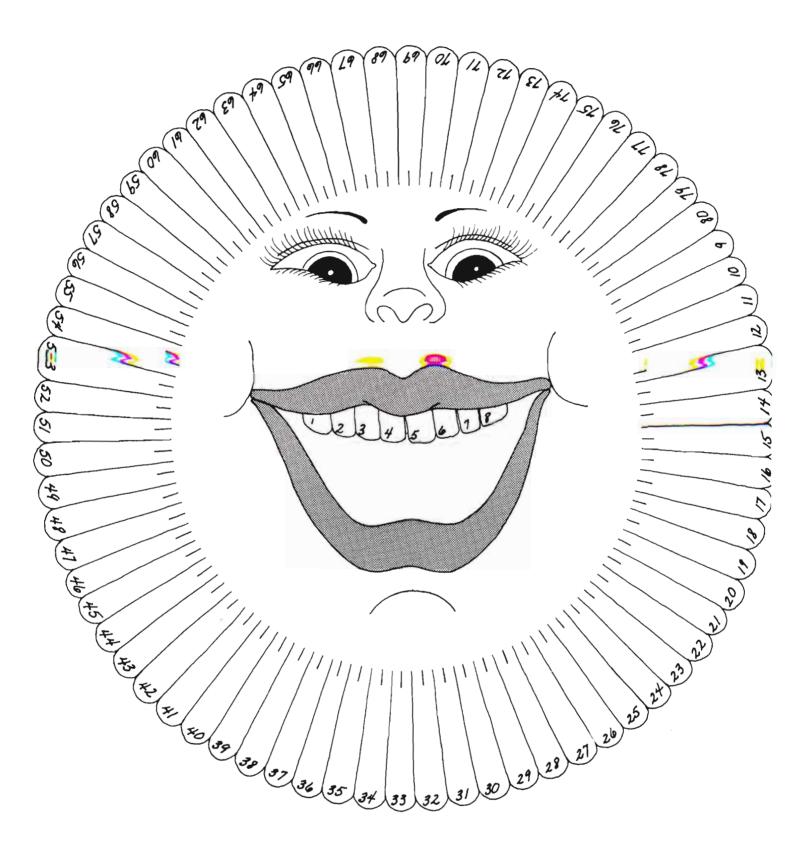
answers & annotations

Individualized Computation a₁





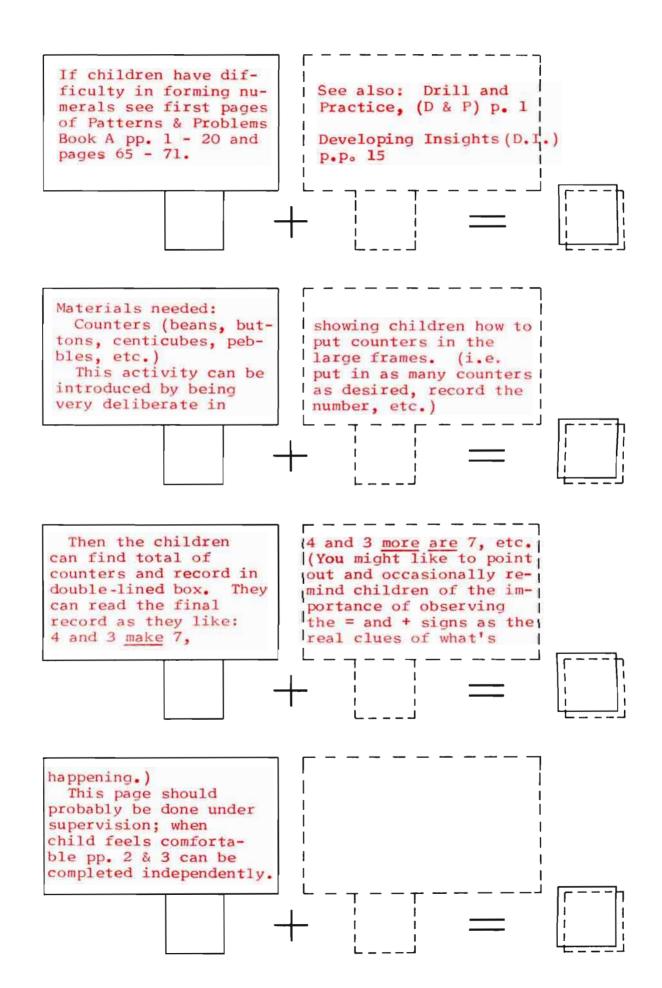


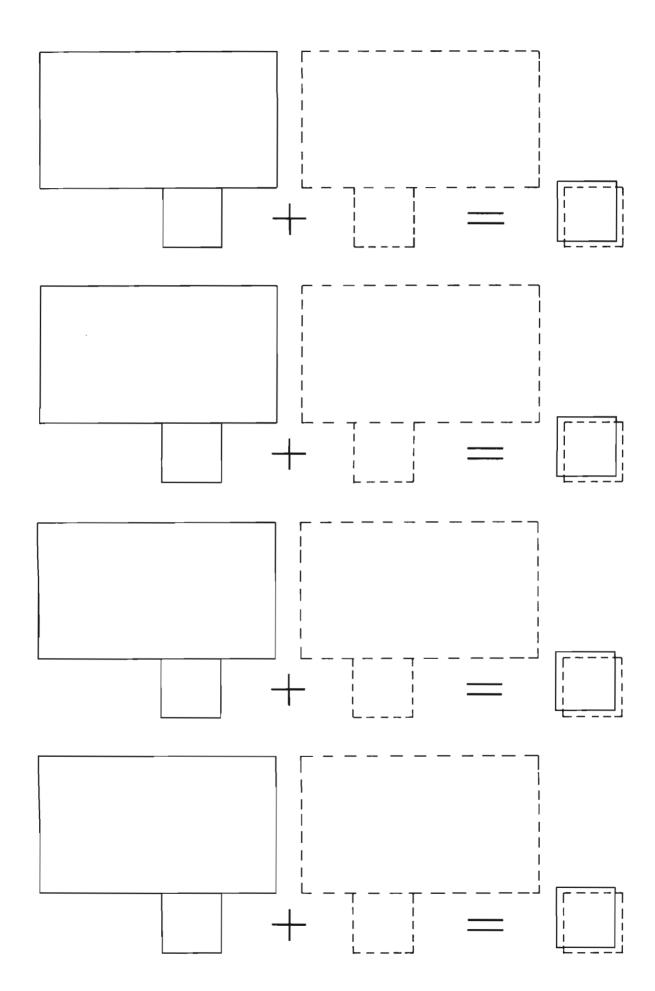
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The Kay Goines Kindergarten class at River School
Art Staff
ROY LIPSCHUTZ

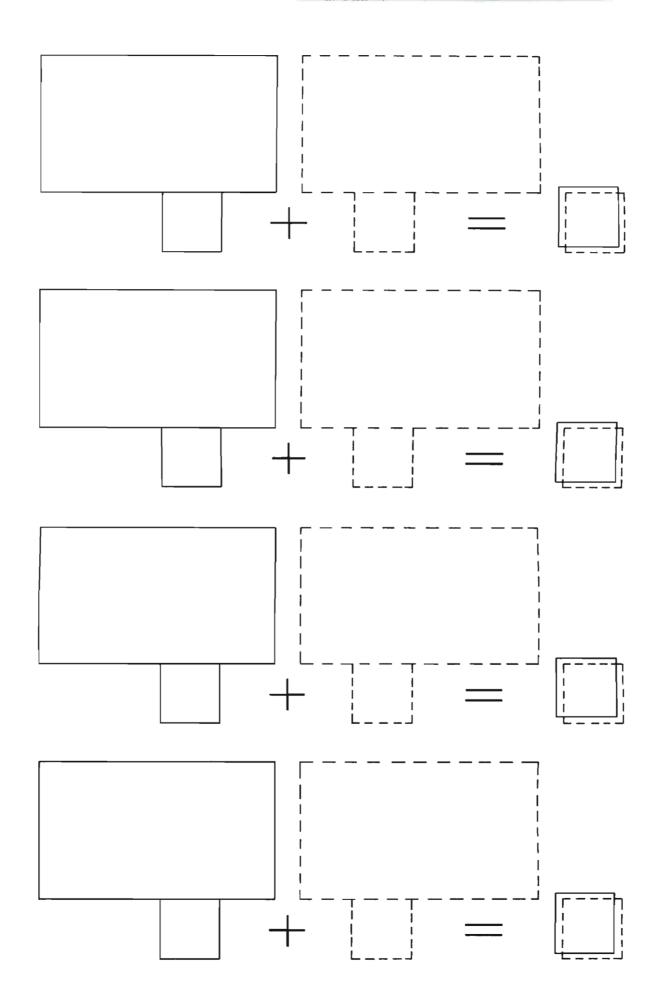
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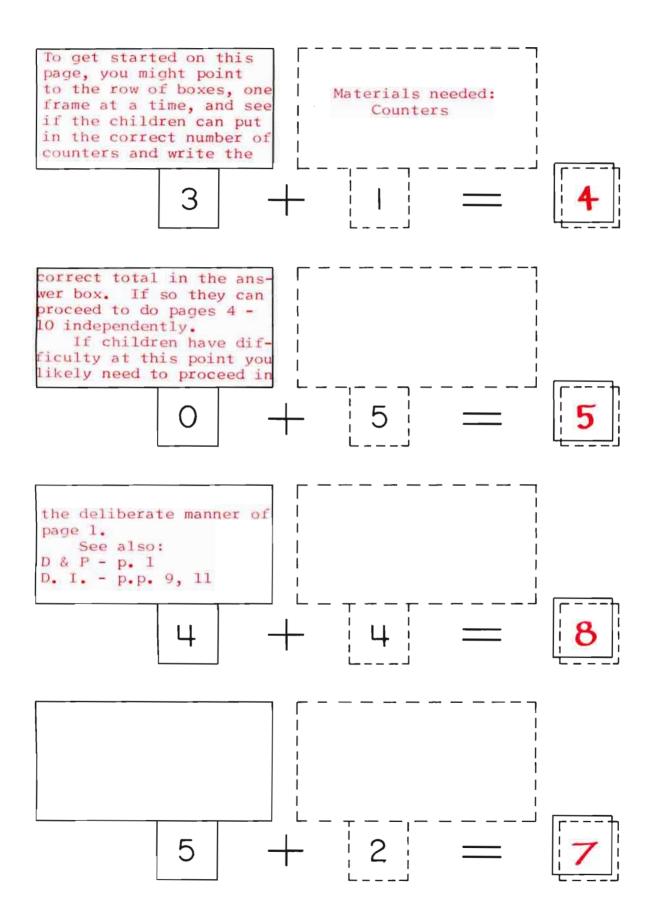
Art and Production Manager LIESELOTTE ESSER

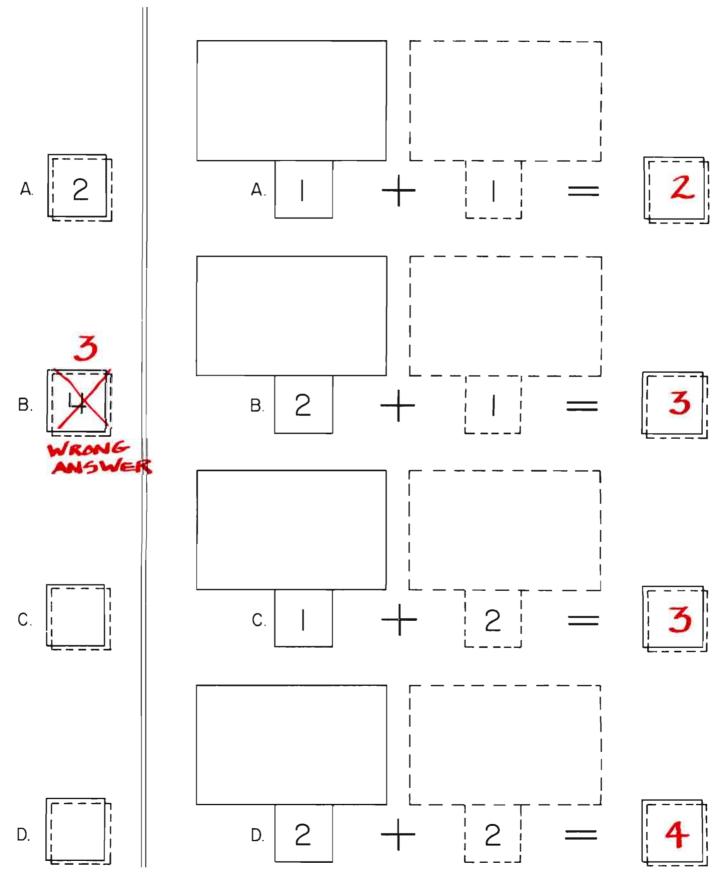
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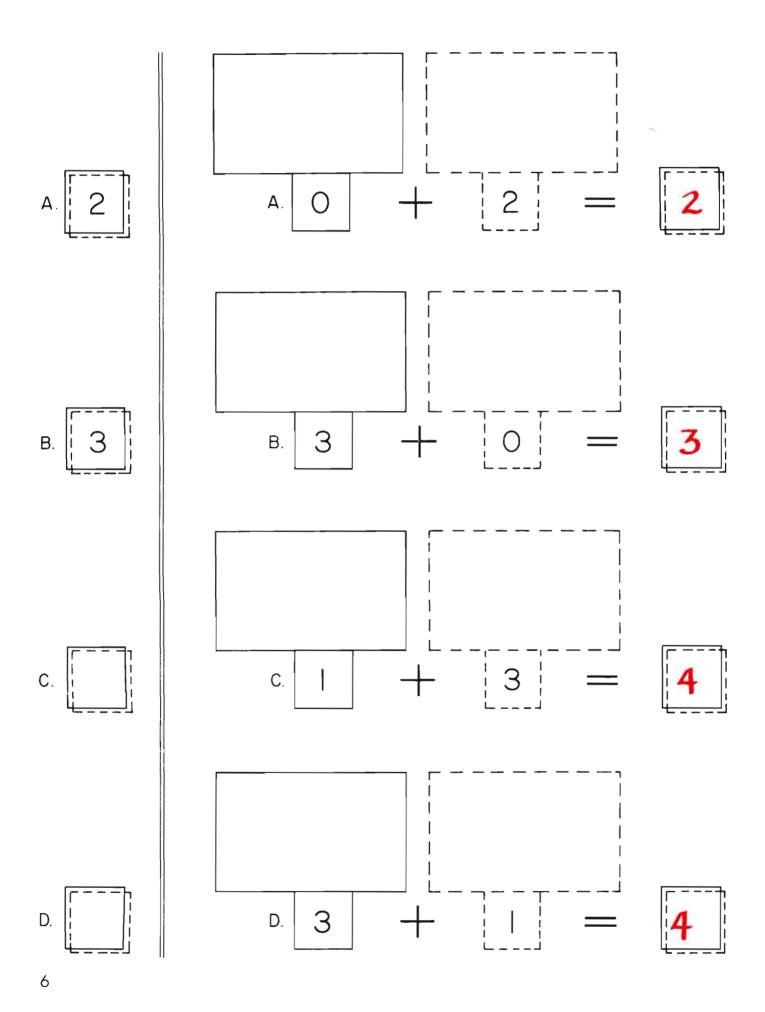


