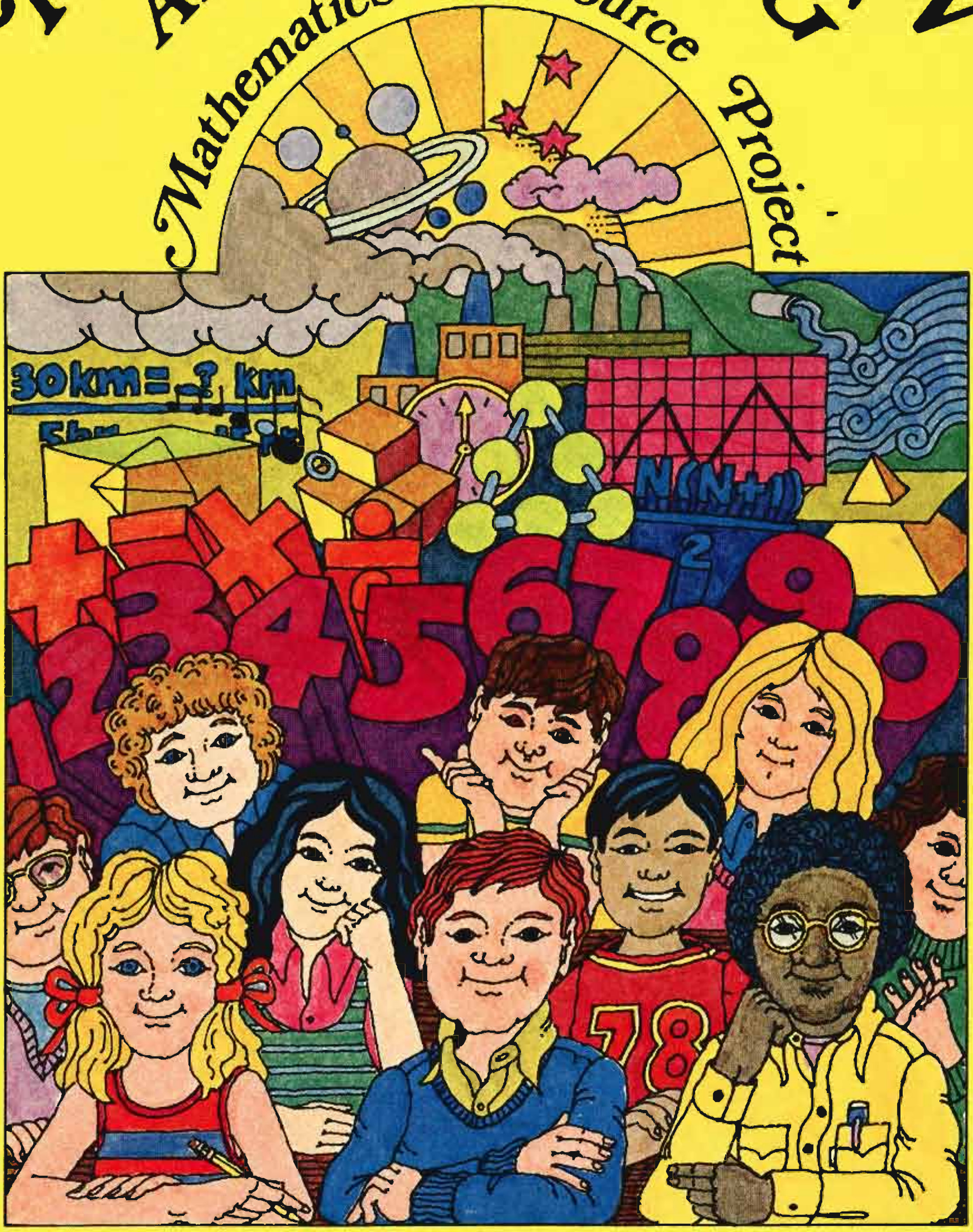
RATIO, PROPORTION AND SCALING

Mathematics Resource Project



Preliminary Edition

RATIO, PROPORTION, AND SCALING

Placement Guide for Tabbed Divisors

RATIO, PROPORTION AND SCALING

MATHEMATICS RESOURCE PROJECT

In this preliminary edition certain references to previously published material have been deleted because the project did not receive permission to use the publishers' materials or a fee requirement prohibited its use by the project.

This material is based upon research supported by the National Science Foundation under Grant No. SED74-15047, hence it is the intent of Congress that:

The resources developed by the Mathematics Resource Project will be made available within the school district using such material for inspection by parents or guardians of children engaged in educational programs or projects of that school district.

Any opinions, findings, and conclusions or recommendations expressed in this publication are those of the author (s) and do not necessarily reflect the views of the National Science Foundation.

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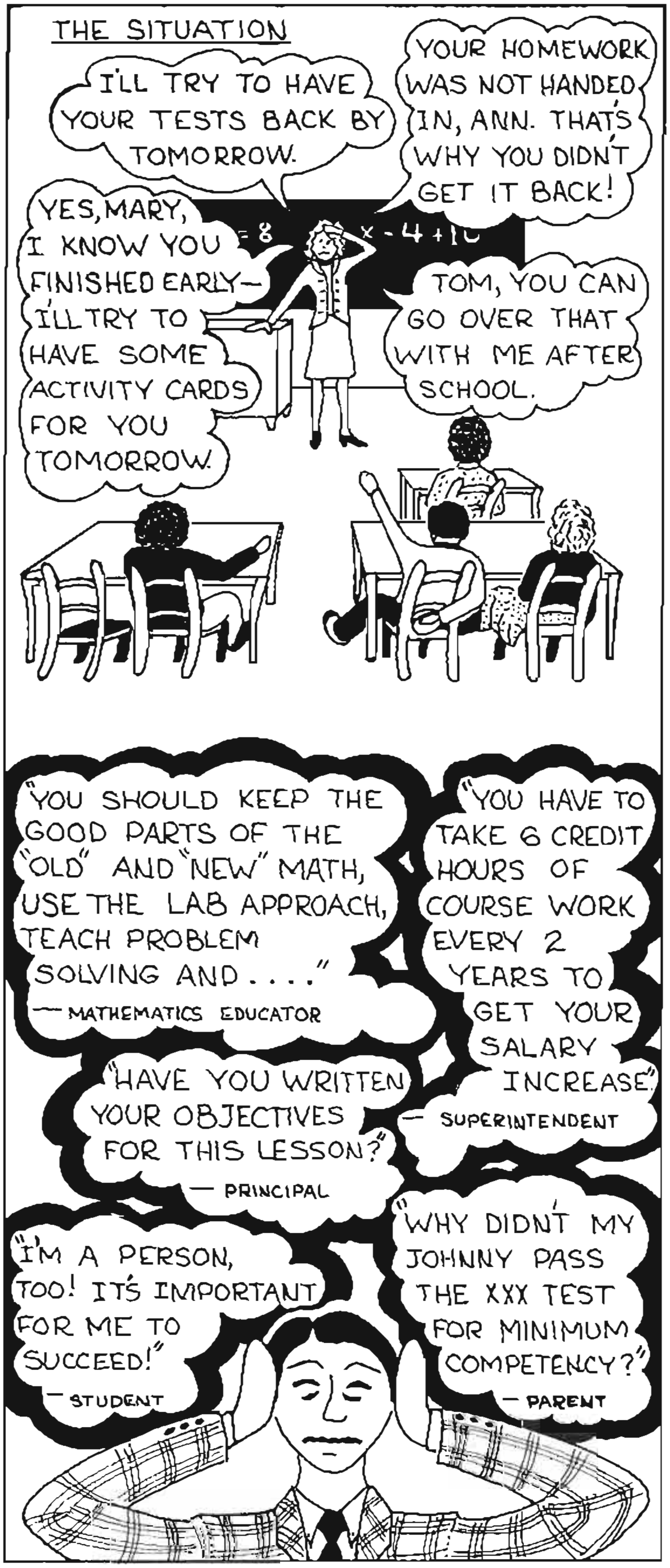
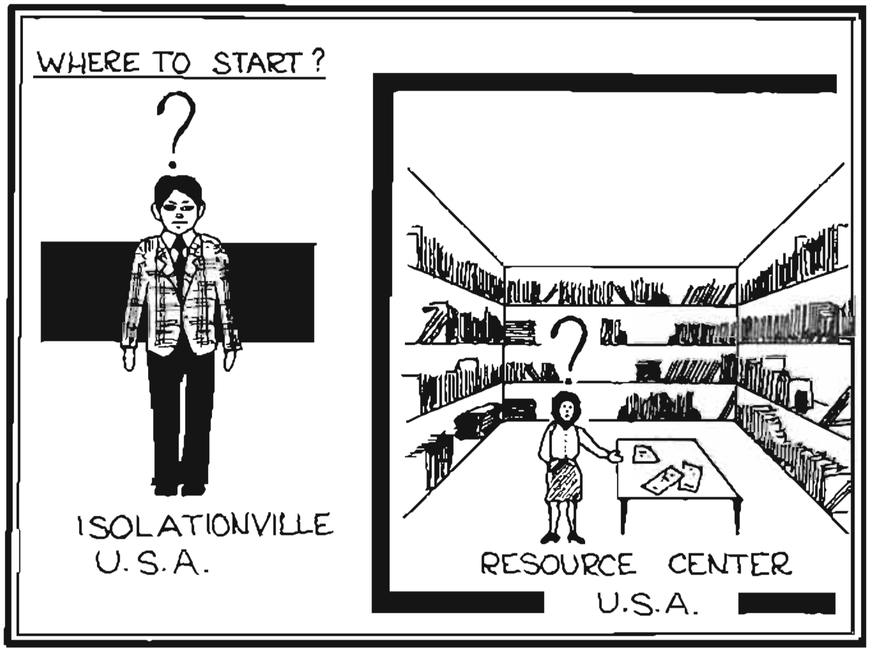
FOREWORD



The demands on teachers are heavy. The fifth or sixth grade teacher with 25 to 30 students is often responsible for covering many subjects besides mathematics. The seventh or eighth grade teacher may be teaching only mathematics but be working with 125 to 150 students each day. Within this assignment the teacher must find time for correcting homework, writing and grading tests, discussions with individual students, parent conferences, teacher meetings and lesson preparations. In addition, the teacher may be asked to sponsor a student group, be present at athletic events or open houses, or coach an athletic team.

Demands are made on the teacher from other sources. Students, parents and educators ask that the teacher be aware of students' feelings, self-images and rights. School districts ask teachers to enlarge their backgrounds in mathematical or educational areas. The state may impose a list of student objectives and require teachers to use these to evaluate each student. There are pressures from parents for students to perform well on standardized tests. Mathematicians and mathematics educators are asking teachers to retain the good parts of modern mathematics, use the laboratory approach, teach problem solving as well as to increase their knowledge of learning theories, teaching strategies, and diagnosis and evaluation.



There is a proliferation of textbooks and supplementary material available. Much of this is related to the demands on teachers discussed above. The teacher in small outlying areas has little chance to see much of this material, while the teacher close to workshop and resource centers often finds the amount of available material unorganized and overwhelming.

The Mathematics Resource Project was conceived to help with these concerns. The goal of this project is to draw from the vast amounts of material available to produce topical resources for teachers. These resources are intended to help teachers provide a more effective learning environment for their students. From the resources, teachers can select classroom materials emphasizing interesting drill and practice, concept-building, problem solving, laboratory approach, and so forth. When completed the resources will include readings in content, learning theories, diagnosis and evaluation as well as references to other sources. A list of the resources is given below. A resource devoted to measurement and another devoted to problem solving have been proposed.

NUMBER SENSE AND ARITHMETIC SKILLS (preliminary edition, 1977)
RATIO, PROPORTION AND SCALING (preliminary edition, 1977)
GEOMETRY AND VISUALIZATION (preliminary edition, 1977)
MATHEMATICS IN SCIENCE AND SOCIETY (preliminary edition, 1977)
STATISTICS AND INFORMATION ORGANIZATION (preliminary edition, 1977)

INTRODUCTION

This is a preliminary edition of RATIO, PROPORTION AND SCALING. The resource is intended to provide teachers with ideas and materials to help them in their important work which involves the minds and personalities of their students.

WHAT IS IN THIS RESOURCE?

- •Didactics
- •Teaching Emphases
- •Classroom Materials
- •Teacher Commentaries
- •Annotated Bibliography

The *Didactics* papers give information on:

Learning Theories

Teaching Techniques

Diagnosis and Evaluation

Goals and Objectives

The titles of the *Didactics* papers in this resource are:

Piaget and Proportions

Reading in Mathematics

Broad Goals and Daily Objectives

Evaluation and Instruction

A list of the *Didactics* papers for all of the resources is given on page 10.

The *Teaching Emphases* section stresses important areas which may help to teach most topics. These include:

Calculators

Applications

Problem Solving

Mental Arithmetic

Estimation and Approximation

Laboratory Approaches

The *Classroom Materials* section includes:

- •Paper and pencil worksheets
- •Transparency masters
- •Laboratory cards and activities
- •Games
- •Teacher directed activities
- •Bulletin board suggestions

The *Teacher Commentaries* which appear before the subsections of the classroom materials intend to:

- •Provide new mathematical information (historical, etc.)
- •Give a rationale for teaching a topic
- •Suggest alternate ways to introduce or develop topics
- •Suggest ways to involve students
- •Highlight the classroom pages
- •Give more ideas on the teaching emphases

The *Annotated Bibliography* lists the sources which were used to develop this resource. These sources contain many additional ideas which can be of help to teachers.

HOW ARE THE IDEAS RELATED?

The classroom materials are keyed to each other within the section, to the teaching emphases, and to the commentaries with symbols and teacher talk as shown on page 9.

The commentaries refer to specific classroom pages (cited in italics) and often a classroom page is shown reduced in size next to the discussion of the page. The commentaries relate the various teaching comphases to the mathematical topic of that subsection.

Each teaching emphasis includes a rationale, highlights from the classroom materials, and a complete list of classroom pages related to that emphasis.

HOW CAN THE RESOURCE BE USED?

Each teacher will decide which material is appropriate for his/her students. The importance of the teacher's role in making these decision cannot be emphasized strongly emough. A teacher might use as few of the paper and pencil worksheets to supplement the textbook, use the laboratory activities to give more "hands-on" experience, or organize a unit around a teaching emphasis. Thus, the resource can serve as a springboard to develop a more flexible mathematics curriculum. More importantly, the teacher can supplement the resource with his/her own ideas to build a dynamic instructional program.

FEATURES OF CLASSROOM PAGES

When a ditto master is made using the thermofax process, the material in blue will not reproduce. Thus, the student's copy will contain only the material printed in black. The corners are designed to describe the content on each page.

The symbols below identify the teaching emphases in this resource. Each of these is discussed in the section *Teaching Emphases*.

These are the topics of the page. The sub-section and section headings are useful for locating and refiling pages.

E *Enrichment* (investigations or extensions)

S *Skill-building* (drill and practice)

I *Introduction* (concepts and meanings)

Calculators

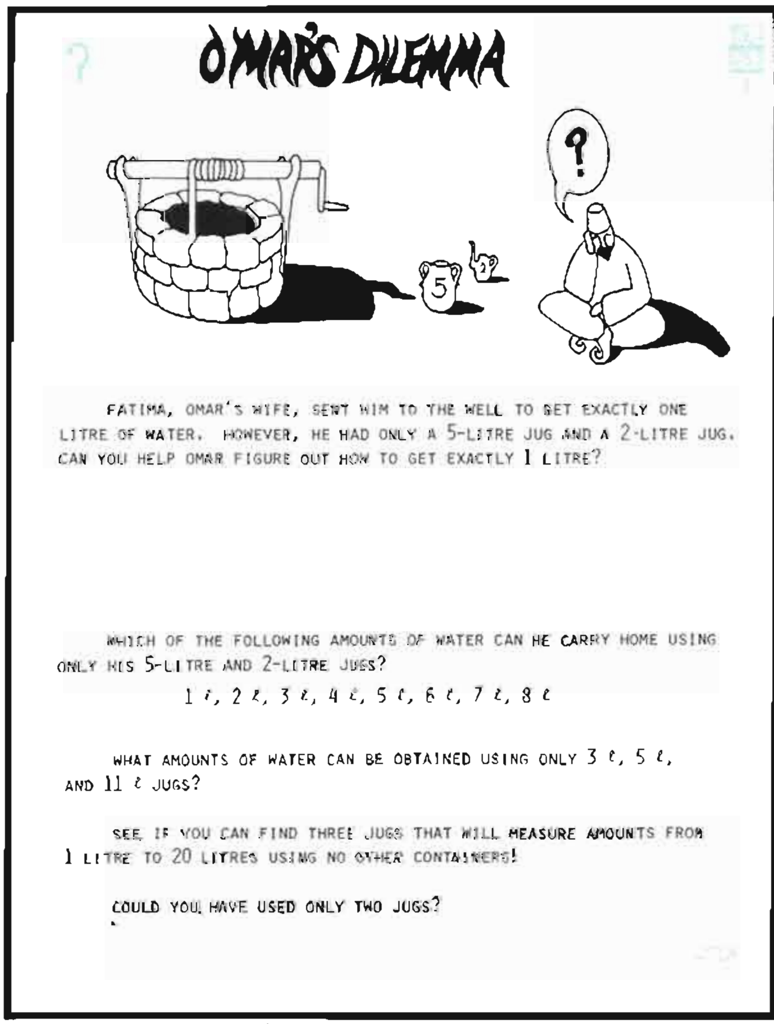
Applications

Problem Solving

Mental Arithmetic

Estimation and Approximation

Laboratory Approaches


OMAR'S DILEMMA

FATIMA, OMAR'S WIFE, SENT HIM TO THE WELL TO GET EXACTLY ONE LITRE OF WATER. HOWEVER, HE HAD ONLY A 5-LITRE JUG AND A 2-LITRE JUG. CAN YOU HELP OMAR FIGURE OUT HOW TO GET EXACTLY 1 LITRE?

WHICH OF THE FOLLOWING AMOUNTS OF WATER CAN HE CARRY HOME USING ONLY HIS 5-LITRE AND 2-LITRE JUGS?

1 ℓ, 2 ℓ, 3 ℓ, 4 ℓ, 5 ℓ, 6 ℓ, 7 ℓ, 8 ℓ

WHAT AMOUNTS OF WATER CAN BE OBTAINED USING ONLY 3 ℓ, 5 ℓ, AND 11 ℓ JUGS?

SEE IF YOU CAN FIND THREE JUGS THAT WILL MEASURE AMOUNTS FROM 1 LITRE TO 20 LITRES USING NO OTHER CONTAINERS!

COULD YOU HAVE USED ONLY TWO JUGS?

Any other blue material on the page is *teacher talk* or *answers*.

If a page is referred to by a didactics paper, one of these symbols is used.

Learning Theories

Teaching Techniques

Diagnosis and Evaluation

Goals and Objectives

Here is the *type* of activity. This refers to the suggested use of the page.

Credit is given here to the *source* if the page is a direct copy. *Ideas from* other sources are also noted.

LIST OF PAPERS ON THE LEARNING THEORY AND THE PLEASURABLE PRACTICE OF TEACHING

NUMBER SENSE AND ARITHMETIC SKILLS

- *Student Self-Concept*
- *The Teaching of Skills*
- *Diagnosis and Remediation*
- *Goals through Games*

RATIO, PROPORTION AND SCALING

- *Piaget and Proportions*
- *Reading in Mathematics*
- *Broad Goals and Daily Objectives*
- *Evaluation and Instruction*

GEOMETRY AND VISUALIZATION

- *Planning Instruction in Geometry*
- *The Teaching of Concepts*
- *Goals through Discovery Lessons*
- *Questioning*
- *Teacher Self-Evaluation*

MATHEMATICS IN SCIENCE AND SOCIETY

- *Teaching for Transfer*
- *Teaching via Problem Solving*
- *Teaching via Lab Approaches*
- *Middle School Students*

STATISTICS AND INFORMATION ORGANIZATION

- *Components of Instruction--an Overview*
- *Classroom Management*
- *Statistics and Probability Learning*

NOTE: A complete collection of all the papers from each resource is available as a separate publication.

GENERAL CONTENTS

DIDACTICS

TEACHING EMPHASES

CLASSROOM MATERIALS

ANNOTATED BIBLIOGRAPHY

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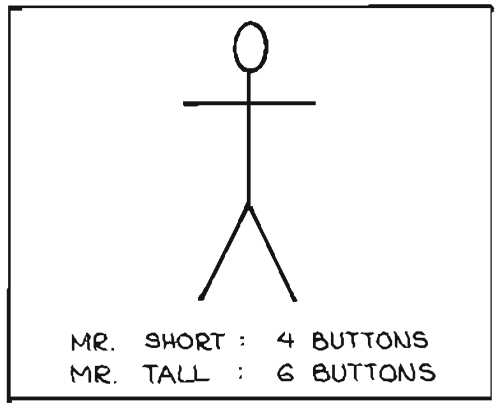
PIAGET AND PROPORTIONS

> The child is not a miniature adult.

How would your students do on this problem?

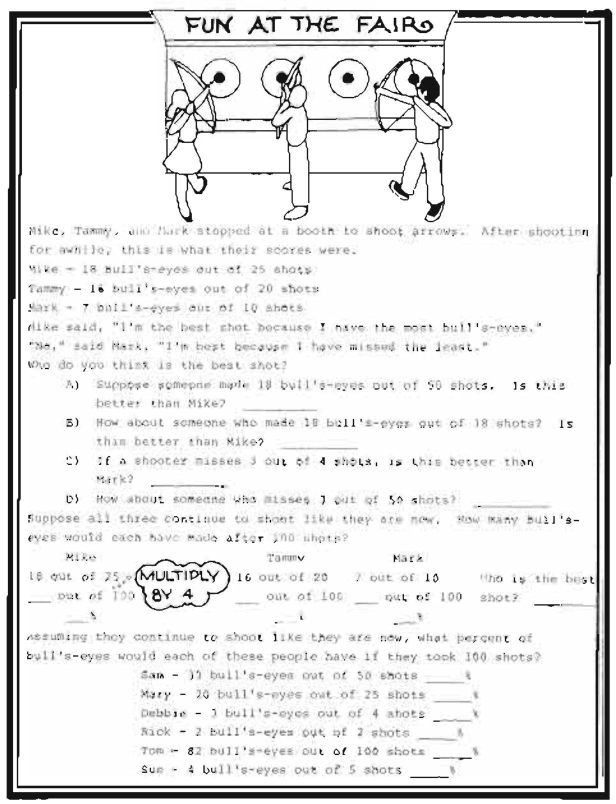


> Mr. Short (given, with a chain of paper clips, to the students) is 4 large buttons in height. Mr. Tall (deliberately not shown to the students) is similar to Mr. Short but is 6 large buttons in height. Measure Mr. Short's height in paper clips (he is 6 paper clips tall) and predict the height of Mr. Tall if you could measure him in paper clips. Explain your prediction.

If your students are like those in a study which used this problem (Karplus, et al., 1974), nearly 30% of the eighth graders and more than 50% of the fourth graders would respond like this:

> Mr. Tall is 8 paper clips high. He is 2 buttons higher than Mr. Short, so I figured he is two paper clips higher.

Perhaps you have tried questions like the one above or like those in the activity to the right (from PERCENT: As a Ratio). If so, you no doubt have noticed that some students do not seem to grasp the idea of proportions even after instruction and even though they may carry out the mechanics all right when the situation is clearly labeled, "Solve with proportions." It is interesting to look at proportional thinking from the framework of the developmental psychology of Jean Piaget and his colleagues. The Piagetians have noted the stages in the natural development of thought patterns--but not how the development can be accelerated or how the patterns can be improved. Indeed, Piaget is usually reluctant to suggest what his


FUN AT THE FAIR

Mike, Tammy, and Mark stopped at a booth to shoot arrows. After shooting for awhile, this is what their scores were.

Mike - 18 bull's-eyes out of 25 shots
Tammy - 16 bull's-eyes out of 20 shots
Mark - 7 bull's-eyes out of 10 shots

Mike said, "I'm the best shot because I have the most bull's-eyes."
"No," said Mark, "I'm best because I have missed the least."
Who do you think is the best shot?

A) Suppose someone made 18 bull's-eyes out of 50 shots. Is this better than Mike? ____
B) How about someone who made 18 bull's-eyes out of 18 shots? Is this better than Mike? ____
C) If a shooter misses 3 out of 4 shots, is this better than Mark? ____
D) How about someone who misses 3 out of 50 shots? ____

Suppose all three continue to shoot like they are now. How many bull's-eyes would each have made after 100 shots?

Mike	Tammy	Mark	
18 out of 25	16 out of 20	7 out of 10	Who is the best
___ out of 100	___ out of 100	___ out of 100	shot? ____
___%	___%	___%	

MULTIPLY BY 4

Assuming they continue to shoot like they are now, what percent of bull's-eyes would each of these people have if they took 100 shots?

Sam - 33 bull's-eyes out of 50 shots ____%
Mary - 20 bull's-eyes out of 25 shots ____%
Debbie - 3 bull's-eyes out of 4 shots ____%
Rick - 2 bull's-eyes out of 2 shots ____%
Tom - 82 bull's-eyes out of 100 shots ____%
Sue - 4 bull's-eyes out of 5 shots ____%

NOTE: Unless otherwise noted, subsections cited are in the RATIO section of the resource Ratio, Proportion and Scaling.

work might dictate to educators. [Hill, 1972, p. 19] When pressed, however, Piaget has offered this comment on the value of his work to teachers:

> *It is essential for teachers to know why particular operations are difficult for children, and to understand that these difficulties must be surmounted by each child in passing from one level to the next. It is not the stages that are important; it is rather what happens in the transition. Teachers must understand, for example, why reversibility cannot be taken for granted with 4-year-olds, and why 12-year-olds have difficulty reasoning from hypotheses. What changes take place from one level to the next, and why does it take so much time?*
>
> *Too many people take the theory of stages to be simply a series of limitations. That is a disastrous view. The positive aspect is that as soon as each stage is reached, it offers new possibilities to the child. There are no "static" stages as such. Each is the fulfillment of something begun in the preceding one, and the beginning of something that will lead on to the next. It is just as disastrous, moreover, to assume that a child has or has not reached a certain stage just because he is a certain age. The ages I have mentioned are only averages.*
> ["Piaget Takes a Teacher's Look," 1973, p. 25]

Let us look at these "stages" to which Piaget refers.

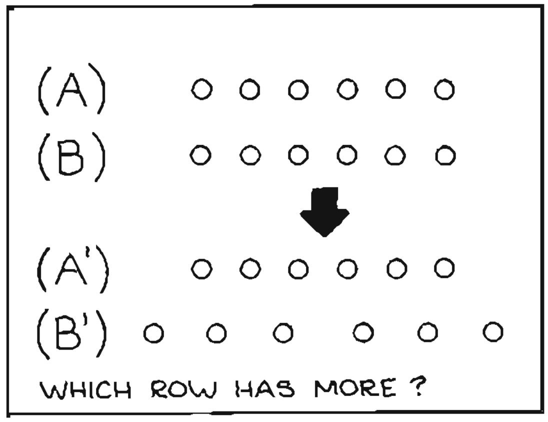
STAGES OF COGNITIVE DEVELOPMENT

Stage	Approximate Ages
Sensorimotor	0 to 2 years
Preoperational	2 to 7 years
Concrete operations	7 to 11 years
Formal operations	11 to 15 years or older

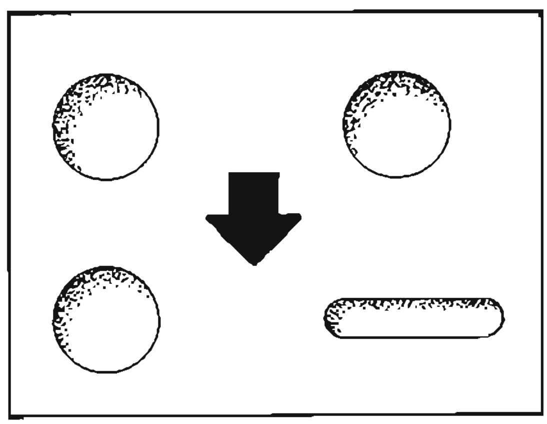
Piaget has identified stages of mental development which can be described in terms of how one thinks during those stages. The existence and order of the stages have been confirmed across cultures. Except as is necessary for illustration, we will consider only the last two stages in the diagram: the concrete operations period and the formal operations period, since virtually all middle school students should be in these two stages. Recall that Piaget warned above about taking ages too literally. The diagram gives the usual age guidelines although considerable data suggest that the age indicated for the start of the formal operations period (11 years) might be too young for many people [Lovell, 1971; Chiappetta, 1975].

The Concrete Operations Period

Concrete operations are parts of an organized structure of mental activities about physically real objects. These objects are not necessarily present. However, if they are not, it is assumed that they have been experienced in the past. Some characteristics of the thinking of the concrete operations period become clearer by considering two of the famous Piagetian tasks that younger children cannot handle. The two examples also show why Piaget is of such interest to teachers in the primary grades.



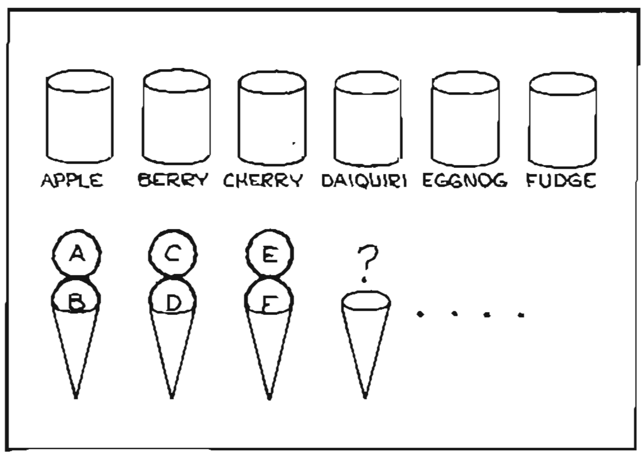
Conservation of number. If preschoolers are given two rows of counters arranged as in (A) and (B), they will agree that the two rows "have the same." If, however, one row is then spread out, as in (A') and (B'), most preschoolers will assert that row (B') has more! Before the concrete operations period, children do not seem to realize that the number in a set remains the same no matter what the rearrangement of the set--they do not "conserve numerousness." Rather, they tend to focus on some dominating perceptual characteristic like the distance between the end counters. Thus, a preschooler may not consider other features, like the greater spacing, which would compensate for the greater apparent "size" of the row. Not until the concrete operations period does a child simultaneously take into account perceptually less compelling aspects. When the child can reverse a mental spreading out of the counters and mentally reposition them in the original display, he can conserve number. This reversibility characteristic is one of the key requirements for a mental activity to become an "operation" in Piagetian terms.



Conservation of amount. Show a preschooler a ball of clay and then have the child make another ball just like the first one (or start with two balls the child agrees have the same amount). If you then roll one of the balls into a wiener shape or flatten it into a pancake shape, the child will likely say that the changed shape now has more than the unchanged shape. He is focusing on the greater length or area and may be unable to reverse (mentally) the rolling-out transformation to roll the wiener or pancake shape back into a ball.

During the concrete operations stage children become increasingly adept at taking into account compensating aspects (greater length may be compensated for by less thickness) and at reversing mental actions like those above. They are much less prisoners of their immediate perceptions. In addition, they become able to deal with classifications, even multiple classifications such as a figure being both a polygon and a triangle. They also become more and more skillful at placing objects in order, based on characteristics like length, size and color intensity. Although they can note a relationship like 4-buttons-to-6-buttons (for Mr. Short to Mr. Tall), they cannot mentally relate that ratio to an equivalent ratio (6-paper-clips-to-how-many). They cannot manipulate relationships between relationships mentally. Further, their thinking seems to be tied to the real world in the sense that they are not comfortable in dealing systematically with what is theoretically possible. The extension of thought from the actual to the potential is a main characteristic of the next period of development, the formal operations period.

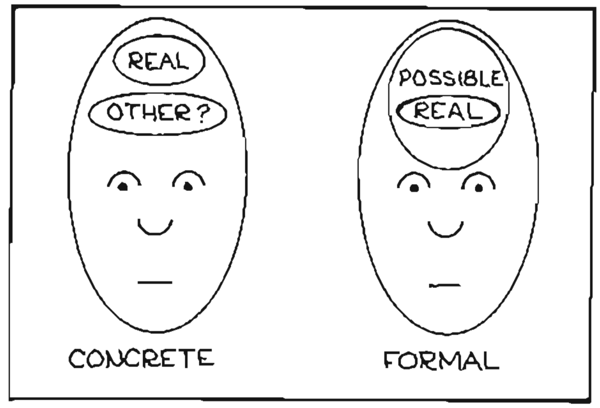
The Formal Operations Period



As an example, suppose that students of various ages are asked to give all the possible choices for a double-dip cone when there are six kinds of ice cream. Children during the concrete period may proceed semi-systematically (e.g., choose all adjacent pairs) but may not find all the possibilities except by a trial-and-error procedure. [Piaget and Inhelder, 1975, ch. 7] One mark of the formal operations stage is that the student can list all the possibilities in a very systematic manner:

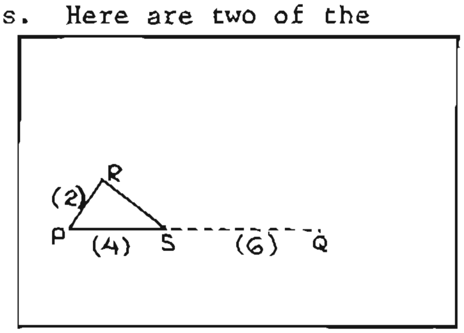
AB, AC, AD, AE, AF, BC, BD, etc.

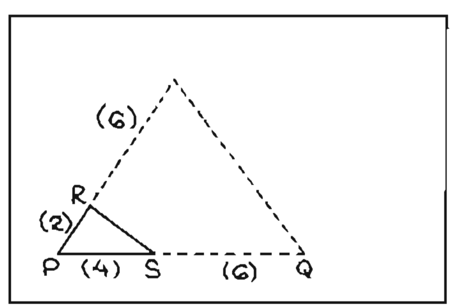
This ice cream cone problem reveals a small facet of a much deeper change. The concrete operations student works primarily to organize and order what is present. What the student cannot do, but what the formal operations student can do, is to list all the possibilities at the start and then try to figure out which of these



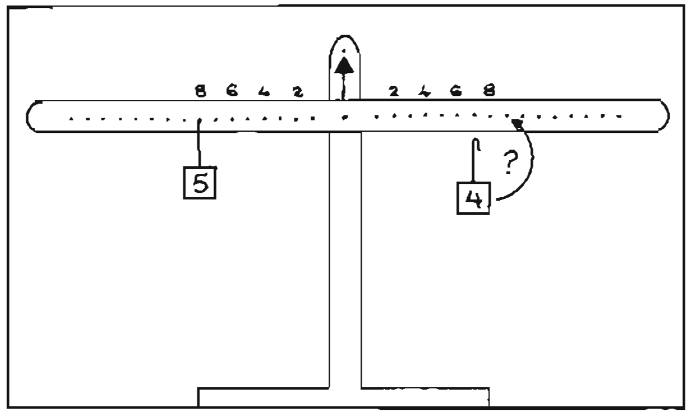
possibilities are all right. ". . . the real becomes a special case of the possible, not the other way around." [Flavell, 1963, p. 205] Piaget regards "this reversal of direction between *reality* and *possibility*" as the fundamental property of formal thought. [Inhelder and Piaget, 1958, p. 255] ". . . the novelty of formal operations is that they bear on hypotheses--that is to say, on statements that are not known, nor supposed to be true at the outset--and on behaviour and properties of objects that cannot be directly observed." [Sinclair, 1971a, p. 9] (Using the label "hypothetical reasoning" for this stage should not, however, be construed to mean that younger students can follow no "if-then" arguments about familiar things.) The student also becomes able to set up experiments to test the effects of variables—for example, with a pendulum, which of the variables length-of-pendulum and weight-of-bob affects the time for a complete swing?

Concrete thought focuses on real objects and events; formal thought can focus on these concrete thoughts (or statements about them) and proceed to work with these thoughts (or the statements). It is this "work with thoughts" that relates to proportions. As was noted for the Mr. Short-Mr. Tall situation, a proportion is a relation (equality) between two relations (ratios). Hence, full understanding of proportions should theoretically require formal thought. The results of the Mr. Short experiment support this assertion, and Piaget has justified the assertion (and may have thought of it) through several experiments. Here are two of the experimental set-ups.





Similar triangles. [Piaget and Inhelder, 1967, ch. 12] Suppose that students in the concrete period are asked to draw a triangle with exactly the same shape as the triangle to the right but with one side being $\overline{PQ}$. Many will focus on the difference of 6 units between PQ and PS and extend $\overline{PR}$ by 6 units to get the triangle. Admittedly, they often recognize that the result "doesn't look right," but the point is that their natural focus is on the difference rather than on a proportional relationship. Recall that this same focus on differences was apparent in the Mr. Short experiment.



Lever-arm balance. (Cf. Inhelder and Piaget, 1958, ch. 11) Suppose students at the concrete stage are asked to place a weight on a lever-arm balance so as to balance a weight already on the balance (as to the right). Some students may count from the ends of the balance arms. Others will have no idea how to predict the balancing position. Once again, some may focus on the difference in weights (1 unit) and feel that the 4-weight should be placed 1 unit farther out (onto the 9-peg) to adjust for this difference.

SO . . . ?

"All very interesting, but what does it mean to a teacher?" you may be thinking. Piaget's reluctance to suggest exactly how teachers should use his findings was noted. However, many Piagetian disciples and interpreters are willing to suggest implications. The following three areas seem to involve safe assertions:

1. Knowledge of stages. As the early quote from Piaget indicates, some knowledge of the developmental stages enables one to be more sensitive (a) to how the student may be viewing things and (b) to topics that the student may not be able to absorb, in particular by strictly verbal means. For example, if teachers know that students may notice the difference relationship in a setting actually involving proportions, they would be likely to use pages like *Comparison 1* in the Getting Started section to develop awareness about ratios and other sorts of relationships. Or, if one knows that reasoning based on hypotheses contrary to fact is difficult for some student, then he would realize that indirect proofs might miss the target. The difficulty, of course, is in determining at what stage a student is. A brief treatment of Piaget may leave the

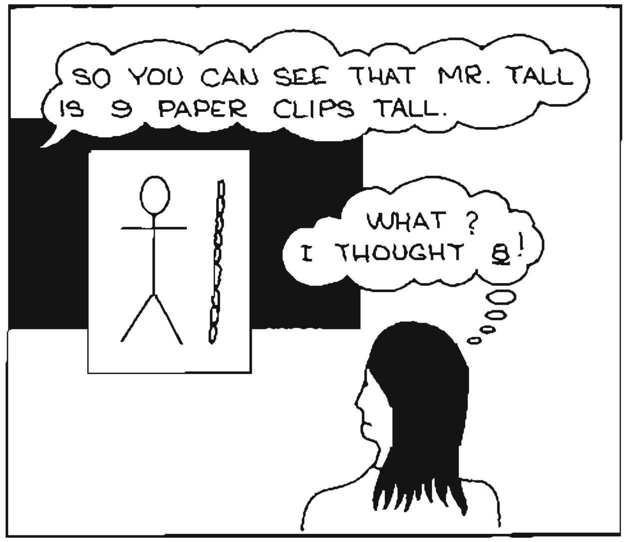
impression that a middle school student can be neatly categorized as being either concrete operational or formal operational. This is not the case. Piaget indicates that one can use formal operations in some familiar area without reaching the formal level in other areas [Piaget, 1972] and quite often students are in a transitional phase between stages.

2. Knowledge of factors in development. Piaget includes these factors in the natural development of the operational stages: maturation, experience, learning from others ("social transmission") and the more technical "equilibration." [1964] The first factor--maturation--reminds us not to expect an overnight development of formal thought even though we might give careful verbal explanations of, say, the proper way to handle proportions. We might be better off to give more attention to the experience factor by providing bases in physical realities for proportions. Pulaski comments that "Piaget feels that children in the classroom suffer from 'cognitive passivity'; they need to take a more active part in discovering facts and relationships." [1971, p. 125]

> FACTORS IN COGNITIVE GROWTH
> --MATURATION
> --EXPERIENCE
> --SOCIAL TRANSMISSION
> --EQUILIBRATION

Piaget's feeling about the importance of social transmission is particularly strong when it comes to a student's learning that there are other points of view than his own (see numbers 6 and 7 on pages 10 and 11). Small group work or discussions seem to offer an excellent way to hear others' viewpoints. Social transmission also, of course, includes information received from the teacher; Piaget warns, however, that the student must be "in a state where he can understand this information." [1964, p. 177] For example, a student who does not yet have a "feel" for the meaning of a ratio has to resort to rote memory or patterns to write equivalent ratios. Piaget is emphatic in insisting that words cannot do everything: "I don't believe in the language factor . . . Language is fundamental for giving precision and clarity to thought. But it is not the source of thought and verbal development is not a source of the . . . development of intelligence." [Hill, 1972, p. 19]

Piaget regards the last factor, equilibration, as the fundamental one since it "explains" how one's mental structures change (if the other factors allow). So long as new experiences fit into an existing way of thinking, there is no need to change that thought pattern. For example, so long as looking at differences enables students



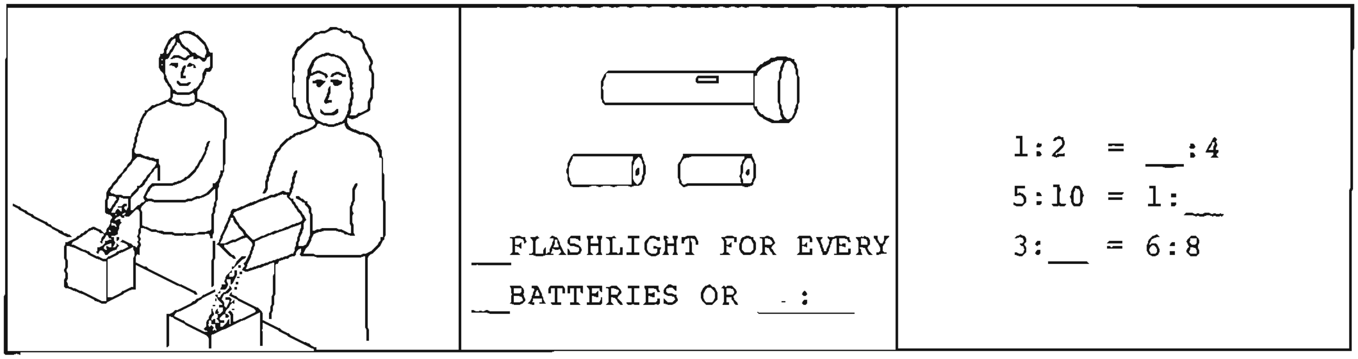
to make correct predictions, then they will continue to look at differences. If, however, a student's way of thinking leads to an erroneous prediction for some new situation, then this discrepancy may provoke a change in how the student thinks. If he makes an incorrect prediction in the Mr. Short experiment, he may be receptive to changing his way of thinking. All the "mays" come in because there can be deficiencies--in maturation, experience or other human sources--which do not allow the thought patterns to change. Hence, if the situation is "ripe" (which is hard to tell), having students work in situations where reality may be contrary to what some of them predict--as in the Mr. Short experiment--may instigate a change in their ways of thinking. If students mispredict about the ratio of the surface areas of different cubes (see *Surface Area and Ratios 1* in the RATIO: Equivalent section), then some may experience a "disequilibrium" and change their thinking to compensate for it.

3. Attention to foundations. As a rule of thumb, instruction for middle school students should start with the concrete operations stage in mind. Making sure that the students do have some concrete bases for their mental manipulations may require providing concrete experiences. Aebli, a Piaget student, gives his interpretation:

> *(The teacher) will arrange the learning materials so that these operations can actually be carried out by the student . . . and then see to it that the student does carry them out. Suppose he wishes to teach the elementary notion of fractions. He would do well to eschew pictures of objects divided into equal parts in favor of actually dividing a concrete object before the class or, better still, getting the student to make the partition himself . . . (Later, get) the student to perform the requisite action with progressively less and less direct support from the external givens.* [Flavell, 1963, p. 368]

This quote supports a practice that many teachers believe in: start a topic as concretely as possible, move to drawings or pictures, and gradually include work with strictly abstract symbols. For example, work with ratio might begin with activities from the section RATIO: Getting Started .

Ratios by Picture I and *Shady Ratios* could provide "semiconcrete" experience before work with numerals only.



There is other research support for the use of non-symbolic aids. Portis [1973] tested upper elementary students on proportions, using either physical aids, pictorial aids or symbols only. His findings? "So low were the mean results . . . when symbolic aids (words only) accompanied the tests that the study raises a question about the appropriateness of arithmetic problem solving tasks . . . when only symbolic aids are used." [p. 5982]

SUMMARY

<u>Concrete Operations Period</u>

The student becomes able to
- --give simple or multiple classifications for objects
- --order objects with respect to one or more attributes
- —deal with number ideas and operations
- --conserve length, area and interior volume

<u>Formal Operations Period</u>

The student becomes able to
- --initiate thought by systematically listing possibilities
- --reason with premises that are not necessarily true
- --use proportions
- --design experiments to control variables

Piaget's experiments surely indicate that the child is not a miniature adult in how he thinks. Middle schoolers will likely be in the concrete operations period, the formal operations period or, in many cases, somewhere between these two. For some topics our teaching may provide not the complete structure but only the underpinnings for a student's full understanding at a later time. The theory (and the successful practices of many) seems to suggest starting instruction with physical realities familiar to the student before working solely with abstract symbols. Better results are likely if the students are active participants in the classroom, not just passive observers. See the reference list for sources for further study of Piaget and developmental psychology.

? ? ? ? ?

1. Try some of the Piaget-type experiments with children of different ages. For examples, with a 5-year-old and an 8-year-old . . .
   a. conservation of number
   b. conservation of amount
   or, with a 9/10-year-old and a 12/13$^+$-year-old . . .
   c. Mr. Short-Mr. Tall
   d. ice cream cones
   e. similar triangles
   f. lever-arm balance

2. Diagnose Jasper's problem and plan how you would remedy it.

   Jasper's work: $\frac{4}{2} = \frac{\overset{7}{\cancel{n}}}{5} \qquad \frac{8}{6} = \frac{\overset{17}{\cancel{n}}}{15} \qquad \frac{\overset{1}{\cancel{n}}}{2} = \frac{7}{8}$

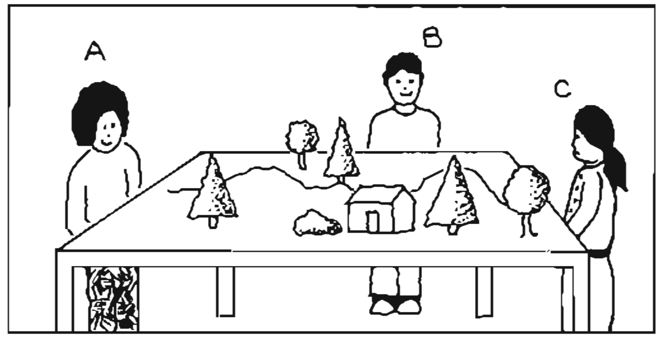
3. (Discussion) Piaget: ". . . there exists a fundamental lacuna in our teaching methods, most of which, in a civilization very largely reliant upon the experimental sciences, continue to display an almost total lack of interest in developing the experimental attitude of mind in our students." (1971, p. 37—The quote actually dates from 1965.)
   What can we do in mathematics classes to develop an "experimental attitude"?

4. "As regards the teaching of the new mathematics, for instance, which constitutes such a notable advance over traditional methods, experience is often falsified by the fact that although the subject is 'modern,' *the way in which it is presented is sometimes psychologically archaic insofar as it rests on the simple transmission of knowledge* . . ." [Piaget, 1973, p. 17, emphasis added]
   a. What alternatives to the "simple transmission of knowledge" might Piaget endorse?
   b. In a recent survey more than 80% of the polled fifth-grade teachers used manipulative materials in less than one-fourth of their lessons. [National Advisory Committee on Mathematical Education, 1975, p. 75] Is this consistent with Piaget's assertion above?

5. Reasoning with verbal statements is not necessarily formal thought. ". . . all verbal thought is not formal and it is possible to get correct reasoning about simple (statements) as early as the 7-8-year level, provided that these (statements) correspond to sufficiently concrete representations." [Inhelder and Piaget, 1958, p. 252] You might try the following two problems on an 8-year-old and a 13-year-old.
   a. Jim is taller than Dale. Matthew is taller than Jim. Who is tallest?
   b. Mary runs faster than Ann. Mary runs slower than Jane. Who runs the fastest?

6. Piaget uses the word <u>egocentrism</u> to refer to the tendency to see things from only one's own point of view. (This use should not carry the usual negative connotation of willful self-centeredness. It is natural; the person is not even aware that other viewpoints or perspectives exist.) Piaget has noted a recurrence of egocentrism at the different stages of development.

6. (continued)
   a. If student A (usually 8-9 years old) views a display as to the right, he may not be able to describe, or pick out a picture showing, how the display would look to student B or C. Relate this phenomenon to egocentrism (you may want to try it on a few of your students). (Cf. Piaget and Inhelder, 1967, Ch.8.)



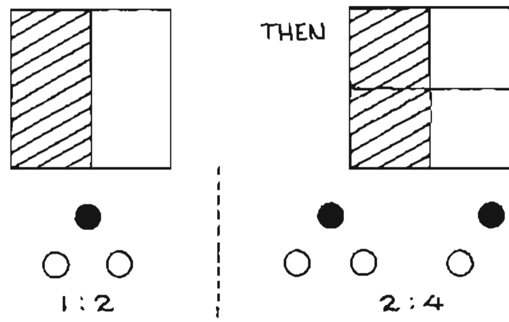
   b. Look at *3 Faces You Should Have Seen* in SCALING: Supplementary Ideas in Scaling in terms of egocentrism.

7. Egocentrism refers to more than the visual perspectives in number 6. For example, egocentrism during the concrete operations period follows these lines: Since the student can now carry on considerable mental activity, his egocentrism takes the form of an "inability to differentiate clearly between what he thinks and what he perceives . . . In the course of . . . reasoning the child often fails to distinguish between his hypotheses and assumptions on the one hand and empirical evidence on the other. *It is this lack of differentiation between assumption and fact that constitutes the egocentrism of the concrete operational period.*" [Elkind, 1970, pp. 54-55] For example, a student may reinterpret or reject facts to fit his idea, rather than use facts to test his idea.
   a. (Discussion) Does this notion of egocentrism account for any behavior you have noticed in students?
   b. Flavell notes that Piaget "has stressed the paramount importance of interaction with peers as the principal vehicle by which the child is liberated from his egocentrism." [1963, p. 369] Why does this make sense?

8. (Discussion) Sinclair, in reporting on the Piagetians' work with memory, notes that excessive emphasis on <u>rote</u> memory could well lead to an underdeveloped self-organized type of memorizing. In addition, ". . . a certain amount of rote learning is, in the present system, inevitable. But as regards mathematics and allied disciplines, it appears that it is the concept formation itself that should be fostered by all possible means . . ." [1971b, p. 134]
   a. What things are, in your opinion, most efficiently learned through rote?
   b. What are some "possible means" to foster the formation of the concepts of ratio, proportion or percent?

9. If a <u>larger</u> unit is used to measure a length, <u>fewer</u> units are required (as in the Mr. Short experiment). Abramowitz [1975] feels that students need to have explicit exposure to such "inverse" situations and to contrast problems in which an <u>increase</u> in one thing is associated with an <u>increase</u> in another (e.g., things bought and money spent, or speed and distance covered, or--usually--time spent on homework and number of exercises finished). Plan how you might handle these inversely related variables:
   a. size of area unit and number of units required to measure a plane region.
   b. size of volume unit and number of units required to measure a space region.
   c. number of teeth in a gear and number of turns, as in *I'm Beat! How About You?* in the PROPORTION: Application subsection.
   d. distance from fulcrum and weight needed to balance a weight, as in a lever-arm experiment.

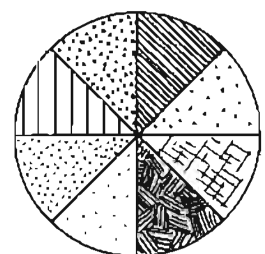
10. It is likely that you would endorse the first row to the right as satisfactorily showing that one-half and two-fourths are equivalent fractions. You might, however, be reluctant to accept the second row of drawings as demonstrating $\frac{1}{2} = \frac{2}{4}$ since the <u>units</u> are different. Yet, the second row does seem acceptable for showing that 1:2 and 2:4 are equivalent ratios. Many writers regard ratios and fractions as being the same (see the commentary and mathematical content section for RATIO); whether they <u>are</u> the same is a minor issue in mathematics education.



   a. What is your view?
   b. Is this issue only an academic question or does it have implications for classroom practices?

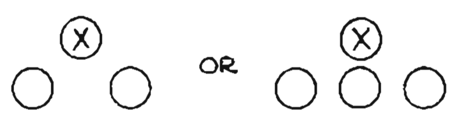
11. (Outside reading) Read the Nelsen [1969] or Van Engen [1960] article for more on the are-ratios-the-same-as-fractions issue. A more technical article is the one by Van Engen and Cleveland [1967].

12. This excerpt dealing with probability may interest you in trying something similar with your students. With respect to a spinner like that to the right, "Which is more probable: two on each color with sixteen spins, or about one hundred on each with eight hundred?"


(REGIONS ARE OF DIFFERENT COLORS)

Chen (11 years, 11 months): "More likely with sixteen spins and two on each color because two is less than one hundred. When there's chance, its (sic) easier (to reach uniformity with 16 tries)." [Piaget and Inhelder, 1975, pp. 76-77] What would you do with a student who responds like Chen?

13. Here are some more quotes from Piaget and Inhelder's work with probability. [1975, pp. 153-156] You may want to try similar questions with your students and plan how you will deal with students who respond in similar fashions. The students are asked which of two situations gives the better chance of drawing a marker with a cross on it. The notation 3:4 means that in one situation 3 out of 4 markers have crosses.
   a. Given 1:3 and 1:4, Ram (9 years, 9 months) chooses 1:3 "because there are only two without crosses." Given 1:2 and 2:4, Ram says, "Here (2:4). No, here (1:2) because there is only one without a cross."



   "Why did you say the other?"
   "Because there were two crosses . . ."
   b. <u>Mey (11 years, 1 month)</u> (1:2 and 2:4) "Here (2:4) it's more certain because there are two crosses."
   "But are there two without crosses also?"
   "Well then it's the same."
   "Which do you choose?"

(continued)

13. (continued)

"This one even so (1:2) because there's only one without a cross . . ."
(2:5 and 1:3) "It's the same because there (2:5) there are three without crosses and two with crosses, and here (1:3) one with the cross and two without."
"And if I put this (1:3 and 2:6)?"
"I prefer this (1:3) . . ."

c. Bis (12 years, 2 months) Bis recognized the equal likelihood in 1:2 and 2:4 situations. For 2:5 and 6:13, ". . . it's the same. There is as much risk on one side as in the other. Here there are six with crosses and seven without crosses, and there two and three [thus a difference of one on each side]."
"And if we compare three piles, 1:2 and 20:21 and 100:101, which is the most likely to give a cross?"
"There where there is one more out of one hundred, there is less chance of getting it than there is with the one more out of twenty (after a long hesitation)."
(See Fischbein, et al. [1970], Lovell [1971] and Romberg and Shepler [1973] for other research on probability with students.)

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Thirty percent or more of the ninth graders could not handle tests designed for aspects of the formal operations period. Sex differences appeared on some tests at some grades.

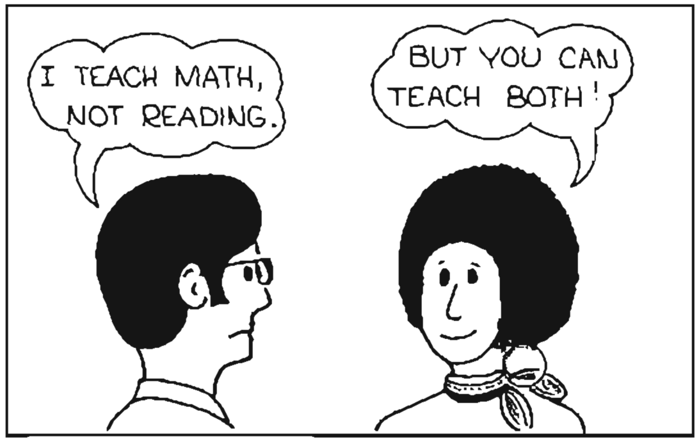
RATIO, PROPORTION, AND SCALING

Placement Guide for Tabbed Divisors

READING IN MATHEMATICS

EVERY TEACHER A READING TEACHER?

Should the mathematics teacher be expected to teach reading? If you live in a state where a course in the teaching of reading is required for secondary certification, you know that some certification boards think the answer is, "Yes." And there seems to be good reason: ". . . student achievement is improved if the reading skills instruction is integrated with the content instruction." [Laffey, 1972, p. vii; see also Aiken, 1972, pp. 369-370] Certainly you will agree that it is unreasonable to expect that the language arts teacher will have much training in mathematics. But by the same token, unless you have had training in the teaching of reading, what can you be expected to do? This section can only skim the surface of this important topic. We have attempted to collect here some recommendations of various experts. Most of the suggestions fall into a "good idea" category and seem usable in mathematics classes.



WHAT CAN WE DO?

1. We need all the help we can get: Befriend the language arts teachers and the reading specialists. They are the experts . . . and their training programs usually encourage them to illustrate word-attack and comprehension skills with vocabulary lists and reading assignments from teachers of other content areas. These teachers can also give you information about reading difficulties of particular students. If you are involved in writing materials for students, a reading specialist can be an invaluable consultant.

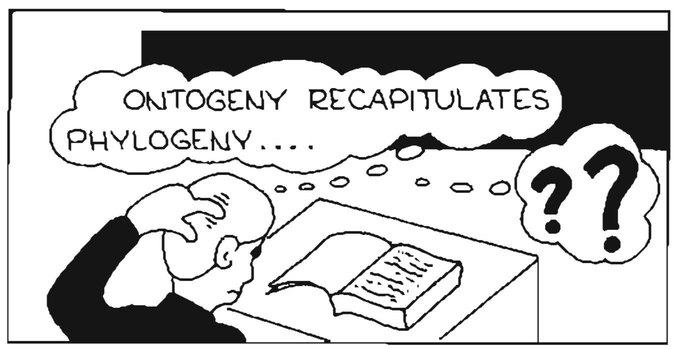
2. Diagnose. "One recommended device in a content class is simply to give, at an early meeting, a reading text assignment with accompanying questions. The teacher calls each individual to his desk to read excerpts quietly from the selection.

NOTE: Unless otherwise noted, subsections cited are in the PROPORTION section of the resource Ratio, Proportion and Scaling.

The teacher can quickly differentiate those who can and cannot cope, at least mechanically, with the reading, and he can obtain some estimate of the range of proficiency within the group." [Burnett, 1974, p. 109] Note the "at least mechanically." Many apparently "good" readers cannot interpret a paragraph they can read smoothly. (Caution: Some poor oral readers understand quite well.) You might wish to check the students' reading scores in their permanent records if you do not want to take time to have them read aloud.

3. Point out features of the book that help to organize the material and enable the reader to review. A student may not know what information is in the table of contents, how the index is used, what and where the glossary is, or where special reference tables are located. Point these features out again as they are needed. Providing an overview of the chapter may remind the students that the parts are related. It may be worthwhile to look at pages to study the organization of explanatory material; the role of underlined, italicized, colored or shaded print; and the use of diagrams to support narrative explanations.

4. Make certain that words and symbols *mean* something. (See also *The Teaching of Concepts* in the section POLYGONS & POLYHEDRA in Geometry and Visualization, Mathematics Resource Project.) You may have had the experience of reading every word in some sentence perfectly, yet having no idea of what the sentence said--and you are a college graduate. We may need to review vocabulary, especially before a reading assignment. Below is a list of terms and phrases and the percents of a large group of seventh and eighth grade students who checked the terms and phrases as "known." Note the results particularly relevant to work in this resource: ratio (37.2% of the students felt they did not know the term), proportion (53.6% unfamiliar), mean (46% unfamiliar), extremes (72.1% unfamiliar), percentage (27.2% unfamiliar), scale drawing (32.9% unfamiliar). And, in a large-scale study with sixth graders, Ebeling found ". . . the average sixth grade



SELECTED TERMS AND PHRASES WITH PERCENTS OF "FAMILIARITY"*

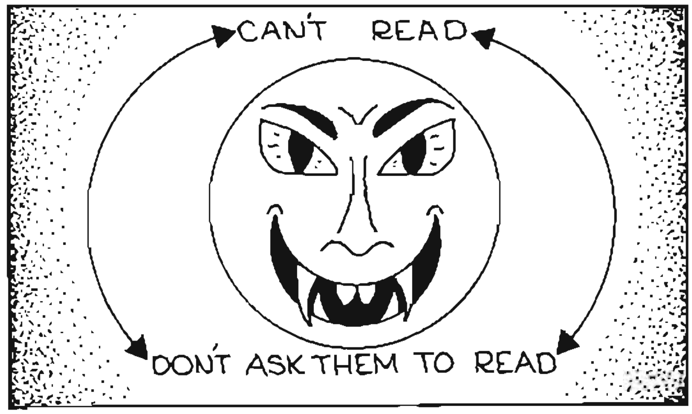
abacus (39.9)
adjacent (25.8)
algorithm (3.8)
alternate exterior angles (30.0)
alternate interior angles (31.8)
arc (48.6)
array (24.5)
associative (76.6)
associativity (39.1)
bisect (27.2)
calculate (49.8)
capacity (41.9)
centroid (6.1)
commutative (71.4)
commutativity (44.8)
complementary angles (37.5)
concentric circles (16.1)
congruent (33.8)
consecutive (22.7)
cross product (36.3)
decagon (24.7)
distributive (67.9)
distributivity (38.1)
dodecahedron (2.5)
ellipse (21.2)
equate (23.6)
equilateral (49.2)
exceed (29.7)
extremes (27.9)
factorization (46.9)
heptagon (24.9)
hyperbola (2.7)
icosahedron (2.4)
integer (33.2)
isosceles (31.1)
magnitude (9.1)
mean (54.0)
midpoint (41.1)
motion (36.5)
obtuse angle (58.7)
octagon (49.7)
octahedron (6.8)
ordered pair (43.8)
origin (45.4)
parabola (3.4)
percentage (72.8)
perpendicular (48.3)
perpendicular bisector (22.4)
pi (36.5)
polygonal region (24.9)
polyhedron (6.3)
probability (32.9)
proportion (46.4)
Pythagorean theorem (3.2)
quantity (68.1)
ratio (62.8)
reciprocal (44.0)
region (49.3)
rhombus (29.3)
scale drawing (67.1)
scalene triangle (36.5)
sides of an equation (43.1)
tangent (8.2)
tetrahedron (5.2)
trapezoid (37.6)
velocity (25.0)

*"Familiarity" was determined by whether the 7th and 8th grade students checked "know" when given the term or phrase. Entries selected from Appendix A in Kane, Byrne and Hater. [1974, pp. 75-90]

student has the ability to associate fewer than half of the algorithms with their mathematical terms." [1974, p. 7515] Certainly, we must give attention to vocabulary and symbolism!

Warn students about words that can be easily misread: tens and tenths, parallelogram and parallel, for example. Look over a reading assignment for the many mathematics words which have everyday (non-mathematical) meanings: point, interest, means, prime, extremes, product, round, foot, yard, scale, set, adjacent, altitude, base, check, cube, and many others. These should be called to students' attention since there is some evidence that readers, particularly poorer readers, give the everyday interpretations to such words. Some mathematics words have multiple meanings, also. For example, students may encounter equivalent sets, equivalent fractions, equivalent ratios, equivalent sentences, logically equivalent, equivalent expressions during their mathematics work.

5. Have the students read! "Although some teachers of mathematics prefer to provide initial explanations themselves in an oral presentation and let the students work alone only on the exercises or problems, it seems unfortunate that pupils should be taught always to rely so heavily on the teacher for new learning." [Catterson, 1974, p. 158] When will the students ever learn to read? The greatest factor in whether students learn something is whether they have the opportunity to learn it. Yet it is so easy to make assignments of exercises from a textbook and never even mention the accompanying explanatory material in the book, let alone give any advice on reading mathematical discourse. Perhaps it should be no surprise when so many students don't "understand" word problems.



A VICIOUS CIRCLE?

Reading experts suggest a few methods. [Catterson, 1974, p. 158]

• Ask the students to read a paragraph or two of narrative silently and then to answer oral questions.

• Prepare a few written questions as a reading study guide for the students.

• Devote a few minutes to silent reading of material as the routine before the exercises are assigned.

• Give students some guidance before a reading assignment: What should they look for? What vocabulary and symbols are in the passage that may need review or explanation? What earlier material is the passage related to? What is the role of accompanying figures and tables, if any? Do the students have paper and pencil ready to fill in gaps or check assertions in the book?

• Form small groups of mixed reading ability occasionally. Have the fluent readers read orally with the others following along. Then the whole group can discuss the passage or written questions.

6. You will probably want to give explicit attention to the reading of charts, diagrams, tables, graphs, measuring tools. Quite often charts, diagrams and tables are not read from left-to-right or top-to-bottom. Learning to make charts, etc., helps in learning to read them. You know that students need instruction in reading rulers, protractors and other measuring devices; they are mentioned here as a reminder of the variety of types of reading required in mathematics.

7. Remind students that reading in mathematics is much slower than most other reading. You cannot get much from a paragraph of mathematics reading by whizzing through at comic-strip or fiction-book speed. Symbols and information-packed sentences cannot be absorbed by fast reading. They require concentration since missing even a part of a symbolic expression or sentence may cause a complete lack of understanding. Skimming a chapter or reading a summary first to get an idea of the "big picture" may be worthwhile, but mathematics reading for understanding requires so many fixation pauses, regressions, intentional re-readings and references to diagrams or parallel numerical work that one's reading rate cannot be great. (Point out that this necessarily slow reading is the reason you assign only a little reading, not because the reading is unimportant.)

8. Try to have available (in the room or in the library) a collection of "recreational" reading books on mathematical themes. For example, Weyl's Men, Ants, and Elephants: Size in the Animal World [1959] might be of interest to a student after work with ratio and scaling. Books of activities, games and puzzles like Patterns and Puzzles in Mathematics [Horne, 1970] or The I Hate Mathematics! Book [Burns, 1975] are easy for a student to browse in even if there are only a few minutes available. The NCTM publication by Hardgrove and Miller [1973] annotates trade books on mathematics. Librarians often have lists of recently-published books and welcome specific requests from mathematics teachers (except perhaps in times of bare budgets). If the librarian does get some of your requested books, be sure they are used by the students!

9. <u>What about the very poor reader</u>, the one who has so much trouble with each word that he has forgotten the beginning of the sentence by the time he gets to the end? Well, you <u>did</u> befriend the reading specialist, didn't you?! . . . General guidelines do include these two: "illuminate, perhaps even magnify, each gain the learner makes, and . . . avoid unfavorable comparisons of the learner's performance with that of other pupils." [Burnett, 1974, p. 104] Besides the reading in small mixed groups mentioned earlier, Burnett also suggests ". . . tape recording a better reader's oral reading of assigned chapters and encouraging disabled readers to read silently along with the tape while listening with headphones." [p. 109] If you have a whole class of poor readers, you might read to them or with them, sentence by sentence. If you have an "individualized" program of some sort, for less severe cases you might be able to find a textbook with a more nearly appropriate reading level.

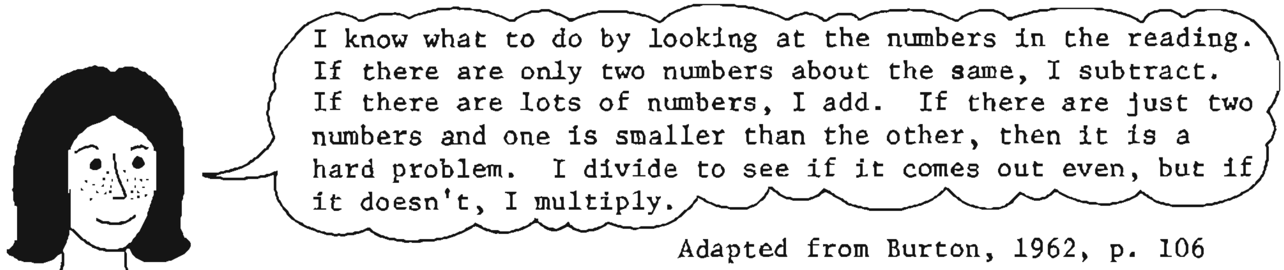
SUMMARY

We can attack the reading problem by . . .

- informally assessing the students' reading ability;
- working with the reading teachers, especially for students with severe reading problems;
- making certain our students know helpful features of texts, understand the vocabulary, have a realistic approach to a mathematics reading rate, and get instruction in reading diagrams, charts, etc.; and
- above all, insisting that the students get some reading practice.

WORD PROBLEMS

One of our most important areas is our students' hardest: word problems. It is no surprise that reading skill is related to success with problem solving (e.g., see Balow, 1964).



Adapted from Burton, 1962, p. 106

"SOLVING" THE WORD-PROBLEM PROBLEM!

1. <u>Students must learn that a single reading of a word problem is not enough</u>. <u>Three</u> readings <u>might</u> be enough: the first a fairly fast one to get the idea of the

situation, the second much more slowly to interrelate the data, the third to go back and pick out specific pieces of information. (Let's call it two readings to keep students from fainting!) Having students paraphrase a word problem is recommended by some authors since paraphrasing requires reading for meaning and not just reciting the words.

2. The students must have a good conceptual base. If a student's "concept" of multiplication consists only of the x-sign and an associated algorithm, he may not see a situation in a word problem as fitting multiplication instead of some other operation. Without a proper foundation, success with word problems involving proportions, say, may depend on the problems being within a sequence of problems dealing with proportions. Time spent on developing meaning for symbolism and vocabulary seems to be worth it. [VanderLinde, 1964]

3. The student may need a reminder that making diagrams, figures or sketches is often a helpful problem-solving aid. Sometimes students seem to think that only numerals should be written, or resist doing things they may not regard as being important for getting an answer. Some teachers make a specific assignment on making diagrams only or require diagrams for most word-problem assignments.

4. Attention to the choice of word-problem situations may help performance. In a very small study with fifth graders, Lyda and Church [1964] found that lower ability students seemed to do better on word problems involving situations like those actually experienced by the students (buying groceries, buying milk in the cafeteria, for example). In some work limited to ninth grade boys, Travers [1967] found that the students preferred "social-economic" situations (selling hot dogs, saving money for a date, organizing school committees) to situations labeled "mechanical-scientific" (repairing tractors, learning to drive a car, testing spark plugs) or "abstract" (solving secret codes, counting imaginary objects like "ooks" and "zooks"). However, performance on preferred types was about the same as performance on non-preferred types. Using students' names or local school situations in word problems seems to help some students.

5. To increase weaker students' chances of some success with word problems, you may need to rewrite some problems, using shorter sentences, easy vocabulary and accompanying diagrams. (You might have some better students rewrite some for you to use next year or with another class this year.) For multi-step problems, having the numerical data presented in the order they are used in the solution makes the problems easier. [Burns and Yonnally, 1964] For example, in "Jo bought 2 loaves of bread

at 69¢ a loaf and a package of cupcakes for 25¢. How much did these items cost?" the data fit (2 x 69) + 25 exactly. Note, however, that the students may "learn" a wrong thing from such problems: "Always use the numbers in order," and would be in trouble with "Jo bought 2 loaves of bread and a package of cupcakes. The cupcakes were 25¢ and the bread 69¢ a loaf. How much did these items cost?" An intermediate step in helping your students might be to have them order the numerical information without actually solving the problem.

6. Just as making graphs helps one learn to read graphs, writing story problems may help students interpret other story problems. You might have students write word problems and prepare an assignment or a bulletin board with their "challenges." Small groups seem to prepare better written, more imaginative problems than some individuals do. Such problems may not be easy!

Similar sorts of activities can be based on these ideas: give data and have students ask questions, or give an equation and have students write a word problem for it. This last activity may reveal some misconceptions. For example, suppose that students are to write a word problem for $\frac{2}{5} = \frac{n}{30}$, and one student writes, "Kay saves \$2 out of every \$5 she gets. She has saved \$30. How much did she get?" The student may have made a careless error, but he might lack "feel" for proportions.

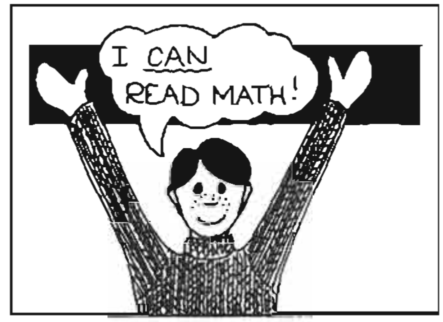
7. Students should have exposure to some word problems which give insufficient information and others which give too much information. Both sorts force the student to read for understanding. Some problems should require the use of "hidden" information (e.g., if the price per kilogram is given and the cost of 135 grams is sought, the "hidden" relationship between kilograms and grams must be used). Each of these three aspects—insufficient data, superfluous data, "hidden" facts—appears in out-of-school problems. Indeed, it may be only in school that word problems present the necessary and sufficient data in a tidy three-line package.

8. A final point is also related to "don't make it easy not to read." If all the word problems in a list are of the same type or require the use of only the

algorithm practiced for the last two days, students can often get correct answers without reading all the problems. Their technique? Just use the recent algorithm or perhaps read the first problem, see what operation is to be performed and then go through each of the other problems, pick out the numbers and perform that same operation. To avoid this version of "problem solving," different operations (or irrelevant data) could be included in every list of word problems.

IN CLOSING

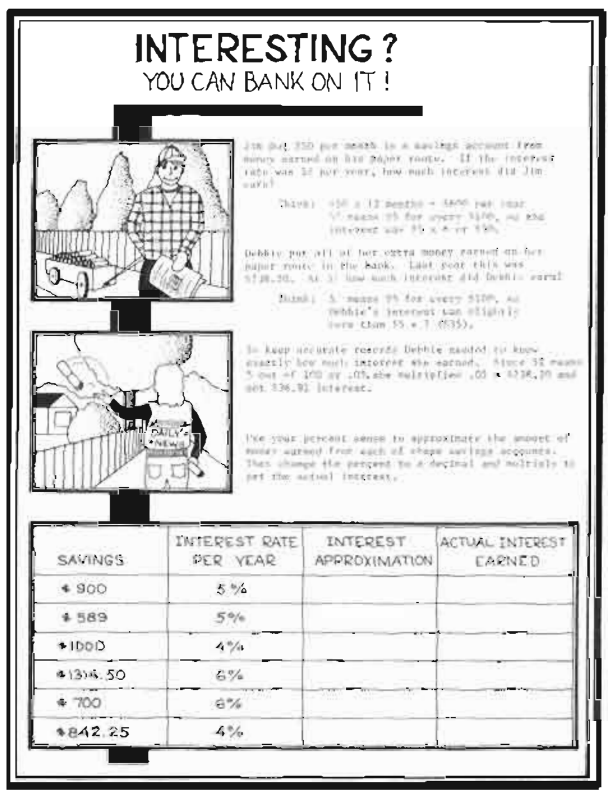
Experience suggests that students can become better readers of mathematics if we give them some help. If instruction on mathematical reading makes the student a more self-sufficient learner and a more confident worker of word problems, the time will be well spent.



????

1. (Discussion) How do you handle the reading of mathematics? Do you give a reading assignment, but then always reteach it?

2. Suppose you were to have students read the narrative portion of the page to the right. Outline what you would say before they started and what questions you would ask after they had read it.



INTERESTING?
YOU CAN BANK ON IT!

SAVINGS	INTEREST RATE PER YEAR	INTEREST APPROXIMATION	ACTUAL INTEREST EARNED
$900	5%		
$589	5%		
$1000	4%		
$1316.50	6%		
$700	6%		
$842.25	4%		

3. a. Talk to the reading specialist (or consultant) in your school about reading in mathematics. Or ask the district reading consultant to talk to the department. Or ask that an inservice day be devoted to reading in the content areas.
   b. Find and study a standardized test which gives reading "levels" for students. Does it give you any ideas for types of reading skills you could foster?

4. Pick a typical textbook passage and plan what you might say as you assign it.

5. a. Does your textbook include a regular review of vocabulary? If not, plan how and when you feel vocabulary review should be worked in.
   b. You might like to poll your classes at the start of the year to see how many students know the word "glossary."
   c. (Discussion) Have students make up sentences using a particular vocabulary word and record them on tape. The whole class, small groups or individuals could listen to the playback (you can expect lots of giggles until the novelty wears off). Do you have other ideas for vocabulary practice? (See Feeman, 1973.)

6. About how much work with a new word does a student need? One (undocumented) claim is that the "average learner must see, hear, and use a word approximately 40 times before he can be expected to decode it and apply it successfully." [Bureau of Elementary Curriculum Development, 1972] When some new term is introduced, audiotape the class session(s) until you feel that the students can work with the term. Listen to the playback to see whether the estimate 40 seems about right.

7. Following are some terms and the corresponding percents of seventh and eighth grade students who claimed to be familiar with them (from Kane, Byrne and Hater, 1974). Account for the low percents.

abacus (39.9)	isosceles (31.1)
bisect (27.2)	probability (32.9)
concentric circles (16.1)	rhombus (29.3)
hypotenuse (12.9)	trapezoid (37.6)

8. It is surprising that some very similar words had very different familiarity percents in the Kane, et al. [1974] lists:

associative (76.6)	commutative (71.4)	divisible (81.9)
associativity (39.1)	commutativity (44.8)	divisibility (59.2)

   What are the implications of such results for us?

9. (Discussion) Have you tried anything to impress upon students the importance of vocabulary (e.g., have them keep a vocabulary-spelling list, have vocabulary quizzes)? Do you try to work new and recent vocabulary into the discussion as much as possible (see number 6)?

10. The results of the Ebeling study [1974] suggest that we give more attention to the meaning of mathematical symbols. Design a worksheet over some collection of symbols, using any of the following modes (or choose your own way).
    a. Write expressions without words for each of these:
       the product of 17 and 4.5 17×4.5
       the sum of 17 and n ________

one-third of 82.2 __________

$1\frac{2}{3}$ increased by 5 __________

the ratio of 6 to 9 __________

8 divided by 2 __________

b. Match:

_____ 6+n	A. some number decreased by 6
_____ 6−n	B. 6 from some number
_____ 6:n	C. the ratio of 6 to some number
	D. 6 increased by some number
	E. the ratio of some number to 6
	F. 6 decreased by some number

c. What symbol is suggested?

add _____ multiply _____ more than _____ is more than _____

difference _____ ratio _____ quotient _____ percent _____

11. Students must learn not to interpret reading piece-meal, but to read for total meaning. For example, the student who thinks that "and" always means "add" should be asked, "What is the product of 5 and 7?" Think of pairs of sentences which would point out to the student the importance of reading for total meaning . . .

a. in contrasting "less than" and "is less than."

b. when a student thinks that "longer than" always cues addition.

c. in being alert to common meanings for technical words--"product" might appear in a word problem involving addition or subtraction.

From the commentary for FRACTIONS: Multiplication/Division, Number Sense and Arithmetic Skills, Mathematics Resource Project.

12. (Discussion) Many teachers (and textbooks) suggest to students that they follow a sequence of steps to organize their approach to word problems.

For example, Read / Re-read } to understand the problem and get the facts

Determine the question asked

Estimate the answer

Write equation

Solve equation

Check the answer by comparing it with the estimate.

Discuss the value (or shortcomings) of such outlines.

13. Suppose you plan to discuss reading in word problems. Outline what you will say (include the problem(s) you would refer to).

14. Suppose you plan to discuss reading in word problems. What would you say for each of these problems?
    a. Nida can buy candy bars at the rate of 3 for 49¢. Estella can buy the same kind at the rate of 6 for a dollar. Who has the better buy?
    b. Jasper took 3 hours to hike 8 miles. At that rate how long will it take him to go 6 miles?

15. Consider adding verbiage to make second versions of *Petite Proportions 1* and *2* in PROPORTION: Getting Started. Use both versions with a class to point out the importance of getting the main ideas from a word problem.

16. In an effort to get students to think about the meaning of a word problem, some teachers emphasize key words and the meanings they connote. For example, "remove" indicates a subtracting action. Students sometimes misinterpret this advice to mean that the word "remove" signals "subtract." This interpretation can, of course, cause errors:

    Dan's father removed 56 kg of junk from the garage on Saturday. If he removed 45 kg Sunday, how many kilograms of junk did he remove that weekend?

    Prepare word problems to convince students that they must think about the total situation and not trust single words to tell them what to do if the students think . . .
    a. "give" always means to subtract.
    b. "all together" or "in all" always means to add.
    c. "cut off" always means to subtract.
    d. "cut up" always means to divide.

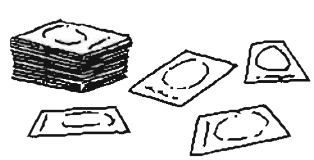
17. This practice is sometimes advocated: Present the students with a word problem without numbers and ask them what information they would need.
    Example: Ellie had some money. Then she bought a record and a bracelet. She put the rest in the bank. How much did she spend?
    Evaluate the practice. If you like the idea, write 5 or 6 such problems and try them out with a class.

18. Since the question in a word problem puts the focus on what is sought, it would seem that putting the question at the start of the problem might improve performance with word problems. You may wish to to try this idea with your classes, especially since many real-life problems do start with a question ("Can we afford a new TV?"). [Williams and McCreight, 1965]

"Gil will trade 4 of his Giants baseball cards for 3 Dodgers cards. How many Dodgers cards can he get for 12 Giants cards?"

OR

"How many Dodgers baseball cards can Gil get for 12 of his Giants cards? He will trade 4 Giants for 3 Dodgers."



19. (Discussion) Some teachers use the following to initiate a discussion about the importance of reading directions. Evaluate and add to the list.
    a. Give a collection of easy calculations (5 - 3, 12 x 2, 100 - 10) but have the directions say, "Change all + signs to -, all - signs to +, (etc.) before giving the answers."
    b. Title of sheet: Can you follow directions?
    Directions: Read all 20 exercises before starting.
    1. Write your name. 2. Add your phone number to your house number.
    3. Multiply your age by 25. (etc.) . . . 20. Do only number 1 and then hand your paper in.
    c. Don't assign any exercises until everyone has (had time to) read the directions. Then have someone tell what the directions say. Then assign the exercises.

20. (Outside reference) If you serve on a textbook selection committee, you may hear references to readability levels. These levels are not subjective opinions but are usually established by one of several "objective" methods. Typical factors might include average sentence length, a measure of vocabulary familiarity, number of question marks, number of long words, ... Study and compare two or three of the methods of arriving at a readability level. (Kane, Byrne and Hater, 1974, outline some of the approaches.)

21. Are there "hidden information" exercises in your textbooks? If not, you may wish to write some and give them to your students.

22. Some teachers prepare dittoes sheets of word problems and have students cross out useless information and underline clues. You might like to try this idea out and evaluate it.

23. One study with second and fifth grade teachers revealed that "students actually read less than one page, or at most one or two pages of textual materials out of five." [National Advisory Committee on Mathematical Education, 1975, p. 72] How much of the reading in the text do you assign to your students?

24. One aspect of vocabulary is that some mathematical words have many synonyms. One of the "worst" is length. For example, distance, perimeter, width, height all refer to the same characteristic that length does. Are there still other synonyms for length?

25. To help students with weak mathematics vocabularies, some teachers . . .
    a. use multiple-choice instead of fill-the-blank test exercises.
    b. list possible answers to a mathematics crossword puzzle in a scrambled list.
    Do you have any additional ideas?

26. Sedlak [1974] used some "modified cloze" word problems in his work with learning-disabled students. The items may suggest a form you could adapt to use as a diagnostic test or to emphasize the importance of reading a word problem. Here are some samples.
    i. The cook had 6 eggs. The cook ___ 3 eggs. The cook now has 9 eggs.
    (a) broke (b) colored (c) borrowed
    ii. (extraneous information) The man had 4 cars. The man bought 2 trucks. The man ___ 3 cars. The man now has 7 cars.
    (a) painted (b) bought (c) sold
    iii. (distracting verb) The baby had 4 dolls. The baby lost 2 toys. The baby ___ 3 dolls. The baby now has 7 dolls.
    (a) touched (b) received (c) lost

27. Manzo [1975] has outlined a "guided reading procedure" which students seem to enjoy, but which requires careful and purposeful reading, gathering information and interpreting the author's message. You might like to try it two or three times a year on reasonably sized text passages or recreational mathematics reading (not more than 5 to 7 minutes reading time).
   - Step 1. Tell the students to read and remember everything.
   - Step 2. With the passage concealed, the students tell what they can remember. The teacher records on the board, probably abbreviating. (Be patient in waiting for students to start talking.) When no one can remember anything else, the class studies the information recalled, looking for incompleteness and inconsistencies.
   - Step 3. The class rereads the passage to fill in the gaps.
   - Step 4. The class suggests an outline (simple or elaborate).
   - Step 5. The teacher asks questions to develop full understanding. (This serves a modeling function.)
   - Step 6. The teacher gives a test of unaided recall (true-false, matching, multiple-choice, etc.).

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RATIO, PROPORTION, AND SCALING

Placement Guide for Tabbed Divisors

BROAD GOALS AND DAILY OBJECTIVES

"Would you tell me, please, which way I ought to go from here?"
"That depends a good deal on where you want to get to," said the Cat.
(Lewis Carroll, Alice in Wonderland)

What are we really after in teaching middle school mathematics? This important question has answers on two different levels: (a) the global "aims of education" level and (b) the day-by-day level.

BROAD GOALS

We shall confine our attention here to those broad goals which are mathematics-based. This restriction should in no way be interpreted to detract from the general, and in many cases most important, middle school goals which cut across subject matter lines: developing the students' views of themselves as maturing individuals, providing maximal opportunity for the students to explore different areas, etc.

If you have never thought about the broad goals of middle school mathematics education, you might take a few minutes to jot down a list--pretend you are preparing a report for the school board . . .

Inservice teachers usually include such goals as the following:

--The student should acquire the mathematical skills needed in everyday living.

--The student has the mathematics background needed for subsequent school work (non-mathematical as well as mathematical).

--The student should be able to think logically.

--The student should be able to solve problems.

--The student can use mathematics in other areas.

--The student can communicate about mathematical ideas and techniques.

--The student likes mathematics.

--The student appreciates the worth and value of mathematics and its relation to other subjects.

There often are references to "mathematics in our cultural heritage," "an inquisitive mind," "the power to visualize" and others, but the list above is fairly typical.

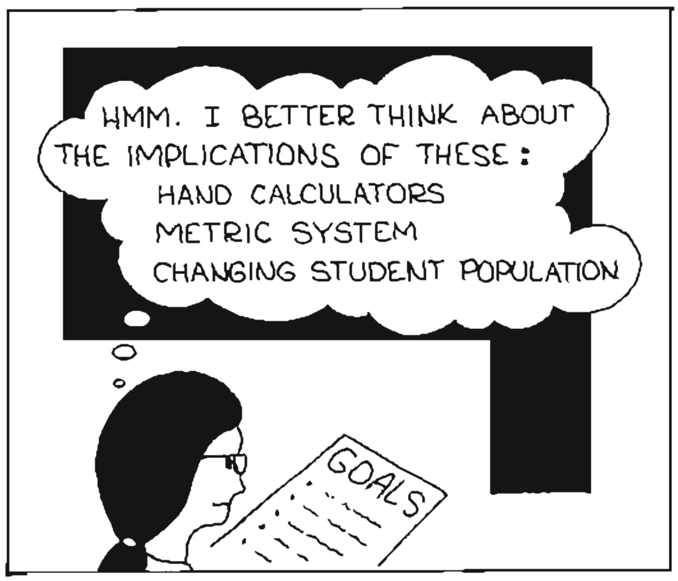
What do other experts say? As times have changed, different groups of distinguished teachers and mathematicians have composed lists (see Osborne and Crosswhite, 1970, for a discussion of several of these recommendations). Such statements of

NOTE: Subsections cited are in the SCALING section of the resource Ratio, Proportion and Scaling.

goals are, of course, quite compatible with the lists of inservice teachers. Goals for School Mathematics [1963], for example, mentions

1. the acquisition of skills,
2. contact with the key ideas of mathematics and the precision offered by mathematical terminology and symbolism,
3. the development of a genuine understanding of mathematical ideas,
4. exposure to the liberal-education aspects (mathematics is a growing, man-made field, marked by hard work and genius),
5. the building of self-confidence in one's analytical power, and
6. an appreciation for the power of mathematics but at the same time an awareness of its limitations.

A statement of goals--either our own or someone else's--gives us direction in choosing what to teach and, to some degree, how to teach. If our statements of goals are not to be merely academic exercises, they should be the basis for our unit plans and daily lessons. Are we giving enough attention to problem solving? Is computation taking too much time? What do we mean by "think logically"? Shouldn't we spend more time on approximation--that's very much needed in everyday living? Frequent re-evaluation of our broad goals--and whether we are doing more than paying them lip-service--is a worthwhile investment of time.

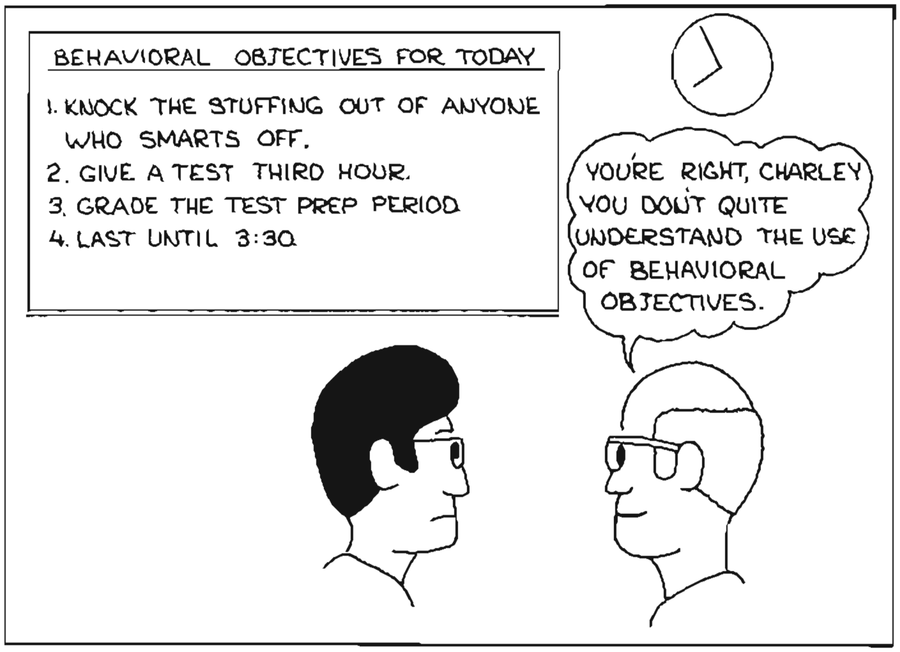


DAILY OBJECTIVES

Broad goals describe general areas and define our direction, but by their very nature broad goals are vague. When we are planning a lesson, we cannot be vague. One development of the last 15 years--the increased use of behavioral objectives--has been both championed and cursed. Let us review this two-edged sword so that we can use it to improve, not impoverish, our teaching.

The Case for Behavioral Objectives

Mr. Denson may jot down only "Pages 158-159" in his plan book, but he probably has more in mind than that. If asked, he might cite as his objective, "To cover proportions," or perhaps, "The student will understand proportions." Although it does emphasize the student rather than the teacher, even this last statement is vague. Does it mean that the student can solve a proportion? . . . Define "proportion"? . . . Solve a word problem using a proportion? . . . Illustrate a proportion with a concrete model? Each of these last aspects is a possible interpretation of "understand"--and that is why "understand" is vague. Phrases like "solve a proportion," "define," "solve a word problem" and "illustrate with a concrete model" are less subject to alternate interpretations than "understand." They describe a clearly observable behavior, whereas "understand" may seem to be an internal sort of thing. These behavioral "actions" are (fairly) unambiguous. A substitute teacher could read the behavioral objective, "The student can make a scale drawing," and have a much better idea of what to do than if confronted with the non-behavioral "The student will know about scale drawings."



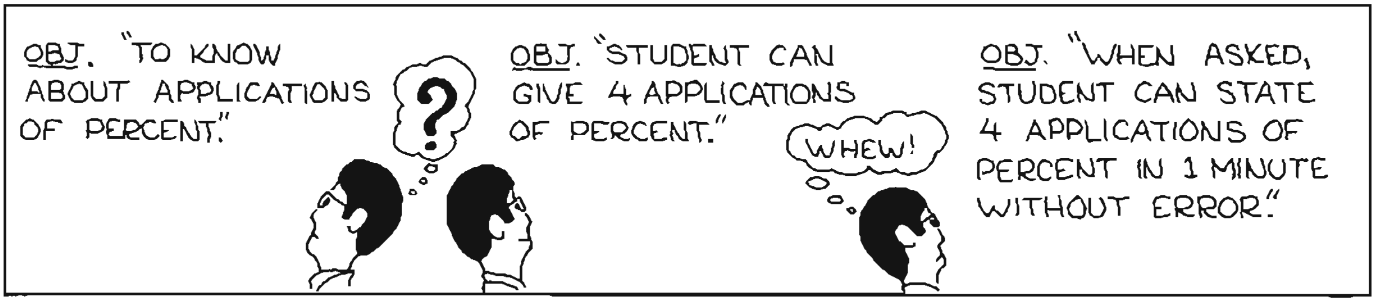
USUALLY VAGUE	"ACTION" ORIENTED	
understand	name	conjecture
know	construct	choose
comprehend	write	give (example)
develop	state	define
appreciate	solve	order
feel	demonstrate	sketch
⋮	justify	calculate
		⋮

To summarize, behavioral objectives meet these requirements:

—Behavioral objectives put the emphasis on the student. They describe what the students, rather than the teacher, will be doing.

--Behavioral objectives describe what observable behavior we are seeking. Verb forms with many interpretations are avoided in favor of "action" verbs with narrow meanings.

Purists usually demand additional conditions for a "true" behavioral objective: a careful description of the conditions under which the student is to act, and a clear specification of time restrictions and proficiency level. Although these are important for evaluation purposes, we will not emphasize them here. See Mager [1962] if you have never been exposed to an extensive treatment of behavioral objectives.



Proponents of behavioral objectives point out these advantages:

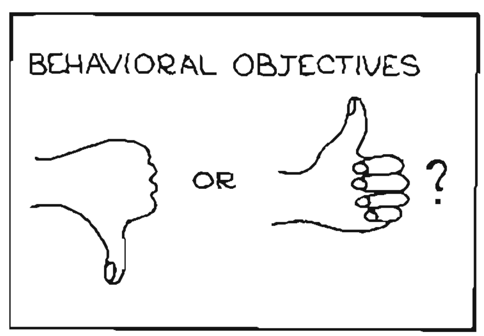
1. The emphasis is where it belongs. Our whole reason for being in the classroom is to help the student to grow in some area.

2. A behavioral objective provides a clear focus for a lesson. Rather than just following the book, the teacher understands exactly what he is trying to achieve and can use that as a guide in planning instruction. (Behavioral objectives are particularly valuable to the beginning teacher. They keep the teacher on track or help to find the track.)

3. Since behavioral objectives are explicit about what will be expected from the students, evaluation is easier. Some teachers have noted that writing sample test items before teaching a unit can help to determine what many of their objectives really are.

4. It may help to tell the students what the objectives are. Then they don't have to guess what is expected of them! Walbesser and Eisenberg [1972] surveyed several studies dealing with the effect on performance of telling students the (behavioral) objectives. They concluded that there was "cautious support" for the positive influence of telling students the objectives on these measures: achievement, rate of acquisition and resistance to forgetting.

What Are the Objections to Behavioral Objectives?



Despite their advantages, the use of behavioral objectives may restrict objectives (and instruction) to things that are easily described in behavioral terms. First, this misuse can result in having only "low-level" objectives. It is easy to write,

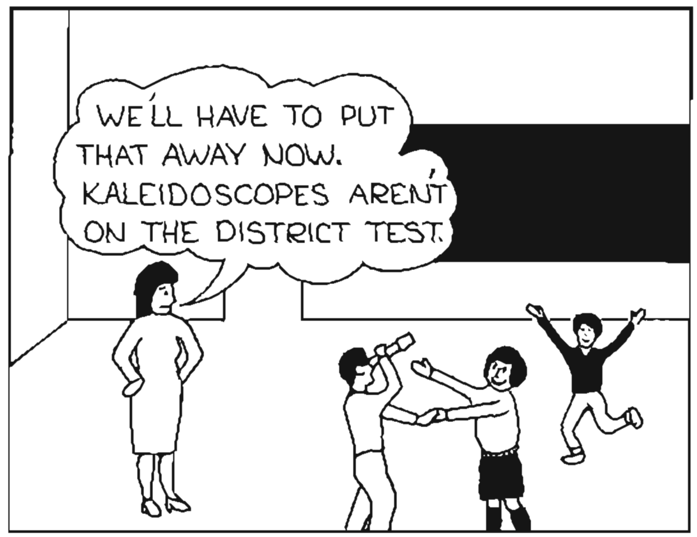
--"The student can give an example of a scale drawing," or

--"The student can use the given scale to find lengths represented in a scale drawing," or even

--"The student can make a scale drawing of, for example, the floor plan of a house."

But it is much harder to put in behavioral terms what we expect from students when we give them nonroutine problems about scale drawings. So, some early efforts at compiling objectives omitted "higher level" concerns. Walbesser [1972], for example, said, "For whatever reason, most behavioral objectives that are available for mathematics curricula name trivial behaviors." [p. 437] And because of this, "Behavioral objectives are now a negative force in contemporary mathematics education." [p. 438]

A second misuse of behavioral objectives may occur if the evaluation of a student (and the teacher) is to be based only on achievement of a narrow list of objectives. Davis expressed this disturbing hypothesis: ". . . most teachers . . . will not 'waste time' on letting students 'mess about' with materials, but will restrict themselves to getting students to memorize rote superficial knowledge, and to testing students for their possession of such knowledge." [1973, p. 36]



Certainly, we do want to let students "mess about," experiment, explore, conjecture, use their imaginations and test ideas. We do not want to cut off a promising student question because its consequences do not fit into an externally imposed list of objectives. We do not want to lose sight of important objectives which are hard to describe in behavioral terms (e.g., "the student appreciates the value of mathematics"). We do not want to confine our students to only "trivial, rote, superficial knowledge." Is there a way to use behavioral objectives yet guard against giving attention only to "low-level" ones? Yes. Make conscious use of an ordered classification system for objectives.

Ordered Classification Systems--Taxonomies

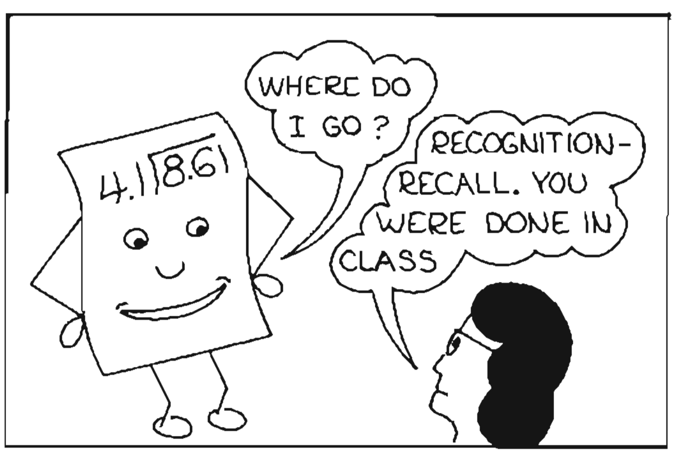
The first part of the best-known taxonomy of educational objectives is usually called "Bloom's taxonomy," [Bloom, et al., 1956], and is devoted to the cognitive domain (objectives having to do with thinking). The second part [Krathwohl, et al., 1964] deals with attitudes, interests, appreciations, awareness, feelings--the affective domain. These general works have given rise to subject-oriented taxonomies which naturally relate better to the pertinent fields (for examples in mathematics, see Epstein, 1968; Romberg and Wilson, 1968; Wilson, 1971). We shall use one suggested by Avital and Shettleworth [1968] to illustrate the nature of such taxonomies. It has three categories (all cognitive), called "levels of thinking." Taxonomies of objectives usually imply an order of complexity; the higher levels are more complex and usually involve thinking from lower levels as well.

Avital-Shettleworth
Levels of Thinking

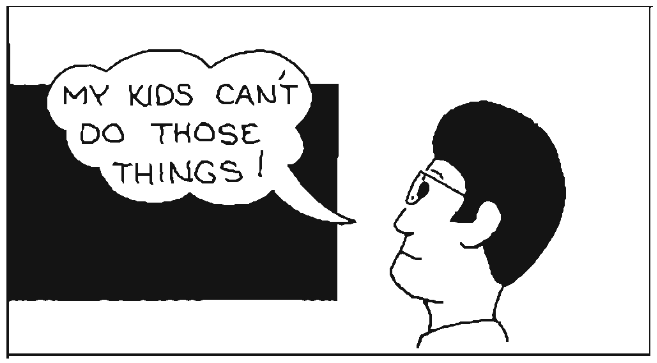
Open search	Productive thought
Algorithmic thinking, generalizations of varying degrees	Reproductive thought
Recognition, recall	

The lowest level of the Avital-Shettleworth classification scheme describes thinking which involves only recognition or recall of learned material. For example, "The student can give an instance of a real-life use of scaling," would require thinking at this level if the teacher/book has given such an example. Working a word problem exactly like one done in class could involve this level of thinking since the student might just repeat a memorized solution. Objectives at this level are quite proper; facts, information and procedures provide the basis for "higher" levels.

Avital and Shettleworth's second level of thinking processes--algorithmic thinking, generalizations--includes all routine applications of learned procedures: Computation with algorithms; generation of slightly different examples; translation among verbal, numerical, graphic forms; word problems with only slight differences from ones done in class; interpreting a scale drawing very similar to one in the book; etc. For example, "The student can draw a figure similar to a given figure, with a given scale factor" could fit into the second level; test items for this objective would involve figures or scale factors not seen by the student but which nonetheless are easily handled by the learned procedure. As an illustration of the gradations within this second level, a scaling activity based on, say, a scale of 2.5:1 would be a bit higher than one based on a whole number not covered in class with whole number scales (as in *Bigger Than Life* in Making a Scale Drawing). Situations at this level are new to the student (otherwise they would belong to the first level) but vary so little from the learned situations that the same step-by-step procedures, the same algorithms, yield the solutions. As the examples or tasks become less and less similar to those covered earlier, they become more and more difficult and occupy a higher gradation within the second level.



Since the thinking required at the first two levels is so close to the original learning, Avital and Shettleworth call these levels reproductive thinking. Their third level—open search--involves productive thinking, the production of something new to the student. A question about a scale drawing of a scale drawing of another scale drawing--unless covered in class--might require a new interrelating of techniques and be "above" the second level. Or, the problem, "Solve $\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \ldots + \frac{1}{99 \times 101} = n$" would be at the open search level for most students. Open search thinking requires the "nonroutine manipulation of previously learned material and, at a higher level, discovery of relationships among previously learned material and, at a (still) higher level, discovery of relationships among previously unrelated concepts and propositions." [Avital and Shettleworth, 1968, p. 19] Certainly thinking at this level is more complex and demanding. Yet, if we never give our students an opportunity to work on open search problems, how will they learn to attack nonroutine problems, to "think logically," to interrelate data? Note, moreover, how much skill work would be likely even in an insightful solution. Working on open search problems exercises lower levels of thinking as well as higher levels.



Three notes should be kept in mind when using any taxonomy.

1. Where a particular objective fits in the taxonomy depends on the background of the student. A very high level task for some sixth graders might be a low level one for eighth graders or even other sixth graders who have had specific instruction on that task.
2. There are "gray" areas between levels of a taxonomy and gradations within levels. Fine distinctions may be important to theorists or evaluators, but our use of taxonomies will probably be only as a reminder to give attention to higher level types of thinking and to affective objectives.
3. The existence of different levels does not mean that each level gets the same amount of attention. What we do want to guard against is ignoring some levels completely.

The Affective Domain

The Avital and Shettleworth taxonomy gives guidance in writing cognitive objectives. Naturally-stated objectives having to do with appreciation or attitudes or other areas of the affective domain do not usually fulfill the condition of describing observable behavior of the student. For example, the following objectives are reasonable--but they are not behavioral:

--"The student will appreciate the compactness of scientific notation."

--"The student will approach a problem with confidence."

--"The student will feel good about mathematics."

To state such objectives behaviorally is possible but seems artificial (see Krathwohl, et al., 1964; Mager, 1968). We could (should?) use objectives like "The students will not have pained looks on their faces when they come to mathematics class" or "Each student will say, 'Hey, neat!' at least once a month." The thing to remember is that affective objectives are legitimate and of great importance for middle school students. Just because measurable affective objectives are awkward to write, we cannot ignore the affective domain and hope it will take care of itself. Objectives devoted to feelings, attitudes and appreciations are necessary whether they are stated behaviorally or not.

USING A TAXONOMY

A taxonomy comes in handy in planning--or in keeping track of--the work for a particular topic. With a taxonomy and charts like the one below, teachers can plan the work so that they give some attention to all the different levels and types of objectives. Checkmarks or brief entries can reveal whether some level is neglected.

Level	Topic: Drawing similar figures (use pages in Making a Scale Drawing section)	
Recognition, recall	Vocabulary: enlargement scale factor, shrink	(use *What's the Point?*)
Algorithmic thinking, generalization	1. Practice technique, with various scale factors. (use *Bigger Than Life* and *A Shrink*)	2. Reverse: give drawing, ask for scale factor.
Open search	Ask: What happens to the area of a region if you make a 3:1 scale drawing of it?	
Affective domain	Drawing usually interest-provoking. Back-up: Use for "art" project.	

A Position

Behavioral objectives, particularly if used with a reasonable taxonomy, can be a valuable tool in planning, organizing and evaluating instruction. The tool must not, however, become the master and result in a narrowed, inflexible curriculum which ignores higher cognitive levels and affective dimensions of learning.

? ? ? ? ?

1. Make your list of the broad goals of middle school mathematics. Which are "need-to-meet" and which are only "nice-to-meet" goals?

2. Choose a couple of the following content areas and describe specifically how each can be relevant to each of your broad goals.
   a. Addition of fractions
   b. Division of fractions
   c. Scientific notation
   d. Proportions
   e. Scale drawings
   f. Percent
   g. Activities like *3 Faces You See* in the Supplementary Ideas in Scaling section.
   h. Activities like those in the Maps section.

3. a. Be your own devil's advocate and attack each of your claims in number 2. For example, if you felt that addition of fractions contributed to a broad goal like "mathematics needed in everyday life," attack that claim by asking how many times a month a person adds fractions or by counterclaiming that estimation and approximation are what people actually do with addition of fractions.
   b. Defend (or alter) your claims.

4. Mr. Youngblood: "I don't get it. Why should I be concerned about broad goals? Let's face it, the people who wrote the book know more than I do. And daily objectives? Why not just use the ones in the teacher's manual?"

   Your response:

5. (Discussion) One's view of what is important about mathematics seems to depend somewhat on the person's mathematics background. Write a short paragraph giving your answer to the question, What is mathematics? Compare your description with others (if you can, find someone who has studied more mathematics than you have, and someone who has studied less).

6. (Discussion) Toffler, in Future Shock [1971], makes the statements below in writing about education. Discuss the merit of the statements and their implications for our broad goals and classroom work.
   a. "Parents look to education to fit their children for life in the future." [p. 398]
   b. Future technology will require people "who can make critical judgments, who can weave their way through novel environments, who are quick to spot new relationships in the rapidly changing reality." [pp. 402-403]
   (continued)

6. (continued)
   c. "The rapid obsolescence of knowledge and the extension of life span make it clear that the skills learned in youth are unlikely to remain relevant by the time old age arrives." [p. 407]
   d. "(N)othing should be included in a required curriculum unless it can be strongly justified in terms of the future." [p. 409]
   e. "Tomorrow's illiterate will not be the man who can't read; he will be the man who has not learned how to learn." [p. 414]

7. The statement was made that our broad goals might influence how we teach (page 2). Explain.

8. Which of the following objectives are not behavioral (in the limited sense of page 3)? Why not?
   a. The student should understand our numeration system.
   b. The student should know the importance of mathematics in society.
   c. The student should be able to appreciate the work saved through abstract thought.
   d. The student should be able to deal with percent.
   e. "Explain the example on p. 134." (From a lesson plan)

9. Clarify the following objectives by making them more behavioral.
   a. The students know how to use ratios.
   b. To introduce proportions.
   c. The student understands scale drawings.
   d. To show how to make and use a hypsometer.
   e. The student will study how changing the dimensions of a cube affects the area and volume of the cube.
   f. The student appreciates the size of the solar system.

10. Do you use behavioral objectives in your teaching? Why or why not?

11. A specific activity may serve different objectives, depending on who is doing it. Write an objective for which one might choose to use *Archie Texs' Ruler* (in the Making a Scale Drawing section) if . . .
    a. the student needs more work with measurement.
    b. the student needs practice at following directions.
    c. the student is already good at something the rest of the class needs more work on.

12. If you are familiar with Bloom's taxonomy [1956], you might like to see Avital and Shettleworth's relation of their thinking processes to five of Bloom's six levels:

Avital and Shettleworth	Bloom, et al.
Recognition, recall	1. Knowledge
Algorithmic thinking, generalization	2. Comprehension 3. Application
Open search	4. Analysis 5. Synthesis

13. Here is another example of a cognitive taxonomy written specifically for secondary school mathematics. Wilson [1971] lists the categories below to define areas for cognitive objectives. Relate these to the Avital and Shettleworth levels.

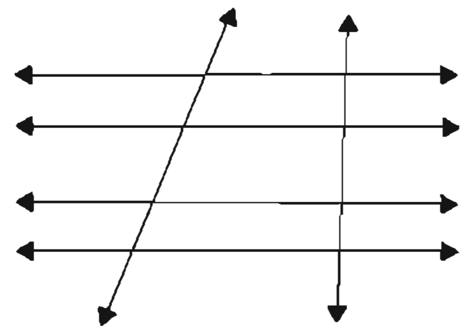
Computation:	Knowledge of specific facts.
	Knowledge of terminology.
	Ability to carry out algorithms.
Comprehension:	Knowledge of concepts.
	Knowledge of principles, rules and generalizations.
	Knowledge of mathematical structure.
	Ability to transform problem elements from one mode to another.
	Ability to follow a line of reasoning.
	Ability to read and interpret a problem.
Application:	Ability to solve routine problems.
	Ability to make comparisons.
	Ability to analyze data.
	Ability to recognize patterns, isomorphisms and symmetries.
Analysis:	Ability to solve nonroutine problems.
	Ability to discover relationships.
	Ability to construct proofs.
	Ability to criticize proofs.
	Ability to formulate and validate generalizations. [pp. 646-647]

14. The National Assessment of Educational Progress [1970] included a section of goals on the "appreciation and use of mathematics," to include the following:
    --recognition of the importance and relevance of mathematics to the individual and to society
    --enjoyment of mathematics
    --use of the content and techniques of mathematics
    --participation in the learning of mathematics beyond the minimum
    What sorts of things can we do in the classroom to meet these goals?

15. Do any of your broad goals (number 1) seem to call for open search or affective-oriented activities?

16. (Discussion) Miss Elto: "You can talk about higher-level objectives all you want--my fifth period class can't even get the <u>low</u>-level ones."
    a. What can we offer Miss Elto besides a sympathetic, "I know what you mean"?
    b. Would you have a different list of broad goals for weaker students?

17. Suppose that the instruction on proportions has covered the meaning of proportions, the usual vocabulary and solving proportions of the form $\frac{n}{\text{whole \#}} = \frac{\text{whole \#}}{\text{whole \#}}$. Where in the Avital-Shettleworth framework would you classify each of these:



a. Asking students to solve $\frac{6}{n} = \frac{21}{48}$.

b. Giving students drawings like the one to the right and having them measure, write ratios and look for proportions.

c. Giving students drawings like the the one to the right, putting the students in small groups, and telling them to see what they can come up with.

d. Asking students to solve $\frac{n}{2\frac{1}{2}} = \frac{3\frac{1}{2}}{5}$ and $\frac{n}{2.1} = \frac{.8}{8.4}$.

e. Asking students what they think $\frac{\frac{2}{3}}{10}$ might mean.

f. Asking students to define "proportion."

18. For one of your classes examine your last few lessons with respect to the chart on page 9. Are all the rows represented?

19. Use a chart something like the one on page 9 to plan some part of your work on percent.

20. (Discussion) Neale [1969] points out that studies indicate that student attitudes account for only a small part of the variation in achievement. Aiken [1972] feels that nonetheless we should be very concerned about attitudes since they may have a bearing on persistence, ambition in class or school, satisfaction, and further work in mathematics. How do you feel about the importance of attitudes?

21. Do you agree with these principles for using objectives? [School Mathematics Study Group, 1972, p. 18]

"II. . . . statements of objectives should be taken as floors, not ceilings. If a teacher or a school can go beyond stated objectives, so much the better."

"IV. If statements (of) objectives are to be taken seriously, then the objectives must be clearly verifiable and feasible . . . Before (an objective) should be advocated, it should have been positively shown to be feasible (and verifiable)."

"VI. Also, to be in conformance with point IV, we advocate at present no affective objectives. There is no evidence available to show that attitudes toward mathematics can be manipulated, so such objectives are not, at present, feasible."

"VII. None of the above should be taken as suggesting that we ignore goals which are, at the moment, not feasible or not verifiable. Indeed, such goals indicate the most important areas in which to concentrate our future research efforts."

22. How would you react to these statements?
    a. "I'm not sure word problems are worth all the effort they take to teach. After all, how many word problems does a person do outside of school?"
    b. Faculty-lounge gadfly: "You math teachers are frauds! If the school board had any sense, they would fire you and use part of the money to buy each kid a good calculator. Teaching math is about as important as teaching soap-making nowadays."

23. a. What educational value might result from these activities?
    --Class project: make a scale model of the town or neighborhood.
    --Individual project: make something artistic, as in *Room Decorations* in Making a Scale Drawing.
    --Individual project: talk to a high school science teacher about the use of ratio and proportion in high school science.
    --Individual project: see how mathematics is used at a gasoline service station.

    Eisner [1969], in attempting to recognize the legitimacy of activities which "seem" right but do not suggest clearly defined behavioral objectives ahead of time, introduced the notion of expressive objectives:

    > ". . . an expressive objective is the outcome of an encounter or learning activity which is planned to provide the student with an opportunity to personalize learning. It is precisely because of the richness of these encounters or activities and the unique character of the outcome that the expressive objective becomes so difficult to describe in advance. . . . engagement, emotional or intellectual immersion, is a better indicator of educational value than achievement test scores." [pp. 130-131]

    For example, the activities in part (a) might result in expressive objectives.

    b. Criticize expressive objectives from the behavioral objectives viewpoint.
    c. Do you endorse the spirit of expressive objectives? Explain.

24. (Outside references) You might want to read one of these more recent reports on middle school mathematics: the Cape Ann conference report [Physical Science Group, 1973], the Orono conference report [Beard and Cunningham], or Intermediate Mathematics Methodology [1968]. Broader looks at the mathematics curriculum are in the Estes Park conference report [Comprehensive Problem Solving in Secondary Schools, 1975] or the Snowmass conference report [Report of the Conference on the K-12 Mathematics Curriculum, 1973].

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RATIO, PROPORTION, AND SCALING

Placement Guide for Tabbed Divisors

EVALUATION AND INSTRUCTION

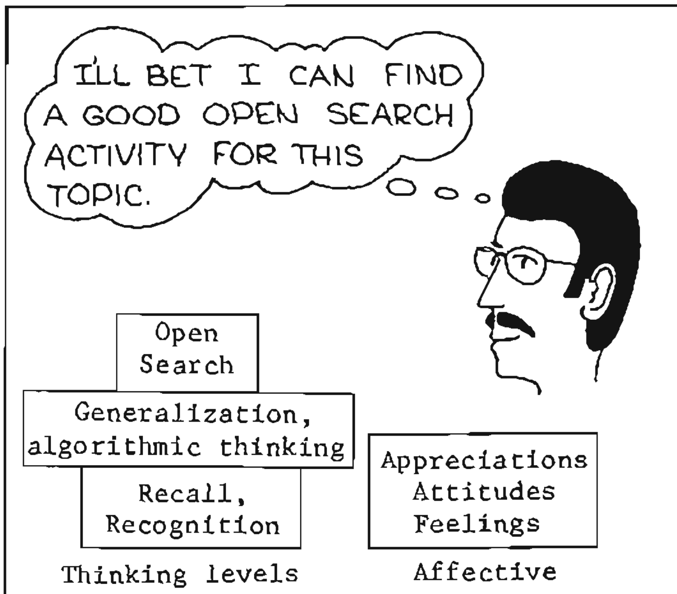
What is the purpose of evaluation? Assigning grades? Motivating students? Perhaps the best viewpoint of evaluation is that the purpose of evaluation is to <u>improve instruction</u>.

EVALUATION AND OBJECTIVES

Since evaluation is built on objectives, let us review the major points about objectives from *Broad Goals and Daily Objectives* (in the SCALING section):

1. <u>Behavioral</u> objectives describe the desired instructional outcomes in terms of <u>observable student</u> behavior.

2. Using an ordered classification system--a taxonomy--can remind us of the variety of objectives we could be working toward. Objectives should include attention to the affective as well as the cognitive domains.



(based on Avital and Shettleworth, 1968)

If our broad goals are to be taken seriously, we must attempt to evaluate the objectives that the goals give rise to. The merit of stating behavioral objectives and using some system to assure attention to several mental levels becomes especially clear when we evaluate. Using behavioral objectives in planning forces us to consider what sort of evidence we can gather about an area of instruction. Behavioral objectives are relatively easy to translate into evaluation items, at least for the lower levels of a taxonomy. Here are two examples.

<u>Objectives</u> The student can . . .	<u>Test items</u>. Fill in the blanks. Work mentally.
Give percent equivalents of common fractions without calculation.	$\frac{1}{2}$ = ___% $\frac{1}{4}$ = ___% $\frac{3}{4}$ = ___%
Express any decimal as a percent.	.57 = ___% 1.2 = ___% .025 = ___%

For higher levels of a cognitive taxonomy, even when we have a "good" behavioral objective, it may not be so easy to find a situation with which to judge attainment of the objective. Avital and Shettleworth, for instance, include as an appendix several items

> In buying a certain object you receive first a reduction of 20%, then an additional reduction of 15% on the new price. What is the total reduction, in percent?
>
> A Grade 8 test was passed by 76% of all students. 80% of the boys passed, and 70% of the girls passed. What is the percentage of boys in Grade 8?
>
> (from Avital and Shettleworth, 1968, pp. 48-49)

NOTE: Unless otherwise noted, subsections cited are in the PERCENT section of the resource <u>Ratio, Proportion and Scaling</u>.

at the open search level of their taxonomy [1968, pp. 45-51] since most teachers have had little experience at making up items for this level.

Evaluating affective objectives is challenging. First of all, such objectives <u>are</u> difficult to state behaviorally. When they can be, the behavioral evidence ("pays good attention," "starts homework at once," "checks out library books on mathematics") is not the sort that one routinely records. Researchers attempt to measure attitudes by student responses to paper-pencil items but usually acknowledge that results with pre-high school students may not be reliable.

A FRAMEWORK FOR EVALUATION ACTIVITIES

It is likely that a wide range of objectives will require a wide variety of evaluation techniques. One way to organize our approach is to list several types of evaluation methods and see which ones fit the different sorts of objectives. The list below gives a start.

EVALUATION TECHNIQUES

<u>Basis</u>: student performance on
- a. teacher-made
  - --pretest
  - --diagnostic test
  - --quiz
- b. standardized test
- c. homework
- d. project
- e. student evaluation report (see below)

<u>Basis</u>: teacher observation
- f. casual observation (in and out of class)
- g. small group
- h. interview

<u>Basis</u>: both student performance and teacher observation
- i. seatwork
- j. boardwork
- k. classroom dialogue

In the list "student evaluation report" refers to asking the students their evaluations of something. For example, they might be asked to write down how well they are doing with percent problems. The information could identify some apprehensive students you would want to check with or at least could give you an idea of how the class as a whole feels about their work on the topic. There do not seem to be reports on whether such information from middle schoolers is reliable. In any case, the reports might help student-teacher rapport and the students' self-concepts ("It matters how <u>I</u> feel").

Evaluating the Cognitive

The use of most of the techniques on page 2 to evaluate lower-level objectives is self-explanatory. Similarly, "higher-level" objectives seem to fit most of the techniques. That is, the techniques seem to fit IF we can come up with good higher-level situations. It is not a trivial matter to come up with problems that require a depth of understanding or some imagination and are accessible to middle schoolers. You can get some ideas from lists of test items (e.g., the Avital and Shettleworth one), other text series, tests of other instructors or your own file of "questions that will see if they really understand." Some teachers keep a file of 3x5 cards with their "best" items on them.

A deeper knowledge of a topic may suggest some facet that can be translated into an open search activity for students. For example, the student pages *Surface Area and Ratios 1 and 2* and *Volume and Ratio 1 and 2* in the RATIO: Equivalent section arose from knowing how doubling the dimensions of a cube affects the area and volume of the cube (and knowing how troublesome these relationships are for students). Although these pages do not themselves present open search activities, the ideas could easily be transformed into higher level tasks: "Investigate to see what happens to the area of a cube if you double all its dimensions." As another example of how a good background can help, one who has studied transformation geometry might think of this open search item: "Suppose you do an enlargement with scale factor 3. Then on the resulting figure do another enlargement with scale factor 2. What can you predict?" Similar questions about combinations of shrinks or "mixtures" of enlargements and shrinks could also be used, although the experience in working out the answer to the first question would put the later questions at a lower level.

Some higher-level objectives may be evaluated most easily by teacher observation during small group work, an interview or classwork. For example, listening to students in a small group or to an individual giving an explanation could help to evaluate whether "the student uses problem-solving strategies in attacking unfamiliar problems." An objective like "the student can justify the answer by using concrete materials" can be evaluated by a paper-pencil item, but seems almost to demand a hands-on exhibition during an interview, a small group discussion or classwork. For some objectives ("the student will be able to use proportions to choose 'best buys'"), confronting the student with a collection of price-marked boxes or cans would be a more nearly lifelike evaluation not complicated by reading problems. We must make it clear to the students that the things they do on paper-pencil tests are not the only important things.

Whenever we write our own test items, whether for lower- or higher-level objectives, we want them to be clear, to cover the material in a fair manner, and to give us valid and reliable information about our instruction. Writing such items takes planning, time and care. NCTM's Evaluation in Mathematics [1961] covers various aspects of evaluation; Merwin's chapter in particular contains several specific suggestions for writing clear, fair items. Suydam [1974a] or more general works like Ebel's [1972] also offer lots of information on writing test items. Whenever possible, it is a good idea to have a colleague read your tests to catch those misleading things that are sometimes so obvious to a "fresh eye."

Evaluating the Affective

Finally, which of the page 2 evaluation techniques seem applicable to objectives in the affective domain?

1. There are standardized tests for attitudes (see Dutton, 1956; Dutton and Blum, 1968) and a teacher could make a "quiz" for testing attitudes, but the dubious reliability of such tests has been noted; will middle school students respond honestly or will they try to give the "right" answer? Asking for anonymous answers may give more reliable information but only for the class as a whole, not for individuals. Corcoran and Gibb [1961] note that ". . . the general rapport between teacher and students has as much bearing on the candor of the (answers) as the question of whether or not they are signed." [p. 116]

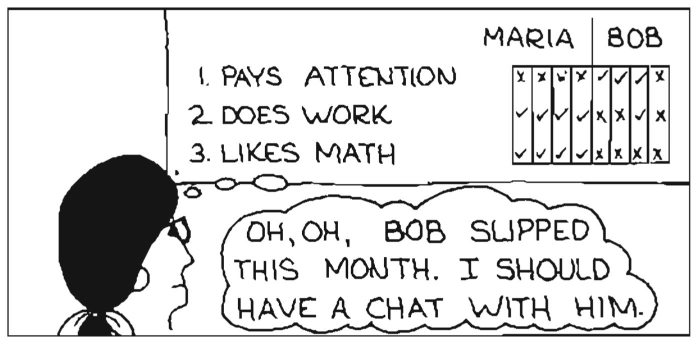
2. Homework can provide some possible information about affective aspects. Is Tom interested enough to do it regularly? Does Ricardo care enough to do it neatly? Does Sherry do extra work? The problem is, do these reflect influences other than interest? Perhaps Tom's parents keep tabs on his work, Ricardo does everything neatly and Sherry wants to get an A.

3. Projects (reports, bulletin boards, art, models . . .), unless required or for extra credit, should indicate which students are doing something because they like it.

4. Student evaluation reports seem promising. The reports could be based on rather specific, recent events ("Write 2 or 3 sentences telling how you felt about

Counting Every Body or "Complete this sentence, 'When we were doing percents, I . . .'") or more general topics ("Write a paragraph on 'What I Think About Mathematics'" or "Jot down your answers to 'Things about mathematics that I like' and 'Things about mathematics I don't like'"). Or you might give the students class time to keep a journal. Topics like "What I liked best this week," "What I found most difficult," "I would like more activities like . . ." could give valuable feedback. If the teacher does not react negatively to the comments and obviously uses some of the suggestions, the students will become accustomed to, and freer in, expressing their feelings. You may wish to keep the report "themes" quite narrowly focused on specific topics to avoid personal remarks that can be unfair (and hard on the ego).

5. Perhaps the most commonly used technique is through observational methods. Smiles, enthusiastic hand-waving, spontaneous questions, diligent seatwork or shared amazement can make a teacher's day--and provide information on the success of a lesson. And scowls, groans, near-tears, nervousness or indifferent slouches also communicate something about interests, anxieties, attitudes and appreciations. All these things, the good and the bad, give us data with which to evaluate. In some cases, anecdotal records or the less time-consuming periodic recording on a checklist might provide a "critical mass" of information.



More on Affective Evaluation

At an advisory conference for the Mathematics Resource Project [Hoffer, 1974, pp. 63-64] teachers and other experts made the following suggestions for getting information about--and even enhancing--the affective dimension. Some of the techniques would naturally spill over into diagnosis of cognitive work also.

6. Observe class attendance. Note the students' behavior and working habits on tasks given in the classroom.

7. Walk around the classroom and listen to comments.

8. Lead a class discussion where students can talk about their feelings and behavior. Encourage the students to ask for as well as lead such discussions.

9. Talk to students in a small group or individually where a teacher can show empathy and compassion for the students.

10. Ask students to identify mathematical topics they understand and enjoy.

11. Note the students' willingness to cooperate and behave acceptably with the class and teacher.

12. Look for students' exhibiting a sense of humor and a willingness to talk about their activities outside of school.

13. Note the students' ambition and initiative in trying mathematical problems and in getting involved in mathematics.

14. Note the correlation between student friendliness to the teacher outside of the classroom and teacher friendliness to the students outside the classroom.

15. Have students write comments and observations on their daily assignments and quizzes as they react to them. For example, when taking a paper-pencil test, the students may circle words, phrases and problems they do not understand, even though they may try the problem.

STANDARDIZED TESTS--TOOLS, NOT TOTEMS

Wilson [1973] points out some of the abuses possible from the use of standardized tests. For example, a test might cover only lower level objectives, neglecting completely some that are regarded as important in a particular school's program but including others that are not considered very important locally. The use of only a single score to "describe" a student ignores the many aspects of mathematics covered by the test (see number 10 on p. 8). Using norm scores makes sense only if we give attention to the nature of the norm group--how similar or different was the norm group from our community? But, above all, Wilson laments the implied belief that a standardized test measures everything we care about. Higher-level objectives, goals like problem-solving ability which take time to realize, and affective objectives may not be adequately treated.

If we keep standardized tests in perspective and remember their shortcomings, however, they can give valuable information. They can provide, depending on the norm group and local curriculum emphasis, measures of how well a school's program stacks up. Analysis of performance on an item-by-item basis can pinpoint local strengths and weaknesses. Used over a long period, growth (or decay) trends can be spotted. So long as the standardized test is chosen carefully, is interpreted properly and is not used to dictate or confine the curriculum, the test can be another valuable evaluation tool.

SUMMARY

To determine whether we are meeting our objectives, we must evaluate. To evaluate a reasonable set of objectives, we will likely require more than paper-and-pencil evaluation tools. Evaluation of higher cognitive levels and affective objectives, in particular, may best be evaluated by techniques which yield only "soft" data.

? ? ? ? ?

1. a. Give at least three more specific statements of objectives, all of which are aspects of "The student will understand percent."
   b. Write a test item for each of your objectives.

2. Write sample test items for these objectives.
   a. The student can express a given fraction as a percent.
   b. The student can compare performances in situations like *More Fun at the Fair* in the As a Ratio section.
   c. Given the tag price and the percent discount, the student can determine the selling price.
   d. Using percent, the student can summarize data in a novel situation.

3. Discuss the use of out-of-class settings like those below as sources of evaluation data.
   a. Four students are to collect information at a basketball game and report the statistics (field goal percent, foul shot percent).
   b. Pairs of students are to shop for the best buys for a given list of items.
   c. Students are to comparison-shop for 10-speed bicycles, trail bikes, records, . . .

4. (Discussion) Would you tape-record a test (a) for use with the whole class? (b) for use with poor readers?

5. Formal tests are also used to guide subsequent instruction, of course. One systematic way to proceed is to perform an item analysis. For example, suppose a unit test includes these three items:

   #1. $\frac{3}{5}$ = ___% #4. $\frac{5}{4}$ = ___% #8. 20 is what percent of 16?

   Part of the tabulation for a class might include wrong answers and look like this:

Item	Bet	Cam	Dee . . .	Class (28 students)
#1	OK	OK	OK	25 right
.				
.				
#4	OK	80%	1.25	15 right
.				
.				
#8	80%	80%	OK	11 right

   a. How do you feel about Dee's performance on these items?
   b. Bet and Dee each missed one of the three items. Can you conclude that they have the same achievement?

c. What misconception might account for the class results on items 4 and 8?
d. It is easy to get the whole-class totals. Call for the hands of those who missed #1, record, call for the hands of those who missed #2, etc. What argument could you give against this procedure?

6. The easiest source of slightly higher level items is a topic which the students will see later in mathematics. For example, students who have worked only with percents up to 100% might be asked what 125% would mean. Give some other examples.

7. Mrs. Washington: "I don't understand all the worry about evaluating affective objectives. Can't you tell just by paying attention to the students?"

While agreeing with Mrs. Washington, suggest some ways that could help
a. to make observational evaluation objective.
b. to keep observational evaluation from being haphazard.

8. One way to get an idea of how students regard mathematics is to use a list of pairs of words like those below and, for each pair, have them check the position on the line which describes how they feel about mathematics.

MATHEMATICS IS

useless-----------------------------useful
fun---------------------------------drudgery
easy--------------------------------difficult

Make up such a checklist and try it out. Include other subjects for comparison.

9. Track down a copy of a standardized test and evaluate the appropriateness for your students of the content covered. Do the items test any higher level objectives (in terms of your students)?

10. Standardized tests usually cover several topics. Hence, it can be very deceptive to use only total scores in planning instruction. For example, Jose may have gotten 4 out of 4 geometry items correct but missed 4 out of 5 percent items. Rose may have missed none of the 5 percent items but all of the geometry items, with the result that both students get the same total score. Obviously, the two students are quite different. How can we avoid this total-score "trap"?

11. (Discussion) Do any of the following have any value with middle school students?
a. take-home tests
b. open-book tests
c. small groups collectively working on a test, with each group member getting the same grade

12. Norm-referenced tests refer to tests for which the passing score is determined by comparison with some group, with the lowest scores receiving the lowest marks. Criterion-referenced tests use a pre-specified level to determine the passing mark. Which sort do you use? Why?

13. (Discussion) What are the pros and cons of basing a student's grade partly on homework?

14. Michael received scores of 75 on Test 1 and 83 on Test 2. You might conclude that he did better on the second test until you find that his score of 75 on Test 1 was tops whereas his 83 on Test 2 was only average. One way to make scores on (norm-referenced) tests comparable is to transform them so that they have the same mean and the same standard deviation. With a hand calculator, what was once an uninviting chore is easy. Use these steps:

    1. Calculate the group mean: $\text{mean} = \dfrac{\text{sum of scores}}{\text{number of scores}}$

    2. Square each score and calculate the <u>standard deviation</u>: $\text{s.d.} = \sqrt{\dfrac{\text{sum of squared scores}}{\text{number of scores}} - \text{square of mean}}$

    3. Suppose you want standard scores for each test to have mean 50 and standard deviation 10. Then calculate

    $$\text{std. score} = 50 + 10 \cdot \frac{\text{actual score} - \text{group mean}}{\text{standard deviation}}$$

    Try this on a couple of tests. (If you report the standard scores to your students, you may decide to choose a different mean score--use 80 or 100 instead of 50 in step 3--to avoid being suspected of "cheating" some students!)

15. It is possible to treat paper chemically in such a way that when a student answers a multiple-choice or true-false item, the treated paper indicates whether the answer is right. If the answer is wrong, the student tries another answer (for fewer points). (Answers are marked with felt-tip pens.) What are the advantages/disadvantages of this technique if . . .
    a. the test covers items that involve the same idea or calculation?
    b. a student gets very nervous during tests?

16. Should students be graded on quality of work, on effort or on some combination of these?

17. Can you think of some examples which show that how one teaches a subject has a bearing on the affective domain?

18. "More educational jargon--taxonomy, cognitive, affective--why don't they just let us teach!?" Your response?

19. Does your school use grade level scores for standardized test results? The National Advisory Committee on Mathematical Education [1975] points out these drawbacks in grade level scores:
    a. Under a norming procedure, half a population will necessarily perform below grade level.
    b. One expects an increase in grade level score of one year after a year of instruction even for a below-grade-level student.
    c. Grade level scores are easily misinterpreted (a sixth grader with a score of 9.5 is not necessarily performing like a mid-year ninth grader).

20. Test 1 (25 possible): Mutt--15 correct (60%); Jeff--20 correct (80%)
    Test 2 (50 possible): Mutt--40 correct (80%); Jeff--30 correct (60%)
    What causes the discrepancy between the averages of the percents for the two boys and their averages based on total points? (See the mathematical content *Ratio*.)

21. (Outside references) Look at The Arithmetic Teacher for the 1975-76 school year for articles on the first National Assessment of Educational Progress.

References and Further Readings

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RATIO, PROPORTION, AND SCALING

Placement Guide for Tabbed Divisors

CALCULATORS

RATIONALE

As early as the seventh century B.C. the counting board or abacus was invented and used for simple whole number computations. Merchants and traders of ancient times probably would have found the abacus cumbersome to carry around in their back pocket. If they were alive today, they could not only have a calculator in their pocket but they might have a computer terminal in their briefcase! Electronic calculators are one of the hottest selling items around the world. They are becoming as popular and inexpensive as watches. They give instantaneous effortless answers to many computations. They are small, quiet and cheap.

Using a calculator is relatively easy. You push a few buttons in sequence and "Voilà!" the keyboard display flashes the answer. "Most of us have so far explored numberland by the very laborious, manual route. The hand calculator lets you travel by automation, and explore far afield effortlessly." [Wallace Judd] Paper and pencil calculations are often slow, inaccurate and tiresome. Interest and enthusiasm for mathematics is often killed by such drudgery. The calculator becomes a fantastic tool that frees us to do investigations and problem solving. Its speed allows us to keep pace with our racing minds as we search for solutions, conjectures, and more questions.

The electronic calculator is NOT a fad; it is here to stay. Like the radio and television, soon everyone may own one (or two or three). The calculator is bound to change our way of life just as other advances in technology have. Already educators are arguing about the use of the calculator in the mathematics classroom. Should the calculator be used when teaching arithmetic skills in elementary schools? Will children need to memorize addition and multiplication facts if they learn to compute using a calculator? Will senior high students need to learn how to use logarithmic tables or should they use an electronic calculator instead? In other words, the whole mathematics curriculum from kindergarten through college will need to make serious adjustments to account for the use of the electronic calculator. Because the calculator is becoming available to all members of our society, including children, educators will need to decide how electronic calculators fit into the school curriculum.

Recently, pocket or desk calculators have been used in mathematics classrooms to motivate students and expand their ability to solve "messy" real-world problems (i.e., stock investments, tax forms, interest on car payments, pollution controls). The calculator provides the immediate feedback of answers and a probelm-solving flexibility that encourages the student to become involved in complex computations. However, one needs to be careful! Most calculators do not retain and display all the numbers or operations entered. If wrong numbers are entered or operations are entered in the wrong order (a faulty algorithm sequence), the incorrect answer must be recognized by the student. To tell a reasonable answer from an unreasonable one, a student needs to know how to compute using the basic arithmetic facts, how to round numbers, how to estimate and approximate answers, and how to place a decimal point. Arithmetic skills and number sense are very important if the hazards of a calculator are to be avoided. The calculator does not replace thought processes. It is a tool that saves time and energy and frees us to think and do mathematics above the computational level.

SUMMARY

I. Calculators fit into the classroom in different ways:

1. Non-electric calculators (abacus, etc.)
   a. teach concepts in counting, place value, and arithmetic computations, and
   b. demonstrate algorithms for solving computational problems.
2. Electronic calculators free the students from tedious pencil and paper calculations. They allow the student to . . .
   a. speed up "messy" calculations, and
   b. investigate and work on mathematical problems and applications that would otherwise involve long, unmanageable calculations.

II. The teacher can prepare students for electronic calculators by . . .

1. Emphasizing estimation and approximation skills which are vital in checking answers and placing the decimal point correctly.
2. Teaching the student to determine the reasonableness of exact answers by approximate calculations.
3. Introducing situations and problems where the hand calculator is an obvious aid to cumbersome, time-consuming calculations.
4. Asking students what types of mistakes can be made while using the calculator.

III. Teachers can prepare themselves for using the electronic calculator in instruction by . . .

1. Experimenting with it themselves. (Let the students see the teacher using a calculator.)
2. Reading current periodicals and checking the mathematics publication companies for new "calculator" books. (There is currently no body of knowledge about how to use a calculator in the classroom.)
3. Having an open mind about the use of the calculator before deciding that the calculators will be a "cure-all" to teaching computation, or that they should be banned from the mathematics curriculum.

Selected Sources for Calculators

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EXAMPLES OF CALCULATOR PAGES FOUND IN THE CLASSROOM MATERIALS

I. Electronic Calculators

I NEED A JOB LIKE THAT!

Fill in this chart of earnings for Mr. Pennypusher. Use a calculator to get each day's wages and the total earnings for all 20 days.

DAY	EARNINGS	DAY	EARNINGS
1	$	11	$
2	$	12	$
3	$	13	$
4	$	14	$
5	$	15	$
6	$	16	$
7	$	17	$
8	$	18	$
9	$	19	$
10	$	20	$
TOTAL FOR 20 DAYS			$

a) What is the average amount Mr. Pennypusher made per day? (Divide the total earnings by 20.)

b) If he worked 8 hours per day, what is the average amount he made per hour?

c) How much did Mr. Pennypusher average per minute?

(d) If you were paid as much as Mr. Pennypusher averaged per minute, how much would you make for the time you are in your mathematics class?

per day? ______

per week? ______

per school year? ______

Multiplying and adding large numbers can be frustrating and painful to do manually. The electronic calculator provides quick answers; then basic conclusions can be drawn from the data.

Simple and compound interest have easy formulas, but messy, repetitive computations that can be handled very efficiently by the electronic calculator.

CERTAIN GROWTHS ARE BENEFICIAL

Many kinds of growth occur and are studied in mathematics. Some involve growth by a fixed amount, some by a fixed rate. These two can produce surprisingly different results.

Have students compute the outcome of depositing $1000 at a bank at a 5% interest rate compounded annually for 20 years and compare it with a deposit of $1000 increased annually by a fixed amount of interest ($50.00 = 5% of $1000) for 20 years.

Tables could be used to organize the results, and a hand calculator would simplify the computation. Interest payments should be rounded to the nearest cent.

COMPOUND INTEREST (fixed rate)

Age of deposit in years	Amount at beginning of year	Interest at 5%	Amount + Interest
1	$1000.00	$50.00	$1050.00
2	$1050.00	$52.50	$1102.50
3	$1102.50	$55.13	$1157.63
4	$1157.63	$57.88	$1215.51

SIMPLE INTEREST (fixed amount)

Age of deposit in years	Amount at beginning of year	Fixed amount of interest credited each year
1	$1000.00	$50.00
2	$1050.00	$50.00
3	$1100.00	$50.00
4	$1150.00	$50.00

Discuss the two outcomes. In the first table the amount of growth each year shares in the growth during the next year.

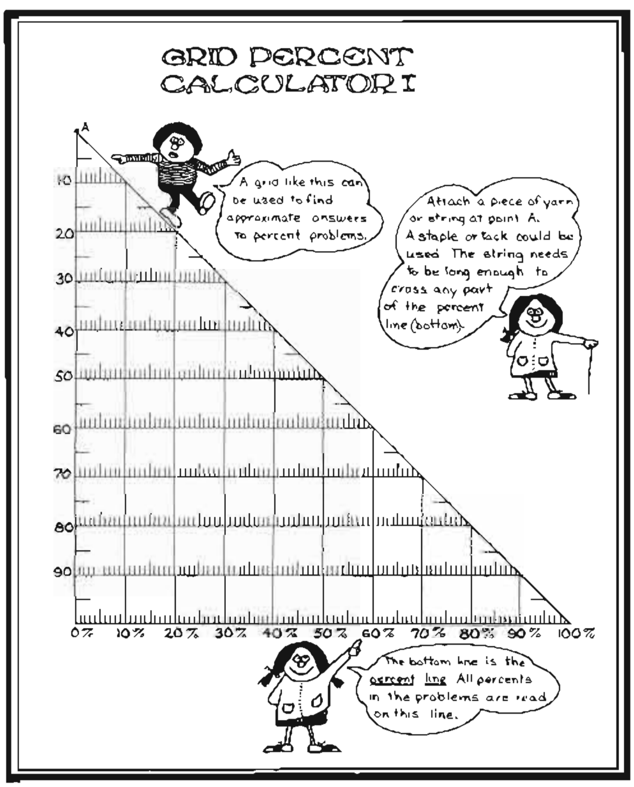
Suppose that the interest is compounded semi-annually or quarterly. What effect would this have? Some banks compound interest continuously. What does this mean? Investigate the savings plans offered at banks and savings and loan. Which would be the best for short term deposits? long term deposits?

In the bank compound interest amounts are calculated from the formula $A = (1 + \frac{r}{m})^{mt}$ where r is the annual rate of simple interest, t is the time period in years, m is the number of compounding periods in a year. By the use of the formula it can be shown that the effective annual yield of a 7% savings certificate compounded daily for a 365-day year is 7.25% ($A = (1 + \frac{.07}{365})^{365 \times 1} = 1.0725$).

If compounded continuously, the formula used is $A = e^{rt}$ where e is the base of natural logarithms. $A = e^{.07 \times 1} = 1.0725$, an effective annual yield of 7.25%. Thus, a 7% certificate could yield 7.25% or 7.25%.

When measuring and recording data, the calculator is a welcome assistant for analyzing results and making quick numerical comparisons.

II. Non-Electronic Calculator



This calculator can be used as a visual learning aid. The student sees percents actually being computed through the use of the simple paper model.

CALCULATORS FOUND IN CLASSROOM MATERIALS

RATIO:

Rate

FIX THAT LEAK	DETERMINING RATES
I NEED A JOB LIKE THAT!	USING RATES TO DETERMINE EARNINGS

Equivalent

I'D WALK A MILE	DETERMINING AND COMPARING

Ratio as a Real Number

PI'S THE LIMIT	APPROXIMATING
CLOSER & CLOSER	RATIO AS A REAL NUMBER

PROPORTION:

Getting Started

THE SOLVIT MACHINE--A DESK TOP PROPORTION CALCULATOR	CROSS PRODUCTS METHOD

Application

CRUISING AROUND	USING PROPORTIONS TO CONVERT MEASURES
WORLD RECORDS	USING PROPORTIONS TO COMPARE MEASURES
I MEAN TO BE MEAN!	DETERMINING MEAN PROPORTIONS

PERCENT:

As a Fraction/Decimal

THE PERCENT PAINTER RETURNS	AS A DECIMAL

Solving Percent Problems

THE ELASTIC PERCENT APPROXIMATOR EXTENDED	USING A PERCENT CALCULATOR
GRID PERCENT CALCULATOR I	USING A PERCENT CALCULATOR

GRID PERCENT CALCULATOR II	USING A PERCENT CALCULATOR
GRID PERCENT CALCULATOR III	USING A PERCENT CALCULATOR
GRID PERCENT CALCULATOR IV	USING A PERCENT CALCULATOR
GRID PERCENT CALCULATOR EXTENSIONS	USING A PERCENT CALCULATOR
PELARGONIUM	FINDING PERCENT OF INCREASE
COUNTING EVERY BODY	FINDING PERCENT OF INCREASE
CERTAIN GROWTHS ARE BENEFICIAL	FINDING AMOUNT OF INTEREST

RATIO, PROPORTION, AND SCALING

Placement Guide for Tabbed Divisors

APPLICATIONS

RATIONALE

Over 2000 years ago man developed number symbols, arithmetic calculations and geometry to describe and record real-world happenings. Mathematics was used to solve the problems of merchants, scientists, builders and priests.

About 600 B.C. Greek mathematicians took a different approach. They began studying numerical patterns and geometry for their aesthetic qualities. Mathematics became an intellectual exercise with no necessary applications in mind. The development of mathematics was soon traveling in two directions: practical or applied mathematics, originating from the Egyptians, and "pure" mathematics, originating from the Greeks.

Practical and "pure" mathematics are not always separable. One often inspires and directs the other; they become interwoven. As a result, applications of mathematics fall into three categories:

1) applications to real-life situations such as business, finance, sports, polls and census taking
2) applications to other disciplines (i.e., science, music, art)
3) applications to other branches of mathematics (problem-solving activities in the realm of "pure" mathematics)

The Egyptians, for example, were interested in learning as much as they could about their environment and how to control it. Today we are also curious about the rapidly changing environment we have created. Because of the complexity of our culture and its emphasis on technology, mathematics is very important to us in our jobs, in our daily living and in our future.

We face many problems in our daily living. Since all problems require the collection of information before solutions can be found and analyzed, mathematics is often a helpful tool in solving problems; yet few people relate mathematics to real-life situations or real-life situations to mathematics.

Many teachers have neglected to teach applications of mathematics for a number of reasons:

1) "I have little background in applications of mathematics."
2) "My students often have little or no background in science, art, music and other disciplines."
3) "Applications require elaborate equipment and preparation."
4) "My students are not interested in applications.
5) "Good applications take too much time to teach. There is plenty to teach in the math textbook."
6) "How can my students apply mathematics when they do not even have basic computational skills?"

Yet educators and the public agree that applications of mathematics are very important and should be taught in the mathematics classroom. Society is demanding accountability and relevancy in our education system. Students need ample opportunity to experience mathematics in a practical sense so that they will be better equipped to apply it as adults.

Even though certain applications of mathematics require special equipment and materials, much of this equipment can be constructed from inexpensive substitutes and common materials. Once the equipment is collected or made, it will last for years. Also, various applications can be adapted to fit available materials and equipment.

Applications should include appropriate topics and activities. Here are a few questions to consider when choosing an application of mathematics:

a) Is it interesting to the students and the teacher?

b) Does it start at the appropriate skill level?

c) Does it extend and develop the computational and/or problem-solving skills of the students?

d) Does it include topics, skills or ideas which might help the students contribute to society and deal with real-life situations?

e) Could it be done as a laboratory activity?

f) What concepts does it imply and develop?

g) How much time would it take to teach?

h) What equipment and materials are needed or available?

SUMMARY

1. Applications of mathematics fall into three categories:
   a) applications to real life situations
   b) applications to other disciplines, and
   c) applications to other branches of mathematics.
2. Down through the centuries, mathematics has been a useful tool for solving real work problems and analyzing our environment.
3. Even though many teachers have neglected to teach applications of mathematics, our complex society demands that public education teach practical mathematics and problem-solving techniques.
4. Mathematics can be used to solve problems in the real world and in other disciplines.
5. Applications to real life situations and other subject areas (i.e., physics, social science, economics, art, music) make abstract mathematics more meaningful and understandable.
6. Applications should include appropriate, interesting topics and activities for students and teachers.

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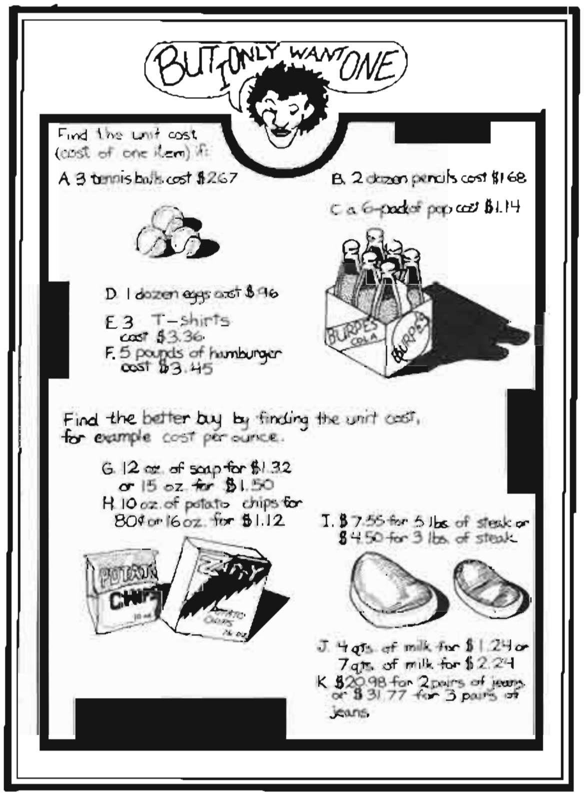
EXAMPLES OF APPLICATIONS FOUND IN THE CLASSROOM MATERIALS

I. Real-World Applications

From sport events to grocery shopping to government spending, we are exposed to applications of mathematics. If we know how to work with numbers and mathematical ideas, we can often use mathematics to help us deal with real-life situations.


BUT I ONLY WANT ONE

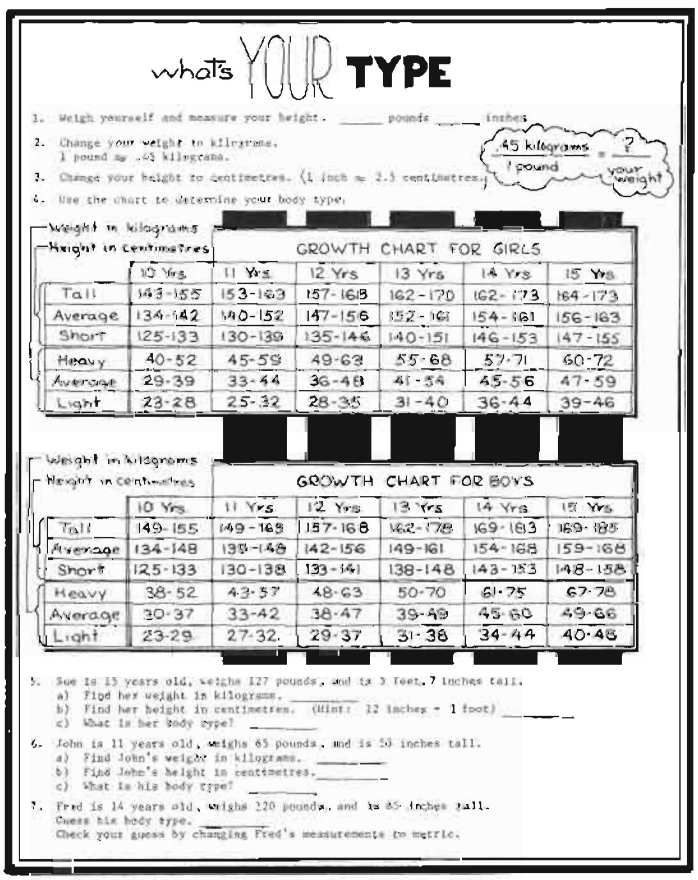
Find the unit cost (cost of one item) if:

A. 3 tennis balls cost $2.67
B. 2 dozen pencils cost $1.68
C. a 6-pack of pop cost $1.14
D. 1 dozen eggs cost $.96
E. 3 T-shirts cost $3.36
F. 5 pounds of hamburger cost $3.45

Find the better buy by finding the unit cost, for example cost per ounce.

G. 12 oz. of soap for $1.32 or 15 oz. for $1.50
H. 10 oz. of potato chips for 80¢ or 16 oz. for $1.12
I. $7.55 for 5 lbs. of steak or $4.50 for 3 lbs. of steak
J. 4 qts. of milk for $1.24 or 7 qts. of milk for $2.24
K. $20.98 for 2 pairs of jeans or $31.77 for 3 pairs of jeans

Unit pricing is frequently posted below the items sold in grocery and department stores for the convenience of the customer. As a consumer we can develop an awareness of prices and quality. Compare prices by finding the unit cost and determine which item is the better buy.


whats YOUR TYPE

1. Weigh yourself and measure your height. _____ pounds _____ inches
2. Change your weight to kilograms. 1 pound = .45 kilograms.
3. Change your height to centimetres. (1 inch = 2.5 centimetres.)
4. Use the chart to determine your body type.

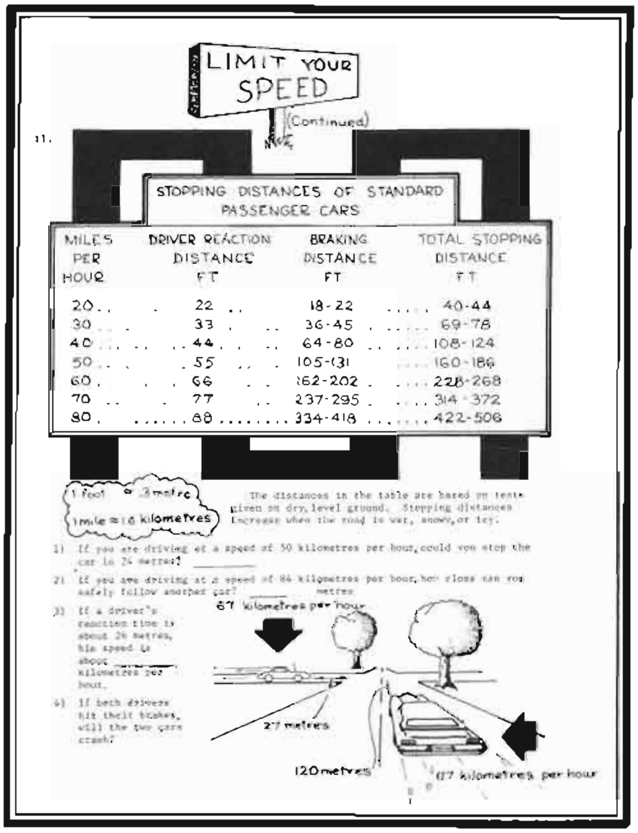
$\frac{.45 \text{ kilograms}}{1 \text{ pound}} = \frac{?}{\text{your weight}}$

GROWTH CHART FOR GIRLS	10 Yrs	11 Yrs	12 Yrs	13 Yrs	14 Yrs	15 Yrs
Height in centimetres: Tall	143-155	153-163	157-168	162-170	162-173	164-173
Average	134-142	140-152	147-156	152-161	154-161	156-163
Short	125-133	130-139	135-146	140-151	146-153	147-155
Weight in kilograms: Heavy	40-52	45-59	49-63	55-68	57-71	60-72
Average	29-39	33-44	36-48	41-54	45-56	47-59
Light	23-28	25-32	28-35	31-40	36-44	39-46

GROWTH CHART FOR BOYS	10 Yrs	11 Yrs	12 Yrs	13 Yrs	14 Yrs	15 Yrs
Height in centimetres: Tall	149-155	149-165	157-168	162-178	169-183	169-185
Average	134-148	139-148	142-156	149-161	154-168	159-168
Short	125-133	130-138	133-141	138-148	143-153	148-158
Weight in kilograms: Heavy	38-52	43-57	48-63	50-70	61-75	67-78
Average	30-37	33-42	38-47	39-49	45-60	49-66
Light	23-29	27-32	29-37	31-38	34-44	40-48

5. Sue is 15 years old, weighs 127 pounds, and is 5 feet, 7 inches tall.
   a) Find her weight in kilograms. _____
   b) Find her height in centimetres. (Hint: 12 inches = 1 foot) _____
   c) What is her body type? _____
6. John is 11 years old, weighs 65 pounds, and is 53 inches tall.
   a) Find John's weight in kilograms. _____
   b) Find John's height in centimetres. _____
   c) What is his body type? _____
7. Fred is 14 years old, weighs 220 pounds, and is 65 inches tall.
   Guess his body type. _____
   Check your guess by changing Fred's measurements to metric.

Physical fitness is measured, in part, by one's body proportions. Once standard growth patterns are tabulated and verified, the average height and weight of a person at a given age provides a measure for comparison.



LIMIT YOUR SPEED

(Continued)

11.

STOPPING DISTANCES OF STANDARD PASSENGER CARS

MILES PER HOUR	DRIVER REACTION DISTANCE FT	BRAKING DISTANCE FT	TOTAL STOPPING DISTANCE FT
20	22	18-22	40-44
30	33	36-45	69-78
40	44	64-80	108-124
50	55	105-131	160-186
60	66	162-202	228-268
70	77	237-295	314-372
80	88	334-418	422-506

1 foot ≈ .3 metre

1 mile ≈ 1.6 kilometres

The distances in the table are based on tests given on dry, level ground. Stopping distances increase when the road is wet, snowy, or icy.

1) If you are driving at a speed of 50 kilometres per hour, could you stop the car in 24 metres? ________

2) If you are driving at a speed of 84 kilometres per hour, how close can you safely follow another car? ______ metres

3) If a driver's reaction time is about 26 metres, his speed is about ______ kilometres per hour.

4) If both drivers hit their brakes, will the two cars crash?

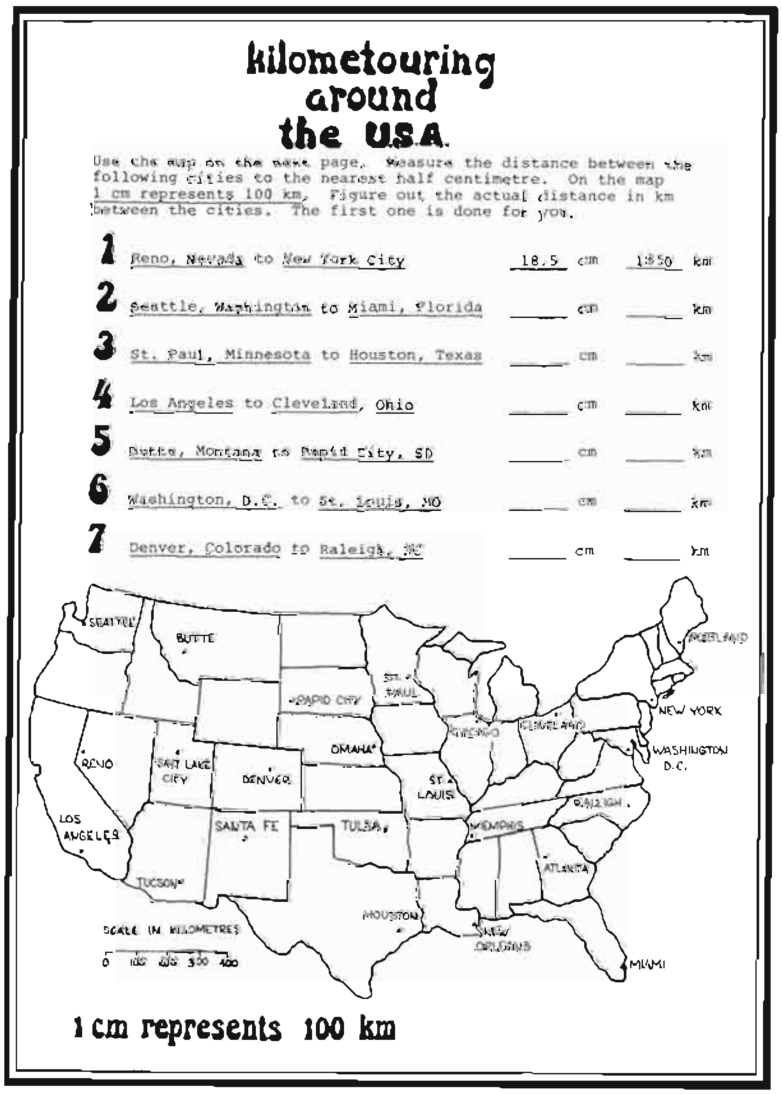
The automobile is one of the main means of transportation. Each state requires that a motorist pass a driver's test and obey certain rules of the road, especially speed limits. Automobiles and the problems they create are frequently discussed by students since riding in a car and being conscious of driving skills are experiences they all have in common.

From Eratosthenes who determined the circumference of the earth to Boy Scouts determining the height of a cliff, the use of indirect measurement is a useful application of mathematics.

II. Applications to Other Subject Areas

A "basic working knowledge" of mathematics is often required for the study and mastery of various subjects. Science, music, art, geography, computer science and many other disciplines use mathematics in the formulation of their research problems and applications.

The geography of the United States and the transportation systems are important to anyone traveling around the U.S.A. Thinking of distance in kilometres is a new experience for most Americans.


kilometouring around the U.S.A.

Use the map on the next page. Measure the distance between the following cities to the nearest half centimetre. On the map 1 cm represents 100 km. Figure out the actual distance in km between the cities. The first one is done for you.

1	Reno, Nevada to New York City	18.5 cm	1850 km
2	Seattle, Washington to Miami, Florida	____ cm	____ km
3	St. Paul, Minnesota to Houston, Texas	____ cm	____ km
4	Los Angeles to Cleveland, Ohio	____ cm	____ km
5	Butte, Montana to Rapid City, SD	____ cm	____ km
6	Washington, D.C. to St. Louis, MO	____ cm	____ km
7	Denver, Colorado to Raleigh, NC	____ cm	____ km

1 cm represents 100 km

III. Miscellaneous Applications

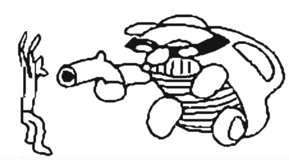
The need for certain mathematical concepts and tools may arise naturally in the context of various situations. The teacher can provide interesting activities that arouse the students curiosity whether they are real-world problems or not.

These short story problems deal with percents that are smaller than 1%. Situations are presented to provide meaning and understanding.

PUNY PERCENTS

2) $\frac{1}{10}$% of all eggs are rejected. 20,000 have been checked. _____ eggs are rejected.

$\frac{1}{10}$ for every 100
1 for every ___
___ for every 20,000



1) In 1973 about 400 auto thefts were reported for every 100,000 people. What percent of the population had cars stolen? _____

400 for every 100,000
4 for every ___
___ for every 100

6) A $\frac{1}{6}$-cup serving of rice has $\frac{1}{2}$% of the minimum daily requirement of Vitamin C. How many cups would you have to cook in order to have enough Vitamin C for one day? _____

7) Many clothing labels say, "Less than 1% shrinkage." If the actual shrinkage is $\frac{1}{2}$%, how much is lost if you wash 100 yds. of cloth? _____

APPLICATIONS FOUND IN CLASSROOM MATERIALS

RATIO:

Rate

RATES ARE RATIOS	IDENTIFYING DIFFERENT RATES
THE FRENCH BREAD PROBLEM:	DETERMINING RATES
FIX THAT LEAK	DETERMINING RATES
AS THE RECORD TURNS	DETERMINING RATES
MY HEART THROBS FOR YOU	USING RATE OF HEARTBEAT TO DETERMINE PHYSICAL FITNESS
STEP RIGHT UP	USING RATE OF HEARTBEAT TO DETERMINE PHYSICAL FITNESS
I BELIEVE IN MUSIC	DETERMINING RATES
WHICH IS A BETTER BUY?	USING RATES TO COMPARE PRICES
WHICH IS BETTER? 1	USING RATES TO COMPARE PRICES
WHICH IS BETTER? 2	USING RATES TO COMPARE PRICES
BUT I ONLY WANT ONE	USING RATES TO COMPARE PRICES
EIGHT HOURS A DAY	USING RATES TO DETERMINE EARNINGS

Equivalent

EQUIVALENT RATIOS BY PATTERNS	CONCEPT, GENERATING
RATIOS IN YOUR SCHOOL	SIMPLIFYING
ONE MAN ONE VOTE	SIMPLIFYING
PEOPLE RATIO	SIMPLIFYING

PROPORTION:

Getting Started

PETITE PROPORTIONS 1	SOLVING PROPORTIONS
PETITE PROPORTIONS 2	SOLVING PROPORTIONS
DID YOU KNOW THAT . . .	SOLVING PROPORTIONS

Application

PROPORTION PROJECTS TO PURSUE	APPLICATIONS
IT'S ONLY MONEY	USING PROPORTIONS TO CONVERT CURRENCY
ONE GOOD TURN DESERVES ANOTHER	USING PROPORTIONS TO DETERMINE DISTANCES
THAT'S THE WAY THE OLD BALL BOUNCES	USING PROPORTIONS TO FIND HEIGHT
GET IN GEAR	USING PROPORTIONS WITH GEARS
WHAT'S YOUR TYPE?	USING PROPORTIONS TO CONVERT MEASURES
LIMIT YOUR SPEED	USING PROPORTIONS TO CONVERT MEASURES
CRUISING AROUND	USING PROPORTIONS TO CONVERT MEASURES
WORLD RECORDS	USING PROPORTIONS TO COMPARE MEASURES
PROPORTIONS WITH A PLANK	USING PROPORTIONS WITH LEVERS INVERSE VARIATION
I'M BEAT! HOW ABOUT YOU?	USING PROPORTIONS WITH GEARS INVERSE VARIATION
MAKING MEANS MEANINGFUL	APPLYING MEAN PROPORTIONS IN A RIGHT TRIANGLE

TEACHING EMPHASES	APPLICATIONS
SCALING:	
Getting Started	
A PERFECT FIT	MOTIVATION
ELEMENTARY, MY DEAR WATSON	MOTIVATION USE OF A SCALE MODEL
Making a Scale Drawing	
BE CREATIVE THIS CHRISTMAS	ENLARGING WITH GRIDS
HOW MUCH IS YOUR GARDEN WORTH?	REDUCING WITH A RULER
STAKE YOUR CLAIM	REDUCING WITH AN INSTRUMENT FINDING LENGTHS USING AN ALIDADE
ANOTHER STAKE OUT	REDUCING WITH AN INSTRUMENT FINDING ANGLES USING A TRANSIT
Supplementary Ideas in Scaling	
MAKE A DIPSTICK	USING A SCALE TO DETERMINE DEPTH
CAREFULLY CONSTRUCTED CARTONS	CONSTRUCTING 3-D MODELS
A SCALE MODEL OF THE SOLAR SYSTEM	MAKING A SCALE MODEL
HOW HIGH THE MOON	MAKING A SCALE MODEL
Maps	
THE GREAT LAKES	USING A SCALE DRAWING TO FIND DISTANCES
KILOMETOURING AROUND THE U.S.A.	USING A SCALE DRAWING TO FIND DISTANCES
AROUND THE U.S.A.	USING A SCALE DRAWING TO FIND DISTANCES

FOREST FIRES ARE A REAL BURN	USING ANGLE READINGS TO LOCATE POINTS ON A SCALE DRAWING
WHERE'S IT AT?	USING A TIME SCALE TO LOCATE POINTS
OUR TOWN	READING A MAP
IT'S ABOUT TIME	USING A SCALE DRAWING TO FIND TRAVEL TIME
DO YOU KNOW THE WAY TO SAN JOSE?	READING A MAP

PERCENT:

As a Ratio

PERCENTS OF SETS-II	PERCENT OF A SET
FUN AT THE FAIR	USING PERCENT TO COMPARE
MORE FUN AT THE FAIR	USING PERCENT TO COMPARE
BE COOL--GO TO SCHOOL	USING PERCENT TO COMPARE
PUNY PERCENTS	PERCENTS LESS THAN 1%

Solving Percent Problems

B-BALL TIME	SOLVING PERCENT PROBLEMS
THE SHADY SALESMAN	SOLVING PERCENT PROBLEMS
INTERESTING? YOU CAN BANK ON IT!	FINDING AMOUNT OF INTEREST
AT THAT PRICE, I'LL BUY IT	FINDING AMOUNT OF DISCOUNT
PERCENT PROBLEMS 1	WORD PROBLEMS
PERCENT PROBLEMS 2	WORD PROBLEMS
PELARGONIUM	FINDING PERCENT OF INCREASE
STATE THE RATE	FINDING AMOUNT OF SALES TAX

TEACHING EMPHASES	APPLICATIONS
COUNTING EVERY BODY	FINDING PERCENT OF INCREASE
CERTAIN GROWTHS ARE BENEFICIAL	FINDING AMOUNT OF INTEREST
HIDDEN COSTS IN A HOME	FINDING AMOUNT OF INTEREST

RATIO, PROPORTION, AND SCALING

Placement Guide for Tabbed Divisors

PROBLEM SOLVING

RATIONALE

Learning to solve problems is probably the most important aspect of one's education. No matter who we are, where we live, or what we do, there will always be problems for us to face and problems for us to solve if we want to solve them. Sometimes it is not easy to determine whether a situation really is a problem for a particular individual. What is a problem to one person may be an exercise to another. Performing or practicing something (a task) that one already knows how to do is an exercise. Therefore, the task may require only a routine procedure which leads to the solution(s). However, if the individual has a clearly defined, desired goal in mind, but the pathway to the goal is blocked, then the individual has a "problem" to solve. "A true problem in mathematics can be thought of as a situation that is novel for the individual called upon to solve it. It requires certain behaviors beyond the routine application of an established procedure." [Troutman and Lichtenberg]

Mathematics teachers should pose and provide problems that have no obvious method or algorithm to follow in reaching a solution. Too often students are given page after page of various computational exercises which use one or more "essential" algorithms the students have "memorized." Once outside the classroom, students rarely use the algorithms they have memorized because the algorithms do not seem applicable. They come across ambiguous, disorganized situations that require considerable thought and skill for making a decision or finding a reasonable solution. Developing the ability to think independently and make wise decisions will help people to solve future problems by themselves.

