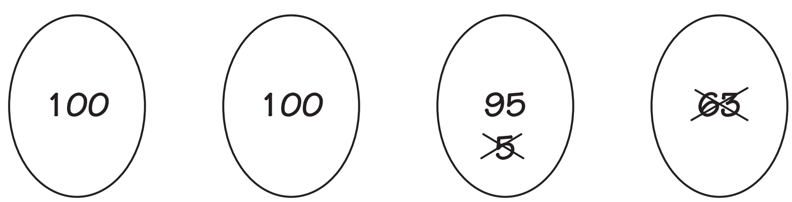


T: ***The sheriff finally draws this picture after he and Mr. Chipper have tried many other numbers that do not work. The sheriff says, "This is a picture of the 363 watermelons that you had yesterday. Today you have only 295 watermelons."***

How could we show in this picture that Mr. Chipper has only 295 watermelons left?

S: ***We could cross out the 63 watermelons and then there would be 300 left. Then cross out 5 more watermelons from one of the 100's, so there are only 295 left.***

$$\begin{array}{r} 365 \\ - \\ \hline 295 \end{array}$$



T: ***Does this picture help us know how many watermelons are missing? How?***

S: ***63 + 5 = 68, so 68 watermelons are missing.***

Record 68 in the space for the missing number.

$$\begin{array}{r} 363 \\ -\ \ 68 \\ \hline 295 \end{array}$$

T: ***Let's check that 363 – 68 = 295.***
Can you suggest an easy way to calculate 363 – 68?

Perhaps your students will suggest one of these methods.

$$\begin{array}{r} 363 \\ -\ 60 \\ \hline 303 \\ -\ \ 8 \\ \hline 295 \end{array} \qquad \begin{array}{r} 363 \\ -\ 63 \\ \hline 300 \\ -\ \ 5 \\ \hline 295 \end{array} \qquad \begin{array}{r} 363 \\ -\ 68 \\ \hline 30\hat{5} \end{array} = 300 + \hat{5} = 295$$

Erase the board before going on to Exercise 3.

Exercise 3

Write this problem on the board.

$$\begin{array}{r} 52 \\ -\ 19 \\ \hline \end{array}$$

T: ***What number is 52 – 19?***
Write the answer on your paper.

Look at many papers before asking a student to answer aloud. Ask several students to explain how they did the calculation. If no one suggests adding 1 to each of the numbers in this subtraction problem, suggest this yourself.

$$\begin{array}{r} 52 \\ -\ 19 \\ \hline \end{array} \qquad \begin{array}{r} 53 \\ -\ 20 \\ \hline \end{array}$$

T: ***Why would we want to calculate 53 – 20 instead of 52 – 19?***

S: ***Both problems have the same answer, but 53 – 20 is easier.***

T: ***What number is 53 – 20?*** (33)
So what number is 52 – 19? (33)
On your paper, write other subtraction problems with 33 as the answer.

As you observe students' work, encourage them to write many subtraction problems, some with large numbers. Invite students who have written correct subtraction problems to write them on the board. Emphasize that when both numbers are increased or decreased by the same amount, the answer is the same as before.

Erase the board and write this problem.

$$534 - 97$$

T: ***What number is 534 – 97? Write the answer on your paper.***

Look at many answers before asking a student to answer aloud. Ask several students to explain how they did the calculation. If no one suggests adding 3 to each of the numbers in this subtraction problem, suggest this yourself. Indicate that you intend to add 3 to each number by writing a small 3 above the ones digit of each number.

If we add 3 to each of these numbers,
what will the new subtraction problem be?

$$53\overset{3}{4} - 9\overset{3}{7}$$

S: ***537 – 100.***

T: ***What number is 537 – 100?***

S: ***437.***

$$53\overset{3}{4} - 9\overset{3}{7} = 537 - 100$$

Emphasize that both problems have the same answer but that the second problem is easier to solve.

Use a similar approach to do these calculations.

$$37\overset{1}{2} - 4\overset{1}{9} = 373 - 50 = 323$$

$$13\overset{2}{2} - 1\overset{2}{8} = 134 - 20 = 114$$

$$1{,}55\overset{4}{5} - 99\overset{4}{6} = 1{,}559 - 1{,}000 = 559$$

Write these problems on the board and ask the students to solve as many of them as they can.

73 – 18	225 – 98	1,463 – 997
95 – 29	865 – 199	2,551 – 995

After 5–10 minutes, solve these problems collectively. A sample dialogue is given here.

T: ***Melanie found an easy way to calculate 73 – 18.***
She added 2 to each of the numbers.
What number is 75 – 20?

$$7\overset{2}{3} - 1\overset{2}{8} = 75 - 20 = 55$$

S: ***55.***

T: ***Frankie found a way to make the second problem easier. He added 1 to each number.***

T: ***What number is 96 – 30?***

$$9\overset{1}{5} - 2\overset{1}{9} = 96 - 30 = 66$$

S: ***66.***

Continue this activity until all the problems have been solved.

$$22\overset{2}{5} - 9\overset{2}{8} = 257 - 100 = 157 \qquad 1{,}46\overset{3}{3} - 99\overset{3}{7} = 1{,}466 - 1{,}000 = 466$$

$$86\overset{1}{5} - 19\overset{1}{9} = 866 - 200 = 666 \qquad 2{,}55\overset{5}{1} - 99\overset{5}{5} = 2{,}556 - 1{,}000 = 1{,}556$$

Your students may prefer to write some of these problems vertically; for example,

$$\begin{array}{r} 2{,}55\overset{5}{1} \\ -\ \ 99\overset{5}{5} \\ \hline \end{array} \qquad \begin{array}{r} 2{,}556 \\ -\ 1{,}000 \\ \hline 1{,}556 \end{array}$$

Home Activity

Suggest that parents/guardians find opportunities to practice subtraction facts (with numbers up to 20) at home.

N6 ONE-THIRD OF 200

Capsule Lesson Summary

Mentally review 2x and ½x (or ÷2), 3x and ⅓x (or ÷3) as opposites. Estimate ⅓ x 135 and then do the calculation on the Minicomputer. Decide how to share $200 between three friends.

Materials

Teacher
- Minicomputer set
- Colored chalk
- Play money (optional)

Student
- Worksheets N6* and **
- Minicomputer set

Description of Lesson

Exercise 1: Mental Arithmetic

Begin this lesson with mental arithmetic involving 2x and ½x (or ÷2), 3x and ⅓x (or ÷3). These are some suggested sequences of calculations. (Read down the columns.)

2 x 10	½ x 50	½ x 10	3 x 10
½ x 20 or 20 ÷ 2	½ x 48	½ x 12	⅓ x 30 or 30 ÷ 3
2 x 14	½ x 52	½ x 11	3 x 13
½ x 28 or 28 ÷ 2	½ x 6	2 x 5½	⅓ x 39 or 39 ÷ 3
2 x 18	½ x 4	½ x 9	3 x 15
½ x 36 or 36 ÷ 2	½ x 5	2 x 4½	⅓ x 45 or 45 ÷ 3

Exercise 2

T: ***What number is 3 x 33?*** (99)
3 x 33 = 99, so what number is ⅓ x 99 (or 99 ÷ 3)? (33)
Approximately, what number is ⅓ x 135?

Write ⅓ x 135 on the board. Use whatever answers students suggest to demonstrate a strategy for estimating ⅓ x 135 or 135 ÷ 3.

S: ***⅓ x 135 is more than 33 because 135 is more than 99.***

$$\tfrac{1}{3} \times 135 > 33$$

S: ***I think ⅓ x 135 is about 42.***

T: ***How can we decide whether or not ⅓ x 135 = 42?***

S: ***Calculate 42 + 42 + 42 (or 3 x 42).***

Write the addition problem on the board and invite a student to solve it.

$$\begin{array}{r} 42 \\ 42 \\ +\ 42 \\ \hline 126 \end{array}$$

T: ***3 x 42 = 126. Is ⅓ x 135 more or less than 42?***

S: ***More.***

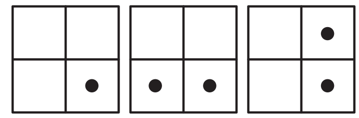
$$\tfrac{1}{3} \times 135 > 42$$

S: ***⅓ x 135 is less than 50, because 3 x 50 = 150 and ⅓ x 150 = 50.***

$$\tfrac{1}{3} \times 135 < 50$$

T: ***We know that ⅓ x 135 is more than 42 and less than 50.***

Continue this activity until your class concludes that ⅓ x 135 = 45. Record this fact on the board and ask a student to put 135 on the Minicomputer.

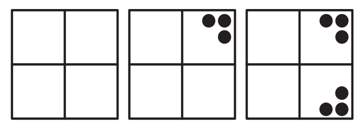


T: ***We already know that ⅓ x 135 is 45, but can we calculate ⅓ x 135 on the Minicomputer?***

S: ***We could make trades until all the checkers are in groups of three.***

Invite students to make trades on the Minicomputer. When they get a group of three checkers on the same square, push the three checkers to a corner of that square. Do not permit a student to break up any group of three checkers. Continue until all the checkers on any square of the Minicomputer are in groups of three.

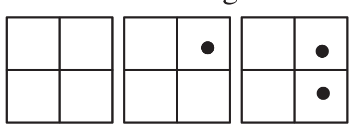
Perhaps you will have this configuration on the Minicomputer.



T: ***All the checkers are in groups of three, so how do we find ⅓ x 135 on the Minicomputer?***

S: ***Take off checkers and leave just one checker from each group of three.***

Ask a volunteer to remove checkers so that ⅓ x 135 will be on the Minicomputer. If necessary, make some trades to obtain the standard configuration for 45.



Erase the board and remove the checkers from the Minicomputer.

Exercise 3

You may like to use play money during this exercise.

T: ***I have $200 in cash. What coins or bills could I have?***

Encourage the class to offer many combinations. For example:

- two $100 bills
- two hundred $1 bills
- five $20 bills and ten $10 bills
- four hundred half-dollars

T: ***I would like to share this $200 among three friends.***

Choose three students to be your friends in this story.

T: ***If I give each of my three friends exactly one-third of the $200, how much will I give to each of them?***

S: ***$50.***

T: ***If I give each friend $50, how much money will I have given away?*** ($150)

I want to give each of my friends one-third of the $200.
Is ⅓ x 200 more or less than 50?

S: *More.*

$\frac{1}{3} \times 200 > 50$

T: *We know that each of my friends will receive at least $50, because ⅓ x 200 is more than 50.*

S: *I think that you can give each of them $70.*

T: *How much is 3 x $70?*

Write an addition problem on the board and ask a student to solve it, or use play money to model and count.

T: *Do I have enough money to give each of my friends $70?*

S: *No, you would need $210 to give each of them $70.*

T: *⅓ x 200 is less than 70.*
We know that ⅓ x 200 is more than 50 and less than 70.

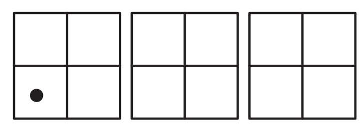
$\frac{1}{3} \times 200 > 50$

$\frac{1}{3} \times 200 < 70$

Continue this activity until your class concludes that each of your friends should receive between $66 and $67.

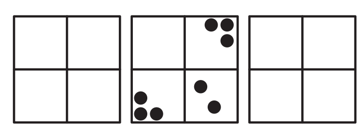
T: *Let's calculate ⅓ x 200 on the Minicomputer.*

Put 200 on the Minicomputer.

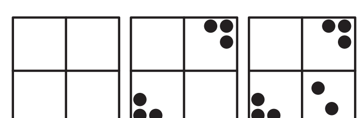


Invite students to make trades. Remind students that they need to get the checkers in groups of three on the same square. Since there are many trades to be made, you can speed up this activity by encouraging each volunteer to make several trades.

Before the jump is made from the tens board to the ones board, mention that all the checkers are in groups of three except the two extra checkers on the 10-square.



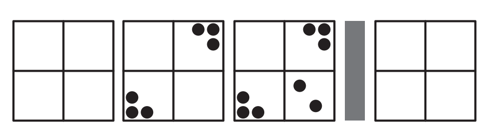
Continue this activity until all the checkers are in groups of three except for two extra checkers on the 1-square. Perhaps you will have this configuration on the Minicomputer.



T: *All the checkers are in groups of three except for two extra checkers on the 1-square. These checkers are for $2 which need to be shared. What could we do now?*

Encourage students to suggest solutions, then move a Minicomputer board to the right of the ones board leaving extra space between these two boards.

If no one mentions drawing a bar between the two boards on the right, ask if you have changed the number on the Minicomputer, and then draw the bar yourself.

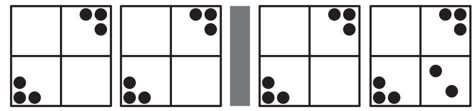


T: *The bar will remind us that the boards on the left are for dollars and the boards on the right are for cents.*

N6

Invite students to continue making trades until all the checkers are in groups of three except for two checkers on the white square of the dimes board. Some students may predict that to continue making trades onto the pennies board will result in the same situation because there will still be two extra checkers on the white square of the pennies board.

Move a Minicomputer board to the right of the dimes board and continue inviting students to make trades until all the checkers are in groups of three except for two extra checkers on the white square of the pennies board. Perhaps you will have this configuration on the Minicomputer.



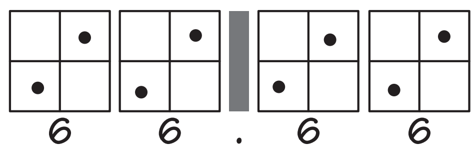
T: ***What would happen if I put another Minicomputer board on the right?***

S: ***If we make trades, we would have two extra checkers on the white square.***

T: ***No matter how many boards we use, there will always be two extra checkers on the white square furthest to the right. Two checkers on this white square*** (point to the extra checkers) ***are for what?*** (Two pennies) ***I don't know how to share two pennies among three people so I will keep them.***

Remove the two extra checkers from the 1¢-square.

Ask a volunteer to remove some checkers so that one-third of this number will be on the Minicomputer. This student should remove two checkers from each group of three. Invite another student to write the number below the Minicomputer.



T: ***How much money should I give each of my three friends?***

S: ***$66.66.***

T: ***If I give each of them $66.66, how much money will I give away altogether?***

Write this problem on the board and ask for a volunteer to solve it.

T: ***Will I be able to give each of my friends $66.66?***

S: ***Yes, because $199.98 is less than $200.***

T: ***How much less … ?***

S: ***2 cents.***

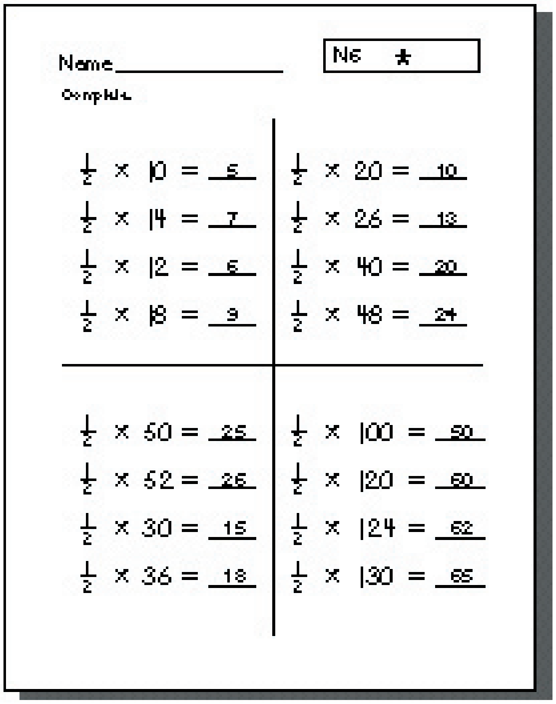
$$\begin{array}{r} {\scriptstyle 1\,1\;\;1} \\ 66.66 \\ 66.66 \\ +\ 66.66 \\ \hline 199.98 \end{array}$$

Write this number sentence on the board.

Worksheets N6* and ** are available for individual work.

$$(3 \times 66.66) \times 0.02 = 200$$

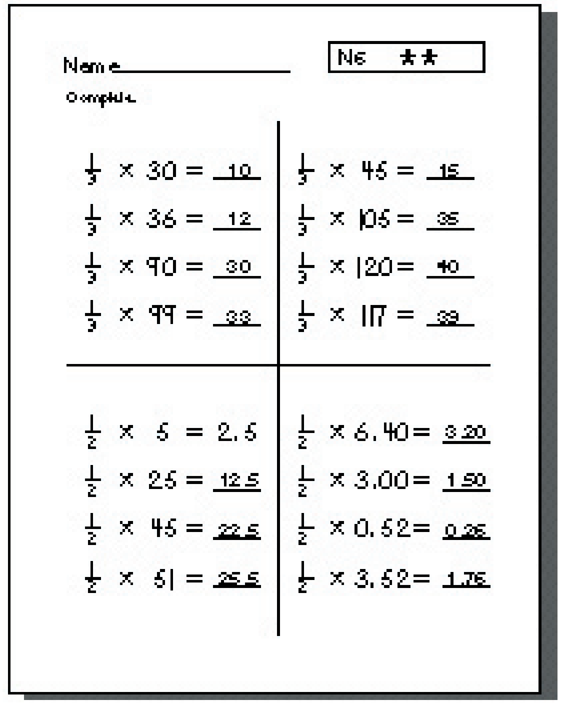
Allow students to use individual Minicomputers.



Name__________ N6 ★

Complete.

$\frac{1}{2} \times 10 = $ 5	$\frac{1}{2} \times 20 = $ 10
$\frac{1}{2} \times 14 = $ 7	$\frac{1}{2} \times 26 = $ 13
$\frac{1}{2} \times 12 = $ 6	$\frac{1}{2} \times 40 = $ 20
$\frac{1}{2} \times 18 = $ 9	$\frac{1}{2} \times 48 = $ 24
$\frac{1}{2} \times 50 = $ 25	$\frac{1}{2} \times 100 = $ 50
$\frac{1}{2} \times 52 = $ 26	$\frac{1}{2} \times 120 = $ 60
$\frac{1}{2} \times 30 = $ 15	$\frac{1}{2} \times 124 = $ 62
$\frac{1}{2} \times 36 = $ 18	$\frac{1}{2} \times 130 = $ 65



Name__________ N6 ★★

Complete.

$\frac{1}{3} \times 30 = $ 10	$\frac{1}{3} \times 45 = $ 15
$\frac{1}{3} \times 36 = $ 12	$\frac{1}{3} \times 105 = $ 35
$\frac{1}{3} \times 90 = $ 30	$\frac{1}{3} \times 120 = $ 40
$\frac{1}{3} \times 99 = $ 33	$\frac{1}{3} \times 117 = $ 39
$\frac{1}{2} \times 5 = $ 2.5	$\frac{1}{2} \times 6.40 = $ 3.20
$\frac{1}{2} \times 25 = $ 12.5	$\frac{1}{2} \times 3.00 = $ 1.50
$\frac{1}{2} \times 45 = $ 22.5	$\frac{1}{2} \times 0.52 = $ 0.26
$\frac{1}{2} \times 51 = $ 25.5	$\frac{1}{2} \times 3.52 = $ 1.76

N7 SUBTRACTION ALGORITHM #1

Capsule Lesson Summary

Introduce one of two standard subtraction algorithms.

Materials

Teacher
- Colored chalk

Student
- Paper
- Colored pencils, pens, or crayons
- *Subtraction Problems* Booklet

Description of Lesson

Exercise 1

Begin this exercise with a few related addition facts such as the following:

3 + 6	2 + 4
30 + 60	20 + 40
300 + 600	120 + 40
	123 + 40

On the board write 123 and a red 4 above the 2.

$1\overset{4}{2}3$

T: ***My friend Nick would write the problem 123 + 40 like this. Why would he write 4 above the 2 in 123?***

S: ***Because 40 is 4 tens. Adding 40 to 123 is the same as adding 4 tens to 123.***

$1\overset{4}{2}3 = 163$

T: ***4 tens + 2 tens is …?*** (6 tens)

On the board write 236 and a red 5 above the 3.

$2\overset{5}{3}6$

T: ***What number is 236 + 50?***

S: ***236 + 50 = 286.***

$2\overset{5}{3}6 = 286$

T: ***5 tens + 3 tens is …?*** (8 tens)

On the board write 352 and a red 5 above the 5.

$3\overset{5}{5}2$

T: ***What number is 352 + 50?***

S: ***10 tens is the same as 1 hundred, so 352 + 50 = 402.***

$3\overset{5}{5}2 = 402$

T: ***This is a problem I saw on Nick's paper. Do you know what calculation Nick is doing? Can you solve it? Write the answer on your paper.***

$4\overset{1}{1}\overset{3}{6}$

Look at many of the students' papers before asking a student to answer aloud. Ask several students to explain the calculation and how to solve it.

$$416 + 103 = 4\overset{1}{1}\overset{3}{6} = 519$$

Continue this activity with these problems. Answers are in boxes.

$5\overset{3}{6}5 = \boxed{595}$ $\qquad$ $4\overset{5}{4}\overset{2}{6} = \boxed{498}$

$1{,}2\overset{5}{3}8 = \boxed{1{,}288}$ $\qquad$ $2{,}\overset{2}{4}5\overset{3}{3} = \boxed{2{,}656}$

$3{,}\overset{1}{4}\overset{6}{0}7 = \boxed{3{,}567}$ $\qquad$ $2{,}\overset{1}{5}\overset{8}{1}\overset{4}{4} = \boxed{2{,}698}$

On the board write 475 and a red 10 above the 5.

$47\overset{10}{5}$

T: ***What number is 475 + 10?***

S: ***475 + 10 = 485.***

T: ***Could Nick have written this problem a different way?***

S: ***Nick could have written a 1 above the 7 in 475 instead.***

Record the equivalent expression and the answer.

$47\overset{10}{5} = 4\overset{1}{7}5 = 485$

T: ***What about this problem?***

$6\overset{10}{7}8$

S: ***10 tens is the same as 1 hundred, so Nick could have written a 1 above the 6 in 678 instead.***

T: ***What number is 678 + 100?***

S: ***778.***

$6\overset{10}{7}8 = \overset{1}{6}78 = 778$

Erase the board before going on to Exercise 2.

Exercise 2

Write this problem on the board and ask the students to solve it on their papers.

$$\begin{array}{r} 986 \\ -\ 352 \\ \hline \end{array}$$

Check several students' papers and then invite someone to do the subtraction calculation at the board.

T: ***Nick would solve this problem the same way we did.***

Pose another problem.

T: ***Is this an easier or a harder problem? Why?***

$$\begin{array}{r} 986 \\ -\ 352 \\ \hline 634 \end{array} \qquad \begin{array}{r} 872 \\ -\ 428 \\ \hline \end{array}$$

Let students comment and then ask them to solve the problem individually. Students may use different methods to solve this problem, for example:

$$\begin{array}{r} 87\overset{2}{2} \\ -\ 42\overset{2}{8} \\ \hline \end{array} = \begin{array}{r} 874 \\ -\ 430 \\ \hline 444 \end{array} \quad \text{or} \quad \begin{array}{r} 8\overset{6}{\cancel{7}}\,{}^{1}2 \\ -\ 428 \\ \hline 444 \end{array}{}^{\dagger} \quad \text{or} \quad \begin{array}{r} 872 \\ -\ 400 \\ \hline 472 \\ -\ \ 20 \\ \hline 452 \\ -\ \ \ 8 \\ \hline 444 \end{array} \quad \text{or} \quad \begin{array}{r} 872 \\ -\ 428 \\ \hline 45\widehat{6} \end{array} = 450 + \widehat{6} = 444$$

You may like to ask a student who added 2 to both numbers to explain this method to the class. If necessary, show this approach yourself.

T: ***Nick solved this problem in another way. He added 10 to both numbers. This is what he wrote. How did Nick show that he added 10 to both 872 and 428?*** (10 ones is the same as 1 ten)

$$\begin{array}{r} 87\overset{10}{2} \\ -\ 4\overset{1}{2}8 \\ \hline \end{array}$$

Pause for a moment to allow students to consider how that might help.

T: ***Then Nick thought, 10 + 2 = 12 and 12 – 8 = 4, and he wrote 4 below the ones column.***

$$\begin{array}{r} 87\overset{10}{2} \\ -\ 4\overset{1}{2}8 \\ \hline 4 \end{array}$$

Nick continued with the tens; 7 tens – 3 tens (1 + 2 = 3) = 4 tens. He wrote 4 below the tens column.

$$\begin{array}{r} 87\overset{10}{2} \\ -\ 4\overset{1}{2}8 \\ \hline 44 \end{array}$$

Nick continued with the hundreds; 8 – 4 = 4, so he wrote 4 below the hundreds column.

$$\begin{array}{r} 87\overset{10}{2} \\ -\ 4\overset{1}{2}8 \\ \hline 444 \end{array}$$

Conclude that 872 – 428 = 444.

Ask the students to solve 8,745 – 5,337 using Nick's method of adding 10 to each of the numbers.

After a few minutes, allow a student to explain how Nick would solve this problem.

$$\begin{array}{r} 8{,}74\overset{10}{5} \\ -\ 5{,}3\overset{1}{3}7 \\ \hline 3{,}408 \end{array}$$

Instruct the students to try Nick's method to solve these problems.

$$\begin{array}{r} 473 \\ -\ 158 \\ \hline \end{array} \qquad \begin{array}{r} 860 \\ -\ 729 \\ \hline \end{array} \qquad \begin{array}{r} 5{,}983 \\ -\ 2{,}657 \\ \hline \end{array}$$

Note: Students may have difficulty knowing which numbers they

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†Accept "borrowing" or "regrouping" as a good method, but do not take the time to present this method to the class.

should subtract. One suggestion is to encircle the digits when you add to both numbers as shown here.

$$\begin{array}{r} 47\overset{10}{3} \\ -\;1\overset{1}{5}8 \\ \hline \end{array}$$

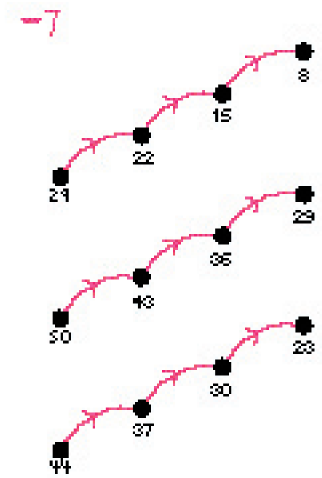
A 16-page booklet, *Subtraction Problems*, contains worksheets to use with the sequence of lessons (N7, N9, and N11) involving a subtraction algorithm. Students should begin on page 2 and progress at their own pace. Allow 5 to 10 minutes today for independent work in this problem book. At the end of the lesson, collect the booklets, check them, and store them for future use in the remaining lessons on the subtraction algorithm.

Subtract.

$56 - 21 = 35$	$45 - 13 = 32$	$57 - 34 = 23$
$73 - 33 = 40$	$87 - 52 = 35$	$69 - 51 = 18$
$159 - 30 = 129$	$268 - 166 = 102$	$100 - 20 = 80$

2

Label the dots.



3

Subtract.

$78 - 32 = 46$	$108 - 91 = 17$	$96 - 50 = 46$
$648 - 325 = 323$	$127 - 117 = 10$	$364 - 162 = 202$
$1{,}463 - 212 = 1{,}251$	$607 - 405 = 202$	$1{,}738 - 637 = 1{,}101$

4

Complete.

$12 - 8 = 4$	$16 - 7 = 9$
$22 - 8 = 14$	$26 - 7 = 19$
$32 - 8 = 24$	$36 - 7 = 29$
$14 - 6 = 8$	$15 - 9 = 6$
$24 - 6 = 18$	$25 - 9 = 16$
$34 - 6 = 28$	$35 - 9 = 26$
$13 - 9 = 4$	$14 - 8 = 6$
$23 - 9 = 14$	$24 - 8 = 16$
$33 - 9 = 24$	$34 - 8 = 26$

5

Subtract.

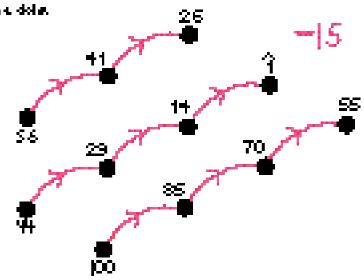
43 − 13 29	72 − 44 28	81 − 23 58
60 − 25 35	87 − 77 10	84 − 27 57
56 − 48 8	80 − 59 21	101 − 50 51

6

Subtract.

50 − 26 24	41 − 24 17	43 − 38 5
426 − 312 114	371 − 222 149	356 − 40 316

Label the dots.



7

Complete.

35 − 45 = 10

263 − 121 = 142

352 − 38 = 314

200 − 99 = 101

460 − 217 = 243

100 − 25 = 75

8

Subtract.

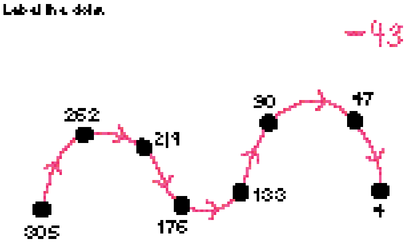
174 − 65 109	365 − 28 337	259 − 132 127
6,152 − 2,048 4,104	7,933 − 5,617 2,316	5,472 − 306 5,166
706 − 524 182	572 − 47 525	629 − 598 31

9

Subtract.

615 − 475 = 140	707 − 332 = 375	462 − 155 = 307	518 − 104 = 414
153 − 63 = 90	663 − 247 = 416	538 − 367 = 171	984 − 576 = 408

Label the dots.

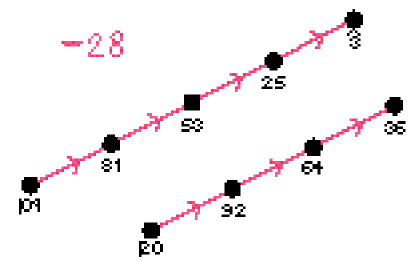


10

Subtract.

1,451 − 339 = 1,112	2,603 − 1,063 = 1,540	3,580 − 436 = 3,144
5,069 − 2,245 = 2,824	1,719 − 359 = 1,360	18,217 − 609 = 17,608

Label the dots.



11

Subtract.

653 − 206 = 447	827 − 754 = 73	147 − 75 = 72
873 − 618 = 255	416 − 284 = 132	496 − 369 = 127
5,829 − 390 = 5,439	1,028 − 414 = 614	2,873 − 2,618 = 255
2,648 − 1,908 = 740	4,627 − 96 = 4,531	7,164 − 5,008 = 2,156

12

Complete.

636 − 386 = 250

503 − 261 = 242

885 − 467 = 418

1,387 − 956 = 431

4,586 − 2,177 = 2,409

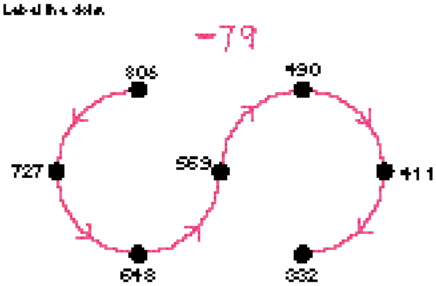
2,163 − 660 = 1,503

13

Subtract.

500 − 174 326	153 − 69 84	876 − 485 391
156 − 59 97	1,860 − 75 1,785	928 − 589 339

Label the dots.

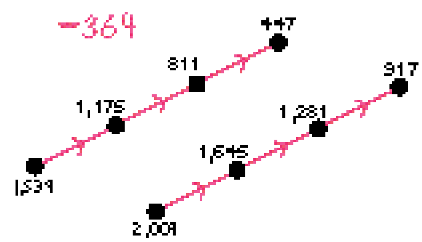


14

Subtract.

3,456 − 1,852 1,604	6,072 − 429 5,643	5,329 − 1,890 3,439
5,900 − 358 5,542	1,397 − 538 859	1,000 − 679 321

Label the dots.



15

Subtract.

9,024 − 3,567 5,457	8,204 − 7,999 205	7,001 − 643 6,358
3,760 − 584 3,176	5,000 − 2,027 2,973	2,546 − 1,494 1,052
3,253 − 1,355 1,898	9,086 − 2,194 6,892	4,800 − 999 3,801
22,270 − 4,631 17,639	14,200 − 8,663 5,537	10,000 − 9,876 124

16

Capsule Lesson Summary

Determine an unknown number by performing a sequence of calculations in reverse. Record the sequence of calculations in an arrow picture. Practice using return arrows in many calculations. Build arrow roads using 2x, +1, and –1 arrows.

Materials

Teacher
- Colored chalk
- Calculator (overhead)

Student
- Paper
- Colored pencils, pens, or crayons
- Calculator

Description of Lesson

Exercise 1

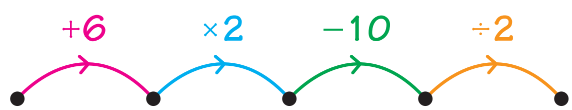
Arrange that each student or pair of students has a calculator.

T: ***Today I am going to try and guess your secret numbers. Each of you choose a number between 1 and 20, but don't tell what it is. Put your number on the display of your calculator. Now follow my directions carefully.***

As you announce a sequence of keys for students to press on their calculators, pause after each number key to draw a corresponding arrow in a picture on the board.

T: ***Press*** $\boxed{+}\ \boxed{6}\ \boxed{\times}\ \boxed{2}\ \boxed{-}\ \boxed{1}\boxed{0}\ \boxed{\div}\ \boxed{2}\ \boxed{=}$.

Now, when I call on you, tell me what number you have on your calculator, and I will tell you what number you started with (your secret number).



Call on several students. Each time, mentally locate the number at the end of the arrow road and calculate the starting number by following the arrows in reverse. For example:

S: ***15.***

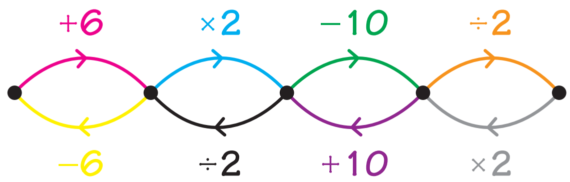
T: ***Your starting (secret) number was 14.***

Note: In this case, you can find the starting number by putting 15 at the end of the arrow road and reversing the operations, or simply by subtracting 1 from the ending number. You may use this information to make the mental calculations easier.

After you have guessed the starting number for several students, the class may say you are "cheating" because you can follow the arrow picture backwards or you just need to subtract 1. Let students explain how to follow the arrow picture backwards.

N8

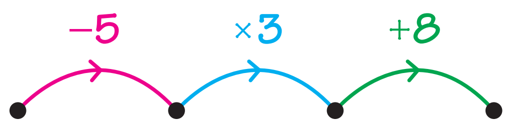
S: ***Draw return (opposite) arrows.***



Draw and label the return arrows. Do a few more examples using new ending numbers that students give you. In these examples, label dots in the arrow picture to show how it is used to find the starting numbers. You may like to do an example on the overhead calculator to show students that if the ending number is entered and the reverse arrows are followed, the starting number will appear on the display.

Repeat Exercise 1 with another sequence of calculations such as suggested below. In this case, ask students to choose a number between 5 and 25.

