COMPREHENSIVE SCHOOL MATHEMATICS PROGRAM



Classroom lessons and games centered around the Papy Minicomputer . . . a source of rich situations that call for mental arithmetic and quick strategic thinking.

TEACHER'S GUIDE

8-76501

CEMREL, Inc. Comprehensive School Mathematics Program 3120 59th Street, St. Louis, Mo. 63139

The Comprehensive School Mathematics Program materials included herein are in the process of development. As a part of our continuing effort to evaluate and improve them, we ask that you comment in detail on the materials and on the way in which you used them.

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INTRODUCTION

A CSMP Mini-Package

The CSMP Mini-packages present parts of the CSMP curriculum through introductory lessons that can be taught by teachers with no prior CSMP training to students with no CSMP background. The purpose of each CSMP Minipackage is twofold:

- to introduce you to one of the three non-verbal languages used in the CSMP elementary curriculum in such a way that you will want to pursue the possibility of adopting the entire curriculum, and
- to provide some mathematically rich activities with that language which you can use immediately in your own classroom.

This CSMP Mini-package introduces the Minicomputer, its language, and two enjoyable games played on the Minicomputer.

The Papy Minicomputer and CSMP

The Papy Minicomputer derives its name from the prominent Belgian mathematician, Georges Papy, who invented and introduced it in the 1950's. The Minicomputer is an abacus with the flavor of a computer in its schematic representation of numbers. Its manner of representation combines decimal notation with binary positional rules, and thus takes advantage of both a well established system and a simple, yet technologically desirable one. See the appendix in this booklet for an explanation of how to display numbers on the Minicomputer. The Minicomputer plays an integral part in the CSMP curriculum beginning with the kindergarten program. In early grades, it is a support for the usual positional system of numeration and an aid in computation. As children become more familiar with place value and more adept at paper-and-pencil calculations, the Minicomputer functions as a pedagogical tool with which to provide new insights into the anatomy of numbers, to support the development of standard algorithms, and to motivate the introduction of negative numbers and decimals. In intermediate grades, when attention focuses more on multiplication and division and extended number systems, the Minicomputer provides a setting in which to pose questions and to practice computation.

Four Lessons and How to Use Them

In 1976 Frédérique Papy developed two mathematically rich games, Minicomputer Tug of War and Minicomputer Golf. The rules of these games are few and the design of the Minicomputer is simple. But together they give rise to an intriguing array of mathematical considerations as each play presents in turn a new problem to be solved. In playing these games, students sharpen their skills in mental arithmetic and employ techniques of estimation. Equally important, students have the opportunity to develop strategic thinking discovering advantageous plays and anticipating their opponents' responses.

In this booklet we describe these two games and how to prepare your students to play them. Four lessons are suggested, each requiring thirty to sixty minutes to complete. You will find that the lessons are most effective if spread out over two to four weeks. Of course, once the games have been introduced, they can be played periodically throughout the rest of the school year.

All of the lessons are designed to be used collectively by one teacher and an entire class. The variety of problem-solving situations that the games provide

makes them appropriate for the spectrum of ability levels in the usual heterogeneous classroom. However, the lessons could be easily adapted for small group use, in which case the lessons might proceed more rapidly.

As the students play the games, you may find it easy to evaluate their strategies. But one of the advantages of using a game as a teaching device is to allow the teacher for a period of time to be a facilitator and not an evaluator. Without constant evaluation of their actions, students have a better chance to explore a situation freely and to discover the effects of the options available. Obviously a simple comment on the progress of a game may be helpful at times!

Now we present the four lessons, written in the standard CSMP format of a dialogue between teacher (T) and students (S). We hope that these games will provide you and your students with the intellectual satisfaction attained from a rich pedagogy of situations.

LESSON 1

NOTE: Read the appendix carefully in preparation for teaching these lessons.

Minicomputer Introduction

On the edge of your chalkboard place four Minicomputer boards, each with the white square in the lower right corner.

T: This is called the "Minicomputer".

Put one magnetic checker * on the purple square of the ones' board.



T: With this checker, I put 4 on the Minicomputer.

Move the checker to the red square of the ones' board.



T: This is 2 on the Minicomputer.

