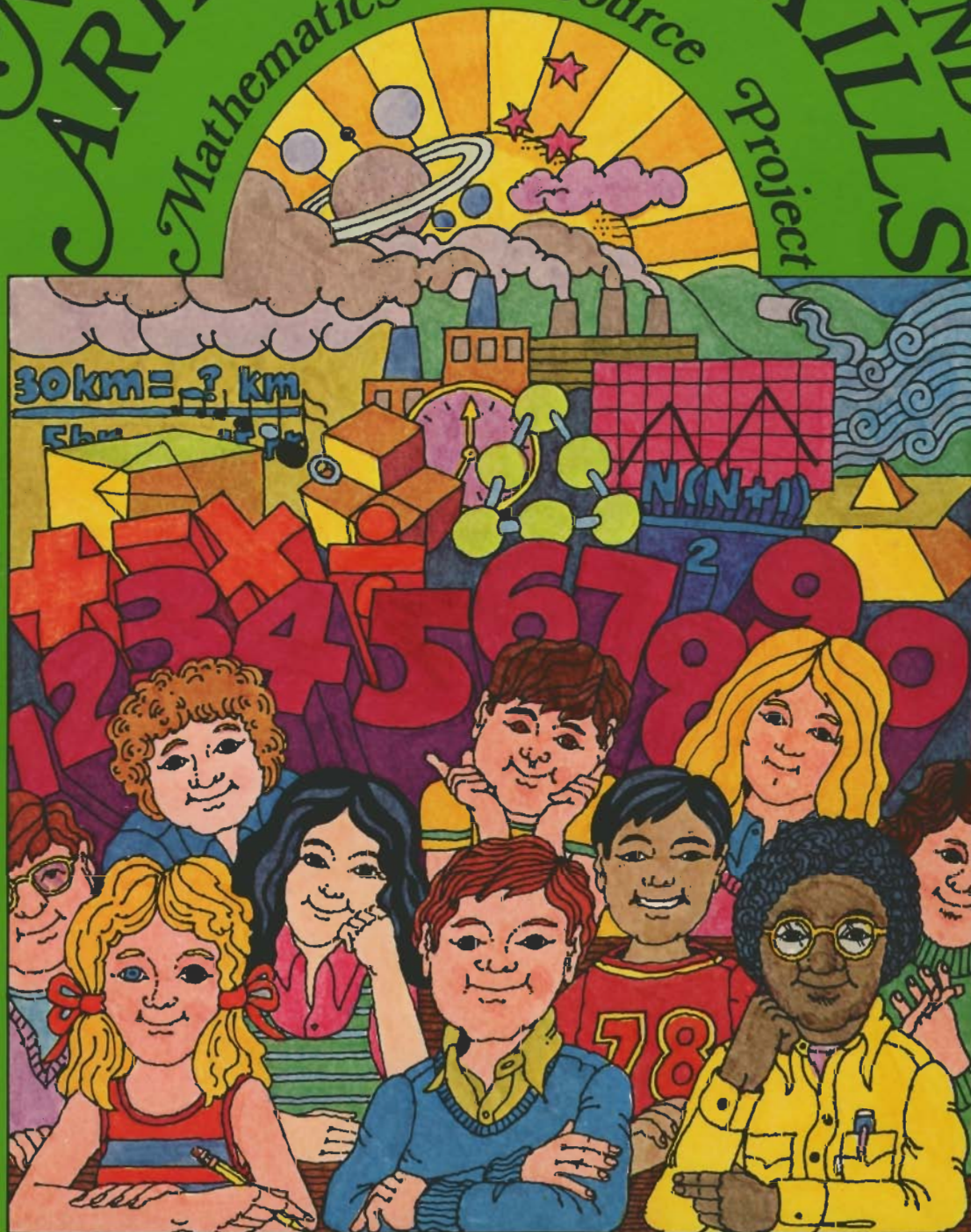


NUMBER SENSE AND ARITHMETIC SKILLS

Mathematics Resource Project



Preliminary Edition

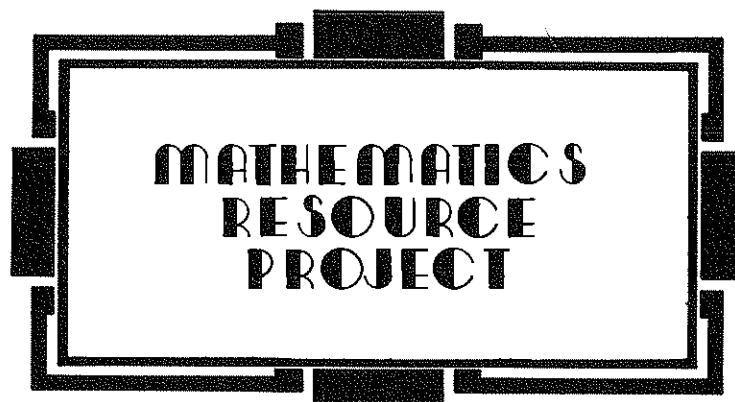
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NUMBER SENSE AND ARITHMETIC SKILLS

Placement Guide for Tabbed Dividers

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NUMBER SENSE AND ARITHMETIC SKILLS



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It is the intent of Congress that the resources developed by the Mathematics Resource Project will be made available within the school district using such material for inspection by parents or guardians of children engaged in educational programs or projects of that school district.

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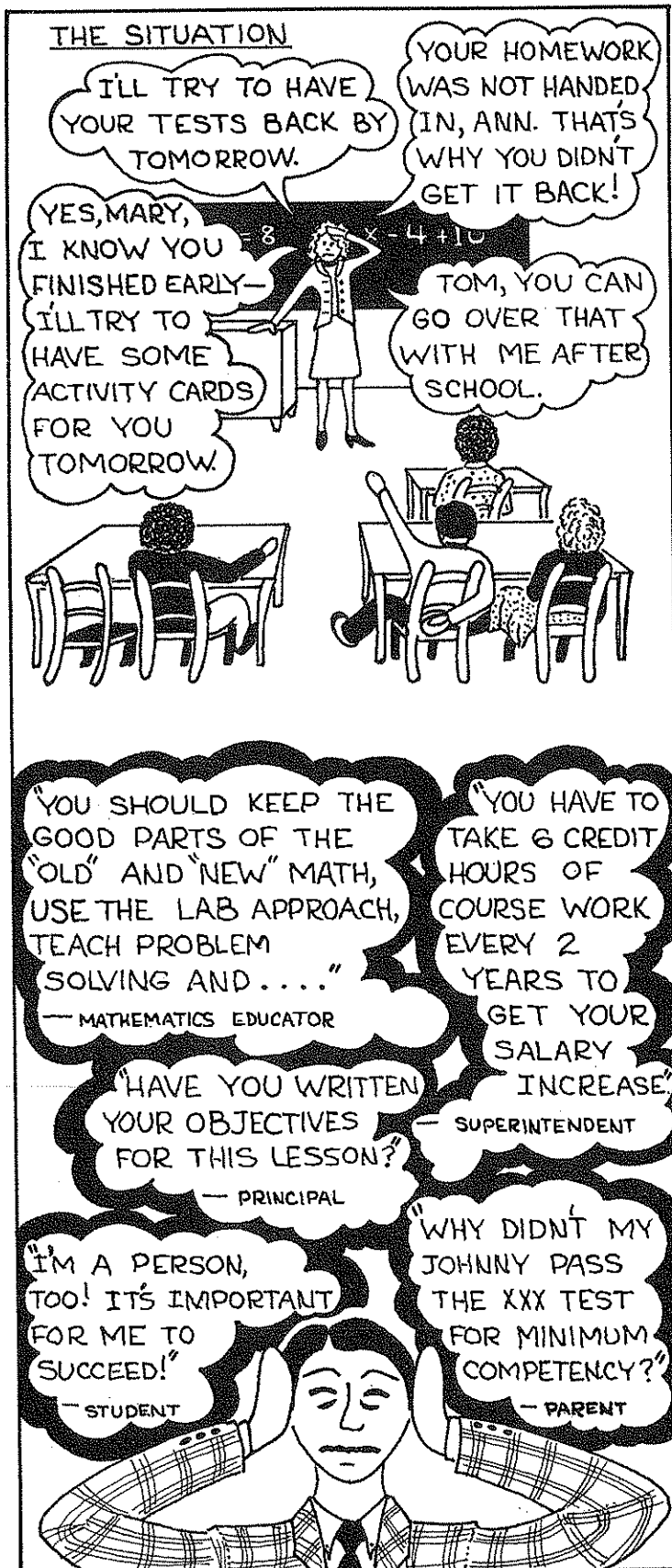
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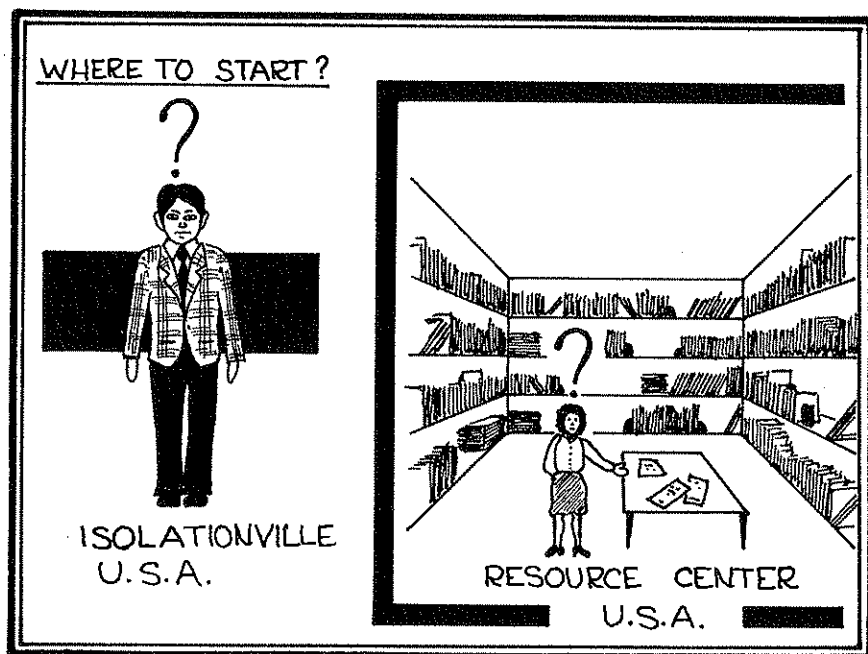
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FOREWORD



The demands on teachers are heavy. The fifth or sixth grade teacher with 25 to 30 students is often responsible for covering many subjects besides mathematics. The seventh or eighth grade teacher may be teaching only mathematics but be working with 125 to 150 students each day. Within this assignment the teacher must find time for correcting homework, writing and grading tests, discussions with individual students, parent conferences, teacher meetings and lesson preparations. In addition, the teacher may be asked to sponsor a student group, be present at athletic events or open houses, or coach an athletic team.

Demands are made on the teacher from other sources. Students, parents and educators ask that the teacher be aware of students' feelings, self-images and rights. School districts ask teachers to enlarge their backgrounds in mathematical or educational areas. The state may impose a list of student objectives and require teachers to use these to evaluate each student. There are pressures from parents for students to perform well on standardized tests. Mathematicians and mathematics educators are asking teachers to retain the good parts of modern mathematics, use the laboratory approach, teach problem solving as well as to increase their knowledge of learning theories, teaching strategies, and diagnosis and evaluation.



There is a proliferation of textbooks and supplementary material available. Much of this is related to the demands on teachers discussed above. The teacher in small outlying areas has little chance to see much of this material, while the teacher close to workshop and resource centers often finds the amount of available material unorganized and overwhelming.

The Mathematics Resource Project was conceived to help with these concerns. The goal of this project is to draw from the vast amounts of material available to produce topical resources for teachers. These resources are intended to help teachers provide a more effective learning environment for their students. From the resources, teachers can select classroom materials emphasizing interesting drill and practice, concept-building, problem solving, laboratory approach, and so forth. When completed the resources will include readings in content, learning theories, diagnosis and evaluation as well as references to other sources. A list of the resources is given below. A resource devoted to measurement and another devoted to problem solving have been proposed.

NUMBER SENSE AND ARITHMETIC SKILLS (preliminary edition, 1977)

RATIO, PROPORTION AND SCALING (preliminary edition, 1977)

GEOMETRY AND VISUALIZATION (preliminary edition, 1977)

MATHEMATICS IN SCIENCE AND SOCIETY (preliminary edition, 1977)

STATISTICS AND INFORMATION ORGANIZATION (preliminary edition, 1977)

INTRODUCTION





This is a preliminary edition of *Number Sense and Arithmetic Skills*. The resource is intended to provide teachers with ideas and materials to help them in their important work which involves the minds and personalities of their students.

WHAT IS IN THIS RESOURCE?





The resource consists of the following components:

- Didactics
- Teaching Emphases
- Laboratory Materials
- Classroom Materials
- Teacher Commentaries
- Annotated Bibliography

The *Didactics* papers give information on:








-  Learning Theories
-  Teaching Techniques
-  Diagnosis and Evaluation
-  Goals and Objectives

The titles of the *Didactics* papers in this resource are:

-  *Student Self-Concept*
-  *The Teaching of Skills*
-  *Diagnosis and Remediation*
-  *Goals Through Games*

A list of the *Didactics* papers for all of the resources is given on page 10.

The *Teaching Emphases* section stresses important areas which may help to teach most topics. These include:

-  Calculators
-  Applications
-  Problem Solving
-  Mental Arithmetic
-  Algorithms
-  Estimation and Approximation
-  Laboratory Approaches

The *Laboratory Materials* section describes various manipulatives and includes:

- Construction information
- Readiness activities

The *Classroom Materials* section includes:

- Paper and pencil worksheets
- Transparency masters

- Laboratory cards and activities
- Games
- Teacher directed activities
- Bulletin board suggestions

The *Teacher Commentaries* which appear before the subsections of the classroom materials intend to:

- Provide new mathematical information (historical, etc.)
- Give a rationale for teaching a topic
- Suggest alternate ways to introduce or develop topics
- Suggest ways to involve students
- Highlight the classroom pages
- Give more ideas on the teaching emphases

The *Annotated Bibliography* lists the sources which were used to develop this resource. These sources contain many additional ideas which can be of help to teachers.

HOW ARE THE IDEAS RELATED?

The classroom materials are keyed to the teaching emphases, to each other within the section and to the commentaries with symbols and teacher talk as shown on the next page.

The commentaries refer to specific classroom page titles, and often a classroom page is shown reduced in size next to the discussion of the page. The commentaries relate the various teaching emphases to the mathematical topic of that subsection.

Each teaching emphasis includes a rationale, highlights from the classroom materials and a complete list of classroom pages related to that emphasis.

HOW CAN THE RESOURCE BE USED?

Each teacher will decide which material is appropriate for his/her students. One cannot emphasize strongly enough the importance of the teacher's role in making these decisions. A teacher might use a few of the pencil and paper worksheets to supplement the textbook, use the laboratory activities to give more "hands-on" experience, or organize a unit around a teaching emphasis. Thus, the resource can serve as a springboard to develop a more flexible mathematics curriculum. More importantly, the teacher can supplement the resource with his/her own ideas to build a dynamic instructional program.

FEATURES OF CLASSROOM PAGES

When a ditto master is made using the thermofax process, the material in blue will not reproduce. Thus, the student's copy will contain only the material printed in black. The corners are designed to describe the content on each page.

The symbols below identify the teaching emphases in this resource. Each of these is discussed in the section *Teaching Emphases*.

These are the topics of the page. The sub-section and section headings are useful for locating and refiling pages.



Enrichment (investigations or extensions)



Skill-building (drill and practice)



Introduction (concepts and meanings)



Calculators



Applications



Problem Solving



Mental Arithmetic



Algorithms

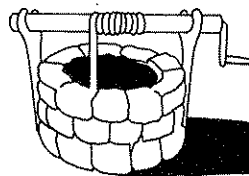


Estimation and Approximation



Laboratory Approaches

OMAR'S DILEMMA



FATIMA, OMAR'S WIFE, SENT HIM TO THE WELL TO GET EXACTLY ONE LITRE OF WATER. HOWEVER, HE HAD ONLY A 5-LITRE JUG AND A 2-LITRE JUG. CAN YOU HELP OMAR FIGURE OUT HOW TO GET EXACTLY 1 LITRE?

WHICH OF THE FOLLOWING AMOUNTS OF WATER CAN HE CARRY HOME USING ONLY HIS 5-LITRE AND 2-LITRE JUGS?

1 l, 2 l, 3 l, 4 l, 5 l, 6 l, 7 l, 8 l

WHAT AMOUNTS OF WATER CAN BE OBTAINED USING ONLY 3 l, 5 l, AND 11 l JUGS?

SEE IF YOU CAN FIND THREE JUGS THAT WILL MEASURE AMOUNTS FROM 1 LITRE TO 20 LITRES USING NO OTHER CONTAINERS!

COULD YOU HAVE USED ONLY TWO JUGS?

Any other blue material on the page is *teacher talk or answers*.

If a page is referred to by a didactics paper, one of these symbols is used.

Learning Theories

Teaching Techniques

Diagnosis and Evaluation

Goals and Objectives

Here is the *type* of activity. This refers to the suggested use of the page.

Credit is given here to the source if the page is a direct copy. *Ideas* from other sources are also noted.

LIST OF PAPERS ON THE LEARNING THEORY AND THE PLEASURABLE PRACTICE OF TEACHING

NUMBER SENSE AND ARITHMETIC SKILLS



Student Self-Concept



The Teaching of Skills



Diagnosis and Remediation



Goals through Games

RATIO, PROPORTION AND SCALING



Piaget and Proportions



Reading in Mathematics



Broad Goals and Daily Objectives



Evaluation and Instruction

GEOMETRY AND VISUALIZATION



Planning Instruction in Geometry



The Teaching of Concepts



Goals through Discovery Lessons



Questioning



Teacher Self-Evaluation

MATHEMATICS IN SCIENCE AND SOCIETY



Teaching for Transfer



Teaching via Problem Solving



Teaching via Lab Approaches



Middle School Students

STATISTICS AND INFORMATION ORGANIZATION

Components of Instruction--an Overview



Classroom Management



Statistics and Probability Learning

NOTE: A complete collection of all the papers from each resource is available as a separate publication.

GENERAL CONTENTS

DIDACTICS



Student Self-Concept



The Teaching of Skills



Diagnosis and Remediation



Goals through Games

TEACHING EMPHASES



Calculators



Applications



Problem Solving



Mental Arithmetic



Algorithms



Estimation and Approximation



Laboratory Approaches

LABORATORY MATERIALS

CLASSROOM MATERIALS

WHOLE NUMBERS

Getting Started

Numeration Systems

Concepts

Addition/Subtraction

Multiplication/Division

Mixed Operations

FRACTIONS

Concepts

Addition/Subtraction

Multiplication/Division

Mixed Operations

DECIMALS

Concepts

Addition/Subtraction

Multiplication/Division

Mixed Operations

LARGE AND SMALL NUMBERS

Awareness

Exponential Notation

ANNOTATED BIBLIOGRAPHY

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STUDENT SELF-CONCEPT

Many teaching problems will be solved in the next few decades. There will be new learning environments and new means of instruction. One function, however, will always remain with the teacher: to create the emotional climate for learning. No machine, sophisticated as it may be, can do this job. [Ginott, 1972, p. 16]

As most teachers know, many students form a negative image of school and of themselves as they move up through the grades. Although a poor self-image may be fostered elsewhere, it can often grow out of school experiences. One large-scale study found a gradual decline in self-regard in students from grade 3 to 11. [Morse, 1964] While 84 percent of the third graders were proud of their work in school, only 53 percent of eleventh graders felt the same way. A person's self-image is a very stable and conservative system of beliefs. For example, when it is negative it will tend to stay negative and it is not easy to change it positively. We have all observed students who when praised for some action react with the attitude that "It was just luck and probably will not happen again."

I asked my students to complete this sentence: "I once had a teacher who . . . " with something positive, something that had made a difference to them. They wrote:

. . . "if the subject didn't fit me he didn't make me feel dumb but looked at me more like a human person than a mark."

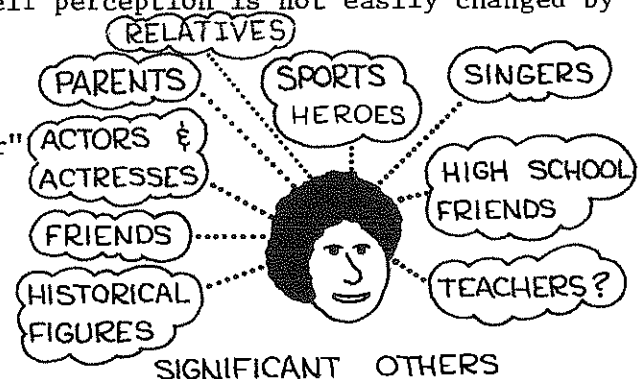
"didn't talk only from the mouth out but from insides."

. . . "when I was young in elementary he loaned me his scarf when it was snowing. It was 100% cashmere."

"this crazy teacher thought I was smarter than I was so I was."

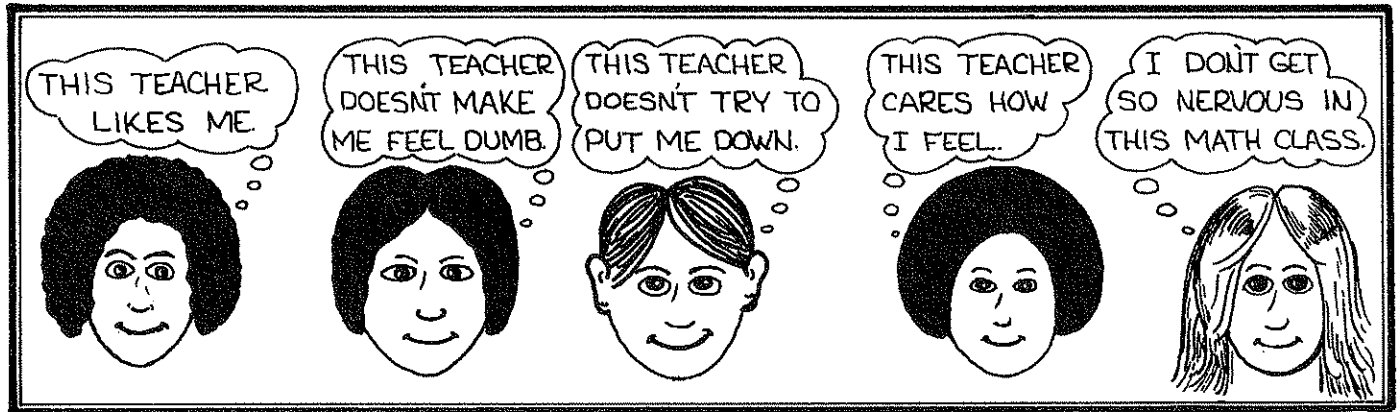
[Kaufman, 1975, pp. 22-23]

Social psychologists point out that our self perception is not easily changed by people we see as unimportant or insignificant. In order for the teacher to influence students positively, he must become a "significant other" in their lives. [Purkey, 1970] Purkey also claims that the teacher becomes a significant other in his students' lives by what he believes (about himself and his students) and what he does (in the classroom). There seems to be a significant interrelationship between self-concept and school achievement. What are some things that a middle school teacher can do to enhance and promote positive student self-concepts?



*This section draws heavily from Chapter 4 in Purkey [1970], which is highly recommended reading.

NOTE: Subsections cited are in the WHOLE NUMBERS section of the resource Number Sense and Arithmetic Skills. 11



ENHANCING AND PROMOTING POSITIVE SELF-CONCEPTS

It seems rather obvious that what the teacher does in the classroom has a profound effect on a student's learning and achievement. It may not seem quite so obvious that the teacher's classroom behavior has an impact on individual self-concepts. Since our students may perceive our actions differently from how we think we are projecting them, it is not always clear just what kind of an image we as teachers present to our students. Purkey* emphasizes the following elements in establishing a classroom atmosphere in which positive self-concepts can develop and thrive. On some days and with some students, the elements may seem to be unrealistically rosy guides, but they do give us an ideal to work toward.

Respecting Students

Each student deserves to be treated like a fellow human being—even the ones about whom we may have occasional doubts! Learning students' names as quickly as possible can help both you and your students relate better as people. Giving each student in class some attention as often as possible reminds him that you are interested in him. Greeting students in hallways and out of school can promote their positive feelings about themselves and about you. This feeling can lead to more trust, better interactions and thus more learning in the classroom. Some teachers make a point of knowing a few of their students' interests outside of the mathematics class. Shooting a few baskets with a student or commenting on a hand-embroidered blouse can put a bright spot in a student's day. Apologizing personally to a student who has been wronged or inadvertently belittled or embarrassed may save a relationship.

There is research evidence that a student's self-concept and his perception of his teacher's feelings toward him are positively related: high self-concepts and



belief that teacher regard is favorable go hand-in-hand. [Davidson and Lang, 1960] This result does not mean that high teacher regard causes high self-concept but it certainly can't hurt.

Being Warm and Supportive

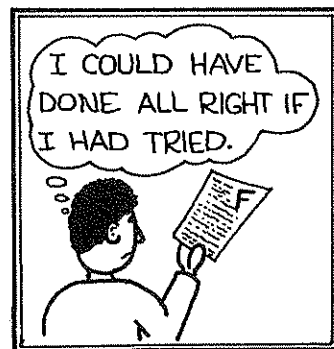
Although we can't get up on the right side of the bed every day, students seem to blossom under considerate treatment from cheerful teachers. Considerate, understanding, friendly, good-humored, courteous, enthusiastic, patient, accepting (within limits), genuine--we would all like to feel that these are adjectives for us. Being human, however, we may sometimes deserve some subset of these: dominating, grim, bossy, sarcastic, threatening, arbitrary, unfair, grouchy. Confessing to being subject to human frailties is all right, so long as it is not used as an excuse for every mistake. One teacher found that she had better results on her "bad" days if she announced at the beginning of class something like, "Last night my son kept me up until two o'clock with his cold--I'm tired and worried, so if I seem upset today, it is not your fault." Surprisingly she found her students were better behaved on those days and that she was less likely to explode from trying to hide all her inner feelings. She did not use the announcement as an excuse to be grouchy but rather as an honest admission of inner feelings which might affect her behavior.

Providing (Fair) Challenges

Demanding the impossible isn't going to bolster anyone's self-concept, of course, but presenting significant challenges with the obvious expectation of success ("This is a tough one, but I'll bet you can figure it out") should do something for one's ego. Purkey warns that such challenges should wait until the chances of success are good. Using pairs or "teams" of students on an activity like *Squaresville Suburbs* in the Getting Started subsection could enable some students to share in successfully meeting a challenge.

Allowing Freedom of Choice and Maximizing Freedom from Threat

Giving a student some freedom to choose ("You can do whichever one looks best for you, *Attic Greek* or *Mayan Numeration System*") shows that you apparently have faith in his judgment. The usual "threat" in the classroom is that of failing. You may have encountered students who refuse to try; they avoid the risk of failing a task (and losing self-esteem) by not undertaking the task. At a less extreme but no less serious level are those students who become so anxious about failing a task that the anxiety hinders their performance--and perhaps does lead to failure. So, there should be value in working to convince students that they can succeed.





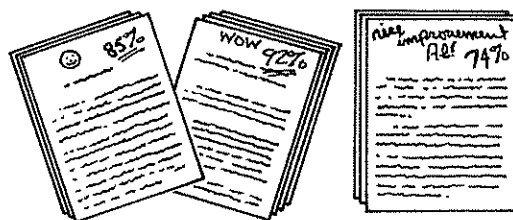
Providing Success Experiences

"People learn that they are able, not from failure, but from success." [Purkey, 1970, p. 56] Unfortunately, our students with the lowest self-concepts often have the least evidence that they are able, since they may come to our classes with much more than the normal amount of failure. Such students must experience some success as soon as possible to establish a "Hey, maybe this year will be different" outlook and to bolster their self-concept. It may help to point out areas of accomplishment ("You really have the times-5 facts down cold"). "Can't fail" activities like *Numbers Around Us* (in the Getting Started subsection) give a student a chance to contribute. Self-checking activities like *Facts in Squares* or *Jigsaw Puzzles* (both in the Multiplication/Division subsection) or *Rounded Line Up* (in the Mixed Operations subsection) give a student a chance to hand in something perfect for a change. Concrete materials (e.g., *Activity Cards--Mathematical Balances*, *Activity Cards--Chip Abacus* in the Addition/Subtraction subsection) often give success (and understanding). You may decide to let some students temporarily bypass obstacles such as lack of mastery of the multiplication facts by allowing use of multiplication tables or a calculator so that "more grown-up" problems, as in *AtTEnSion* (in the Multiplication/Division subsection) or *Curiosities* (in Mixed Operations), can be tackled. Some teachers start assignments with easy problems that everyone can do.

Celebrating the small victories may help: on quizzes we might mark the rights, and not the wrongs (let's hope that marking the rights is more work than marking the wrongs!); some teachers write supportive comments at the top of student papers; you may try to use as much positive verbal feedback as you can. In general, praise is more productive than blame.

Social psychologists point out one

danger about praise: it may backfire if it is not genuine and seems to be offered to a student just to manipulate him. In some classes, praise of individuals might better be kept private. And praise might also be given privately to students who are uneasy at being praised.



It is important that the successes be genuine. Some research has indicated that while assurances and praise may boost self-concept, they should not be given for low-quality work. [Massey, Scott & Dornbusch, 1975] To praise work which is actually poor can mislead the student into thinking that his work is satisfactory. Praising effort, however, may lead to greater effort.



Maintaining Classroom Control

"It is yet another way of telling the student that the teacher cares about him and what he does." [Purkey, 1970, p. 54] Consistent, courteous but firm handling of "incidents"; obvious and careful preparation for class (including a repertoire of 5-minute "fillers"); keeping up with grading and promises; clear explanations of why certain guidelines or policies must be observed--all these say, "You are important."

WATCH OUT, SELF-CONCEPT!

Because of the difficulty of changing students' self-concepts, it is very important to avoid instilling low self-concepts in students. Doing the opposite of all the suggestions above would certainly tear down students' self-concepts. There are two areas of classroom behavior that deserve special note: teacher-student interaction patterns and the self-fulfilling prophecy phenomenon. Both of these can very easily result in unintentional behavior which tells the student, "You don't have much to offer." *Fortunately, just being aware of them seems to enable teachers to avoid them.*

Interaction Patterns

If we let them, only a few students can dominate all class discussions with the result that some students never contribute. An ideal is to have every student contribute something to the class every day, even if it turns out to be passing out papers or erasing the board. Some teachers run a spot check every so often by going over a class list or seating chart and noting whether every student (ideally) was somehow involved during the last class or two. If a teacher can't manage to get to everyone, he might wish to plan lab lessons or small group work so that there is more student interaction during class.

The Self-fulfilling Prophecy

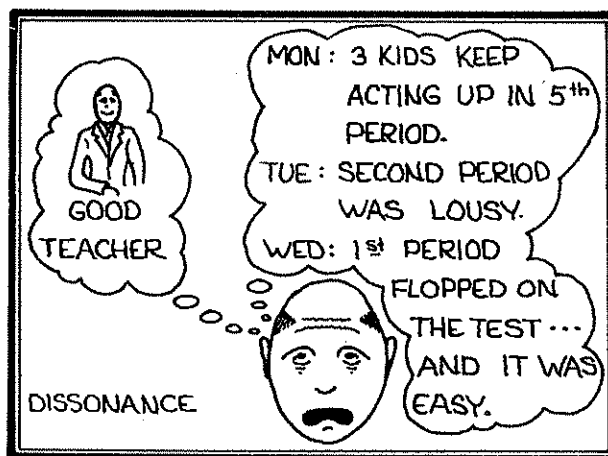
"You get what you expect" is a short, over-simplified summary of some striking findings (as in Rosenthal and Jacobson, 1968, or Brophy and Good, 1974). The work indicates that a teacher's expectations about a student's potential performance may unconsciously affect how the teacher interacts with the student, both in quantity and in "quality" of interaction. For example, in one study students for whom the teachers had high expectations "were asked more questions, received more extended teacher feedback, and received proportionally more praise and less criticism." [Jeter, 1975, p. 163] Low achievers, on the other hand, received much less teacher contact and when teachers did interact with them, the contact was briefer and less



encouraging. The net effect may be that the students begin to respond in ways which confirm the teacher's expectations. Some researchers believe that many students fail to reach their potential because their teachers, not expecting much from them, are satisfied with less than average performance from them. [Jeter, 1975, p. 163]

YOUR SELF-CONCEPT

It is very easy to pledge allegiance to the Golden Rule. It's another matter to act accordingly when three of your students seem to be competing for the Nobel Prize for disrupting class. Unless you are very talented or have only "good" classes, there will be days when your self-concept as a teacher doesn't quite fit what went on in school--you'll experience a form of "dissonance" (see Aronson, 1972).



There are three ways to handle this dissonance. One way would be to change your self-concept. Another way is to convince yourself that perhaps the "bad" days weren't really so bad (which is likely the case--don't forget all the good things that happened). Perhaps the best thing to do is to keep track of the causes (control problems? didn't get across? poor explanation? lost temper? ...) and note whether one sort pops up quite often. Then you have something specific to attack (with the result, it is hoped, being a success experience!). In the same way, identifying your strengths may enable you to build on them. Relax, expect to make a few mistakes--and resolve to learn from them.

Fortunately, most teachers are sensitive to the needs and feelings of their students. All teachers should be concerned about achievement, but we should also pay close attention to student attitudes and personal problems. In fact we must recognize that in the in loco parentis sense we have some general responsibility for the lives of our students. Let us hope, for example, that our students will go on to write poems, but poems that do not reflect feelings like those expressed in this verse written by one young person:

*I am neither a sacrilege or a privilege
I may not be competent or excellent
But I am present.
My happiness is me, not you,
Not only because you may be temporary, but also because you want
me to be what I am not.*



*I cannot be happy when I change merely to satisfy your selfishness,
Nor can I feel content when you criticize me for not
 thinking your thoughts,
Or for seeing like you do.
You call me a rebel.
And yet each time I have rejected your beliefs, you have
 rebelled against mine.*

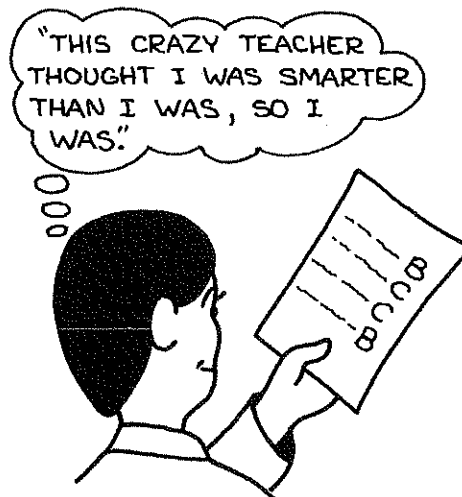
*I do not try to mould your mind.
I know you are trying hard enough to be just you.
And I cannot allow you to tell me what to be, for I am
 concentrating on being me.
You said that I was transparent and easily forgotten.
But, why then did you try to use my lifetime to prove to
 yourself who you are?*

--Michelle

SUMMARY

We teachers should be watchful for middle schoolers who are developing negative self-concepts. We should help all students to develop healthier self-images by communicating our own regard for them and by giving them evidence that they are indeed worthy of regard and trust.

- Show them respect and warmth.
- Maintain class control to provide a comfortable setting in which to work.
- Provide a challenge in the class work.
- Allow the students some freedom in choices when possible.
- Especially, provide opportunities for success experiences.



?????

1. Purkey [1970, ch. 4] lists several questions we can ask ourselves as a check on the classroom atmosphere we may be developing. The questions are slanted toward Purkey's ideal; even a "teacher of the century" would be hard-put to answer "yes" to all these questions all the time for all the students.
 - a. Am I projecting an image that tells the student I am here to build, rather than to destroy, him as a person?
 - b. Do I let the student know that I am aware of and interested in him as a unique person?
 - c. Do I provide well-defined standards of values, demands for competence, and guidance toward solutions to problems?
 - d. By my behavior, do I serve as a model of authenticity for the student (i.e., . . . serve as a model of genuineness, without "front")?



- e. Do I teach in as exciting and interesting a manner as possible?
 - f. Do I arrange some time when I can talk quietly alone with students?
 - g. Do I notice and comment favorably on the things that are important to students?
 - h. Do I remember to see small disciplinary problems as understandable, and not as personal insults?
 - i. Do I have, and do my students have, a clear idea of what is and what is not acceptable in my class?
 - j. Do I permit my students some opportunity to make mistakes without penalty?
 - k. Do I give extra support and encouragement to slower students?
 - l. Do I recognize the successes of students in terms of what they did earlier?
2. (Discussion) Can a teacher honestly believe that every student can succeed?
 3. Folk wisdom: Nothing succeeds like success. You might wish to pick out your two lowest-achieving students and try to think of three honest tasks at which each could succeed.
 4. (Discussion) What influence might grades have on self-concepts? Are there viable alternatives to grades?
 5. Starting the school year with success experiences for students with low self-esteem may seem a good idea--but the beginning of the year is when we know the least about our students. Evaluate this list of sources of information, and add to it if you can:
 - a. teachers in the previous grades, even if they're in other schools
 - b. permanent records
 - c. counselors
 - d. articulation conferences with feeder schools
 - e. the first day, play *Glances and Blows* in the Concepts subsection (perhaps first the whole class, then in pairs), or *Digit Ideas* or *Fresno* in Mixed Operations, and watch the students
 - f. assignments where there are no right or wrong answers--e.g., *Numbers Around Us* (in Getting Started)
 - g. personal information sheets filled out by students the first day and including open-response items like these:

My teachers think I am _____.

When I look at other students and then look at myself, I _____.

When I think about myself, I _____.
 6. Most of us have to work at learning students' names. Here are some procedures. Can you give another idea?
 - a. "I seat them alphabetically until I've learned their names."
 - b. "I meet students at the door and greet them, asking for their names as they enter."
 - c. "I tell them to stop me in the hall and ask me what their names are."
 - d. "I take Polaroid pictures of sections of the class the first day, note the names, and study them." (Other teachers ask students to bring a small picture from home.)
 - e. "I play some drill games early in the year--computation drill for the kids and name drill for me. When I go down a row and can't give a student's name, it's his turn."



- f. "I make a seating chart and study it during seatwork. Once you learn a few names, it gets faster."
 - g. "I play the Name Game--the first student gives his name, the next student gives his name and the first student's, the third student gives all three names, and so on. It helps the students coming in from the different schools as well as me."
7. Evaluate these classroom practices with respect to their effects on student self-concept.
 - a. Reading student test scores aloud as you pass the tests back.
 - b. Reporting only the range and mean of test scores.
 - c. Having students check homework or quizzes and report their scores aloud.
 - d. Posting the best papers on the bulletin board. Posting the worst papers.
 8. Thinking of something positive to say to a low self-concept student who gives an incorrect response isn't easy. Evaluate this list and add to it:
 - a. "Right idea, but something isn't quite perfect somewhere."
 - b. "How about that answer?" (said to invite reaction from the class)
 - c. "Close, but not quite."
 - d. "You've got the big things, but you made a little error someplace."
 - e. "Check over that step."
 - f. "That's interesting--why do you think that?"
 - g. "Good as far as you've gone."
 9. Give examples of times, if any, when each of the following might be (i) productive and (ii) counter-productive.
 - a. Reprimand a student in front of the whole class.
 - b. Reprimand a student in private.
 - c. Compliment a student in front of the whole class.
 - d. Compliment a student in private.
 10. (Discussion) How can you lessen a student's anxiety when you're convinced it is hindering the student's performance? Evaluate the following, and give additional ideas.
 - a. Talk with the student about it. Be sympathetic.
 - b. Offer to give an alternative form of examination--e.g., an oral exam--for a make-up if the student does poorly.
 - c. Point out that a test is only a bunch of homework problems. If homework has been all right, the test should present no difficulties. (See number 16.)
 - d. Advise the student to stop and take a deep breath if he feels like he's getting too nervous.
 11. Having a readily-available collection of 5-minute fillers can be a life-saver for those restless moments before the dismissal bell (or for openers to get everyone's attention). You may want to flag the student pages (e.g., *ESP Puzzle*, *Game of 50* in the Getting Started subsection) which can be fit into a few minutes. Find several more ideas in sources like Sobel and Maletsky [1975] and record them on 3x5 cards.
 12. Folk wisdom: An ounce (30 grams?) of prevention is worth a pound (half-kilogram?) of cure. Evaluate these ways of inhibiting classroom misbehavior; add to the list.
 - a. Using an overhead so you're facing the class.
 - b. Developing positive relationships with students outside of class.



- c. Being interested in students.
 - d. Giving a stern glance.
 - e. Giving a thumbs-down signal.
 - f. Moving close to a point of possible trouble.
 - g. Approaching a student and privately requesting that he see you after class (or school).
 - h. Publicly requesting that a student see you after class (or school).
 - i. Smiling and shaking your head.
 - j. Frowning and shaking your head.
 - k. Always having an alternate activity to change to.
13. (Discussion) "The students just don't seem interested." Count yourself lucky if you've never thought that. Interest and enthusiasm can, however, be contagious. Teacher educators have noticed that one striking difference between experienced teachers and student teachers is this: experienced teachers seem more enthusiastic in the classroom and seem genuinely interested in the material. Their approaches include the ones below; can you add to the list?
- a. Introducing a new topic as an exciting challenge (even decimal division!).
 - b. Hamming it up occasionally.
 - c. Finding interesting contexts in which to present topics.
 - d. Getting excited about student performance.
14. a. Listen to an audiotape of one of your classes and study the interaction patterns. Are there neglected areas of the room? Do two or three students dominate things?
- b. In particular, are your responses to slower and faster students similar? Check for these variables that communicate low expectations [Brophy and Good, 1974, pp. 330-332].
1. Waiting less time for low achievers to answer (for questions of the same difficulty as the high achiever gets).
 2. Not giving any help or hints to a low achiever in a failure situation.
 3. Rewarding inappropriate behavior of a low achiever (e.g., praising a poor/marginal answer).
 4. Criticizing low achievers more frequently than high achievers.
 5. Praising low achievers less frequently than high achievers.
 6. Not giving feedback to classroom responses of low achievers.
 7. Paying less attention to low achievers (e.g., eye contact, smiles, casual talk).
 8. Calling on low achievers less often.
 9. Differing quality of interaction for low achievers and high achievers.
 10. Seating low achievers farther from teacher (check seating chart).
 11. Demanding less from low achievers.
- c. We often use terms like low achiever, weaker student, poorer students, and others not to be written here. Why might that be risky?
15. Observe a class of a popular colleague to take notes on how she/he seems to affect self-concepts.
16. Some statements, intended to be supportive or encouraging, may not get the desired result. Tell why the following might back-fire.
- a. "You could do it if you tried."
 - b. "You can do this problem. It's not hard at all."
 - c. "Don't be nervous. The test is easy."



17. (Discussion) Adolescents, particularly those with a low self-concept, often look to peer leaders to guide their behavior, that is, they may adopt ideas because they are the ideas of esteemed classmates. What are some ways to identify and "win over" these leaders?
18. Social psychologists have some evidence that people may develop great liking for those who have dissimilar attitudes but who like them. [Aronson, 1972, p. 222] Assuming that being "liked" can be a part of being a "significant other," decide whether each of the following might be worth the effort.
 - a. You attend an athletic event---even though the students know you don't like sports.
 - b. You inquire about popular singers or attend a sock-hop---even though the students know you don't particularly like that kind of music.
19. If someone does a favor for you, and the favor has a successful outcome, then that person may like you better. [Aronson, 1972, pp. 211-212]
 - a. You might consider planning some gambit to get a key semi-hostile student to do a favor for you (plan some way to be sure that after the deed, he knows you regard it as a favor).
 - b. Mr. Salome: "I object to this part (a) idea on a matter of principle. It is not professional to plan special treatments for some kids just to make it easier on yourself."
Your reaction to Mr. Salome's position?
 - c. Ms. Teek: "I agree. All this manipulation type of thing reminds me of Brave New World and 1984. I must deal with each of my students as a fellow human, not as an antagonist to be outwitted."
You:
20. Recall that dissonance exists when two conflicting views, opinions, mental states, etc., occur at the same time. Explain the "won't-try" student from the dissonance viewpoint. ("I want to feel like I'm smart." "Failing makes me feel dumb.")
21. Page [1958] found that writing brief comments at the top of student tests had a positive effect on subsequent performance (why?). Although not all teachers have found the same effect, it nonetheless seems to be a good idea. What are some supportive comments you could write
 - a. for a weak student who is showing improvement?
 - b. for a weak student who is continuing to do poorly?
 - c. for a strong student who slips?
22. (Discussion) "Funny about teachers. It's hard to think of them as people. Sometimes when you see them after school they talk like human beings. But in class they walk around like robots, waiting for you to do something bad so they can yell at you. Except for Mrs. D. . . . She is a person even when she is a teacher." [Ginott, 1972, p. 119] Are teachers different in the classroom from their out-of-classroom selves? Are you?
23. The warning was given above that insincere praise may not get the desired result. It is also true that too lavish or unwarranted praise may lead to being not liked too much. [Aronson, 1972, p. 208] What reactions might be produced in these situations? How would you follow Bill's statement and Donna's help?
 - a. Bright Bill: ". . . and then two-eighths and three-eighths give five-eighths."
Teacher: "Great! That's really outstanding!"

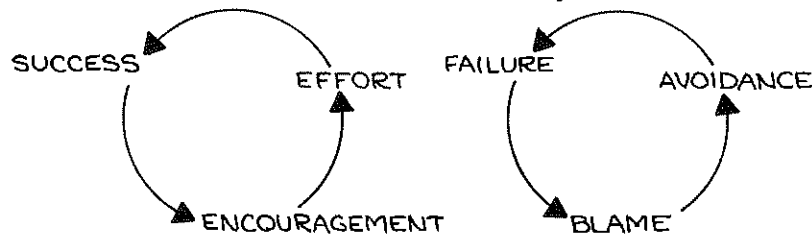


- b. Teacher: "Donna, I really appreciate your plugging in the overhead and making a big contribution to the class by doing it."

24. "Satisfaction with one's teacher is an important facilitative condition for a student's academic performance. Students are attracted to teachers who provide them with a boost of status in the peer group and who grant them security. Teachers who reward frequently and who do not rebuke or demean students in the eyes of their peers are attractive The continual rejection of an overtly aggressive student by both classroom peers and the teacher feeds the negative cycle of low self-esteem, unfriendly overtures to others, and poor performance in academic work." [Schmuck and Schmuck, 1975, p. 106]

What are some ways a teacher can

- boost a student's status in the peer group?
 - grant students security against loss of face in the classroom?
 - find something favorable about a "trying" student?
 - involve "isolated" students with the peer group, if they seem to wish to be?
25. What is the teacher's role in these two cycles?



References and Further Readings

Aronson, Elliot. The Social Animal. San Francisco: W. H. Freeman, 1972.

Chapters 1, 2, 4, 5 and 7 are easy to apply to classroom settings. The book does not require background in social psychology.

Brophy, Jere and Good, Thomas. Teacher-student Relationships: Causes and Consequences. New York: Holt, Rinehart and Winston, 1974.

Canfield, Jack and Wells, Harold. 100 Ways to Enhance Self-Concept in the Classroom. Englewood Cliffs, New Jersey: Prentice-Hall, 1976.

This book offers an excellent collection of activities for fostering self-esteem. The ideas could be used in the homeroom and, as they fit, in the mathematics classroom.

Curwin, Richard and Fuhrmann, Barbara. Discovering Your Teaching Self. Englewood Cliffs, New Jersey: Prentice-Hall, 1975.

Subtitled "Humanistic Approaches to Effective Teaching," this book gives several activities you might consider for improving your teacher-student relationships.

Davidson, Helen and Lang, Gerhard. "Children's perceptions of their teachers' feelings toward them related to self-perception, school achievement, and behavior." Journal of Experimental Education, Vol. 29 (December, 1960), pp. 107-118.



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This book is not technical and includes several suggestions on how best to interact with students, especially when the student is angry or in need of help.

Jeter, Jan. "Can teacher expectation function as self-fulfilling prophecies?" Contemporary Education, Vol. XLVI, No. 3 (Spring, 1975), pp. 161-165.

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This is a good book for your "must read" list.

Rosenthal, Robert and Jacobson, Lenore. Pygmalion in the Classroom. New York: Holt, Rinehart and Winston, 1968.

The results of this widely publicized study were later questioned on statistical grounds. The Brophy and Good reference gives the recent status of the research on the influence of teacher expectations.

Schmuck, Richard and Schmuck, Patricia. Group Processes in the Classroom. 2nd ed. Dubuque, Iowa: Wm. C. Brown, 1975.

Sobel, Max and Maletsky, Evan. Teaching Mathematics: A Sourcebook of Aids, Activities, and Strategies. Englewood Cliffs, New Jersey: Prentice-Hall, 1975.

This is available in either hardback or paperback.

NUMBER SENSE AND ARITHMETIC SKILLS

Placement Guide for Tabbed Dividers

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THE TEACHING OF SKILLS

From a sample of 30,000 13-year-olds from across the country:

32% missed at least one of
$$\begin{array}{r} 38 \quad 36 \quad 38 \quad 5 \overline{)125} \\ +19 \quad -19 \quad \times 9 \end{array}$$

39% missed "If 23.8 is subtracted from 62.1, the result is . . ."

58% missed $\frac{1}{2} + \frac{1}{3}$. (30% thought $\frac{1}{2} + \frac{1}{3} = \frac{2}{5}$.) . . .

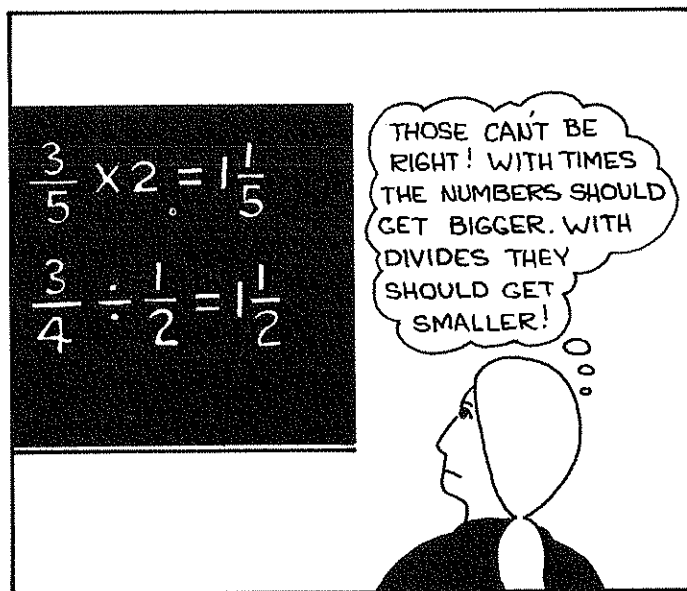
Selected results from the National
Assessment of Educational Progress [1975]

Despite our best efforts, disappointingly many students never master computational skills. If only we did not spend so much time trying to develop skill with calculation, such poor results might be of less concern in an age of the hand-held calculator. This section presents some ideas about teaching skills more effectively. The examples will be largely related to computational skills, but many of the principles may be applied to the teaching of other mathematical skills: measuring, constructing, using tables, developing vocabulary, reading, etc.

WORK FOR MEANING BEFORE TEACHING AN ALGORITHM

One thing we surely must do is make certain the students understand the concepts involved. Some middle-school students are vague about the meaning of, say, $\frac{3}{4}$ or $1\frac{2}{3}$; the vagueness increases as the ideas become more remote from everyday experience: 0.0035 or $\sqrt{8}$, for example. This problem is compounded by the fact that fractions and decimals can appear in so many different settings (see the commentary to FRACTIONS: Concepts).

Even students who have a fair grasp of the numbers involved may associate no meaning to the operation we may be discussing. For fractions and decimals this failure is more pronounced with multiplication and division than with addition and subtraction. Perhaps this is because the meanings learned for whole number multiplication and division may not carry over so naturally in the student's mind (2×3 means 2 sets of 3, but $2\frac{1}{2} \times 3\frac{1}{3}$ means . . .?) or because we



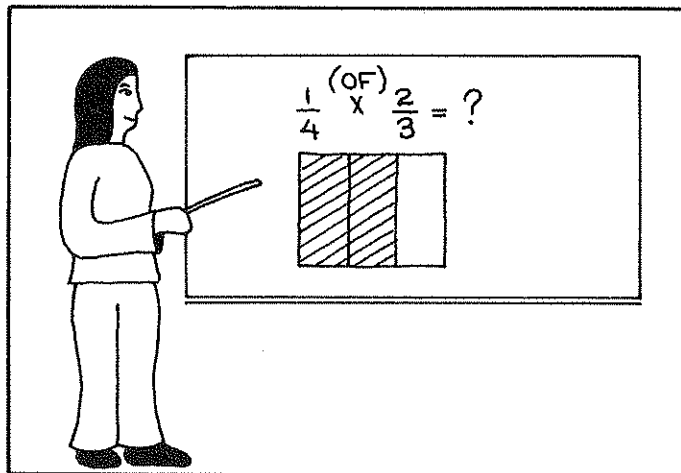
NOTE: Unless otherwise noted, subsections cited are in the FRACTIONS section of the resource Number Sense and Arithmetic Skills.



may have introduced the algorithm too soon without enough emphasis on meaning.

Several studies indicate that in general at least half the class time should be spent on developmental activities [Shipp and Deer, 1960; Shuster and Pigge, 1965; Zahn, 1966]. During these activities, meanings and understandings can be stressed through work with manipulatives, selected laboratory activities, teacher demonstrations or group discussion. For example, one way to develop and check for number and operation concepts is to use lots of concrete materials. Why not use apples, colored rods, geoboards, tangrams, circular and rectangular regions, number lines, colored cubes, . . . , to build up and diagnose our students' understanding and to support our development of algorithms? One of the most compelling arguments for student use of concrete objects is that working with concrete objects helps students to attach meanings to symbols. *In general, material that has meaning is learned faster and retained longer than material that is learned only by rote.* (Note the many developmental activities available in, for example, the FRACTIONS: Concepts subsection.)

Another value of student work with concrete materials is that we can then challenge the class to figure out how to carry out an operation with "new" numbers. Note that without concrete materials to work with, responses to such challenges are likely to be little more than symbol-based guesswork. Unfortunately, for some middle grades almost all of our "thunder" may have been stolen since the students may already have some exposure to all the algorithms. But when the topic is new, approaches like "I wonder if we could use colored rods to figure out $\frac{3}{4} \div \frac{1}{2}$?" or "Does anyone see a way to figure out $\frac{1}{4} \times \frac{2}{3}$ with rectangular regions?" may stimulate creative responses. Note the difference in outlook that such an approach can foster: "Maybe I don't have to be told how to do something; maybe I can figure it out myself."



In any case, concrete materials can provide valuable background for a student. If Ann has worked $1.63 - .2$ on an abacus, she should be less likely to ignore place values when she is doing similar problems symbolically. Concrete work provides something to "hook" the symbols to. However, transfer from concrete materials to symbols does not always take place automatically, so instruction should be planned to link the two.

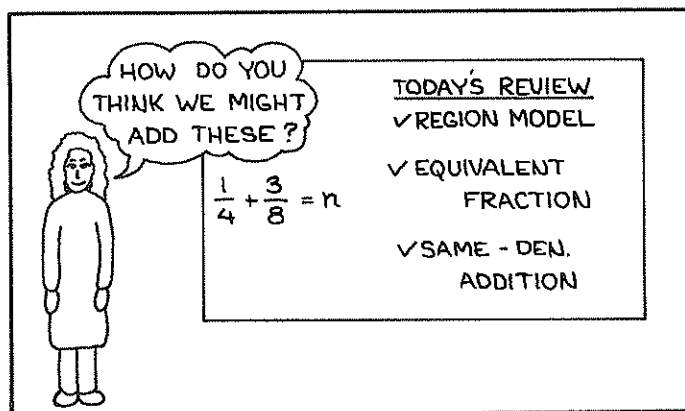


*Though I figure with the skills of men and computers and have not understanding, I am become as a mechanical toy, or a lifeless robot.
And though I have the gift of memory, and know the multiplication tables, and all the number facts, and though I know all algorithms, so that I can grind out all answers, and have not understanding, I am not free....
Understanding lasts forever, and is always with me, understanding faileth not; understanding makes no false promises, does not make me overly confident....
And now abideth right answers, rote memory, and understanding, these three; but the greatest of these is understanding.*

[Crittenden, 1975, p. 203]

CHECKING FOR OTHER PREREQUISITES

Textbook treatments usually are designed to make sure the prerequisites have been developed. Since textbook development and student grasp are not the same thing, it is a good idea to check that needed computational and vocabulary skills are still present, particularly if some time has lapsed since the student's last exposure. If you are designing your own materials, you will want to check that the prerequisites have been planned for. A student can't go very far in addition of fractions if he hasn't dealt with equivalent fractions, for example. Although they may recognize material as



having been covered, students are sometimes reluctant to admit that they have forgotten important details. Many teachers review the prerequisites for a day's lesson at the start. Using the review as a subtle means of orienting the student's train of thought, they then ask "how do you think . . .?" sorts of questions.

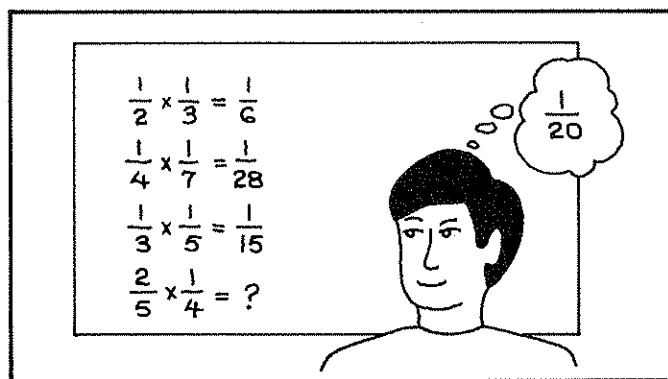
DEMONSTRATING AN ALGORITHM

Psychologists have done quite a bit of research on the development of psychomotor skills (e.g., how can a pilot be trained to handle the controls? how should typing be taught?). Some of their general findings [Biehler, 1971; Cronbach, 1963] seem applicable to the teaching of mathematical skills. Here are some considerations for planning lessons which will include demonstrating an algorithm:



a. It may help to go through the algorithm slowly before class, looking at the steps and the less familiar vocabulary from the viewpoint of a naive learner ("How did I know that 12 was the denominator that worked?" "Why didn't I just put the decimal point under the rest of the decimal points?" "In going from horizontal to vertical form, how did I know . . .?"). Such a rehearsal may remind you of critical or confusing points and enable you to plan some of your questions ahead of time.

b. Choosing the examples carefully may save confusion. Although you may choose "nasty" numbers to illustrate the need for an algorithm, in the first student exercises "nice" numbers keep the intermediate computations from taking so much attention that the student can't concentrate on the process. On the other hand, examples which are too special can be misleading. (To the beginning teacher: This point is more important than it may seem. It is a good idea to choose and work through your examples before class rather than rely on your ability to come up with them spontaneously. Who wants to find out too late that the example is too complicated to illustrate concretely, that a zero in a "bad" place detracts from the thrust of the lesson, or that the examples are giving an incorrect idea?)



c. Psychologists advise demonstrating the whole process completely so students have the total picture. Of course, you may want to base the development on earlier work with concrete materials.

d. Then describe the steps, emphasizing the essential ones and the ones likely to cause trouble. You may wish to emphasize some steps by giving practice on them in isolation (see, for examples, *Dotman's Decimal Notes* and *Lots of Dots Shortcut* in the DECIMALS: Multiplication/Division subsection).

e. You may need to demonstrate again (and again and again . . .). Many teachers ask for student help at various steps because of the danger of wandering attention. If you are experienced, you know the risks associated with your talking too long.

f. Get the students actively involved as soon as possible. Have them doing something mentally or physically. Some teachers might ask their students to copy the second problem and to try to anticipate the next step. Other teachers might have the students vocalize the steps as they are demonstrated. In some situations, having the students parallel steps with a concrete representation might be in order.

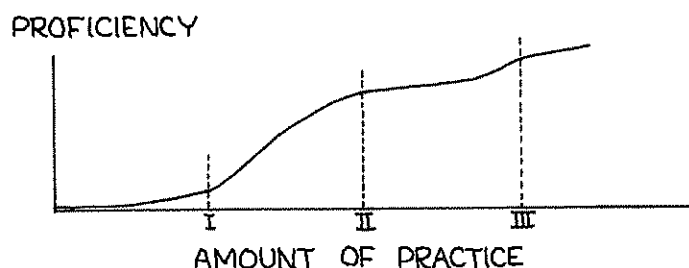


g. Try to get the students practicing immediately. Everyone trying a problem at his desk, one row working at the board with others working at their desks, each row doing a different problem--use any means to confront all the students with the computation. Make answers to the first few exercises available so students can get some quick feedback even if you are busy elsewhere or there is no time left at the end of the class for checking work.

h. Move about the room and give guidance which allows the student to make the decisions. Compare "Put the common denominator underneath" with "And what is the denominator?" Or "Here, let me show you" vs. "Where do you start?" This time is crucial for correcting wrong technique, reinforcing right technique, and getting feedback on how well you "got across." The feedback may suggest that you demonstrate another example to the whole class or to several students. Moving about the room also makes it less easy for the same student to monopolize your time, gives the shy student a chance to stop you as you go by, and gives you opportunities to catch error patterns as they are developing.

Learning Curves

The theoretical learning curve to the right also implies some things to keep in mind. [Cronbach, 1963]
In the early work, a student may not be very successful.



Hence, we should be patiently supportive (and available) during this time to provide encouragement, assistance and reinforcement. Since this period will last longer for slower students, we will want to be especially attentive to their needs. From I to II, the student masters all the steps in the algorithm, and the computation requires less and less pondering. The long plateau between II and III may be less common with computational skills than with motor skills. However, it is plausible that a student working with multiplication of decimals, for example, might reach such a plateau and not be able to improve further until he has become more accurate with the basic multiplication facts. Your mention of such plateaus might encourage a slower student who has been working diligently but without seeming to get any better, and it might prompt him to work on the skill holding him back. It is also encouraging at the plateau stage to point out how much better a student is performing than he was at the beginning of the learning.



It is so easy to get all wrapped up in the steps of an algorithm that students may forget that the answer should make sense. It is a good idea always to ask, "Does that make sense?" as a reminder that answers should be looked at for sensibility. Work with estimation, approximation and mental arithmetic should provide a basis for judging whether an answer is sensible. For ideas, see *Can You Get Close Up* (in the Multiplication/Division subsection), *Where's Your Head At?* (in Mixed Operations), *Getting 'Round to Calculating*, and *Approx-Appraisals* (both in DECIMALS: Mixed Operations).

"Meaning" hasn't been mentioned in the last few paragraphs. Trafton and Suydam [1975] remind us: ". . . learning why a computational procedure works helps a child to retain the procedure and apply it in new situations. Many teachers know this from experience, and it has been affirmed by numerous research studies across the past 40 years." [p. 532, emphasis added]

SKILL CONSOLIDATION AND MAINTENANCE

One promise of the hand-held calculator is that students will very likely spend less time multiplying or dividing many-digit numerals by hand. On the other hand, it is difficult to conceive of the day when all calculations in middle-school mathematics classes are carried out by machine. Most students should know the basic $+$, $-$, \times , \div facts by heart and be able to carry out algorithms for these operations.

Attitudes

One of the criticisms of the drill-drill-drill days of mathematics teaching was the negative effect of such teaching on student attitude. But students do need practice to gain confidence and a mastery of the basic facts and techniques. How can we handle drill to achieve these and yet build positive attitudes?

Variety of Settings

Perhaps the drill-drill-drill treatment was deadly because of how it was presented: long lists of computation problems and long lists of word problems using a particular type of computation. Presenting the drill in a variety of contexts seems to be a profitable approach. This resource contains samples of the many clever ways in which writers have disguised drill. Most of these illustrate what Cronbach [1963] says: "The student will generally be interested in doing well when the act is necessary to a larger undertaking." [p. 285, emphasis added] For example, *Fraction*

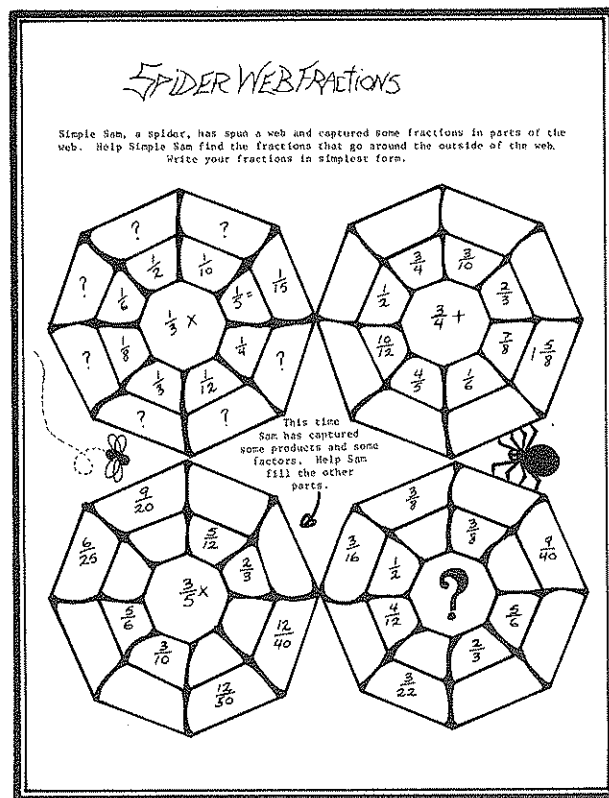


Recipes (in the Mixed Operations subsection) might be attacked with enthusiasm if the shortbread cookies are in the plan. *Weird Ruler*, *Fraction Magic Squares* and *Can You Find a Path?* (all in the Addition/Subtraction subsection) are samples in which the "larger undertakings" are to do things other than just carry out a calculation. Perhaps part of the popularity of games as a means of disguising drill lies in the game being a "larger undertaking." In *Two-A-Part* (in the Addition/Subtraction subsection) a student is trying to win a game, but in doing so he practices lots of fraction addition and subtraction. The larger undertaking for many students could be, "Beat my personal record."

Another source of variety is within the mathematics curriculum itself. Using topics like number theory, probability or matrix arithmetic, say, requires that the student practice computational skills in a larger context. Working with applications (e.g., *Picture Problems* in the Mixed Operations subsection or units from *Mathematics in Science and Society*, Mathematics Resource Project) entails calculation as a tool, not as an end in itself.

Planning Practice and Reviews

The consensus seems to be that if we are interested in long-term retention, we should spread the practice out rather than concentrate it in a shorter time span. Camp [1973] suggests, "... for some mathematics topics, distributed practice may be no more efficient for (immediate) learning than massed practice, but ... it may produce a greater resistance to forgetting." [p. 2456] Short, daily periods of mental arithmetic or paper-pencil work is likely a more effective approach than staying with an algorithm "until everyone gets it." Some teachers make frequent use of a few review problems as a warm-up while they are taking roll or passing back papers. Other teachers build a pile of



WARM-UP	NAME _____
1. $23\overline{)775}$	2. $\frac{2}{3} + \frac{1}{4} =$
3. $\frac{1}{4} = \underline{\hspace{1cm}}\%$	4. KIM PAID \$2.98,
$\frac{2}{5} = \underline{\hspace{1cm}}\%$	\$4.¢ \$1.29.
	HOW MUCH IN ALL?



flash-cards, adding items to the pile as additional topics are covered. They shuffle the cards and use them occasionally as a class opener (or closer).

If students are to work efficiently with algorithms, they must know the basic facts at a stimulus-response level. Hence, many teachers use basic-facts speed drills in which the student does not have time to count on fingers or to use other "immature" methods. The student can compete either against himself or against some standard. Games which emphasize speed could be used with opponents of equal ability (see *Goals Through Games* in the LARGE & SMALL NUMBERS section).

Exactly how to distribute the practice after the initial learning period likely depends on so many factors (topic, students, intervening events) that no one can provide a recipe detailing a universal practice schedule. With one group of eighth graders reviews scheduled one day and seven days after the initial learning gave better retention than reviews concentrated in the two days after the initial learning. [Gay, 1973] How do you schedule your reviews?

Laing and Peterson [1973; see also Peterson, 1971] suggest that daily assignments (for seat work, supervised study or homework) should build in work for "yesterday, today and tomorrow." The work for "yesterday" would involve a carefully planned review of old topics, the "today" portion would provide practice in the topic of the day, and the "tomorrow" exercises would include exploratory activities for an upcoming topic. Too often assignments are oriented only toward "today," with the result that the student may not learn to discriminate between similar-looking exercises. For example, you may have noticed how some students seem to suffer a "relapse" with adding fractions after work with multiplying fractions.

HELP FOR SLOWER STUDENTS

Few things can be more frustrating--to both student and teacher--than continued lack of mastery of basic facts or unusual difficulty in performing some algorithm. Perhaps some of the following may be helpful in responding to this problem.

Success is Important

Maximizing success requires careful attention to pacing and assignments. During the difficult early period of learning a new algorithm, slower students may need much more encouragement and reinforcement since their progress is likely to be slower. Once again, they will see that others are "better" than they are. Building in some change-of-pace activity that is likely to result in success may be a good idea during such a "rough" period. Using some self-checking activities like *Picture Fractions*



or *What's the Message?* (in the Concepts subsection) gives the student immediate feedback and the chance to hand in a perfect paper. Try to avoid excessively stressful situations; Cronbach [1963] points out that the performance of a non-expert usually deteriorates under stress. Sometimes slower students can be filled in earlier than the rest of the class on some shortcut, discovery lesson, game or manipulative. Then they can gain some self-esteem by being able to help other students when the material is introduced to the rest of the class.

Use Concrete Aids

The slower student, especially, may need the support of concrete referents. Otherwise, he must rely on his memory (which may not be too good) to keep all the symbolic processes straight. *Meaning is most crucial to our less academically-endowed students.*

Allow the Use of Tables

Anomalously, this seems to help some students finally master the basic facts. Some 7th-8th-9th-grade teachers routinely allow students to use their own copy or a posted copy of the tables. Carrying out algorithms is then not obstructed by struggles with 7×8 , and more attention can be given to the procedures. If a student has his own table, he can mark off the facts he does know (usually most of them); on seeing how few are left, he may feel encouraged enough to work on the remaining ones.

Allow the Use of Calculators

When (a) students have demonstrated some degree of competence with algorithms or (b) you are convinced that their attitude or ability will never allow them to learn how to compute, you might consider allowing them to use calculators on occasion. Perhaps learning how to use a calculator and practicing with it in problem situations will be the best preparation we can give them for becoming adult users of mathematics.

Maintain a File of Examples

Many slower students need repeated reminders of procedures and techniques. Some teachers make available a file of worked-out examples (computations and word problems). Students are allowed to refer to the file when they need to refresh their memories.

Allow the Use of Flowcharts for Algorithms

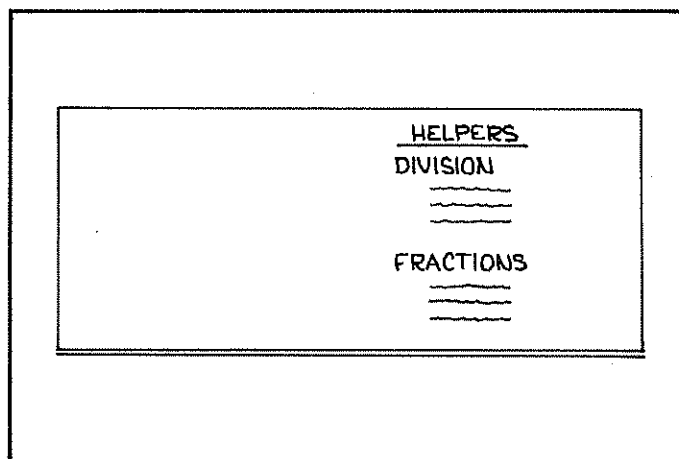
Like tables, flowcharts give the very weak student a chance for some success. Many algorithms are complicated, and it may not be reasonable to expect all students to remember exactly how to proceed, even though our demonstrations may have been



brilliant. Sample flowcharts are given in *Reducing Flowcharts*, *Renaming Flowcharts* (both in the Concepts subsection), *LCD Flowcharts I and II*, and *Fraction Flowchart* (all in Addition/Subtraction).

Use Peer Tutors, or Tutors from Upper Grades

Some teachers report good results with tutors. It can be handled casually within a class or can involve an organized selection and use of students from outside the class. One study with tutored eighth graders found that tutors from grades 8, 10 and 12 were all helpful but that the 12th graders were most effective. [Linton, 1973] Tutors should be carefully coached. Untrained tutors usually concentrate on how to do something, so if your objectives call for something more than that, be sure to let the tutors know. Give peer tutors a little instruction on working with others; they may be impatient and tend to make the helped students feel "dumb." You can establish a less elaborate tutoring set-up by keeping a chalkboard list of volunteers willing to help other students; students add their names to the list if they are willing to help others.



Consider Approaches Different from Those of Earlier Grades

a. It may help some students to see a concrete model which they have not seen in earlier grades. The commentaries in this resource give several ideas for models. Look at the textbooks for previous grades to see what models the students have likely seen already. Watch for students who think that a new model requires a new algorithm!

b. Some students may never have been exposed to, or may have forgotten, more sophisticated ways of thinking about the basic facts. For example, students who still calculate $9 + 7$ by counting on fingers might be able to think " $9 + 7$ would be 1 less than $10 + 7$." Or a student who counts by sixes to find 7×6 might be able to think " 7×6 would be 5 sixes plus 2 sixes" or " 7×6 is 6 more than 6×6 ."

c. You may wish to introduce alternate algorithms (e.g., the common denominator algorithm for division of fractions--see *Other Ways* in the Multiplication/Division subsection). A student who has failed repeatedly with the standard algorithm may react more positively to a new, fresh-start approach. However, proceed cautiously. Some teachers have found that bringing up a new algorithm causes more confusion instead



of helping the student. For example, there is some evidence that introducing two algorithms in a short period of time may result in their interfering with each other. [Barszcz and Gentile, 1976] The commentary to the Multiplication/Division subsection, the mathematical content sections and Hutchings [1976] offer more information about alternate algorithms. Without bringing up radically different algorithms, you might allow or even encourage less "mature" forms of the conventional algorithms, as to the right.

OKAY

$$\begin{array}{r} 395 \\ 3 \overline{)1185} \\ \underline{9} \\ 28 \\ \underline{27} \\ 15 \\ \underline{15} \\ 0 \end{array}$$

OKAY TOO

$$\begin{array}{r} 395 \\ 5 \\ 80 \\ 10 \\ 300 \\ 3 \overline{)1185} \\ \underline{900} \\ 285 \\ 30 \\ 255 \\ 240 \\ 15 \\ 15 \\ 0 \end{array}$$

Finger multiplication can help a student who cannot master all the "higher" basic multiplication facts. See the diagrams below. The student can use the method by putting his hands in his lap and thereby concealing from other students that he is using this crutch.

<p>ASSOCIATE NUMBERS WITH FINGERS</p>	<p>TO FIND 8×7 PUT FINGERS LIKE THIS</p>	<p>THEN...</p> <ul style="list-style-type: none"> - MULTIPLY # OF FINGERS (2, 3) <u>ABOVE</u> THE TOUCHING FINGERS $2 \times 3 = 6$ - ADD # OF FINGERS TOUCHING AND BELOW THEN $\times 10$ $(3+2) \times 10 = 50$ - ADD 56 <p style="text-align: center;">SO $8 \times 7 = 56$</p>
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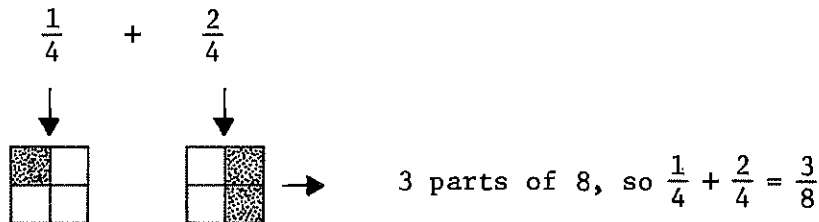
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1. (Discussion) Give arguments for and against the statement, "most students should know the basic +, -, \times , \div facts by heart and be able to carry out algorithms for these operations."
2. (Discussion) Mr. Ivory: "I don't use concrete materials because I want my students to think. They don't do much thinking when they're playing with something."

You:



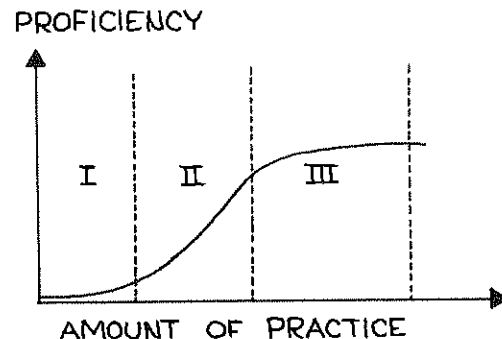
3. (Discussion) Discuss these positions and questions.
 - a. Computation with fractions can be deferred until the seventh grade and even then can be drastically curtailed except for those who will take algebra.
 - b. "Why teach division of common fractions?" [Price, 1969]
 - c. "Every seventh-grade mathematics student should be provided with an electronic calculator for his personal use throughout secondary school." [from a poll cited in Editorial Panel, 1974]
 - d. "... it appears to us that the case for decreased classroom emphasis on manipulative skills is stronger now than ever before." [National Advisory Committee on Mathematical Education, 1975, p. 24]
4. (Discussion) Following are three things to do with basic addition or multiplication facts. Evaluate them and, if you think they are all appropriate, give the order in which you think they should be used.
 - a. Point out regularities (e.g., by filling in a table or part of a table, or by using commutativity and identity properties).
 - b. Focus on meaning (e.g., cut graph paper or make arrays to show 3×4 , 5×6 , etc.).
 - c. Provide random exposure (e.g., a game in which a throw of the dice determines what combination is revealed).
 - d. Use speed drills.
5. Middle school students may have learned too well how to play the "game of school." If we never ask them on a test to show the meaning of an operation or explain a step in an algorithm, they may "learn" that such things are not important (the only thing that is important is to be able to calculate answers!). Swart [1974] argues cogently that we should ask students to demonstrate meanings, to illustrate with concrete materials, and to provide rationales for steps. Think of test items in which you could ask students
 - a. to show the meaning of a specific fraction.
 - b. to illustrate in some concrete mode that two fractions are equivalent.
 - c. to justify the multiplication-of-fractions algorithm.
6. A concrete representation must be interpreted correctly!
 - a. What is wrong in this "proof" that $\frac{1}{4} + \frac{2}{4} = \frac{3}{8}$?



- b. How would you avoid this misinterpretation during your demonstration?
7. Give your plan for a demonstration of addition or multiplication of fractions. Include attention to prerequisites, meaning, concrete materials, and the suggestions given above for a demonstration.



8. a. Design an activity card for division of fractions with Cuisenaire rods. Emphasize the "how many $\frac{1}{3}$'s in $\frac{3}{4}$?" meaning for $\frac{3}{4} \div \frac{1}{3}$.
- b. Which algorithm below most accurately reflects the work with the rods?
- i. $\frac{3}{4} \div \frac{1}{3} = \frac{3}{4} \times \frac{3}{1} = \frac{9}{4} = 2\frac{1}{4}$ ii. $\frac{3}{4} \div \frac{1}{3} = \frac{9}{12} \div \frac{4}{12} = 9 \div 4 = 2\frac{1}{4}$
9. Romberg [1968] notes that in a nation-wide testing, many students multiplied fractions correctly but missed the multiple-choice items by failing to simplify answers or by doing so incorrectly.
- a. Outline how you would handle the topic of simplifying fractions. Note what concrete representations (e.g., circle fractions, colored rods), approach (as exemplified in the two parts of *Reducing Flowcharts* in FRACTIONS: Concepts), drill games (if any), and rationale for the student you would use.
- b. How can "cancelling" in multiplication of fractions be handled to make it meaningful to students?
10. Projects can serve as "larger undertakings" in which to provide practice. *Mini-Projects for Fractions* and *Art Projects for Fractions* (both in the Concepts subsection) give some samples of such projects. Design a project which requires some computational skill with fractions or decimals.
11. Suppose a learning curve for some algorithm looks like the one to the right. During which period (I, II or III), if any, would you use each of the following?
- a. a game involving groups of 3
- b. supervised practice
- c. concrete materials
- d. timed drills
- e. applications using the algorithm
- f. a quiz
12. Sketch likely learning curves for a strong student and for a weaker student on the same graph. What do these curves imply for instruction, for student attitude, for planning variety, for diagnosis, for evaluation, . . .?
13. Sketch a learning curve which shows your idea of the influence of spaced reviews on learning. How do you think that curve would differ from one which reflects review given soon after the initial learning?
14. (Discussion) Many teachers report that the frequent use of separate worksheets (like those in these resources) seems to get slower students working and helps their attitude. Why? Refine, and add to, the following list of possible reasons.
- a. Such sheets are more entertaining than much textbook material.
- b. Such sheets are more appropriate for the students than textual material.
- c. It's something different.
- d. It shows that the teacher cares.
- e. The sheets have a clear "the-end-is-in-sight" finiteness.
- Do you see any shortcomings in the use of such worksheets?





15. Sam can't seem to master the finding of least common denominators in fraction addition. You are considering letting him use the product of the denominators as the common denominator:

$$\frac{1}{6} + \frac{2}{9} = \frac{9}{54} + \frac{12}{54} = \dots; \frac{3}{4} + \frac{5}{6} = \frac{18}{24} + \frac{20}{24} = \dots; \text{etc.}$$
 - a. What are some arguments pro and con letting Sam do this?
 - b. What prerequisite skills are vital to Sam's succeeding with this algorithm?
 - c. Will you insist that Sam simplify his answers? Justify.
 - d. Would you allow Sam to use factor boards (*Factor Boards-I* in the Concepts subsection)? Justify.
16. Position 1: "A slow learner is not helped by more repetition. An enlarged view, carefully developed, is better for him than endless review and repetition." [Junge, 1972, p. 147]
 Position 2: "Drill is exactly what slower students need. They will only get confused if you give them too many ways to think about things."
 Evaluate each position in light of your experience.
17. Write the exercises you would include in the assignment for the day you introduce calculations like $2\frac{1}{2} \times 1\frac{3}{5}$. Use part of the assignment to show analogous calculations encountered earlier (e.g., 25×16) and to force the student to discriminate (e.g., $2\frac{1}{2} + 1\frac{3}{5}$).
18. Design a homework assignment which takes into account "yesterday, today and tomorrow."
19. What does $2\frac{1}{2} \times 3\frac{1}{3}$ mean?
20. A common sequence of topics in teaching fractions is meaning(s), equivalent fractions, ordering, adding, subtracting, multiplying, and dividing. Is this the only reasonable sequence?
21. Which models for fractions should be taught, and when?
 - a. Survey the textbooks to which your students have been, and will be, exposed to see their answer to the question.
 - b. Does your department have a plan (beyond the choice of textbooks) which addresses the question?
22. (Outside reference) Brownell [1956] describes the extremes of repetitive practice and varied practice.
 - a. Describe these two types.
 - b. Illustrate each of the types for . . .
 - 1) simplifying fractions
 - 2) adding fractions
 - 3) dividing fractions
23. (Outside reference) One careful way to determine prerequisites is to task analyze a procedure to obtain a learning hierarchy [Gagné, 1970, ch. 9 or Gagné, 1962]. Give a learning hierarchy for some algorithm (e.g., adding fractions or multiplying mixed numbers).
24. (Research critique of outside reference) Critique the Koch [1965] study.



25. Mrs. Ippi: "I rather like the idea of letting slower students use flowcharts, but I have a mixed-ability class. How can I let some use flowcharts and others not?"

You:

26. The tutor often learns more than the tutored! Evaluate these possibilities:
- Letting weaker students serve as tutors for lower grade level.
 - Having students in a pair take turns teaching each other or asking "Why?" questions of each other.
27. S. Mason: "I prefer to start with the problem and then have the kids help me work it out. The usual lead-them-by-the-hand technique is short-sighted: They never experience how to figure something out on their own!"
- How would Ms. Mason approach the unequal-denominators case of adding fractions?
28. (Discussion) Many teachers think that handheld calculators should be used in developing algorithms. How could the calculator be used
- in developing the whole number division algorithm?
 - decimal multiplication?
 - decimal division?

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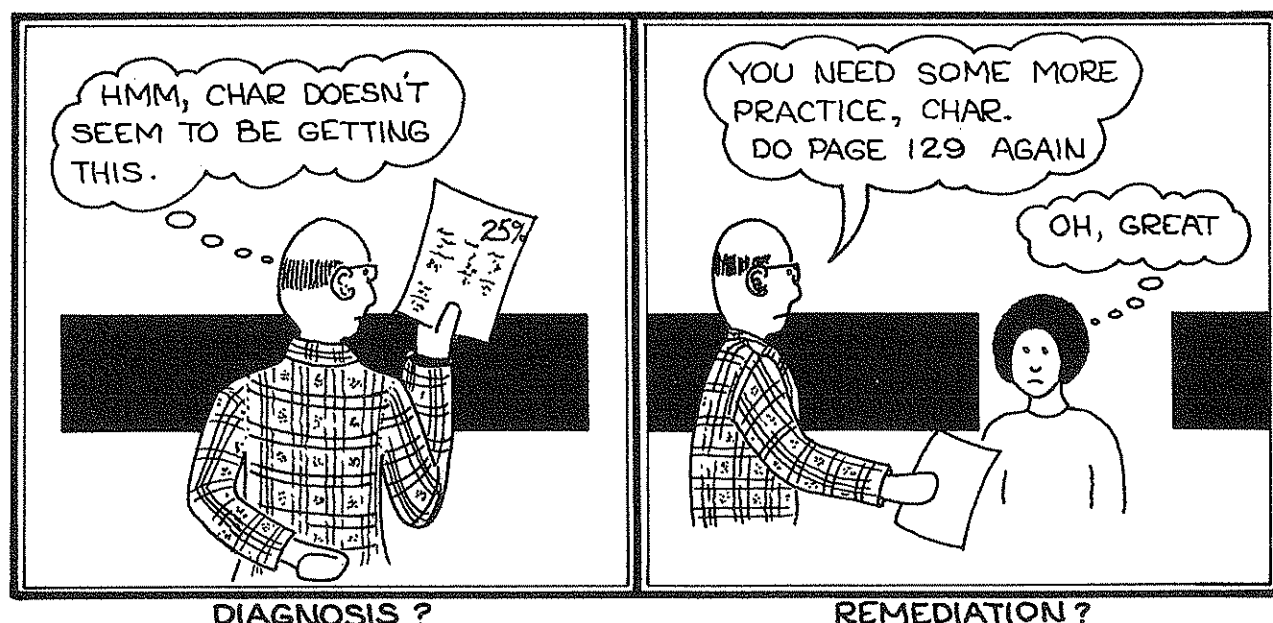
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NUMBER SENSE AND ARITHMETIC SKILLS

Placement Guide for Tabbed Dividers

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DIAGNOSIS AND REMEDIATION



DIAGNOSIS ?

REMEDICATION ?

The marks, numerals and erasures that students have put on their papers may tell a lot more than whether the answer is correct. Consider the following work done by Cecil and Marty:

Cecil

$\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$	$\begin{array}{r} 134 \\ \times 3 \\ \hline 122 \end{array}$	$\begin{array}{r} 528 \\ \times 7 \\ \hline 496 \end{array}$
--	--	--

Marty

$\begin{array}{r} 43 \\ 4 \overline{)136} \\ \underline{12} \\ 16 \\ \underline{16} \\ 0 \end{array}$	$\begin{array}{r} 71 \\ 5 \overline{)85} \\ \underline{5} \\ 35 \\ \underline{35} \\ 0 \end{array}$	$\begin{array}{r} 45 \\ 8 \overline{)432} \\ \underline{40} \\ 32 \\ \underline{32} \\ 0 \end{array}$
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Apparently Cecil is adding the number he "carries" (his crutch) to the tens' digit before he multiplies units times tens. When he doesn't need to carry, he gets the correct answer. Marty's answers are all incorrect and one might consider Marty a near-hopeless cause unless it is noticed that the partial quotients are all correct and, in fact, the answers are correct if the tens' digit and units' digit are reversed. Marty is recording from right to left. But why not? We record from right to left when we add, subtract and multiply. Whatever the reason for such an error pattern, our job is to pinpoint the underlying problem (to find the error pattern or to diagnose) and then to provide help in rectifying the trouble (to remediate). Merely assigning "more practice" to the Chars, Cecils and Martys is not likely to make them aware of incorrect concepts or procedures and help them to improve.

NOTE: Unless otherwise noted, subsection cited are in the DECIMALS section of the resource Number Sense and Arithmetic Skills.

DIAGNOSING CAN BE FUN (BUT CHALLENGING)

Many student errors, of course, are due to carelessness or lack of mastery of basic facts. Since such causes are so common, they complicate the identification of a systematic procedural error and perhaps even hide the fact that a student may be using such a pattern. To complicate the matter, students are extremely versatile in the ways in which they can miss a problem. For example, Lankford [1972] identified 42 different ways in which 54 seventh-grade students missed 75×58 ! Isn't that a challenge? Finding where the students are in error is essential if any real relearning is to take place.

Here is an error pattern with fractions.

$$\frac{1}{2} + \frac{2}{3} = \frac{3}{5} \quad \frac{3}{8} + \frac{1}{4} = \frac{4}{12} = \frac{1}{3} \quad \frac{2}{5} + \frac{1}{3} = \frac{3}{8}$$

This error pattern seems to be one of students' favorites: add the numerators, then add the denominators. (Doesn't that sound reasonable?)

This one is a little harder to diagnose:

Chris' Work

$$\begin{array}{r} 1.4 \\ \times 1.3 \\ \hline 42 \\ 14 \\ \hline 18.2 \end{array}$$

$$\begin{array}{r} 7.4 \\ \times 3.1 \\ \hline 74 \\ 222 \\ \hline 2294 \end{array}$$

$$\begin{array}{r} 45 \\ \times .7 \\ \hline 3.15 \end{array}$$

$$\begin{array}{r} 794 \\ \times .2 \\ \hline 1588 \end{array}$$

(Chris was lucky.)

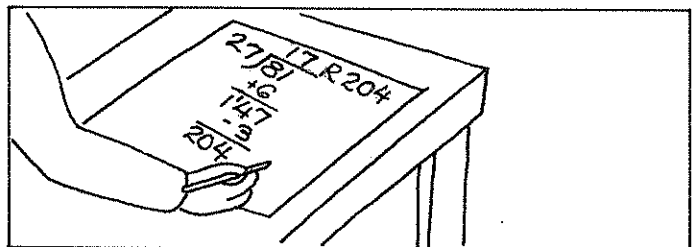
Did you get this one? Chris remembered something about counting decimal places but forgot which "end" of the answer to start counting from. Note the 3.1×7.4 exercise. Chris' incorrect procedure nonetheless gave a correct answer--another reason that diagnosis can be difficult. When only some of the answers are incorrect, you may suspect carelessness rather than a systematic error pattern.

How about the one to the right?!!!

(If you can analyze this error, you can skip the rest of this section.) This work was actually given by a seventh-grader. Here is the student's "explanation":

" $7 \times 1 = 7$, so put 7 up; $2 \times 8 = 16$ " (student put 6 below 1 of 81 and 1 above 8 of 81), " $6 + 1 = 7$; $8 + 6 = 14$. . . (counting) . . . for 147; $6 + 7 = 13$ " (student wrote 3 under 7 of 147 and carried 1 above 4 of 147); " $7 - 3 = 4$; $4 - 3 = 1$, - 1" (student carried above 4) " $= 0$; 3 take away 1 is 2." [Lankford, 1972, p. A-13]

Impressed? Or depressed?