$\boxed{-}\ \boxed{5}\ \boxed{\times}\ \boxed{3}\ \boxed{+}\ \boxed{8}\ \boxed{=}$

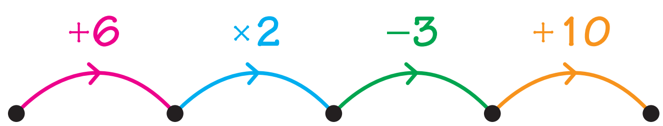


Exercise 2

T: ***Now, all of you are going to see if you can guess secret numbers.***

Organize the class in groups of three or four students. Put a sequence of calculations and an arrow picture on the board. For example,

$\boxed{+}\ \boxed{6}\ \boxed{\times}\ \boxed{2}\ \boxed{-}\ \boxed{3}\ \boxed{+}\ \boxed{1}\boxed{0}\ \boxed{=}$

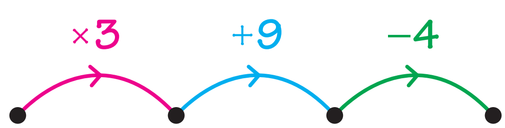


Give the following roles to group members:

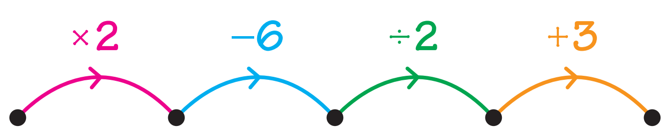
- One person (A) copies the arrow picture and gives directions to the person (B) who chooses a secret number. You may like to give groups copies of Blackline N8 so person A need only color and label arrows.
- One person (B) chooses a secret number between 0 and 20 and puts it on a calculator. Person A gives B directions to press keys on the calculator, following the sequence of calculations in the arrow picture.
- The other group members record the result on the calculator at the end of the arrow road. Then they work together to discover B's secret number. Encourage these students to draw return (opposite) arrows and label dots to find the secret number.

As groups finish one activity, assign another sequence of calculations and arrow pictures. Direct students in a group to change roles.

$\boxed{\times}\ \boxed{3}\ \boxed{+}\ \boxed{9}\ \boxed{-}\ \boxed{4}\ \boxed{=}$



$\boxed{\times}\ \boxed{2}\ \boxed{-}\ \boxed{6}\ \boxed{\div}\ \boxed{2}\ \boxed{+}\ \boxed{3}\ \boxed{=}$

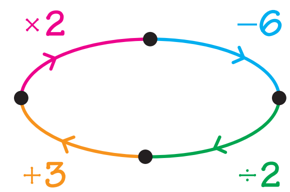


With this last example, students should discover that the starting number and the ending number are always the same.

T: ***How could we draw the arrow picture to show that the starting number and the ending number are the same?***

S: ***Draw it like a circle.***

S: ***Make a loop of arrows.***



If there is time remaining, direct students to think of a number and then perform a sequence of calculations, one at a time, as you draw the corresponding arrow picture. Call on students to announce their ending numbers, and then quickly respond with their starting numbers. Ask the class to explain how you can be so fast at figuring their starting numbers. Use one or more of these sequences:

- +5, –8, +1, +12 which is equivalent to +10
- +1, x2, –2 which is equivalent to x2
- +8, ÷2, –4 which is equivalent to ÷2

N9 SUBTRACTION ALGORITHM #2

Capsule Lesson Summary

Use the subtraction algorithm introduced in N7 to solve various problems.

Materials

Teacher		Student	
	• Colored chalk		• Paper • *Subtraction Problems* Booklet

Description of Lesson

Students should have scratch paper or a math notebook available to them during this lesson.

T: ***What number is 465 + 30?***

S: ***495.***

T: ***How would Nick show adding 30 to 465?***

S: ***Nick would put a 3 above the 6.***

$$4\overset{3}{6}5$$

T: ***The 3 tells us that we want to add 3 tens to the 6 tens in 465. 3 + 6 = 9, so the answer has 9 tens.***

$$4\overset{3}{6}5 = 495$$

Erase the board and then write this problem.

$$2{,}3\overset{10}{4}8$$

T: ***What addition problem is Nick doing here?***

S: ***Adding 10 tens to 2,348.***

T: ***Nick wants to add 10 tens to 2,348. Write the answer on your paper.***

Look at several answers before asking a student to answer aloud.

S: ***2,348 + 100 = 2,448.***

T: ***Could Nick have written this problem another way?***

S: ***He could have put 1 above the 3.***

On the board write the equivalent expression and the answer.

$$2{,}3\overset{10}{4}8 = 2{,}\overset{1}{3}48 = 2{,}448$$

T: ***10 tens is the same as 100, so writing 10 above the number of tens or writing 1 above the number of hundreds are just two different ways to show adding 100.***

Write these problems on the board and direct students to copy and solve as many of them as they can. Answers are in boxes.

$57\overset{4}{3} = \boxed{577}$	$1{,}2\overset{7}{3}8 = \boxed{1{,}308}$	$\overset{1}{6}\overset{7}{2}\overset{7}{4} = \boxed{801}$
$\overset{2}{4}52 = \boxed{652}$	$2{,}3\overset{10}{4}0 = \boxed{2{,}440}$	$4\overset{9}{7}\overset{8}{3} = \boxed{571}$
$5\overset{2}{9}4 = \boxed{614}$	$4\overset{5}{4}\overset{3}{6} = \boxed{499}$	$2{,}5\overset{10}{7}\overset{4}{6} = \boxed{2{,}680}$

When a few students have solved all the problems, call on students to put solutions on the board.

You may wish to go over a particular problem collectively if many students are having difficulty with that problem.

Erase the board and then write this subtraction problem.

$$\begin{array}{r} 362 \\ -\ 135 \\ \hline \end{array}$$

T: ***How would Nick solve this subtraction problem?***

S: ***He would add 10 ones to 362 (write 10 above the 2) and 1 ten to 135 (write 1 above the 3).***

T: ***Yes, Nick adds 10 to each of these numbers*: *adding 10 ones is the same as adding 1 ten.***

$$\begin{array}{r} 36\overset{10}{2} \\ -\ 1\overset{1}{3}5 \\ \hline \end{array}$$

Do this problem on your paper.

Observe the students' work. Ask students who finish quickly to solve another problem using Nick's method; for example, 1,396 – 209.

After a few minutes, solve the first problem collectively.

T: ***Nick thinks 10 + 2 = 12 and 12 – 5 = 7, and then he writes 7 below the ones column.***

Then Nick thinks 6 tens – 4 tens (1 + 3 = 4) = 2 tens, and he writes 2 below the tens column.

$$\begin{array}{r} 36\overset{10}{2} \\ -\ 1\overset{1}{3}5 \\ \hline 227 \end{array}$$

T: ***3 hundreds – 1 hundred = 2 hundreds, so he writes 2 below the hundreds column.***

Conclude that 362 – 135 = 227.

Then ask a student to solve the second problem and to explain each step.

$$\begin{array}{r} 1{,}39\overset{10}{6} \\ -\ 2\overset{1}{0}9 \\ \hline 1{,}187 \end{array}$$

Write the following subtraction problems on the board, and direct students to copy and solve as many of them as they can.

$$\begin{array}{r} 71 \\ -\ 36 \\ \hline \end{array} \qquad \begin{array}{r} 150 \\ -\ 34 \\ \hline \end{array} \qquad \begin{array}{r} 926 \\ -\ 418 \\ \hline \end{array} \qquad \begin{array}{r} 608 \\ -\ 427 \\ \hline \end{array}$$

When a few students have solved all four problems, call on students to solve the problems at the board and to explain each step.

$$\begin{array}{r} 7\overset{10}{1} \\ -\ \overset{1}{3}6 \\ \hline 35 \end{array} \qquad \begin{array}{r} 15\overset{10}{0} \\ -\ \overset{1}{3}4 \\ \hline 116 \end{array} \qquad \begin{array}{r} 92\overset{10}{6} \\ -\ 4\overset{1}{1}8 \\ \hline 508 \end{array} \qquad \begin{array}{r} 60\overset{10}{8} \\ -\ \overset{1}{4}27 \\ \hline 181 \end{array}$$

Emphasize that in the last problem (608 – 427), Nick does not add 10 to each of the numbers because 8 is more than 7. The difficulty lies in the tens column because 0 tens is less than 2 tens; that is why Nick adds 100 to each of the numbers. To explain, say, "Adding 10 tens is the same as adding 1 hundred."

Write these subtraction problems on the board, and direct students to copy and solve as many of them as they can.

$$\begin{array}{r} 62 \\ -\ 46 \\ \hline \end{array} \qquad \begin{array}{r} 154 \\ -\ 44 \\ \hline \end{array} \qquad \begin{array}{r} 237 \\ -\ 165 \\ \hline \end{array} \qquad \begin{array}{r} 1{,}342 \\ -\ 67 \\ \hline \end{array}$$

When a few students have solved all the problems, call on students to solve the problems at the board and to explain each step.

$$\begin{array}{r} 6\overset{10}{2} \\ -\ \overset{1}{4}6 \\ \hline 16 \end{array} \qquad \begin{array}{r} 154 \\ -\ 44 \\ \hline 110 \end{array} \qquad \begin{array}{r} 2\overset{10}{3}7 \\ -\ \overset{1}{1}65 \\ \hline 72 \end{array} \qquad \begin{array}{r} 1{,}3\overset{10}{4}\overset{10}{2} \\ -\ \overset{1}{}\overset{1}{6}7 \\ \hline 1{,}275 \end{array}$$

Distribute students' copies of the *Subtraction Problems* Booklet and allow 10 to 15 minutes for individual work. Encourage students to correct any errors they made previously before starting new pages. At the end of the lesson, collect the booklets, check them, and have them ready for use in Lesson N11.

Note: You may allow students to use whatever method they find most comfortable to solve these subtraction problems. However, even students who prefer other methods should be encouraged to try Nick's method for some problems. They may find it interesting and easy to use.

Home Activity

This is a good time to send a letter to parents/guardians about subtraction and Nick's method (algorithm) for subtraction. Blackline N9 has a sample letter.

Capsule Lesson Summary

Do some mental arithmetic involving 10x, and examine the effect of 10x on the Minicomputer. Find the ending number of all possible roads that start at 0, and have exactly two 10x arrows and ten +1 arrows.

Materials

Teacher
- Minicomputer set
- ⑩-checkers
- Colored chalk

Student
- Unlined paper
- Colored pencils, pens, or crayons

Advance Preparation: Before the lesson begins, magnetize each ⑩-checker by sticking a piece of magnetic material to the back. You may also want to prepare the arrow road for Exercise 2 before starting the lesson.

Description of Lesson

Exercise 1: 10x

Begin this exercise with mental arithmetic involving 10x.

4 x 10 (40)	8 x 10 (80)	10 x 5 (50)	10 x 10 (100)
10 x 4 (40)	10 x 8 (80)	10 x 0 (0)	10 x 11 (110)

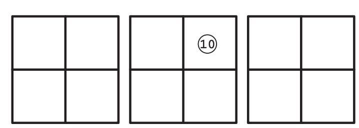
T: ***How do you know that 10 x 11 = 110?***

S: ***10 x 10 = 100, so 10 x 11 = 100 + 10 = 110.***

S: ***I thought about 11 on the Minicomputer and moved it over one board to the left.***

S: ***I just put a 0 after the 11.***

Put a ⑩-checker on the 40 square of the Minicomputer and write the 10x calculation to one side of the Minicomputer.

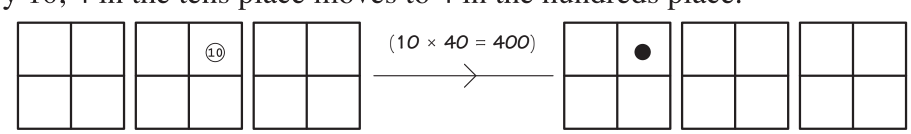


= 10 × 40

T: ***What number is 10 x 40?***

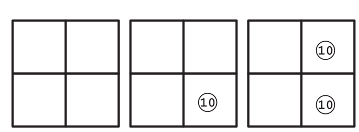
S: ***400.***

Make a 10 x 40 = 400 trade and complete the number sentence on the board. Emphasize that when you multiply by 10, 4 in the tens place moves to 4 in the hundreds place.



Erase the board.

Put 10 x 15 on the Minicomputer and write the calculation to one side.



= 10 × 15

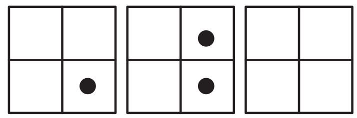
T: ***What number is 10 x 15?*** (150)
How do you know?

S: ***I imagined the trades. There will be one hundred and five tens.***

S: ***10 x 10 = 100 and 10 x 5 = 50, so 10 x 15 = 100 + 50 = 150.***

S: ***I just put a 0 after the 15; 10 x 15 = 150.***

Ask a student to make the trades. Observe that when you multiply by 10, the digits 1 and 5 move to the next place. Complete a number sentence.



= 10 × 15 = 150

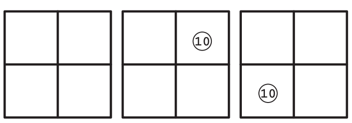
T: ***Let's record this information in a 10x table.***

Begin a table on the board and ask students to make a similar table on their papers.

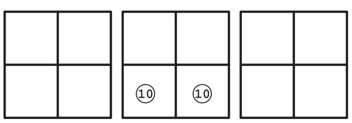
Start	10×
15	150

T: ***Start with the number on the left.***
On the right is 10x the starting number.

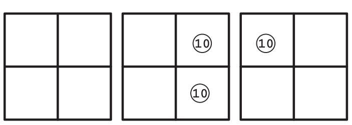
Continue this activity with these calculations or with 10x calculations suggested by the class.



= 10 × 42 = 420



= 10 × 30 = 300



= 10 × 58 = 580

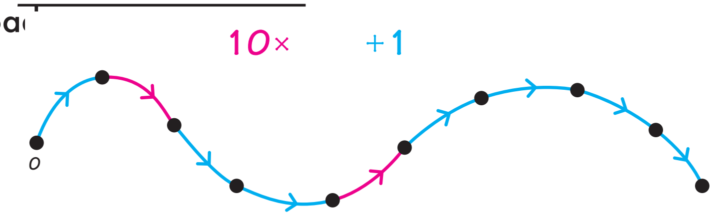
Start	10×
15	150
42	420
30	300
58	580
158	1,580
613	6,130

When you have six or more pairs of numbers in your table, ask students if they see any patterns. Encourage the class to notice that all the numbers listed in the 10x column end in 0, and when you multiply by 10 the digits move over a place.

Erase the board before going on to Exercise 2.

Exercise 2: A 10x and +1 Arrow Road

Draw this arrow picture on the board.

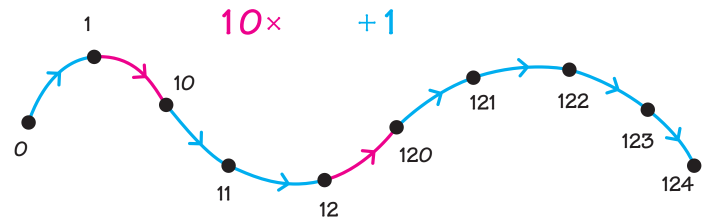


T: ***What number is the ending dot of this arrow road? Write your answer on your paper (or whisper it to a neighbor).***

Check several responses before asking a student to answer aloud.

S: ***The ending dot is for 124.***

Invite students to label the dots on the board.



T: ***The starting number of this arrow road is 0 and the ending number is 124.***
How many 10x arrows are there in this arrow road? (Two)
How many +1 arrows? (Seven)
Would it be possible to draw a different arrow road that also starts at 0, and has two 10x arrows and seven +1 arrows? (Yes)

Encourage students to describe at least one other arrow road with two 10x arrows and seven +1 arrows. Ask if the ending number of this road is 124, and allow students to explain why it is different.

Instruct students to draw a different arrow road on their papers. Each road should start at 0 and have exactly two 10x arrows and seven +1 arrows. Also, suggest that a road start with at least one +1 arrow.

Students can work with a partner on this task, but you may still need to help some students get started. When you observe students who have completed a road that is different from the road on the board, record the ending number of this road on the board. Encourage those who finish quickly to build another such arrow road starting at 0 with the greatest (least) possible ending number.

Soon you should have a list of about 15 to 18 possible ending numbers on the board. There are 28 possible ending numbers[†]; some of which are listed here.

124	106	610	502
421	700	520	205
430	214	511	250
304	223	322	331

T: ***What is the greatest number in our list of ending numbers?***
Would it be possible to have an ending number greater than this one?
What is the greatest possible ending number?

S: ***700.***

T: ***In what order should we draw the 10x and the +1 arrows if we want the ending number as great as possible?***

S: ***We should draw the seven +1 arrows first and then the two 10x arrows.***

If necessary, tell your class that the greatest possible ending number is 700.

[†]For your information, the possible ending numbers are three-digit numbers in which the sum of the digits is 7. For example, 124 (1 + 2 + 4 = 7) is a possible ending number.

T: ***What is the least number in our list of ending numbers?***
Would it be possible to have an ending number less than this one?
What is the least possible ending number (assuming the road starts with a +1 arrow)?

S: ***106.***

T: ***In what order should we draw the 10x and the +1 arrows if we want the ending number as small as possible?***

S: ***We should draw one +1 arrow first, then two 10x arrows, and then the other six +1 arrows.***

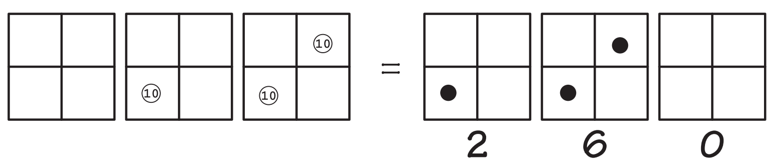
If necessary, tell your class that the least possible ending number is 106.

Perhaps a student will notice that the sum of the digits of each number is 7. You may wish to point this out yourself if none of the students mentions it.

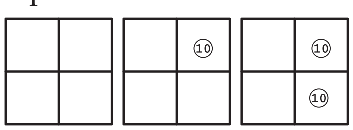
Home Activity

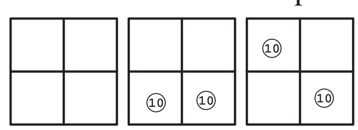
Suggest that parents/guardians practice multiplying by 10 with their child.

1) Do some mental arithmetic involving 10x. Use one- and two-digit numbers to multiply by 10.
2) Make ⑩-checkers to use on home Minicomputers. Write a 10x calculation, represent it on the Minicomputer, make the trades, and write the result on a paper below the boards. For example:

10 × 26 = 

3) Put multiplies of 10 on the Minicomputer using the ⑩-checkers. For example:

450 = 

390 = 

Capsule Lesson Summary

Do some multiplication calculations on the Minicomputer using positive and negative checkers, for example:

$$3 \times 198 = 3 \times (200 - 2) = 3 \times (200 + \widehat{2})$$

Label the dots in an arrow road, using a subtraction algorithm to do the necessary calculations.

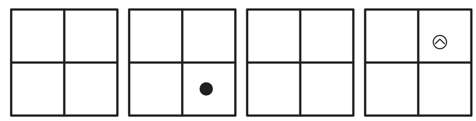
Materials

Teacher	• Minicomputer set • Colored chalk	**Student**	• Paper • Colored pencils, pens, or crayons • *Subtraction Problems* Booklet

Description of Lesson

Exercise 1

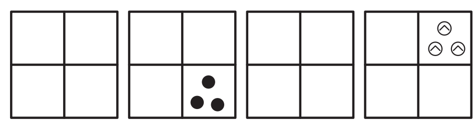
Put this configuration on the Minicomputer.



T: ***What number is this? How do you know?***

S: ***96, because 100 – 4 = 96.***

T: ***What number is 3 x 96? Write the answer on your paper.***



Look at many answers before asking a student to give and explain the answer.

S: ***288. 96 is 4 less than 100, so I subtracted 12 (3 x 4) from 300.***

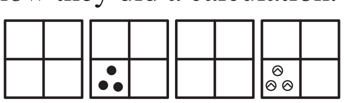
S: ***288. 3 x 96 = 300 + $\widehat{12}$ = 300 – 12 = 288.***

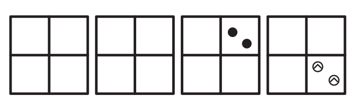
$$\begin{array}{r} {\scriptstyle 10\,10} \\ 300 \\ {\scriptstyle 1\;\,1} \\ -\ \ 12 \\ \hline 288 \end{array}$$

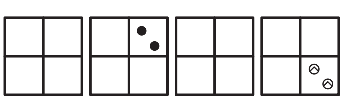
300 – 10 = 290
300 – 12 = 288

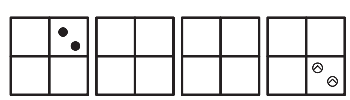
You may want to write the subtraction problem on the board and ask a student to solve it, using Nick's method or another method of choice.

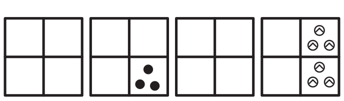
Continue this activity with the following configurations. Frequently ask several students to explain how they did a calculation.

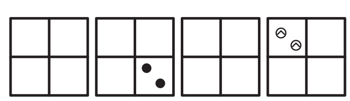

= 3 x 198 = 594

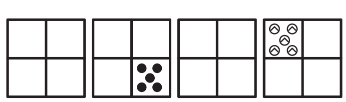

= 2 x 39 = 78


= 2 x 399 = 798


= 2 x 3,999 = 7,998

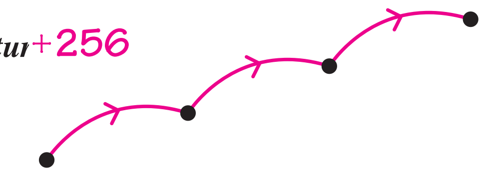

= 3 x 195 = 585


= 2 x 92 = 184


= 5 x 92 = 460

Exercise 2

Draw this arrow picture on the board and ask the students to copy it on their papers.



T: ***Where is the greatest number in this arrow picture?***

The students should point to the ending dot of the road.

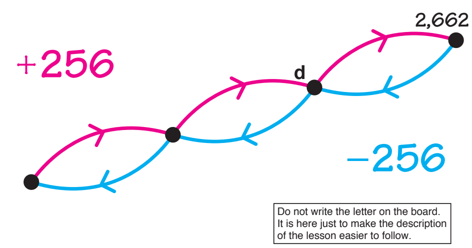
T: ***Label that dot 2,662 on your papers.***

Invite a student to label the dot in the picture on the board.

T: ***I would like to label the other dots in this arrow picture, but the arrows are going in the wrong direction. What could I do?***

S: ***Draw return arrows.***

Instruct students to draw return arrows on their papers, and invite one or more students to draw the return arrows on the board.



T: ***What are the return arrows for?***

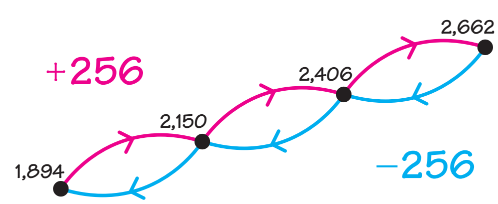
S: ***–256.***

T (pointing to **d**)**:** ***What calculation do we need to do before we can label this dot?***

S: ***2,662 – 256.***

Direct students to do the calculation on their papers, and help students who have difficulty. Then ask a student to write the problem on the board, to solve it explaining each step, and finally to label the dot (**d**).

Continue this activity until all of the dots are labeled.



Your students will have done the following calculations in order to label the dots.

$$\begin{array}{r} 2{,}66\overset{10}{2} \\ -\ \ 2\overset{1}{5}6 \\ \hline 2{,}406 \end{array} \qquad \begin{array}{r} 2{,}40\overset{10}{6} \\ -\ \ 2\overset{1}{5}6 \\ \hline 2{,}150 \end{array} \qquad \begin{array}{r} 2{,}\overset{10}{1}\overset{10}{5}\overset{10}{0} \\ -\ \overset{1}{}\ \overset{1}{2}\overset{1}{5}6 \\ \hline 1{,}894 \end{array}$$

When calculating 2,150 – 256, emphasize that ten hundreds is the same as one thousand.

Distribute students' copies of the *Subtraction Problems* Booklet and allow 10 to 15 minutes for individual work. Encourage students to correct errors that they made previously before starting new pages. At the end of the lesson, collect the booklets for your review. This is the last lesson using the *Subtraction Problems* Booklet; however, you may want to use it again for subtraction practice.

N12 DIVISION PROBLEMS #1

Capsule Lesson Summary

Evenly divide quantities of fruit between two classes. Write several number sentences to record the results.

Materials

Teacher	• Colored chalk	**Student**	• Paper • Worksheets N12*, **, ***, and ****

Description of Lesson

Exercise 1

Begin the lesson with a short mental arithmetic activity involving ½x.

½ x 16 (8)	½ x 24 (12)	½ x 32 (16)	½ x 42 (21)
½ x 20 (10)	½ x 30 (15)	½ x 38 (19)	½ x 52 (26)

Occasionally ask students to explain how they did a calculation such as ½ x 38 or ½ x 52. You may also sometimes rephrase a calculation using ÷2 rather than ½x.

Exercises 2, 3, and 4 have similar division stories. As you tell these or comparable stories, you may like to choose other teachers' classes in your school as well as your own to star in them. You may also like to use props, such as bags and counters, while you tell the stories even though you will draw pictures on the board.

Exercise 2

T: ***Let's pretend that I brought 536 cherries to school with me today. Since there are so many cherries I would like to keep half of them for our class and give half of the cherries to Ms. Briggs' class. I brought two bags and I want to put half of the cherries in each bag.***

Is ½ x 536 more or less than 100? (More)
Is ½ x 536 more or less than 200? (More)
How can you be sure that ½ x 536 is more than 200?

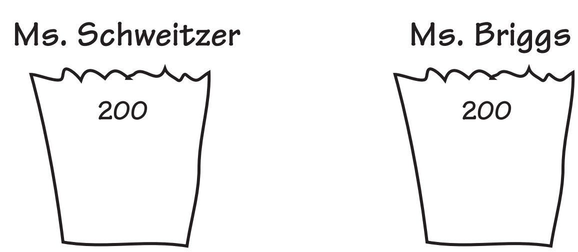
S: ***2 x 200 = 400 and 536 is more than 400.***

T: ***Is ½ x 536 more or less than 300?*** (Less) ***How do you know?***

S: ***2 x 300 = 600 and 536 is less than 600.***

T: ***We know that ½ x 536 is between 200 and 300. Therefore we can give each class at least 200 cherries. Let's begin by putting 200 cherries in each of the bags.***

Draw two bags on the board and put 200 in each.



T: ***Now there are 200 cherries in each of these bags. How many cherries have we distributed so far?***

S: ***400.***

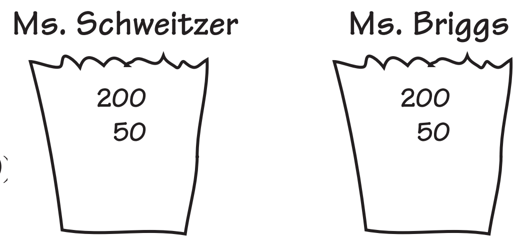
T: ***How many cherries do we still need to divide between the two classes?*** (136)
How could we do this?

Perhaps a student will suggest putting 50 more in each.

S: ***We could put 50 more cherries in each of these bags.***

T: ***2 x 50 = 100 and 100 is less than 136, so let's put 50 more cherries in each bag.***

How many cherries are in each bag now? (250)
How many cherries have been distributed altogether? (500)
How many cherries do we still need to share between our class and Ms. Briggs' class? (36)
What number is ½ x 36?

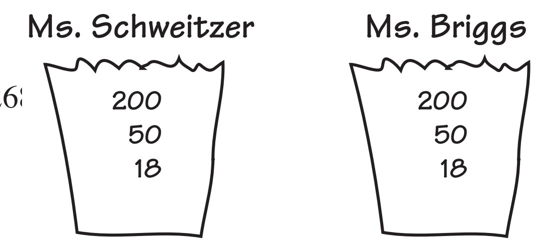


S: ***We can put 18 more cherries in each bag.***

T: ***How many cherries will each class receive?*** (26
Are there any cherries left over? (No)
How can we be check that ½ x 536 = 268?

S: ***We could calculate 2 x 268 and see if the answer is 536.***

S: ***Do 536 ÷ 2 on the calculator.***



Ask students to check 268 + 268 on their papers. Conclude that ½ x 536 = 268.

T: ***What number sentences can we write about this problem?***

Record number sentences on the board as they are suggested. If a division sentence is not suggested, suggest it yourself.

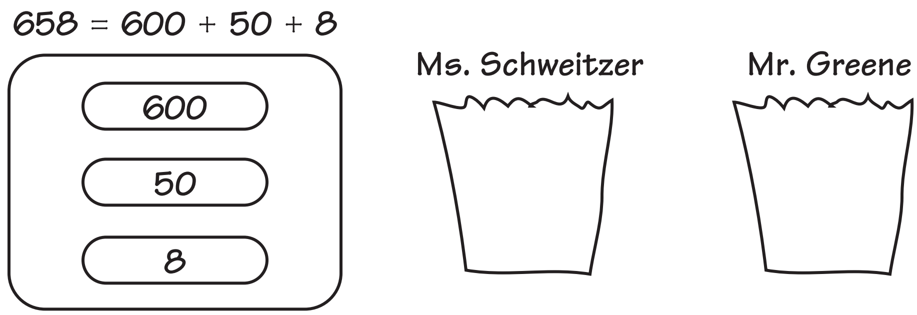
$$2 \times 268 = 536 \qquad 268 \times 268 = 536$$
$$\tfrac{1}{2} \times 536 = 268 \qquad \tfrac{1}{2} \times 536 = 200 \times 50 \times 18$$
$$536 \div 2 = 268$$

Erase the board before continuing with the next story.

Exercise 3

T: ***Let's pretend that I have 658 peaches to share with Mr. Greene's class. We will keep half of the peaches for our class, and give half of them to Mr. Greene's class. How many peaches will each class receive? What number is ½ x 658? Try to solve this problem by yourself before we solve it together.***

Draw this picture on the board while the students are working.



Look at several students' papers before continuing with a class discussion.

T: ***There are 658 peaches inside. I wrote 658 as 600 + 50 + 8. Let's divide the 600 peaches first. What number is ½ x 600?*** (300) ***Each class will get 300 peaches.***

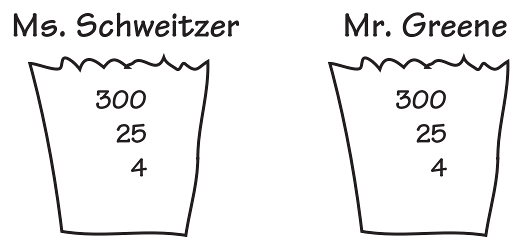
Put 300 in each bag.

T: ***Now let's divide the 50 peaches. How many of these peaches should each class receive? What number is ½ x 50?*** (25)

Put 25 in each bag.

T: ***Now let's divide the 8 peaches. How many of these peaches should we give to each class? What number is ½ x 8?*** (4)

Put 4 in each bag.



T: ***How many peaches will each class receive altogether?***

S: ***329.***

Ask the students to check by calculating 2 x 329 or 329 + 329. Conclude that ½ x 658 = 329.

Erase the board before continuing with the next story.

Exercise 4

T: ***Tomorrow I am going to bring 375 apples to school. I have promised to give half of the apples to Ms. Jackson's class. How many apples will be left for our class?***

Encourage students to try to solve this problem on their own while you draw a picture of the bags.

T: ***Would we be able to give each class at least 100 apples?*** (Yes)
2 x 100 = 200, so each class can receive at least 100 apples.
Could we give each class 200 apples ? (No) ***Why not?***

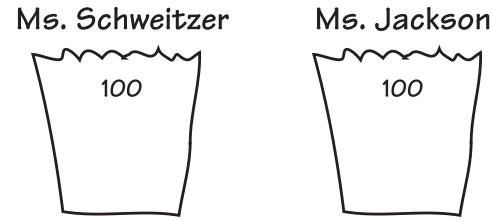
S: ***2 x 200 = 400 and 375 is less than 400.***

T: ***Let's begin by giving each class 100 apples.***

T: *How many apples do we have left to distribute?*

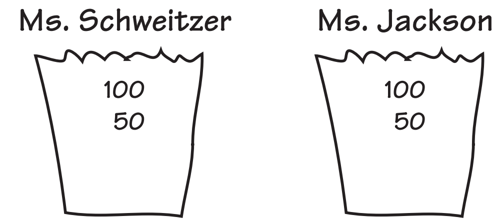
S: *175.*

T: *What do you suggest we do now?*



S: *We can give each class 50 more apples because 2 x 50 = 100 and 100 is less than 175.*

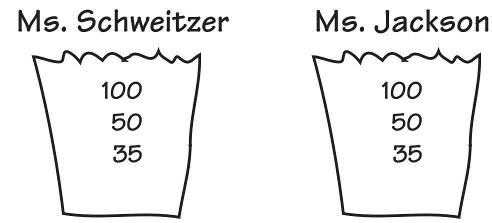
T: *How many apples have been distributed?* (300) *How many apples do we still need to divide between the two classes?* (75)



S: *½ x 70 = 35, so let's give each class 35 more apples.*

T: *How many apples do we still need to share between our class and Ms. Jackson's class?*

S: *5.*



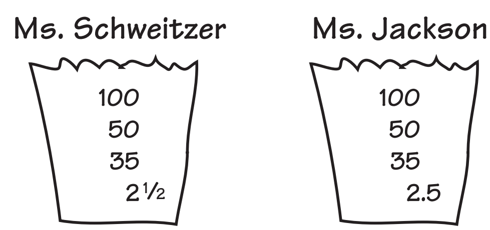
T: *What do you suggest?*

S: *We give Ms. Jackson's class 2 apples and we keep the other 3.*

S: *We need to cut one of the apples in half.*

S: *Each class should get 2½ more apples because ½ x 5 = 2½.*

Invite a student to write 2½ on the board. Ask if anyone knows a different way to write this number. If no one suggests 2.5 or 2.50, suggest 2.5 yourself. Emphasize that 2.5 and 2½ are the same number.



T: *How many apples will each class receive?*

Write these calculations on the board and invite two students to solve them.

100	100
50	50
35	35
+ 2½	+ 2.5
187½	187.5

T: *Are 187½ and 187.5 the same number?* (Yes) *Each class will receive 187½ apples. Does ½ x 375 = 187 ½? Check this answer on your paper.*

Write these calculations on the board while the students are working.

187½	187.5
+ 187½	+ 187.5

T: *What number is ½ + ½?* (1) *We write 1 above the ones column.*

Invite a student to complete the calculation on the board.

Ask another student to complete the addition problem on the right.

$$\begin{array}{r} {\scriptstyle 1\,1\,1} \\ 187\tfrac{1}{2} \\ +\ 187\tfrac{1}{2} \\ \hline 375 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\,1\,1} \\ 187.5 \\ +\ 187.5 \\ \hline 375.0 \end{array}$$

Conclude that 2 x 187½ = 375. Ask students to suggest other number sentences about this problem. Several possibilities are listed below. If a number sentence with division is not suggested, suggest one yourself.

$$2 \times 187.5 = 375 \qquad 375 \div 2 = 187\tfrac{1}{2}$$
$$\tfrac{1}{2} \times 375 = 187\tfrac{1}{2} \qquad 375 - 187.5 = 187.5$$

Worksheets N12*, **, ***, and **** are available for individual work.

