Workbooks

WORKBOOKS TABLE OF CONTENTS

WORKBOOKS INTRODUCTION

There are many opportunities for students to work individually during the course of the lessons described in the other content strands. In the Workbooks strand, however, it is this individualized work that becomes the chief end of the majority of lessons. The goal in this strand is to provide students with opportunities

- to review many of the ideas they have met in other content strands;
- to apply their acquired knowledge to new situations requiring various kinds of strategic thinking; and
- to learn how to read and use mathematics workbooks.

The following workbooks are provided:

- *Galaxy of Problems #1*
- *Galaxy of Problems #2*
- *Galaxy of Problems #3*
- *Galaxy of Problems #4*
- *Fishing for Numbers, Part IV*

One story-workbook is provided:

- *To Picture*

Each workbook contains problems of varying levels of difficulty. Approximately the first ten pages of each workbook are easy problems, the next ten to twelve pages are average level difficulty, and the last ten pages are more challenging problems. For each workbook, we suggest that all students start work at the easiest level (i.e., on page 2) and then work through as many pages as they can handle during the two lessons scheduled for that workbook. We estimate that, in a typical class, about two-thirds of the students will correctly finish the first ten pages, about one-third will finish the first twenty pages, and a few will finish all or most of the workbook. These proportions will vary from class to class.

This guide contains an answer key for each workbook. The key follows an introduction to the workbook and a suggested collective lesson. The lesson either presents the workbook to the whole class or provides a warm-up activity on a problem similar to one found in the workbook.

Ten of the 17 lessons in the strand have the individual work in the workbooks as their main activity. In the story-workbook *To Picture*, used in two lessons, students locate eight numbers in an arrow picture and in related arrow pictures. The other five lessons present detective stories where there are one to three secret numbers and several clues that lead to their discovery. The clues review many ideas from the other content strands as well as involve students in new situations involving patterns and counting techniques.

Use of the Workbooks Strand for Evaluation Purposes

The workbooks provide an excellent instrument to assess the progress of your students on a regular basis. You may not feel it is necessary to check every page and problem for each student, but you should develop a procedure for checking students' work with which you are comfortable. This may include checking one or more specific pages, discussing some particular mistakes with individual students and letting them correct their work, or just looking carefully at a few pages to be sure the students have understood the general idea of the problems in that particular book.

In the Blacklines, you will find a record-keeping tool for each workbook to help you assess student progress in the various strands. This tool may also assist you in parent conferences and in filling out periodic progress reports.

Here are some important points to bear in mind for workbooks.

- Always read the introductory material for each workbook and give the short introductory collective lesson(s).
- All students should start at the beginning of each workbook and progress as far as they can.
- All students should begin a new title on the same day, even if some students have not finished work on the previous title.

Capsule Lesson Summary

Complete a subtraction table. Begin the workbook *Galaxy of Problems #1*.

Materials

Teacher	• None	Student	• *Galaxy of Problems #1* Workbook • Colored pencils, pens, or crayons

Description of Lesson

Draw this subtraction table on the board.

–	9	6	3	0
9				
7				
5				

T: ***The minus sign here*** (point to – in the upper left corner) ***tells us that this is a subtraction table. When this table is complete, there will be a number in each square. To decide which numbers are in the table, we start with a number in the left column*** (point to 9, 7, and 5) ***and then subtract one of the numbers in the top row*** (point to 9, 6, 3, and 0)***.***

Point to 7, –, and 6 in the table as you say,

T: ***For example, one of the numbers in this table is 7 – 6. What number is 7 – 6?*** (1) ***Where should we put 1?***

Some of your students may know how to complete an operation table like this. Invite a student to show where to put 1 for 7 – 6 = 1 or, if necessary, do so yourself. Simultaneously trace the 7-row with your left index finger and the 6-column with your right index finger as you say,

T: ***7 – 6 = 1.***

Write 1 in the square where your fingers cross. Emphasize that the 1 is in the same row as 7 and in the same column as 6.

–	9	6	3	0
9			m	
7		1		
5				k

Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T (pointing to **m**)**:** ***What number goes here? What calculation should we do?***

S: ***9 – 3 = 6, so 6 is in that square.***

T (pointing to **k**)**:** ***What number goes in this square? Whisper your answer to a neighbor.***

Check answers from several students before letting a student write the answer in the table.

–	9	6	3	0
9			6	
7		1		
5				5

T: ***Who can fill in another square of this table?***

Continue inviting students to fill a square of the table until it is complete. Emphasize that only the numbers in the column under – and in the row to the right of – are used to decide which numbers go in the table. Frequently trace the appropriate row and column and say the subtraction calculation corresponding to a number entered in the table.

The completed table is shown here.

–	9	6	3	0
9	0	3	6	9
7	2	1	4	7
5	4	1	2	5

Distribute copies of the workbook *Galaxy of Problems* #1, and allow students to work independently for the rest of the class period. Emphasize that the students should read instructions on each page carefully. You may like to read the instructions on pages 2 and 3 collectively before the students begin working individually.

At the end of the lesson, collect the workbooks for your review. They will be used again in Lesson W2.

Capsule Lesson Summary

Continue working in the workbook *Galaxy of Problems #1*. (This is the second of two lessons using this workbook.) Play The Number Line Game.

Materials

Teacher	• None	Student	• *Galaxy of Problems #1* Workbook • Colored pencils, pens, or crayons

Description of Lesson

Distribute students' copies of the *Galaxy of Problems #1* Workbook. Ask students first to correct or complete pages from their previous work on this workbook. Your review of the workbooks may indicate that a short collective discussion about a particular page in the workbook is needed.

Allow approximately 30–40 minutes for students to work on their workbooks. Then collect the workbooks for your review. Use any remaining class time to play The Number Line Game as described on the following page.

Assessment Activity

An individual student progress report for the workbook is available on Blackline W2(a). You may like to use this form to monitor student work.

Home Activity

If you choose to send workbooks home with students, you may want to include a letter (reminder) to parents/guardians with this workbook. Blackline W2(b) has a sample letter.

THE NUMBER LINE GAME

The Number Line Game may be played frequently throughout the year whenever you finish a lesson early or have some extra time. Each game takes approximately five to ten minutes. There are several variations of this game; use these variations when your class is ready for them.

T: ***Today we are going to play The Number Line Game. I'm thinking of a number between 500 and 1,000. I'll show you where it is on the number line; my number is hidden in the box.***

Draw a line on the board and position a box for the secret number near the center of your picture. Begin with the box near the center of your line. For your convenience in locating numbers, you may wish to extend the line during the game.

The following is a description of a possible game. Suppose the secret number is 827.

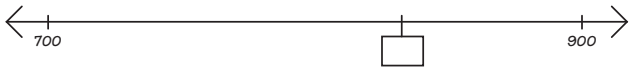
S: ***700.***

T: ***I'll show you where 700 is on this number line. It might not be in exactly the right place, but I'll try to make it as close as possible.***

700 is less than my number.

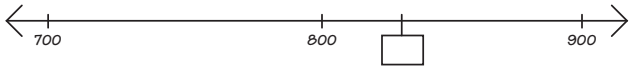
S: ***900.***

T: ***900 is more than my number.***



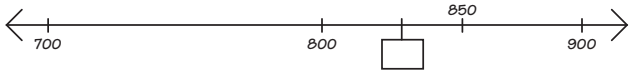
S: ***800.***

T: ***800 is less than my number.***



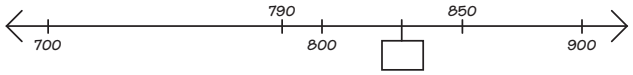
S: ***850.***

T: ***850 is more than my number.***



S: ***790.***

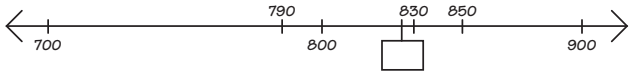
T: ***790 is less than my number.***



Note: Although a guess may give no new information, such as 790 in this sample game, do not criticize or comment except to say that it is more or less than your number.

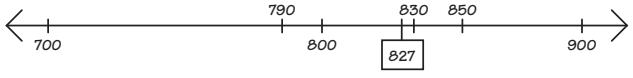
S: ***830.***

T: ***830 is more than my number.***



S: ***827.***

T: ***You have discovered my secret number!***



Play the game a few more times. Let the student who guesses the correct number choose the secret number for the next game and whisper it to you. Ask that student to keep the number a secret and not to play in that game. You may like to allow the student to help judge the game. (See Variation 4 of The Number Line Game).

VARIATIONS OF THE NUMBER LINE GAME

Variation 1

As your class becomes familiar with The Number Line Game, expand the range of possible numbers. Possibilities include the following:

- numbers between 1,000 and 1,500
- numbers between M100 and 100
- numbers between 1,900 and 2,100
- numbers between M200 and M100

Variation 2

As the students become familiar with numbers such as $3\frac{1}{2}$, occasionally choose a non-integer as your secret number. It is best to limit the range of possible numbers to those between 0 and 20 when you use this variation.

Variation 3

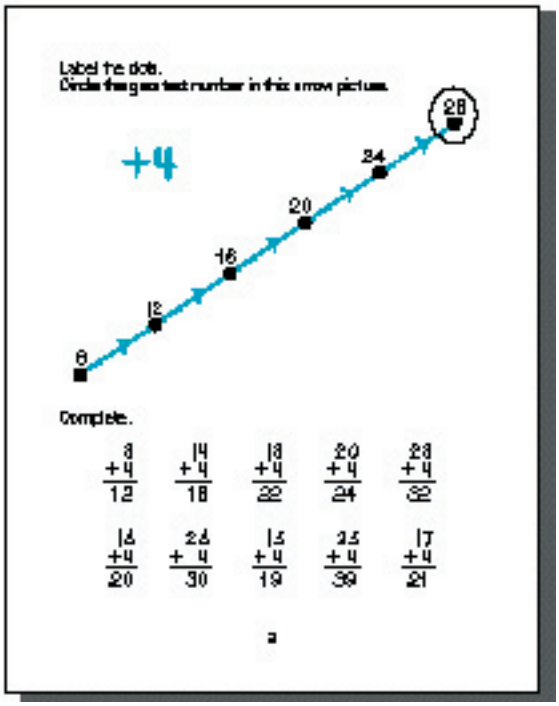
As the students become familiar with numbers such as 0.58, occasionally choose a non-integer decimal number as your secret number. Again, it is best to limit the range of possible numbers to those between 0 and 20 when you use this variation.

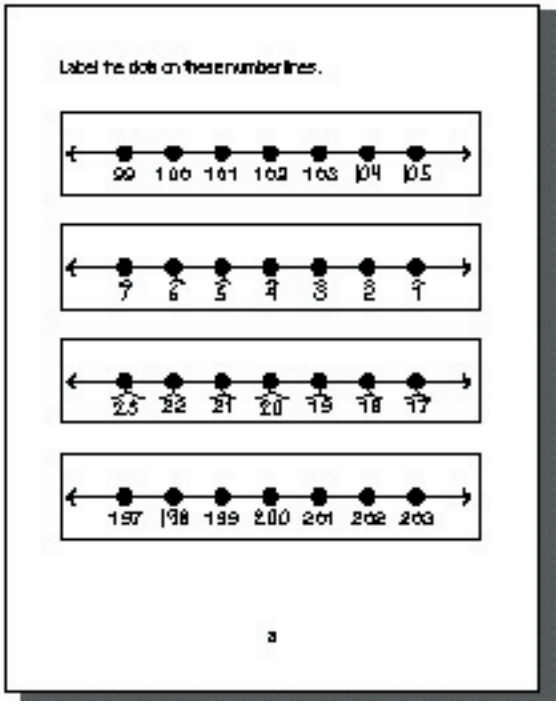
Variation 4

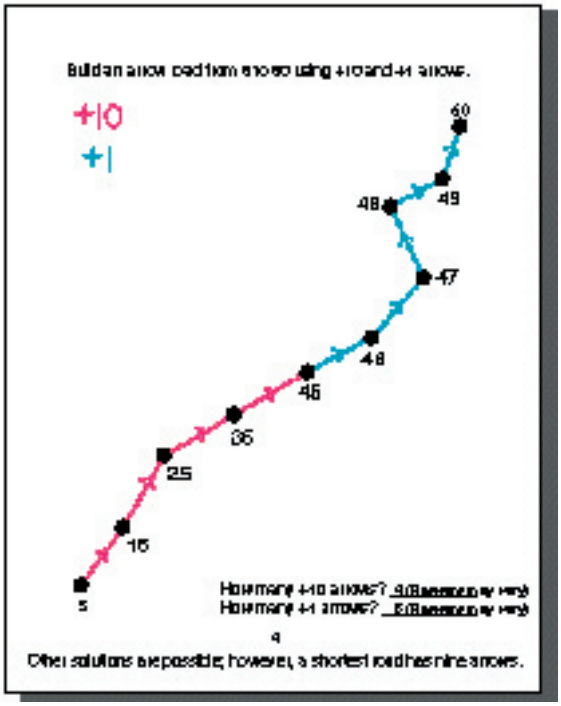
When you are using only the whole numbers for your range of possible numbers, ask a student to choose a secret number and to judge the guesses of the other students. The student who chooses the secret number tells the class whether a guess is more or less than the secret number. You label a mark on the number line for each guess, and if necessary, help the student to judge. Remind student judges not to give additional clues.

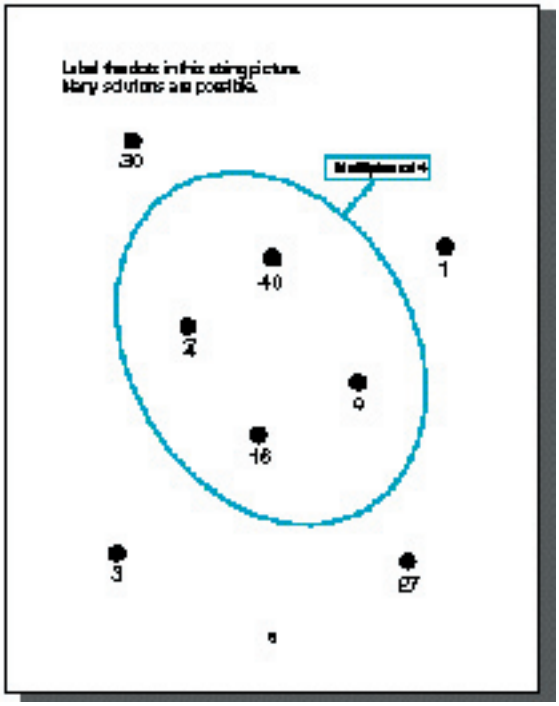
Variation 5

Divide the class into two teams. Let teams alternate turns and let the members of each team take turns guessing what the secret number is.


Label the dots.
+4
28
24
20
16
12
8
Complete.
8 +4 12
14 +4 18
18 +4 22
20 +4 24
28 +4 32
16 +4 20
26 +4 30
15 +4 19
35 +4 39
17 +4 21


Label the dots on these number lines.
99 100 101 102 103 104 105
7 6 5 4 3 2 1
23 22 21 20 19 18 17
197 198 199 200 201 202 203


+10
+1
60
49
48
47
46
36
25
15
5
How many +10 arrows? 4 Guarantee by very
How many +1 arrows? 5 Guarantee by very
Other solutions are possible; however, a shortest road has nine arrows.


Label the dots in this string picture.
Many solutions are possible.
30
40
1
2
9
16
3
87

Complete this addition table.

+	10	9	7	8
10	20	19	17	18
8	18	17	15	16
9	19	18	16	17
7	17	16	14	15

9 + 9 = 18 | 7 + 8 = 15 | 8 + 9 = 17 | 7 + 7 = 14 | 10 + 9 = 19

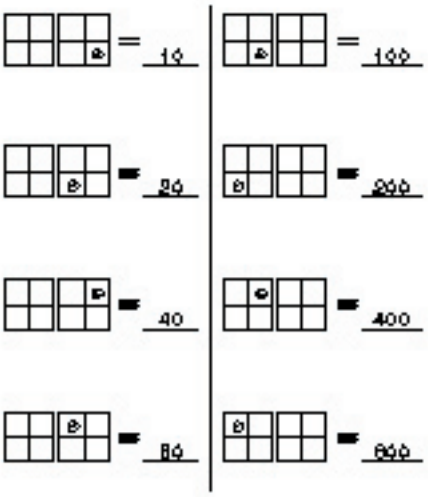
6 + 8 = 14 | 7 + 11 = 18 | 12 + 9 = 21 | 15 + 8 = 23 | 15 + 10 = 25

6

7 is the least number in this arrow picture. Label the dots.

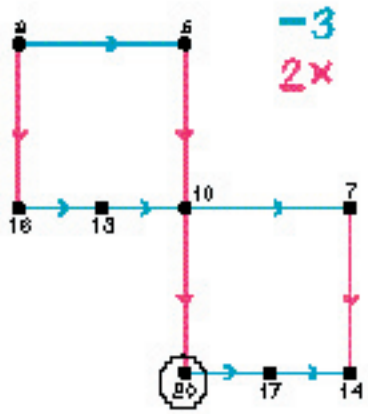
7

What number is on the Mini computer?

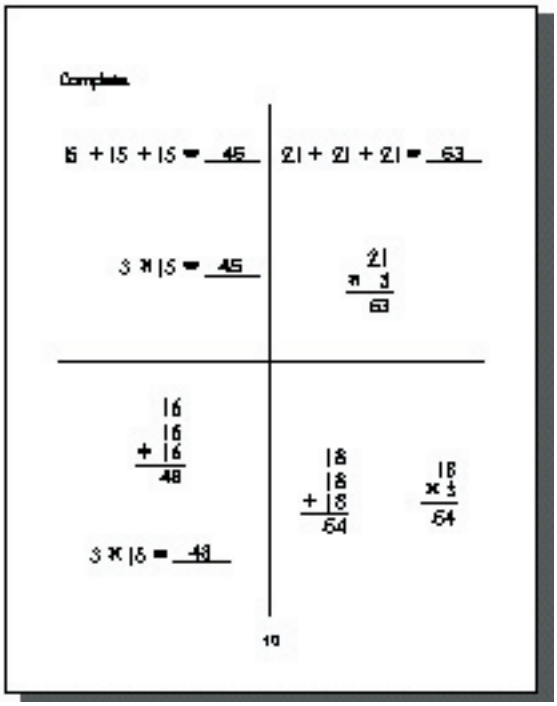


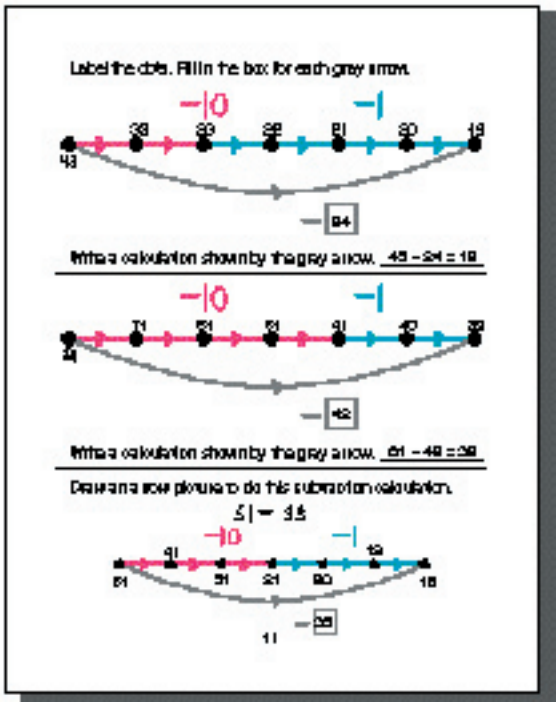
8

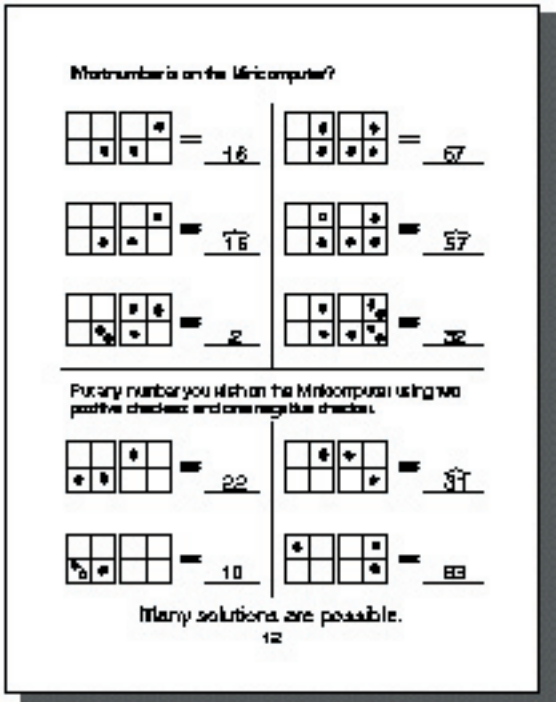
Label the dots.
Circle the greatest number in this arrow picture.

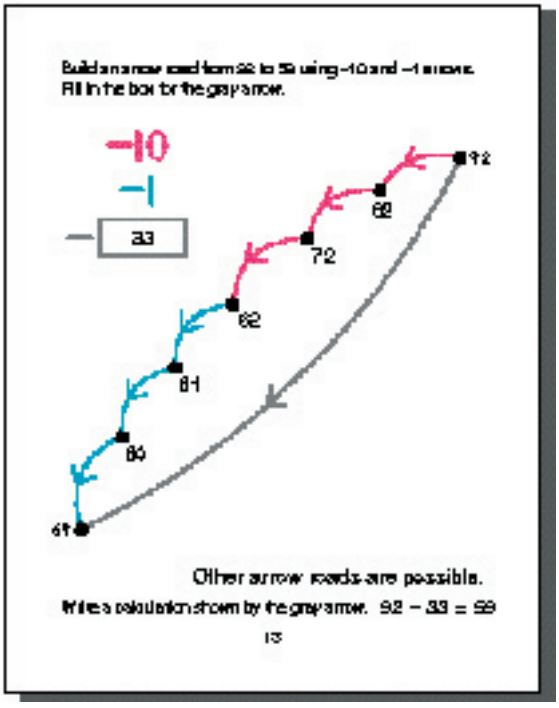


9


Complete.
15 + 15 + 15 = 45
21 + 21 + 21 = 63
3 × 15 = 45
21
× 3
63
16
16
+ 16
48
16
16
+ 16
64
16
× 4
64
3 × 16 = 48
10


Label the dots. Fill in the box for each gray arrow.
−10
−1
93
88
82
81
80
79
73
84
Write a calculation shown by the gray arrow. 93 − 84 = 19
−10
−1
71
61
51
41
42
32
42
Write a calculation shown by the gray arrow. 61 − 42 = 28
Draw an arrow picture to do this subtraction calculation.
51 − 15 = 36
−10
−1
41
19
51
31
21
20
16
36
11


What number is on the Mini computer?
= 16
= 57
= 15
= 57
= 2
= 32
Put any number you wish on the Mini computer using two positive checkers and one negative checker.
= 22
= 31
= 10
= 88
Many solutions are possible.
12


Build an arrow road from 92 to 59 using −10 and −1 arrows.
Fill in the box for the gray arrow.
−10
−1
33
92
82
72
62
61
60
59
Other arrow roads are possible.
Write a calculation shown by the gray arrow. 92 − 33 = 59
13

Complete this subtraction table.

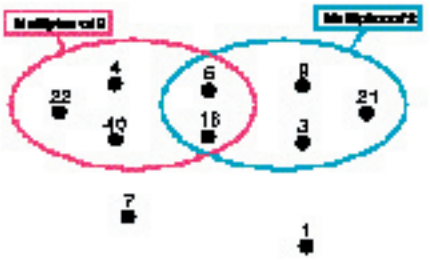
−	5	7	3	8
10	5	3	7	2
15	10	8	12	7

Complete this multiplication table.

×	6	4	9	10
2	12	8	18	20
3	18	12	27	30

14

Label the dots in this string picture.
Many solutions are possible.



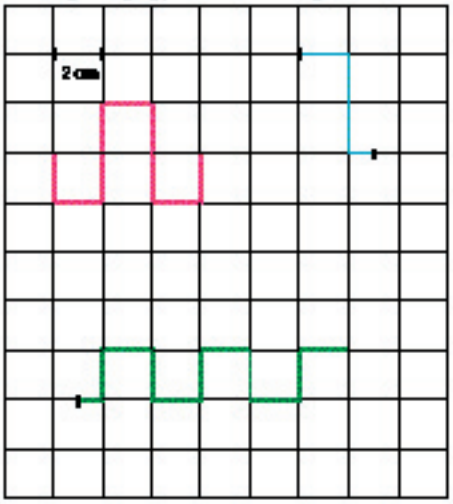
13

Rook is a special number.
Rook is in this arrow picture and in this string picture.

Who is Rook? 21

15

Draw a red zigzag path that is 18 cm long.
Draw a blue zigzag path that is 7 cm long.
Draw a green zigzag path that is 21 cm long.



17

Many solutions are possible.

Complete these number sentences.

$(6 + 5) \times 2 =$ 22

$6 + (5 \times 2) =$ 16

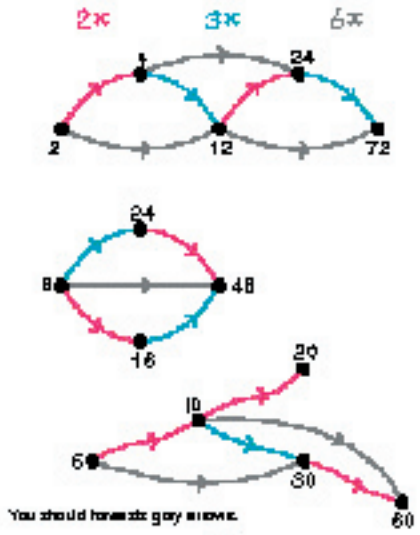
$(7 - 2) \times 3 =$ 15

$7 - (2 \times 3) =$ 1

$(7 - 3) \times 2 =$ 8

18

Label the dots. Draw all the possible 6x (gray) arrows.



You should have six gray arrows.

19

Put these numbers on the Mini computer.

4.05 =

8.60 =

0.79 =

3.00 =

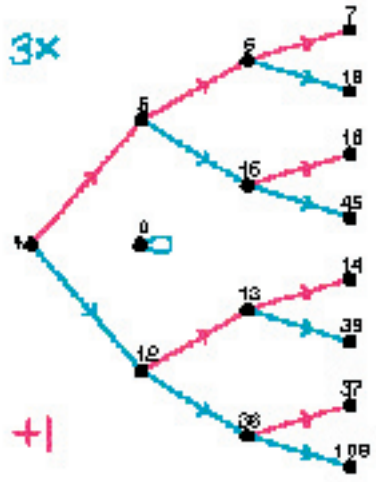
2.0 =

1.4 =

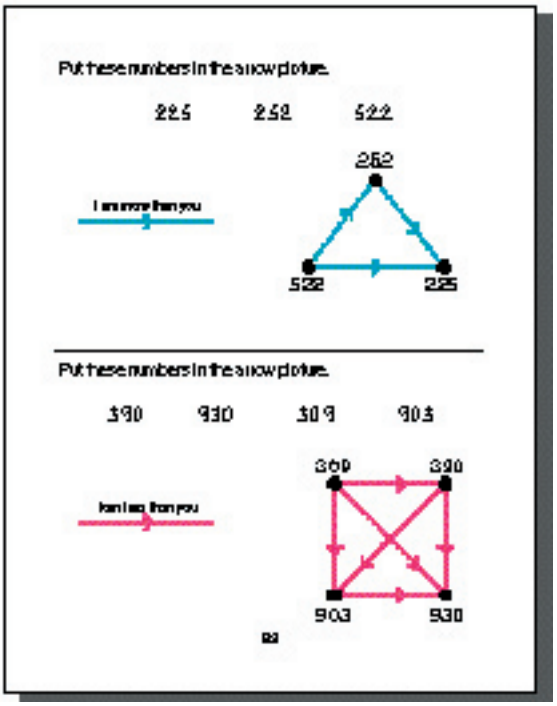
Numbers are shown in standard configurations.
Other solutions are possible.

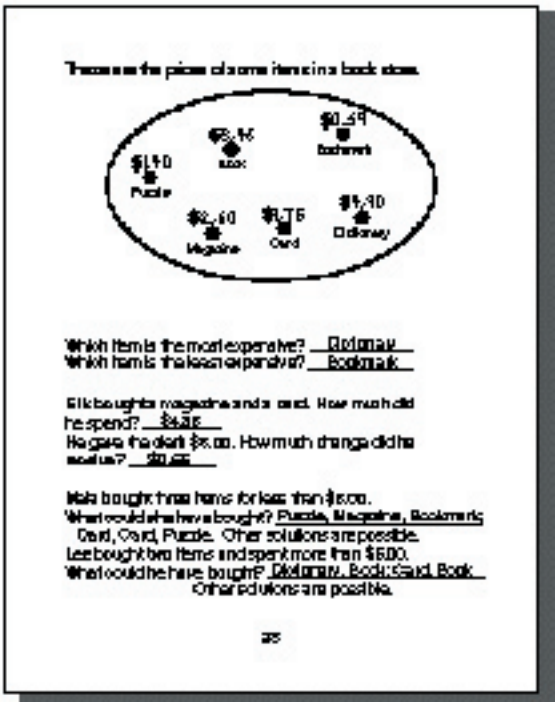
20

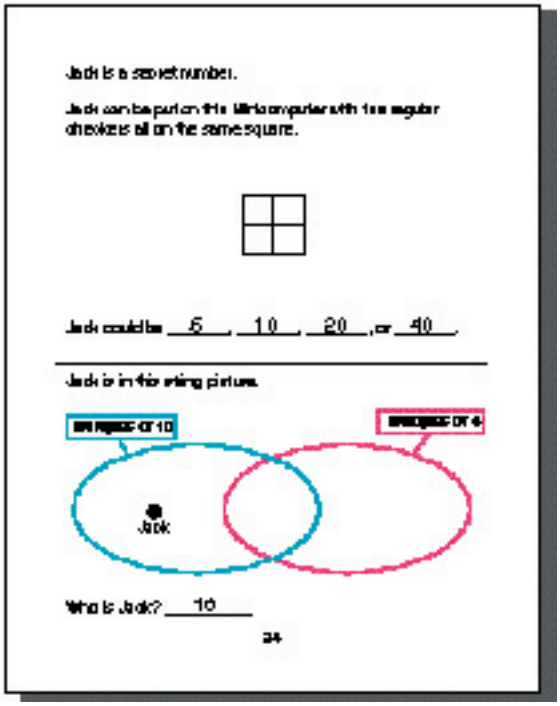
Label the dots.