Several of the items at the end are designed to give you practice at diagnosing other error patterns. (See also the commentaries to FRACTIONS: Addition/Subtraction and FRACTIONS: Multiplication/Division.) All the error patterns are drawn from classroom observation or interview reports; you will likely recognize some of them immediately. Some of the "weirder" ones are not too common, fortunately. When you do encounter one of the weird ones with a student of yours, what should you do? Just what Lankford did

Puzzled?--Look in the Horse's Mouth

The only certain way to find out whether your diagnosis is correct is to go directly to the source: the student whose work you are studying. This course of action is advised even for a diagnosis you may feel confident about. For the following example, pretend that the student did all the intermediate computation in his head or on scratch paper:

$$\begin{array}{r} 8 \frac{2}{5} \\ -4 \frac{3}{10} \\ \hline 3 \frac{9}{5} \end{array}$$

You may have a plausible diagnosis in mind. Lankford's seventh-grade students provided three different explanations [Lankford, 1972, p. B-7]:

Student 1--"10 will not go into 5 so change 8 to a 7 and 2 (of 2/5) to 12. Then I borrow one from 12 making it 11, and making 5 (of 2/5) a 15. So $7 \frac{11}{15} - 4 \frac{3}{10} = 3 \frac{9}{5}$. (4 from 7 = 3, 10 from 15 = 5, 3 from 11 = 9.)"

Student 2--"Borrowed 1 from 8, made it 7 and 2 of 2/5 into 12. Then $7 \frac{12}{5} - 4 \frac{3}{10} = 3 \frac{9}{5}$. (4 take away 7 is 3, 3 take away 12 is 9, and 10 take away 5 is 5.)"

Student 3--"8 take away 4 is 4, you can't take 2 from 3 so you borrow from 4 (the difference number). Then 12 take away 3 is 9 and 5 take away 10 is 5."

Since all three students obtained the same answer it might have been assumed before their explanations that their difficulties were the same. As you can see, however, each student had idiosyncrasies which you would like to know before deciding what remedial action to take. *How could the teacher discover the nature of these different erroneous strategies without having each student relate his thinking process?*

Finding time for diagnostic interviews is a problem, whether you are teaching in a self-contained classroom or teaching from five to eight sections a day. A further complication is that some interviews should be fairly private to avoid distractions and self-consciousness on the part of the student. Perhaps interviews will necessarily be restricted to those students who have the most serious problems as identified through paper-and-pencil work. Teachers fortunate enough to have teacher aides may be able to use them to monitor the rest of the class while interviews are carried out (this may be a very important reason to appeal for teacher aides or for volunteer help from the community). Frequent, well-conceived classroom laboratory lessons or small-group games may provide free time for the teacher to work closely with one or two individuals in a diagnostic interview. Supervised study periods, on the rare occasions when demands from other students allow, might also be used for brief diagnostic sessions. Some cases may seem so drastic that you will want to schedule a free period or a before- or after-school session with the student.

Conducting a Diagnostic Interview

Suppose you do find time to talk with a student for several minutes without interruption. Experienced interviewers offer some suggestions that might not be immediately obvious or that bear emphasis (see Ashlock, 1972, or Lankford, 1974). The primary object of the interview is to find out how the student approaches tasks and what erroneous procedures and concepts he uses.

- i. If possible, secure a private setting for the interview. The student must believe that you are interested in him as a person. It must be clear that you are not punishing the student but are attempting to help him with his difficulties. He must not be afraid.

Afraid of failing, afraid of being kept back, afraid of being called stupid, afraid of feeling themselves stupid . . . Even in the kindest and gentlest of schools, children are afraid, many of them a great deal of the time, some of them almost all of the time. [Holt, How Children Fail]

- ii. Become an objective data collector and not a teacher. The urge will be to instruct and point out errors, but remediation will come later after all the facts are gathered.
- iii. Look for patterns and strategies; try to avoid being distracted by isolated events. For example, students will often say the wrong words but do the correct thing. The more you learn about the student's procedures, the easier it will be to remedy the situation.



REMEDIATION

Cecil

$$\begin{array}{r} 58 \\ \times 7 \\ \hline 496 \end{array}$$

Marty

$$\begin{array}{r} 45 \\ 8 \overline{) 432} \\ \underline{40} \\ 32 \\ \underline{32} \\ 0 \end{array}$$

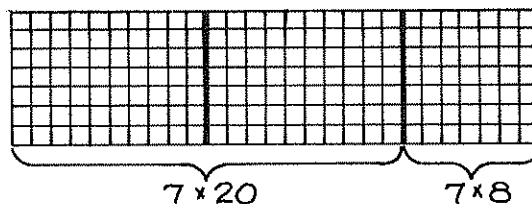
Johnny

$$\begin{array}{r} 8 \frac{2}{5} \\ - 4 \frac{3}{10} \\ \hline \end{array} \rightarrow \begin{array}{r} 7 \frac{12}{5} \\ - 4 \frac{3}{10} \\ \hline 3 \frac{9}{5} \end{array}$$

Let's look again at Cecil, Marty and Johnny (formerly Student 2). It is not likely that merely having Cecil, Marty and Johnny do more problems will lead to correct practices. What can we do? Suppose that we have talked with the students and have confirmed our earlier diagnoses. Let's use their errors to illustrate some possible remedial practices.

Cecil (who adds the carried digit before multiplying units and tens) Cecil, with a little prompting, should be able to use his number sense to see that his answer can't be right. 28 is less than 30, and $7 \times 30 = 210$, nowhere close to 496. Student estimation with whole numbers (e.g. *Rounded Line Up*, *A Slick Slip Stick*, *Rounded Results*, all in WHOLE NUMBERS: Mixed Operations), fractions (e.g., *Between*, *Can You Get Close Up*, both in FRACTIONS: Multiplication/Division) and decimals (e.g., *Approx-Appraisals*, in DECIMALS: Mixed Operations, or *Squared Off* and *Never Hold a Grudge*, both in DECIMALS: Multiplication/Division) should have pay-off in avoiding obviously incorrect answers. Giving explicit attention to estimation is almost a necessity since middle-school students sometimes have acquired the idea that only the "exact" answer is a legitimate concern. Let's get back to Cecil. Do we want to leave Cecil only with "In times problems you add it after you multiply"? That might satisfy Cecil at the moment, but he might not retain it. Probably we would prefer that Cecil have an idea of why it is done that way, rather than having him view the procedure only as some mysterious step to be memorized. It may be necessary to ask Cecil to show what 7×28 means with say, graph paper. If he doesn't know, that's significant information and suggests that he needs some work with the meaning of multiplication. If he does, then he can be asked to show where his computational steps come in (see the diagram to the right). Looking at a less condensed version of the algorithm may fill a gap between the graph paper and the standard algorithm.

EACH ROW HAS 28 SQUARES



$$\begin{array}{r} 28 \\ \times 7 \\ \hline 56 \\ 140 \\ \hline 196 \end{array} \rightarrow \begin{array}{r} 528 \\ \times 7 \\ \hline 196 \end{array}$$

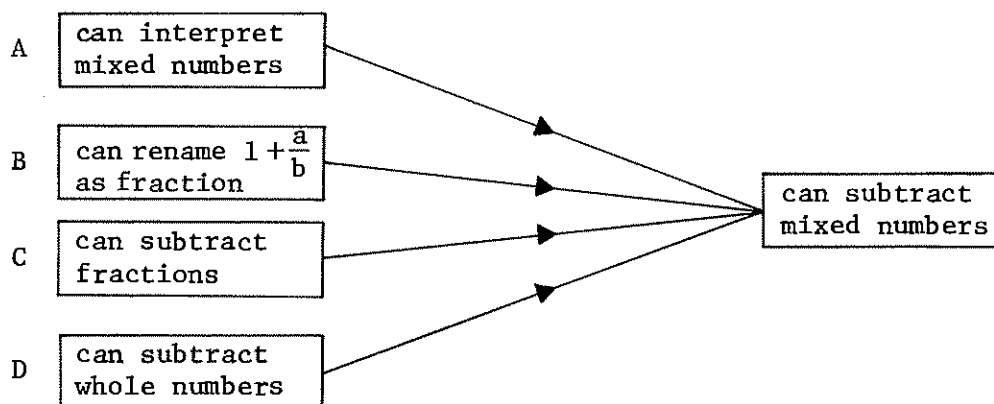
Marty (who records the quotient from the right rather than the left) We might ask Marty what the 40 in the work means. It may be that an earlier teacher skipped an extended form of the division algorithm, and Marty does not realize that the 40 (and the 5) refer to a number of tens. If she is exposed to an "unabbreviated" form like that to the right, it should be easy to get her to record her digits in the quotient correctly. Checking by multiplying or going to a concrete aid (as in *Activity Cards-10 Multibase-Blocks V* in *WHOLE NUMBERS: Multiplication/Division*, for example) may be necessary.

$$\begin{array}{r} 45 \\ 8 \overline{)432} \\ \underline{40} \\ 32 \\ \underline{32} \\ 0 \end{array}$$

$$\begin{array}{r} 4 \\ 50 \\ 8 \overline{)432} \\ \underline{400} \\ 32 \\ \underline{32} \\ 0 \end{array}$$

$$\begin{array}{r} 8 \frac{2}{5} \\ -4 \frac{3}{5} \\ \hline 3 \frac{9}{5} \end{array}$$

Johnny is not ready to subtract mixed numbers because he has not mastered the necessary prerequisite skills. A brief breakdown of the prerequisites might look as follows:



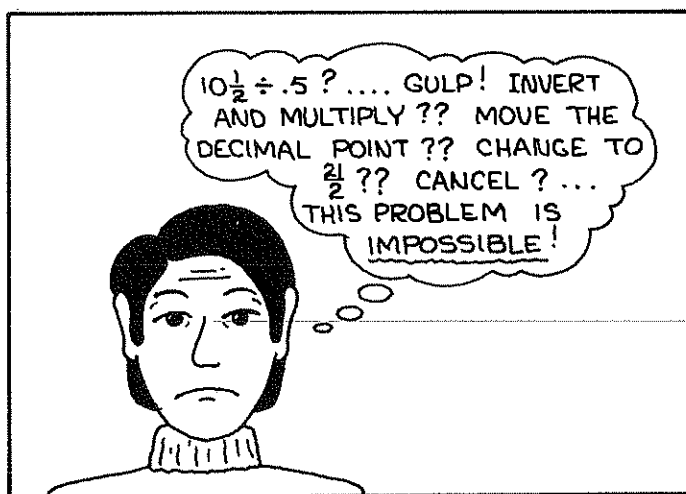
Johnny certainly has troubles with respect to prerequisites B and C (the one problem does not give information about A). Our job will be to fill in Johnny's background. It also appears that he may need concrete support (e.g., as in *Fraction Subtraction* in *FRACTIONS: Addition/Subtraction*). As in most remedial situations, our most important task may be to convince Johnny that his method is incorrect *because it does not give true results*. Sometimes a student thinks that his (wrong) method is all right, too, but that this teacher just wants the exercise done a certain way. [Erlwanger, 1973]

Rundown of Possible Remediation Steps

1. Does the student need bolstering? Middle school students who are still having computational difficulties or who regress to earlier error patterns often have a history of failure in mathematics. Above all else these students may need to believe that they are worth-while persons and capable of learning the needed skill. Whatever remedial action is chosen, let the students experience frequent reinforcement and success.

2. Does the student need to be convinced that he *is* wrong? Sometimes the student genuinely believes that his way is right, not realizing that his memory of last week or last year or Dad's way may be in error. Appealing to an estimate may work. It may require a concrete model--arrays, circular regions, number line, paper folding, graph paper, money--to convince the student that something about his procedure needs to be changed.

3. Are prerequisites the problem? Even a superficial task analysis like the one above for subtracting mixed numbers may be useful in planning efficient remediation. Students who have relied solely on their memories to keep algorithms straight may become overwhelmed by all the separate "rules" to keep in mind for whole numbers, for fractions and mixed numbers, for decimals, and for percents.



4. Does the student have any "number sense"? Is he using it? Frequently the student who needs remedial work does not have number sense. Those who do have number sense often get so wrapped up in carrying out the algorithm that they forget to check their answers for reasonableness.

5. Would elaborating on an algorithm help to build an understanding of the process? Some of our algorithms are remarkably concise. To speed up calculations, we omit zeros in partial products, use shortcuts for finding least common denominators, and move the decimal points in division. However, these actions often obscure the meaning associated with the algorithm. There may be a pay-off in reviewing the process at a more basic level, as in Marty's case.

6. Can concrete materials help? Using concrete materials to convince the student that something is awry has been mentioned. Concrete representations give meaning to the operations, so the student can resort to a basic understanding instead of memory alone. Some concrete aids are especially good for specific problems; they might supply just the "missing link" needed by a faltering student. There are many examples: the abacus for place-value, multibase blocks for powers (see *Multibase Blocks-I* in LARGE & SMALL NUMBERS: Exponential Notation), regions for adding or multiplying fractions, colored rods for dividing fractions, number lines for mixed-number-to-equivalent-fraction work.

7. Can the student be involved in planning the remedial work? The value of asking the student for ideas or to choose among options may be more attitudinal than anything else. [Cooney, et al., 1975] Having to do remedial work may be very discouraging, especially when it's for the n th time (n large). Ask the student to suggest ideas and to choose among options for remediation. "What do you think would fix up the trouble? Would you rather use an abacus or multibase blocks?" Answering such questions might make the student feel more in control of the situation. Since he has chosen the course of action, he may be more committed to following it and to completing it. Pick the options carefully, of course, since you want the student to benefit from whatever he does choose.

8. Is my attitude positive? We must convince the student that he can learn the material. One way is through our attitude. "I'm sure you can learn this and achieve success. You just haven't been given an approach that will work for you." The over-all student-teacher relationship must convey an optimistic outlook. Words alone won't do it if the student feels that the teacher really believes that the student is "dumb."

CLOSING REMARK

One of the greatest challenges to a middle-school teacher is the diagnosis and remediation of student misconceptions and error patterns. The time and energy we devote to these activities should result in greater student achievement and perhaps more importantly, in improved student self-concept.

?????

1. Some students may have "inherited" an I-never-was-any-good-in-math attitude from parents. What can be done to change such an attitude?

2. Here are some sample error patterns with decimals to analyze.

Abe:

$$\begin{array}{r} .7 \\ +.5 \\ \hline .12 \end{array}$$

$$\begin{array}{r} .6 \\ +.9 \\ \hline .15 \end{array}$$

$$\begin{array}{r} .2 \\ .4 \\ .5 \\ \hline .11 \end{array}$$

(Note: Abe would probably say each sum correctly as he writes it.)

Bert: $1.7 + 1.4 + 3 = 3.4$ $1.28 + .7 + .54 = 1.89$ $.28 + .7 = .35$

Clarisse:

$$\begin{array}{r} 1.4 \\ \times .3 \\ \hline 4.2 \end{array}$$

$$\begin{array}{r} 1.7 \\ \times .6 \\ \hline 10.2 \end{array}$$

$$\begin{array}{r} 3.2 \\ \times .3 \\ \hline 9.6 \end{array}$$

David:

$$\begin{array}{r} .56 \\ .3 \overline{)1.68} \\ \underline{15} \\ 18 \\ \underline{18} \\ 0 \end{array}$$

$$\begin{array}{r} 1.2 \\ 1.4 \overline{)16.8} \\ \underline{14} \\ 28 \\ \underline{28} \\ 0 \end{array}$$

$$\begin{array}{r} .831 \\ .7 \overline{)5.817} \\ \underline{56} \\ 21 \\ \underline{21} \\ 07 \end{array}$$

3. The work below refers to the students in number 2. One sample exercise is repeated, with at least part of the student's explanation given. Choose the students you find most interesting and plan remediation for them.

Abe:

$$\begin{array}{r} .7 \\ +.5 \\ \hline .12 \end{array}$$

"7 tenths and 5 tenths are 12 tenths."

Bert: $1.7 + 1.4 + 3 = 3.4$

Bert's work:

$$\begin{array}{r} 1.7 \\ 1.4 \\ \hline 3 \\ \hline 3.4 \end{array}$$

Clarisse:

$$\begin{array}{r} 1.4 \\ \times .3 \\ \hline 4.2 \end{array}$$

". . . Then put the decimal point right under the others."

David:

$$\begin{array}{r} .56 \\ .3 \overline{)1.68} \\ \underline{15} \\ 18 \\ \underline{18} \\ 0 \end{array}$$

"First, the point goes right over the point. Then 3 into 16 is . . ."

4. Diagnosing errors on specific basic facts with middle-schoolers may take some effort. There are, of course, the "demons" ($7 \times 8 = 54$, $6 \times 9 = 56$), but ones unique to a student may slip by unnoticed or be credited to "slips of the mind" (e.g., $3 \times 3 = 6$). Evaluate these ways of more systematically identifying trouble spots with basic facts.
- In a game based on digits (e.g., *Fresno* in *WHOLE NUMBERS: Mixed Operations*) use just a few digits.
 - Regularly have a short speed (?) drill, with the exercises being either written or read out loud. Students should note which ones they miss.
5. Mr. Bunyan: "Hoo-ray! I'm going to have an aide who will check homework and tests and will record grades! I'll never have to look at a paper!"
- Let us share Mr. Bunyan's elation, but what can he and his students gain by his looking at the marked papers?
 - These are the usual sources of help: paid aides, parent volunteers, students in an upper grade, high school students in a future teachers' club or service club (or just volunteers), students from a nearby college, senior citizens. Are any of these reasonable for your situation?
6. Sometimes a student will reason along unconventional lines and come up with a correct answer which, alas, we might not recognize as correct. Try to re-create these two students' thinking. What would you say to them?
- $$\frac{1}{2} + \frac{1}{5} = \frac{3\frac{1}{2}}{5} \qquad \frac{1}{3} + \frac{3}{4} = \frac{4\frac{1}{3}}{4}$$
 - $$.75 \times .25 = \frac{3}{16} \qquad .5 \times 1.06 = \frac{53}{100}$$
7. Any time a student does something (or doesn't, for that matter), we have an opportunity to evaluate. For example, what might one diagnose as a result of using the student page, "The Answer," in the Mixed Operations subsection?
8. (Discussion) It is certainly worthwhile to ask a student to help in diagnosing his problem. What can be gained by doing this, even when you are already nearly certain of the trouble?
9. (Discussion) This section has centered diagnosis on a single student. Another common use of the term "diagnosis" is in connection with analyzing a short "pre-test" given before instruction on the topic(s) of the pre-test.
- Report your experience with such diagnostic tests.
 - What sort of useful information can be obtained in a similar way from chapter or unit tests given after instruction?
10. Give a breakdown of the prerequisites for each of the following:
- fraction addition
 - fraction multiplication
 - fraction division
 - mixed number addition
 - mixed number multiplication
 - decimal division
 - fraction-to-decimal conversion

11. How can you handle remediation activity without jeopardizing a student's self concept . . .
- when he is one of 5 with the same difficulty?
 - when he is the only student with the difficulty?
12. (Discussion) Evaluate these possible ways of dealing with the student who can't seem to learn the basic facts. Add to the list.
- Don't make him; let him use tables.
 - Have him help a younger student who doesn't know them. (See NCTM, 1972, p. 159, for a testimonial.)
 - Promise an apple for each fact he learns and remembers for two weeks (after finding out which ones he knows).
 - Give him 5 minutes of undisguised drill daily (oral, filling in tables, audiotape, . . .).
 - Frequently use games which require knowledge of basic facts to win.
 - Let him check final answers with a calculator, with the understanding he must completely correct any wrong exercises.
 - Have him give a fact when he asks for a favor.
13. (Discussion) Mr. Callust: "Let's face it, if a kid hasn't learned how to do whole number arithmetic by the 7th grade, he's not going to. I have them use the calculator for any whole number stuff."
- Your reaction?
14. (Discussion) Developing a number sense in our students should be a high priority objective. Several student pages in this resource should help foster this development. What are some other ideas? (E.g., design a "How's Your Number Sense?" bulletin board; practice mental arithmetic thrice-weekly; in discussing chalk-board work, always ask, "Is that reasonable?" . . .)
15. "Carelessness" gets a lot of blame for student errors. Perhaps thinking about possible causes of carelessness might in turn suggest ways to avoid the causes. See what you can add to these lists.

<u>Cause</u>	<u>To avoid cause</u>
a. student hurries too much	a1. tell class "Haste makes waste" (ha!) a2. give only part of assignment, let work 5 minutes, then give another part, etc. a3. say, "Today speed is not important. Answers <u>must</u> be correct. Check by using inverse operations."
b. assignments too long for student	b1. give "reasonable" assignments b2. let the student choose the assignment ("You can do the first 10 or the first 15, whichever you feel is best for you.") b3. give "conditional" assignments: "If you get exercises 1, 5, 10, 15 correct, then go on. If you miss 10 and 15, go back to 6." b4. set up student contracts

c. sloppy work

- c1. make a big to-do about neatness in your own work
- c2. give a "neatness" grade occasionally
- c3. post neat papers
- c4. give no credit if you can't read it or have to hunt for it

16. (Outside reference) Hutchings [1975; 1976] suggests that using "low-stress" algorithms can be an interesting diversion for capable students and a big help to the less-capable ones. From the samples below, notice how useful this form would be for finding errors with basic facts. Check the references for "low-stress" subtraction [1975 or 1976], multiplication and division [1976] algorithms.

Sample (low-stress addition):

Exercise 1.
$$\begin{array}{r} 74 \\ 68 \\ 39 \\ \hline \end{array}$$

Diagram showing the low-stress addition process with arrows indicating the flow of digits and carries:

$$\begin{array}{r} 7^2 4 \\ 6^1 8^2 \\ 3^0 9^1 \\ \hline 1 \quad 1 \quad 8 \quad 1 \end{array}$$

Exercise 2.
$$\begin{array}{r} 4^2 9 \\ 7^6 8 \\ 1^3 1^7 \\ 6^4 4 \\ 9^1 1 \\ \hline 1^5 4^4 5^6 \\ \hline 2 \quad 4 \quad 6 \end{array}$$

17. (Outside reference) Read Lankford [1974], Ashlock [1972] or the 35th Yearbook [NCTM, 1972, pp. 308-309, pp. 514-516] for more ideas on interviewing a student. Then interview one. Plan the exercises you will present (so that, for example, you could reasonably ask the student to "prove" his answer with some concrete aid) but be flexible; student responses often suggest other questions to ask.
18. (Outside reference) Even better-than-average students sometimes develop some mysterious procedures or concepts. Read Erlwanger [1973; 1975] for an interesting account of what ideas a student can come up with, especially in an individualized program without careful teacher monitoring. The report may inspire you to carry out an in-depth interview with one of your above average students.

Below are several error pattern diagnosis and remediation exercises. Sample them to suit your taste.

19. Younger students also display error patterns, of course. Occasionally some of these endure into the middle grades. Analyze the error patterns in each of the following parts and complete the fourth exercises as the students would. (Patterns are based on Cox, 1975.)

a.
$$\begin{array}{r} 519 \\ + 82 \\ \hline 511 \end{array} \quad \begin{array}{r} 345 \\ + 76 \\ \hline 511 \end{array} \quad \begin{array}{r} 483 \\ + 57 \\ \hline 711 \end{array} \quad \begin{array}{r} 657 \\ + 85 \\ \hline \end{array}$$

d.
$$\begin{array}{r} 43 \\ - 7 \\ \hline 44 \end{array} \quad \begin{array}{r} 51 \\ - 29 \\ \hline 38 \end{array} \quad \begin{array}{r} 30 \\ - 12 \\ \hline 22 \end{array} \quad \begin{array}{r} 64 \\ - 27 \\ \hline \end{array}$$

b.
$$\begin{array}{r} 46 \\ + 3 \\ \hline 79 \end{array} \quad \begin{array}{r} 21 \\ + 8 \\ \hline 109 \end{array} \quad \begin{array}{r} 17 \\ + 2 \\ \hline 39 \end{array} \quad \begin{array}{r} 24 \\ + 5 \\ \hline \end{array}$$

e.
$$\begin{array}{r} 57 \\ - 23 \\ \hline 314 \end{array} \quad \begin{array}{r} 74 \\ - 51 \\ \hline 113 \end{array} \quad \begin{array}{r} 25 \\ - 13 \\ \hline 12 \end{array} \quad \begin{array}{r} 49 \\ - 15 \\ \hline \end{array} \quad \text{(Note that this answer is correct.)}$$

c.
$$\begin{array}{r} 54 \\ + 7 \\ \hline 511 \end{array} \quad \begin{array}{r} 64 \\ + 8 \\ \hline 612 \end{array} \quad \begin{array}{r} 38 \\ + 5 \\ \hline 313 \end{array} \quad \begin{array}{r} 67 \\ + 8 \\ \hline \end{array}$$

f.
$$\begin{array}{r} 53 \\ - 14 \\ \hline 49 \end{array} \quad \begin{array}{r} 82 \\ - 36 \\ \hline 56 \end{array} \quad \begin{array}{r} 46 \\ - 39 \\ \hline 17 \end{array} \quad \begin{array}{r} 72 \\ - 48 \\ \hline \end{array}$$

20. Here are some more illustrations of error patterns with whole-number operations to analyze. Most of these were observed by Lankford [1972] with second-semester seventh graders.

Al:

$$\begin{array}{r} 19 \\ \times 20 \\ \hline 20 \end{array}$$

$$\begin{array}{r} 41 \\ \times 36 \\ \hline 126 \end{array}$$

$$\begin{array}{r} 15 \\ \times 71 \\ \hline 75 \end{array}$$

$$\begin{array}{r} 62 \\ \times 74 \\ \hline 428 \end{array}$$

$$\begin{array}{r} 58 \\ \times 75 \\ \hline 390 \end{array}$$

$$\begin{array}{r} 42 \\ \times 26 \\ \hline \end{array}$$

Bea:

$$\begin{array}{r} 304 \\ \times 506 \\ \hline 1824 \\ 000 \\ 1520 \\ \hline 17024 \end{array}$$

$$\begin{array}{r} 145 \\ \times 202 \\ \hline 290 \\ 000 \\ 290 \\ \hline 3190 \end{array}$$

$$\begin{array}{r} 245 \\ \times 403 \\ \hline 735 \\ 000 \\ 980 \\ \hline 10535 \end{array}$$

$$\begin{array}{r} 474 \\ \times 502 \\ \hline \end{array}$$

Carl:

$$\begin{array}{r} 15 \overline{) 7590} \\ \underline{75} \\ 90 \\ \underline{90} \end{array}$$

$$\begin{array}{r} 23 \overline{) 9246} \\ \underline{92} \\ 46 \\ \underline{46} \end{array}$$

$$\begin{array}{r} 32 \overline{) 6496} \\ \underline{64} \\ 96 \\ \underline{96} \end{array}$$

$$17 \overline{) 5134}$$

Dan:

$$\begin{array}{r} 8 \overline{) 4824} \\ \underline{48} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

$$\begin{array}{r} 14 \overline{) 5647} \text{ R5} \\ \underline{56} \\ 47 \\ \underline{42} \\ 5 \end{array}$$

$$\begin{array}{r} 12 \overline{) 3627} \text{ R3} \\ \underline{36} \\ 27 \\ \underline{24} \\ 3 \end{array}$$

$$18 \overline{) 7227}$$

Denise:

$$\begin{array}{r} 43 \\ \times 35 \\ \hline 13115 \end{array}$$

$$\begin{array}{r} 56 \\ \times 24 \\ \hline 11424 \end{array}$$

$$\begin{array}{r} 34 \\ \times 75 \\ \hline 23970 \end{array}$$

$$\begin{array}{r} 28 \\ \times 54 \\ \hline \end{array}$$

Derrold:

$$\begin{array}{r} 3 \overline{) 4784} \text{ R5} \\ \underline{1} \\ 1 \\ \underline{1} \\ 2 \\ \underline{2} \\ 1 \end{array}$$

$$\begin{array}{r} 5 \overline{) 7575} \text{ R4} \\ \underline{2} \\ 0 \\ \underline{2} \\ 0 \end{array}$$

$$\begin{array}{r} 3 \overline{) 419} \text{ R2} \\ \underline{1} \\ 1 \\ \underline{1} \\ 0 \end{array}$$

$$7 \overline{) 7418}$$

21. Lankford [1972] also observed error patterns with fractions. Sample these and complete the unfinished exercises as the students would.

Emma:

$$\frac{3}{8} + \frac{7}{8} = \frac{10}{16}$$

$$\frac{2}{3} + \frac{1}{2} = \frac{3}{5}$$

$$\frac{4}{5} + \frac{1}{3} = \frac{5}{8}$$

$$\frac{2}{9} + \frac{1}{3} =$$

Francie:

$$\frac{3}{4} + \frac{5}{2} = \frac{8}{4} = 2$$

$$\frac{1}{6} + \frac{2}{3} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{2}{5} + \frac{7}{10} = \frac{9}{10}$$

$$\frac{3}{4} + \frac{1}{8} =$$

Gil:

$$\frac{1}{2} + \frac{1}{3} = \frac{2}{6} = \frac{1}{3}$$

$$\frac{2}{3} + \frac{4}{5} = \frac{6}{15}$$

$$\frac{3}{4} + \frac{2}{3} = \frac{5}{12}$$

$$\frac{5}{6} + \frac{1}{3} =$$

Helen: $\frac{1}{3} + \frac{1}{8} = \frac{2}{4} = \frac{1}{2}$ $\frac{1}{5} + \frac{4}{8} = \frac{5}{8}$ $\frac{7}{10} + \frac{1}{6} = \frac{8}{8} = 1$ $\frac{2}{3} + \frac{6}{7} =$

Ira: $\frac{1}{2} + \frac{3}{4} = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$ $\frac{5}{8} + \frac{1}{2} = \frac{5}{8} + \frac{4}{8} = \frac{9}{8} = \frac{3}{4}$

$\frac{3}{10} + \frac{2}{5} = \frac{3}{10} + \frac{4}{10} = \frac{7}{10} = \frac{1}{2}$ $\frac{1}{3} + \frac{2}{9} =$

Jane:
$$\begin{array}{r} 5\frac{1}{4} \\ -3\frac{3}{4} \\ \hline 2\frac{2}{4} = 2\frac{1}{2} \end{array}$$

$$\begin{array}{r} 2\frac{1}{7} \\ -1\frac{6}{7} \\ \hline 1\frac{5}{7} \end{array}$$

$$\begin{array}{r} 10\frac{3}{5} \\ -4\frac{4}{5} \\ \hline 6\frac{1}{5} \end{array}$$

$$\begin{array}{r} 6 \\ -3\frac{1}{3} \\ \hline 3\frac{1}{3} \end{array}$$

$$\begin{array}{r} 4\frac{1}{3} \\ -1\frac{2}{3} \\ \hline 3\frac{1}{3} \end{array}$$

Kelly:
$$\begin{array}{r} 4\frac{1}{4} \\ -2\frac{3}{4} \\ \hline 2\frac{8}{4} = 4 \end{array}$$

$$\begin{array}{r} 5\frac{1}{2} \\ -1\frac{4}{15} \\ \hline 4\frac{8}{15} \end{array}$$

$$\begin{array}{r} 1\frac{1}{10} \\ -5\frac{7}{10} \\ \hline 6\frac{6}{10} \end{array}$$

$$\begin{array}{r} 3\frac{5}{16} \\ -1\frac{7}{16} \\ \hline 2\frac{1}{8} \end{array}$$

Len:
$$\begin{array}{r} 6\frac{1}{5} \\ -4\frac{2}{5} \\ \hline 2\frac{3}{5} \end{array}$$

$$\begin{array}{r} 4\frac{1}{3} \\ -2\frac{1}{6} \\ \hline 2\frac{3}{6} = 2\frac{1}{2} \end{array}$$

$$\begin{array}{r} 8\frac{3}{8} \\ -5\frac{3}{8} \\ \hline 3\frac{6}{8} = 3\frac{3}{4} \end{array}$$

$$\begin{array}{r} 4\frac{1}{2} \\ -3\frac{1}{4} \\ \hline 1\frac{1}{4} \end{array}$$

Marcy: $\frac{1}{2} \times \frac{3}{2} = \frac{3}{2}$ $\frac{2}{3} \times \frac{1}{4} = \frac{8}{12} \times \frac{3}{12} = \frac{24}{12} = 2$

$\frac{7}{8} \times \frac{1}{5} = \frac{35}{40} \times \frac{8}{40} = \frac{280}{40} = 7$ $\frac{3}{4} \times \frac{5}{7} =$

Nate: $\frac{2}{5} \times \frac{1}{2} = \frac{4}{5}$ $\frac{4}{9} \times \frac{3}{2} = \frac{8}{27}$ $\frac{3}{8} \times \frac{1}{3} = \frac{9}{8}$ $\frac{2}{5} \times \frac{5}{8} =$

Ozzie: (How many answers are wrong?)

$\frac{2}{3} \times \frac{3}{5} = \frac{2}{15} \times \frac{3}{15} = \frac{6}{15} = \frac{2}{5}$ $\frac{1}{4} \times \frac{8}{3} = \frac{1}{12} \times \frac{8}{12} = \frac{8}{12} = \frac{2}{3}$

$\frac{3}{2} \times \frac{4}{5} = \frac{3}{10} \times \frac{4}{10} = \frac{12}{10} = \frac{6}{5} = 1\frac{1}{5}$ $\frac{1}{2} \times \frac{5}{6} =$

Peg: $2\frac{1}{2} \times \frac{1}{5} = 2\frac{1}{10}$ $1\frac{2}{3} \times 3\frac{1}{7} = 3\frac{2}{21}$ $2\frac{1}{2} \times 5\frac{4}{5} = 10\frac{2}{5}$ $4 \times 2\frac{1}{7} =$

Quint: $\frac{3}{4} \div \frac{1}{4} = \frac{3}{4}$ $\frac{9}{10} \div \frac{3}{10} = \frac{3}{10}$ $\frac{8}{9} \div \frac{2}{9} = \frac{4}{9}$ $\frac{9}{8} \div \frac{3}{8} =$

Rae: $\frac{8}{5} \div \frac{2}{8} = \frac{1}{5} \times \frac{1}{2} = \frac{1}{10}$ $\frac{6^3}{7} \div \frac{1}{4_2} = \frac{3}{7} \times \frac{2}{1} = \frac{6}{7}$ $\frac{4}{9} \div \frac{3}{5} =$

Sam: $1\frac{2}{3} \div \frac{1}{2} = 1\frac{2}{3} \times \frac{2}{1} = 1\frac{4}{3}$ $2\frac{3}{4} \div \frac{3}{2} = 2\frac{3}{4} \times \frac{2}{3} = 2\frac{1}{2}$ $5\frac{1}{2} \div \frac{3}{4} =$

Tiny: $6\frac{9}{10} \div 3 = 2\frac{9}{10}$ $8\frac{5}{6} \div 2 = 4\frac{5}{6}$ $4\frac{1}{3} \div 2 = 2\frac{1}{3}$ $6\frac{3}{4} \div 2 =$

Ursula: $\frac{2}{3} \div \frac{1}{2} = \frac{4}{6} \div \frac{3}{6} = 1\frac{1}{6}$ $\frac{7}{8} \div \frac{2}{3} = \frac{21}{24} \div \frac{16}{24} = 1\frac{5}{24}$ $\frac{3}{4} \div \frac{1}{5} =$

Vince:
$$\begin{array}{r} 1.57 \text{ R2} \\ 3 \overline{) 4.73} \\ \underline{3} \\ 17 \\ \underline{15} \\ 23 \\ \underline{21} \\ 2 \end{array}$$

$$\begin{array}{r} 2.2 \text{ R4} \\ 2.1 \overline{) 4.66} \\ \underline{4.2} \\ 46 \\ \underline{42} \\ 4 \end{array}$$

$$\begin{array}{r} .39 \text{ R2} \\ 5 \overline{) 1.97} \\ \underline{1.5} \\ 47 \\ \underline{45} \\ 2 \end{array}$$

Walt: For $.7 \div 1.4$, works $.7 \overline{) 1.4}$
For $5 \div 1.25$, works $5 \overline{) 1.25}$

22. Here are a few more error patterns with decimals.

a. $1.24 \times .6$
$$\begin{array}{r} 1.24 \\ \times .60 \\ \hline 0.00 \\ 74.40 \\ \hline 74.40 \end{array}$$

b. $6 - 1.37$
$$\begin{array}{r} 6 \\ - 1.37 \\ \hline 5.37 \end{array}$$

c. $3 \overline{) 8} \frac{2.2}{2}$

23. Finally, here are some common exponent and square root errors. Can you figure them out?

a. $2^3 \times 2^5 = 4^8$ $3^2 \times 3^3 = 9^5$ $10^2 \times 10^4 = 100^6$

b. $10^2 \times 10^3 = 10^6$ $2^4 \times 2^3 = 2^{12}$ $3^2 \times 3^5 = 3^{10}$

c. $2^3 = 6$ $10^2 = 20$ $4^3 = 12$ $5^2 = 10$

d. $3 \times 10^2 = 900$ $2 \times 10^3 = 8000$ $5 \times 10^2 = 2500$

e. $4^3 + 4^2 = 4^5$ $10^2 + 10 = 10^3$ $2^3 + 2^4 = 2^7$

f. $\frac{10^6}{10^2} = 10^3$ $\frac{2^8}{2^4} = 2^2$ $\frac{3^{10}}{3^2} = 3^5$

g. $\sqrt{9 + 16} = 3 + 4 = 7$ $\sqrt{12^2 + 8^2} = 20$ $\sqrt{25 + 64} = 13$

24. Are there other error patterns that are not mentioned above but that you have noticed? (For example, no illustrations of ordering fractions or decimals have been given here.)
25. These exercises refer to the students who used the error patterns in exercises 20-21. One sample exercise is repeated, with at least part of the student's explanation given. The number in parentheses is the answer to the incomplete exercise for the student in number 20 or 21. Choose the students you find most interesting (please don't skip Emma and Ozzie) and plan remediation for them.

Al:
$$\begin{array}{r} 41 \\ \times 36 \\ \hline 126 \end{array}$$
 "6 times 1 is 6; 3 times 4 is 12." (92)

Bea:
$$\begin{array}{r} 304 \\ \times 506 \\ \hline 1824 \\ 000 \\ 1520 \\ \hline 17024 \end{array}$$
 "... the 0 in 1520 doesn't matter. You can put it in or leave it out, just like the 000 row. . ." (24648)

Carl:
$$\begin{array}{r} 56 \\ 15 \overline{) 7590} \\ \underline{75} \\ 90 \\ \underline{90} \\ 0 \end{array}$$
 "... 15 won't go into 9, so you bring down the 0 . . ." (32)

Dan:
$$\begin{array}{r} 630 \\ 8 \overline{)4824} \\ \underline{48} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

"... You've got to put a 0 in the answer to fill it up even with below . . ."

(410 R9)

Denise:
$$\begin{array}{r} 12 \\ 43 \\ \times 35 \\ \hline 1315 \end{array}$$

"... carry the 1. 5 x 4 is 20, plus 1 is 21. Write down the 1, carry the 2 . . ."

(14112)

Derrold:
$$\begin{array}{r} 103 R2 \\ 3 \overline{)419} \\ \underline{1} \\ 10 \end{array}$$

"I do short division. 3 into 4 once, 1 left. 3 into 1 won't go, 1 left. 3 into 9, 3, none left over. 1 and 1 is 2. Remainder = 2."

(1001 R6)

Emma:
$$\frac{3}{8} + \frac{7}{8} = \frac{10}{16}$$

"3 and 7 is 10, over 8 and 8 is 16." $(\frac{3}{12})$

Francie:
$$\frac{3}{4} + \frac{5}{2} = \frac{8}{4} = 2$$

"5 + 3 = 8. You don't add the bottom numbers because 2 will go into 4."

[Lankford, 1972, p. B-1]

$$(\frac{4}{8} = \frac{1}{2})$$

Gil:
$$\frac{1}{2} + \frac{1}{3} = \frac{2}{6} = \frac{1}{3}$$

"The bottom is 2 times 3. 1 and 1 is 2 . . ."

$$(\frac{6}{18})$$

Helen:
$$\frac{1}{3} + \frac{1}{3} = \frac{2}{4} = \frac{1}{2}$$

"Well, you cancel the 2s. Then 1 and 1 gives 2 and 3 and 2, no 1, gives 4 . . ."

$$(\frac{4}{8} = \frac{1}{2})$$

Ira:
$$\frac{1}{2} + \frac{3}{4} = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$

"The l.c.d. is 4. Got to take 2 times the 2 to get 4, then add 1 and 3 . . ."

$$(\frac{3}{9} = \frac{1}{3})$$

Jane:

$$\begin{array}{r} 5\frac{1}{4} \\ -3\frac{3}{4} \\ \hline 2\frac{2}{4} = 2\frac{1}{2} \end{array}$$

" $\frac{1}{4}$ from $\frac{3}{4}$ is $\frac{2}{4}$. . ." $(3\frac{1}{3})$

Kelly:

$$\begin{array}{r} 5\cancel{1}2 \\ \overline{15} \\ -1\overline{15} \\ \hline 4\overline{8} \\ \overline{15} \end{array}$$

"You borrow 1 from the 6, that gives 12 . . ."

$$(1\frac{8}{16} = 1\frac{1}{2})$$

Len:

$$\begin{array}{r} 6\frac{1}{5} \\ -4\frac{2}{5} \\ \hline 2\frac{3}{5} \end{array}$$

" $\frac{1}{5}$ and $\frac{2}{5}$ is $\frac{3}{5}$. 6 take away 4 is 2." $(1\frac{3}{4})$

Marcy:

$$\frac{1}{2} \times \frac{3}{2} = \frac{3}{2}$$

"The bottoms are the same, so you 1 times 3 and put that over 2."

$$(\frac{420}{28} = 15)$$

Nate:

$$\frac{2}{5} \times \frac{1}{2} = \frac{4}{5}$$

"2 times 2 is 4 and 5 times 1 is 5." Note: Nate's teacher had mentioned "cross multiplying" when discussing equivalent fractions.

$$(\frac{16}{25})$$

Ozzie:

$$\frac{2}{3} \times \frac{3}{5} = \frac{2}{15} \times \frac{3}{15} = \frac{6}{15} = \frac{2}{5}$$

"The l.c.d. is, uh, 15 . . . Then 2 times 3 is 6, over 15 . . ."

$$(\frac{5}{12})$$

Peg:

$$1\frac{2}{3} \times 3\frac{1}{7} = 3\frac{2}{21}$$

" $\frac{2}{3}$ times $\frac{1}{7}$ is 2 over 21, 1 times 3 is 3." $(8\frac{1}{7})$

Quint:

$$\frac{3}{4} \div \frac{1}{4} = \frac{3}{4}$$

"3 divided by 1 is 3, over 4." $(\frac{3}{8})$

Rae:

$$\frac{\cancel{1}}{5} \div \frac{2}{\cancel{1}} = \frac{1}{5} \times \frac{1}{2} = \frac{1}{10}$$

"Cancel the 3s first . . . Nothing else to cancel, so . . ."

$$(\frac{20}{3})$$

Sam:

$$1\frac{2}{3} \div \frac{1}{2} = 1\frac{2}{3} \times \frac{2}{1} = 1\frac{4}{3}$$

"Invert the 1 over 2 and multiply . . ."

$$(5\frac{2}{3})$$

Tiny: $6\frac{9}{10} \div 3 = 2\frac{9}{10}$

"3 goes into 6 twice." $(3\frac{3}{4})$

Ursula: $\frac{2}{3} \div \frac{1}{2} = \frac{4}{6} \div \frac{3}{6} = 1\frac{1}{6}$

"Get the denominators the same . . . then
3 goes into 4 once with 1 left over. $1\frac{1}{6}$."
 $(3\frac{3}{20})$

Vince and Walt do not have to speak.

References and Further Readings

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- Lankford suggests that audio-taping an interview can help the student interviewed, or other students, by listening to the playback.*
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Rising, Gerald. "What are the basic skills of mathematics?" The NIE Conference on Basic Mathematical Skills and Learning, Volume I: Contributed Position Papers. HEW/NIE, October, 1975, pp. 148-160.

In particular, Professor Rising feels that central to mathematics is mathematical trust--"student belief in the consistency of mathematics, in the fact that today's ideas will last overnight, and in their ability to figure things out for themselves." (p. 150) Note the implications for remediation!

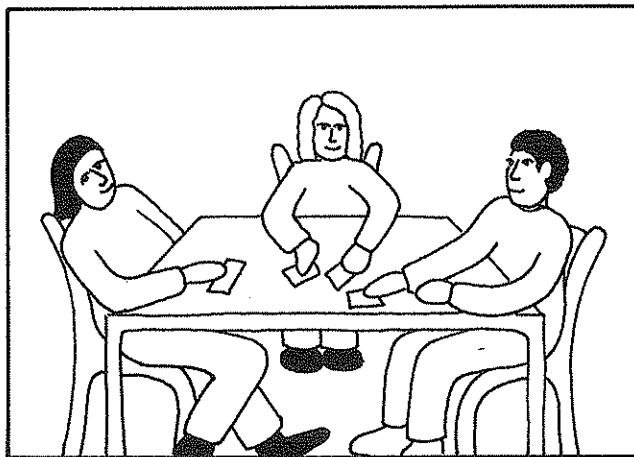
NUMBER SENSE AND ARITHMETIC SKILLS

Placement Guide for Tabbed Dividers

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GOALS THROUGH GAMES

Educational games have become increasingly popular over the last decade. Although simulation games, prompted by military, business training and college use, get most of the popular-press attention, nonsimulation games are much more common in mathematics classrooms. (See *Mom & Pop Steel Co.* in *WHOLE NUMBERS: Mixed Operations* for a sample of a simulation activity in mathematics.) This section considers uses for games, summarizes some promising research on games and teams, lists some desirable features to look for in a game, and adds a few points to keep in mind when using games.



USES OF GAMES

Disguise Practice

One problem for many arithmetic topics in middle school is that students feel that they "already know that stuff" and aren't overly interested in knowing it better. Games can disguise drill by making it part of a larger, desirable context. Many games can be played at home, offering an excellent opportunity for "fun" out-of-school practice and, with luck (and your skillful promotion), parent participation.

Variety

Many teachers use games for a change of pace. Anything that can add to your repertoire of variety-producers safeguards against the homework-lecture-assignment rut. Students can make clever adaptations of commercial games, television quiz shows or sports to mathematical themes. Teachers also report success in asking students to make up games for younger students, an activity which can give some built-in review (you would, of course, ask them to try the game to work out the bugs!).

Enrichment

Many of the games described in this resource are oriented toward computational drill. On closer look, you will see that more than computation may be involved, especially in games where you can thwart your opponent by looking ahead (e.g., *Game of 50* in *WHOLE NUMBERS: Getting Started*). Hence, carefully chosen games may serve objectives like problem solving in addition to providing some drill and practice. Having a wide selection of games may enable you to pick and choose to meet individual differences better or to help early finishers find something worthwhile to do.

NOTE: Unless otherwise noted, subsections cited are in the *LARGE & SMALL NUMBERS* section of the resource Number Sense and Arithmetic Skills.



Opportunity for Diagnosis

During many games you can be relatively free to observe students in action. The plays they make, their confidence, and their relationships with their fellow players can give valuable information about their achievement.

Attitudes

Since you have probably used a game in class, you already know the high interest level that games seem to produce. It may sometimes seem to be interest only in winning but, given the interest, perhaps we can take advantage of it. In some classes having everyone pay attention might be reward enough! A game may also stimulate a student's interest in learning the skills or facts needed in the game.

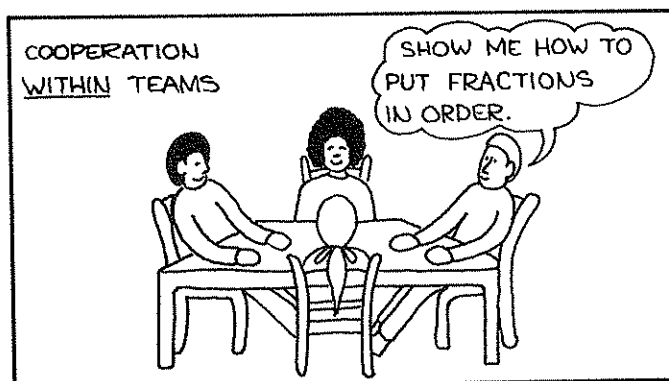
COMPETITION AND COOPERATION?

Mr. Denson: *Wow! That seemed to perk everyone up, even Charley Dinkle. I made up two teams, the left three rows against the right three rows, and then just had a competition, one player from each side working at the board on a division exercise. The kids really liked it . . . It was hard to keep them from shouting out hints, though. And most of the kids didn't work on the exercises at their seats, as I told them to . . . And poor Nancy Kinkle didn't want to take her turn and was mortified when everybody laughed--she didn't get a single digit written before Jane Daly beat her . . . And I thought Jimbo Smith was going to fight Don Adams when Don got on him for losing to Jon Fish . . . Maybe we didn't accomplish all that much after all.*

Using games is not a panacea of course; they can be over-used or misused, as can most teaching tools. Like Mr. Denson, you may have had some second thoughts about the over-competitiveness that may develop during a game and interfere with the learning you had planned--or that may produce side-effects you cannot tolerate. Competition clearly is a motivator (except, perhaps, for the Nancy Kinkles). How can we best use it?

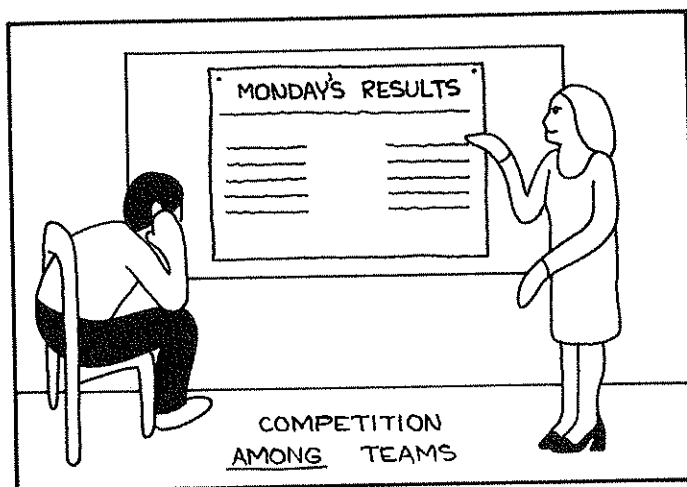


Here is one possible answer: Use an extended competition among small teams. The team-versus-team aspect uses the competition motivator, but the team structure encourages team members to cooperate with each other to learn techniques, facts and strategies.



How the teams are made up is important, of course. Four students on a team seem to work well; teams should be heterogeneous in ability. There will be practice periods during which team members can "tutor" each other, so having homogeneous abilities or too many on a team would likely detract from the tutoring.

The students should first meet with their teams and have some practice periods both to learn from each other and also to establish a number-1-player, number-2-player, etc., order for the official competition. When the "real" competition starts, all the number-1-players play among themselves in groups of three or four, as do all the number-2-players, etc. (Note that most of the games in this resource can be played in small groups.) To keep the competitors as equal as possible, for the next day of play the winner in each group moves to a group of greater ability and the loser in each group to a group of lower ability (with obvious exceptions in the highest and lowest groups), other players staying in their groups.



Some games have a built-in scoring system; if not, it is usually no trouble to design one. For example, in a four-player group the winner of the greatest number of rounds or hands could get 4 points, the winner of the second greatest number could get 3 points, and so on. At the conclusion of a round of competition, each person's score is handed in, each team's score is calculated, and cumulative team standings are determined (the results and standings might even be reported in a "newsletter" the next day). Interspersing within-team practice periods with rounds of competition provides a chance for more peer tutoring.



There are several things you'll have to play by ear. You may want to be flexible in scheduling the rounds of competition; perhaps start with half a period of practice one day a week, then half a period of competition another day of the week. Relating the particular game that is to be played to current or recent class work should not be too hard. You may wish to have different groups playing different games. If, for example, a few students are not good enough at fractions to play a fractions game, they could be assigned a whole numbers game. You may want to change teams around if, say, one team is super-strong or super-weak, if the team members cannot work with each other, or if you want to get a greater student "mix" periodically. Although the noise level may rise since several students may be talking at once, discipline seems not to be a problem because the students become so involved. You may need to keep a closer eye on the plays of the lower-ability groups as they tend to be less critical of an opponent's plays. You will likely have to play down the competitiveness, perhaps by emphasizing the teamwork ("The Skew-balls team must be helping each other a lot since they have been coming up in the standings lately") or by emphasizing the luck that enters into most games ("The cards that the Euclid's Angels were getting weren't very helpful"). Although some students may thrive on competition, others--because of their abilities or personalities--may shrink from it. Such students should not, of course, be forced to compete with others but can work with solitaire games or self-competition games while the others are competing.

What Does All This Do?

(a) The extended competition among teams provides a team spirit feeling: students won't want to let the team down, and stronger team members can help the weaker ones improve during the practice sessions. (b) Using small groups for competition increases the amount of time each student is working (compare Mr. Denson's setting) and, partly because of the matching of abilities, increases each student's chances of winning since he is now competing against 2 or 3 opponents of about equal ability, not 30 with some of the 30 unbeatable. (c) Each student competes individually and is accountable. (d) Since in most games each player is subject to evaluation by the opponents, the player gets immediate feedback on his thinking and perhaps some additional instruction in being shown that the play was incorrect. He is usually actively evaluating his opponent's plays also.

In some carefully monitored settings, teams and games have produced (a) greater achievement, (b) more student-student help, (c) a "rub-off" in increased attention during regular instruction [Edwards, DeVries & Snyder, 1972; DeVries and



Edwards, 1973, 1974], and (d) even improved attendance [Allen and Main, 1976]. The extra work in keeping the records and making the assignments to tables may be shared by the teams--and can involve work with percent and statistics.

There can be other benefits, too. DeVries and Edwards [1973] note that "individuals express concern for each other when placed in small groups." [p. 316] There certainly will be lots of opportunities for student-student interaction. And, if your situation warrants, the use of teams may be of help in establishing more cross-racial or cross-ethnic helping and friendship [DeVries and Edwards, 1974].



FEATURES GAMES SHOULD HAVE

1. Your students should like the game. If they don't but you see value in it, ask for suggestions on how to change the rules to make the game more interesting. Perhaps something can be salvaged.
2. The game should serve an objective of the course. Playing checkers, say, just to fill in the time or to entertain the students would be difficult to justify (even on "success experience" grounds). If the game is to provide practice with a skill, the skill should be necessary for winning at the game and the students should have enough background in the skill to make their playing the game a reasonable undertaking.
3. Chance should play a role in the game. The "luck of the draw" can give the less-skilled player a chance to win--or an excuse if he doesn't. An element of chance may be less important if the players are of equal ability, but it still supplies an alibi for the loser as well as an added degree of excitement.
4. All students should be participating during every play. Some "whole-class" games involve only one or two students at a time, with a premium being placed on speed. For example, "Travel" (or "Around the World," in which a student stands by someone's desk, the teacher presents a problem, the first of the two to answer correctly "wins" and gets to stand by the next desk, etc.) may involve only two students at a time, with the faster player always winning. Games which can involve the whole



class (e.g., *2 Place-Value Games* in WHOLE NUMBERS: Concepts) or several students in small groups (e.g., *Fraction Dominoes* in FRACTIONS: Concepts) are more efficient in terms of practice provided; these examples also give less emphasis to speed, although there may be times when developing speed is the intent. Maximal participation can be approached in small-group games by building in a reward for evaluation of an opponent's play. In large-group games (like Mr. Denson's competition), getting everyone doing something may be difficult. Sometimes allowing challenges to the opponents or "saving" your team with a right answer gets more students involved. Better yet would be to plan the game so that the problem is given before the choice of the particular player(s)—see also number 8 on page 8.

5. Adaptable means more useful. This ideal has several aspects: adaptability to students of different abilities, adaptability to other topics in the curriculum, and flexibility in playing time required. The first aspect is obviously desirable. The second aspect can mean a saving in equipment or in time required to explain the rules. The third aspect reminds us that materials for some games take time to pass out or that the length of a "round" may not fit the time available.

MISCELLANEOUS POINTS

1. It is a good idea to play any game yourself before introducing it to the class. This helps you see whether it does fit your objectives, eases your explanation to the players, and familiarizes you with the game so you can put the rules down and watch the students while they are playing.

2. If the class needs to be separated into teams, you might avoid having 5 or 6 students choose their teams. You know the students' abilities better (unless it is early in the year) and you can save some student the ignominy of being the last one chosen. If you feel that your assigning students to teams would result in too many miffed feelings, see number 10 on page 9 for some alternatives.

3. Settle ahead of time how you will handle cheating. Many times having the students decide on how cheating should be dealt with can result in a worthwhile discussion. Ordinarily, loss of turn, penalty points or expulsion from the game is a sufficient deterrent. The students usually do a thorough job of monitoring the other team for cheating! Penalties should be clearly defined before play is started.

4. Do whatever you can to avoid the over-competitiveness. You may have to choose only games (like variations of Bingo) which usually do not get out of hand. Careful remarks before the game starts may help establish some guides: "Remember



that this is just a game we're playing for fun and to practice . . . (whatever). We'll have to stop if it gets too noisy or if there's bad sportsmanship. People are going to make mistakes just because they're excited, so don't make fun of anyone. Don't get down on yourself if you goof up . . ."

5. Give encouragement to the perpetual loser. Unless he--and you--are very unlucky, a game with an element of chance and roughly equal opponents should result in his winning every now and then. If not, be ready to give some encouragement ("you've really improved a lot," "your team is right up there today"), blame the fates, or offer him an escape (to another game, to a puzzle, to being your lieutenant, . . .).

6. Get whatever mileage you can from a popular game that doesn't fit your current objectives. Invite the students to take the game home to play with parents or siblings.

7. Be on the lookout for new games. You might read journals, attend professional meetings and visit exhibits, talk to other teachers, or find a workshop that spends some time on games. If you buy a game, keep the receipt--it is tax-deductible.

?? ??

1. (Discussion) Describe your experience with games in your classes (mention flops or disappointments, if any, also).
2. What objective(s) might be served by playing *Cooperation* in FRACTIONS: Concepts?
3. Mr. Denson tries again. Evaluate the teaching strategy described in this vignette:

"It was just like an old-fashioned spelling bee. I lined everyone up around the room and started going around, giving each student a basic fact or a mental computation. If he missed, he sat down. When there was a small enough number left to fit at the board space, I started using harder calculations and let them use the board. Joey Dorr did not win--made a silly mistake. Claire Flagg started taking too much time, so I had to put a time limit on. No cheating this time, but the ones at their seats really had to be reminded to work on the problems too. Wish I could figure out a way to get them working . . . Claire won, which I could have predicted after Joey went down."

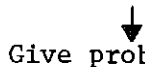
4. a. Adapt one or more of the following to a game usable in one of your classes:
 - i. a card game (e.g., *Fraction Blackjack* in FRACTIONS: Addition/Subtraction)
 - ii. a board game (e.g., Monopoly, Scrabble, Bingo)
 - iii. a TV quiz show (see Homan [1973] for ideas)
 - iv. a sport (see Karau [1956] for a version of football)
- b. Give the assignment above to your students.



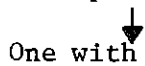
5. The faculty lounge guru: "Your problem, Miss Nodgen, is that you worry too much about shielding the dumb kids. They need to face up to the fact that they aren't going to be academic whizzes. They might as well play your games against the best students right now rather than have you lead them into thinking they're hot stuff."
Your reaction:
6. Sometimes there seems to be little value in using your time and energy or in taking a portion of a class period to make a particular card deck or other materials that might be needed. Evaluate the following sources to see whether they fit your situation.
 - a. An enthusiastic audio-visual department
 - b. Student volunteers
 - c. Students who need "extra credit"
 - d. Earlier finishers on tests
 - e. Students from a study hall or from detention
 - f. Teacher aides or paraprofessionals
 - g. Parents
 - h. An inservice day
 (Note: Well-made, "professional-looking" material seems to get better care from the students.)
7. Suppose you are involved in an "individualized" program and see games as one means of providing some variety. Give merits and demerits of each of these:
 - a. Adopt a team-competition, with team practice for half a period on Wednesdays and competition all period on Fridays.
 - b. Have a weekly game day, with students free to choose whatever game they wish to play.
 - c. Have an occasional game day, with students assigned to a particular game (different games would likely be played simultaneously with game assignments based on individual needs).
 - d. Have every Friday be game day, but only for those students who are caught up in their work or who have "worked hard" all week.
 - e. Let the class set goals and decide when to have games and what games to play.
 - f. Your idea.
8. a. Contrast these two procedures for large-team competition (e.g., half of the class versus the other half).

I

Choose competitor from each team.



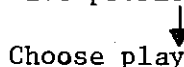
Give problem.



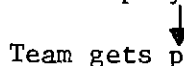
One with first correct response gets a point for the team.

II

Give problem. Let work.



Choose player from one team.



Team gets point if player gives correct answer; if not, player chosen from second team to try. (Other team judges whether an answer is correct.)

- b. Contrast these methods for choosing the next player(s).
 - I. Use the next student in the seating order.
Toss a marker on a seating chart or in the grade book.
 - II. Then call on the student indicated.
 - II'. Call on whomever your judgment indicates, no matter where the marker lands.
 - III. Choose from volunteers.

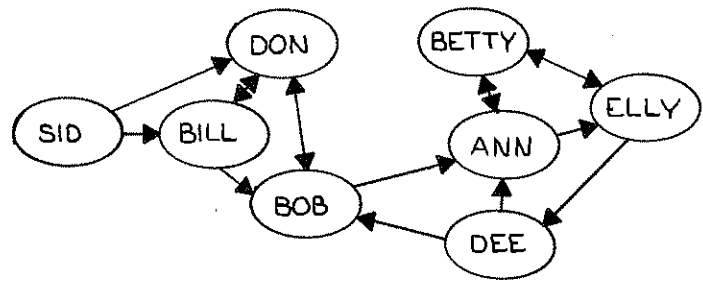


9. Suppose you are running a 2-team competition as in number 8. Evaluate these ways of deciding what problem to give next.
 - a. As they come on your list.
 - b. Matched to the ability of the student currently playing.
 - c. Level chosen by the student (e.g., in a "quiz-show" game, the student could ask for a 5-point, 10-point, . . . question; in a baseball adaptation, there could be home-run level questions, 3-base hit exercises, etc.).
10. What are some advantages/disadvantages of these ways of separating a class into teams?
 - a. Count off. If there are to be 6 teams, have students count off: 1,2,3,4,5, 6,1,2,. . . . All the 1's make up one team, all the 2's another, etc.
 - b. Have the slowest students pick the teams. (Caution: will they pick only their friends?)
 - c. Have 6 students of equal ability pick even teams. Then you assign each "picker" to one of the teams.
11. (Discussion) After 5 weeks of competition, here are the team standings:

Googols	69	
Multipliers	35	
Skew-balls	33	
Hexagons	27	What would you do?
Primes	26	
Euclid's Angels	11	
12. (Discussion) Suppose that each class at your school has set up a team competition. Would you . . .
 - a. have challenges between class sections at the same grade level?
 - b. have challenges between the champions of each grade?
 - c. form teams of "all-stars" for each grade?
13. Suppose you decide to try a team competition arrangement with one of your classes.
 - a. Decide which students should go on which teams. Should good friends be on the same team? Should "enemies"?
 - b. Choose the first game you will use.
 - c. Plan at least the schedule you will use for the first practice period(s) and the first round of competition. (Allow some time during the first practice session for teams to choose their names.)
 - d. Plan your explanation to the students about the competition, the schedule, how the first game is played, the scoring system for the first game.
14. Suppose you are using a team competition. You are thinking of reporting the results in a newsletter or the school paper. What are some of the arguments for and against doing this? Would you report the scores of individual students or only team scores and standings? (See *Student Self-Concept* in the WHOLE NUMBERS section.)
15. There might be valuable information in making a sociogram for a class. In your (smallest, for starters) class, ask the students to put down their names and to list 3 classmates they would like to work with if you choose to set up some small groups or some teams for games. First, draw arrows in a class list to



choices. The result will be messy and usually needs reorganizing so that mutual choices are close to each other. The sample to the right shows the common way of reorganizing the information. Compare the final results to compare your predictions and to note "isolates." The results might help you to get a more cohesive class.



16. Having heard that a team competition may help attendance, you are surprised that Nickie seems to be absent nearly every day of competition. Evaluate these courses of action.
 - a. Establish a rule: If a team member is absent, the team gets 0 points for the missing person.
 - b. Talk to Nickie to see what the problem is.
17. How would you handle these two situations in a competition setting?
 - a. Myra, one of the stronger students, gets very frustrated when she or her team does not win.
 - b. In a team competition, the "last" table is called the "stupidos" by some students who are always at higher-ability tables.
18. (Outside reference) See what you can find about the pros and cons of competition for middle school students.
19. (Research critique, outside reference) Critique the Allen, Allen and Ross study (1970), in which the game-playing group gained an average of 20.9 points in nonlanguage IQ score. Then read the "Letters to the Editor" section in the March, 1971, issue of Simulation and Games.

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Stanley and Schild criticize the Allen et al. [1970] study, and Allen et al. [1971] reply. A more complete version of the data appear as promised in the June, 1971, issue of Simulation and Games, pp. 250-251.

NUMBER SENSE AND ARITHMETIC SKILLS

Placement Guide for Tabbed Dividers

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CALCULATORS

RATIONALE

As early as the seventh century B.C. the counting board or abacus was invented and used for simple whole number computations. Merchants and traders of ancient times probably would have found the abacus cumbersome to carry around in their back pocket. If they were alive today, they could not only have a calculator in their pocket but they might have a computer terminal in their briefcase! Electronic calculators are one of the hottest selling items around the world. They are becoming as popular and inexpensive as watches. They give instantaneous effortless answers to many computations. They are small, quiet and cheap.

Using a calculator is relatively easy. You push a few buttons in sequence and "Voilà!" the keyboard display flashes the answer. "Most of us have so far explored numberland by the very laborious, manual route. The hand calculator lets you travel by automation, and explore far afield effortlessly." [Wallace Judd] Paper and pencil calculations are often slow, inaccurate and tiresome. Interest and enthusiasm for mathematics is often killed by such drudgery. The calculator becomes a fantastic tool that frees us to do investigations and problem solving. Its speed allows us to keep pace with our racing minds as we search for solutions, conjectures, and more questions.

The electronic calculator is NOT a fad; it is here to stay. Like the radio and television, soon everyone may own one (or two or three). The calculator is bound to change our way of life just as other advances in technology have. Already educators are arguing about the use of the calculator in the mathematics classroom. Should the calculator be used when teaching arithmetic skills in elementary schools? Will children need to memorize addition and multiplication facts if they learn to compute using a calculator? Will senior high students need to learn how to use logarithmic tables or should they use an electronic calculator instead? In other words, the whole mathematics curriculum from kindergarten through college will need to make serious adjustments to account for the use of the electronic calculator. Because the calculator is becoming available to all members of our society, including children, educators will need to decide how electronic calculators fit into the school curriculum.

Recently, pocket or desk calculators have been used in mathematics classrooms to motivate students and expand their ability to solve "messy" real-world problems (i.e., stock investments, tax forms, interest on car payments, pollution controls). The calculator provides the immediate feedback of answers and a problem-solving flexibility that



encourages the student to become involved in complex computations.

However, one needs to be careful!

Most calculators do not retain and display all the numbers or operations entered. If wrong numbers are entered or operations are entered in the wrong order (a faulty algorithm sequence), the incorrect answer must be recognized by the student. To tell a reasonable answer from an unreasonable one, a student needs to know how to compute using the basic arithmetic facts, how to round numbers, how to estimate and approximate answers, and how to place a decimal point. Arithmetic skills and number sense are very important if the hazards of a calculator are to be avoided. The calculator does not replace thought processes. It is a tool that saves time and energy and frees us to think and do mathematics above the computational level.

SUMMARY

I. Calculators fit into the classroom in different ways:

1. Non-electric calculators (abacus, etc.)
 - a. teach concepts in counting, place value, and arithmetic computations, and
 - b. demonstrate algorithms for solving computational problems.
2. Electronic calculators free the students from tedious pencil and paper calculations. They allow the student to . . .

- a. speed up "messy" calculations, and
- b. investigate and work on mathematical problems and applications that would otherwise involve long, unmanageable calculations.

II. The teacher can prepare students for electronic calculators by . . .

1. Emphasizing estimation and approximation skills which are vital in checking answers and placing the decimal point correctly.
2. Teaching the student to determine the reasonableness of exact answers by approximate calculations.
3. Introducing situations and problems where the hand calculator is an obvious aid to cumbersome, time-consuming calculations.
4. Asking students what types of mistakes can be made while using the calculator.

III. Teachers can prepare themselves for using the electronic calculator in instruction by . . .

1. Experimenting with it themselves. (Let the students see the teacher using a calculator.)
2. Reading current periodicals and checking the mathematics publication companies for new "calculator" books. (There is currently no body of knowledge about how to use a calculator in the classroom.)
3. Having an open mind about the use of the calculator before deciding that the calculators will be a "cure-all" to teaching computation, or that they should be banned from the mathematics curriculum.

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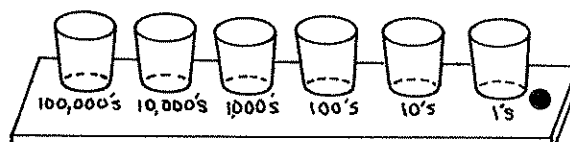
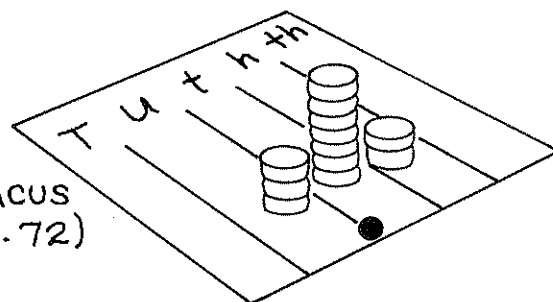


EXAMPLES OF CALCULATOR ACTIVITIES FOUND IN THE CLASSROOM MATERIALS

I. Non-electronic Calculators

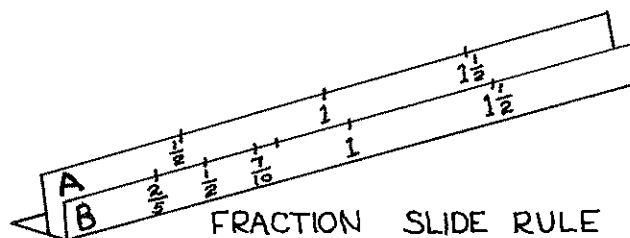
The abacus is a concrete model for place value, addition and subtraction, multiplication and division by powers of ten, and expanded notation.

CHIP ABACUS
(SHOWING 3.72)



CUP ABACUS

Here a slide rule is used as a physical model for addition and subtraction of fractions.





FRACTION SLIDE RULE

II. Electronic Calculators

This activity is an introduction to using the calculator with some problems that relate addition and multiplication; subtraction and division.

CALCULATOR CAPERS I

WHAT IS THE LARGEST NUMBER YOU CAN SHOW ON THE CALCULATOR?
 ADD $792 + 792 + 792$. WHAT IS YOUR ANSWER?
 DO $792 + 792 + 792$ AS A MULTIPLICATION PROBLEM.
 DO $192 + 192 + 192 + 192 + 192$ AS A MULTIPLICATION PROBLEM.

DIVIDE 45 BY 15.
 SUBTRACT 15 FROM 45 AS MANY TIMES AS POSSIBLE. HOW MANY?
 DID YOU GET THE SAME ANSWER FOR THE LAST TWO PROBLEMS?



CALCULATED CODES

USE ME TO FIND THE DECIMAL
VALUE OF EACH OF THE FRACTIONS, THEN
CRACK THE CODE BY ORDERING FROM
THE SMALLEST TO THE LARGEST
FROM LEFT TO RIGHT.

A calculator, a partner,
and a difficult task
becomes more enjoyable.

Ever wonder how many days,
minutes, seconds you have been
alive? A calculator can assist
you in problem-solving investi-
gations and explorations.

CALCULATOR CAPERS II

TAKE YOUR
AGE AT
YOUR LAST
BIRTHDAY
(IN YEARS)
AND FIGURE ON
THE CALCULATOR
THE FOLLOWING
INFORMATION.

1. THE NUMBER OF MONTHS YOU'VE BEEN ALIVE _____
2. THE NUMBER OF WEEKS YOU'VE BEEN ALIVE _____
3. THE NUMBER OF DAYS YOU'VE BEEN ALIVE _____
4. THE NUMBER OF HOURS YOU'VE BEEN ALIVE _____
5. THE NUMBER OF MINUTES YOU'VE BEEN ALIVE _____
6. THE NUMBER OF SECONDS YOU'VE BEEN ALIVE _____

PERSISTENT NUMBERS

LOOK FOR A PATTERN IN THESE EXAMPLES:

34 → 12 → 2

78 → 56 → 30 → 0

77 → 49 → 36 → 18 → 8

IF YOU HAVE DISCOVERED THE RULE FOR THESE EXAMPLES, YOU SHOULD BE
ABLE TO COMPLETE THE FOLLOWING SEQUENCES. (HINT: THINK MULTIPLICATION.)

53 → _____ → _____

437 → _____ → _____ → _____

549 → _____ → _____

THE ABOVE PATTERNS WERE COMPLETED BY COMPUTING PRODUCTS OF THE DIGITS
OF ONE NUMBER TO FIND THE NUMBER TO ITS RIGHT. THE NUMBER OF ARROWS
REQUIRED TO OBTAIN A SINGLE-DIGIT NUMBER IS CALLED THE PERSISTENCE OF
THE NUMBER.

99 → 81 → 8 PERSISTENCE (NUMBER OF ARROWS) = 2

FIND THE PERSISTENCE OF EACH OF THE FOLLOWING NUMBERS ON A SEPARATE
PIECE OF PAPER. (HINT: YOUR ANSWER SHOULD FORM TWO FAMOUS DATES.)

1. a) 91 b) 77 c) 2688999 d) 25
2. a) 321 b) 68889 c) 839844 d) 6788

TWO VERY PERSISTENT NUMBERS ARE 3,778,888,999 AND 277,777,788,888,999.
ARE YOU PERSISTENT ENOUGH TO FIND THEIR PERSISTENCE? MAYBE YOU CAN
FIND A HAND CALCULATOR TO HELP YOU.

Problem-solving activities
done with a calculator
encourage students to look
for patterns and answers
instead of grumbling about
all the "work."



CALCULATORS FOUND IN CLASSROOM MATERIALS

WHOLE NUMBERS:

Addition/Subtraction

PALINDROMES!

ADDITION

REVERSING DIGITS

ADDITION/SUBTRACTION

Multiplication/Division

PERSISTENT NUMBERS

MULTIPLICATION

FOUR INVESTIGATIONS

PATTERNS

REVERSALS

MULTIPLICATION

REVERSAL-LIKE PAIRS

MULTIPLICATION

STRETCH YOUR CALCULATOR

MULTIPLICATION

SUM NUMBERS

MULTIPLICATION

Mixed Operations

CALCULATOR CAPERS I

CALCULATOR

CALCULATOR CAPERS II

CALCULATOR

CURIOSITIES

PATTERNS

FRACTIONS:

Addition/Subtraction

FRACTION SLIDE RULE

NUMBER LINE MODEL

DECIMALS:

Multiplication/Division

CALCULATED CODES

RELATION TO FRACTIONS

9-TIME

RELATION TO FRACTIONS

SQUARE ROOT GAME

SQUARE ROOTS

Mixed Operations

"THE ANSWER"

COMPUTATION $+$, $-$, \times , \div



THAT'S JUST ABOUT THE
SIZE OF IT!

WORD PROBLEMS

UP AND DOWN WITH THE
CALCULATOR

ROUNDING

LARGE AND SMALL NUMBERS:

Awareness

STRETCHING YOUR DOLLARS

COMPUTATION-LARGE

YOUR HEARTBEATS

COMPUTATION-LARGE

SECONDS TIMELINE

COMPUTATION-LARGE

A FREE SUNDAE

COMPUTATION-LARGE

Exponential Notation

CHEERY SEQUENCES

POWERS OF TWO

SIMPLE SUMS

POWERS OF TWO

RUMORS

POWERS OF THREE

NUMBER SENSE AND ARITHMETIC SKILLS

Placement Guide for Tabbed Dividers

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Goals through Games	62
TEACHING EMPHASES	74
Calculators	74
Applications	82
Problem Solving	92
Mental Arithmetic	104
Algorithms	112
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Concepts	448
Addition/Subtraction	514
Multiplication/Division	558
Mixed Operations	584
DECIMALS	610
Concepts	610
Addition/Subtraction	664
Multiplication/Division	686
Mixed Operations	734
LARGE & SMALL NUMBERS	756
Awareness	756
Exponential Notation	780
ANNOTATED BIBLIOGRAPHY	814

APPLICATIONS

RATIONALE

Over 2000 years ago man developed number symbols, arithmetic calculations and geometry to describe and record real-world happenings. Mathematics was used to solve the problems of merchants, scientists, builders and priests.

About 600 B.C. Greek mathematicians took a different approach. They began studying numerical patterns and geometry for their aesthetic qualities. Mathematics became an intellectual exercise with no necessary applications in mind. The development of mathematics was soon traveling in two directions: practical or applied mathematics, originating from the Egyptians, and "pure" mathematics, originating from the Greeks.

Practical and "pure" mathematics are not always separable. One often inspires and directs the other; they become interwoven. As a result, applications of mathematics fall into three categories:

- 1) applications to real-life situations such as business, finance, sports, polls and census taking
- 2) applications to other disciplines (i.e., science, music, art)
- 3) applications to other branches of mathematics (problem-solving activities in the realm of "pure" mathematics)

The Egyptians, for example, were interested in learning as much as they could about their environment and how to control it. Today we are also curious

about the rapidly changing environment we have created. Because of the complexity of our culture and its emphasis on technology, mathematics is very important to us in our jobs, in our daily living and in our future.

We face many problems in our daily living. Since all problems require the collection of information before solutions can be found and analyzed, mathematics is often a helpful tool in solving problems; yet few people relate mathematics to real-life situations or real-life situations to mathematics.

Many teachers have neglected to teach applications of mathematics for a number of reasons:

- 1) "I have little background in applications of mathematics."
- 2) "My students often have little or no background in science, art, music and other disciplines."
- 3) "Applications require elaborate equipment and preparation."
- 4) "My students are not interested in applications."
- 5) "Good applications take too much time to teach. There is plenty to teach in the math textbook."
- 6) "How can my students apply mathematics when they do not even have basic computational skills?"

Yet educators and the public agree that applications of mathematics are very important and should be taught in the mathematics classroom. Society is demanding accountability and relevancy in our education system. Students need



ample opportunity to experience mathematics in a practical sense so that they will be better equipped to apply it as adults.

Even though certain applications of mathematics require special equipment and materials, much of this equipment can be constructed from inexpensive substitutes and common materials. Once the equipment is collected or made, it will last for years. Also, various applications can be adapted to fit available materials and equipment.

Applications should include appropriate topics and activities. Here are a few questions to consider when choosing an application of mathematics:

- a) Is it interesting to the students and the teacher?
- b) Does it start at the appropriate skill level?
- c) Does it extend and develop the computational and/or problem-solving skills of the students?
- d) Does it include topics, skills or ideas which might help the students contribute to society and deal with real-life situations?
- e) Could it be done as a laboratory activity?
- f) What concepts does it imply and develop?

- g) How much time would it take to teach?
- h) What equipment and materials are needed or available?

SUMMARY

1. Applications of mathematics fall into three categories:
 - a) applications to real life situations
 - b) applications to other disciplines, and
 - c) applications to other branches of mathematics.
2. Down through the centuries, mathematics has been a useful tool for solving real work problems and analyzing our environment.
3. Even though many teachers have neglected to teach applications of mathematics, our complex society demands that public education teach practical mathematics and problem-solving techniques.
4. Mathematics can be used to solve problems in the real world and in other disciplines.
5. Applications to real life situations and other subject areas (i.e., physics, social science, economics, art, music) make abstract mathematics more meaningful and understandable.
6. Applications should include appropriate, interesting topics and activities for students and teachers.

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The Man-Made World, McGraw-Hill Book Company, 1972.

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SRA Math Applications Kit, Science Research Associates, Inc., 1971.

USMES (Unified Science and Mathematics for Elementary Schools), Education Development Center, Inc., 1973.

The World Almanac and Book of Facts, Newspaper Enterprise Association, Inc., 1975 (or current yearbook).



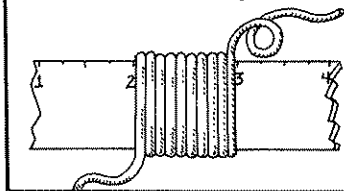
EXAMPLES OF APPLICATIONS FOUND IN THE CLASSROOM MATERIALS

I. Word and Wordless Problems


Word problems illustrated with pictures can be found in several sections of the classroom materials.

PICTURE PROBLEMS

3. Here is a picture of a piece of string wrapped around a ruler. How thick is the string?




4. If a ring is 18 carat gold, this means 18/24 of the ring is gold. Express this in lowest terms. What part of the ring is not gold?



THAT'S JUST ABOUT THE SIZE OF IT!

1. MUSCLES MAKE UP ABOUT $\frac{1}{4}$ OF A PERSON'S BODY WEIGHT. ABOUT HOW MUCH DO YOUR MUSCLES WEIGH?



2. YOUR INTESTINES (LARGE & SMALL) ARE ABOUT 7.5 m (25 FT.) LONG. HOW DOES THIS COMPARE WITH YOUR HEIGHT?

WORDLESS PROBLEMS

WRITE ONE OR MORE PROBLEMS FOR EACH PICTURE BELOW

1. City A, City B, City C. Distances: 219 km, 375 km.



2. JOE'S MARKET. Items: 2.99, 1.49, 1.99, 2.49, 3.99, 4.99, 5.99, 6.99, 7.99, 8.99, 9.99.



3. Perimeter = 54. Dimensions: 18, 18.



4. 53 km/h. Distance: 971 km. Points A and B.




There are wordless problems which imply various problems to be solved.

II. Mini-Activities

Many introductory class and individual activities provide questions which students can relate to.

NUMBER NAMES
CLASS ACTIVITIES



How do I know who you are? What are some of your I.D. numbers?

1. PEOPLE NUMBERS

How are you identified by numbers? How are your students? As a quick class activity a list is started of how people are identified and/or described by numbers. This can become a growing list on a poster and/or a list kept by each student. Here are some examples:

SOCIAL SECURITY NUMBER	DATE OF BIRTH	HEIGHT
LICENSE NUMBER (bike or car)	PHONE NUMBER	WEIGHT
STREET OR AVENUE NUMBER	ZIP CODE	SPORTS RECORDS
HOUSE NUMBER	CREDIT CARDS	SEAT LOCATION IN CLASS



What would it cost to hire someone to do all the work that a housewife does?

III. Maxi-Activities

THE AMERICAN HOUSEWIFE

A survey conducted by economists for a large bank revealed the worth of an average American housewife. Here are 12 of the tasks a housewife is called upon to perform daily. Calculate her earnings per week for each job. ROUND OFF ANSWERS TO THE NEAREST WHOLE CENT.

JOB	HOURS PER WEEK	RATE PER HOUR	VALUE PER WEEK
Nursemaid	44.5	\$2.00	

Chauffeur	2.0	\$3.12	
WHAT IS THE TOTAL EARNED IN ONE WEEK →			
WHAT WOULD BE HER YEARLY SALARY (52weeks=1year) →			

SHOPPING WITH A NEWSPAPER

MATERIALS: DAILY NEWSPAPERS FROM LOCAL AREA

PREPARATION: PLACE THE FULL PAGE ADVERTISEMENTS FROM LOCAL GROCERY STORES AROUND THE ROOM. USE ADS FROM 3 TO 5 DIFFERENT STORES. (WITH A LARGE CLASS YOU MAY WISH TO HAVE MORE THAN ONE COPY OF EACH AD.)

INSTRUCTIONS TO STUDENTS:



1. PLAN A PARTY FOR 10 OR 20 OF YOUR FELLOW STUDENTS. YOU MAY USE ONLY THE ITEMS LISTED IN ANY OF THE ADS. YOU MAY SHOP AT MORE THAN ONE STORE.
2. WRITE THE MENU FOR YOUR PARTY.
3. MAKE A LIST OF EVERYTHING YOU WILL NEED AND HOW MUCH OR MANY OF EACH ITEM.
4. FIND THE COST OF YOUR PARTY - BE SURE TO CHECK THE ADS AND GET THE BEST PRICE ON EACH ITEM.

The daily newspaper can be used as a source for many activities using everyday applications.



The daily lives of the students can be used as the basis for data collection and open-ended questions.

MAXIPROJECTS FOR FRACTIONS (CONTINUED)

HOW DO YOU SPEND YOUR TIME?



WHAT FRACTIONAL PART OF YOUR DAY DO YOU SPEND SLEEPING, PLAYING SPORTS, WATCHING TELEVISION, TALKING, ETC.?

MAKE A LIST OF THE THINGS YOU DO EACH DAY AND EITHER ESTIMATE HOW MUCH TIME YOU SPEND DOING THEM OR KEEP A LOGBOOK FOR A COUPLE OF DAYS. YOU WILL BE ABLE TO COMPARE WHAT YOU DO WITH YOUR FRIENDS, DISPLAY YOUR RESULTS.

IT WOULD BE VERY INTERESTING TO ESTIMATE WHAT PART OF A YEAR (OR YOUR LIFETIME) YOU SPEND DOING VARIOUS THINGS.

LET'S SPLIT HAIRS

THIS IS A THICKNESS OF HAIR MAGNIFIED 600 TIMES.
THE OTHER ITEMS ON THE HAIR ARE DRAWN TO SCALE.

GET A SCIENCE BOOK OR ENCYCLOPEDIA TO FIND THE SIZE AND SHAPE OF OTHER THINGS. DRAW PICTURES MAGNIFYING YOUR THINGS IN PROPORTION TO THIS HAIR.

HERE ARE SOME SUGGESTIONS:

- 1) RED CORPUSCLES
- 2) VARIOUS SMALL INSECTS (LOUSE, FLEA, ETC.)
- 3) GRAIN OF SAND (OR ANY OTHER CRYSTALS)
- 4) ALGAE (VARIOUS DIFFERENT TYPES)
- 5) FUNGI (MUSHROOMS, SMALLER FUNGI)
- 6) DIATOMS
- 7) CELLS (COMPARE HUMAN OR ANIMAL CELLS TO PLANT CELLS)
- 8) MOLECULES
- 9) PROTOZOA
- 10) VIRUSES

MAKE A BULLETIN BOARD OF SMALL THINGS.

Applications may relate to other classes the students are taking.



APPLICATIONS FOUND IN CLASSROOM MATERIALS

WHOLE NUMBERS:

Getting Started

NUMBERS AROUND US

NUMBER USES

NUMBER NAMES

NUMBER USES

Concepts

POPULATION

ORDERING

TIME FLIES

ORDERING

Mixed Operations

ABOUT RIGHT

WORD PROBLEMS

WORDLESS PROBLEMS

WORD PROBLEMS

ALTITUDES

AVERAGE

WEATHER GRAPH GAME

AVERAGE

MOM & POP STEEL CO.

MIXED OPERATIONS

FRACTIONS:

Concepts

NUMBER PAIRS

DEFINITION

MORE NUMBER PAIRS

A PART OF THE ACTION

FRACTIONAL PARTS

READING YOUR RULER

EQUIVALENT

Addition/Subtraction

WORLD TRACK RECORDS

ADDITION/SUBTRACTION

Multiplication/Division

TRACK RECORDS IN SPACE???---

MULTIPLICATION



Mixed Operations

PICTURE PROBLEMS

WORD PROBLEMS

WORDLESS PROBLEMS

WORD PROBLEMS

MAXIPROJECTS FOR FRACTIONS

MIXED OPERATIONS

FRACTION RECIPES

MIXED OPERATIONS

IT'S A "FULLER" WORLD

WORD PROBLEMS

DECIMALS:

Concepts

DEWEY DECIMALS

MOTIVATION

PLACING A POINT

PLACE VALUE

DECIMAL

PLACE VALUE

MAKING CHANGE

PLACE VALUE

MATH TOOLS

ROUNDING

BASEBALL AUTO RACING RECORD

ROUNDING

DENSITY

ROUNDING-ORDERING

CHANGE UP

RELATION TO FRACTIONS

Addition/Subtraction

ODOMETER DECIMALS

ADDITION

MONEY EXCHANGE RATES

ADDITION/SUBTRACTION

DECIMAL TRIPS

ADDITION

Multiplication/Division

THE AMERICAN HOUSEWIFE

MULTIPLICATION

GIANT ECONOMY SIZE

DIVISION

SPEED AND SOUND

MULTIPLICATION/DIVISION

STOCKS - INVESTMENTS FUNDS
INVESTMENT FUNDS

RELATION TO FRACTIONS



Mixed Operations

WORLD TRACK RECORDS

COMPUTATION $+$, $-$, \div

APPROX-APPRAISALS

WORD PROBLEMS

OPERATION PLEASE!

WORD PROBLEMS

WORDLESS PROBLEMS

WORD PROBLEMS

SIMPLIFY THE NUMBERS

WORD PROBLEMS

THAT'S JUST ABOUT THE
SIZE OF IT!

WORD PROBLEMS

SHOPPING WITH A NEWSPAPER

MIXED OPERATIONS

LARGE AND SMALL NUMBERS:

Awareness

BULLETIN BOARD IDEA

NUMBER SENSE

ONE-MILLIONTH

COMPUTATION

Exponential Notation

I DIDN'T PLANET THAT WAY

SCIENTIFIC NOTATION

SCIENTIFIC FACTS

SCIENTIFIC NOTATION

GRAINS OF SAND AND TURNING
WHEELS

SCIENTIFIC NOTATION

NUMBER SENSE AND ARITHMETIC SKILLS

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PROBLEM SOLVING

RATIONALE

Learning to solve problems is probably the most important aspect of one's education. No matter who we are, where we live, or what we do, there will always be problems for us to face and problems for us to solve if we want to solve them. Sometimes it is not easy to determine whether a situation really is a problem for a particular individual. What is a problem to one person may be an exercise to another. Performing or practicing something (a task) that one already knows how to do is an exercise. Therefore, the task may require only a routine procedure which leads to the solution(s). However, if the individual has a clearly defined, desired goal in mind, but the pathway to the goal is blocked, then the individual has a "problem" to solve. "A true problem in mathematics can be thought of as a situation that is novel for the individual called upon to solve it. It requires certain behaviors beyond the routine application of an established procedure." [Troutman and Lichtenberg]

Mathematics teachers should pose and provide problems that have no obvious method or algorithm to follow in reaching a solution. Too often students are given page after page of various computational exercises which use one or more "essential" algorithms the students have

"memorized." Once outside the classroom, students rarely use the algorithms they have memorized because the algorithms do not seem applicable. They come across ambiguous, disorganized situations that require considerable thought and skill for making a decision or finding a reasonable solution. Developing the ability to think independently and make wise decisions will help people to solve future problems by themselves.

Problem solving is a structured process. George Polya, in his book How to Solve It, divides the problem solving process into four steps:

- 1) Understanding the problem.
- 2) Devising a plan.
- 3) Carrying out the plan.
- 4) Looking back and checking the results.

Other authors have discussed the problem solving process with similar steps that match or fit into Polya's four steps (see Selected Sources for Problem Solving). These steps provide a structure which guides the problem solver through a search for the solution(s) to a problem. In the discussion which follows, several questions to answer and "things to try" are given under each of the four steps.

Understanding the Problem:

1. State the problem in your own words. (If the student cannot read the problem well enough to understand its meaning, the teacher may need to



read it to him. If the student can read but does not understand the problem, the teacher could rephrase the problem. The teacher should check for stumbling blocks. If the student has read the problem but seems bothered, ask what he thinks about the problem. Perhaps the student sees the situation as unrealistic, inconsistent or incomplete.)

2. What are you trying to find out? What is the unknown?
3. What relevant information do you get from the problem?
4. Is there any information that is not needed to solve the problem?
5. Are there any missing data that you need to know to solve the problem?
6. Are there any diagrams, pictures or models that may provide additional information about the problem?
7. Can you try some numerical examples?
8. Is it possible to recreate, dramatize, or make a drawing of the problem?
9. Can you make an educated guess as to what the solution(s) might be?