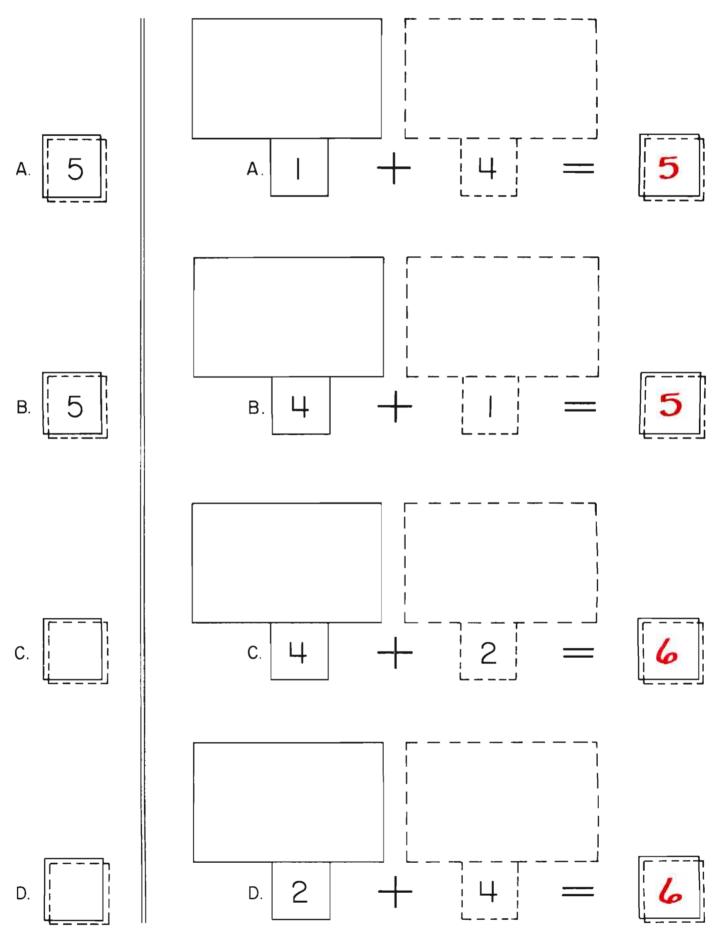




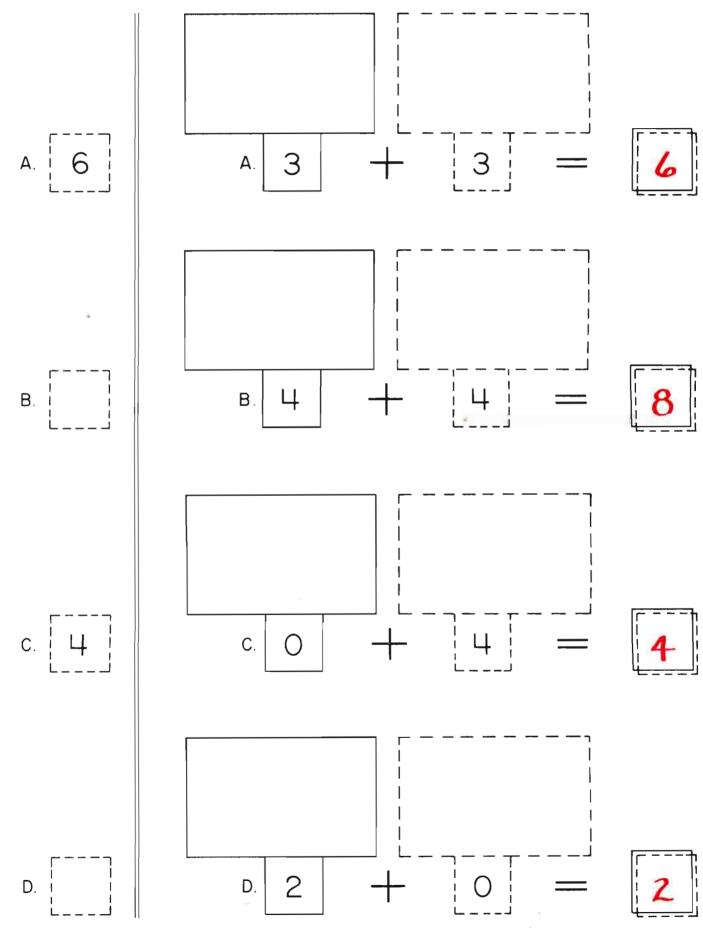


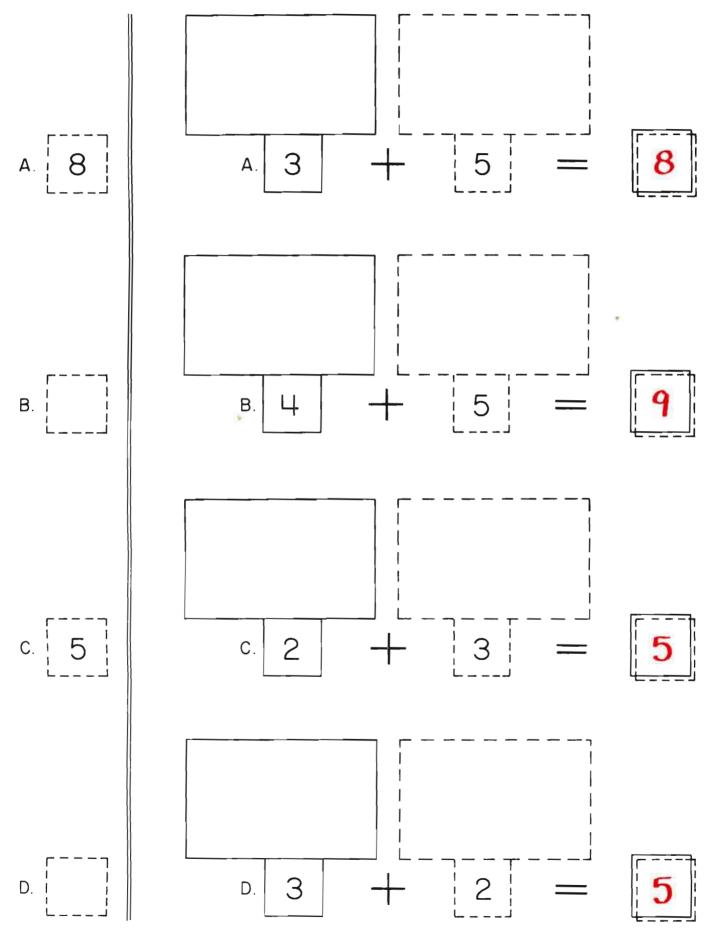


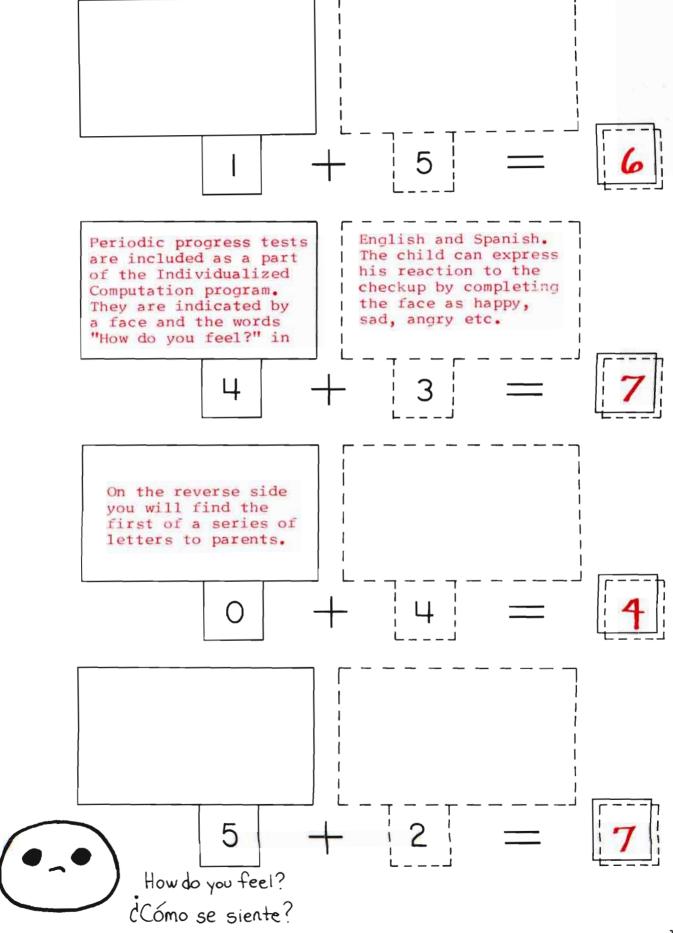




5 В. C. 3 D.







Dear Parent.

All of us know your child has already clearly demonstrated a fantastic ability to learn whatever seems worthwhile - to him or her. Every child is a born learner.

Perhaps the most complex task children face is to "learn to speak the native language", and before entering school your child has learned a native language without much conscious outside help; this in addition to tying shoelaces, telling the difference between dogs and cats . . . and all children have already learned to read every facial expression adults can make.

We - parents and teachers - will be most successful if we keep children's matured learning systems "turned on" . . . remembering they will learn everything they want to learn.

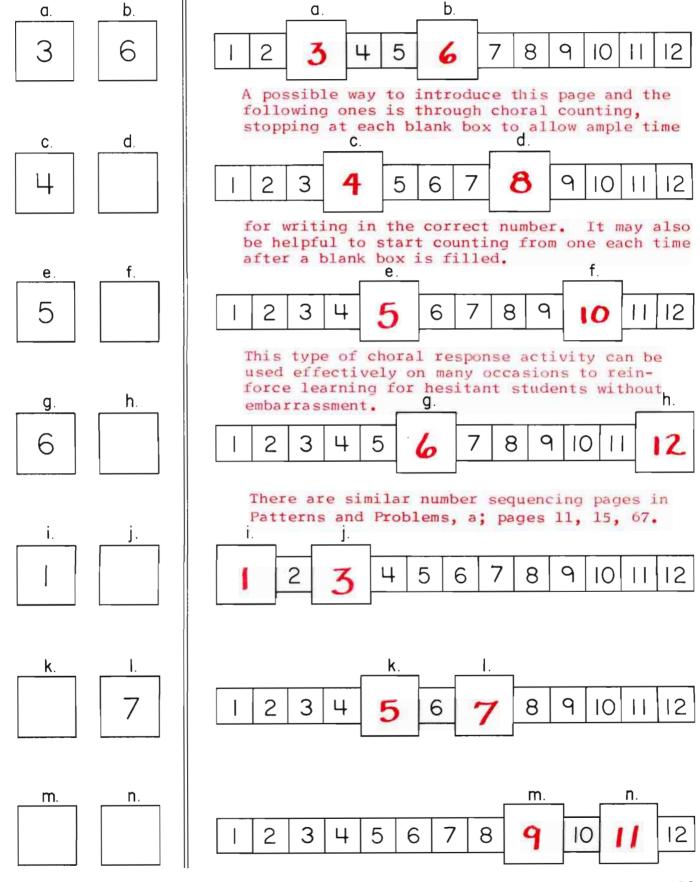
We are aware of the fact that all children coming to school already have a firm grasp on such ideas as "putting together", "take away", "the same number for everyone", "sharing" (even though sharing has its problems, sometimes). In other words, they have the ideas we call "addition", "subtraction", "multiplication", and "division" with remainders. Together let's help children learn to write about these common experiences in a way everyone can understand - the shorthand of arithmetic.

In the exercise on the other side the children "add" by putting groups of beans or other counters in the large boxes, and then combining. We are introducing the socially accepted way of reporting the combination, such as 3+4=7. Children have "put groups together" many, many times. We hope we can interest them in learning to report those experiences or experiments. That's the way we look at arithmetic.

You will receive other letters like this as your child progresses in arithmetic. We will often have suggestions about things you can do at home to help. For instance some television programs such as Sesame Street can be valuable. The game of dominoes (using the set to six dots) is excellent, especially if you keep score, encouraging your child to count and add.

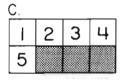
The most important thing to remember when working with your child at home is to stop if it is no longer fun. Pleasure and self-confidence assist math learnings more than any other ingredient.

Sincerely,





В.		
ı	2	3
4		



D.		_		
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6				

E		
	2	3
4	5	
7		

F.			
	2	3	4
5	6	7	

Α	
	2
3	4

This might be a good place to emphasize left to right sequencing. Choral response may be used again if any difficulties persist.

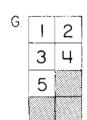
В.		
	2	3
4	5	6

C			
	2	3	+
5	6	7	8

D				
1	2	3	4	5
6	7	8	9	10

<u>E.</u>		
	2	3
4	5	6
7	8	9

F			
	2	3	4
5	6	7	8
9	10		12



Η.	I	2	3
	4	5	
	7		

1	2	3
4	5	6

J.			
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5	6		
9	l .		

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	7	8

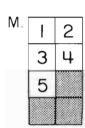
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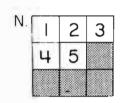
Κ.	
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5	6
7	ω
9	10

H.		2	3
	4	5	6
	7	8	9

1	2	3	4
5	6	7	8
9	10	11	12

1	2	3	4	5
6	7	8	9	10





1	2	3
4	5	6
7	8	

P.			
1	2	3	4
5	6	7	8

ર 1	2
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5	6
7	

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L.	2	3	#	5

M.		2
	3	4
	5	6
	7	A

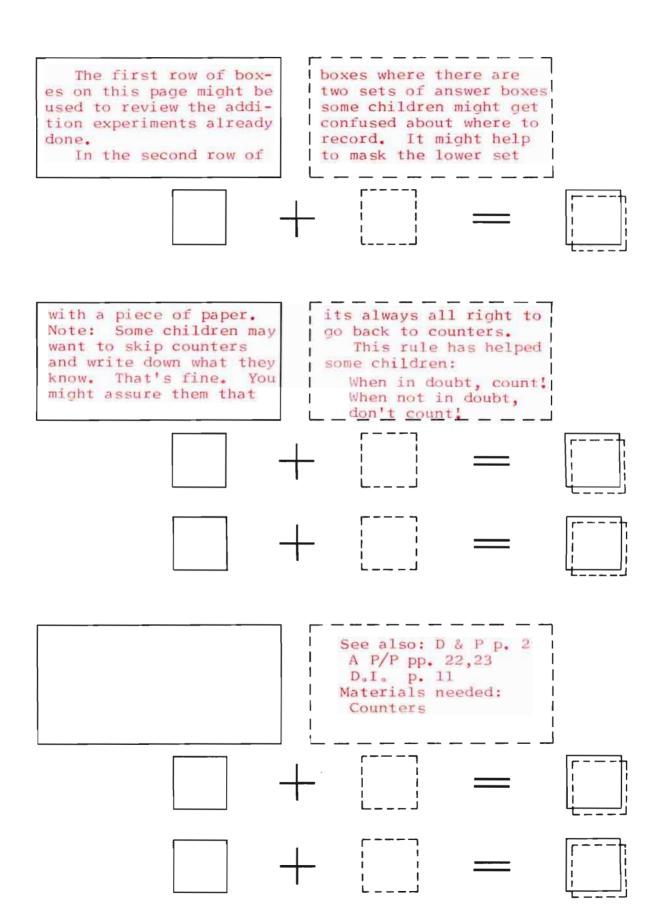
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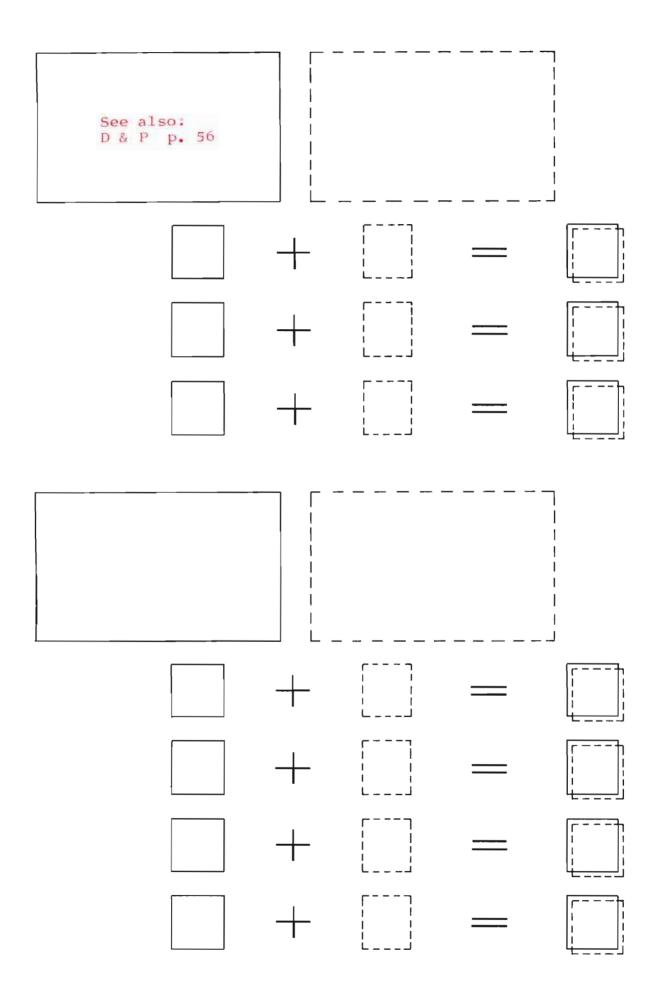
(Q.	
	1	2
	3	4
	5	6
	7	8
	9	10