Problem solving is a structured process. George Polya, in his book How to Solve It, divides the problem solving process into four steps:

1) Understanding the problem.
2) Devising a plan.
3) Carrying out the plan.
4) Looking back and checking the results.

Other authors have discussed the problem solving process with similar steps that match or fit into Polya's four steps (see Selected Sources for Problem Solving). These steps provide a structure which guides the problem solver through a search for the solution(s) to a problem. In the discussion which follows, several questions to answer and "things to try" are given under each of the four steps.

Understanding the Problem:

1. State the problem in your own words. (If the student cannot read the problem well enough to understand its meaning, the teacher may need to

read it to him. If the student can read but does not understand the problem, the teacher could rephrase the problem. The teacher should check for stumbling blocks. If the student has read the problem but seems bothered, ask what he thinks about the problem. Perhaps the student sees the situation as unrealistic, inconsistent or incomplete.)

2. What are you trying to find out? What is the unknown?
3. What relevant information do you get from the problem?
4. Is there any information that is not needed to solve the problem?
5. Are there any missing data that you need to know to solve the problem?
6. Are there any diagrams, pictures or models that may provide additional information about the problem?
7. Can you try some numerical examples?
8. Is it possible to recreate, dramatize, or make a drawing of the problem?
9. Can you make an educated guess as to what the solution(s) might be?

Devising a Plan:

1. Make a diagram, number line, chart, table, picture, model or graph to organize and structure the data.
2. Guess and check. Organize the trial and error investigations into a table.
3. Look for patterns.
4. Translate the phrases of the problem into mathematical symbols and sentences.
5. Try to solve one part of the problem at a time (i.e., break the problem into cases).
6. Have you worked a problem like this before? What method did you use?
7. Can you solve a simpler but related or analogous problem?
8. Keep the goal in sight at all times.

Carrying Out the Plan:

1. Keep a record of your work.
2. Perform the steps in your plan; check each step carefully.
3. Complete your diagram, chart, table or graph.
4. Follow patterns; organize and generalize them.
5. Compare your estimates and guesses with your work.
6. Solve the mathematical sentence; record the calculations and answer.
7. Work out any simpler but related or analogous problems. Compare the solutions.

Looking Back:

1. Can you check your result? Is the answer reasonable?
2. What does the result tell you? What conclusions can be made?
3. Is there another solution? Is there another way of finding the answer?
4. Make up some problems like the one you worked. Is there a rule or generalization that can be used to solve similar problems?
5. What method(s) helped you get the answer(s)?

Teaching Problem Solving

"The best way to learn problem solving is by working problems and studying the processes we used in working them." [Hints for Problem Solving] If a person is going to become a problem solver, he/she will need to be involved in a variety of problem-solving experiences. Before any problem can be tackled, there has to be the desire to solve the problem. The teacher can motivate the students by giving them problems within their range of experience and interests. Stimulating questions can guide the students through the problem-solving process. Getting the students to the point where they WANT to solve the problem is the most important step that will lead to successful problem solving. To further insure the success of a problem-solving activity, the teacher should stress a thorough understanding of the problem and encourage students to devise and carry out their own plan for finding the solution. It is important to provide all students with enough time to arrive at the solution independently without the faster students blurting out their solutions.

In the beginning the teacher should realize that most students are NOT problem solvers. They become frustrated quickly and tend to give up easily. They often make incorrect conjectures and fail to check the reasonableness of their answers. They lack a knowledge of problem-solving techniques and the ability to use them. Some students have not acquired the necessary computational skills or reading/comprehension skills needed to carry out the problem-solving process.

No teacher or student has to memorize Polya's four steps and its list of "things to try," but there are specific skills from the list that can be the focus of a lesson. Some activities, such as *Patterns for Introducing Ratio*, *Ratio of Ages* and *Proportions with a Plank*, have specific patterns to follow when finding the solution and then finally arriving at a generalized solution. Other activities like *Poppin' Wheelies in a Ring*, *Surface Area and Ratios 2*, *Percent with Cubes*, *The Percent Painter Returns* and *Scaling a Skyscraper* all use manipulatives or cubes to build models of each situation. These activities using visual aids encourage active participation by the students who often have little confidence in their ability to tackle a problem-solving situation. Many of the specific problem-solving suggestions discussed earlier can be tried and applied while working the problem-solving activities found in the classroom materials.

Why Teach Problem Solving?--A Final Argument

". . . In the minds of all but a few college freshmen, problem solving is not a process by which one ascertains the truth. Rather, it is a process by which one gets the answer in the back of the book by a sequence of steps, each of which has been authorized by the teacher." (Edwin E. Moise, Siam News, Feb. 1975) Indeed, too many mathematics assignments do require rote procedures to be followed while finding the same answer as the "answer in the back of the book," but this is really drill and practice, not problem solving, and the students are doing exercises, not problems.

If our students are to become independent thinkers and problem solvers, it is important that we give them many situations which cannot be routinely solved. It is important that we as educators provide guidance and examples that involve a variety of problem-solving techniques. Problem solving is a process of thinking that "emancipates us from merely routine activity."

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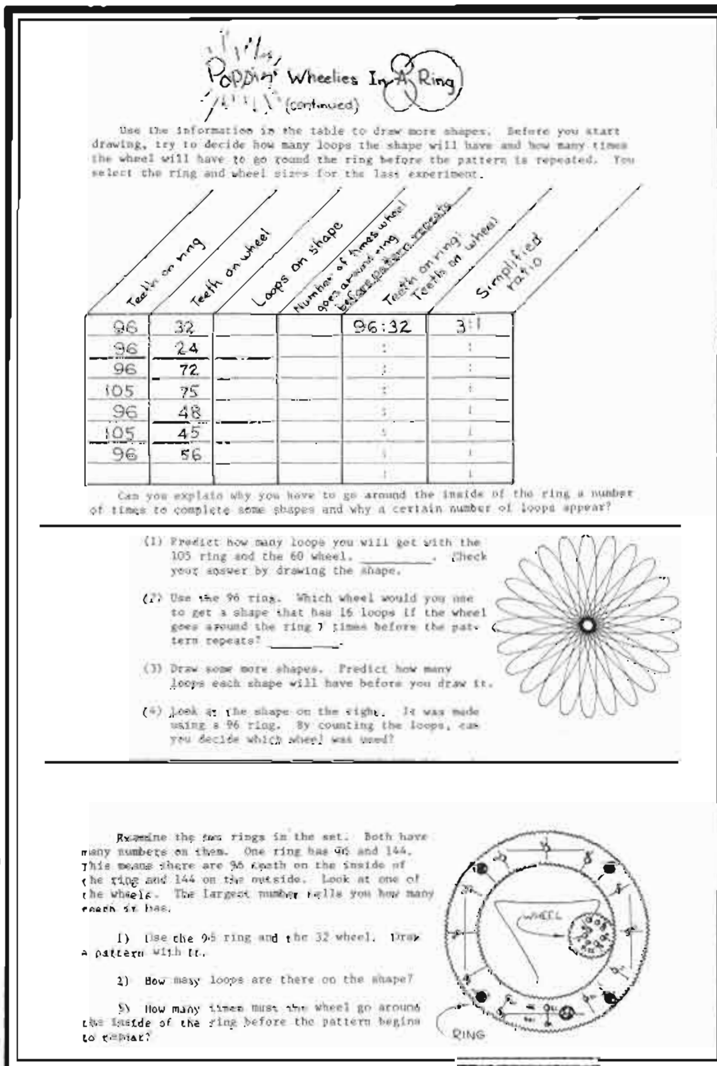
EXAMPLES OF PROBLEM SOLVING FOUND IN THE CLASSROOM MATERIALS

I. Manipulatives and Models

Manipulatives and models enhance the understanding of the problem. They provide a representation of the situation, creating visual and physical feedback that is often necessary in the search for a solution.

The Spirograph creates many exciting patterns. How does it work? The wheels and rings move together in ratios to create intricate designs.



Poppin' Wheelies In A Ring (continued)

Use the information in the table to draw more shapes. Before you start drawing, try to decide how many loops the shape will have and how many times the wheel will have to go round the ring before the pattern is repeated. You select the ring and wheel sizes for the last experiment.

Teeth on ring	Teeth on wheel	Loops on shape	Number of times wheel goes around ring before pattern repeats	Teeth on ring: Teeth on wheel	Simplified ratio
96	32			96:32	3:1
96	24			:	:
96	72			:	:
105	75			:	:
96	48			:	:
105	45			:	:
96	56			:	:
				:	:

Can you explain why you have to go around the inside of the ring a number of times to complete some shapes and why a certain number of loops appear?

(1) Predict how many loops you will get with the 105 ring and the 60 wheel. ________. Check your answer by drawing the shape.

(2) Use the 96 ring. Which wheel would you use to get a shape that has 16 loops if the wheel goes around the ring 7 times before the pattern repeats? ________.

(3) Draw some more shapes. Predict how many loops each shape will have before you draw it.

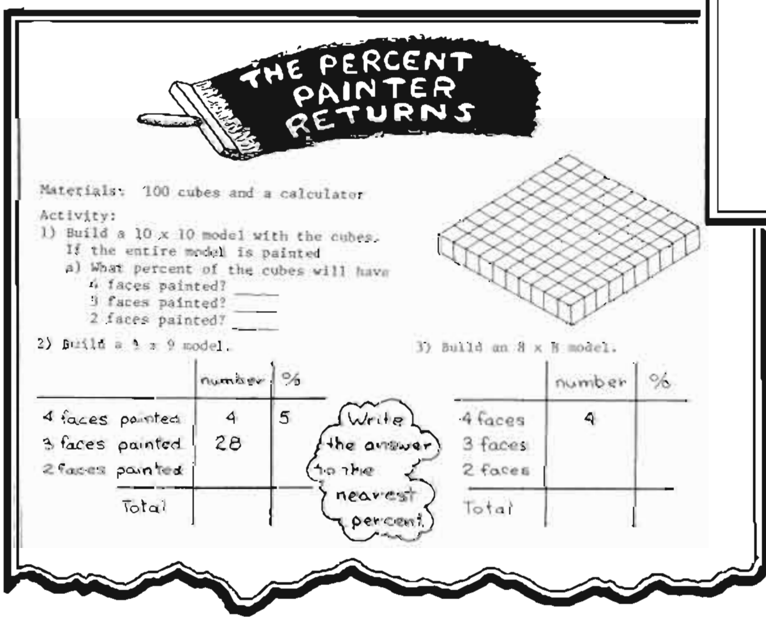
(4) Look at the shape on the right. It was made using a 96 ring. By counting the loops, can you decide which wheel was used?

Examine the two rings in the set. Both have many numbers on them. One ring has 96 and 144. This means there are 96 teeth on the inside of the ring and 144 on the outside. Look at one of the wheels. The largest number tells you how many teeth it has.

1) Use the 96 ring and the 32 wheel. Draw a pattern with it.

2) How many loops are there on the shape?

3) How many times must the wheel go around the inside of the ring before the pattern begins to repeat?



THE PERCENT PAINTER RETURNS

Materials: 100 cubes and a calculator

Activity:
1) Build a 10 x 10 model with the cubes. If the entire model is painted
a) What percent of the cubes will have
4 faces painted? ____
3 faces painted? ____
2 faces painted? ____

2) Build a 9 x 9 model.

	number	%
4 faces painted	4	5
3 faces painted	28	
2 faces painted		
Total		

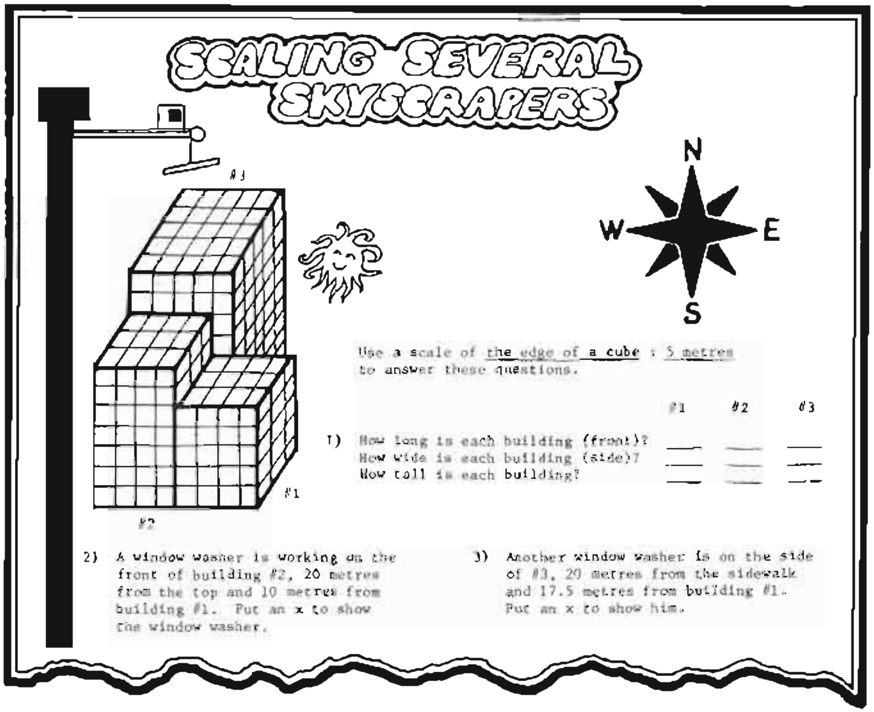
Write the answer to the nearest percent.

3) Build an 8 x 8 model.

	number	%
4 faces	4	
3 faces		
2 faces		
Total		

Using 100 cubes and a calculator, a percent model can be investigated. By observing the patterns found, one can predict and perhaps generalize what happens in similar problems.

Students learn to interpret a model or drawing by experiencing problem situations that involve its use. Sometimes students will solve a problem more readily if they build a scale model or look at a drawing of the situation.

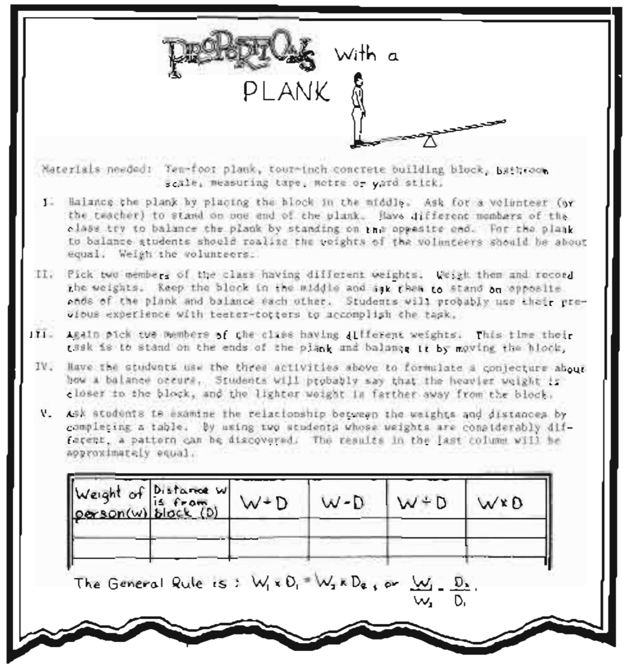


II. Research Problems

Research is a fundamental process that all disciplines use to gain and expand knowledge in their fields. Situations are encountered where the answer is not known. Unless one performs some experiments, gathers data and, in general, does some research, the answer may never be clear, not even with educated guessing.

Which letters of the alphabet occur most frequently in printed materials? How can we find out?--do a little research and compile the results. Use the information to create your own "Morse code" and compare it to the real Morse code.

Set up an experiment and record weight and distance in a table. What patterns are noticed after the data is recorded? Is there a relationship between the weights and distances?



PROPORTIONS With a PLANK

Materials needed: Ten-foot plank, four-inch concrete building block, bathroom scale, measuring tape, metre or yard stick.

I. Balance the plank by placing the block in the middle. Ask for a volunteer (or the teacher) to stand on one end of the plank. Have different members of the class try to balance the plank by standing on the opposite end. For the plank to balance students should realize the weights of the volunteers should be about equal. Weigh the volunteers.

II. Pick two members of the class having different weights. Weigh them and record the weights. Keep the block in the middle and ask them to stand on opposite ends of the plank and balance each other. Students will probably use their previous experience with teeter-totters to accomplish the task.

III. Again pick two members of the class having different weights. This time their task is to stand on the ends of the plank and balance it by moving the block.

IV. Have the students use the three activities above to formulate a conjecture about how a balance occurs. Students will probably say that the heavier weight is closer to the block, and the lighter weight is farther away from the block.

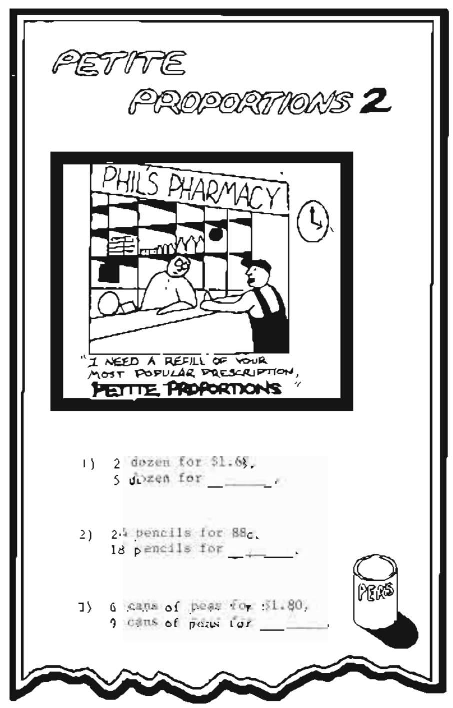
V. Ask students to examine the relationship between the weights and distances by completing a table. By using two students whose weights are considerably different, a pattern can be discovered. The results in the last column will be approximately equal.

Weight of person (W)	Distance W is from block (D)	W+D	W-D	W÷D	W×D

The General Rule is: $W_1 \times D_1 = W_2 \times D_2$, or $\frac{W_1}{W_2} = \frac{D_2}{D_1}$.

III. Miscellaneous Problem Solving

These "petite" story problems give common situations that use numbers and proportions. The format of the problem makes the proportion easy to identify.

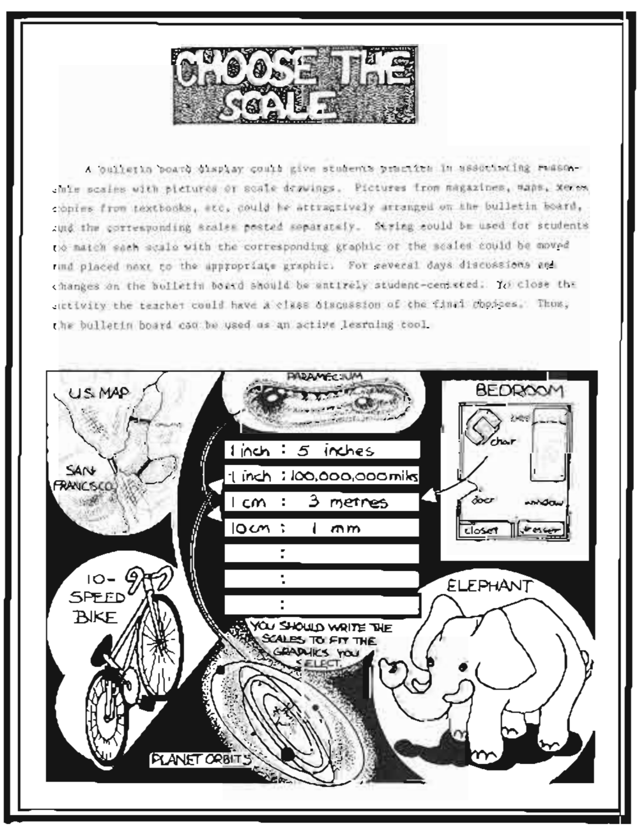


PETITE PROPORTIONS 2

1) 2 dozen for $1.68,
5 dozen for ______.

2) 24 pencils for 88c.
18 pencils for ______.

3) 6 cans of peas for $1.80,
9 cans of peas for ______.



CHOOSE THE SCALE

A bulletin board display could give students practice in associating reasonable scales with pictures or scale drawings. Pictures from magazines, maps, xerox copies from textbooks, etc. could be attractively arranged on the bulletin board, and the corresponding scales posted separately. String could be used for students to match each scale with the corresponding graphic or the scales could be moved and placed next to the appropriate graphic. For several days discussions and changes on the bulletin board should be entirely student-centered. To close the activity the teacher could have a class discussion of the final choices. Thus, the bulletin board can be used as an active learning tool.

This sample bulletin board display associates a reasonable scale with its corresponding drawing or picture. A brief discussion centered around the display may increase the students' understanding of scaling and its relationship to their visual world.

PROBLEM SOLVING FOUND IN CLASSROOM MATERIALS

RATIO:

Getting Started

CAN YOU FIND THE PATTERN?	USING PATTERNS
PATTERNS FOR INTRODUCING RATIO	USING PATTERNS
CONSTANT COMMENTS	USING PATTERNS
ROWS AND RATIOS	DETERMINING RATIOS FROM PATTERNS
WHAT'S IN A RATIO?	INTERPRETING RATIO STATEMENTS
RATIO OF AGES	USING RATIOS TO COMPARE CHANGE IN AGE

Rate

FIX THAT LEAK	DETERMINING RATES
WHICH IS A BETTER BUY?	USING RATES TO COMPARE PRICES

Equivalent

POPPIN' WHEELIES IN A RING	SIMPLIFYING
SURFACE AREA AND RATIOS 2	SIMPLIFYING

PROPORTION:

Getting Started

PERSONALIZED PROPORTIONS	SOLVING PROPORTIONS
PETITE PROPORTIONS 1	SOLVING PROPORTIONS
PETITE PROPORTIONS 2	SOLVING PROPORTIONS
COUNTEREXAMPLE	RECOGNIZING INCORRECT PROPORTIONS

Application

IT'S ONLY MONEY	USING PROPORTIONS TO CONVERT CURRENCY

PROPORTIONS WITH A PLANK	USING PROPORTIONS WITH LEVERS INVERSE VARIATION
SCALING:	
Getting Started	
YOUR MOD BOD	USING SCALES TO REPRESENT HEIGHTS
CHOOSE THE SCALE	CHOOSING A REASONABLE SCALE
Making a Scale Drawing	
ROOM DECORATIONS	ENLARGING WITH A COMPASS AND RULER
Supplementary Ideas in Scaling	
THE PERPLEXING PENTOMINOES	WORKING WITH SHAPES
HOW WELL DO YOU STACK UP THIS TIME?	BUILDING 3-D MODELS FROM SKETCHES
3 FACES YOU SHOULD HAVE SEEN	IDENTIFYING 3-D MODELS FROM SCALE DRAWINGS
SCALING A SKYSCRAPER	USING A SCALE TO LOCATE POINTS
SCALING SEVERAL SKYSCRAPERS	USING A SCALE TO LOCATE POINTS
BUILDING A SKYSCRAPER	CONSTRUCTING 3-D MODELS
PERCENT:	
Percent Sense	
DOLLAR$ AND PER¢ENTS 2	REFERENCE SET OF 100* MONEY MODEL
PERCENT WITH CUBES	REFERENCE SET OF 100* SET MODEL
THE PERCENT PAINTER	REFERENCE SET OF 100 SET MODEL
PERCENTS: BACKWARDS AND FORWARDS 4	MODELS*

*Indicates percents greater than 100% are used on the page.

THE WHOLE THING	SET MODEL
FINDING 100% FROM BELOW	AREA MODEL
FINDING 100% FROM ABOVE	AREA MODEL*
PEACE-N-ORDER	AREA MODEL*
As a Ratio	
PERCENT PICTURES - II	GRID MODEL
PUNY PERCENTS	PERCENTS LESS THAN 1%
As a Fraction/Decimal	
THE PERCENT PAINTER RETURNS	AS A DECIMAL
Solving Percent Problems	
A SIGN OF THE TIMES	SOLVING PERCENT PROBLEMS
PERCENT PROBLEMS 1	WORD PROBLEMS
PERCENT PROBLEMS 2	WORD PROBLEMS
CERTAIN GROWTHS ARE BENEFICIAL	FINDING AMOUNT OF INTEREST

*Indicates percents greater than 100% are used on the page.

RATIO, PROPORTION, AND SCALING

Placement Guide for Tabbed Divisors

MENTAL ARITHMETIC

RATIONALE

Many of our day-to-day calculations are done mentally. Without using pencil and paper or a hand calculator, we often think about answers to such questions as: Did the clerk give me the right amount of change? How long will it take me to travel across town? How many boxes of candy will have to be sold for a fund-raising project needing $500?

Mental arithmetic is an important basic skill which can be applied to many situations. One might perform mental checks on routine computations. Mental arithmetic can help students develop a better number sense and a better feeling about their ability to calculate answers. It may also improve their knowledge of basic facts and motivate them to move on to more advanced or applied mathematics. People can use mental arithmetic to improve the process of estimation and approximation by . . .

1) Checking for reasonableness and correctness of answers.
2) Getting "ball-park" estimates.
3) Rounding.
4) Computing with simplified numbers.
5) Multiplying and dividing by powers of ten.

The use of mental arithmetic can quicken the problem-solving process--especially for those problems which involve trial and error.

Just as any skill must be developed through practice, the ability to do arithmetic mentally can be improved with drill and mental calculations. These can be short and part of the daily routine (such as a five-minute warm-up activity). Or the activities can be longer and stressed early in the school year to develop the habit of using mental arithmetic. Encourage the students to do mental calculations whenever they are involved in checking pencil and paper calculations, calculator activities, and problem solving.

<u>Selected Sources for Mental Arithmetic</u>

Cutler, Ann and Rudolph McShane. <u>The Trachtenberg Speed System of Basic Mathematics</u>, Doubleday, 1960.

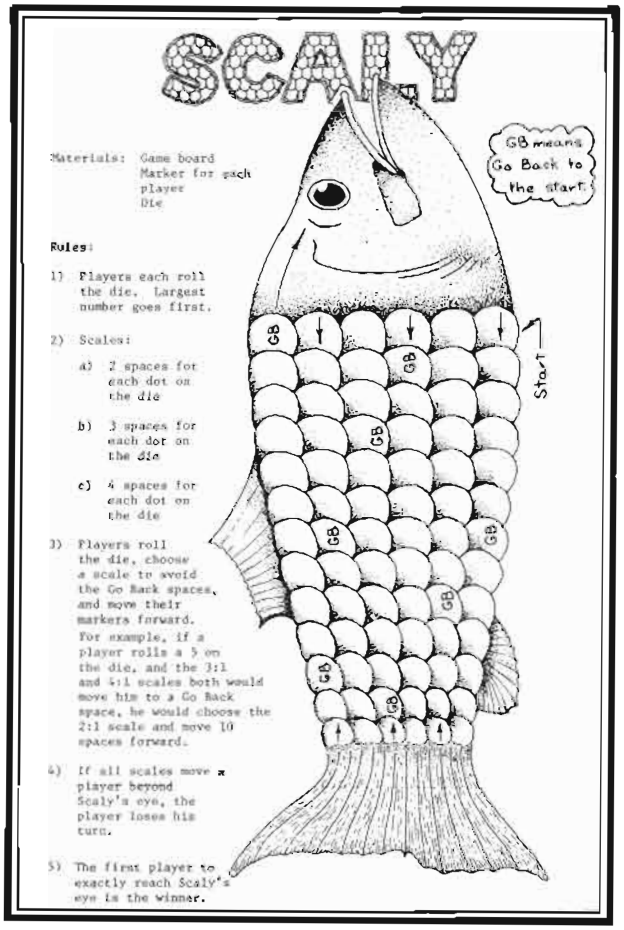
Garvin, Alfred E. <u>Shortcuts, Checks and Approximations in Mathematics</u>, J. Weston Walch, 1973.

Kramer, Klass. <u>Mental Computation</u>, Science Research Associates.

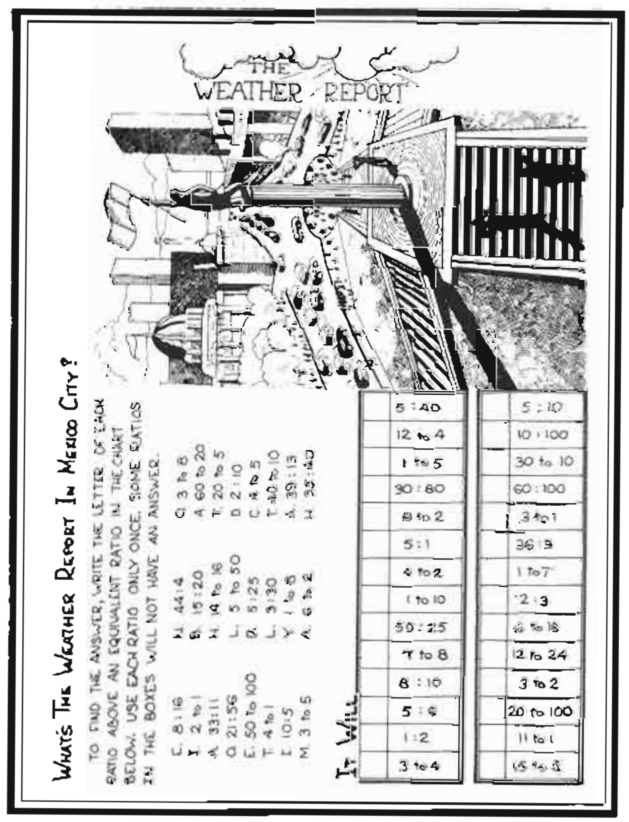
EXAMPLES OF MENTAL ARITHMETIC PAGES FOUND IN THE CLASSROOM MATERIALS

I. Games and Puzzles

Games and puzzles often require quick thinking. Figuring on paper or using a calculator is not always necessary or convenient.



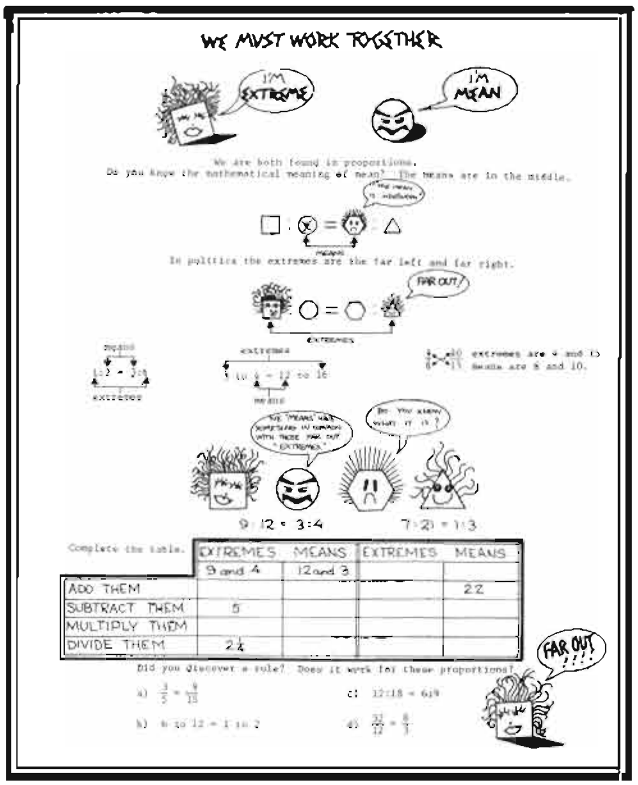
This game involves chance and strategy. The player makes an educated guess between a scale of 2:1, 3:1, or 4:1 for each toss of the die. This requires a quick mental evaluation of the position on the playing board, the number on the die, and the best choices of a scale.



Equivalent ratios form patterns that can be assimilated mentally. This puzzle matches equivalent ratios and displays self-correcting answers.

II. Concepts and Patterns

Once a basic concept is understood, one can use mental arithmetic and shortcuts to cut down computation. Patterns often lead to the answers and mentally following a pattern can reveal the final answer with minimal effort.

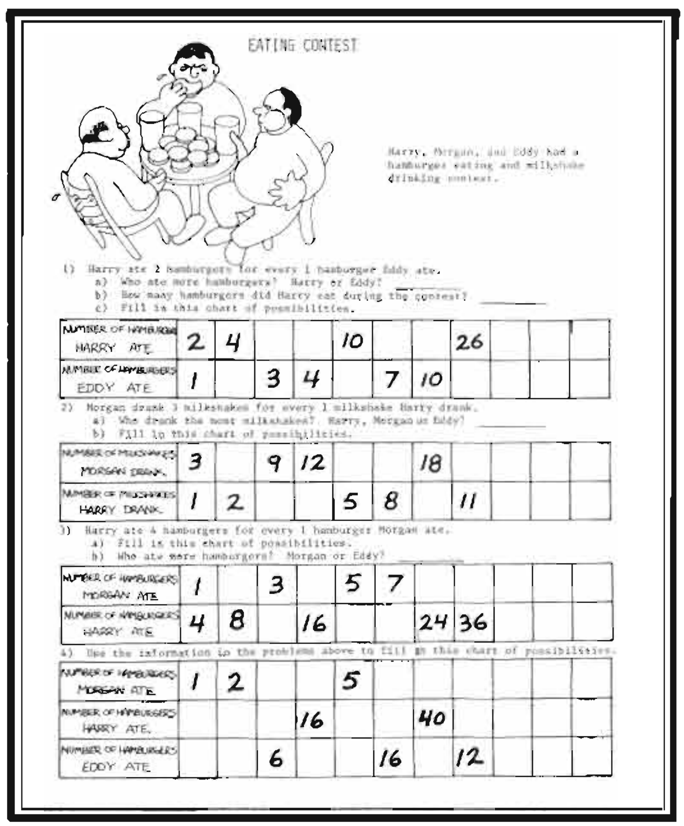


Complete the table.

	EXTREMES	MEANS	EXTREMES	MEANS
	9 and 4	12 and 3		
ADD THEM				22
SUBTRACT THEM	5			
MULTIPLY THEM				
DIVIDE THEM	$2\frac{1}{4}$			

Did you discover a rule? Does it work for these proportions?

Proportions can be solved by following a special pattern. In any proportion the product of the means equals the product of the extremes.



1) Harry ate 2 hamburgers for every 1 hamburger Eddy ate.
   a) Who ate more hamburgers? Harry or Eddy? ______
   b) How many hamburgers did Harry eat during the contest? ______
   c) Fill in this chart of possibilities.

NUMBER OF HAMBURGERS HARRY ATE	2	4			10			26			
NUMBER OF HAMBURGERS EDDY ATE	1		3	4		7	10				

2) Morgan drank 3 milkshakes for every 1 milkshake Harry drank.
   a) Who drank the most milkshakes? Harry, Morgan or Eddy? ______
   b) Fill in this chart of possibilities.

NUMBER OF MILKSHAKES MORGAN DRANK	3		9	12			18				
NUMBER OF MILKSHAKES HARRY DRANK	1	2			5	8		11			

3) Harry ate 4 hamburgers for every 1 hamburger Morgan ate.
   a) Fill in this chart of possibilities.
   b) Who ate more hamburgers? Morgan or Eddy? ______

NUMBER OF HAMBURGERS MORGAN ATE	1		3		5	7					
NUMBER OF HAMBURGERS HARRY ATE	4	8		16			24	36			

4) Use the information in the problems above to fill in this chart of possibilities.

NUMBER OF HAMBURGERS MORGAN ATE	1	2			5						
NUMBER OF HAMBURGERS HARRY ATE				16			40				
NUMBER OF HAMBURGERS EDDY ATE			6			16		12			

Quick mental computation discloses the simple patterns and comparisons displayed in the charts.

MENTAL ARITHMETIC FOUND IN CLASSROOM MATERIALS

RATIO:

Getting Started

CAN YOU FIND THE PATTERN?	USING PATTERNS
PATTERNS FOR INTRODUCING RATIO	USING PATTERNS

Equivalent

EQUIVALENT RATIOS BY PATTERNS	CONCEPT, GENERATING
EATING CONTEST	GENERATING
A LOVELY DESIGN	RECOGNIZING
SPIDER TO FLY RATIOS	RECOGNIZING
SPICY RATIOS	RECOGNIZING
A STATEMENT OF PRIME IMPORTANCE	RECOGNIZING
THE WEATHER REPORT	RECOGNIZING

PROPORTION:

Getting Started

GETTING BULLISH ON PROPORTIONS	MULTIPLICATION METHOD
WE MUST WORK TOGETHER	CROSS PRODUCTS METHOD
AN EXTREME TOOL	CROSS PRODUCTS METHOD
A STEWED SURPRISE	SOLVING PROPORTIONS

SCALING:

Getting Started

SCALY	CHOOSING AN APPROPRIATE SCALE

PERCENT:

As a Ratio

WHAT DO A CAT AND A SKUNK HAVE IN COMMON WITH %?	EQUIVALENT FORMS

Solving Percent Problems

HOLLYWEED SQUARES	REVIEWING SKILLS
A SIGN OF THE TIMES	SOLVING PERCENT PROBLEMS

RATIO, PROPORTION, AND SCALING

Placement Guide for Tabbed Divisors

ESTIMATION and APPROXIMATION

RATIONALE

Why estimate and approximate? Why should we be concerned with educated guesses (estimation) or a process to improve the accuracy of an educated guess (approximation)?

Today, according to some authorities, 75% of adult non-occupational uses of arithmetic is mental. If we are concerned about students having a number sense, then we need to work on such things as: mental computation, rounded results, reasonableness of answers, a feel for large and small numbers, and numbers representing measures.

In our daily lives we use inexact numbers every time we measure. News sources frequently use approximations when discussing large numbers. Exact results are often not necessary, and they often obscure the issue. (Which would be better--49,717 people attended the football game, or "about 50,000," The family income is $11,978 vs. The family income is $12,000?)

For example, we make many educated guesses every time we

a) plan a trip (How long will it take, when will we arrive, how much will it cost, what should we take?)

b) determine a budget (I think we can go out for dinner and a show once this year.)

We make a life and death estimation when we decide if it is safe to cross the street, or if we can stop a car or bike in time.

The reasonableness of calculated results can mean a difference of many dollars to each of us, whether it be in checking the change at the supermarket, figuring taxes, or making time payments on large purchases.

Often we need to locate the decimal point in computations by hand, with a slide rule, with a calculator, or in using square root tables. Even when we do long division problems we usually use some type of "guess and check" method.

We make "ball park" estimates for

a) how many (hot dogs to order for a football game)

b) how things compare (can 1,000 people fit into the ballroom?)

c) personal information (if we could spend a dollar a second, how long would it take to spend a billion dollars?)

d) functioning effectively in our daily lives.

Before anyone can make an estimation that is more than just a guess, he must first of all have a familiarity with certain reference points for measures of length, weight, time, area, volume, cost, and so on. Most of these come from experiences in the person's day to day world. They can be extended through development of measuring skills, arithmetic skills, and a number sense for large and small numbers. To obtain a "good" estimate, it is also useful to

$\approx$

have a knowledge of counting methods. (For additional information see <u>Peas and Particles</u>.)

Before a person can quickly check the reasonableness of an answer he must have already developed a wide variety of arithmetic skills. These must include:

a) ability to perform accurately single-digit operations (9 million x 7 million requires 9 x 7 = 63)

b) ability to multiply and divide by powers of ten

c) ability to perform operations with multiples of powers of ten--mentally if possible

d) being comfortable with inequalities and other relationships

e) ability to round whole numbers and decimals to one or two significant digits.

It is also helpful for more difficult approximations if a person has a familiarity with exponential notation.

Here is an example which illustrates most of these points: About how long is a billion seconds?

$$\frac{1{,}000{,}000{,}000}{60 \times 60 \times 24 \times 365} = \text{years} \approx$$

$$\frac{1 \times 10^9}{60 \times 60 \times 20 \times 400} = \frac{1 \times 10^9}{3600 \times 8000}$$

$$\approx \frac{1 \times 10^9}{4 \times 10^3 \times 8 \times 10^3} = \frac{1 \times 10^9}{32 \times 10^6}$$

$$\approx \frac{1 \times 10^9}{3 \times 10^7} = \frac{1}{3} \times 10^2 \approx 33 \text{ years}$$

There is much to be said for knowing when to estimate and when to approximate, when to use an estimation or approximation, and when to use an exact answer. The use of estimation and approximation should help all persons to deal with exact numbers, understand and perform operations with numbers arising from measurement, deal comfortably with numbers through approximate calculations and rounding off, and in general develop a number sense. Finally, it would seem most worthwhile if teaching the techniques of estimation and approximation helped to eliminate the "exact answer" syndrome.

SUMMARY

These are the key points to be emphasized when teaching estimation and approximation:

1. When do we need to estimate and approximate to find a rough answer?
2. When do we need exact answers?
3. We often estimate "how many" (e.g., objects, people, items) or "how much" (e.g., money, air, water).
4. We often estimate the dimensions, capacity or amount of something we would measure. (Measurements are always approximate.)
5. Problem solving and computation is aided by the use of estimation and approximation to . . .
   a) check the reasonableness of answers
   b) narrow the scope of your investigations
   c) simplify computations

≈

6. The students need a sound background in arithmetic skills, number sense, and finding reference points.

Selected Sources for Estimation and Approximation

Garvin, Alfred D. Shortcuts, Checks and Approximations in Mathematics, J. Weston Walch, 1973.

Herrick, Marian, et al. Mathematics for Achievement/Individualized Course 2, Book 5, Houghton Mifflin, 1972.

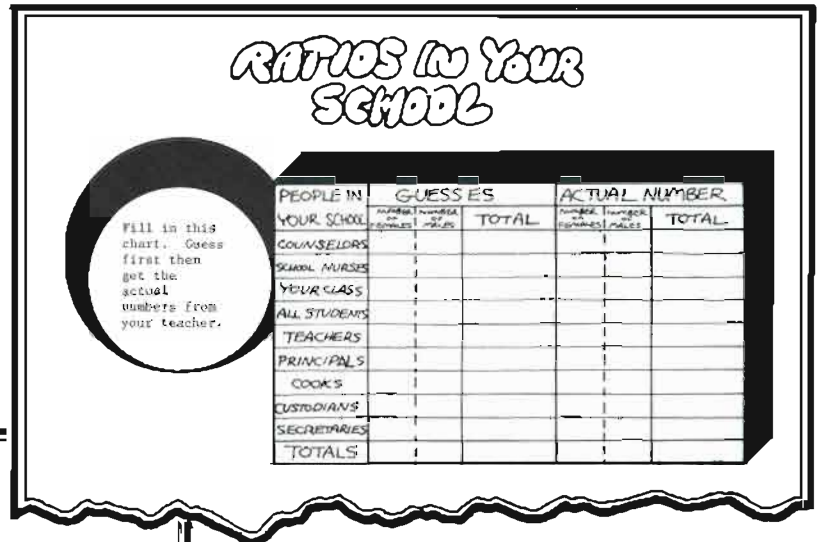
Mathex Book 5 Measurement and Estimation, Encyclopedia Britannica, 1970

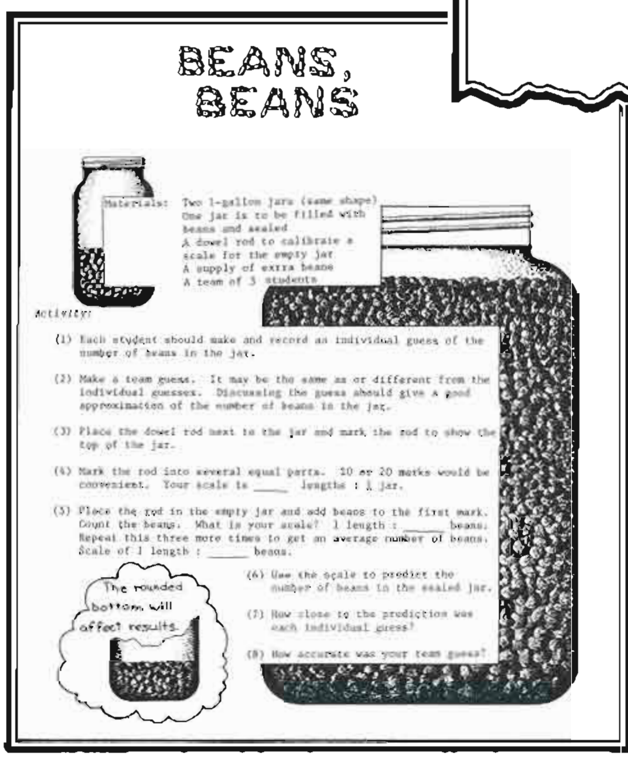
Peas and Particles (Teacher's Guide), Elementary Science Study, Webster/McGraw-Hill, 1969.

EXAMPLES OF ESTIMATION AND APPROXIMATION IN THE CLASSROOM MATERIALS

I. Estimating "How Many"

Estimate the number of people that work at your school. How is the student-teacher ratio determined?

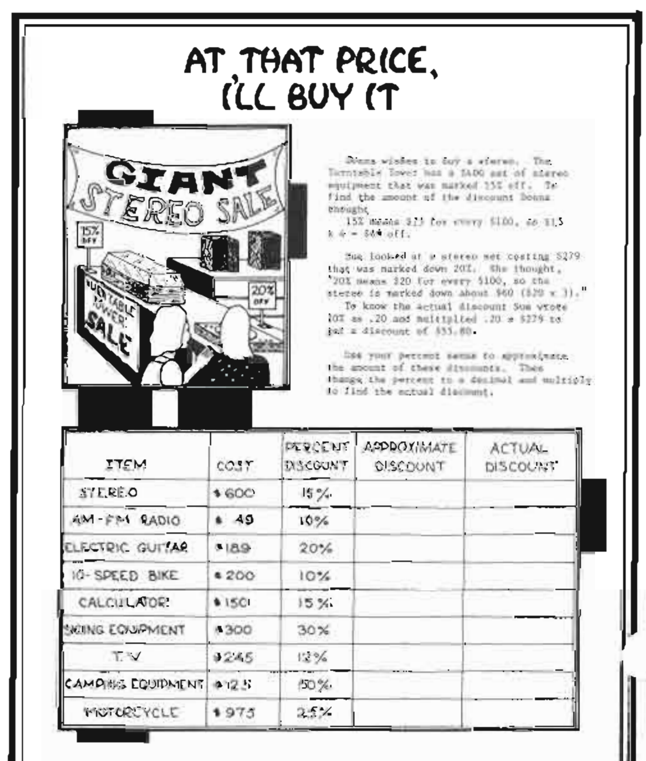




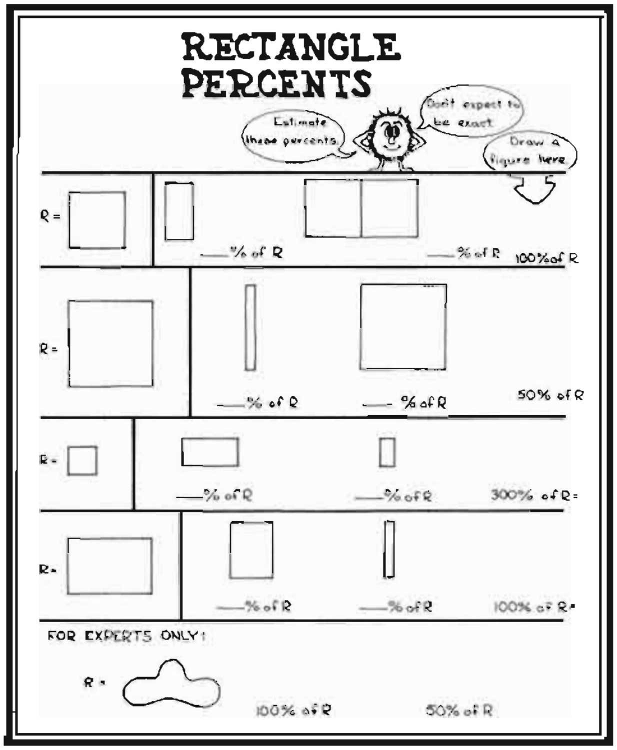
How many beans in the jar? The container's volume plays an important part in finding a reasonable estimate for the number of beans.

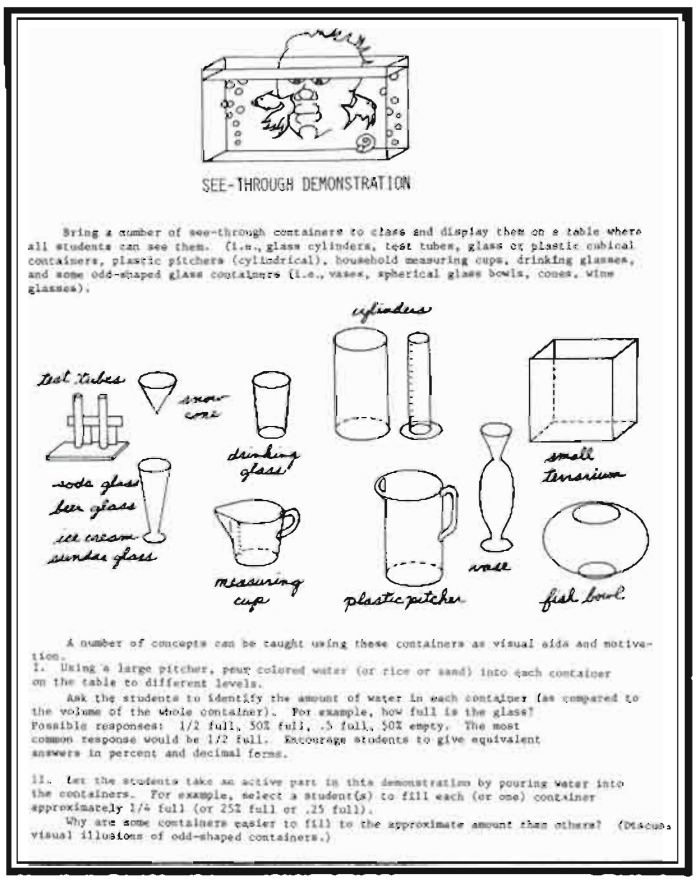
II. Estimating "How Much"

How much money can you save on sales? Approximate your savings on various items that are discounted a given percent.



Draw the amount of area that represents the given percent. A reference set is always necessary before an area can be compared and then drawn.




SEE-THROUGH DEMONSTRATION

Bring a number of see-through containers to class and display them on a table where all students can see them. (i.e., glass cylinders, test tubes, glass or plastic cubical containers, plastic pitchers (cylindrical), household measuring cups, drinking glasses, and some odd-shaped glass containers (i.e., vases, spherical glass bowls, cones, wine glasses).

A number of concepts can be taught using these containers as visual aids and motivation.

I. Using a large pitcher, pour colored water (or rice or sand) into each container on the table to different levels.

Ask the students to identify the amount of water in each container (as compared to the volume of the whole container). For example, how full is the glass? Possible responses: 1/2 full, 50% full, .5 full, 50% empty. The most common response would be 1/2 full. Encourage students to give equivalent answers in percent and decimal forms.

II. Let the students take an active part in this demonstration by pouring water into the containers. For example, select a student(s) to fill each (or one) container approximately 1/4 full (or 25% full or .25 full).

Why are some containers easier to fill to the approximate amount than others? (Discuss visual illusions of odd-shaped containers.)

Estimate the amount of water it will take to fill each container $\frac{1}{2}$ or 50% full. How can you tell the real volume of each odd-shaped container? These experiments with volume test spatial relationships and the ability to estimate three-dimensional quantities.

ESTIMATION AND APPROXIMATION FOUND IN CLASSROOM MATERIALS

RATIO:

Rate

MATH IS A FOUR-LETTER WORD	DETERMINING RATES

Equivalent

RATIOS IN YOUR SCHOOL	SIMPLIFYING
ONE MAN ONE VOTE	SIMPLIFYING

PROPORTION:

Application

I MEAN TO BE MEAN!	DETERMINING MEAN PROPORTIONS

SCALING

Getting Started

BEANS, BEANS	USING A SCALE TO MAKE PREDICTIONS
CHOOSE THE SCALE	CHOOSING A REASONABLE SCALE

Making a Scale Drawing

PACE OUT THE SPACE	REDUCING WITH A GRID OR RULER

Maps

THE GREAT LAKES	USING A SCALE DRAWING TO FIND DISTANCES

PERCENT

Percent Sense

GUESS AND CHECK	REFERENCE SET OF 100 GRID MODEL

THE TRANSPARENT HUNDRED	REFERENCE SET OF 100* GRID MODEL
ELASTIC PERCENT APPROXIMATOR	REFERENCE SET OF 100 NUMBER LINE MODEL
PERCENTS OF LINE SEGMENTS	REFERENCE SET OF 100* NUMBER LINE MODEL
PERCENTING: LINE SEGMENTS	REFERENCE SET OF 100 NUMBER LINE MODEL
STRINGING ALONG WITH PERCENTS	REFERENCE SET OF 100* NUMBER LINE MODEL
PERCENTS OF RECTANGLES	AREA MODEL*
RECTANGLE PERCENTS	AREA MODEL*
GEOBOARD PERCENTS	AREA MODEL
PEACE-N-ORDER	AREA MODEL
As a Ratio	
THAT'S "ABOUT" RIGHT	AS A RATIO
BE COOL--GO TO SCHOOL	USING PERCENT TO COMPARE
As a Fraction/Decimal	
PERCENT WITH RODS & METRES - III	AS A FRACTION/DECIMAL* NUMBER LINE MODEL
THE PERCENT BAR SHEET	AS A FRACTION/DECIMAL* NUMBER LINE MODEL
HALLELUJAH I'VE BEEN CONVERTED	AS A FRACTION/DECIMAL NUMBER LINE MODEL
SEE-THROUGH DEMONSTRATION	AS A FRACTION/DECIMAL VOLUME MODEL
Solving Percent Problems	
THE ELASTIC PERCENT APPROXIMATOR EXTENDED	USING A PERCENT CALCULATOR
GRID PERCENT CALCULATOR I	USING A PERCENT CALCULATOR

*Indicates percents greater than 100% are used on the page.

GRID PERCENT CALCULATOR II	USING A PERCENT CALCULATOR
GRID PERCENT CALCULATOR III	USING A PERCENT CALCULATOR
GRID PERCENT CALCULATOR IV	USING A PERCENT CALCULATOR
GRID PERCENT CALCULATOR EXTENSIONS	USING A PERCENT CALCULATOR
THE OLD OAK TREE	SOLVING PERCENT PROBLEMS
ENORMOUS ESTIMATE	SOLVING PERCENT PROBLEMS
LOVE IS WHERE YOU FIND IT	SOLVING PERCENT PROBLEMS
INTERESTING? YOU CAN BANK ON IT!	FINDING AMOUNT OF INTEREST
AT THAT PRICE, I'LL BUY IT!	FINDING AMOUNT OF DISCOUNT
COUNTING EVERY BODY	FINDING PERCENT OF INCREASE

RATIO, PROPORTION, AND SCALING

Placement Guide for Tabbed Divisors

LABORATORY APPROACHES

RATIONALE

What is the Laboratory Approach?

For many decades, learning, instead of just memorization and training, has been the primary emphasis of education. Each society or community decides what should be learned. We are required to learn mathematics, reading, science and other subjects. Yet our schools have been organized for teachers to teach and not necessarily for children to learn. The laboratory approach is a philosophy which emphasizes "learning by doing" and breaks free from formal teaching methods. "It is a system based on active learning and focuses on the learning process rather than on the teaching process." [Kidd, et al.] Experiences are devised to help the student learn mathematics by seeing, touching, hearing and feeling. An environment--the math lab--emerges where the teacher and the students work and communicate with each other to plan activities and learn by doing. At the level of their abilities and interests, the students discover relationships and study real-world problems which utilize specific mathematical skills.

A laboratory approach breaks the monotony of straight textbook teaching. It extends and reinforces the students' understandings and skills while providing background experiences for later development of abstract concepts. It also offers a unique, concrete way to learn mathematics. The laboratory approach can be integrated into the classroom and used along with, not in place of, many other equally valuable teaching strategies.

Lab activities help to eliminate the unrealistic one-method syndrome so characteristic of mathematics classes. A variety of methods of attacking a problem can be explored. Open-ended activities encourage students to make discoveries, formulate and test their own generalizations (i.e., problem solving). Lab assignments can be used to challenge the students by providing them with opportunities for developing self-confidence, habits of independent work, and enjoyment of mathematics. The relaxed atmosphere can encourage student involvement and positive attitudes toward mathematics. By direct observation, the teacher can assess the student's skills in problem solving and computing while the student's attitude and work habits can also be evaluated.

The Mathematics Laboratory

The math lab is an environment that provides for active learning and encourages active participation. In terms of physical organization, three basic kinds of mathematics laboratories are most often discussed.

1. A centralized laboratory--a room

especially designed (or adapted) and equipped for use as a permanent math lab. Classes are usually brought into the lab room on a rotating schedule that allows each mathematics class to use the lab materials several times a week as needed.

2. A rolling or movable laboratory--a set of lab materials placed on a cart, stored in a central location, and wheeled from classroom to classroom as needed.

3. A decentralized laboratory--a self-contained set of lab materials stored in the teacher's classroom and readily available for the students to use.

For most schools, the decentralized laboratory is the most practical and desirable math lab. Lab materials can be collected and organized at a modest rate as they are constructed, donated or purchased.

Eventually a set of lab materials will grow to a size large enough to be quite versatile. The classroom environment needs to be versatile as well. Flat tables, bookcases, movable carts and other furniture can be added to provide work areas for the students and storage space for the lab activities.

What is a Laboratory Activity?

A laboratory activity is a task or mathematical exercise that emphasizes "learning by doing." It can be a game, a puzzle, a paper and pencil exercise, a set of manipulatives with a task card, or an experiment using apparatus and instruments to take measurements. A game involving two or more students might review the concept of equivalent fractions. A challenging puzzle could require a student to apply several problem-solving techniques. A lab activity could use Cuisenaire Rods to illustrate decimal concepts, or multibase blocks to show place value, or wooden cubes to demonstrate spatial relationships, or factor boards to clarify an algorithm. Manipulative objects often provide physical models that can introduce or clarify a mathematical concept to the student. There are also experiments which can be performed to take measurements and gather data. Students learn how to use certain equipment and tools in their search for solutions. Laboratory activities can directly involve students in "hands-on" assignments, often with group participation. Lab activities encourage the student to take an active role in learning mathematics rather than the passive role of "you teach me."

Getting Started

There are many ways to implement the lab approach. The descriptions below provide several suggestions to consider when starting to use the laboratory approach.

Mr. Langford has a class of thirty seventh graders. He was not sure about using lab materials, so he decided to start small. He set up an "activity corner" in the room. Three lab cards with the necessary equipment (e.g., squared paper, ceramic tile, measuring tape, metric wheel) were set up in the "activity corner." Each day for a week a different group of six students were allowed to work in pairs using the lab materials. The rest of the class worked on related paper and pencil exercises. All week was spent on the study of area. All thirty students had a chance to do the lab activities, and the activities integrated well with the week's mathematics concept of area. Mr. Langford wants to collect or write task cards that mix well with his established curriculum. Later, he might try other ways of using the lab activity cards.

Ms. Wilkins decided to assign each Friday as a "lab day" for her eighth-grade class of 28 students. She had watched several classes using a "lab day" once a week and decided to try it herself. She prepared two sets of seven lab cards covering seven different mathematic topics. Each student was assigned a partner, and the pair would work together for each of the seven "lab days." For seven weeks the students rotated to a new lab activity each Friday. They were asked to keep a record of their results and follow the planned rotation schedule. Ms. Wilkins found that this seven-week period with one "lab day" a week coincided well with the nine-week term. She developed a second set of lab materials for another seven weeks. This time there were 14 task cards put into 14 shoe boxes along with manipulatives, paper, or other materials needed for each activity. Each card was written on the topic of measurement and contained various levels of abstraction and enrichment options for the students.

Mr. Jeffreys and Ms. Slone had adjoining sixth-grade rooms. They had been team teaching a number of units in mathematics. They decided to try the lab approach for their unit on Base 10 and Other Bases. Their school had recently purchased two Chip Trading Math Lab Sets. Mr. Jeffreys and Ms. Slone picked out several chip trading activities to be used every other day for two weeks. They divided the class into groups of 3 or 4 students. For each "chip trading day" one student in each group was responsible for picking up and distributing the manipulatives to each member of the group. The days between each "chip trading day" were used for discussions, board work, and worksheets that emphasized paper and pencil computation in base 10 and other bases.

The above are examples of teachers who were willing to support an active approach to learning. They prepared for using the lab approach by collecting and organizing materials and deciding on the content of lab activities. It helps to gain the support of other teachers; their contributions and ideas can rapidly increase the number of lab activities developed.

Most difficulties that arise in the math lab result from students not knowing what to do. The teacher needs to find, organize and store lab materials for easy use; tell students where lab materials are, what to do with them and how to schedule their use; prepare task cards or directions for the lab activities; instruct students in problem-solving methods of attack and investigation; interact enthusiastically with students and share in their experiences; and evaluate each student's attitudes, work habits and accomplishments.

Start small--in no way can most teachers and students survive a complete change of program. Students who have become passive learners need time to adapt to the role of active learners. They need supervision and guidance from the teacher as they learn to function in the lab environment. Eventually, the students should be able to select materials for each lab activity and return materials to the proper storage area when finished. By keeping a work record, the students can evaluate their progress and try to improve their skills and understanding. The students need to develop inquisitive attitudes that motivate them to keep at a problem and not give up. Small groups or pairs of students will require the cooperation of each individual and the sharing of ideas.

Initially, when selecting material and equipment to use in the math lab, find readily available materials in the school. As time goes on, you will be able to buy, make or scrounge other materials as they are needed for particular activities.

Ideas for laboratory activities can be found in any of the sources listed in the selected sources. Many periodicals (such as The Arithmetic Teacher or The Mathematics Teacher) include sections in each issue which contain ideas for activities that require a minimum of preparation and materials. Notice the interests of the students. Be creative and use your own ideas or their ideas as a source of lab activities. Discuss and exchange ideas about math labs with other teachers.

Begin with a lab activity that everyone can do at the same time. Later on, the students can separate into groups or small teams (students usually work best in small groups of 2 or 3). Experiment with the size and the make-up of the groups. In the beginning it is a good idea to provide activities where each group member has a specific role. Provide several lab activities and let each group move from one activity to another. Have specific objective(s) in mind for each activity, and have a clear idea of its mathematical content. Go through the lab activity to find what background concepts or skills the students will need to tackle it. Check for any difficulties the students might encounter as they do the activity.

SUMMARY

1. The laboratory approach is a system that emphasizes learning by doing; it involves the student in multi-sensory experiences that often require social interaction as well as physical participation and problem-solving skills.
2. There are several types of math labs--even math lab is versatile; each includes lab materials; each requires careful organization and upkeep.
3. A laboratory activity is a task or mathematical exercise that provides an active role in learning for the student.
4. One can implement the lab approach in various ways:
   a) Set up an activity corner and allow a few students each day to work on assigned lab activities.
   b) Declare a lab day; perhpas once a week the whole class will be involved in lab activities.
   c) Pick out a particular topic or unit in mathematics; develop a number of lab activities for the specific topic and have the students work through the various activities each day or every other day.
   d) Be brave; try the laboratory approach and plan your own creative schedule and activities for the students.
5. Most difficulties that arise in the math lab result from students not knowing what to do.
6. Start small--there are many materials and ideas to use in a math lab. Do not be overwhelmed, but collect lab materials gradually, adding manipulatives, games, task cards, etc. as you have time to make and/or develop them.

Selected Sources for Laboratory Approaches

The Arithmetic Teacher, National Council of Teachers of Mathematics.

Biggs, Edith and James MacLean. Freedom to Learn, Addison-Wesley (Canada) Ltd., 1969.

Hamilton, Schmeltzer and Schmeltzer. "The Mathematics Laboratory," Teaching Mathematics in the Junior High.

Kidd, et al. The Laboratory Approach to Mathematics, Science Research Associates, Inc., 1970.

Krulik, Stephen. A Mathematics Laboratory Handbook for Secondary Schools, W.B. Saunders Co., 1972.

The Mathematics Teacher, National Council of Teachers of Mathematics.

Reys, Robert F. and Post, Thomas R. The Mathematics Laboratory: Theory to Practice. Prindle, Weber and Schmidt, Inc., 1973.

Sobel, Max and Maletsky, Evan. Teaching Mathematics: A Sourcebook of Aids, Activities and Strategies, Prentice Hall, Inc., 1975.

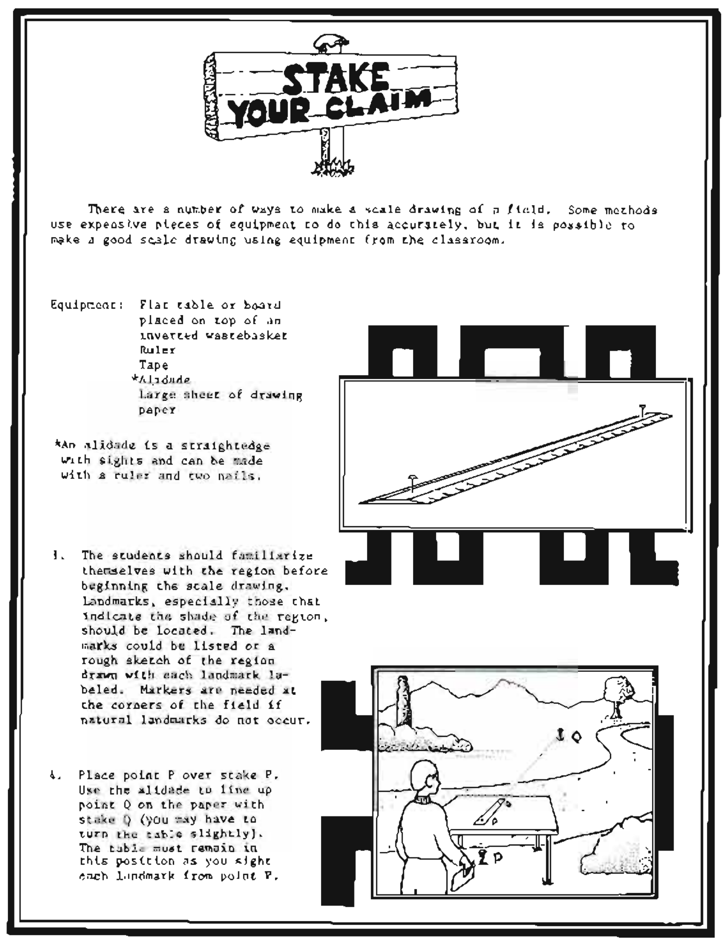
Teacher-Made Aids for Elementary School Mathematics, Readings from the Arithmetic Teacher, National Council of Teachers of Mathematics.

EXAMPLES OF LABORATORY ACTIVITIES FOUND IN THE CLASSROOM MATERIALS

I. "Homemade" Materials

When selecting materials and equipment to use for lab activities, it is relatively inexpensive and simple to use available materials in the school. Apparatus or equipment can often be made by the students. Active participation in measurement activities helps to build concepts through visual, concrete experiences.

A measuring instrument (in this case, an alidade) is often used to record mathematical data and to analyze our environment.



STAKE YOUR CLAIM

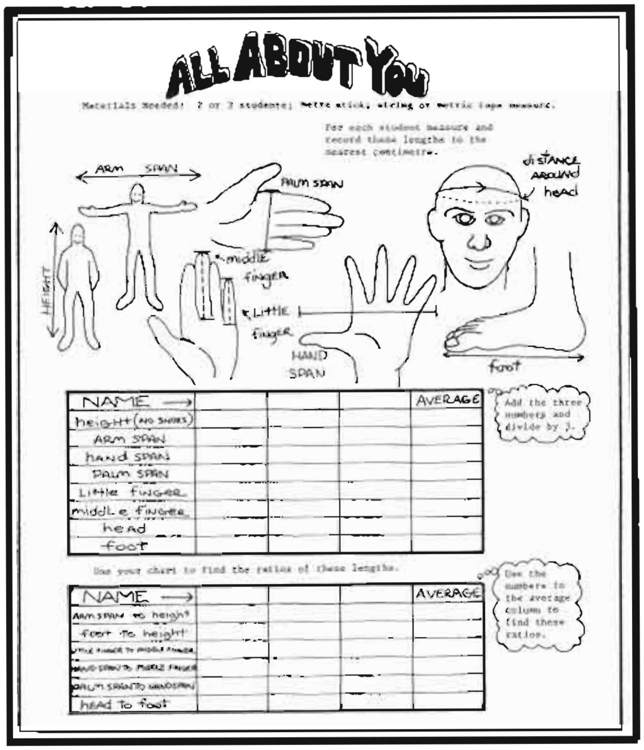
There are a number of ways to make a scale drawing of a field. Some methods use expensive pieces of equipment to do this accurately, but it is possible to make a good scale drawing using equipment from the classroom.

Equipment: Flat table or board placed on top of an inverted wastebasket
Ruler
Tape
*Alidade
Large sheet of drawing paper

*An alidade is a straightedge with sights and can be made with a ruler and two nails.

1. The students should familiarize themselves with the region before beginning the scale drawing. Landmarks, especially those that indicate the shade of the region, should be located. The landmarks could be listed or a rough sketch of the region drawn with each landmark labeled. Markers are needed at the corners of the field if natural landmarks do not occur.

4. Place point P over stake P. Use the alidade to line up point Q on the paper with stake Q (you may have to turn the table slightly). The table must remain in this position as you sight each landmark from point P.

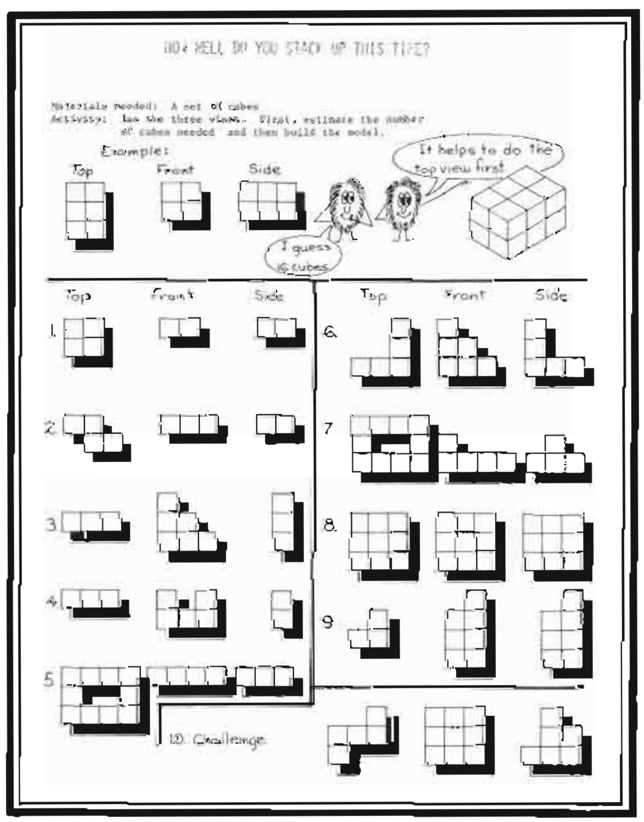


The students develop an awareness of their body and how it can be described and compared using mathematics.

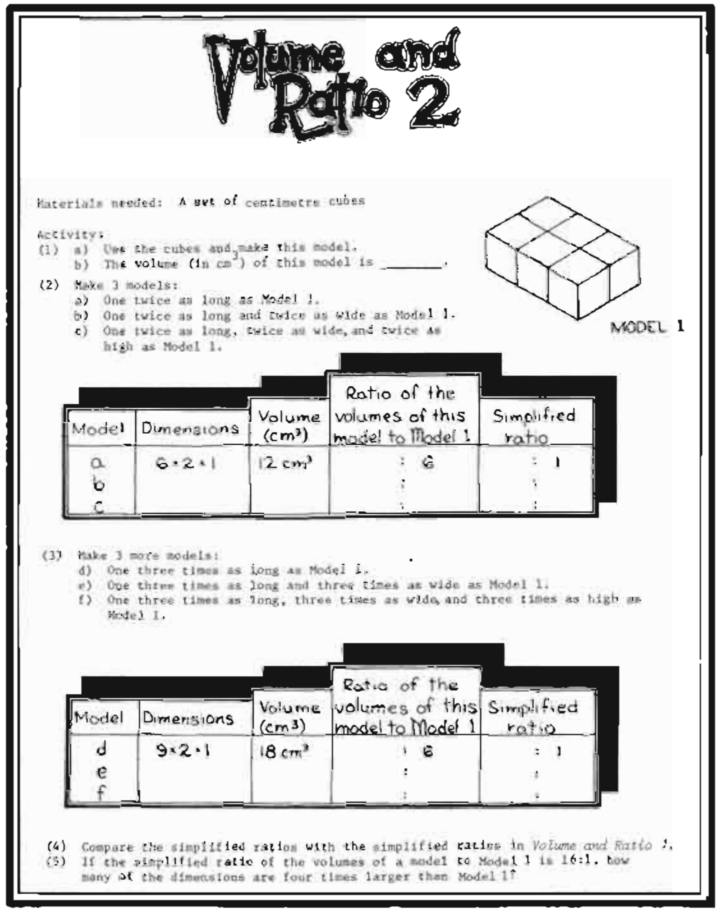
II. The Cube as a Lab Manipulative

Cubes are versatile, "hands-on" objects. They can be used to bridge the gap between abstract thinking, scale models and physical reality.

The students look at the abstract two-dimensional drawings of a solid and then construct the corresponding three-dimensional figures using cubes.



Students build physical models to clarify the problem and help them understand the concepts of volume and ratio.



Volume and Ratio 2

Materials needed: A set of centimetre cubes

Activity:

(1) a) Use the cubes and make this model.
b) The volume (in cm^3) of this model is _______.

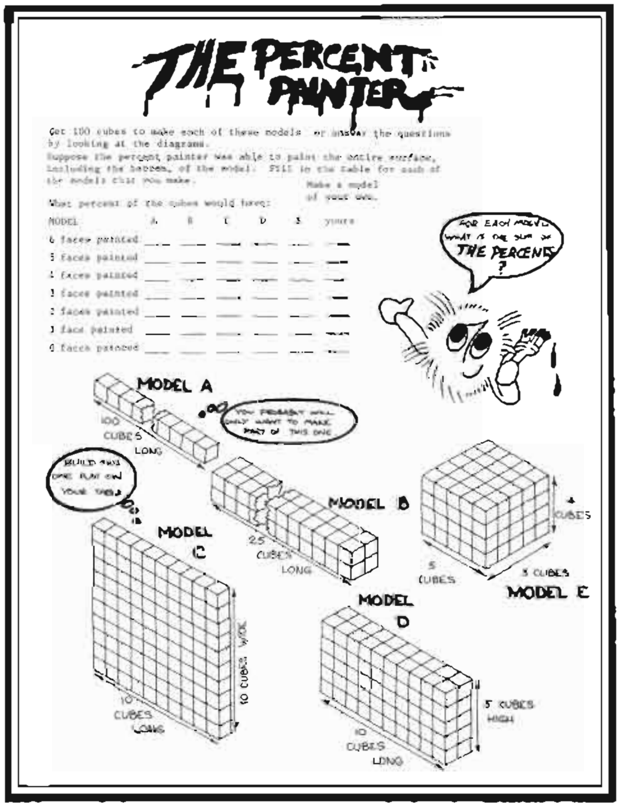
(2) Make 3 models:
a) One twice as long as Model 1.
b) One twice as long and twice as wide as Model 1.
c) One twice as long, twice as wide, and twice as high as Model 1.

Model	Dimensions	Volume (cm^3)	Ratio of the volumes of this model to Model 1	Simplified ratio
a	6 × 2 × 1	12 cm^3	: 6	: 1
b			:	:
c			:	:

(3) Make 3 more models:
d) One three times as long as Model 1.
e) One three times as long and three times as wide as Model 1.
f) One three times as long, three times as wide, and three times as high as Model 1.

Model	Dimensions	Volume (cm^3)	Ratio of the volumes of this model to Model 1	Simplified ratio
d	9 × 2 × 1	18 cm^3	: 6	: 1
e			:	:
f			:	:

(4) Compare the simplified ratios with the simplified ratios in *Volume and Ratio 1*.
(5) If the simplified ratio of the volumes of a model to Model 1 is 16:1, how many of the dimensions are four times larger than Model 1?



The Percent Painter

Get 100 cubes to make each of these models or answer the questions by looking at the diagrams.
Suppose the percent painter was able to paint the entire surface, including the bottom, of the model. Fill in the table for each of the models that you make.

Make a model of your own.

What percent of the cubes would have:

MODEL	A	B	C	D	E	yours
6 faces painted						
5 faces painted						
4 faces painted						
3 faces painted						
2 faces painted						
1 face painted						
0 faces painted						

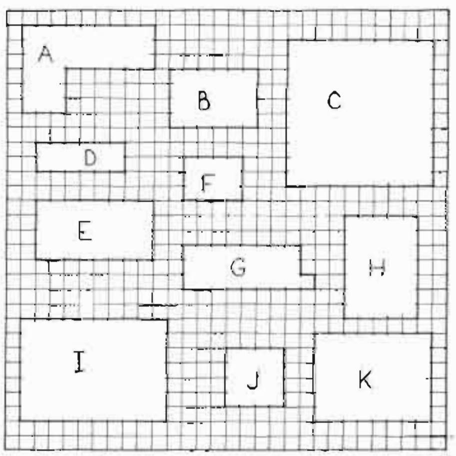
To fill in the table, the students can make each model or look at the diagrams, depending on their ability to abstract the situation.

III. Grid Activities

Grids and grid paper are used as two-dimensional models that pictorially represent many concepts in ratio, percent and scaling. Construction activities that involve making models, scale drawings or geometric figures often utilize grid, isometric paper or squared paper.

This activity features several puzzles, such as fitting together all the pentominoes to cover a given area, and a game with pentominoes. Puzzles and games entertain yet provide important practice with shapes and ideas.

LAKE & ISLAND BOARD



This is a scale drawing of a "Lake and Island" board. To construct the board cut a 30 cm square from colored railroad board. Enlarge the pattern 2 to 1. Clip the enlargement to the board and perforate the corners of each island with a compass point. Cut the islands from poster paper of contrasting color and use the compass marks to help glue the islands to the board. For durability laminate.

Students can determine the size of each island by using centimetre cubes or a transparent centimetre grid.

I. Use Island C as the reference set. What percent of C is each of the following? Estimate first.

Island A ____	Island E ____	Island I ____
Island B ____	Island F ____	Island J ____
Island C ____	Island G ____	Island K ____
Island D ____	Island H ____	

Change the reference set. If Island J is the reference set, what percent of J is Island K? E? F? D? A?

II. Use the entire board as the reference set.
(a) What percent of the board is water, islands?
(b) If a woman parachutes from a plane over the area what are her chances of landing on Island A?

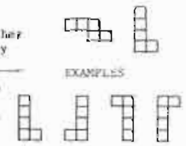
THE PERPLEXING PENTOMINOES

Materials needed: Five squares, 3 centimetres on a side, and centimetre grid paper or five 1-inch tiles and inch grid paper.

Activity:

1) A pentomino is a pattern made by joining 5 squares together so that each shares a common side with another. How many different pentominoes do you think there are? ________

2) Take the 5 squares and make all the pentominoes that you can. Copy each pentomino pattern on the grid paper and cut out the shape. If one of the patterns can be turned or flipped to exactly fit another one, the two patterns are the same pentomino.



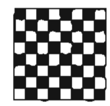
These four patterns are the same pentomino.

3) Check with your teacher to see if you have found all the pentominoes.

4) Try to arrange the pentominoes so that they make the rectangle. Do not overlap the pieces. There are more than 2000 ways to do this!

5) Play a game using the pentominoes.

Needed: 2 players
Game mat is an 8 by 8 square constructed out of the grid paper with alternate squares shaded.



a) Players alternate picking pentomino pieces until all the pieces have been selected.

b) Each player in turn then places a pentomino on the mat. Play continues until it is impossible for a player to place on that mat a pentomino that doesn't overlap another pentomino or lie completely on the mat.

c) The winner is the last person to successfully place a pentomino on the mat.

The Lake and Island Board can be constructed for use with a number of lab activity cards. Here the students use the board to do percent exercises.

LABORATORY ACTIVITIES FOUND IN CLASSROOM MATERIALS

RATIO:

Getting Started

ALL ABOUT YOU	DETERMINING RATIOS FROM STUDENT DATA
M & M'S	DETERMINING RATIOS

Rate

MATH IS A FOUR-LETTER WORD	DETERMINING RATES
SPY ON THE EYE	DETERMINING RATES
LET YOUR FINGERS DO THE WALKING	DETERMINING RATES
FIX THAT LEAK	DETERMINING RATES
AS THE RECORD TURNS	DETERMINING RATES
MY HEART THROBS FOR YOU	USING RATE OF HEARTBEAT TO DETERMINE PHYSICAL FITNESS
STEP RIGHT UP	USING RATE OF HEARTBEAT TO DETERMINE PHYSICAL FITNESS
I BELIEVE IN MUSIC	DETERMINING RATES

Equivalent

RATIOS AND CUBES 1	CONCEPT, GENERATING
RATIOS AND CUBES 2	CONCEPT, GENERATING
I'D WALK A MILE	DETERMINING AND COMPARING
POPPIN' WHEELIES IN A RING	SIMPLIFYING
SURFACE AREA AND RATIOS 1	SIMPLIFYING

TEACHING EMPHASES	LABORATORY APPROACHES
SURFACE AREA AND RATIOS 2	SIMPLIFYING
VOLUME AND RATIO 1	SIMPLIFYING
VOLUME AND RATIO 2	SIMPLIFYING
CUBISM	SIMPLIFYING
Ratio as a Real Number	
A VERY SPECIAL RATIO	APPROXIMATING
PI'S THE LIMIT	APPROXIMATING
BUFFON'S PI	APPROXIMATING
CLOSER & CLOSER	RATIO AS A REAL NUMBER
PROPORTION:	
Getting Started	
AS THE SQUARE TURNS	RECOGNIZING PROPORTIONS
THE BOB AND RAY SHOW	GEOMETRIC MODEL
THE SOLVIT MACHINE--A DESK TOP PROPORTION CALCULATOR	CROSS PRODUCTS METHOD
Application	
ONE GOOD TURN DESERVES ANOTHER	USING PROPORTIONS TO DETERMINE DISTANCES
THAT'S THE WAY THE OLD BALL BOUNCES	USING PROPORTIONS TO FIND HEIGHTS
GET IN GEAR	USING PROPORTIONS WITH GEARS
I'M BEAT! HOW ABOUT YOU?	USING PROPORTIONS WITH GEARS INVERSE VARIATION

TEACHING EMPHASES	LABORATORY APPROACHES
SCALING:	
Getting Started	
YOUR MOD BOD	USING SCALES TO REPRESENT HEIGHTS
ELEMENTARY, MY DEAR WATSON	MOTIVATION USE OF A SCALE MODEL
BEANS, BEANS	USING A SCALE TO MAKE PREDICTIONS
HAVE YOU GOT SPLIT ENDS?	USING A MICROSCOPE TO ENLARGE
Making a Scale Drawing	
GEOBOARD DESIGNS	COPYING DESIGNS
BE CREATIVE THIS CHRISTMAS	ENLARGING WITH GRIDS
PACE OUT THE SPACE	REDUCING WITH A GRID OR RULER
ARCHIE TEXS' RULER	ENLARGING WITH A RULER
PROJECTING THROUGH A PINHOLE	DEMONSTRATION OF PERSPECTIVE
A SNAPPY SOLUTION TO SCALE DRAWINGS	ENLARGING/REDUCING WITH RUBBER BANDS
STAKE YOUR CLAIM	REDUCING WITH AN INSTRUMENT FINDING LENGTHS USING AN ALIDADE
ANOTHER STAKE OUT	REDUCING WITH AN INSTRUMENT FINDING ANGLES USING A TRANSIT
Supplementary Ideas in Scaling	
MAKE A DIPSTICK	USING A SCALE TO DETERMINE DEPTH

TEACHING EMPHASES	LABORATORY APPROACHES
THE PERPLEXING PENTOMINOES	WORKING WITH SHAPES
HOW WELL DO YOU STACK UP?	DRAWING SKETCHES OF 3-D MODELS
HOW WELL DO YOU STACK UP THIS TIME?	BUILDING 3-D MODELS FROM SKETCHES
3 FACES YOU SAW	MAKING SCALE DRAWINGS OF 3-D MODELS
3 FACES YOU HAVE SEEN	MAKING SCALE DRAWINGS OF 3-D MODELS
CAREFULLY CONSTRUCTED CARTONS	CONSTRUCTING 3-D MODELS
BUILDING A SKYSCRAPER	CONSTRUCTING 3-D MODELS
BUILDING SEVERAL SKYSCRAPERS	CONSTRUCTING 3-D MODELS
A SCALE MODEL OF THE SOLAR SYSTEM	MAKING A SCALE MODEL
HOW HIGH THE MOON	MAKING A SCALE MODEL
Maps	
THE GREAT LAKES	USING A SCALE DRAWING TO FIND DISTANCES
PERCENT:	
Percent Sense	
STICKING TOGETHER WITH PERCENTS	REFERENCE SET OF 100* GRID MODEL
YOUR BODY PERCENTS	REFERENCE SET OF 100* NUMBER LINE MODEL
PERCENT WITH CUBES	REFERENCE SET OF 100* SET MODEL
THE PERCENT PAINTER	REFERENCE SET OF 100 SET MODEL
HUNDREDS BOARD PERCENT	REFERENCE SET OF 100 SET MODEL

*Indicates percents greater than 100% are used on the page.

PERCENT WITH RODS & SQUARES - I	REFERENCE SET OF 100 GRID MODEL
PERCENT WITH RODS & METRES - I	REFERENCE SET OF 100* NUMBER LINE MODEL
ACTIVITY CARDS - NUMBER LINE	NUMBER LINE CONCEPTS
STRINGING ALONG WITH PERCENTS	REFERENCE SET OF 100* NUMBER LINE MODEL
PERCENTS OF AN ORANGE ROD	REFERENCE SET OF 100* NUMBER LINE MODEL
As a Fraction/Decimal	
BE A REAL CUTUP	AS A FRACTION/DECIMAL* GRID MODEL
PERCENTS WITH RODS & SQUARES - II	AS A FRACTION/DECIMAL* GRID MODEL
PERCENTS WITH RODS & SQUARES - III	AS A FRACTION* GRID MODEL
PERCENT WITH RODS & METRES - II	AS A FRACTION/DECIMAL* NUMBER LINE MODEL
PERCENT WITH RODS METRES - III	AS A FRACTION/DECIMAL NUMBER LINE MODEL
Solving Percent Problems	
LAKE & ISLAND BOARD	USING A MODEL

*Indicates percents greater than 100% are used on the page.

RATIO, PROPORTION, AND SCALING

Placement Guide for Tabbed Divisors

RATIO

Ratio is one of the most useful ideas in everyday mathematics. Here are a few examples of the use of ratio in newspapers and magazines.

TEL AVIV

Israel has the highest ratio of physicians. There is one physician to every 420 people.

In the last year of the Civil War the North had 4 soldiers for every soldier from the South.

In 1973 1 out of every 25 homes in Eugene, Oregon was burglarized.

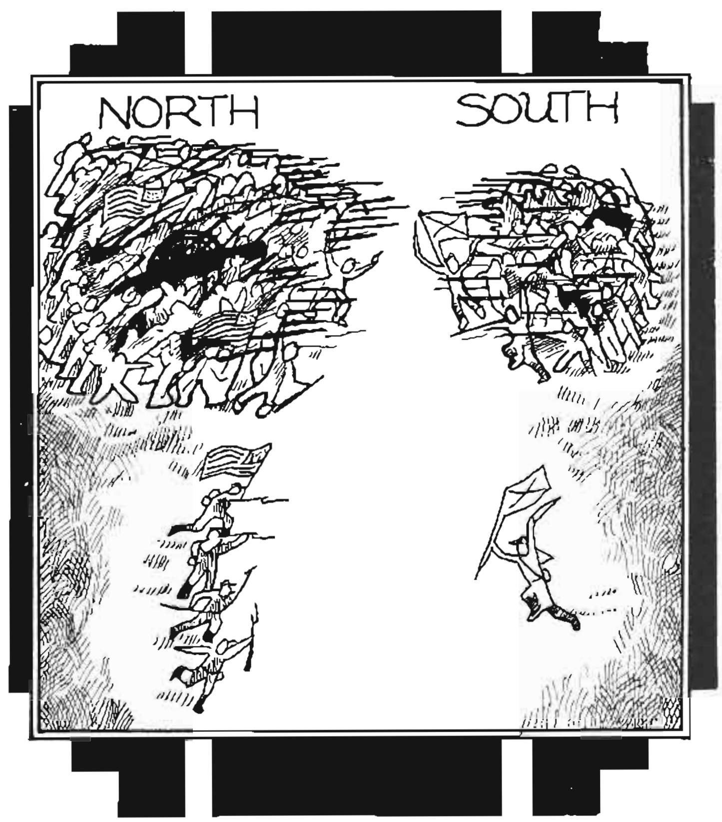
SPORTS

LONDON

Jack Nicklaus is a 1-4 favorite to capture the British Open which starts Wednesday at Carnoustie, Scotland.

DURHAM, NEW HAMPSHIRE VOTED 14 to 1 AGAINST PROPOSED OIL REFINERY

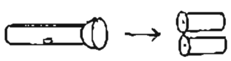
A ratio is an ordered pair of measures. The ratio of Northern soldiers to Southern soldiers in the last year of the Civil War was 4 to 1. This tells us that for every 4 soldiers from the North there was only 1 soldier from the South. From this ratio we know the relative size of the two sets, but we are not given the numbers of soldiers. This is the essence of the idea of ratio; it gives relative measures which can be used for comparisons.



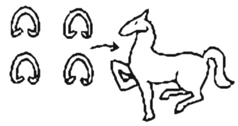
INTRODUCING YOUR CLASS TO RATIOS

Each of the pictures from the student page *Ratios by Picture II* in the section RATIO: Equivalent illustrates a ratio. For each ratio there is a corresponding list of pairs of numbers which are in the given ratio.

A



B



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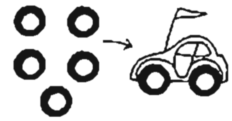
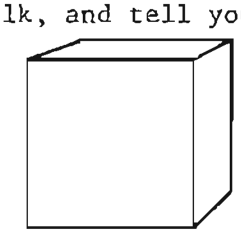


Flashlights to Batteries	Shoes to Horses	Tires to Cars
1 for every 2	4 for every 1	5 for every 1
2 for every 4	8 for every 2	10 for every 2
3 for every 6	12 for every 3	15 for every 3
4 for every 8	16 for every 4	20 for every 4
. .	. .	. .
. .	. .	. .
. .	. .	. .

With these lists of pairs of numbers the student can answer such questions as: If there were 6 cars, how many tires would there be? If there are 12 flashlights, how many batteries would there be?

Guessing Game

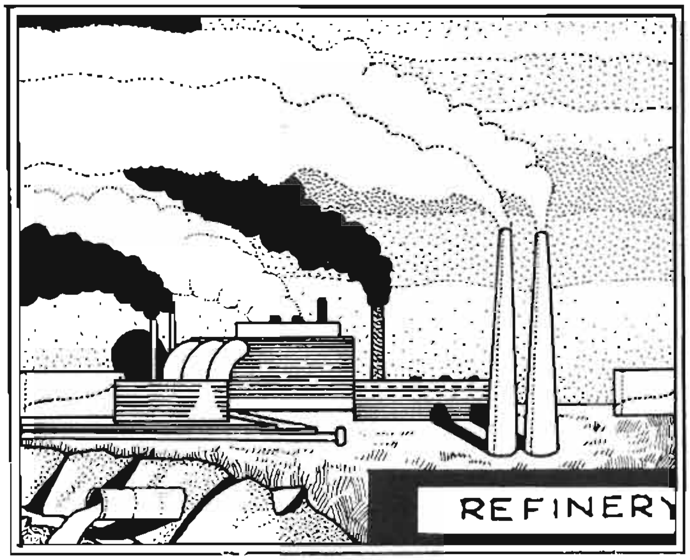
This game can help your students develop the idea of ratio. Place two kinds of objects in a box, for example, pencils and chalk, and tell your class the ratio. Suppose the ratio of pencils to chalk is 2 to 3. You may wish to explain this means there are 2 pencils for every 3 pieces of chalk. Now the class, or possibly teams from the class, try to guess the number of pencils and chalk. For example, 8 pencils and 12 pieces of chalk would be one possibility. Ten pieces of chalk would not be possible. What are the possibilities for the total number of pencils and pieces of chalk?



Pencils	Chalk	Total
2	3	5
4	6	10
6	9	15
8	12	20
.	.	.
.	.	.
.	.	.

SIMPLIFYING RATIOS

Ratios involving whole numbers are usually stated with the smallest possible whole numbers. In the example of the Durham, New Hampshire voters there were 14 against the refinery to every 1 for the refinery. The ratio is also 1190 to 85, since for every 1190 votes against the refinery there were 85 votes for the refinery. However, the smaller numbers, 14 to 1, are preferred. Conveying the relative size of large sets by small numbers is one of the advantages of the idea of ratio.

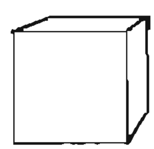


In the tables of ratios shown on the previous page, each pair of numbers is a multiple of the first pair. Therefore, dividing any pair of numbers in a table by a common factor will produce a smaller pair of numbers which are also in the table. When the two whole numbers in a ratio have no common factors other than 1, the ratio is said to be a simplified ratio.

Number Against	to Number For	
1190	85	Divide by 5
238	17	Divide by 17
14	1	Simplified Ratio

Activities for Simplifying Ratios

Play the Guessing Game described above by placing a number of pieces of chalk and pencils in a box. This time tell the students the number of each kind and ask them for the simplified ratio. Suppose, for example, there are 18 pencils and 30 pieces of chalk. When a ratio is given, have them check by listing its equivalent ratios.



Pencils	to Chalk	
3	5	Guess Simplified Ratio
6	10	
9	15	
12	20	
15	25	
18	30	Multiply numbers in the simplified ratio by 6.

There are some tables in the student text where the students complete the data and compute the corresponding ratios. Here are some examples.

	number	ratio	simplified ratio
Students that are left-handed			
Students that are right-handed			

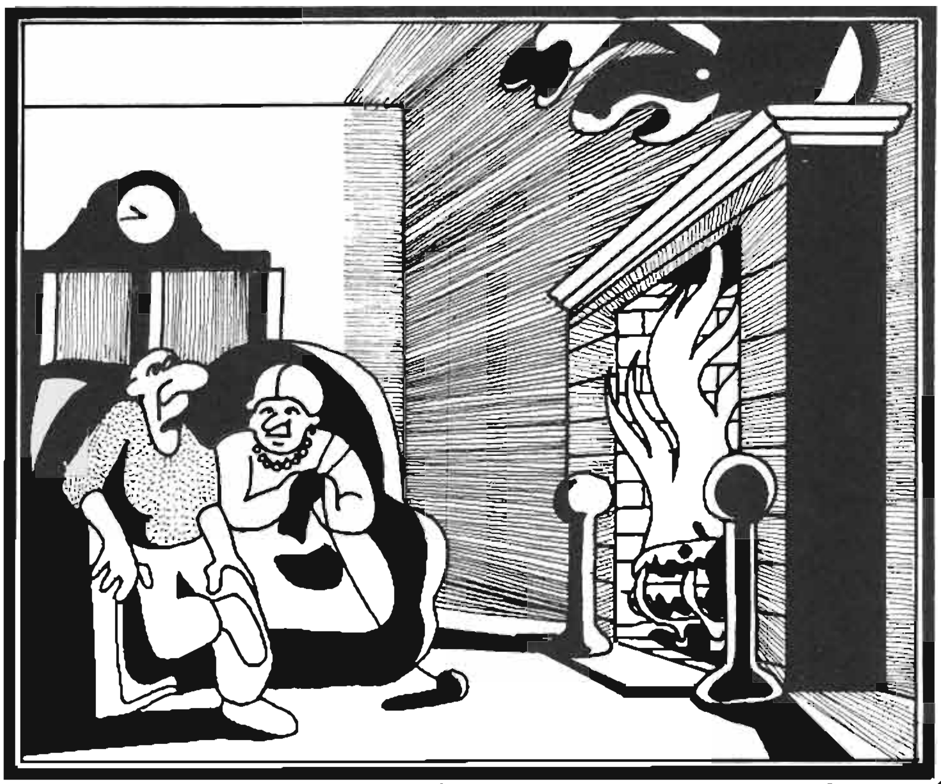
	number	ratio	simplified ratio
Students that ride a bike to school			
Students that do not ride a bike to school			

RATES ARE RATIOS

A rate is a special kind of a ratio in which the two sets being compared have different units of measure. Some texts call such a ratio a rate pair.

The two units in this cartoon are dollars and cords. The rate, \$95 per cord, is a ratio between number of dollars and number of cords and gives rise to the pairs of numbers shown in this table.

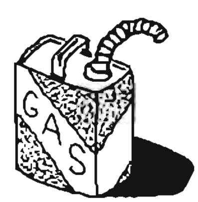
Dollars	to Cords
95	1
190	2
285	3
.	.
.	.
.	.



"You ask me what I see in the dancing flames? I see logs that cost ninety-five dollars a cord, that's what."

Suggested Activities

Start a bulletin board of rates. Have each student bring in an example of a particular rate. Rates, such as miles per hour, cost per hour, births per day, accidents per month, gallons per mile, etc., will be easy to find in newspapers and magazines.

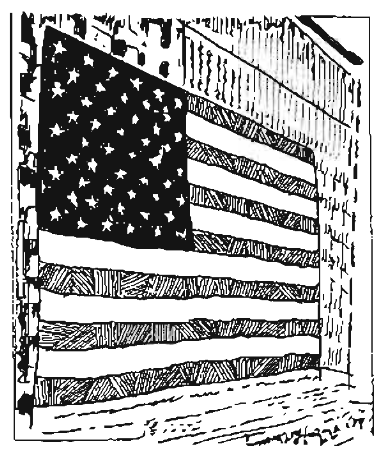


Gallons	Miles
1	22
2	44
3	66
4	88

The Guinness Book of World Records and almanacs are valuable sources of rates. Your students might be interested in finding out which countries have: the highest birth rate; the greatest income per person; the lowest infant mortality rate; the greatest density of people per square mile; and the highest death rate. There are speed records for people, animals, birds, planes and cars where the rates usually involve a unit of length and a unit of time.

USING REAL NUMBERS TO REPRESENT RATIOS

Sometimes the first number of a ratio is divided by the second number, and the resulting quotient is used to represent the ratio. For example, Federal law says that the ratio of the length to width of the official United States flag must be 1.9. This means that no matter what the size of the flag, the length divided by the width should be 1.9. The largest flag in the world is the Stars and Stripes displayed annually on the side of J. L. Hudson's store in Detroit, Michigan. Its length is 235 feet and its width is 104 feet. Does the number 1.9 represent the ratio of the length to the width of this flag?

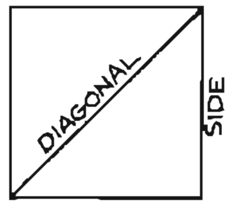


Students often have difficulty solving ratio problems when a single real number is used to represent a ratio. The same difficulty often occurs with rates. To eliminate this problem the classroom materials of this resource use the ratio notation (1.9:1) whenever practical.

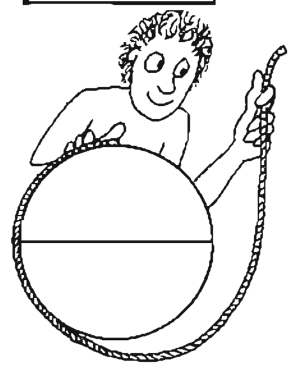
Suggested Student Activities

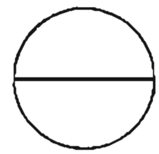
1. Measure the length and width of your school flag. Divide the length by the width and compare this number with the official ratio represented by 1.9.

2. Draw several different squares and compute the real number which represents the ratio of the length of a diagonal to the length of a side. Compute this number to one decimal place. Will this number always be the same? See the student page *A Special Ratio in all Squares* in the section RATIO: Ratio as a Real Number.



3. Draw several circles of different sizes and find the ratio of the circumference to the diameter. Computing the related real number to one decimal place, will this number always be the same? See student pages: *Pi's the Limit, A Very Special Ratio* and *Buffon's Pi* in the section RATIO: Ratio as a Real Number.

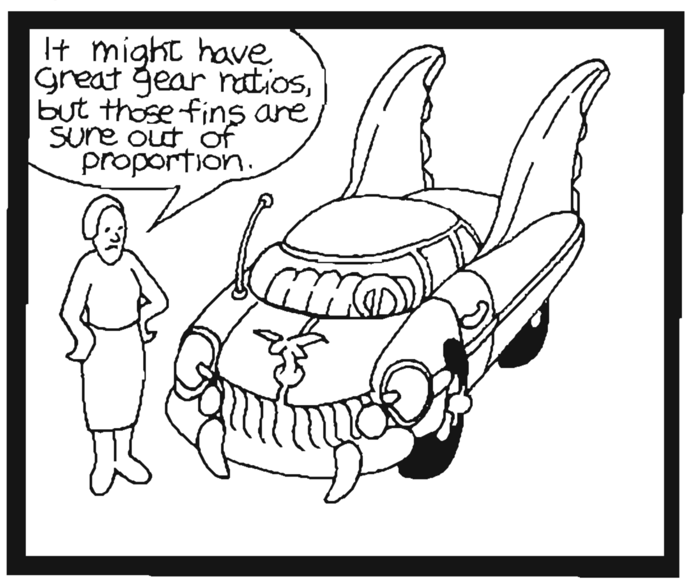




Terminology

The word "ratio" has never been a favorite outside the mathematics classroom. In newspapers, books and magazines the word "ratio" and notations for ratios are usually avoided by such expressions as: 4 to 3; 2 out of 5; 9 for every 1; etc.

Ratio is a Latin word for the verb reri (past participle, ratus) which means to think or estimate. In the Middle Ages it was commonly used to mean computation. To express the idea of ratio as we use it today, the medieval Latin writers used the word "proportio," and most mathematical works of the Renaissance times used the word "proportion." This language has by no means died out as can be seen in such expressions as: "Mix the sand and water in the proportion of 3 to 1;" or "Divide this in the proportion of 2 to 3." The use of the word proportion for ratio was never universal, and over the years ratio has become the accepted term in mathematics.



Notation

It is pedagogically sound to introduce students to a concept before bringing in notation. The examples and activities up to this point have not required the use of ratio notation, and yet the basic idea of ratio has been introduced and used. When a notation for ratios is used, two of the most common are

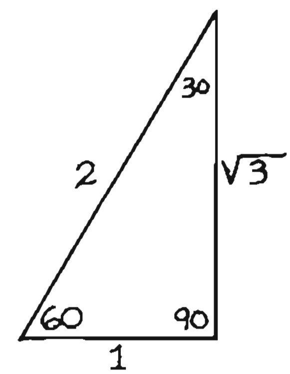
$$a:b \quad \text{and} \quad \frac{a}{b}$$

Both of these are read as: "the ratio of a to b." These notations can be avoided in the introductory stages of using ratios and perhaps should be avoided by merely writing out the expression "a to b." The fraction notation $\frac{a}{b}$ is especially confusing to students when it is used as a ratio to compare two disjoint sets. This will be examined further in the next section.

RELATIONSHIP OF RATIOS TO FRACTIONS

In some cases, the same situation may be described by either a fraction or a ratio. Although not all authors agree, this resource uses the terms "ratio" and "fraction" in the following way.

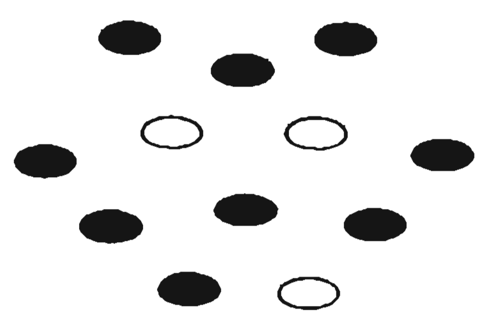
Ratio: A ratio is an ordered pair of measures. Any two positive real numbers may be used in a ratio. These numbers may be whole numbers, fractions, or irrational numbers. For example, in any 30-60-90 right triangle the ratio of the length of the hypotenuse to the length of the longest side is always 2 to $\sqrt{3}$.



Fraction: A fraction is a number represented by an ordered pair of integers, written $\frac{a}{b}$ for $b \neq 0$. Fractions are often used to describe part of a whole as shown by the diagram at the right.

$\frac{7}{10}$ of the rectangle is not shaded.

Fractions are also used to compare part of a set to the whole set. In the example shown here $\frac{3}{12}$ or $\frac{1}{4}$ of the balls are white. The fraction $\frac{1}{4}$ compares part of the set (a subset) to the whole set. A ratio is often, though not always, used to compare two disjoint sets. For example, the ratio of white balls to black balls is 3 to 9 (1 to 3 or $\frac{1}{3}$).



In this example both the fraction and the ratio tell the relative sizes of the two sets, but neither gives the actual size. The fraction $\frac{1}{4}$ compares a subset with a set, and the ratio 1 to 3 compares two disjoint sets. This example shows how the use of fractions to represent a ratio can be confusing. The ratio of white balls to black balls is $\frac{1}{3}$, and yet only $\frac{1}{4}$ of the balls are white.

Sometimes a ratio is used to compare a subset to a set. Using the 12 balls above, the ratio of white balls to the total number of balls is 1 to 4 or $\frac{1}{4}$. In this case, the idea of ratio is being used like a fraction, that is, part of a set is being compared to the whole set.

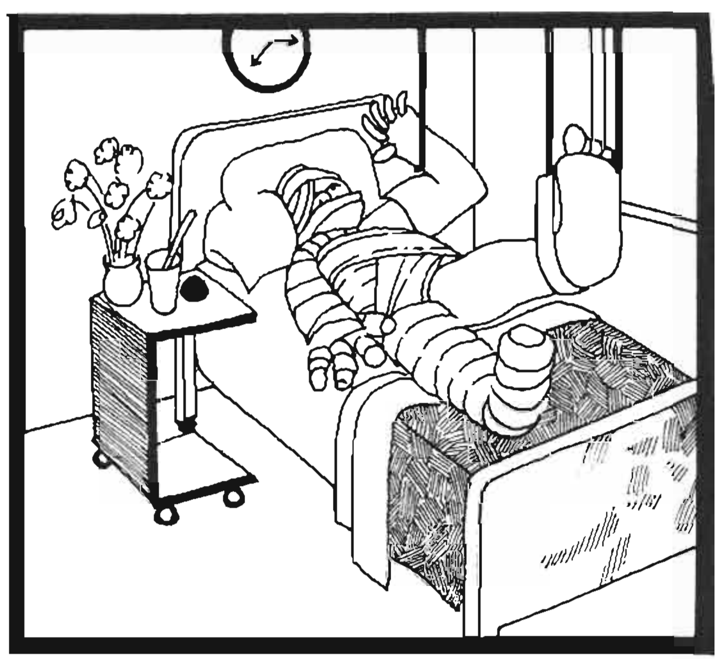
Here are four examples of the use of ratio. The first two of these examples compare disjoint sets; the third compares a subset and set. How would you interpret the fourth example?

a) Durham, New Hampshire voted 14 to 1 against a proposed oil refinery.

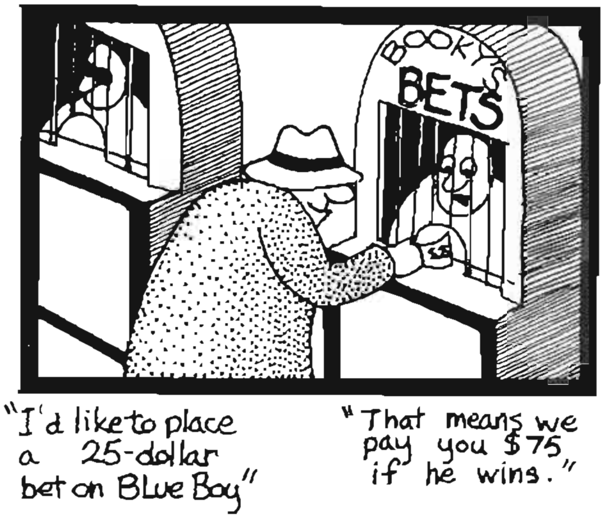
b) In the last year of the Civil War the North had 4 soldiers to every soldier from the South.

c) In 1973 1 out of every 25 homes in Eugene, Oregon was burglarized.

d) Israel has the highest ratio of physicians. There is 1 physician to every 420 people.



Ratio statements can often be replaced by fraction statements. To do this it is necessary to look at the sets being compared. Suppose, for example, that the ratio of hospital patients with type O blood to those without type O blood is 3 to 2. In this case, two disjoint sets are being compared. We can use fractions and say that $\frac{3}{5}$ of the patients have type O blood or that $\frac{2}{5}$ do not have type O blood.



Sometimes we wish to convert ratio statements given by odds into fraction statements. Suppose the odds on Blue Boy winning were 1 to 3. This means that for every dollar that is bet on Blue Boy the odds makers will put up 3 dollars. In terms of fractions Blue Boy has $\frac{1}{4}$ (not $\frac{1}{3}$) of a chance of winning.

Ratio is one of the most fundamental and important ideas in mathematics; yet it is not given much attention in many elementary or secondary classrooms. We could increase students' abilities to understand many word problems and applications involving rates and ratios if we would provide them with a better intuitive idea of ratio. Using tables and the "for every" phrase seems to make ratio much more understandable. Writing a rate such as 30 g/cc as 30 grams for every 1 cubic centimetre or 30 g for every 1 cc can help students start a table. Answers to rate problems can be seen as logical when they occur in such a table. Let's give students a chance to use their intuition and logic on ratio problems before they learn to solve them formally.

RATIO, PROPORTION, AND SCALING

Placement Guide for Tabbed Divisors

CONTENTS

RATIO: GETTING STARTED

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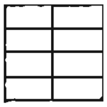
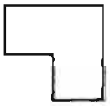
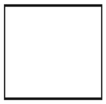
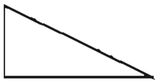
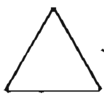
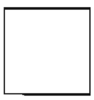
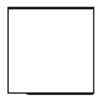
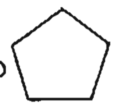
A.  to  is the same as  to [redacted]

B.  to  is the same as  to [redacted]

C.  to  is the same as  to [redacted]

D. A to B is the same as M to ______ is the same as ______ to T [redacted]

E.  to  is the same as  to  is the same as  to ______

F. 217 to 712 is the same as 564 to ______ [redacted]

G. 594 to 945 is the same as ______ to 954 [redacted]

H. 961 to 691 is the same as 861 to ______ [redacted]

I. 123 to 234 is the same as ______ to 789 [redacted]

J. ABC to XYZ is the same as DEF to ______ [redacted]

K. TEN to NET is the same as ______ to NAP [redacted]

L. TRAP to ART is the same as DULL to ______ [redacted]

COMPARISON 1

It is often useful to compare numbers or measurements. These are some phrases that are used for making comparisons.

$40 more than

$5 less than

10 inches shorter than

3 centimetres taller than

20 pounds fatter than

$1\frac{1}{2}$ kilograms heavier than

2 sizes smaller than

6 inches larger than

2 floors higher than

4 metres lower than

23 years older than

8 times as long as

Example #1:

Write the numbers 2000 and 20 on the chalkboard. How can we compare these two numbers?

I) 2000 > 20 (greater than)

II) 2000 is 1980 more than 20 (difference)

III) 2000 has two more digits (zeros) than 20

IV) 2000 is 100 times as much as 20 (times)

V) Be receptive to other student responses.

Example #2:

Write the measurements "100 cm" and "3 metres" on the chalkboard. How can these measurements be compared?

I) 3 metres > 100 centimetres (Note that 100 > 3, but we are not comparing the numbers.)

II) 3 metres is 200 centimetres longer than 100 centimetres.

III) 3 metres is a shorter way of writing 300 centimetres.

IV) 3 metres is 3 times as long as 100 centimetres.

V) Any other student answers?

PATTERNS FOR INTRODUCING RATIO

There are many patterns all around us. Some are difficult to see or understand. Other patterns seem obvious and are taken for granted.

Show the students various patterns. By making some easier, some harder, you can set the pace, reinforce responses and challenge the class. Have students continue the patterns.

a) 1, 2, 3, . . .

b) 1, 2, 4, 8, . . .

c) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

d) 1, 4, 9, 16, . . .

e) 1, 1, 2, 3, 5, 8, . . .

Continue the patterns by writing the next two pairs of numbers.

a)	b)	c)	d)	e)
(1, 2)	(1, 2)	(1, 1)	(0, 5)	$(\frac{1}{2}, 2)$
(2, 3)	(2, 4)	(2, 4)	(1, 4)	$(\frac{1}{3}, 3)$
(3, 4)	(3, 6)	(3, 9)	(2, 3)	$(\frac{1}{4}, 4)$
(4, 5)	(4, 8)	(4, 16)	(3, 2)	$(\frac{1}{5}, 5)$
______	______	______	______	______
______	______	______	______	______

Each set of number pairs is related by a constant (same) sum, difference, product, or quotient. Discuss these relationships carefully with the students.

Students should identify the pattern and write three more number pairs in each problem.

a)	b)	c)	d)
(10,5)	(3, 4)	(70, 10)	(2, 5)
(8, 3)	$(\frac{1}{2}, 24)$	(28, 4)	$(\frac{1}{4}, 6\frac{3}{4})$
(16, 11)	(12, 1)	(21, 3)	(4, 3)
______	______	______	______
______	______	______	______
______	______	______	______

Constant Comments

Look at the following sets of number pairs. The pairs in each set are related by a constant (same) sum (+), constant difference (-), constant product (x), or constant quotient (÷). How are the pairs related? Write three more number pairs which fit the pattern.

Example: The pattern is

(10, 2) 10÷2=5
(50,10) 50÷10=5
(25, 5) 25÷5=5
40,8
100,20
37,7

5 is the constant quotient

1
(6, 6)
(36, 1)
(2, 18)
______
______
______

2
(12, 0)
(2, 10)
(8, 4)
______
______
______

3
(18, 9)
(14, 7)
(6, 3)
______
______
______

4
(2, 2)
(7, 7)
($\frac{1}{4}$, $\frac{1}{4}$)
______
______
______

5
(6, 4)
(12, 2)
($\frac{1}{2}$, 48)
______
______
______

6
(8, 2)
($\frac{1}{2}$, $9\frac{1}{2}$)
(5, 5)
______
______
______

7
(15, 3)
(20, 8)
(13, 1)
______
______
______

8
(12, 2)
(18, 3)
(6, 1)
______
______
______

9
(4, 1)
(20, 5)
(12, 3)
______
______
______

10
(15, 65)
(79, 1)
(3, 77)
______
______
______

11
(3, 11)
(6, 14)
(4, 12)
______
______
______

12
(2, 3)
(6, 9)
(16, 24)
______
______
______

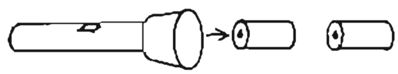
13
(25, 65)
(79, 11)
(13, 77)
______
______
______

14
(17, 34)
(48, 65)
(3, 20)
______
______
______

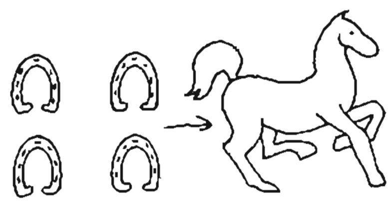
15
($\frac{1}{2}$, 96)
(6, 8)
(2, 24)
______
______
______

RATIOS BY PICTURE I

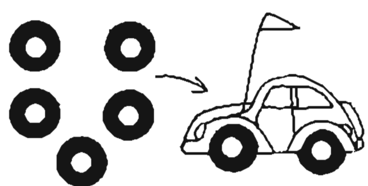
WRITE THE RATIO THAT IS SUGGESTED BY EACH OF THESE PICTURES.

A) 

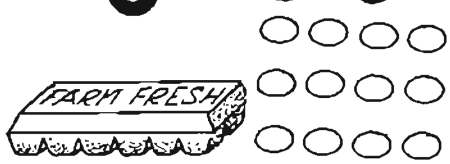
___ FLASHLIGHT FOR EVERY ___ BATTERIES OR 1:2

B) 

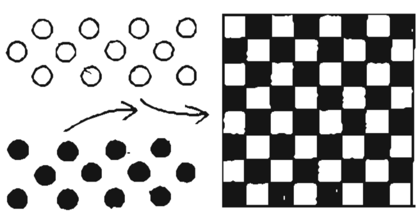
___ HORSESHOES FOR EVERY ___ HORSE OR ___:___

C) 

___ TIRES FOR EVERY ___ CAR OR ___:___

D) 

___ EGG CARTON FOR EVERY ___ EGGS OR ___:___

E) 

___ CHECKERS FOR EVERY ___ SQUARES ON A CHECKERBOARD OR ___:___

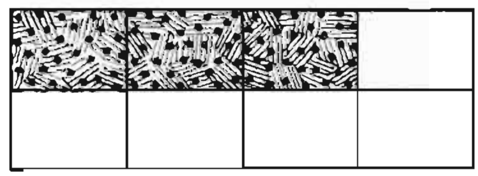
DRAW A DIAGRAM AND WRITE A RATIO FOR EACH OF THESE STATEMENTS.

F) 1 SINGLE DIP ICE CREAM CONE FOR EVERY 15¢
G) 6 CANDY BARS FOR 79¢
H) 3 TENNIS BALLS FOR 1 CAN
I) 25¢ FOR EVERY 3 PACKS OF GUM
J) 5 BATS FOR EVERY 9 BASEBALL PLAYERS

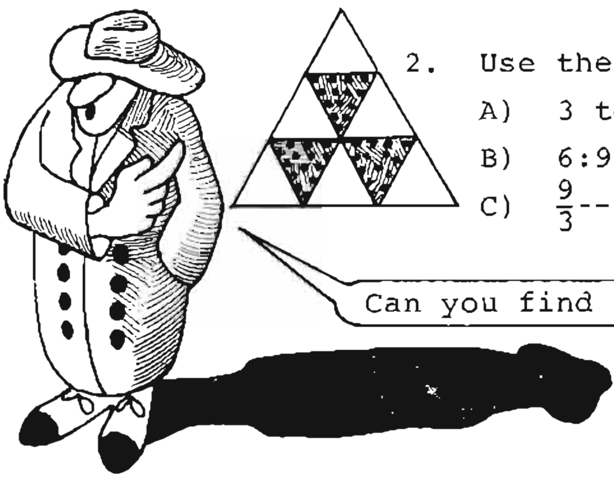
SHADY RATIOS

1. A) The ratio of the number of shaded rectangles to the number of unshaded rectangles is 3 to 5. This ratio may be written 3 to 5, 3:5, or $\frac{3}{5}$

   B) The ratio of shaded rectangles to small rectangles is 3 to 8, 3:8, or $\frac{3}{8}$.



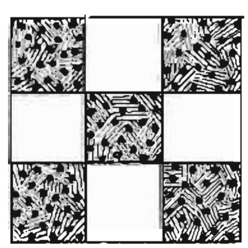
   C) The ratio of small rectangles to unshaded rectangles is 8 to 5, 8:5, or $\frac{8}{5}$.



2. Use the figure to describe a ratio of:
   A) 3 to 6--3 shaded triangles to ____________.
   B) 6:9--____________ to small triangles.
   C) $\frac{9}{3}$--Small triangles to ____________.

Can you find a ratio of 9:1?

3. Use this figure to describe a ratio of:
   A) 5 to 4 ____________________
   B) $\frac{4}{9}$ ____________________
   C) 9:5 ____________________
   D) 1 to 9 ____________________



The ratio is all small triangles to one <u>large</u> triangle.

IDEA FROM: *Activities with Ratio and Proportion*
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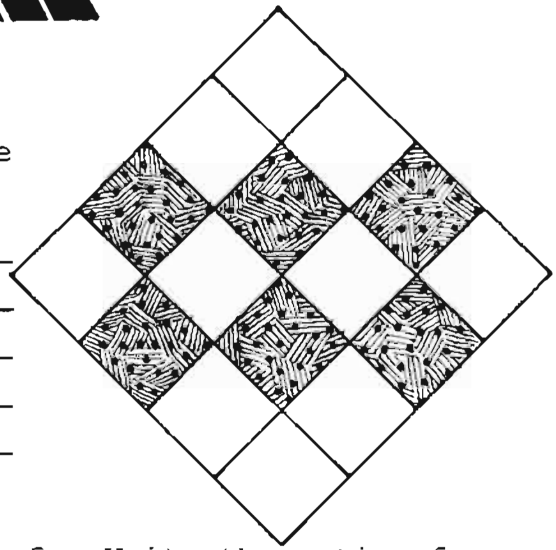
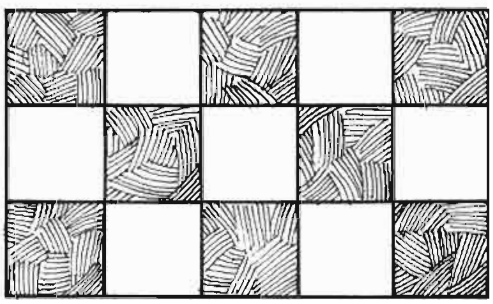
SHADY RATIOS (CONTINUED)

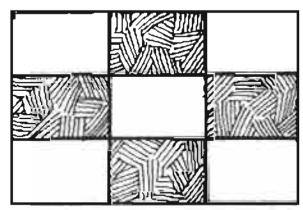


4. For the figure on the right write and describe <u>at least</u> 3 ratios.
   A) ______________________________
   B) ______________________________
   C) ______________________________
   D) ______________________________
   E) ______________________________

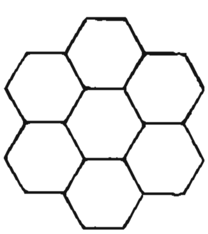


5. Write the ratio of:
   A) Number of shaded squares to number of unshaded squares.
      ____
   B) Number of shaded squares to number of small squares.
      ____
   C) Number of small squares to number of unshaded squares.
      ____

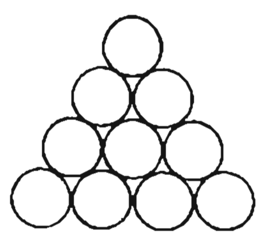
6. Write the ratio of:
   A) Shaded rectangles to unshaded rectangles.
      ____
   B) Small rectangles to shaded rectangles.
      ____
   C) Unshaded rectangles to shaded rectangles.
      ____



7. Shade to show a ratio of 6 shaded hexagons to 1 unshaded hexagon, 6:1. In how many different ways can this be done?



8. Shade the circles to show a ratio of 7 to 10. In how many different ways can this be done?



IDEA FROM: *Activities with Ratio and Proportion*
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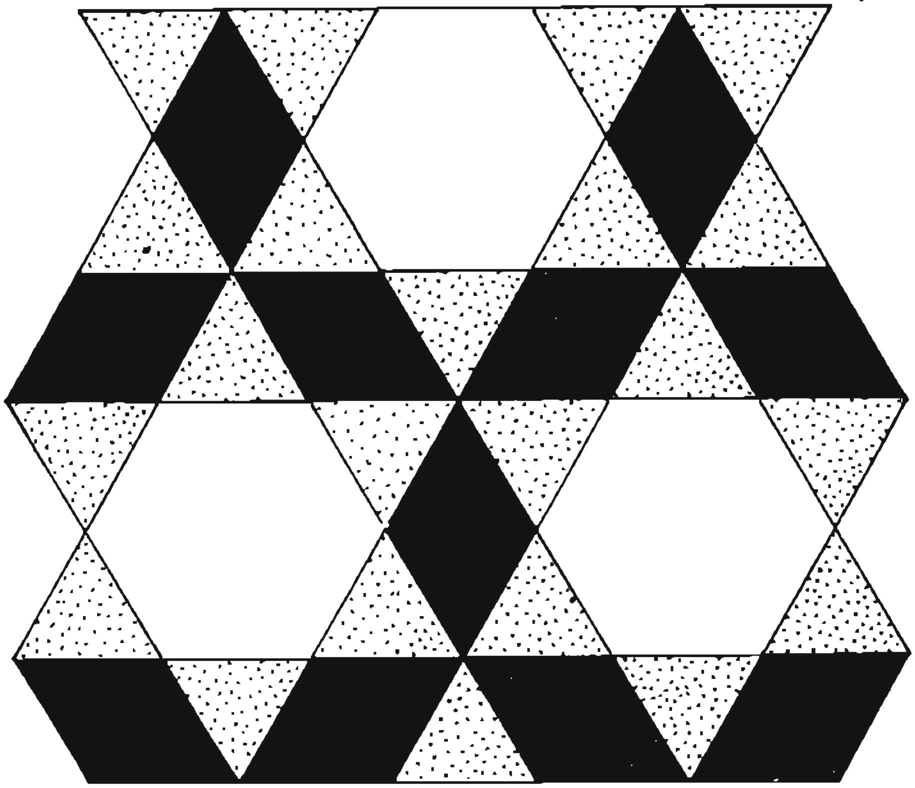
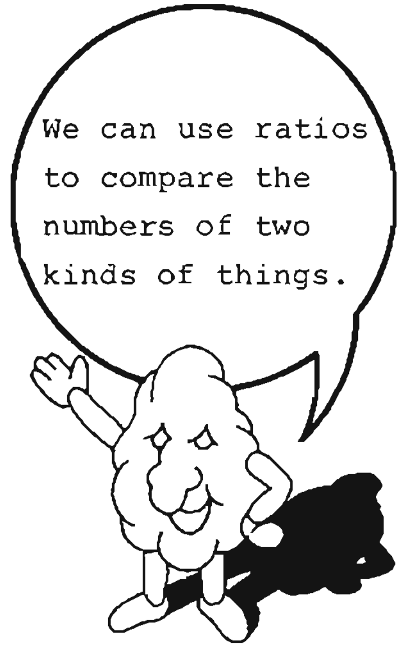
RECKONING RATIOS





The ratio of small triangles to rhombuses is 22 to 11.

We can write this as 22:11.

Determine these ratios:

1) Rhombuses to Small Triangles

_____ to _____ or _____:_____

2) Hexagons to Rhombuses

_____ to _____ or _____:_____

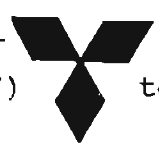
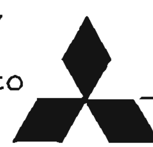
3) Small Triangles to Hexagons

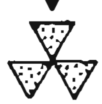
_____ to _____ or _____:_____

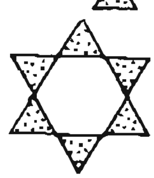
4)  to  ___ to ___ or ___:___

5)  to  ___ to ___ or ___:___

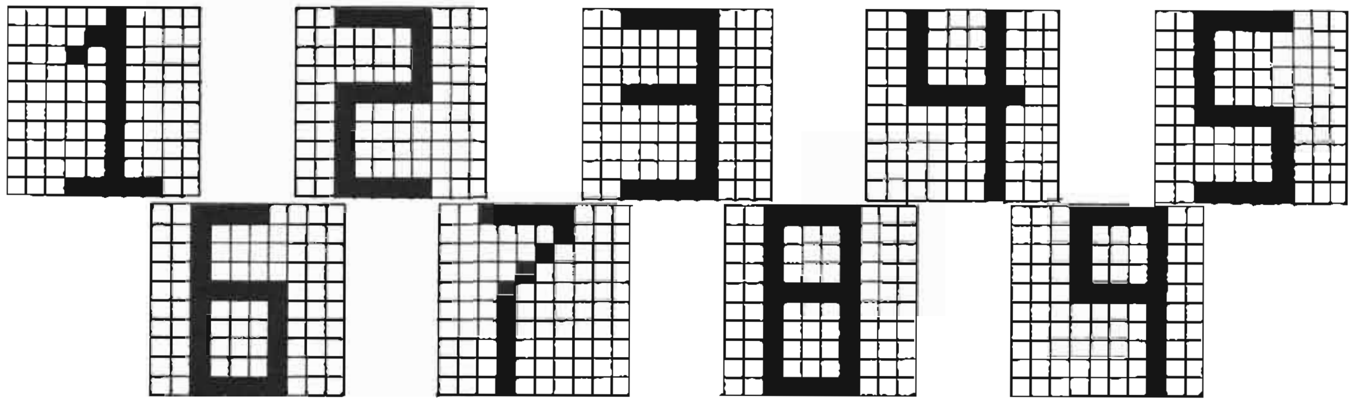
6)  to  ___ to ___ or ___:___

7)  to  ___ to ___ or ___:___

8)  to  ___ to ___ or ___:___

9)  and  to  ___ to ___ or ___:___

SHADY NUMERAL RATIOS



THESE GRIDS EACH HAVE 100 SMALL SQUARES.

WITHOUT COUNTING, GUESS:

A) WHICH NUMERAL SHADES THE MOST SQUARES? ________

B) WHICH NUMERAL SHADES THE LEAST SQUARES? ________

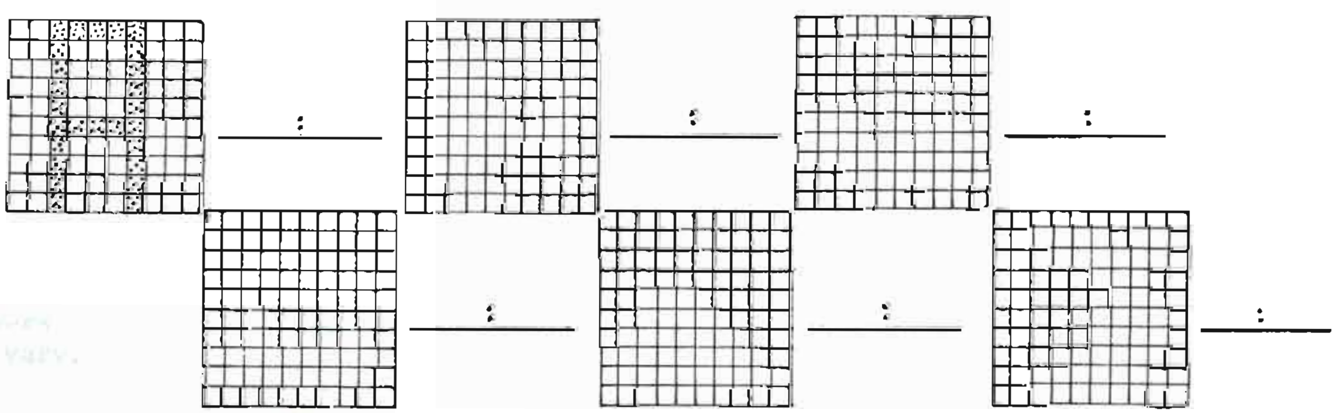
C) WHICH 3 NUMERALS SHADE THE SAME NUMBER OF SQUARES. ________

FOR EACH NUMERAL COUNT THE SHADED SQUARES AND WRITE THE RATIO OF THE NUMBER OF SHADED SQUARES TO THE TOTAL NUMBER OF SQUARES.

1 ______ 2 ______ 3 ______ 4 ______ 5 ______

6 ______ 7 ______ 8 ______ 9 ______

IN THESE GRIDS SHADE THE VOWELS OF THE ALPHABET.
THEN WRITE THE RATIO OF SHADED SQUARES TO UNSHADED SQUARES.



? WHICH LETTER OF THE ALPHABET DO YOU THINK WOULD SHADE THE MOST SQUARES?

IDEA FROM: *Activities with Ratio and Proportion*
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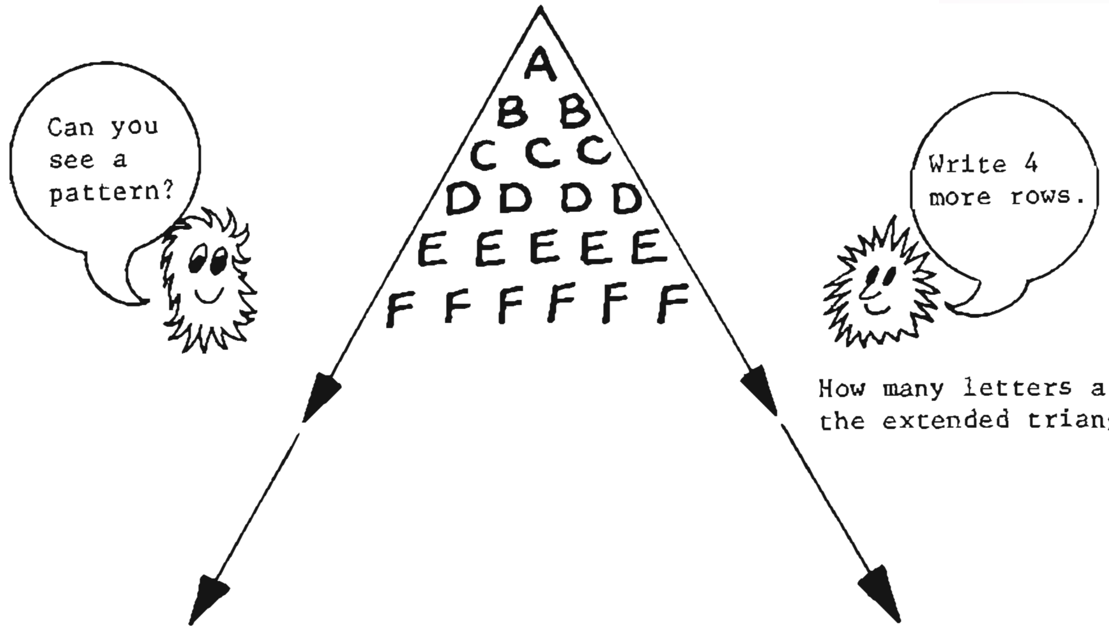
ROWS AND RATIOS



How many letters are in the extended triangle? ________

1. Write the ratio of the number of:

   a) A's to B's ____ b) E's to D's ____ c) C's to F's ____ d) J's to G's _____

   e) H's to I's ____ f) C's to all the letters _____ g) letters in the top 5 rows to letters in the bottom 5 rows ____ h) letters in the top row to letters in the bottom row. ____

2. a) Could the triangle be extended past 10 rows? _____
   b) What letter would be in the 24th row? _____
   c) How many of this letter would be in the 24th row? _____
   d) How many rows would be in the completed triangle? _____
   e) How many total letters would be in the completed triangle? Study the chart below. You might see a way to do it without adding each row. The total is _____.

NUMBER OF LETTERS PER ROW	1	2	3	4	5	…	20	21	23	23	24	25	26

3. If you had the completed triangle of letters write the ratio of the number of:
   a) different letters to total letters. ___________
   b) letters in the top 3 rows to letters in the bottom 3 rows. _____________
   c) letters in the 5th row to letters in the 15th row. ______________

☆ HAPPY RATIO ☆ ☆ DAY

In a group of 24 people there is about a 50 percent chance that 2 people in the group will have the same birthday (month and day, not necessarily year). This interesting fact can be the lead-in to using the birthdays of the students in your class to study ratios. (See Probability and Statistics for Everyman by Irving Adler.)

Record the birthdays of your students on the overhead or chalkboard. Be sure to include your own birthday. A chart or table will help to organize the data.

JAN.	MAY	SEPT
FEB.	JUNE	OCT.
MAR.	JULY	NOV.
APR	AUG.	DEC.

J	F	M	A	M	J	J	A	S	O	N	D

Older students are sometimes hesitant about revealing personal information. You may have to record the data with a show of hands or record a birthday with no reference to a name.

Questions such as these can be used.

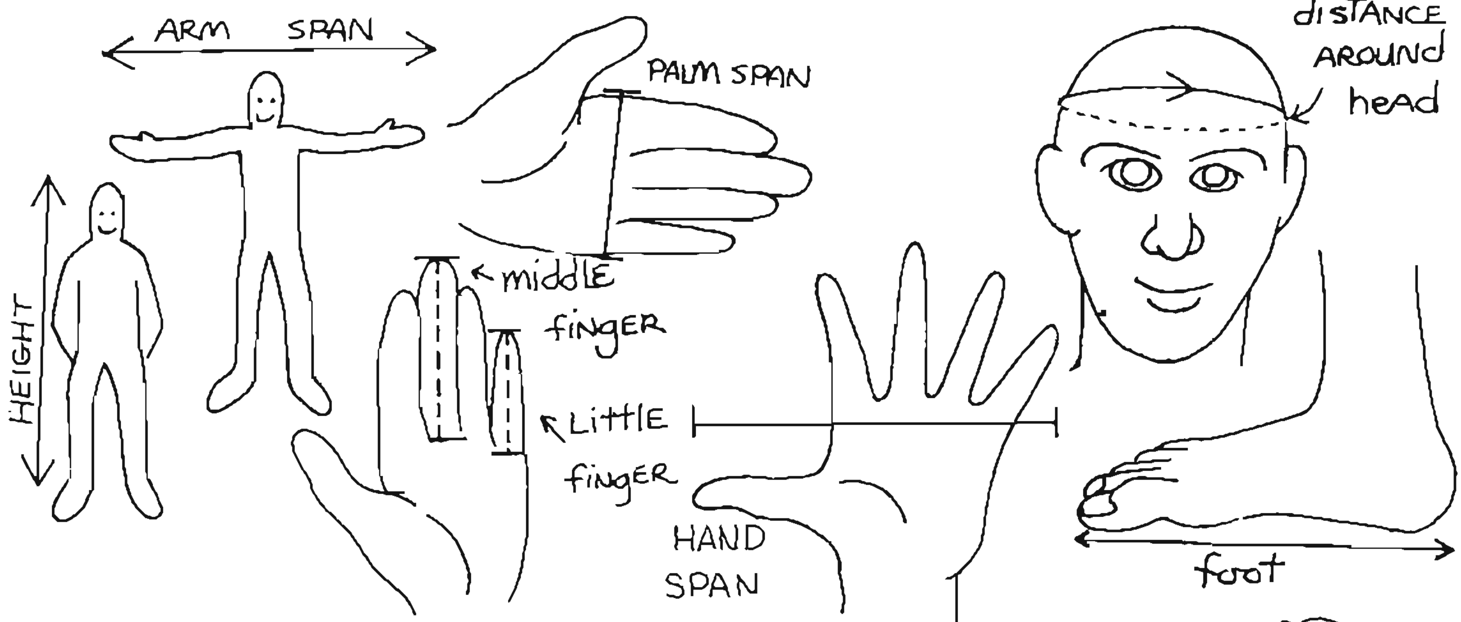
1) What is the total number of birthdays recorded?
2) Which month has the most birthdays? The fewest?
3) What is the ratio of birthdays in (May) to the total number of birthdays? (Fill in any of several months.)
4) What is the ratio of birthdays in (March) to birthdays in (May)?
5) What is the ratio of the number of birthdays in the first half of (March) to the birthdays in the second half of (May)?
6) What is the ratio of birthdays in the first 6 months to the birthdays in the second 6 months?
7) What is the ratio of birthdays that are holidays to the total number of birthdays?
8) What is the ratio of people having a birthday on the same day as another person to the total number of birthdays?

Note: This is a nice way to get information about your students so you can personalize your class and wish them a Happy Birthday.

ALL ABOUT YOU

Materials Needed: 2 or 3 students; metre stick; string or metric tape measure.

For each student measure and record these lengths to the nearest centimetre.



NAME →				AVERAGE
height (NO SHOES)				
ARM SPAN				
hand SPAN				
PALM SPAN				
Little finger				
middle finger				
head				
foot				

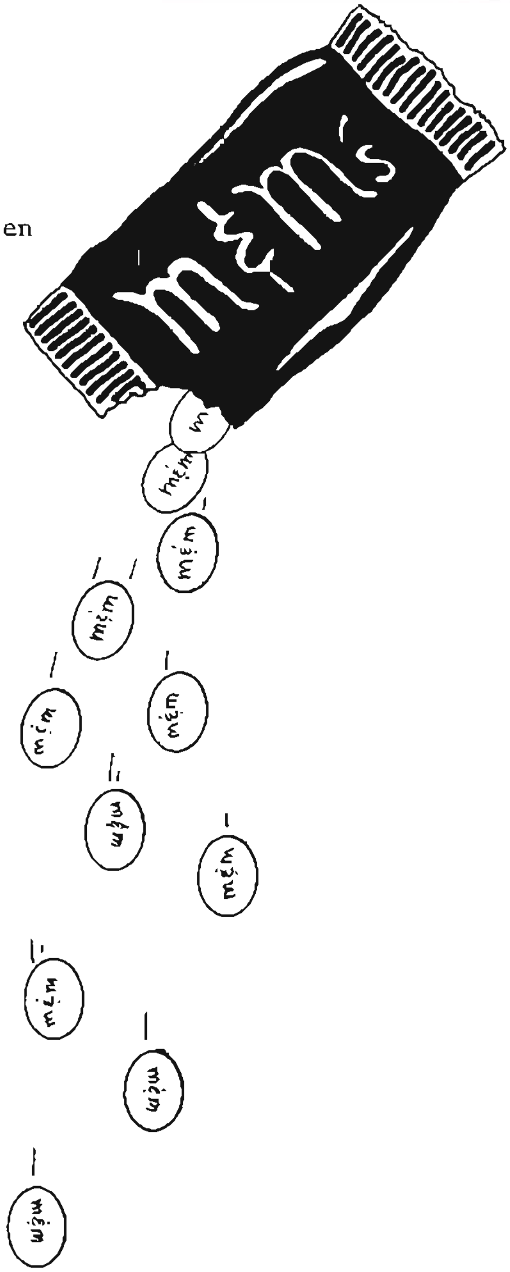
Add the three numbers and divide by 3.

Use your chart to find the ratios of these lengths.

NAME →				AVERAGE
ARM SPAN to height				
foot to height				
LITTLE FINGER TO MIDDLE FINGER				
HAND SPAN TO MIDDLE FINGER				
PALM SPAN TO HANDSPAN				
head to foot				

Use the numbers in the average column to find these ratios.

IDEA FROM: *Activities with Ratio and Proportion*



Get a package of M & M's from your teacher. Carefully open the package and put the candy on your table.

DON'T EAT ANY YET

How many M & M's are in your package? ________

How many different colors do you have? ________

COLOR	NUMBER OF M & M's
Brown (B)	________
Tan (T)	________
Red (R)	________
Orange (O)	________
Yellow (Y)	________
Green (G)	________

Use your numbers and write these ratios in fraction notation.

a) G to O _____

b) R to B _____

c) T to Y _____

d) Y to O _____

e) B to G _____

f) R to T _____

g) (R + Y) to total _____

h) (G + R + B) to (Y + O + T) _____

Write these ratios in fraction notation.

1) a) T to total _____
   b) B to total _____
   c) (T + B) to total _____

2) d) O to B _____
   e) R to Y _____
   f) (O + R) to (B + Y) _____

3) g) G to R _____
   h) G to Y _____
   i) G to (R + Y) _____

* NOW YOU CAN EAT THE CANDY

WHAT'S IN A RATIO?

The answers to some questions can not be determined from the information conveyed by ratios.

Interpreting Ratio Statements
Getting Started
RATIO

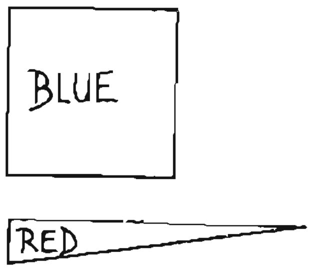
1) The ratio of the length of Lucy's hair to the length of Sharon's hair is 3 to 1.

A) __________ has the longer hair.

B) Lucy's hair is _____ times longer than Sharon's hair.

C) Sharon's hair is very short. T or F? _____

D) The ratio of the length of Sharon's hair to Lucy's hair is __________.



2) The ratio of the area of the red triangle to the area of the blue square is 1 to 2.

A) The color of the shape with the greater area is ________.

B) The area of the red triangle is _______ square centimetres.

C) If the area of the red triangle is 10 square centimetres then the area of the blue square is ________ square centimetres.

D) The ratio of the area of the blue square to the area of the red triangle is ________.

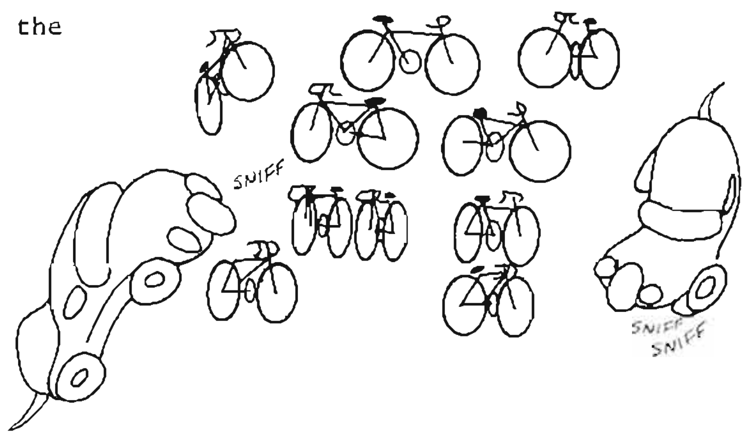
E) The ______________ has the greater perimeter.

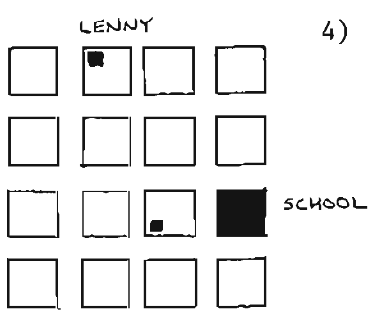
3) In this picture the ratio of the the number of bikes to the number of cars is 10 to 2.

A) There are ________ times as many bikes as cars.

B) A car is ________ times longer than a bike.

C) If there were 150 bikes there would be ________ cars. (Assume the ratio is the same as in the picture.





4) The ratio of the distance Lenny Lightfoot lives from school to the distance Sally Speedball lives from school is 4 to 1.

A) ______________ lives farther from school.

B) If they bike to school at the same speed, what can you say about the time each takes? ____________________

C) If they bike to school in the same amount of time what can you say about their speeds? ____________________

D) The ratio of Sally's height to Lenny's height is _____.

TYPE: Paper & Pencil/Discussion

IDEA FROM: *The School Mathematics Project*, Book D
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RATIO OF AGES

TEACHER DIRECTED ACTIVITY — Getting Started RATIO

Part I:

There are two brothers; Jon is 13 years old, Ron is only 1 year old. As the two grow older the ratio of their ages will change. Neatly organize a chart so the students can compare the ages. Do a few lines and suggest that they continue the pattern until Jon is 24 years old.

Jon	Ron	Ratio of Ages	Times As Old As
13	1	13:1	Jon is 13 times as old as Ron
14	2	14:2	Jon is 7 times as old as Ron
15	3	15:3	Jon is 5 times as old as Ron
16	4	16:4	Jon is 4 times as old as Ron
.	.	.	.
.	.	.	.
.	.	.	.
24	12	24:12	Jon is 2 times as old as Ron

Ask the students if they see any patterns in the chart.

a) How old will Jon and Ron be when Jon is 2 times as old as Ron?

b) When will Jon be $1\frac{1}{2}$ times as old as Ron?

c) If Jon is 100 years old, then he is _____ times as old as Ron.

d) If Jon is 500 years old, then he is _____ times as old as Ron.

e) When will Jon be 1 times older than Ron?

If students fail to see that the ratio of their ages approaches but doesn't equal 1 another example may be needed. Personalize the activity by selecting a student with a younger brother or sister.

IDEA FROM: *Synchro-Math/Experiences*

RATIO OF AGES (CONTINUED)

Part II:

The ratio of ages pattern can be investigated by working backwards in age. Suppose Lynn is 12 years old and Mark is 8 years old.

Lynn	Mark	Ratio of Ages	Times As Old As
12	8	12:8	$1\frac{1}{2}$ times as old as
11	7	11:7	
10	6	10:6	
9	5	9:5	
8	4	8:4	
7	3	7:3	
6	2	6:2	
5	1	5:1	
.	.		
.	.		
.	.		
4	0	nonsense	

Extend the table using months. If calculations become too burdensome use calculators.

60	12	60:12	5 times as old as
59	11	59:11	
58	10	58:10	
57	9	57:9	
56	8	56:8	7 times as old as
55	7	55:7	
54	6	54:6	9 times as old as
53	5	53:5	
52	4	52:4	13 times as old as
51	3	51:3	17 times as old as
50	2	50:2	25 times as old as
49	1	49:1	49 times as old as

Change Lynn's and Mark's ages to days and continue the countdown. When will Lynn be 100 times as old as Mark? . . . 1000 times as old as Mark? . . . 10,000 times? 1 million times? (Try hours and minutes.)

IDEA FROM: *Synchro-Math/Experiences*

RATIO, PROPORTION, AND SCALING

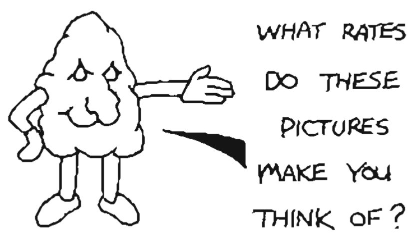
Placement Guide for Tabbed Divisors

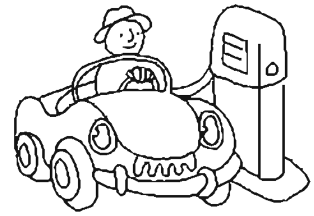
CONTENTS

RATIO: RATE

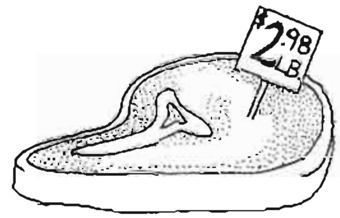
RATES ARE RATIOS

A rate is a special kind of ratio. With the rate the two measures being compared have different units, and the units cannot be converted to one another. One common rate is the rate of speed, 55 miles per hour. This means the ratio of miles traveled to the number of hours spent traveling is 55 miles to 1 hour, or 55 mph.

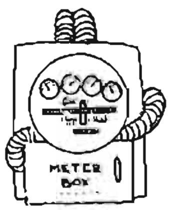




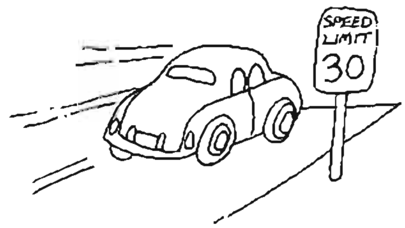
RATE ______________



RATE ______________



RATE ______________



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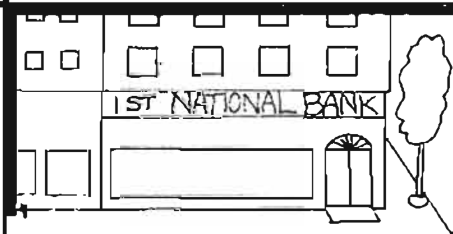

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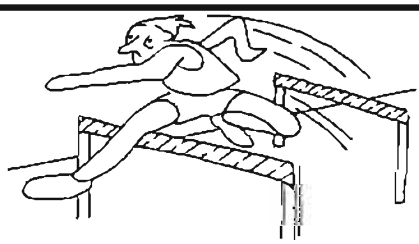
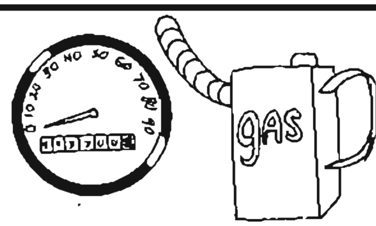
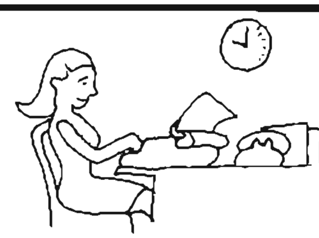


RATE ______________

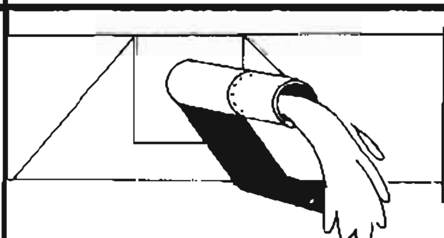


RATE ______________

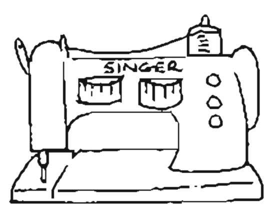


RATE ______________

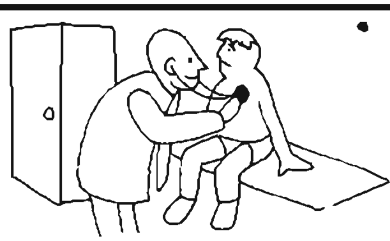
RATE ______________



RATE ______________



RATE ______________



RATE ______________

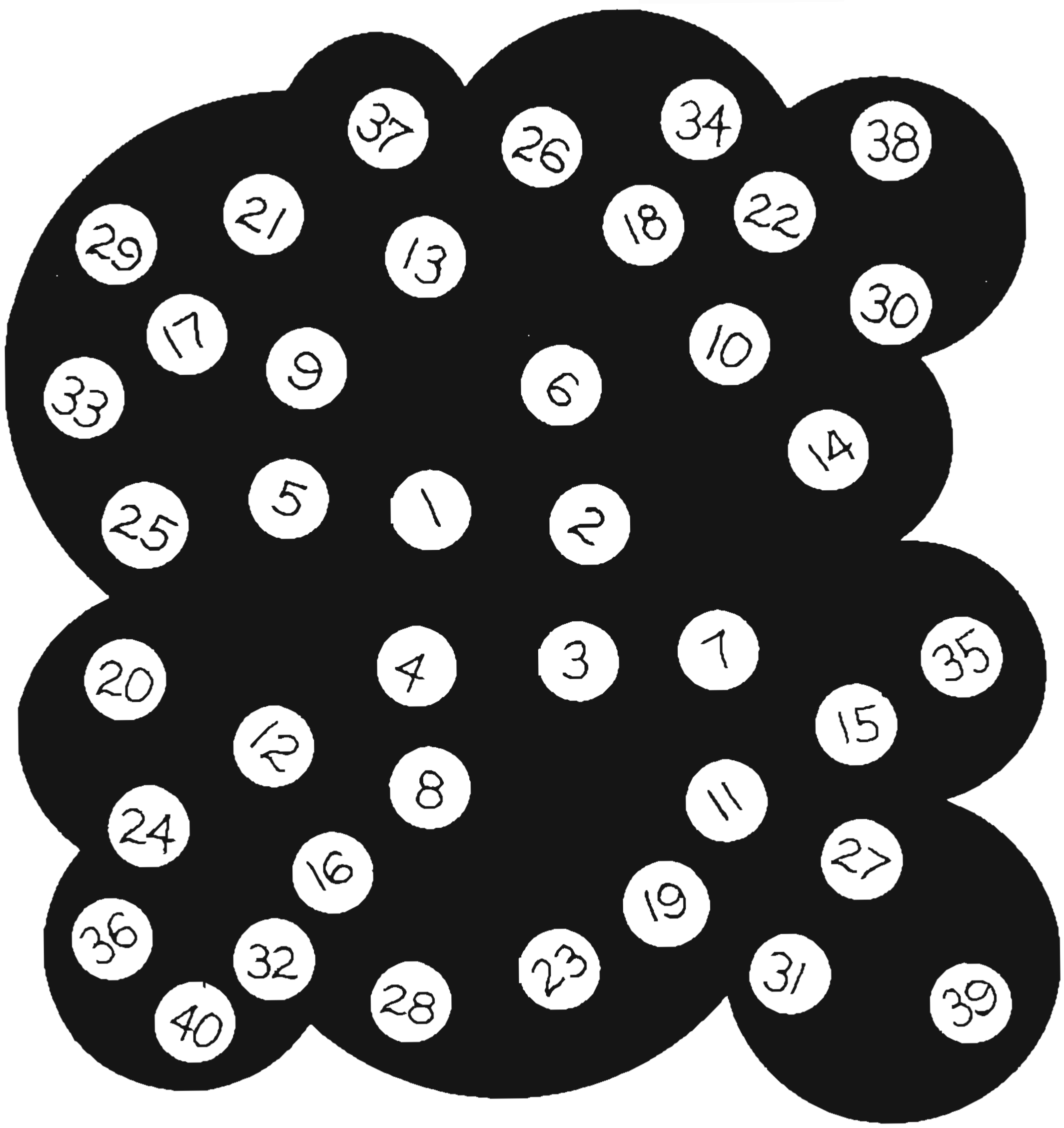
RATE ______________

PATTERN GAMES

I

At the bottom of the page are 40 numbers. By placing your finger on each circle touch each number in order starting at 1. You will have at most 1 minute. Don't start until you hear "Go" and stop immediately when you hear "Time's Up." In the table record the number you finish on and the time in seconds. If you finish before one minute, stop and record your time. Write your rate in the table. See if you can improve your rate with each trial.

Trial	Number	Seconds	Rate = Number : Seconds
1			
2			
3			

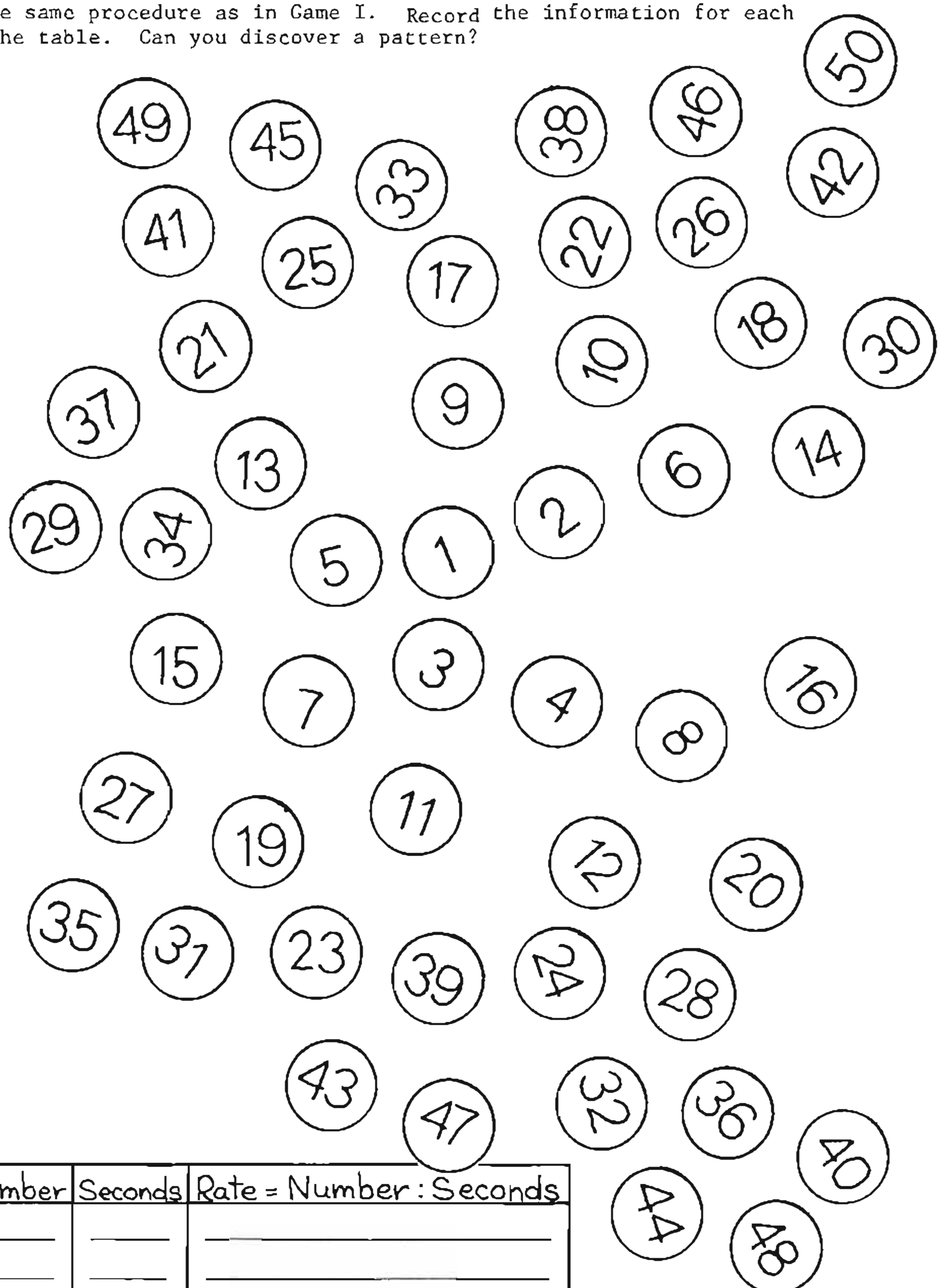


IDEA FROM: *Ideas and Investigations in Science: Life Science,* by Harry Wong, Leonard Bernstein, and Edward Shevick. © 1973, Idea 6/Investigation 6. Reprinted by permission of Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

PATTERN GAMES

II

Use the same procedure as in Game I. Record the information for each trial in the table. Can you discover a pattern?



Trial	Number	Seconds	Rate = Number : Seconds
1			
2			
3			

IDEA FROM: *Ideas and Investigations in Science: Life Science,* by Harry Wong, Leonard Bernstein, and Edward Shevick. © 1973, Idea 6/Investigation 6. Reprinted by permission of Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

MATH IS A FOUR-LETTER WORD

A. 1. Look at the four-letter words and 4-digit numerals on the next page.

2. Record in the table an estimate of how many words you could copy by printing for 15 sec., 30 sec., 1 minute.

3. Check your estimate by copying words as your partner times you. Don't copy the same word twice.

4. Record your results in the table. Write your rate.

5. Are the three rates for 15 sec., 30 sec. and 1 minute equivalent? ____

____

6. How many words could you copy in 5 minutes? ________________

Time in seconds	Estimate of words copied by printing	Number of words copied by printing	Rate = Number copied : time in seconds	Estimate of words copied in cursive	Number of words copied in cursive	Rate = Number copied : time in seconds	Estimate of numerals copied	Number of numerals copied	Rate = Number copied : time in seconds
15 sec.	___	___	______ : ______	___	___	______ : ______	___	___	______ : ______
30 sec.	___	___	______ : ______	___	___	______ : ______	___	___	______ : ______
60 sec			:			:			:

B. 1. Repeat A, except this time copy in cursive writing.

2. Record in the table.

3. How does your printing rate compare to your cursive rate? ____________

C. 1. Repeat the same procedure, except copy from the list of 4-digit numerals.

2. Record in the table.

3. For which of the three activities is your rate the best? ____________

4. Why do you think your rates differ? ________________________

DID YOU KNOW . . . Monks used to copy the Bible by hand. Using your rate, how long would it take you to copy the Bible?

IDEA FROM: *New Oxford Junior Mathematics,* Book 4

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MATH IS A FOUR-LETTER WORD
(CONTINUED)

math	boat	four
word	mice	post
bike	love	swim
time	farm	come
golf	hike	nice
kite	kick	much
some	from	slip
date	mate	late
name	rate	bill
rest	play	here
sail	take	find
tail	time	same
coat	many	know
they	with	hand
foot	ring	shoe
knee	like	snow
fill	rain	pill
your	game	help
self	home	that
hail	what	less
soup	salt	grid
iron	ball	bent
toad	code	cold
warm	half	mean
bite	only	over
meat	body	fair
seat	toss	tall
able	cope	card
came	work	fail
dear	pass	heat
sick	this	past
year	copy	sent
back	deer	note
colt	were	case
term	face	wave
once	pond	path
best	draw	life
long	shop	rail
mail	five	hear
talk	nine	fall
pike	pipe	tape
hate	goat	mate
baby	pair	palm
seek	mile	drip
taxi		

4159	5268	3085
3917	3045	4921
6134	8898	4822
7751	2506	4537
4887	6586	5252
3221	3531	4517
4945	2543	1132
1036	3031	1993
4924	8454	8793
5074	3283	2004
9973	6561	7397
9614	3254	9664
8123	1504	8815
3425	9054	1930
8754	8093	3425
2494	3425	2254
3425	5054	3114
5064	8612	7935
7349	5243	5002
2935	5204	1364
2763	6531	7461
1173	4328	7938
2554	7639	4253
3986	4104	8114
5243	9061	3425
1123	7946	1871
2793	4103	4926
7728	1749	3384
4196	4085	7683
5877	7415	8315
7351	2663	5671
8924	6223	5073
4605	8604	3053
1084	3335	2213
7037	3425	5133
8773	4141	5243
3154	3080	4515

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SPY ON THE EYE

Materials Needed: Clock with a second hand
2 secretive, spying students

Activity:

(1) For 1 minute count the number of blinks your partner makes and record blinks per minute in the table. Ask your partner to blink in a natural way.

(2) Have your partner find your rate of blinking.

(3) Move to a bright area--near the window, near a lamp, in the sunshine--and find out if the blinking rates increase.

(4) Secretly find the blinking rate of 4 other students, 2 girls and 2 boys. Record.

(5) Do the same for your teacher.

(6) Find and record the blinking rate of someone wearing contact lenses.

