CD A math
ROBERT W. WIRTZ

patterns & problems

(Jumping around in mathematics)

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JUMPING AROUND IN MATHEMATICS

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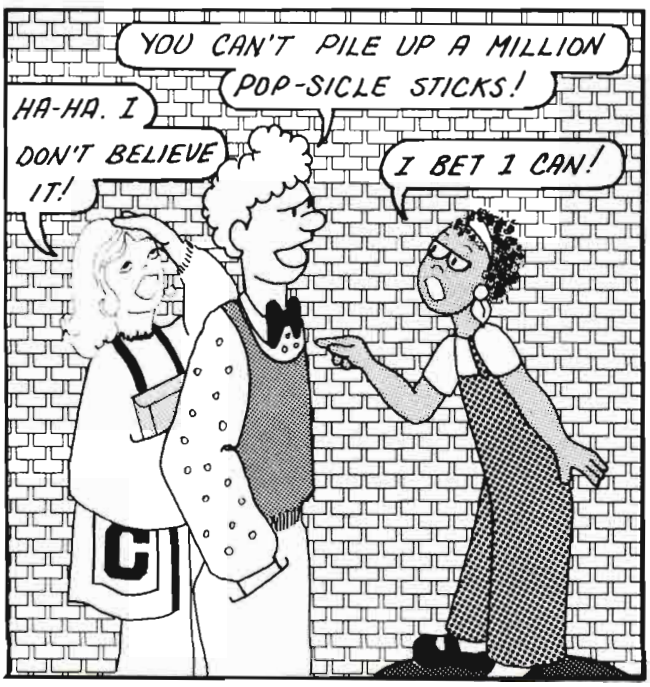
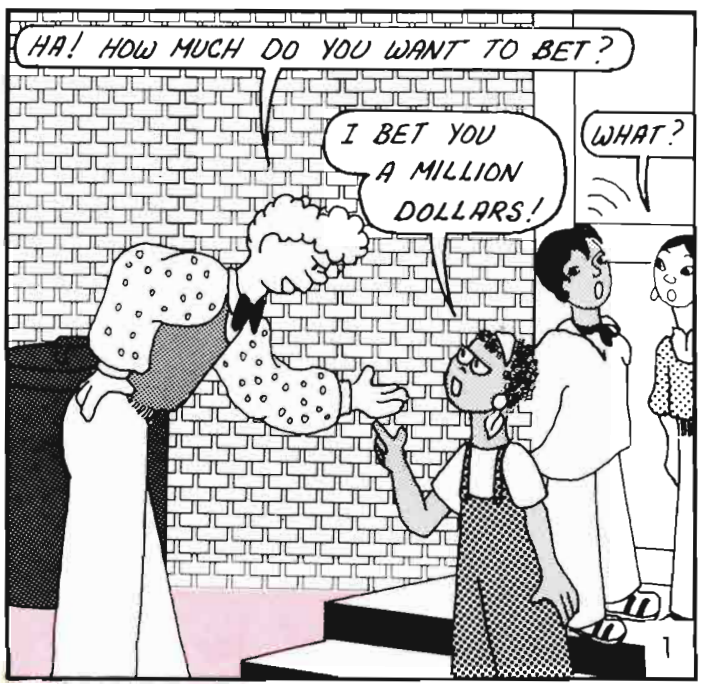
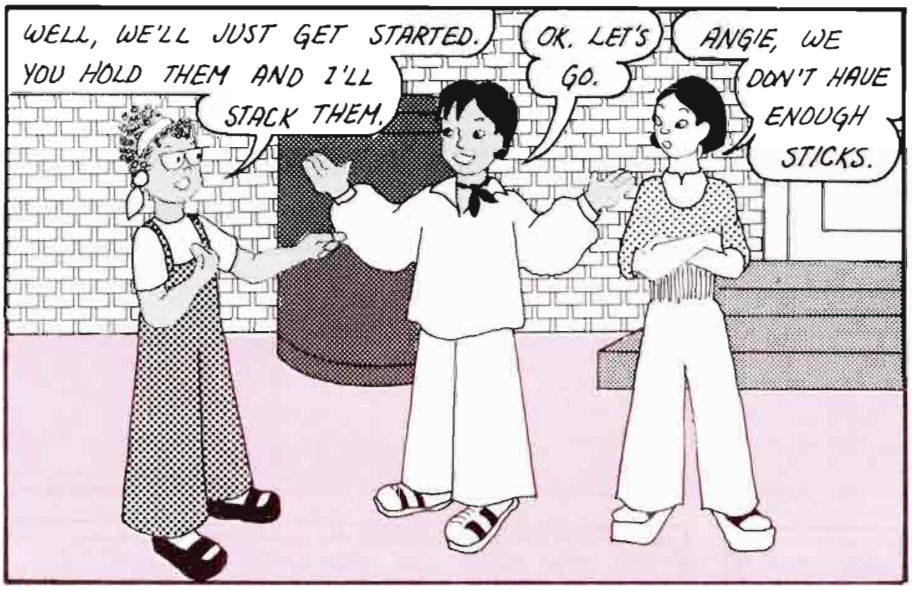
"A MILLION HIGH"
HEY! WHAT'S GOING ON HERE?
WHAT'S IT LOOK LIKE?
WE'RE STACKING A MILLION POP-SICLE STICKS!

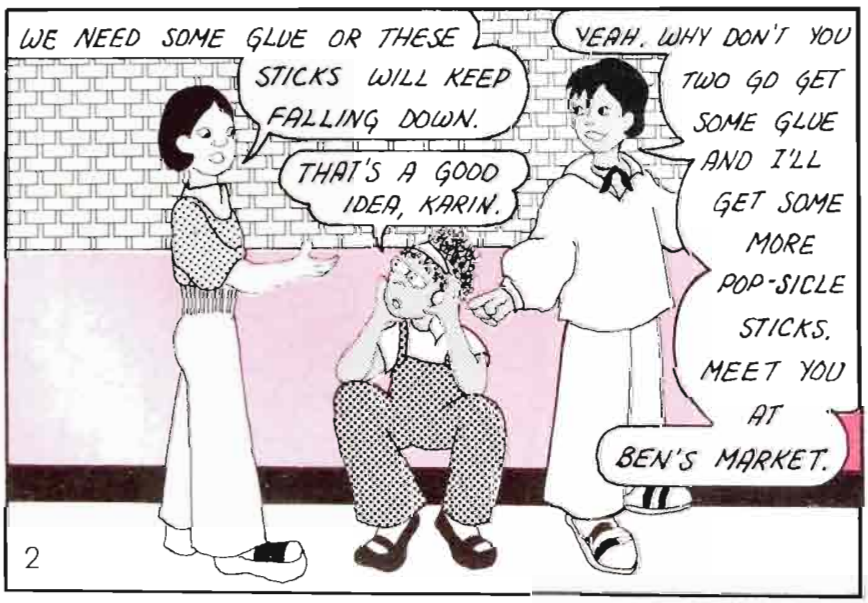

YOU CAN'T PILE UP A MILLION POP-SICLE STICKS!
HA-HA. I DON'T BELIEVE IT!
I BET I CAN!


HA! HOW MUCH DO YOU WANT TO BET?
I BET YOU A MILLION DOLLARS!
WHAT?


OH, FLIP, I KNOW YOU'LL WIN.
WELL, YOU'RE ON. SEE YOU GUYS NEXT YEAR.
ANGIE, WHAT HAVE YOU DONE?
I DON'T KNOW... BUT WE'D BETTER GET TO WORK- OR WE'LL BE HERE ALL NIGHT.
WHAT DO YOU MEAN BY WE? WE DON'T EVEN HAVE A MILLION POP-SICLE STICKS TO STACK!


WELL, WE'LL JUST GET STARTED. YOU HOLD THEM AND I'LL STACK THEM.
OK. LET'S GO.
ANGIE, WE DON'T HAVE ENOUGH STICKS.


15 MINUTES LATER...
KARIN!
OH-OH, THEY'RE FALLING!
THIS IS SILLY.

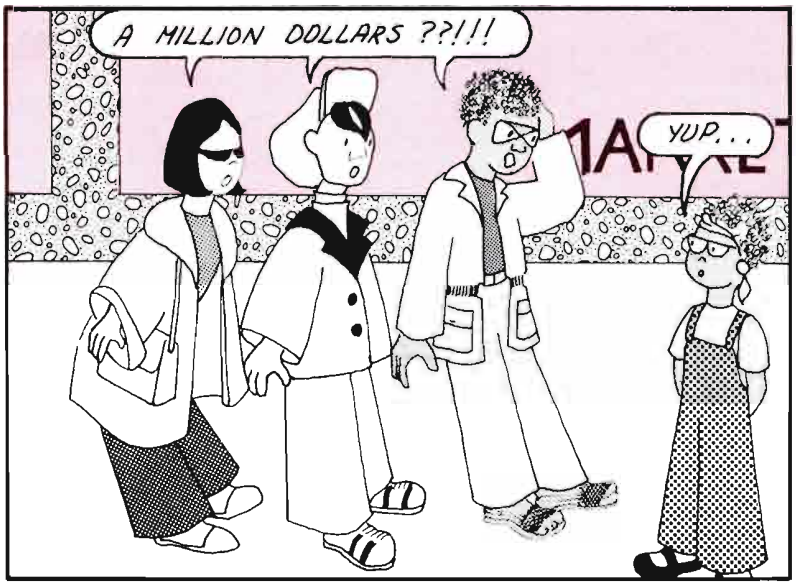

WE NEED SOME GLUE OR THESE STICKS WILL KEEP FALLING DOWN.
YEAH. WHY DON'T YOU TWO GO GET SOME GLUE AND I'LL GET SOME MORE POP-SICLE STICKS. MEET YOU AT BEN'S MARKET.
THAT'S A GOOD IDEA, KARIN.


HOW HIGH IS A STACK OF A MILLION POP-SICLE STICKS, FLIP?
WHO KNOWS. BUT I'LL MAKE SURE THEY WON'T FIND OUT.

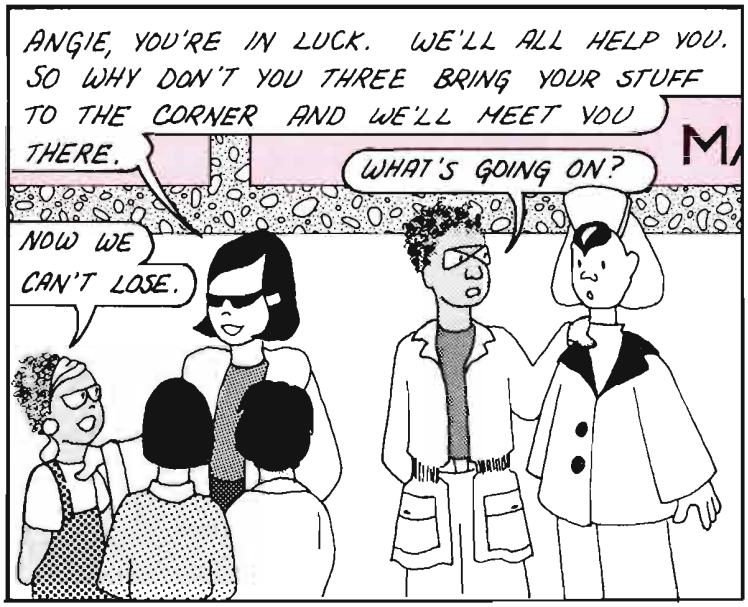

ANGIE, KARIN AND TONY MEET BACK AT BEN'S MARKET...
WE GOT THE GLUE, TONY. FIND ANY MORE STICKS?
LITTER
YEAH. I FOUND SOME MORE... BUT I DON'T THINK THAT WE HAVE ENOUGH.
YUK! LOOK AT THIS STICK. OH, HI, ROSS.
WHAT ARE YOU GUYS DOING IN THE LITTER?
WE'RE LOOKING FOR POP-SICLE STICKS.
WELL, AT LEAST IT KEEPS THEM OFF THE STREETS.
LITTER
HEY, LOOK, THERE'S HAROLD. AND LOOK WHAT HE HAS!
A POP-SICLE STICK! COME ON. LET'S GET IT!
HELP!
CRASH!

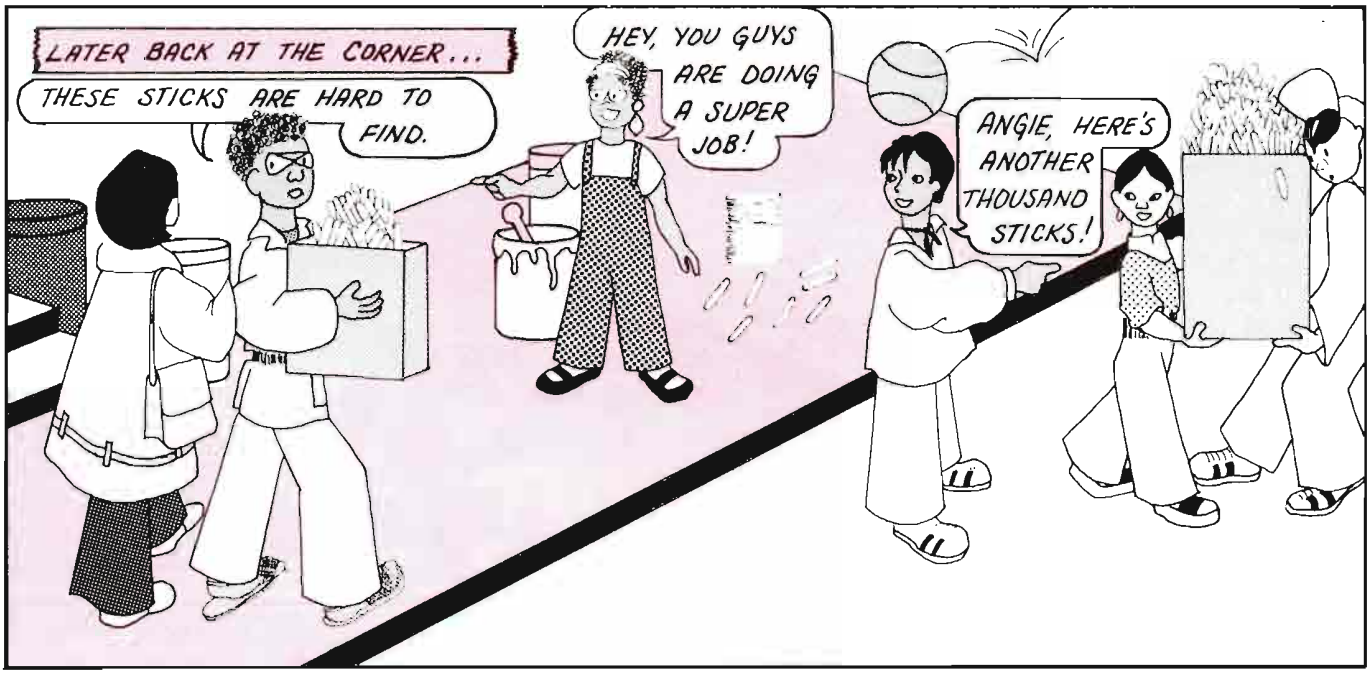

WHAT DID YOU DO TO HAROLD'S POP-SICLE?
NOTHING. WE JUST NEED THE STICK. HERE HAROLD- HERE'S YOUR POP-SICLE.
OH, THANKS.
BUT WHY DO YOU JUST NEED THE STICK?
I BET THIS GUY THAT I COULD STACK A MILLION!

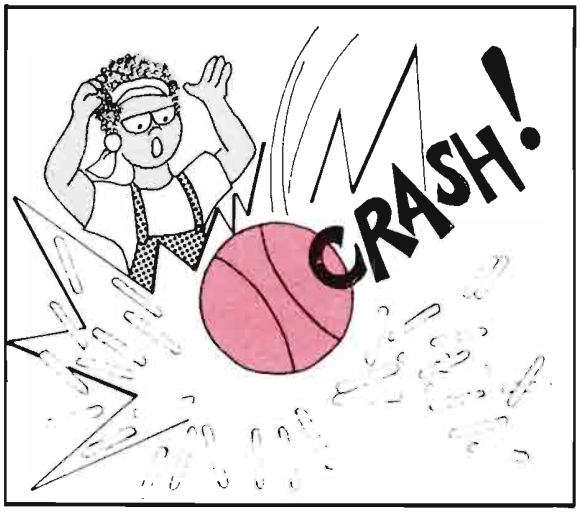

WHAT DID YOU BET HIM, ANGIE?
OH... WELL... A- A MILLION DOLLARS.

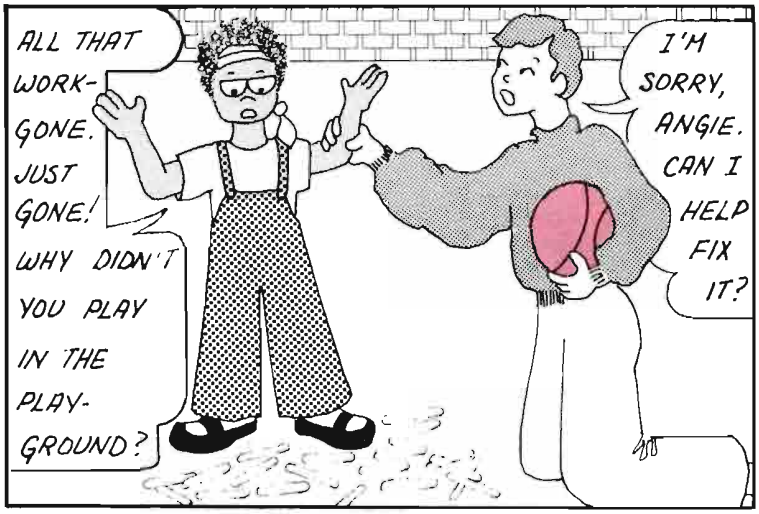

A MILLION DOLLARS ??!!!
YUP...


LET ME SEE. WHY, THAT'S A DOLLAR A POP-SICLE STICK!
IT IS?

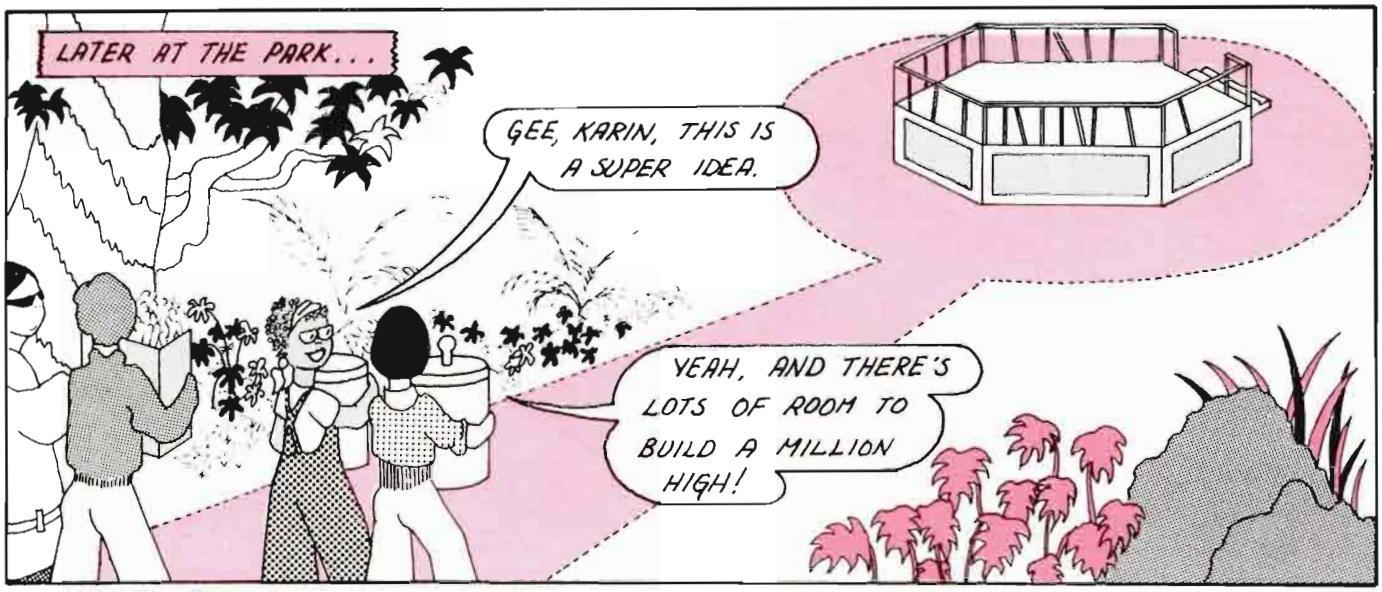

ANGIE, YOU'RE IN LUCK. WE'LL ALL HELP YOU. SO WHY DON'T YOU THREE BRING YOUR STUFF TO THE CORNER AND WE'LL MEET YOU THERE.
WHAT'S GOING ON?
NOW WE CAN'T LOSE.


LATER BACK AT THE CORNER...
THESE STICKS ARE HARD TO FIND.
HEY, YOU GUYS ARE DOING A SUPER JOB!
ANGIE, HERE'S ANOTHER THOUSAND STICKS!


CRASH!

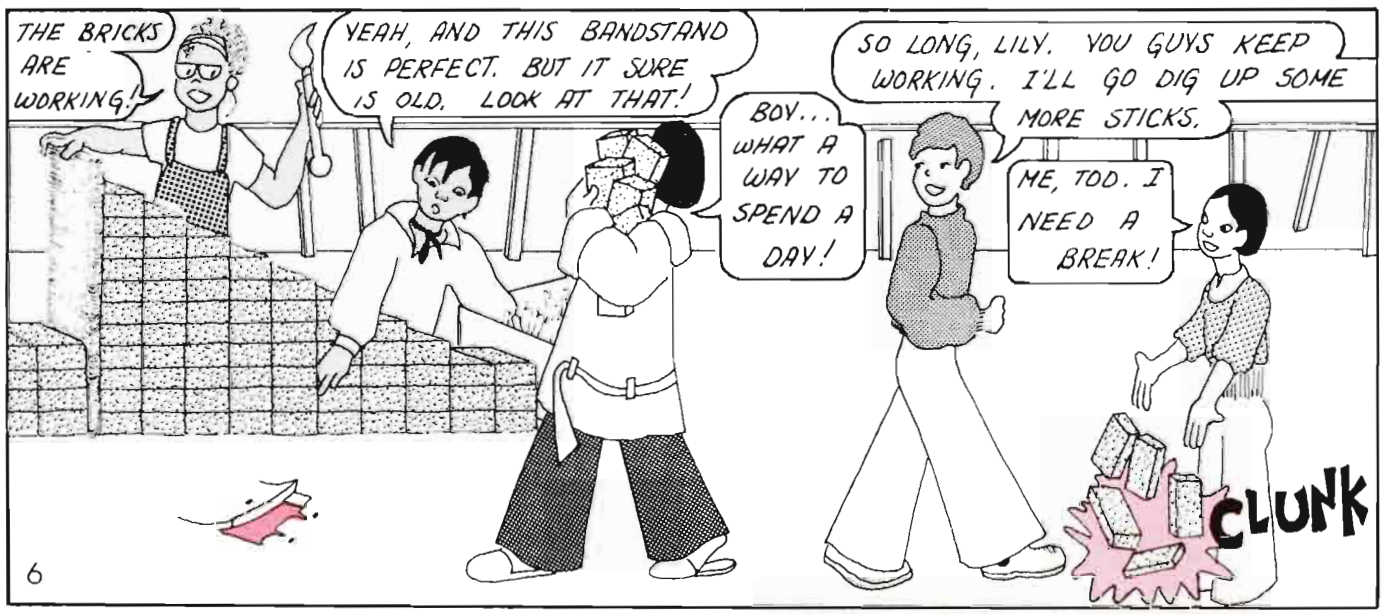

ALL THAT WORK- GONE. JUST GONE! WHY DIDN'T YOU PLAY IN THE PLAY- GROUND?
I'M SORRY, ANGIE. CAN I HELP FIX IT?

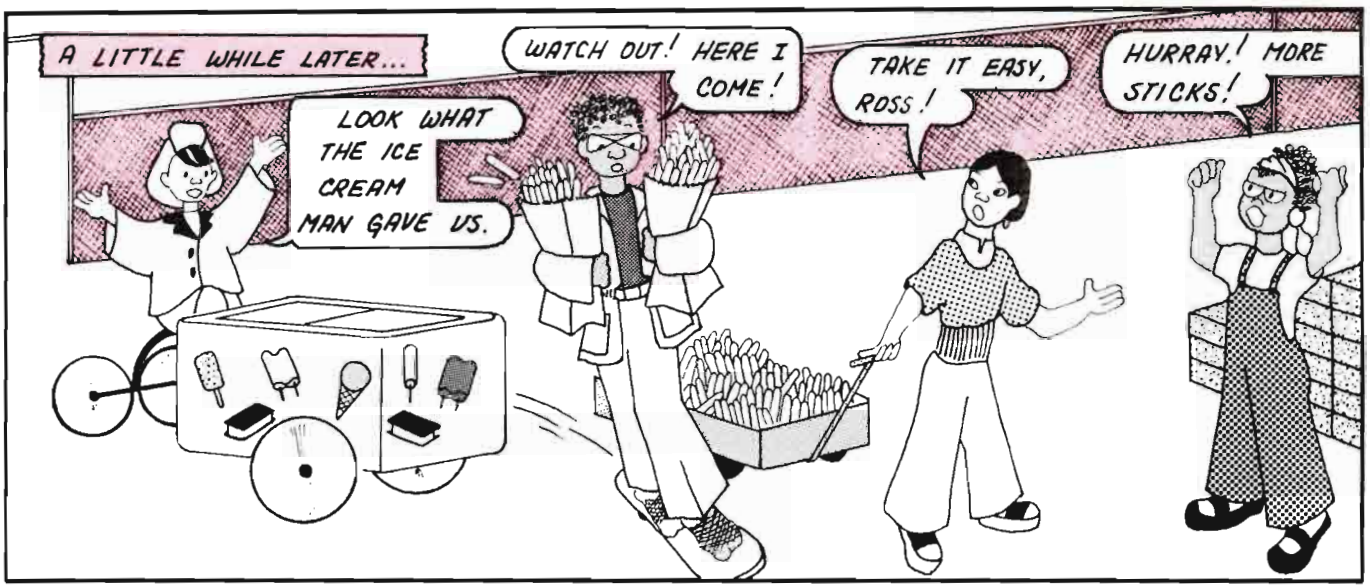

I GIVE UP! I QUIT! I'M THROWING IN MY STICK.!!!
CUT IT OUT, TONY. WILLY SAID HE'D HELP US.
LILY, WHY ARE YOU DOING THIS HERE?
GOT A BETTER IDEA?
YEAH, NOBODY USES THE BANDSTAND ANYMORE.


LATER AT THE PARK...
GEE, KARIN, THIS IS A SUPER IDEA.
YEAH, AND THERE'S LOTS OF ROOM TO BUILD A MILLION HIGH!


LOOK! TOO HIGH!
LOOKS GOOD, ANGIE.
BUT IT LOOKS BAD HERE. IT'S FALLING.
IT IS FALLING!
MAYBE WE SHOULD GET SOME BRICKS TO HOLD UP THE STACK.


I DON'T KNOW IF THIS IS SUCH A GOOD IDEA?

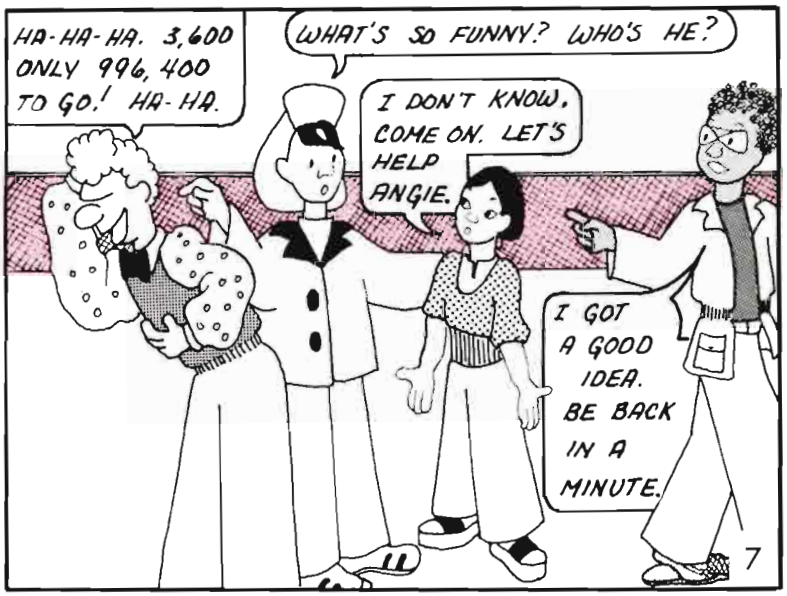

THE BRICKS ARE WORKING!
YEAH, AND THIS BANDSTAND IS PERFECT. BUT IT SURE IS OLD. LOOK AT THAT!
BOY... WHAT A WAY TO SPEND A DAY!
SO LONG, LILY. YOU GUYS KEEP WORKING. I'LL GO DIG UP SOME MORE STICKS.
ME, TOO. I NEED A BREAK!
CLUNK

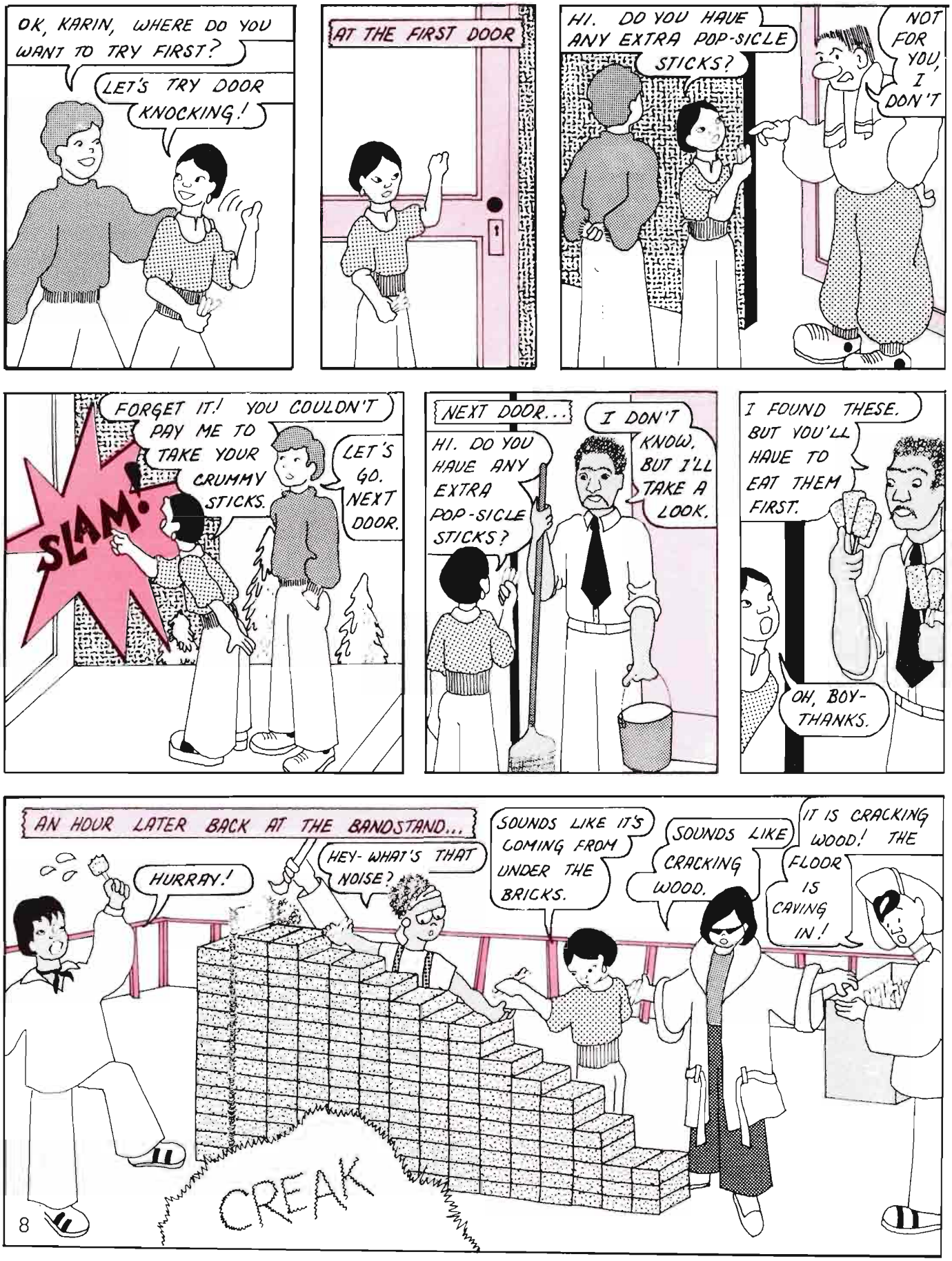

A LITTLE WHILE LATER...
LOOK WHAT THE ICE CREAM MAN GAVE US.
WATCH OUT! HERE I COME!
TAKE IT EASY, ROSS!
HURRAY! MORE STICKS!

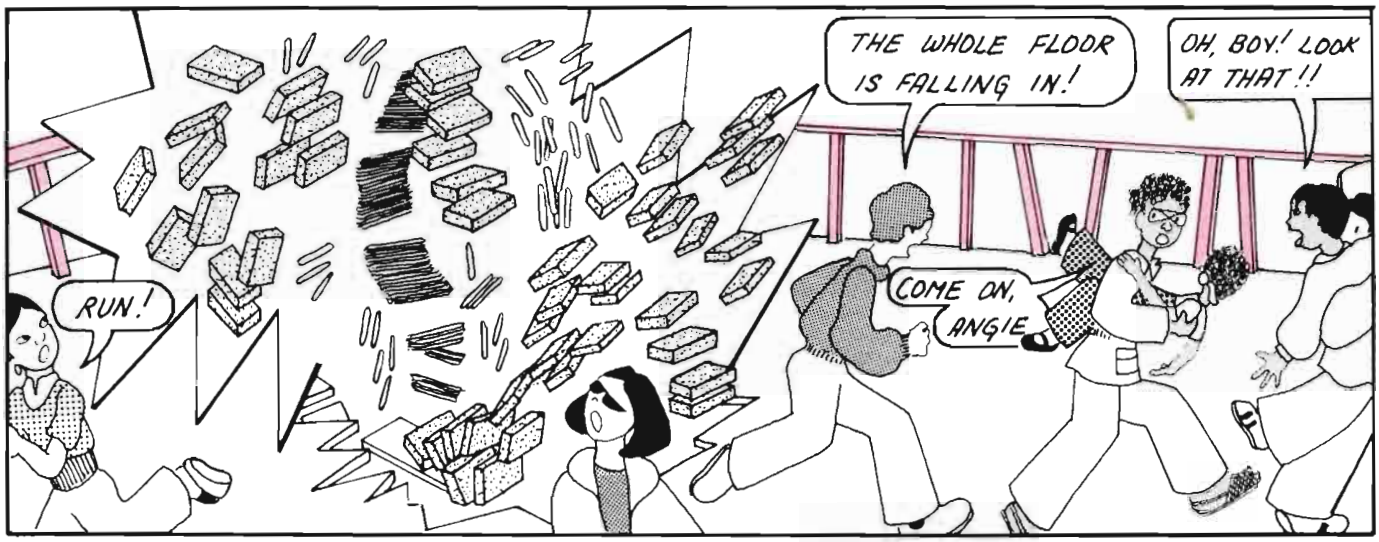

NOW WE CAN REALLY GO TO WORK!
ANGIE - WHAT IS IT?


HI, ROSS. YOU GUYS GOT BACK JUST IN TIME. WE RAN OUT OF STICKS.
ISN'T IT SUPER? WE STACKED AND STACKED - BUT THEN WE RAN OUT OF BRICKS!
WELL... HOW HIGH ARE YOU?


WELL, 3,600 NOT COUNTING THESE TWO.
SAY, ROSS, HOW ABOUT SENDING UP SOME MORE STICKS. IT'S TRICKY GOING UP AND DOWN.

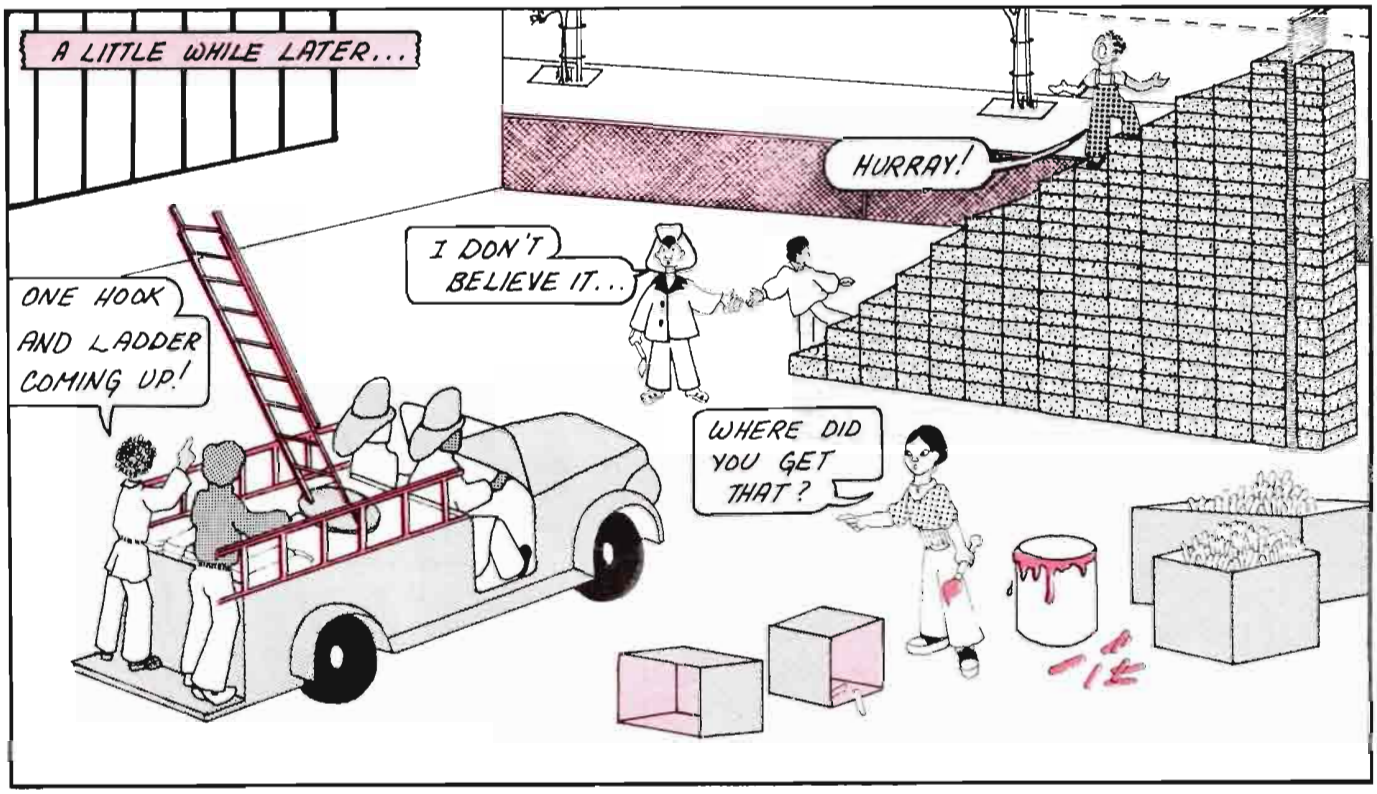

HA-HA-HA. 3,600 ONLY 996,400 TO GO! HA-HA.
WHAT'S SO FUNNY? WHO'S HE?
I DON'T KNOW. COME ON. LET'S HELP ANGIE.
I GOT A GOOD IDEA. BE BACK IN A MINUTE.


OK, KARIN, WHERE DO YOU WANT TO TRY FIRST?
LET'S TRY DOOR KNOCKING!
AT THE FIRST DOOR
HI. DO YOU HAVE ANY EXTRA POP-SICLE STICKS?
NOT FOR YOU, I DON'T
SLAM!
FORGET IT! YOU COULDN'T PAY ME TO TAKE YOUR CRUMMY STICKS.
LET'S GO. NEXT DOOR.
NEXT DOOR...
HI. DO YOU HAVE ANY EXTRA POP-SICLE STICKS?
I DON'T KNOW, BUT I'LL TAKE A LOOK.
I FOUND THESE. BUT YOU'LL HAVE TO EAT THEM FIRST.
OH, BOY- THANKS.
AN HOUR LATER BACK AT THE BANDSTAND...
HURRAY!
HEY- WHAT'S THAT NOISE?
SOUNDS LIKE IT'S COMING FROM UNDER THE BRICKS.
SOUNDS LIKE CRACKING WOOD.
IT IS CRACKING WOOD! THE FLOOR IS CAVING IN!
CREAK

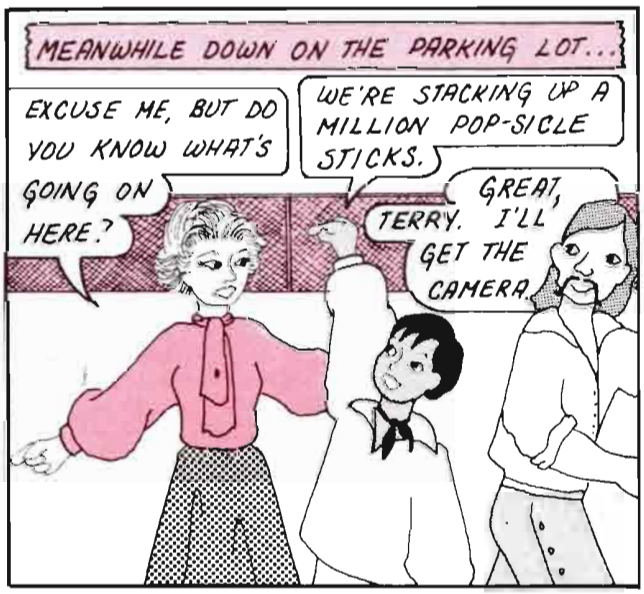

THE WHOLE FLOOR IS FALLING IN.!
OH, BOY.! LOOK AT THAT!!
RUN.!
COME ON, ANGIE


WHY NO- WE'LL JUST FIND SOLID GROUND THIS TIME. LIKE THE 14TH STREET PARKING LOT.!
WE AREN'T GIVING UP ARE WE.?
I GUESS THOSE BRICKS WERE TOO HEAVY.
YOU?
I DON'T BELIEVE THIS.
WE ALMOST WENT DOWN WITH THOSE STICKS.
TOO BAD WE DIDN'T BUILD ON SOLID GROUND.

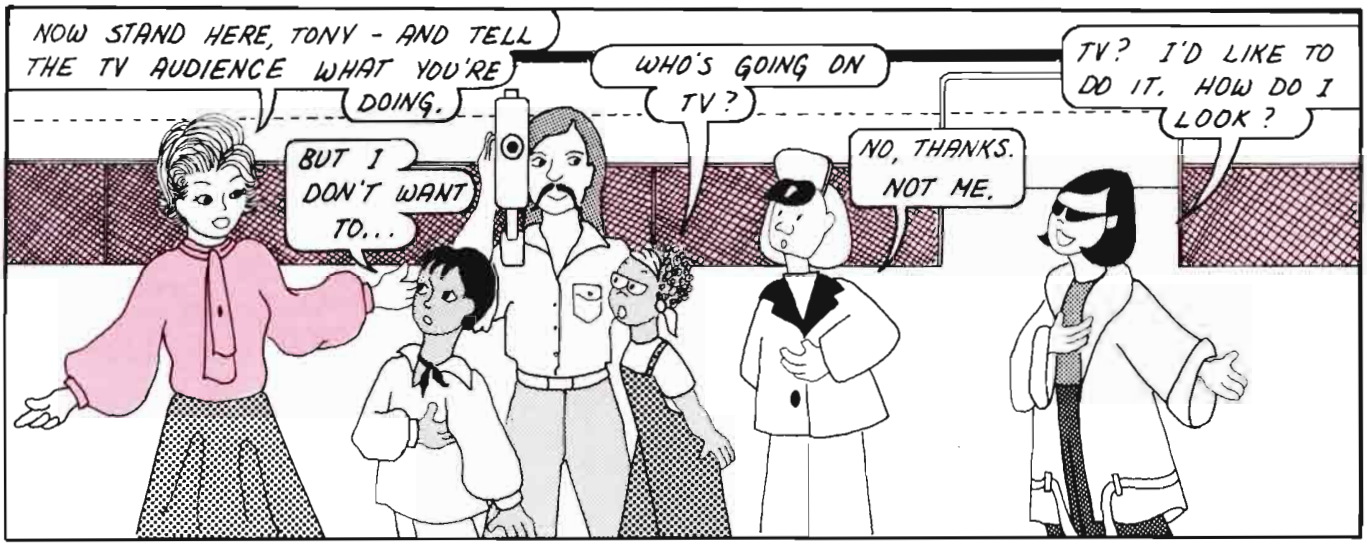

OH, HERE WE GO AGAIN...
BUT THIS TIME WE KNOW WHAT WE'RE DOING.
HEY, WHEN WE GET TO THE PARKING LOT - WHY DON'T YOU GUYS GO LOOK FOR MORE STICKS.