N12

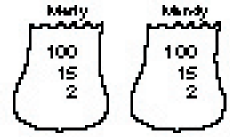
Name________________ N12 ★

Complete.

$\frac{1}{2} \times 20 =$ 10	$60 \div 2 =$ 30
$\frac{1}{2} \times 24 =$ 12	$68 \div 2 =$ 34
$\frac{1}{2} \times 40 =$ 20	$80 \div 2 =$ 40
$\frac{1}{2} \times 46 =$ 23	$82 \div 2 =$ 41
$\frac{1}{2} \times 50 =$ 25	$30 \div 2 =$ 15
$\frac{1}{2} \times 52 =$ 26	$32 \div 2 =$ 16
$\frac{1}{2} \times 54 =$ 27	$34 \div 2 =$ 17
$\frac{1}{2} \times 56 =$ 28	$36 \div 2 =$ 18

Name________________ N12 ★★

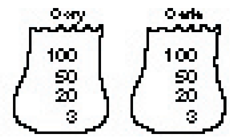
Share 234 marbles between Marty and Mandy.



Complete.

$234 \div 2 =$ 117 $\frac{1}{2} \times 234 =$ 117

Share 346 cards between Cory and Carla.



Complete.

$346 \div 2 =$ 173 $\frac{1}{2} \times 346 =$ 173

Other ways to do the sharing are possible.

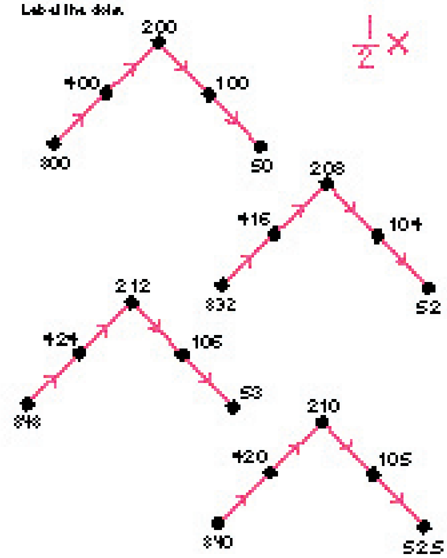
Name________________ N12 ★★★

Complete.

$4 \div 2 =$ 2	$\frac{1}{2} \times 600 =$ 300
$5 \div 2 =$ 2.5	$\frac{1}{2} \times 640 =$ 320
$6 \div 2 =$ 3	$\frac{1}{2} \times 642 =$ 321
$7 \div 2 =$ 3.5	$\frac{1}{2} \times 650 =$ 325
$100 \div 2 =$ 50	$\frac{1}{2} \times 500 =$ 250
$30 \div 2 =$ 15	$\frac{1}{2} \times 90 =$ 45
$8 \div 2 =$ 4	$\frac{1}{2} \times 6 =$ 3
$138 \div 2 =$ 69	$\frac{1}{2} \times 596 =$ 298

Name________________ N12 ★★★★

Label the dots.



Capsule Lesson Summary

Introduce the symbol ÷ and do several division calculations on a calculator. Using only the keys 2, 8, +, −, ×, ÷, and =, put various numbers on the display of a calculator with exactly five key presses. Lifting the restriction on the number of presses but with the same limit on keys, put some given numbers on the display.

Materials

Teacher
- Calculator (overhead)
- Paper

Student
- Calculator

Description of Lesson

Note: You may want to read the "Role and Use of Calculators" in Section One: Notes to the Teacher. This lesson assumes the use of calculators that do chain operations and have an automatic constant feature. Some calculators may operate differently than described here.

Arrange for each student or pair of students to have a calculator. Ask the students to listen very carefully and to follow your directions exactly.

Exercise 1

Draw a division symbol on the board and instruct students to look for the division key (÷) on their calculators.

T: ***This symbol is read "divided by." Put 100 on your calculator display and then press ÷ 2 =. What number is on the display?***

S: ***50.***

Students who do not have 50 on their display should press C and then put 50 on the display.

T: ***Press ÷ 2 =.What number is on the display?***

S: ***25.***

T: ***What does the calculator do when we press ÷ 2 = ?***

S: ***It divides the number on the display by 2.***

S: ***It finds one-half of the number.***

T: ***Dividing by 2 is the same as taking one-half of a number.***

Direct students to press C and then to display 80.

T: ***What number is ½ x 80?*** (40) ***Press ÷ 2 =. What number is on the display?***

S: ***40.***

T: ***What number is ½ x 40?*** (20) ***Press ÷ 2 =. What number is on the display?***

S: ***20.***

Continue this halving sequence until 5 is on the display.

T: ***If we divide by 2 again, what number will we get next?*** (2½)
Press ÷ 2 =. What number is on the display? (2.5)
Do you know another name for 2.5? (2½)
What does 2.5 dollars mean?

S: ***2 dollars and 50 cents.***

T: ***One-half of $5 is $2.50. What is one-half of $2.50?*** ($1.25)
Press ÷ 2 =. What number is on the display?

S: ***1.25.***

Direct students to press C and then to display 60.

T: ***What number is 60 ÷ 2?*** (30)
Press ÷ 2 =. What number is on the display?

S: ***30.***

T: ***What number is 30 ÷ 3?*** (10)
Press ÷ 3 =. What number is on the display?

S: ***10.***

T: ***What does the calculator do when we press ÷ 3 =?***

S: ***It divides the number on the display by 3.***

S: ***It finds one-third of the number.***

T: ***Dividing by 3 is the same as finding one-third of a number.***

Use an overhead calculator or one of the classroom calculators for the following mental arithmetic activity. Students should turn off their calculators for a few minutes. Put 38 on the calculator display and then cover the display with your hand being careful not to cover the light source.

T: ***What number is on the display?*** (38)
Do the calculations in your head just as you think the calculator does the calculations when I press the keys: + 3 + 5 ÷ 2 =.

Announce clearly the keys you press and pause after each number key so students will have an opportunity to do the calculation mentally.

T: ***What number should be on the display?*** (23)

Allow several students to answer before showing the class the calculator display.

Continue this activity with the following or similar sequences of calculations.

- Start with 23 on the display; press + 1 0 = = = (53)
- Start with 53 on the display; press + 2 − 5 ÷ 2 = (25)
- Start with 25 on the display; press − 1 0 = = (5)
- Start with 5 on the display; press × 2 + 4 ÷ 2 = (7)

Exercise 2

T: ***Today we are going to solve some calculator puzzles.***
The puzzles require that you only use a few of the keys on the calculator.

Write these key symbols on the board. $\boxed{2}\ \boxed{8}\ \boxed{+}\ \boxed{-}\ \boxed{\times}\ \boxed{\div}\ \boxed{=}$

T: ***You may choose any of these keys*** (point to the list)***, and you may press a key more than once. For this first puzzle, start with 0 on the display and press keys exactly five times. What numbers can you put on the display following these rules?***

Encourage students to use their calculators and to record the sequence of keys they press along with the resulting numbers on the display. Be patient! Students may have difficulty restricting their use of the calculators to fit the rules of the game. As necessary, remind the class that they may use only the keys you have listed on the board and that they must press exactly five times.

As students find numbers that can be put on the calculator with these restrictions, record the sequence of keys that were pressed to get a number on the board. For example:

$\boxed{8}\ \boxed{2}\ \boxed{+}\ \boxed{8}\ \boxed{=}$	(90)	$\boxed{8}\ \boxed{2}\ \boxed{\div}\ \boxed{8}\ \boxed{=}$	(10.25)
$\boxed{8}\ \boxed{8}\ \boxed{\times}\ \boxed{8}\ \boxed{=}$	(704)	$\boxed{8}\ \boxed{\times}\ \boxed{8}\ \boxed{=}\ \boxed{=}$	(512)
$\boxed{+}\ \boxed{=}\ \boxed{\times}\ \boxed{=}\ \boxed{=}$	(0)	$\boxed{8}\ \boxed{-}\ \boxed{2}\ \boxed{\times}\ \boxed{=}$	(36)

When decimal numbers are found, ask where (i.e., between which two integers) each number is on the number line. For example, 10.25 is between 10 and 11 on the number line.

When your class has suggested several numbers, encourage students to find as great a number as they can on their calculators following these rules. On the board, continue recording sequences of keys that were pressed, but ask students to read the resulting numbers. When a new sequence is suggested, determine whether the new number is the greatest number found so far. Several sequences that result in large numbers are given below. Do not worry that your students actually find the greatest possible number.

$\boxed{8}\ \boxed{8}\ \boxed{\times}\ \boxed{8}\ \boxed{=}$	(704)	$\boxed{8}\ \boxed{2}\ \boxed{\times}\ \boxed{=}\ \boxed{=}$	(551,368)
$\boxed{8}\ \boxed{\times}\ \boxed{=}\ \boxed{=}\ \boxed{=}$	(4,096)	$\boxed{8}\ \boxed{8}\ \boxed{\times}\ \boxed{=}\ \boxed{=}$	(681,472)
$\boxed{8}\ \boxed{8}\ \boxed{8}\ \boxed{8}\ \boxed{8}$	(88,888)	$\boxed{8}\ \boxed{8}\ \boxed{8}\ \boxed{\times}\ \boxed{=}$	(788,544)

Erase everything on the board except the list of keys: $\boxed{2}$, $\boxed{8}$, $\boxed{+}$, $\boxed{-}$, $\boxed{\times}$, $\boxed{\div}$, $\boxed{=}$.

Exercise 3

Announce t~~o the class that the~~y are going to do some different puzzles, still using just these seven keys.

T: ***You may use just these keys (*** $\boxed{2}$, $\boxed{8}$, $\boxed{+}$, $\boxed{-}$, $\boxed{\times}$, $\boxed{\div}$, $\boxed{=}$ ***) and you may use them in any way you like. In fact, you can press as many keys as you like. Start with 0 on your display*** (press $\boxed{C}$) ***and then try to put 100 on the display.***

You may need to remind students that they may use only the keys in the list on the board. To help them remember, suggest students record the sequence of keys they press to get 100.

When many students have found at least one solution, begin to record some of their suggestions on the board. For example:

100:

$\boxed{8}\ \boxed{2}\ \boxed{+}\ \boxed{8}\ \boxed{+}\ \boxed{8}\ \boxed{+}\ \boxed{2}\ \boxed{=}$

$\boxed{+}\ \boxed{8}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{+}\ \boxed{2}\ \boxed{=}\ \boxed{=}$

$\boxed{+}\ \boxed{2}\ \boxed{2}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{-}\ \boxed{8}\ \boxed{-}\ \boxed{2}\ \boxed{=}$

$\boxed{8}\ \boxed{\times}\ \boxed{8}\ \boxed{\times}\ \boxed{2}\ \boxed{-}\ \boxed{2}\ \boxed{8}\ \boxed{=}$

$\boxed{8}\ \boxed{\times}\ \boxed{8}\ \boxed{+}\ \boxed{2}\ \boxed{2}\ \boxed{=}\ \boxed{=}\ \boxed{-}\ \boxed{8}\ \boxed{=}$

$\boxed{8}\ \boxed{8}\ \boxed{+}\ \boxed{2}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}$

$\boxed{8}\ \boxed{2}\ \boxed{+}\ \boxed{2}\ \boxed{2}\ \boxed{-}\ \boxed{2}\ \boxed{=}\ \boxed{=}$

$\boxed{8}\ \boxed{+}\ \boxed{2}\ \boxed{\times}\ \boxed{=}$

Try to get a variety of solutions. Sometimes one student's solution will result in several similar solutions from other students.

Repeat this exercise asking students to put 7 on the display with the same restrictions. This problem may be more difficult for students because it requires that they use $\boxed{\div}$ at some time. Several possible solutions are given below.

7:

$\boxed{8}\ \boxed{+}\ \boxed{2}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{\div}\ \boxed{2}\ \boxed{=}$

$\boxed{2}\ \boxed{\div}\ \boxed{2}\ \boxed{+}\ \boxed{2}\ \boxed{=}\ \boxed{=}\ \boxed{=}$

$\boxed{2}\ \boxed{8}\ \boxed{\div}\ \boxed{2}\ \boxed{=}\ \boxed{=}$

$\boxed{8}\ \boxed{8}\ \boxed{\div}\ \boxed{8}\ \boxed{-}\ \boxed{2}\ \boxed{=}\ \boxed{=}$

Center Activity

Create calculator puzzles for students to work on in a center.

Extension Activity

Make Exercise 3 more challenging by asking students to find solutions that use fewer than a given number of key presses, or as few key presses as possible. For example, pretend it costs a dollar to press a key on the calculator. Try to find a solution spending as little as possible.

Home Activity

Create calculator puzzles for students to work on at home with family members. For example:

- The only keys you may use are $\boxed{2}$, $\boxed{8}$, $\boxed{+}$, $\boxed{-}$, $\boxed{\times}$, $\boxed{\div}$, and $\boxed{=}$ but you may use them in any way you like. Start at 0 and try to put 50 (or 42 or 15) on the display.

Capsule Lesson Summary

Build arrow roads using 2x, +1, and –1 arrows

Materials

Teacher
- Colored chalk

Student
- Worksheets N14*, **, ***, and ****
- Colored pencils, pens, or crayons

Description of Lesson

Exercise 1

Begin this lesson with some mental arithmetic involving 2x.

T: ***What number is 2 x 7?*** (14)
2 x 7 = 14, so what number is 2 x 17? (34)
How did you calculate 2 x 17?

Encourage several students to explain their calculation methods. Continue this activity with these or similar problems.

2 x 8	(16)	2 x 9	(18)
2 x 18	(36)	2 x 19	(38)
2 x 28	(56)	2 x 109	(218)
2 x 58	(116)	2 x 1,009	(2,018)

Exercise 2

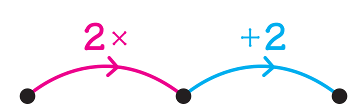
Present the following or a similar problem-solving situation and let students work with partners to answer the questions. Provide students with play money or other props if they wish to use them. Also, you can suggest that an arrow picture might help them think about the problem.

T: ***Caleb needs $10.00 to go to an amusement park. Caleb's grandfather says he will double whatever money Caleb has, and Caleb's grandmother offers to give him $2.00. Who should Caleb go to see first? Why? How much money does Caleb need to have before visiting his grandparents so that he will end with exactly $10.00?***

After several student pairs think they have some answers, hold a collective discussion of the problem. As students report, use arrow pictures to display their suggestions.

S: ***We think Caleb should go to his grandfather first, because grandfather doubles his money.***

T: ***This dot is for the money Caleb has to start. A 2x arrow shows that Grandfather doubles his money.***

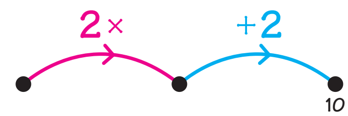


S: ***Then, Caleb goes to his grandmother and gets $2.00 more. Draw a +2 arrow following the 2x arrow.***

Point to the appropriate dots and trace the arrows as you reiterate the sequence of events.

T: ***Caleb starts with this amount of money. Grandfather doubles it, and then Grandmother adds $2.00 more. Now Caleb has this amount of money. How much does Caleb want to have at the end?***

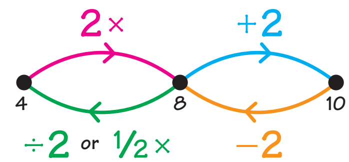
S: ***$10.***



T: ***Did you find how much money Caleb needs to start?***

S: ***$4.***

Check that $4 works as the starting amount of money. Then draw return arrows in the picture to show how students might use the picture to find $4 at the start.

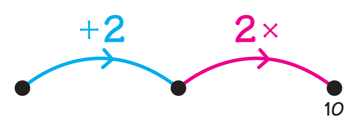


You may need to ask what would happen if Caleb went to Grandmother first.

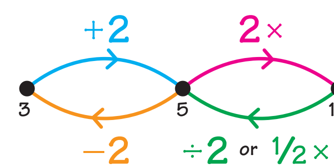
S: ***We think Caleb should go to his grandmother first, because then he could start with less money.***

T: ***Let's draw an arrow picture to show what happens if Caleb goes first to Grandmother and then to Grandfather.***

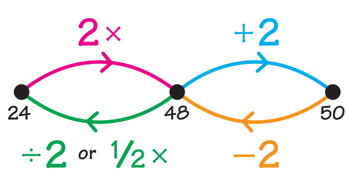
S: ***Draw a dot for the money he has to start. Draw a +2 arrow for the $2 he gets from Grandmother. Then draw a 2x arrow to show Grandfather doubles his money. Put 10 at the ending dot.***

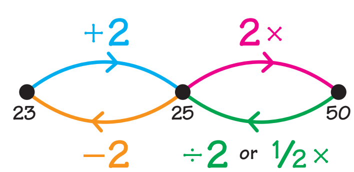


Call on students to label the other dots in this picture to find that Caleb needs only $3 at the start in this case.



You may like to compare the two starting numbers in these arrow pictures for different ending numbers, for example, 50.





Exercise 3

Distribute Worksheets N14* and ** to students.

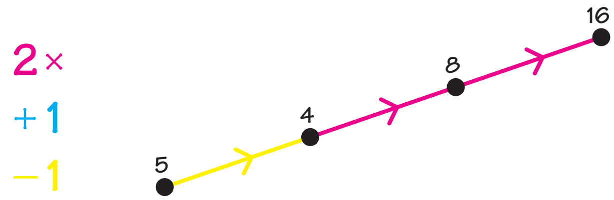
T: ***We are going to build arrow roads using 2x, +1, and –1 arrows.***
On the first worksheet, build a road between 5 and 16, but try to make your road as short (as few arrows) as possible.

While the students are working independently (or if you prefer, in pairs) draw the picture from Worksheet N14* on the board.

As you observe students' work, count with them how many arrows they used in a road and suggest they try to find a road with fewer arrows. You may like to challenge students to find even shorter roads by telling the class how many arrows are in the shortest road you have seen so far.
When most of the students have built at least one road between 5 and 16, call on a student with the

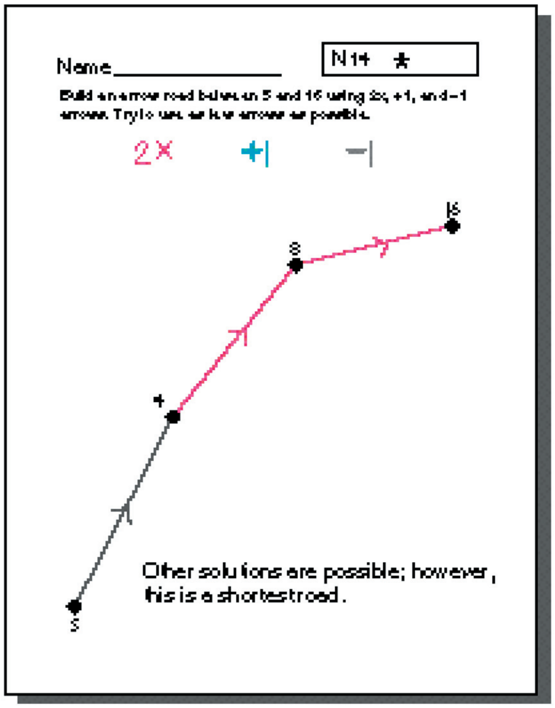
shortest road to draw it on the board. Discuss the road briefly; if it has more than three arrows, tell the class that there is an even shorter road between 5 and 16.

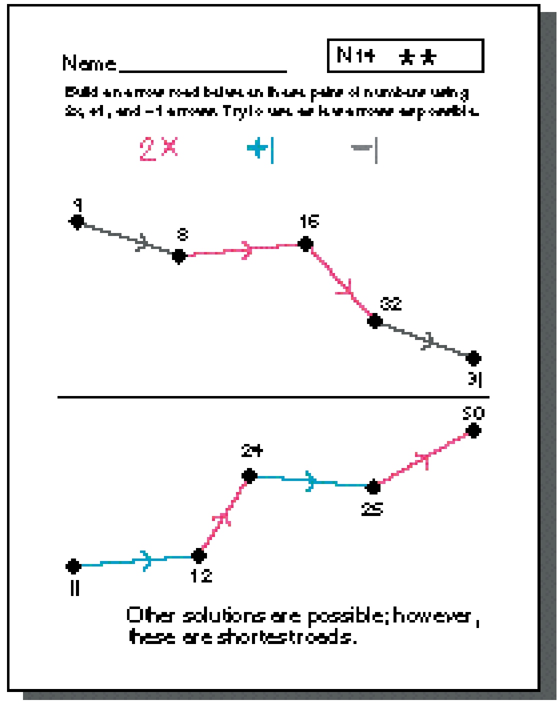
The shortest possible road from 5 to 16 has three arrows and is illustrated below.

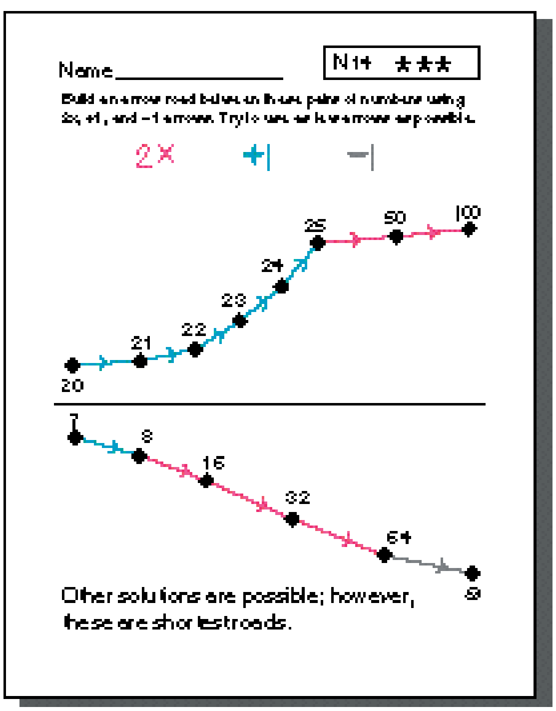


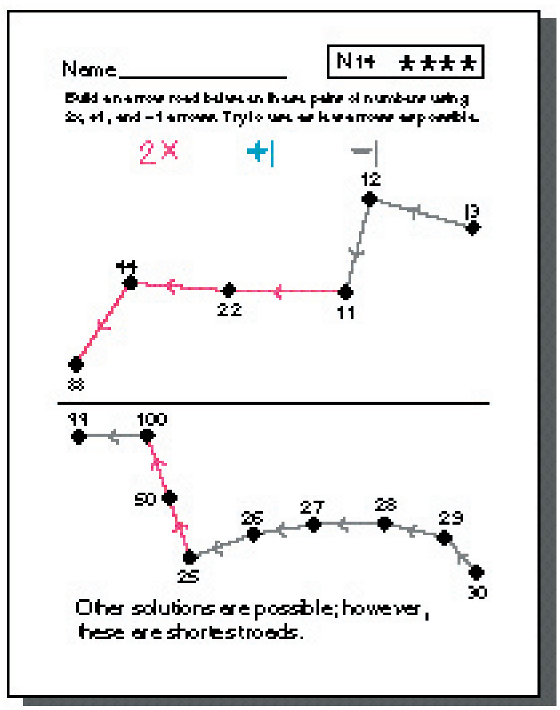
Repeat this activity with Worksheet N14**. Encourage students to find shortest possible roads, but remind them that their primary objective is to build a road between each pair of numbers. You might challenge students to use six or fewer arrows in each road. Those who finish Worksheet N14** quickly can continue with Worksheets N14*** and ****.

The shortest possible road between each pair of numbers is shown on the answer key page. There are many ways to build a road between each pair of numbers.


Name
N14 ★
2×
+|
−|
5
4
8
16
Other solutions are possible; however, this is a shortest road.


Name
N14 ★★
2×
+|
−|
9
8
16
32
31
11
12
24
25
50
Other solutions are possible; however, these are shortest roads.


Name
N14 ★★★
2×
+|
−|
20
21
22
23
24
25
50
100
7
8
16
32
64
63
Other solutions are possible; however, these are shortest roads.


Name
N14 ★★★★
2×
+|
−|
88
44
22
11
12
13
99
100
50
25
26
27
28
29
30
Other solutions are possible; however, these are shortest roads.

Capsule Lesson Summary

Explore the effects of moving various checkers in a configuration on the Minicomputer; after a move, is the number on the Minicomputer more, less, or the same as the previous number? Follow an arrow road with moves on the Minicomputer, and then follow the arrow road in reverse with moves on the Minicomputer.

Materials

Teacher
- Minicomputer set
- Colored chalk

Student
- Index card
- Paper
- Minicomputer set

Description of Lesson

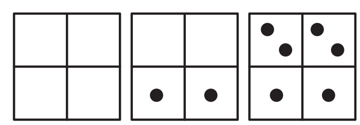
Exercise 1

Write these words on the board and instruct students to write them with large letters on their index card for use later in the lesson.

More
Same
Less

T: ***I am going to put a number on the Minicomputer. See if you can figure out what number it is.***

Gradually put this configuration on the Minicomputer, starting with the checkers on the squares of greatest value. Pause frequently so your students can do mental calculations.



Direct students to write the number on their papers or whisper it to a neighbor before letting someone answer aloud.

S: ***57.***

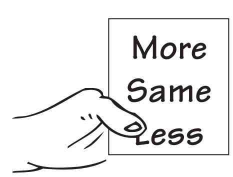
Invite several students to explain how they know this number is 57.

T: ***I am going to move one of these checkers to another square. Tell me if the new number is more than, less than, or the same as the number on the Minicomputer now.***

Move a checker from the 2-square to the 1-square. Instruct students to hold up their cards with fingers pinching on the word that describes the new number. The students should indicate that the new number is less than before.

T: ***How much less?***

S: ***1 less.***

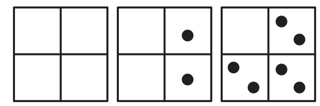


Repeat this activity several times as suggested below. Do not return checkers to their original positions. Each move will start from a new number on the Minicomputer.

Move a checker

- from the 4-square to the 1-square (3 less)
- from the 20-square to the 40-square (20 more)
- from the 10-square to the 2-square (8 less)
- from the 1-square to the 10-square (9 more)
- from the 8-square to the 4-square (4 less)
- from the 8-square to the 2-square (6 less)

Check that this configuration is on the Minicomputer.



T: ***Who can move exactly one checker and make the number 2 more than it is now?***

A student should move a checker from the 2-square to the 4-square.

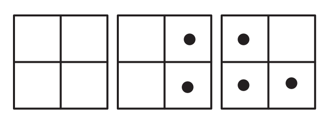
Continue this activity by asking for volunteers to make these changes. Again, do not return checkers to their original positions; otherwise, some changes may be impossible.

- 9 more (from 1-square to the 10-square)
- 19 more (from the 1-square to the 20-square)
- 10 less (from the 20-square to the 10-square)
- 3 less (from the 4-square to the 1-square)
- 30 less (from the 40-square to the 10-square)
- 6 more (from the 4-square to the 10-square, or from the 2–square to the 8–square)
- 99 more (from the 1-square to the 100-square)

Exercise 2

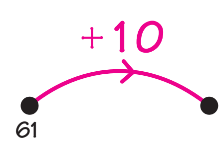
Pair students and distribute individual Minicomputers for each pair.

T: ***I am going to put a number on the Minicomputer.***
I want you to put the same number on your Minicomputer.
See if you can figure out what number it is.



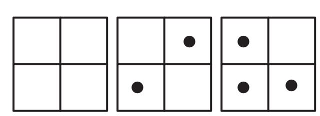
Invite several students to explain how they know the number is 61.

T: ***As I draw an arrow picture you will follow the arrows by making moves on your Minicomputer. Let's try some together.***



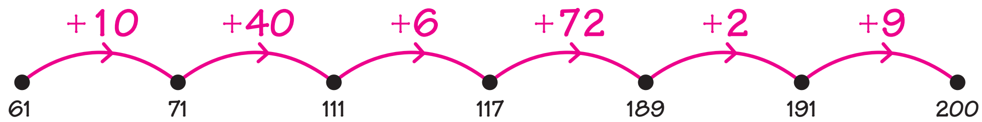
Can you show how to make the number on the Minicomputer 10 more? (Move a checker from the 10-square to the 20-square)

What should I label my new dot? (71)



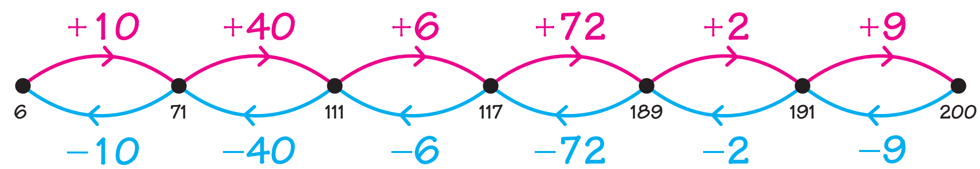
Direct students to make the move you just demonstrated on their individual Minicomputers.

Continue as above, one arrow at a time, until the following picture is completed.

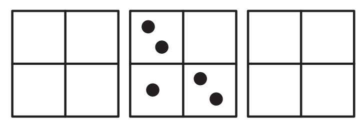


Note: You can use just one color for the arrows and label each arrow.

T: ***What number is on the Minicomputer?*** (200)
Now I want to move checkers so that 61 is on the Minicomputer again.
Let's draw return arrows and show those moves on the Minicomputer.



T: ***Your Minicomputer should look like this.***
Now follow the return arrows with moves
on your Minicomputer.



Capsule Lesson Summary

Build an arrow road from one number to another using only 10x, +1, and –1 arrows.

Materials

Teacher
- Colored chalk

Student
- Unlined paper
- Colored pencils, pens, or crayons
- Worksheets N16*, **, ***, and *****

Description of Lesson

Exercise 1: Mental Arithmetic

Begin this lesson with a short mental arithmetic activity involving the function 10x.

10 x 4	(40)	10 x 0	(0)	10 x 20	(200)
10 x 5	(50)	10 x 10	(100)	10 x 23	(230)
10 x 2	(20)	10 x 12	(120)	10 x 200	(2000)
10 x 1	(10)	10 x 16	(160)	10 x 223	(2230)

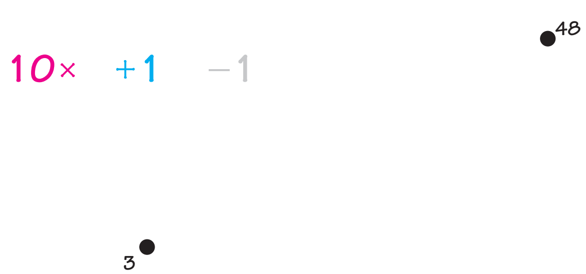
Ask students to suggest several numbers that are multiples of 10. With each suggestion, ask the student to tell you an appropriate 10x number fact. For example, if 670 is suggested, the corresponding number fact would be 10 x 67 = 670.

Erase the board before continuing with Exercise 2.

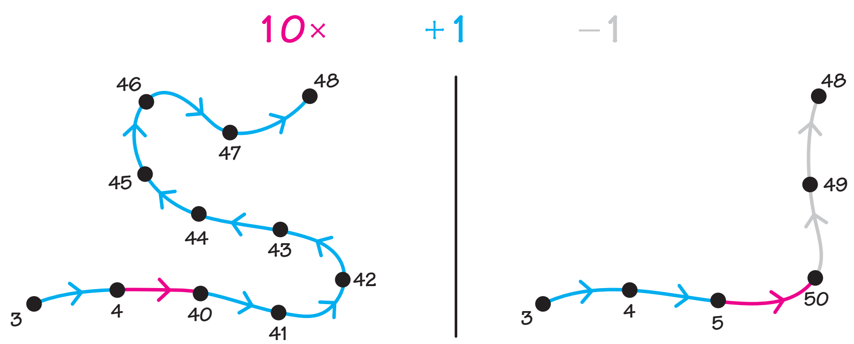
Exercise 2

Put this information on the board.

Direct students to copy what you have drawn on the board and to build a road between 3 and 48. The problem is solved when one road has been built, but encourage students to try to build as short a road as possible. Observe students' work, and challenge students to find shorter roads by telling the class how many arrows are in the shortest road you have seen so far.



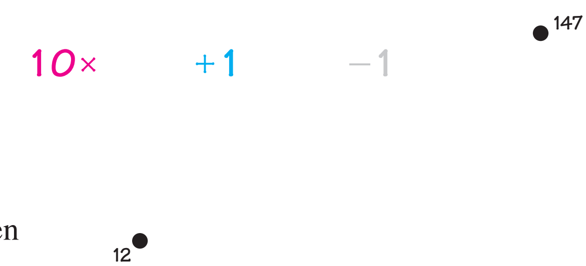
When most everyone has built a road between 3 and 48, call on students with the following arrow roads to draw them on the board. You may need to draw one of these roads yourself if none of your students has built it.



Briefly discuss these roads with your class. Encourage students to observe which multiple of 10 is closest to 48 (50 is closer than 40).

Exercise 3

Erase the board and put on this information. Instruct students to copy the information on their papers.

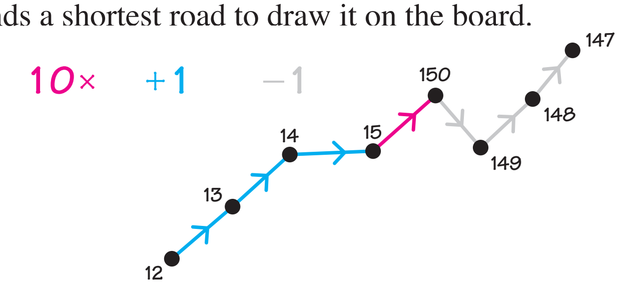


Tell the class that they are going to build a road using 10x, +1, and –1 again, but this time between 12 and 147.

T: ***Which multiple of 10 is closest to 147?***

Students may suggest both 140 and 150. Observe that 150 is closer, and then draw a dot for 150 near the one for 147. Instruct students to complete a road between 12 and 147 and to try to make it as short as possible.

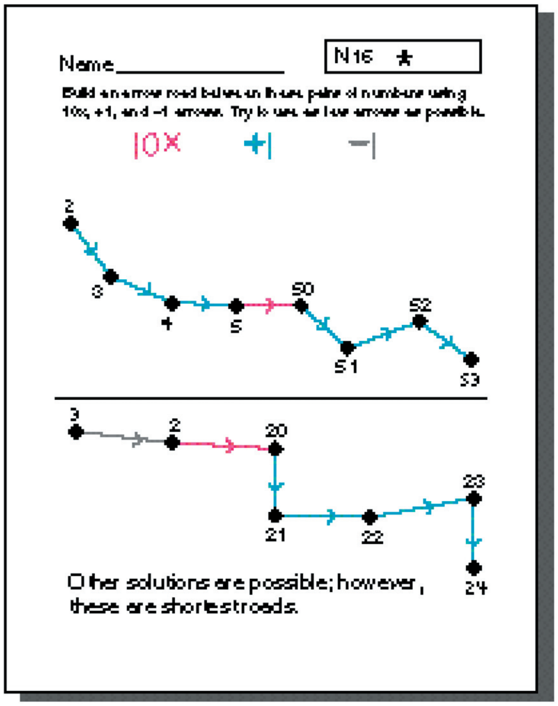
Call on a student who finds a shortest road to draw it on the board.