^{*} NOTE: The ten white checkers included with your equipment (and bearing the mark " Λ ") are used for a special purpose in the CSMP curriculum. CSMP uses these markers as <u>negative</u> checkers; i.e., a negative checker placed on the 2-square of the Minicomputer denotes the number $\hat{2}$ (read "negative two"). We suggest you avoid using them for the moment, since you may wish to use them in this special way later on.

Move the checker to the brown square of the ones' board.



T: This is 8 on the Minicomputer.

Move the checker to the white square of the ones' board.



T: Which number do you think this is?

S: 1.

You may need to tell your students that the number is 1.

Review very briefly the placement of the checker for 1, 2, 4, and 8. Remove the checker from the Minicomputer.

T: Who can put 3 on the Minicomputer?

Invite a student to put 3 on the Minicomputer.



T (lifting each checker momentarily): Yes, 2 + 1 = 3.

This is also 3 on the Minicomputer. If this way of putting 3 on the Minicomputer is given, accept it but then ask for another way, using fewer checkers.



Follow a similar procedure for 5, 9, 6, and 7. There are several correct configurations for each of these numbers. The standard configuration ^{\$} for each number is shown below.



* The standard configuration of a number on the Minicomputer has at most one checker on a square and has a digit 9 or less on each board. Each time a number is put on the Minicomputer, determine whether or not it is the number asked for by adding the values of the checkers used, each checker assuming the value of the square it is on. Checking the representation of 7 shown on the preceding page, you would say:

T: Yes, 4 + 2 + 1 = 7.

Put two checkers on the ones' board, one on the brown square and one on the red square.



T: What number is this?

S: 10; 8 + 2 = 10.

T: That's right. Now I will show you another way to put 10 on the Minicomputer.

Pick up the checker on the brown square with one hand and the checker on the red square with the other. Then put one of these checkers on the white square of the tens' board and take the other checker away (put it in the chalk tray).



This is also 10.

T:

Write "1" and "0" below the boards as shown below.



Move the checker back and forth several times from the 1-square to the 10square as the students tell you each time which number is on the Minicomputer.



T: Who can put 14 on the Minicomputer?



If another configuration for 14 is put on the Minicomputer, for example,



... agree that it is 14 and then ask someone to put 14 on using fewer checkers. If a number other than 14 is put on, with the class compute the number and then ask again for 14.

Follow a similar procedure for 16, 19, 13, and 12. (Note: For 16 there is a non-standard configuration with fewer than three checkers). The standard configurations for these numbers are shown below. In each case, write the numerals below the boards and to the right of them.



Who can put 20 on the Minicomputer?

Τ:

If someone suggests putting two checkers on the 10-square, agree that this is 20, and then ask a student to put 20 on the Minicomputer using one checker fewer. You may need to show the standard configuration for 20 yourself.



Move the checker back and forth as shown, each time asking the students which number is represented.



Similarly introduce the 40-square and the 80-square.





Who can put 54 on the Minicomputer?



T: Yes, 40 + 10 = 50 and 4 more is 54.

If 54 is put on in another way, check with the class that it is 54 and then ask for 54 using fewer checkers.

Follow a similar procedure for 36, 90 and 72. (Note: 36 can be put on with three checkers (20 + 8 + 8), but this is not the standard configuration for 36.) The standard configurations for all three numbers are shown below and on the next page. Each time a number is put on the Minicomputer, write the numerals below the boards.



Τ:



T: How do you think we can put 100 on the Minicomputer?

Students often anticipate the value of the white square of the third board to the left and excitedly suggest putting a checker there.



You may need to put a checker on the 100-square and to tell the students its value yourself.

Move the checker to the 200-square, then to the 400-square, and then to the 800-square, each time asking the students which number is represented.

Dealing with incorrect answers and non-standard representations as you did earlier, ask the students to put the following numbers (on the next page) on the Minicomputer. Each time, write the numerals below the boards.











Minicomputer Tug of War

T: Now we are going to play a game called "Minicomputer Tug of War."

Remove one Minicomputer board, allowing the other three to remain. Position two red checkers and two blue checkers on the Minicomputer as in the illustration below.

Divide the class into two teams (the RED team and the BLUE team).

T: The BLUE team will play with the blue checkers, and the RED team will play with the red checkers.

The number of the RED team is the number that is on the Minicomputer with red checkers. What is the starting number of the RED team?

- S: 1,000.
- T: The number of the BLUE team is the number that is on the Minicomputer with the blue checkers. What is the starting number of the BLUE team?
- S: 5.