A LITTLE WHILE LATER...
HURRAY!
I DON'T BELIEVE IT...
ONE HOOK AND LADDER COMING UP!
WHERE DID YOU GET THAT?

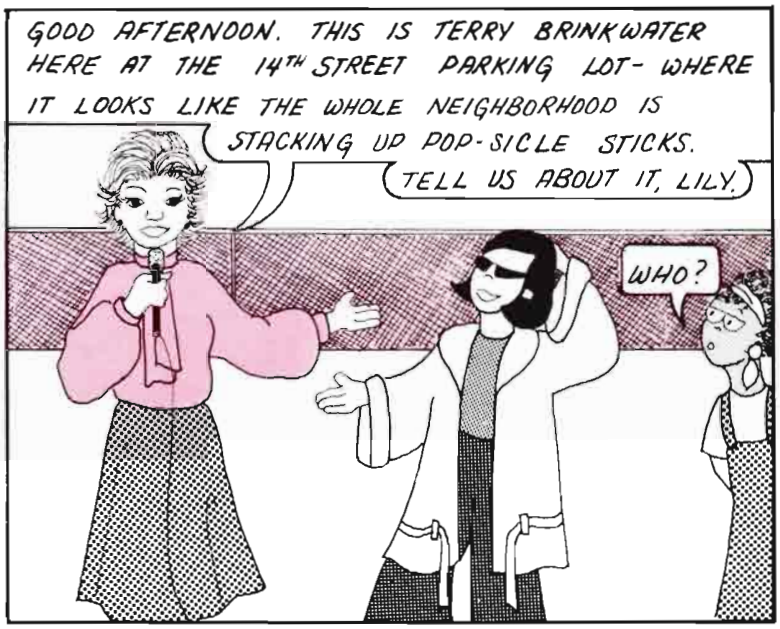

KEEP THEM COMING!
HOW HIGH IS THE STACK NOW, WILLY?
THIS MAKES AN EVEN 6,000.
6,004! STILL EVEN.


MEANWHILE DOWN ON THE PARKING LOT...
EXCUSE ME, BUT DO YOU KNOW WHAT'S GOING ON HERE?
WE'RE STACKING UP A MILLION POP-SICLE STICKS.
GREAT, TERRY. I'LL GET THE CAMERA.


NOW WE'RE GOING TO PUT YOU ON TV. SO JUST LISTEN TO TERRY.
WHAT? TV? ANGIE!


NOW STAND HERE, TONY - AND TELL THE TV AUDIENCE WHAT YOU'RE DOING.
BUT I DON'T WANT TO...
WHO'S GOING ON TV?
NO, THANKS. NOT ME.
TV? I'D LIKE TO DO IT. HOW DO I LOOK?

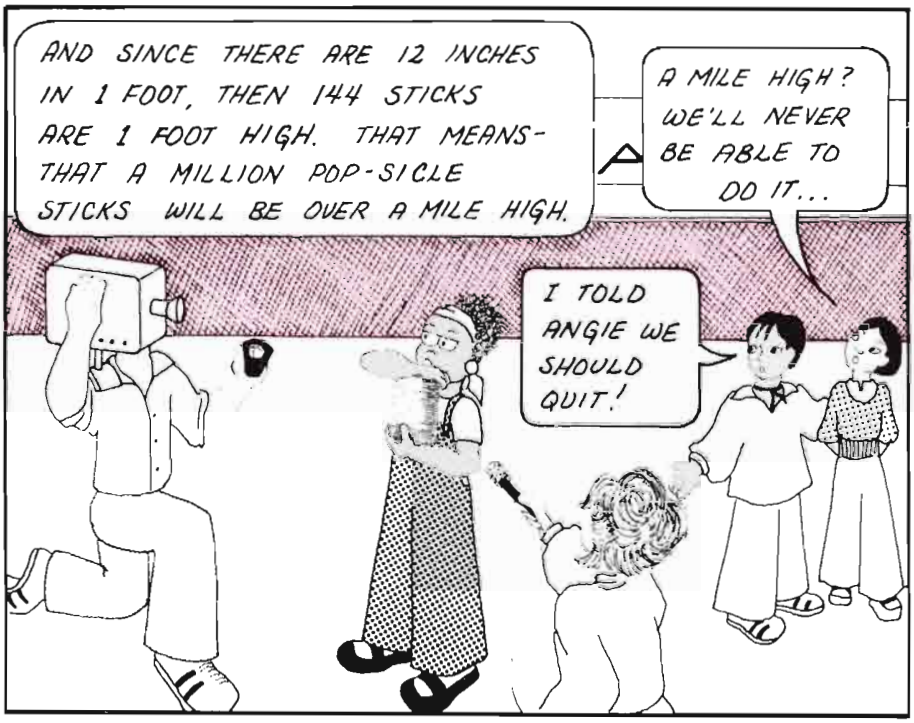

FINE. THEN YOU STAND HERE. READY?
RIGHT HERE, HUH? O.K. READY.

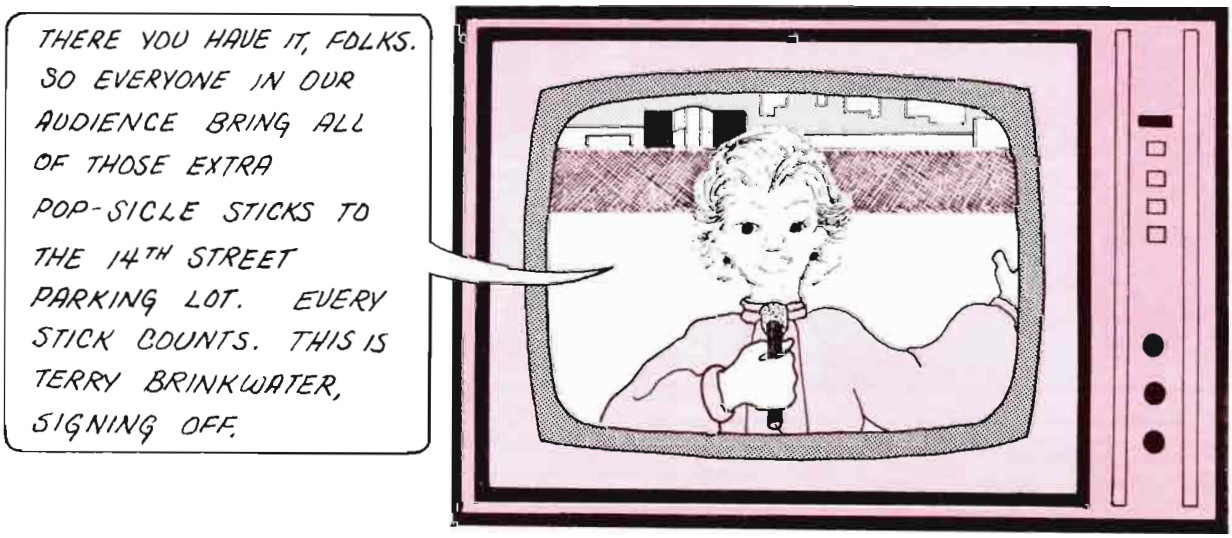

GOOD AFTERNOON. THIS IS TERRY BRINKWATER HERE AT THE 14TH STREET PARKING LOT- WHERE IT LOOKS LIKE THE WHOLE NEIGHBORHOOD IS STACKING UP POP-SICLE STICKS.
TELL US ABOUT IT, LILY.
WHO?

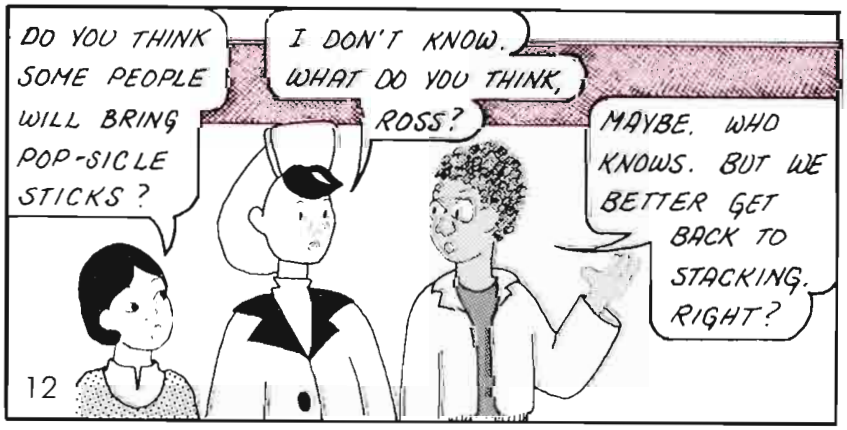

I BET THAT I COULD STACK UP A MILLION POP-SICLE STICKS AND NOBODY THOUGHT I COULD DO IT. BUT, AS YOU SEE, I CAN.
YES. JUST HOW HIGH WILL THIS STACK BE?
WHAT?!

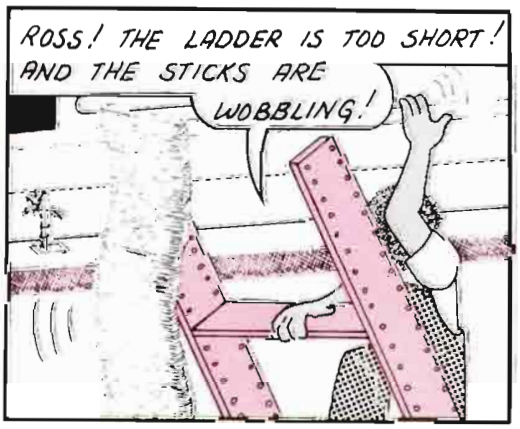

WELL... A... LET ME SEE. HOW HIGH IS IT GOING TO BE? HUH?
I KNOW HOW TO FIND OUT.

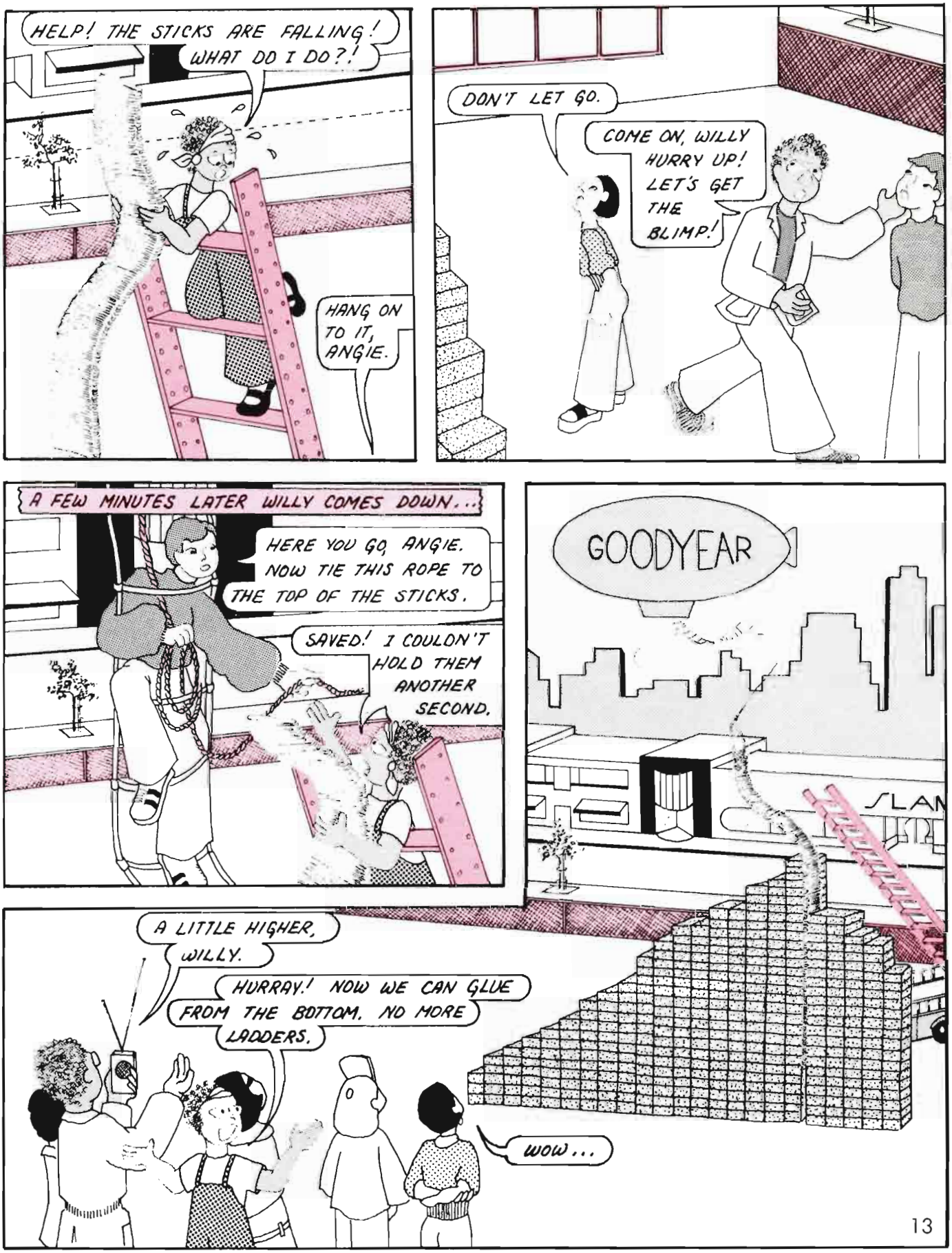

YOU SEE 1 POP-SICLE STICK IS 1/12 INCH THICK. SO 12 STICKS ARE 1 INCH THICK. LIKE THIS.


AND SINCE THERE ARE 12 INCHES IN 1 FOOT, THEN 144 STICKS ARE 1 FOOT HIGH. THAT MEANS- THAT A MILLION POP-SICLE STICKS WILL BE OVER A MILE HIGH.
A MILE HIGH? WE'LL NEVER BE ABLE TO DO IT...
I TOLD ANGIE WE SHOULD QUIT!


THERE YOU HAVE IT, FOLKS. SO EVERYONE IN OUR AUDIENCE BRING ALL OF THOSE EXTRA POP-SICLE STICKS TO THE 14TH STREET PARKING LOT. EVERY STICK COUNTS. THIS IS TERRY BRINKWATER, SIGNING OFF.

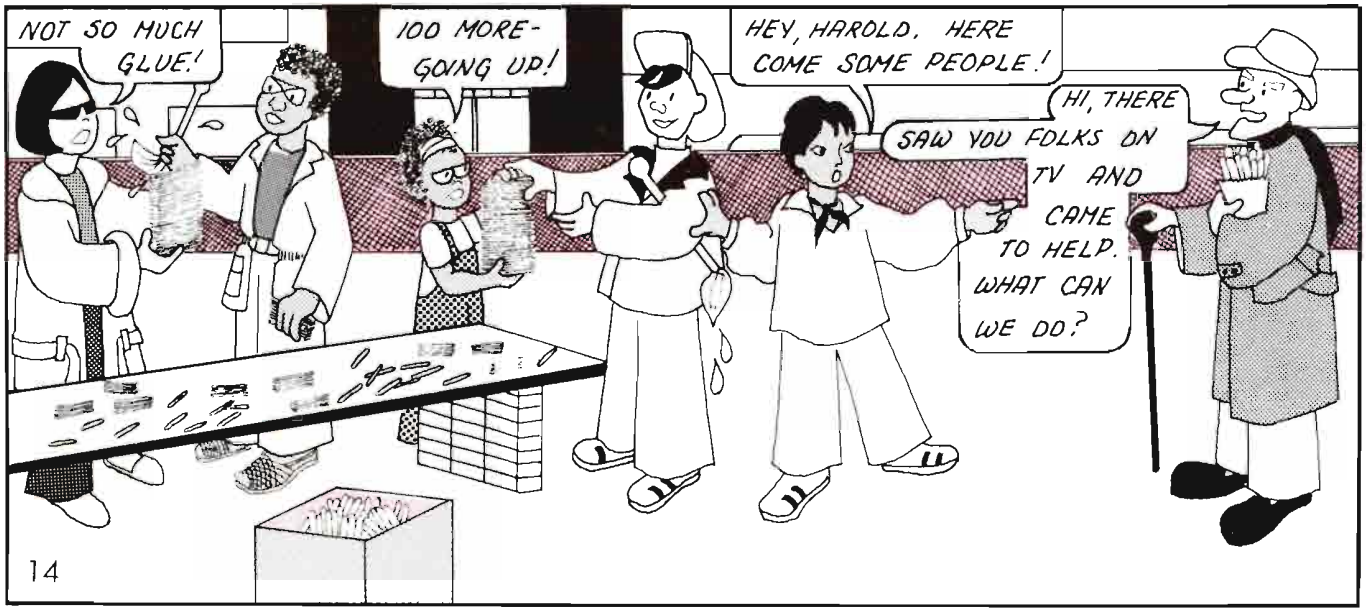

DO YOU THINK SOME PEOPLE WILL BRING POP-SICLE STICKS?
I DON'T KNOW. WHAT DO YOU THINK, ROSS?
MAYBE. WHO KNOWS. BUT WE BETTER GET BACK TO STACKING. RIGHT?

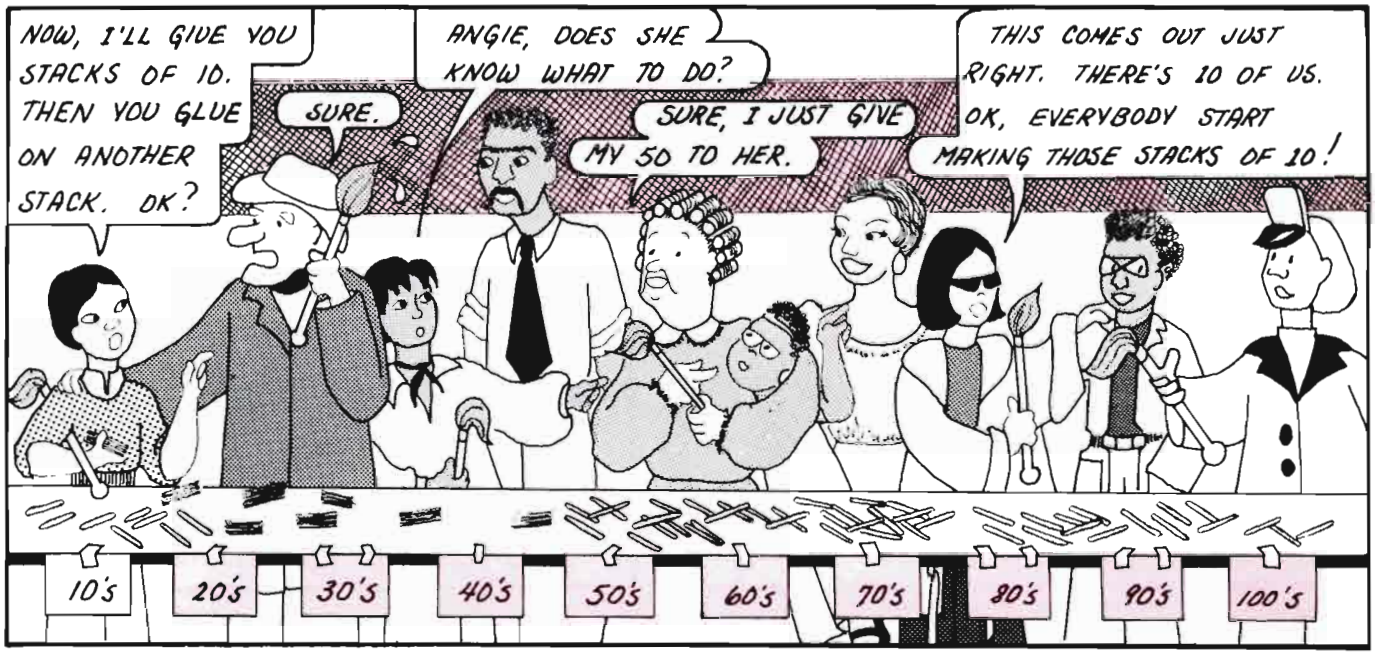

ROSS! THE LADDER IS TOO SHORT! AND THE STICKS ARE WOBBLING!


HELP! THE STICKS ARE FALLING!
WHAT DO I DO?!
HANG ON TO IT, ANGIE.
DON'T LET GO.
COME ON, WILLY HURRY UP! LET'S GET THE BLIMP!
A FEW MINUTES LATER WILLY COMES DOWN...
HERE YOU GO, ANGIE. NOW TIE THIS ROPE TO THE TOP OF THE STICKS.
SAVED! I COULDN'T HOLD THEM ANOTHER SECOND.
GOODYEAR
A LITTLE HIGHER, WILLY.
HURRAY! NOW WE CAN GLUE FROM THE BOTTOM. NO MORE LADDERS.
WOW...


DROP IT DOWN JUST A LITTLE MORE, WILLY.
HOW MUCH LONGER WILL THIS TAKE, ANGIE?
WELL, IF WE GLUE 30 STICKS A MINUTE - THAT'S 1800 STICKS AN HOUR. IF WE WORK DAY AND NIGHT - IT'LL TAKE US ABOUT - 3 WEEKS.
OH-NO
I GOT AN IDEA!
ANOTHER IDEA.


SEE - I GIVE YOU A STACK OF 10 STICKS GLUED TOGETHER. THEN YOU ADD YOUR STACK OF 10 TO MINE.
I GET IT. EACH OF US GLUE STACKS OF 10. I GLUE YOUR 10 TO MINE AND PASS THE 20 TO LILY.
THEN I ADD 10 AND PASS THE 30 TO ROSS...
WE ALL ADD 10 - SO WHEN HAROLD GETS THE LAST STACK HE CAN GLUE ON A STACK OF 100 AT A TIME.
BUT THERE'S ONLY 6 OF US. I GUESS I'LL DO THE EXTRA 4 STACKS.
WELL, LET'S GO TO WORK.


NOT SO MUCH GLUE!
100 MORE - GOING UP!
HEY, HAROLD. HERE COME SOME PEOPLE!
HI, THERE SAW YOU FOLKS ON TV AND CAME TO HELP. WHAT CAN WE DO?


NOW, I'LL GIVE YOU STACKS OF 10. THEN YOU GLUE ON ANOTHER STACK. OK?
SURE.
ANGIE, DOES SHE KNOW WHAT TO DO?
SURE, I JUST GIVE MY 50 TO HER.
THIS COMES OUT JUST RIGHT. THERE'S 10 OF US. OK, EVERYBODY START MAKING THOSE STACKS OF 10!
10's
20's
30's
40's
50's
60's
70's
80's
90's
100's


MEANWHILE, OUTSIDE THE LOT...
HEY, LOOK! LOOK HOW THEY'RE GETTING THOSE STICKS UP! TIE THESE STRINGS AROUND YOUR LEGS AND LET'S STEP IN.
OK, FLIP.


CAN I HELP?
YES, NOW YOU GLUE THESE 10 ON THE 50 I GIVE YOU.


FLIP, I DON'T HAVE ANY MORE ROOM.


HEY... HOW COME YOU'RE SLOWING UP, KARIN?

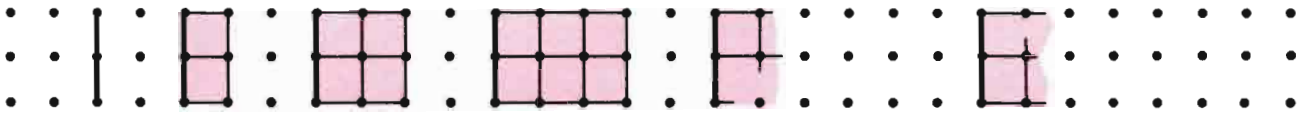

I'M NOT SLOW. I GIVE HIM MY 10's.
BUT THERE AREN'T ANY 60's.
WHO'S HE?
HE'S THE GUY I GIVE MY 50's TO.
SEE YOU LATER, FLIP.


NOW, WAIT A SECOND. IF HE GOT THE 50'S . . .
HAROLD, LOOK AT HIS PANTS!
YES, LOOK.
SO THAT'S WHERE THE 50's ARE!
COME ON!
WHAT ARE YOU DOING?
HELP!
HE TOOK ALL THE STICKS, ANGIE.
HEY, HE'S THE GUY WHO MADE THE BET WITH ME.
GEE, CAN'T YOU TAKE A JOKE?
HA!
JUST GET YOUR MILLION DOLLARS READY!
DAYS LATER . . .
999,900 - 1,000,000! WE DID IT! A MILLION.
BRAVO!
YIPPEE!
HOORAY!
WE WON! WELL . . .
WILL YOU TAKE A CHECK?
OH, NO!

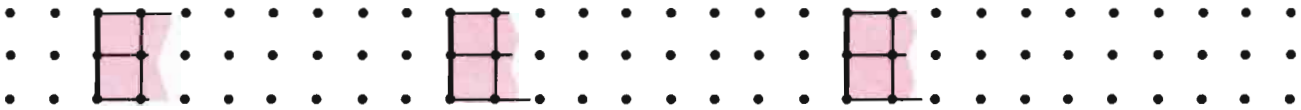
Plans by which Numbers Grow

TWO CLOSELY RELATED FAMILIES . . . odd and even

Please complete the sketches below.



0 2 ___ ___ ___ ___



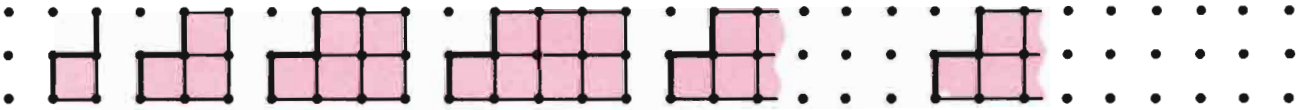
___ ___ ___

Please list the above numbers and extend the list.

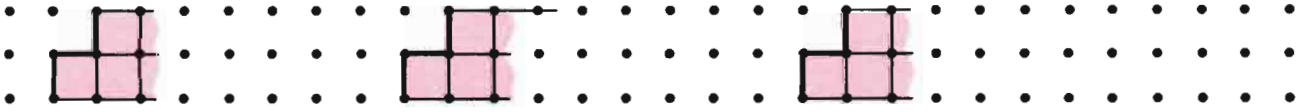
0, 2, 4, ___, ___, ___, ___, ___, ___, ___, ___, ___, ___, ___.... etc.

These numbers are called **EVEN numbers.**

Please complete the sketches below.



1 3 ___ ___ ___ ___



___ ___ ___

Please list the above numbers and extend the list.

1, 3, ___, ___, ___, ___, ___, ___, ___, ___, ___, ___, ___, ___.... etc.

These numbers are called **ODD numbers.**

Please color in all of the EVEN numbers in this chart. →

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
90	91	92	93	94	95	96	97	98	99
100	101	102	103	104	105	106	107	108	109
110	111	112	113	114	115	116	117	118	119
190	191	192	193	194	195	196	197	198	199
200	201	202	203	204	205	206	207	208	209
210	211	212	213	214	215	216	217	218	219
990	991	992	993	994	995	996	997	998	999
1000	1001	1002	1003	1004	1005	1006	1007	1008	1009

Do you believe that . . .

1. Every number in this chart is an EVEN or ODD number?

2. Every EVEN number has a 0, 2, 4, 6 or 8 in the units place?

3. Every ODD number has a 1, 3, 5, 7 or 9 in the units place?

Let's play "from the list" with EVEN and ODD numbers.

EVEN numbers

0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20...

4 + 14 = 18

___ − ___ = ___

___ x ___ = ___

___ ÷ ___ = ___

___ + ___ = ___

___ − ___ = ___

___ x ___ = ___

___ ÷ ___ = ___

___ + ___ = ___

___ − ___ = ___

___ x ___ = ___

___ ÷ ___ = ___

___ + ___ = ___

___ − ___ = ___

___ x ___ = ___

___ ÷ ___ = ___

___ + ___ = ___

___ − ___ = ___

ODD numbers

1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21...

___ + ___ = ___

___ − ___ = ___

} AHAH! You can't.

3 x 7 = ___

15 ÷ 5 = ___

} AHAH! You can.

___ x ___ = ___ ___ x ___ = ___

___ ÷ ___ = ___ ___ ÷ ___ = ___

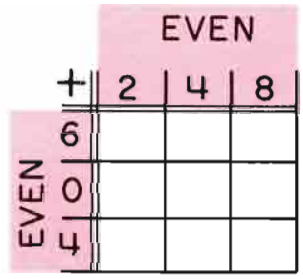
1 + 3 + 5 = ___

___ + ___ + ___ = ___

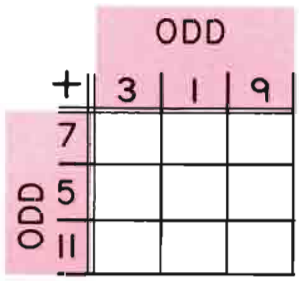
___ + ___ + ___ = ___

} AHAH! You can.

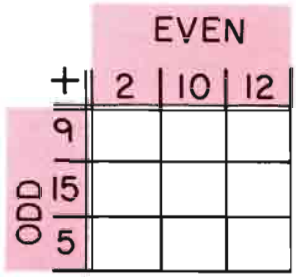
Adding EVEN numbers and ODD numbers.



All sums are _____

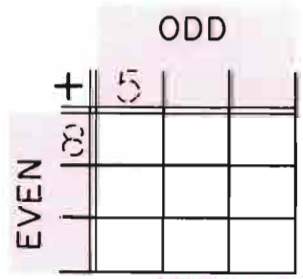


All sums are _____



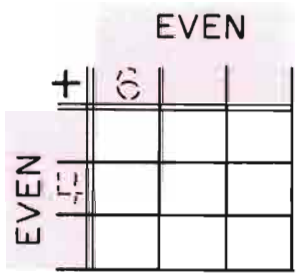
All sums are _____

Please make up your own examples.

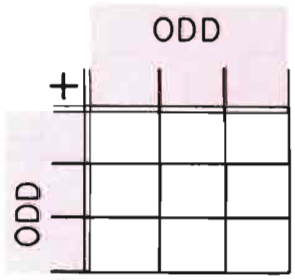


(EVEN or ODD)

All sums are _____

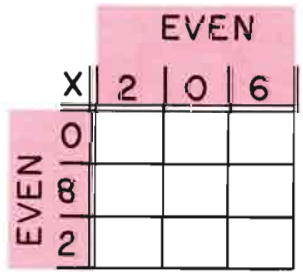


All sums are _____

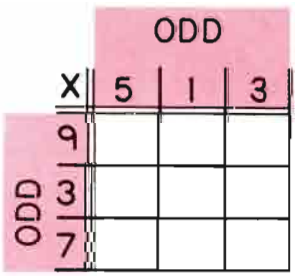


All sums are _____

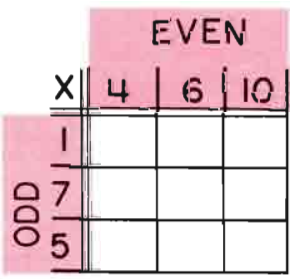
Multiplying EVEN numbers and ODD numbers.



All products are _____

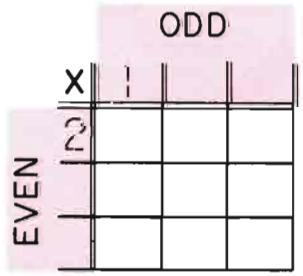


All products are _____

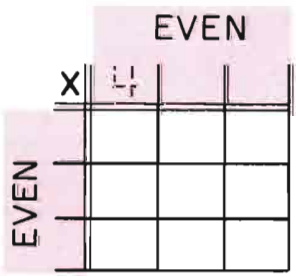


All products are _____

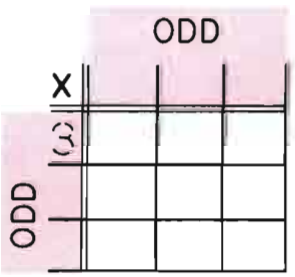
Please make up your own examples.



All products are _____



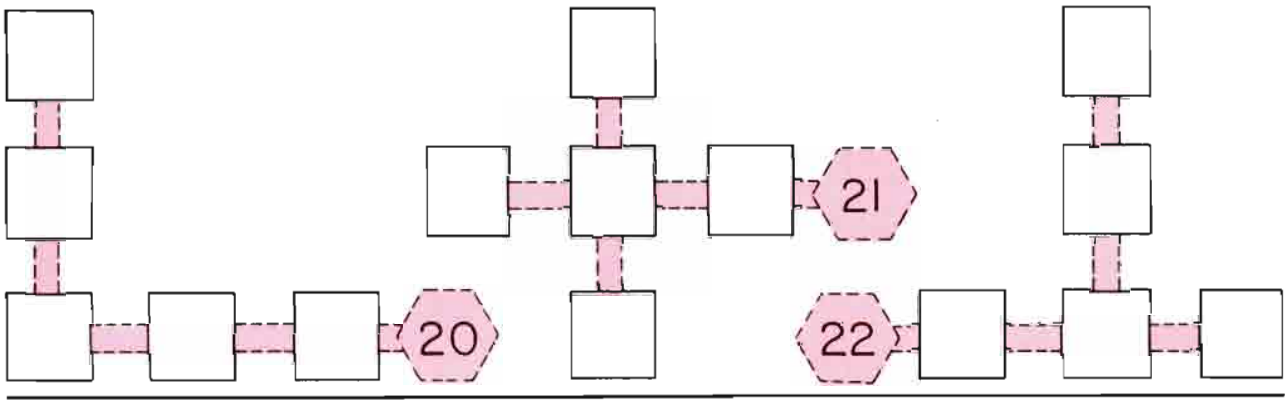
All products are _____

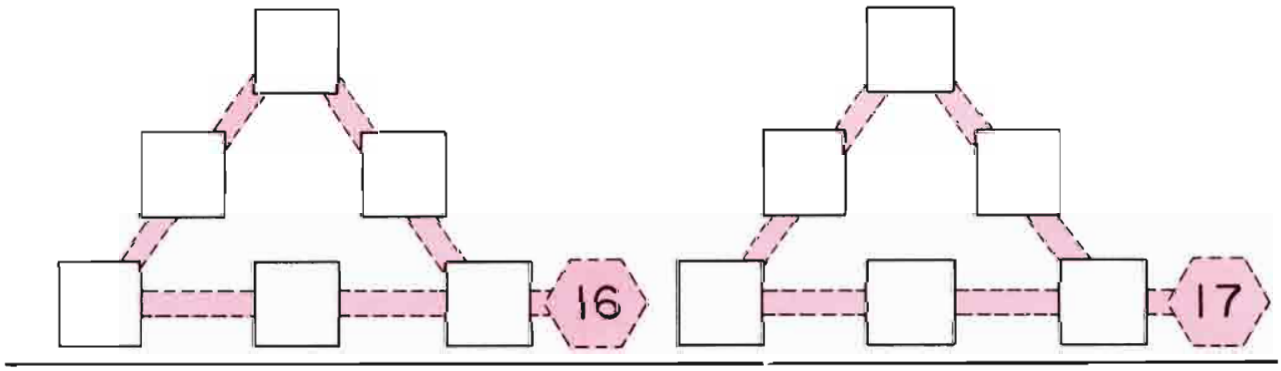


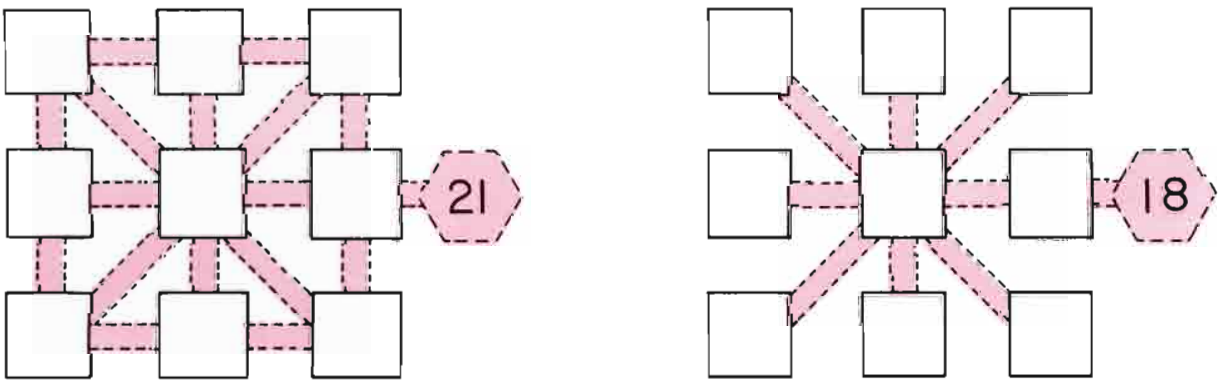
All products are _____

Arrangement PUZZLES with Small Numbers.

Please arrange the numbers on this page so each row of 3 numbers adds to the number shown in the hexagon.



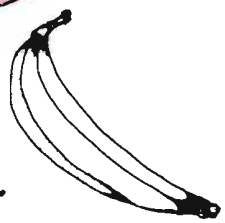




cut-outs may help find the solution

0	1	2	3	4	5	6	7	8	9	10

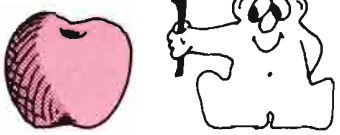
MEASURING WITH FRUIT



There are _____ sides on a banana.
How long is the banana? _____ cm
Do all bananas have the same number of sides?

There are _____ seeds in an apple.
There are _____ seeds in 13 apples.

How many twists to take off the apple's stem? ____



How many bites to eat half of an apple? _____

Please measure other things you eat.

What are your discoveries?

_____

_____

_____

Carla's "Quickies"

"I make up examples so I can do them almost without thinking. I take a little fact such as 8 + 4 = 12 and write a whole string of examples that don't require any more memory. Here are some strings I made up."

8 + 4 = 12	18 + 4	28 + 4	38 + 4	48 + 4	58 + 4	68 + 4	78 +14
12 - 4 = 8	22 - 4	32 - 4	42 - 4	42 -14	52 -14	62 -24	82 -34
6 + 7 = 13	16 + 7	26 + 7	26 +17	36 + 7	46 + 7	56 + 7	66 +17

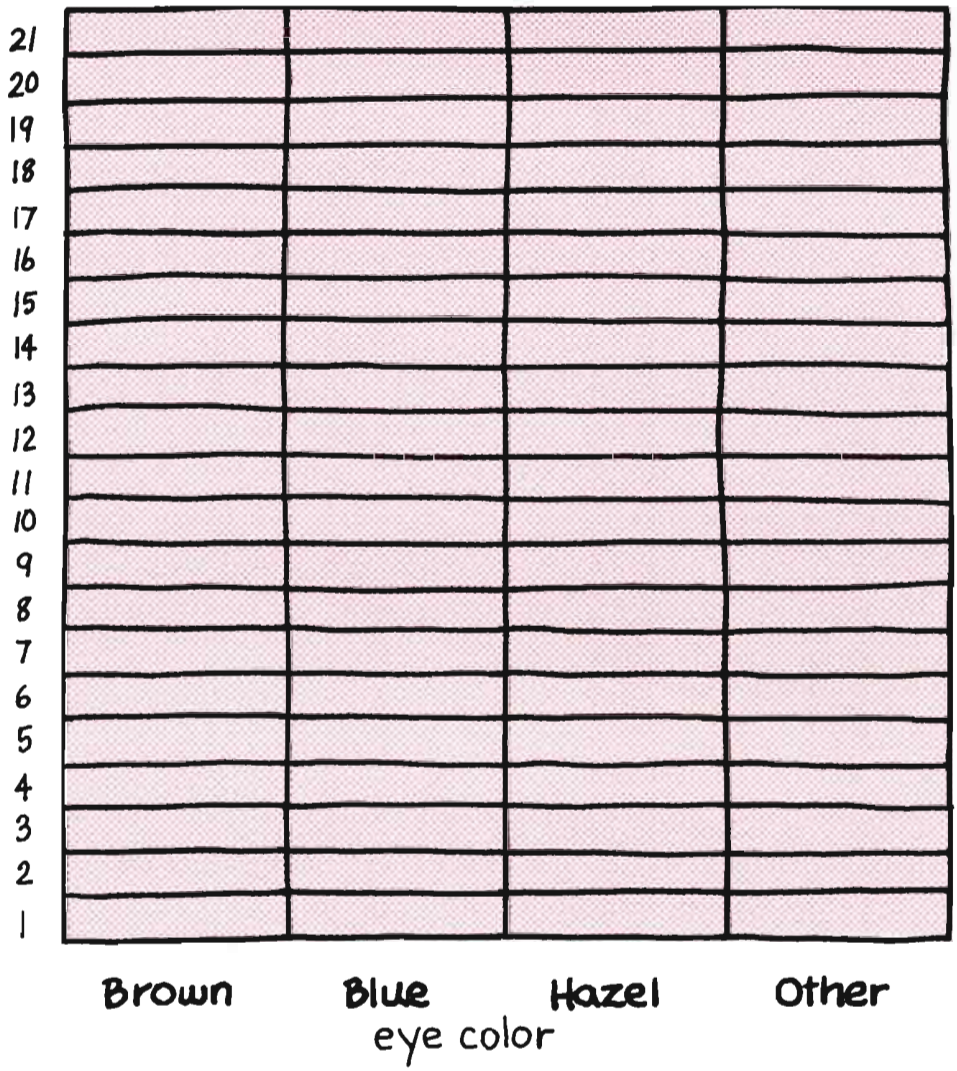
Nat's "9 examples out of 3 doubles"

"Doubles are easiest. I pick 3 doubles and make 9 examples from those doubles. I can write down answers as fast as I can write. I'll show you what I mean. It's fun."

starters

3 + 3	4 + 4	9 + 9	33 +33	2 + 2	5 + 5	8 + 8	22 +22
34 +34	39 +39	43 +43	44 +44	25 +25	28 +28	52 +52	55 +55
49 +49	93 +93	94 +94	99 +99	58 +58	82 +82	85 +85	88 +88

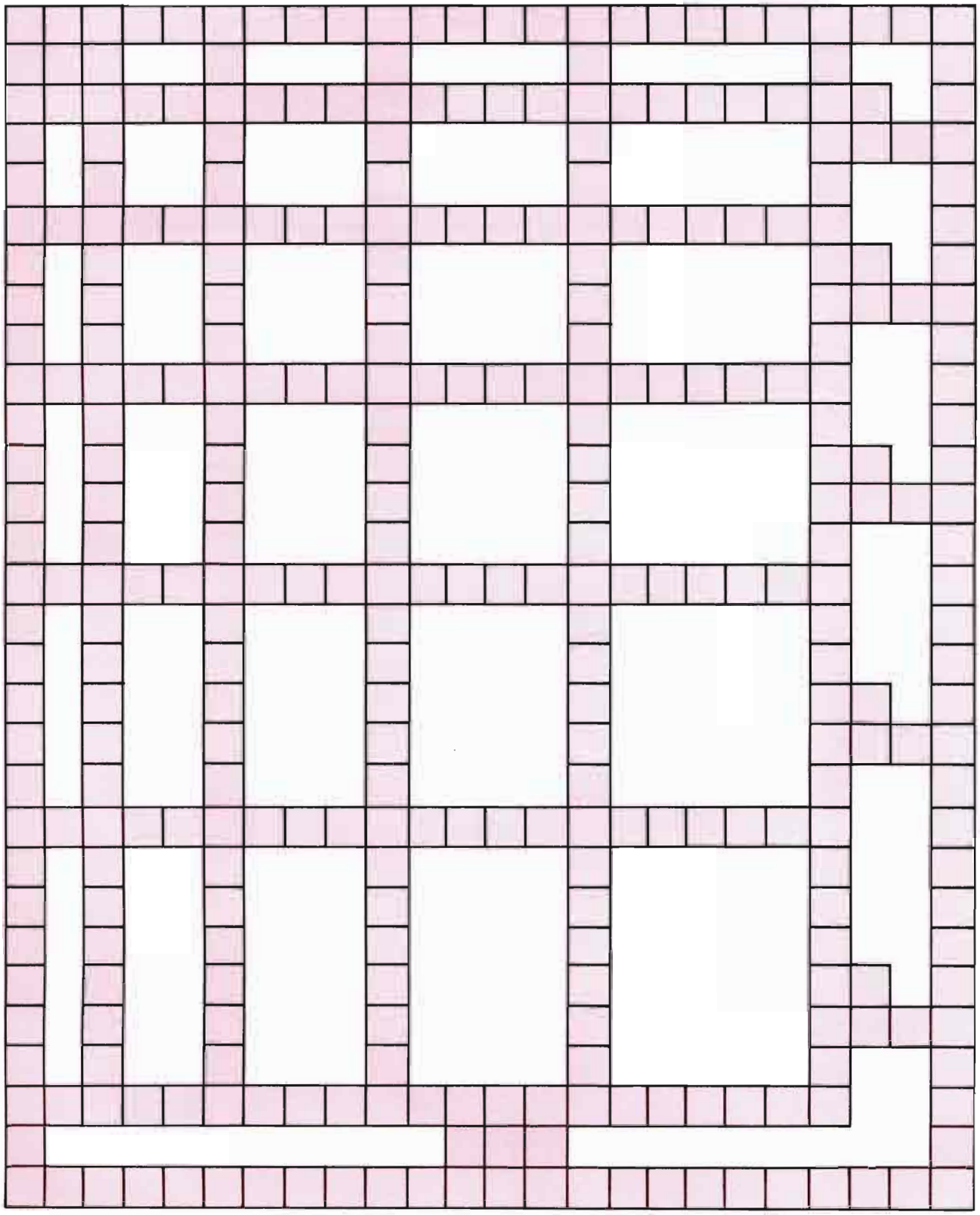
EYE COLORS IN MY CLASS



What eye color is most common in your class? _____

What other information about your class can you gather and graph?