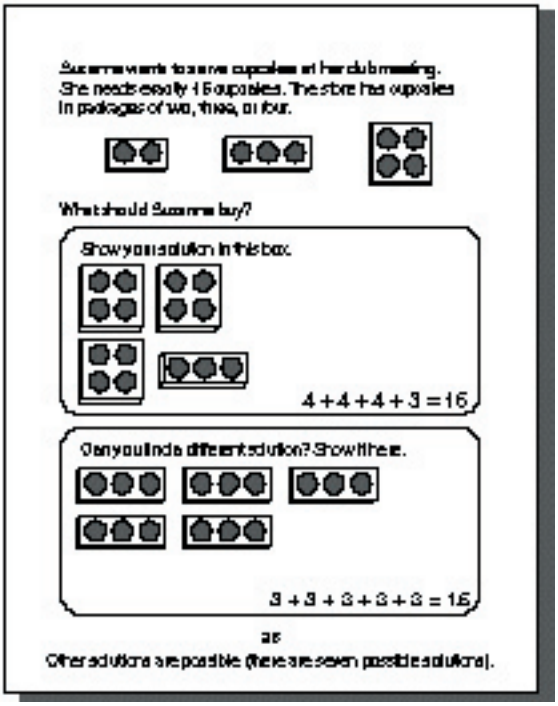


21

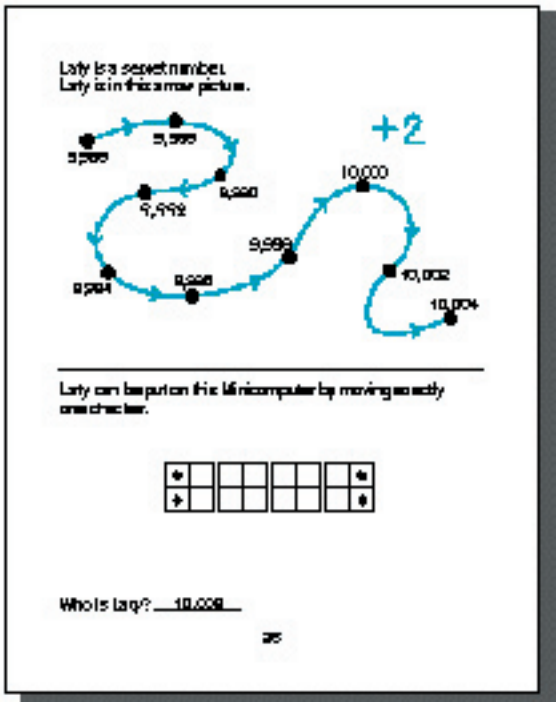


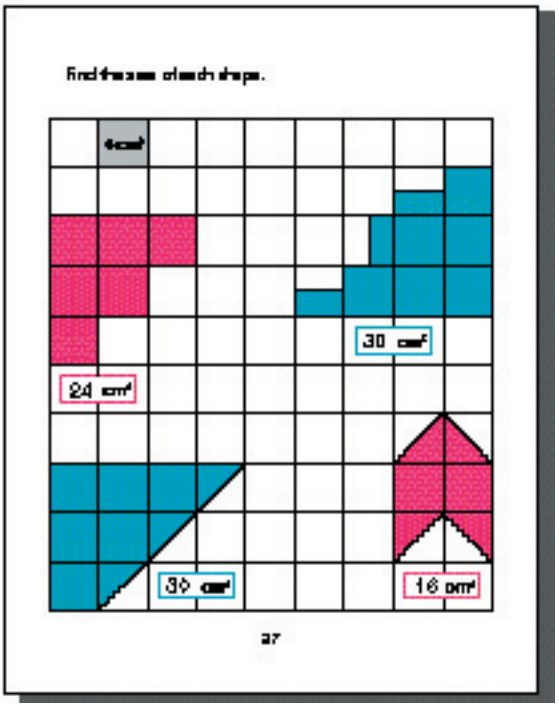


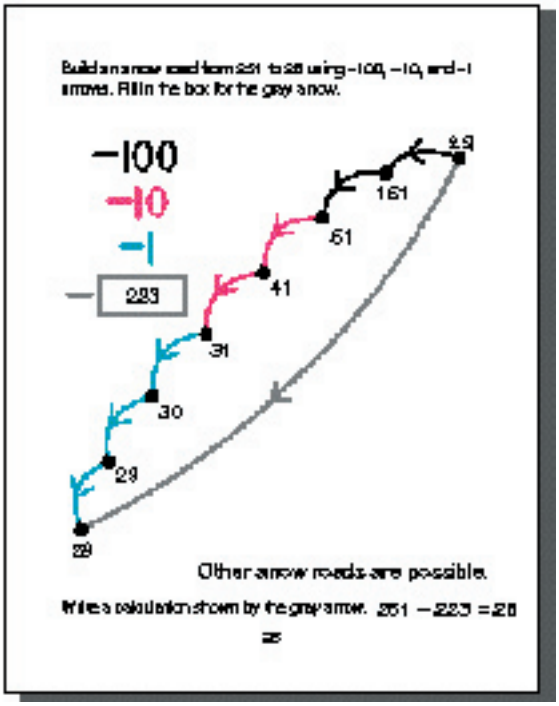


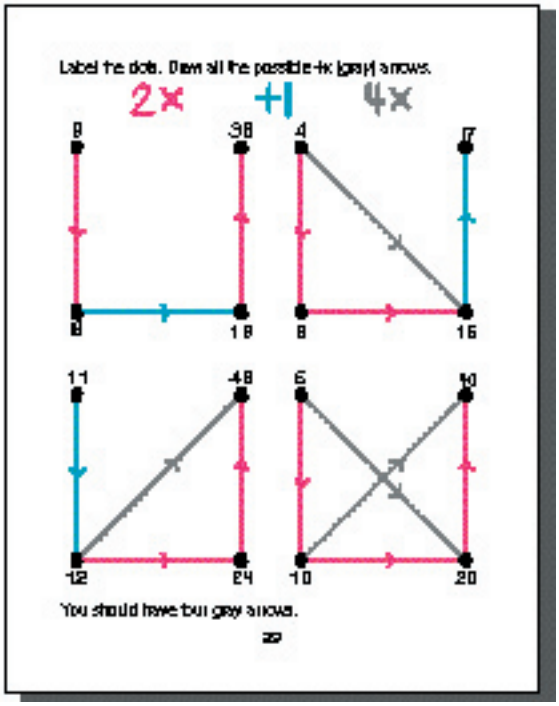


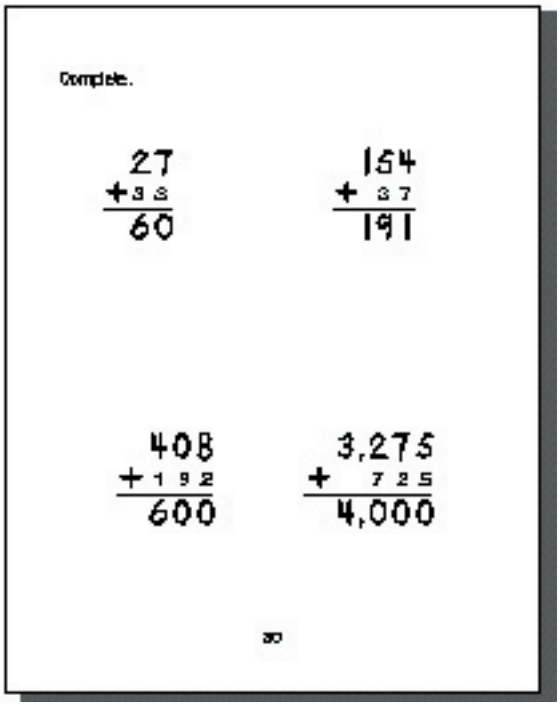
Other solutions are possible (there are seven possible solutions).

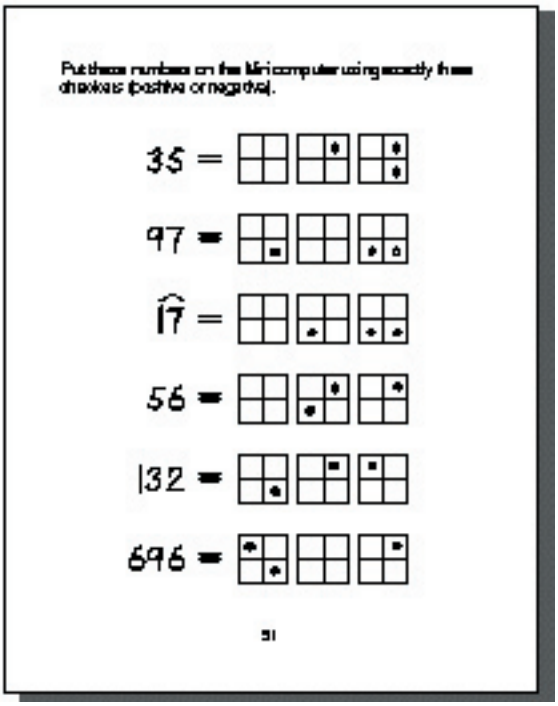

+2
10,000
10,002
10,004

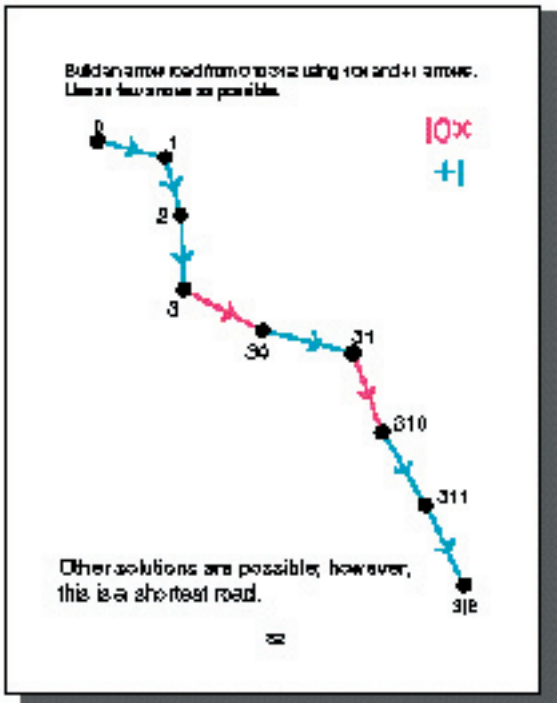

30 cm²
24 cm²
16 cm²


−100
−10
−1
223
161
51
41
31
30
29
29


2×
+1
4×
8
38
8
18
4
17
8
16
11
48
12
24
6
10
10
20


Complete.
27
+33
60
164
+ 37
191
408
+192
600
3,275
+ 725
4,000


Put these numbers on the Mini computer using exactly these strokes (positive or negative).
35 =
97 =
17 =
56 =
132 =
696 =


10×
+1
0
1
2
3
30
31
310
311
318
Other solutions are possible, however,
this is a shortest road.

Capsule Lesson Summary

Solve a detective story with clues involving regular and s-checkers on the Minicomputer, a string picture, and an arrow picture.

Materials

Teacher	Student
• Colored chalk • Minicomputer set • s-checkers	• Minicomputer set • Paper • Colored pencils, pens, or crayons • Worksheets W3*, **, ***, and ****

Description of Lesson

You may like to allow students to work with a partner during this lesson.

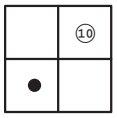
T: ***Today you will have another opportunity to be detectives. Ply is the secret number. Follow the clues carefully and you will be able to discover who Ply is.***

Clue 1

Display one Minicomputer board, one regular checker, and one s-checker. Student pairs should have the same in their individual Minicomputer sets.

T: ***Ply can be put on this Minicomputer using exactly these two checkers (one regular and one s-checker.) What numbers could be Ply?***

Invite students to put numbers on the Minicomputer using the two given checkers. When one student puts a number on the Minicomputer, ask another student to identify the number. For example, suppose a student puts this configuration on the Minicomputer.



T (pointing to the s-checker)**:** ***10 x 4 = …?***

S: ***40.***

T (pointing to the regular checker)**:** ***… plus 2 more? What number is on the Minicomputer?***

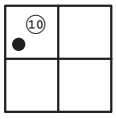
S: ***42.***

Continue this activity until three or four possible numbers are found. Record these possibilities on the board.

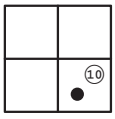
T: ***How many different numbers can we put on this Minicomputer using just these two checkers?***

Allow several students to predict the number of possibilities.

T: ***What is the greatest such number?*** (88)
What is the least such number? (11)



= 88



= 11

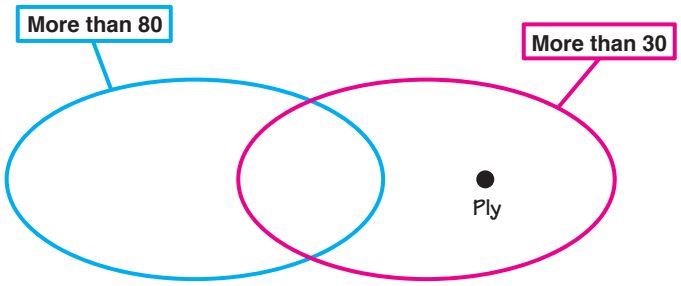
Continue asking students to put numbers on the Minicomputer until your list is complete.

11	12	14	18
21	22	24	28
41	42	44	48
81	82	84	88

You may wish to commend students who predicted there were sixteen numbers that could be put on the Minicomputer using exactly one regular and one s-checker.

Clue 2

Draw this string picture on the board and ask students to copy it on their papers.



T: ***What does this string picture tell us about Ply?***

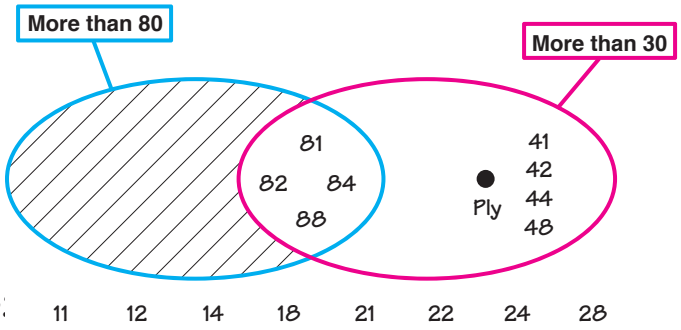
S: ***Ply is more than 30.***

S: ***Ply is less than 80.***

Encourage several students to comment. Then suggest students find several numbers from Clue 1 that could be Ply and write them on their papers. You may like to ask students to put all the numbers from Clue 1 into the string picture.

After a few minutes, invite students to put the numbers from Clue 1 into the string picture on the board.

Observe with the class that no number goes in the region of the blue string outside the red string because all numbers that are more than 80 are also more than 30. Hatch the region.



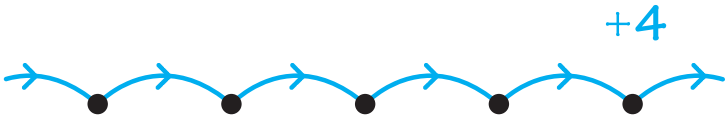
T: ***Now, which numbers still can be Ply?***

S: ***41, 42, 44, and 48.***

Erase everything on the board except the four numbers left for Ply?

Clue 3

Draw this arrow picture on the board and ask students to copy it on their papers.



T: ***What do the unfinished arrows tell us?***

S: ***The arrow picture continues in both directions.***

T: ***What can we say about all the numbers on this +4 arrow road?***

S: ***All of the numbers on this arrow road could be multiples of 4.***

T: ***If one of the numbers on this arrow road is a multiple of 4, then all the numbers are multiples of 4. What if one of the numbers on this road is not a multiple of 4?***

S: ***Then none of the numbers would be multiples of 4.***

T: ***What if one of these numbers is an odd number?***

S: ***Then all of the numbers would be odd numbers.***

T: ***… and if one of them is an even number?***

S: ***Then all of them would be even numbers.***

T: ***Ply is one of the numbers on this arrow road. I will put one number in this arrow picture. Label the dots in your picture and try to find Ply. You will need to think about numbers the arrow road meets when you extend it.***

Label any one of the dots in the arrow picture 6. Allow several minutes for students to work, and check many of their papers before asking a student to reveal Ply's identity. (42) Discuss with the class why 42 is the only number left in the list after Clue 2 that would also be on this +4 arrow road.

T: ***Let's check the other numbers before we agree that Ply is 42. Could 41 be the secret number?***

S: ***No, because 41 is an odd number.***

T: ***Could 44 be Ply?***

S: ***44 is a multiple of 4, and 6 and is not a multiple of 4, so they will not be on the same +4 arrow road.***

T: ***Could Ply be 48?***

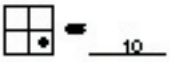
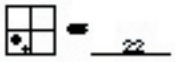
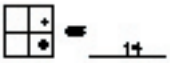
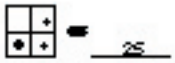
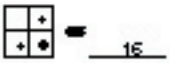
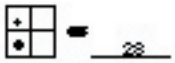
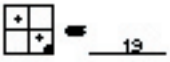
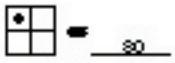
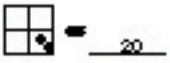
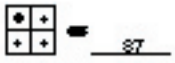
S: ***48 is also a multiple of 4.***

Conclude that Ply is 42.

Worksheets W3*, **, ***, and **** are available for individual work.

Name _____ W3 ★

What number is on the Minicomputer?

 = 10	 = 22
 = 14	 = 25
 = 16	 = 28
 = 19	 = 80
 = 20	 = 87

Name _____ W3 ★★

Put each number on the Minicomputer. Use at least one ⊕ checker for each number.

20 = 	30 = 
21 = 	34 = 
40 = 	70 = 
42 = 	79 = 
45 = 	100 = 

Other solutions are possible.

Name _____ W3 ★★★

Gobb is a secret number.

Clue 1

Gobb can be put on the ones board of the Minicomputer with exactly two ⊕ checkers.

Gobb could be 20, 30, 40, 50, 60, 80, 90, 100, 120, and 180.

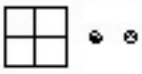
Clue 2

Who is Gobb? 90

Name _____ W3 ★★★★

Flobb is a secret number.

Clue 1 Flobb can be put on the ones board of the Minicomputer with one ⊕ checker and one negative checker.

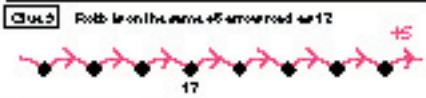


Flobb could be 2, 6, 8, 9, 12, 16, 18, 19, 28, 36, 38, 39, 78, 76, 78, and 72.

Clue 2

Flobb could be 38, 36, 38, and 32.

Clue 3 Flobb is on the same +5 arrow as 17



Who is Flobb? 38

Capsule Lesson Summary

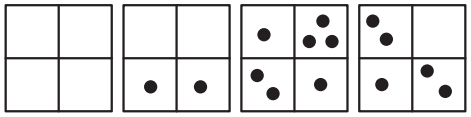
Explore the effect of moving, removing, or adding checkers to a configuration on the Minicomputer—in each case, does the numerical value increase, decrease, or stay the same? Estimate a number on the Minicomputer and then make trades, stopping periodically to get better estimates, until standard configuration is obtained. Begin the workbook *Galaxy of Problems #2*.

Materials

Teacher	Student
• Minicomputer set	• Index card • *Galaxy of Problems #2* Workbook • Colored pencils, pens, or crayons

Description of Lesson

Put this configuration on the Minicomputer.



Lead the class to estimate the number.
A possible dialogue is given here.

T: ***Is the number on the Minicomputer more than 200? How do you know?***

S: ***Yes, 300 is on the hundreds board and 300 is more than 200.***

T: ***Is this number more than 400? How do you know?***

S: ***The number on the hundreds board is 300 and there is more than 100 on the tens board (80 + 20 = 100), so this number is more than 400.***

T: ***Is this number more than 500? How do you know?***

A student should explain that this number is more than 500 by indicating checkers for still another hundred on the tens board. Eventually the class may estimate this number to be between 500 and 600, and perhaps closer to 600.

T: ***We know that this number is more than 500. We do not need to know exactly what number is on the Minicomputer to compare it to other numbers.***

Write these words on the board close to the Minicomputer, and ask students to write them on their index card.

More
Same
Less

T: ***I am going to move, remove, or add some checkers. Each time, tell me if the number on the Minicomputer is more than, the same as, or less than before.***

Move a checker from the 10-square to the 2-square. Decide on a method for students to show whether they believe the new configuration is for a number more than, the same as, or less than before. For example, they could hold up the index card pinching it on their choice.

Repeat the move very obviously if many students do not know that the number on the Minicomputer is less than the previous number.

T: ***How much less is this number than the number we had before? How do you know?***

S: ***It is 8 less, because you moved the checker from the 10-square to the 2-square.***

Return the checker to its original position. Continue this activity with the following or similar moves. After each move return the checkers to their original positions.

- Make an 80 + 20 = 100 trade. (Same)
- Move a checker from the 1-square to the 10-square. (9 more)
- Move a checker from the 8-square to the 2-square. (6 less)
- Move a checker from the 80-square to the 20-square. (60 less)
- Move a checker from the 200-square to the 800-square. (600 more)
- Remove two checkers from the 8-square, and put one checker on the 10-square. (6 less)

After you make several moves yourself, invite students to transform the number on the Minicomputer. After each move, return the checkers to their original positions.

At the end of this activity you should have the same configuration you started with on the Minicomputer. Remind students that they estimated the number to be between 500 and 600. Allow students to guess what the number is, and record their guesses on the board. Then invite some students to make trades that will make the number easier to read. After a couple of trades are made, guide the class to make a closer estimate. Continue making trades until the standard configuration is obtained. Invite a student to write the number below (or above) the Minicomputer, and decide which guess is the closest.

Distribute copies of the workbook *Galaxy of Problems #2* and let students work independently for the rest of the class period. If many students are having difficulty with a particular page, you may wish to have a collective discussion about that page.

At the end of the lesson, collect the workbooks for your review. They will be used again in Lesson W5.

Capsule Lesson Summary

Display some decimals on the Minicomputer and look at different decimal names for the same number. Decide how to share $10 equally among two, among four, and among eight people. Continue working in the workbook *Galaxy of Problems #2*. (This is the second of two lessons using this workbook.)

Materials

Teacher	• Minicomputer set • Colored chalk	Student	• *Galaxy of Problems #2* Workbook • Colored pencils, pens, or crayons

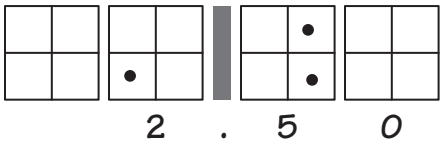
Description of Lesson

Exercise 1

Display four Minicomputer boards. Draw a bar between the second and third Minicomputer boards.

T: ***Who can put 2.5*** (read as "two point five") ***on the Minicomputer?***
Who can write this number below (above) the Minicomputer?

Perhaps the volunteer will write 2.50 below the Minicomputer. If the student writes 2.5, ask what could be written below the pennies board and write the 0 yourself.



T: ***Is this number 2.5?***

S: ***Yes, we can erase the 0 below the pennies board.***

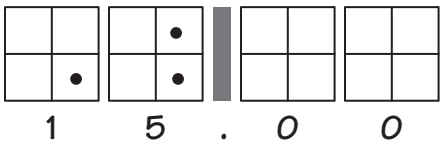
$$2.5 = 2.50$$

T: ***2.5 and 2.50 are the same number.***

Remove the checkers and erase the board.

T: ***Who can put 15 on the Minicomputer?***
Who can write this number below the Minicomputer?

Perhaps the volunteer will write 15.00.



T: ***Is this number 15?*** (Yes) ***What other names does this Minicomputer suggest for 15?***

S: ***15.00*** (read as "fifteen point zero zero")***.***

S: ***15.0.***

Record an equality expression on the board.

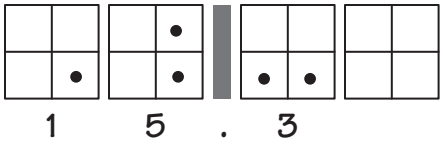
$$15 = 15.0 = 15.00$$

T: ***15, 15.0, and 15.00 are all names for the same number.***

Erase the board.

T: ***The number on the Minicomputer is 15. Who can put 15.3 on the Minicomputer by just adding some more checkers?***

Write 15.3 below (above) the Minicomputer.



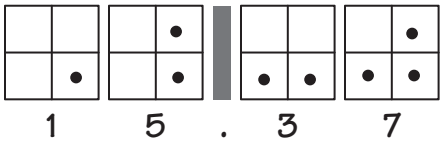
T: ***What is another name for 15.3?***

S: ***15.30.***

T: ***This 3*** (point to 3 in 15.3) ***is for three dimes, and three dimes is the same as 30¢.***

Erase the board.

T: ***The number on the Minicomputer is 15.3. Who can put 15.37 on the Minicomputer by just adding some more checkers?***



Remove the checkers and erase the board. Ask students to put these numbers on the Minicomputer and to write the appropriate numerals below the Minicomputer.

26	2.06	20.06
2.6	0.26	20.6

Remove the checkers and erase the board before going on to Exercise 2.

Exercise 2

T: ***Imagine that we have $10 and want to share it equally between two people. How much money should each person receive?***

S: ***$5 each because $\frac{1}{2} \times 10 = 5$.***

T: ***Suppose we want to share $10 equally among four people. How much for each person?***

S: ***$2.50 each because $2.50 + $2.50 = $5, and $2.50 + $2.50 = $5, and $5 + $5 = $10.***

Invite a student to write $2.50 on the board.

$2.50

T: ***The point separates the dollars and the cents. Now suppose we want to share $10 equally among eight people. How much money should each person receive?***

Your students might suggest several incorrect answers. The following dialogue assumes a student answers $1.50.

S: ***$1.50***

T: ***How much money do we need to give $1.50 each to eight people?***

Write the multiplication problem on the board. 8 x $1.50

T: ***Suppose we give each person a $1 bill and a 50¢ piece (half-dollar).***
How many $1 bills do we need? (Eight)
How much money is eight $1 bills? ($8) 8 x $1.50 = $8 +
How many half-dollars do we need? (Eight)
How much money is eight half-dollars?

S: ***$4, because two half-dollars make $1.*** 8 x $1.50 = $8 + $4

T: ***To give $1.50 to eight people, how much money do we need?***

S: ***$12.***

Complete the number sentence on the board. 8 x $1.50 = $8 + $4 = $12

T: ***$1.50 is too much to give each person because we have just $10 to share.***

S: ***Give each of them $1.25.***

T: ***How did you decide that each person should receive $1.25?***

S: ***When we share $10 among four people, each person gets $2.50. So I divided each of four shares in half. $1 is one-half of $2, and 25¢ is one-half of 50¢.***

Write the multiplication problem on the board. 8 x $1.25

T: ***Suppose we give each person a $1 bill and a quarter.***
How many $1 bills do we need? (Eight)
How much money is eight $1 bills? ($8) 8 x $1.25 = $8 +
How many quarters do we need? (Eight)
How much money is eight quarters?

S: ***Four quarters are $1, so eight quarters are $2.*** 8 x $1.25 = $8 + $2

T: ***To give $1.25 each to eight people, how much money would we need?***

S: ***$10.*** 8 x $1.25 = $8 + $2 = $10

Conclude that when $10 is shared equally among eight people, each person receives $1.25.

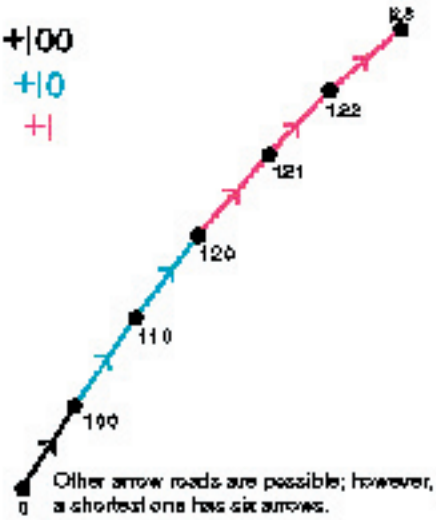
Distribute the students' copies of the *Galaxy of Problems #2* Workbook. Ask students first to correct or complete pages from their previous work, and then to continue. At the end of the lesson, collect the workbooks for your review.

Assessment Activity

An individual student progress report for the workbook is available on Blackline W5. You may like to use this form to monitor student work.

Build an arrow road from 0 to 189 using +100, +10, and +1 arrows.

+100
+10
+1



Other arrow roads are possible; however, a shortest one has six arrows.

c

Complete this addition table.

+	10	20	30	40
10	20	30	40	50
9	19	29	39	49
18	28	38	48	58

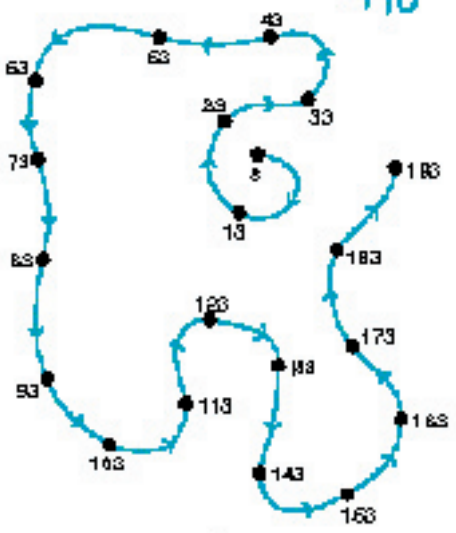
Complete this subtraction table.

−	7	10	5	6
11	4	1	6	5
15	8	5	10	9

a

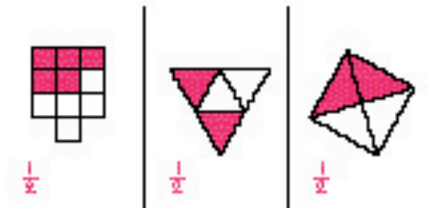
Label the dots.

+10

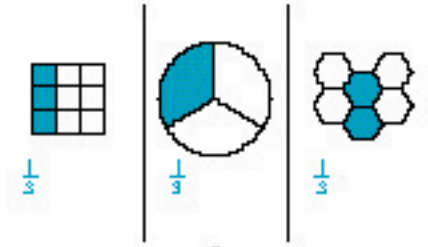


4

Color one-half of each shape red.

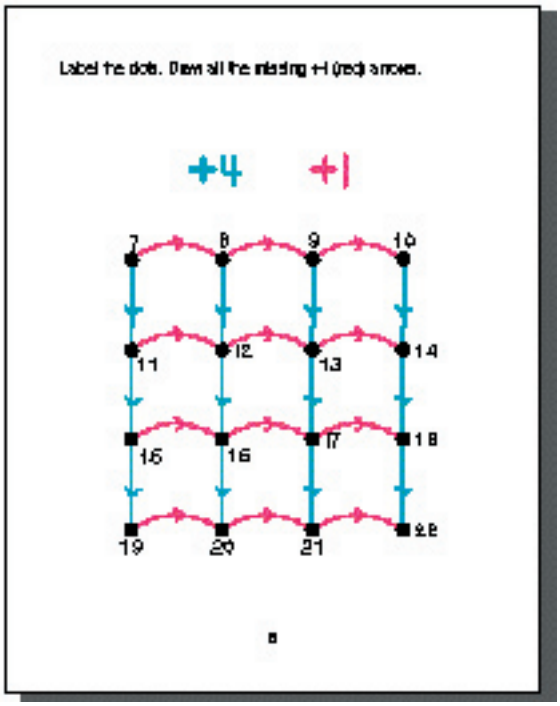


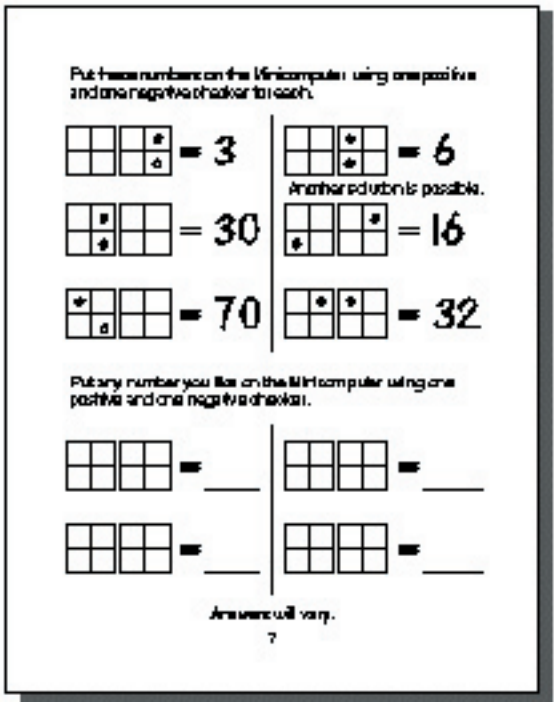
Color one-third of each shape blue.

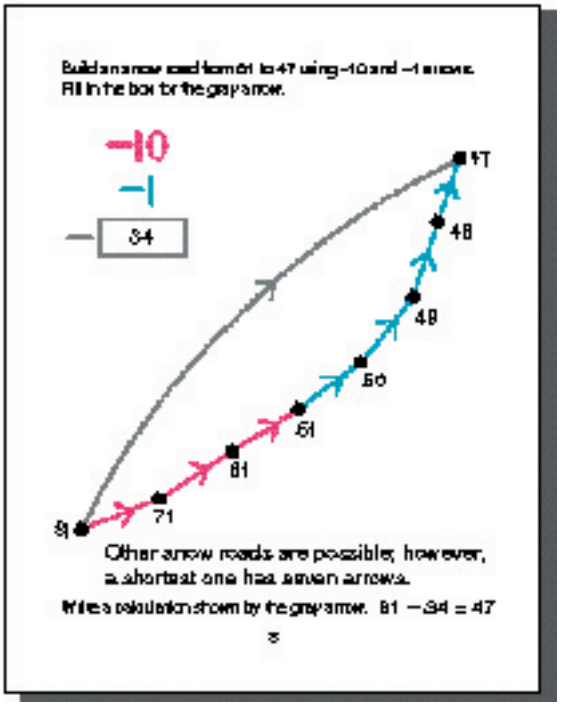


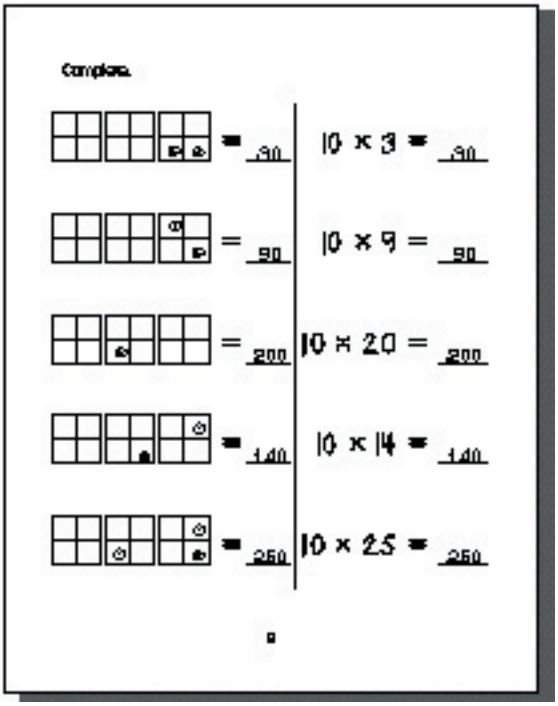
6

Other colorings are possible.

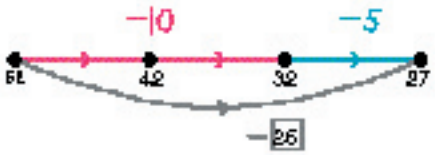

Label the dots. Draw all the missing +1 (red) arrows.
+4
+1
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
8


Put these numbers on the Mini-computer using one positive and one negative shaker for each.
= 3
= 6
Another solution is possible.
= 30
= 16
= 70
= 32
Put any number you like on the Mini-computer using one positive and one negative shaker.
=
=
=
=
Answers will vary.
7

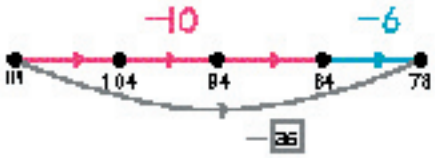

Build a snow road from 81 to 47 using −10 and −1 arrows.
Fill in the box for the gray arrow.
−10
−1
−34
81
71
61
51
50
48
48
47
Other snow roads are possible, however, a shortest one has seven arrows.
Write a calculation shown by the gray arrow. 81 − 34 = 47
9


Complete.
= 30
10 × 3 = 30
= 90
10 × 9 = 90
= 200
10 × 20 = 200
= 140
10 × 14 = 140
= 250
10 × 25 = 250
8

Label the dots. Fill in the box for each gray arrow.



Write a calculation shown by the gray arrow. 32 − 26 = 27



Write a calculation shown by the gray arrow. 114 − 36 = 78

10

Five children are playing a game. They record their scores with tally marks.

Gary	Emanuel	Doreen	Melodie	Otis
𝍸 𝍸	𝍸 𝍸	𝍸 𝍸	𝍸 𝍸	𝍸 𝍸
𝍸 𝍸	𝍸 𝍸	𝍸 𝍸	𝍸 𝍸	𝍸 𝍸
|		𝍸 ||	𝍸 𝍸	
			|	

List the children in order from lowest score to highest score.

Emanuel, Otis, Gary, Doreen, Melodie
Lowest — Highest

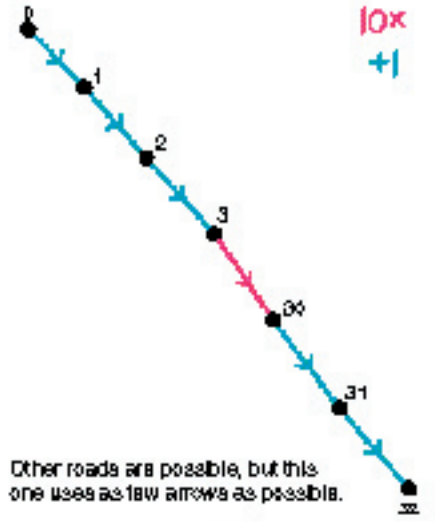
Suppose the children play with two teams, boys (Gary, Emanuel, Otis) and girls (Doreen, Melodie).