Devising a Plan:

1. Make a diagram, number line, chart, table, picture, model or graph to organize and structure the data.
2. Guess and check. Organize the trial and error investigations into a table.
3. Look for patterns.
4. Translate the phrases of the problem into mathematical symbols and sentences.
5. Try to solve one part of the problem at a time (i.e., break the problem into cases).
6. Have you worked a problem like this

before? What method did you use?

7. Can you solve a simpler but related or analogous problem?
8. Keep the goal in sight at all times.

Carrying Out the Plan:

1. Keep a record of your work.
2. Perform the steps in your plan; check each step carefully.
3. Complete your diagram, chart, table or graph.
4. Follow patterns; organize and generalize them.
5. Compare your estimates and guesses with your work.
6. Solve the mathematical sentence; record the calculations and answer.
7. Work out any simpler but related or analogous problems. Compare the solutions.

Looking Back:

1. Can you check your result? Is the answer reasonable?
2. What does the result tell you? What conclusions can be made?
3. Is there another solution? Is there another way of finding the answer?
4. Make up some problems like the one you worked. Is there a rule or generalization that can be used to solve similar problems?
5. What method(s) helped you get the answer(s)?

Teaching Problem Solving

"The best way to learn problem solving is by working problems and studying the processes we used in working them."

[Hints for Problem Solving] If a person is going to become a problem solver, he/she will need to be involved in a



variety of problem-solving experiences. Before any problem can be tackled, there has to be the desire to solve the problem. The teacher can motivate the students by giving them problems within their range of experience and interests. Stimulating questions can guide the students through the problem-solving process. Getting the students to the point where they WANT to solve the problem is the most important step that will lead to successful problem solving. To insure further the success of a problem-solving activity, the teacher should stress a thorough understanding of the problem and encourage students to devise and carry out their own plan for finding the solution. It is important to provide all students with enough time to arrive at the solution independently without the faster students blurting out their solutions.

In the beginning the teacher should realize that most students are NOT problem solvers. They become frustrated quickly and tend to give up easily. They often make incorrect conjectures and fail to check the reasonableness of their answers. They lack a knowledge of problem-solving techniques and the ability to use them. Some students have not acquired the necessary computational skills or reading/comprehension skills needed to carry out the problem-solving process.

No teacher or student has to memorize Polya's four steps and its list of "things to try," but there are specific skills from the list that can be the focus of a lesson. *Ten Men in a Boat* is a good introductory problem for students who have little confidence in their ability to tackle a problem-solving situation. The activity gives the teacher an opportunity to guide the student through "things to try" and finally arrive at a generalized solution. Other introductory activities include *Enchanted Alphabet*, *Magic Perimeters*, and *Glances and Blows*. Each of these can be used to illustrate some of the specific problem-solving suggestions discussed earlier.

Why Teach Problem Solving? - A Final Argument

". . . In the minds of all but a few college freshmen, problem solving is not a process by which one ascertains the truth. Rather, it is a process by which one gets the answer in the back of the book by a sequence of steps, each of which has been authorized by the teacher." (Edwin E. Moise, SIAM News, Feb., 1975) Indeed, too many mathematics assignments do require rote procedures to be followed while finding the same answer as the "answer in the back of the book," but this is really drill and practice, not problem solving, and the students are doing exercises,



not problems.

If our students are to become independent thinkers and problem solvers, it is important that we give them many situations which cannot be routinely solved. It is important that we as

educators provide guidance and examples that involve a variety of problem-solving techniques. Problem solving is a process of thinking that "emancipates us from merely routine activity."

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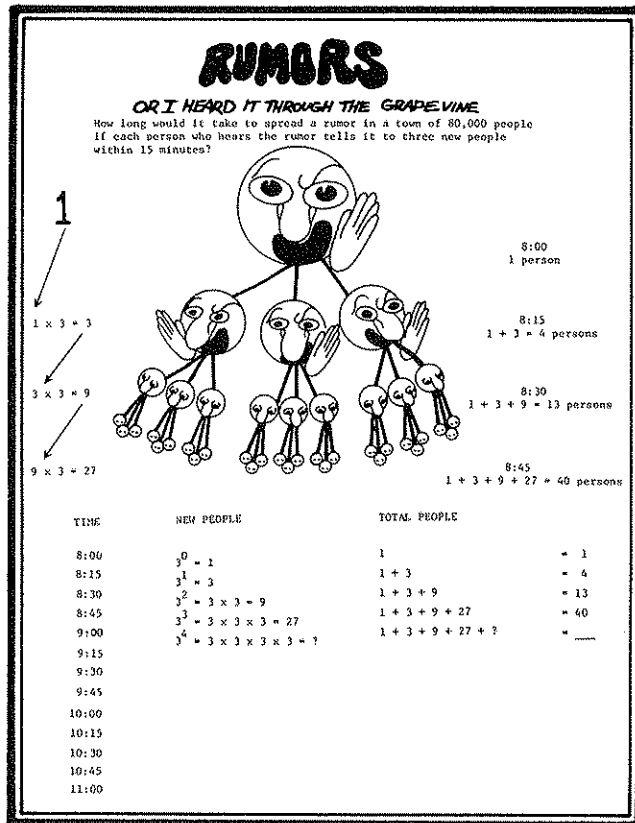
Schaaf, Oscar. "Problem-Solving Approach to Mathematics Instruction," unpublished mimeograph.

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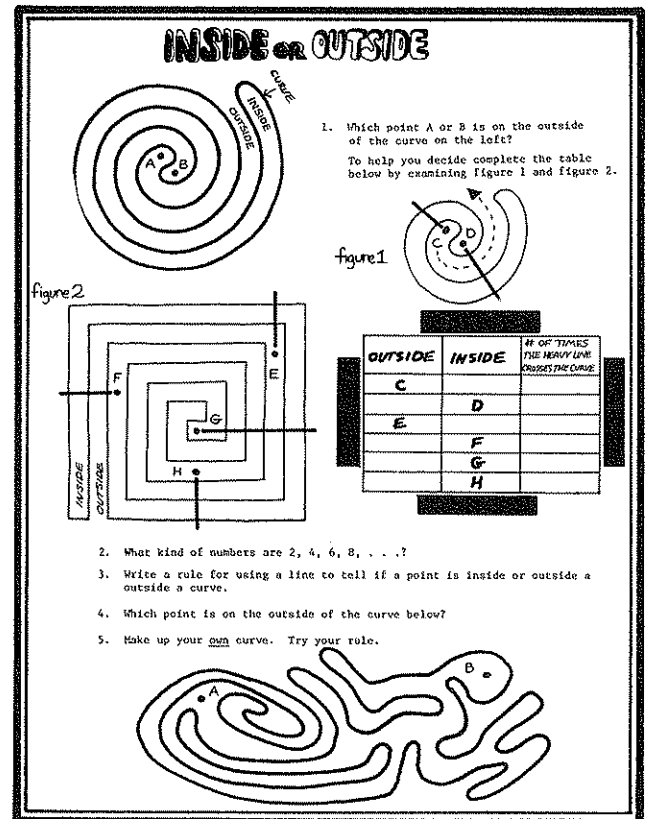
EXAMPLES OF PROBLEM SOLVING FOUND IN THE CLASSROOM MATERIALS

Each of the following pages from the resource book has a number of steps that can be carried out in the problem-solving process. There are many steps and many combinations of steps which make each problem unique and thought-provoking.



State the problem in your own words. Are there any diagrams, pictures or models that may provide additional information about the problem? Make a table to organize and structure the data. Look for patterns. Complete the table. Follow patterns; organize and generalize them. What does the result tell you? What conclusions can be made? Make up some problems like the one you worked. Is there a rule or generalization that can be used to solve similar problems?

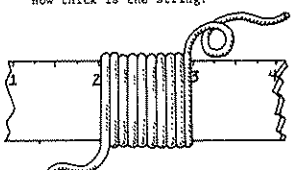
What are you trying to find out? Are there any diagrams or pictures or models that may provide additional information about the problem? Make a table to organize and structure the data. Look for patterns. Organize and generalize them. What conclusions can be made? Make up some problems like the one you worked. Is there a rule or generalization that can be used to solve similar problems?






PICTURE PROBLEMS


3 Here is a picture of a piece of string wrapped around a ruler. How thick is the string?



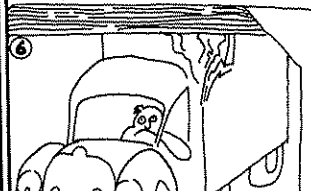
4 If a ring is 18 carat gold, this means $\frac{18}{24}$ of the ring is gold. Express this in lowest terms. What part of the ring is not gold?



5 Lony Ludwig weighs 30 kilograms on earth but only 5 kilograms on the moon. His moon weight is what fraction of his earth weight?



6 A truck is going under a bridge with clearance of 14 feet. The truck is 16 feet high. What part of the truck is in trouble?



What are you trying to find out? Are there any diagrams, pictures or models that may provide additional information about the problem? Can you make an educated guess as to what the solution might be? Translate the phrases of the problem into mathematical symbols and sentences. Solve the mathematical sentence; record the calculations and answer. Compare your estimates and guesses with your work. Can you check your result? Is the answer reasonable?

Make a concrete model of the problem. What relevant information do you get from the problem? Guess and check. Translate the problem into a mathematical sentence; record the calculations and answer. Can you check your result? Is there another solution? Is there another way of finding the answer?

MYSTERY STACKS

Use the clues to answer the questions below. There may be one answer, no answer possible, or many answers.

1. CLUES:



- There are two stacks of cubes hidden behind these three stacks.
- Both hidden stacks have the same number of cubes.
- 16 cubes in all.

How many cubes are in each of the two stacks?

2. CLUE:



- 45 cubes in all.

How many cubes in the unlabeled stack?

5. CLUES:



- Total number of cubes is 70.
- The unlabeled stacks all have the same number of cubes.

How many cubes in each unlabeled stack?

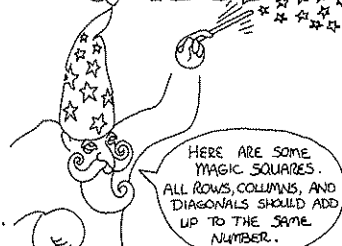
6. CLUES:



- 5 stacks.
- The tallest stack is 5 cubes high.
- No two stacks have the same height.

How many cubes in all?

FRACTION Magic SQUARES



USE THESE NUMBERS
 $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}, \frac{1}{11}, \frac{1}{12}$.
 BE CAREFUL.

MAGIC $8\frac{1}{8}$ SQUARE

$2\frac{1}{2}$			1	
	$\frac{5}{8}$	$\frac{7}{8}$	$1\frac{1}{4}$	
		$1\frac{3}{8}$	$2\frac{1}{2}$	$2\frac{3}{8}$
$1\frac{1}{4}$		$2\frac{3}{8}$		$\frac{3}{8}$
$1\frac{3}{8}$	$2\frac{1}{4}$			$1\frac{1}{8}$

MAGIC $1\frac{1}{2}$ SQUARE

$\frac{2}{3}$		$\frac{1}{2}$
		$\frac{7}{12}$

MAGIC $2\frac{1}{2}$ SQUARE

$5\frac{1}{2}$		$10\frac{3}{2}$
$6\frac{1}{2}$		

MAGIC $5\frac{3}{8}$ SQUARE


$2\frac{3}{8}$	$\frac{1}{3}$		$1\frac{2}{3}$	$1\frac{1}{3}$
$\frac{3}{2}$	$\frac{7}{6}$			$\frac{1}{6}$
	$\frac{7}{3}$			

What relevant information do you get from the problem? Guess the placement of the numbers and check. Where do you start? Try to solve one part of the problem at a time. Keep a record of your work. Can you check your result? Is there another solution? Make up some problems like the one you worked. Is there a rule or generalization that can be used to solve similar problems?



State the problem in your own words. Are there any diagrams, pictures or models that may provide additional information about the problem? Can you solve a simpler analogous problem? Translate the phrases of the problem into mathematical symbols and sentences. Keep a record of your work. What conclusions can be made? Make up some problems like the one you worked. Is there a rule or generalization that can be used to solve similar problems?

THE PEG GAME OR TEN MEN IN A BOAT




Ten men are fishing in a boat. One seat in the center of the boat is empty. The five men in the front of the boat want to change seats and fish in the back of the boat, and the five men in the back of the boat want to fish from the front of the boat. A man may move from his seat to the next empty seat, or he may step over one man without capsizing the boat. What is the minimum number of moves it will take to exchange the five men in front with the five in back?

Materials:

- a) 10 markers for each student, 2 different kinds or colors - 5 each (Wooden cubes, 1" ceramic tile, or beans may be used.)
- b) Mat for making the moves (See sample.)
- c) Answer sheet (See sample.)
- d) Overhead projector

Procedure: Use a handout or overhead to present the problem to the students. Supply the markers and let them struggle for a minute or two, then suggest an easier problem - 2 men in the boat, 1 on each side. Supply students with the mat at this time and tell them to place 1 marker of each type in Figure 1. After another minute or two demonstrate the answer to the 2 men in a boat on the overhead.

SOLUTION - TWO MEN IN BOAT - RLR = 3 MOVES



(NOTE: THE PROBLEM MAY BE DONE IN EXACTLY THE OPPOSITE MOVES - LRL.)

Suggested Mat - Be sure the squares are a little bigger than the markers.



Figure 1.  Figure 2. 



Figure 3. 

Figure 4. 



PROBLEM SOLVING FOUND IN CLASSROOM MATERIALS

WHOLE NUMBERS:

Getting Started

MYSTERY STACKS

COUNTING

THE PEG GAME OR TEN
MEN IN A BOAT

LOOKING FOR PATTERNS

THE PAINTED CUBE

LOOKING FOR PATTERNS

STRICTLY SQUARESVILLE

LOOKING FOR PATTERNS

SQUARESVILLE SUBURBS

LOOKING FOR PATTERNS

ESP PUZZLE

LOOKING FOR PATTERNS

GAME OF 50

LOOKING FOR STRATEGIES

THE GAME OF NIM

LOOKING FOR STRATEGIES

A CROSSNUMBER PUZZLE

MOTIVATION

RIVERS AND RELATIVES

LOGIC PROBLEMS

Numeration

ATTIC-GREEK

FINDING PATTERNS

ROMAN NUMERATION SYSTEM

FINDING PATTERNS

MATCHSTICK EQUATIONS

ANCIENT NUMERALS

MAYAN NUMERATION SYSTEM

FINDING PATTERNS

BABYLONIAN NUMERATION
SYSTEM

FINDING PATTERNS

Concepts

ACTIVITY CARDS - PAN
BALANCE

PLACE VALUE

GLANCES AND BLOWS

PLACE VALUE

GREATER THAN OR LESS
THAN

ORDERING



IN-BETWEEN

ORDERING

Addition/Subtraction

SUM COMBINATIONS

ADDITION

PICTURE PROBLEMS 1

WORD PROBLEMS

DOMINO DONUTS

ADDITION

ENCHANTED ALPHABET

ADDITION

MAGIC PERIMETERS

ADDITION

TINKER TOTALS

ADDITION

MAGIC SQUARES

ADDITION

CALENDAR MAGIC

ADDITION

MAGIC HEXAGON

ADDITION

MAGIC CUBE

PATH SUMS

ADDITION

OMAR'S DILEMMA

ADDITION/SUBTRACTION

Multiplication/Division

GRID MULTIPLICATION WITH
INTERSECTIONS

MULTIPLICATION

ATTENTION

SQUARES

PATH PRODUCTS

MULTIPLICATION

FOUR INVESTIGATIONS

PATTERNS

2 x 3

 GOOD TIMES

MULTIPLICATION

THE YOUNG GENIUS

PATTERNS

Mixed Operations

FILL IN THE WHOLES

COMPUTATION

WORDLESS PROBLEMS

WORD PROBLEMS

BLANK SQUARES

COMPUTATION



WHAT'S MY RULE I

PATTERNS

WHAT'S MY RULE II

PATTERNS

SEEING SEQUENCES

PATTERNS

ARROW MATH

PATTERNS

FRESNO

ORDER OF OPERATION

DIGIT IDEAS

ORDER OF OPERATION

MOM & POP STEEL CO.

MIXED OPERATIONS

CHANGE FOR A QUARTER

MIXED OPERATIONS

FRACTIONS:

Concepts

A SHADY DEAL

WHOLE MODEL

GRENDDEL'S GAUGES

FRACTIONAL PARTS

NAME A POINT AT THE LINE UP
FIND A POINT!

IMPROPER-MIXED

Addition/Subtraction

FRACTION MAGIC SQUARES

ADDITION/SUBTRACTION

CAN YOU FIND A PATH

ADDITION/SUBTRACTION

MORE PATHS

ADDITION/SUBTRACTION

CHALLENGE PATHS . . .

Mixed Operations

FRACTIONS FOREVER

PATTERNS

OF WHAT'S LEFT

PATTERNS

PICTURE PROBLEMS

WORD PROBLEMS

WORDLESS PROBLEMS

WORD PROBLEMS



IT'S A "FULLER" WORLD

WORD PROBLEMS

DECIMALS:

Concepts

CROSS-NUMBER PUZZLE

MOTIVATION

Addition/Subtraction

LATTICE MATHEMATICS - I

ADDITION/SUBTRACTION

DECIMAL ADD-A-BOX

ADDITION

ROUNDED ADD-A-BOX

ADDITION

Multiplication/Division

LATTICE MATHEMATICS - II

MULTIPLICATION/DIVISION

Mixed Operations

"THE ANSWER"

COMPUTATION

NOTHING IS FOREVER

PATTERNS

WORDLESS PROBLEMS

WORD PROBLEMS

SIMPLIFY THE NUMBERS

WORD PROBLEMS

THAT'S JUST ABOUT THE
SIZE OF IT!

WORD PROBLEMS

ONE-LINERS

MIXED OPERATIONS

LARGE & SMALL NUMBERS:

Awareness

BEANS IN A JUG

COMPUTATION - LARGE

A FREE SUNDAE

COMPUTATION - LARGE

MORE INVESTIGATIONS

COMPUTATION



Exponential Notation

CHEERY SEQUENCES

POWERS OF TWO

GREAT-GREAT-GREAT- ...
GRANDPARENTS

POWERS OF TWO

RUMORS

POWERS OF THREE

NUMBER SENSE AND ARITHMETIC SKILLS

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MENTAL ARITHMETIC

RATIONALE

Many of our day-to-day calculations are done mentally. Without using pencil and paper or a hand calculator, we often think about answers to such questions as: Did the clerk give me the right amount of change? How long will it take me to travel across town? How many boxes of candy will have to be sold for a fund-raising project needing \$500?

Mental arithmetic is an important basic skill which can be applied to many situations. One might perform mental checks on routine computations. Mental arithmetic can help students develop a better number sense and a better feeling about their ability to calculate answers. It may also improve their knowledge of basic facts and motivate them to move on to more advanced or applied mathematics. People can use mental arithmetic to improve the process of estimation and approximation by . . .

- 1) Checking for reasonableness and correctness of answers.
- 2) Getting "ball-park" estimates.
- 3) Rounding.
- 4) Computing with simplified numbers.
- 5) Multiplying and dividing by powers of ten.

The use of mental arithmetic can quicken the problem-solving process--especially for those problems which involve trial and error.

Just as any skill must be developed through practice, the ability to do arithmetic mentally can be improved with drill and mental calculations. These can be short and part of the daily routine (such as a five-minute warm-up activity). Or the activities can be longer and stressed early in the school year to develop the habit of using mental arithmetic. Encourage the students to do mental calculations whenever they are involved in checking pencil and paper calculations, calculator activities, and problem solving.

Selected Sources for Mental Arithmetic

Cutler, Ann and Rudolph McShane. The Trachtenberg Speed System of Basic Mathematics, Doubleday, 1960.

Garvin, Alfred E. Shortcuts, Checks and Approximations in Mathematics, J. Weston Walch, 1973.

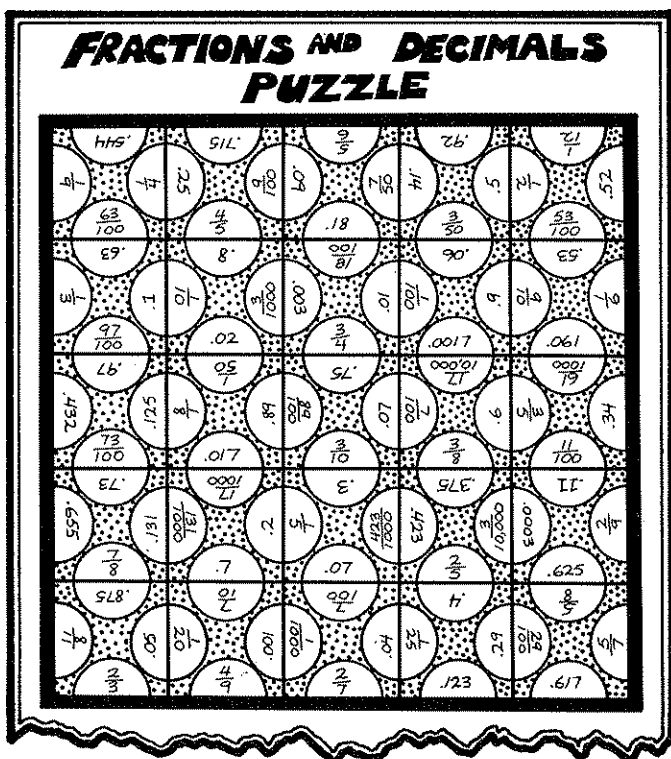
Kramer, Klass. Mental Computation, Science Research Associates.



EXAMPLES OF MENTAL ARITHMETIC FOUND IN THE CLASSROOM MATERIALS

I. Developing Mental Arithmetic Skills Through Drill and Practice

Number patterns can be used as starter activities which extend into problem solving.



Personalized activities which students help to construct can motivate them to tackle a variety of trial and error problem-solving questions.

SEEING SEQUENCES

1, 2, 3, —, —, —

2, 4, 6, —, —, —

1, 3, 5, —, —, —

1, 4, 7, 10, —, —, —

1, 2, 4, 8, —, —, —

7, 14, 21, —, —, —

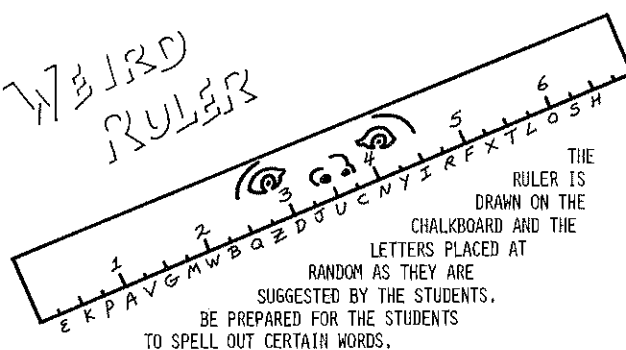
1, 2, 4, 7, 11, 16, —, —, —

A, B, C, —, —, —

0, 1, 1, 2, 3, 5, 8, —, —, —

Pieces of the puzzle are cut out and handed to the student who puts the puzzle back together.

WEIRD
RULER



The following questions are examples which can be answered using the ruler.

1. How long is CAT? ($3\frac{3}{4} + 1 + 5\frac{1}{2} = 9\frac{1}{4}$)
2. Which student has the longest name?
3. Which is longer: forever, eternity, infinity?
4. Can you find a word exactly 3 units long?
5. Which is longer: MINUTE or HOUR?
6. What is the longest three letter word you can find?



Teacher directed activities help students to see patterns and easier methods of doing arithmetic calculations.

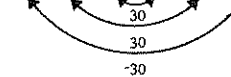
MIND OVER MATH

- I. Answer Only Please - Place a series of problems on the board one at a time. Encourage students to look for a short way, then write only the answer to the problem.

Examples: $5 \times 22 = (5 \times 2) \times 11 = 10 \times 11 = 110$

$(53 \times 114) - (53 \times 104) = 53 \times 10 = 530$

$11 + 12 + 13 + 17 + 18 + 19 = 30 + 30 + 30 = 90$



WHERE'S YOUR HEAD AT?

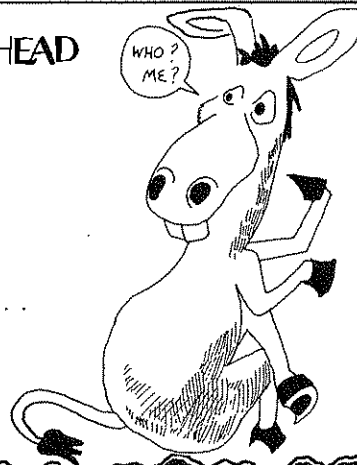
I can do most any fraction problem in my head. For example: $7 + 7\frac{1}{2}$ is easy!

Just add $7 + 7 = 14$
and $14 + \frac{1}{2} = 14\frac{1}{2}$

Now for a couple quick subtractions, watch this . . .

$40\frac{1}{2} - 20$ $40\frac{1}{2}$ means $40 + \frac{1}{2}$

so . . . $40 + \frac{1}{2} - 20 = 20 + \frac{1}{2} = 20\frac{1}{2}$.



Patterns and shortcuts can help students to understand addition and subtraction with mixed numbers.

II. Applying Mental Arithmetic Skills

These questions ask the students to check the reasonableness of an answer.

ABOUT RIGHT

- ① The mad bicyclist can pedal at least 80 kilometres in one day. At this pace he should be able to travel 8000 kilometres in 10 days.



Approx-Appraisals

6. About how much change is left from two \$20 bills if you pay \$19.89 for a tennis racket and \$3.49 for tennis balls?

\$12.00

\$16.00

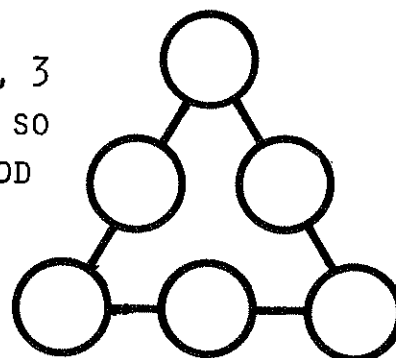
\$20.00



Solutions found by trial and error can be done more quickly when mental skills are applied.

MAGIC PERIMETERS

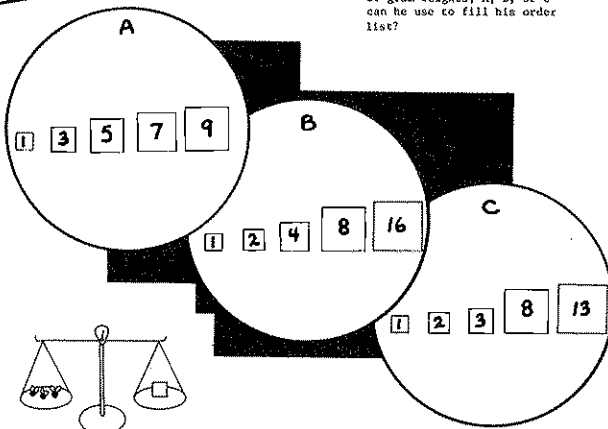
PLACE THE NUMBERS 1, 2, 3
4, 5, 6 IN THE CIRCLES SO
THAT ALL THREE SIDES ADD
TO THE SAME NUMBER.



TOTAL = 10



ORDER		LBS.		
GRAMS	A	B	C	
1	✓			
2				
4	✓			
5				
7				
13				
19				
25				



Al, the apiarist, sells his bees by the gram. Assuming he can get the bees to rest on one pan while he places the weights on the other pan, which of the following sets of gram weights, A, B, or C can he use to fill his order list?

No pencil and paper allowed here!
Students are encouraged to use
rounding and then compute with
simplified numbers.

DON'T BE AFRAID TO GUESS

I. APPROXIMATION GAME - Place problems on the board, one at a time. Encourage students to guess the answer without working the problem. Make a contest out of this. See whose guess comes closest to the correct answer. Use problems having numbers with two or more digits.

Examples: 217	4215		
391	3982		
682	1475		
395	8613	207	584
702	9813	$\times 58$	$\times 217$

Estimate answer to nearest hundred . Estimate to nearest thousand.

NEVER

HOLD A GRUDGE...

FOR EACH OF THE SIXTEEN PROBLEMS CIRCLE THE MOST REASONABLE ANSWER. (THERE ARE NO CORRECT ANSWERS GIVEN, SO YOU SHOULDN'T HAVE TO WORK THE PROBLEMS.) PLACE THE LETTER IN THE CIRCLED BOXES IN THE APPROPRIATE SPACE BELOW.

1	$\begin{array}{r} 45 \\ \times 6 \\ \hline \end{array}$	280 _P	2.7 _U	24 _T	11	$\begin{array}{r} 9.27 \\ \times 6 \\ \hline \end{array}$	55.6 _J	5.6 _R	.562 _Y
2	$\begin{array}{r} 6138 \\ \div 24 \\ \hline \end{array}$	24 _C	2.2 _H	.21 _D	12	$\begin{array}{r} 339 \\ \div 9 \\ \hline \end{array}$.03 _N	27 _R	2.7 _Z
3	$\begin{array}{r} 1.4 \\ \times 1.8 \\ \hline \end{array}$.32 _A	3 _E	35 _G	13	$\begin{array}{r} 75.3 \\ \times 94 \\ \hline \end{array}$	707 _D	7.078 _C	71.5 _I
4	$\begin{array}{r} 8325 \\ \div 25 \\ \hline \end{array}$	47 _H	.4 _J	4.6 _Y	14	$\begin{array}{r} 9813 \\ \div 13 \\ \hline \end{array}$.9 _B	9 _F	91 _H
5	$\begin{array}{r} 29.3 \\ \times 4 \\ \hline \end{array}$	120 _S	1002 _K	9 _T	15	$\begin{array}{r} 58.2 \\ \times 1.1 \\ \hline \end{array}$	57 _M	62 _L	640 _S
6	$\begin{array}{r} 83386 \\ \div 1083 \\ \hline \end{array}$	1083 _V	16 _H	25 _O	16	$\begin{array}{r} 249423 \\ \div 201 \\ \hline \end{array}$	201 _P	2.1 _Q	20 _Y
7	$\begin{array}{r} 5.08 \\ \times 2.4 \\ \hline \end{array}$	13 _E	121 _B	1.4 _C					
8	$\begin{array}{r} 7562 \\ \div 80 \\ \hline \end{array}$	80 _P	.803 _S	81 _D					
9	$\begin{array}{r} 35.6 \\ \times 3.9 \\ \hline \end{array}$	14 _X	1452 _N	143.2 _T					
10	$\begin{array}{r} 1751 \\ \div 29 \\ \hline \end{array}$	29 _E	300 _K	3 _F					



1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
PROBLEM NUMBERS



MENTAL ARITHMETIC FOUND IN CLASSROOM MATERIALS

WHOLE NUMBERS:

Concepts

ACTIVITY CARDS - PAN BALANCE

PLACE VALUE

MAKE A MILLION

READING & WRITING

Addition/Subtraction

MAKING IT ADD UP

ALTERNATE METHOD

SUBTRACTION IS SUBTRACTION
AND ADDITION

ALTERNATE METHOD

FILL IN THE DOMINOES

ADDITION

DOUBLE DIFFERENCES

SUBTRACTION

ADD-A-BOX

ADDITION

DIFFY

SUBTRACTION

HUMAN COMPUTER

ADDITION

PATH SUMS

ADDITION

OMAR'S DILEMMA

ADDITION/SUBTRACTION

Multiplication/Division

JIGSAW PUZZLE

BASIC FACTS

SQUARING 2-DIGIT NUMBERS

SQUARES

ATTENSION

SQUARES

AMAZING UNITS' DIGIT

PATTERNS

Mixed Operations

MIND OVER MATH

COMPUTATION $+, -, \times, \div$

BACKWARDS & FORWARDS

COMPUTATION $+, -, \times$

FILL IN THE WHOLES

COMPUTATION $+, -, \times, \div$

RULE RUMMY

COMPUTATION $+, -, \times, \div$



ROUNDED RESULTS

APPROXIMATION

ROUNDED LINE UP

APPROXIMATION

DON'T BE AFRAID TO GUESS

APPROXIMATION

MAGIC TRICKS

COMPUTATION $+, -, \times, \div$

WHAT'S MY RULE I

PATTERNS

WHAT'S MY RULE II

PATTERNS

SEEING SEQUENCES

PATTERNS

ARROW MATH

PATTERNS

DIGIT IDEAS

COMPUTATION $+, -, \times, \div$

FRACTIONS:

Concepts

PICTURE FRACTION

REDUCING

ESR - EVERY STUDENT RESPONDS

IMPROPER-MIXED

Addition/Subtraction

WEIRD RULER

ADDITION

CAN YOU FIND A PATH

ADDITION/SUBTRACTION

ALMOST GAMES

ADDITION/SUBTRACTION

Multiplication/Division

CAN YOU GET CLOSE UP

APPROXIMATION

Mixed Operations

WHERE'S YOUR HEAD AT?

COMPUTATION $+, -$

IN YOUR HEAD . . . AGAIN?

COMPUTATION \times

EGYPTIAN FRACTION TILES

COMPUTATION $+, -, \times$

SPIDER WEB FRACTIONS

COMPUTATION $+, \times$



MORE WEB FRACTIONS

COMPUATION $+, -, \times$

FRACTIONS FOREVER

PATTERNS

DECIMALS:

Addition/Subtraction

DECIMAL ADD-A-BOX

ADDITION

ROUNDED ADD-A-BOX

ADDITION

Multiplication/Division

NEVER HOLD A GRUDGE . . .

APPROXIMATION

Mixed Operations

THE DIAMOND GAME

COMPUTATION $+, -, \times, \div$

APPROX-APPRAISALS

WORD PROBLEMS

LARGE AND SMALL NUMBERS:

Awareness

GOING BIG TIME

COMPUTATION-LARGE

Exponential Notation

TREASURE HUNT

SCIENTIFIC NOTATION

NUMBER SENSE AND ARITHMETIC SKILLS

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Awareness	756
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ALGORITHMS

RATIONALE

Have you ever followed the directions on a bank statement or a tax return? Have you ever changed a mixed number to an improper fraction? Have you written a set of directions (or flowchart) to show the steps followed in a certain process? If so, then you have been using an algorithm--a systematic procedure for obtaining a desired result.

Algorithms help us in many ways. Some algorithms give us a set of instructions that lead us through a sequence of computations to a solution. The long division algorithm involves repeated multiplication and subtraction. The steps of the algorithm provide the student with a clearly defined, orderly process. When a consistent algorithm is used to guide students to a solution(s), it builds confidence and a feeling of security as well as increasing speed and accuracy. In the process of adding unlike fractions, finding a common denominator is an important step in reaching a solution. Students often guess or use their intuition to find a common denominator, but if that becomes an impossible or impractical task, it helps to have an exact recipe to follow (i.e., multiply the denominators together to find a common denominator).

Such new and different algorithms can provide students with fresh approaches.

Students who repeatedly fail to solve problems using the standard algorithms we teach will sometimes respond to nonstandard algorithms. They offer new and interesting problem-solving techniques by illustrating the possibilities of alternative methods and emphasizing the importance of the rules of arithmetic. For example, long division can be performed by first making and using a table where the divisor is doubled several times.

The divisor is 26. Make
this simple times table to
use when dividing by 26.

1	26
2	52
4	104
8	208

Answer: 87 r 13

1
2
84
26)2275
-208
195
-104
91
-52
39
-26
13

Special algebraic formulas can be used to find the sums of consecutive whole numbers. For example,

$$1 + 2 + 3 + \dots + 100 = \frac{100 \times 101}{2} = 5050$$

and more generally

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

Nonstandard algorithms have other advantages. Some can be fun and easy; others may show the benefits of using a standard algorithm which is more efficient and easier to understand. Sometimes



students develop their own nonstandard methods of solving a problem which are correct, yet dissimilar to the traditional algorithm. The teacher needs to recognize a correct algorithm invented by a student and avoid judging it as wrong only because it differs from the usual algorithm. As students create and explore algorithms and patterns, they learn to express themselves and gain confidence.

Nonstandard algorithms also have disadvantages. Some might work in only a limited number of cases. Others may be hard to follow logically--tricks may pop up suddenly to make calculations simpler (e.g., $\frac{18}{84} = \frac{1}{4}$). Parents and teachers will not accept some nonstandard algorithms, claiming they are unnecessary and deviate too much from the traditional methods. Nonstandard algorithms might cause an incorrect transfer of ideas. Children may become confused and frustrated if too many processes are introduced at one time.

Students often invent incorrect algorithms. For example, they may add numerators and denominators of fractions (e.g., $\frac{3}{4} + \frac{1}{4} = \frac{4}{8} = \frac{1}{2}$) or find a common denominator before multiplying two fractions. (e.g., $1\frac{1}{5} \times \frac{3}{4} = \frac{6}{5} \times \frac{3}{4} = \frac{24}{20} \times \frac{15}{20} = \frac{360}{20}$) When working story problems, students may multiply when they should divide;

add when they should multiply. These errors can stem from misunderstanding how and when to use a certain algorithm. Steps and operations from various algorithms become confused in a child's mind. Children reveal their lack of comprehension and number sense as they pursue the use of incorrect algorithms in the anxious search for answers. To help students overcome their difficulties, concrete models can be used to demonstrate an algorithm and provide a clear understanding of how and why the algorithm works. The subtraction algorithm can be illustrated by Multibase Blocks, Chip Trading activities, Cuisenaire Rods, and Bean Sticks.

Algorithms have changed and evolved throughout the centuries just as scientific theories have been disproved and remolded when new discoveries are made. New and better processes replace old methods. New algorithms are being invented to solve problems. The age of computers has brought a new dimension to the use and purpose of algorithms. Without specific rules and exact, consistent algorithms to follow, computers cannot be programmed. Meanwhile, hand calculators silently question the need for slide rules, log tables, and other slow algorithmic methods of calculating as they flash instantaneous answers after mere seconds of button-pushing. Algorithms, standard or nonstandard, play a dynamic



and vital role in problem-solving. Without them we would lack the many processes, methods, directions and procedures for coping with our complex society.

SUMMARY

1. An algorithm is a systematic procedure for obtaining a desired result.
2. Algorithms help us compute and solve problems by providing orderly, time-saving processes to follow.
3. Nonstandard algorithms provide:
 - a) different approaches and alternative methods for finding answers,
 - b) clarification of standard algorithms,
 - c) incentive for drill and practice of computational skills, and
 - d) creativity and confidence in problem-solving techniques.
4. A nonstandard algorithm can hamper the student when . . .

a) it works in only a limited number of cases.

b) it has tricks that are not logical or consistent.

c) it deviates too much from the traditional approach and causes misunderstanding of the concept.

5. Incorrect algorithms are invented and used by children who . . .

a) do not understand how to solve a problem,

b) confuse steps and operations from various algorithms,

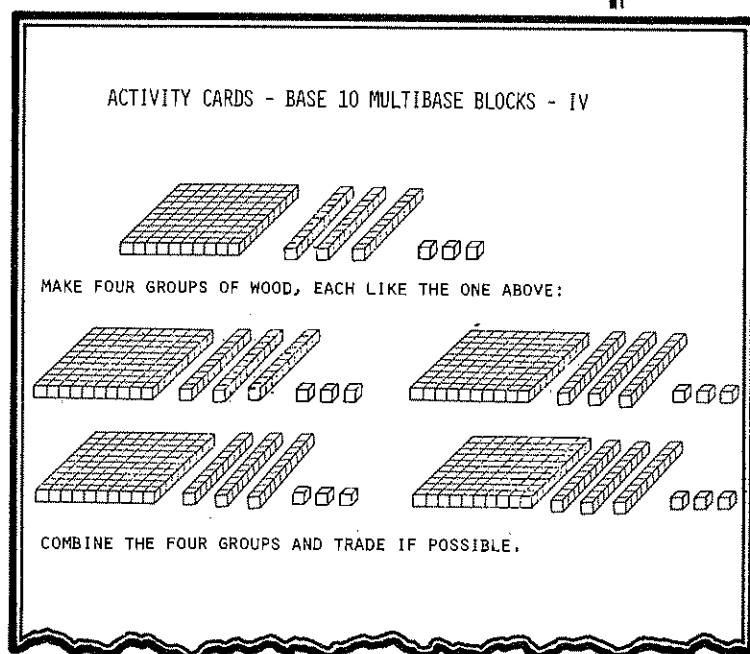
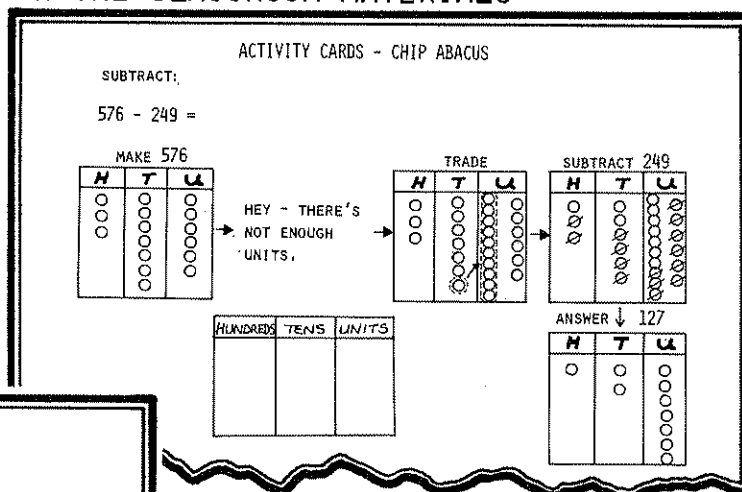
c) lack a number sense and fail to check the reasonableness of their answers.

6. Algorithms play an important role in the function of our society. Our technology today relies heavily upon many processes, methods, instructions and procedures.

EXAMPLES OF ALGORITHMS FOUND IN THE CLASSROOM MATERIALS

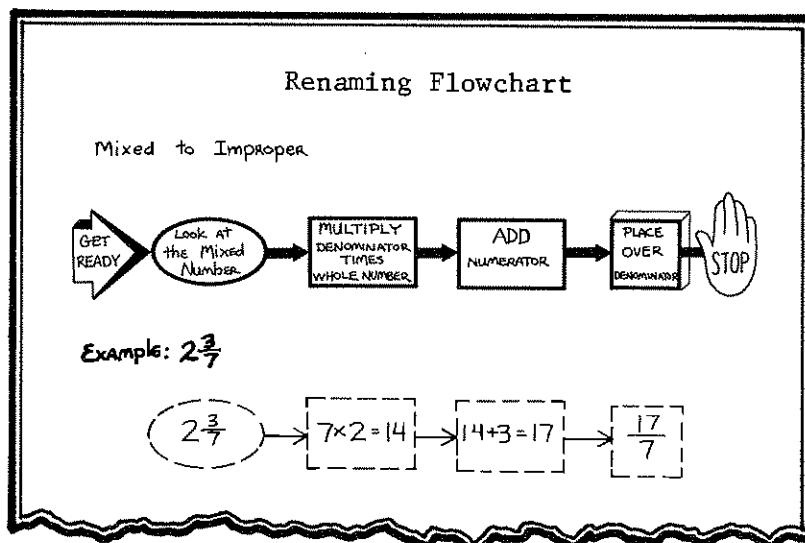
I. Standard Algorithms

Manipulatives can be used to develop an understanding of the basic operations with whole numbers.



Various algorithms are developed through activities which encourage the use of concrete physical models.

A flowchart placed on a bulletin board or used as a transparency can motivate an explanation of the steps involved in expressing a mixed number as an improper fraction.



II. Nonstandard Algorithms

Formulas can be used to simplify long and tedious calculations. Gauss' solution tells us that the sum of the first N counting numbers is $\frac{N(N+1)}{2}$.

MIXED NUMBER MULTIPLICATION

FIRST, CAN YOU FIGURE OUT HOW WE GET THE NUMBERS IN THE LATTICE?

$\frac{1}{3} \times 6$

$5\frac{1}{3} \times 6\frac{4}{5} = 30 + 4 + 2 + \frac{4}{15} = 36\frac{4}{15}$

A teacher demonstration of these equalities should provide a starting point for a lively class discussion.

THE YOUNG GENIUS

②

When the world's greatest mathematician, C. F. Gauss, was a young boy, he was asked to add the first hundred numbers. Here's his surprising solution.

$$\begin{array}{cccccccccccc} 1 & + & 2 & + & 3 & + & \dots & + & 98 & + & 99 & + & 100 \\ + & 100 & + & 99 & + & 98 & + & \dots & + & 3 & + & 2 & + & 1 \\ \hline 101 & + & 101 & + & 101 & + & \dots & + & 101 & + & 101 & + & 101 \end{array}$$

101 added 100 times or

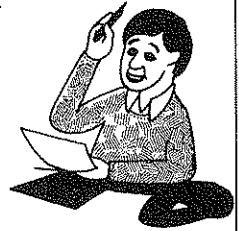
$$100 \times 101$$

But I must divide this product by two.

(Why?) Therefore,

$$1 + 2 + 3 + \dots + 99 + 100 = \frac{100 \times 101}{2}$$

or 5050.



No need for improper fractions here! Simple multiplication and then addition gets us the answer.

INCREDIBLE EQUALITIES

Have the following statement written on the board when the students come to class.

.9999... is equal to 1

Allow student to debate the issue, then show them each of the following:

ADDITION		MULTIPLICATION	
$\frac{1}{3} = .3333\ldots$	But $\frac{2}{3} = 1$	$\frac{1}{9} = .1111\ldots$	But $\frac{9}{9} = 1$
$\frac{1}{3} = .3333\ldots$		$9 \times \frac{1}{9} = \frac{9}{9}$	
$\frac{1}{3} = .3333\ldots$		$9 \times .1111\ldots = .9999\ldots$	
$+\frac{1}{3} = .3333\ldots$			
$\frac{2}{3} = .9999\ldots$			
PATTERN		DIVISION	
$\frac{1}{9} = .1111\ldots$	Have students continue the pattern.	$\frac{9}{9} = 1$, and $\frac{9}{9}$ means $9 \div 9$	Before trying this, be sure students will accept remainders equal to the divisor.
$\frac{2}{9} = .2222\ldots$			
$\frac{3}{9} = .3333\ldots$			
$\frac{4}{9} = .4444\ldots$			
$\frac{5}{9} = .5555\ldots$			
$\frac{6}{9} = .6666\ldots$			
$\frac{7}{9} = .7777\ldots$			
$\frac{8}{9} = .8888\ldots$			
$\frac{9}{9} = .9999\ldots$			
	But $\frac{9}{9} = 1$		

ALGORITHMS FOUND IN CLASSROOM MATERIALS

WHOLE NUMBERS:

Addition/Subtraction

ACTIVITY CARDS - BEAN STICKS - I	REGROUPING TO ADD
ACTIVITY CARDS - BEAN STICKS - II	REGROUPING TO SUBTRACT
MAKING IT ADD UP	ALTERNATE METHOD
SUBTRACTION IS SUBTRACTION AND ADDITION	ALTERNATE METHOD
THERE IS NO ONE WAY	ALTERNATE METHOD

Multiplication/Division

ACTIVITY CARDS - BEAN STICKS - III	MULTIPLICATION
ACTIVITY CARDS - BEAN STICKS - IV	DIVISION
HOW MANY GROUPS - HOW MANY LEFT?	DIVISION
FAIR SHARE	DIVISION
I HAVE A BETTER WAY!	ALTERNATE METHOD
SQUARING 2-DIGIT NUMBERS	SQUARES

FRACTIONS:

Concepts

REDUCING FLOWCHARTS	REDUCING
RENAMING FLOWCHARTS	IMPROPER-MIXED

Addition/Subtraction

LCD FLOWCHART I	LCD
LCD FLOWCHART II	LCD
FRACTION FLOWCHART	ADDITION/SUBTRACTION

Multiplication/Division

FACTOR BOARDS - II	MULTIPLICATION/DIVISION
--------------------	-------------------------



FRACTION FLOWCHART

MULTIPLICATION/DIVISION

OTHER WAYS

DIVISION

MIXED NUMBER MULTIPLICATION

MULTIPLICATION

DECIMALS:

Multiplication/Division

DOTMAN AND BOBBIN

MULTIPLICATION

DOTMAN'S DECIMAL NOTES

MULTIPLICATION

LOTS OF DOTS SHORTCUT

DIVISION

SQUARE ROOT GAME

SQUARE ROOTS

Mixed Operations

"THE ANSWER"

COMPUTATION

LARGE AND SMALL NUMBERS:

Exponential Notation

LET'S GENERALIZE

POWER SERIES

LET'S GET SCIENTIFIC

SCIENTIFIC NOTATION

NUMBER SENSE AND ARITHMETIC SKILLS

Placement Guide for Tabbed Dividers

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Concepts	610
Addition/Subtraction	664
Multiplication/Division	686
Mixed Operations	734
LARGE & SMALL NUMBERS	756
Awareness	756
Exponential Notation	780
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ESTIMATION and APPROXIMATION

RATIONALE

Why estimate and approximate? Why should we be concerned with educated guesses (estimation) or a process to improve the accuracy of an educated guess (approximation)?

Today, according to some authorities, 75% of adult non-occupational uses of arithmetic is mental. If we are concerned about students having a number sense, then we need to work on such things as: mental computation, rounded results, reasonableness of answers, a feel for large and small numbers, and numbers representing measures.

In our daily lives we use inexact numbers every time we measure. News sources frequently use approximations when discussing large numbers. Exact results are often not necessary, and they often obscure the issue. (Which would be better--49,717 people attended the football game, or "about 50,000," The family income is \$11,978 vs. The family income is \$12,000?)

For example, we make many educated guesses every time we

- a) plan a trip (How long will it take, when will we arrive, how much will it cost, what should we take?)
- b) determine a budget (I think we can go out for dinner and a show once this year.)

We make a life and death estimation when we decide if it is safe to cross the street, or if we can stop a car or

bike in time.

The reasonableness of calculated results can mean a difference of many dollars to each of us, whether it be in checking the change at the supermarket, figuring taxes, or making time payments on large purchases.

Often we need to locate the decimal point in computations by hand, with a slide rule, with a calculator, or in using square root tables. Even when we do long division problems we usually use some type of "guess and check" method.

We make "ball park" estimates for

- a) how many (hot dogs to order for a football game)
- b) how things compare (can 1,000 people fit into the ballroom?)
- c) personal information (if we could spend a dollar a second, how long would it take to spend a billion dollars?)
- d) functioning effectively in our daily lives.

Before anyone can make an estimation that is more than just a guess, he must first of all have a familiarity with certain reference points for measures of length, weight, time, area, volume, cost, and so on. Most of these come from experiences in the person's day to day world. They can be extended through development of measuring skills, arithmetic skills, and a number sense for large and small numbers. To obtain a "good" estimate, it is also useful to



have a knowledge of counting methods.
(For additional information see Peas and Particles.)

Before a person can quickly check the reasonableness of an answer he must have already developed a wide variety of arithmetic skills. These must include:

- a) ability to perform accurately single-digit operations (9 million x 7 million requires $9 \times 7 = 63$)
- b) ability to multiply and divide by powers of ten
- c) ability to perform operations with multiples of powers of ten--mentally if possible
- d) being comfortable with inequalities and other relationships
- e) ability to round whole numbers and decimals to one or two significant digits.

It is also helpful for more difficult approximations if a person has a familiarity with exponential notation.

Here is an example which illustrates most of these points: About how long is a billion seconds?

$$\frac{1,000,000,000}{60 \times 60 \times 24 \times 365} = \text{years} \approx$$

$$\frac{1 \times 10^9}{60 \times 60 \times 20 \times 400} = \frac{1 \times 10^9}{3600 \times 8000}$$

$$\approx \frac{1 \times 10^9}{4 \times 10^3 \times 8 \times 10^3} = \frac{1 \times 10^9}{32 \times 10^6}$$

$$\approx \frac{1 \times 10^9}{3 \times 10^7} = \frac{1}{3} \times 10^2 \approx 33 \text{ years}$$

There is much to be said for knowing when to estimate and when to approximate, when to use an estimation or approximation, and when to use an exact answer. The use of estimation and approximation should help all persons to deal with exact numbers, understand and perform operations with numbers arising from measurement, deal comfortably with numbers through approximate calculations and rounding off, and in general develop a number sense. Finally, it would seem most worthwhile if teaching the techniques of estimation and approximation helped to eliminate the "exact answer" syndrome.

SUMMARY

These are the key points to be emphasized when teaching estimation and approximation:

1. When do we need to estimate and approximate to find a rough answer?
2. When do we need exact answers?
3. We often estimate "how many" (e.g., objects, people, items) or "how much" (e.g., money, air, water).
4. We often estimate the dimensions, capacity or amount of something we would measure. (Measurements are always approximate.)
5. Problem solving and computation is aided by the use of estimation and approximation to . . .
 - a) check the reasonableness of answers
 - b) narrow the scope of your investigations
 - c) simplify computations



6. The students need a sound background in arithmetic skills, number sense, and finding reference points.

Selected Sources for Estimation and Approximation

Garvin, Alfred D. Shortcuts, Checks and Approximations in Mathematics, J. Weston Walch, 1973.

Herrick, Marian, et al. Mathematics for Achievement/Individualized Course 2, Book 5, Houghton Mifflin, 1972.

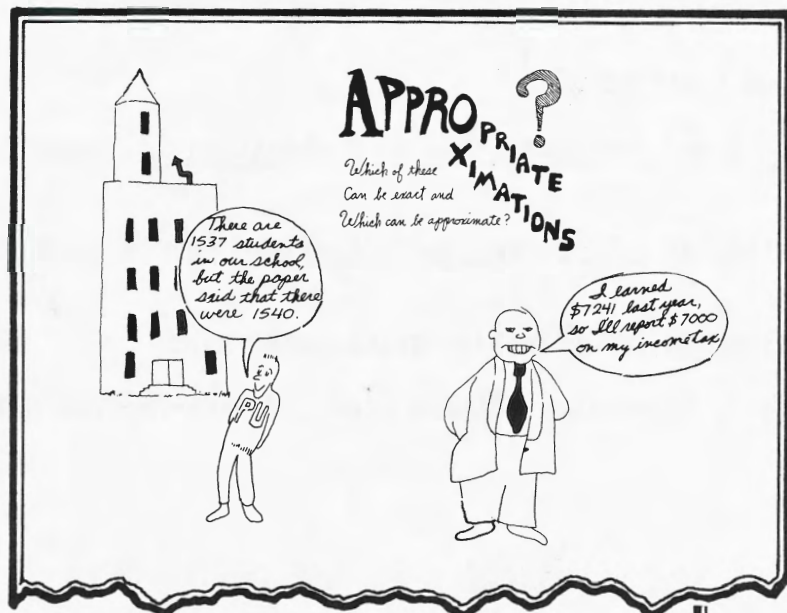
Mathex Book 5 Measurement and Estimation, Encyclopedia Britannica, 1970

Peas and Particles (Teacher's Guide), Elementary Science Study, Webster/McGraw-Hill, 1969.



EXAMPLES OF ESTIMATION AND APPROXIMATION IN THE CLASSROOM MATERIALS

I. When to Estimate and Approximate



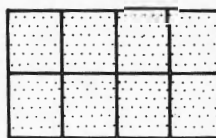
When is it appropriate to approximate? When do you need exact figures?

Guess the number of items in each picture. Can you think of a systematic way to estimate the number of dots?

"GUESS AGAIN"

MAKE A QUICK GUESS AND AN EDUCATED GUESS FOR THESE. EXPLAIN YOUR EDUCATED GUESS.

HOW MANY DOTS?



QUICK GUESS _____

EDUCATED GUESS _____

HOW MANY BIRDS?

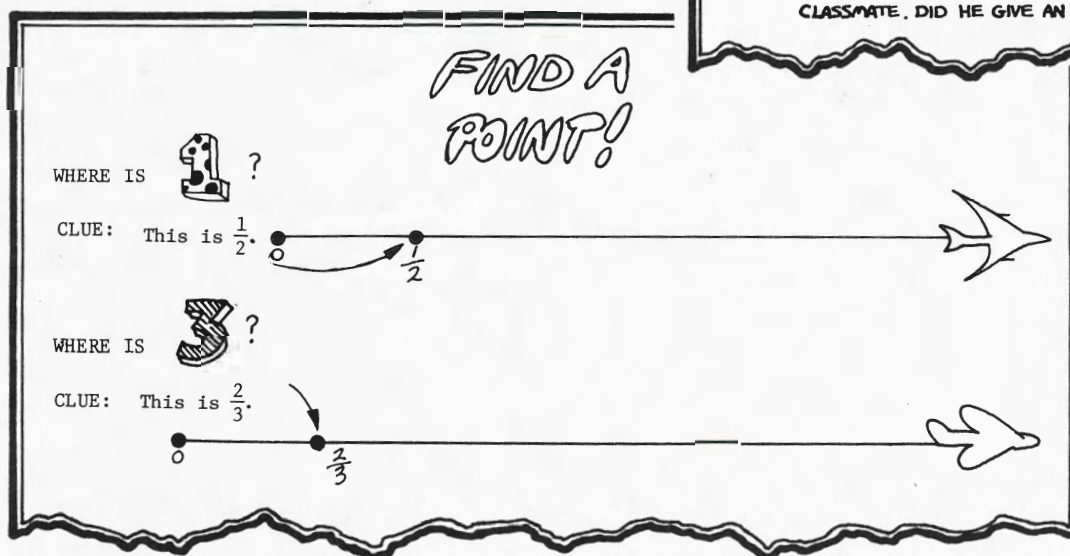


QUICK GUESS _____

EDUCATED GUESS _____

MAKE UP A PROBLEM. GIVE IT TO A CLASSMATE. DID HE GIVE AN EDUCATED GUESS?

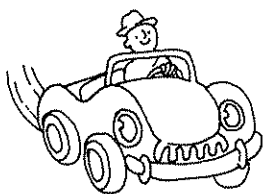
II. Applying Estimation and Approximation Skills



Use a given measure to estimate a larger or smaller quantity.



PLACING A POINT



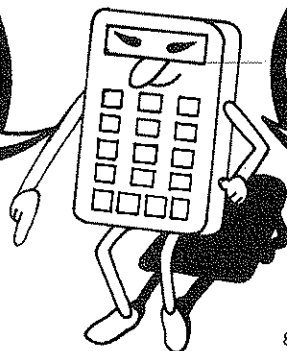
On the average
Mr. Brown's car
travels 182
miles on 1 gallon
of gasoline.



The patient's
temperature
in the morning
was 100.8
degrees.

A SLICK SLIP STICK

I am a hand calculator.
I was used to do these
problems. Approximate
to see if these answers are
reasonable.



If the answer
is not reasonable
what could have
happened?

1. $378 + 594 = 682$
2. $178 \times 342 = 60876$

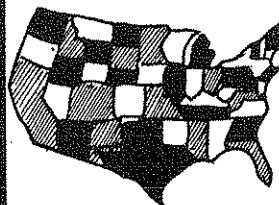
8. $9178 - 3904 = 5274$
9. $189 \div 20 = 9.45$

Do these answers make
sense? What would be
a reasonable answer?
Are your calculator
answers reasonable?

Simplify problems with involved
computations by rounding the large
numbers and approximating the
answers.

ONE-MILLIONTH ONE-MILLIONTH

- 6) Find one-millionth of the
total area of your state;
your city.



- 7) Find one-millionth of the
distance across the United
States.



You often estimate in everyday situations. Look up prices in your newspaper to see which store has the best buy.

SHOPPING WITH A NEWSPAPER

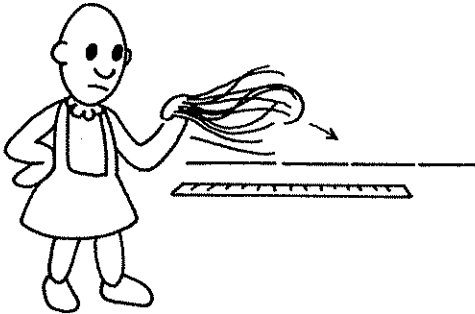
PLAN A PARTY FOR 10 OR 20 OF YOUR FELLOW STUDENTS. YOU MAY USE ONLY THE ITEMS LISTED IN ANY OF THE ADS. YOU MAY SHOP AT MORE THAN ONE STORE.

EXAMPLE :

ITEM	STORE A	STORE B	STORE C	STORE D	STORE E	BEST BUY
2 POUNDS GROUND ROUND						
3 POUNDS HOT DOGS						
6 SMALL CANS FROZEN ORANGE JUICE						
3 POUNDS COFFEE						
1 CAN TOMATO SOUP						
TOTAL						

MORE INVESTIGATIONS

Estimate in metres the total length of hair on your head. If you were to have it cut, how much total length would be taken off?



Estimate how much it would cost to buy a mathematics textbook for every student in your school. Estimate how much money is spent on paper for worksheets, quizzes, etc. in mathematics classes.



However you approach these problems, they involve guess work, estimation and approximation.



ESTIMATION AND APPROXIMATION FOUND IN CLASSROOM MATERIALS

WHOLE NUMBERS:

Getting Started

EXACT OR APPROXIMATE	COUNTING
PATTERNS FOR COUNTING	PATTERNS
SEALS ON MY ICEBERG	COUNTING
EDUCATED GUESS GUESS AGAIN	COUNTING
25-MORE OR LESS	COUNTING

Concepts

APPROPRIATE APPROXIMATION	ROUNDING
EARTH FACTS	ROUNDING
GIVE OR TAKE	APPROXIMATION

Mixed Operations

ROUNDED RESULTS	COMPUTATION
ROUNDED LINE UP	COMPUTATION
DON'T BE AFRAID TO GUESS	COMPUTATION
A SLICK SLIP STICK	COMPUTATION
ABOUT RIGHT	WORD PROBLEMS
ALTITUDES	AVERAGE

FRACTIONS:

Concepts

COOPERATION	WHOLE MODEL
COLOR CLASH	WHOLE MODEL
DIAL A FRACTION	WHOLE MODEL

Addition/Subtraction

UP-DOWN APPROXIMATION	ADDITION/SUBTRACTION
-----------------------	----------------------



ALMOST GAMES

ADDITION/SUBTRACTION

Multiplication/Division

CAN YOU GET CLOSE UP

MIXED NUMBERS

BETWEEN

MIXED NUMBERS

DECIMALS:

Concepts

PLACING A POINT

PLACE VALUE

DECIMAL

PLACE VALUE

BOXING IT

PLACE VALUE

IT'S CLOSER THAN YOU THINK!!

ROUNDING

MATH TOOLS

ROUNDING

BASEBALL AUTO RACING RECORD

ROUNDING

EYEBALLING DECIMALS

RELATION TO FRACTIONS

Addition/Subtraction

ROUNDED ADD-A-BOX

ROUNDING

Multiplication/Division

SEEING'S BELIEVING-I

MULTIPLICATION

SEEING'S BELIEVING-II

DIVISION

NEVER HOLD A GRUDGE . . .

MULTIPLICATION/DIVISION

NOBODY KNOWS BUT THE ZEROS

MULTIPLICATION

SQUARED OFF

SQUARES

GRASS ROOTS

SQUARE ROOTS

SQUARES

SQUARES

SQUARE ROOTS

SQUARE ROOTS

SQUARE ROOT GAME

SQUARE ROOTS



Mixed Operations

APPROX-APPRAISALS

WORD PROBLEMS

OPERATION PLEASE!

WORD PROBLEMS

GETTING 'ROUND TO CALCULATING

COMPUTATION

UP AND DOWN WITH THE
CALCULATOR

ROUNDING

SHOPPING WITH A NEWSPAPER

MIXED OPERATIONS

LARGE AND SMALL NUMBERS:

Awareness

ONE MILLION CUBES

ONE MILLION

BEANS IN A JUG

COMPUTATION-LARGE

STRETCHING YOUR DOLLARS

COMPUTATION-LARGE

YOUR HEARTBEATS

COMPUTATION-LARGE

SECONDS TIMELINE

COMPUTATION-LARGE

ONE-MILLIONTH

COMPUTATION

MORE INVESTIGATIONS

COMPUTATION

Exponential Notation

GRAINS OF SAND AND TURNING
WHEELS

SCIENTIFIC NOTATION

NUMBER SENSE AND ARITHMETIC SKILLS

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LABORATORY APPROACHES

RATIONALE

What is the Laboratory Approach?

For many decades, learning, instead of just memorization and training, has been the primary emphasis of education. Each society or community decides what should be learned. We are required to learn mathematics, reading, science and other subjects. Yet our schools have been organized for teachers to teach and not necessarily for children to learn. The laboratory approach is a philosophy which emphasizes "learning by doing" and breaks free from formal teaching methods. "It is a system based on active learning and focuses on the learning process rather than on the teaching process." [Kidd, et al.] Experiences are devised to help the student learn mathematics by seeing, touching, hearing and feeling. An environment--the math lab--emerges where the teacher and the students work and communicate with each other to plan activities and learn by doing. At the level of their abilities and interests, the students discover relationships and study real-world problems which utilize specific mathematical skills.

A laboratory approach breaks the monotony of straight textbook teaching. It extends and reinforces the students' understandings and skills while providing background experiences for

later development of abstract concepts. It also offers a unique, concrete way to learn mathematics. The laboratory approach can be integrated into the classroom and used along with, not in place of, many other equally valuable teaching strategies.

Lab activities help to eliminate the unrealistic one-method syndrome so characteristic of mathematics classes. A variety of methods of attacking a problem can be explored. Open-ended activities encourage students to make discoveries, formulate and test their own generalizations (i.e., problem solving). Lab assignments can be used to challenge the students by providing them with opportunities for developing self-confidence, habits of independent work, and enjoyment of mathematics. The relaxed atmosphere can encourage student involvement and positive attitudes toward mathematics. By direct observation, the teacher can assess the student's skills in problem solving and computing while the student's attitude and work habits can also be evaluated.

The Mathematics Laboratory

The math lab is an environment that provides for active learning and encourages active participation. In terms of physical organization, three basic kinds of mathematics laboratories are most often discussed.

1. A centralized laboratory--a room



especially designed (or adapted) and equipped for use as a permanent math lab. Classes are usually brought into the lab room on a rotating schedule that allows each mathematics class to use the lab materials several times a week as needed.

2. A rolling or movable laboratory--a set of lab materials placed on a cart, stored in a central location, and wheeled from classroom to classroom as needed.
3. A decentralized laboratory--a self-contained set of lab materials stored in the teacher's classroom and readily available for the students to use.

For most schools, the decentralized laboratory is the most practical and desirable math lab. Lab materials can be collected and organized at a modest rate as they are constructed, donated or purchased.

Eventually a set of lab materials will grow to a size large enough to be quite versatile. The classroom environment needs to be versatile as well. Flat tables, bookcases, movable carts and other furniture can be added to provide work areas for the students and storage space for the lab activities.

What is a Laboratory Activity?

A laboratory activity is a task or mathematical exercise that emphasizes "learning by doing." It can be a game, a puzzle, a paper and pencil exercise, a set of manipulatives with a task card, or an experiment using apparatus and instruments to take measurements. A

game involving two or more students might review the concept of equivalent fractions. A challenging puzzle could require a student to apply several problem-solving techniques. A lab activity could use Cuisenaire Rods to illustrate decimal concepts, or multi-base blocks to show place value, or wooden cubes to demonstrate spatial relationships, or factor boards to clarify an algorithm. Manipulative objects often provide physical models that can introduce or clarify a mathematical concept to the student. There are also experiments which can be performed to take measurements and gather data. Students learn how to use certain equipment and tools in their search for solutions. Laboratory activities can directly involve students in "hands-on" assignments, often with group participation. Lab activities encourage the student to take an active role in learning mathematics rather than the passive role of "you teach me."

Getting Started

There are many ways to implement the lab approach. The descriptions below provide several suggestions to consider when starting to use the laboratory approach.

Mr. Langford has a class of thirty seventh graders. He was not sure about using lab materials, so he decided to start small. He set up an "activity (



corner" in the room. Three lab cards with the necessary equipment (e.g., squared paper, ceramic tile, measuring tape, metric wheel) were set up in the "activity corner." Each day for a week a different group of six students were allowed to work in pairs using the lab materials. The rest of the class worked on related paper and pencil exercises. All week was spent on the study of area. All thirty students had a chance to do the lab activities, and the activities integrated well with the week's mathematics concept of area. Mr. Langford wants to collect or write task cards that mix well with his established curriculum. Later, he might try other ways of using the lab activity cards.

Ms. Wilkins decided to assign each Friday as a "lab day" for her eighth-grade class of 28 students. She had watched several classes using a "lab day" once a week and decided to try it herself. She prepared two sets of seven lab cards covering seven different mathematical topics. Each student was assigned a partner, and the pair would work together for each of the seven "lab days." For seven weeks the students rotated to a new lab activity each Friday. They were asked to keep a record of their results and follow the planned rotation schedule. Ms. Wilkins found that this seven-week period with

one "lab day" a week coincided well with the nine-week term. She developed a second set of lab materials for another seven weeks. This time there were 14 task cards put into 14 shoe boxes along with manipulatives, paper, or other materials needed for each activity. Each card was written on the topic of measurement and contained various levels of abstraction and enrichment options for the students.

Mr. Jeffreys and Ms. Slone had adjoining sixth-grade rooms. They had been team teaching a number of units in mathematics. They decided to try the lab approach for their unit on Base 10 and Other Bases. Their school had recently purchased two Chip Trading Math Lab Sets. Mr. Jeffreys and Ms. Slone picked out several chip trading activities to be used every other day for two weeks. They divided the class into groups of 3 or 4 students. For each "chip trading day" one student in each group was responsible for picking up and distributing the manipulatives to each member of the group. The days between each "chip trading day" were used for discussions, board work, and worksheets that emphasized paper and pencil computation in base 10 and other bases.

The above are examples of teachers who were willing to support an active approach to learning. They prepared for using the lab approach by collecting and organizing



materials and deciding on the content of lab activities. It helps to gain the support of other teachers; their contributions and ideas can rapidly increase the number of lab activities developed.

Most difficulties that arise in the math lab result from students not knowing what to do. The teacher needs to find, organize and store lab materials for easy use; tell students where lab materials are, what to do with them and how to schedule their use; prepare task cards or directions for the lab activities; instruct students in problem-solving methods of attack and investigation; interact enthusiastically with students and share in their experiences; and evaluate each student's attitudes, work habits and accomplishments.

Start small--in no way can most teachers and students survive a complete change of program. Students who have become passive learners need time to adapt to the role of active learners. They need supervision and guidance from the teacher as they learn to function in the lab environment. Eventually, the students should be able to select materials for each lab activity and return materials to the proper storage area when finished. By keeping a work record, the students can evaluate their progress and try to improve their skills

and understanding. The students need to develop inquisitive attitudes that motivate them to keep at a problem and not give up. Small groups or pairs of students will require the cooperation of each individual and the sharing of ideas.

Initially, when selecting material and equipment to use in the math lab, find readily available materials in the school. As time goes on, you will be able to buy, make or scrounge other materials as they are needed for particular activities.

Ideas for laboratory activities can be found in any of the sources listed in the selected sources. Many periodicals (such as The Arithmetic Teacher or The Mathematics Teacher) include sections in each issue which contain ideas for activities that require a minimum of preparation and materials. Notice the interests of the students. Be creative and use your own ideas or their ideas as a source of lab activities. Discuss and exchange ideas about math labs with other teachers.

Begin with a lab activity that everyone can do at the same time. Later on, the students can separate into groups or small teams (students usually work best in small groups of 2 or 3). Experiment with the size and the make-up of the groups. In the beginning it is a good idea to provide activities where each group member has a specific role. Provide several lab activities and let ea



group move from one activity to another. Have a specific objective(s) in mind for each activity, and have a clear idea of its mathematical content. Go through the lab activity to find what

background concepts or skills the students will need to tackle it. Check for any difficulties the students might encounter as they do the activity.

Selected Sources for Laboratory Approaches

The Arithmetic Teacher, National Council of Teachers of Mathematics.

Biggs, Edith and James MacLean. Freedom to Learn, Addison-Wesley (Canada) Ltd., 1969.

Hamilton, Schmeltzer & Schmeltzer. "The Mathematics Laboratory," Teaching Mathematics in the Junior High.

Kidd, Kenneth, et al. The Laboratory Approach to Mathematics, Science Research Associates, Inc., 1970.

Krulik, Stephen. A Mathematics Laboratory Handbook for Secondary Schools, W.B. Saunders Co., 1972.

The Mathematics Teacher, National Council of Teachers of Mathematics.

Reys, Robert F. and Thomas R. Post. The Mathematics Laboratory: Theory to Practice, Prindle, Weber & Schmidt, Inc., 1973.

Sobel, Max and Evan Maletsky. Teaching Mathematics: A Sourcebook of Aids, Activities and Strategies, Prentice Hall, Inc., 1975.

Teacher-Made Aids for Elementary School Mathematics, Readings from the Arithmetic Teacher, National Council of Teachers of Mathematics.

Laboratory and instructional materials can be obtained from the following publishing companies:

Creative Publications, Inc.
P. O. Box 10328
Palo Alto, CA 94303

Herder and Herder, Inc.
232 Madison Avenue
New York, New York 10016

Cuisenaire Company of America, Inc.
12 Church Street
New Rochelle, New York 10805

Ideal School Supply Company
11000 South Laverne Avenue
Oaklawn, Illinois 60453

Educational Teaching Aids Division
159 West Kinzie Street
Chicago, Illinois 60611

Midwest Publications Company, Inc.
P.O. Box 307
Birmingham, Michigan 48012

TEACHING EMPHASES

LABORATORY APPROACHES



Mind/Matter Corporation
P.O. Box 345
Danbury, Connecticut 06810

Ohaus Scale Corporation
29 Hanover Road
Florham Park, New Jersey 07932

Scott Resources, Inc.
1900 E. Lincoln
Box 2121
Fort Collins, Colorado 80521

Selective Educational Equipment, Inc. (C)
3 Bridge Street
Newton, Massachusetts 02195

Walker Educational Book Corporation
720 Fifth Avenue
New York, New York 10019

Webster Division
McGraw-Hill Book Company
330 West 42nd Street
New York, New York 10036



LABORATORY APPROACHES FOUND IN CLASSROOM MATERIALS

WHOLE NUMBERS:

Concepts

ACTIVITY CARDS - PAN BALANCE	PLACE VALUE
ACTIVITY CARDS - BASE 10 MULTIBASE BLOCKS - I	PLACE VALUE
FOUR-PLACE NUMBERS	EXPANDED NOTATION

Addition/Subtraction

ACTIVITY CARDS - CUISENAIRE RODS - I	ADDITION
ACTIVITY CARDS - CUISENAIRE RODS - II	SUBTRACTION
SUM COMBINATIONS	ADDITION
ACTIVITY CARDS - BEANSTICKS - I	REGROUPING TO ADD
ACTIVITY CARDS - BEANSTICKS - II	REGROUPING TO SUBTRACT
ACTIVITY CARDS - MATHEMATICAL BALANCE - I	ADDITION/SUBTRACTION
ACTIVITY CARDS - BASE 10 MULTIBASE BLOCKS - II	REGROUPING TO ADD
ACTIVITY CARDS - BASE 10 MULTIBASE BLOCKS - III	REGROUPING TO SUBTRACT
ACTIVITY CARDS - CHIP ABACUS	REGROUPING TO ADD
FILL IN THE DOMINOES	ADDITION

Multiplication/Division

ACTIVITY CARDS - CUISENAIRE RODS - III	MULTIPLICATION
ACTIVITY CARDS - MATHEMATICAL BALANCE - II	MULTIPLICATION
ACTIVITY CARDS - MATHEMATICAL BALANCE - III	DIVISION



ACTIVITY CARDS - BASE 10 MULTIBASE BLOCKS - IV	MULTIPLICATION
ACTIVITY CARDS - BASE 10 MULTIBASE BLOCKS - V	DIVISION
ACTIVITY CARDS - BEANSTICKS - III	MULTIPLICATION
ACTIVITY CARDS - BEANSTICKS - IV	DIVISION
EVEN STEVEN ODD ROD	ODD & EVEN
NUMBER TYPES	MULTIPLICATION
A SQUARE DEAL	MULTIPLICATION

ACTIONS:

Concepts

ACTIVITY CARDS - CUISENAIRE RODS - I	WHOLE MODEL
FRACTION APPLE-CATION	WHOLE MODEL
ACTIVITY CARDS - CIRCLE FRACTIONS - I	WHOLE MODEL
ACTIVITY CARDS - GEOBOARDS - I	WHOLE MODEL
ACTIVITY CARDS - TANGRAMS - I	WHOLE MODEL
DIAL A FRACTION	WHOLE MODEL
ACTIVITY CARDS - COLOR CUBES - I	SET MODEL
DIVIDE ME UP	QUOTIENT MODEL
ACTIVITY CARDS - FRACTION BARS - I	EQUIVALENT
ACTIVITY CARDS - CIRCLE FRACTIONS - II	EQUIVALENT
ACTIVITY CARDS - COLOR CUBES - II	EQUIVALENT
READING YOUR RULER	NUMBER LINE MODEL



FACTOR BOARDS - I

REDUCING

ACTIVITY CARDS - COLOR
CUBES - III

IMPROPER-MIXED

ACTIVITY CARDS - CLOCK
FRACTIONS - I

IMPROPER-MIXED

ACTIVITY CARDS - CUISENAIRE
RODS - II

IMPROPER-MIXED

MULTIPLE BOARDS - I

ORDERING

Addition/Subtraction

ACTIVITY CARDS - FRACTION
BARS - II

ADDITION/SUBTRACTION

ACTIVITY CARDS - CUISENAIRE
RODS - III

ADDITION/SUBTRACTION

ACTIVITY CARDS - CLOCK
FRACTIONS - II

ADDITION/SUBTRACTION

ACTIVITY CARDS - TANGRAMS - III

ADDITION/SUBTRACTION

ACTIVITY CARDS - GEOBOARDS - II

ADDITION/SUBTRACTION

FRACTION SLIDE RULE

ADDITION/SUBTRACTION

MULTIPLE BOARDS - II

ADDITION

Multiplication/Division

ACTIVITY CARDS - FRACTION
BARS - III

MULTIPLICATION/DIVISION

ACTIVITY CARDS - GEOBOARDS - III

MULTIPLICATION/DIVISION

ACTIVITY CARDS - TANGRAMS - IV

MULTIPLICATION/DIVISION

GRAPH PAPER MULTIPLICATION

MULTIPLICATION

FACTOR BOARDS - II

MULTIPLICATION/DIVISION

Mixed Operations

PIZZA PUZZLE

ESTIMATION

DECIMALS:

Concepts



ACTIVITY CARDS - CUISENAIRE
RODS - I

PLACE VALUE

BOXING IT

PLACE VALUE

ACTIVITY CARDS - DECIMAL
ABACUS - I

PLACE VALUE

MAKING A DECIMAL RULER

PLACE VALUE

ACTIVITY CARDS - CUISENAIRE
RODS - II

ROUNDING

ACTIVITY CARDS - CUISENAIRE
RODS - III

RELATION TO FRACTIONS

ACTIVITY CARDS - CUISENAIRE
RODS - IV

RELATION TO FRACTIONS

ACTIVITY CARDS - CUISENAIRE
RODS - V

ORDERING

Addition/Subtraction

ACTIVITY CARDS - CUISENAIRE
RODS - VI

ADDITION/SUBTRACTION

ACTIVITY CARDS - DECIMAL
ABACUS - II

ADDITION/SUBTRACTION

Multiplication/Division

MULTIPLICATION PAPER FOLDING

MULTIPLICATION

ACTIVITY CARDS - DECIMAL
ABACUS - III

MULTIPLICATION

ACTIVITY CARDS - DECIMAL
ABACUS - IV

POWERS OF TEN

HANDS ON DECIMALS

DIVISION

LARGE AND SMALL NUMBERS:

Exponential Notation

MULTIBASE BLOCKS - I

EXPANDED NOTATION

MULTIBASE BLOCKS - II

EXPANDED NOTATION

LARGE NUMBERS ON THE ABACUS - I

POWERS OF TEN

TEACHING EMPHASES

LABORATORY APPROACHES



SMALL NUMBERS ON THE ABACUS - I

POWERS OF TEN

LARGE NUMBERS ON THE ABACUS - II

EXPANDED NOTATION

SMALL NUMBERS ON THE ABACUS - II

EXPANDED NOTATION

NUMBER SENSE AND ARITHMETIC SKILLS

Placement Guide for Tabbed Dividers

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GEOBOARDS

MATHEMATICAL BALANCE

MULTIBASE BLOCKS

MULTIPLE BOARDS

TANGRAMS

LIST OF MANIPULATIVES

The name Cuisenaire and the color sequence of the rods are trademarks of the Cuisenaire Company of America, Inc.

ABACUS

An abacus is a tool for representing numbers with some sort of markers. The markers are placed in the columns of each place value. Abaci are useful for working with whole number and decimal concepts and operations. There are many different kinds of abaci, many of which can be easily constructed. They can be purchased from Creative Publications and Scott Resources.

Each of the abaci described here can be extended to any number of places.

Construction Information:

Chip Abacus

Materials and Construction:

Rule six columns on a sheet of tagboard in such a way that 10 chips will fit vertically in each column.

You can make several different label-strips to head the columns. Here are some possibilities:

Th	H	T	U	t	h

RED	GREEN	BLUE	YELLOW
-----	-------	------	--------

(Use colored chips in this case.)

Millions	Hundred Thous	Ten Thous	Thous	Hund	Tens	Units	Tenths	Hundth	Thsth	Ten Thsth	Hund Thsth
----------	------------------	--------------	-------	------	------	-------	--------	--------	-------	--------------	---------------

M	H-Th	T-Th	Th	H	T	U	t	h	th	t-th	h-th
---	------	------	----	---	---	---	---	---	----	------	------

10^6	10^5	10^4	10^3	10^2	10^1	10^0	$\frac{1}{10^1}$	$\frac{1}{10^2}$	$\frac{1}{10^3}$	$\frac{1}{10^4}$	$\frac{1}{10^5}$
--------	--------	--------	--------	--------	--------	--------	------------------	------------------	------------------	------------------	------------------

Slide these strips to fit over the abacus with the units place wherever you want it. It can overlap on both sides or be folded back. Fasten to the abacus with paper clips.

If your chip abacus is laminated, you could write the labels with a grease pencil.

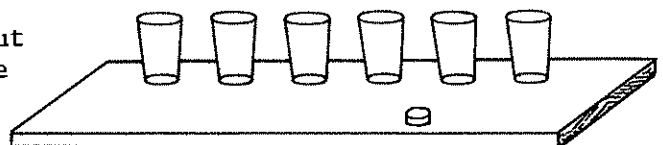
Cup Abacus

Materials:

12 dixie cups, a 25" x 6" board (heavy cardboard or wood), beans or markers (not less than 100) and 1 black checker.

Construction:

Glue 6 cups to the board as shown. Put the other 6 cups inside each of these cups. (This simplifies dumping the beans out of separate cups.) The checker may be placed between any cups to designate the units place. Label the place value with strips like the Chip Abacus.

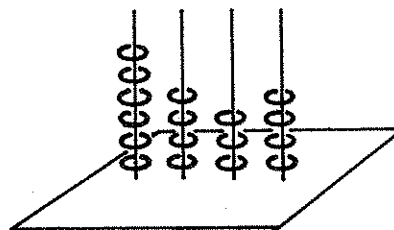




Bead Abacus

Materials:

Block of wood, 1" by 3" by 6"
Four 11" pieces of steel rod or
wood doweling
80 beads or discs which will fit
on the wire or doweling
Glue



Construction:

Drill four holes in the wood.
Finish (make smooth) the upper end of each rod.
Glue one rod into each hole.
Put 20 beads or discs on each rod.
A clothes pin can be used to separate beads on each rod before performing addition.

Readiness Activities:

- 1) A small checker could be used as a physical model of the decimal point. It can be moved to various parts of the abacus, thus varying the position of the units' place. Have the class figure out how the checker determines the units' place by moving the checker. Ask for guesses where the units' place is for all the possible positions of the checker.
- 2) Have the students practice different trading tasks, and decide on one to show the class.
- 3) Show a trade on one type of abacus. Have the class do the same trade on a different type of abacus.
- 4) See *Chip Trading* in WHOLE NUMBERS: Concepts for a variety of place value games that could be played with an abacus.

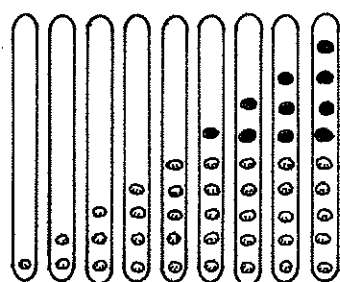
BEANSTICKS

Beansticks are helpful models in understanding place value and operations with whole numbers.

Construction Information:

Make a set of beansticks using dried beans of various kinds, wooden popsicle sticks and glue. There is no standard set of sticks--the size of a set varies with its uses. You will find a set such as the one illustrated below useful.

To make the bean sticks last longer without repair, run some glue over the top of the beans after the beans are glued on.



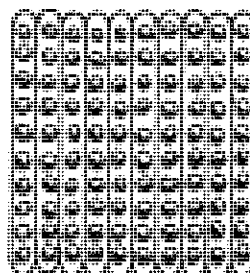
MAKE 3 OF EACH



MAKE 10

10 STICK

10x10
RAFT



MAKE 2

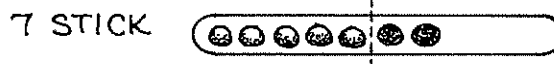


(HOLD 10 STICKS
TOGETHER WITH 2
STICKS GLUED ON
THE BACK.)

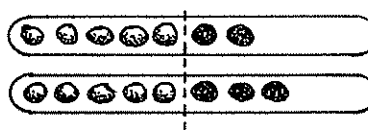
Beansticks can be constructed by the children at almost no cost. Used popsicle sticks can be collected and run through a dishwasher, or new popsicle sticks could be purchased through a restaurant supply house. Your PTA may be persuaded to find enough beans or have the students bring some. There is already a supply of "white glue" in most school warehouses.

Beansticks can also be made by drawing circles on the popsicle sticks with felt pens. Small checkers or poker chips could also substitute for loose beans.

The most efficient method of showing numbers greater than 5 is to place 5 beans on one end and the rest on the other. To make the number even clearer, two colors may be used:



Then when you add $7 + 8$, it looks like this:

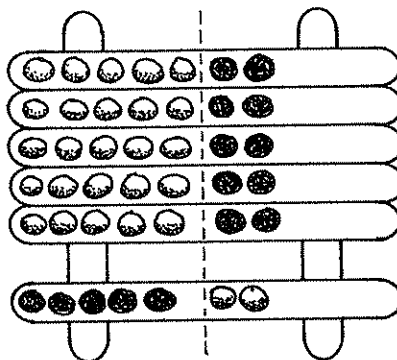


It is easy to see 10 and 5 are 15.



Rafts are beansticks with the same number of beans, attached together to illustrate a product. Rafts can be made so that the total is easy to see without counting:

Here is a 6 x 7 raft:

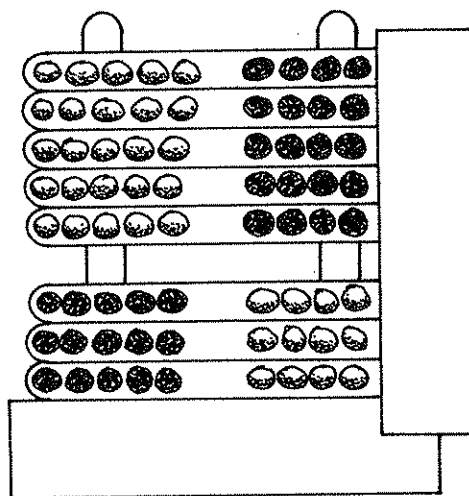


$$25 + 10 + 5 + 2 = 42$$

$$\text{so } 6 \times 7 = 42$$

A ten x ten raft can be used instead of other rafts. A student can use two cards to fence off the combination that he needs:

Here is a fenced 10 x 10 raft showing 8 x 9:



Readiness Activities:

- 1) Have students construct the beansticks.
- 2) Have students sort the beansticks and play trading games.
- 3) Have students show various numbers using the beansticks.
- 4) See *Chip Trading* in WHOLE NUMBERS: Concepts for a variety of place value games that could be played with beansticks.

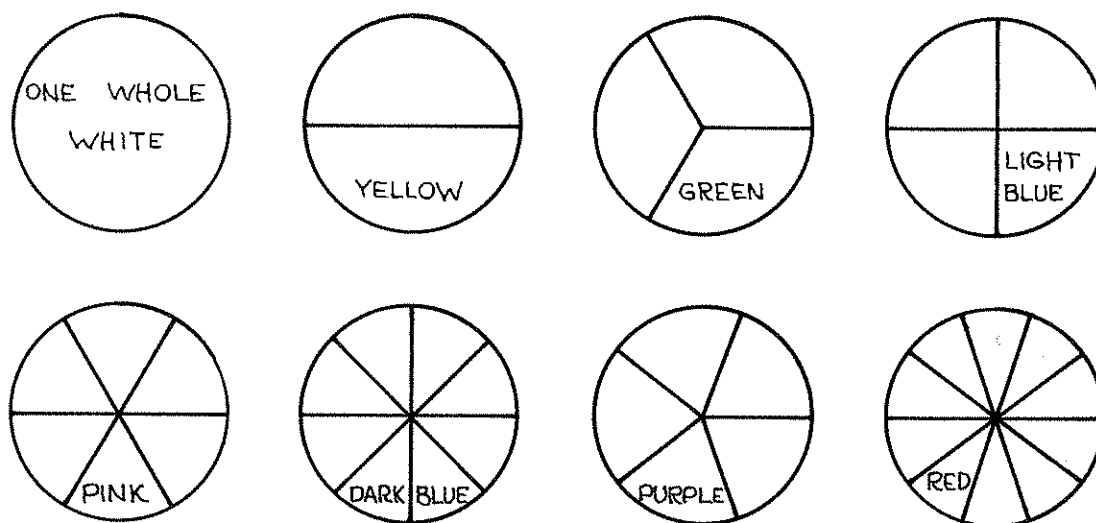
CIRCLE FRACTIONS

Circle fractions are circular fraction disks used for showing order, equivalence and basic operations with fractions.

Construction Information:

Cut eight 15 cm circles from different colored tagboard or construction paper. (The circles could be cut from masonite, $\frac{1}{4}$ " plywood or floor tile and then painted. Perhaps you have a student who could cut them in the school shop.)

The colors shown are only suggestions. If you use other colors, make the appropriate changes in the activities and games relating to the circle fractions.



Label the white circle with "ONE WHOLE" and cut the remaining circles into $\frac{1}{2}$'s, $\frac{1}{3}$'s, $\frac{1}{4}$'s, $\frac{1}{6}$'s, $\frac{1}{8}$'s, $\frac{1}{5}$'s and $\frac{1}{10}$'s (for metric). These fractional parts are not to be labeled.

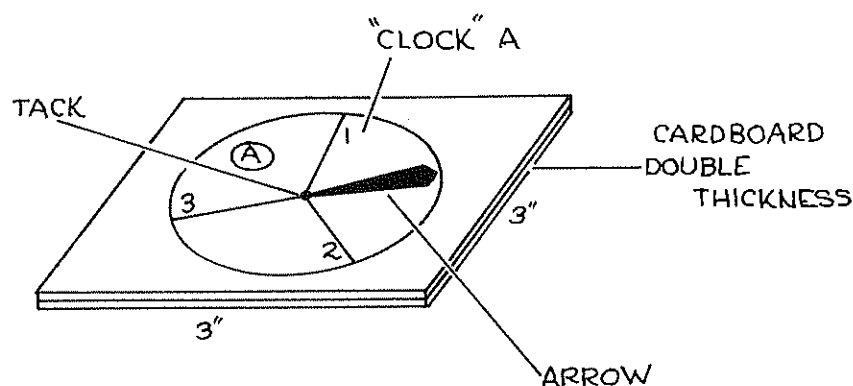
Readiness Activities:

When doing the activities, encourage the students to physically place the pieces on top of each other to compare them.