How many blocks are missing?



What is Your Name Worth?

LET A=1, B=2, C=3, ETC.

A 1	B 2	C 3	D 4	E 5	F 6	G 7
H 8	I 9	J 10	K 11	L 12	M 13	N 14
O 15	P 16	Q 17	R 18	S 19	T 20	U 21
V 22	W 23	X 24	Y 25	Z 26		?

G = 7
O = 15
R = 18
F = +6
46

ADD UP THE VALUE OF THE LETTERS IN YOUR NAME. _____

HOW MUCH IS IT WORTH? _____

WHO HAS THE MOST VALUABLE FIRST NAME IN THE ROOM? _____

WHO HAS THE MOST VALUABLE LAST NAME IN THE ROOM? _____

THINK OF A NAME THAT HAS THE LEAST VALUE POSSIBLE. _____

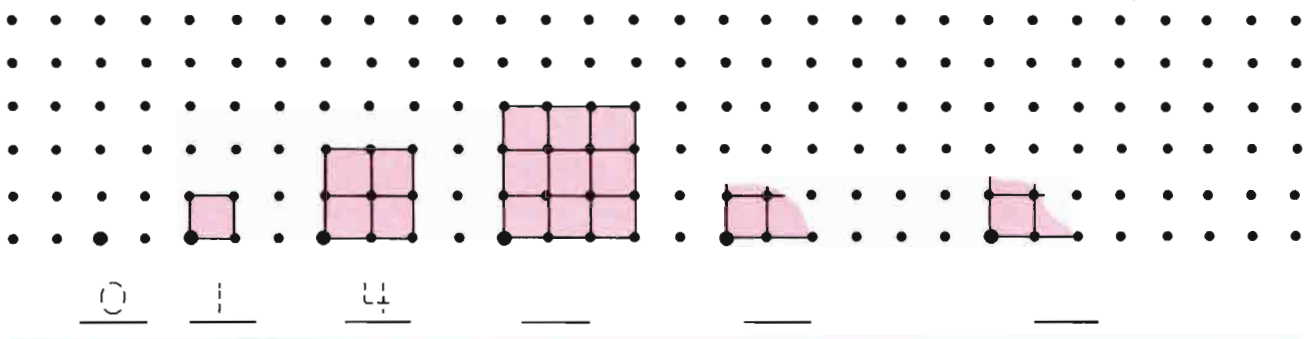
THINK OF A NAME THAT HAS THE GREATEST VALUE POSSIBLE.

_____

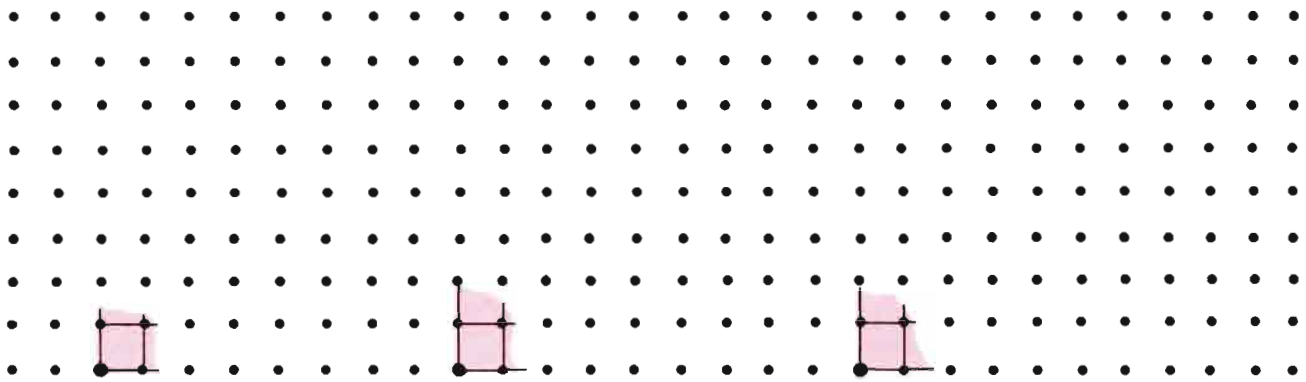
The Family of SQUARE NUMBERS

What's My Rule?

Please complete the sketches below.



0 1 4 ___ ___ ___



___ ___ ___

Please list the above numbers and extend the list.
Then, on the line below, show the differences (diff.) between neighbors.

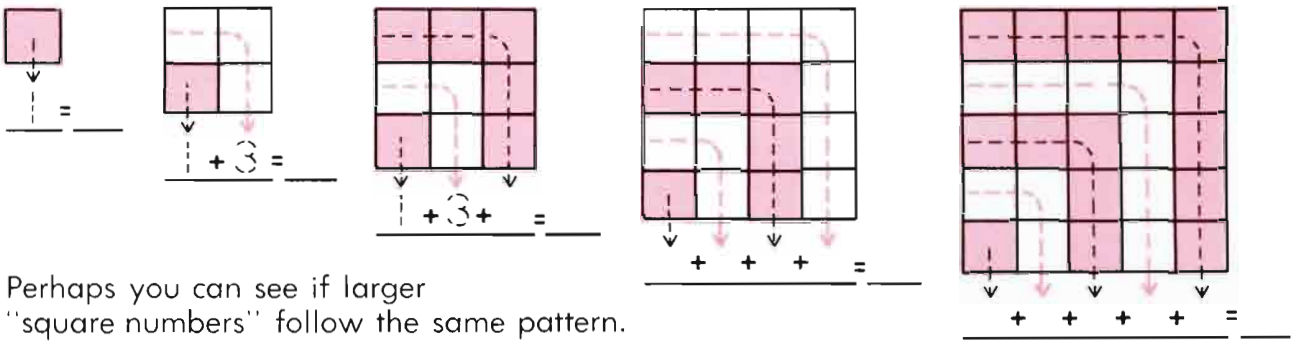
0, 1, 4, ___, ___, ___, ___, ___, ___, ___, ___, ___ ...

diff. 1 3 _____ _____ _____ _____ _____ _____ _____ _____ _____ _____ _____ _____ ...

The numbers in the top list are called SQUARE numbers.

The numbers in the "difference" list are called _____ numbers.

1 = 1	1 + 3 + 5 + 7 = ____	1 + 3 + 5 + 7 + 9 + 11 + 13 = ____
1 + 3 = 4	1 + 3 + 5 + 7 + 9 = ____	1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = ____
1 + 3 + 5 = ____	1 + 3 + 5 + 7 + 9 + 11 = ____	1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 = ____



Perhaps you can see if larger "square numbers" follow the same pattern.

On the top line below, write all the SQUARE numbers that are EVEN.
On the bottom line below, write all the SQUARE numbers that are ODD.
Please list them in order with the smallest on the left.

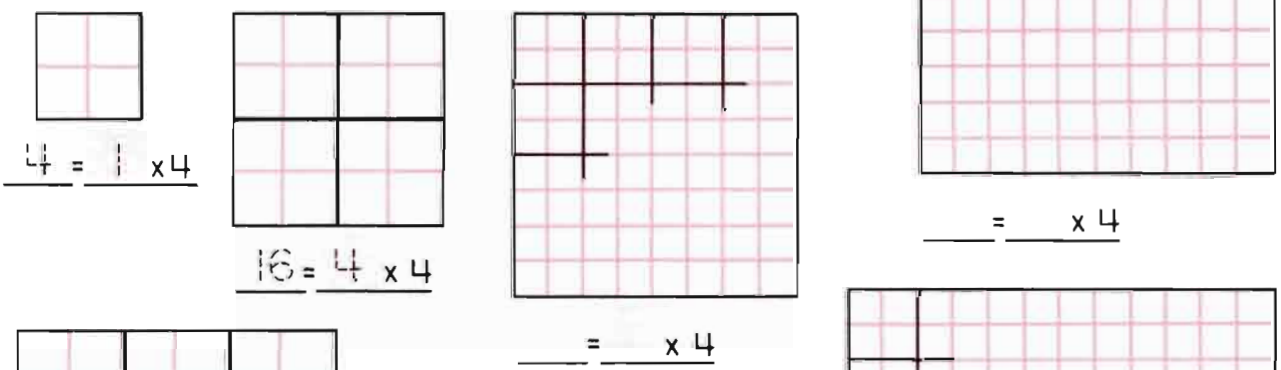
EVEN 0 4 ____ ____ ____ ____ 144 196

ODD ____ ____ ____ ____ ____ ____ 169

n x 4 or n x 4 + 1

Helen said: "I notice something about the sketches showing SQUARE numbers—4 or more—that are EVEN. I can divide them into groups of four."

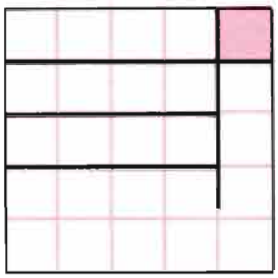
Can you divide each sketch into groups of four?



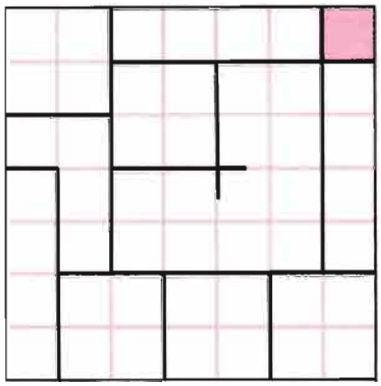
What do you notice about the numbers you wrote in the shaded boxes?

Al said: "When I try Helen's idea on SQUARE numbers that are ODD, I always end up with 1 left over."

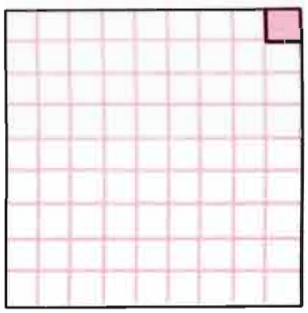
9 = (2 x 4) + 1



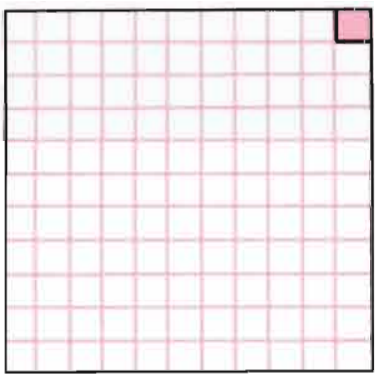
25 = (x 4) + 1



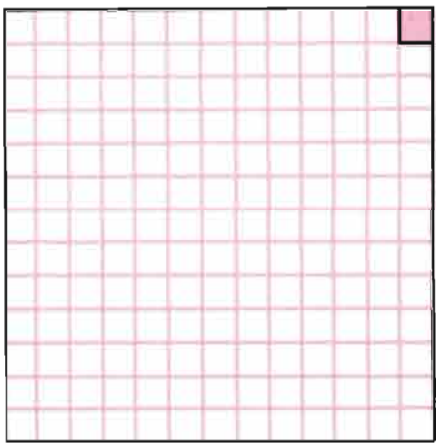
49 = (x 4) + 1



81 = (x 4) + 1

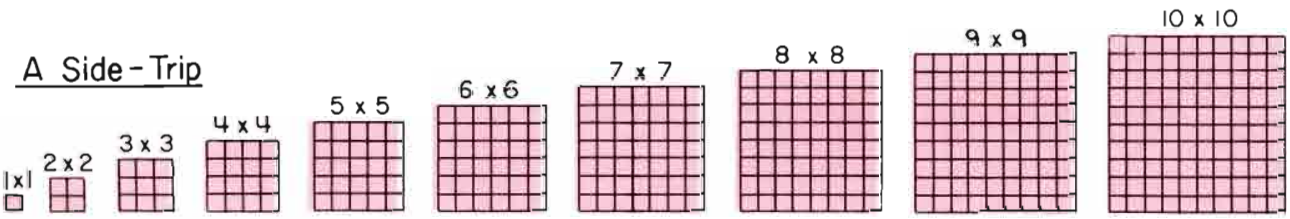


121 = (x 4) + 1



169 = (x 4) + 1

A Side-Trip



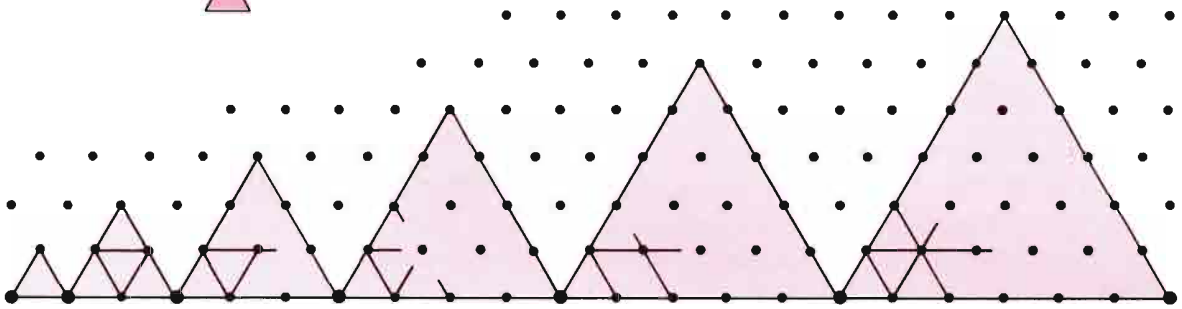
1 4 9 ___ ___ ___ ___ ___ ___ ___

Please fill in the blanks below from the list of "square numbers."

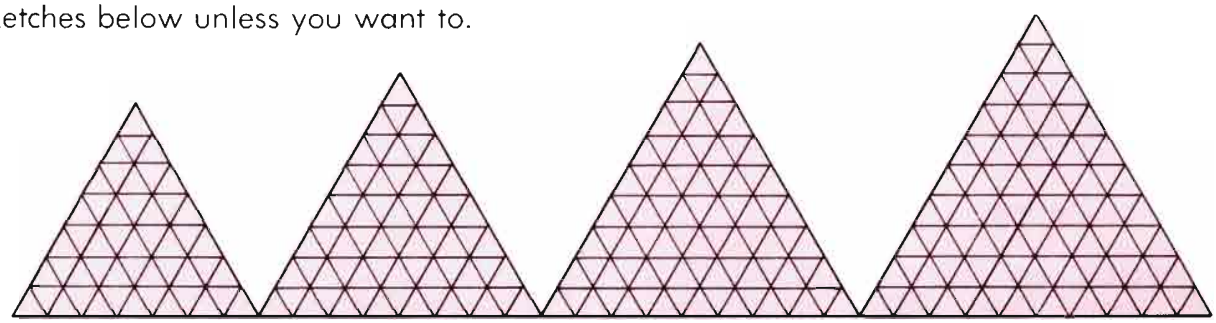
___ = 3	___ = 8	___ = 15	___ = 24
___ = 5	___ = 12	___ = 21	___ = 32
___ = 7	___ = 16	___ = 27	___ = 40
___ = 9	___ = 20	___ = 33	___ = 48
___ = 11	___ = 24	___ = 39	___ = 56

Please complete the pattern and find out how many triangles like this △ are in each.

A SURPRISE

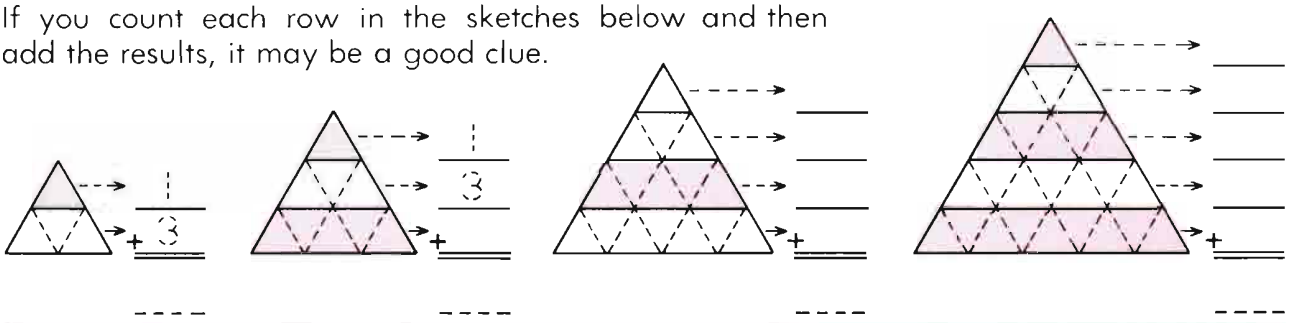


SURPRISED? You don't need to count the sketches below unless you want to.

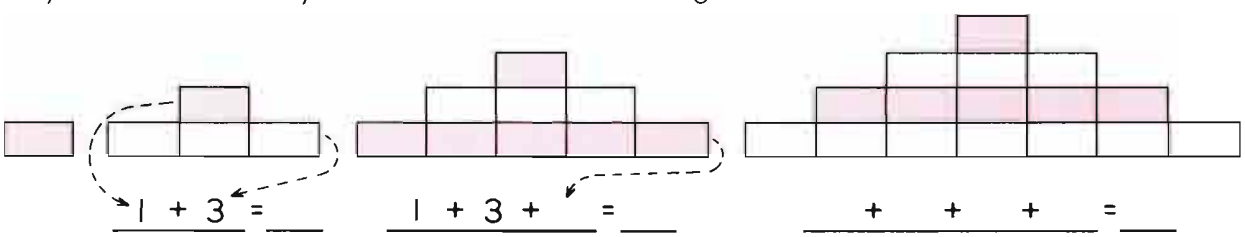


Why is it the "square numbers" again?

If you count each row in the sketches below and then add the results, it may be a good clue.



Here are sketches of bricks stacked in a special way. Can you tell how many there are without counting?



Please list the "square numbers."
The "difference between neighbors" may help.

RATE OF GROWTH

Square Numbers	0	1	4	___	___	___	___	___	___
Rate of Growth (differences)	1	3	___	___	___	___	___	___	___

Problem: Which numbers can be shown as the sum of 2 "square numbers"?

Please fill in the blanks below with only "square numbers" from the list above. (Numbers can be repeated.) If there are some you can't do, please loop them.

1 = 0 + 1	11 = ___ + ___	21 = ___ + ___	31 = ___ + ___
2 = 1 + 1	12 = ___ + ___	22 = ___ + ___	32 = ___ + ___
3 = can't be done	13 = ___ + ___	23 = ___ + ___	33 = ___ + ___
4 = 0 + 4	14 = ___ + ___	24 = ___ + ___	34 = ___ + ___
5 = ___ + ___	15 = ___ + ___	25 = ___ + ___	35 = ___ + ___
6 = can't be done	16 = ___ + ___	26 = ___ + ___	36 = ___ + ___
7 = ___ + ___	17 = ___ + ___	27 = ___ + ___	37 = ___ + ___
8 = 4 + ___	18 = ___ + ___	28 = ___ + ___	38 = ___ + ___
9 = 0 + ___	19 = ___ + ___	29 = ___ + ___	39 = ___ + ___
10 = ___ + ___	20 = ___ + ___	30 = ___ + ___	40 = ___ + ___

We couldn't do 20 of the examples ..... with a pair of "square numbers." Let's see how many of those we can do with 3 "square numbers."

3 = 1 + 1 + 1	19 = ___ + ___	30 = ___ + ___
6 = 1 + 1 + 4	21 = ___ + ___	31 = ___ + ___
7 = ___ + ___	22 = ___ + ___	33 = ___ + ___
11 = ___ + ___	23 = ___ + ___	35 = ___ + ___
12 = 4 + 4 + 4	24 = ___ + ___	38 = ___ + ___
14 = ___ + ___	27 = ___ + ___	39 = ___ + ___
15 = ___ + ___	28 = ___ + ___	41 = ___ + ___

We still couldn't write 6 of the numbers 40 or less as the sum of 3 "square numbers." Let's see if we can do them with 4 "square numbers."

7 = ___ + ___ + ___	23 = ___ + ___ + ___	31 = ___ + ___ + ___
15 = ___ + ___ + ___	28 = ___ + ___ + ___	39 = ___ + ___ + ___

They can all be shown as the sum of 4 "square numbers."

Let's go a little beyond 41. Maybe you would like to go further . . . on your own.

42 = 0 + 1 + 16 + 25	45 = ___ + ___ + ___	48 = ___ + ___ + ___
43 = ___ + ___ + ___	46 = ___ + ___ + ___	49 = 0 + 0 + 0 + 49
44 = ___ + ___ + ___	47 = ___ + ___ + ___	50 = ___ + ___ + ___

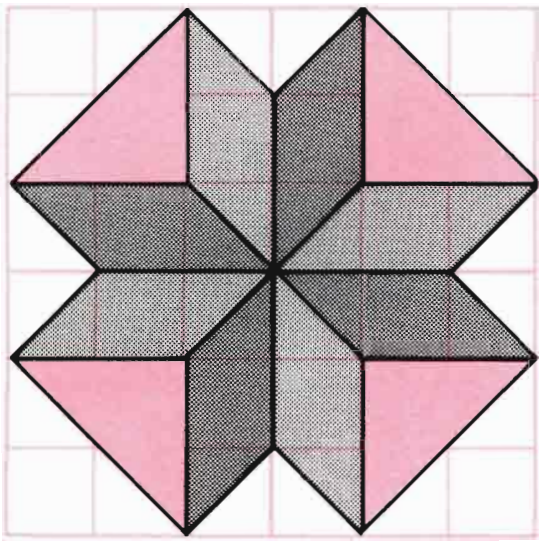
(Mathematicians claim you will never need more than 4 for any number. Of course you need a long list of "square numbers." Do you believe it?)

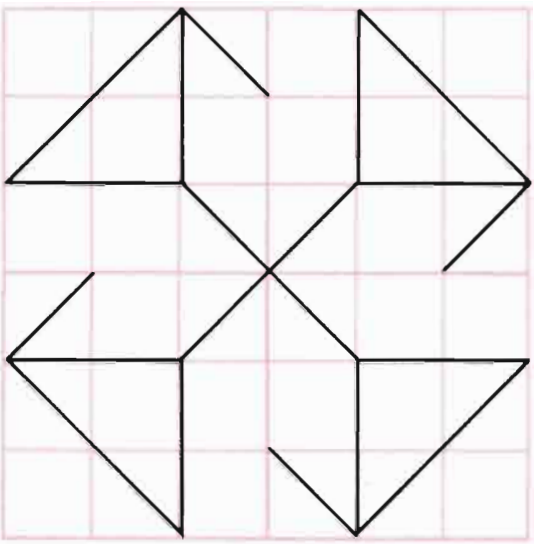
n = ___ + ___ + ___ in more than one way

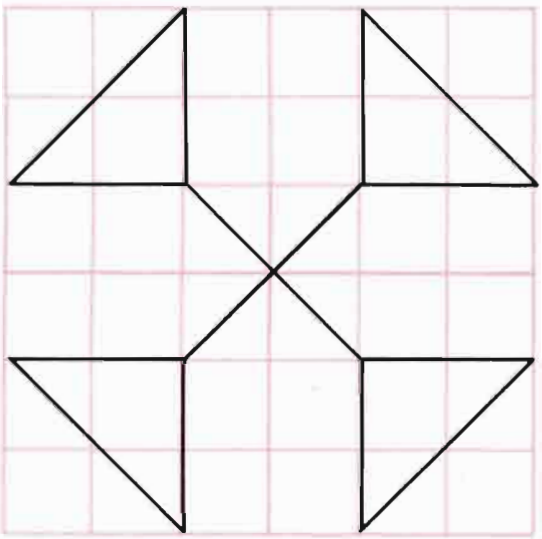
0	1	2	3	4	5	6	7	8	9	10	11
x 0	x 1	x 2	x 3	x 4	x 5	x 6	x 7	x 8	x 9	x 10	x 11
0	1	4	____	____	____	____	____	____	____	____	121

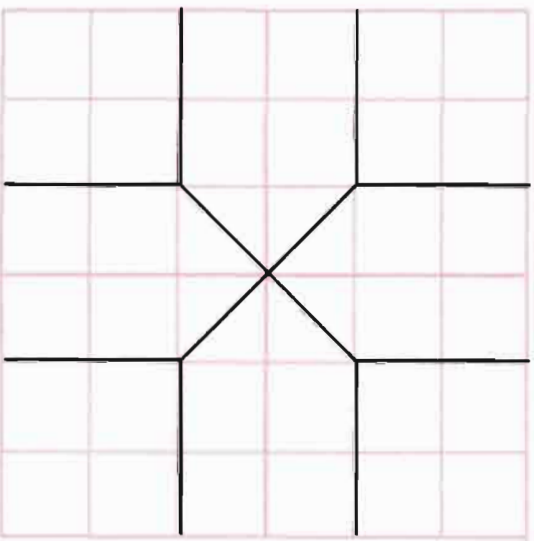
We found we could write all numbers 0 through 50 as the sum of four or less "square numbers"—0, 1, 4, 9, etc. Some can be written in this form ___ + ___ + ___ in more than one way—with different combinations. Let's try it:

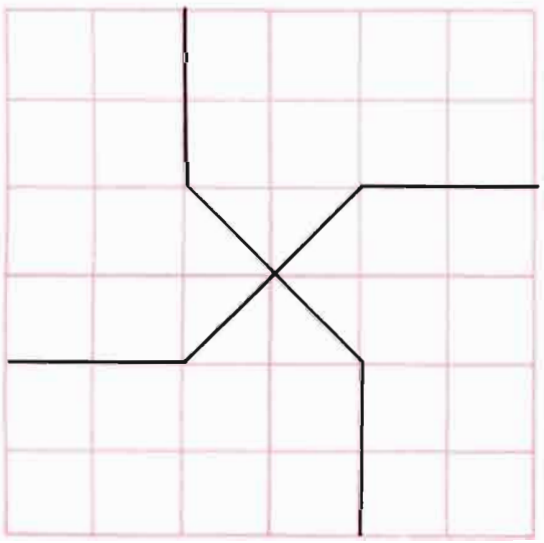
10 = 0 + 0 + 1 + ___	29 = ___ + ___ + ___	42 = ___ + ___ + ___
10 = 1 + 1 + 4 + ___	29 = ___ + ___ + ___	42 = ___ + ___ + ___
13 = ___ + ___ + ___	31 = ___ + ___ + ___	43 = ___ + ___ + ___
13 = ___ + ___ + ___	31 = ___ + ___ + ___	43 = ___ + ___ + ___
19 = ___ + ___ + ___	34 = ___ + ___ + ___	45 = ___ + ___ + ___
19 = ___ + ___ + ___	34 = ___ + ___ + ___	45 = ___ + ___ + ___
20 = ___ + ___ + ___	38 = ___ + ___ + ___	50 = ___ + ___ + ___
20 = ___ + ___ + ___	38 = ___ + ___ + ___	50 = ___ + ___ + ___

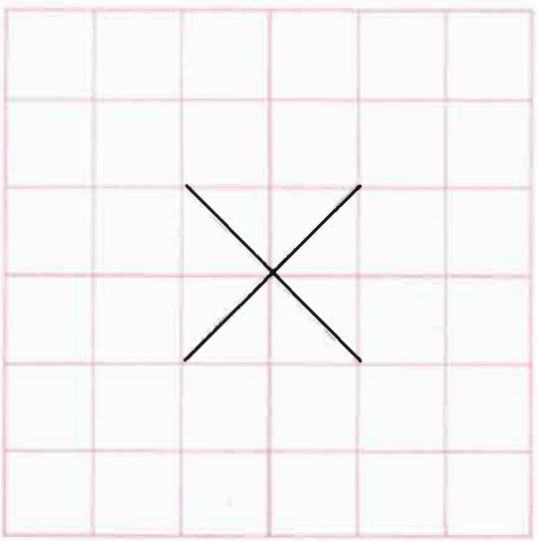






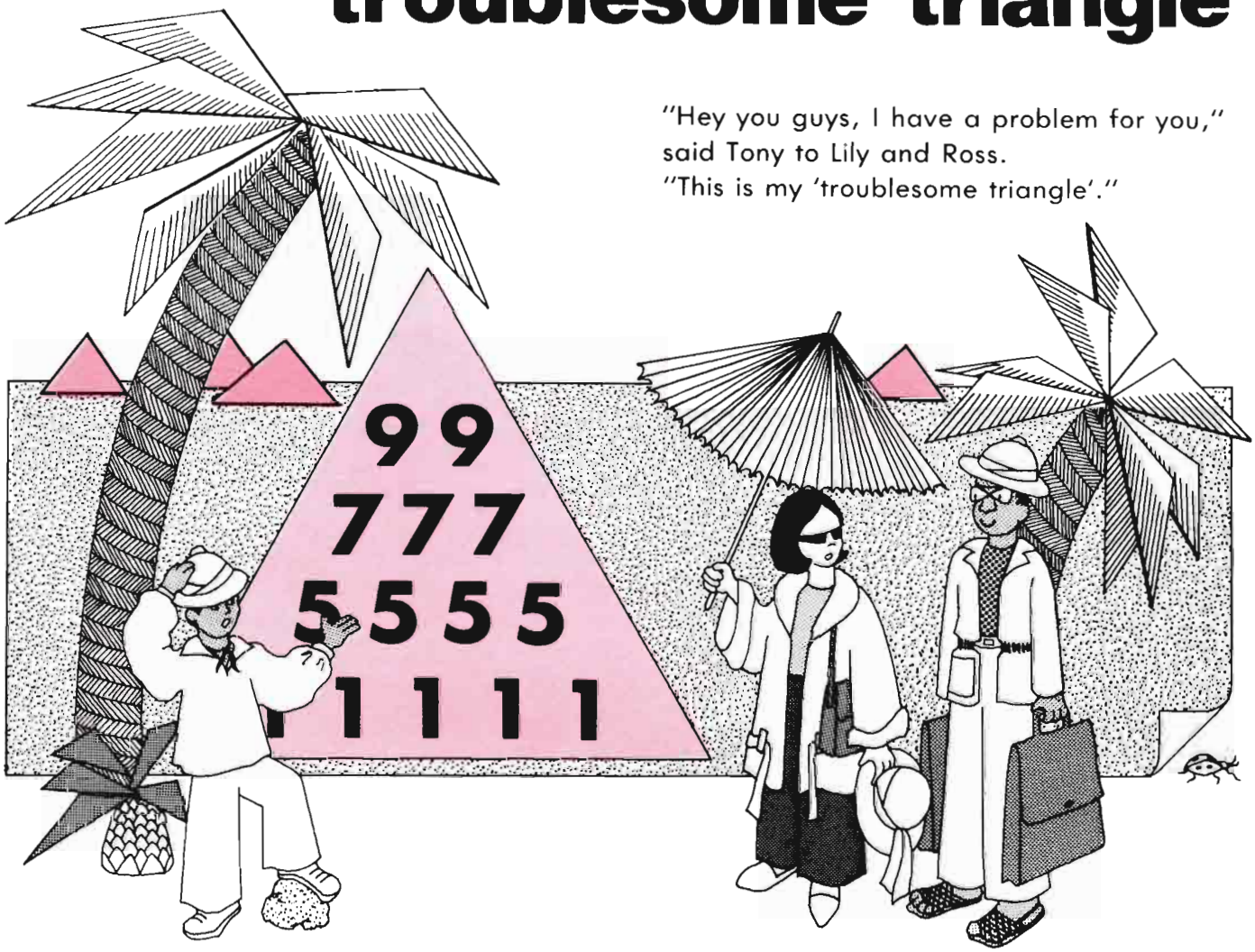






the case of the troublesome triangle

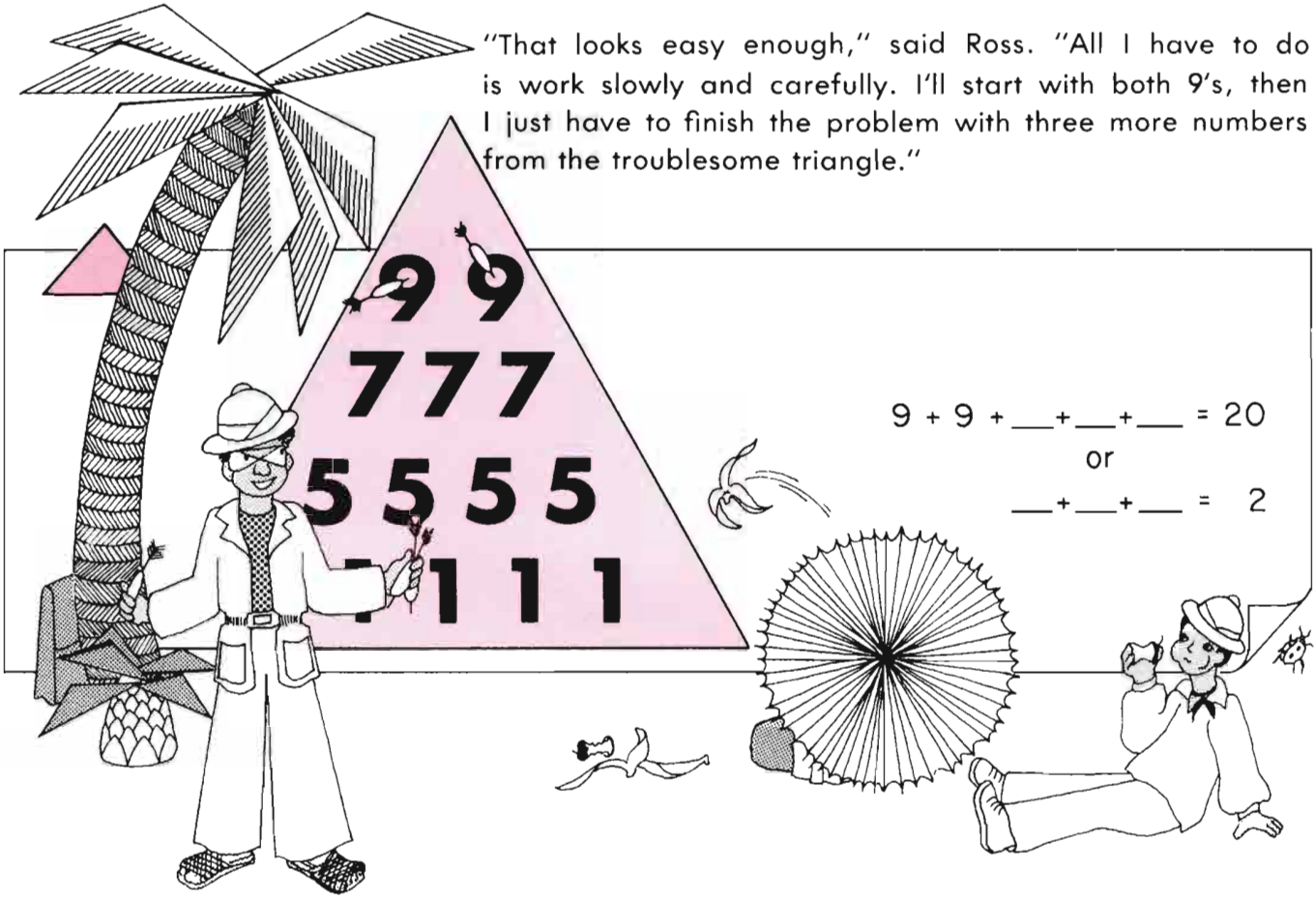
"Hey you guys, I have a problem for you," said Tony to Lily and Ross.
"This is my 'troublesome triangle'."



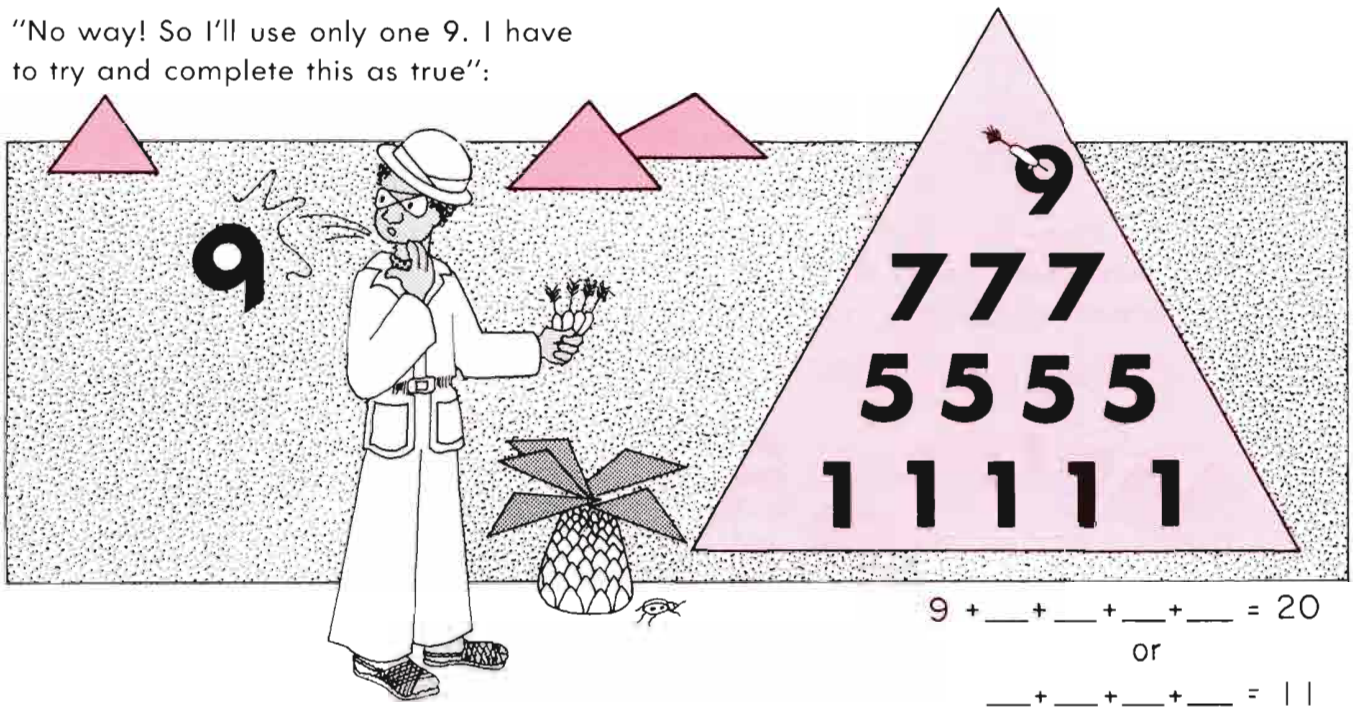
"Two 9's, three 7's, four 5's and five 1's. See if you can throw darts at five of those numbers so they'll add up to exactly 20—or find numbers from the troublesome triangle to make this a true statement."

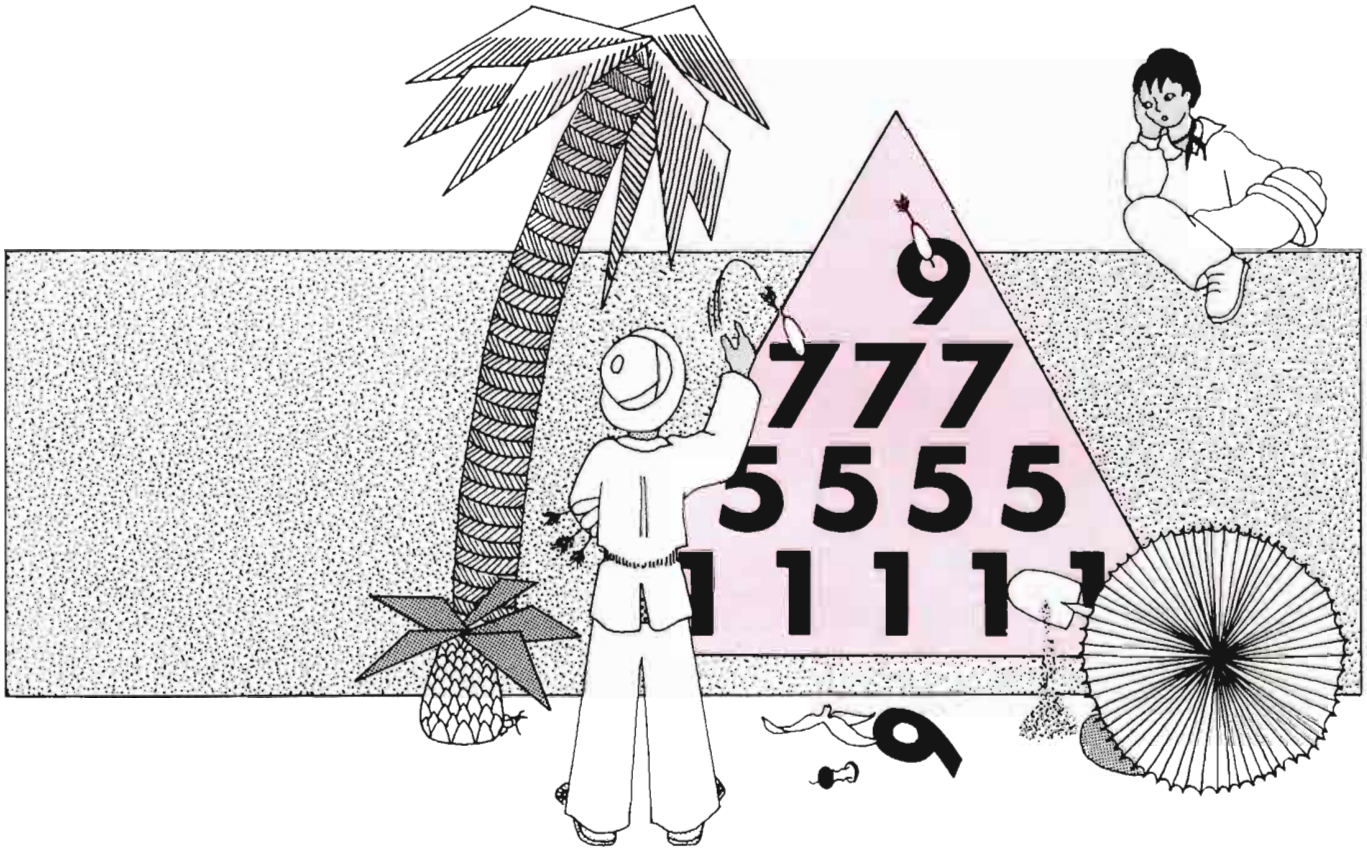
$$\frac{(\quad + \quad + \quad + \quad + \quad)}{(1)\ (2)\ (3)\ (4)\ (5)} = 20$$

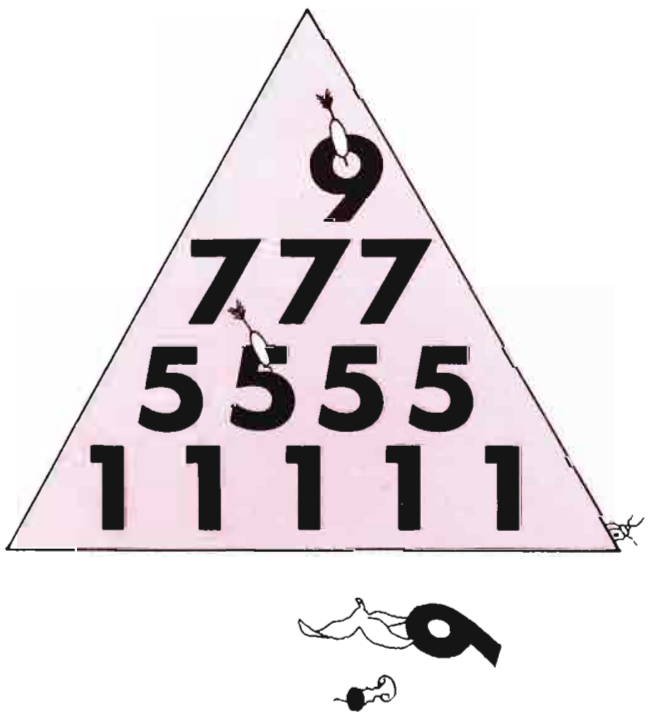
"That looks easy enough," said Ross. "All I have to do is work slowly and carefully. I'll start with both 9's, then I just have to finish the problem with three more numbers from the troublesome triangle."



