## Independent Exploration Materials

## Weights and Springs

Bob, Joan and Mary Francis were examining the contents of this box one day. Joan said, "I've seen a picture of equipment like this in my brother's physics book; it usually is set up this way."


CAN YOU SET UP THE EQUIPMENT AS IN THE DIAGRAM ABOVE?

Bob asked, "What can you do with this material now that it is set up?" Mary Frances asked, "What are the washers for?"

WHAT DO YOU THINK?
"The more washers I hang on the spring, the more it stretches," said Joan.
DO YOU AGREE?

Mary Francis suggested they keep track of the number of washers (since they all weigh the same) and the distance stretched. Bob said,"l'll tape the ruler on the wall so that the " 0 " point on the ruler is behind the bottom of the hook. Then as we add washers we can measure the distance the spring stretches."


CAN YOU SET UP THE RULER AS $\mathbb{N}$ THE DIAGRAM?
(see next card)
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Joan said, "I'll set up a table. Let the-number be the number of washers and the $\triangle$ number be the distance the spring stretches."


Mary Francis said, "Let's graph the pairs of numbers in the table to see what it looks like."


CAN YOU GRAPH THE FIVE PAIRS OF NUMBERS IN THE TABLE? (on the paper included in the box)

CAN YOU PREDICT THE TOTAL AMOUNT OF STRETCH USING SIX WASHERS?
CAN YOU CHECK YOUR PREDICTION?
(see next card)

Joan said, "This is something like the game of guessing functions: I think I know a rule which. approximately relates the $\square$ number and the $\Delta$ number."

CAN YOU MAKE UP A RULE WHICH APPROXIMATELY WORKS?

## Questions

1. Can you think of any other experiments you could set up?
2. What would happen if you used centimeters instead of inches to measure the distance stretched?
3. What would happen if you set the bottom of the hook opposite " 2 " on the ruler instead of " 0 "?
4. What things could cause "errors" in this experiment?

See your teacher for additional materials to go with this box.

Using a different spring but the same length as before:
Will this new spring stretch (1) the same as (2) more than (3) less than the first spring (used in cards 1-7)?

HOW CAN YOU CHECK YOUR PREDICTION?

Using the same spring as in cards 1-7 but $1 / 2$ the length, answer the same questions as on card 10.

## Independent Exploration Materials

## Discs

Two students were examining the contents of this box one day. Steve asked Joyce, "How many distances can you measure on each dise with this ruler and string? " WHAT DO YOU THINK? Jane answered, "Three." DO YOU AGREE? WHERE ARE THESE DISTANCES THAT WE CAN MEASURE?

Joyce made up a table using two of the distances. She let the diameter (the distance across the disc through the center) be the $\square$-number and the circumference (distance around the disc) be the $\Delta$-number.


CAN YOU FILL IN THE TABLE?
(on the paper included in the box)

Joyce suggested they use other circular objects around school or at home to obtain other pairs of numbers. WHAT OBJECTS CAN YOU FIND TO OBTAIN MORE DATA FOR THE TABLE?

Steve then made a graph by plotting points for each pair of numbers in the table. CAN YOU DO THE SAME? (On the paper included in the box).


Joyce said she could write an open sentence or rule which approximately relates the-number and $\triangle$-number. CAN YOU FIND A RULE?

CAN YOU PREDICT ABOUT WHAT THE CIRCUMFERENCE WOULD BE IF THE DIAMETER OF A CIRCULAR OBJECT WAS 6"? 41/2"? 1"?

CAN YOU PREDICT ABOUT WHAT THE DIAMETER OF A CIRCULAR OBJECT WOULD BE IF THE CIRCUMFERENCE IS $14^{\prime \prime} 2^{\prime \prime} ?$

## Independent Exploration Materials

## Geoboard

How many different shapes and sizes can you make by stretching a rübber band around some of the nails on this geoboard?

Let's try something --. Stretch a rubber band around four nails, like this:


If this is said to have an area of "one square," can you make a shape that you think would have an area of " 2 squares"? " 3 squares"? " $11 / 2$ squares"?

Don says that this shape has an area of $21 / 2$ squares. DO YOU AGREE?


That this shape has an area of $21 / 2$ squares. IS HE RIGHT?


Frank says that this shape has an area of $21 / 2$ squares. WHAT DO YOU THINK?