What is the score of the boy's team? 68
What is the score of the girl's team? 39

Otis has to leave the game and wants to share his points equally among the other four players. How many points should he give to each other person? 6

11

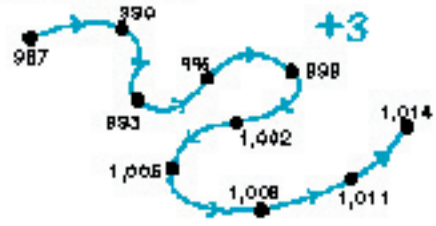
Build an arrow road from 0 to 1 using ×10 and +1 arrows.



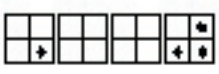
Other roads are possible, but this one uses as few arrows as possible.

12

Ling is a secret number.
Ling is in this arrow picture.

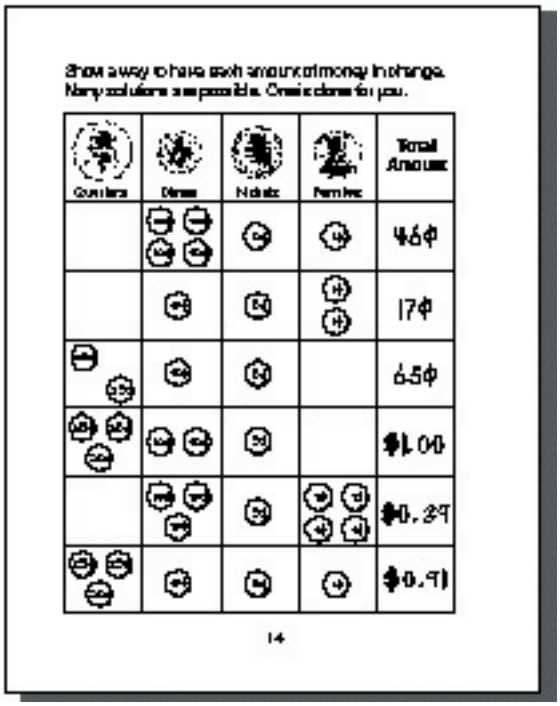


Ling can be put on this Minicomputer by taking off exactly one checker.



What is Ling? 1,005

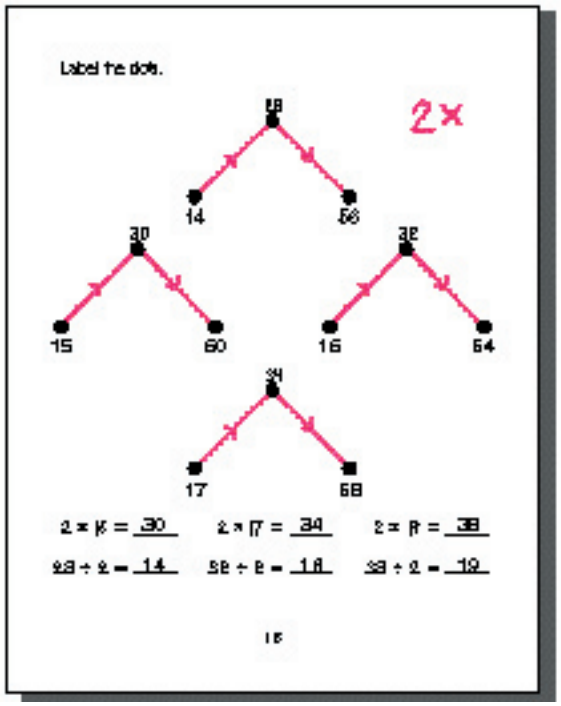
13



Show a way to have each amount of money in change.
Many solutions are possible. One is done for you.

Quarters	Dimes	Nickels	Pennies	Total Amount
	2	1	1	46¢
	1	1	2	17¢
2	1	1		65¢
4	2	1		$1.00
	3	1	4	$0.29
3	1	1	1	$0.91

14



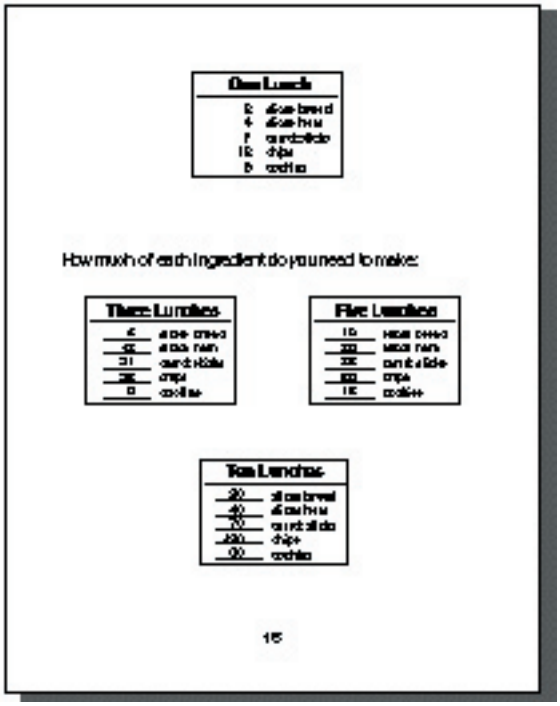
Label the dots.

2×

2 × 15 = 30 2 × 17 = 34 2 × 16 = 32

68 ÷ 2 = 14 56 ÷ 2 = 28 64 ÷ 2 = 32

15



Class Lunch

2	slices bread
4	slices ham
1	sandwich
10	chips
5	cookies

How much of each ingredient do you need to make:

Three Lunches

6	slices bread
12	slices ham
3	sandwiches
30	chips
15	cookies

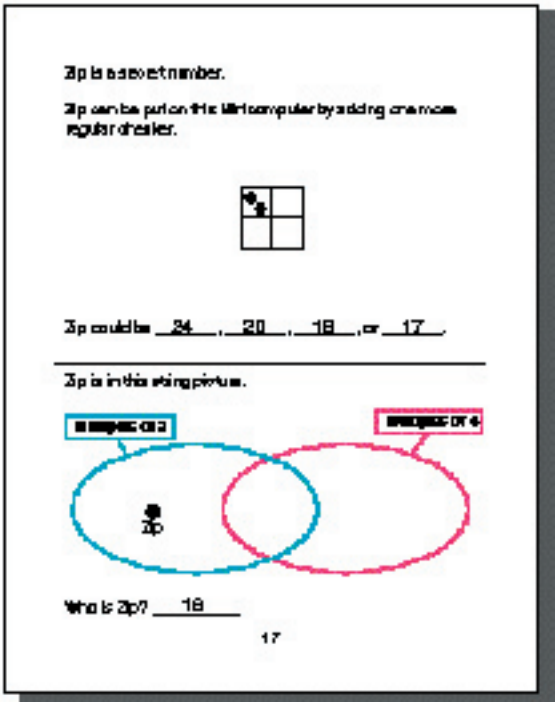
Five Lunches

10	slices bread
20	slices ham
5	sandwiches
50	chips
25	cookies

Ten Lunches

20	slices bread
40	slices ham
10	sandwiches
100	chips
50	cookies

16



Zip is a secret number.

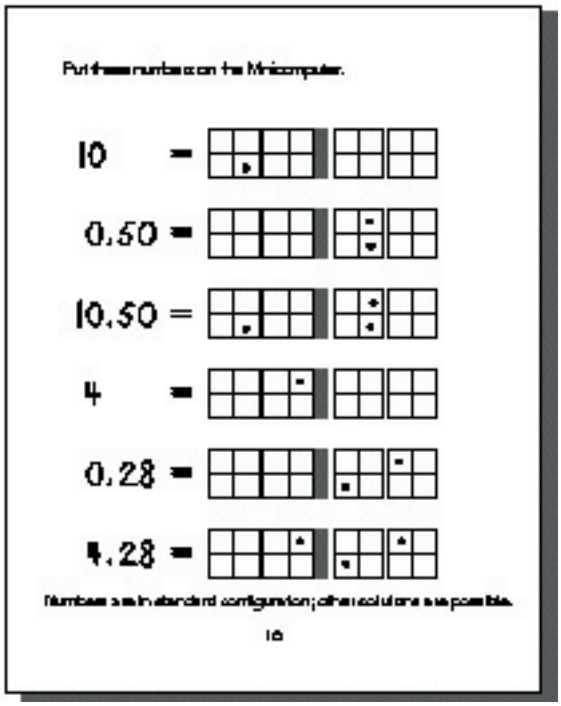
Zip can be put on this Venn computer by asking one more yes/no question.

Zip could be 24 , 20 , 18 , or 17 .

Zip is in this Venn picture.

Who is Zip? 18

17


Put these numbers on the Minicomputer.

10 =

0.50 =

10.50 =

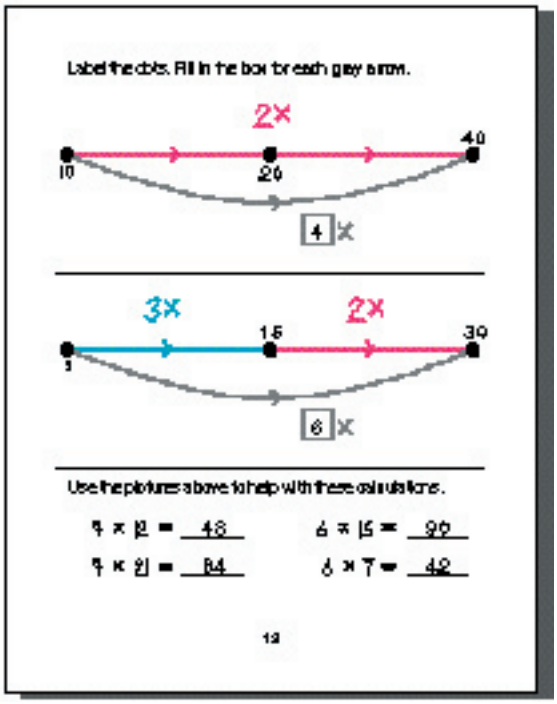
4 =

0.28 =

4.28 =

Numbers are in standard configuration; other solutions are possible.

16


Label the dots. Fill in the box for each gray arrow.

2×: 10 → 20 → 40; 4×

3×: 3 → 15; 2×: 15 → 30; 6×

Use the pictures above to help with these calculations.

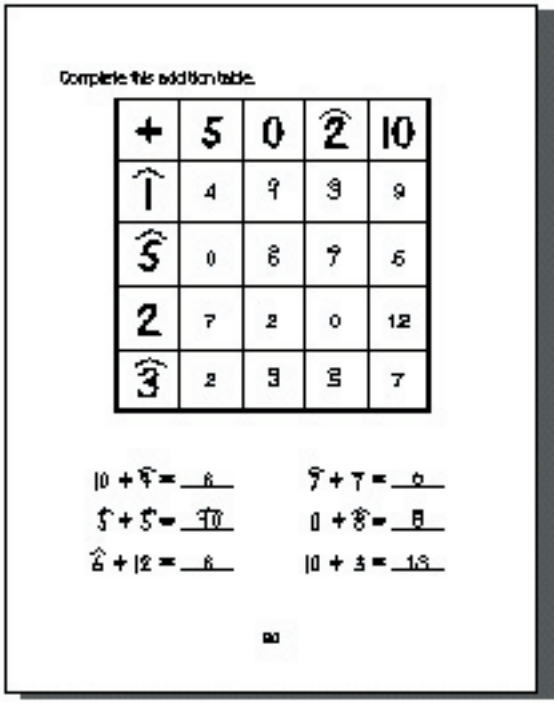
9 × 12 = 48

6 × 15 = 90

9 × 21 = 84

6 × 7 = 42

19


Complete this addition table.

+	5	0	2	10
1	4	9	3	9
5	0	5	7	5
2	7	2	0	12
3	2	3	5	7

10 + 9 = 6

7 + 7 = 0

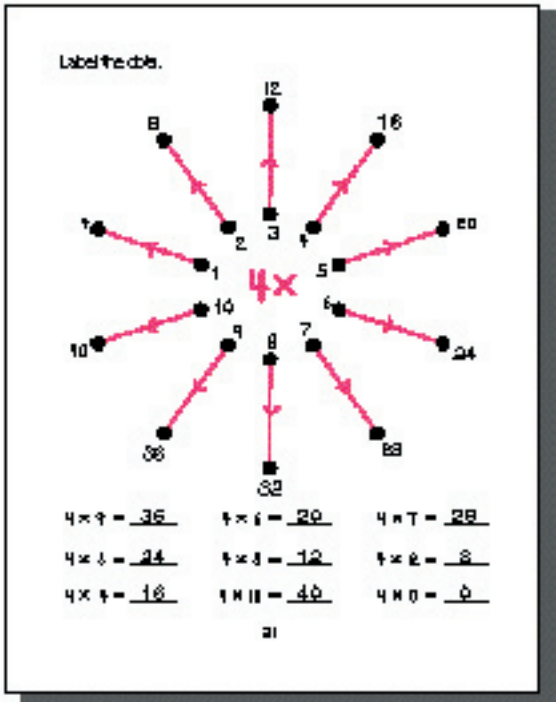
5 + 5 = 10

0 + 8 = 8

2 + 12 = 6

10 + 3 = 13

20


Label the dots.

4×: 1 → 4, 2 → 8, 3 → 12, 5 → 20, 6 → 24, 7 → 28, 8 → 32, 9 → 36, 10 → 40

4 × 9 = 36

4 × 6 = 20

4 × 7 = 28

4 × 4 = 24

4 × 3 = 12

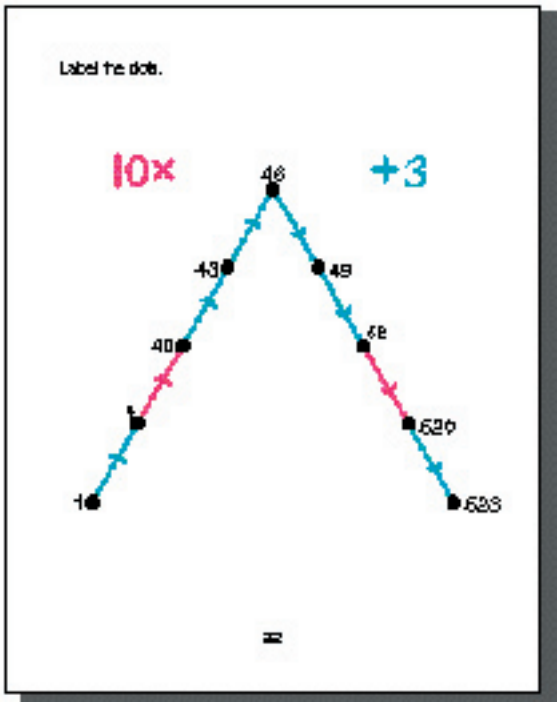
4 × 2 = 8

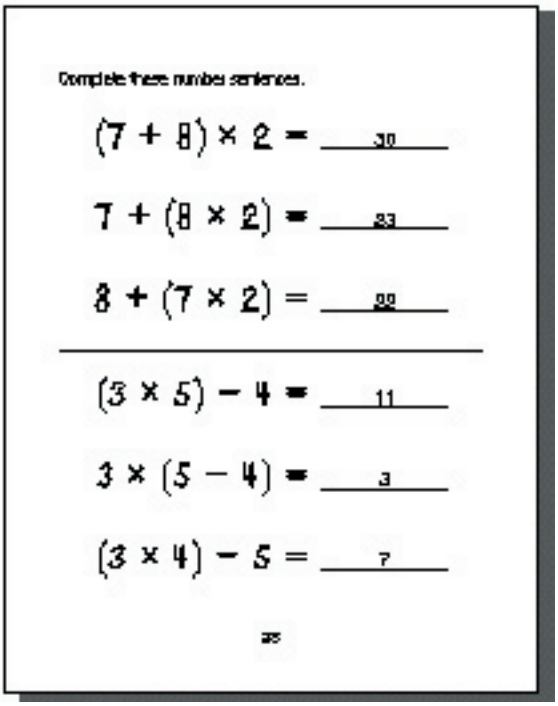
4 × 4 = 16

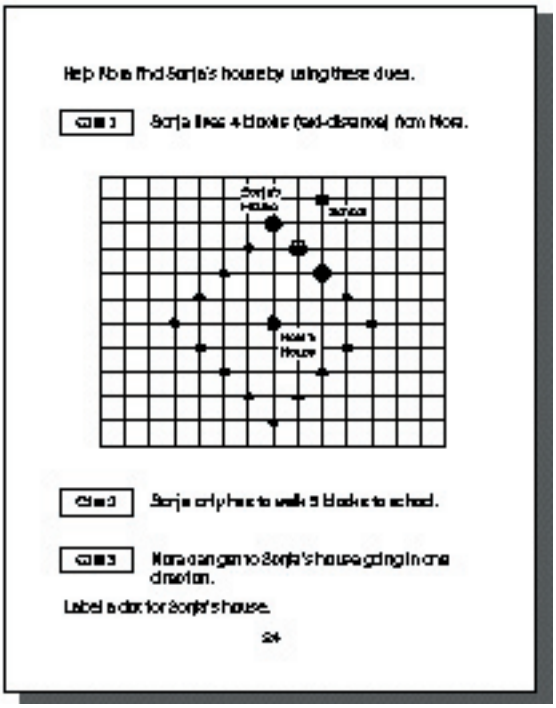
4 × 10 = 40

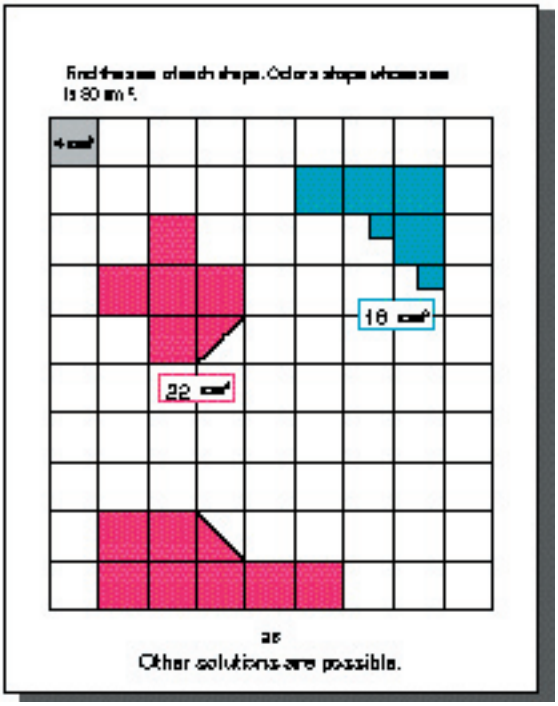
4 × 0 = 0

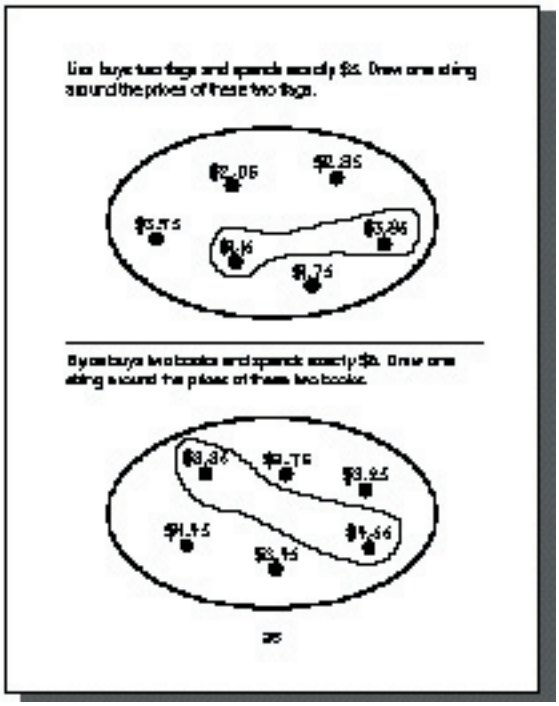
21

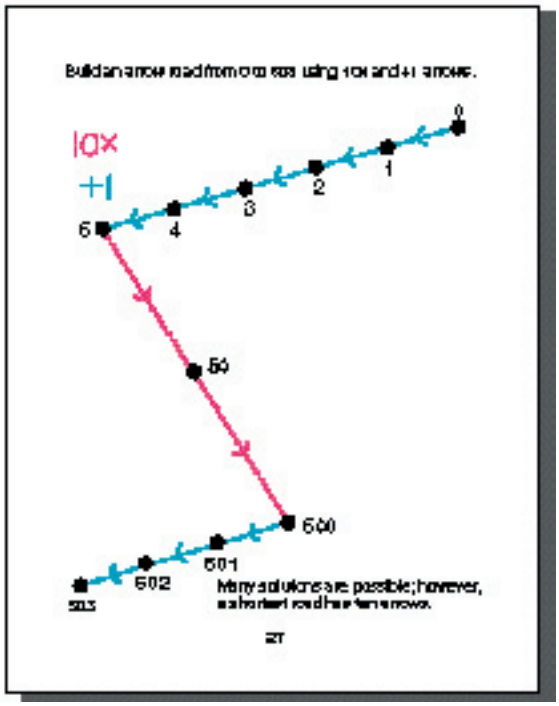

Label the dots.
10×
+3
46
43
49
40
52
529
533
1

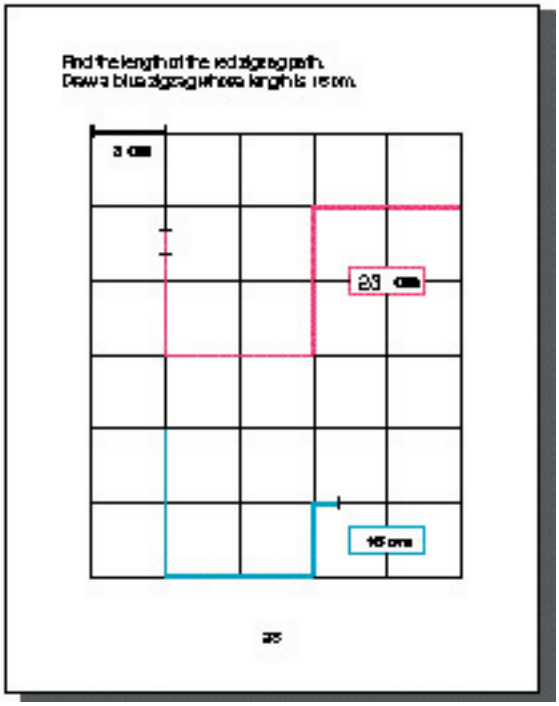

Complete these number sentences.
(7 + 8) × 2 = 30
7 + (8 × 2) = 23
8 + (7 × 2) = 22
(3 × 5) − 4 = 11
3 × (5 − 4) = 3
(3 × 4) − 5 = 7

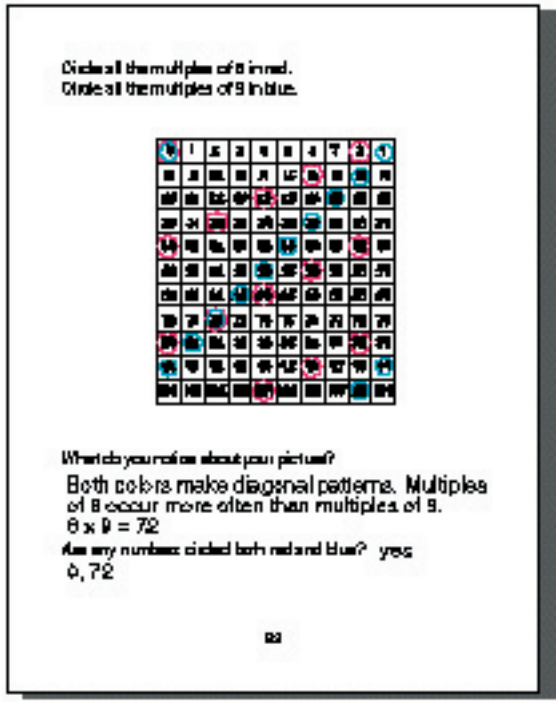

Help Nora find Sonja's house by using these clues.
Sonja lives 4 blocks (tail-distance) from Nora.
Sonja's House
School
Nora's House
Sonja only has to walk 3 blocks to school.
Nora can go to Sonja's house going in one direction.
Label a dot for Sonja's house.


Find the area of each shape. Color a shape whose area is 80 cm².
4 cm²
18 cm²
22 cm²
Other solutions are possible.


Lisa buys two bags and spends exactly $5. Draw one string around the prices of these two bags.
$2.05
$2.95
$3.75
$3.95
$1.25
$1.75
Ryan buys two books and spends exactly $5. Draw one string around the prices of these two books.
$3.35
$1.75
$3.95
$1.15
$1.65
$3.15
25


Build an arrow road from 0 to 606 using 10x and +1 arrows.
10x
+1
0
1
2
3
4
5
50
600
601
602
603
Many solutions are possible; however, a shortest road has ten arrows.
27

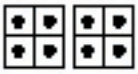

Find the length of the red zigzag path.
Draw a blue zigzag whose length is 15 cm.
3 cm
23 cm
15 cm
25


Circle all the multiples of 8 in red.
Circle all the multiples of 9 in blue.
What do you notice about your picture?
Both colors make diagonal patterns. Multiples of 8 occur more often than multiples of 9.
8 x 9 = 72
Are any numbers circled both red and blue? yes
0, 72
83

Laph is a secret number.
Laph is in this sting picture.

What is the greatest whole number Laph could be? 129
What is the least whole number Laph could be? 100

Laph can be put on this Mini computer by taking off exactly one checker.



What is Laph? 125

30

Put in parentheses to get as small a number as possible.

$5 + (3 \times 2) =$ 11

$(4 \times 4) + 6 =$ 22

$7 \times (2 - 1) =$ 7

$8 - (4 \times 2) =$ 0

$9 + (3 \times 5) =$ 24

31

Put the six number cards 0 2 3 5 6 8 in the spaces of this subtraction problem. Use all the cards, each card once.

What is the greatest difference you can get? 531
Explain.

What is the least positive difference you can get? 47
Explain.

Can you get a difference between 300 and 400? Yes
Explain.

613 − 254 = 359

Many answers are possible.

Can you get a difference of 405? Yes
Explain.

841 − 335 = 406

32

Capsule Lesson Summary

Name some even numbers less than 5,000; include the greatest such number. Solve a detective story with clues involving a string picture and two arrow pictures.

Materials

Teacher	• Colored chalk	Student	• Paper • Colored pencils, pens, or crayons

Description of Lesson

Exercise 1

Ask the class to tell you what they know about even numbers and to give several examples including

- an even number less than 0;
- an even number greater than 1,000; and
- an even number between 4,000 and 5,000.

Starting with the first choice of an even number between 4,000 and 5,000, begin a sequence of inequalities on the board. For example, suppose 4,098 is mentioned first.

T: ***4,098 is more than 4,000. What is an even number more than 4,098 but less than 5,000?***

S: ***4,910.***

$$4{,}000 < 4{,}098 < 4{,}910$$

T: ***4,910 is more than 4,098 and less than 5,000. What is the greatest even number less than 5,000?***

Possibly the first number suggested will not be the greatest, but record it anyway; for example,

S: ***4,980.***

$$4{,}000 < 4{,}098 < 4{,}910 < 4{,}980$$

T: ***4,980 is more than 4,910 and less than 5,000, but is it the greatest even number less than 5,000?***

S: ***4,998 is the greatest.***

Record 4,998 and 5,000 in the sequence of inequalities on the board.

$$4{,}000 < 4{,}098 < 4{,}910 < 4{,}980 < 4{,}998 < 5{,}000$$

T: ***4,998 is the greatest even number less than 5,000. Is 5,000 an even number?***

S: ***Yes, because it ends in 0.***

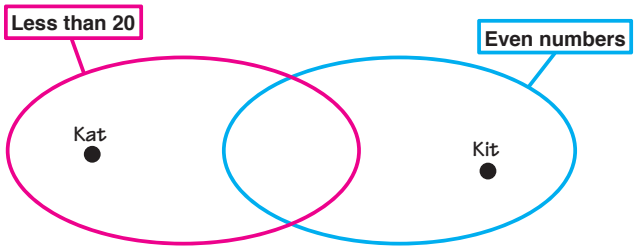
Exercise 2: Detective Story

You may like to allow students to work with partners during this exercise.

T: ***Today I have two secret numbers for you to discover. The secret numbers are Kit and Kat, and both are integers[†]. I'll give you some clues.***

Clue 1

Draw this string picture on the board, and ask students to copy it on their papers.



Suggest student pairs try to find some possible numbers for Kit and Kat. After a few minutes, begin a class discussion.

T: ***Kit is in the blue string, but not in the red string. So what do we know about Kit?***

S: ***Kit is an even number.***

S: ***Kit is not less than 20.***

S: ***Kit could be 20.***

T: ***What are some other numbers Kit could be?*** (Even numbers 20 or more than 20)
What do we know about Kat?

S: ***Kat is less than 20.***

S: ***Kat is not an even number.***

S: ***Kat is an odd number.***

T: ***What are some numbers Kat could be?*** (Odd numbers less than 20)

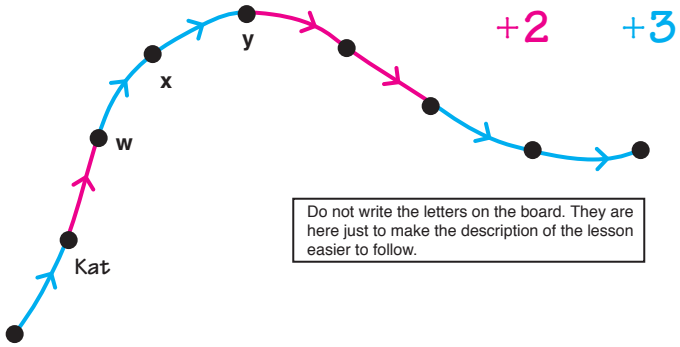
You may like to make a list on the board of some possible numbers for Kit and Kat. Include 20 in the list for Kit; include 19 and some negative odd numbers in the list for Kat.

T: ***Which number is greater, Kit or Kat?***

S: ***Kit.***

Clue 2

Draw this arrow picture on another part of the board, but do not erase the string picture.



T: ***Kit and Kat are in this arrow picture, but for the moment only Kat is located. Are there any odd numbers in this arrow picture?***

S: ***Yes, Kat is an odd number.***

Point to the dot for Kat.

[†]If necessary, tell the class that integers can be positive or negative, but cannot be fractions like $\frac{1}{3}$ or $\frac{1}{4}$ nor decimals like 3.5 or 1.73.

T: ***We know that Kat is an odd number, so we know that this number*** (point to Kat) ***is an odd number.***

Write *odd* near the dot for Kat.

T (pointing to **w**)**:** ***Is this number even or odd?***

S: ***Odd, because an odd number (Kat) plus 2 is an odd number.***

Write *odd* near **w**.

T (pointing to **x**)**:** ***Is this number even or odd?***

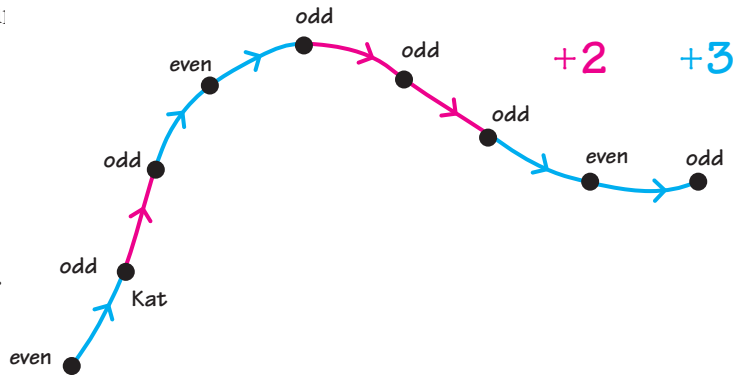
S: ***Even. If this number*** (pointing to **w**) ***were 11, then this number*** (**x**) ***would be 14. 14 is an even number.***

Ask students to repeat such a check several times by assigning various odd numbers to **w** and deciding if the number at **x** is an odd or an even number. Conclude that the number at **x** is even. Write *even* near **x**. Point to **y**.

T: ***Is this number even or odd?***

S: ***Odd, because an even number plus 3 equals an odd number.***

Write *odd* near **y**. Continue this activity u
either odd or even is noted for each dot.



T: ***Kit is in this arrow picture. Where could Kit be?***

Students should observe that one of the three dots for even numbers could be Kit.

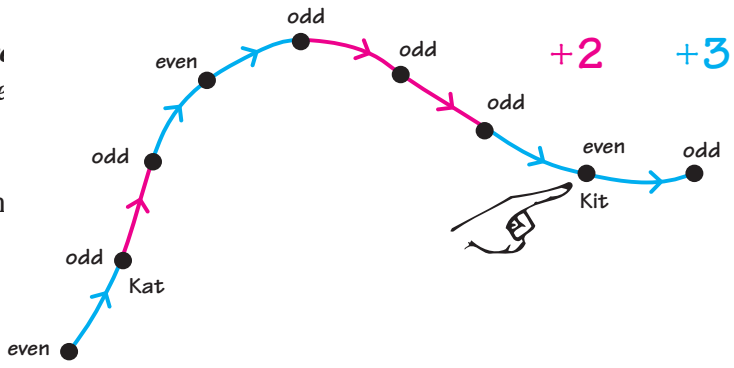
T: ***How do you know that Kit is one of those numbers?***

S: ***Kit is an even number.***

Perhaps someone will notice that Kit cannot be the first number in the arrow road because Kit is greater than Kat and the blue arrow adds 3.

Clue 3

T: ***Here is another clue. Kit is the gr
even number in this arrow picture
Which of these dots is for Kit?***



Ask a student to point to the dot for Kit an

T: ***How do you know that this is the greatest even number in this arrow picture?***

S: ***All the arrows are for +2 or +3, so the numbers increase as you move to the right.***

T: ***Let's try to find some pairs of numbers that could be Kit and Kat. Give me a number that Kat could be.***

Accept any odd number less than 20.