You	Partner	You (bright)	Partner (bright)	Girl #1	Girl #2	Boy #1	Boy #2	Contacts	Teacher
___ bpm	___ bpm	___ bpm	___ bpm	___ bpm	___ bpm	___ bpm	___ bpm	___ bpm	___ bpm

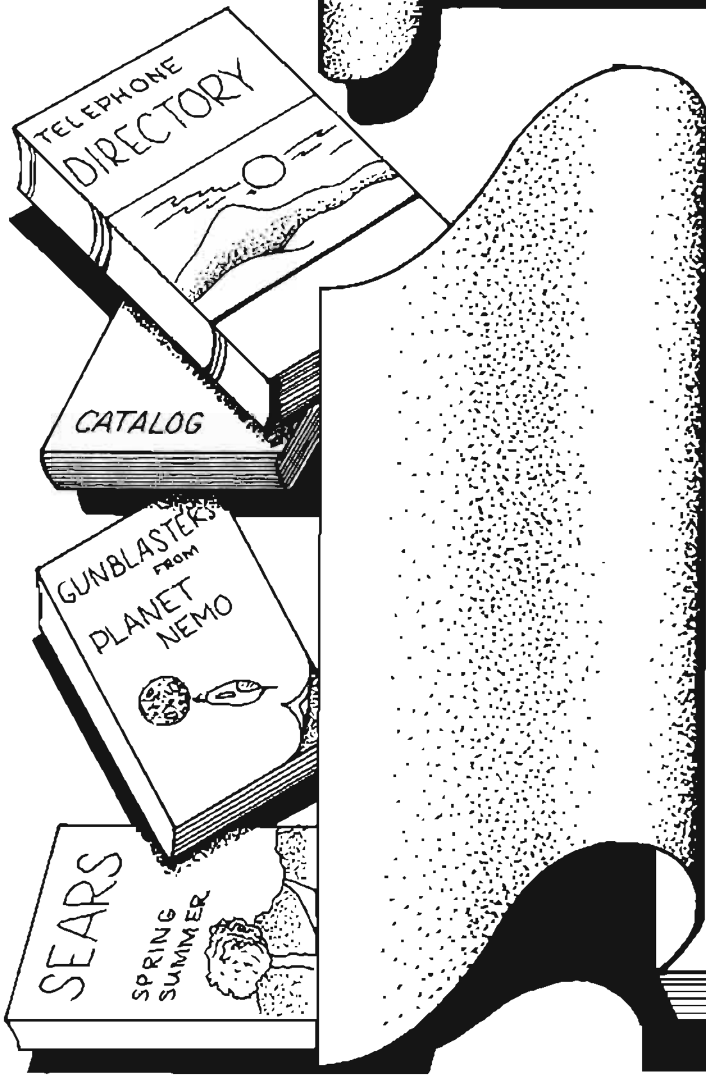
(7) Is there any difference in the blinking rate of boys and girls?

(8) Is the blinking rate of the student wearing the contact lenses faster than the other rates? Why?

(9) Use your blinking rate to find the number of blinks you will make in a day? a year?

(10) If your eyelids move 2 cm in a blink (1 cm in closing and 1 cm in opening), how far will your eyelids move in a day?

LET YOUR FINGERS DO THE WALKING



Materials needed: Telephone book, paperback novel, catalog, stopwatch or clock with a second hand.

Activity:

(1) Turn the pages in the telephone book one page at a time. Have a partner time you for 1 minute. Record the number of pages turned per minute in the table below. Now time your partner as he/she turns the pages. Record in the table.

(2) Repeat the activity with the paperback novel. Record both rates in the table.

(3) Repeat the activity with the catalog.

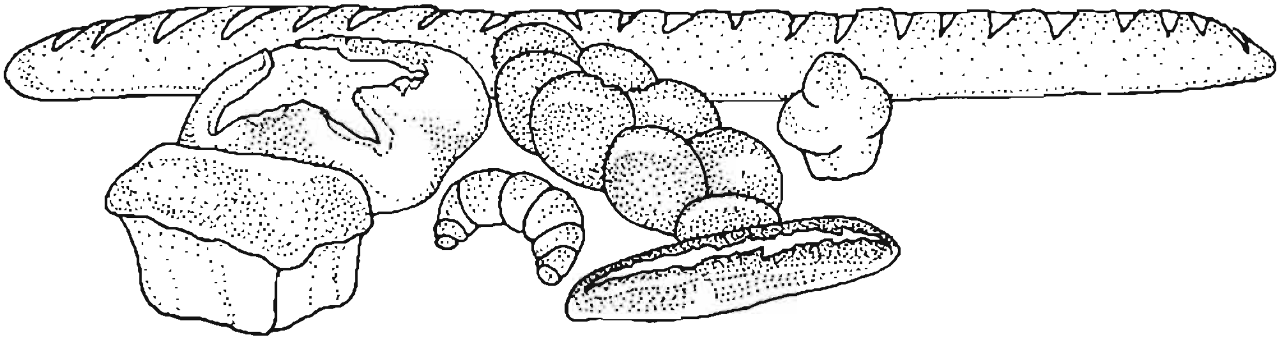
(4) With which book did you get the fastest rate? ____________ your partner? ____________

(5) What are some reasons why the rates might differ?

	You	Partner
Telephone book	______ pages per min.	______ pages per min.
Paperback book	______ p.p.m.	______ p.p.m.
Catalog	______ p.p.m.	______ p.p.m.

Students will have to agree on a method for turning pages.

THE FRENCH BREAD PROBLEM:



French bread comes in many sizes and shapes. Some loaves are fat and round. Some loaves are braided or odd-shaped. Other loaves are long and thin, about 1 to 2 metres long.

Bring a 60 cm loaf of French bread to class. Cut the loaf into two equal pieces. How long would each piece be? (30 centimetres)

How long would each piece be if the loaf was divided into three equal pieces? (20 centimetres)

Make a chart on the chalkboard (or overhead transparency) and list the answers that students give.

Number of Pieces	Rate	
2	$\frac{60}{2}$	30 centimetres per piece
3	$\frac{60}{3}$	20 centimetres per piece
4	$\frac{60}{4}$	15 centimetres per piece
5	$\frac{60}{5}$	12 centimetres per piece
6	$\frac{60}{6}$	10 centimetres per piece
.	.	.
.	.	.
.	.	.

Have students continue
the pattern for a while.

1. How many pieces of bread will there be in the loaf if each piece is two centimetres thick?

2. Bread is sliced about one centimetre thick to fit into a toaster. How many pieces of toast could be made from the loaf if each piece was one-centimetre thick? $\frac{1}{2}$-centimetre thick? (Have students follow the pattern in the chart if they do not know how to find the answer.)

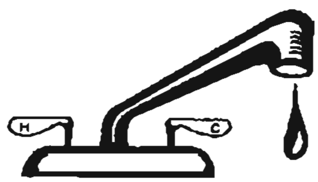
3. 150 pieces of toast are needed for a large breakfast. About how many 60-centimetre loaves of French bread would be needed?

4. 1 out of every 5 pieces of toast is too dark to serve. How many pieces out of the 150 slices of toast are too dark? _____ How many more loaves will be needed to get enough toast? _____

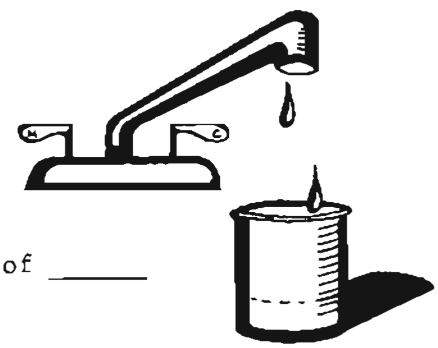
FIX THAT LEAK

Materials needed: Graduated beaker
Clock with a second hand
Calculator, if available

Activity: (1) (a) Turn on a faucet so it drips at a steady rate.
(b) Count the number of drops falling from the faucet in 30 seconds. Repeat the count for accuracy.
The water is dripping at a rate of ______ drops per minute.
______ drops per hour.
______ drops per day.
______ drops per year.



(2) (a) Measure the volume of 100 drops of water in millilitres.
(b) Use the data above. The faucet is dripping at a rate of ______ millilitres per minute.
______ millilitres per hour.
______ millilitres per day.
______ millilitres per year.

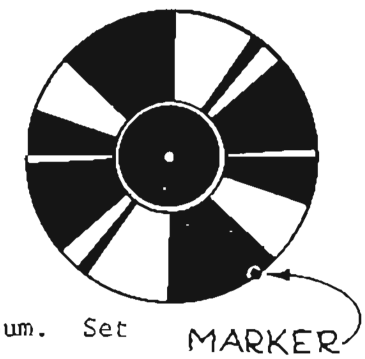


(c) The last rate is equivalent to a rate of ______ litres per year.

(3) Call your local water board to find the rate charged for residential water use. (The rate will probably be dollars per 1000 gallons of water. 1 litre ≈ .2624 gallons.) How much money is wasted by this dripping faucet in one year?

AS THE RECORD TURNS

Materials needed: Record player with variable speeds
Several popular $33\frac{1}{3}$ albums, 45 singles and a 78 record (if available)
Stopwatch or clock with a second hand.



Questions: How fast does a record turn?
What do the speeds on a record player mean?

I. Place a small marker on the outer edge of a $33\frac{1}{3}$ record album. Set the record player to $33\frac{1}{3}$ and carefully count the number of revolutions the marker makes in 1 minute. Make a table like the one below and record the number. Repeat the count two more times for accuracy.

	Number of revolutions
minute 1	______
minute 2	______
minute 3	

Find the sum of the revolutions. ____
Find the average by dividing by 3. ____

II. Repeat the activity again using a 45 record.

III. If you have a 78 record, repeat the activity again.

IV. Place a small marker on the label of a $33\frac{1}{3}$ album. Repeat the activity. Find the average for the marker on the label.

AS THE RECORD TURNS (CONTINUED)

The following are additional activities and questions that can be used as a follow-up to the *As the Record Turns* student page.

(a) Play a $33\frac{1}{3}$ album at 45 rpm. How is the sound distorted? How many times faster is the record revolving compared to its normal speed?

(b) Play a 45 single at $33\frac{1}{3}$ rpm. How is the sound distorted? How many times slower is the record revolving compared to its normal speed?

Note: Most record players have a different needle setting for 78 rpm records, so you should not try to play one of these records at a different speed.

(c) Measure the time a song plays at its normal speed. (This figure can also be found on the label.) Compute the time of the same song played at a slower or faster speed.

(d) If a song takes 3:30 minutes ($3\frac{1}{2}$ minutes) to play at 45 rpm, how many revolutions does the record make?

(e) If it takes 21 minutes to play one side of an album at $33\frac{1}{3}$ rpm, how many revolutions does the turntable make?

Did you know that . . .

- -- the first needle used to play records was a cactus needle?
- -- 78 rpm was the first speed used for records because it seemed like a convenient speed?
- -- RCA tried to corner the market on records when they patented the 45 rpm single record with the large center hole?
- -- with the invention of more refined and sharper needles, records could be made with finer grooves which played best at $33\frac{1}{3}$ rpm?
- -- some commercials played on radio stations run at 16 rpm and start from the center and play to the outside?

MY HEART THROBS FOR YOU

Materials Needed: 2 students
Stopwatch or clock with a second hand

Activity: 1. On your paper draw a chart like the one below.

Name	Inactive Pulse		Active Pulse		Recovery Pulse	
	Self	Partner	Self	Partner	Self	Partner

2. a) Guess how many times your heart beats in one minute. _____ bpm (beats per minute)

   b) Have your partner take your pulse and record it in the inactive column as _____ bpm.

   c) Take and record your partner's pulse.

3. a) Run in place for two minutes.

   b) Record your pulse rate in the active column.

   c) Have your partner run in place for two minutes.

   d) Record your partner's pulse rate.

REST 5 MINUTES

4. a) Record both of your pulse rates in the recovery column.

   b) Have your pulses returned to normal?

   c) Is your recovery rate faster than your partner's?

STEP RIGHT UP

Numerous ads to eat wisely and exercise regularly encourage students to think about their physical condition which, in turn, affects the pulse and recovery time following exercise. In general conditioned persons have a slower resting pulse and a slower pulse during exercise. Their pulse will recover to the resting rate quicker following strenuous exercise than persons who are in poor condition. Because of heredity some persons inherit efficient hearts with slower rates, while others are born with relatively inefficient hearts. However, both types can be improved.

Since the physical condition of an individual affects his heartbeat, pulse tests can be used to measure physical fitness. Four pulse tests are described below, and tables to interpret the results are provided. Better results could be obtained from the first two tests if they are done at home with parental help.

I. Pulse Lying:

The pulse lying is the slowest, resting pulse of a person. The student can find this rate by taking her pulse for 30 seconds before she gets out of bed in the morning. If done in class, have the student lie down and attempt to completely relax for ten minutes. In the lying position count her heartbeats for 30 seconds. The student should continue to rest in the lying position for 2 more minutes and repeat the count. If it is the same double the count to get the pulse lying, and record the number. If less the student should rest longer and repeat the count.

II. Pulse Standing:

To obtain the slowest, resting, standing pulse have the student rise slowly after finishing the pulse lying test and remain standing for two minutes. Count the heartbeats for 30 seconds and double the number to get the pulse standing.

Have the student subtract the pulse lying from the pulse standing. This number is the pulse difference. By checking Table A the student can find her physical fitness rating.

TABLE A

Physical Fitness Rating	Excellent			Very Good		Above Average			Average			Below Average			Poor			Very Poor	
Pulse Lying	40	54	57	58	60	63	66	69	71	73	75	77	78	79	80	82	84	86	105
Pulse Standing	46	63	67	68	70	74	77	80	83	85	87	90	91	92	94	96	98	101	123
Pulse Difference	6	9	10	10	10	11	11	11	12	12	12	13	13	13	14	14	14	15	18

IDEA FROM: *Physical Fitness Workbook,* by Thomas Cureton. Copyright © 1944 by Stipes Publishing Company.

STEP RIGHT UP

(CONTINUED)

III. Simplified Pulse Ratio Test:

(a) While sitting, have the student count and record her heartbeats for one minute.

(b) Have the student face a chair (approximately 45 cm high) and step up with the left foot, up with the right, down with the left, and down with the right. The student should do 30 of these steps in one minute. In order to set the cadence the teacher or another student can call out "up, up, down, down" at the required speed or play a taped recording of the cadence.

(c) Immediately after completing the 30 steps, the student should sit, count and record her heartbeats for two minutes.

(d) Have the student write her pulse ratio. Pulse Ratio = Heartbeats for 2 minutes following the exercise : Heartbeats for 1 minute before exercise. Simplify the ratio by dividing the first number by the second, correct to one decimal place. Check Table B to find the physical fitness rating.

TABLE B

Pulse Ratio	Physical Fitness Rating
1.5 - 1.7	Excellent
1.8 - 2.0	Very Good
2.1 - 2.3	Above Average
2.4 - 2.5	Average
2.6 - 2.8	Below Average
2.9 - 3.1	Poor
3.2 - 3.4	Very Poor

IV. Three Minute Step Test:

This test is administered like the previous test, except the student steps for three minutes, and the cadence is 24 steps per minute. In addition wait one minute after the student completes the exercise and count the heartbeats for only 30 seconds. The efficiency score is the ratio of

$$\frac{\text{number of seconds stepping} \times 100}{\text{pulse for 30 seconds} \times 5.6}$$

Divide and check Table C to find the physical fitness rating.

TABLE C*

Efficiency Score	Physical Fitness Rating
72 - 100	Excellent
62 - 71	Very Good
51 - 61	Good
41 - 50	Fair
31 - 40	Poor
0 - 30	Very Poor

*This table is accurate for junior high girls. The efficiency scores may need to be raised for junior high boys. At the grade school level there is not much difference between boys and girls.

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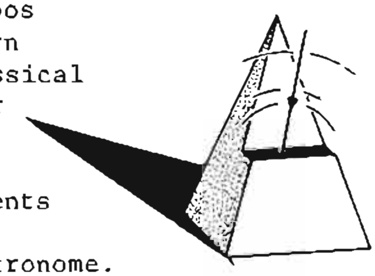
I believe in music

Materials Needed: Records and record player, clock with second hand, metronome, piano, drums, guitar, flute or other instruments, sheet music.

I. (a) Select several musical pieces that have different tempos (beats per minute), for example, a slow country western song, a Sousa march, a rock and roll piece, and a classical arrangement. Ask your students to bring some of their records to play.



(b) Have the students determine the tempo of the song by counting the number of beats in 10 seconds. The students can keep time with the music and count the beats by tapping their feet or hands, dancing, or setting a metronome.

(c) Have the students repeat the count to check for accuracy and record the results in the table as a rate; number of beats : 10 seconds.

(d) Rewrite the rate and express it in the table as number of beats : 60 sec.

(e) Look at the record to find the total time of the song and record the time in the table.

(f) Estimate and record the total number of beats in the song.

Musical Selection	Number of beats: 10 sec.	Number of beats: 60 sec.	Total time of song	Estimate of total number of beats
1.	: 10 sec.	: 60 sec.		
2.	:	:		
3	:	:		
4.	:	:		
5.	:	:		
6.	:	:		

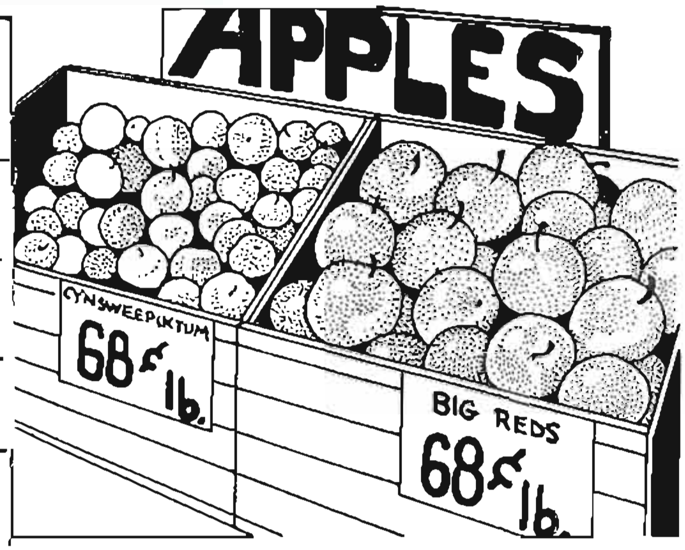
II. Have a student or the music teacher play a selection at various tempos. Use a metronome to determine the tempo.

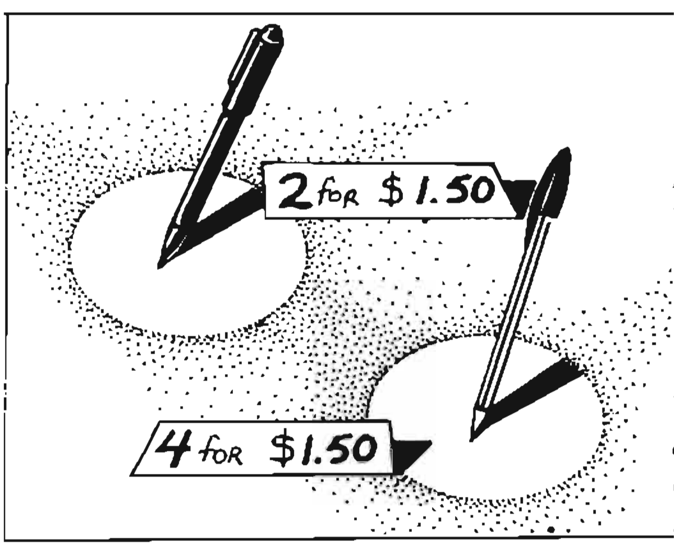
A student with a set of drums could keep the beat. Time a song at a specified tempo and record the time in the table. Select a new tempo and estimate the new time for the song. Check the estimate by having the musician play the song at the new tempo.

Musical Selection	Tempo = beats : 1 minute	Total time of song	New tempo = beats : 1 minute	Estimated time of song
1.	:		:	
2.	:		:	
3.	:		:	
4.	:		:	

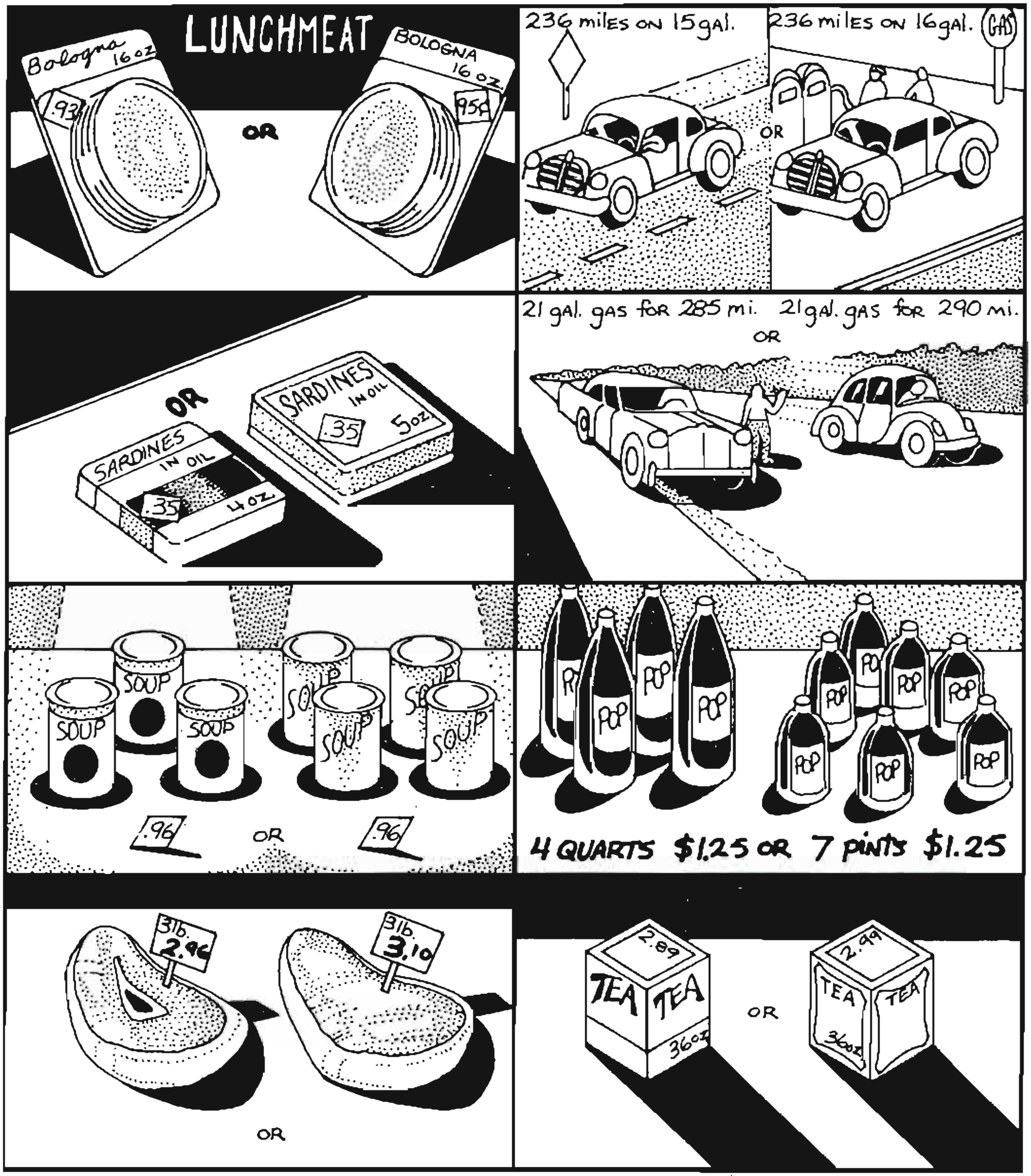
III. Select some sheet music. Read the tempo suggested on the music. Have a student estimate the tempo by tapping his foot. Check the estimate with the metronome. Have the musician play the selection.

WHICH IS A BETTER BUY?

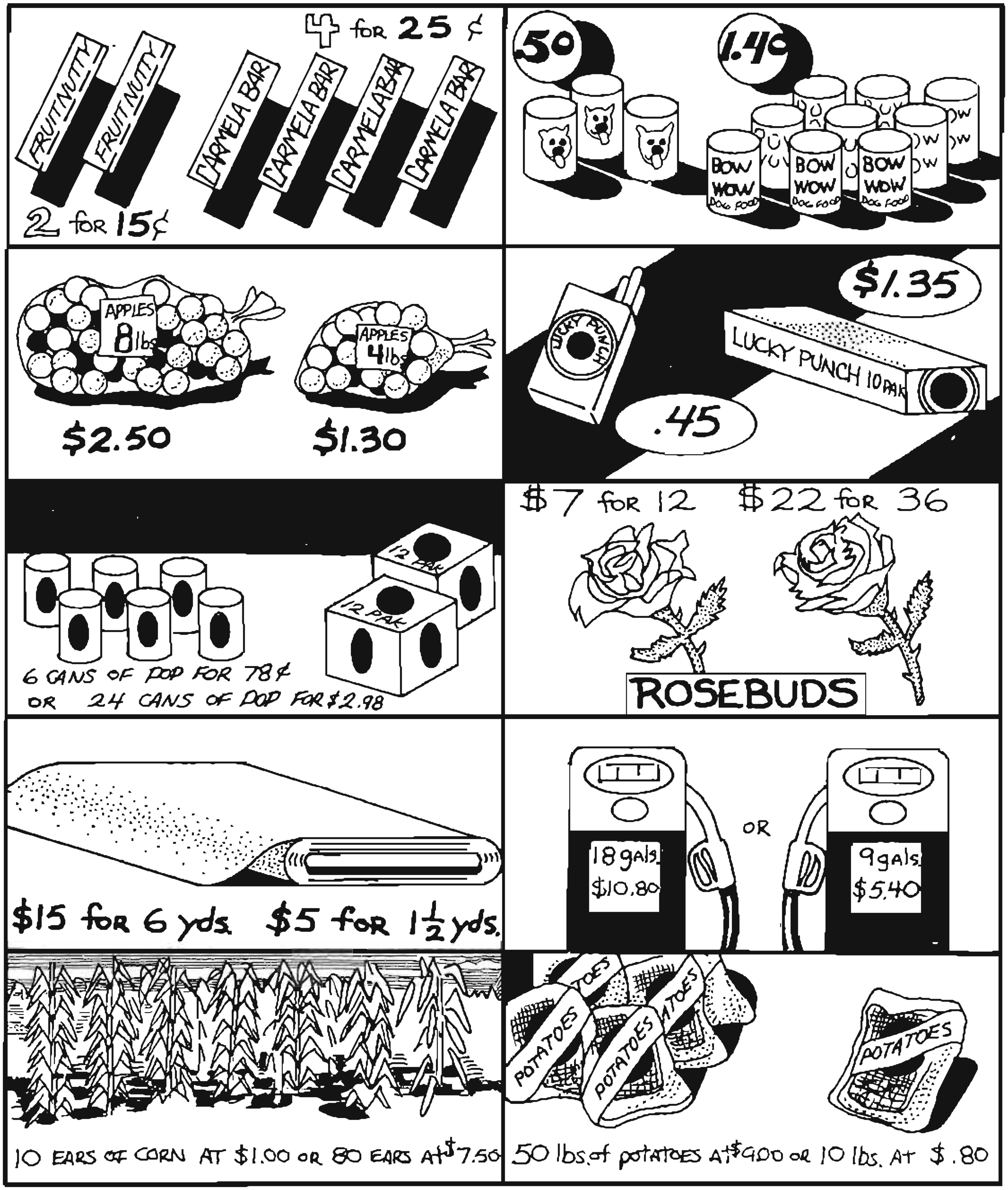




WHICH IS BETTER? 1



WHICH IS BETTER? 2



BUT I ONLY WANT ONE

Find the unit cost (cost of one item) if:

A. 3 tennis balls cost $2.67

B. 2 dozen pencils cost $1.68

C. a 6-pack of pop cost $1.14

D. 1 dozen eggs cost $.96

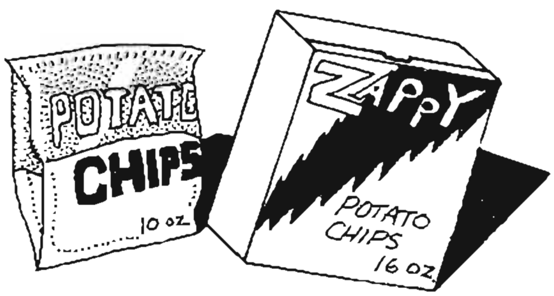
E. 3 T-shirts cost $3.36

F. 5 pounds of hamburger cost $3.45

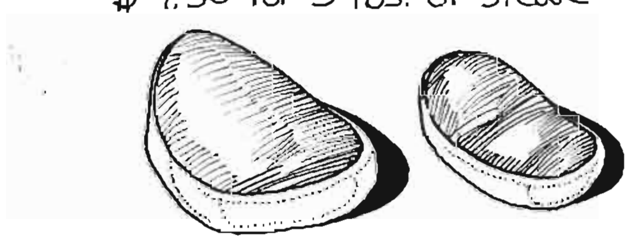
Find the better buy by finding the unit cost, for example cost per ounce.

G. 12 oz. of soap for $1.32 or 15 oz. for $1.50

H. 10 oz. of potato chips for 80¢ or 16 oz. for $1.12



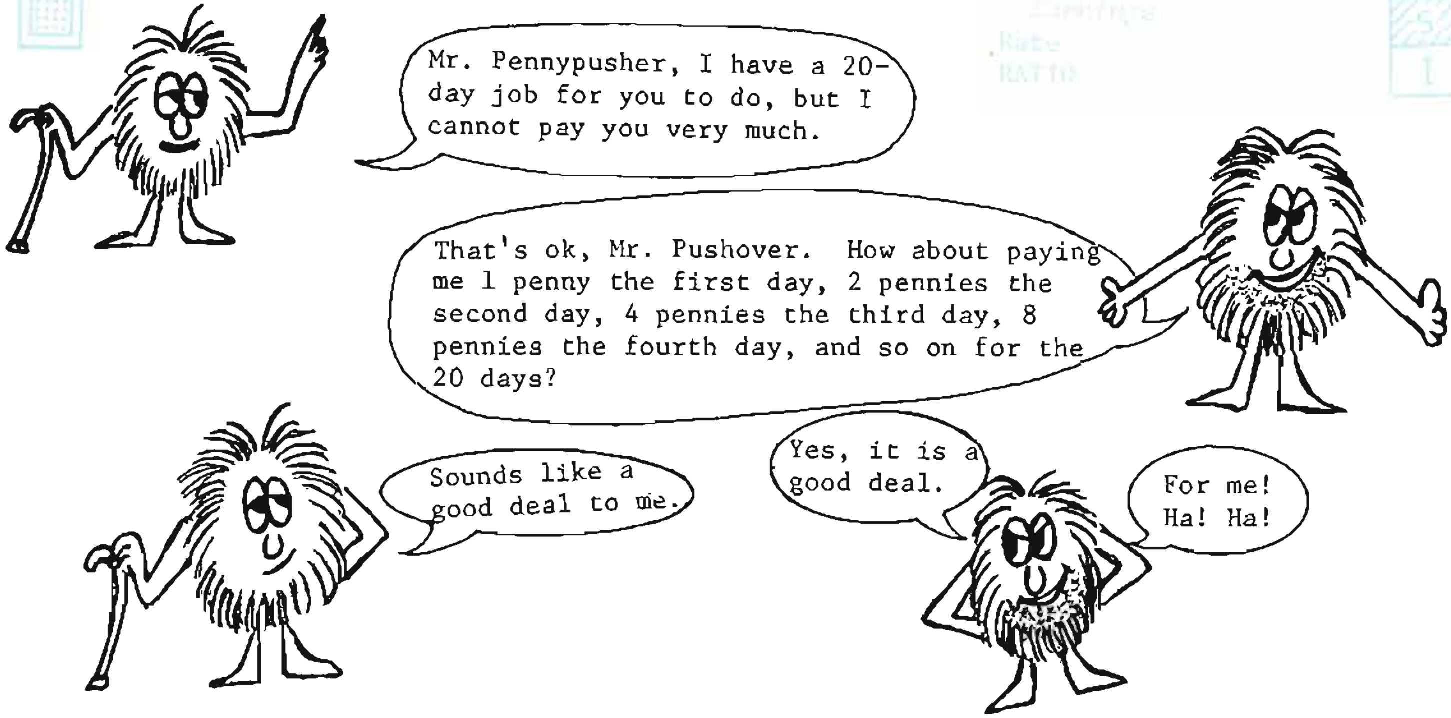
I. $7.55 for 5 lbs. of steak or $4.50 for 3 lbs. of steak



J. 4 qts. of milk for $1.24 or 7 qts. of milk for $2.24

K. $20.98 for 2 pairs of jeans or $31.77 for 3 pairs of jeans

I NEED A JOB LIKE THAT!



Fill in this chart of earnings for Mr. Pennypusher. Use a calculator to get each day's wages and the total earnings for all 20 days.

DAY	EARNINGS	DAY	EARNINGS
1	$.	11	$.
2	$.	12	$.
3	$.	13	$.
4	$.	14	$.
5	$.	15	$.
6	$.	16	$.
7	$.	17	$.
8	$.	18	$.
9	$.	19	$.
10	$.	20	$.
TOTAL FOR 20 DAYS			$.

a) What is the average amount Mr. Pennypusher made per day? (Divide the total earnings by 20.)

____

b) If he worked 8 hours per day, what is the average amount he made per hour?

____

c) How much did Mr. Pennypusher average per minute?

____

(d) If you were paid as much as Mr. Pennypusher averaged per minute, how much would you make for the time you are in your mathematics class?

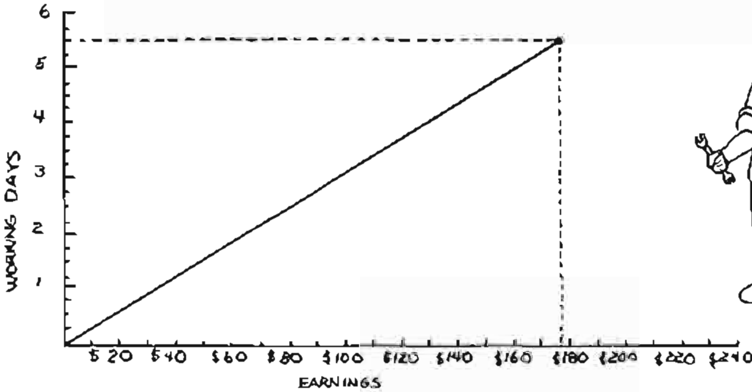
per day? ________

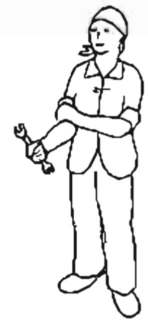
per week? ________

per school year? ________

EIGHT HOURS A DAY

TERRY APPLIED FOR A JOB AS A MECHANIC. THE BOSS SAID SHE COULD EARN $176 FOR $5\frac{1}{2}$ DAYS OF WORK.





A) FROM THE GRAPH FIND ABOUT HOW MUCH TERRY WILL EARN IN:
   1) 2 DAYS ________.
   2) 3 DAYS ________.
   3) 1 DAY ________.

B) TERRY WORKS 8 HOURS PER DAY. ESTIMATE FROM THE GRAPH HOW MUCH SHE EARNS PER HOUR. ________

C) AFTER 3 MONTHS OF GOOD WORK, TERRY RECEIVED A $1.00 PER HOUR RAISE. FILL IN THE CHART TO SHOW TERRY'S CUMULATIVE EARNINGS AFTER HER RAISE.

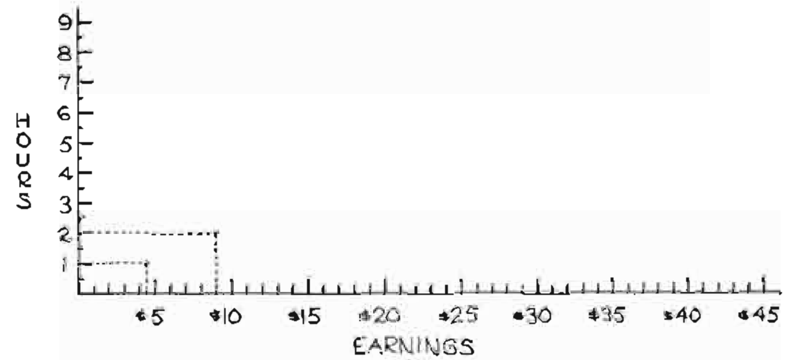
DAYS	1	2	3	4	5	$5\frac{1}{2}$
EARNINGS						

D) PLOT TERRY'S EARNINGS ON THE GRAPH ABOVE.

SAM'S JOB PAYS $4.50 PER HOUR. FILL IN THE CHART TO SHOW SAM'S RATE OF EARNINGS FOR AN EIGHT-HOUR DAY.

HOURS	1	2	3	4	5	6	7	8
EARNINGS								

PLOT SAM'S CUMULATIVE WAGES ON THE GRAPH BELOW. SEE THE EXAMPLE.



A) WHAT DO YOU NOTICE ABOUT ALL THE POINTS?

B) CONNECT THEM.

C) USE THE GRAPH TO FIND SAM'S EARNINGS FOR:
   1) SATURDAY WHEN HE WORKS $5\frac{1}{2}$ HOURS.
   2) SUNDAY WHEN HE WORKS 3 HOURS, 30 MINUTES. ________
   3) FRIDAY WHEN HE WORKS $9\frac{1}{2}$ HOURS. ________

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RATIO, PROPORTION, AND SCALING

Placement Guide for Tabbed Divisors

CONTENTS

RATIO: EQUIVALENT

TITLE	PAGE	TOPIC	TYPE
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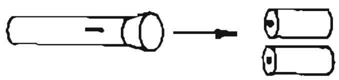
One Man One Vote	203	Simplifying	Paper and pencil
Poppin' Wheelies in a Ring	205	Simplifying	Activity Manipulative
People Ratio	207	Simplifying	Activity
Surface Area and Ratios 1	208	Simplifying	Activity
Surface Area and Ratios 2	209	Simplifying	Activity
Volume and Ratio 1	210	Simplifying	Activity
Volume and Ratio 2	211	Simplifying	Activity
Cubism	212	Simplifying	Activity

RATIOS BY PICTURE II

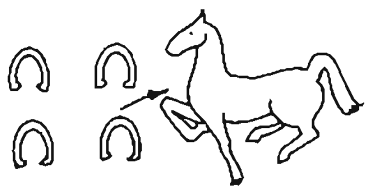
Write the equivalent ratios suggested by each of these pictures.

(A)



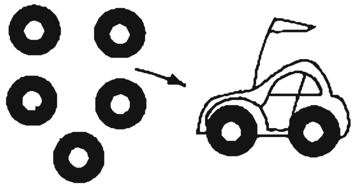
_______ flashlights for every 4 batteries
_______ flashlights for every 6 batteries
_______ flashlights for every 8 batteries
_______ flashlights for every 20 batteries

(B)



8 horseshoes for every _______ horses
12 horseshoes for every _______ horses
16 horseshoes for every _______ horses
28 horseshoes for every _______ horses

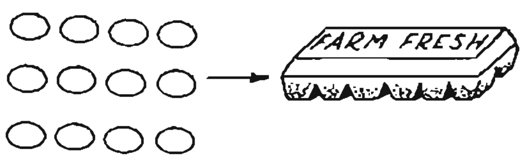
(C)



_______ tires for every 6 cars

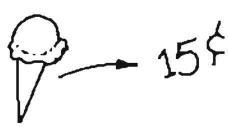
_______ tires for every 50 cars

(D)



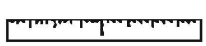
_______ egg cartons for every 144 eggs

(E)



_______ ice cream cones for every 30¢
_______ ice cream cones for every 45¢
_______ ice cream cones for every 60¢
_______ ice cream cones for every 90¢

(F)



24 inches → 2 feet

_______ inches for every 1 foot
_______ inches for every 3 feet
_______ inches for every 5 feet

(G)

25¢ for 3

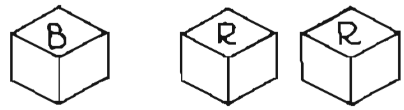
50¢ for every _______ candy bars
75¢ for every _______ candy bars
$1.25 for every _______ candy bars
$2.00 for every _______ candy bars

RATIOS AND CUBES 1

Materials: 24 blue cubes and 24 red cubes

Activity:

B means blue cube.
R means red cube.



A. Use 1 B and 2 R.

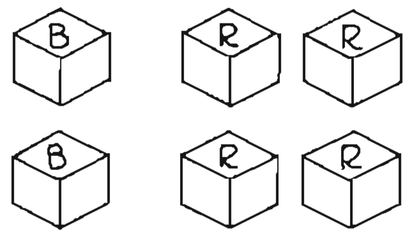
1. The ratio of B to R is ___:___.

Use another group of 1 B and 2 R.

2. The ratio of B to R is now _2_:___.

Use another group of 1 B and 2 R.

3. The ratio of B to R is now ___:_6_.

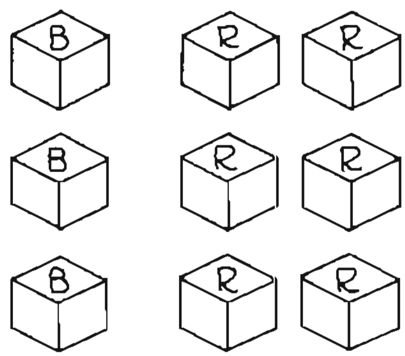


4. If you continued using groups of 1 B and 2 R, write the ratio of B to R that you would get.

_6_:___ , ___:_22_ , _10_:___ , ___:_18_

Each ratio was formed using groups of <u>1 B and 2 R</u>.
The ratios are <u>equivalent ratios</u>.

B. Use 4 R and 3 B.



1. The ratio of R to B is ___:___.

Use another group of 4 R and 3 B.

2. The ratio of R to B now is ___:___.

Use another group.

3. The ratio of R to B is now ___:___.

4. Continue using groups of 4 R and 3 B.

Write the ratio of R to B.

_16_:___ , ___:_15_ , ___:_24_ , _24_:___

Each ratio was formed using groups of <u>3 R and 4 B</u>. The ratios are <u>equivalent ratios</u>.

C. Use these groups and write equivalent ratios.

1. 3:5 , _9_:___ , ___:_30_
2. 1:4 , _5_:___ , ___:_12_
3. 8:3 , ___:_15_ , _32_:___

IDEA BASED ON: *The Laboratory Approach to Mathematics* by Kenneth P. Kidd, Shirley S. Myers, and David Cilley. © 1970, Science Research Associates, Inc.

RATIOS AND CUBES 2

Materials: 24 blue cubes and 24 red cubes

B means blue cube.
R means red cube.

Activity:

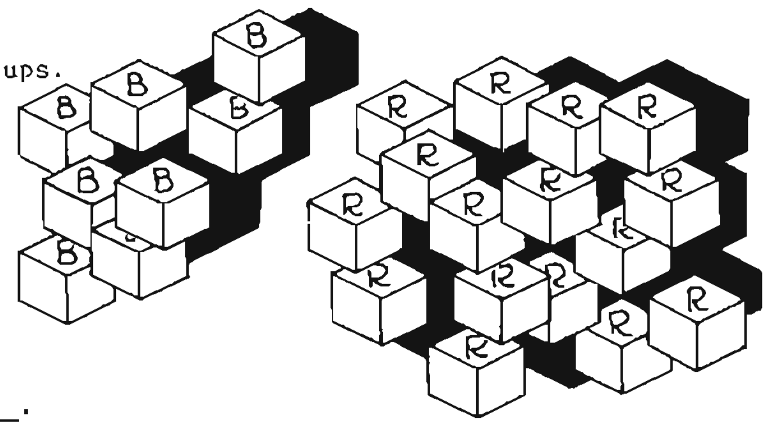
A. Use 8 B and 16 R.

1. The ratio of B to R is ___:___.
Separate each color into two equal groups.

2. Using one group of each color,
the ratio of B to R is ___:_8_.
Separate each group into two smaller groups.

3. Using one smaller group of each color, the ratio of B to R is _2_:___.
Separate again.

4. Using one group of each color,
the **simplest** ratio of B to R is ___:___.
The ratios above are <u>equivalent ratios</u>.
Each can be formed using ___B and ___R.



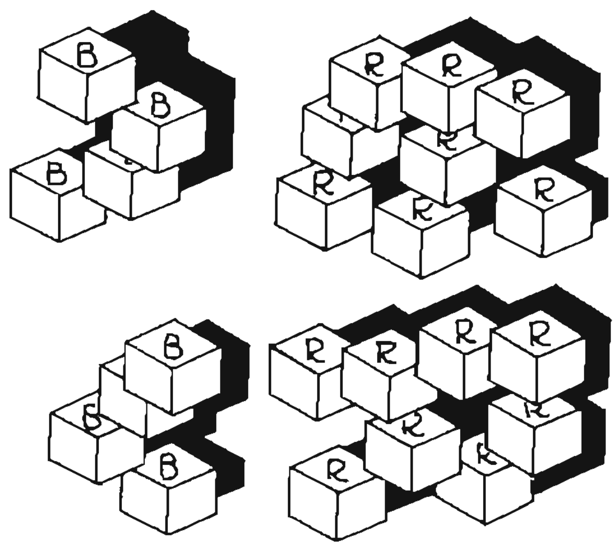
B. Use 24 R and 18 B.

1. The ratio of R to B is ___:___.
Separate each color into three equal groups.

2. Using one group of each color,
the ratio of R to B is ___:___.
Separate each group into two smaller groups.

3. Using one smaller group of each color, the ratio of R to B is ___:___.
Separate again.

4. Using one group of each color, the simplest ratio of R to B is ___:___.
The ratios above are <u>equivalent ratios</u>.
Each can be formed using ___ R and ___ B.

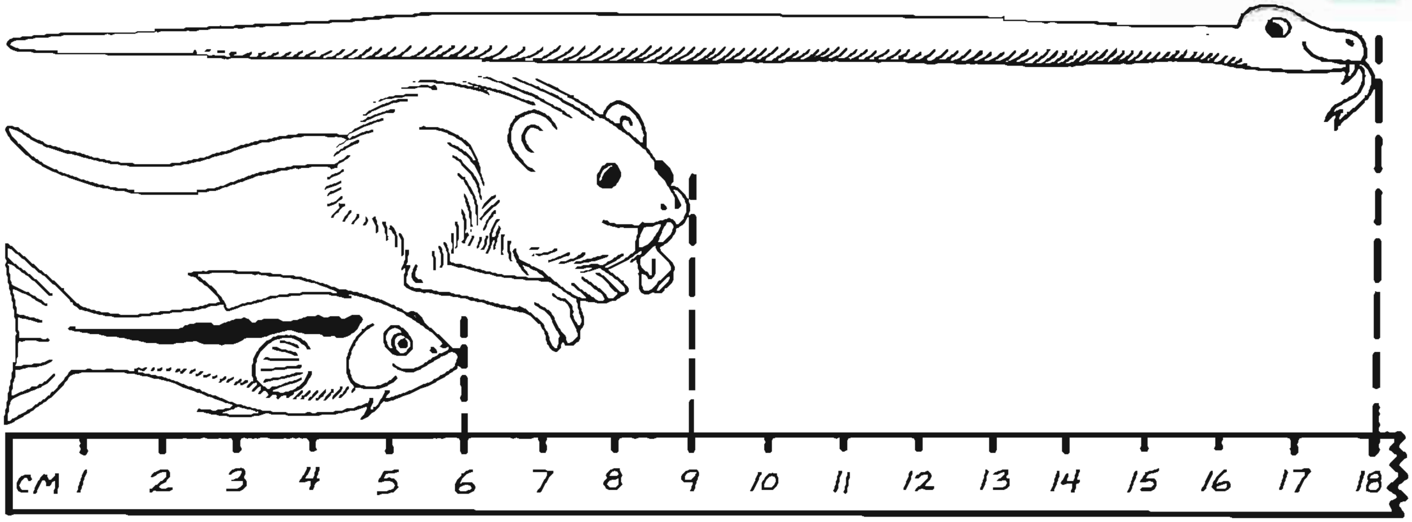


C. Find the simplest ratio that could be used to form these equivalent ratios.

1. 8:4
2. 18:30
3. 9:27
4. 25:40
5. 30:100
6. 180:150

IDEA BASED ON: *The Laboratory Approach to Mathematics* by Kenneth P. Kidd, Shirley S. Myers, and David Cilley.

ANIMAL RATIO



1. a) How long is the fish? __________
   b) How long is the snake? __________
   c) Write a ratio to compare the length of the snake to the length of the fish _____:_____.
   d) How many fish placed end-to-end would be needed to have the same length as the snake? ________________
   e) The snake is _____ times as long as the fish.
   f) Find two lengths on the ruler, so the longer length is 3 times the shorter length. _____ and _____
   g) What are the two shortest lengths (whole number of centimetres) on the ruler, so that the longer is 3 times the shorter? _____ and _____
   h) Write the ratios from (e) _____:_____, (f) _____:_____ and (g) _____:_____. These ratios are called <u>equivalent ratios</u>. Can you see why?
   i) Write another ratio equivalent to the ratios in (h) _____:_____.

2. a) Write a ratio to compare the lengths of the gerbil to the snake _____:_____.
   b) The gerbil is _____ times as long as the snake.
   c) Find two lengths on the ruler, so the shorter length is $\frac{1}{2}$ of the longer length and write their ratio _____:_____.
   d) Find the two shortest lengths, so the shorter length is $\frac{1}{2}$ of the longer length. _____ and _____
   e) Write the ratios from (a) _____:_____, (c) _____:_____, (d) _____:_____.
      These ratios are equivalent ratios because the shorter length is $\frac{1}{2}$ of of the longer length in each case.
   f) Write another ratio equivalent to the ratios in (e) _____:_____.

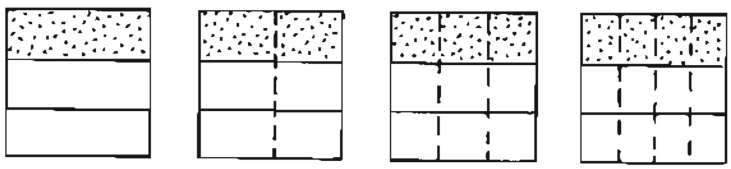
3. a) Write a ratio to compare the length of the gerbil to the length of the fish. _____:_____
   b) The gerbil is _____ times as long as the fish.
   c) Find two lengths, so the longer is 1 1/2 times the shorter. _____ and _____
   d) Find the two shortest lengths, so the longer is 1 1/2 times the shorter. _____ and _____
   e) Write the ratios from (a) _____:_____, (c) _____:_____, (d) _____:_____. These ratios are <u>equivalent ratios</u> because the longer is 1 1/2 times the shorter.
   f) Write another ratio equivalent to the ratios in (e) _____:_____.

EQUIVALENT RATIOS WITH GEOMETRIC MODELS

TEACHER PAGE

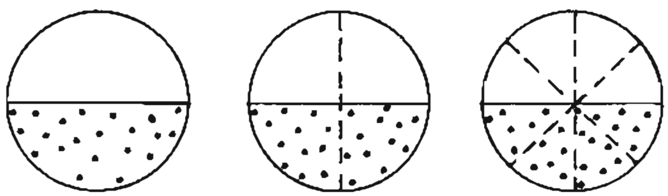
A series of equivalent ratios can be generated by keeping the shaded area and total area of a figure constant but changing the number of parts.

EXAMPLE 1



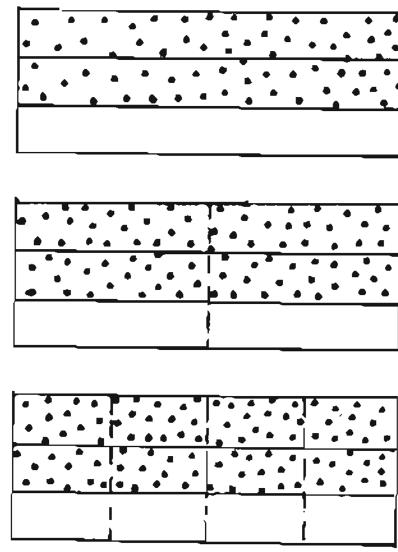
1:3   2:6   3:9   4:12

EXAMPLE 2



1:2   2:4   4:8

EXAMPLE 3



2:3

4:6

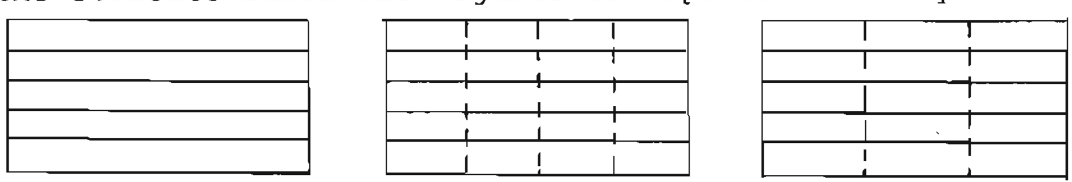
8:12

Sample questions in example 1 might be:

1) Has the shaded area changed?
2) Has the size of the square changed?
3) Can you see a ratio of 1:3 in each diagram?

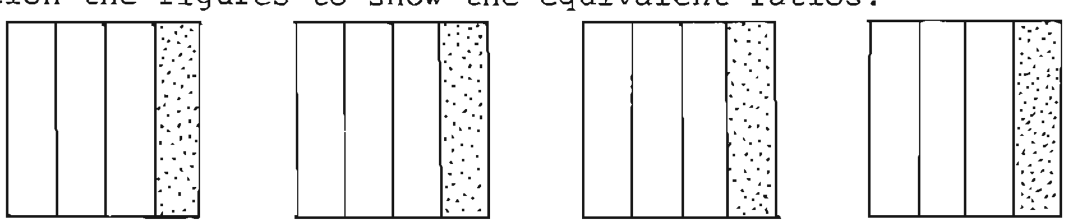
Student worksheets can be developed to:

a) have the students shade the figures to represent the equivalent ratios.



1:5   4:20   3:15

b) partition the figures to show the equivalent ratios.



1:4   2:8   4:16   5:20

These activities can give an intuitive feeling for generating equivalent ratios by the algorithm of multiplying both terms of the ratio by some number. The models show how the number of shaded parts and total parts are both doubled, tripled, etc.

TYPE: Paper & Pencil/Transparency

IDEA FROM: *Activities with Ratio and Proportion*

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EQUIVALENT RATIOS BY PATTERNS

Developing the concept of equivalent ratios by using patterns can be accomplished as a bell work, warmup, or mental arithmetic drill at the beginning of class. These activities can be started in advance of the introduction of ratio notation by presenting problems with different word phrases in place of the colon.

Examples of problems which can be prepared for an overhead or the blackboard might be:

1) $ 4 for 1 hour's work.
   $ 8 for ___ hour's work.
   $12 for ___ hour's work.
   ___ for 4 hour's work.
   ___ for ___ hour's work.
   ___ for ___ hour's work.

2) 30 students for every 1 class.
   ___ students for every 2 classes.
   ___ students for every 5 classes.
   300 students for every ___ classes.
   ___ students for every ___ classes.
   ___ students for every ___ classes.

Various ratio notations and ratio tables can be used. The problems might start with the "unit quantity" given (see 3, 4, 5) and then develop into ones where the "unit quantity" must be found (see 6, 7, 8).

3) 1 to 4
   2 to ___
   ___ to 12
   4 to ___
   5 to ___
   ___ to ___
   ___ to ___

4) 2:1
   4:___
   ___:3
   ___:4
   10:___
   ___:___
   ___:___

6) 2:16
   1:___
   3:___
   8:___
   ___:___
   ___:___

7) 18 to 6
   ___ to 1
   12 to ___
   ___ to 5
   ___ to ___
   ___ to ___

5) $\frac{1}{5}, \frac{\_\_}{10}, \frac{4}{\_\_}, \frac{\_\_}{40}, \frac{16}{\_\_}, \_\_, \_\_$

8) $\frac{10}{20}, \frac{1}{\_\_}, \frac{\_\_}{8}, \frac{12}{\_\_}, \_\_, \_\_$

Rates that students would be familiar with can be given in the form of ratio tables (see 9, 10, 11).

9)

miles	50	100		200	250		
hours	1		3				

10)

typed words	43	172		86			
minutes	1		10		5		

11)

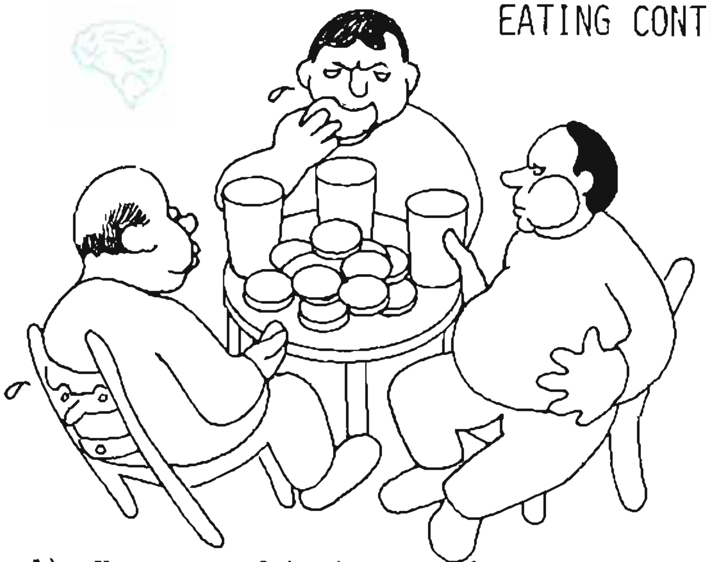
pounds of meat	4	1	2				
cost	$6.00			$4.50	$12.00		

In each problem provide a few blanks for students to write their own equivalent ratios. The development of this technique will be useful in the solution of simple proportion problems, such as: A bagger in a supermarket works 2 1/2 hours and earns $8. How much would the bagger make in 10 hours? 1 hour?

This type of solution avoids work with cross products and division using fractions.

Possible Solutions: 2 1/2 hours at $8
5 hours at $16
10 hours at $32
1 hour at $3.20

EATING CONTEST



Harry, Morgan, and Eddy had a hamburger eating and milkshake drinking contest.

1) Harry ate 2 hamburgers for every 1 hamburger Eddy ate.
   a) Who ate more hamburgers? Harry or Eddy? ____________
   b) How many hamburgers did Harry eat during the contest? ____________
   c) Fill in this chart of possibilities.

NUMBER OF HAMBURGERS HARRY ATE	2	4			10			26			
NUMBER OF HAMBURGERS EDDY ATE	1		3	4		7	10				

2) Morgan drank 3 milkshakes for every 1 milkshake Harry drank.
   a) Who drank the most milkshakes? Harry, Morgan or Eddy? ____________
   b) Fill in this chart of possibilities.

NUMBER OF MILKSHAKES MORGAN DRANK	3		9	12			18				
NUMBER OF MILKSHAKES HARRY DRANK	1	2			5	8		11			

3) Harry ate 4 hamburgers for every 1 hamburger Morgan ate.
   a) Fill in this chart of possibilities.
   b) Who ate more hamburgers? Morgan or Eddy? ____________

NUMBER OF HAMBURGERS MORGAN ATE	1		3		5	7					
NUMBER OF HAMBURGERS HARRY ATE	4	8		16			24	36			

4) Use the information in the problems above to fill in this chart of possibilities.

NUMBER OF HAMBURGERS MORGAN ATE	1	2			5						
NUMBER OF HAMBURGERS HARRY ATE				16			40				
NUMBER OF HAMBURGERS EDDY ATE			6			16		12			

I'D WALK A MILE

Equivalent
RATIO

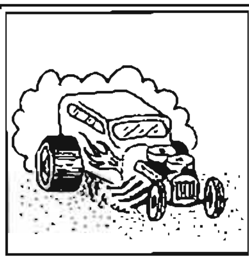
Materials needed: Stopwatch
Tape measure
Bicycle

An extension of this activity could be to find the rate of walking up stairs, one step at a time or two steps at a time.

15 sec.	30 sec.	1 minute
_____ feet	_____ feet	_____ feet

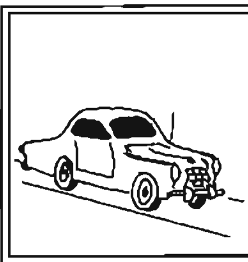
(1) Walk in a straight line for 15 seconds, 30 seconds and 1 minute. Record your rate in the chart above.

(2) Find the speed for each of the below in feet per minute. A calculator can help with the computation.



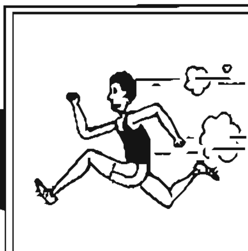
200 mph

_______ fpm



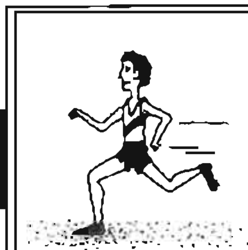
55 mph.

_______ fpm



100 yds. in 10 sec.

_______ fpm



1 mile in 4 min.

_______ fpm

(3) The speed of the sprinter is _____ times faster than your walking rate.

(4) If the speed of the dragster is 200 times an adult's walking rate what is the adult's speed? _______________

(5) Outside, find your speed for riding a bicycle 15 seconds, 30 seconds and 1 minute. Record.

15 sec.	30 sec.	1 minute
_____ feet	_____ feet	_____ feet

(6) How does your bicycle speed compare to your walking speed?

TYPE: Activity

IDEA FROM: *New Oxford Junior Mathematics*, Book 4
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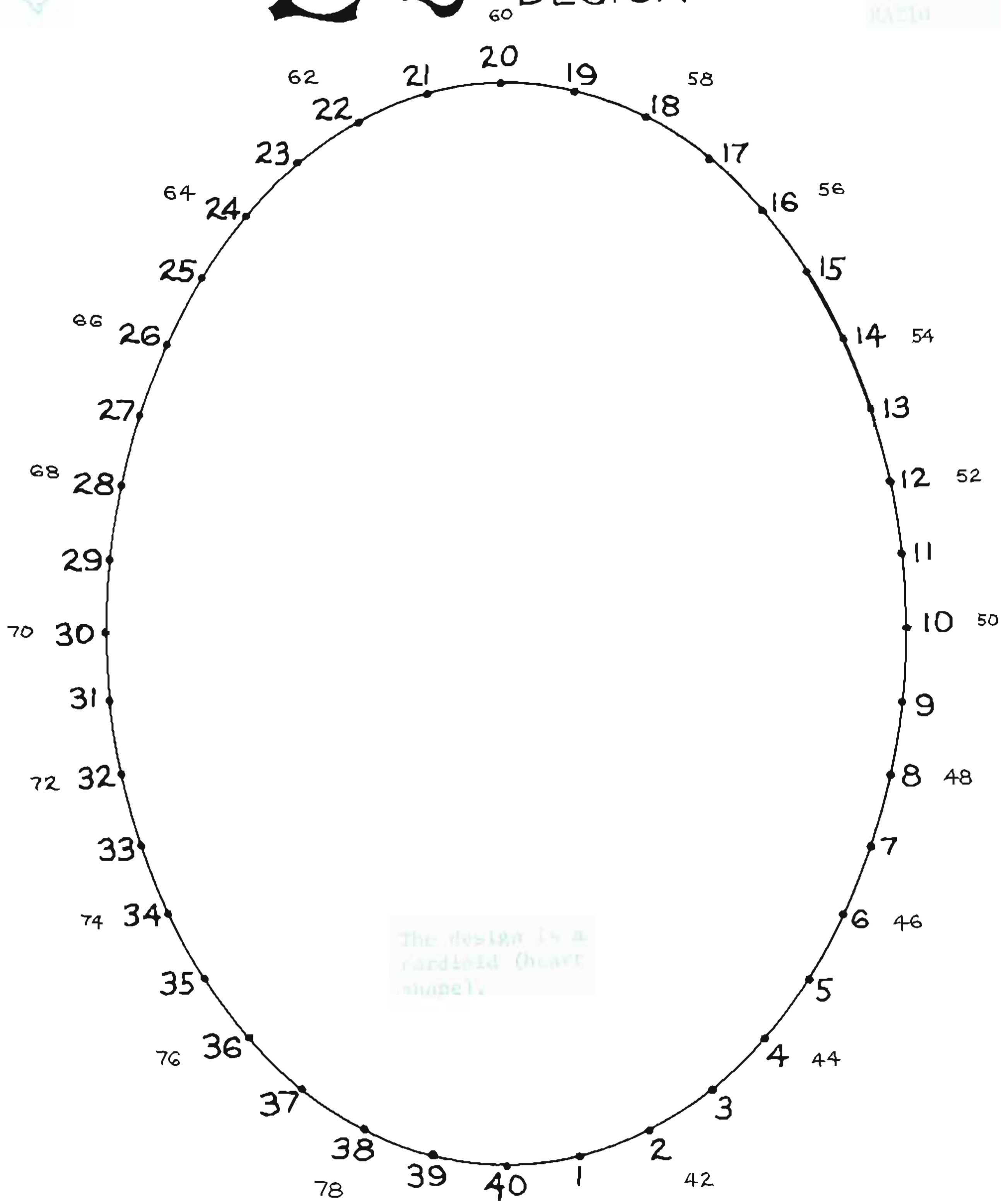
A Lovely DESIGN

Equivalent RATIO



Start with 1 and use a line segment to connect each point with another point, so that the numbers joined are in the ratio 1:2. For example, connect 18 to 36 and connect 37 to 74.

IDEA FROM: *Aftermath*, Book 4

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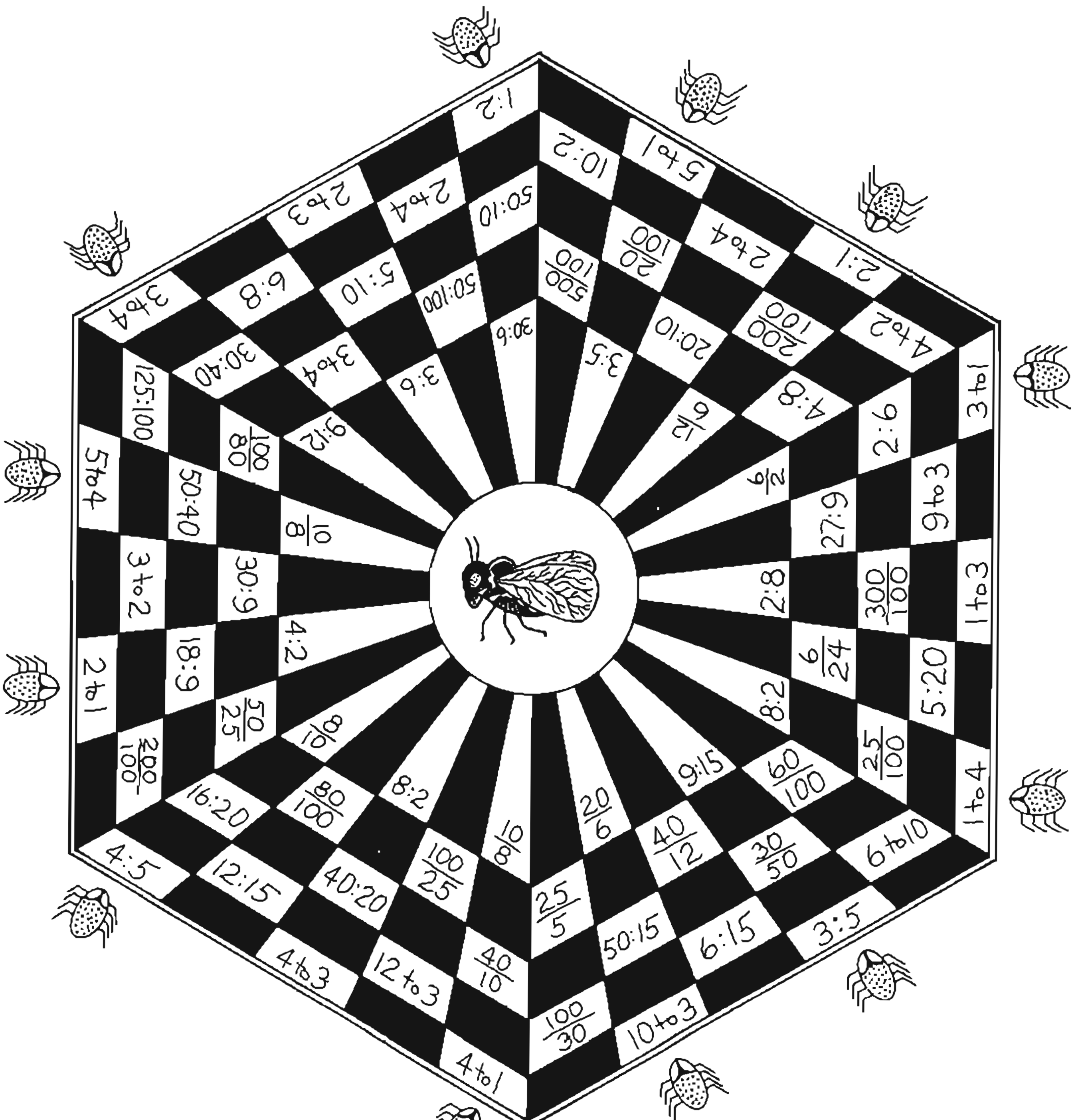
SPIDER TO FLY RATIOS

START AT EACH SPIDER AND FIND A PATH TO THE FLY USING EQUIVALENT RATIOS.



IDEA FROM: *Patterns in Space*

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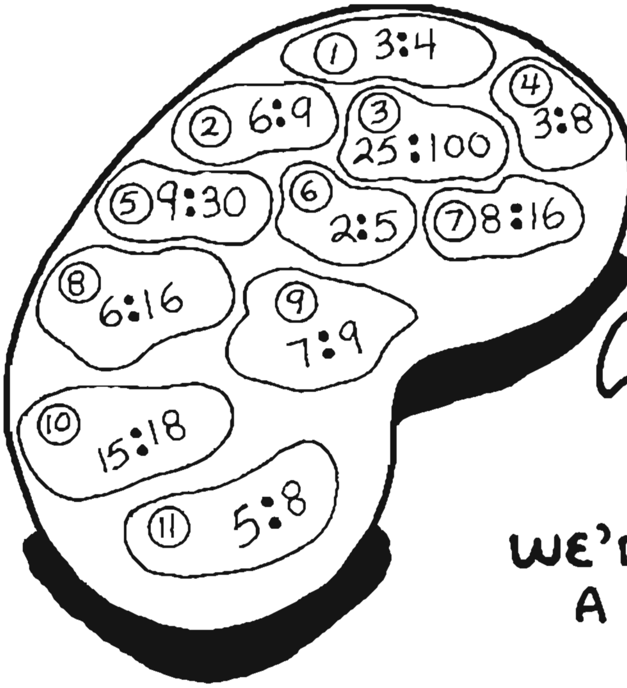
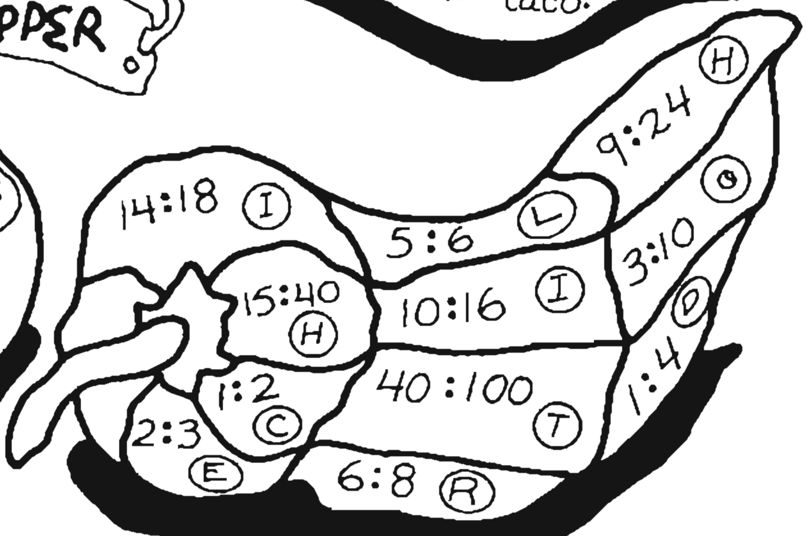
SPICY RATIOS

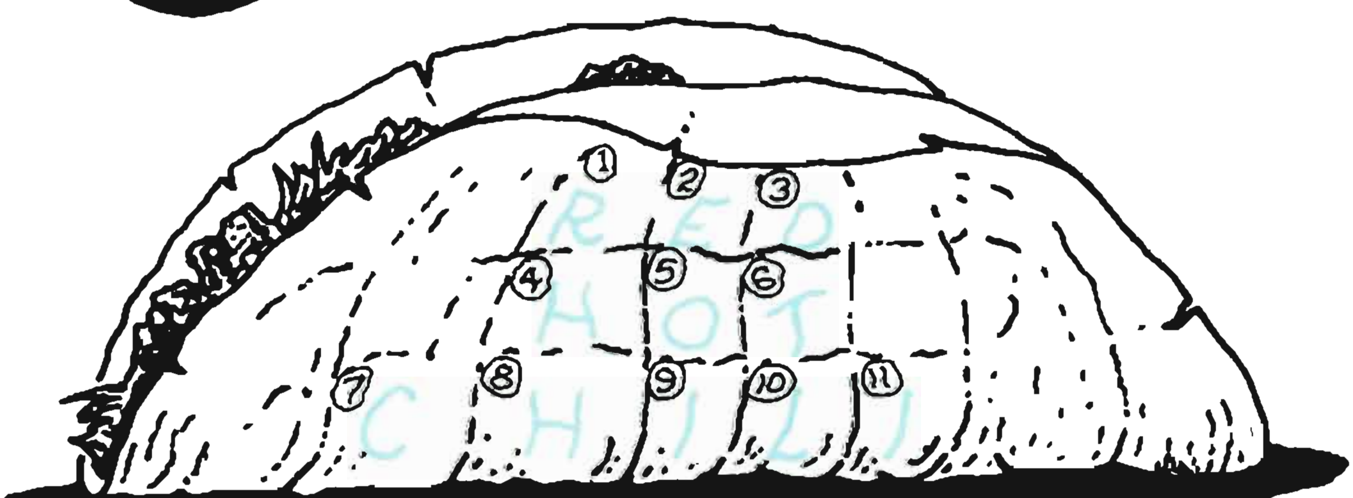
Hola, Amigos!
Match each 'bean' ratio with an equivalent 'pepper' ratio. Write the letter in the appropriate part of the taco.

WHAT DID THE BEAN SAY TO THE PEPPER?

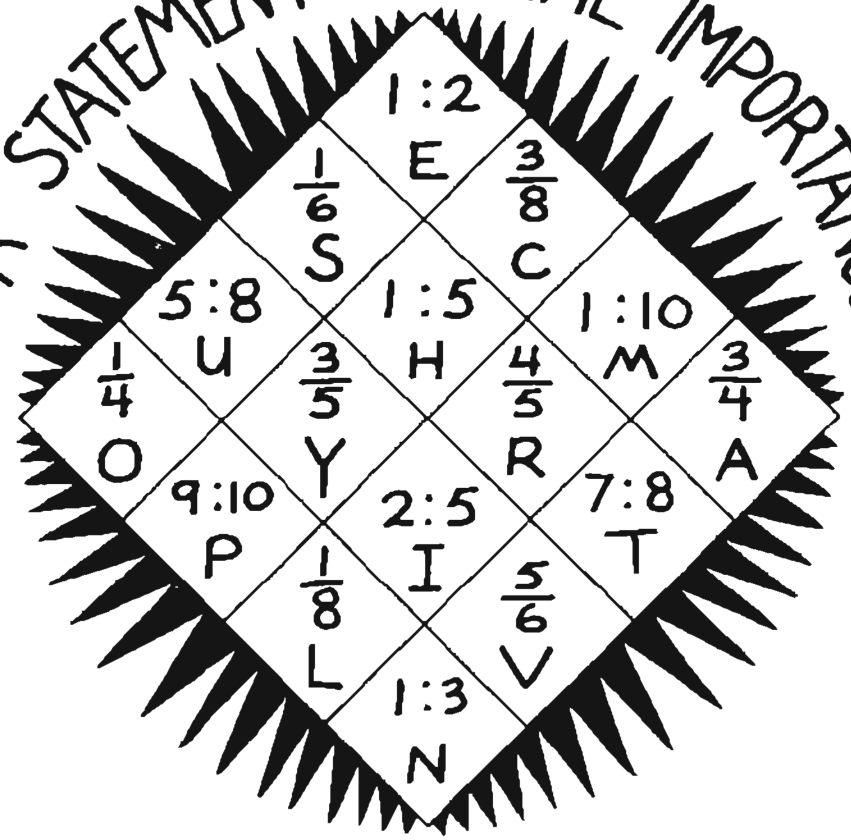




WE'D MAKE A GREAT ...



A STATEMENT OF PRIME IMPORTANCE



___ $\frac{5}{20}$ ___ $\frac{4}{12}$ ___ $\frac{15}{40}$ ___ $\frac{7}{14}$ ___ 60:100 ___ 2:8 ___ 25:40

___ 10 to 50 ___ 6 to 8 ___ 15 to 18 ___ 4 to 8   ___ 5:30 ___ 9:18 ___ 50:100 ___ 6:18

___ $\frac{25}{100}$ ___ $\frac{5}{15}$ ___ $\frac{2}{4}$ ___ 12 to 24 ___ 50 to 60 ___ 5 to 10 ___ 3 to 9

___ 18:20 ___ 20:25 ___ 40:100 ___ 5:50 ___ 18:36 , ___ 6 to 10 ___ 3 to 12 ___ 10 to 16

___ $\frac{2}{10}$ ___ $\frac{75}{100}$ ___ $\frac{10}{12}$ ___ $\frac{10}{20}$ ___ 3 to 18 ___ 8 to 16 ___ 6 to 12 ___ 2 to 6

___ 21:24 ___ 20:100 ___ 3:6 ___ 10:100   ___ $\frac{12}{16}$ ___ $\frac{4}{32}$ ___ $\frac{2}{16}$.

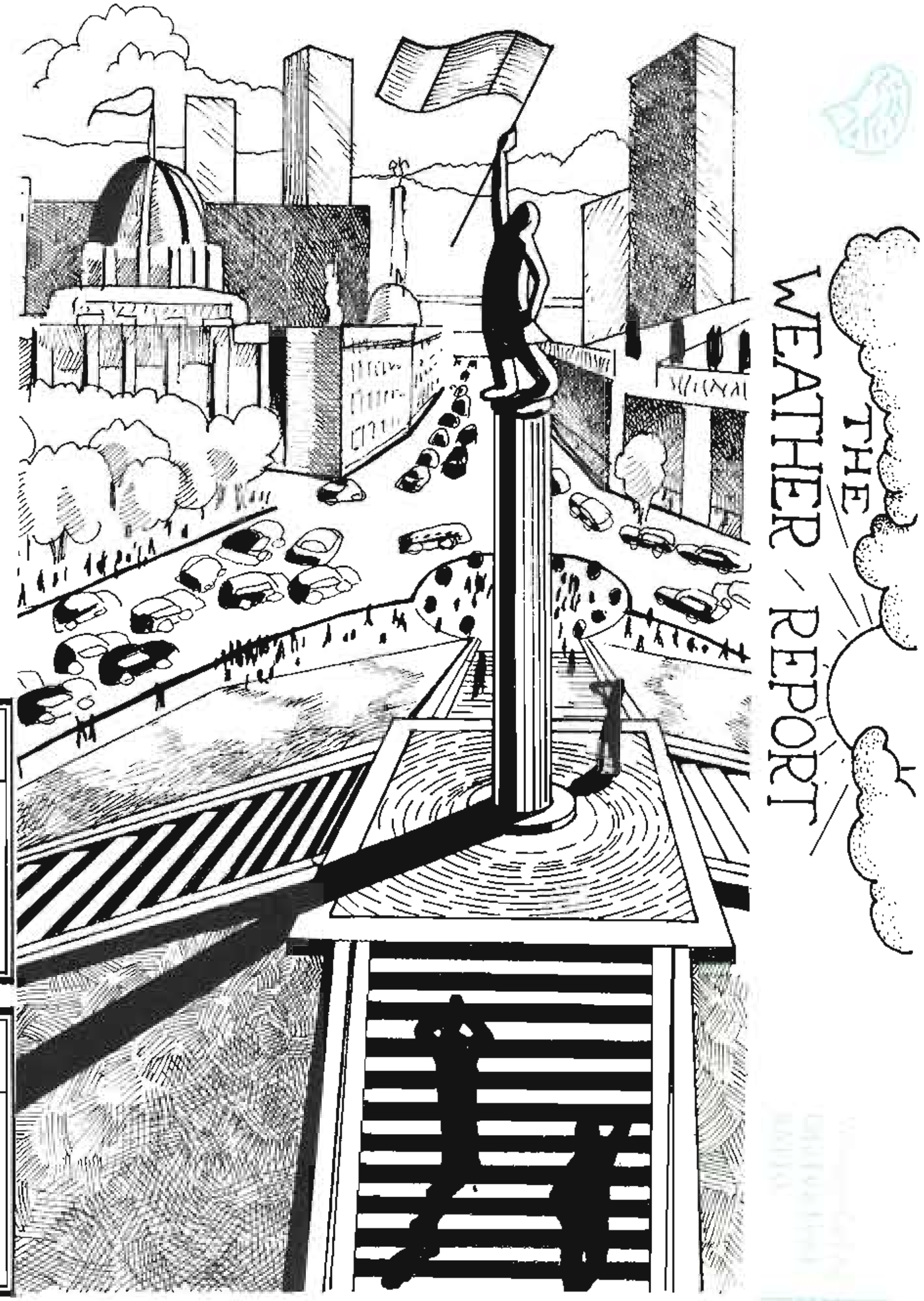
WHAT'S THE WEATHER REPORT IN MEXICO CITY?

TO FIND THE ANSWER, WRITE THE LETTER OF EACH RATIO ABOVE AN EQUIVALENT RATIO IN THE CHART BELOW. USE EACH RATIO ONLY ONCE. SOME RATIOS IN THE BOXES WILL NOT HAVE AN ANSWER.

E. 8 : 16
I. 2 to 1
A. 33 : 11
O. 21 : 56
E. 50 to 100
T. 4 to 1
I. 10 : 5
M. 3 to 5

N. 44 : 4
B. 15 : 20
H. 14 to 16
L. 5 to 50
D. 5 : 25
L. 3 : 30
Y. 1 to 8
A. 6 to 2

O. 3 to 8
A. 60 to 20
T. 20 to 5
D. 2 : 10
C. 4 to 5
T. 40 to 10
A. 39 : 13
H. 35 : 40



IT WILL

B	E		C	H	I	L	I		T	O	D	A	Y
3 to 4	1 : 2	5 : 6	8 : 10	7 to 8	50 : 25	1 to 10	4 to 2	5 : 1	8 to 2	30 : 80	1 to 5	12 to 4	5 : 40

A	N	D		H	O	T		T	A	M	A	L	E
15 to 5	11 to 1	20 to 100	3 to 2	12 to 24	6 to 16	12 : 3	1 to 7	36 : 9	3 to 1	60 : 100	30 to 10	10 : 100	5 : 10

RATIO DOMINOES

Wanted:
- 2 or more players
- Set of ratio dominoes
- 1 gallon of enthusiasm

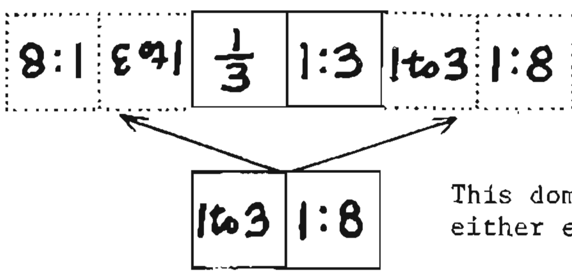
These are the rules:

a) All dominoes are placed face down on the table.

b) Each player draws 5 dominoes one at a time.

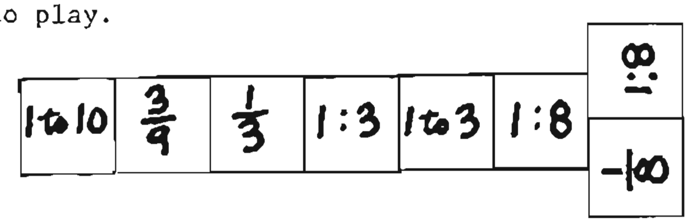
c) The player drawing the largest double (equivalent ratio on both parts of the domino) plays it in the middle of the table. If no player gets a double in the first 5 draws all players continue to draw in turn until someone gets a double.

d) The next player to the left tries to play a domino on the end of the double. If a play cannot be made, the player draws extra dominoes until he can play. Play continues to the left.



This domino can be played on either end of the double.

e) All doubles (except the first) are placed at right angles to give more places to play.



f) The first player to play all of his dominoes is the winner.

RATIO DOMINOES (CONTINUED)
CONSTRUCTION HINTS

a) A suggested size is 3 cm by 6 cm.

b) The dominoes can be made from note cards, tagboard, poster board, or scraps of lumber. (Perhaps you have a student that would like to cut the lumber as an extra project.)

c) The following are suggestions for a set of 45 dominoes made using the ratios 1:2, 1:3, 1:4, 1:5, 1:6, 1:7, 1:8, 1:9, 1:10. Of course, you may choose any set of ratios that you want. Each is paired with an equivalent ratio to form nine doubles. Then each is paired with all other ratios to form the remainder of the dominoes.

d) For each ratio the three ratio notations should be used, 1 to 2, 1:2, and $\frac{1}{2}$, and also two equivalent ratios are needed, say 5:10 and 50 to 100.

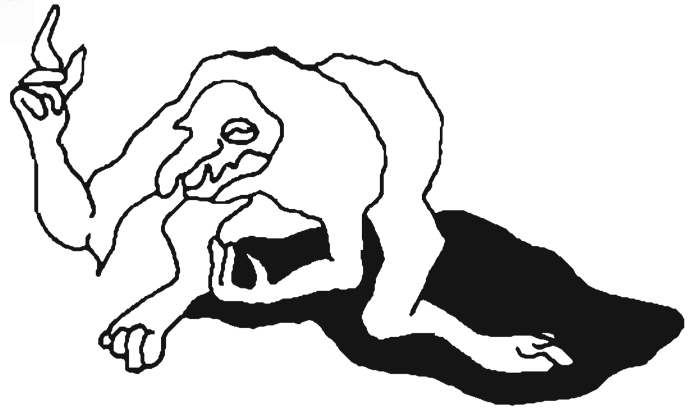
e) Suggested pairings for the 45 dominoes.

1:2	1:3	1:4	1:5	1:6
$\frac{1}{2}$, 1 to 2	1:3, $\frac{1}{3}$	$\frac{1}{4}$, 2 to 8	1 to 5, $\frac{20}{100}$	1:6, $\frac{2}{12}$
$\frac{50}{100}$, $\frac{1}{3}$	$\frac{3}{9}$, 1 to 4	$\frac{25}{100}$, $\frac{20}{100}$	1 to 5, 1 to 6	$\frac{6}{36}$, 1 to 7
1 to 2, $\frac{1}{4}$	1:3, 1 to 5	1:4, 1 to 6	$\frac{2}{10}$, 1:7	$\frac{1}{6}$, 7 to 56
3:6, 2:10	$\frac{1}{3}$, $\frac{2}{12}$	1 to 4, 1:7	$\frac{20}{100}$, 1 to 8	1 to 6, 1:9
1 to 2, 5:30	1:3, 2 to 14	2 to 8, $\frac{3}{24}$	$\frac{1}{5}$, $\frac{1}{9}$	1:6, 1 to 10
$\frac{1}{2}$, $\frac{4}{28}$	4 to 12, 1:8	$\frac{6}{54}$, 2:8	1:5, 1 to 10	
5:10, $\frac{1}{8}$	$\frac{1}{3}$, 1 to 9	1:4, $\frac{10}{100}$		
1:2, 1:9	1 to 3, $\frac{10}{100}$			
1:10, $\frac{50}{100}$				

1:7	1:8	1:9	1:10
$\frac{1}{7}$, 1:7	1:8, 1 to 8	1 to 9, 2:18	$\frac{5}{50}$, 10:100
$\frac{4}{28}$, $\frac{1}{8}$	$\frac{1}{8}$, $\frac{1}{9}$	1:9, $\frac{1}{10}$	
1:7, 1 to 9	1 to 8, 5:10		
1:7, 1:10			

MONSTER RATIO

For information on the construction of ratio cards see next page.

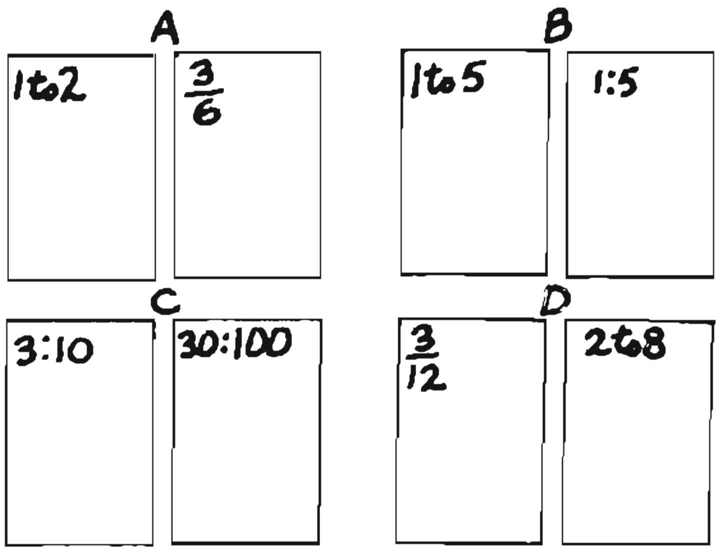


You will need the Monster Ratio cards and 2 or more players.

Rules:

a) The dealer deals out all of the cards, one at a time, to the players.

b) All players lay down the matches in their hands. A match is two cards showing equivalent ratios. Some examples of matches are shown.



c) When all matches have been laid down from the players' hands, the dealer draws a card from the hand of the player to the left and, using the card, tries to make a match. If no match can be made, the player keeps the card in his hand.

d) The player to the left then draws a card from the next player, and so on.

e) The player that finishes the game holding the Monster card is the loser.

MONSTER RATIO

(CONTINUED)

DIRECTIONS FOR MAKING RATIO RUMMY
AND MONSTER RATIO CARDS.

a) The deck consists of 54 cards with an additional Monster card.

b) There should be 6 cards for each ratio used, 3 cards using the 3 ratio notations, and 3 cards using equivalent ratios. For example,

1 to 2, 1:2, $\frac{1}{2}$, 4 to 8, $\frac{2}{4}$, 50:100.

c) If possible, one of the equivalent ratios should be expressed with 100 as the second term as readiness for percent.

d) The choice of ratios is left to the teacher with these suggestions. Perhaps two decks, Ratio 1 and Ratio 2, could be made with 1:2, 1:4, 3:4, 1:5, 4:5, 1:10, 3:10, 7:10, 9:10 (these all easily convert to hundredths) as the first deck, and 1:3, 2:3, 2:5, 3:5, 1:6, 1:8, 3:8, 5:8, 7:8 as the) second deck.

e) The cards can be made from 3 x 5 note cards. By cutting the note cards into two 3 x $2\frac{1}{2}$ cards you will have convenient sized cards. Blank cards with rounded corners may be purchased at the rate of $3.30 per 500 cards. These cards must be ordered on an order form that can be obtained from:

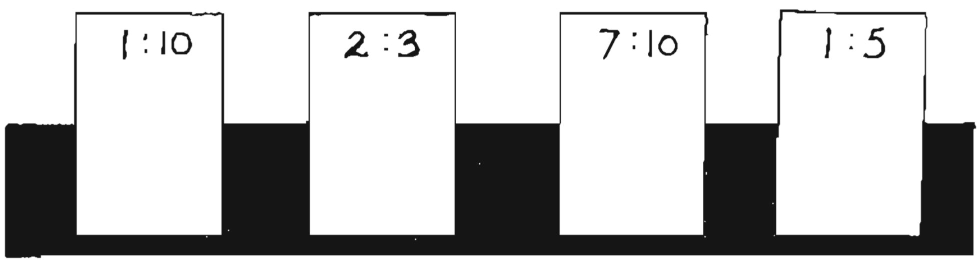
Personalized Instruction Center
NCEBOCS
830 South Lincoln
Longmont, Colorado 80501

Ratio Rummy

a) 2-5 players are needed.
b) Each player is dealt 7 cards.
c) The remaining cards are placed face down to form a stack with the top card turned up to form the discard pile.
d) The player to the left of the dealer draws the top card, either from the stack or the discard pile. A player must discard each turn.
e) Each player tries to lay his cards on the table by:

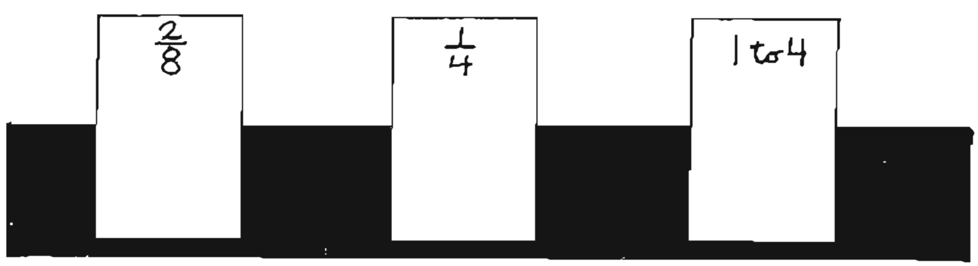
1) Making a Type I book of 4 simplified ratios having the same ratio notation.

Type I



2) Making a Type II book of 3 or more cards that show equivalent ratios.

Type II

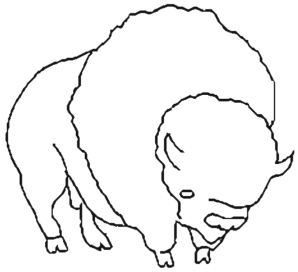


3) Playing a card on a Type II book already played by someone else. 1:4 or $\frac{3}{12}$ could be played on the above book.

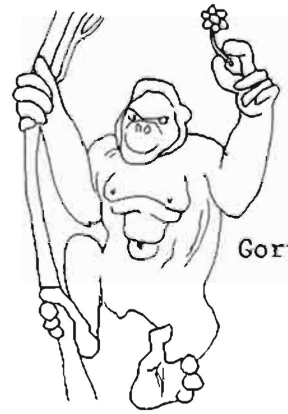
f) Scoring
1) Score 5 points for the first person to lay down all his cards.
2) Score 1 point for each card laid down.
3) Subtract 1 point for each card not laid down.
4) First player to get 30 points wins the game.

ANIMAL AGES

Average Life Span (in years)

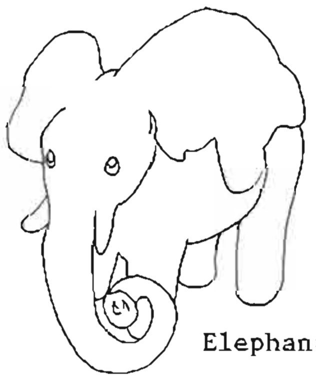


Buffalo - 20



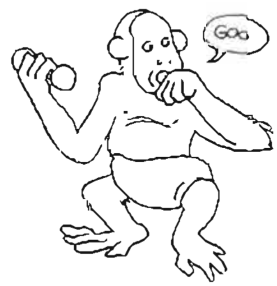
Gorilla - 25

Rabbit - 5



Elephant - 50

Cat - 15



Chimpanzee - 30

Write a ratio comparing the average life spans of these animals. Then simplify each ratio.

a) Buffalo to Rabbit _____:_____ = _____:_____

b) Gorilla to Elephant _____:_____ = _____:_____

c) Chimpanzee to Cat _____:_____ = _____:_____

d) Rabbit to Buffalo _____:_____ = _____:_____

e) Chimpanzee to Elephant _____:_____ = _____:_____

f) The ratio of the average life span of the Buffalo to Man is 1:3. About how many years can Man expect to live? __________

g) The ratio of the average life span of a Gorilla to a Gerbil is 5:1. About how long can a Gerbil be expected to live? __________

h) The ratio of the average life span of a Buffalo to a Guinea Pig is 5:1. About how long can a Guinea Pig be expected to live? __________

i) The ratio of the average life span of a Cat to a Dog is 1:1. What is the expected life span of a Dog? __________

j) The ratio of the average life spans of a Bat, a Cat, and a Grizzly Bear is 1:3:6. About how long is the average life span of a Bat and a Grizzly Bear? _____ _____

IDEA FROM: *Activities with Ratio and Proportion*
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RATIOS IN YOUR SCHOOL

Fill in this chart. Guess first then get the actual numbers from your teacher.

PEOPLE IN YOUR SCHOOL	GUESSES			ACTUAL NUMBER		
	NUMBER OF FEMALES	NUMBER OF MALES	TOTAL	NUMBER OF FEMALES	NUMBER OF MALES	TOTAL
COUNSELORS						
SCHOOL NURSES						
YOUR CLASS						
ALL STUDENTS						
TEACHERS						
PRINCIPALS						
COOKS						
CUSTODIANS						
SECRETARIES						
TOTALS						

I. Use the chart of actual numbers to find these ratios. If possible write each ratio in simplest form.

The ratio of:

a) Males to females in your class is ___ to ___ or ___:___.

b) Students in your class to all students is ___ to ___ or ___:___.

c) Principals to all students is ___ to ___ or ___:___.

d) Cooks to everyone in school is ___ to ___ or ___:___.

e) Female teachers to male teachers is ___ to ___ or ___:___.

f) Female teachers to total teachers is ___ to ___ or ___:___.

g) Male cooks to female cooks is ___ to ___ or ___:___.

h) Male principals to female principals is ___ to ___ or ___:___.

i) Female secretaries to male secretaries is ___ to ___ or ___:___.

j) All teachers to all students is ___ to ___ or ___:___.

Pupil-Teacher Ratio is the closest whole number of students for each teacher. For example, in a school with 224 students and 10 teachers the pupil-teacher ratio is 224 to 10 or about 22 to 1.

k) Find the pupil-teacher ratio for your school. ___:___

l) Find the pupil-counselor ratio for your school. ___:___

ONE MAN ONE VOTE

From a world almanac find
- (a) the number of senators in Congress ____________
- (b) the number of representatives in Congress ____________
- (c) the ratio of senators to representatives is ____________:____________ or about ____________:____________.

How many senators does your state have in Congress? __________
How many representatives does your state have in Congress? __________
Write the ratio of senators to representatives in your state _______:_______.
Are the two ratios equivalent?

In the 94th Congress the House of Representatives has ________ Democrats and ________ Republicans. The ratio of Democrats to Republicans is about _____:_____
What is the ratio of Democrats to Republicans in the House of Representatives in your state? _______:_______

There are 16 women in the 94th Congress. All serve in the House of Representatives. The ratio of women to men is ________:________, or about 1 woman for every _____ men.

The number of representatives each state has is based on the population of the state. Oregon has 4 representatives out of 435 or about 1:100. If the population of the United States is about 200,000,000 people, approximate the population of Oregon. ____________________

Check in the almanac to see how close your approximation is.

Use 2,000,000 people as the population of Oregon and approximate the population of these states.

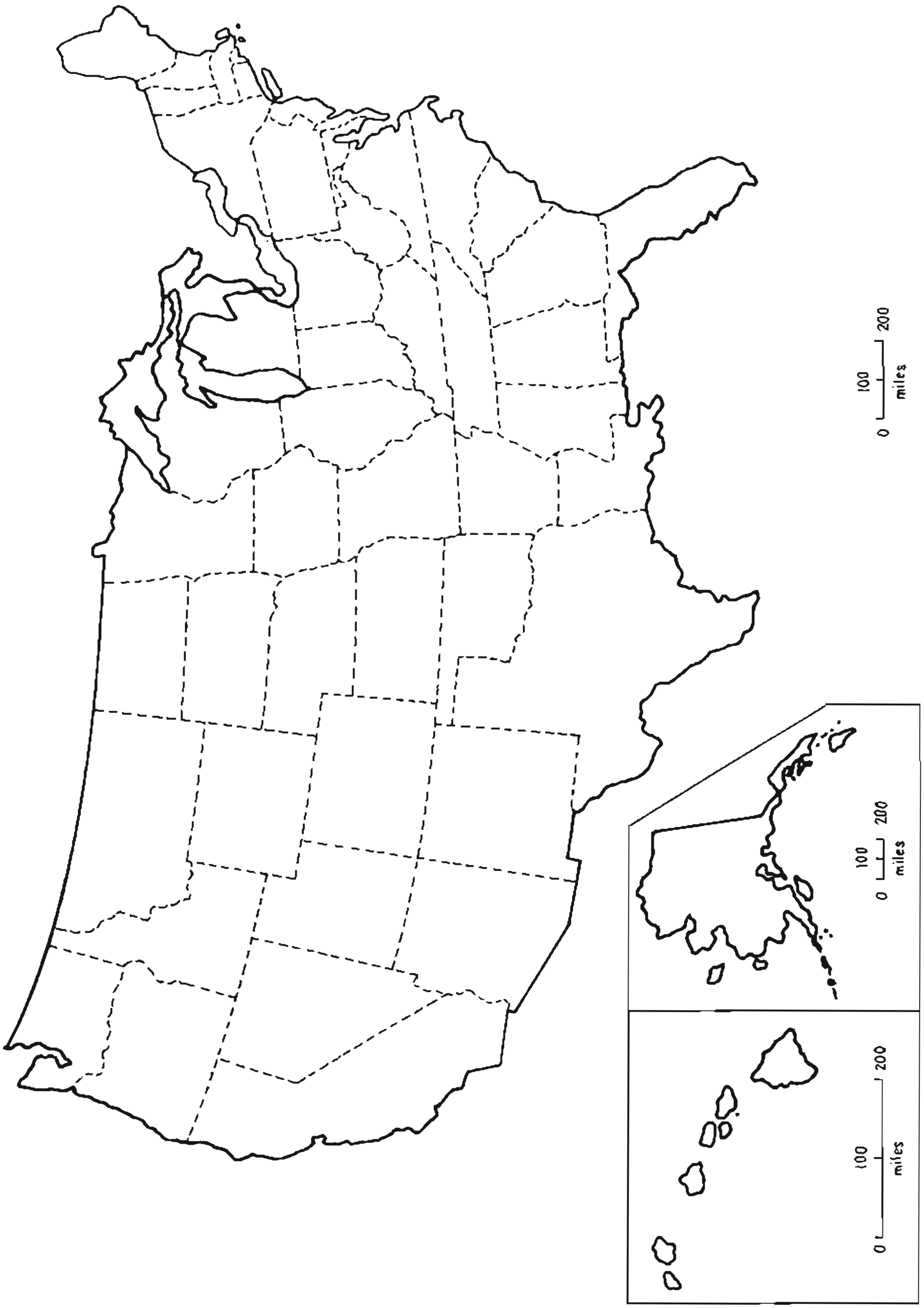
State	Number of Representatives	Approximate Population	Actual Population
Tennessee	________	________	________
Maine	________	________	________
Massachusetts	________	________	________
Nevada	________	________	________
California			

On a map of the United States color the states according to the number of representatives.

1 - 5	Red	21 - 25	Blue
6 - 10	Orange	26 - 30	Purple
11 - 15	Yellow	31 - 35	Violet
16 - 20	Green	36 - more	Black

Can you see an area of the United States that has a large population? a small population?

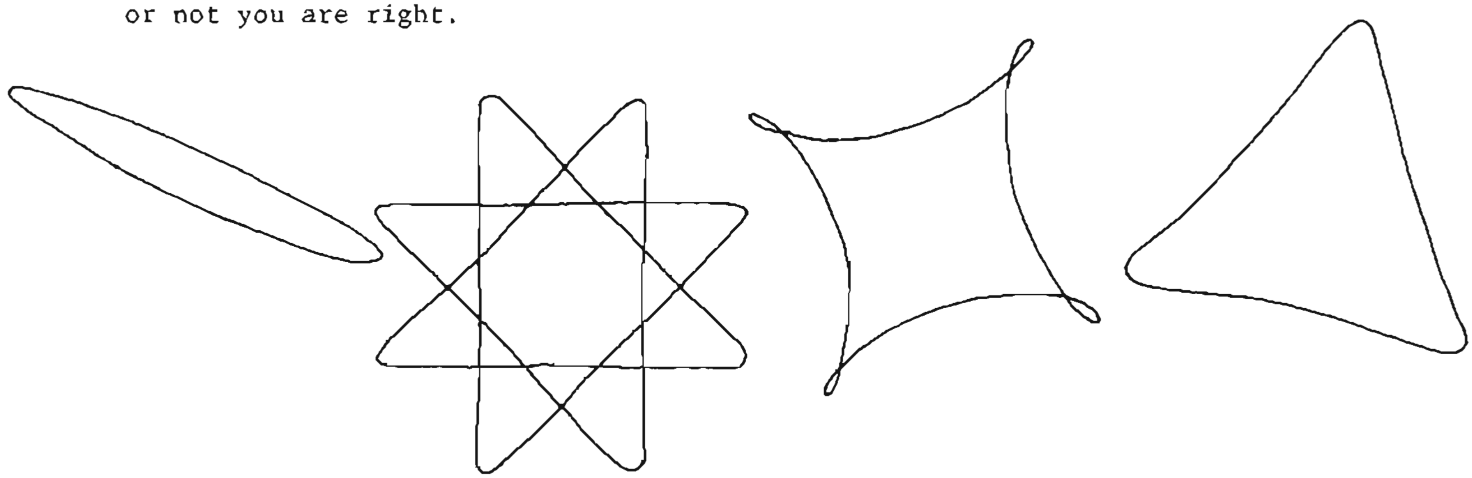
ONE MAN ONE VOTE (continued)



Poppin' Wheelies In A Ring

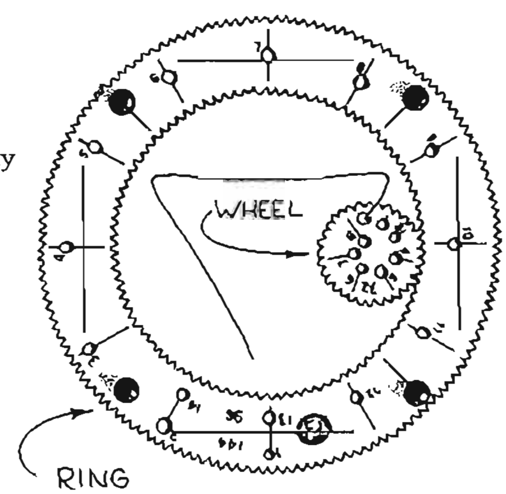
Use Spirograph rings and wheels for your experiment.

Look at the patterns below. Can you decide which rings and wheels have been used to draw each of them? If you think you know, try drawing them to see whether or not you are right.



Look at the numbers on the ring and wheel for each pattern. Do these help you decide what patterns you can get?

Examine the two rings in the set. Both have many numbers on them. One ring has 96 and 144. This means there are 96 teeth on the inside of the ring and 144 on the outside. Look at one of the wheels. The largest number tells you how many teeth it has.



1) Use the 96 ring and the 32 wheel. Draw a pattern with it.

2) How many loops are there on the shape?

3) How many times must the wheel go around the inside of the ring before the pattern begins to repeat?

Poppin' Wheelies In A Ring (continued)

Use the information in the table to draw more shapes. Before you start drawing, try to decide how many loops the shape will have and how many times the wheel will have to go round the ring before the pattern is repeated. You select the ring and wheel sizes for the last experiment.

Teeth on ring	Teeth on wheel	Loops on shape	Number of times wheel goes around ring before pattern repeats	Teeth on ring: Teeth on wheel	Simplified ratio
96	32			96:32	3:1
96	24			:	:
96	72			:	:
105	75			:	:
96	48			:	:
105	45			:	:
96	56			:	:
				:	:

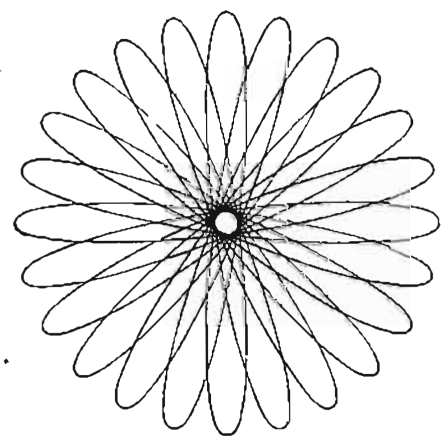
Can you explain why you have to go around the inside of the ring a number of times to complete some shapes and why a certain number of loops appear?

(1) Predict how many loops you will get with the 105 ring and the 60 wheel. __________. Check your answer by drawing the shape.

(2) Use the 96 ring. Which wheel would you use to get a shape that has 16 loops if the wheel goes around the ring 7 times before the pattern repeats? __________.

(3) Draw some more shapes. Predict how many loops each shape will have before you draw it.

(4) Look at the shape on the right. It was made using a 96 ring. By counting the loops, can you decide which wheel was used?



PEOPLE RATIO

Population density is the average number of people for each square mile of land considered. It is computed by comparing the total population to the total land area. To make this ratio meaningful to students modified population densities can be investigated.

I. Gather data from the students informally during class discussion.
   a) Find the average number of people per household for the class. Example: There are 28 students in the class, each from a different household. Each student reports the number of people living at home. These numbers are added to obtain the total population, say 105. The ratio 105:28 is about equivalent to 4:1 or 4 people per 1 household.
   b) Find the average number of pets per household for the class.
   c) Find the average number of dogs (cats) per household for the class.

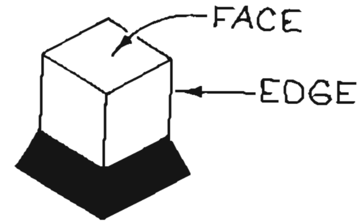
II. Use local census data and an almanac.
   a) Have students find the population density of their city (county). (Population figures can be obtained from census data which is usually filed in city libraries.) Has the population density increased or decreased in the last fifty years? What are some of the changes that occur in the community when the population increases (decreases)? How are jobs, transportation systems and housing affected?
   b) Have students find the population density of their state. Compare this density to the population density of the city or county. Compare the densities of nearby states. What conclusions can one make?
   c) Ask if there are students who have visited or lived in foreign countries. Have students make a chart of these countries which includes the population, area and population density for each country.

Country	Population (1970)	Area (sq. miles)	Population Density
U.S.	203,235,298	3,536	$\frac{203}{4} \rightarrow \frac{51 \text{ people}}{1 \text{ sq. mile}}$
Canada			
Mexico			
.			
.			
.			

Compare and discuss the population density of the different countries. What problems result from high population density?

III. Have students use the almanac to find the birth rate for the foreign countries listed in II. (c).
Does the birth rate have any relationship to the population density? What effects does the "population explosion" have on population density? Why do some countries have higher birth rates than others?
Discuss the "population explosion" problem. Predict the population of various countries by the year 2000. How will the predictions affect the population density?

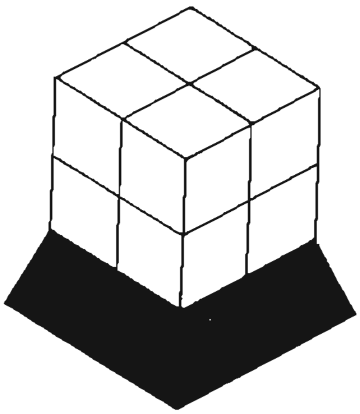
SURFACE AREA & RATIOS 1



Materials needed: A set of 125 centimetre cubes

Activity:

1 a) Use 1 cube. Call it the unit cube.
b) How many faces on the cube?
c) Each edge is 1 cm, so each face is 1 cm by 1 cm or 1 cm^2.
d) The unit cube has a surface area of ___ x 1 cm^2 or ___ cm^2.



2 a) Build a model of a larger cube, so all edges are 2 cm. You should have used 8 cubes.
b) What is the area of each face? ________
c) What is the area of this cube? 6 x ______ or _____

3 Continue to make larger cubes with edges of 3 cm, 4 cm, 5 cm . . . until you run out of cubes. Find the surface area of each cube and record it in this chart. See if you can finish the chart up to a cube that has an edge of 10 cm.

EDGE OF CUBE (CM)	TOTAL CUBES USED	AREA OF 1 FACE (cm^2)	SURFACE AREA OF CUBE (cm^2)	RATIO OF SURFACE AREAS OF LARGE CUBE TO SMALL CUBE	SIMPLIFIED RATIO
1	1	1	6	6 : 6	1 : 1
2	8	4	24	24 : 6	4 : 1
3	27				
4	64				
5					
6					
7					
8					
9					
10					

4 Predict the ratio of the surface area of a large cube to the surface area of the unit cube if the large cube has an edge of:

a) 20 cm ____:____
b) 12 cm ____:____
c) 1.5 cm ____:____
d) n cm ____:____

IDEA FROM: *New Oxford Junior Mathematics,* Book 5

SURFACE AREA & RATIOS 2

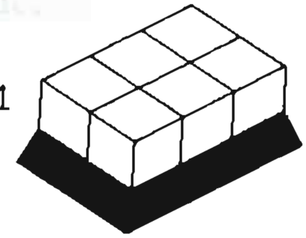
E
S
I

Equivalent
RATIO

The student should have worked [illegible] before this page and should have the information from that page available.

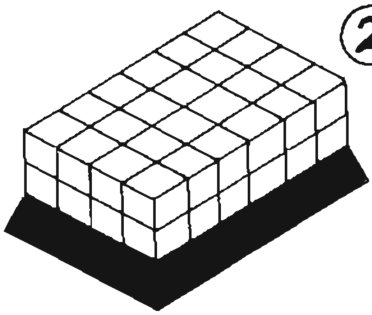
Materials: A set of centimetre cubes (at least 200)

MODEL 1



Activity:

1
a) Make a 3 x 2 x 1 model from the cubes.
b) Each face of each cube has a surface area area of 1 cm^2
c) The model has 6 faces. What is the surface area of the top? ____ cm^2, bottom? ____ cm^2, front? ____ cm^2, back? ____ cm^2, left? ____ cm^2, right? ____ cm^2, ____
d) The surface area of the entire model is ____ cm^2.



2
a) Enlarge Model 1, so it is twice as long, twice as wide and twice as high.
b) Model 2 is now ___ x ___ x ___
c) In Model 2 what is the surface area of the top? ____ cm^2, bottom? ____ cm^2, front? ____ cm^2, back? ____ cm^2, left? ____ cm^2, right? ____ cm^2
d) The surface area of Model 2 is ____ cm^2.
e) The ratio of the surface areas of model 2 to model 1 is ___ : ___ or ___ : ___

3
a) Enlarge Model 1, so it is three times as long, three times as wide, and three times as high.
b) Model 3 is now ___ x ___ x ___.
c) The surface area of Model 3 is ____ cm^2.
d) The ratio of the surface areas of Model 3 and Model 1 is ____:___ or ____:____.

4 Use the results of *Surface Area & Ratio 1* to complete this chart for models that have dimensions 4 times the dimensions of Model 1; 5 times; 6 times.

MODEL	SIZE	SURFACE AREA (cm^2)	RATIO OF THE SURFACE AREA OF THIS MODEL TO THE SURFACE AREA OF MODEL 1
1	3 x 2 x 1		
2	6 x 4 x 2		88 : 22 = 4 : 1
3	9 x 6 x 3		: 22 = : 1
4			: 22 = 16 :
5			: = :
6			: = :

5 Can you predict the ratio of the surface area of a model with dimensions that are 10 times the dimensions of Model 1? ____:____
that are n times the dimensions of Model 1? ____:____

IDEA FROM: *New Oxford Junior Mathematics*, Book 5
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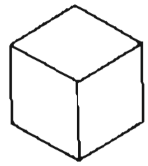
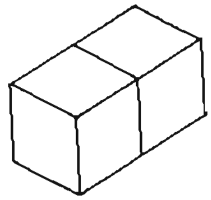
VOLUME AND RATIO 1



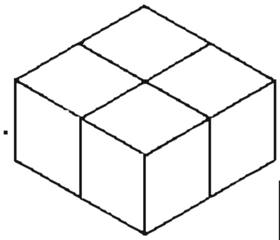
Materials needed: A set of centimetre cubes.

1 Use a centimetre cube as the unit cube. The volume of this unit cube is 1 cm x 1 cm x 1 cm = 1 cm^3.

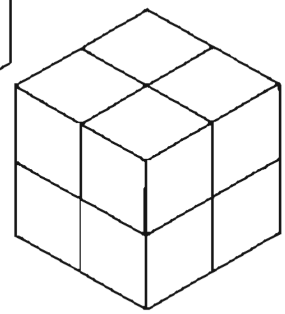


2 Make 3 different models:

a) one twice as long as the unit cube.



b) one twice as long and twice as wide as the unit cube.



c) one twice as long, twice as wide and twice as high as the unit cube.

MODEL	DIMENSIONS	VOLUME (cm^3)	RATIO OF THE VOLUME OF THIS MODEL TO THE VOLUME OF THE UNIT CUBE
A	2 x 1 x 1		
B	2 x 2 x 1		
C	2 x 2 x 2		

3 Make 3 different models:

d) one three times as long as the unit cube.
e) one three times as long and three times as wide as the unit cube.
f) one three times as long, three times as wide and three times as high as the unit cube.

MODEL	DIMENSIONS	VOL (cm^3)	RATIO OF THE VOLUME OF THIS MODEL TO THE VOLUME OF THE UNIT CUBE
D			
E			
F			

4 Make 3 different models:

g) one four times as long as the unit cube.
h) one four times as long and four times as wide as the unit cube.
i) one four times as long, four times as wide and four times as high as the unit cube.

Model	DIMENSIONS	Vol (cm^3)	RATIO OF THE VOLUME OF THIS MODEL TO THE VOLUME OF THE UNIT CUBE
G			
H			
I			

5 The Jones have a swimming pool that is 2 metres deep, 4 metres wide and 7 metres long. Mr. Smith, who lives next door, wants to build a larger pool. How many times as much water will Mr. Smith need if he builds a pool twice as long and twice as wide? ____________________

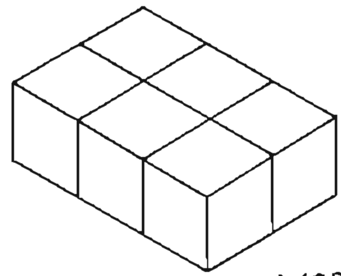
IDEA FROM: *New Oxford Junior Mathematics*, Book 5

Volume and Ratio 2

Materials needed: A set of centimetre cubes

Activity:

(1) a) Use the cubes and make this model.
b) The volume (in cm^3) of this model is ________.



MODEL 1

(2) Make 3 models:
a) One twice as long as Model 1.
b) One twice as long and twice as wide as Model 1.
c) One twice as long, twice as wide, and twice as high as Model 1.

Model	Dimensions	Volume (cm^3)	Ratio of the volumes of this model to Model 1	Simplified ratio
a	6 × 2 × 1	12 cm^3	: 6	: 1
b			:	:
c			:	:

(3) Make 3 more models:
d) One three times as long as Model 1.
e) One three times as long and three times as wide as Model 1.
f) One three times as long, three times as wide, and three times as high as Model 1.

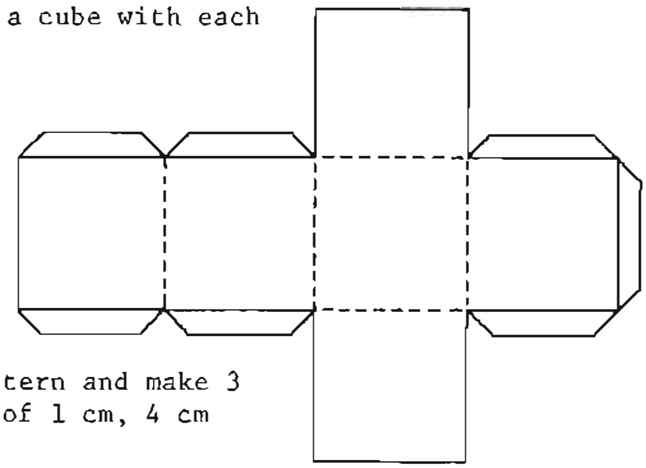
Model	Dimensions	Volume (cm^3)	Ratio of the volumes of this model to Model 1	Simplified ratio
d	9 × 2 × 1	18 cm^3	: 6	: 1
e			:	:
f			:	:

(4) Compare the simplified ratios with the simplified ratios in *Volume and Ratio 1*.
(5) If the simplified ratio of the volumes of a model to Model 1 is 16:1, how many of the dimensions are four times larger than Model 1?

CUBISM

Materials needed: Construction paper, scissors and paste, glue, or tape.

Activity: (1) Copy this pattern on the construction paper. Cut it out and fold it on the dotted lines to make a cube with each edge 2 cm long.



(2) Use a similar pattern and make 3 cubes with edges of 1 cm, 4 cm and 8 cm.

(3) Make a table like this and write the simplified ratios of lengths of the edges, areas of the faces and volumes of the cubes.

	1 cm to 2 cm	2 cm to 4 cm	4 cm to 8 cm
Length of edges	1 : 2		
Area of faces	1 : 4		
Volume of cubes			

(4) Make cubes and a chart to show the simplified ratios of 3 cubes with edges of 1 cm, 3 cm, 27 cm.

(5) Compare the 1 cm cube to the 4 cm cube;
1 cm cube to the 8 cm cube;
2 cm cube to the 8 cm cube.
Do you see any patterns?

IDEA FROM: *New Oxford Junior Mathematics*, Book 2
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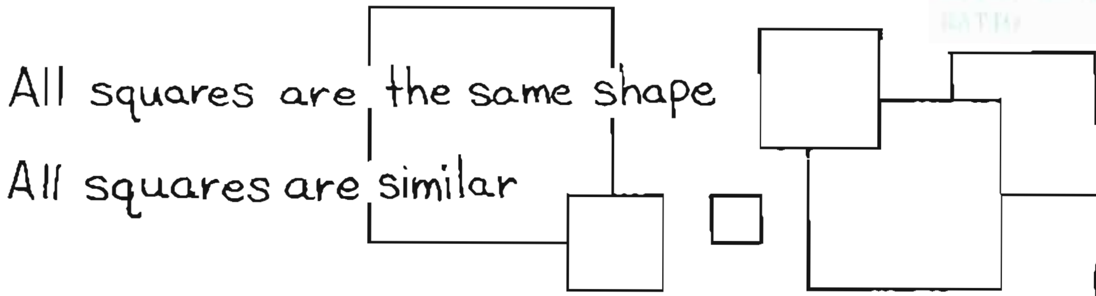
RATIO, PROPORTION, AND SCALING

Placement Guide for Tabbed Divisors

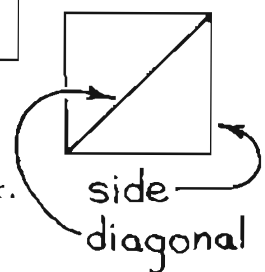
CONTENTS

RATIO: RATIO AS A REAL NUMBER

A SPECIAL RATIO IN ALL SQUARES



Materials needed: Metric ruler and centimetre (or $\frac{1}{2}$ cm) grid paper.



Activity: (1) On the centimetre grid paper draw a square 4 cm on a side. Measure the diagonal of the square. Did you get about 5.6 cm?

(2) Draw a square 7 cm on a side. Measure the diagonal. Is it about 9.8 cm?

(3) Draw squares with the sides shown in the table, and measure the diagonals.

Record the results on your paper.

Side (cm)	Diagonal (cm)	Ratio of diagonal to side	Simplified ratio
1			___ : 1
2			___ : 1
3			___ : 1
4	5.6	5.6:4	1.4 : 1
5			___ : 1
6			___ : 1
7	9.8	9.8:7	___ : 1
8			___ : 1

Divide both terms by the length of the side.

(4) For each square write the ratio of the length of the diagonal to the length of the side. Then simplify each of the ratios by dividing the length of the diagonal by the length of the side.

(5) The simplified ratio is always about ___:1. This means the diagonal of a square is about ___ times the length of a side.

(6) Use the above fact to find the length of the diagonal of a square if the side measures

(a) 1.5 cm ______ (d) 3.8 cm ______

(b) 1.8 cm ______ (e) 5.7 cm ______

(c) 2.5 cm ______ (f) 6.5 cm ______

Check these by drawing the squares and measuring the diagonals.

Challenge: What is the length of a side of a square if the diagonal is:

(a) 1.4 cm? ________ (b) 4.2 cm? ________ (c) 7 cm? ________

IDEA FROM: *New Oxford Junior Mathematics*, Book 3

A Very Special Ratio

Approximating π
Ratio as a Real Number
RATIO

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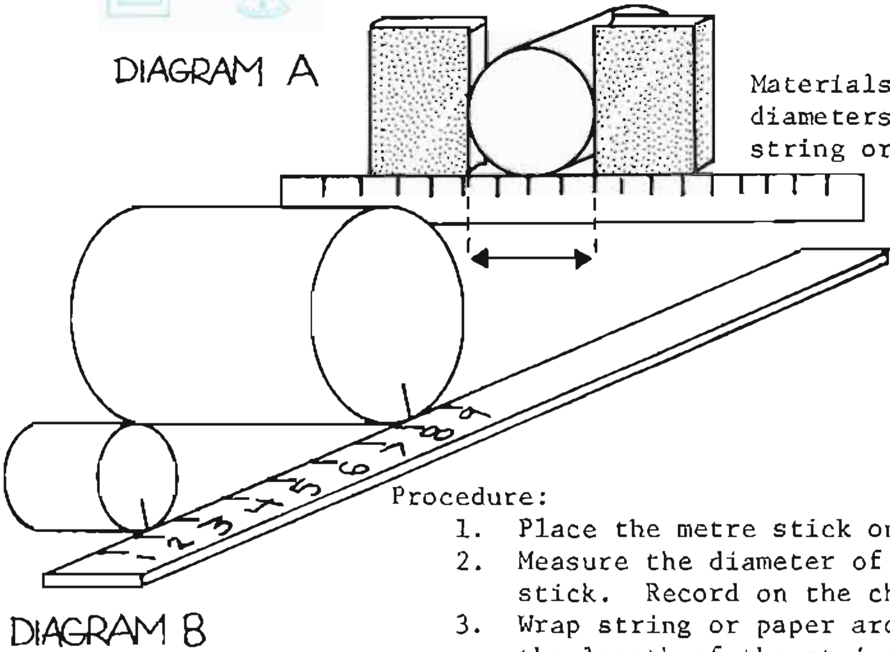


Materials: Metre stick, cans of varying diameters, rolls of tape, small wheels, string or paper to wrap around objects

You may wish to use cardboard discs cut with diameters of 2 cm, 4 cm, 7 cm, 8 cm and 10 cm.

Procedure:

1. Place the metre stick on a level surface.
2. Measure the diameter of a can by placing it on the metre stick. Record on the chart. (See Diagram A.)
3. Wrap string or paper around the can one time and measure the length of the string or paper.
4. Record the measurement in the chart.
5. Carefully roll a can along the metre stick for one complete turn to check for accuracy in step 3. (See Diagram B.)
6. Complete the chart. Use a calculator to find the values correct to two decimal places.

DIAMETER OF CAN	LENGTH OF STRING	LENGTH + DIAMETER	LENGTH − DIAMETER	LENGTH × DIAMETER	LENGTH ÷ DIAMETER

In which column are the numbers nearly the same? ___________

If you were careful in carrying out your experiments, you found that the circumference (length of the string) divided by the diameter of the can is about 3.1 or 3.2. This can be expressed as the ratio, circumference : diameter = 3.1:1, which is approximately 3:1.

To represent this ratio we use the Greek letter π (pi). π is pronounced "pie."

π cannot be exactly expressed as a decimal, no matter how many decimal places are used.

π is approximately

3.1415926535897932384626433832795028840971693993751O

TYPE: Activity

IDEA FROM: *Math Workshop*, Level F

PI'S THE LIMIT

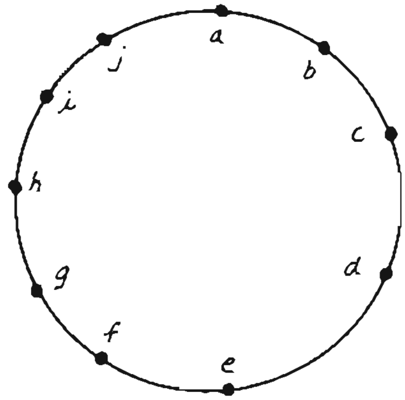


DIAGRAM I

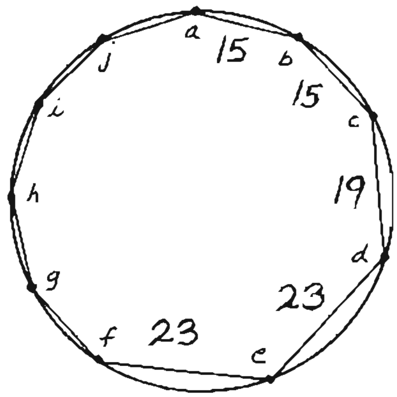


DIAGRAM II

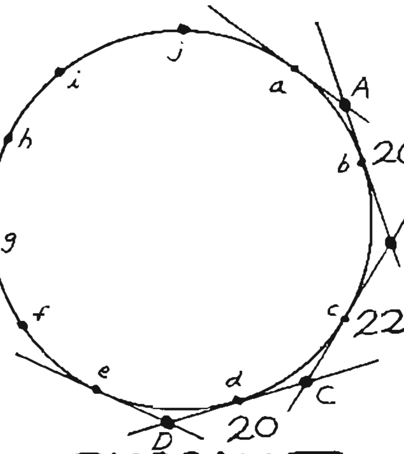


DIAGRAM III

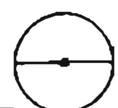
I. Draw a circle with a radius of at least 6 cm. Mark between 8 and 15 points on the circle any distance apart. They need not be equally spaced. Label the points with lower case letters. (See Diagram I.)

II. Connect consecutive points with line segments. Measure each segment and record the length in millimetres next to the segment. Add the measures and record the total in the top part of the table below. (See Diagram II.)

Record the diameter of the circle in millimetres.

III. Outside the circle draw line segments that touch the circle only at the points you have already marked (a, b, c, etc.). Label with capital letters the points where the line segments cross. You should have the same number of capital letters as lower case letters. Measure each new line segment and record its length as before. Add these measures and record in the table below. (See Diagram III.)

IV. Record the diameter of the circle in millimetres. ____________



Total of measures of segments inside of the circle	
Total of measures of segments outside of the circle	

Average of totals ____________

Hint: To compute the average, add the two totals and divide by 2.

V. Compute: $\dfrac{\text{Average of totals}}{\text{Diameter of circle}}$ = ____________

Repeat the experiment with a larger circle.

IDEA FROM: *Math Workshop*, Level F

PI's THE LIMIT (CONTINUED)

Purpose and Use:

This activity provides the student with an alternative and historical method for approximating π. In addition, the activity could be used as an exercise for measurement with a ruler. A calculator would facilitate the computation.

Suggested Procedure:

After the activity, introduce the term circumference as the distance around the circle. Some sample discussion questions could be:

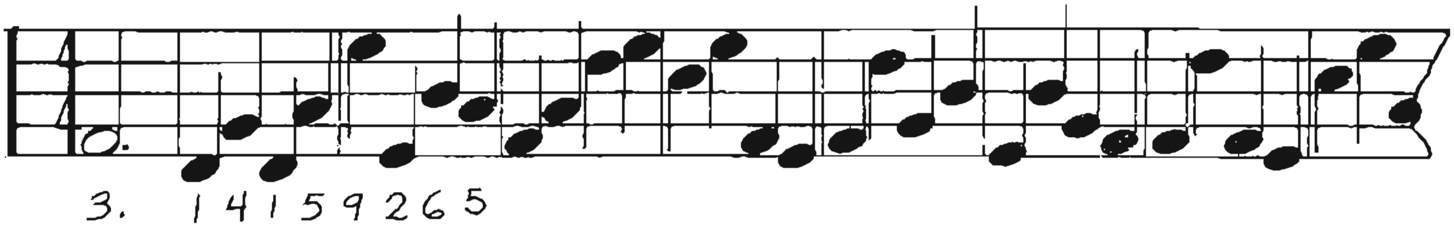
1) Do you understand why the circumference is smaller than the total of outside measures and larger than the total of inside measures of segments?
2) Is the average of totals a good approximation to the circumference?
3) How does the number of points on the circle affect the accuracy?

Content:

The ratio $\frac{\text{average of totals}}{\text{diameter of circle}}$ closely approximates the ratio $\frac{\text{circumference of a circle}}{\text{diameter of the circle}}$, which is about 3:1. The Greek letter π is used to express the ratio $\frac{\text{circumference}}{\text{diameter}}$, since the ratio is a constant and cannot be exactly expressed as a fraction or decimal.

Historical Facts & Curiosities:

1) Archimedes (287-212 B.C.), a great mathematician and scientist of ancient Greece, used a method similar to the one performed by the students to estimate that π was between 3.140845 and 3.142857.
2) In China Tsu Chung-Chih (470 A.D.) gave π = 3.1415924 which is correct to seven decimal places.
3) Today with the help of computers π has been found to more than 500,000 places. π correct to twenty places is 3.14159265358979323846.
4) The symbol π was first used in the 17th century.
5) In 1873 using a formula and making the computations with paper and pencil William Shanks of England computed pi to 707 decimal places. His representation of pi was used until 1948 when two men, using a computer, discovered that Shanks had made an error in the 528th decimal place.



6) Another method is to set π to music. The music shown above represents π in the key of C, with F having a value of 3, D the value of 1, and so on.

An excellent source for information about π is A History of π by Petr Beckmann published by the Golem Press.

IDEA FROM: *Math Workshop*, Level F

BUFFON'S PI

Approximating π
Ratio as a Real Number
RATIO

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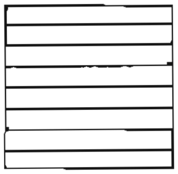
Comte de Buffon, a 18th century Frenchman, is known for this "Buffon needle problem" in the theory of probability.

1 METRE SQUARE CLOTH



Equipment:
- 10 toothpicks
- 1 metre square cloth
  Mark the cloth with parallel lines. The distance between the lines should be twice the length of a toothpick. The cloth will keep the toothpicks from rolling.

2-4 people

Have students work in pairs or small groups so that some can drop toothpicks and others can tally. Calculators will speed up the computation and motivate students to consider a large number of tosses. Each trial takes about 2 minutes.

											Total number of picks dropped	T Total number of picks touching	R
A											100		
B											200		
C											300		
D											400		
E											500		
F											600		
G											700		
H											800		
I											900		
J											1000		

Place the cloth on a flat surface. Select a person to record the data and one to drop the picks. You may wish to exchange positions.

I. Hold the 10 toothpicks a metre above the cloth and carefully drop them. Repeat this ten times. Each time record in Chart A the number of toothpicks touching or crossing a line. Total the number of picks touching and record in Column T. Divide 100 by this total. Round to two decimal places and record in Column R.

II. Repeat the experiment. Record in Chart B. Total the number of picks touching and add this to the previous total of toothpicks touching. (See Column T.) Record the new total in Column T. Divide 200 by this total and record in Column R.

III. The ratio Total toothpicks dropped : Total toothpicks touching a line should approximate π whose value is about 3.14. Continue the experiment and check the ratio after each trial. In 1901 Lazzerini, an Italian, found π correct to 6 decimal places or 3.141592 from 3408 drops.

TYPE: Activity

IDEA FROM: *Exploring Mathematics on Your Own*

CLOSER & CLOSER

Ratio as a Real Number
RATIO

Investigation I:

A) Make a sequence of counting numbers by selecting two numbers and writing them in the first two blanks below. Each number after the first two is obtained by adding the two previous numbers.

Example: 5 , 7 , $\underset{(5+7)}{12}$, $\underset{(7+12)}{19}$ . . .

Choose your own numbers.

_____, _____, _____, _____, _____, _____, _____, _____,

_____, _____, _____, _____, _____, _____, _____, . . .

B) Now use the sequence above to write a sequence of ratios. The ratios are obtained by comparing each number to the number on the right.

If you used: 5 , 7 , 12 , 19 , . . .

The ratios are (1) $\frac{5}{7}$, (2) $\frac{7}{12}$, (3) $\frac{12}{19}$, . . .

(1)_____, (2)_____, (3)_____, (4)_____, (5)_____, (6)_____, (7)_____, (8)_____,

(9)_____, (10)_____, (11)_____, (12)_____, (13)_____, (14)_____

C) Use your calculator to change each ratio to a decimal.

1.	8.
2.	9.
3.	10.
4.	11.
5.	12.
6.	13.
7.	14.

D) What do you notice about the sequence of ratios?

E) If each decimal is rounded to three places, the ratios get close to what number? (.618)

F) Pick two different numbers and repeat the steps. What do you notice about this sequence of ratios?

G) Suppose your friend picks any two counting numbers and repeats the steps to get a sequence of ratios. If these ratios are written as three place decimals, what number do they get close to?

TYPE: Paper & Pencil

The ratios get close to the Golden Ratio which equals $\frac{\sqrt{5}-1}{2} \approx .618$.

CLOSER & CLOSER

Investigation II: (CONTINUED)

A) Make a sequence of counting numbers by selecting three numbers and writing them in the first three blanks below. Each number after the first three is obtained by adding the three previous numbers.

Example: 5, 7, 2, $\frac{14}{(5+7+2)}$, $\frac{23}{(7+2+14)}$,

Choose your own numbers.

_____, _____, _____, _____, _____, _____, _____, _____,

_____, _____, _____, _____, _____, _____, _____, ...

B) As in Investigation I, write a sequence of ratios by comparing each number to the number on the right.

If you used: 5, 7, 2, 14, . . .

The ratios are: (1) $\frac{5}{7}$, (2) $\frac{7}{2}$, (3) $\frac{2}{14}$

(1)_____, (2)_____, (3)_____, (4)_____, (5)_____, (6)_____, (7)_____, (8)_____,

(9)_____, (10)_____, (11)_____, (12)_____, (13)_____, (14)_____

C) Use your calculator to change each ratio to a decimal.

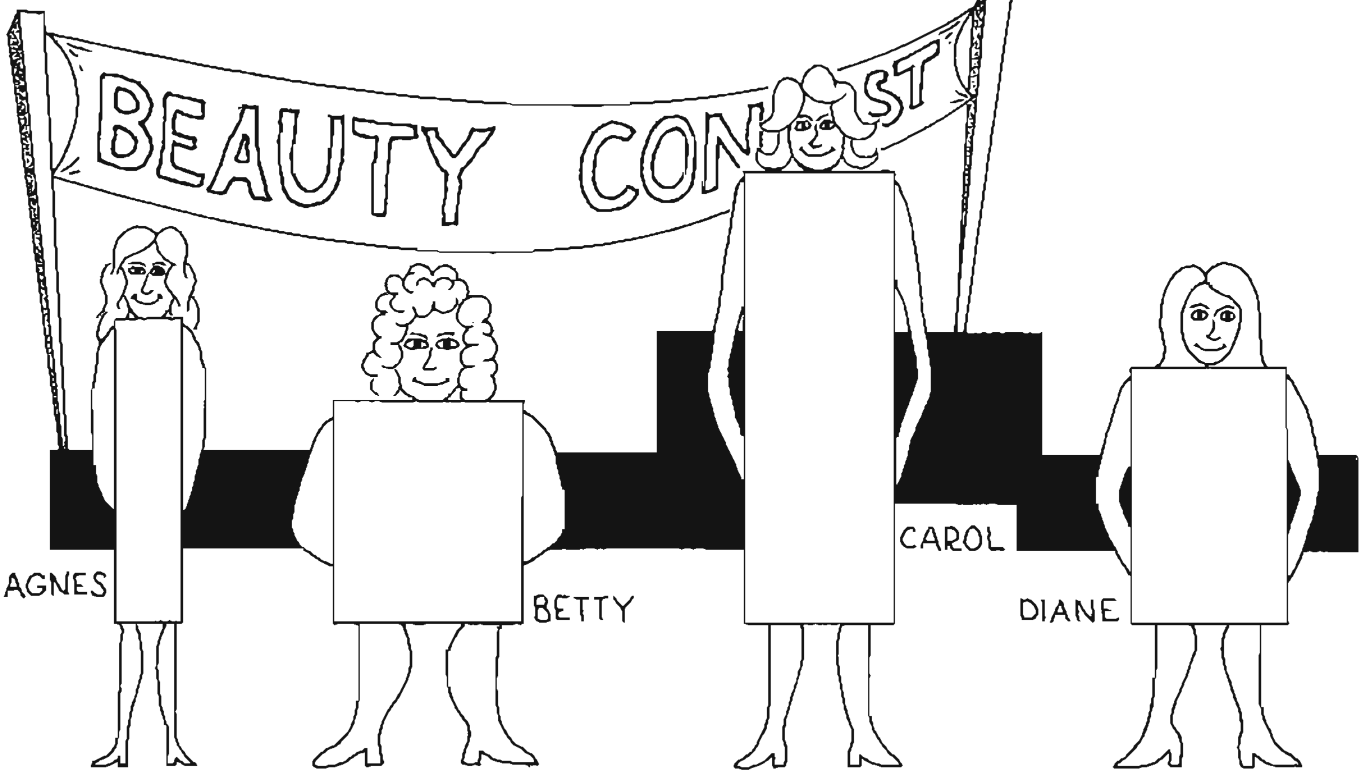
1. ________	8. ________
2. ________	9. ________
3. ________	10. ________
4. ________	11. ________
5. ________	12. ________
6. ________	13. ________
7. ________	14. ________

D) What do you notice about the sequence of ratios?

E) If each decimal is rounded to three places, the ratios get close to what number?

F) Pick three different numbers and repeat the steps. What number do these ratios get close to?

ACTIVITY III



Agnes, Betty, Carol, and Diane are competing in a beauty contest. You are the judge. Who has the best shape? ________________. Use a ruler to find each girl's measurements in millimetres. Complete the table.

Divide the width by the length and express as a three-place decimal.

Contestant	Width	Length (height)	Ratio of width to length	
AGNES			:	:1
BETTY			:	:1
CAROL			:	:1
DIANE			:	:1

The Golden Ratio equals $\frac{\sqrt{5}-1}{2} \approx .618$.

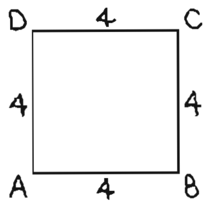
Is the ratio of Diane's width to height about .618:1?

The ratio .618:1 is called the Golden Ratio. In a rectangle if the ratio of the width to the height is the Golden Ratio the rectangle is a Golden Rectangle. Many examples of the Golden Rectangle can be found in both art and architecture--the United Nations building, the Parthenon at Athens. Find pictures of these buildings and check to see if they are Golden Rectangles. Can you find examples of Golden Rectangles in the classroom?

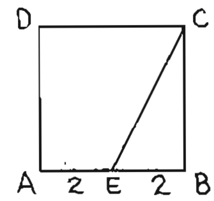
ACTIVITY IV

Can you make a Golden Rectangle?

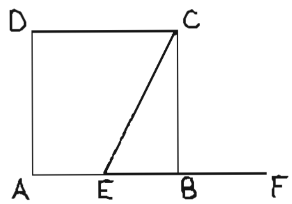
First, draw a 4 cm square.



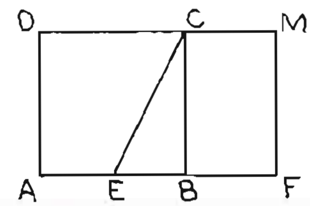
Next, locate point E, the midpoint of segment AB. Join C to E.



Extend side AB and mark point F so that EF is the same length as segment CE.



Complete the rectangle.

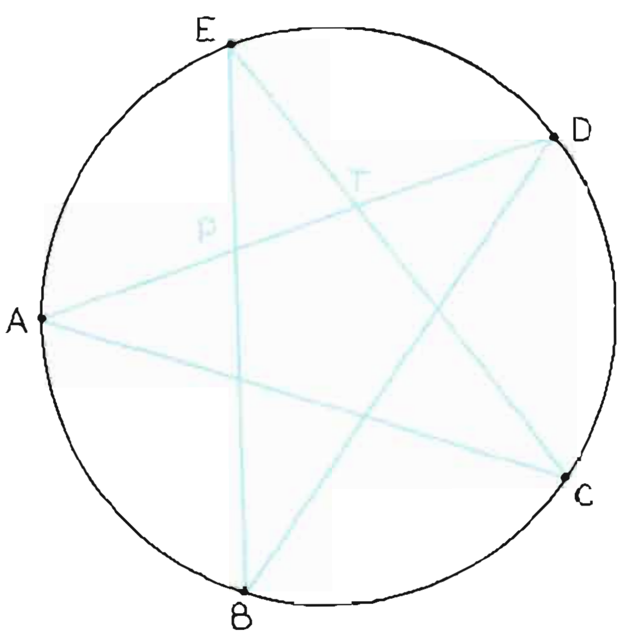


To check your drawing, find the ratio:

$\frac{\text{length AD}}{\text{length AF}}$ and express it as a three-place decimal.

Write the ratio: $\frac{\text{length BF}}{\text{length CB}}$ and express it as a three-place decimal. What can you say about rectangle CBFM?

What's In A Circle ?



The circle to the left has five equally spaced points marked on it.

Join points A and D, D and B, B and E, E and C, C and A. You have just drawn a five-pointed star.

Locate the point where line segments EC and AD cross and label it T.

Measure the segments TD and AT. Find the ratio $\frac{TD}{AD}$ = ____ and express it as a three-place decimal. What do you notice? Do you see a way to draw a smaller five-pointed star?

RATIO, PROPORTION, AND SCALING

Placement Guide for Tabbed Divisors

Tab Label **Follows page:**

PROPORTION

In 1973 the ratio of juveniles to adults prosecuted for burglary was 11 to 9. This ratio can be represented by any of the pairs of numbers in the table below. These pairs of numbers are called equivalent or equal ratios.

Juvenile Burglars	Adult Burglars
11	9
22	18
33	27
44	36
.	.
.	.
.	.

"It says here, 'You have permanently lost your picture – the Midnite Phantom.'"

A proportion is a statement of equality between two ratios. Here are two ways of writing a proportion:

$$a:b = c:d \qquad \text{or} \qquad \frac{a}{b} = \frac{c}{d}$$

These proportions are both read as "the ratio of a to b is equal to the ratio of c to d." Sometimes the expression "a is to b as c is to d" is also used.

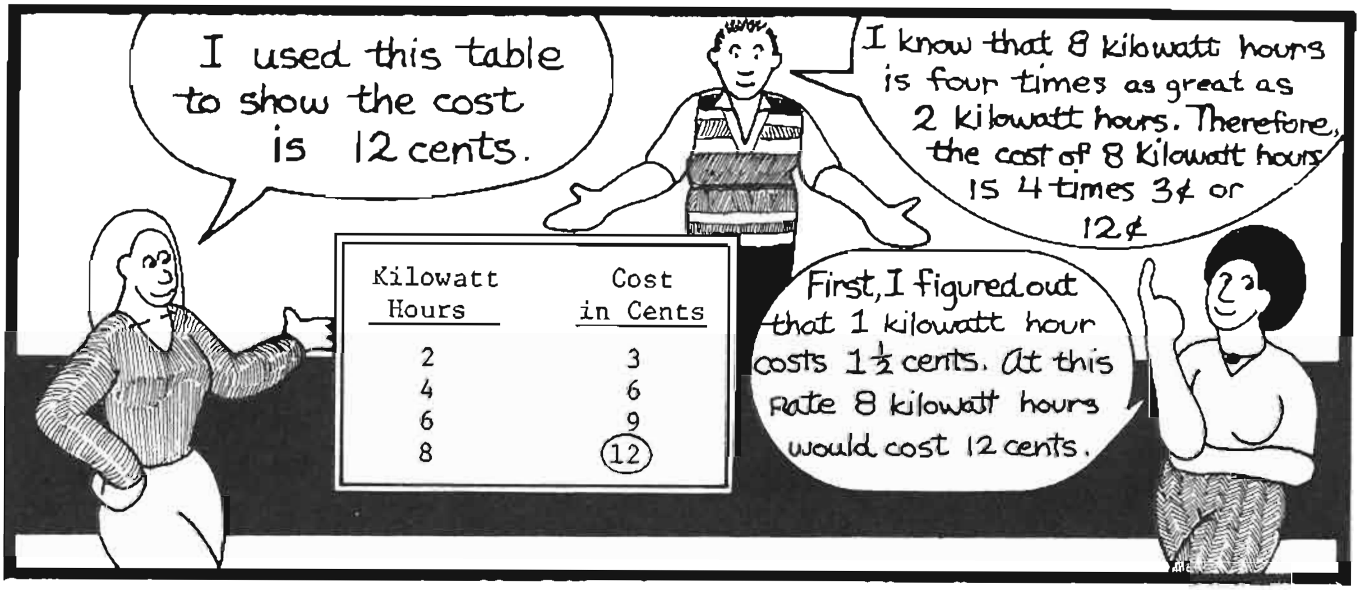
INTRODUCING YOUR CLASS TO PROPORTIONS

Two equal ratios contain 4 numbers. When 3 of these numbers are given, it is possible to determine the fourth number. For example, using the ratio of juvenile to adult burglars, 11 to 9, how many juvenile burglars would there be for every 90 adult burglars? Your students will be able to answer this question by extending the above table to the tenth row, which is the row containing the 90 adult burglars. Most students will understand this use of tables, and given any 3 numbers, they will be able to use multiples of a given ratio to find the fourth number of the proportion.

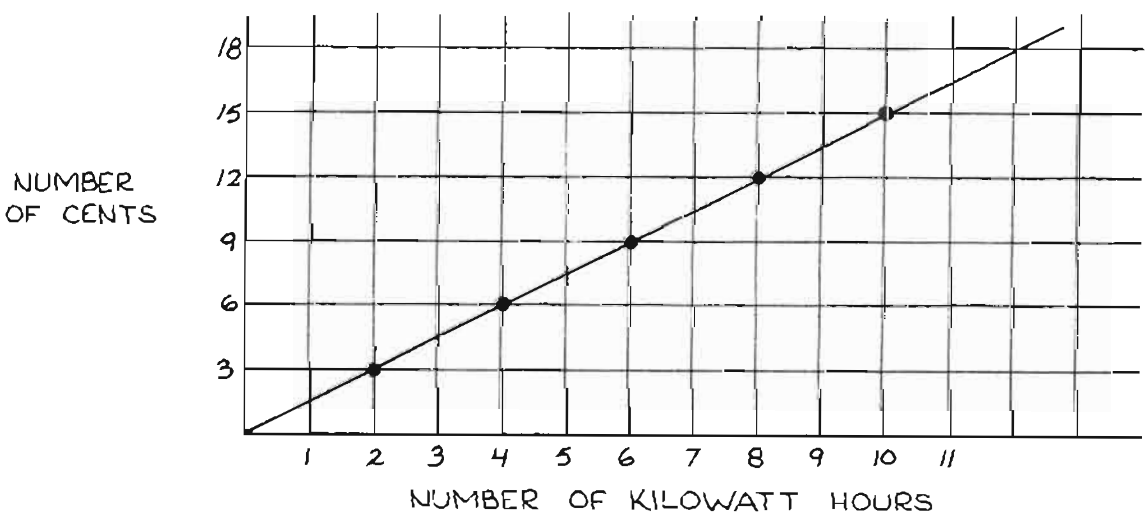
Proportions occur naturally when dealing with rates. Here's a familiar kind of question. If the cost of electricity is 3 cents for 2 kilowatt hours, how much will 8 kilowatt hours cost?

Cost in Cents	Number of Kilowatt Hours
3	2
6	4
9	6
.	.
.	.
.	.

Your students will find a variety of ways to answer such questions. Here are a few examples of sound reasoning which all produce the correct answer.



You may wish to have your students graph some rates. These graphs will always be straight lines if the rate stays constant. The following graph shows the rate of 3 cents for every 2 kilowatt hours. Some of your students will be able to use this graph to determine the cost of 5 kilowatt hours.



A TEST FOR EQUALITY

When a proportion is written in the form a:b = c:d, the first and fourth numbers are called the extremes, and the second and third numbers are called the means. Two ratios are equal whenever the product of the extremes is equal to the product of the means.

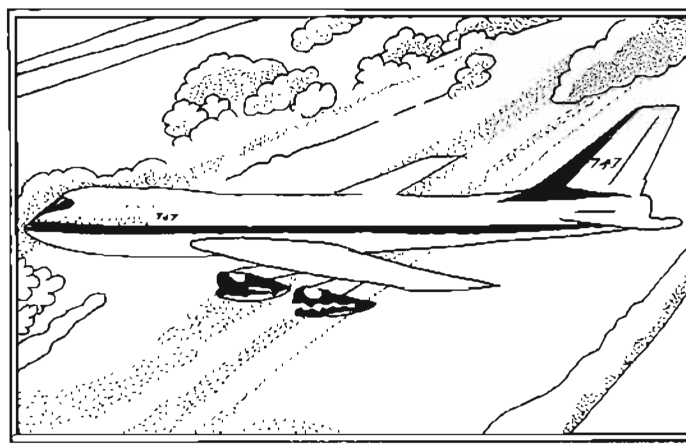
$$a : \underbrace{b = c}_{\text{MEANS}} : d \quad \text{whenever } ad = bc$$

EXTREMES

When the proportion is expressed by fractions, we have:

$$\frac{a}{b} = \frac{c}{d} \quad \text{whenever} \quad ad = bc$$

The products ad and bc are commonly called cross products. Students can remember this with the following aid. $\frac{a}{b} \times \frac{c}{d}$



The test for equality of ratios is useful for finding the fourth number of a proportion when 3 of its numbers are known. Here is a typical rate problem which can be solved by using cross products. The Boeing 747 has a cruising speed of 595 miles per hour. How long will it take to travel 1500 miles at this rate?

It will be instructive for the students to try solving this problem with a list of numbers. The student will be able to see that the answer is between 2 and 3 hours. Some students will estimate that the answer is close to $2\frac{1}{2}$ hours.

Hours	Miles
1	595
2	1190
3	1785

Letting T be the unknown time, the given information can be expressed by the following equation. It must be emphasized to the student that the ratios are hours to miles on both sides of this equation.

$$\frac{1}{595} = \frac{T}{1500}$$

Using cross products, (1) x (1500) = 595T, and so T = 2.52.

It is somewhat remarkable that proportions can be set up in so many different ways and still produce the correct answer. Look at the examples below. The same value of T = 2.52 satisfies each of the equations.

$\frac{1 \text{ hr.}}{595 \text{ mi.}} = \frac{T \text{ hr.}}{1500 \text{ mi.}}$	$\frac{595 \text{ mi.}}{1 \text{ hr.}} = \frac{1500 \text{ mi.}}{T \text{ hr.}}$	$\frac{595 \text{ mi.}}{1500 \text{ mi.}} = \frac{1 \text{ hr.}}{T \text{ hr.}}$	$\frac{1500 \text{ mi.}}{595 \text{ mi.}} = \frac{T \text{ hr.}}{1 \text{ hr.}}$

Could we use the expression $\frac{1 \text{ hr.}}{595 \text{ mi.}} = \frac{1500 \text{ mi.}}{T \text{ hr.}}$ to solve this problem? If we examine the cross products 1 hr. x T hr. and 595 mi. x 1500 mi., we see the unit of measure in the first product is hr. x hr. and in the second is mi. x mi. The units of measure of the cross products are not the same; we cannot use the expression above to solve the problem.

Often students try to apply proportions without noticing the units of measure given in the problem. Consider this problem: "A worm travels 12 cm every 4 seconds. How many metres does he travel in 48 seconds?" A student might set up the problem as $\frac{12}{4} = \frac{Y}{48}$, ignore the units and give 144 as the answer. If the units are included and their cross products checked, $\frac{12 \text{ cm}}{4 \text{ sec.}} = \frac{Y \text{ m}}{48 \text{ sec.}}$, it can be seen that cm x sec. is not the same as sec. x m. The problem can be solved by changing 12 cm to metres or by finding an answer in centimetres and then converting it to metres.

$$\frac{.12 \text{ m}}{4 \text{ sec.}} = \frac{Y \text{ m}}{48 \text{ sec.}} \quad \text{or} \quad \frac{12 \text{ cm}}{4 \text{ sec.}} = \frac{Y \text{ cm}}{48 \text{ sec.}}, \text{ where } \frac{Y}{100} = \text{the number of metres.}$$

Being conscious about the units of measure will not guarantee that the proportion is set up correctly. A student might try to solve the airplane problem discussed above with this proportion: $\frac{T \text{ hr.}}{1 \text{ hr.}} = \frac{595 \text{ mi.}}{1500 \text{ mi.}}$. The units of the cross products are the same, but this is certainly not a correct proportion. To avoid this confusion teachers often have students form proportions in a standard way, say miles to hours on both sides of the equation.

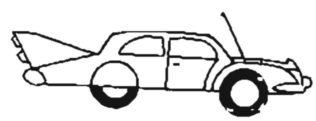
Suggested Exercises

Some interesting proportion problems can be solved by using cross products. If your students use the Guinness Book of World Records, they will find that frequently three of the four numbers of a proportion are given.

The numbers in the Guinness Book of World Records may be too large for some of your students. These proportion exercises from the student page *Petite Proportions 2* contain some common rate questions with smaller numbers.

9) 2 pantsuits for $35.
7 pantsuits for ________.

11) Car goes 10 km on 2 litres of gas.
Car goes ________ km on 16 litres of gas.



There are many interesting proportion ideas and applications in the classroom materials. Here are a few examples: measuring heights of objects by using shadows; determining gear ratios on 10-speed bikes; using the Golden Ratio; placing weights on balance beams or teeter-totters; computing driver reaction times and braking distances for cars; using money exchange tables; and comparing your weight and height to standard growth charts.

INEQUALITIES OF RATIOS

There are times when it is useful to determine the greater of two ratios.

2 out of 5 households watched the Super Bowl.
1 out of 3 households watched the World Series.
7 out of 25 households watched the Riggs-King Tennis Match.

One way to compare two ratios is to write them in fraction notation and find the greater fraction. Since $\frac{2}{5}$ is greater than $\frac{1}{3}$, more households were tuned into the 1973 Super Bowl than the 1973 World Series. Another approach is to represent each ratio by a real number. Since $2 \div 5 = .4$ and $1 \div 3 \approx .33$, the Super Bowl had the greater audience. How did the T.V. audience for the Riggs-King Tennis Match compare with that for the World Series?

You may have noticed that in the above examples the ratios were used to compare part of a set to the whole set. The following examples use ratios to compare disjoint sets.

For every 8 men there were 5 women who watched the Super Bowl.
For every 5 men there were 4 women who watched the World Series.
For every 9 men there were 11 women who watched the Riggs-King Tennis Match.

Was there a greater ratio of men to women watching the Super Bowl or the World Series? To answer this question we may use the same approach as above. The ratio 8 to 5 is greater than the ratio 5 to 4 because $\frac{8}{5}$ is greater than $\frac{5}{4}$.

Let's use our test for proportion to determine the cost per ounce of the 69¢ package of Complete Buttermilk Pancake Mix.

$$\frac{40\text{ oz.}}{69\text{ cents}} = \frac{1\text{ oz.}}{Y\text{ cents}}$$

By cross products, $40y = 69$, so $y = 1.7$¢. In a similar manner we can find that the cost per ounce of the 85¢ package is 1.5¢. The contents of the first package sells for 27¢ per pound, and the second package sells for 24¢ per pound. How does the price per pound of the Variety Baking Mix compare with the price per pound of the Buckwheat Mix?

There is an abundance of practice with proportions in computing unit prices. Your students could collect information (prices and weights) of different brands for unit pricing comparisons. Using a calculator will simplify the computations and allow students to focus on the proportions and comparisons.

For additional ideas in using proportions to compare rates see *Proportion Projects to Pursue* in the section PROPORTION: Application.

RATIO, PROPORTION, AND SCALING

Placement Guide for Tabbed Divisors

CONTENTS

PROPORTION: GETTING STARTED

2:3 = 4:6 ●   ● 3:6 = 5:10

1 to 3 = 2 to 6 ●   ● 3 to 5 = 6 to 10

3 to 6 = 1 to 2 ●   ● 3:6 = 1:2

I LIKE YOUR FORM

$\frac{3}{5} = \frac{6}{10}$ ●   ● $\frac{1}{3} = \frac{2}{6}$

Use line segments to connect the dots that name the same proportion.

The result is startling!

$\frac{3}{6} = \frac{5}{10}$ ●   ● 2 to 3 = 4 to 6

$\frac{2}{3} = \frac{4}{6}$ ●   ● 3 to 6 = 5 to 10

1 to 3 = 2 to 6 ●   ● 3:5 = 6:10

●
$\frac{3}{6} = \frac{1}{2}$

Write the other forms for these proportions.

$\frac{4}{5} = \frac{12}{15}$		
	2:7 = 8:28	
		12 to 2 = 6 to 1

Use the four numbers 2, 6, 9, 27 to write a proportion. Then write the other forms.

________ ________ ________

Did you get the same proportion as your neighbor? Compare and see.

Can you make another proportion from the four numbers?

PROP OR SHUN

In order to introduce or diagnose a student's concept of equivalent ratios, one could approach the subject intuitively. The presentation of various methods of checking for equivalent ratios can come later. Students need to be shown equivalent ratios in various forms such as 2:3 and 6:9, 1 to 5 and 4 to 20, or $\frac{3}{4}$ and $\frac{6}{8}$.

A first activity might be as follows: Several pairs of ratios can be written with open frames. The student fills in the frames with the appropriate value and determines if the ratios are equivalent. For example, let:

$\square = 4$

$\bigcirc = 12$

$\triangle = 8$

⬡ = 6

Set up several pairs of ratios,

$\frac{\square}{⬡}$ and $\frac{\triangle}{\bigcirc}$, $\square : \bigcirc$ and ⬡ : $\triangle$

and ask the students to determine which pairs are equivalent.

The activity could be done as a student worksheet or as a class activity on the overhead.

A second activity might be to present students with one of the ratios and ask them to supply an equivalent ratio.

For example let: a = 3, b = 5, c = 9, d = 15

Sample questions could include:

1. What ratio is represented by $\frac{a}{b}$? ______
2. Write a ratio equivalent to the above ratio. ______
3. Represent this ratio using the letters. ______

Write a ratio equivalent to $\frac{a}{c}$ ______; $\frac{b}{d}$ ______; etc.

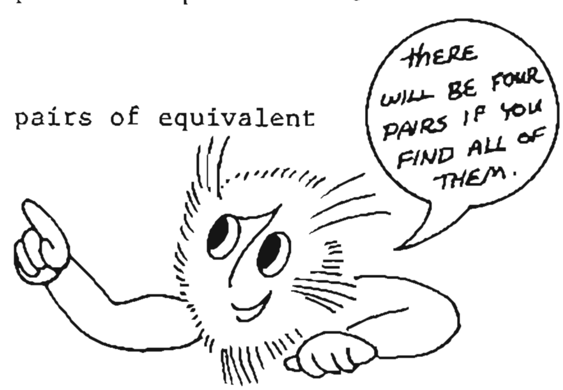
At this time you may also wish to acquaint students with the terminology "a is to b as c is to d."

A third activity could then ask students to supply pairs of equivalent ratios given four numbers.

For example:
Use the numbers 1, 12, 4, 3 and write as many pairs of equivalent ratios as you can.

Extension: Have students write 3 equivalent ratios using these numbers. 2, 9, 6, 3, 4 (The numbers may be used more than once.)

__________ = __________ = __________.



IDEA FROM: *Similarity and Congruence*
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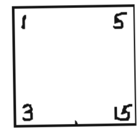
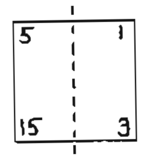
AS THE SQUARE TURNS

This could be done as a teacher-directed activity with two square transparencies.

(1) Make a square. Label it like this.

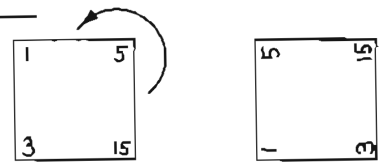


(2) Flip the square over and label the back like this.



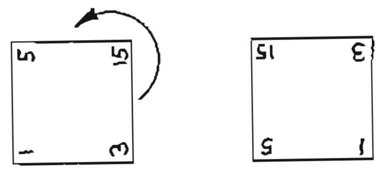
(3) Look at the original square. See two ratios $\frac{1}{3}$ and $\frac{5}{15}$. Do the ratios form a proportion? (Yes or No) ______

(4) Rotate the original square.
Ratios are $\frac{5}{1}$ and $\frac{15}{3}$.



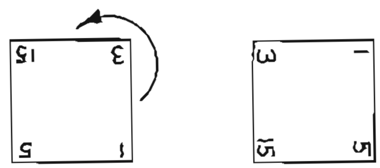
Proportion? Yes $\frac{5}{1} = \frac{15}{3}$

(5) Rotate again. Ratios $\frac{15}{5}$ and $\frac{3}{1}$.



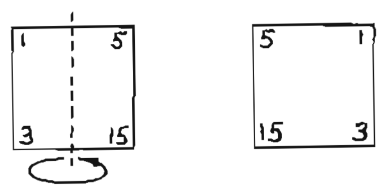
Proportion? ______ ______

(6) Rotate again.



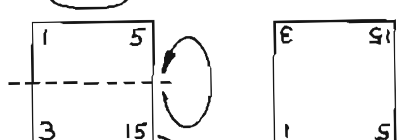
Proportion? ______ ______

(7) Flip the original square.



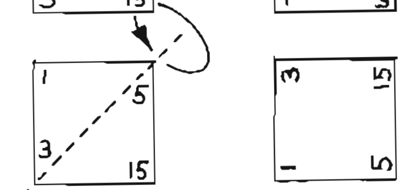
Proportion? ______ ______

(8) Flip the original square.



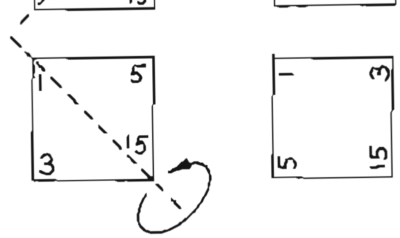
Proportion? ______ ______

(9) Flip the original square.



Proportion? ______ ______

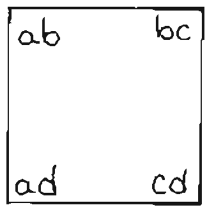
(10) Flip the original square.



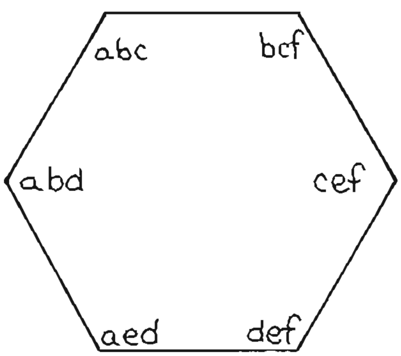
Proportion? ______ ______

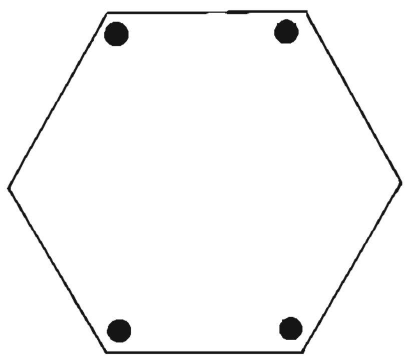
AS THE SQUARE TURNS
(CONTINUED)

Similar activities can be developed using a hexagon, octagon, etc. A general form can be generated using non-zero values to a, b, c, etc.

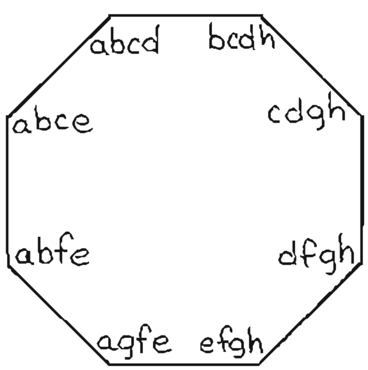


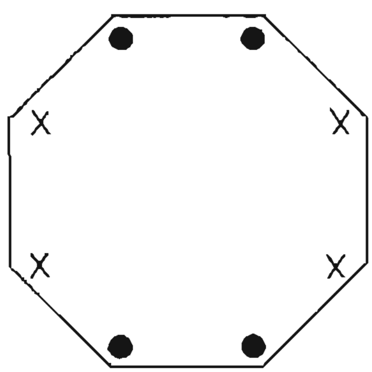
In the example on the student page
a = 1, b = 1, c = 5, and d = 3.





Using the six rotations and six reflections of a regular hexagon, twelve proportions occur. The placement of the numbers that form the proportion are shown in the second diagram.





Using the eight rotations and eight reflections of a regular octagon, thirty-two proportions occur in each position of the octagon. The placement of the numbers that form the proportion are shown in the second diagram.

GETTING BULLISH ON PROPORTIONS

Some proportions can be solved by multiplying. Study these examples.