Frank's friend Martha says Frank is wrong. She says the area is 2 squares. WHO IS RIGHT? (see next card)

Can you figure out the areas within these shapes?
a)

b)


d)


Can you make up some interesting shapes and find the areas within them?
Can a friend of yours make up a shape that is too hard for you to find the area of?
Can you make up one that he can't find the area of?
(see next card)

Have you ever played the game "Guessing Functions?" If you have, you might like to try this. Suppose you were told a certain number of nails, and you were told that you must touch all the nails with a rubber band .- CAN YOU PREDICT WHAT THE LARGEST POSSIBLE AREA WITHIN THE FIGURE WILL BE (Nails within the figure are illegal!)

Pat says that if you tell her "3 nails," the area will be $1 / 2$. DO YOU AGREE?
Bill says that if you tell him " 5 nails," the area will be $11 / 2$. IS HE RIGHT?
Louise says that if you tell her " 6 nails," the area will be $21 / 2$. WHAT DO YOU THINK?

Gerry and Bob made this table for their answers.

| (Nails) | (Area) |
| :---: | :--- |
| 4 | 1 |
| 5 | $11 / 2$ |
| 6 | 2 |

For example, if you were told " 6 nails"


This is illegal!


This is legal!


This is legal!

DO YOU THINK THEY ARE RIGHT?
CAN YOU FILL $\mathbb{N}$ ANY MORE PAIRS OF NUMBERS?
CAN YOU GRAPH YOUR PAIRS OF NUMBERS?
CAN YOU GUESS THE RULE?
(see next card)

WHAT HAPPENS IF YOU ALLOW ONE NAIL WITHIN the FIGURE, NOT TOUCHED BY THE RUBBER BAND?

DOES IT MAKE ANY DIFFERENCE HOW MANY NAILS ARE WITHIN A FIGURE, NOT TOUCHED BY THE RUBBER BAND?

CAN YOU WRITE A NEW RULE TO FIT THIS IDEA?

## Tower Puzzle

Three students were examining the contents of this box one day. Marilyn said, "This is a puzzle l've seen before. The object of the puzzle is to transfer the discs from the center peg to either of the other two pegs, ending with the discs arranged in the same order as at the start (smaller dises on top of larger discs).


There are only two rules in moving the discs: 1) only one disc may be moved at a time and 2) a larger disc may never be placed on top of a smaller disc." Frank and Mark said they would like to try it.

CAN YOU DO IT STARTING WITH 5 DISCS?

CAN YOU DO THE PUZZLE STARTING WITH ONLY 3 DISCS?
To transfer 3 discs, Frank said it took him 11 moves, Mark did it in 7 moves, and Marilyn said it took her 10 moves. They each tried again. Mark said, "This is the shortest way to do the puzzle with 3 discs; it should take 7 moves." DO YOU AGREE?
is there a way to tell whether you have the minimum (Smallest) Number of MOVES OR NOT?

CAN YOU DO THE PUZZLE STARTING WITH 2 DISCS? 4 DISCS? 5 DISCS?

Frank asked,, "Is there any relation between the number of discs and the minimum (smallest) number of moves needed to transfer a pile?" Mark suggested they make a table to keep track of the numbers. "Let the number of discs be the $\square$-number and let the minimum number of moves to transfer all the discs be the $\triangle$-number," he added. Marilyn said, "This is something like the game Guessing Functions, when you put a number in and get a number out and figure out a rule that works."

Number of discs $\rightarrow$| $\square$ | $\triangle \leftarrow$ Minimum number of moves needed to transfer the discs |
| :--- | :--- | :--- |
| 1 |  |
| 2 |  |
| 3 | 7 |
| 4 |  |
| 5 |  |

CAN you complete the table above? fill in the number on the paper included IN THE BOX.

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(see next card)
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CAN YOU FIGURE OUT ONE RULE THAT WORKS FOR ALL THE PAIR OF NUMBERS IN THE TABLE?

WRITE YOUR RULE (USING $\square$ and $\triangle$ ), THEN CHECK IT BY TRYING VARIOUS PAIRS OF NUMBERS FROM THE TABLE?
"I think the graph of these pairs of numbers will lie along a straight line," said Mark. DO YOU AGREE? CAN YOU MAKE A GRAPH OF THE PAIRS OF NUMBERS IN THE TABLE?


MAKE YOUR GRAPH ON THE GRAPH PAPER INCIUDED IN THE BOX.

IF YOU STARTED WITH 64 DISCS AND IT TOOK ONE SECOND FOR EACH MOVE, APPROXIMATELY HOW LONG WOULD IT TAKE TO TRANSFER THE PILE?