S: ***Kat could be 5.***

Write 5 near the dot for Kat, and then point to the dot for Kit in the arrow picture.

T: ***If Kat were 5, what number would Kit be?***

Suggest students write their answers on paper before asking a student to answer aloud.

S: ***20.***

Write 20 near the dot for Kit in the arrow picture.

T: ***How did you know that Kit would be 20 if Kat were 5?***

S: ***5 + 2 = 7; 7 + 3 = 10; 10 + 3 = 13; 13 + 2 = 15; 15 + 2 = 17; and 17 + 3 = 20.***

T: ***This arrow picture tells us that if Kat were 5, then Kit would be 20.*** (Point to the string picture.) ***Could Kit be 20?***

S: ***Yes, 20 is an even number and 20 is not less than 20.***

Begin a table of possibilities for Kat and Kit on the board.

Kat	Kit
5	20

T: ***What are some other numbers Kat and Kit could be?***

Instruct students to work with their partners to find a few other possibilities for Kat and Kit. Then invite students to make suggestions, and collectively check each one. If a student makes an incorrect suggestion for Kat, ask what number Kit would be and where that number belongs in the string picture. For example, suppose a student suggests that Kat could be 1.

S: ***1.***

T: ***If Kat were 1, what would Kit be?***

When the class determines that Kit would be 16, refer to the string picture.

T: ***Could Kit be 16?***

S: ***No, because 16 is less than 20 and is in the red string.***

T: ***If Kit cannot be 16, then Kat cannot be 1.***

Do not record an incorrect pair, such as 1 and 16, in the table.

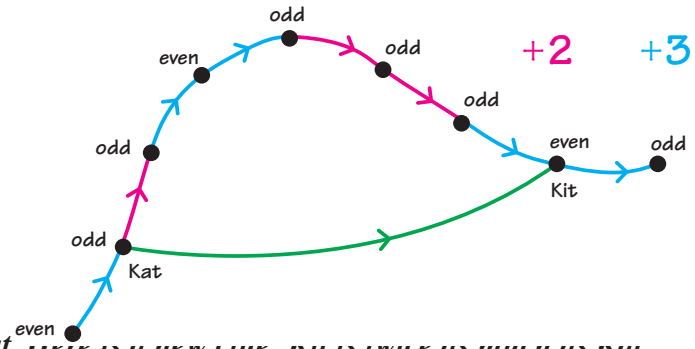
Continue this activity until several pairs of numbers have been recorded in the table. All of the possible pairs are recorded in this table for your information.

Kat	Kit
5	20
7	22
9	24
11	26
13	28
15	30
17	32
19	34

Draw a green arrow from Kat to Kit in the arrow picture.

T: ***What could this arrow be for?***

S: ***+15.***

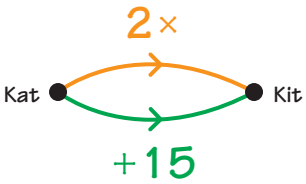


Label the green arrow +15.

Clue 4

T: ***We know that Kit is 15 more than Kat. Here is a new clue: Kit is twice as much as Kat.***

Draw this arrow picture on the board and ask students to copy it on their papers.



T: ***2 x Kat = Kit and Kat + 15 = Kit.***
Which numbers are Kat and Kit?

Allow student partners to investigate, and then to write their answers in the arrow picture. Do not be too quick to confirm correct answers.

T: ***Someone tried 9 for Kat. Could Kat be 9?***

Write 9 near the dot for Kat.

T (tracing the 2x arrow)**:** ***If Kat were 9, then Kit would be 2 x 9. What number is 2 x 9?***

S: ***18.***

Write 18 near the dot for Kit.

T (tracing the +15 arrow)**:** ***Does 9 + 15 = 18?***

S: ***No.***

Erase 9 and 18 from the arrow picture.

T: ***This arrow picture tells us that 2 x Kat is the same number as Kat + 15. 2 x 9 is not the same number as 9 + 15, so Kat cannot be 9.***

If 9 and 24 are recorded in your table, cross them out. Explain that if Kat cannot be 9, then Kit cannot be 24.

Continue this activity until someone suggests that Kat is 15. Write 15 near the dot for Kat.

T: ***If Kat is 15, then 2 x 15 is the same number as 15 + 15.***
What number is 2 x 15? What number is 15 + 15?

S: ***30.***

Label the dot for Kit 30, and conclude that Kat is 15 and Kit is 30.

Capsule Lesson Summary

In the story-workbook *To Picture*, find where eight numbers belong in an arrow picture. Discover that a second arrow picture is just a disguise of the first picture and use this to help label the dots.

Materials

Teacher
- *To Picture* Story-Workbook
- Colored chalk

Student
- *To Picture* Story-Workbook

Description of Lesson

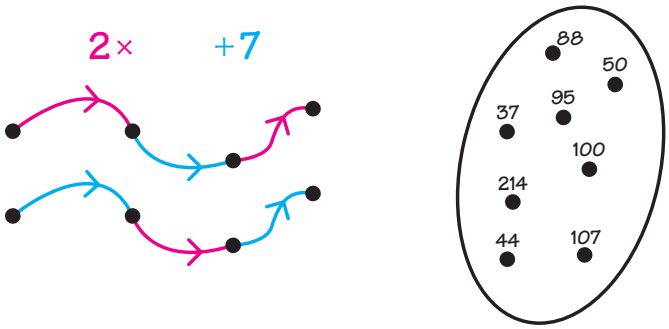
Distribute copies of the story-workbook *To Picture* to the students. You may like to pair the students and give a story-workbook to each pair. Ask them to follow along as you or students read the story, and not to read ahead.

Encourage your class to discuss the problems confronted by the number friends and to explain the solutions that they propose. Additional comments on specific pages of the story-workbook are provided here.

Pages 3, 4, and 5

Each of the numbers on page 5 belongs in the arrow picture on page 4. This problem is similar to problems the students have solved in *Fishing for Numbers* activities. Ask the students to label the dots on page 4, but emphasize that they should only use the numbers on page 5.

While the students are working, copy the arrow picture from page 4 on the board. Near the arrow picture draw a string and put the eight numbers in it as on page 3.



Pages 6–10

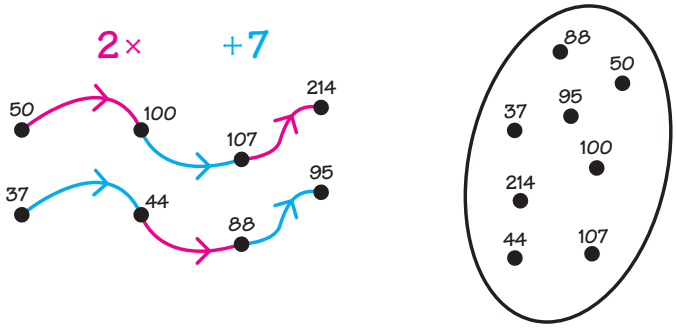
Encourage students to comment about errors they find on page 7.

After page 10 has been read, ask where the least number is in each arrow road and why. Students very likely will tell you that the starting dot of each arrow road is for the least number because multiplying by 2 gives a greater number and adding 7 gives a greater number.

Note: Doubling a number does not necessarily result in a greater number; for example, 2 x B5 = N10 and B5 > N10. In this arrow picture, however, all the numbers are positive, so the starting numbers must be less than their ending numbers. (This is for your information only.)

Pages 11 and 12

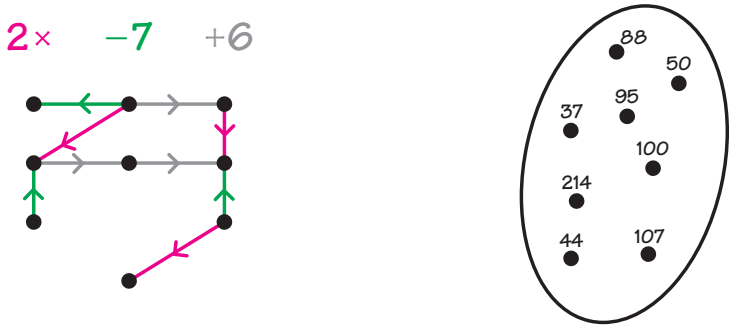
Allow a few minutes for students to label the dots on page 12. After a few minutes, invite a student to label the dots in the arrow picture on the board. Check that each of the numbers in the string picture is in the arrow picture.



Ask if this is the only way that the eight number friends in the story can arrange themselves in the arrow picture. Consider any different arrangements that are proposed and conclude that there is only one correct way to label the dots in the arrow picture with the eight numbers in the story.

Pages 13, 14, and 15

While students are labeling the dots on page 14, erase the first arrow picture and copy the arrow picture from page 14 on the board near the string picture. Use white chalk to draw the arrows shown in gray below.



Pages 16 and 17

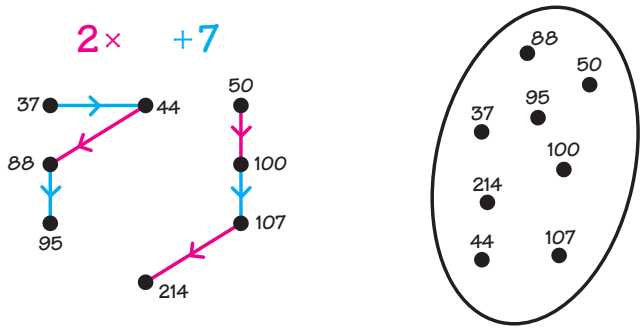
Erase the gray (white) arrows in the picture on the board. Trace one of the green arrows.

T: ***If this –7 arrow points this way, which way should a +7 arrow point?***

S: ***The opposite way.***

Erase the –7 arrow and replace it with a blue +7 arrow. Then, upon agreement, replace all the –7 arrows with +7 arrows. The arrow picture on the board should look like the picture on page 17. Ask students to compare the arrow picture from page 4 with the arrow picture on the board. Emphasize that both arrow pictures have an arrow road with a blue arrow between two red arrows and an arrow road with a red arrow between two blue arrows.

Students can use page 4 to help them label the dots on page 17. After a few minutes, invite a student to label the dots in the arrow picture on the board.



If students have difficulty labeling the dots on page 14, direct them to return to the preceding page and label the dots before continuing.

Pages 18–21

Page 21 is a good place to end the lesson. Collect the story-workbooks and keep them for use in Lesson W8.

Capsule Lesson Summary

Finish the story-workbook *To Picture*. Discover that making one backward trade and one forward trade does not alter the number of checkers in a configuration on the Minicomputer and, of course, does not change the number displayed.

Materials

Teacher
- *To Picture* Story-Workbook
- Minicomputer set
- Paper

Student
- *To Picture* Story-Workbook

Description of Lesson

Distribute the students' copies of the *To Picture* Story-Workbook. If students were paired for Lesson W7, pair them again for this lesson. Ask everyone to turn to page 22.

Pages 22 and 23

Read or call on students to read pages 22 and 23 aloud; then ask students to determine which number is on each Minicomputer. Students who finish quickly may continue until they complete page 25.

When most everyone has completed pages 22 and 23, compare the numbers on these pages with the numbers in the arrow picture on page 20.

Pages 24 and 25

Read or call on students to read pages 24 and 25 aloud.

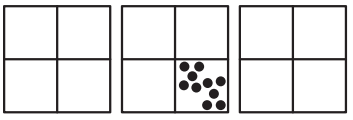
You may need to emphasize that students are to put 100 on the Minicomputer using exactly ten regular checkers. Allow about five minutes for individual or partner work on page 25. Students who finish quickly can try to find other configurations for 100 with ten checkers and record them on a separate piece of paper.

Invite students, one at a time, to put 100 on the demonstration Minicomputer using exactly ten checkers. Continue this activity until four or five configurations have been shown.

Pages 26 and 27

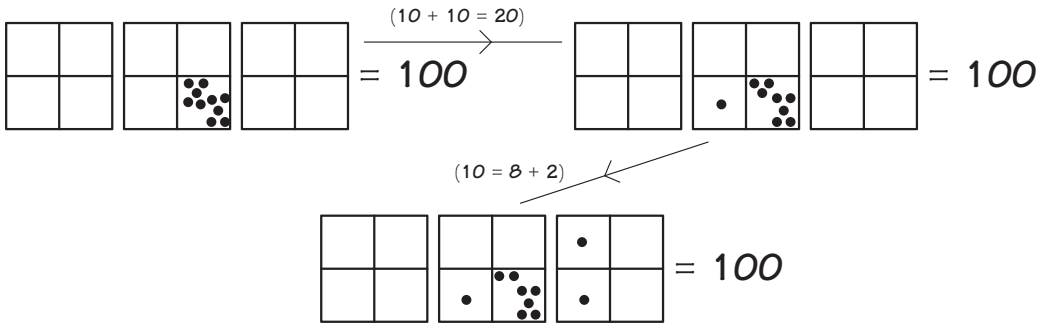
Suggest students look carefully at pages 26 and 27 to see if they can determine what game 100 is playing.

Put this configuration on the Minicomputer.



T: ***Who can make some trades so our Minicomputer will have checkers on the same squares as the second Minicomputer on page 26?***

The volunteer should make one forward trade and one backward trade. Either trade may be made first.

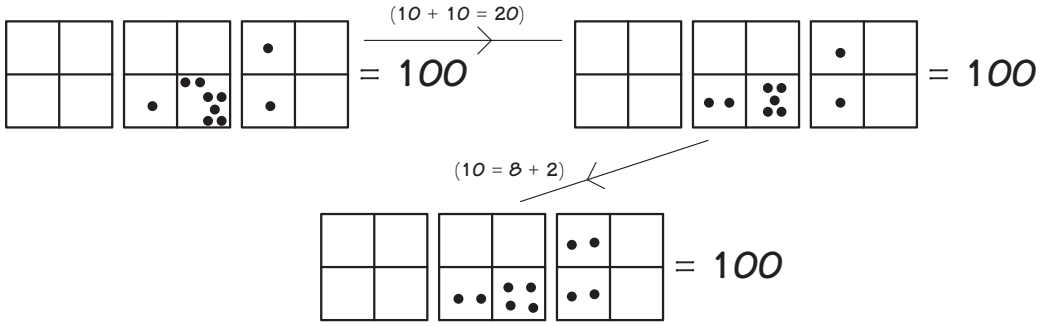


T: ***100 made one forward trade and one backward trade. Are there still ten checkers on the Minicomputer?***

S: ***Yes.***

T: ***Who can make some trades so our Minicomputer will have checkers on the same squares as the third Minicomputer on page 26?***

The volunteer should make one forward and one backward trade.



T: ***So 100 made one forward trade and one backward trade. Are there still ten checkers on the Minicomputer?***

S: ***Yes.***

Continue this activity with the configurations on pages 26 and 27.

T: ***Do you understand 100's game?***

S: ***100 always makes one forward trade and one backward trade so that there are still ten checkers on the Minicomputer.***

T: ***Could we continue this game?***

S: ***Yes.***

Invite students to play 100's game several times on the Minicomputer. Each student should make one forward and one backward trade.

Ask students to find several ways to put 88 on the Minicomputer using exactly twelve regular checkers and to show them on page 27. Students who finish quickly can draw other configurations with twelve checkers for 88 on a separate piece of paper. Allow a few minutes for individual or partner work and then invite students to put some of these configurations on the demonstration Minicomputer. Continue this activity until four or five configurations have been shown.

Page 28

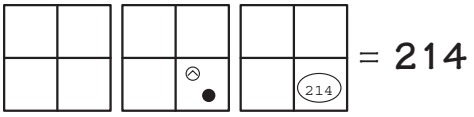
88 is playing 100's game. Put the first configuration on the demonstration Minicomputer, and invite a student to make trades so that the demonstration Minicomputer has checkers on the same squares as the second Minicomputer on page 28. The student should make a 10 + 10 = 20 trade and a 2 = 1 + 1 trade.

Continue this activity for the other configurations on page 28. If your students understand and enjoy 100's game, they may wish to continue playing the game a few more times.

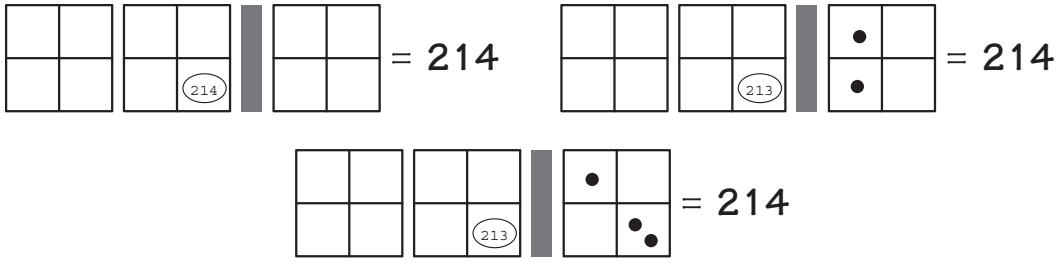
Pages 29–32

Ask students to read and complete pages 29–32 independently or with their partners. If your students are interested, you may like to explore the possibility of putting 214 on the Minicomputer with more than 214 checkers.

Note: There are two ways your students might suggest to use more than 214 checkers to put 214 on the Minicomputer. One way is to use negative checkers such as was done in the *Rollerskating 37* Story-Workbook (see Lessons W11 and W12 in *UPG-III*), and as illustrated here.

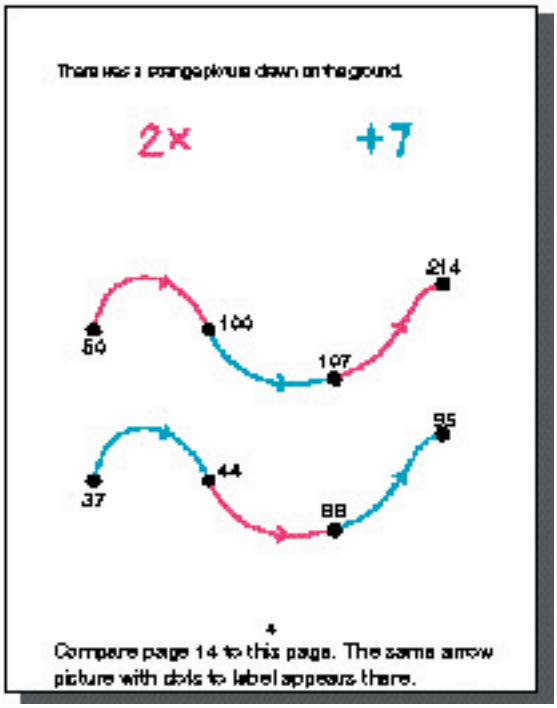


The other way is to draw a bar, add boards to the right of it, and make backward trades. For example, 214 can be shown as 213 + 0.8 + 0.2 or as 213 + 0.8 + 0.1 + 0.1.



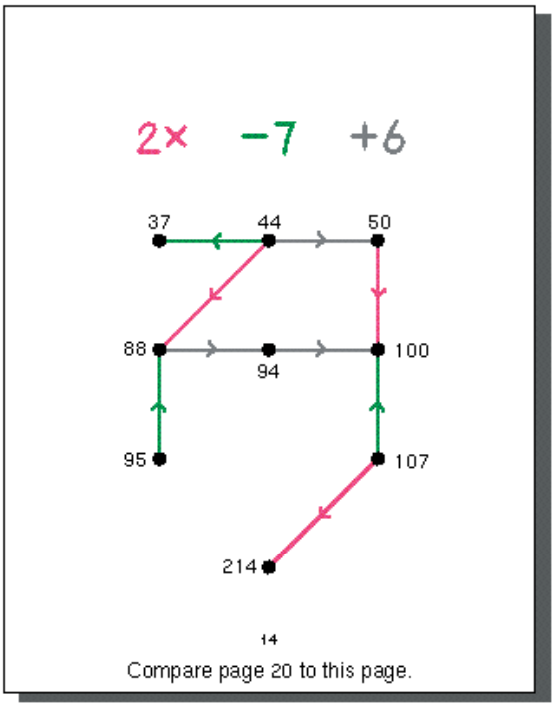
Home Activity

Suggest that students explain 100's game to someone at home and challenge them to play it.



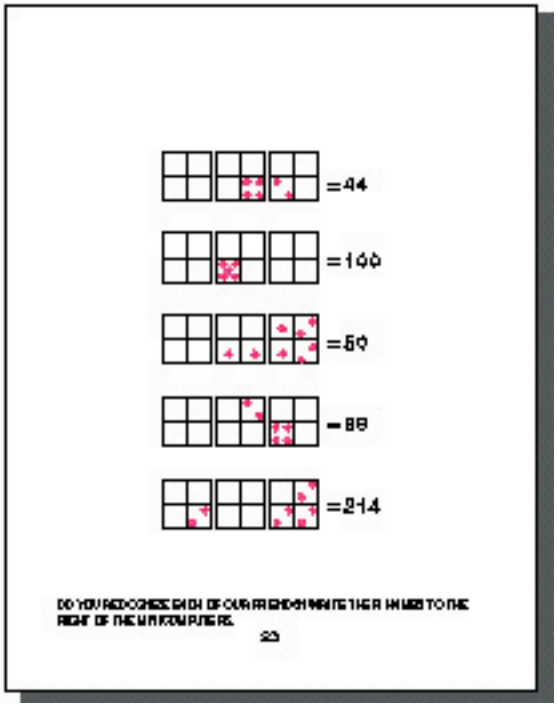
4

Compare page 14 to this page. The same arrow picture with dots to label appears there.

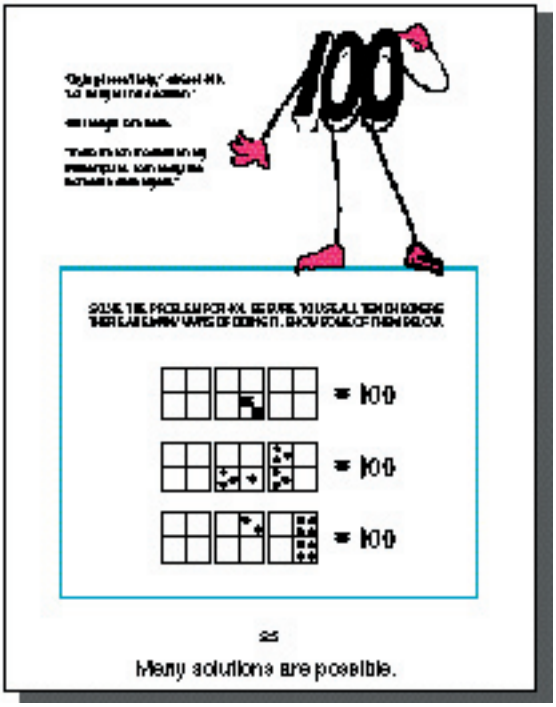


14

Compare page 20 to this page.

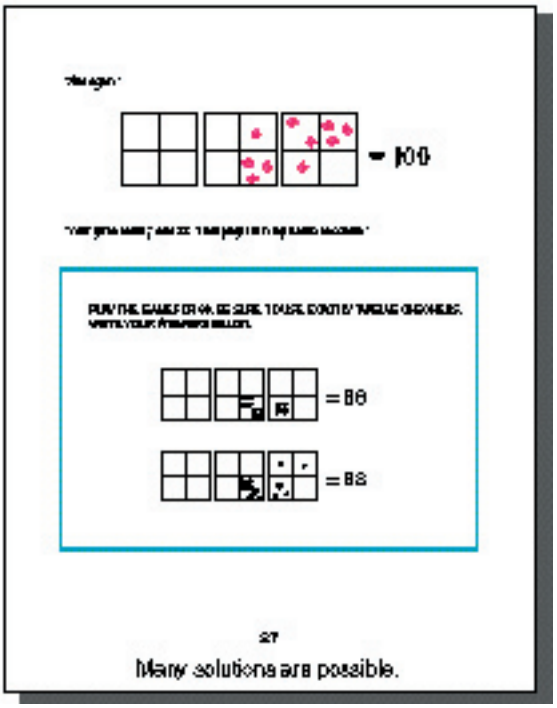


23

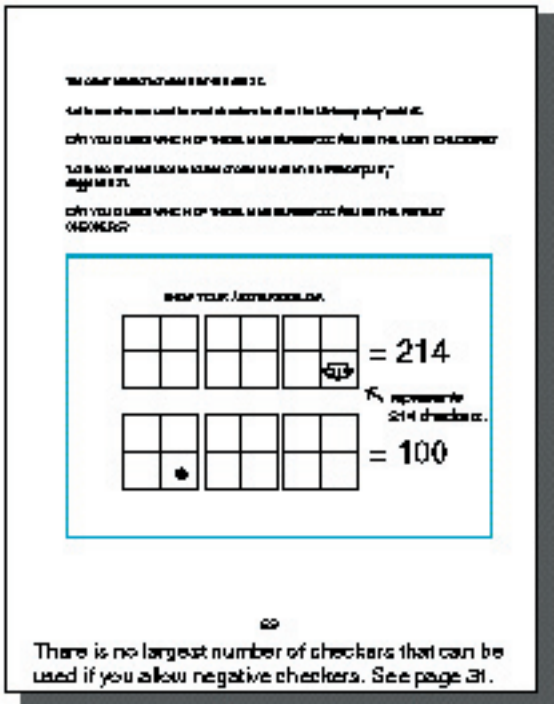


25

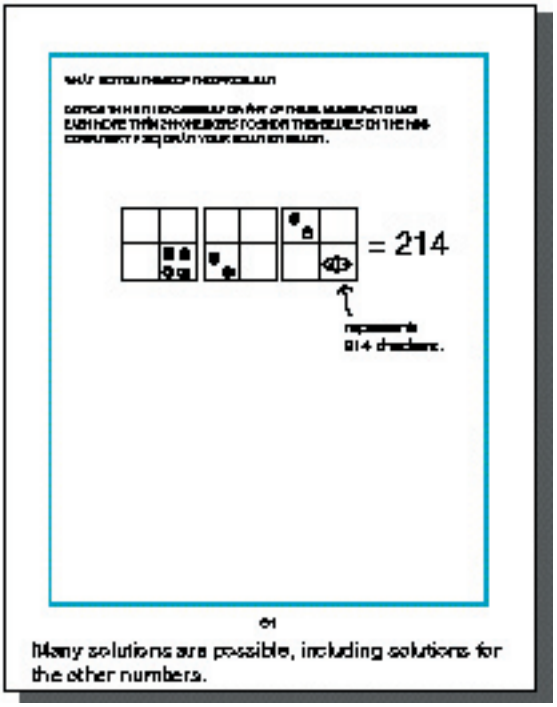
Many solutions are possible.



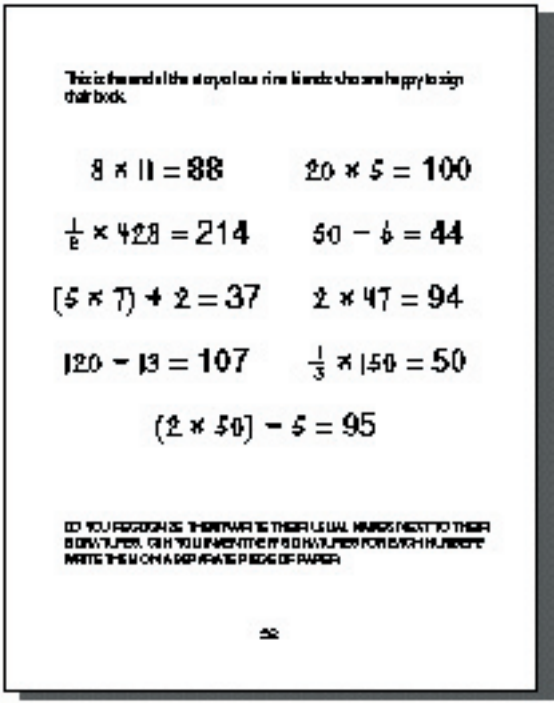
Many solutions are possible.



There is no largest number of checkers that can be used if you allow negative checkers. See page 31.



Many solutions are possible, including solutions for the other numbers.



Capsule Lesson Summary

Find how much spending money Stanley brings from home to the zoo if he has 98¢ after his grandfather doubles the original amount and his grandmother adds 9¢. Begin the workbook *Galaxy of Problems #3*.

Materials

Teacher	• Colored chalk • Collection of coins	Student	• *Galaxy of Problems #3* Workbook • Paper • Colored pencils, pens, or crayons

Description of Lesson

Tell the following or a similar story to your class. Choose one of your students to star in this story.

T: ***Last Sunday Stanley's grandparents took him to the zoo. Stanley brought some spending money from home. At the zoo, Grandfather gave Stanley some more money so he had twice as much as he came with, and then Grandmother gave Stanley a dime. Stanley had a total of 98¢ to spend at the zoo.***

Invite students to choose coins to show some possible ways to make 98¢.

T: ***How much money do you think Stanley brought with him from home?***

Suggest students write their guesses on a piece of paper.

T: ***Can we use an arrow picture to solve this problem?***

Encourage students to tell you which arrows to draw. Ask them to draw the arrow picture on their papers as you do it on the board.

T (drawing a dot on the board)**:** ***This dot is for the money Stanley brought from home. Stanley's grandfather gave him enough money to double this amount. How can we show this in the picture?***

S: ***Draw a 2x arrow.***

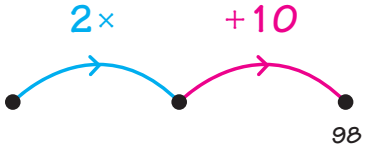
T: ***Then Stanley's grandmother gave him a dime. How much money is a dime?*** (10¢) ***How can we show this in the picture?***

S: ***Draw a +10 arrow.***

T: ***How much money did Stanley have after his grandmother gave him the dime?***

S: ***98¢.***

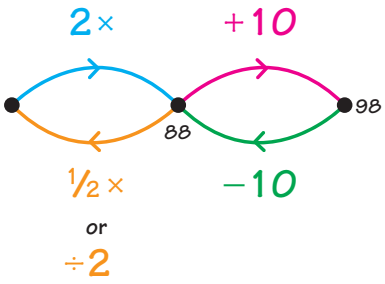
Label the ending dot 98.



T: ***How does this arrow picture help us find how much money Stanley brought from home?***

S: ***We can draw the return arrows and label the dots.***

Invite students to draw and label the return arrows.



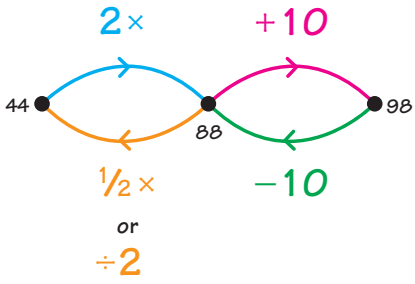
T: ***How much money did Stanley bring with him from home?***

Suggest students check their previous guesses in the arrow picture and change them if they need to. Then discuss labeling the dots with the class.

T (tracing the –10 arrow)**:** ***What number is 98 – 10?***

S: ***88.***

Label the dot for 88 and trace the $\frac{1}{2}$x (or ÷2) arrow.



T: ***What number is $\frac{1}{2}$ x 88? How do you know?***

S: ***44. $\frac{1}{2}$ x 80 = 40 and $\frac{1}{2}$ x 8 = 4, so $\frac{1}{2}$ x 88 = 44.***

Label the dot for 44.

T: ***How much money did Stanley take with him from home?*** (44¢)
How much money did Stanley's grandfather give him? (44¢)

Distribute copies of the workbook *Galaxy of Problems #3,* and let the students work independently for the rest of the class period. If many students have difficulty with a particular page, you may wish to have a collective discussion about that page.

At the end of the lesson, collect the workbooks for your review. They will be used again in Lesson W10.

Capsule Lesson Summary

Play a cooperative game with the calculator in which you start with a number on the display and try to reach a target number. Players can press å, ß, ∂, or ƒ any one-digit number, followed by ≠, to reach the target. Continue working in the workbook *Galaxy of Problems #3*. (This is the second of two lessons using this workbook.)

Materials

Teacher	• Calculator (overhead) • Colored chalk	Student	• Calculator • *Galaxy of Problems #3* Workbook • Colored pencils, pens, or crayons

Description of Lesson

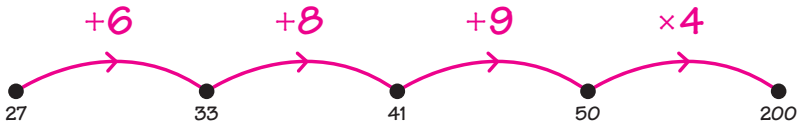
Using an overhead calculator or a class calculator, put 27 on the display.

T: ***27 is on the calculator and our target is 200. We are going to work together to reach the target. When you take a turn you can press*** å, ß, ∂, ***or*** ƒ ***any one-digit number you want, and then ≠. Let's see how quickly we reach the target.***

Call on students one at a time. After each turn, announce where you are and repeat the target as well as the rule. For example:

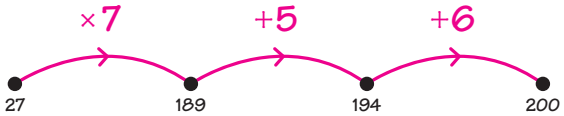
T: ***Anthony pressed*** å § ≠. ***The number on the calculator is now 33 and our target is 200. The next person can press*** å, ß, ∂, ***or*** ƒ ***any one-digit number and then ≠. Remember, we are trying to reach 200.***

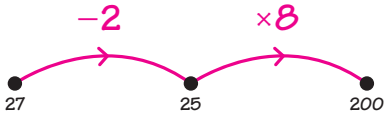
Continue this activity, letting different students take turns until the class gets 200 (the target) on the calculator. If appropriate, you may like to keep track of how many turns it takes in an arrow picture and challenge the class to reach this same target in fewer steps. For example:



T: ***It took us four steps (turns) to reach the target. Do you think we could reach the target in fewer steps?***

In this case there are many three-step solutions and one two-step solution.