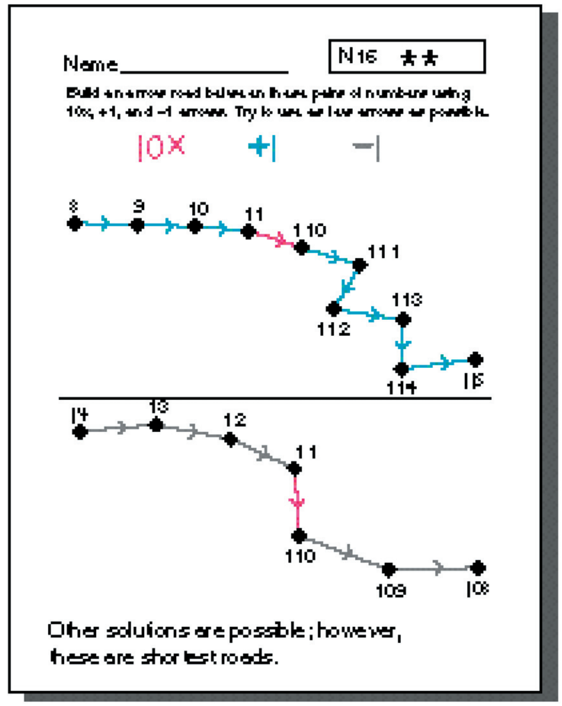


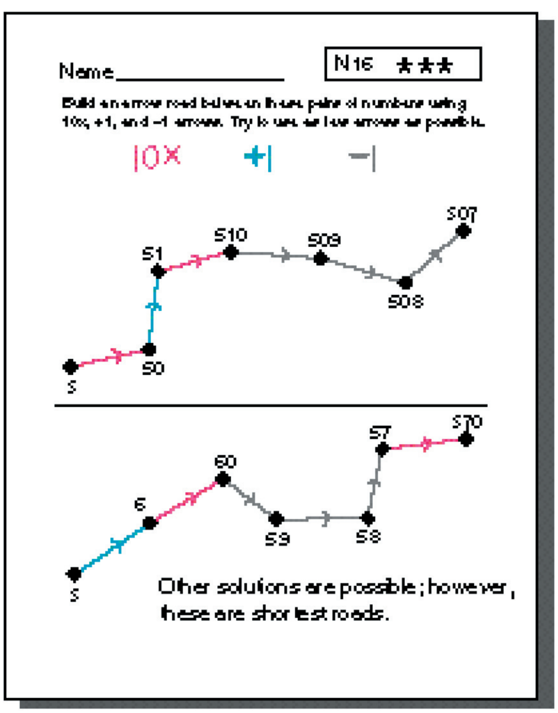
Worksheets N16*, **, ***, and **** are available for individual or small group work. Encourage students to find shortest possible roads, but remind them that they have solved the problem when they build a road between the two numbers, no matter how long the road. There are many ways to build a road between each pair of numbers on the worksheets, although there is only one shortest road in each case.

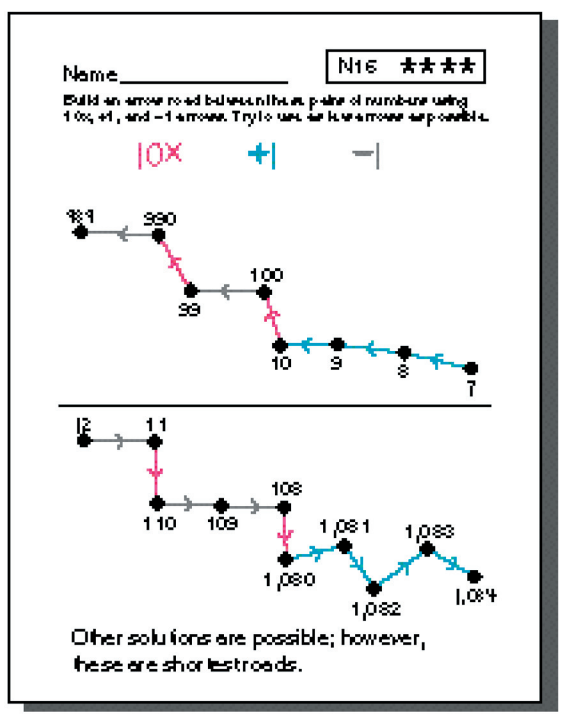
Home Activity

Suggest that parents/guardians work with their child to build a shortest road from 17 to 195, using 10x, +1, and –1 arrows.


Name
N16 ★
Build an arrow road between these pairs of numbers using 10x, +1, and −1 arrows. Try to use as few arrows as possible.
10x
+1
−1
2
3
4
5
50
51
52
53
3
2
20
21
22
23
24
Other solutions are possible; however, these are shortest roads.


Name
N16 ★★
Build an arrow road between these pairs of numbers using 10x, +1, and −1 arrows. Try to use as few arrows as possible.
10x
+1
−1
8
9
10
11
110
111
112
113
114
115
14
13
12
11
110
109
108
Other solutions are possible; however, these are shortest roads.


Name
N16 ★★★
Build an arrow road between these pairs of numbers using 10x, +1, and −1 arrows. Try to use as few arrows as possible.
10x
+1
−1
5
50
51
510
509
508
507
5
6
60
59
58
57
570
Other solutions are possible; however, these are shortest roads.


Name
N16 ★★★★
Build an arrow road between these pairs of numbers using 10x, +1, and −1 arrows. Try to use as few arrows as possible.
10x
+1
−1
989
990
99
100
10
9
8
7
12
11
110
109
108
1,080
1,081
1,082
1,083
1,084
Other solutions are possible; however, these are shortest roads.

Capsule Lesson Summary

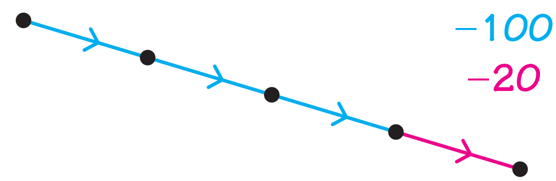
Find the composite of several subtraction arrows. Build arrow roads to record ways of subtracting 650 – 298.

Materials

Teacher	• Colored chalk	**Student**	• Unlined paper • Colored pencils, pens, or crayons

Description of Lesson

Exercise 1



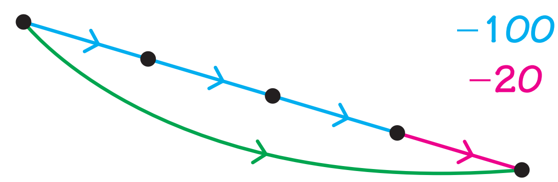
Draw this arrow picture on the board.

T: ***Point to the least number in this arrow picture.***

Students should point to the ending dot of the road.

T: ***How do you know that this is the least number?***

S: ***Because when we subtract the result is less.***†



Draw an arrow from the starting dot to the ending dot.

T: ***What could this green arrow be for? How do you know?***

S: ***There are three –100 arrows and one –20 arrow, so the green arrow is for –320.***

Label the green arrow –320 and the starting dot 575.

T: ***If this dot were for 575, what would the ending number be?*** (255)

Let a student explain the answer by pointing to the dots in the arrow picture.

S: ***These numbers would be 575, 475, 375, 275, and 255.***

Erase all the dot labels and label the starting dot 837.

T: ***If the starting dot were for 837, what would the ending dot be? How do you know?***

S: ***517, because 800 – 300 = 500 and 37 – 20 = 17.***

Repeat this activity with 915, 306, and 1,200 as starting numbers of the road. Erase the board before going on to Exercise 2.

†This is true when a positive number is subtracted.

Exercise 2

At the beginning of this exercise, do not tell the students what the arrows are for.

T: ***Draw an arrow road on your paper with several blue arrows (less than ten) and just one red arrow. When you finish, draw a green arrow from the starting dot to the ending dot.***

Help individual students as necessary. When many have finished this task, instruct the students to write on their papers –100 in blue and –30 in red.

T: ***Blue arrows are for –100 and the red arrow is for –30. Since you have drawn different arrow pictures, your green arrows may be for different things. Who can tell us what his or her green arrow could be for?***

S: ***My green arrow is for –630, because I drew 6 blue arrows and one red arrow. 6 x 100 = 600 and 600 + 30 = 630.***

T: ***Shawna's green arrow is for –630, so she should write –630 on her paper in green (to label the green arrow). Whose green arrow is for something different than Shawna's?***

S: ***My green arrow is for –930. I drew nine blue arrows and one red arrow.***

Direct students to label their green arrows. Students who are having difficulty may seek the help of the class by describing their arrow roads. Continue this activity until all students have labeled their green arrows.

T: ***Choose any number you like to be your starting number. Write that number near your starting dot and then label all the other dots.***

As you observe students' papers, undoubtedly you will find some with negative numbers in their arrow pictures. Students who finish quickly may be paired with other students who need assistance in labeling their dots. Student pairs can check each other's work.

T: ***Write a number sentence about your green arrow, and the starting and ending numbers in your picture.***

Invite several students to write their number sentences on the board. For example:

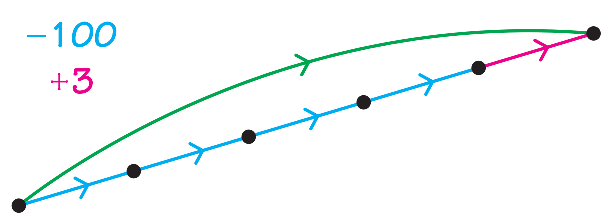
$$1{,}000 - 630 = 370 \qquad 400 - 630 = \widehat{230}$$

$$350 - 230 = 120 \qquad 1{,}050 - 530 = 520$$

Erase the board before going on to Exercise 3.

Exercise 3

Draw this arrow picture on the board.



T: ***What could this green arrow be for? How do you know?***

S: ***There are four –100 arrows and one +3 arrow. The green arrow is the same as –400 and then +3, so the green arrow is for –397.***

Write –397 in green near the arrow picture.

T (pointing to the starting dot)**:** ***If this dot were for 1,000, what number would be at the ending dot?*** (603)

Let a student explain the answer by pointing to dots in the arrow picture. Repeat this activity with 500, 750, and 1,500 as starting numbers.

Erase the board before going on to Exercise 4.

Exercise 4

T: ***If there are 650 students in an elementary school and 298 walk to school, how many students ride to school?***

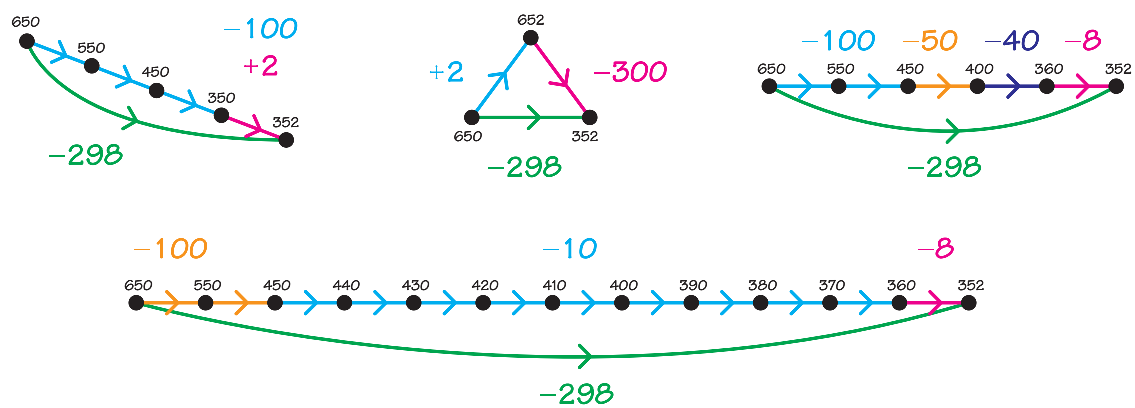
Write this subtraction problem on the board.

650 – 298 = ?

T: ***Solve this problem using any method you like. Draw an arrow picture to show how you do the calculation.***

Some students may need help drawing an arrow picture. If these students are able to explain to you how they do the calculation, describe for them a corresponding arrow picture. Suggest to students who have trouble finding a method to calculate 650 – 298 that they start at 650 and draw two –100 arrows, nine –10 arrows, and one –8 arrow.

Ask students who finish quickly to draw their arrow pictures on the board. Try to get at least three different methods. A few of the many possibilities are shown below.



Discuss each arrow picture briefly and conclude that 650 – 298 = 352. Emphasize that there are many ways to solve this problem.

Home Activity

Ask students to choose their favorite method of doing the calculation 650 – 298 and to draw the corresponding arrow picture to take home and share with family members. Suggest other similar subtraction problems for students to solve at home using a similar method; for example:

221 – 98   73 – 49   152 – 97   312 – 195

Capsule Lesson Summary

Evenly divide 108 books among three classes. Do a similar activity with 192 balloons.

Materials

Teacher		Student	
	• None		• Paper • Worksheets N18*, **, ***, and ****

Description of Lesson

Exercise 1: Mental Arithmetic

Begin this lesson with a short mental arithmetic activity involving the functions 3x and $\frac{1}{3}$ x.

3 x 10 (30)	3 x 12 (36)	$\frac{1}{3}$ x 60 (20)
$\frac{1}{3}$ x 30 (10)	$\frac{1}{3}$ x 36 (12)	$\frac{1}{3}$ x 69 (23)

Occasionally ask students to explain how they did a calculation.

Note: Exercises 2 and 3 have division stories. As you tell these or similar stories, you may like to choose other teachers' classes as well as your own to star in them. You may also like to use props while you tell the stories even though you will draw charts on the board.

Exercise 2

T: ***Let's pretend that someone donated 108 books to our school and we need to share them equally among three classes. We would like to give one-third of the books to Ms. Briggs' class, another one-third to Mr. Moore's class, and keep one-third of them for our class.***

Draw a chart on the board.

Ms. Briggs	Mr. Moore	Ms. Schweitzer

T: ***How many books should we give to each of these three classes? Remember that we want to give one-third of the books to each class. Write an answer on your paper.***

As you observe students' answers, do not tell them yet whether or not they are correct.

T: ***Will each classroom receive at least 100 books? How do you know?***

S: ***No, because if we give 100 books to Ms. Briggs' class, there will be only eight books left for the other two classes.***

T: ***Could we give 50 books to each class?*** (No) ***Why not?***

S: ***3 x 50 = 150 and 150 is more than 108.***

T: ***Could we give 25 books to each class?***

S: ***Yes, because 3 x 25 = 75 and 75 is less than 108.***

Note: Certainly other choices besides 25 could be made. Feel free to follow a student's suggestion.

Record 25 in each column of the chart.

T: ***After we give 25 books to each class, how many books will we have left to share? How do you know?***

S: ***33. 108 – 75 = 33 (or 75 + 33 = 108).***

T: ***What number is $^1/_3$ x 33?***

S: ***11.***

Ms. Briggs	Mr. Moore	Ms. Schweitzer
25	25	25
11	11	11

Write 11 in each column.

T: ***How many books should we give to each class?***

S: ***36; 25 + 11 = 36.***

T: ***What number sentence can we write about this problem?***

Record number sentences on the board as they are suggested. For example,

$$36 \times 36 \times 36 = 108 \qquad \tfrac{1}{3} \times 108 = 36$$
$$3 \times 36 = 108 \qquad 108 \div 3 = 36$$

T: ***This is another way to record a division calculation. 108 divided by 3 equals 36.***

$$\begin{array}{r} 36 \\ 3\overline{)108} \end{array}$$

Erase the board before continuing with the next story.

Exercise 3

T: ***The~~re are 192 balloon~~s left over from the school carnival. Suppose we want to divide them equally among the three first grade classes.***

We are sharing 192 balloons. See if you can figure out how many balloons each class should receive. Do the problem on your paper.

As you observe students' work, do not tell them yet whether or not they are correct. Do, however, comment on good sharing strategies.

T: ***How should we begin to share the 192 balloons?***

Follow the suggestions of your students.

S: ***Give 50 balloons to each class.***

T: ***Is it possible to give 50 balloons to each class ?*** (Yes)

Write 50 in each column of a chart.

T: ***What number is 3 x 50?***

Ms. Powers	Mr. Hickel	Mr. Kenfeld
50	50	50

S: ***150.***

T: ***How many balloons do we still have to distribute?***

S: ***42.***

T: ***What should we do now?***

S: ***Give 10 more balloons to each class.***

T: ***3 x 10 = 30, so that's possible.***

Write 10 in each column of the chart.

Ms. Powers	Mr. Hickel	Mr. Kenfeld
50 10	50 10	50 10

T: ***How many balloons are left now?***

S: ***12.***

T: ***What number is $^1/_3$ x 12?***

S: ***4.***

Write 4 in each column.

Ms. Powers	Mr. Hickel	Mr. Kenfeld
50 10 4	50 10 4	50 10 4

T: ***How many balloons did we give to each class?***

S: ***64; 50 + 10 + 4 = 64.***

T: ***How can we check that $^1/_3$ x 192 = 64?***

S: ***Calculate 64 + 64 + 64.***

Ask students to calculate 3 x 64 on their papers. Conclude that $^1/_3$ x 192 = 64, so each class receives 64 balloons. Direct students to write number sentences about this situation on their papers. Invite several students to record their number sentences on the board. A few of the possible number sentences are shown here.

$192 \div 3 = 64$

$64 \times 3 = 192$

$\frac{1}{3} \times 192 = 64$

$3 \times (50 + 10 + 4) = 192$

$$3\overline{)192}\quad\text{quotient } 64$$

$$\begin{array}{r} 64 \\ \times\ 3 \\ \hline 192 \end{array} \qquad \begin{array}{r} 64 \\ 64 \\ +\ 64 \\ \hline 192 \end{array}$$

Worksheets N18*, **, ***, and **** are available for individual work.

Name________________ N18 ★

Share 26 cards fairly between Dick and Nina.

For Dick	For Nina
10	10
3	3
13	13

Write a number sentence about this sharing.

$13 + 13 = 26$

Share 27 pencils fairly among Andrea, Sheila, and Rob.

For Andrea	For Sheila	For Rob
5	5	5
4	4	4
9	9	9

Write a number sentence about this sharing.

$27 \div 3 = 9$

Other ways to do the sharing and other number sentences are possible.

Share 34 pennies fairly between Pat and Gary.

For Pat	For Gary
10	10
5	5
2	2
17	17

Write a number sentence about this sharing.

$\frac{1}{2} \times 34 = 17$

Share 54 dimes fairly among Bill, Stanley, and Lisa.

For Bill	For Stanley	For Lisa
10	10	10
4	4	4
4	4	4
18	18	18

Write a number sentence about this sharing.

$3 \times 18 = 54$

Other ways to do the sharing and other number sentences are possible.

Name________________ N18 ★★

Share 114 pictures fairly between Arthur and Marla.

For Arthur	For Marla
50	50
7	7
57	57

Write a number sentence about this sharing.

$$2\overline{)114} = 57$$

Share 81 candies fairly among Nora, Brad, and Mark.

For Nora	For Brad	For Mark
20	20	20
5	5	5
2	2	2
27	27	27

Write a number sentence about this sharing.

$3 \times 27 = 81$

Other ways to do the sharing and other number sentences are possible.

Share 186 stamps fairly between Andy and Pam.

For Andy	For Pam
50	50
40	40
3	3
93	93

Write a number sentence about this sharing.

$93 + 93 = 186$

Share 129 marbles fairly among John, Ann, and Cathy.

For John	For Ann	For Cathy
40	40	40
3	3	3
43	43	43

Write a number sentence about this sharing.

$\frac{1}{2} \times 129 = 43$

Other ways to do the sharing and other number sentences are possible.

Name________________ N18 ★★★

Share 483 stickers fairly among Paula, Stacey and Joy.

For Paula	For Stacey	For Joy
100	100	100
50	50	50
11	11	11
161	161	161

Write a number sentence about this sharing.

3 x 161 = 483

Share 732 cards among Wally, James, Amy, and Jade.

For Wally	For James	For Amy	For Jade
100	100	100	100
50	50	50	50
20	20	20	20
10	10	10	10
3	3	3	3
183	183	183	183

Write a number sentence about this sharing.

732 ÷ 4 = 183

Other ways to do the sharing and other number sentences are possible.

Name________________ N18 ★★★★

Share 819 seeds fairly among Mike, Ellen, and Eric.

For Mike	For Ellen	For Eric
200	200	200
50	50	50
20	20	20
3	3	3
273	273	273

Write a number sentence about this sharing.

$$3\overline{)819}^{\,273}$$

Share 1,935 books among Sandra, Lee, Christy, Sharona, and Mala.

For Sandra	For Lee	For Christy	For Sharona	For Mala
200	200	200	200	200
100	100	100	100	100
80	80	80	80	80
7	7	7	7	7
387	387	387	387	387

Write a number sentence about this sharing.

$\frac{1}{5}$ x 1,935 = 387

Other ways to do the sharing and other number sentences are possible.

N19 DECIMALS ON THE NUMBER LINE

Capsule Lesson Summary

Multiply non–integer decimals by 10 on the Minicomputer. List the corresponding number sentences and look for patterns. Locate decimals between 0 and 1 on a number line using money as a support.

Materials

Teacher

- Minicomputer set
- Colored chalk
- ⑩–checkers
- Coins

Student

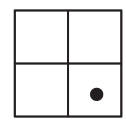
- Paper
- Worksheet N19 (no star), *, and **

Description of Lesson

During this lesson, you may like as well to display amounts of money with actual coins, also.

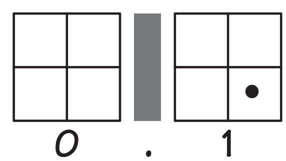
Exercise 1

Display one Minicomputer board and put a checker on the 1-square.

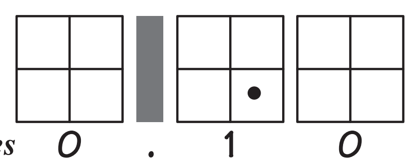


T: ***This checker is for $1. How could you show one dime on the Minicomputer?***

Students should suggest putting up another board and drawing a bar between the boards. Follow this suggestion and then ask a student to put one dime on the Minicomputer. Write the number below (above) the Minicomputer.



Add another new board on the right.



T: ***When I put another Minicomputer board on the right, the number is not changed. It is still one dime, but does this suggest another way to write this number?***

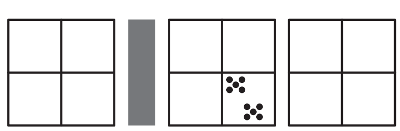
S: ***You can write 0.10.***

Record this information on the board.

$$0.1 = 0.10$$

T: ***The number on the Minicomputer is 0.1 or one dime.***

Put ten checkers on the 0.1-square and write the corresponding calculation to the right of the Minicomputer.

 $= 10 \times 0.10$

T: ***How many dimes are on the Minicomputer?***

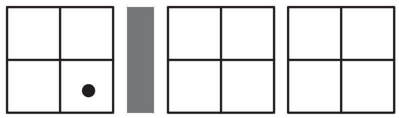
S: ***Ten.***

T: ***How much money is this?***

S: ***$1.***

T: ***Yes, ten dimes is the same as $1.***

Call on students to make trades on the Minicomputer until the standard configuration for 1.00 is obtained. Ask a student to write the number below (above) the Minicomputer.



$= 10 \times 0.10 = 1.00$

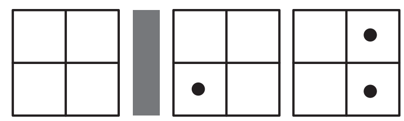
1 . 0 0

T: ***Do you know any other ways to write this number?†***

If necessary, suggest 1.0 and 1 yourself and put this information in a list of equalities.

$$10 \times 0.10 = 1.00 = 1.0 = 1$$

Remove the checker from the Minicomputer. Ask a student to put a quarter on the Minicomputer, and then write the number below (above) it.



0 . 2 5

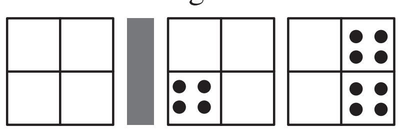
T: ***One quarter is on the Minicomputer.***

Put three more quarters on the Minicomputer in the same way.

T: ***Now how many quarters are on the Minicomputer?***

S: ***Four.***

Write the calculation 4 x 0.25 to the right of the Minicomputer.



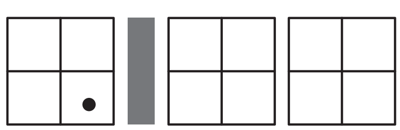
$= 4 \times 0.25$

T: ***How much money is this?***

S: ***$1.***

T: ***Yes, four quarters is also the same as $1.***

Call on students to make trades on the Minicomputer until the standard configuration for 1.00 is obtained. Ask a student to write the number below (above) the Minicomputer.



$= 4 \times 0.25 = 1.00$

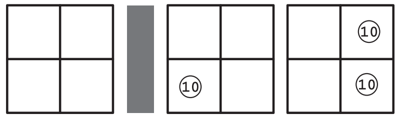
1 . 0 0

T: ***How much money is ten quarters?***

†There are many expressions, including 01 and 1.000, that can be used for this number.

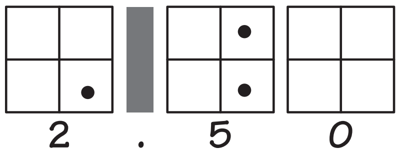
Write the calculation 10 x 0.25, and let students write the answer on their papers or whisper their answer to a neighbor.

Put 10 x 0.25 on the Minicomputer with ⑩-checkers.



$= 10 \times 0.25$

Call on a volunteer to make trades with the ⑩-checkers and to write the number below (above) the Minicomputer.



$= 10 \times 0.25 = 2.50$

T: ***How much money is ten quarters?*** ($2.50) ***10 x 0.25 = 2.50. What is another name for 2.50?***

S: ***2.5.***

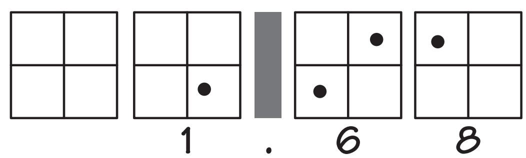
$10 \times 0.10 = 1.00 = 1.0 = 1$

$10 \times 0.25 = 2.50 = 2.5$

Write this number sentence in the list forming on the board.

Erase the board except for the list of number sentences and remove the checkers from the Minicomputer. Put a fourth Minicomputer board on the left.

Ask a student to put $1.68 on the Minicomputer and to write the number below (above) the Minicomputer.

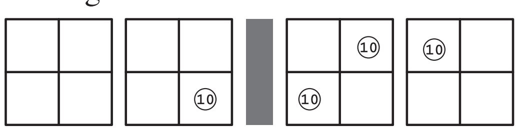


Write this problem on the board.

10×1.68

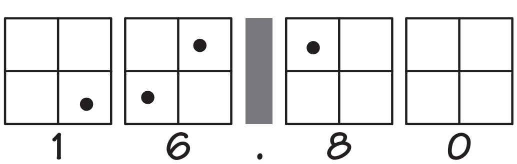
T: ***What number is 10 x 1.68?***

Put 10 x 1.68 on the Minicomputer with ⑩-checkers, and ask students to write their answers on paper or whisper to a neighbor. Write the calculation to the right of the Minicomputer.



$= 10 \times 1.68$

Call on volunteers to make trades with the ⑩-checkers and to write the number below (above) the Minicomputer.



$= 10 \times 1.68 = 16.80$

T: ***How much money is 10 x $1.68?*** ($16.80)

10 x 1.68 = 16.80.
What is another name for 16.80?

S: ***16.8.***

$$10 \times 0.10 = 1.00 = 1.0 = 1$$
$$10 \times 0.25 = 2.50 = 2.5$$
$$10 \times 1.68 = 16.80 = 16.8$$

Add this number sentence to the list on the board.

T: ***Do you notice any patterns we can use when we multiply by 10?***

Encourage students to express their ideas, especially comparing the positions of the decimal points in the answers to their positions in the problems.

Write this problem on the board.

$$10 \times 0.50$$

T: ***What number is 10 x 0.50?***

S: ***5.00.***

Complete the number sentence and pose another problem.

$$10 \times 0.43$$

T: ***What number is 10 x 0.43?***

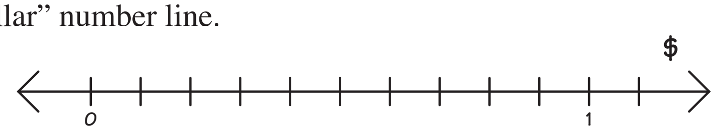
S: ***4.30.***

Complete the number sentence. You should have this list on the board.

$$10 \times 0.10 = 1.00 = 1.0 = 1$$
$$10 \times 0.25 = 2.50 = 2.5$$
$$10 \times 1.68 = 16.80 = 16.8$$
$$10 \times 0.50 = 5.00 = 5.0 = 5$$
$$10 \times 0.43 = 4.30 = 4.3$$

Exercise 2

Draw a number line on the board with each segment approximately 10 cm in length. Explain to the class that this is a "dollar" number line.

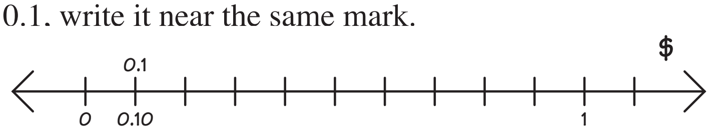


T: ***How many spaces are there between \$0 and \$1?*** (Ten)
What could these marks be for? (Dimes)
Where is the mark for one dime?

Students should point to the first mark to the right of the mark for 0. Label it 0.1.

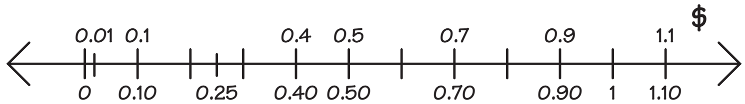
T: ***What is another name for 0.10?***

When someone suggests 0.1. write it near the same mark.



Ask students to point to and label marks for these amounts.

- forty cents
- nine dimes
- seventy cents
- the amount exactly halfway between $0 and $1 on the number line
- $1.10
- one penny
- one quarter



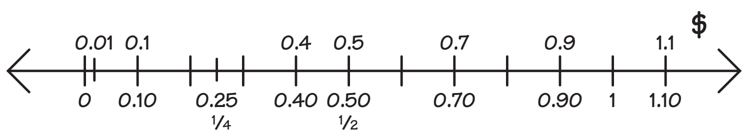
T: ***What part of a dollar is 50¢?***

S: ***Half a dollar.***

T: ***We can also use ½ for the mark already labeled 0.5 and 0.50. This mark is halfway between 0 and 1.***

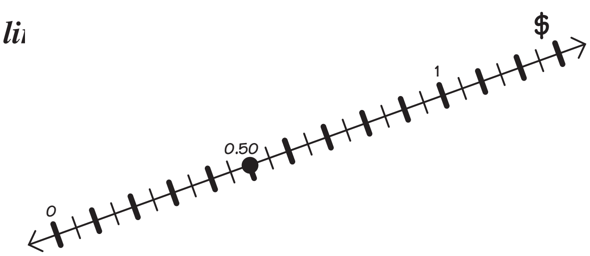
What part of a dollar is $0.25? (One-fourth)
How many quarters are there in one dollar? (Four)
So one quarter is one-fourth of a dollar.

T: ***We can also use ¼ for the mark already labeled 0.25. This mark is one-fourth of the way from 0 to 1.***



Distribute copies of Worksheet N19 (no star). Erase the board and draw a number line similar to the number line on the worksheet.

T: ***What could these heavy marks be for?*** (Dimes)
What could the light marks be for? (Nickels)
Draw and label a dot for 50¢ on your number li



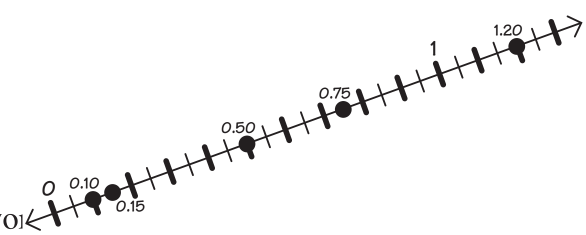
Invite a student to draw and label a dot for 50¢ on the number line on the board. Ask students to correct their papers, if necessary.

T: ***How did you know that this is the place for 50¢ on this number line?***

S: ***The heavy marks are for dimes, so I just counted 10, 20, 30, 40, 50.***

S: ***50¢ is halfway between 0 and 1.***

Direct students to draw and label dots on their number lines for 10¢, 15¢, 75¢, and $1.20. After a few minutes, call on students to draw and label the dots on the number line on the board.

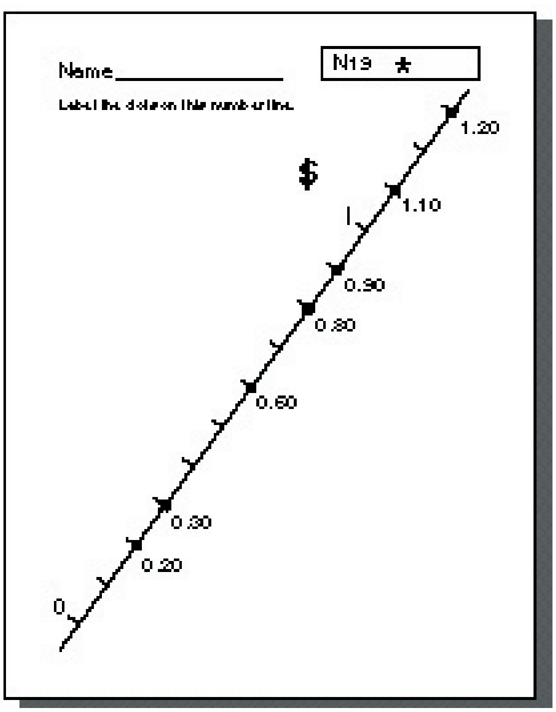


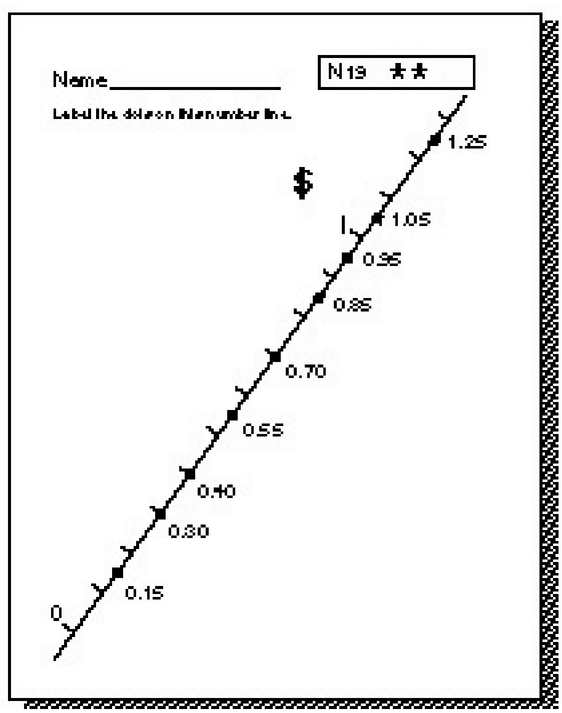
Worksheets N19* and ** are available for individual wo

Home Activity

N19

Suggest that parents/guardians work with their child to count collections of coins from someone's pocket or purse. They can then draw a number line and locate amounts of money (less than $2.00).