- 1) Mix two sets of circle fractions together and give them to a group of students to determine the fractional value of each piece. (Since the value can't be found by counting the number of pieces of a color, the pieces must be compared to one whole.)
- 2) Arrange one piece of each color in order from largest to smallest.
- 3) How many pinks fit on a whole?
- 4) How many reds fit on a yellow?
- 5) Find different ways to combine the pieces to cover a whole.
- 6) Find different ways to cover a yellow.

CLOCK FRACTIONS

Fraction "clocks" are manipulatives used to study fraction concepts and equivalence of fractions. They could also be used for the study of decimals and modular systems.



Construction Information:

Fraction "clocks" are put together using the materials mentioned below in this way:

The "clocks" on the following page are duplicated on tagboard (old file folders cut in half), and cut out. Then, different "clocks" are put together on top of a double thickness of cardboard (used as backing). The arrows (hands) are attached to the center of the "clocks" with a thumbtack.

To have your students make their own fraction "clocks" each student will need:

A tagboard duplicate of the "clocks" (next page)

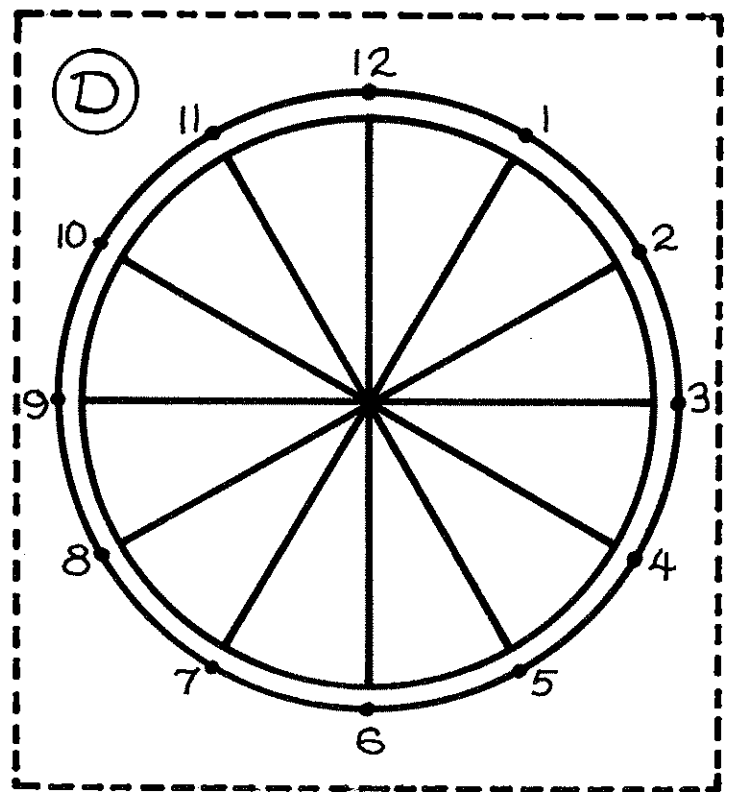
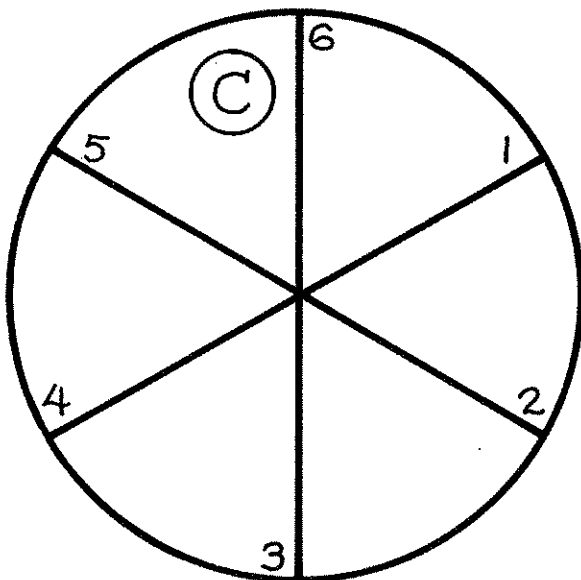
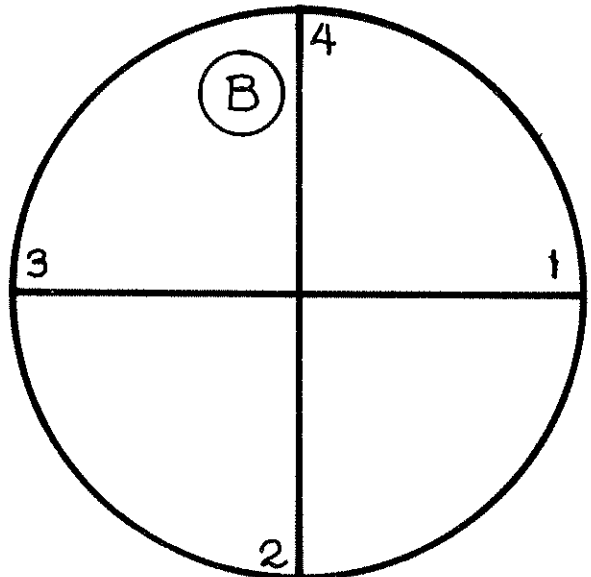
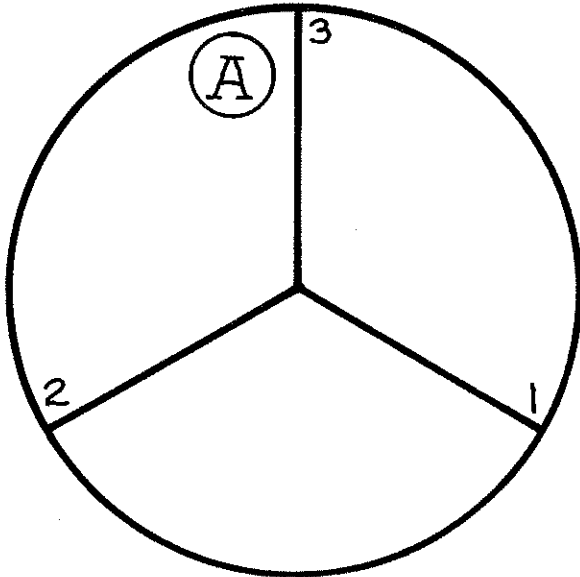
Scissors

1 Thumbtack

A 3" square double thickness of cardboard (for backing)

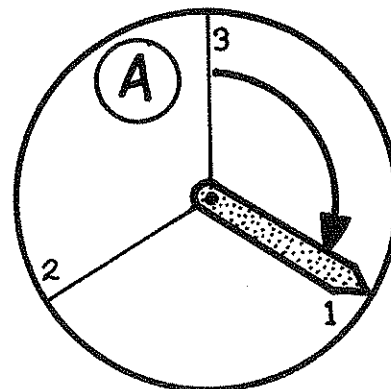
Directions:

Cut out "clocks" A, B, and C, and cut D along the dotted lines. Cut out the arrows.



Readiness Activities:

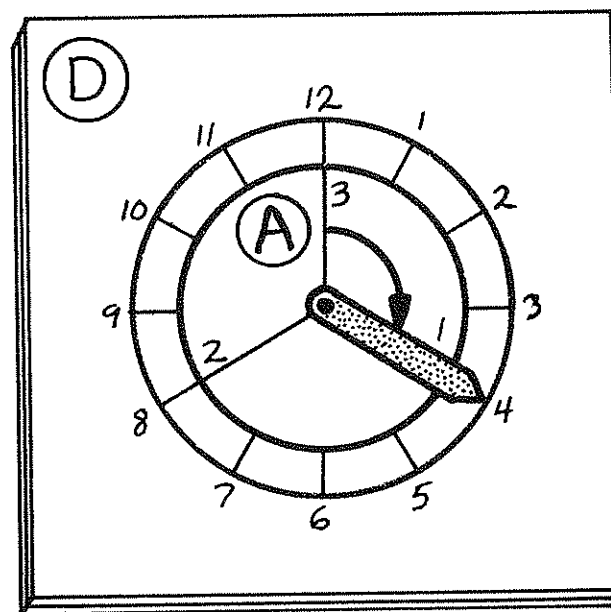
1) Use your thumbtack to attach "clock" A and one arrow to the backing. Rotate the arrow in a clockwise direction from 3 to 1 and fill in the following blanks:



In A there are parts. The arrow was rotated across of these parts. We might say that the arrow was rotated through $\frac{1}{3}$ of A.
 Rotating clockwise from 3 to 2 covers _____ of A.
 Rotating clockwise from 3 to 3 covers _____ of A.

Now have the students attach "clocks" B, C and D and one arrow to the backing and ask similar questions.

2) Place A on top of D. Place an arrow on top of A and attach everything with a thumbtack to the backing. Line up the 3 on A with the 12 on D. Point the arrow to the 12.



Holding A firmly in place with your thumb, rotate the arrow across one of the parts of A. How much of A is this? _____
 In D the arrow was rotated from 12 to 4. This was across how many parts? _____
 How much of D is this? _____ Can you see why this suggests $\frac{1}{3} = \frac{4}{12}$?

Now have the students rotate from 12 to 8 and see what equivalence it suggests.

Similar equivalences can be discovered using "clocks" B and D, and "clocks" C and D.

COLOR CUBES

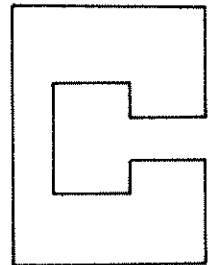
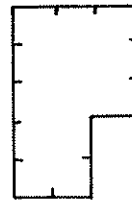
Color Cubes are colored blocks (usually wooden) which can be used for finding volume, area and patterns as well as for working with whole number operations and concepts.

Color cubes are not easy to make, but there are many manufacturers who produce them. A few are listed below.

- Linking cubes -- Creative Publications
 - two sizes: 1 cm or 1.7 cm
- 2 cm color cubes -- Creative Publications
- Unifix cubes -- Educational Teaching Aids, or
Mind/Matter Corporation
- 1" color cubes -- Ideal School Supply Company, or
Midwest Publications
- Cube-o-grams -- Ohaus Scale Corporation
 - linking cubes 1 cm
 - each cube weighs exactly one gram

Readiness Activities:

- 1) Count the number of cubes needed to cover a pattern.

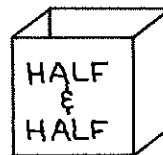


R	Y	B	G
G	R	Y	B
B	G	R	Y
?	θ	?	R

R B B R ?

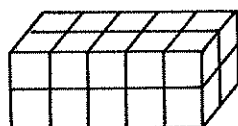
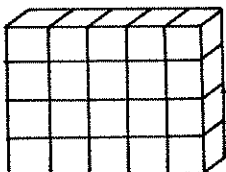
- 2) Extend or complete some sequences or patterns.

- 3) Estimate the number of cubes needed to fill boxes or containers. Check your results with the cubes.



3	2	4	9	6	2	3	3
4	5	8	7	2	1	1	1

- 4) Build stacks of cubes, then rearrange them to find the number of cubes without counting.

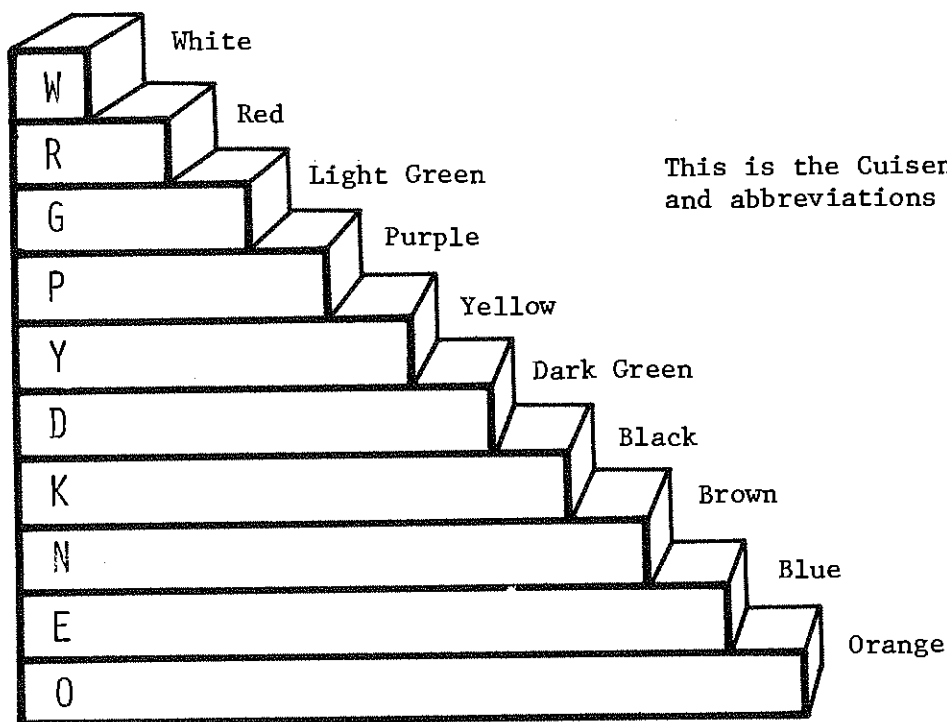


- 5) Build rectangular solids with varying dimensions.

CUISENAIRE RODS

Cuisenaire rods are colored wooden rods one centimetre high, one centimetre thick, and one to ten centimetres long. Each length is coded by a different color. Cuisenaire rods are useful for work with fractions, decimals, and whole number operations and concepts.

Cuisenaire rods can be purchased from the Cuisenaire Company of America.



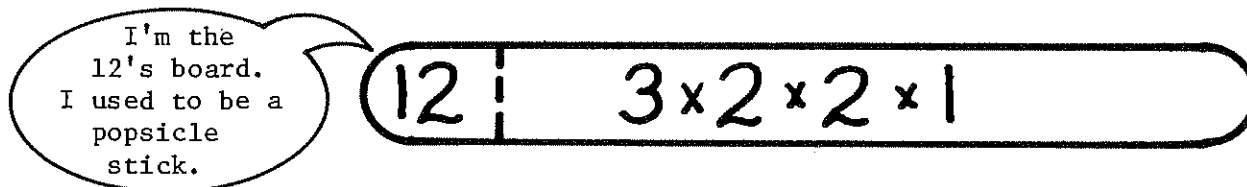
Readiness Activities:

- 1) Find the shortest rod and the longest rod.
- 2) Find a rod longer than the red and shorter than the purple.
- 3) Make a staircase using a rod of each color.
- 4) Name the rods by color; agree on names. (Example: light green → lime.)
- 5) Put the white, yellow, light green, purple and yellow rods in a box and shake. Without looking, find the white rod. How did you do it?
- 6) Put one light green and one red rod end to end like a train. Find one rod as long as your "train."
- 7) Take a blue rod. Place a yellow rod on top. What rod fits next to the yellow rod to make a train the same size as the blue rod?
- 8) If a red rod = 1, which rod = 2?
- 9) Take the light green rod. How many trains can you make as long as the light green rod?

The name Cuisenaire and the color sequence of the rods are trademarks of the Cuisenaire Company of America, Inc.

FACTOR BOARDS

Factor boards can be used for working with factors and primes, for reducing fractions and for multiplying and dividing fractions.



Construction Information:

The construction of factor boards provides practice with factors and primes.

Students can use any of the following to make factor boards: tongue depressors, popsicle sticks, tagboard or railroad board.

Factor boards can be made for all numbers from 1 to 20 (or you could go higher or only to 10 depending on the needs of your students).

Always include 1 on the factor board (even though it is not a prime factor) since it is sometimes needed in calculations.

Letting students work in pairs or small groups will enable them to pool the boards.

When using factor boards to reduce fractions, use markers to cover the like factors so you don't mark up the boards.

Readiness Activities:

- 1) Find all factor boards with only two numbers as factors.
- 2) Find all boards with 2 as a factor.
- 3) Find all boards with 2 and 5 as factors.

FRACTION BARS

A set of fraction bars consists of 64 colored bars divided into 2, 3, 4, 6 and 12 equal parts. This serves as an introductory model of fractions. The denominators of the fractions are represented by the number of parts to a bar and the numerators by the number of shaded parts.

Fraction bars extend the relationships and the four operations for whole numbers in a natural way to the corresponding concepts for fractions.

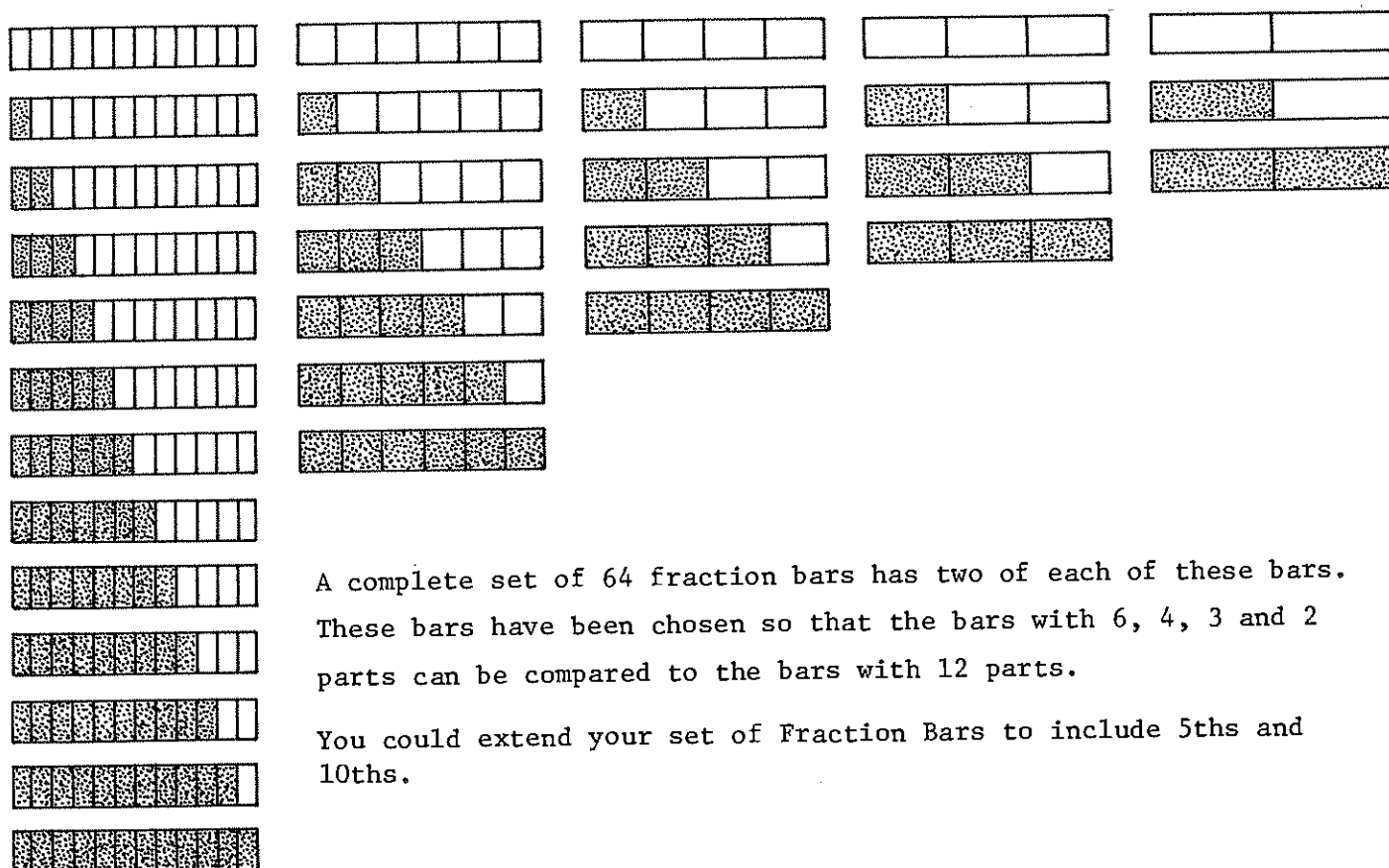
Laminated sets of fraction bars can be purchased through Scott Resources, Inc.

Construction Information:

You can make your own set of fraction bars from colored poster board, orange for 12ths, red for 6ths, blue for 4ths, yellow for 3rds and green for halves.

Each fraction bar should be the same size. 1" x 6" or 2 cm x 12 cm both work well.

There are 32 different fraction bars.



A complete set of 64 fraction bars has two of each of these bars. These bars have been chosen so that the bars with 6, 4, 3 and 2 parts can be compared to the bars with 12 parts.

You could extend your set of Fraction Bars to include 5ths and 10ths.

GAMES AND PUZZLES

Rationale:

A game or puzzle is a teaching device used to motivate and involve the students. Commercial or teacher-made games and puzzles can provide drill and practice of basic skills and review of familiar concepts. "Discovering" the rules of an algebra game may reinforce a new mathematical concept. Solving a puzzle may require the application of several problem-solving techniques. Above all, games and puzzles should be fun and challenging for the students.

A mathematical game usually involves a group of two or more people. The students interact and teach each other the rules of the game--winners and losers are determined by a combination of chance, strategy and skill. One or more mathematical concepts should be reinforced by playing the game.

When selecting a game to be used by the students, there are a number of factors to consider. Is the game worthwhile and enjoyable to at least some of the students? Should the game be purchased or could it be constructed quickly and inexpensively from available materials? Is the game flexible--can it be used with small or large groups; over a long or short period of time; is it easily adapted to a wide range of ability levels? Are the rules easy to read and understand? Will the game be too easy or will it be too difficult and frustrating for most of the students? Do the students like the game and want to play it?

In selecting a math puzzle, one should consider its cost, its flexibility and its purpose (or mathematical objective). A student can usually work individually on a puzzle. Often the puzzle is self-correcting; the student knows when he is finished and if the answer is correct (e.g., Tangram, Soma®, or Cross-number puzzles).

Games and puzzles should be used in the classroom when they are the best teaching strategy available for a specific objective. If two strategies are about equal, use the one which will provide the students with a change of pace or a more enjoyable, interesting lesson. In any activity, the teacher's involvement and enthusiasm influences the students' attitudes and willingness to become involved in their mathematics assignments. The teacher needs to play the mathematics games and solve the puzzles before using them in the classroom. This is the best way to be prepared for the difficulties students may have with an activity.

Materials:

Games and puzzles require a variety of materials. Many can be purchased ready-made. However, if you find yourself with little or no money to buy commercial games, puzzles and equipment, the following suggestions may be helpful.

Cards:

Unmarked playing cards are available from most local printing shops. They often come in a variety of colors, with rounded corners, and in boxes of 500. Be sure that the ones you get have a good resiliency factor. (Will bend and spring back.)

3 x 5 notecards cut in half are also suitable, but they do not last very long.

For heavier cards, use railroad or tagboard cut into 2 x 3 sizes. These work



well for cards that do not need shuffling.

Regular playing cards can sometimes be used as they are (*Game of 50*) or modified for use in mathematics games.

On page 4 you will find instructions for making a deck of rummy cards.

Dice:

Stick-on labels can be placed on regular dice to provide more flexible use of the dice.

Wooden Cubes can be made in the school shop or purchased from commercial companies. They come plain or in a variety of colors. (See the Creative Publications catalog.)

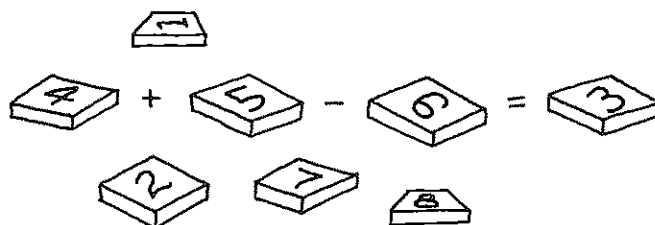
Use of styrofoam or foam rubber cubes can help reduce "rolling-the-dice" noise during a game.

On page 5 you will find a model for dodecahedron dice (12-sided dice). The top model is for dice that are to be used with the *Fraction Bar Football* game. The dice are cut out and glued or taped together. (The blank model can be numbered in the same way, or you may wish to use other numbers so the die can be used for a different activity such as a probability experiment.)

Puzzles:

For durable border puzzles use 2" x 2" size squares of heavy tagboard or railroad board. Write the numbers or figures on the squares and make an obvious border design on the outside pieces for their immediate identification. To make the puzzle more difficult, eliminate the border design. (See highlights for example.)

For "trial and error" puzzles, use numbered ceramic tile for easy manipulation without a record of wrong answers. These tiles come in various sizes. They can be obtained from floor covering stores for a minimal cost (usually less than \$1 per 100). Ask for fragmented sections because stores may have parts of sections which they will give to the school free.



Selected Sources for Games and Puzzles

The Arithmetic Teacher, National Council of Teachers of Mathematics.

Becker, Jan, et al. Enhance Chance, Activity Resources Company, Inc., 1973.

Beefing Up Basic Skills. The Math Group, Inc., 1974.



Brandes, Louis Grant. 4 the Math Wizard, J. Weston Walch, 1962.

____ Yes, Math Can Be Fun, J. Weston Walch, 1960.

Henderson, George L. and Lowell D. Glenn. Let's Play Games in General Mathematics, National Textbook Co., Skokie, Illinois, 1970.

Johnson, Donovan A. Games for Learning Mathematics, J. Weston Walch, 1973.

Let's Play Games in Mathematics, Volumes K-8, National Textbook Co., Skokie, Illinois, 1970.

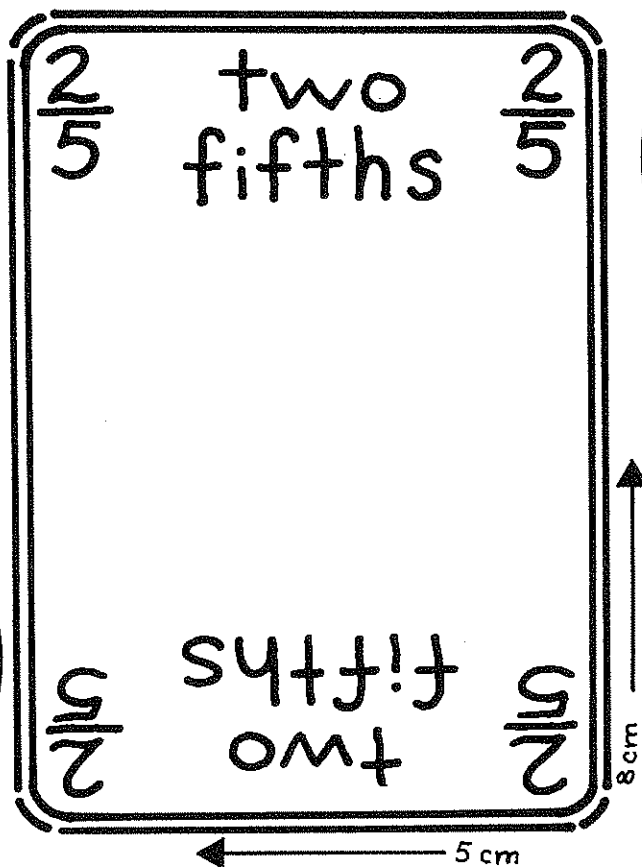
Marcy, Steve and Janis. Mathimagination, Books A-F, Creative Publications, 1973.

Schreiner, Nikki Bryson. Games and Aids for Teaching Math, Activity Resources Co., Inc., 1972.

Seymour, Dale, et al. Aftermath, I-IV, Creative Publications, 1975.



TENTHS AND TWELFTHS RUMMY CARDS CONSTRUCTION



Put fractions in all four corners for left-handed students.



The 10ths rummy deck has two each of $\frac{0}{2}$, $\frac{1}{2}$, $\frac{2}{2}$, $\frac{0}{5}$, $\frac{0}{10}$, $\frac{1}{10}$, ..., $\frac{10}{10}$ for a total of forty cards.



Twelfths rummy cards have the same dimensions and consist of two each of $\frac{0}{2}$, $\frac{1}{2}$, $\frac{2}{2}$, $\frac{0}{3}$, ..., $\frac{3}{3}$, $\frac{0}{4}$, ..., $\frac{4}{4}$, $\frac{0}{6}$, ..., $\frac{6}{6}$, $\frac{0}{12}$, ..., $\frac{12}{12}$ for a total of sixty-four cards.

Blank, round-cornered cards can be purchased from Creative Publications, Inc., P. O. Box 10328, Palo Alto, CA 94303.



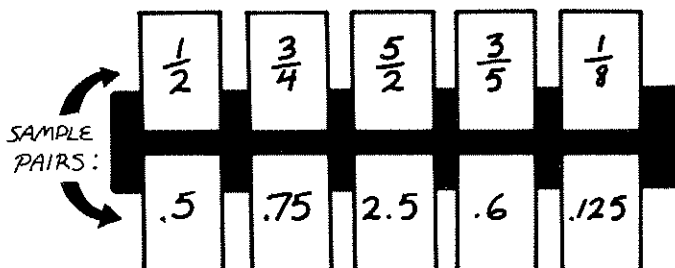
GAMES AND PUZZLES FOUND IN THE CLASSROOM MATERIALS

I. Games

Games can be used to teach a variety of problem-solving techniques and at the same time give needed practice in mental arithmetic.

FRACTIONS AND DECIMALS CONCENTRATION

For this game you will need 20 cards in pairs. Each pair will contain one fraction card and one decimal card.



The first time students play this game be sure to use only fractions that have terminating decimal forms.

Do not use fractions such as $\frac{1}{3}$ or $\frac{1}{6}$.

RULES: Place the cards facedown in to 4 rows of 5 cards each.

The first player turns any two cards faceup. If the two cards match (are equivalent), the player keeps them and turns over two more cards. If the two cards do not match, they are turned facedown again in the original dealt position.

Play proceeds to the left until all cards have been paired.

WINNER: The player with the most cards at the end of the game.

STRATEGY: Players should try to remember the placements of the cards as they are turned faceup.

GAME of 50

PLAYERS: 2

MATERIALS: 24 cards numbered 1, 2, 3, 4, 5, 6 (four of each)

PROCEDURE:

1. Spread all 24 cards face up on a table.
2. The first player picks up any card and calls its number. Then, the other player picks up a card, adds it to the number called by the first player, and gives the total. Repeat the process, always adding to the last number called by the opponent.

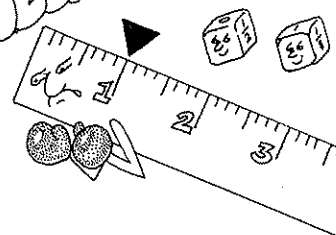
WINNER: The first player to call exactly 50.

NOTE: Once a card is picked up, it may be replaced on the table.

The playing of card games and dice games can reinforce specific mathematical skills.

BEAT THE RULER

NEEDED: 2 OR MORE PLAYERS
A 12" RULER FOR EACH PLAYER
A MARKER FOR EACH PLAYER
A PAIR OF DICE, EACH MARKED WITH 0, $\frac{1}{16}$, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, 1



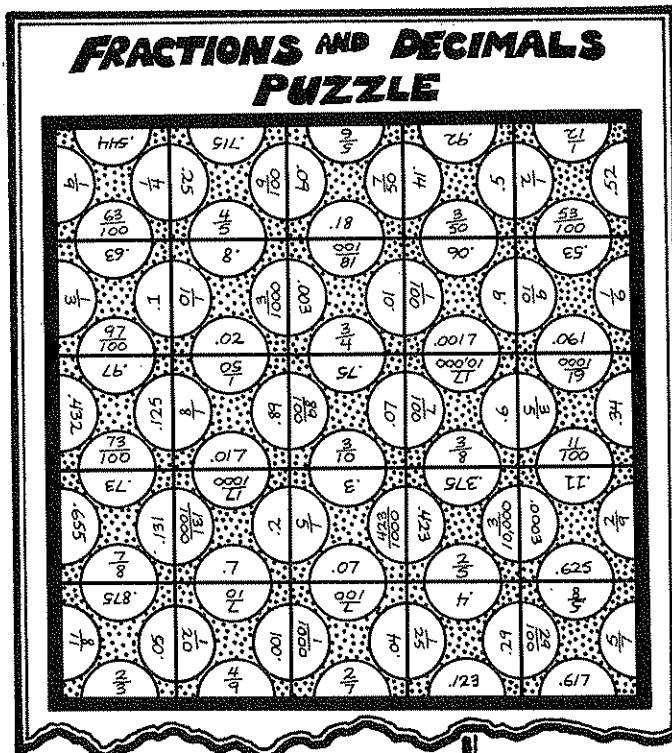
RULES:

- 1) PLAYERS ROLL DICE, SMALLEST SUM GOES FIRST.
- 2) EACH PLAYER ROLLS THE DICE AND, STARTING FROM ZERO, MOVES HIS MARKER A DISTANCE EQUAL TO THE SUM OF THE FRACTIONS ON THE DICE.
- 3) IF DOUBLES ARE ROLLED, THE PLAYER GETS ANOTHER TURN.
- 4) IF A PLAYER ADDS WRONG, HE LOSES THAT TURN, AND THE DISTANCE GOES TO THE PLAYER THAT FINDS THE MISTAKE.
- 5) FIRST PLAYER TO MOVE HIS MARKER TO THE 12" MARK OR BEYOND IS THE WINNER.



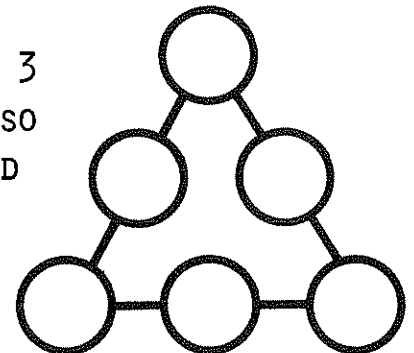
II. Puzzles

Self-correcting puzzles aimed at reinforcing a particular skill can be used as a warm-up for the day's lesson. A clever riddle may bring a few boos and hisses from your students.



Students can be provided with more challenging puzzles which require them to apply problem-solving skills in the finding of a solution.

PLACE THE NUMBERS 1, 2, 3
4, 5, 6 IN THE CIRCLES SO
THAT ALL THREE SIDES ADD
TO THE SAME NUMBER.



TOTAL = 12

NEVER

HOLD A GRUDGE...

FOR EACH OF THE SIXTEEN PROBLEMS CIRCLE THE MOST REASONABLE ANSWER. (THERE ARE NO CORRECT ANSWERS GIVEN, SO YOU SHOULDN'T HAVE TO WORK THE PROBLEMS.) PLACE THE LETTER IN THE CIRCLED BOXES IN THE APPROPRIATE SPACE BELOW.

1	$\frac{45}{\times 6}$	280	P	2.7	U	24	T	11	$\frac{9.27}{\times 6}$	55.6	J	5.6	R	562	K
2	$\frac{13.8}{\times 3}$	24	C	2.2	H	.21	D	12	$\frac{33}{\times 9}$.03	N	27	R	2.7	Z
3	$\frac{1.4}{\times 1.8}$.32	A	3	E	35	G	13	$\frac{75.3}{\times 94}$	707	D	7078	C	71.5	I
4	$\frac{8}{\times 3.25}$	47	H	.4	J	4.6	Y	14	$\frac{9813}{\times 13}$.9	B	9	F	91	H
5	$\frac{29.3}{\times 4}$	120	S	100	K	9	T	15	$\frac{28.2}{\times 1.1}$	57	M	62	L	640	S
6	$\frac{83}{\times 386}$	1083	Y	16	H	25	O	16	$\frac{24423}{\times 1}$	201	P	2.1	Q	20	Y
7	$\frac{5.08}{\times 2.4}$	13	E	121	B	1.4	C								
8	$\frac{756.2}{\times 3.9}$	80	P	.803	S	81	D								
9	$\frac{35.6}{\times 3.9}$	14	X	1452	N	143.2	T								
10	$\frac{17}{\times 51}$	29	E	300	K	3	F								



1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
PROBLEM NUMBERS

Jigsaw and border puzzles offer another way of getting students to practice basic skills. They can be made with almost any objective in mind and in various sizes and shapes. Hand the cutout pieces to the students and have them put the puzzle together.

MAGIC PERIMETERS

GEOBOARDS

Geoboards are commonly used in geometry for finding area and comparing geometric shapes. The geoboard is used also as a model for fractions. For example, the large square of a 5 x 5 geoboard can be considered one whole, and each small square $1/16$ of the whole. This is not a simple model, and it is not suggested for use in this way by all students.

Geoboards can be purchased from Scott Resources or from Walker Educational Book Corporation.

Construction Information:

Have your woodshop take a 4' x 8' x $5/8$ " piece of plywood or particle board and cut out thirty-six 10" x 10" squares. Sand sides and edges, and spray the top with dark paint.

Provide each student in your class with:

A hammer (try to borrow from the school shop).

A 10 x 10 board.

Twenty-five $3/4$ " round-headed nails (brass escutcheon pins work well).

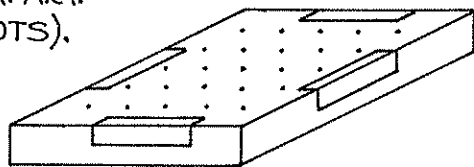
A pattern sheet with 25 dots.

Have the students center the pattern sheet on the board and hammer the nails through the dots. Be sure the nails are pounded in far enough to be firm. They should all be the same height. (You could use a spacer, empty bobbin, or large nut to place around the nail when hammering.)

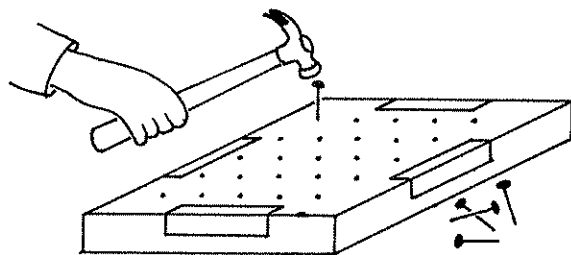
When all 25 nails are in, have the student tear off the sheet.



DITTOED PAPER
WITH DOTS 2" APART.
(ARRAY OF 25 DOTS).



TAPE PAPER ON
THE BOARD.



USE THE PAPER PATTERN OF
DOTS TO PLACE THE NAILS.

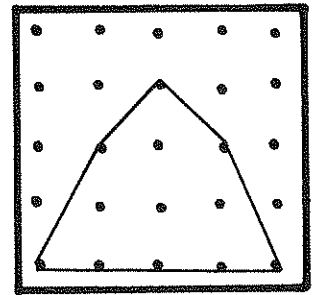
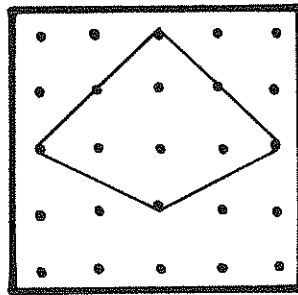
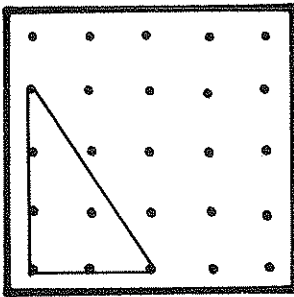
Each student needs a geoboard, about ten colored rubber bands, a supply of dot paper and a pencil to record the results. Sometimes it is helpful to have students work in pairs or small groups.



Readiness Activities:

The first time students work with geoboards, let them experiment with them. Free play is very important. Students need a few minutes to make designs with rubber bands and discover some patterns on their own. After 5 or 10 minutes, direct the students by having them:

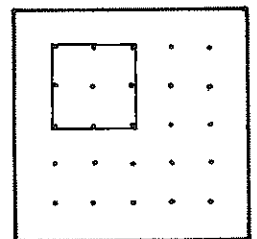
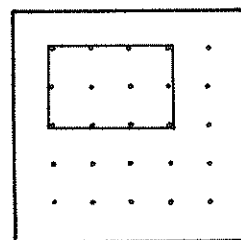
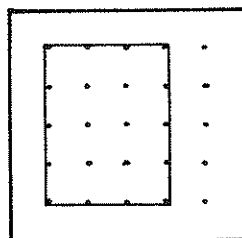
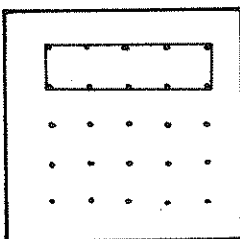
- 1) Copy some designs on their geoboards as shown below.



- 2) Make a figure resembling a STOP sign.
- 3) Make a four-pointed star.
- 4) Make different letters of the alphabet.
- 5) Make the number 471.
- 6) Make the largest five-digit number you can.

Fractions could be introduced with the following activities.

- 1) Make the smallest square you can.
- 2) How many of these squares are there on your geoboard?
- 3) Encircle eight of these squares with a rubber band.
- 4) Encircle two of these squares.
- 5) How many of these squares are encircled in each diagram below?



NOTE: Students usually need some practice in recording results. They can record some or all of their answers on dot paper. (See next page.)

GEOBOARD RECORD SHEET



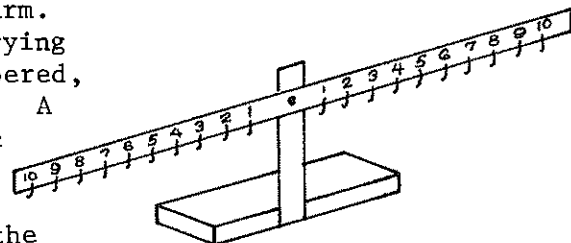
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Source: *Math Activity Worksheet Masters*

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MATHEMATICAL BALANCE

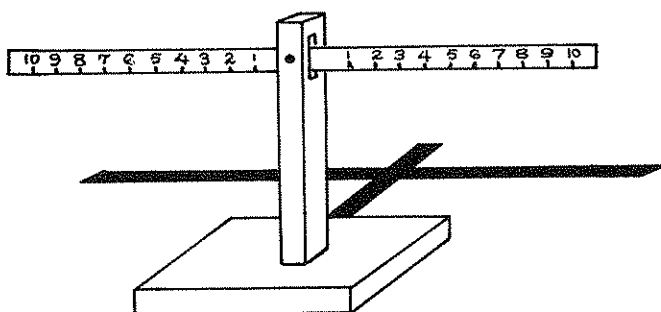
The mathematical balance is a beam that is pivoted on its center, with equally spaced hooks on each arm. The beam can be balanced by placing weights at varying distances on each arm. The hooks are usually numbered, showing the distance of each hook from the center. A mathematical balance can be used to show the basic whole number operations and properties.



Mathematical balances can be purchased from the General Learning Corporation, Creative Publications, Mind/Matter Corporation, and Selective Educational Equipment, Inc. (SEE).

Construction Information:

A mathematical balance can be constructed with a board about 42 cm long and not less than 1 cm thick to use as the arm. Label the board as shown. Attach long nails at each number to hold many washers. Attach the arm onto a stand (as shown) with a bolt and nut. Be sure that it swings freely. For best results use two boards for the vertical part of the stand, with the arm between them.



Readiness Activities:

- 1) Place tape over the numbers on the scale, and using washers of the same weight, have the students discover where the washers will balance and where they won't.
- 2) Try balancing with two washers on one arm and one on the other. How many different ways can this be done?
- 3) Use 5 washers and balance the scale in as many ways as possible so that there is at least one washer on a side and all the washers are used.

MULTIBASE BLOCKS

One important aspect of multibase blocks is that they are equivalent in quantity as well as in value. A long (Base 10) is exactly the size of 10 units, and a block is exactly 10 longs or 100 units. This eliminates the abstract exchange one encounters with one dime for 10 pennies. (The dime is smaller in size and yet worth more than a penny.)

Units for all bases are the same size (usually 1 cm^3). The varying powers of a number are in this order:

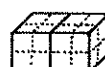
(For Base 2)


 UNIT (1_2)

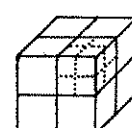
 LONG (10_2)

 FLAT (100_2)

 BLOCK (1000_2)

 LONG-BLOCK ($10,000_2$)

 FLAT-BLOCK ($100,000_2$)

 BLOCK-BLOCK ($1,000,000_2$)

Multibase blocks can be purchased from Webster/McGraw-Hill, the Dick Blick Company, and Creative Publications.

Construction Information:

In making your own set of multibase blocks, you need a set of Base 10 blocks and perhaps one or two sets of other bases. For Base 5 and up, "blocks" are the largest pieces needed.

Here are a few materials that can be used as a substitute for multibase blocks:

Interlocking Cubes--1 cm plastic cubes in a variety of colors and made so they can be linked together (1000 cubes, 10 colors).

Graph Paper-- $\frac{1}{2}$ inch or 1 cm. Units, longs and flats (see first page of readiness activities which follow) can all be cut out. Flats can be stapled together to make a block. (Hint: Laminate, it lasts longer that way.)

Mathmats--heavy plastic sheets with centimetre cubes--these will last longer than graph paper. (Creative Publications.)

Beansticks--For more information see the Beansticks material in the Laboratory Materials Section.

If you choose to use graph paper or mathmats, it is recommended that at least one set of Multibase Blocks be available for demonstration purposes. One way to make a demonstration set is to begin with about 200 wooden cubes and glue them together.

A multibase cutout sheet is provided following the readiness activities. This sheet can be dittoed so that you can make some activity cards of your own.

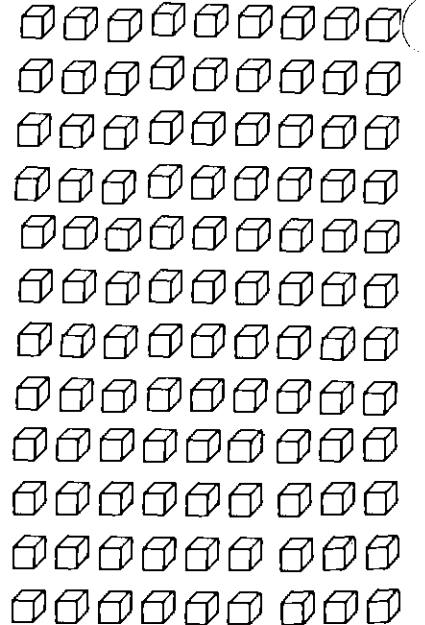
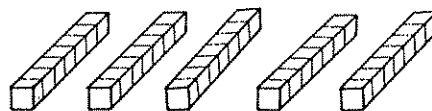
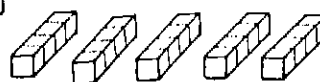
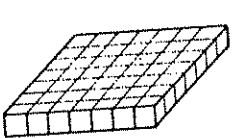
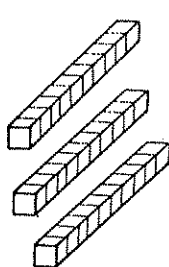
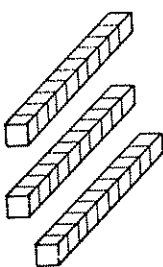
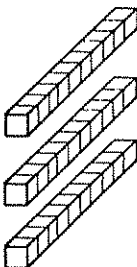
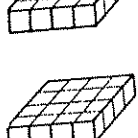
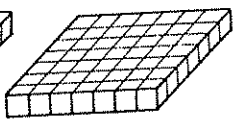
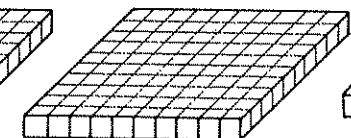
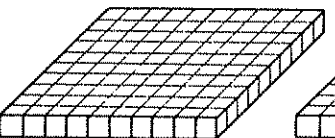
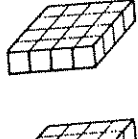
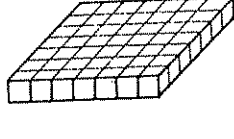
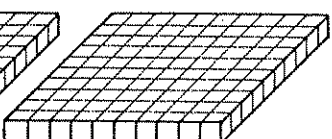
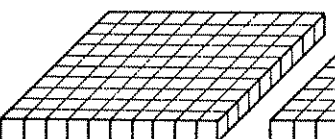
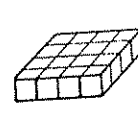
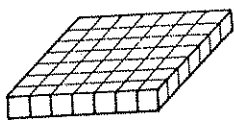
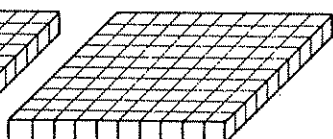
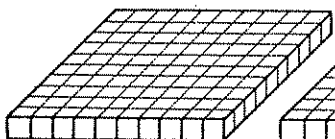
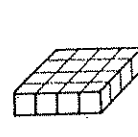
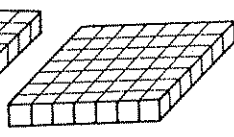
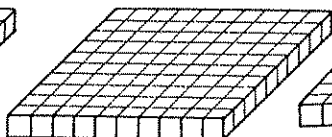
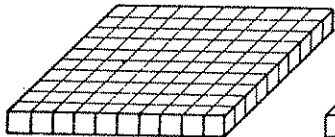
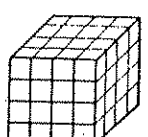
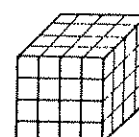
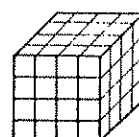
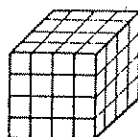
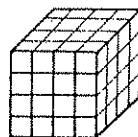
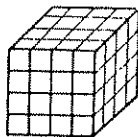
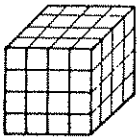
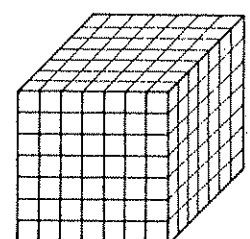
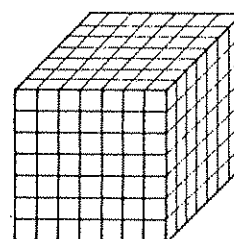
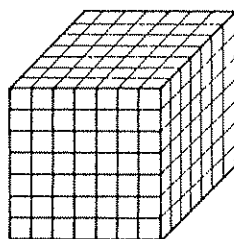
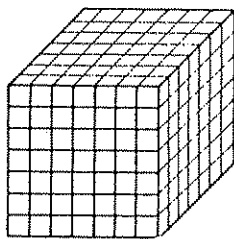
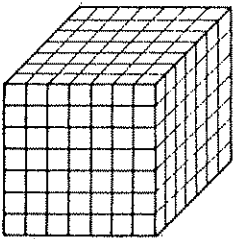
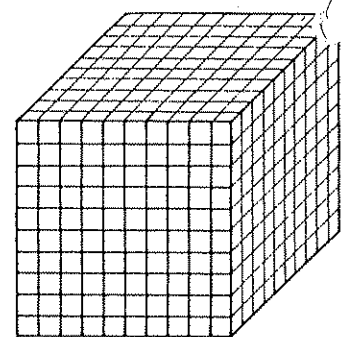
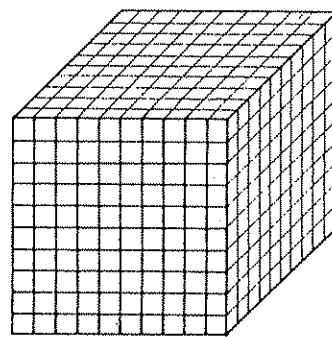
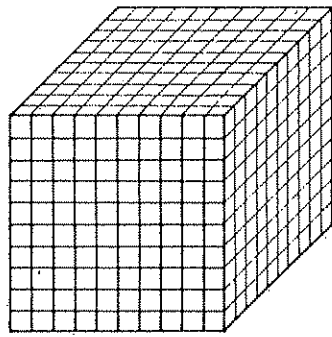
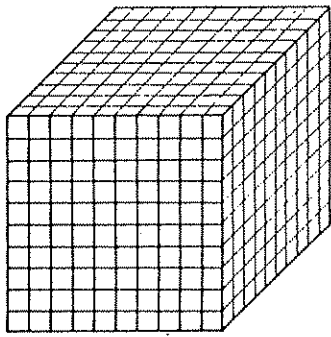
Readiness Activities:

- 1) Have students name the pieces (i.e., block, flat, long, unit).
- 2) Have the students build a long using only units. How many units are in a long? How many longs are in a flat? How many flats in a block? How many longs in a block? Etc.
- 3) Have students count with the blocks.
- 4) Have students show various numbers with the multibase blocks. Record how many of each piece was used. Use the fewest number of pieces possible for each problem.

<u>NUMERAL</u>	<u>BLOCKS</u>	<u>FLATS</u>	<u>LONGS</u>	<u>UNITS</u>
74				
341				
795				
2107				
3040				

- 5) See *Chip Trading* in WHOLE NUMBERS: Concepts for a variety of place value games that could be played with multibase blocks.

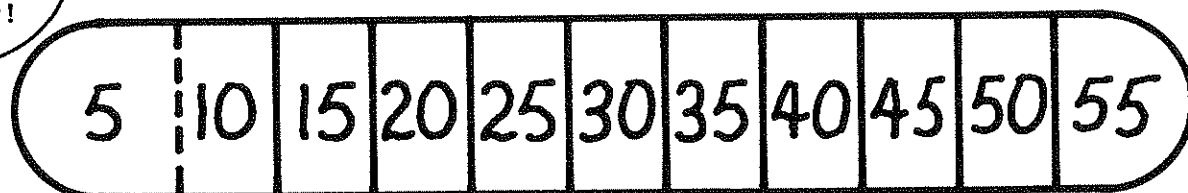
(BASES 10, 7 AND 4)



MULTIPLE BOARDS

Multiple boards can be used to show multiples of whole numbers and equivalent fractions, to compare fractions, and to add and subtract fractions.

I'm the
5's board.
I used to be
a tongue
depressor!



Construction Information:

The construction of multiple boards provides practice with multiples.

Students can use any of the following to make multiple boards: tongue depressors, popsicle sticks, tagboard or railroad board. Multiplication tables could be cut into strips and glued on the popsicle sticks to make a quick set.

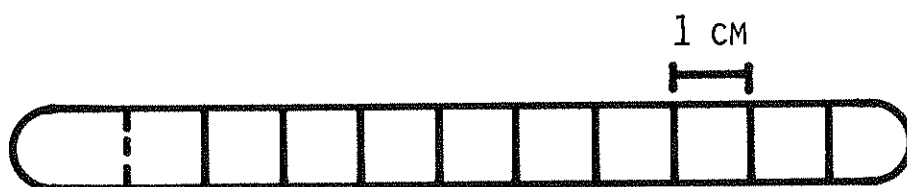
Multiple boards can be made for all numbers from 1 to 15 (or you could go higher or only to 10 depending on the needs of your students).

Be sure each board is divided in the same way so the numbers will line up. The patterns below should help.

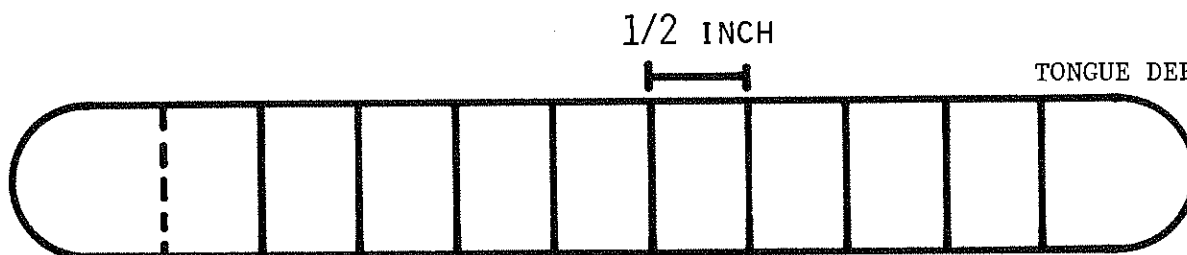
You might want a set of transparent multiple boards for the overhead projector.

Letting students work in pairs or small groups will enable them to pool the boards.

Patterns for Multiple Boards



POPSICLE STICKS



TONGUE DEPRESSORS

Readiness Activities:

- 1) Get a 6's and an 8's multiple board. What "common" multiples do they have?
- 2) Play a game. Select a multiple board. Tell one of the numbers on it. See who can guess which board it is. Keep telling numbers on the board until the board is guessed. The guesser can pick the next multiple board.

TANGRAMS

The tangram originated in China. Its exact age is unknown. Some sources date it as early as 4,000 years ago, while other sources refute this and say that no evidence of this kind of puzzle can be found dating earlier than the 19th century.

A tangram is a seven-piece puzzle which can be cut from a single square. The pieces comprise two small triangles, one medium sized triangle, two large triangles, a square, and a parallelogram. The charm of this puzzle lies in the extraordinary variety of ways in which these pieces can be put together.

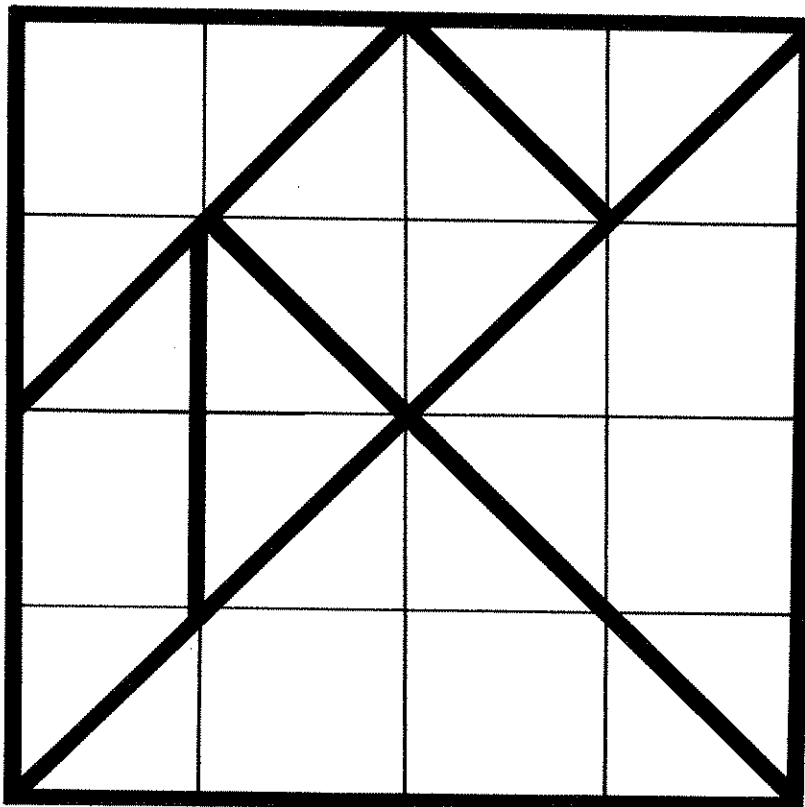
Each of the pieces bears a mathematical relation to the others. For example, the medium size triangle, the square, and the parallelogram are all twice the area of one of the small triangles and each large triangle is four times the area of the small triangle. All angles of the tangram pieces are either 45° , 90° or 135° . Thus, tangrams are useful for the study of fractions, geometry and area.

Tangrams can be purchased from Selective Educational Equipment, Inc. (SEE), World Wide Games, or Creative Publications.

Construction Information:

Method #1: Print 'n Cut

Duplicate this tangram and have the students cut it out along the heavy black lines. You can use graph paper to enlarge or reduce tangrams to any size you desire. However, the activity cards in this book use pieces that fit into a square 10 cm by 10 cm as above. For durable tangram pieces use railroad board, tagboard or wood as a backing.

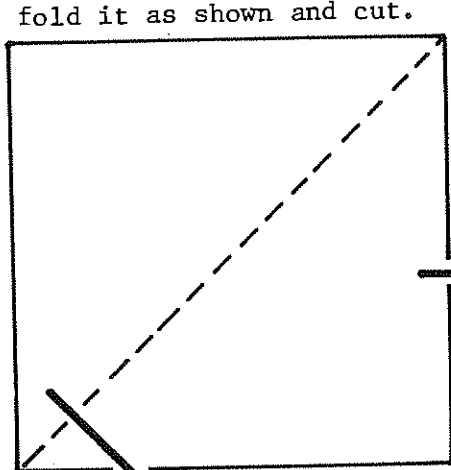




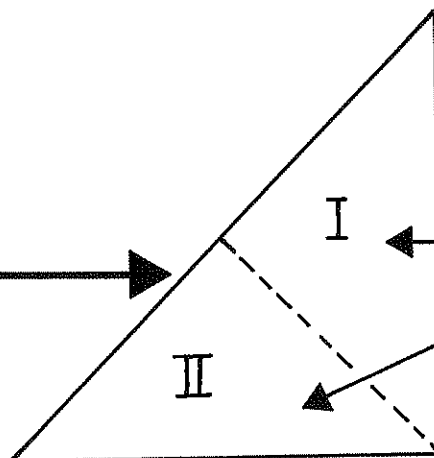
Method #2: Fold 'n Cut

The seven final pieces are labeled with Roman Numerals.

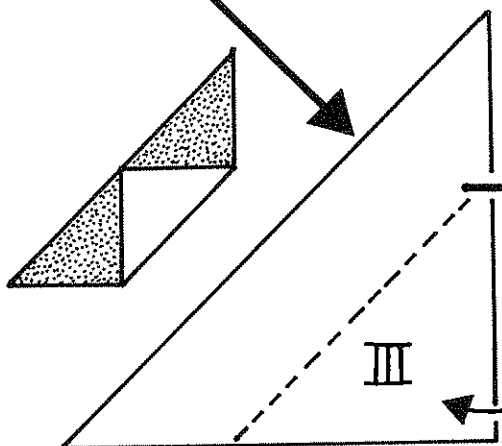
1. Start with a square---
fold it as shown and cut.



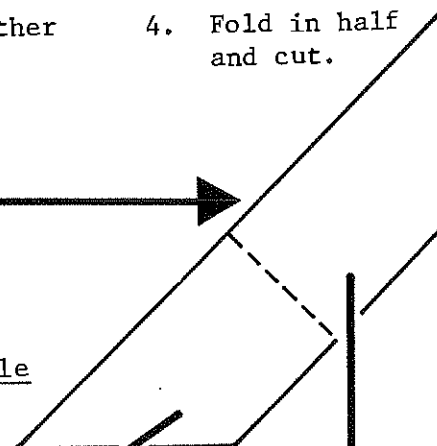
2. Fold one half as
shown and cut.



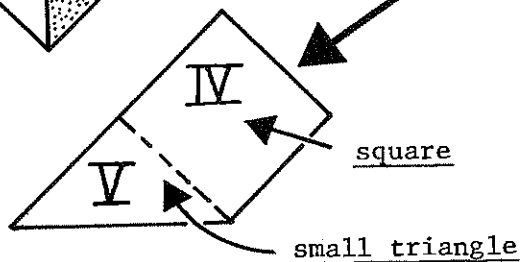
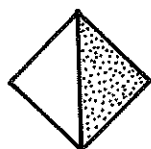
3. Fold and cut the other
half as shown.



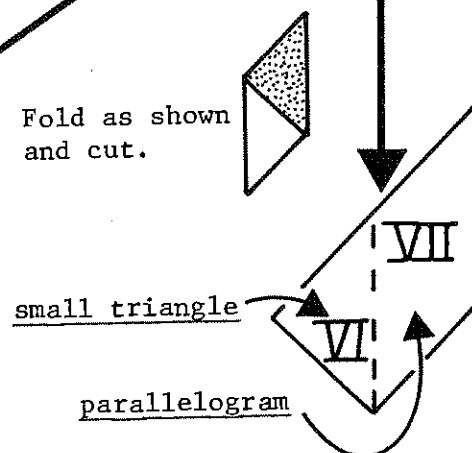
4. Fold in half
and cut.



5. Fold as shown
and cut.



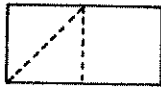
6. Fold as shown
and cut.





Readiness Activities:

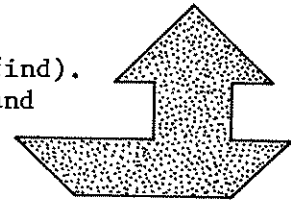
- 1) Put the pieces back together to make the original square.
- 2) Place the two small triangles so they exactly cover (a) the square;
(b) the middle-size triangle; and (c) the parallelogram.
- 3) Cover the large triangle using
 - a. the square and two small triangles
 - b. the middle-size triangle and two small triangles
 - c. the parallelogram and two small triangles.

- 4) Use the square and the two small triangles to make these figures:






- 5) Try to make a large  using all seven pieces.
- 6) Try to make a large  using all seven pieces.

- 7) Make pictures with the tangrams (samples are easy to find).
You might try drawing in the pieces on the first few and just have the students duplicate them.



- 8) This is an activity that could involve quite a bit of time and effort---for your inquisitive students.

See how much of the table you can fill in. Record your answers. Which ones cannot be made?

# OF PIECES	SQUARE	TRIANGLE	RECTANGLE	TRAPAZOID	PARALLELOGRAM
1					
2					
3					
4					
5					
6					
7					

LIST OF MANIPULATIVES

LIST OF MANIPULATIVES FOUND IN CLASSROOM MATERIALS

Abacus

WHOLE NUMBERS:

Addition/Subtraction

ACTIVITY CARDS - CHIP ABACUS

REGROUPING TO ADD

DECIMALS:

Concepts

ACTIVITY CARDS - DECIMAL
ABACUS - I

PLACE VALUE

Addition/Subtraction

ACTIVITY CARDS - DECIMAL
ABACUS - II

ADDITION/SUBTRACTION

Multiplication/Division

ACTIVITY CARDS - DECIMAL
ABACUS - III

MULTIPLICATION

ACTIVITY CARDS - DECIMAL
ABACUS - IV

POWERS OF TEN

LARGE AND SMALL NUMBERS:

Exponential Notation

LARGE NUMBERS ON THE ABACUS - I

POWERS OF TEN

SMALL NUMBERS ON THE ABACUS - I

POWERS OF TEN

LARGE NUMBERS ON THE ABACUS - II

EXPANDED NOTATION

SMALL NUMBERS ON THE ABACUS - II

EXPANDED NOTATION

Beansticks

WHOLE NUMBERS:

Addition/Subtraction

ACTIVITY CARDS - BEANSTICKS - I

REGROUPING TO ADD

ACTIVITY CARDS - BEANSTICKS - II

REGROUPING TO SUBTRACT

Multiplication/Division

ACTIVITY CARDS - BEANSTICKS - III MULTIPLICATION

ACTIVITY CARDS - BEANSTICKS - IV DIVISION

Circle Fractions

FRACTIONS:

Concepts

ACTIVITY CARDS - CIRCLE WHOLE MODEL
FRACTIONS - IACTIVITY CARDS - CIRCLE EQUIVALENT
FRACTIONS - IIClock Fractions

FRACTIONS:

Concepts

ACTIVITY CARDS - CLOCK IMPROPER-MIXED
FRACTIONS - I

Addition/Subtraction

ACTIVITY CARDS - CLOCK ADDITION/SUBTRACTION
FRACTIONS - IIColor Cubes

FRACTIONS:

Concepts

ACTIVITY CARDS - COLOR CUBES - I SET MODEL
ACTIVITY CARDS - COLOR CUBES - II EQUIVALENT
ACTIVITY CARDS - COLOR CUBES - III IMPROPER-MIXEDCuisenaire Rods

WHOLE NUMBERS:

Addition/Subtraction

ACTIVITY CARDS - CUISENAIRE ADDITION
RODS - IACTIVITY CARDS - CUISENAIRE SUBTRACTION
RODS - II

The name Cuisenaire and the color sequence of the rods are trademarks of the Cuisenaire Company of America, Inc.



Multiplication/Division

ACTIVITY CARDS - CUISENAIRE
RODS - III

MULTIPLICATION

EVEN STEVEN
ODD ROD

ODD AND EVEN

FRACTIONS:

Concepts

ACTIVITY CARDS - CUISENAIRE
RODS - I

WHOLE MODEL

ACTIVITY CARDS - CUISENAIRE
RODS - II

IMPROPER-MIXED

Addition/Subtraction

ACTIVITY CARDS - CUISENAIRE
RODS - III

ADDITION/SUBTRACTION

DECIMALS:

Concepts

ACTIVITY CARDS - CUISENAIRE
RODS - I

PLACE VALUE

ACTIVITY CARDS - CUISENAIRE
RODS - II

ROUNDING

ACTIVITY CARDS - CUISENAIRE
RODS - III

RELATION TO FRACTIONS

ACTIVITY CARDS - CUISENAIRE
RODS - IV

RELATION TO FRACTIONS

ACTIVITY CARDS - CUISENAIRE
RODS - V

ORDERING

Addition/Subtraction

ACTIVITY CARDS - CUISENAIRE
RODS - VI

ADDITION/SUBTRACTION

Factor Boards

FRACTIONS:

Concepts

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FACTOR BOARDS - I

REDUCING

Multiplication/Division

FACTOR BOARDS - II

MULTIPLICATION/DIVISION

Fraction Bars

FRACTIONS:

Concepts

ACTIVITY CARDS - FRACTION
BARS - I

EQUIVALENT

Addition/Subtraction

ACTIVITY CARDS - FRACTION
BARS - II

ADDITION/SUBTRACTION

Multiplication/Division

ACTIVITY CARDS - FRACTION
BARS - III

MULTIPLICATION/DIVISION

Geoboards

FRACTIONS:

Concepts

ACTIVITY CARDS - GEOBOARDS - I

WHOLE MODEL

Addition/Subtraction

ACTIVITY CARDS - GEOBOARDS - II

ADDITION/SUBTRACTION

Multiplication/Division

ACTIVITY CARDS - GEOBOARDS - III

MULTIPLICATION/DIVISION

Mathematical Balance

WHOLE NUMBERS:

Addition/Subtraction

ACTIVITY CARDS - MATHEMATICAL
BALANCE - I

ADDITION/SUBTRACTION

Multiplication/DivisionACTIVITY CARDS - MATHEMATICAL
BALANCE - II

MULTIPLICATION

ACTIVITY CARDS - MATHEMATICAL
BALANCE - III

DIVISION

Multibase Blocks

WHOLE NUMBERS:

Concepts

ACTIVITY CARDS - BASE 10
MULTIBASE BLOCKS - I

PLACE VALUE

Addition/Subtraction

ACTIVITY CARDS - BASE 10
MULTIBASE BLOCKS - II

REGROUPING TO ADD

ACTIVITY CARDS - BASE 10
MULTIBASE BLOCKS - III

REGROUPING TO SUBTRACT

Multiplication/Division

ACTIVITY CARDS - BASE 10
MULTIBASE BLOCKS - IV

MULTIPLICATION

ACTIVITY CARDS - BASE 10
MULTIBASE BLOCKS - V

DIVISION

LARGE AND SMALL NUMBERS:

Exponential Notation

MULTIBASE BLOCKS - I

EXPANDED NOTATION

MULTIBASE BLOCKS - II

EXPANDED NOTATION

Multiple Boards

FRACTIONS:

Concepts

MULTIPLE BOARDS - I

ORDERING

Addition/Subtraction

MULTIPLE BOARDS - II

ADDITION



Tangrams

FRACTIONS:

Concepts

ACTIVITY CARDS - TANGRAMS - I

WHOLE MODEL

Addition/Subtraction

ACTIVITY CARDS - TANGRAMS - III

ADDITION/SUBTRACTION

NUMBER SENSE AND ARITHMETIC SKILLS

Placement Guide for Tabbed Dividers

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Whole Numbers Commentary: Getting Started (pages 181-186)

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Numbers Around Us	188	Number uses	Activity
Number Names	190	Number uses	Activity Transparency Bulletin board
Numerology	191	Motivation	Activity
Exact or Approximate	194	Counting Approximation	Paper and pencil Transparency
Patterns for Counting	195	Using patterns	Paper and pencil Transparency
Seals on my Iceberg	196	Estimation Counting	Activity Transparency
Educated Guess/Guess Again	197	Estimation Counting	Paper and pencil Transparency
25--More or Less	198	Estimation Counting	Paper and pencil Transparency
Counting Cubes	200	Counting	Paper and pencil Transparency
Mystery Stacks	201	Counting	Paper and pencil Manipulative
The Peg Game or Ten Men in a Boat	202	Counting Looking for patterns	Activity
The Painted Cube	205	Counting Looking for patterns	Paper and pencil Manipulative
Strictly Squaresville	206	Counting Looking for patterns	Paper and pencil Transparency
Squaresville Suburbs	207	Counting Looking for patterns	Paper and pencil
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<u>TITLE</u>	<u>PAGE</u>	<u>TOPIC</u>	<u>TYPE</u>
Game of 50	209	Motivation Looking for strategies	Game
The Game of Nim	210	Motivation Looking for strategies	Game
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Rivers and Relatives	213	Logic problems	Paper and pencil

WHOLE NUMBERS: GETTING STARTED

MATHEMATICAL GRAFFITI

Have you ever felt like you were turning into a number? Maybe we never will change all of our conversation into numbers but we certainly are confronted with lots of them. Our clocks, books, newspapers, maps and street signs are filled with number symbols. News, sports, politics and advertisements are flooded with numbers. Whole numbers from zero to the trillions along with fractions, decimals and percents are used to convey information in our everyday lives. As a means of understanding and utilizing (even as a defense against) this deluge of numbers, we all need a background in number sense and arithmetic skills.

Since numbers are in such abundant supply and free of charge, why not use them in the classroom? The type of introductory activities you pick up could enhance NUMBER AWARENESS and at the same time be FUN, provide MOTIVATION, and serve as a NON-THREATENING way to introduce or reintroduce number ideas (especially to students who have not been successful in the past).

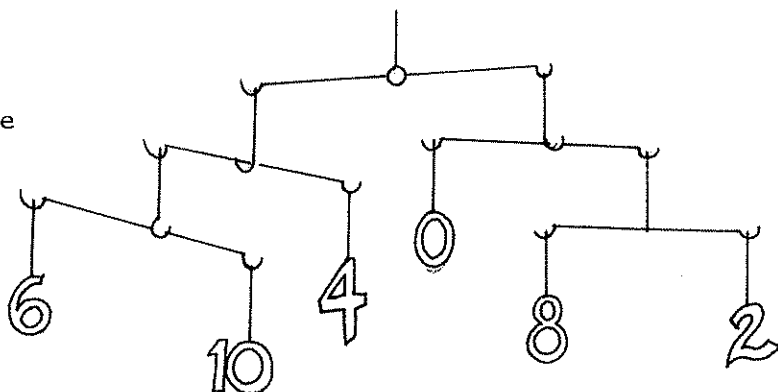


POSSIBLE STARTING POINTS

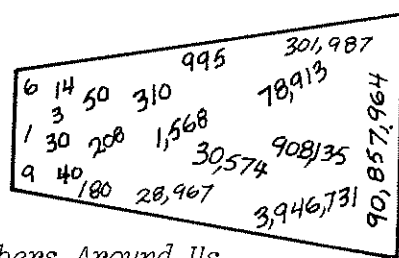
Have students bring interesting pictures, advertisements, newspaper articles, cartoons, etc., which illustrate numbers. Place these pictures on a number graffiti bulletin board. Some of them could be placed along the wall of the room with the largest at one end and the smallest at the other--how about a contest to see who can bring in the largest and the smallest?

Call on students by number names for a period of time. For example, "Two, five" might refer to the student in the second row from the door and five seats back. (Have the students decipher your code.) Group students, call on students, or excuse them for activities by the number of letters in their name (or last name). Invent your own system!

Students interested in art may want to design a number characterization, mobile or number collage to put on the walls. Some schools have permitted students to paint number graphics on the walls. Some example graphics are given on the page *Number Graphics*.



Activities of this nature provide experiences in which every student can be successful, provide group interaction, and may create a classroom "climate" in which it is easier to study mathematics. Isn't it possible that if students contribute to the decoration of their classroom via "number graffiti" and designs that the room may be more conducive to learning mathematics--doesn't the "atmosphere" of a room affect our mood?



See *Numbers Around Us*



WHAT ARE NUMBERS GOOD FOR?

An examination of number graffiti reveals the multitude of number uses. One mathematician succinctly summarizes number uses as follows:

Numbers are an indispensable tool of civilization, serving to whip its activities into some sort of order. In their most primitive applications they serve as identification tags: telephone numbers, car licenses, ZIP-code numbers. At this level we merely compare one number with another; the numbers are not subjected to arithmetical operations. (We would not expect to arrive at anything significant by adding the number of Leonard Bernstein's telephone to Elizabeth Taylor's.) At a somewhat higher level we make use of the natural order of the positive integers: in taking a number for our turn at the meat counter or in listing the order of finish in a race. There is still no need to operate on the numbers; all we are interested in is whether one number is greater or less than another. Arithmetic in its full sense does not become relevant until the stage at which we ask the question: How many? It is then that we must face up to the complexities of addition, subtraction, multiplication, division, square roots and the more elaborate dealings with numbers.*

Numbers have many uses that students are familiar with and any of these uses can become the basis for a discussion. The everyday use of numbers can be dramatized effectively by giving a number profile. For example, the teacher might recite his/her profile in this manner:

BIRTH DATE	11/27/42
HEIGHT	5'7"
WEIGHT	140 LBS.
SHOE SIZE	8
SHIRT	15 $\frac{1}{2}$
HAT	7 $\frac{3}{8}$
ADDRESS	14672 LANE DRIVE
TELEPHONE	779-226-6227
SOCIAL SECURITY	485-00-2796
DRIVER'S LICENSE	N4627592
CREDIT CARD NO.	375200946567
AUTO LICENSE	254-68


See *Number Names*

This list can grow impressively long and may give you the feeling that you are completely described by numbers. Remember, most students are interested in their teachers so this makes a nice starting point. Students can also write number profiles of themselves -- who can write the longest, legitimate number profile?

*Philip J. Davis, "Number," *Scientific American* (September, 1964), p. 51.

Most students—even those who claim not to like mathematics—use and even enjoy mathematics in certain situations. If those situations can be identified they may serve as convenient starting points. In particular, there are many interesting number puzzles, games, counting and estimating activities. These activities can be fun, motivational, and yet have a mathematical payoff.

Puzzles have an element of mystery about them. In the teacher-directed *ESP Puzzle*, the teacher is able to guess any number a student chooses. How is this possible? The students may work individually or in groups to find the secret.



GAME of 50

PLAYERS: 2

MATERIALS: 24 cards numbered 1, 2, 3, 4, 5, 6 (four of each)

PROCEDURE:

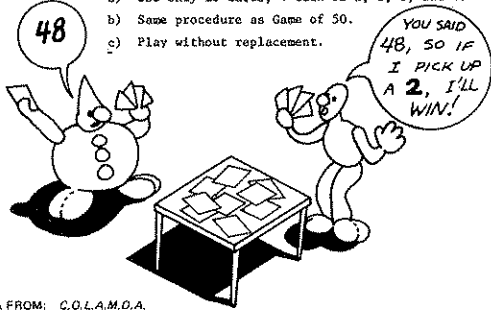
1. Spread all 24 cards face up on a table.
2. The first player picks up any card and calls its number. Then, the other player picks up a card, adds it to the number called by the first player, and gives the total. Repeat the process, always adding to the last number called by the opponent.

WINNER: The first player to call exactly 50.

NOTE: Once a card is picked up, it may be replaced on the table.

VARIATIONS:

1. Play the game without replacing the cards.
2. **GAME of 22**
 - a) Use only 16 cards, 4 each of 1, 2, 3, and 4.
 - b) Same procedure as Game of 50.
 - c) Play without replacement.

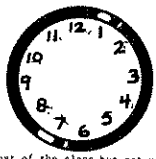


IDEA FROM: C.O.L.A.M.O.A.
Permission to use granted by Northern Colorado Educational Board of Cooperative Services

Many mathematical investigations begin with the question, "How many?" The page *Strictly Squaresville* asks us to find the number of squares of all sizes in Sevensville. After deciding what things are to be counted, and after organizing them, we might not have to count one by one. We may look for patterns to simplify counting. Sometimes we are not looking for an exact count but only an estimate as illustrated in *Educated Guess - Guess Again* and *25 - More or Less*.

ESP PUZZLE

DRAW A CLOCK FACE ON THE CHALKBOARD SIMILAR TO THE ONE ON THE RIGHT.
(OR USE THE ROOM CLOCK)



Have a student choose a number and show the rest of the class but not you. Now the students are to count quietly to themselves as you tap and point to numbers on the face of the clock. They must start counting with the number they choose. When they reach 21 they tell you to stop. You are now pointing to the number originally chosen. For example: if they chose 7, then for the first number you tapped they would think 7; the next tap or number pointed to would be counted as 1; etc.

EXAMPLE:

TAP	STUDENTS COUNT (SILENTLY)	TEACHER (POINTS TO) (THINKS)	
1ST	7	ANY	1
2ND	8	ANY	2
3RD	9	ANY	3
4TH	10	ANY	4
5TH	11	ANY	5
6TH	12	ANY	6
7TH	13	ANY	7
8TH	14	ANY	8
9TH	15	ANY	9
10TH	16	12	10
11TH	17	11	11
12TH	18	12	12
13TH	19	13	13
14TH	20	14	14
15TH	21	7	7

The secret lies in the fact that you count also, only you start with 1, and randomly tap nine numbers. The tenth tap must be 12, then you tap the numbers 11, 10, 9, 8, etc. until told to stop by the students. If all has gone correctly when they reach 21 and tell you to stop, you should be on the number they chose.

When someone thinks they have the pattern, let them try their solution on the class. Solutions are to be kept secret once discovered.

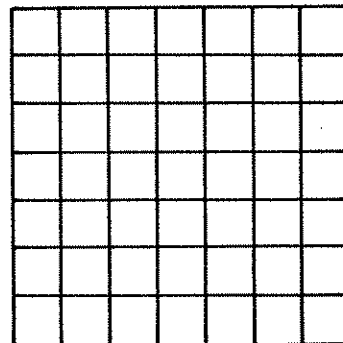
VARIATIONS:

- A. Use a clock numbered 1-9. Tap out 6 random numbers. Have students count to 15 and then tell you to stop.
- B. After the secret is known, suggest other ways to number the clock. Ask students to determine how many random numbers to tap out and what number to have the audience count to.

IDEA FROM: *Mathematics Laboratory Handbook for the Junior High School*, San Diego City Schools
Permission to use granted by San Diego City Schools

Games are fun. The *Game of 50* provides practice in mental arithmetic and is a good activity to introduce students to developing strategies.

Sevensville



TEACHING EMPHASES



Application



Algorithms



Estimation and Approximation



Mental Arithmetic



Problem Solving




Lab Activity



Calculator

The seven teaching emphases stressed throughout the resource are indicated by a symbol on the upper lefthand corner of the classroom materials. A rationale for each teaching emphasis is given in the appendix. The emphases will be found in every section of the resource.

Many times it will be the teacher's responsibility to develop the idea behind a teaching emphasis on a given page. Consider for example, the page *Strictly Squaresville* which is marked . By completing the table on this page, predicting the population of Sevensville, and moving on to the next assignment the essential problem-solving ideas behind this page might be missed. What is really important about Squaresville is that it stresses very basic problem-solving strategies (heuristics). The strategies used in the solution of Squaresville will be important in the solution of many other problems. Some of these strategies are:

- *simplify the problem--instead of counting all squares in Sevensville start with Onesville, Twosville, etc.
- *use tables to organize data
- *look for patterns that occur in the tables of numbers
- *generalize--looking at the patterns in the table, can you predict the population of Sevensville?

Strictly Squaresville

I'm Sid the Census taker. I have to count the squares of all sizes in each town. Can you help me finish the census?


Onesville
Twosville
Threesville
Foursville
Fivesville

Now - Threesville has nine 1×1 squares, four 2×2 squares, and one 3×3 square.

SID'S CENSUS LIST					
	SIZES OF SQUARES				SQUARE POPULATION
	1×1	2×2	3×3	4×4	
ONESVILLE	1				1
TWOSVILLE	4	1			5
THREESVILLE					
FOURSVILLE					
FIVESVILLE					

Can you predict the square population for Sevensville?

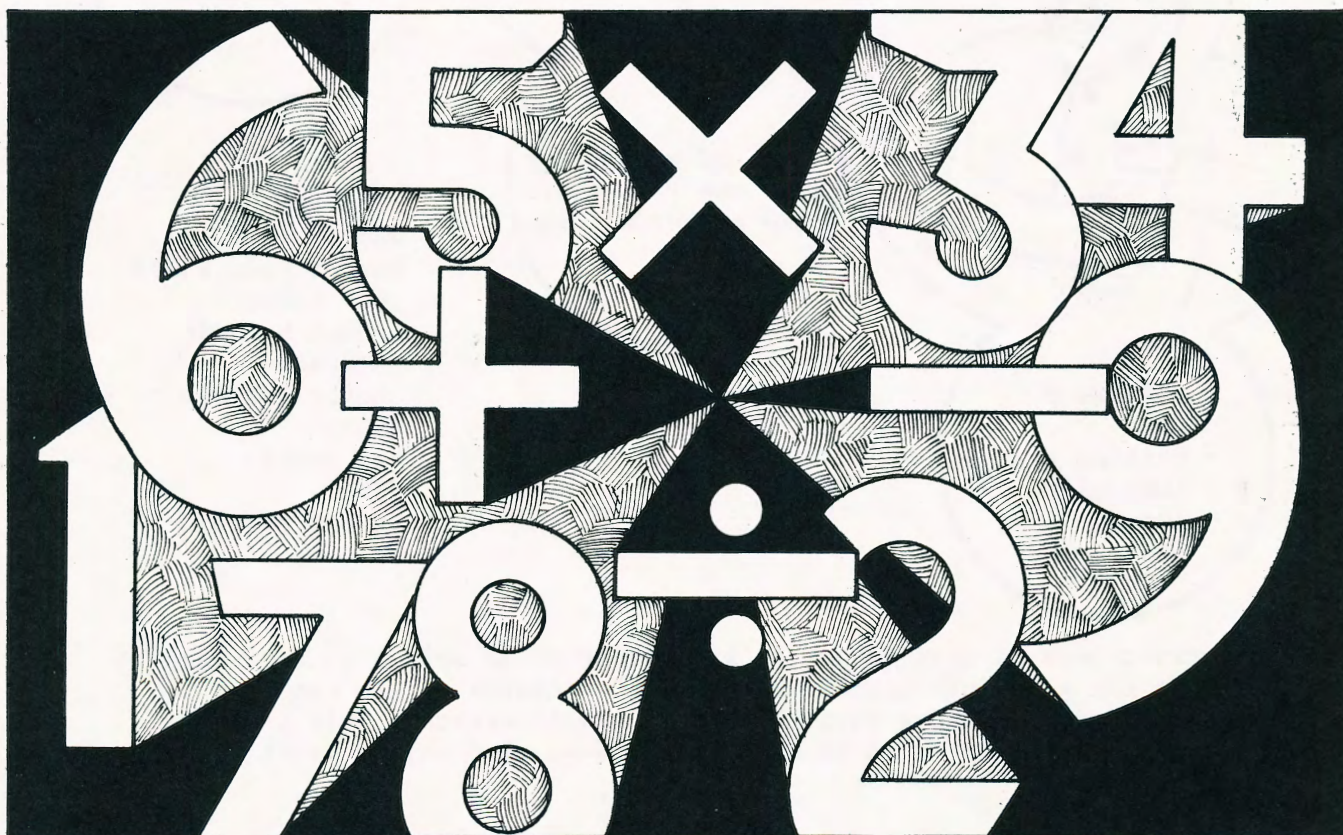
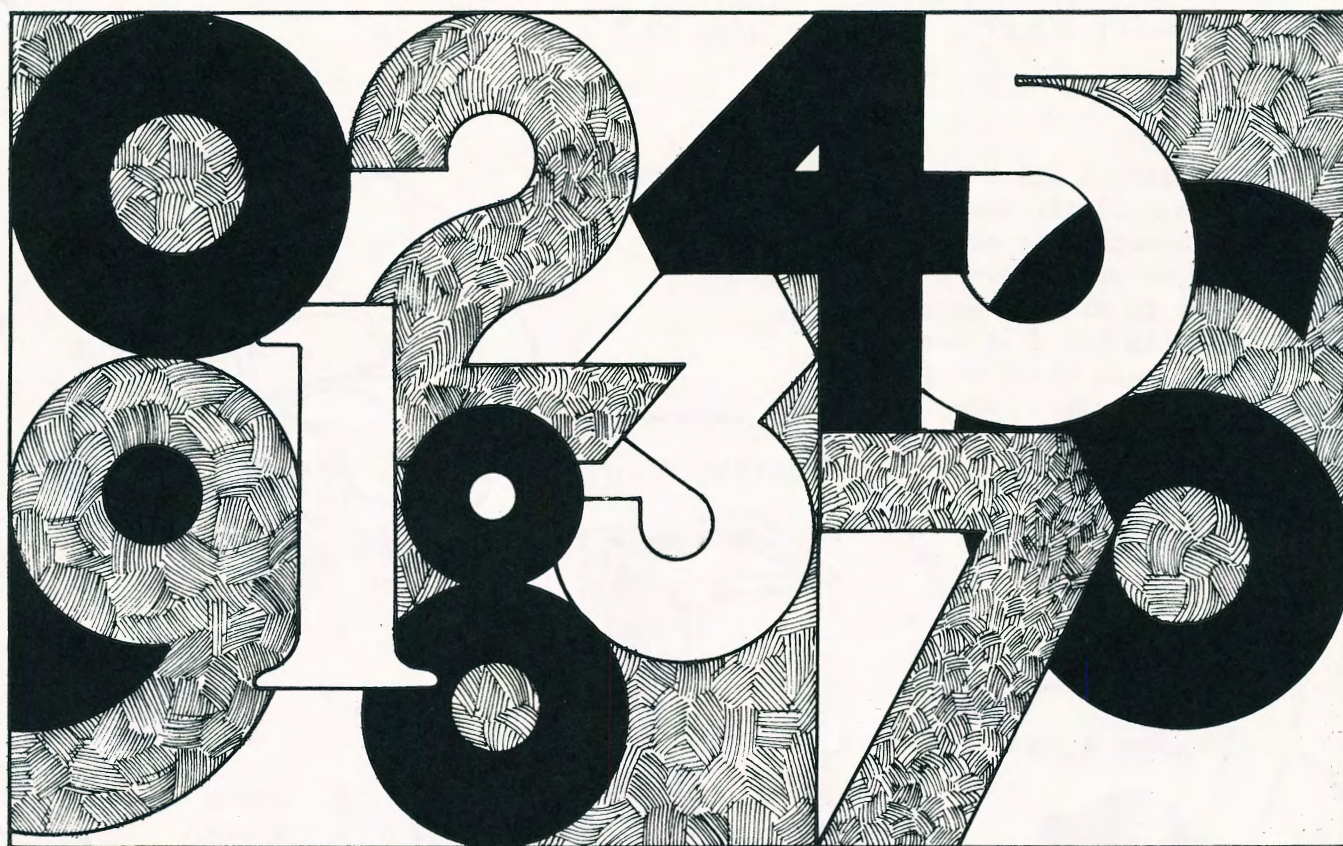
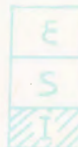
IDEA FROM: Problems--Green Set, Nutfield Mathematics Project
Permission to use granted by John Wiley and Sons, Inc.

The *Game of 50* is also marked . Here the problem is how to win the game. One problem-solving heuristic that can be applied is to work backwards. "I can win the game if my opponent says 44, 45, 46, 47, 48, or 49 (all I have to do is add 1, 2, 3, 4, 5, or 6 to get 50). I can make my opponent say any of those numbers if I say 43. I can say 43 if my opponent says 42, 41, 40, 39, 38, or 37," etc. Another problem-solving strategy would involve playing a simpler game first and then working up to the *Game of 50*. Thus, while the game provides the motivation it is the problem-solving process and not the winning that is really important.

The comments made above for problem solving also apply to the other teaching emphases. The importance of the teacher in focusing on the significant characteristics of a page cannot be understated.

NUMBER GRAPHICS

Motivation
Getting Started
WHOLE NUMBERS



TYPE: Bulletin Board

This can be colored and placed around the room, or enlarged and painted on the wall to make an attractive mural.