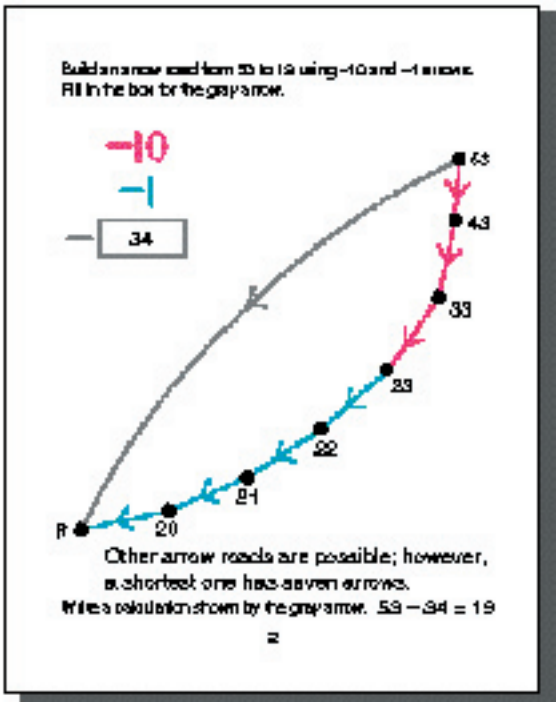


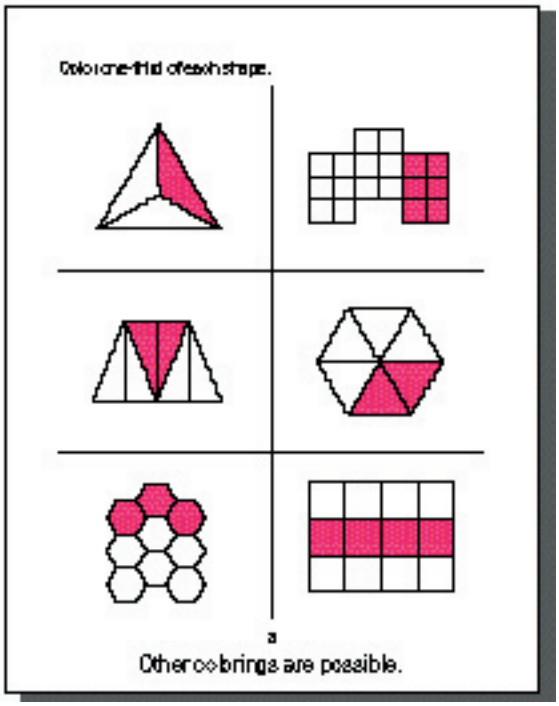
If your class enjoys this activity, provide calculators and a couple similar target problems for the students to work on with a partner. You may challenge students with targets such as 175 or 231.

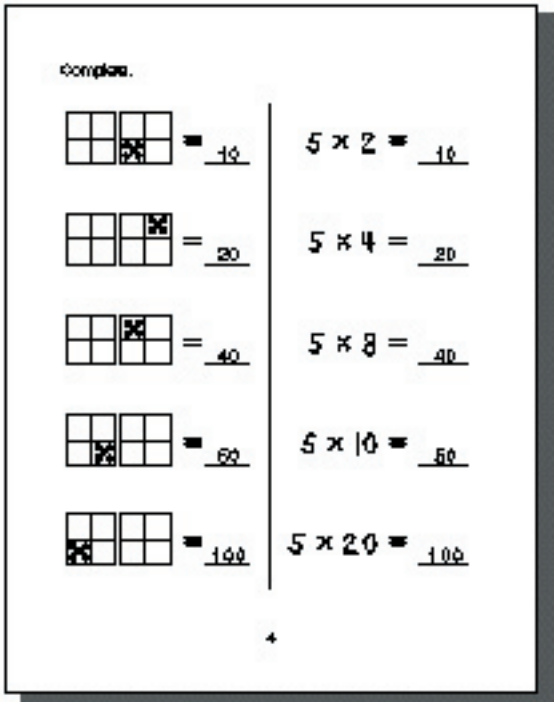
Distribute the students' copies of the *Galaxy of Problems #3* Workbook. Ask students first to correct pages from their previous week's work and then to continue. At the end of the lesson, collect the workbooks for your review.

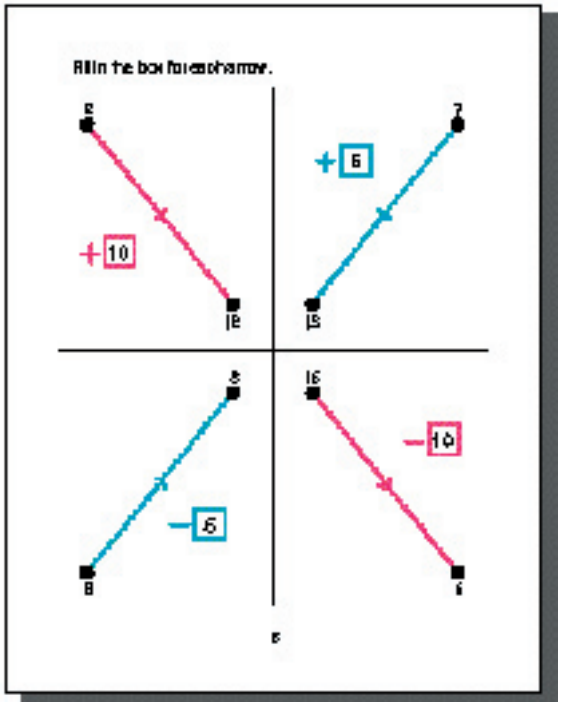
Assessment Activity

An individual student progress report for the workbook is available on Blackline W10. You may like to use this form to monitor student work.


−10
−1
34
43
33
23
22
21
20
Other arrow roads are possible; however, a shortest one has seven arrows.
Write a subtraction shown by the gray arrow. 53 − 34 = 19


Other colorings are possible.


Complete.
= 10
5 × 2 = 10
= 20
5 × 4 = 20
= 40
5 × 8 = 40
= 60
5 × 10 = 60
= 100
5 × 20 = 100


Fill in the box for each arrow.
+10
+6
−5
−10

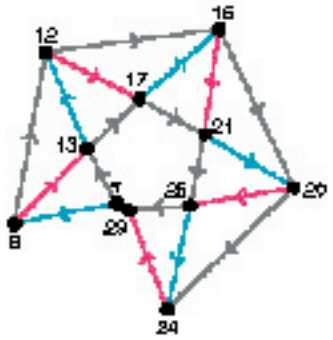
Complete.

10 − 5 = ☐ 6	5 − 4 = ☐ 1
9 − 4 = ☐ 5	6 − 4 = ☐ 2
8 − ☐ 3 = 5	7 − ☐ 4 = 3
☐ 7 − 2 = 5	☐ 8 − 4 = 4
15 − 5 = ☐ 10	12 − 2 = ☐ 10
15 − 6 = ☐ 9	14 − 4 = ☐ 10
☐ 15 − 7 = 8	16 − ☐ 6 = 10
15 − ☐ 8 = 7	☐ 18 − 8 = 10

8

Label the dots. Draw all the missing +4 arrows in gray.

+5 −1 +4



There should be nine gray arrows.

7

Zip is a secret number.

Zip can be put on this Write-on/Wipe-off by removing one shape.

Zip could be 7, 11, 13, or 14.

Zip is in this string picture.

Who is Zip? 14

8

Share 38 cards fairly between Tony and Juan.

For Tony	For Juan
10	10
6	6
4	4

Write a number sentence about this sharing.

38 ÷ 2 = 19

Share 42 bows fairly among Linda, Wanda, and Erin.

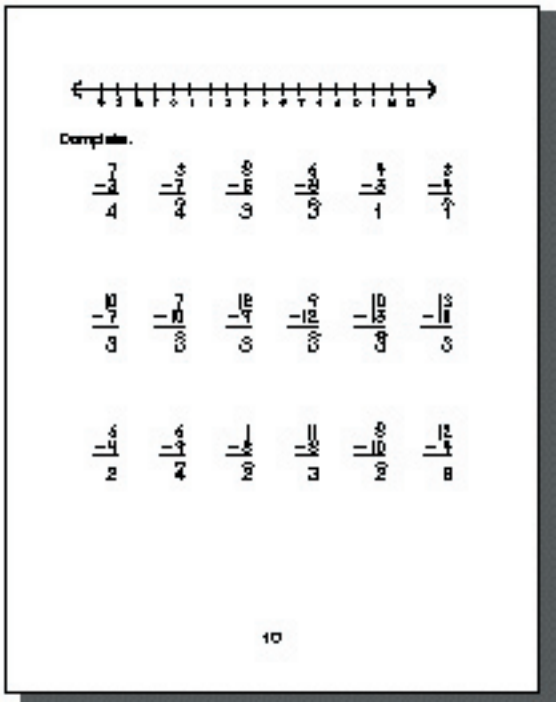
For Linda	For Wanda	For Erin
10	10	10
4	4	4

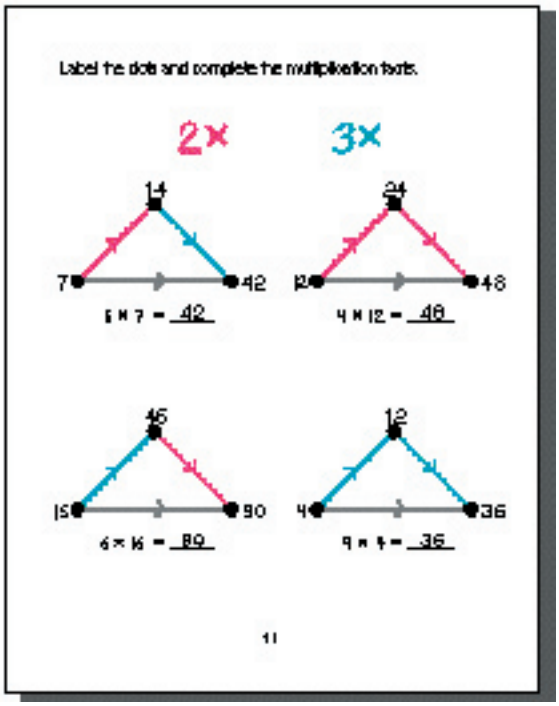
Write a number sentence about this sharing.

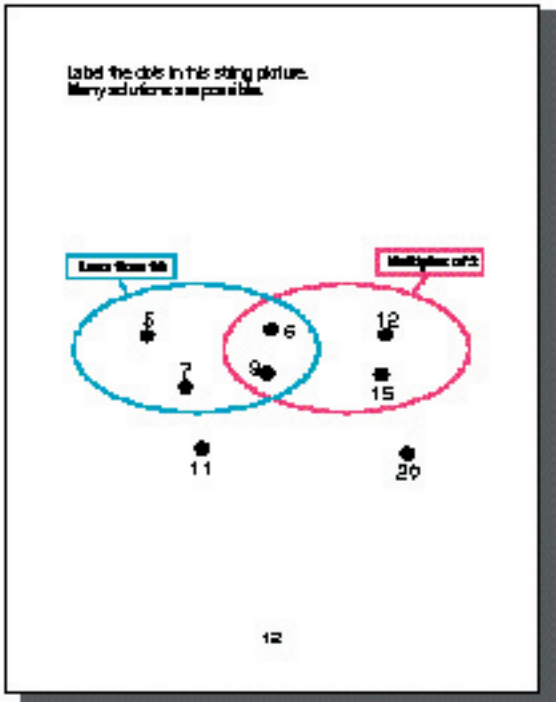
3 × 14 = 42

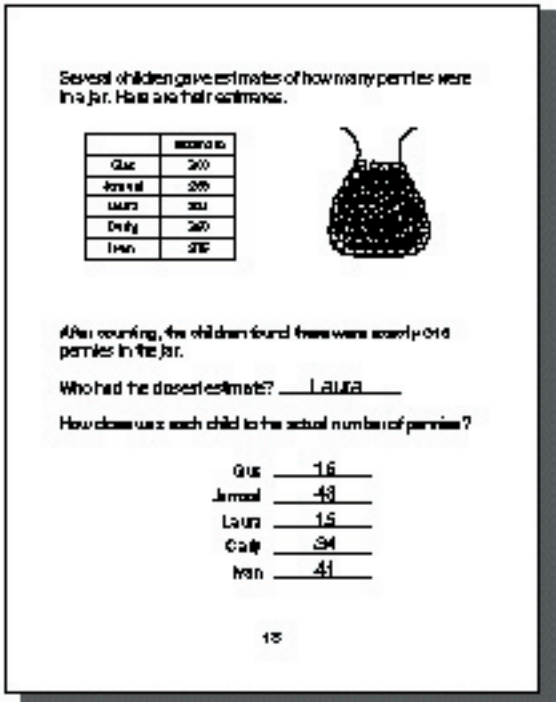
8

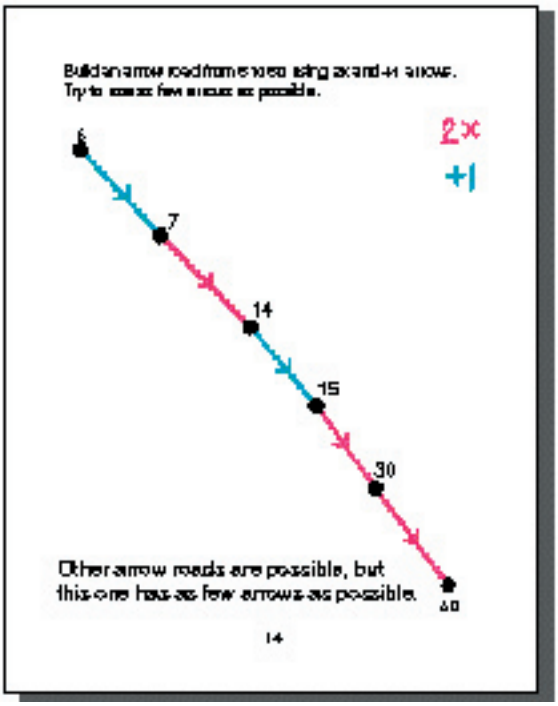
Other sharing methods and other number sentences are possible.

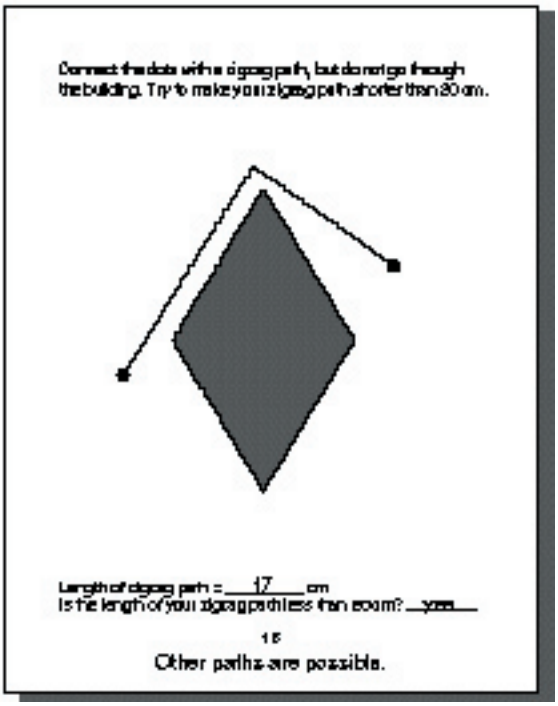

Complete.
10

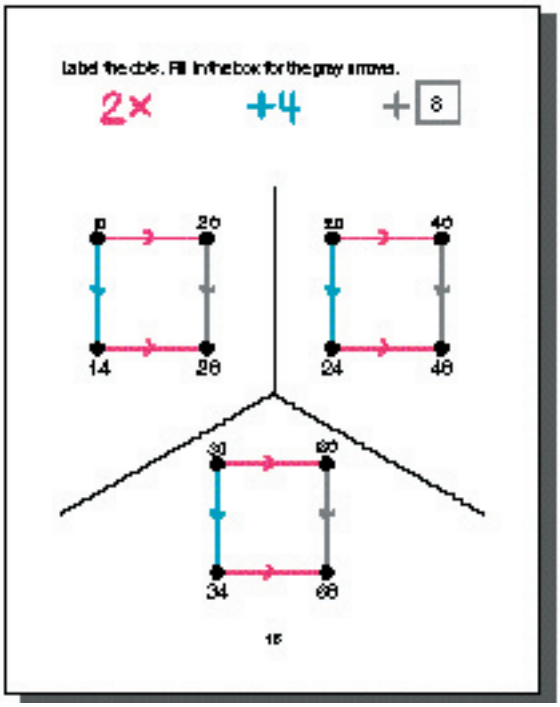

Label the dots and complete the multiplication facts.
2×
3×
14
7
42
6 × 7 = 42
24
8
48
4 × 12 = 48
45
15
90
6 × 15 = 90
12
4
36
9 × 4 = 36
11

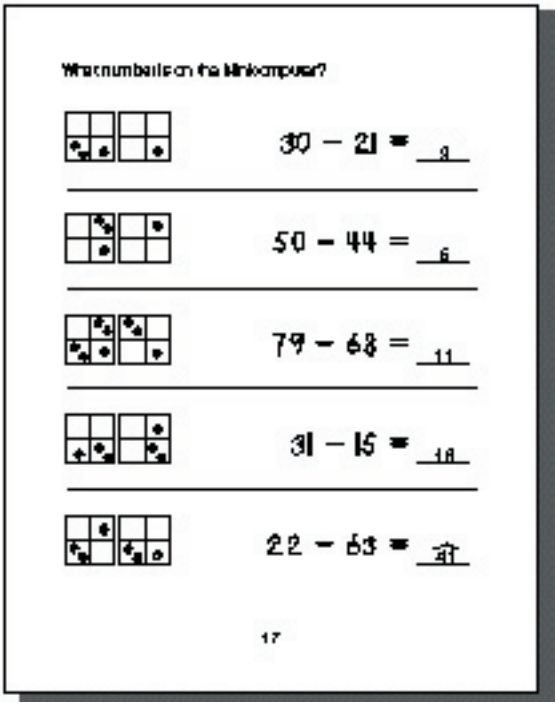

Label the dots in this string picture.
Many solutions are possible.
5
6
12
7
9
15
11
20
12


Several children gave estimates of how many pennies were in a jar. Here are their estimates.
After counting, the children found there were exactly 64 pennies in the jar.
Who had the closest estimate? Laura
How close was each child to the actual number of pennies?
Gus 16
Jamal 48
Laura 15
Cady 34
Ivan 41
13


2×
+1
7
14
15
30
Other arrow roads are possible, but this one has as few arrows as possible.
14

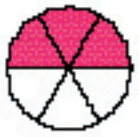

Length of zigzag path = 17 cm
15
Other paths are possible.


2×
+4
+ 8
14
24
34
16


30 − 21 = 9
50 − 44 = 6
79 − 68 = 11
31 − 15 = 16
17

Color one half of each shape red. In each case, write another name for $\frac{1}{2}$ as suggested by the picture.

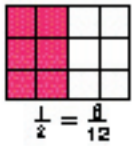
Example



$\frac{1}{2} = \frac{3}{6}$

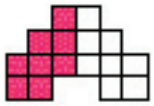
$\frac{1}{2} = \frac{2}{4}$

$\frac{1}{2} = \frac{4}{8}$



$\frac{1}{2} = \frac{6}{12}$

$\frac{1}{2} = \frac{5}{10}$



$\frac{1}{2} = \frac{8}{16}$

18

Other colorings are possible.

A great miracle to many numbers in these problems. Cross out one number in each problem to make the addition correct.

$\cancel{36} + 78 + 43 = 121$

$84 + \cancel{38} + 56 = 140$

$35 + 47 + \cancel{36} = 82$

184	~~246~~ 236	527
~~256~~		~~384~~
+245	+326	+387
429	562	914

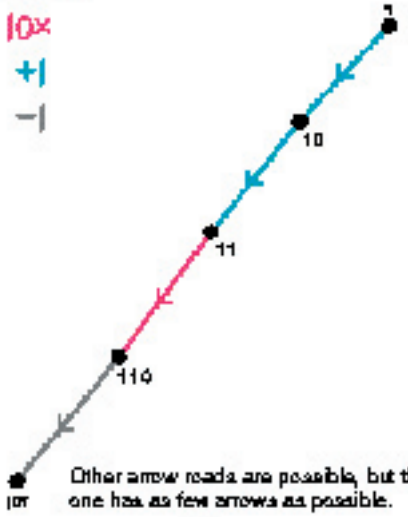
19

Build an arrow road from 1 to 100. Try to use as few arrows as possible.

10×

+1

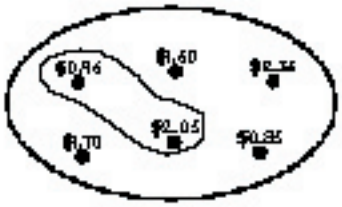
−1



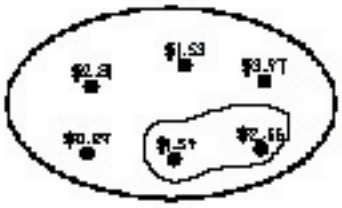
Other arrow roads are possible, but this one has as few arrows as possible.

20

Dana buys two models and spends exactly $5.
Draw one string around the prices of these two models.

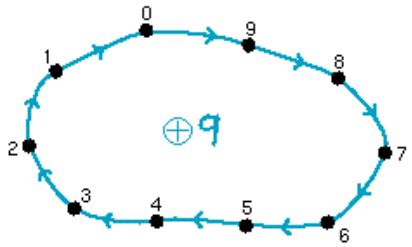


Alex buys two kites and spends exactly $4.
Draw one string around the prices of these two kites.



21

Ten number friends—0, 1, 2, 3, 4, 5, 6, 7, 8, 9—are playing the ⊕9 game. Put these numbers in the picture.



Complete these number sentences about addition with the ten number friends.

$6 \oplus 6 = \boxed{2}$

$\boxed{5} \oplus 8 = 3$

$9 \oplus \boxed{5} = 4$

$2 \oplus 4 \oplus 6 \oplus 8 = \boxed{0}$

$5 \oplus \boxed{7} \oplus 9 = 1$

22

Jon wants to weigh his dog Tapper but Tapper won't stand still on the scale. Jon gets on the scale holding Tapper. Together they weigh 186 pounds. Jon weighs 112 pounds. How much does Tapper weigh? 74 pounds

Ellen has 60 jellybeans. She gives 5 jellybeans to each other 3 friends. How many jellybeans does Ellen have left? 35

Will works for seven hours. Each hour he earns $5.00 and each hour he spends $1.00. How much money does he get? $28.00

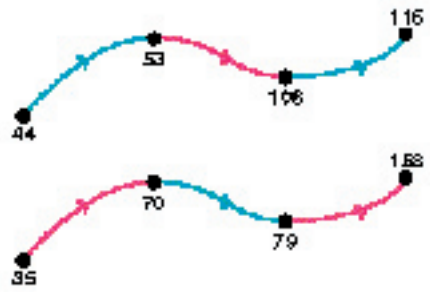
Alice went shopping. She spent half her money on lunch. Then she bought a hat for $6.00. Now she has $2.00. How much did Alice have to start with? $16.00

35

Put these numbers in this snow picture.

36 44 63 70 79 106 116 168

+9 2×



34

Put parentheses in these number sentences to make them TRUE.

$3 + (5 \times 4) = 23$

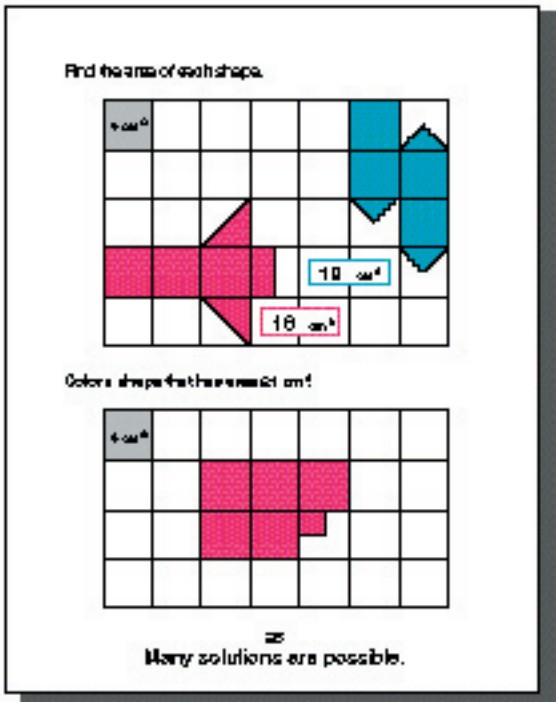
$(3 \times 5) - 4 = 11$

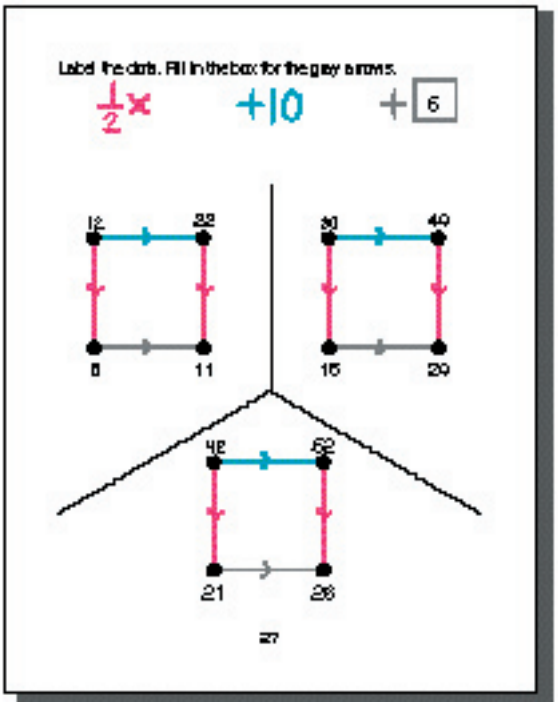
$(3 \times 5) + 4 = 19$

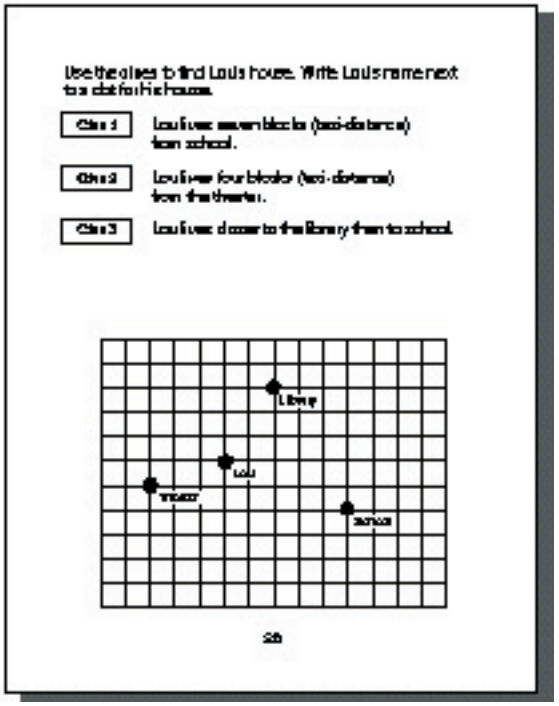
$(3 + 5) \times 4 = 32$

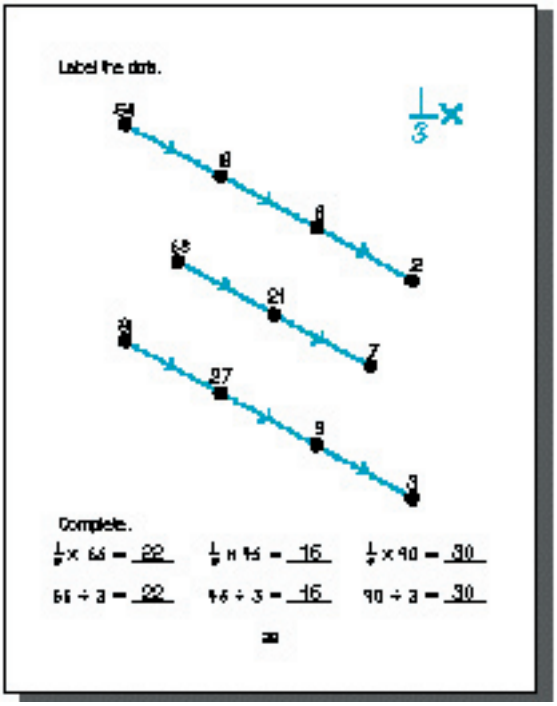
$3 \times (4 + 5) = 27$

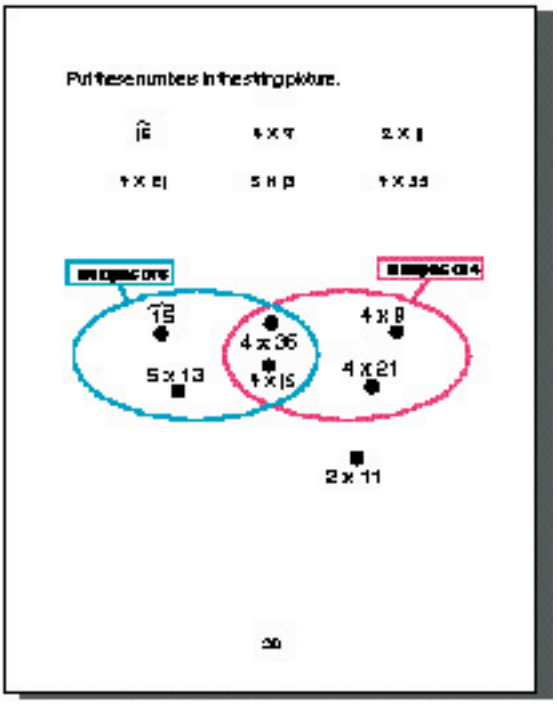
36

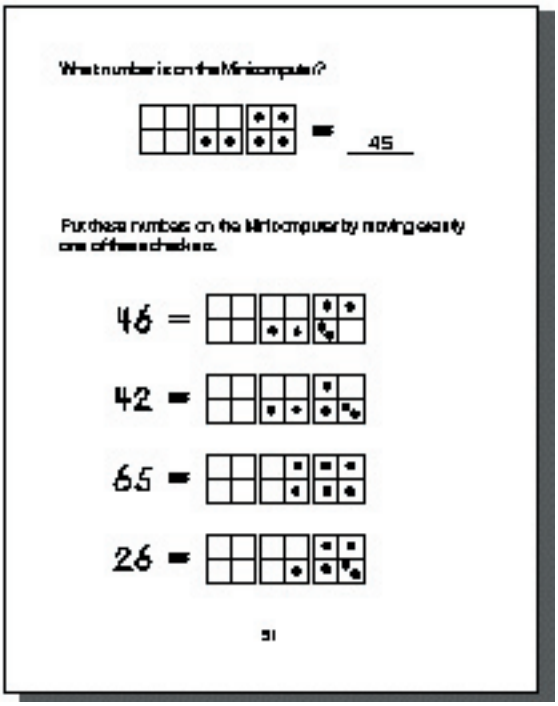

Find the area of each shape.
Many solutions are possible.

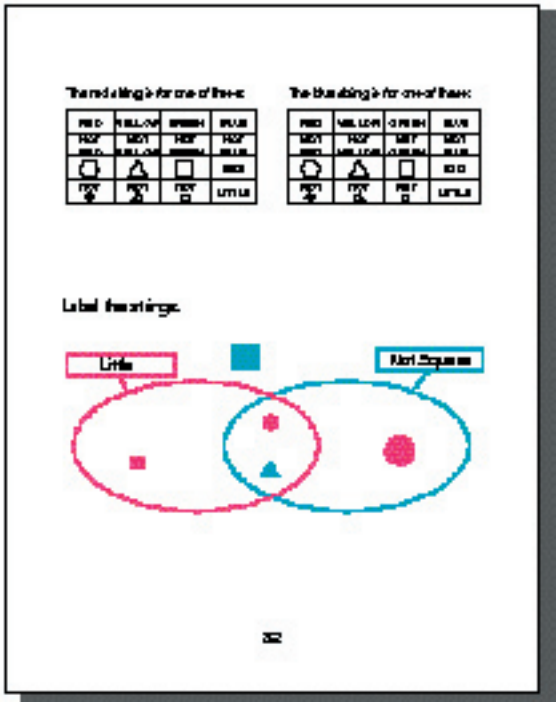

Label the dots. Fill in the box for the gray arrows.
+10
+ 6


Use the clues to find Louis's house. Write Louis's name next to the dot for his house.
Louis lives seven blocks (taxi-distance) from school.
Louis lives four blocks (taxi-distance) from the theater.
Louis lives closer to the library than to school.
Library
Louis
Theater
School


Label the dots.
Complete.


Put these numbers in the string picture.
15
4 x 36
4 x 8
5 x 13
4 x 15
4 x 21
2 x 11


What number is on the Minicomputer?
45
Put these numbers on the Minicomputer by moving exactly one of these checkers.
46 =
42 =
65 =
26 =


Label the strings.
Little
Not Square

Capsule Lesson Summary

Review how to play a spinner game. Solve a detective story in which the clues involve the Minicomputer, a string picture, an arrow picture, and a spinner game.

Materials

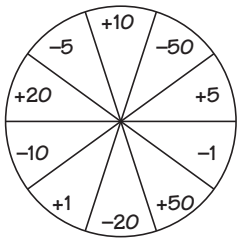
Teacher	• Spinner face • Colored chalk • Minicomputer set	• Colored pencils, pens, or crayons • Minicomputer set
Student	• Paper	

Advance Preparation: Use Blackline W11 to make a spinner face for display during Exercise 1. You may want to put the spinner face on an overhead transparency, or to draw it directly on the chalkboard, making it bigger so everyone can see easily.

Description of Lesson

Exercise 1

Display this spinner face on the board.



Note: Try to keep this part of the lesson moving quickly so your class will have enough time for the detective story in Exercise 2.

T: ***Do you remember the spinner game? What can you tell me about this game?***

Encourage students to recall the important facts about the spinner game.

- Starting at 0, spin four times, and follow the results of the four spins to get your score.
- The highest possible score is 200.
- The lowest possible score is M200.
- Some scores between M200 and 200 are possible to get and some are impossible to get when you play this game.