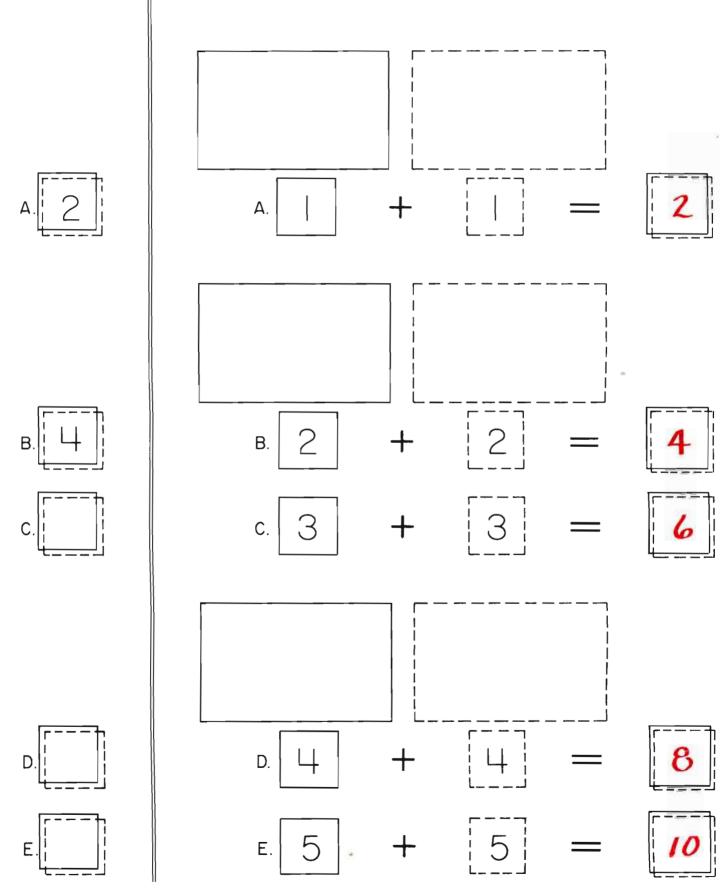
N.		2	3
	4	15)	6
	7	8	9

P			
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5	6	7	8
9	10	11	12

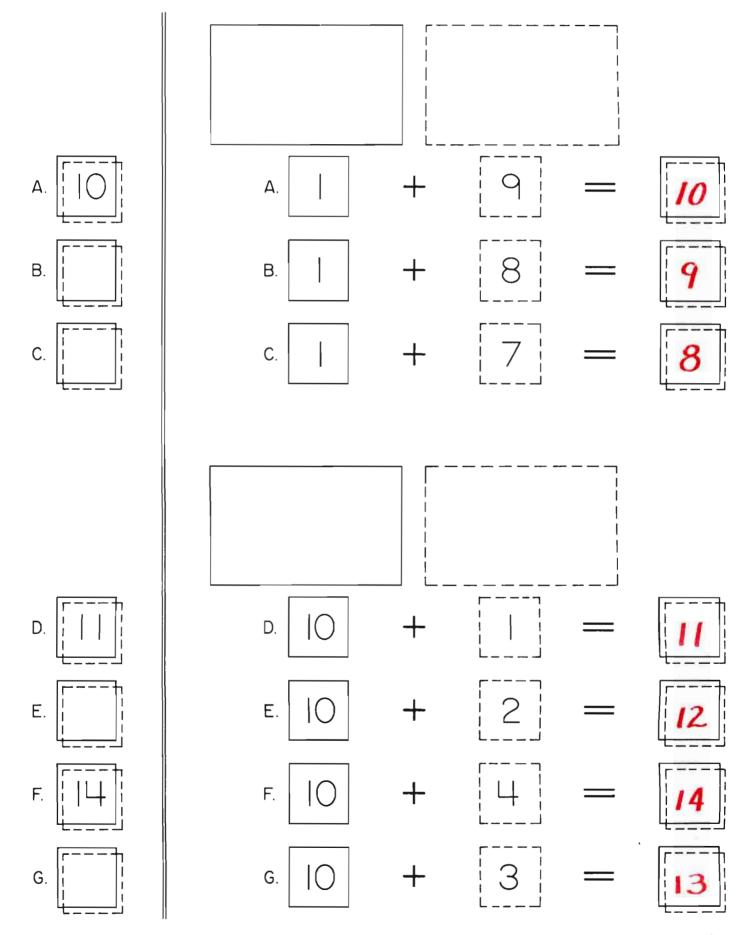
R				
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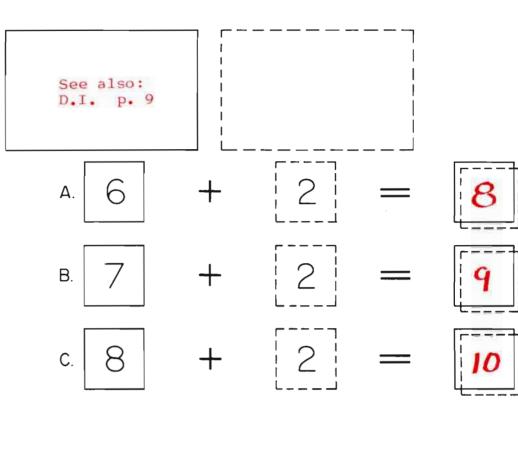


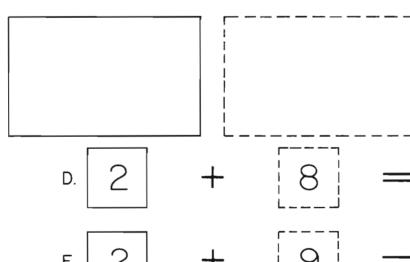


6 Α. B. C. D. D. Ε. Ε. 20





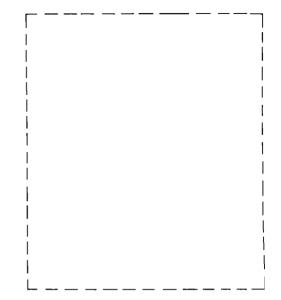




$$F = \begin{bmatrix} 2 \\ + \\ 9 \end{bmatrix} = \begin{bmatrix} 11 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ + \\ \boxed{8} \end{bmatrix} + \begin{bmatrix} 9 \\ \boxed{11} \end{bmatrix} = \begin{bmatrix} 12 \\ \boxed{11} \end{bmatrix}$$

See also: D & P p. 4



$$A. \begin{bmatrix} 4 \\ + \end{bmatrix} + \begin{bmatrix} 1 \\ \end{bmatrix} = \begin{bmatrix} 5 \\ \end{bmatrix}$$

c.
$$I + 5 = 6$$

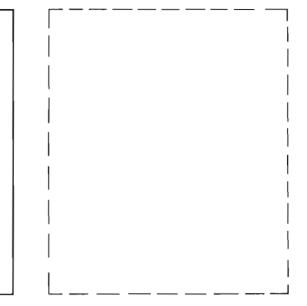
D.
$$9 + 1 = 10$$

F.
$$| + 6 | = 7$$

$$G. 7 + I = 8$$

$$H. _{1} + 10 = 11$$

See also: D & P p. 5



A.
$$\begin{bmatrix} 9 \\ + \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

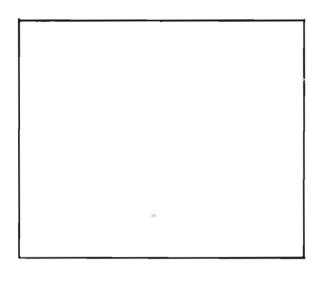
c.
$$3 + 9 = 12$$

E.
$$2 + 8 = 10$$

$$F. 8 + 4 = 12$$

G.
$$8 + 6 = 14$$

$$H. 8 + 8 = 16$$





$$7 + 3 = 10$$

$$3 + 9 = 12$$

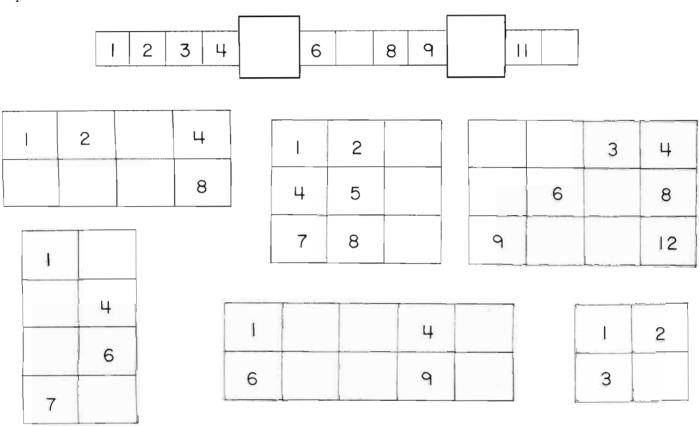
$$10 + 3 = 13$$



How do you feel? ¿Cómo se siente? Dear Parent.

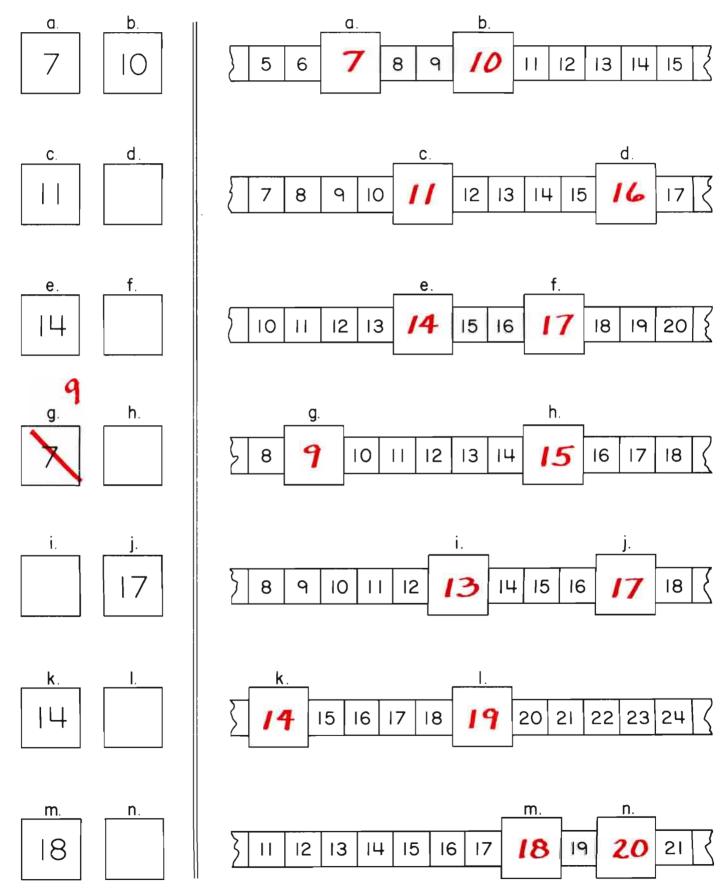
As you can see on the back of this page, your child can success-fully count and write numbers at least to twelve, and has the ability to record addition in a more independent way -- without the small boxes -- but still using objects to count, if necessary.

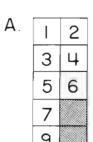
Notice the different patterns developed as the child gets to twelve. These will become an important preparation for greater skill development in addition and for learning multiplication later on. Maybe you would like to have your child show you how he works with the patterns below:



If you would like to help your child at home, and still keep math learning enjoyable, we suggest that Bingo is a good commercial game for number recognition. You can easily make your own Bingo cards. If recognizing the shapes of written numbers is hard for your child, it is often helpful to engage the sense of touch. In shaping numerals from clay, or in feeling the numerals cut from sandpaper, the child learns much faster than when the eye alone is used. Again, this kind of activity is much more enjoyable for children than practicing with paper and pencil.

Sincerely,





Α.		2
	3	4
	5	6
	7	8
	9	10

В.		
	2	3
4	5	6
7	8	9
10		12
13	14	15

В.		
	2	3
4	5	6
7	8	9
10	11	
13	14	

С.			
	2	3	4
5	6	7	8
9		11	
	14	15	

D.		2	3	4	5
	6	7		9	
	11		13	14	

Ε.							
1	2	3	4	5	6	7	8
9		11		13		15	

<u>C</u> .				
		2	3	4
	5	6	7	8
	9	10		12
	13	14	15	16

D.		2	3	4	5
	6	7	8	9	10
		12	13	1	15

Ε.							
	Z	3	4	5	6	7	8
9	10	11	12	13	14	15	16

F.	I	2	3	4	5	6
	7	8	9	10	П	12
	13			16	17	

(G.		
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	4	5	6
	7	8	9
	10	11	
	13		15
		17	18

Н.			
1	2	3	4
5	6	7	8
9	10	П	
13	1	15	
17		19	20

I.		2	3	4	5
	6	7	8	9	10
		12	13	14	15
	16				

F.		2	3	4	5	6
	7	8	9	10	11	12
	13	14	15	16	17	18

H			
	2	3	4
5	6	7	∞
9	10		12
13	14	15	16
17	18	19	20

	2	3
4	5	6
7	8	9
0		12
3	14	15
6	17	18
	1 7 0 3	4 5 7 8 0 11 3 14

I.		2	3	4	5
	6	7	8	9	10
		12	13	14	15
	16	17	18	19	20

J							
1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16

K.	1	2	3	4	5	6
	7	8	9	10	П	
	13		15		17	

L .		
1	2	З
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7	8	9
10		12
	14	
16		18

Μ.			
	2	3	4
5	6	7	8
9	10	Н	
13	14		
	18	19	

N.		2	3	4	5
	6	7	8	9	Ю
	П	12	13	14	
	16	17	18	19	

0.							
1	2	3	4	5	6	7	8
9	Ю	11					

K. '		2	3	4	5	6
	7	8	9	10		12
	13	14	15	16	17	18

Μ.			
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٧.		2	3	4	5
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	16	17	18	19	20

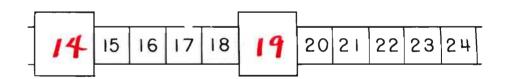
<u>O</u> .							
1	2	3	4	5	6	7	8
9	10		12	13	14	15	16

	2	3
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8	9	10	Ξ	12	13	±	15	16	17

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	12	13	14	15	16	17	18	19	20 21

	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16



	1 2		4	
5	6	7	8	
9	10	1/	12	
13	14	15	16	
17	18	19	20	

	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20



. How do you feel? d Cómo se siente? Dear Parent.

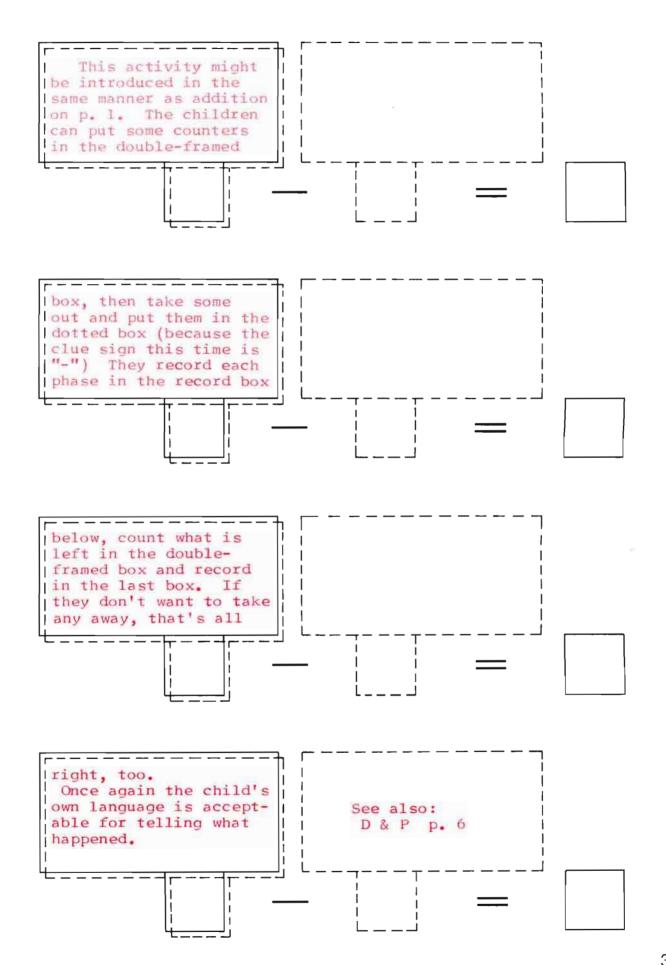
This test requires writing numbers through twenty, and, as you can see, your child can successfully do this now.