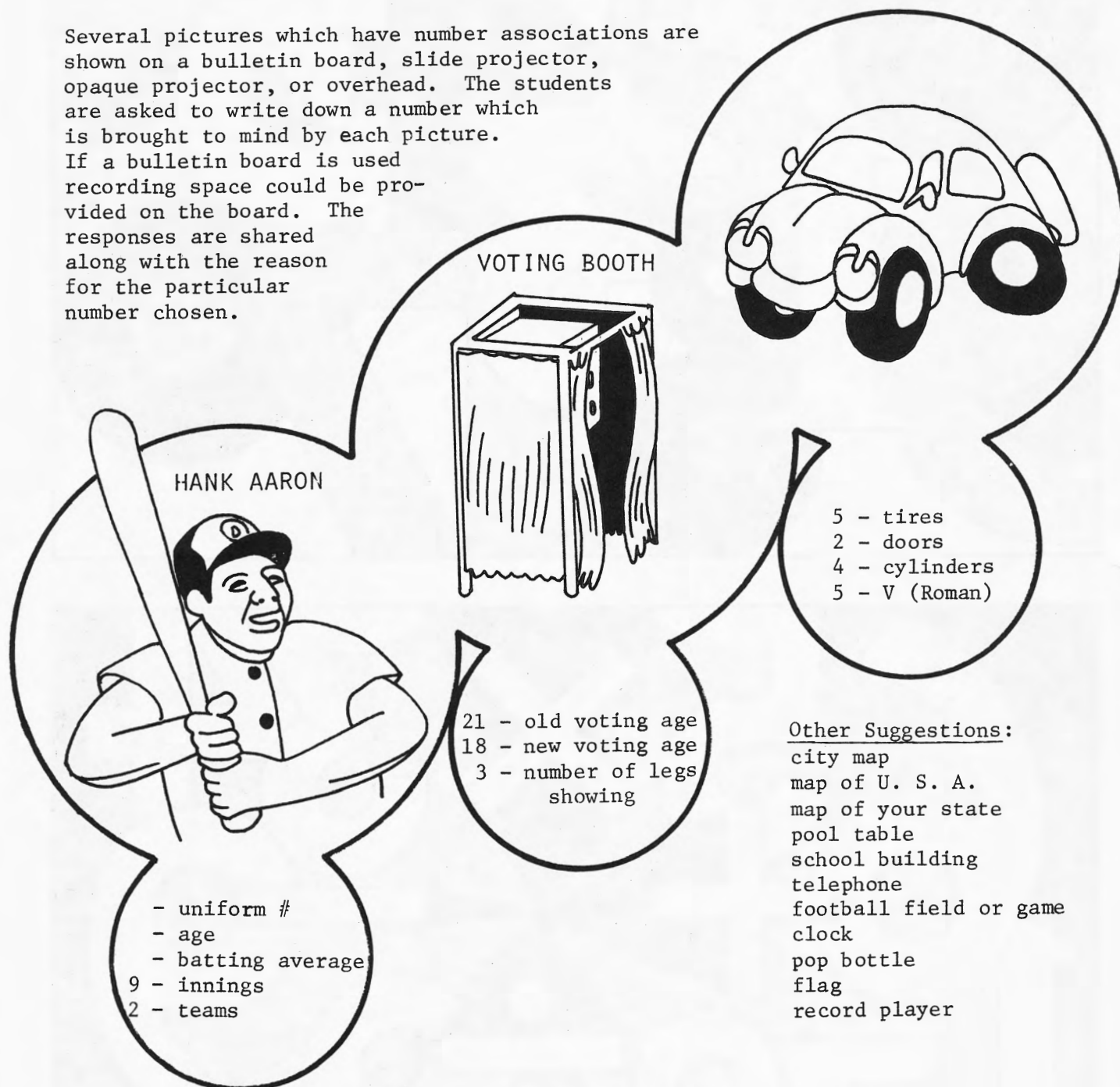
NUMBERS AROUND US - SOME ACTIVITIES AND IDEAS

Number Lines
Counting Started
Whole Numbers



ACTIVITY 1. WHAT NUMBERS DO YOU THINK OF?

Several pictures which have number associations are shown on a bulletin board, slide projector, opaque projector, or overhead. The students are asked to write down a number which is brought to mind by each picture. If a bulletin board is used recording space could be provided on the board. The responses are shared along with the reason for the particular number chosen.

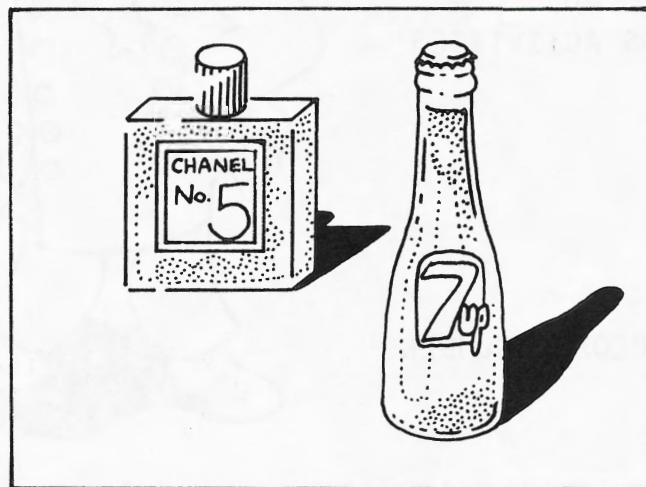


This activity can be used to get to know students or help them know each other. It can give information about student interests. It can give students a chance to share their knowledge and interests. This is a nonthreatening activity with no wrong answers--a good way to start the year.

NUMBERS AROUND US (CONTINUED)

ACTIVITY 2. NUMBERS WE SEE

The students are asked to bring pictures, advertisements, etc. which show numbers. The pictures are then made into a bulletin board or display. Pictures of 7-up, Seagram's 7, Chanel No. 5, etc. can be found. An alternative to finding pictures is having students create their own. This could range from the slogan: "We're no. 1!" to a number graphic or cartoon.



ACTIVITY 3. THE CALENDAR

Have students make a list of questions about the calendar. The list might include such things as:

- A. If the 1st of the month is on a Wednesday, what day of the week is each of these dates: 8th, 10th, 21st, 30th, 31st?
- B. If October is the 10th month why does an octagon have only 8 sides?
- C. Why doesn't a month have 4 weeks?
- D. What is the history of the calendar?

ACTIVITY 4. SPOTTING NUMBERS

For a week have students keep a record of the numbers they see. Have them:

- A. Note WHERE the numbers are seen.
- B. Note WHAT the numbers are used for.
- C. Note the LARGEST number seen in the week.

Keep a record of the largest number for the class to see--change it as new numbers come in.

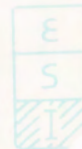


NUMBER NAMES CLASS ACTIVITIES

TEACHER DIRECTED



Number Uses
Getting Started
WHOLE NUMBERS



1. PEOPLE NUMBERS

How are you identified by numbers? How are your students? As a quick class activity a list is started of how people are identified and/or described by numbers. This can become a growing list on a poster and/or a list kept by each student. Here are some examples:

SOCIAL SECURITY NUMBER
LICENSE NUMBER (bike or car)
STREET OR AVENUE NUMBER
HOUSE NUMBER

DATE OF BIRTH
PHONE NUMBER
ZIP CODE
CREDIT CARDS

HEIGHT
WEIGHT
SPORTS RECORDS
SEAT LOCATION IN
CLASS

2. HOUSE NUMBERS (INDIVIDUALIZED PROJECT)

How are the houses on your street numbered?
Is there a pattern?
Is this pattern followed on other streets?
Could you come up with a better pattern? Why is it better?

3. CAR NUMBERS (CLASS DISCUSSION)

How are cars numbered?
Why are cars numbered?
Do cameras, radios, bicycles, and motors have number names? Why?

4. ROAD NUMBERS (CLASS PROJECT - YOU WILL NEED CITY, STATE AND COUNTY MAPS)

How are the streets in your home town named?
How are the highways in your state named? Is there a pattern?
How are the highways in the country named? Is there a pattern?

TYPE: Activity/Transparency/Bulletin Board

NUMEROLOGY

OR

WHAT'S YOUR NUMBER

Motivation
Getting Started
WHOLE NUMBERS



TEACHER DIRECTED ACTIVITY

Numerology is the study of numbers. People have been exploring the meaning of numbers to describe things about themselves and the world around them for over 2500 years. Pythagoras, who is sometimes called the "Father of Numerology," is credited with its development. Numerologists believe all things are numbers and all worldly happenings can be explained with numbers. Each number has a special meaning. These meanings are interpreted into human traits as you will see on the following pages. Here is the meaning behind the digits 1-9:

1 - REASON (capable of producing a consistent whole) AND CREATION. It is the beginning of our number system. Without it no other numbers could be found ($1 + 1 = 2$). It is also associated with the sun, the positive and the male.

2 - OPINION AND THE UNLIMITED. Since $1 + 1 = 2$, its presence assured man of indefinite numbers and infinity, as you can always add one more. It symbolized the moon, the negative and the female.

3 - SELF-EXPRESSION AND THE SACRED. Religion tells of many threefold deities. Greeks considered it the first odd number.

4 - JUSTICE. Four is the first square number. Ever heard of a "square" deal or a "square shooter?" It is also the product and sum of equals.

5 - MARRIAGE AND RECREATION. It was the union of their first odd and even numbers, or the first male and female link.

6 - SOCIAL CONSCIENCE. It's also the first (and only single digit) perfect number.

7 - HEALTH AS WELL AS INTELLECTUAL AND SPIRITUAL DEVELOPMENT. It's also considered sacred because of its frequent use in the Bible.

8 - LOVE AND FRIENDSHIP AND MATERIAL ACCOMPLISHMENT. Since the Greeks considered one the unity and not a quantity, it was the first and only single digit cube ($2 \times 2 \times 2$).

9 - SELFLESSNESS AND UNIVERSAL AWARENESS. It is the one number that recreates itself, in every set of factors: $2 \times 9 = 18$ ($1 + 8 = 9$), $3 \times 9 = 27$ and ($2 + 7 = 9$) etc.

Numerologists study vibrations of numbers and how these vibrations influence your personality, emotions and future. The system on the next page reduces every number or collection of numbers into a single life cycle number. Certain traits, characteristics, and personalities are then attributed to each number.

TYPE: Activity

IDEA FROM: *Drilling 4 Fun* and "Numerology" Oregon Council of Teachers of Mathematics Monograph
Nov., 1974

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NUMEROLOGY PAGE 2

HOW TO FIND WHAT NUMBER YOU ARE

Your number is known as your Life Cycle Number. It can be figured out in this way:

1. Write your full name given at birth.
2. Use the table below and find the number value of each letter.
3. Place the proper number under each letter.
4. Add up all the numbers in your name.
5. Add the digits of your answer and continue adding digits in your answer until it is reduced to one digit.
6. Look on Life Cycle Number Characteristics and see what this tells you about your personality and future.

1	2	3	4	5	6	7	8	9
A	B	C	D	E	F	G	H	I
J	K	L	M	N	O	P	Q	R
S	T	U	V	W	X	Y	Z	

EXAMPLE: Frank Lloyd Wright:

$$\begin{array}{ccccccc} F & R & A & N & K & L & L & O & Y & D & W & R & I & G & H & T \\ 6 & 9 & 1 & 5 & 2 & 3 & 3 & 6 & 7 & 4 & 5 & 9 & 9 & 7 & 8 & 2 \end{array} = 86$$
$$86 \rightarrow 8 + 6 = 14 \rightarrow 1 + 4 = 5$$

Frank Lloyd Wright's Life Cycle Number is 5.

IDEA FROM: *Drilling 4 Fun* and "Numerology" Oregon Council of Teachers of Mathematics Monograph, Nov., 1974

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NUMEROLOGY PAGE 3

LIFE CYCLE NUMBER CHARACTERISTICS

1

ONE people are pioneers and innovators. They are strong, capable, unchanging, dignified and independent. They are born leaders and organizers. They are outgoing and high-spirited. They like activity and have a keen sense of humor and a quick temper. Orange is the best color (related to the sun), and Sundays are the best days for 1 people.

2

TWO people are tactful and considerate. They are gentle, helpful, adaptable, quiet and reserved. They cooperate with people and resort to anything to avoid an unpleasant situation. They are ruled by their hearts not their heads. They make friends easily and other people turn to them in time of trouble. They are good at handling money and have common sense. Green is their best color, and Mondays their best day.

3

THREE people are enthusiastic, talented, popular, independent, ambitious, optimistic and full of energy. They can be selfish as they enjoy the finer things of life. They are destined to achieve success by learning to accept the good with the bad. 3's often like red and have good days on Tuesdays.

4

FOUR people are hard working, steady, reliable, studious and agile. They are thoughtful and considered the builders who get things done. They are usually quiet, straightforward and honest. They respond quickly to injustice and inequalities. They enjoy sports and bodily activity. In life they are destined to seek security. Their best day is Wednesday.

5

FIVE people are impetuous, and quick-tempered. They love adventure, people and parties. They are destined to seek fame and fortune. They hate to make plans in advance. For them all life is a romantic adventure. Their best day is Thursday; the best color is blue.

6

SIX people are dependable, patient, even-tempered and soft-spoken with a strong sense of fair play and a love of beauty. In life they are destined to search for and/or create beauty. Their best day is Friday.

7

SEVEN people are interested in world affairs. They are thoughtful, analytical, critical, serious, curious and original thinkers. They enjoy traveling but tend to be shy. They are destined to use their abilities for the good of mankind rather than their own gain. They like gray, and Saturday is their best day.

8

EIGHT people have intense concentration and drive themselves hard. They are energetic and active both physically and mentally. They are loyal and trustworthy. In life they are destined to be energetic workers and leaders who offer help and courage to those they work with. They like blues and do well on Saturdays.

9

NINE people are passionate, sensitive, emotional, perceptive, and compassionate. They are moody and have hot tempers. They are devoted to worthy causes. In life they are destined to work hard to seek perfection. They like lavender and purples and do well on Thursdays.

IDEA FROM: *Drilling 4 Fun and "Numerology" Oregon Council of Teachers of Mathematics Monograph,*
Nov., 1974

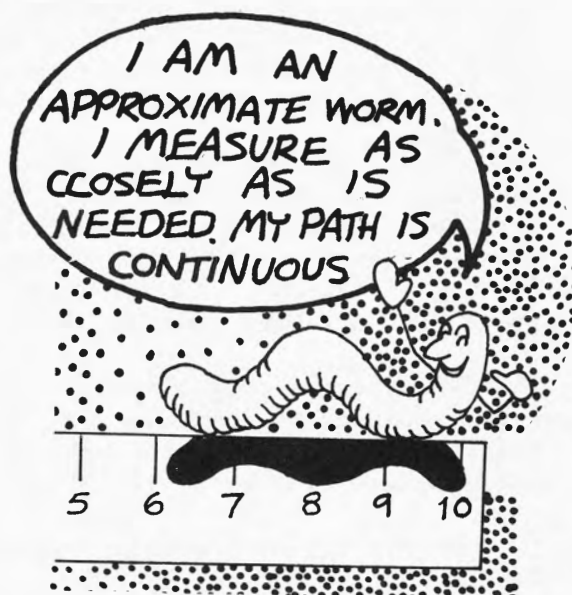
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Gettling Start
WHOLE NUMBERS



EXACT OR APPROXIMATE



Counting is exact. If a recipe lists 4 eggs it means to use 4 eggs, not 3 or 5.

Measuring uses approximation. A recipe listing 3 cups of sugar means as close to 3 cups of sugar as you can reasonably measure.

Classify each of the following as exact or approximate:

1. I live 2 kilometres from school. _____
2. You are 5 minutes late. _____
3. Slice 6 apples. _____
4. Add 1 litre of milk. _____
5. I have 10 fingers. _____
6. Net contents: 520 grams. _____
7. I am 150 centimetres tall. _____
8. Together we have 10 records. _____
9. Find more examples that are exact.
10. Find more examples that are approximate.



PATTERNS FOR COUNTING

Using Patterns
Getting Started
WHOLE NUMBERS

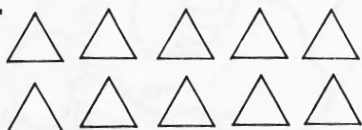


1. WHICH GROUP HAS MORE TRIANGLES, A OR B?

A.



B.



4. HOW MANY HERE?



WHAT PATTERNS DO YOU SEE?

$2 + 2 + 2$ $3 + 3$ 2×3 3×2

2. WHICH GROUP HAS MORE CIRCLES, A OR B?

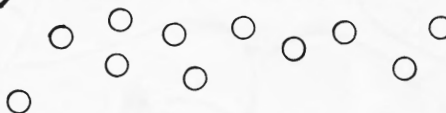
A.



B.



3. HOW MANY CIRCLES ARE THERE?



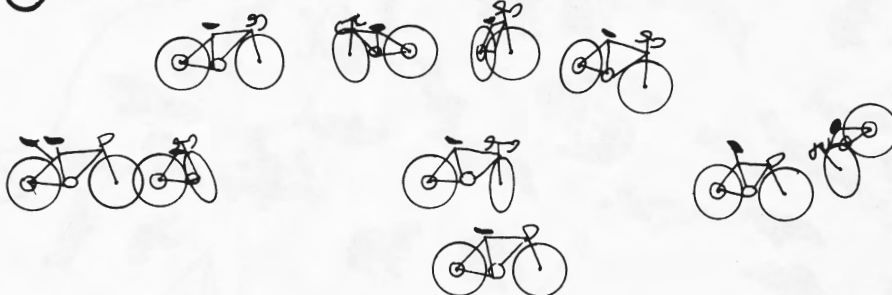
5. HOW MANY HERE?



WHAT PATTERNS DO YOU SEE?

$3 + 3 + 3$ 3×3 $1 + 2 + 3 + 2 + 1$

6. WITHOUT COUNTING, HOW MANY BIKES ARE THERE? USE PATTERNS.



FIND THE NUMBER OF UNIT SQUARES. FIND PATTERNS AND HUNT FOR FAST WAYS TO GET THE ANSWER. DON'T JUST COUNT THEM "ONE," "TWO," "THREE," . . .

7.

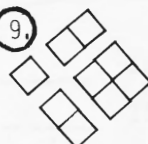


HOW MANY SQUARES?

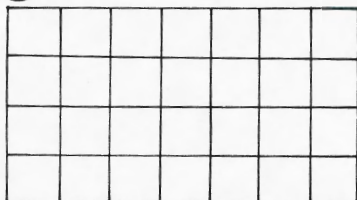
8.



9.

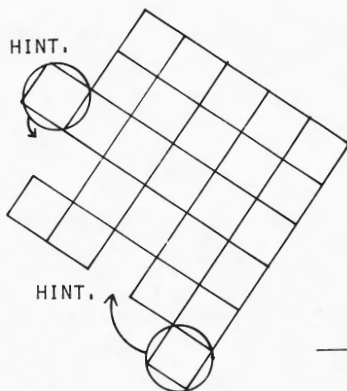


10.

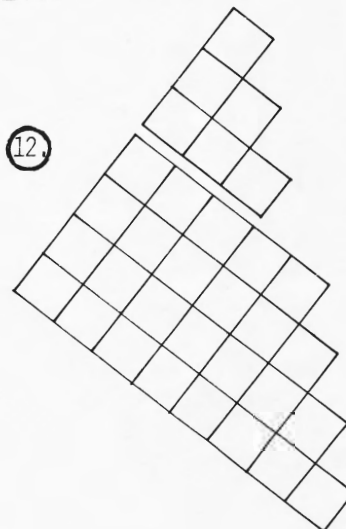


HINT: (4 ROWS, 7 IN EACH ROW,
SO $4 \times 7 = ?$)

11.



12.



JEA FROM: *Ways to Find How Many*



SEALS ON MY ICEBERG

TEACHER DIRECTED ACTIVITY

Estimation
Counting
Getting Started
Whole Numbers



INDIVIDUAL GUESS _____ TEAM GUESS _____
INDIVIDUAL COUNT _____ TEAM COUNT _____

Use the activity sheet above. Students, working in pairs, estimate the number of seals and write their guesses down. They then discuss the guess with their partner and write down a team guess. (They must agree on the team guess.) Next, each person counts the seals and records this individual count. Again, they confer, recount if they wish, and then write down the team count.

Class Discussion:

Write the highest and lowest guesses on the board. Ask several students about their procedure for individual guesses.

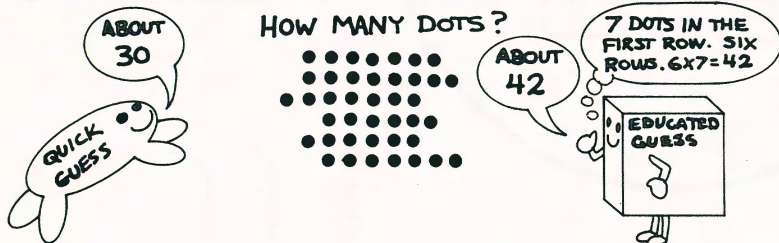
Discuss actual counts; if there is a disagreement, decide on the actual figure. Ask about methods for counting. Discuss the advantages and disadvantages of each one. You may wish to have an overhead transparency of the seals. As the students discuss their procedures, you could trace their methods on the overhead.

TYPE: Activity/Transparency

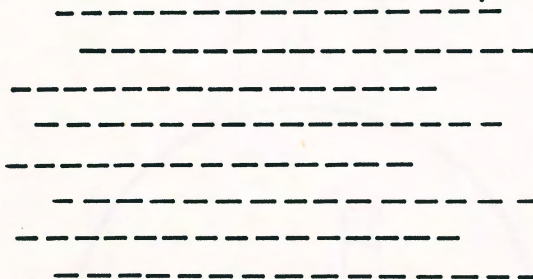
Place mats from restaurant chains could also be used for this activity.

EDUCATED GUESS

QUICK GUESS AND EDUCATED GUESS ARE HAVING A CONTEST. LOOK AT THEIR GUESSES. WHO IS THE BETTER GUESSESS?

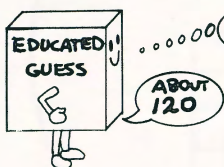


HOW MANY DASHES?



EXPLAIN THE METHOD USED BY EDUCATED GUESS.

USING THIS METHOD, WHAT IS THE WORST ESTIMATE THAT COULD BE MADE? WHY?

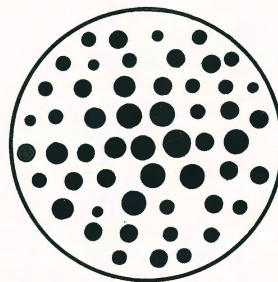


FIND OUT HOW MANY BOOKS ARE ON TEN SHELVES OF YOUR SCHOOL LIBRARY. EDUCATED GUESS _____. CAN YOU GUESS OTHER AMOUNTS AROUND THE CLASSROOM? NAME THEM.

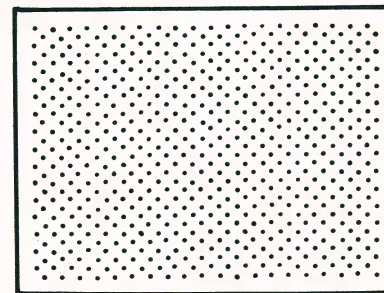
"GUESS AGAIN"

WHICH OF THE THREE ESTIMATES IS CLOSEST? (DON'T COUNT.)

HOW MANY DOTS IN THE CIRCLE?
20 50 100

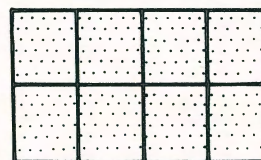


HOW MANY DOTS IN THE RECTANGLE?
150 450 600



MAKE A QUICK GUESS AND AN EDUCATED GUESS FOR THESE. EXPLAIN YOUR EDUCATED GUESS.

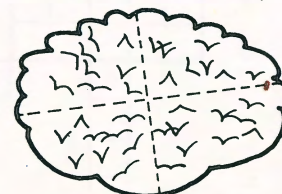
HOW MANY DOTS?



QUICK GUESS _____

EDUCATED GUESS _____

HOW MANY BIRDS?



QUICK GUESS _____

EDUCATED GUESS _____

MAKE UP A PROBLEM. GIVE IT TO A CLASSMATE. DID HE GIVE AN EDUCATED GUESS?

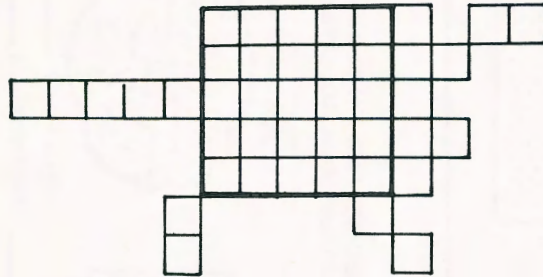


25 - MORE OR LESS

Estimation
Counting
Getting Started
WHOLE NUMBERS



Look at each diagram for about 5 seconds. Estimate whether the number of squares is more than 25 or less than 25. Record your estimate. (Don't erase it, even if it's wrong--that's O.K.) Check your estimate using the fastest method that you know.



Estimate Check
_____ more than 25 _____
_____ less than 25 _____

It is enough to see that there is a 5 x 5 square in the middle to show whether your estimate is correct.

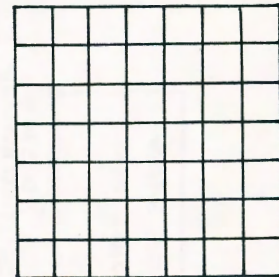
A.

Try to do these diagrams by looking at each for only 3 seconds.



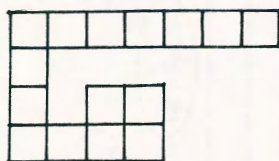
Estimate Check
_____ more than 25 _____
_____ less than 25 _____

B.



Estimate Check
_____ more than 25 _____
_____ less than 25 _____

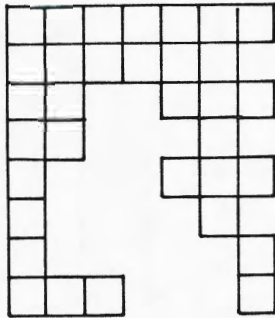
C.



Estimate Check
_____ more than 25 _____
_____ less than 25 _____

25 - MORE OR LESS (CONTINUED)

D.



Estimate

Check

_____ more than 25 _____

_____ less than 25 _____

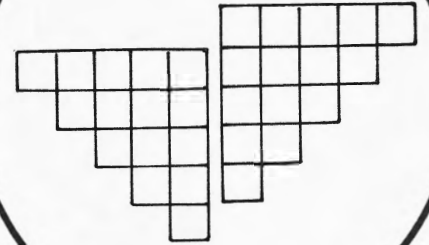
Estimate

Check

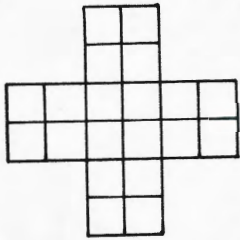
_____ more than 25 _____

_____ less than 25 _____

E.



F.



Estimate

Check

_____ more than 25 _____

_____ less than 25 _____

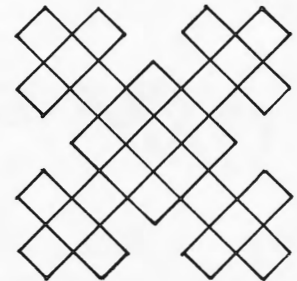
Estimate

Check

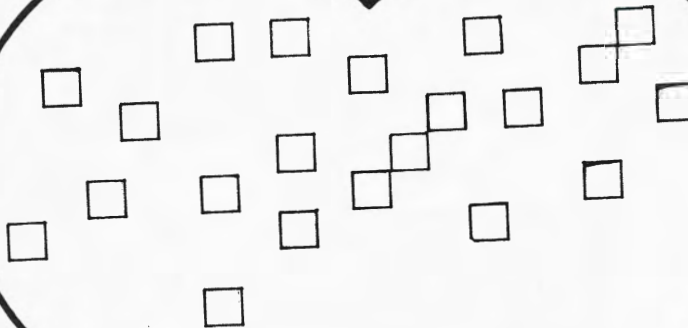
_____ more than 25 _____

_____ less than 25 _____

G.



H.



Estimate

Check

_____ more than 25 _____

_____ less than 25 _____

Extensions:

Have two students work together with the centimetre cubes. One student builds a stack, and the other student estimates whether the stack is more than or is less than 8 or 27 by trying to compare it to a $2 \times 2 \times 2$ stack or a $3 \times 3 \times 3$ stack.

IDEA FROM: *Ways to Find How Many*

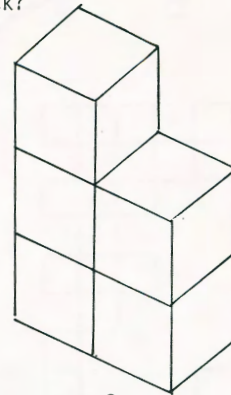
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COUNTING CUBES

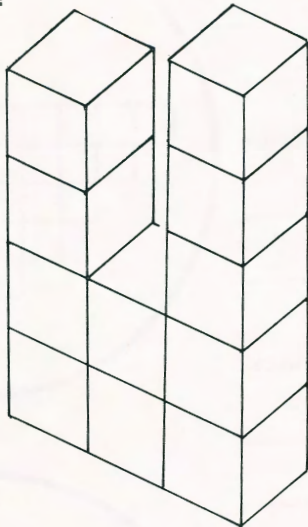
Counting
Getting Started
WHOLE NUMBERS



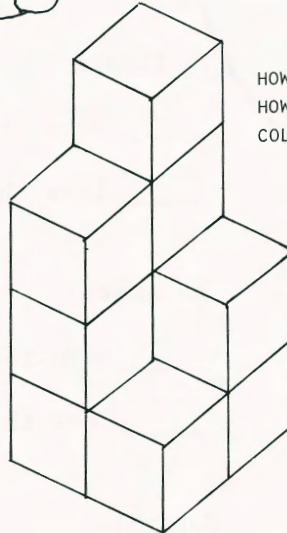
HOW MANY CUBES
IN THIS STACK?



HOW MANY
HERE?



HOW MANY HERE?
HOW MANY IN EACH
COLUMN?



HERE'S AN AIRPLANE'S VIEW
(TOP VIEW) OF THESE CUBES.

CAN YOU SEE
WHAT THE
NUMBERS REPRESENT?

4	2
3	1

**AIRPLANE'S
VIEW**

3	3
1	2

HOW MANY WOULD
BE IN THIS STACK?

4		2
3	3	1
4	1	

HOW ABOUT THIS STACK?
IF YOU HAVE CUBES, MAKE
A MODEL TO CHECK!

4	3	3
3	4	3
3	3	4

IS THERE AN
EASY WAY TO
FIND THE
NUMBER OF
CUBES IN
THESE STACKS?



5	5	5	5	5
6	6	6	6	6
7	7	7	7	7

CAN YOU SEE 15 SIXES? HOW ABOUT 5 EIGHTEENS?

7	7	5	5
7	7	5	5
7	7	5	5
7	7	5	5



This activity is a good
introduction to *Mystery
Stacks*.

CAN YOU MAKE MORE STACKS AND FIND
OTHER INTERESTING WAYS TO COUNT
THE CUBES?

TYPE: Paper & Pencil/Transparent

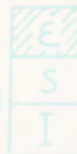
IDEA FROM: *Ways to Find How Many*

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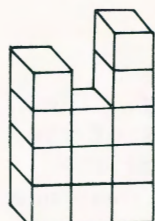
MYSTERY STACKS

Counting
Getting Started
WHOLE NUMBERS



Use the clues to answer the questions below. There may be one answer, no answer possible, or many answers.

1. CLUES:



- There are two stacks of cubes hidden behind these three stacks.
- Both hidden stacks have the same number of cubes.
- 16 cubes in all.

How many cubes are in each of the two stacks?

2. CLUE:

8	8	8
8	8	

- 45 cubes in all.

How many cubes in the unlabeled stack?

3. CLUE:

4	4	4
4	4	

- Exactly 5 stacks.

How many cubes in all?

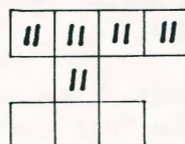
4. CLUE:

7	7	
7		
7	7	7

- 40 cubes in all.

How many cubes in the unlabeled stack?

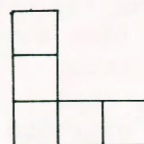
5. CLUES:



- Total number of cubes is 70.
- The unlabeled stacks all have the same number of cubes.

How many cubes in each unlabeled stack?

6. CLUES:



- 5 stacks.
- The tallest stack is 5 cubes high.
- No two stacks have the same height.

How many cubes in all?

7. CLUES:

- 5 stacks.
- None taller than 3.
- Total not more than 15.
- No stack 2 cubes high.
- Not all stacks have the same height.

What is the most number of cubes possible? _____
What is the least number of cubes possible? _____
Which clue don't you need? _____

8. CLUES:

- Tallest stack is one cube higher than the lowest stack.
- Exactly 2 stacks are one cube high.
- 18 cubes in all.

How many stacks?

TYPE: Paper & Pencil/Manipulative

IDEA FROM: *Ways to Find How Many*



THE PEG GAME OR TEN MEN IN A BOAT

TEACHER DIRECTED ACTIVITY

Counting
Looking for Patterns
Getting Started
WHOLE NUMBERS



Ten men are fishing in a boat. One seat in the center of the boat is empty. The five men in the front of the boat want to change seats and fish in the back of the boat, and the five men in the back of the boat want to fish from the front of the boat. A man may move from his seat to the next empty seat, or he may step over one man without capsizing the boat. What is the minimum number of moves it will take to exchange the five men in front with the five in back?

- Materials:
- a) 10 markers for each student, 2 different kinds or colors - 5 each (Wooden cubes, 1" ceramic tile, or beans may be used.)
 - b) Mat for making the moves (See sample.)
 - c) Answer sheet (See sample.)
 - d) Overhead projector

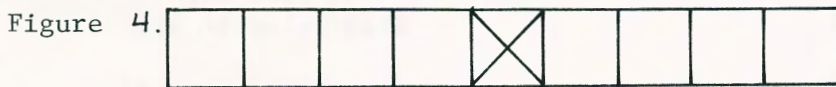
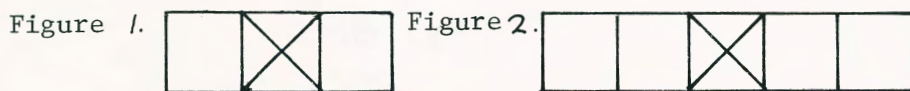
Procedure: Use a handout or overhead to present the problem to the students. Supply the markers and let them struggle for a minute or two, then suggest an easier problem - 2 men in the boat, 1 on each side. Supply students with the mat at this time and tell them to place 1 marker of each type in Figure 1. After another minute or two demonstrate the answer to the 2 men in a boat on the overhead.

SOLUTION - TWO MEN IN BOAT - RLR = 3 MOVES



(NOTE: THE PROBLEM MAY BE DONE IN EXACTLY THE OPPOSITE MOVES - LRL)

Suggested Mat - Be sure the squares are a little bigger than the markers.



A similar problem-solving strategy involving reducing to a simpler case, making a chart to organize the data, building backup, and looking for a pattern can be used on the next three activities, *The Painted Cube*, *Strictly Squareville*, and *Squareville Suburbs*. Each activity could be started by asking a question which seems impossible to solve.

TYPE: Activity

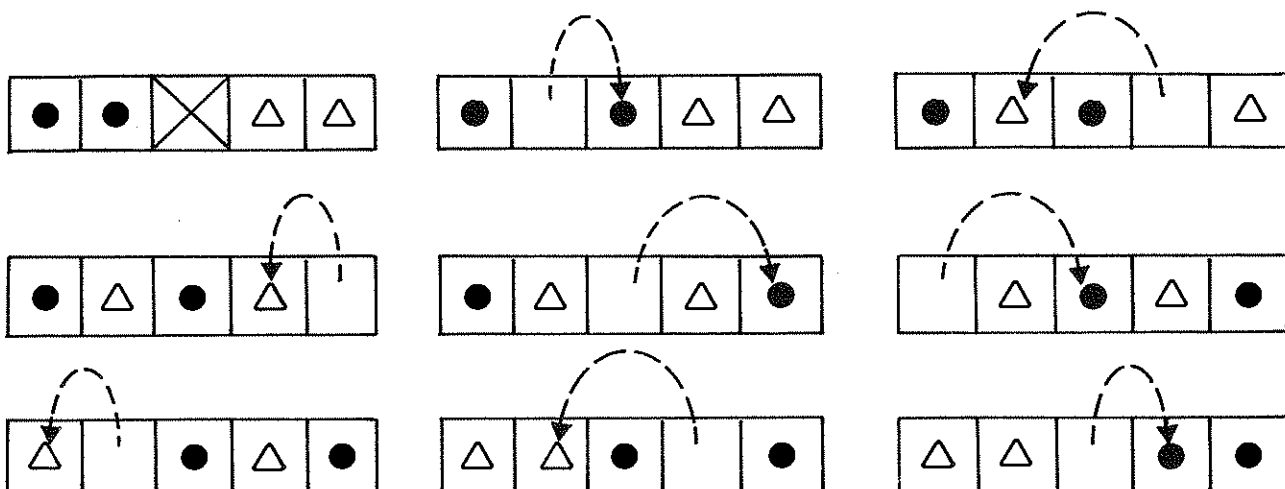
IDEA FROM: *Finite Differences*

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THE PEG GAME (PAGE 2)

Now have the students try the problem with 4 men in the boat, using Figure 2 on the mat. Again, allow them to struggle for a short while, then go through the moves on the overhead. Do it quickly at first, then repeat until everyone can do the 8 moves.

SOLUTION - 4 MEN IN BOAT - RLLRRLLR = 8 MOVES



Now on to 6 men in the boat, using Figure 3 on the mat. After 3 - 5 minutes of struggle time, go through the 15 moves rapidly, using the overhead and saying out loud the words right and left as you make the moves.

Hand Out The Answer Sheet At This Time.

(See the next page for an explanation of this answer sheet.)

SUGGESTED ANSWER SHEET

Men in a Boat	Minimum Moves	Movement Code	Triangle Code
2			
4			
6			
8			
10			

Complete the table above. Find a pattern that will help you find the minimum number of moves it will take for 12 men; for 20 men.

12 MEN _____

20 MEN _____

THE PEG GAME (PAGE 3)

Before giving away the answer, have the students go back to the 2 men problem and observe that the 3 moves can be done this way: RIGHT - LEFT - RIGHT. Have them put this movement code on the answer sheet as RLR. Ask the question: What is the movement code for the 8 moves with 4 men in the boat? (RLLRLLLR) Now back to the 6 men problem. Be sure they find both the movement code and the minimum number of moves before going on to the 8 men problem. Again, be sure everyone can do the 15 moves for the 6 men. Repeat the process several times on the overhead as they continue to struggle. After several students have found the 24 moves for the 8 men in the boat and found the movement code of RLL RRR LLL RRR LLR, introduce the class to the triangle code. Place the Δ 's on the board and have students put it on their answer sheet.

2 men	RLR	or	1 1 1
4 men	RLLRLLR	or	1 2 2 2 1
6 men	RLLRRRLLLRLLR	or	1 2 3 3 3 2 1
8 men	RLLRRRLLLL . . .	or	1 2 3 4 4 4 3 2 1
10 men		or	1 2 3 4 5 5 5 4 3 2 1

Now let the students finish the activity and answer the last two questions. Do not give them any more markers - they should be able to find the answers by using one of the codes.

Note: Be sure to give the students early and continuous reinforcement but try to let most of them discover the moves and codes for themselves. Students should be able to discover that a marker on the left only moves right, and a marker on the right only moves left.

This activity can be a good way of getting the uninvolved students involved and introduced to problem-solving techniques, but it may be used in any math class. If it is used in algebra or geometry, you may wish to have the students find the general formula. A suggested table would be:

MEN IN A BOAT	MEN ON 1 SIDE	MINIMUM # OF MOVES
2	1	3
4	2	8
6	3	15
8	4	24
10	5	35
12	6	48
.	.	.
.	.	.
.	.	.
20	10	120
.	.	
.	.	
.	.	
2n	n	$n(n + 2)$ or $(n + 1)^2 - 1$, where n = number of men on one side.

IDEA FROM: *Finite Differences*



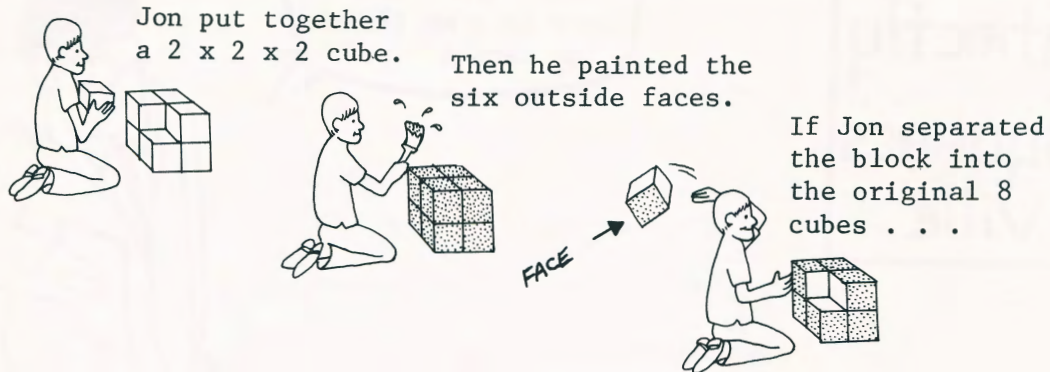
THE PAINTED CUBE

Counting
Looking for Patterns
Getting Started
WHOLE NUMBERS



Materials: about 100 wooden cubes

Use the cubes to answer the questions following the story.



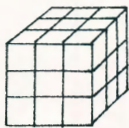
- I. How many small cubes would have exactly . . .
- 4 faces painted? _____
 - 3 faces painted? _____
 - 2 faces painted? _____
 - 1 face painted? _____
 - 0 faces painted? _____

You could introduce this activity by:

1) Painting a $3 \times 3 \times 3$ cube and giving the 27 pieces to students with instructions to put it together so only painted faces are showing.

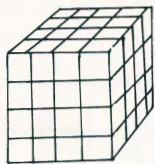
2) Asking the following question. Suppose you had a $10 \times 10 \times 10$ painted cube. How many of the cubes would have exactly two faces painted?

- II. Repeat for a $3 \times 3 \times 3$ cube. How many small cubes would have exactly . . .
- 4 faces painted? _____
 - 3 faces painted? _____
 - 2 faces painted? _____
 - 1 face painted? _____
 - 0 faces painted? _____



- III. How many cubes are in the large cube at the left? _____

- A. How many of the smaller cubes are painted on four faces? _____
- B. How many on just three faces? _____
- C. How many on just two faces? _____
- D. How many on just one face? _____
- E. How many on zero faces? _____



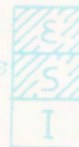
- F. What is the sum of your answers to questions A, B, C, D, and E? _____

How does this compare with your first answer to Part III?

- IV. Extend your investigations to larger cubes. Put your result in the following table. Are there any patterns?

No. of CUBES ON A SIDE	NUMBER OF PAINTED SIDES					TOTAL No. OF CUBES
	0	1	2	3	4	
2						
3						
4						
5						
6						
8						
10						
20						
50						

TYPE: Manipulative



Strictly Squares Ville

I'm Sid the Census Taker. I have to count the squares of all sizes in each town. Can you help me finish the census?

You could introduce this activity by showing students a 5 x 5 grid and asking them to find the total number of squares. After some guesses and discussion hand out the student page.



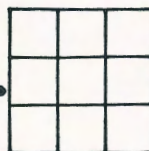
Onesville



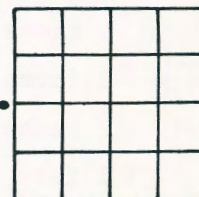
Twosville



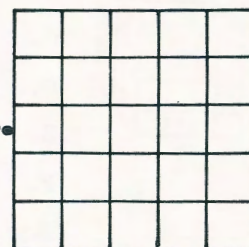
Threesville



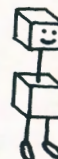
Foursville



Fivesville



Hmmm - Threesville has nine 1 x 1 squares, four 2 x 2 squares, and one 3 x 3 square.



Can you predict the square population for Sevensville? 140
For Tensville? 185

SID'S CENSUS LIST						
	SIZES OF SQUARES					SQUARE POPULATION
	1x1	2x2	3x3	4x4	5x5	
ONESVILLE	1					1
TWOSVILLE	4	1				5
THREES...						14
FOURS...						30
FIVES...						55
...						

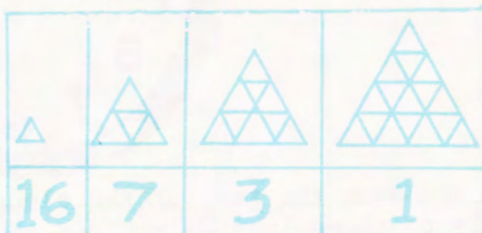


SQUARESVILLE SUBURBS

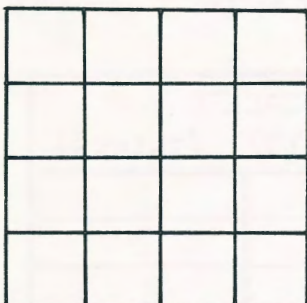
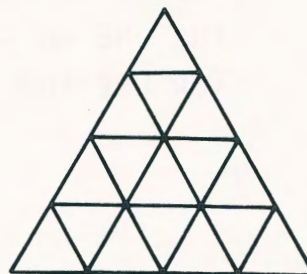
Counting
Looking for Patterns
Getting Started
WHOLE NUMBERS



How many triangles can you find in this figure? Don't forget to look for different sizes!



$$= 27$$



How many rectangles can you find? Make a chart. Remember, a square is a rectangle too.

1x1	1x2	1x3	1x4	2x2	2x3	2x4	3x3	3x4	4x4
16	24	16	8	9	12	6	4	4	1

$$= 100$$



How many of this shape



are in this figure? 36

How many are there of this shape



? 24

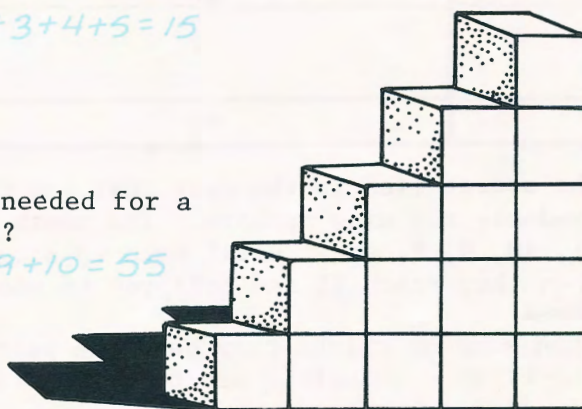
How many cubes are needed to build this staircase?

$$1+2+3+4+5=15$$



How many cubes would be needed for a staircase with ten steps?

$$1+2+3+4+5+6+7+8+9+10=55$$





ESP PUZZLE

TEACHER DIRECTED ACTIVITY

Motivation
Looking for Patterns
Getting Started
WHOLE NUMBERS



DRAW A CLOCK FACE ON THE
CHALKBOARD SIMILAR TO
THE ONE ON THE RIGHT.
(OR USE THE ROOM CLOCK)



Have a student choose a number and show the rest of the class but not you. Now the students are to count quietly to themselves as you tap and point to numbers on the face of the clock. They must start counting with the number they choose. When they reach 21 they tell you to stop. You are now pointing to the number originally chosen. For example: if they chose 7, then for the first number you tapped they would think 7; the next tap or number pointed to would be counted as 8; etc.

EXAMPLE:

TAP	STUDENTS COUNT (SILENTLY)	TEACHER	
		(POINTS TO)	(THINKS)
1ST	7	ANY	1
2ND	8	ANY	2
.	.		.
.	.		.
9TH	15	ANY	9
10TH	16	12	10
11TH	17	11	
.	.	.	
.	.	.	
.	21	7	

If the students picked 12 they would be at 20 on the 9th tap.

The secret lies in the fact that you count also, only you start with 1, and randomly tap nine numbers. The tenth tap must be 12, then you tap the numbers 11, 10, 9, 8, etc. until told to stop by the students. If all has gone correctly when they reach 21 and tell you to stop, you should be on the number they chose.

When someone thinks they have the pattern, let them try their solution on the class. Solutions are to be kept secret once discovered.

VARIATIONS:

- Use a clock numbered 1-9.
Tap out 6 random numbers.
Have students count to 15 and then tell you to stop.
- After the secret is known, suggest other ways to number the clock.
Ask students to determine how many random numbers to tap out and what number to have the audience count to.

TYPE: Activity



GAME of 50

Motivation
Looking for Strategies
Getting Started
WHOLE NUMBERS

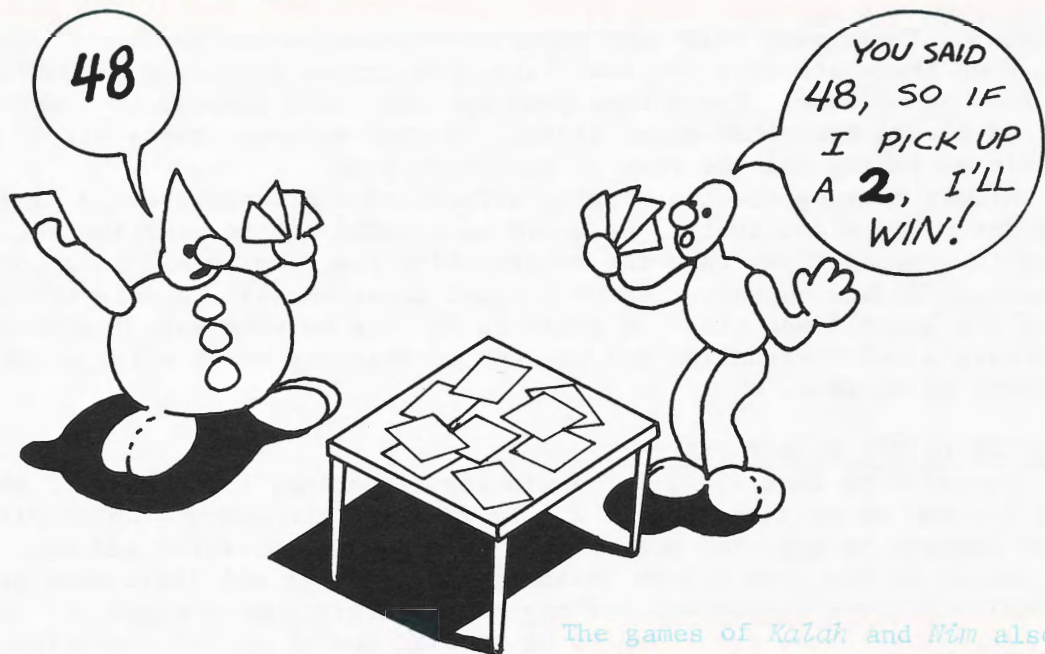


- PLAYERS: 2
- MATERIALS: 24 cards numbered 1, 2, 3, 4, 5, 6 (four of each)
- PROCEDURE:
1. Spread all 24 cards face up on a table.
 2. The first player picks up any card and calls its number. Then, the other player picks up a card, adds it to the number called by the first player, and gives the total. Repeat the process, always adding to the last number called by the opponent.
- WINNER: The first player to call exactly 50.

N O T E: Once a card is picked up, it may be replaced on the table.

- VARIATIONS:
1. Play the game without replacing the cards.
 2. G A M E of 22
 - a) Use only 16 cards, 4 each of 1, 2, 3, and 4.
 - b) Same procedure as Game of 50.
 - c) Play without replacement.

The game of 50 is a good game to use when introducing students to the idea of finding winning strategies. The key numbers in this game are 43, 36, 29, 22, 15, 8 and 1. The first player can always win if he picks up a card with a number one.



The games of *Kalah* and *Wim* also present students with problem-solving and strategy challenges.

TYPE: Game

IDEA FROM: C.O.L.A.M.D.A.

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THE GAME OF NIM

TEACHER PAGE

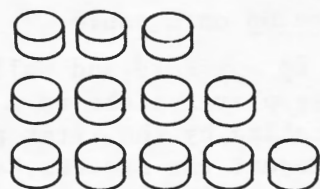
Motivation
Looking for Strategies
Getting Started
WHOLE NUMBERS



Nim is a two-person game. It may be used to:

- 1) Motivate the students,
- 2) Develop problem-solving techniques.

The simplest form of Nim is played with 3 rows of markers as illustrated:



Each player in turn takes any number of markers from exactly one row. (At least one marker must be removed each turn.) The winner is the player who takes the last marker.

Have the students play and try to develop some winning strategies.

AN EXTENSION OF NIM:

Nim can be played with three rows using any number of markers in each row. The strategies for both the simplest form and this extension of Nim are outlined below.

STRATEGIES FOR NIM:

There are many winning situations for Nim and hopefully the players will discover some as they are playing. One example of a winning situation is 1-2-3 (one marker in one row, 2 markers in another row, and 3 markers in the third row). To win you try and create this situation during your turn. No matter what the opponent does at this point, you will win if you use the correct strategy. Experiment with some markers and see for yourself.

When there are only two rows left, you create a winning situation when you form equal rows. Every time your opponent must respond to 2 equal rows you can always make them equal again. If your opponent takes all of one row, you win by taking all the rest in the other row.

Simply knowing the two winning situations mentioned above, a player could play 3-row Nim effectively. He would be careful not to make two equal rows, since the opponent may take the entire third row, leaving him in a losing situation. If his opponent leaves 2 equal rows, he will be able to take the third row himself and win. In addition to this he will wait for an opportunity to create a 1-2-3 situation and try not to make any moves which would allow his opponent to do this.

RELATION TO THE BINARY SYSTEM:

In order to describe all the winning situations the number of markers in each row can be written in base 2. To find all the 3-row winning situations the three numbers in base two are written in a column, as if for adding. The sum of all digits in the same column is the digit-sum. If all digit-sums are even, it is called an even situation. If any of the digit sums are odd, it is called an odd situation. Here are examples of an even and of an odd situation.

10001	1100
1101	11000
11100	1110
22202	13210
even situation,	odd situation.

TYPE: Game

THE GAME OF NIM (CONTINUED)

There is one simple rule: the even situations are winning, and the odd situations are losing. An odd situation can always be changed to an even situation in one move, and an even situation can never be changed to an even situation in one move. Therefore, to win at Nim simply make an even situation each time it's your turn. Request to begin whenever an odd situation is set at the beginning and allow your opponent to begin whenever the beginning situation is even.

THE SIMPLEST WAY TO PLAY:

It may seem necessary to use a pencil and paper to convert everything to base 2 in order to win. This is not so. A player can find even situations simply by discreetly arranging each row into small groups of powers of 2. For example, a row of 23 markers could be grouped as such:

..... (16, 4, 2, 1). This would be the same as 10111 in base two. When all the rows are grouped in this way, it is easy to find an even or odd situation. It is a winning situation if the number of groups of each power is even. In a player's turn he looks at the groups of markers and does what is necessary to make the number of groups of each power even.

Example 1

Here is an even number of groups of 8 and 4, but an odd number of 2's and 1's. To make an even number of groups of each power remove one from the bottom row, making 2 eights, 2 fours, and 2 ones.

Example 2

In this example there is an odd number of 8's, 4's and 1's. If the top row contained just a 4 and a 2 it would be an even situation. Therefore, remove 5 markers from that row.

Example 3

In this example there is an odd number of 8's, 4's and 1's. If the top row contained just a 4 and a 2 it would be an even situation. Therefore, remove 5 markers from that row.

EXTENSION OF THE GAME OF NIM TO MORE THAN THREE ROWS:

The general case of Nim is played with any number of rows having any number of markers in each row. There is no essential change in strategy when we begin with more than three rows of markers. To find the winning situations use the binary system to arrange the markers in the various rows. The winning situations are simply all the even situations.

NOTE: Some people play with a rule that the person who is forced to take the last marker loses. All of the above strategies also apply to this rule with the exception of the last few moves. Examine this for yourself to discover what changes are necessary at the end of the game.

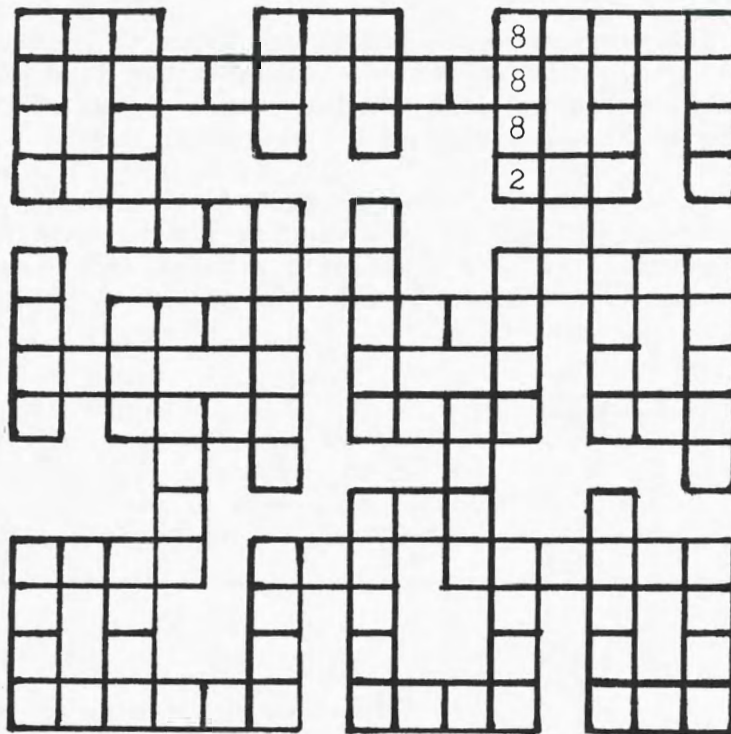
IDEA FROM: *The Master Book of Mathematical Recreations*

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A CROSSNUMBER PUZZLE

Directions: Fill in each of the blanks below from the list of numbers given. All the entries in the table must tie together vertically and horizontally.



3	4	5	6
027	0456	5000	26733
072	0764	6274	31196
192	0933	6789	39821
228	1972	7635	49390
256	3140	7843	81850
260	3171	7863	93112
289	3207	8176	99533
296	4040	8193	
409	4368	8882	
496	4392	9038	
589	4486	9193	
623		9310	
739			
999			



RIVERS AND RELATIVES

TEACHER IDEAS

Logic Problems
Getting Started
WHOLE NUMBERS

E
S
I

- Two fathers and two sons divide three apples among themselves, each receiving exactly one apple. How was this possible? (grandpa, father and son)
- A farmer has to get a fox, a goose, and a bag of corn across a river in a boat which is only large enough for her and one of these three items. If she leaves the fox alone with the goose, the fox will eat the goose. If she leaves the goose alone with the corn, the goose will eat the corn. How does she get all the items across the river? (Use 3 different markers to experiment.)



Solution: The farmer takes the goose across and comes back. The farmer takes the fox across and brings the goose back. The farmer takes the corn across and comes back; finally the farmer takes the goose across.

- Two couples want to get across a river. All that was available was a boat which could carry only 200 pounds. The women weighed 100 pounds each, and the men 200 pounds each. How did they all manage to cross the river? (Use markers to experiment)

Solution: Two women go across, one woman comes back. One man goes across, the other woman comes back. Two women go across, one woman comes back. The other man goes across, the other woman comes back. Finally, both women go across.



- What are the fewest number of people in the car if there are two grandmothers, one great-grandmother, one great-grandchild, two grandchildren, three mothers, one son, and two daughters? (4; ggm, gm, m, ggs)
- Three cannibals and three missionaries are traveling through the jungle. The six reach a stream which can only be crossed by means of the rowboat they are carrying. But the boat can hold only two men. Each of the three missionaries knows how to row, but two of the cannibals do not. Moreover, at no time would the missionaries permit a situation to occur in which some of them would be alone with a greater number of cannibals. In other words, the cannibals revert to their old habits if they are left alone in a situation where they outnumber the missionaries. How did all manage to get across the river?

(Use markers to experiment.)

Solution: Let M_1, M_2, M_3 represent the missionaries, and C_1, C_2, C_r (rowing cannibal) be the cannibals.

C_r & C_1 go across; C_r comes back.
 C_r & C_2 go across; C_r comes back.
 M_1 & M_2 go across; M_1 & C_1 come back.
 M_3 & C_r go across; M_2 & C_2 come back.
 M_1 & M_2 go across; C_r comes back.
 C_r & C_1 go across; C_r comes back.
 Finally, C_r & C_2 go across.



TYPE: Paper & Pencil

NUMBER SENSE AND ARITHMETIC SKILLS

Placement Guide for Tabbed Dividers

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Body Counting Systems	223	Motivation	Bulletin board Transparency
Attic-Greek	224	Finding patterns Ancient numerals	Paper and pencil
Roman Numeration System	225	Finding patterns Ancient numerals	Paper and pencil
Secret Roman Message	226	Ancient numerals	Paper and pencil Puzzle
Matchstick Equations	227	Ancient numerals	Paper and pencil Puzzle
Mayan Numeration System	228	Finding patterns Ancient numerals	Paper and pencil
Babylonian Numeration System	229	Finding patterns Ancient numerals	Paper and pencil
Creating a Numeration System	230	Devising numerals	Activity

WHOLE NUMBERS: NUMERATION SYSTEMS

Most children enter school counting--well, at least chanting certain sounds which they associate with sets of objects. In school they soon learn to represent these sounds with certain visual symbols and begin to see patterns and rules which govern the use of these symbols. This collection of symbols, patterns and rules for representing numbers is their NUMERATION SYSTEM. The numeration system that they are learning is a simple and efficient system employing ingenious techniques that have evolved over many centuries. While there is no doubt of the importance of learning and understanding our numeration system, there does seem to be some question as to whether or not the study of earlier numeration systems is important.



WHY OTHER NUMERATION SYSTEMS?

Various reasons are given for studying other numeration systems. Three of these are given below:

- To motivate the study of our own system
- For cultural and historical perspective
- To develop appreciation for our own system

The attainment of these goals is related to the way students encounter these other systems.

ATTIC GREEK

Consider, for example, this student page which encourages students to break the code. Here the students are actively involved in discovering the missing symbols based on patterns (much like amateur archaeologists). They also must discover rules for using these symbols. In addition, they are asked to be creative--"How do you think the Greeks would have written 500?" There may be several answers, and students could discuss them. In a sense, the numeration system becomes partly theirs.

Can you break this code?

Hindu-Arabic	Attic-Greek
1	I
3	III
6	VI
17	ΔVII
19	ΔΔI
24	ΔΔΔII
51	LI
53	LVIII
73	LXXIII
89	LXXXIX
216	CCXVI
347	CCXLVII
402	CDII

COMPLETE THE TABLE BELOW


Hindu-Arabic	Attic-Greek
	I
	II
	III
500	D
1000	M



MAYAN
NUMERATION SYSTEM

HINDU-ARABIC	MAYAN
0	⊙
1	•
4	...
5	—
6	—•
8	—••
10	==
13	—•••
15	—••••
17	—•••••
20	⊙
28	—•••••
	—••••••
	—•••••••
	—••••••••

FILL IN THE MISSING NUMERALS



HINDU-ARABIC	MAYAN
	••
	•••
85	—•••••
100	⊙
173	—•••••
	—••••••
246	—••••••
	—•••••••
360	⊙
	—•••••••
	—••••••••

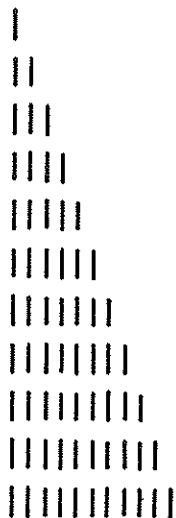
CAN YOU EXPLAIN HOW THE MAYAN NUMERATION SYSTEM WORKS?
YOU MAY WANT TO DO SOME RESEARCH.

After a similar treatment of one or more other systems, students are now in a position to discuss the attributes of the various systems. "Which system do you like best?" "Why?" "What advantage does the Mayan system have over the Greek system?" "In which system would it be easier to write large numbers?" Students could construct a system of their own. They may want to know about the people who developed these systems and should be encouraged to do library projects. (What kind of mathematics did these cultures need and use?)

CHARACTERISTICS OF EARLIER NUMERATION SYSTEMS

To do a thorough study of ancient numeration systems would be an enormous undertaking. However, by sampling a few of the better known systems we can see how some of the qualities of our system developed over the centuries.

One-to-One Correspondence



It seems logical to assume that one of the earliest numeration systems was the tallying system. The symbols consist of scratches or notches cut in sticks, and the rule is very simple—make a scratch for each object counted. This system is based on the concept of one-to-one correspondence between scratches and objects. Simple, but not very efficient for large numbers.





Simple Grouping

A diagram consisting of 15 vertical bars of increasing height, followed by a horizontal line with a right-pointing arrow and a semi-circular arc above it.

Position

▼
▼▼
▼▼▼
.....
▼▼▼▼▼▼▼▼▼▼
<
<▼
<▼▼
.....
<<<<<<▼▼▼▼▼▼▼▼▼▼
▼
▼▼▼
▼▼▼
.....
▼<<<▼

The Egyptian Hieroglyphic Numeration System dates back to about 3400 B.C. While this system is based on tallying, certain abbreviations are introduced. For example, ten scratches (staves) are replaced by one new symbol (heel bone), and ten heel bones are replaced by a scroll. While the system is still cumbersome, this simple grouping technique does permit a more efficient way of writing large numbers. It is interesting that the Egyptians chose their new symbols to represent powers of ten (10, 100, 1000,...).

The Babylonians (about 3000 B.C.) introduced some very modern features in their numeration system. They employed two symbols,  and , representing what we know as 10 and 1, respectively. Every number from 1 to 59 was written with a combination of these symbols and would look much like the Egyptian numerals if the symbols  and  were substituted for the Egyptian heelbone and staff.

The Babylonians decided to organize their system around groups of sixty. Instead of inventing a new symbol for sixty, they used the same symbol that they used for one, namely ▼. To distinguish between one and sixty, they invented a positional system. This positional notation was accomplished by leaving a space between the symbols for groups of sixty and the symbols for numbers less than sixty.

The symbols for 75, 692, and 3081 were written like this:



one group of sixty
and fifteen



eleven groups of sixty
and thirty-two

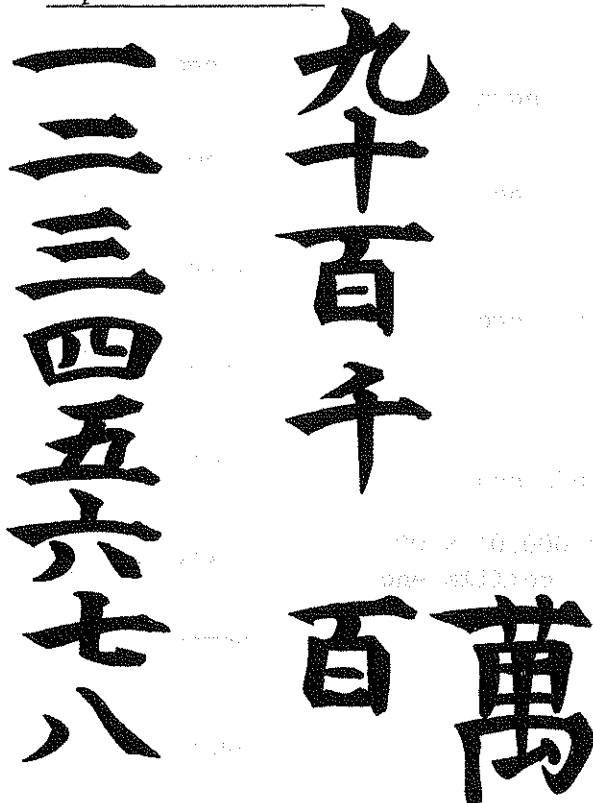


fifty-one groups of sixty
and twenty-one

Using this positional notation and grouping by sixty, they could write very large numbers. For example, the symbol $\nabla\nabla \lll \nabla\nabla\nabla$ would mean two groups of 3600 (sixty groups of sixty) and twenty-one groups of sixty and three, or what we would call 8463.

Ingenious as the Babylonian system was, it was not without its drawbacks. For example, the symbol $\nabla\nabla\nabla$ could represent three or 180 (three groups of sixty) or 10,800 (three groups of sixty groups of sixty), etc. Their system lacked a symbol equivalent to our zero. They had no way of writing, say, three groups of sixty and no units in their system. They had to decide upon symbol meanings by the context in which the number occurred. (The Babylonians did invent a symbol which corresponds to our zero by 600 B.C.)

Expanded Notation



The Chinese numeration system uses symbols which date back to 300 B.C. It was adopted by the Japanese at a later date. This system differs from the previous systems because it employs nine basic symbols and special symbols for ten and powers of ten. While this Chinese-Japanese system groups by tens, it does not, strictly speaking, use a positional notation. This system uses what we might call expanded notation. It would be like ours if instead of writing 473, for example, we wrote

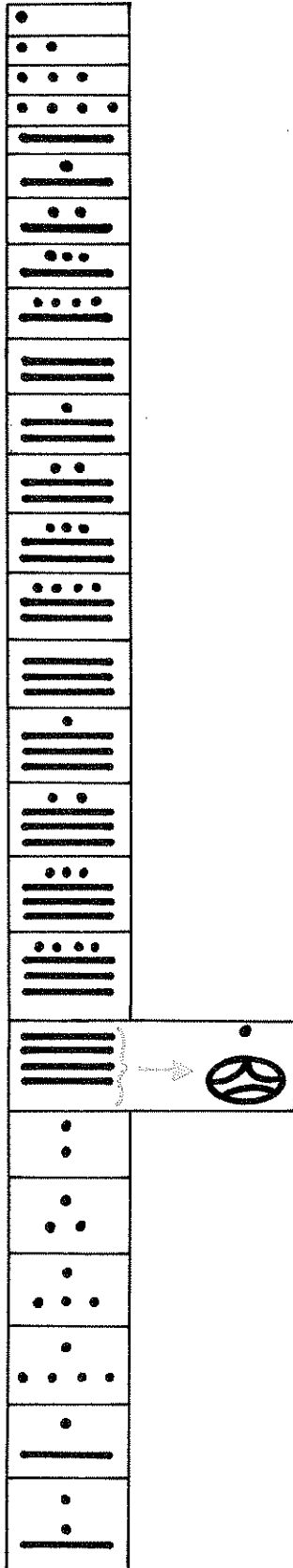
$$\begin{array}{r} 4 \times 100 \\ 7 \times 10 \\ 3 \end{array}$$

They would write 473 as




$$\left. \begin{array}{c} \text{四} \\ \text{百} \\ \text{七} \\ \text{十} \end{array} \right\} \begin{array}{l} 4 \times 100 \\ 7 \times 10 \end{array}$$


$$\text{三} \quad 3$$

Position and Zero



etc.

The Mayan culture of Yucatan and Central America began its great advance around 400 B.C. Mathematics was important to their society, and they invented a sophisticated numeration system which utilized positional notation and a special symbol for zero. By using combinations of dots (ones) and lines (fives) they wrote the numbers from one to nineteen. They then grouped by twenty and wrote their symbols in a vertical fashion like the Chinese-Japanese system. Unlike the oriental system, however, they did not need new symbols to write larger numbers because the position of their symbols had certain values. They represented the numbers 20, 43, and 187 as , , and , respectively.

The symbol in the top position represents groups of twenty, while the bottom symbol represents the numbers zero through nineteen. Notice the importance of zero, , as a placeholder.

The Mayans developed a numeration system to facilitate calendar calculations. The Mayan "year," eighteen months of twenty days each, affected their place values. For example, the symbol below meant six groups of 360 (or 18×20) plus zero groups of twenty plus three.



The Mayan system used place value and a zero symbol. For calendar notation they grouped by 20, 18×20 , 18×20^2 , ... instead of by 20, 20^2 , 20^3 , ...

In a sense it is unfair to treat these numeration systems so briefly. Each of the systems took many centuries to develop, and each system did become more sophisticated as its shortcomings were realized, and appropriate changes were made. They do serve to illustrate, however, what an ingenious system we have today-- a system which developed over centuries and had many impressive

predecessors. Interestingly enough, very little is known about the origins of our Hindu-Arabic system, the evolution of its symbols, or the influence of the other ancient numeration systems on its development. Historians have placed the final development of the Hindu-Arabic system, with use of zero and place value, between 400 A.D. and 700 A.D.--only 1200 to 1500 years ago.

COUNTING SHEEP

Motivation
Numeration
WHOLE NUMBERS



Students are divided into groups with differing tools for counting.
Group 1--15 small pebbles and 10 big ones.
Group 2-- Modeling clay and a knife.
Group 3--2 pieces of rope for knotting.
Group 4--15 seeds of 2 kinds, or colors.

NOTE: 52 could be shown in several ways:

or $\bigcirc = 50, \bigcirc = 10, 5 = 1$, etc.

Group 5--A stick and a felt tip pen.

Group 6--A large bundle of sticks and rubber bands.

Problem I: Show how many sheep using the materials you were given.
Demonstrate your method to the class.

Problem II: Decide which polygon has the most vertices. Demonstrate your method to the class.

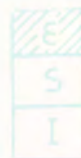
Follow up activities:

- Make up other counting projects. Have the students change groups, so each student tries each method.
- Discuss advantages and disadvantages of different "counting" methods.

BODY COUNTING SYSTEMS

TEACHER PAGE

Motivation
Numeration Systems
WHOLE NUMBERS



BODY COUNTING SYSTEMS OF NUMERATION ARE USED BY A LARGE NUMBER OF PAPUAN AND NEW GUINEAN GROUPS. EACH NUMBER IS INDICATED BY THE COUNTER POINTING TO, TOUCHING, OR CALLING THE NAME OF A BODY PART. MOST SEEM TO HAVE AN ODD NUMBER AS THEIR BASE WITH THE SAME BODY PARTS ON EITHER SIDE OF THE NOSE OR FOREHEAD BEING INDICATED (IN REVERSE ORDER) ON THE WAY UP AND THE WAY DOWN.

THE EAST AND WEST KEWA SYSTEM WHICH USES BASE 47 IS GIVEN BELOW:



1	little finger	47
2	ring finger	46
3	middle finger	45
4	index finger	44
5	thumb	43
6	heel of thumb	42
7	palm	41
8	wrist	40
9	forearm	39
10	large arm bone	38
11	small arm bone	37
12	above elbow	36
13	lower upper arm	35
14	upper upper arm	34
15	shoulder	33
16	shoulder bone	32
17	neck muscle	31
18	neck	30
19	jaw	29
20	ear	28
21	cheek	27
22	eye	26
23	inside corner of eye	25
	between eyes	
	..	
	24	



IN THIS SYSTEM YOU SHOW NUMBERS GREATER THAN 47 IN THIS WAY: 50 WOULD BE ONCE AROUND THE BODY (47) AND 3 MORE, OR IN KEWAN THEY WOULD SAY, "ONE PAAPU AND MIDDLE FINGER."

SPE: Bulletin Board/Transparency

IDEA FROM: "The Original Counting Systems of Papua and New Guinea," *Arithmetic Teacher*, Feb. 1971

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ATTIC GREEK

Finding Patterns
Ancient Numeral
Numeration Systems
WHOLE NUMBERS



NUMERATION SYSTEM--A DISCOVERY APPROACH

Can you break this code?

Hindu-Arabic	Attic-Greek
1	I
3	III
4	IIII
6	ΓI
8	ΓIII
17	ΔΓII
19	ΔΓIII
26	ΔΔΓI
34	ΔΔΔIII
37	ΔΔΔΓII
51	ϞI
53	ϞIII
57	ϞΓII
66	ϞΔΓI
73	ϞΔΔIII
89	ϞΔΔΔΓIII
216	HHΔΓI
325	HHHΔΔΓI
347	HHHΔΔΔΓII
402	HHHHII



COMPLETE THE TABLE BELOW

Hindu-Arabic	Attic-Greek
1	I
5	Γ
10	Δ
50	Ϟ
100	H
500	ϞϞ
1000	X

Before giving the correct answer,
let students use their imagination.

The correct answer is listed in blue
in the table.

Here are some possible student
suggestions for 500:

ϞϞ or ϞϞϞ

IDEA FROM: "A Discovery Approach with Ancient Numeration Systems," *Arithmetic Teacher*,
Nov. 1972

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ROMAN NUMERATION SYSTEM

A DISCOVERY APPROACH

CAN YOU BREAK THIS CODE?

Finding Patterns
Ancient Numerals
Numeration Systems
WHOLE NUMBERS



HINDU-ARABIC	ROMAN
1	I
3	III
4	IV
5	V
7	VII
9	IX
10	X
11	XI
18	XVIII
39	XXXIX
45	XLV
50	L
56	LVI
72	LXXII
97	XCVII
99	XCIX
100	C
219	CCXIX
359	CCCLIX
370	CCCLXX
423	CDXXIII
500	D
611	DCXI
755	DCCLV
879	DCCCLXXIX
974	CMLXXIV
1000	M
1327	MCCCXXVII
1975	MCMLXXV
2498	MMCDXCVIII
5000	\overline{V}
10 000	\overline{X}
50 000	\overline{T}



COMPLETE THE TABLE BELOW

HINDU-ARABIC	ROMAN
1	I
5	V
10	X
50	L
100	C
500	D
1000	M
5000	\overline{V}
10 000	\overline{X}

SINCE 5000 EQUALS \overline{V} ,

WHAT DO YOU THINK

\overline{T} WOULD EQUAL? _____

WHAT SYMBOL IS USED

FOR 1000? _____

SECRET ROMAN MESSAGE

Ancient Numerals
Numeration Systems
WHOLE NUMBERS



A = XVI = 16	M = MCMLXXIV = 1974
C = CLXXVI = 176	N = LV = 55
D = XLIX = 49	O = CCLVII = 257
E = XCIX = 99	Q = LIII = 53
G = XLIV = 44	R = CDXIX = 419
H = MCCCXXIV = 1324	S = DCCXLIII = 743
I = DCCCXLII = 842	T = CMXCIX = 999
K = LXXIX = 79	U = CDLXII = 462
L = XXXVI = 36	W = MMCDXCIV = 2494

WHO WAS JULIUS CAESAR?

DECODE THIS MESSAGE USING THE CODE ABOVE:

999	1324	99
T	H	E

419	257	1974	16	55
R	O	M	A	N

176	257	55	53	462	99	419	257	419
C	O	N	Q	U	E	R	O	R

2494	1324	257
W	H	O

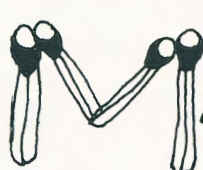
2494	16	743
W	A	S

1324	16	842	36	99	49
H	A	I	L	E	D

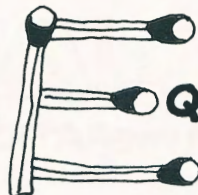
2494	1324	842	36	99
W	H	I	L	E

1324	99
H	E

419	99	842	44	55	99	49
R	E	I	G	N	E	D

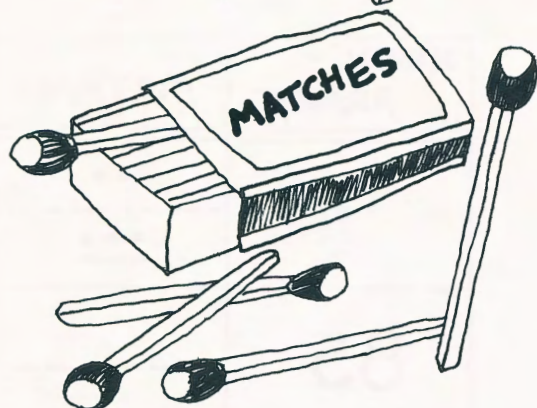


MATCHSTICK



QUATIONS

Ancient Numerals
Numeration Systems
WHOLE NUMBERS



HERE ARE SEVERAL EQUATIONS WHICH ARE ALL FALSE AS THEY STAND. EACH ONE MAY BE MADE TO BE CORRECT BY CHANGING THE POSITION OF ONE AND ONLY ONE MATCHSTICK.

① $III + II = V$

III + II = V

③ $VI + IV = X$

VI + IV = X

⑤ $VI = X - IV$

VI = X - IV

⑦ $IX - VI = III$

IX - VI = III

② $I = III - II$

I = III - II

④ $IV + I = V$

IV + I = V

⑥ $XI = II$ (Tricky)

XI = II (Tricky)

⑧ $V = IV + I$

V = IV + I

⑨

MOVE TWO MATCHES TO FORM A TRUE EQUATION.

$XI - V - I = V$

XI - V - I = V

⑩

CORRECT THIS ONE WITHOUT MOVING ANY MATCH.

Turn page upside down.

$XI + I = X$

Students might use toothpicks instead of matches.



MAYAN

NUMERATION SYSTEM

A DISCOVERY APPROACH

Finding Patterns
Ancient Numerals
Numeration Systems
WHOLE NUMBERS



HINDU-ARABIC	MAYAN
0	
1	.
4
5	—
6	— .
8	— . .
10	==
13	— . .
15	— . . .
17	— . . .
20	—
28	—
36	—
42	—

FILL
IN
THE
MISSING
NUMERALS



HINDU-ARABIC	MAYAN
48	— . .
85	— . .
100	— . .
173	— . .
227	— . .
246	— . .
356	— . .
360	— . .
757	— . .

CAN YOU EXPLAIN HOW THE MAYAN
NUMERATION SYSTEM WORKS?
YOU MAY WANT TO DO SOME RESEARCH.

TYPE: Paper & Pencil

This is a challenging
numeration system. For more
information see the commentary
to WHOLE NUMBERS: Numeration
Systems.



BABYLONIAN NUMERATION SYSTEM

A DISCOVERY APPROACH

FILL IN THE MISSING NUMERALS

Finding Patterns
Ancient Numerals
Numeration System
WHOLE NUMBERS



HINDU-ARABIC	BABYLONIAN
0	⌘
1	▼
7	▼▼▼▼▼
10	◁
24	◁◁ ▼▼▼
27	◁◁ ▼▼▼▼▼
30	⌘◁
43	⌘⌘ ▼▼▼
48	◁◁ ▼▼▼▼▼
52	◁◁ ◁▼▼
59	◁◁ ◁▼▼▼▼



HINDU-ARABIC	BABYLONIAN
60	▼ ⌘
73	▼ ◁▼▼
95	◁◁ ▼▼▼
174	▼▼ ⌘◁◁ ▼▼▼
200	▼▼▼ ◁◁
321	▼▼▼ ◁◁▼
683	◁▼ ◁◁ ▼▼▼
732	◁▼▼ ◁▼▼
1365	◁◁▼ ⌘◁◁ ▼▼▼
3600	▼ ⌘ ⌘
8411	▼▼ ◁◁ ◁▼
9000	▼▼ ◁◁ ⌘

WHAT IS THE BASE FOR THIS PLACE VALUE SYSTEM? 60
HOW MANY DIFFERENT SYMBOLS DID THE BABYLONIANS USE?

TYPE: Paper & Pencil

CREATING A NUMERATION SYSTEM

TEACHER IDEA

Devising Numerals
Numeration Systems
WHOLE NUMBERS



Think about the various numeration systems used by people before us such as: Egyptian, Greek, Roman. Develop some symbols of your own so that you can have your own numeration system.

HOW ORIGINAL CAN YOU BE?

HINDU-ARABIC	YOUR SYMBOL
1	
5	
10	
50	
100	

HINDU-ARABIC	YOUR SYMBOL
500	
1000	
5000	
10,000	
(other)	

Use your symbols to show the following numbers:

2 = _____ 6 = _____ 17 = _____
103 = _____ 796 = _____ 3,624 = _____

MAKE A SECRET TEN-WORD MESSAGE FOR A FRIEND

These directions are a bit involved. You may need to review them with the class.

On a separate sheet of paper make boxes like the ones below. Write a message for a friend in your boxes, one word in each box. Select any number at random to write below each word. (Be sure not to use the same number twice.)

WORD	→					
NUMBER	→					
(Hindu-Arabic)						

Now, on another sheet of paper make boxes like the ones above to give to your friend.

In the bottom part of the boxes write the numbers you selected in YOUR OWN NUMERATION SYSTEM. Leave out the words.

Directly below this, make more boxes. In these boxes you will write your message, each word with its corresponding HINDU-ARABIC number, but don't write the message in order.

Give your message to a friend with these directions:

1. Study "my" numeration system on the top of this page.
2. Match the numbers in my numeration system with the Hindu-Arabic numbers. Write in the appropriate word for that number.

NUMBER SENSE AND ARITHMETIC SKILLS

Placement Guide for Tabbed Dividers

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Chip Trading	244	Place value Bases 2-10	Manipulative Game
Activity Cards - Base 10 Multibase Blocks - I	247	Place value Expanded notation	Manipulative
Chip Toss	249	Place value Base 10	Game
2-Place Value Game	250	Place value Base 10	Game
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Counting in Japanese	258	Reading and writing	Activity Transparency
Appropriate Approximation	261	Rounding	Transparency Discussion
Rounding Cubes	262	Rounding	Game Manipulative

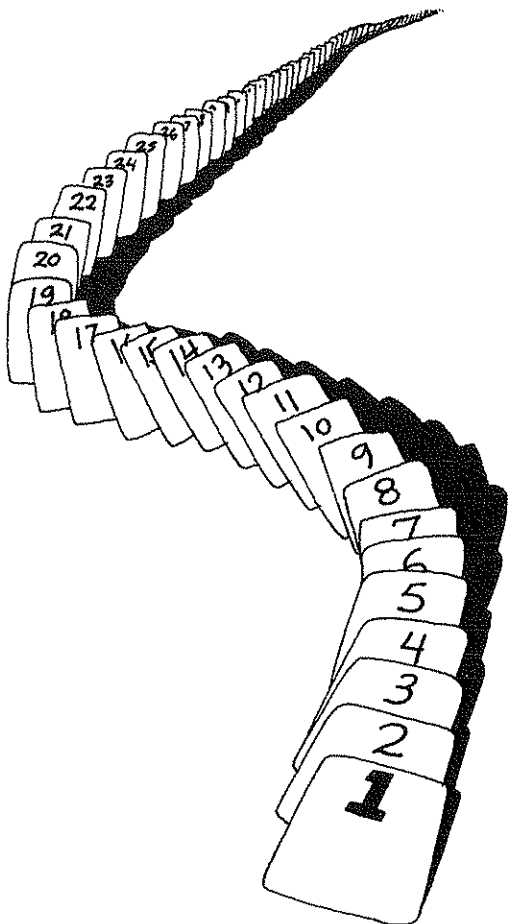
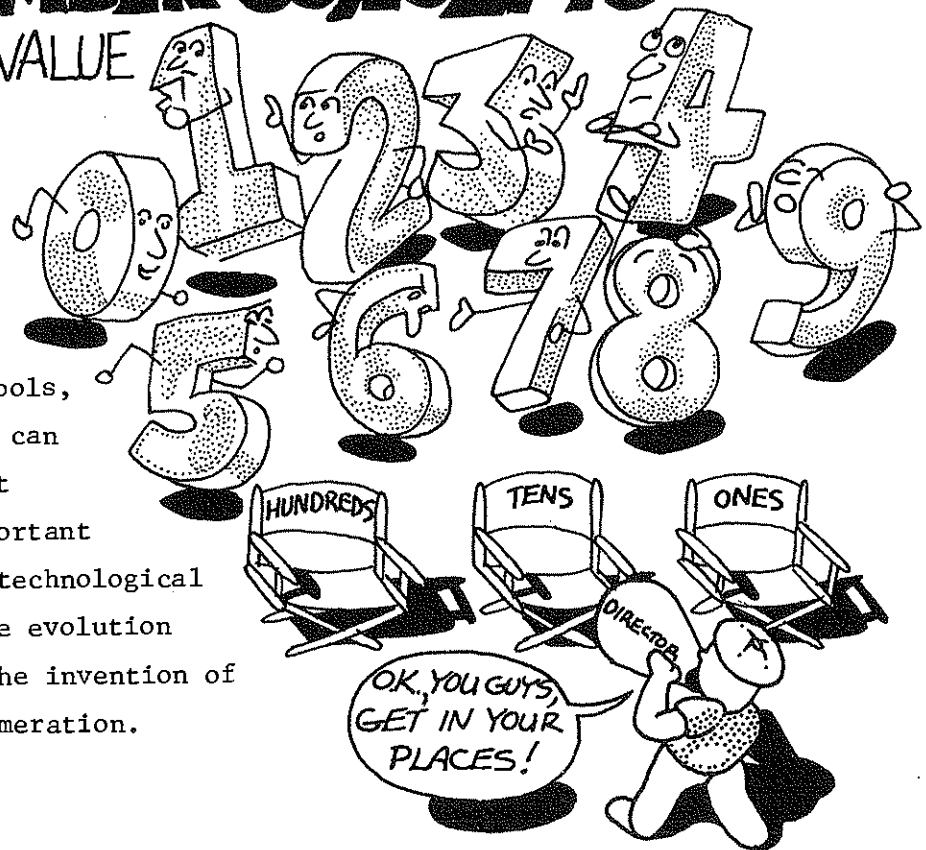
<u>TITLE</u>	<u>PAGE</u>	<u>TOPIC</u>	<u>TYPE</u>
Earth Facts	263	Rounding	Paper and pencil
Deorr is in "Order"	267	Ordering	Activity
Population	268	Ordering	Paper and pencil
Laying it on the Line	272	Ordering Number line	Activity
Time Flies	273	Ordering Number line	Activity Bulletin board
Greater Than or Less Than	274	Ordering	Activity
In-between	275	Ordering	Paper and pencil Transparency
Give or Take	276	Approximation	Paper and pencil

WHOLE NUMBER CONCEPTS:

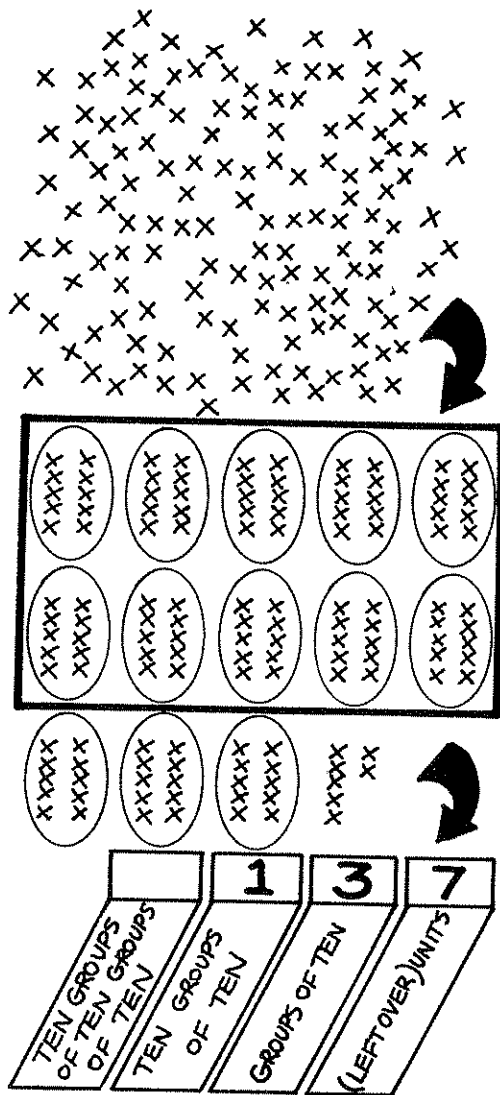
PLACE VALUE

The Hindu-Arabic system of numeration was an ingenious invention. With only ten basic symbols and the proper positioning of those symbols, any number can be named. There can be no doubt that this convenient numeration system played an important role in the development of our technological society. As we have seen in the evolution of numeration systems, it was the invention of place value that streamlined numeration.

WHAT IS PLACE VALUE?



Most people go through two phases in learning numeration. The first begins at home before children go to school. This might be called the "what comes next" approach. Children begin by learning to chant a sequence of unrelated words (one, two, three, . . . twelve). The next few words in the sequence have a pattern slightly related to the first words they learned (thirteen, fourteen, fifteen, . . . nineteen). Finally, a more reliable pattern emerges (twenty-one, twenty-two, . . . thirty-one, thirty-two, . . .). Later on they learn to replace these words with number symbols. When counting objects, students using this approach might announce that there are "137." When asked about the significance of the "3" in "137," there most likely will be no response. They can count and use the correct numerals, yet do not understand place value.