T: ***What are some of the final scores we can get when we play this game?***

Record the students' suggestions on the board. Some of the possibilities are the following:

	Score
0 + 10 + 20 – 10 + 50 =	70
0 + 50 + 50 + 50 – 50 =	100
0 – 50 – 50 – 50 + 50 =	$\widehat{100}$

	Score
0 + 20 + 20 + 20 – 20 =	40
0 + 20 + 20 + 10 + 10 =	60
0 – 20 – 20 – 20 + 50 =	$\widehat{10}$

T: ***Is it possible to get a score of 150?***

S: ***We can get 150 in five spins; 0 + 50 + 50 + 50 + 50 – 50 = 150.***

S: ***I can get 151 in four spins; 0 + 50 + 50 + 50 + 1 = 151.***

S: ***We need +30 on the spinner; 0 + 50 + 50 + 20 + 30 = 150.***

S: ***We cannot get a score of 150 with four spins.***

T: ***Is it possible to get a score of 149?***

S: ***Yes, 0 + 50 + 50 + 50 – 1 = 149.***

T: ***I am going to write some numbers on the board. I want you to try to find a way to get each of them as a score in the spinner game.***

Write these numbers on the board. 16 29 51 52 53

Ask the students to record on their papers a sequence of spins that would result in each score. Allow a few minutes for individual work, and then invite some students to record on the board one possible sequence of spins to obtain a score of 16, 29, 51, and 53; 52 is not a possible score.

16 = 0 + 5 + 5 + 5 + 1
29 = 0 + 10 + 10 + 10 – 1
51 = 0 + 50 + 1 + 1 – 1
52
53 = 0 + 50 + 1 + 1 + 1

T: ***Is it possible to get a score of 52?*** (No)
Why not?

Perhaps a student can explain why a score of 52 is impossible, but do not expect a well-formed explanation.

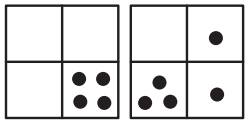
Note: It is possible to get a score of 50 after only one spin or after three spins, but it is not possible to increase that score by 2 in exactly three more spins or in one more spin.

Exercise 2: Detective Story

Students may work with a partner or in groups to solve the detective story.

Clue 1

Put this configuration on the Minicomputer.

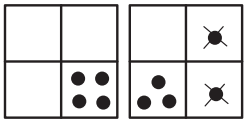


T: ***What number is on the Minicomputer?***

S: ***51, because 4 x 10 = 40 and 4 + 6 + 1 = 11; 40 + 11 = 51.***

T: ***Today you will have another opportunity to be detectives. The first clue is that the secret number can be found on this Minicomputer by removing exactly two checkers. What could the secret number be?***

Invite a student to remove two checkers from the configuration on the Minicomputer. The student might remove the checker on the 4-square and the checker on the 1-square.



An "×" through a checker indicates that the checker is removed.

T: ***What number is this?***

S: ***46, because 4 x 10 = 40 and 3 x 2 = 6.***

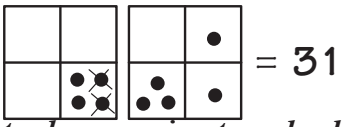
T: ***The secret number could be 46, because we can get 46 on this Minicomputer by removing exactly two checkers.***

Record this possibility for the secret number, and then put the checkers back in the original positions.

T: ***What is the least number we can get on this Minicomputer by removing two checkers?***

S: ***31.***

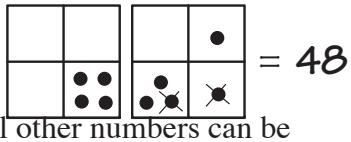
Invite someone to put 31 on the Minicomputer by removing exactly two checkers.



T: ***What is the greatest number we can get on this Minicomputer by removing two checkers?***

S: ***48.***

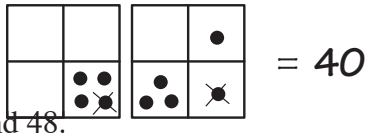
Invite someone to put 48 on the Minicomputer by removing exactly two checkers.



Record 31 and 48 on the board, leaving enough space so that several other numbers can be recorded between them. Return the checkers to their original positions.

T: ***The least number we can get by removing exactly two checkers is 31 and the greatest is 48. We can also get 46. Are there any other numbers between 31 and 48 that we can get? Can we get 40?*** (Yes)

Invite a student to show 40 by removing two checkers, and add it to the list of possibilities,



Let students experiment to find the other possibilities between 40 and 48.

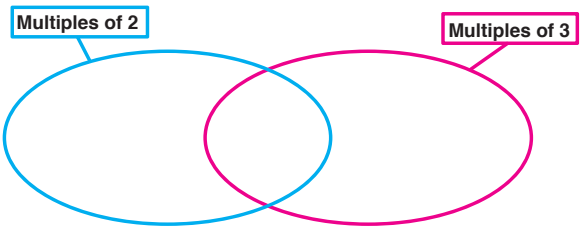
Note: Removing two checkers from the tens board gives 31 as a possibility. Removing two checkers from the ones board gives possibilities greater than 40. Removing one checker from each board gives the other possibilities (37, 39, 40).

31 37 39 40 45 46 47 48

T: ***The secret number is one of these numbers. We need another clue.***

Clue 2

Draw this string picture on the board.



T: ***Which numbers belong inside the blue string?***

S: ***The multiples of 2.***

S: ***0, 2, 4, 6, 8, 10, ….***

S: B*2* , B*4* , B*6* , B*8*, ….

T: ***Which numbers belong inside the red string?***

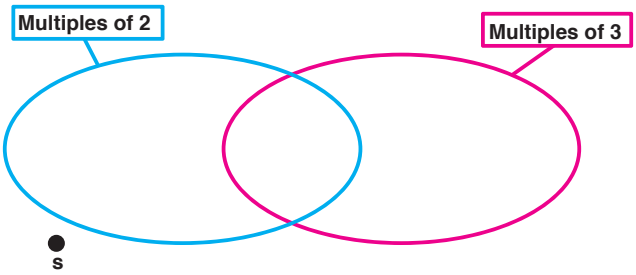
S: ***The multiples of 3.***

S: *0, 3, 6, 8, 12, …and* B*3* , B*6* , B*9* , N*12, ….*

T: ***Which numbers are in both the blue string and the red string?***

Encourage students to suggest several numbers that are both multiples of 2 and multiples of 3. Perhaps a student will comment that the numbers in the middle region are the multiples of 6.

Draw and label a dot for the secret number outside of both strings.



T: ***What information does this give us about the secret number?***

S: ***The secret number is not a multiple of 2.***

S: ***The secret number is an odd number.***

S: ***The secret number is not a multiple of 3.***

T: ***Which of the numbers in the list can we cross out because they are multiples of 2?***

S: ***40, 46, and 48.***

T: ***Which of these numbers can we cross out because they are multiples of 3?***

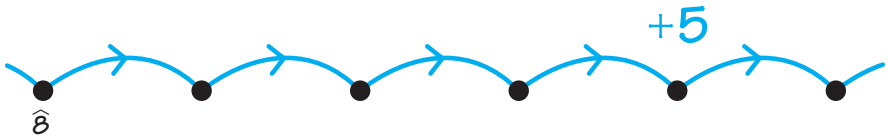
S: ***39, 45, and 48.***

Note: 48 is a multiple of 3 and a multiple of 2.

Your class should conclude that the secret number could be 31, 37, or 47. Erase the other numbers from the list and erase the string picture.

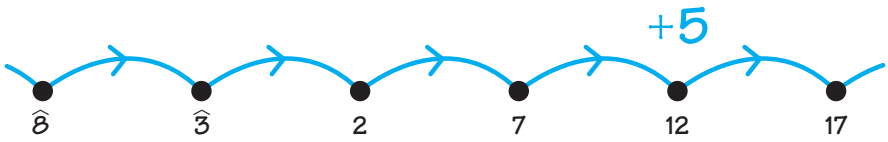
Clue 3

Draw this arrow picture on the board and ask students to copy it.



T: ***The secret number is one of these numbers: 31, 37, or 47. I will give you another clue. The secret number is on the same +5 arrow road as* B*8. Label the dots in this arrow picture, and try to decide which of these numbers could be the secret number.***

Walk around the room and look at several of the students' papers. Invite a student to label the dots in the arrow picture on the board. If necessary, remind students that the unfinished arrows indicate that the +5 arrow road continues in both directions and that only part of the road is drawn on the board.



Point to the list of possible numbers and ask which of them are on this +5 arrow road.

S: ***37 and 47 are on this arrow road because they both end in 7.***

T: ***All whole numbers that end in 2 or 7 are on this arrow road, so 37 and 47 could both be the secret number. Is 31 on this arrow road?***

S: ***No, 31 is on the +5 arrow road that meets the number 1.***

Erase 31 and conclude that the secret number is either 37 or 47.

Clue 4

T: ***The last clue is that the secret number is one of the scores we can get when we play the spinner game. Remember that we spin exactly four times when we play the game. What is the secret number? Write it on your paper.***

Look at several of the students' papers and then ask someone to answer aloud.

S: ***47 is the secret number because 0 + 50 –1 – 1 – 1 = 47.***

T: ***Can we get 37 as a score when we play the spinner game?*** (No)

Perhaps one of the students will be able to explain why it is not possible to get a score of 37 when you play the spinner game, but do not expect a well-formed answer.

Note: A score of 37 is impossible because neither +30 nor +40 is on the spinner face (so it takes at least two spins to get a score of 30 or 40). It takes at least three spins to increase a score of 30 to 37 or to decrease a score of 40 and 37.

Conclude that 47 is the secret number.

Capsule Lesson Summary

Begin the workbook *Galaxy of Problems #4* . Play The Number Line Game with non-integer decimal numbers as secret numbers.

Materials

Teacher	• None	Student	• *Galaxy of Problems #4* Workbook • Colored pencils, pens, or crayons

Description of Lesson

Distribute copies of the workbook *Galaxy of Problems #4* and let students begin working individually. If the questions you receive indicate that many students are having difficulty with a particular page, hold a short collective discussion of the problems presented on that page.

Allow 30 to 40 minutes for individual work in the workbook; then collect the workbooks for your review. They will be used again in Lesson W13. Spend any remaining time playing The Number Line Game with secret numbers that are non-integer decimal numbers. An example of a possible game is given here. The secret numbers is 0.57.

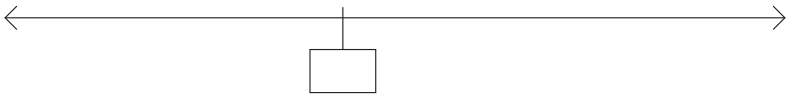
T: ***I am thinking of a number between 0 and 1. My number is hidden in this box.***

Draw a line on the board and position a box for the secret number near the center of your picture.

T: ***Guess what my secret number is. It may help you to think of the $ number line between $0 and $1.***

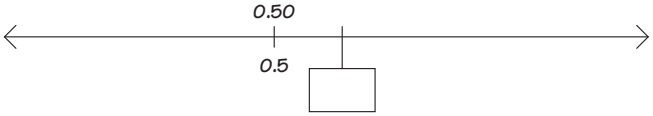
S: ***0.5.***

T: ***That's the same number as ½ and 0.50. I'll show you where 0.5 is on this number line. 0.5 is less than my number.***



S: ***0.9.***

T: ***What is another name for 0.9?***

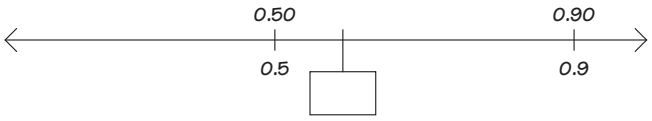


S: ***0.90.***

T: ***0.9 might not be in exactly the right place, but I've tried to make it as close as possible. 0.9 is more than my number.***

S: ***0.30 (or 0.3).***

T: ***How much money would that be?***

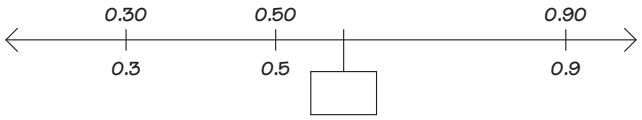


S: ***30 cents.***

T: ***0.30 is less than my secret number.***

Note: Although a guess may give no new information, such as 0.30 in this sample game, do not criticize or comment except to say that it is more or less than your number.

The game continues with students guessing 0.60; 0.55; and 0.57.



T: ***0.57 is my secret number.***

You may like to continue this activity with a secret number between 0 and 2.

Capsule Lesson Summary

Continue working in the workbook *Galaxy of Problems #4*. (This is the second of two lessons using this workbook.)

Materials

Teacher	• None	Student	• *Galaxy of Problems #4* Workbook • Colored pencils, pens, or crayons

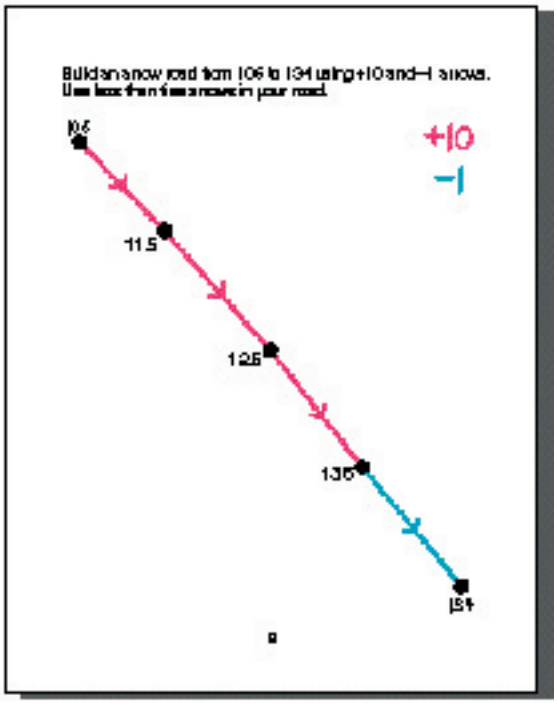
Description of Lesson

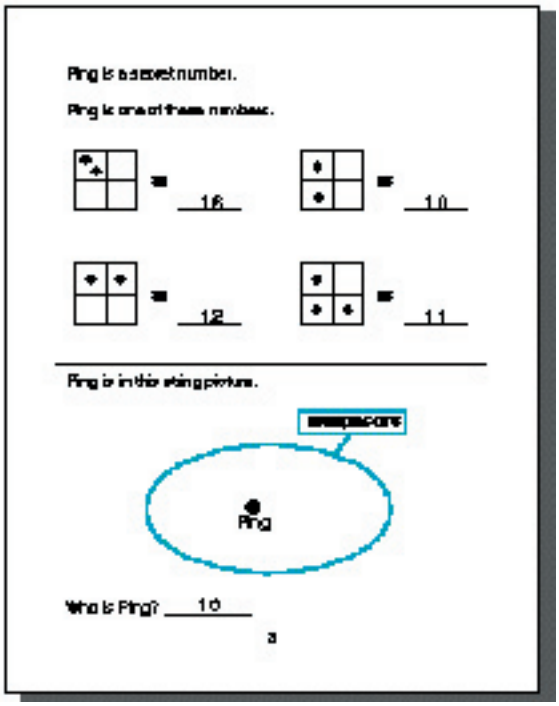
Your review of the students' workbooks may indicate that a short collective discussion is needed about a particular page. If appropriate, solve a problem similar to the problems on that page before distributing the students' copies of the workbook *Galaxy of Problems #4*.

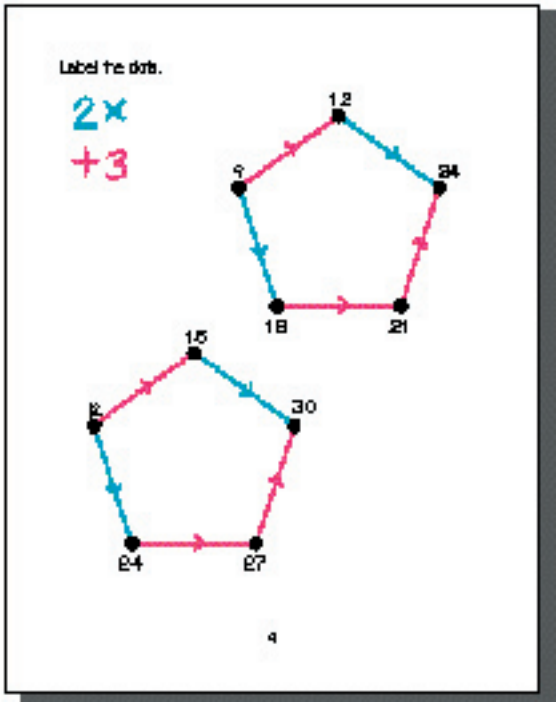
After 30–40 minutes of individual work, collect the workbooks for your review. Spend any time that remains playing Minicomputer Golf (see Lesson N25 for a description of this game).

Assessment Activity

An individual student progress report for the workbook is available on Blackline W13. You may like to use this form to monitor student work.

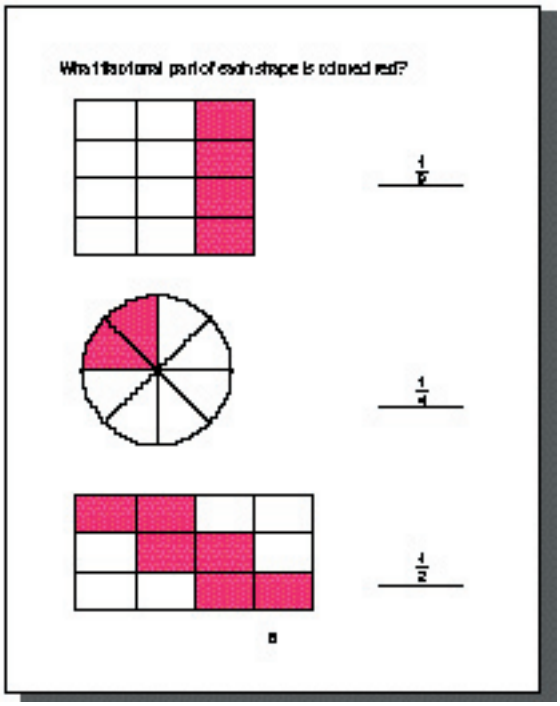


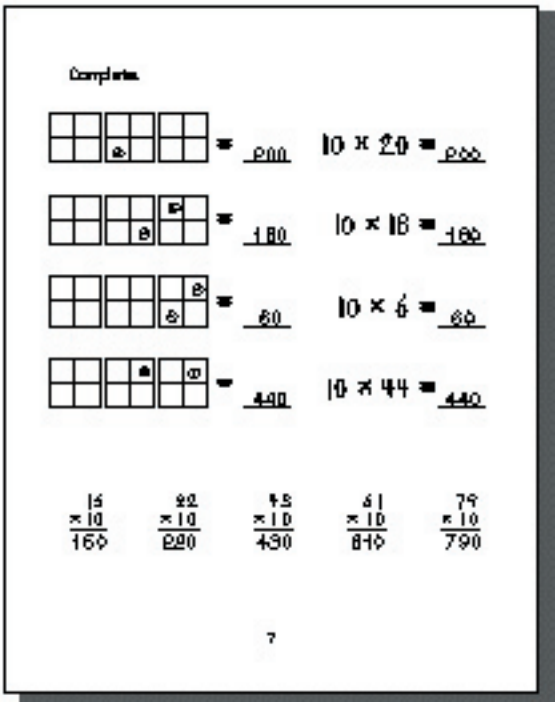


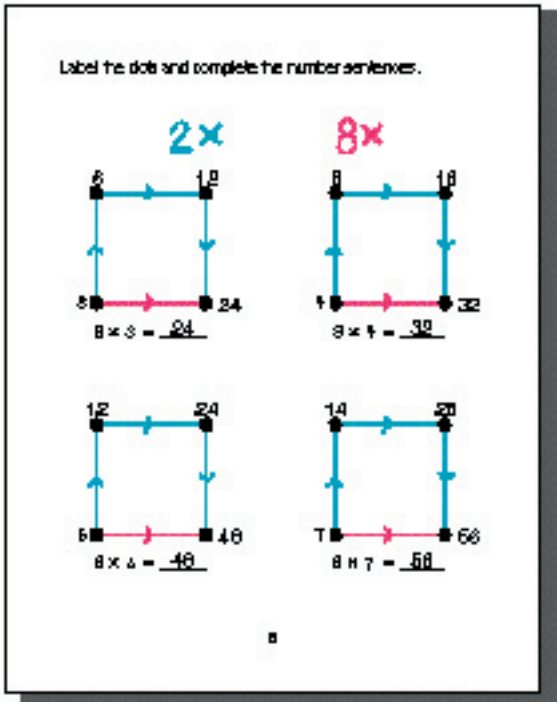


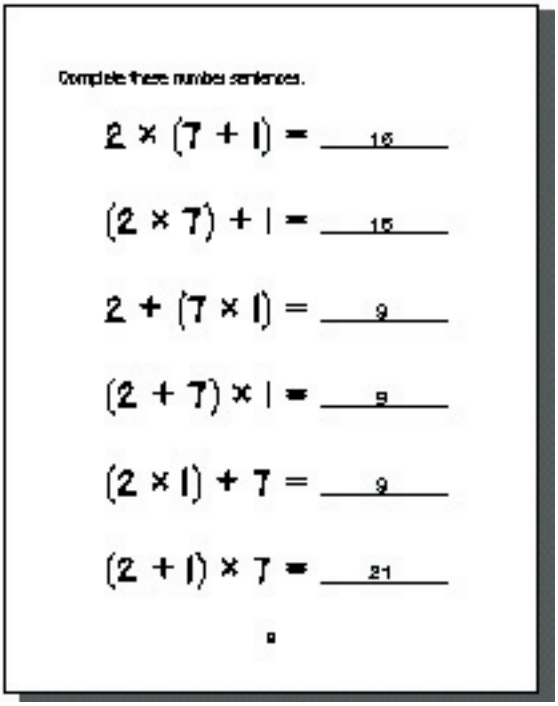
Complete this subtraction table.

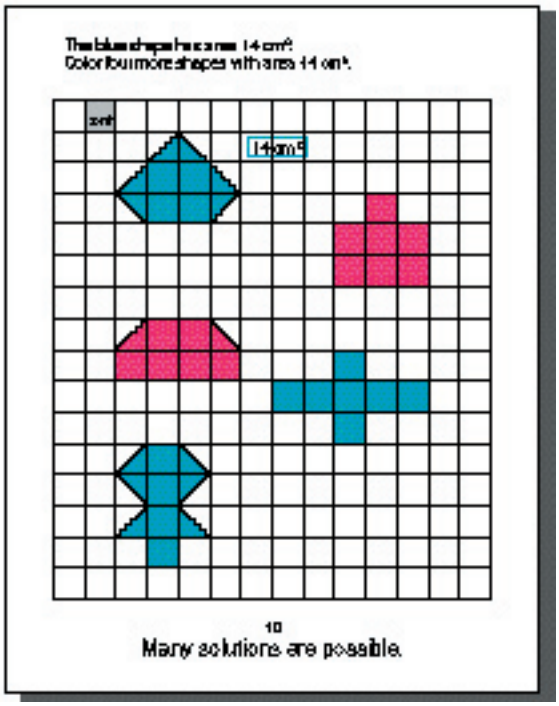
−	3	5	9
10	7	5	1
20	17	15	11
30	27	25	21
40	37	35	31
50	47	45	41

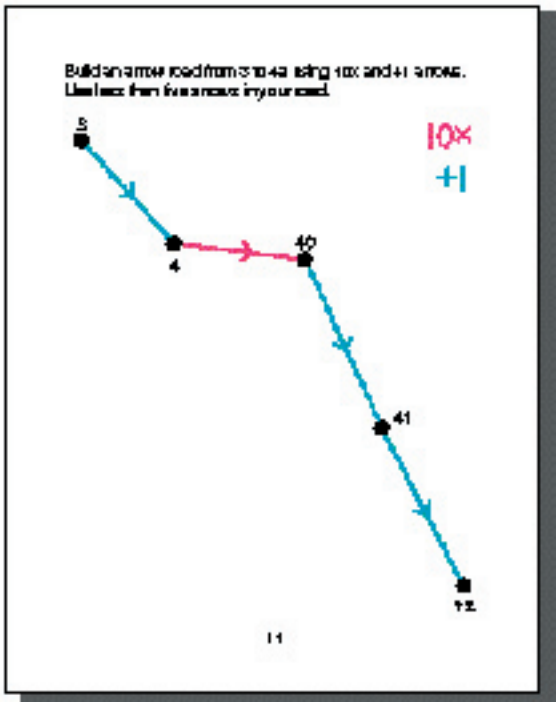

What fractional part of each shape is colored red?
$\frac{1}{4}$
$\frac{1}{8}$
$\frac{1}{2}$
8

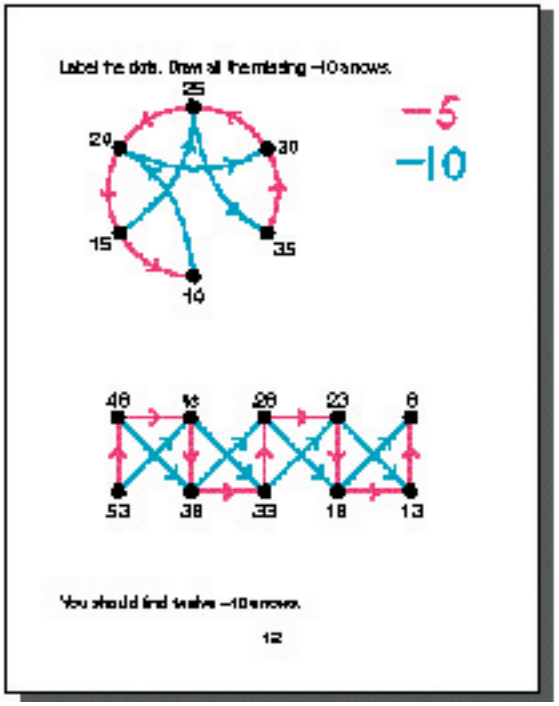

Complete.
= 200
10 × 20 = 200
= 180
10 × 18 = 180
= 60
10 × 6 = 60
= 440
10 × 44 = 440
15 × 10 = 150
22 × 10 = 220
43 × 10 = 430
61 × 10 = 610
79 × 10 = 790
7

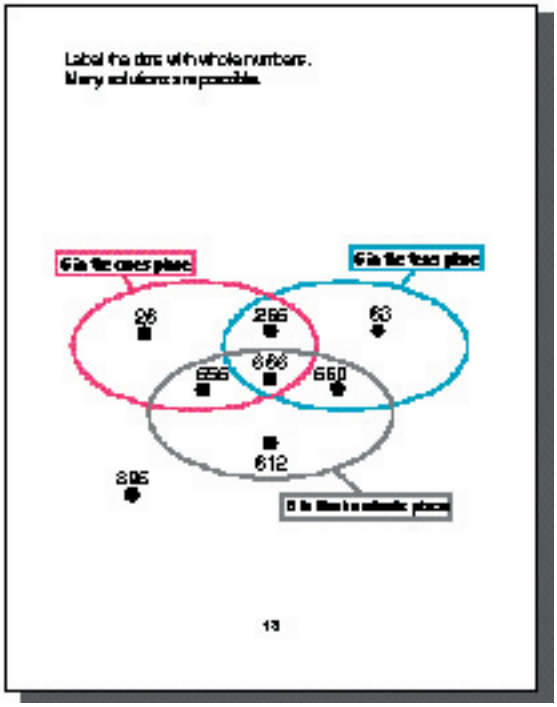

Label the dots and complete the number sentences.
2×
8×
8 × 3 = 24
8 × 4 = 32
8 × 6 = 48
8 × 7 = 56
8

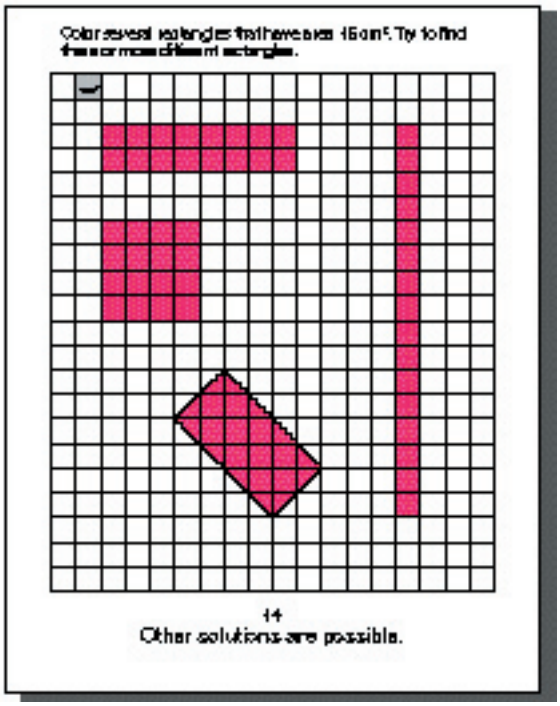

Complete these number sentences.
2 × (7 + 1) = 16
(2 × 7) + 1 = 15
2 + (7 × 1) = 9
(2 + 7) × 1 = 9
(2 × 1) + 7 = 9
(2 + 1) × 7 = 21
8

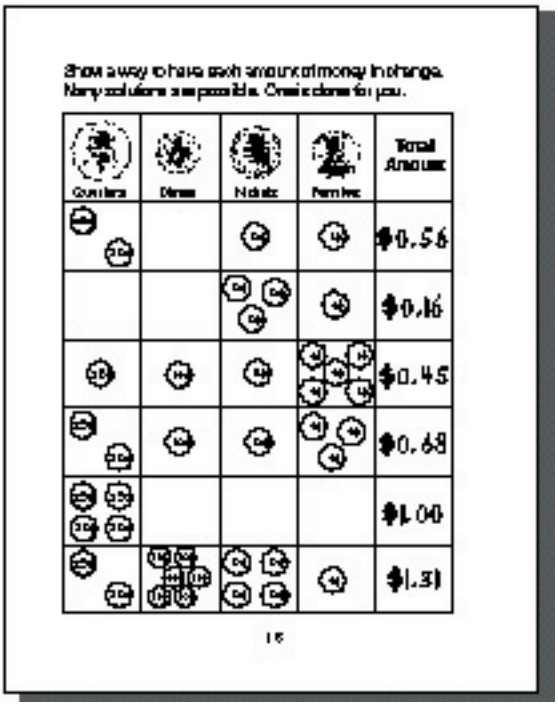

14 cm²
10
Many solutions are possible.

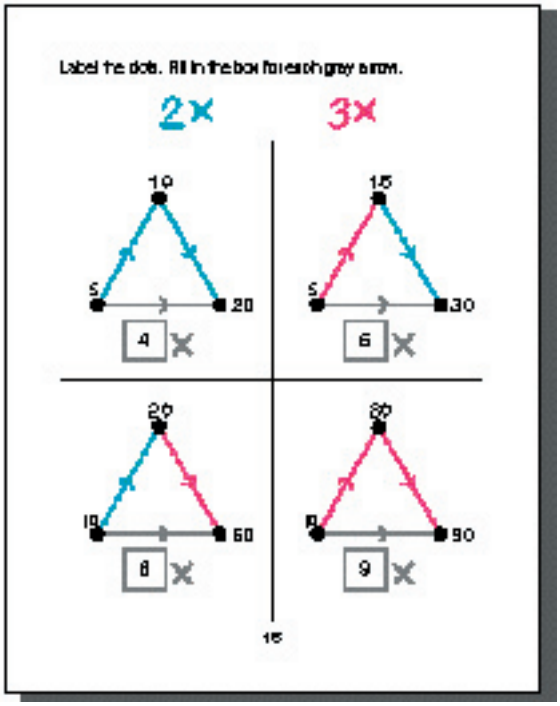

10×
+1
11

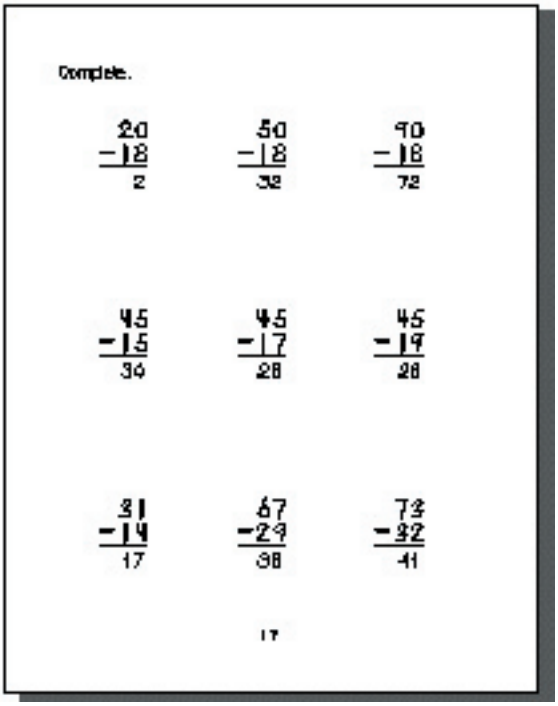

Label the dots. Draw all the missing −10 arrows.
−5
−10
12


Label the dots with whole numbers.
Many solutions are possible.
13

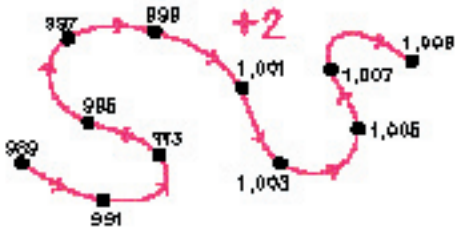

14
Other solutions are possible.


Total Amount
$0.56
$0.16
$0.45
$0.68
$1.00
$1.31
15


Label the dots. Fill in the box for each gray arrow.
2×
3×
10
5
20
4
15
5
30
6
20
10
60
8
30
10
90
9
16


Complete.
20 − 18 = 2
50 − 18 = 32
90 − 18 = 72
45 − 15 = 30
45 − 17 = 28
45 − 19 = 26
31 − 14 = 17
67 − 29 = 38
73 − 32 = 41
17

Pang is a secret number.
Pang is in the snow picture.



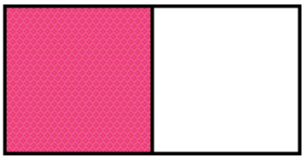
Pang can be put on this Minicomputer using exactly one regular checker and one negative checker.

Who is Pang? 999

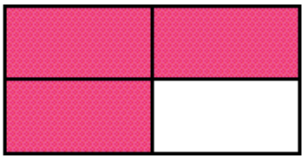
Put Pang on the Minicomputer.

18

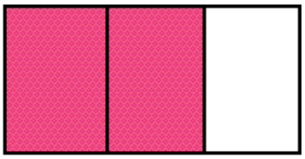
Divide this rectangle and color one-half ($\frac{1}{2}$).



Divide this rectangle and color three-fourths ($\frac{3}{4}$).