① $\frac{3}{5} = \frac{\square}{15}$ ② $\frac{8}{28} = \frac{2}{\square}$ ③ 4 TO $\square$ = 8 TO 18 ④ $\square : 6 = 9 : 2$

$\frac{3 \times 3}{3 \times 5} \dashrightarrow \frac{\square}{15}$ $\quad$ $\frac{8 \dashleftarrow 2 \times 4}{28 \quad \square \times 4}$ $\quad$ $\frac{2 \times 4 \dashrightarrow 8}{2 \times \square \quad 18}$ $\quad$ $\frac{\square}{6} = \frac{9 \times 3}{\dashleftarrow 2 \times 3}$

THE ARROWS ALWAYS GO FROM THE SMALLER TO THE LARGER NUMBERS

$\frac{3 \times 3 \dashrightarrow \boxed{9}}{3 \times 5 \dashrightarrow 15}$ 3× $\quad$ $\frac{8 \dashleftarrow 2 \times 4}{28 \dashleftarrow \boxed{7} \times 4}$ $\quad$ $\frac{2 \times 4 \dashrightarrow 8}{2 \times \boxed{9} \dashrightarrow 18}$ $\quad$ $\frac{\square \dashleftarrow 9 \times 3}{6 \dashleftarrow 2 \times 3}$

Solve the proportions to discover the answer to this feed problem.

IF PAPA BULL (1200 POUNDS) CAN EAT 80 POUNDS OF HAY IN 4 DAYS, AND BABY BULL (200 POUNDS) CAN EAT 80 POUNDS OF HAY IN 24 DAYS, HOW LONG WILL IT TAKE MAMA BULL (600 POUNDS) TO EAT 80 POUNDS OF HAY?

$\frac{E}{35} = \frac{5}{7}$ $\quad$ $\frac{3}{5} = \frac{B}{25}$ $\quad$ $\frac{N}{5} = \frac{12}{20}$ $\quad$ $\frac{3}{T} = \frac{9}{12}$

U TO 8 = 25 TO 40 $\quad$ 7 TO O = 28 TO 24

9 TO 8 = 63 TO H $\quad$ 32 TO 40 = R TO 10

3:8 = I : 24 $\quad$ 9:7 = 27 : M

30 : L = 15 : 20 $\quad$ A : 20 = 3 : 5

4	56	25	8	25		12	9	3	4		3	6		21	12	21	12		15	5	40	40	
T	H	E	R	E		A	I	N	'T		N	O		M	A	M	A		B	U	L	L	!

THE BOB AND RAY SHOW

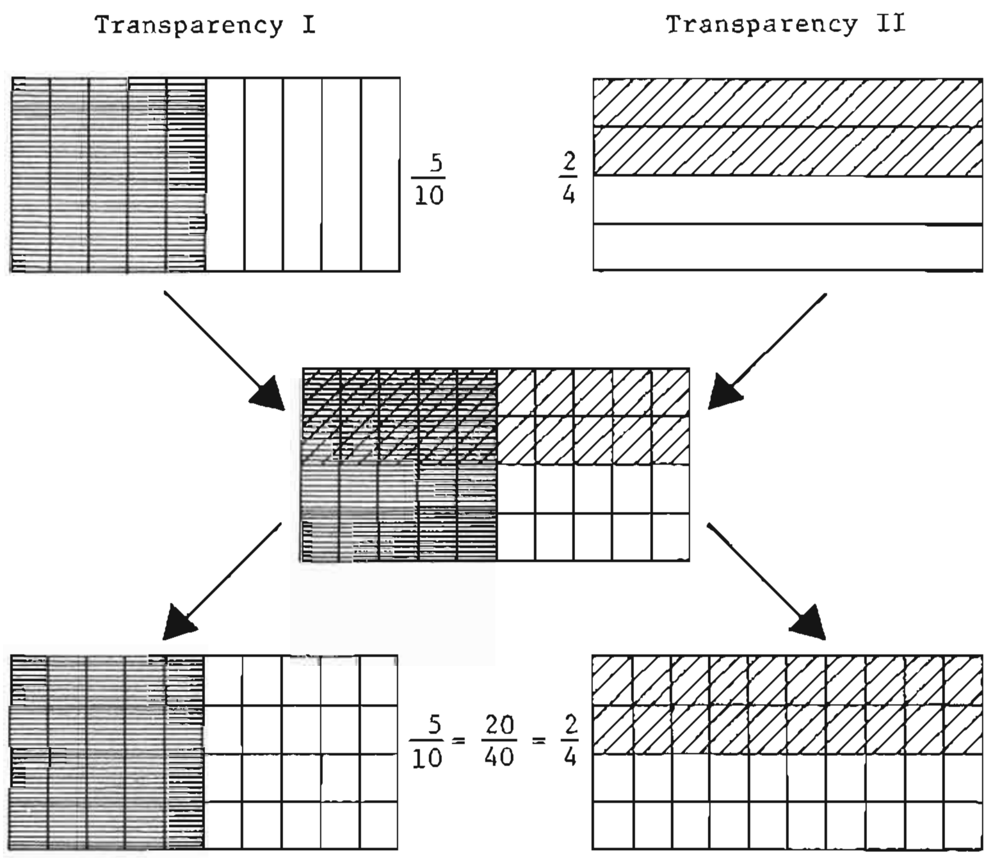
This activity uses geometric models to determine equivalent ratios and can be used to solve simple proportions. Using the included script, taping the lesson in advance and/or having students present the lesson could provide a unique experience for your class.

Included in this activity are (1) a teacher page indicating the steps used to determine if two ratios are equivalent, (2) a transparency master for a demonstration of these steps, (3) a sample script that could be used with the demonstration and (4) a student page to follow up the activity.

A. Is the ratio $\frac{5}{10}$ equivalent to the ratio $\frac{2}{4}$?
Use two rectangles that are the same size. Divide one vertically and one horizontally as shown and shade the appropriate parts, $\frac{5}{10}$ of one and $\frac{2}{4}$ of the other.

Place one rectangle on top of the other and draw in the dividing lines of each rectangle on the other.

Slide the rectangles apart and restate the ratios in terms of the new subdivisions. Since $\frac{5}{10} = \frac{20}{40}$, and $\frac{2}{4} = \frac{20}{40}$, the ratio $\frac{5}{10}$ is equivalent to the ratio $\frac{2}{4}$.



B. Is the ratio $\frac{2}{5}$ equivalent to the ratio $\frac{1}{4}$?
The same transparency master can be used for this demonstration.

Since $\frac{2}{5} = \frac{8}{20}$ and $\frac{1}{4} = \frac{5}{20}$, $\frac{2}{5} \neq \frac{1}{4}$.

IDEA FROM: *Activities with Ratio and Proportion*
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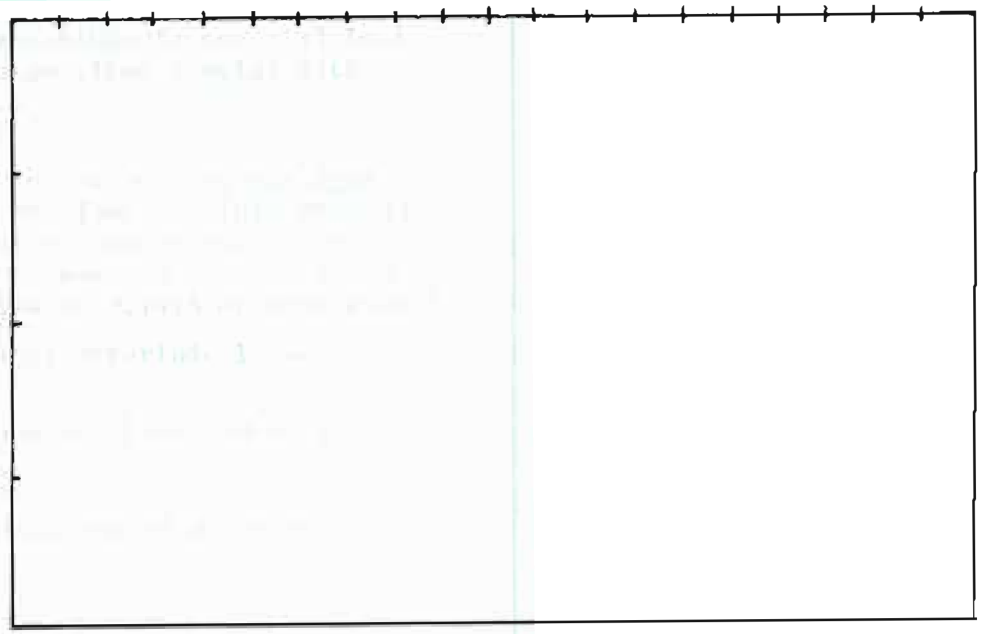
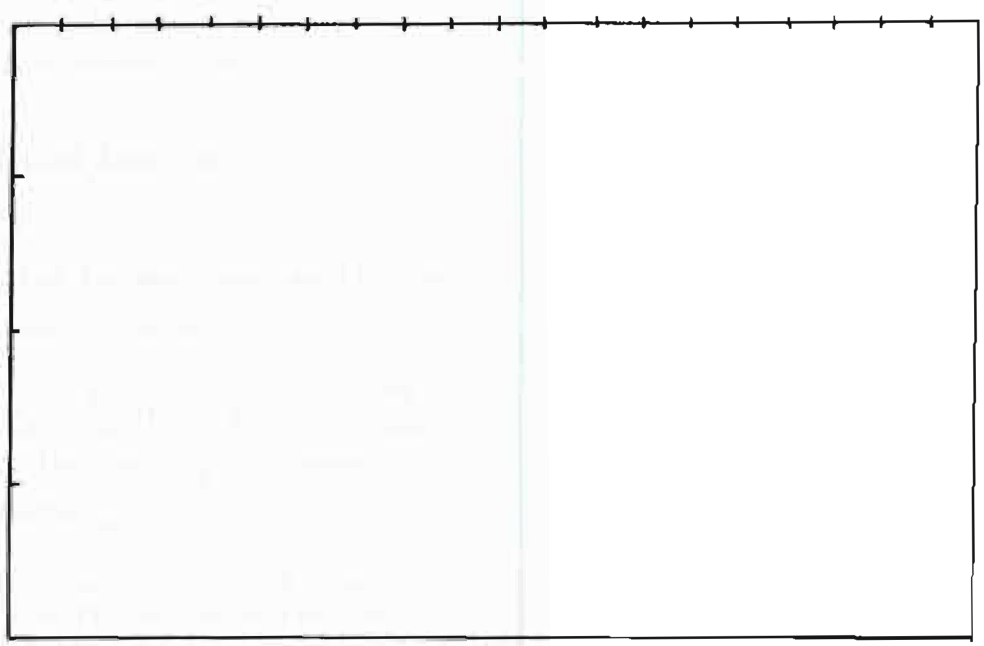
THE BOB AND RAY SHOW (PAGE 2)

Transparency Master for Geometric Models

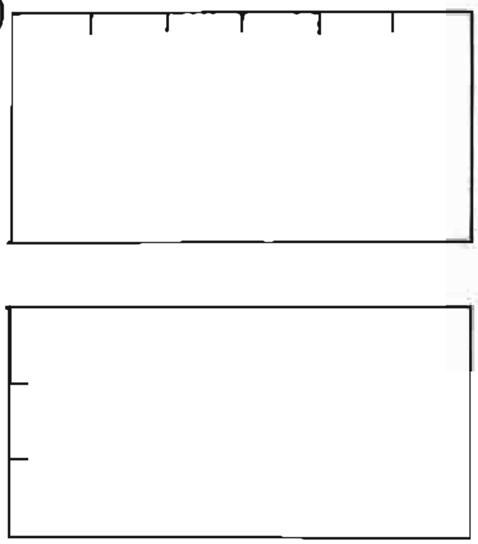




IDEA FROM: *Activities with Ratio and Proportion*

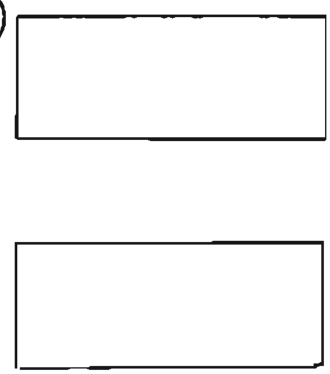
Use these rectangles to decide if the ratios are equivalent. Remember to divide one rectangle horizontally and the other vertically.

(1)



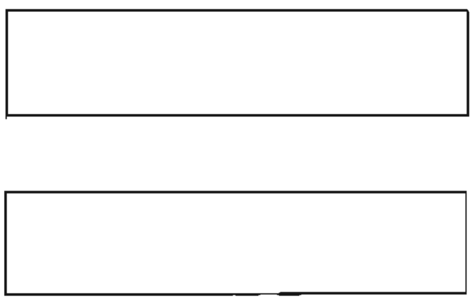
$\frac{4}{6}$ $\frac{2}{3}$

(2)



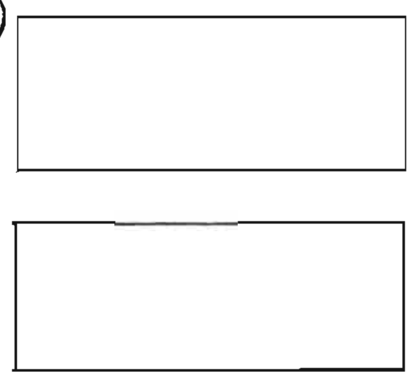
$\frac{4}{8}$ $\frac{3}{6}$

(3)



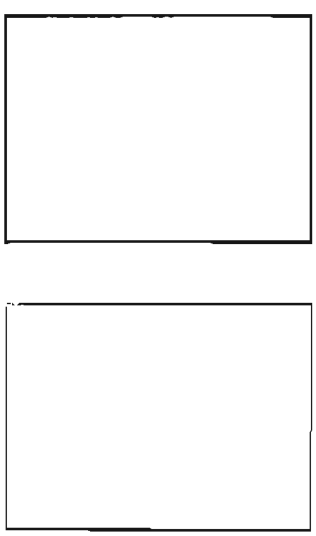
$\frac{5}{12}$ $\frac{1}{2}$

(4)



$\frac{6}{10}$ $\frac{3}{5}$

? See if you can use rectangles to solve this proportion:



$\frac{?}{8}$ $\frac{3}{6}$

$$\frac{\square}{8} = \frac{3}{6}$$

IDEA FROM: *Activities with Ratio and Proportion*

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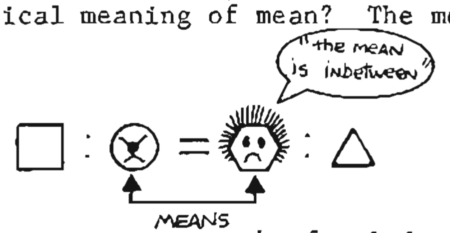
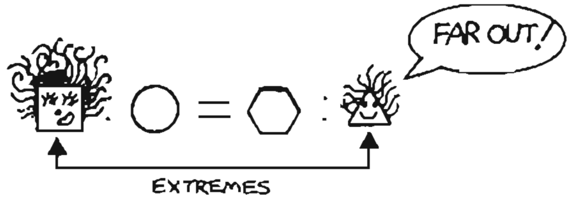
WE MUST WORK TOGETHER

We are both found in proportions.
Do you know the mathematical meaning of mean? The means are in the middle.



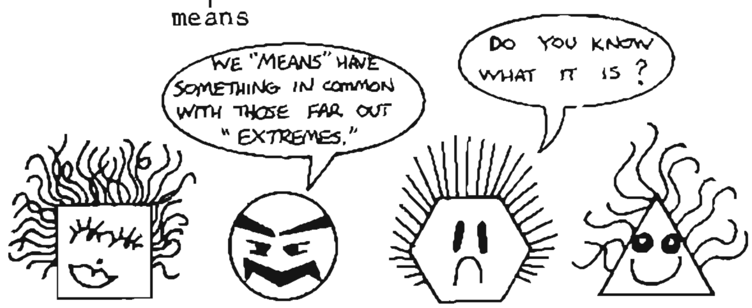
In politics the extremes are the far left and far right.



means
1:2 = 3:6
extremes

extremes
3 to 4 = 12 to 16
means

$\frac{4}{6} \times \frac{10}{15}$ extremes are 4 and 15
means are 6 and 10.



9 : 12 = 3 : 4   7 : 21 = 1 : 3

Complete the table.

	EXTREMES	MEANS	EXTREMES	MEANS
	9 and 4	12 and 3		
ADD THEM				22
SUBTRACT THEM	5			
MULTIPLY THEM				
DIVIDE THEM	$2\frac{1}{4}$			

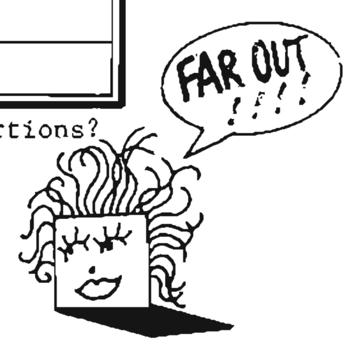
Did you discover a rule? Does it work for these proportions?

a) $\frac{3}{5} = \frac{9}{15}$

b) 6 to 12 = 1 to 2

c) 12:18 = 6:9

d) $\frac{32}{12} = \frac{8}{3}$



AN EXTREME TOOL

The Cross Products Rule can be used to check if two ratios are equivalent or used to solve proportions. Rule: In a proportion the product of the <u>extremes</u> equals the product of the <u>means</u>.

Examples: $\frac{2}{4} = \frac{5}{10}$ 3:4 = 18:24   15 to 10 = 6 to 4

$2 \times 10 = 4 \times 5$ $3 \times 24 = 4 \times 18$ $15 \times 4 = 10 \times 6$

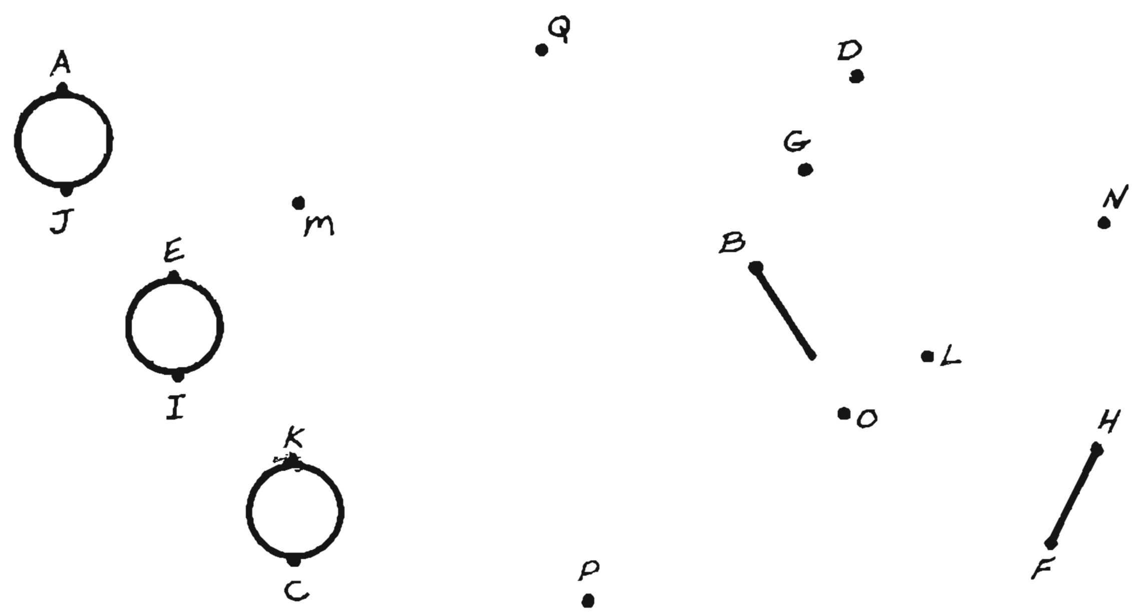
$20 = 20$ $72 = 72$ $60 = 60$

Do these ratios form proportions?

1. 5:8 = 10:16 < if Yes, connect A to D; if No, connect B to J
2. 12 to 3 = 36 to 9 < if Yes, connect I to L; if No, connect C to G
3. $\frac{4}{9} = \frac{21}{45}$ < if Yes, connect M to N; if No, connect G to L
4. 14:4 = 35:10 < if Yes, connect H to K; if No, connect F to N

Solve these proportions.

5. 3:9 = 4:$\square$, if $\square$ = < 12, connect B to E; 16, connect E to J
6. $\frac{4}{9} = \frac{\square}{18}$, if $\square$ = < 8 connect D to H; 72, connect L to M
7. $\frac{\square}{6} = \frac{12}{24}$, if $\square$ = < 72, connect D to O; 3, connect G to J
8. 9:4 = $\square$:20, if $\square$ = < 80, connect Q to D; 45, connect C to F
9. 5 to $\square$ = 15 to 9, if $\square$ = < 15, connect M to P; 3, connect B to G



THE SOLVIT MACHINE -
A DESK TOP PROPORTION CALCULATOR

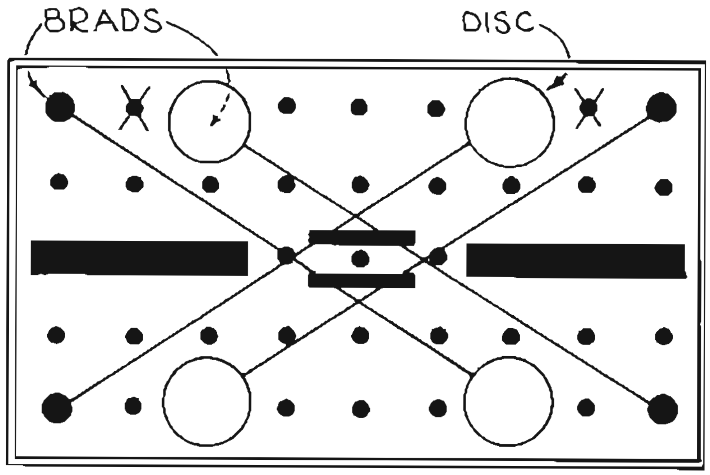
TEACHER DIRECTED ACTIVITY

Needed for Construction:
1) A piece of pegboard, 5 holes by 9 holes.
2) 4 wooden discs - about $1\frac{1}{4}''$ in diameter.
3) String and 8 brads.
4) Chalkboard paint

a) Paint the discs and the symbols with chalkboard paint.

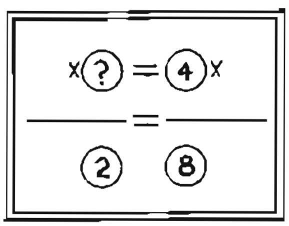
b) Put a staple on the back of each disc, loose enough to allow the disc to move freely along the string.

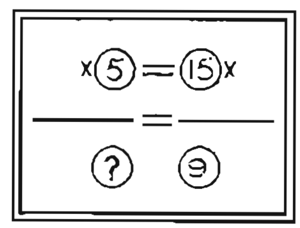
c) Place the brads and strings in the positions shown. Thread a disc on each string.



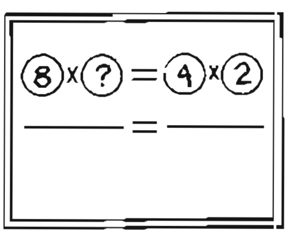
EXAMPLE 1: $\frac{?}{2} = \frac{4}{8}$

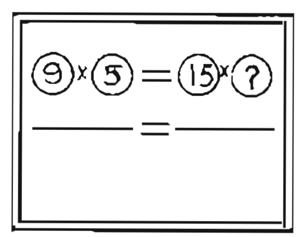
EXAMPLE 2: $\frac{5}{?} = \frac{15}{9}$



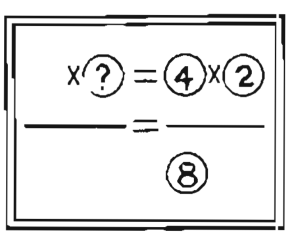


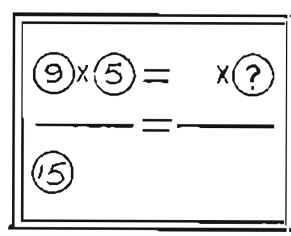
a) The student writes the numbers on the discs.





b) By sliding the lower discs carefully along the strings, the student can show the cross products.





c) By sliding the coefficient of the unknown down the string, the student can see and compute the solution to the problem.

IDEA FROM: *C.O.L.A.M.D.A.*

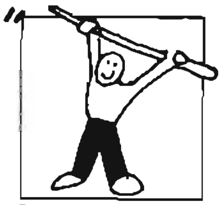
PERSONALIZED PROPORTIONS

The names and interests of students in your class can be a viable source of material for word problems. By directly involving the student (and teacher), story problems can be more interesting than those typically found in textbooks. Textbook problems, however, can be adapted simply by using the students' names. Personalizing word problems is a neat way for humanizing instruction and establishing teacher-student rapport. Here are some sample problems to be used in a proportion unit.

YOUR CLASS "TALKER"

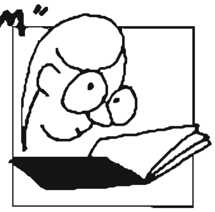
1. Mark can talk at the rate of 16 words every 5 seconds. How many words can he say in a minute and a half?

THE CLASS "ATHLETE"



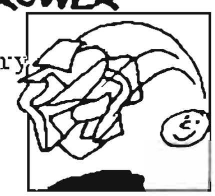
2. Brooks lifts a 4-ft. steel pipe that weighs 15 lbs. How much does he lift with a similar pipe that is 7 ft. long?

THE CLASS "BOOKWORM"



3. Doris can read 4 pages of a novel in 7 minutes. At this rate how long will it take her to read a chapter which is 26 pages long?

THE CLASS "PAPERTHROWER"

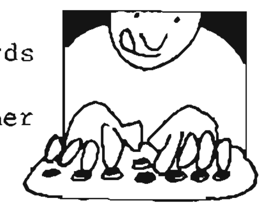


4. Paul hits the wastebasket 4 times for every 6 wads of paper he throws. At the end of the week how many hits will he have if he tosses 30 wads of paper?

"GOOD MATH STUDENT"

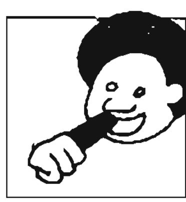
5. June gets the top score on a math test: 29 out of 30. If she always does about the same, how many points would she get on a 100-point math test?

"TYPIST"



6. Amy can type 155 words in 5 minutes. How long would it take her to type a 2500-word English theme?

"EATER"



7. Derek's favorite candy bars cost 25¢ for 3. How much does he pay for 20 of them?

THROW ONE IN PER LESSON THAT CAN'T BE SOLVED

8. Cindy saved $27 in 4 weeks. At that rate how much does she weigh if she is 5 feet, 2 inches?

9. __________ can correct math papers at the rate of 2 assignments every 3 minutes. How long will it take her to correct the papers from this class today. (There are ______ students here.)

THE STUDENT WHO ALWAYS "TRIES HARD"

10. Julie can work 3 math problems in 14 minutes. How long will it take her to do this worksheet?

Have the students make up some problems about each other.

PETITE PROPORTIONS 1

Tired of those large numbers creeping into your problems? Want to avoid straining your brain? Pounce on the problem. Try **PETITE PROPORTIONS**, our most popular prescription, and become a positively perfect and proficient problem solver.

These exercises can be solved in many ways. Encourage students to use mental arithmetic and simplify to make the computation easier.

Sample solutions:

1. 60 seconds is 6 times as long, so there would be 6 times as many heartbeats. $6 \times 13 = 78$ heartbeats

2. $\frac{13 \text{ heartbeats}}{10 \text{ sec.}} = \frac{? \text{ heartbeats}}{60 \text{ sec.}}$

$13 \times 60 = 10 \times ?$

$\frac{13 \times \cancel{60}^{6}}{\cancel{10}_{1}} = ?$

$78 = ?$

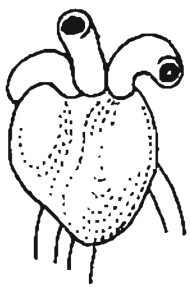
3. $\frac{10 \text{ sec.}}{60 \text{ sec.}} = \frac{13 \text{ beats}}{? \text{ beats}}$

$\frac{1}{6} = \frac{13}{?}$

$? = 6 \times 13$

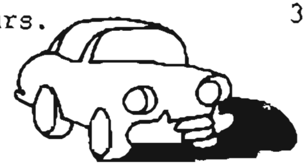
$? = 78$ heartbeats

1) 13 heartbeats in 10 seconds.
How many in 60 seconds?



Try these **PETITE PROPORTIONS**.
Divide before you find the final product.

2) 100 kilometres in 2 hours.
How many in 3 hours?

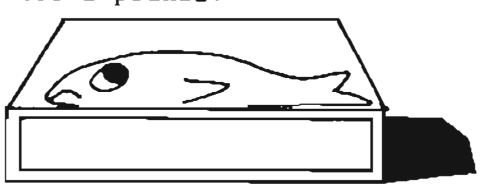


3) 5 hits every 15 times at bat.
How many hits in 75 times at bat?

4) 5 candy bars cost 59¢.
How much for 20 candy bars?

5) $3.50 for 5 magazines.
How much for 10 magazines?

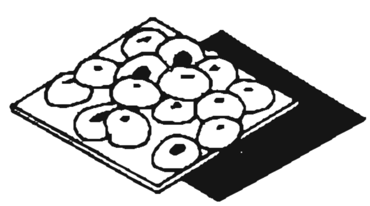
6) $3.98 for 4 pounds.
How much for 2 pounds?



7) 4 cans of beans for $1.00.
How much for 6 cans?

8) Run 50 metres in 8 seconds.
How many in 4 seconds?

9) 6 donuts for 53¢.
How much per dozen?



10) 21 problems solved in 3 minutes.
How many solved in 24 minutes?

PETITE PROPORTIONS 2

1) 2 dozen for $1.68.
   5 dozen for ________.

2) 24 pencils for 88¢.
   18 pencils for ________.

3) 6 cans of peas for $1.80.
   9 cans of peas for ________.

4) A drill turns 240 times in 3 seconds.
   A drill turns ________ times in 60 seconds.

5) 192 cm of pipe weighs 8 kg.
   ________ cm of pipe weighs 2 kg.

6) 100 metres of fencing cost $89.50.
   20 metres of fencing cost ________.

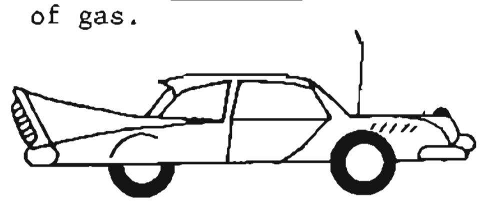
7) 3 records for $11.94.
   2 records for ________.

8) 20 minutes to do 30 math problems.
   50 minutes to do ________ math problems.

9) 2 pantsuits for $35.
   7 pantsuits for ________.

10) 2 cm on a map represents 100 km.
    5.3 cm represents ________ km.

11) Car goes 10 km on 2 litres of gas.
    Car goes ________ km on 16 litres of gas.



12) Check 14 cars in 30 minutes.
    Check ________ cars in 75 minutes.

Paper & Pencil

No particular method is suggested for solving these problems. Mental arithmetic is to be encouraged. The cross-products rule can be used to check the solutions.

DID YOU KNOW THAT . . .

There is an 8-word, 32-letter sentence that uses all 26 letters of the alphabet. The sentence is a great typing exercise.

Solve the proportions to discover the sentence. Write the problem letter under the answer to the problem in the table below.

A 8 correct out of 15.
___ correct out of 75.

B 1 inch represents ___ miles.
10 inches represents 500 miles.

C 5000 revolutions per minute.
___ revolutions per 15 minutes.

D 45 words per minute.
225 words per ___ minutes.

E 17 miles per gallon.
238 miles on ___ gallons.

F ___ yards per pass.
162 yards on 9 passes.

G ___ heartbeats per minute.
19 heartbeats per 15 seconds.

H 10 feet in 20 seconds.
___ feet in 60 seconds.

I 12 apples for 60¢.
___ apples for $1.00

J ___ for 1 pound.
$1.95 for 3 pounds.

K 46 hits out of 200 times at bat.
___ hits out of 1000 times at bat.

L ___ for 3 cans.
$1.55 for 5 cans.

M 55 miles in 1 hour.
385 in ___ hours.

N $.74 for 1 dozen.
___ for 6 dozen.

O ___ pounds per cubic foot.
150 pounds per 6 cubic feet.

P ___ made out of 56 tries.
5 made out of 8 tries.

Q 250 kilometres on 40 litres.
50 kilometres on ___ litres.

R 240 kilometres in ___ hours.
80 kilometres in 1 hour.

S 3 centimetres represents 90 kilometres.
___ centimetres represents 180 kilometres.

T $4.00 per hour.
___ for 8 hours.

U 100 won out of 150 played.
10 won out of ___ played.

V $200 per month.
$2400 per ___ months.

W 3° rise in 1 hour.
12° rise in ___ hours.

X 4 beats per measure.
36 beats in ___ measures.

Y ___ for 5 yards of fabric.
$2.50 for 1 yard.

Z 3 tennis balls per can.
___ tennis balls in a dozen cans.

35	40	75,000	230		7	$1250		50	25	9		4	20	$32	30		18	20	12	14
		5	25	36	14	$4.44		$.93	20	8	15	25	3		$.65	15	76	6		

A STEWED SURPRISE

Solving Proportions
Getting Started
PROPORTION

E
5
I

SOLVE THE PROPORTIONS BELOW AND FIND YOUR ANSWERS IN THE CODE AT THE BOTTOM OF THE PAGE. FOR EACH ANSWER IN THE CODE WRITE THE LETTER IN THE PROPORTION ABOVE IT.

KEEP WORKING UNTIL YOU HAVE DECODED THE LIMERICK.

1. 1 : 2 = T : 8
   T = ____
2. 8 : W = 2 : 3
   W = ____
3. 2 : 5 = 10 : C
   C = ____
4. 6 : S = 3 : 7
   S = ____
5. L : 15 = 6 : 5
   L = ____
6. 7 : 2 = B : 12
   B = ____
7. V : 14 = 12 : 7
   V = ____
8. 5 : 7 = A : 14
   A = ____
9. 4 : 3 = H : 6
   H = ____
10. 7 : 5 = 28 : G
    G = ____
11. M : 6 = 18 : 36
    M = ____
12. 36 : 9 = F : 4
    F = ____
13. 3 : 8 = 30 : O
    O = ____
14. 3 : 11 = 6 : D
    D = ____
15. 3 : 5 = 21 : U
    U = ____
16. 3 : 2 = Q : 20
    Q = ____
17. 9 : 10 = 45 : R
    R = ____
18. 8 : 72 = 9 : I
    I = ____
19. 15 : 27 = 5 : E
    E = ____
20. 7 : 4 = N : 16
    N = ____

A GENTLEMAN DINING AT CREWE
10 20-9-28-4-18-9-3-10-28 22-81-28-81-28-20 10-4 25-50-9-12-9

FOUND QUITE A LARGE MOUSE IN HIS STEW
16-80-35-28-22 30-35-81-4-9 10 18-10-20-50-9 3-80-35-14-9 81-28 8-81-14 14-4-9-12

SAID THE WAITER DON'T SHOUT
14-10-81-22 4-8-9 12-10-81-4-9-50 22-80-28-4 14-8-80-35-4

AND WAVE IT ABOUT
10-28-22 12-10-24-9 81-4 10-42-80-35-4

OR THE REST WILL BE WANTING ONE TOO
80-50 4-8-9 50-19-14-4 12-81-18-18 42-9 12-10-28-4-81-28-20 80-28-9 4-80-80

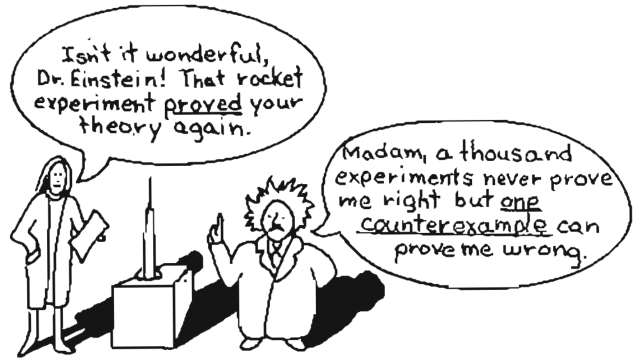
COUNTEREXAMPLE

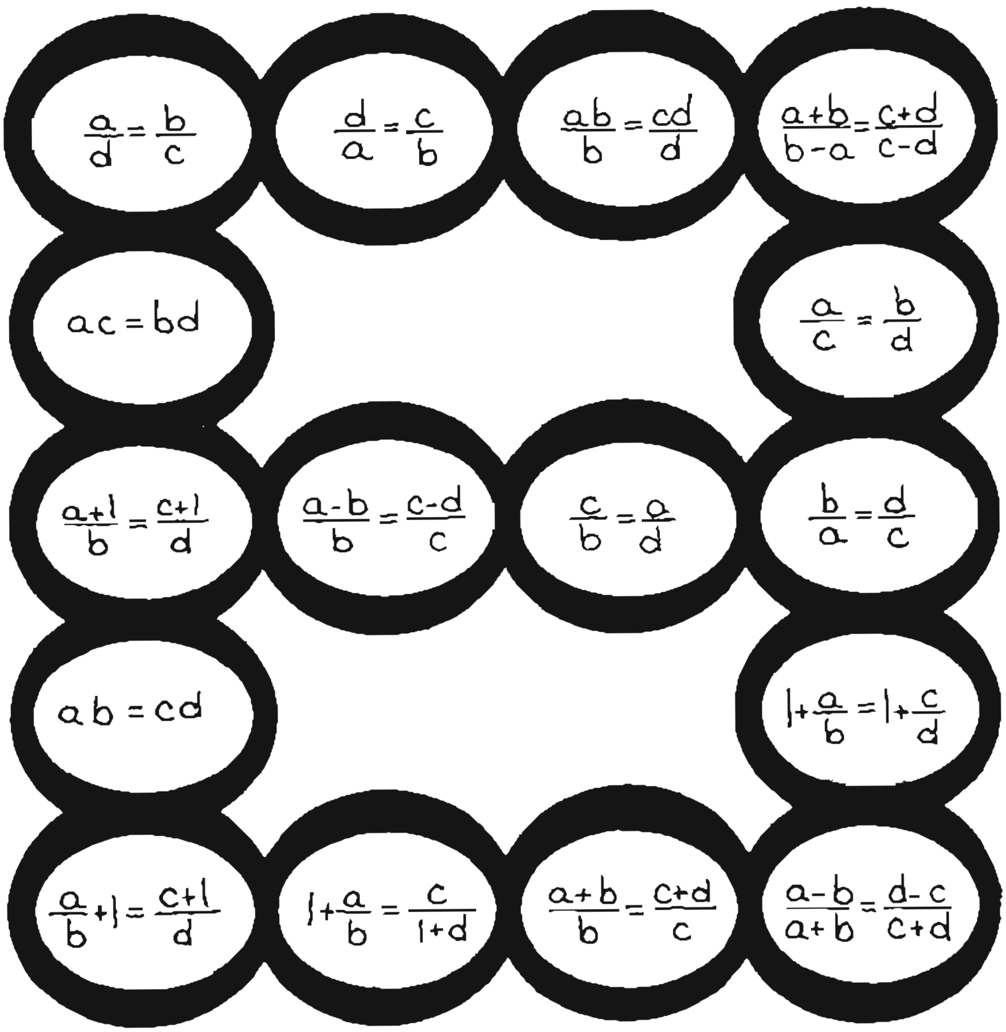
TEACHER DIRECTED ACTIVITY

Much emphasis is given in mathematics to finding the correct solution to a problem. This activity is one designed to have the student find a counterexample by substituting values to make the statement false. Several of these examples could be tried with a group of students, and the remaining problems could be used as a competition between two groups of students.

Two alternative methods are (1) making a bulletin board display of the problems and allowing students to write the counterexample on the display when one is discovered or (2) giving an individual student the problems to work on.

As well as finding counterexamples, students should also be encouraged to find at least one set of values that makes the problem true. Students should know that some of the problems are true, and no counterexample exists.





Assume $\frac{a}{b} = \frac{c}{d}$ and none of a, b, c, or d are zero. Try to find counterexamples for each of the problems. Shade the problems that have counterexamples to find a letter in the alphabet.

Solution strategy—If $\frac{a}{b} = \frac{c}{d}$, does $\frac{a-d}{b} = \frac{c-b}{d}$? Use a simple proportion $\frac{1}{2} = \frac{2}{4}$ where a = 1, b = 2, c = 2, d = 4. Then $\frac{a-d}{b} = \frac{c-b}{d}$ becomes $\frac{1-4}{2} \neq \frac{2-2}{4}$ and this set of values is a counterexample.

TYPE: Paper & Pencil/Puzzle

RATIO, PROPORTION, AND SCALING

Placement Guide for Tabbed Divisors

CONTENTS

PROPORTION: APPLICATION

PROPORTON PROJECTS TO PURSUE

Application
PROPORTION
I

TEACHER IDEAS

1) In an almanac find the world records for the 100-metre dash, 400-metre dash, 1500-metre run, and the 3000 metre run. Are the rates of distance to time for each race proportional? If a 6000-metre run were a track event, predict the world record time.

2) Go to the supermarket and find several sizes of the same product. Record the prices and the net weights (weight of contents only) of the different sizes. Are the rates of price to net weight proportional? Investigate cereals, soap powders, shampoos, hamburger, and sugar.

3) Check the phone book and approximate the number of Smiths living in your city and surrounding area. Choose several other cities and approximate the number of Smiths living in these cities. (Most public libraries have phone books of other cities.) Compare the ratios of number of Smiths to total population for each city. (Remember to use the total population of the city and surrounding area.) Are the ratios proportional? Use the ratio of your city to predict the number of Smiths living in San Francisco; New York; your state, the United States.

Paper & Pencil

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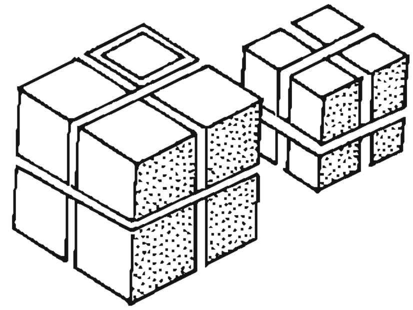
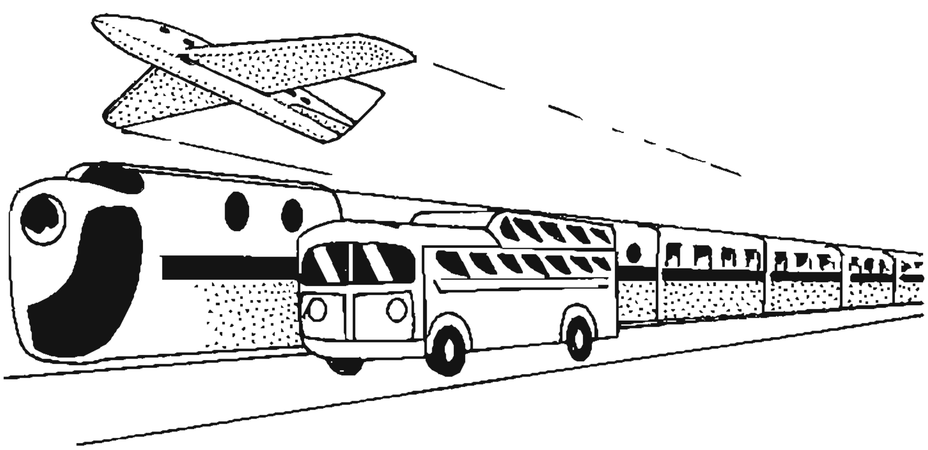
PROPORTION PROJECTS TO PURSUE
(CONTINUED)

4) From the post office get the rates for mailing letters and packages. Is the cost of mailing a light package proportional to the cost of mailing a large package? Is the cost of mailing a package a short distance equivalent to the cost of mailing the same package a long distance?



5) From a catalog of Montgomery Ward, Penney, Sears Roebuck, or Spiegel, find the shipping rates for orders. Is the cost of shipping a light package proportional to the cost of shipping a heavy package? Is the cost of shipping a package a short distance proportional to the cost of shipping the same package a long distance?

6) Find out the cost of train fare from your nearest railroad station to four other stations. Are the rates of cost to distance traveled proportional for the trips?



Find the same information for buses and airplanes. Which of the three types of transportation has the most consistent rate?

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IT'S ONLY MONEY

Foreign Exchange

NEW YORK (AP) — Tuesday Foreign exchange in dollars and decimals of a dollar. New York prices.

	Tues.	Fri.
Argentina (peso)	.0900	.0900
Australia (dollar)	1.3525	1.3500
Austria (schilling)	.0615	.0610
Belgium (franc)	.028675	.028900
Brazil (cruzeiro)	.1310	.1310
Britain (pound)	2.3260	2.3260
30 Day Futures	2.3175	2.3170
60 Day Futures	2.3090	2.3090
90 Day Futures	2.3010	2.3000
Canada (dollar)	.9760	.9750
Colombia (peso)	.0340	.0340
Denmark (krone)	.1860	.1850
France (franc)	.2525	.2515
Holland (guilder)	.4168	.4165
Hong Kong (dollar)	.2050	.2050
Israel (pound)	.1800	.1800
Italy (lira)	.001615	.001610
Japan (yen)	.003440	.003440
Mexico (peso)	.0801	.0801
Norway (krone)	.2040	.2035
Portugal (escudo)	.0425	.0415
South Africa (rand)	1.4750	1.4750
Spain (peseta)	.0180	.0181
Sweden (krona)	.2560	.2560
Switzerland (franc)	.4050	.4030
Venezuela (bolivar)	.2340	.2340
W. Germany (dchmark)	.4300	.4295

This chart is taken from the financial page of a newspaper. It can be used as a source of many proportion exercises. Students can bring the chart from home. Before starting an activity, explanation of the chart may be necessary; necessary, i.e., 1 peso = \$.09, 1 Sweden krona = \$.256, and 1 South Africa rand = \$1.475. Explanation of the decimal part of a cent may also be necessary.

Exercises could be developed like the ones that follow.

(a) A family planning a trip to the United States wishes to exchange 1000 dchmark. What country is the family from? _______________ How much American money will the family receive according to the Tuesday exchange rate? __________________

(b) An American businesswoman will be visiting a factory in Hong Kong. She wishes to exchange \$2000 into Hong Kong dollars. About how many will she get? _______________

(c) An investor in Belgium has 1,000,000 francs to exchange. Which would have been the better day to make the exchange? _______________ How many more American dollars would the investor receive by choosing the better day? _______________

(d) An unlucky investor waited until Friday to exchange 700,000 Australian dollars to American dollars. How much did he lose by waiting? _______________ Should he keep his money and wait until next week? __________

(e) A livestock buyer will be going from the United States to Spain and then to Argentina. To avoid exchanging the peseta back to American dollars and then to peso, find the exchange rate between peseta and peso.

One solution strategy:

$$\left.\begin{array}{l} 1 \text{ peso} = \$.09 \rightarrow 200 \text{ peso} = \$18.00 \\ 1 \text{ peseta} = \$.018 \rightarrow 1000 \text{ peseta} = \$18.00 \end{array}\right\} \rightarrow \begin{array}{l} 200 \text{ peso} = 1000 \text{ peseta, so} \\ \quad 1 \text{ peso} = 5 \text{ peseta} \end{array}$$

(f) Investigate the different coins of a country. Is the value of each coin related to the decimal system or to some other place value system?

(g) Investigate the monetary system of countries not listed on the exchange table. Can you find the exchange rate according to an American dollar?

STRETCH SMITH

"Stretch" Smith, a basketball star, predicts his age and height will remain in the same ratio. At 12 years "Stretch" was 160 centimetres tall.
Age : Height = 12 yrs. : 160 cm

Complete the tables:

How Tall Will "Stretch" Be?

A	H
12 Yrs.	160 cm
15 Yrs.	
18 Yrs.	
24 Yrs.	
30 Yrs.	
36 Yrs.	

How Old Was "Stretch"?

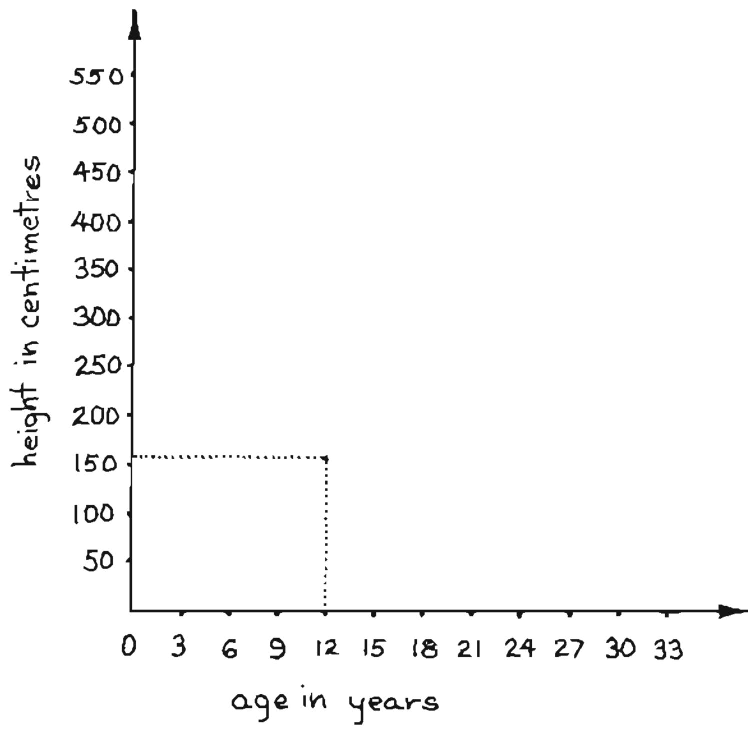
A	H
12 Yrs.	160 cm
Yrs.	120 cm
Yrs.	80 cm
Yrs.	40 cm
Yrs.	20 cm
Yrs.	10 cm

a) Graph the information from the tables.
b) What do you notice about the graph?
c) Use the graph to approximate his height at these ages.

1. A = 0 yrs., H = ______
2. A = 33 yrs., H = ______
3. A = 4 yrs., H = ______
4. A = 50 yrs., H = ______

d) Use the graph to approximate his age when his height has these values.

1. A = ______, H = 30 cm
2. A = ______, H = 100 cm
3. A = ______, H = 600 cm
4. A = ______, H = 0 cm



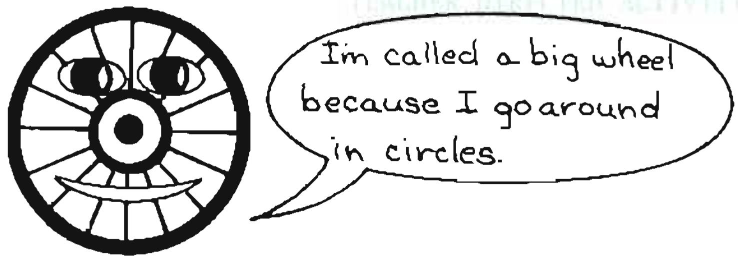
Are your age and height proportional, that is, do they stay in the same ratio?

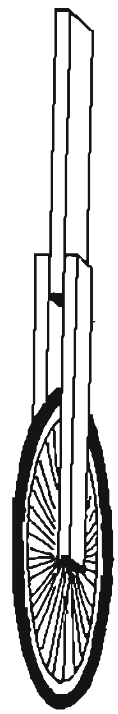
Does "Stretch" know what he is talking about?

Can you think of any two things about you that are proportional?

ONE GOOD TURN DESERVES ANOTHER

TEACHER DIRECTED ACTIVITY





Materials needed: 1 bicycle wheel or a round piece of wood
1 tape measure
1 piece of rope
1 roll of masking tape

A bicycle and/or tricycle wheel could be used. If both are used, students could compare the results.
Mounting the wheel in a simple frame will make it easy to roll.

Activity 1: Place a marker on the wheel. Tape a straight line on the floor approximately 4 metres long. Place the wheel so the marker is at one end of the tape. Carefully roll the wheel 1 turn along the tape. Measure this distance in centimetres or feet. Repeat this to check your measurement. Write the ratio that compares 1 turn of the wheel to the distance measured (the circumference of the wheel).

1 turn: ________ (feet or centimetres)

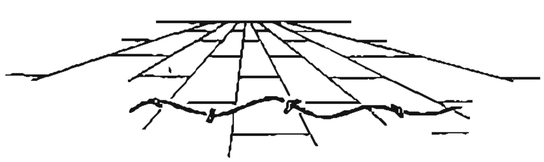
This ratio can now be used in the following activities to measure distances. Students should be encouraged to first estimate the distances to be measured. The students can place markers on the wheel to show $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of a turn to increase their accuracy.

Activity 2: Find the length and width of your classroom.

Activity 3: Find the length of the sideline and baseline of your basketball floor.

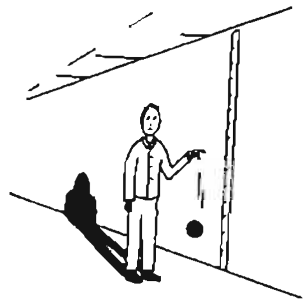
Activity 4: Use the ratio to find the number of turns needed to go 50 metres. Check the answer with the wheel.

Activity 5: Tape the rope to the floor in a curved line and use the wheel to find its length.



TYPE: Activity

THAT'S THE WAY THE OLD BALL BOUNCES



Materials needed: Tennis ball
Metre stick
Strip of paper or tape
3 metres long

(1) Use the metre stick to mark the strip of paper (tape) into decimetres. Mount the strip of paper on the wall. Be sure the zero mark is at the base of the wall.

(2) Drop the ball from the heights listed in the table. Each time write down the height of the first bounce. Repeat the drops to check the accuracy of your readings. Select four different heights for the last four trials.

Height of bounce in decimetres	____	____	____	____	____	____	____	____
Height of drop in decimetres	10dm	15dm	5dm	20dm				

(3) Examine the table and compare the ratio: $\frac{\text{height of bounce in decimetres}}{\text{height of drop in decimetres}}$ for the various drops.

If you measured carefully, the eight ratios should be nearly equivalent. Since the ratio of the height of the bounce to the height of the drop is nearly the same, we can say that "the bounce is proportional to the drop."

(4) Use this information to complete the following:
If the height of the drop is 40 decimetres, the bounce will be about ________________.
If the height of the bounce is 7 decimetres, the ball was dropped from a height of ________________.

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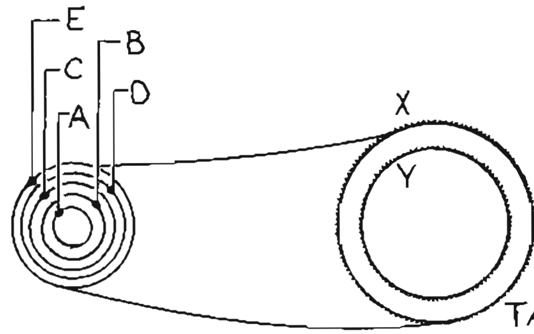
GET IN GEAR

Have a student bring a 5 or 10 speed bicycle to class. Turn the bike upside down so that the gears can be shifted. Put a piece of tape on the rear wheel of the bicycle. Have the students count the teeth in each gear and record in Table 1. (The number is not standard. The front gears vary from 52 to 39 teeth and the rear gears from 34 to 14.)



Write the gear ratios and simplify. Record in Table 2.

The following activities are suggested for student investigation:

TABLE 1

Gear	Number of teeth
X	
Y	
A	
B	
C	
D	
E	

TABLE 2

Gear Ratio	Ratio of number of teeth	Simplified Teeth Ratio
X to A		
X to B		
X to C		
X to D		
X to E		
Y to A		
Y to B		
Y to C		
Y to D		
Y to E		

1. Select a simple gear ratio, for example, 13 to 4, and set the gears to correspond. Check the gear ratio by slowly turning the pedals. The pedals should turn four times and the wheel thirteen. (Hold the rear tire lightly to aid in counting the turns of the wheel.) Check some other gear ratios by counting pedal and rear wheel turns.

2. Select a back gear and use the small front gear. Turn the pedals slowly and shift to the large front gear. Continue turning the pedals at the same rate. What change do you notice in the back wheel? Can you explain? What are the corresponding gear ratios?

3. Move the gearshifts so the chain is on the smallest back and front gears. Turn the pedals at a constant rate. Shift only the back gear so that the chain travels from the smallest to the largest gear wheel. What change occurs in the back wheel? Can you explain. What are the corresponding gear ratios?

4. If the pedals were turned at a constant rate, which ratio would cause the back wheel to turn the fastest? Order the simplified gear ratios from largest to smallest. Students could use a calculator to change each ratio to a decimal and then order the decimals.

5. In riding the bicycle, which gear setting is the easiest to pedal? the most difficult? Experiment on the playground. Which gear setting allows you to travel the farthest for one turn of the pedal? Devise a method for checking your prediction.

6. Select a gear setting. Suppose you pedal at a constant rate (one turn per second, thirty turns per minute, etc.). How far would you travel in 20 minutes?

7. Select a gear setting. How many turns of the pedal are needed for the bike to travel a distance of one mile?

what's YOUR TYPE

1. Weigh yourself and measure your height. ______ pounds ______ inches
2. Change your weight to kilograms.
   1 pound ≈ .45 kilograms.
3. Change your height to centimetres. (1 inch ≈ 2.5 centimetres.)
4. Use the chart to determine your body type.

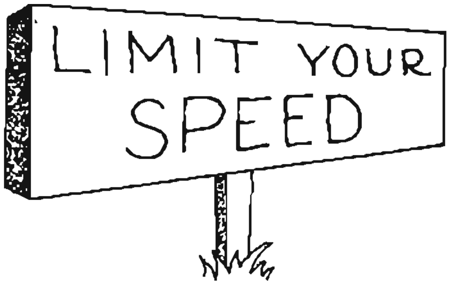
$$\frac{.45 \text{ kilograms}}{1 \text{ pound}} = \frac{?}{\text{your weight}}$$

GROWTH CHART FOR GIRLS	10 Yrs.	11 Yrs.	12 Yrs.	13 Yrs.	14 Yrs.	15 Yrs.
Height in centimetres						
Tall	143-155	153-163	157-168	162-170	162-173	164-173
Average	134-142	140-152	147-156	152-161	154-161	156-163
Short	125-133	130-139	135-146	140-151	146-153	147-155
Weight in kilograms						
Heavy	40-52	45-59	49-63	55-68	57-71	60-72
Average	29-39	33-44	36-48	41-54	45-56	47-59
Light	23-28	25-32	28-35	31-40	36-44	39-46

GROWTH CHART FOR BOYS	10 Yrs.	11 Yrs.	12 Yrs.	13 Yrs.	14 Yrs.	15 Yrs.
Height in centimetres						
Tall	149-155	149-163	157-168	162-178	169-183	169-185
Average	134-148	139-148	142-156	149-161	154-168	159-168
Short	125-133	130-138	133-141	138-148	143-153	148-158
Weight in kilograms						
Heavy	38-52	43-57	48-63	50-70	61-75	67-78
Average	30-37	33-42	38-47	39-49	45-60	49-66
Light	23-29	27-32	29-37	31-38	34-44	40-48

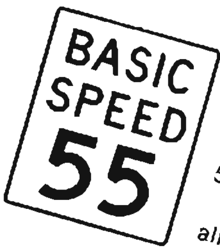
5. Sue is 15 years old, weighs 127 pounds, and is 5 feet, 7 inches tall.
   a) Find her weight in kilograms. ________
   b) Find her height in centimetres. (Hint: 12 inches = 1 foot) ________
   c) What is her body type? ________
6. John is 11 years old, weighs 65 pounds, and is 53 inches tall.
   a) Find John's weight in kilograms. ________
   b) Find John's height in centimetres. ________
   c) What is his body type? ________
7. Fred is 14 years old, weighs 120 pounds, and is 65 inches tall.
   Guess his body type. ________
   Check your guess by changing Fred's measurements to metric.

Some students may be sensitive about revealing their body type. For this reason you might want to make the first four questions optional.



The state of Oregon has the following speed laws.

20 MILES PER HOUR
- In any business district.
- When passing school grounds or crosswalks when children are present and signs are posted.



25 MILES PER HOUR
- In any residential area.
- On ocean beaches, where motor vehicles are permitted. (A maximum limit, or as posted.)
- In city public parks, unless otherwise posted.

55 MILES PER HOUR
- On the open highway in locations other than those already mentioned, unless otherwise posted.

I. Find out the speed laws for your state or use those for Oregon to answer these questions.

1 mile ≈ 1.6 kilometres

1) What is the speed limit in kilometres per hour in front of your school?

2) What is the speed limit in kilometres per hour in front of your home?

3) What would be a reasonable speed limit in kilometres per hour for freeway driving in your state?

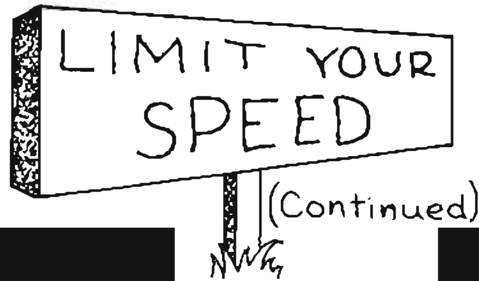
4) If a trailer is being towed by a pickup or truck, the maximum speed limit is 50 miles per hour. What is the speed in kilometres per hour?

5) 

40 mph

This curve can be safely driven at __________ kilometres per hour.

6) What speed in kilometres per hour will cars be going in the Indianapolis 500? Use an almanac to help you.



II.

STOPPING DISTANCES OF STANDARD PASSENGER CARS

MILES PER HOUR	DRIVER REACTION DISTANCE FT.	BRAKING DISTANCE FT.	TOTAL STOPPING DISTANCE FT.
20	22	18-22	40-44
30	33	36-45	69-78
40	44	64-80	108-124
50	55	105-131	160-186
60	66	162-202	228-268
70	77	237-295	314-372
80	88	334-418	422-506

1 foot ≈ .3 metre

1 mile ≈ 1.6 kilometres

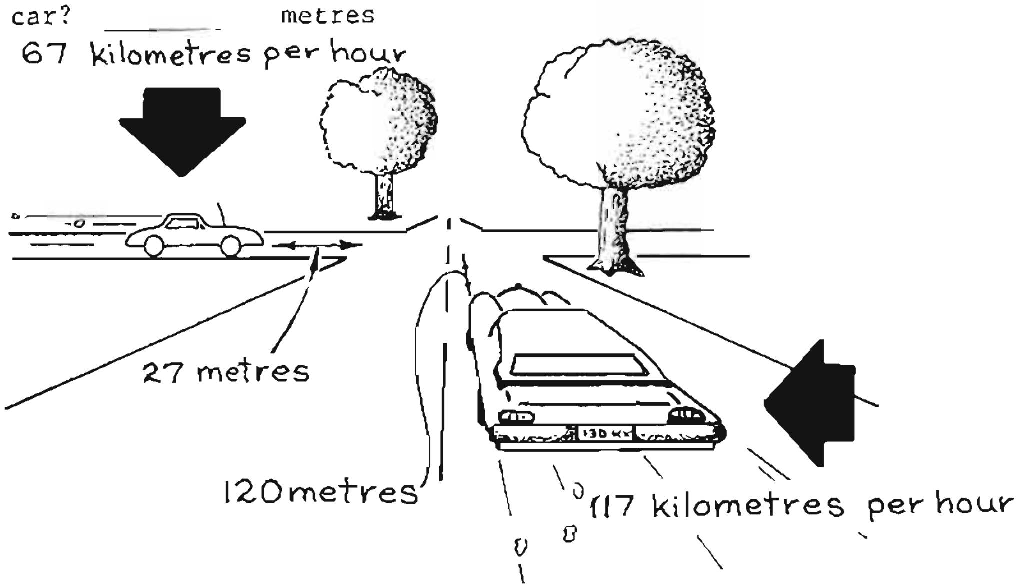
The distances in the table are based on tests given on dry, level ground. Stopping distances increase when the road is wet, snowy, or icy.

1) If you are driving at a speed of 50 kilometres per hour, could you stop the car in 24 metres? ________

2) If you are driving at a speed of 84 kilometres per hour, how close can you safely follow another car? ______ metres

3) If a driver's reaction time is about 26 metres, his speed is about ________ kilometres per hour.

4) If both drivers hit their brakes, will the two cars crash?



CRUISING AROUND

	Bakersfield	Bangor	Birmingham	Des Moines	Olympia	San Antonio	Santa Fe	Pittsburgh
Bakersfield								
Bangor	2760							
Birmingham	1900	1300						
Des Moines	1440	1310	800					
Olympia	860	2660	2160	1540				
San Antonio	1320	1980	760	920	1880			
Santa Fe	760	2180	1260	820	1250	730		
Pittsburgh	2200	640	660	780	2240	1340	1500	

The table to the left is a mileage chart for air routes.

Change each distance, to kilometres and record in the table below.

Remember: 1 mile = $\frac{10}{6}$ kilometres

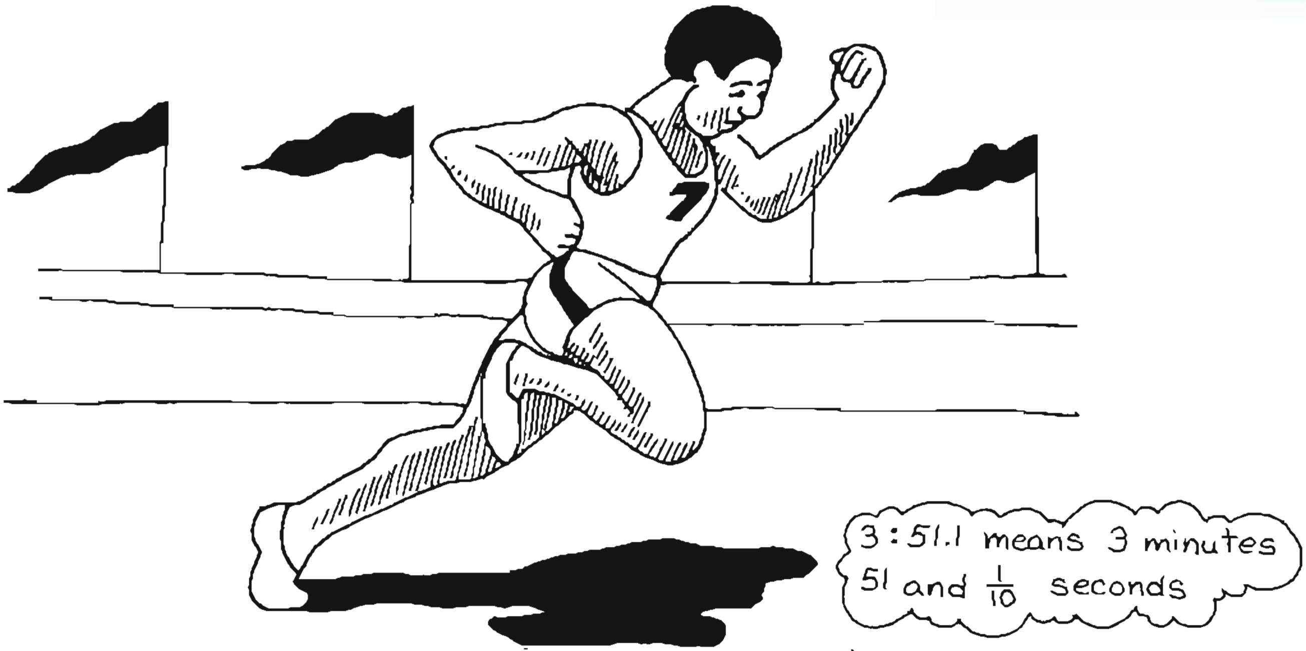
$$\frac{1 \text{ mile}}{\frac{10}{6} \text{ kilometres}} = \frac{2760 \text{ miles}}{? \text{ kilometres}}$$

$$\frac{10}{6} \times 2760 = 1 \times ?$$

	Bakersfield	Bangor	Birmingham	Des Moines	Olympia	San Antonio	Santa Fe	Pittsburgh
Bakersfield								
Bangor								
Birmingham								
Des Moines								
Olympia								
San Antonio								
Santa Fe								
Pittsburgh								

If a plane cruises at a rate of 570 miles per hour the rate in kilometres per hour is ______.

WORLD RECORDS



WORLD RECORDS

1) 1 mile - 3:51.0
   2 miles - 8:13.8
   3 miles - 12:47.8

   Each of these three records is roughly proportional to running a mile every ______ minutes.

2) Steve Williams of the U.S. ran the 100-metre dash in 10.1 seconds and the 200-metre dash in 20.6 seconds. His speed was about _____ metres per second.

3) Tommie Smith of the U.S. ran both the 200-yard dash and the 200-metre dash. The time for both is 19.5 seconds. Does this mean he ran the same speed for each race? _____ If one race has a faster speed which race is it? _____

4) These are world records. Rank them from slowest to fastest based on the time taken to go 1 kilometre (1000 metres).

   a) Canoeing (1000 m) 3:48.06
   b) Swimming (1500 m) Men 15:31.85
   c) Running (1500 m) Women 4:01.4
   d) Ice Skating (1500 m) Men 1:58.7
   e) Running (1000 m) 2:16
   f) Swimming (1500 m) Women 16:49.9
   g) Cycling (1000 m) 1:7.51
   h) Ice Skating (1500 m) Women 2:15.8

5) Is the world record of 9.9 seconds for 100 metres faster or slower than the world record of 9.1 seconds for 100 yards?
   (1 yd. $\approx$ .914 m or 1 m $\approx$ 1.1 yd.)

6) The world land speed record for a jet-propelled automible is about 622 miles per hour. At this rate how long would it take to drive to the moon (if there was a road) 240,000 miles away? ____________________

Based on information taken from *The Official Associated Press Sports Almanac 1974.*
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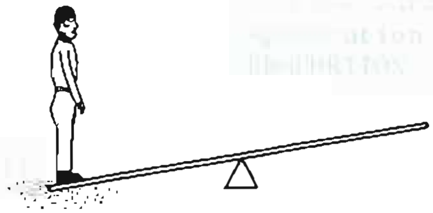
PROPORTIONS With a PLANK



Materials needed: Ten-foot plank, four-inch concrete building block, bathroom scale, measuring tape, metre or yard stick.

I. Balance the plank by placing the block in the middle. Ask for a volunteer (or the teacher) to stand on one end of the plank. Have different members of the class try to balance the plank by standing on the opposite end. For the plank to balance students should realize the weights of the volunteers should be about equal. Weigh the volunteers.

II. Pick two members of the class having different weights. Weigh them and record the weights. Keep the block in the middle and ask them to stand on opposite ends of the plank and balance each other. Students will probably use their previous experience with teeter-totters to accomplish the task.

III. Again pick two members of the class having different weights. This time their task is to stand on the ends of the plank and balance it by moving the block.

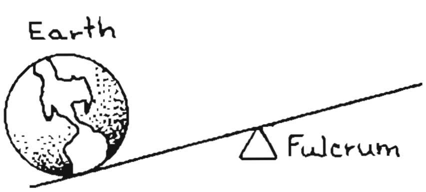
IV. Have the students use the three activities above to formulate a conjecture about how a balance occurs. Students will probably say that the heavier weight is closer to the block, and the lighter weight is farther away from the block.

V. Ask students to examine the relationship between the weights and distances by completing a table. By using two students whose weights are considerably different, a pattern can be discovered. The results in the last column will be approximately equal.

Weight of person(W)	Distance W is from block (D)	W+D	W−D	W÷D	W×D

The General Rule is : $W_1 \times D_1 = W_2 \times D_2$, or $\dfrac{W_1}{W_2} = \dfrac{D_2}{D_1}$.

VI. Students can apply the general rule to solve problems: For example, John weighs 90 lbs. and stands 4 feet from the block. Tim balances the plank by standing 3 feet from the block. How much does Tim weigh?

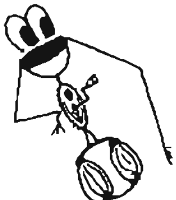


"Give me a place to stand, and I will move the Earth." This is what the famous Greek scientist Archimedes (287-212 B.C.) was supposed to have boasted after discovering the law of the lever: $W_1 \times D_1 = W_2 \times D_2$. Assume that Archimedes weighs 150 lbs., and the fulcrum of the lever is 4,000 miles from the Earth. How far from the fulcrum would he have to stand in order to move the Earth? The Earth weighs 13,176,000,000,000,000,000,000,000 lbs.

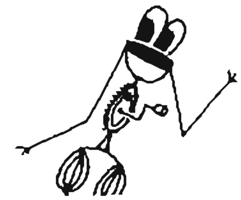
TYPE: Activity

If a plank is not available, a 1-metre board can be used to balance several objects.

I'M BEAT!
HOW ABOUT YOU?



Materials: 1 hand eggbeater



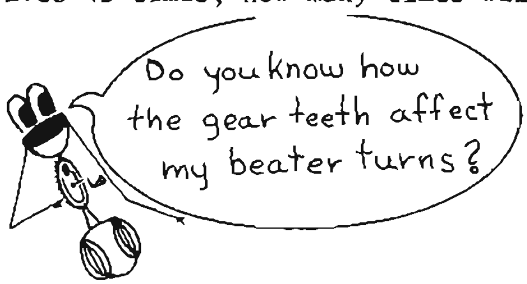
(1) Turn the crank one complete turn and have your partner count the number of turns of the beater. ________

(2) How many beater revolutions are there when you turn the crank 4 times? ________ 6 times? ________

(3) Write a ratio showing the number of beater revolutions for 1 turn of the crank. ________

(4) Predict the number of beater revolutions if the crank makes 8 turns. ________ Check your prediction by turning the crank and counting.

(5) If the beater revolves 45 times, how many times will the crank turn? ________



(6) Count the teeth in each gear and record your answer. ________, ________ Write a ratio that compares the number of teeth in the large gear to the number of teeth in the small gear. ________

(7) There are ________ teeth in the large gear for each one tooth in the small gear. Write this ratio. ________ This ratio should be equivalent to the ratio in question 6.

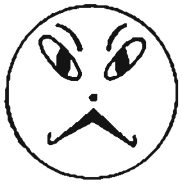
(8) Compare the ratio in question 3 to the ratio in question 7. (Beater revolutions : 1 turn of the crank = number of teeth in large gear : 1 tooth in small gear)

(9) Is the ratio of beater revolutions to turns of the crank always equivalent to the ratio of the teeth in the large gear to the number of teeth in the small gear? Use the information in questions 4 and 6 to help you decide. ______ beater revolutions : 8 turns of the crank = ______ teeth in large gear : ______ teeth in small gear.

(10) An egg beater has gears with 64 and 14 teeth each. If the crank is turned 28 times, how many revolutions will each beater make?

(11) In making the meringue for a lemon meringue pie, you must beat the egg whites until they are stiff. This may take 4 minutes of rapid beating. If you turned the crank 100 times a minute, how many times would each beater revolve during the 4 minutes?

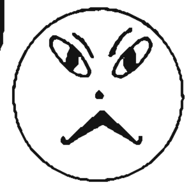
IDEA FROM: *Activities with Ratio and Proportion*



I mean to be MEAN!

A proportion is a <u>mean</u> proportion when the two means are equal.

$$\frac{⬡}{☹} = \frac{☹}{△}$$



I'm a <u>mean</u> proportional between ⬡ and △

Circle the proportions that are <u>mean</u> proportions.

$$\frac{1}{7} = \frac{7}{49} \qquad \frac{6}{5} = \frac{6}{5} \qquad \frac{4}{2} = \frac{2}{1}$$

EXAMPLES:

A) Find a mean proportional between 3 and 12.

Solution:

$$\frac{3}{☹} = \frac{☹}{12}$$

$3 \times 12 = ☹ \times ☹$

$36 = ☹ \times ☹$

$6 = ☹$

B) Find an approximate mean proportional between 12 and 4.

Solution:

$$\frac{12}{☹} = \frac{☹}{4}$$

$12 \times 4 = ☹ \times ☹$

$48 = ☹ \times ☹$

$6.9 \approx ☹$

7×7=49, so I'm a little less than 7, about 6.9

Find the mean proportional between:

a) 4 and 16
b) 5 and 45
c) 2 and 50
d) 8 and 2
e) 100 and 4

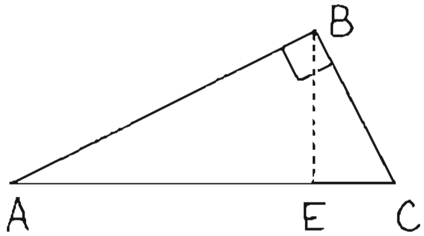
Approximate the mean proportional between:

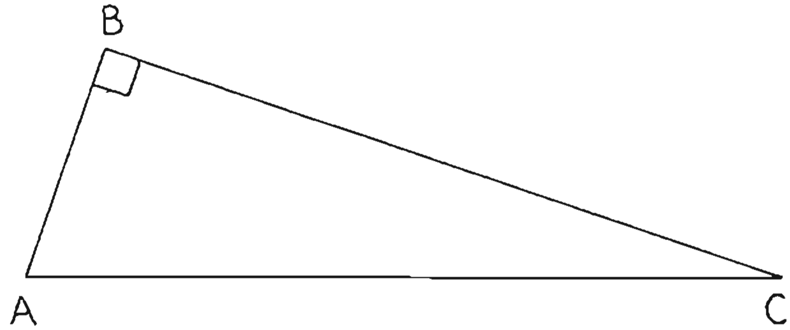
a) 3 and 16
b) 5 and 7
c) 8 and 4
d) 5 and 3
e) 12 and 4

Challenge: Find three pairs of numbers for which 8 is the mean proportional.

MAKING MEANS MEANINGFUL

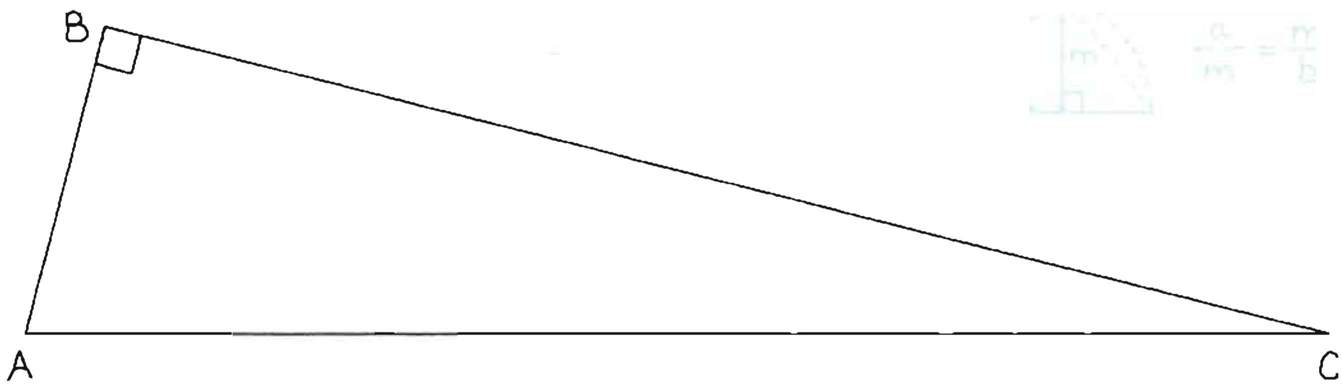
In each right triangle below draw a line through B perpendicular to the line through A and C. Where the two lines cross, label the point E. Measure the line segments $\overline{AE}$, $\overline{EC}$, and $\overline{BE}$ to the nearest centimetre and record below to complete each ratio.



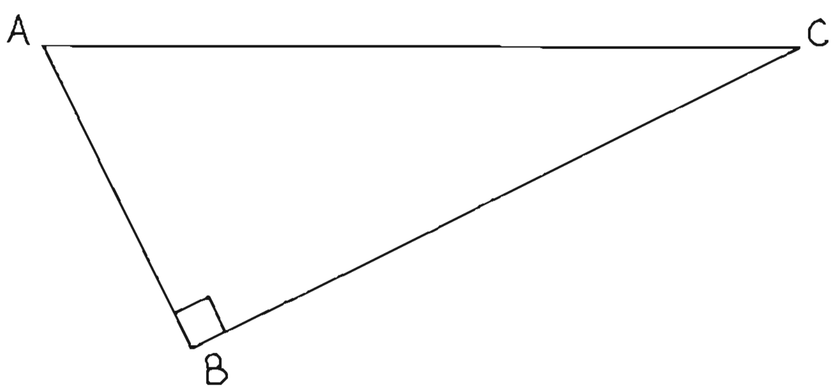


① $\frac{AE}{BE} = \frac{4cm}{2cm}$ $\frac{BE}{EC} =$ ____

② $\frac{AE}{BE} =$ ____ $\frac{BE}{EC} =$ ____



③ $\frac{AE}{BE} =$ ____ $\frac{BE}{EC} =$ ____



④ $\frac{AE}{BE} =$ ____ $\frac{BE}{EC} =$ ____

In each problem what do you notice about the ratios

$\frac{AE}{BE}$ and $\frac{BE}{EC}$

When two ratios are equal, they form a proportion.

$$\frac{AE}{BE} = \frac{BE}{EC}$$

The extremes are <u>AE</u> and ____.

The means are ____ and ____.

Since the means are equal, the proportion is called a <u>mean proportion</u>.

BE is the <u>mean proportional</u>.

RATIO, PROPORTION, AND SCALING

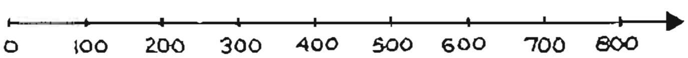
Placement Guide for Tabbed Divisors

SCALING

THE MEANING AND USES OF SCALING

The words scale and scaling are used in many different ways.

There are pay scales, musical scales, and scales for comparing weights and temperatures. In this resource the word scale will refer to a ratio. A scale of 1 cm : 2 km can be interpreted as the ratio 1 cm for every 2 km. The scale might be useful on a city map where 1 cm on the map represents 2 km in the actual city. A scale (ratio) of 1 cm : 100 people might be used as a basis for a number line graph.



Scaling means to make use of a scale. Some examples of scaling are: finding distances with a map using the given scale, scaling a recipe up or down according to a given ratio, and making a scale enlargement or reduction of a drawing.

Scale Drawings

Scale drawings are indispensable in the design and construction of objects. The huge tire shown in the picture at the right was designed from small scale drawings. Most manufactured objects were initially drawn to a scale. The dimensions of a scale drawing may be smaller than, equal to, or larger than the dimensions of the object.

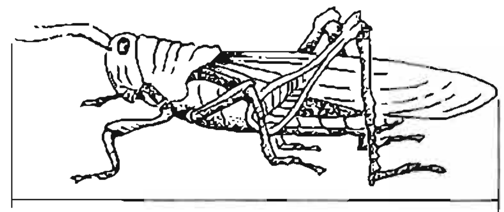
If a 1,996-pound tire falls on a 25-pound dog--

Being "dog tired" was evidently taken literally by this little beagle who was photographed recently in Zanesville, Ohio, taking a morning rest in the center of a huge earth mover tire. The account went on to point out that the tire weighs 1,996 pounds and the dog weighs 25 pounds, the importance of which isn't exactly clear unless the tire falls on the dog. Then the dog's dimensions are going to be about three feet long, three feet wide and an eighth of an inch thick.

Small objects, such as those found in clocks, transistors, radios and miniature calculators, are scaled up so they can be conveniently designed. Buildings, cars, furniture, clothing and other relatively large objects are scaled down to fit on blueprint and drawing paper.

When the dimensions of the scale drawing are equal to the dimensions of the object the scale drawing is said to be to actual size. This scale drawing of the grasshopper is drawn to size with a scale of 1 to 1 (1:1).

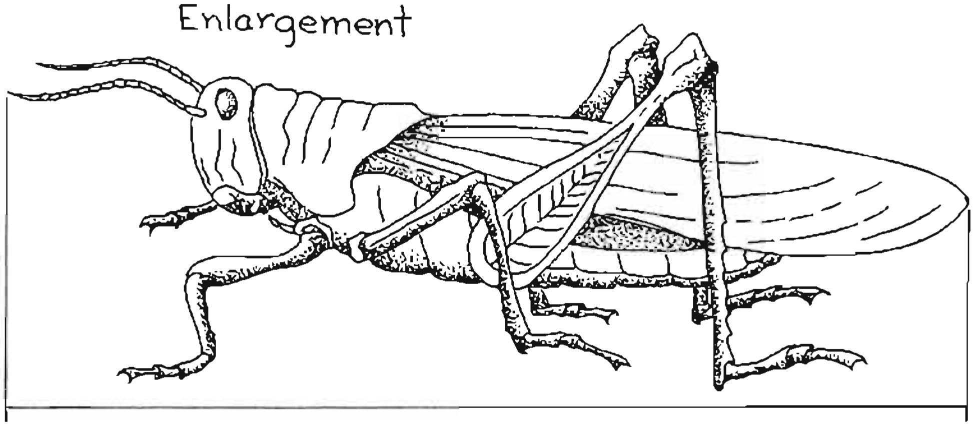
Drawn to Actual Size



Scale of 1:1

When the dimensions of the scale drawing are greater than the dimensions of the object, the scale drawing is called an enlargement. The scale drawing shown below is a 2 to 1 enlargement. Using ratio notation, this can be written as 2:1. Using fraction notation, we can write $\frac{2}{1}$ and say that the scale factor is two. (A few textbooks reverse the notation for scales and instead of writing 2:1 enlargement as we have here, they will write 1:2 enlargement.)

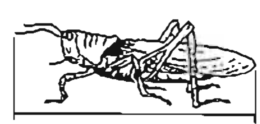
Enlargement



Scale of 2:1

When the dimensions of the scale drawing are smaller than the dimensions of the object, the scale drawing is called a reduction. This scale drawing of the grasshopper is a 1 to 2 (1:2) reduction. In this case, we can say that the scale factor is $\frac{1}{2}$.

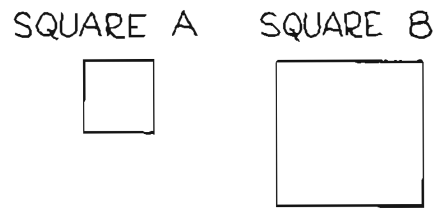
Reduction



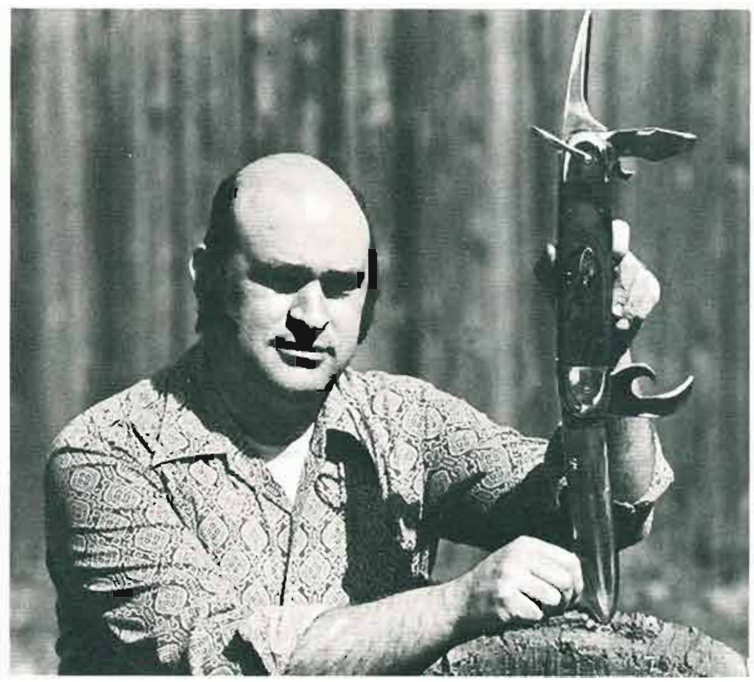
Scale of 1:2

"Twice as Large"

An exerpt from a magazine states, "If you want to make the original design twice as high and twice as wide, make the square twice as large." While it is common to speak of something as being twice as large when its linear dimensions are doubled, this practice can be a source of confusion. For example, the sides of square B are twice as long as the sides of square A, and yet its area is four times as great.



This newspaper clipping says that the big knife is three times larger than the conventional Scout knife. This means that the length and width are three times as great. As in the above examples, the comparison of sizes refers to the linear dimensions and not to the surface area, volume or weight. For example, the weight of the large knife is $4\frac{1}{4}$ pounds, and this is much more than 3 times the weight of a conventional Scout knife.



Prepared for anything

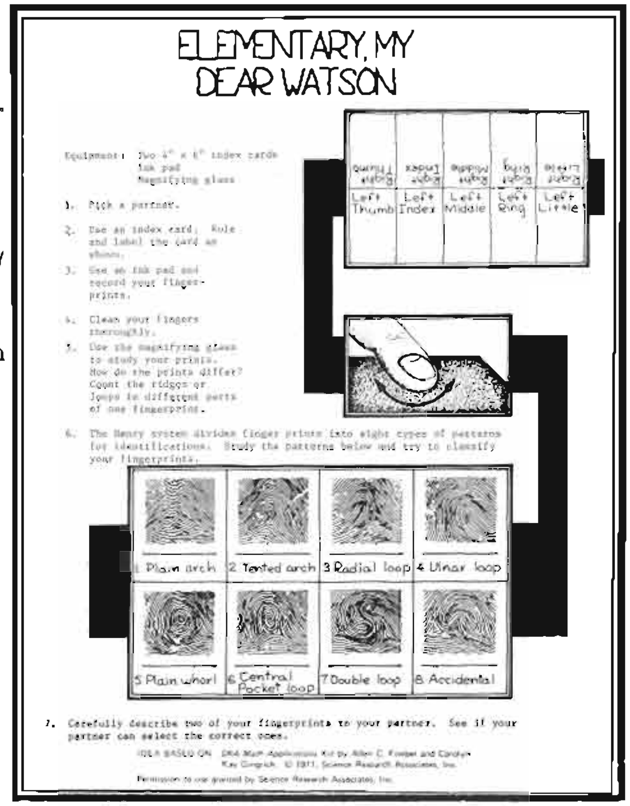
What could be the world's largest Scout-type knife is ready for the world's largest potato. Wayne Goddard, a professional knife-maker who works at his home at 473 Durham St., Eugene, turned this one out for Dennis and Raymond Ellingsen, Eugene knife collectors. Completely functional, the knife is $24\frac{1}{2}$ inches long when opened. It weighs $4\frac{1}{4}$ pounds and is three times larger than the conventional Scout knife.

While such expressions as, "twice as large," "three times as large," "half as big," etc., usually refer to lengths, there are exceptions. If a farmer speaks of one plot of land as being twice as large as another, he is referring to the area or acreage and not the length and width. If he wants a silo which is twice as large, then he is referring to a volume which is twice as large and not the height or width of the silo. The change in area and volume as related to a scale is discussed further in this commentary under "Supplementary Ideas in Scaling."

GETTING STARTED ON SCALING

Representation

Representation is very important in the study of scaling. Bar graphs use a given scale to represent information. Maps represent geographic areas based on the scale given in the legend of the map. Since scaling is often concerned with representing information and/or objects, you might like to begin a unit on scaling with some discussion of representations. Students can be asked to think of pictorial ways to represent or identify people. They might think of snapshots, shadow profiles, fingerprints or sketches. The student page *Elementary, My Dear Watson* would be appropriate here.



ELEMENTARY, MY DEAR WATSON

Equipment: Two 4" x 6" index cards
Ink pad
Magnifying glass

1. Pick a partner.
2. Use an index card. Rule and label the card as shown.
3. Use an ink pad and record your fingerprints.
4. Clean your fingers thoroughly.
5. Use the magnifying glass to study your prints. How do the prints differ? Count the ridges or loops in different parts of one fingerprint.
6. The Henry system divides finger prints into eight types of patterns for identifications. Study the patterns below and try to classify your fingerprints.

7. Carefully describe two of your fingerprints to your partner. See if your partner can select the correct ones.

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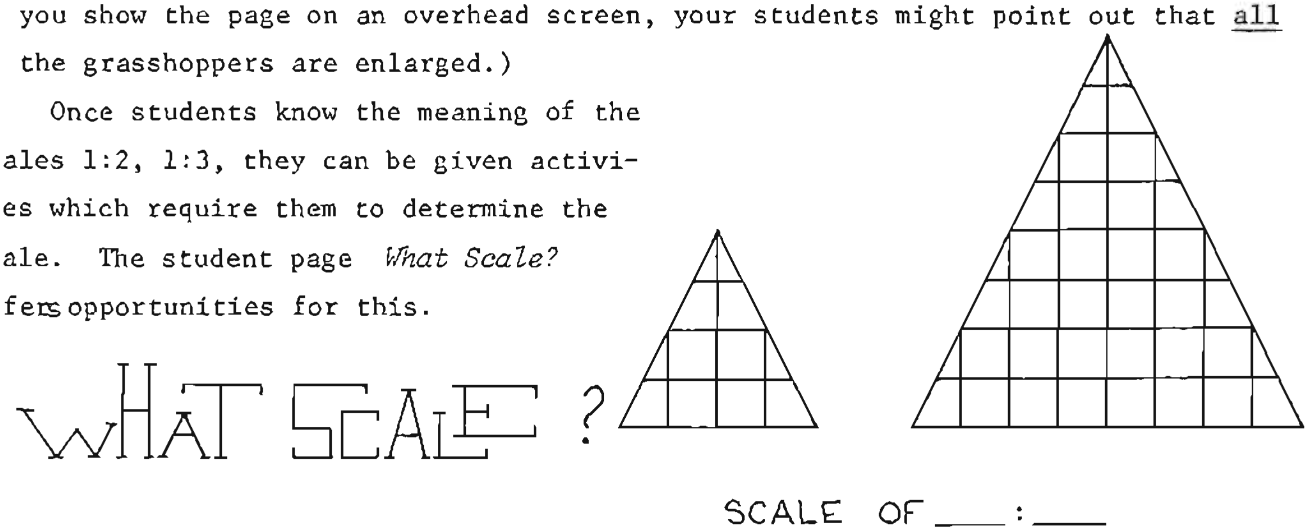
The page *What Am I?* can be used to begin a discussion on identifying objects from their outlines. When do we need to know more than size and outline to identify something?

Scales on Enlargements or Reductions

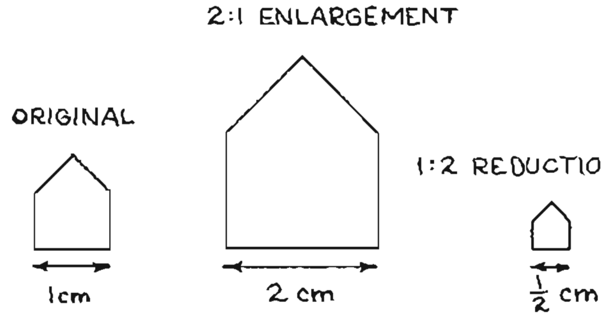
Representations of objects can be the same size, smaller than or larger than the objects. This idea can be introduced along with the use of ratio notation for scales with the page *Bug Off!* The page shows three scale drawings of a grasshopper, one of which is identified as actual size. (Note: if you show the page on an overhead screen, your students might point out that all of the grasshoppers are enlarged.)

Once students know the meaning of the scales 1:2, 1:3, they can be given activities which require them to determine the scale. The student page *What Scale?* offers opportunities for this.



MAKING A SCALE DRAWING

Students have already made many 1:1 scale drawings. Common examples are tracing the outline of a hand or figure, or making fingerprints which are mirror 1:1 scale representations of the patterns on fingertips. Usually the scale drawings we want to make are enlargements or reductions. A 2:1 enlargement means that each linear measurement on the scale drawing will be twice as long as the corresponding linear measurement on the original. A 1:2 reduction will have linear measurements one-half the corresponding measurements on the original.



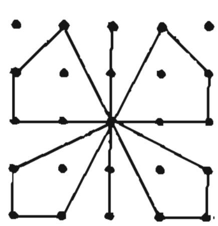
Snapshots, television shows, and billboards are examples of scale representations. Most of these are made with the aid of cameras, projectors and other technical devices, but there are several useful methods for making scale drawings by hand. These methods are discussed below.

Using Grids to Make Scale Drawings

Grids can be used to make scale drawings in several ways. Since each of these ways involves transferring a design from one grid to another, some practice in copying designs is helpful. The student page *Border Designs* asks students to continue geometric patterns with a 1:1 scale. These same patterns could later be enlarged or reduced in size.

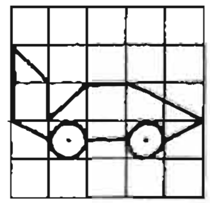

A BORDER DESIGN

The grid of nails on a geoboard can be the basis for a rubber band pattern. This pattern can be transferred onto a paper grid of dots where each dot on the page represents a nail on the geoboard. The dot grid can be any size. If it is smaller than the geoboard, the scale drawing will be a reduction. The butterfly shown at the left is a reduction of a geoboard design. Any two nails which are joined by a rubber band on the geoboard are represented by two dots joined by a line segment. In both of the activities *Border Designs* and *Geoboard Designs* students are involved in counting squares or dots, finding corresponding points and checking to see that their scale drawings really look proportional to the originals.

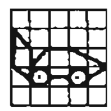

A GEOBOARD DESIGN

To make an enlargement or reduction to a specific scale, place the original design on a grid of squares. To enlarge the design to a scale of 2:1, make a grid with squares twice as long and wide. Copy the design one square at a time onto the new grid. To make a 1:2 reduction, make a grid with squares half as long and wide. Copy the design. The classroom pages *I Have Designs on You*, *Grid Graphs*, and *Paint Your Wagon* use these ideas with grids. Sometimes it is helpful to number the lines of the grid as shown in *Grid Graphs*.

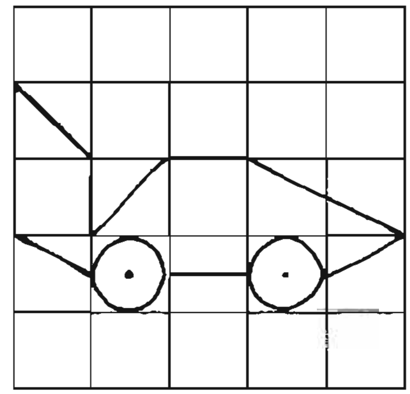
ORIGINAL



1:2 REDUCTION



2:1 ENLARGEMENT



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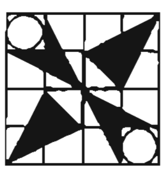
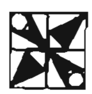
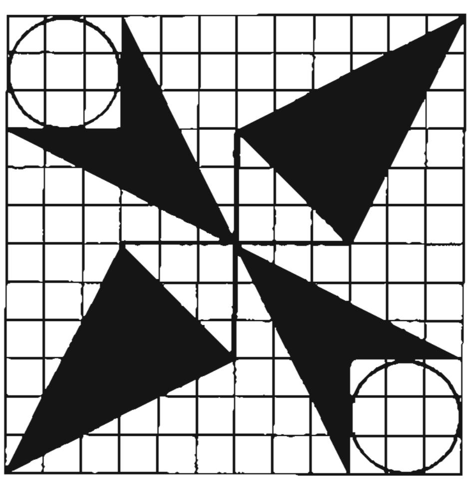


1:2 REDUCTION

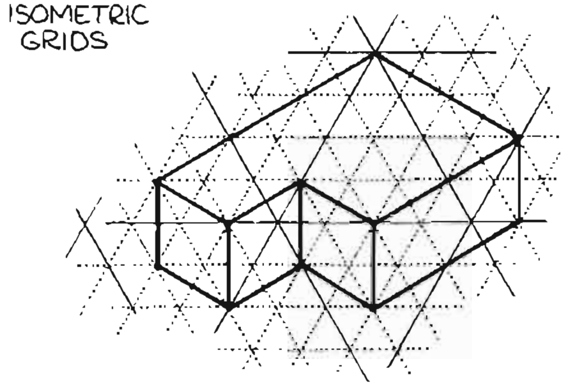


3:1 ENLARGEMENT



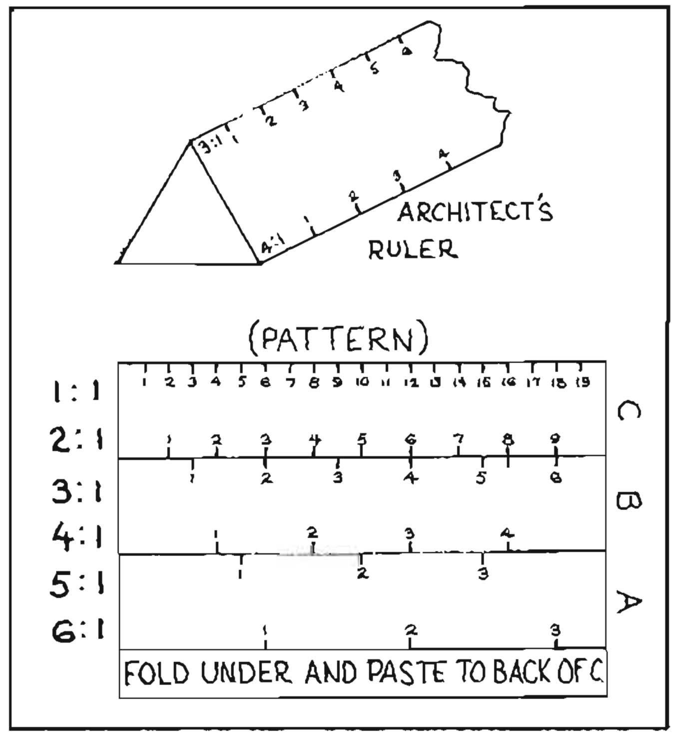
Another way to make a grid enlargement is to use the same size grid for the scale drawing as for the original. For a 3:1 enlargement the <u>edge</u> of one square on the original will correspond to the <u>edge</u> of 3 squares on the enlargement. Notice on the scale drawings at the left that the circle on the original occupies one square, but on the 3:1 enlargement it occupies 3^2 or 9 squares and on the 1:2 reduction it occupies $(\frac{1}{2})^2$ or $\frac{1}{4}$ squares. The student page *The Parthenon* has students make a 1:3 reduction using the same size grid paper. Your students can probably draw other designs to enlarge or shrink.

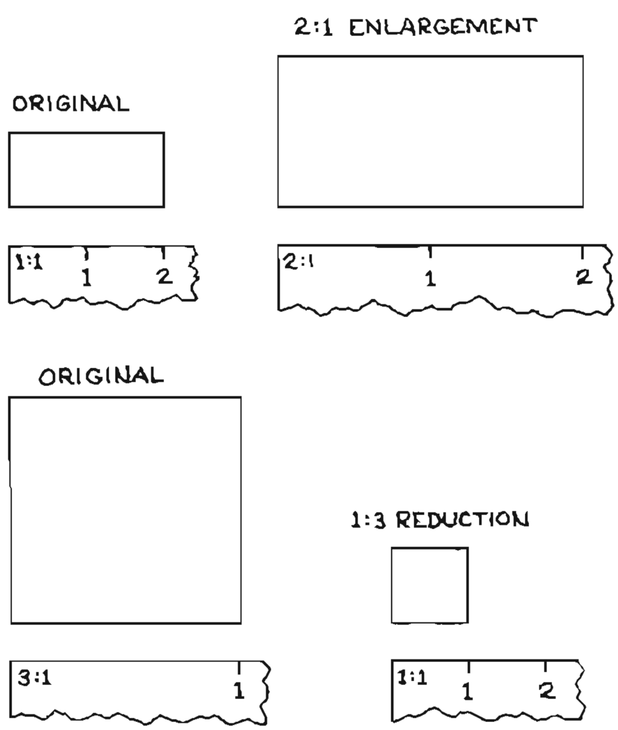
Students who have enjoyed making scale drawings in two dimensions might like to try scale drawings of three-dimensional objects. Isometric grids are useful for three-dimensional scale drawings. The edges of the figure at the right are not on the grid lines themselves, but they do connect vertices of the grid. To draw a 2:1 enlargement of this figure, count the horizontal and vertical spaces for each edge of the figure and double these lengths on an isometric grid.



<u>Using a Ruler to Make Scale Drawings</u>

Perhaps you have seen a triangular ruler like the one shown here. Rulers like this are used by architects and engineers for scaling. There is a pattern for making an architect's ruler on the student page *Archie Texs' Ruler*. The page can be run on tagboard to make a sturdy ruler. The students are asked to complete the six number lines according to the given scales as shown at the right.





<u>Enlargement</u> To make a 2:1 enlargement, measure each side of the original using the 1:1 scale, then reproduce the figure using the 2:1 scale. Notice that the <u>units</u> change when making the scale drawing, but the <u>number</u> of units read on the ruler stays the same.

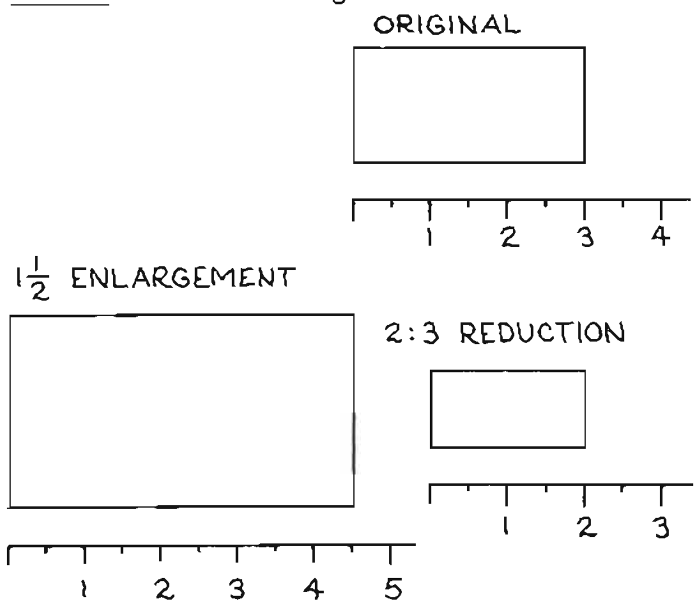
<u>Reduction</u> To make a 1:3 reduction, measure each side of the original using the 3:1 scale and then reproduce the figure using the 1:1 scale.

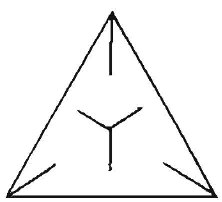
Combinations of these scales can be used. For example, for a 3:2 enlargement first reproduce the original figure by a 3:1 enlargement and then reproduce the resulting second figure by a 1:2 reduction.

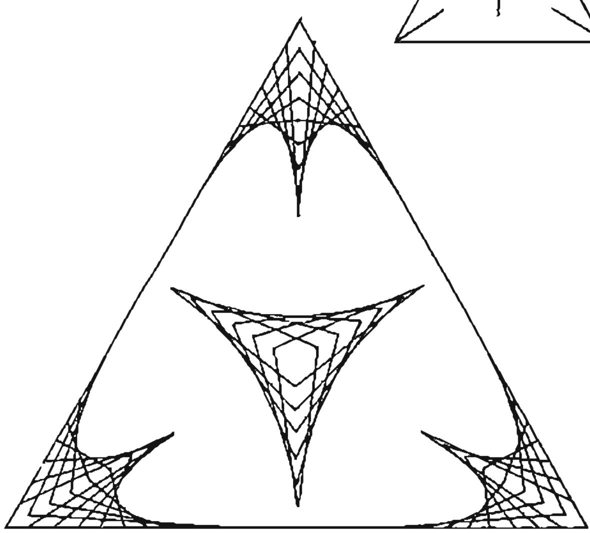
Architect's rulers are useful, but most of our measuring is done with common inch or metric rulers. Using a ruler with one number line to make a scale drawing involves a different process than using the architect's ruler which has several number lines.

To make a 2:1 enlargement with a centimetre ruler, the <u>number</u> of units for each linear measurement must be doubled. 2 cm ➔ 4 cm, 5 cm ➔ 10 cm, ... In other words, the <u>units</u> stay the same, but the <u>number</u> of units changes.

The rectangle at the right has width 3 cm. To make a $1\frac{1}{2}$:1 enlargement, each centimetre in the original must be stretched to $1\frac{1}{2}$ centimetres in the enlargement. This is equivalent to multiplying each linear measurement by $1\frac{1}{2}$. For the 2:3 reduction 3 centimetres on the original is shrunk to 2 centimetres in the reduction. This is equivalent to multiplying each length by $\frac{2}{3}$.







LINE DESIGN

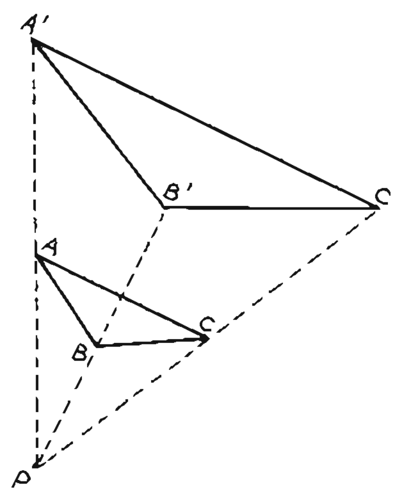
The ability to enlarge with a ruler is useful for making home decorations. The geometric design at the left is the basis for an elaborate line design. To make a wall-size line design, the geometric pattern must be enlarged to the desired size. Nails or holes are spaced at equal distances along all the edges. The design is then sewn or wrapped with thread. To enlarge this design with a ruler, the basic geometric shapes must be identified and scaling techniques applied. You can find patterns for line designs in <u>Line Designs</u> by Dale Seymour.

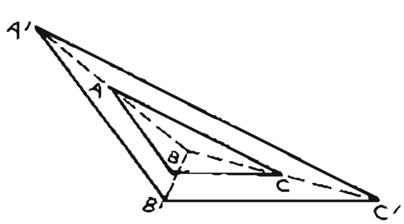
The classroom page in this resource which involves making scale drawings using a ruler is *Take Me Out to the Ball Game*.

Using Projection Points to Make Scale Drawings

The Renaissance painters were interested in depicting the natural world. The specific problem they coped with was that of painting three-dimensional scenes on canvas. The solution was the creation of a new system of mathematical perspective. The most influential of the artists who wrote on perspective was Albrecht Durer. Durer thought of the artist's canvas as a glass window through which the scene to be painted is viewed. From one fixed point lines of sight are imagined to go through the artist's canvas to each point of the scene. This set of lines is called a projection.

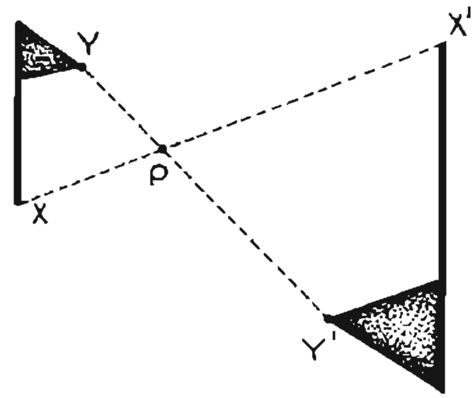
Durer's method is very handy for reproducing figures for a given scale factor. In the figure shown to the right, triangle A'B'C' is a 2:1 enlargement of triangle ABC. To obtain this enlargement, the points A, B and C are projected (pushed out) from projection point P so that the points A', B' and C' are twice as far from point P as the corresponding points A, B and C. By this method the sides of $\triangle$ A'B'C' are reproduced twice as long as the sides of $\triangle$ABC.

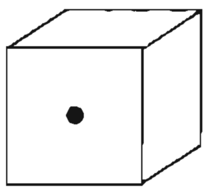




Surprisingly, it does not matter where the projection point is placed. If we place the projection point P inside $\triangle$ ABC as shown at the left and then project the points A, B and C out twice as far from P, we again obtain a reproduction which is a 2:1 enlargement.

The scale factor for a projection may be a fraction or a negative number. A scale factor of -2 has been used here to enlarge the smaller flag. For a negative scale factor the original figure and its reproduction will be on opposite sides of the projection point. For example, Y' is twice as far from the projection point P as Y, but in the opposite direction.

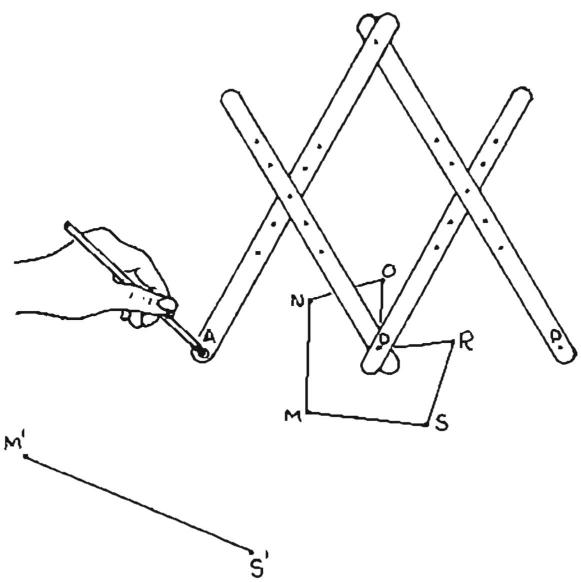



PINHOLE CAMERA

The lenses of our eyes and of cameras invert the images of scenes much like a projection with a negative scale factor. The scene is reproduced upside down on the retinas of our eyes and on the film of a camera. Your class might like to make a pinhole camera. You can find plans for such a camera in World Book Encyclopedia. The following student pages use perspective points to make scale drawings: *What's the Point, Bigger Than Life, A Shrink, A Negative Feeling* and *Projecting Through the Pinhole*.

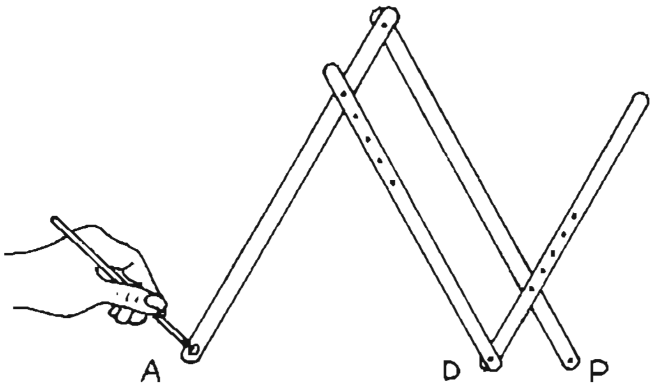
Using a Pantograph to Make Scale Drawings

A pantograph is a mechanical device for enlarging and reducing figures. It can be easily constructed from four strips of cardboard or from an erector set. These four strips are connected so the strips move freely. Point P acts like the projection point and should be held fixed. As point D is placed over each vertex of a polygon, a pencil at point A can be used to mark each vertex of the enlargement.

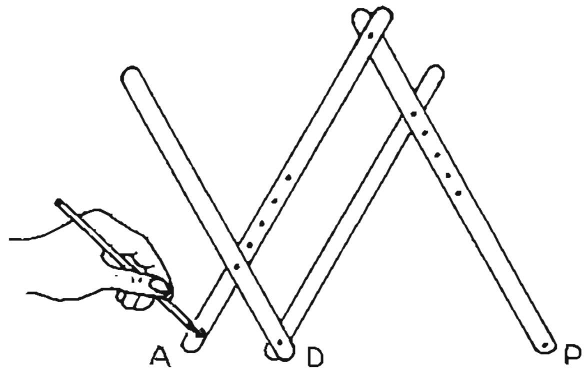


As the new points are found, the ratio PA:PD remains the same. In the picture above $\overline{PA}$ is twice as long as $\overline{PD}$, so the scale factor is 2. In order to use the pantograph for a reduction, the pencil should be placed at point D, and point A should be placed over various points of the original.

The pantograph on the student pages has just one set of holes for enlargements with a scale factor of two. There are several holes in the arms of the pantograph shown below to allow for different ratios of PA to PD. The following illustrations show two more settings of the pantograph.



As the pantograph is changed so that D moves closer to P, the ratio $\overline{PA}:\overline{PD}$ gets larger. In this illustration the scale factor is 4.

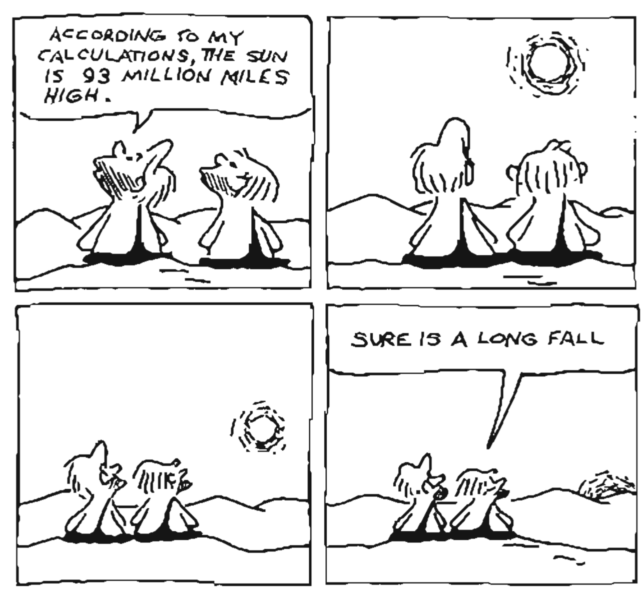


As D moves farther from P, the ratio $\overline{PA}:\overline{PD}$ gets smaller. In this illustration the scale factor is $\frac{4}{3}$.

Pantographs are not always made out of rigid material. Your students might enjoy using the rubber band pantograph described in *A Snappy Solution to Scale Drawings*.

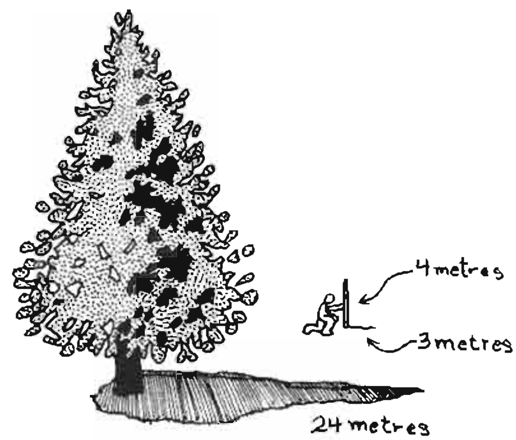
Using Indirect Measurement to Make Scale Drawings

Often we cannot measure distances directly--can you imagine a tape measure stretching to the sun? The heights of trees, buildings, mountains, etc. can be determined from scale drawings which are reductions of the actual scene. The Greeks created and applied methods of indirect measurement. They found the circumferences of the earth, moon and sun and computed the distances to the moon and the sun. Such things, which at first seem incredible, can be accomplished with only a knowledge of scales and proportions.



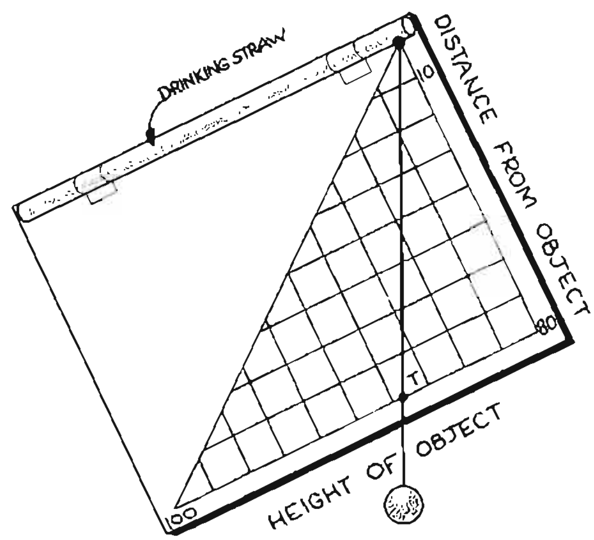
B.C. by permission of Johnny Hart and Field Enterprises, Inc.

Measuring with Shadows: The use of a stick and its shadow to measure the heights of objects is very old. As long ago as 600 B.C. the Greek mathematician Thales (thā′lēz) used this method to measure the heights of pyramids. The method is simple and uses proportions. Suppose a stick of height 4 metres is held perpendicular to the ground and has a shadow of length 3 metres. Then the ratio of height to length is 4:3. If the length of the tree's shadow is 24 metres, then the height of the tree can be found by solving the following proportion.



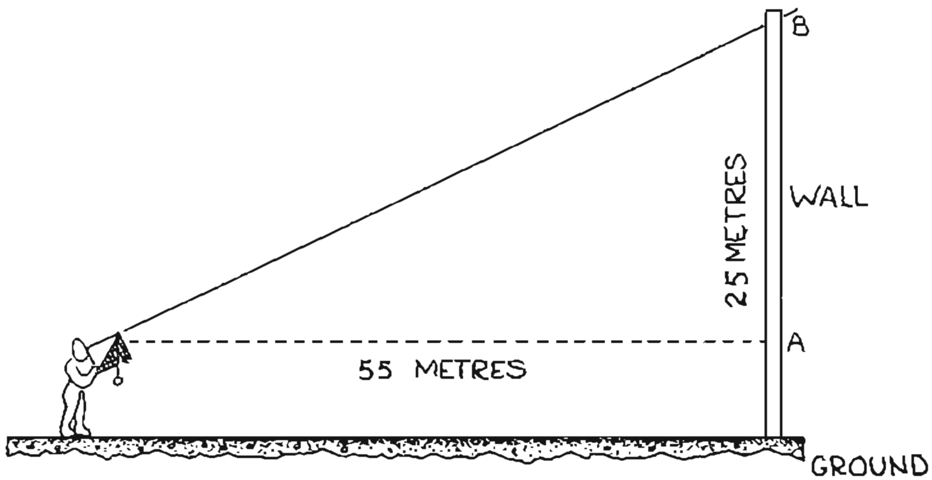
$$\frac{x}{24} = \frac{4}{3}$$

Measuring with a Hypsometer: The hypsometer is a simplified version of the quadrant, an important instrument in the Middle Ages, and the sextant, an instrument for locating the positions of ships. The grid on the hypsometer is used to set up a scale. For example, if you are 55 metres from the base of an object, this distance can be located on the right-hand side of the hypsometer by representing ten metres as one unit on the edge of the grid. Following the dotted line on this hypsometer to the string of the plumb line and then down to the lower edge of the grid shows that the height of the object above eye level is 25 metres. The units on the grid may represent feet, yards, centimetres, metres or any other convenient measure.



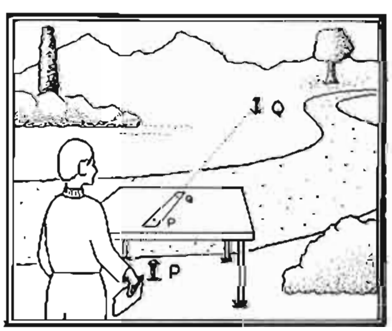
HOMEMADE HYPSOMETER

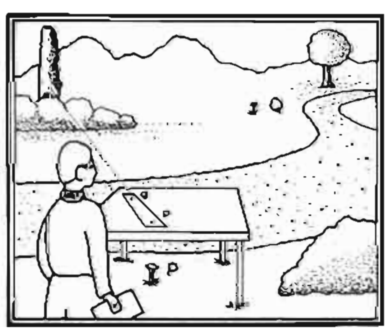
A common mistake in using the hypsometer is forgetting to add the distance from the ground up to eye level. For the position of the hypsometer which is shown above, the height of the wall from A to B in the following diagram is 25 metres. To get the total height of the wall, the distance of the eye above the ground must be added to 25 metres.

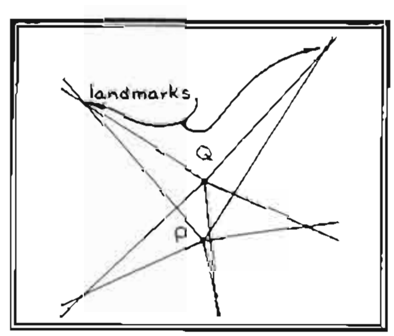


Using a Plane Table to Make a Scale Drawing

A plane table is one of the simplest ways of making a scale drawing of a small region, such as a room, backyard or field. With this device it is unnecessary to measure the angles or distances between objects. Only one distance needs to be known. The following series of pictures show the plane table being used to map the location of objects onto a piece of paper. In the second picture the line $\overleftrightarrow{PQ}$ has been represented on the paper by the line $\overleftrightarrow{P'Q'}$. The use of a plane table is illustrated on the student page *Stake Your Claim*.

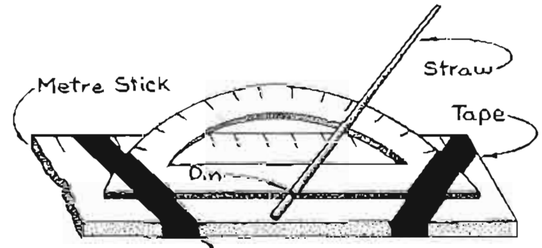






Using a Transit to Make Scale Drawings

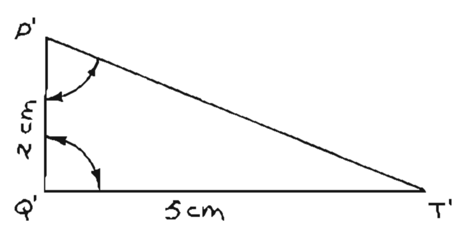
When making a scale drawing of a geographic area, it is often necessary to know the angles formed by imaginary lines joining trees, buildings and other landmarks. The transit is an important instrument for measuring horizontal and vertical angles in civil engineering. Like the early transit, the homemade transit shown here and developed on the student page *Another Stake Out* is capable of measuring only horizontal angles.



HOMEMADE TRANSIT

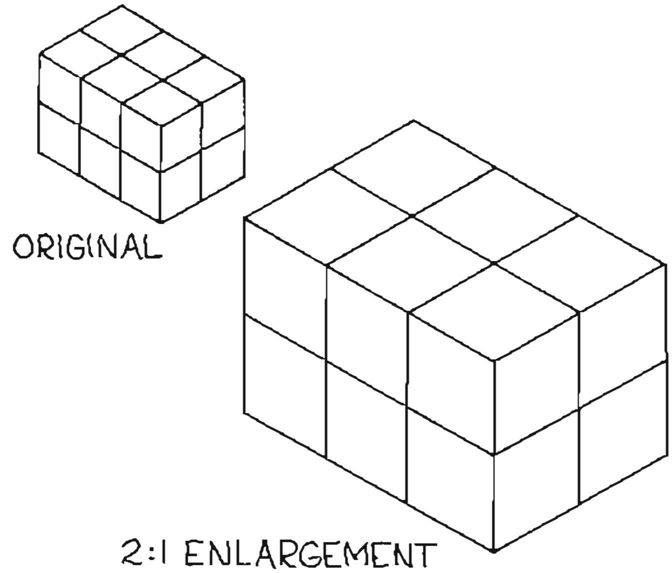
Suppose we wish to find the distance between points Q and T shown in the diagram below. If we had a scale drawing of the area, we could easily determine the distance. A stake can be placed at point Q and another stake placed at an arbitrary point P. The distance from P to Q is measured. The transit is used to measure the angles P and Q of the triangle PTQ. In each case the metre stick part of the transit should be held parallel to the line through stakes P and Q.

In the classroom a scale drawing of the triangle can be drawn on paper with angles P' and Q' equal to angles P and Q, respectively. The length of $\overline{P'Q'}$ can be chosen conveniently to set up a scale between the lengths of $\overline{PQ}$ and $\overline{P'Q'}$. For example, 1 centimetre might represent 10 metres. Since $\overline{Q'T'}$ is 5 cm long, $\overline{QT}$ has length 50 m.

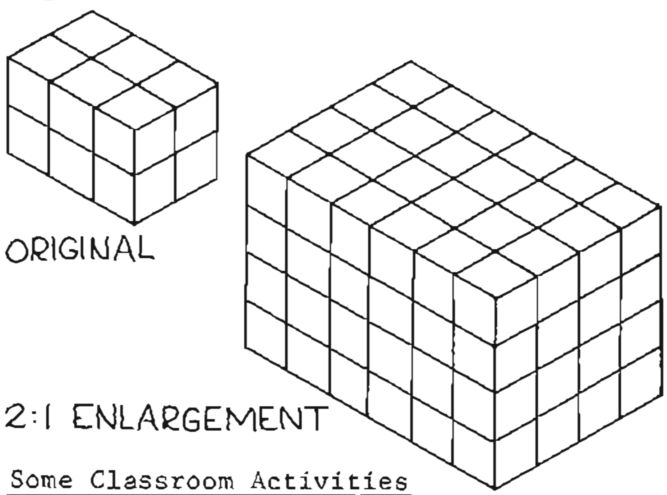


SUPPLEMENTARY IDEAS IN SCALING

Scale drawings and maps are common topics in scaling, but there are many other scaling activities which involve important mathematical content. Students might use a given scale to make a dip stick (see *Make a Dip Stick*) for measuring the volume of irregular containers. The effect of different scales on graphs can be tested These and other topics in scaling are developed in the classroom materials in this section.



Many of the student pages in this section are devoted to 3-dimensional scaling. One of the simplest ways of introducing scaling in three dimensions is through the use of cubes or building blocks. The larger of the two figures shown here is a 2:1 enlargement of the smaller figure. This means that the length, width and height of the larger figure are each twice as long as the length, width and height of the smaller figure.

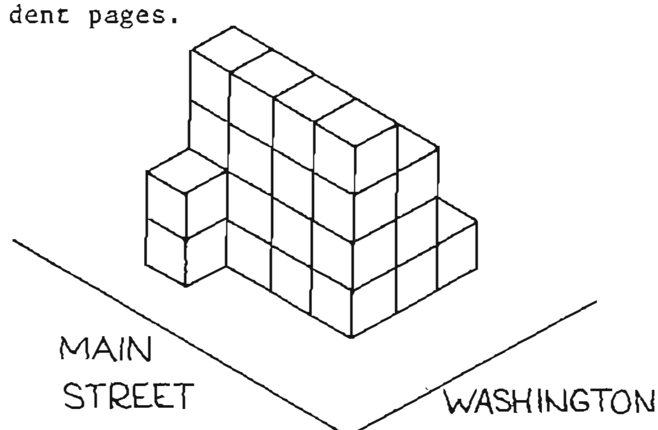


A 2:1 enlargement can also mean that <u>each</u> cube or building block is to be replaced by a 2:1 enlargement. In this case, the 2:1 enlargement of the smaller figure would have bigger cubes.

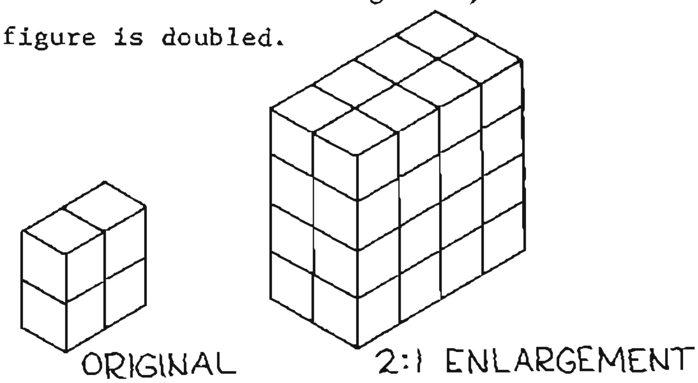
Since different size cubes are normally not available, enlargements with the same size cubes are used in the following illustrations and on the student pages.

Some Classroom Activities

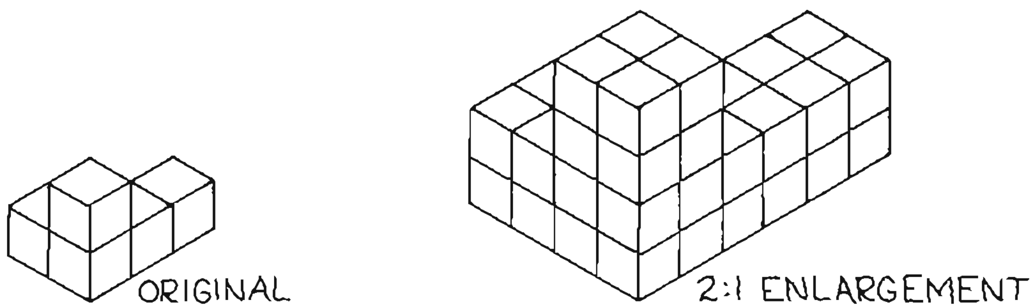
a. Build some skyscrapers out of cubes. Set up a scale and pose some questions. For example, if the edge of a cube represents 15 feet, how long is the building on Main Street? For similar questions see the student pages *Scaling a Skyscraper* and *Building a Skyscraper*.

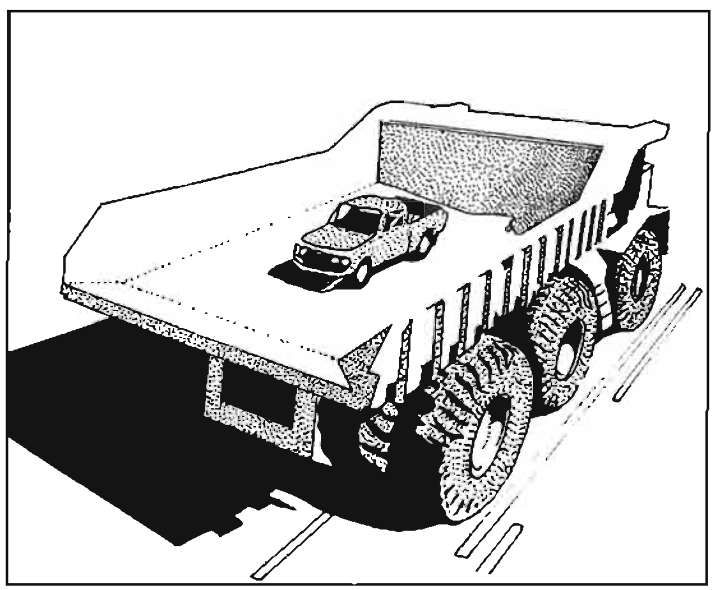


b. Build a box-shaped figure of cubes and then build an enlargement for some given scale factor. For a 2:1 enlargement, as shown here, each linear dimension of the original figure is doubled.



c. Build some irregular figures, such as the one shown here, and then build a 2:1 enlargement. This activity is much more difficult than enlarging box-shaped figures as in Activity #2 and will probably generate discussions.



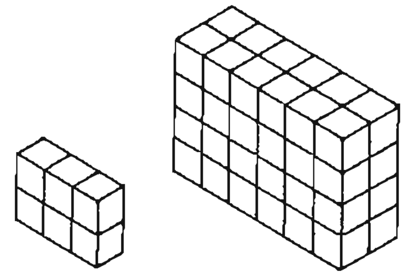


General Motors has unveiled the world's largest truck, this 67-foot-long Terex Titan. The off-highway hauler is 25 feet wide and can carry more than 350 tons, GM says. The truck is scheduled to undergo a minimum of 12 months of testing at a mining site in southern California. For size comparison, that's a Chevrolet Luv pickup on the Titan's bed.

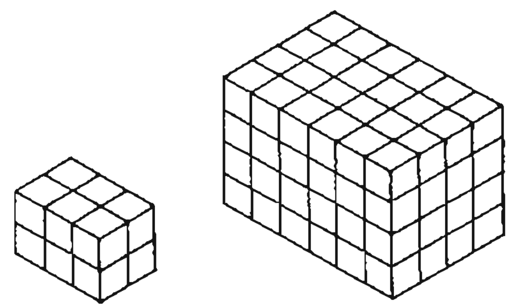
Scales, Area and Volume

Imagine meeting the Terex Titan on a highway. This truck is so wide that it would require three regular road lanes. It is 4.6 times larger and 4.8 times wider than the Chevrolet Luv pickup which is shown on its dumpbox. Needless-to-say, the Terex Titan does not cost just 4.8 times more than the Chevrolet pick-up nor is its capacity only 4.8 times greater.

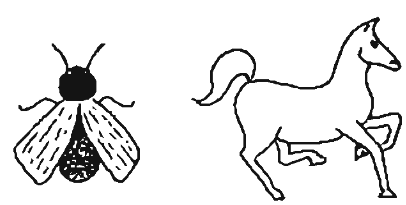
The relationships between scale enlargements and the increase in area and volume can be discovered by your students through the use of cubes or building blocks as suggested in the above activities. These relationships are discussed in the following paragraphs.



Area. If we refer to the figures shown here as buildings and the faces of the cubes as windows, the area of each side of the smaller building can be compared to the area of the corresponding side of the larger building in terms of windows. For example, there are four times as many windows on each side of the larger building. By varying the scale factors and comparing areas, your students will be able to see that the area of the enlargement is always the product of the square of the scale factor and the area of the original figure.



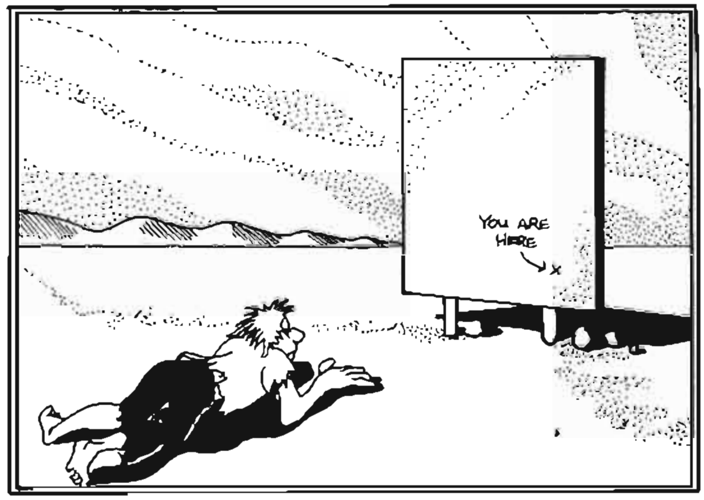
Volume. In the enlargement activities suggested on the previous page your students will quickly discover that if the original figure has too many cubes, they may not have enough cubes for the enlargement. For the 2:1 enlargement shown here there are 8 times as many cubes in the large building as in the smaller one. By varying the scale factors and comparing the numbers of cubes needed in a building and its enlargement, your students will become acquainted with the fact that the volume of the enlargement is always the product of the cube of the scale factor and the volume of the original figure.



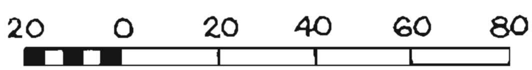
The relationships of linear dimensions with area and volume are responsible for governing the sizes of living things. For example, it would be impossible for a fly to be the size of a horse, or a rabbit to be the size of a hippopotamus. For an interesting discussion on this topic read "On Being the Right Size" by J.B.S. Haldane. This essay can be found in Readings in Mathematics, Volume 2, edited by Irving Adler, Ginn and Co. or in The World of Mathematics, Volume 2, edited by James R. Newman, Simon and Schuster.

MAPS

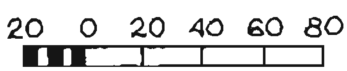
Maps have been indispensable since the beginning of recorded time. The first maps may have been directions drawn on the ground. Today there are many types of maps which represent the earth's surface and parts of this surface. There are maps of towns, states, regions and countries, all of which can be easily obtained for use in the classroom. Maps show the relative locations of objects, and it is the lack of objects which is causing the difficulty for the cartoon character at the right.



Scales on maps are often indicated by a line segment and the distance which the line segment represents. Here is an example which was taken from an American Automobile Association map of Western United States. The five small spaces to the left of 0 can be used for smaller subdivisions of the 20-unit intervals.



Scale in Miles



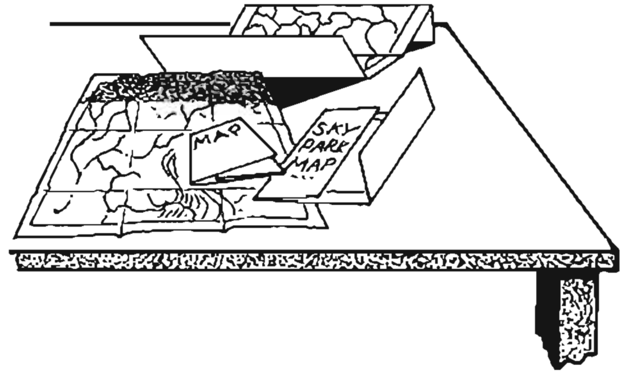
Scale in Kilometres

One inch represents approximately 40 miles or 64 kilometres.

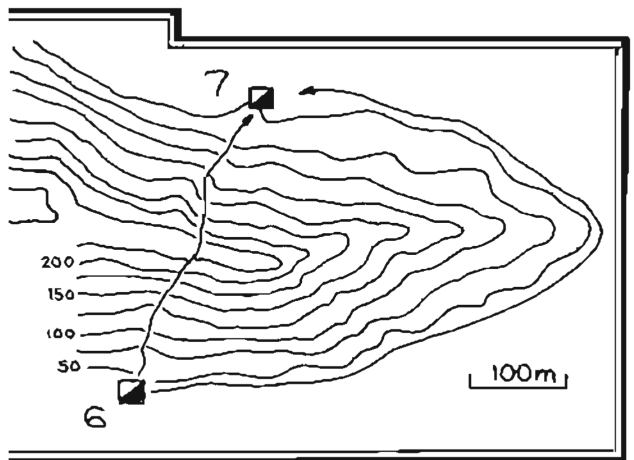
Since scales are ratios, it is common to see scales written in the following two ways.

1 inch to 40 miles or 1 inch : 40 miles

While the use of equality, such as 1 inch = 40 miles, is mathematically incorrect, it is frequently found on maps. Students will need to realize that 1 inch on the map represents 40 miles on the corresponding geographic region.

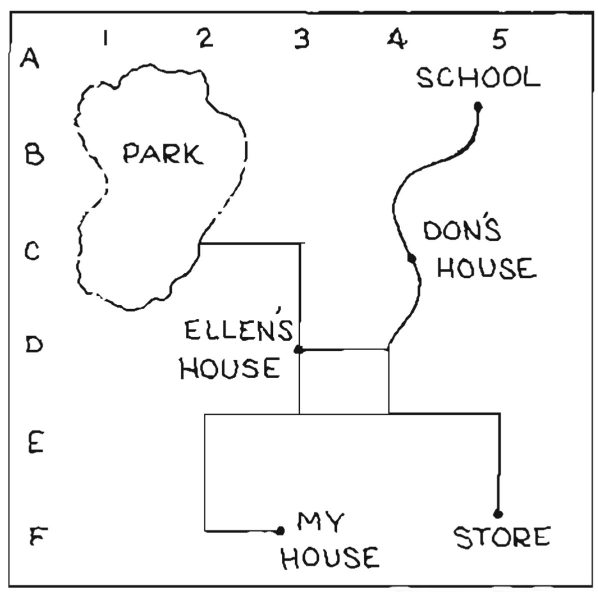


Local, state and regional maps are available for classroom use. The mileage to various points of interest can be computed along with the travel costs to such points. The bus, train or automobile costs can be approximated, including gas, oil and tolls. The scales on different maps can be compared. What happens to scales for larger and larger regions of the earth? Most of the student pages on maps contain examples and questions for developing and reading maps.



You could obtain some contour maps of your region and have your students find the highest and lowest points of elevation. All the points which have a given elevation are shown with a contour line. For example, the points with 150-foot elevation on the contour map shown here are on the heavy line.

Your students might enjoy making a map of an area of their own choice. They can measure the distance to each landmark by counting blocks, paces or turns of a trundle wheel. A homemade transit can be used to measure angles between objects and a convenient scale chosen to fit the map onto paper. Some students might like to make a treasure map and have other classmates use the map to find the treasure.



RATIO, PROPORTION, AND SCALING

Placement Guide for Tabbed Divisors

CONTENTS

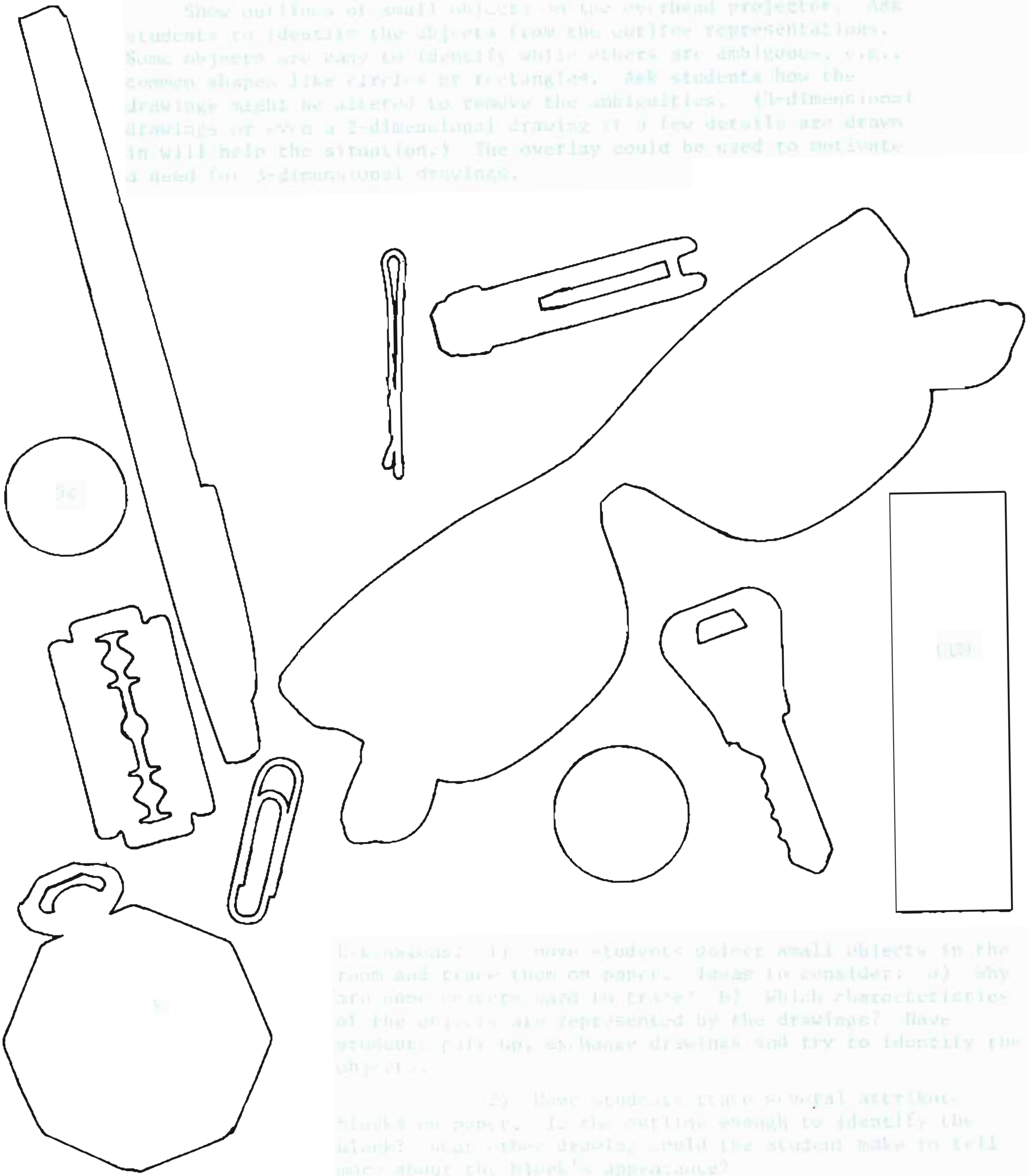
SCALING: GETTING STARTED

WHAT AM I?

Identifying by outlines
Getting Started
SCALING

TEACHER DIRECTED ACTIVITY

Show outlines of small objects on the overhead projector. Ask students to identify the objects from the outline representations. Some objects are easy to identify while others are ambiguous, e.g., common shapes like circles or rectangles. Ask students how the drawings might be altered to remove the ambiguities. (3-dimensional drawings or even a 2-dimensional drawing if a few details are drawn in will help the situation.) The overlay could be used to motivate a need for 3-dimensional drawings.



Extensions: 1) Have students select small objects in the room and trace them on paper. Ideas to consider: a) Why are some objects hard to trace? b) Which characteristics of the objects are represented by the drawings? Have students pair up, exchange drawings and try to identify the objects.

2) Have students trace several attribute blocks on paper. Is the outline enough to identify the block? What other drawing could the student make to tell more about the block's appearance?

IDEA FROM: *Minnemast,* Unit 18, Scaling and Representation

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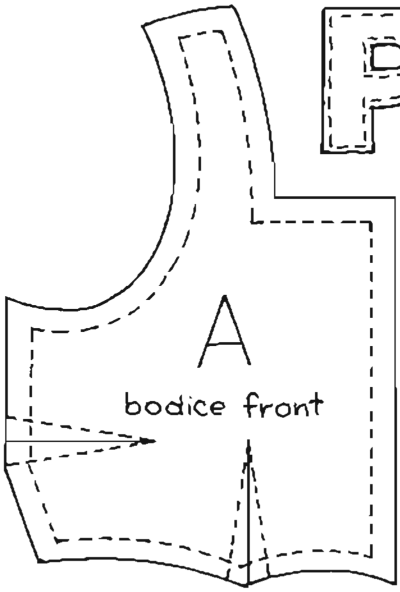
A PERFECT FIT



Project:

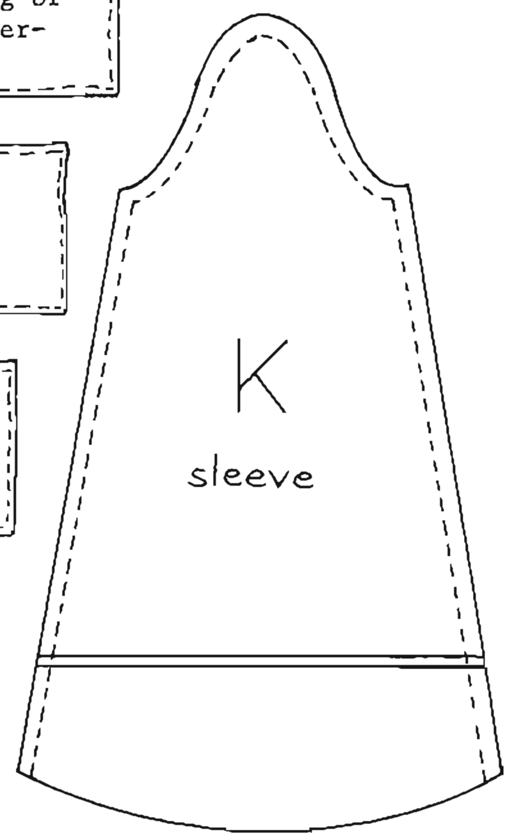
Have a student bring in a clothing pattern to be displayed on the bulletin board. The actual pattern, the cutting layouts, and the pattern package should all be displayed. A background of actual fabric would be attractive. The bulletin board display could serve as an emphasis and motivation for several concepts.

1. The pattern is a 1:1 scale drawing of the pieces needed to construct the garment and also is a scale drawing of the size of the garment needed to fit a particular person.

2. The pattern package shows the amount of material needed for the garment according to the size and to the width of the material.

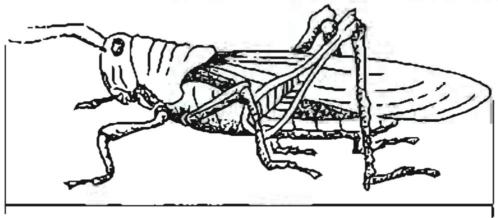
3. The cutting layout is a representation (scale drawing) showing how the pattern should be laid out on the material.

4. A lab activity could be developed where students actually lay out a pattern on material. This could show a student how the left and right sides of garments are cut (also, how to eliminate a seam by laying out the material along the fold). If several widths of material are available, the student can see how the arrangement of the pattern is changed to waste the least amount of material



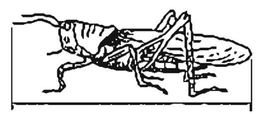
BUG OFF!

1. The same size as the object.



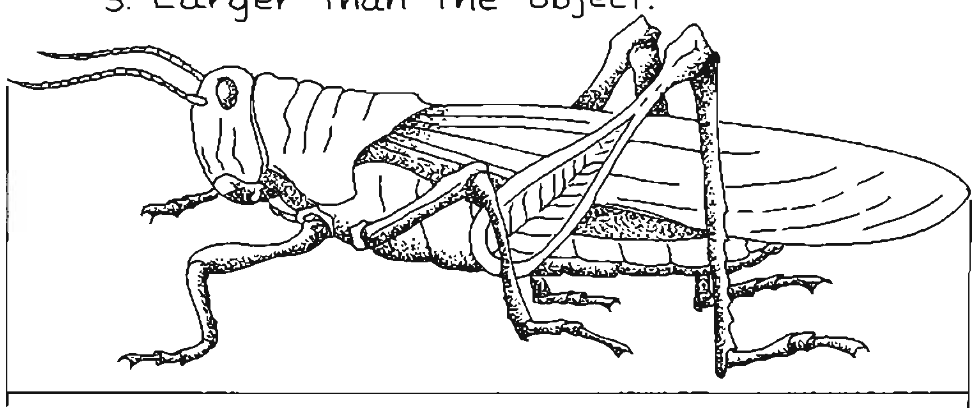
Scale of 1:1

2. Smaller than the object.



Scale of 1:2

3. Larger than the object.



Scale of 2:1

IDEA FROM: *Greater Cleveland Mathematics Program*, 6

Permission to use granted by Educational Research Council of America

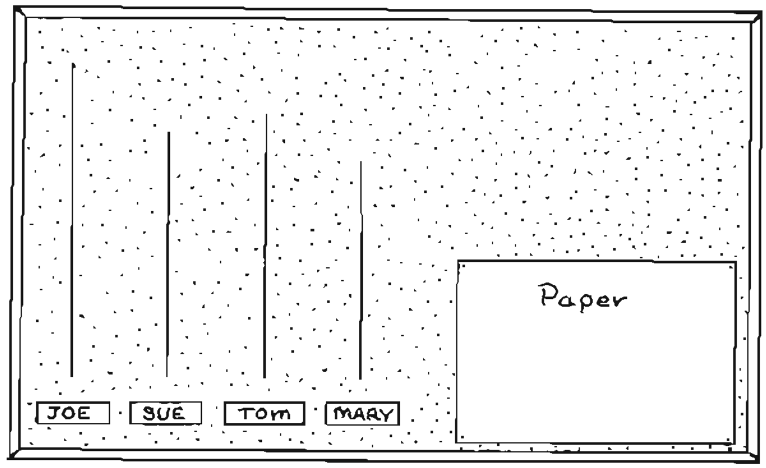
Getting Started
SCALING

E
S
I

YOUR MOD BOD

Materials needed: Long piece of string, scissors, name label, stapler or thumbtack.

Activity:



(1) Have a classmate measure your height with a piece of string.

(2) Cut the string to represent your height.

(3) Fold the string into two equal pieces and cut.

(4) Attach <u>one</u> piece of the string to the bulletin board. Label it with your name.

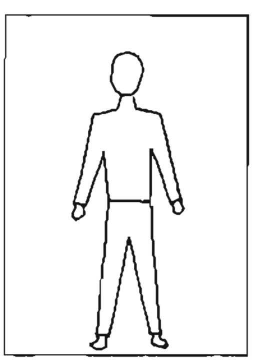
(5) Save the other piece of string.

When all of your classmates have placed their strings on the bulletin board, you will have a scale representation of everyone's height.

If at this introductory level you wish to talk about the scale, it is a scale of 1:2. That is, 1 unit of string (scale) represents 2 units of string (height of the student).

(6) Use the other piece of string; fold it into equal pieces to make a scale representation of your height that will fit on the piece of paper on the bulletin board. How many times did you fold the string?

(7) How are the two scale representations different? How are they the same?



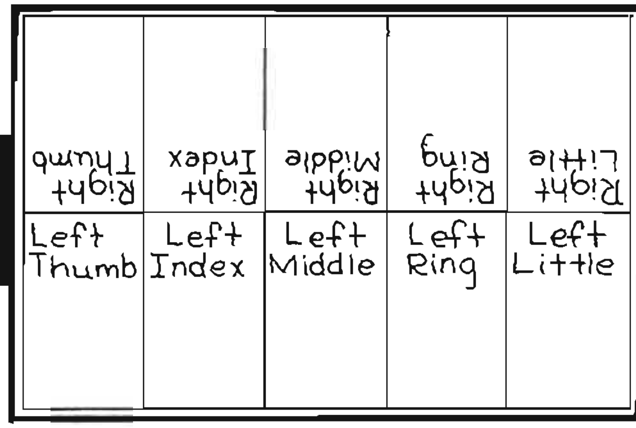
(8) Use another piece of string for measuring and draw a scale representation of yourself that will fit lengthwise on a piece of notebook paper.

The instructions for this activity are purposely open-ended to encourage problem solving.

Extension: Using this same idea a scale representation of the heights of several items in the classroom could be made. The students could then try to match the item to the representation.

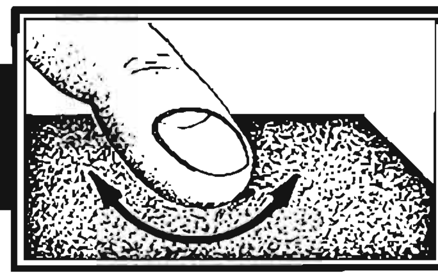
TYPE: Activity

ELEMENTARY, MY DEAR WATSON

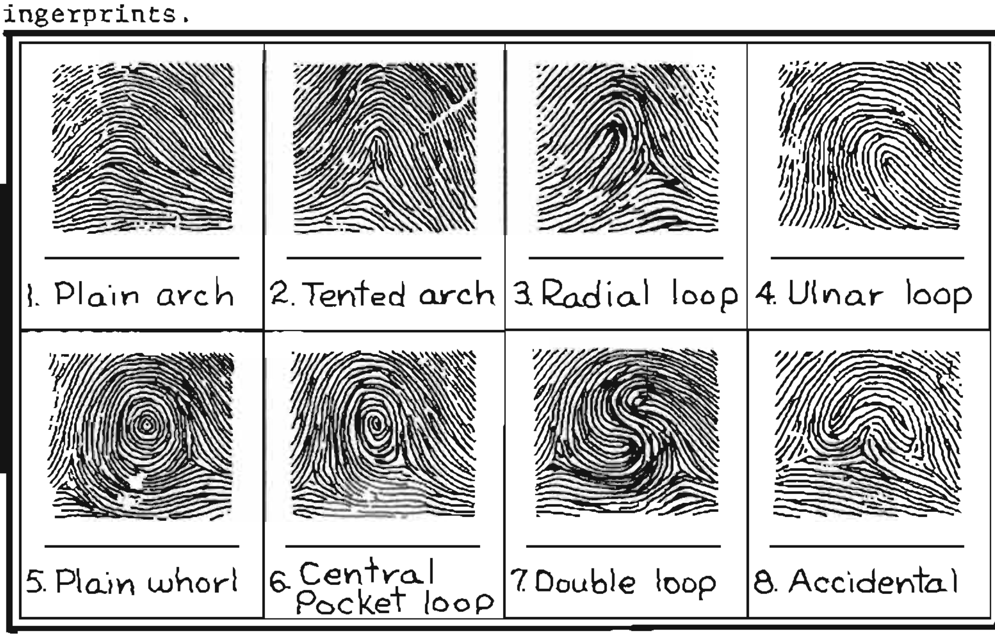


Equipment: Two 4" x 6" index cards
Ink pad
Magnifying glass

1. Pick a partner.
2. Use an index card. Rule and label the card as shown.
3. Use an ink pad and record your fingerprints.



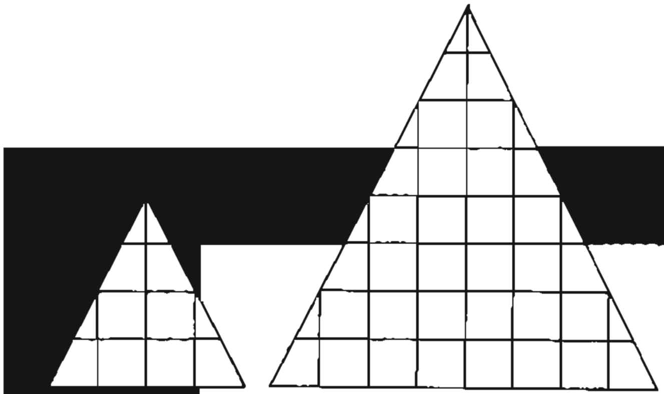
4. Clean your fingers thoroughly.
5. Use the magnifying glass to study your prints. How do the prints differ? Count the ridges or loops in different parts of one fingerprint.
6. The Henry system divides finger prints into eight types of patterns for identifications. Study the patterns below and try to classify your fingerprints.

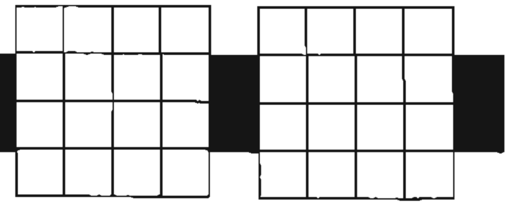


7. Carefully describe two of your fingerprints to your partner. See if your partner can select the correct ones.

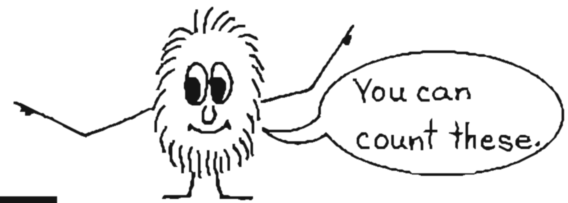
IDEA BASED ON: *SRA Math Applications Kit* by Allen C. Friebel and Carolyn Kay Gingrich. © 1971, Science Research Associates, Inc.

WHAT SCALE?

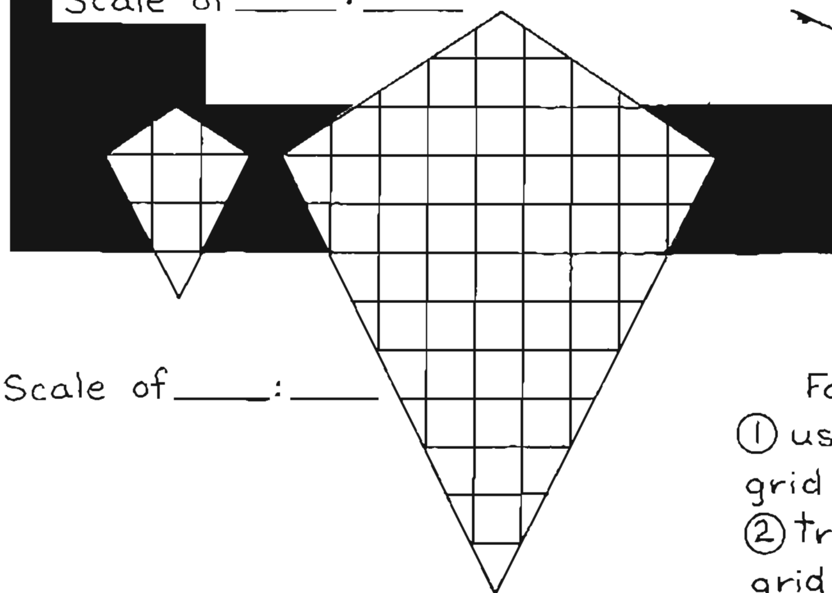




Scale of ____ : ____



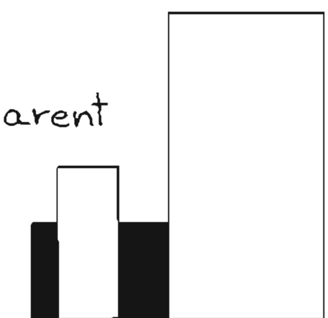
Scale of ____ : ____

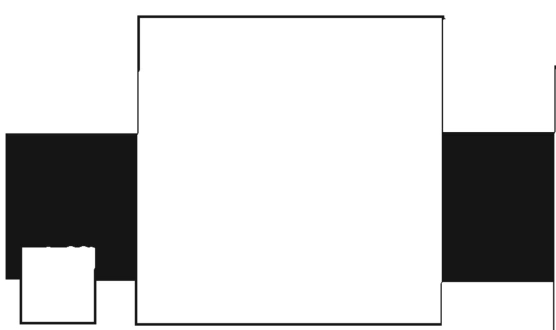


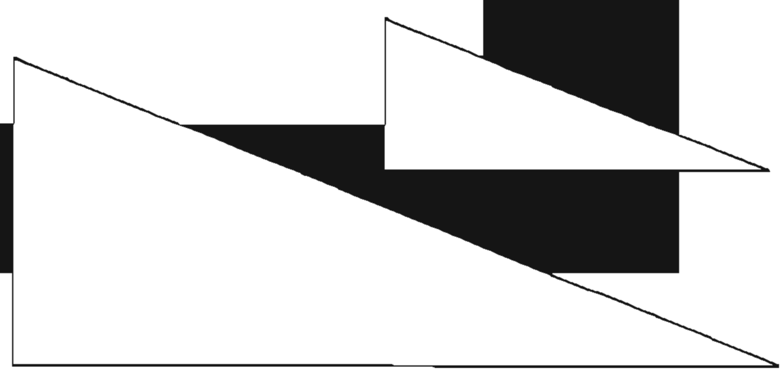
Scale of ____ : ____

Scale of ____ : ____

For these:
① use a transparent grid, or
② trace on grid paper, or
③ measure with a ruler.







Scale of ____ : ____

Scale of ____ : ____

SCALY

Materials: Game board
Marker for each player
Die

Rules:

1) Players each roll the die. Largest number goes first.

2) Scales:

   a) 2 spaces for each dot on the die

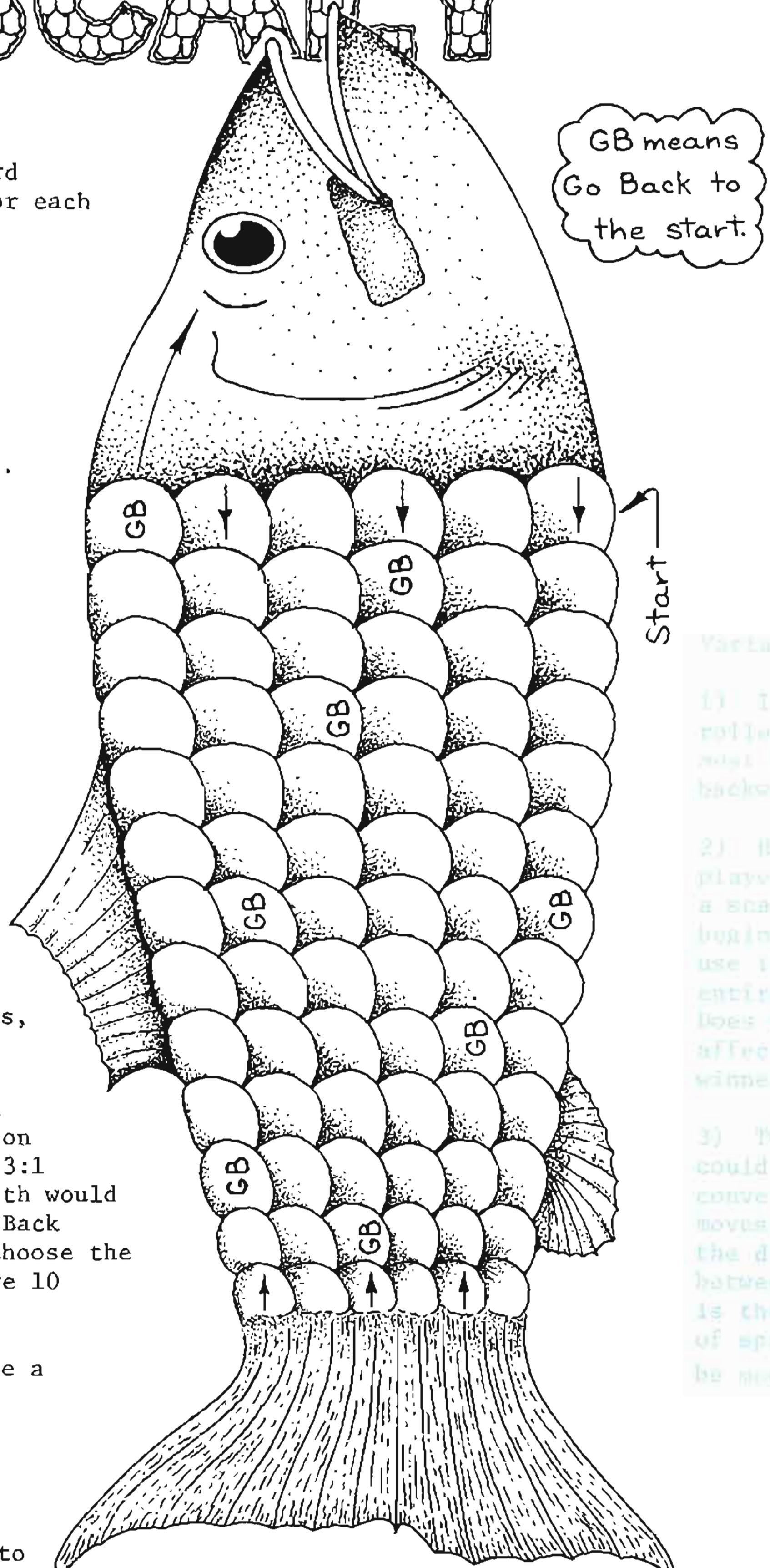
   b) 3 spaces for each dot on the die

   c) 4 spaces for each dot on the die

3) Players roll the die, choose a scale to avoid the Go Back spaces, and move their markers forward.
For example, if a player rolls a 5 on the die, and the 3:1 and 4:1 scales both would move him to a Go Back space, he would choose the 2:1 scale and move 10 spaces forward.