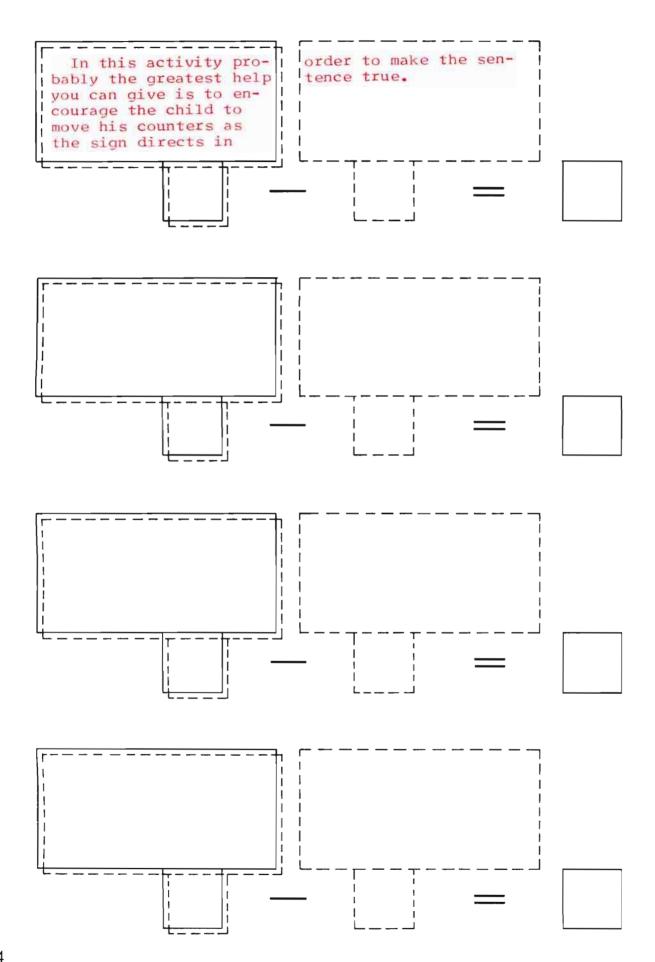
This is a good time for us to talk with you about a very common problem among young children -- writing numbers "backwards" or even upside down. This happens because the eye is one of the last things to mature, and the message from the eye through the brain to the writing hand can get mixed up on the way. Please do not scold or make your child feel stupid because of this kind of "mistake". It has been well established that young children will reverse numbers (and letters) when under pressure.

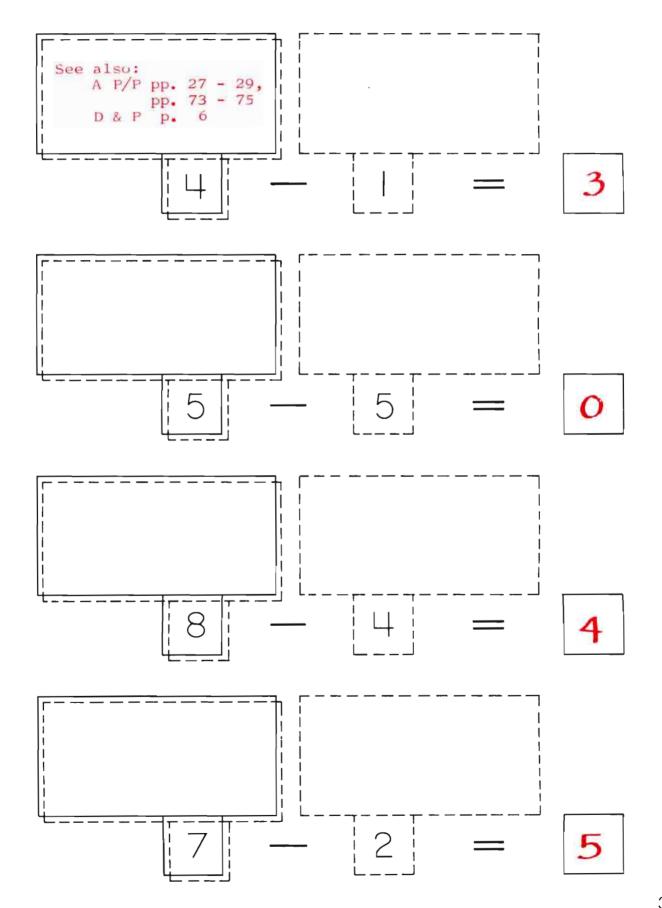
By the age of eight or nine the muscle system should be developed enough that the child will naturally turn the numbers around the "right" way. In the meantime, a gentle reminder from you, and practice with the sense of touch suggested earlier will help some, but maturity will eventually solve the problem.

Subtraction, or "take away" is coming next. Again, you can help by just talking to your child like this: "Will you count to see how many slices of bread are left in this loaf? If we use two of them now, how many will be left?" But please -- remember to do this kind of thing only when you feel like making it a game, and when you are able to deal with mistakes in a friendly, supportive manner!

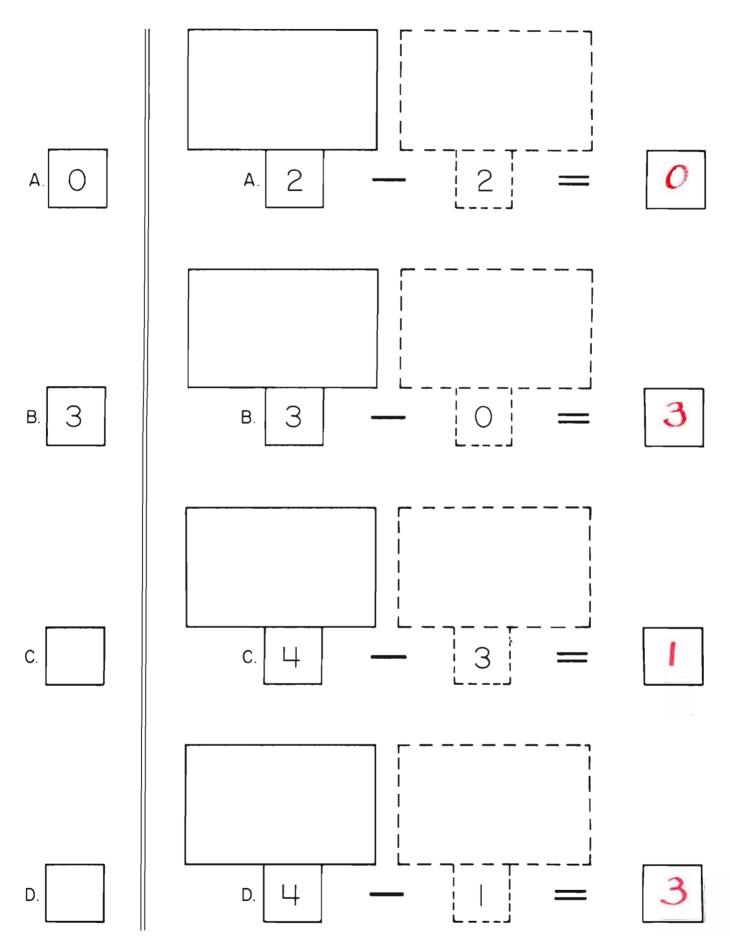
We, here at school will try to keep math enjoyable, too! Sincerely,



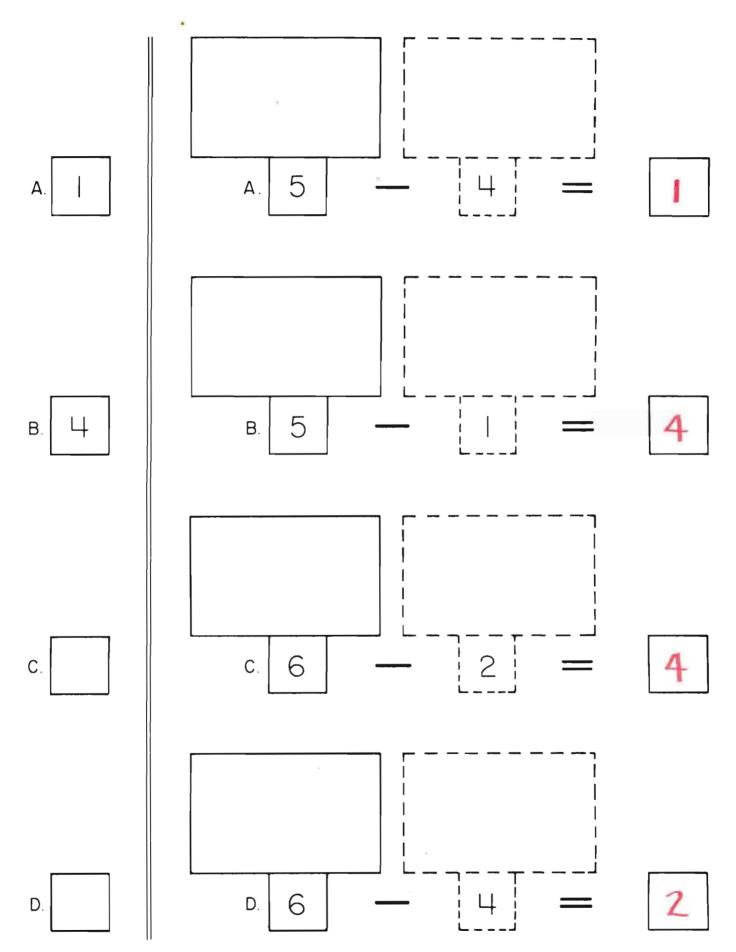


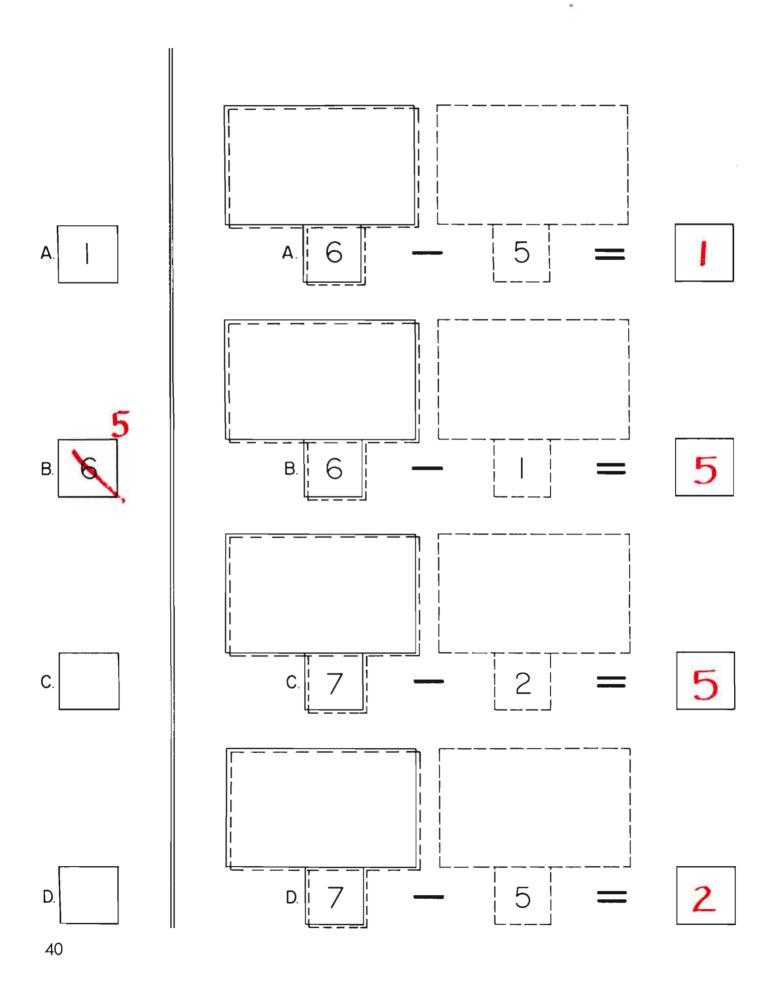


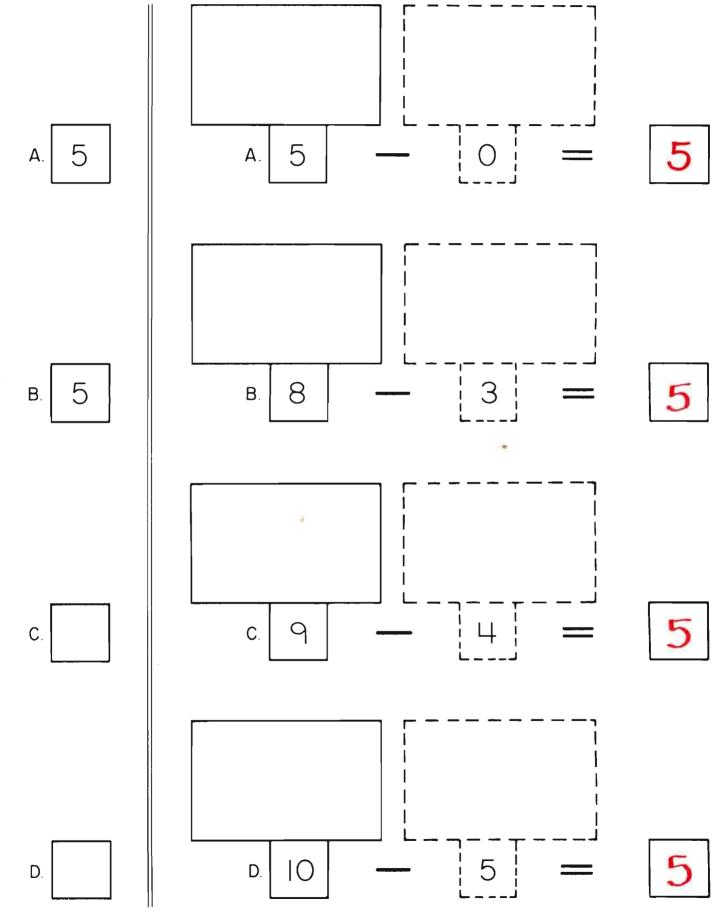
Α. 2 В. B. С. D. D. 36

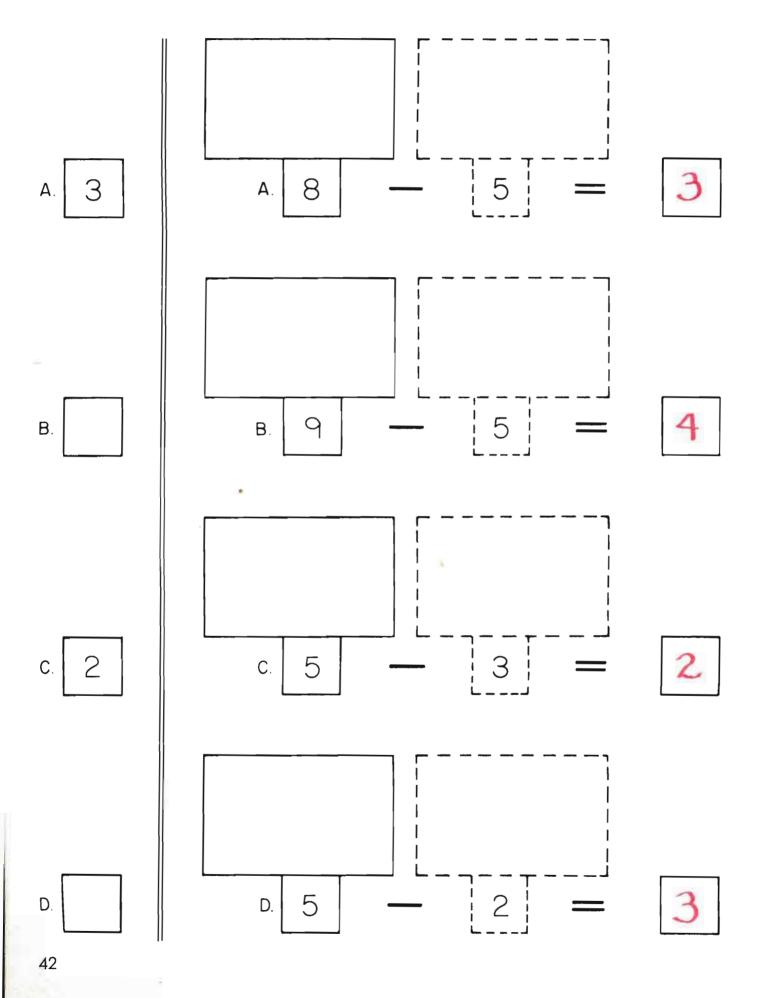


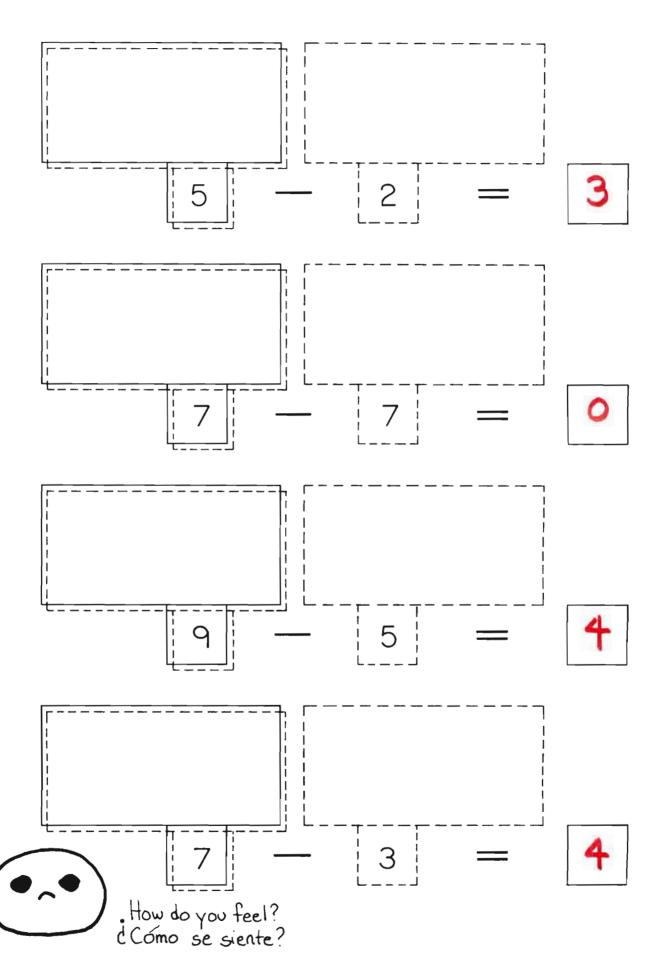
5 В. B. C. D. D. 38









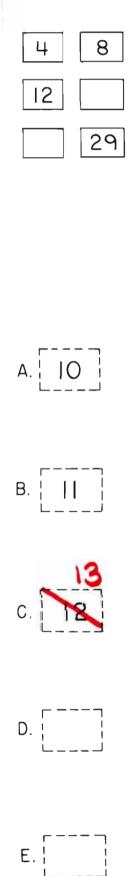


Dear Parent,

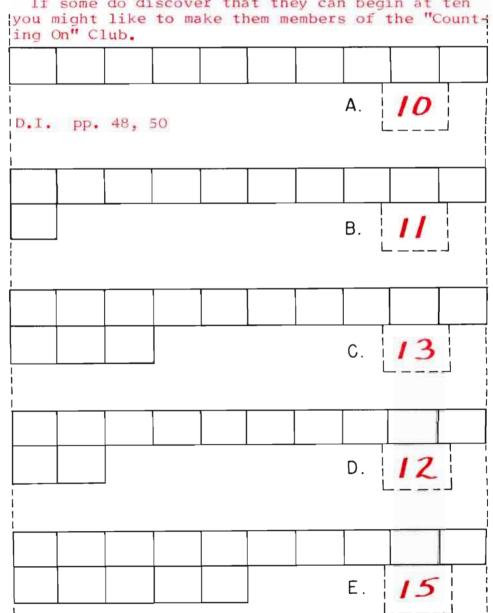
Your child has demonstrated the ability to write down the results of a "take away" situation, really just making a record of what was already known about subtraction even before coming to school: given a group of objects, taking some away, how many are left? Yes, and your child can do it. We will work on this, using things to count and writing down the results. Then we go on to the next step, which is using pictures of things to count and take away.