Write the two "starting" numbers on the board and use colored chalk to frame them



T: Teams take turns during the game. Players on the RED team must move one red checker to a square that has a <u>lower</u> value. Players on the BLUE team must move <u>one</u> blue checker to a square that has a <u>higher</u> value.

So the RED team's number will get smaller and smaller; the BLUE team's number will get larger and larger. The first team to <u>tie</u> or <u>pass</u> the other team LOSES the game.

In other words, the RED team loses by making the red number LESS THAN or EQUAL TO the blue number. Similarly, the BLUE team loses by making the blue number GREATER THAN or EQUAL TO the red number.

Begin playing the game. Insist on silence when a player is deciding where to move a checker. Also, do not let a team forfeit its turn during the game.

NOTE: We suggest letting students volunteer to make moves during the first game rather than asking them to play in some order. This will speed up the game and allow students who are unsure of the rules to become more familiar with the game before playing themselves.

We present the description of two games to give you an idea of what to expect. Of course, your students are unlikely to make the same sequence of moves shown here. The first records the experience of a class new to the game. The second follows the same class after six games.

A student from the RED team played first, moving one red checker from the 800square to the 8-square. The student determined that the RED team's number was then 208 and changed the number in the red box.



Next, a student from the BLUE team moved a blue checker from the 4-square to the 40-square. The student determined that the BLUE team's number was then 41 and changed the number inside the blue box.



The game continued as shown below.



At this point in our game, the student whose turn it was smiled happily and exclaimed "I can make them lose!" She moved in this way.



The BLUE team surveyed the situation and quickly conceded that she was right; their smallest possible move (from the 1-square to the 2-square) would have made the number 102, which was the same as the RED team's number.

By studying the preceding game, you will see that the BLUE team could have won after the RED team's first move (by moving a checker from the 1-square to the 200-square). However, the students were not playing at such a sophisticated level, nor were they expected to. Such situations occur frequently. Simply note them without comment and allow the students to enjoy the game. Although students can easily determine each new number, often they are able to see and verbalize what is happening and to find a winning move only when the red and the blue numbers are close. This is particularly true during their early experiences with the game.

We conclude with the record of a game played after the students had experienced six games earlier. Here the BLUE team won the game because the RED team was forced to make a move putting their number below the BLUE team's number. The BLUE team began.



LESSON 2

Putting Numbers on the Minicomputer

Put this sequence of numbers on the Minicomputer, displaying one configuration at a time. Ask students to identify the numbers.



Ask students to put the following numbers on the Minicomputer. The standard configuration of each number is shown below; but there are, of course, many other correct configurations.



Transforming a Number

T: I will put a number on the Minicomputer. As I'm putting on checkers, see if you can figure out which number.

Put checkers on the Minicomputer gradually, allowing the students to calculate the number mentally, until you have this configuration displayed.



S: 50!

T: Let's check to be sure.

Point to the checkers one by one as the class adds the values to total 50. Gradually put additional checkers on the Minicomputer to obtain this configuration.



Let several students whisper the new number (100) to you and then ask someone to answer aloud.

T: I will move one checker. Tell me if the new number is larger or smaller.

Move a checker from the 4-square to the 2-square.



S: Smaller.

T: ... from 4 to 2. How much smaller?

S: 2 smaller.

T: We started with 100. What is the number now?

S: 98.

Move a checker from the 20-square to the 40-square.



- S: Larger!
- S: 20 larger!

T: We had 98. The new number is ... ?

S: 118.

Continue making moves and ask how much larger or smaller the new number is. At this point, it is not necessary to keep track of what the number <u>is</u>, only how it differs from the previous number as a result of your moving a checker.

Minicomputer Tug of War

T: Now let's play "Minicomputer Tug of War" again.

Play Minicomputer Tug of War as described in Lesson 1. If you feel the students are ready for a more complex game, play with <u>three</u> checkers per team.



Our experience has been that children are enthusiastic about this game and enjoy it. One factor that may interfere with their enjoyment, however, is too much caution before making moves; such caution may result in long and dull games. If your students begin to slip into this habit, challenge them to a game (you vs. the entire class). Take the opportunity to demonstrate some bold, dramatic moves; the class will see how much more fun the game is when played this way. You may even lose a game or two, but do not worry --- the excitement of your class will more than compensate!