4) If all scales move a player beyond Scaly's eye, the player loses his turn.

5) The first player to exactly reach Scaly's eye is the winner.



Variations:

1) If a 1 is rolled the move must be made backwards.

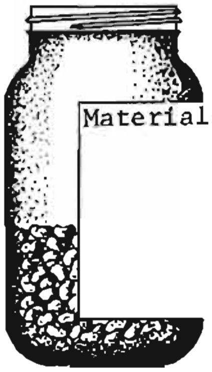
2) Have each player choose a scale at the beginning and use it for the entire game. Does this affect the winner?

3) Two dice could be rolled, converted to moves, and then the difference between the two is the number of spaces to be moved.

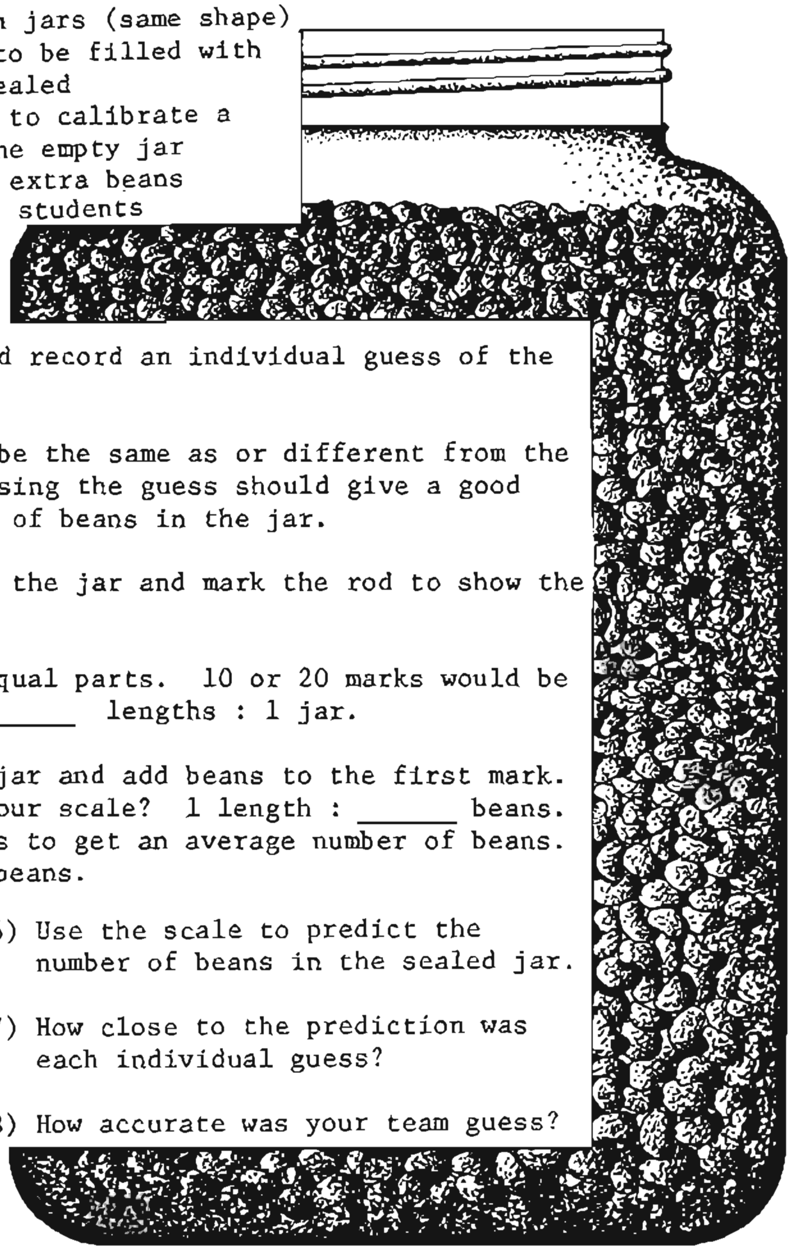
TYPE: Game

BEANS, BEANS

This is an activity that uses scaling to obtain information and make predictions.



Materials: Two 1-gallon jars (same shape)
One jar is to be filled with beans and sealed
A dowel rod to calibrate a scale for the empty jar
A supply of extra beans
A team of 3 students



Activity:

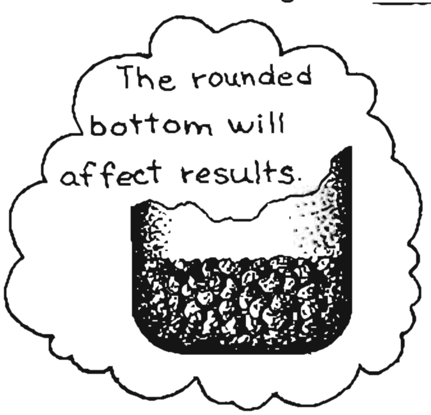
(1) Each student should make and record an individual guess of the number of beans in the jar.

(2) Make a team guess. It may be the same as or different from the individual guesses. Discussing the guess should give a good approximation of the number of beans in the jar.

(3) Place the dowel rod next to the jar and mark the rod to show the top of the jar.

(4) Mark the rod into several equal parts. 10 or 20 marks would be convenient. Your scale is _____ lengths : 1 jar.

(5) Place the rod in the empty jar and add beans to the first mark. Count the beans. What is your scale? 1 length : ______ beans. Repeat this three more times to get an average number of beans. Scale of 1 length : ______ beans.



(6) Use the scale to predict the number of beans in the sealed jar.

(7) How close to the prediction was each individual guess?

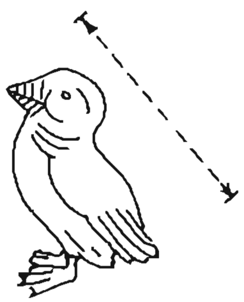
(8) How accurate was your team guess?

? Before beginning this activity, have students develop their own method for determining the number of beans in the jar.

A PICTURE'S WORTH 1000 WORDS

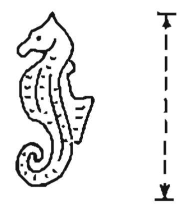
The dictionary uses pictures to illustrate its definitions. Sometimes a scale is given near the picture to indicate the size of the picture compared to the real thing.

Measure each picture in mm or cm and use the scale to figure the size of the real thing. Choose your answer from the bottom of the page by taking the measure closest to your answer.



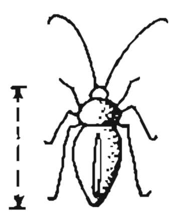
Puffin Scale of 1:10

(a) A puffin is __________ cm.



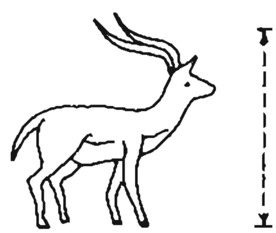
Sea Horse Scale of 1:4

(b) A sea horse is __________ mm.



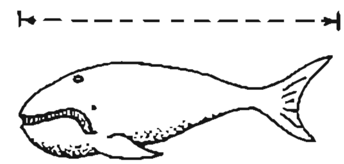
Cockroach Scale of 1:2

(c) A cockroach is __________ cm.



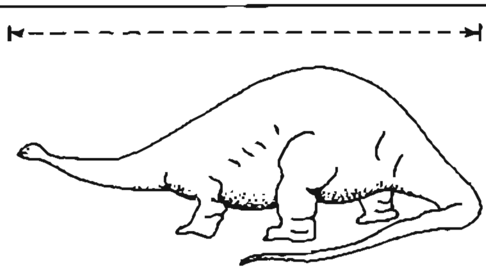
Antelope Scale of 1:50

(d) An antelope is __________ m.



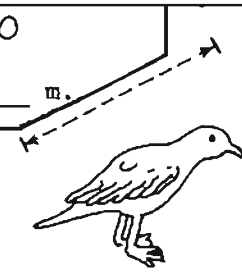
Whale Scale of 1:300

(e) A whale is __________ m.



Dinosaur Scale of 1:250

(f) A dinosaur is __________ m.



Albatross Scale of 1:35

(g) An albatross is __________ cm.

98, 30, 100, 12, 3.0, 15, 1.25

IDEA FROM: *The Metric System of Measurement*

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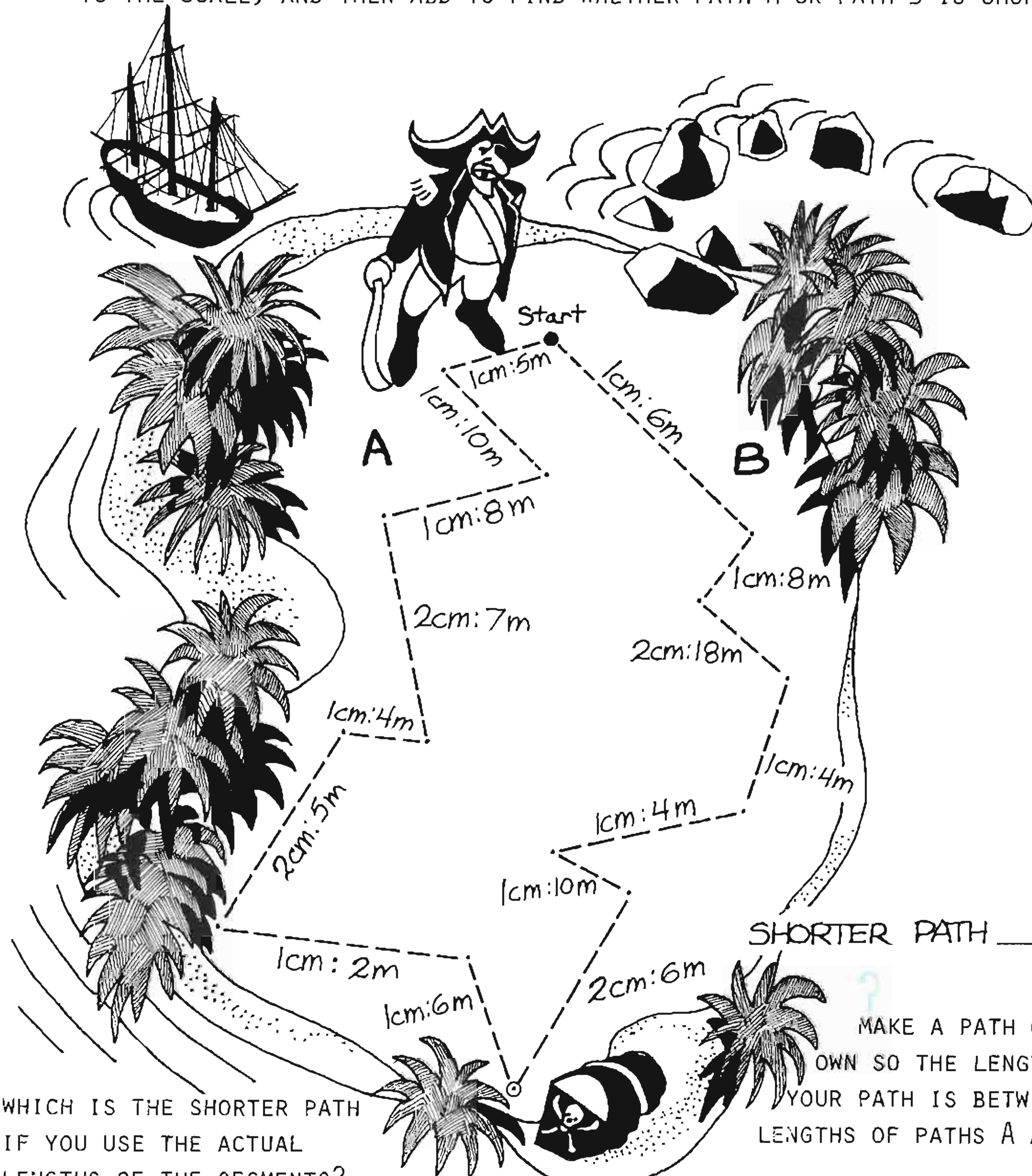
THE PIRATE'S DREAM

HELP BLACKBEARD FIND THE SHORTER DISTANCE TO THE TREASURE. USE YOUR METRIC RULER TO MEASURE EACH LENGTH, CONVERT THE LENGTH ACCORDING TO THE SCALE, AND THEN ADD TO FIND WHETHER PATH A OR PATH B IS SHORTER.



SHORTER PATH _____

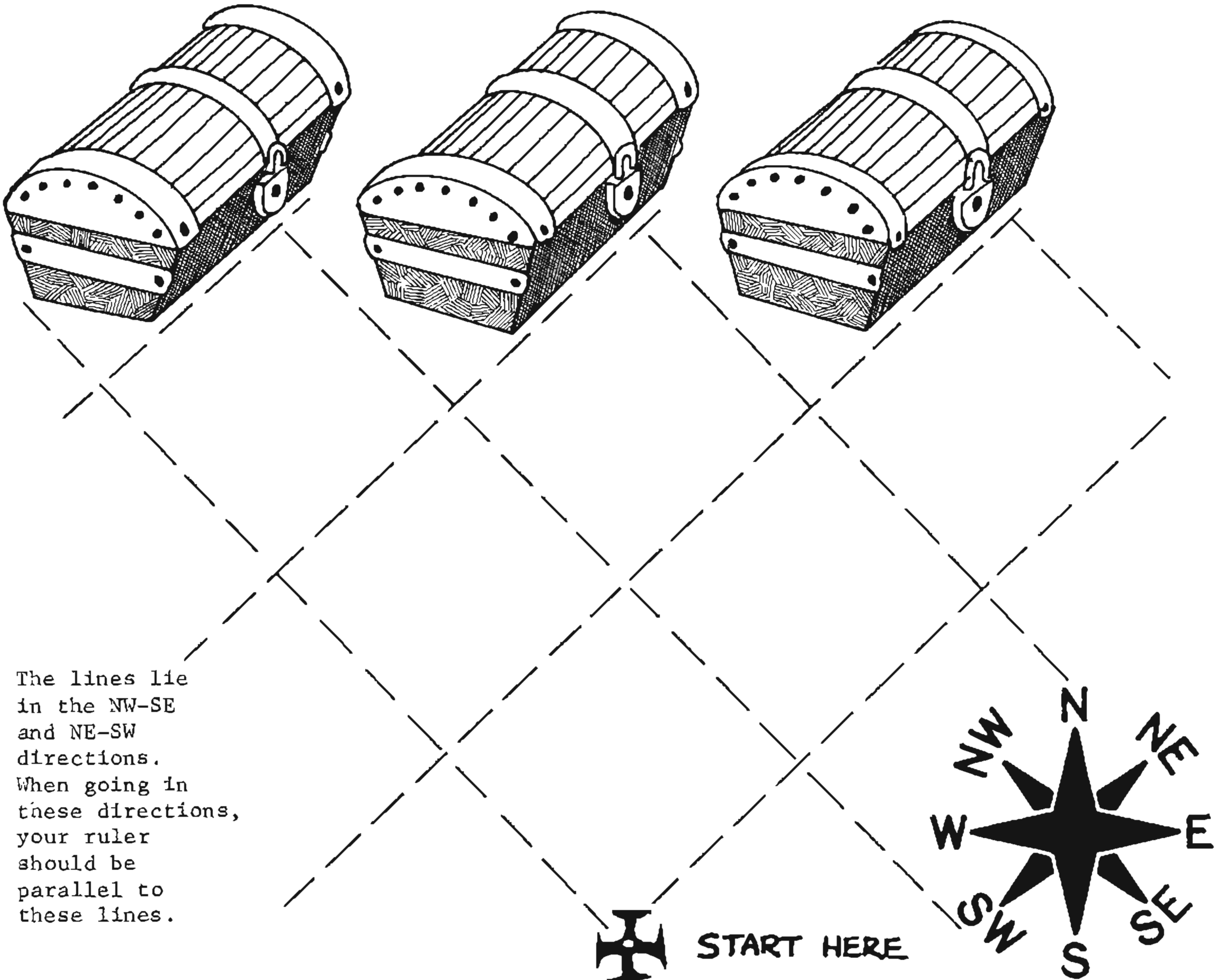
WHICH IS THE SHORTER PATH IF YOU USE THE ACTUAL LENGTHS OF THE SEGMENTS? _____

MAKE A PATH OF YOUR OWN SO THE LENGTH OF YOUR PATH IS BETWEEN THE LENGTHS OF PATHS A AND B

BEWARE the COBRAS!

Two of these chests contain deadly cobras and the third contains a treasure to make a person rich beyond their wildest dreams.

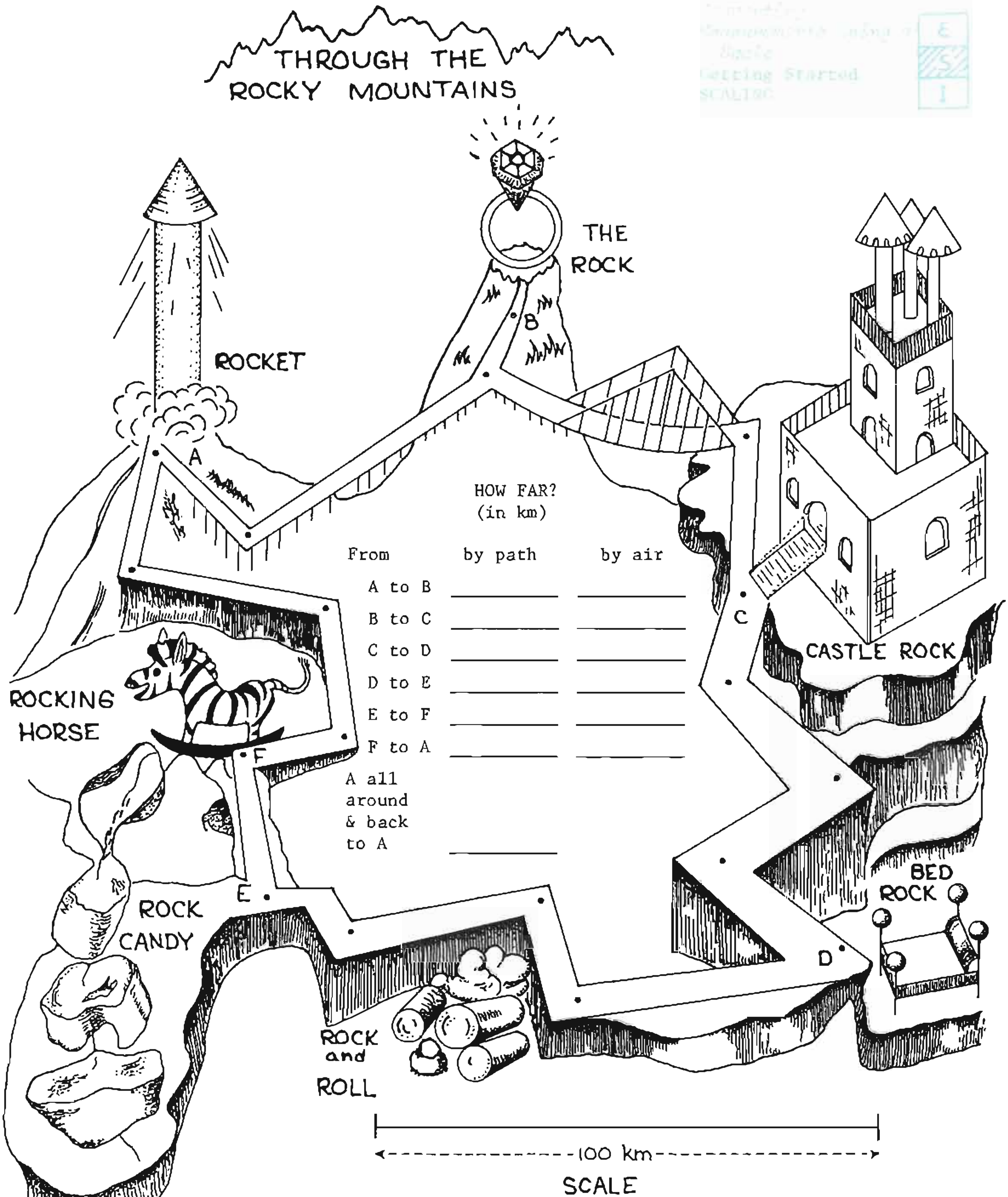
Use a metric ruler and find the treasure by following the directions on the parchment.



The lines lie in the NW-SE and NE-SW directions. When going in these directions, your ruler should be parallel to these lines.

Scale: 1cm : 10 paces

N 10 paces, NE 40 paces, S 20 paces, NW 25 paces, E 10 paces, SW 35 paces, N 40 paces, E 75 paces, N 20 paces, NW 30 paces, SW 45 paces, W 60 paces, SE 15 paces, N 35 paces, E 50 paces, SW 25 paces, S 5 paces, NE 40 paces, NW 35 paces.



HOW FAR?
(in km)

From	by path	by air
A to B		
B to C		
C to D		
D to E		
E to F		
F to A		
A all around & back to A		

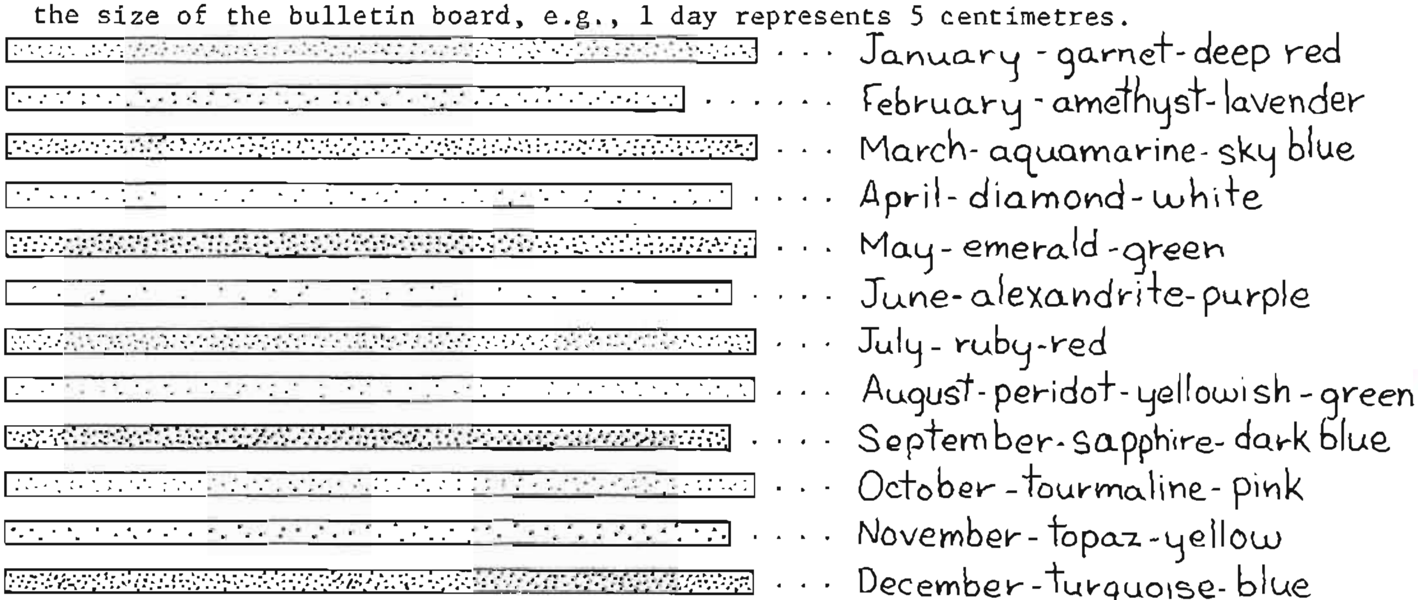
SOURCE: *The Metric System of Measurement*

CLASSY CALENDAR

TEACHER DIRECTED ACTIVITY

Getting Started
SCALING

On the bulletin board arrange colored strips of paper to represent the twelve months of the year. The color of each strip could correspond to the color of the birthstone for that month, and the length of each strip should be proportional to the number of days in the month. Select a scale that is suitable for the size of the bulletin board, e.g., 1 day represents 5 centimetres.



(1) Have a couple of students measure the strips and determine the scale used.

(2) Each student should then use the scale to locate his birthday on a strip. After each has colored in his birthday, these questions could be asked.

(a) Which month is the most popular month for birthdays? Least popular?

(b) Number of birthdays in a month : Total number of students?

(c) Number of birthdays in 1st half of year : Number of birthdays in 2nd half of year?

(d) Number of boys having a birthday in a month : Number of girls having birthdays in the same month?

(e) Number of months with 31 days : Number of months with 30 days?

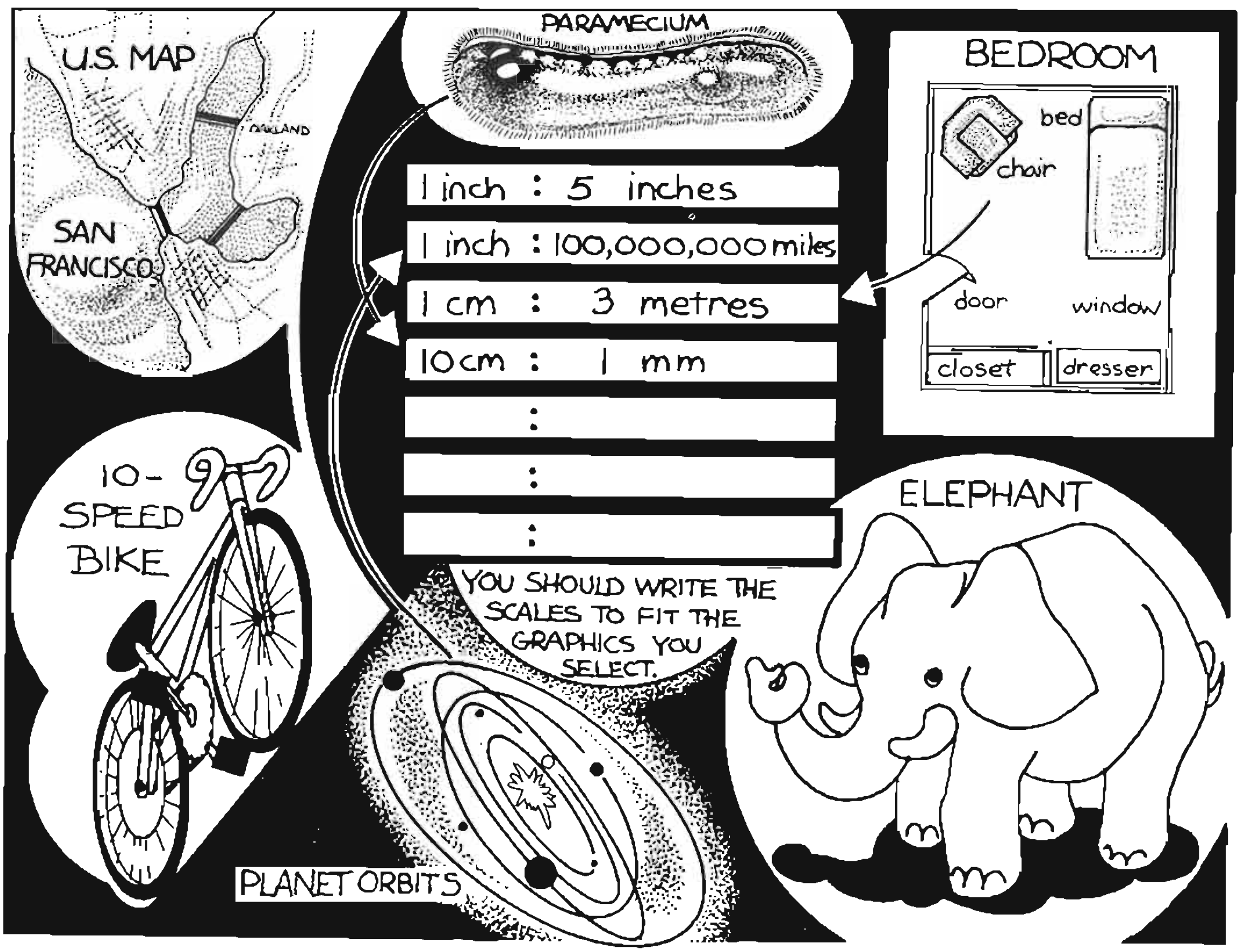
(f) Number of months starting with the same letter of the alphabet : Total number of months?

(3) Vacation times, weekends and/or holidays could be colored in on the strips. Special events, such as the World Series, state fairs or Mardi Gras, could also be shown.

(4) Specify a scale, e.g., 1 cm represents 1 day and have students draw a model of the bulletin board calendar on their papers. Have them locate their birthdays and the birthdays of those in their families. Holidays and days of special importance to each student could also be marked.

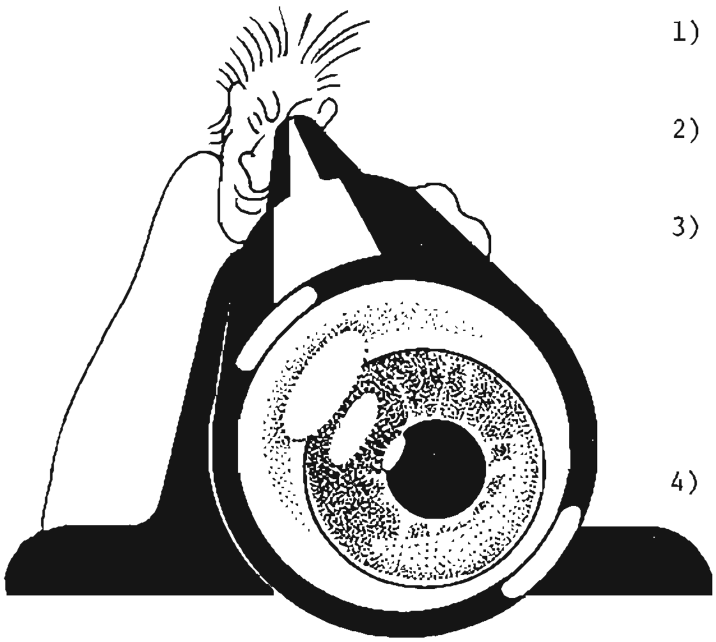
CHOOSE THE SCALE

A bulletin board display could give students practice in associating reasonable scales with pictures or scale drawings. Pictures from magazines, maps, xerox copies from textbooks, etc. could be attractively arranged on the bulletin board, and the corresponding scales posted separately. String could be used for students to match each scale with the corresponding graphic or the scales could be moved and placed next to the appropriate graphic. For several days discussions and changes on the bulletin board should be entirely student-centered. To close the activity the teacher could have a class discussion of the final choices. Thus, the bulletin board can be used as an active learning tool.



HAVE YOU GOT SPLIT ENDS?

Materials: Microscope
Several slides of small objects
Ruler



Activity:

1) Guess (in millimetres) the width of a hair from your head.

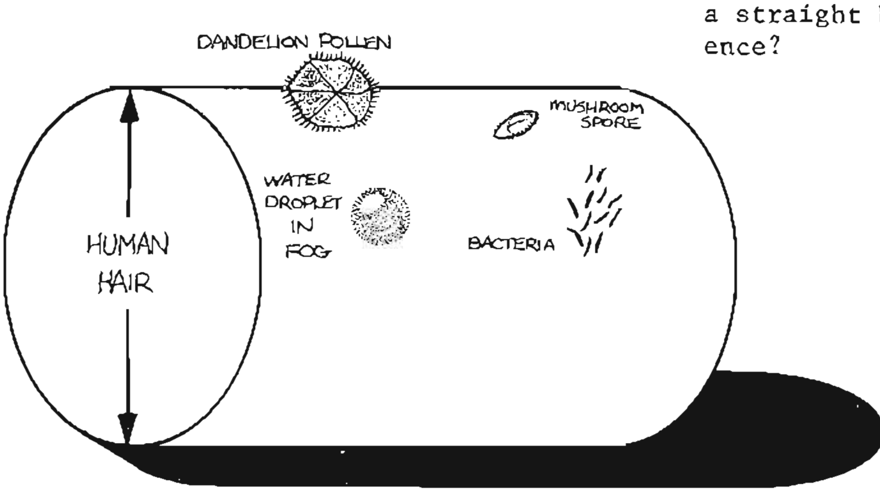
2) Pull out a hair and try to measure it with the ruler.

3) Place the hair on the stage of the microscope. What scale enlargements can you see under the microscope? The numbers are usually written on the lenses.

   a) ___:___ b) ___:___ c) ___:___.

4) Compare the width of your hair with a hair from another person with a different color of hair. Is one color of hair wider than the other? If so, which color is the widest?

5) Compare the widths of a curly hair and a straight hair. Is there a difference?



6) If you have other slides, compare the width of your hair with the width of the other objects. In each case which is wider?

7) This picture shows the width of a hair drawn to a scale of 600:1. If this hair is 45 mm wide, how wide is the actual hair?

8) Using the same scale, about how long are each of the other objects on the picture above?

   a) Pollen
   b) Spore
   c) Water
   d) Bacteria

IDEA FROM: *Arithmetical Excursions: An Enrichment of Elementary Mathematics*
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RATIO, PROPORTION, AND SCALING

Placement Guide for Tabbed Divisors

Tab Label **Follows page:**

CONTENTS

SCALING: MAKING A SCALE DRAWING

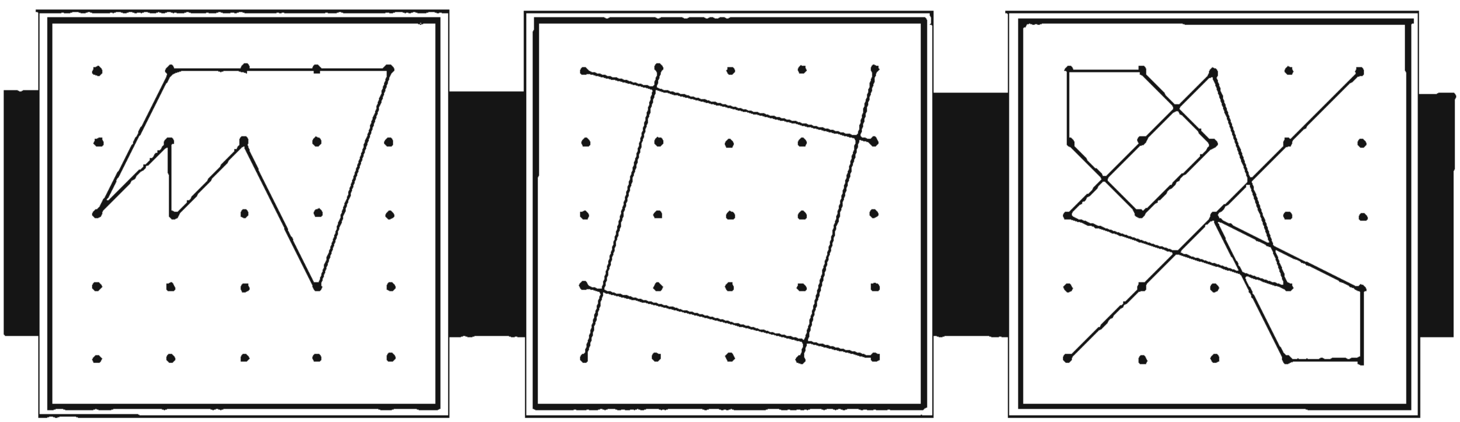
TITLE	PAGE	TOPIC	TYPE
A Negative Feeling	322	Enlarging/reducing using similar figures and a perspective point	Paper and pencil
Projecting Through the Pinhole	323	Demonstration of perspective	Activity
A Snappy Solution to Scale Drawings	324	Enlarging/reducing with rubber bands	Paper and pencil Activity
Stake Your Claim	326	Reducing with an instrument Finding lengths using an alidade	Activity
Another Stake Out	328	Reducing with an instrument Finding angles using a transit	Activity

GEOBOARD DESIGNS

Use only 4 or 5 rubber bands on the geoboard.

1) Make each design on your geoboard.

a. b. c.



2) Make a design of your own on the geoboard. Copy your design on dot paper.

3) Make a stop sign on the geoboard. Copy the sign on dot paper.

4) Make the largest number you can on the geoboard. Copy the number on dot paper.

5) Make your name on the geoboard. Copy each letter on dot paper.

6) Make a house, a boat or an airplane on the geoboard. Copy each design on dot paper.

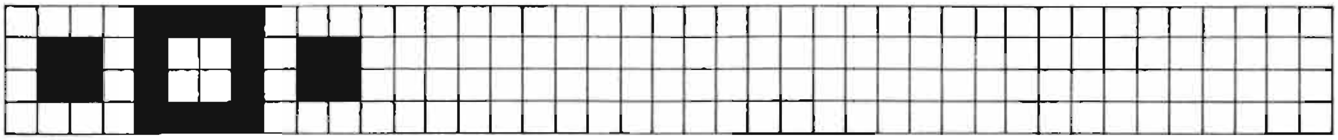
7) Make a triangle on the geoboard. Copy the triangle on dot paper. Is your triangle the same as your neighbor's? How many different triangles do you think you could make?

BORDER DESIGNS

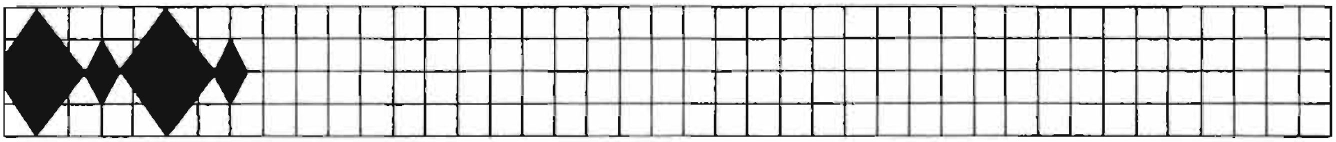
Making a Scale Drawing
SCALING

This worksheet serves as a readiness activity for grid enlargements and reductions. The students create designs by copying a pattern. The scale for each drawing is 1:1.

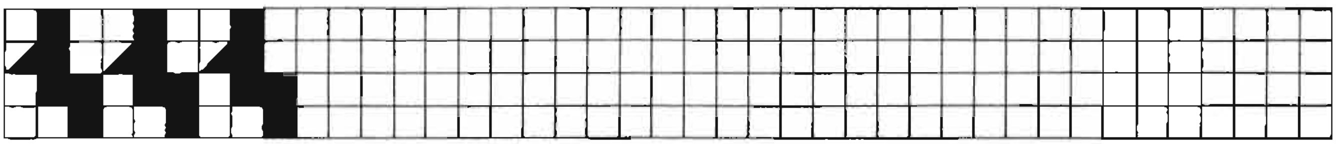
Continue these border designs. You could use colored pencils.

1. 

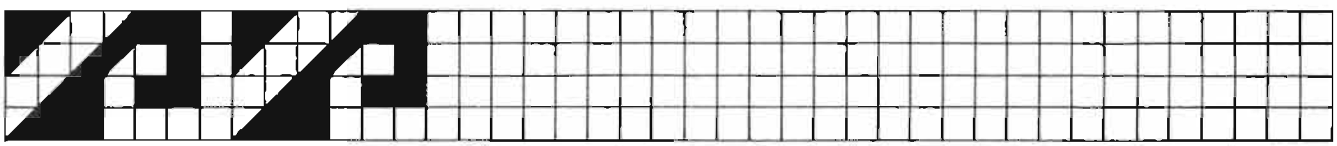
[redacted]

2. 

[redacted]

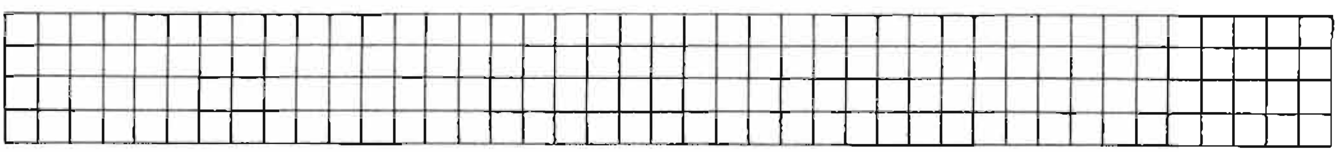
3. 

[redacted]

4. 

[redacted]

Create a design of your own. Repeat the pattern.

5. 

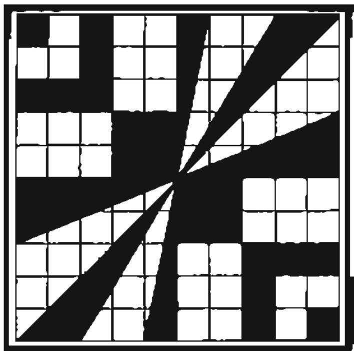
[redacted]

This activity can be done by duplicating grid designs (some easy, some difficult) for students to copy on grid paper.

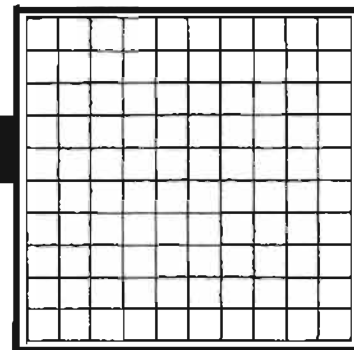
I HAVE DESIGNS ON YOU

Enlarging/Reducing with Grids
Making a Scale Drawing
SCALING

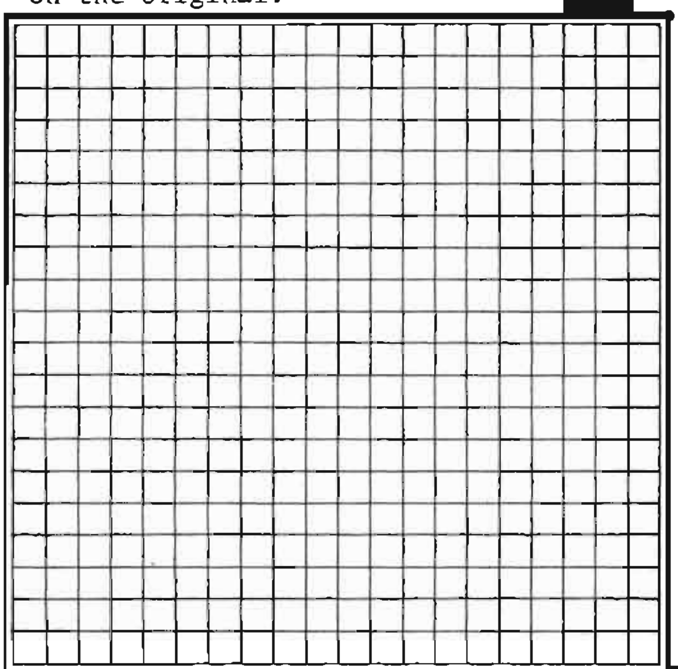
E
S
I



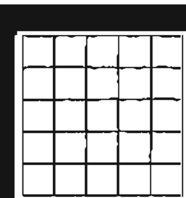
1. Copy the design on the grid below. The scale is 1:1.



2. Make an enlarged copy of the design that fills the grid below. On the enlargement the scale is 2:1. That is, 2 lengths on the enlargement represents 1 length on the original.



3. Make a reduced copy of the design on the grid below. On the reduction the scale is 1:2.



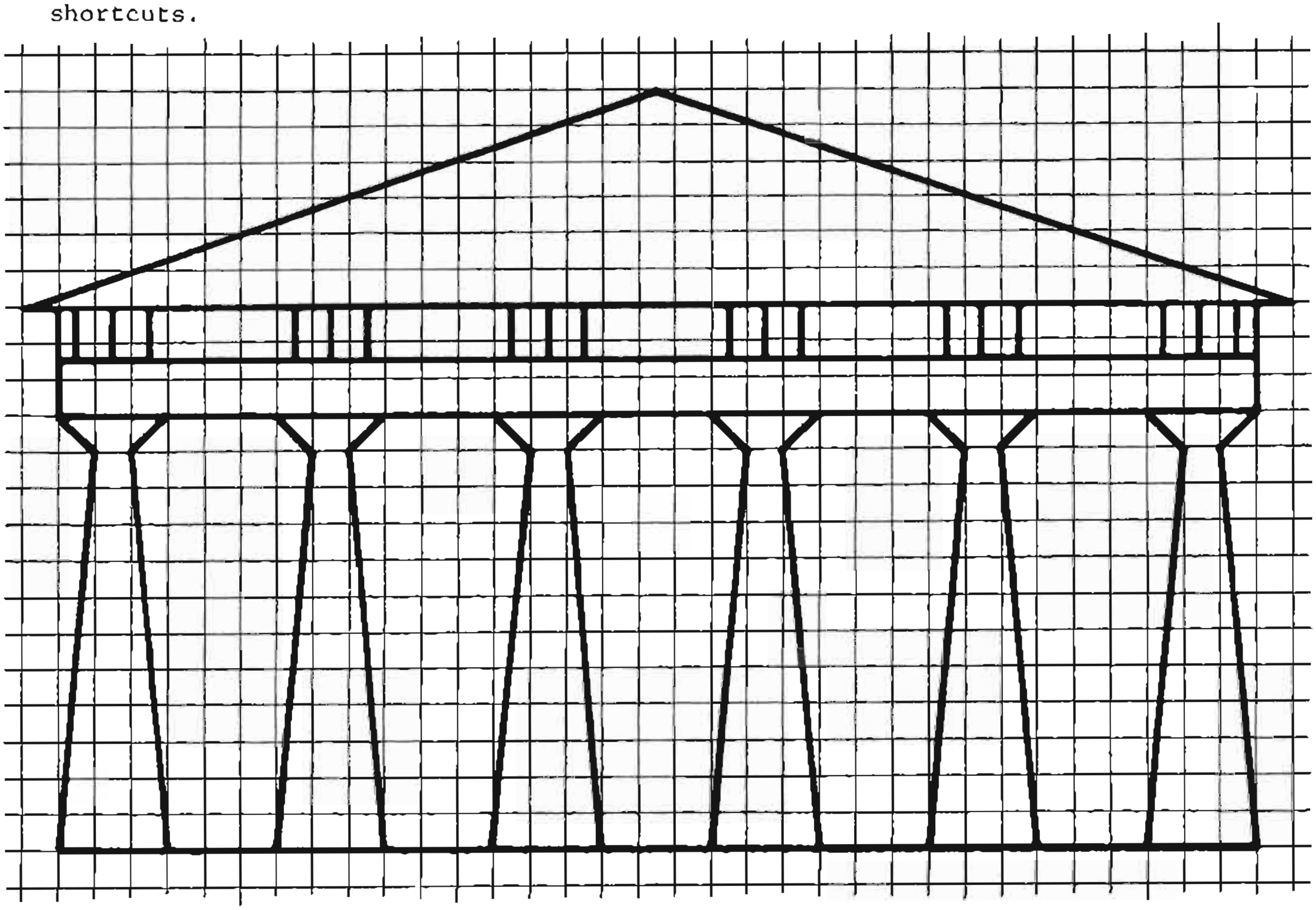
For the enlargement every other line could be darkened, and for the reduction a line between each pair of lines could be drawn.

4. Make a design of your own on graph paper. Have a friend make a 2 to 1 enlargement of the design.

TYPE: Paper & Pencil

THE PARTHENON

Use the $\frac{1}{2}$-centimetre grid provided to make a scale drawing of the Parthenon below. Reduce the dimensions of the drawing to one-third their present length. Look for shortcuts.



GRID GRAPHS

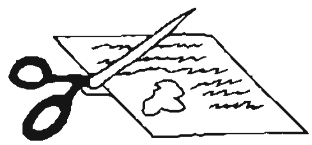
How to make grid graphs and distorted graphs:

Ask students to bring comic books, newspaper comic strips, Mad magazines, and picture magazines for use in the classroom. The school library often has old copies of newspapers and magazines. Used-book stores are another source of such materials.

Let students choose a cartoon character, a comic strip character, a real life photograph, or a real life drawing. The first pictures should be simple.

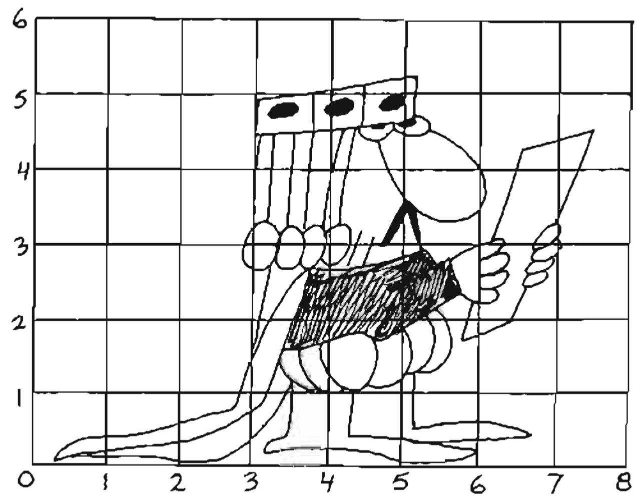
Instruct them to make an enlargement, a reduction, and/or a distortion of the picture they select.

STEPS TO FOLLOW:

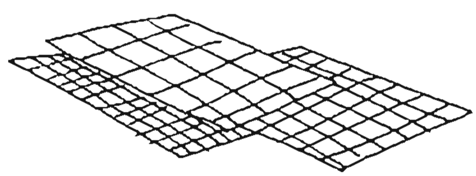


a. Cut out the picture.

b. Using a ruler and a pencil, draw a grid over the picture. Make the squares a standard size (i.e., square centimetres, square inches, square half-centimetres--whatever seems the most appropriate size.)



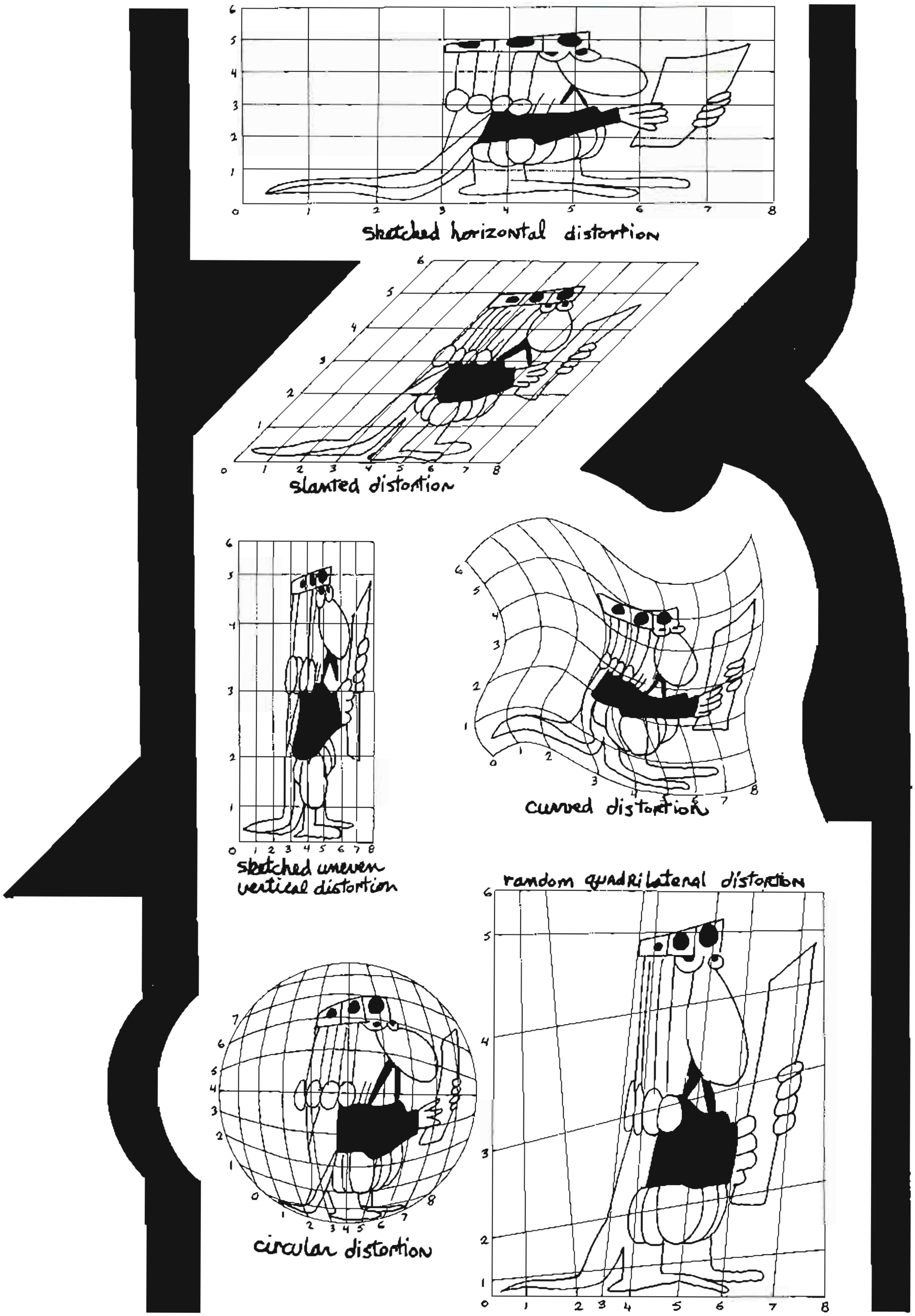
c. Use graph paper sizes provided, or create your own grids to enlarge, reduce, and/or distort the picture.



d. The following grids show various distortions of the original figure. Students can use these illustrations for ideas. Students could draw an original distorted grid and give it to another student to complete. These scaling assignments make a nice bulletin board display.

IDEA FROM: *Art 'N' Math*

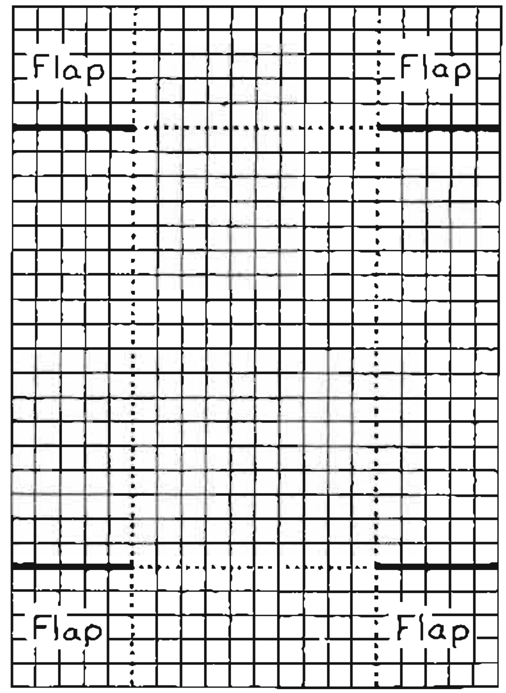
GRID GRAPHS
CONTINUED



IDEA FROM: *Art 'N' Math*

PAINT YOUR WAGON

Below are instructions for making a wagon.
Make a wagon twice as long and twice as wide.

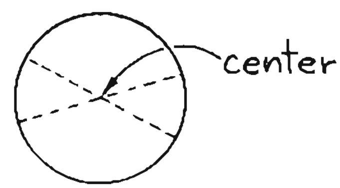


A. The Body

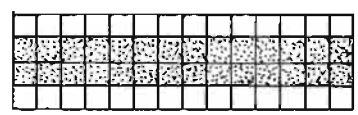
1. Copy this shape on squared paper. Count the spaces you need for each line.
2. Cut on the heavy solid lines.
3. Fold along the dotted lines.
4. Use the square flaps to fasten the body together.

B. The Wheels

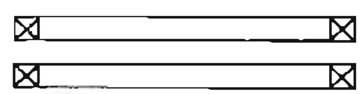
1. Use a poker chip, 50¢ piece, or small lid. Place it on an index card and draw around it.
2. Cut out the circle.
3. Make three more wheels like this.
4. To find the center of each wheel, draw another circle, cut it out, and fold it in half. Open it out and fold it again in a different place. Open it out. The center of the circle is where the fold lines cross. Fit the circle on each of your wheels and use a pin to make a hole through the center.

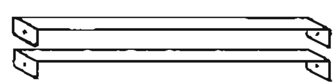


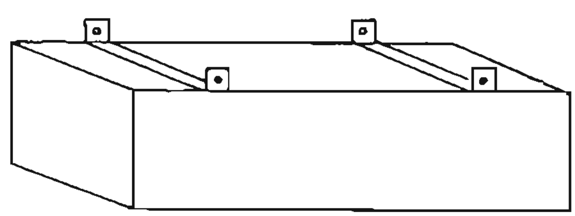
C. The Axles

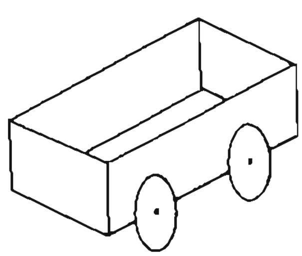


1. Use squared paper to mark out two strips of index card, each 12 spaces long and 1 space wide. Cut out the strips.
2. At each end mark off one square.
3. Find the center of each square by drawing the diagonals. Make a small hole at each center.
4. Bend down the end squares.
5. Turn the body of the wagon upside down and stick the axles to it.
6. Put a pin through the center of each wheel and fasten the wheels to the axles. You may need to tape the pins to the bottom of the wagon to keep the wheels from coming off.



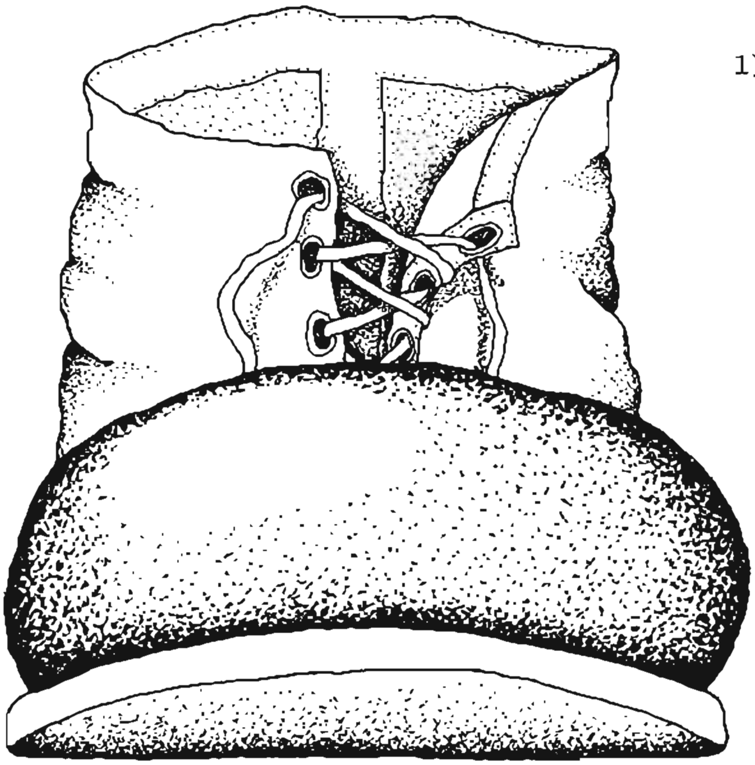






IDEA FROM: *New Oxford Junior Mathematics*, Book 1
Permission to use granted by Oxford University Press

PACE OUT THE SPACE



1) Count the number of paces needed to pace

_____ a) the length of the room.

_____ b) the width of the room.

_____ c) the length of the black boards.

_____ d) the length of the heating units.

_____ e) the length of the teacher's desk.

_____ f) the width of the door.

_____ g) the distance from the corner of the room to the door.

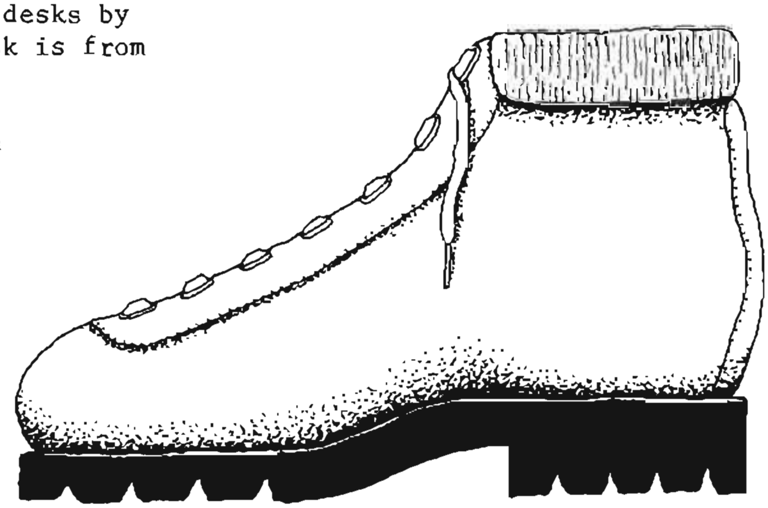
_____ h) the width of the windows.

_____ i) the width of other prominent objects in your classroom such as large tables, bookshelves, or filing cabinets.

2) On a piece of grid paper using a scale of 1 unit of length : 1 pace or on a piece of plain paper using a scale of 1 cm: 1 pace, make a scale drawing of your classroom. Include the arrangement of the desks by pacing the distance the 1st desk is from the front and side wall.

3) Compare your scale drawing to a classmate's drawing. Are the drawings similar? Why might the drawings be different?

4) With a tape measure or metre stick find out how many centimetres long your pace is. Measure several times to get an accurate answer.



5) Use your scale drawing and the length of your pace to find the approximate lengths (in metres and centimetres) of the objects in part (1). Measure with the tape measure to check your approximations.

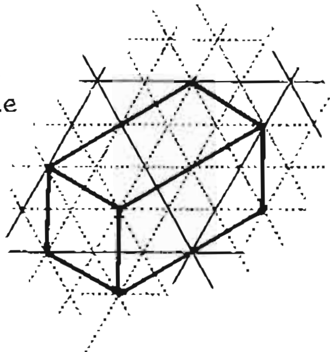
TYPE: Activity

WHAT'S YOUR ANGLE?

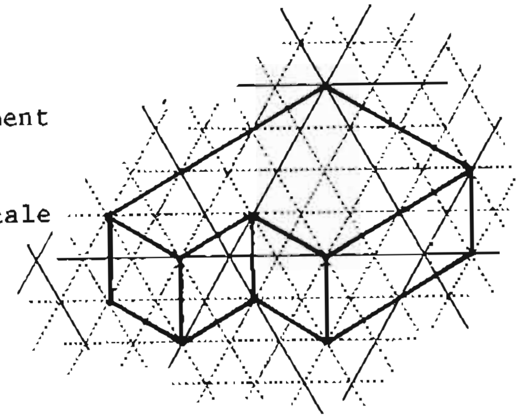
Materials needed: Isometric grid paper, straightedge

Activity:

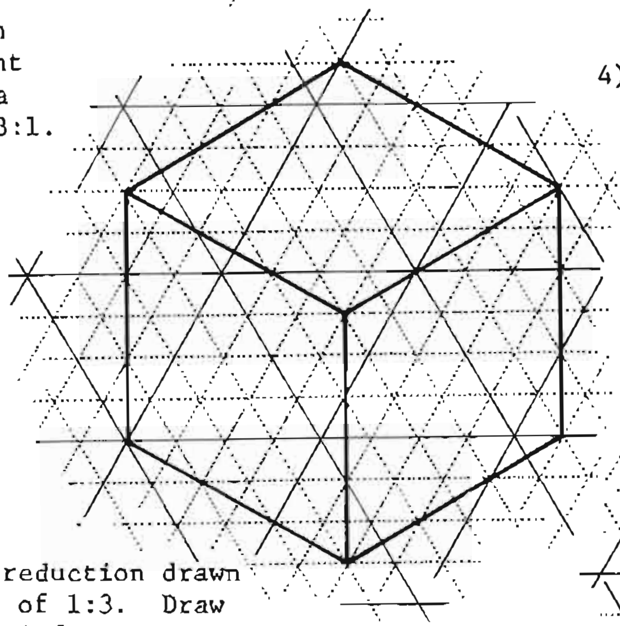
1) Draw a copy of this figure. Use a scale of 1:1.



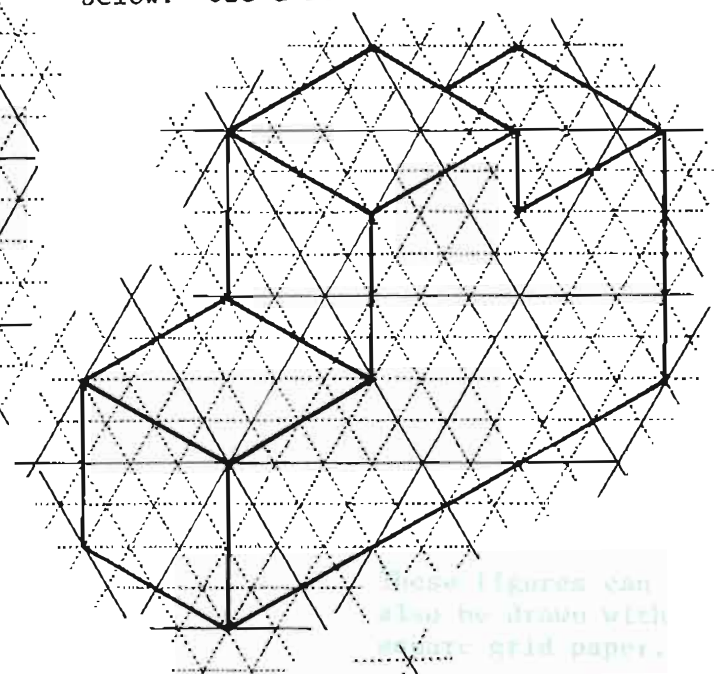
2) Draw an enlargement of this figure. Use a scale of 2:1.



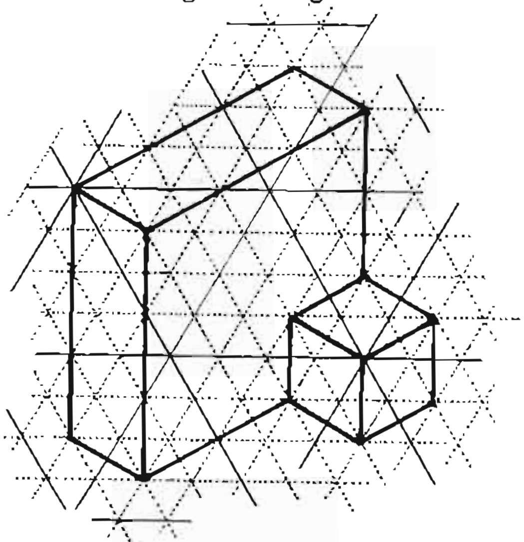
3) This is an enlargement drawn to a scale of 3:1. Draw the original figure.



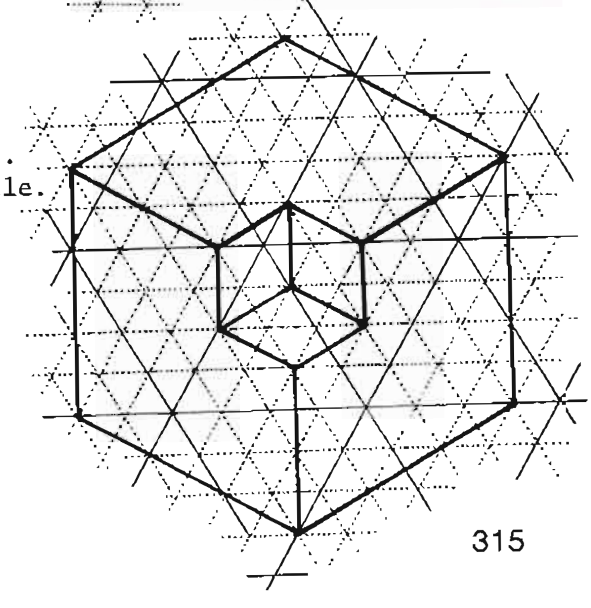
4) Draw a reduction of the figure below. Use a scale of 1:2.



5) This is a reduction drawn to a scale of 1:3. Draw the original figure.



6) What do you see? Draw an enlargement. Choose your own scale.



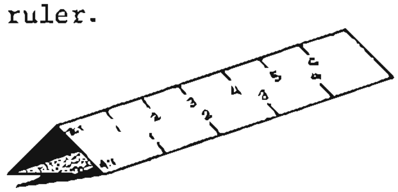
ARCHIE TEXS' RULER

Making a Scale Drawing
SCALING

Activity:

(1) Complete the ruler by marking sides B and C to show the given scale.

(2) Cut out this chart, fold on the lines, and paste the flap under to make your architect's ruler.



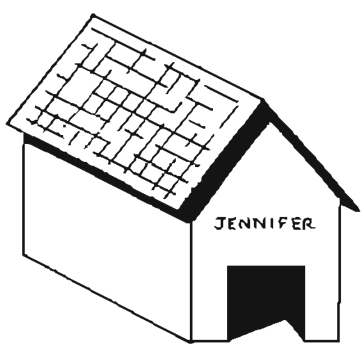
(3) On another paper use the 1:1 scale to draw a rectangle 2 units wide and 4 units long.

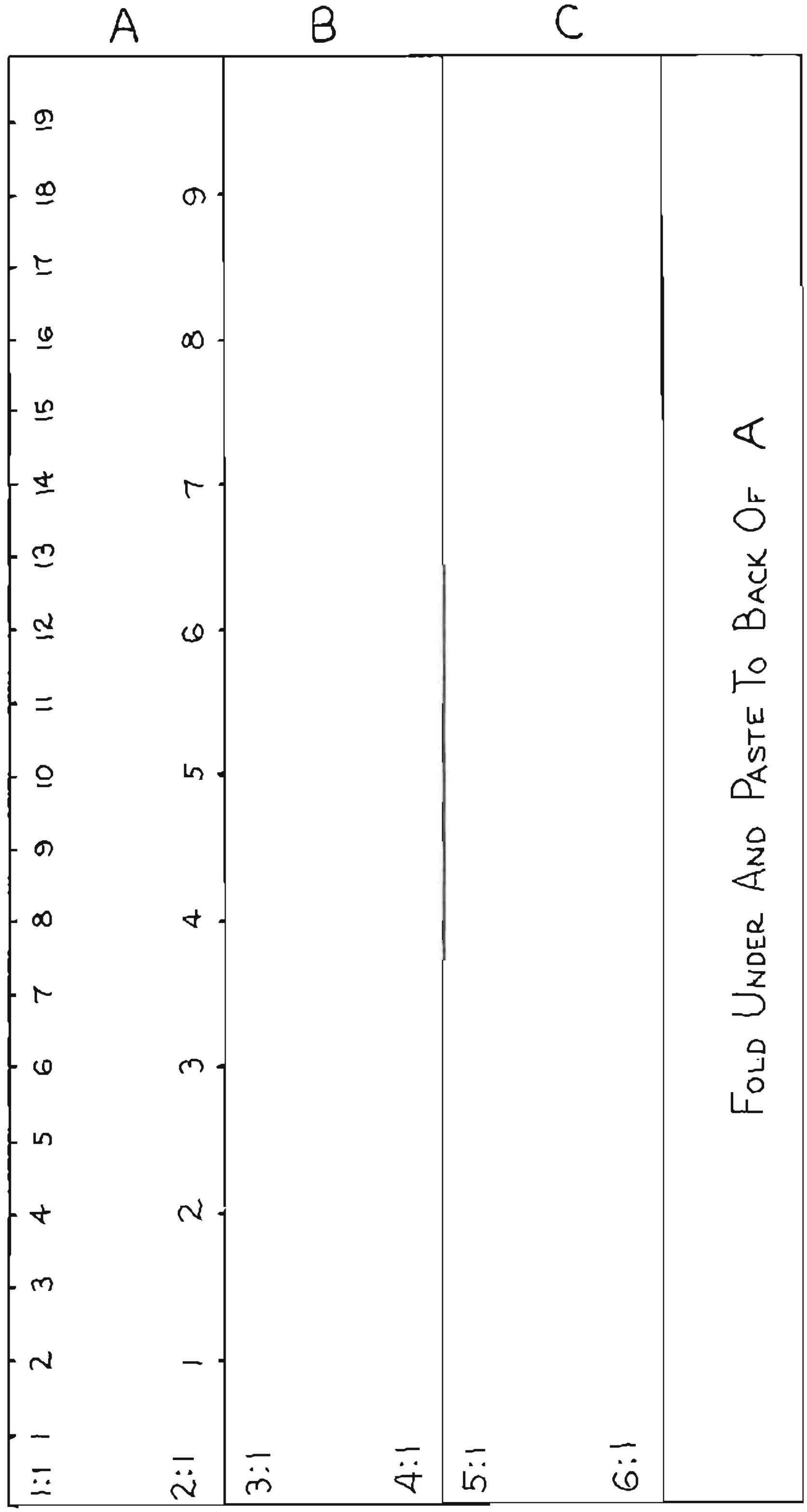
(4) Now use the 2:1 scale to make an enlargement of the rectangle that is 2 times as wide and 2 times as long.

(5) Use the 1:1 scale and draw another rectangle. Make a 3 to 1 enlargement.

(6) Use the 1:1 scale and draw a square 3 units on a side. Make a 6 to 1 enlargement of the square.

Challenge: Make a 4:1 enlargement of Jennifer's doghouse.





Activities could be developed for using an actual architect's ruler, an instrument used primarily for making reductions.

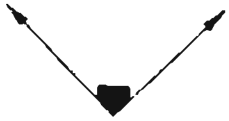
TYPE: Activity

IDEA FROM: *Minnemast*, Unit 18, Scaling and Representation
Permission to use granted by Minnesota Mathematics and Science Teaching Project

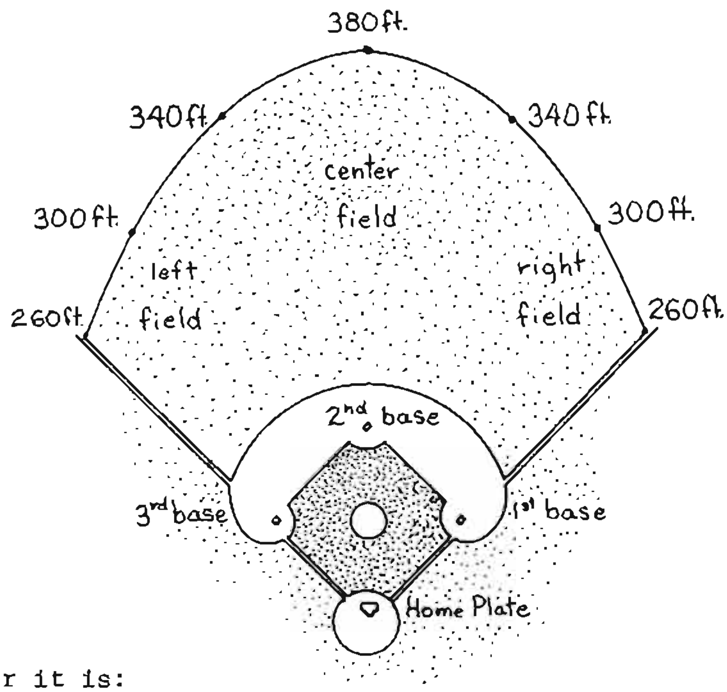
TAKE ME OUT TO THE BALL GAME

Using a scale of 1 cm : 20 ft., make a scale drawing of this baseball field.

Place this diagram of home plate at the bottom of your paper and trace over these lines to help you get started.



A major league baseball diamond is a square 90 feet on each side. Outfields do not have a standard size. Using a ruler, draw this baseball field so that the distances from home plate to the outfield fence are the same as shown in the diagram.



1) Use your scale drawing to find how far it is:
   a) if you run around the bases after hitting a home run. ________ ft.
   b) across the infield from home plate to 2nd base. ________ ft.
   c) across the infield from 1st base to 3rd base. ________ ft.

2) The pitcher's mound is located approximately in the middle of the diamond. Put the pitcher's mound on your drawing. How far is it from the mound to home plate? ________ ft.

3) Hank Aaron smashes a 250-foot hit into left field. Mark an "X" in your drawing to show where the ball might hit if the left fielder misses the catch. Compare your answer with a friend.

4) Reggie Jackson hits a towering fly ball 310 feet that is not a home run. With an "O" mark three possible spots where the ball can be caught.

5) The longest measured home run was hit in 1953 by Mickey Mantle. It traveled 565 feet. On the scale drawing this home run would have landed ____________ centimetres from home plate. Can you show this on your drawing? ____________

6) How far will the ball travel from the pitcher to the first baseman if the batter hits a line drive to the third baseman, who catches the ball while standing on third base and relays the ball to first base? __________

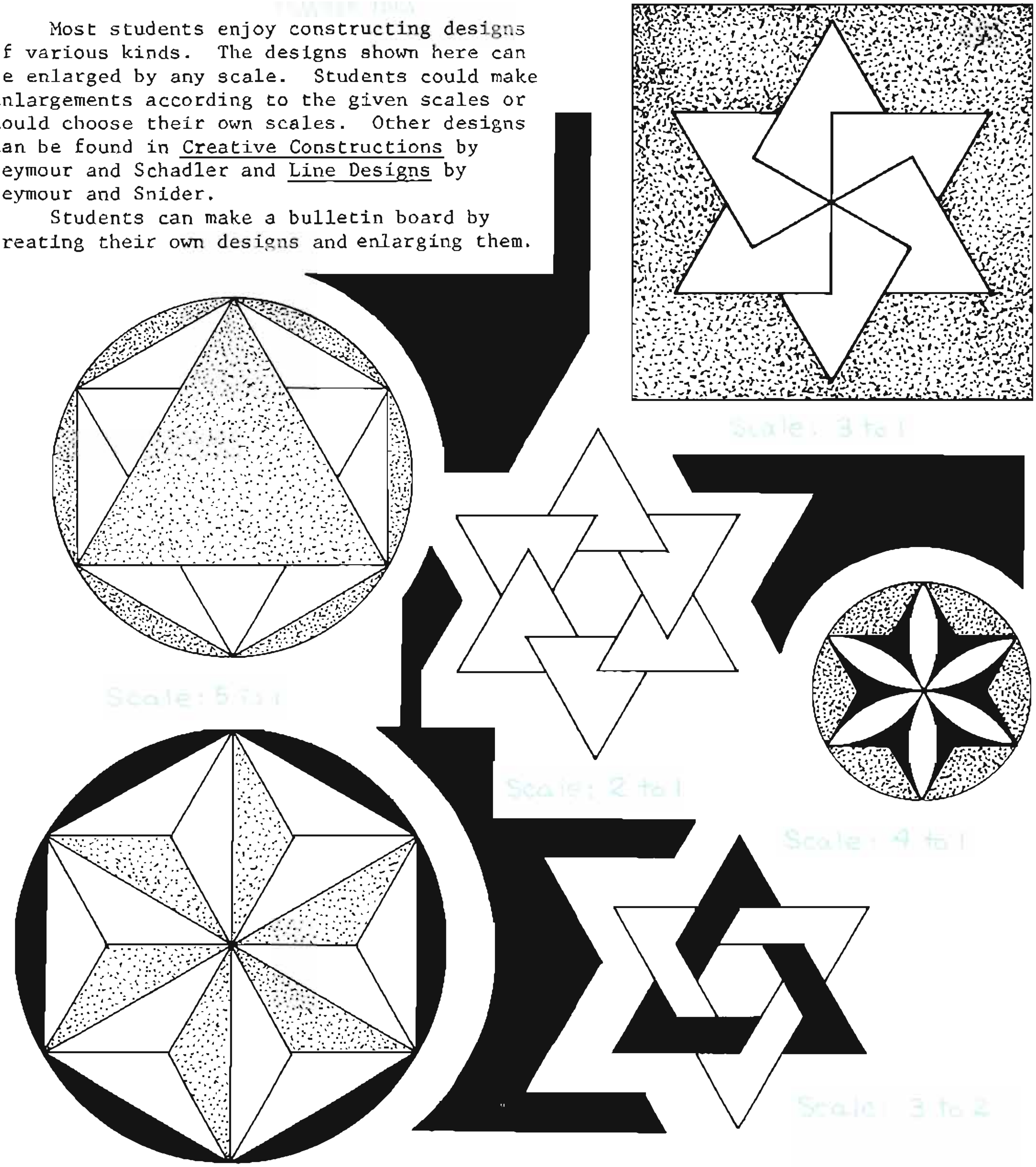
7) The batter hits a Texas leaguer (a short fly ball) into center field 190 feet from home plate. The second baseman receives the throw from the center fielder at second base. How far did the ball travel? __________

IDEA BASED ON: *Fraction Factory* and *SRA Math Applications Kit* by Allen C. Friebel and Carolyn Kay Gingrich. © 1971, Science Research Associates, Inc.

ROOM DECORATIONS

Most students enjoy constructing designs of various kinds. The designs shown here can be enlarged by any scale. Students could make enlargements according to the given scales or could choose their own scales. Other designs can be found in Creative Constructions by Seymour and Schadler and Line Designs by Seymour and Snider.

Students can make a bulletin board by creating their own designs and enlarging them.



IDEA FROM: *Creative Constructions*

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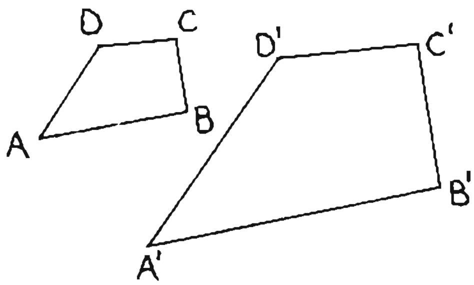
WHAT'S THE POINT?

Making a Scale Drawing
SCALING

Students will probably need a teacher demonstration of the first problem.



1. Use a metric ruler to measure the sides of each figure to the nearest $\frac{1}{2}$- cm. Write these ratios.

A'B' : AB = ______ : ______
B'C' : BC = ______ : ______
C'D' : CD = ______ : ______
D'A' : DA = ______ : ______

2. Do all your ratios simplify to about 2:1?
3. Draw lines to connect A to A', B to B'. C to C', D to D'. Extend these lines until they cross. Label this point P.
4. With a metric ruler measure these line segments and write the ratios.

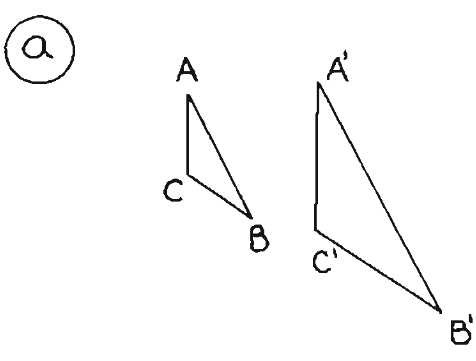
PA' : PA = ______ : ______
PB' : PB = ______ : ______
PC' : PC = ______ : ______
PD' : PD = ______ : ______

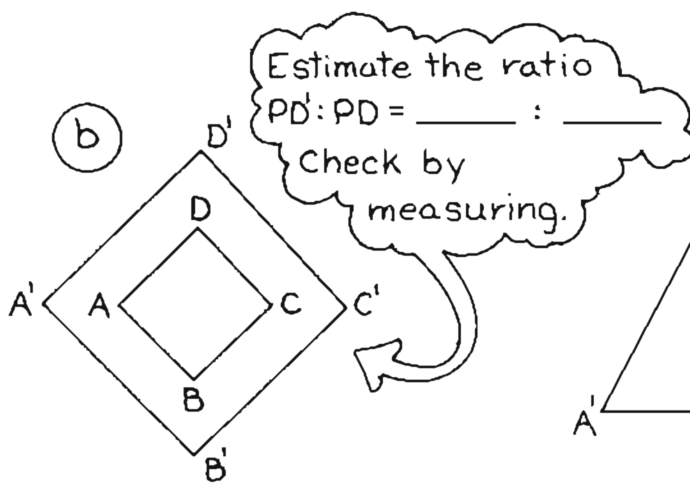
These should all simplify to about 2:1.

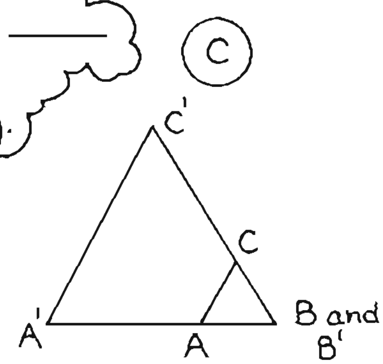
A'B'C'D' is an enlargement of ABCD by a scale factor of 2.
P is the starting point for the enlargement.

5. Find P (the starting point for the enlargement). Measure the sides of the figures to find the scale factor of each enlargement.

Point P may be located outside, inside or on the figures.







TYPE: Paper & Pencil

IDEA FROM: *The School Mathematics Project*, Book D
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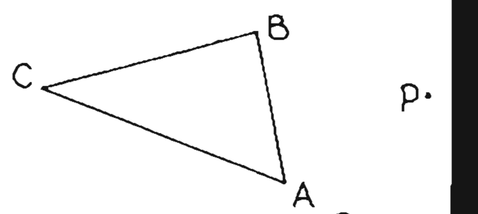
Making a Scale Drawing
SCALING

E S I

BIGGER THAN LIFE

Demonstrate the first problem for your class.

To make an enlargement of triangle ABC using a scale factor of 2 do the following:



1) Draw lines from P through A, B and C.
2) On line PA mark point A' so PA':PA = 2:1.
   On line PB mark point B' so PB':PB = 2:1.
   On line PC mark point C' so PC':PC = 2:1.
3) Use a metric ruler to measure the sides of the two triangles. Write these ratios.

Hint: Mark the length of segment PA on the edge of a piece of paper. Use these marks to find A'.

A'B' : AB = ______ : ______

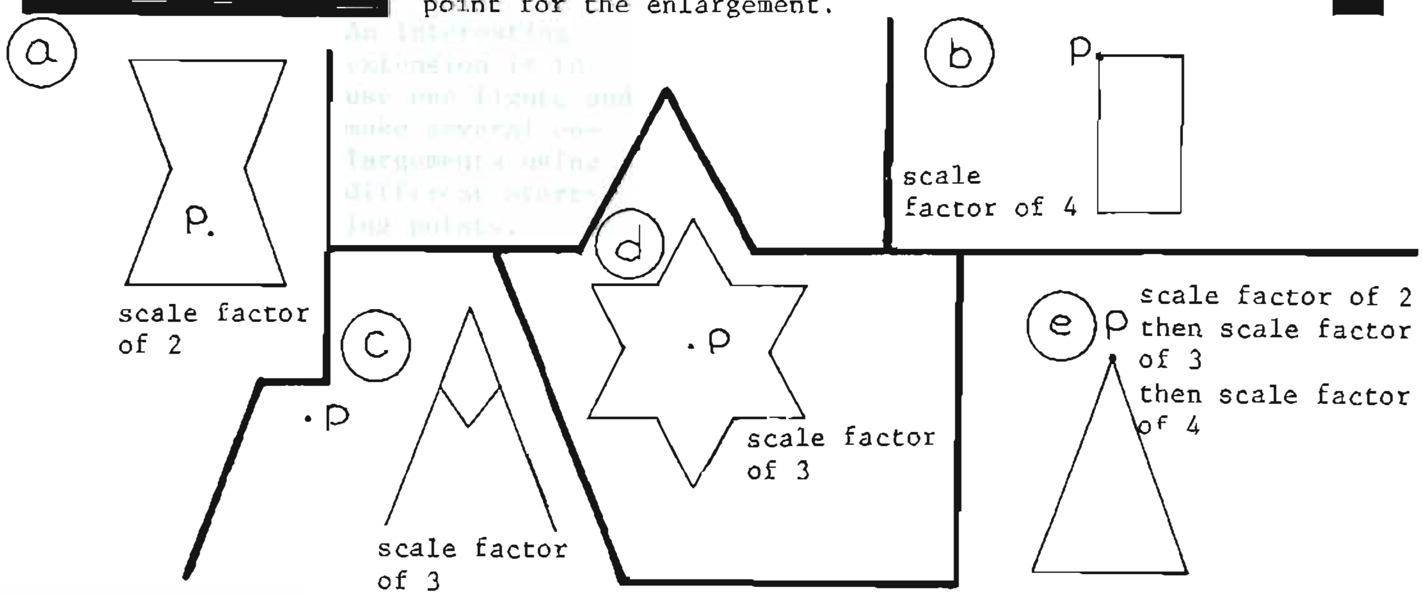
B'C' : BC = ______ : ______

C'A' : CA = ______ : ______

Is each side in the new triangle about twice as long as its corresponding side in the original?

Trace each figure on another sheet of paper and use the scale factor to make an enlargement. P is the starting point for the enlargement.

An interesting extension is to use one figure and make several enlargements using different starting points.



PPE — Paper & Pencil

IDEA FROM: *The School Mathematics Project*, Book D
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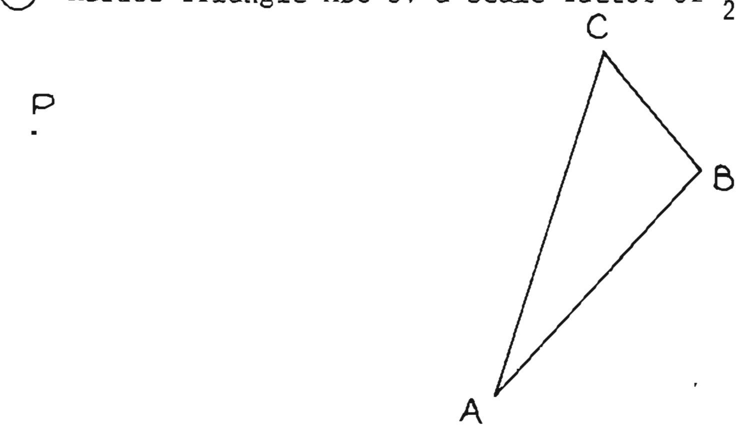
A SHRINK

If the scale factor is less than 1, the drawing actually becomes a reduction or shrink.

ⓐ Reduce triangle ABC by a scale factor of $\frac{1}{2}$. P is the starting point for the reduction.



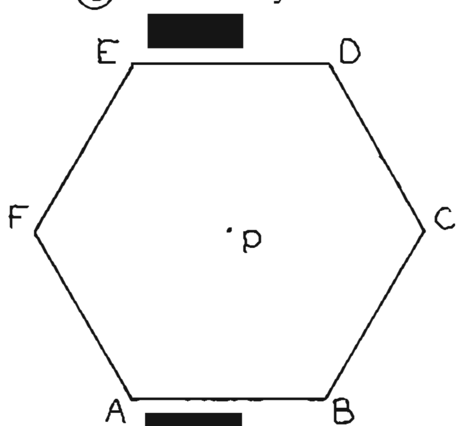
1) Draw segments $\overline{PA}$, $\overline{PB}$, $\overline{PC}$.

2) Find A' so that PA':PA = 1:2.
   Find B' so that PB':PB = 1:2.
   Find C' so that PC':PC = 1:2.

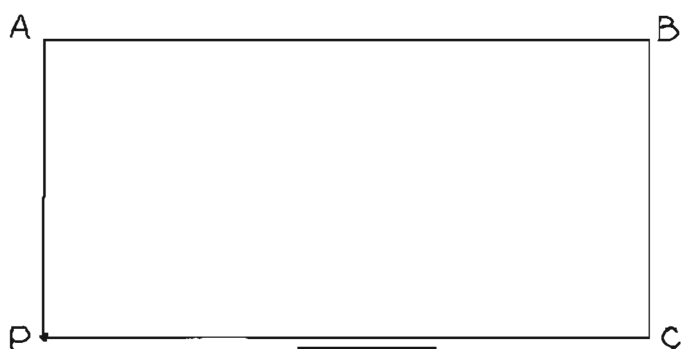
3) Measure the sides of triangle A'B'C'. Is each side in a 1:2 ratio with its corresponding side in triangle ABC?

On another piece of paper trace each of the figures and do the reduction. In each proplem P is the starting point for the reduction.

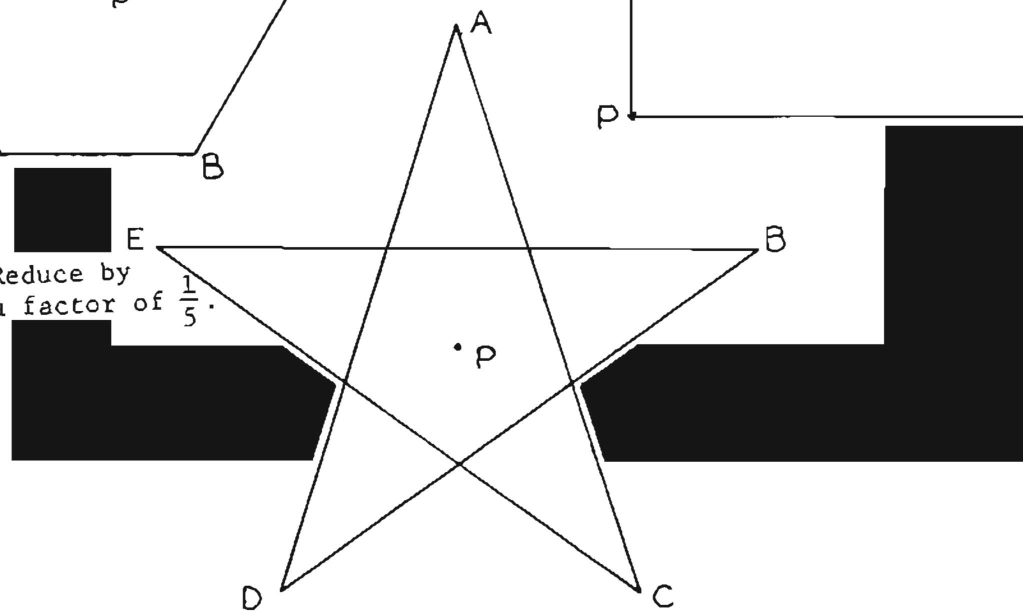
ⓑ Reduce by a factor of $\frac{1}{3}$.



ⓒ Reduce by a factor of $\frac{1}{4}$.



ⓓ Reduce by a factor of $\frac{1}{5}$.



IDEA FROM: *The School Mathematics Project*, Book D
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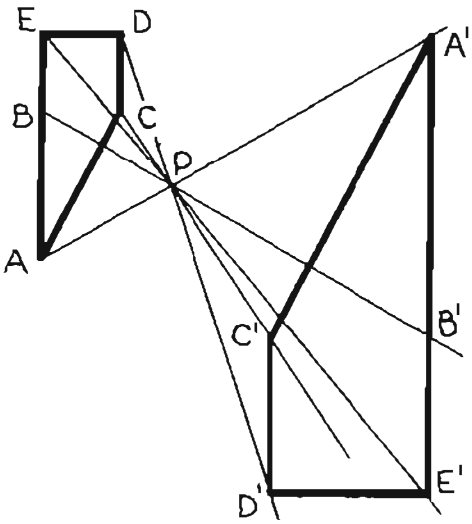
A NEGATIVE FEELING

Making a Scale Drawing
SCALING

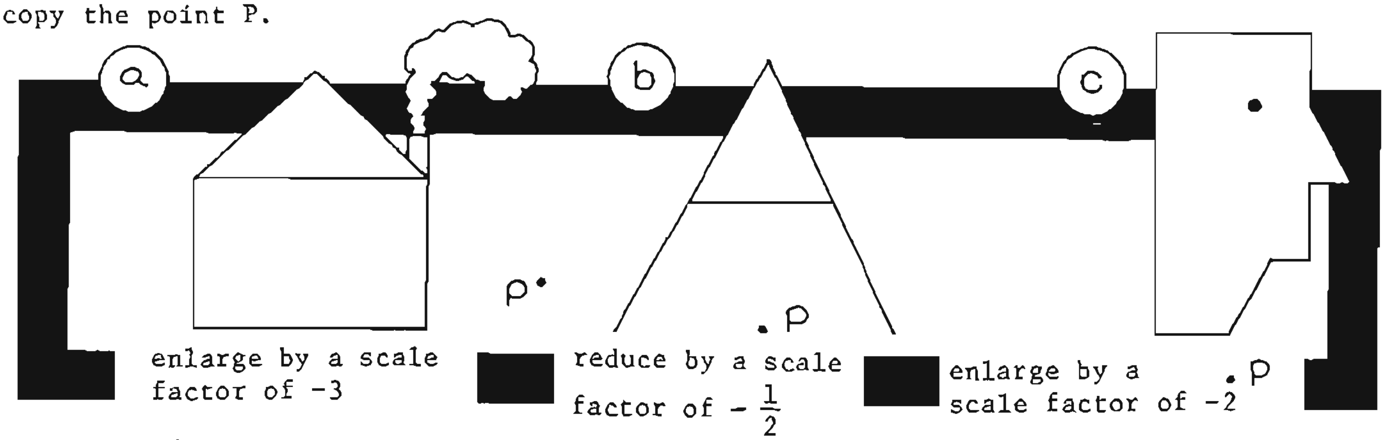
When the starting point of an enlargement or reduction is between the original design and the new design, the new design will be upside down. The scale factor is written as a negative number.

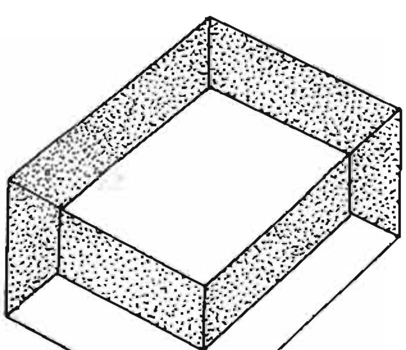
For example: Enlarge this design by a scale factor of -2. P is the starting point for the enlargement.



1) Draw lines $\overline{PA}$, $\overline{PB}$, $\overline{PC}$, $\overline{PD}$, $\overline{PE}$.
2) On line $\overline{PA}$ locate A' so that P is between A' and A, and PA':PA = 2:1. Do the same for lines $\overline{PB}$, $\overline{PC}$, $\overline{PD}$, and $\overline{PE}$.
3) Measure the sides of the new design to see if each side is in the ratio of 2 to 1 with a corresponding side of the original design.
4) Is the new design upside down?

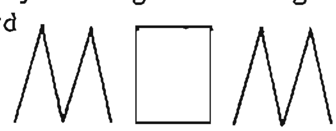
Copy each design on another sheet of paper and make the new design. Be sure to copy the point P.





Challenge: Stand the box on its end by making an enlargement with a scale factor of -2.

Challenge: Find out what I think of my mother by making an enlargement of the word



Use a scale factor of -4.

TYPE: Paper & Pencil

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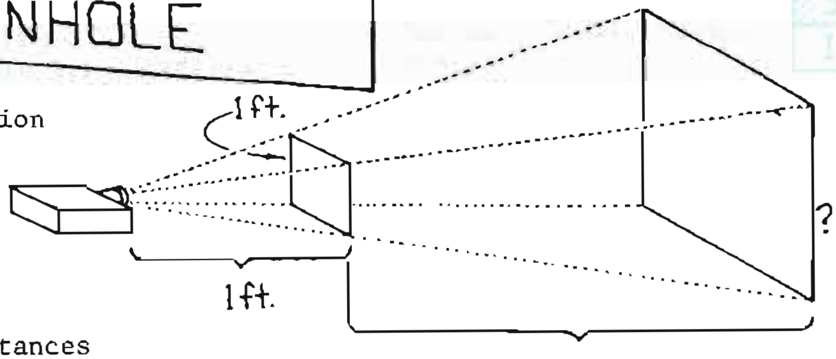
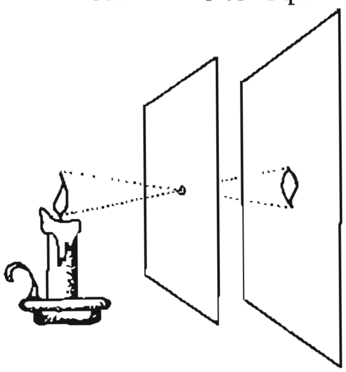
PROJECTING THROUGH THE PINHOLE



1) This is a "practical" demonstration of the concept in *Bigger Than Life*.

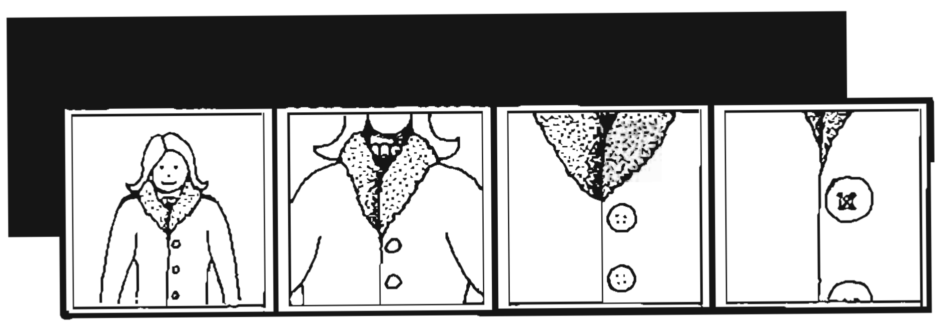
Construct a 6-in. square and place it 1 ft. in front of a projector. Locate a screen behind the square at various distances from the projector, e.g., 2 ft., 3 ft. or 4 ft. Have students estimate the length of the shadow at each distance and then measure to check. Students should discover that length of square : length of shadow = distance of square from the projector : distance of screen from the projector. A discussion can be held on the ratio of the areas of the square and the shadow at each distance.



2) This is a "practical" demonstration of the concept in *A Negative Feeling*.

Make a small pinhole in a piece of heavy paper. Hold the paper about 4 inches from the wall and hold a lighted candle in front of the pinhole. The image of the flame projected onto the wall will be inverted. A diagram of how a simple camera works also illustrates this concept.

3) Use a slide projector and a 1-foot square frame to generate a series of enlargements with a constant scale factor, e.g., 2.

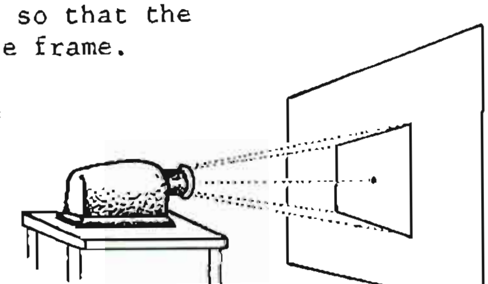


a. Select a slide. Mark the center (dot with pen) for a reference point.

b. Mount the frame on a wall.

c. Position the projector about 2 feet from the wall so that the dot on the slide is projected in the center of the frame.

d. Observe the portion of the slide projected inside the frame. Select an object(s) near the center of the frame (like the button above) and measure its length.



e. Double the distance of the projector from the wall (keep the reference dot in the center of the frame).

f. Note the image in the frame. Remeasure the object(s).

g. Repeat. Students may predict new lengths of object(s) for new distances.

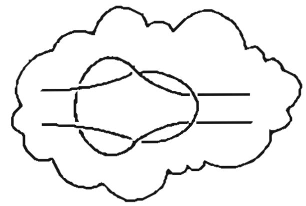
Making a Scale Drawing
SCALING

I

A SNAPPY SOLUTION TO SCALE DRAWINGS

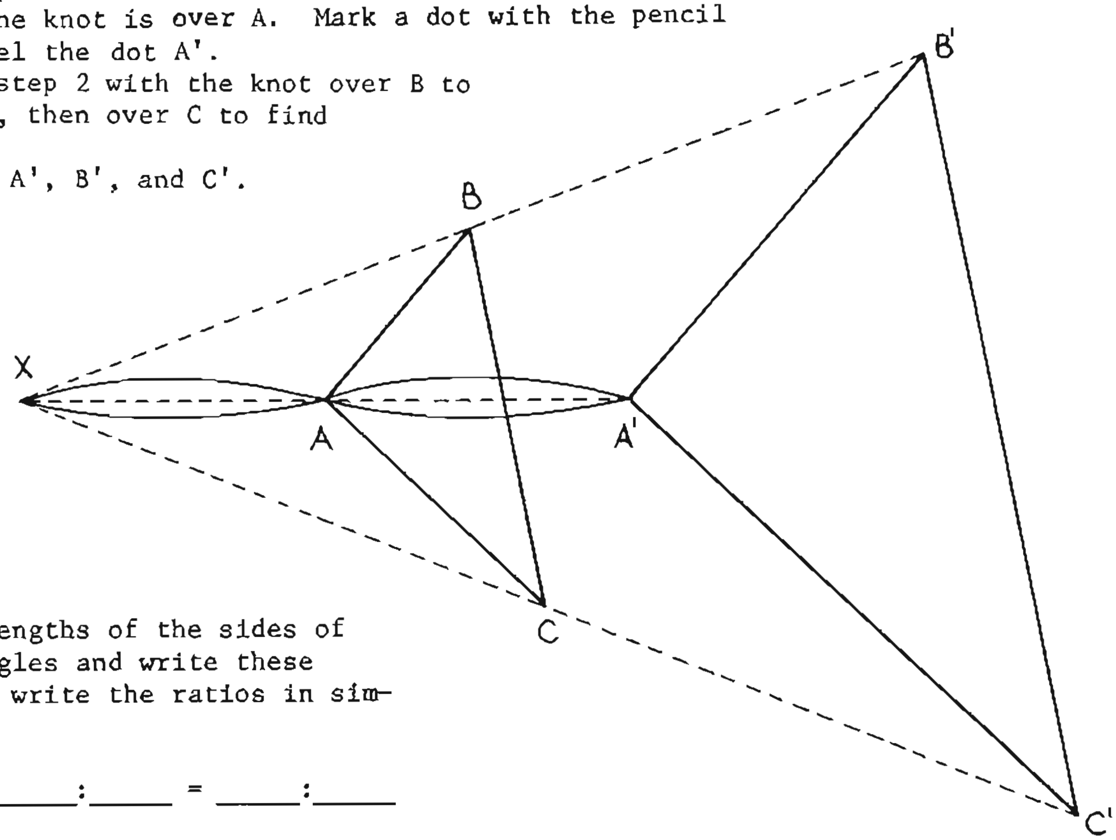
Materials needed: Several identical rubber bands, a thumbtack, a centimetre ruler, butcher paper, large table.

Activity: Loop two identical rubber bands together to form a knot in the middle.



To enlarge triangle ABC:

1) Pick a point X so the distance from X to A is longer than the length of a rubber band.
2) Hold one end of the rubber band on point X with your thumb or the thumbtack.
3) With a pencil in the other end, stretch the rubber bands until the knot is over A. Mark a dot with the pencil and label the dot A'.
4) Repeat step 2 with the knot over B to find B', then over C to find C'.
5) Connect A', B', and C'.



Measure the lengths of the sides of the two triangles and write these ratios. Then write the ratios in simplest form.

A'B':AB = _____:_____ = _____:_____

A'C':AC = _____:_____ = _____:_____

B'C':BC = _____:_____ = _____:_____

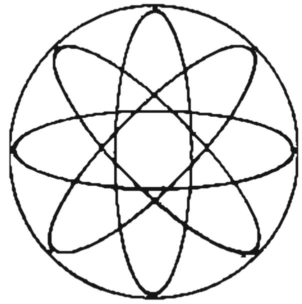
The rubber bands have helped you make a 2 to 1 enlargement. Do one of your own.

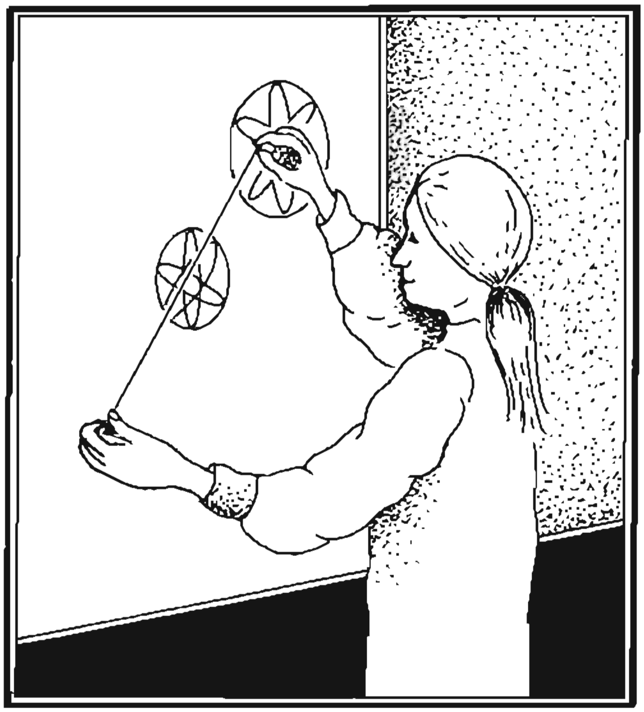
TYPE: Paper & Pencil/Activity

IDEA BASED ON: *Activities with Ratio and Proportion*, and *The Laboratory Approach to Mathematics* by Kenneth P. Kidd, Shirley S. Myers, and David Cilley, © 1970, Science Research Associates, Inc.

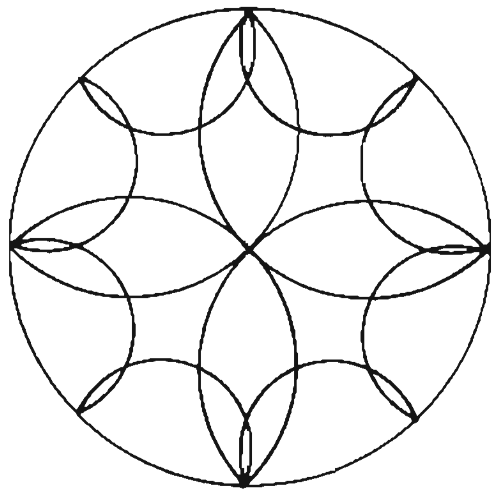
A SNAPPY SOLUTION TO SCALE DRAWINGS (CONTINUED)

1) How many rubber bands would you use to make a 3 to 1 enlargement? Could you make a 1 to 3 reduction?

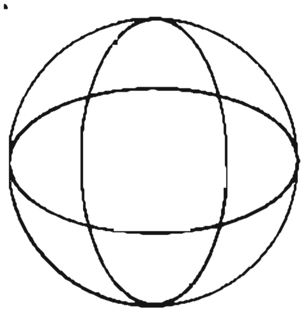




2) Designs with curved lines can be enlarged by watching just the knot and moving the pencil so the knot traces over the design.



3) Find a design that you like and make a 3 to 1 enlargement. A large, simple design is easier to enlarge.



4) Use an enlargement done by a classmate and make a reduction of the design. Compare your reduction to the original design.

5 rubber bands are needed. The second knot from X traces the original design.

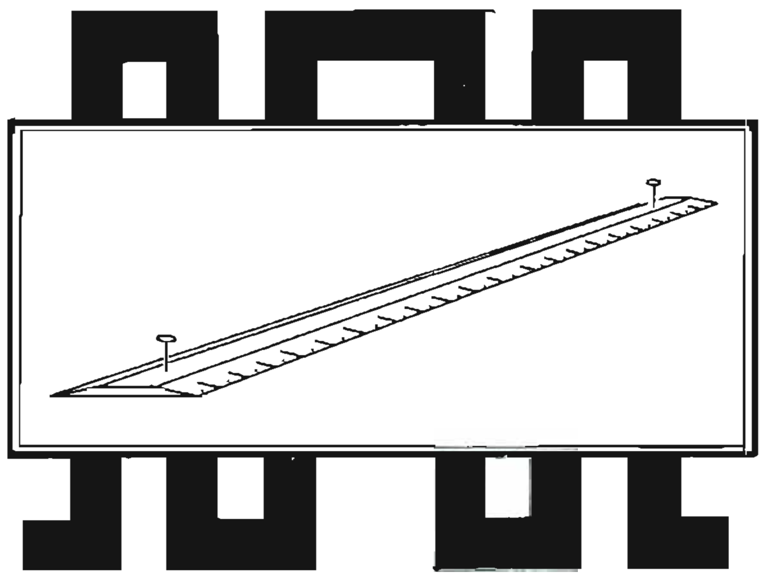
Challenge: Make a 5 to 2 enlargement of a design of your choice.

IDEA BASED ON: *Activities with Ratio and Proportion,* and *The Laboratory Approach to Mathematics* by Kenneth P. Kidd, Shirley S. Myers, and David Cilley, © 1970, Science Research Associates, Inc.

There are a number of ways to make a scale drawing of a field. Some methods use expensive pieces of equipment to do this accurately, but it is possible to make a good scale drawing using equipment from the classroom.

Equipment: Flat table or board placed on top of an inverted wastebasket
Ruler
Tape
*Alidade
Large sheet of drawing paper

*An alidade is a straightedge with sights and can be made with a ruler and two nails.



1. The students should familiarize themselves with the region before beginning the scale drawing. Landmarks, especially those that indicate the shade of the region, should be located. The landmarks could be listed or a rough sketch of the region drawn with each landmark labeled. Markers are needed at the corners of the field if natural landmarks do not occur.

2. Label two wooden stakes P and Q and place them ten metres apart in the middle of the field. Be careful that the stakes are not in line with any of the landmarks.

3. Tape the large sheet of paper to the table. Select a suitable scale so that the drawing will fit on the paper. Near the center of the paper, mark and label two points corresponding to the stakes in the field, i.e., if a scale of 1 cm : 1 m is chosen, draw the two points 10 cm apart.

IDEA FROM: *Making Mathematics, A Secondary Course,* Book 4
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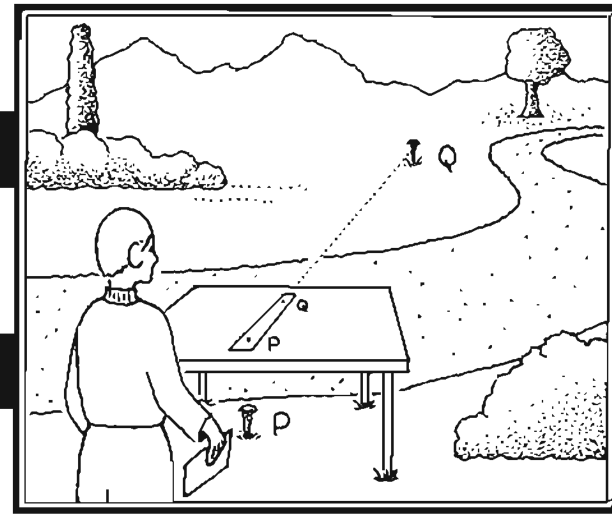
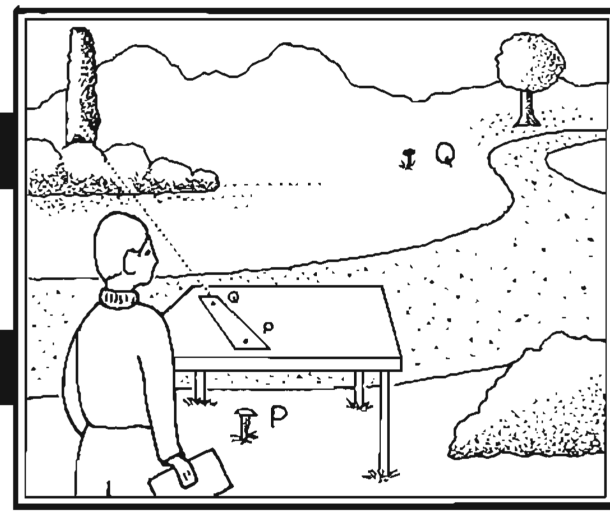
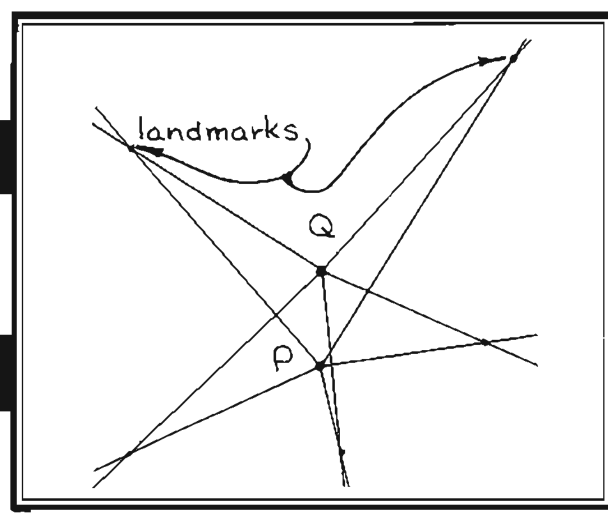


4. Place point P over stake P. Use the alidade to line up point Q on the paper with stake Q (you may have to turn the table slightly). The table must remain in this position as you sight each landmark from point P.



5. To sight a landmark from point P place one edge of the alidade against point P. Line up the landmark and draw a line to the edge of the paper. Repeat for each landmark.

6. To complete the activity move the table over stake Q. Line up point P with stake P. As above, use the alidade to sight each landmark from point Q. On the scale drawing each landmark is represented by the intersection of a line from P and a line from Q.



7. The field can now be represented by connecting the appropriate intersection points.
The students should write the scale at the bottom of the drawing. Students may wish to check the accuracy of the drawing by actually measuring the distances between landmarks.

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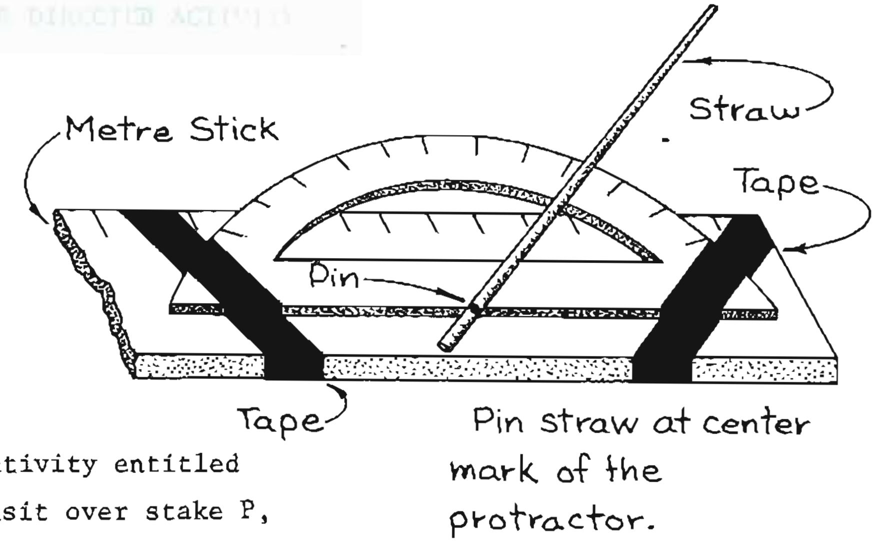
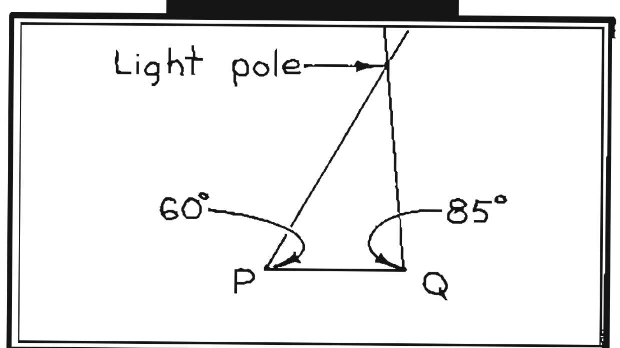
ANOTHER STAKE OUT



A homemade or commercial transit can be used to make a scale drawing of a field or playground.

Stakes P and Q should be positioned as they were in the activity entitled *Stake Your Claim*. Place the transit over stake P, record the transit readings for each landmark, and then repeat for stake Q. To make the commercial transit readings easier to interpret: place the transit over stake P, sight stake Q, and set the transit at 0° before sighting any landmarks. After moving the transit to stake Q, be sure to sight stake P and set the transit at 0°. A table will help students organize the results so that each landmark is paired with the appropriate transit reading.

Landmark	Reading at P	Reading at Q
light pole	60°	85°



In the classroom select a suitable scale. Use the scale to label two points, P and Q, i.e., if a scale of 1 cm : 1 m is chosen, P and Q are 10 cm apart. Connect P and Q with a line segment. The scale drawing can be completed by using the table of angle measurements, a protractor and a straightedge.

RATIO, PROPORTION, AND SCALING

Placement Guide for Tabbed Divisors

CONTENTS

SCALING: SUPPLEMENTARY IDEAS IN SCALING

TITLE	PAGE	TOPIC	TYPE
A Scale Model of the Solar System	348	Making a scale model	Activity
How High the Moon	349	Making a scale model	Activity

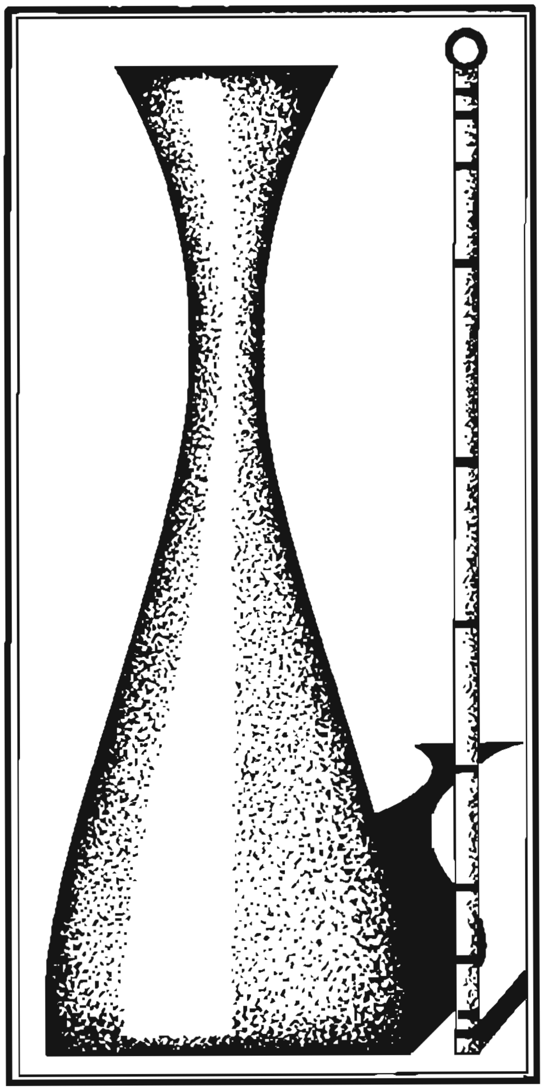
MAKE A DIP STICK

This activity calibrates a dowel which can then be used as a dipstick to check the level of fluid in a container.

Equipment: Eight to ten containers approximately the same height but having different shapes, e.g., detergent bottle, starch bottle, pop bottle, catsup bottle, milk carton, vase, bubble bath containers

Eight to ten thin wooden dowels

Eight to ten graduated cylinders that measure in ml (medicine cups from a hospital work nicely)



1. (a) Use an irregularly shaped bottle for a classroom demonstration. Let the students make conjectures about where the marks will appear. Pour 50 ml of water into the bottle. Carefully lower a thin dowel into the bottle until it touches the bottom. Lift the dowel out and mark the water level. Repeat the procedure until the bottle is full.
   (b) The dowel is now calibrated to measure fluid levels in the bottle to the nearest 50 ml. The dipstick represents a scale for the bottle just as a legend represents a scale for a map.
   (c) Discuss how the spacing of the marks is related to the shape of the bottle.
2. (a) Divide the class into groups. Give each group a bottle and have them make a dipstick for their container.
   (b) Collect the bottles and the dipsticks. Have each group try to match the dipsticks with the appropriate containers.
3. (a) Ask students if they know of any uses for dipsticks.
   (b) Suggest that each student check the oil and/or transmission fluid level in the family car.
   (c) How does the gas station operator measure the fuel in the station's tanks? Suggest that each student check at their neighborhood station. Perhaps the attendant will demonstrate the use of the dipstick.

Supplementary Ideas in Scaling
SCALING

E
S
I

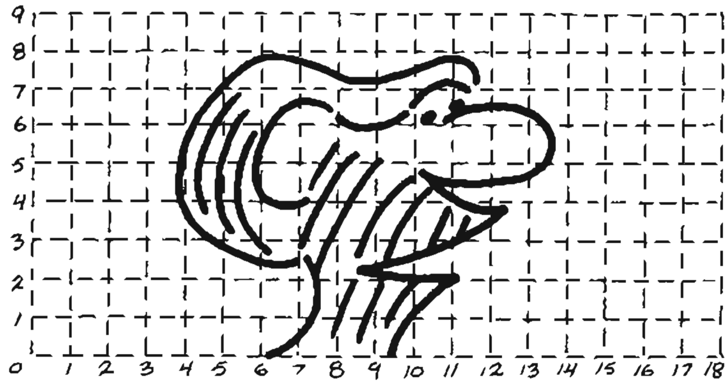
WRAP-A-ROUNDS OR Graphing for a Cylindrical "Mirror"

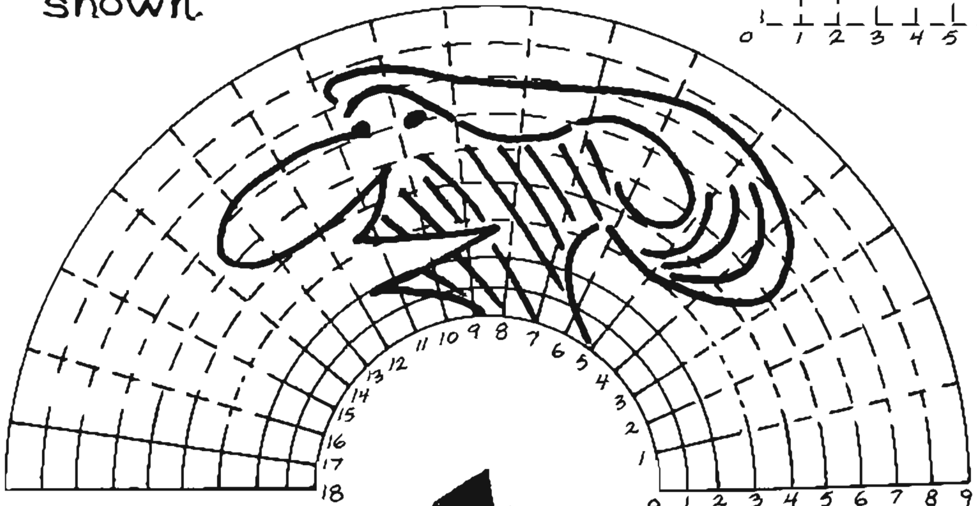
Materials:

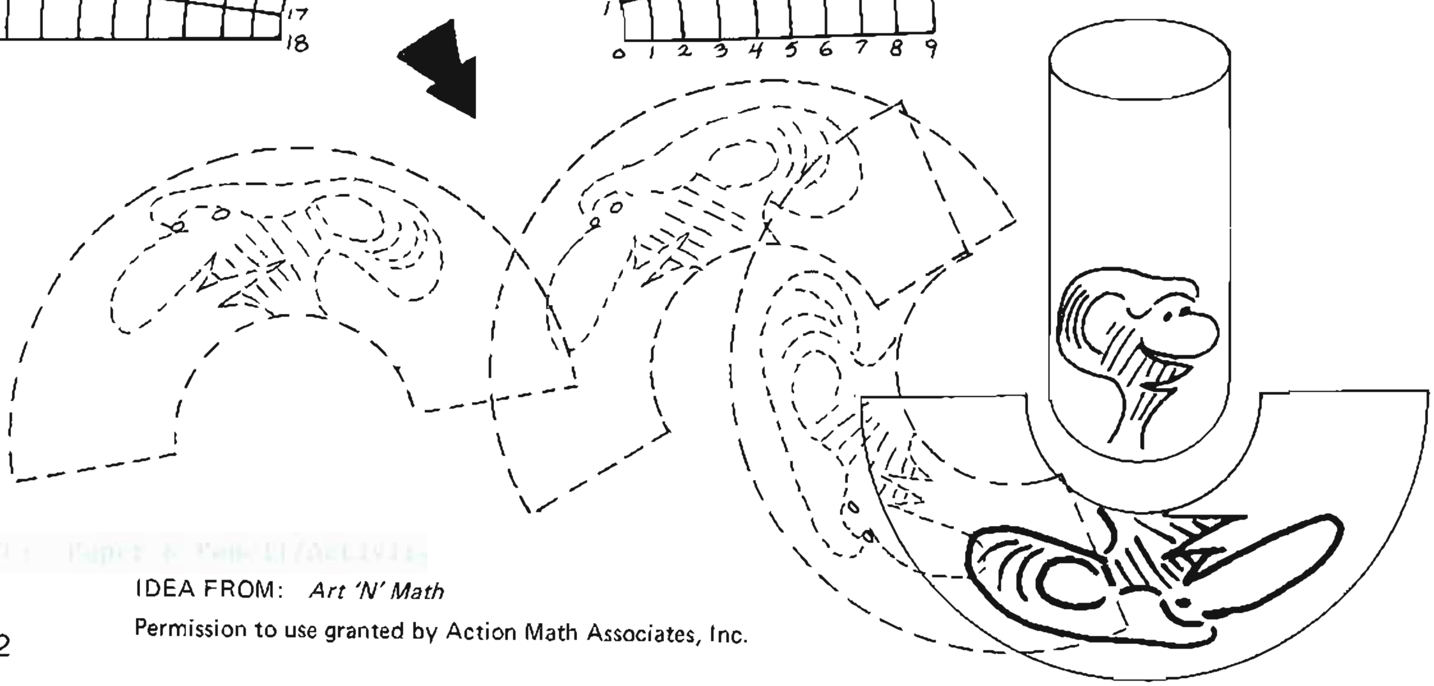
1. Several pieces of chrome tubing (15 cm high; 2–4 cm in diameter) from a local hardware store. (Tin cans also work).
2. Cylindrical graph paper

A. Center picture on 9×18 grid of squares.

B. The drawing done on the cylindrical graph paper is a reversed, distorted image of the original drawing. Number the coordinate axis' on the cylindrical graph paper as shown.







IDEA FROM: *Art 'N' Math*

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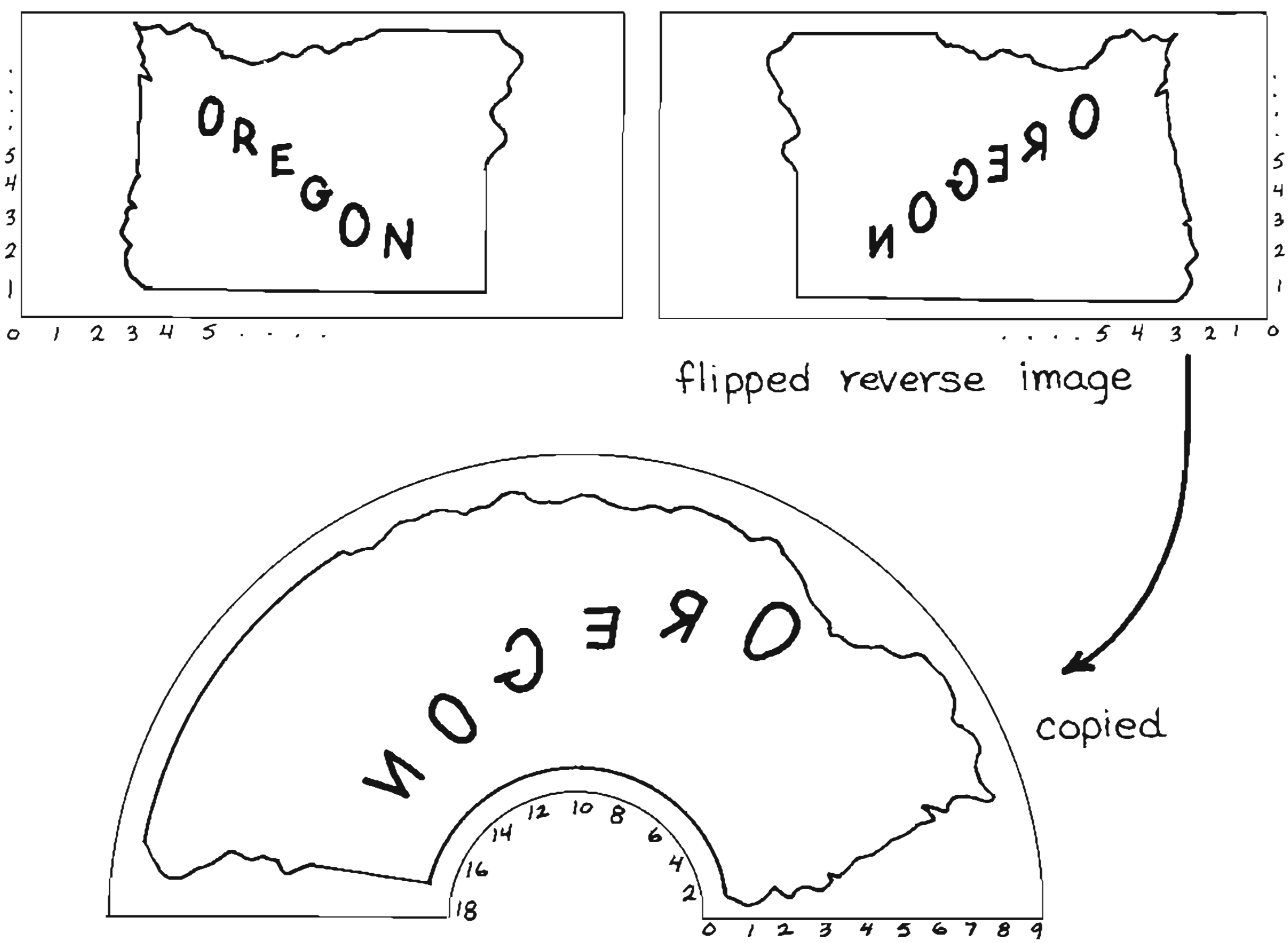
WRAP-A-ROUNDS
(PAGE 2)

Here is another example.

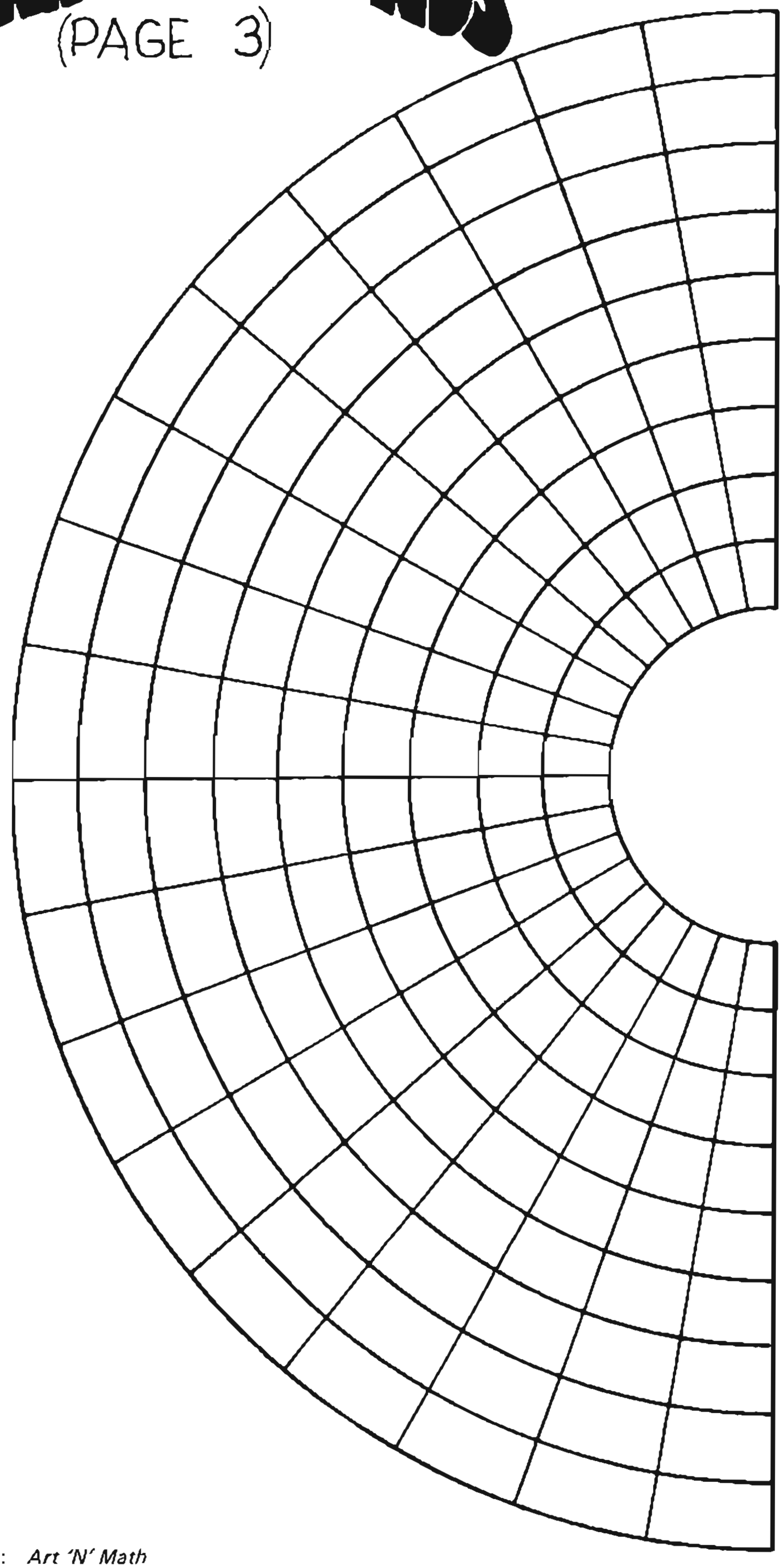
The original drawing can be drawn on tracing paper, flipped over for the reverse image, and copied onto the cylindrical graph paper.



IDEA FROM: *Art 'N' Math*

WRAP-A-ROUNDS
(PAGE 3)

. . . . CYLINDER GRAPH PAPER . . . .



IDEA FROM: *Art 'N' Math*

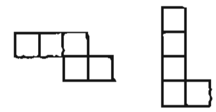
THE PERPLEXING PENTOMINOES

Materials needed: Five squares, 3 centimetres on a side, and centimetre grid paper or five 1-inch tiles and inch grid paper.

The grids should be duplicated on heavy construction paper

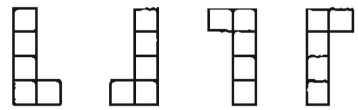
Activity:

1) A pentomino is a pattern made by joining 5 squares together so that each shares a common side with another. How many different pentominoes do you think there are? ____________



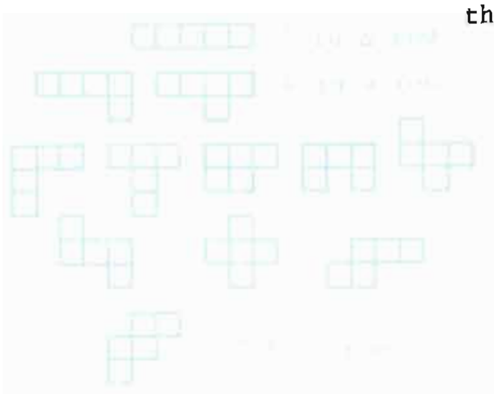
EXAMPLES

2) Take the 5 squares and make all the pentominoes that you can. Copy each pentomino pattern on the grid paper and cut out the shape. If one of the patterns can be turned or flipped to exactly fit another one, the two patterns are the same pentomino.



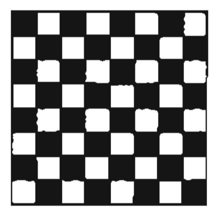
These four patterns are the same pentomino.

3) Check with your teacher to see if you have found all the pentominoes.



4) Try to arrange the pentominoes so that they make the rectangle. Do not overlap the pieces. There are more than 2000 ways to do this!

5) Play a game using the pentominoes.



Needed: 2 players
Game mat is an 8 by 8 square constructed out of the grid paper with alternate squares shaded.

a) Players alternate picking pentomino pieces until all the pieces have been selected.

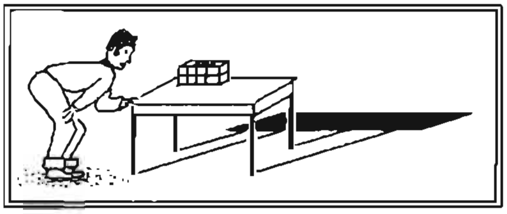
b) Each player in turn then places a pentomino on the mat. Play continues until it is impossible for a player to place on that mat a pentomino that doesn't overlap another pentomino or lie completely on the mat.

c) The winner is the last person to successfully place a pentomino on the mat.

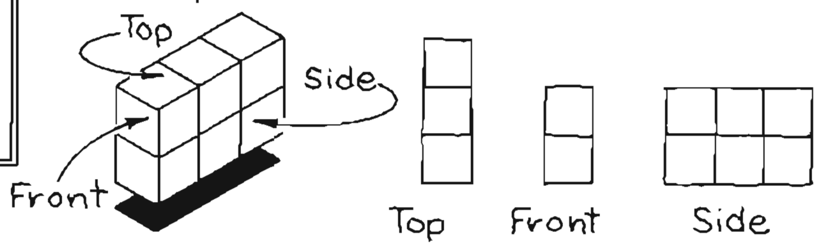
HOW WELL DO YOU STACK UP?

Materials needed: A set of cubes

Activity: Make each of these models with cubes. On your paper draw a sketch of each model that shows the top, front, and side views.



Example:



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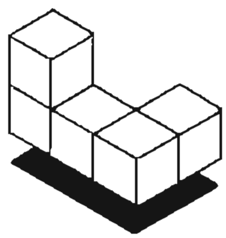

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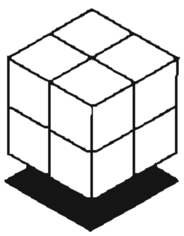

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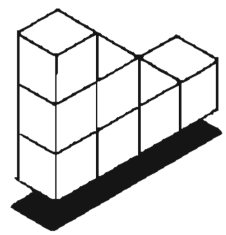

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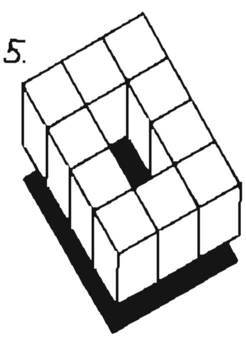

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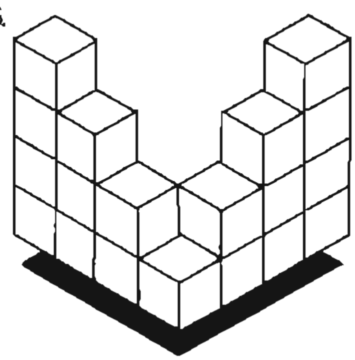

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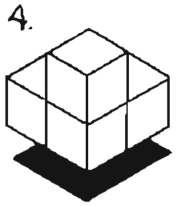

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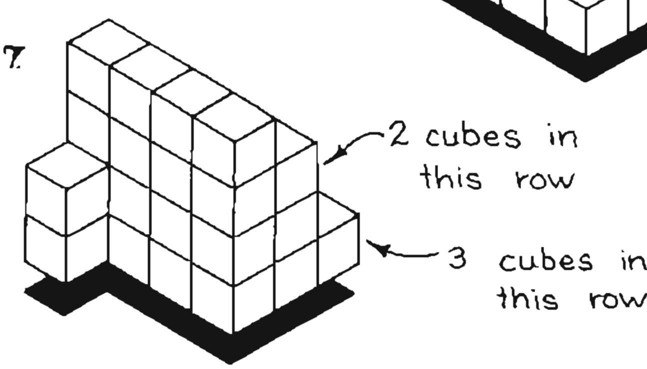
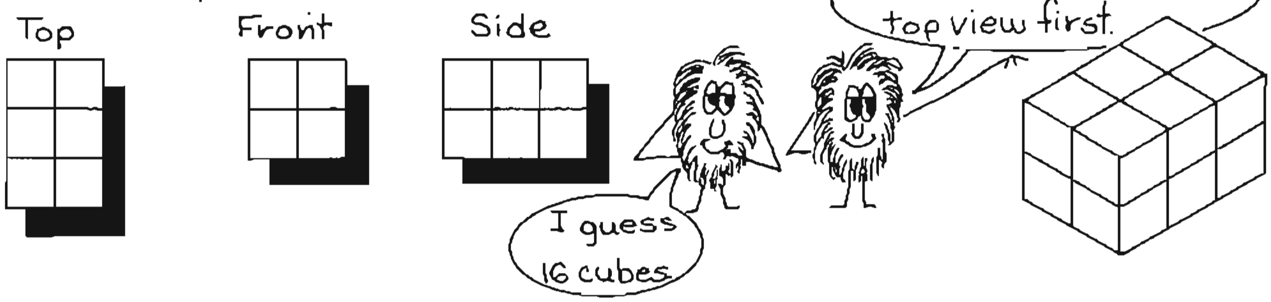
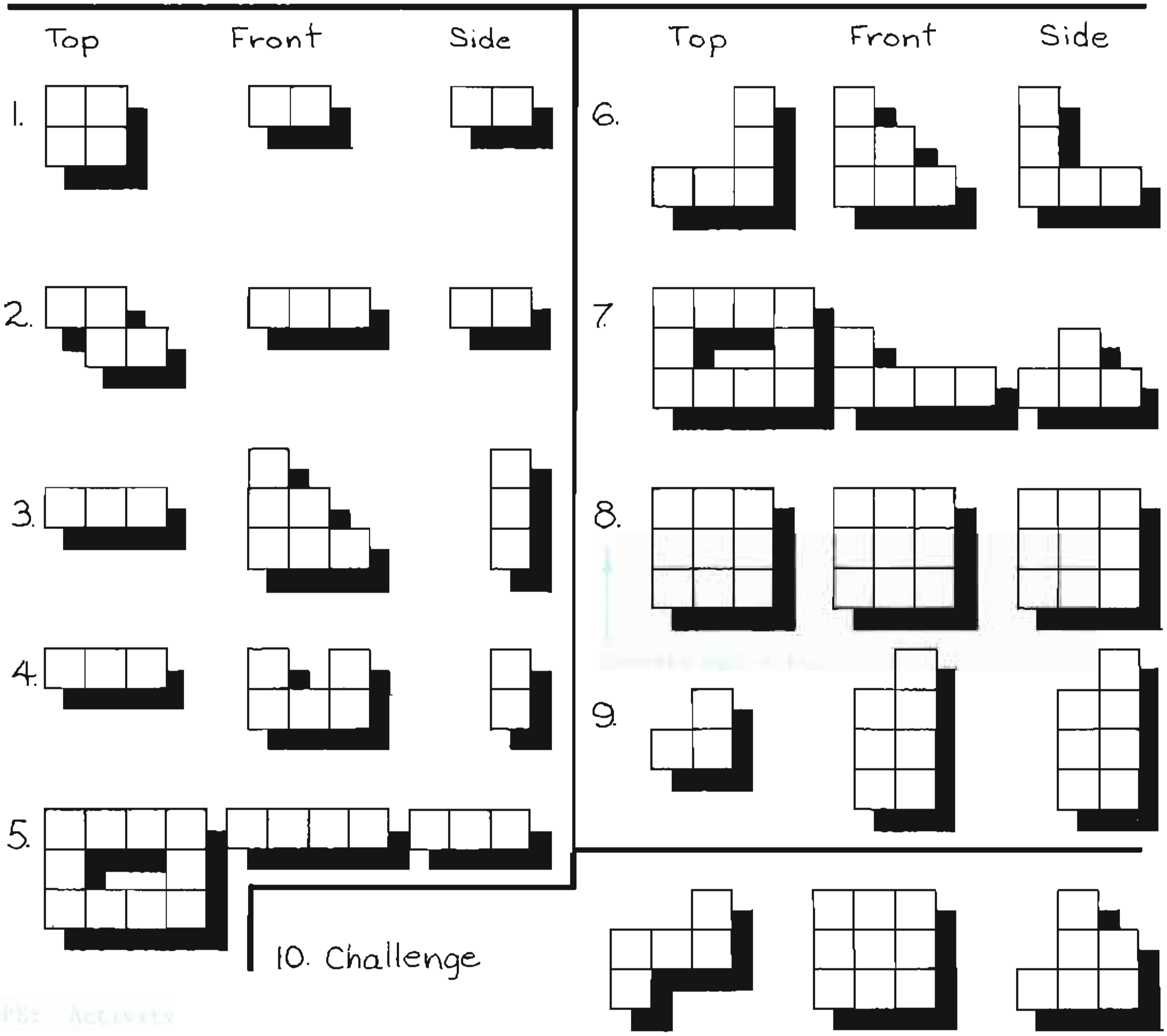


HOW WELL DO YOU STACK UP THIS TIME?

Materials needed: A set of cubes
Activity: Use the three views. First, estimate the number of cubes needed and then build the model.

Example:





3 FACES YOU SEE

Supplementary Ideas
In Scaling
SCALING

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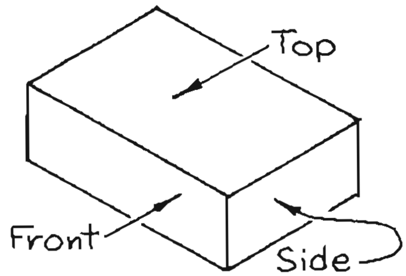
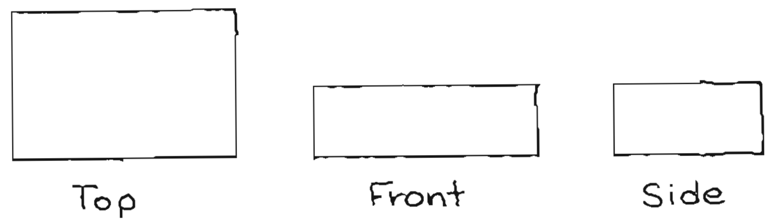
These sketches show the outlines of this block.

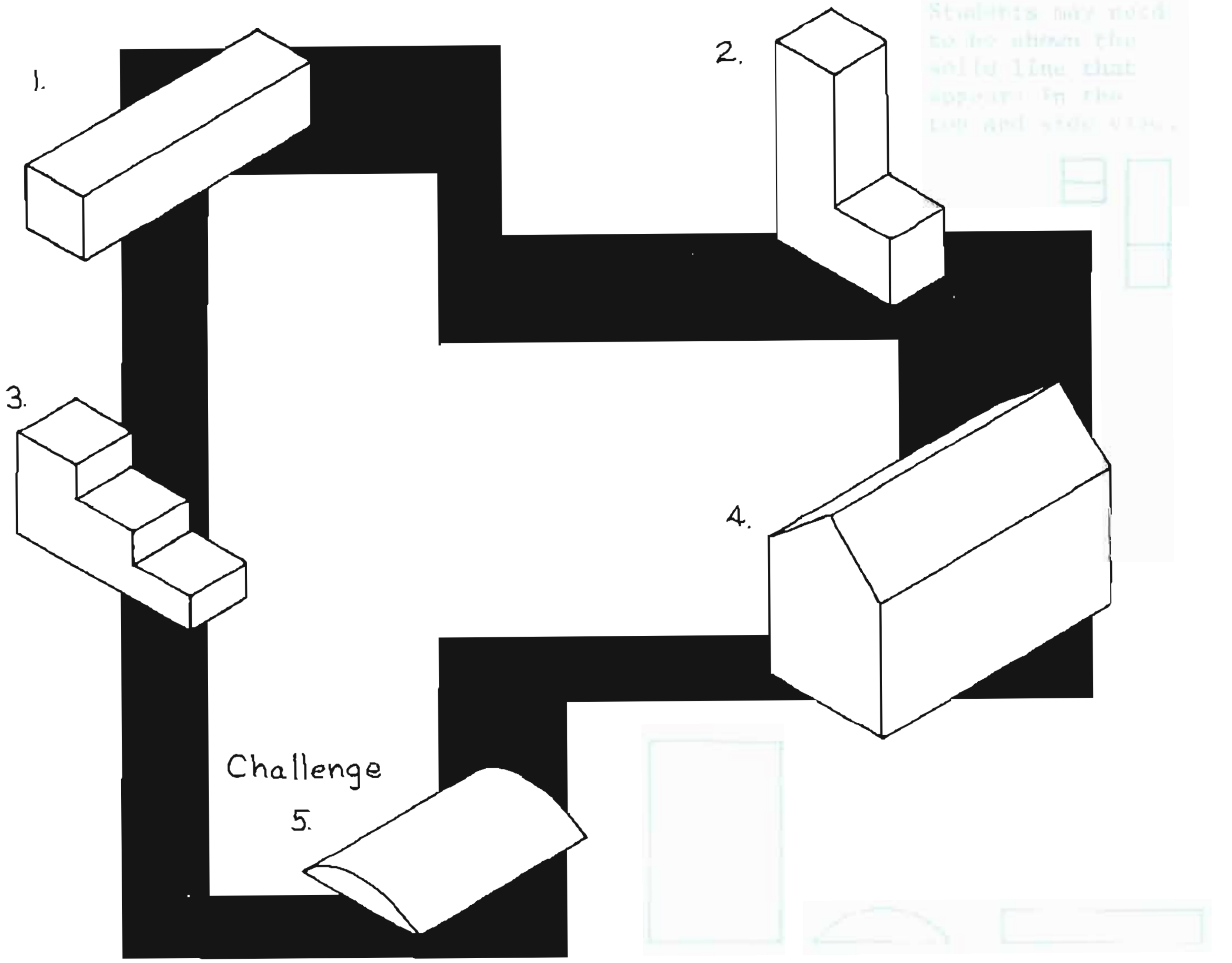




These drawings are only rough sketches and are not drawn to scale.

On another piece of paper sketch the top, front, and side of these blocks like the example.

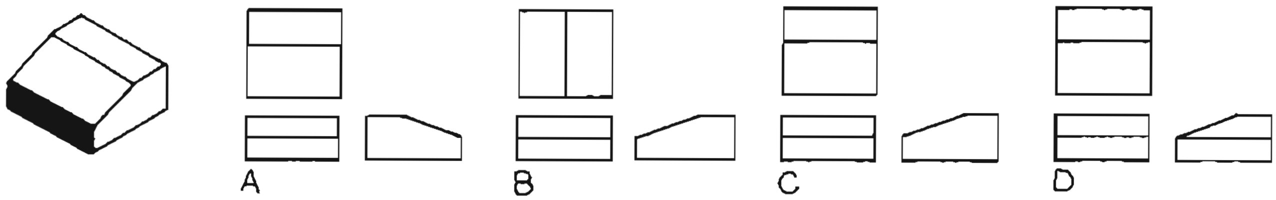
Students may need to be shown the solid line that appears in the top and side view.

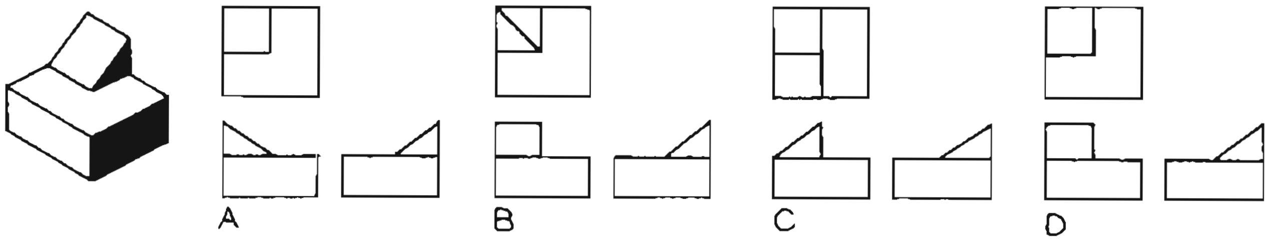


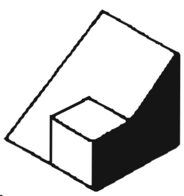
TYPE: Paper & Pencil

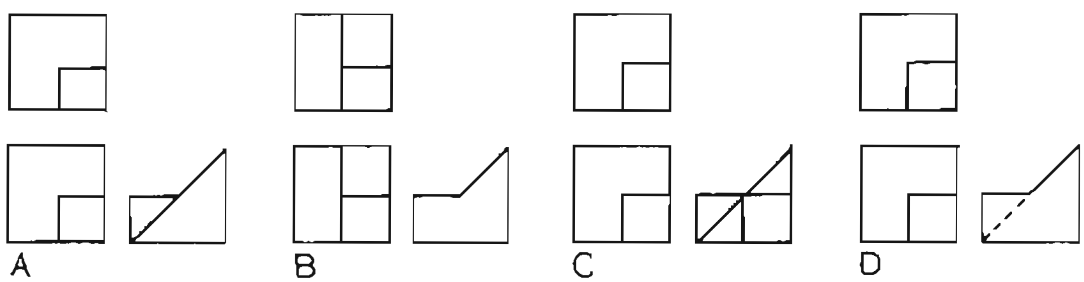
3 FACES YOU SHOULD HAVE SEEN

Circle the letter that shows the correct top, front, and side views.

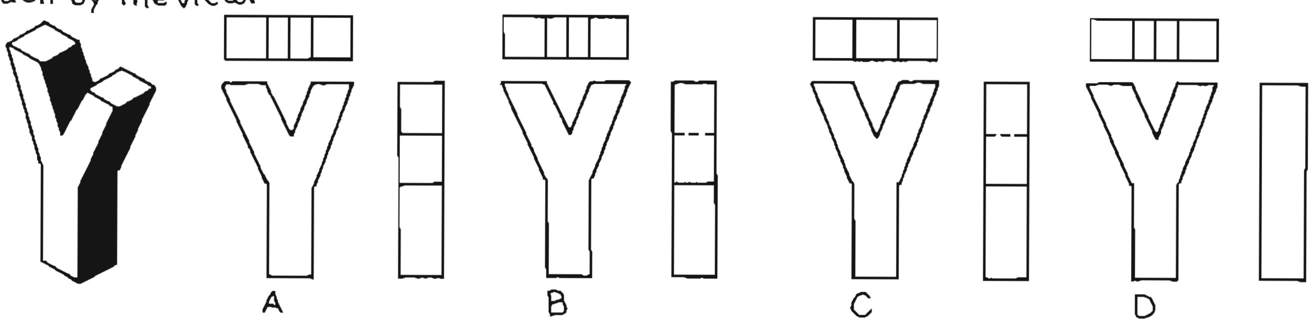


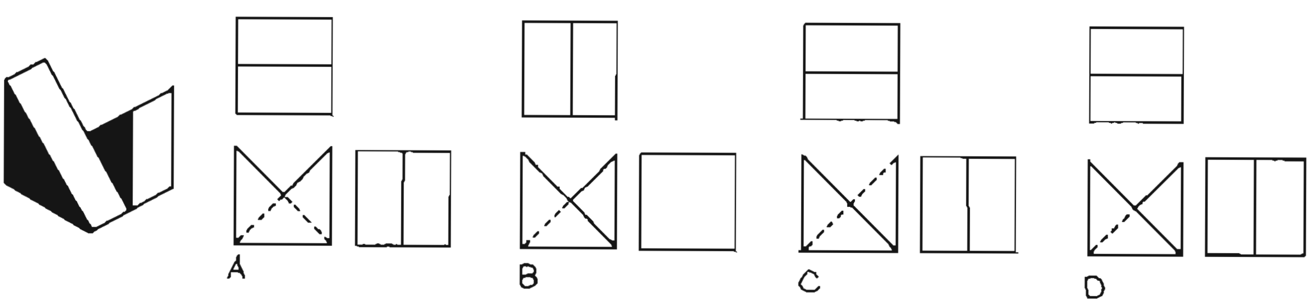






Dotted lines stand for edges hidden by the view.





IDEA FROM: *Seeing Shapes*

3 FACES YOU SAW

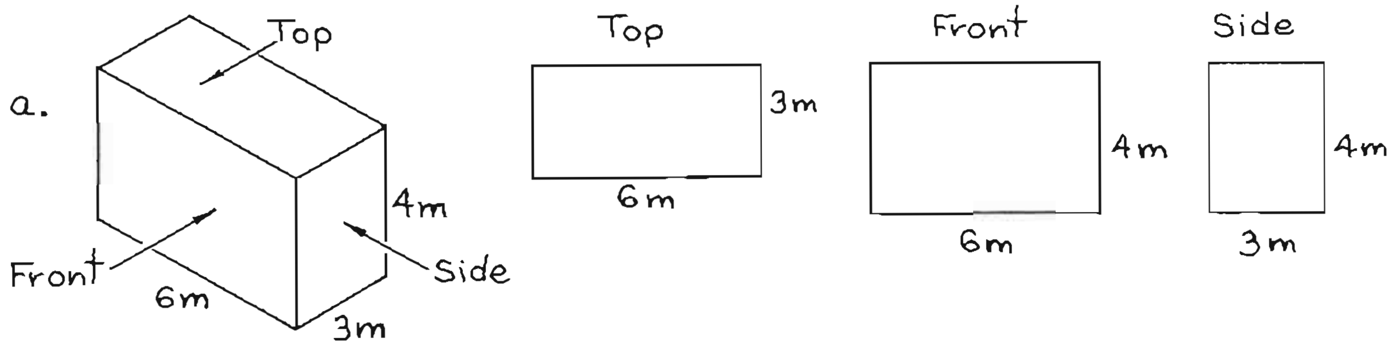
Supplementary Ideas in Scaling
SCALING

Materials needed: Metric ruler

Activity: Make a scale drawing of the top, front, and side of each model. Use a scale of $\frac{1}{2}$ cm : 1 m.

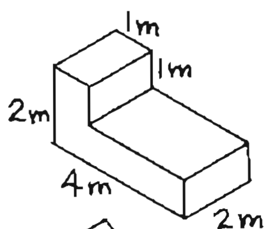
Example:

a.

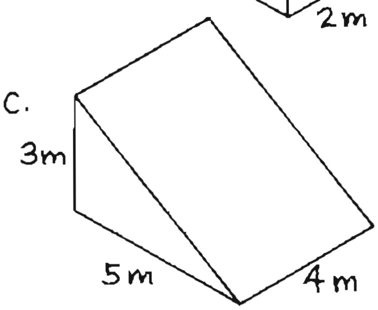


The scale drawings could be done on $\frac{1}{2}$-cm grid paper or on this page using the metric ruler.

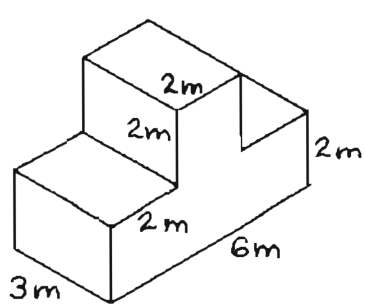
b.



c.



d.



? Estimate the number of 1-metre cubes needed to construct each model. Check your estimate by building each model with cubes.

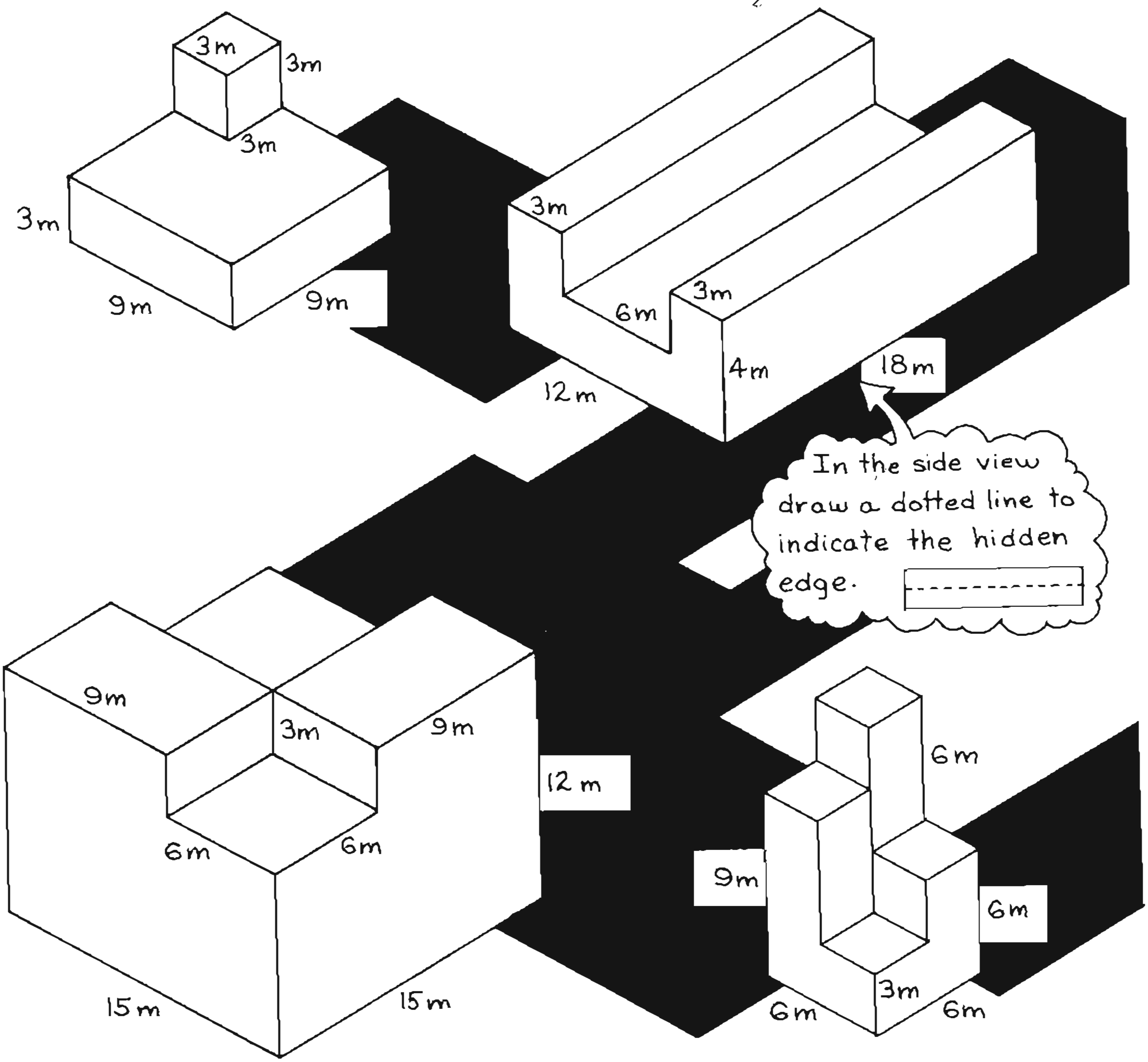
IDEA FROM: *Seeing Shapes*

Exercise c is half of a 5 x 4 x 3 rectangular solid.

3 FACES YOU HAVE SEEN

Materials needed: Metric ruler

Activity: On another piece of paper make a scale drawing of the top, front and side of each model. Use a scale of $\frac{1}{2}$ cm : 1 m.