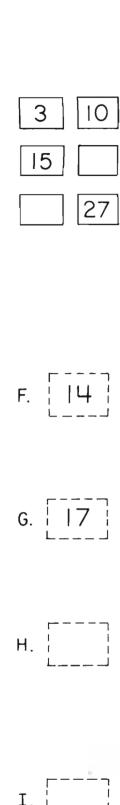
The next learning step is complicated. It may seem simple to us because we can't remember when we didn't know how to do it. It involves being able to "count on". For instance, given a picture of ten connected blocks and three loose blocks, will your child figure out that it is quicker and easier to "count on" from ten to thirteen? Or will every block have to be counted to reach thirteen? This is very hard for children to learn, and many won't understand it even when it's explained. Some won't catch on to this trick until age eight or older.

Dealing with money can help children with the "counting on" learning, but you may be hard pressed to find situations involving money that are real for your child. "Leggo" is a good commercial game that can help in developing this concept. Watch carefully the next series of math papers your child brings home and you will easily see what we are talking about.



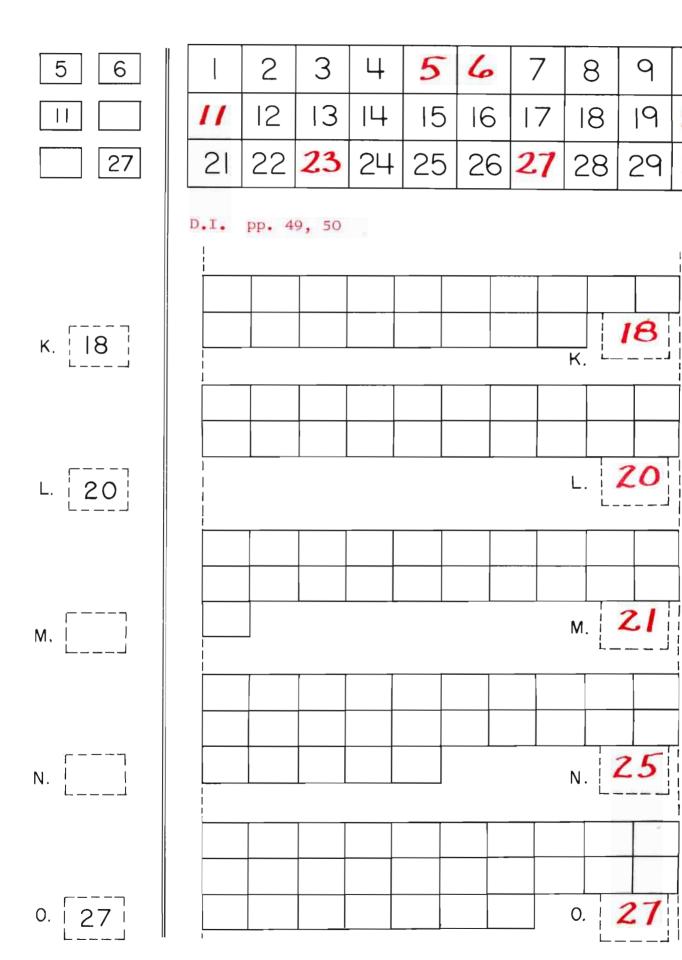
	2	3	4	5	6	7	8	9	10	
11	12	13	14	15	16	17	18	19	20	
21	22	23	24	25	26	27	28	29	30	
At this point you might want to see if any children have discovered that the long rows of blocks are all 10 units long. A mention of this is probably helpful but some children will still need to count from one. If some do discover that they can begin at ten you might like to make them members of the "Counting On" Club.										

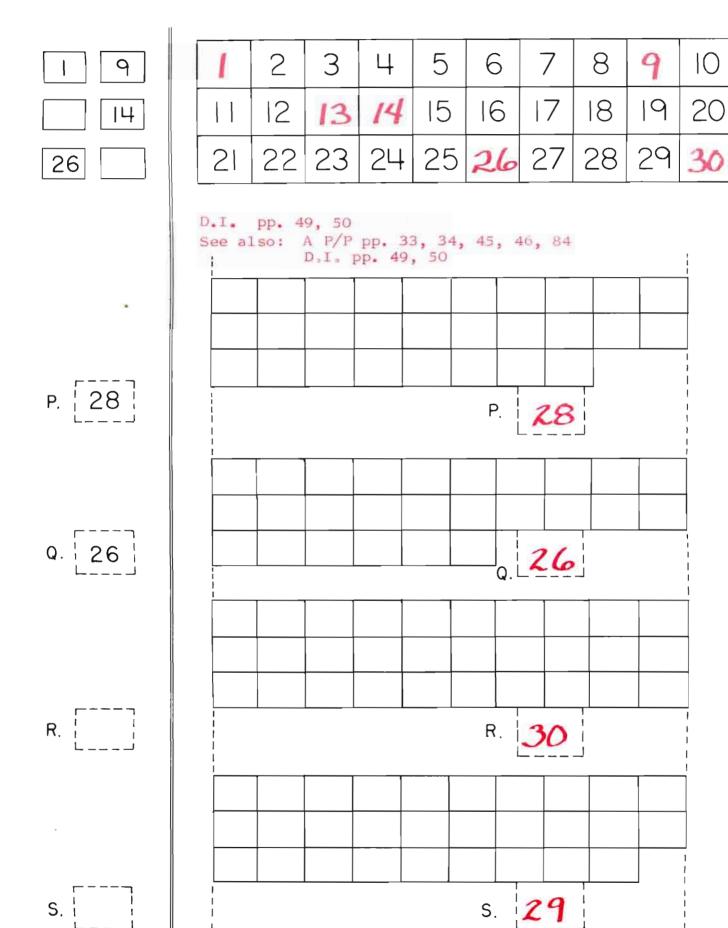




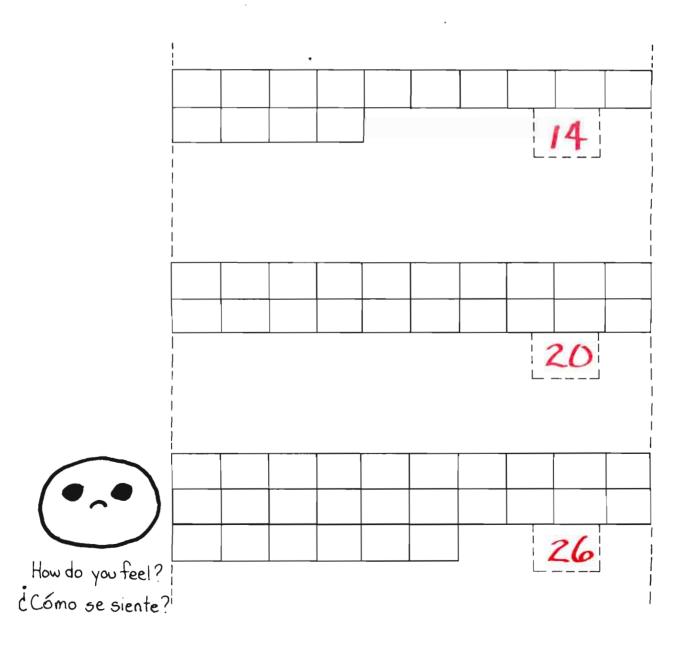
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J. [20]





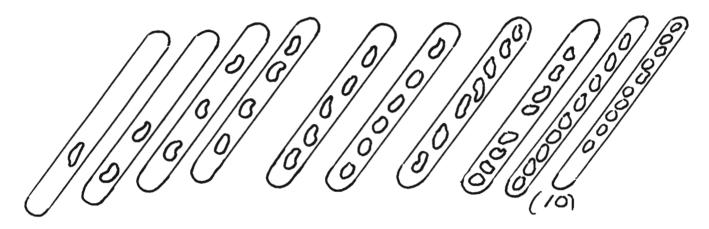
1	2	3	4	5	6	7	8	9	10
	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30



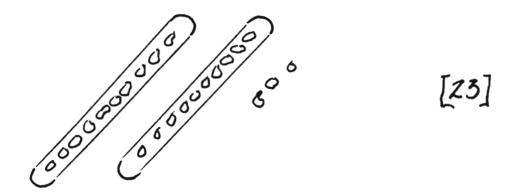
Dear Parent,

Your child can now write numbers to 30 and can successfully count blocks pictured on a page, recording how many there are. We hope that the discovery was made that it is faster and easier to count by tens first, then to count on from there. This is one of the first short cuts a child can discover in math, and children will wisely use short cuts wherever they find them!

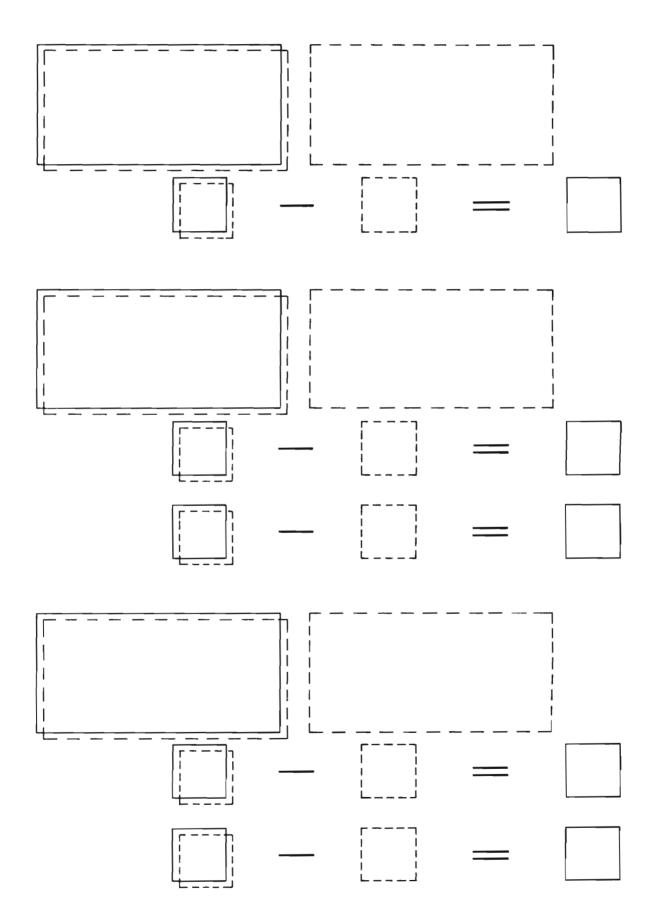
We suggest that if you want to continue helping at home, you make "bean sticks" like the ones we use at school. You will need quite a few popsicle sticks and dried beans. Your child can show you how they are made. They will look like this:

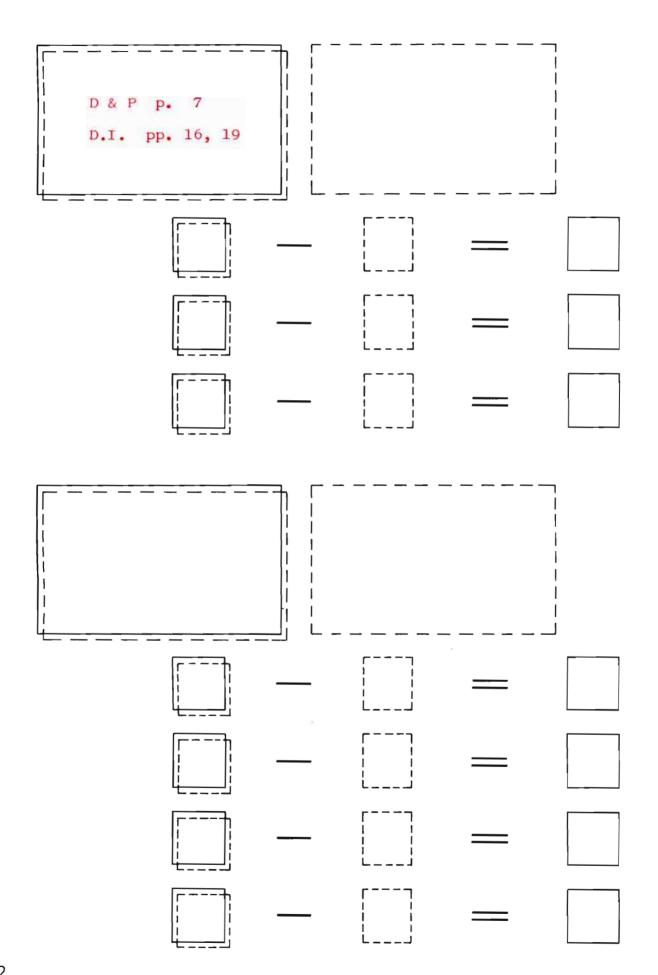


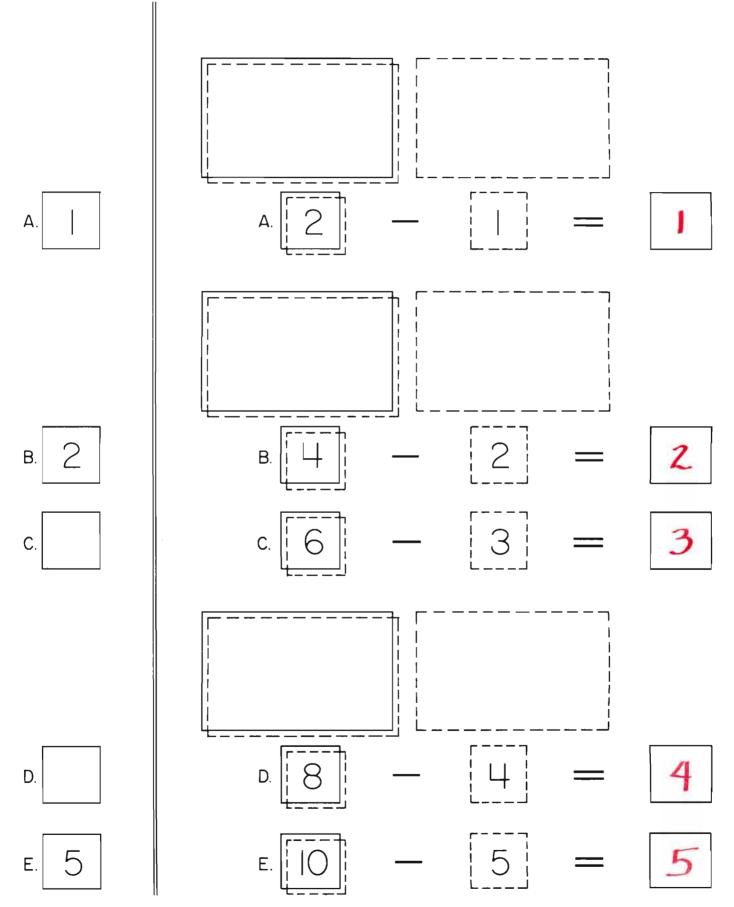
To strengthen the idea of counting on from ten, or twenty, or thirty, use the "ten-sticks" and loose beans:

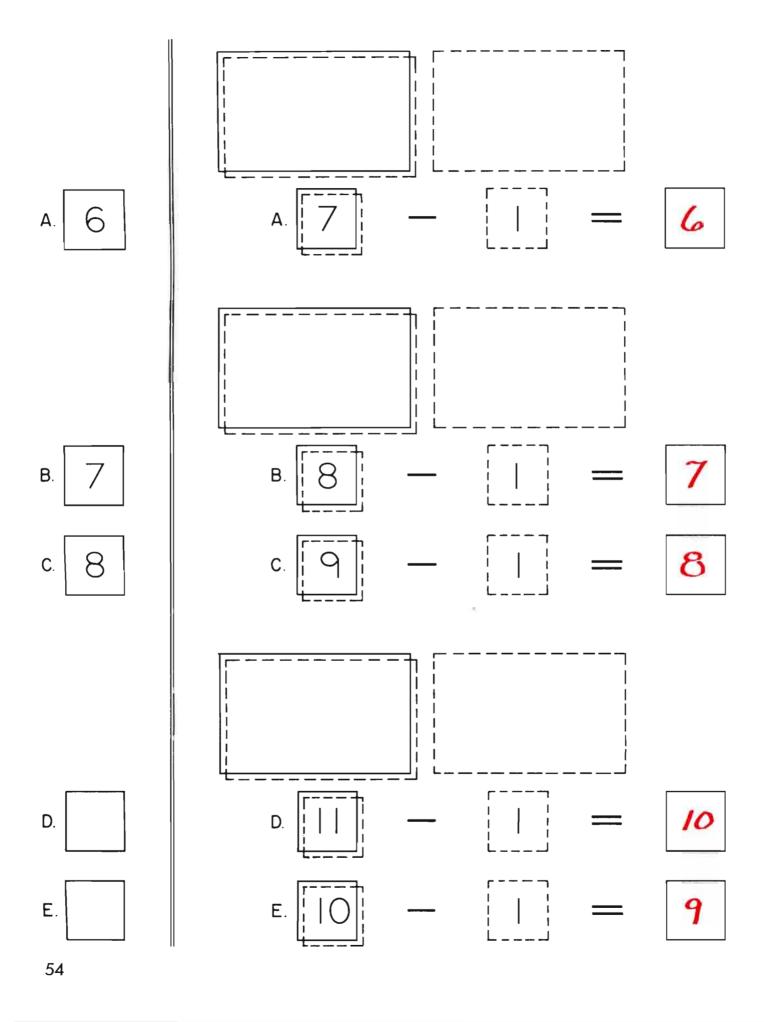


Encourage your child to make up the problems and show you how to "count on".

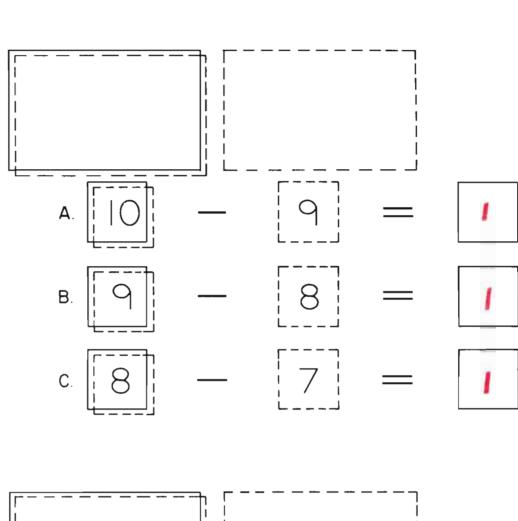


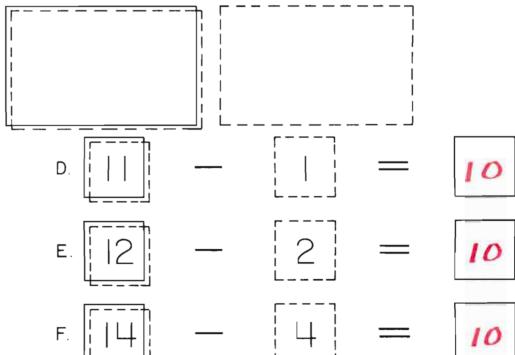


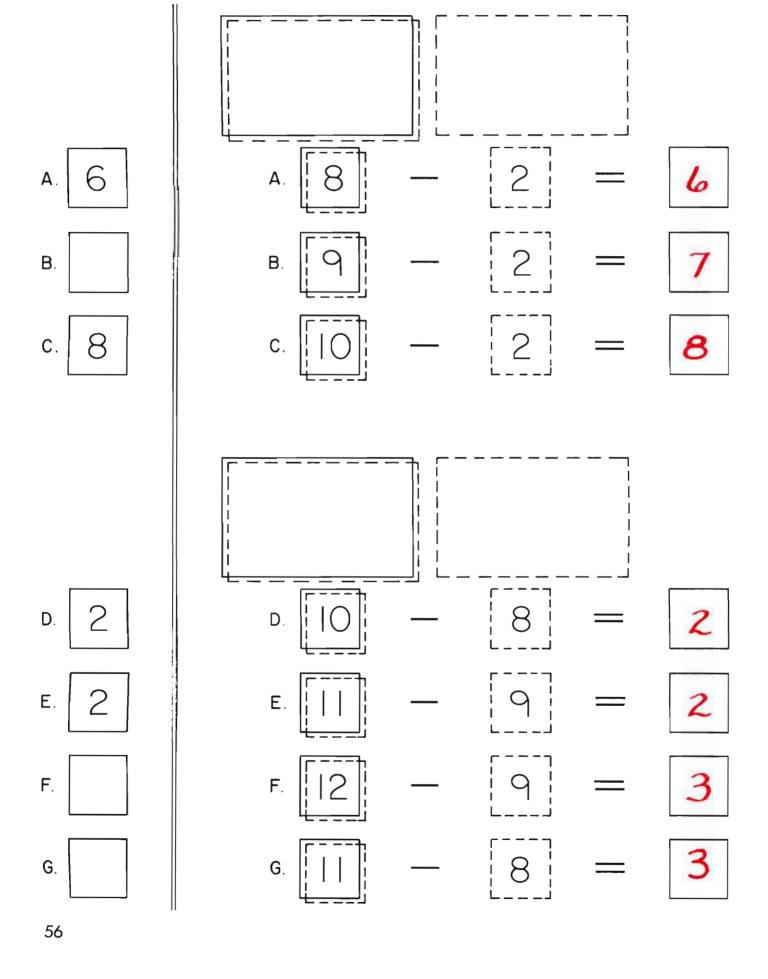


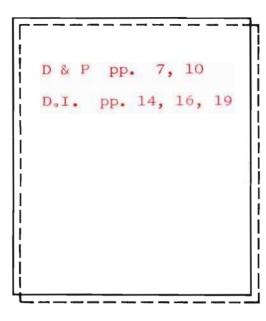


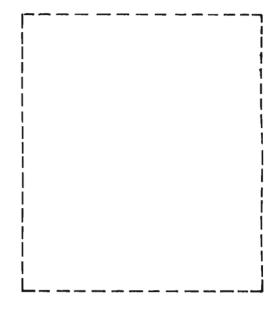
A. B. F. G.











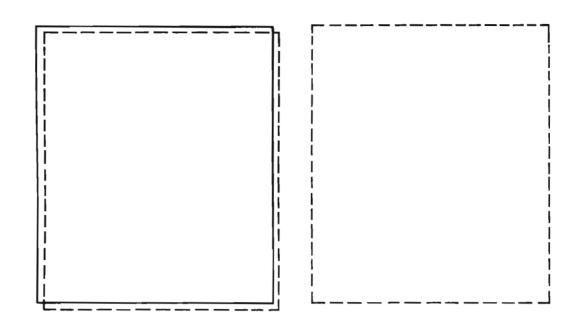
c.
$$6 - 5 = 1$$

d.
$$10 - 1 = 9$$

$$f. 7 - 6 = 1$$

g.
$$8 - 1 = 7$$

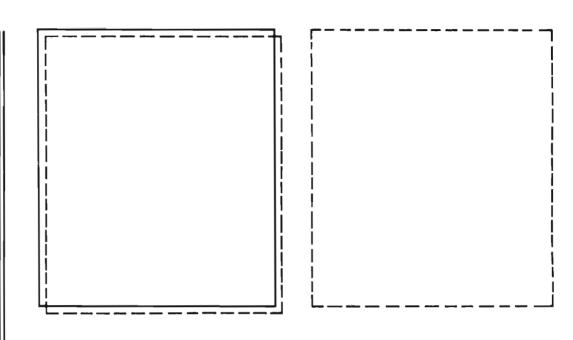
$$h. 11 - 10 = 1$$



d.
$$9 - 7 = 2$$

g.
$$10 - 2 = 8$$

h.
$$12 - 10 = 2$$



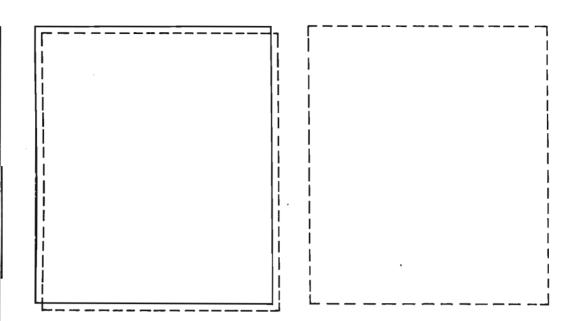


d.
$$10 - 3 = 7$$

e.
$$7 - 5 = 2$$

f.
$$10 - 9 = 1$$

h.
$$10 - 7 = 3$$



b.
$$\begin{bmatrix} 7 \\ - \end{bmatrix} - \begin{bmatrix} 6 \\ \end{bmatrix} = \begin{bmatrix} 1 \\ \end{bmatrix}$$

c.
$$11 - 1 = 10$$

d.
$$12 - 4 = 8$$

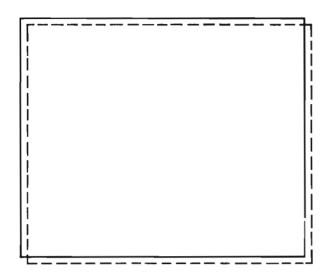
e.
$$9 - 2 = 7$$

f.
$$12 - 9 = 3$$

g.
$$9 - 8 = 1$$

h.
$$13 - 3 = 10$$

i.
$$12 - 8 = 4$$







$$12 - 6 = 6$$

$$12 - 5 = 7$$

$$16 - 8 = 8$$

$$|2 - 7| = 5$$



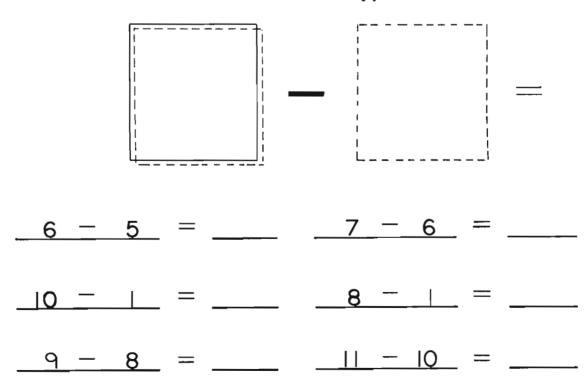
How do you feel? ¿Cómo se siente? Dear Parent.

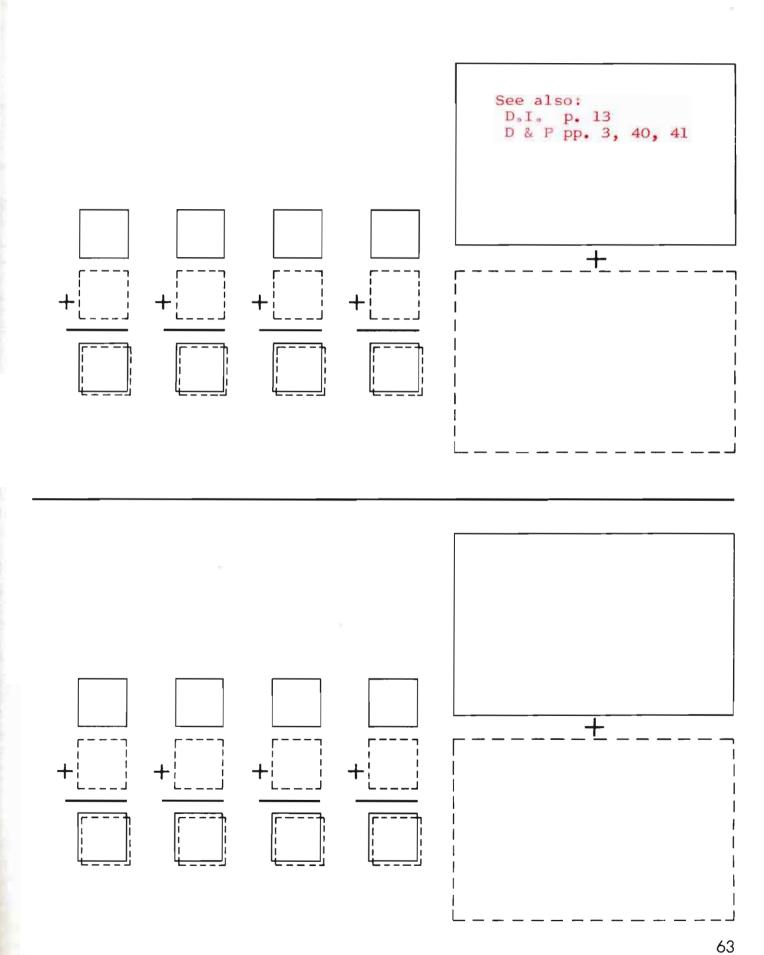
The exercise on the back of this page shows that your child can record subtraction now without having to have the boxes for guidance.

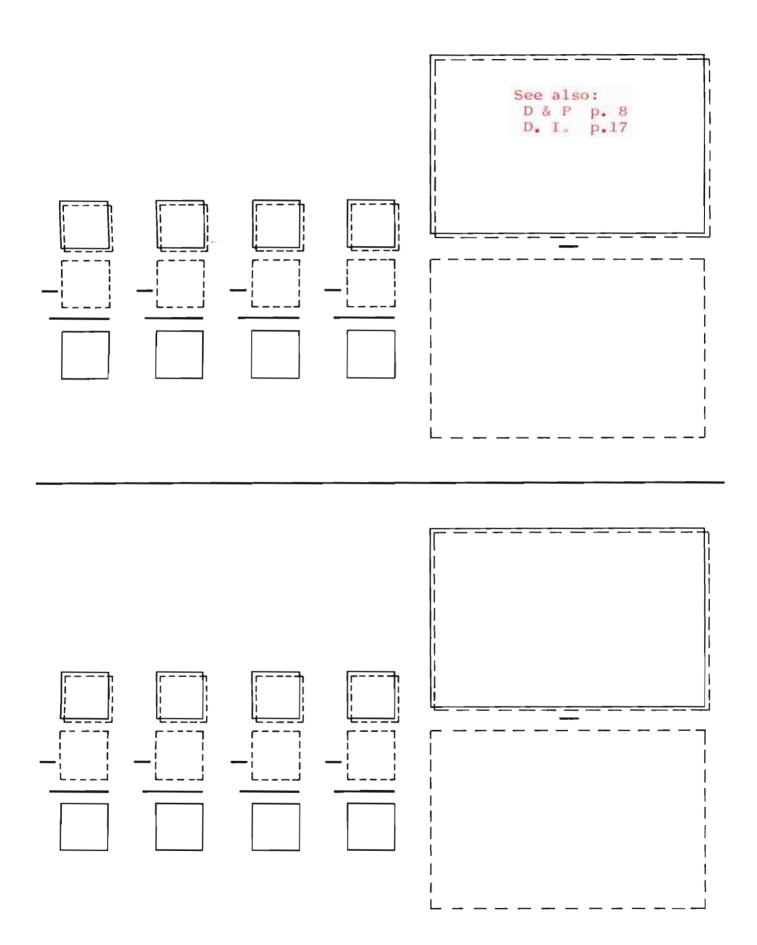
Below are some sample problems. You might want to ask your child to show you how to work them. Remember to have a supply of beans or buttons, something to count, to help if necessary.