"No way! So I'll use only one 9. I have to try and complete this as true":







"First I'll use a 7":

9 + 7 + ___ + ___ + ___ = 20

or

7 + ___ + ___ + ___ = 11

or

___ + ___ + ___ = 4

"No way! So I'll try a 5":

9 + 5 + ___ + ___ + ___ = 20

14 + ___ + ___ + ___ = 20

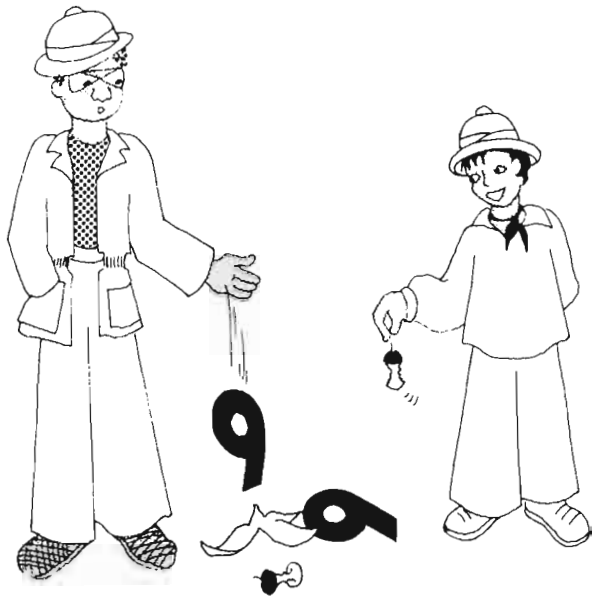
___ + ___ + ___ = 6

Then Ross wrote:

5 + 1 + 1 = 7

1 + 1 + 1 = 3

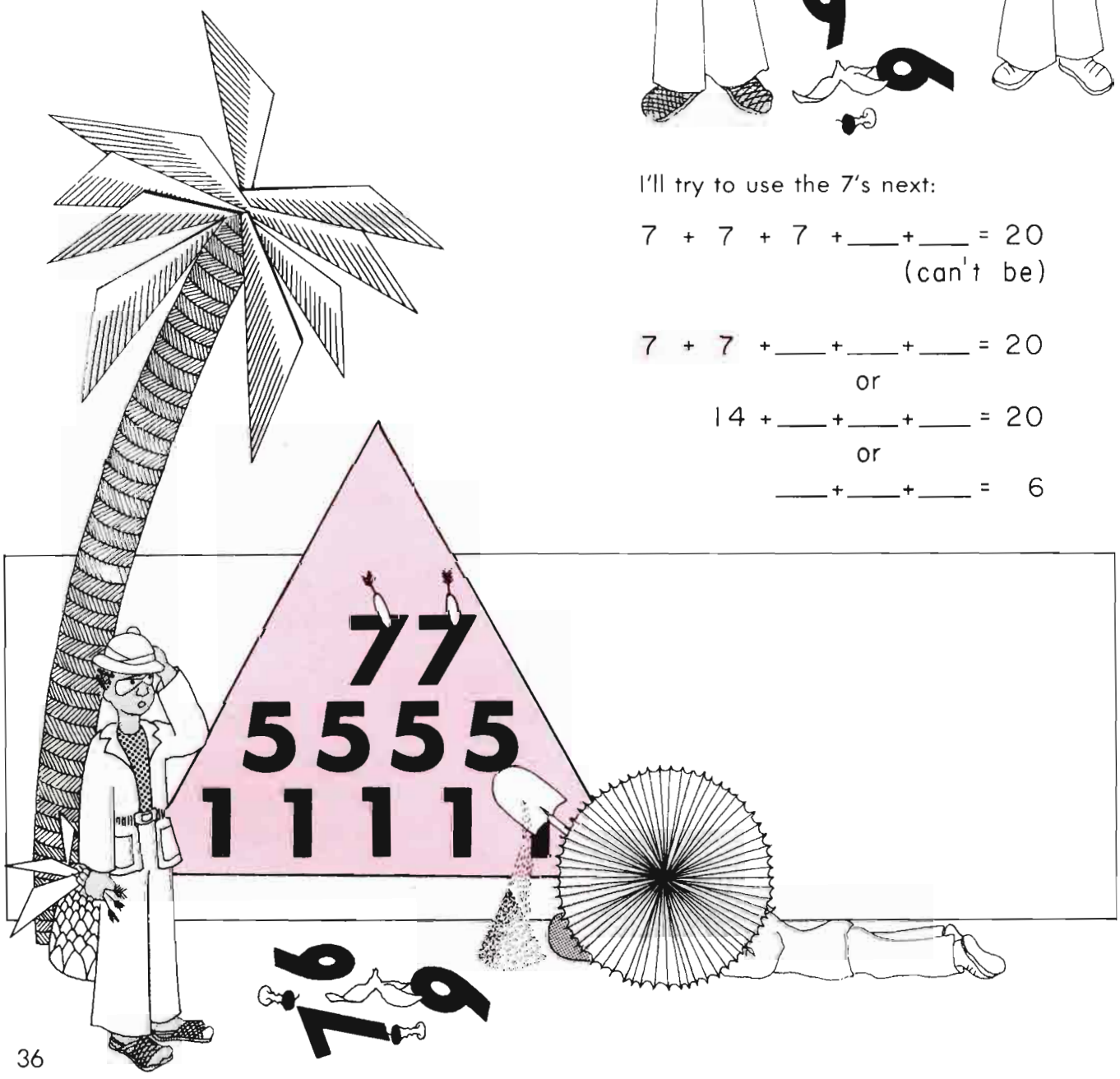
"You see that I can't make 6 with three of those numbers. So I'll just get rid of the 9's and forget about them.



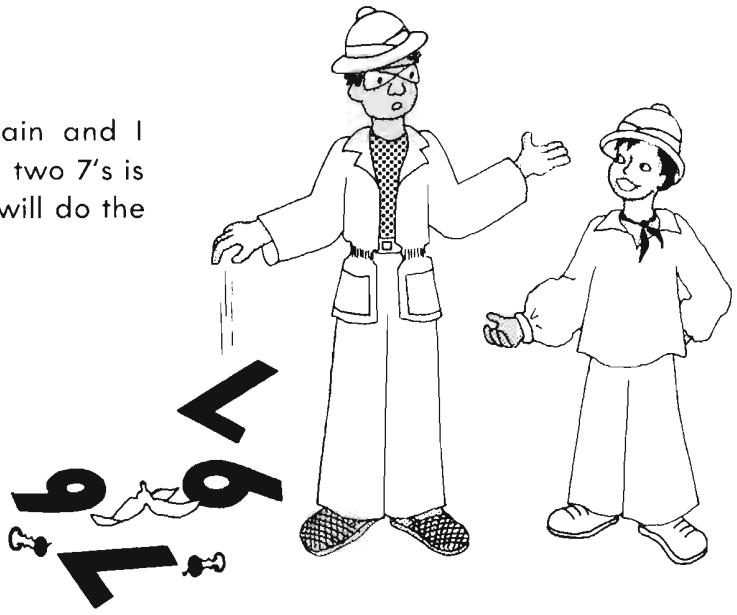
I'll try to use the 7's next:

7 + 7 + 7 + ___ + ___ = 20
(can't be)

7 + 7 + ___ + ___ + ___ = 20
or
14 + ___ + ___ + ___ = 20
or
___ + ___ + ___ = 6



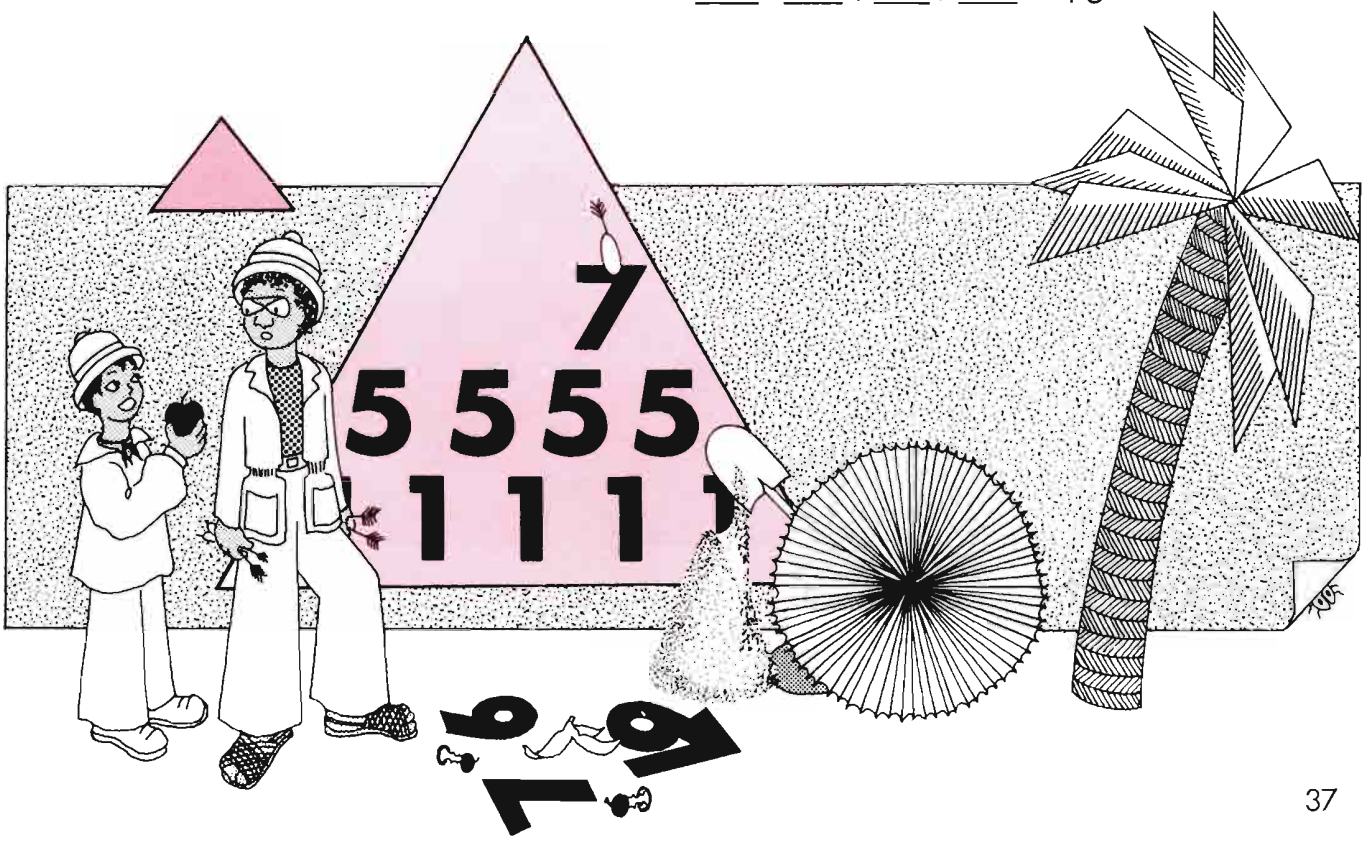
"I'm stuck with that 6 again and I know that I can't get it. So, two 7's is a bad start. Maybe one 7 will do the job."



7 + ___ + ___ + ___ + ___ = 20

or

___ + ___ + ___ + ___ = 13



"First I'll try two 5's with the 7."

7 + 5 + 5 + ___ + ___ = 20

or

17 + ___ + ___ = 20

or

___ + ___ = 3

"No way!
Maybe a 7 and a 5 will work."

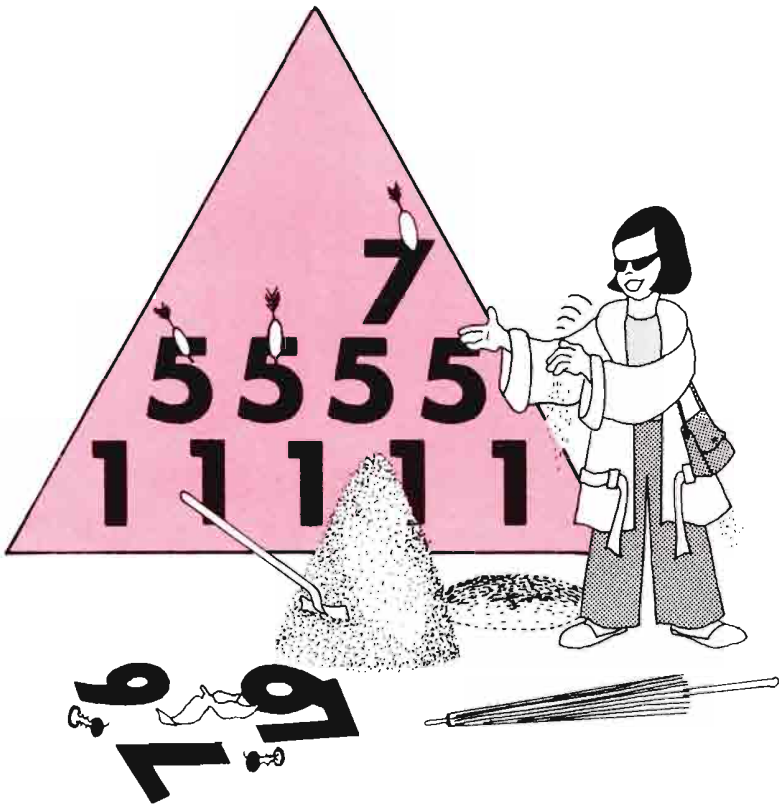
7 + 5 + ___ + ___ + ___ = 20

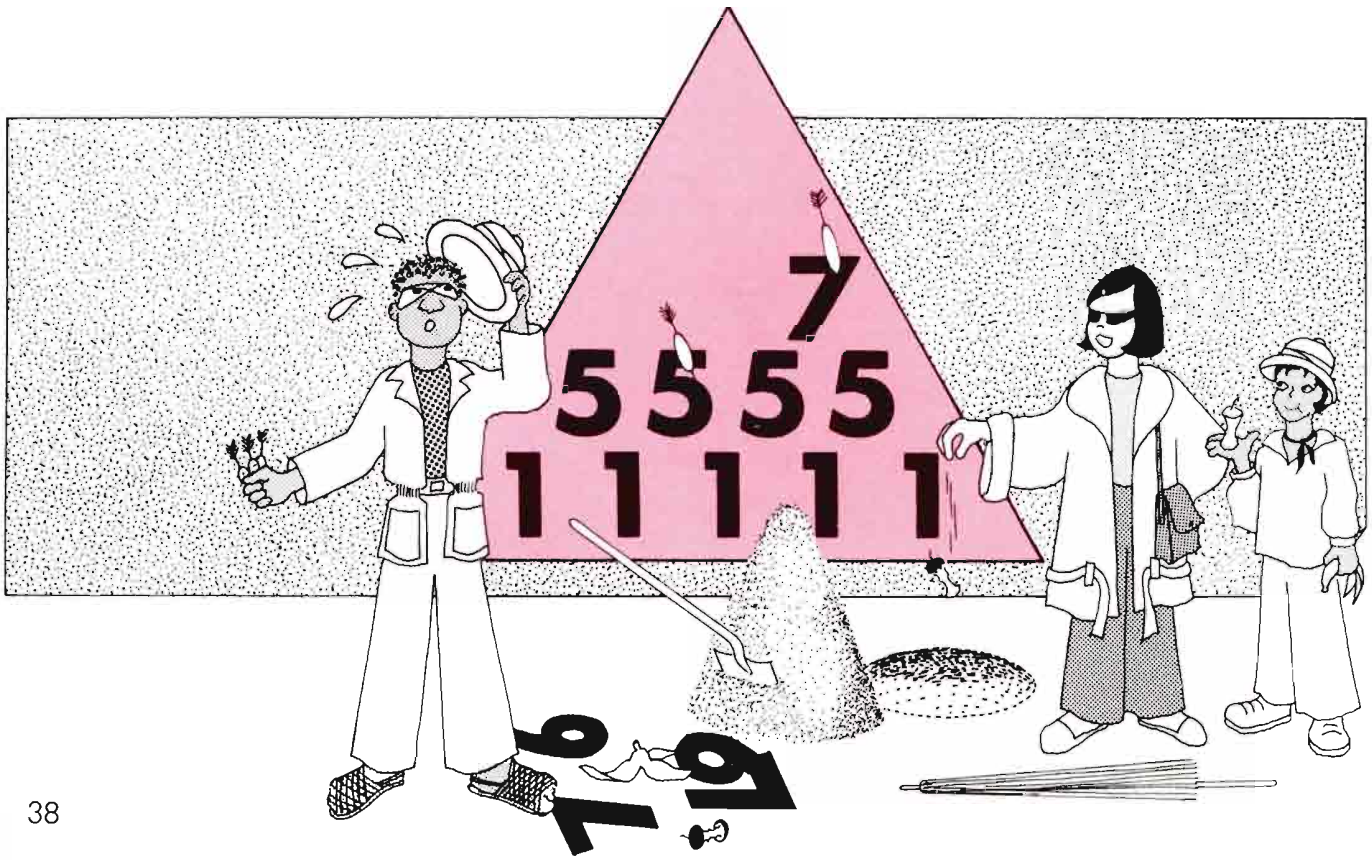
or

12 + ___ + ___ + ___ = 20

or

___ + ___ + ___ = 8

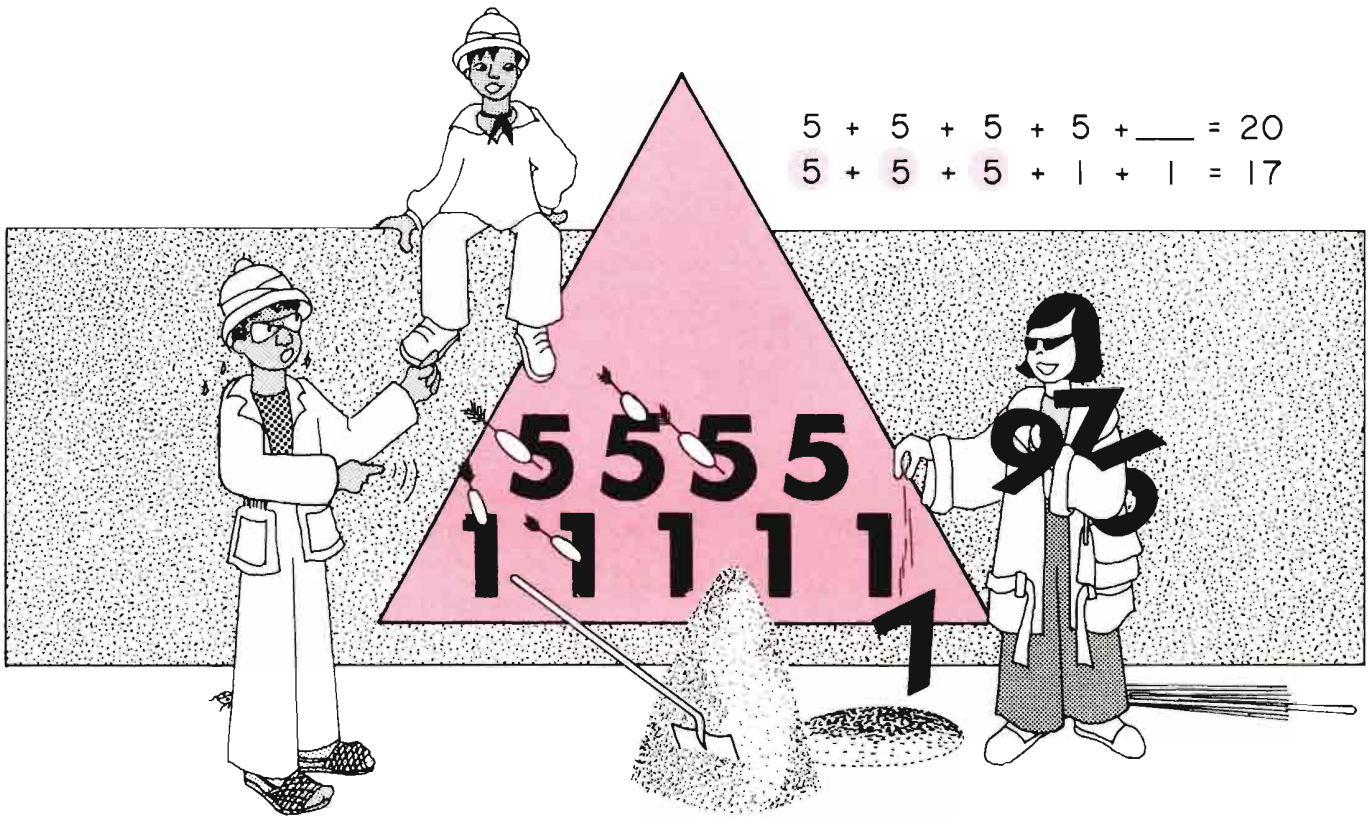




Ross went to work trying to get 8.

$$5 + 5 + 1 = 11$$
$$5 + 1 + 1 = 7$$

"I can't use three numbers in the list that will add up to 8. So one 7 and one 5 won't work either. That means that we're done with 7's. The 5's should be easy."



"Hey, Tony, I think that I've just proved it can't be done. There's no way to find five numbers in your troublesome triangle that add up to exactly 20. And I'm worn out just like your problem." "Good work, Ross, but Lily found the answer in less than a minute and I knew it already," Tony said.
"Less than a minute? That's hard to believe. Show me!"

"Well, if you notice that all the numbers in Tony's troublesome triangle are odd numbers," Lily explained, "the rest is very easy." "It isn't very hard to see that if you add up four odd numbers, you'll get an even number."

$$3 + 5 = 8 \qquad 9 + 7 = 16$$
$$7 + 7 = 14 \qquad 7 + 5 = 12$$
$$\text{or} \quad \underline{\text{odd + odd}} = \underline{\text{even}}$$

$$\underline{3 + 5} + \underline{7 + 9} = 8 + 16 = 24$$
$$\underline{\text{odd + odd}} + \underline{\text{odd + odd}} = \text{even}$$
$$\underline{4 \text{ odds}} = \underline{\text{even}}$$

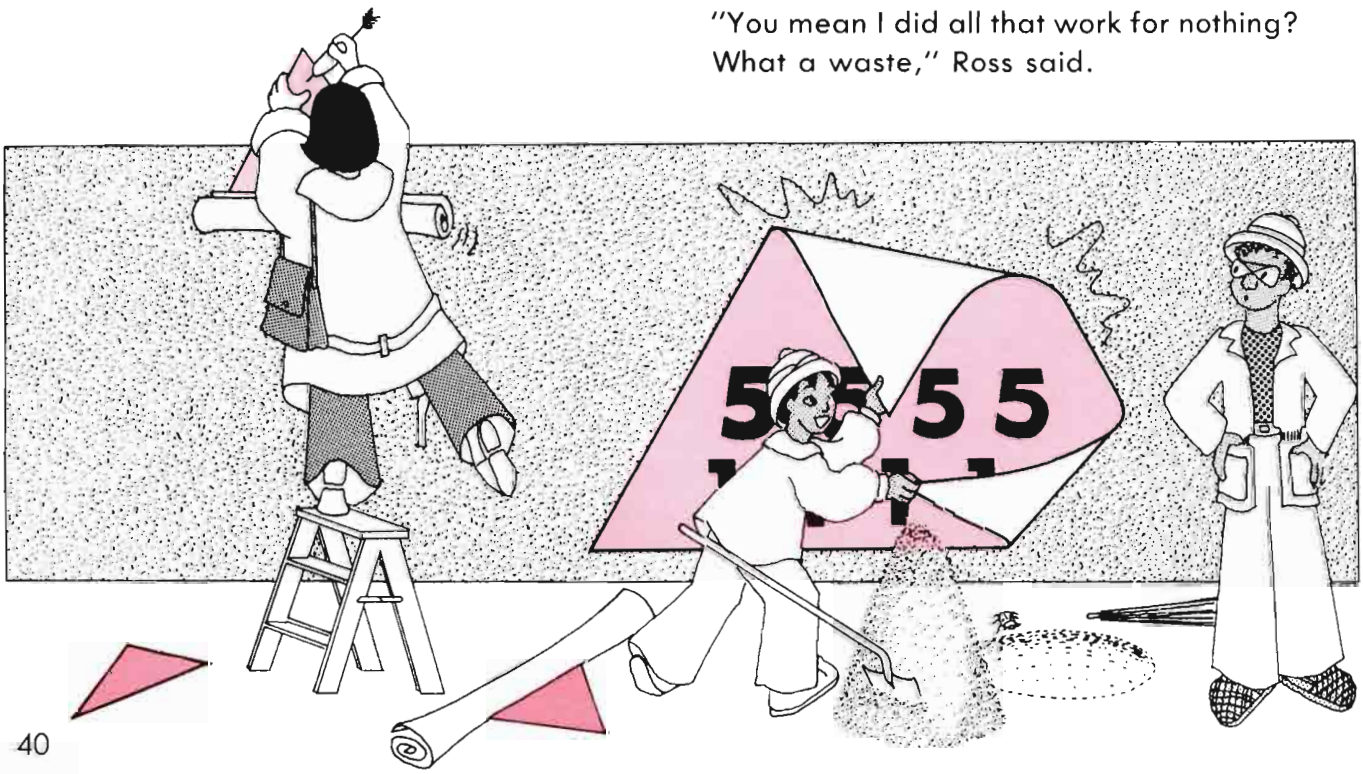
"Your last choice from the triangle has to be 1, 5, 7 or 9—and they are all odd numbers, and"

$$4 \text{ odd} + \text{odd} =$$
$$\underline{\text{even + odd}} = \underline{\text{odd}}$$

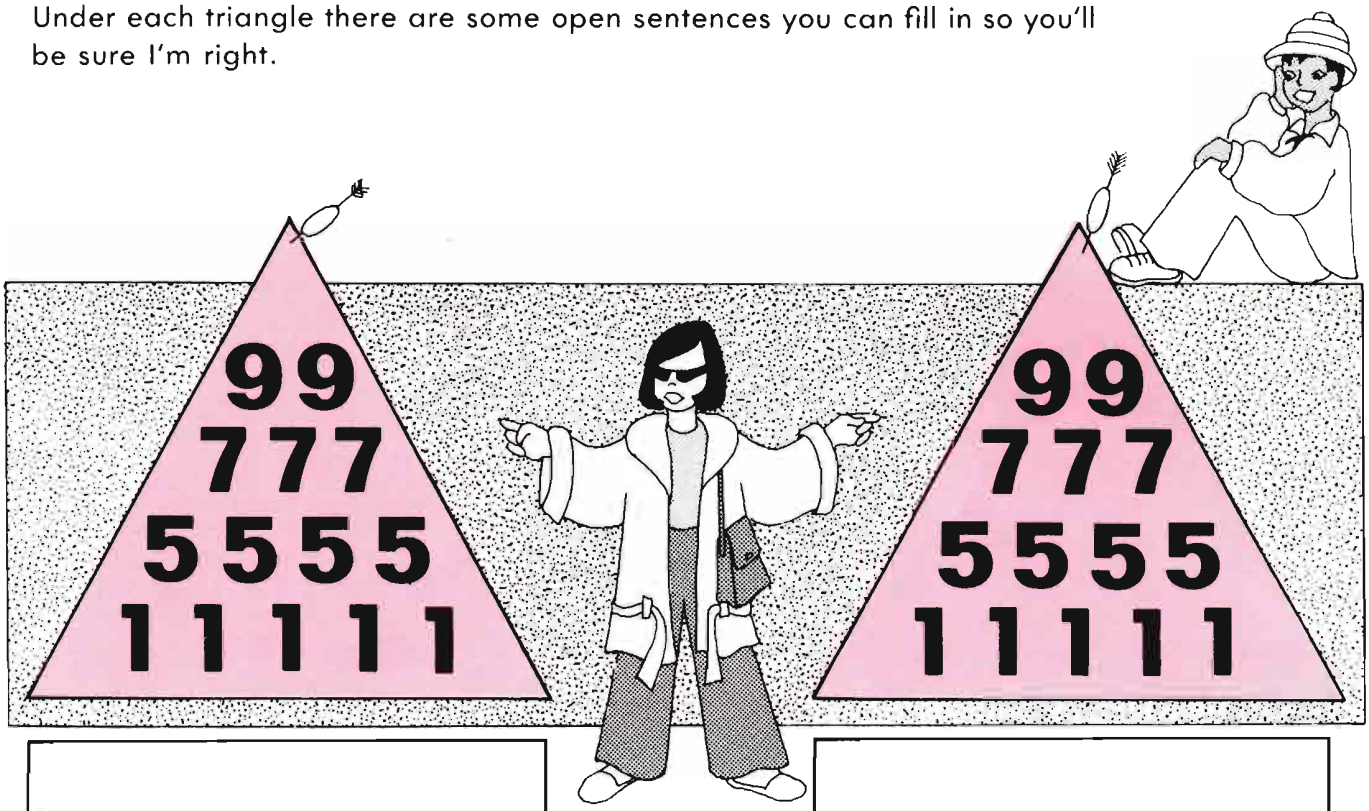
"no matter what five numbers you pick from the triangle, the sum of the first four will be even; and when you add an odd number to the even number that you had added together, you'll end up with another odd number."

$$5 \text{ odds} = \text{odd}$$

"But 20 is an even number, so you can't possibly get the sum of five of Tony's odd numbers in the troublesome triangle to work."
"You mean I did all that work for nothing? What a waste," Ross said.



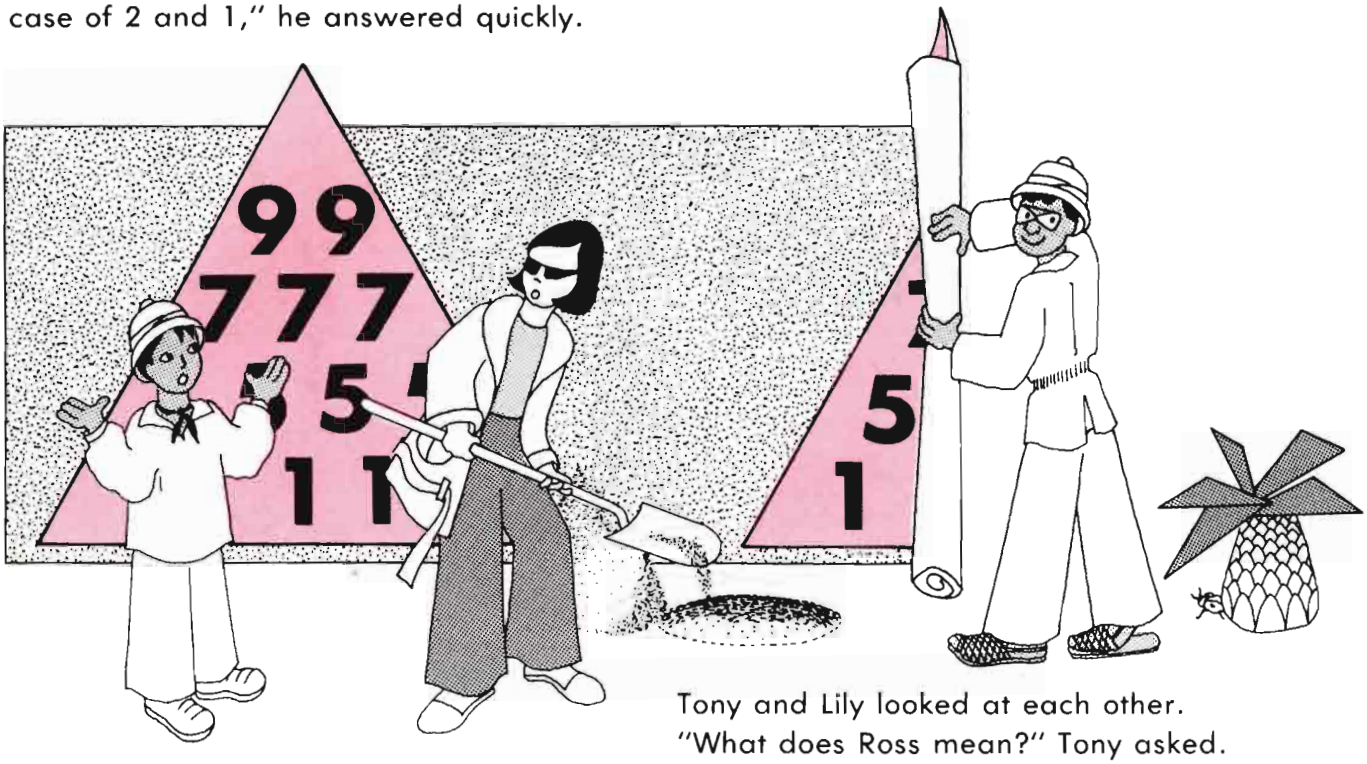
"Look, Ross, I made twin triangles just like Tony's troublesome triangle. Under each triangle there are some open sentences you can fill in so you'll be sure I'm right.



____ + ____ + ____ = 20	____ + ____ + ____ = 21
____ + ____ + ____ + ____ = 20	____ + ____ + ____ + ____ = 21
____ + ____ + ____ + ____ + ____ = 20	____ + ____ + ____ + ____ + ____ = 21

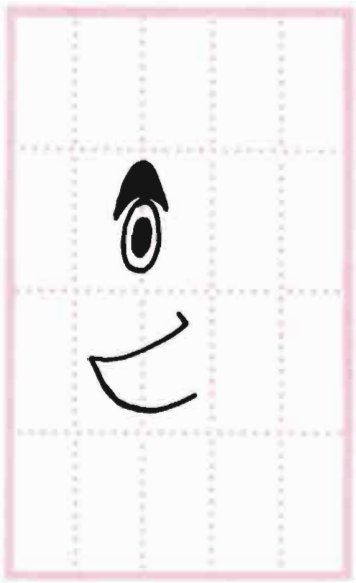
"You can only use numbers from one triangle to fill in the sentences under that triangle to make true statements. In which cases can you get a true statement and in which cases is it impossible?" Lily asked.

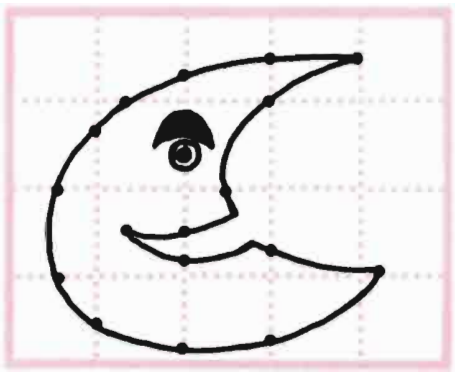
Ross looked over the examples. "It's a case of 2 and 1," he answered quickly.

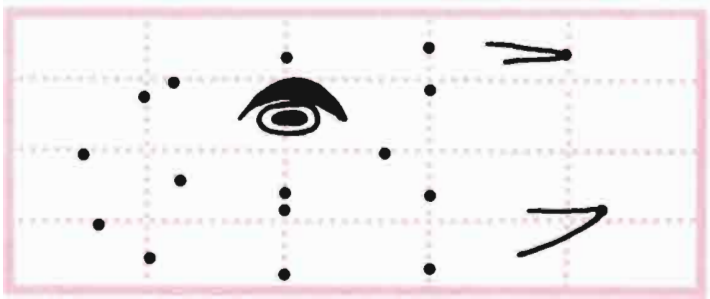


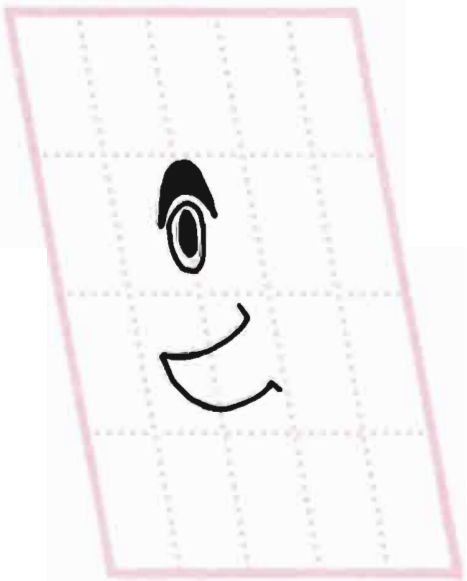
Tony and Lily looked at each other.
"What does Ross mean?" Tony asked.
"I don't know yet," Lily said. "A case of 2 and 1! Really, that Ross is impossible!"

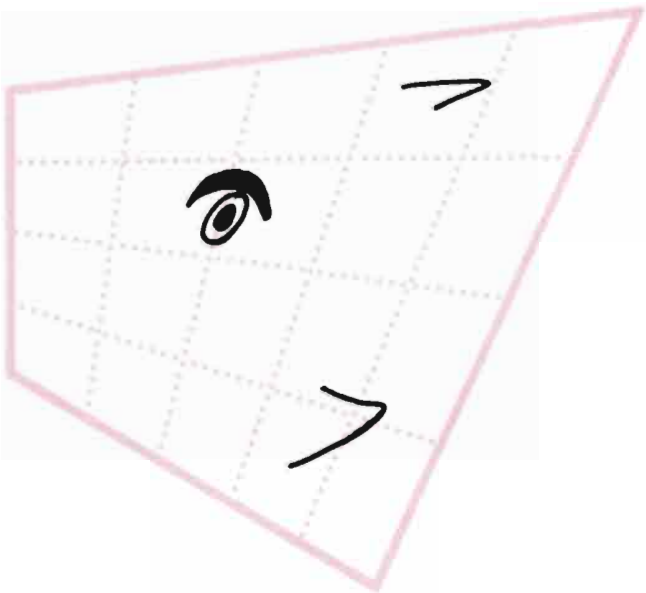
Changing Shapes











FLIPPING COINS

Flip a penny 10 times and record the time it comes up heads or tails.

_____ heads _____ tails

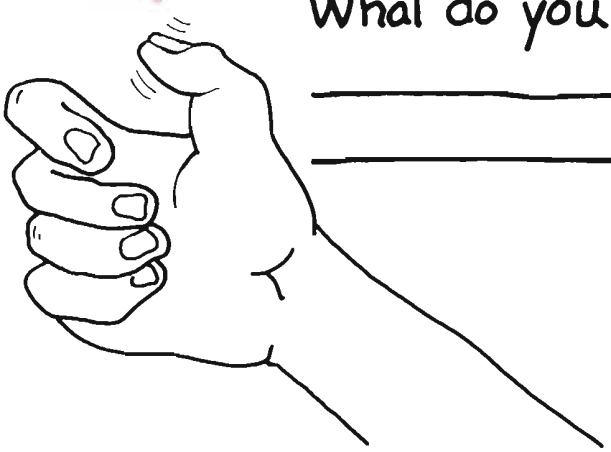
Now try it with 2 pennies.

_____ heads _____ tails

What do you see ?