Capsule Lesson Summary

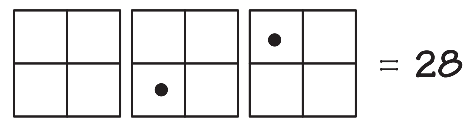
With the support of the Minicomputer, decide that the composition 2x followed by 2x followed by 2x is 8x. Draw a corresponding arrow picture. Do several 8x calculations using the composition method suggested by the arrow picture.

Materials

Teacher	• Colored chalk • Minicomputer set	• Colored pencils, pens, or crayons • Worksheets N20*, **, ***, and ****
Student	• Paper	

Description of Lesson

Ask a student to put 28 on the Minicomputer.



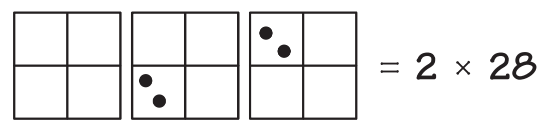
T: ***What number is $\frac{1}{2}$ x 28? How do you know?***

S: ***$\frac{1}{2}$ x 28 = 14, because $\frac{1}{2}$ x 20 = 10 and $\frac{1}{2}$ x 8 = 4.***

T: ***What number is $\frac{1}{4}$ x 28? How do you know?***

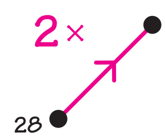
S: ***7, because 7 + 7 = 14 and 14 + 14 = 28.***

Ask a student to double the number on the Minicomputer.

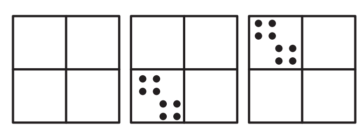


T: ***Let's show this in an arrow picture. We start with 28 and then we double it.***

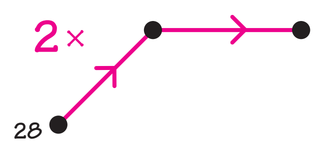
The number on the Minicomputer is 2 x 28. How can we double the number now on the Minicomputer?



S: ***Put two more checkers on the 20-square and on the 8-square.***



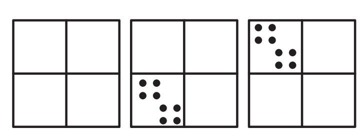
T: ***There were two checkers on both of these squares and now there are four. Let's show doubling again with another 2x arrow.***



Point to the dot at the end of the second 2x arrow.

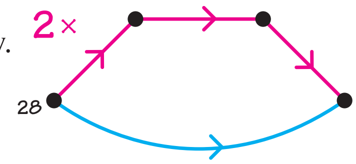
T: ***This is the number on the Minicomputer now. We doubled 28 and then we doubled again. Let's double the number on the Minicomputer again. Who can do this for me?***

There were four checkers on each of these squares, and now there are eight.



N20

Draw another 2x arrow, and then draw a blue arrow from the starting dot of the first arrow to the ending dot of the last arrow.



T: ***How many times do you see 28 on the Minicomputer?*** (Eight)
What could the blue arrow be for?

S: ***8x.***

Label the blue arrow 8x. Trace the appropriate arrows as you say,

T: ***2x followed by 2x followed by 2x is the same as 8x.***

What number is on the Minicomputer? Write your answer on paper (or whisper it to a neighbor).

Check several students' answers before asking students to explain how they did the calculation.

S: ***I counted by twenties to find the number on the 20-square, and then I found the number on the 8-square by multiplying 8 x 8.***

Record the calculations on the board.

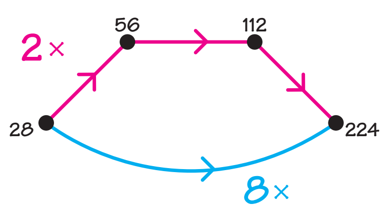
$$\begin{array}{r} 8 \times 20 = 160 \\ 8 \times 8 = 64 \\ \hline 8 \times 28 = 224 \end{array}$$

T: ***What number is 8 x 20?*** (160)
8 x 2 = 16, so 8 x 20 = 160.
What number is 8 x 8? (64)

8 x 20 = 160 and 8 x 8 = 64, so what number is 8 x 28?

S: ***8 x 28 = 224, because 160 + 64 = 224.***

S: ***I followed the arrows in the arrow picture.***
2 x 28 = 56, 2 x 56 = 112, and 2 x 112 = 224.



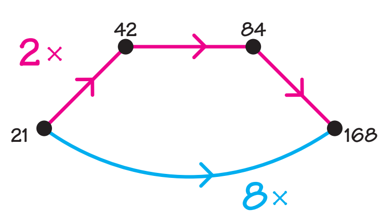
Label the dots in the arrow picture.

If no one suggests labeling the dots in the arrow picture to find the number on the Minicomputer, suggest this yourself. Conclude that the number on the Minicomputer is 224.

Remove the Minicomputer and erase everything on the board except the arrow picture. Label the starting dot of the first 2x arrow 21.

T: ***What number is 8 x 21? How do you know?***

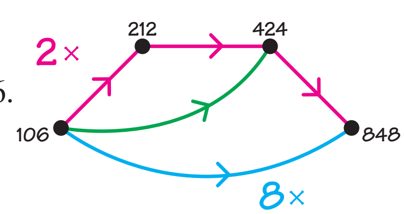
S: ***2 x 21 = 42, 2 x 42 = 84, and 2 x 84 = 168.***



Label the dots in the arrow picture.

S: ***8 x 20 = 160 and 8 x 1 = 8, so 8 x 21 = 160 + 8 = 168.***

Erase the labels for the dots and then label the starting dot 106.



T: ***What number is 8 x 106? How do you know?***

Encourage several students to explain how they determined that 8 x 106 = 848. If labeling the dots in the arrow picture is not suggested, suggest this yourself. Draw a green arrow from 106 to 424.

T: ***What could this green arrow be for?***

S: ***4x.***

Label the green arrow 4x, and trace the appropriate arrows as you say,

T: ***A 2x arrow followed by a 2x arrow is the same as a 4x arrow.***

Draw this table on the board near the arrow picture and instruct students to copy it on their papers. Direct them to enter numbers in the table as you do so on the board.

Start	2×	4×	8×
106			

T: ***The number in the first column is the starting number.***

Point to the dot for 106 in the arrow picture.

T: ***The number in the second column is two times the starting number. What number is 2 x 106?***

S: ***212.***

Trace the red arrow from 106 to 212 and write 212 in the second column.

T: ***The number in the third column is four times the starting number. What number is 4 x 106?***

S: ***424.***

Trace the green arrow from 106 to 424 and write 424 in the third column.

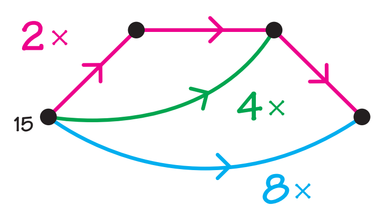
T: ***The number in the fourth column is eight times the starting number. What number is 8 x 106?***

S: ***848.***

Trace the blue arrow from 106 to 848 and write 848 in the fourth column.

Erase the labels for the dots. Label the starting dot 15 and write 15 in the first column of the table.

Start	2×	4×	8×
106	212	424	848
15			



T: ***What number is 8 x 15? Write your answer (or whisper it to a neighbor).***

N20

Check several students' answers before filling in the table. Conclude that 8 x 15 = 120. As you fill in the table, emphasize the doubling; that is, emphasize that the entry in the next column is 2x that in the preceding column.

Repeat this activity starting at 1,002; at 25; and at 51. Your table should look like this one.

Start	2×	4×	8×
106	212	424	848
15	30	60	120
1,002	2,004	4,008	8,016
25	50	100	200
51	102	204	408

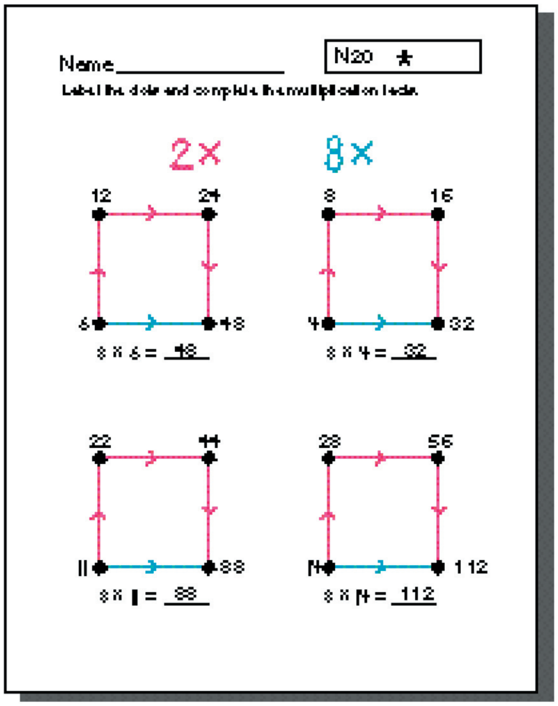
Worksheets N20*, **, ***, and **** are available for individual work.

Home Activity

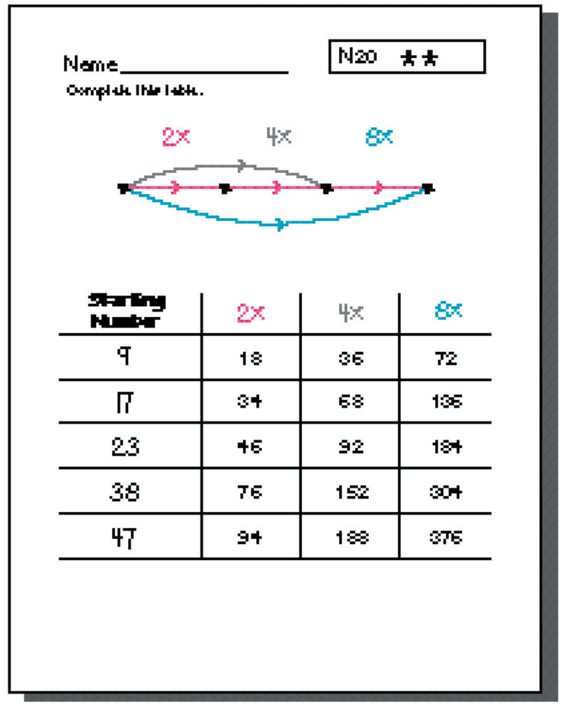
Pose the following question for parents/guardians to answer with their child.

> A recipe for cookies makes 15. If the recipe is doubled and doubled again, how many cookies will be made?

Draw an arrow road to show what happens. Can you double the recipe once more?

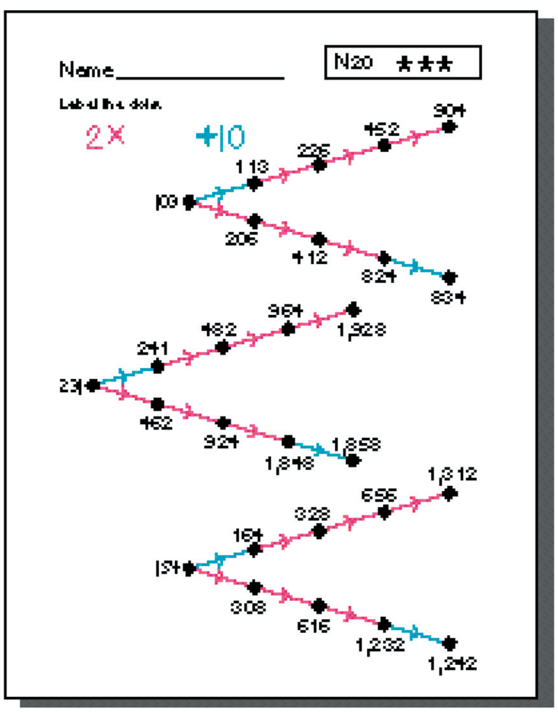

Name_______________ N20 ★

Label the dots and complete the multiplication facts.

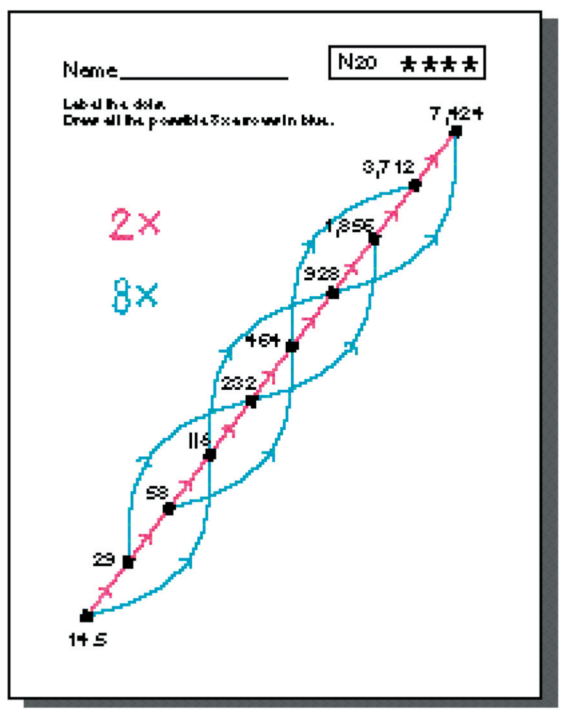

Name_______________ N20 ★★

Complete this table.

Starting Number	2×	4×	8×
9	18	36	72
17	34	68	136
23	46	92	184
38	76	152	304
47	94	188	376


Name_______________ N20 ★★★

Label the dots.


Name_______________ N20 ★★★★

Label the dots.
Draw all the possible 8× arrows in blue.

N21 MULTIPLICATION PROBLEMS #1

Capsule Lesson Summary

Put three checkers on the 4-square; then move the checkers to the 40-square, to the 400-square, and to the 4,000-square. Each time, ask what number is on the Minicomputer and record appropriate number sentences. Repeat this activity with other configurations. Multiply 3 x 148 by putting 148 on the Minicomputer three times and figuring out the number on each board, and also by computing 3 x (150 – 2). Multiply 2 x 257 by doubling the ones, the tens, and the hundreds in 257.

Materials

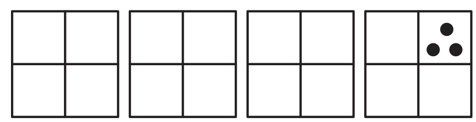
Teacher • Minicomputer set

Student • Paper

Description of Lesson

Write corresponding number sentences on the board as you do this activity.

Put this configuration on the Minicomputer.



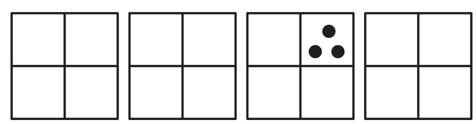
T: ***What calculation is on the Minicomputer?***

S: ***3 x 4.***

T: ***What number is 3 x 4?***

S: ***12.***

Move the checkers to the 40-square.



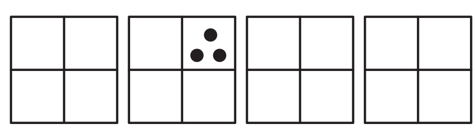
T: ***What calculation is on the Minicomputer?***

S: ***3 x 40.***

T: ***What number is 3 x 40?***

S: ***120.***

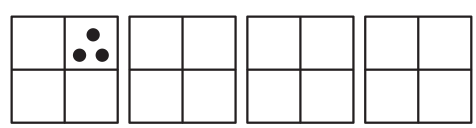
Move the checkers to the 400-square.



T: ***And now…?***

S: ***3 x 400; 3 x 400 = 1,200.***

Move the checkers to the 4,000-square.



T: ***And now…?***

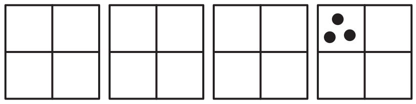
S: ***3 x 4,000; 3 x 4,000 = 12,000.***

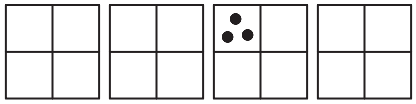
T: ***What patterns do you see in these number sentences?***

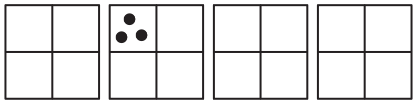
$$3 \times 4 = 12$$
$$3 \times 40 = 120$$
$$3 \times 400 = 1{,}200$$
$$3 \times 4{,}000 = 12{,}000$$

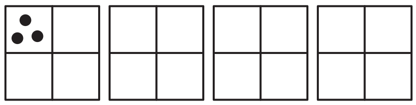
Let students make observations. Someone very likely will comment on the number of zeros.

Erase the board and repeat the activity with these configurations.

 $= 3 \times 8 = 24$

 $= 3 \times 80 = 240$

 $= 3 \times 800 = 2{,}400$

 $= 3 \times 8{,}000 = 24{,}000$

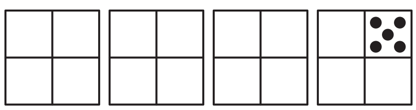
If students are unsure about any of these numbers, suggest that they do the corresponding addition problem on their papers. For example, 800 + 800 + 800 = 2,400 and 3 x 800 = 2,400.

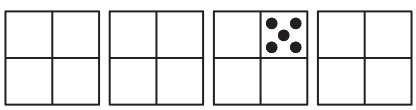
T: ***What number is 3 x 8,000,000?***

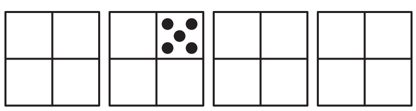
Invite a student to record the answer and to read it aloud.

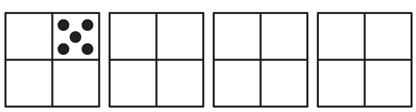
$$3 \times 8{,}000{,}000 = 24{,}000{,}000$$

Erase the board and repeat the activity with these configurations.

 $= 5 \times 4 = 20$

 $= 5 \times 40 = 200$

 $= 5 \times 400 = 2{,}000$

 $= 5 \times 4{,}000 = 20{,}000$

T: ***What number is 5 x 4,000,000?***

Invite a student to record the answer and to read it aloud.

$$5 \times 4{,}000{,}000 = 20{,}000{,}000$$

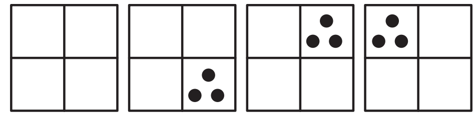
Erase the board and repeat the activity to generate another sequence of number sentences on the board.

$$3 \times 12 = 36$$
$$3 \times 120 = 360$$
$$3 \times 1{,}200 = 3{,}600$$

Continue this sequence with a couple more similar calculations.

$$3 \times 12{,}000 = 36{,}000$$
$$3 \times 12{,}000{,}000 = 36{,}000{,}000$$

Erase the board and put this configuration on the Minicomputer.



T: ***What calculation is on the Minicomputer?*** (3 x 148)
What number is this? Write it on your paper.

Look at many answers before discussing the problem collectively.

T: ***What number is on the hundreds board? How do you know?***

S: ***300, 3 x 100 = 300.***

T: ***What number is on the tens board? How do you know?***

S: ***120, because 40 + 40 = 80 and 80 + 40 = 120.***

T: ***What number is on the ones board? How do you know?***

S: ***24, because 3 x 8 = 24.***

T: ***What number is 3 x 148? How do you know?***

S: ***444, because 300 + 120 + 44 = 444.***

T: ***Did anyone calculate 3 x 148 another way?***

$$\begin{array}{r} 3 \times 100 = 300 \\ 3 \times 40 = 120 \\ \underline{3 \times 8 = 24} \\ 3 \times 148 = 444 \end{array}$$

Encourage students to explain their calculations. If no one suggests calculating 3 x (150 – 2), suggest this method yourself.

T: ***148 is almost 150.***

S: ***148 = 150 – 2.***

$$148 = 150 - ?$$

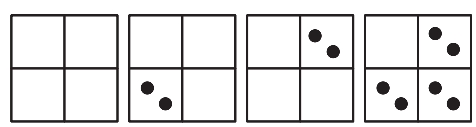
Write 3 x (150 – 2) on the board as you ask,

T: ***How can we complete this number sentence?***

S: ***3 x 150 = 450 and 450 – 6 = 444.***

$$3 \times (150 - 2) = 450 - 6 = 444$$

Erase the board and put this configuration on the Minicomputer.



T: ***What number is this? Write it on your paper.***

Look at many answers before asking students to answer aloud and to explain how they did the calculation. Write calculations following students' explanations on the board.

S: ***I added 247 + 247 and got 494.***

S: ***I calculated 2 x 7 = 14, 2 x 40 = 80, and 2 x 200 = 400. Then I added 14 + 80 + 400 = 494.***

T: ***Here is a shorter way to write your calcul***

$$\begin{array}{r} 247 \\ \underline{+\ 247} \\ 494 \end{array} \qquad \begin{array}{r} 2 \times 7 = 14 \\ 2 \times 40 = 80 \\ \underline{2 \times 200 = 400} \\ 2 \times 247 = 494 \end{array}$$

	What number is 2 x 7?
S:	***14.***
T:	***What number is 2 x 40?***
S:	***80.***
T:	***What number is 2 x 200?***
S:	***400.***
T:	***14 + 80 + 400 = 494, so 2 x 247 = 494.***
S:	***I calculated 2 x 250 = 500 and 500 – 6 = 494.***
T:	***You used the fact that 247 is 3 less than 250.***

$$\begin{array}{r} 247 \\ \times\quad 2 \\ \hline 14 \\ 80 \\ 400 \\ \hline 494 \end{array}$$

$$247 = 250 - 3$$
$$2 \times (250 - 3) = 500 - 6 = 494$$

Erase the board and then write this problem.

T: ***What number is 3 x 356? Do this calculation on your paper.***

$$\begin{array}{r} 356 \\ \times\quad 3 \\ \hline \end{array}$$

Observe the methods students are using before inviting them to explain how they did the problem. When appropriate, record their calculations on the board. For example:

$$\begin{array}{r} 356 \\ 356 \\ +\ 356 \\ \hline 1{,}068 \end{array} \qquad \begin{array}{r} 356 \\ \times\quad 3 \\ \hline 18 \\ 150 \\ 900 \\ \hline 1{,}068 \end{array}$$

Home Activity

This would be a good time to send home a review letter about the Minicomputer explaining how it supports work on multiplication by a single digit number. Blackline N21 has a sample letter.

N22 RETURN ARROWS #2

Capsule Lesson Summary

Perform sequences of calculations on a calculator and predict results before observing them. With a secret number displayed on the calculator, perform a sequence of operations one at a time. Record the sequence of operations in an arrow picture and use the picture to help discover the secret number.

Materials

Teacher
- Colored chalk
- Calculator (overhead)

Student
- Paper
- Colored pencils, pens, or crayons
- Calculator
- Worksheets N22*, **, ***, and ****

Description of Lesson

In Exercise 1, use an overhead calculator or a classroom calculator; in Exercise 2, every student or student pair should have a calculator, if possible.

Exercise 1

Tell the class that they are going to do mental calculations like a calculator. Display 10 on the overhead or classroom calculator and be sure everyone knows you are starting with 10 on the display. Then cover the display with your hand.

T: ***Do the calculations in your head, just as you think the calculator does the calculations when I press the keys:*** $\boxed{\times}$ $\boxed{3}$ $\boxed{+}$ $\boxed{6}$ $\boxed{=}$ ***.***

When you announce what keys you press, pause after each number key so students have an opportunity to do mental calculations.

T: ***What number should be on the display?*** (36)

Allow several students to answer before showing the class the calculator display. Be sure everyone knows that 36 is on the display and cover it again.

T: ***36 is on the display. I'll press*** $\boxed{-}$ $\boxed{1}$ $\boxed{0}$ $\boxed{\div}$ $\boxed{2}$ $\boxed{=}$ ***.***
What number should be on display? (13)

Allow several students to answer before showing the calculator display. Continue this activity with the following (or similar) sequences of calculations.

- Start with 13 on the display; press $\boxed{-}$ $\boxed{2}$ $\boxed{\times}$ $\boxed{3}$ $\boxed{=}$ (33)
- Start with 33 on the display; press $\boxed{+}$ $\boxed{4}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ (45)
- Start with 45 on the display; press $\boxed{-}$ $\boxed{5}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ (30)
- Start with 30 on the display; press $\boxed{\times}$ $\boxed{2}$ $\boxed{\div}$ $\boxed{1}\boxed{0}$ $\boxed{=}$ (6)

N22

Exercise 2

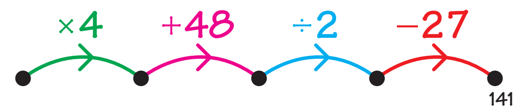
Invite a student to choose a secret number, a whole number between 50 and 100. Direct the student to write the secret number on a piece of paper, and then to put it on the display of the overhead or class calculator without revealing it to the other students. Ask the student to follow your instructions carefully. You may like to ask the rest of the class to copy the arrow picture recording the sequence of calculations as you do so on the board.

T: ***Press*** $\boxed{\times}$ $\boxed{4}$ ***.***

Now press $\boxed{+}$ $\boxed{4}\boxed{8}$ (read as "plus forty-eight")***.***

Press $\boxed{\div}$ $\boxed{2}$***.***

Finally, press $\boxed{-}$ $\boxed{2}\boxed{7}$ $\boxed{=}$ ***.***
What number is on the display?



Label the ending dot with the number given; for example, 141.

Distribute calculators and instruct students (possibly working with a partner) to try to figure out what the secret (starting) number is. They should write it on their papers. As students work on the problem, suggest that return (opposite) arrows might be helpful. Check papers to see if anyone knows the starting number (in this example, 72). Ask students to put the ending number (141 in this example) on the display of their calculators.

T: ***Let's label all the dots in this arrow picture with the help of our calculators.***

Trace the return (opposite) arrow for the –27 arrow.

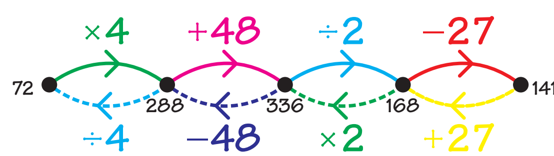
T: ***What is the return (opposite) of –27?*** (+27)

Draw and label a +27 arrow.

T: ***Press*** $\boxed{+}$ $\boxed{2}\boxed{7}$ $\boxed{=}$***. What number is on the display?***

S: ***168.***

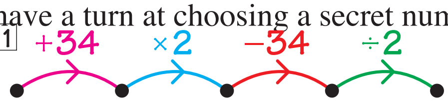
Continue this activity until all the return arrows are drawn and all the dots are labeled.



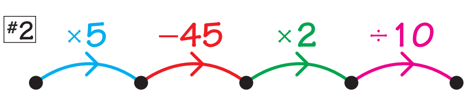
Check that the starting (secret) number is the same as the number written on the student's paper.

Erase the board and repeat this activity with these sequences of calculations. Let several students have a turn at choosing a secret number.

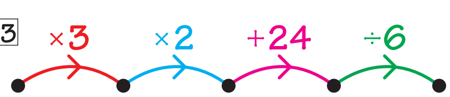
#1



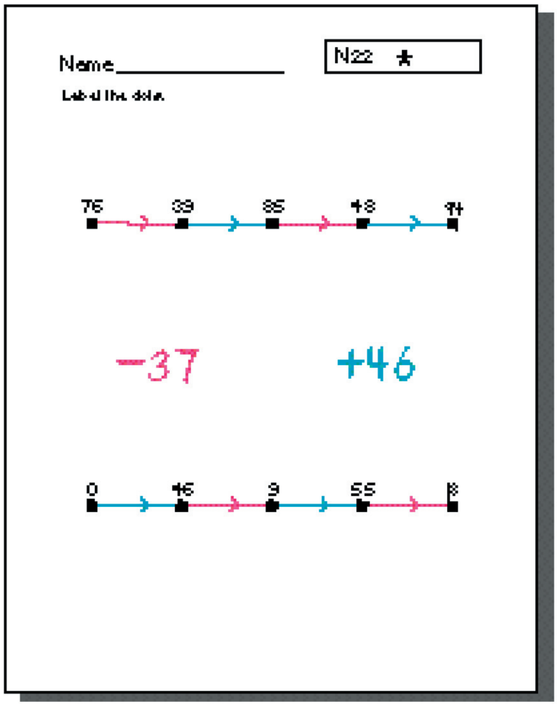
#2

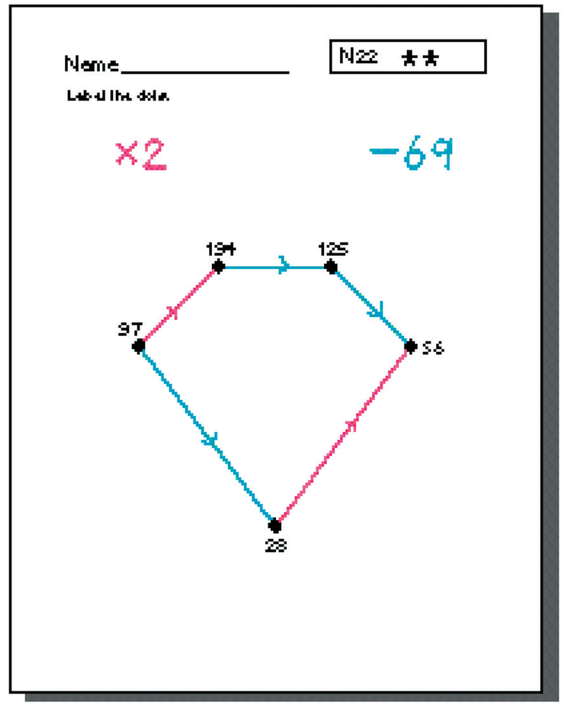


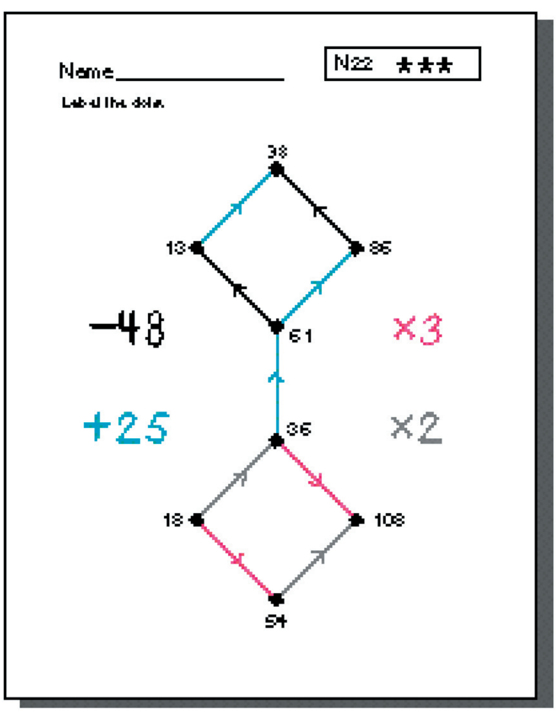
#3

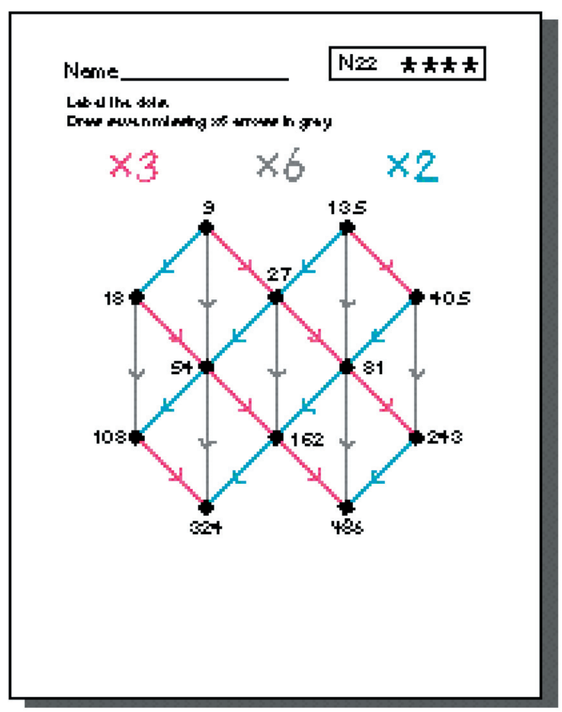


Worksheets N22*, **, ***, and **** are available for individual work. Allow students to use calculators while doing these worksheets and encourage them to think about using return arrows.


Name
N22 ★
Label the dots.
76
39
85
48
94
−37
+46
0
46
9
55


Name
N22 ★★
Label the dots.
×2
−69
194
125
97
56
28


Name
N22 ★★★
Label the dots.
38
13
86
61
−48
×3
+25
×2
36
18
108
54


Name
N22 ★★★★
Label the dots.
Draw every missing ×6 arrow in grey.
×3
×6
×2
9
13.5
27
18
40.5
54
81
108
162
243
324
486

N23 COMPARING PRICES #1

Capsule Lesson Summary

Compare prices of items packaged in different quantities. Then, find the cost of a minimal number of the items that can be purchased with either packaging.

Materials

Teacher
- Colored chalk
- Popsicle sticks and rubber bands (optional)

Student
- Paper
- Colored pencils, pens, or crayons

Description of Lesson

You may like to let students work in groups during this lesson. You may also want to use popsicle sticks and rubber bands to model the packaging in each story.

Exercise 1

Choose one of your students to be the star of this or a similar story.

T: ***Patty is going to buy a lot of paintbrushes. She goes to two stores to price paintbrushes. The two stores sell exactly the same paintbrushes, but one store sells a package of three paintbrushes for 40¢ and the other store sells a package of two paintbrushes for 24¢.***

Record this information on the board.

Store #1   3 brushes for 40¢
Store #2   2 brushes for 24¢

T: ***Patty wants to know which store offers the better price. Where should she buy the paintbrushes?***

Let students discuss this problem in their groups. Very likely some students will observe that brushes are cheaper in the second store, because there each paintbrush costs 12¢ and so three paintbrushes cost 36¢. Three paintbrushes at the first store cost 40¢.

Perhaps some students will object that in the second store you must buy the paintbrushes in packages of two, so you cannot buy exactly three paint brushes there. If necessary, mention this yourself.

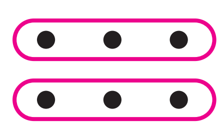
T: ***Could you buy the same number of paintbrushes at both stores?*** (Yes)
What is the least such number?

S: ***Six.***

T: ***To get six brushes from the first store, how many packages would she buy?***

S: ***Two packages, because there are three brushes in each package.***

Draw this picture on the board.



T: ***How much would it cost to buy six paintbrushes from the first store? How do you know?***

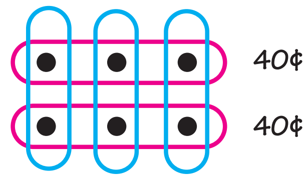
S: ***80¢, because each package costs 40¢ and 2 x 40 = 80.***

Record this information on the board.