The second phase in numeration usually begins in the early grades and stresses the idea of grouping. It is this approach that clearly exemplifies the attributes of the Hindu-Arabic system (organizing by groups of ten, utilizing only ten basic symbols and place value). Using this approach, we can write the symbol for any number of objects without counting beyond ten. By partitioning objects into groups of ten, ten groups of ten, ten groups of ten groups of ten, etc. and always recording the results in a certain position, we obtain the same result as we would with the counting approach. Now, when asked the significance of the "3" in "137," we can see that it refers to the number of groups of ten. That is, each digit in a number symbol takes on a value because of its position in that symbol.

Both of these views of numeration are learned, and both are important. Students seem to be more adept at using the first view because it occurs more frequently in everyday life. However, an understanding of the second is essential for extending learning in mathematics. Could many students write the number seven hundred thousand six without understanding place value? Could they read the number 60,060 without the concept of place value?

It has been found that many, maybe even a majority, of middle grade students use counting when working addition, subtraction, and multiplication exercises. For example, to add 7 and 9 many students start with the 7 and count "8, 9, 10, 11, 12, 13, 14, 15, 16." They keep track of their counting by using their fingers, pencil marks, or making motions in the air. Why don't they regroup so that $7 + 9$ becomes $6 + 1 + 9$ or six and one group of ten? Is it because their counting approach is more practical and familiar than the grouping approach?

The algorithms for addition, subtraction, multiplication and division depend heavily on grouping and place value concepts. These four computations (performed by seventh grade students) are not untypical of the kind of errors commonly found.

Can you diagnose the errors?

$$\begin{array}{r} 37 \\ +18 \\ \hline 415 \end{array}$$

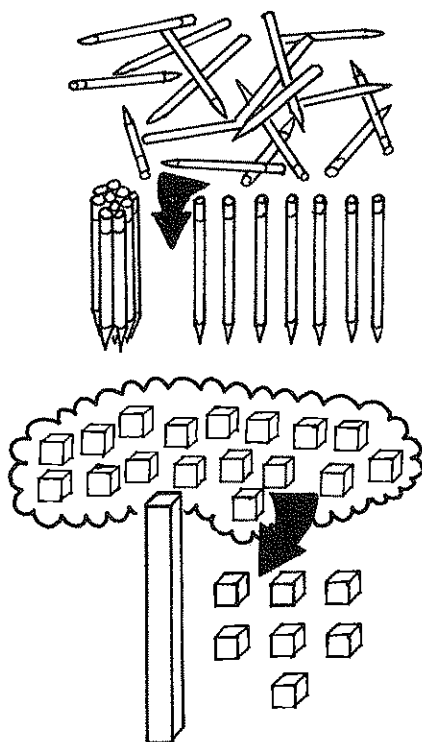
$$\begin{array}{r} 181 \\ \times 3 \\ \hline -32 \\ \hline 151 \end{array}$$

$$\begin{array}{r} 528 \\ \times 7 \\ \hline 496 \end{array}$$

$$\begin{array}{r} 40 \text{ R } 1 \\ 27 \overline{) 81} \\ \underline{8} \\ 01 \\ \underline{00} \\ 1 \end{array}$$

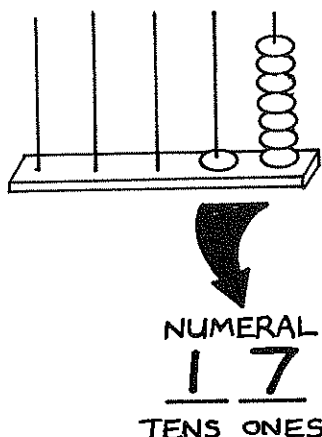
If students understood place value, they would make fewer errors of this kind. Faulty place value concepts also come back to haunt us when teaching decimals. How many students when comparing .23 and .8 write .23 > .8?

HOW DO WE HELP STUDENTS UNDERSTAND PLACE VALUE?



Many objects need to be physically grouped into bunches of ten.

A group of ten objects is traded for one object the same size as ten objects.



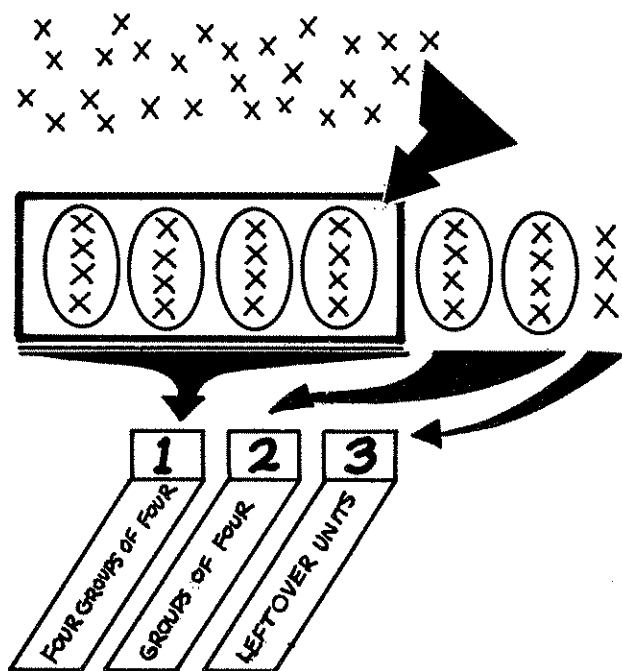
A group of ten objects is traded for one object the same size as a single object but in a different position.

What can be done to help students who have an incomplete understanding of place value? In some cases, a short paper and pencil activity will be sufficient, but more often than not the teacher will probably have to return to a concrete model to thoroughly remedy the situation. The choice of a model may depend upon the individual student--but which model should be chosen? Even concrete models vary in their degree of abstraction. The simplest models have students physically group objects (pencils, lima beans, etc.) into groups of ten, ten groups of ten, etc. The more abstract models (abacus, chip trading, etc.) have students group by ten, but then each group of ten is exchanged for a single object which represents it. These higher level models have a nice transition to writing numerals and also help bring together the counting and grouping approaches to numeration.

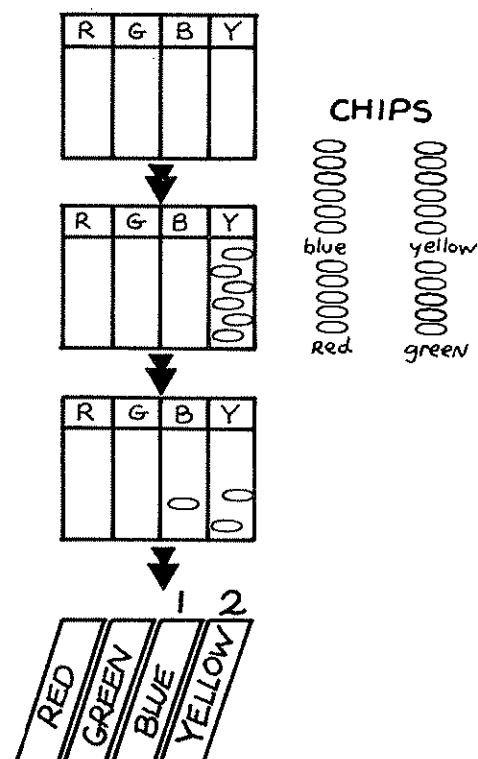
When using any one of these concrete aids to learn place value, we begin to see that place value is dependent upon how we group (by 3's, by 5's, by 10's, etc.). Grouping by different numbers is usually called "other bases," and, unfortunately, this topic has been abused by being studied and taught as an end-in-itself rather than as a facilitator to the understanding of place value.

WHY OTHER BASES?

The main goal for the use of modern nondecimal systems (systems like ours, only grouping by some number other than ten) is to give students a better understanding of their own base ten system. By approaching different bases through interesting activities which focus on those properties important to the base ten system (namely grouping and place value), this goal may be attained. However, if nondecimal systems become an end in themselves--just one more isolated topic to learn--they may just add confusion.



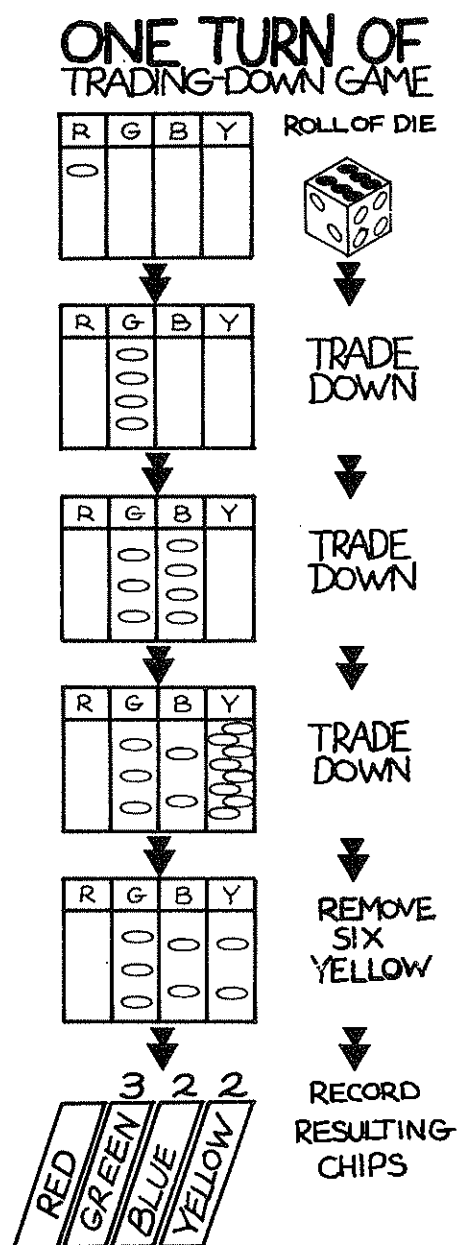
Consider, for example, the approach to different bases through chip trading activities. Using a four-colored mat, matching colored chips, and a die, a game can be played to see who gets a red chip first. The rules are simple: a trading ratio is determined (say 4 to 1); a player rolls the die and places that many yellow chips in the yellow column. Anytime a player accumulates four yellow chips, he trades them in for one blue chip (likewise, four blue are traded for one green, and four green for one red).



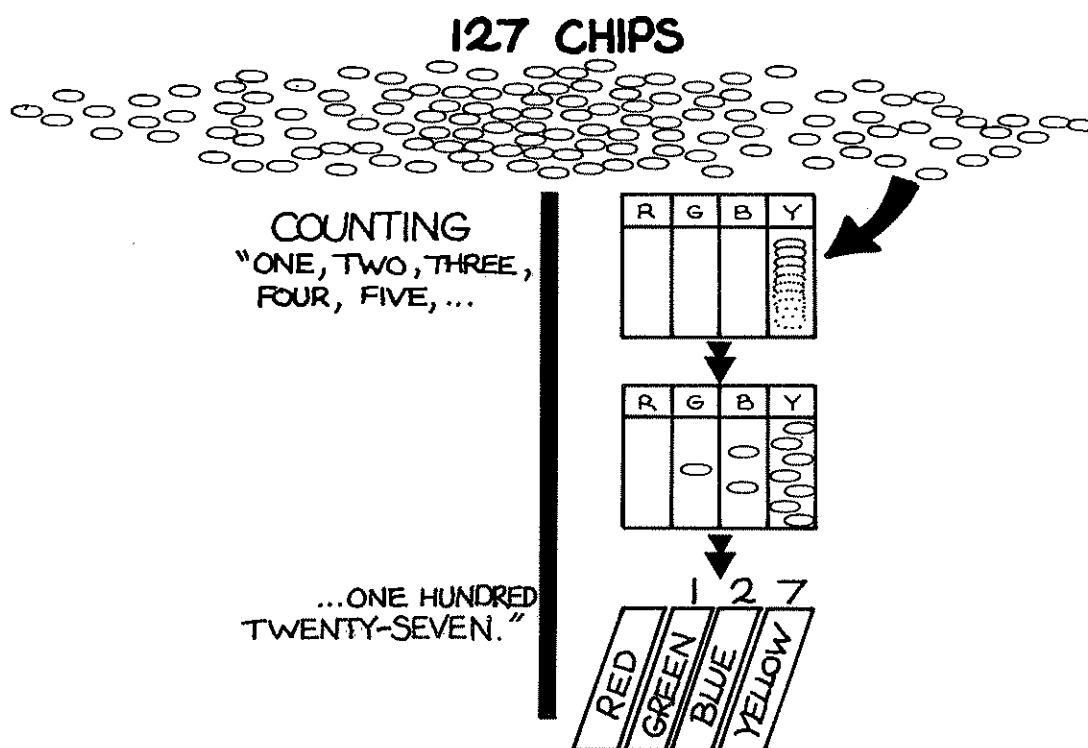
Notice the constant regrouping according to the given ratio (it could be 2 to 1, 7 to 1, 10 to 1, or any ratio). After the game rules have been established, students could be asked to record their chips after each move. This leads naturally into a place value notation for any given base. The fact that students are working in other bases does not even need to be mentioned. The important ideas are regrouping and the place value notation that arise naturally in a game context.

An extension of the first game (after it has been mastered using different ratios) is to reverse it. Start with one red chip and a given trading ratio (say 4 to 1), roll the die, and remove that many yellow chips. Of course, yellow chips cannot be removed until they have been created by trading down. After each move students could be asked to record their chips. Not only are they regrouping and using different base value notation, they are subtracting.

Thus, this simple and fun activity is utilizing different bases (very subtly), focusing on important numeration properties (regrouping and place value), and serving as a readiness activity for addition and subtraction. You may already see its implications for addition and subtraction.



The immediate payoff for this and similar activities, however, should be an understanding of regrouping and place value in our base ten system. In fact, this is a place where the "what comes next" or counting approach can be reconciled with the grouping approach in base ten numeration. Given a set of yellow chips, students can first be asked to count them and record the number. Then they can be asked to place the set of chips in the yellow column of their chip board, regroup using a ten to one ratio until they can no longer regroup, and record their totals in each column.



When counting, the students are reciting a pattern of number names which they associate in a one-to-one fashion with each of the chips. The final number name they recite is the number of chips in the set. When using the chip board, they need only count to ten, then trade. Thus, the focus is on groups of ones, tens, hundreds, etc. and the corresponding numeral. Each place in the numeral refers to one size grouping (10's, 100's, 1000's, ...). The digit occupying that place tells how many groups of that size.

A "HUMAN" COUNTER

TEACHER DIRECTED ACTIVITY

Place Value
Base 2
Concepts
WHOLE NUMBERS



Six students line up beside one another close enough to nudge the person to their right with their elbow. Each student will need a piece of cardboard or tagboard with a 0 on one side and a 1 on the other.

The activity begins with all students displaying a 0 on their cards.

The counter works on three principles.

- (1) If a student is flipping from 0 to 1, she should flip her card and then make some sound (let the students decide). Everytime the student in the ones' position hears the sound or says it herself, she knows it is time for her to flip again.
- (2) If a student is flipping from 1 to 0 she should flip and then nudge the student to her right so that student will know it is time for him to flip.
- (3) The 'counter' will proceed slowly at first, with the person in the one's position flipping only when they hear the sound, and the other five flipping only when they are nudged.

The first few flips would go like this:

(If the students are facing the class, the action would proceed from right to left.)

0	0	0	0	0	1	flip ding		
0	0	0	0	1	flip ding	0	flip nudge	
0	0	0	0	1	1	flip ding		
0	0	0	1	flip ding	0	flip nudge	0	flip nudge
0	0	0	1	0	1	flip ding		

TYPE: Activity

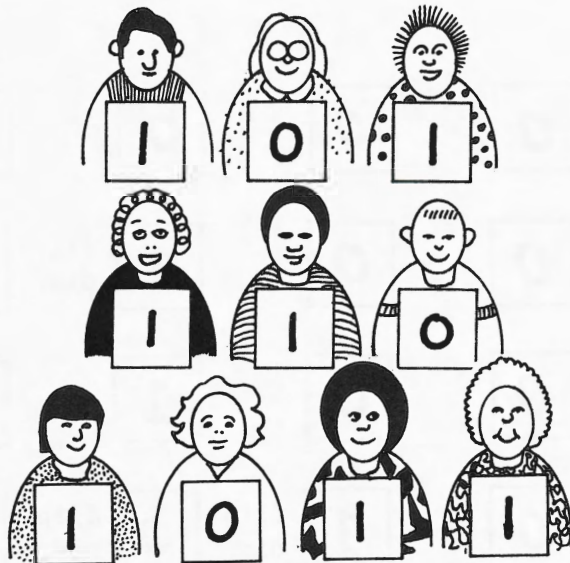
A "HUMAN" COUNTER (CONTINUED)

A CLASSROOM LESSON MIGHT INVOLVE:

1. Having the 'counter' slowly count with emphasis on 'carrying' to the next column - you may wish to write down the numbers on the board as they are being displaced.
2. Having other groups of students simulate the counting process by flipping the cards.
3. A contest to see which group can correctly count to the highest possible number.
4. When the students are familiar with the counting process, the place value concept of each column can be explained. This is a good introduction to base 2.
5. Have the 'counter' count and have a student call 'stop'. The class members then have to determine the value displayed on the counter.
6. Have the counter randomly flip with the class determining the number that is displayed.
7. Each class member can write a number on paper, figure out how the counter would display it.
8. Class members could call out their numbers and check to see if the counter can display it correctly.

EXTENSIONS:

1. The cards could be used to perform addition in base two. For example, three tall students could display 101_2 and three



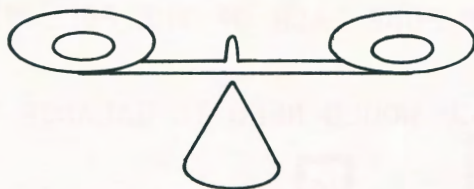
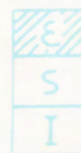
shorter students display 110_2 . Then four kneeling students could add and display the answer. Subtraction is somewhat more difficult than addition.

2. Since the machine language of a computer is base two, discussions on computers could follow or maybe precede this activity.



ACTIVITY CARDS - PAN BALANCE

Place Value
Base 2, 3, 5
Concepts
WHOLE NUMBERS



SEVERAL OBJECTS ARE TO BE WEIGHED USING A PAN BALANCE. THE ONLY WEIGHTS YOU HAVE AVAILABLE ARE ONE EACH OF THE FOLLOWING: 1 GRAM, 2 GRAMS, 4 GRAMS, AND 8 GRAMS.

WHAT IS THE HEAVIEST OBJECT YOU COULD BALANCE?

WHAT WEIGHTS WOULD YOU NEED TO BALANCE AN OBJECT WEIGHING 10 GRAMS?

FILL OUT THE TABLE BELOW TO SHOW HOW YOU WOULD BALANCE OBJECTS WITH THE GIVEN WEIGHTS.

8g

4g

2g

1g

WEIGHT OF
OBJECT (GRAMS)

				1
				2
				3
				4
				5
				6
				7
				8
				9
				10
				11
				12
				13
				14
				15

CARD 1

TYPE: Manipulative

IDEA FROM: *The Balance Book*

Permission to use granted by Activity Resources Company, Inc.

ACTIVITY CARDS - PAN BALANCE (PAGE 2)

FOR THIS ACTIVITY YOU HAVE FOUR EACH OF THE FOLLOWING WEIGHTS:
5 GRAMS AND 1 GRAM

RECORD HOW MANY OF EACH YOU WOULD NEED TO BALANCE THE
FOLLOWING OBJECTS,

5g

1g

WEIGHT OF OBJECT (GRAMS)

		1
		2
		3
		4
		5
		6
		7
		8
		9
		10
		11
		12
		13
		14
		15

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CARD 2

GIVEN: TWO EACH OF THE FOLLOWING WEIGHTS:
9 GRAMS, 3 GRAMS, AND 1 GRAM.

CARD 3

9g

3g

1g

WEIGHT OF OBJECT (GRAMS)

		1
		3
		5
		7
		9
		11
		13
		15
		17
		19
		21
		23
		25

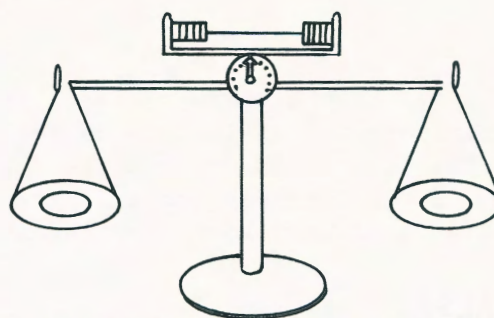
{ BALANCE
THESE
OBJECTS

IDEA FROM: *The Balance Book*

Permission to use granted by Activity Resources Company, Inc.

ACTIVITY CARDS - PAN BALANCE (PAGE 3)

A PAN BALANCE IS MADE UP OF TWO PANS OF EQUAL WEIGHT SUSPENDED FROM THE ENDS OF A FREE-SWINGING ARM.



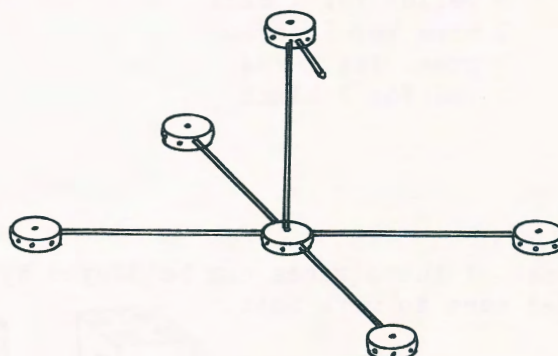
PAN BALANCES CAN BE USED TO INTRODUCE OTHER BASES AS WELL AS SHOW THE BASIC OPERATIONS ON WHOLE NUMBERS, FRACTIONS, AND DECIMALS.

PAN BALANCES CAN BE PURCHASED FROM THE DICK BLICK CO., CREATIVE PUBLICATIONS, AND SELECTIVE EDUCATIONAL EQUIPMENT (SEE), INC.

CONSTRUCTION INFORMATION:

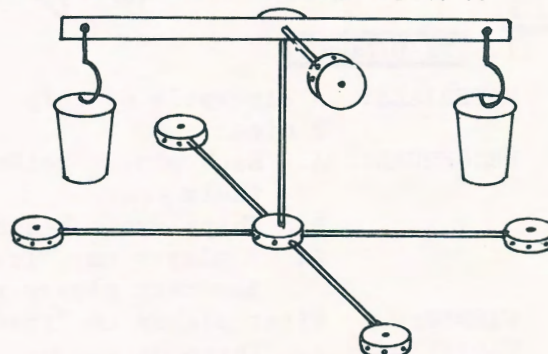
A PAN BALANCE CAN BE SIMPLY CONSTRUCTED A NUMBER OF WAYS. HERE IS ONE WAY THAT USES TINKER TOYS, S-HOOKS, A RULER (FOR A 3-RING BINDER), AND 2 PAPER CUPS.

ASSEMBLE A TINKER TOY STAND AS SHOWN:



PUT THE RULER ON THE STAND THROUGH ITS CENTER HOLE AND CLOSE WITH A JOINT (⊙).

PUT A S-HOOK THROUGH THE TWO END HOLES AND ATTACH A PAPER CUP TO EACH HOOK.



SAMPLE READINESS ACTIVITIES:

- 1) PLACE AN OBJECT IN ONE CUP. THEN MAKE THE "ARMS GO STRAIGHT" BY PLACING OTHER OBJECTS IN THE OTHER CUP.
- 2) PUT ONE OBJECT IN EACH CUP. DETERMINE WHICH OBJECT IS HEAVIER.
- 3) TRY TO FIND TWO OBJECTS WITH THE SAME WEIGHT. CHECK TO SEE IF THEY WILL BALANCE.

IDEA FROM: *The Balance Book*

Permission to use granted by Activity Resources Company, Inc.

CHIP TRADING TEACHER IDEAS

Place Value
Base 2-10
Concepts
WHOLE NUMBERS



All of the games and activities outlined below can be adapted to other laboratory materials. These include multibase blocks, lima bean cups and a chip abacus which are discussed in the section on Lab Materials.

3 for 1 trades (base 3) are used in all the games and activities, but all could be adapted for use with any convenient base, including base 10.

MATERIALS:

- 50-100 yellow markers
- 50-100 blue markers
- 25-50 green markers
- 10-25 red markers
- 5-10 black markers



This is for a classroom set. Markers can be poker chips or made from colored tagboard. (Do not use lightweight paper.)

INITIAL INSTRUCTIONS:

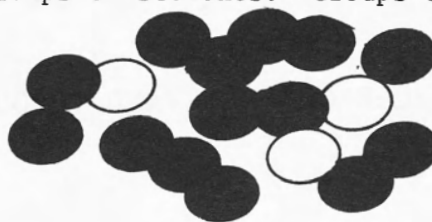
Announce to the students that they will be making 3 for 1 trades:

- 3 yellow for 1 blue
- 3 blue for 1 green
- 3 green for 1 red
- 3 red for 1 black



GROUPING:

Most of these games can be played by small groups of students. Groups of 3-5 seem to work best.



GAMES

I. **TRADING-UP**

MATERIALS: A stockpile of chips
2 dice

PROCEDURE: A. Each player rolls the dice and uses the product to determine their score.
B. Chips are taken from the stockpile equal in value to the score.
C. A player may "trade-up" only when it is his turn (before the next player rolls).

WINNER: First player to "trade-up" for a black chip.

VARIATIONS: A. Three dice - use sum or product of all three to determine score.
B. Two dice numbered 4-9 (use product).
C. Three dice, 2 red, 1 white. Use sum of red times white.
D. Three dice - same color.
i) take sum of any two times third.
ii) take product of any two minus third.
iii) use numbers other than 1-6.

TYPE: Manipulative/Game

IDEA FROM: *Chip Trading Activities*, Sets I-IV

Chip Trading Activities is a comprehensive program stressing whole number concepts and operations.

CHIP TRADING (PAGE 2)

II. **TRADING-DOWN**

PROCEDURE: Same as trading up except that:

- A. Each player begins with a black chip.
- B. A player's score determines how much he/she may return to the stockpile.

WINNER: First player to go out - return all his chips to the stockpile.

FOLLOW UP QUESTIONS FOR GAMES I & II.

- A. How many yellow could you get for a red chip; 2 green; a black?
- B. How many blue could you get for a red chip; a black chip?
- C. What is the total value in terms of yellow?
 - 1) 2 blue and 1 yellow
 - 2) 1 green and 2 yellow
 - 3) 3 red and 1 blue
 - 4) 1 black, 1 red, 1 green, 1 blue, and 1 yellow
 - 5) 1 black, 2 green, and 2 yellow



III. **WHAT'S IN THE BAG**

This is a game for 2 or more teams with 3-5 players on each team.

- PROCEDURE:
1. Choose one person to be the leader.
 2. The leader selects a few chips (10 or less) and places them in a bag making sure no one else sees them.
 3. The leader announces the total value of the chips in terms of yellow.
 4. Each team in turn makes a guess as to how many of each color chip are in the bag. Play continues until the actual chips have been guessed.

SCORING: 1 point - A guess with the same value.
3 points - The perfect guess.

EXAMPLE: The leader selects the following chips: 1 green, 3 blue, 4 yellow.
The value in yellows is 22.

GUESS	VALUE	SCORE
2 green, 4 yellow	22	1
3 green	27	0
7 blue, 1 yellow	22	1
2 green, 1 blue, 1 yellow	22	1
8 blue	24	0
1 green, 3 blue, 4 yellow	22	3



WINNER: Team with highest score at the end or, if you choose not to do any scoring, the team which makes the perfect guess.

VARIATION: Only 3 players - one chooses chips, one guesses, one checks values.

IDEA FROM: *Chip Trading Activities, Sets I-IV*

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CHIP TRADING (PAGE 3)

ACTIVITIES

I. FINDING ALL THE WAYS

The trades are 3 for 1. Have students find all the ways to have groups of chips with a value of:

- 17 yellow
- 31 yellow
- 45 yellow
- 70 yellow



Encourage students to record their answers and also to record the total number of chips used for each way.

EXAMPLE: 17 yellow

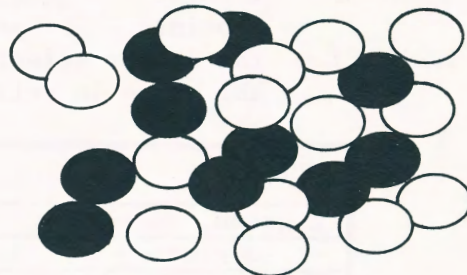
BLACK	RED	GREEN	BLUE	YELLOW	TOTAL CHIPS
				17	17
			1	14	15
			2	11	13
			3	8	11
		1	0	8	9
			4	5	9
		1	1	5	7
			5	2	7
		1	2	2	5

Students should be encouraged to observe that in each case the way that uses the least number of chips is also the only way which allows no further "trading-up." The tables also provide a ready means for a variety of patterns if the data is arranged in an order similar to the example.

II. FINDING CERTAIN WAYS (Trade 3 for 1)

Have students find:

- a) 4 chips worth 8 yellow
- b) 3 chips worth 13 yellow
- c) 2 chips worth 10 yellow
- d) 3 chips worth 21 yellow
- e) 2 chips worth 30 yellow
- f) 6 chips worth 52 yellow



III. FINDING LEAST NUMBER OF CHIPS (Trade 3 for 1)

Have students find the fewest number of chips with the following values and record their answers.

- a) 7 yellow
- b) 13 yellow
- c) 19 yellow
- d) 25 yellow
- e) 50 yellow
- f) 100 yellow



ACTIVITY CARDS - BASE 10 MULTIBASE BLOCKS - I

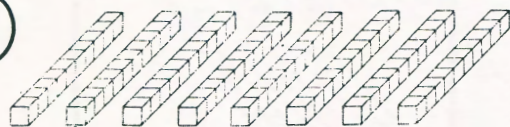
Place Value
Expanded Notation
Base 10
Concepts
WHOLE NUMBERS

E
S
I

MB-I-1

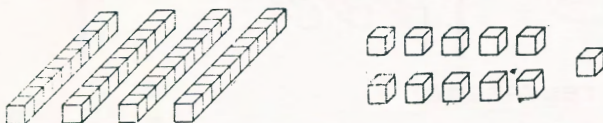
For information on the construction, use and readiness activities for Multibase Blocks see the section on Lab Materials.

A

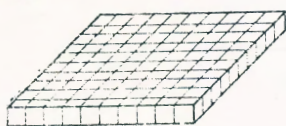


Fill in the blanks.

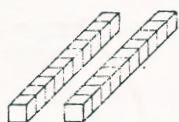
_____ LONGS + _____ UNITS



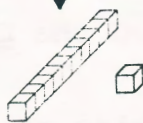
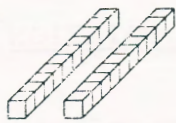
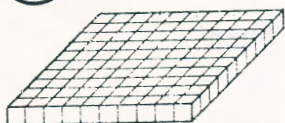
B



_____ FLAT + _____ LONGS + _____ UNITS



C



_____ FLAT + _____ LONGS + _____ UNITS =



A, B, and C are different ways of showing the same number. Can you explain why people say C is the "best" way?

Find three ways to represent 237 with the blocks. Record the number of flats, longs and units you used each time.

It uses the fewest possible pieces.

D

_____ FLATS + _____ LONGS + _____ UNITS

E

Answers will vary.

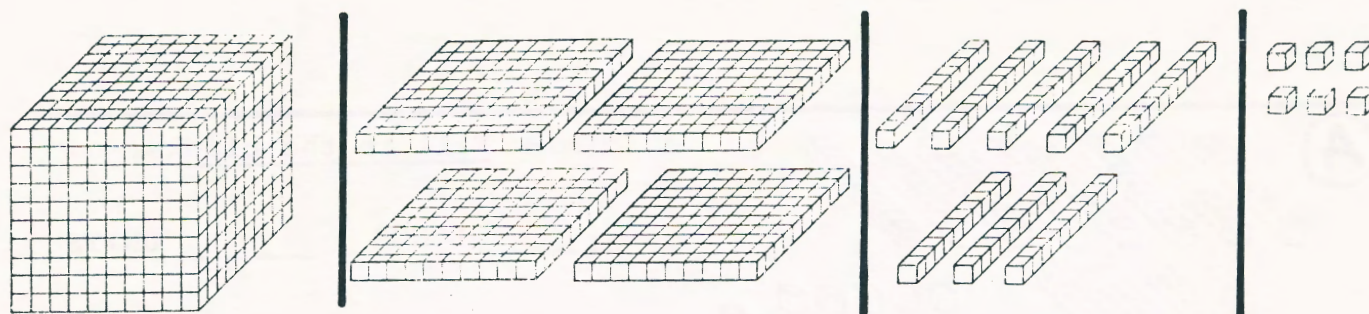
F

On a separate sheet record different ways to show each of these: 419, 65 and 132. Try some of your own.

TYPE: Manipulative

ACTIVITY CARDS - BASE 10 MULTIBASE BLOCKS - I (CONTINUED)

MB-I-2



What number do the above blocks represent?

1 block + 4 flats + 8 longs + 6 units = _____

Here is another way to write the number:

1 thousand + 4 hundreds + 8 tens + 6 units = _____

or

$(1 \times 1000) + (4 \times 100) + (8 \times 10) + (6 \times 1) = \underline{\hspace{2cm}}$

The last expression is called the EXPANDED NOTATION form of the number 1486.

Represent each of the following using the Multibase blocks. On another sheet of paper record your answers in two ways:

- (1) Write the number of blocks, flats, longs and units used to show the number. (Be sure you don't have more than 9 of any one shape.)
- (2) Write the number in EXPANDED NOTATION.

EXAMPLE: 3209

3 blocks + 2 flats + 0 longs + 9 units = 3209
or
 $(3 \times 1000) + (2 \times 100) + (0 \times 10) + (9 \times 1) = 3209$

(1) 452

(5) 1900

(2) 570

(6) 2487

(3) 300

(7) 4000

(4) 2040

(8) 9030

Try some more of your own.

CHIP TOSS

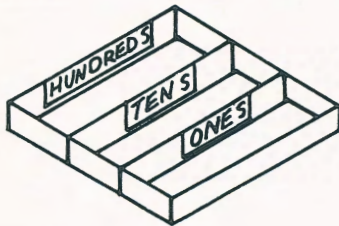
TEACHER PAGE

Place Value
Base 10
Concepts
WHOLE NUMBERS



MATERIALS: Shoe boxes, glue, chips (Any small object to throw will do: blocks, paper clips, etc.), tagboard, magic marker.

CONSTRUCTION: Glue together 3 shoe boxes. Make labels from tagboard to place over the edges of each box to label them. The labels should not be fixed permanently, so the game can be used with fractions, decimals, and integers as well as place value.



LABEL:



(Fix in place with paper clips.)

PLAYERS: 2-3

PROCEDURE: Set up a point from the boxes where all the players will stand when it is their turn. Each player throws 10 chips toward the boxes and totals up his score.

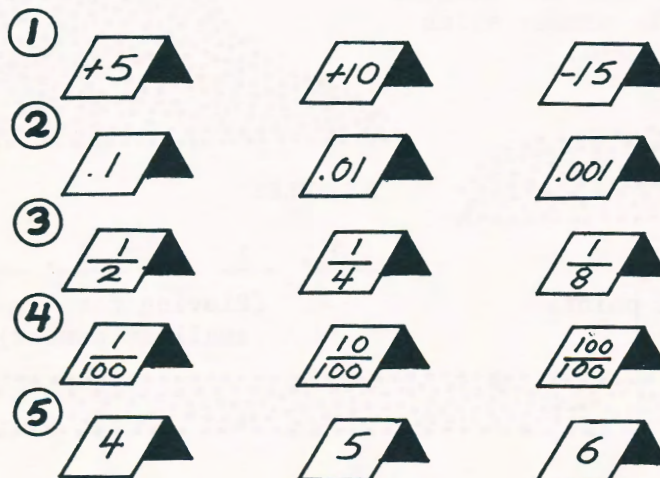
(Each chip that fails to fall in a box is counted as zero.)



The game ends in any of these ways: 1) one player reaches 1,000, 2) a set time limit is up, 3) a set number of throws has been completed by each player.

WINNER: The player with the highest score.

EXTENSIONS: This game may also be used for practice in multiplication and addition of integers, fractions, decimals, and whole numbers. For example:



The winner would be the closest to zero after a set number of throws.

TYPE: Game

IDEA FROM: C.O.L.A.M.D.A.

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2-PLACE VALUE GAMES

TEACHER IDEAS

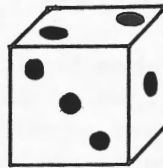
Place Value
Base 10
Concepts
WHOLE NUMBERS



GAME 1 Closest to 100

- 1 Tens Ones Have students make a chart like this on their paper.

2



The first player rolls the die seven times. After each roll the other players place the number shown on the die in one of their two columns.

- 3 The winner is the person whose sum is closest to 100. The sum must not go over 100. The winner rolls the die for the next game.

EXAMPLE:

Tens	Ones
	1
3	5
	6
2	
3	
<hr/>	
80	+ 18 = 98

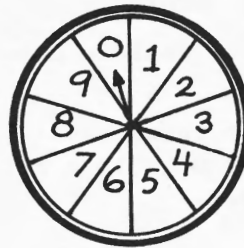
EXTENSIONS:

- 1) Decimals - Use columns labeled tenths and hundredths. Play closest to 1.
- 2) Use more than two columns and roll the die more than seven times.

GAME 2 Spin To Win

- 1 Have students make seven lines on their papers like this:

____, _____, _____, _____



2

Pick a student to spin the spinner seven times. After each spin the other students place that number on one of the seven spaces on their papers.

- 3 The winner is the person with the largest (or smallest) number. The winner spins the spinner for the next game.

EXTENSIONS: Put in a decimal point.

EXAMPLE:

0, 1 3 4, 2 4 9

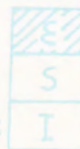
(Playing for smallest number)



GLANCES AND BLOWS

TEACHER IDEA

Place Value
Base 10
Concepts
WHOLE NUMBERS



Here is a logic game with many variations. It can be played by two people or used by the teacher with the entire class. The game is also called PICA-CENTRO.

PROCEDURE: Player A or the teacher writes down a 4-digit number on a piece of paper. Each digit must be different. The object of the game is for player B to guess the number in as few guesses as possible.

Each guess must be a 4-digit number.

After each guess by player B, player A must give certain clues. If one of the digits of the guess is in the correct position, player A must respond "1 blow." If one of the digits of the guess is correct but in the wrong position the response by player A is "1 glance."

Player B continues to guess until he has the exact number.

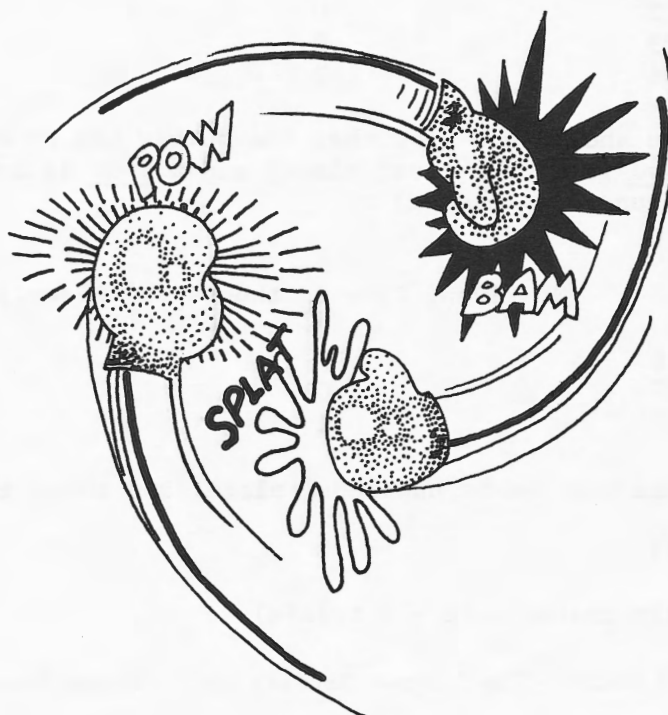
EXAMPLE: Player A writes the number 4178.

Player B guesses:

2369
2345
2145
1489
4187
4178

Player A responds:

No glances, no blows
1 glance, no blows
1 glance, 1 blow
3 glances, no blows
2 glances, 2 blows
4 blows



TYPE: Game

IDEA FROM: "Pica-Centro—A Game of Logic," *Arithmetic Teacher*, May 1972

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GLANCES AND BLOWS (CONTINUED)

- VARIATIONS:
- A) Present the game to the class and let them discover what is meant by a GLANCE or a BLOW. Begin by saying, "I'm thinking of a 4-digit number. Can you guess the number?" Respond to each guess by giving only the correct number of glances and blows. You may wish to start with a 3-digit or a 2-digit number.
 - B) Once students clearly understand the game, have them pair off. Give each player A the same 4-digit number and see which player B can find the number in the fewest guesses.
 - C) Have students try to find a strategy for guessing the number in 10 or fewer guesses. (Hint: First guess is 1111.)
 - D) Allow digits to be repeated.
 - E) Allow zero in any position.
 - F) Try with 2, 3, 5, 6, or more-digit numbers.

EXAMPLE OF STRATEGY:

Suppose the number you are trying to guess is 4729.

	GUESSES	GLANCES	BLOWS
1.	1111	0	0
2.	2222	0	1
3.	2223	0	1
4.	2244	2	0

(You should now know that the number has no ones or threes; a two is in the tens' place; and a four is in the thousands' or hundreds' place.)

5.	4525	0	2
	(No fives, four in thousands' place.)		
6.	4626	0	2
7.	4727	0	3
8.	4827	1	2

(Seven must be in hundreds' place, and there is no eight.)

9.	4729	0	4
----	------	---	---

(Only choice left - 9 trials)

IDEA FROM: "Pica-Centro—A Game of Logic," *Arithmetic Teacher*, May 1972

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Four-Place Numbers

GET 2 sets of Expanded Notation Cards

DO

- Use more than one card to-
 - make any 2-place number.
 - make any 3-place number.
 - make any 4-place number.

Record the cards used to make each number.

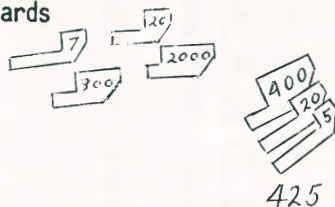
- Make the following numbers: 28 304 1742 2040 4006
Tell what cards you used for each.

- Make the largest 4-place number.
Make the smallest 4-place number.
Record cards used for each.

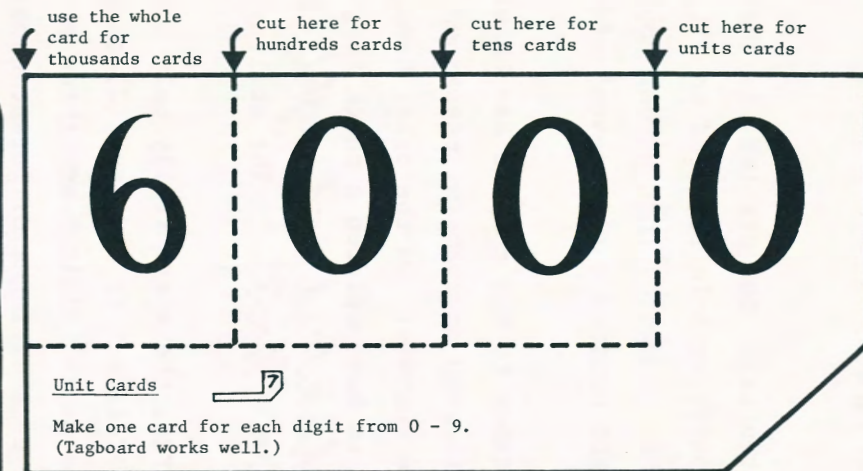
- Make these numbers and record the results:
 - the largest 2-place number with a 3 in the tens place.
 - the largest 3-place number with a 0 in the tens place.
 - the smallest 4-place number with a 2 in the ones place.

CHALLENGE

Select 2 "ones" cards, 2 "tens" cards, and 2 "hundreds" cards.
How many different numbers can you make using three cards at a time?
Record.



MAKING A SET OF EXPANDED NOTATION CARDS



Tens Cards

Make one card for each TEN from 10 - 90.

Hundreds Cards

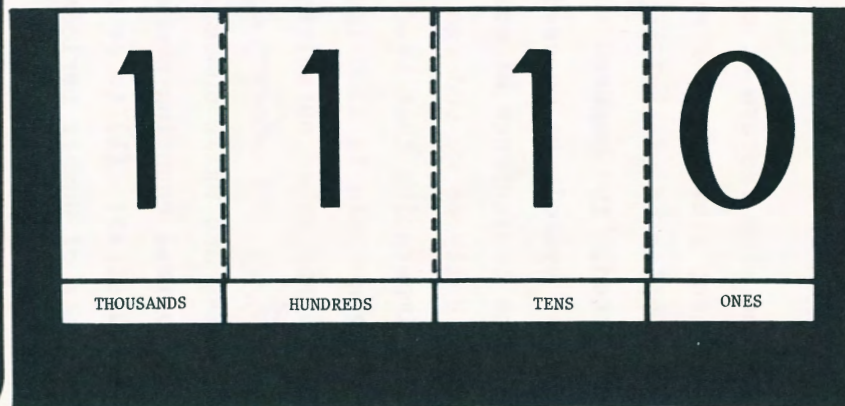
Make one card for each HUNDRED from 100 - 900.

Thousands Cards

Make one card for each THOUSAND from 1000 - 9000.

A CASE FOR YOUR CARDS

- Cut out a cardboard back $7\frac{1}{2}'' \times 3\frac{1}{4}''$.
- Cardboard front, $7\frac{1}{2}'' \times 1\frac{1}{2}''$, bend over the bottom $\frac{1}{4}''$ so the cards will fit in.
- Tape the front to the back, with a $\frac{1}{4}''$ margin of tape on each side.
- Mark off place values every $1\frac{3}{4}''$ as shown:



Tape

Expanded Notation
Base 10
Concepts
WHOLE NUMBERS



WHOLE NUMBER CONCEPTS:

READING AND WRITING

One nice feature of our number symbols, often taken for granted, is their convenient brevity. For example, a recent newspaper article referred to 231,887 square miles of mineral productive land in Alaska. In words that number would be written, "two hundred thirty-one thousand eight hundred eighty-seven"--fifty alphabet symbols as opposed to six digits!

The convenience of numbers and their importance to our culture can be witnessed daily as we undergo number bombardments through newspapers, magazines, etc. It is imperative that students learn how to read numbers! On the other hand, writing numbers is also important. How many of us have written a check for, say \$208.73 and asked ourselves, "Now, is it *two hundred and eight and $\frac{73}{100}$ or two hundred eight and $\frac{73}{100}$?"* The ability to both READ and WRITE numbers is important.

To read counting numbers, no matter how large, there are really only two prerequisites: (1) to be able to read any number from 1 to 999, and (2) to know the names of number periods. Beginning with the units' digit every three consecutive places in a numeral constitute a period, each having a different name. In numbers consisting of five or more digits the periods are separated by commas.

Given number  2 3 7, 1 4 6, 9 0 1
 2 3 7, 1 4 6, 9 0 1

Know period names  million thousand unit

Read the number (from 1-999)
 in each period followed by  (237) million (146) thousand 901
 the period name

Verbalize 
 "Two hundred thirty-seven million one hundred forty-six thousand nine hundred one."

Notice that the period name "unit" is not attached and that there are no "ands" in verbalization. Technically, "and" is used to read the decimal point, although in everyday language people often use "ands" in the middle of number names. Would you be tempted to use "and" while reading this number?

Write:

Number in each period:  seventy-two two hundred eight one

Adjoin period name: seventy-two million two hundred eight thousand one

one	one hundred one
two	.
.	.
.	.
nineteen	one hundred twenty
twenty	one hundred twenty-one
twenty-one	.
twenty-two	.
.	.
.	one hundred ninety-nine
.	two hundred
thirty-one	two hundred one
.	.
.	.
.	.
ninety-nine	
one hundred	

The rules for writing number names are quite easy after one learns how to spell and read numbers. Compound number names from twenty-one to ninety-nine are the only names that are hyphenated.

Many times students are interested in what comes after a million or what comes after a billion. This partial list of period names may be of interest to inquisitive students.

undecillions, decillions, nomillions, octillions, septillions, sextillions, quintillions, quadrillions, trillions, billions, millions, thousands, units,



Make a Million

Reading & Writing
Concepts
WHOLE NUMBERS



TEACHER IDEA

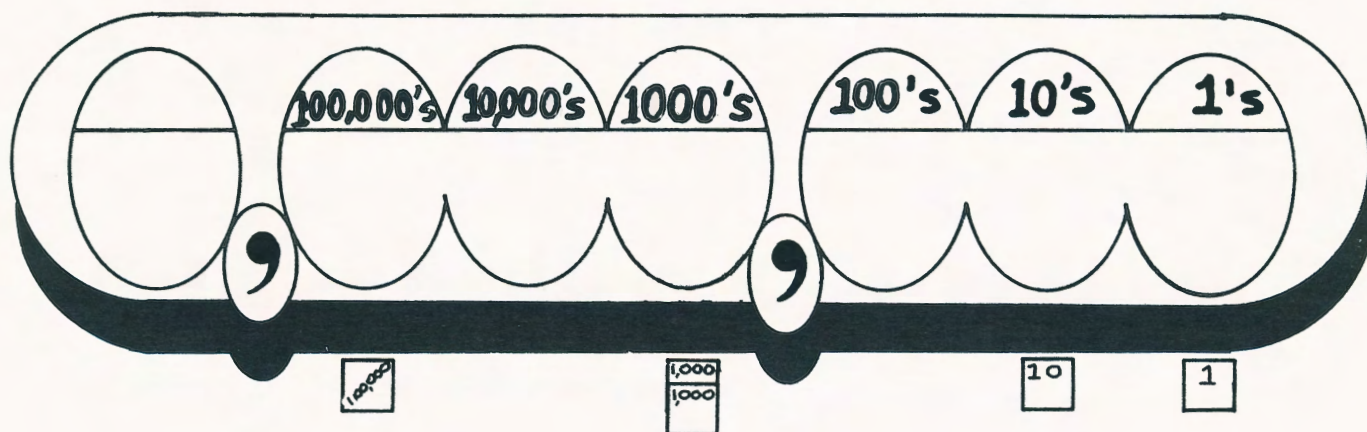
NUMBER OF PLAYERS: 2 TO 4

EQUIPMENT: BOX FOR A DICE CUP,

5 FOAM CUBES, EACH CUBE HAS A SIDE LABELED 1; 10;
100; 1,000; 10,000; 100,000

PROCEDURE: TO SEE WHO PLAYS FIRST EACH ROLLS ONE CUBE. THE PLAYER WITH THE HIGHER NUMBER GOES FIRST. IN CASE OF A TIE, THE PLAYERS WHO TIED ROLL AGAIN. EACH PLAYER IN TURN PLACES THE 5 CUBES IN THE DICE CUP, THEN EMPTIES THEM OUT ON THE TABLE. HE RECORDS THE TOTAL OF THE NUMBERS ON HIS PAPER (100,000 + 1,000 + 1,000 + 1,000 + 1 WOULD BE WRITTEN 103,001). THAT PLAYER MUST READ THE NUMBER. PLAY PASSES AROUND THE TABLE TO THE LEFT. ON THE FOLLOWING PLAYS EACH PLAYER RECORDS THE TOTAL AND ADDS IT TO HIS OTHER SCORE. FIRST PLAYER TO REACH OR PASS 1,000,000 WINS THE GAME.

A CHART LIKE THIS WILL HELP STUDENTS WHO NEED ASSISTANCE IN WRITING THE NUMERALS:



RECORDED AS 102,011

THE EMPTY PLACES SHOW WHERE TO WRITE 0.

TYPE: Game

IDEA FROM: MathLab

Permission to use granted by Action Math Associates, Inc.

THIRTY-TWO WORDS

TEACHER DIRECTED ACTIVITY

Reading & Writing
Concepts
WHOLE NUMBERS



- MATERIALS:** 1) Place the following 32 word names on a large chart, or display on the overhead one, two, three, . . . , nine, ten, eleven, twelve, thirteen, . . . , nineteen, twenty, thirty, forty, . . . ninety, hundred, thousand, million, billion, trillion.
- 2) Write each name on a 3 x 5 index card with large felt tip markers. Prepare the following amounts:



You will probably want to have students make these cards.

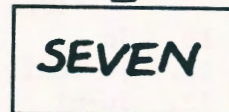
One, two, . . . , nine —————→ 10 cards each
 ten, eleven, . . . , nineteen, twenty, } —→ 5 cards each
 thirty, . . . , ninety, hundred }
 Thousand, million, billion, trillion —→ 1 card each

PROCEDURE: This activity can be for two students, a small group, or the entire class.

- 1) Place word names in a handy array.
- 2) Student A writes down a single digit number (not zero) on the chalkboard.
- 3) Student B chooses the correct word name. (Have the name displayed in a place where the whole class can see it.) The cards will stand up in a chalk tray.
- 4) Student B then adds one digit to the previous number. (It may be placed on either side - zero may now be used.)
- 5) Student A (or C) chooses the correct word name and displays the answer.

Example

7



73



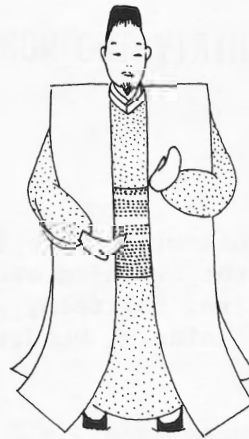
- 6) Keep going to 100 trillion, then start over.

TYPE: Activity

IDEA FROM: C.O.L.A.M.D.A.

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counting in
japanese



		(Pronounced)
1.	Ichi	ee-Chee
2.	Ni	Nee
3.	San	Sahn
4.	Shi	Shee
5.	Go	Go
6.	Roku	Ro-Koo
7.	Shichi	Shee-Chee
8.	Hachi	ha-Chee
9.	Ku	Koo

(Pronounced)								
10.	Ju	Joo	20.	Ni-Ju	30.	San-Ju	31.	San-Ju-Ichi
11.	ju-ichi		21.	Ni-Ju-Khi	40.	Shi-Ju	41.	Shi-Ju-Ichi
12.	Ju-ni		22.	Ni-Ju-Ni	50.	Go-Ju		.
13.	Ju-San		23.	Ni-Ju-San		.		.
14.	Ju-Shi		24.	Ni-Ju-Shi		.		.
15.	Ju-go			.		.		.
16.	Ju-Roko			.		.		.
17.	Ju-Shichi			.		.		.
18.	Ju-Hachi			.		.		.
19.	Ju-Ku			.		.		.

The top part of this page can be used as a transparency for motivation and general interest. To familiarize students with the words, you might ask questions such as: How much is Go plus Ju; Ju-san plus Ni-ju; Roku times Go; Hachi times San.

Suggest some changes which could be made in our own numbering system from 1 to 99 so that it might be as logical as the Japanese numerals above.

0	Zero	10	_____	Ty	20	_____	Twoty
1	One	11	_____	Ty-one	30	_____	Threety
2	Two	12	_____	Ty-two	40	_____	Fourty
3	Three	13	_____	.	50	_____	Fivety
4	Four	14	_____	.	60	_____	Sixty
5	Five	15	_____	.	70	_____	Seventy
6	Six	16	_____	The blue	80	_____	Eighty
7	Seven	17	_____	suggests only	90	_____	Ninety
8	Eight	18	_____	one possible			
9	Nine	19	_____	answer.			

WHOLE NUMBER CONCEPTS:

ROUNDING

Auto firms' engine choice
may cost buyer \$147 billion

Auto plant on the West Coast
in the South, and that once turned
out to be a mistake. Nearly 2,000
workers have been let go.

49,000,000 adult
Americans never exercise at all-
not even walking.

Airplanes top \$21 billion overrun
Weapons costs

with a \$4 million deficit
faced

Have you ever wondered why so many of the numbers you read and hear each day are so conveniently "rounded"? Why weren't 1,989 workers laid off? How did the county manage a \$4 million deficit and not \$4,121,242.37? Of course, we realize that most of these numbers are not exact but only approximations. In some cases the exact figures are not known, and in other cases it is really not necessary to know the exact number, even though that information is available. Rounded numbers give us an idea of magnitudes when greater accuracy is not necessary (\$21 billion as opposed to \$20,987,394.29). They are also much easier to comprehend and remember. Thus, for various reasons many of the numbers we see are rounded.

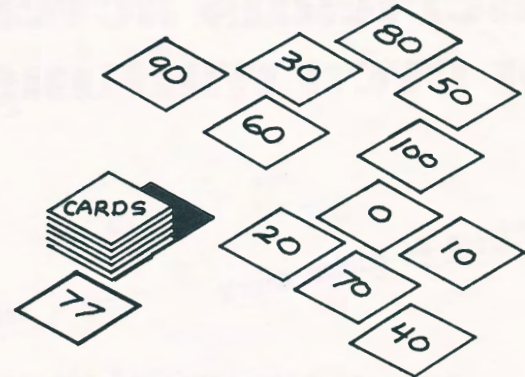
WHY TEACH ROUNDING?

Students should know that many numbers seen in print are obtained by rounding, but there are other good reasons why students should learn to round numbers. Rounding is a skill essential to making approximate calculations. (If we expect students to see that the answer to 42×28 is about 40×30 , they must know how to round.) Rounding is helpful in making quick comparisons. (There are about 215 million people in the United States, and 49 million don't "exercise at all." Thus, about $\frac{1}{4}$ of the population, $\frac{50 \text{ million}}{200 \text{ million}}$, doesn't exercise.) The process of rounding is used frequently when making measurements and learning to operate with numbers arising from measurement. If students have an understanding of rounding before the more difficult ideas of approximation and measurement are introduced, they will be able to learn these new concepts more readily.

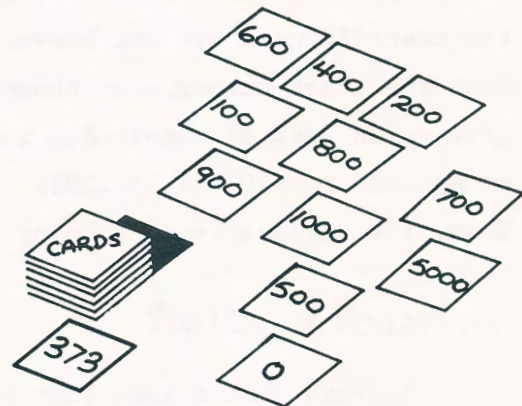
Interesting discussions can arise from questions about rounding. In everyday life, when is it permissible to round? Here are some possible starting points for discussion: Do you use rounding when planning for a party? How exact do we have to be when reporting school attendance to the office? Can you round on your income tax? For more such ideas see *Appropriate Approximations*.

A STARTING POINT

One way to introduce rounding is through a simple rounding game. In the Rounding to Tens game shown here, eleven cards are numbered by tens (0 through 100). A deck of cards with numbers between 1 and 100 is shuffled and placed facedown. When the top card is turned over, the players touch the tens' card which is the closest, lower multiple of ten, the closest, upper multiple of ten, or the closest multiple of ten (depending upon whether you are playing round down, round up, or round to nearest ten).^{*} The first player to touch the correct tens' card gets a point (or he may keep the card turned over from the top of the deck). A player touching an incorrect card forfeits a point to the player touching the correct card. After all cards have been played, the player with the most points or cards is the winner.



ROUNDING TO TENS



ROUNDING TO HUNDREDS

This game can be extended to Rounding to Hundreds as well as rounding fractions and decimals (see *The Almost Game* in FRACTIONS: Addition/Subtraction). The following student pages illustrate some ideas for extending, applying, and practicing rounding techniques.

^{*}There may be some active discussion when playing Rounding to Tens, and a card labelled, say, 25 is turned over. This may be a good time to have students decide what would be the best way to treat a number "in the middle."



APPROXIMATE?

APPROPRIATE XIMATIONS

Which of these
Can be exact and
Which can be approximate?

B+O LOW CLEARANCE 11'5"⁰⁴

They told
me that this
truck was about
11 feet high.
What happened?

Ladies and Gentlemen
-Tonight's attendance
is about
50,000 !!

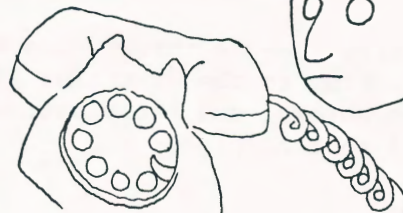
No! there
are 49,687
people in
attendance

I earned
\$7241 last year,
so I'll report \$7000
on my income tax

Lady, I realize
that it's 9:00, but
the train always
leaves at 8:59.

There are
1537 students
in our school,
but the paper
said that there
were 1540.

Let's see, my
phone number is
753-2143 (approximately
753 0000), so I'll dial
753-0000



Find some situations where it's appropriate to approximate. **F**ind some where you must be exact.

ROUNDING CUBES

Rounding
Concepts
WHOLE NUMBERS

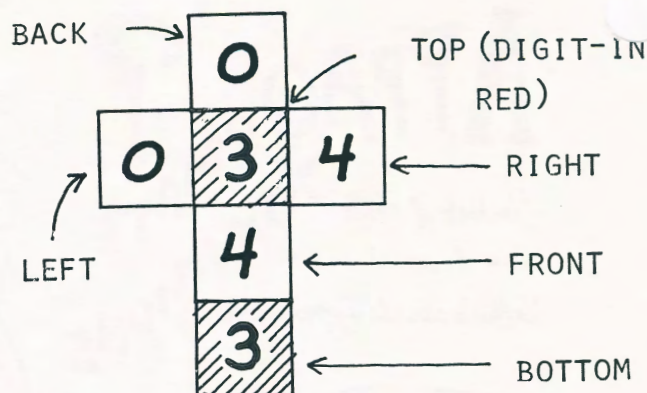


MATERIALS:

20 cubes (2 for each digit)

The cubes are to be marked as follows:

- 2 Faces - write the digit in red.
- 2 Faces - write the next higher digit in black (zero if the digit is 9).
- 2 Faces - write the digit zero in black.



EXAMPLE:



PROCEDURE: A student draws a specified number of cubes and arranges them with the red digits up to form a number. Another student or the teacher gives him a place value. The student manipulates the cubes so that his number is rounded to that place and reads the answer aloud.

EXAMPLE: Round to nearest 10



387

ROUNDS TO



390 (NEAREST 10)

- EXTENSIONS:
1. Extend to decimals by adding another cube with a decimal point on each face.
 2. Make two additional cubes marked as follows:

PLACE VALUE CUBE

- 2 faces - write 10
- 2 faces - write 100
- 2 faces - write 1000

SIZE CUBE

- 1 face - largest
- 1 face - 2nd largest
- 1 face - 3rd largest
- 1 face - smallest
- 1 face - 2nd smallest
- 1 face - 3rd smallest

A player draws 4 cubes. The size cube is thrown, and the player forms a number with the red digits up, according to the direction given on the size cube. The place value cube is then thrown, and the player manipulates his cubes so that his number is rounded to that place.

EXAMPLE: Player draws 7, 4, 1, 8.

Size cube comes up 2nd largest.

The player forms the number 8714 (the second largest number he can make with his cubes).

Place value cube comes up 100.

Player now shows 8700 with his cubes.

IDEA FROM: C.O.L.A.M.D.A.



EARTH FACTS

Rounding
Concepts
WHOLE NUMBERS



Earth's average diameter (distance through) is about 12,755 kilometres (km). Round this number to the:

- A. Nearest hundred. _____
- B. Nearest thousand. _____

If you went from New York through the center of the earth you would land just southwest of Perth, a small city in the southwest corner of Australia.

The distance around the earth from New York City to Perth, Australia is about 18,700 kilometres. Round this number to the:

- C. Nearest thousand. _____
- D. Nearest ten thousand. _____

Why is it shorter through the earth than halfway around it?



Earth's average circumference (distance around) is about 40,071 kilometres. Round this number to the:

- E. Nearest thousand. _____
- F. Nearest ten thousand. _____

What is the distance from New York to Perth and back to New York? _____

Round this number to the nearest ten thousand. _____

How does your result compare with the Earth's circumference, rounded to the nearest ten thousand? _____

Paper & Pencil

EARTH FACTS (CONTINUED)

The average distance from Earth to the Moon is 384,393 km. Round this number to the:

- G. Nearest hundred. _____
- H. Nearest thousand. _____
- I. Nearest hundred thousand. _____

What is the distance ten times around the circumference of the Earth?

Round this number to the nearest hundred thousand. _____

About how far is it to the Moon?

The average distance from Earth to the Sun is 149,593,690 kilometres.
Round this number to the:

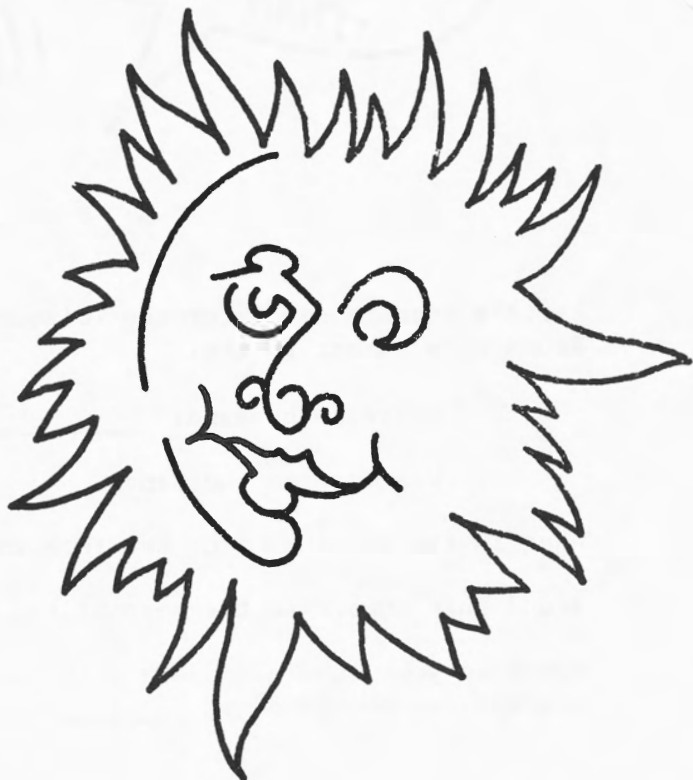
- J. Nearest hundred. _____
- K. Nearest hundred thousand. _____
- L. Nearest million. _____

The distance from Baltimore, MD. to Philadelphia, PA. is 154 kilometres.
Round this to the nearest ten.

Use your rounded figure to calculate the approximate distance you would travel in 500,000 round trips between the cities:

_____ x 1,000,000 =

Now, about how far is it to the Sun?



WHOLE NUMBER CONCEPTS:

ORDERING

Have you ever stopped to think about the ordered world in which we live? What is not ordered alphabetically is ordered numerically, and sometimes, as in shoe sizes, both systems are used. We order by size, weight, position and even by intangible attributes like beauty and quality. Even the letters of the alphabet have been ordered numerically according to their usage. (The nine most used letters in descending order of frequency are E T A O N I S R H.) We also use order in making everyday comparisons: "That piano won't fit through the door." "You're going over the speed limit!" "My dad's bigger than your dad." Order is very important in the study of mathematics. At this introductory level the foundations for ordering can be developed from the everyday ordering experiences of students.



USES OF ORDERING

Students should be encouraged to find examples of ordering in their "world." How are the "top forty" popular songs determined each week? How are basketball or football standings determined? Do house addresses make use of the order of numbers? How about telephone numbers? Students might work in groups and see which group can compile the longest list of ordering usages.

Other ideas that could be used to introduce ordering are in the classroom materials.

Introductory activities *Deorr is in Order*

Applications *Population*

Ordering and number line *Time Flies*

ORDER PROPERTIES

The TRICHOTOMY PROPERTY says that if m and n are whole numbers, then exactly one of the following holds: m is equal to n , m is greater than n , or m is less than n . In mathematical shorthand: $m = n$, $m > n$, or $m < n$. The message in this rather formal statement can be developed and understood in an interesting and enjoyable way. Suppose you start by playing the game of twenty questions. The teacher chooses an object in the classroom, and students must identify the object by asking no more than twenty questions--to which the teacher answers "yes" or "no." After a short discussion of game strategies it is natural to move to a similar game with numbers. (See the student sheet *Greater Than or Less Than*.) Someone picks a number between 1 and 100, and the others must identify that number in less than twenty questions. This game not only exemplifies the trichotomy property but provides an opportunity for problem solving. Is it possible to identify any number in less than 20 questions? less than 10 questions? What is the fewest number of questions needed to identify any number between 1 and 100? Between 1 and 1000?

Another property of ordering, the TRANSITIVITY PROPERTY, says that if one whole number is greater than a second whole number, and the second is greater than a third, then the first number is also greater than the third. In mathematical shorthand: if $p > q$ and $q > r$ then $p > r$. An awareness of this property can be developed easily by using relationships other than "greater than" or "less than." For example, replace ">" by "taller than": if person A is taller than person B, and person B is taller than person C, then A is taller than C. Try other relationships like "is the brother of," "is equal to" (for numbers), "is heavier than," or "is not equal to." Do all of these relationships satisfy the transitive property? Now, how about whole numbers and the relationship "greater than"?

At this level we are concerned with developing an awareness of the ordering properties through interesting activities and experiences. The names and formal statements of the properties would probably serve no useful purpose. If this awareness is started now and further developed when studying fractions, the students should have little trouble later on when, and if, it is necessary to formalize the ideas.

DEORR IS IN "ORDER"

TEACHER DIRECTED ACTIVITIES

Ordering
Concepts
WHOLE NUMBERS



- I. A group of students might enjoy ordering themselves by height, length of hair, age, etc.

If they choose something like ages, you might have them arrange themselves before giving their birthdates and compare this with the actual results.

- II. Numbers can be ordered by using a variety of different rules.

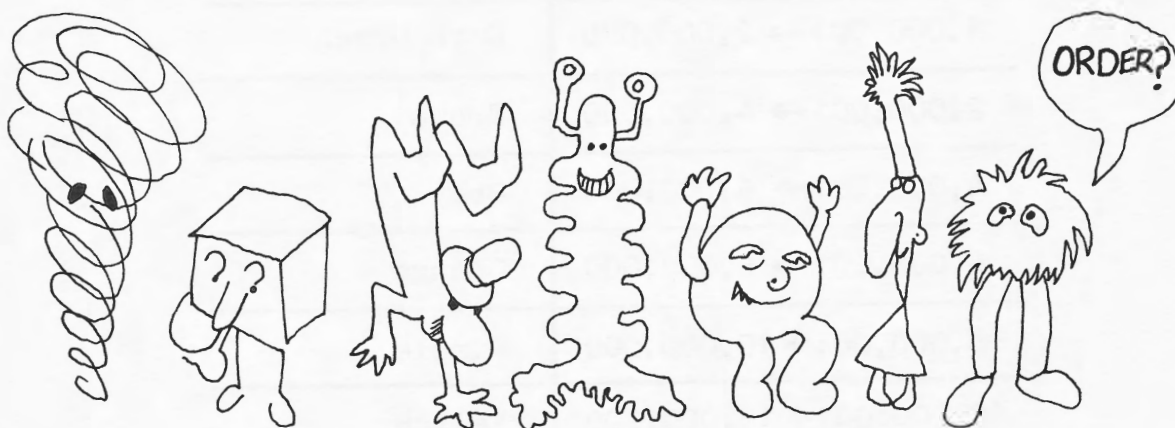
For the set 17, 12, 10, 42, 9 order according to:

- a) size - smallest to largest (9, 10, 12, 17, 42)
- b) alphabetically (42, 9, 17, 10, 12)
- c) number of letters in the corresponding words (10, 9, 12, 42, 17)
- d) sum of digits (10, 12, 42, 17, 9)

- III. Give your class a set of numbers arranged in various orders and have them discover the rule.

- a) 11, 15, 17, 22, 36 (size)
- b) 11, 15, 17, 36, 22 (alphabetically)
- c) 11, 22, 15, 17, 36 (sum of digits)
- d) 11, 22, 15, 36, 17 (reverse digits and order by size)

- IV. Have students select their own numbers, find their own rule, and then order their own numbers. Lists may be exchanged or presented to the class.





POPULATION

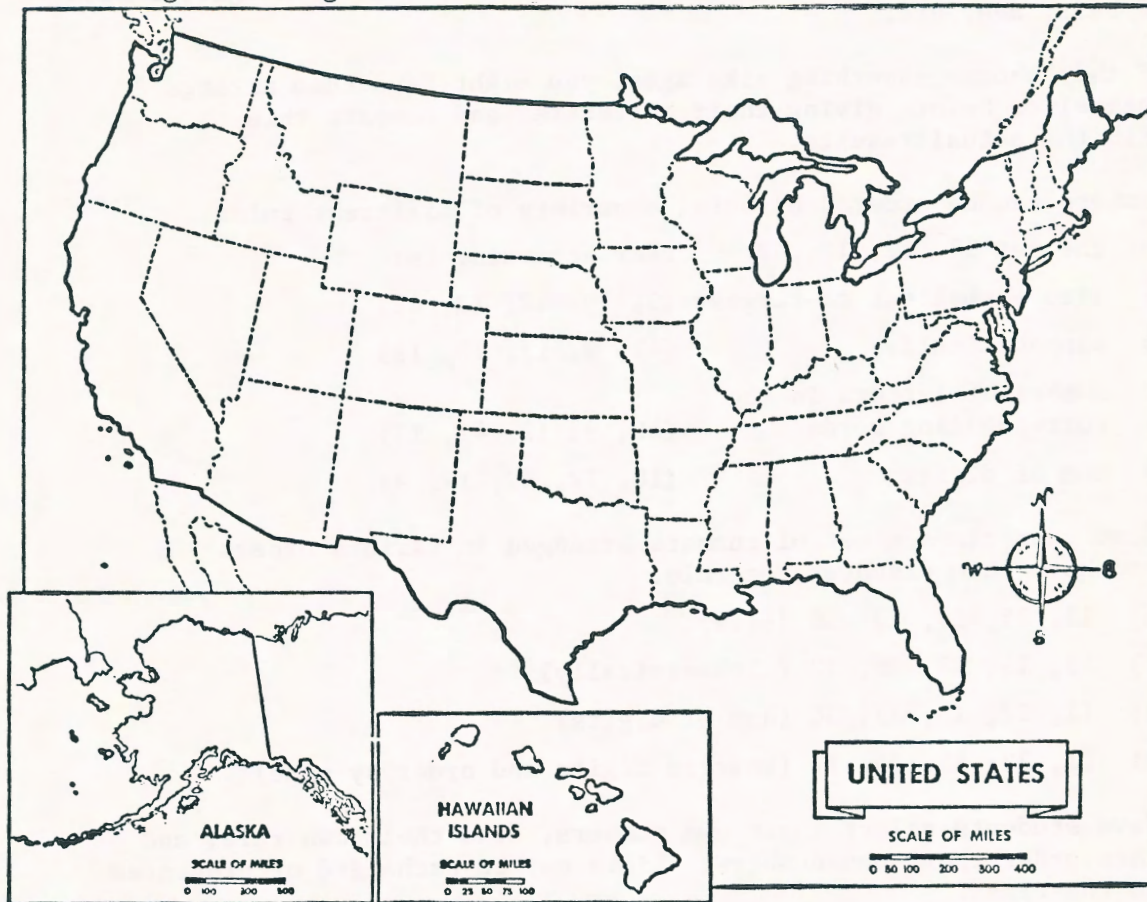
Ordering
Concepts
WHOLE NUMBERS



To do this assignment, you will need an Almanac.

Use the index of the Almanac: locate the page for POPULATION (U.S.--by state).

Using crayons, colored pencils, or felt pens, color the map according to this guide.

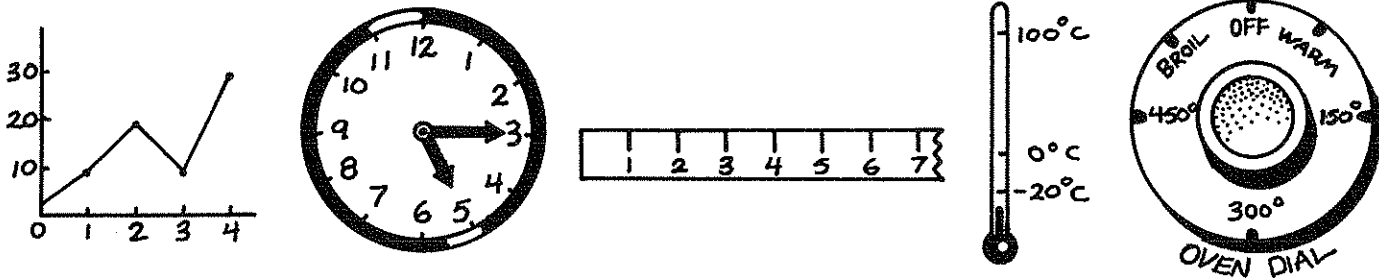


POPULATION OF STATE	COLOR OF STATE
0 → 1,000,000	Light Green
1,000,001 → 2,000,000	Dark Green
2,000,001 → 4,000,000	Brown
4,000,001 → 6,000,000	Red
6,000,001 → 8,000,000	Orange
8,000,001 → 10,000,000	Purple
10,000,001 → 15,000,000	Yellow
15,000,001 → 20,000,000	Pink
over 20,000,000	Blue

WHOLE NUMBER CONCEPTS:

NUMBER LINES

Number lines are an important concept in mathematics and in everyday life. They form the basis of most graphs, rulers, thermometers and many other measuring devices. They show up in disguise on clocks, speedometers and dials.

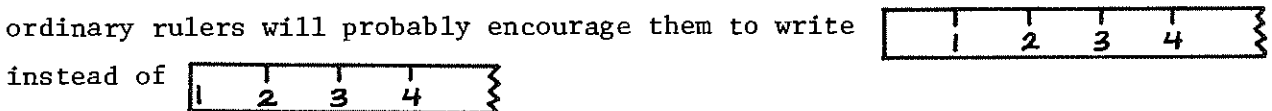


In the above examples number lines are used to describe and understand relationships and quantities. To understand this usage we must know how to read number lines and how to make or label number lines. Our concern here is with representing and comparing whole numbers on number lines; later they can be used for addition, subtraction, multiplication and division of numbers.

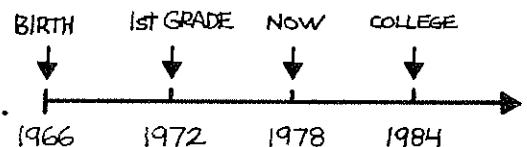
SUGGESTED BEGINNING ACTIVITIES

a. Students could be encouraged to cut out or describe situations where they see numbers written in order. Besides the situations above there are odometer checks on highways, highway mileage signs, house numbers, etc.

b. Students could make their own rulers with their own units. Familiarity with ordinary rulers will probably encourage them to write



c. Each student could make his own life line. This is an opportunity to capture the student's interest (since the topic involves the student's life) and to generate discussion of the student's future, hopes and self-concept. The teacher's life line could also add to the discussion.



CONCEPTUAL PROBLEMS

a. Sarah was asked to make a number line to use for measuring. Here is what she drew:



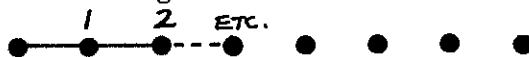
Sarah's mistake is a common one. She is probably counting the dots and placing that number above the last dot counted. Fortunately, she is counting from left to right (the conventional choice), and she does have her numerals in their correct order. Now, if we could convince her to count line segments and place the number of the line segments over the last endpoint, she would be labeling correctly.

It is hard to see line segments on a continuous line with "dots" on it. This method might help her.

1. Make the equally spaced "dots" first.



2. Now make the segments between the dots from left to right. Label the point after each segment with the number of segments to the left.



3. What shall we name the first dot? How many segments come before the first dot?



b. Jerry was given 80 and 100 and was asked to find places for 120, 40 and 0 on the number line.



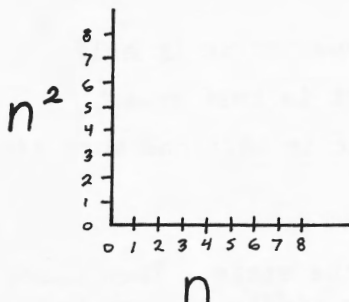
Instead of extending the line to the left to provide for equal spacing for multiples of 20, Jerry pushed the numbers together to fit on the provided line. He can be encouraged to extend the line whenever necessary to keep the spacing of numbers consistent. (It is not always, however, most convenient to space numbers evenly on a number line. A logarithmic scale on an engineer's slide rule is an example.)

c. A class was asked to make a graph showing the relationship between the whole numbers 0, 1, 2, . . . 8 and their squares. They were given graph paper and this table:

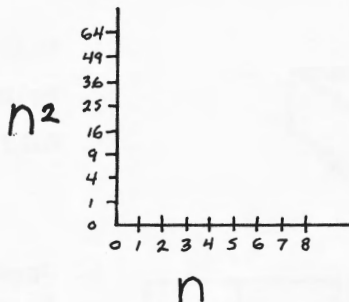
n	0	1	2	3	4	5	6	7	8
n^2	0	1	4	9	16	25	36	49	64

Here are some of the number lines they used to label the graph paper.

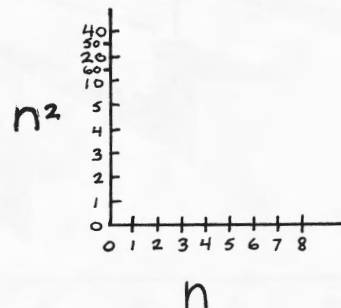
MANUEL



JOHN

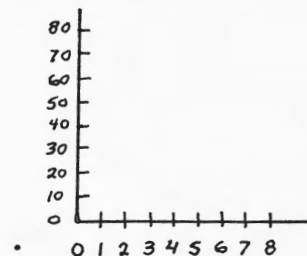


DOLORES



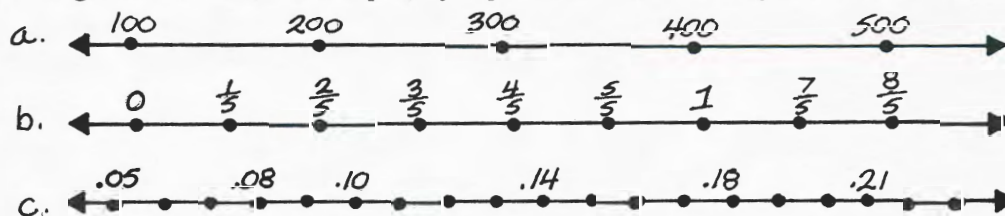
Manuel is labeling correctly, but he will not have room to show 9, 16, . . . 64 on his graph. John has used the square numbers in order and equally spaced. His first space shows a difference of 1 and the last space a difference of 15! His graph will be readable but will show a distorted "picture." Dolores saw she needed to place more numbers on her line and squashed them together. When she still did not have enough room, she tucked 50 and 60 in at random. (This is an actual example from a university class.)

These students were asked to look at the largest number (64) which must "fit" on the graph. It was pointed out that counting and labeling by tens would place 70 near the top. Most students arrived at the useful labeling shown at the right.



A FINAL NOTE

Number lines are an extremely useful model for the concepts and operations of whole numbers, fractions and decimals. They will be used extensively in this resource. These number lines use numbers as labels for points on a line. Multiples of a given number are equally spaced. Some examples are below.



LAYING IT ON THE LINE

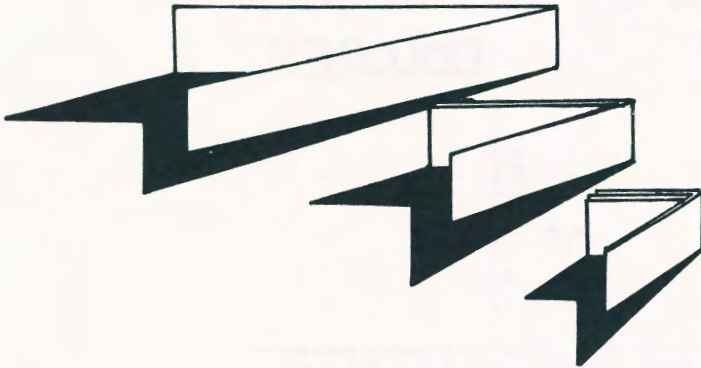
TEACHER IDEA

Ordering
Number Line
Concepts
WHOLE NUMBERS



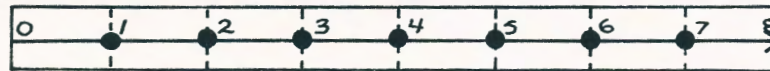
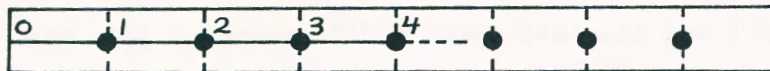
Making a Number Line:

- Materials for each student:
1. A strip of paper about 16 cm long - adding machine tape would be fine.
 2. A straightedge.




Directions to the student:

1. Fold your strip in half.
2. Fold it in half again.
3. Fold it in half one more time.
4. Open the strip. Then place a dot in the middle of each fold and at each end.
5. Use the straightedge to draw segments connecting the dots. Work from left to right. Label the point after each segment with the number of segments drawn.
6. To label the first dot ask the question, "How many segments are before the first dot?"
7. Put an arrow at the right end to show this number line can continue. (You may want an arrow at each end.)



USING THE NUMBER LINE:

Here are some student activities using number lines:

1. By looking at a number line, find out how you can tell which of two numbers is larger. (Which is larger, A or B? )
2. Locate pairs of numbers and find how many whole numbers are between them. (How many numbers between 6 and 9?)
3. Have the students consider questions about an extended number line. (If you were to extend your number line using strips the same length as your original strip, what would you name the middle dot on the second strip? How many strips would you have to put together to show each of the following? A ____, B ____, C ____.)

You may wish to use the number line to perform whole number and fraction operations.

TYPE: Activity

273



GREATER THAN OR LESS THAN

TEACHER DIRECTED ACTIVITY

Ordering
Concepts
WHOLE NUMBERS



THE TEACHER CHOOSES A STUDENT TO BE THE "CALLER."

THE CALLER WRITES A WHOLE NUMBER BETWEEN 1 AND 1000 ON A PIECE OF PAPER AND HANDS IT TO THE TEACHER.

THE CLASS IS TO GUESS THE NUMBER. THE CALLER CALLS ON STUDENTS WHO ASK HIM QUESTIONS ABOUT THE NUMBER. ALL THE CALLER MAY ANSWER TO THEIR QUESTIONS IS YES OR NO.

HERE ARE SOME SAMPLE QUESTIONS: "IS IT GREATER THAN 500?" "IS IT LESS THAN 250?"

THE STUDENT WHO GUESSES THE NUMBER BECOMES THE NEW "CALLER."

Variations:

- (1) Form teams of 3 or 4, have the teams record their guess, and show it to the caller in turn. The team that gets the number first wins.
- (2) Divide the class into two teams. Each team guesses in turn. If the guess is out-of-bounds, the other team gets two guesses.
- (3) The caller records the "no" answers on the board. 10 no's mean the caller wins and may choose a new number.
- (4) The caller may answer saying, "Too large," or "Too small." The students then guess numbers from the start and the caller's responses to their guesses will lead them to the number.

COMMENTS:

- (1) Note that if the number is 50, the answer to, "Is it greater than 50?" is no!
- (2) The students will discover more efficient methods for finding the number as several games are played. Hopefully, they will see how to use "binary search." In the case of 43 the guess would go as follows:

Is it > 50?	(between 1 and 100)	no
Is it < 25?	(between 1 and 50)	no
Is it > 37?	(between 25 and 50)	yes
Is it > 46?	(between 37 and 50)	no
Is it > 41?	(between 37 and 46)	yes
Is it 43?	(range = 5 numbers)	yes

Have the students consider the question: "What is the minimum number of guesses necessary if the number is between:

1 and 50
1 and 100
1 and 200
etc.

TYPE: Activity



Ordering
Concepts
WHOLE NUMBERS



How many whole numbers between

- | | |
|---------------------|-----------------------|
| 1) 5 and 12? _____ | 5) 16 and 11? _____ |
| 2) 3 and 15? _____ | 6) 46 and 75? _____ |
| 3) 7 and 13? _____ | 7) 123 and 124? _____ |
| 4) 47 and 37? _____ | 8) 98 and 1000? _____ |

These are change of pace problems which might be used to help students view order and betweenness in a more challenging way.

There are 17 whole numbers between.

- | | |
|-----------------|-------------------|
| 1) 75 and 93 | 3) _____ and 135 |
| 2) 16 and _____ | 4) _____ and 1001 |

There are no whole numbers between

- | | |
|-----------------|-------------------|
| 1) 7 and _____ | 3) _____ and 13 |
| 2) 46 and _____ | 4) _____ and 2001 |

Ask students if similar questions could be answered for fractions. (How many fractions between $\frac{1}{2}$ and $\frac{1}{3}$, or between 0 and 1?)

When can you always find at least one whole number between two different whole numbers? When can't you?



There are 21 numbers between 17 and \square .

There are \triangle numbers between \square and 60.

What are \triangle and \square ?

$\square = \underline{\hspace{2cm}}$ $\triangle = \underline{\hspace{2cm}}$

There are \square numbers between \triangle and 135.

There are 51 numbers between \triangle and 160.

What are \square and \triangle ?

$\square = \underline{\hspace{2cm}}$ $\triangle = \underline{\hspace{2cm}}$

There are 17 numbers between \square and \triangle , 21 numbers between \triangle and \square , and 43 numbers between \square and \square . How many numbers between \square and \square ?

How many odd numbers between 23 and 144? _____

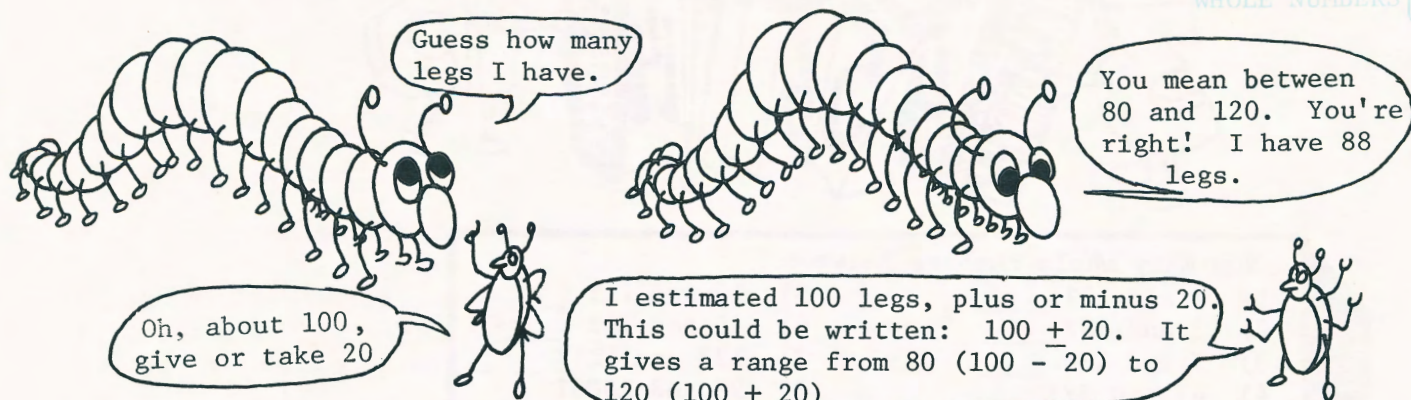
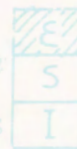
How many even numbers between 15 and 109? _____

Make up some more in-between problems.



GIVE OR TAKE

Approximation
Concepts
WHOLE NUMBERS



650 ± 50 gives a range from $650 - 50 = \underline{\hspace{2cm}}$ to $650 + 50 = \underline{\hspace{2cm}}$.

or simply $\underline{\hspace{2cm}}$ to $\underline{\hspace{2cm}}$.

Give the range for each of these:

1) $2,400 \pm 100$ $\underline{\hspace{2cm}}$ to $\underline{\hspace{2cm}}$

2) 700 ± 25 $\underline{\hspace{2cm}}$ to $\underline{\hspace{2cm}}$

3) 350 ± 30 $\underline{\hspace{2cm}}$ to $\underline{\hspace{2cm}}$

4) $7,450 \pm 450$ $\underline{\hspace{2cm}}$ to $\underline{\hspace{2cm}}$

5) 10 ± 10 $\underline{\hspace{2cm}}$ to $\underline{\hspace{2cm}}$

Estimate answers to each of the questions below. Using your estimation, give the range of your answer. Look up the answers in the almanac and see how close you came.