Divide this rectangle and color two-thirds ($\frac{2}{3}$).



19

Other divisions and colorings are possible.

Share 105 marbles fairly among Nick, Cathy, and Julie.

for Nick	for Cathy	for Julie
25	25	25
10	10	10
5	5	5

Write a number sentence about this sharing.

105 ÷ 3 = 35
35 x 3 = 105

Share 152 marbles fairly among Kyle, Max, Chris, and Liam.

for Kyle	for Max	for Chris	for Liam
25	25	25	25
10	10	10	10
3	3	3	3

Write a number sentence about this sharing.

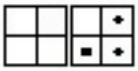
4 x 38 = 152
152 ÷ 4 = 38

20

Other sharing methods and other number sentences are possible.

Tip is a secret number.

Clue 1 Tip can be put on this Minicomputer by moving one of these checkers to the ones board.



Which numbers could Tip be? 13, 23, 43, 33, 15, 25, 41, 51, 18, 28, 48, or 88.

Clue 2

Tip could be 28, 38, or 48.

Clue 3 Tip is on the same snow road as 8.

Who is Tip? 28

21

Label the dots in this picture with these numbers:

3 × 7 7 × 11 99

5 × 13 3 × 25 33 + 9

School Carnival

Tickets cost 10¢ each

Booths		Food	
Face Painting	—3 tickets	Hotdog	—3 tickets
Fortune Telling	—5 tickets	Drink	—1 ticket
Ride the Pony	—5 tickets	Cookie	—2 tickets
Grab Bag	—1 ticket	Chips	—1 ticket

Farkle goes to the carnival with $20. How many tickets can he buy? 200

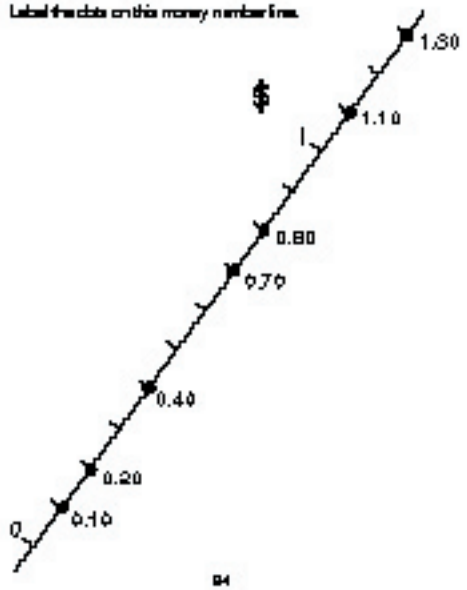
Farkle wants to use all his tickets. He decides to use half of his tickets on booths and half on food. What booths could he go to? What food could he get?

Booths:
Face Painting and Grab Bag
or
Ride the Pony and Grab Bag
or
Twice on the Fortune Telling

Food:
Hotdog and Drink
or
Hotdog and two Cookies
or
Drink and two Chips
or
Five Cookies

Other solutions are possible.

Label the dots on this money number line.



2× 4× 8×

Complete this table.

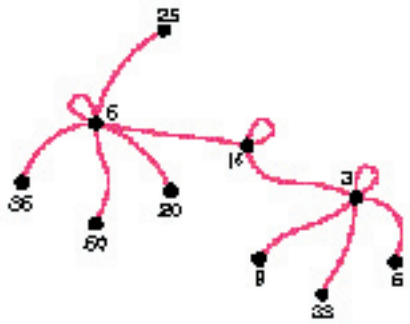
Starting Number	2×	4×	8×
12	24	48	96
16	32	64	128
25	50	100	200
50	100	200	400
61	122	244	488

Complete.

$$\begin{array}{r} 154 \\ +\ 505 \\ \hline 659 \end{array} \qquad \begin{array}{r} 39 \\ +\ 52 \\ \hline 91 \end{array}$$

$$\begin{array}{r} 408 \\ +\ 357 \\ \hline 765 \end{array} \qquad \begin{array}{r} 273 \\ +\ 127 \\ \hline 400 \end{array}$$

Label the dots. Many solutions are possible.



Rona wants to buy 12 pencils. Pencils are in packages of 3 for 4 and in packages of 8 for 9. What should Rona buy? Explain your answer.

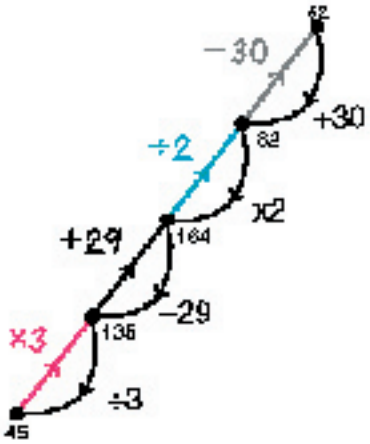
Rona should buy four packages of 3 because that would cost her the least.

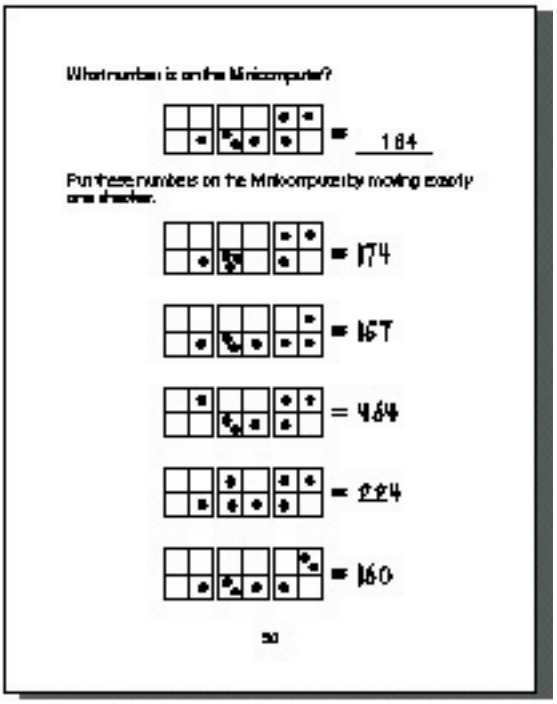
How much will 12 pencils cost? $2.60

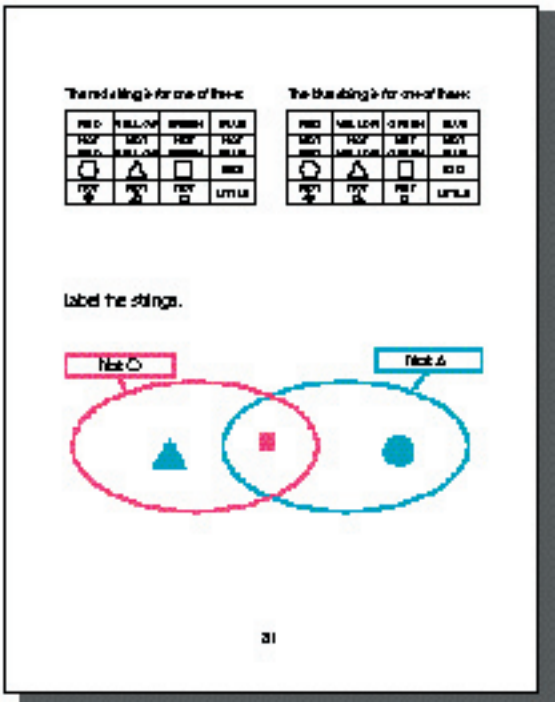
Rona has $3.00. Can she buy 12 pencils? Yes
If yes, how much change will she get? $2.40

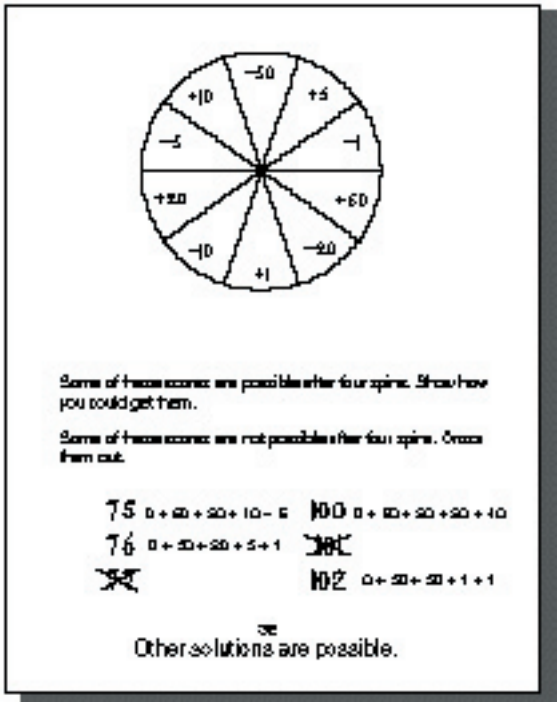
Answers may vary if students give other explanations of what Rona should buy.

Draw and label all the return arrows.
Label the dots.




What number is on the Minicomputer?
= 184
Put these numbers on the Minicomputer by moving exactly one checker.
= 174
= 167
= 464
= 224
= 160
30


Label the strings.
31


−50
+5
+10
−1
−5
+50
+50
−10
−50
+1
Some of these scores are possible after four spins. Show how you could get them.
Some of these scores are not possible after four spins. Cross them out.
75 0 + 50 + 50 + 10 − 5
100 0 + 50 + 50 + 50 + 50
76 0 + 50 + 50 + 5 + 1
102 0 + 50 + 50 + 1 + 1
32
Other solutions are possible.

Capsule Lesson Summary

Solve a detective story with two secret numbers using clues that involve the Minicomputer, a string picture, and an arrow picture.

Materials

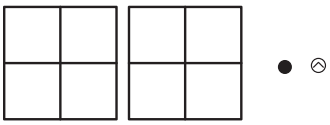
Teacher	• Minicomputer set • Colored chalk	Student	• Paper • Colored pencils, pens, or crayons

Description of Lesson

T: ***Today we are going to investigate two secret numbers. Their code names are Zin and Zan.***

Clue 1

Display two Minicomputer boards, one regular checker, and one negative checker.



T: ***Your first clue is that Zin and Zan can each be put on this Minicomputer using exactly one regular checker and one negative checker. Which numbers could Zin and Zan be?***

Invite several students to put numbers on the Minicomputer using exactly two checkers, one regular and one negative. For each number, ask if it is even or odd and record it on the board.

After several possibilities for Zin and Zan have been found, ask for the following:

- the greatest possible number (79)
- the least possible number (N79)
- an even number less than 50
- the greatest possible even number (78)
- an odd number between 30 and 40 (39)
- an odd number less than 0

Continue this activity until your list includes ten to fifteen numbers.

T: ***The greatest number we can put on this Minicomputer using these two checkers is 79 and the least number is*** N***79. Therefore, we know that Zin and Zan are both between 80 and*** N***80.***

Zin and Zan are between N80 and 80.

Record this information on the board.

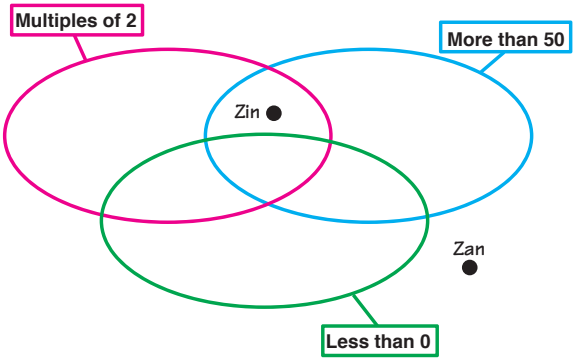
T: ***Some of the numbers between*** N***80 and 80 that could be Zin and Zan are in our list. How many numbers do you think would be in our list if we found all the numbers that can be put on this Minicomputer using one regular checker and one negative checker?***

Let the students guess but do not try to reach any conclusions about the number of possibilities. For your information, there are 53 numbers that can be put on two Minicomputer boards using exactly two checkers, one regular and one negative.

T: ***Remember, only a few of the numbers that could be Zin or Zan are in our list. I will give you another clue.***

Clue 2

Draw this string picture on the board.



T (tracing the red string)**:** ***This string is for multiples of 2. What is another name for those numbers?***

S: ***Even numbers.***

T: ***What does this string picture tell us about Zin and Zan?***

S: ***Zin is an even number more than 50.***

S: ***Zan is an odd number between 0 and 50.***

Note: Since Zan is an odd number, Zan cannot be 0 or 50.

T: ***Which of the secret numbers (Zin or Zan) is greater?***

S: ***Zin.***

T: ***Let's make a list of numbers Zin could be.***

Point to the list of numbers resulting from the first clue.

T: ***Now that we know where Zin is in this string picture, could Zin be any of these numbers?***

With your students, identify numbers in the list that could be Zin and record them on the board. Perhaps 78 is the only number in your original list that could be Zin. Ask for other numbers that Zin could be. When a number is suggested, first ask if it is an even number greater than 50. If the answer is yes, invite a volunteer to put the number on the Minicomputer using exactly one regular checker and one negative checker. Record any numbers that could be Zin on the board. Continue until you have this list for Zin.

Zin	
78	$(80 + \widehat{2})$
76	$(80 + \widehat{4})$
72	$(80 + \widehat{8})$
70	$(80 + \widehat{10})$
60	$(80 + \widehat{20})$

T: ***Could Zan be any of these numbers? Remember that the string picture tells us that Zan is an odd number between 0 and 50.***

With your students, identify numbers in the list that could be Zan and record them on the board. Perhaps 39 will be the only number in the original list that could be Zan. Ask for other numbers that could be Zan. When a number is suggested, ask first if it is an odd number between 0 and 50. If the answer is yes, invite a student to try to put the number on the Minicomputer using exactly one regular checker and one negative checker. On the board, record any numbers that could be Zan.

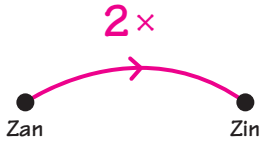
Note: It is possible that a student will observe that for Zan to be an odd number and to be on the Minicomputer, the negative checker must be on the 1-square. This observation, if made, will help your students find all the possibilities for Zan.

Continue until your students have found all possibilities for Zan. Erase everything except the list of numbers for Zin and Zan.

Zin	
78	(80 + $\widehat{2}$)
76	(80 + $\widehat{4}$)
72	(80 + $\widehat{8}$)
70	(80 + $\widehat{10}$)
60	(80 + $\widehat{20}$)

Clue 3

Draw this arrow picture on the board, and ask students to copy it on their papers.



T: ***2 x Zan = Zin. Zin is twice as much as Zan. If you think you know which numbers are Zan and Zin, write them on your paper.***

Look at several of the students' papers, and then ask someone to answer aloud.

S: ***Zan is 39 and Zin is 78.***

Find that 39 is in the list for Zan and 78 is in the list for Zin, label the dots in the arrow picture, and calculate 2 x 39 = 78. Conclude that Zan could be 39 and Zin could be 78.

T: ***Could any of these other numbers be Zan?***

S: ***39 is the greatest number in the list for Zan and 19 is the next greatest. 2 x 19 is less than 50, so none of the numbers except 39 can be Zan.***

T: ***What number is 2 x 19?***

S: ***38.***

T: ***All the numbers Zin could be are 50 or more, so it would be impossible to draw a 2x arrow from any of these numbers*** (point to numbers in the list for Zan) ***to any of the numbers that Zin could be.***

Conclude that Zan is 39 and Zin is 78.

Capsule Lesson Summary

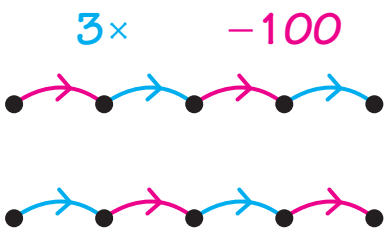
Put a set of ten numbers in their proper places in an arrow picture with 3x and –100 arrows. Begin the workbook *Fishing for Numbers, Part IV*.

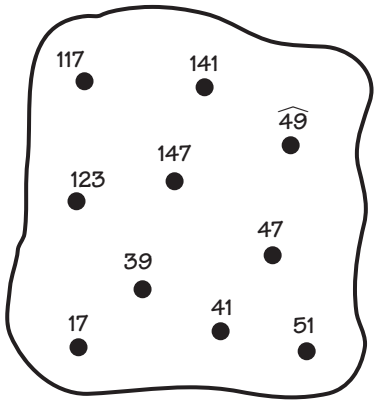
Materials

Teacher	Student
• Colored chalk	• Paper • Colored pencils, pens, or crayons • *Fishing for Numbers, Part IV* Workbook

Description of Lesson

You may like to let students work in pairs for this warm-up activity. Draw these pictures on the board and ask students to copy the pictures on their papers. In pairs, one student can copy the arrow picture while the other copies the numbers.





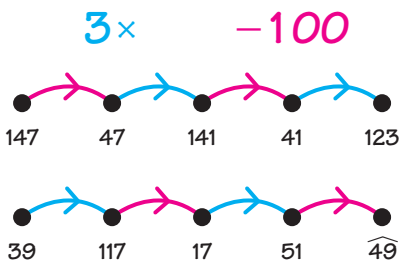
Remind students of the *Fishing for Numbers* activities done previously. They should recall that the problem is to put all of the numbers shown inside the string in their proper places in the arrow picture. Direct students to work with their partners on this problem.

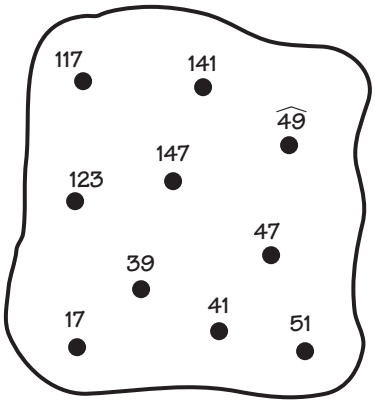
When many students have finished a problem, ask someone to suggest a number for the starting dot of one of the arrow roads. Pursue this choice and see where it leads. If it leads to an incorrect situation, let the students discover this and discuss why that choice for a starting dot won't work.

In the course of solving this problem collectively, the opportunity to discuss various strategies should arise. Allow ample opportunity for the students to discover some good strategies themselves. A student may suggest identifying the least (greatest) number in one of the arrow roads, but this would be difficult because multiplying a whole number by 3 results in a greater number and subtracting 100 will always result in a lesser number.

Students might suggest crossing out numbers inside the string as they place them in the arrow picture. This procedure is questionable since in a trial and error method mistakes may be made and new starts may be necessary.

Continue until all the dots in the arrow picture are labeled.





Distribute copies of the workbook *Fishing for Numbers, Part IV,* and allow students to work independently for the rest of the class period. Emphasize that it is very likely they will need to use their erasers, and that they should make several tries before asking for help.

At the end of the lesson, collect the workbooks for your review. These workbooks will be used again in Lesson W16.

Capsule Lesson Summary

Use context clues to put the numbers in a string picture in their proper places in a story. Continue working in the workbook *Fishing for Numbers, Part IV*. (This is the second of two lessons using this workbook.)

Materials

Teacher
- Colored chalk

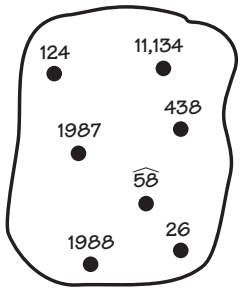
Student
- Paper
- *Fishing for Numbers, Part IV* Workbook

Advance Preparation: Before starting the lesson, write the story for the warm-up activity on the board or on an overhead transparency.

Description of Lesson

Allow students to work with a partner for this warm-up activity.

Put this story and set of numbers on the board.



Sarah Covington–Fulcher made the longest run by an individual. She ran _____ miles around the United States starting July 21, _____. She ran every day for _____ days ending on Oct. 2, _____. During the run, Sarah experienced temperatures from _____° F (coldest) to _____° F (hottest) and wore out _____ pairs of running shoes.

Tell the class that the problem is to put all the numbers inside the string into the blanks of the story so that the story makes sense. Invite student partners to work on this problem for a few minutes.

When many students have finished the problem, conduct a collective discussion. As students tell the class how to fill in the blanks, ask them to explain their reasoning. A solution is given below.

Sarah Covington–Fulcher made the longest run by an individual. She ran 11,134 miles around the United States starting July 21, 1987. She ran every day for 438 days ending on Oct. 2, 1988. During the run, Sarah experienced temperatures from 58° F (coldest) to 184° F (hottest) and wore out 26 pairs of running shoes.

Distribute the students' copies of the workbook *Fishing for Numbers, Part IV* and allow about 30–40 minutes for individual work. Ask students first to correct or complete pages from the previous week's work and then to continue. At the end of the lesson, collect the workbooks for your review.

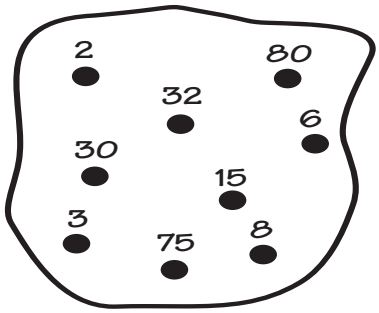
Assessment Activity

An individual student progress report for the workbook is available on Blackline W16. You may like to use this form to monitor student work.

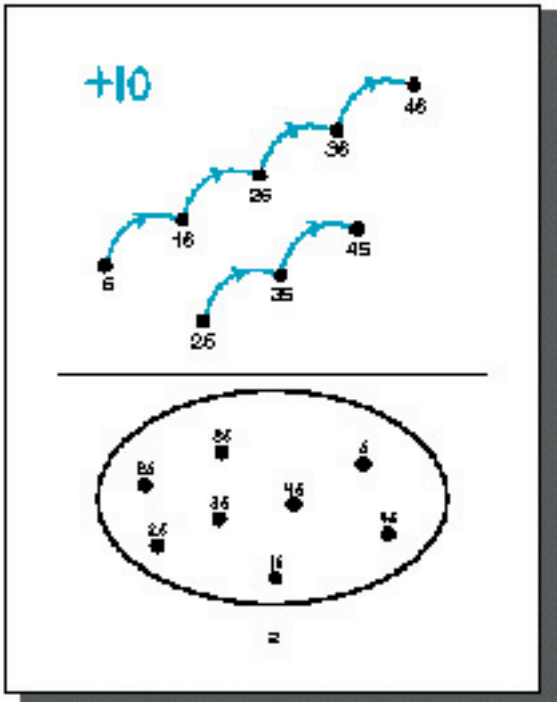
Home Activity

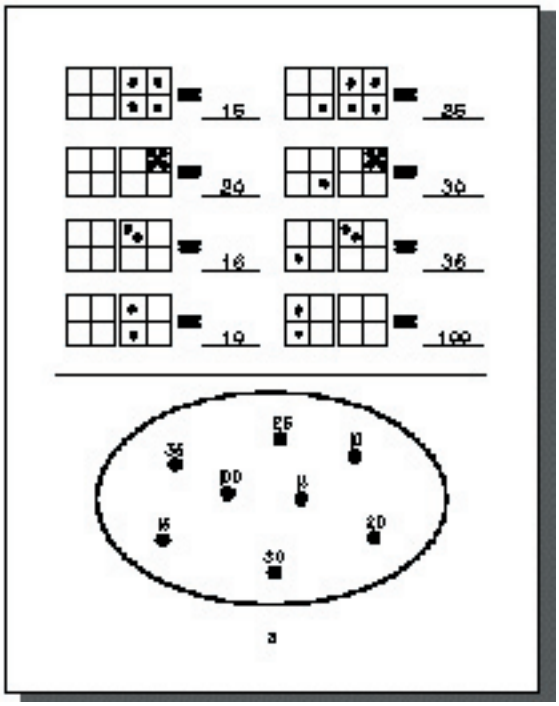
Prepare a story similar to the warm-up activity for students to do at home with family participation. The following example is a money puzzle.

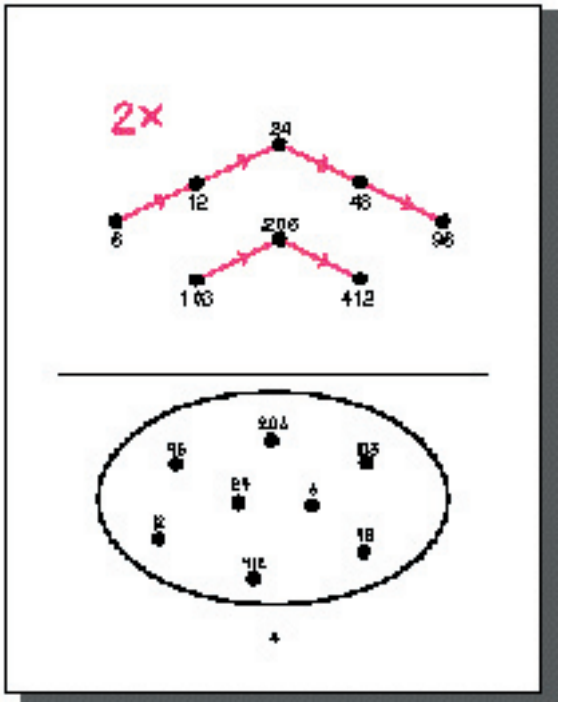
Ted has a lot of coins but not too much money.
Ted has _____ pennies.
He has _____ nickels, or ____ ¢ in nickels.
He has _____ dimes, or ____ ¢ in dimes.
He has _____ quarters, or ____ ¢ in quarters.
Altogether, Ted has _____ coins, and $ _____.

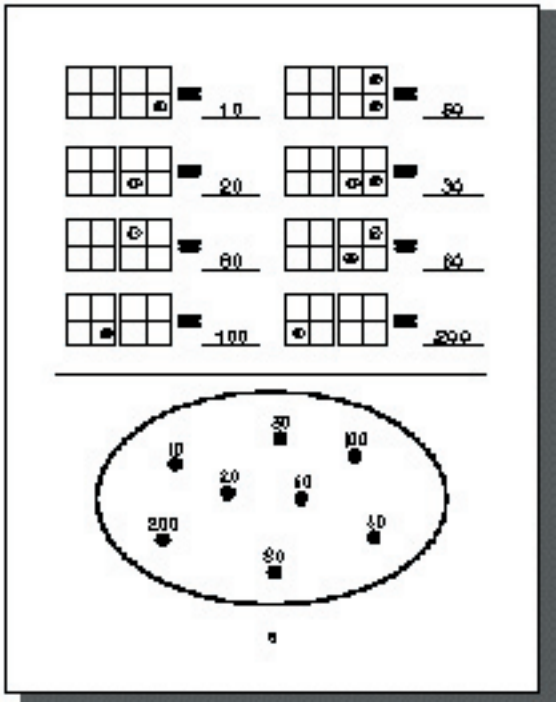


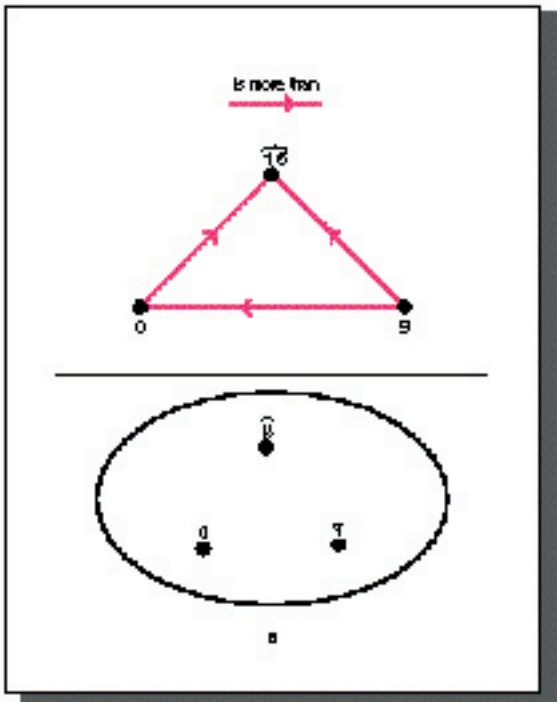
Key:
Ted has a lot of coins but not too much money.
Ted has 6 pennies. 30¢
He has 8 nickels, or 80¢ in nickels.
He has 3 dimes, or 75¢ in dimes.
He has 2 quarters, or 50¢ in quarters.
Altogether, Ted has 19 coins, and $ 1.61.