T: ***To get six paintbrushes from the second store, how many packages would she buy?***

S: ***Three packages, because there are only two paintbrushes in each package at that store.***

Draw blue strings around pairs of dots on the board.

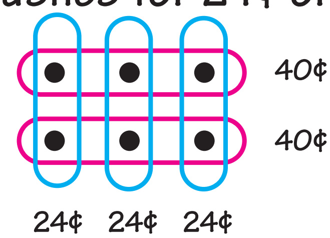


T: ***How much would it cost to buy six paintbrushes from the second store? How do you know?***

S: ***72¢, because each package costs 24¢ and 24 + 24 + 24 = 72.***

Record this information on the board.

Store #1 3 brushes for 40¢ or 6 brushes for 80¢
Store #2 2 brushes for 24¢ or 6 brushes for 72¢



T: ***Which store has the best price if Patty is buying six paintbrushes?***

S: ***The second store.***

T: ***How much will she save if she buys six paintbrushes at the second store instead of at the first store?***

S: ***8¢.***

T: ***Patty wants to buy a lot of paintbrushes. Which store offers the better deal? Why?***

S: ***The second store, because the price (per brush) is lower there.***

T: ***When would you ever buy paintbrushes from the first store?***

S: ***…if I only needed three brushes.***

S: ***…if I had more than 40¢ but less than 48¢.***

S: ***If I wanted to buy five brushes, I would buy three from one store and two from the other.***

Erase the board before going on to Exercise 2.

Exercise 2

Choose one of your students to be the star of this or a similar story.

T: ***Matthew wants to order scissors for the craft center. He looks at two catalogs and finds their prices for scissors. Both catalogs offer exactly the same scissors, but one catalog charges $1.20 for a package of five scissors and the other catalog charges $2.10 for a package of eight scissors.***

Record this information on the board.

Catalog #1 5 scissors for $1.20
Catalog #2 8 scissors for $2.10

T: ***Which catalog offers the best price?***
From which catalog should Matthew order the scissors?

Let students discuss this problem in their groups. Students may not be able to figure out the price of one pair of scissors.

T: ***The first catalog sells the scissors in packages of five and the second catalog sells them in packages of eight. Could Matthew buy the same number of scissors from both catalogs?*** (Yes) ***What is the least such number?*** (40)

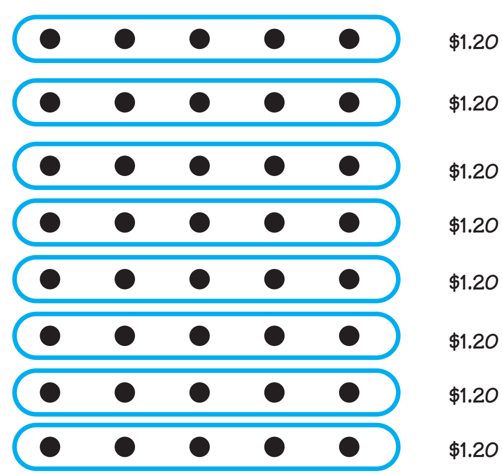
If necessary, encourage students to think of numbers that are multiples of 5 and multiples of 8. You might suggest they list the quantities he could buy from each catalog; i.e., list the multiples of 5 and of 8. Then look for quantities Matthew could buy from both.

Catalog #1: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, ...
Catalog #2: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, ...

T: ***To get 40 scissors, how many packages would he order from the first catalog?***

S: ***Eight packages of five scissors.***

Draw this picture on the board.



T: ***How much would 40 scissors cost if he orders them from the first catalog? How do you know?***

S: ***$9.60. Five scissors cost $1.20, so I calculated 8 x $1.20. 8 x $1 = $8 and 8 x 20¢ = $1.60; $8 + $1.60 = $9.60.***

Record this information on the board.

T: ***To get 40 scissors, how many packages would he orders from the second catalog?***

S: ***Five packages of eight scissors.***

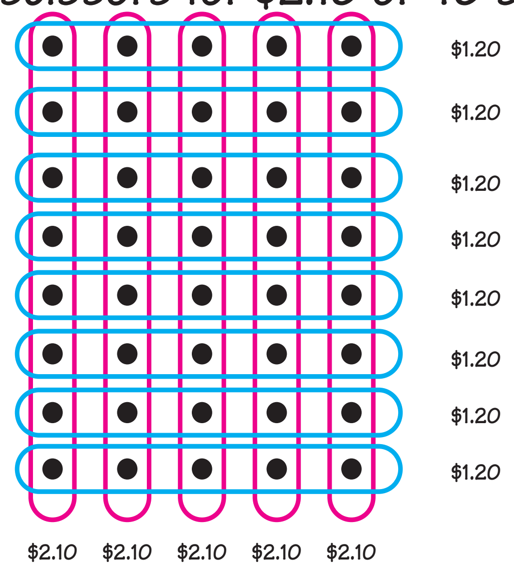
Draw red strings around groups of eight dots. (See the next illustration.)

T: ***How much would 40 scissors cost if he orders them from the second catalog? How do you know?***

S: ***$10.50. He would buy five packages of eight scissors, so I calculated 5 x $2.10; 5 x $2 = $10 and 5 x 10¢ = 50¢; $10 + 50¢ = $10.50.***

Record this information on the board.

Catalog #1 5 scissors for $1.20 or 40 scissors for $9.60
Catalog #2 8 scissors for $2.10 or 40 scissors for $10.50



T: ***Which catalog offers the better price if Matthew buys 40 pairs of scissors?*** (Catalog#1)
How much would he save if he orders 40 scissors from the first catalog instead of from the second catalog? (90¢)
From which catalog should Matthew order the scissors for the craft center? Why?

S: ***The first catalog, because its price (per pair of scissors) is lower.***

S: ***But if he needs just eight scissors, he should buy them from the second catalog.***

S: ***If he needs 13 scissors, he should buy five from the first catalog and eight from the second.***

Erase the board before going on to Exercise 3.

Exercise 3

Choose one of your students to be the star of this or a similar story.

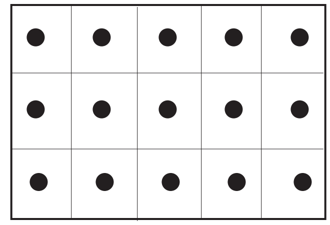
T: ***Rusty often goes to see movies at the city library. The library sells cards good for three admissions and cards good for five admissions. Each time Rusty goes to the library to see a movie, the librarian punches his card. Since he goes so often, Rusty would like to know which card is a better deal. A three-punch card costs 80¢ and a five-punch card costs $1.30.***

Record this information on the board.

3 admissions for $0.80
5 admissions for $1.30

Direct students to try to solve this problem for Rusty in their groups.

As you observe group work, you may like to make manipulatives (sticks and rubber bands, or small cards to punch) available to them. After a few minutes, show the class the paper of a group that has drawn a three by five array.



T: ***How many dots did this group draw on their paper?*** (Fifteen) ***How could these dots help solve Rusty's problem?***

S: ***We can find out how much it costs for fifteen admissions with each type of card.***

3 admissions for $0.80......15 admissions for ______
5 admissions for $1.30 ....15 admissions for ______

Allow the groups to continue working on this problem for a few more minutes, and then ask students to explain how they calculated the price of fifteen admissions with the two types of cards. Illustrate the students' calculations on the board with a dot picture. A possible dialogue is given here.

S: ***It costs $4.00 to buy 15 admissions with three-punch cards.***

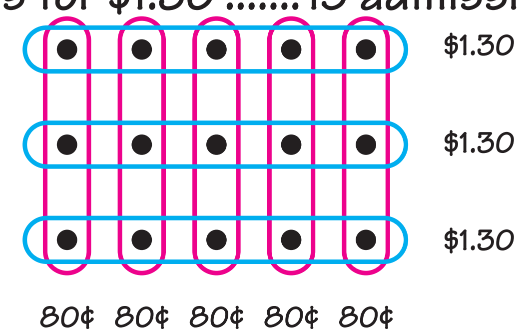
T: ***How do you know?***

S: ***Five three-punch cards are good for 15 admissions and 5 x $0.80 = $4.00.***

T: ***How much does it cost to buy 15 admissions with the five-punch cards? How do you know?***

S: ***$3.90, because three five-punch cards are good for 15 admissions and 3 x $1.30 = $3.90.***

3 admissions for $0.80......15 admissions for $4.00
5 admissions for $1.30 ....15 admissions for $3.90



T: ***Which card is the better deal?***

S: ***The five-punch card.***

T: ***When would you ever buy a three-punch card instead of a five-punch card?***

S: ***…if I only wanted to go to the movies a couple of times.***

S: ***…if I only had enough money for a three-punch card.***

S: ***If I wanted to go to the movie six times, I would buy two three-punch cards.***

Erase the board before going on to Exercise 4.

Exercise 4

You may like to use this exercise as a group assessment activity.

T: ***Carolyn frequently buys pencils for school. At her favorite store, they sell four pencils for 13¢ or six pencils for 20¢. Carolyn would like to know which is the better deal.***

Write this information on the board.

4 pencils for 13¢
6 pencils for 20¢

Direct students to try to solve this problem in their groups. Make manipulatives (sticks or pencils and rubber bands) available to groups.

Note: You may like to observe whether groups try to find the cost of 12 or 24 (or some other multiple of 12) pencils at each price. However, they should conclude that four pencils for 13¢ is the better deal.

4 pencils for 13¢ .... 12 pencils for 39¢
6 pencils for 20¢...... 12 pencils for 40¢

Home Activity

Send home a problem similar to one in this lesson for parents/guardians to work on with their child.

N24 COMPOSITION OF FUNCTIONS #3

Capsule Lesson Summary

With the support of the Minicomputer, decide that the composition 10x followed by $\frac{1}{2}$x is 5x. Draw a corresponding arrow picture. Do several 5x calculations using the method suggested by the arrow picture. In a more elaborate arrow picture with 10x and $\frac{1}{2}$x arrows, label the dots and draw 5x arrows.

Materials

Teacher
- Minicomputer set
- Colored chalk

Student
- Paper
- Worksheets N24 (no star), *, **, and ***
- Colored pencils, pens, or crayons

Description of Lesson

Exercise 1

Begin the lesson with a short mental arithmetic activity involving the function 10x. Write these problems on the board, asking for and recording the answer to a problem before posing the next. Answers are in boxes.

$10 \times 10 = \boxed{100}$	$10 \times 59 = \boxed{590}$
$10 \times 12 = \boxed{120}$	$10 \times 73 = \boxed{730}$
$10 \times 17 = \boxed{170}$	$10 \times 90 = \boxed{900}$
$10 \times 25 = \boxed{250}$	$10 \times 89 = \boxed{890}$
$10 \times 48 = \boxed{480}$	$10 \times 146 = \boxed{1{,}460}$

Continue this activity with the following calculations. Suggest students write their answers on paper before completing the number sentence on the board. Answers are in boxes.

$10 \times 203 = \boxed{2{,}030}$	$10 \times 465 = \boxed{4{,}650}$
$10 \times 300 = \boxed{3{,}000}$	$10 \times 1{,}697 = \boxed{16{,}970}$

T: ***How do you calculate 10x a number so quickly? Do you know a shortcut?***

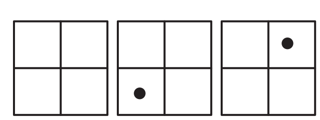
S: ***When you multiply a number by 10, all you do is put 0 to the right of the number.*** †

S: ***When you calculate 10x a number, you just move all the digits over a place (to the left).***

Erase the board before going on to Exercise 2.

Exercise 2

Call on a student to put 24 on the Minicomputer.

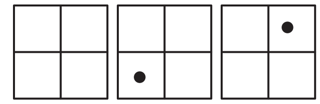


T: ***How could we put 10 x 24 on the Minicomputer?***

† This is true only in the case of integers, because $10 \times 0.37 \neq 0.370$ and $2.5 \times 10 \neq 2.50$.

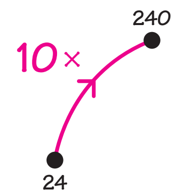
N24

Accept correct suggestions and then put this configuration on the Minicomputer.

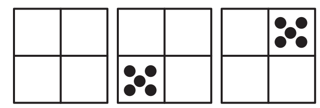


T: ***Is this number 10 x 24?*** (Yes)
What number is 10 x 24? (240)

I'm going to show that we multiplied 24 by 10 in an arrow picture.



By removing some checkers from the Minicomputer, how can we show one-half of the number?



S: ***Take off half of the checkers; that is, take off five checkers from each group of ten checkers.***

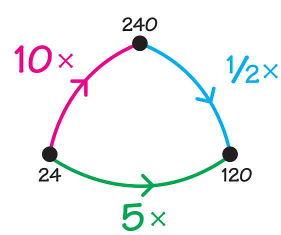
Add a $^1/_2$x arrow to the picture

T: ***What number is on the Minicomputer? How do you know?***

S: ***120, because 5 x 20 = 100 and 5 x 4 = 20.***

S: ***I know it's 120 because $^1/_2$ x 240 = 120.***

Label the dot for 120 and draw a green arrow from 24 to 120.



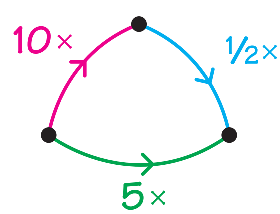
T: ***What could this green arrow be for? How do you know?***

S: ***5x, because there are five 24's on the Minicomputer.***

Label the green arrow 5x and trace the appropriate arrows as you say,

T: ***10x followed by $^1/_2$x is the same as 5x .***

Erase the dot labels. Point to the starting dot of the green arrow.



T: ***What number is 5 x 14? How do you know?***

S: ***Calculate 10 x 14 and then $^1/_2$x that number.***

T: ***What number is 10 x 14?*** (140)
So what number is 5 x 14? (70)
10 x 14 = 140 and $^1/_2$ x 140 = 70, so 5 x 14 = 70.
What number is 5 x 26?

Suggest students write their answers on paper before asking a student to answer aloud.

T: ***How do you know that 5 x 26 = 130?***

S: ***10 x 26 = 260; $^1/_2$ x 200 = 100 and $^1/_2$ x 60 = 30; so $^1/_2$ x 260 = 130.***

Continue this activity with these calculations.

5 x 12 (60)	5 x 42 (210)
5 x 45 (225)	5 x 81 (405)

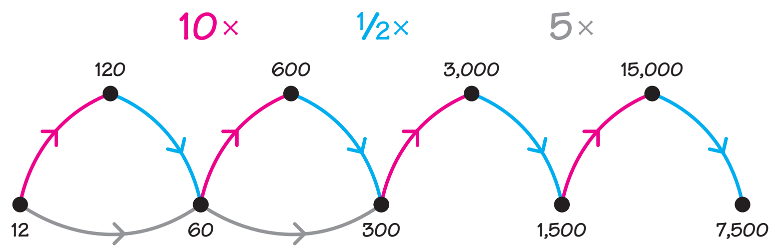
Erase the board and distribute copies of Worksheet N24 (no star). Copy the arrow picture from the worksheet onto the board.

T: ***Where could we draw 5x arrows in this arrow picture?***

Invite students to trace arrows and, if correct, draw them in gray. Encourage all students to draw the same 5x arrows on their worksheets.

After a couple 5x arrows have been drawn in the picture, label the starting dot 12; then ask students to label all the dots on their worksheets. Students who finish quickly should draw all the possible 5x arrows.

After several minutes of individual work, invite students to label the dots in the arrow picture on the board and to explain how they did the calculations. Continue until all the dots are labeled. Perhaps your arrow picture will look like this one.



T: ***Where could we draw some more 5x arrows in this arrow picture?***

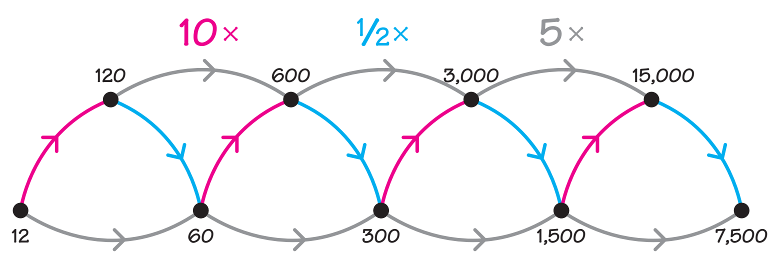
When a 5x arrow is drawn in the lower part of the picture, trace the appropriate arrows as you say,

T: ***10x followed by $^1/_2$x is the same as 5x.***

When a 5x arrow is drawn in the upper part of the picture, trace the appropriate arrows as you say,

T: ***$^1/_2$x followed by 10x arrow is the same as 5x.***

Continue until all the possible 5x arrows have been drawn.



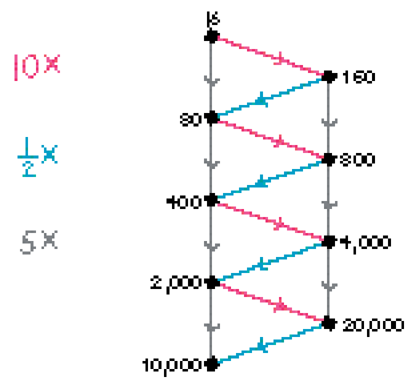
Worksheets N24 *, **, and *** are available for individual work.

Note: If students have trouble with the chart format on Worksheet N24 **, review it with small groups of students who are ready to begin the worksheet.

N24

Name________________ N24 ★

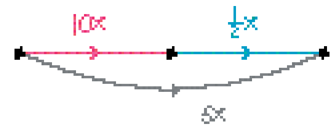
Label the dots.
Draw all the possible 5x arrows in gray.



You should find seven 5x arrows.

Name________________ N24 ★★

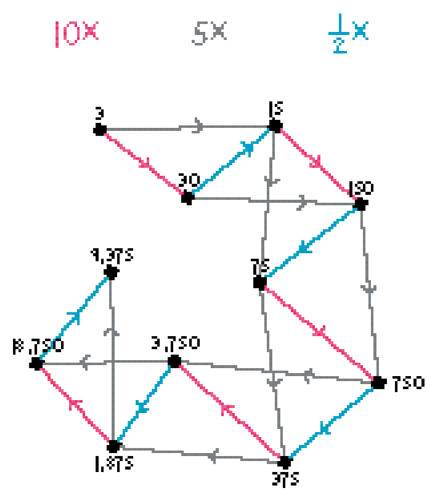
Complete the table.



Starting Number	10x	5x
25	250	125
82	820	410
41	410	205
63	630	315
85	850	425
94	940	470

Name________________ N24 ★★★

Draw all the possible 10x, 5x, and ½x arrows.



N25 MINICOMPUTER GOLF #1

Capsule Lesson Summary

Introduce a version of the Minicomputer Golf game in which checkers are moved from a starting configuration on the Minicomputer in order to reach a specified goal.

Materials

Teacher		Student	
	• Minicomputer set • Colored chalk		• Paper • Colored pencils, pens, or crayons

Description of Lesson

Begin this lesson with some discussion about the game of golf.

T: ***What do you know about playing golf?***

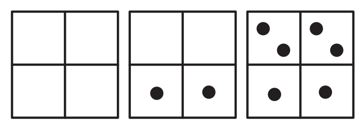
S: ***You play with a ball and clubs.***

S: ***You try to get the ball in a hole.***

S: ***Sometimes you drive the ball a long distance; sometimes you putt.***

T: ***We are going to learn a game called Minicomputer Golf.***

Put this configuration on the Minicomputer.



T: ***What number is this?***

S: ***57.***

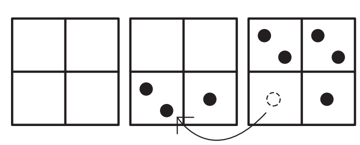
T: ***Our goal is to reach 200. When we get exactly 200 on the Minicomputer, it is like getting the ball in the hole in golf.***

Draw and label a dot for 57 and another dot for 200.

T: ***Do we need to make the number on the Minicomputer more or less?*** (More)
The way we play in this game is to move a checker. We cannot put on more checkers or take off checkers.

Invite a student to move exactly one checker from any square to another square of the Minicomputer. After moving a checker, ask the student how much more or less the new number is. You may like to require students to be able to tell how much change was made, and otherwise make a different move. Also, you may require that when the number on the Minicomputer is less than the goal, a move must increase the number, or vice-versa. Continue in this way until the goal is reached. The move that reaches the goal is the winning move. A sample game is described below:

The first volunteer moves a checker from the 2-square to the 20-square.



T: ***Did your move make the number more or less than before?***

S: ***More.***

T: ***How much more?***

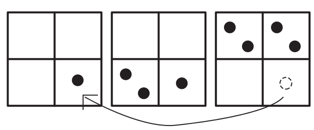
S: ***18 more.***

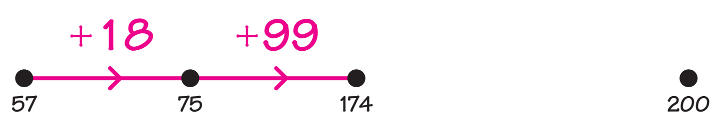
Draw a +18 arrow starting at 57 to record the increase.

T: ***What number is 57 + 18?*** (75)

Some students might look at the Minicomputer to calculate the number (75); others might do the addition mentally or on paper. If necessary, write the addition problem on the board and solve it collectively. Label the dot for 75.

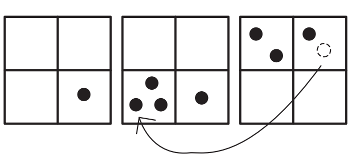
The next volunteer moves a checker from the 1-square to the 100-square and tells the class that the number is now 99 greater. The class calculates 75 + 99 = 174 to find that the new number on the Minicomputer is 174.

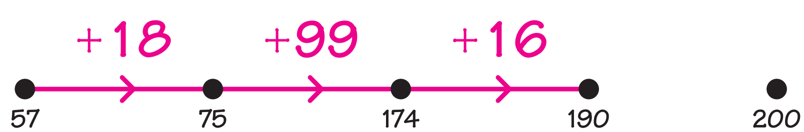




T: ***Do we need to make the number on the Minicomputer more or less?*** (More)
How much more …? (26)

A student moves a checker from the 4-square to the 20-square and tells the class that the number is now 16 greater. The class calculates 174 + 16 = 190 to find that the new number on the Minicomputer is 190.

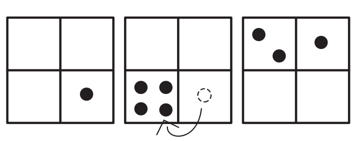


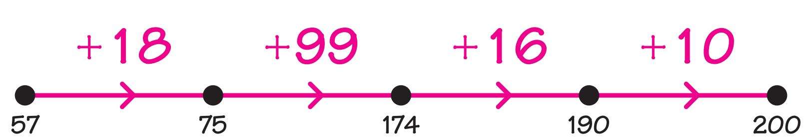


T: ***What do we need to do to get 200 on the Minicomputer?***

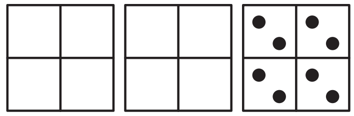
S: ***Move a checker to increase by 10; 190 + 10 = 200.***

A student moves a checker from the 10-square to the 20-square and the goal is reached.





Play the game again, but start with a different configuration and possibly a different goal. For example:



= 30

You may like to challenge the class to reach the goal with as few moves as possible.

At this time, if you play other games of Minicomputer Golf, we suggest that you start with any number representable by eight to ten positive checkers on the Minicomputer and choose numbers less than 1,000 as your goal.

Writing Activity

Write a letter to an absent classmate explaining the Minicomputer Golf game.

Capsule Lesson Summary

Do several 10x calculations on the Minicomputer using ⑩-checkers. Individually practice other 10x calculations. Build an arrow road between 5 and 29 using 10x, +1, and –1 arrows, and with fewer than ten arrows.

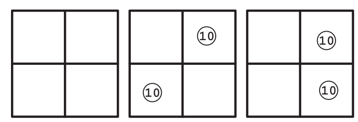
Materials

Teacher
- Minicomputer set
- ⑩-checkers
- Colored chalk

Student
- Paper
- Colored pencils, pens, or crayons
- Worksheets N26 *, **, ***, and ****

Description of Lesson

Put this configuration on the Minicomputer.



T: ***What calculation is on the Minicomputer?***

S: ***10 x 65.***

T: ***What number is 10 x 65?***

$10 \times 65 = 650$ or $\begin{array}{r} 65 \\ \times\ 10 \\ \hline 650 \end{array}$

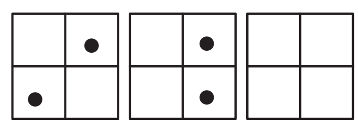
S: ***650.***

T: ***How do you know?***

S: ***When you multiply a number by 10, the answer ends in zero. Just put 0 on 65.***†

S: ***When you multiply a number by 10, the digits move over a place.***

Invite a student to make trades on the Minicomputer to put the number in standard configuration.



= 650

T: ***The ones board has no checkers on it, so the ones digit in this number is 0.***

Continue this activity with several more 10x calculations, such as the following:

10 x 93 (930)   10 x 71 (710)   10 x 208 (2,080)   10 x 280 (2,800)

Write the following calculations on the board, and ask students to copy and solve them on their papers.

10 x 35	10 x 300	10 x 24	10 x 200
10 x 305	10 x 3,005	10 x 204	10 x 2,004
10 x 350	10 x 3,000	10 x 240	10 x 2,400

†This comment is true for the integers; however, it is not true in general. For example, 10 x 8.8 = 88, not 8.80.

N26

When a few students have solved all the problems, begin letting students complete and read number sentences on the board. Emphasize that commas sometimes make numbers easier to read, and that a comma is put between the thousands and the hundreds digit.

Erase the board. Then draw this picture and direct students to copy it on their papers.

T: ***Build an arrow road between 5 and 29 using these kinds of arrows: 10x or +1 or –1. Try to use less than ten arrows.***

As you observe students' work, help those who have trouble getting started. Students who finish quickly may begin working on Worksheets N26 * and **. Students who build a road with ten or more arrows should try again. After a few minutes, point to the picture on the board.

T: ***Which multiple of 10 is closest to 29?***

S: ***30.***

Draw a dot for 30 near the dot for 29.

T: ***Will it be easy for us to draw arrows between 30 and 29?***

S: ***Yes, all we need is a –1 arrow, because 30 – 1 = 29.***

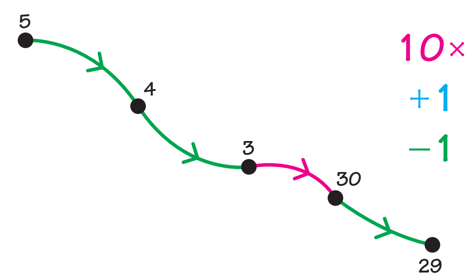
T: ***How can we build an arrow road between 5 and 30?***

S: ***We can go from 5 to 3 and then from 3 to 30.***

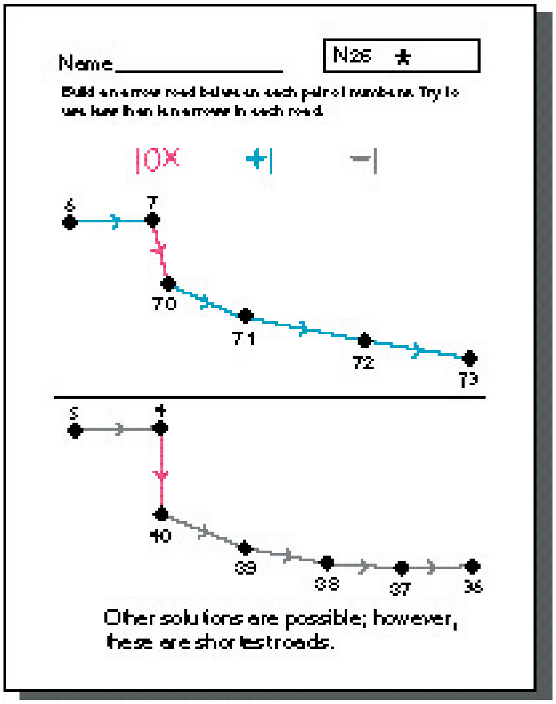
T: ***Which arrows should we use?***

S: ***Use two –1 arrows from 5 to 3, and then use a 10x arrow from 3 to 30.***

Do not draw the arrow road on the board at this time, but help students having trouble getting started by telling them to draw a dot for 30 on their papers and to follow the suggestions that were given by other students. When most everyone has an arrow road between 5 and 30, call on a student to put a solution on the board.



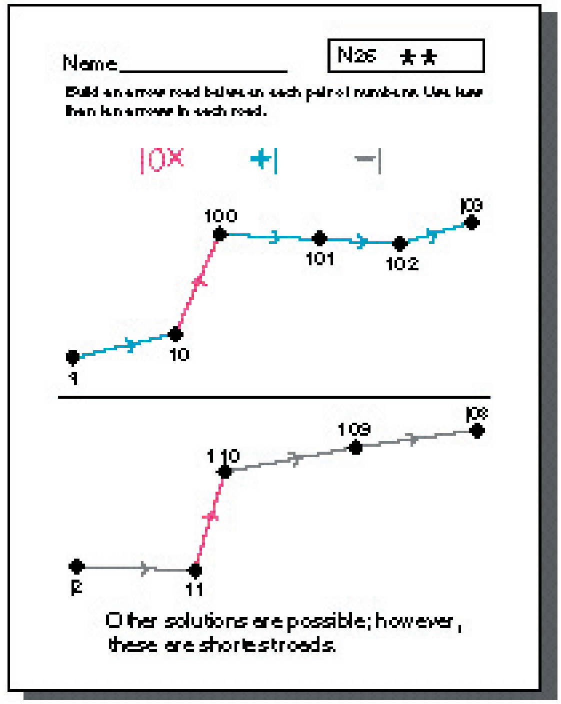
Worksheets N26*, **, ***, and **** are available for individual work. Emphasize to students that they are to use less than ten arrows to build each road. If a student has difficulty with the * or ** problems, you may wish to give a hint by asking which multiple of 10 is closest to the greater of the two numbers in the problem. Suggest that the student draw a dot for the closest multiple of 10 and then build an arrow road from the smaller number to that number.



Name_________________ N26 ★

Build an arrow road between each pair of numbers. Try to use fewer than ten arrows in each road.

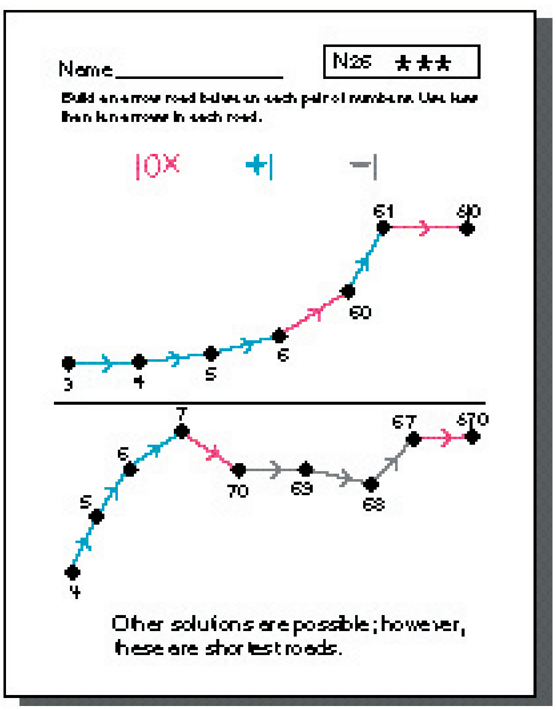
Other solutions are possible; however, these are shortest roads.



Name_________________ N26 ★★

Build an arrow road between each pair of numbers. Use fewer than ten arrows in each road.

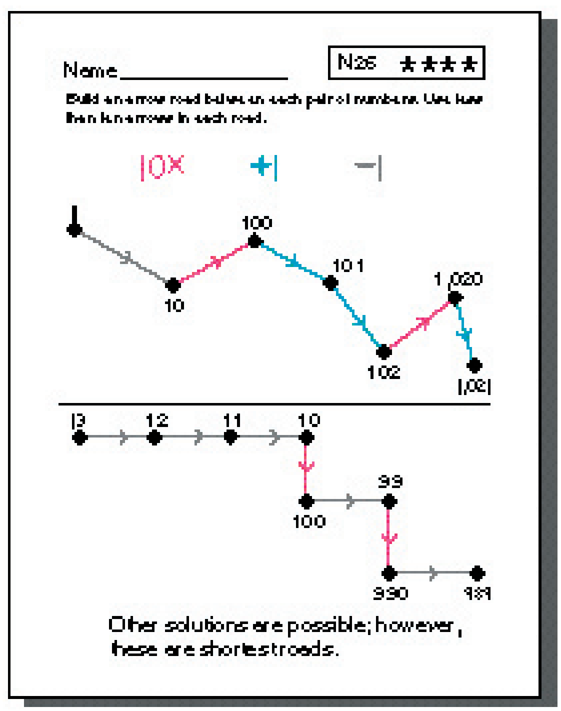
Other solutions are possible; however, these are shortest roads.



Name_________________ N26 ★★★

Build an arrow road between each pair of numbers. Use fewer than ten arrows in each road.

Other solutions are possible; however, these are shortest roads.



Name_________________ N26 ★★★★

Build an arrow road between each pair of numbers. Use fewer than ten arrows in each road.

Other solutions are possible; however, these are shortest roads.

N27 MYSTERY ARROWS

Capsule Lesson Summary

Do some multiplication calculations on the Minicomputer using positive and negative checkers; for example, 2 x 38 = 2 x (40 – 2) = 2 x (40 + $\hat{2}$) = 80 + $\hat{4}$ = 76. Find possible labels for the two arrows in an arrow road given its starting and ending numbers.

Materials

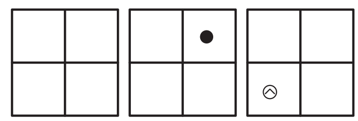
Teacher
- Minicomputer set
- Colored chalk

Student
- Paper
- Colored pencils, pens, or crayons

Description of Lesson

Exercise 1

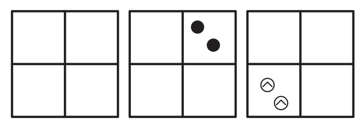
Put this configuration on the Minicomputer.



T: ***What number is this? How do you know?***

S: ***38, because 40 – 2 = 38.***

Double the number on the Minicomputer.

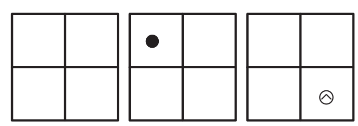


T: ***What number is 2 x 38? How did you know?***

S: ***76. I calculated 80 – 4 = 76.***

If necessary, point out 80 and $\hat{4}$ on the Minicomputer.

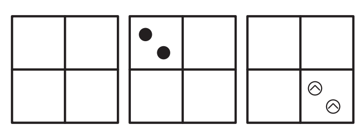
Put this configuration on the Minicomputer.