The following is a record of the sixth game played by the class mentioned in Lesson 1 and was their first using three checkers per team.









T: What do you think has happened?

S (from the RED team): We lost!

- T: Yes, that's right. Let's put the last checker back where it was before, and let's see if there was a better move to make. (The teacher repositioned the checkers as they were at 128 and 120.) What do you think?
- S (from the RED team): We could have moved a checker from the 8-square to the 4-square and then they (the BLUE team) would have lost!

The student demonstrated the move.



In this more complex game, the students were less able to anticipate where their moves were leading. Actually, at first none of them was aware that the RED team had lost. No matter! The teacher simply asked a question to direct their thoughts to what had taken place, and some understood. A further question helped one student project another possible outcome.

Later in the year, after the students also have had experience with "Minicomputer Golf" (see Lessons 3 and 4), you could return to this game. Very likely your students will play with a greater awareness of how the game is proceeding. At that time you may want to play the game with this starting configuration.



LESSON 3

Transforming A Number

T: I will put a number on the Minicomputer. As I'm putting on checkers, see if you can figure out which number.

Put checkers on the Minicomputer gradually, allowing the students to calculate the number mentally, until you have this configuration displayed.



S: 100!

Check by pointing to the checkers, one at a time, as the class adds to find the total. Write "100" to the right of the boards.

T: Now I will move one checker. Tell me whether the new number is smaller or larger and by how much. Also tell me the new number.

Move a checker from the 4-square to the 8-square.



S: Larger by 4.

S: Now we have 104.

Erase "100" and write "104" beside the Minicomputer.



Continue this activity with the moves illustrated below.









T: Who can move one checker and make a new number, 3 smaller?

A volunteer should move a checker from the 4-square to the 1-square.



T: Who can move one checker and make a new number, 20 smaller?

A volunteer should move the checker from the 40-square to the 20-square.



T: We started with 100. Could we move just one checker and get 100? (Yes)

Let a student make a move. There are two possible ways to arrive at 100: to move a checker from the 10-square to the 8-square or from the 4-square to the 2-square. Agree with whichever is offered; then return the checker to its original position and ask for another way.

Minicomputer Golf

T: Now let's play a new game called "Minicomputer Golf".

Put this configuration on the Minicomputer.



T: What number is on the Minicomputer?

S:

49.

T: In Minicomputer Golf we start at one number, and by moving checkers we try to get another number. Today we'll start at 49 and our goal will be to reach 200.

> We play with two teams, the RED team and the BLUE team; the two team take turns. On your turn, you move exactly <u>one</u> checker from one square to another square. The new number can be larger or smaller than the number before. Also, the color of the checkers does not matter in this game.

A team wins if one of its players reaches the goal.

On the board record "200" as the goal.



Goal: 200

Divide the class into two teams.

T: I will use red and blue arrows to keep a record of each team's moves.

To explain further how a teacher conducts and records a game of Minicomputer Golf, we describe a sample game.

Suppose that the first player on the RED team moves a checker from the 20-square to the 40-square.

T: Is the number on the Minicomputer larger or smaller than before?

S: Larger.

T: From 20 to 40 ... how much larger?

S: 20 larger.

T: What number is now on the Minicomputer?

S: $69; 49 + 20 = 69.^{\$}$

On the board, a red arrow records the result of the RED team's move.



Suppose the first player from the BLUE team moves a checker from the 2-square to the 20-square.

T: Is the number on the Minicomputer larger or smaller than before?

S: Larger.

- T: From 2 to 20 ... how much larger?
- S: 18 larger.
- T: What number is on the Minicomputer now?

S: 87;69 + 18 = 87.

A blue arrow records the result of the BLUE team's move.



Some of the addition problems that arise in this game may be difficult. Check students' addition or ask a student to use a hand-calculator for checking.

The game continues in this manner until the goal (200) is reached — the RED team wins. This red-blue "road" describes the game from start to finish.



Play the game two or three times. Refrain from making comments on the quality of the moves, letting students enjoy the game as they gradually improve their strategies. Insist on silence when a player is deciding where to move a checker. Draw the arrow picture large so that it is clearly visible throughout the room.

LESSON 4

Minicomputer Golf

Play Minicomputer Golf as described in Lesson 3, except this time begin with a different configuration of checkers.