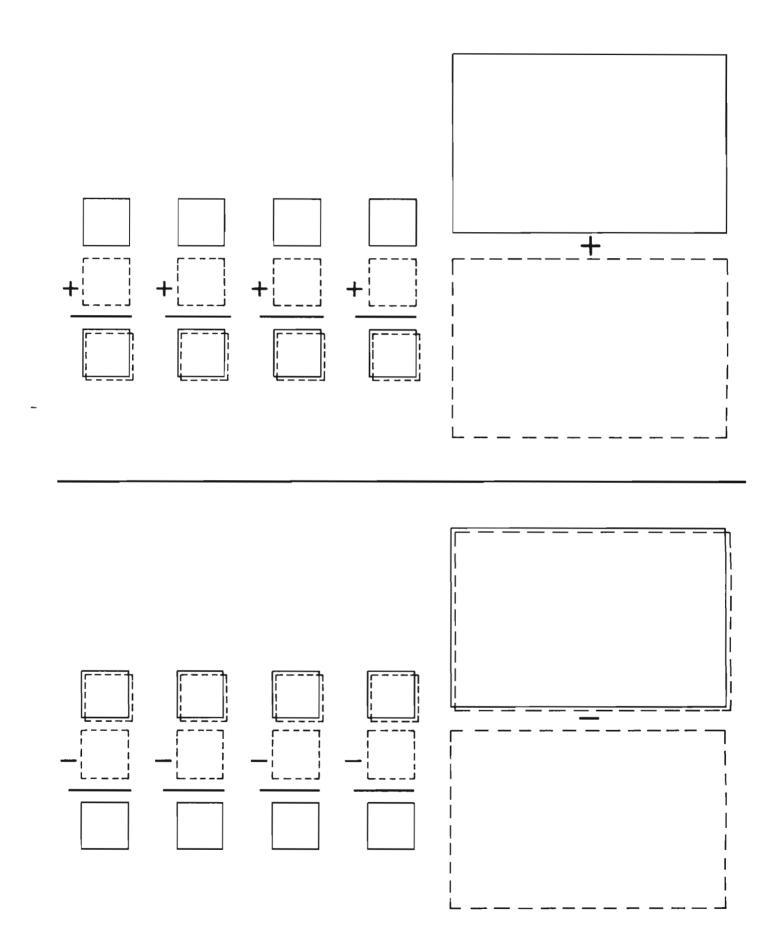
We encourage the child to use counters for accuracy as long as needed. This builds the confidence so necessary for success in any learning situation. You will see the day come, sooner or later it matters not, when your child will say, "I can do this faster without counting. I know the answer!" We have seen it happen eventually with every child. It occurs when children have learned the real value of numbers. There is no shortcut to this place without a risk to understanding.

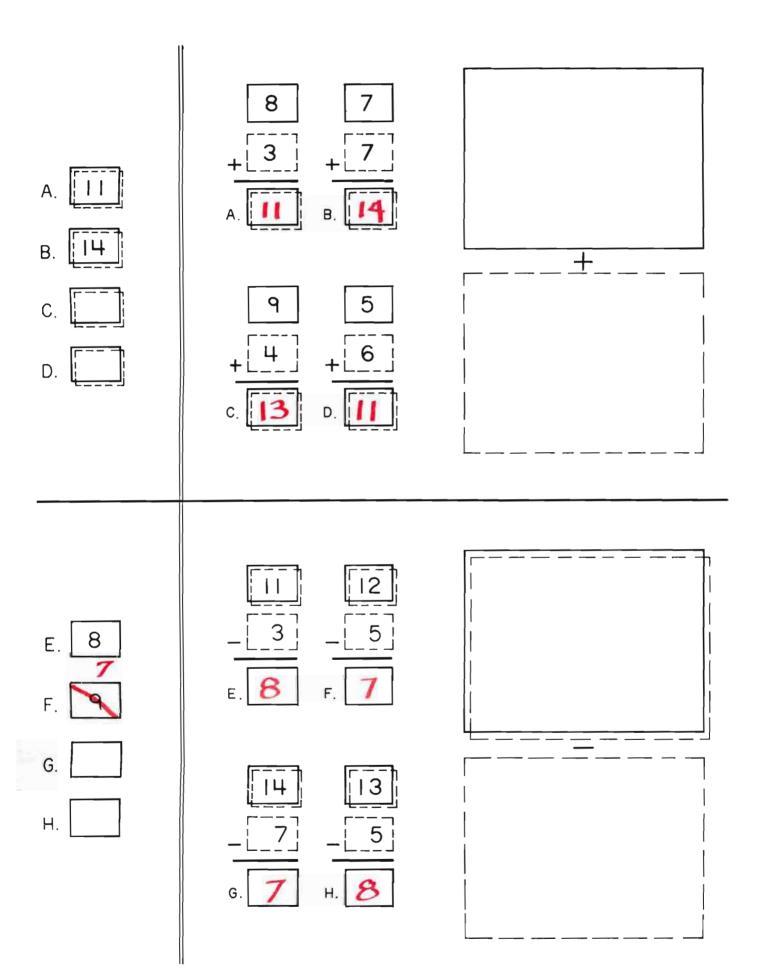
Your help at home has been most valuable. By now you should be seeing in your child a self-confidence with numbers, an attitude of "I can do!" Keep up all the home games we have suggested, especially dominoes. We're sure you can invent some games of your own, too! (If you do, and have success, let us in on your secrets!)

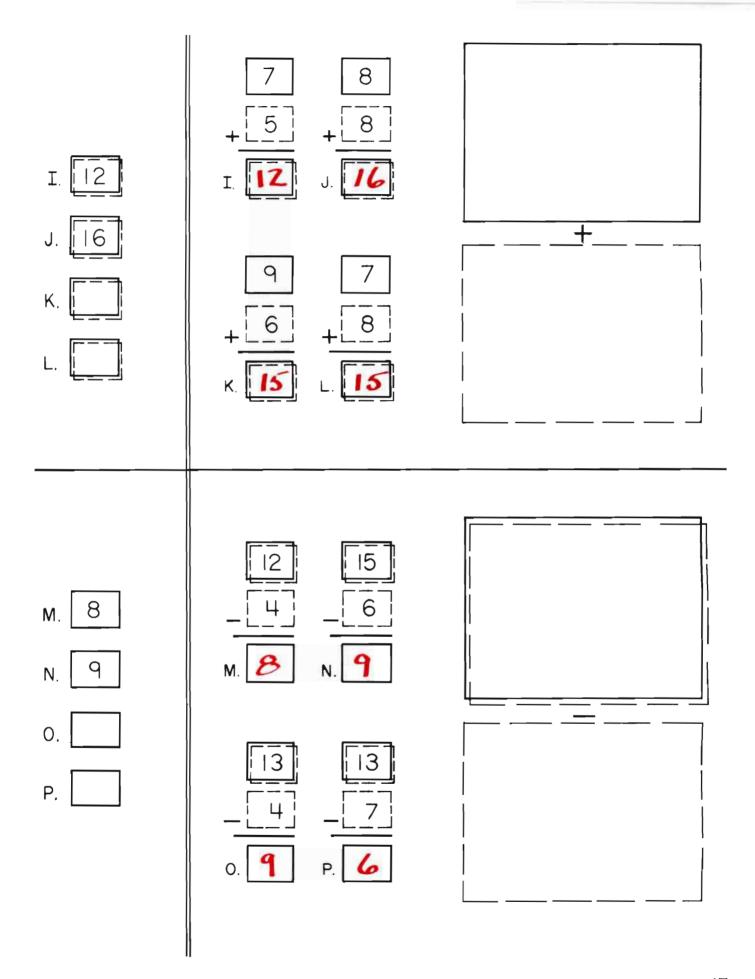


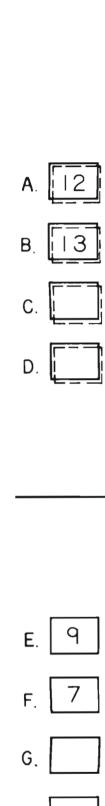


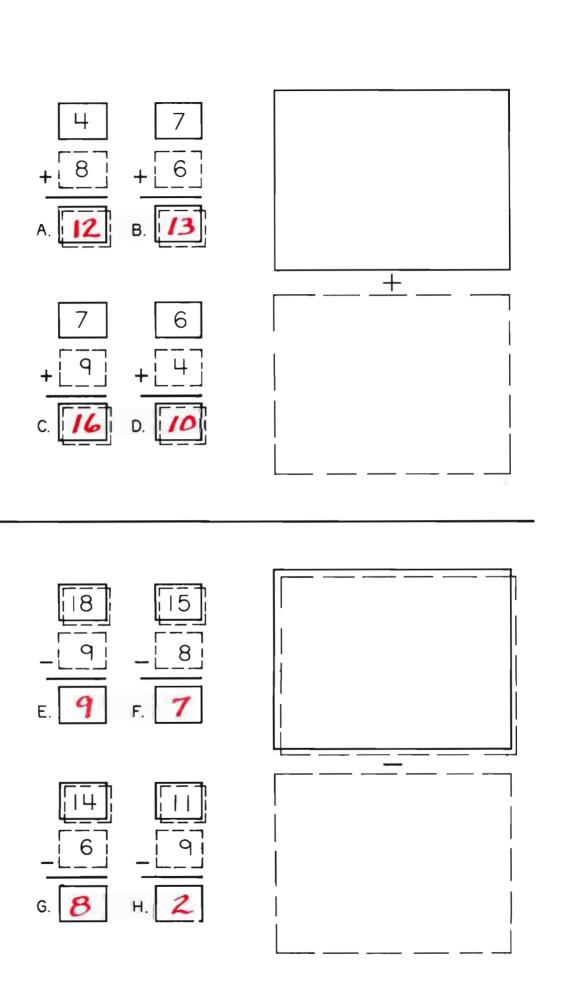


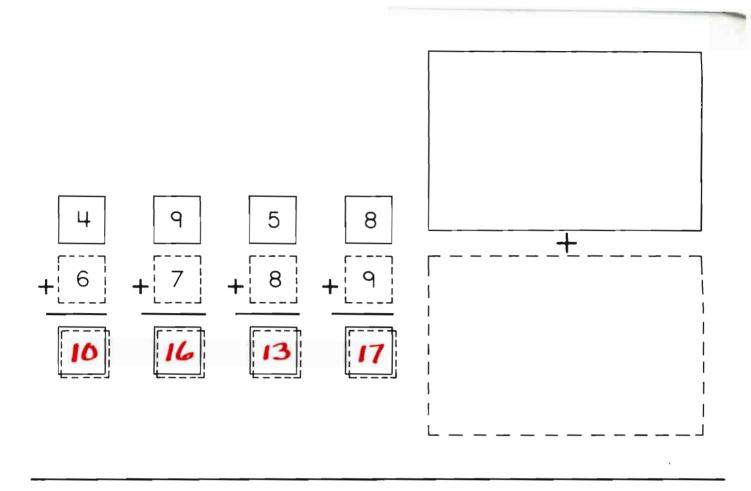


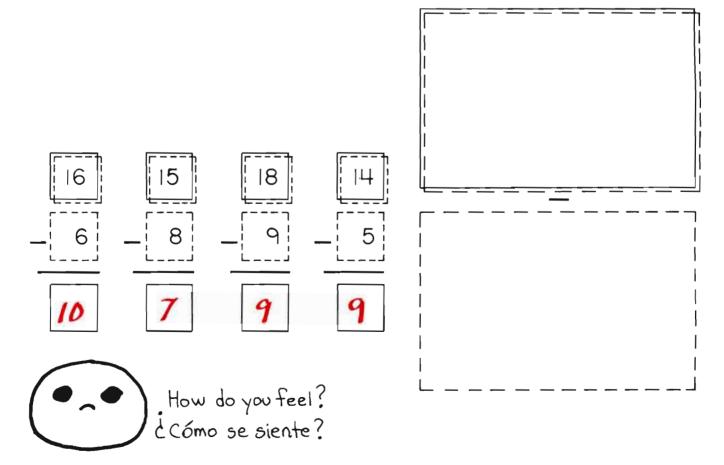








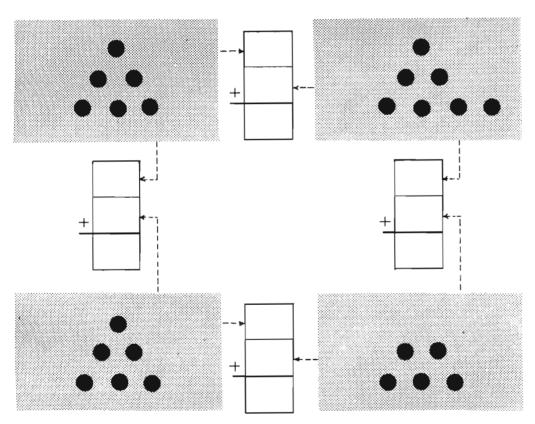




Dear Parent,

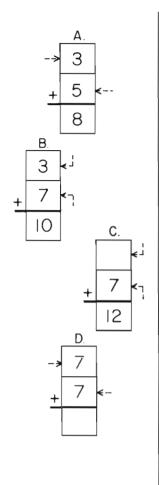
Your child has learned to record addition and subtraction vertically, still using real things to count. Encourage a demonstration if you can use the experience to express your admiration for this accomplishment. Your approval and acceptance is probably the most important ingredient in the success of this program.

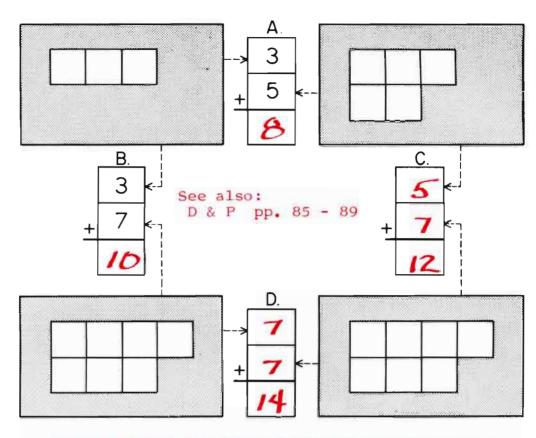
The last step in this book will be adding and subtracting using pictured things to count on the page, rather than real things to touch and move about. Below is an example:



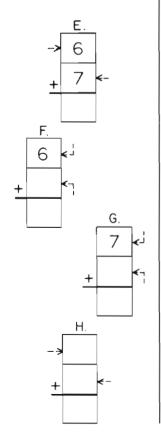
This represents a new sophistication in your child -- the move from using things to feel and touch to count, to counting pictured things. This was the way human beings kept track of things for centuries before someone found a short cut by inventing numerals to show the value of number. If it took humanity that long to figure out the short cut to tallying, we must resist the temptation to hurry children over the first learnings that are the foundation for understanding.

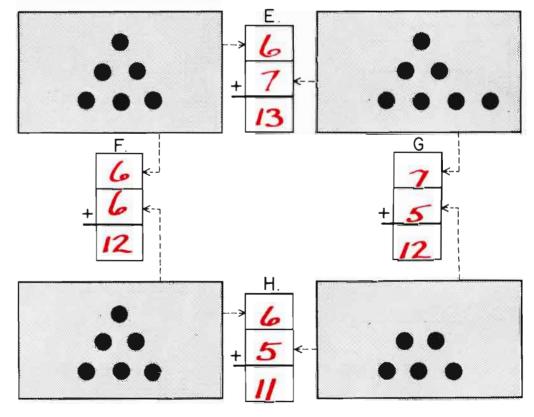
Keep up the support at home! We appreciate your help more than you can know.