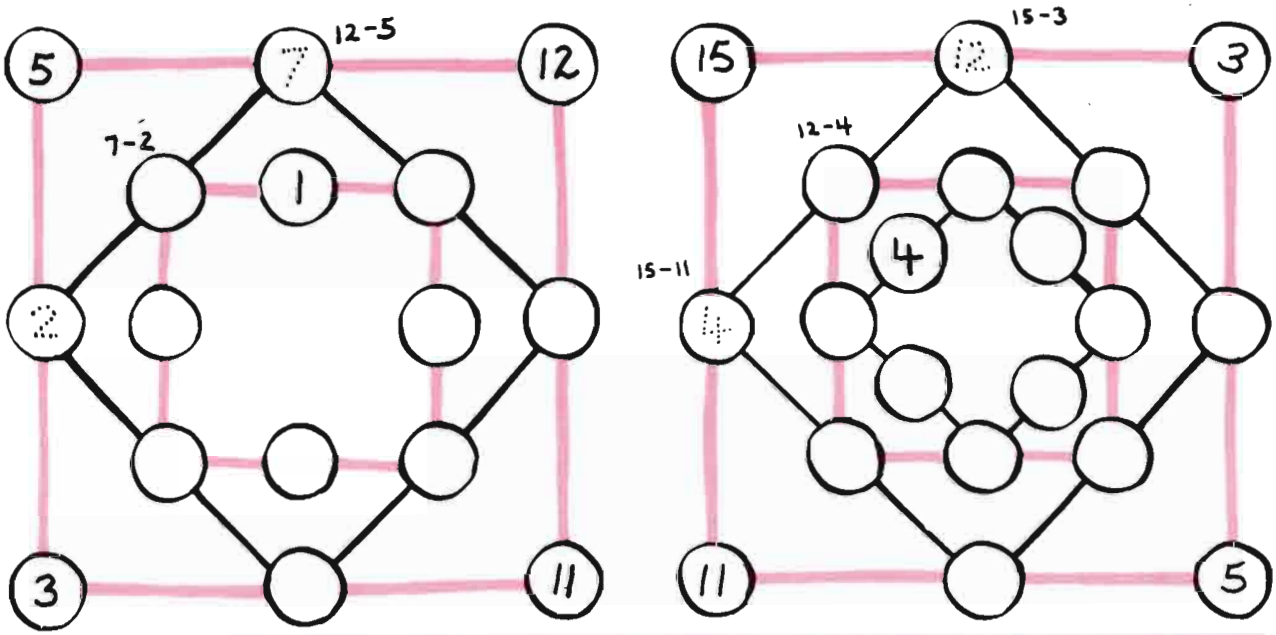
_____

_____

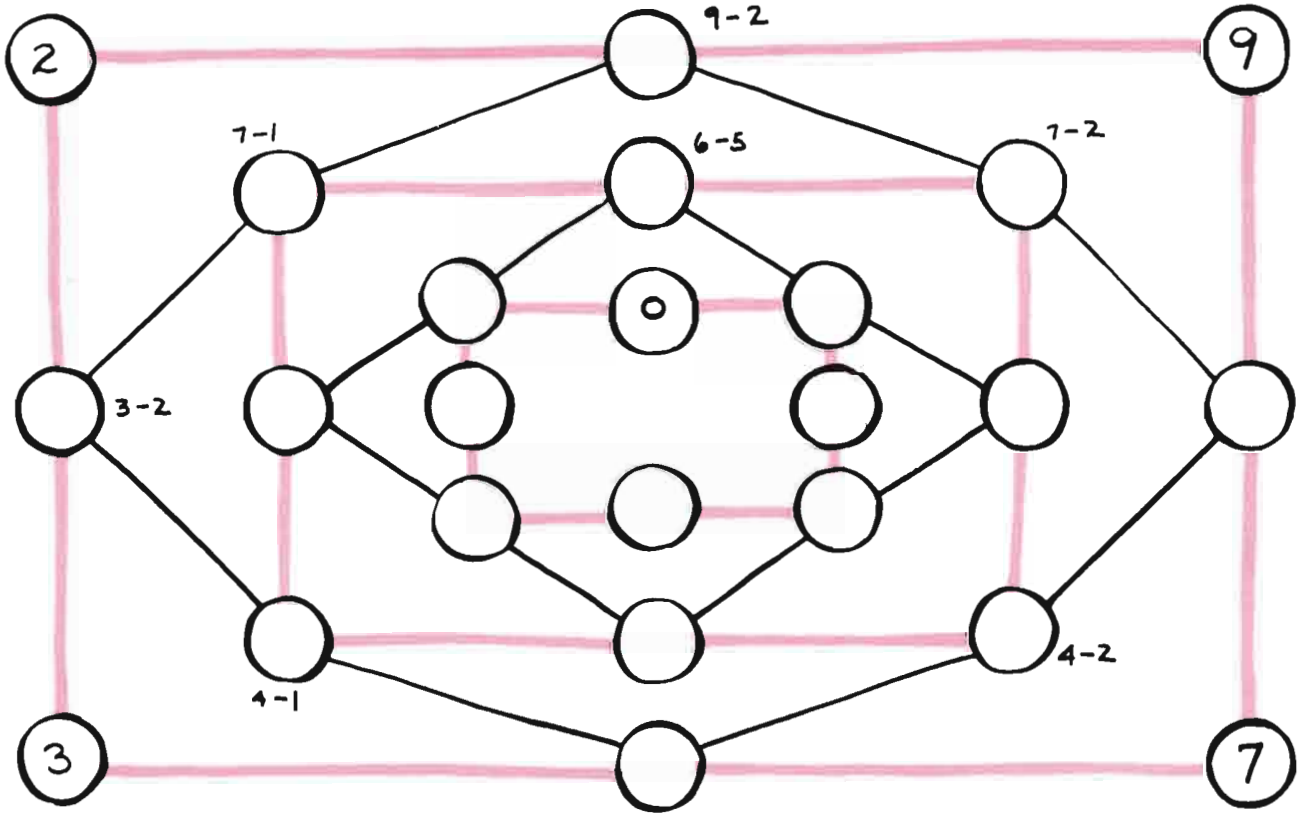


A Study of DIFFERENCES
. . . the "DIFFY" game

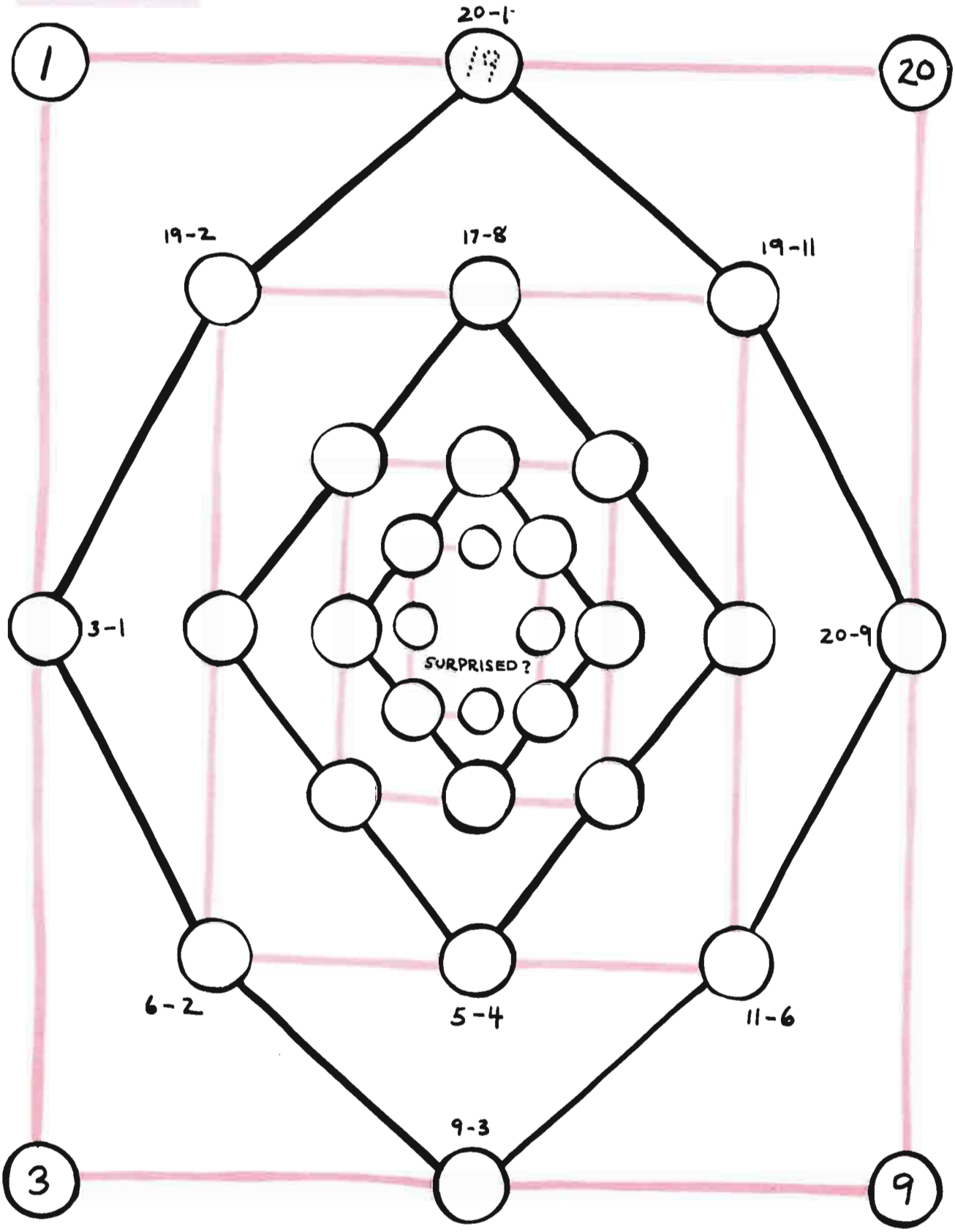
Start with a number in each corner and work toward the center.



Imagine the next example as a square that was "squashed down" to save space.



A "BIG DIFFY"



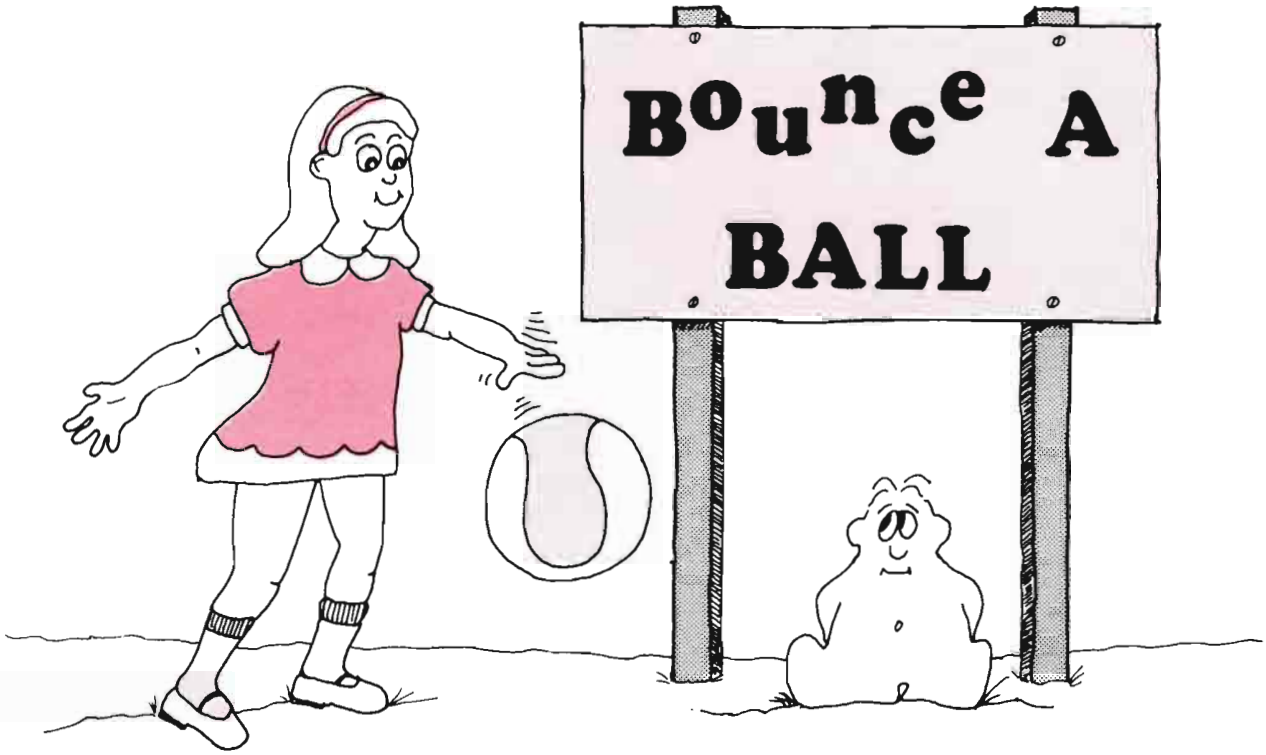
Carla's "Quickies"

3 + 7 = 10	13 + 7	17 + 3	23 + 7	27 + 3	33 + 7	33 +17	33 +57
9 + 4 = 13	19 + 4	29 + 4	24 + 9	34 + 9	39 +14	34 +19	44 +49
14 - 8 = 6	24 - 8	34 - 8	44 - 8	64 - 8	84 - 8	84 -18	84 -38
16 - 9 = 7	26 - 9	36 - 9	36 -19	46 - 9	56 - 9	56 -19	56 -49

Nat's "9 examples out of 3 doubles"

starters	3 +3 = 6	5 +5 = 10	7 +7 = 14
	33 +33	37 +37	53 +53
	35 +35	55 +55	73 +73
	57 +57	75 +75	77 +77

starters	6 +6 = 12	7 +7 = 14	9 +9 = 18
	76 +76	67 +67	66 +66
	96 +96	77 +77	69 +69
	99 +99	97 +97	79 +79



How many times can you bounce
a ball in 30 seconds? _____

... in 1 minute? _____

try bouncing the ball as few
times as possible in 30 seconds.

What is the difference between
your two scores? _____

Hidden Numbers

In the chart below, please color in all multiples of 2 larger than 2, multiples of 3 larger than 3, multiples of 5 larger than 5, and multiples of 7 larger than 7. Then circle all numbers not colored in.

Multiples of

(2), 4, 6, 8, 10, 12, ___, ___, ___, ___, ___ ....

(3), 6, 9, 12, ___, ___, ___, ___, ___, ___, ___ ....

(5), 10, 15, 20, ___, ___, ___, ___, ___, ___, ___ ....

(7), 14, 21, 28, ___, ___, ___, ___, ___, ___, ___ ....

	(2)	(3)	4	(5)	6	(7)	8	9	10
(11)	12	(13)	14	15	16	(17)	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Please list all numbers not colored in . . . from smallest to largest.

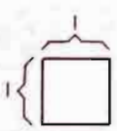
2, 3, 5, 7, 11, 13, 17, ___, ___, ___, ___, ___, ___,

___, ___, ___, ___, ___, ___, ___, ___, ___, ___, ___, ___ ...

"factors" "product"
2 x 5 = 10

Please circle those numbers below that can be expressed as the product of pairs of factors in only 2 different ways.

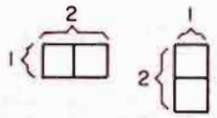
1



1 x 1

less than 2 ways

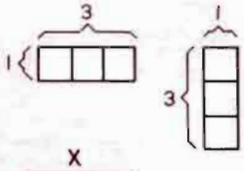
(2)



1 x 2 2 x 1

exactly 2 ways

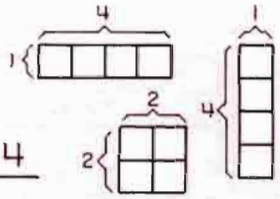
(3)



x x

exactly 2 ways

4



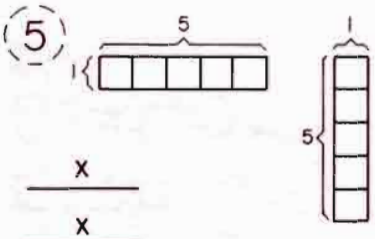
1 x 4

4 x

2 x

more than 2 ways

(5)

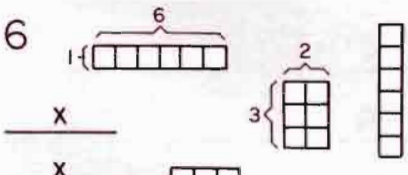


x

x

exactly 2 ways

6



x

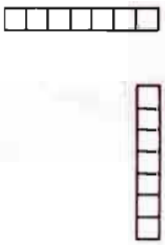
x

x

x

more than 2 ways

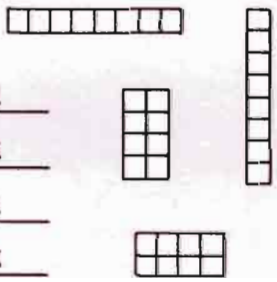
7



x

x

8



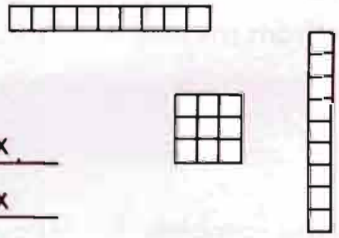
x

x

x

x

9

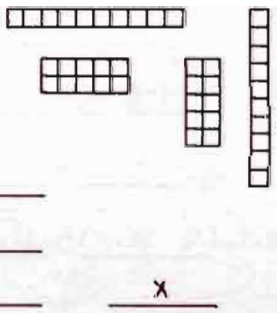


x

x

x

10



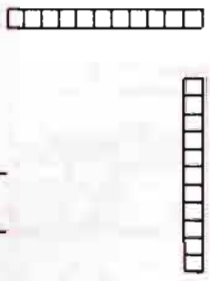
x

x

x

x

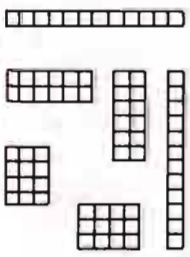
11



x

x

12



x

x

x

x

x

x

Please list the numbers circled: 2, 3, ____, ____, ____

These numbers are called "Prime Numbers" . . . and there are many more that can be expressed as the product of pairs of factors in only 2 different ways.

Please continue the search. When in doubt, make your own sketches.
If you find more than 2 pairs of factors then the number is not "prime."

(13)	=	1 x 13	= 13 x 1	=	x		
14	=	1 x 14	= 14 x 1	=	2 x 7		
15	=	x	=	x	=	x	
16	=	x	=	x	=	x	
17	=	x	=	x	=	x	
18	=	x	=	x	=	x	
19	=	x	=	x	=	x	
20	=	x	=	x	=	x	
21	=	x	=	x	=	x	

22	=	x	=	x	=	x	
23	=	x	=	x	=	x	
24	=	x	=	x	=	x	
25	=	x	=	x	=	x	
26	=	x	=	x	=	x	
27	=	x	=	x	=	x	
28	=	x	=	x	=	x	
29	=	x	=	x	=	x	
30	=	x	=	x	=	x	

Please list the "prime numbers" you have found on these pages
. . . from smallest to largest:

2 , 3 , ____ , ____ , ____ , ____ , ____ , ____ , ____ , 29

Please complete the following as true statements
. . . using only numbers "from the list" . . . "prime numbers."

+	=		+	=		+	=	
−	=		−	=		−	=	
+	=		−	=		+	=	
+	=	+	+	=	+	+	=	+
−	=	−	−	=	−	−	=	−
+	=	−	−	=	+	+	=	−

x = ____ ÷ = ____ Can you find even one example?

RUNNING AROUND

FIND A BASEBALL DIAMOND!

It takes _____ leaps to reach 1st base.

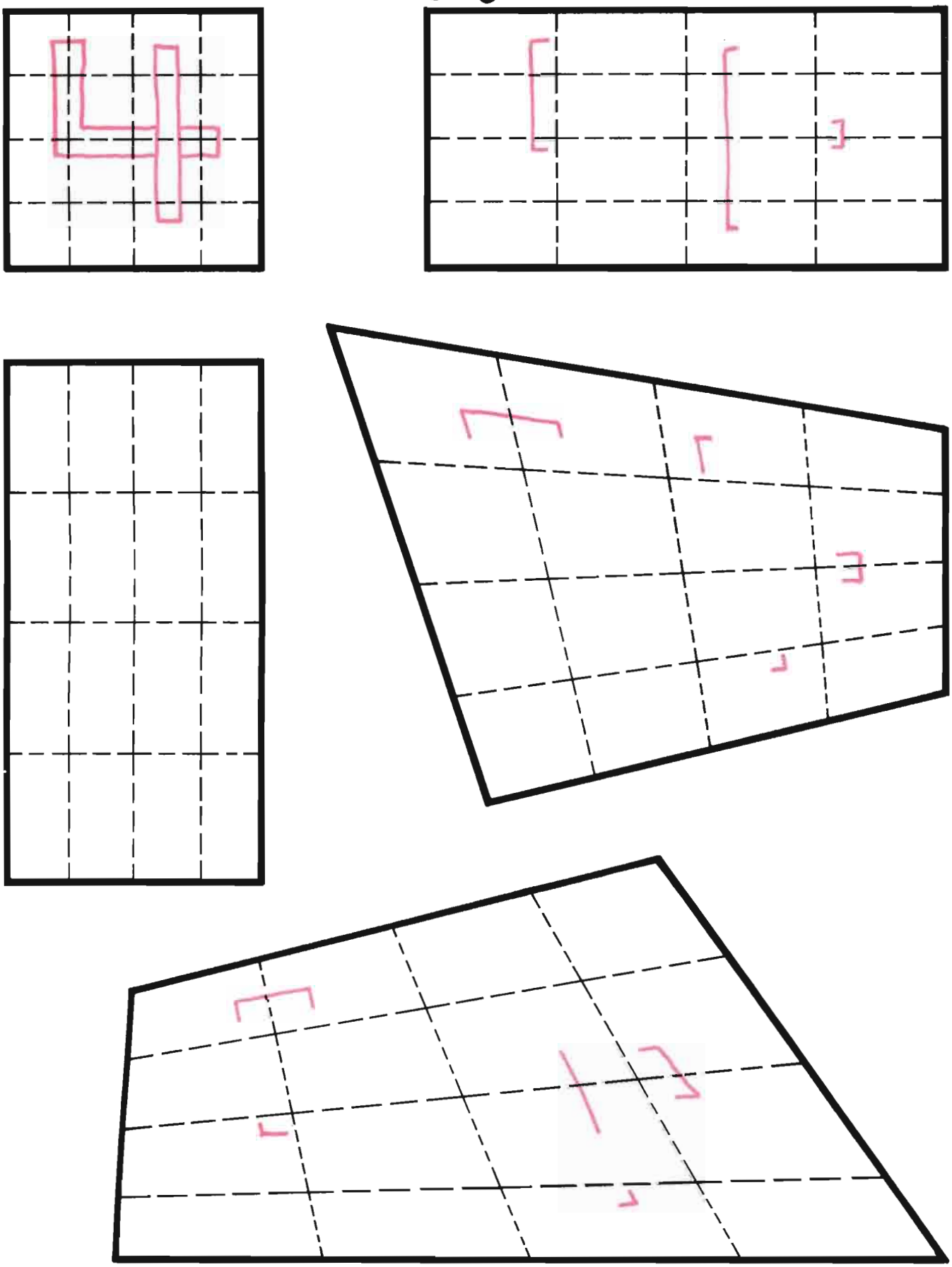
It takes _____ leaps to make a homerun.

Running backwards: _____ steps to make a homerun.

It is _____ feet from home plate to the pitcher's mound.

What other discoveries can you make?

Changing Shapes



In the chart below, please circle and color in all the "Prime Numbers." (Color in 2, 3, 5, 7, 11, 13, etc.)

	(2)	(3)	4	(5)	6	(7)	8	9	10
(11)	12	(13)	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Whole numbers larger than 1 that are "not prime numbers" are called COMPOSITE numbers. Each composite number can be written as the product of 2 or more primes. Please fill in the blanks below with "primes" only.

4 = 2 x 2

6 = 2 x ___

8 = 2 x 2 x ___

9 = x

10 = x

12 = 2 x 2 x ___

14 = x ___

15 = x ___

16 = 2 x 2 x x

18 = 2 x x ___

20 = x x

21 = x ___

22 = x ___

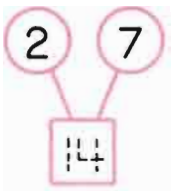
24 = x x x

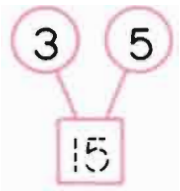
25 = x ___

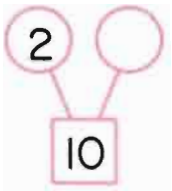
Primes: 2 3 5 7 11 13 17 19 23 29 etc.

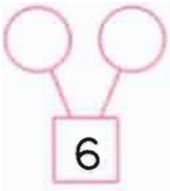
Composites: 4 6 8 9 10 12 14 15 16 18 etc.

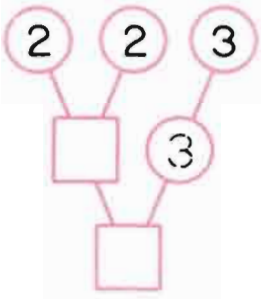
What's My Rule?

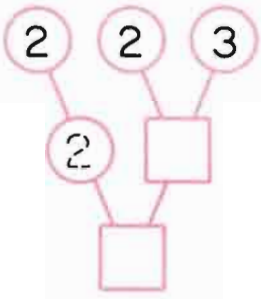


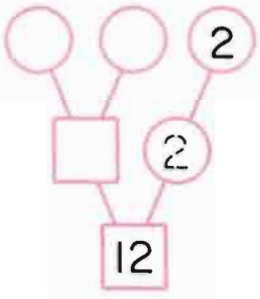


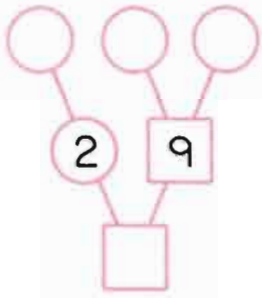


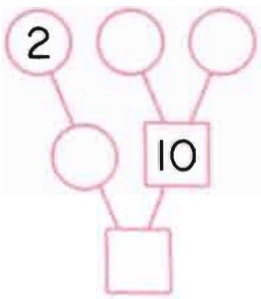


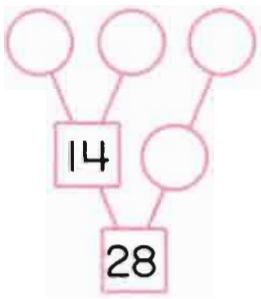


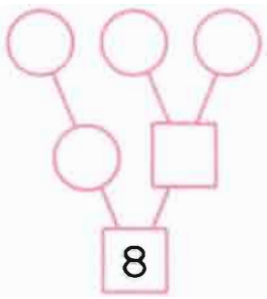


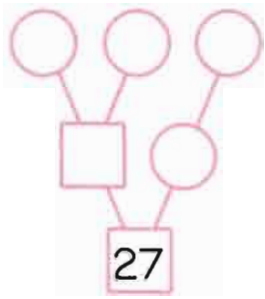


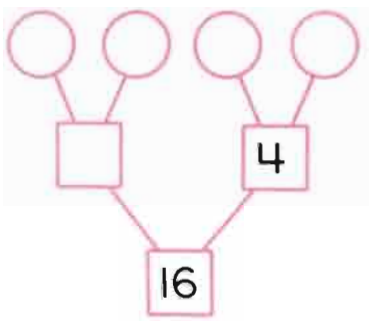


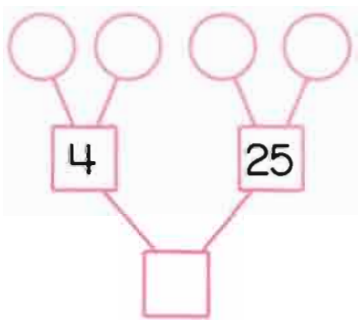


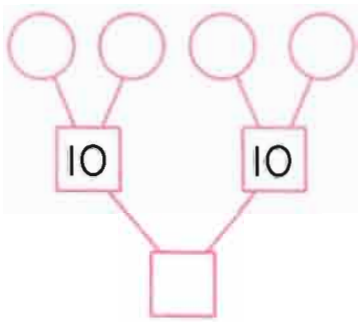


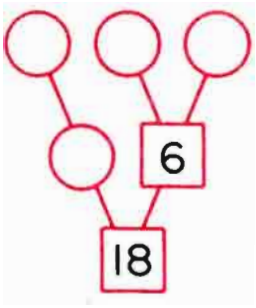


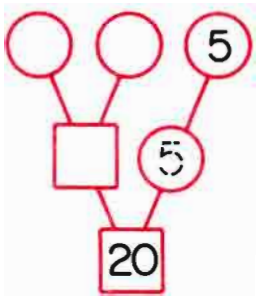


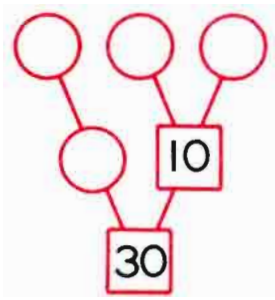


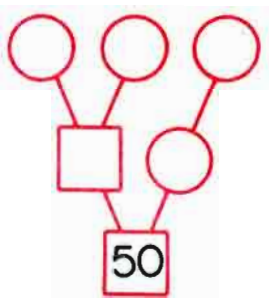


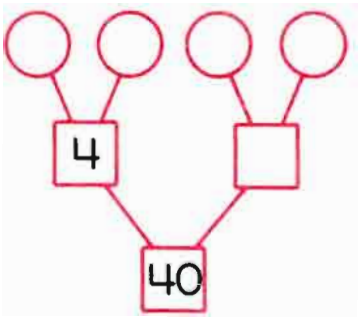


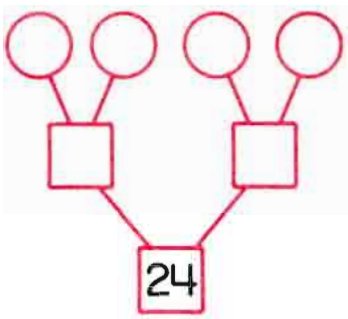


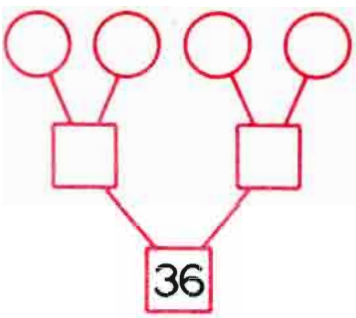




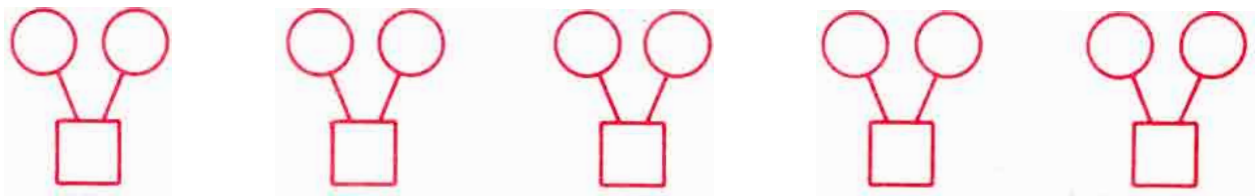


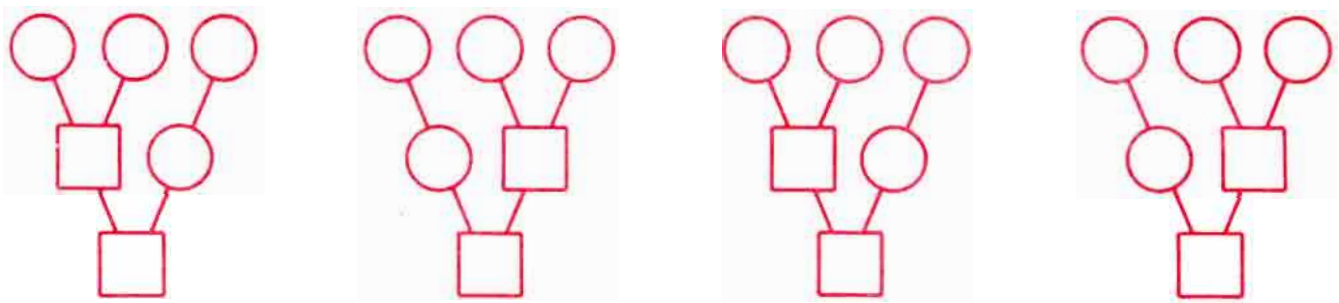


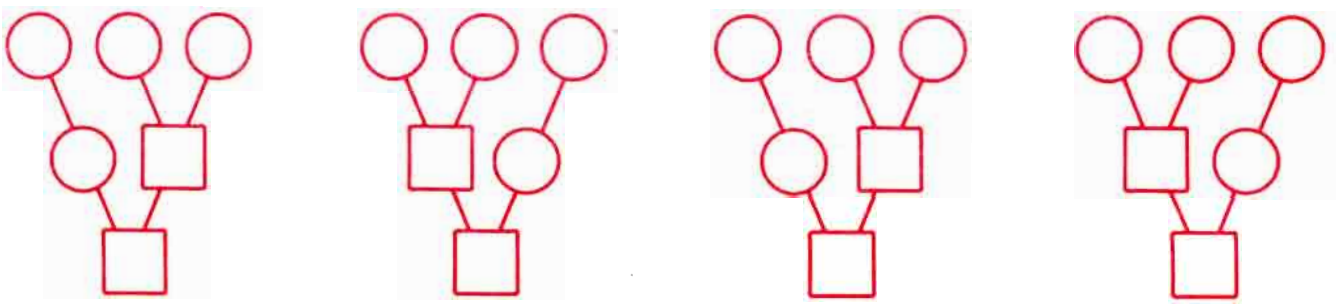




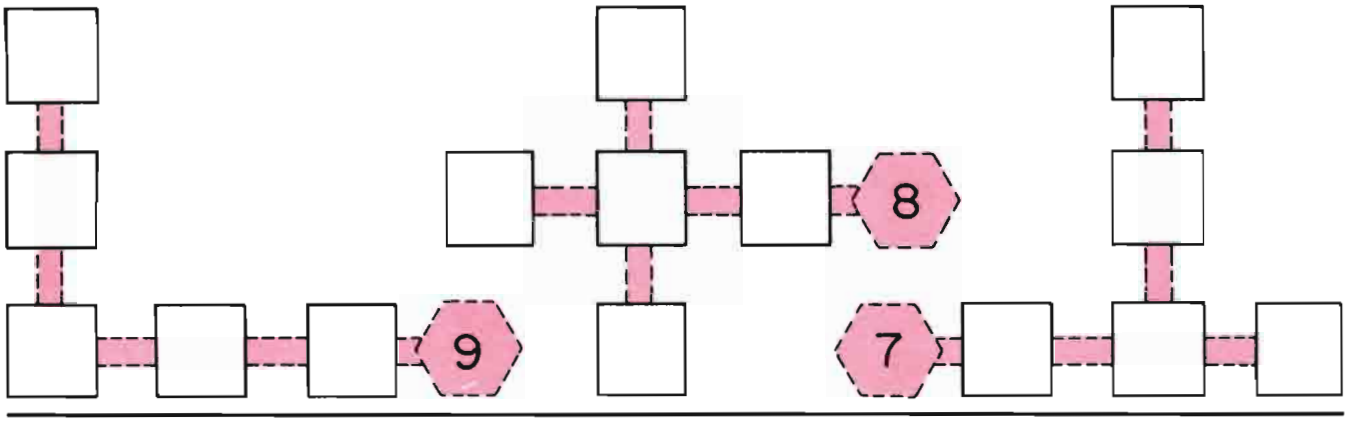
Please make up some of your own examples.

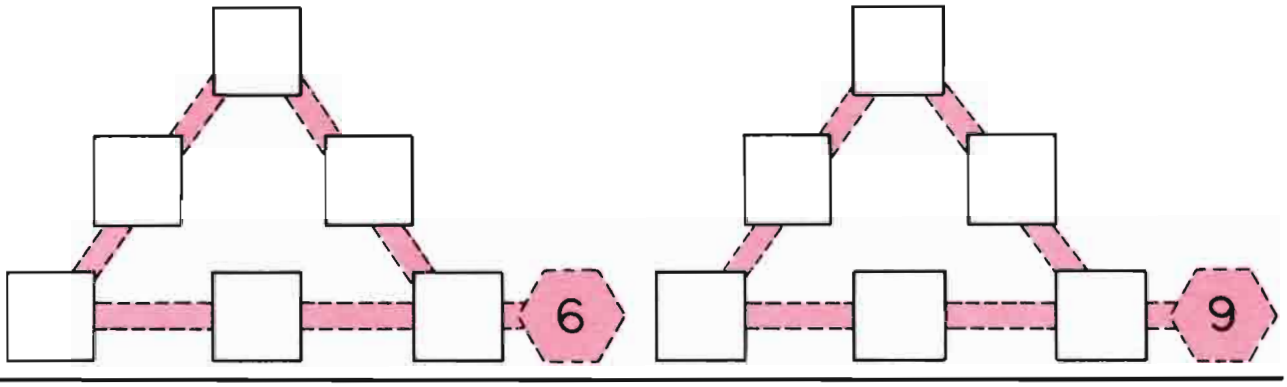


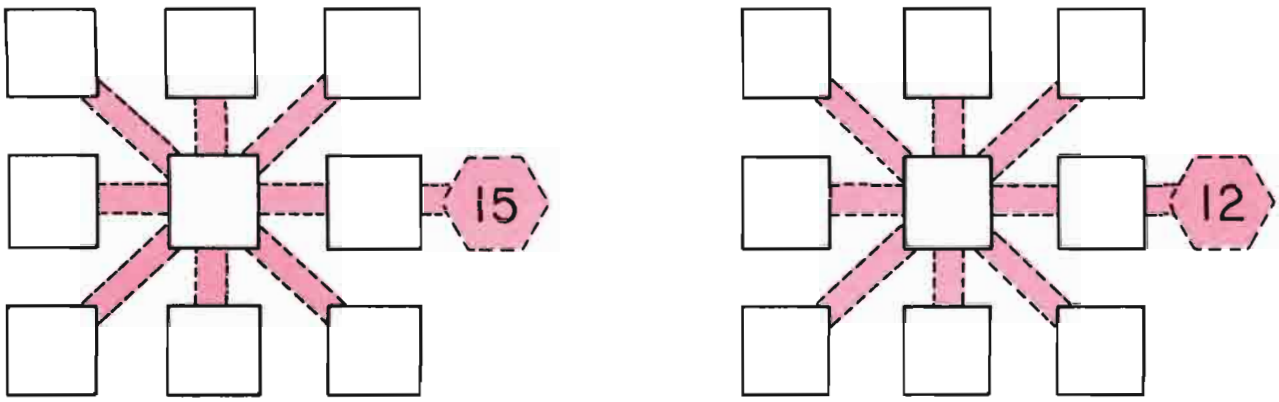




Please arrange different numbers in each example so each line of 3 numbers adds to the numbers given in the hexagon.





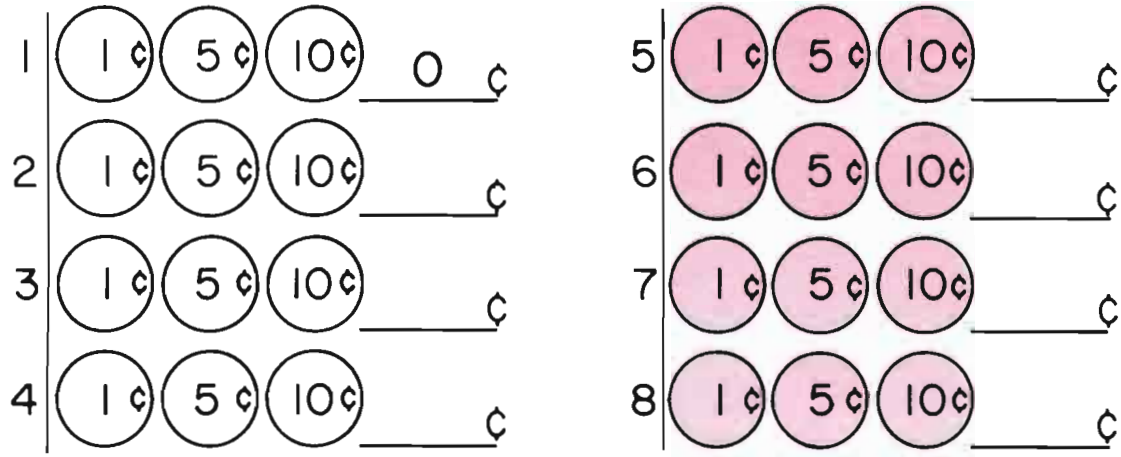


cut-outs may help find the solution

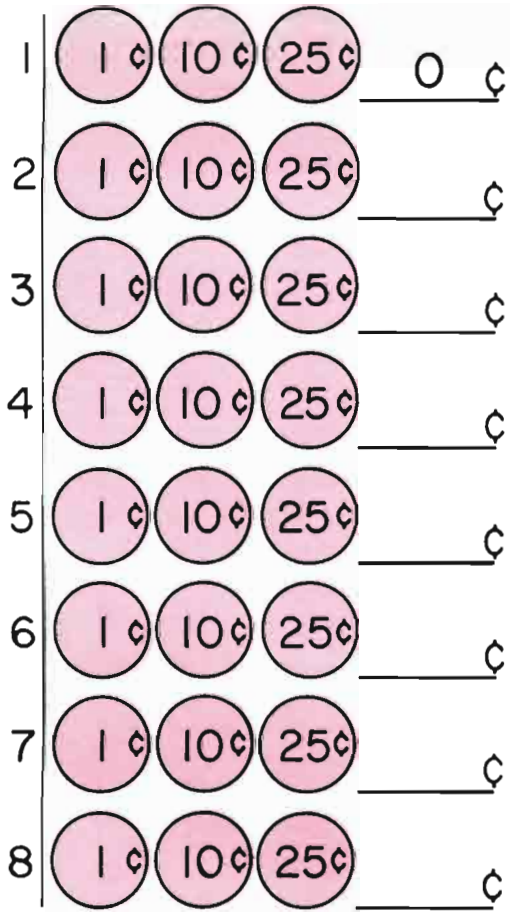
0	1	2	3	4	5	6	7	8	9	10

VENDING MACHINE ARITHMETIC

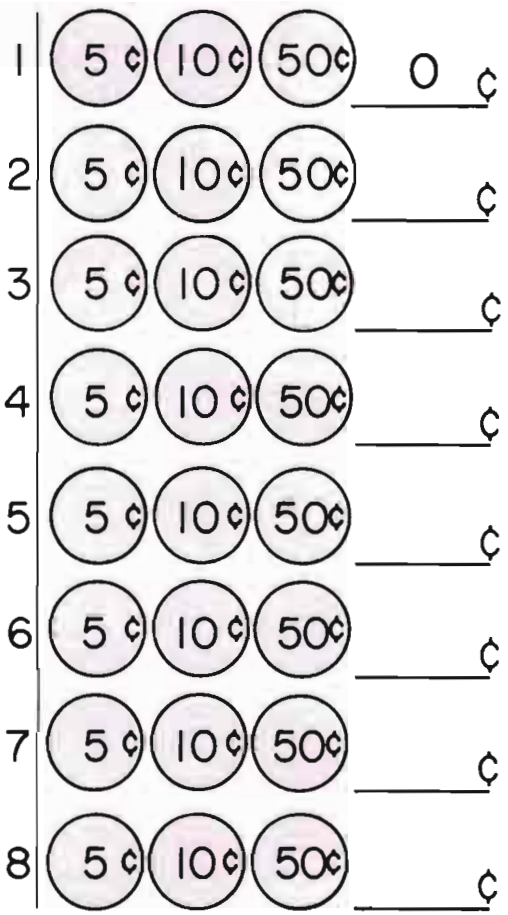
Given a combination of coins, how many different purchases can you make?