Announce a goal of 400. Below is an "arrow-road" record of two games played by a fourth-grade class. These two examples were chosen to demonstrate how much two games can differ, even with the same goal.

Game #1



Game #2



Play another game, either with the same goal (400) or with a goal of 500 or even 1,000.

We recommend that your class play collectively several times so that the students are comfortable with the rules. Then, you might arrange for some games to be played in small groups ^{\$%}; for example, groups of four (with two students per team) work well. The games are most effective when each group is composed of children whose numerical abilities are approximately the same. Most students can play the game with little supervision.

Experience in playing the games yourself may lead you to discover the inadequacy of the rules of Minicomputer Golf as they are presented in Lesson 3. The difficulty is that the rules allow the possibility of never-ending games.

Students soon learn that even if they do not know a winning move, they can prevent opponents from winning by making the number as far from the goal as is possible. The result is a continuing game whose conclusion remains out of reach for both teams. For this reason, introduce this extra rule (see the next page) after students have played many times.

[✤] Your materials for playing the game in groups could consist of small paper Minicomputers (which students could easily make), and paper clips, etc. for checkers Student Minicomputer sets (with checkers) are available from CSMP and may be ordered using the form on page 45 of this booklet.

EXTRA RULE

If a player increases the number and passes the goal, the resulting number must be smaller than any previously played number higher than the goal.

If a player decreases the number and passes the goal, the resulting number must be larger than any previously played number lower than the goal.

Using the extra rule, the teams must come closer and closer to the goal as the game progresses.

We present a description of a game using this extra rule.



At this point the goal is exceeded. The next numbers must be smaller than 597 but larger than 477.



Since the number is now less than 500, subsequent moves must increase the number, but the number must remain less than 537. In fact, there is a winning move.



NOTE: During a game with the extra rule being used, a situation could arise in which there is no possible legal move. For example,



According to the extra rule, the next move must make the number smaller than 1,003, but must keep the number larger than 995. No such move is possible with the present configuration.

If such a situation occurs during a game you and your students are playing, suspend the extra rule for one move (e.g., in the game described above, allow a student to decrease the number to one smaller than 995). However, subsequent moves must again follow the extra rule, whenever possible.

CONCLUDING REMARKS

Having completed this CSMP Mini-package, your experience with the Minicomputer is in the context of two games, Minicomputer Tug of War and Minicomputer Golf, and some preparatory exercises. We hope that not only did you find the games exciting, but that you can begin to envision a wide spectrum of challenging activities in which the Minicomputer would play a central role. In the CSMP program, the Minicomputer is used carefully and gradually to examine negative numbers and decimals, to develop standard algorithms, and to provide problem-solving situations. In fact, Minicomputer Golf and Minicomputer Tug of War can be played using negative numbers and decimals, but the preparatory exercises need to be more extensive than those for the versions of the games presented in this package.

The Minicomputer provides only one of the non-verbal languages for the CSMP curriculum, and this CSMP Mini-package details only some game situations involving the Minicomputer. It gives you a preview of CSMP's unique approach to mathematics at the elementary-school level. If you would like more information, contact:

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APPENDIX

The Minicomputer consists of a set of brightly colored boards (see the cover of this booklet) and a set of checkers. Each square has a numerical value $\stackrel{\text{*}}{\overset{\text{*}}}$. These are the values on the ones' board.



As you move to the next board to the left, you have the tens' board with corresponding values; the next board, the hundreds' board with corresponding values; and so on.



A number is put on the Minicomputer by placing checkers on its squares. A checker assumes the value of the square it is on. If several checkers are on the Minicomputer, the number is the sum of the values of the checkers. A number can be put on the Minicomputer in a variety of ways, but the representation which uses at most one checker on each square and uses checkers to represent a digit 9 or less on each board is usually the easiest to read. In this case, we say that the number is in <u>standard configuration</u>. Standard configurations for the numbers 1-9 become as familiar to the students as the usual numerals so that they no longer need to do any mental calculations for such configurations.

^{*} The values of the squares are not written on the boards; learning them is part of becoming acquainted with the Minicomputer.

STANDARD MINICOMPUTER CONFIGURATIONS OF THE NUMBERS 1 - 10

(Each board is oriented so that the white square is in the lower right-hand corner.)



T liked your games. Tliked your Mini-Computer Tt isn't every day you see a man walking around with a neat thing like a Mini-Comp uter. Drug

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