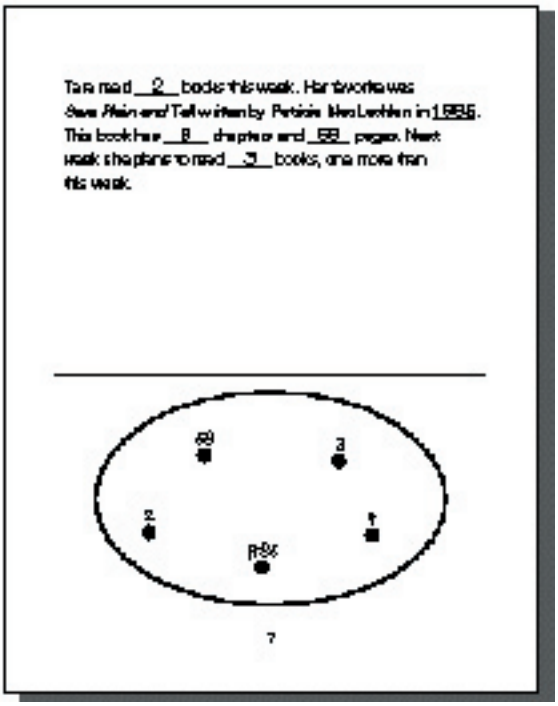

+10
5
16
26
36
46
25
35
45

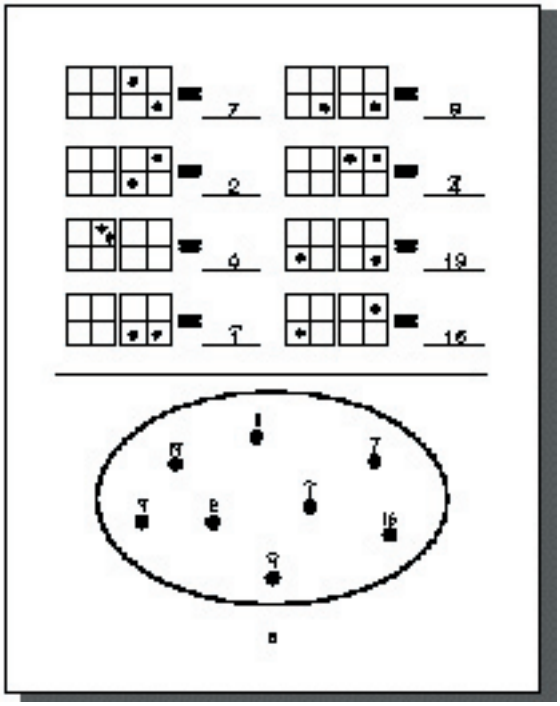

15
25
20
30
10
30
10
100

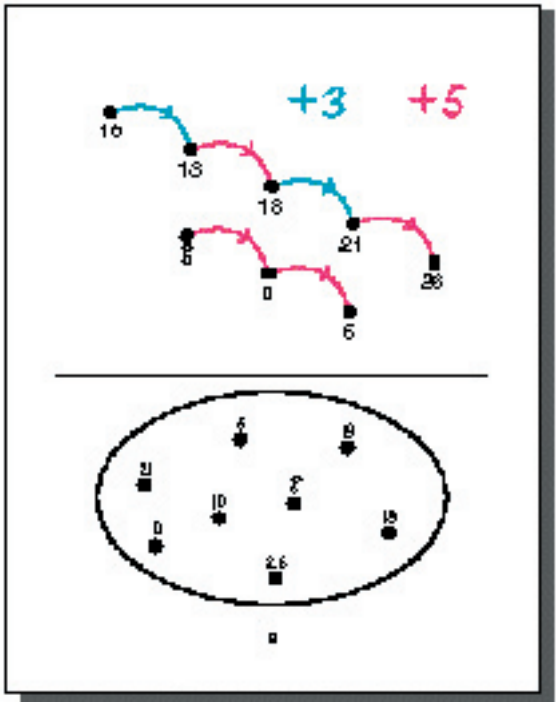

2×
6
12
24
48
96
206
103
412

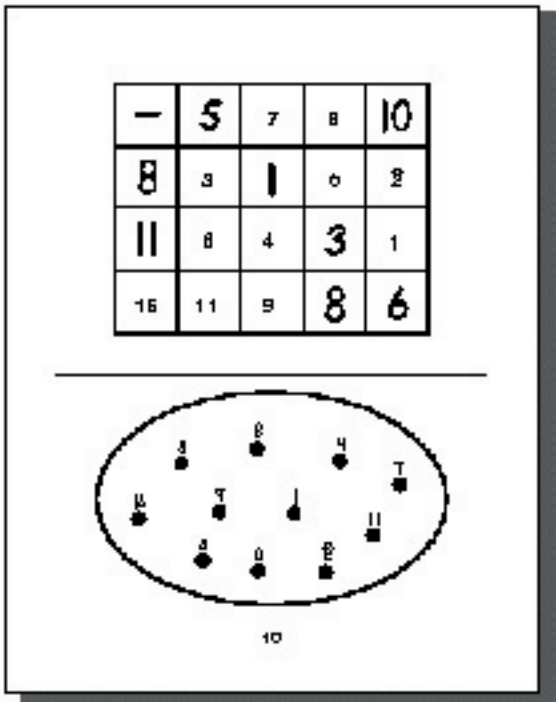

10
20
60
80
100
200

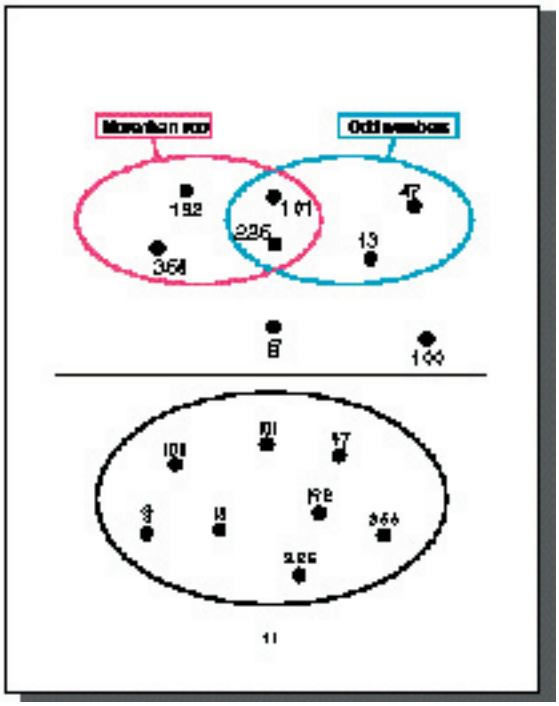

is more than






+3
+5


10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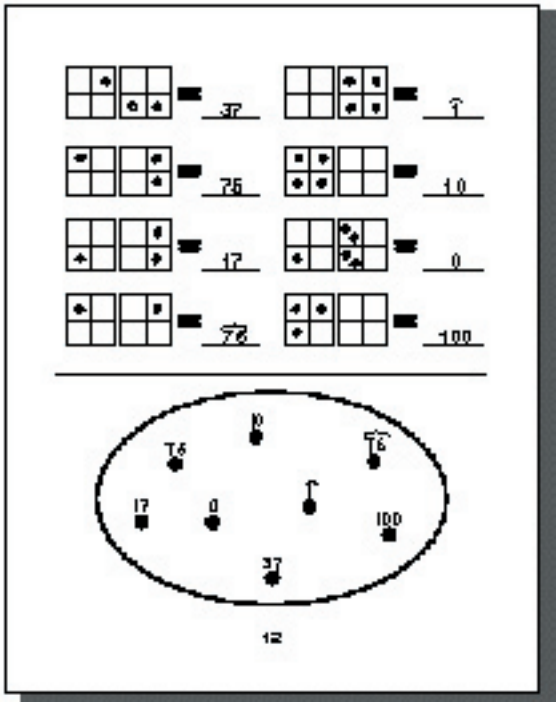
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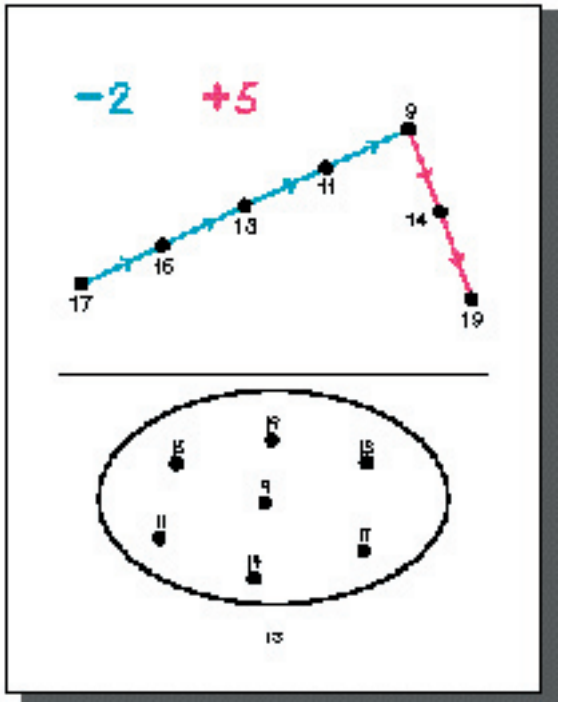
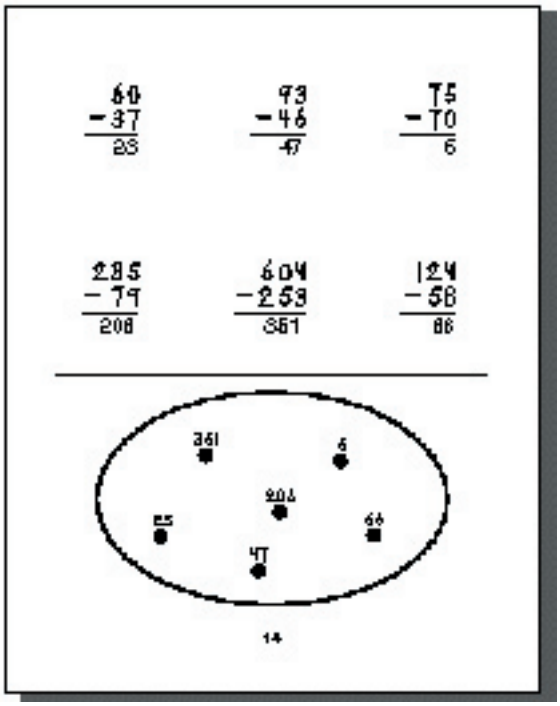
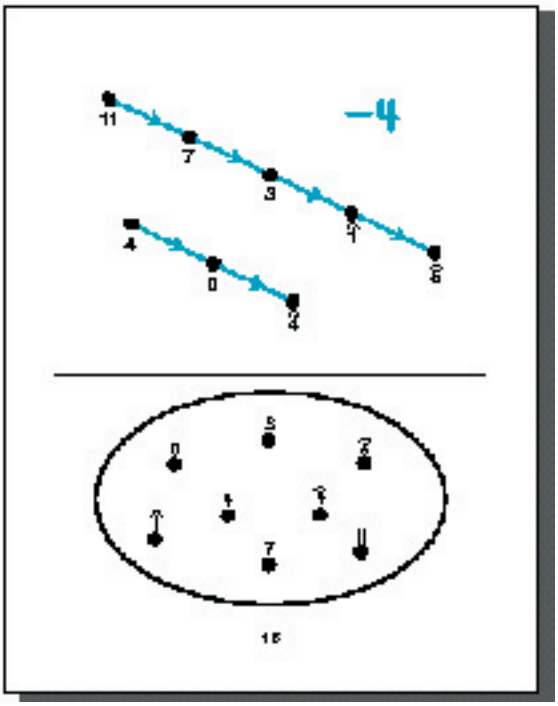
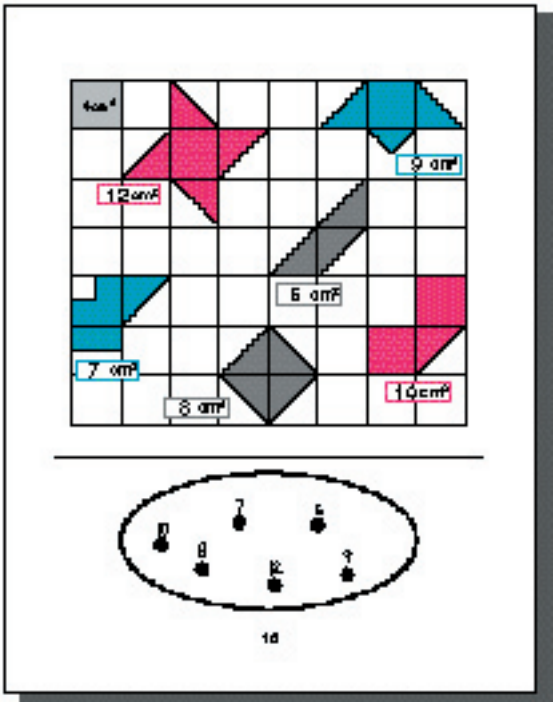
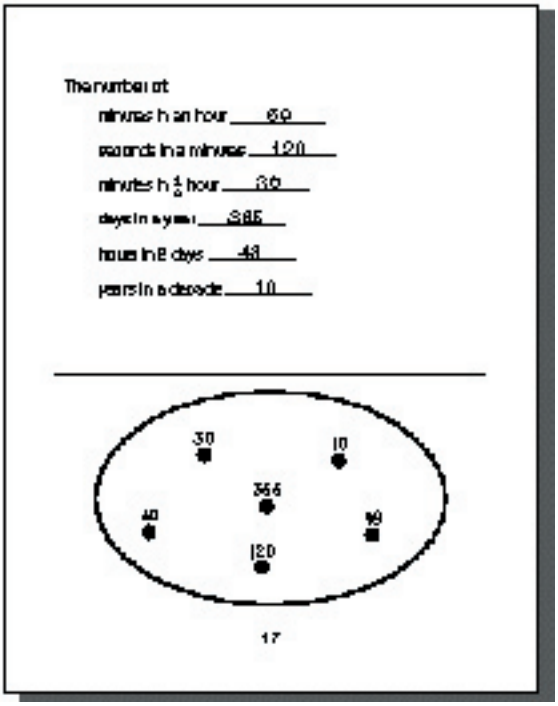
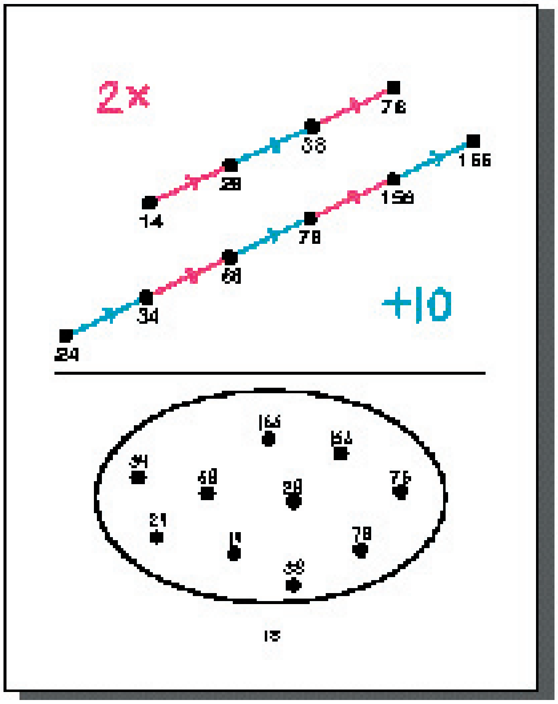

-2
+5
13

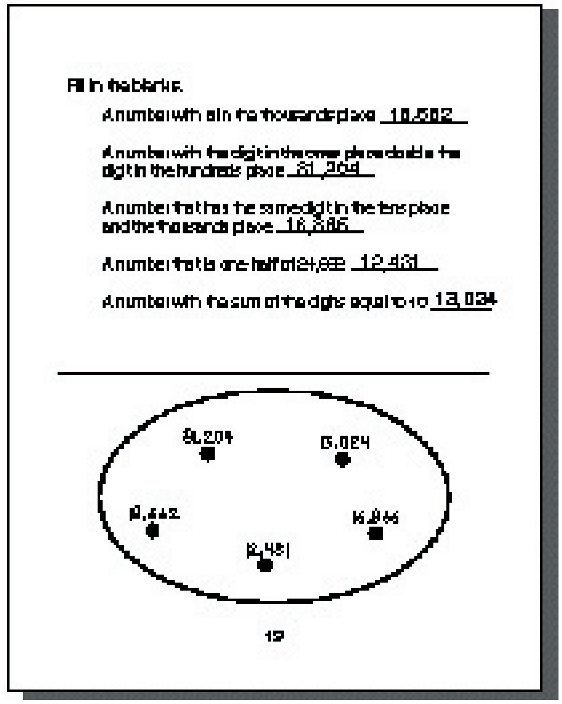

60 − 37 = 23
93 − 46 = 47
75 − 70 = 5
285 − 79 = 206
604 − 253 = 351
124 − 58 = 66
351
5
204
65
66
47
14

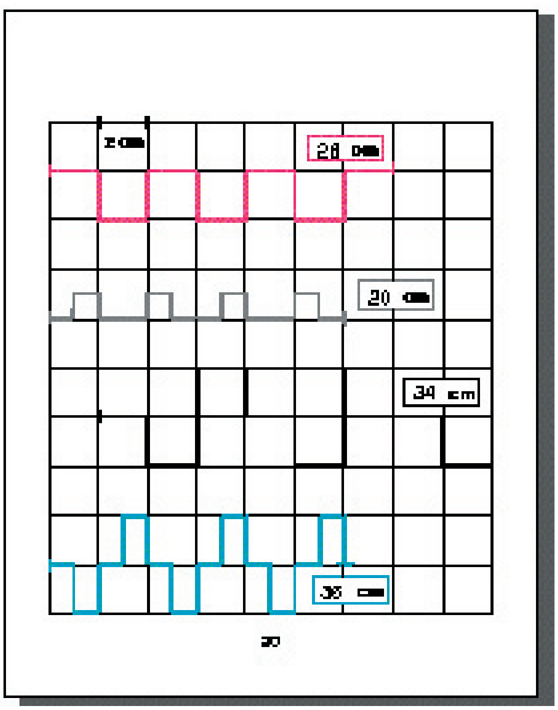

−4
11
7
3
−1
−5
4
0
−4
15

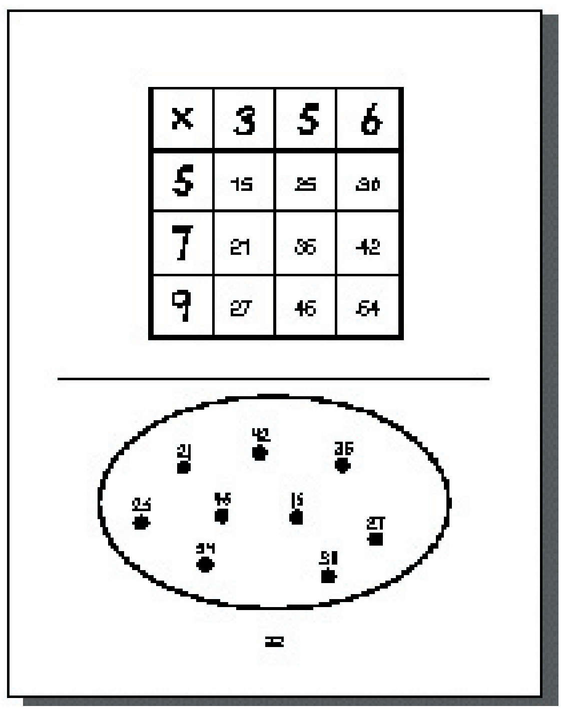

12 cm²
9 cm²
6 cm²
7 cm²
8 cm²
10 cm²
16

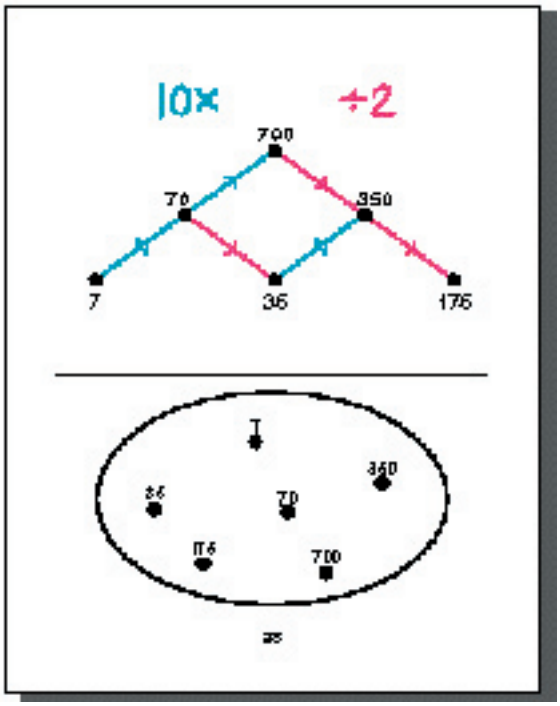

There are
minutes in an hour 60
seconds in a minute 120
minutes in ½ hour 30
days in a year 365
hours in 2 days 48
years in a decade 10
30
10
365
40
60
120
17

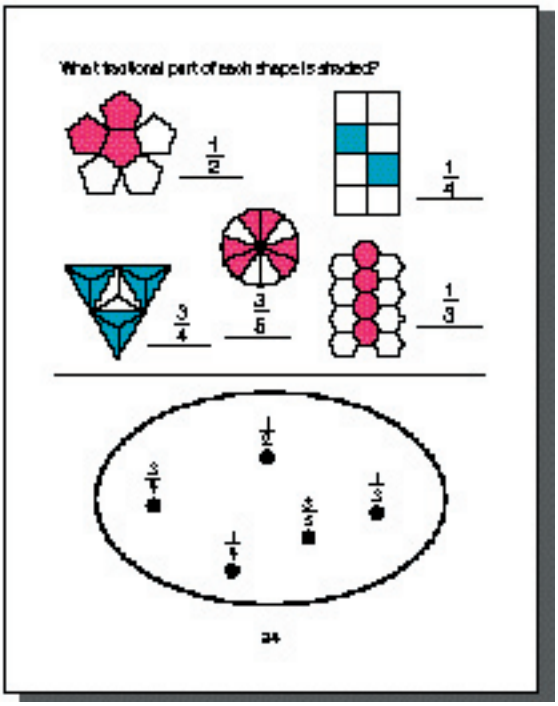

2×
14
28
38
78
24
34
68
78
158
156
+10

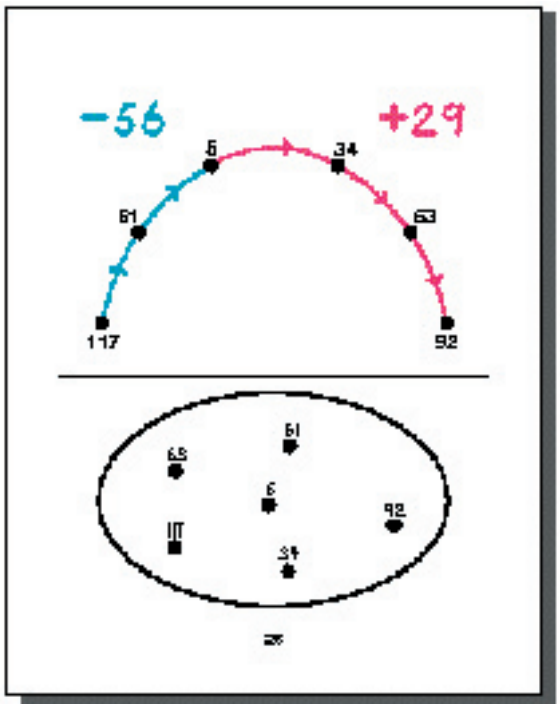

Fill in the blanks.
A number with 0 in the thousands place 10,502
A number with the digit in the ones place double the digit in the hundreds place 31,204
A number that has the same digit in the tens place and the thousands place 16,386
A number that is one half of 24,862 12,431
A number with the sum of the digits equal to 10 13,024

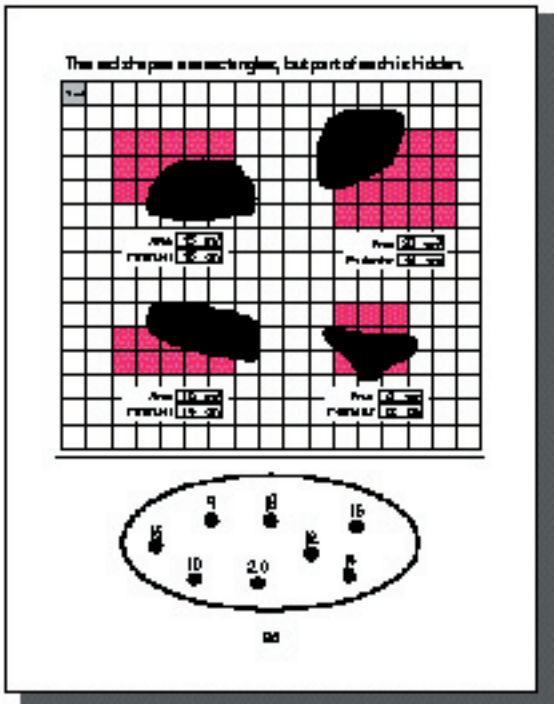

2 cm
28 cm
20 cm
34 cm
36 cm

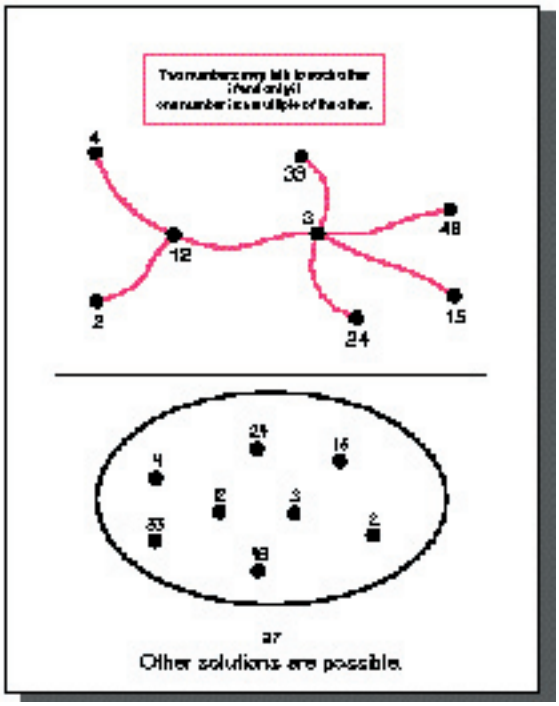

× 3 5 6
5 15 25 30
7 21 35 42
9 27 45 54

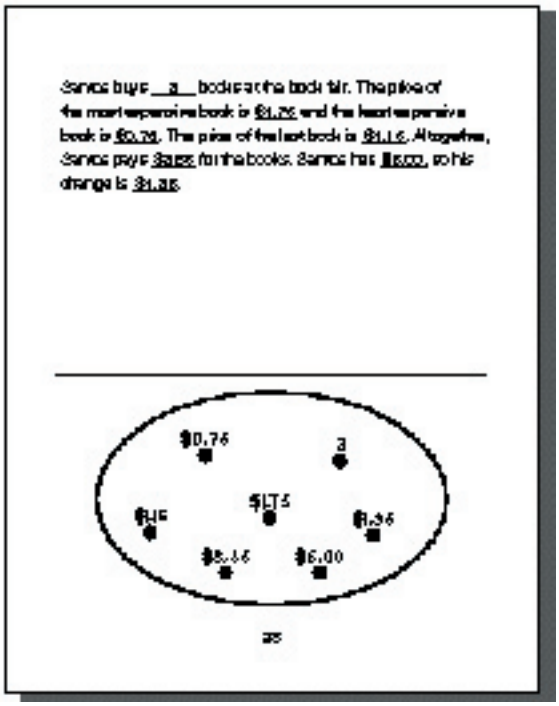

10×
÷2
700
70
350
7
35
175

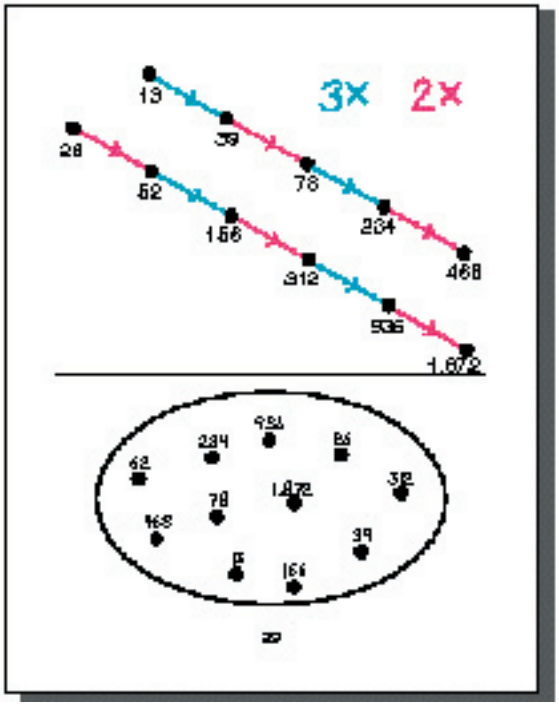

What fractional part of each shape is shaded?
1/2
1/4
3/4
3/5
1/3

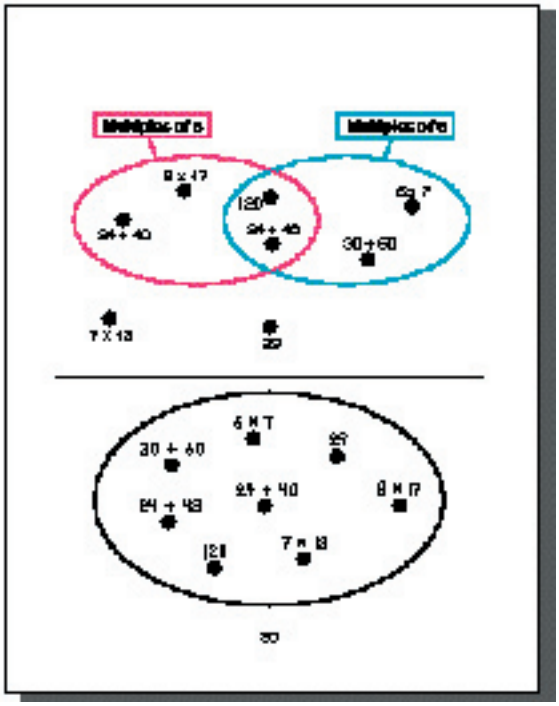

−56
+29
5
34
61
63
117
92

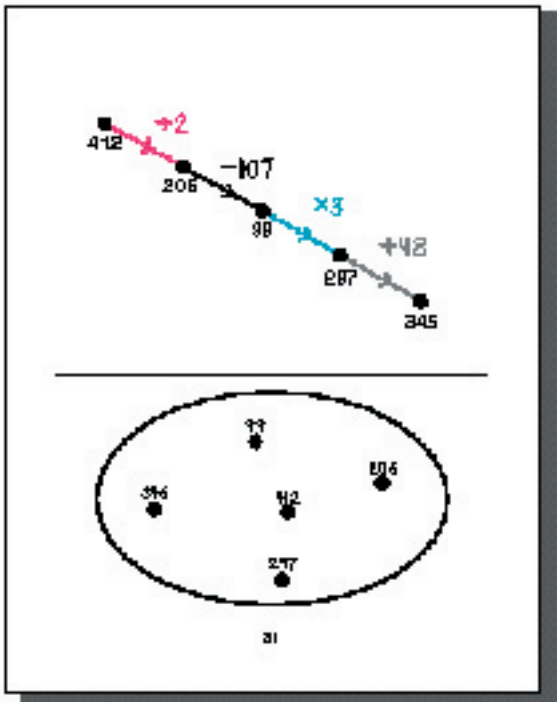

The red shapes are rectangles, but part of each is hidden.

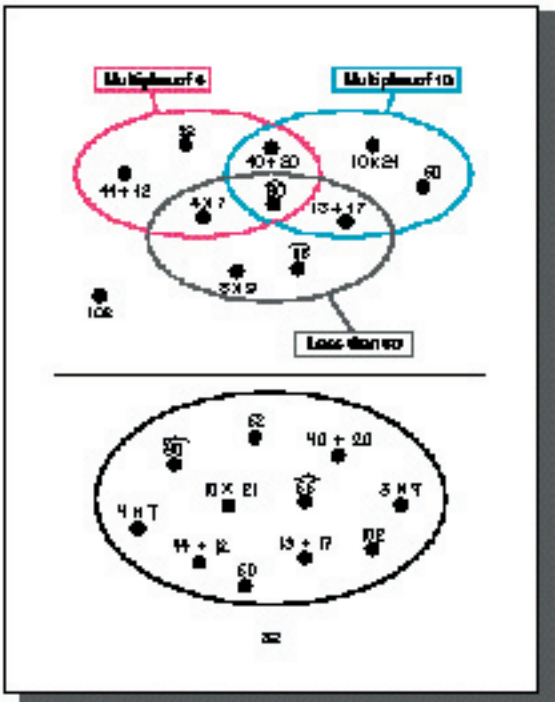

4
39
3
48
12
2
15
24
24
16
4
12
3
35
2
18
27
Other solutions are possible.


$0.76
3
$1.15
$1.56
$5.46
$6.00
35


3×
2×
13
39
78
234
468
28
52
156
312
936
1,872
952
234
86
62
1,872
312
78
168
39
8
156
39


Multiples of 8
Multiples of 6
8 x 17
120
24 + 40
24 + 48
30 + 60
7 X 13
39
6 X 7
30 + 60
95
24 + 40
8 X 17
84 + 48
121
7 x 13
35


+2
−107
×3
+42
412
206
98
287
345



Capsule Lesson Summary

Solve a detective story for three secret numbers. A first clue gives their positions in a +2 and a +3 arrow picture. A second clue says that one of the three numbers is 3x another. Play The Number Line Game with non-integer decimal numbers as the secret numbers.

Materials

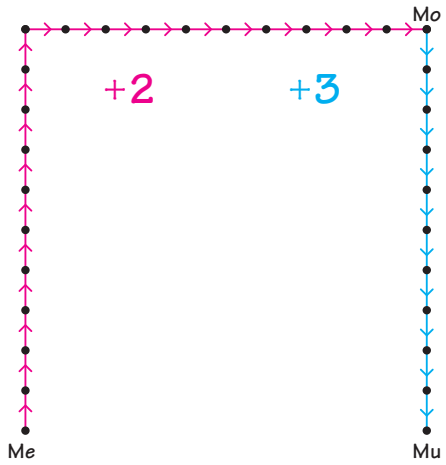
Teacher • Colored chalk

Student • Worksheet W17

Description of Lesson

Exercise 1: Detective Story

Distribute copies of Worksheet W17, and draw the arrow picture from the worksheet on the board.



T: ***Today you are going to be detectives again. Your assignment is to discover the identities of three whole numbers whose code names are Me, Mo, and Mu. The first clue is this arrow picture. What does this arrow picture tell us about the three secret numbers?***

Students should observe that Me is the least and Mu is the greatest of the three secret numbers. If necessary, ask where the greatest and least numbers are in the arrow picture.

Perhaps a student will explain that Mo is 40 more than Me, and Mu is 30 more than Mo. Hence, Mu is 70 more than Me. Commend any student who makes this observation but do not suggest it yourself.

T: ***This is the same arrow picture as on your worksheet. How many +2 arrows are there? How many +3 arrows?***

S: ***There are twenty +2 arrows and ten +3 arrows.***

T: ***If Me were 100, what would Mo be and what would Mu be?***

Draw a table for the three numbers on the board near the arrow picture.

S: ***Mo would be 140 and Mu would be 170.***

Me	Mo	Mu
100	140	170

Record this information in the table on the board, and ask students to record it in the table on their worksheets.

T: ***If Me were 30, what numbers would Mo and Mu be?***

Let students write their answers in the table on their worksheets. Look at several of the students' answers before letting someone answer aloud.

S: ***If Me were 30, then Mo would be 70 and Mu would be 100.***

Record this information in the table on the board. Repeat this activity asking for Mo and Mu when Me is 52, 201, and 18, respectively.

Me	Mo	Mu
100	140	170
30	70	100
52	92	122
201	241	271
18	58	88

T: ***What do you notice about Me, Mo, and Mu?***

Allow the students to express themselves freely.
Your students very likely will observe that

- Me + 40 = Mo;
- Mo + 30 = Mu;
- Me + 70 = Mu; and
- the ones digit of all three numbers is the same.

Trace an arrow from Me to Mo.

T: ***What could this arrow be for? How do you know?***

S: ***It could be for + 40, because Me + 40 = Mo.***

S: ***Ten +2 arrows are the same as one +20 arrow, so twenty +2 arrows are the same as one +40 arrow.***

Trace an arrow from Mo to Mu.

T: ***What could this arrow be for? How do you know?***

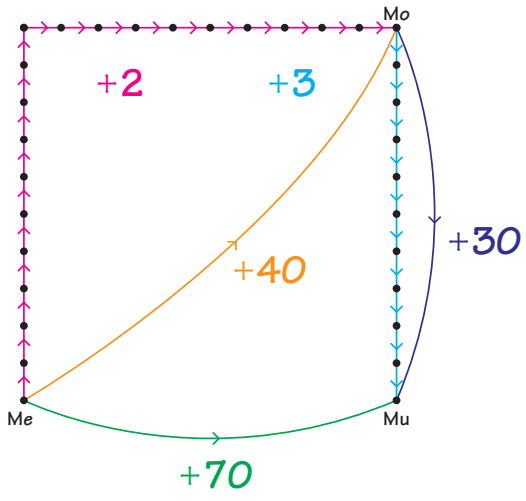
S: ***+ 30, because there are ten + 3 arrows.***

Draw an arrow in green from Me to Mu.

T: ***What could this green arrow be for?***
How do you know?

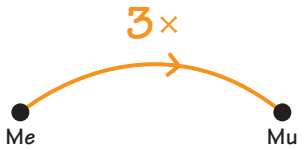
S: ***+ 70, because 40 + 30 = 70.***

T: ***If Me were 10, what number would Mu be?*** (80)
If Me were 105,…? (175)
If Me were 37,…? (107)
If Me were 99,…? (169)



Ask students to explain how to calculate 99 + 70.
If no one suggests 99 + 70 = (100 + 70) – 1,
suggest this yourself.

Draw this arrow picture on another part of the board.

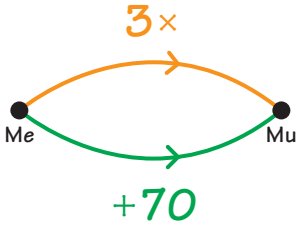


T: ***What does this picture tell us about Me and Mu?***

S: ***Mu is three times Me.***

T: ***If Me were 30, what number would Mu be?*** (90)
If Me were 25, …? (75)
If Me were 40, …? (120)
If Me were 50, …? (150)
If Me were 75, …? (225)

Draw a green +70 arrow from Me to Mu.



T: ***Our first clue told us that Mu is 70 more than Me.***
Now we know that Mu is also three times Me.
Could Me be 20? (No)
Why not?

S: ***Because 3 x 20 = 60 and 20 + 70 = 90. Mu cannot be both 60 and 90 at the same time.***

T: ***Could Me be 30?*** (No)
Why not?

S: ***Because 3 x 30 = 90 and 30 + 70 = 100.***

T: ***Could Me be 40?*** (No)
Why not?

S: ***Because 3 x 40 = 120 and 40 + 70 = 110.***

T: ***Who are Me and Mu? Write your answer on your worksheet.***

Look at many of the students' answers before asking a student to answer aloud.

S: ***Me is 35 and Mu is 105.***

T: ***What number is 3 x 35?*** (105)
What number is 35 + 70? (105)

Label the dots for Me and Mu in both arrow pictures.

T: ***If Me is 35 and Mu is 105, what number is Mo?***

S: ***75.***

Label the dot for Mo, and conclude that 35, 75, and 105 are the three secret numbers.

Exercise 2: Number Line Game

Play The Number Line Game with your class. Choose non-integer decimal numbers to be your secret numbers. Begin with a secret number between 0 and 1. If you play a second game, choose a secret number between 0 and 10. Adjust the difficulty of the game to the abilities of your students.