T: ***What number is this? How do you know?***

S: ***79, because 80 – 1 = 79.***

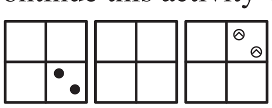
Double the number on the Minicomputer.

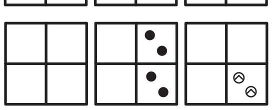


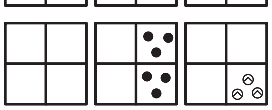
T: ***What number is 2 x 79? How do you know?***

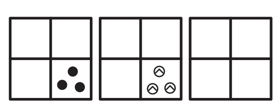
S: ***2 x 80 = 160 and 160 – 2 = 158, so 2 x 79 = 158.***

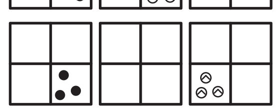
Continue this activity with these configurations.

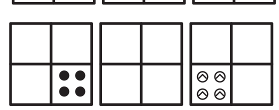
 = 2 × 96 = 192

 = 2 × 49 = 98

 = 3 × 49 = 147

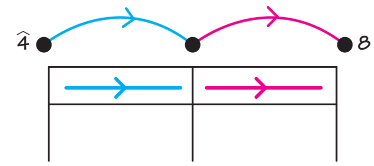
 = 3 × 90 = 270

 = 3 × 98 = 294

 = 4 × 98 = 392

Exercise 2

You may like to let students work in pairs during this exercise.



Draw this picture on the board and instruct students to copy it on their papers. Ask students (pairs) to find possible labels for the arrows. As you observe students' work, you may occasionally want to give a student a label for one of the arrows and ask the student to label the other arrow. After about five minutes, invite students to share their findings with the class. Record appropriate addition, subtraction, multiplication, and division relations in the table. If students correctly label the arrows with relations such as *is less than*, *is greater than*, *is a multiple of*, and *is a divisor of*, accept these as correct suggestions but do not record them in the table. A sample dialogue follows.

T: ***In this arrow picture, what could the blue arrow and the red arrow be for?***

S: ***The blue arrow could be for +4.***

T: ***If the blue arrow were for +4, what could the red arrow be for?***

S: ***+8.***

$\rightarrow$	$\rightarrow$
+4	+8

T: ***$\widehat{4} + 4 = 0$ and $0 + 8 = 8$. Is this the only solution?*** (No)

S: ***The blue arrow could be +8 and the red arrow +4.***

T: ***$\widehat{4} + 8 = 4$ and $4 + 4 = 8$. Are there other solutions?*** (Yes)

S: ***The blue arrow could be +8 and the red arrow 2x.***

T: ***$\widehat{4} + 8 = 4$ and $2 \times 4 = 8$.***

$\rightarrow$	$\rightarrow$
+4	+8
+8	+4 or 2x

Select a student's paper and begin to read other solutions.

T: ***Melinda says the blue arrow could be for +6.***
In this case, what could the red arrow be for? (+6)
$\widehat{4} + 6 = \ldots$? (2) ***And $2 + 6 = \ldots$?*** (8)
Is there another possibility for the red arrow when the blue arrow is +6? (4x)

Jerome says the blue arrow could be +20.
What number is $\widehat{4} + 20$? (16)
In this case, what could the red arrow be? (–8) ***16 – 8 = 8.***

S: ***The red arrow could also be $^1/_2$x (÷2) because $^1/_2 \times 16 = 8$.***

$\rightarrow$	$\rightarrow$
+4	+8
+8	+4 or 2x
+6	+6 or 4x
+20	–8 or ½x

T: ***Sheila thinks the red arrow could be +2. Is this possible?***
What would the blue arrow be?

S: ***The middle dot would be for 6, so the blue arrow could be +10.***

T: ***Anthony is stuck. Let's see if we can help him.***
Anthony's blue arrow is +30, but he can't figure out what the red arrow is. Do you know?
Write a solution on your paper.

T: ***What number is $\widehat{4} + 30$?*** (26)

Now we need to go from 26 to 8.

S: *We could subtract 18.*

Note: Students might suggest subtraction or multiplication relations for the blue arrow, although it is unlikely, since subtracting or multiplying $\widehat{4}$ by whole numbers gives numbers less than $\widehat{4}$ and they are trying to get to 8. Students are unlikely to suggest division relations simply because they have no experience with division involving a negative number.

If there is time, you may present students (pairs) with another mystery arrows problem. For example:

+4	+8
+8	+4 or 2x
+6	+6 or 4x
+20	−8 or ½x
+10	+2
+30	−18

Home Activity

Suggest a problem similar to that in Exercise 2 for students to work on at home with a family member.

Capsule Lesson Summary

Solve division problems suggested by story situations. Use string pictures to find and record solutions.

Materials

Teacher	• Colored chalk	**Student**	• Paper • Colored pencils, pens, or crayons

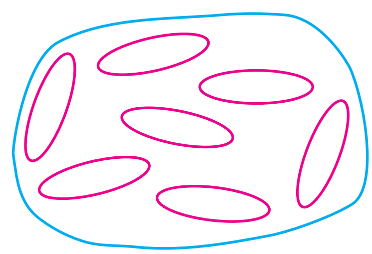
Description of Lesson

Exercise 1

Draw a large string on the board.

T: ***This string is for 100 tulips that are to be shared equally among seven people.***

Draw seven smaller strings inside the large string and write the corresponding division problem.



7)100

T: ***Each of these red strings is for one person's share of the tulips. How many tulips does each person get?***

Let students think for awhile. A possible dialogue is given here.

S: ***Each person gets at least 10 tulips.***

Put 10 inside each of the red strings and indicate giving 10 to each person in the division problem.

T: ***How many tulips did we give out?*** (70)
How many tulips are left to share? (30)

Record this as well in the division problem.

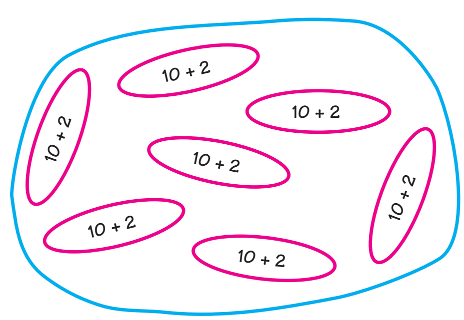
7)100	
− 70	10 each
30	

T: ***Can we give five more tulips to each person?***

S: ***No, 7 x 5 = 35, so five is too much.***

S: ***Give two more tulips to each person.***

Put 2 more inside each of the red strings, and record this in the division problem.



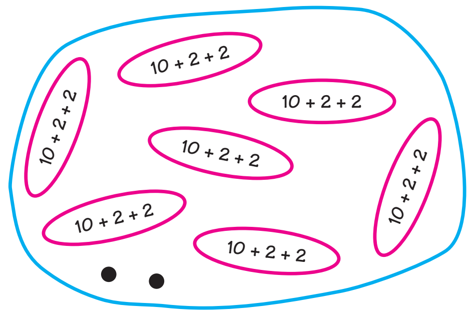
7)100	
− 70	10 each
30	
− 14	2 each
16	

T: ***How many more tulips did we give out?*** (14)
Now each person has twelve tulips.
How many tulips are left? (16)

Note: Of course, if students suggest giving four more tulips to each person, the next part of the dialogue would be eliminated and the record in the division problem would be shorter.

S: ***Give each person two more tulips. Then there will be two tulips left over.***

Put another 2 inside each of the red strings and record two more each in the division problem.



$$\begin{array}{r|l} 7\overline{)100} & \\ -\ 70 & \text{10 each} \\ \hline 30 & \\ -\ 14 & \text{2 each} \\ \hline 16 & \\ -\ 14 & \text{2 each} \\ \hline 2 & \end{array}$$

S: ***We could cut up the two extra tulips.***

S: ***That's no good. No one wants part of a flower.***

T: ***There are two extra tulips. I'll just draw dots for them in the string picture. How many tulips does each person get? How many tulips left over?***

S: ***Fourteen and two tulips left.***

T: ***What number sentence can we write about this problem?***

Perhaps a student will suggest this number sentence.

$$7 \times 14 = 98$$

T: ***This number sentence does not tell everything. There are 100 tulips.***

Continue letting students write number sentencess on the board. For example:

$$\begin{array}{r} 70 \\ 14 \\ 14 \\ +\ 2 \\ \hline 100 \end{array}$$

$$\begin{array}{r} (7 \times 14) + 2 = 100 \\ 7 \times (10 + 2 + 2) + 2 = 100 \\ 100 - (7 \times 14) = 2 \\ (7 \times 10) + (7 \times 4) + 2 = 100 \end{array}$$

$$\begin{array}{r} 100 \\ -\ 70 \\ \hline 30 \\ -\ 28 \\ \hline 2 \end{array}$$

T: ***We can also record the problem like this: 100 divided by 7 is 14 with a remainder of 2. We use the word "remainder" to show what is left over. We usually abbreviate remainder by just writing "R."***

$$\begin{array}{r|l} 14 & \text{R} = 2 \\ 7\overline{)100} & \\ -\ 70 & 10 \\ \hline 30 & \\ -\ 14 & 2 \\ \hline 16 & \\ -14 & 2 \\ \hline 2 & \end{array}$$

Erase the board before going on to Exercise 2.

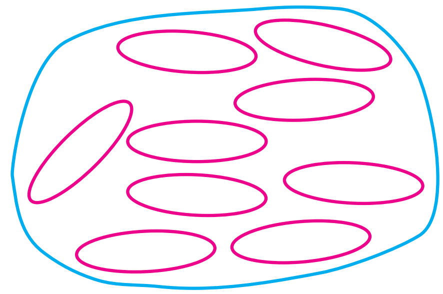
Exercise 2

Draw a large string on the board.

T: ***This string is for 200 baseball cards that are to be shared equally among nine children. How many small strings should we draw inside this large string?***

S: ***Nine.***

Instruct students to draw this string picture on their papers and to try to determine how many baseball cards each of the children should receive.



9)200

When you observe that all students have at least copied the string picture, interrupt them for a brief discussion of the problem.

T: ***We are trying to find out how many baseball cards each of the nine children should receive. What are your suggestions?***

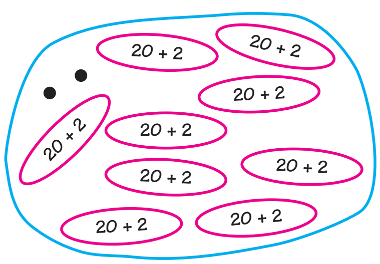
S: ***9 x 20 = 180, so let's begin by giving each child 20 baseball cards.***

Indicate that each child gets 20 baseball cards in the string picture as well as in the division problem.

S: ***Now there are 20 baseball cards left.***

T: ***Try to finish this problem on your own. When you have solved the problem, write a number sentence about it.***

The students who finish quickly may be given the problem of sharing 200 cookies among seven children while other students continue to work on the original problem. After a few minutes, call
on students to complete the string picture on the board and to write their number sentences.




```
   22  R = 2
9)200
 - 180 | 20 each
    20 |
    18 | 2 each
     2 |
```

$(9 \times 22) + 2 = 200$
$200 - (9 \times 22) = 2$

180	22
18	× 9
+ 2	198
200	+ 2
	200

Conclude that each of the nine children should receive 22 baseball cards and that there will be two cards left over.

Erase the board and then write these three problems. Direct students to solve as many of them as they can. Some students will continue to draw string pictures while others may prefer to just record the sharing in a division problem.

Share 100 marbles among 9 children.
Share 200 cookies among 7 children.
Share 300 walnuts among 8 cakes.

Writing Activity

Suggest students write to the baker about how to share 300 walnuts equally among the batter for 8 cakes.

Capsule Lesson Summary

Describe the make up of clubs with second, third, and fourth grade members when certain conditions are given. Solve a problem to decide how many buses to order when 300 students are going on a trip and each bus is to carry the same number of students.

Materials

Teacher
- Colored chalk
- Counters or blocks in three different colors

Student
- Paper
- Problem cards
- Counters or blocks in three different colors.

Advance Preparation: Use Blackline N29 to make problem cards (different clubs) for student groups.

Description of Lesson

Organize the class in pairs or small groups for this lesson. In each exercise, use props or manipulatives, such as counters in three different colors, to make a story more realistic or interesting. Props or manipulatives may also help students to act out a situation and solve the problems.

Exercise 1

As you pose this problem situation, identify different color counters or blocks to represent each grade level of students.

T: ***At Greenview Elementary School there are several clubs. Each club has members from second, third, and fourth grade. We are going to try to figure out how many students of each grade level are in a club.***

Here is the information we have about the Stamp Club
(i) There are 12 members in all.
(ii) There are two more third graders than second graders.
(iii) There is one less fourth grader than third grader.

Let students use the counters or blocks to represent the club. When a possible club is formed check that it satisfies the three conditions. Students may like to begin a list of possibilities that satisfy conditions (ii) and (iii), and then look for one that has 12 students total. For example:

2nd Graders	3rd Graders	4th Graders
1	3	2
2	4	3
3	5	4
4	6	5

Distribute problem cards (for different clubs) one at a time to student groups. You may like to arrange that different groups are working on a different problem at any one time. As a group completes one problem and writes its solution on the card, give the group another card. Encourage group members to use counters or blocks to display a solution to a problem, and let everyone in the group check that it fits the requirements on the card.

N29

Solutions for the six cards provided on Blackline N29 are given below.

CHESS CLUB

10 members total
2 more 3rd graders than 2nd graders
Same number of 3rd and 4th graders

2nd	3rd	4th
2	4	4

SPANISH CLUB

18 members total
Same number of 2nd and 3rd graders
3 less 4th graders than 3rd graders

2nd	3rd	4th
7	7	4

COMPUTER CLUB

One-half of members are 3rd graders
5 4th graders
Twice as many 3rd graders as 4th

2nd	3rd	4th
5	10	5

BOWLING CLUB

12 members total
Twice as many 2nd graders as 3rd
2 more 4th graders than 2nd graders

2nd	3rd	4th
4	2	6

MATH CLUB

Between 15 and 20 members
1 more 3rd grader than 2nd
1 more 4th grader than 3rd

2nd	3rd	4th
5	6	7

WRITING CLUB

16 members total
Twice as many 3rd graders as 2nd
More than 2 2nd graders
Less than 5 4th graders

2nd	3rd	4th
4	8	4

Exercise 2

Pose a different kind of problem to the class.

T: ***Ms. Gaither is getting ready to order buses for a school field trip. There are 300 students going on the trip. Ms. Gaither wants to put the same number of students on each bus. How many buses should she order?***

Let students think for a couple minutes about the problem and then ask for suggestions. When it appears there are many solutions, organize them in a table. The table organization may help students find other solutions.

300 students

Number of buses	Number of students on each bus	Number of buses	Number of students on each bus
1	300	20	15
2	150	25	12
3	100	30	10
5	60	50	6
6	50	60	5
10	30	100	3
12	25	150	2
15	20	300	1

Discuss with the class which of these answers are reasonable. For example, consider a reasonable range of possible numbers of students to put on one bus. It is not likely Ms. Gaither would order 300 buses and put one student on each bus, and buses usually do not seat as many as 300 students.

Writing Activity

You may like to challenge some students to write a club description that has only one solution like those in Exercise 1.

Capsule Lesson Summary

Solve division problems suggested by story situations. Use string pictures and arrow pictures to find and record solutions.

Materials

Teacher
- Colored chalk
- Package of M&M's®, counted before class (optional)

Student
- Paper
- Colored pencils, pens, or crayons

Description of Lesson

This lesson description assumes a class of 26 students. You should determine and use the actual number of students in your class.

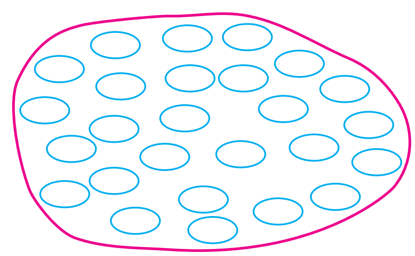
T: ***Suppose our class is going on a picnic and we have 137 sandwiches. I would like to share these 137 sandwiches equally among the students in our class. How many students are in our class?*** (26)

How many sandwiches will each of you get and how many sandwiches will be left over? Draw a picture on your paper to show your solution to this problem.

Allow a few minutes for individual work. When you observe that several students have solved the problem, begin letting students share their solutions with the class.

T: ***Randy has drawn a string picture.***

Draw a large string on the board.



T: ***Randy, is this big string for the 137 sandwiches?*** (Yes) ***How many small strings did you draw inside of the big string?***

S: ***26, because there are 26 students in our class.***

T: ***What will we put in the small strings?***

S: ***The sandwiches each person gets.***

T: ***How many sandwiches did you give to each person?***

S: ***Five.***

T: ***How can we be sure this is correct?***

S: ***We could count by fives.***

S: ***We could calculate 5 x 26.***

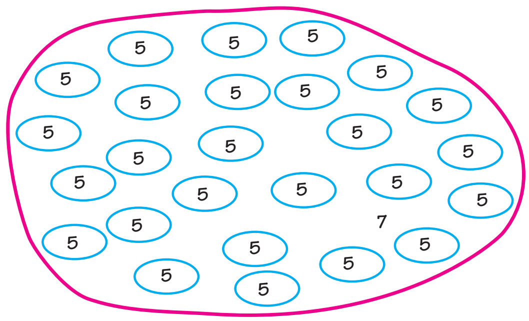
T: ***What number is 5 x 26?*** (130)

T: ***Can we give each of you five sandwiches?*** (Yes)
How many sandwiches will be left over? (Seven)
Could we give each of you one more sandwich?

S: ***No, because there are 26 of us and there are only seven sandwiches left over.***

T: ***Each of you will receive five sandwiches and there will be seven sandwiches left over.***

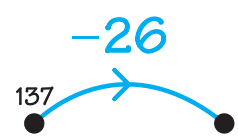
Complete the string picture and record the result in a division problem.




```
      5 R = 7
26)137 |
 - 130 | 5 each
     7 |
```

T: ***Rosa drew an arrow picture about this problem. She drew a dot for 137 and then she drew –26 arrows.***

On the board draw a –26 arrow starting at 137.



T: ***Why did you draw a –26 arrow?***

S: ***I gave one sandwich to each of us.***

T: ***How did you find how many sandwiches were left?***

S: ***I calculated 137 – 26 = 111.***

Invite a student to write this problem on the board and to solve it.

$$\begin{array}{r} 137 \\ -\ 26 \\ \hline 111 \end{array}$$

Label the dot for 111.

T: ***Rosa, what did you do next?***

S: ***I drew another –26 arrow.***

T: ***How many –26 arrows are in your arrow picture?*** (Five)

Draw the corresponding arrow picture on the board.

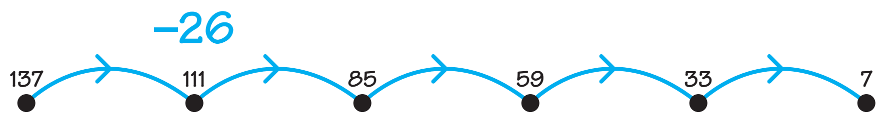
T: ***What calculation do we need to do to label the next dot?***

S: ***111 – 26.***

Invite a student to do the subtraction calculation at the board while others do it on their papers.

$$\begin{array}{r} {\scriptstyle 10\,10} \\ 111 \\ {\scriptstyle 1\ 1\ \ } \\ -\ 26 \\ \hline 85 \end{array}$$

Continue until all the dots are labeled. Use this opportunity to review subtraction methods.



$$\begin{array}{r}137\\ -\ 26\\ \hline 111\end{array}\qquad \begin{array}{r}111\\ -\ 26\\ \hline 85\end{array}\qquad \begin{array}{r}85\\ -\ 26\\ \hline 59\end{array}\qquad \begin{array}{r}59\\ -\ 26\\ \hline 33\end{array}\qquad \begin{array}{r}33\\ -\ 26\\ \hline 7\end{array}$$

T: ***Look carefully at this arrow picture. How many sandwiches will each person receive and how many sandwiches will be left over? How do you know?***

S: ***Each person will get five sandwiches because there are five blue arrows. There will be seven sandwiches left over, because the ending number is 7.***

T: ***Did anyone draw a different picture to show how they solved this problem?***

If students have solved this problem in different ways, draw their pictures on the board.

T: ***What number sentences can we write about this problem?***

Perhaps some of the following number sentences will be suggested. Be sure to include the division format.

$(26 \times 5) + 7 = 137$

$137 - (26 \times 5) = 7$

$137 - 130 = 7$

$$\begin{array}{r}26\\ 26\\ 26\\ 26\\ 26\\ +\ 7\\ \hline 137\end{array}$$

$$\begin{array}{r}5 \text{ R} = 7\\ 26\overline{)137}\end{array}$$

Erase the board and then present the following problem to your class

T: ***Suppose that our class helps clean up the park and collects 105 soda bottles. If we put all the empty bottles into cartons that hold eight bottles each, how many cartons would we fill?***

Record the information on the board.

T: ***Try to solve this problem on your own, and then draw a string picture or an arrow picture to show your solution.***

105 soda bottles
8 bottles per carton

As you observe students' work, suggest to those having trouble that they draw a dot for 105 and then draw –8 arrows (one for each carton filled) until they reach a number less than 8. The number of –8 arrows tells how many cartons are filled. While the students are working, briefly describe some methods to the class. For example:

T: ***Mindy started at 105 and is drawing –16 arrows. Why would Mindy draw –16 arrows?***

S: ***A –16 arrow is the same as two –8 arrows; a –16 arrow shows filling two cartons.***

T: ***Frankie drew +40 arrows. Why did you use +40, Frankie?***

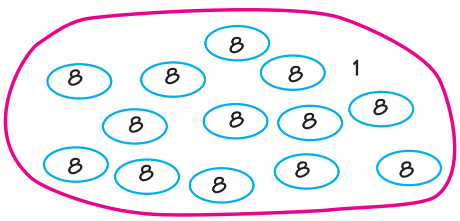
S: ***5 x 8 = 40, so 40 bottles will fill five cartons and 40 is easier to add than 8.***

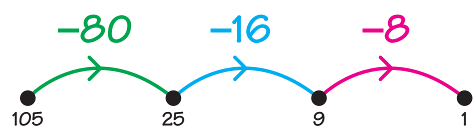
T: ***Nicole has a –80 arrow. Why do you think she chose a –80 arrow?***

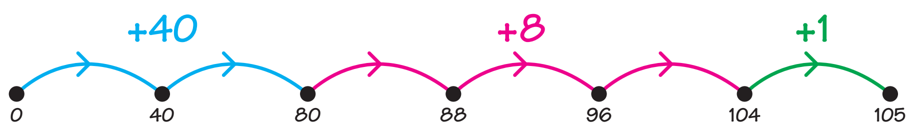
S: ***10 x 8 = 80, so ten cartons will hold 80 bottles.***

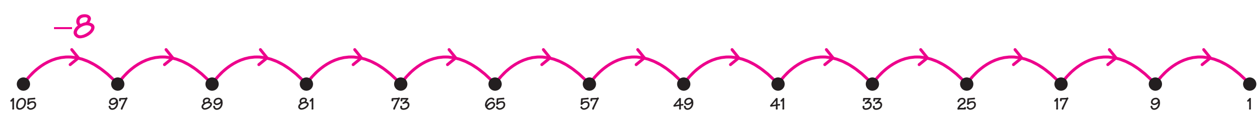
Encourage students to solve the problem in their own ways, to show their solutions in pictures, and to write number sentences about the problem. Students who finish quickly can be asked to determine how many cartons would be needed for 307 bottles.

When most students have solved the problem of putting 105 soda bottles into cartons that hold eight bottles each, ask two or three students to show their solutions on the board. A few of the possible arrow pictures and string pictures are shown below.









Conclude that 105 bottles will fill 13 cartons and there will be one bottle left over. Record several of the students' number sentences and the division problem on the board.

Optional: Purchase a large package of M&M's®. Pose this (or a similar) problem:

> This package contains 402 M&M's®. (Numbers will vary.) I would like to share the M&M's® equally among the students in our class. How many M&M's® will each of you get? How many will be left over? Share 402 M&M's® among 26 students.

Center Activity

In a center, place task cards posing division problems similar to those in this lesson.

Capsule Lesson Summary

With the display of the calculator hidden, anticipate the effect of a sequence of operations on a starting number. Go from one number to another on the calculator using the operations +, –, x, and ÷, and using one-digit positive integers.

Materials

Teacher	• Colored chalk • Calculator (overhead)	• Paper • Colored pencils, pens, or crayons
Student	• Calculator	

Description of Lesson

Exercise 1

Use an overhead calculator or a classroom calculator to do a mental arithmetic activity similar to Exercise 1 in Lesson N22. A possible sequence of calculations is suggested below.

- Start with 0 on the display; press $\boxed{4}$ $\boxed{\times}$ $\boxed{3}$ $\boxed{+}$ $\boxed{8}$ $\boxed{-}$ $\boxed{2}$ $\boxed{=}$ (18)
- Start with 18 on the display; press $\boxed{+}$ $\boxed{1}\boxed{0}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ (48)
- Start with 48 on the display; press $\boxed{\div}$ $\boxed{2}$ $\boxed{=}$ $\boxed{=}$ (12)
- Start with 12 on the display; press $\boxed{-}$ $\boxed{6}$ $\boxed{\times}$ $\boxed{5}$ $\boxed{=}$ (30)
- Start with 30 on the display; press $\boxed{\div}$ $\boxed{3}$ $\boxed{\times}$ $\boxed{1}\boxed{5}$ $\boxed{=}$ (150)

Exercise 2

Provide each student or pair of students with a calculator for this exercise.

Ask students to recall some of what they remember about golf and the Minicomputer Golf game.

T: ***Today we are going to play a game called Calculator Golf. We start with a number on the display of the calculator and then set a goal.***

Draw two dots on the board. Label one of them 7 and the other 100.

T: ***We might start with 7*** (put 7 on the display) ***and make 100 be the goal. When you play this golf game, you can press any operation key (*** $\boxed{+}$, $\boxed{-}$, $\boxed{\times}$, ***or*** $\boxed{\div}$***) followed by a one-digit number (1 through 9) and then*** $\boxed{=}$***. Play continues until 100 is on the display.***

Put 7 on your display. Try to get to 100. You can add, subtract, multiply, or divide by any of the one-digit numbers 1 through 9.

Suggest that students press $\boxed{=}$ after pressing a number key so that they can see the result before deciding which operation to use next.

Note: Some students may want to keep track of each step, but do not require it. If your students have calculators, you need not require that they record their steps on paper. Some will want to keep track of their steps while others will find working with pencil and calculator simultaneously awkward and inhibiting.

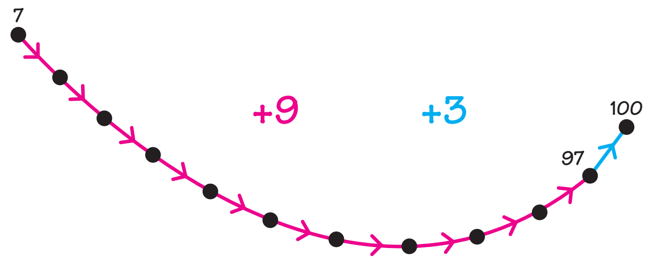
N31

Allow a few minutes for students to work on this problem before asking students to share solutions with the class. As a student describes a solution, draw the corresponding arrow road on the board. For example:

S: ***I pressed*** $\boxed{+}$ $\boxed{9}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ ***… until I got 97; then I pressed*** $\boxed{+}$ $\boxed{3}$ $\boxed{=}$ ***.***

T: ***How many times did you add 9?*** (10 times)

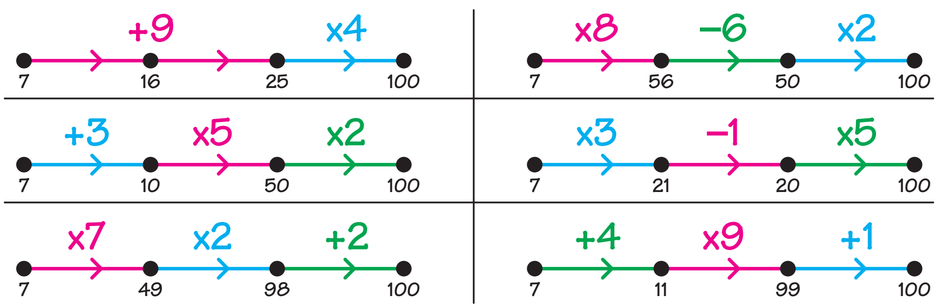
How many arrows are in this arrow road? (11) ***Did anyone get to 100 in another way?***



S: ***I pressed*** $\boxed{+}$ $\boxed{3}$ ***and then*** $\boxed{\times}$ $\boxed{1}\boxed{0}$ ***.***

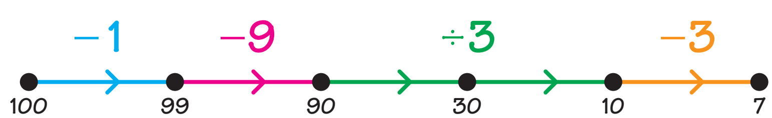
T: ***But 10 is not a one-digit number. Try ag***

Continue this activity until several solutions are (
with three steps (arrows). Six of the many three-step solutions are shown here for your information.

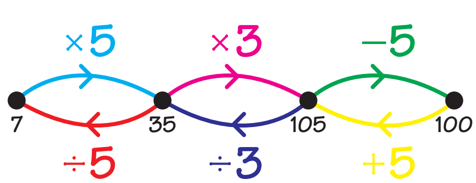


Encourage students to try to find other solutions that use fewer steps (arrows) than those on the board. Perhaps your class will see that it is not possible to go from 7 to 100 using only one or two such arrows.

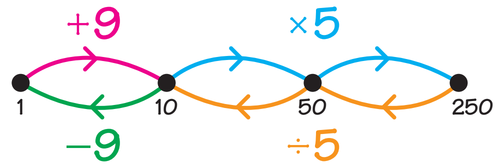
Do not erase the arrow pictures. Ask students to go from 100 to 7 with the same restrictions; that is, play the game starting at 100 and making 7 the goal. Allow students to work independently on this problem for a few minutes before sharing solutions with the class. As a solution is described, draw the corresponding arrow picture on the board. One possible solution is shown here.



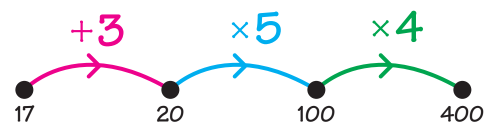
Encourage students to find solutions that use fewer steps (arrows) than those on the board. Students should notice that to build an arrow road from 100 to 7, you only need to go backward on a road from 7 to 100. For example, if this arrow road was suggested from 7 to 100, then the return arrows form a road from 100 to 7.



Erase the arrow pictures and play Calculator Golf going from 1 to 250 and from 250 to 1. For your information, solutions with the fewest steps are shown below.



You may wish to ask students who finish quickly to play the game going from 17 to 400, and to try to use as few steps as possible. For example:



Extension Activity

Present a version of the Calculator Golf game where using one or two of the keys (for example, the $\boxed{1}$ and $\boxed{5}$ keys) is not allowed.

Capsule Lesson Summary

Begin the lesson with some subtraction calculations. Label the dots in an arrow road that starts at 200 and has –48 and –8 arrows. Decide how many cartons 200 eggs will fill by using the arrow picture. Ask how many adults should accompany the class to a swimming pool if must be at least one adult for every five students.

Materials

Teacher	• Colored chalk	• Colored pencils, pens, or crayons • Worksheets N32* and **
Student	• Paper	

Description of Lesson

The first exercise in this lesson is for subtraction practice. You may prefer to make time for such practice other than with this lesson.

Exercise 1________________

Write these problems on the board. Instruct students to copy and solve them one at a time.

$$\begin{array}{r} 782 \\ -\ 526 \\ \hline \end{array} \qquad \begin{array}{r} 8{,}429 \\ -\ 156 \\ \hline \end{array} \qquad \begin{array}{r} 9{,}587 \\ -\ 2{,}964 \\ \hline \end{array} \qquad \begin{array}{r} 4{,}603 \\ -\ 1{,}345 \\ \hline \end{array}$$

Allow several minutes for individual work. When you help a student who is having difficulty with a problem, try to respect his or her method for subtraction. When several students have solved all the problems, call on students to solve the problems at the board and to explain each step.

$$\begin{array}{r} 78\overset{10}{2} \\ -\ 5\overset{1}{2}6 \\ \hline 256 \end{array} \qquad \begin{array}{r} 8{,}4\overset{10}{2}9 \\ -\ \overset{1}{1}56 \\ \hline 8{,}273 \end{array} \qquad \begin{array}{r} 9{,}\overset{10}{5}87 \\ -\ \overset{1}{2}{,}964 \\ \hline 6{,}623 \end{array} \qquad \begin{array}{r} 4{,}6\overset{10}{0}\overset{10}{3} \\ -\ 1{,}\overset{1}{3}\overset{1}{4}5 \\ \hline 3{,}258 \end{array}$$

Erase the board before going on to Exercise 2.

Exercise 2________________

Pose the following or a similar division problem.

T: ***Pretend that we have a case of 200 eggs and we want to put them into egg cartons. How may eggs do we put in each carton?***

S: ***Twelve.***

T: ***Each carton holds one dozen or 12 eggs. How many cartons can we fill? Will there be any eggs left over?***

Allow students to work on this problem for a few minutes.

T: ***Did anyone think about drawing an arrow picture to help?***

N32

Let students share their ideas for solving this problem and discuss their methods.

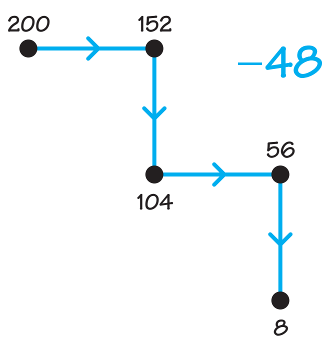
S: ***I started at 200 and subtracted 12 (filling one carton) many times.***

T: ***That's a lot of –12 arrows to draw. Maybe we could think of filling several cartons at a time.***

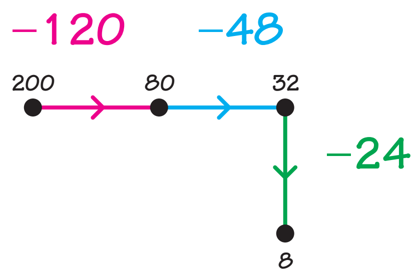
S: ***We could subtract 24 (filling two cartons) or 48 (filling four cartons).***

S: ***If we think about filling 10 cartons, we could subtract 120.***

Choose one or two methods suggested by students, draw the corresponding arrow pictures, and record the results in a division problem. For example:



$$
\begin{array}{r|l}
\multicolumn{2}{l}{\quad 16 \text{ R} = 8} \\
12\overline{)200} & \\
-\ 48 & \text{4 cartons} \\ \hline
152 & \\
-\ 48 & \text{4 cartons} \\ \hline
104 & \\
-\ 48 & \text{4 cartons} \\ \hline
56 & \\
-\ 48 & \text{4 cartons} \\ \hline
8 &
\end{array}
$$



$$
\begin{array}{r|l}
\multicolumn{2}{l}{\quad 16 \text{ R} = 8} \\
12\overline{)200} & \\
-\ 120 & \text{10 cartons} \\ \hline
80 & \\
-\ 48 & \text{4 cartons} \\ \hline
32 & \\
-\ 24 & \text{2 cartons} \\ \hline
8 &
\end{array}
$$

T: ***What number sentences can we write about this problem?***

Record several number sentences on the board. Try to include one or two that involve multiplication.

$$(16 \times 12) \times 8 = 200 \qquad 200 - 120 - 72 = 8$$
$$(4 \times 48) \times 8 = 200 \qquad 200 - (4 \times 48) = 8$$

Exercise 3

Pose the following or a similar problem to the class.