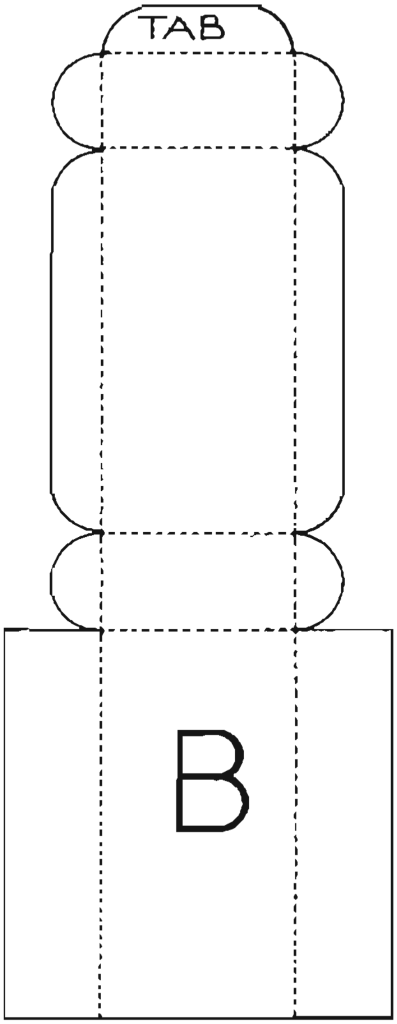


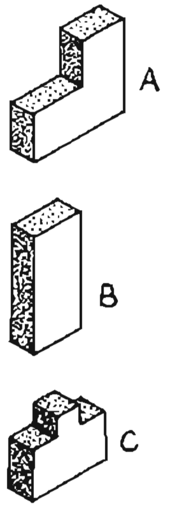
Estimate the number of 1 metre cubes needed to construct each model. Check your estimate by building each model.

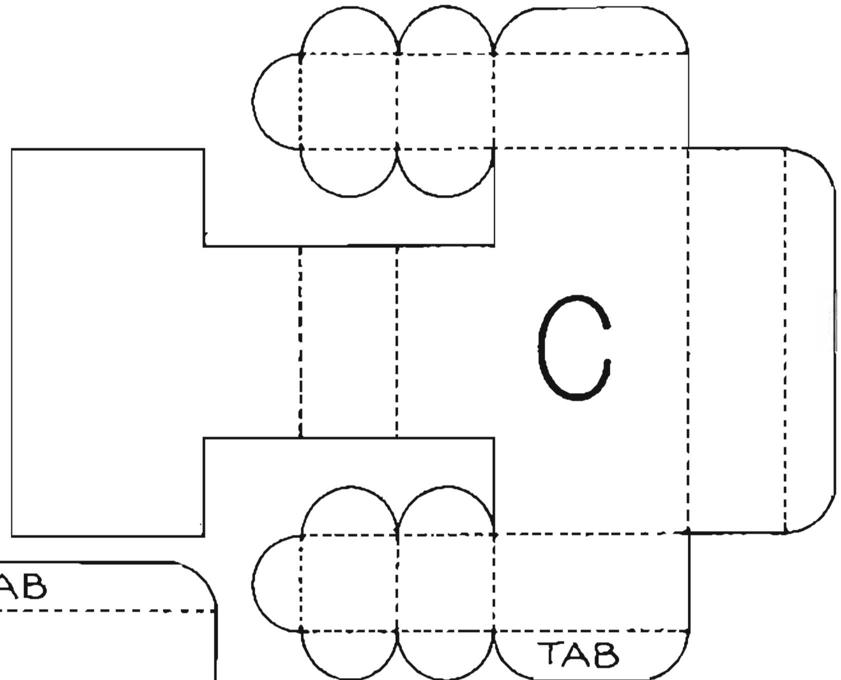
TYPE: Paper & Pencil/Activity

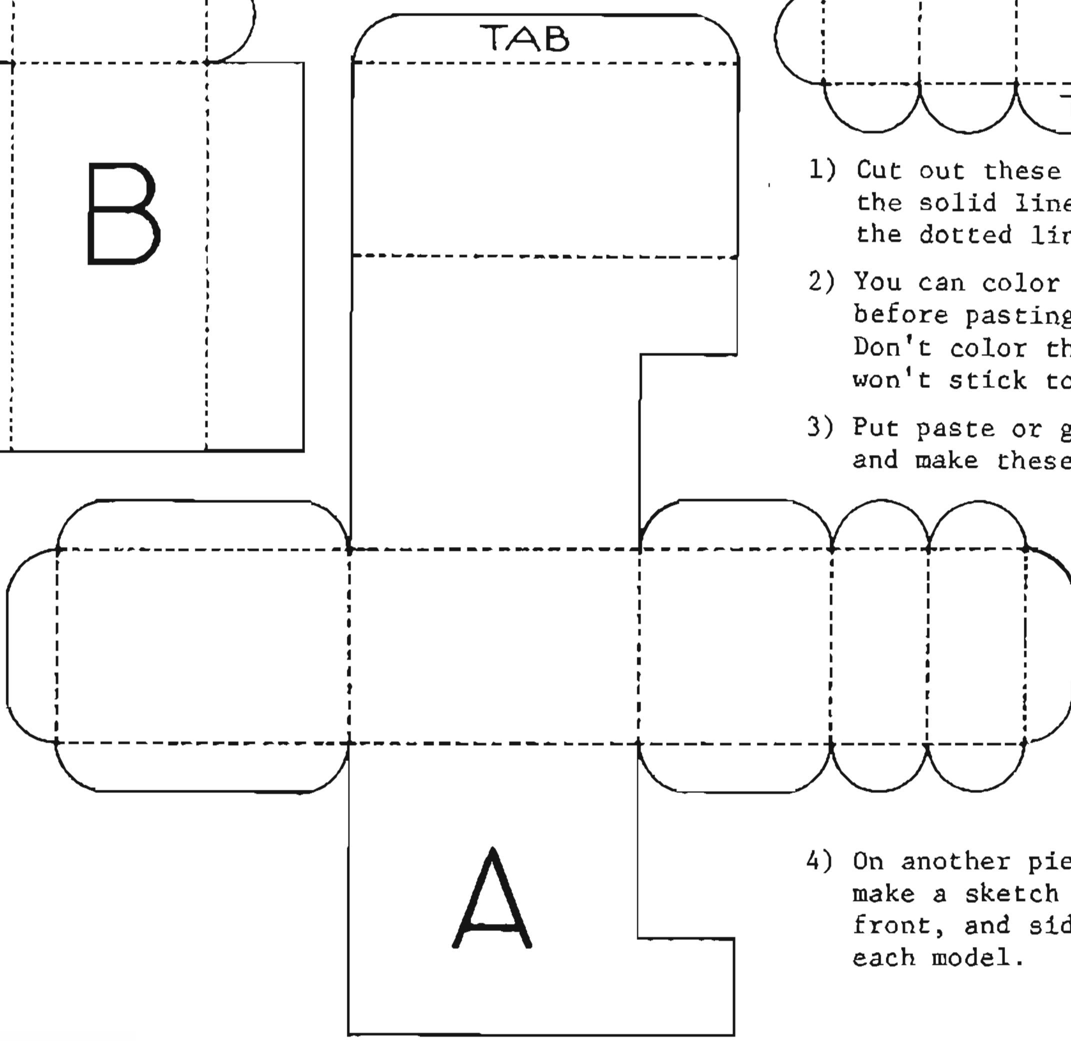
IDEA FROM: *Seeing Shapes*

CAREFULLY CONSTRUCTED CARTONS









1) Cut out these patterns along the solid lines and fold along the dotted lines.

2) You can color the pattern before pasting it together. Don't color the tabs. They won't stick together.

3) Put paste or glue on the tabs and make these models.

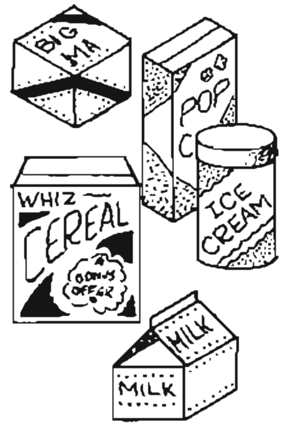
4) On another piece of paper make a sketch of the top, front, and side views of each model.

IDEA FROM: *Open-Ended Task Cards*

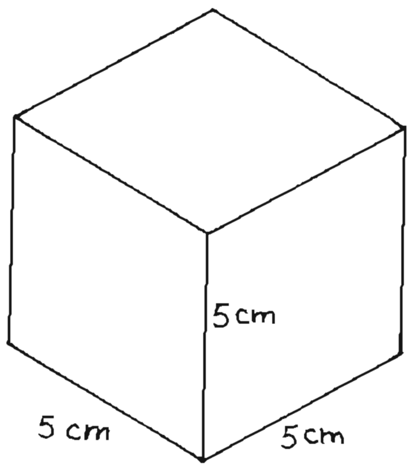
CAREFULLY CONSTRUCTED CARTONS (CONTINUED)

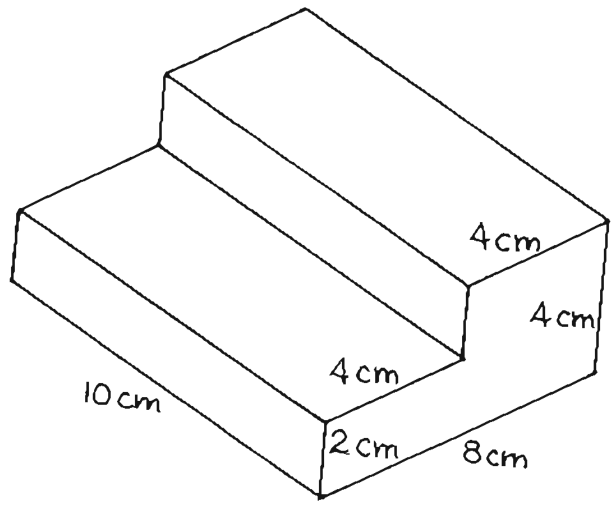
TEACHER PAGE

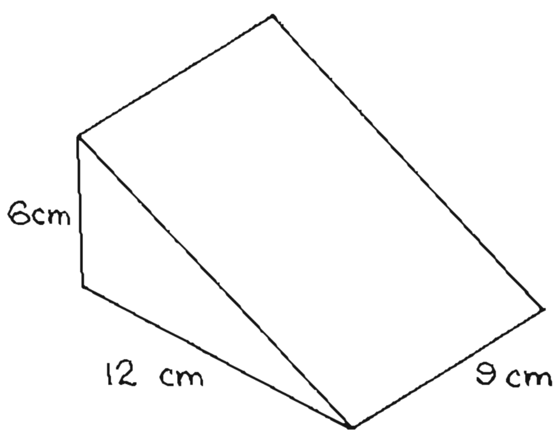
1) Bring several cardboard containers for the students to take apart to see the patterns used to construct the container. Some suggested containers are shown on the right. Students could pick a pattern and use butcher paper to find an arrangement of the pattern that minimizes wasted space.

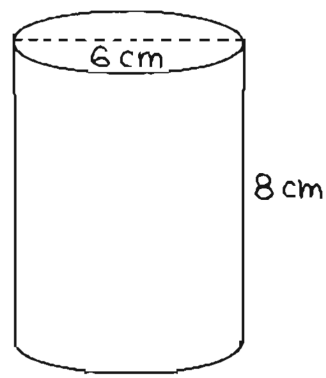


2) Have students draw the pattern for each figure below. The patterns could be checked by cutting them out and folding them back together.





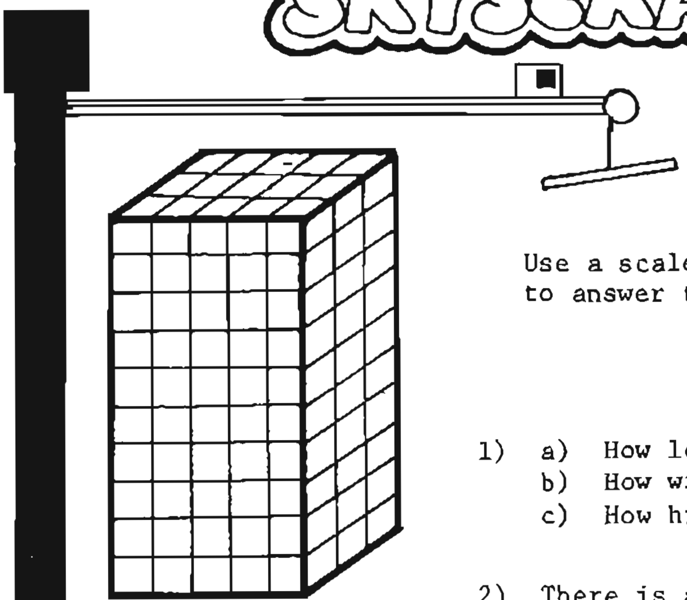




IDEA FROM: *Open-Ended Task Cards*

Permission to use granted by Teachers Exchange of San Francisco

SCALING A SKYSCRAPER



Use a scale of the edge of a cube : 10 feet to answer these questions about the skyscraper.

1) a) How long is the building along the front? _____
   b) How wide along the side? _____
   c) How high? _____

2) There is a broken window on the front of the building, 50 feet up from the bottom and 20 feet from the left side. Put an x on the broken window.

3) A window washer is working on the right side of the building, 10 feet from the back and 30 feet from the top. Put a small * where he is working.

4) The flag pole carrying the company flag is in the middle of the front of the building, 35 feet from the sidewalk. Put a ● where the flagpole is.

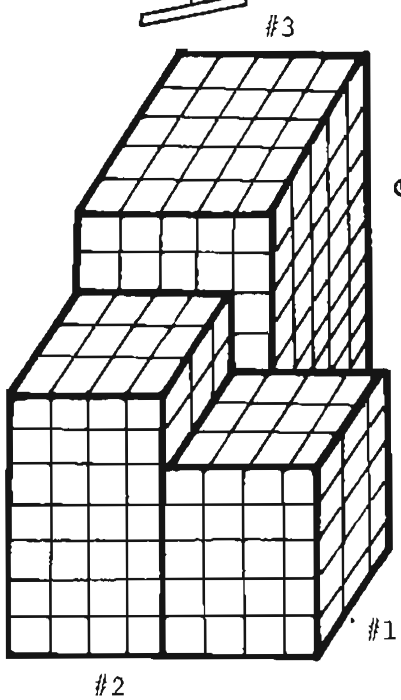
5) At night the company's neon sign is turned on. It is a sign 20 feet long and 5 feet high. The upper left hand corner of the sign is 15 feet from the top and 15 feet from the left side. Draw a rectangle in the position of the sign.

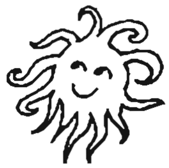
6) The building has a ventilating unit on the roof. If the unit is 15 ft. from the front and 15 ft. from a side, put a V on all places where the unit could be located.

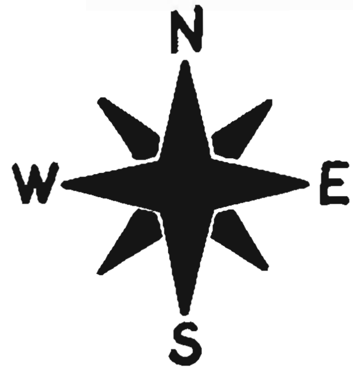
7) Lobbies, hallways, restrooms, storage areas, etc. take up 1/3 of the skyscraper. If an office is 10' by 10' by 10', how many offices are in the skyscraper? _______________

8) How much office rent is collected each month if the offices on floors 1, 2, 3 rent for $150 per month, floors 4, 5, 6, 7 rent for $175 per month and floors 8, 9, 10 rent for $200 per month?

SCALING SEVERAL SKYSCRAPERS







Use a scale of the edge of a cube : 5 metres to answer these questions.

		#1	#2	#3
1)	How long is each building (front)?	____	____	____
	How wide is each building (side)?	____	____	____
	How tall is each building?	____	____	____

2) A window washer is working on the front of building #2, 20 metres from the top and 10 metres from building #1. Put an x to show the window washer.

3) Another window washer is on the side of #3, 20 metres from the sidewalk and 17.5 metres from building #1. Put an x to show him.

4) Lobbies, hallways, restrooms, etc. take up $\frac{1}{3}$ of each building. If each cube represents one office, and each office has one parking space, how many spaces are needed for

building #1 ______
building #2 ______
building #3 ______

5) Mr. Jones has an office on the 3rd floor of building #2. If his office is 15 metres from building #3 and 10 metres from building #1, put a J on Mr. Jones office.

6) Mr. Smith has an office 15 metres higher, 10 metres to the right of, and 30 metres behind Mr. Jones office. Put an S on Mr. Smith's office.

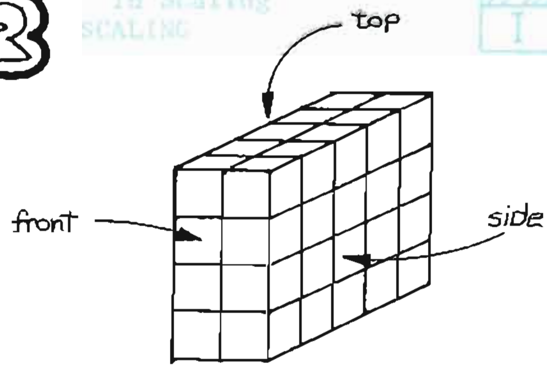
7) There is just one elevator for all 3 buildings. Shade the place that would be the most convenient place for the elevator to be located.

8) Which buildings could you see if you were standing far away with your back to the:

a) South ________
b) West ________
c) North ________
d) East ________

BUILDING A SKYSCRAPER

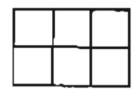
Materials: A set of 100 centimetre cubes.



Activity:

1) a) Use a scale of <u>the edge of a cube</u> : <u>20 metres</u>. Make a model of a building 60 metres long (front), 40 metres wide (side), and 100 metres high.

b) Does this sketch show the top of your model? _____



c) How many cubes are in your model? __________

d) How many cubes would be in the model if you used this scale, <u>the edge of a cube</u> : <u>10 metres</u>? __________

2) a) Use a scale of <u>the edge of a cube</u> : <u>5 metres</u>.
Make a model: 20 m long (front), 10 m wide (side), 50 m high.

b) Draw a sketch of the front of your model.

c) If this scale was changed to <u>the edge of a cube</u> : <u>2 metres</u>, how many cubes would be needed? __________

3) a) Use a scale of <u>the edge of a cube</u> : <u>10 metres</u>.
Make a model: 20 metres long (front), 40 metres wide (side), 80 metres high.

b) Draw a sketch of the side of your model.

c) How many cubes are in your model? __________

d) If you changed the scale to <u>the edge of a cube</u> : <u>5 metres</u>, how many cubes would be needed? __________

4) a) You choose a scale to make this model.
30 m long (front), 30 m wide (side), 30 m high and a tower on top 10 m wide (front), 20 m wide (side), 30 m high.
Scale ______________________________

b) Draw a sketch of the front, the side, and the top of your model.

c) How many cubes are in your model? __________

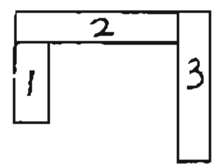
d) Compare the scale you chose to the scale chosen by a friend. If different, how does the number of cubes needed to make the model compare? ______________________________

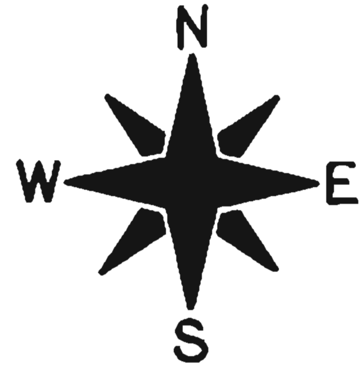
BUILDING SEVERAL SKYSCRAPERS

Materials Needed: A set of cubes, a metric ruler.

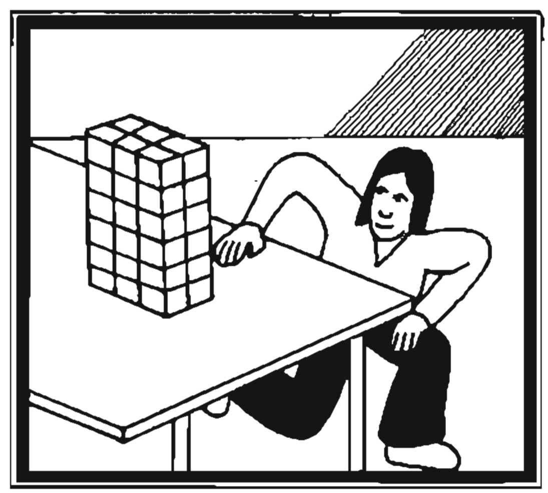
Activity: Use the cubes to make models of the buildings below using a scale of the edge of a cube : 50 m. Fit the buildings together like this sketch.

TOP VIEW





	Length (front)	Width (side)	Height
Skyscraper 1	50 m	50 m	150 m
Skyscraper 2	400 m	50 m	200 m
Skyscraper 3	50 m	250 m	300 m



a) Draw a <u>sketch</u> of what you would see if you were far away with your back to the East.

b) Draw a <u>sketch</u> of the view with your back to the South.

c) Make a <u>scale drawing</u> of the East view. Use a scale of <u>the edge of a cube</u> : <u>5 cm</u>.

d) Make a <u>scale drawing</u> of the South view. Use a scale of <u>the edge of a cube</u> : <u>1 cm</u>.

Challenge:

Make your own skyscrapers, decide on a scale, and make a sheet like this for a classmate to do.

A Scale Model Of The Solar System

Materials needed:

Basketball, grain of sand, several peas, large straight pin, orange, peach, plum (or objects similar in size), metric tape measure.

Activity:

(1) Look up the actual sizes and distances of the planets from the sun.

(2) Take your class outside. Have one student stand at home plate of a ball field (or goal line of a football field) holding the basketball to represent the sun.

(3) Have the students estimate the positions and sizes of the planets.

(4) Place students holding the objects at the appropriate distances (until space runs out).

(5) Refer to other distances as homes where students in your class live, i.e. Uranus would be the size of a small plum located at Nancy's house.

(6) Some student(s) may wish to find the scale used for this activity by using the actual distances of the planets from the sun. The scale is about 1 m : 2,400,000,000 m or 1 m : 2,400,000 kilometres.

(7) This would be a good activity to be done in cooperation with the science teacher during the study of the solar system.

If the sun is the size of a basketball,

Mercury is the size of a grain of sand 25 metres away.

Venus is the size of a pea 43 metres away.

Earth is the size of a pea 65 metres away.

Mars is the size of a large pinhead 99 metres away.

The asteroids are specks of dust averaging 366 metres away.

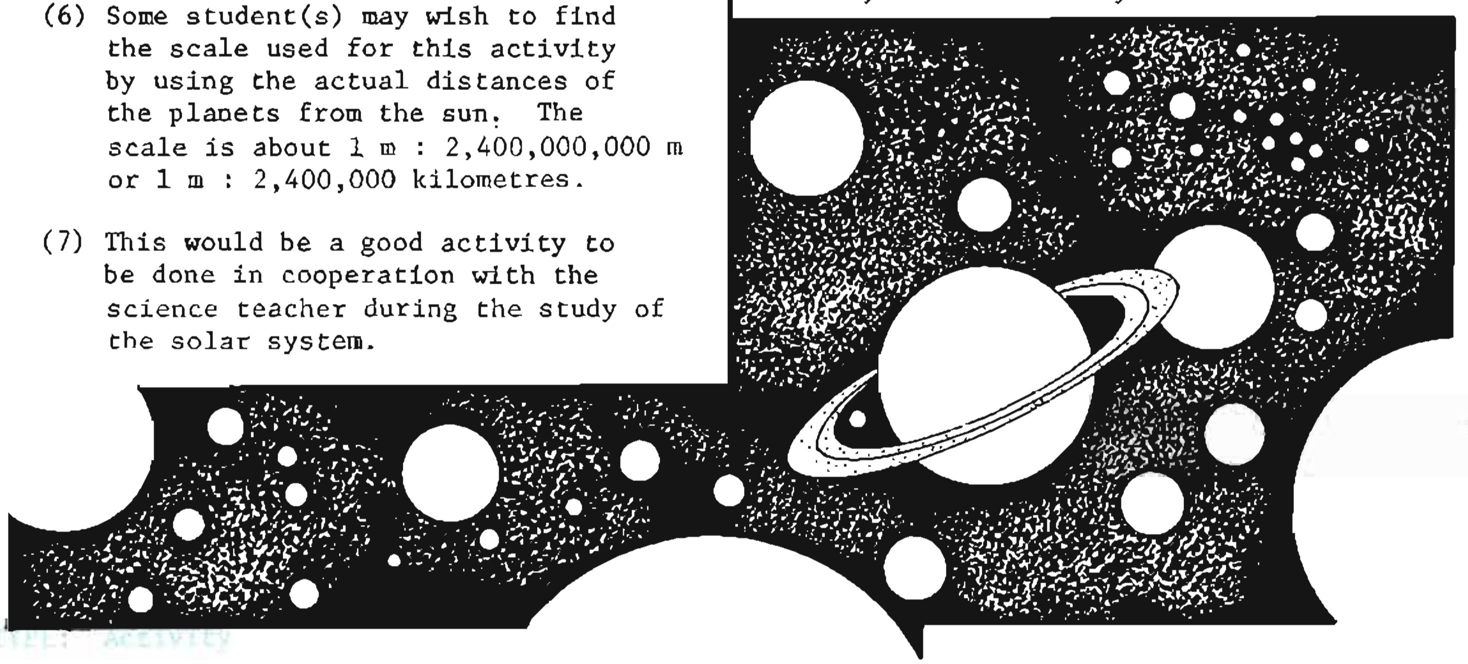
Jupiter is the size of an orange 402 metres away.

Saturn is the size of a peach 644 metres away.

Uranus is the size of a small plum 1 kilometre, 207 metres away.

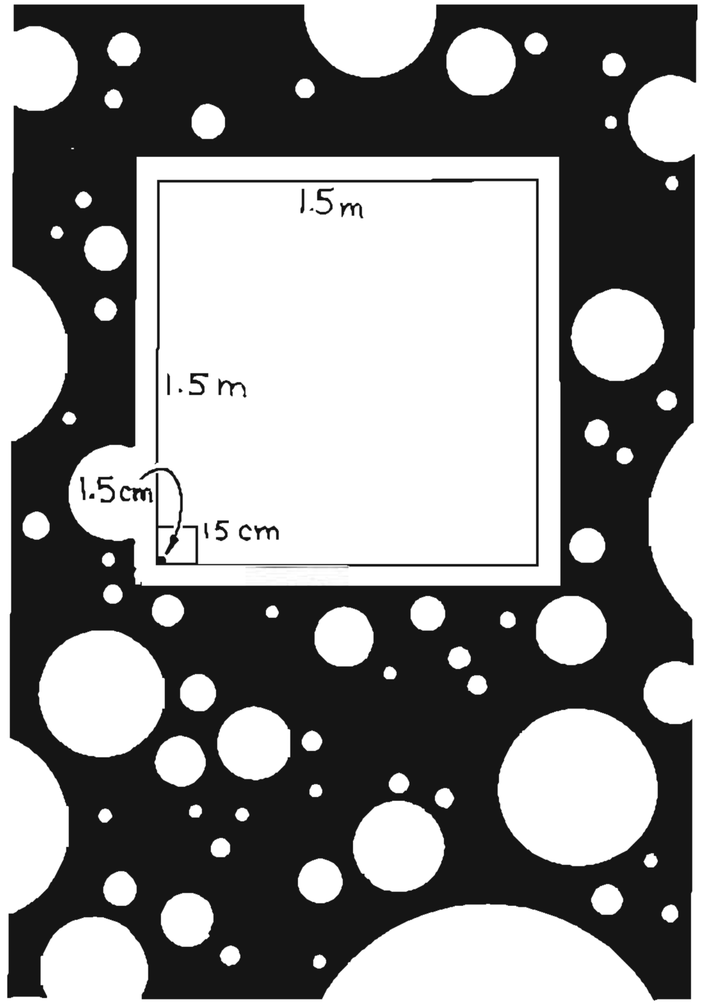
Neptune is the size of a smaller plum 2 kilometres away.

Pluto is the size of a pea 2 kilometres, 414 metres away.



TYPE: Activity

TEACHER DIRECTED ACTIVITY



One of two movies, Powers of Ten or Cosmic Zoom, or the book, Cosmic View by Kees Boeke, can be used to emphasize the immense size of the solar system and the universe. If the book is used, the concept can be made more relevant by having students construct a square 1.5 metres on a side. In one corner draw a series of squares 15 cm, 1.5 cm and .15 cm on a side. These sides will show four successive powers of ten. The measurement of 15 cm is being used because it corresponds to measures used in the book.

Outside have students measure off a 15 metre square and place the 1.5 m square in one corner. If the school ground is large enough, measure off a 150-metre square.

Then, on a city map a 1500-metre square can be drawn with the school in one corner. By relating the series of squares to the pictures in the book numbered -2 through 4, students might get a "sense of scale." The .15 cm square will be similar to the picture numbered -2, and the city map square will be similar to the picture numbered 4.

The films are available from:

POWERS OF TEN (8 min. color)
1968 Producer: Charles Eames
The University of Southern California
Division of Cinema
Film Distribution Section
University Park
Los Angeles, Ca 90007
Rental @ 10.00

COSMIC ZOOM (8 min. color)
1970 Producer: National Film Board of Canada
Contemporary/McGraw Hill Films
Western Regional Ofc.
1714 Stockton Street
San Francisco, Ca 94133
Rental @ 12.50

TYPE: Activity

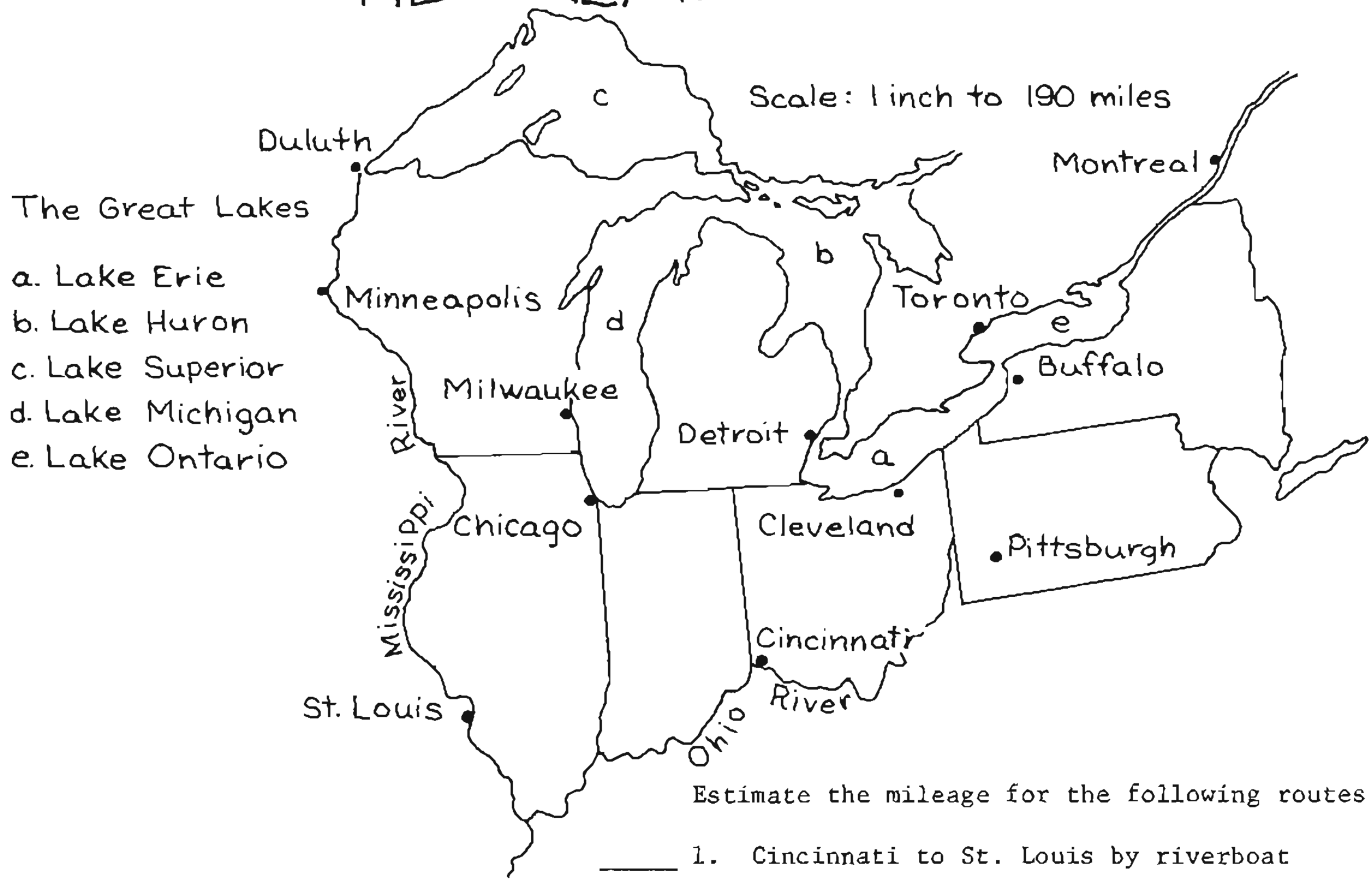
RATIO, PROPORTION, AND SCALING

Placement Guide for Tabbed Divisors

CONTENTS

SCALING: MAPS

THE GREAT LAKES



Estimate the mileage for the following routes

_____ 1. Cincinnati to St. Louis by riverboat

_____ 2. Duluth to Cleveland by the shortest water route

_____ 3. Chicago to Cleveland by lake streamer

_____ 4. Detroit to Chicago by air

_____ 5. Montreal to Milwaukee by water

_____ 6. Buffalo to Minneapolis by land

SOURCE: *Greater Cleveland Mathematics Program*, 6

kilometouring around the U.S.A.

Use the map on the next page. Measure the distance between the following cities to the nearest half centimetre. On the map 1 cm represents 100 km. Figure out the actual distance in km between the cities. The first one is done for you.

1	Reno, Nevada to New York City	18.5	cm	1850	km
2	Seattle, Washington to Miami, Florida	______	cm	______	km
3	St. Paul, Minnesota to Houston, Texas	______	cm	______	km
4	Los Angeles to Cleveland, Ohio	______	cm	______	km
5	Butte, Montana to Rapid City, SD	______	cm	______	km
6	Washington, D.C. to St. Louis, MO	______	cm	______	km
7	Denver, Colorado to Raleigh, NC	______	cm	______	km
8	Tucson, Arizona to Atlanta, Georgia	______	cm	______	km
9	Santa Fe, New Mexico to Salt Lake City	______	cm	______	km
10	Tulsa, Oklahoma to Portland, Maine	______	cm	______	km
11	Omaha, Nebraska to Chicago, Illinois	______	cm	______	km
12	Memphis, Tennessee to New Orleans, LA	______	cm	______	km

✖ Debbie flew on a business trip from Washington, D.C. to Los Angeles, and then to Miami and back to Washington D.C. How far did she travel? ______ cm which represents ______ km

✖ John lives in Los Angeles and is flying to Washington, D.C. for a vacation. He can either fly from Los Angeles to Chicago and then to Washington, D.C., or from Los Angeles to Atlanta and then to Washington, D.C. Which is shorter?

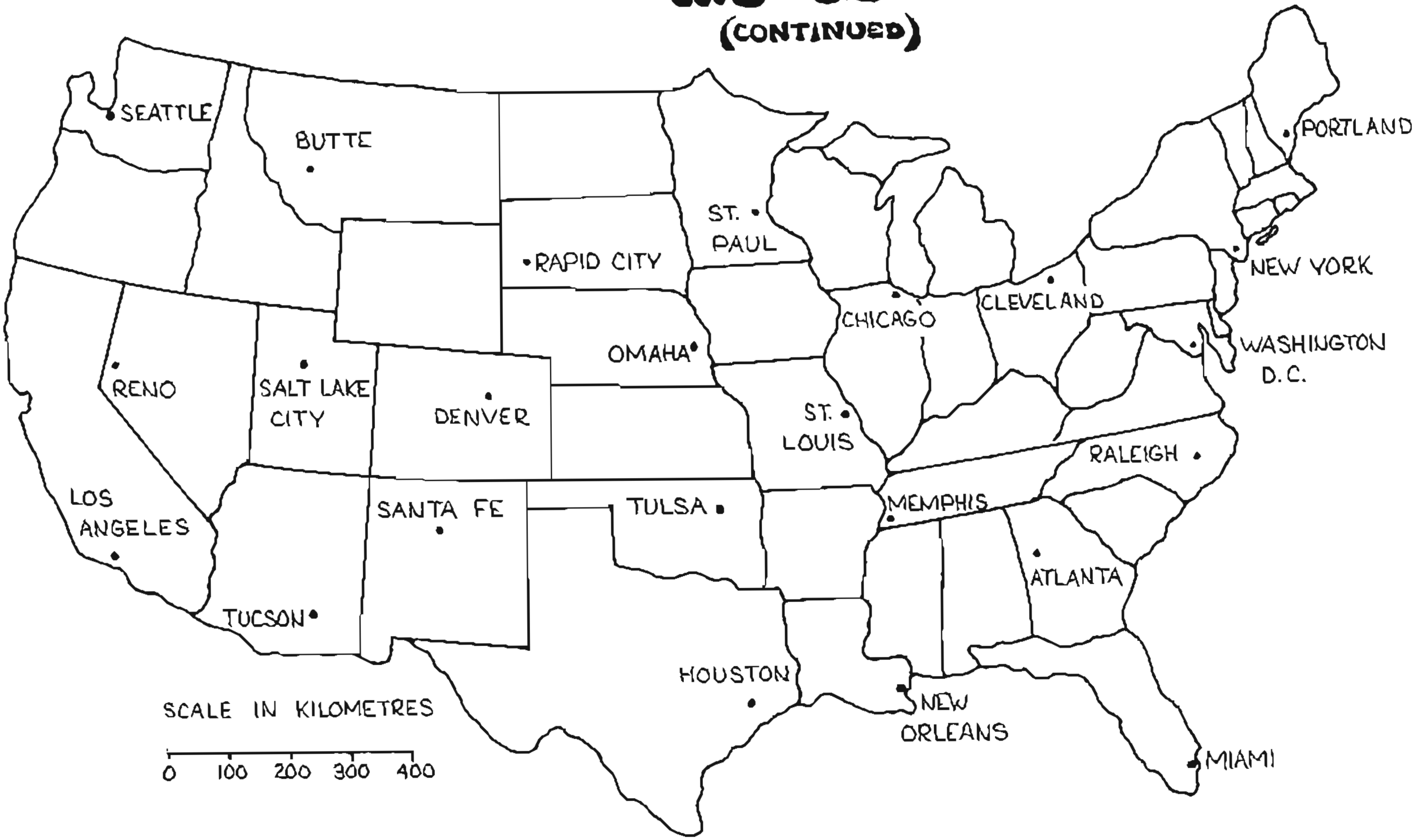
Los Angeles → Chicago → Washington ______ cm ______ km

Los Angeles → Atlanta → Washington ______ cm ______ km

SOURCE: *Metric Measurement: Activities in Linear Measurement*

kilometouring around the U.S.A

(CONTINUED)



1 cm represents 100 km

AROUND THE U.S.A.

Using a Scale Drawing to Find Distances
Maps
SCALING

E
S
I

This activity could extend over two days by doing parts 1 and 2 on the first day, collecting the papers to check the table, and then using the table to do part 3.

1) Find and record the distances between these cities. (Measure to the nearest $\frac{1}{4}$-inch.)

Obtain some airplane flight schedules from a travel agent. United, TWA and Northwest Orient have fares listed.

Allow students time to examine the flight schedule and route map and interpret the reference marks.

Have students investigate to see if distance is proportional to cost. Compare the costs of direct to non-direct flights, e.g., N.Y. - L.A. vs. N.Y. - St. Louis - L.A.

	DENVER	LOS ANGELES	MIAMI	MINNEAPOLIS	NEW ORLEANS	NEW YORK	SEATTLE	ST. LOUIS
DENVER								
LOS ANGELES								
MIAMI								
MINNEAPOLIS								
NEW ORLEANS								
NEW YORK								
SEATTLE	1000							
ST. LOUIS								

2) Use a two-rubber band pantograph to help you draw a rough sketch of the United States from the picture at the top of the page. Locate the cities. The map scale is now ___________:___________. How will the measured distances between cities change? Will the mileage between the cities change?

?

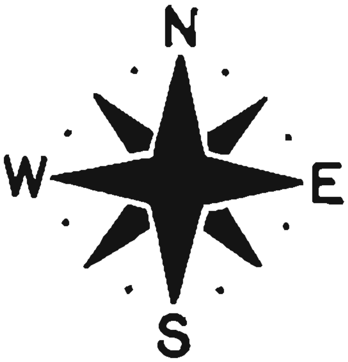
3) Plan a trip that starts and finishes in New York and includes stops at all the cities listed on the map. Write down the trip and the mileage between cities. What is the total mileage? ___________ Could you find a shorter way to make the trip? Compare with a friend.

Is the shortest trip necessarily the cheapest?

.PE: Paper & Pencil

IDEA FROM: *Operating with Mathematics*

FOREST FIRES ARE A REAL BURN

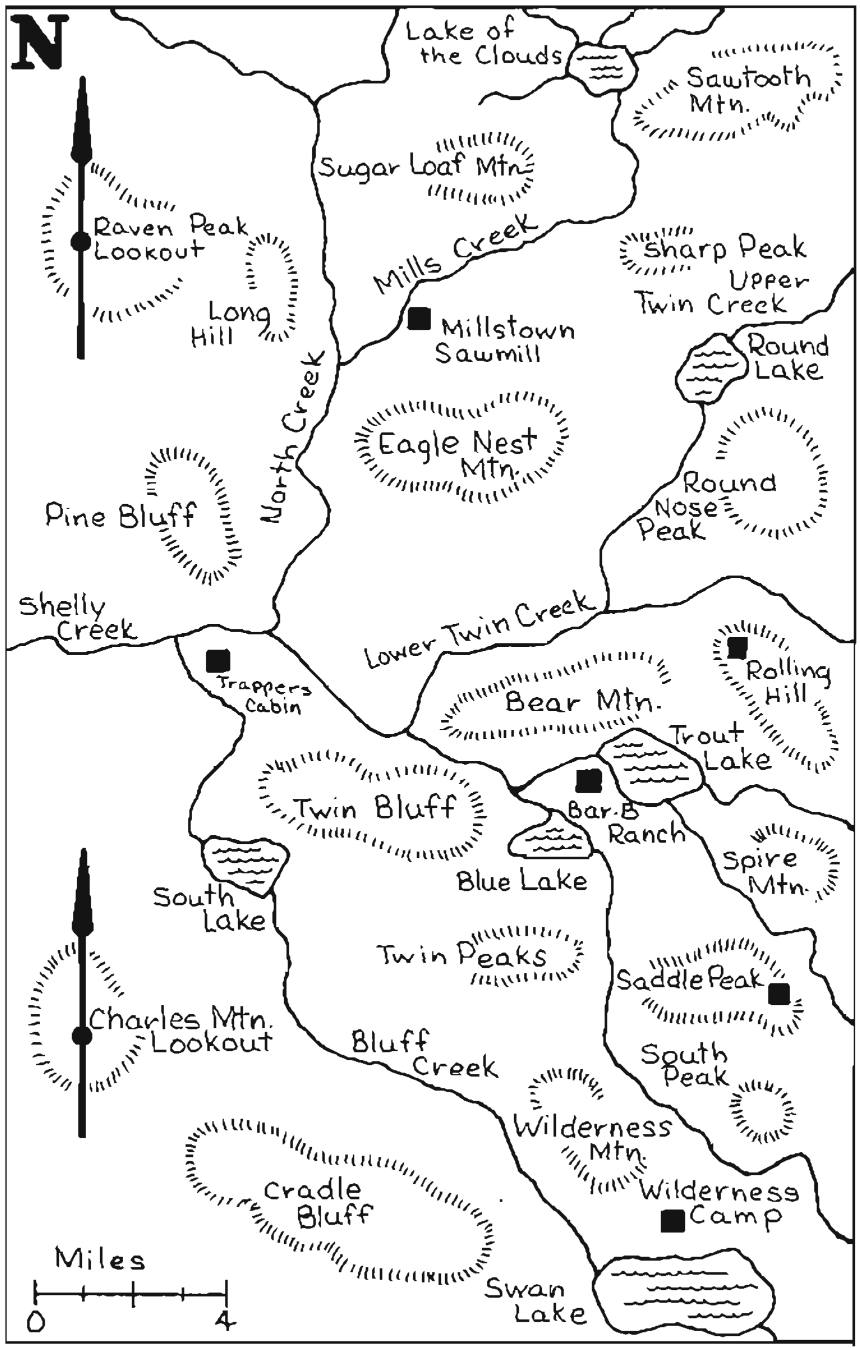


Materials needed: Ruler and compass

Activity: Draw a line connecting Raven Peak Lookout and Charles Mountain Lookout.

1. A fire breaks out on the eastern slope of Saddle Peak. Draw lines from the fire to each of the lookouts. Place the center of the compass on a lookout with North (0°) on the line between the lookouts. Read the angle from North to South. What compass readings would each of the lookouts report? Raven Peak __________ Charles Mountain __________.

2. Raven Peak reports a fire at 113° and Charles Mountain reports the fire at 53°. Where is the fire located? ______________ How far is the fire from Raven Peak? Measure to the nearest mile. _______



FOREST FIRES ARE A REAL BURN

(CONTINUED)

3. Raven Peak reports a column of smoke at 150° and Charles Mountain reports this at 100°. Find the location of the fire. ____________________

4. A fire breaks out on the northwest tip of Rolling Hill. What compass readings will each lookout report? Raven Peak __________ Charles Mountain ________. Which lookout is nearer to the fire?

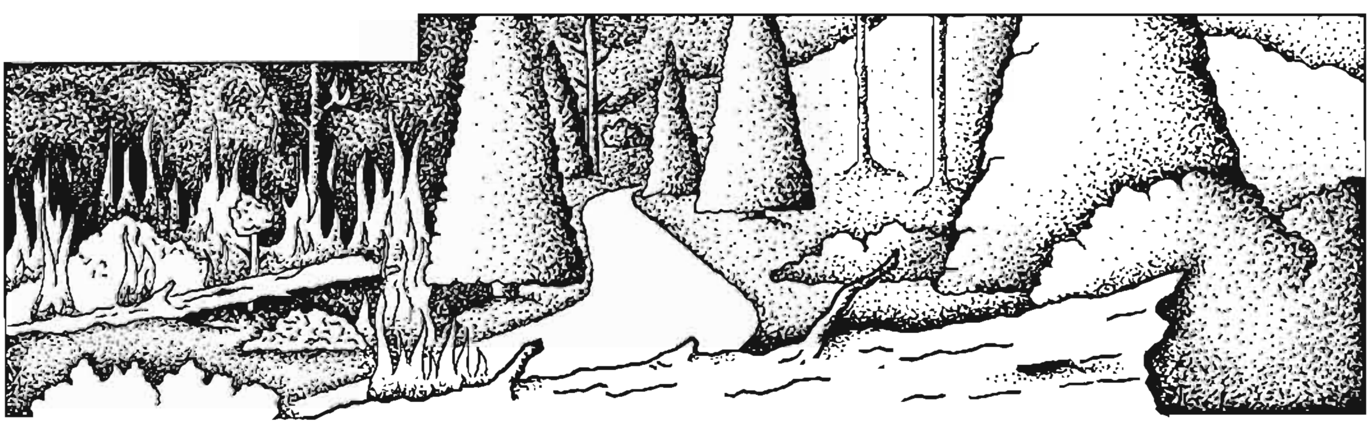
5. A hunter is reported missing in the Upper Twin Creek area. A flare is seen during the night at 101° from Raven Peak and 42° from Charles Mountain. Where did the flare originate? ____________________. How far is it from Raven Peak? Measure to the nearest mile. ____________________

6. Since Millstown Sawmill is always burning scraps, what smoke readings should both lookouts ignore? Raven Peak __________ Charles Mountain __________.

7. Locate the following fires.

	Raven Peak Reading	Charles Mountain Reading	Fire Location
a.	79°	35°	____________
b.	165°	120°	____________
c.	107°	49°	____________
d.	158°	13°	____________

8. The ranger at Charles Mountain Lookout has to deliver supplies. His route will take him to Raven Peak Lookout, Millstown Sawmill, Trapper's Cabin, Bar-B Ranch, Wilderness Camp, and then back to Charles Mountain Lookout. Describe the route. Record the distance and compass reading from each stop to the next stop. ____________________

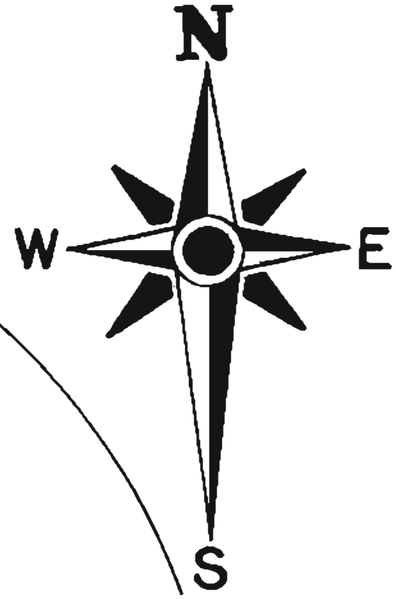


Using a Time Scale
to Locate Points
Maps
SCALING

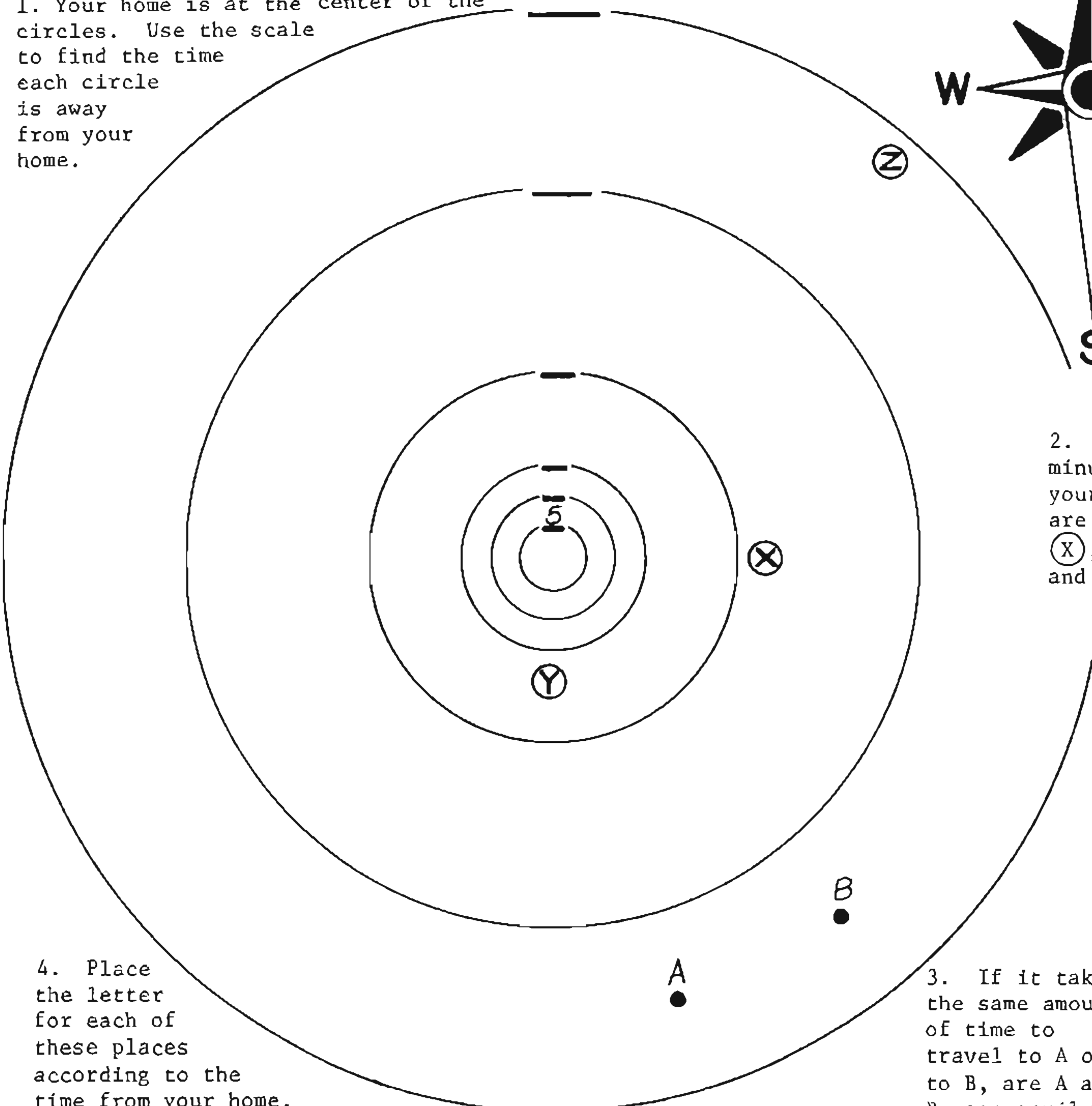
E | S | I

WHERE'S IT AT?

Scale of $\frac{1}{2}$ cm : 5 minutes



1. Your home is at the center of the circles. Use the scale to find the time each circle is away from your home.



2. How many minutes from your home are points X, Y, and Z?

3. If it takes the same amount of time to travel to A or to B, are A and B necessarily the same time distance from your home?

4. Place the letter for each of these places according to the time from your home. Try to determine the correct direction.

a) school	b) grocery store	c) church
d) gas station	e) camp ground	f) movie
g) zoo	h) swimming pool	i) clothing store
j) public library	k) boating area	l) tennis court
m) bowling alley	n) playground	

 TYPE: Paper & Pencil

IDEA FROM: *The Nature of Recreation*

OUR TOWN

Reading a Map
Maps
SCALING

E
S
I

Local maps can be used to develop map reading skills. A map of LaGrande, Oregon and sample student questions are provided. If a map of your town is not available, this map may be copied for class use.

As readiness for the activity, have students sketch the route they take from home to school. The sketch should include streets crossed and landmarks passed. A follow-up discussion should point out that even though these sketches are not scale drawings, they communicate valid information. The technique could be further developed by having students draw sketches for routes between two landmarks, i.e., the post office and the high school.

Students should be given time to familiarize themselves with the map before answering the worksheet.

1) How many schools does LaGrande have? ________________

2) The railroad depot is located on ________________ Street.

3) The postoffice is located on the corner where ________________ Street and ________________ Avenue meet.

4) The location of the library is ____.

5) The main highway from Pendleton to LaGrande to Baker is U.S. Route Number ______.

6) The highway from LaGrande to Wallowa Lake is State Route Number ______.

7) Locate the homes of three of your friends. Name the locations by writing one or two streets.

   a) ____

   b) ____

   c) ____

   Hand your descriptions to another student and see if he or she can find the houses.

8) Start at Eastern Oregon College and describe a route to the Union County Fairgrounds.

9) Use the scale of the map to estimate the length of your route in question #8.

   ____

10) Give your directions in question #8 to another student and see if he or she is able to follow your route to the fairgrounds.

11) Describe a route from your home to school. Estimate the distance.

12) Describe a route from your school to Pioneer Park. Estimate the distance.

13) The bridge on 2nd Street has been closed because of an accident. Describe an alternate route from the library to the Union County Fairgrounds.

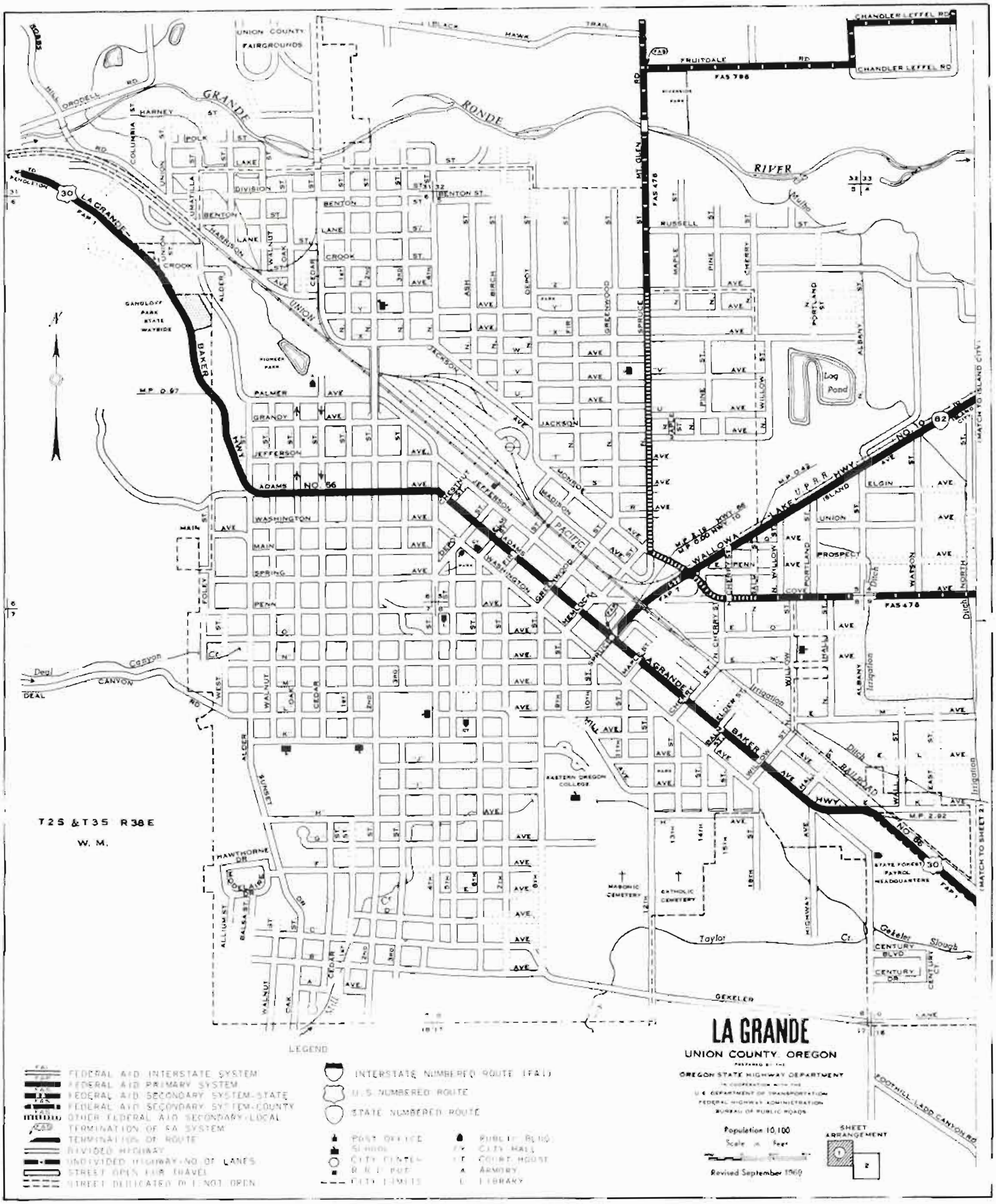
14) Check with the fire department for a description of the fire routes. Sketch the route on your map if a fire is spotted on the corner of Greenwood Street and "X" Avenue.

15) Ask your mailman for a description of his route. How many miles does he travel in one day?

16) Check with the newspaper office or ask a friend with a paper route for a description of his route. Sketch the route on the map. Estimate the distance.

17) Plan a ten-mile benefit walk-a-thon through LaGrande.

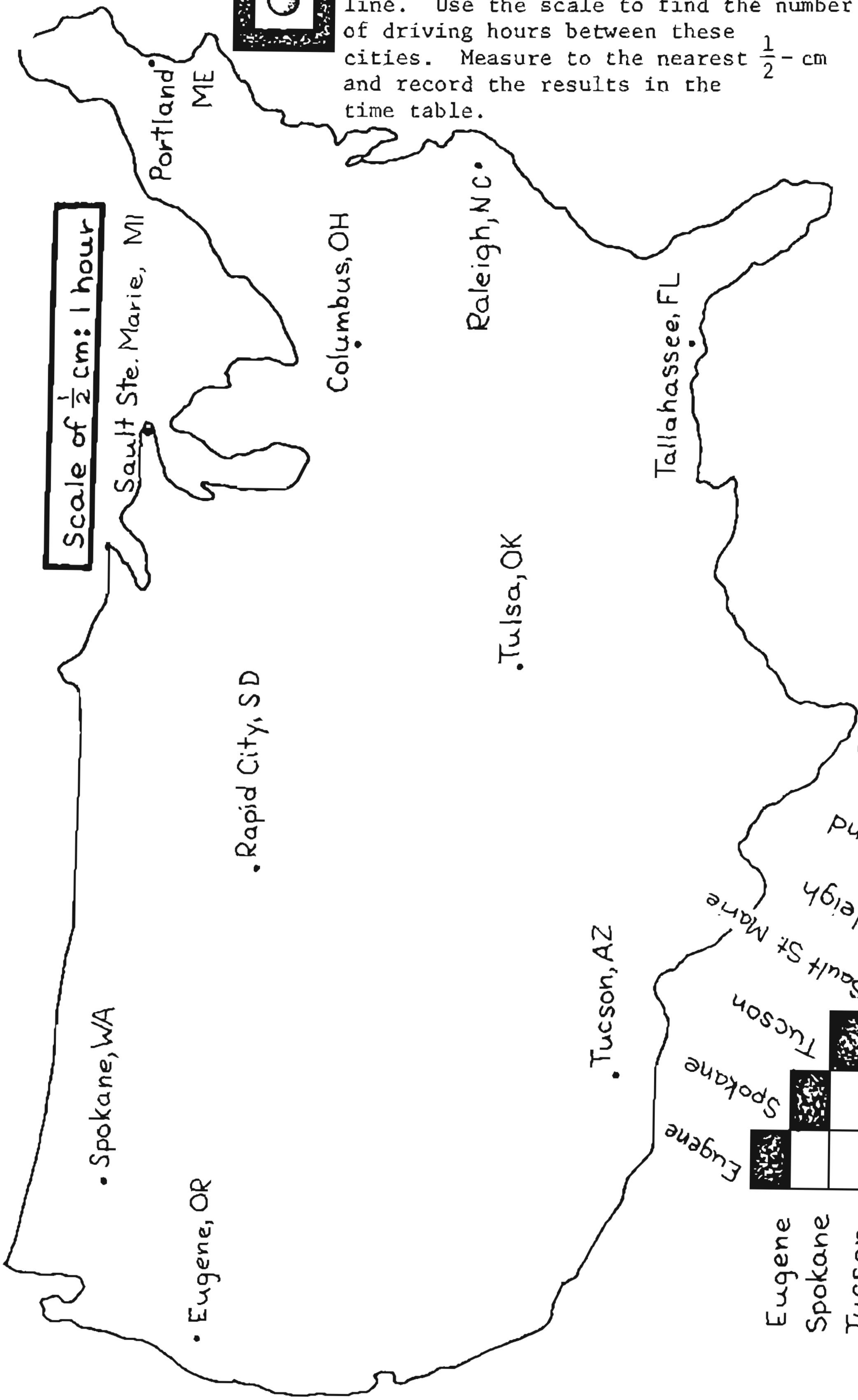
TYPE: Paper & Pencil/Activity



LA GRANDE-UNION COUNTY
CHAMBER OF COMMERCE

IT'S ABOUT TIME

Assume you could drive in a straight line. Use the scale to find the number of driving hours between these cities. Measure to the nearest $\frac{1}{2}$ - cm and record the results in the time table.



	Eugene	Spokane	Tucson	Sault St Marie	Raleigh	Portland	Columbus	Tallahassee	Rapid City	Tulsa
Eugene										
Spokane										
Tucson										
Sault Ste. Marie										
Raleigh										
Portland										
Columbus										
Tallahassee										
Rapid City										
Tulsa										

DO YOU KNOW THE WAY TO SAN JOSE ?

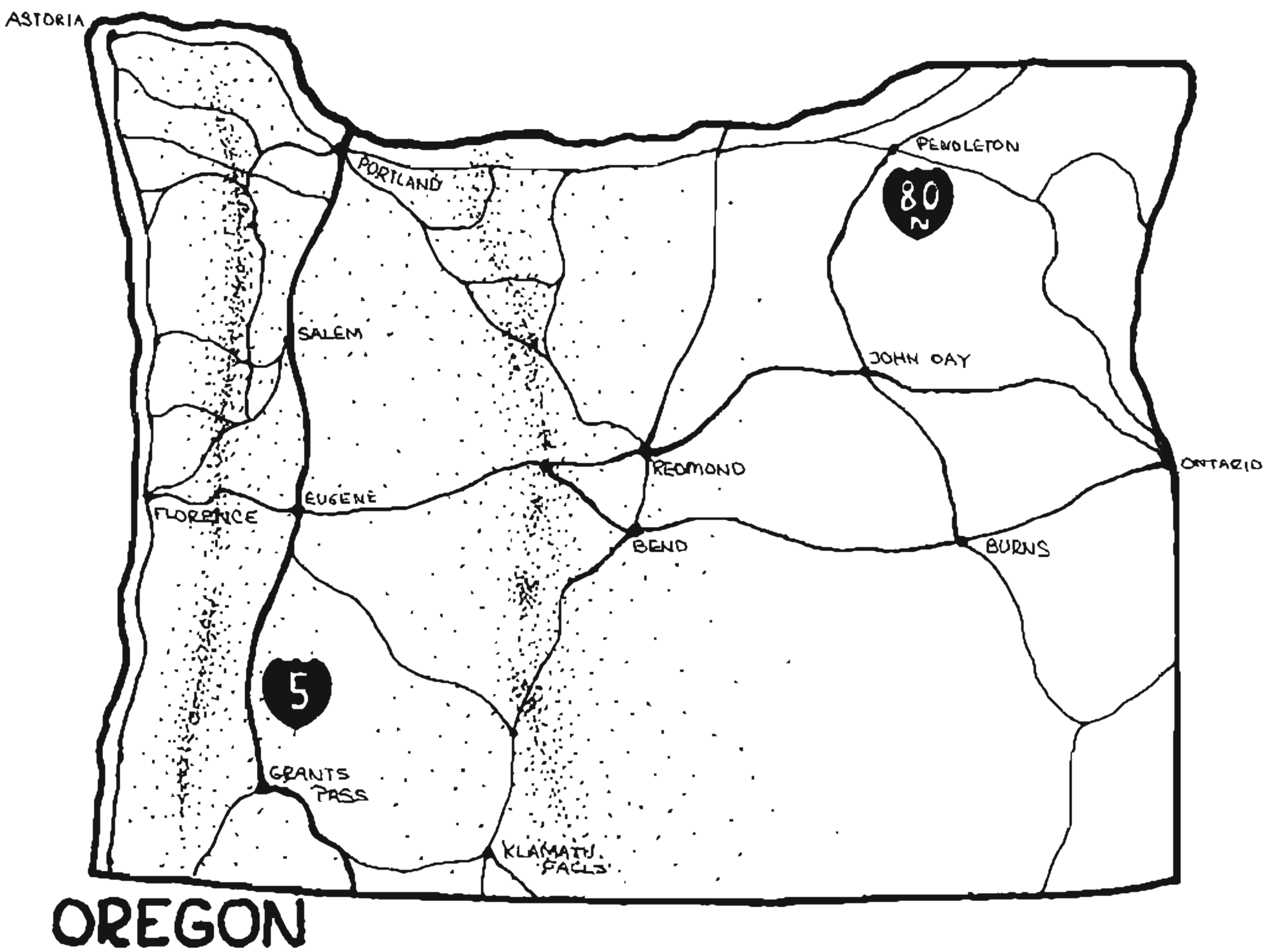
TEACHER IDEAS

A state road map can provide students with a variety of interesting and practical activities. If done at the beginning of the school year, the road map activity could be a diagnostic tool to use in ascertaining students' computational and problem-solving skills. These maps can be obtained from your State Highway Division or from oil company service stations. You may wish to obtain two or three different maps as each has some features of interest not found on the others.

To prepare the students for map reading, use the map's coordinate system to name your students' seats. Indicate the rows by letter and the columns by number. Refer to each student by his coordinate. "Who is student A-5?" "Who is sitting next to student B-2?" "What answer does student C-2 have to Problem #4?"

When students first receive the map, they should be given time to investigate the map. Refer to the chart of symbols and ask students to find examples on the map. The back of the map should also be investigated. City maps, mileage tables, and park information are usually provided.

On the following page are three sample student pages based on an Oregon map and a teacher idea for an extended activity using maps.



TYPE: Paper & Pencil/Activity

IDEA FROM: "Road Maps in the Mathematics Classroom," *Oregon Council of Teachers of Mathematics* Monograph, December, 1973

DO YOU KNOW THE WAY TO SAN JOSE? (CONTINUED)

LOCATIONS

1) The largest city in coordinate square H-2 is ________________.
2) Is there a mountain over 10,000 feet in square D-3? __________ If so, what mountain? ________________
3) Find the location of each town and its population.

TOWN	LOCATION	POPULATION
Eugene		
Baker		
Medford		
Bend		
Portland		

4) Give the coordinate squares where ski areas can be found.
5) What counties are in square F-7? ____________________
6) Is there a fish hatchery in square D-4? _________
7) Find the location of Crater Lake National Park. ______________
8) In section J-2 what type of road surface would you drive on in traveling from Imnaha to Hat Point? ______________________
9) How many interstate freeways pass through the state? __________ Name them. ____
10) Does Roseburg have an airport? ____________
11) Is there a game refuge near Lakeview? ____________
12) What is the county seat of Wallowa County? ______________
13) Are there any state parks with overnight camping facilities in square F-9? __________
14) Can you drive on the beach at Lincoln City? __________
15) What national monument is located near the southern border? ___ ______________
16) In the winter would it be wise to travel from Springfield to Sisters on Highway 242? ______________

DISTANCE AND AREA

1) How far is LaGrande from Eugene by paved highway? ___________
2) How far from La Grande to Eugene by air? (Use the scale on your map.) __________
3) Find the distances to fill in the table below.

	Use the mileage table	Add numbers given on highways. (Use the shortest paved route).	Use a string and the map scale on the shortest paved route.
Bend to Burns			
Corvallis to Seaside			

4) How many miles long is the southern boundary of Oregon? _______
5) List the state parks within 15 miles of Redmond.
6) What is the airline distance from Brookings to Astoria? _______ What is the distance along Highway 101? __________
7) Determine which route is shorter: Ontario - Burns - Bend or Ontario - John Day - Redmond - Bend. ________________
8) Find the distance up the Rogue River from Gold Beach to Agnes. __________
9) Lake Owyhee near Ontario has a perimeter of ________ miles.
10) Find the difference in elevation between Klamath Falls and Ashland. __________
11) Can you find two towns whose difference in elevation is the same as the air distance between them? ________________ For example, Scio is 142 feet lower than Lostine and 142 miles away.
12) What is the largest lake in Oregon? __________ What is its coordinate section? __________ Use transparent grid paper to estimate the area of the lake in square miles. __________
13) Suppose you wish to build an airport in the center of the state. Find and describe its location. ________________ The flying distance from the airport to Portland International Airport will be ________ miles.

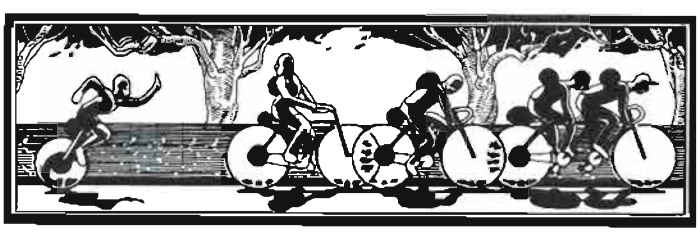
A BICYCLE TRIP

A person is planning a 6-day bicycle trip. The route will be as follows:

FIRST DAY: Eugene to Florence to Newport
SECOND DAY: Newport to Astoria
THIRD DAY: Astoria to Portland on Hwy. 30
FOURTH DAY: Portland to Madras on Hwy. 26
FIFTH DAY: Madras to the junction of Hwy. 97 and Hwy. 58
SIXTH DAY: The junction back to Eugene

Based on the information related to this route, answer the following questions:

1. Could this person be you? ____________
2. What will be the farthest distance for one day? ___________
3. What will be the shortest distance for one day? ___________
4. What will be the total distance for the trip? ___________
5. What will be the average distance for one day? ___________
6. If you could maintain this pace, could you bicycle from Eugene to New York City in 37 days? ____________
7. Explain your answer to Question 6.
____
____
____



SEE THE SIGHTS

Students can be given an amount of money, $500 apiece, and told to design a vacation trip lasting from 4 days to two weeks. Travel brochures, motel guides, sight-seeing fliers, and road maps can help them choose a destination and plan a route. During each class period the student can record the distance and expenses incurred for one day's journey of the trip on a log sheet.

Start of day odometer reading _____
Finish of day odometer reading _____
"0" at the beginning of the trip.
miles driven _____
mph for day _____
furnished by teacher
driving time _____
Amount of money at beginning of day _______
Amount of money left at end of day _______

EXPENSES FOR DAY

Meals	Gas and oil	Lodging	Miscellaneous
	1 can of oil for every 1,000 miles		(fares)

Several students may decide to travel together and pool their resources. "Hazard" cards can be provided to give the students practice in planning ahead and budgeting for the unexpected. Each day the student draws a card that might cause him/her to have a flat tire, find $20, pay a traffic violation, lose a wallet in a restaurant, etc. Students should also budget enough money to return home from the vacation. At the end of the trip students could give a written or oral account of their vacation to the class.

This activity could be developed as a long-range class and/or individual project. A contest could be made between groups of students, the winner being the group who took the "best" trip for the money.

As an introductory or final activity, invite a travel agent to speak to the class.

The game "Mille Bornes" by Parker Brothers is a nice extension.

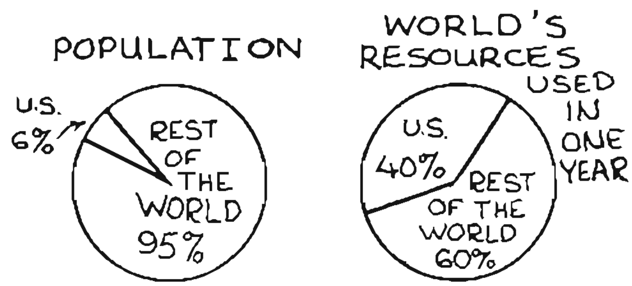
IDEA FROM: "Road Maps in the Mathematics Classroom," *Oregon Council of Teachres of Mathematics* Monograph, December 1973

RATIO, PROPORTION, AND SCALING

Placement Guide for Tabbed Divisors

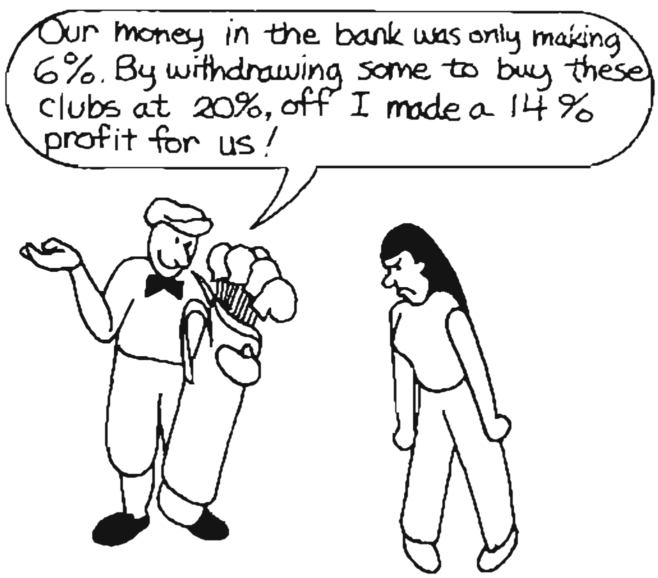
PERCENT

Percents are a very useful way to convey information. The graphics and percents at the right tell us quickly that the United States has a very small part of the world's population, but uses almost half of the world's resources. Substituting percent for actual data gives us a much more efficient way of making comparisons. Besides being a convenient way of conveying and comparing information, percents are constantly being quoted by newspapers, news announcers and businesses as rates of discount or increase.



IS UNCLE SAM A GLUTTON?

The U.S., with only 5.7% of the world population, consumes 40% of the resources the world uses in a year. If all other peoples were raised to our standard of living, the known resources of the world would be exhausted in decades.

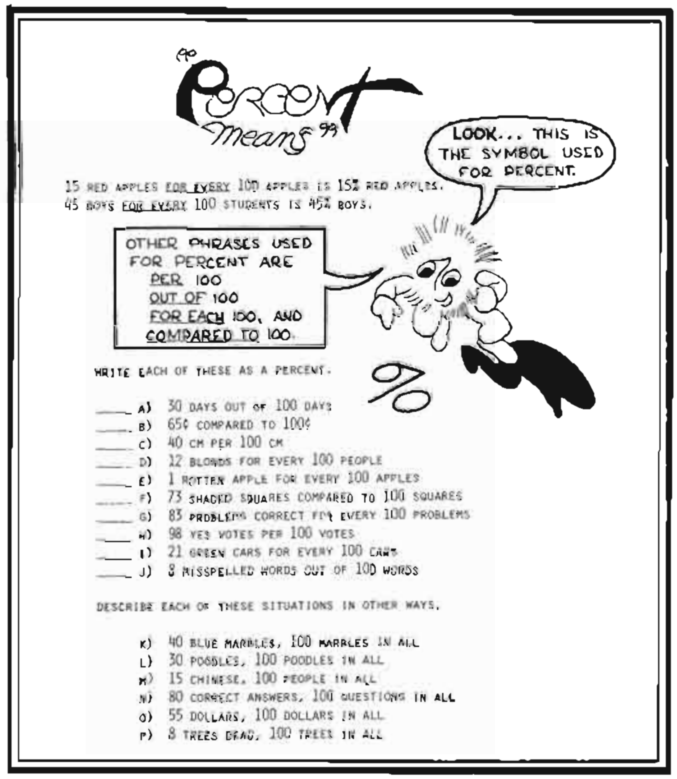


Banks advertise their interest rates and stores promote sales with "X% off." Many important questions come out of the percents we confront daily. With the cost of living rising as it has, will that 10% raise make your salary worth as much as last year's? Would you be money ahead by borrowing money for one month rather than taking money from a savings account and losing a quarter's interest?

Many students acquire notions about percent before they formally study it in school. They hear about a 50% chance of rain, and many can even compute a 15% tip at a restaurant. Before beginning a unit on percent, why not have an informal discussion with students to see how extensive their intuitive grasp of percents is? Can they compute a tip? What does 10% chance of rain mean? 100% chance? What does 100% mean? Can anything be over 100%?

PERCENT SENSE

When teaching percent we tend to rush toward fraction, decimal or proportion computation. Students move the decimal point two places, then multiply or divide without knowing whether their answer is sensible or whether they could have solved the exercise in their heads. There are activities which can help students focus on understanding percent without reverting to decimals, fractions or proportions. In this resource we have placed such activities under the topic Percent Sense. Pages from the Percent Sense section can supplement the learning of percent in many ways; they are not intended to be taught first in <u>all</u> cases. You might choose to use some of these activities after percent has been introduced as a ratio or rational number. Some of the specific ideas stressed in this section are discussed below.


"PERCENT MEANS"

15 RED APPLES FOR EVERY 100 APPLES IS 15% RED APPLES.
45 BOYS FOR EVERY 100 STUDENTS IS 45% BOYS.

WRITE EACH OF THESE AS A PERCENT.

_____ A) 30 DAYS OUT OF 100 DAYS
_____ B) 65¢ COMPARED TO 100¢
_____ C) 40 CM PER 100 CM
_____ D) 12 BLONDS FOR EVERY 100 PEOPLE
_____ E) 1 ROTTEN APPLE FOR EVERY 100 APPLES
_____ F) 73 SHADED SQUARES COMPARED TO 100 SQUARES
_____ G) 83 PROBLEMS CORRECT FOR EVERY 100 PROBLEMS
_____ H) 98 YES VOTES PER 100 VOTES
_____ I) 21 GREEN CARS FOR EVERY 100 CARS
_____ J) 8 MISSPELLED WORDS OUT OF 100 WORDS

DESCRIBE EACH OF THESE SITUATIONS IN OTHER WAYS.

K) 40 BLUE MARBLES, 100 MARBLES IN ALL
L) 30 POODLES, 100 POODLES IN ALL
M) 15 CHINESE, 100 PEOPLE IN ALL
N) 80 CORRECT ANSWERS, 100 QUESTIONS IN ALL
O) 55 DOLLARS, 100 DOLLARS IN ALL
P) 8 TREES DEAD, 100 TREES IN ALL

<u>Percent Means Per Hundred</u>

Percent is closely tied to the word hundred. Here are some typical statements included in introductions or definitions of percent.

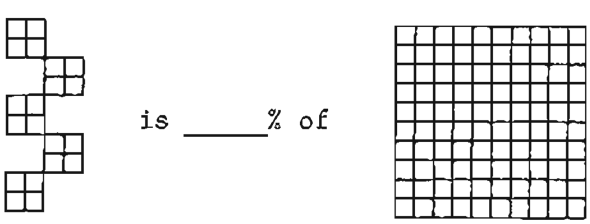
a) Percent; by the hundred; in the hundred.

b) A special ratio which compares a number to 100 is called a percent.

c) It is reasonable for students to think of 5% as meaning 5 for every hundred.

d) ...percent means per hundred. Thus, 61% means 61 per hundred.

The everyday phrases shown in the above student page can be used in building the concepts of percent. If 25 for every 100 can be written as 25% and if there are 25 seniors for every 100 students, then 25% of the students are seniors. Hundred grids of various sizes and shapes can be used to represent the 100 part of percent. A pattern of squares can be shown aside from the 100 grid, and students may be asked, "What percent of the reference grid is shown by the design?" Since the design is made of 20 squares and the grid has 100 squares, the design is 20% of 100. This suggestion and other such ideas are developed in the classroom materials.



Questions based on the idea of 100 should be answered and understood before going to more complicated work with percent. The so-called three types of percent problems may be included.

a) 17% of 100 = ___ b) 80 is ___% of 100 c) 21 is 21% of ___

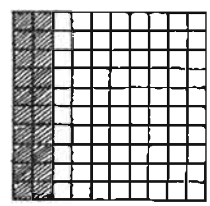
d) 32 is what percent of 100? e) What is 57% of 100?

f) Sam answered 67% of the questions correctly. If he answered 67 questions right, how many questions were there?

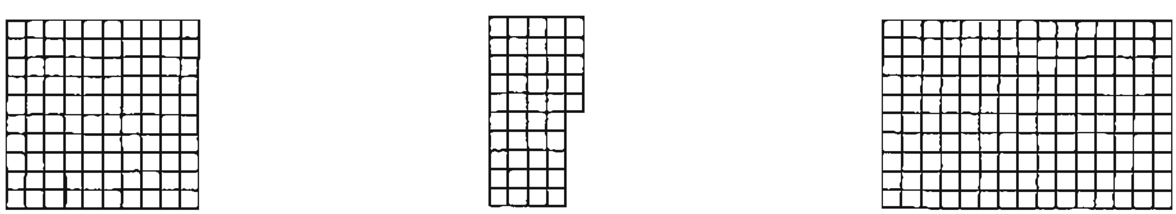
Exercises like these require no computation, but they do focus on the close relationship of percent to 100 and on word phrases which are used to relate pairs of quantities and percent. Later these phrases will be used in more complicated settings.

<u>But 100% is Everything!</u>

Students often think the idea of 150% is absurd, since 100% of something is all there is. Perhaps we encourage their objections by overconcentration on phrases like "20 <u>out of</u> 100" and diagrams like the one shown at the right. It seems ridiculous to say "150 out of 100," and how could we shade 150% of the squares? To avoid this problem the phrases "for every 100, per hundred or compared to 100" could be used instead of "out of 100." Percents over 100 can be used when introducing percents--not reserved for later. The reference 100-grid can be kept to the side and various percents of the grid shown.



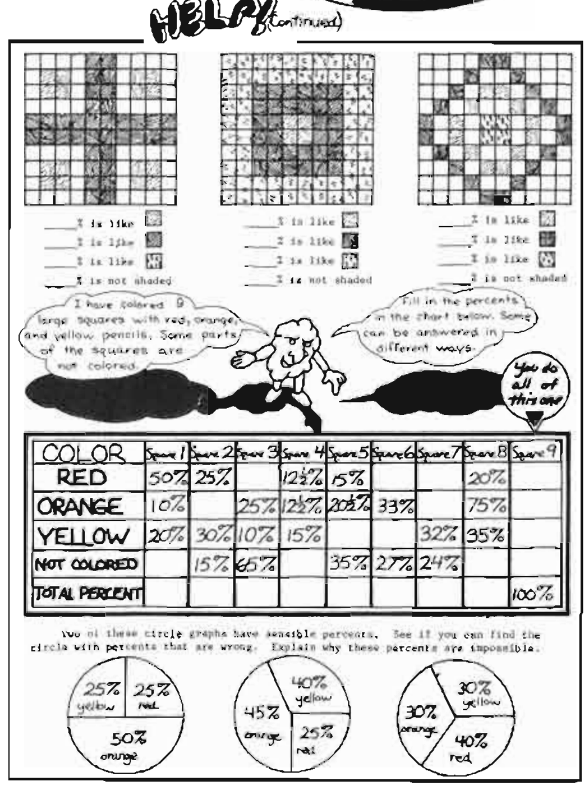
20% of the square is shaded



R   45% of R   150% of R

Number patterns can be used to make percents over 100 plausible. A sequence of exercises leading to percents greater than 100 can be given. 60 = ___% of 100, 80 = ___% of 100, 100 = ___% of 100, 120 = ___% of 100. Those students who understand when a glass is 100% full can be asked, "How full is a glass of mounded-up slushes or ice cream?"

There is an important idea about percent which sounds very much like, "But 100% is everything!" The combined parts of a whole are 100% of the whole. The page at the right addresses these ideas and is a good readiness activity for making percent circle graphs. If 25% of the money is spent, 75% is left. (A question for discussion is, "If 25% of the money is spent, is there any left over? Can you think of any cases where 125% of the available money was spent?)

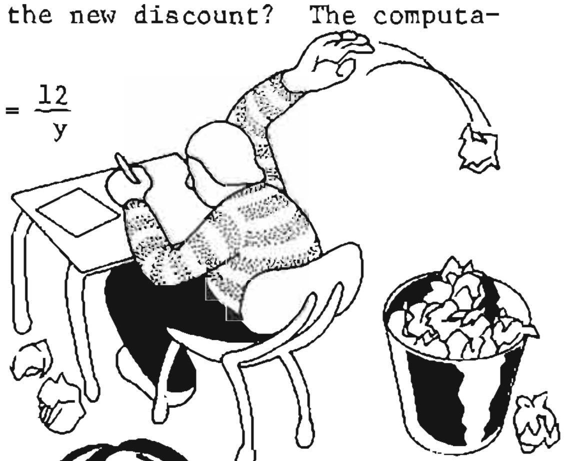


COLOR	Square 1	Square 2	Square 3	Square 4	Square 5	Square 6	Square 7	Square 8	Square 9
RED	50%	25%		$12\frac{1}{2}$%	15%			20%	
ORANGE	10%		25%	$12\frac{1}{2}$%	$20\frac{1}{2}$%	33%		75%	
YELLOW	20%	30%	10%	15%			32%	35%	
NOT COLORED		15%	65%		35%	27%	24%		
TOTAL PERCENT									100%

Two of these circle graphs have sensible percents. See if you can find the circle with percents that are wrong. Explain why these percents are impossible.

<u>If You Know 10%, You Know a Lot!</u>

If an item is advertised at 10% off for a \$12 savings and later it is marked 25% off the original price, what is the dollar savings for the new discount? The computation and method-oriented person might write:



10% of y = \$12, so (by the proportion method) $\dfrac{10}{100} = \dfrac{12}{y}$

Now $10y = 100 \times 12$, so $y = \dfrac{100 \times 12}{10} = 120$

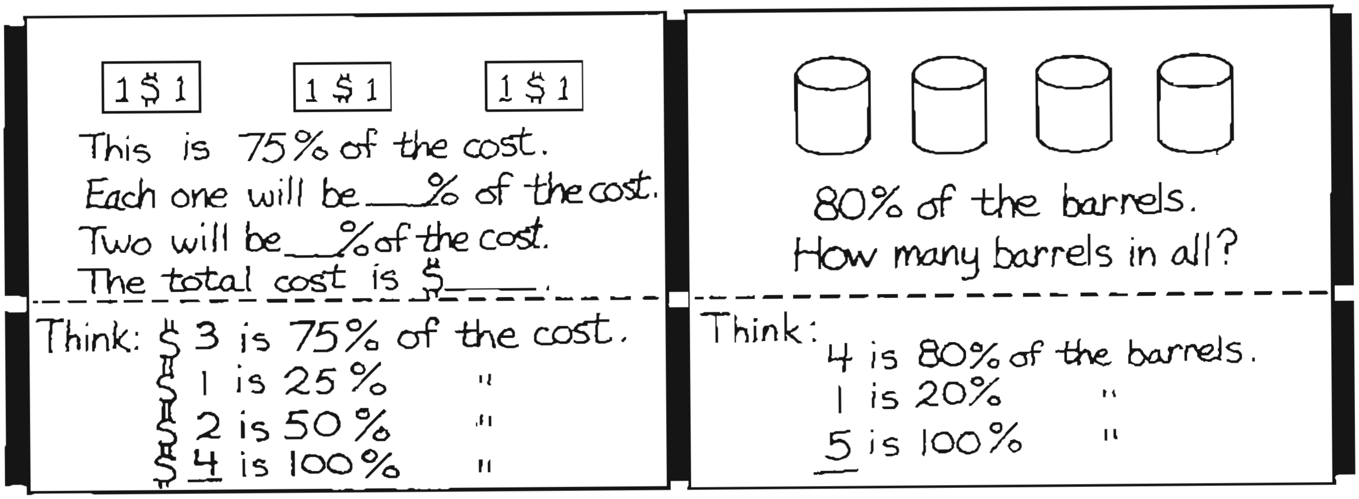
25% of y = 25% of 120 and $\dfrac{25}{100} = \dfrac{z}{120}$

$100z = 25 \times 120$, so $z = \dfrac{\overset{1}{\cancel{25}} \times \overset{30}{\cancel{120}}}{\underset{\underset{1}{\cancel{4}}}{\cancel{100}}} = 30$

A person using his percent sense can reason like this:

10% of the price was \$12.
20% of the price is twice as much or \$24.
5% of the price is half as much or \$6.
\$24 + \$6 = \$30.

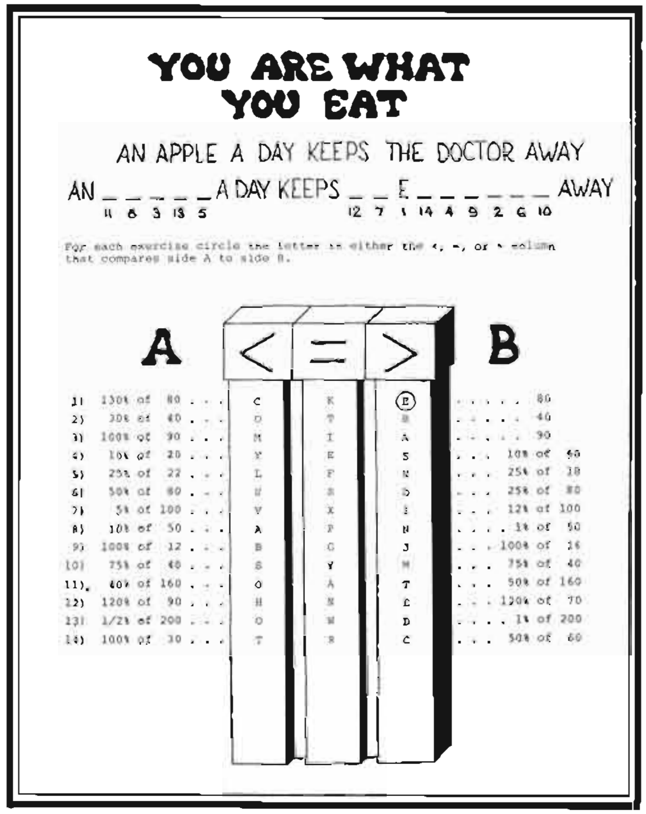
The computations required for the first type of solution are much more complex than for the second type. Some people develop the ability to solve problems mentally; they are fortunate. We can encourage more mental computation by providing appropriate exercises. Some questions from the student page *The Whole Thing* are given below with a way of solving each. Students might find other reasonable ways to solve these.



The strategy here is to multiply (or divide) both numbers by the same factor. The same idea can be developed using geometric figures. If is 50% of an object, what might 100% of the object look like? Possible answers: or or .... Activities which incorporate this strategy are *Percents of Line Segments*, *Percents of Rectangles*, *Finding 100% From Above*, *Finding 100% From Below*, *Percents: Backwards and Forwards (1, 2, 3, 4)* and *Peace-N-Order*.

Comparison

A useful part of percent sense is knowing how N% of A compares with A. Is N% of A less than, greater than or equal to A? When a student computes 85% of 20 and obtains 170, his percent sense can catch the error if he knows that 85% of "something" is less than the "something." This kind of percent sense can also be used to catch keypunch mistakes on a calculator. The skill-building page shown at the right includes comparing a percent of a number to the number, and it also asks the student to compare numbers like 50% of 80 to 25% of 80 or 120% of 90 to 120% of 70. How do the values compare when the base number is kept the same and the percent is changed? How do they compare when the percent is kept the same and the base number changed? Other pages covering this concept are *A Sign of the Times*, *Enormous Estimate*, *Love Is Where You Find It* and *Smile*.



Percents Backwards and Forwards

On the student page *Percents: Backwards and Forwards 1* students determine the percent one geometric shape is of another. When A is 20% of B, B is 500% of A. The completed table from this student page is shown at the right. What is the relationship between the two columns of percent? Students being introduced to percents might notice that as the percents increase on the left, they decrease on the right. A more advanced class which can change the percents to decimals could discover that the product of the two decimals is always 1.0000.

A is __% of B	B is __% of A
20	500
25	400
50	200
100	100
200	50
400	25
500	20

PERCENT AS RATIOS

How would a student find 5% of 400? If he had been told 5% means 5 for every hundred, he might reason that there are 4 hundreds in 400 and then multiply 4 x 5 to find 20. This contrasts to the decimal method of dividing the percent number by 100 and then multiplying times 400. Even though percents were historically developed as another form for fractions and decimals, the treatment of percent as a ratio is desirable and mathematically sound. If a percent is written as a ratio (a pair of numbers) as shown at the right, both numbers can be multiplied or divided by the same number, and the same percent number can be used to relate the new pair of numbers. Approximation can also be used with percents or ratios. The exercise below is from *That's "About" Right*.

75 % MEANS

75 FOR EVERY 100	75 % OF 100 IS ____.
DIVIDE BY 25	
3 FOR EVERY 4	75 % OF 4 IS 3.
MULTIPLY BY 5	
15 FOR EVERY ____	75 % OF ____ IS 15.
30 FOR EVERY ____	75 % OF ____ IS ____.

11% MEANS

DIVIDE BY 11	11 FOR EVERY 100	11 % OF 100 IS ____ .
ABOUT	1 FOR EVERY 9	11 % OF 9 IS ABOUT ____ .
ABOUT	7 FOR EVERY 63	11 % OF 63 IS ABOUT ____ .
MULTIPLY BY 7	14 FOR EVERY ____	11 % OF ____ IS ABOUT ____ .

In late middle school the treatment of percents as ratios can be supplemented by the use of formal proportions for solving percent problems. A proportion is a statement of equality of two ratios. When using proportions to solve percent problems, the ratios are usually written in fraction form. Instead of writing 30% as 30 for every 100, we write it as $\frac{30}{100}$. To find 30% of 40 we need only to find the missing term in the proportion 30:100 = A:40 or in fraction form $\frac{30}{100} = \frac{A}{40}$. Using cross products we have 30 x 40 = 100A or 1200 = 100A and A = 12. Some basic properties of equality are applied in such a solution, but this is often less difficult for students then the traditional decimal methods. Students need to know that they can multiply or divide both sides of an equation by a nonzero number and that equality is reversible ($4 = A \longrightarrow A = 4$). Some preliminary work with simple equations like 10 = 3A would be beneficial.

The proportion method unifies the three types of percent problems to the problem of finding the missing term of a proportion. The student will still have the task of deciding how the numbers in the problems relate so that a correct proportion can be written. See *Solving Percent Exercises by the Proportion Method* for more ideas on this. Below are three word problems and their proportion solutions. Students will probably use more steps in solving the proportions.

a) *A sale advertises 15% off. How much is saved on a $10 shirt?*

$$\frac{15}{100} = \frac{A}{10}$$

$$15 \times 10 = 100A$$

$$A = \frac{150}{100} = \frac{3}{2}$$

answer: $1.50

b) *A car was bought for $3000 but is now worth only $1800. What percent of the original price is the current value?*

$$\frac{N}{100} = \frac{1800}{3000}$$

$$3000N = 100 \times 1800$$

$$N = \frac{180000}{3000} = 60$$

answer: 60%

c) *A tent is marked 80% of the original price. What was the original price if it costs $96 now?*

$$\frac{80}{100} = \frac{96}{B}$$

$$80B = 100 \times 96$$

$$B = \frac{9600}{80} = 120$$

answer: $120

Of course, there are other percent problems which do not fit into the three basic types of problems. Two such problems are given below.

a) *25% of the class is done with their work. What percent of the class is not done?*

No proportion is needed here, but it is necessary to have a good understanding of percent to see that 75% of the class is not done. The student page *Help*! in Percent Sense covers this concept.

b) *A car was bought for $3000 and is worth $2100 a year later. What was the percent of depreciation?*

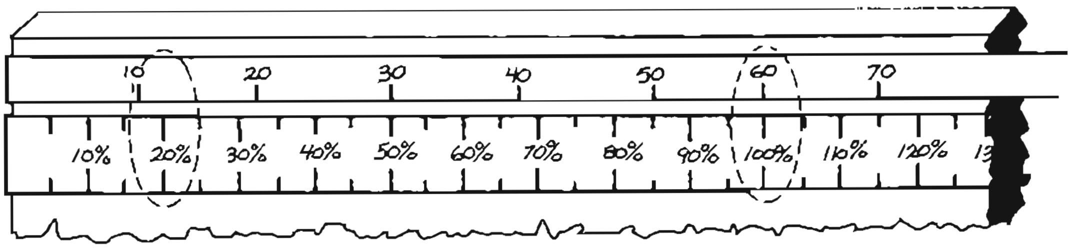
This is a 2-step problem, a variation of the car problem above. It must be seen that subtraction is necessary.

$\$3000 - \$2100 = \$900$ or $\frac{M}{100} = \frac{2100}{3000}$, so $M = 70\%$

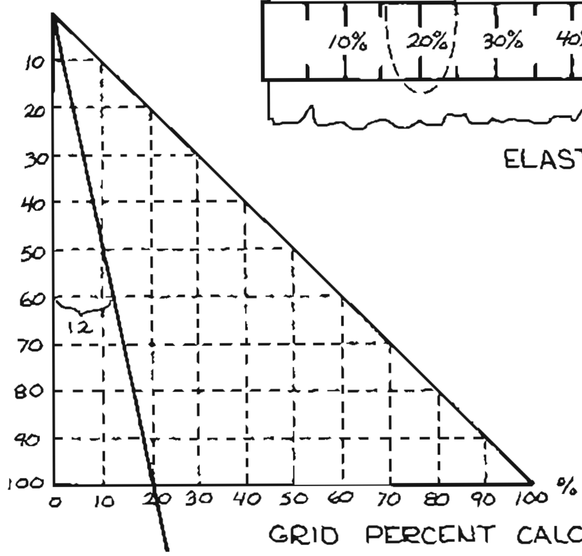
$\frac{N}{100} = \frac{900}{3000}$, so $N = 30$ $\qquad 100\% - 70\% = 30\%$

answer: 30%

The *Elastic Percent Approximator Extended* and the *Grid Percent Calculator* in the section on Solving Percent Problems both use proportions to solve percent exercises. To find 20% of 60 the proportion $\frac{20}{100} = \frac{x}{60}$ would be used. Solving the proportion gives x = 12. Can you see how the calculators below relate the numbers 12, 60, 20 and 100?



ELASTIC PERCENT APPROXIMATOR



GRID PERCENT CALCULATOR

PERCENTS AS RATIONAL NUMBERS

Most of us learned about percents through fractions and decimals. 25% means $\frac{25}{100}$ or .25 and 4.6% = $\frac{4.6}{100} = \frac{46}{1000}$ = .046.

The rule for changing a percent to a decimal is, "Move the decimal point two places to the left and drop the %" and for changing a decimal to a percent, "Move the decimal point two places to the right and add the %." Now, if <u>all</u> the computation with decimals is clear, a percent exercise reduces to a decimal computation. The understanding of percents as decimals is useful when using a calculator. The percent key on a calculator usually moves the decimal point two places to the left. To enter 35.7% push $\boxed{3}$, $\boxed{5}$, $\boxed{\cdot}$, $\boxed{7}$ then $\boxed{\%}$. The calculator will read .357. The question "What percent of 80 is 50?" translates to <u>N</u> x 80 = 50 or N = 50 ÷ 80. A calculator will give 50 ÷ 80 as .625000. This number must then be changed to a percent: .625000 ⟶ 62.5%.

To change a percent to a fraction, we write the percent number over 100 and drop the percent sign. Changing a fraction to a percent is not so simple. We usually convert the fraction to a decimal ($\frac{2}{3} \approx .667$) and then to a percent (66.7%).

Knowing the fraction equivalent for certain percents is very helpful. If 25% is exchanged for the fraction $\frac{1}{4}$ and approximation is used, the question 25% of 83 is __?__ becomes an easier mental exercise: 25% of 83 $\approx \frac{1}{4}$ of 84 = 21.

We can help students learn the decimal and fraction equivalents for percents with manipulatives, 100-grids, number lines and various skill-building activities. See the classroom activities *The Percent Bar Sheet, Hallelujah I've Been Converted, Fractions → Percent 1* and *Fractions → Percent 2.*

SOLVING PERCENT PROBLEMS

Word problems involving percent are usually confusing to students. They can work a series of problems when an example is given and the problems are of the same type as the example, but when word problems of different types are mixed, they can't decide what operation or proportion to use. If we encourage students to spend time understanding the problems before manipulating the given numbers to find "answers," their ability to solve problems might improve.

Suggested Activities

1. Put two simple percent word problems on the board or overhead. Ask the students if the problems could be worked in the same way or not. The problems (a) and (b) at the right could both be solved by taking the given percent of the given number.

a) 50 math problems. Got 90% correct. How many correct?

b) 30 girls. 20% are blond. How many are blond?

Now add a few more simple problems like (c), (d) and (e) as shown. Ask if any of these can be solved like problems (a) and (b). Some students might pick problem (d) but notice the extra step in (d); subtraction is also involved.

c) Played 10 games. Won 6. What percent of the games were won?

d) Regular price \$9. Sale is 10% off. What is the sale price?

e) 40 trading cards. 12 are baseball cards. What percent are baseball cards?

Are any two of the new problems solved in the same way? Yes. (c) and (e) are both solved by finding what percent one given number is of the other.

2. After comparing and discussing problems of various types and complexity, students can be asked to match problems which would be worked in the same way. There are 6 types of percent problems given below; two of each type. Each problem on the left can be matched with one on the right. This is <u>not</u> an introductory activity--it is necessary that students have previously solved and discussed each of these kinds of problems. Whether students have learned to solve percent problems with the proportion method or by changing the percent to a decimal or fraction should not affect their matching of the problems. To keep confusion to a minimum you could sort the problems into 2 groups with 3 types per group. The "matches" are given below.

a. If the annual interest rate is 7%, find a year's interest on $1200.

b. Chris spent 40% of her savings for a bike. If the bike cost $60.00, how much was in her savings before she bought the bike?

c. A school has 5500 books. 45% are being used. How many are <u>not</u> being used?

d. A city has 12,000 people. Its population is expected to increase 15% in 10 years. What is its expected population in 10 years?

e. There are 30 teachers for a junior high. 18 are women. What percent of the teachers are women?

f. A radio was originally $60. Its reduced price is $48. What is the percent of the reduction?

_____ 1. Jon took $8 to the fair and came home with $3. What percent of his money did he spend?

_____ 2. A class has 30 students. 55% are girls. How many are boys?

_____ 3. A babysitter receives a 10% raise. He was making 80¢ an hour. How much does he make per hour now?

_____ 4. A basketball team won 60% of its games. If 12 games were won, how many were played?

_____ 5. A record is regularly $5.98. It is on sale at 15% off. How much is saved by buying it on sale?

_____ 6. Jay was given 8 problems for homework. He got 6 problems right. What percent of the problems were right?

Match a to 5, b to 4, c to 2, d to 3, e to 6 and f to 1.

RATIO, PROPORTION, AND SCALING

Placement Guide for Tabbed Divisors

CONTENTS

PERCENT: PERCENT SENSE

*Indicates percents greater than 100% are used on the page.

TITLE	PAGE	TOPIC	TYPE
Percent with Cubes	397	Reference set of 100* Set model	Manipulative
The Percent Painter	398	Reference set of 100 Set model	Manipulative
Hundreds Board Percent	399	Reference set of 100 Set model	Manipulative
Percent with Rods & Squares - I	400	Reference set of 100 Grid model	Manipulative
Percent with Rods & Metres - I	401	Reference set of 100* Number line model	Manipulative
Elastic Percent Approximator	402	Reference set of 100 Number line model	Manipulative
Percents of Line Segments	404	Reference set of 100* Number line model	Chalkboard Transparency Paper and pencil
Activity Cards - Number Line	405	Number line concepts	Manipulative
Percenting: Line Segments	409	Reference set of 100 Number line model	Paper and pencil
Stringing Along with Percents	410	Reference set of 100* Number line model	Manipulative
Percents of Rectangles	411	Area model*	Paper and pencil Chalkboard
Rectangle Percents	412	Area model*	Transparency Paper and pencil
Percents of an Orange Rod	413	Reference set of 100* Number line model	Manipulative
Percents: Backwards and Forwards 1	414	Number line model*	Paper and pencil Transparency
Percents: Backwards and Forwards 2	415	Area model*	Paper and pencil Transparency

*Indicates percents greater than 100% are used on the page.

TITLE	PAGE	TOPIC	TYPE
Percents: Backwards and Forwards 3	416	Volume model*	Paper and pencil Transparency
Percents: Backwards and Forwards 4	417	Models*	Paper and pencil Transparency
Geoboard Percents	418	Area model	Manipulative
The Whole Thing	420	Set model	Paper and pencil
Finding 100% From Below	421	Area model	Paper and pencil Transparency
Finding 100% From Above	422	Area model*	Paper and pencil Transparency
Peace-N-Order	423	Area model*	Paper and pencil Transparency
You Are What You Eat	424	Percent sense*	Paper and pencil Puzzle
Changing Percent Shapes	425	Other reference sets	Activity

*Indicates percents greater than 100% are used on the page.

REFERENCE SET

Prepared transparencies are a convenient means for an introduction to percent. The same transparencies can provide a review of the basics of percent. The transparencies that follow and the suggestions below can be used for this introduction.

Teacher Strategy

Tell students that percents are easiest to understand when they are based on a reference set of 100. Write REFERENCE SET on the board (or overhead) and say that this reference set might be the number 100, 100 squares, 100 pennies, or 100 centimetres, or any other convenient set.

Square Grid Transparency

Use the first transparency, showing only the 10 x 10 grid. Talk about a 10 x 10 array as a convenient (but not the only) way of arranging 100 little squares. This 10 x 10 grid will be the reference set (which we'll call R) for this transparency. Uncover the entire top row. Tell the class that each of the figures is a certain percent of the Reference Set R. Write "20% of R" under the first figure and ask the students to raise their hands if they know what should be written under the next figure.

Students then volunteer phrases to place under the remaining figures in the top row. Ask them to describe what 50%, 60%, 90% of R looks like. At this point discuss counting the squares to determine the percent of R shown. Emphasize that when the reference set (R) is 100 objects, N% of R is N of the objects.

Ask them if they know what 100%, 120%, and 200% of R looks like. Have students look at the second row and discuss and label each of the figures. The last problem brings out different arrangements of 20% of R. Emphasize that 20% of R is less than R, 100% of R is the same as R, and 120% of R is greater than R.

The third and then fourth rows of figures can then be uncovered and labeled with percents of R.

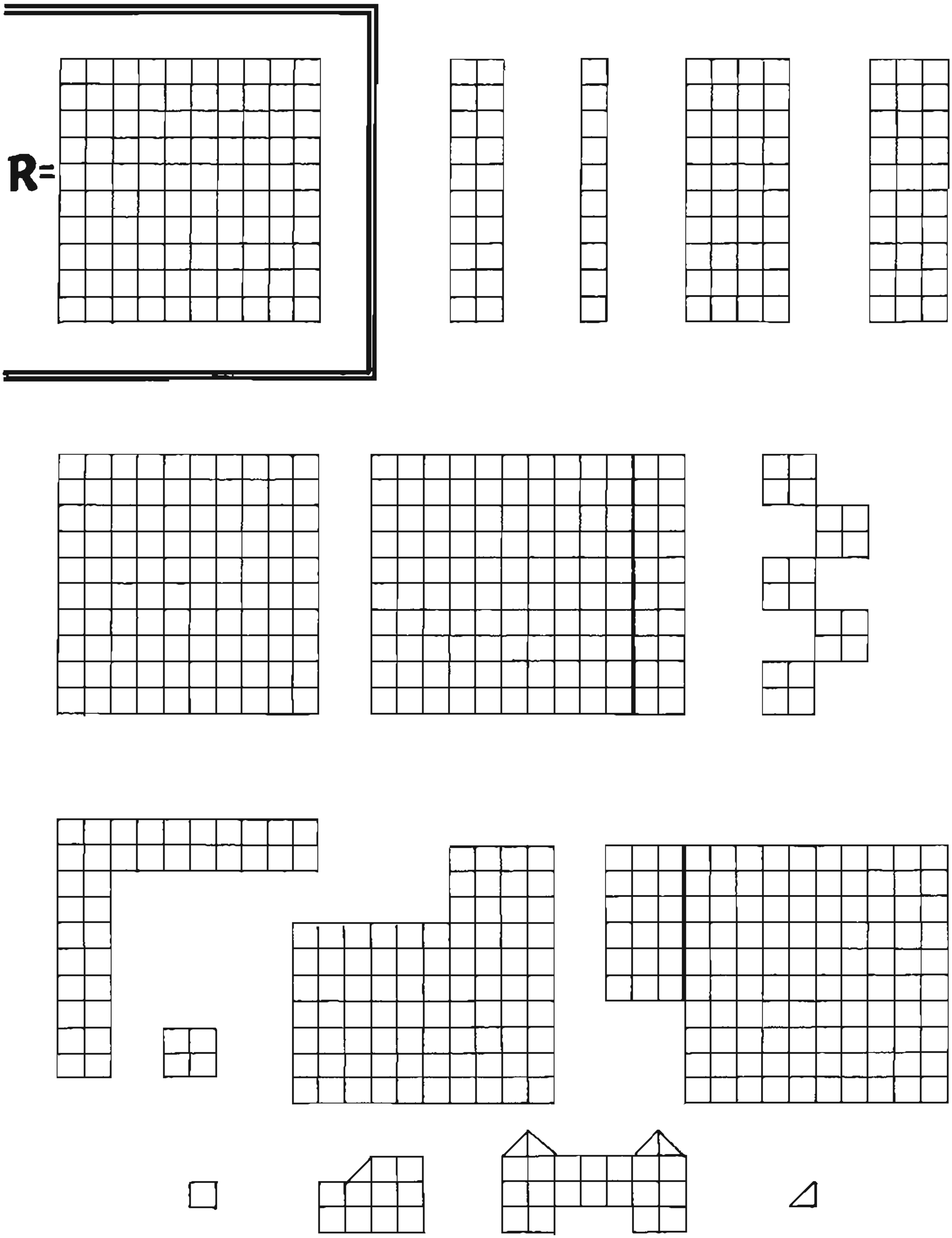
Line Segment and Dollar Transparency

To give students other models for percent look at the line segment and dollar transparency. Ask students to describe how to draw N% of the line segment R for various values of N (see transparency).

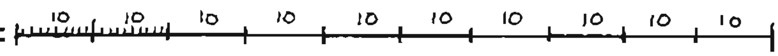
Other Grids Transparency

Look at the unusual 100 grids briefly to reinforce the idea that the reference set of 100 could be things other than squares, line segments, or pennies.

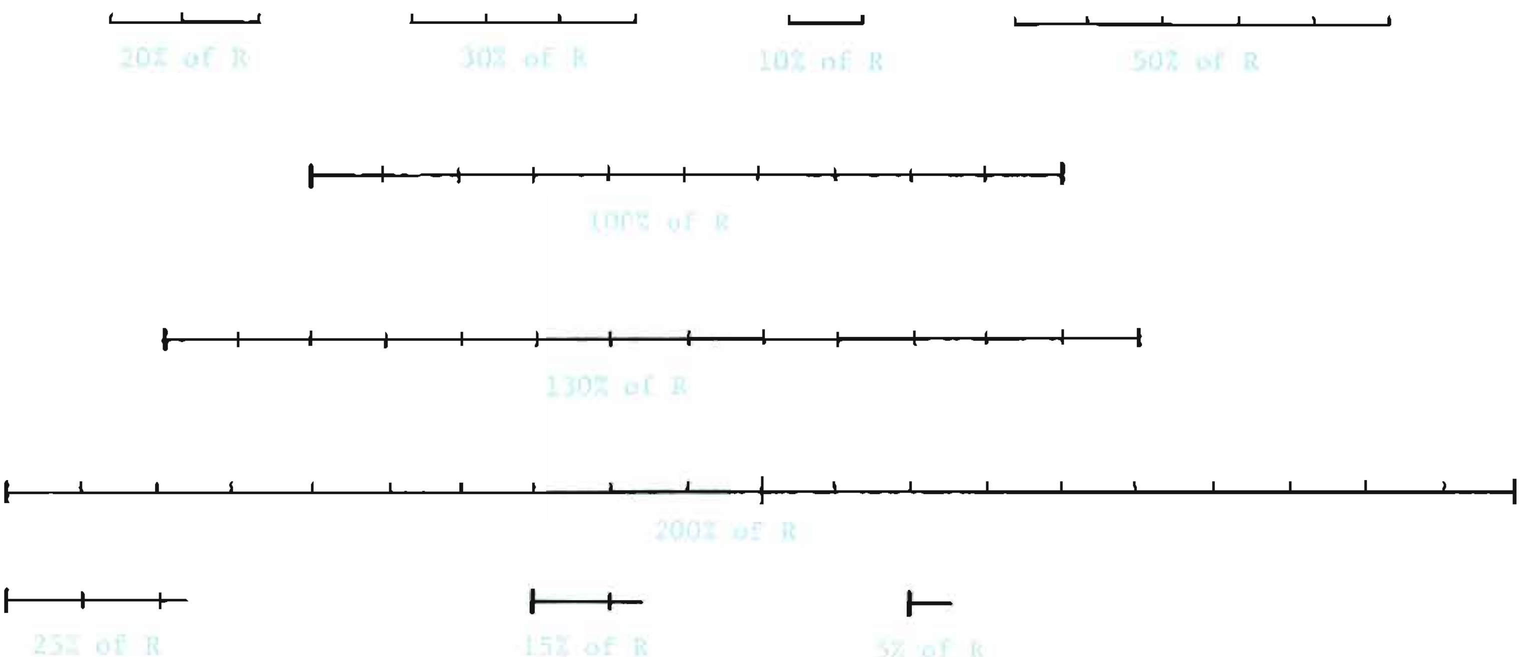
REFERENCE SET (PAGE 2)



REFERENCE SET (PAGE 3)

R = 

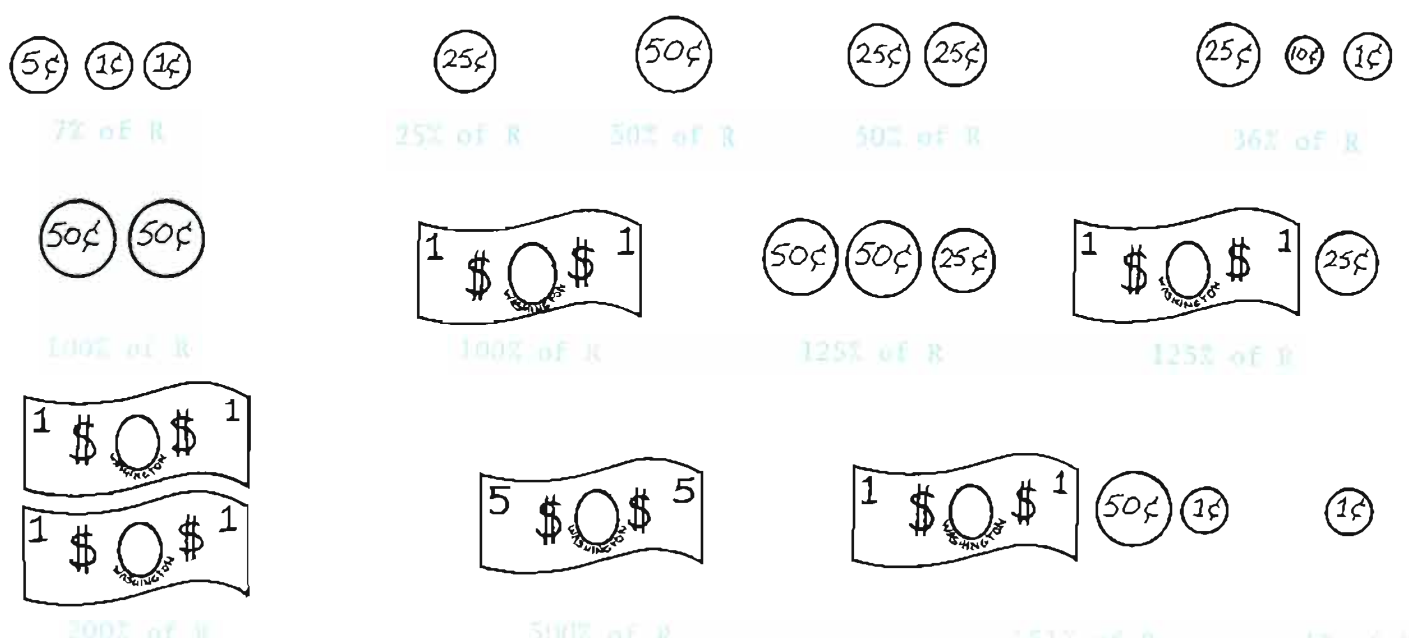
100 UNITS



REFERENCE SET

R =  = ONE DOLLAR

(100 PENNIES)



"PERCENT MEANS"

Word Phrases
Percent Sense
PERCENT

E S I

15 RED APPLES FOR EVERY 100 APPLES IS 15% RED APPLES.
45 BOYS FOR EVERY 100 STUDENTS IS 45% BOYS.

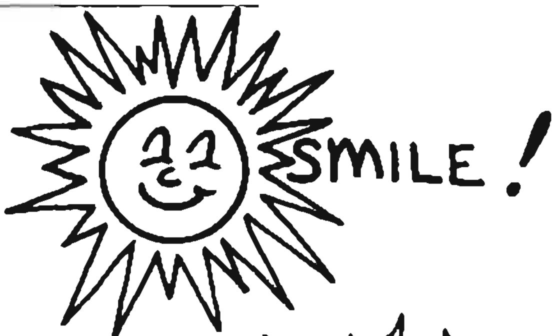
WRITE EACH OF THESE AS A PERCENT.

_____ A) 30 DAYS OUT OF 100 DAYS
_____ B) 65¢ COMPARED TO 100¢
_____ C) 40 CM PER 100 CM
_____ D) 12 BLONDS FOR EVERY 100 PEOPLE
_____ E) 1 ROTTEN APPLE FOR EVERY 100 APPLES
_____ F) 73 SHADED SQUARES COMPARED TO 100 SQUARES
_____ G) 83 PROBLEMS CORRECT FOR EVERY 100 PROBLEMS
_____ H) 98 YES VOTES PER 100 VOTES
_____ I) 21 GREEN CARS FOR EVERY 100 CARS
_____ J) 8 MISSPELLED WORDS OUT OF 100 WORDS

DESCRIBE EACH OF THESE SITUATIONS IN OTHER WAYS.

K) 40 BLUE MARBLES, 100 MARBLES IN ALL
L) 30 POODLES, 100 POODLES IN ALL
M) 15 CHINESE, 100 PEOPLE IN ALL
N) 80 CORRECT ANSWERS, 100 QUESTIONS IN ALL
O) 55 DOLLARS, 100 DOLLARS IN ALL
P) 8 TREES DEAD, 100 TREES IN ALL

..PE: Transparency/Bulletin Board

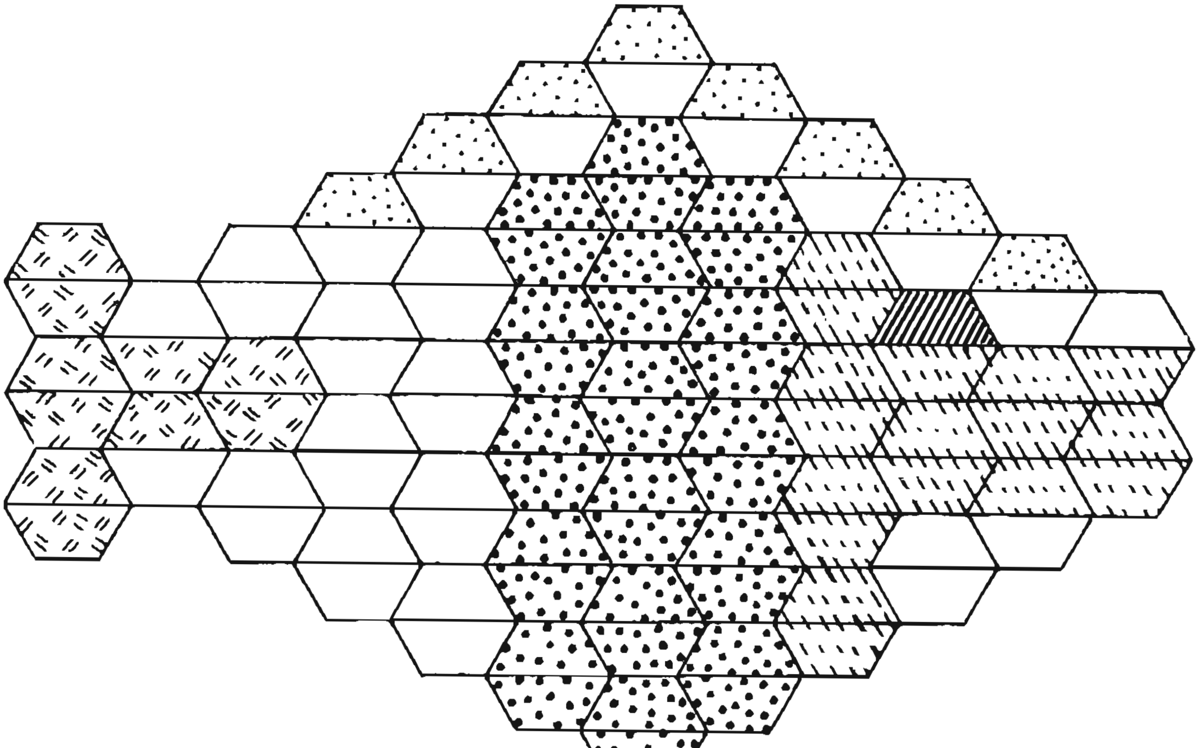


START EACH DAY WITH A SMILE AND GET...

I	T		O	V	E	R		W	I	T	H
5	8		4	10	1	6		7	2	9	3

	less than R	equal to R	greater than R
1. 30 % of R is	E	T	W
2. 85 % of R is	I	R	S
3. 100 % of R is	E	H	R
4. 150 % of R is	M	P	O
5. 13 % of R is	I	N	B
6. 225 % of R is	D	H	R
7. 100 % of R is	K	W	L
8 500 % of R is	S	C	T
9. $66\frac{2}{3}$ % of R is	T	Y	F
10. 101 % of R is	G	J	V

UNUSUAL 100 GRIDS - I

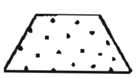


What percent of the grid is shaded:

a)  ? ______%

b)  ? ______%

c)  ? ______%

d)  ? ______%

e)  ? ______%

f)  ? ______%

Without counting, what percent of all the shapes are in this position  ?

________%

Color each percent of the grid as indicated.

a) 19% blue

b) 12% red

c) 21% green

d) 8% brown

e) 28% orange

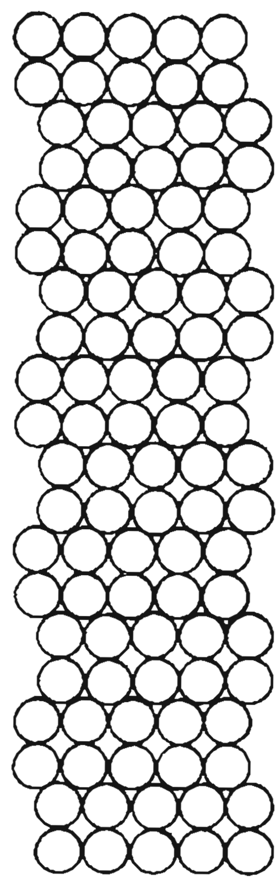
f) 3% purple

g) 8% yellow

h) 1% black

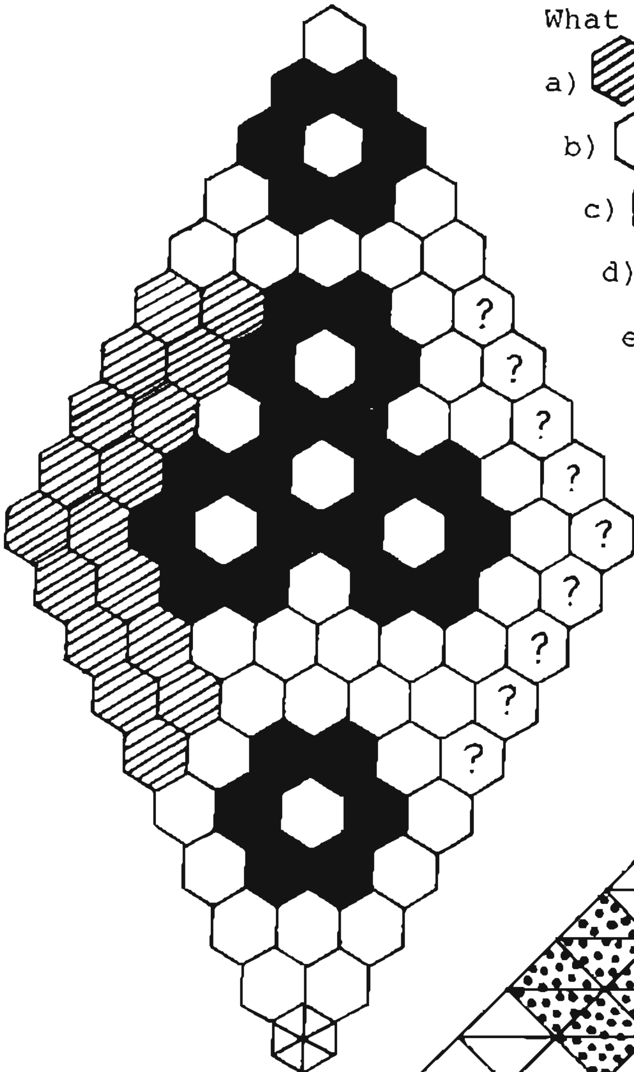
i) What % of the grid is colored? _______%

j) What % of the grid is <u>not</u> colored? ______%



GRIDS FROM: *The Metric System of Measurement*

UNUSUAL 100 GRIDS - II



What percent of the hexagons are:

a) ? ______%

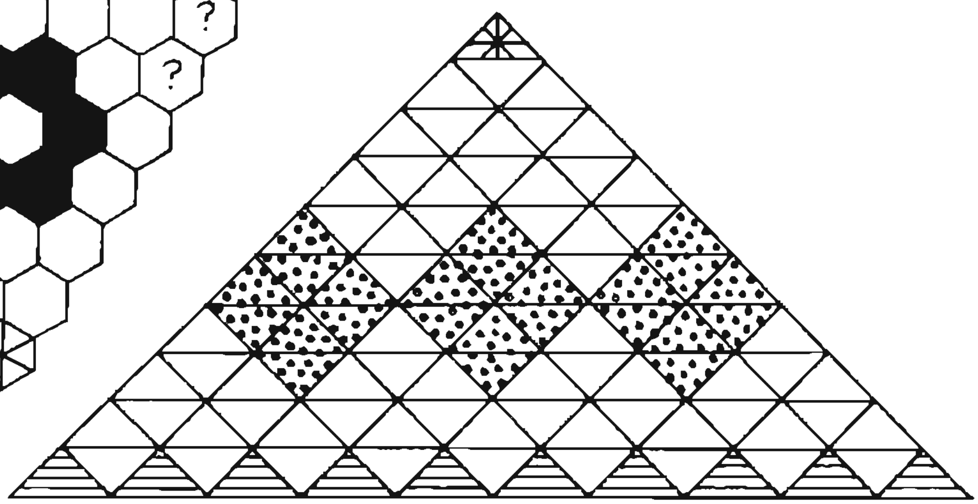
b)  ? ______%

c)  ? ______%

d)  ? ______%

e) touching a  on all 6 sides? ______%

f) touching a  on only 4 sides? ______%



What percent of the triangles look like:

a)  ? ______%

b)  ? ______%

c)  ? ______%

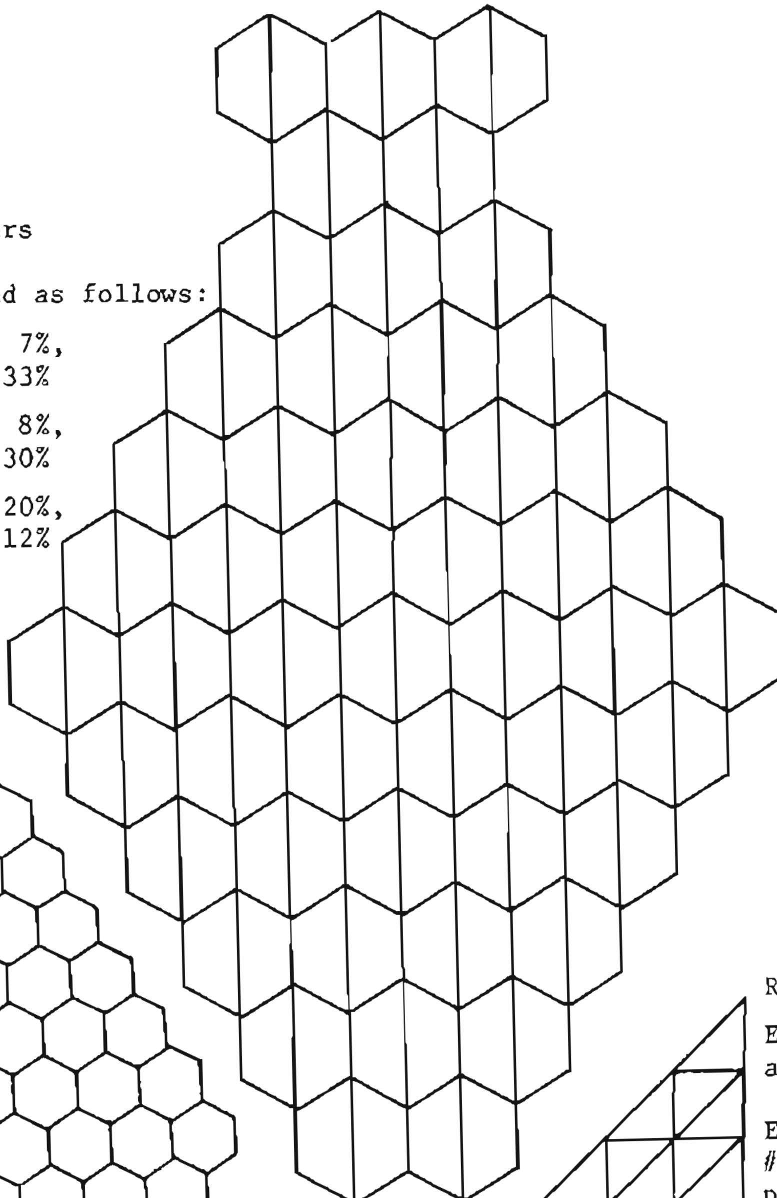
d)  ? ______%

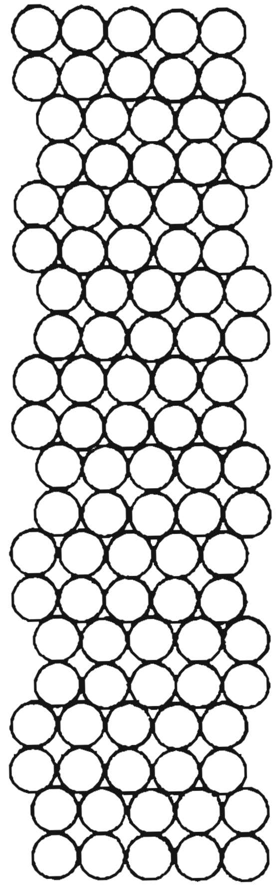
Needed: 2 to 4 players
Game grids
3 dice marked as follows:

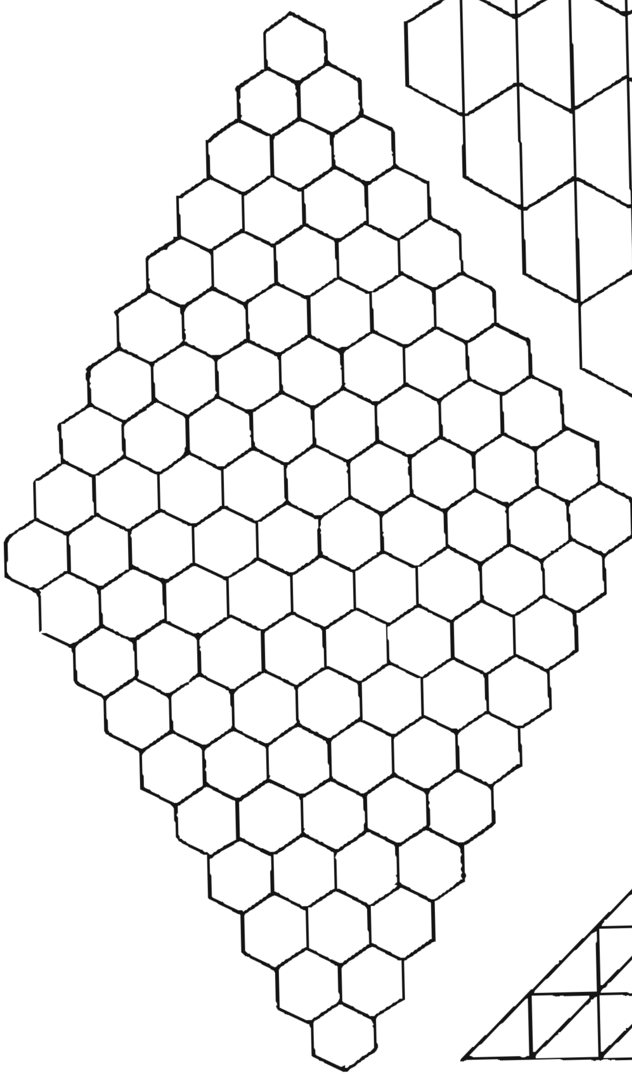
#1 marked 1%, 3%, 7%, 18%, 21%, 33%

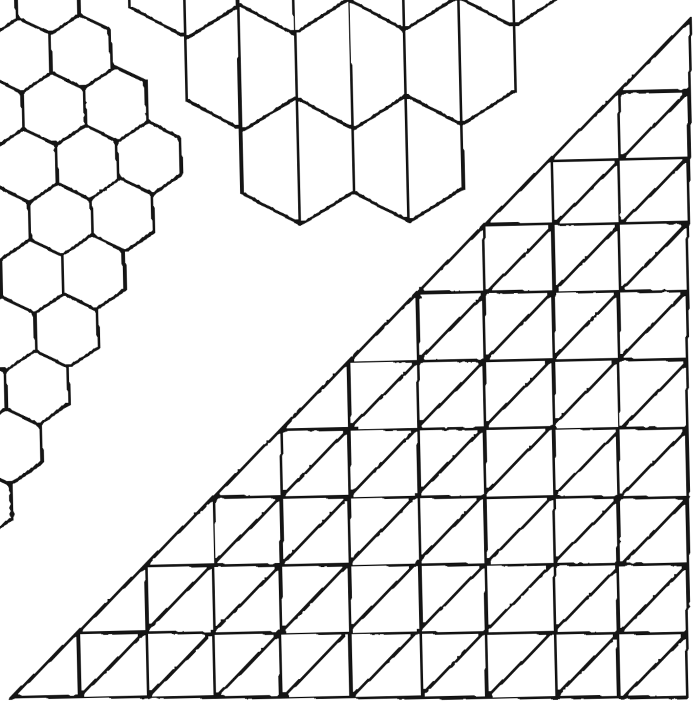
#2 marked 2%, 4%, 8%, 19%, 27%, 30%

#3 marked 5%, 9%, 20%, 13%, 6%, 12%









Rules:

Each player selects a grid to fill up.

Each player rolls #1 die. Highest percent goes first.

Each player rolls all dice and selects one amount to shade on his grid.

If all amounts are too large, no part may be shaded.

Winner is the first player to exactly fill his grid.

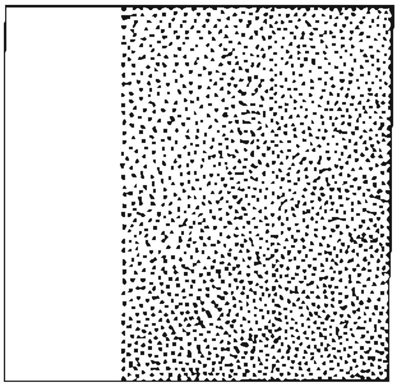
IDEA FROM: *Enhance Chance*

GRIDS FROM: *Metric System of Measurement*

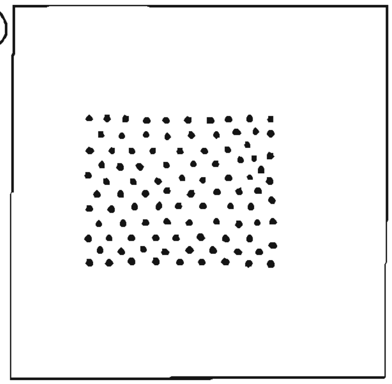
GUESS AND Check

In each problem the REFERENCE SET (R) is the large square.

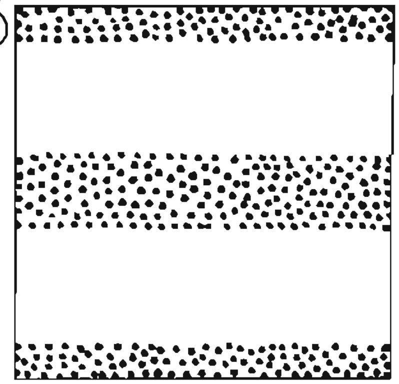
First, approximate the percent of R that is shaded. Then, using the transparent 100 grid, find the exact percent of R that is shaded.

1


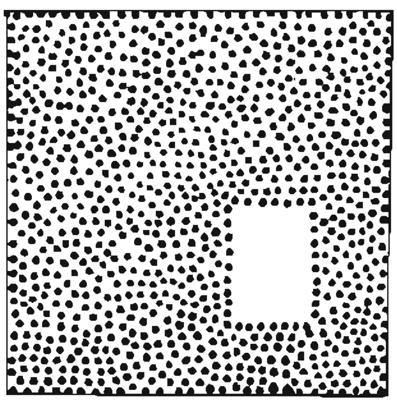
Guess: _____ % of R shaded
Exact: _____ % of R shaded

2


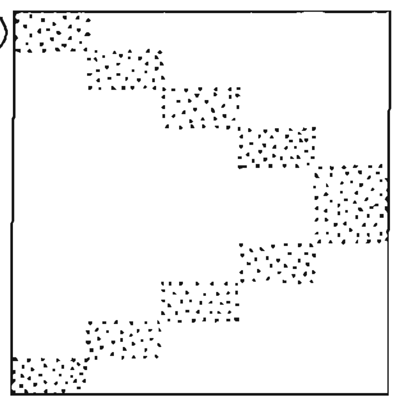
Guess: _____ % of R shaded
Exact: _____ % of R shaded

3


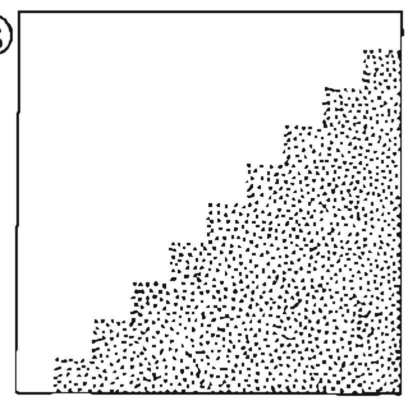
Guess: _____ % of R shaded
Exact: _____ % of R shaded

4


Guess: _____ % of R shaded
Exact: _____ % of R shaded

5


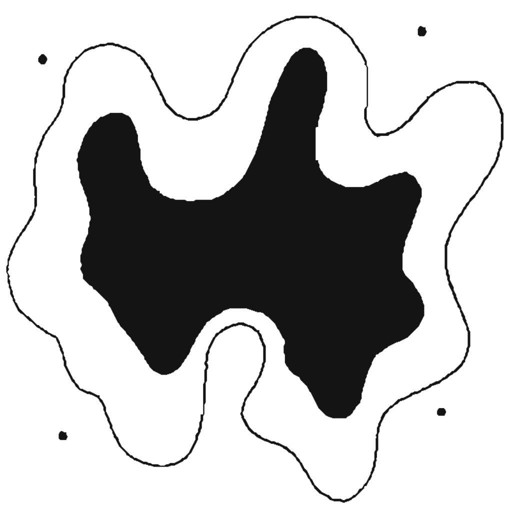
Guess: _____ % of R shaded
Exact: _____ % of R shaded

6


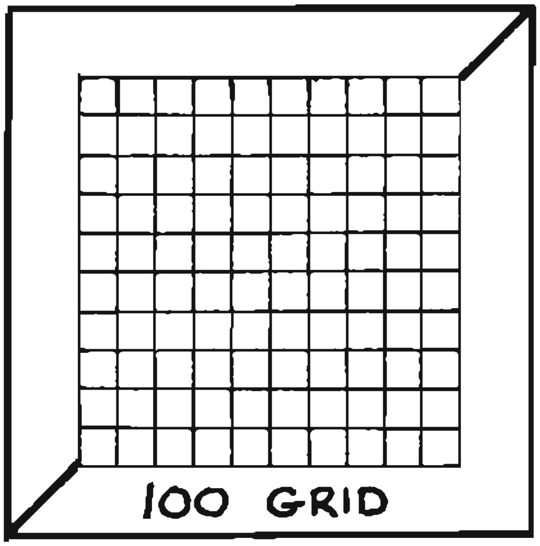
Guess: _____ % of R shaded
Exact: _____ % of R shaded

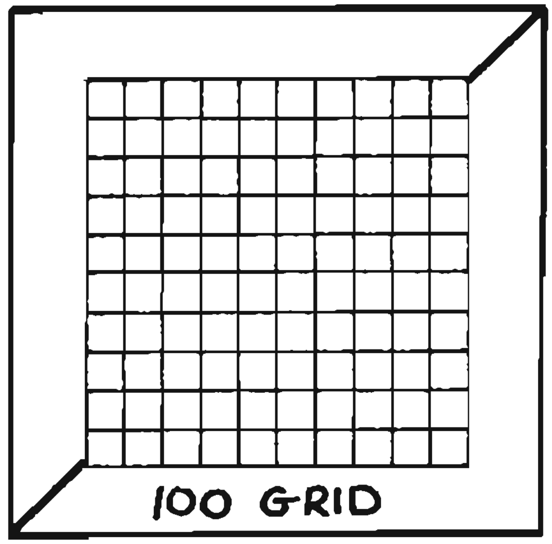
7 On how many of problems 1-6 was the % you guessed within 5 of the exact %? _______

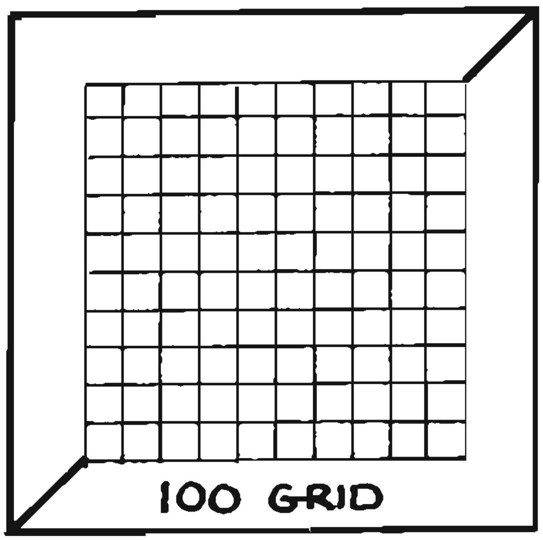
8 This blob has about the same area as the reference sets above. Approximate the percent of the blob that is shaded.
_______%

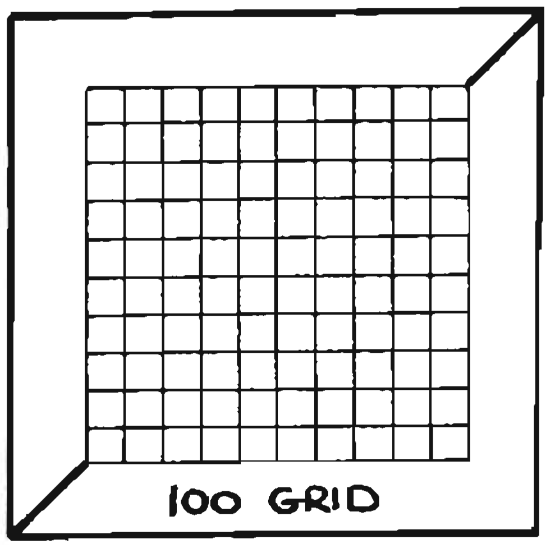


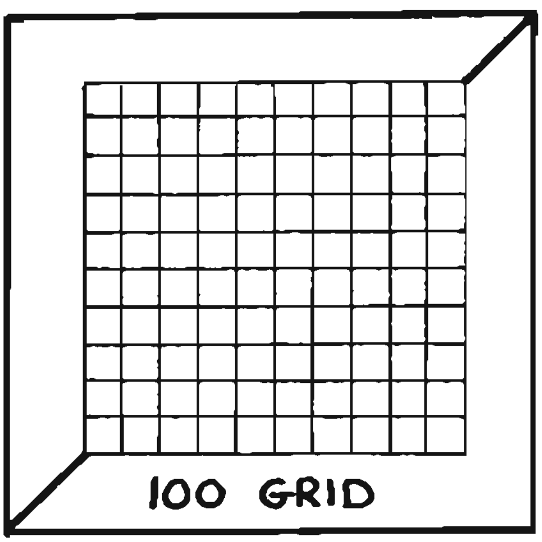
TRANSPARENT 100 GRIDS

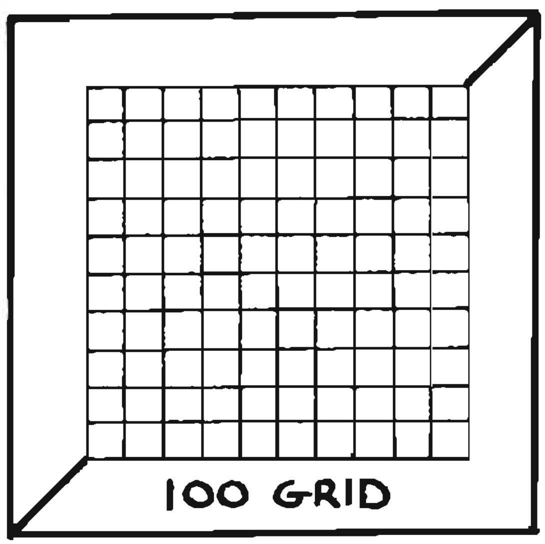






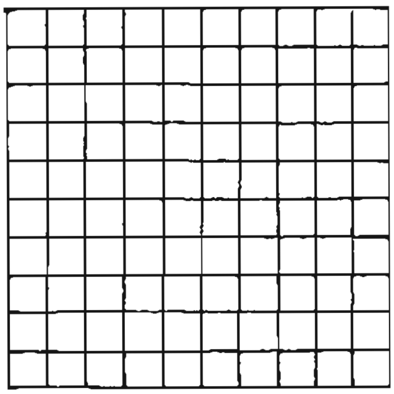




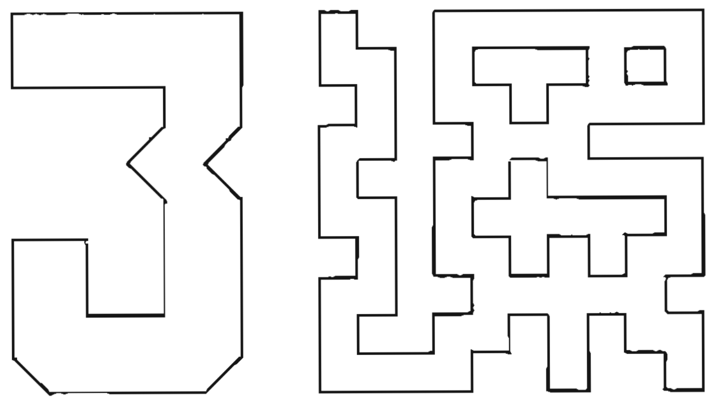


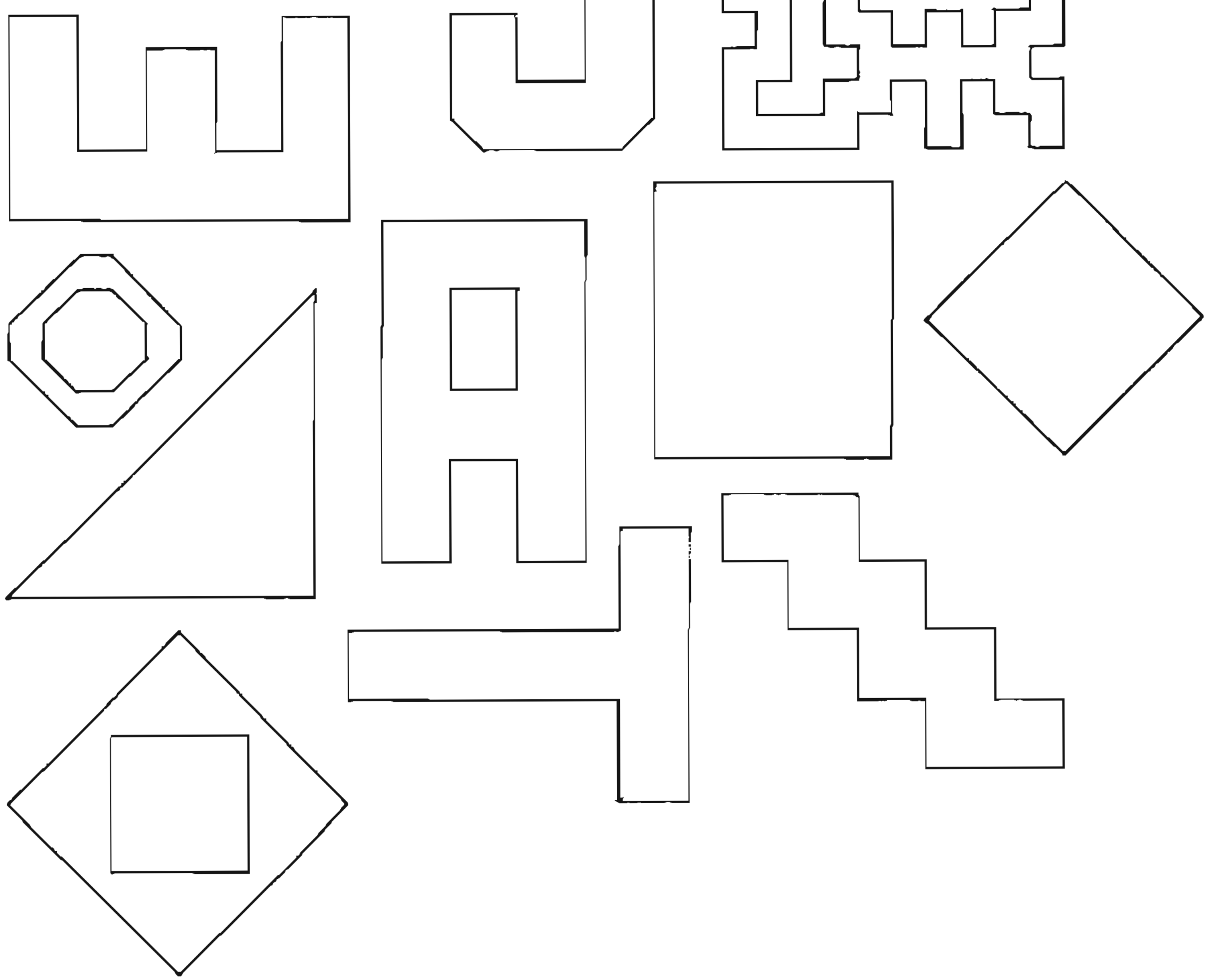
THE TRANSPARENT HUNDRED

THIS IS A HUNDRED GRID.

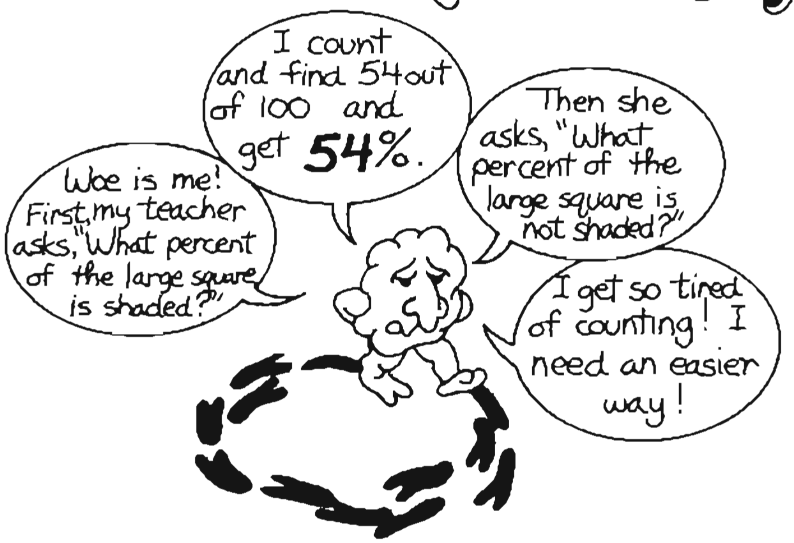


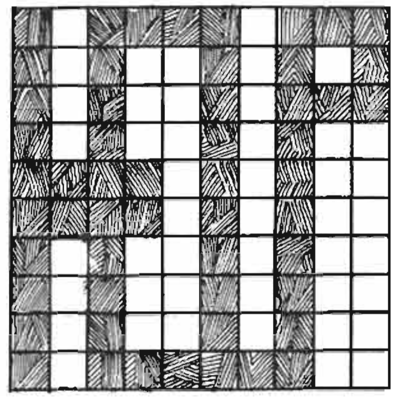
GUESS WHAT PERCENT OF THE HUNDRED GRID EACH SHAPE BELOW REPRESENTS. WRITE YOUR GUESS INSIDE THE SHAPE. THEN USE THE TRANSPARENT 100 GRID TO FIND THE ACTUAL PERCENT.





HELP!!

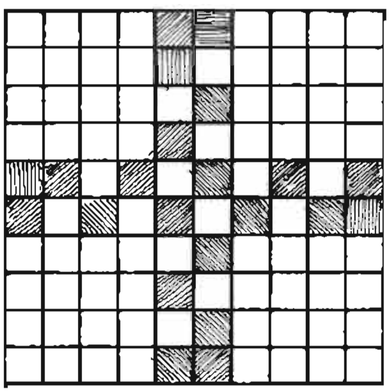




_____% of the square is shaded

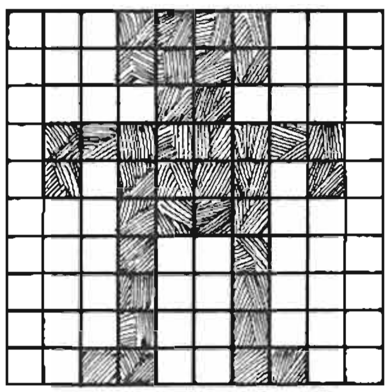
_____% of the square is not shaded

Solve these problems. Try to find the easy way.



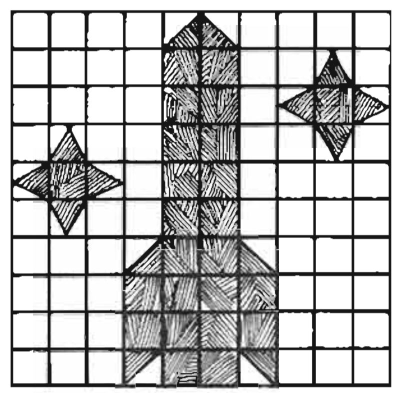
_____% is shaded

_____% is not shaded



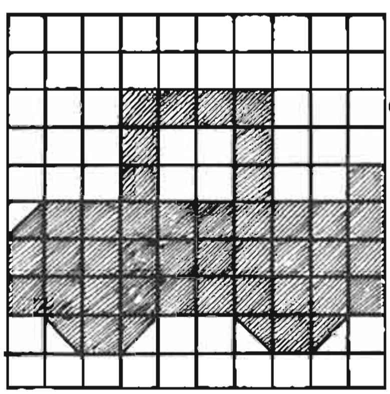
_____% is shaded

_____% is not shaded



_____% is shaded

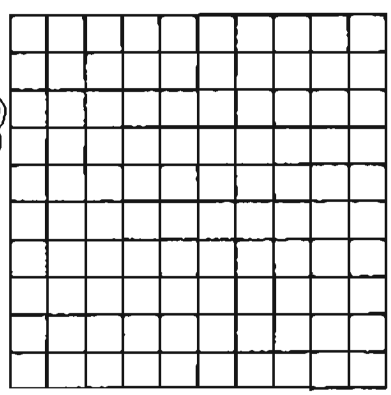
_____% is not shaded



_____% is shaded

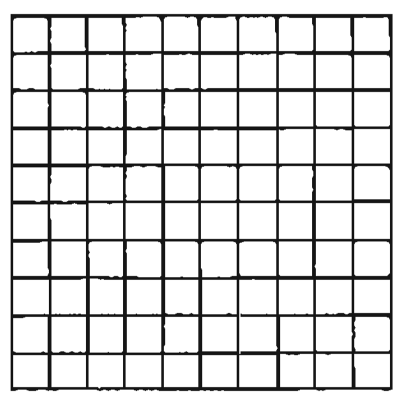
_____% is not shaded

Make your own designs on these



_____% is shaded

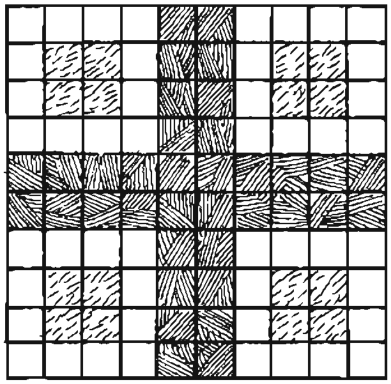
_____% is not shaded



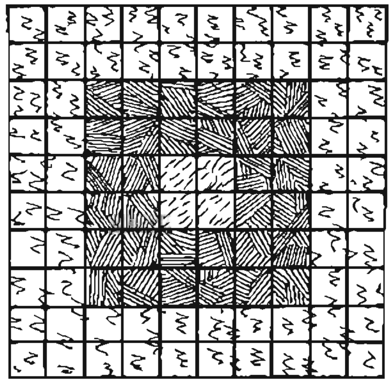
_____% is shaded

_____% is not shaded

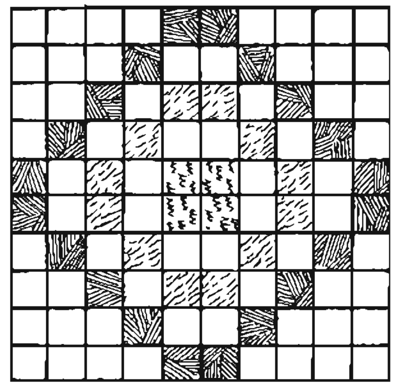
HELP! (Continued)



_____% is like [shading 1]
_____% is like [shading 2]
_____% is like [shading 3]
_____% is not shaded



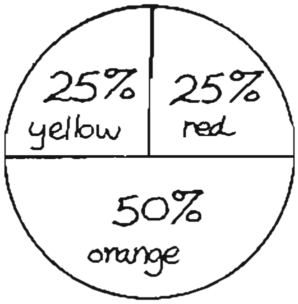
_____% is like [shading 1]
_____% is like [shading 2]
_____% is like [shading 3]
_____% is not shaded

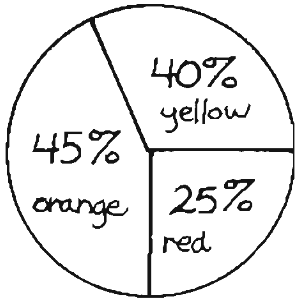


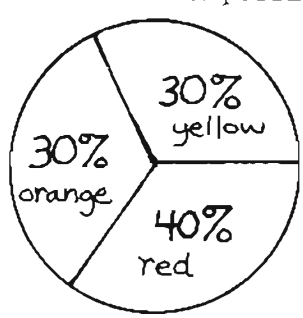
_____% is like [shading 1]
_____% is like [shading 2]
_____% is like [shading 3]
_____% is not shaded

COLOR	Square 1	Square 2	Square 3	Square 4	Square 5	Square 6	Square 7	Square 8	Square 9
RED	50%	25%		$12\frac{1}{2}$%	15%			20%	
ORANGE	10%		25%	$12\frac{1}{2}$%	$20\frac{1}{2}$%	33%		75%	
YELLOW	20%	30%	10%	15%			32%	35%	
NOT COLORED		15%	65%		35%	27%	24%		
TOTAL PERCENT									100%

Two of these circle graphs have sensible percents. See if you can find the circle with percents that are wrong. Explain why these percents are impossible.





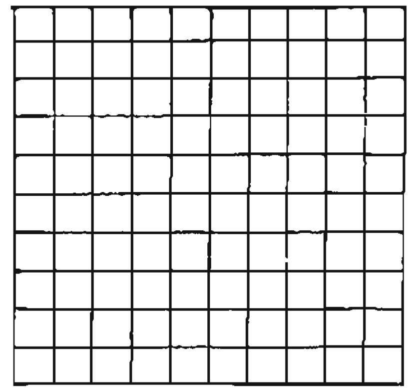


STICKING TOGETHER WITH PERCENTS

EQUIPMENT: 1 SHEET OF GRID PAPER, SCISSORS, PASTE

ACTIVITY:

R =



1. A) CUT A PIECE OF THE GRID PAPER TO SHOW 100% OF R. CUT A PIECE OF THE GRID PAPER TO SHOW 50% OF R.
   B) SLIDE THE 2 PIECES TOGETHER. HOW MANY SQUARES DO YOU HAVE IN ALL? _____ WHAT PERCENT OF THE 100 SQUARES OF R DO YOUR 2 PIECES SHOW? _____
   C) ON ANOTHER PIECE OF PAPER PASTE THE 2 PIECES BESIDE EACH OTHER AND LABEL THIS 150% OF R.
   D) THE SENTENCE BELOW SHOWS WHAT YOU HAVE DONE.
   100% OF R + 50% OF R = 150% OF R.

2. A) CUT A PIECE TO SHOW 100% OF R. CUT A PIECE TO SHOW 35% OF R.
   B) HOW MANY SQUARES IN ALL? ____
   C) PASTE THE PIECES AND LABEL.
   D) WRITE A SENTENCE TO DESCRIBE THIS.

3. A) CUT AND PASTE 2 PIECES TO SHOW 110% OF R.
   B) WRITE A SENTENCE TO DESCRIBE THIS.
   C) DO THIS AGAIN BUT DO NOT USE A 100% PIECE OF R.
   D) WRITE A SENTENCE TO DESCRIBE THIS.

4. A) CUT AND PASTE 2 PIECES TO SHOW 200% OF R.
   B) WRITE A SENTENCE TO DESCRIBE THIS.
   C) CUT AND PASTE 3 PIECES TO SHOW 200% OF R.
   D) WRITE A SENTENCE TO DESCRIBE THIS.

YOUR BODY PERCENTS

Let a metre (100 cm) be your reference set.

What percent of a metre is:

a) your span  ____________

b) your palm  ____________

c) your little finger length  ____________

d) your foot length (without shoes)  ____________

e) your foot length (with shoes) ____________

f) the distance between the centers of your eyes  ____________

g) your arm span  ____________

h) your height ____________

i) distance around your waist  ____________

j) distance around your head  ____________

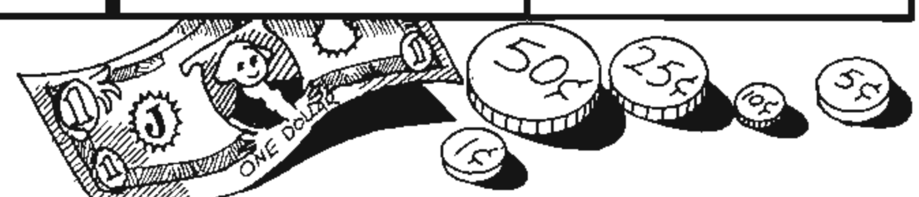
PERCENTS IN YOUR CLASSROOM

Let a metre (100 cm) be your reference set.

Find:	in cm	% of a metre
a) the height of the wastepaper basket	________	________
b) the distance across the wastepaper basket	________	________
c) the length of the flag (or door)	________	________
d) the width of the flag (or door)	________	________
e) the width of your teacher's desk.	________	________
f) the length of the gradebook (or light switch plate)	________	________
g) the width of the gradebook (or light switch plate)	________	________
h) the length of a piece of chalk	________	________
i) the width of the filing cabinet (or bulletin board)	________	________

DOLLAR$ AND PER¢ENTS 1

COINS	PERCENT OF 1 DOLLAR	COINS	PERCENT OF 1 DOLLAR
1 dime		3 half-dollars	
1 quarter		5 quarters	
1 penny		13 dimes	
1 half-dollar		29 nickels	
1 nickel		214 pennies	
3 nickels		3 quarters, 6 dimes, and 4 pennies	
4 dimes		6 dimes, 4 pennies, and 3 quarters	
22 pennies		4 pennies, 3 quarters, & 6 dimes	
3 quarters		2 of each coin shown	
2 half-dollars		3 of each coin shown	
2 dimes & 3 nickels		5 of each coin shown	
3 quarters and 2 pennies			
6 dimes and 1 quarter			
1 half-dollar, 3 dimes, & 1 nickel			
1 of each coin shown			



DOLLAR$ AND PERCENTS 2

Show three different ways to make 30% of a dollar.

____

____

____

Show four different ways to make 120% of a dollar.

____

____

____

____

What are the fewest number of coins needed to make:

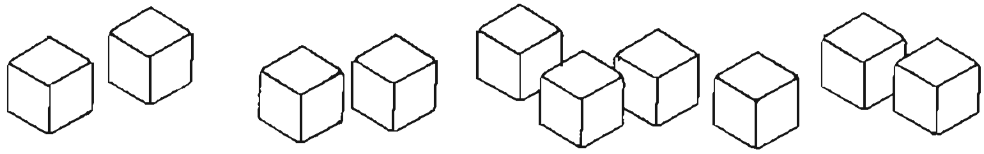
4% of a dollar ________	20% of a dollar ________	137% of a dollar ________
55% of a dollar ________	82% of a dollar ________	98% of a dollar ________

CHALLENGE:

I have items to sell that cost from 1¢ up to 99¢. A customer gives me a $1 bill. What is the fewest number of coins I must have in the cash register to give change to the customer regardless of what she buys? What is the total value of these coins?