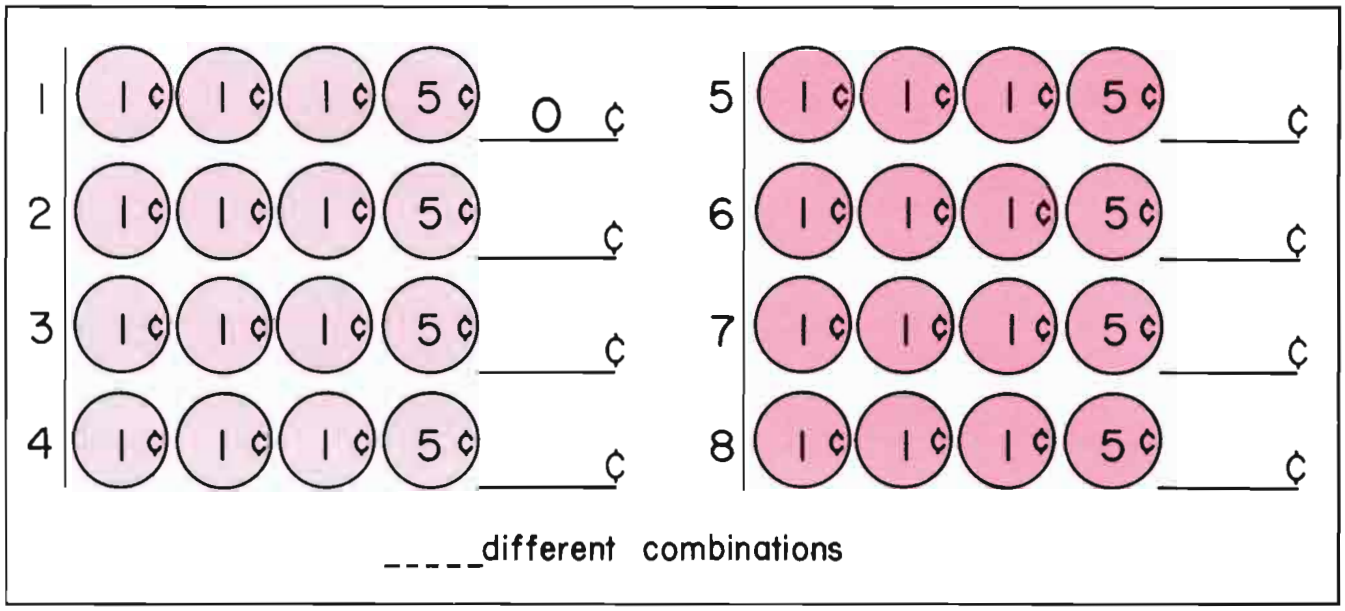
____different combinations

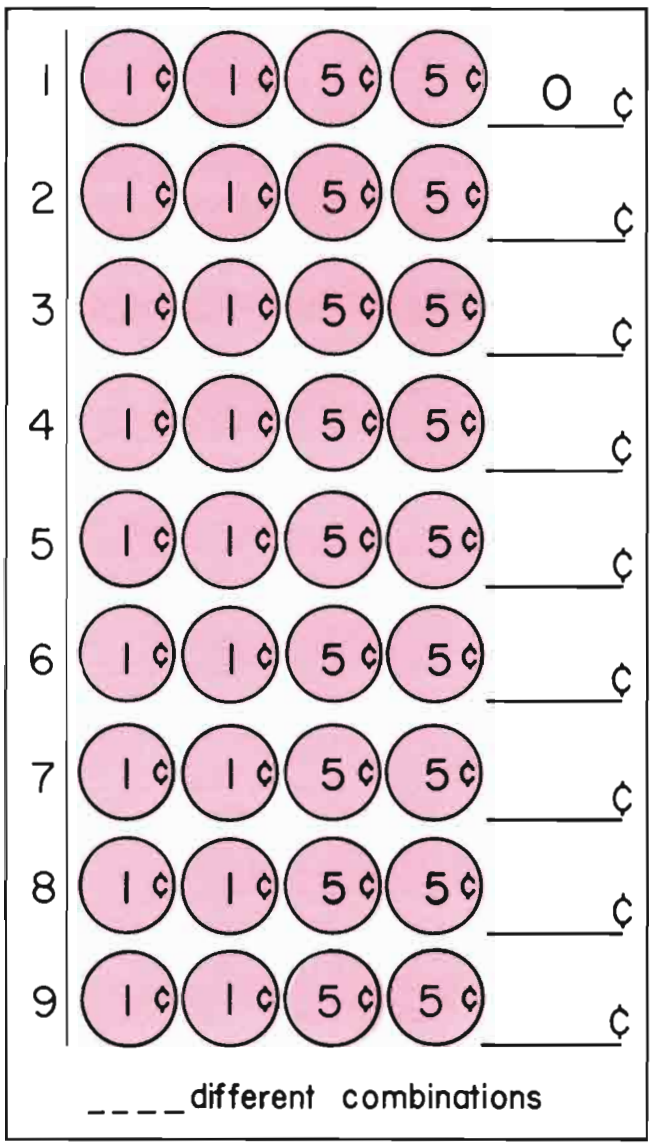


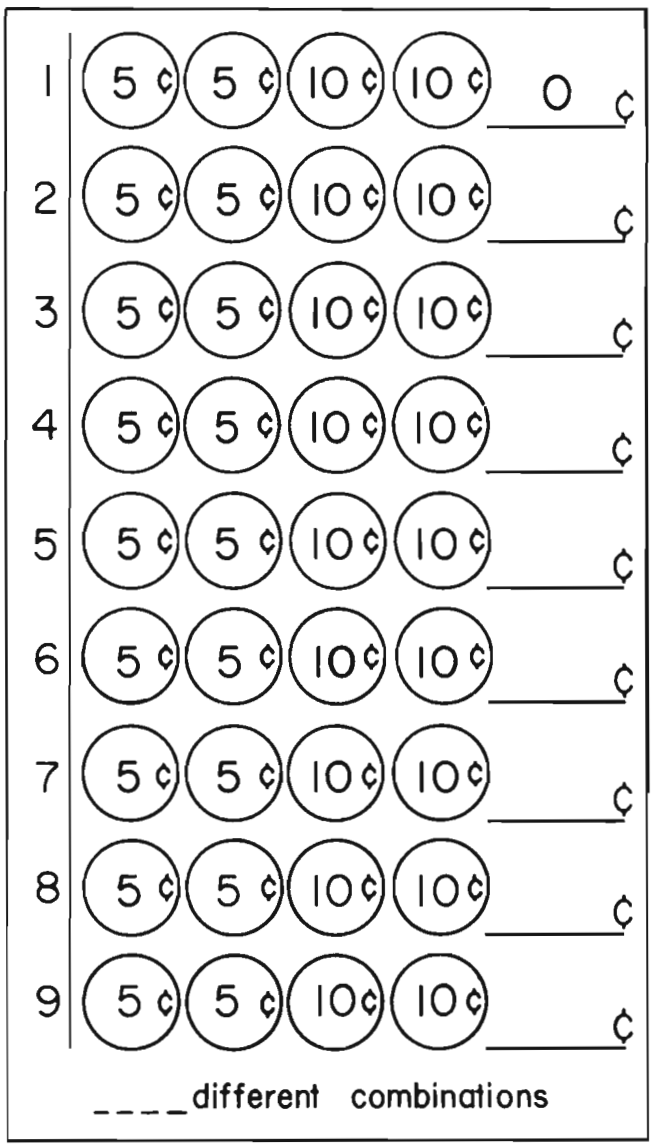
____different combinations

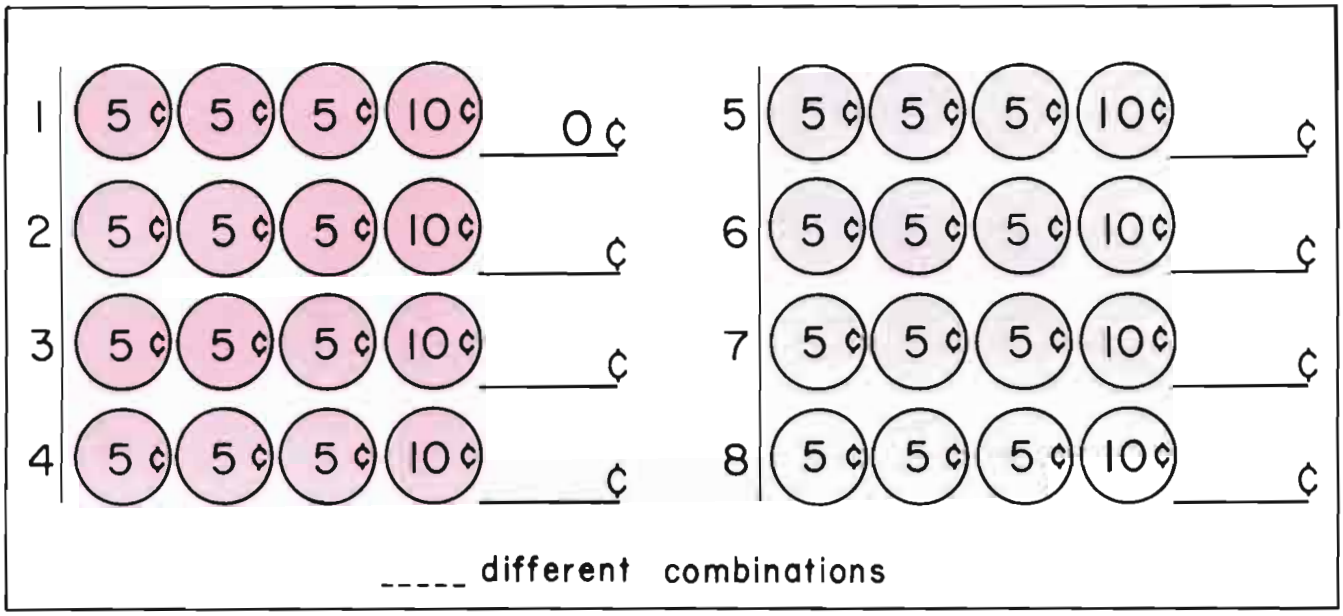


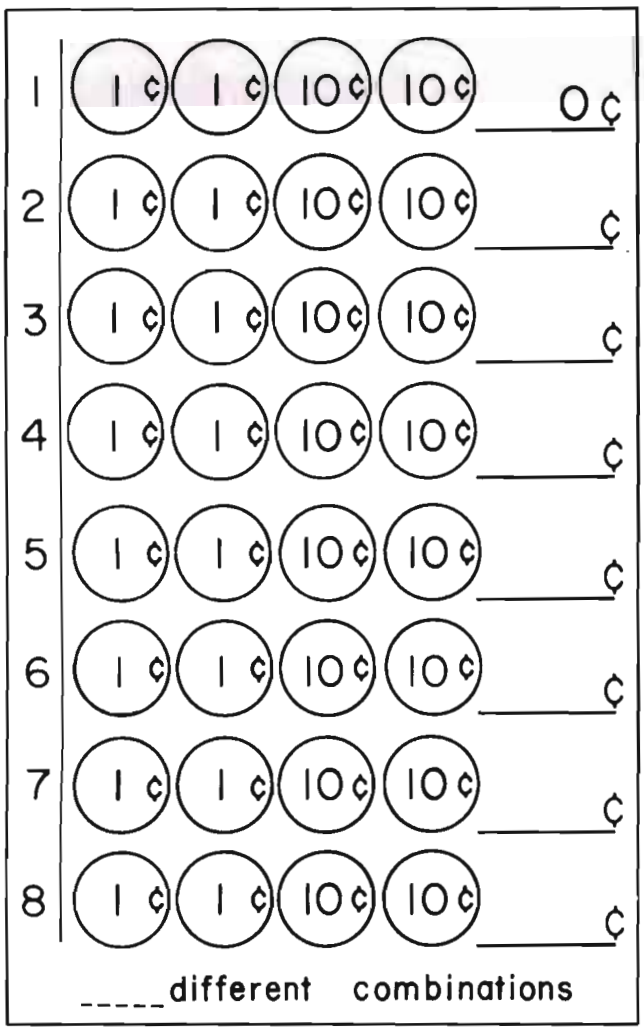
____different combinations

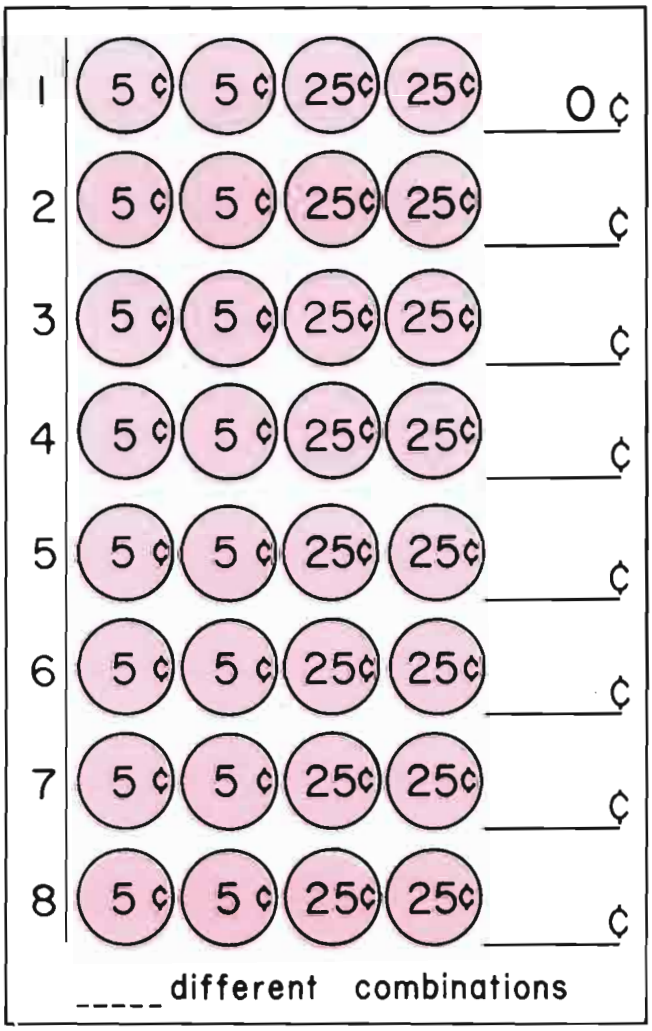

1 1¢ 1¢ 1¢ 5¢ 0 ¢
2 1¢ 1¢ 1¢ 5¢ ¢
3 1¢ 1¢ 1¢ 5¢ ¢
4 1¢ 1¢ 1¢ 5¢ ¢
5 1¢ 1¢ 1¢ 5¢ ¢
6 1¢ 1¢ 1¢ 5¢ ¢
7 1¢ 1¢ 1¢ 5¢ ¢
8 1¢ 1¢ 1¢ 5¢ ¢
_____different combinations

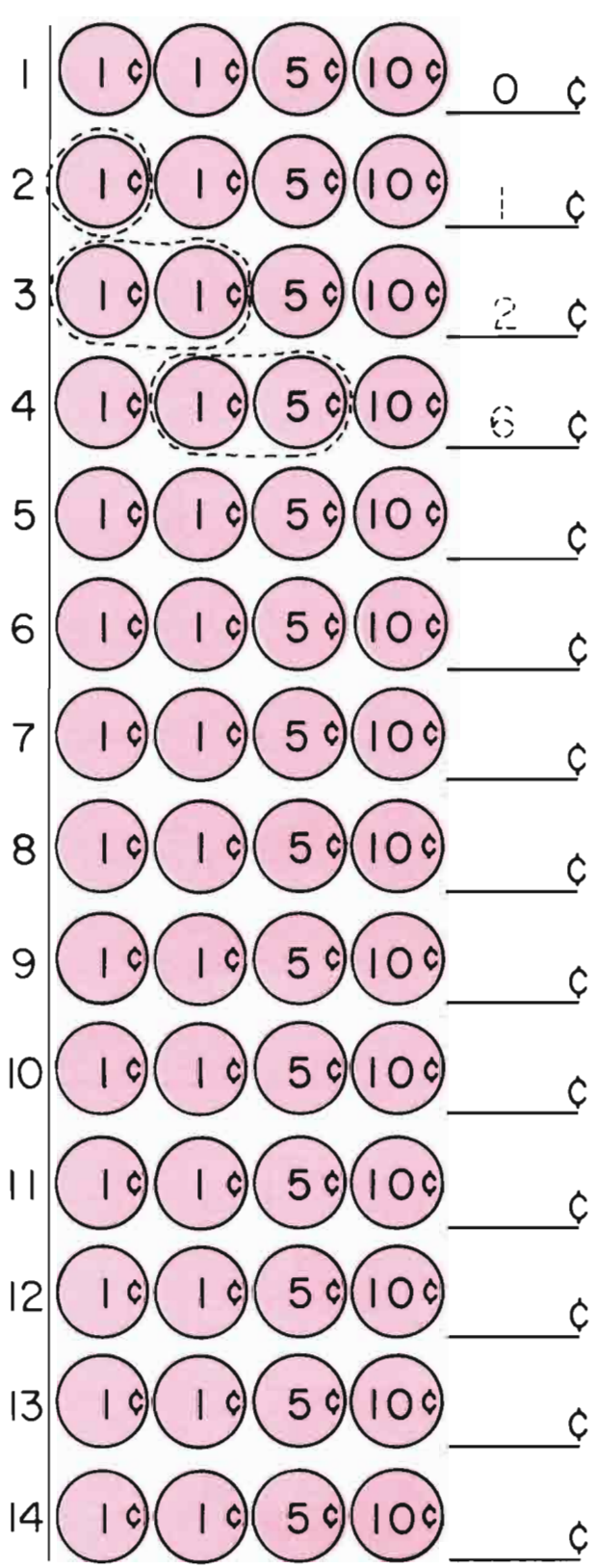

1 1¢ 1¢ 5¢ 5¢ 0 ¢
2 1¢ 1¢ 5¢ 5¢ ¢
3 1¢ 1¢ 5¢ 5¢ ¢
4 1¢ 1¢ 5¢ 5¢ ¢
5 1¢ 1¢ 5¢ 5¢ ¢
6 1¢ 1¢ 5¢ 5¢ ¢
7 1¢ 1¢ 5¢ 5¢ ¢
8 1¢ 1¢ 5¢ 5¢ ¢
9 1¢ 1¢ 5¢ 5¢ ¢
____different combinations


1 5¢ 5¢ 10¢ 10¢ 0 ¢
2 5¢ 5¢ 10¢ 10¢ ¢
3 5¢ 5¢ 10¢ 10¢ ¢
4 5¢ 5¢ 10¢ 10¢ ¢
5 5¢ 5¢ 10¢ 10¢ ¢
6 5¢ 5¢ 10¢ 10¢ ¢
7 5¢ 5¢ 10¢ 10¢ ¢
8 5¢ 5¢ 10¢ 10¢ ¢
9 5¢ 5¢ 10¢ 10¢ ¢
____different combinations

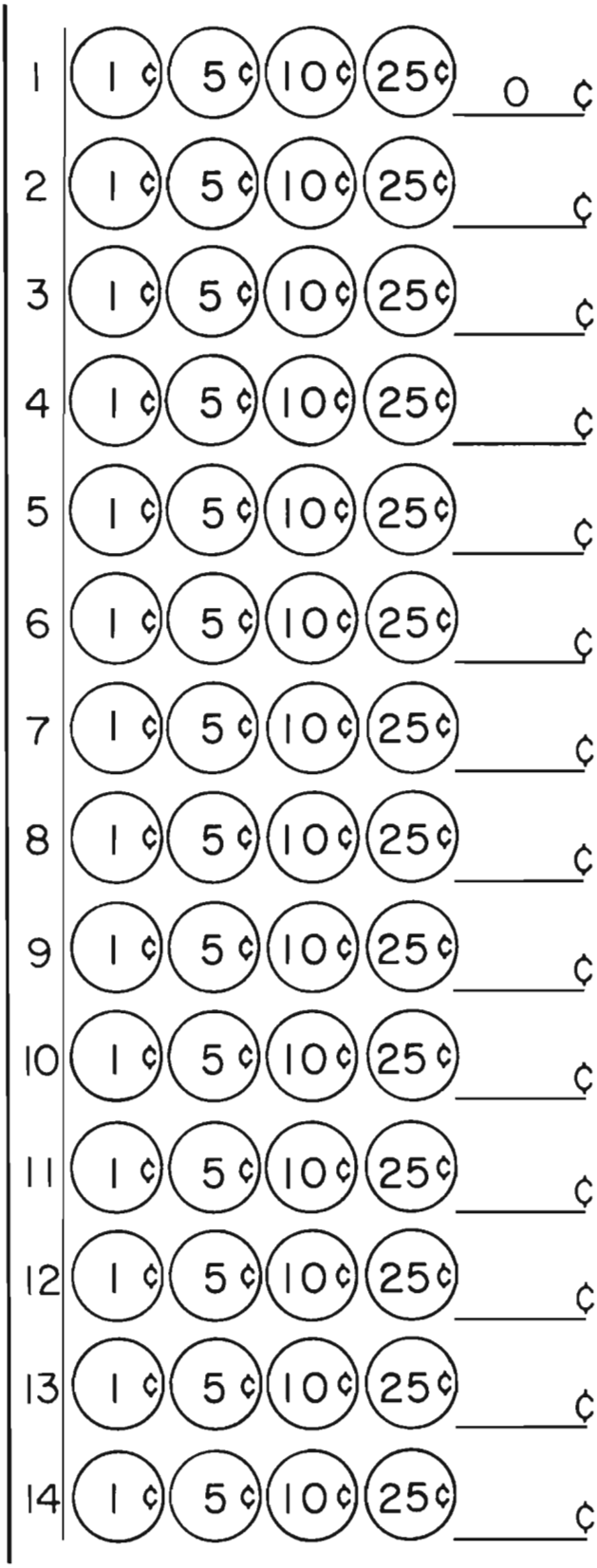

1 5¢ 5¢ 5¢ 10¢ 0¢
2 5¢ 5¢ 5¢ 10¢ ¢
3 5¢ 5¢ 5¢ 10¢ ¢
4 5¢ 5¢ 5¢ 10¢ ¢
5 5¢ 5¢ 5¢ 10¢ ¢
6 5¢ 5¢ 5¢ 10¢ ¢
7 5¢ 5¢ 5¢ 10¢ ¢
8 5¢ 5¢ 5¢ 10¢ ¢
_____ different combinations


1 1¢ 1¢ 10¢ 10¢ 0¢
2 1¢ 1¢ 10¢ 10¢ ¢
3 1¢ 1¢ 10¢ 10¢ ¢
4 1¢ 1¢ 10¢ 10¢ ¢
5 1¢ 1¢ 10¢ 10¢ ¢
6 1¢ 1¢ 10¢ 10¢ ¢
7 1¢ 1¢ 10¢ 10¢ ¢
8 1¢ 1¢ 10¢ 10¢ ¢
_____different combinations


1 5¢ 5¢ 25¢ 25¢ 0¢
2 5¢ 5¢ 25¢ 25¢ ¢
3 5¢ 5¢ 25¢ 25¢ ¢
4 5¢ 5¢ 25¢ 25¢ ¢
5 5¢ 5¢ 25¢ 25¢ ¢
6 5¢ 5¢ 25¢ 25¢ ¢
7 5¢ 5¢ 25¢ 25¢ ¢
8 5¢ 5¢ 25¢ 25¢ ¢
_____different combinations

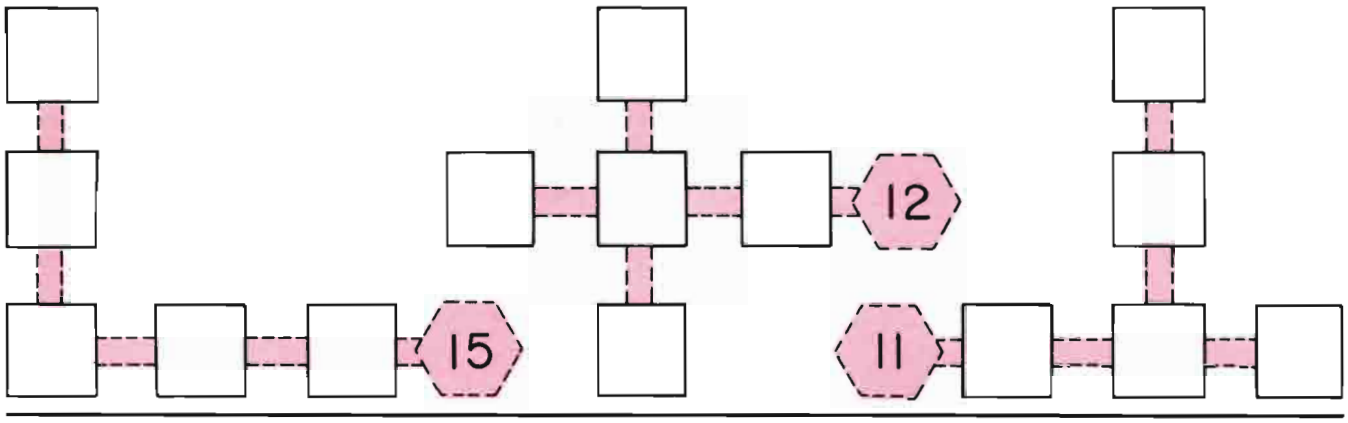


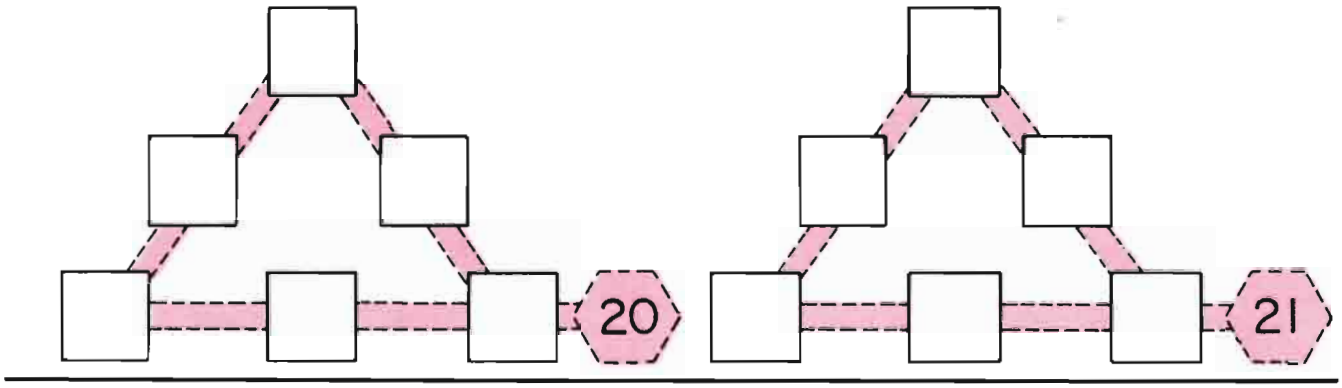
number of
different combinations _____

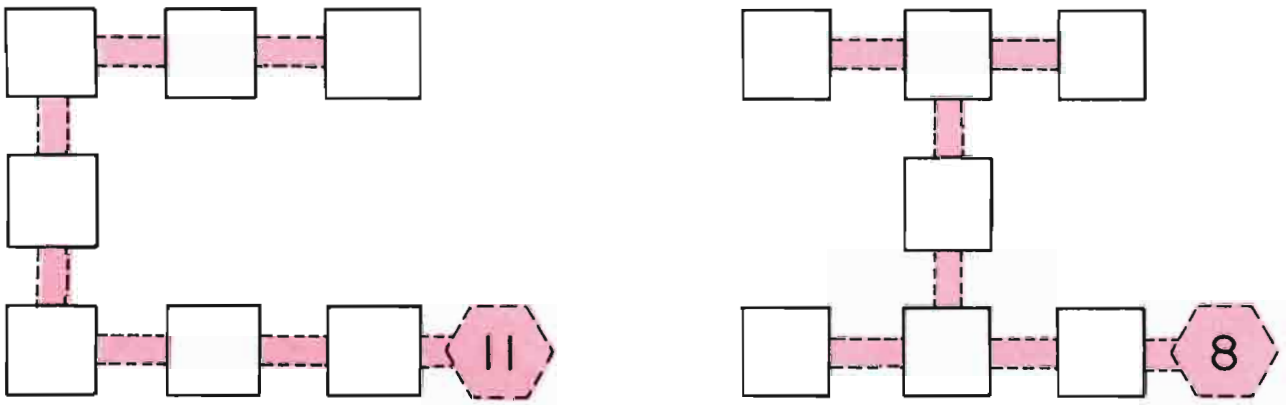


number of
different combinations _____

Please arrange different numbers in each example so each line of 3 numbers adds to the numbers given in the hexagon.



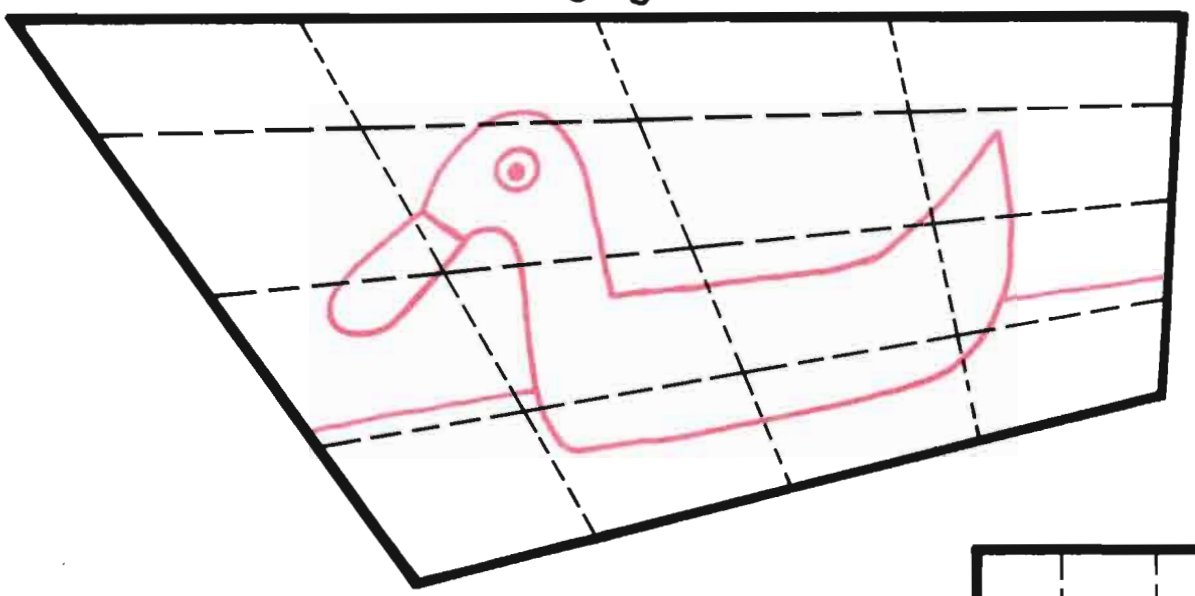


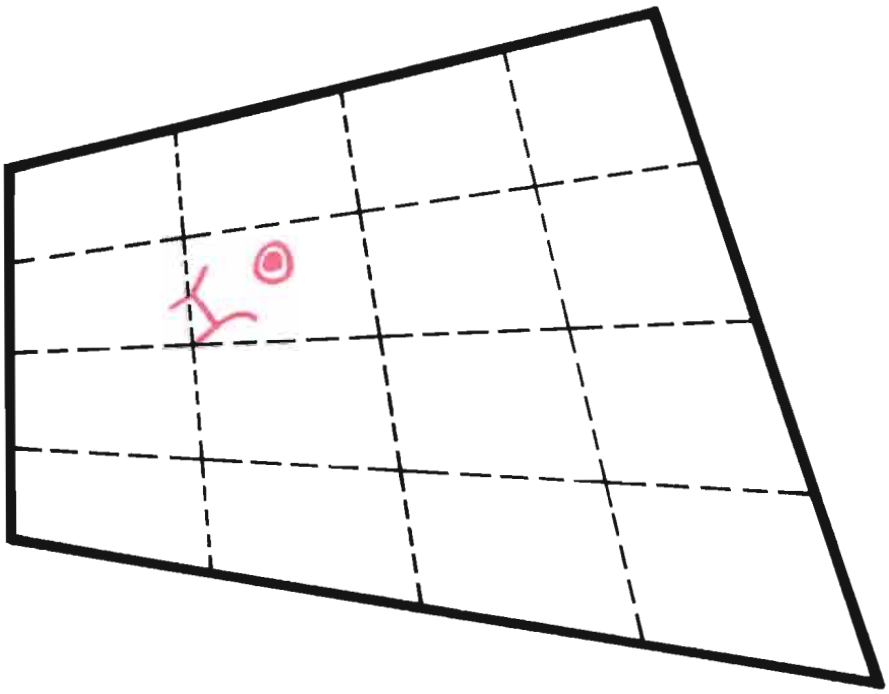


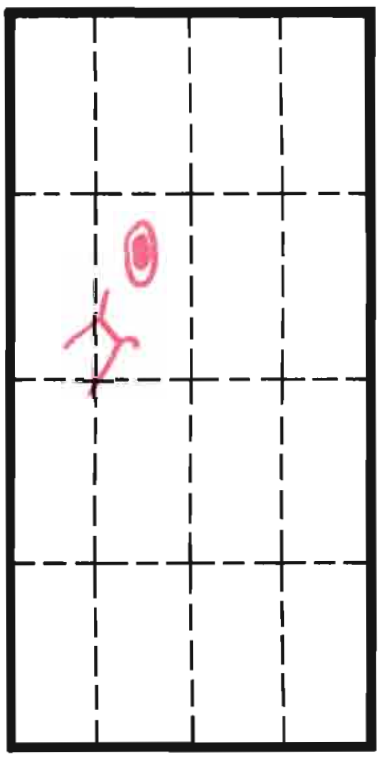
cut-outs may help find the solution

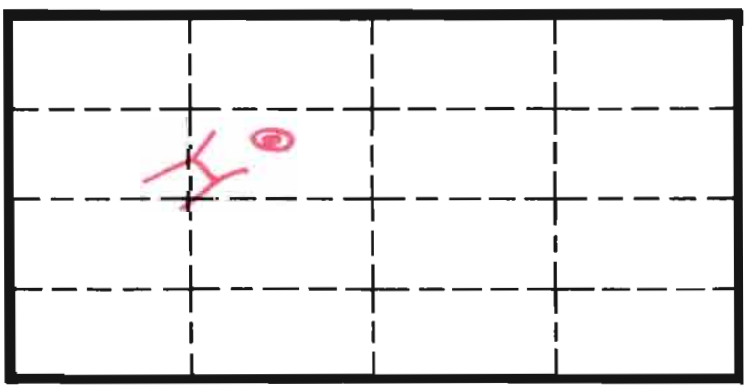
0	1	2	3	4	5	6	7	8	9	10

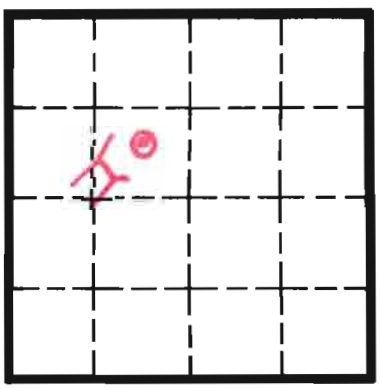
Changing Shapes











Carla's "Quickies"

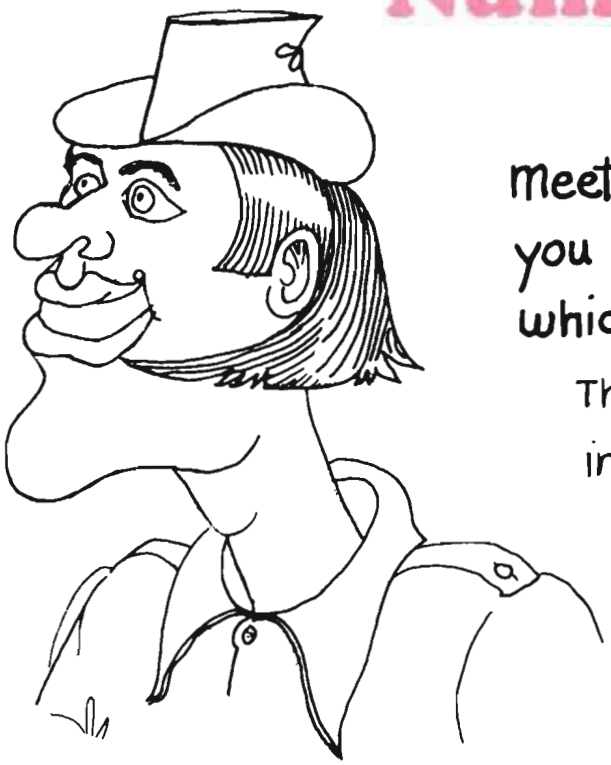
7 + 7 = 14	17 + 7	27 + 7	37 + 7	47 + 7	67 + 7	77 + 7	77 + 17
14 - 7 = 7	24 - 7	34 - 7	54 - 7	84 - 7	84 - 17	74 - 17	94 - 7
6 + 3 = 9	16 + 3	26 + 3	26 + 13	26 + 53	36 + 33	56 + 43	43 + 56
17 - 9 = 8	27 - 9	37 - 9	47 - 9	67 - 9	67 - 19	67 - 29	87 - 29

Nat's "9 examples out of 3 doubles"

starters	2 + 2 = 4	1 + 1 = 2	7 + 7 = 14
	21 + 21	11 + 11	17 + 17
	12 + 12	72 + 72	22 + 22
	27 + 27	71 + 71	77 + 77

starters	4 + 4 = 8	5 + 5 = 10	6 + 6 = 12
	44 + 44	45 + 45	46 + 46
	54 + 54	64 + 64	55 + 55
	65 + 65	66 + 66	56 + 56

Drawing from Numbers



Meet Mr. Numberface. Can you find the number from which he was drawn?

The answer may be found in one of the numbers below.

9

7

5

You can make drawings from the other numbers.

8

3

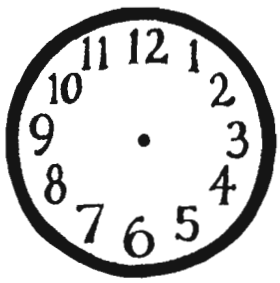
2

2

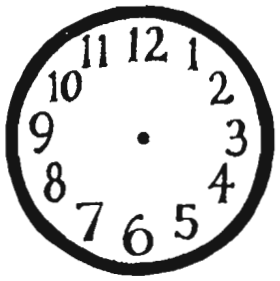
4

Time In Other Places

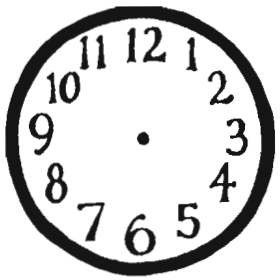
[:] PM
Chicago, Illinois



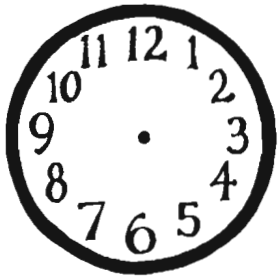
[:]
New York, New York



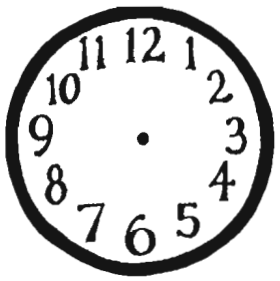
[:]
Los Angeles, California



[:]
Denver, Colorado



[:]
Honolulu, Hawaii



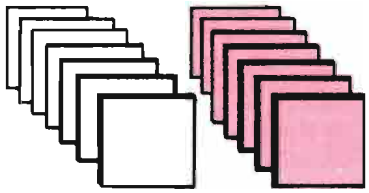
[:]
London, England

If it is 8:00 PM in New York it is:

_____ in New Orleans, Louisiana

_____ in Fairbanks, Alaska

When it is noon in Honolulu, what would you be doing at home? _____



1 tile long:

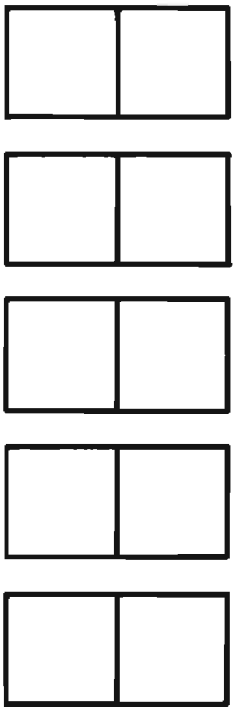
2 different ways

A Problem from Mrs. Burg — who calls herself "Tanya the Tile Setter"

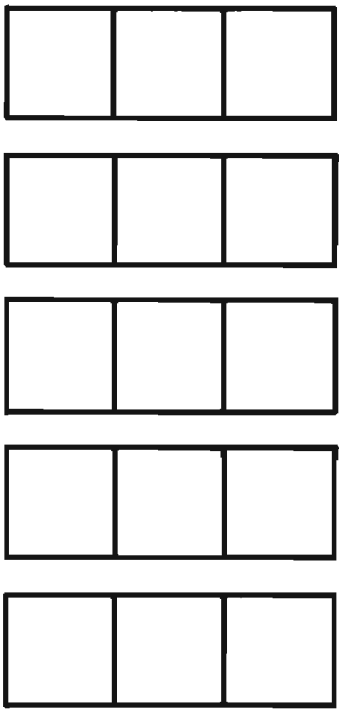
Suppose you have a large supply of white and of colored tile. The rule is that there are never two colored tiles next to each other.

How many different ways can tiles be arranged in rows 1, 2, 3, 4, 5, 6 . . . tiles long.

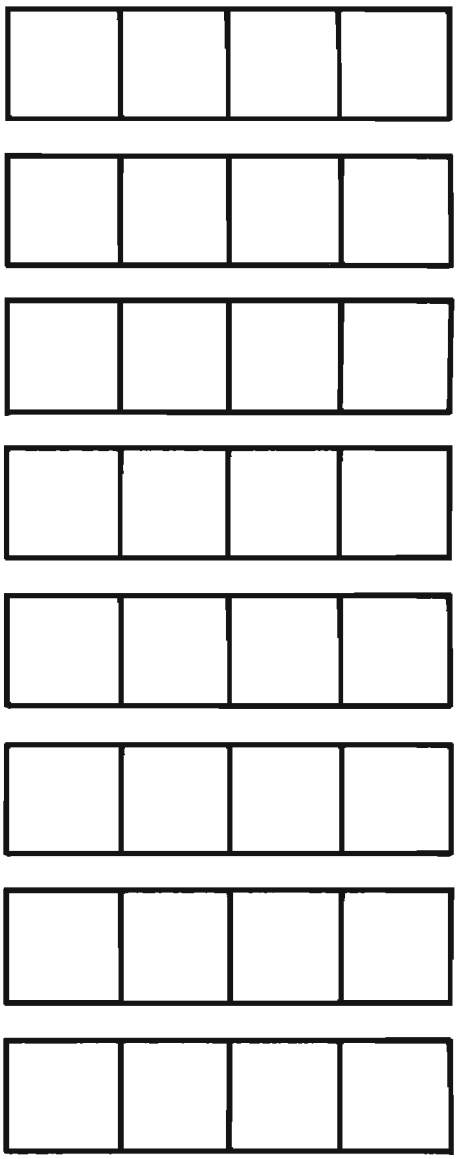
Remember: All white is always an arrangement!



____ different ways



____ different ways



____ different ways

Please summarize your results in this table:

Length of row	1	2	3	4	5
Different ways	2				

Have you a prediction about 5 in a row?

U.S. DEPARTMENT OF COMMERCE
National Bureau of Standards
Washington, D.C. 20234

Letter Circular 1052
February 1974

All You Will Need to Know About Metric

(For Your Everyday Life)

10

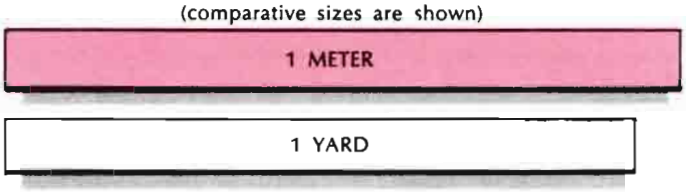
Metric is based on Decimal system

The metric system is simple to learn. For use in your everyday life you will need to learn only ten new units. You will also need to get used to a few new temperatures. There are even some metric units with which you are already familiar: those for time and electricity are the same as you use now.