1. What is the population of the United States?

estimate: $\underline{\hspace{2cm}} \pm \underline{\hspace{2cm}}$ range: $\underline{\hspace{2cm}}$ to $\underline{\hspace{2cm}}$

almanac figure: $\underline{\hspace{2cm}}$

2. How many TV stations are there in the United States?

estimate: $\underline{\hspace{2cm}} \pm \underline{\hspace{2cm}}$ range: $\underline{\hspace{2cm}}$ to $\underline{\hspace{2cm}}$

almanac figure: $\underline{\hspace{2cm}}$

3. How many people are born each year in your state?

estimate: $\underline{\hspace{2cm}} \pm \underline{\hspace{2cm}}$ range: $\underline{\hspace{2cm}}$ to $\underline{\hspace{2cm}}$

almanac figure: $\underline{\hspace{2cm}}$

4. Make up your own question: $\underline{\hspace{10cm}}$

estimate: $\underline{\hspace{2cm}} \pm \underline{\hspace{2cm}}$ range: $\underline{\hspace{2cm}}$ to $\underline{\hspace{2cm}}$

almanac figure: $\underline{\hspace{2cm}}$

NUMBER SENSE AND ARITHMETIC SKILLS

Placement Guide for Tabbed Dividers

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Whole Numbers Commentary: Addition/Subtraction (pages 281-288)

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Activity Cards - Cuisenaire Rods - II	290	Basic facts Model Subtraction	Manipulative
Sum Combinations	291	Basic facts Model Addition	Manipulative
Activity Cards - Bean Sticks - I	293	Regrouping to add Model Addition	Manipulative
Activity Cards - Bean Sticks - II	295	Regrouping to subtract Model Subtraction	Manipulative
Activity Cards - Mathematical Balance - I	297	Model Addition/subtraction	Manipulative
Activity Cards - Base 10 Multibase Blocks - II	299	Regrouping to add Model Addition	Manipulative
Activity Cards - Base 10 Multibase Blocks - III	300	Regrouping to subtract Model Subtraction	Manipulative
Activity Cards - Chip Abacus	301	Regrouping to add/ subtract Model Addition/subtraction	Manipulative

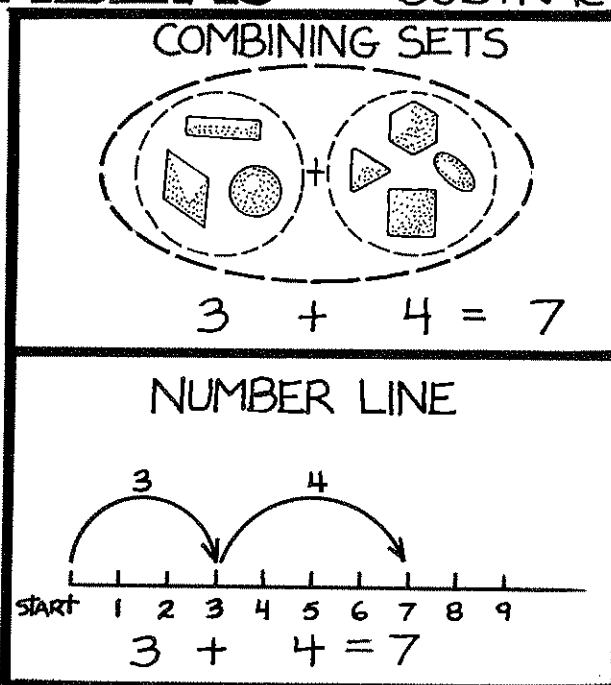
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Subtraction is Subtraction and Addition	303	Alternate method Subtraction	Paper and pencil
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Enchanted Alphabet	326	Addition	Paper and pencil Puzzle
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Path Sums	334	Addition	Paper and pencil
Omar's Dilemma	335	Addition/subtraction	Paper and pencil Transparency

WHOLE NUMBERS: ADDITION / SUBTRACTION

MEANINGS

In the early elementary grades the foundation for addition begins. Sets of objects are combined to form a new set of objects. For example, when a set with four elements is joined to a set with three elements, the resulting set has seven elements. The corresponding number sentence is $3 + 4 = 7$. Besides the use of physical objects, other devices like the number line are used to reinforce the concept of addition.



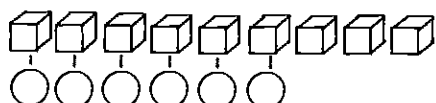
a. "Take Away"



If 7 blocks are removed
how many blocks are left?

$$10 - 7 = 3$$

b. Comparing



How many more blocks
are there than balls?

$$10 - 7 = 3$$

c. Missing Addend



How many blocks must
be added to give 10 blocks?

$$10 - 7 = 3 \quad (7 + \quad = 10)$$

Learning the concept of subtraction somewhat parallels that of addition. A group of objects is removed or "taken away" from a set, and a corresponding number sentence describes the transaction. The same subtraction sentence can be used to answer questions about other situations. The illustrations at the left use the subtraction sentence $10 - 7 = 3$ to solve a "take-away" question, a comparing question and a missing addend question. A student now has a choice. If he wants to solve the problem $10 - 7 = \square$, he can:

- start with 10 objects, remove 7 and see how many are left.
- take 10 blocks and 7 circles, pair blocks and circles and see how many blocks are leftover.
- start with 7 blocks, add blocks until there are 10 and see how many were added.

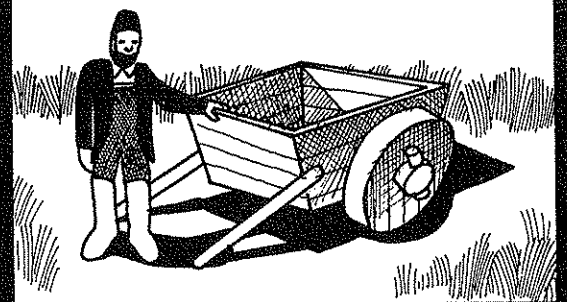
The same choices are made by teachers and textbook writers when they explain subtraction to students. Models for subtraction can use any of these interpretations. You might want to revise a student activity to embody a different view of subtraction.

It is not enough to be able to find sums and differences using physical objects and other devices. To become proficient in addition and subtraction students must know basic facts, use number properties (like $3 + 4 = 4 + 3$), be able to write number sentences and be able to translate verbal statements into correct arithmetic symbols. Eventually, they must master the addition and subtraction algorithms. Although some middle school students may still be having trouble with their basic facts, the predominate difficulty involves the addition and subtraction algorithms.

An algorithm is a systematic procedure for obtaining a desired result. The algorithm illustrated at the right is a rather uncommon multiplication procedure that always works. We could memorize the steps in this algorithm and use it every time we multiply without knowing why it works. If we use it infrequently, we may forget some of the steps in the procedure. However, if we understand why it works, as well as how it works, it may become a more permanent part of our knowledge that can be quickly recalled.

Because students must learn several different algorithms in mathematics, learning them meaningfully becomes very important.

RUSSIAN PEASANT ALGORITHM FOR MULTIPLICATION



ALGORITHM	MULTIPLY 78×168	
Step 1: Head two columns with the numbers being multiplied.	HALVE	DOUBLE
	78	168
Step 2: Halve the entry in one column and double the entry in the other column. (If result of halving yields a remainder, then discard the remainder and record the whole number.)	39	336
	19	672
	9	1344
	4	2688
	2	5376
	1	10752
Step 3: Repeat Step 2 until 1 is obtained in the HALVE column.		13104
Step 4: Cross out entries in the DOUBLE column that are opposite <u>even</u> numbers in the HALVE column.		
Step 5: Add entries in DOUBLE column that have not been crossed out. The sum will be the desired product.		

Addition Algorithm

If students lack the necessary prerequisite understanding, then learning the addition algorithm becomes a rote process at best. In order to understand this algorithm, they need a reasonable command of:

- i. place value
- ii. renaming numbers
- iii. basic addition facts
- iv. certain number properties

It may be helpful to have students invent their own way to add numbers. In fact, a discussion of different ways to find a sum, say $27 + 36$, may lead to many interesting solutions and culminate in the addition algorithm itself. If encouraged to experiment, students may devise ingenious methods. Here are the ways six students thought out the problem $27 + 36$.

Scott: $27 + 36 \rightarrow 20 + 30 + 6 + 7 \rightarrow 50 + 13 \rightarrow 50 + 10 + 3 \rightarrow 63$

Jan: $27 + 36 \rightarrow 27 + 3 + 33 \rightarrow 30 + 33 \rightarrow 63$

Kay: $27 + 36 \rightarrow 23 + 4 + 36 \rightarrow 23 + 40 \rightarrow 20 + 3 + 40 \rightarrow 63$

Steve: $27 + 36 \rightarrow 27 + 30 + 6 \rightarrow 57 + 6 \rightarrow 57 + 3 + 3 \rightarrow 60 + 3$

Tom: $27 + 36 \rightarrow 20 + 36 + 7 \rightarrow 56 + 7 \rightarrow 56 + 4 + 3 \rightarrow 60 + 3$

Jean: $27 + 36 \rightarrow 25 + 2 + 25 + 11 \rightarrow 50 + 2 + 11 \rightarrow 63$

The exercise above provided experiences in mental arithmetic and prerequisite skills for the addition algorithm. One of the strategies--adding the units, adding the tens, and then combining these sums as in (a)--leads naturally to the standard algorithm. Individual teachers may have a preference for forms (b) or (c), but ultimately, either of those forms is condensed to the shortened form (d).

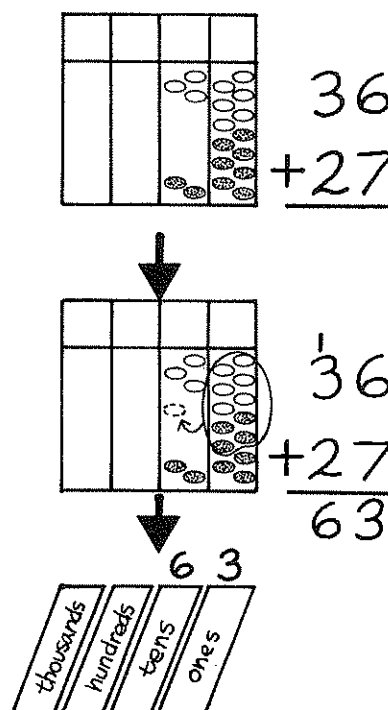
(a) $27 + 36 \rightarrow 20 + 30 + 6 + 7$

(b)
$$\begin{array}{r} 27 \\ +36 \\ \hline \end{array} \rightarrow \begin{array}{r} 20 + 7 \\ 30 + 6 \\ \hline 50 + 13 \end{array}$$

(c)
$$\begin{array}{r} 27 \\ +36 \\ \hline \end{array} \rightarrow \begin{array}{r|l} \text{tens} & \text{ones} \\ \hline 2 & 7 \\ 3 & 6 \\ \hline 5 & 13 \end{array}$$

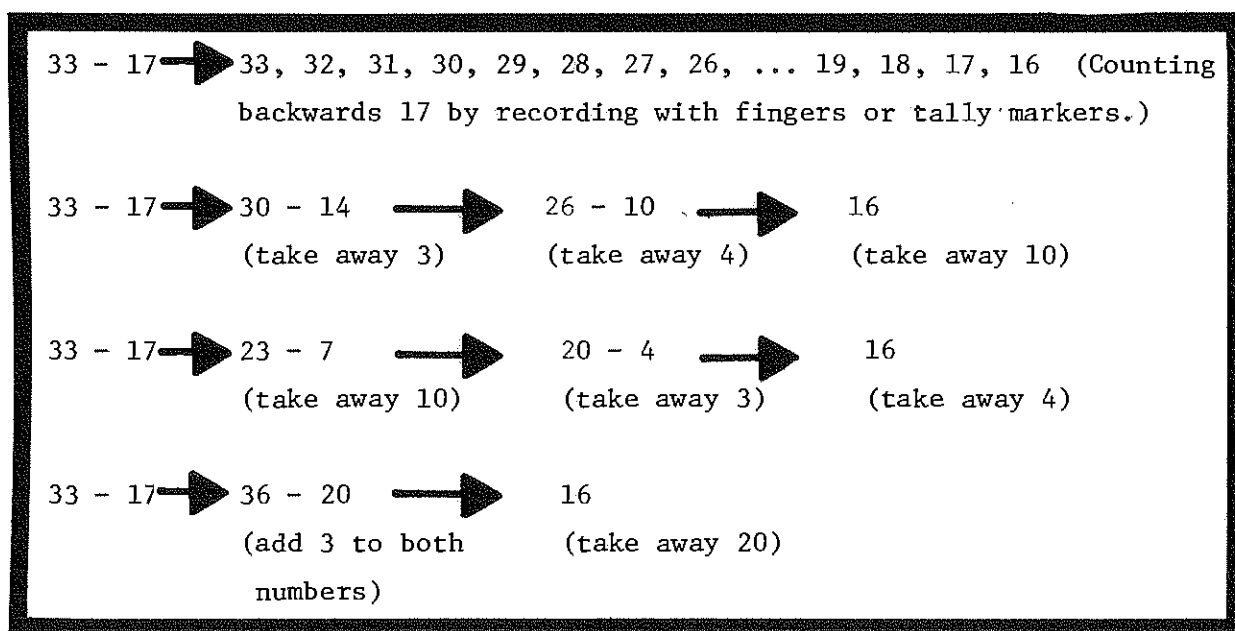
(d)
$$\begin{array}{r} 27 \\ +36 \\ \hline \end{array} \xrightarrow{1} \begin{array}{r} 127 \\ +36 \\ \hline \end{array} \xrightarrow{1} \begin{array}{r} 127 \\ +36 \\ \hline 63 \end{array}$$

It may be necessary for some students to go back to concrete objects in order to conceptualize addition. The activities involving Chip Trading, Bean Sticks, Multibase Blocks or an Abacus show combining of objects, necessary regrouping and answer recording. Each step of the activity can be described by a number statement.



Subtraction Algorithm

The development of the subtraction algorithm could easily parallel the approach to addition. Students could be challenged to invent subtraction techniques.



With encouragement students may devise some other ingenious methods. Notice that in the last example the student realized that when the same number is added to 36 and 20, the difference remains the same. (This student always adds the number which makes the subtrahend a multiple of ten, e.g., $43 - 25 \rightarrow 48 - 30$.)

For those students who need a more developmental approach we can utilize Chip Trading, Bean Sticks, Multibase Blocks, or an Abacus. The development of the algorithm can parallel the use of a manipulative. A physical interpretation of subtraction is chosen when using a manipulative. Usually, the "take away" view is chosen as shown in the Chip Trading example below; however, some materials make use of the missing addend or comparing view. The idea of comparing or "matching up" is used in the Multibase Blocks example below.

**33 - 17 with Chip Trading
(Take away View)**

33
(Task-to remove 17)

Regroup 33
as 2 tens
and 13 units

Remove 17

Record
Answer

16

1 6
tens ones

**124 - 53 with Multibase Blocks
(Comparing View)**

124
- 53

Pair like objects.
WARNING!
Students might
just remove the
5 longs and 3 units.

The problem is now
101-30. A long needs
to be traded for
10 units.

Pair again.

71

After working a series of problems with regrouping, it is a good idea to revert to a few problems like $37 - 23$. Some students may have a tendency to regroup when it is not necessary and to write:

$$\begin{array}{r} 57 \\ - 23 \\ \hline \end{array} \rightarrow \begin{array}{r} 1 \\ 47 \\ - 23 \\ \hline 24 \end{array}$$

This could lead to a correct answer, but more likely will result in a place value error.

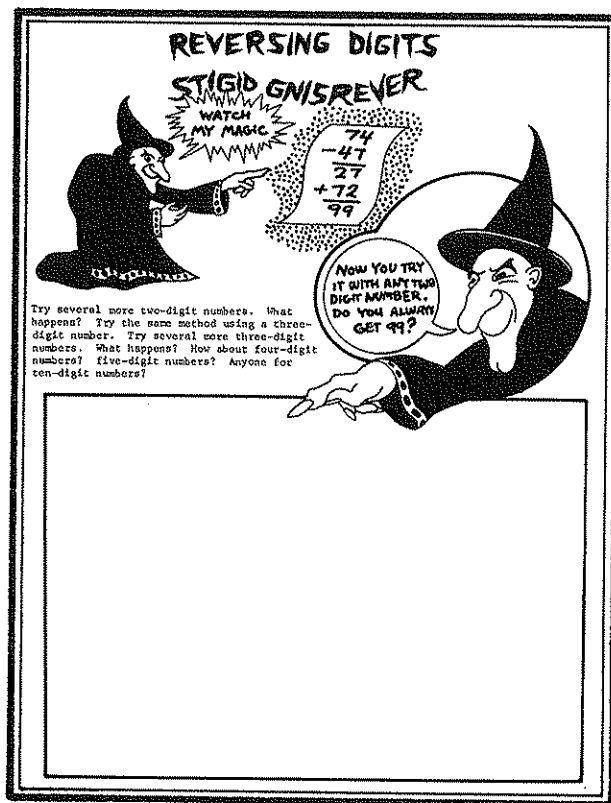
MEANINGFUL DRILL AND PRACTICE

Hopefully, a creative approach to the addition and subtraction algorithms will not result in a meaningless procedure. But what comes next? How can we provide practice and not revert to pages of meaningless computational exercises with meaningless answers? Perhaps we can find a clue by looking at the teaching of other subjects. After a few reading skills are learned, a child is given a story to read. Reading the story is enjoyable (usually), and it also improves skills. Later, the child is asked to use his reading skills to read signs, newspapers and encyclopedias--all practical applications. Would teachers, students or parents be satisfied with skill building in phonics, spelling and writing if students were never given a book to read (for fun), a story to write (for creativity), or an encyclopedia to use (for information)? Drill and practice in mathematics can be made just as enjoyable and practical by providing mathematical recreations, problem solving and applications.

Mathematical Recreations

For centuries people have been fascinated with number puzzles, tricks and games. Paperback books on crossnumber puzzles and other mathematical diversions are usually found on newsstands. Many of these puzzles and tricks provide enjoyable skill building in addition and subtraction of whole numbers.

At the right is a copy of the student page *Reversing Digits*. Will the procedure described by this page always result in 99? Students can practice their addition and subtraction of 2-digit numbers while trying to find out. Self-correction is built-in here. A student whose calculations don't give 99 needs to go over his computation. An algebra student can be challenged to figure out why it works.



IF LETTERS WERE DOLLARS

SUPPOSE LETTERS WERE MONEY. USE THE CHART BELOW TO SEE WHO HAS THE MOST EXPENSIVE NAME IN CLASS.

A = \$1	E = \$5	I = \$9	M = \$13	O = \$17	U = \$21
B = \$2	F = \$6	J = \$10	N = \$14	R = \$18	V = \$22
C = \$3	G = \$7	K = \$11	P = \$15	S = \$19	W = \$23
D = \$4	H = \$8	L = \$12	T = \$16	X = \$20	Z = \$24

BILL COMPUTES HIS NAME THIS WAY.

B I L L C O L L E C T O R
 $2 + 9 + 12 + 12 + 3 + 15 + 12 + 12 + 5 + 3 + 20 + 15 + 18 = \138

HOW MUCH IS THE NAME OF YOUR SCHOOL WORTH?

HOW ABOUT YOUR TEACHER'S NAME? YOUR PRINCIPAL'S NAME?

WHAT'S THE MOST EXPENSIVE 3 LETTER WORD YOU CAN THINK OF?
 4 LETTER WORD? 5 LETTER WORD?

WHAT'S THE LEAST EXPENSIVE 3 LETTER WORD YOU CAN FIND? 4 LETTER WORD?
 TRY SOME MORE.

USELESS IS AN EXAMPLE OF A \$100 WORD. CAN YOU FIND MORE \$100 WORDS?

IDEA FROM: WYOMOLAMP, Secondary Materials
 Permission to use granted by Fremont County Unified School District #25

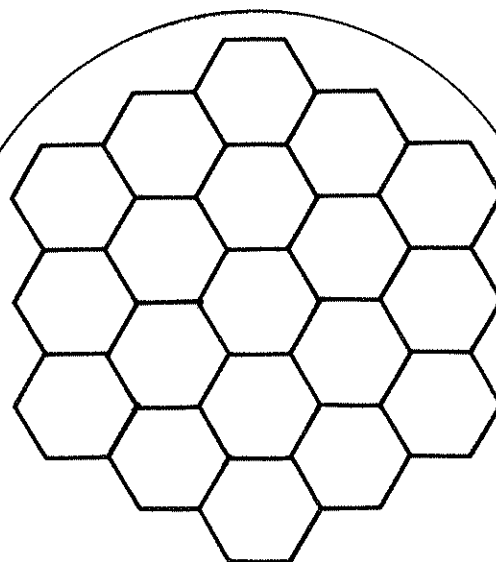
The student page *If Letters Were Dollars* provides for enjoyable skill building. What student could resist figuring what his/her name, or a friend's name, is worth? (Did you know that "skill building" is a \$141 word?) This type of activity not only provides practice, but it also includes mental arithmetic and problem solving. Other student pages, like *Guided Mazes* and *Shady Squares*, are self-correcting with activities that are interesting to students.

		9
10	6	2
	8	

Problem Solving

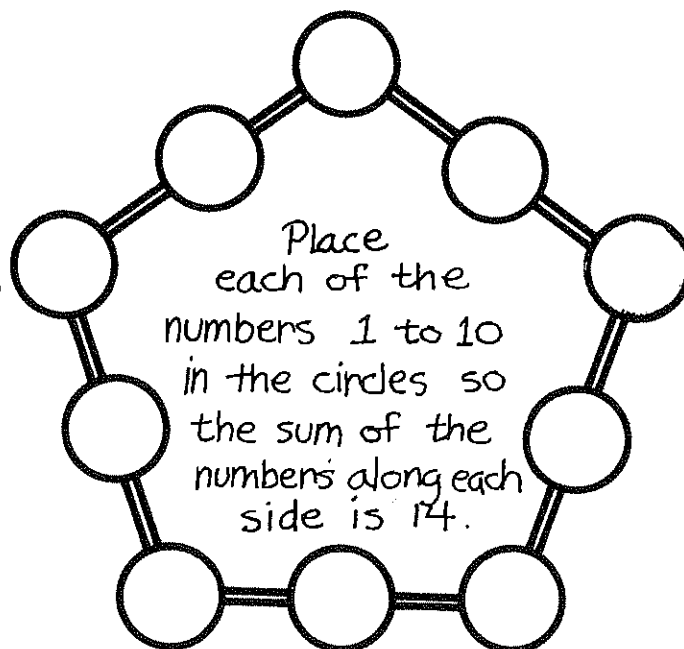
Magic squares provide a problem to solve, while computational skills are improved. "How are the numbers placed so all the columns, rows and diagonals have the same sum? How can you get started solving this problem? How do you know certain numbers can't go in specific places?" Methods of attacking such problems can be discussed in class. A teacher page following the student page *Magic Squares* explains how to make magic squares.

The idea behind magic squares can be varied in many ways. Several extensions are given in the student pages *Calendar Magic* and *Magic Hexagons*. The magic hexagon at the right is difficult to solve. To help students begin, the student page has several numbers already



Write the numbers 1 to 19 in the hexagon so that the sum of any straight line of numbers is 38.

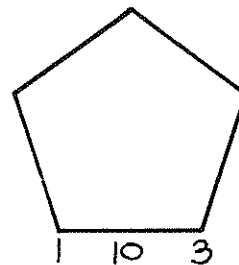
The problem at the right is from the student page *Magic Perimeters*. Try to solve the problem yourself while keeping track of your method of attack. Most people begin with trial and error and find that they need to devise an organized plan. Your students might use trial and error, or you can help them use good problem solving techniques. One person's reasoning on this problem is given below. Perhaps you have a better method for attacking this problem.



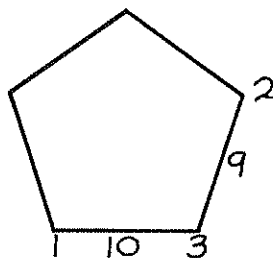
"Where shall I start? It seems that the small numbers will fit in many places, so maybe I should start with the big numbers. I'll make a list of combinations whose sum is 14."

$$\begin{array}{r}
 10 + 1 + 3 \\
 \hline
 9 + 1 + 4 \\
 9 + 2 + 3 \\
 \hline
 8 + 1 + 5 \\
 8 + 2 + 4 \\
 \hline
 7 + 1 + 6 \\
 7 + 2 + 5 \\
 7 + 3 + 4
 \end{array}$$

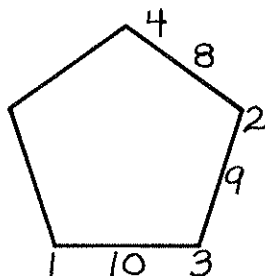
"It looks like 10 has to go with 1 and 3. Can 10 go on a corner? No, since it doesn't fit into 2 combinations. 10 has to go in the middle of a side. So far, I have this:"



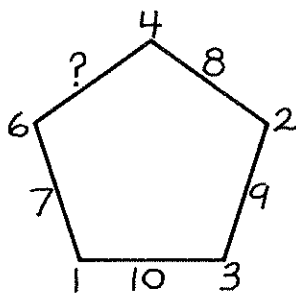
"Where should 9 go? 9 has to be in a line with 1 or 3. It can't go in a corner because 1 and 3 are both placed. Try 9 with 3. (If I get stuck I'll come back and try it with 1.) Now I have this:"



8 has to go with 1 or 2. Try 2.
Putting 8 in, I have this:



7 can't go with 4 because 3 is already used, so it has to go with 1 to give this:



Now, I'm stuck! The only number remaining to be used is 5 and that makes the sum along the last side 15. I'd better go back and change my choices at one of the previous steps.

What would happen if a teacher worked this problem, or one like it, in front of the students without having solved it beforehand? The teacher could verbalize his/her reasoning as the problem is being solved. This would provide a real example of problem solving. The teacher might even get stuck and need help from students. It would certainly provide an opportunity for students to see strategies, blind alleys, starting over and persistence. Wouldn't a few such instances be more helpful than a polished solution that has been worked out beforehand?

Drill and practice can be useful, fun and productive. It doesn't have to be a "meaningless procedure to get meaningless answers to meaningless questions."



ACTIVITY CARDS - CUISENAIRE RODS - I

Basic Facts
Model
Addition
WHOLE NUMBERS



For information and readiness activities
for Cuisenaire Rods see the section on
Lab Materials.

Elementary activity for students having difficulty with basic facts

IF THE WHITE ROD EQUALS ONE WHAT ARE THE VALUES OF THE OTHER COLORS?

RED = 2

YELLOW =

BROWN =

LIGHT GREEN =

DARK GREEN =

BLUE =

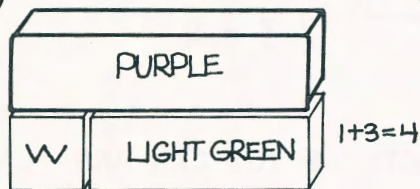
PURPLE =

BLACK =

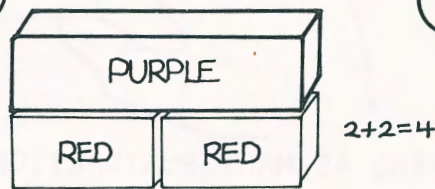
ORANGE =

HERE ARE SOME EXAMPLES OF SENTENCES THAT ADD UP TO 4. THEY ARE FILLED
IN THE TABLE BELOW. CAN YOU FIND SOME MORE?

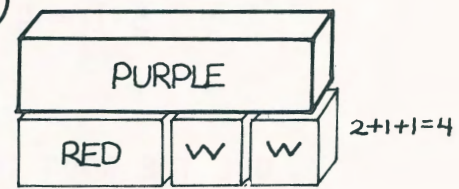
A



B



C



USE THE RODS TO MAKE SOME ADDITION FACTS FOR EACH SUM IN THE TABLE BELOW.

SUM	ADDITION SENTENCES	# OF SENTENCES
2		
3		
4	$1 + 3 = 4, 2 + 2 = 4, 2 + 1 + 1 = 4.$	
5		
6		
7		
8		
9		
10		

TYPE: Manipulative



ACTIVITY CARDS - CUISENAIRE RODS - II

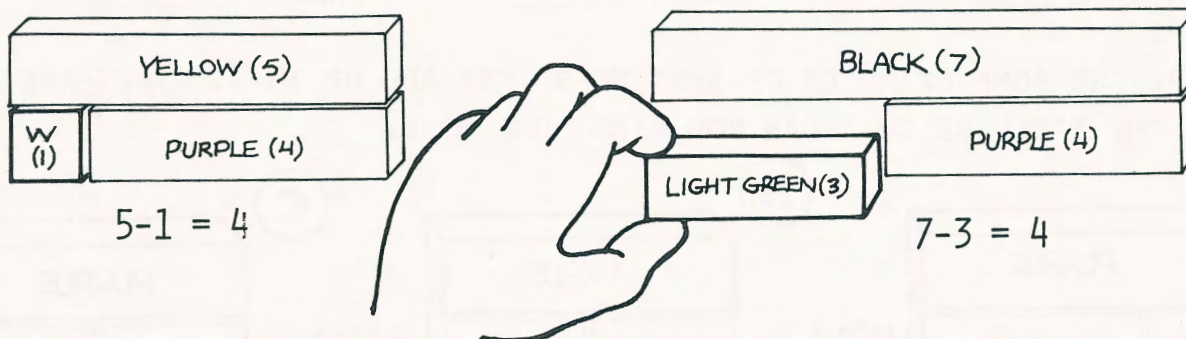
For information and readiness activities
for Cuisenaire Rods see the section on
Lab Materials.

Basic Facts
Model
Subtraction
WHOLE NUMBERS



YOU WILL NEED ONE SET OF CUISENAIRE RODS. YOU CAN USE THE CUISENAIRE RODS TO MAKE SUBTRACTION SENTENCES.

HERE ARE TWO EXAMPLES OF SENTENCES WITH A DIFFERENCE OF 4.
THEY ARE FILLED IN THE TABLE BELOW. CAN YOU FIND SOME MORE?



USE THE RODS TO FIND AS MANY SUBTRACTION FACTS AS YOU CAN AND PLACE THEM IN THE TABLE BELOW.

DIFFERENCE	SUBTRACTION SENTENCES	# OF SENTENCES
0		
1		
2		
3		
4	$5-1 = 4$, $7-3 = 4$,	
5		
6		
7		
8		
9		
10		

TYPE: Manipulative

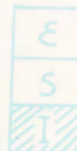
The name Cuisenaire and the
color sequence of the rods are
trademarks of the Cuisenaire
Company of America, Inc.



SUM COMBINATIONS

TEACHER IDEAS

Basic Facts
Model
Addition
WHOLE NUMBERS



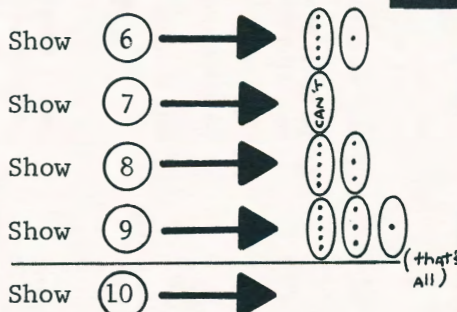
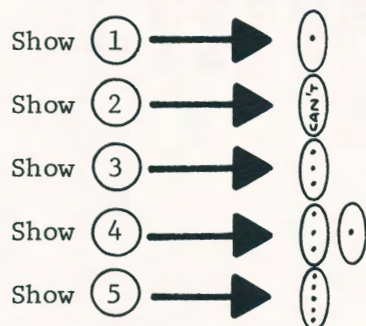
This is a series of low level problem-solving activities which use beansticks. For information on the construction, use and readiness activities for Beansticks see the section on Lab Materials.

Activity I. SHOW THE NUMBER

The student chooses 3 or 4 different beansticks that are less than 8 and records the choices on his recording sheet.

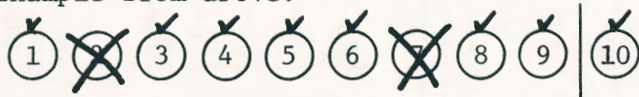
Now the student tries to use the chosen beansticks to exactly show each of the numbers listed on the bottom of his recording sheet.

Example: Using a 1-stick, a 3-stick and a 5-stick



To record the results, check the numbers that can be shown, cross out the ones that can't be shown, and draw a line to mark off the highest possible number.

Example from above:



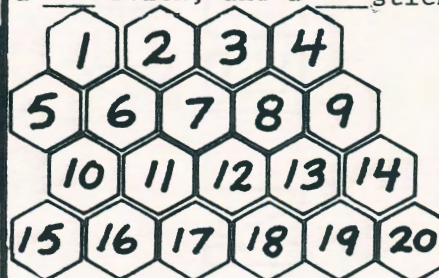
- Variations:
- (1) Put 2 checks on any numbers that can be made in more than one way.
 - (2) Choose 2 or 3 sticks that are the same.
 - (3) The teacher writes down the numbers that can be shown using an unknown combination of 3 or 4 sticks; the students decide which sticks were chosen.

SAMPLE RECORDING SHEETS

Select 3 beansticks
a ____-stick, a ____-stick
and a ____-stick.



Select 4 beansticks
a ____-stick, a ____-stick
a ____-stick, and a ____-stick



PE: Manipulative

IDEA FROM: *Drill and Practice at the Problem Solving Level*, Activity Pages

Permission to use granted by Curriculum Development Associates, Inc.

SUM COMBINATIONS

(CONTINUED)

ACTIVITY II. HOW MANY DIFFERENT COMBINATIONS?

FOR THIS ACTIVITY THE STUDENT NEEDS 5 BEANSTICKS: A 1-STICK, A 2-STICK, A 3-STICK, A 4-STICK, AND A 5-STICK.

HE SHOULD USE THESE BEANSTICKS TO SHOW EACH NUMBER AT THE LEFT OF HIS RECORDING SHEET IN AS MANY DIFFERENT WAYS AS POSSIBLE. THE SUM OF THE BEANS SHOULD EQUAL THE NUMBER GIVEN.

FOR EXAMPLE, SHOW 8:



IN THIS EXAMPLE THE BEANSTICKS HAVE BEEN GROUPED IN THREE DIFFERENT WAYS, EACH SHOWING 8.

RECORD THE RESULTS AS FOLLOWS:



A SAMPLE STUDENT SHEET COULD LOOK LIKE THIS:

BEANSTICKS TO WORK WITH: ○ ○ ○ ○ ○	
6	11
7	12
8	13
9	14
10	15
...	...

FOR VARIETY ALTER THE "BEANSTICKS TO WORK WITH" BY:

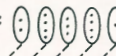
- USING MORE BEANSTICKS, AND/OR
- USING DIFFERENT BEANSTICKS

ACTIVITY III. COMBINATIONS OF NEIGHBORS

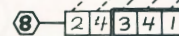
HAVE THE STUDENT PICK ANY FIVE BEANSTICKS.

THE OBJECT OF THE ACTIVITY IS TO TRY TO FIND THE SUM AT THE LEFT, USING ONLY NEIGHBORING STICKS.

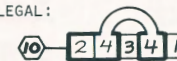
USING THIS ARRANGEMENT:



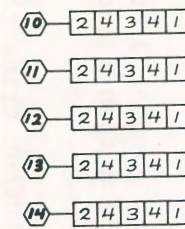
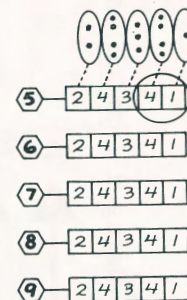
THIS IS LEGAL:



THIS IS NOT LEGAL:



HERE IS A SAMPLE PROBLEM. CIRCLE THE NUMBERS USED TO SHOW EACH NUMBER AT THE LEFT.



EXTENSION: CAN THE STICKS BE REARRANGED SO THERE ARE FEWER IMPOSSIBLE SUMS? FEWER POSSIBLE SUMS?



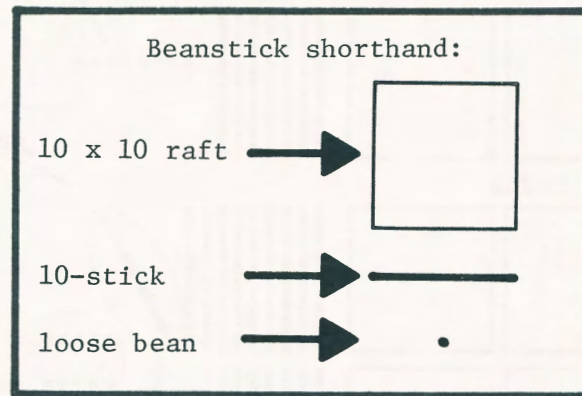
ACTIVITY CARDS - BEANSTICKS - I

Regrouping to Add
Model
Addition
WHOLE NUMBERS



This activity may be performed using any multibase materials. For information on the construction, use and readiness for Beansticks or Multibase Materials see the section on Lab Materials. You may need to guide your students through this to be sure they follow each step and are getting a feeling for the written method.

ADDITION IN BEANSTICK SHORTHAND



① Show 37 with 10-sticks and loose beans. WRITE:

② Show 25.

③ Combine.

④ Trade 10 loose beans for one 10-stick.

⑤ Find the total.

NEW TEN

LOOSE BEANS LEFT

TOTAL TENS

37
+ 25

1
37
+ 25

2
1
37
+ 25

62

USE YOUR 10-STICKS AND LOOSE BEANS TO DO THESE PROBLEMS.
RECORD YOUR WORK HERE.

①	48	②	59	③	27	④	53	⑤	76
	+ 37		+ 26		+ 45		+ 19		+ 18

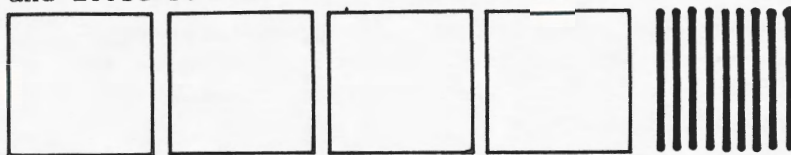
TYPE: Manipulative

ACTIVITY CARDS - BEANSTICKS - I (CONTINUED)

TO ADD $496 + 278$:

Show 496 with 10 x 10 rafts, 10-sticks, and loose beans.

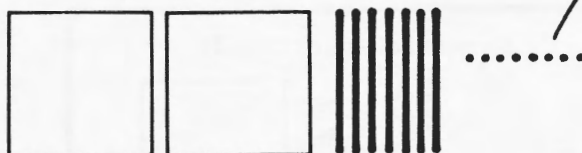
①



$$\begin{array}{r} 496 \\ + 278 \\ \hline \end{array}$$

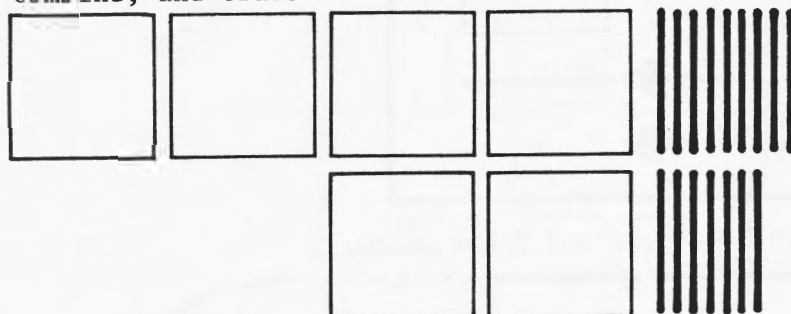
②

Show 278.



③

Combine, and trade loose beans



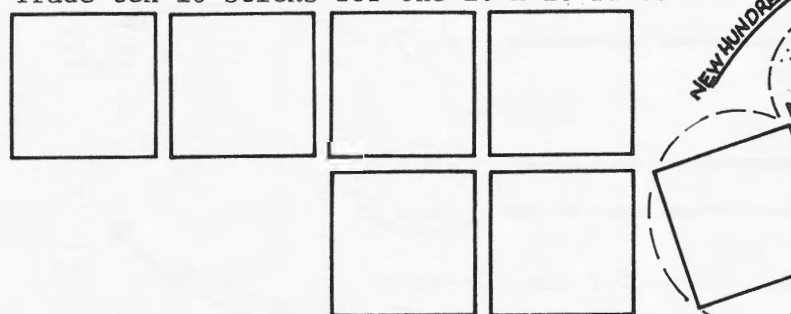
$$\begin{array}{r} 496 \\ + 278 \\ \hline \end{array}$$

NEW TEN

LOOSE BEANS LEFT

④

Trade ten 10-sticks for one 10 x 10 raft.



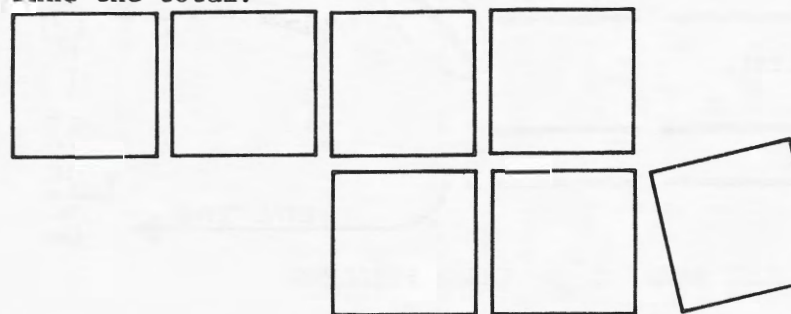
$$\begin{array}{r} 496 \\ + 278 \\ \hline \end{array}$$

NEW HUNDRED

10-STICKS LEFT

⑤

Find the total.



$$\begin{array}{r} 496 \\ + 278 \\ \hline 774 \end{array}$$

TOTAL HUNDREDS

USE YOUR RAFTS, STICKS, AND LOOSE BEANS TO DO THESE.
RECORD YOUR WORK HERE.

①

$$\begin{array}{r} 589 \\ + 163 \\ \hline \end{array}$$

②

$$\begin{array}{r} 452 \\ + 184 \\ \hline \end{array}$$

③

$$\begin{array}{r} 376 \\ + 289 \\ \hline \end{array}$$

④

$$\begin{array}{r} 251 \\ + 449 \\ \hline \end{array}$$

⑤

$$\begin{array}{r} 627 \\ + 259 \\ \hline \end{array}$$



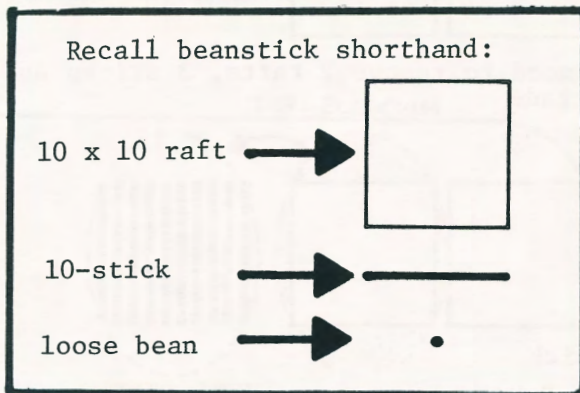
ACTIVITY CARDS - BEANSTICKS - II

Regrouping to Subtract
Model
Subtraction
WHOLE NUMBERS

ε
5
1

This activity may be performed using any multibase materials. For information on the construction, use and readiness for Beansticks or Multibase Materials see the section on Lab Materials. You may need to guide your students through this to be sure they follow each step and are getting a feeling for the written method.

SUBTRACTION IN BEANSTICK SHORTHAND



TO SUBTRACT 43 - 28:

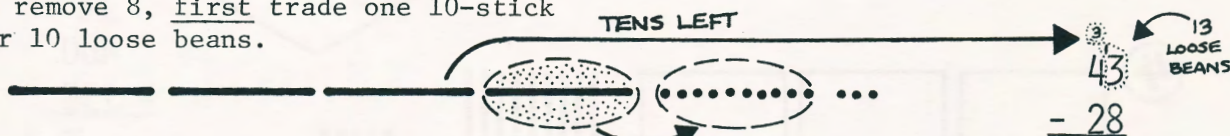
WRITE:

- ① Show 43 with 10-sticks and loose beans.

_____ ...

To subtract 28 you need to remove 2, 10-sticks and 8 loose beans. $\begin{array}{r} 43 \\ - 28 \end{array}$

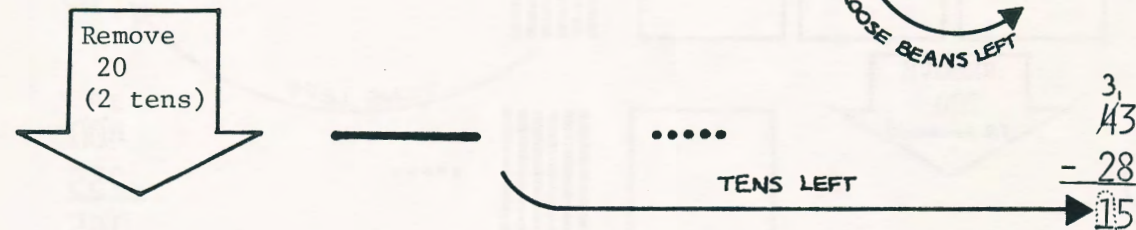
- ② To remove 8, first trade one 10-stick for 10 loose beans.



- ③



- ④



USE YOUR BEANSTICKS AND BEANS TO DO THESE PROBLEMS.
Record your work here.

- | | | | | |
|---|---|---|---|---|
| ① | ② | ③ | ④ | ⑤ |
| $\begin{array}{r} 71 \\ - 25 \end{array}$ | $\begin{array}{r} 34 \\ - 16 \end{array}$ | $\begin{array}{r} 32 \\ - 47 \end{array}$ | $\begin{array}{r} 56 \\ - 39 \end{array}$ | $\begin{array}{r} 77 \\ - 49 \end{array}$ |

TYPE: Manipulative

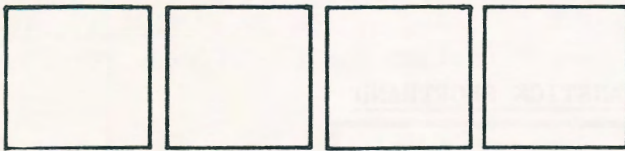
This is an example of a take-a-way method for subtraction. See the commentary to WHOLE NUMBERS: Addition/Subtraction.

ACTIVITY CARDS - BEANSTICKS - II (CONTINUED)

TO SUBTRACT 400 - 235:

①

Show 400 (with 10 x 10 rafts)

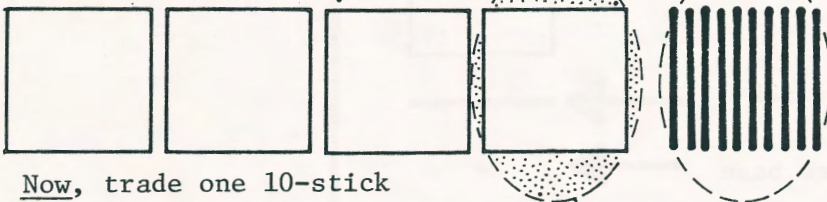


WRITE:

$$\begin{array}{r} 400 \\ - 235 \\ \hline \end{array}$$

To subtract 235 you need to remove 2 rafts, 3 sticks and 5 beans.
To remove 5, first trade
one 10 x 10 raft
for ten 10-sticks.

②



$$\begin{array}{r} 39 \\ 400 \\ - 235 \\ \hline \end{array}$$

10 LOOSE BEANS

Now, trade one 10-stick
for 10 loose beans.

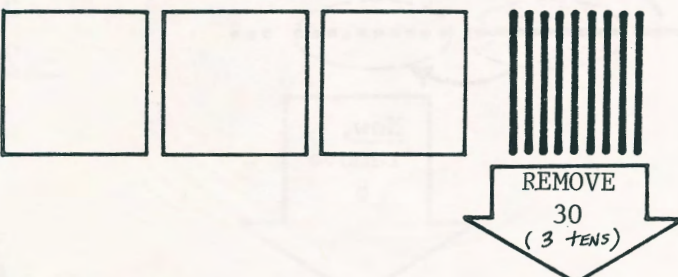
③



$$\begin{array}{r} 39 \\ 400 \\ - 235 \\ \hline \end{array}$$

REMOVE
5

④



$$\begin{array}{r} 39 \\ 400 \\ - 235 \\ \hline 5 \end{array}$$

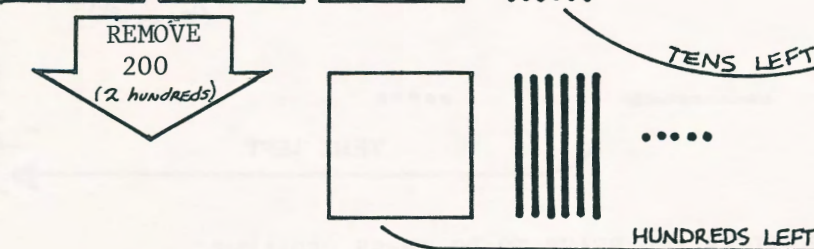
LOOSE BEANS LEFT

⑤



$$\begin{array}{r} 39 \\ 400 \\ - 235 \\ \hline 65 \end{array}$$

⑥



$$\begin{array}{r} 39 \\ 400 \\ - 235 \\ \hline 165 \end{array}$$

USE YOUR BEANSTICKS, RAFTS, AND BEANS TO DO THESE PROBLEMS.
Record your work here.

① $\begin{array}{r} 327 \\ - 483 \\ \hline \end{array}$

② $\begin{array}{r} 401 \\ - 167 \\ \hline \end{array}$

③ $\begin{array}{r} 700 \\ - 279 \\ \hline \end{array}$

④ $\begin{array}{r} 920 \\ - 264 \\ \hline \end{array}$

⑤ $\begin{array}{r} 900 \\ - 416 \\ \hline \end{array}$



ACTIVITY CARDS - MATHEMATICAL BALANCE - I

Model
Addition/Subtraction
WHOLE NUMBERS



MA-I-1

For information and readiness activities for the Mathematical Balance see the section on Lab Materials.

- 1) PLACE 11 ON THE LEFT SIDE OF THE BALANCE SCALE, (1 WASHER ON THE 10 AND 1 WASHER ON THE 1's HOOK).
FIND ANOTHER NAME FOR 11 BY PLACING WASHERS ON THE RIGHT SIDE TO BALANCE THE ARM.
WHAT DID YOU USE? _____
FIND OTHER WAYS. _____
- 2) RENAME 18 IN THE SAME WAY YOU DID PROBLEM 1.
WHAT DID YOU USE? _____
- 3) RENAME 25 USING ONLY WASHERS ON THE 6 HOOK AND THE 7 HOOK.
YOU USED _____ 6's AND _____ 7's.
- 4) RENAME 12 USING ONLY THE 3 HOOK AND THE 2 HOOK.
YOU USED _____ 3's AND _____ 2's.
- 5) RENAME 23 USING ONLY THE 2 HOOK AND THE 5 HOOK. RECORD THE DIFFERENT WAYS IN CHART I. HOW MANY WAYS DID YOU FIND? _____
- 6) RENAME 33 USING ONLY THE 4 HOOK AND THE 3 HOOK. RECORD THE DIFFERENT WAYS IN CHART II. HOW MANY WAYS DID YOU FIND? _____
- 7) RENAME 29 USING ONLY THE 7 HOOK AND THE 4 HOOK. RECORD THE DIFFERENT WAYS IN CHART III. HOW MANY WAYS DID YOU FIND? _____
- 8) CAN YOU RENAME 29 USING 7's AND 6's? _____
- 9) IF THE 5 HOOK AND ONE OTHER HOOK ARE USED TO RENAME 31 WHAT ARE CHOICES FOR THE SECOND HOOK? _____

23 CHART I		33 CHART II		29 CHART III	
2's	5's	4's	3's	7's	4's
1.		1.		1.	
:		:		:	
:		:		:	
:		:		:	
:		:		:	
:		:		:	
:		:		:	
:		:		:	
:		:		:	
:		:		:	

E: Manipulative

IDEA FROM: C.O.L.A.M.D.A.

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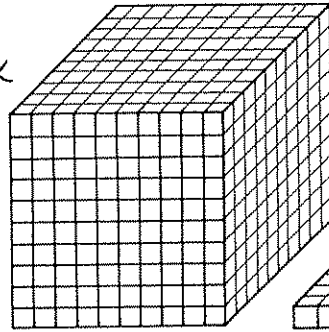


ACTIVITY CARDS - BASE 10 MULTIBASE BLOCKS - II

regrouping to Add
Model
Addition
Whole Numbers

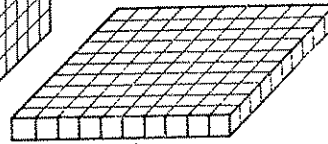


BLOCK

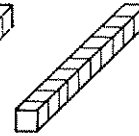


For information and readiness activities
for Multibase Blocks see the section on
Lab Materials.

FLAT



LONG



UNIT

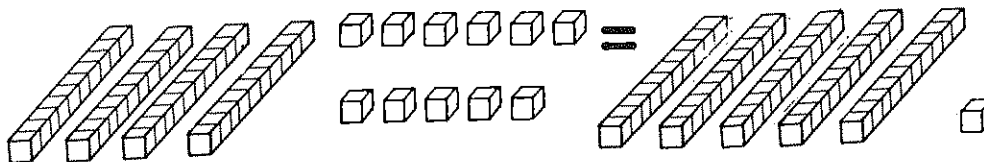
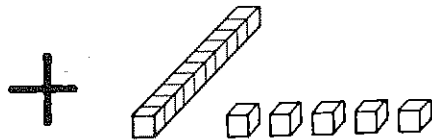
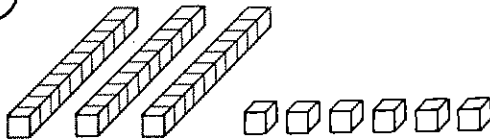


FIND THE SUMS. RECORD YOUR ANSWERS USING THE FEWEST
NUMBER OF PIECES.

EXAMPLE:

①

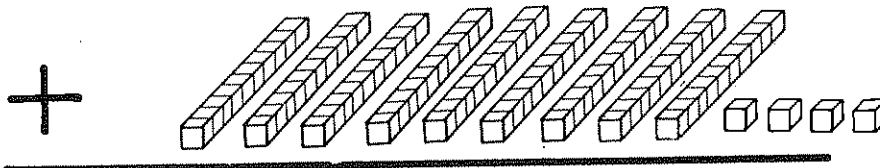
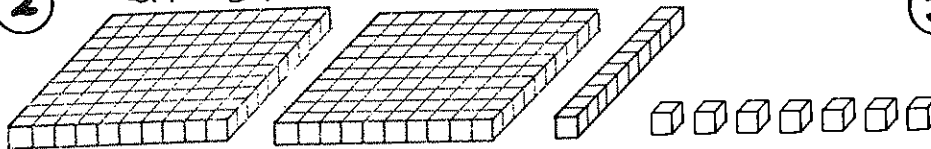
$$36 + 15$$



= 51

②

$$217 + 94$$



③

TRY THESE

$$37 + 58$$

$$49 + 53$$

$$311 + 915$$

$$257 + 354$$

$$874 + 347$$

See Appendix

IDEA FROM: *Multibase Activities, Base 10*

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ACTIVITY CARDS - MATHEMATICAL BALANCE - I (CONTINUED)

TO ADD 13 AND 28 PLACE 13 ON THE LEFT SIDE OF THE BALANCE (1 WASHER ON THE 10 HOOK AND 1 WASHER ON THE 3 HOOK).
NOW PLACE 28 ON THE SAME SIDE (2 WASHERS ON THE 10 HOOK, AND 1 WASHER ON THE 8 HOOK).
PLACE WASHERS ON THE RIGHT SIDE TO BALANCE THE ARM,
USING ONLY TENS AND ONES.

HOW MANY 10'S DID YOU USE? _____

HOW MANY 1'S DID YOU USE? _____

$13 + 28 =$ _____

ADD THESE NUMBERS ON YOUR BALANCE SCALE. FOLLOW THE ABOVE PROCEDURE.

A) $16 + 31 =$

E) $46 + 27 =$

B) $21 + 23 =$

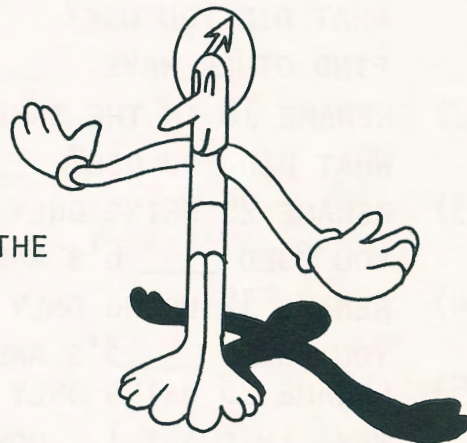
F) $79 + 16 =$

C) $19 + 15 =$

G) $13 + 17 + 14 =$

D) $17 + 34 =$

H) $17 + 9 + 22 =$



MA-I-2

MA-I-3

TO SUBTRACT 16 FROM 29 PLACE 29 ON THE RIGHT SIDE (2 WASHERS ON THE 10 HOOK, AND 1 WASHER ON THE 9 HOOK).

PLACE 16 ON THE LEFT SIDE OF THE BALANCE, (1 WASHER ON THE 10, AND 1 WASHER ON THE 6 HOOK).

PLACE WASHERS ON THE LEFT SIDE TO BALANCE THE ARM, USING ONLY 10'S AND 1'S.

HOW MUCH DID YOU ADD ON TO BALANCE THE ARM? _____ TENS AND _____ ONES.

$26 - 16 =$ _____

SUBTRACT THESE NUMBERS ON YOUR BALANCE SCALE. FOLLOW THE ABOVE PROCEDURE.

A) $33 - 14 =$

E) $28 - 22 =$

B) $25 - 9 =$

F) $30 - 12 =$

C) $19 - 15 =$

G) $62 - 47 =$

D) $22 - 17 =$

H) $50 - 27 =$

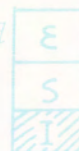
CAN YOU FIGURE OUT ANOTHER WAY TO SUBTRACT ON THE BALANCE SCALE?

This is an example of an additive method for subtraction. See the commentary to WHOLE NUMBERS: Addition/Subtraction.

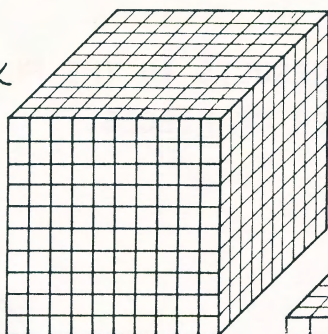


ACTIVITY CARDS - BASE 10 MULTIBASE BLOCKS - II

Regrouping to Add
Model
Addition
WHOLE NUMBERS

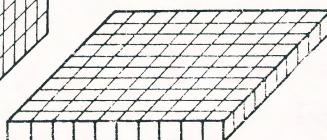


BLOCK

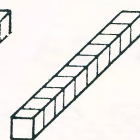


For information and readiness activities for Multibase Blocks see the section on Lab Materials.

FLAT



LONG



UNIT

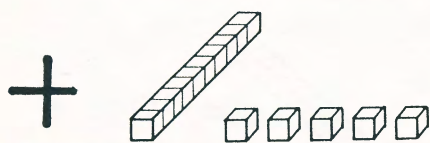
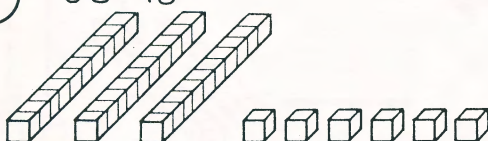


FIND THE SUMS. RECORD YOUR ANSWERS USING THE FEWEST NUMBER OF PIECES.

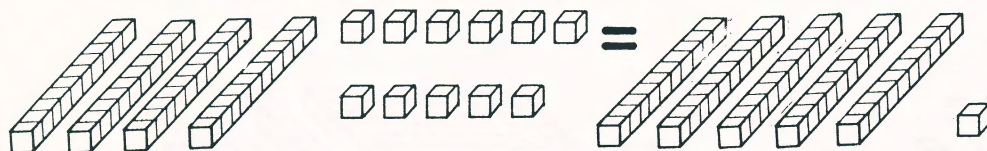
EXAMPLE:

①

$$36 + 15$$



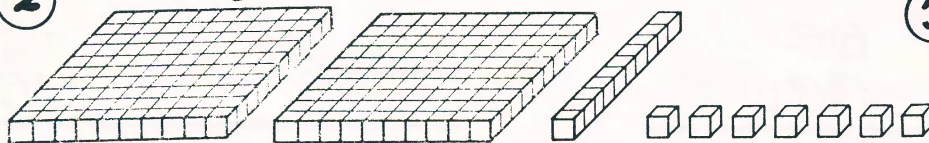
+



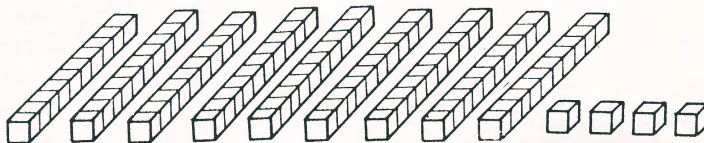
= 51

②

$$217 + 94$$



+



③

TRY THESE

$$37 + 58$$

$$49 + 53$$

$$311 + 915$$

$$257 + 354$$

$$874 + 347$$

TYPE: Manipulative

IDEA FROM: *Multibase Activities, Base 10*

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ACTIVITY CARDS -

BASE 10 MULTIBASE BLOCKS - III

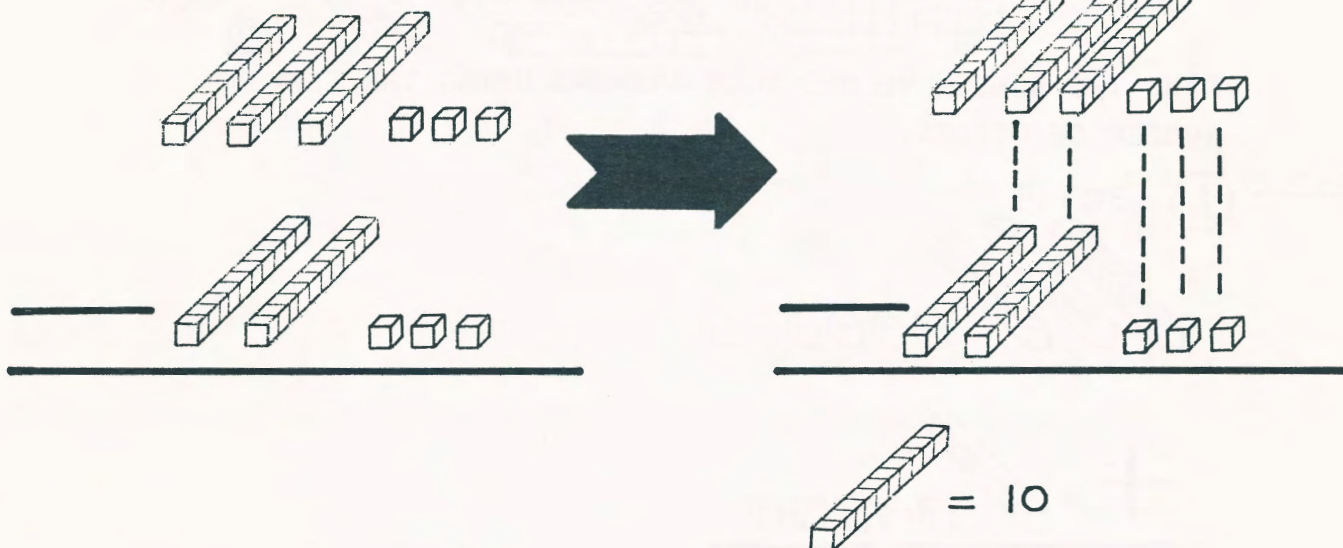
For information and readiness activities for
Multibase Blocks see the section on Lab Materials.

Regrouping to Subtract
Model
Subtraction
WHOLE NUMBERS

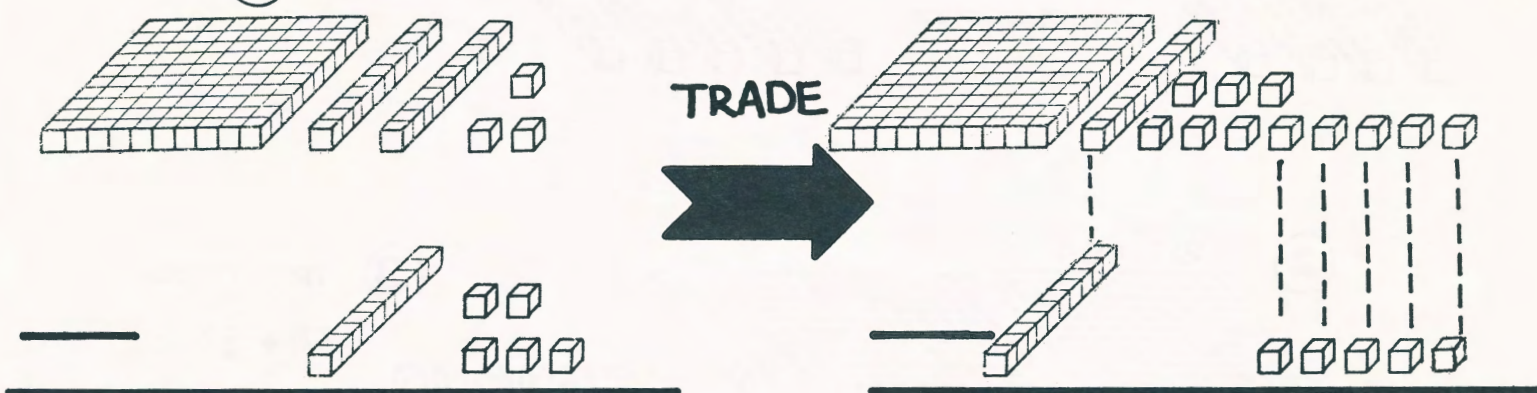


FIND THE DIFFERENCES. EXPRESS YOUR RESULTS IN THE MOST EFFICIENT FORM; THAT IS, USING THE FEWEST NUMBER OF PIECES.

① 33-23



② 123-15



Be sure students match each object in the subtrahend to a corresponding one in the minuend.

③ TRY THESE.

$$75 - 32$$

$$107 - 84$$

$$1001 - 345$$

$$75 - 38$$

$$3121 - 2876$$

$$2040 - 1435$$

TYPE: Manipulative

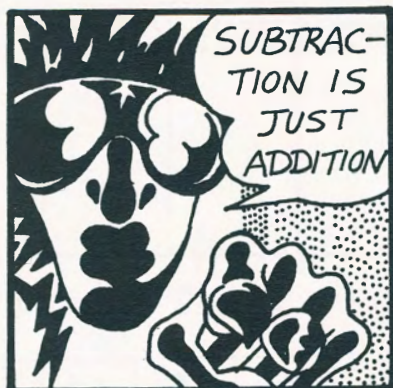
IDEA FROM: *Multibase Activities, Base 10*

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This is an example of a comparing to find the difference method. See the commentary to WHOLE NUMBERS: Addition and Subtraction

MAKING IT ADD UP

Alternate Method
Subtraction
WHOLE NUMBERS



1. $74 - 28$

$$\begin{array}{r} 28 \\ + 2 \\ \hline 30 \\ + 40 \\ \hline 70 \\ + 4 \\ \hline 74 \end{array} \quad \begin{array}{r} 2 \\ 40 \\ 4 \\ \hline 46 \end{array}$$

so $74 - 28 = 46$

2. $135 - 86$

$$\begin{array}{r} 86 \\ + 4 \\ \hline 90 \\ + 40 \\ \hline 130 \\ + 5 \\ \hline 135 \end{array} \quad \begin{array}{r} 4 \\ 40 \\ 5 \\ \hline 49 \end{array}$$

so $135 - 86 = 49$

3. $317 - 49$

$$\begin{array}{r} 49 \\ + 1 \\ \hline 50 \\ + 50 \\ \hline 100 \\ + 200 \\ \hline 300 \\ + 10 \\ \hline 310 \\ + 7 \\ \hline 317 \end{array} \quad \begin{array}{r} 1 \\ 50 \\ 200 \\ 10 \\ 7 \\ \hline 268 \end{array}$$

so $317 - 49 = 268$

ANSWERS WILL VARY, FOR EXAMPLE
PROBLEM NUMBER THREE WOULD BE DONE

$49 + 51 = 100$

$100 + 200 = 300$

$300 + 17 = 317$

FIND THE ANSWERS TO EACH OF THE FOLLOWING BY "ADDING TO SUBTRACT."

$37 - 13 =$

$$\begin{array}{r} 192 \\ - 36 \\ \hline \end{array}$$

800

$$\begin{array}{r} 800 \\ - 247 \\ \hline \end{array}$$

$254 - 127 =$

$4500 - 33 =$

$$\begin{array}{r} 5008 \\ - 273 \\ \hline \end{array}$$

$10,043$

$$\begin{array}{r} 10,043 \\ - 4,519 \\ \hline \end{array}$$

$251 - 128 =$



?
**SUBTRACTION
IS
SUBTRACTION
AND
ADDITION
?**

**SURE.
WATCH
THIS!**

$$\begin{array}{r} 342 \\ - 89 \\ \hline \end{array}$$

$89 = 100 - 11$

$$\begin{array}{r} 342 \\ - 100 \\ \hline 242 \\ + 11 \\ \hline 253 \end{array}$$

**HEY, that's
NEAT! DOES
IT ALWAYS
WORK?**

**HERE'S
ANOTHER
PROBLEM**

Find $754 - 96$.

$96 = 100 - 4$

Calculate $(754 - 100) + 4$, or $654 + 4$, or 658.
Therefore, $754 - 96 = 658$. Easy, isn't it.

**TRY
THIS
METHOD**

$$612 - 95 = (612 - 100) + \underline{\quad} = \underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$751 - 87 = (\underline{\quad} - \underline{\quad}) + \underline{\quad} = \underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$892 - 197 = (\underline{\quad} - 200) + \underline{\quad} = \underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$531 - 286 = (\underline{\quad} - \underline{\quad}) + \underline{\quad} = \underline{\quad} + \underline{\quad} = \underline{\quad}$$

**DO
THESE
MENTALLY**

$$546 - 99 = \underline{\quad}$$

$$1121 - 796 = \underline{\quad}$$

$$453 - 299 = \underline{\quad}$$

$$513 - 391 = \underline{\quad}$$

$$751 - 687 = \underline{\quad}$$

$$1276 - 899 = \underline{\quad}$$

$$811 - 589 = \underline{\quad}$$

$$5843 - 3996 = \underline{\quad}$$

**CAN YOU
EXPLAIN
WHY THIS
WORKS?**

Try to "Subtract and Add" these problems.

$$\begin{array}{r} 882 \\ - 197 \\ \hline \end{array}$$

$$\begin{array}{r} 731 \\ - 83 \\ \hline \end{array}$$

$$\begin{array}{r} 1234 \\ - 571 \\ \hline \end{array}$$

$$\begin{array}{r} 1032 \\ - 175 \\ \hline \end{array}$$



THERE IS NO...



Alternate Method
Subtraction
WHOLE NUMBERS



TEACHER PAGE

PEOPLE USE MANY DIFFERENT WAYS TO SUBTRACT NUMBERS.
THIS METHOD IS CALLED AUSTRALIAN SUBTRACTION...

... MAYBE YOU'LL LIKE IT!! CAN YOU FIGURE OUT
HOW IT WORKS?

LOOK AT THESE EXAMPLES:

$$\begin{array}{r} 88 \\ -23 \\ \hline \end{array}$$

$$\begin{array}{l} \rightarrow 3 + \boxed{?} = 8 \\ \rightarrow 2 + \boxed{?} = 8 \end{array}$$

$$\begin{array}{r} 72 \\ -35 \\ \hline \end{array}$$

$$\begin{array}{l} \rightarrow 5 + \boxed{?} = 2 \\ \rightarrow 5 + \boxed{?} = 12 \\ \rightarrow 3 + 1 + \boxed{?} = 7 \\ \rightarrow 4 + \boxed{?} = 7 \end{array}$$

I CAN'T
DO IT. I WILL
ADD A 10 TO
THE 2. AND A
1 TO THE 3.

$$\begin{array}{r} 6001 \\ -2756 \\ \hline \end{array}$$

$$\begin{array}{l} \rightarrow 6 + \boxed{?} = 1000 \\ \rightarrow 6 + \boxed{?} = 11 \\ \rightarrow 5 + 1 = 6, 6 + \boxed{?} = 0, 6 + \boxed{?} = 10 \\ \rightarrow 7 + 1 = 8, 8 + \boxed{?} = 0, 8 + \boxed{?} = 10 \\ \rightarrow 2 + 1 = 3, 3 + \boxed{?} = 6 \end{array}$$

CAN'T DO IT?
ADD 10 to 1 = 11
ADD 1 to 5 =

TRY

THESE:

$$\begin{array}{r} 48 \\ -14 \\ \hline \end{array}$$

$$\begin{array}{r} 61 \\ -27 \\ \hline \end{array}$$

$$\begin{array}{r} 80 \\ -36 \\ \hline \end{array}$$

$$\begin{array}{r} 732 \\ -158 \\ \hline \end{array}$$

$$\begin{array}{r} 826 \\ -174 \\ \hline \end{array}$$

$$\begin{array}{r} 701 \\ -327 \\ \hline \end{array}$$

$$\begin{array}{r} 300 \\ -163 \\ \hline \end{array}$$

$$\begin{array}{r} 7531 \\ -2486 \\ \hline \end{array}$$

$$\begin{array}{r} 692 \\ -586 \\ \hline \end{array}$$

HAVE YOU DISCOVERED THE
METHOD? DO YOU KNOW
WHY IT WORKS?

TYPE: Paper & Pencil/Transparency

Students may be interested
in seeing that there is
more than one correct
algorithm for subtraction.

COUNT THE CONE



I'm Lee the Big Three.
I play baseball. Count me up
and see how many homers I hit.








I'm the Mysterious Mathematical Lady. I started teaching math so long ago, I have forgotten how long I've been teaching. Count me up and find out.

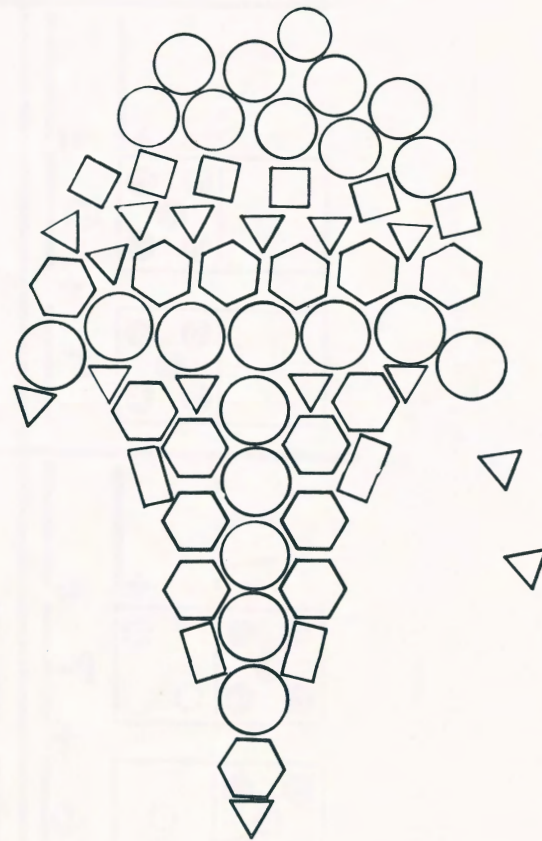


I'm Miss Martha Matics.
All of my students want to
know how old I am. Count me
up and find out.



I'm Matte Matics.
I forgot to study for my
math test. If you count
me up you can find the
score I got.

 = 1¢  = 2¢  = 5¢  = 10¢  = 25¢





FILL IN THE DOMINOES

Addition
WHOLE NUMBERS



FILL IN THE MISSING SPOTS THEN ADD BOTH WAYS.

Beans or dominoes may be used to solve these problems, so that there would be no record of error.

 $6 + 7 =$	 $9 + 5 =$	 $9 + 7 =$
 $8 + 1 =$	 $8 + 6 =$	 $8 + 11 =$
 $9 + 6 =$	 $4 + 10 =$	 $8 + 8 =$
 $9 + 5 =$	 $8 + 5 =$	 $9 + 6 =$

TYPE: Paper & Pencil/Manipulative

IDEA FROM: *Drill and Practice at the Problem Solving Level*, Activity Pages

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DOUBLE DIFFERENCES

SUBTRACT TWO WAYS:
GOING DOWN AND GOING
ACROSS. THEN SUB-
TRACT THE ANSWERS
GOING DOWN AND GOING
ACROSS. WHAT HAPPENS?

Subtraction
WHOLE NUMBERS



HERE'S
AN
EXAMPLE

9	6	3
5	4	1
4	2	2

12	8	
6	5	

13	9	
6	3	

17	9	
8	6	

19	7	
6	3	

27	9	9
8	3	2
5	2	1

ARE YOU
READY FOR A
HARD ONE?

SEE IF
YOU CAN
MAKE ONES
THAT WORK
!!!

Algebraic solution:

a	c	a-c
b	d	b-d
a-b	c-d	a-b-c+d

Some students may get
negative solutions.

TYPE: Paper & Pencil

IDEA FROM: Drill and Practice at the Problem Solving Level, Activity Pages

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ADD-A-BOX

Addition
WHOLE NUMBERS



Find a path from the START to the FINISH that adds up to the FINISH NUMBER.
Cross no number twice. No corner crossings allowed, either. Each group of two, three or four numbers adds up to 10 if you are following the right path.

START

6	7	4	5	1
6	1	4	3	2
5	1	6	3	7
2	1	3	3	3
4	7	3	1	6

Harder add-a-boxes can be made using positive or negative numbers. Another possible variation is to have the paths not add up to tens.

You can make an add-a-box for your student using this blank box. Or perhaps students would like to make their own add-a-box.

FINISH ► 70

START

8	7	3	6	4
3	4	1	5	9
6	5	3	7	1
5	2	8	5	6
3	1	1	2	3

FINISH ► 60

START

FINISH ►

TYPE: Paper & Pencil/Puzzle

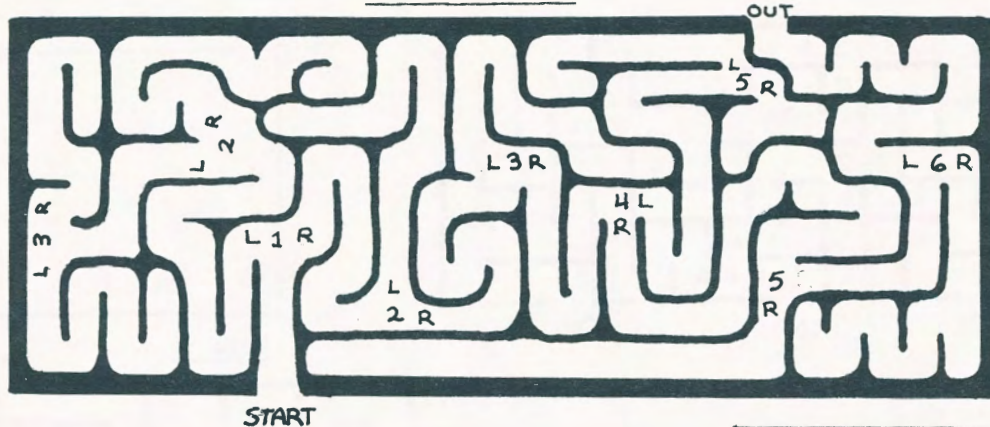
GUIDED MAZES

Addition/Subtraction
WHOLE NUMBERS



Directions: Work a problem and select the correct answer from one of the columns. If the answer is in the column marked Right travel the maze until the first intersection and then go right. Continue in the maze until the next intersection. Work the problem to decide which way to go.

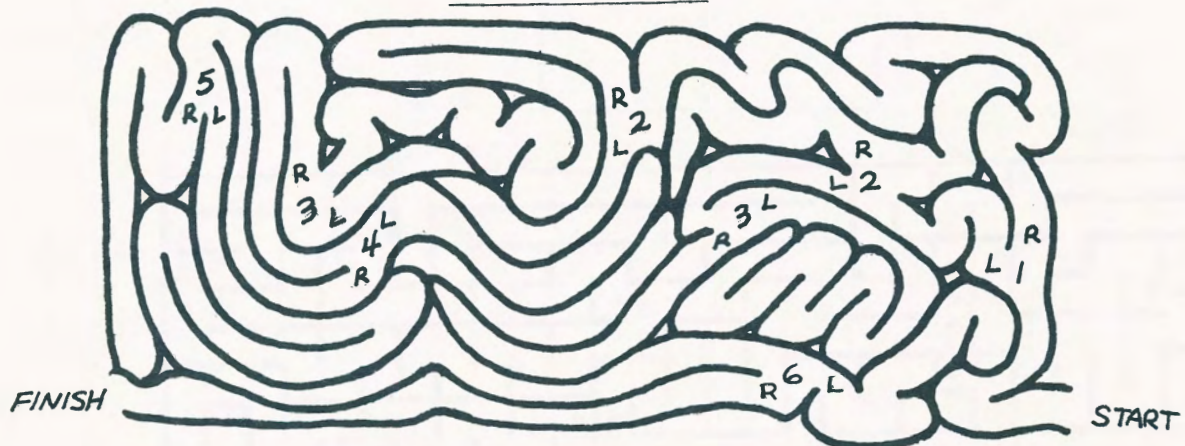
Addition Maze



- | | | |
|--|--|--|
| 1. $\begin{array}{r} 37 \\ + 19 \\ \hline \end{array}$ | 2. $\begin{array}{r} 48 \\ + 15 \\ \hline \end{array}$ | 3. $\begin{array}{r} 56 \\ + 91 \\ \hline \end{array}$ |
| 4. $\begin{array}{r} 73 \\ + 16 \\ \hline \end{array}$ | 5. $\begin{array}{r} 125 \\ + 396 \\ \hline \end{array}$ | 6. $\begin{array}{r} 732 \\ + 985 \\ \hline \end{array}$ |

Left	Right
1717	89
521	63
1617	56
99	159
621	147
156	79

Subtraction Maze



- | | | |
|---|--|---|
| 1. $\begin{array}{r} 76 \\ - 39 \\ \hline \end{array}$ | 2. $\begin{array}{r} 45 \\ - 27 \\ \hline \end{array}$ | 3. $\begin{array}{r} 183 \\ - 46 \\ \hline \end{array}$ |
| 4. $\begin{array}{r} 239 \\ - 51 \\ \hline \end{array}$ | 5. $\begin{array}{r} 587 \\ - 489 \\ \hline \end{array}$ | 6. $\begin{array}{r} 111 \\ - 22 \\ \hline \end{array}$ |

Left	Right
88	18
27	137
28	89
97	37
147	29
98	188

TYPE: Paper & Pencil/Puzzle

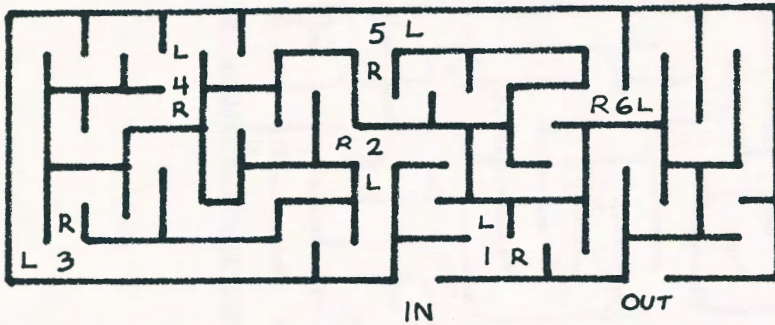
IDEA FROM: C.O.L.A.M.D.A.

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GUIDED MAZES (CONTINUED)

The three mazes below are provided for you to prepare more personalized mazes for your class. You can use the mazes for all four whole number operations as well as extend it to fractions, decimals, etc., using various other computations.

Maze A

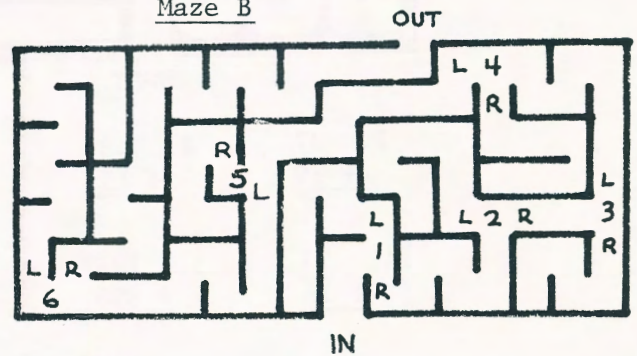


- | | |
|------|------|
| 1. L | 4. L |
| 2. L | 5. L |
| 3. L | 6. L |

Left	Right

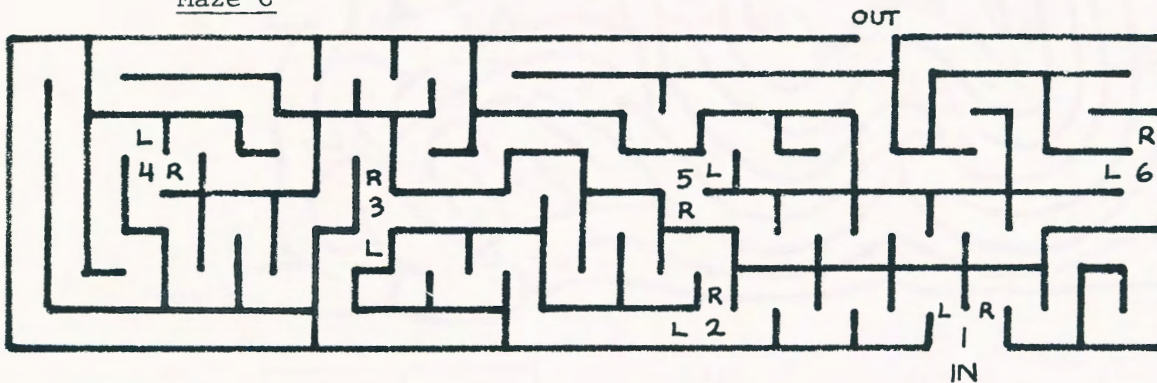
Left	Right

Maze B



- | | |
|------|------|
| 1. R | 4. L |
| 2. R | 5. L |
| 3. L | 6. R |

Maze C



- | | | |
|------|------|------|
| 1. L | 2. R | 3. R |
| 4. R | 5. R | 6. L |

Left	Right

PICTURE PROBLEMS 1



How many feet of string? _____

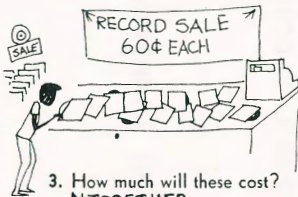


How tall is the stack of bricks? _____



By how many points did the home team win? _____

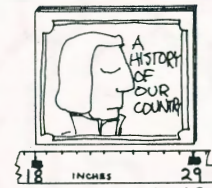
John weighs 112 pounds. His father is 161. How much heavier is the father? _____



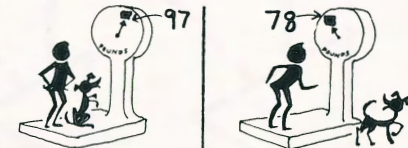
3. How much will these cost? **ALTOGETHER** _____



Total cost = _____

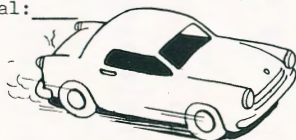


How wide is the book? _____

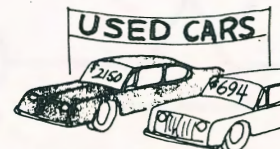


How much does the dog weigh? _____

On a 3-day trip, a salesman drove 243 the first day, 319 the second day, and 186 the third day. How far total: _____



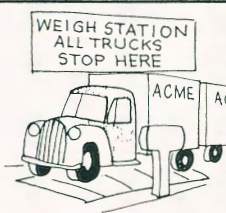
Joe and his friends ate 34 B.L.T. sandwiches, 27 P&J sandwiches, and 18 salami & cheese sands. How many sands did they eat altogether? _____



How much more does the **LEFT** car cost? _____



The glove that Carlos wants to buy costs \$7.89. If he has \$4.91, how much more does he need? _____



Here are the weight tickets:

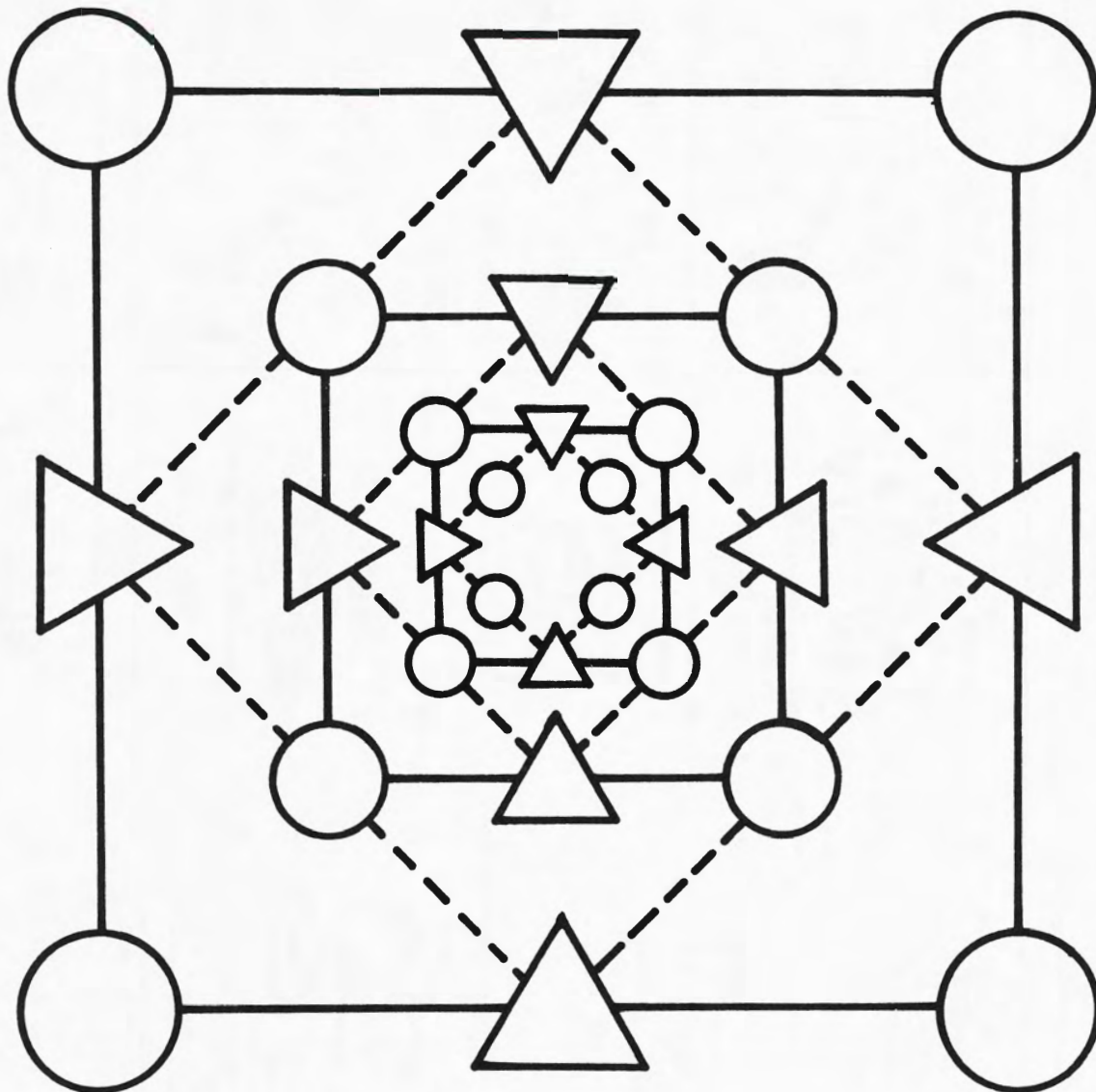


What is the total weight? _____



2. How far has he traveled? _____

DIFFY



Choose any four numbers.