T: ***Pretend that this weekend our class is going swimming. The place where we are going requires that there be at least one adult for each group of five students. If everyone in our class is going, how many adults need to go with us?***

Encourage students to discuss the problem. The calculations involved are fairly simple, but students may have some difficulty agreeing on how many adults are needed. A sample dialogue is given here for a class of 27 students.

S: ***We need five adults.***

S: ***There are extra students.***

T: ***How many students may go if there are five adults?*** (25)
Remember, there are 27 students and all are going.

S: ***We need six adults so that everyone can go swimming.***

S: ***If there were six adults, we could invite three more students to come with us.***

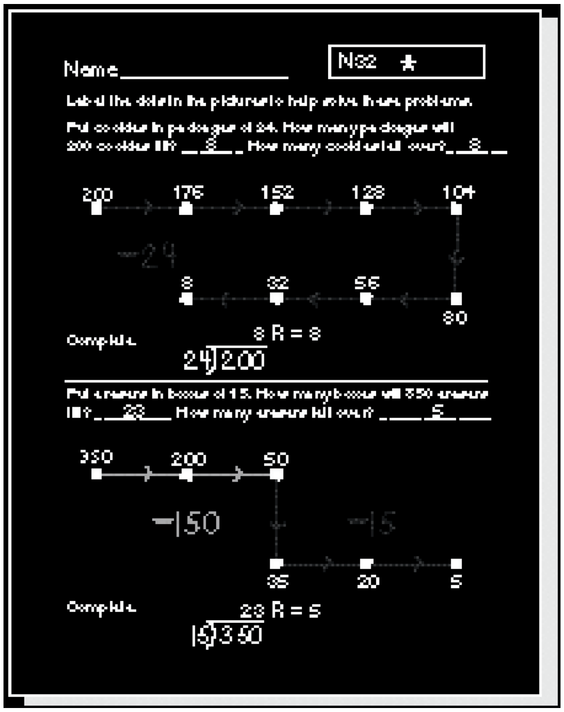
A class of 27 students should conclude that they will need at least six adults to accompany them swimming.

Worksheets N32* and ** are available for individual work.

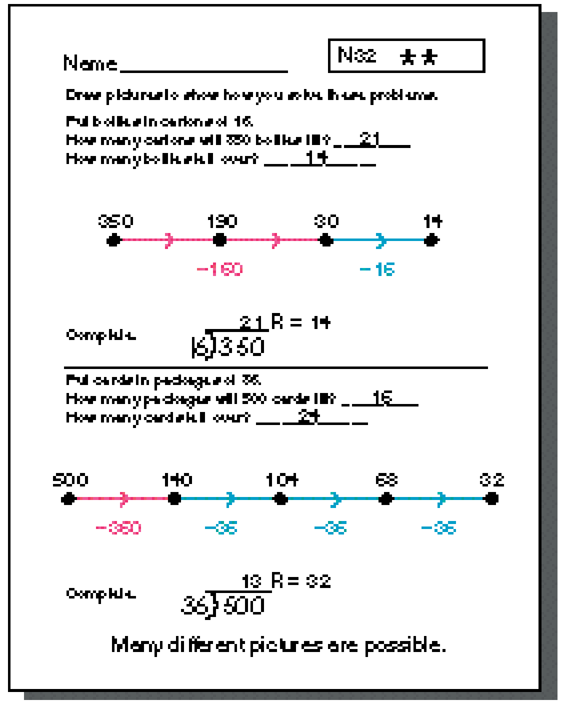
Home Activity

This is a good time to send a letter to parents/guardians about division. Blackline N32 has a sample letter.

N32



Name
N32 ★
200
176
152
128
104
−24
8
32
56
80
Complete.
8 R = 8
24)200
23
5
350
200
50
−150
−15
35
20
5
Complete.
23 R = 5
15)350



Name
N32 ★★
21
14
350
190
30
14
−160
−16
Complete.
21 R = 14
16)350
13
24
500
140
104
68
32
−360
−36
−36
−36
Complete.
13 R = 32
36)500
Many different pictures are possible.

N33 COMPARING PRICES #2

Capsule Lesson Summary

Compare prices of items packaged in two different quantities by finding the cost of a specified number of the items that can be purchased with either packaging. Do a similar activity that involves comparing four prices.

Materials

Teacher	• Colored chalk	**Student**	• Paper • Colored pencils, pens, or crayons

Description of Lesson

You may like to let students work in cooperative groups during this lesson. Provide groups with paper, colored pencils, and manipulatives such as cards to use to solve the problems.

Exercise 1

Tell the following or a similar story to the class.

T: ***I have a friend named Jeremy who lives near a stable. Do you know what a stable is?***

Discuss with students that a stable is a place where horses live and are cared for. Many people who own horses pay a fee to keep their horses at a stable either because they do not have enough land or they do not have enough time to care for a horse properly.

T: ***Jeremy loves to go horseback riding, and the stable has several horses they will rent by the hour to a person like Jeremy who doesn't own a horse. The stable sells three-punch cards and five-punch cards to people who come to the stable often. Each time Jeremy rents a horse for one hour, the stable punches his card once. Since he goes so often Jeremy would like to know which card is a better deal. The three-punch card costs $7 and the five-punch card costs $11.***

Record this information on the board.

3 one-hour rides $7
5 one-hour rides $11

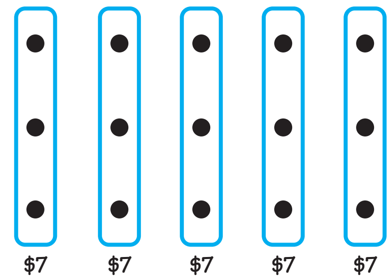
T: ***Which card is the better deal?***

Let student groups work on this problem for awhile. Some groups may want to make three-punch and five-punch cards. When several groups believe they have an answer for Jeremy, discuss the problem collectively. If no one suggests finding the cost of the same number of rides at each price, suggest this approach yourself. Ask the students for a good number to use. Any common multiple of 3 and 5 is an appropriate choice. In the dialogue that follows, the cost of 15 rides at each price is computed.

T: ***How many three-punch cards would Jeremy need to buy in order to get 15 rides? How do you know?***

S: ***He would need to buy five three-punch card, because 3 + 3 + 3 + 3 + 3 = 15.***

Draw this picture on the board.



T: ***Each three-punch card costs $7. How much does it cost to buy enough three-punch cards to get 15 rides? How do you know?***

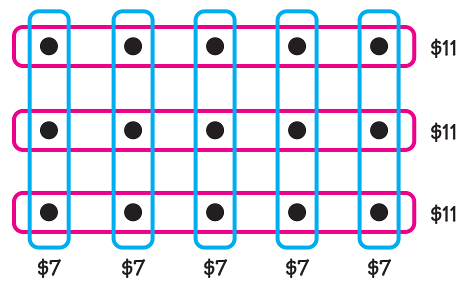
S: ***5 x 7 = 35, so 15 rides cost $35.***

Record this information on the board.

T: ***How many five-punch cards would Jeremy need to buy in order to get 15 rides?***

S: ***Three.***

Draw red cards around groups of five dots.



T: ***Each five-punch card costs $11. How much does it cost to buy enough five-punch cards to get 15 rides? How do you know?***

S: ***3 x 11 = 33, so 15 rides cost $33.***

Record this information on the board.

3 rides for $7 or 15 rides for $35
5 rides for $11 or 15 rides for $33

T: ***Which card is the better deal?***

S: ***The five-punch card.***

T: ***When might Jeremy ever buy a three-punch card instead of a five-punch card?***

S: ***…if he only wanted to ride three times.***

S: ***…if he had enough money to buy a three-punch card but not enough to buy a five-punch card.***

S: ***If he wanted to ride eight times, he could buy one five-punch card and one three-punch card.***

Exercise 2

Tell the following or a similar story to the class. If possible, use rates from a local facility.

T: ***This summer the municipal swimming pool is going to sell four-punch tickets and ten-punch tickets. Each admission uses one punch. There are many children who would like to know which ticket is a better deal. The four-punch ticket sells for $2.50, and the ten-punch ticket sells for $5.50.***

Record this information on the board.

4 admissions for $2.50
10 admissions for $5.50

Direct groups to try to solve this problem. After several minutes, let groups share their approaches. Perhaps someone will recognize that if ten admissions cost \$5.50, then one costs 55¢. So four admissions at this rate would cost \$2.20, since 55 + 55 + 55 + 55 = 220. Otherwise, compute the cost of the same number of rides at each price, choosing a number that is a common multiple of 4 and 10. In the following dialogue, the cost of 20 admissions at each price is computed.

4 admissions for \$2.50 or 20 admissions for ______
10 admissions for \$5.50 or 20 admissions for ______

Allow groups to continue working on this problem for a few more minutes while you draw a four by five array of dots on the board. Ask students to explain how they calculated the price of 20 admissions with the two types of tickets. Illustrate students' calculations with the dot picture on the board. A possible dialogue is given here.

T: ***How much does it cost to buy 20 admissions with four-punch tickets? How do you know?***

S: ***\$12.50, because five four-punch tickets are good for 20 admissions and 5 x \$2.50 = \$12.50.***

T: ***How did you calculate 5 x \$2.50?***

S: ***We added \$2.50 + \$2.50 + \$2.50 + \$2.50 + \$2.50.***

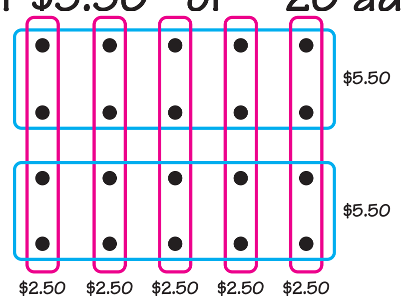
S: ***5 x \$2 = \$10 and 5 x \$0.50 = \$2.50, so 5 x \$2.50 = \$12.50.***

T: ***How much does it cost to buy 20 admissions with ten-punch tickets? How do you know?***

S: ***\$11, because two ten-punch tickets are good for 20 admissions and 2 x \$5.50 = \$11.00.***

Complete the dot picture and the information on the board.

4 admissions for \$2.50 or 20 admissions for \$12.50
10 admissions for \$5.50 or 20 admissions for \$11.00



T: ***Which ticket is a better deal?***

S: ***The ten-punch ticket.***

T: ***How much do you save on 20 admissions if you buy ten-punch tickets instead of four-punch tickets?***

S: ***\$1.50.***

Erase the board before going on to Exercise 3.

Exercise 3

Tell the following or a similar story to the class.

T: ***When I went to the meat market recently, they had the following prices for quantities of ground beef.***

Record this information on the board. Point out that “lb” is an abbreviation for pound.

1 lb for \$2.00
2 lb for \$3.50
3 lb for \$5.00
4 lb for \$6.75

T: ***Sometimes the more you buy the less you pay for each pound of meat. Is that true if these are the prices?***

Let student groups work to answer this question. After a short while, discuss it collectively. Very likely students will observe that the price for one pound of ground beef is the highest; at the one pound price, two pounds cost \$4, three pounds cost \$6, and four pounds cost \$8. If no one mentions this, ask if the price for one pound of ground beef is the best one. When the class decides that the price for one pound is the most expensive, erase this price from your list.

T: ***Is it possible that the price for two pounds of ground beef is the least expensive of these prices?***

S: ***If you bought four pounds at that price, it would cost \$7 because 2 x \$3.50 = \$7, so that price is more expensive than the price for four pounds of ground beef.***

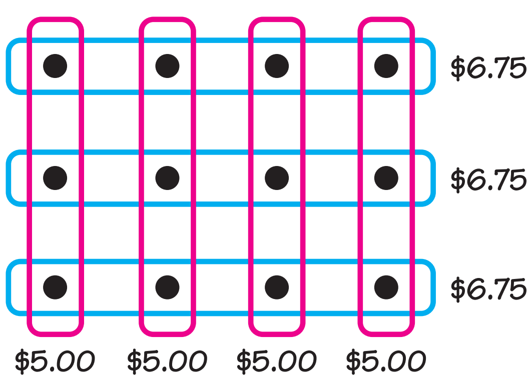
Erase the price for two pounds of ground beef.

T: ***Which of these prices (for 3 pounds or 4 pounds) is the better deal?***

As students express their opinions, very likely someone will suggest calculating the price of 12 pounds of ground beef at each of the prices so a comparison can be made. If necessary, suggest this yourself.

Continue as in Exercise 2. Conclude that the lowest price (per pound) is three pounds for \$5.00. Point out that buying the largest package is not the best deal in this case. You should have the following information on the board.

3 lb for \$5.00 or 12 lb for \$20.00
4 lb for \$6.75 or 12 lb for \$20.25



Capsule Lesson Summary

Consider two relations: (i) pair two numbers if and only if one number is 1 more than the other, and (ii) pair two numbers if and only if one number is the double of the other. Label the dots in pictures involving both of these relations.

Materials

Teacher
- Colored chalk

Student
- Paper
- Colored pencils, pens, or crayons
- Worksheets N34 *, **, ***, and ****

Description of Lesson

Draw a blue cord connecting two dots on the board.

T: ***These two dots are for numbers. Two numbers can be connected by a blue cord if and only if one of the numbers is 1 more than the other.***

Label one of the dots 13.

T: ***If this dot were for 13, what could the other dot be for?***

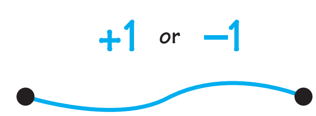
S: ***14.***

T: ***13 + 1 = 14. Is there another possibility?***

S: ***12.***

T: ***13 – 1 = 12. The blue cord is for +1 or –1.***

Indicate this key for the blue cord.



Erase 13 and label one of the dots 100.

T: ***If this dot were for 100, what could the other dot be for?***

S: ***101, because 100 + 1 = 101.***

S: ***99, because 100 – 1 = 99.***

Repeat this activity with several other numbers at one of the dots; for example, 8; 360; 1,000; 1,010; and $\hat{5}$.

Draw a red cord on the board.

T: ***Two numbers can be connected by a red cord if and only if one of the numbers is the double of the other number.***

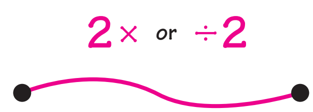
Label one of the dots 14.

T: *If this dot were for 14, what could the other dot be for?*

S: *28, because 2 x 14 = 28.*

S: *7, because 2 x 7 = 14.*

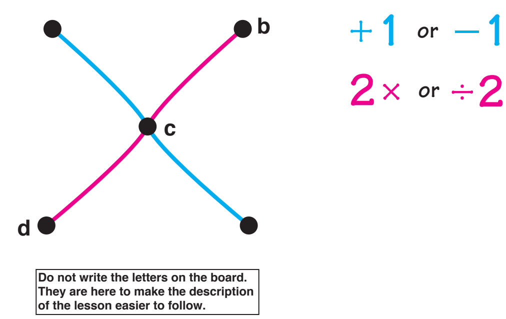
T: *The red cord is for 2x or ÷2.*



Indicate this key for the red cord.

Repeat this activity with several other numbers at one of the dots; for example: 22, 30, 50, and 1,000.

Draw this picture on the board.

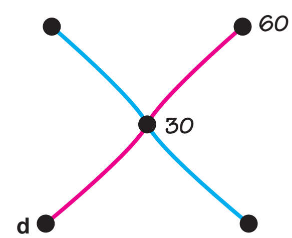


T: *Who would like to label one of these dots?*
You may choose any whole number.

Suppose a student puts 60 at **b**.

T: *If this dot* (**b**) *were for 60, what could the other dots be for?*

Invite students to label the other dots and discuss their choices.



T: *60 ÷ 2 = 30. Are there any other numbers this dot* (**c**) *could be for?*

S: *120.*

T: *Right, this dot could also be for 120 because 2 x 60 = 120. So this number* (**c**) *could be 30 or 120, and we have chosen 30.*

Point to the dots at the other ends of the blue cords as you ask,

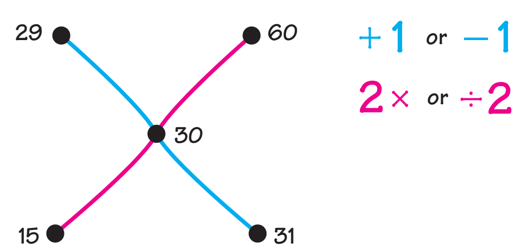
T: *What numbers could these be?*

S: *31, because 30 + 1 = 31.*

S: *29, because 30 – 1 = 29.*

T: *There is one dot left to label.*
What number could this (**d**) *be?*

S: *15.*

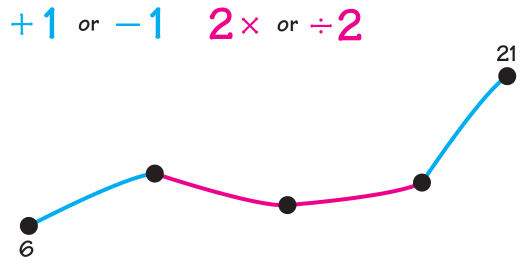


If a student suggests 60, remind the class that 60 is already in the picture.

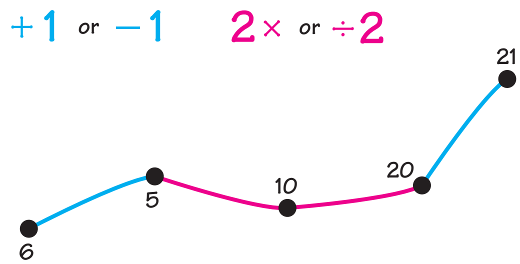
Erase the numbers from the cord picture on the board. Instruct students to copy the picture on their papers and to label the dots in another way. Suggest they choose a whole number to start at one dot and then label the other dots. Some students may be able to complete the cord picture in several ways.

Erase the board. Draw this picture on the board and direct students to copy it on their papers.

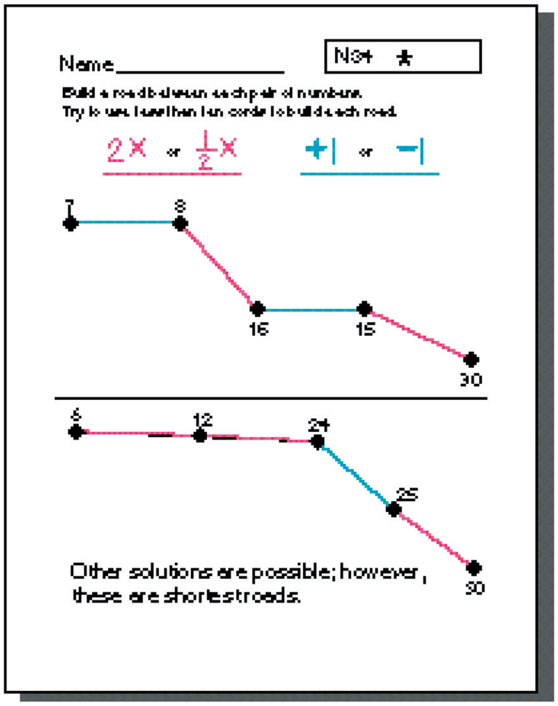
T: ***This is a road between 6 and 21. Try to label the other dots.***

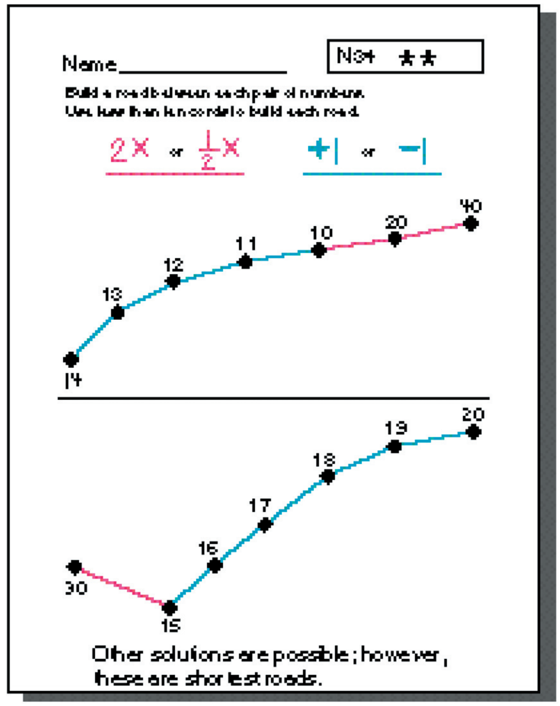


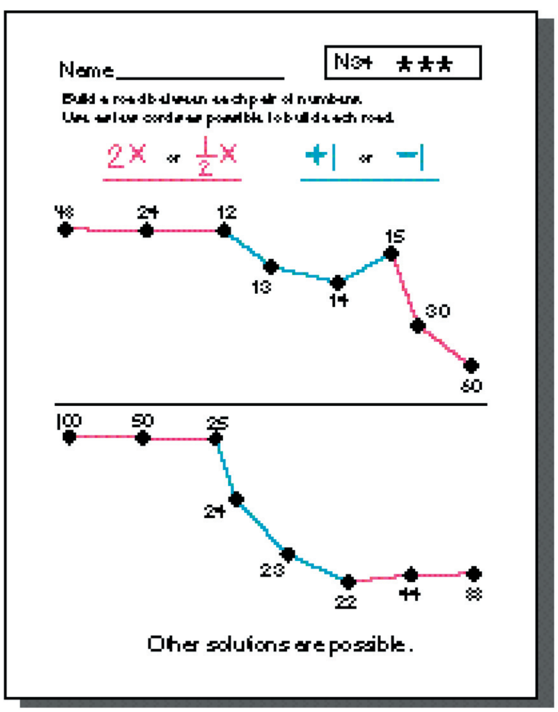
Allow students to work independently or with a partner. Students who finish labeling the dots correctly can begin working on the worksheets. When most students have labeled the dots in this cord picture, call on a student to label the dots on the board.

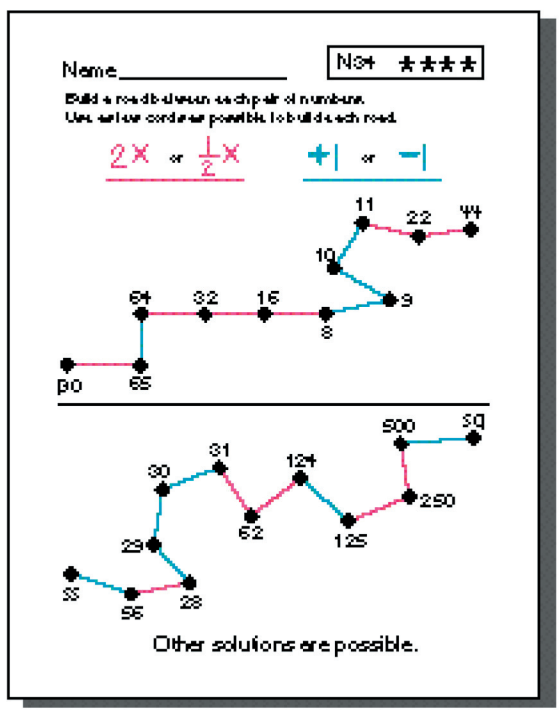


Worksheets N34 *, **, ***, and **** are available for individual work. Emphasize to students that they are to use less than ten cords to build each road on the * and ** worksheets.


Name
N34 ★
2× or ½×
+1 or −1
7
8
16
15
30
6
12
24
25
50
Other solutions are possible; however, these are shortest roads.


Name
N34 ★★
2× or ½×
+1 or −1
14
13
12
11
10
20
40
30
15
16
17
18
19
20
Other solutions are possible; however, these are shortest roads.


Name
N34 ★★★
2× or ½×
+1 or −1
48
24
12
13
14
15
30
60
100
50
25
24
23
22
44
88
Other solutions are possible.


Name
N34 ★★★★
2× or ½×
+1 or −1
130
65
64
32
16
8
9
10
11
22
44
55
56
28
29
30
31
62
124
125
250
500
50
Other solutions are possible.

N35 MINICOMPUTER GOLF #2

Capsule Lesson Summary

Explore the effect of moving various checkers on the minicomputer in a configuration that has both regular and negative checkers. After a move, is the number on the Minicomputer more, less, or the same number as before? Play Minicomputer Golf, possibly with a negative checker in the starting configuration of a game.

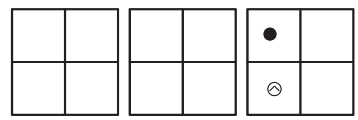
Materials

Teacher		Student	
	• Minicomputer set • Colored chalk		• None

Description of Lesson

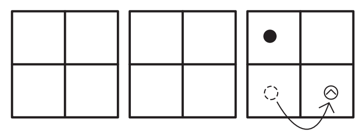
Exercise 1

Put this configuration on the Minicomputer.



T: ***What number is on the Minicomputer?*** (6)

Move the negative checker to the 1–square.



T: ***What number is this?*** (7)

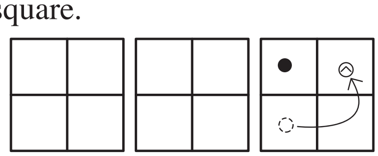
By moving the negative checker from the 2-square to the 1-square, did we get more or less than the original number?

S: ***More.***

T: ***How much more?***

S: ***1.***

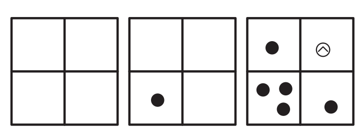
Move the negative checker back to the 2-square. Repeat the activity, only this time move the negative checker to the 4-square.

 = 4 (2 less)

Clear the Minicomputer.

T: ***I'll put a number on the Minicomputer. As I'm putting on checkers, see if you can figure out what number it is.***

Put checkers on the Minicomputer gradually, allowing students to calculate mentally until you have this configuration displayed.



T: ***What number did I put on the Minicomputer?***

S: ***31.***

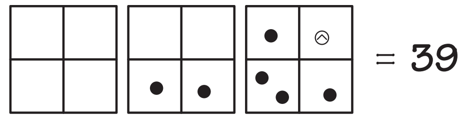
Check that the number is 31 by pointing to the checkers, one or two at a time, as the class computes.

T: ***Now I'll move a checker. Tell me whether the new number is less or more than 31 and by how much. Also tell me the new number.***

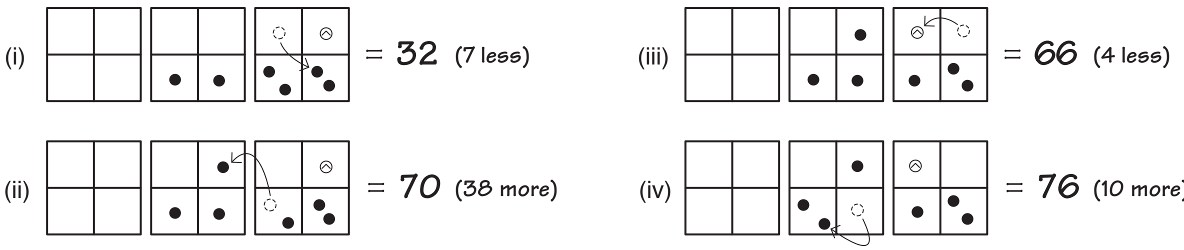
Move a checker from the 2-square to the 10-square.

S: ***8 more.***

S: ***39 is on the Minicomputer.***

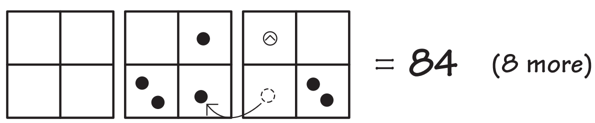


Continue this activity with the moves illustrated below.



T: ***76 is on the Minicomputer. Can we move one checker and get a new number that is 8 more than this number?***

A volunteer should move a checker from the 2-square to the 10-square.



T: ***76 is on the Minicomputer. Can we move one checker and get a new number that is 6 more than this number?***

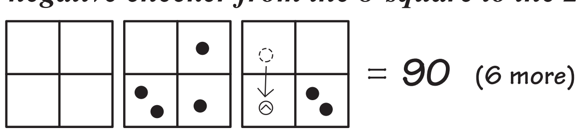
Your students may first think about moving a regular checker. They should find that there is no move with a regular checker that will increase the number by 6. This should focus their attention on the negative checker.

S: ***We can't do it because we don't have a checker on the 2-square (or the 4-square).***

T: ***Where would we move it if we did?***

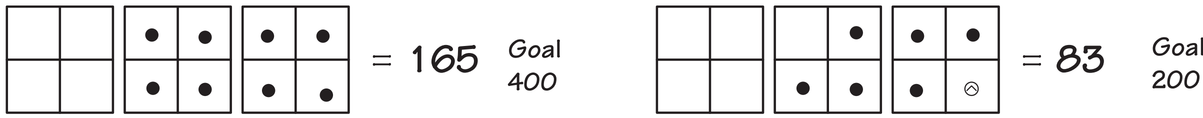
S: ***To the 8-square (or to the 10-square).***

S: ***We could move the negative checker from the 8-square to the 2-square.***

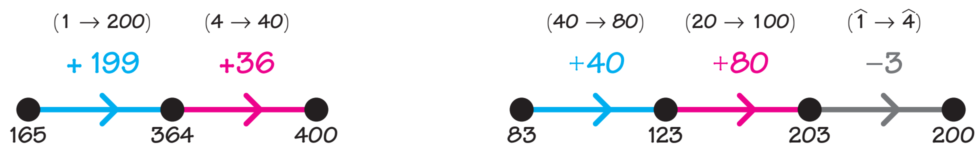


Exercise 2: Minicomputer Golf

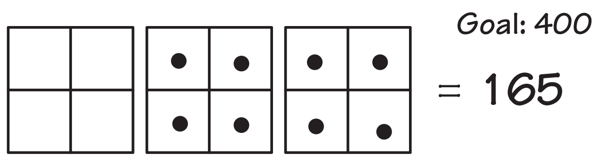
Play Minicomputer Golf as described in Lesson N15. Starting configurations and goals for two games are suggested below.

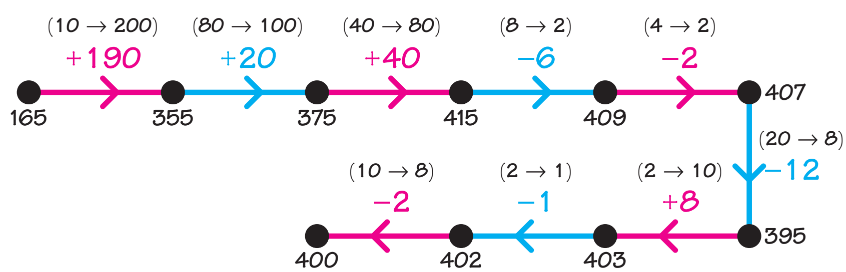


As a cooperative game, you could challenge the class to reach the goal with as few moves as possible. Sample games with minimal numbers of moves are recorded below.



You may wish to play Minicomputer Golf as a competition between two or more teams. Teams can take turns making moves, and the first team to reach the goal wins the game. It's a good idea to require that a move must take the number towards the goal. That is, when the number on the Minicomputer is below the goal, a move should increase the number, and when it is above the goal, a move should decrease the number. A sample game with a Red Team and a Blue Team is recorded in the arrow picture below.





The Red Team wins!

N36 MULTIPLICATION PROBLEMS #2

Capsule Lesson Summary

Put four checkers on the 8-square; then move the checkers to the 80-square, to the 800-square, and to the 8000-square. Each time, ask what number is on the Minicomputer and record appropriate number sentences. Repeat this activity with another configuration. Use the Minicomputer to help solve several multiplication problems.

Materials

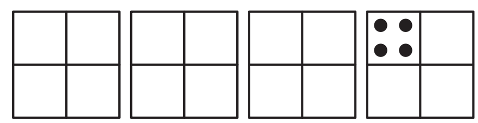
Teacher	• Minicomputer set	• Paper
Student	• Minicomputer set	

Description of Lesson

Exercise 1

Write corresponding number sentences on the board as you do this activity.

Put this configuration on the Minicomputer.



T: ***What calculation is on the Minicomputer?***

S: ***4 x 8.***

T: ***What number is 4 x 8?***

S: ***32.***

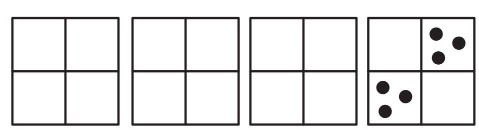
Move the four checkers first to the 80-square, then to the 800-square, and finally to the 8,000-square, each time asking the same questions.

$$4 \times 8 = 32$$
$$4 \times 80 = 320$$
$$4 \times 800 = 3{,}200$$
$$4 \times 8{,}000 = 32{,}000$$

T: ***What patterns do you see in these number sentences?***

Let students make observations. Some will likely comment on same digits and on the number of zeros. You may like to add 4 x 8,000,000 to your list and ask students to complete the number sentence.

Repeat this activity starting with this configuration on the Minicomputer.



Again, generate a list of similar number sentences.

$$3 \times 6 = 18$$
$$3 \times 60 = 180$$
$$3 \times 600 = 1{,}800$$
$$3 \times 6{,}000 = 18{,}000$$

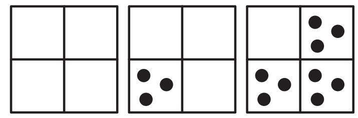
N36

Exercise 2

Announce to the class that you would like to use the Minicomputer to help answer some multiplication problems.

T: ***Alberto bought three big packs of chewing gum. Each pack has 27 pieces of gum. How many pieces of gum does Alberto have? How can I show this problem on the Minicomputer?***

S: ***Put on 3 x 27.***



Direct students to put 3 x 27 on their desk Minicomputers, and call on a student to put it on the demonstration Minicomputer. Ask students to observe 3 x 27 on the Minicomputer, and suggest they write the calculation on their papers.

T: ***What number is on the ones board?***

S: ***3 x 7 = 21.***

T: ***What number is on the tens board?***

S: ***3 x 20 = 60.***

T: ***What number is 3 x 27?***

S: ***81.***

$$\begin{array}{r} 3 \times 7 = 21 \\ 3 \times 20 = 60 \\ \hline 3 \times 27 = 81 \end{array}$$

Show students a shorter way to write this multiplication calculation. Ask students to complete the calculation on their papers.

$$\begin{array}{r} 27 \\ \times\ 3 \\ \hline 21 \\ +\ 60 \\ \hline 81 \end{array}$$

Repeat this exercise to calculate the following:

- How many eggs in five dozen?
- How many wheels on 46 tricycles?
- How many horns on 53 triceratops?
- How many legs on 14 puppies?
- How many hours in five days?

You may like to solicit multiplication problems like these from the students. That is, invite a student to pose a problem for everyone to solve.