Percent With Cubes

Your reference set for this activity is 100 unit cubes.



Build models with these dimensions (if you can).
What percent of the 100 cubes do you use in each case?

Build models with these dimensions.
What percent of the 100 cubes do you use in each case?

a) 4 cubes long, 3 cubes wide, 2 cubes high _____%

b) 6 cubes long, 5 cubes wide, 3 cubes high _____%

c) 5 cubes long, 10 cubes wide, 2 cubes high _____%

d) 5 cubes long, 6 cubes wide, 5 cubes high _____%

e) 2 cubes long, 1 cube wide, 1 cube high _____%

Percent of 100 cubes used	Dimensions of the model		
	Length	Width	Height
50% (all dimensions different)	____	____	____
64% (all dimensions same)	____	____	____
36% (two dimensions the same)	____	____	____
60%	____	____	____
60% (another way)	____	____	____
110%	____	____	____
81% (all dimensions different)	____	____	____
1%	____	____	____
$\frac{1}{2}$%	____	____	____

THE PERCENT PAINTER

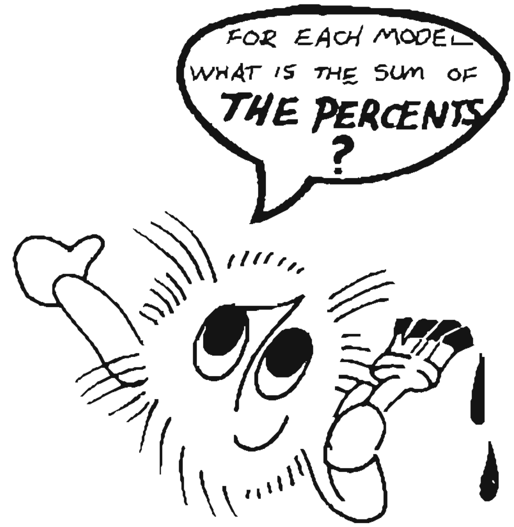
Get 100 cubes to make each of these models or answer the questions by looking at the diagrams.

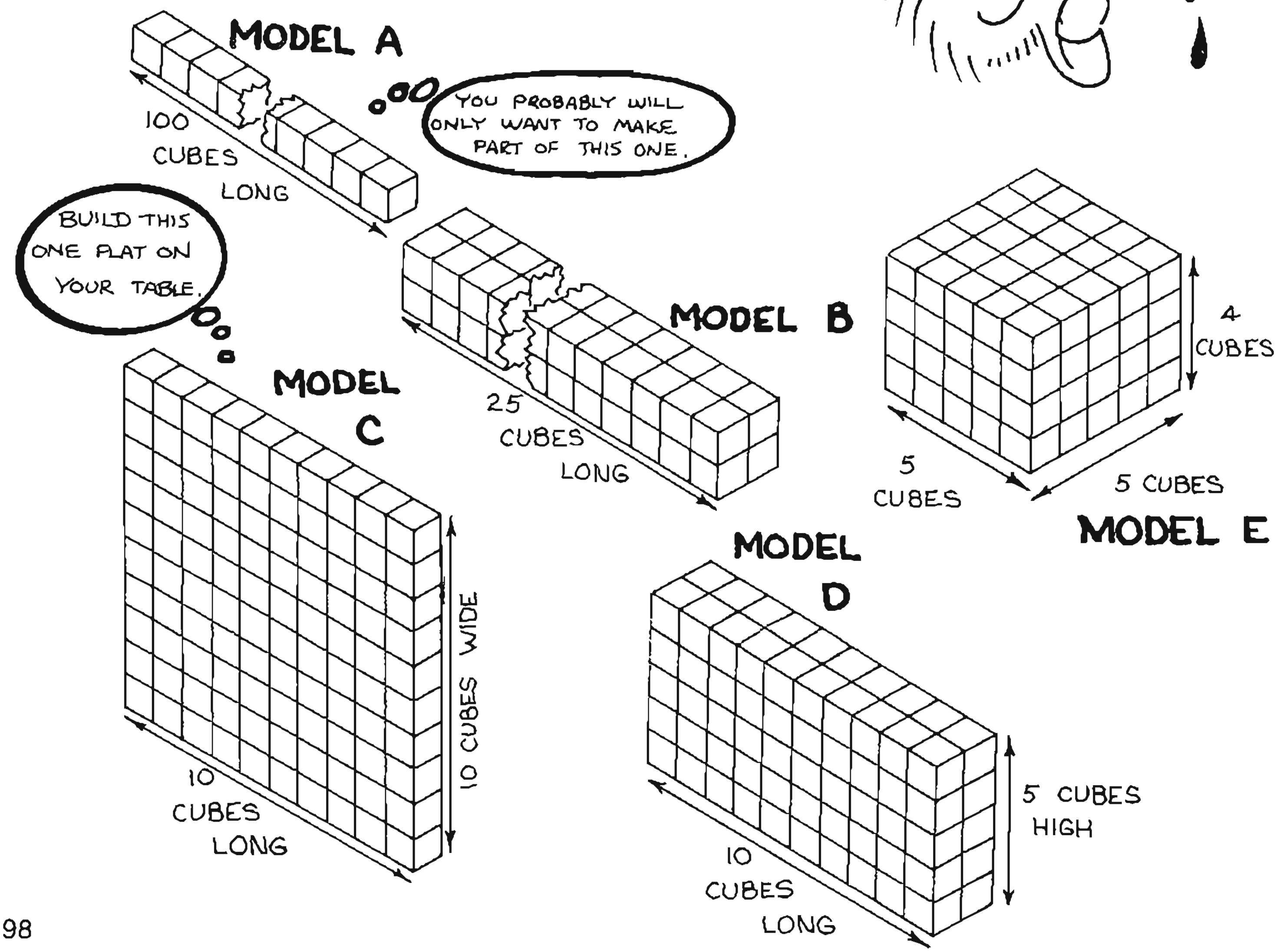
Suppose the percent painter was able to paint the entire surface, including the bottom, of the model. Fill in the table for each of the models that you make.

Make a model of your own.

What percent of the cubes would have:

MODEL	A	B	C	D	E	yours
6 faces painted						
5 faces painted						
4 faces painted						
3 faces painted						
2 faces painted						
1 face painted						
0 faces painted						





HUNDREDS BOARD PERCENT

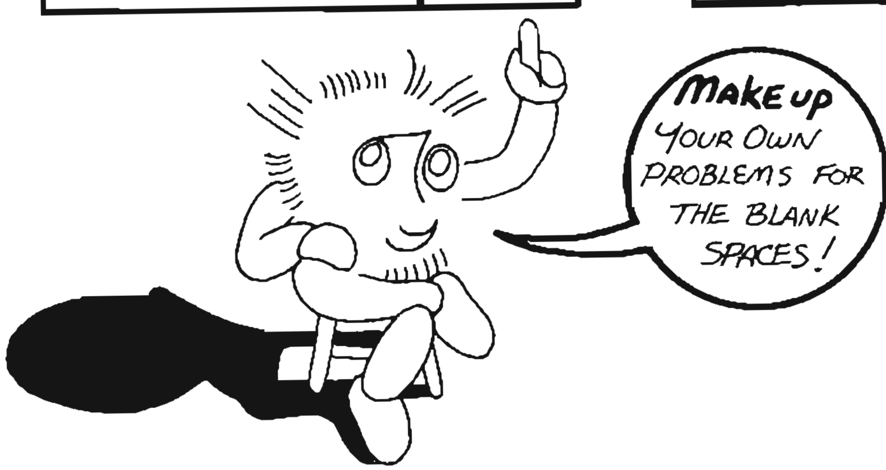
Equipment: Hundreds Board or hundreds chart
Number tiles, 1 to 100

Activity: Put the numbers in order on the board.

Make these charts on your paper.
Find the percent each set of
numbers is of the 100 numbers.

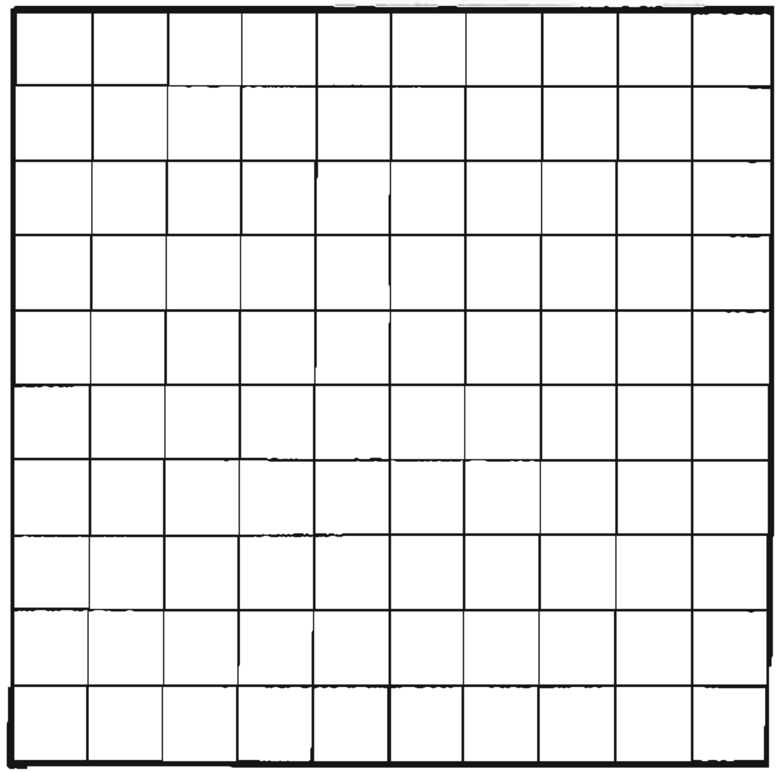
Type of Number	%

Type of Number	%



PERCENT WITH RODS & SQUARES - I

EQUIPMENT: ORANGE CUISENAIRE RODS (10 TO 15 IF POSSIBLE)
WHITE RODS



ACTIVITY:

1. HOW MANY ORANGE RODS ARE NEEDED TO COVER THE SQUARE?

2. HOW MANY WHITE RODS IN AN ORANGE ROD? HOW MANY WHITE RODS ARE NEEDED TO COVER THE SQUARE?

3. EACH WHITE ROD COVERS WHAT PERCENT OF THE SQUARE?

4. EACH ORANGE ROD COVERS WHAT PERCENT OF THE SQUARE?

5. COPY THIS CHART ON YOUR PAPER. FILL IN THE BLANKS.

RODS	PERCENT OF SQUARE COVERED
3 white	%
7 white	
15 white	
____ white	35%
1 orange	
5 orange	

RODS	PERCENT OF SQUARE COVERED
10 orange	%
100 white	
1 orange + 1 white	
2 orange + 5 white	
7 orange + 5 white	
	67%

PERCENT WITH RODS & METRES - I

EQUIPMENT: METRE STICK
ORANGE AND WHITE CUISENAIRE RODS

ACTIVITY:

1. HOW MANY ORANGE RODS ARE NEEDED TO MAKE THE LENGTH OF A METRE STICK?

2. HOW MANY WHITE RODS IN AN ORANGE ROD? HOW MANY WHITE RODS ARE NEEDED TO MAKE THE LENGTH OF A METRE STICK?

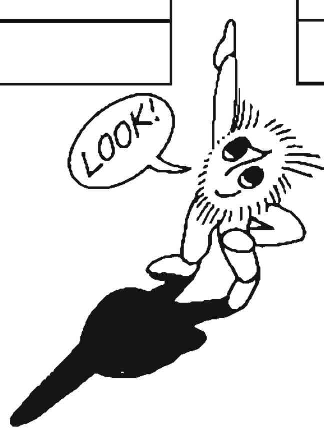
3. THE LENGTH OF A WHITE ROD IS WHAT PERCENT OF A METRE?

4. THE LENGTH OF AN ORANGE ROD IS WHAT PERCENT OF A METRE?

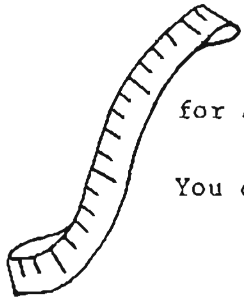
5. MAKE THIS CHART ON YOUR PAPER. FILL IN THE BLANKS.

ROD	PERCENT OF A METRE
1 white	
5 white	
10 white	
37 white	
100 white	
1 orange	
3 orange	
5 orange	
10 orange	
15 orange	

ROD	PERCENT OF A METRE
2 orange + 5 white	
7 orange + 5 white	
10 orange + 5 white	
20 orange	



ELASTIC
PERCENT APPROXIMATOR

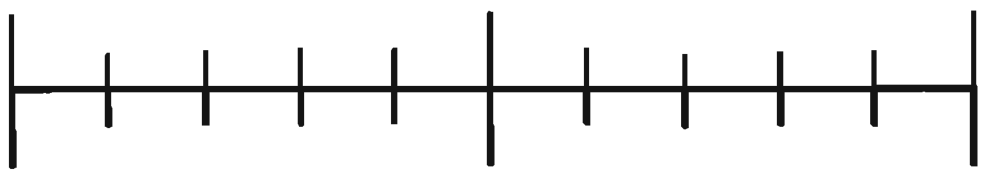


A piece of elastic or a rubber band can be made into a percent calculator for approximating.

You can use:

a) a 3" piece of $\frac{3}{16}$" elastic (the smaller the width, the more the stretch)

b) a $2\frac{1}{2}$" - 3" piece of a rubber band that is $\frac{1}{8}$" - $\frac{1}{2}$" wide



Two students work together to mark the elastic (rubber band). One stretches the material along the scale at the top, while the other marks the divisions on the elastic (rubber band). If the material is wide enough, the left end can be labeled 0%, the middle 50%, and the right end 100%. Note: These labels assume that the part of the elastic with the marks is the reference set (100% quantity).

At this point, the students should experiment with the elastic to see that the marks remain evenly spaced regardless of how much it is stretched. They should be reminded that their answers will be approximate and that each segment represents 10% of the reference set because the reference set (100%) was divided into 10 equal parts.

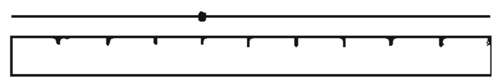
The next page shows examples of student problems. Depending on your students, you may want to supply separate worksheets on the length, area, and volume concepts or include all three on the same worksheet. It is hoped that students will see that n% of a quadrilateral with opposite sides congruent can be shown in two ways and that n% of a 6-sided polyhedron with opposite faces congruent can be shown in three ways.

IDEA FROM: *Activities with Ratio and Proportion*

ELASTIC
PERCENT APPROXIMATOR
(CONTINUED)

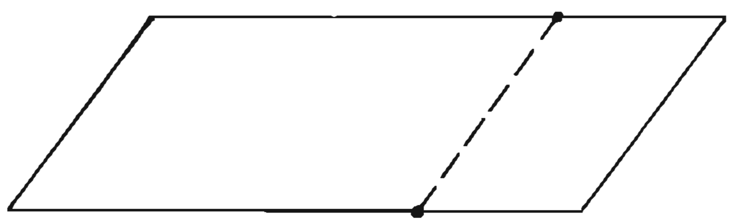
Example 1: Divide this line segment so the left-hand part represents 40% of the entire line segment.

a) Place 0% on the left-hand endpoint.
b) Stretch the elastic until 100% falls on the right-hand endpoint.
c) Mark a point to represent 40% of the line segment.



Example 2: Divide a parallelogram so the left-hand part represents 75% of the parallelogram.

a) 0% on left endpoint - 100% on right endpoint
b) Approximate 75% and mark.
c) Repeat on top segment.
d) Connect points.

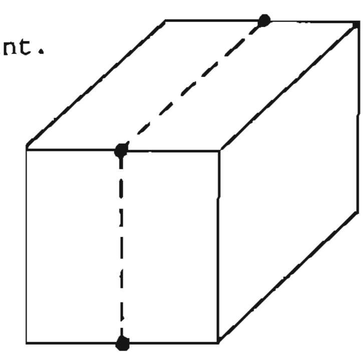


Note: If the side dimension is less than the length of the elastic, the elastic could not be used to find 75% of the parallelogram.

Example 3: Divide a cube so the left side shows 50% of the cube.

a) 0% on left endpoint - 100% on right endpoint.
b) Mark 50%.
c) Repeat on other edges.
d) Connect points.

Note: Similarly the front 50% and the bottom 50% can be found.



These three concepts could be developed into a series of lab activities using 1) different lengths of string for the lines, 2) transparent quadrilaterals and a felt tip pen for marking, and 3) transparent commercial polyhedron models with a felt tip pen for marking.

IDEA FROM: *Activities with Ratio and Proportion*

Percents of Line Segments

In some cases it is possible to split an object or number (R) into into 100 equal pieces and by taking N of the pieces determine N% of R. This N could be 20%, 100%, 200%, or even $\frac{1}{2}$%. In other cases it is easier to split R into 10 equal parts. In these cases, one part is 10% of R so 20% of R would be 2 of the parts, 75% of R would be $7\frac{1}{2}$ parts and so on. In still other cases a split into 2, 3... parts is more helpful.

The suggestions below apply these concepts to line segments. These suggestions could be developed on a blackboard, overhead, or on dittoed sheets. Some students may need to review the number line concepts covered in *Number Lines I - VII.* If student worksheets are written, the *Elastic Percent Approximator* or a prepared key can be used by the student to check his work.

I. Two line segments are given (R and _% of R). The student is asked to estimate what percent one is of the other.

Example: **R** = 10 10 10 10

___% of **R** =

Solution Strategy: Split R into 10 approximately equal parts. Each part is 10% of R. ___% of R is about 4 parts. A good guess is 40.

Problem Suggestions:
a) Cover percents <, = and > 100.
b) Some problems can be included where the 2 line segments do not "line up" at the left end. (See example below.)
c) Vary R from the first to second line segment.

Example: ___% of **R** =

R =

II. A line segment (R) is given. The student is asked to draw N% of the line segment. Do not expect exact measures.

Example: **R** =

30% of **R** =

Solution Strategy: Split R into 10 approximately equal parts. 30% of R is 3 of the parts.

30% of R =

Problem Suggestions: 30%, 50%, 80%, etc. first, then move to 25%, 75%, 5%. Include 100% and then percents over 100.

III. A line segment (N% of R) is given. The student is asked to draw R.

Example: 20% of **R** =

R =

Solution Strategy: 40% of R is 2 of the segments shown.
60% of R is 3 of the segments shown.
100% of R is 5 of the segments shown.

Note: An alternate strategy is to determine 10% of R and then find 100% of R. This (or some other alternate strategy is necessary when 80% of R is given).

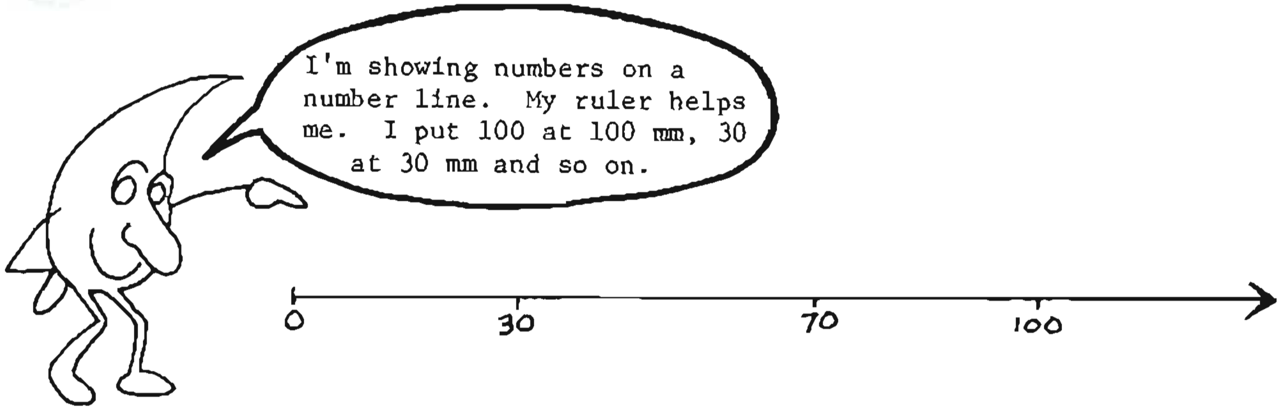
Problem Suggestions: Include percents <, =, > 100.

Example: 300% of **R** =

R =

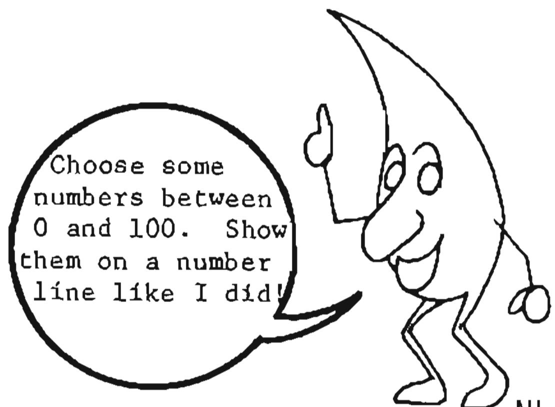
Solution Strategy: The segment shown is 300 pieces. Split this into 3 segments. One of them is 100% of R.

ACTIVITY CARDS - NUMBER LINE



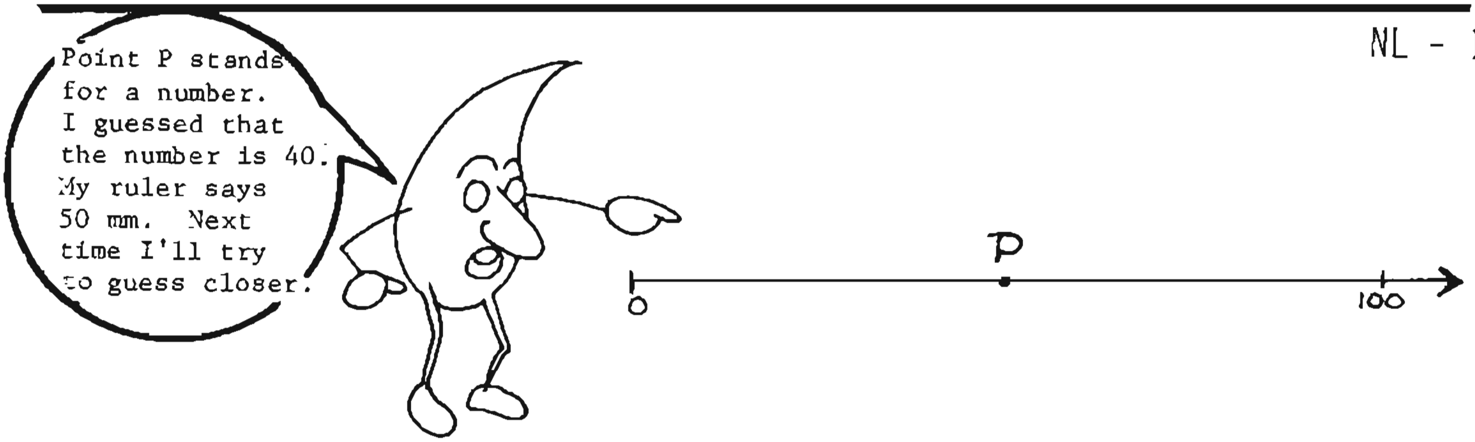
On your paper draw 5 lines. Mark 100 mm on each. Show two numbers on each line.

1) 20, 60
2) 90, 50
3) 95, 35
4) 25, 75
5) 27, 63

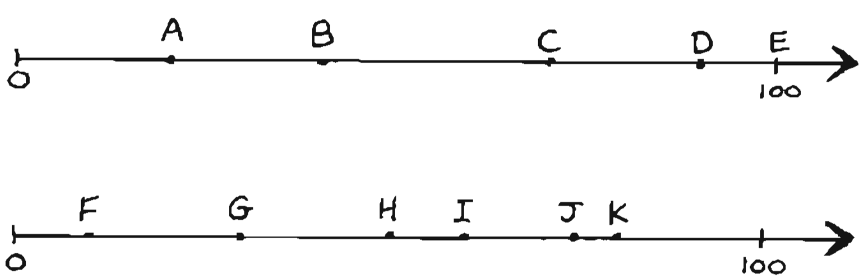


NL - I

NL - II

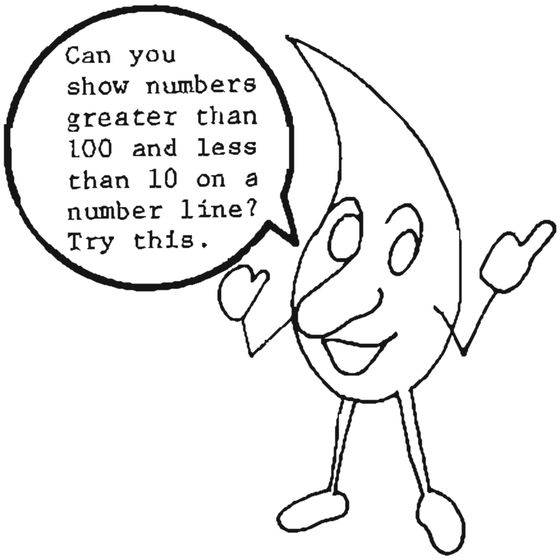


Guess and check the numbers for these letters. Record in a chart like this



LETTER	GUESS NUMBER	RULER NUMBER
A		
B		
C		

ACTIVITY CARDS - NUMBER LINE (PAGE 2)

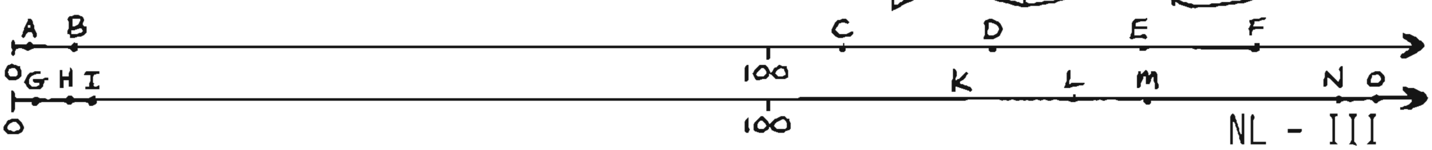


Draw five lines on your paper.
Make 0 and 100 on each.
Show two numbers on each line.

1) 120, 5
2) 150, 3
3) 200, 8
4) 105, 1
5) 125, 6

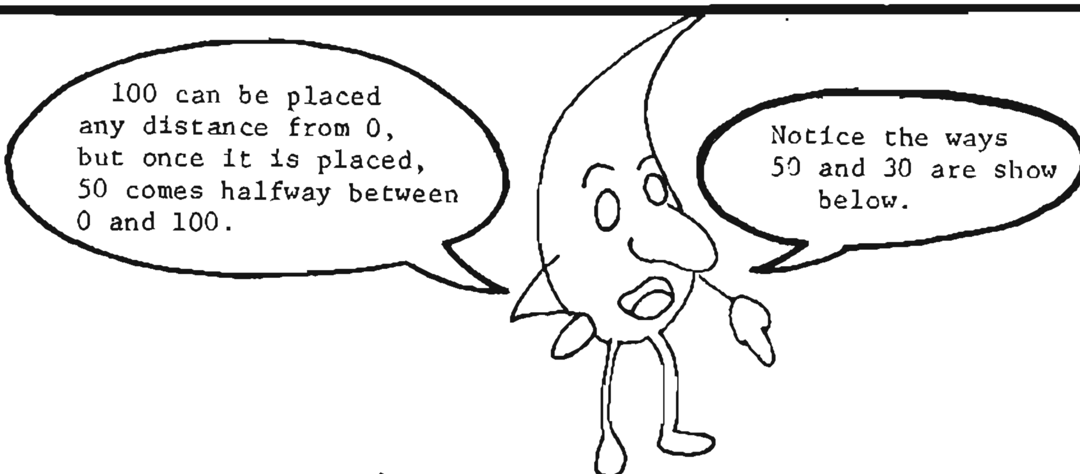
Guess and check the numbers for the letters below. Record in a chart.

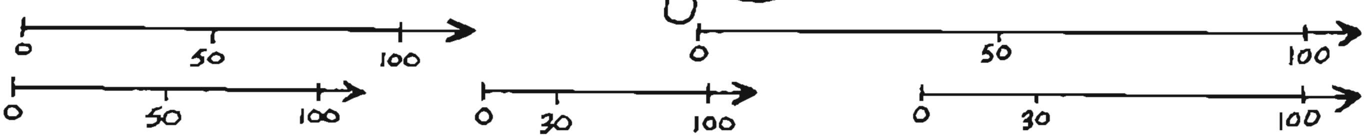
LETTER	GUESS NUMBER	RULER NUMBER
A		
B		



NL - III

NL - IV

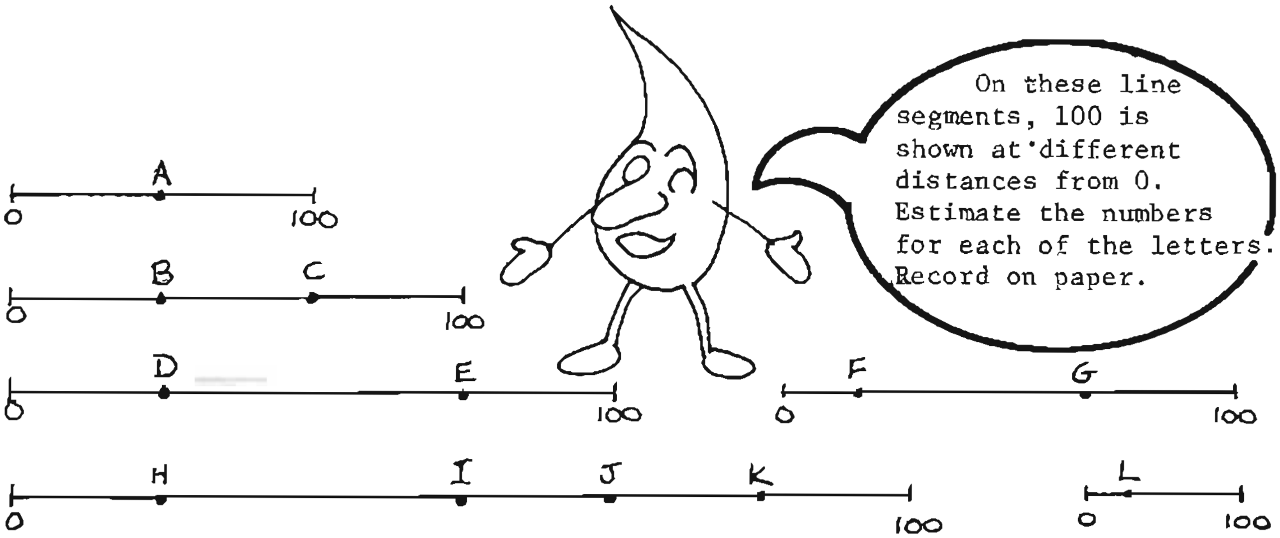




Draw five lines. Place 0 and 100 on each line so there are five different distances from 0 to 100.

Show 25, 50 and 75 on each line.

ACTIVITY CARDS - NUMBER LINE (PAGE 3)



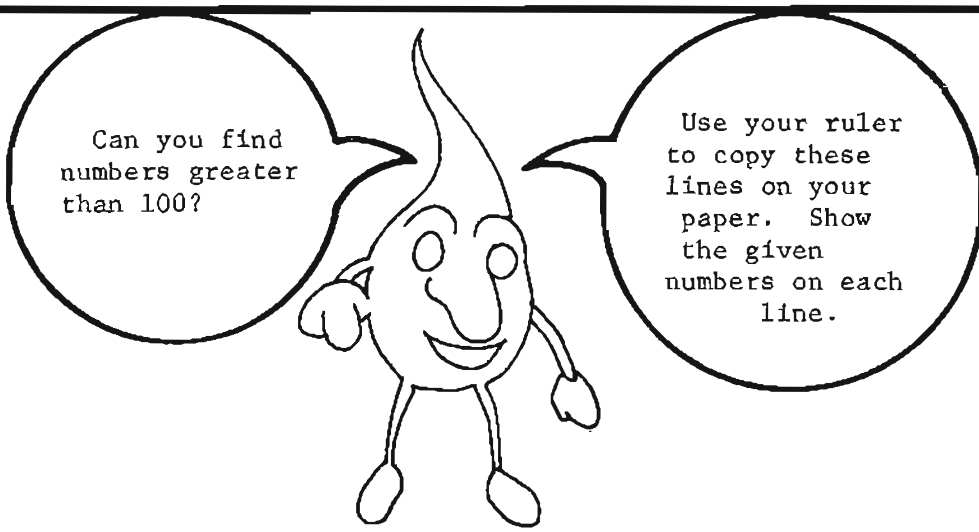
Compare your estimates with someone else's. How well do you agree?

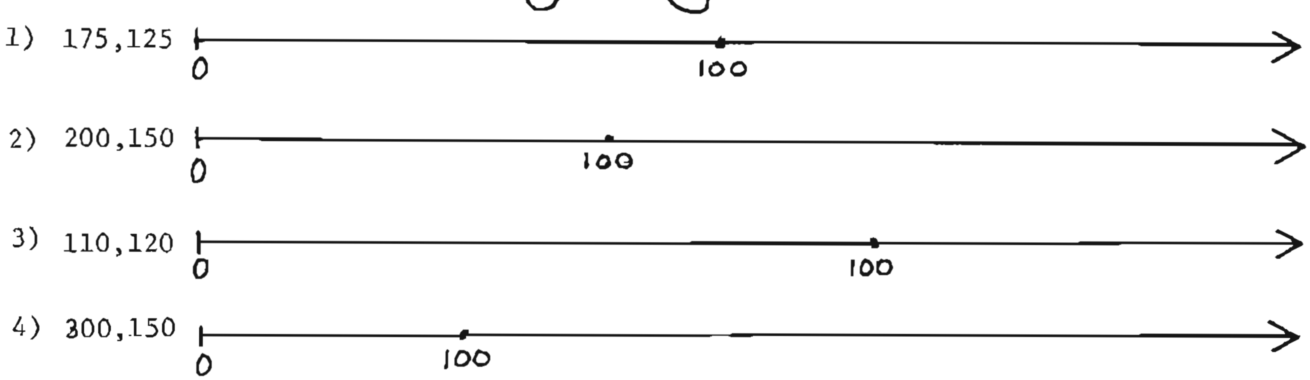
Make your own line segments like these. Put letters on the line segments. Give them to your teacher or a classmate.

How well can he guess?

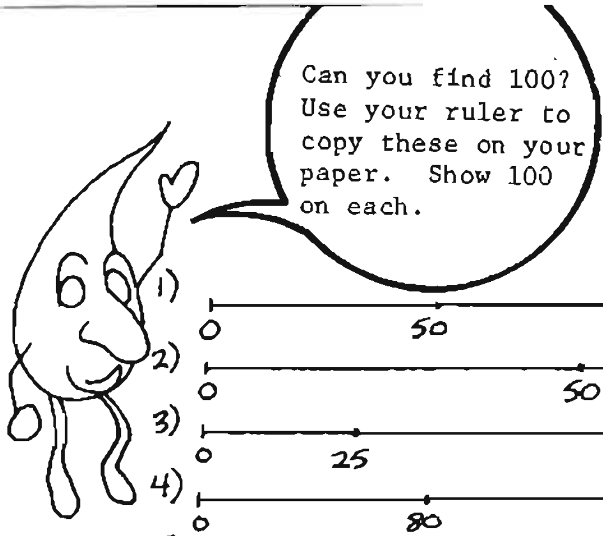
NL - V

NL - VI

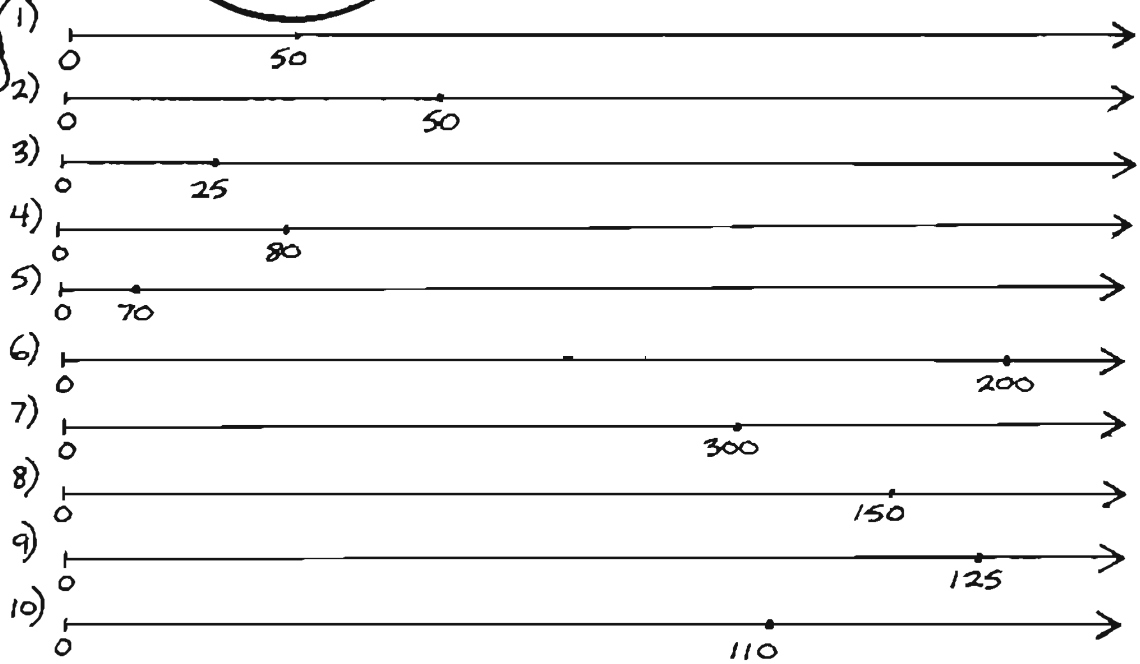




Make up a problem.
Give it to a classmate.



ACTIVITY CARDS - NUMBER LINE (PAGE 4)

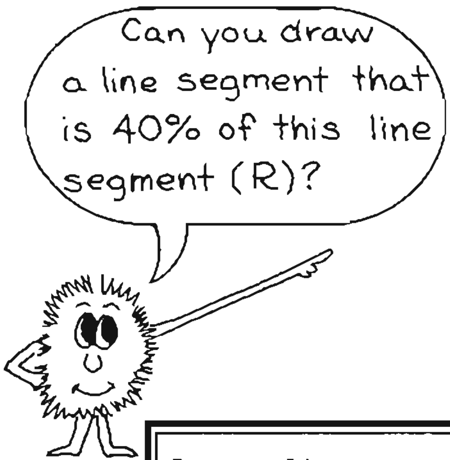


Make up a hard one for your teacher!

NL - VII

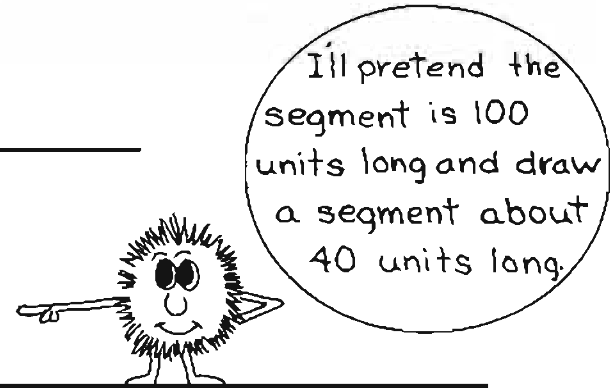
PERCENTING :

LINE SEGMENTS



R = ____________________

40% of R= __________



Draw a line segment the given percent of each shown line segment (R). Don't expect to be exact.

HINT

R = 10 10 10 10 ____________________

70% of R = __________

R= ______________

50% of R = __________

R= ____________________

30% of R= __________

R= ____________________

90% of R= __________

R = ____

25 % of R = __________

R = ____________________

75 % of R= __________

R = ____________________

100 % of R= __________

Stringing Along With Percents

Equipment: String
Scissors

Activity: 1) Cut a piece of string as long as the line segment below. Lay it on the line segment. Call this string R.

R = ____

2) Cut pieces of string so that their lengths are about the following percents of R. Put each piece of string next to its percent.

50 % of R =

20 % of R =

80 % of R =

100 % of R =

200 % of R =

150 % of R =

3) Write the answers on your paper:

a) Which string is the longest? _______% of R

b) Which string is the shortest? _______% of R

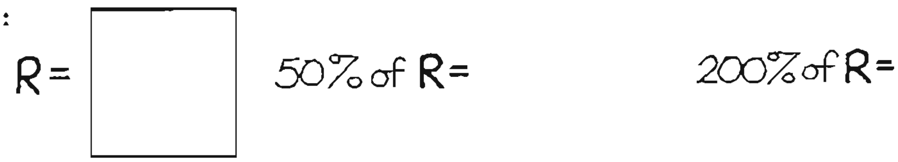
c) Are there two strings which are the same length? If yes, which two? _______, _______

Percents of Rectangles

The suggestions below apply the ideas used in *Percents of Line Segments* to rectangles. The suggestions can be developed on a blackboard, overhead or on dittoed sheets.

I. A rectangle (R) is given. The student is asked to draw N% of the rectangle. Reasonable (but not exact) answers are expected.

Example:



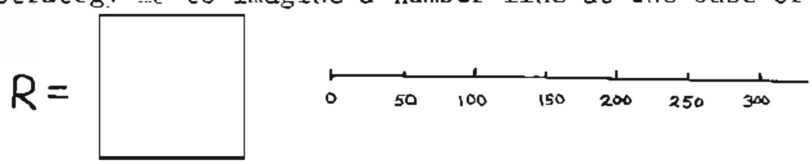
Solution Strategies:

a) One possibility is to split R into 10 equal, vertical parts. Each part is 10% of R (since each part would contain 10 of the 100 equal pieces). 5 of the parts are needed for 50% of R. 20 are needed for 200% of R.

b) Some students might reason as follows: "If R were 100 equal pieces, 50% of R would be 50 of those pieces. Therefore, 50% of R is $\frac{1}{2}$ of R. 200% of R would be 200 of the pieces. So, 200% of R is two R's.

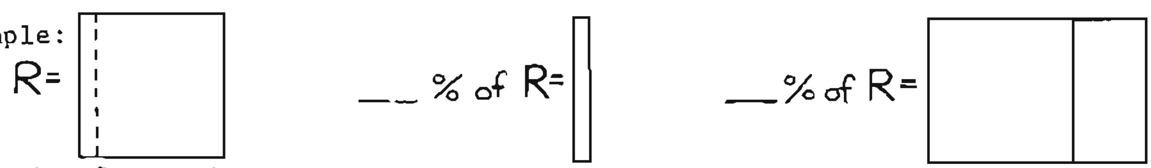
c) Another strategy is to imagine a number line at the base of the rectangle.

Example:



II. A reference rectangle is given (R). The student is asked to estimate what percent various rectangles are of R.

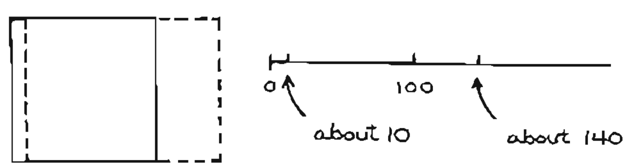
Example:



Solution Strategies:

a) Split R into 10 equal, vertical parts. Each part is 10% of R. The first rectangle is about 1 of these parts. Guess 10% (a rough but reasonable guess). The second has about 14 parts--guess 140% (actually 150%).

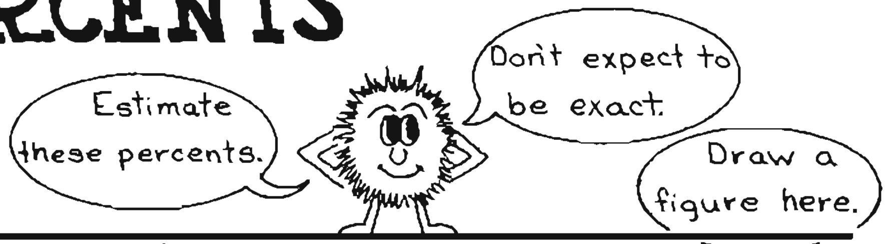
b) Sketch a number line at the base of the rectangle. Place 0 and 100 at the edges of the rectangle. Estimate the number corresponding to the edges of the rectangles in Question 1.



General Suggestions:

a) Keep the percents relatively simple. 83% of R is difficult to estimate.
b) Accept reasonable answers--or let students give a range.
c) Vary the size of the reference square.
d) A key could be provided so work can be checked by students.
e) See *Rectangle Percents* for a sample student page.

RECTANGLE PERCENTS



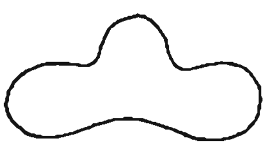
R = ___% of R ___% of R 100% of R

R = ___% of R ___% of R 50% of R

R = ___% of R ___% of R 300% of R =

R = ___% of R ___% of R 100% of R =

FOR EXPERTS ONLY:

R = 

100% of R 50% of R

PERCENTS OF AN ORANGE ROD

Equipment: Container of Cuisenaire Rods

Activity:

Find an orange rod. It will be 100 units for this activity.

ORANGE = 100 UNITS

Record your answers on your own paper.

1. How many white rods are needed to make the length of an orange rod?
   How many units are in one white rod?
   One white rod is _____% of an orange rod.

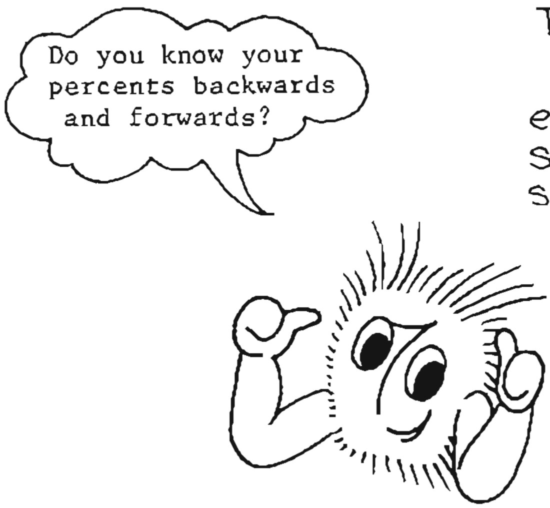
2. How many units in one red rod?
   One red rod is _____% of an orange rod.

3. Make these charts on your paper. Use rods to help you fill the blanks.

RODS	NUMBER OF UNITS	PERCENT OF AN ORANGE ROD
1 LIGHT GREEN		
2 RED		
	30	
		50%
1 BLUE		
1 BLACK		
1 ORANGE		

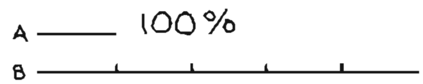
RODS	NUMBER OF UNITS	PERCENT OF AN ORANGE ROD
1 Yellow + 1 white		
1 Purple + 1 light green		
1 brown + 1 red		
1 orange + 1 Yellow		
2 black		
	140	
		150%

PERCENTS: BACKWARDS AND FORWARDS 1



Think:

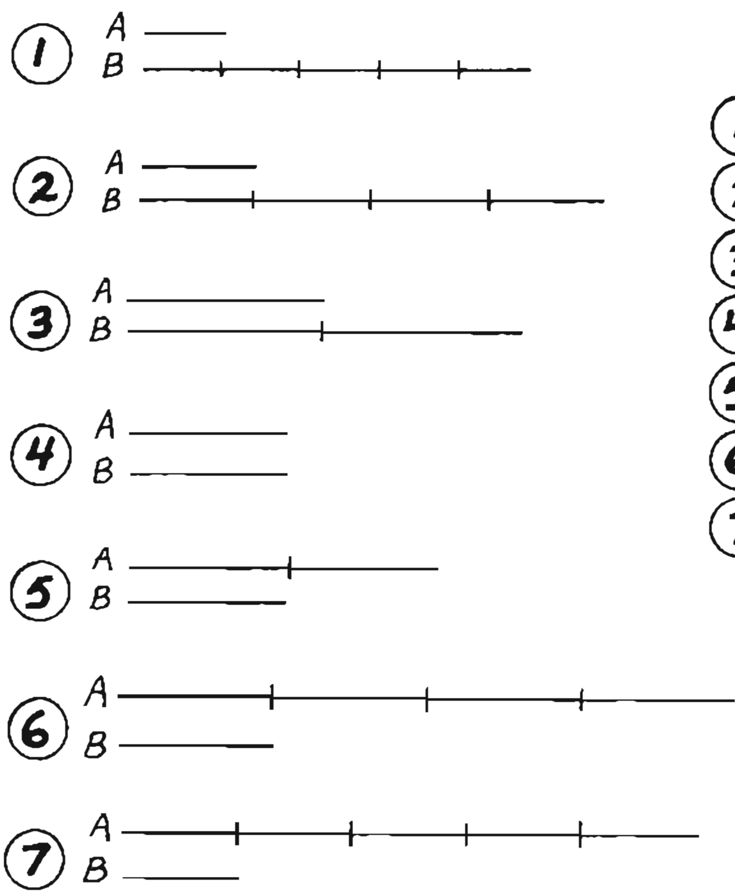
B is 100% and B has five equal sections. The first section of B is 20% (100% ÷ 5), so A is 20% of B.



Think:

Now A is 100%. B is five times as long as A, so B is 500% of A (100% × 5).

USE THE DIAGRAMS BELOW TO FILL IN THE CHART.

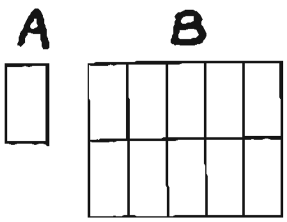


	A is ___ % of B	B is ___ % of A
1		
2		
3		
4		
5		
6		
7		

a) Do you see any pattern between the percents in the first column and the percents in the second column? Explain.

b) What percent would go in the second column if 10% were in the first column? What if 5% were in the first column?

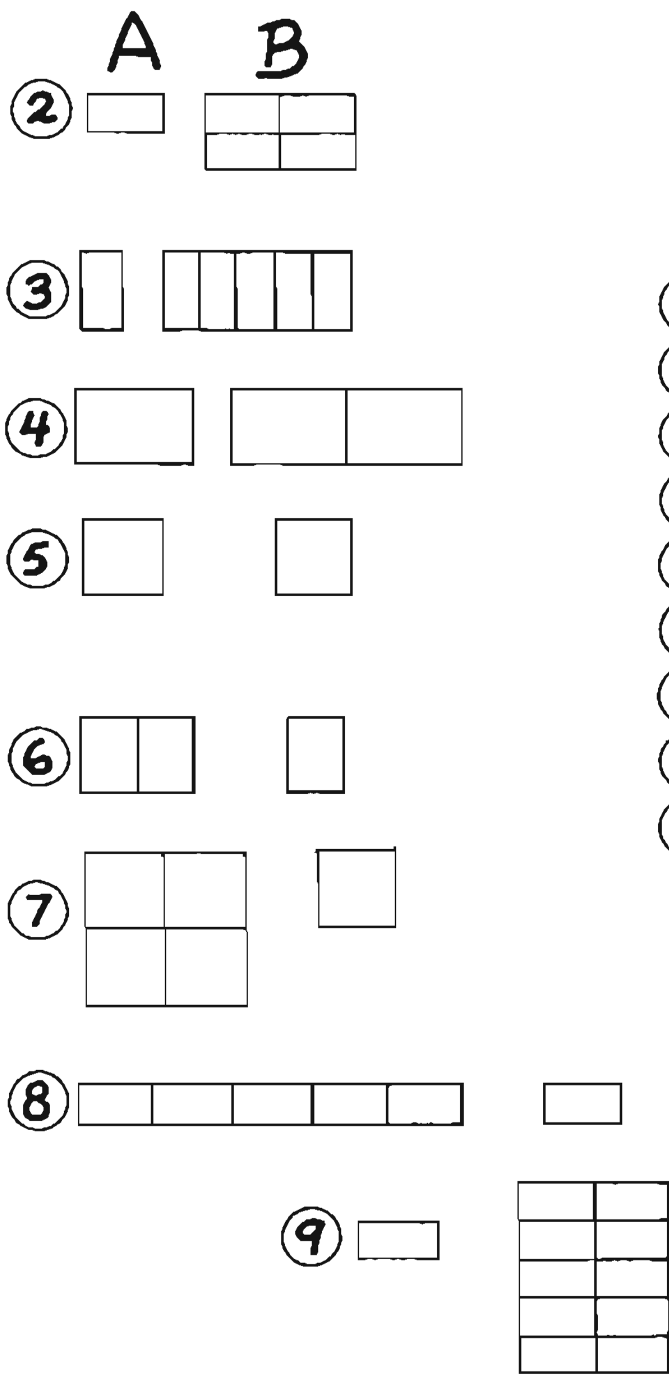
PERCENTS: BACKWARDS AND FORWARDS 2

1 

Think:
B is 100% and B has ten equal sections. The first section of B is 10% (100% ÷ 10), so A is 10% of B.

Think:
Now A is 100%. B is 10 times as big as A, so B is 1000% of A (100% x 10).

USE THESE DIAGRAMS TO FILL IN THE CHART.



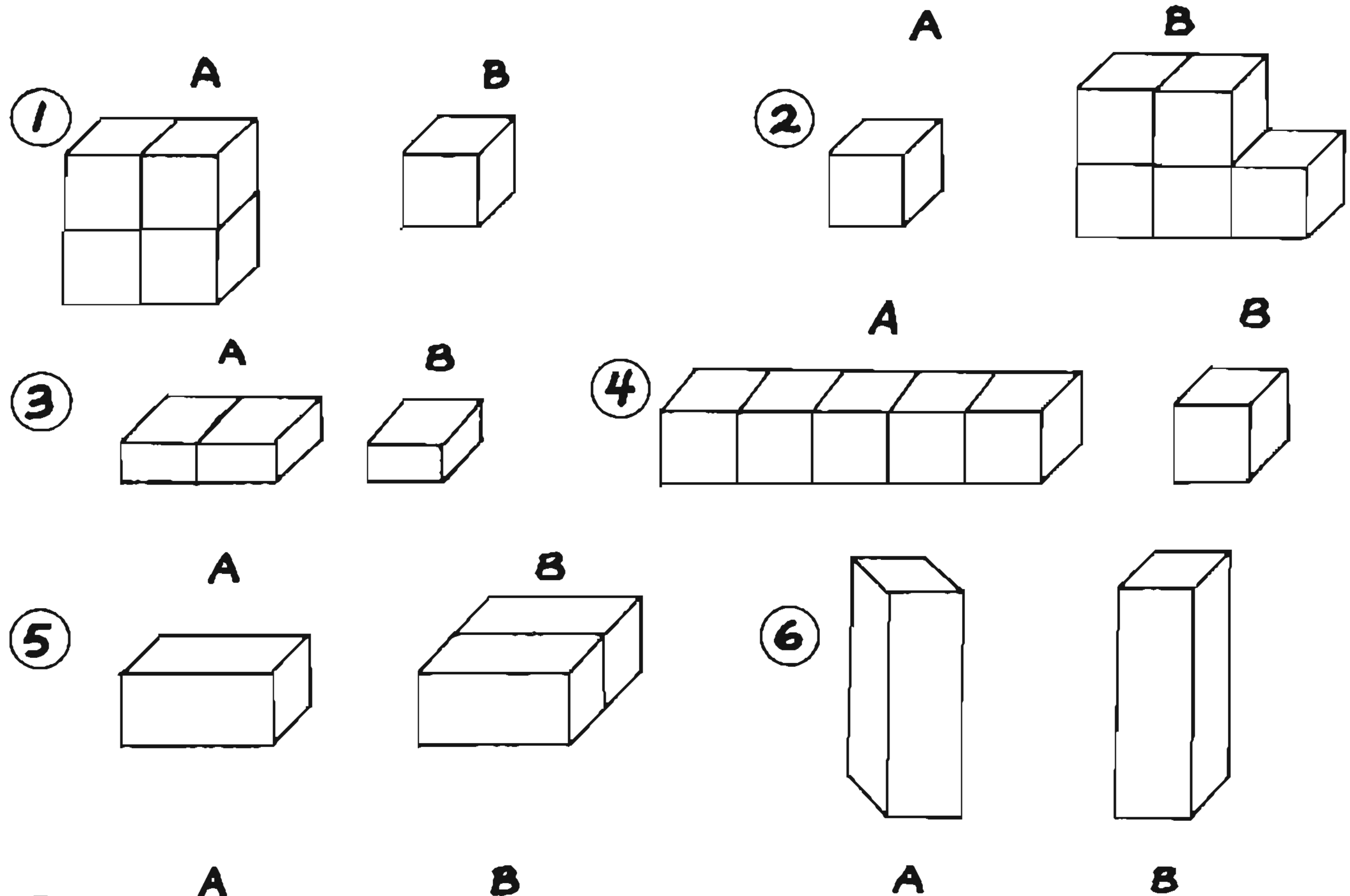
	A is ___% of B	B is ___% of A
1	10	1000
2		
3		
4		
5		
6		
7		
8		
9		

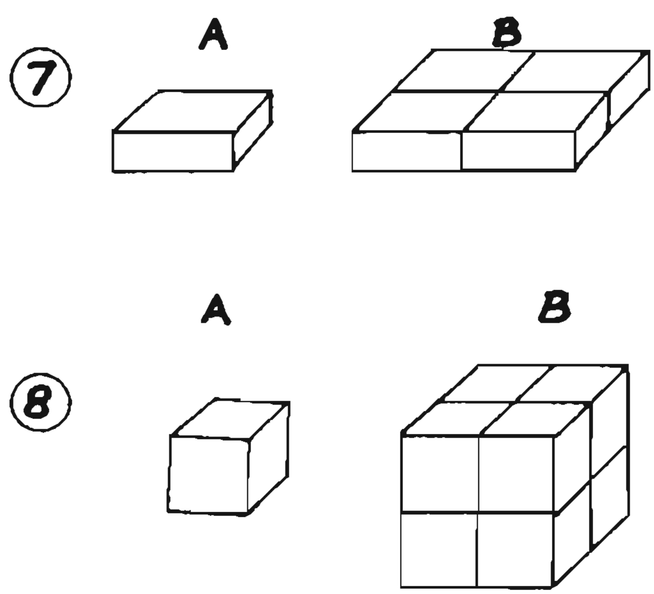
a) What % would go in the first column if 5% were in the second column? ___________

b) What % would go in the first column if 1% were in the second column? ___________

PERCENTS: BACKWARDS AND FORWARDS 3

USE THESE DIAGRAMS TO FILL IN THE CHART



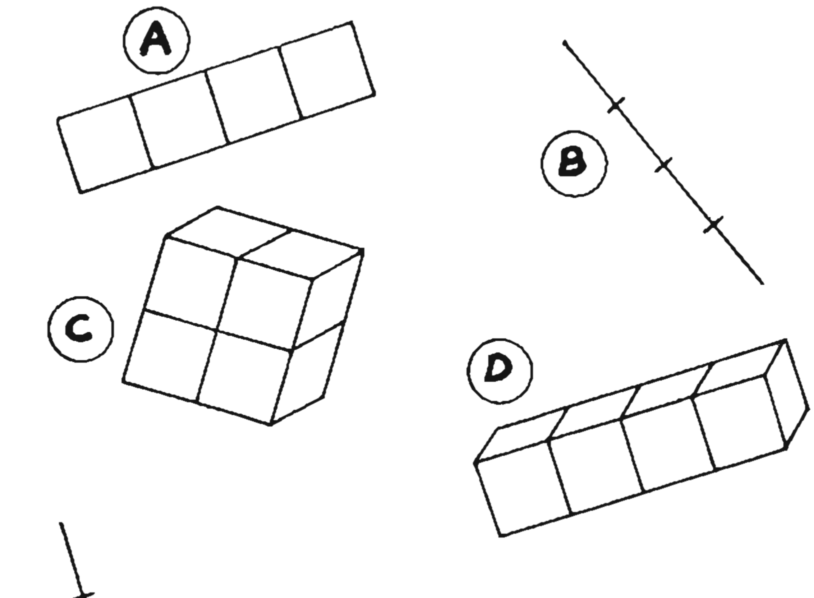


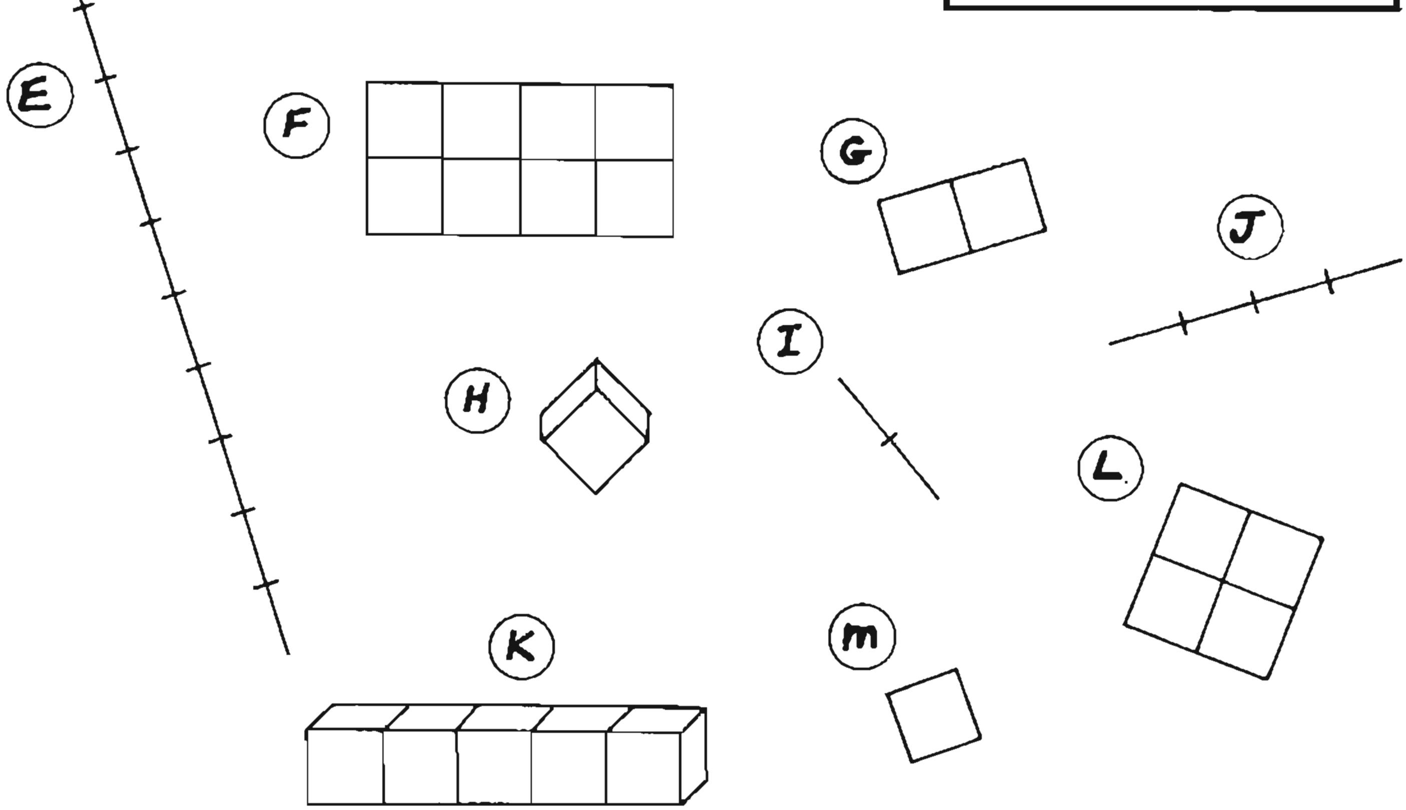
	A is __% of B	B is __% of A
1		
2		
3		
4		
5		
6		
7		
8		

PERCENTS: BACKWARDS AND FORWARDS 4

USE THE DIAGRAMS BELOW TO HELP YOU FILL IN THE CHART.
SOME PROBLEMS HAVE MORE THAN ONE CHOICE FOR THE ANSWER.

K	is	500% of	____
I	is	20% of	____
H	is	25% of	____
J	is	100% of	____
L	is	50% of	____
D	is	400% of	____
M	is	25% of	____
C	is	100% of	____
A	is	200% of	____
I	is	50% of	____



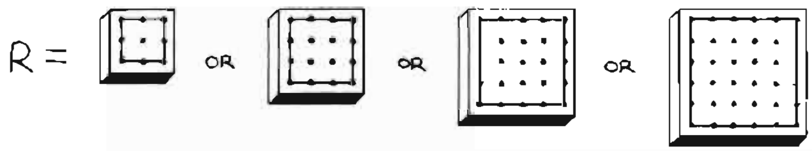


GEOBOARD PERCENTS

TEACHER DIRECTED ACTIVITY

The geoboard can be used to motivate and reinforce the concept of 50% (4x4 nail geoboard); 50%, 25%, and 75% (3x3 nail and 5x5 nail geoboards); and 20%, 40%, 60%, and 80% (6x6 nail geoboard). If students are not familiar with geoboards, they should first do the readiness activities found in the section on Lab Materials in the resource book, Number Sense and Arithmetic Skills.

1) 100% Each student could make the largest square possible on the geoboard. This square will be the REFERENCE SET (100% quantity). For example:

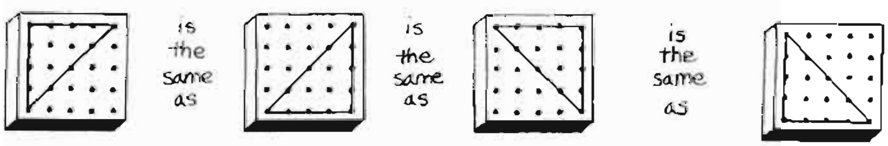


2) 50% Each student could be asked to make as many different polygons as possible that represent 50% of the geoboard and record the results on dot paper or geoboard record paper.

Note: If the large square represents 100% of the geoboard, and if this square is divided into two congruent polygons, then each polygon will represent 50% of the geoboard.

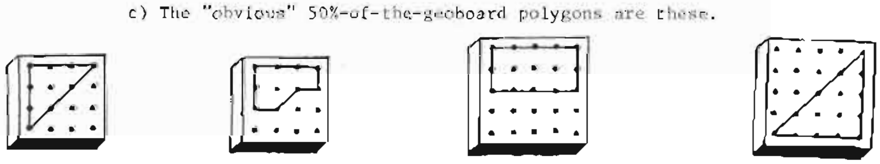
a) Polygon

b) Different implies polygons that are not congruent.

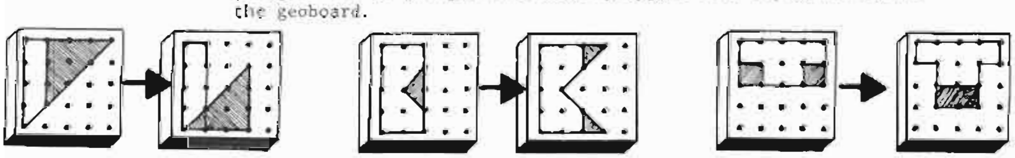


Only one of these shapes should be recorded

c) The "obvious" 50%-of-the-geoboard polygons are these.



d) Students could now be shown a demonstration of how the "obvious" polygons can be changed into other polygons that represent 50% of the geoboard.



e) Students will now have many different polygons that represent 50% of the geoboard. An easy way for teachers to check these polygons is to apply PICK'S LAW. This states that AREA $= \frac{B}{2} + I - 1$ where B is the number of nails in the boundary of the polygon, and I is the number of nails in the interior of the polygon. Using the first example in (d) above,

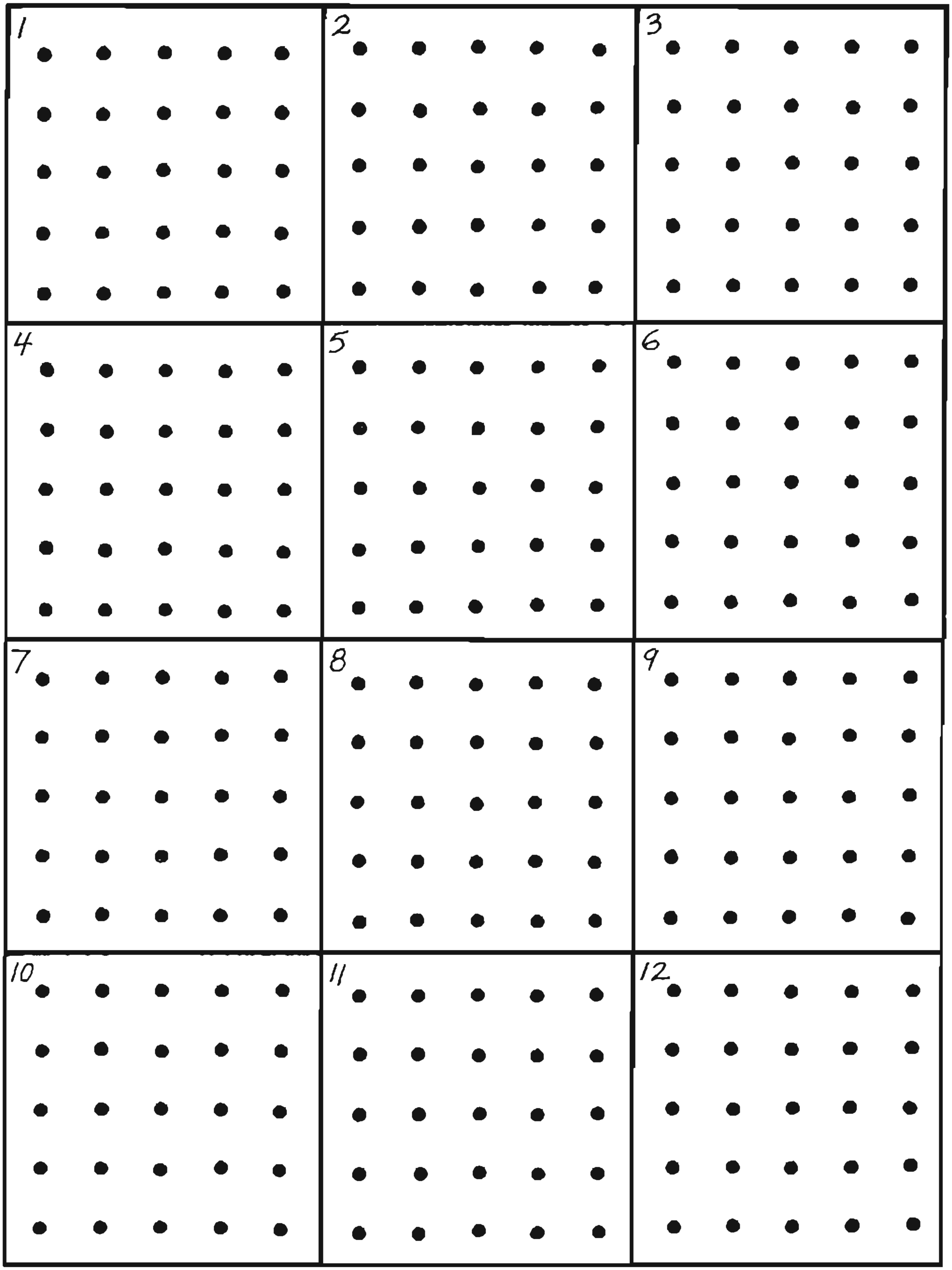
$B = 12$, $I = 3$, so AREA $= \frac{12}{2} + 3 - 1 = 8$ (50% of the 5x5 nail geoboard),

then $B = 16$, $I = 1$, so AREA $= \frac{16}{2} + 1 - 1 = 8$ (50% of the 5x5 nail geoboard).

[A student discovery activity of PICK'S LAW can be found in The Mathematics Teacher, May, 1974, p. 431.]

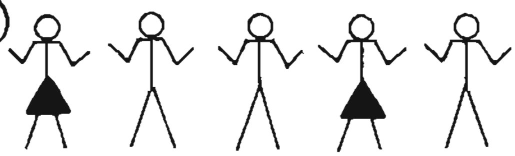
3) A similar development can be done with the other percents listed in the opening paragraph.

GEOBOARD PERCENTS (CONTINUED)
RECORD SHEET



THE WHOLE THING

(A)

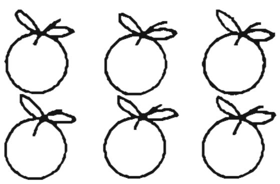


This is 20% of a class.
Show 40% of the class.
How many in the entire class?

(B)

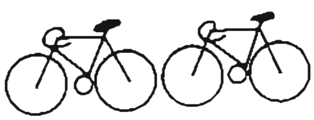
This is 75% of the cost.
Each one will be ___% of the cost.
Two of them will be ___% of the cost.
The total cost is $___.

(C)



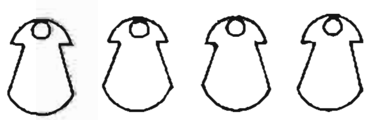
This is 30% of the apples in a bag.
Show 10% of the apples in the bag.
Show 50% of the apples in the bag.
How many apples in the bag?

(D)



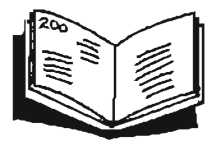
This is 1% of the bikes.
How many bikes altogether? ______

(E)



These are 10% of the dresses on a dress rack.
How many are 20% of the dresses on the dress rack? _____
How many are 50% of the dresses on the dress rack? _____
How many are 100% of the dresses on the dress rack? _____

(F)

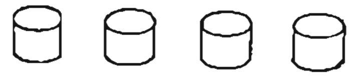


Mary has read 50% of her book.

How many pages are in the book?

______

(G)



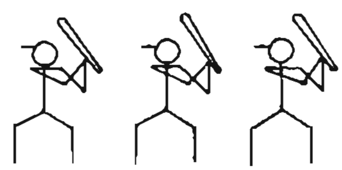
These are 80% of the barrels.
How many barrels in all?

(H)

$\begin{array}{r}1\\+1\\\hline\end{array}$	$\begin{array}{r}2\\+2\\\hline\end{array}$	$\begin{array}{r}3\\+3\\\hline\end{array}$	$\begin{array}{r}4\\+4\\\hline\end{array}$	$\begin{array}{r}5\\+5\\\hline\end{array}$	$\begin{array}{r}6\\+6\\\hline\end{array}$
$\begin{array}{r}1\\-1\\\hline\end{array}$	$\begin{array}{r}2\\-2\\\hline\end{array}$	$\begin{array}{r}3\\-3\\\hline\end{array}$	$\begin{array}{r}4\\-4\\\hline\end{array}$	$\begin{array}{r}5\\-5\\\hline\end{array}$	$\begin{array}{r}6\\-6\\\hline\end{array}$

These are 60% of the problems on a test.
How many test problems in all? _____

(I)



This is $33\frac{1}{3}\%$ of the players.

Show $66\frac{2}{3}\%$ of the players.

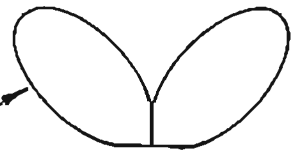
Show 100% of the players.

How many players on the team?

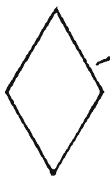
FINDING 100% FROM BELOW

This is 25% of a larger rectangle.

Draw 50% of the larger rectangle.
Draw 75% of the larger rectangle.
Draw 100% of the larger rectangle.



This is about 66% of a design.
Draw about 33% of the design.
Draw 100% of the design.

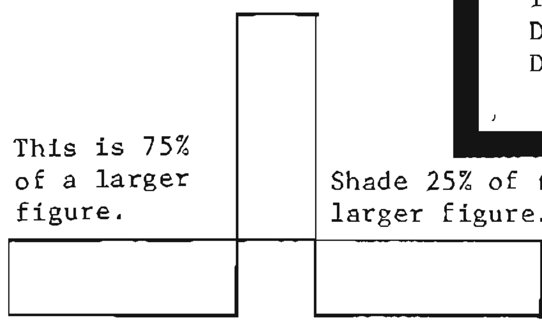


25% of a larger diamond

Draw the larger diamond.

This is 75% of a larger figure.

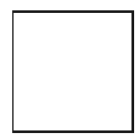
Shade 25% of the larger figure.



Draw 100% of the larger figure.

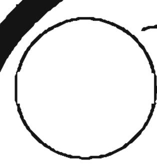
10% of a larger rectangle.

Draw 20% of the larger rectangle.
Draw 100% of the larger rectangle.



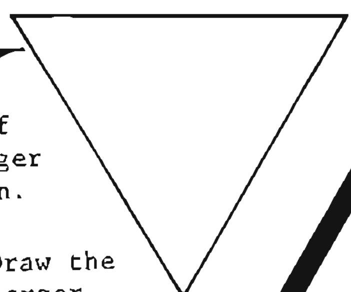
25% of a larger square.

Draw the larger square.



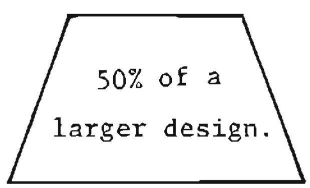
10% of a larger design.

Draw the larger design.

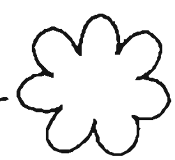


20% of a larger design.

Draw the larger design.



Draw the larger design.



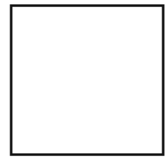
This is 50% of a larger design.

Draw the larger design around this part.

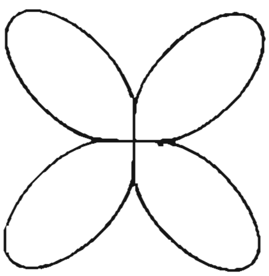
IDEA FROM: *C.O.L.A.M.D.A.*
Permission to use granted by Northern Colorado Educational Board of Cooperative Services

FINDING 100% FROM ABOVE

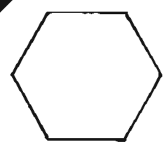
This is 150% of a smaller rectangle.
Shade 50% of the smaller rectangle.
The unshaded part is 100% of the smaller rectangle.



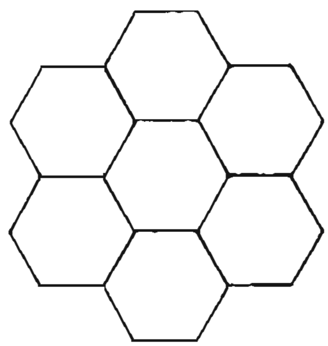
This is 100% of a square.
Draw 25% of the square on another paper.
Draw 50% of the square.
Draw 100% of the square.



This is 200% of a smaller figure.
Shade the smaller figure.

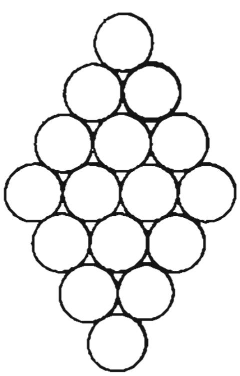


This is 120% of a smaller figure.
Can you show 20% of the smaller figure?
Shade 100% of the smaller figure.



This is 700% of a smaller design.

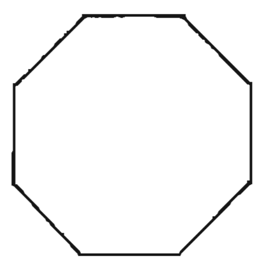
Shade 100% of the smaller design.



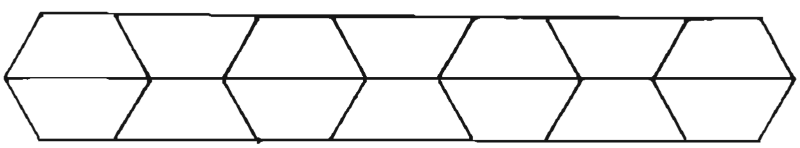
This is 160% of a smaller figure.

Draw 10% of the smaller figure.

Shade 100% of the smaller figure.



This is 160% of a smaller figure.
Shade the smaller figure.



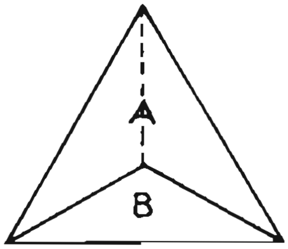
This is 140% of a smaller design. Draw 10% of the smaller design.
Shade 100% of the smaller design.

IDEA FROM: *C.O.L.A.M.D.A.*

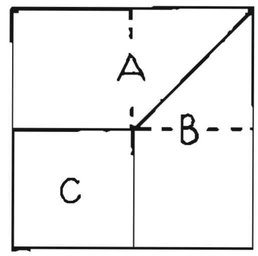
PEACE-N-ORDER

For each of these diagrams use the letters and
- a) list the pieces in order from smallest to largest.
- b) let the smallest piece be the reference set R (100% quantity).
- c) approximate the percent of R each of the other pieces will be.

It will help to draw dotted lines to divide the pieces into convenient sections. 1 and 2 are done for you.

1. 

B - 100% of B
A - 200% of B

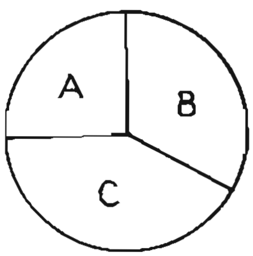
2. 

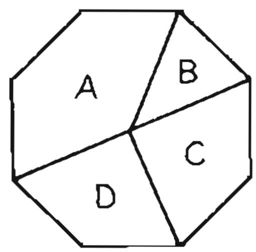
C - 100% of C
A - 150% of C
B - 150% of C

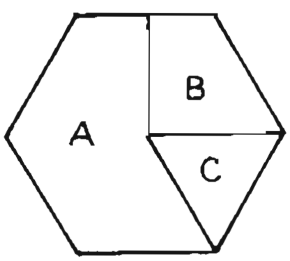
This activity would be a good teacher directed activity using this page as an overhead transparency. Stress that percent depends upon defining the reference set (100% quantity).

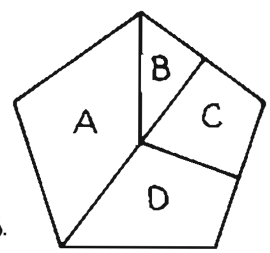
Extension:

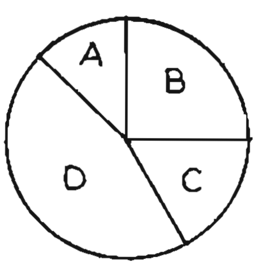
Assign various pieces of the Tangram puzzle as the 100% quantity, and relate the other pieces to the 100% quantity.

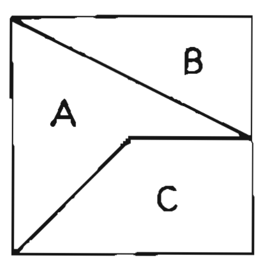
3. 

4. 

5. 

6. 

7. 

8. 

Use: Paper & Pencil/Transparency

IDEA FROM: *Seeing Shapes*

YOU ARE WHAT YOU EAT

AN APPLE A DAY KEEPS THE DOCTOR AWAY

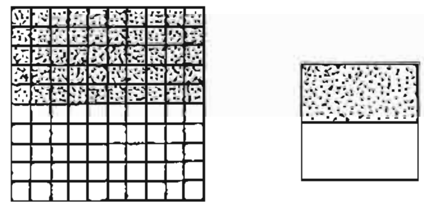
AN _ _ _ _ _ A DAY KEEPS _ _ E _ _ _ _ _ _ AWAY

11 8 3 13 5   12 7 1 14 4 9 2 6 10

For each exercise circle the letter in either the <, =, or > column that compares side A to side B.

A	<	=	>	B
1) 130% of 80 . . .	C	K	E	 80
2) 30% of 40 . . .	O	T	B	 40
3) 100% of 90 . . .	M	I	A	 90
4) 10% of 20 . . .	Y	E	S	. . . 10% of 50
5) 25% of 22 . . .	L	F	N	. . . 25% of 18
6) 50% of 80 . . .	U	R	D	. . . 25% of 80
7) 5% of 100 . . .	V	X	I	. . . 12% of 100
8) 10% of 50 . . .	A	P	N	. . . . . 1% of 50
9) 100% of 12 . . .	B	G	J	. . . 100% of 16
10) 75% of 40 . . .	S	Y	M	. . . 75% of 40
11) 40% of 160 . . .	O	A	T	. . . 50% of 160
12) 120% of 90 . . .	H	N	E	. . . 120% of 70
13) 1/2% of 200 . . .	O	W	D	. . . . . 1% of 200
14) 100% of 30 . . .	T	R	C	. . . 50% of 60

CHANGING PERCENT SHAPES



Which of the 2 squares at the right have 50% shaded? Teachers will probably assume that both are 50% shaded but students might not be so sure. The two squares aren't the same size. The shaded areas are not the same. One square is divided into more parts than the other and the numbers of shaded parts are not equal. The next four pages are masters for transparencies which can be used to help students make the transition from a 100 grid as a reference set to percents of figures with different sizes and shapes. The transparencies can be used as a teacher directed activity with the students deciding what needs to be shaded and what numbers to place in the blanks.

<u>50% Transparency</u>

The squares on this transparency are the same size. The first square has 100 equal parts, so shading 50% of the square means to shade 50 of the parts. The other squares do not have 100 parts, but since they are the same size, 50% of each square is the same area as was shaded in the first square. After shading 50% of each square and counting the parts, students can see that 50% of 40 is 20, 50% of 20 is 10, 50% of 10 is 5, etc. The statements at the bottom can be answered by referring to an appropriate square above.

<u>10% Transparency</u>

This master is similar to the 50% transparency. The same area is shaded to show 10% of each square and the number of divisions varies from 100 to 20.

<u>30% Transparency</u>

This transparency makes the transition from squares of the same area to figures of different area and shape. The first square has 30 of its 100 equal parts shaded. To shade 30% of the second square the same area can be shaded or 3 out of 10 equal parts. The third figure is a different shape but it has 10 equal parts, so it is logical to shade 3 of the parts to show 30% of the figure. 30% of each of the other figures can be shaded by shading 3 out of 10 equal parts.

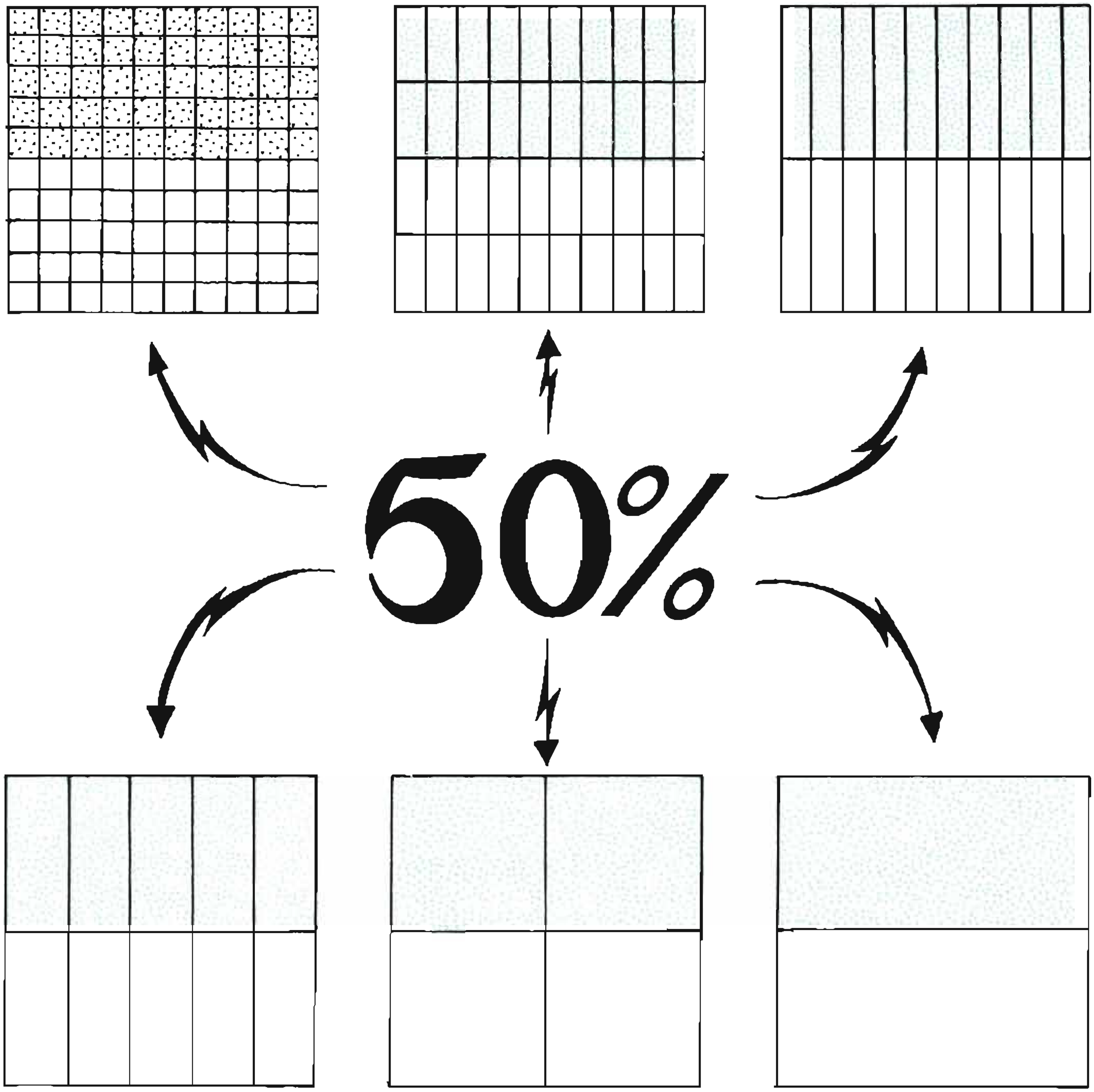
<u>55% Transparency</u>

Three percents, 55%, 40%, and 25% are carried through the same transition as described for 30%. The transitions follow this outline:

To shade 55% of a figure
shade 55 out of 100
  or 11 out of 20 (same area)
  or 11 out of 20 (different figures).

To shade 40% of a figure
shade 40 out of 100
  or 4 out of 10 (same area)
  or 4 out of 10 (different figures).

CHANGING PERCENT SHAPES (PAGE 2)



A) 50% OF 20 IS ____

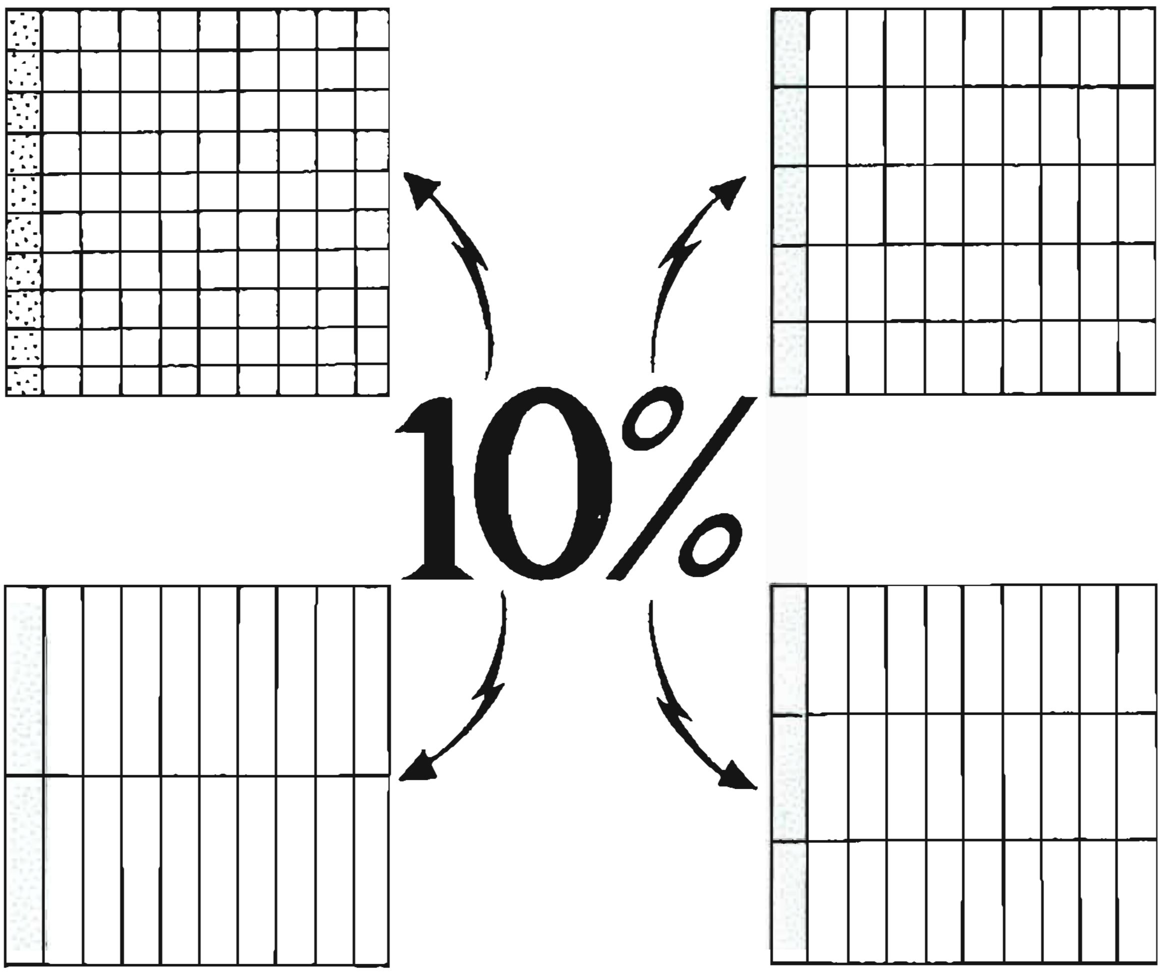
B) 50% OF ____ IS 1

C) ____% OF 10 IS 5

D) 50% OF 40 IS _____

E) ____ IS 50% OF 100

F) 2 IS ____% OF 4



A) 10% OF 30 IS _____

B) 5 IS 10% OF _____

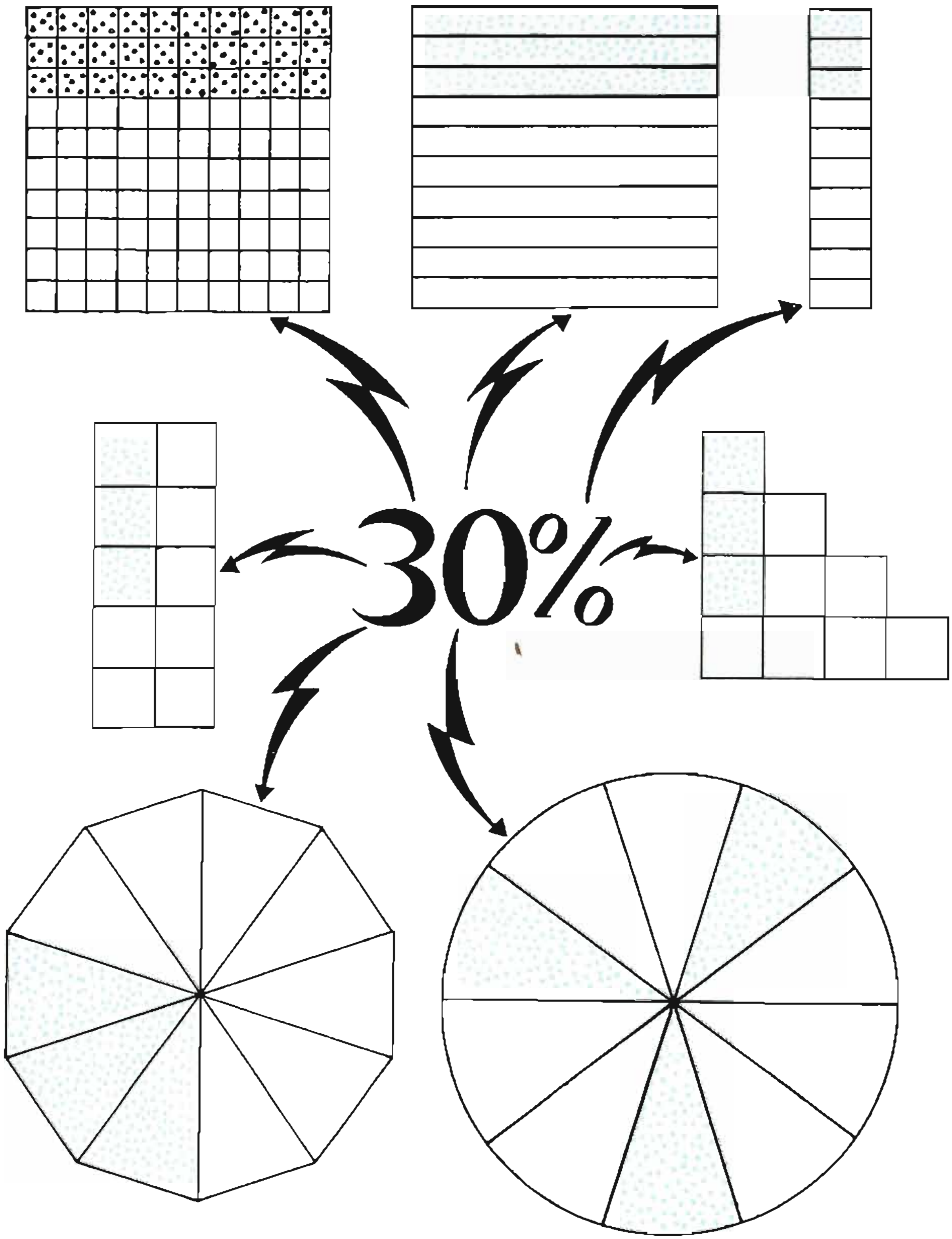
C) 10% OF ____ IS 2

D) 10 IS ____% OF 100

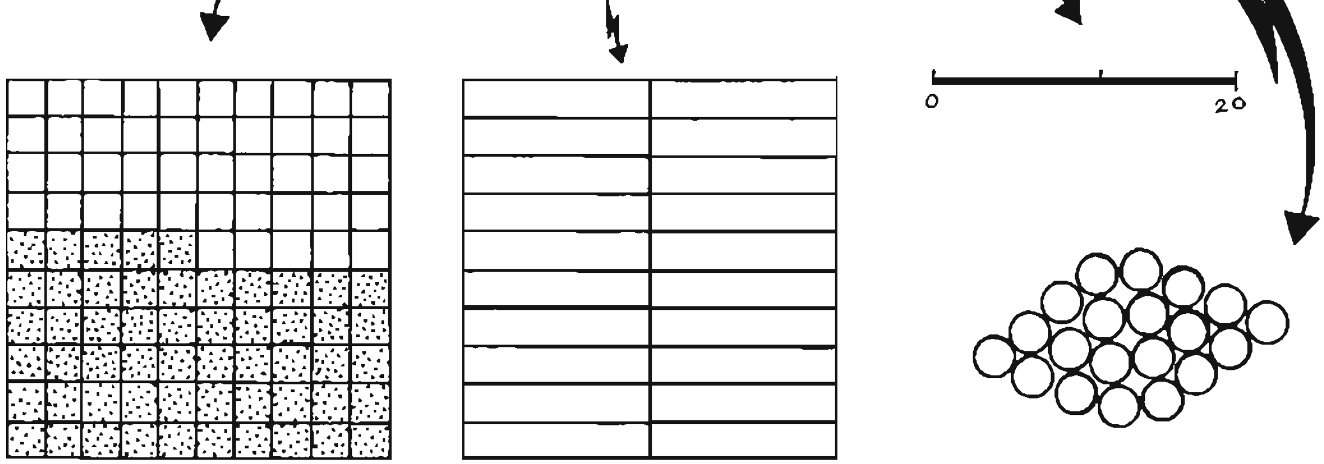
E) 10% OF 40 IS _____

F) 6 IS 10% OF _____

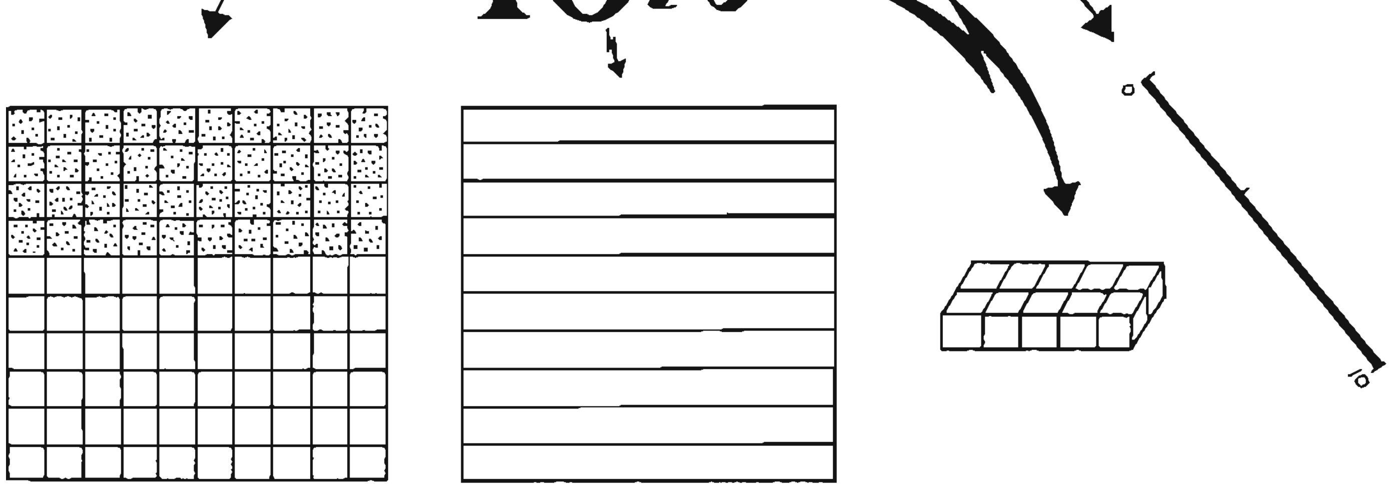
CHANGING PERCENT SHAPES (PAGE 4)



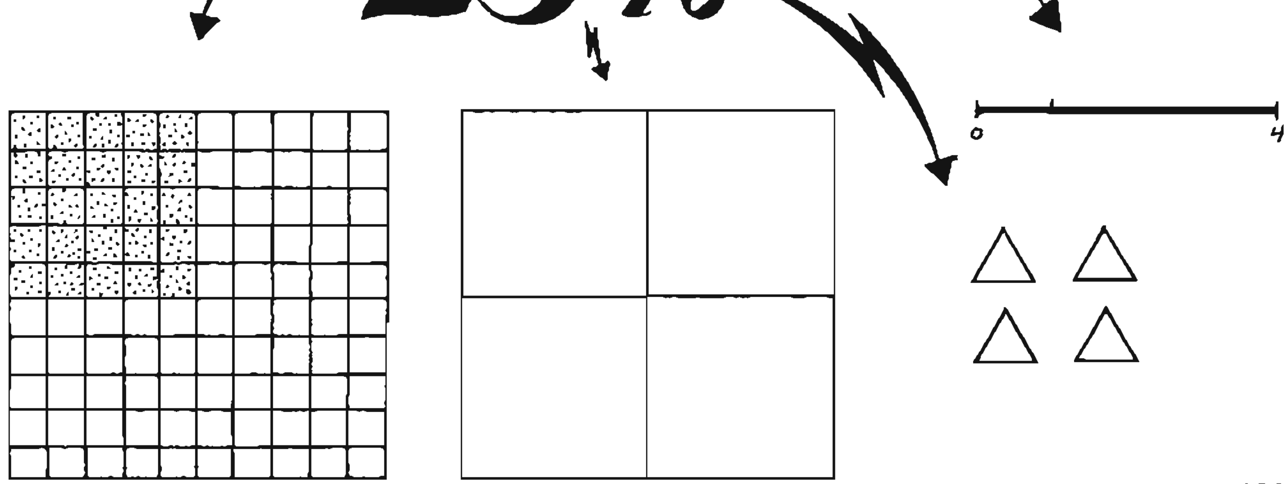
55%



40%



25%



RATIO, PROPORTION, AND SCALING

Placement Guide for Tabbed Divisors

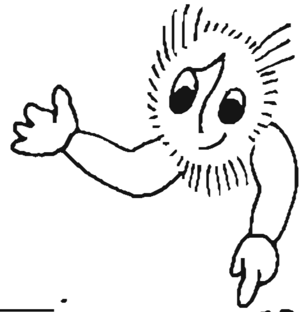
Tab Label **Follows page:**

CONTENTS

PERCENT: AS A RATIO

PERCENTS - NO MODEL NEEDED

Can't you do percent problems without using a picture?



Sure! Look at this.

20% of 25 is _____.

20% → 20 for every 100.

5 for every 25

so 20% of 25 is 5.

Divide both numbers by 4 because 100 ÷ 25 = 4

Wow! That's neat. Show me some more.

30% of ___ is 15.

30% → 30 for every 100.

DIVIDE BY 2.

15 for every 50.

so 30% of 50 is 15.

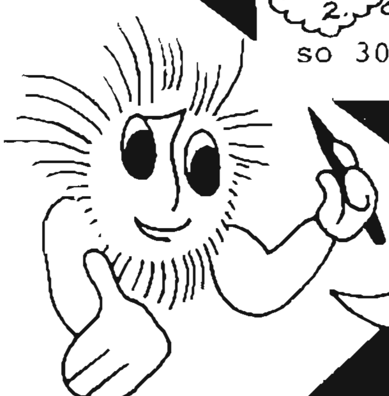
___% of 20 is 8.

MULTIPLY BY 5.

8 for every 20

40 for every 100 → 40%

so 40% of 20 is 8.



Of course, you could draw a picture to see if your answers look right.

Do these problems.

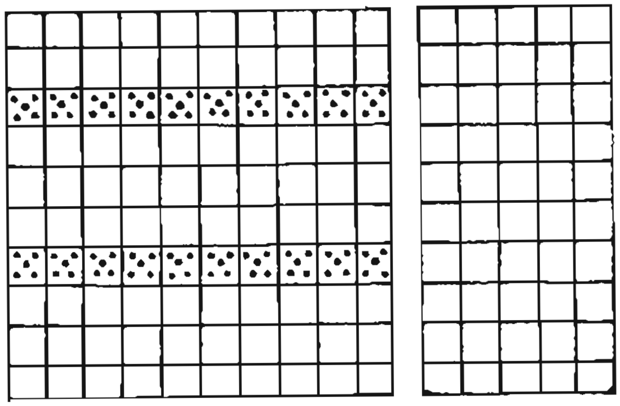
A) 48% of 25 is ___.
B) 83% of 100 is ___.
C) 2% of 50 is ___.

D) 50% of ___ is 1.
E) 40% of ___ is 8.
F) 90% of ___ is 9.

G) ___% of 10 is 8.
H) ___% of 50 is 21.
I) ___% of 4 is 3.

Percent Pictures-I

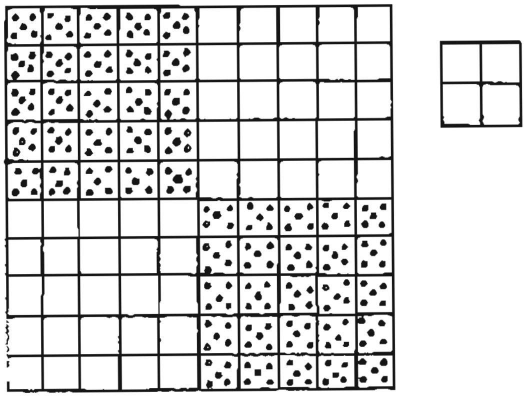
Shade the smaller grid to show the same percent of shaded squares.



20 out of 100

____ out of 50

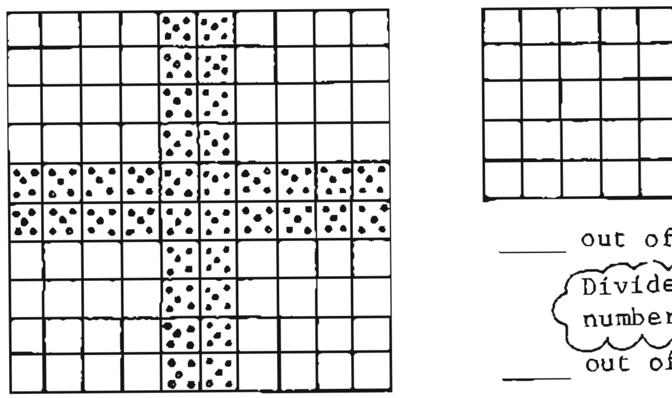
____%



____ out of 100

____ out of 4

____%

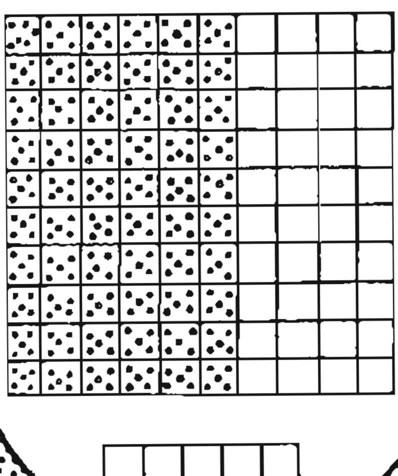


____ out of 100

Divide both numbers by 4.

____ out of ____

____%

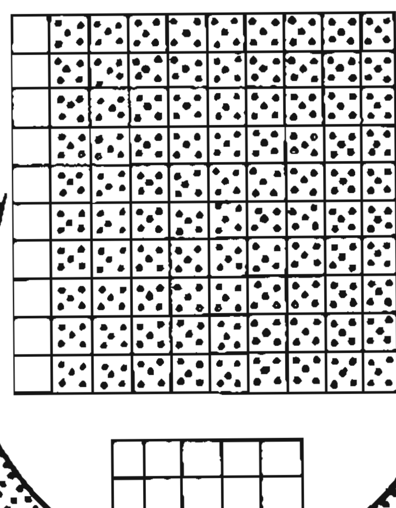


____ out of 100

Divide both numbers by 20.

____ out of ____

____%



____ out of 100

Divide by ____.

____ out of 10

____%

PERCENT PICTURES–I (CONTINUED)

1. 20 OUT OF 100 AND 10 OUT OF 50

20%

SO 20% OF 50 IS ___, ___% OF 50 IS 10, AND 20% OF ___ IS 10

2. 36 OUT OF 100 AND 9 OUT OF 25

36%

SO ___% OF 25 IS 9, 36% OF ___ IS 9, AND 36% OF 25 IS ___

3. 60 OUT OF 100 AND 3 OUT OF 5

60%

SO 60% OF ___ IS 3, 60% OF 5 IS ___, AND 3 IS ___% OF 5

4. 90 OUT OF 100 AND 9 OUT OF 10

90%

SO 90% OF ___ IS 9, ___% OF 10 IS 9, AND ___ IS 90% OF 10

5. 50 OUT OF 100 AND 2 OUT OF 4

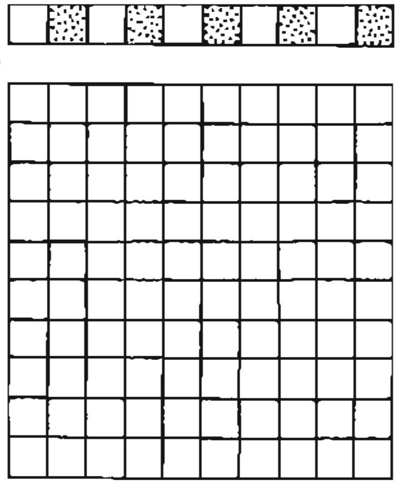
50%

SO 2 IS ___% OF 4, 50% OF 4 IS ___, AND 2 IS 50% OF ___

PERCENT PICTURES - II

Shade the larger grid to show the same percent of shaded squares.

A

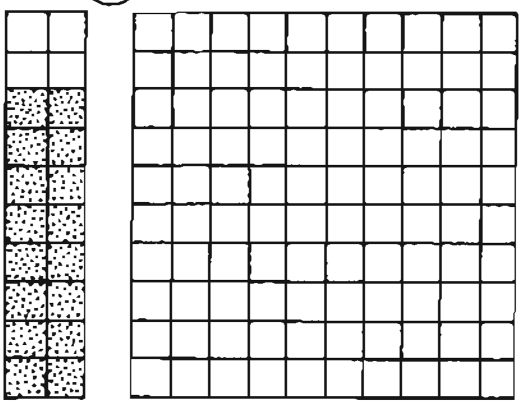


5 out of 10

____ out of 100

So 5 is ____% of 10.

B

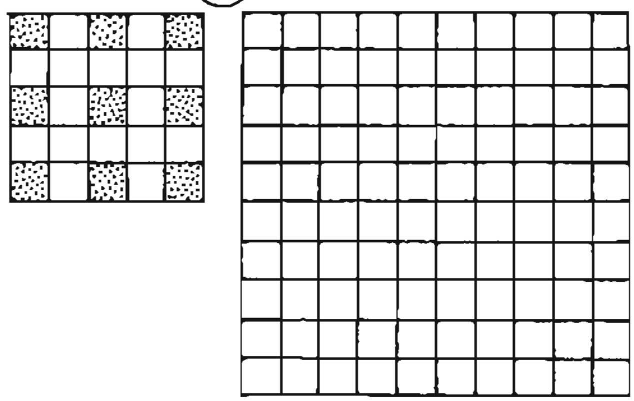


16 out of ____

____ out of 100

So ____% of 20 is 16.

C



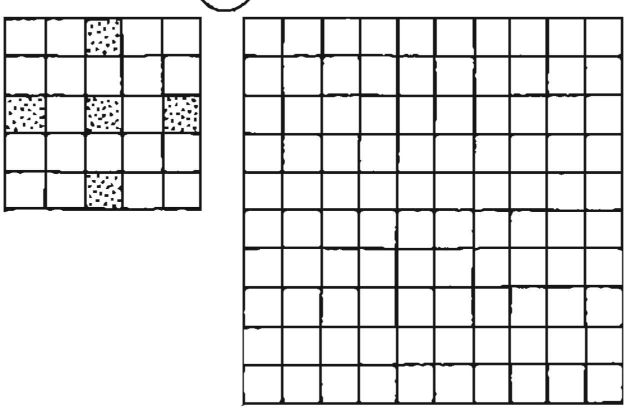
Multiply both numbers by 4.

____ out of ____

____ out of 100

So 9 is ____% of 25.

D

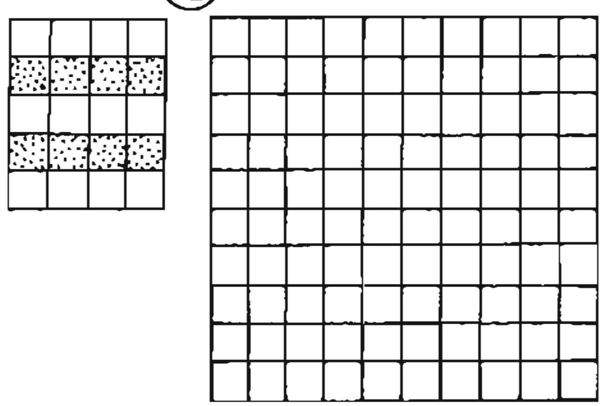


____ out of ____

____ out of 100

So 5 is ____% of 25.

E



Multiply both numbers by 5.

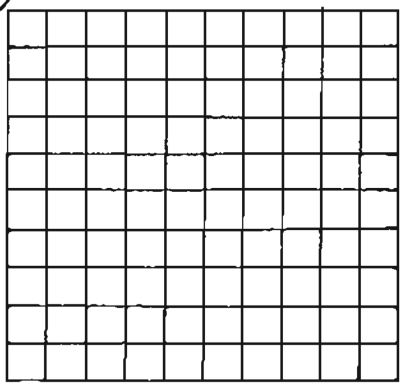
____ out of ____

____ out of 100

So ____% of 20 is 8.

SHADY PERCENTS

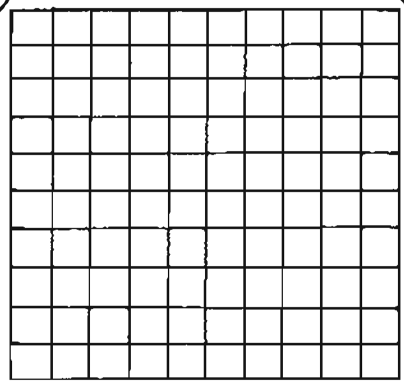
A



Shade 40% of A.

___ out of 100

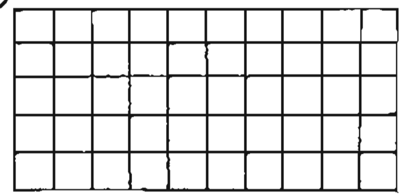
B



Shade 10% of B.

___ out of 100

C

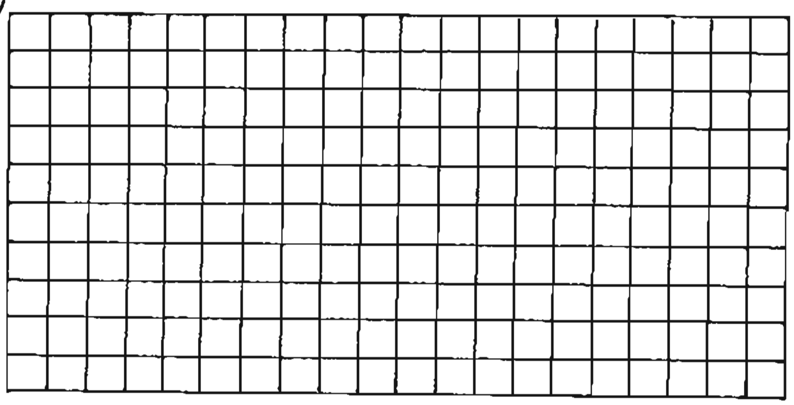


Shade 50% of C.

___ out of 100

25 out of 50

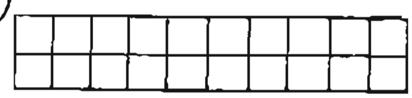
D



Shade 25% of D.

___ out of 100 or ___ out of ___

E

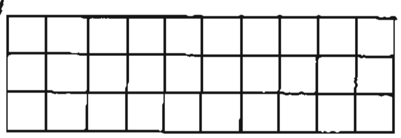


Shade 5% of E.

___ out of 100

___ out of ___

F

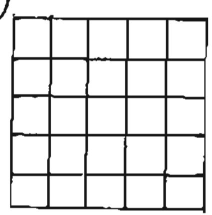


Shade 10% of F.

___ out of 100

___ out of ___

G



Shade 80% of G.

___ out of 100

___ out of ___

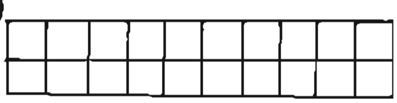
H

Shade 20% of H.

___ out of 100

___ out of ___

I



Shade 90% of I.

___ out of 100

___ out of ___

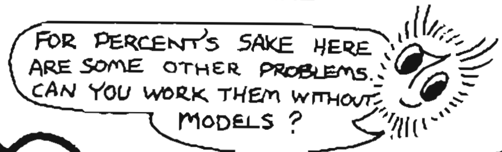
J

Shade 90% of J.

___ out of 100

___ out of ___

FOR PERCENT'S SAKE



25% means

25 for every 100 (DIVIDE BOTH 25 AND 100 BY 25.)

1 for every 4 (MULTIPLY BOTH 1 AND 4 BY 6.)

6 for every 24

___ for every 48

So 25% of 100 is ___.

25% of 4 is ___.

25% of ___ is 6.

25% of ___ is ___.

2% means

2 for every 100 (DIVIDE BY 2.)

___ for every ___

8 for every ___ (MULTIPLY BY 8.)

So 2% of 100 is ___.

2% of ___ is ___.

2% of ___ is 8.

100% means

100 for every 100 (DIVIDE BY 100.)

1 for every ___ (MULTIPLY BY ?)

___ for every ___

So 100% of 100 is ___.

100% of ___ is 1.

100% of ___ is ___.

50% means

50 for every 100

1 for every ___

18 for every ___

___ for every ___

___ for every ___

So 50% of 100 is ___.

50% of ___ is 1.

50% of ___ is 18.

50% of ___ is ___.

50% of ___ is ___.

10% means

10 for every 100 (DIVIDE BY 10.)

___ for every 10 (MULTIPLY BY 9.)

9 for every ___

___ for every ___

___ for every ___

So 10% of 100 is ___.

10% of 10 is ___.

10% of ___ is 9.

10% of ___ is ___.

10% of ___ is ___.

THAT'S "ABOUT" RIGHT
(OR PRETTY CLOSE PERCENTS)

11% means

11 for every 100 ...... 11% of 100 is ___.

Divide by 9 — about 1 for every 9 ...... 11% of 9 is about ___

OR Multiply by 3 — about 3 for every 27 ...... 11% of ___ is about 3.

OR Multiply by 7 — about 7 for every 63 ...... 11% of 63 is about ___

OR You choose — about ___ for every ___ ...... 11% of ___ is about ___

OR You choose — about ___ for every ___ ...... 11% of ___ is about ___

48% means

48 for every 100 ..... 48% of 100 is ___.

Divide by 48 — about 1 for every 2 ..... 48% of 2 is about ___

OR Multiply by 15 — about 15 for every 30 ..... 48% of ___ is about 15.

OR Multiply by 8 — about 8 for every ___ ...... 48% of ___ is about 8.

OR You choose — about ___ for every ___ ...... 48% of ___ is about ___

OR You choose — about ___ for every ___ ...... 48% of ___ is about ___

35% means

35 for every 100 ..... 35% of 100 is ___.

Divide by 35 — about 1 for every 3 ..... 35% of 3 is about ___

OR Multiply by 20 — about 20 for every ___ ..... 35% of ___ is about 20.

OR Multiply by 6 — about 6 for every ___ ..... 35% of ___ is about 6.

OR You choose — about ___ for every ___ ..... 35% of ___ is about ___

OR You choose — about ___ for every ___ ..... 35% of ___ is about ___

OTHER CONVENIENT PERCENTS

With some percents it is convenient to convert to the ratio "1 for every ___," but for other percents it is not.

75% means

	75 for every 100	75% of 100 is _____.
Divide by 25.	3 for every 4	75% of 4 is 3.
or Multiply by 12.	36 for every 48	75% of 48 is _____.
or Multiply by 5.	15 for every ___	75% of ___ is 15.
or You choose.	___ for every ___	75% of ___ is _____.
or You choose.	___ for every ___	75% of ___ is _____.

90 % means

	90 for every 100	90 % of 100 is _____.
Divide by 10.	9 for every ___	90 % of ___ is 9.
or Multiply by 7.	63 for every ___	90 % of 70 is _____.
or Multiply by 12.	108 for every ___	90% of ___ is 108.
or You choose.	___ for every ___	90 % of ___ is _____.
or You choose.	___ for every ___	90 % of ___ is _____.

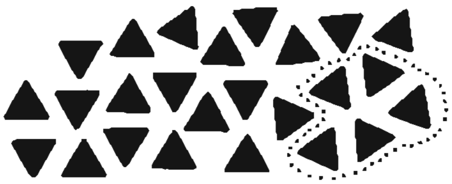
60% means

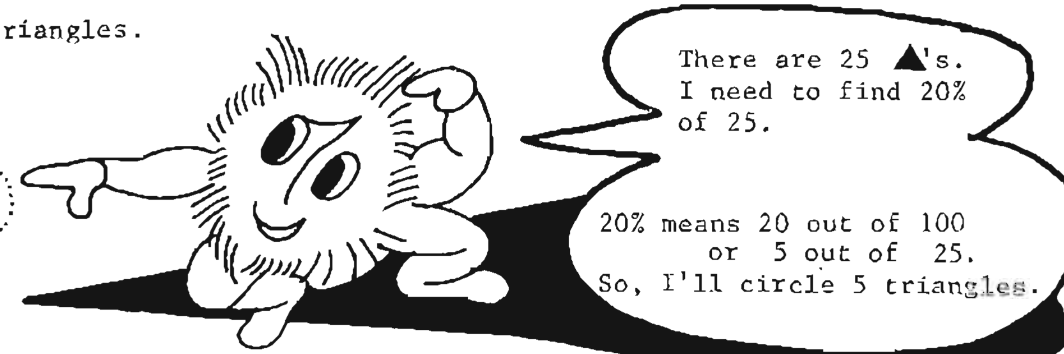
	60 for every 100	60% of 100 is _____.
Divide by 20.	3 for every 5	60% of 5 is _____.
or Multiply by 16.	48 for every ___	60% of ___ is 48.
or Multiply by 9.	___ for every 45	60% of 45 is _____.
or you choose.	___ for every ___	60% of ___ is _____.
or you choose.	___ for every ___	60% of ___ is _____.

Percents of Sets - I

The next page is a lab activity that could be used with this concept.

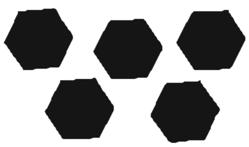
Circle 20% of this set of triangles.





Circle the given percent of each set below.

(A)

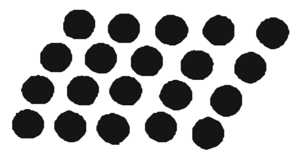


20% of this set

(B)

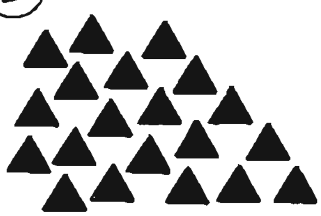
75% of this set

(C)



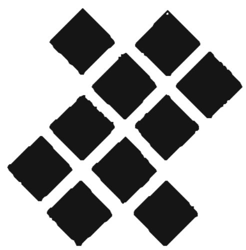
25% of this set.

(D)



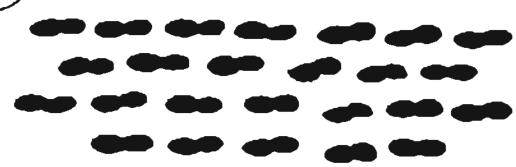
10% of this set

(E)



80% of this set

(F)



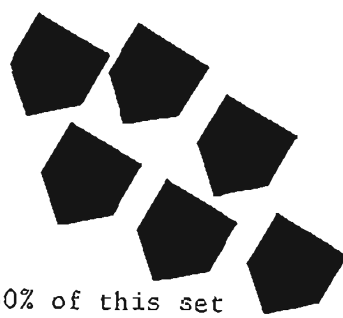
12% of this set

(G)

25% of the set

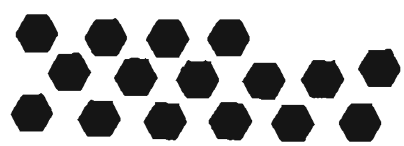
(Hint: 25 out of 100
or ___ out of 4
or ___ out of 12)

(H)



50% of this set

(I)



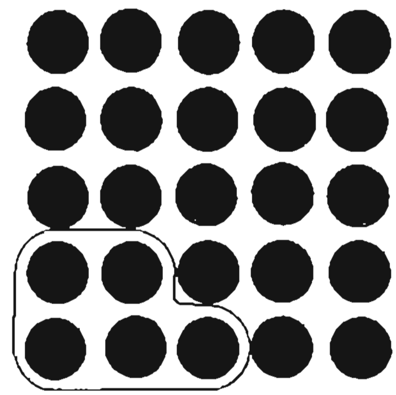
75% of this set

(Hint: 75 out of 100
or ___ out of 4
or ___ out of 16)

PERCENTS OF SETS - I (CONTINUED)

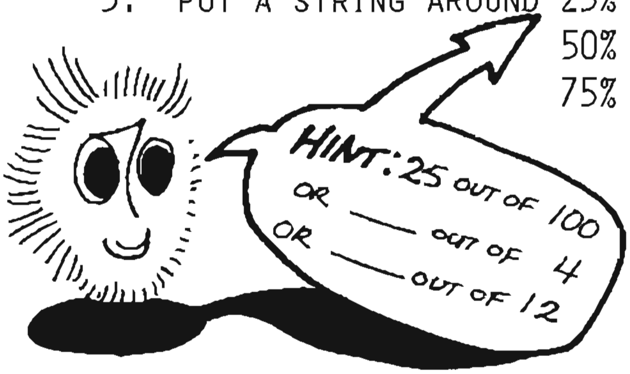
EQUIPMENT: 30 OBJECTS (LIMA BEANS, PAPER CLIPS, BLOCKS, ETC.)
STRING

ACTIVITY: 1. COUNT OUT 25 OBJECTS. PUT A STRING AROUND 20% OF THE 25 OBJECTS. RECORD WITH A PICTURE.



2. PUT A STRING AROUND 30% OF 10 OBJECTS. RECORD BY PICTURE.
20% OF 5 OBJECTS. RECORD BY PICTURE.
12% OF 25 OBJECTS.
25% OF 20 OBJECTS.
75% OF 4 OBJECTS.
10% OF 20.

3. PUT A STRING AROUND 25% OF 12 OBJECTS. RECORD
50% OF 6 OBJECTS.
75% OF 80 OBJECTS.



This activity gives the student a chance to make several trials before recording their answers.

PERCENTS OF SETS - II

Percent of a Set
As a Ratio
PERCENT

E
S
I

(1) A SCHOOL'S ENROLLMENT IS 1000 STUDENTS (GRADES 7, 8, & 9).

30% ARE 7th GRADERS → ____ OF THE 1000 STUDENTS ARE 7th GRADERS.
45% ARE 8th GRADERS → ____ OF THE 1000 STUDENTS ARE 8th GRADERS.
____% ARE 9th GRADERS → ____ OF THE 1000 STUDENTS ARE 9th GRADERS.

(2) 5% SALES TAX

____¢ FOR EVERY $1.00
____¢ FOR $9.00
____¢ FOR $15.00
____¢ FOR $149.00

(3) 10% INTEREST RATE

10¢ ON EVERY ____
$1.20 ON EVERY ____
$5.00 ON EVERY ____

(4) 6% SALES TAX

____¢ ON $1.00
____¢ ON $0.50
____¢ ON $2.50
____¢ ON $6.50

(5) SAVE 24%

$____ OF EVERY $100 EARNED
$____ OF EVERY $50 EARNED
$____ OF EVERY $225 EARNED
$____ OF EVERY $950 EARNED

(6) 88% OF THE FLOWER SEEDS WILL GROW.

____ OF EVERY 100 SEEDS
____ OF EVERY 750 SEEDS
____ OF EVERY 20,000 SEEDS
____ OF EVERY 55,000 SEEDS

TYPE: Paper & Pencil

Equivalent Forms
As a Ratio
PERCENT

E
S
I

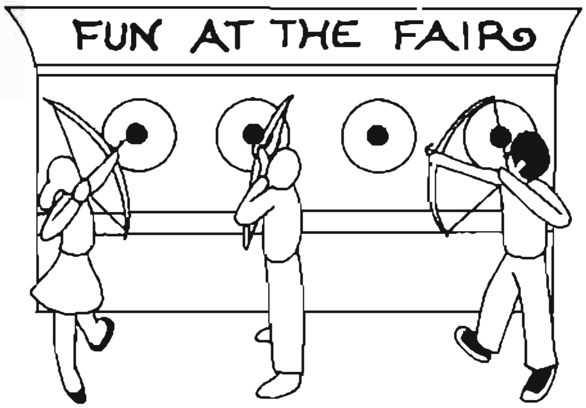
WHAT DO A CAT AND A SKUNK HAVE IN COMMON WITH %?

CIRCLE THE ANSWERS THAT HAVE THE SAME MEANING AS THE PERCENT WRITTEN IN THE FIRST COLUMN. THEN WRITE THE LETTER IN THE CORRECT BOX. THERE ARE FOUR CORRECT ANSWERS FOR EACH PROBLEM.

10%	6 FOR EVERY 60	28 T	1 WRONG FOR EVERY 10 PROBLEMS	13 N	10 PER 100	1 A	5 OUT OF 25	18 O	35 FOR EVERY 350	3 A	$10. FOR EACH 1 SHIRT	14 E
25%	2 ENDS FOR AN 11 MAN TEAM	19 A	50 HITS FOR 200 SHOTS	4 T	25 OUT OF 75	20 R	25¢ FOR EACH DOLLAR	18 U	1 OUT OF 4	26 E	12 FOR EVERY 48	7 S
50%	5 DIMES COMPARED TO $1.00	22 A	1 FOR EVERY 2	10 R	50¢ FOR 200 PINS	17 H	50 FOR EVERY 100	19 N	23 PER 25	16 K	60 HITS FOR 120 SWINGS	12 A
75%	5 NO'S FOR EACH 6 VOTES	3 A	15 OUT OF 50	23 T	75 FOR EVERY 100	14 D	THREE OUT OF FOUR	5 H	150 COMPARED TO 200	20 K	3 25¢ FOR EACH DOLLAR	21 H
100%	4 HITS FOR 4 SWINGS	23 S	63 OUT OF 63	16 S	12 EGGS PER DOZEN	2 C	4 BIRTHS PER 100 PEOPLE	13 C	100 OUT OF 100	11 R	FIVE PER FIFTY	1 N
MORE THAN 100%	2 FOR EVERY ONE	24 S	85 FOR EACH 100	22 A	105 COMPARED TO 100	9 U	999 FOR EVERY 1	15 A	1 OUT OF 100.	27 A	25 FOR EACH 20	17 K
LESS THAN 1%	1 OUT OF 150	6 A	3 PER 50	10 S	5 OUT OF 600	8 P	1 CHANCE OUT OF A MILLION	27 N	10 FOR EVERY 100	15 T	2 COMPARED TO 250	25 C

1	2	3	4	5	6	7	8	9	10	11	12	13	14
A	C	A	T	H	A	S	P	U	R	R	A	N	D
15	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**
A	S	K	U	N	K	H	A	S	S	C	E	N	T

TYPE: Paper & Pencil/Puzzle



Mike, Tammy, and Mark stopped at a booth to shoot arrows. After shooting for awhile, this is what their scores were.

Mike - 18 bull's-eyes out of 25 shots

Tammy - 16 bull's-eyes out of 20 shots

Mark - 7 bull's-eyes out of 10 shots

Mike said, "I'm the best shot because I have the most bull's-eyes."

"No," said Mark, "I'm best because I have missed the least."

Who do you think is the best shot?

A) Suppose someone made 18 bull's-eyes out of 50 shots. Is this better than Mike? ________

B) How about someone who made 18 bull's-eyes out of 18 shots? Is this better than Mike? ________

C) If a shooter misses 3 out of 4 shots, is this better than Mike? ________

D) How about someone who misses 3 out of 50 shots? ________

Suppose all three continue to shoot like they are now. How many bull's-eyes would each have made after 100 shots?

Mike		Tammy	Mark	
18 out of 25	MULTIPLY BY 4	16 out of 20	7 out of 10	Who is the best
___ out of 100		___ out of 100	___ out of 100	shot? ________
___%		___%	___%	

Assuming they continue to shoot like they are now, what percent of bull's-eyes would each of these people have if they took 100 shots?

Sam - 33 bull's-eyes out of 50 shots _____%

Mary - 20 bull's-eyes out of 25 shots _____%

Debbie - 3 bull's-eyes out of 4 shots _____%

Rick - 2 bull's-eyes out of 2 shots _____%

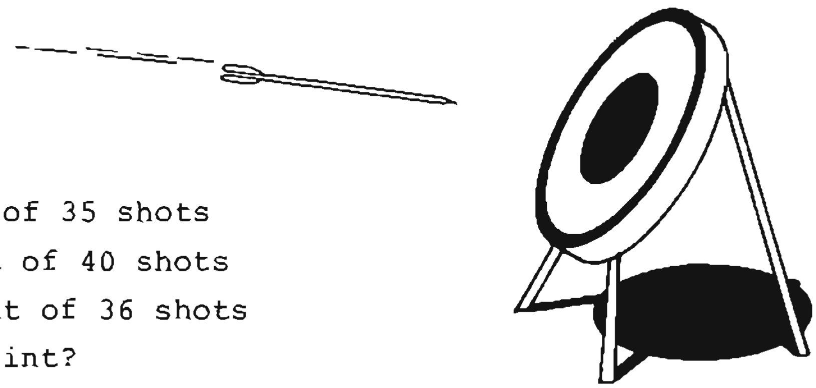
Tom - 82 bull's-eyes out of 100 shots _____%

Sue - 4 bull's-eyes out of 5 shots _____%

IDEA FROM: *Synchro-Math/Experiences*

MORE FUN AT THE FAIR

On Saturday afternoon a contest was held at the archery booth. During the contest these were some of the results.



Tom - 21 bull's-eyes out of 35 shots
Dick - 28 bull's-eyes out of 40 shots
Harry - 27 bull's-eyes out of 36 shots
Who is leading at this point?

If they continue to shoot like this, who will be leading after 100 shots are taken? ______________________

Tom

21 for every 35 DIVIDE BY 7
___ for every 5
___ for every 100 MULTIPLY BY 20
___%

Dick

28 for every 40 DIVIDE BY 4
___ for every 10
___ for every 100
___%

Harry

27 for every 36
___ for every 4
___ for every 100
___%

Predict what percent of bull's-eyes each of these archers will have a after taking 100 shots.

Cindy - 42 bull's-eyes for every 70 shots _____%
Tony - 60 bull's-eyes for every 80 shots _____%
Ben - 36 bull's-eyes for every 72 shots _____%
Kathy - 44 bull's-eyes for every 55 shots _____%
Terry - 12 bull's-eyes for every 48 shots _____%
Barb - 40 bull's-eyes for every 60 shots _____%

Of all nine shooters who will probably win the contest? ____________
Who probably need shooting lessons and more practice? ______________

BE COOL - GO TO SCHOOL

Ted, Sally, and Phil moved into the school district at different times. The chart shows their attendance so far.

NAME	DAYS PRESENT	DAYS ENROLLED
TED	20	33
SALLY	11	17
PHIL	7	11

a) Who has been present the most days? ____________

b) Who has been present the fewest days? __________

c) Who has been absent the most days? ____________

d) Who has been absent the fewest days? __________

If the attendance pattern of all three students remains about the same, who will have the highest percent of days present in school? Work the examples below.

Ted

20 out of 33

about ___ out of 100

about ___%

Multiply by 3 because $100 \div 33 \approx 3$

Sally

11 of 17

about ___ of 100

about ___%

Multiply by 6 because $100 \div 17 \approx 6$

Phil

7 for every 11

about ___ for every 100

about ___%

Multiply by 9

Find the approximate percent of days present for these students.

BETTY	13	19	≈ %
MEL	27	35	≈ %
TERRY	15	26	≈ %
HELEN	6	7	≈ %
CLARA	12	15	≈ %
DAN	7	9	≈ %

Can you pick a period of ten consecutive school days that shows you would have

1) an excellent attendance record

2) a poor attendance record

PUNY PERCENTS

Kathy, John, Eric and Lucy each sent an entry to the McDuffs' hamburger contest. McDuffs advertised it would award prizes to 1% of the total entries. It was reported that 1600 entries were received. How many prizes were awarded?

1% means
1 for every 100
______ for every 1600

What percent of the total were the four students' entries?

4 for every 1600
1 for every 400
$\frac{1}{4}$ for every 100
So, 4 = $\frac{1}{4}$% of 1600

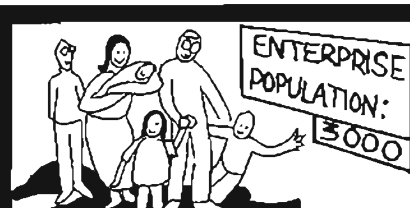
1) In 1973 about 400 auto thefts were reported for every 100,000 people. What percent of the population had cars stolen?

______

400 for every 100,000
4 for every ___
___ for every 100

2) $\frac{1}{10}$% of all eggs are rejected. 20,000 have been checked. ______ eggs are rejected.

$\frac{1}{10}$ for every 100
1 for every ___
___ for every 20,000



3) Mark's family has 6 members. Mark's family is _____% of Enterprise.

6 out of 3000
1 out of ___
___ out of 100

4) Rhode Island is _____% of Alaska.

1000 sq. miles for every 600,000 sq. miles
1 sq. mile for every ___
___ sq. miles for every 100

5) In counting the letters in a paragraph Jack found 5 z's in 2000 letters. What percent of the letters were z's? _____

5 for every 2000
1 for every ___
___ for every 100

6) A $\frac{1}{6}$-cup serving of rice has $\frac{1}{2}$% of the minimum daily requirement of Vitamin C. How many cups would you have to cook in order to have enough Vitamin C for one day? _____

7) Many clothing labels say, "Less than 1% shrinkage." If the actual shrinkage is $\frac{1}{2}$%, how much is lost if you wash 100 yds. of cloth? _____

SOLVING PERCENT EXERCISES
BY THE PROPORTION METHOD

Almost all exercises involving percent can be solved by using a proportion format, $\frac{\quad}{\quad} = \frac{\quad}{100}$. Many words can be used to describe the terms of the proportion, but these pages will emphasize the use of "is," "of" and "percent." So the proportion format to be used is

$$\frac{\text{is number}}{\text{of number}} = \frac{\text{percent number}}{100}$$

One advantage of this method is that only one format is needed to solve percent exercises, rather than three; $p = br$, $b = \frac{p}{r}$, $r = \frac{p}{b}$. Another advantage is that the percent need not be converted to a fraction or a decimal. The use of the form $\frac{\text{percent number}}{100}$ allows a student to write 3.4% as $\frac{3.4}{100}$. Obviously, students will need the skills for computing with fractions and decimals.

For those teachers who have used the ratio method of the previous pages for solving percent exercises, the proportion method can be motivated by examining these examples.

(1) 50% of 40 is __?__.

50% means 50 out of 100

or 1 out of 2

or 20 out of 40

$$\text{of} \rightarrow \frac{20}{40} = \frac{50}{100} \leftarrow \text{of}$$

(2) 15 is what percent of 60?

15 out of 60

5 out of 20

25 out of 100

$$\text{of} \rightarrow \frac{15}{60} = \frac{25}{100} \leftarrow \text{of}$$

These examples emphasize that the "of terms" become the denominators of the proportion.

Students will probably need practice in converting exercises to the proportion method. A worksheet of "set 'em up, but don't solve 'em" would be appropriate. It is easiest to first write the "percent number," then the "of number" and finally the "is number." For example,

What number is 30% of 90? $\frac{?\ (3\text{rd})}{90\ (2\text{nd})} = \frac{30\ (1\text{st})}{100}$

SOLVING PERCENT EXERCISES
BY THE PROPORTION METHOD (CONTINUED)

While providing this practice, show students many different forms of the exercise.

(a) 50% of 60 is ___?___

(b) ___?___ is 50% of 60

(c) What number is 50% of 60?

(d) 50% of 60 is what number?

(e) What is the discount if a $60 pantsuit is marked down 50%?

$$\frac{?}{60} = \frac{50}{100}$$

Emphasize that the "of number" is always written behind the word "of," but the "is number" may be written before or after the word "is." For example, 35 is 50% of 70 or 50% of 70 is 35.

Examples of percent exercises solved using the proportion method.

(1) What percent of 25 is 20? $\frac{\text{is } \#}{\text{of } \#} = \frac{\text{percent } \#}{100} \longrightarrow \frac{20}{25} = \frac{?}{100}$

$$20 \times 100 = 25 \times ?$$
$$2000 = 25 \times ?$$
$$80 = ?$$

(2) Find $4\frac{1}{2}\%$ of 200. $\frac{\text{is } \#}{\text{of } \#} = \frac{\text{percent } \#}{100} \longrightarrow \frac{?}{200} = \frac{4\frac{1}{2}}{100}$

$$? \times 100 = 200 \times 4\tfrac{1}{2}$$
$$? \times 100 = 900$$
$$? = 9$$

(3) 3.3% of what number is 99? $\frac{\text{is } \#}{\text{of } \#} = \frac{\text{percent } \#}{100} \longrightarrow \frac{99}{?} = \frac{3.3}{100}$

$$99 \times 100 = ? \times 3.3$$
$$9900 = ? \times 3.3$$
$$3000 = ?$$

(4) An airline ticket costs $400 not including tax. Find the tax if the tax rate is 5%.

Restated: 5% of $400 is ___?___. $\frac{\text{is } \#}{\text{of } \#} = \frac{\text{percent } \#}{100} \longrightarrow \frac{?}{400} = \frac{5}{100}$

$$? \times 100 = 400 \times 5$$
$$? \times 100 = 2000$$
$$? = 20$$

RATIO, PROPORTION, AND SCALING

Placement Guide for Tabbed Divisors

CONTENTS

PERCENT: AS A FRACTION/DECIMAL

*Indicates percents greater than 100% are used on the page.

TITLE	PAGE	TOPIC	TYPE
See-Through Demonstration	468	As a fraction/decimal Volume model	Activity
Only the Names Have Been Changed	470	As a fraction/decimal	Game Paper and pencil
Games Students Play	473	As a fraction/decimal	Game
Search & Circle	475	As a fraction/decimal*	Paper and pencil
The Percent Painter Returns	476	As a decimal	Manipulative

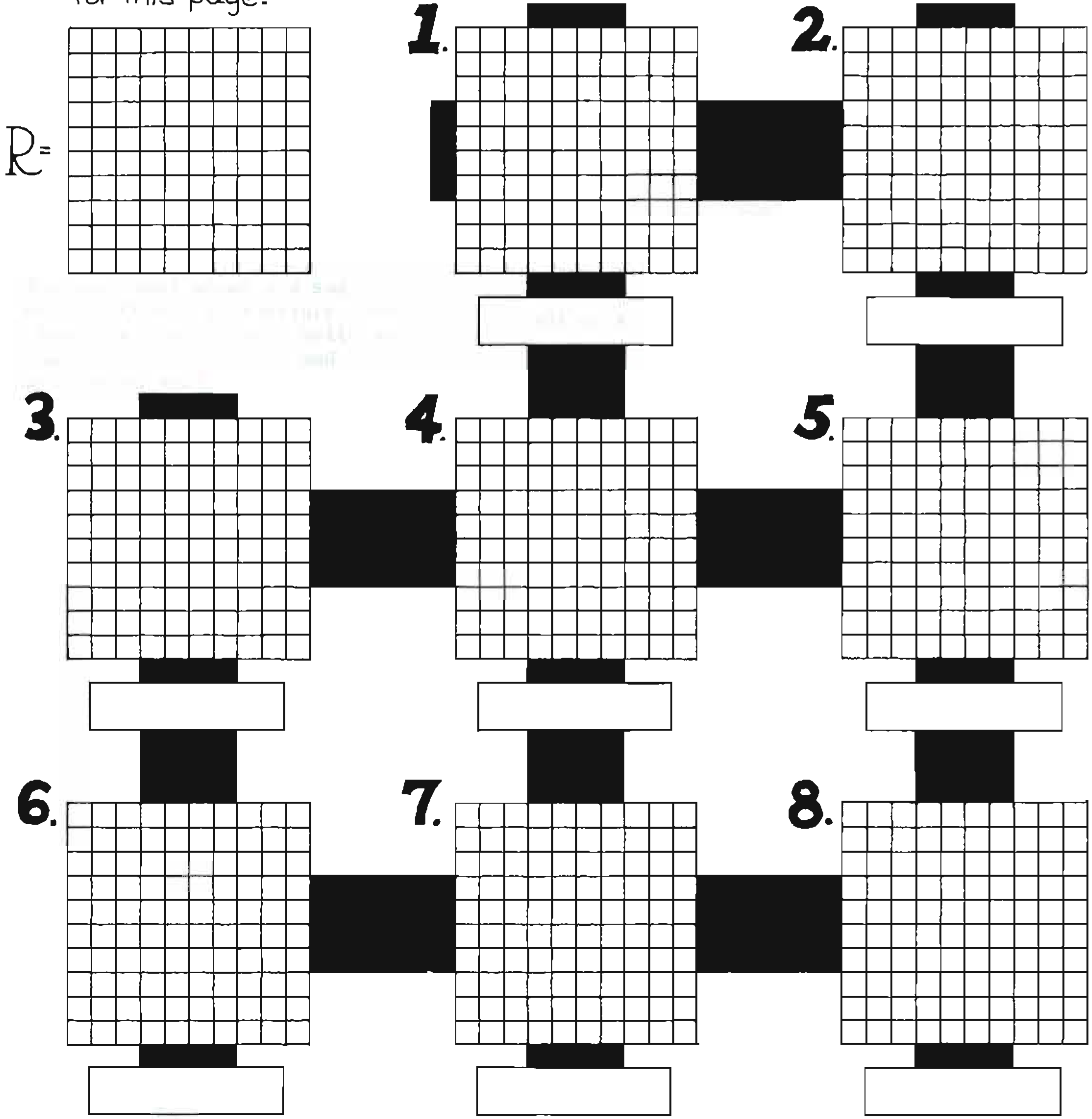
*Indicates percents greater than 100% are used on the page.

F, D & P

THE SHADY TRIO

Reference set for this page.

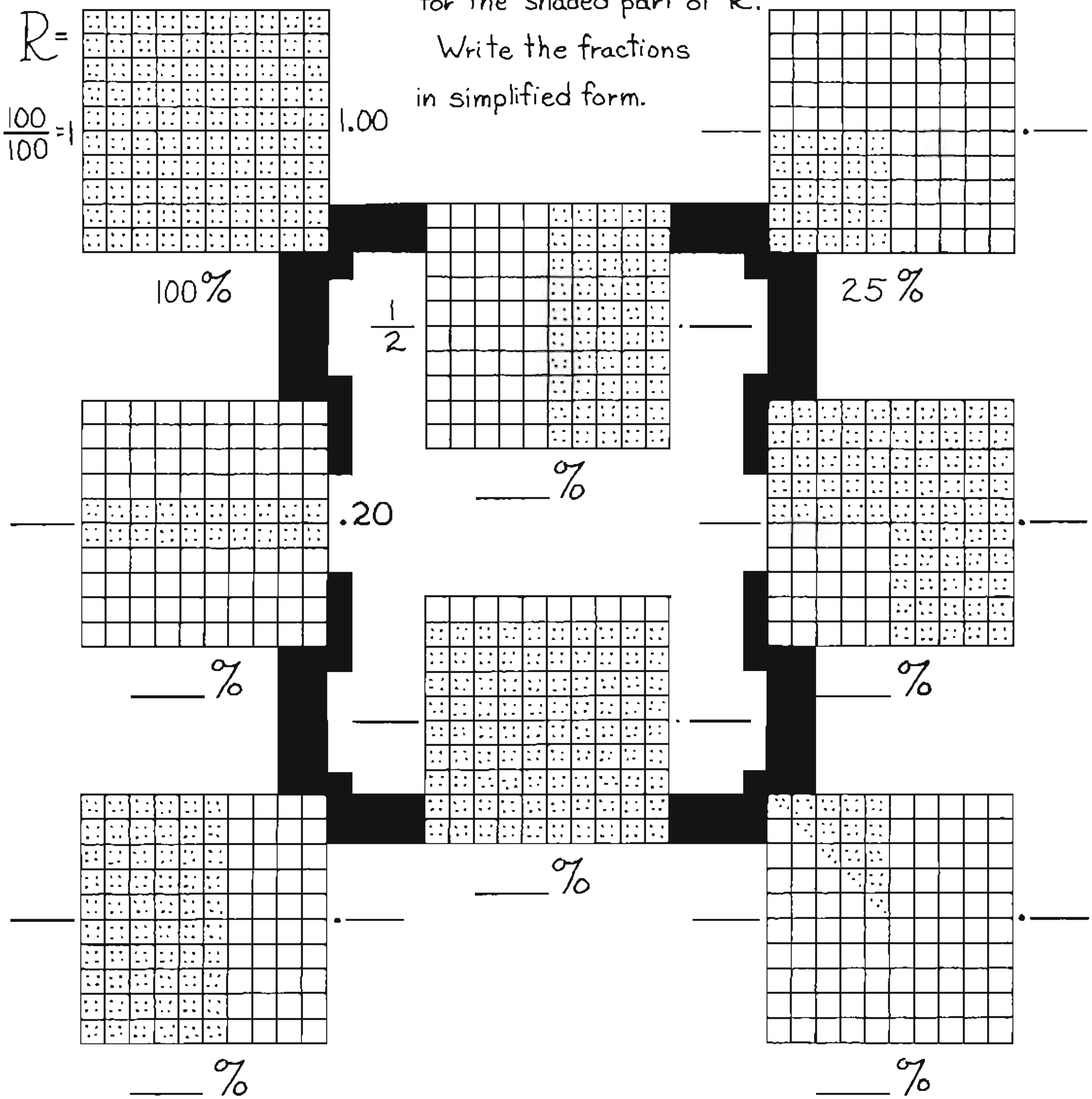
In each grid shade the amount shown below.



F, D & P

A SHADED TRIO

Write equivalent forms for the shaded part of R. Write the fractions in simplified form.



TYPE: Paper & Pencil

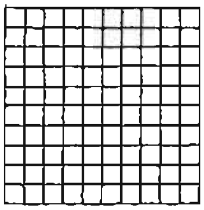
IDEA FROM: *Project R-3*
Permission to use granted by E.L. Hodges

BE A REAL CUTUP

Grid Model
as a Fraction/Decimal/
PERCENT

Provide each student with a piece of $\frac{1}{10}$" grid paper.

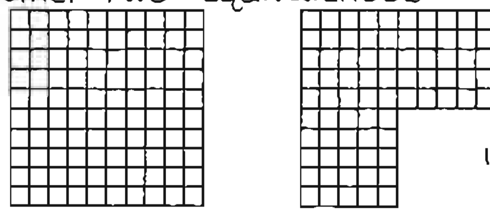
Reference Set for this page.

R = 

Cut and paste the grid paper to show each amount. Write the other two equivalences.

EXAMPLE :

$1\frac{3}{4}$ R



1.75

175%

1. 12% of R

2. .87 R

3. .35 R

4. $\frac{61}{100}$ of R

5. 1.7 R

6. 130 % of R

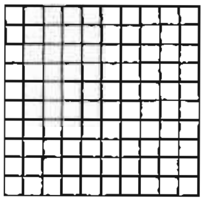
7. $3\frac{1}{20}$ R

8. 2.01 R

TYPE: Manipulative

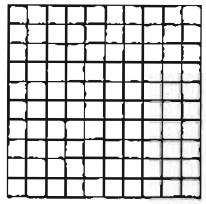
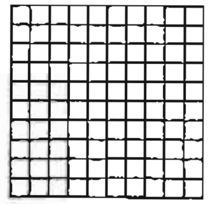
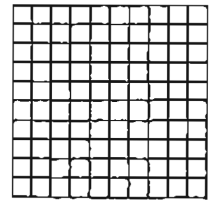
FRACTION ⟶ PERCENT 1

REFERENCE SET

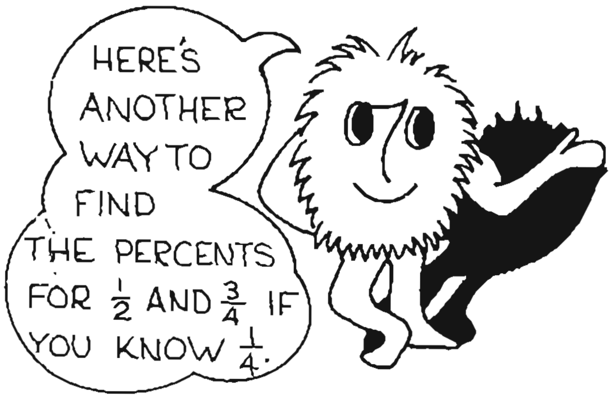
R = 

SHADE

$\frac{1}{4}$ of R | $\frac{1}{2}$ of R | $\frac{3}{4}$ of R

WHAT PERCENT OF R IS SHADED? ____% ____% ____%



$\frac{1}{2} = \frac{2}{4} = \frac{1}{4} + \frac{1}{4}$

$\frac{1}{2} = 25\% + 25\%$

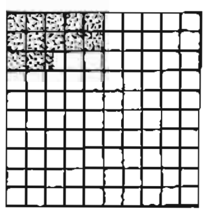
$\frac{1}{2} = 50\%$

$\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

$\frac{3}{4} =$ ____% + ____% + ____%

$\frac{3}{4} =$ ____%

This diagram shows that $\frac{1}{8}$ of R is shaded. What percent of R is shaded? ____%



ANOTHER WAY

$\frac{1}{4} = \frac{2}{8} = \frac{1}{8} + \frac{1}{8}$

$\frac{1}{4} = 25\% = 12\frac{1}{2}\% +$ ____%

$\frac{1}{8} =$ ____%

$\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$

$\frac{3}{8} =$ ____% + ____% + ____%

$\frac{3}{8} =$ ____%

$\frac{5}{8} =$

$\frac{5}{8} =$

$\frac{5}{8} =$ ____%

$\frac{7}{8} =$

$\frac{7}{8} =$

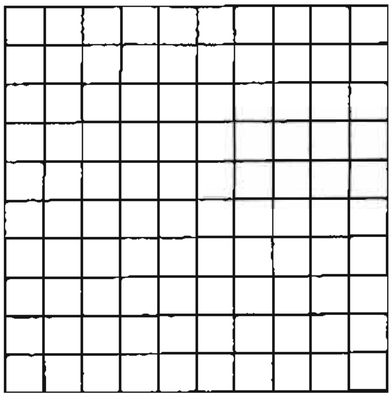
$\frac{7}{8} =$ ____%

Can you see shortcuts for doing these problems?

Challenge: Find the percent equivalences for $\frac{1}{16}, \frac{3}{16}, \frac{5}{16}, \frac{7}{16}, \frac{9}{16}, \frac{11}{16}, \frac{13}{16}, \frac{15}{16}$.

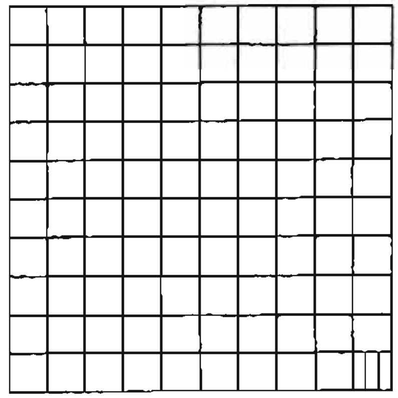
FRACTION → PERCENT 2

REFERENCE SET

R = 

Shade $\frac{1}{3}$ of R by shading 1 out of every 3 squares. The last square is divided into 3 parts to help you.

What percent of R is shaded? ________%



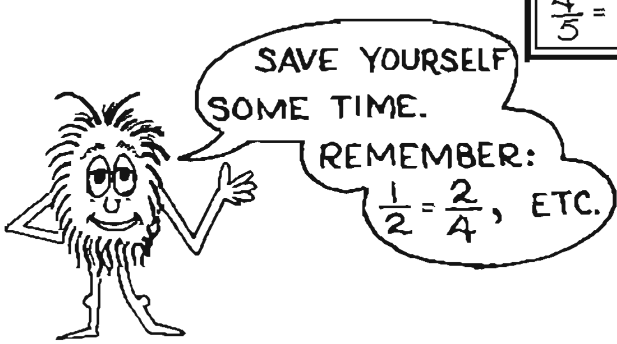
$\frac{1}{3}$ of R = ____% of R

If $\frac{2}{3} = \frac{1}{3} + \frac{1}{3}$, what percent of R is represented by $\frac{2}{3}$? __________

Challenge: If $\frac{1}{3} = \frac{2}{6} = \frac{1}{6} + \frac{1}{6}$, find the percent of R represented by $\frac{1}{6}$; by $\frac{5}{6}$.
$\frac{1}{6}$ = ______%; $\frac{5}{6}$ = ______%

Use grid paper to help you complete this chart. Save it for future use.

$\frac{1}{2}$ = %	$\frac{1}{3}$ = %	$\frac{1}{4}$ = %	$\frac{1}{5}$ = %	$\frac{1}{6}$ = %	$\frac{1}{8}$ = %	$\frac{1}{10}$ = %
	$\frac{2}{3}$ = %	$\frac{2}{4}$ = %	$\frac{2}{5}$ = %	$\frac{2}{6}$ = %	$\frac{2}{8}$ = %	$\frac{2}{10}$ = %
		$\frac{3}{4}$ = %	$\frac{3}{5}$ = %	$\frac{3}{6}$ = %	$\frac{3}{8}$ = %	$\frac{3}{10}$ = %
			$\frac{4}{5}$ = %	$\frac{4}{6}$ = %	$\frac{4}{8}$ = %	$\frac{4}{10}$ = %
				$\frac{5}{6}$ = %	$\frac{5}{8}$ = %	$\frac{5}{10}$ = %
					$\frac{6}{8}$ = %	$\frac{6}{10}$ = %
					$\frac{7}{8}$ = %	$\frac{7}{10}$ = %
						$\frac{8}{10}$ = %
						$\frac{9}{10}$ = %



PERCENTS WITH RODS & SQUARES - II

Equipment: Orange and white Cuisenaire Rods.

Activity:

1. Use the orange rods to cover $\frac{1}{2}$ of the square.

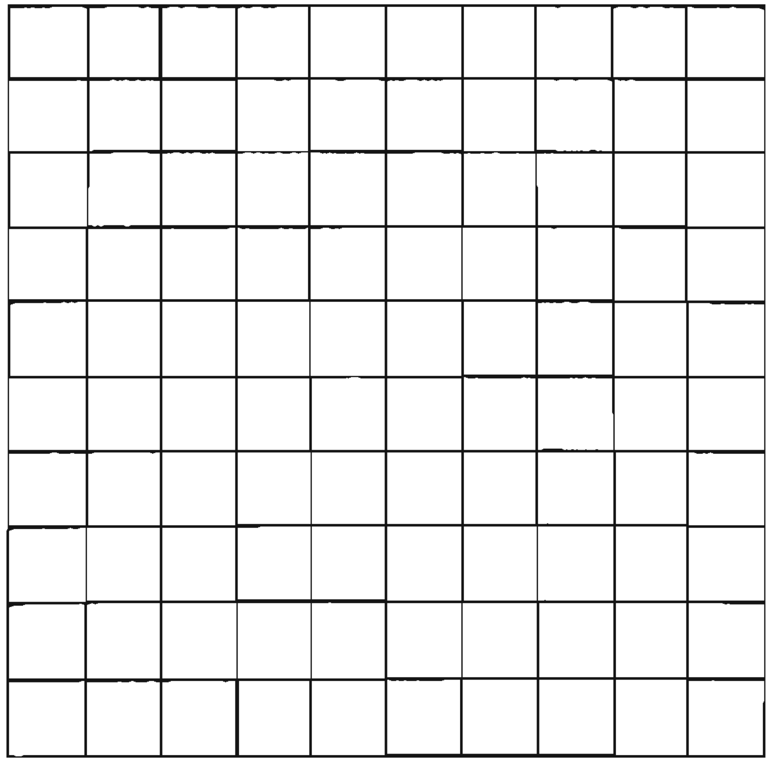
   How many did you use? ________

   What percent of the square did you cover? ________

2. Use the white rods to cover $\frac{3}{100}$ of the square.

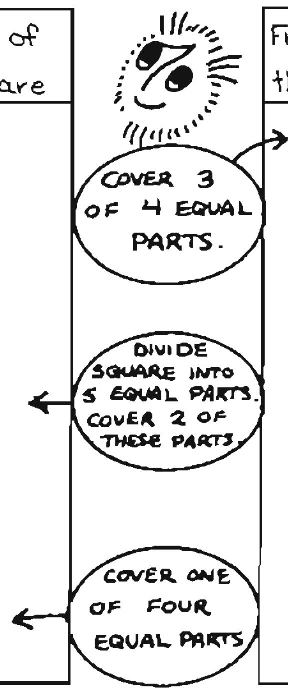
   How many did you use? ________

   What percent of the square did you cover? ________



3. Use orange and white rods to find the corresponding percents and decimals for these fractions.

Fraction of the square	Percent of the square	Decimal of the square
$\frac{1}{10}$		
$\frac{9}{10}$		
$\frac{3}{10}$		
$\frac{2}{5}$		
$\frac{4}{5}$		
$\frac{1}{2}$		
$\frac{1}{4}$		



Fraction of the square	Percent of the square	Decimal of the square
$\frac{3}{4}$		
$\frac{1}{50}$		
$\frac{1}{20}$		
$\frac{1}{25}$		
$\frac{13}{100}$		
$\frac{3}{1}$		
$\frac{217}{100}$		

PERCENTS WITH RODS & SQUARES - III

Equipment: 100 white Cuisenaire Rods
or 100 cubes

Activity:

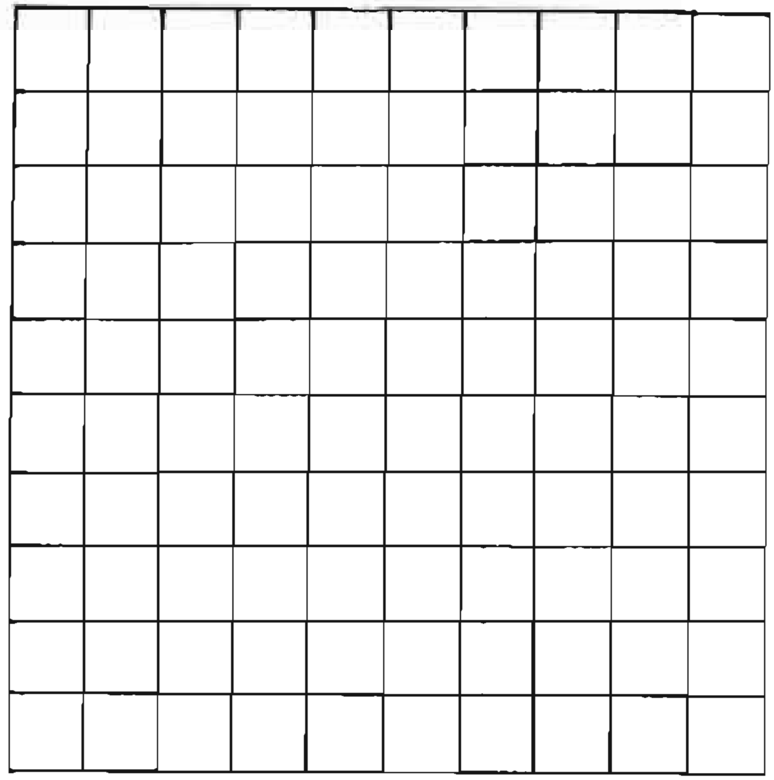
1. a) Use the rods to completely cover the square.
   1 rod will cover ___% of the square.

   b) Can you use the rods to exactly cover $\frac{1}{3}$ of the square? ______

   c) Use the rods to cover approximately $\frac{1}{3}$ of the square.
   Hint: Divide the 100 rods into three equal groups.

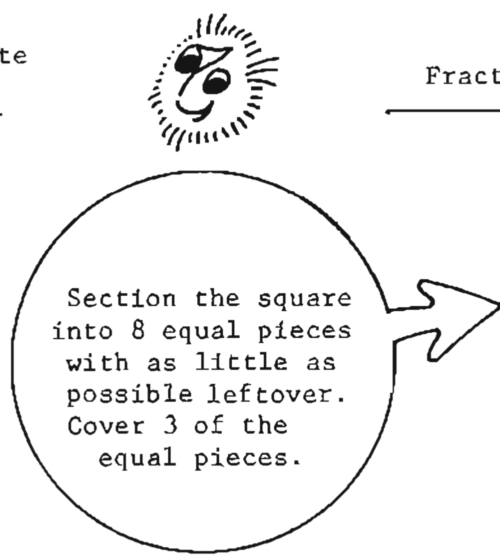
   d) How many rods in each group?
   Are there any rods left over? ___

   e) $\frac{1}{3}$ of the square $\approx$ ___% of the square.