BASIC UNITS

METER: a little longer than a yard (about 1.1 yards)
LITER: a little larger than a quart (about 1.06 quarts)
GRAM: a little more than the weight of a paper clip

(comparative sizes are shown)



25 DEGREES FAHRENHEIT

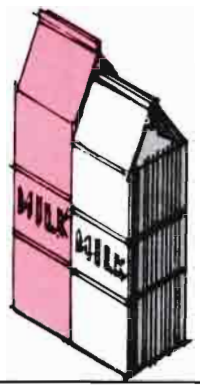
25 DEGREES CELSIUS

COMMON PREFIXES
(to be used with basic units)

milli: one-thousandth (0.001)
centi: one-hundredth (0.01)
kilo: one-thousand times (1000)

For example:
1000 millimeters = 1 meter
100 centimeters = 1 meter
1000 meters = 1 kilometer

1 LITER



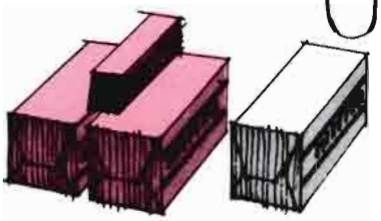
1 QUART

OTHER COMMONLY USED UNITS

millimeter:	0.001 meter	diameter of paper clip wire
centimeter:	0.01 meter	a little more than the width of a paper clip (about 0.4 inch)
kilometer:	1000 meters	somewhat further than ½ mile (about 0.6 mile)
kilogram:	1000 grams	a little more than 2 pounds (about 2.2 pounds)
milliliter:	0.001 liter	five of them make a teaspoon

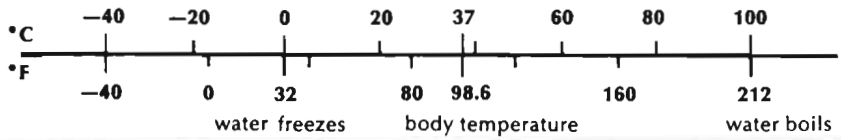
OTHER USEFUL UNITS

hectare: about 2½ acres
tonne: about one ton



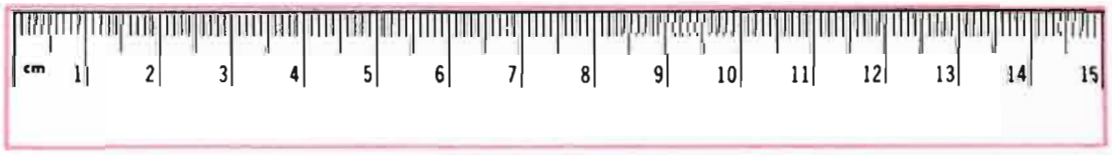
1 KILOGRAM 1 POUND

TEMPERATURE
degrees Celsius are used

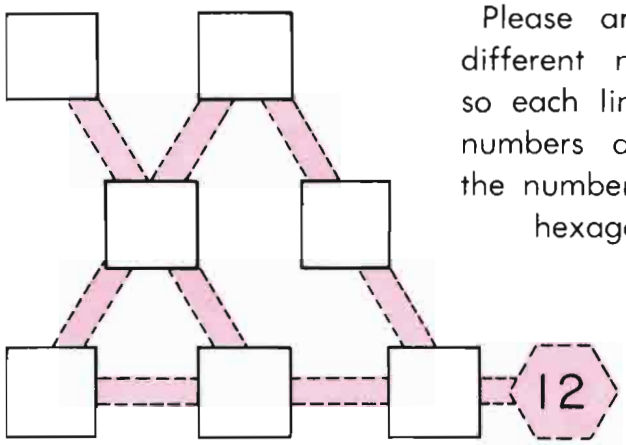


METRIC ME

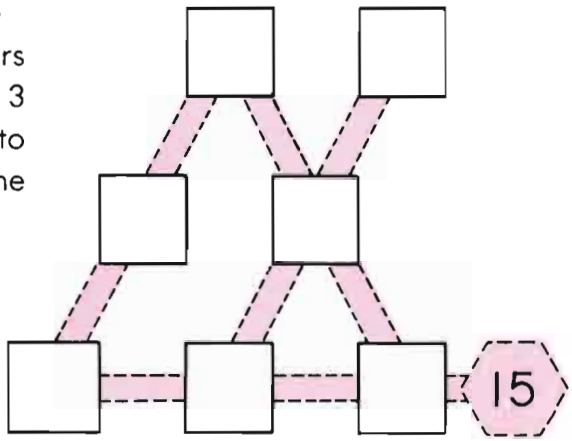
MY LEFT cm	PARTS OF THE BODY	MY RIGHT cm
	length of arm	
	around arm	
	around thumb	
	around index finger	
	length of thumb	
	length of thumbnail	
	around wrist	
	length of leg	
	around leg	
	around ankle	
	length of foot	
	length of big toe	
	length of ear	
	total	

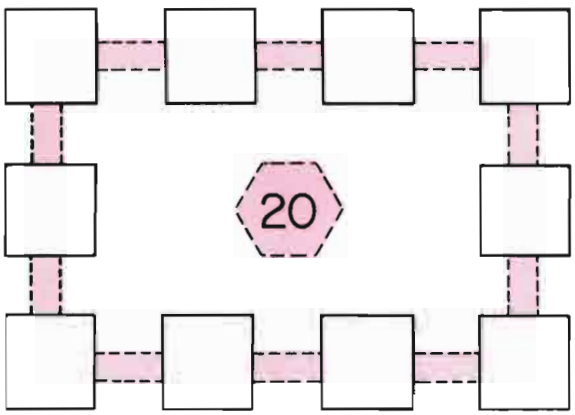


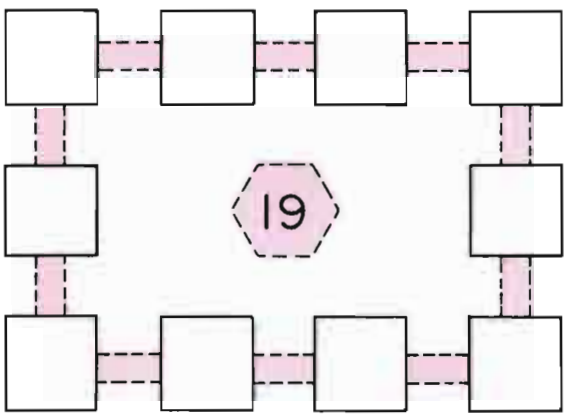
Arrangement PUZZLES with Small Numbers

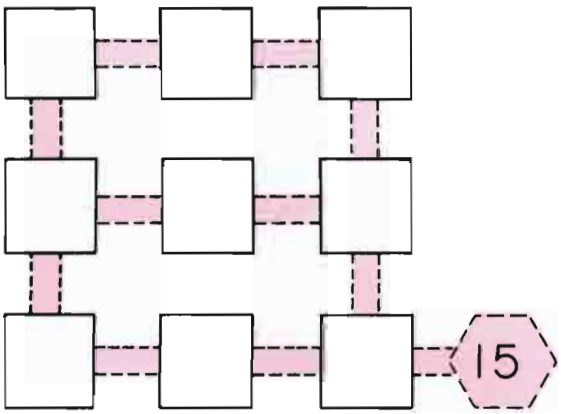


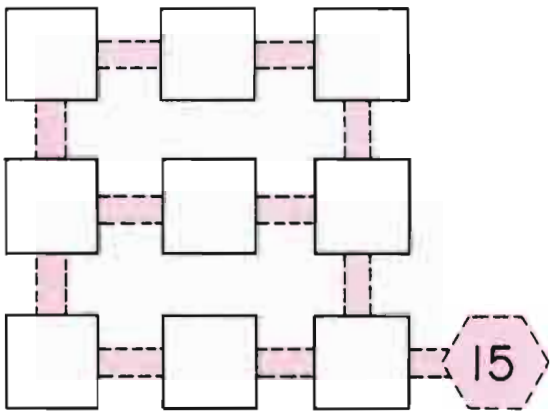
Please arrange different numbers so each line of 3 numbers adds to the number in the hexagon.











0	1	2	3	4	5	6	7	8	9	10	11

cut-outs may help

Time Yourself!

Mark off 100 feet on the playground.
Pick a partner to time you.

NOW RUN as fast as you can!

1st try _____ seconds
2nd " _____ "
3rd " _____ "

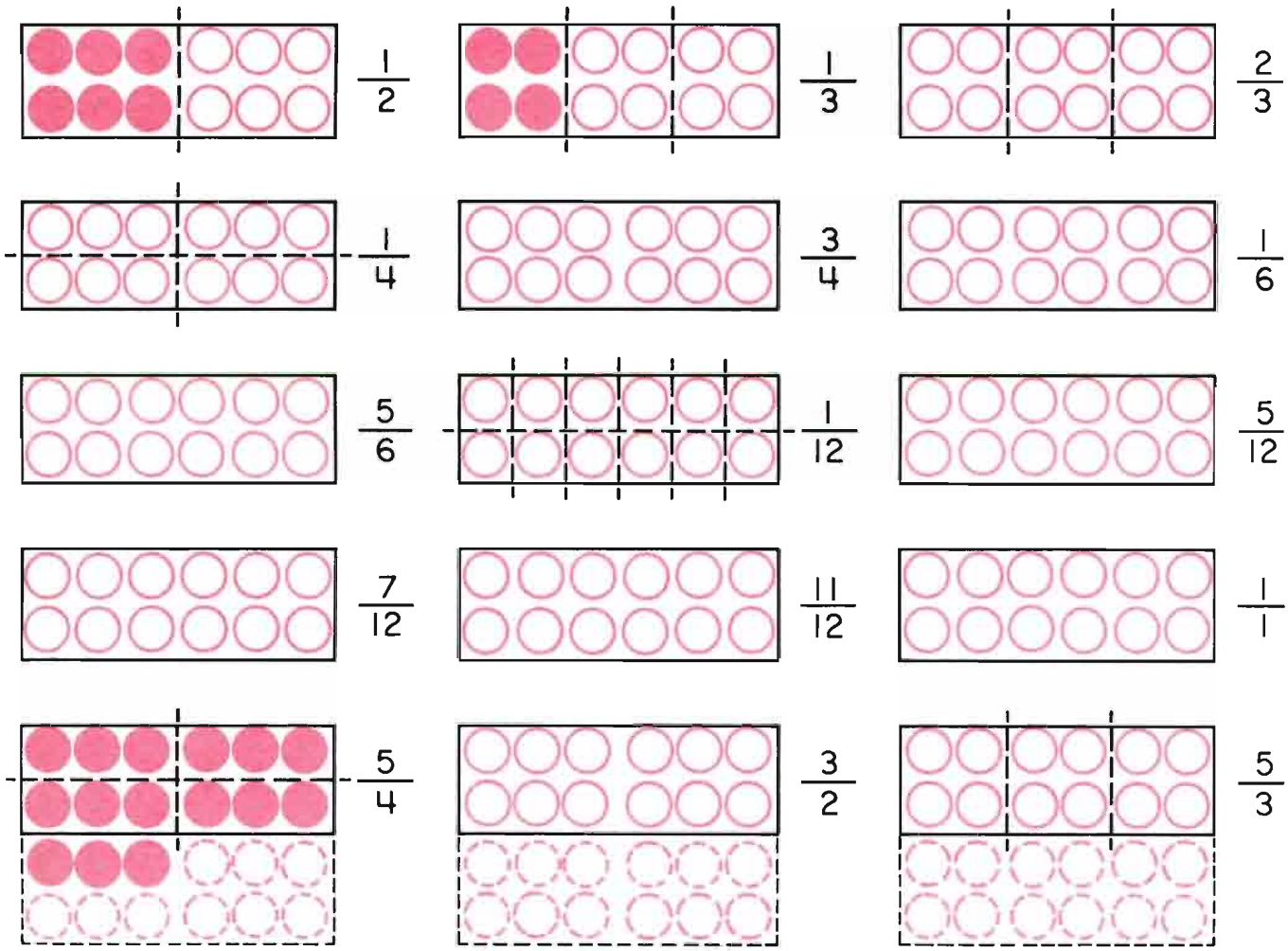
Now try walking 100 feet as fast as you can.
Please note: Walking means one foot on the ground at all times.

1st try _____ seconds
2nd " _____ "
3rd " _____ "

The important thing is not <u>who</u> is fastest
but how much you improve.

All about a dozen eggs.

Please divide each dozen in this many parts → $\frac{3}{4}$ ← Then color in this many parts.



There are ____ eggs in one-half of a dozen eggs.

There are ____ eggs in two-thirds of a dozen eggs.

There are ____ eggs in five-fourths of a dozen eggs . . . or 1 and ¼ dozen.

There are ____ eggs in one dozen eggs. (1/1 dozen can be read as "one dozen").

Please complete this summary:

eggs	6	4										12
dozen(s) of eggs	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{7}{12}$	$\frac{11}{12}$	1

Please complete the list and use it to do the examples below:

eggs	1	2	3	4	5	6	7	8	9	10	11	12	13
doz.	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{3}$		$\frac{1}{2}$							$1\frac{1}{12}$

All about eggs and dozens of eggs:

$3 + 3 = 6$ $\quad$ $4 + \quad = \quad$ $\quad + 6 = \quad$

$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ $\quad$ $\frac{1}{3} + \frac{1}{3} = \quad$ $\quad \frac{1}{4} + \quad = \quad$

$5 + 2 = \quad$ $\quad + 1 = \quad$ $\quad 4 + \quad = \quad$

$\quad + \quad = \quad$ $\quad \frac{2}{3} + \quad = \quad$ $\quad + \frac{1}{6} = \quad$

Please watch the signs!

$6 - 3 = \quad$ $\quad - \quad = \quad$ $\quad 8 - \quad = \quad$

$\frac{1}{2} - \frac{1}{4} = \quad$ $\quad \frac{3}{4} - \frac{1}{4} = \quad$ $\quad - \frac{1}{3} = \quad$

$\quad - \quad = \quad$ $\quad 10 - \quad = \quad$ $\quad 12 - \quad = \quad$

$\frac{7}{12} - \frac{1}{12} = \quad$ $\quad - \frac{1}{4} = \quad$ $\quad 1 - \frac{1}{12} = \quad$

All about dozens of eggs:

$\frac{1}{2} + \frac{1}{4} = \quad$ $\quad \frac{5}{12} + \frac{1}{12} = \quad$ $\quad \frac{1}{4} + \frac{3}{4} = \quad$

$1 - \frac{1}{2} = \quad$ $\quad \frac{2}{3} - \frac{1}{3} = \quad$ $\quad \frac{5}{6} - \frac{1}{2} = \quad$

Please complete the list.

eggs	14	15	16	17	18	19	20	21	22	23	24	25	26
doz.	$1\frac{1}{6}$	$1\frac{1}{4}$		$1\frac{5}{12}$							2	$2\frac{1}{12}$	

All about eggs and dozens of eggs:

4 x 2 = 8	9 x 2 =	x 2 =
$\frac{1}{3}$ x 2 = $\frac{2}{3}$ ("multiplied by 2")	$\frac{3}{4}$ x 2 =	$\frac{5}{12}$ x 2 =
x 3 =	x 3 =	x 4 =
$\frac{1}{6}$ x 3 =	$\frac{1}{3}$ x 3 =	$\frac{1}{3}$ x 4 =

Please watch the signs!

6 ÷ 2 =	÷ 2 =	÷ 2 =
$\frac{1}{2}$ ÷ 2 = ("divided by 2")	$\frac{2}{3}$ ÷ 2 =	$\frac{5}{6}$ ÷ 2 =
÷ 3 =	8 ÷ 4 =	24 ÷ 3 =
$1\frac{1}{2}$ ÷ 3 =	÷ 4 =	÷ 3 =

All about dozens of eggs:

$\frac{1}{4}$ x 2 =	$\frac{2}{3}$ x 2 =	$\frac{1}{2}$ x 3 =
$1\frac{1}{3}$ ÷ 2 =	$\frac{1}{2}$ ÷ 3 =	1 ÷ 4 =

Please complete the list and use it to do the examples below:

eggs	12											
doz. of eggs	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{7}{12}$	$\frac{11}{12}$

All about eggs and dozens of eggs:

3 + 6 =

$\frac{1}{4}$ + $\frac{1}{2}$ = ☐

5 + 1 =

☐ + ☐ = ☐

9 + 3 =

☐ + ☐ = ☐

9 − 6 =

☐ − ☐ = ☐

6 − 3 =

☐ − ☐ = ☐

12 − 8 =

☐ − ☐ = ☐

3 x 2 =

☐ x 2 = ☐

("multiplied by 2")

5 x 2 =

☐ x 2 = ☐

1 x 3 =

☐ x 3 = ☐

6 ÷ 2 =

☐ ÷ 2 = ☐

("divided by 2")

10 ÷ 2 =

☐ ÷ 2 = ☐

8 ÷ 4 =

☐ ÷ 4 = ☐

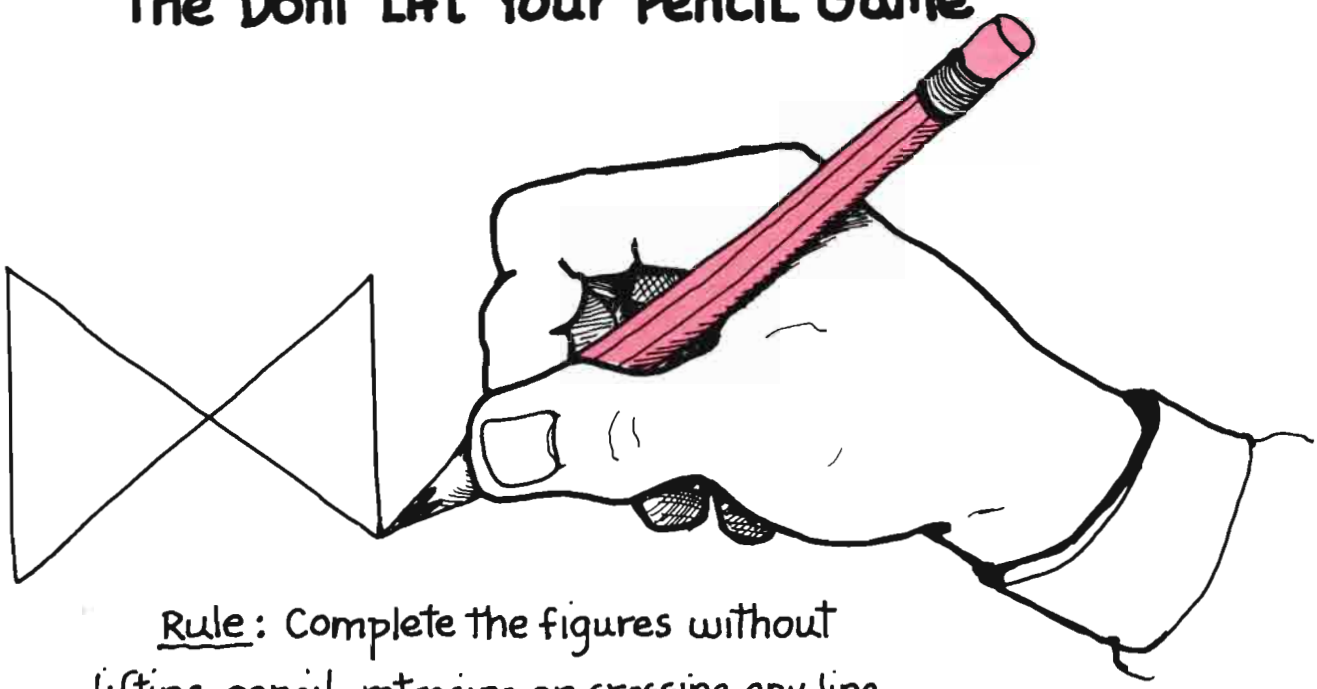
All about dozens of eggs:

$\frac{1}{2}$ + $\frac{1}{4}$ = ☐

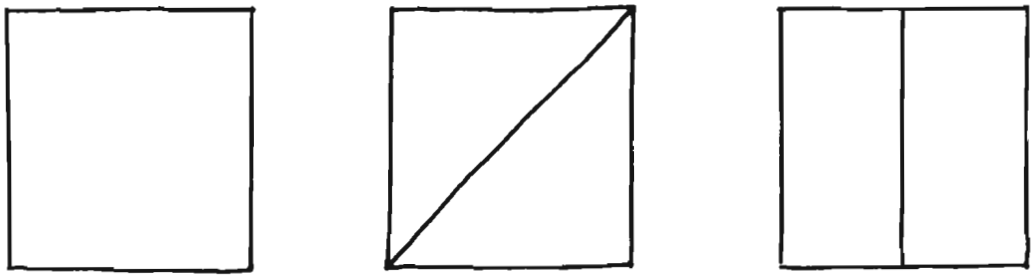
$\frac{2}{3}$ − $\frac{1}{6}$ = ☐

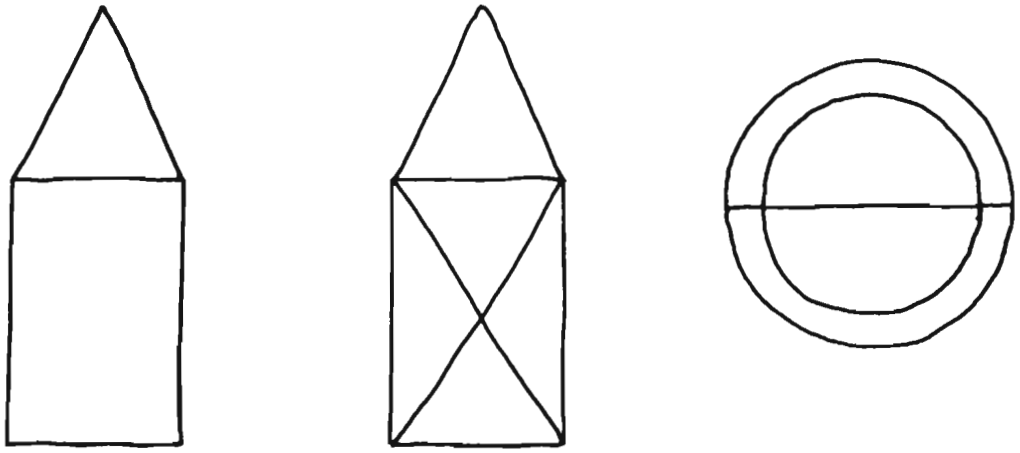
$\frac{1}{6}$ x 3 = ☐

The "Don't Lift Your Pencil" Game



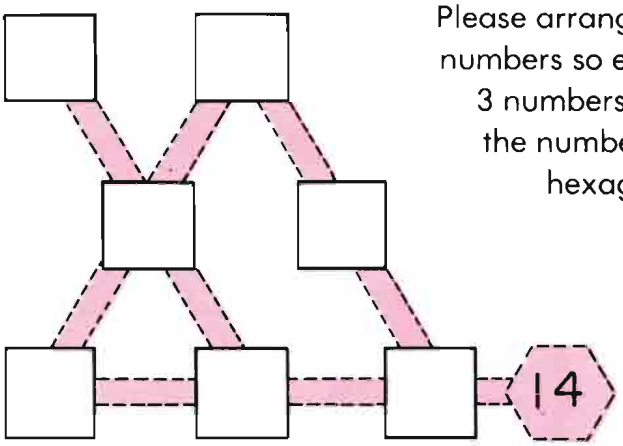
Rule: Complete the figures without lifting pencil, retracing or crossing any line.

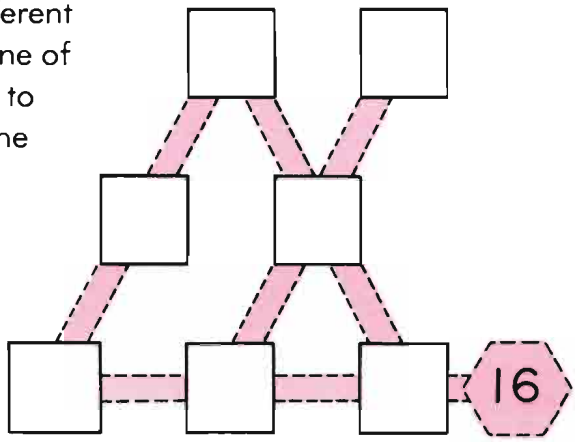


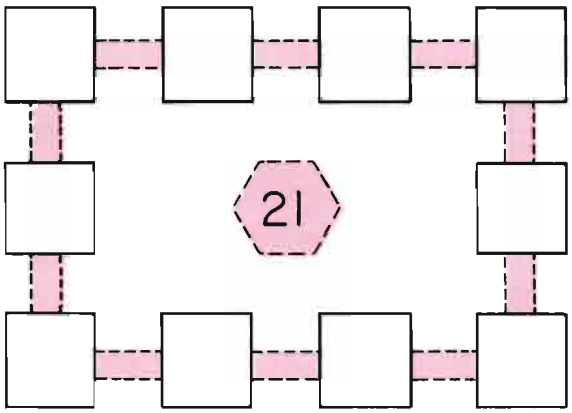


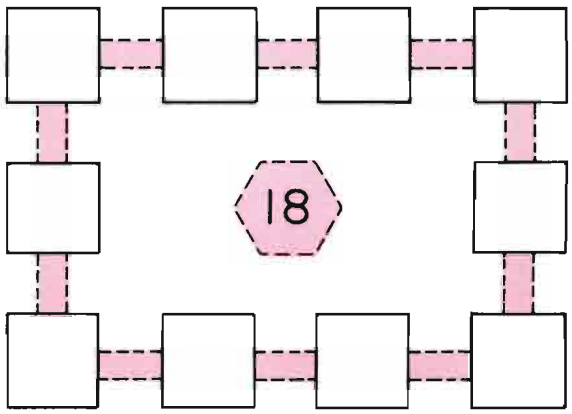
Arrangement PUZZLES with Small Numbers

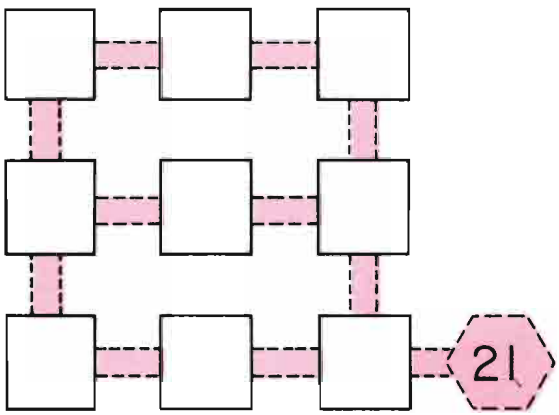
Please arrange different numbers so each line of 3 numbers adds to the number in the hexagon.

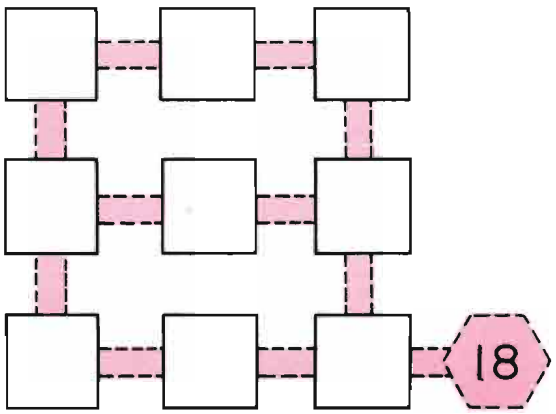


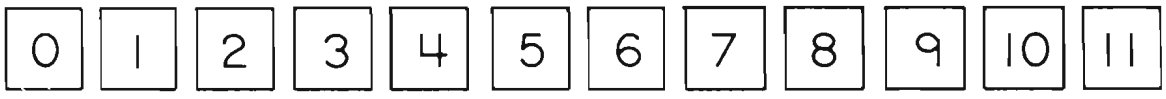


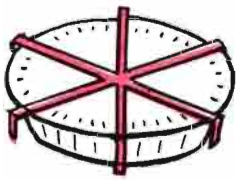










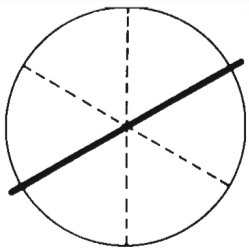


All About Pies Cut with a 6-Piece Cutter.

Please divide each pie into this many parts. → $\frac{2}{3}$ ← Then color in this many parts.

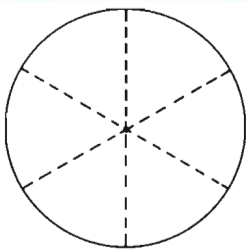
X

4 ← Number of pieces of pie colored in.



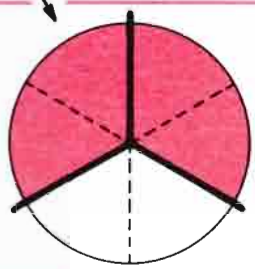
$\frac{1}{2}$ one half of a pie

3 pieces



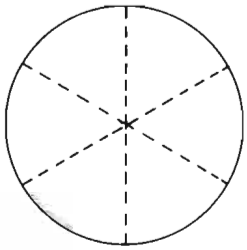
$\frac{1}{3}$ one third of a pie

____ pieces



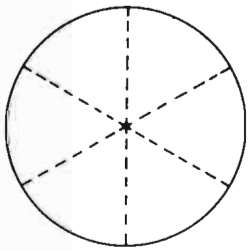
$\frac{2}{3}$ two thirds of a pie

4 pieces



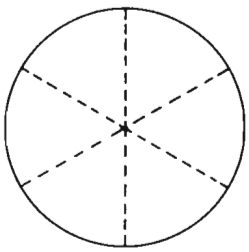
$\frac{1}{6}$ one sixth of a pie

____ piece



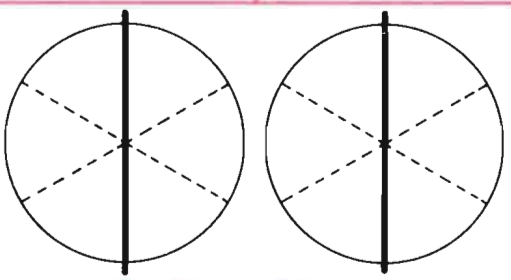
$\frac{5}{6}$ five sixths of a pie

____ pieces



$\frac{1}{1}$ or 1 one pie

____ pieces



$\frac{3}{2}$ or $1\frac{1}{2}$ three halves of a pie—or one and one half pies

____ pieces

Please complete the chart. It may help with the 3 examples that follow:

parts of a pie (s)	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{5}{6}$	1	$1\frac{1}{2}$	$1\frac{1}{3}$	$1\frac{2}{3}$	$1\frac{1}{6}$	$1\frac{5}{6}$
pieces	3										

$\frac{2}{3} + \frac{1}{6} = \frac{\quad}{\quad}$

4 + 1 = 5

$\frac{5}{6} - \frac{1}{2} = \frac{\quad}{\quad}$

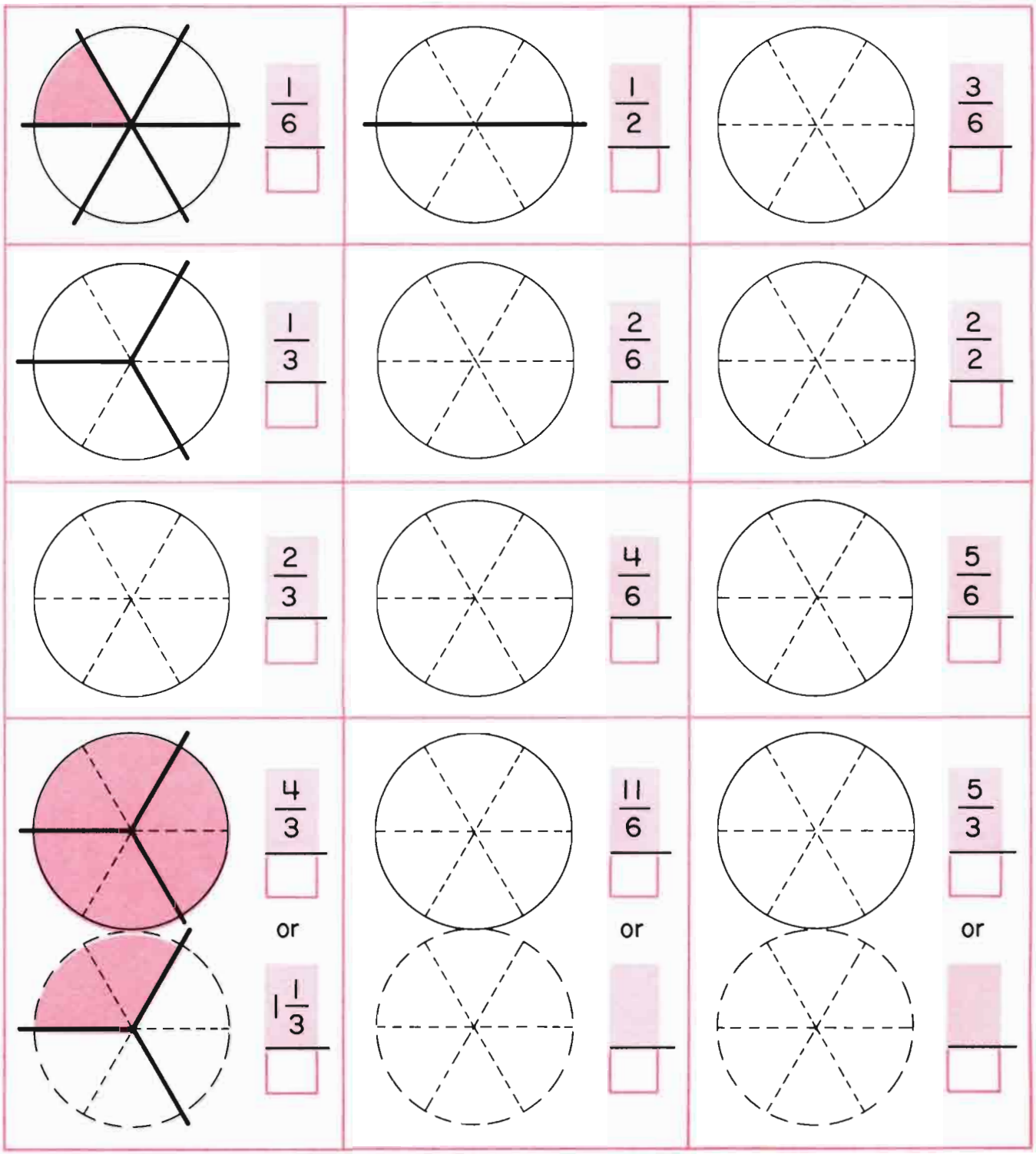
____ − ____ = ____

$\frac{1}{6} \times 2 = \frac{\quad}{\quad}$

____ x 2 = ____

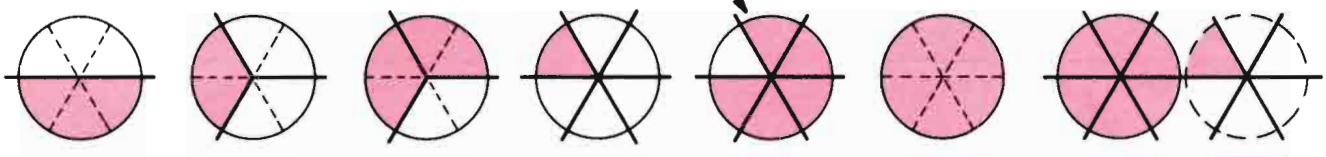
Please divide each pie into this many parts. → ___ ← Then color in this many parts.

___ ← Number of pieces of pie colored in.



Please divide each pie into this many parts → $\frac{5}{5}$ ← Then color in this many parts.

5 ← Number of pieces of pie colored in.



$\frac{1}{2}$ $\frac{\quad}{3}$ $\frac{2}{3}$ $\frac{\quad}{\quad}$ $\frac{\quad}{\quad}$ $\frac{1}{5}$ $\frac{7}{6}$ or $1\frac{1}{\quad}$

$\frac{1}{6} + \frac{1}{6} = \frac{\quad}{\quad}$ $\quad \frac{1}{5} + \frac{1}{5} = 2$

$\frac{1}{6} + \frac{2}{3} = \frac{\quad}{\quad}$ $\quad \frac{\quad}{\quad} + \frac{\quad}{\quad} = \frac{\quad}{\quad}$

$\frac{1}{6} + \frac{1}{3} = \frac{\quad}{\quad}$ $\quad \frac{\quad}{\quad} + \frac{\quad}{\quad} = \frac{\quad}{\quad}$

$1 - \frac{1}{6} = \frac{\quad}{\quad}$ $\quad \frac{\quad}{\quad} - \frac{\quad}{\quad} = \frac{\quad}{\quad}$

$\frac{2}{3} - \frac{1}{6} = \frac{\quad}{\quad}$ $\quad \frac{\quad}{\quad} - \frac{\quad}{\quad} = \frac{\quad}{\quad}$

$1\frac{1}{6} - 1 = \frac{\quad}{\quad}$ $\quad \frac{\quad}{\quad} - \frac{\quad}{\quad} = \frac{\quad}{\quad}$

$\frac{1}{6} \times 3 = \frac{\quad}{\quad}$ $\quad \frac{\quad}{\quad} \times 3 = \frac{\quad}{\quad}$

$\frac{1}{6} \times 7 = \frac{\quad}{\quad}$ $\quad \frac{\quad}{\quad} \times 7 = \frac{\quad}{\quad}$

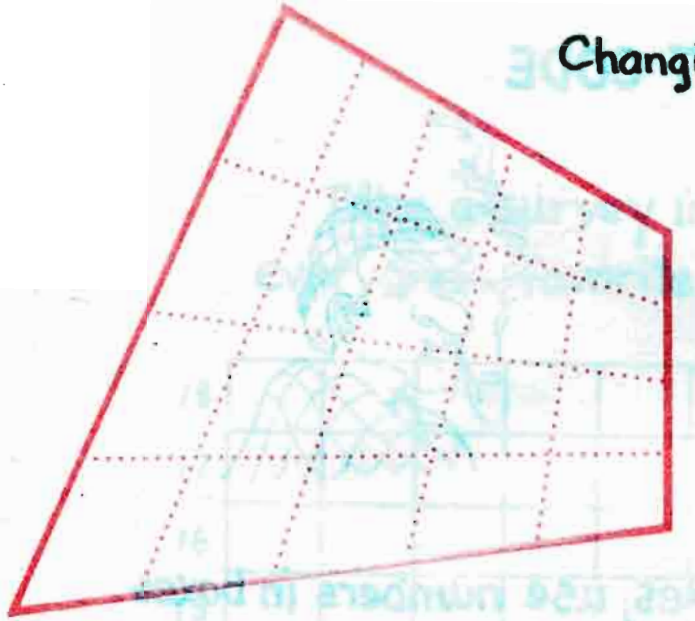
$\frac{2}{3} \div 2 = \frac{\quad}{\quad}$ $\quad \frac{\quad}{\quad} \div 2 = \frac{\quad}{\quad}$

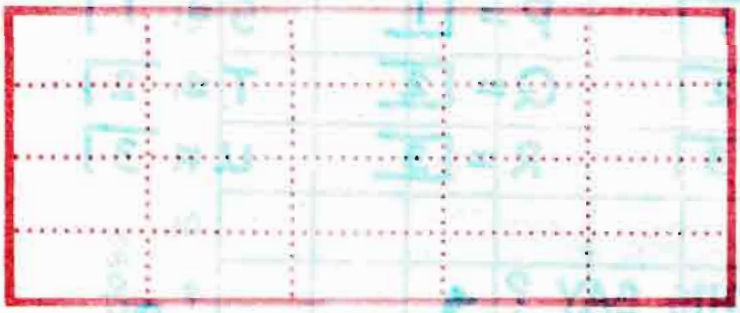
$\frac{1}{3} + \frac{1}{3} = \frac{\quad}{\quad}$ $\quad \frac{\quad}{\quad} + \frac{\quad}{\quad} = \frac{\quad}{\quad}$

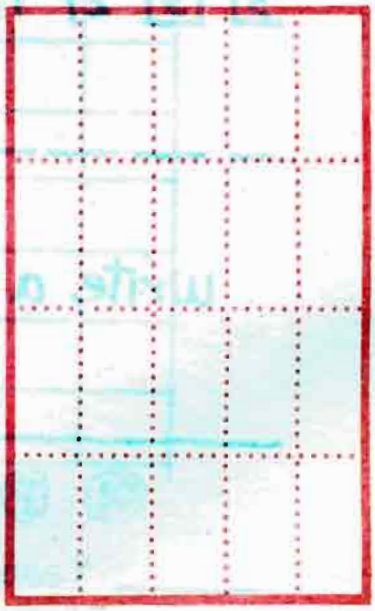
$1 - \frac{1}{2} = \frac{\quad}{\quad}$ $\quad \frac{\quad}{\quad} - \frac{\quad}{\quad} = \frac{\quad}{\quad}$

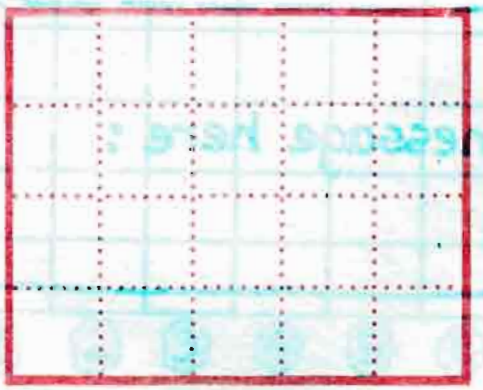
$\frac{1}{3} \times 2 = \frac{\quad}{\quad}$ $\quad \frac{\quad}{\quad} \times \quad = \frac{\quad}{\quad}$

Changing Shapes









SECRET CODE

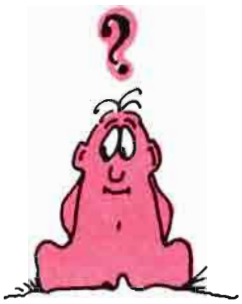
ABC	DEF	GHI
JKL	MNO	PQR
STU	VWX	YZ.

Instead of letters in the boxes, use numbers in boxes.

A = 1	V = 1	P = 1	S = 1
B = 2	W = 2	Q = 2	T = 2
C = 3	X = 3	R = 3	U = 3

WHAT DOES THIS SAY ?

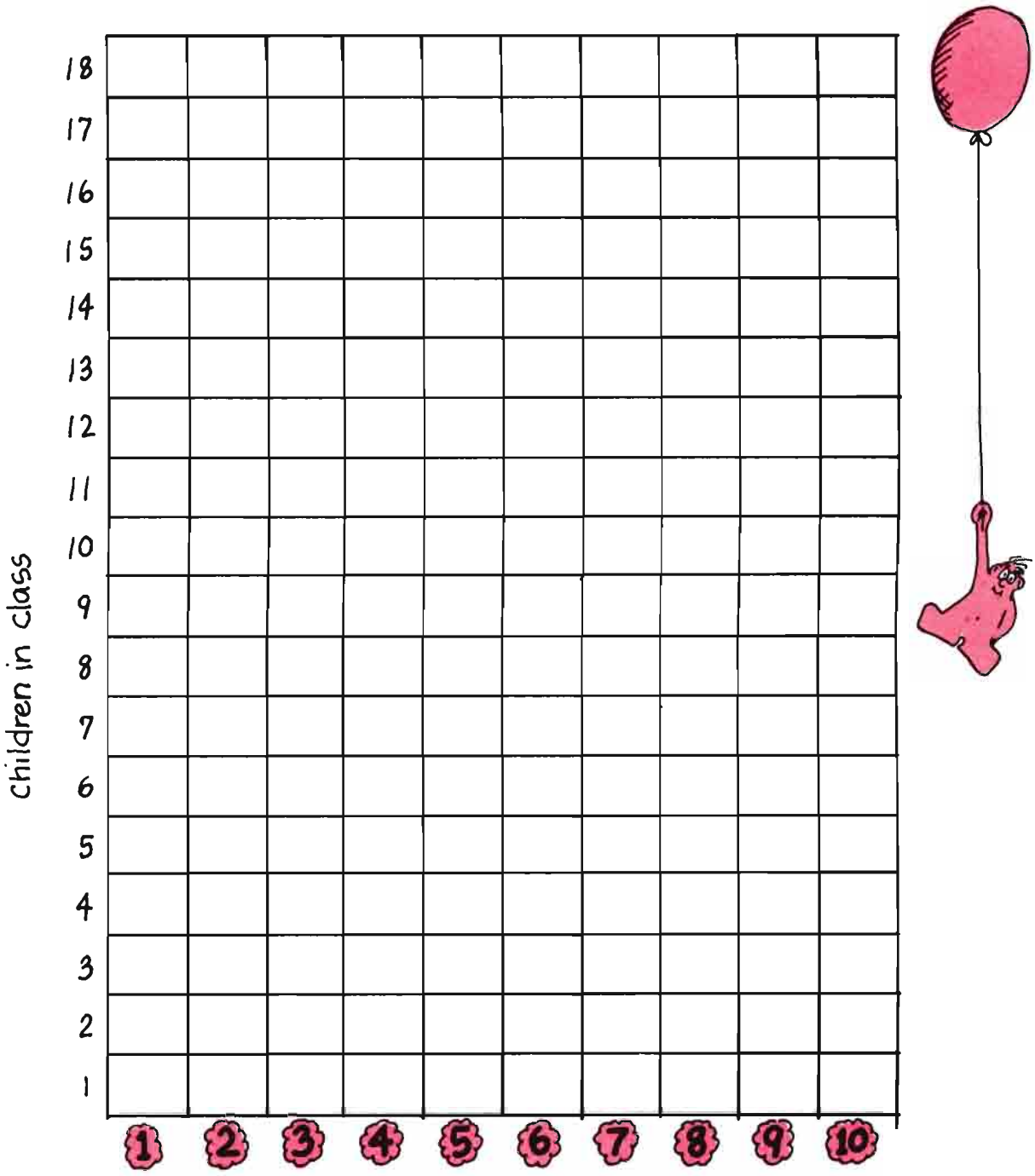
3 3 2 1 3 3 3 3 3 2 3



Write a secret message here :

LUCKY NUMBERS

Take a survey in your class of everyone's favorite number between 1 and 10.



What number is the favorite of your class? _____

Carla's "Quickies"

3 + 8 = 11	13 + 8	18 + 3	23 + 8	13 + 18	33 + 8	33 + 18	73 + 8
14 - 9 = 5	24 - 9	34 - 9	44 - 9	64 - 9	64 - 19	64 - 39	74 - 39
6 + 6 = 12	16 + 6	26 + 6	46 + 6	76 + 6	76 + 16	66 + 16	46 + 26
7 - 5 = 2	17 - 5	27 - 5	27 - 15	47 - 35	87 - 5	77 - 25	67 - 45

Nat's "9 examples out of 3 doubles"

starters	2 + 2 = 4	3 + 3 = 6	6 + 6 = 12
	23 + 23	33 + 33	32 + 32
	26 + 26	36 + 36	62 + 62
	63 + 63	22 + 22	66 + 66

starters	1 + 1 = 2	4 + 4 = 8	9 + 9 = 18
	41 + 41	11 + 11	14 + 14
	44 + 44	19 + 19	91 + 91
	49 + 49	99 + 99	94 + 94

allowance arithmetic

Angie and Lenny had been talking for a long time one afternoon in August. Just before dinner, they came in the house with wide smiles that spelled trouble for their parents.