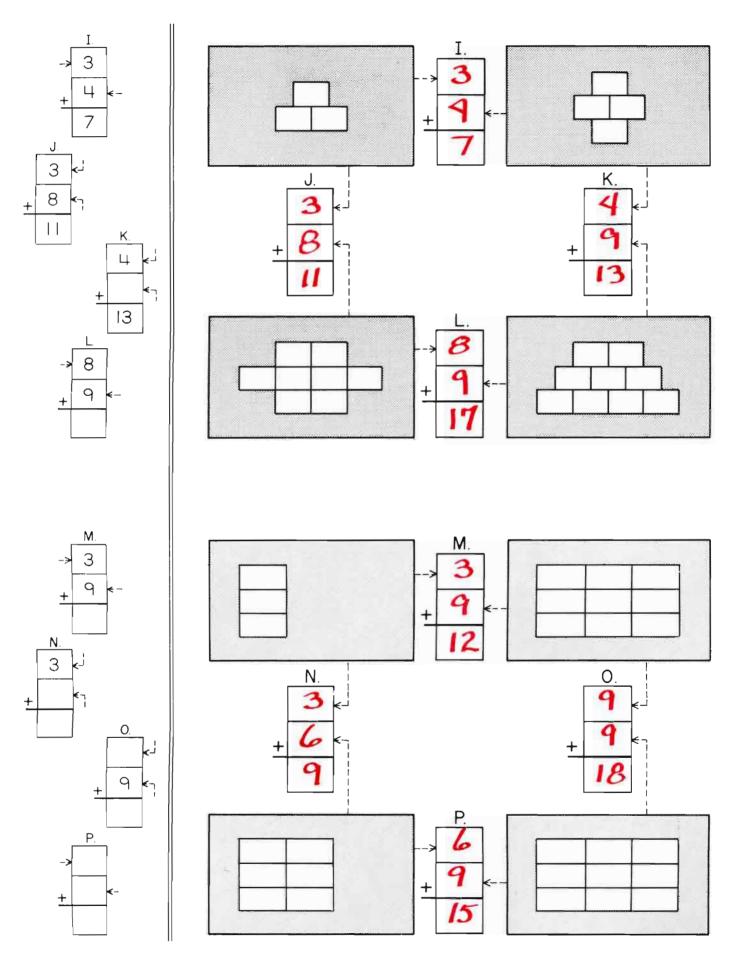


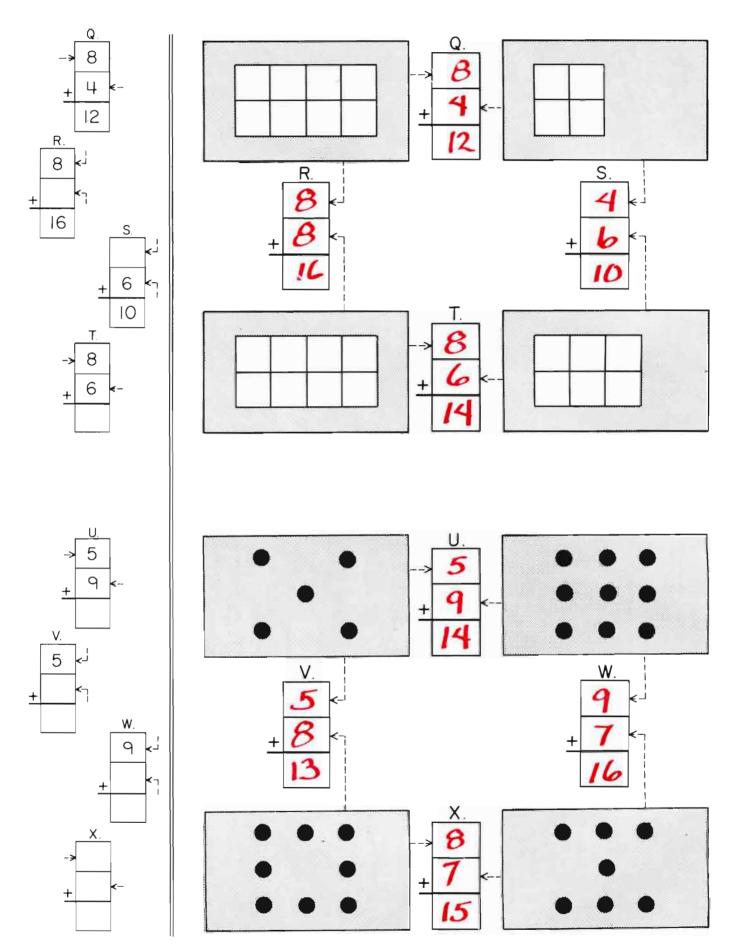


Some children may have difficulty at this first activity at the representational level. If so you will likely want to give them more experience at manipulative levels.

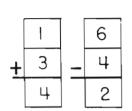


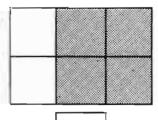


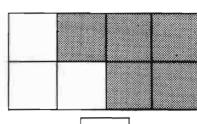


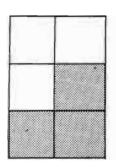


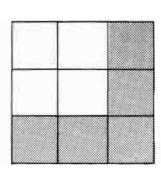
You might want to remind the children that the shading simply separates the boxes into two groups and does not necessarily indicate subtraction as in other examples.

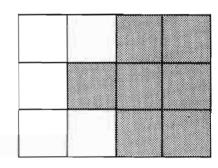


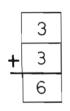




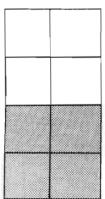


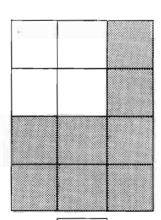


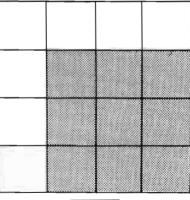


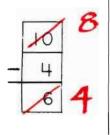


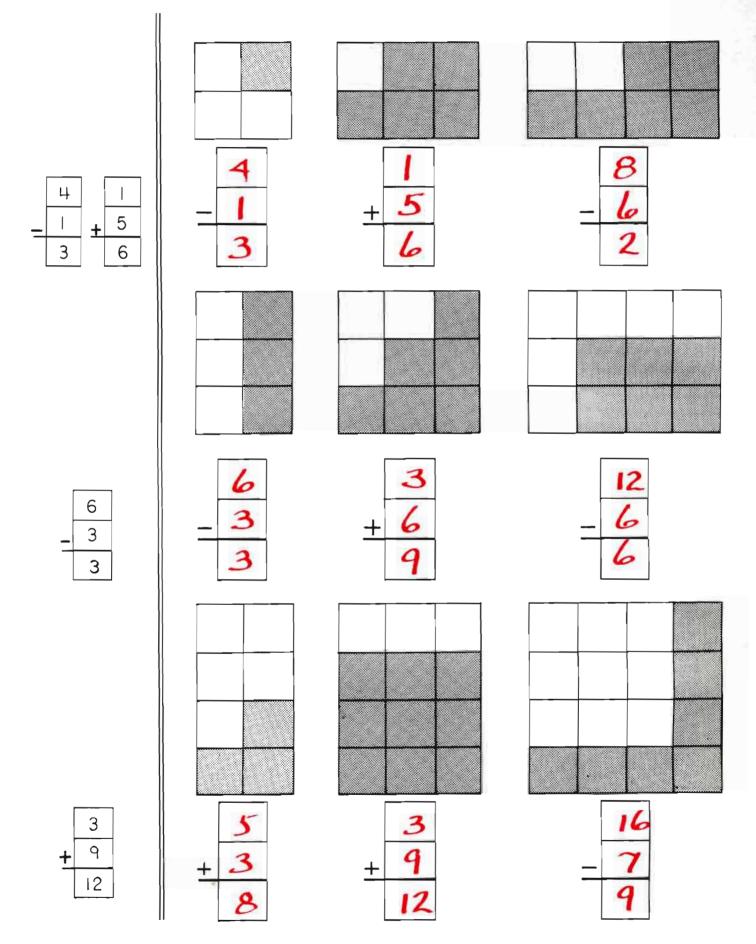
	5
+	7
	12



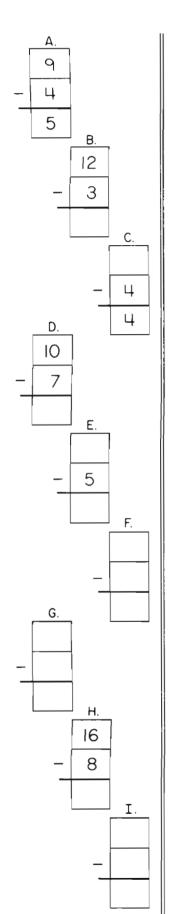


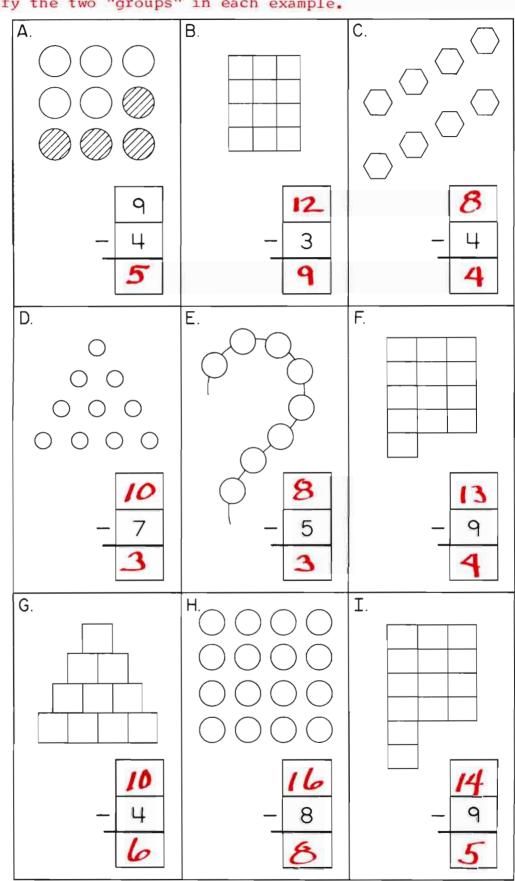


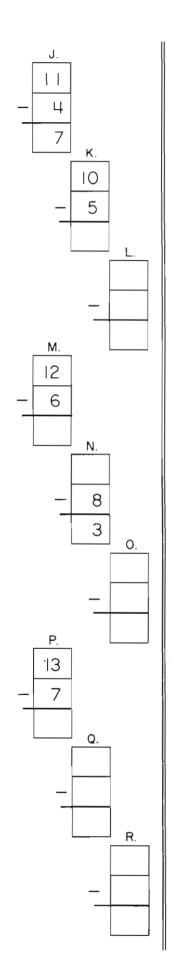


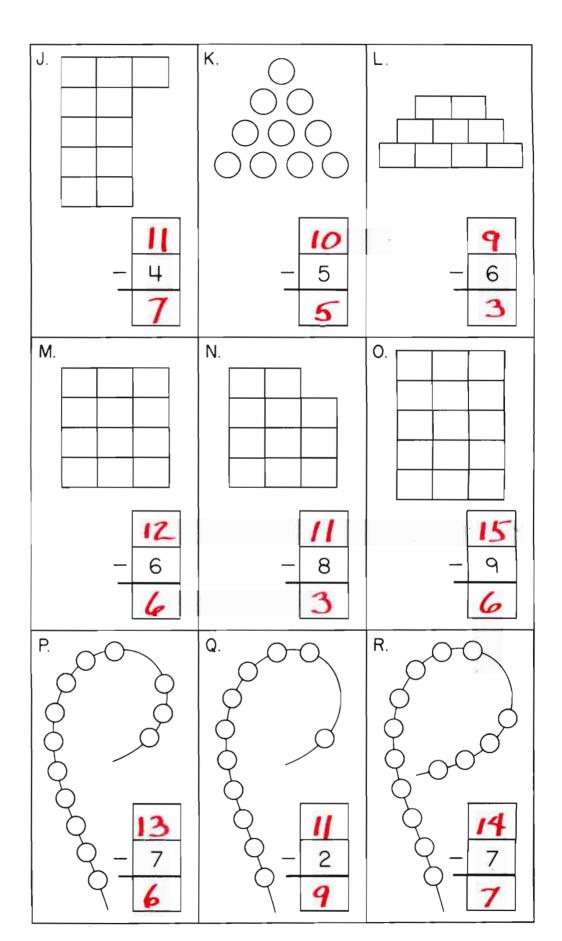


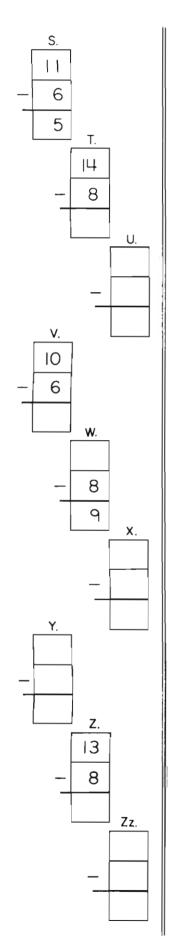
If the child needs to, he may still use shading to identify the two "groups" in each example.

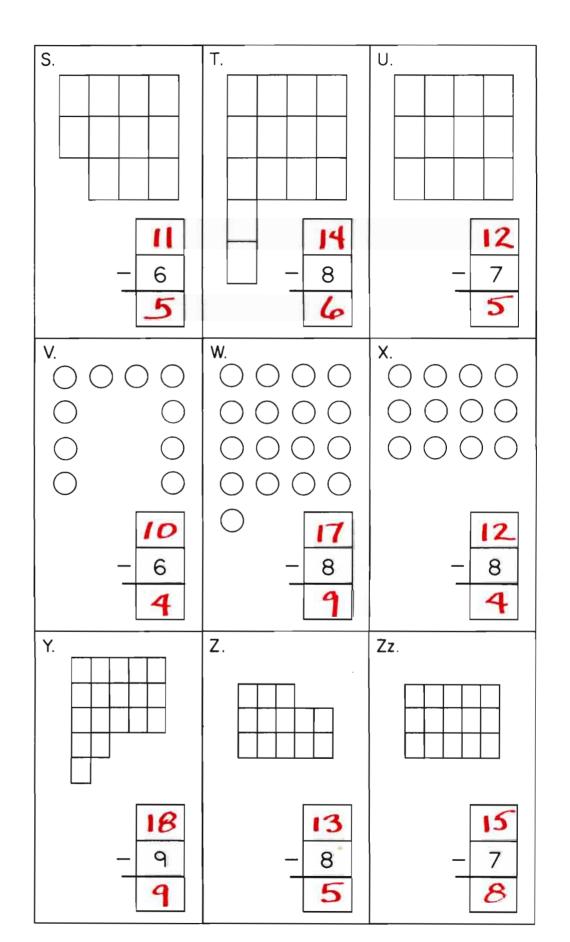


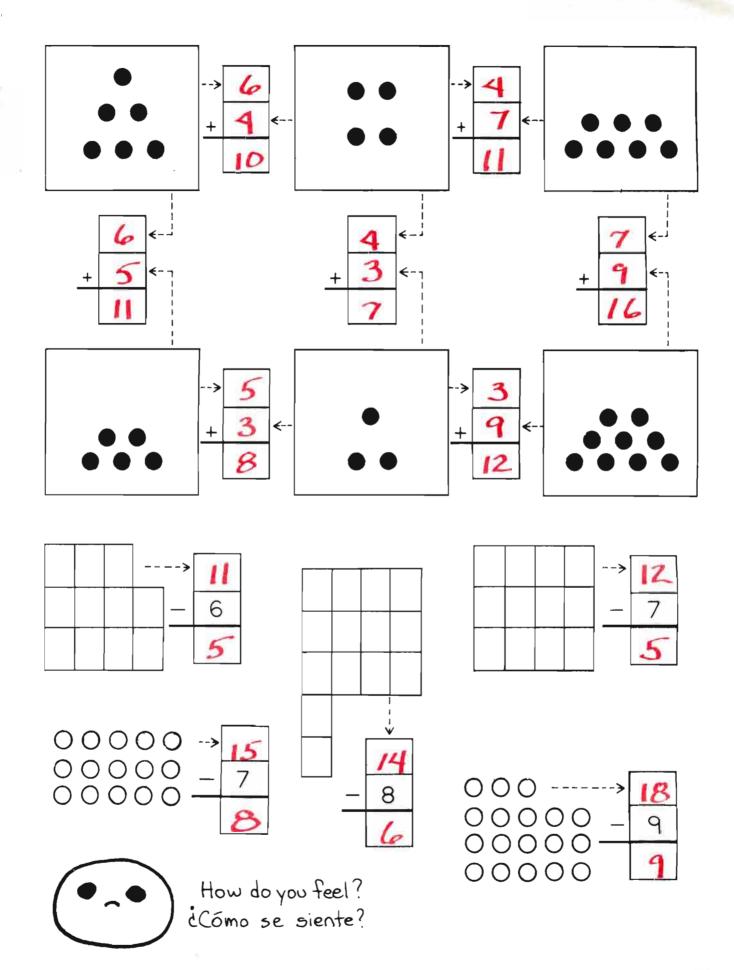












Dear Parent,

The exercise on the back of this page demonstrates your child's success in adding and subtracting pictured groups of objects and recording the results with accuracy. This is the final step in Book A-1.

Now, let us go over your child's learnings since we began: recording adding and subtraction two different ways, first by counting real things, then by counting pictures of things; "counting on" from ten, twenty, or thirty; and knowing how to write in the missing numbers in a series up to thirty. This is quite a bit of achievement for one so young! However, at this stage, what was learned is not as important as how your child feels about math. Is pride and joy still there? Are there any feelings of insecurity that you have noticed?

Again, we can't tell you how important your help is, and we hope that these letters help clear up questions you may have about the math program and keep you informed of your child's progress. You can see the high degree of success with which your child operates by the little "check-tests" on the back of the letters. We trust that both you and your child are feeling friendly with numbers.

Sincerely.

Since this is the end of A-Level Computation we have left space in case you wish to add your own note to a parent.



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