2. Find an approximate percent to correspond to each fraction.

Fraction	Approximate Percent
$\frac{1}{6}$	
$\frac{1}{8}$	
$\frac{1}{11}$	
$\frac{1}{7}$	
$\frac{1}{9}$	



Fraction	Approximate Percent
$\frac{2}{3}$	
$\frac{3}{8}$	
$\frac{5}{8}$	
$\frac{4}{3}$	
$\frac{5}{3}$	

PERCENT WITH RODS & METRES - II

EQUIPMENT: METRE STICK
ORANGE AND WHITE CUISENAIRE RODS

ACTIVITY:

1. AN ORANGE ROD IS WHAT PERCENT OF A METRE? ________
   WHAT DECIMAL PART OF A METRE? ________
   WHAT FRACTIONAL PART OF A METRE? $\frac{1}{10}$

2. A WHITE ROD IS WHAT PERCENT OF A METRE? ________
   WHAT DECIMAL PART OF A METRE? .01
   WHAT FRACTIONAL PART OF A METRE? ________

3. MAKE THIS CHART ON YOUR PAPER. FILL IN THE BLANKS.

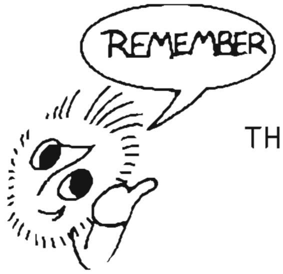
ROD	PERCENT OF A METRE	DECIMAL PART OF A METRE	FRACTION OF A METRE
1 WHITE			
3 WHITE			
10 WHITE			
50 WHITE			
85 WHITE			
100 WHITE			
125 WHITE			
1 ORANGE			
4 ORANGE			
5 ORANGE			

ROD	PERCENT OF A METRE	DECIMAL PART OF A METRE	FRACTION OF A METRE
10 orange			
15 orange			
2 orange + 5 white			
7 orange + 5 white			
6 orange + 2 white			
12 orange + 3 white			
			45%
		.37	
	$\frac{3}{10}$		
	$\frac{3}{5}$		

PERCENTS WITH RODS & METRES - III

EQUIPMENT: METRE STICK
ORANGE AND WHITE CUISENAIRE RODS

ACTIVITY: THE LENGTH OF AN ORANGE ROD IS $\frac{1}{10}$ OR .1 OR 10% OF A METRE.



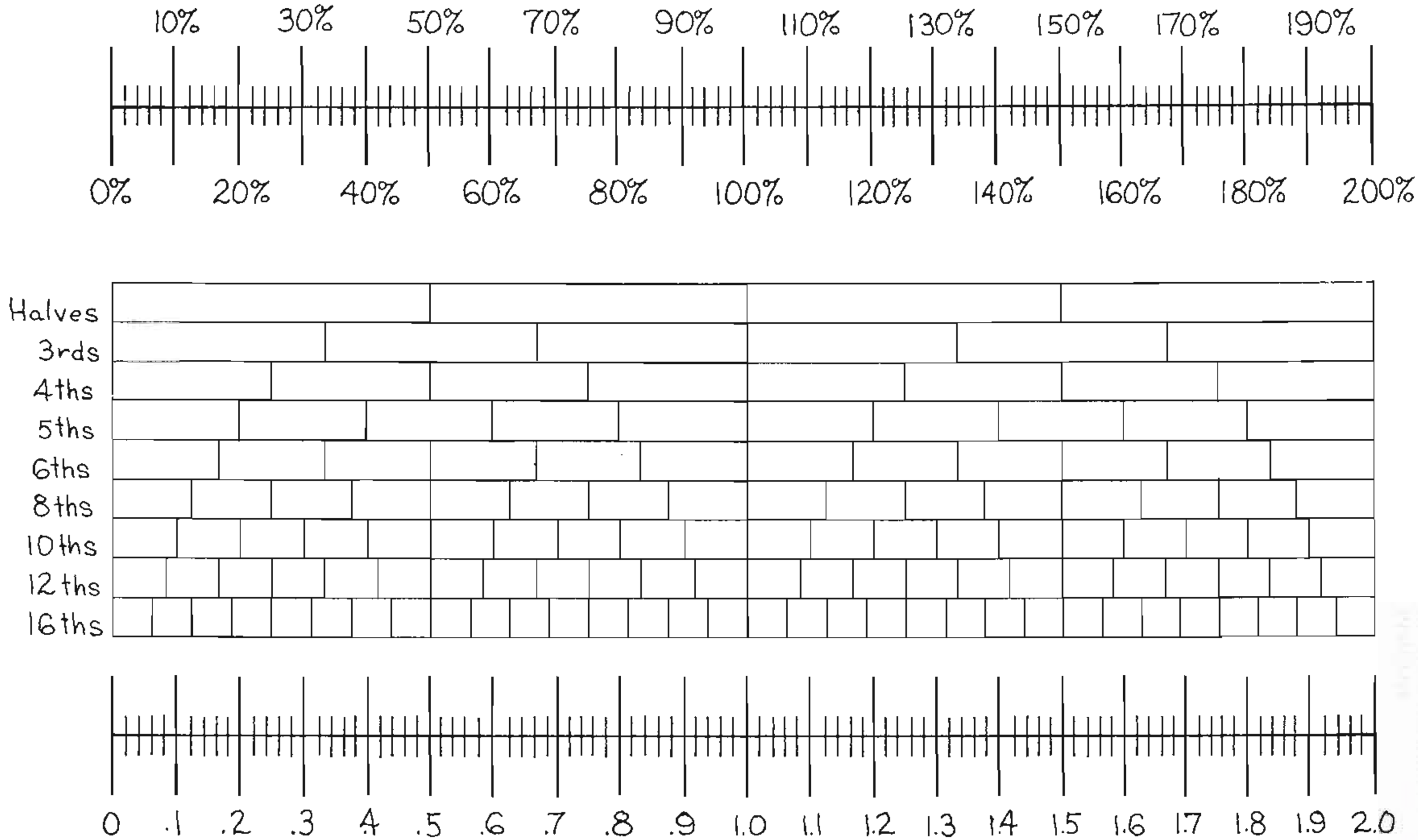
THE LENGTH OF A WHITE ROD IS $\frac{1}{100}$ OR .01 OR 1% OF A METRE.

$$\frac{1}{10} = \frac{10}{100} \quad \text{AND} \quad .1 = .10$$

MAKE THIS CHART ON YOUR PAPER. FILL IN THE BLANKS. THE RODS CAN HELP THE DECIMALS, FRACTIONS AND PERCENTS MAKE SENSE!

	FRACTION OF A METRE	# OF ORANGE RODS USED	# OF WHITE RODS USED	DECIMAL PART OF A METRE	PERCENT OF A METRE
EXAMPLE:	$\frac{1}{10}$	1	0	.1	10%
	$\frac{3}{10}$				
	$\frac{7}{10}$				
	$\frac{1}{2}$				
	$\frac{1}{5}$				
	$\frac{2}{5}$				
	$\frac{3}{5}$				
	$\frac{1}{4}$				
	$\frac{3}{4}$				
	$\frac{1}{20}$				
	$\frac{1}{25}$				
	$\frac{2}{1}$				
	$\frac{3}{2}$				

THE PERCENT BAR SHEET



IDEA FROM: C.O.L.A.M.D.A.
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HALLELUJAH I'VE BEEN CONVERTED

Use *The Percent Bar Sheet* and a straightedge to make these conversions.

a) percent ⟶ fraction

30% ⟶ 75% ⟶ 67% ⟶

b) fraction ⟶ percent

$\frac{7}{10}$ ⟶ $\frac{4}{10}$ ⟶ $\frac{11}{16}$ ⟶

c) percent ⟶ decimal

45% ⟶ 78% ⟶ 11% ⟶

d) decimal ⟶ percent

.9 ⟶ .35 ⟶ .06 ⟶

Fill in the blanks with the other two forms.

50%, _____, _____

_____, $\frac{7}{8}$, _____

_____, _____, .16

8%, _____, _____

Let the length of the percent line be the REFERENCE SET R. Circle the longer of these three lengths.

50% of R, $\frac{1}{3}$ of R, .4 of R

55% of R, $\frac{5}{8}$ of R, .7 of R

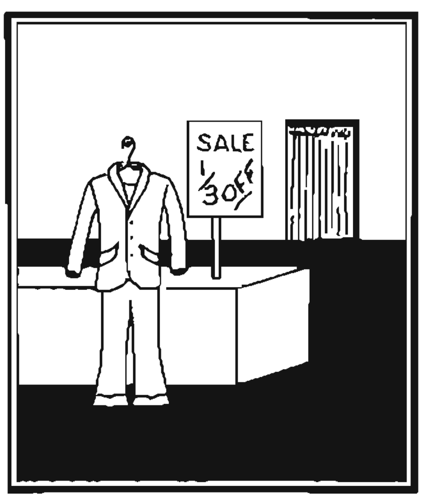
15% of R, $\frac{1}{8}$ of R, .09 of R

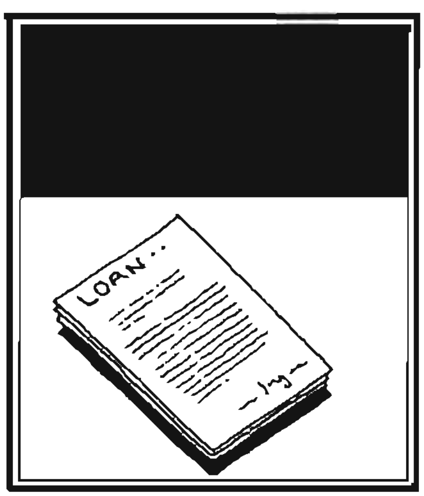
75% of R, $\frac{3}{4}$ of R, .8 of R

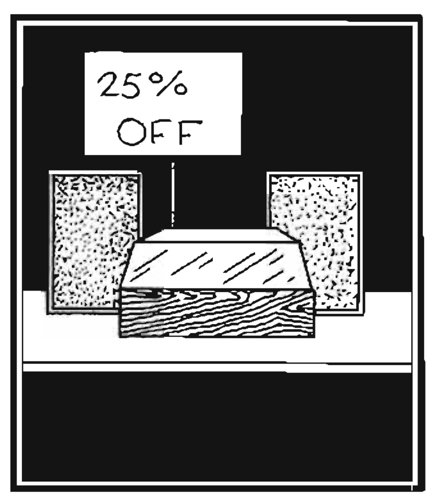
A pantsuit is marked $\frac{1}{3}$ off. What percent is this? ______

A loan has an annual interest rate of 18%. What decimal is this? ______

The purchase of a new stereo requires 25% down. What fraction is this? ______







IDEA FROM: *C.O.L.A.M.D.A.*

PIANOS ARE HEAVY!

Decode the riddles by writing the letter above the correct answer.

(1) Change each of these to a fraction in simplest form.

A = 50% = ______ L = 4% = ______ A = 5% = ______

T = 30% = ______ O = 10% = ______ R = 90% = ______

M = 75% = ______ I = 25% = ______ N = 35% = ______

F = 20% = ______

What do you get when you push a piano down a mine shaft?

A	F	L	A	T	M	I	N	O	R
$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{25}$	$\frac{1}{20}$	$\frac{3}{10}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{7}{20}$	$\frac{1}{10}$	$\frac{9}{10}$

(2) Change each of these to a percent.

R = $\frac{2}{5}$ = ______ A = $\frac{3}{25}$ = ______ F = $\frac{17}{20}$ = ______

A = $\frac{41}{50}$ = ______ T = $\frac{1}{100}$ = ______ A = $\frac{1}{50}$ = ______

M = $\frac{4}{5}$ = ______ O = $\frac{6}{6}$ = ______ L = $\frac{3}{5}$ = ______

J = $\frac{13}{100}$ = ______

What do you get when you push a piano through an officer's club?

A	F	L	A	T	M	A	J	O	R
12%	85%	60%	82%	1%	80%	2%	13%	100%	40%

SUSPENDED FOR TEN DAYS

Why was the "A" student in a cannibal school suspended for ten days?

To find the answer circle the true equations in each row. Over each equation is a number and a letter. The number tells you in which box at the bottom of the page to put the letter. Each row contains only three correct statements.

6 - T	15 - A	20 - C	23 - I	14 - H
4% = .04	2.9% = .29	8% = .08	77.3% = 77.3	18.6% = .186
21 - H	7 - E	13 - P	4 - B	21 - O
16.3% = .163	5% = .5	40% = .4	2.5% = .025	3% = .003
9 - S	11 - D	20 - L	9 - R	24 - S
2% = .2	1% = .01	92% = 9.2	98.9% = .989	35% = .35
3 - E	17 - T	10 - A	12 - U	19 - H
3.72% = .0372	.9% = .009	9% = .9	15.2% = .152	1.5% = .15
8 - E	16 - S	1 - S	18 - J	5 - U
67% = .67	10% = .01	123% = 1.23	16.3% = 1.63	.8% = .008
24 - P	18 - E	10 - E	23 - R	17 - M
77.3% = 77.3	5% = .05	.2% = .002	97% = .97	2.9% = .29
13 - T	22 - E	14 - W	2 - H	19 - A
128% = 12.8	.9% = .009	3.2% = .0032	150% = 1.5	.4% = .004
15 - E	7 - T	12 - A	11 - N	16 - R
7.9% = .079	20% = .2	3% = .3	12.5% = 1.25	256% = 2.56

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
S	H	E	B	U	T	T	E	R	E	D	U	P	H	E	R	T	E	A	C	H	E	R	S

IDEA FROM: *Mathimagination*, Book E

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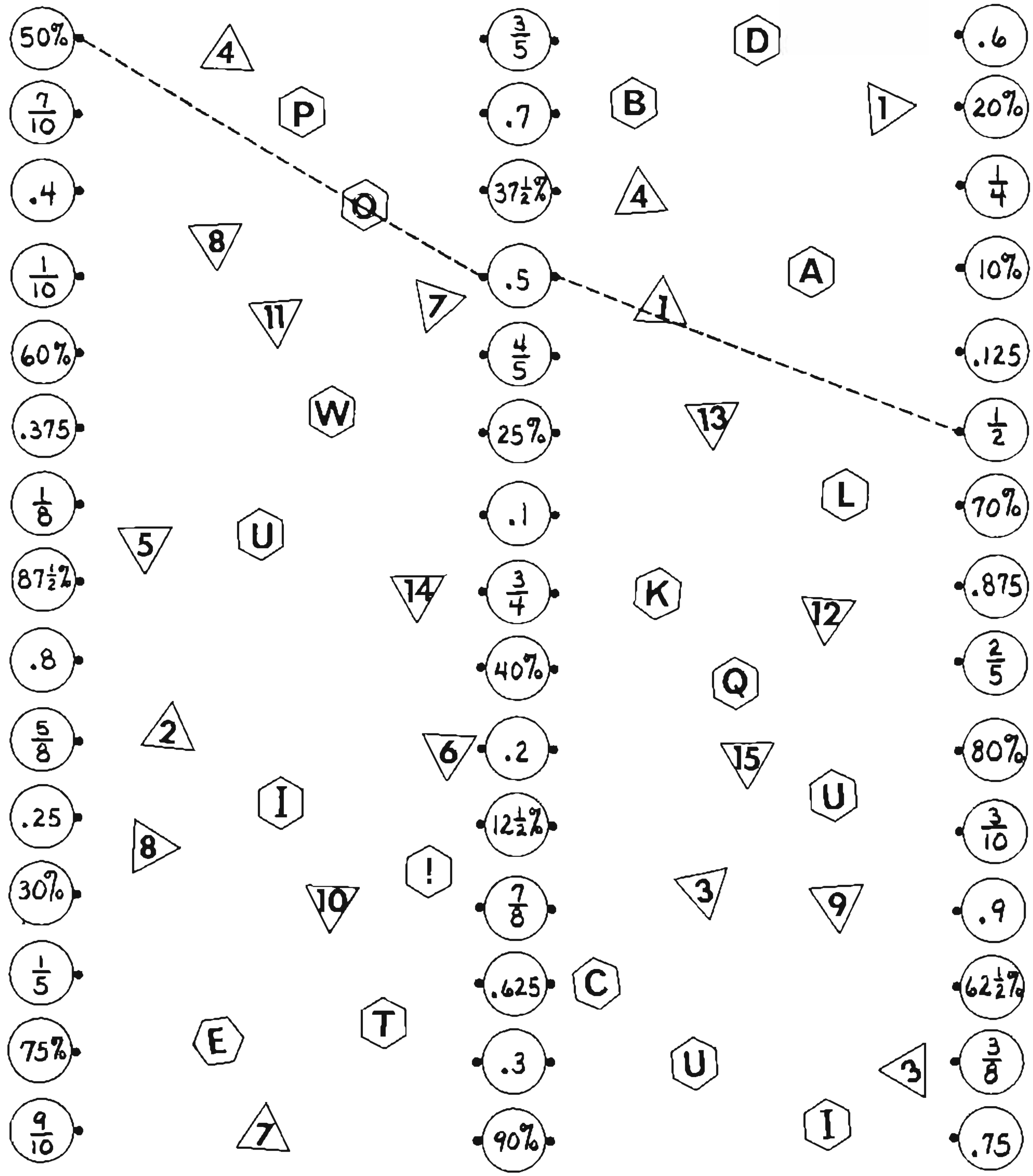
DUCK SOUP

Connect each pair of equivalents in the first and second columns and each pair of equivalents in the second and third columns. The connecting lines tell which letter should go above each number at the bottom of the page.



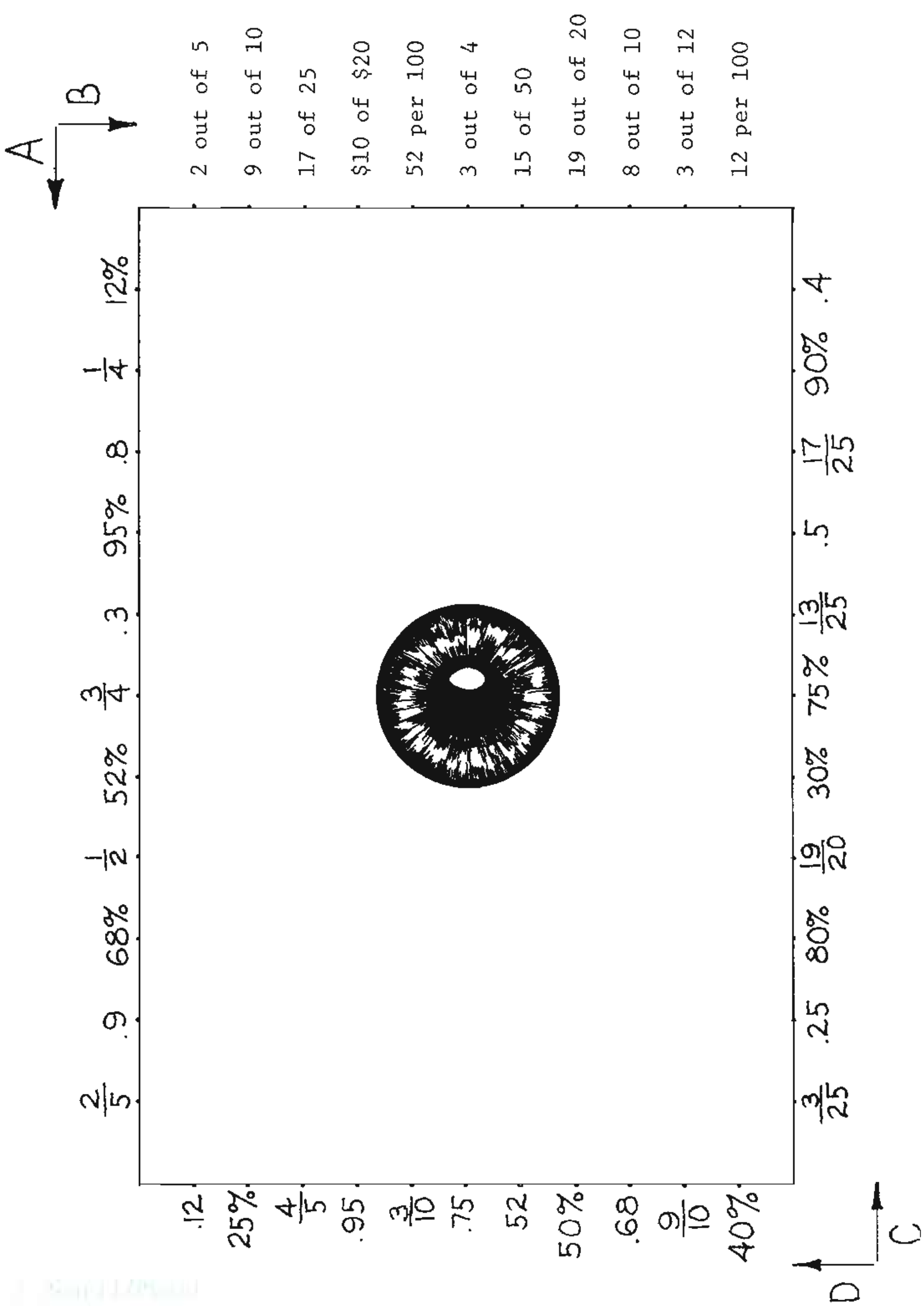
What would happen if a duck flew upside down?

___	___		___	O	___	___	___		___	___	___	___	___	___	___
3	9		13	1	11	5	8		14	7	2	10	6	12	4	15

I SEE IT

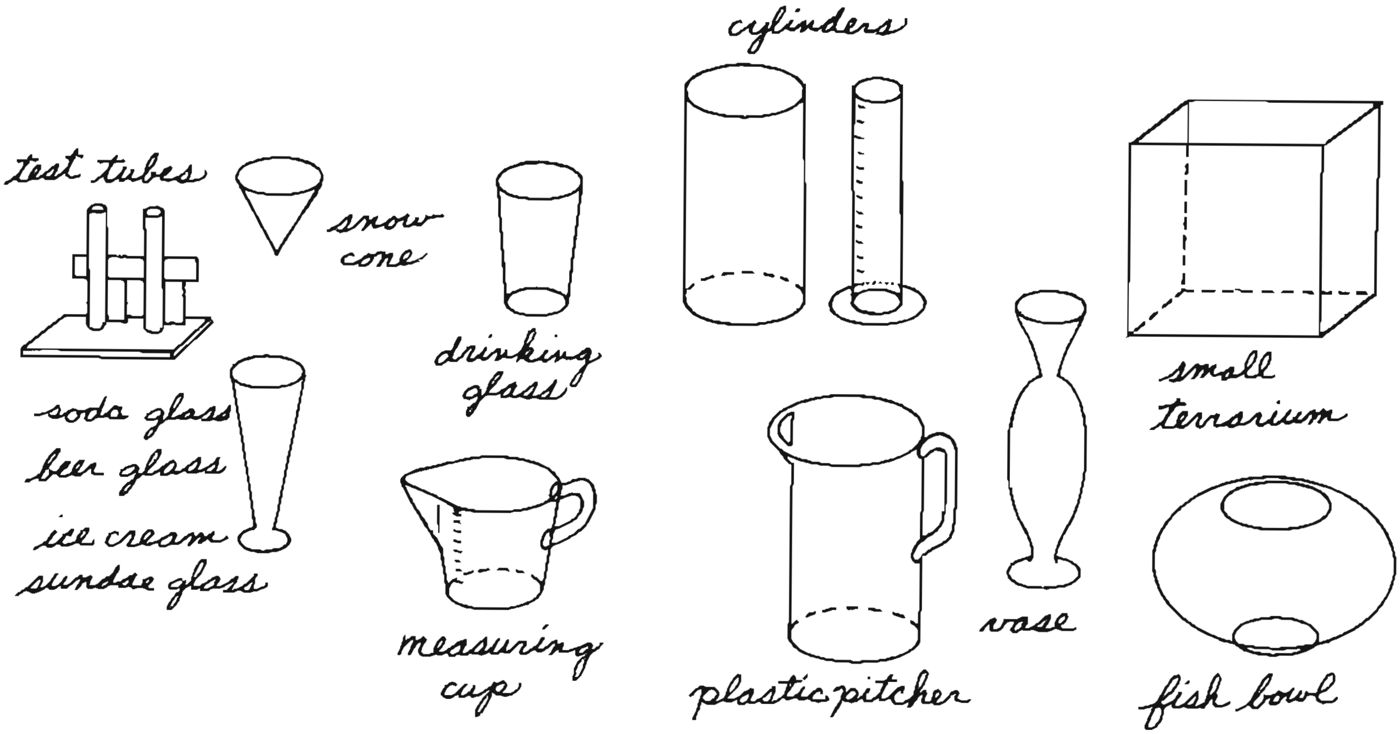
Connect equivalent statements.

Connect points on side A to points on side B, side B to side C, side C to side D, and side D to side A.



SEE-THROUGH DEMONSTRATION

Bring a number of see-through containers to class and display them on a table where all students can see them. (i.e., glass cylinders, test tubes, glass or plastic cubical containers, plastic pitchers (cylindrical), household measuring cups, drinking glasses, and some odd-shaped glass containers (i.e., vases, spherical glass bowls, cones, wine glasses).



A number of concepts can be taught using these containers as visual aids and motivation.

I. Using a large pitcher, pour colored water (or rice or sand) into each container on the table to different levels.

Ask the students to identify the amount of water in each container (as compared to the volume of the whole container). For example, how full is the glass? Possible responses: 1/2 full, 50% full, .5 full, 50% empty. The most common response would be 1/2 full. Encourage students to give equivalent answers in percent and decimal forms.

II. Let the students take an active part in this demonstration by pouring water into the containers. For example, select a student(s) to fill each (or one) container approximately 1/4 full (or 25% full or .25 full).

Why are some containers easier to fill to the approximate amount than others? (Disc visual illusions of odd-shaped containers.)

Continue to select students to take part in the demonstration,
i.e., fill the glass cylinder 50% full.
fill the test tube 1/3 full.
fill the plastic pitcher .75 full.
fill the glass cubical container 90% full.
fill the measuring cup 2/3 full.

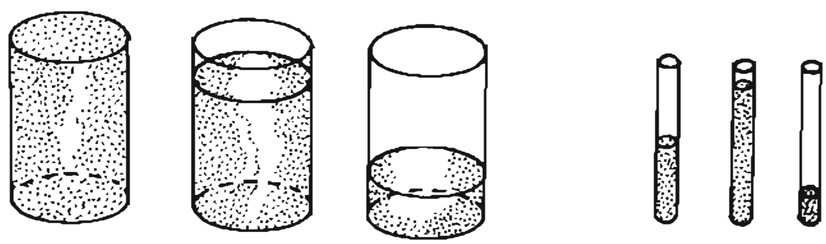
If a student fills a container approximately 75% full, and other students disagree with the approximation, it may be necessary for students to "check" the approximation by other means than "eye-balling" (guessing by looking). Students can check actual volume of odd-shaped containers (vases, cones, spherical bowls) by using standard containers that display volume measurements in cups, 1/4 cup, tablespoons, millilitres or litres. Other strategies for checking answers: 1. Put masking tape along side of container and mark intervals on the tape with a ruler. 2. Use the elastic percent approximation and stretch to find the percent of water in the container. 3. Make a "dipstick" to measure level of the water compared to the height of the container. See *Make a Dipstick* in Scaling. The demonstration could be reversed by having students pour from full containers to leave them x% empty.

ALTERNATIVES TO THIS TEACHER DEMONSTRATION:

The teacher can provide an <u>overhead transparency</u> with outlines of empty glasses (or test tubes or aquarium tanks or other see-through containers). The demonstration is in 3-dimensions; the overhead transparency would abstract the concept to 2-dimensions.

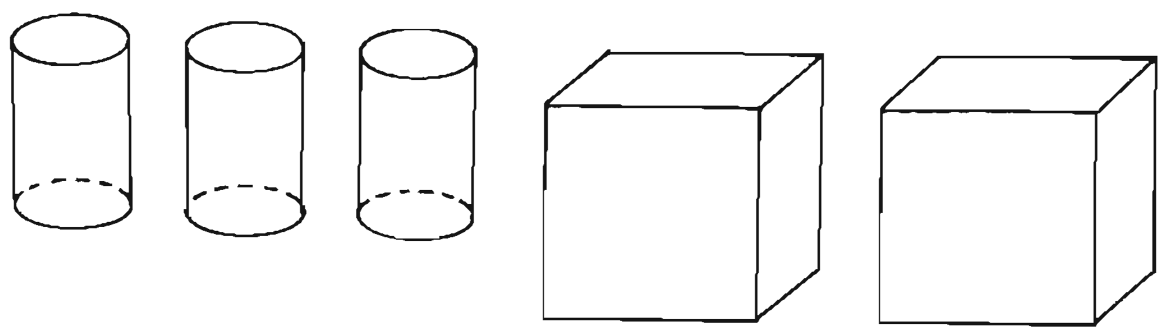
Using 2-D pictures of containers, students can approximate the fraction and corresponding percent of liquid in the container in two ways:

A.



Let students guess the fraction, percent, and decimal amount of liquid in each container pictured.

B.



$25\% = \frac{1}{4} = .25$ $\frac{9}{10} = .9 = 90\%$ $60\% = .6 = \frac{3}{5}$ $\frac{1}{2} =$ ____ $\frac{1}{5} =$ ____

Let students color in the amounts on an overhead transparency or shade drawings on the chalkboard. A worksheet or an activity card could provide a number of pictures of containers to be "filled" to the given amount. Ask students to identify the amount of water "filled in" with fraction, percent, and decimal equivalencies.

ONLY THE NAMES HAVE BEEN CHANGED

Each activity below was presented earlier to help students develop a "sense of percent." However, the activities can be adapted to develop informal equivalences between fractions, decimals and percents. Some suggestions for each page are provided.

1. *Fill it Up!*

Alternative dice may be substituted.

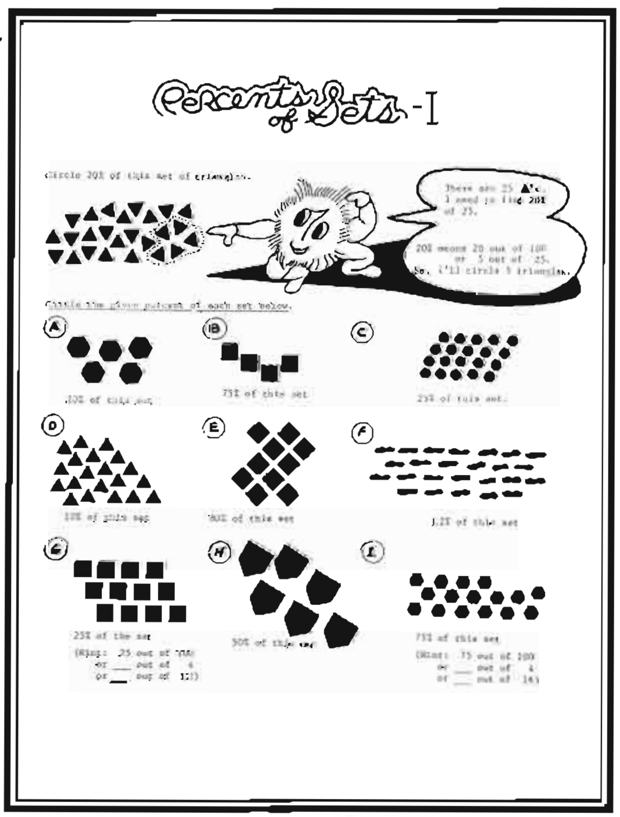
#4 marked $\frac{1}{100}$, $\frac{1}{5}$, $\frac{1}{4}$, $\frac{1}{25}$, $\frac{1}{20}$

#5 marked .01, .05, .13, .10, .03, .09

2. *Percents of Sets - I*

A similar page could be designed which asks students to circle the given part of each set. The part may be expressed as a decimal, fraction or percent.

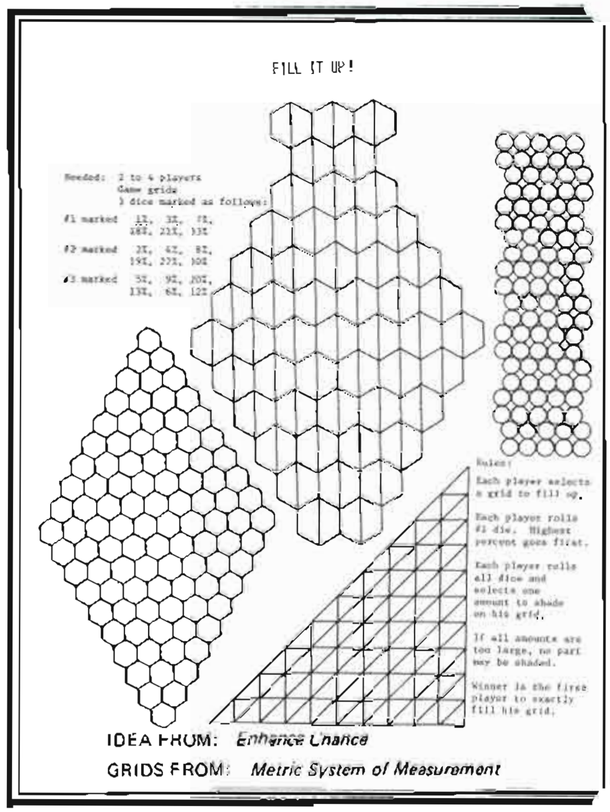
2.



3. *Changing Percent Shapes*

For each picture the students could be asked to express the number of shaded parts compared to the total number of parts as a fraction and as a decimal.

1.



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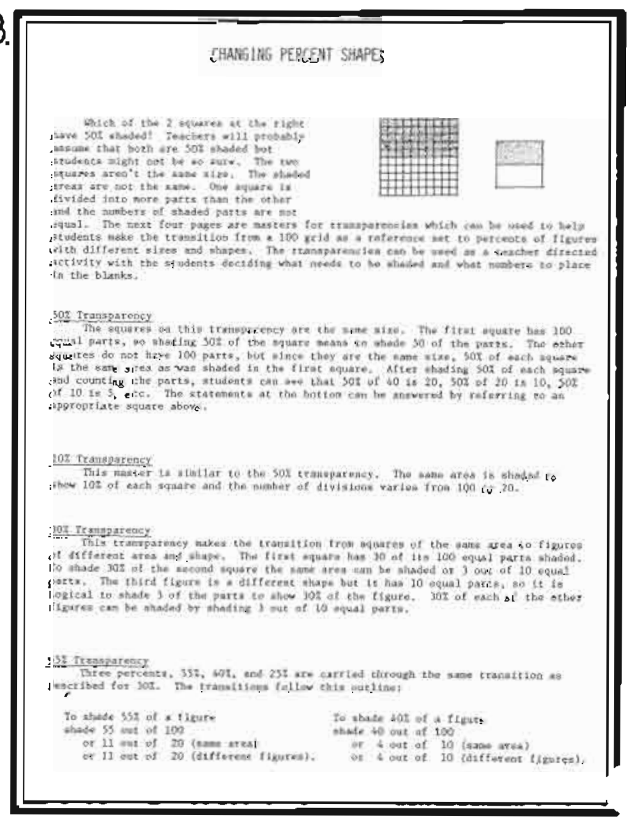
3.



CHANGING PERCENT SHAPES

Which of the 2 squares at the right have 50% shaded? Teachers will probably assume that both are 50% shaded but students might not be so sure. The two squares aren't the same size. The shaded areas are not the same. One square is divided into more parts than the other and the numbers of shaded parts are not equal. The next four pages are masters for transparencies which can be used to help students make the transition from a 100 grid as a reference set to percents of figures with different sizes and shapes. The transparencies can be used as a teacher directed activity with the students deciding what needs to be shaded and what numbers to place in the blanks.

50% Transparency

The squares on this transparency are the same size. The first square has 100 equal parts, so shading 50% of the square means to shade 50 of the parts. The other squares do not have 100 parts, but since they are the same size, 50% of each square is the same area as was shaded in the first square. After shading 50% of each square and counting the parts, students can see that 50% of 40 is 20, 50% of 20 is 10, 50% of 10 is 5, etc. The statements at the bottom can be answered by referring to an appropriate square above.

10% Transparency

This master is similar to the 50% transparency. The same area is shaded to show 10% of each square and the number of divisions varies from 100 to 20.

30% Transparency

This transparency makes the transition from squares of the same area to figures of different area and shape. The first square has 30 of its 100 equal parts shaded. To shade 30% of the second square the same area can be shaded or 3 out of 10 equal parts. The third figure is a different shape but it has 10 equal parts, so it is logical to shade 3 of the parts to show 30% of the figure. 30% of each of the other figures can be shaded by shading 3 out of 10 equal parts.

55% Transparency

Three percents, 55%, 40%, and 25% are carried through the same transition as described for 30%. The transitions follow this outline:

To shade 55% of a figure	To shade 40% of a figure
shade 55 out of 100	shade 40 out of 100
or 11 out of 20 (same area)	or 4 out of 10 (same area)
or 11 out of 20 (different figures).	or 4 out of 10 (different figures).

ONLY THE NAMES HAVE BEEN CHANGED (PAGE 2)

4. *Guess & Check*

Decimals as well as percent could be used to estimate the shaded portion of R. The transparent 100 grid could be used to find the exact decimal

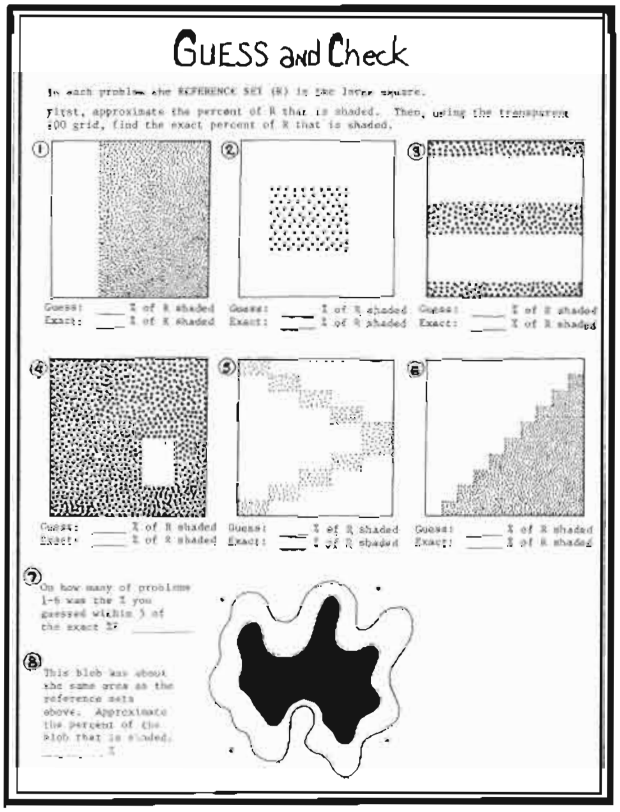
5. *The Transparent Hundred*

The teacher may wish to create new shapes for the student to measure.

6. *Dollar$ & Per¢ents 1*

Decimals could be used by adding a column labeled "Value of coins in ¢" or fractions could be used by adding a column that asks for the value of the coins as a fractional part of a dollar.

4.



GUESS and Check

In each problem the REFERENCE SET (R) is the large square.

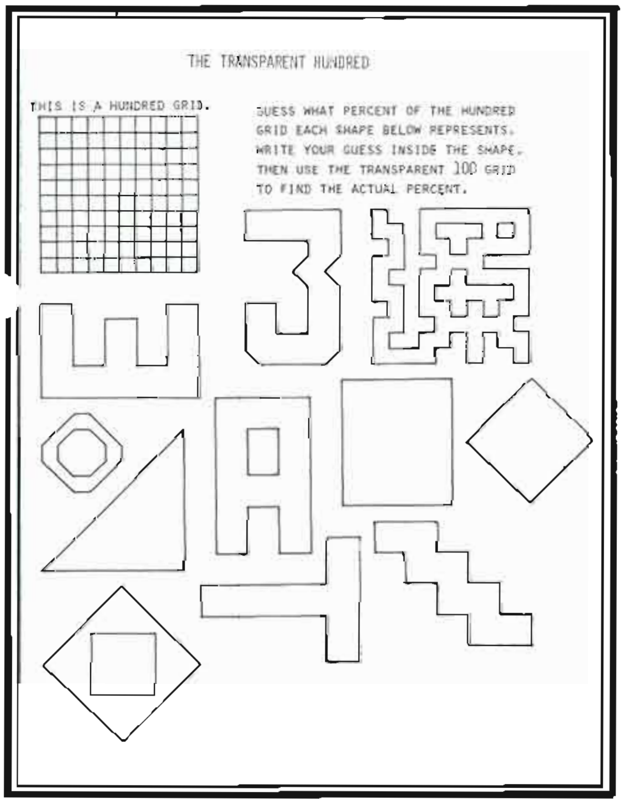
First, approximate the percent of R that is shaded. Then, using the transparent 100 grid, find the exact percent of R that is shaded.

① Guess: ____ % of R shaded Exact: ____ % of R shaded
② Guess: ____ % of R shaded Exact: ____ % of R shaded
③ Guess: ____ % of R shaded Exact: ____ % of R shaded
④ Guess: ____ % of R shaded Exact: ____ % of R shaded
⑤ Guess: ____ % of R shaded Exact: ____ % of R shaded
⑥ Guess: ____ % of R shaded Exact: ____ % of R shaded

⑦ On how many of problems 1-6 was the % you guessed within 5 of the exact %? ____

⑧ This blob has about the same area as the reference sets above. Approximate the percent of the blob that is shaded. ____ %

5.



THE TRANSPARENT HUNDRED

THIS IS A HUNDRED GRID.

GUESS WHAT PERCENT OF THE HUNDRED GRID EACH SHAPE BELOW REPRESENTS. WRITE YOUR GUESS INSIDE THE SHAPE. THEN USE THE TRANSPARENT 100 GRID TO FIND THE ACTUAL PERCENT.

6.

DOLLAR$ AND PER¢ENTS 1

COINS	PERCENT OF 1 DOLLAR	COINS	PERCENT OF 1 DOLLAR
1 dime		3 half-dollars	
1 quarter		5 quarters	
1 penny		13 dimes	
1 half-dollar		29 nickels	
1 nickel		214 pennies	
3 nickels		3 quarters, 6 dimes, and 4 pennies	
4 dimes		6 dimes, 4 pennies, and 3 quarters	
22 pennies		4 pennies, 3 quarters, & 6 dimes	
3 quarters		2 of each coin shown	
2 half-dollars		3 of each coin shown	
2 dimes & 3 nickels		5 of each coin shown	
3 quarters and 2 pennies			
6 dimes and 1 quarter			
1 half-dollar, 2 dimes, & 1 nickel			
1 of each coin shown			

IDEA FROM: *C.O.L.A.M.D.A.*

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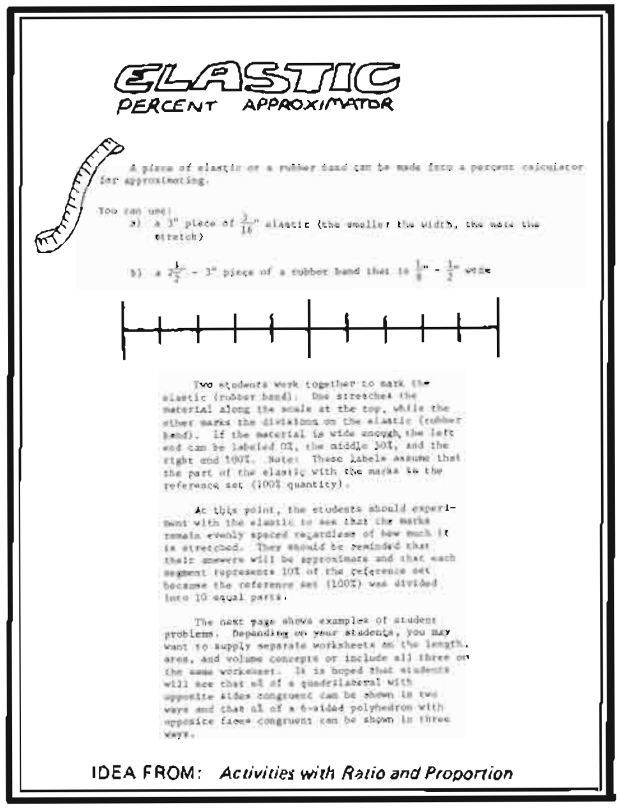
ONLY THE NAMES HAVE BEEN CHANGED (PAGE 3)

7. *Elastic Percent Approximator*

Mark a second piece of elastic with the fractions $\frac{0}{1}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, $\frac{4}{10}$, $\frac{6}{10}$, $\frac{7}{10}$, $\frac{8}{10}$, $\frac{9}{10}$, $\frac{1}{1}$ so that $\frac{0}{1}$ corresponds with 0% and $\frac{1}{1}$ corresponds with 100%. Use both pieces of elastic to measure the examples.

7.



ELASTIC PERCENT APPROXIMATOR

A piece of elastic or a rubber band can be made into a percent calculator for approximating.

You can use:

a) a 3" piece of $\frac{3}{16}$" elastic (the smaller the width, the more the stretch)

b) a $2\frac{1}{2}$" - 3" piece of a rubber band that is $\frac{1}{8}$" - $\frac{1}{2}$" wide

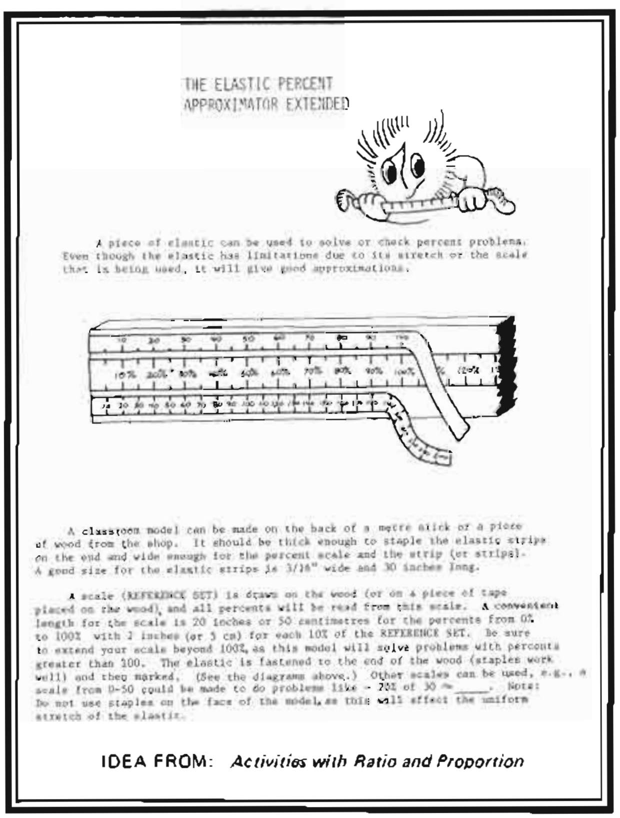
Two students work together to mark the elastic (rubber band). One stretches the material along the scale at the top, while the other marks the divisions on the elastic (rubber band). If the material is wide enough, the left end can be labeled 0%, the middle 50%, and the right end 100%. Note: These labels assume that the part of the elastic with the marks is the reference set (100% quantity).

At this point, the students should experiment with the elastic to see that the marks remain evenly spaced regardless of how much it is stretched. They should be reminded that their answers will be approximate and that each segment represents 10% of the reference set because the reference set (100%) was divided into 10 equal parts.

The next page shows examples of student problems. Depending on your students, you may want to supply separate worksheets on the length, area, and volume concepts or include all three on the same worksheet. It is hoped that students will see that a1 of a quadrilateral with opposite sides congruent can be shown in two ways and that a1 of a 6-sided polyhedron with opposite faces congruent can be shown in three ways.

IDEA FROM: *Activities with Ratio and Proportion*

8.



THE ELASTIC PERCENT APPROXIMATOR EXTENDED

A piece of elastic can be used to solve or check percent problems. Even though the elastic has limitations due to its stretch or the scale that is being used, it will give good approximations.

A classroom model can be made on the back of a metre stick or a piece of wood from the shop. It should be thick enough to staple the elastic strips on the end and wide enough for the percent scale and the strip (or strips). A good size for the elastic strips is 3/16" wide and 30 inches long.

A scale (REFERENCE SET) is drawn on the wood (or on a piece of tape placed on the wood), and all percents will be read from this scale. A convenient length for the scale is 20 inches or 50 centimetres for the percents from 0% to 100% with 2 inches (or 5 cm) for each 10% of the REFERENCE SET. Be sure to extend your scale beyond 100%, as this model will solve problems with percents greater than 100. The elastic is fastened to the end of the wood (staples work well) and then marked. (See the diagrams above.) Other scales can be used, e.g., a scale from 0-50 could be made to do problems like - 70% of 30 = ____. Note: Do not use staples on the face of the model, as this will effect the uniform stretch of the elastic.

IDEA FROM: *Activities with Ratio and Proportion*

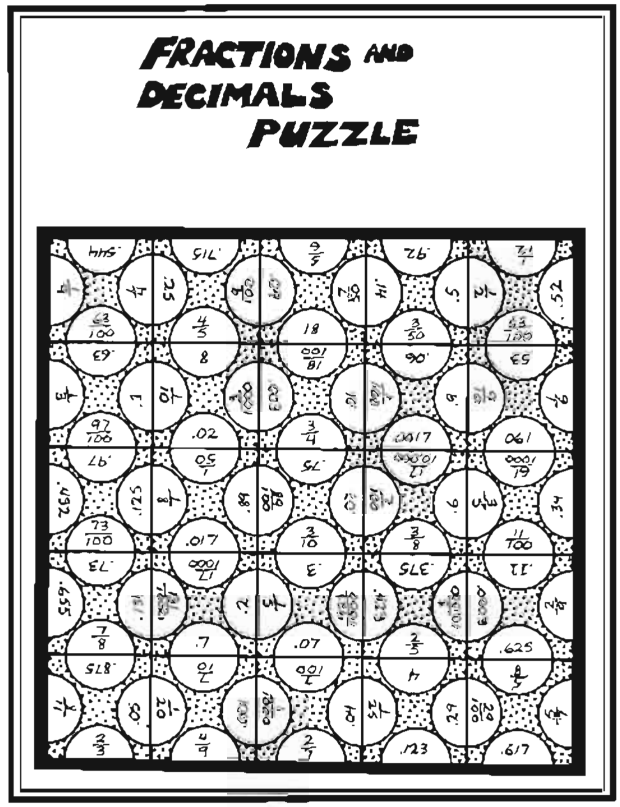
8. *The Elastic Percent Approximator Extended*

This manipulative can be used to approximate answers to problems like $\frac{5}{8}$ of 140. Using the *Percent Bar Sheet*, $\frac{5}{8}$ can be converted to about 62%. Then 62% of 140 can be found on the percent approximator. In a similar way, .34 x 98 could be approximated.

GAMES STUDENTS PLAY

The following are examples of games found in the resource, Number Sense and Arithmetic Skills, that can be adapted for use in this section of the Ratio, Proportion and Scaling resource. These games will provide drill on fraction, decimal, and percent equivalences.

1.

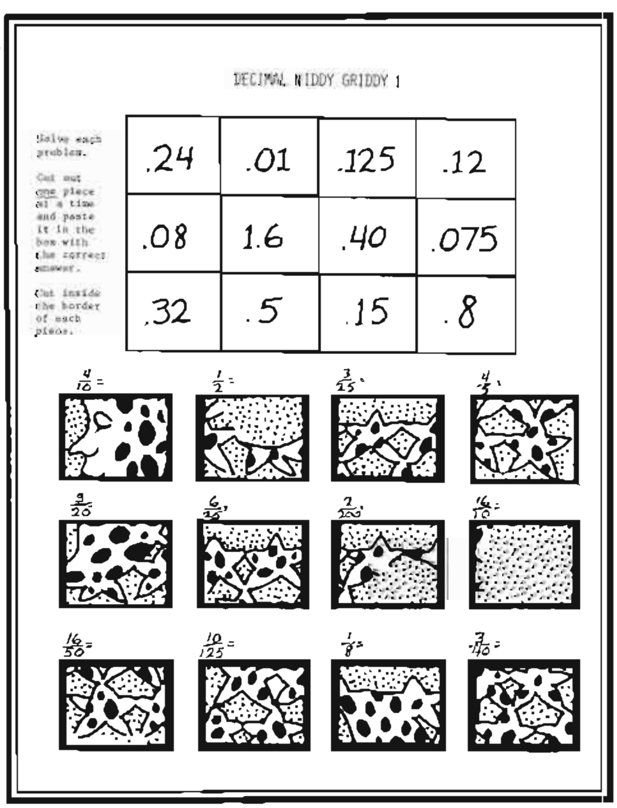


IDEA FROM: *Math Amusement in Developing Skills*, Volume I
Permission to use granted by Midwest Publications Co., Inc.

1. *Fractions and Decimals Puzzle*

   Three games, one with fraction-percent equivalences, one with decimal-percent equivalences and one with all three equivalences, could be constructed using this idea. In order to save the game for future use, the page can be dry mounted and laminated before the page is cut into sections.

2.



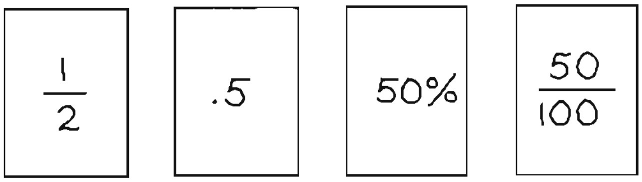
2. *Decimal Niddy Griddy I*

   This self-checking game could be adapted by using either percent-fraction equivalences or percent-decimal equivalences. Again, dry mounting and laminating the pieces before cutting them out will preserve the game for future use.

GAMES STUDENTS PLAY (CONTINUED)

3. *Fraction and Decimal Concentration*

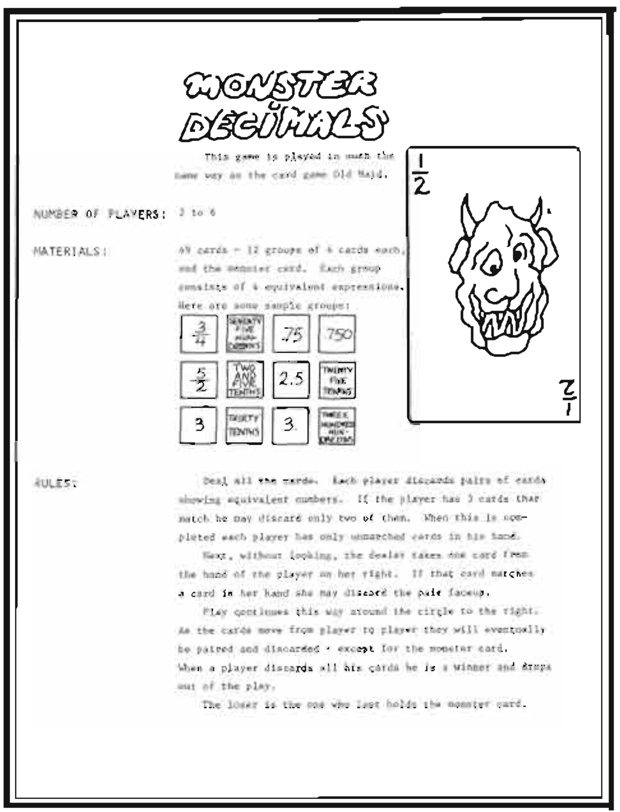
This game can be played with 5 sets of cards, each set showing 4 equivalent expressions, e.g.,



4. *Monster Decimals*

The game deck includes 12 sets of 4 cards. One of the 4 cards can be written as a percent expression. The other rules will remain the same.

4.



MONSTER DECIMALS

This game is played in much the same way as the card game Old Maid.

NUMBER OF PLAYERS: 2 to 6

MATERIALS: 49 cards – 12 groups of 4 cards each, and the monster card. Each group consists of 4 equivalent expressions. Here are some sample groups:

3/4	SEVENTY FIVE HUNDREDTHS	.75	.750
5/2	TWO AND FIVE TENTHS	2.5	TWENTY FIVE TENTHS
3	THIRTY TENTHS	3.	THREE HUNDRED HUNDREDTHS

RULES: Deal all the cards. Each player discards pairs of cards showing equivalent numbers. If the player has 3 cards that match he may discard only two of them. When this is completed each player has only unmatched cards in his hand.

Next, without looking, the dealer takes one card from the hand of the player on her right. If that card matches a card in her hand she may discard the pair faceup.

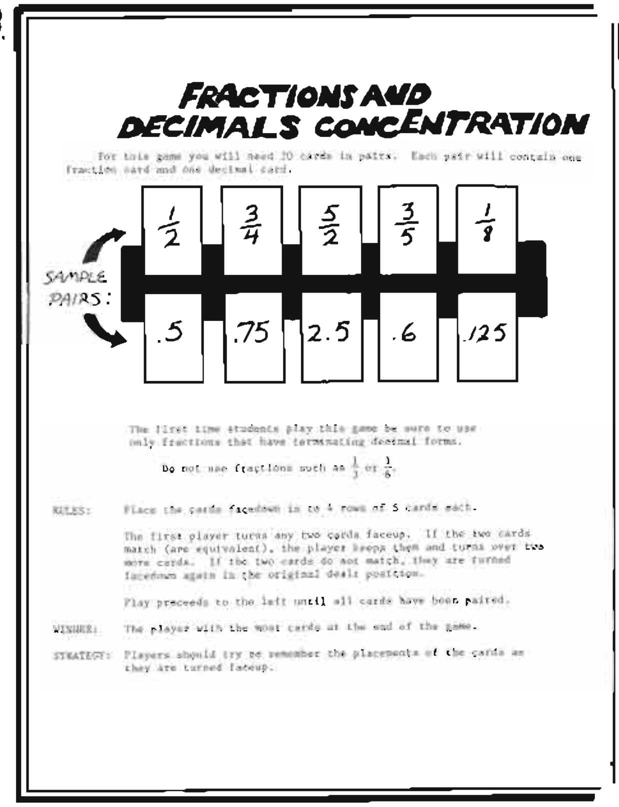
Play continues this way around the circle to the right. As the cards move from player to player they will eventually be paired and discarded – except for the monster card. When a player discards all his cards he is a winner and drops out of the play.

The loser is the one who last holds the monster card.

5. *More Dominoes*

Percent equivalences can be substituted on one end of several dominoes.

3.



FRACTIONS AND DECIMALS CONCENTRATION

For this game you will need 20 cards in pairs. Each pair will contain one fraction card and one decimal card.

The first time students play this game be sure to use only fractions that have terminating decimal forms.

Do not use fractions such as $\frac{1}{3}$ or $\frac{1}{6}$.

RULES: Place the cards facedown in to 4 rows of 5 cards each.

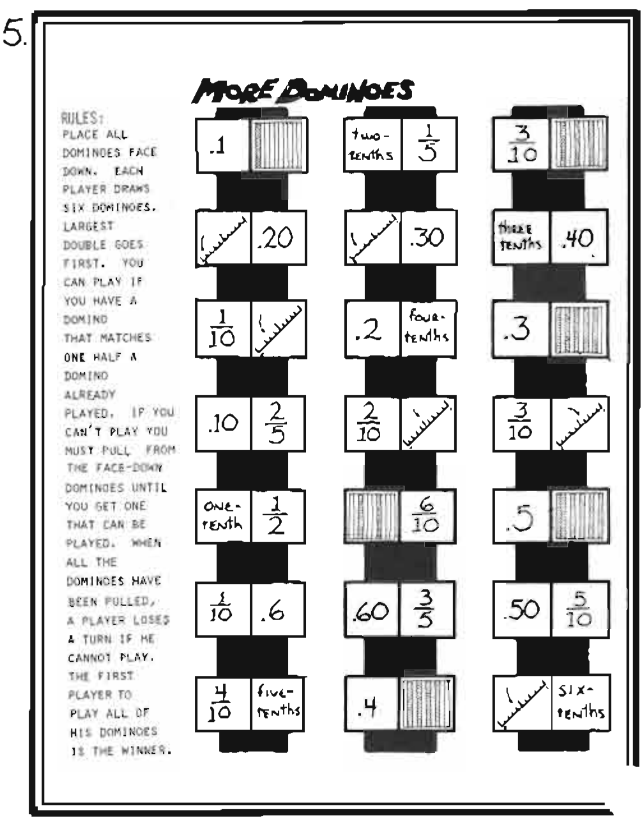
The first player turns any two cards faceup. If the two cards match (are equivalent), the player keeps them and turns over two more cards. If the two cards do not match, they are turned facedown again in the original deal position.

Play preceeds to the left until all cards have been paired.

WINNER: The player with the most cards at the end of the game.

STRATEGY: Players should try to remember the placements of the cards as they are turned faceup.

5.

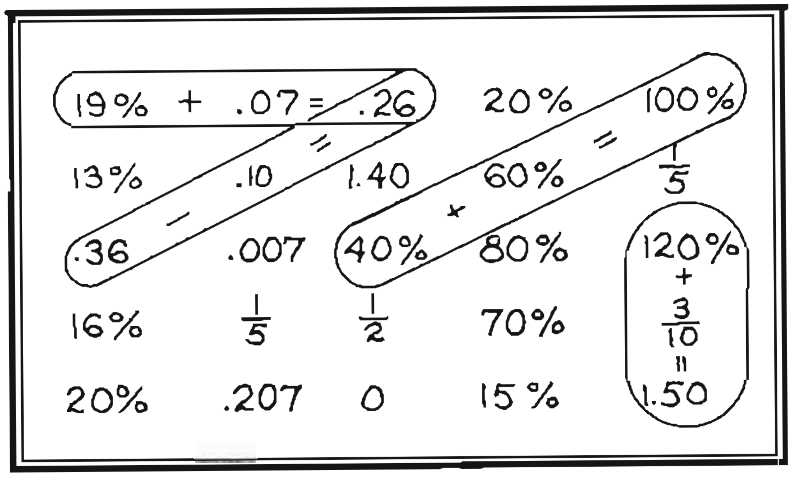


MORE DOMINOES

RULES: PLACE ALL DOMINOES FACE DOWN. EACH PLAYER DRAWS SIX DOMINOES. LARGEST DOUBLE GOES FIRST. YOU CAN PLAY IF YOU HAVE A DOMINO THAT MATCHES ONE HALF A DOMINO ALREADY PLAYED. IF YOU CAN'T PLAY YOU MUST PULL FROM THE FACE-DOWN DOMINOES UNTIL YOU GET ONE THAT CAN BE PLAYED. WHEN ALL THE DOMINOES HAVE BEEN PULLED, A PLAYER LOSES A TURN IF HE CANNOT PLAY. THE FIRST PLAYER TO PLAY ALL OF HIS DOMINOES IS THE WINNER.

SEARCH & CIRCLE

For each grid the reference set is the same.



The student is to construct any true mathematical sentence within the grid by putting in an +, −, x, or ÷ sign and an equal sign, and by enclosing the sentence in a bubble. Statements may be made vertically, horizontally, or diagonally.

Construction of the grid: (see Grids #1, #2 below).

a) Identify the concept(s) you wish to review:

PERCENT, FRACTION, DECIMAL EQUIVALENCES

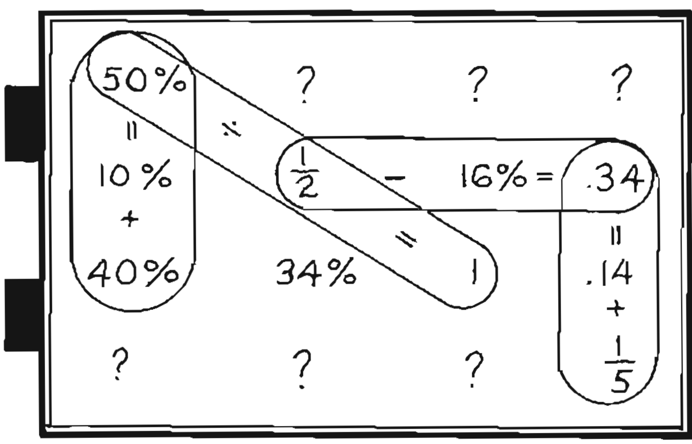
b) Select specific sentences to be used:

$10\% + 40\% = 50\%$ $50\% \div \frac{1}{2} = 1$

$\frac{1}{5} + .14 = 34\%$ $\frac{1}{2} - 16\% = .34$

c) Construct a grid using each sentence (see Grid #1).

d) Fill in the remaining cells with appropriate quantities (see Grid #2).



#1

50%	70%	$\frac{3}{4}$	.19
10%	$\frac{1}{2}$	16%	.34
40%	34%	1	.14
1	.18	$\frac{2}{3}$	$\frac{1}{5}$

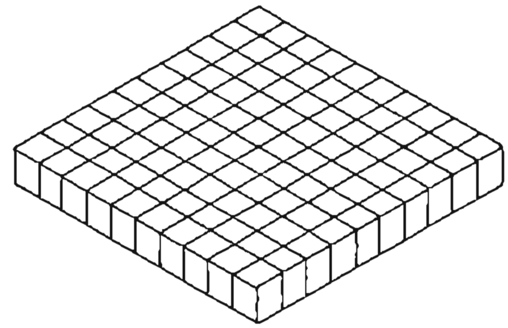
#2

Purpose: Number grids can be used to stimulate thought and enjoyment on the part of the student and also provide drill. Students who are reluctant to do homework could be challenged by an assignment. "Find as many true sentences as you can using the grid," or "Construct a grid of your own and exchange with a friend."

TYPE: Paper & Pencil

IDEA FROM: "If We Just Turn the Crank," *Mathematics Teacher,* Jan., 1975

Materials: 100 cubes and a calculator



Activity:

1) Build a 10 x 10 model with the cubes.
   If the entire model is painted
   a) What percent of the cubes will have
      4 faces painted? _____
      3 faces painted? _____
      2 faces painted? _____

2) Build a 9 x 9 model.

	number	%
4 faces painted	4	5
3 faces painted	28	
2 faces painted		
Total		

Write the answer to the nearest percent.

3) Build an 8 x 8 model.

	number	%
4 faces	4	
3 faces		
2 faces		
Total		

4) Build these models.

7 x 7

	number	%
4		
3		
2		
Total		

6 x 6

	number	%
4		
3		
2		
Total		

5 x 5

	number	%
4		
3		
2		
Total		

4 x 4

	number	%
4		
3		
2		
Total		

3 x 3

	number	%
4		
3		
2		

2 x 2

	number	%
4		
3		
2		
Total		

5) Predict the numbers and find the percents on your calculator for each of these models.
   a) 12 x 12
   b) 15 x 15
   c) 20 x 20
   d) 1 x 1
   e) n x n

RATIO, PROPORTION, AND SCALING

Placement Guide for Tabbed Divisors

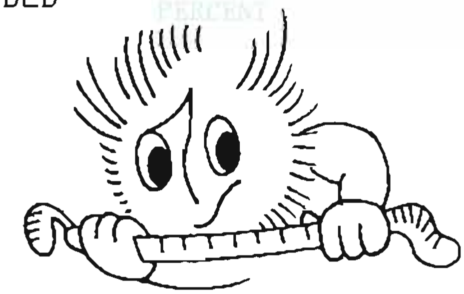
CONTENTS

PERCENT: SOLVING PERCENT PROBLEMS

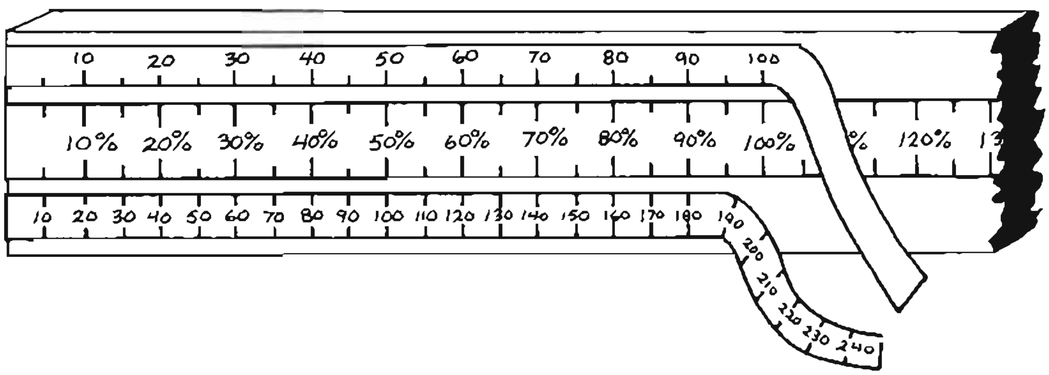
*Indicates percents greater than 100% are used on the page.

TITLE	PAGE	TOPIC	TYPE
At That Price I'll Buy It	495	Finding amount of interest	Paper and pencil
Percent Problems 1	496	Word problems	Paper and pencil
Percent Problems 2	497	Word problems	Paper and pencil
Pelargonium	498	Finding percent of increase	Paper and pencil
State the Rate	499	Finding amount of sales tax	Paper and pencil
Counting Every Body	500	Finding percent of increase	Paper and pencil
Certain Growths Are Beneficial	501	Finding amount of interest	Paper and pencil
Hidden Costs in a Home	502	Finding amount of interest	Paper and pencil

THE ELASTIC PERCENT APPROXIMATOR EXTENDED



A piece of elastic can be used to solve or check percent problems. Even though the elastic has limitations due to its stretch or the scale that is being used, it will give good approximations.



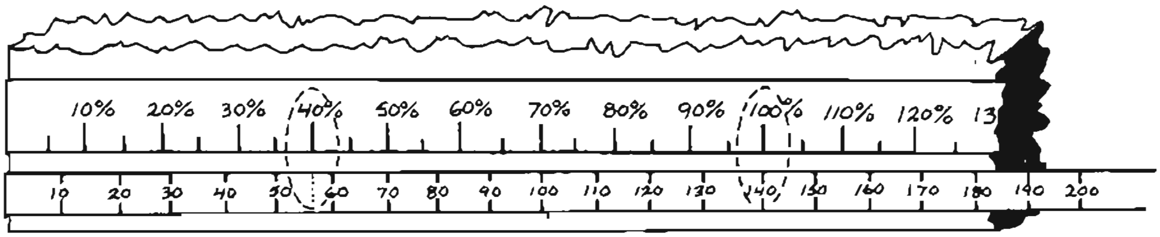
A classroom model can be made on the back of a metre stick or a piece of wood from the shop. It should be thick enough to staple the elastic strips on the end and wide enough for the percent scale and the strip (or strips). A good size for the elastic strips is 3/16" wide and 30 inches long.

A scale (REFERENCE SET) is drawn on the wood (or on a piece of tape placed on the wood), and all percents will be read from this scale. A convenient length for the scale is 20 inches or 50 centimetres for the percents from 0% to 100% with 2 inches (or 5 cm) for each 10% of the REFERENCE SET. Be sure to extend your scale beyond 100%, as this model will solve problems with percents greater than 100. The elastic is fastened to the end of the wood (staples work well) and then marked. (See the diagrams above.) Other scales can be used, e.g., a scale from 0-50 could be made to do problems like - 20% of 30 $\approx$ _____. Note: Do not use staples on the face of the model, as this will affect the uniform stretch of the elastic.

THE ELASTIC PERCENT APPROXIMATOR
EXTENDED (CONTINUED)
SAMPLE PROBLEMS

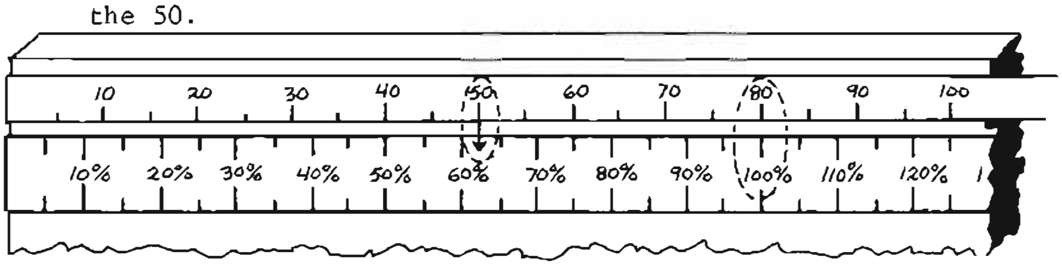
A) 40% of 140 ≈ ___

1) Use the bottom scale when the number > 100.
2) Stretch the elastic until 140 is located opposite 100.
3) Find 40% and read the answer 56 opposite the 40%.



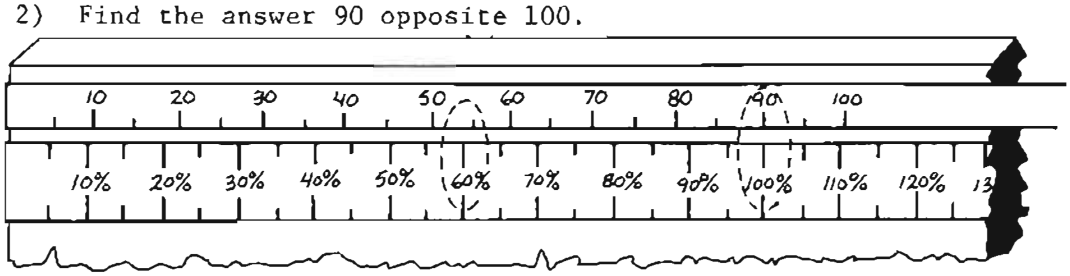
B) ____% of 80 ≈ 50

1) Use the bottom scale when the number ≤ 100.
2) Stretch the elastic until 80 is located opposite 100.
3) Find 50 on the elastic and read the answer 62% opposite the 50.



C) 60% of ____ ≈ 54

1) Stretch the elastic until 54 is located opposite 60%.
2) Find the answer 90 opposite 100.



Special Notes:

1) Emphasis to the students on why this model works is important. It should be stressed that when a number is placed opposite 100, the distance from 0 to the number has, in effect, been divided into 100 equal parts.
2) By setting up one problem, many others are also set up, e.g., 50% of 140 ≈ 70 also sets up 60% of 140 ≈ 85, 120% of 140 ≈ 168, etc.
3) A problem that can't be solved because the elastic will not stretch might be solved by using patterns. 25% of 20 can't be done but 25% of 100 ≈ 25, 25% of 80 ≈ 20, 25% of 60 ≈ 15, so 25% of 40 ≈ ___ and 25% of 20 ≈ ___.

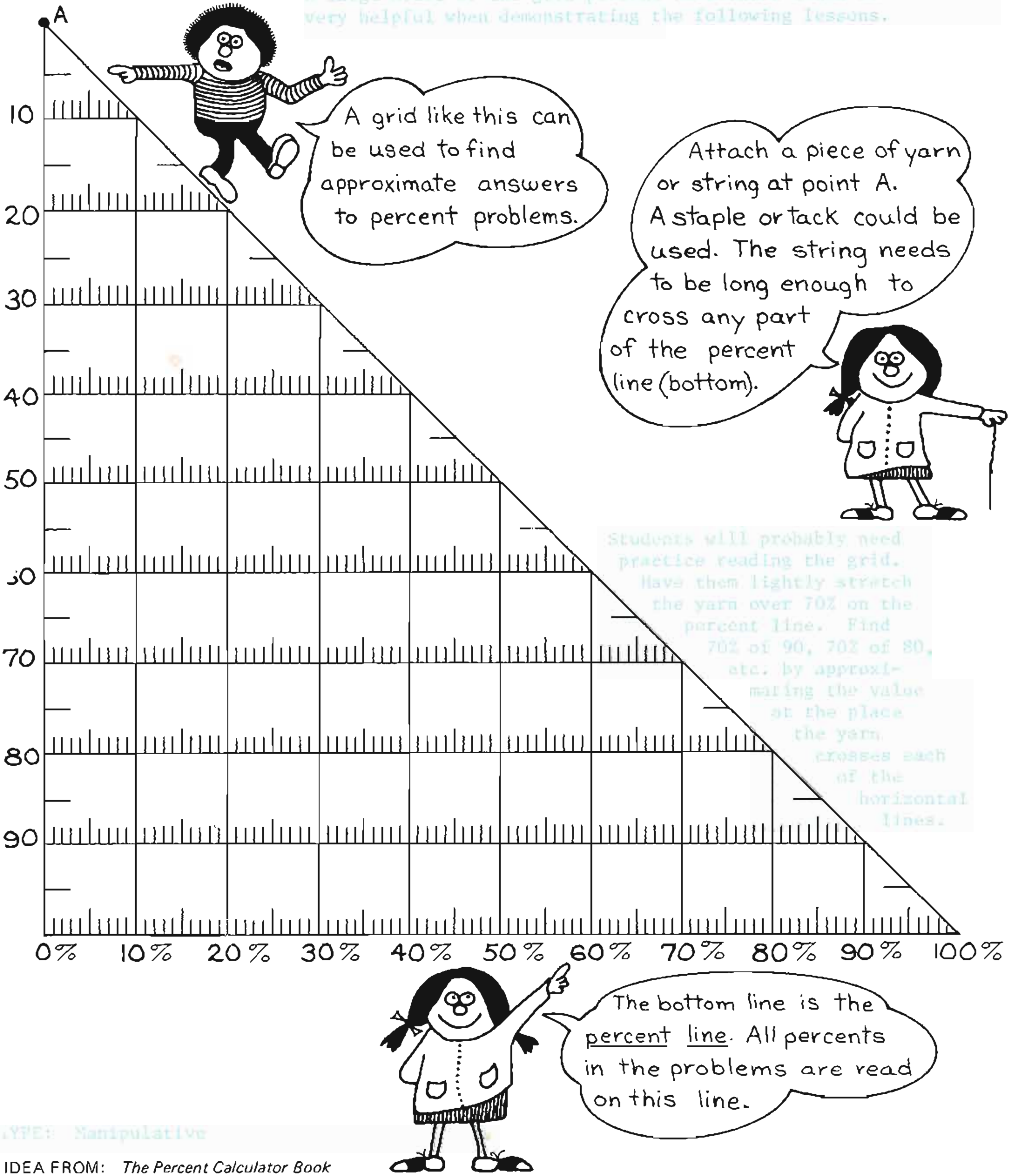
IDEA FROM: *Activities with Ratio and Proportion*

GRID PERCENT CALCULATOR I

Using a Percent Calculator
Solving Percent Problems
PERCENT

E S I

A large model of the grid percent calculator could be very helpful when demonstrating the following lessons.

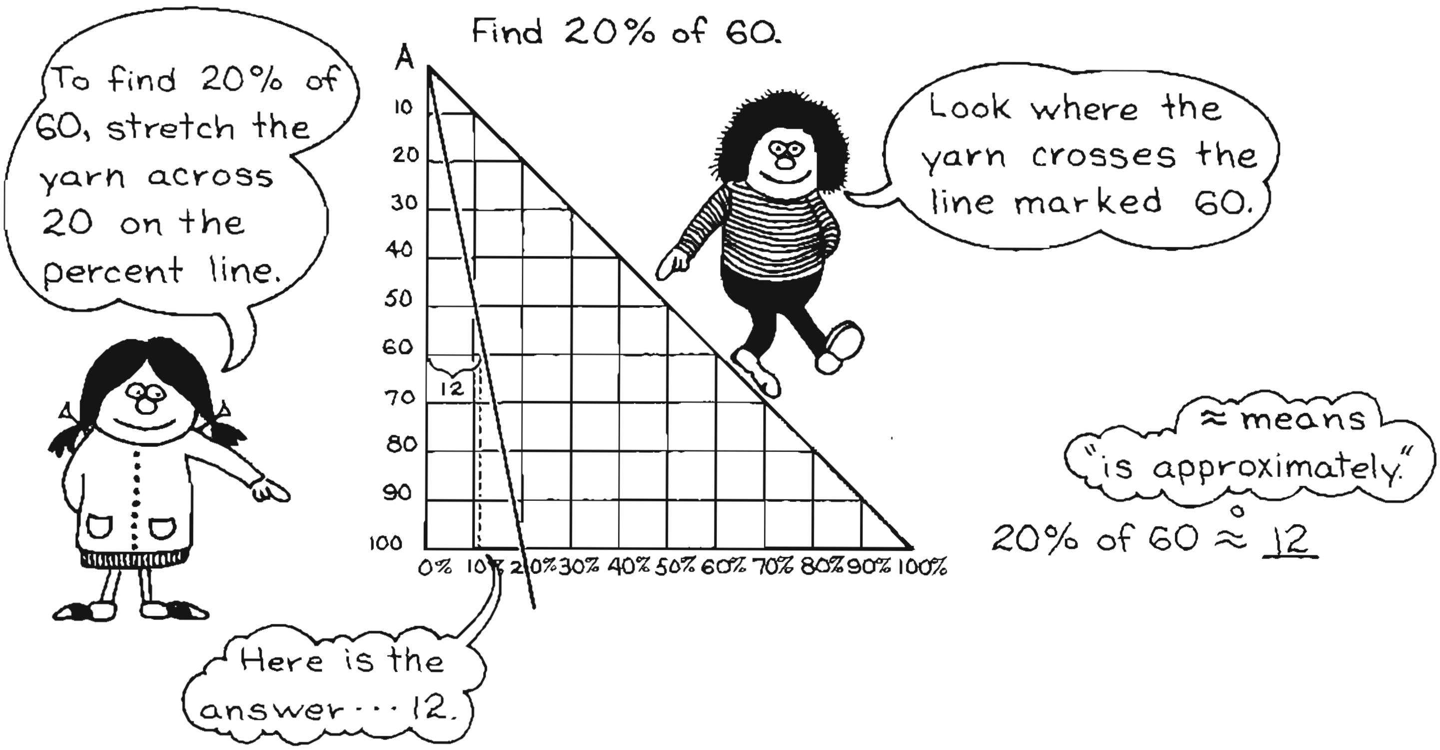


Students will probably need practice reading the grid. Have them lightly stretch the yarn over 70% on the percent line. Find 70% of 90, 70% of 80, etc. by approximating the value at the place the yarn crosses each of the horizontal lines.

TYPE: Manipulative

IDEA FROM: *The Percent Calculator Book*

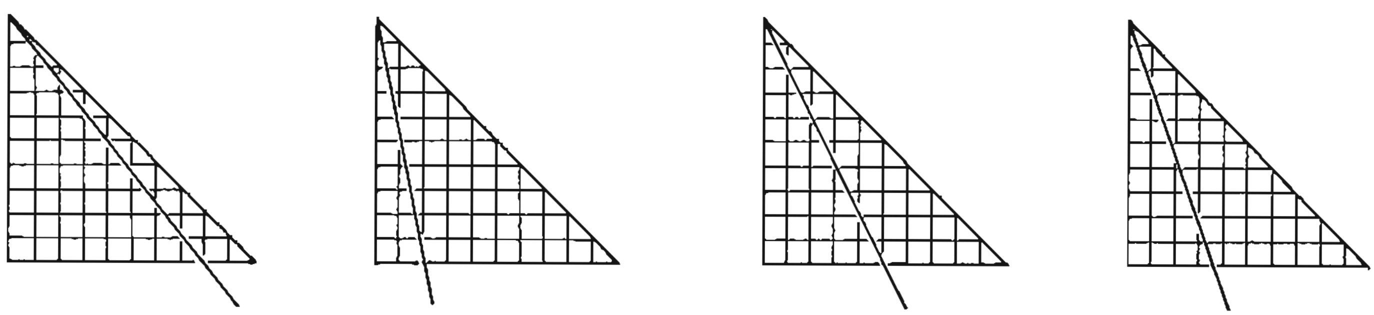
GRID PERCENT CALCULATOR IX



From the example above you can also find
a) 20% of 50 ≈ ___ b) 20% of 20 ≈ ___ c) 20% of 90 ≈ ___

Are each of the problems below set up correctly to find the answer?

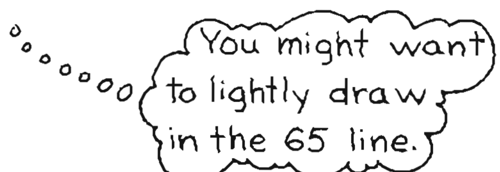
80% of 40 _____ 20% of 40 _____ 70% of 20 _____ 35 % of 50 _____



Use your grid percent calculator to approximate

a) 30% of 60 ≈ ___ c) 40% of 50 ≈ ___ e) 75% of 50 ≈ ___

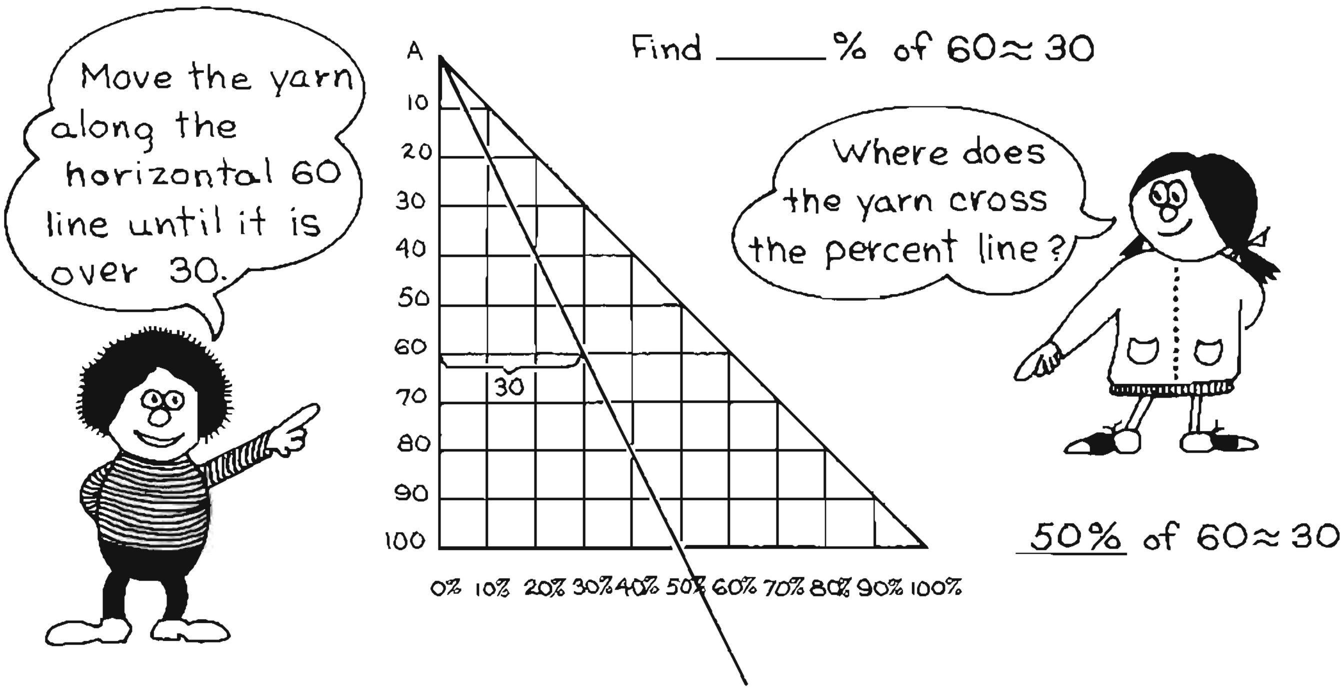
b) 50% of 70 ≈ ___ d) 80% of 65 ≈ ___ f) 62% of 85 ≈ ___



IDEA FROM: *The Percent Calculator Book*

GRID PERCENT CALCULATOR III

Using a Percent Calculator
Solving Percent Problems
PERCENT

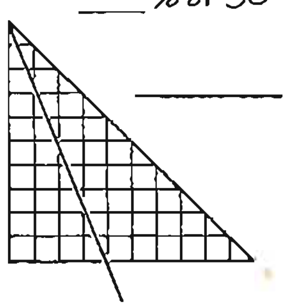


From the example above you can also find

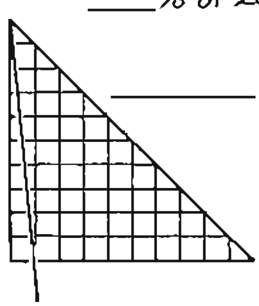
a) ___% of 40 ≈ 20   b) ___% of 90 ≈ 45   c) ___% of 65 ≈ 32

Are each of the problems below set up correctly to find the answer?

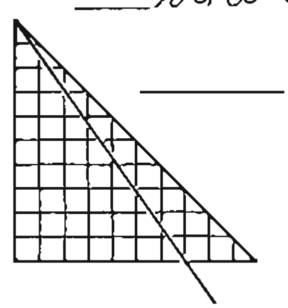
____% of 50 ≈ 20



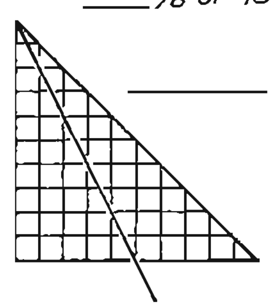
____% of 20 ≈ 10



____% of 80 ≈ 60



____% of 90 ≈ 45



Use your grid percent calculator to approximate

a) ___% of 80 ≈ 20   c) ___% of 50 ≈ 15   e) ___% of 75 ≈ 25

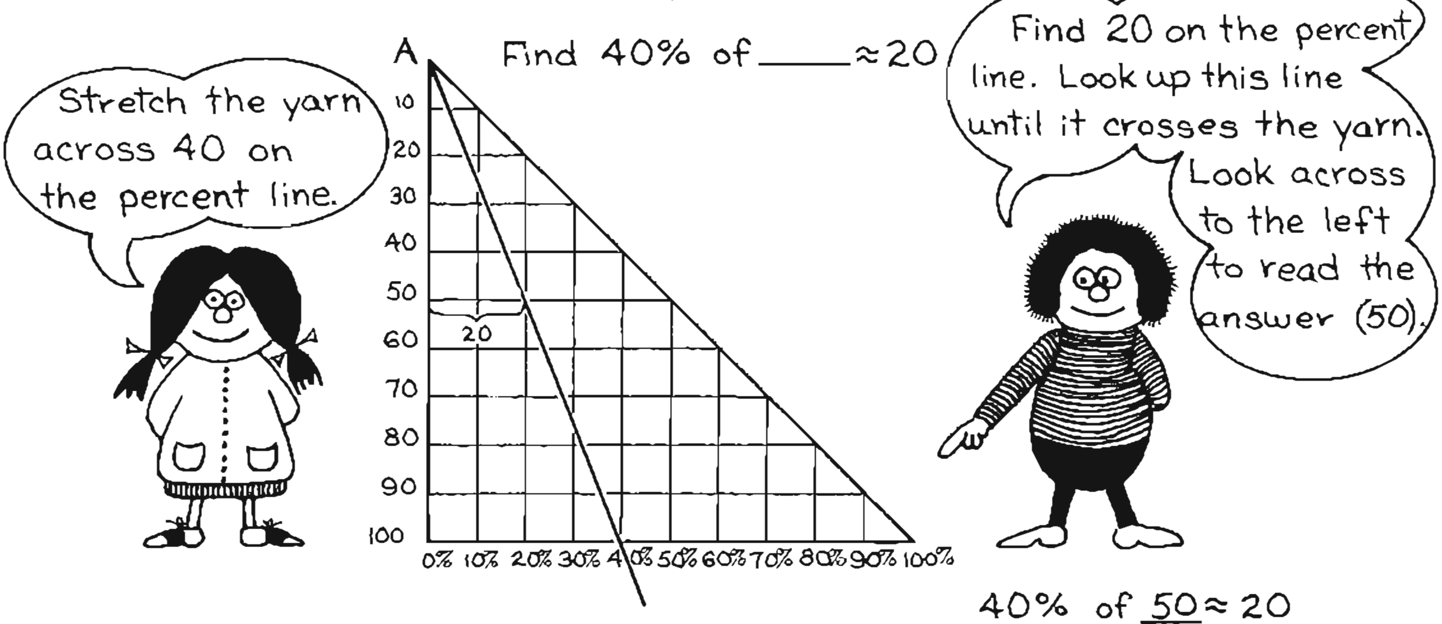
b) ___% of 90 ≈ 30   d) ___% of 100 ≈ 10   f) ___% of 35 ≈ 24

TYPE: Manipulative

You might have your students check their answers using the method on

IDEA FROM: *The Percent Calculator Book*

GRID PERCENT CALCULATOR IV

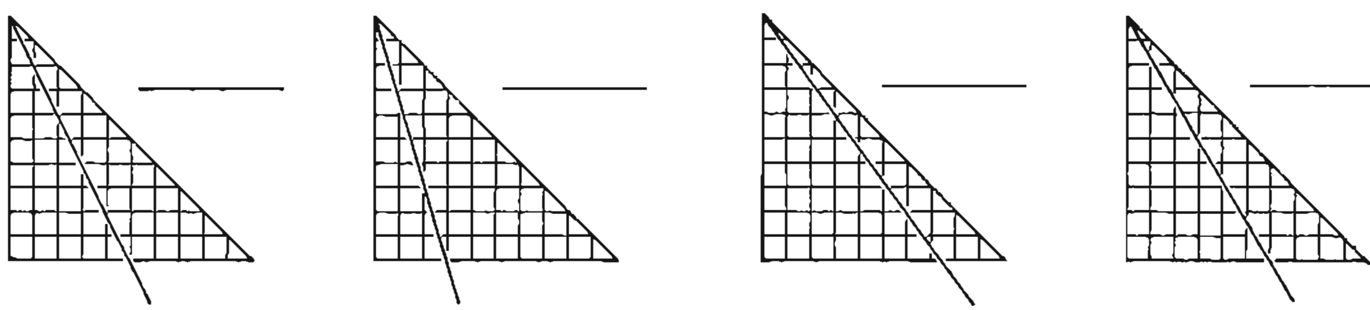


40% of <u>50</u> ≈ 20

From the example above you can also find
a) 40% of ___ ≈ 28 b) 40% of ___ ≈ 12 c) 40% of ___ ≈ 40

Are each of the problems below set up correctly to find the answer?

50% of ____ ≈ 40 30% of ____ ≈ 15 75% of ____ ≈ 60 60% of ____ ≈ 15



Use your grid percent calculator to approximate

a) 50% of ___ ≈ 30 c) 75% of ___ ≈ 30 e) 48% of ___ ≈ 27

b) 20% of ___ ≈ 10 d) 80% of ___ ≈ 55 f) 85% of ___ ≈ 63

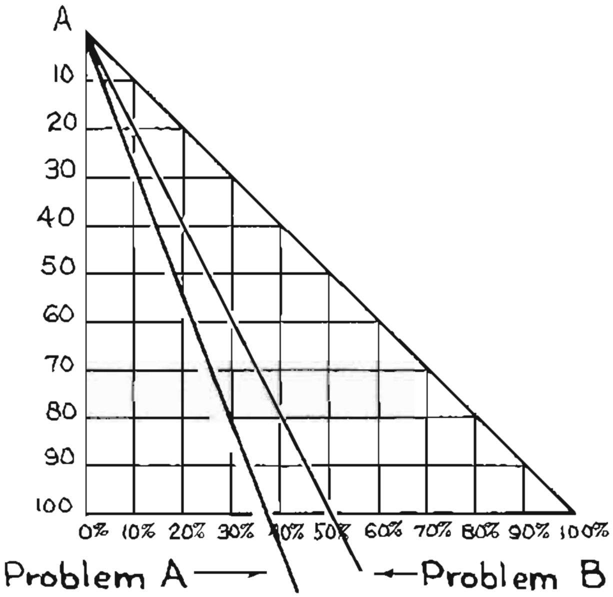
TYPE: Manipulative

IDEA FROM: *The Percent Calculator Book*

GRID PERCENT CALCULATOR EXTENSIONS

Attaching another string at A allows comparisons of two problems, e.g., which has the larger percent for an answer?

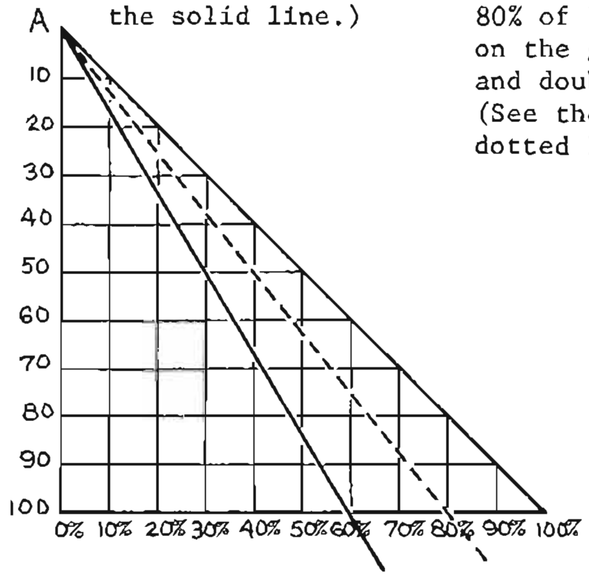
A) ___% of 80 ≈ 30   B) ___% of 50 ≈ 25



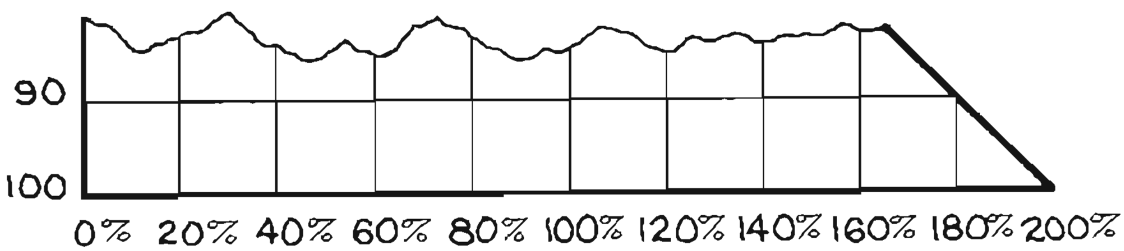
2) Percents greater than 100% can be worked by two methods.

A) 160% of 70 ≈ ___. 100% of 70 is 70, so the student can find 60% of 70 on the grid and add. (See the solid line.)

B) 160% of 70 will be twice 80% of 70, so the student can find 80% of 70 on the grid and double. (See the dotted line.)



3) Percents greater than 100% can also be done by changing the percent scale. You could either extend the percent base line or relabel the percent line using intervals of 20.



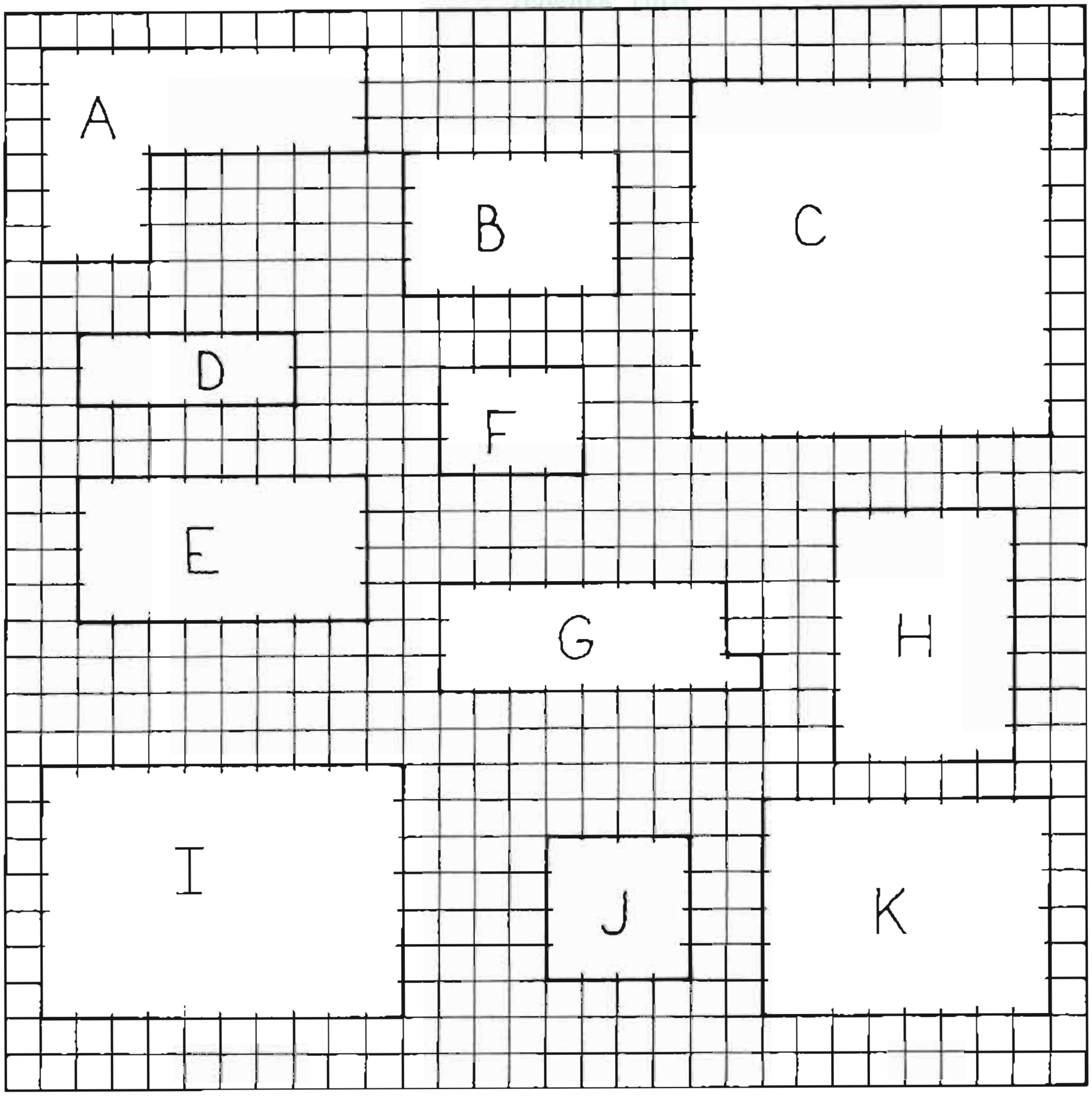
4) Percents less than 1% can be done by relabeling the percent line using intervals of .1.

5) Save the grid percent calculators to be used in later activities as an aid in checking the reasonableness of answers.

TYPE: Manipulative

IDEA FROM: *The Percent Calculator Book*

LAKE & ISLAND BOARD



This is a scale drawing of a "Lake and Island" board. To construct the board cut a 30 cm square from colored railroad board. Enlarge the pattern 2 to 1. Clip the enlargement to the board and perforate the corners of each island with a compass point. Cut the islands from poster paper of contrasting color and use the compass marks to help glue the islands to the board. For durability laminate.

Students can determine the size of each island by using centimetre cubes or a transparent centimetre grid.

I. Use Island C as the reference set. What percent of C is each of the following? Estimate first.

Island A	_____	Island E	_____	Island I	_____
Island B	_____	Island F	_____	Island J	_____
Island C	_____	Island G	_____	Island K	_____
Island D	_____	Island H	_____		

Change the reference set. If Island J is the reference set, what percent of J is Island K? E? F? D? A?

II. Use the entire board as the reference set.
(a) What percent of the board is water, islands?
(b) If a woman parachutes from a plane over the area what are her chances of landing on Island A?

HOLLYWEED SQUARES

Materials: 1) Overhead projector or chalkboard
2) Prepared list of problems
3) Spinners (dice)
4) Coordinated seating chart

PERCENT APPLICATION	FRACTIONS	FRACTION-PERCENT EQUIVALENCES
FRACTION-DECIMAL EQUIVALENCES	PERCENT CALCULATIONS	GUESS WHAT ?
DECIMALS	PERCENT SENSE	WHOLE NUMBERS

I. To prepare the game sheet construct a tic-tac-toe grid on an $8\frac{1}{2}$" by 11" transparency. Select nine categories depending on which concepts you wish to review and write them in the squares. The game sheet pictured to the right might be used for percent drill and a review of fractions, decimals and whole numbers.

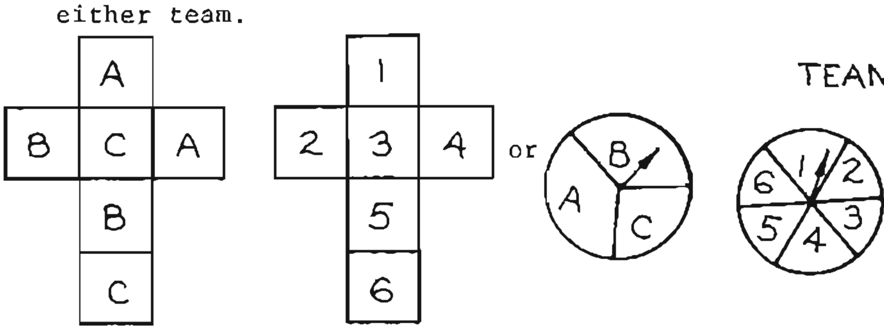
Construct twelve small transparent squares, six labeled with an "X" and six labeled with an "O", to be used as markers on the game sheet.

II. For each of the nine categories listed on the game sheet prepare five or six problems on 3 by 5-inch cards. The "guess what" category could be a non-math related question on current events.

III. Use the coordinated seating chart to divide the class into two teams. The sketch on the right depicts a classroom with 36 students arranged in rows.

Construct two spinners or dice so that you can randomly select students on either team.

TEAM 1	A1	A2	A3	A4	A5	A6
	B1	B2	B3	B4	B5	B6
	C1	C2	C3	C4	C5	C6
TEAM 2	A1	A2	A3	A4	A5	A6
	B1	B2	B3	B4	B5	B6
	C1	C2	C3	C4	C5	C6



IV. To begin the game each team chooses a captain. The captain's job is to select the category from which the team will be presented a problem. Spinners (dice) determine the person on the team to answer the problem. If the answer is incorrect, the same problem is given to a member of the opposing team. When someone does correctly answer the problem, one of the team's markers is placed over the appropriate square on the game board. The captain for the opposing team then selects another category and the play continues.

The game is won by the team that gets three of their markers in a row or has the most markers on the board when all categories are covered.

IDEA FROM: "Games in the Classroom" *Oregon Council of Teachers of Mathematics* Monograph, December, 1973

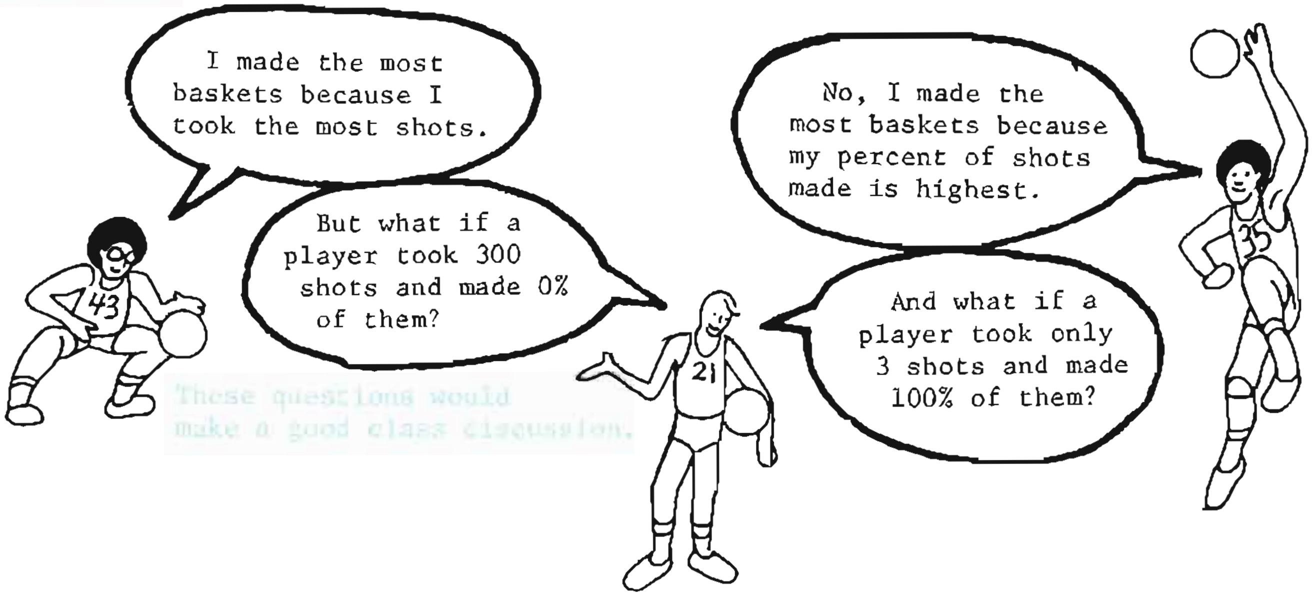
B-BALL TIME

Solving Percent Problems
PERCENT

E
S
I

You can use names of students on the school basketball team to personalize this activity.

Basketball Statistics — First 6 games — East Jr. High School			
Name Number	Shots Taken	Percent Made	Baskets Made
Jones - 43	60	30%	
Smith - 21	50	40%	
Payne - 35	36	50%	



These questions would make a good class discussion.

Who did make the most baskets? ___________

Complete the statistics for the rest of the team.

Hodge — 15	16	50%	
Briggs — 22	20	25%	
Dotson — 41	30	30%	
Taylor — 12	50	40%	
Khan — 31	10	60%	
Fowler — 44	40	25%	
Bielawski — 33	6	50%	
Lopez — 14	20	40%	
Williams — 23	15	20%	
Team Totals			

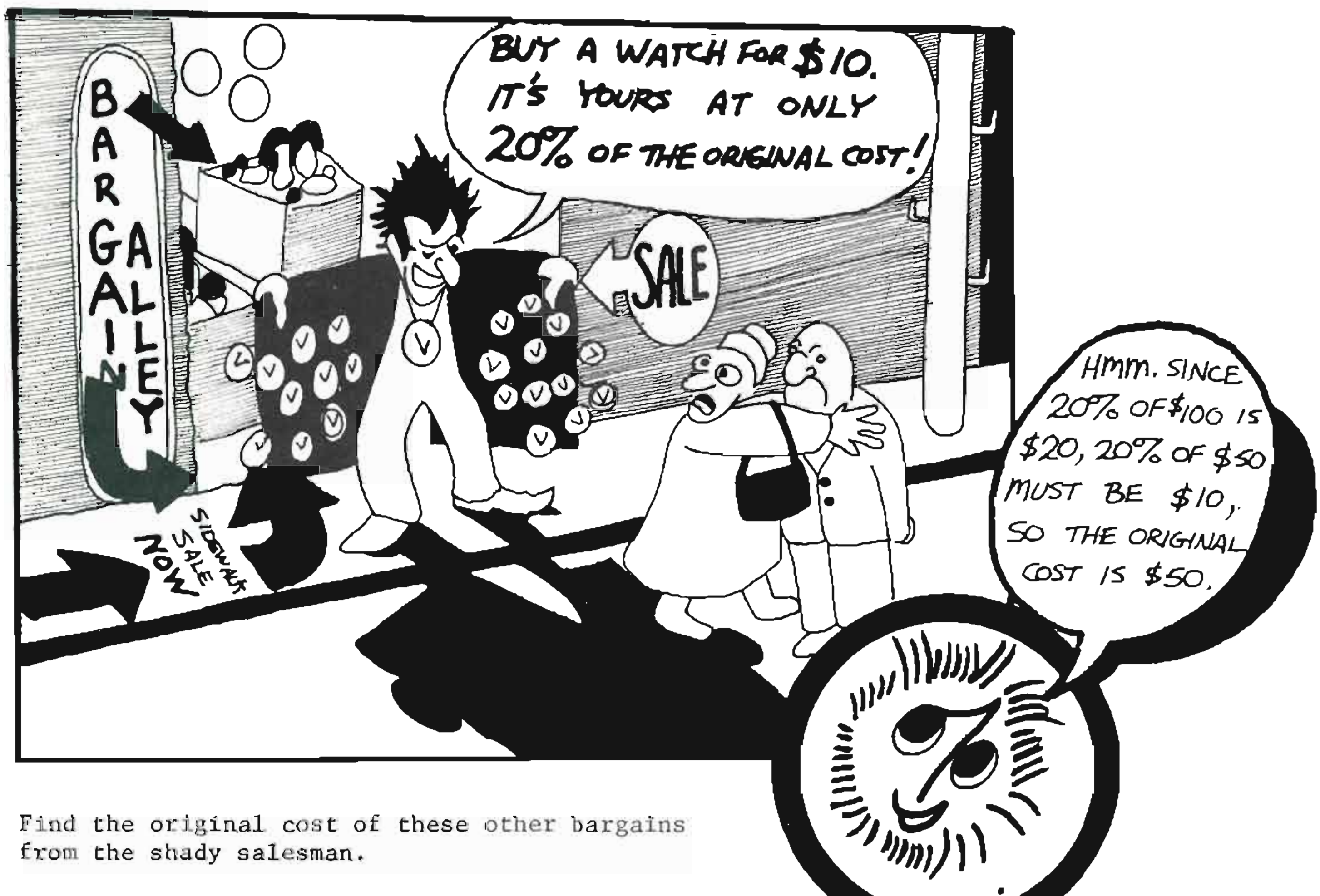
Find the team totals.

Based on these statistics, what five players would you pick to be the starting lineup for the next basketball game?

TYPE: Paper & Pencil

IDEA FROM: *Synchro-Math/Experiences*

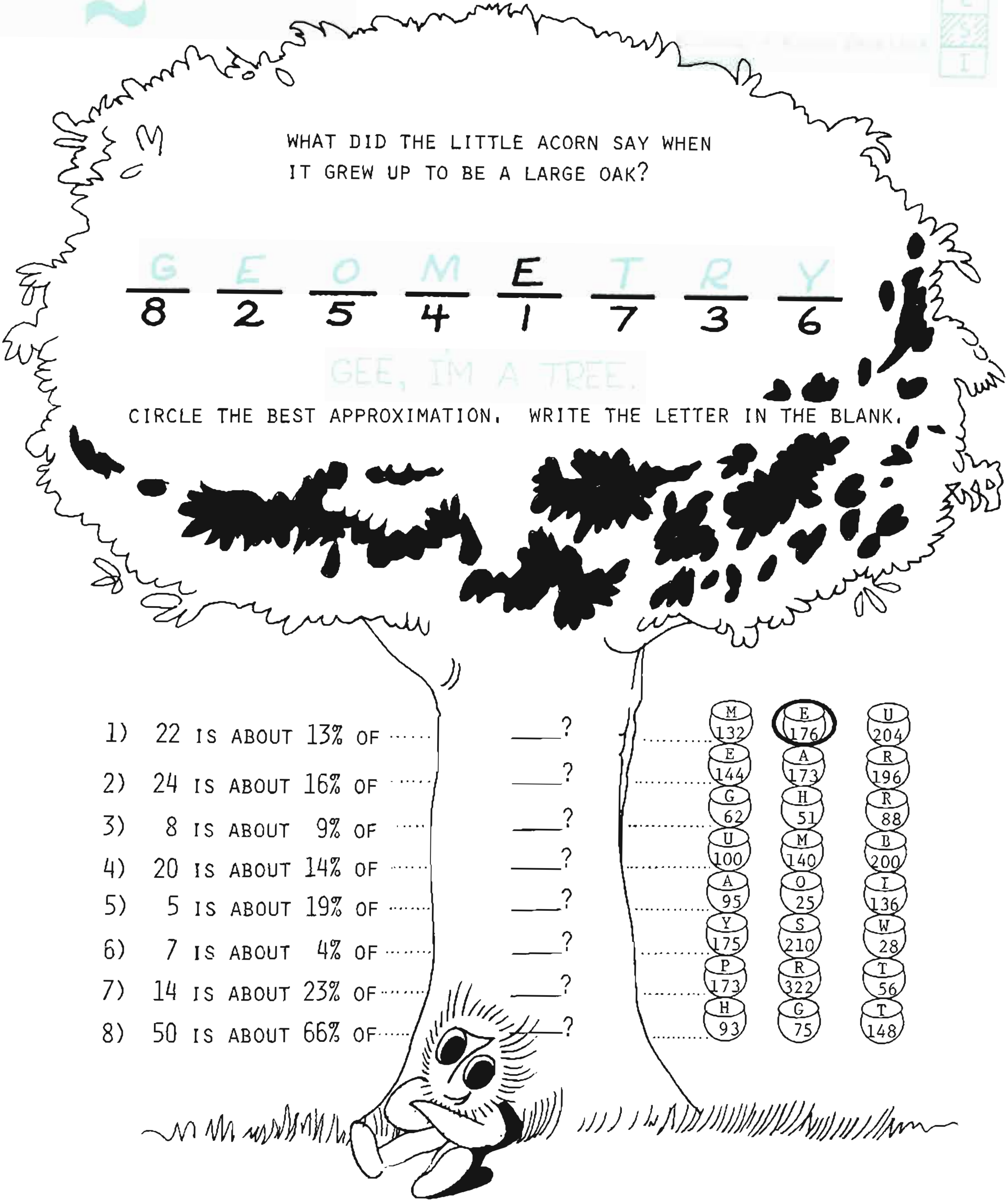
THE SHADY SALESMAN



Find the original cost of these other bargains from the shady salesman.

Pants Now $8, or 80% of the original cost __________	AM-FM Radio Only $15, 60% of the original cost __________	Deluxe Hair Dryer $24, 50% of the original cost __________
Camera $30, 30% of the original cost __________	Stereo Now $80, reduced to 25% of the original cost __________	10-speed Bike Now $105, 70% of the original cost __________
110-pound set of Weights Now $14, reduced to 70% of the original cost __________	Bikini $12, 75% of the original cost __________	Bargain of the week Electric Guitar Only $100, reduced to 40% of the original cost __________

THE OLD OAK TREE



WHAT DID THE LITTLE ACORN SAY WHEN IT GREW UP TO BE A LARGE OAK?

G	E	O	M	E	T	R	Y
8	2	5	4	1	7	3	6

GEE, I'M A TREE.

CIRCLE THE BEST APPROXIMATION. WRITE THE LETTER IN THE BLANK.

1) 22 IS ABOUT 13% OF ____?	M 132	E 176	U 204
2) 24 IS ABOUT 16% OF ____?	E 144	A 173	R 196
3) 8 IS ABOUT 9% OF ____?	G 62	H 51	R 88
4) 20 IS ABOUT 14% OF ____?	U 100	M 140	B 200
5) 5 IS ABOUT 19% OF ____?	A 95	O 25	I 136
6) 7 IS ABOUT 4% OF ____?	Y 175	S 210	W 28
7) 14 IS ABOUT 23% OF ____?	P 173	R 322	T 56
8) 50 IS ABOUT 66% OF ____?	H 93	G 75	T 148

A SIGN OF THE TIMES

This is not a sign of the zodiac, but it may be your lucky sign today in mathematics class.

For each problem on the left shade the boxes on the right that contain a correct answer.

Some problems have more than one correct answer.

1% of 60 >	2% of 60	1% of 60	.5% of 60	$\frac{2}{3}$% of 60	.1% of 60
.7% of 5 >	1% of 5	.8% of 5	.3% of 5	$2\frac{2}{3}$% of 5	$\frac{1}{4}$% of 5
10% of 98 <	.20% of 98	5% of 98	15% of 98	$10\frac{1}{2}$% of 98	50% of 98
5% of 100 =	1% of 50	.5% of 200	1% of 200	.5% of 50	5% of 100
1% of 117 <	1% of 100	$1\frac{3}{4}$% of 117	$\frac{2}{3}$% of 117	.4% of 117	1% of 117
1% of 60 =	.1% of 60	.5% of 60	$\frac{1}{2}$% of 120	$\frac{1}{2}$% of 30	1% of 80
$\frac{1}{2}$% of 341 >	1% of 341	.9% of 341	$2\frac{1}{2}$% of 341	$\frac{1}{4}$% of 341	6% of 341
$1\frac{1}{4}$% of 800 =	2.5% of 800	$2\frac{1}{2}$% of 1600	1% of 800	.25% of 800	$2\frac{1}{2}$% of 400
$\frac{7}{8}$% of 138 <	1% of 138	5% of 138	$3\frac{2}{5}$% of 138	$\frac{1}{8}$% of 138	$\frac{1}{2}$% of 138
$5\frac{2}{5}$% of 575 >	$1\frac{3}{4}$% of 575	10% of 575	1% of 575	6% of 575	$9\frac{1}{2}$% of 575
$3\frac{3}{4}$% of \$700 <	5% of \$700	20% of \$700	4% of \$700	1% of \$700	$3\frac{3}{4}$% of \$700

Now turn the paper sideways to see the sign.

ENORMOUS ESTIMATE

Circle the best ESTIMATE for each problem. Put the letter from your answer above the problem number.

What is big and green and has a trunk?

___	___	___	___	___	___	___	E	___	___	___	___	___	___	___	___
5	9	13	3	7	11	15	1	14	8	6	12	16	2	10	4

1)	40% of 60	100	A	24	E	60	R
2)	100% of 316	415	T	31	B	316	A
3)	1% of 85	.85	N	8500	Y	85	C
4)	320% of 10	32	T	3	D	10	F
5)	22.2% of 25	55	S	25	L	5.55	A
6)	100% of 8	16	V	2	J	8	E
7)	6.5% of 80	80	E	5.2	R	100	X
8)	15% of 220	300	M	220	B	33	L
9)	125% of 448	560	N	448	G	5.6	P
10)	75.8% of 50	50	Z	37.9	N	75.8	S
11)	8% of 5225	418	I	5225	Q	41800	U
12)	82% of 5	5	F	6.3	X	4.1	P
13)	480% of 15	10	H	72	U	15	I
14)	100% of 16.5	.0165	G	16.5	E	165	D
15)	3.2% of 75	2.4	P	102	W	75	K
16)	208% of 92.5	92.5	N	52.6	A	192.4	H

SOURCE: *Beefing Up Basic Skills,* Decimals and Percents
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LOVE IS WHERE YOU FIND IT

CIRCLE THE BEST ESTIMATE FOR EACH PROBLEM. PUT THE LETTER FROM YOUR ANSWER ABOVE THE PROBLEM NUMBER TO COMPLETE THE MESSAGE BELOW.

1	6% of 48 is ____	12 B	3 A	240 C
2	____ % of 10 is 10	200% D	10% F	100% E
3	35% of ____ is 25	70 I	6.5 Y	195 S
4	460% of 8 is ____	3.2 T	75 J	36 O
5	6.5% of 241 is ____	135 I	15 U	2 Q
6	100% of ____ is 87	8 R	830 H	87 Y
7	60% of 48 is ____	30 L	50 G	70 K
8	350% of ____ is 50	15 M	107 H	21000 K
9	36 is ____ % of 6	12% G	120% U	600% N
10	72 is ____ % of 25	30% J	300% S	3% E
11	125% of ____ is 320	378 F	253 V	117 L
12	____ % of 70 is 78	75% I	95% W	110% T
13	24 is ____ % of 700	4% R	96% V	140% N
14	5% of ____ is 892	44 M	18000 C	983 D
15	____ % of 117 is 24	20% W	40% B	100% O
16	100% of 2341 is ____	23410 A	23.41 X	2341 P
17	6 is ____ % of 8	100% P	125% C	75% G

S A V E   T I M E !
10 1 11 2   12 3 8 2

C O N S U L T   Y O U R   Y E L L O W
14 4 9 10 5 7 12   6 4 5 13   6 2 7 7 4 15

P A G E S
16 1 17 2 10

INTERESTING ? YOU CAN BANK ON IT !

Jim put $50 per month in a savings account from money earned on his paper route. If the interest rate was 5% per year, how much interest did Jim earn?

Think: $50 x 12 months = $600 per year
5% means $5 for every $100, so the interest was $5 x 6 or $30.

Debbie put all of her extra money earned on her paper route in the bank. Last year this was $738.20. At 5% how much interest did Debbie earn?

Think: 5% means $5 for every $100, so Debbie's interest was slightly more than $5 x 7 ($35).

To keep accurate records Debbie needed to know exactly how much interest she earned. Since 5% means 5 out of 100 or .05, she multiplied .05 x $738.20 and got $36.91 interest.

Use your percent sense to approximate the amount of money earned from each of these savings accounts. Then change the percent to a decimal and multiply to get the actual interest.

SAVINGS	INTEREST RATE PER YEAR	INTEREST APPROXIMATION	ACTUAL INTEREST EARNED
$ 900	5 %		
$ 589	5%		
$1000	4%		
$1314.50	6%		
$ 700	6%		
$842.25	4%		

IDEA FROM: *Synchro-Math/Experiences*
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AT THAT PRICE, I'LL BUY IT

Donna wishes to buy a stereo. The Turntable Tower has a $400 set of stereo equipment that was marked 15% off. To find the amount of the discount Donna thought

15% means $15 for every $100, so $15 x 4 = $60 off.

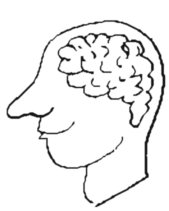
Sue looked at a stereo set costing $279 that was marked down 20%. She thought, "20% means $20 for every $100, so the stereo is marked down about $60 ($20 x 3)."

To know the actual discount Sue wrote 20% as .20 and multiplied .20 x $279 to get a discount of $55.80.

Use your percent sense to approximate the amount of these discounts. Then change the percent to a decimal and multiply to find the actual discount.

ITEM	COST	PERCENT DISCOUNT	APPROXIMATE DISCOUNT	ACTUAL DISCOUNT
STEREO	$600	15%		
AM-FM RADIO	$ 49	10%		
ELECTRIC GUITAR	$189	20%		
10-SPEED BIKE	$200	10%		
CALCULATOR	$150	15%		
SKIING EQUIPMENT	$300	30%		
T. V.	$245	12%		
CAMPING EQUIPMENT	$125	50%		
MOTORCYCLE	$975	25%		

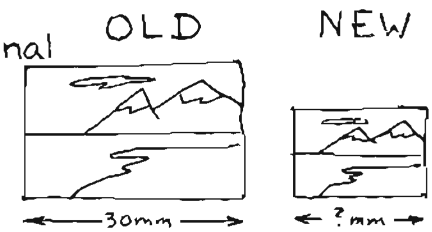
PERCENT PROBLEMS 1



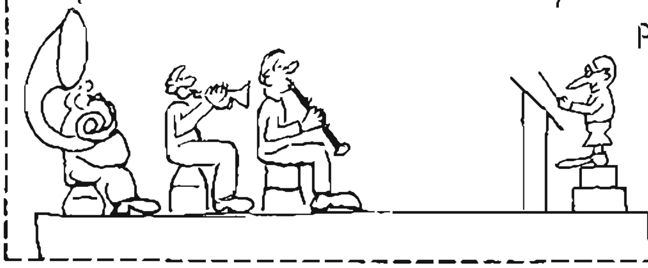
The weight of the brain is about 2.5% of the total body weight. How much would the brain of a 120 pound person weigh?

____ pounds

Jean had a picture that was 30 mm long. She asked the photo shop to reduce the picture to 65% of its original length and width. How long will the new picture be? ____ mm



210 people attended the school's band concert last year. This year's attendance is expected to be 140% of last year's. How many people are expected this year?



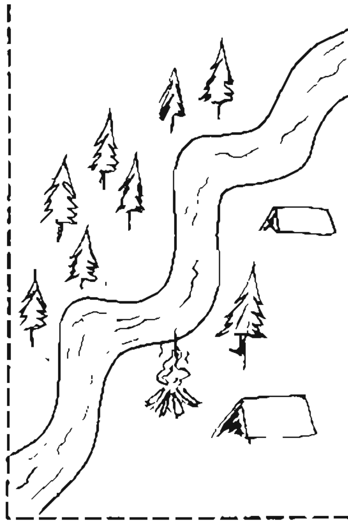
____

Ed has owed a store \$150 for one month. He must pay the store 1% of the amount he owes as a finance charge. How much is the finance charge?

\$____

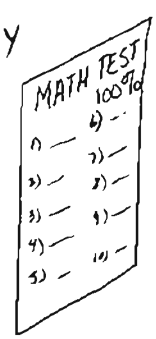
Meandering Creek is 112 kilometres long. $37\frac{1}{2}$% of the creek is inside a park. How much of the creek is inside the park?

____ kilometres



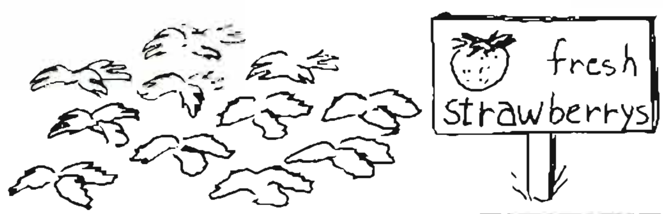
There were 30 problems on the math test. Candy got 100% of the problems correct. How many problems did she get correct?

____



SOURCE: *Beefing Up Basic Skills,* Decimals and Percents

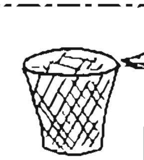
PERCENT PROBLEMS 2



Fred planted 80 strawberry plants. 68 of the plants grew. What percent of the plants grew? ____%

Our school collected $140 for the Red Cross fund. Our goal was to collect $80 What percent of our goal did we collect?

____%



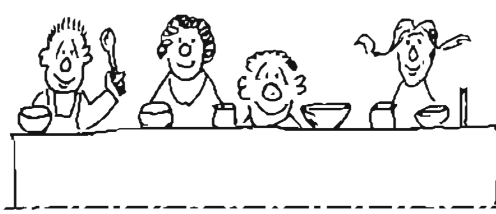
During a basketball game the Dunkers made 48% of the shots they tried. They made 36 shots. How many shots did they try? ____

Martha bought a mini-bike for 60% of the original price. She paid $96. What was the original price?

$____

This year we had 24 inches of snowfall during January. This is 120% of the snow for last January. How many inches of snow fell last January? ____ inches

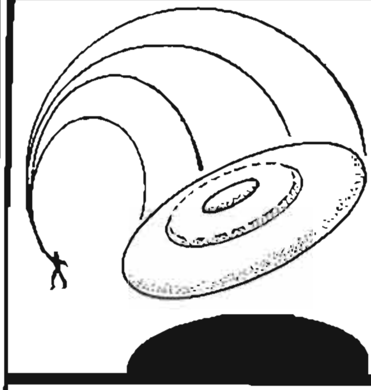
The cooking class made 175 cookies for the Open House. 14 cookies burned and couldn't be used. What percent couldn't be used? ____%



SOURCE: *Beefing Up Basic Skills,* Decimals and Percents

PELARGONIUM
1 2 3 4 5 6 7 8 9 0

The code tells what the store paid for the frisbee.



PRE
$2.69

Store cost ______

Markup ______

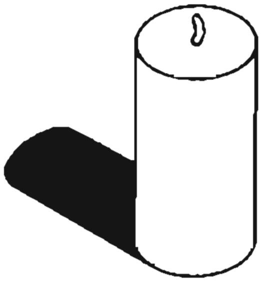
Percent of markup ______

LERG
$68.50

Store cost ______

Markup ______

Percent of markup ______

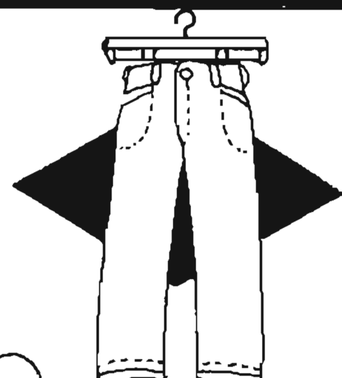


PLN
$2.75

Store cost ______

Markup ______

Percent of markup ______



MOPU
$11.95

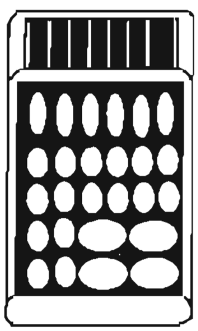
"M" means no digits

Store cost ______

Markup ______

Percent of markup ______

"M" means the previous digit is repeated

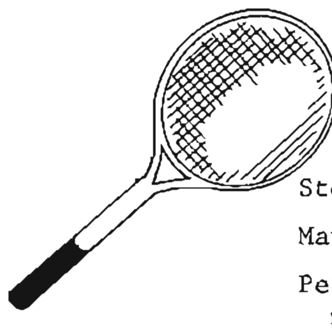


EPUM
$42.50

Store cost ______

Markup ______

Percent of markup ______



MIGU
$15.95

Store cost ______

Markup ______

Percent of markup ______

PLUM
$22.95

Store cost ______

Markup ______

Percent of markup ______

LRU
$6.98

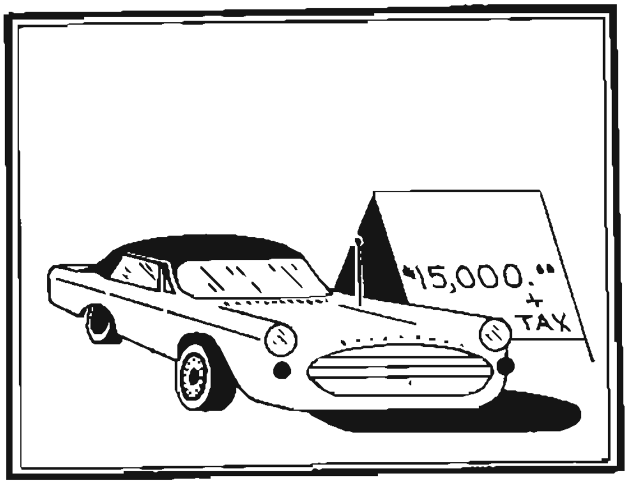
Store cost ______

Markup ______

Percent of markup ______

Check some stores in your area. Do they use codes in pricing merchandise? Can you discover the code word and break the code?

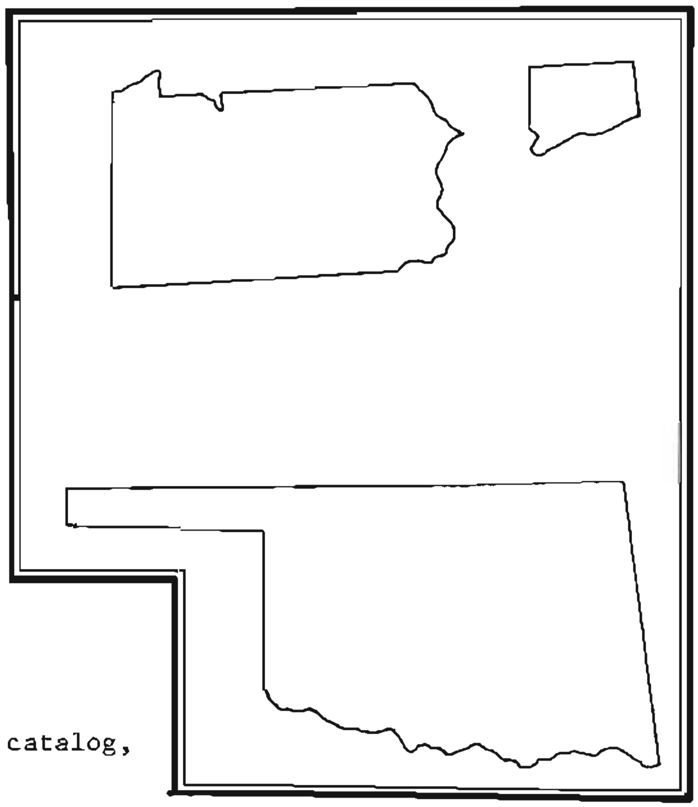
STATE THE RATE



You will need a 1975 or newer almanac and some advertisements from a local newspaper or catalog to do this assignment.

What is the year of your almanac? __________

Use the index to find the page for Taxes (State) -- Sales.



1) How many states have a sales tax? _____ (D.C. is not a state.)

2) How many states do not have a sales tax? _____

3) What percent of the 50 states have a sales tax? _____

4) Does your state have a sales tax? _____ If so, what is the rate? _____

5) What is the highest rate listed? _____ What state(s)? __________ ______________

6) What is the lowest rate listed? _____ What state(s)? __________ ______________

7) Select ten items from the newspaper or catalog, and find their total cost.

   a) Use the tax rate from #5 to find the sales tax on the total cost. __________

   b) Use the tax rate from #6 to find the sales tax on the total cost. __________

   c) Your answer to (a) is how much larger than your answer to (b)? __________

   d) If the tax rate in your state is not the highest or lowest rate, use it to find the sales tax for the ten items.

IDEA FROM: Project R-3
Permission to use granted by E.L. Hodges

COUNTING EVERY BODY

CENSUS YEAR	(a) POPULATION	(b) POPULATION ROUNDED TO NEAREST MILLION	(c) CHANGE FROM PRIOR CENSUS	(d) PERCENT CHANGE FROM PRIOR CENSUS
1790	3,929,214	4,000,000	-------	--- --
1800	5,308,483	5,000,000	1,000,000	25.0%
1970				

You will need a 1971 or newer almanac. Use the index to find the page location for United States population.

The national census was first taken in 1790.

COMPUTE TO THE NEAREST .1%

Since then a census has been taken every 10 years.

a) From the almanac copy the population for each ten-year period from 1790 to 1970.
b) Round each figure to the nearest million.
c) Find the change in population for each period--use the numbers in Column (b) to get your answer.
d) Find the percent change for each period by dividing the change--Column (c)--by the rounded population <u>of the previous census year</u>--Column (b).

Which column gives you more information--Column (c) or Column (d)?

____

Why? ____________

What major trend do you see? ________

____

IDEA FROM: *Project R-3*

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CERTAIN GROWTHS ARE BENEFICIAL

Many kinds of growth occur and are studied in mathematics. Some involve growth by a fixed amount, some by a fixed rate. These two can produce surprisingly different results.

Have students compute the outcome of depositing \$1000 at a bank at a 5% interest rate compounded annually for 20 years and compare it with a deposit of \$1000 increased annually by a fixed amount of interest (\$50.00 = 5% of \$1000) for 20 years.

Tables could be used to organize the results, and a hand calculator would simplify the computation. Interest payments should be rounded to the nearest cent.

COMPOUND INTEREST (fixed rate)

Age of deposit in years	Amount at beginning of year	Interest at 5%	Amount + Interest
1	\$1000.00	\$ 50.00	\$1050.00
2	\$1050.00	\$ 52.50	\$1102.50
3	\$1102.50	\$55.13	\$1157.63
4	\$1157.63	\$ 57.88	\$1215.51

SIMPLE INTEREST (fixed amount)

Age of deposit in years	Amount at beginning of year	Fixed amount of interest credited each year
1	\$1000.00	\$50.00
2	\$1050.00	\$50.00
3	\$1100.00	\$50.00
4	\$1150.00	\$50.00

Discuss the two outcomes. In the first table the amount of growth each year shares in the growth during the next year.

Suppose that the interest is compounded semi-annually or quarterly. What effect would this have? Some banks compound interest continuously. What does this mean? Investigate the savings plans offered at banks and savings and loan. Which would be the best for short term deposits? long term deposits?

In the bank compound interest amounts are calculated from the formula $A = (1 + \frac{r}{m})^{mt}$ *where r is the annual rate of simple interest, t is the time period in years, m is the number of compounding periods in a year. By the use of the formula it can be shown that the effective annual yield of a 7% savings certificate compounded daily for a 365-day year is 7.25%* $(A = (1 + \frac{.07}{365})^{365 \times 1} = 1.0725)$.

If compounded continuously, the formula used is $A = e^{rt}$ *where e is the base of natural logarithms.* $A = e^{.07 \times 1} \approx 1.735$, *an effective annual yield of 7.35%. Thus, a 7% certificate could yield 7.25% or 7.35%.*

IDEA FROM: "Mathematics Applied in the Modern Bank," *The Mathematics Teacher*, November, 1974
Permission to use granted by the National Council of Teachers of Mathematics

HIDDEN COSTS IN A HOME

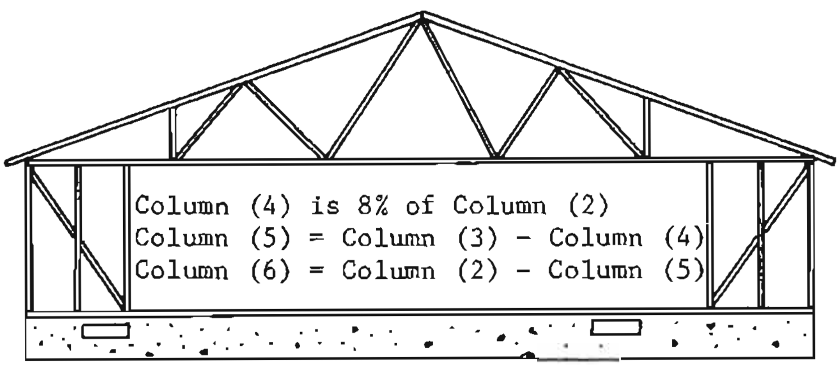
Time payments allow the consumer the use of an article before he has completely paid for it. In exchange for the convenience the consumer must pay a service charge.

In buying a home few people realize that the interest (service charge) they will pay on the mortgage may amount to more than the money they borrowed. In addition, long term payment plans can increase the total service charge considerably.

Pose the following situation to your class:

You are borrowing $10,000 to buy a home. The interest rate is 8% each year on the unpaid balance. Only one payment each year is made on the loan. How large would the yearly payment have to be in order to cover the service charge (interest) the first year? If the loan is paid off in 20 years, how much money do you think is paid in interest? If the loan is paid in 30 annual payments, how much interest is paid?

Two tables are provided that give a year by year breakdown of the payment of the loan--one, a twenty-year plan, the other, a thirty-year plan. All amounts are rounded to the nearest dollar.



Hand out the tables or make a transparency for the overhead. The following questions are suggested for discussion.

1) For each plan how much is the yearly payment?
2) How much of the first payment in each table is used for interest? How much money is still owed at the end of the first year?
3) After the tenth payment how much money is still owed?
4) Students could draw a bar graph showing the balance owed for each year.
5) In paying off the 20-year loan how much money is spent? What is the amount of interest?
6) Which loan is the most costly? By how much?
7) Why would someone select the more costly plan?

Extension:

Have students select an item(s) they would like to purchase, e.g., stereo, 10-speed, skiing equipment from a local store or mail-order catalogue. Investigate the time payment plan(s) of the store and/or catalogue. Students could organize the results in a table similar to the two mortgage tables. Suppose a credit card were used for the purchase. Discuss the interest charge. How long would it take to pay for the item if $10 a month was paid? What would the total service charge be?

HIDDEN COSTS IN A HOME (CONTINUED)

$10,000 Loan at 8%
Repaid in 21 years

Age of loan in years (1)	Unpaid balance from previous year (2)	Payment made (3)	Interest at 8% (4)	Reduction of mortgage (5)	Balance owed (6)
1	10,000	1000	800	200	9,800
2	9,800	1000	784	216	9,584
3	9,584	1000	767	233	9,351
4	9,351	1000	748	252	9,099
5	9,099	1000	728	272	8,827
6	8,827	1000	706	294	8,533
7	8,533	1000	683	317	8,216
8	8,216	1000	657	343	7,873
9	7,873	1000	630	370	7,503
10	7,503	1000	600	400	7,103
11	7,103	1000	568	432	6,671
12	6,671	1000	534	466	6,205
13	6,205	1000	496	504	5,701
14	5,701	1000	456	544	5,157
15	5,157	1000	413	587	4,570
16	4,570	1000	366	634	3,936
17	3,936	1000	315	685	3,251
18	3,251	1000	260	740	2,511
19	2,511	1000	201	799	1,712
20	1,712	1000	137	863	849
21	849	917	68	849	0
	Total	20,917	10,917	10,000	

$10,000 Loan at 8%
Repaid in 29 Years

Age of loan in years (1)	Unpaid balance from previous year (2)	Payment made (3)	Interest at 8% (4)	Reduction of mortgage (5)	Balance owed (6)
1	10,000	900	800	100	9,900
2	9,900	900	792	108	9,792
3	9,792	900	783	117	9,675
4	9,675	900	774	126	9,549
5	9,549	900	764	136	9,413
6	9,413	900	753	147	9,266
7	9,266	900	741	159	9,107
8	9,107	900	729	171	8,936
9	8,936	900	715	185	8,751
10	8,751	900	700	200	8,551
11	8,551	900	684	216	8,335
12	8,335	900	667	233	8,102
13	8,102	900	648	252	7,850
14	7,850	900	628	272	7,578
15	7,578	900	606	294	7,284
16	7,284	900	583	317	6,967
17	6,967	900	557	343	6,624
18	6,624	900	530	370	6,254
19	6,254	900	500	400	5,854
20	5,854	900	468	432	5,422
21	5,422	900	434	466	4,956
22	4,956	900	396	504	4,452
23	4,452	900	356	544	3,908
24	3,908	900	313	587	3,321
25	3,321	900	266	634	2,687
26	2,687	900	215	685	2,002
27	2,002	900	160	740	1,262
28	1,262	900	101	799	463
29	463	500	37	463	0
	Total	25,700	15,700	10,000	

Tables adapted from *Readings in Mathematics, Book 2*, edited by Irving Adler, © Copyright, 1972, by Ginn and Company (Xerox Corporation). Used with permission.

RATIO, PROPORTION, AND SCALING

Placement Guide for Tabbed Divisors

ANNOTATED BIBLIOGRAPHY

The following is a list of sources used in the development of this resource. It is not a comprehensive listing of materials available. In some cases, good sources have not been included simply because the project did not receive permission to use the publisher's materials or a fee requirement prohibited its use by the project.

ACTIVITIES IN MATHEMATICS, Second Course. Donovan A. Johnson, Viggo P. Hanson, Wayne H. Peterson, Jesse A. Rudnik, Ray Cleveland, and L. Carey Bolster. Glenview, Illinois: Scott, Foresman and Company, 1973. (Scott, Foresman and Company, 1900 East Lake Ave., Glenview, IL 60025)

432 pp; cloth; teacher's guide; color

A collection of 85 activities covering the areas of graphs, statistics, proportions, and geometry are contained in this book.

ACTIVITIES WITH RATIO AND PROPORTION. Oakland County Mathematics Project. Pontiac, Michigan: Oakland Schools, 1970. (Oakland Schools, 2100 Pontiac Lake Rd., Pontiac, MI 48054)

paper; b/w; workbooks; medium reading level

This is one of a series of 23 topical workbooks covering such general areas as geometry, statistics, probability, measurement, ratio and proportion.

AFTERMATH, Volumes I-IV. Dale Seymour, Mary Laycock, Verda Holmberg, Ruth Heller, and Bob Larsen. Palo Alto, California: Creative Publications, Inc., 1971. (Creative Publications, Inc., P.O. Box 10328, Palo Alto, CA 94303)

97 pp in each; paper; b/w; teacher reference

A collection of four booklets each including reproducible student worksheets on a variety of topics.

THE ARITHMETIC TEACHER. Reston, Virginia: The National Council of Teachers of Mathematics. (National Council of Teachers of Mathematics, 1906 Association Dr., Reston, VA 22091)

This magazine, published eight times a school year, January through May and October through December, contains a wealth of ideas and activities for elementary and middle school mathematics teachers.

ARITHMETICAL EXCURSIONS: AN ENRICHMENT OF ELEMENTARY MATHEMATICS. Henry Bowers and Joan E. Bowers. New York: Dover Publications, Inc., 1961. (Dover Publications, Inc., 180 Varick St., New York, NY 10014)

320 pp; paper; b/w; teacher reference

This is a book of enrichment topics in mathematics ranging from counting through the arithmetic operations to figurate, perfect and amicable numbers and then on to mysteries and folklore of numbers. Exercises and answers are provided for each of the 27 topics.

ART 'N' MATH. Karen Billings, Carol Campbell, and Alice Schwandt. Eugene, Oregon: Action Math Associates, Inc., 1975. (Action Math Assocates, Inc., 1358 Dalton Dr., Eugene, OR 97404)

63 pp; paper; workbook; b/w; medium reading level

This is a collection of 19 activities that combine math skills with an art project.

BEEFING UP BASIC SKILLS. Minneapolis: The Math Group Inc., 1974. (The Math Group, Inc., 396 E. 79th St., Minneapolis, MN 55420.

40 pp each booklet; paper; workbook; b/w; low reading level

The series includes three booklets: Whole Numbers, Fractions, and Decimals and Percents. Each booklet contains puzzle pages for drilling basic skills.

C.O.L.A.M.D.A. Longmont, Colorado: Personalized Instruction Center, Northern Colorado Educational Board of Cooperative Services, n.d. (Northern Colorado Educational Board of Cooperative Services, 830 South Lincoln, Longmont, CO 80501)

374 activities; workbook; b/w; unbound; medium reading level

The collection of materials provides instruction for reluctant learners in grades 7-12 through a laboratory approach. Included are games, disguised drill worksheets, activity cards and a guide for recommended use.

COSMIC VIEW: THE UNIVERSE IN FORTY JUMPS. Kees Boeke. New York: John Day Company, Inc., 1957. (John Day Company, Inc., 666 Fifth Ave., New York, NY 10019)

CREATIVE CONSTRUCTIONS. Dale Seymour and Reuben Schadler. Palo Alto, California: Creative Publications, Inc., 1974. (Creative Publications, Inc., P.O. Box 10328, Palo Alto, CA 94303)

62 pp; paper; b/w; workbook; medium reading level

This is a workbook containing many designs that can be created from inscribed triangles, squares, pentagons, hexagons and octagons.

THE DIVINE PROPORTION. H.E. Huntley. New York: Dover Publications, Inc., 1970. (Dover Publications, Inc., 180 Varick St., New York, NY 10014)

186 pp; paper; teacher reference; b/w

The golden ratio, Ø, is the basis for many seemingly unrelated topics in mathematics. This book covers those topics in a sophisticated manner.

Chapter XIII: Spira Mirabilis (pp. 164-176) emphasizes the golden triangle, (a 72°, 72°, 36°, isosceles triangle) the golden ratios to be found, and the logarithmic spiral that can be drawn by connecting vertices of a nested sequence of similar golden triangles.

ELEMENTARY SCHOOL MATHEMATICS. Second Edition. Robert E. Eicholz and Phares G. O'Daffer. Menlo Park, California: Addison-Wesley Publishing Company, 1968. (Addison-Wesley Publishing Company, 2725 Sand Hill Rd., Menlo Park, CA 94025)

cloth; textbook series; color

This textbook series covers grades K-8. The usual topics of an elementary curriuclum are covered. The series was written to provide students with many opportunities to think mathematically.

ENCYCLOPAEDIA BRITANNICA. Montreal, Canada: Encyclopaedia Britannica Publications, Ltd. (Encyclopaedia Britannica Publications, Ltd., 2 Bloor St. W., Suite 1100, Toronto, Ontario, M4W 3J1, Canada)

THE ENCYCLOPEDIA AMERICANA, International Edition. New York: Americana Corporation, 1973. (Americana Corporation, Subsidiary, Grolier, Inc., 575 Lexington Ave., New York, NY 10022)

cloth; teacher/student reference; b/w

Much useful information is found in this (and other) sets of encyclopedia.

ENHANCE CHANCE. Jan Becker, Mary Laycock, and Genevieve Waring. Los Gatos, California: Contemporary Ideas, 1973. (Activity Resources Company, Inc., P.O. Box 4875, Hayward, CA 94540)

58 pp; paper; teacher reference; b/w

The book contains 45 games that use dice to reinforce and extend mathematical concepts. Game mats are provided and rules are concisely explained.

EXPLORING MATHEMATICS ON YOUR OWN. Donovan A. Johnson and Glenn William. London: John Murray (Publishers), 1965. Copyright is held by John Murray (Publishers) and Donovan A. Johnson. John Murray (Publishers) Ltd., 50 Albermarle St., London, England, WIX 4BD. Donovan A. Johnson, 4360 Brookside Ct., 213, Minneapolis, MN 55436

303 pp; paper; teacher reference; b/w

The book contains many interesting ideas and activities that can be presented in the classroom. Special emphasis is given to numeration systems, number patterns, the Pythagorean theorem, sets, sentences, operations, topology and fun activities.

FIBONACCI AND LUCAS NUMBERS. Verner E. Hoggatt, Jr. Boston: Houghton-Mifflin Company, 1969. (Houghton-Mifflin Company, Educational Division, 1 Beacon St., Boston, MA 02107)

92 pp; paper; teacher reference; b/w

Many interesting properties of Fibonnaci and Lucas numbers are explored in this booklet which is one of a series of eleven booklets in the Houghton-Mifflin Mathematics Enrichment Series.

FRACTION FACTORY. Minneapolis: The Math Group, Inc., 1973. (The Math Group, Inc., 396 E. 79th St., Minneapolis, MN 55420)

21 activity cards; b/w

Twenty-one activity cards presenting situations where fractions are needed for solution of a problem make up this set.

FREEDOM TO LEARN. Edith E. Biggs and James R. MacLean. Canada: Addison-Wesley (Canada) Ltd., 1969. (Addison-Wesley (Canada) Ltd., P.O. Box 580, 36 Prince Andrew Place, Don Mills, Ontario, M3C 2T8, Canada)

206 pp; cloth; teacher reference; color

This book emphasizes the active approach to learning in elementary mathematics and gives many suggestions for organizing lab lessons, classrooms, materials, etc. to promote an active approach.

"Games in the classroom." Charles Henderer. *OCTM MONOGRAPH*, (December, 1973), p. 3. (Oregon Council of Teachers of Mathematics, 4015 S.W. Canyon Rd., Portland, OR 97221)

GREATER CLEVELAND MATHEMATICS PROGRAM. Chicago: Science Research Associates, Inc., 1968. (Educational Research Council of America, Rockefeller Building, Cleveland, OH 44113)

cloth; textbook series; b/w

This set of standard textbooks for grades one through six places emphasis on the understanding of basic mathematical ideas. It is now out of print.

GUIDE TO ADVENTURES IN MATHEMATICS. B. Hewitt. Langley, Bucks: St. Paul Publications, 1970. (St. Paul Publications, Middlegreen, Slough, SL3 6BT England)

144 activity cards; looseleaf, b/w

An activity approach to the concepts of geometry, weight, area, time, volume, numeration, length and proportion is promoted in this set of 144 cards.

HISTORY OF PI. Peter Beckman. New York: St. Martin's Press, Inc., 1964. (St. Martin's Press, Inc., 175 Fifth Ave., New York, NY 10010)

HOW TO LIE WITH STATISTICS. Darrell Huff. New York: W.W. Norton and Company, 1954. (W.W. Norton and Company, 500 Fifth Ave., New York, NY 10036)

142 pp; paper; b/w; teacher reference

A delightful book illustrating how statistics can be used to demonstrate any point of view. Although published in 1954, the examples of distortion and suggestions for critical examination of data are still relevant.

IDEAS AND INVESTIGATIONS IN SCIENCE: LIFE SCIENCE. Harry Wong, Leonard Bernstein, and Edward Shevick. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1973. (Prentice-Hall, Inc., Englewood Cliffs, NJ 07632)

410 pp; cloth; color; textbook; low reading level

This textbook uses a laboratory-investigation approach to life science. Students are asked to discover concepts and build several small concepts into a big idea of how things work.

"If we just turn the crank." James Hutcheson. *THE MATHEMATICS TEACHER*, Vol. 68, No. 1 (January, 1975), pp. 33-35. Reston, Virginia: National Council of Teachers of Mathematics. (National Council of Teachers of Mathematics, 1906 Association Dr., Reston, VA 22091)

INVESTIGATING SCHOOL MATHEMATICS, Second Edition. Robert E. Eicholz, Phares G. O'Daffer, and Charles R. Fleenor. Menlo Park, California: Addison-Wesley Publishing Company, Inc., 1976. (Addison-Wesley Publishing Company, Inc., 2725 Sand Hill Rd., Menlo Park, CA 94025)

cloth; textbook series; color

This is a textbook series for grades 1-6. The books are colorfully illustrated and include the usual topics in mathematics. A spiral approach is used for most topics and many thought-provoking extensions are included.

THE LABORATORY APPROACH TO MATHEMATICS. Kenneth P. Kidd, Shirley S. Myers, and David M. Cilley. Chicago: Science Research Associates, Inc., 1970.

282 pp; paper; teacher reference; b/w

This is an excellent resource that details the organization, planning and facilities needed for using a mathematics laboratory. A series of activities concerning the use of ratio is presented.

LINE DESIGNS. Dale G. Seymour and Joyce Snider. Palo Alto, California: Creative Publications, Inc., 1968. (Creative Publications, Inc., P.O. Box 10328, Palo Alto, CA 94303)

MAKING MATHEMATICS, A Secondary Course. D. Paling, C.S. Banwell, and K.D. Saunders. London: Oxford University Press, 1970. (Oxford University Press, 200 Madison Ave., New York, NY 10016)

6 topical books; textbooks; b/w; medium reading level

This is a series of six topical books designed to cover a five-year secondary course.

MATH ACTIVITY CARDS, Levels A-E. (Macmillan Elementary Mathematics). David M. Clarkson. New York: The Macmillan Publishing Company, Inc., 1969, 1970. (Macmillan Publishing Company, Inc., 866 Third Ave., New York, NY 10022)

48 cards in each level; color; activity cards; low reading level

A series of five sets of 48 activity cards basically for grades 2-6 which cover the concepts of graphs, shapes, measurement, patterns and reasoning.

MATH AMUSEMENTS IN DEVLOPING SKILLS, Vol. I. Alice A. Clack, and Carol H. Leitch. Troy, Michigan: Midwest Publications Company, Inc., 1972. (Midwest Publications Company, Inc., P.O. Box 129, Troy, MI 48099)

87 pp; paper; workbook; b/w

The book contains fifty-seven puzzles for disguised drill in whole number, fraction and decimal operations. The pages can be reproduced for classroom use.

MATH WORKSHOP, Level F. Robert Wirtz, Morton Botel, May Beberman, and W.W. Sawyer. Chicago: Encyclopaedia Britannica Educational Corporation, 1967. (Encyclopaedia Britannica Educational Corporation, 425 N. Michigan Ave., Chicago, IL 60611)

320 pp; cloth; textbook series; color

This sixth book in a series designed for use in grades 1-6 covers the main topics of structure, sets, number and counting, numeration, addition and subtraction, multiplication and division, functions and relations, geometry, and measurement.

MATHEMATICS A HUMAN ENDEAVOR. Harold R. Jacobs. San Francisco: W.H. Freeman and Company, Publishers, 1970. (W.H. Freeman and Company, Publishers, 660 Market St., San Francisco, CA 94104)

529 pp; cloth; textbook; b/w; high reading level

This text for a liberal arts course is an excellent resource book. Many of the ideas are suitable for or could be adapted for middle school students.

"Mathematics applied in the modern bank." Lee E. Boyer, Philip J. Hippensteel, and Robert J. Lutz. *THE MATHEMATICS TEACHER*, Vol. 67, No. 7 (November, 1974), pp. 611-614. Reston, Virginia: National Council of Teachers of Mathematics. (National Council of Teachers of Mathematics, 1906 Association Dr., Reston, VA 22091)

THE MATHEMATICS TEACHER. Reston, Virginia: The National Council of Teachers of Mathematics. (National Council of Teachers of Mathematics, 1906 Association Dr., Reston, VA 22091)

This magazine, published eight times a school year, January through May and October through December, contains a wealth of ideas and activities for middle school and secondary mathematics teachers.

MATHEMATICS: THE STORY OF NUMBERS, SYMBOLS AND SPACE. (The Golden Library of Knowledge). Irving Adler. New York: Golden Press, 1958. (Western Publishing Company, Inc., 1220 Mound Ave., Racine, WI 53404)

56 pp; paper; student reference; medium reading level

The book contains easy-to-read sections on number concepts, geometry, mathematics in nature, navigation, probability, the slide rule, etc.

MATHEX. Montreal, Canada: Encyclopaedia Britannica Publications, Ltd., 1970. (Encyclopaedia Britannica Publications, Ltd., 2 Bloor Street West, Suite 1100, Toronto, Ontario, Canada, M4W 3J1)

approx. 45 pp in each; paper workbook; b/w; medium reading level

MATHEX is a series of ten student workbooks with corresponding Teacher Resource books. Books 1-5 are designed for primary grades and books 6-10 are designed for grades 4-6. The topics covered in books 1-5 are matching and graphing, numeration, operations, geometry and probability, numeration, operations and problem solving, geometry, and measurement.

MATHIMAGINATION, Books A-F. Steve Marcy and Janis Marcy. Palo Alto, California: Creative Publications, Inc., 1973. (Creative Publications, Inc., P.O. Box 10328, Palo Alto, CA 94303)

48 pp in each; paper; workbooks; b/w

Each book provides drill in the form of a puzzle on basic concepts and skills of a mathematics topic; Book A: beginning multiplication and division; Book B: operations with whole numbers; Book C: number theory; Book D: fractions; Book E: decimals and percents; Book F: geometry, measurement and cartesian coordinates.

METRIC MEASUREMENT: ACTIVITIES IN LINEAR MEASUREMENT. Diana Hestwood, Royce Helmbrecht, Anne Bartel, Earl Orf, and Ed Harter. Minneapolis: The Math Group, Inc., 1974. (The Math Group, Inc., 396 E. 79th St., Minneapolis, MN 55420)

36 pp; paper; workbook; b/w

The book presents 36 pages, all of which involve linear measurement. Many pages are puzzle types with clever messages about the metric system.

THE METRIC SYSTEM OF MEASUREMENT. Verda Holmberg. Hayward, California: Activity Resources Company, Inc., 1973. (Activity Resources Company, Inc., P.O. Box 4875, Hayward, CA 94540)

98 pp; paper; workbook; b/w

The book is a collection of metric activities; linear area, volume, weight and temperature measurements are included.

MINNEMAST, Unit 18: SCALING AND REPRESENTATION. Minnesota Mathematics and Science Teaching Project. Minneapolis: University of Minnesota, 1971. (Minnesota School Mathematics and Science Center, 720 Washington Ave., S.E., Minneapolis, MN 55414)

80 pp; paper; b/w; teacher's guide

This title is one of 29 units for grades K-3. The unit covers activities and observations students can do to investigate the idea of scale representation.

THE NATURE OF RECREATION. Richard Saul Wurman, Alan Levy, and Joel Katz. Cambridge, Massachusetts: Massachusetts Institute of Technology Press, 1973. (Massachusetts Institute of Technology Press, 28 Carleton St., Cambridge, MA 02142)

76 pp; paper; color; teacher reference

This is an interesting book with several topics that can be used to illustrate mathematical concepts in the field of recreation.

NEW OXFORD JUNIOR MATHEMATICS, BOOK 1-5. E.M. Williams and E.J. James. Ely House, London: Oxford University Press, 1971. (Oxford University Press, 200 Madison Ave., New York, NY 10016)

approx. 150 pp in each; paper; color; textbook; medium reading level

These student textbooks, which use a discovery approach to a variety of mathematical concepts, provide real-world applications and include some drill and practice pages to check basic computations.

NEW WAYS IN GEOGRAPHY, Introductory Book and Books 1 & 2. J.P. Cole and N.J. Beynon. Oxford, England: Basil Blackwell and Mott, Ltd., 1968. (Basil Blackwell and Mott, Ltd., 108 Cowley Road, Oxford, OX4 1JF, England)

46 pp (Introductory Book); 61 pp (Book 1); 64 pp (Book 2); paper; color; workbook; medium reading level

The three books stress the mathematical aspects of geography. Much use is made of drawing maps, reading maps, collecting data, and discovering relationships.

OCTM MONOGRAPH. Eugene, Oregon: Oregon Council of Teachers of Mathematics. (Oregon Council of Teachers of Mathematics, 4015 S.W. Canyon Rd., Portland, OR 97221)

The OCTM MONOGRAPH is now part of the Oregon Mathematics Teacher, a magazine published eight times a school year that contains many useful ideas for teachers of mathematics at all levels.

THE OFFICIAL ASSOCIATED PRESS SPORTS ALMANAC 1974. Keith Fuller. New York: Dell Publishing Company, Inc., 1974. (Associated Press, 50 Rockefeller Plaza, New York, NY 10020)

927 pp; paper; student reference; b/w

The book contains an extensive collection of sports records and facts.

OPEN-ENDED TASK CARDS. J. Risk. San Francisco: J. Risk, 1971. (Teachers Exchange of San Francisco, 600 35th Ave., San Francisco, CA 94121)

21 cards; color; activity cards; low reading level

This is a series of 18 teacher-written activities using supplies available in the classroom to supplement regular mathematics lessons.

OPERATING WITH MATHEMATICS, Teacher's Annotated Edition. Robert B. Kane, Robert A. Oesterle, James L. Fejfar, and James W. Goodfellow. New York: American Book Company, 1969. (American Book Company, Division of Litton Educational Publishing, 450 W. 33rd St., New York, NY 10001)

555 pp; cloth; color; textbook

This textbook is the seventh in a series for low achievers in mathematics. The reading level is lower and the pace of the material is significantly slower than for most mathematics series.

PAPER FOLDING FOR THE MATHEMATICS CLASS. Donovan A. Johnson. Reston, Virginia: The National Council of Teachers of Mathematics, 1957. (The National Council of Teachers of Mathematics, 1906 Association Dr., Reston, VA 22091)

32 pp; paper; teacher reference

This book is comprised of a collection of exercises which use paper folding to illustrate and help students discover relationships of lines and angles. This technique can add realism and interest to your mathematics teaching.

PATTERNS IN SPACE. Colonel Robert S. Beard. Palo Alto, California: Creative Publications, Inc., 1973. (Creative Publications, Inc., P.O. Box 10328, Palo Alto, CA 94303)

THE PERCENT CALCULATOR BOOK. Oakland County Mathematics Project. Pontiac, Michigan: Oakland Schools, 1970. (Oakland Schools, 2100 Pontiac Lake Rd., Pontiac, MI 48054)

PHYSICAL FITNESS WORKBOOK. Thomas Cureton. Champaign, Illinois: Stipes Publishing Company, 1944. (Stipes Publishing Company, 10-12 Chester St., Champaign, IL 61820)

150 pp; hardback; teacher reference; b/w

This book was written as a manual for students in physical fitness classes.

PROJECT R-3. E.L. Hodges, ed. San Jose, California: T.M.T.T., 1974. (E.L. Hodges, T.M.T.T., 990 Asbury, San Jose, CA 95126)

looseleaf; worksheets; b/w

Four packets of student pages and a packet of answer sheets and suggested forms for grading, record keeping, etc. make up the materials. The four packets cover whole numbers, fractions, decimals and percents.

READINGS IN MATHEMATICS, Book 2. Edited by Irving Adler. Lexington, Massachusetts: Ginn and Company (Xerox Corporation), 1972. (Ginn and Company [Xerox Corporation], 191 Spring St., Lexington, MA 02173)

188 pp; paper; teacher reference; b/w

Many selections written by both contemporary authors and by great scientists and mathematicians of the past are included in this book. The selections relate mathematics to everyday life, science and literature. The book will be interesting reading for the lay person as well as the mathematician.

"Road maps in the mathematics classroom." Albert Gardner. *OCTM MONOGRAPH*, (December, 1973), p. 1. Eugene, Oregon: Oregon Council of Teachers of Mathematics. (Oregon Council of Teachers of Mathematics, 4015 S.W. Canyon Rd., Portland, OR 97221)

THE SCHOOL MATHEMATICS PROJECT, Book A-H. The School Mathematics Project. Great Britain: Cambridge University Press, 1971. (Cambridge University Press, 32 East 57th St., New York, NY 10022)

approx. 300 pp in each; paper; textbook; b/w; medium reading level

The series is intended for students in grades 7-10. The books present a spiral acitivity-oriented approach that interweaves arithmetic, algebra and geometry with an attempt to emphasize the practical applications of each topic.

THE SCHOOL MATHEMATICS PROJECT, Books 1-5 [Metric]. The School Mathematics Project. Great Britain: Cambridge University Press, 1969. (Cambridge University Press, 32 East 57th St., New York, NY 10022)

approx. 300 pp in each; cloth; textbook; b/w; high reading level

This is the first series of textbooks produced by the School Mathematics Project. Like Books A-H, it presents a spiral activity-oriented approach, but the coverage of topics is more comprehensive and the reading level is higher.

SEEING SHAPES. Ernest Ranucci. Palo Alto, California: Creative Publications, Inc., 1973. (Creative Publications, Inc., P.O. Box 10328, Palo Alto, CA 94303)

94 pp; paper; b/w; teacher reference

An excellent collection of materials designed for use from grade 7 to the junior college level, this book provides exercises to strengthen visual perception and the understanding of spatial relationships. Emphasis is on discovery and an attempt is made to provoke imaginative thinking.

SIMILARITY AND CONGRUENCE. Oakland County Mathematics Project. Pontiac, Michigan: Oakland Schools, 1971. (Oakland Schools, 2100 Pontiac Lake Rd., Pontiac, MI 48054)

SRA MATH APPLICATIONS KIT. Allen C. Friebel and Carolyn K. Gingrich. Chicago: Science Research Associates, 1971. (Science Research Associates, Inc., 259 East Erie St., Chicago, IL 60611)

270 cards; color; acitivity cards; medium reading level

This is an excellent collection of 270 activity cards that presents a large number of problems and activities in science, sports and games, occupations, social studies, and everyday things that students, using elementary school mathematics, can explore. Also included are 10 reference cards and student and teacher handbooks.

SYNCHRO-MATH/EXPERIENCES. Oscar Schaaf, Scott McFadden, and Klaas Kramer. Chicago: Lyons and Carnahan, 1972. (Copyright now with Action Math Associates, Inc., 1358 Dalton Dr., Eugene, OR 97404.)

469 pp; cloth; b/w; textbook; medium reading level

This is a textbook that emphasizes a discovery technique of learning.