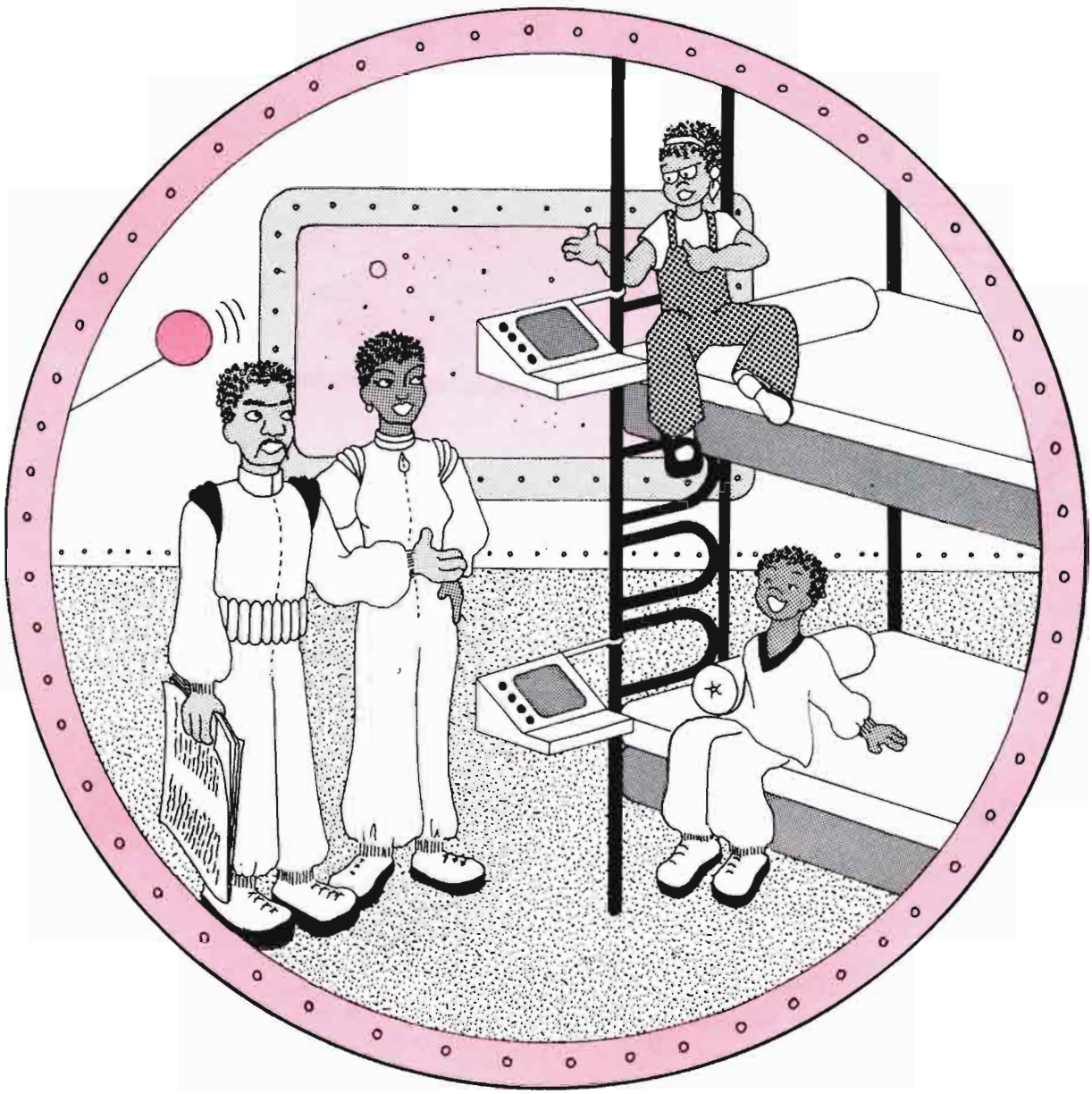
After dinner, Angie said, "Can we talk about our allowances for the next school year?"

"Okay," said Dad as he folded his newspaper.

"Well," Lenny began, "we've saved some money over the summer, so we won't need much in the beginning of the school year."

"So our idea is this," interrupted Angie. "On the 1st Monday in September you pay us only 2¢. That's 1¢ for Lenny and 1¢ for me. That's all."

"Then, on the next Monday, you pay us twice as much—4¢. So that's 2¢ for each of us. Then twice as much the next Monday—8¢; that's 4¢ for each of us. On the fourth Monday you pay us twice as much again—16¢; or 8¢ for each of us."



Lenny broke in. "And that's all we would get during the whole month of September."

2¢	the first week
4¢	the second week
8¢	the third week
16¢	the fourth week
30¢	for the whole month

"Of course we'll need more than that," Angie added, "But we'll use some of the money we saved doing odd jobs this summer."

Mom smiled. "It sounds like you're really trying to help us," Mom began, "but you don't really have to give up so much just because you've saved something."

Dad wasn't quite sure. "Do you mean you wouldn't ask for anything else—not for lunches or parties or movies or candy or presents or holidays?"

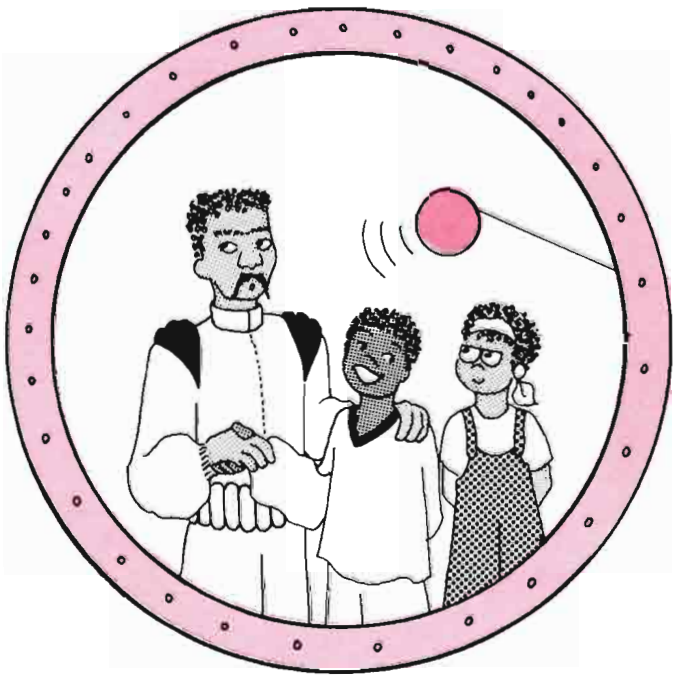
"Not only that Dad," Lenny said, "we'd even like to start buying some of our own clothes."

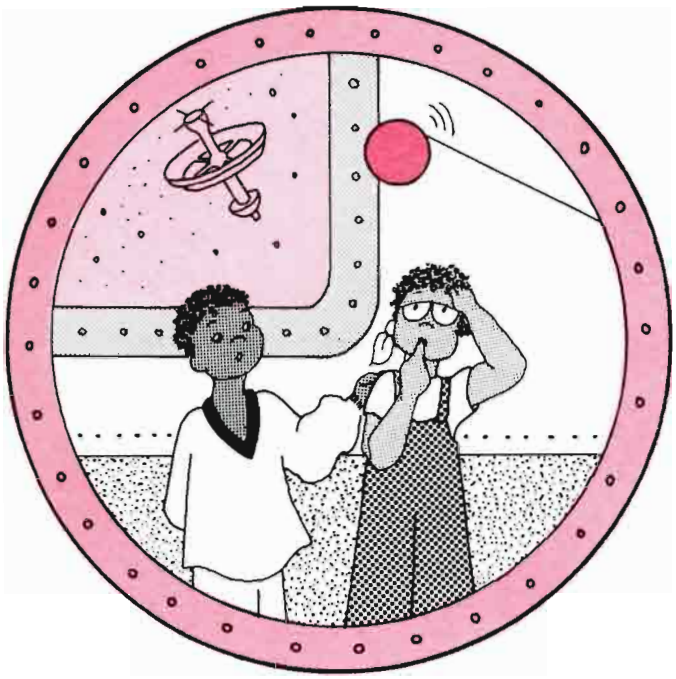
Mom and Dad looked at each other and smiled.

Dad was pleased, but he felt that maybe Angie and Lenny didn't realize how fast their savings would be used up. "We can try it," he agreed, "but if you begin finding that you don't have enough money come to us and we'll talk about it again."

"Is it a deal?" Lenny asked.

"It's a deal," said Dad and they shook hands. Mom and Dad left the room.





Angie and Lenny smiled at each other . . . but Lenny's smile turned into a bit of a frown.

"Are you sure we were right when we figured it out this afternoon, Angie?"

"I think so," Angie said, but she didn't look too sure.

Lenny and Angie went over the figuring that they did before dinner: "Only 15¢ for the whole month of September. That won't even buy a single lunch."

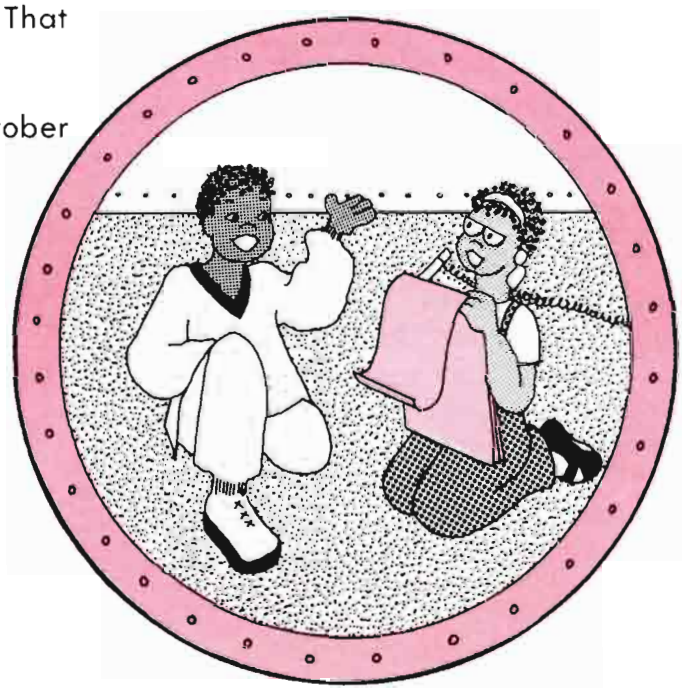
"Well,"Angie said, "let's figure October out again."

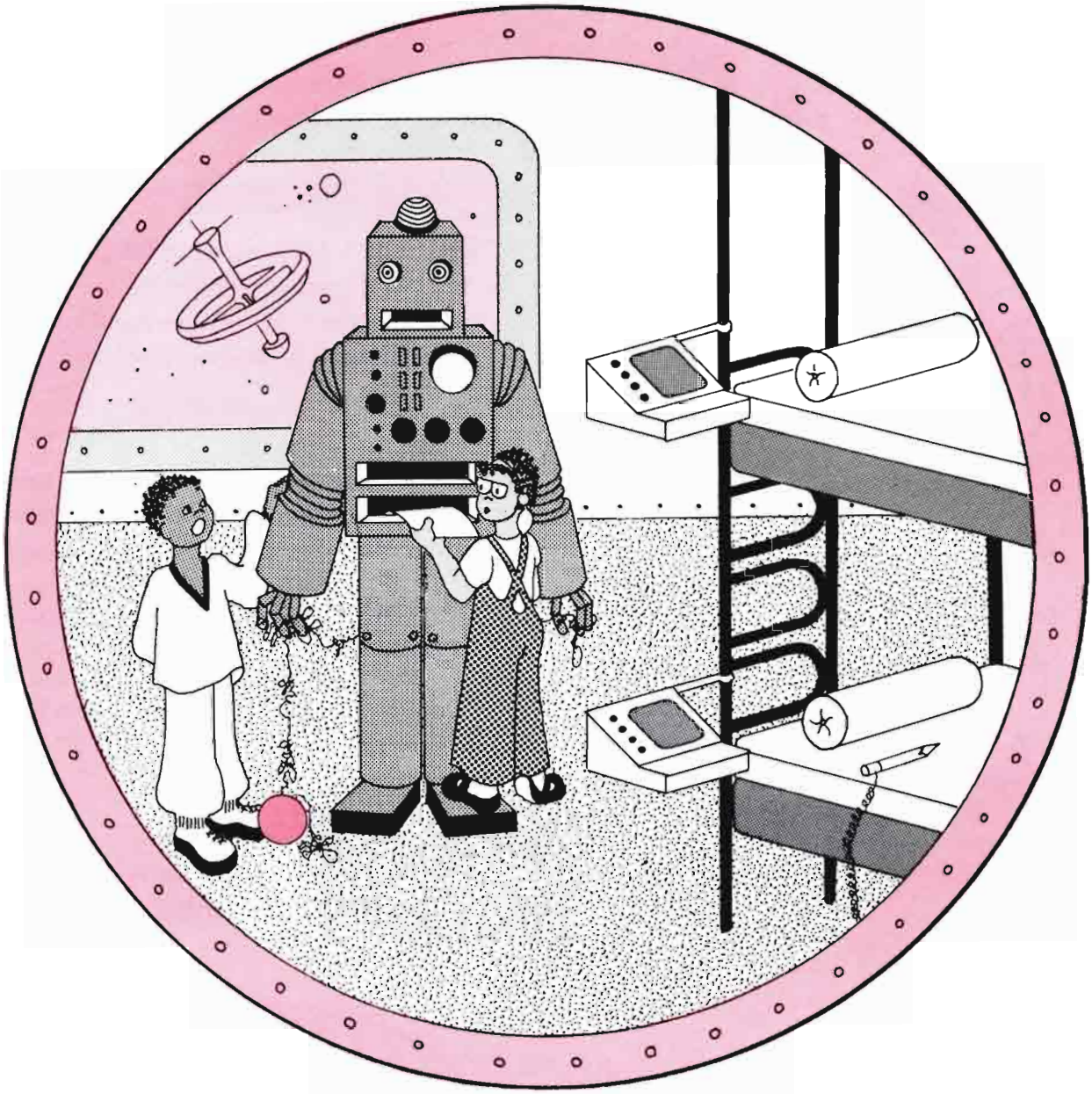
(Please do the arithmetic for the month of October. Everyone can get together and keep a record.)

1st week—	16¢	each
2nd week—	____	each
3rd week—	____	each
4th week—	____	each
Total	$2.40	each

"Not bad," said Lenny. "Let's go on with November, just to be sure; there are 5 Mondays in November."

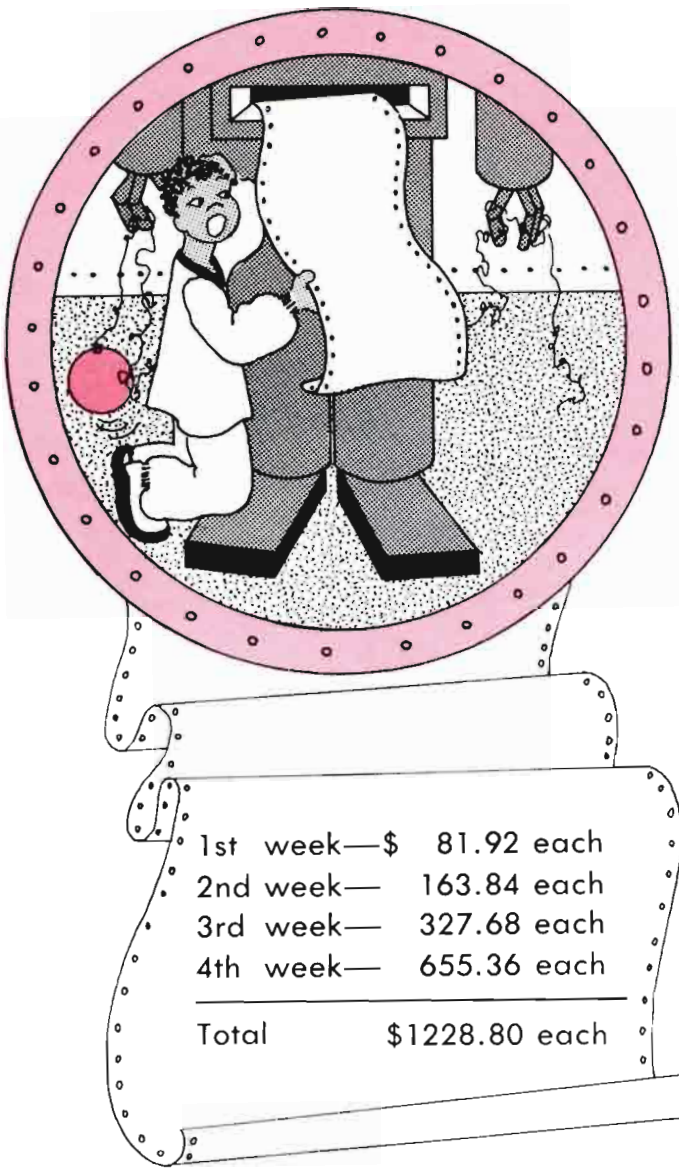
1st week—	$ 2.56	each
2nd week—	5.12	each
3rd week—	10.24	each
4th week—	20.48	each
5th week—	40.96	each
Total	$79.36	each





"Wow!" Angie exclaimed, "We would each get almost $80.00 in November."

Lenny was excited. "Let's feed the information to Howie so we can hurry up and see what we'll get in December."



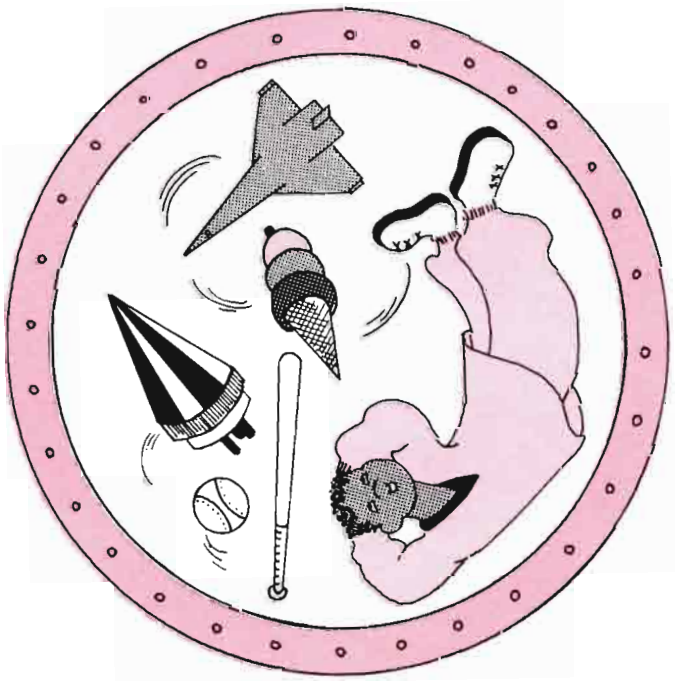
1st week—	$ 81.92	each
2nd week—	163.84	each
3rd week—	327.68	each
4th week—	655.36	each
Total	$1228.80	each

"Double wow . . . we'll each get over $1000 in December." Angie's doubts had flown away.

"Boy, what a deal," Lenny added with a dreamy smile on his face.

"Do you want to see what we'll get in January?" asked Angie. "Or are you happy now?"

Lenny didn't answer—he was already dreaming about how he would spend his allowance for December.



How would you end this story?

How long do you think it will take for Dad and Mom to realize they had been tricked all the way from 15¢ a month to $1000 a month in only four months?

One happy ending might find Angie and Lenny's family working together to see just what could happen the next year if they were millionaires, which of course they weren't.

January

1st	week—	$ 1,310.72 each
2nd	week—	2,621.44 each
3rd	week—	5,242.88 each
4th	week—	10,485.76 each
Total		$19,660.80 each

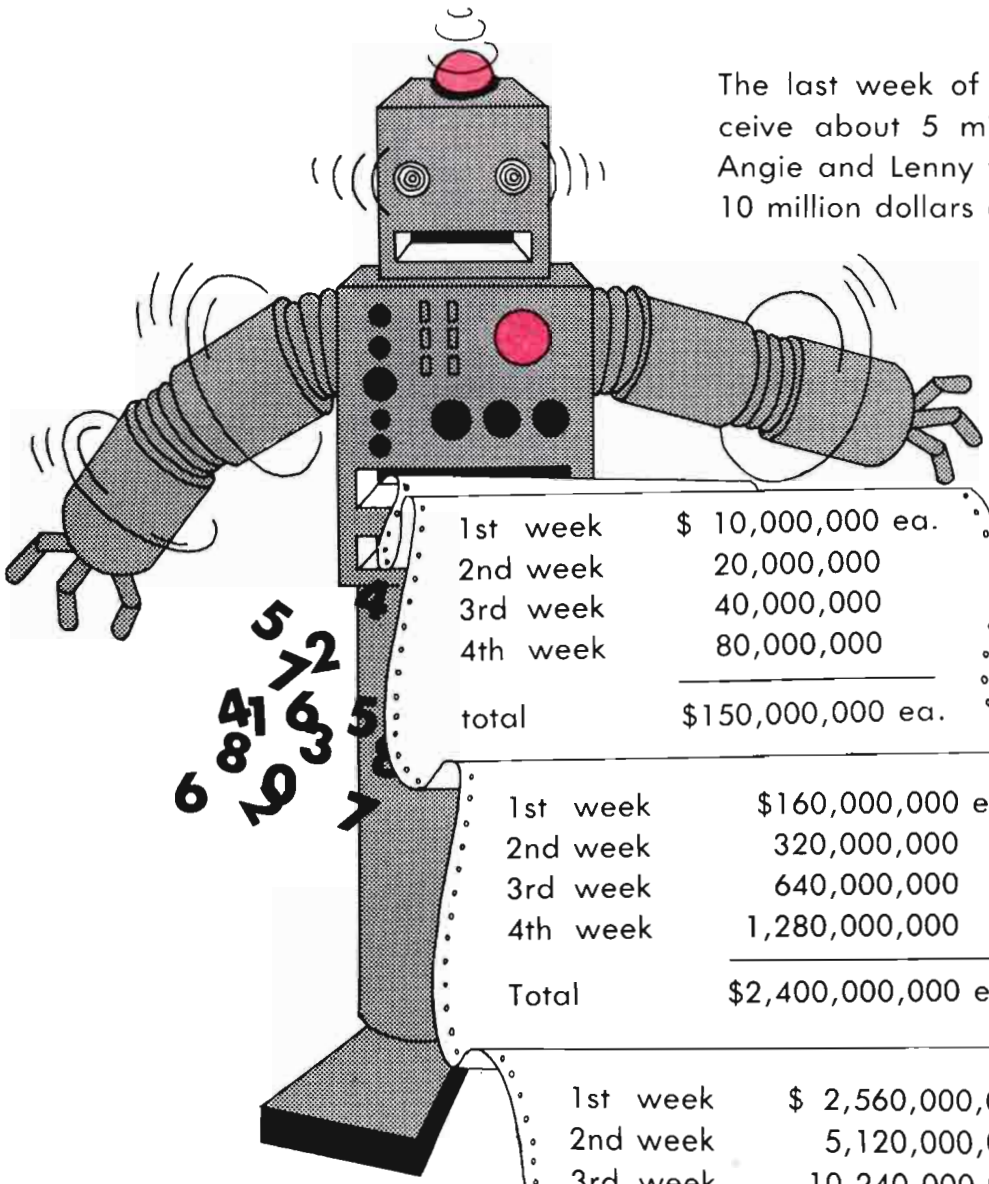
That's about $300,000 for both Angie and Lenny and that is only for the 1st 6 months, ½ year. To make the arithmetic for March–another 5 Monday month–easier, let's use double that "rounded off" amount for the 1st week:

March

1st	week—	$ 320,000 each
2nd	week—	640,000 each
3rd	week—	1,280,000 each
4th	week—	2,560,000 each
5th	week—	5,120,000 each
Total		$9,920,000 each

February

1st	week—	$ 20,971.52 each
2nd	week—	41,943.04 each
3rd	week—	83,886.08 each
4th	week—	167,772.16 each
Total		$314,572.80 each



The last week of March they would receive about 5 million dollars each. So Angie and Lenny would begin April with 10 million dollars each:

April

1st week	$ 10,000,000 ea.
2nd week	20,000,000
3rd week	40,000,000
4th week	80,000,000
total	$150,000,000 ea.

May

1st week	$160,000,000 ea.
2nd week	320,000,000
3rd week	640,000,000
4th week	1,280,000,000
Total	$2,400,000,000 ea.

June

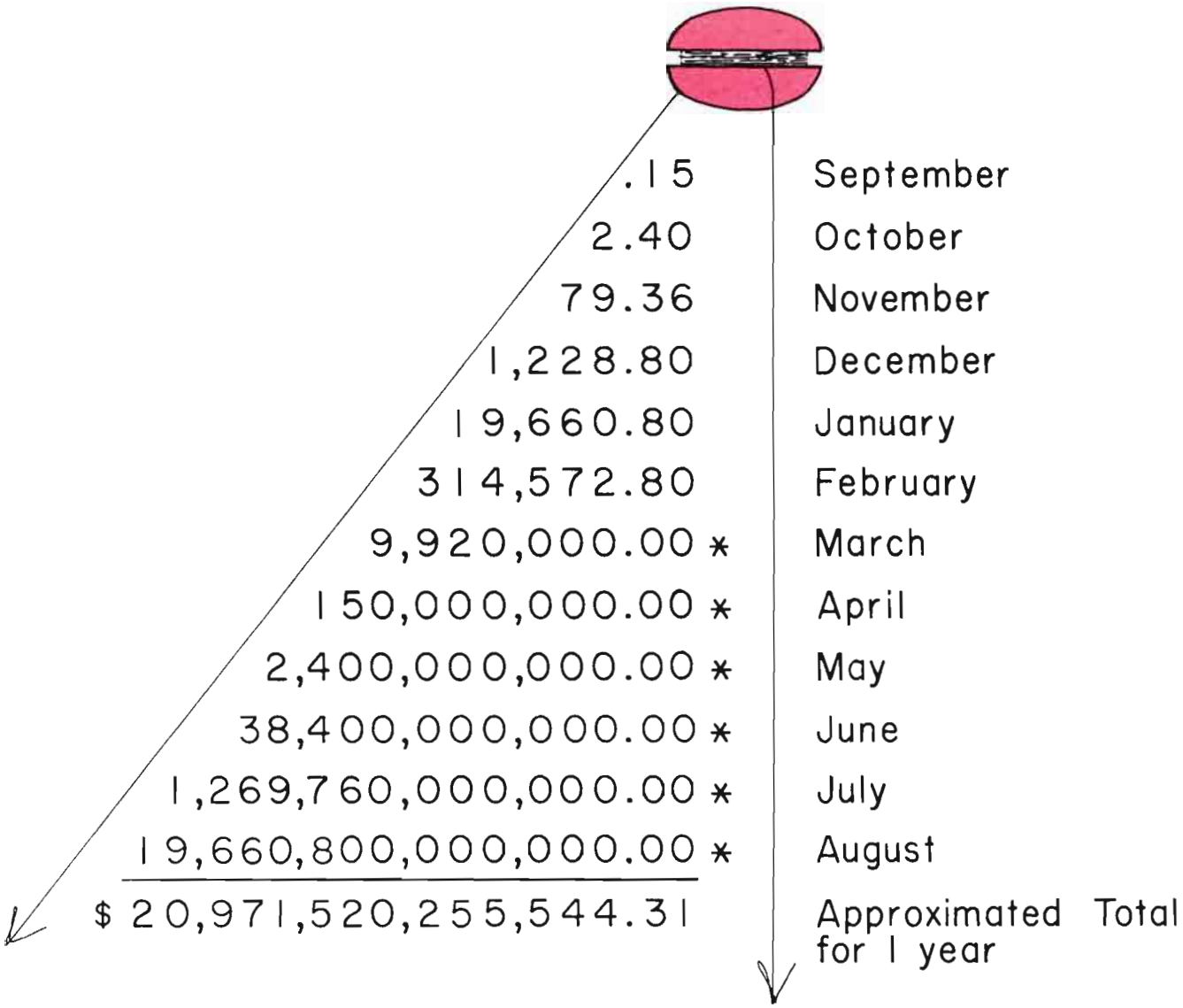
1st week	$ 2,560,000,000 ea.
2nd week	5,120,000,000
3rd week	10,240,000,000
4th week	20,480,000,000
Total	$38,400,000,000 ea.

July

2nd week	163,[illegible]
3rd week	327,680,000,000
4th week	655,360,000,000
5th week	[illegible]
Total	1,269,760,000,000 ea.

August

1st week	1,310,720,000,000 ea.
2nd week	2,621,440,000,000
3rd week	5,242,880,000,000
4th week	10,485,760,000,000
Total	$19,660,800,000,000 ea. (about 20 trillion dollars)

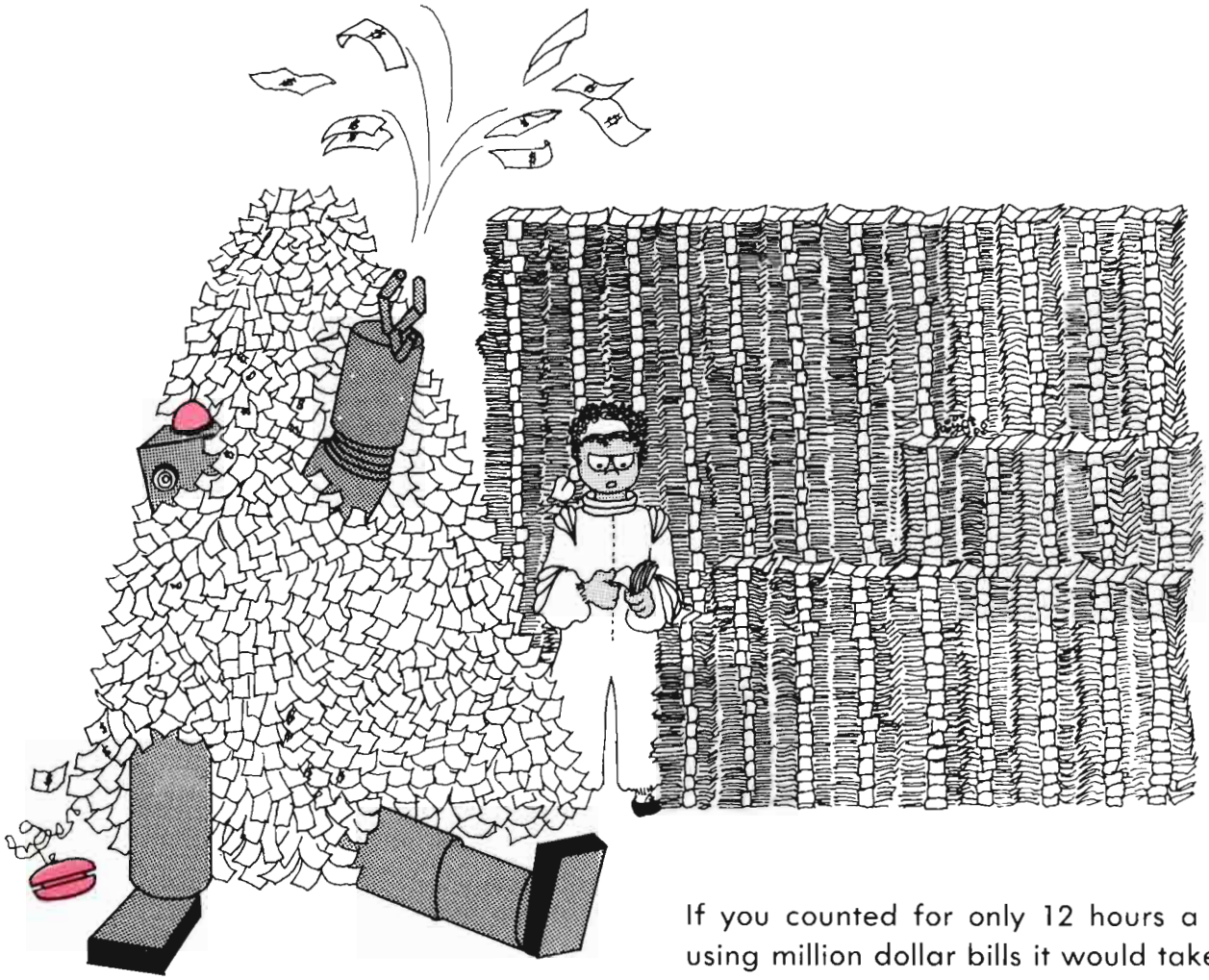


Almost 21 trillion dollars for Angie and 21 trillion dollars for Lenny . . . and all this from 1¢ the first week and twice as much each week after that for a year—52 weeks.

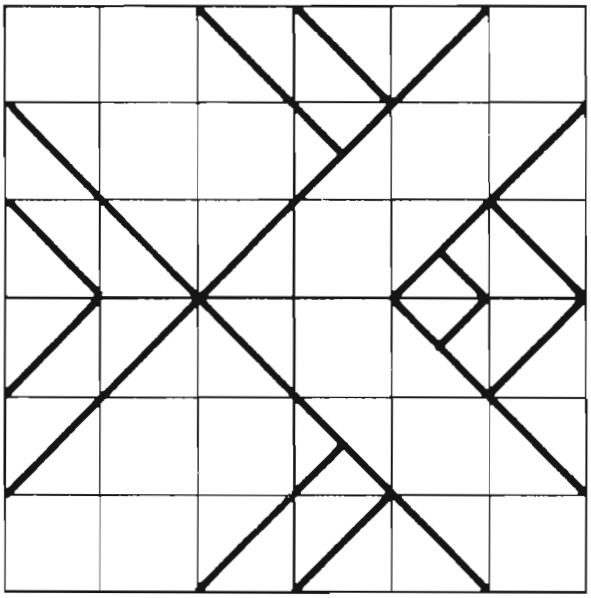
Do you think there is that much money in the world?

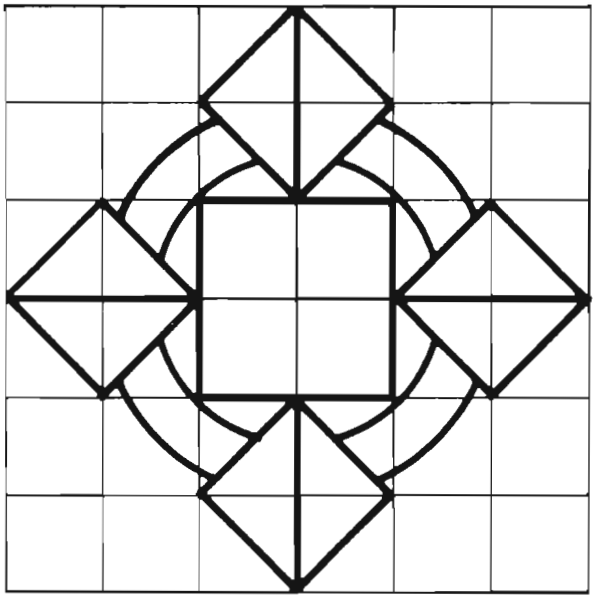
If all that money were in $10.00 bills, how long do you think it would take you to count it at one $ 10.00 bill per second? (about 75,000 years)

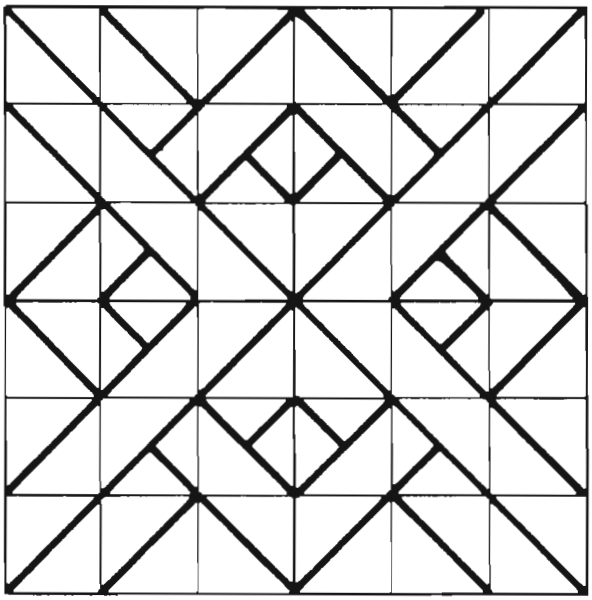
75,000 years in $10.00 bills
7,500 years in $100.00 bills
750 years in $1000.00 bills
75 years in $10,000.00 bills
7½ years in $100,000.00 bills
¾ year in $1,000,000.00 bills

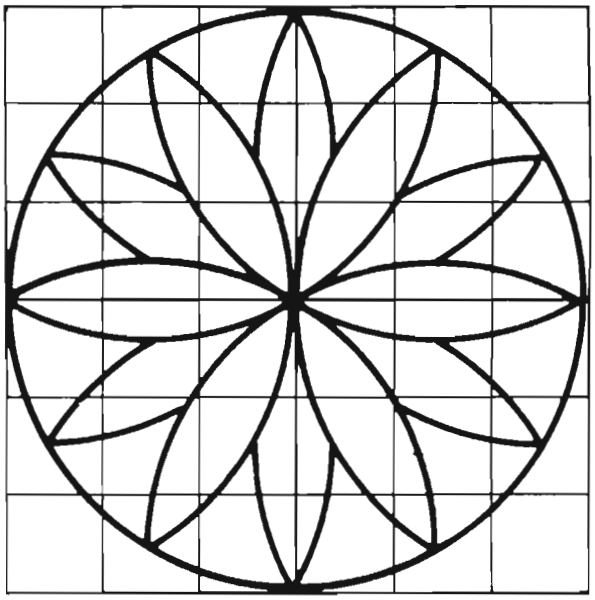


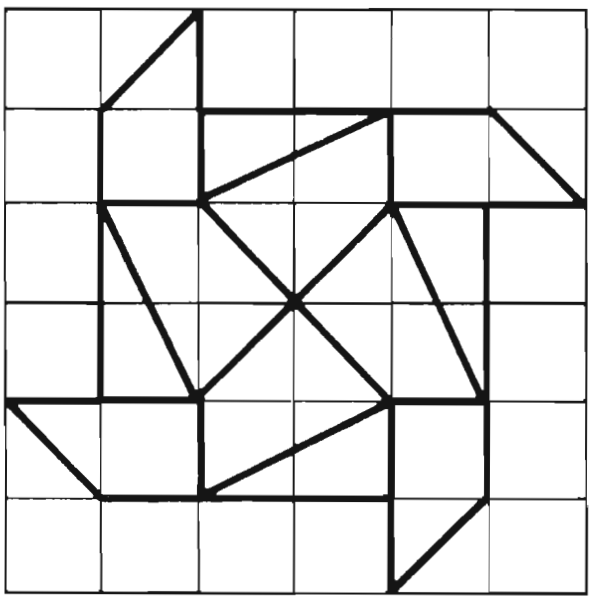
If you counted for only 12 hours a day, using million dollar bills it would take you about 1½ years to count out the allowance for Angie and Lenny, unless of course, you had Howie to help you.

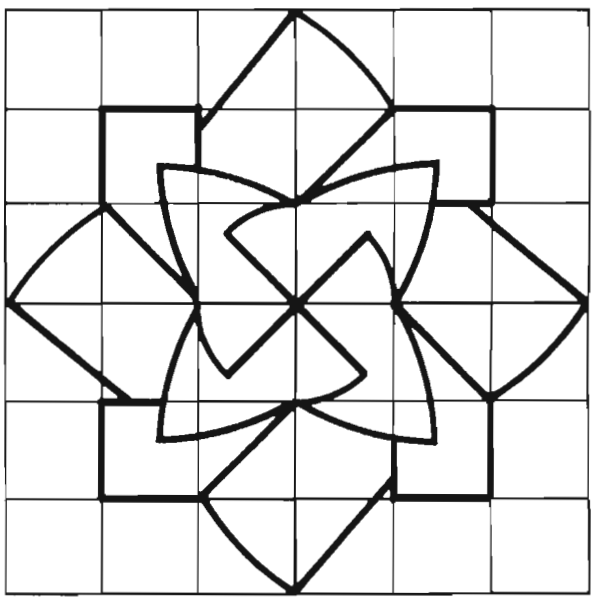


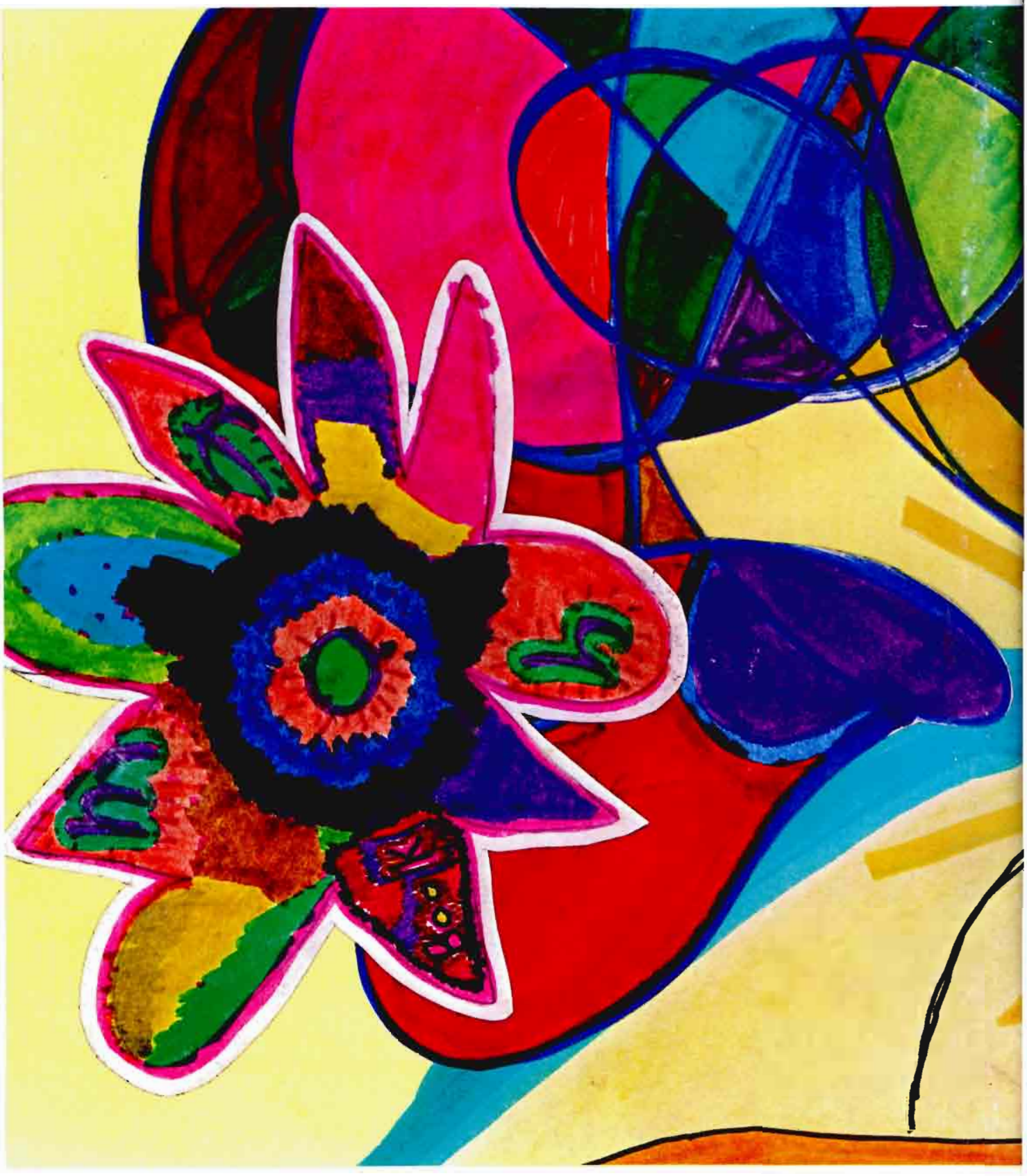












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