Place them in the four outside circles.

Record the differences of two adjacent circles in the triangle between them.

Continue until you have to stop.

You win if you are able to reach the innermost circles without writing zeros in them.

IDEA FROM: "Diffy," *Arithmetic Teacher*, October 1971

Permission to use granted by the National Council of Teachers of Mathematics



DIFFY (CONTINUED)

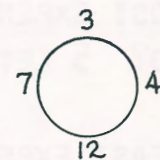
TEACHER PAGE

A game ends when either all zeros are obtained, or when the innermost set of circles are reached. The game given on the student page is a 6 move game. Variations could include:

1. a 7, 8, or 9 move game.
2. finding four numbers which take exactly 6 moves to get all zero's; exactly 5 moves; 4 moves; and so forth.
3. finding winners to the 6 move game using only single digits in the outside circles.
4. using fractions or decimals for the starting numbers. Diffy provides a good vehicle for drill and practice in subtracting fractions and also gives practice in comparing them since the rule does not permit a larger number to be subtracted from a smaller.
5. using division rather than subtraction (DIVVY). Here we should end with all ones and do not want to use zero for a starting number. In Divvy never divide a smaller by a larger. Also, disregard any remainder and record only the whole number in the quotient.
6. having students compare 2 games where the starting entries of the second double those of the first game. Compare using triples and other multiples.
7. having students compare 2 games where the starting entries of a second game are 5 more than these of the first game.

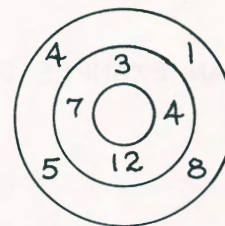
ANOTHER FORMAT

Place 4 whole numbers around a circle.



Enclose these numbers in a circle and place the differences obtained by subtracting the smaller from the larger in each adjacent pair. Continue until 4 zeros appear.

Variation: See when zeros can be obtained with
3 numbers . . . with 5 numbers,
. . . with 8 numbers.



IDEA FROM: "Diffy," *Arithmetic Teacher*, October 1971

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IF LETTERS WERE DOLLARS

Addition
WHOLE NUMBERS



SUPPOSE LETTERS WERE MONEY. USE THE CHART BELOW TO SEE WHO HAS THE MOST EXPENSIVE NAME IN CLASS.

A = \$1	E = \$5	I = \$9	M = \$13	Q = \$17	U = \$21
B = \$2	F = \$6	J = \$10	N = \$14	R = \$18	V = \$22
C = \$3	G = \$7	K = \$11	O = \$15	S = \$19	W = \$23
D = \$4	H = \$8	L = \$12	P = \$16	T = \$20	X = \$24
					Y = \$25
					Z = \$26

BILL COMPUTES HIS NAME THIS WAY.

$$\begin{array}{cccccccccccc} B & I & L & L & & C & O & L & L & E & C & T & O & R \\ 2 & + & 9 & + & 12 & + & 12 & + & 3 & + & 15 & + & 12 & + & 12 & + & 5 & + & 3 & + & 20 & + & 15 & + & 18 & = & \$138 \end{array}$$

HOW MUCH IS THE NAME OF YOUR SCHOOL WORTH?

HOW ABOUT YOUR TEACHER'S NAME? YOUR PRINCIPAL'S NAME?

WHAT'S THE MOST EXPENSIVE 3 LETTER WORD YOU CAN THINK OF?
4 LETTER WORD? 5 LETTER WORD?

WHAT'S THE LEAST EXPENSIVE 3 LETTER WORD YOU CAN FIND? 4 LETTER WORD?
TRY SOME MORE.

USELESS IS AN EXAMPLE OF A \$100 WORD. CAN YOU FIND MORE \$100 WORDS?

Variations: Values can be changed to generate different answers.
Values can be chosen at random or letters may have values assigned according to their probability of occurrence.

TYPE: Paper & Pencil/Transparency

IDEA FROM: WYMOLAMP, Secondary Materials

Permission to use granted by Fremont County Unified School District #25

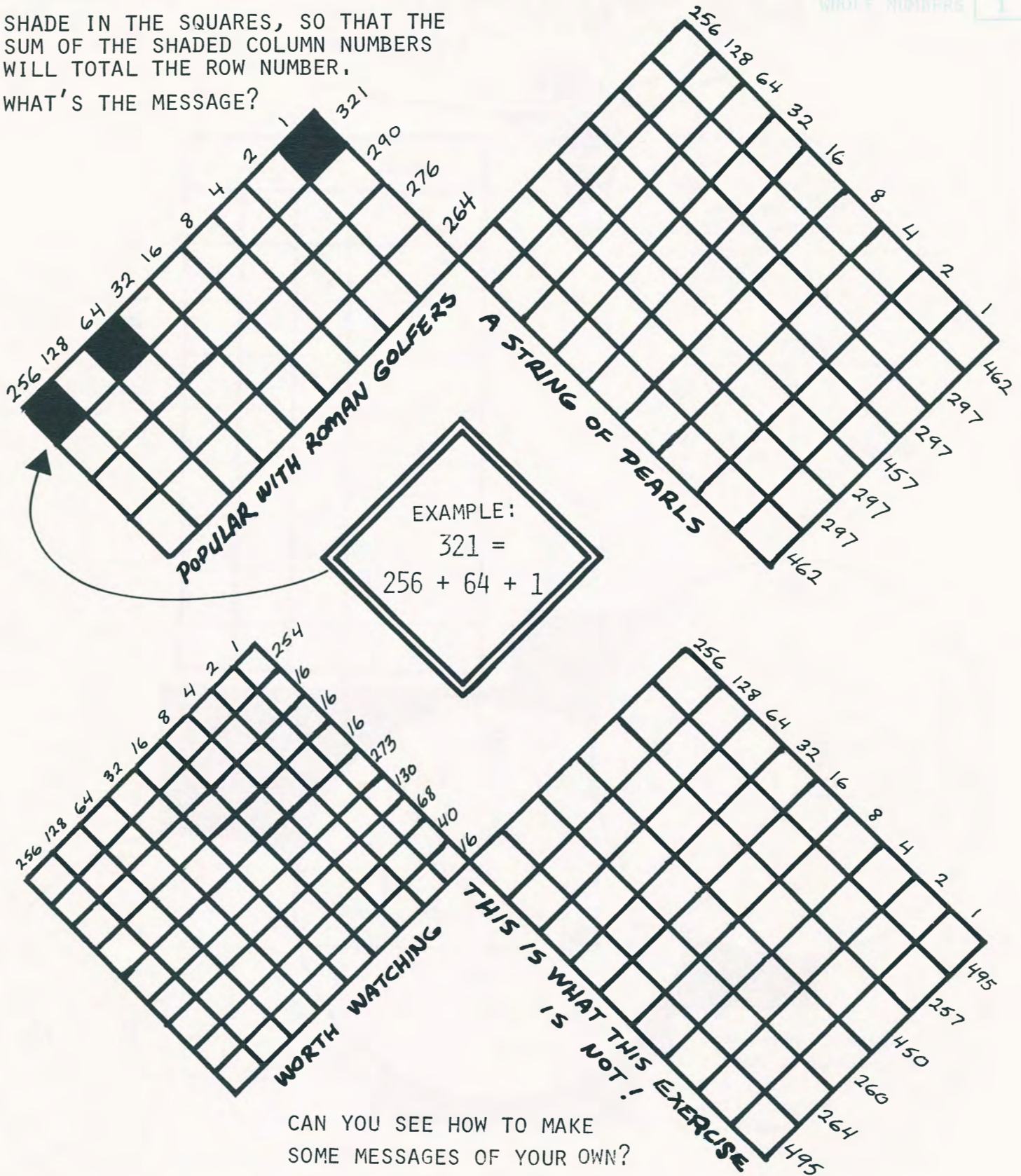
SHADY SQUARES

ADDITION
WHOLE NUMBERS



SHADE IN THE SQUARES, SO THAT THE
SUM OF THE SHADED COLUMN NUMBERS
WILL TOTAL THE ROW NUMBER.

WHAT'S THE MESSAGE?



CAN YOU SEE HOW TO MAKE
SOME MESSAGES OF YOUR OWN?
TRY IT.

TYPE: Paper & Pencil/Puzzle



Addition
WHOLE NUMBERS



Al, the apiarist, sells his bees by the gram. Assuming he can get the bees to rest on one pan while he places the weights on the other pan, which of the following sets of gram weights, A, B, or C can he use to fill his order list?



ORDER	LIST		
GRAMS	A	B	C
1	✓		
2			
4	✓		
5			
7			
13			
19			
25			

Check which weights can be measured with Set A, Set B, and Set C. Which set can be used to do them all?

A

1 3 5 7 9

B

1 2 4 8 16

C

1 2 3 8 13

? Suppose Al wants to weigh bees in amounts of 1 gm., 2 gm., 3 gm., . . . , 31 gm. Find the fewest number of weights he could use.

TYPE: Paper & Pencil



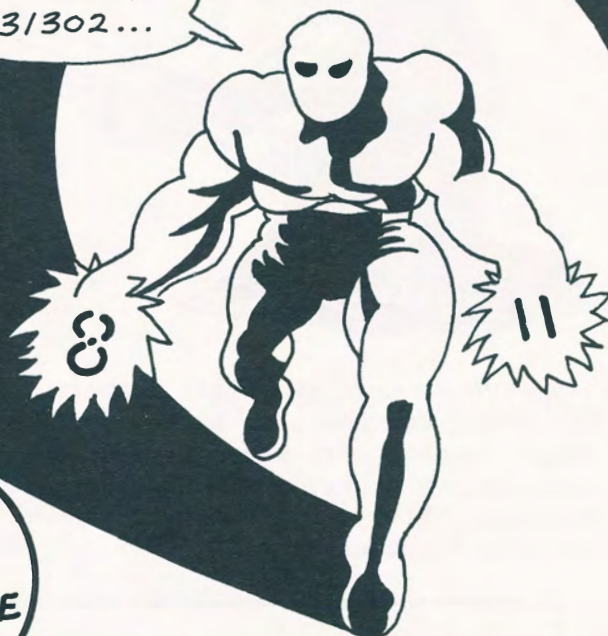
PALINDROMES!

STARRING
LETTERMAN +
NUMERALMAN



HAH! PEP,
SIS, TOOT,
MOM, POP.

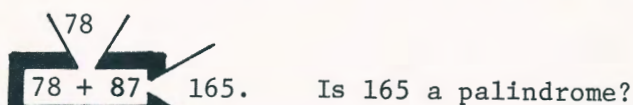
121, 2442,
15751, **5335**,
2031302...



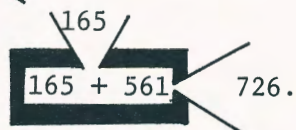
CAN
YOU GUESS
WHAT A
PALINDROME
IS ?!!

You can make palindromes from any number using the Reverse and Add box.

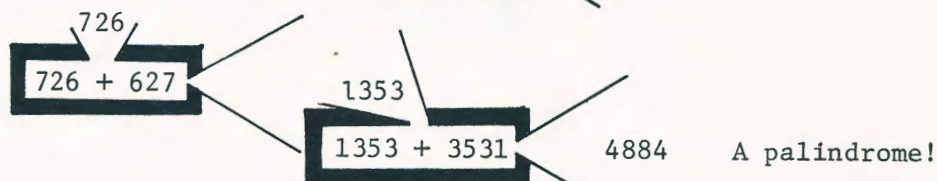
Is 78 a palindrome? No. Then drop it in the Reverse and Add box.



No. Then into the R and A box.



Repeat:



Can you make palindromes from these numbers?

57 588 238 1924

Can you find a number that will produce a palindrome after 5 steps? 6 steps? more?

TYPE: Paper & Pencil

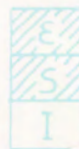
IDEA FROM: *Aftermath*, Volume 2

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REVERSING DIGITS



Addition/Subtraction
WHOLE NUMBERS



STIGID GNISREVER



WATCH
MY MAGIC

$$\begin{array}{r} 74 \\ - 47 \\ \hline 27 \\ + 72 \\ \hline 99 \end{array}$$

NOW YOU TRY
IT WITH ANY TWO
DIGIT NUMBER.
DO YOU ALWAYS
GET 99?

Try several more two-digit numbers. What happens? Try the same method using a three-digit number. Try several more three-digit numbers. What happens? How about four-digit numbers? five-digit numbers? Anyone for ten-digit numbers?



Here's why the two-digit reversing magic works:

Choose any two-digit number, say

If $a = b$, we obtain zero

If $a > b$, then we have

tens	ones
a	b

tens	ones
a	b
- b	a
(a - 1) - b	(b + 10) - a
+ (b + 10) - a	(a - 1) - b
9	9

If $b > a$, then we compute

tens	ones
b	a
- a	b
	as above.

The justification for the three-digit problem is similar, except it is slightly more complicated.

For 3 digits, you should always get 1089.

LARGER — SMALLER ETC.

Addition/Subtraction
WHOLE NUMBERS



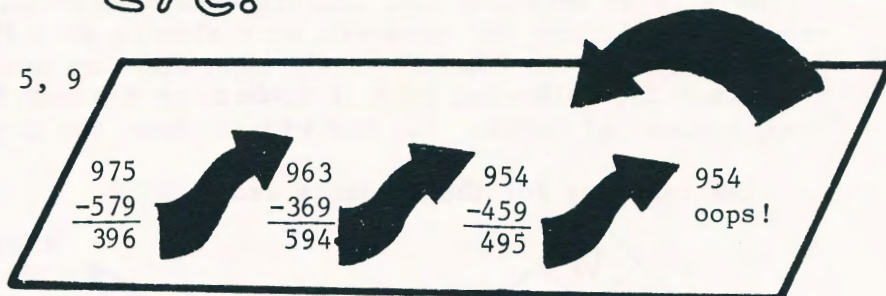
Choose 3 different digits. 7, 5, 9

Write the greatest and
smallest numbers. 975

$$\begin{array}{r} 975 \\ - 579 \\ \hline \end{array}$$

Subtract.

$$\begin{array}{r} 975 \\ - 579 \\ \hline 396 \end{array}$$



Repeat the steps using the answer.

Try some more three-digit numbers. What happens? Does this work for two-digit numbers? Try some. How about four-digit numbers? Try several.



FOR THE
VERY **BRAVE.**

Try six-digit numbers.

When you use 4-digit numbers you will always arrive at 6174, the Jordanian number.

An extension of this activity is to have the students see if they can find a four-digit number that arrives at 6174 after one subtraction; ...after two subtractions; ...etc. Actually, 6174 should appear after, at most, seven subtractions. Can the students find such a number? Similar classifying activities could be done in the two digit and three digit cases.

Two digits is easy.

$$\begin{array}{r} 75 \\ - 57 \\ \hline 18 \end{array}$$

$$\begin{array}{r} 81 \\ - 18 \\ \hline 63 \end{array}$$

$$\begin{array}{r} 63 \\ - 36 \\ \hline 27 \end{array}$$

$$\begin{array}{r} 72 \\ - 27 \\ \hline 45 \end{array}$$

$$\begin{array}{r} 54 \\ - 45 \\ \hline 9 \end{array}$$

For 6 digits: 631764 or 840852 or 549945

For 6-digit numbers there are 384 different results possible at the end of the first subtraction. Thirty of these lead to the "repeater 631764, one leads to 549943 and the remaining lead to the cycle (840852, 860832, 862632, 642645, 420876, 851742, 075084.)

Students can do this activity in pairs—one using a calculator and the other recording results.

TYPE: Paper & Pencil

IDEA FROM: *Ingenuity in Mathematics*

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SQUARE SUMS

TEACHER DIRECTED ACTIVITY

Addition

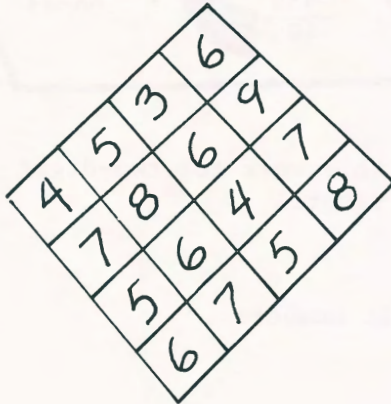
WHOLE NUMBERS



Here is an activity that all students seem to enjoy. The only basic skill needed is addition, but students must also be able to understand and follow directions. Why it "works" should generate many questions.

Place the following grid of numbers on the board and have students copy it onto a sheet of paper. You may wish to have the grid on a handout.

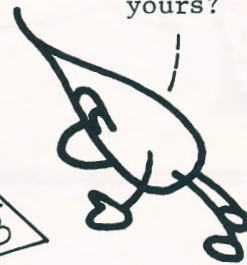
Instructions for the students are:



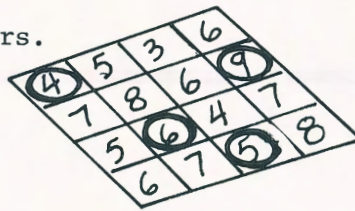
Circle any number.

Now circle another number in a different row and different column. Circle two more numbers in different rows and columns.

My sum is 24. What's yours?



Add the four circled numbers.



Here are several more grids. Students can be given one of these each day for a few days. Encourage them to try and discover why it works. (For the solution see the next page.)

22	20	16	24
17	15	11	19
20	18	14	22
19	17	13	21

5	9	7	12	17
11	15	13	18	23
3	7	5	10	15
1	5	3	8	13
8	12	10	15	20

15	13	9	17
20	18	14	22
23	21	17	25
19	17	13	21

14	20	3	12	7
11	17	0	9	4
32	38	21	30	25
17	23	6	15	10
19	25	8	17	12

SQUARE SUMS (PAGE 2)

HOW IT WORKS:

Why does everyone get 24? Examine the following addition facts table.

+	2	3	1	4
2	④	5	3	6
5	7	8	6	⑨
3	5	⑥	4	7
4	6	7	⑤	8

Adding the 4 numbers from each row and column has the net effect of adding the 8 outside numbers.

$$\begin{array}{c} (2+2) \\ \swarrow \quad \searrow \\ 4 \end{array} + \begin{array}{c} (5+4) \\ \swarrow \quad \searrow \\ 9 \end{array} + \begin{array}{c} (3+3) \\ \swarrow \quad \searrow \\ 6 \end{array} + \begin{array}{c} (4+1) \\ \swarrow \quad \searrow \\ 5 \end{array} = 24$$

$$4 + 9 + 6 + 5 = 24$$

Let's construct a 3 x 3 grid.

+	6	9	8
9			
7			
8			



15	18	17
13	16	15
14	17	16

The square sum will be the sum of the numbers chosen for the outside.

EXTENSIONS:

- I. Once students understand how it works, you might give them these grids and ask them to fill in the outside numbers.

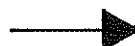
+			
	7	9	3
	16	18	12
	11	13	7

+					
	14	20	3	12	7
	32	38	21	30	25
	19	25	8	17	12
	11	17	0	9	4
	17	23	6	15	10

- II. The grids could be filled with fractions or decimals, or a combination.

EXAMPLES:

+	$\frac{1}{4}$	$\frac{5}{6}$	$\frac{2}{3}$
$\frac{1}{2}$			
$\frac{1}{3}$			
$\frac{1}{6}$			



$\frac{3}{4}$	$\frac{11}{12}$	$\frac{7}{6}$
$\frac{7}{12}$	$\frac{7}{6}$	1
$\frac{5}{12}$	1	$\frac{5}{6}$

+	.7	.75	.4
$\frac{1}{2}$			
$\frac{1}{4}$			
$\frac{1}{5}$			



1.2	$\frac{5}{4}$.9
.95	1	.65
$\frac{9}{10}$	$\frac{19}{20}$	$\frac{3}{5}$

SQUARE SUMS (PAGE 3)

Have each student make a 5 x 5 grid on a piece of paper. Next, have each student place her favorite digit in the upper left-hand square then complete the grid by numbering the squares consecutively.

Example:

6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30

Favorite digit is 6.

Your Directions For The Students:

Again, circle 5 numbers, each representing a different row and column. Add the 5 numbers together. Tell me the sum of the numbers.

To find the student's favorite digit merely subtract 60 from the total he gives you and then divide that number by 5.

$$\frac{\text{SUM} - 60}{5} = \text{FAVORITE DIGIT}$$

In the example above the $\text{SUM} = 10 + 12 + 16 + 23 + 29 = 90$;

$$\frac{90 - 60}{5} = \frac{30}{5} = 6.$$

Algebra students should be able to discover why this works by using the following grid:

a	$a+1$	$a+2$	$a+3$	$a+4$
$a+5$	$a+6$	$a+7$	$a+8$	$a+9$
$a+10$	$a+11$	$a+12$	$a+13$	$a+14$
$a+15$	$a+16$	$a+17$	$a+18$	$a+19$
$a+20$	$a+21$	$a+22$	$a+23$	$a+24$

When we add 5 entries, each from a different row and column, we get $5a + 60$.

If the sum = $5a + 60$ then
 $\text{sum} - 60 = 5a$ and

$$\frac{\text{sum} - 60}{5} = a.$$

SEEING INTO THE FUTURE

Addition
WHOLE NUMBERS



TEACHER DIRECTED ACTIVITY

ANNOUNCE THAT YOU ARE GOING TO ADD UP SEVERAL 3-DIGIT NUMBERS, SOME PROVIDED BY THE CLASS, SOME THAT YOU WILL SELECT, BUT THAT YOU WILL WRITE DOWN THE ANSWER AFTER KNOWING ONLY THE FIRST NUMBER.

PROCEDURE:

STUDENTS PROVIDE FIRST NUMBER.

YOU ANNOUNCE: THE TOTAL WILL

BE **2817**

EXAMPLE

819

STUDENTS PROVIDE ONE NUMBER.

193

806

YOU SELECT THE NEXT NUMBER.

275

STUDENTS PROVIDE ANOTHER NUMBER.

724

YOU SELECT THE LAST NUMBER.

TOTAL **2817**

HAVE STUDENTS FIND THE TOTAL.

819

THE SECRET:

$$2817 = 819 + 2000 - 2 \text{ AND } 2000 - 2 = 999 + 999$$

$$\begin{array}{r} 193 \\ 806 \end{array} \begin{array}{l} > \\ > \end{array} 999$$

EACH TIME STUDENTS PROVIDE A NUMBER, WRITE DOWN A NUMBER TO MAKE THE SUM OF THE TWO NUMBERS EQUAL TO 999.

$$\begin{array}{r} 275 \\ 724 \\ \hline 2817 \end{array} \begin{array}{l} > \\ > \end{array} 999$$

IF STUDENTS PROVIDE:

999

17

974

YOU SELECT:

0

982

25

YOU MAY WANT TO TRY VARIATIONS SUCH AS: SEVEN NUMBERS, ALLOWING 2 OR 4-DIGIT NUMBERS, KEYING ON 499 INSTEAD OF 999, ETC.



HUMAN COMPUTER

Addition
WHOLE NUMBERS



TEACHER DIRECTED ACTIVITY

GIVE THE FOLLOWING INSTRUCTIONS TO YOUR STUDENTS:

1. WRITE DOWN THE LETTERS A-J IN A COLUMN.
2. PUT YOUR FAVORITE DIGIT NEXT TO A.
3. PUT YOUR SECOND FAVORITE DIGIT NEXT TO B.
4. ADD THE TWO NUMBERS TOGETHER AND PUT THE SUM NEXT TO C.
5. ADD THE NUMBERS NEXT TO B AND C AND PUT THE SUM NEXT TO D.
6. CONTINUE THIS PROCESS.

NOW WALK AROUND THE ROOM, AND AS SOON AS THE STUDENT HAS WRITTEN THE 10TH NUMBER, YOU LOOK AT HIS PAPER AND IMMEDIATELY WRITE DOWN A NUMBER WHICH IS THE SUM OF ALL TEN NUMBERS. (MULTIPLY THE NUMBER NEXT TO G BY 11.)

7. ADD ALL TEN NUMBERS.

EXAMPLE:

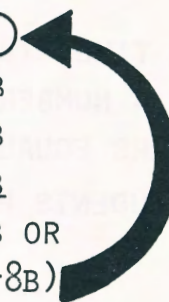
A	2	(FAVORITE DIGIT)
B	7	(SECOND FAVORITE DIGIT)
C	9	(2 + 7)
D	16	(7 + 9)
E	25	(9 + 16)
F	41	
G	66	
H	107	
I	173	
J	280	

TOTAL = 726 OR 11×66

WHY IT WORKS

$$\begin{aligned}
 &A \\
 &B \\
 &A + B \\
 &A + 2B \\
 &2A + 3B \\
 &3A + 5B \\
 &5A + 8B \\
 &8A + 13B \\
 &13A + 21B \\
 &21A + 34B \\
 &55A + 88B \text{ OR} \\
 &11(5A + 8B)
 \end{aligned}$$

Have algebra students try to develop this proof!

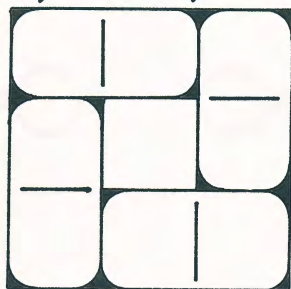
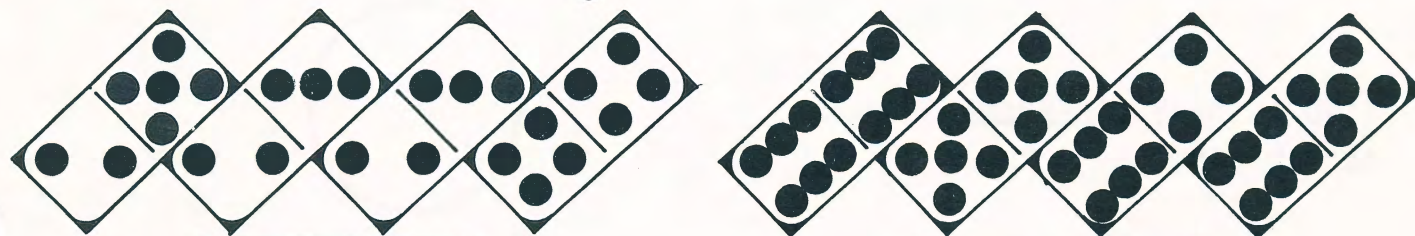
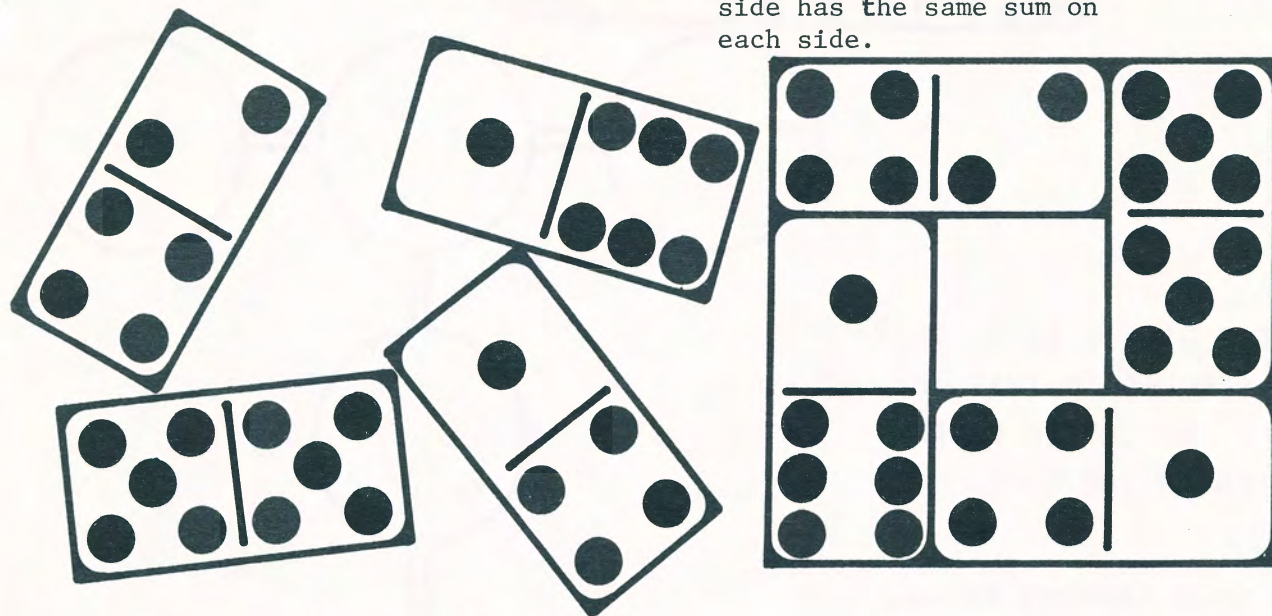


DOMINO DONUTS

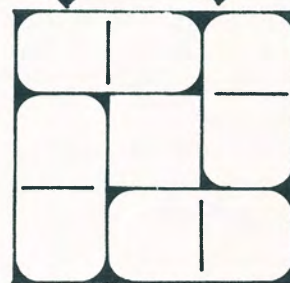
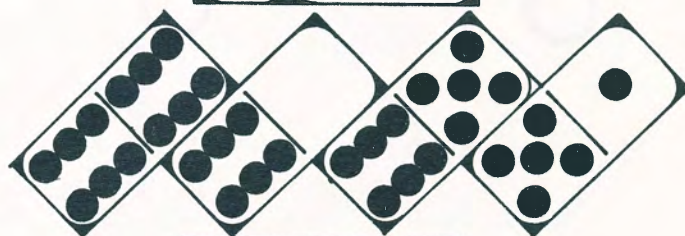
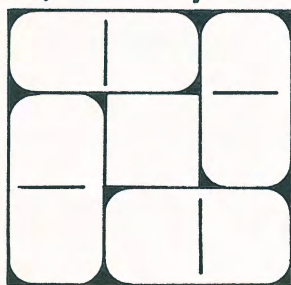
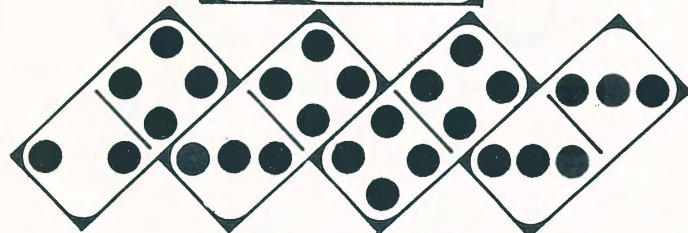
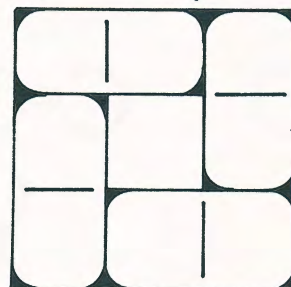


The four dominoes have been arranged so that the donut side has the same sum on each side.

Addition
WHOLE NUMBERS



Make donuts so each side has the same sum.
Record your answers in the blank donuts.



TYPE: Paper & Pencil/Manipulative/Wuzzle

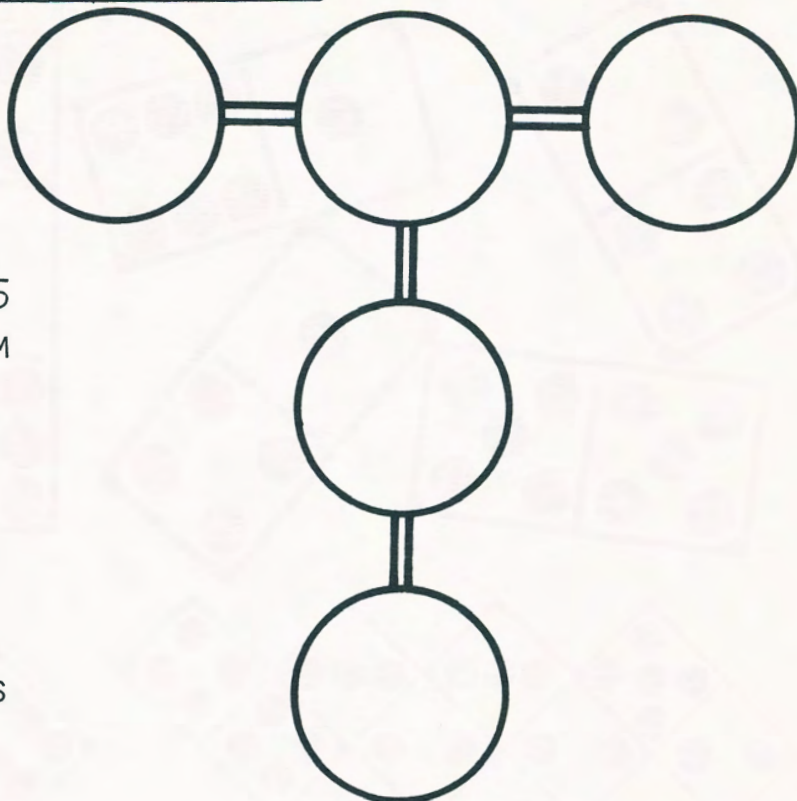
IDEA FROM: *Figures for Fun*

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ENCHANTED ALPHABET

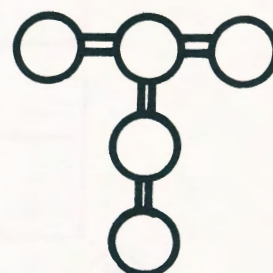
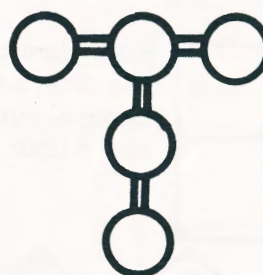
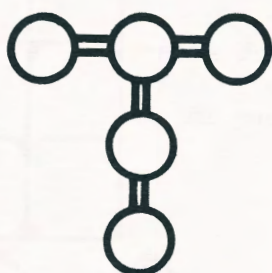
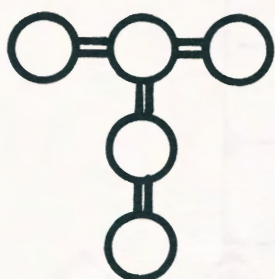
Addition
WHOLE NUMBERS



PLACE THE DIGITS 1, 2, 3, 4, 5
IN THE CIRCLES SO THAT THE SUM
OF THREE NUMBERS IN EACH
DIRECTION IS THE SAME.

RECORD YOUR ANSWERS BELOW.

FIND AS MANY DIFFERENT ANSWERS
AS YOU CAN.



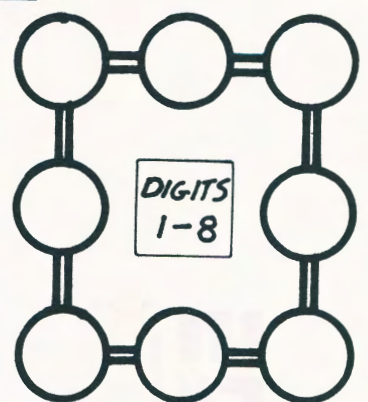
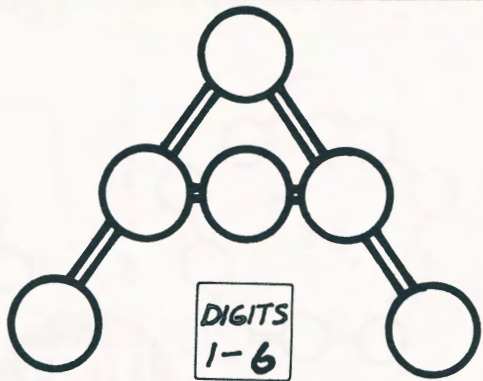
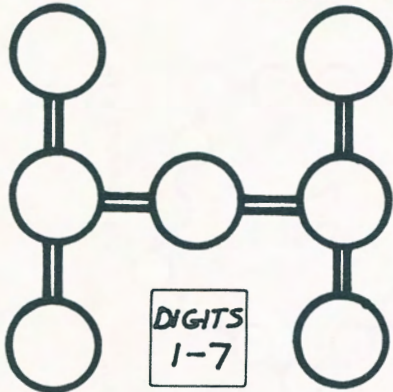
Before doing these problems, each student can be given markers with a single digit written on each one. (Small ceramic tiles work well.) The student can then place the markers in the circles of the large T or H and by trial and error come up with the solutions.

This technique can also be used with many of the activities that follow. The students now have the freedom to make mistakes . . . Only the right answers are recorded, and no erasure marks are left as reminders of wrong answers.

TYPE: Paper & Pencil/Puzzle

ENCHANTED ALPHABET

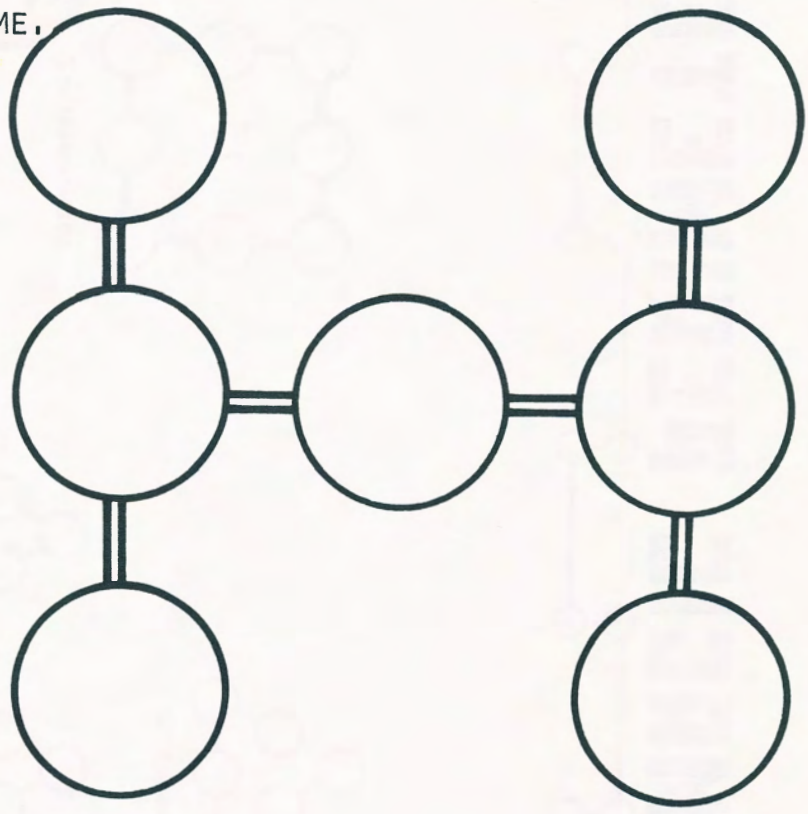
CONTINUED



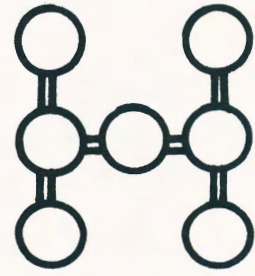
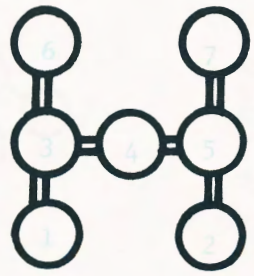
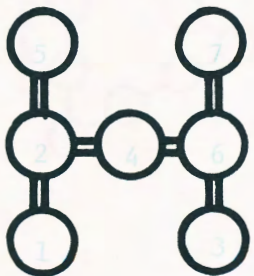
NOW TRY THESE. FOR EACH LETTER, THE SUM OF ANY THREE NUMBERS IN A LINE MUST BE THE SAME.

PLACE THE DIGITS 1-7 IN THE CIRCLES SO THAT FOR ANY THREE IN A ROW THE SUM OF THE TWO END NUMBERS MINUS THE MIDDLE NUMBER IS THE SAME.

RECORD YOUR ANSWERS BELOW.



TWO POSSIBLE SOLUTIONS:



Note: $(6 + 1) - 3 = 4$
 $(7 + 2) - 5 = 4$
 $(3 + 5) - 4 = 4$



MAGIC PERIMETERS



PLACE THE NUMBERS 1, 2, 3
4, 5, 6 IN THE CIRCLES SO
THAT ALL THREE SIDES ADD
TO THE SAME NUMBER.

TOTAL = 10



TOTAL = 12



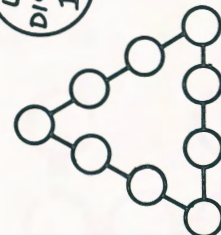
TOTAL = 11



TOTAL = 9

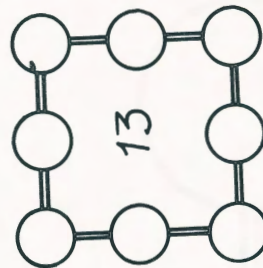


TOTAL = 17

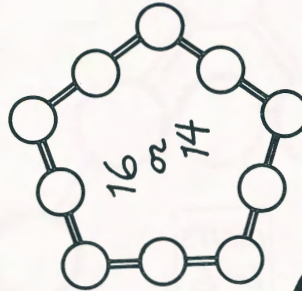


TOTAL = 20

USE
DIGITS
1-9



USE NUMBERS 1-7

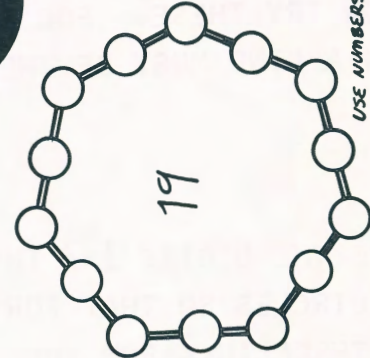


USE NUMBERS 1-10

ALL SIDES
OF POLYGONS
ADD TO
THE SAME
NUMBER



USE NUMBERS 1-15



USE NUMBERS 1-14

27

19

TYPE: Paper & Pencil/Puzzle

IDEA FROM: *Aftermath* and "Perimeter-Magic Polygon," *Journal of Recreational Mathematics*, Winter, 1974

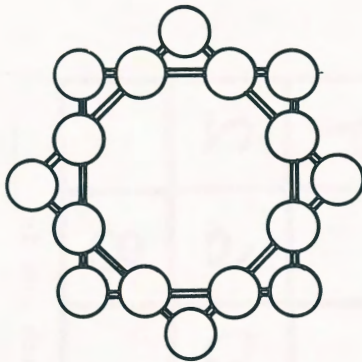


TINKER TOTALS



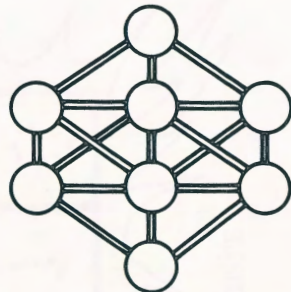
Place first five even numbers in the circles to the right so that any three numbers in a line have the same sum.

Next try problems A, B & C. For each problem any three numbers in a line must have the same sum.



FILL THE CIRCLES TO THE LEFT WITH THE NUMBERS 1 TO 16 SO THAT THE SUM OF EACH SIDE OF EACH SQUARE IS **34**

<p>A.</p> <p>Use first seven odd numbers so that the sum of each line is the same.</p>	<p>B.</p> <p>Use first nine even numbers so that the sum of each line is 30.</p>
<p>C.</p> <p>Use first seven counting numbers so that the sum of each line is 12.</p>	<p>D.</p> <p>Place the numbers 1-5 so that no number is connected to its successor. (3 must not be connected to 2 or 4.)</p>



PLACE THE NUMBERS 1-8 IN THE CIRCLES TO THE RIGHT SO THAT NO TWO CONSECUTIVE NUMBERS ARE IN CIRCLES CONNECTED BY LINES.

Addition
WHOLE NUMBERS





MAGIC SQUARES

Addition
WHOLE NUMBERS



A magic square is a square array of numbers in which the sum of the numbers in every row, column and diagonal is the same.

Complete each of the following 3 x 3 magic squares.

EXAMPLE

8	1	6
3	5	7
4	9	2

Each sum is 15.

		9
10	6	2
	8	

Each sum is ____.

17	10	
	14	
13	18	

Each sum is ____.

5	12	
10	8	6
9		

Each sum is ____.

	8	
2	6	10
	4	

Each sum is ____.

	5	
11	7	3
	9	

Each sum is ____.

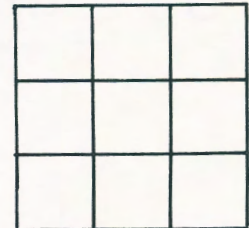
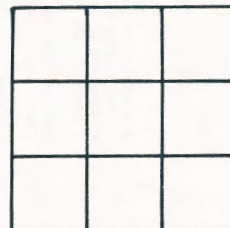
Examine all six cases above.

For each magic square write all nine numbers in order from smallest to largest.

Which number appears in the middle of the magic square?

How does the magic sum relate to the sum of the smallest, largest, and middle number?

Make a magic square using the first nine multiples of 3; of 5.



TYPE: Paper & Pencil/Puzzle

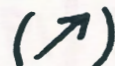
MAGIC SQUARES (CONTINUED)

TEACHER PAGE

HERE IS A METHOD FOR MAKING ODD-CELLED MAGIC SQUARES.

8	1	6
3	5	7
4	9	2

- 1) PLACE THE FIRST NUMBER IN THE MIDDLE CELL OF THE TOP ROW. PLACE EACH SUBSEQUENT NUMBER IN THE CELL TO THE NORTHEAST OF THE PREVIOUS CELL.



- 2) WHEN TRAVELING IN A NORTHEASTERLY DIRECTION, IF THE NEXT NUMBER FALLS OUTSIDE THE GRID, PLACE IT IN THE CELL DIRECTLY ON THE OPPOSITE SIDE OF THE GRID AND CONTINUE AS BEFORE. (SEE PLACEMENT OF THE #2 IN THE EXAMPLE.)
- 3) IF A CELL HAS BEEN PREVIOUSLY OCCUPIED, THEN PLACE THE NEW NUMBER DIRECTLY BELOW THE LAST CELL COMPLETED. (SEE PLACEMENT OF #4 IN THE EXAMPLE)
- 4) IF THE NEXT NUMBER FALLS OUTSIDE THE GRID AT THE INTERSECTION OF THE NORTHEAST CORNER, PLACE THE NEXT NUMBER IN THE AVAILABLE CELL DIRECTLY BENEATH THE CELL AT THE N.E. CORNER. (SEE PLACEMENT OF #7 ABOVE.)

THIS PROCEDURE WORKS FOR ALL ODD-CELLED MAGIC SQUARES; 3 x 3, 5 x 5, 7 x 7, . . .

HERE IS A 5 x 5 MAGIC SQUARE WHICH USES THE FIRST 25 COUNTING NUMBERS.

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

CALENDAR MAGIC

JUNE

Addition
WHOLE NUMBERS



				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	



SELECT A 4 X 4 GRID OF 16 NUMBERS.

SUM OF DIAGONALS:

$$5 + 13 + 21 + 29 = 68$$

$$8 + 14 + 20 + 26 = 68$$

SUM OF FOUR CORNERS:

$$5 + 8 + 26 + 29 = 68$$

SUM OF FOUR INSIDE NUMBERS:

$$13 + 14 + 20 + 21 = 68$$

For any 4x4 square on the calendar,

n	$n+1$	$n+2$	$n+3$
$n+7$	$n+8$	$n+9$	$n+10$
$n+14$	$n+15$	$n+16$	$n+17$
$n+21$	$n+22$	$n+23$	$n+24$

The sum of the diagonal is $4n + 48$;

The sum of the four corners is $4n + 48$;

The sum of the four inside numbers is

$4n + 48$; etc.

For the example shown, $n=5$, so the magic sum is 68.

CAN YOU FIND OTHER COMBINATIONS OF FOUR NUMBERS WHOSE SUM IS THE MAGIC SUM?

$$6 + 7 + 27 + 28 = 68$$

$$7 + 15 + 19 + 27 = 68$$

$$12 + 19 + 15 + 22 = 68$$

$$6 + 12 + 22 + 28 = 68$$

$$12 + 20 + 14 + 22 = 68$$

$$5 + 27 + 7 + 29 = 68$$



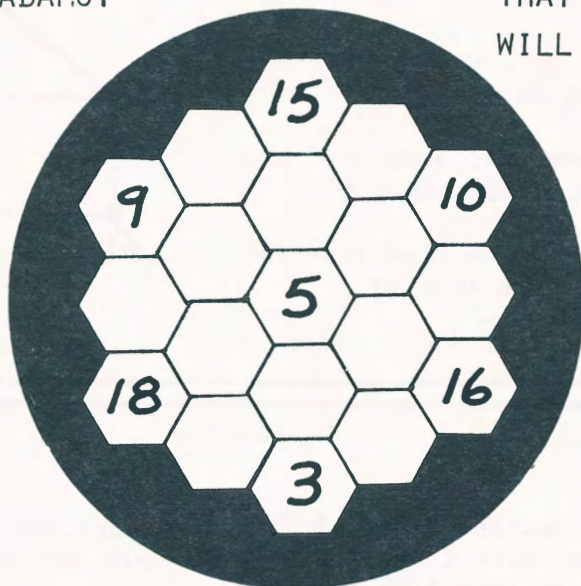
MAGIC HEXAGON

Addition
WHOLE NUMBERS



THIS PUZZLE WAS CREATED
IN 1910 BY 19-YEAR-OLD
CLIFFORD ADAMS.

THE OBJECT OF THE PUZZLE IS TO WRITE
THE NUMBERS 1 TO 19 IN THE CELLS SO
THAT ANY STRAIGHT LINE OF NUMBERS
WILL ADD TO 38.

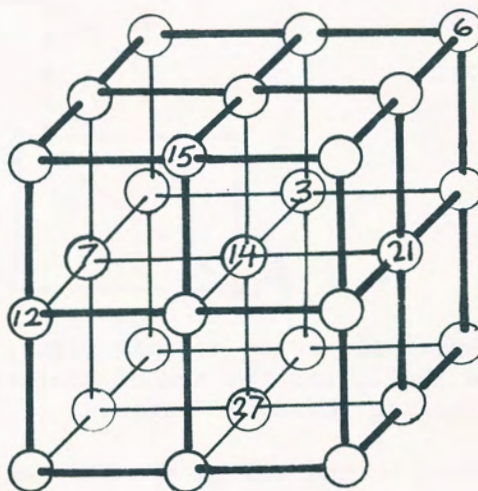


SOME OF THE NUMBERS ARE
ALREADY IN PLACE.

MAGIC CUBE

COMPLETE THE MAGIC CUBE AT THE
RIGHT SO THAT THE SUM OF EACH
ROW, EACH COLUMN, EACH PILLAR
AND EACH 3-DIMENSIONAL DIAGONAL
EQUALS 42.

USE THE NUMBERS 1-27.



TYPE: Paper & Pencil/Puzzle

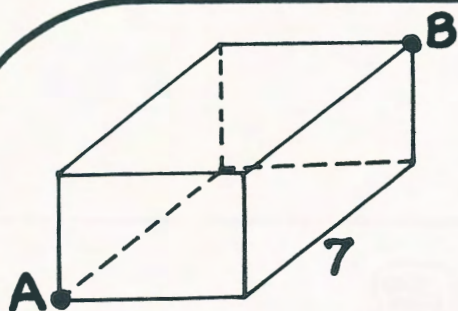
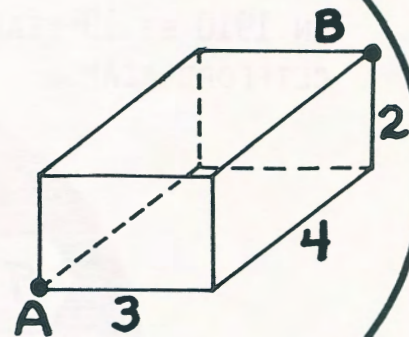
IDEA FROM: *Journal of Recreational Mathematics*

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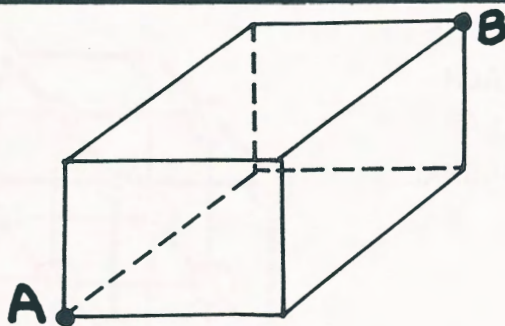
PATH SUMS

- 1) Can you find a path from A to B so that the sum of the segments is 9?
- 2) Write a number sentence to show your path.
- 3) Find as many "9" paths as you can. Write a number sentence for each.
- 4) Can you find a path from A to B so that the sum of the segments is greater than 9?
- 5) Can you find a path from A to B so that the sum of the segments is less than 9?



Label the edges of this figure so that the sum of one path from A to B is 10, and the sum of another path from A to B is 26.

Write a number sentence showing these two paths.



Label the edges of this figure so that the sum of one path from A to B is 12, and the sum of another path is 30. Write number sentences showing these two paths.

What is the sum of the longest path from A to B in this figure without going over any segment twice?

TYPE: Paper & Pencil

IDEA FROM: *Computation and Structure*, Nuffield Mathematics Project

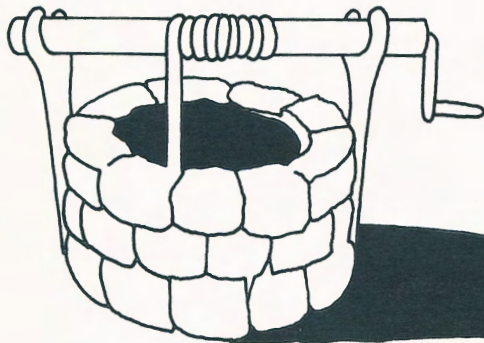
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This activity can be used to demonstrate the commutativity and associativity of addition.



OMAR'S DILEMMA

Addition/Subtraction
WHOLE NUMBERS



FATIMA, OMAR'S WIFE, SENT HIM TO THE WELL TO GET EXACTLY ONE LITRE OF WATER. HOWEVER, HE HAD ONLY A 5-LITRE JUG AND A 2-LITRE JUG. CAN YOU HELP OMAR FIGURE OUT HOW TO GET EXACTLY 1 LITRE?

There are at least two solutions. Here is one: Fill 5-litre jug, pour 2 litres in the 2-litre jug; empty 2-litre jug and fill it again from the 5-litre jug, leaving 1 litre in the 5-litre jug.

The problem can also be done using a 5-litre jug and a 3-litre jug or other combinations.

WHICH OF THE FOLLOWING AMOUNTS OF WATER CAN HE CARRY HOME USING ONLY HIS 5-LITRE AND 2-LITRE JUGS?

1 l, 2 l, 3 l, 4 l, 5 l, 6 l, 7 l, 8 l

WHAT AMOUNTS OF WATER CAN BE OBTAINED USING ONLY 3 l, 5 l, AND 11 l JUGS?

SEE IF YOU CAN FIND THREE JUGS THAT WILL MEASURE AMOUNTS FROM 1 LITRE TO 20 LITRES USING NO OTHER CONTAINERS!

COULD YOU HAVE USED ONLY TWO JUGS?

This could be a good small group activity in which students can interact and discuss possible solutions.

TYPE: Paper & Pencil/Transparency

NUMBER SENSE AND ARITHMETIC SKILLS

Placement Guide for Tabbed Dividers

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Activity Cards - Mathematical Balance II	351	Model Multiplication	Manipulative
Activity Cards - Mathematical Balance III	352	Model Division	Manipulative
Activity Cards - Base 10 Multibase Blocks IV	353	Model Multiplication	Manipulative
Activity Cards - Base 10 Multibase Blocks V	354	Model Division	Manipulative
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WHOLE NUMBERS:

MULTIPLICATION & DIVISION

MULTIPLICATION

As every teacher knows students enter the middle grades at all levels of proficiency in multiplication and leave the middle grades at different levels. Have you ever seen students multiply like this?

$$\begin{array}{r} 42 \\ \times 23 \\ \hline 86 \end{array}$$

multiplying units times units and tens times tens

$$\begin{array}{r} 34 \\ \times 7 \\ \hline 218 \end{array} \quad \begin{array}{r} 34 \\ \times 7 \\ \hline 358 \end{array}$$

carried digit, either not included or included before multiplication of tens digit

$$\begin{array}{r} 206 \\ \times 405 \\ \hline 1030 \\ 000 \\ 824 \\ \hline 9270 \end{array}$$

place value in partial products is confused

$$\begin{array}{r} 69 \\ \times 7 \\ \hline 476 \end{array}$$

basic multiplication facts are recalled incorrectly

In a recent testing of a large group of seventh grade students over one-third, 54 students, obtained the incorrect answer to 58×75 . What is even more interesting is that those 54 students obtained the incorrect answer for 42 different reasons. Most errors in multiplication can be traced to one of the following causes:

The multiplication algorithm was introduced before addition was mastered.

The basic multiplication facts have not been learned.

Place value errors are being made in: the regrouping process or the placement of partial product.

Correct algorithmic procedure has been forgotten.

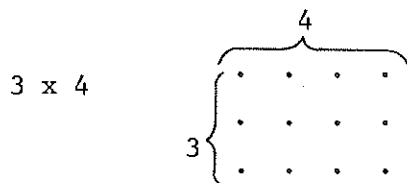


Also, many students simply make accidental errors but fail to notice them because they can't, or don't, approximate their answers.

Thus, the middle grade teacher is faced with improving skills, providing applications and trying to remedy the many deficiencies. Skill building and practice are important, but all the skill building practice in the world will not help the student who does not understand place value, the meaning of multiplication or some other essential prerequisite. For many students the teacher will have to diagnose the difficulties and supply the necessary remediation before they are ready for skill building, problem solving and applications. By the middle grades most students have been prepared for multiplication, introduced to the multiplication algorithm and are ready for, or have already started, the adult performance level of the multiplication algorithm. That is, they are multiplying multi-digit numbers by multi-digit numbers. They have gone through many stages to reach this level, and some may need to go back to relearn earlier stages. A simplified map of their course might look like the following.

Basic Facts

Students were introduced to multiplication as repeated addition ($3 \times 4 = 4 + 4 + 4$) or in terms of arrays



The 100 basic facts were compiled in an orderly table. By the time the easy multiples were entered (multiples of 0, 1, 2 and 5) much of the table was filled. The remaining "holes" could be filled in a variety of ways. For example,

X	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6			15				
4	0	4	8			20				
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12			30				
7	0	7	14			35				
8	0	8	16			40				
9	0	9	18			45				

1. Doubling $4 \times 6 = \text{double } 2 \times 6, 8 \times 7 = \text{double } 4 \times 7$

2. Commuting $4 \times 6 = 6 \times 4, 8 \times 7 = 7 \times 8$

3. Renaming $4 \times 7 = 4 \times (5 + 2) = 20 + 8 = 28$

4. Patterns $9 \times 2 = 18$
 sum is 9
 one left

$9 \times 3 = 27$
 sum is 9
 one left

$9 \times 4 = 36$
 sum is 9
 one left

To maintain the basic facts periodic practice is necessary. This practice can be gotten in a variety of interesting ways. In this box-multiplication the object is to find how many of each number add to the total in the bottom box.

$$\begin{array}{|c|c|c|} \hline 3 & \times 6 & 18 \\ \hline 8 & \times 1 & 8 \\ \hline & & 26 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 4 & & \\ \hline 7 & & \\ \hline & & 39 \\ \hline \end{array}$$

Trial and error methods of solution give practice in multiplication. Some students may be challenged to see if there is more than one answer. Other useful ideas can be found on the pages *Facts in Squares*.

Multiples of 10, 100

Because most students could count by tens, and even hundreds, at an early age, it was easy to see that $6 \times 10 = 60$ and $3 \times 100 = 300$. The next step was to find

$$2 \times 30 \longrightarrow 2 \times 3 \text{ tens} \longrightarrow 6 \text{ tens} \longrightarrow 60 \quad \text{and}$$

$$\begin{array}{r} 30 \\ \times 2 \\ \hline 60 \end{array}$$

$$7 \times 400 \longrightarrow 7 \times 4 \text{ hundreds} \longrightarrow 28 \text{ hundreds} \longrightarrow 2800$$

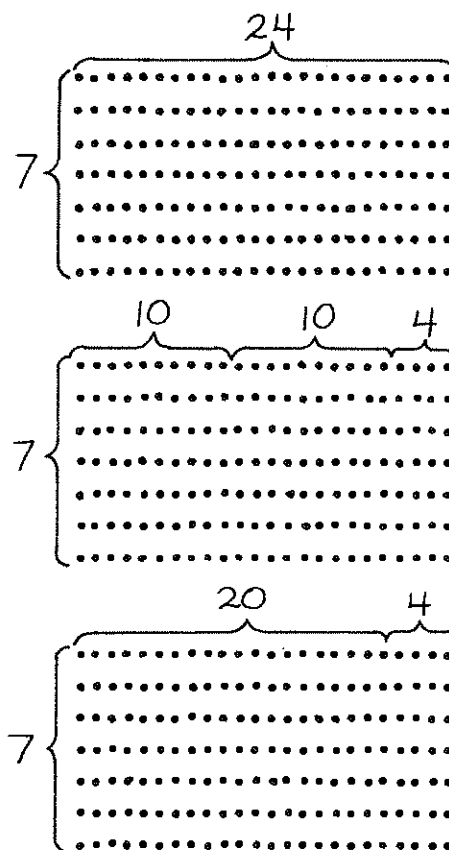
$$\begin{array}{r} 400 \\ \times 7 \\ \hline 2800 \end{array}$$

Learning to multiply numbers which are multiples of ten is an essential prerequisite to the multiplication algorithm.

Multiplication Algorithm

Now we are ready for problems like: $\begin{array}{r} 24 \\ \times 7 \\ \hline \end{array}$ Before algorithmic procedures are taught students should have the opportunity to devise some way to multiply based on their previous experiences. For some this may be adding 7 twenty-fours, others may be insightful and think of 24 as $10 + 10 + 4$ or $20 + 4$ and then multiply to get $70 + 70 + 28$ or $140 + 28$. With this particular exercise someone may suggest 7 quarters take away 7 pennies, $7(25 - 1) = (7 \times 25) - (7 \times 1)$. The suggestion to rename 24 as $20 + 4$ leads naturally to the algorithm that needs to be learned:

$$\begin{array}{r} 24 \\ \times 7 \\ \hline \end{array} \longrightarrow \begin{array}{r} 20 + 4 \\ \times 7 \\ \hline \end{array} \longrightarrow \begin{array}{r} 20 + 4 \\ \times 7 \\ \hline 140 + 28 \end{array}$$



Later, a suggested notational shortcut leads to:

Finally, another notational shortcut based on

place value leads to "carrying."

$$\begin{array}{r} 20 + 4 \quad 24 \\ \times \quad 7 \quad \times \quad 7 \\ \hline 140 + 28 \text{ ----} \rightarrow 28 \\ \hline 168 \end{array}$$

$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \\ \hline \textcircled{2} \quad \quad \quad \\ \times \quad \quad \quad \\ \hline \end{array} \quad \begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \\ \hline \textcircled{2} \quad \quad \quad \\ \times \quad \quad \quad \\ \hline 1 \quad 6 \quad 8 \end{array}$$

This last step is conceptually difficult and needs to be thoroughly mastered before moving to more difficult computations. The *Activity Cards - Beansticks - III* describe an approach to the multiplication algorithm that uses a physical model. This might be a good alternative approach for some students who have "had" the multiplication algorithm before but still can't multiply.

Extending the Algorithm

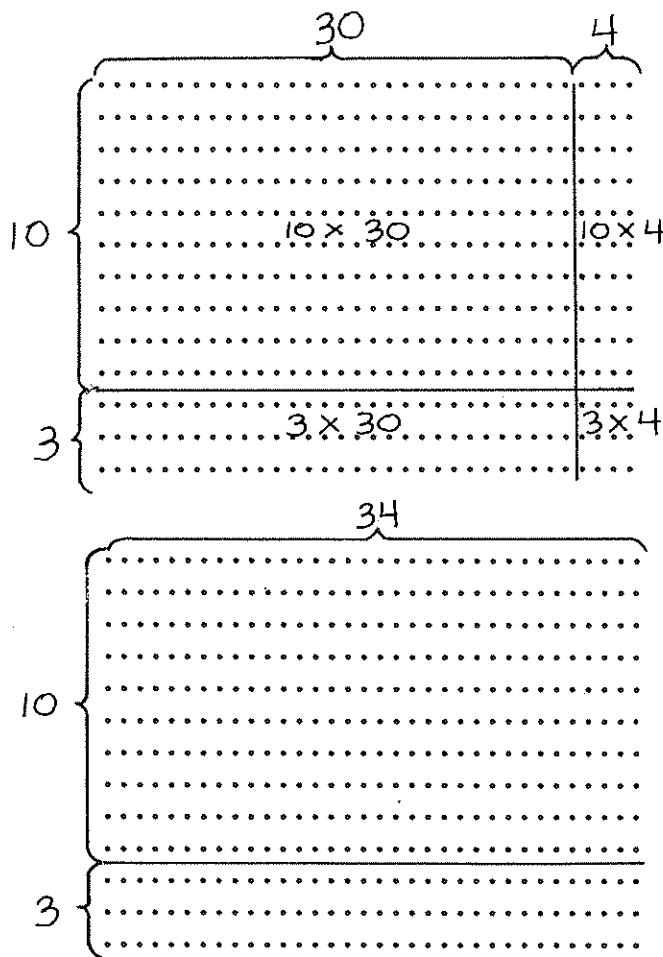
At the next higher level, multiplying a two-digit number times a two-digit number, there are two additional prerequisites: ability to find the products of multiples of 10 (e.g., 30×40) and the product of any two-digit number and a multiple of 10 (e.g., 26×30).

One method of approaching this algorithm uses four partial products as illustrated by the diagram and below.

$$\begin{array}{r} 34 \\ \times 13 \\ \hline \end{array} \rightarrow \begin{array}{r} 30+4 \\ \times 10+3 \\ \hline \end{array} \rightarrow \begin{array}{r} 30+4 \\ \times 10+3 \\ \hline 12 \quad (3 \times 4) \\ 90 \quad (3 \times 30) \\ 40 \quad (10 \times 4) \\ 300 \quad (10 \times 30) \\ \hline 442 \end{array}$$

A second method utilizes two partial products. This method leads more directly to the standard algorithm.

$$\begin{array}{r} 34 \\ \times 13 \\ \hline \end{array} \rightarrow \begin{array}{r} 34 \\ \times 10+3 \\ \hline \end{array} \rightarrow \begin{array}{r} 34 \\ \times 10+3 \\ \hline 102 \quad (3 \times 34) \\ 340 \quad (10 \times 34) \\ \hline 442 \end{array}$$



The algorithm is extended to multi-digit multiplication in a similar manner. One common area of confusion for students arises in an exercise involving a "0" in the middle digit of the multiplier as in:

$$\begin{array}{r} 254 \\ \times 307 \\ \hline \end{array}$$

If this exercise is thought of as $\begin{array}{r} 254 \\ \times 307 \end{array} \longrightarrow \begin{array}{r} 254 \\ \times 300+7 \end{array} \longrightarrow \begin{array}{r} 254 \\ \times 300+7 \\ \hline 1778 \\ 76200 \\ \hline 77978 \end{array}$ then the

problem does not even arise. However, the older rote method of teaching the algorithm where the emphasis is on the rule (and indenting the partial product)

$$\begin{array}{r} 254 \\ \times 307 \\ \hline 1778 \\ 000 \\ 762 \\ \hline 77978 \end{array}$$

rather than expanded notation and place value, often results in a serious error when the "middle zeros" are involved.

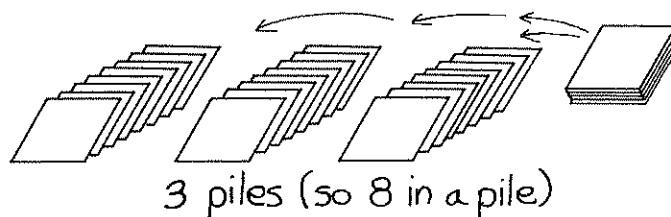
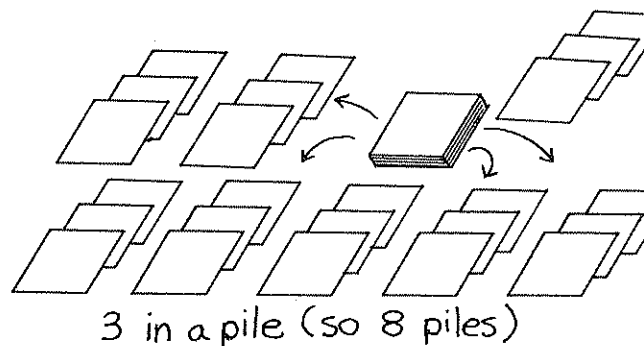
The road to learning the multiplication algorithm is a long one. There are lots of places for errors to occur, and it is easy to see why some students still have trouble multiplying in the middle grades. The importance of diagnosing student errors cannot be overstated. The classroom pages 2x3 *Good Times* and *Hexed Products* are examples of skill building activities which involve problem solving and allow the teacher to watch students use their multiplication knowledge and perform "over-the-shoulder" diagnosis.



DIVISION

Suppose you were asked to write a number sentence which describes each of the following situations.

1. There are 24 playing cards and each person who plays gets three cards. How many people can play?
2. There are 24 playing cards to be distributed among three players. How many cards does each player get?



The number sentence you would write in both cases is $24 \div 3 = 8$. Suppose, however, that you solved the problem physically. In the first situation you would probably lay down consecutive piles of three cards and then count the number of piles. In the second you would lay one card in each of three locations, then add a second to each location, then a third, etc. When all cards are exhausted you would count the number in each pile. Thus, although you described both situations with the same number sentence, the physical solutions are quite different. In the first case you knew the number in each subgroup, while in the second case you knew the number of subgroups.

The process or arithmetic operation which corresponds to partitioning a set into a number of equivalent subsets is division. In a division problem the number you are dividing by is called the divisor and the result of the division operation is called the quotient. When division is used to describe a physical situation the divisor could represent the number of subgroups (or the number in each subgroup) and then the quotient would represent the number in each subgroup (or the number of subgroups).

Like multiplication, the path leading to an efficient division algorithm is a long one. Introductory grouping ideas start in the primary grades, are refined and extended into intermediate grades and mastery of the division algorithm is usually expected by the fifth or sixth grade. Division can be considered more difficult than multiplication because students must know how to multiply before they can divide. Because many students enter the middle grades deficient in multiplication skills and understandings it is no wonder that division skills are also lacking. A brief sketch of the development of division might look like as follows.

Basic Facts

Once division readiness activities with physical objects and diagrams have been presented, the basic division facts are related to the basic multiplication facts. For example, the array that was used to discuss $4 \times 5 = 20$ can now be used to show $20 \div 5 = 4$ and $20 \div 4 = 5$. In this way a family of facts is established.

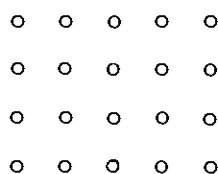
Because $\boxed{5} \times 4 = 20$ we have $20 \div 4 = \boxed{5}$

Because $\boxed{4} \times 5 = 20$ we have $20 \div 5 = \boxed{4}$

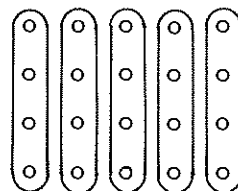
That is, the question "twenty divided by four is what," ($20 \div 4 = \square$) is equivalent to the question "what do we multiply times 4 to get 20," ($\square \times 4 = 20$).

Remainders

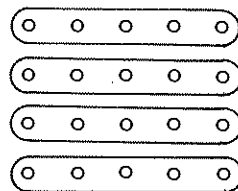
The next step is to consider division situations where there is a remainder. Appealing to the actual physical setting makes this easier to understand (e.g., 20 pieces of candy for 3 people). Students learn to estimate quotients by trial use of multiplication facts.



$$4 \times 5 = 20$$



$$20 \div 4 = 5$$



$$20 \div 5 = 4$$

$$20 \div 3 =$$

$$\boxed{5} \times 3 = 15 \quad \text{too small}$$

$$\boxed{6} \times 3 = 18$$

$$\boxed{7} \times 3 = 21 \quad \text{too large}$$

$$20 \div 3 = 6 \quad \text{with remainder 2}$$

$$\begin{array}{r} 5 \\ 3 \overline{)20} \\ \underline{15} \\ 5 \end{array}$$

$$\begin{array}{r} 6 \\ 3 \overline{)20} \\ \underline{18} \\ 2 \end{array}$$

$$\begin{array}{r} 7 \\ 3 \overline{)20} \\ \underline{21} \end{array}$$

Division Algorithm

There are many different approaches to the development of a meaningful algorithm for division. In any approach it may be useful to state an interesting question and see if students can devise their own technique for answering the question. An interesting question may provide a reason for dividing as well as a reference to which the answer can be related. In the beginning, partial quotients may be obtained in an informal way. In example a) ten groups of three are formed leaving 58, then ten more groups of three are formed leaving 28, etc.

Ultimately, however, the algorithm should become more efficient. In example b) we see that a quick mental test indicates that 20 groups of 3 can be formed (but not 30).

After subtracting 60 from 88 to obtain 28 we can use basic division facts to see that there are nine groups of 3 in 28 and we are left with a remainder of 1.

Eighty-eight students in a gym class need to organize in groups of three for physical fitness tests. Each group needs a stopwatch. How many stopwatches will we need?

$$\begin{array}{r}
 \begin{array}{l} 4 \\ 5 \\ 10 \\ 10 \end{array} \left. \vphantom{\begin{array}{l} 4 \\ 5 \\ 10 \\ 10 \end{array}} \right\} 29 \quad \begin{array}{l} \text{sum of} \\ \text{partial} \\ \text{quotient} \end{array} \\
 \text{a) } 3 \overline{)88} \\
 \underline{30} \\
 58 \\
 \underline{30} \\
 28 \\
 \underline{15} \\
 13 \\
 \underline{12} \\
 1 \quad \text{remainder}
 \end{array}$$

$$\begin{array}{ll}
 \text{b) } 3 \times 10 = 30 & \text{too small} \\
 3 \times 20 = 60 & \\
 3 \times 30 = 90 & \text{too large}
 \end{array}$$

$$\begin{array}{r}
 9 \\
 20 \\
 3 \overline{)88} \\
 \underline{60} \\
 28 \\
 \underline{27} \\
 1 \quad \text{remainder}
 \end{array}$$

Using similar procedures we can extend this method to multi-digit dividends.

7 $\overline{)2278}$

Think:

$7 \times 10 = 70$
 $7 \times 100 = 700$
 $7 \times 1000 = 7000$

and

$7 \times 200 = 1400$
 $7 \times 300 = 2100$
 $7 \times 400 = 2800$

7 $\overline{)2278}$

300

$\underline{2100}$

178

Think:

$7 \times 10 = 70$
 $7 \times 20 = 140$
 $7 \times 30 = 210$

7 $\overline{)2278}$

300

$\underline{2100}$

178

$\underline{140}$

38

Basic Facts:

$7 \times 5 = 35$
 $7 \times 6 = 42$

7 $\overline{)2278}$

300

$\underline{2100}$

178

$\underline{140}$

38

$\underline{35}$

3 remainder

The last, and most difficult, stage of algorithm development involves dividing a multi-digit number by a multi-digit number. This stage calls for approximation and rounding. An example will illustrate the difficulties.

- Dividing by two-digit number
- Round the divisor to the nearest ten
- Find the first partial quotient (7) in terms of the rounded number (approximation)
- Use that partial quotient ($7 \times 36 = 252$)
- Try the next larger partial quotient (8)

a) $36 \overline{)289}$

b) $36 \approx 40$

c) $7 \times 40 = 280$

d) $36 \overline{)289}$

$\underline{252}$

37

e) 37 is larger than 36

f) $36 \overline{)289}$

$\underline{288}$

1 remainder

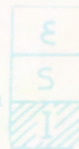
Difficulties like those above are not to be avoided. Rounding, approximations and practice will be needed to become efficient. By using lined paper to stress the place value in the quotient it will be possible to shorten the procedure by not "stacking" the partial quotients. Some students may discover by themselves that the trial digit can be found by a shorter procedure. That is, instead of thinking $7643 \div 38$ they may notice that the trial hundreds digit may be obtained by $7 \div 3$.

$$\begin{array}{r} 200 \\ 38 \overline{) 7643} \\ \underline{7600} \\ 43 \end{array}$$

$$\begin{array}{r} | 2 | | \\ 38 \overline{) 7643} \\ \underline{7600} \\ | 43 | \end{array}$$

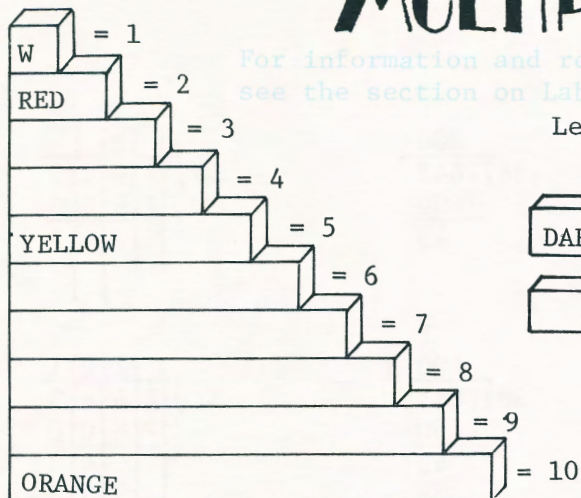
$$\begin{array}{r} 1 \\ 200 \\ 38 \overline{) 7643} \\ \underline{7600} \\ 43 \\ \underline{38} \\ 5 \text{ remainder} \end{array}$$

$$\begin{array}{r} | 2 | 0 | 1 | \\ 38 \overline{) 7643} \\ \underline{7600} \\ | 43 | \\ \underline{38} \\ | 5 | \end{array}$$

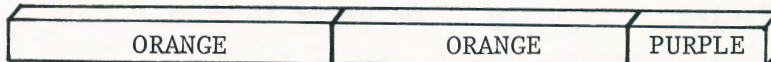


MULTIPLICATION

For information and readiness activities for Cuisenaire Rods see the section on Lab Materials.



Let's multiply 4×6 . Use dark green 4 times.



$$10 + 10 + 4 = 24$$

TRY THESE: 5×3 = Light green 5 times =
or (yellow 3 times)

The name Cuisenaire and the color sequence of the rods are trademarks of the Cuisenaire Company of America, Inc.

$2 \times 7 =$

$9 \times 4 =$

$8 \times 4 =$

$9 \times 3 =$

$3 \times 8 =$

$6 \times 7 =$

$7 \times 4 =$

$8 \times 7 =$

CR-III-1

DIVISION

CR-III-2

$18 \div 3$

Make 18.

Use as many orange rods as you can.



How many 3's (light green) in 18?

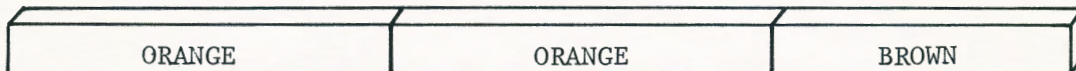
Is anything left over?



$18 \div 3 =$

$28 \div 6$

Make 28.



How many 6's (dark green) in 28?
What is left over?



$28 \div 6 = \text{ } r \text{ }$

TRY THESE:

$28 \div 9 =$

$40 \div 8 =$

$56 \div 8 =$

$36 \div 9 =$

$32 \div 8 =$

$43 \div 7 =$

$83 \div 10 =$

$63 \div 7 =$

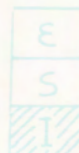
TYPE: Manipulative

The name Cuisenaire and the color sequence of the rods are trademarks of the Cuisenaire Company of America, Inc.



ACTIVITY CARDS - MATHEMATICAL BALANCE - II

Model
Multiplication
WHOLE NUMBERS



For information and readiness activities for the Mathematical Balance see the section on Lab Materials.

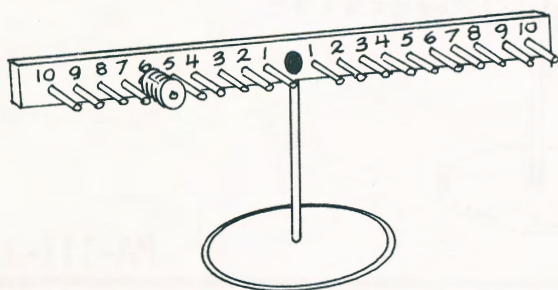
To multiply 6×4 place 4 washers on the 6 hook on the left side of the scale. (Could you show 6×4 a different way?)

Balance the scale using only tens and ones on the right side.

How many 10's did you use? _____

How many 1's did you use? _____

$6 \times 4 =$ _____



Multiply these numbers on your balance:

a) $4 \times 7 =$

b) $3 \times 6 =$

c) $9 \times 2 =$

d) $7 \times 8 =$

e) $6 \times 7 =$

f) $8 \times 8 =$

g) $7 \times 9 =$

MA-II-1

MA-II-2

To multiply 24×3 place 24 on the left side of the scale 3 times. (24 can be made with 2 washers on the 10 hook and 1 washer on the 4 hook.)

Place washers on the right side to balance the arm, using only tens and ones.

How many 10's did you use? _____

How many 1's did you use? _____

$24 \times 3 =$ _____

Multiply these numbers on your balance:

a) $14 \times 4 =$

b) $29 \times 2 =$

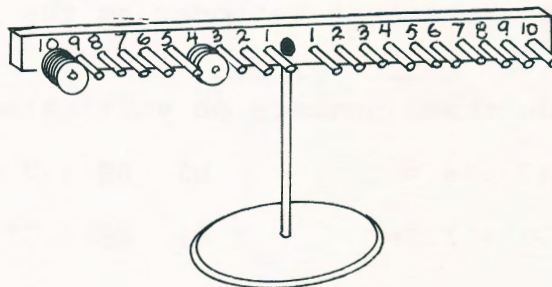
c) $13 \times 5 =$

d) $17 \times 3 =$

e) $12 \times 6 =$

f) $26 \times 3 =$

g) $38 \times 2 =$

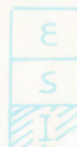


TYPE: Manipulative



ACTIVITY CARDS - MATHEMATICAL BALANCE - III

Model
Division
WHOLE NUMBERS



For information and readiness activities for the Mathematical Balance see the section on Lab Materials.

To divide 45 by 9 put 45 on the left side of the scale (4 washers on the 10 hook and 1 washer on the 5 hook).

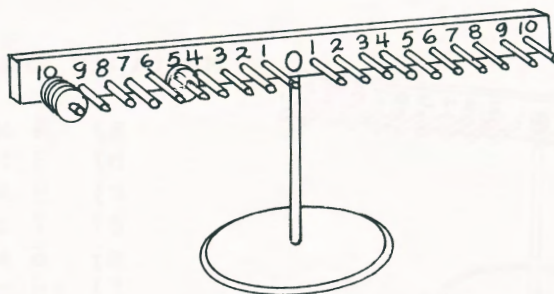
To find out how many 9's there are in 45, on the right side of the scale place washers on the 9 hook until the arm is balanced.

How many washers did you put on the 9 hook? _____

$$45 \div 9 = \underline{\hspace{2cm}}$$

Divide these numbers on your balance:

- a) $18 \div 6 =$
- b) $36 \div 9 =$
- c) $25 \div 5 =$
- d) $24 \div 3 =$
- e) $56 \div 8 =$
- f) $72 \div 9 =$
- g) $56 \div 8 =$



MA-III-1

MA-III-2

To divide 36 by 12, put 36 on the left side of the scale (3 washers on the 10 hook and 1 washer on the 6 hook).

To find out how many 12's there are in 36, on the right side of the scale place 12's until the arm is balanced. (12 can be made with one washer on the _____ and one washer on the _____.)

How many 12's in 36? _____ $36 \div 12 =$ _____

To find $25 \div 6$ put 25 on the left side, and put 6 on the right side.

Did it balance? _____

If not add washers to the 1 hook on the right until it balances.

There are _____ 6's in the 25 with _____ 1's leftover.

Write the amount leftover as the remainder (r).

$$25 \div 6 = \underline{\hspace{1cm}} r \underline{\hspace{1cm}}$$

Divide these numbers on your balance:

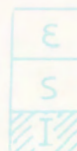
- | | | |
|-------------------|-------------------|-------------------|
| a) $47 \div 6 =$ | b) $60 \div 7 =$ | c) $52 \div 12 =$ |
| d) $39 \div 12 =$ | e) $86 \div 21 =$ | |

TYPE: Manipulative

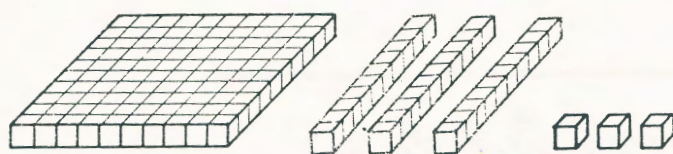


ACTIVITY CARDS - BASE 10 MULTIBASE BLOCKS - IV

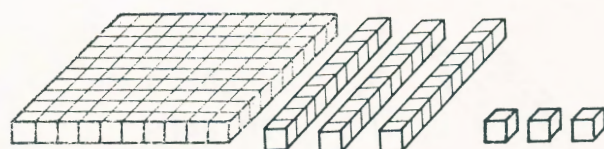
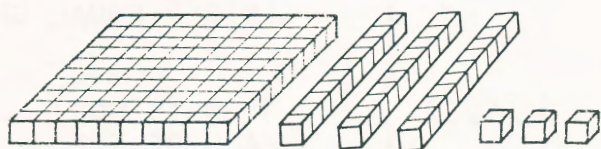
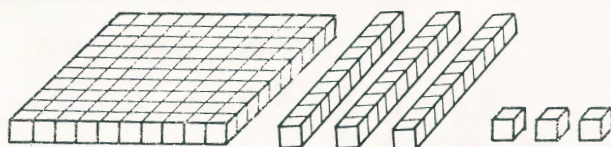
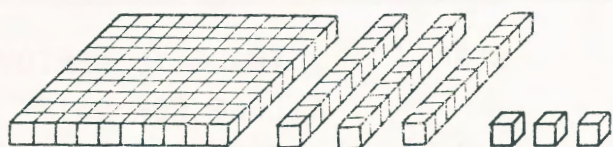
Model
Multiplication
WHOLE NUMBERS



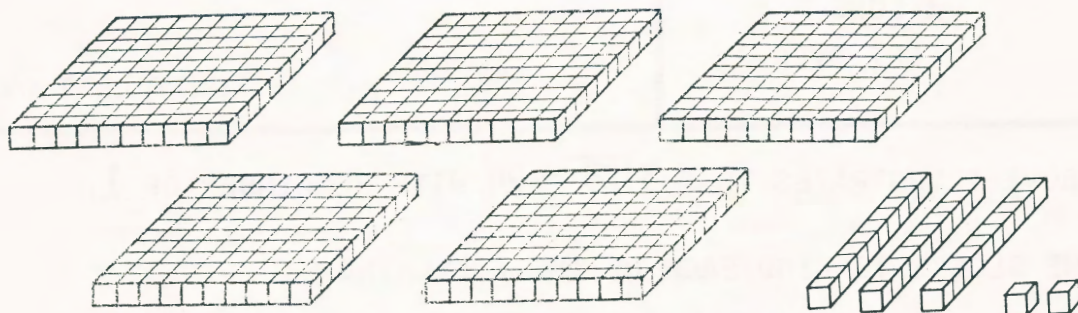
For information and readiness activities for Multibase Blocks see the section on Lab Materials.



MAKE FOUR GROUPS OF WOOD, EACH LIKE THE ONE ABOVE:



COMBINE THE FOUR GROUPS AND TRADE IF POSSIBLE.
YOU SHOULD GET THIS RESULT.



THE ABOVE ILLUSTRATES THAT $133 \times 4 = 532$.
USE THE BLOCKS TO FIND EACH OF THE FOLLOWING.

1. 7×2

5. 305×4

2. 25×2

6. 23×9

3. 33×5

7. 275×4

4. 107×2

8. 23×10

E: Manipulative

IDEA FROM: *Multibase Activities, Base 10*

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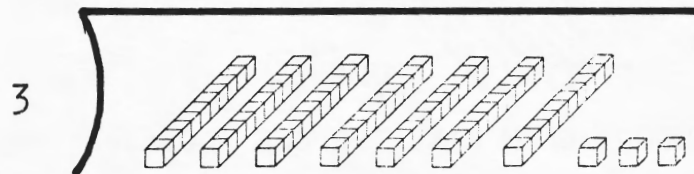
ACTIVITY CARDS - BASE 10 MULTIBASE BLOCKS - V

Model
Division
WHOLE NUMBERS

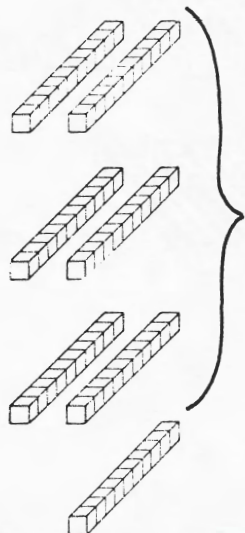


For information and readiness activities for Multibase Blocks see the section on Lab Materials.

EXAMPLE:



STEP 1 - PUT LONGS INTO
3 EQUAL GROUPS.

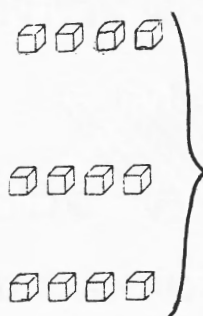


2 LONGS
IN EACH
GROUP.

1 LONG
LEFT OVER.

STEP 2 -

TRADE IN LEFTOVER
LONG FOR UNITS AND
THEN PUT ALL UNITS
INTO 3 EQUAL GROUPS.



4 UNITS FOR
EACH OF THE
THREE GROUPS.



1 UNIT LEFT OVER.

THE ABOVE ILLUSTRATES THAT $3\overline{)73} = 24$ WITH REMAINDER OF 1.

USE THE BLOCKS TO FIND EACH OF THE FOLLOWING.

① $2\overline{)46}$

⑥ $3\overline{)1008}$

② $2\overline{)56}$

⑦ $3\overline{)426}$

③ $5\overline{)241}$

⑧ $4\overline{)1273}$

④ $2\overline{)517}$

⑨ $4\overline{)1002}$

⑤ $3\overline{)324}$

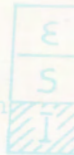
⑩ $7\overline{)1561}$

TYPE: Manipulative



ACTIVITY CARDS - BEANSTICKS - III

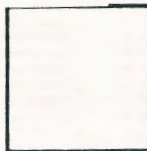
Model
Multiplication
WHOLE NUMBERS



MULTIPLICATION IN BEANSTICK SHORTHAND

Recall beanstick shorthand:

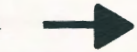
10 x 10 raft



10-stick



loose bean



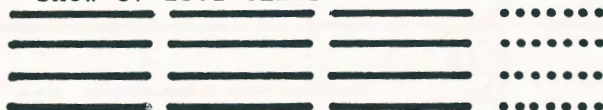
.

This activity may be performed using any multibase materials. For information on the construction, use and readiness for Beansticks or Multibase Blocks see the section on Lab Materials. You may need to guide your students through this to be sure they follow each step and get a feeling for the written method.

To MULTIPLY 37×4

①

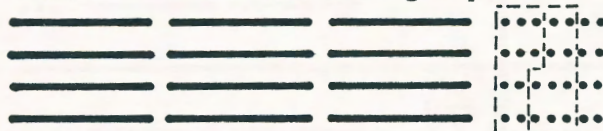
Show 37 four times



$$\begin{array}{r} 37 \\ \times 4 \\ \hline \end{array}$$

②

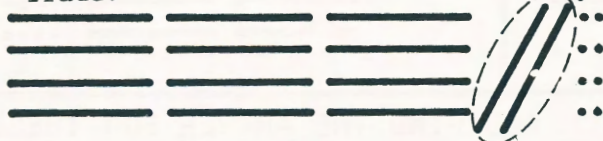
Combine loose beans into groups of 10.



At this point we are multiplying 7×4 .

③

Trade.



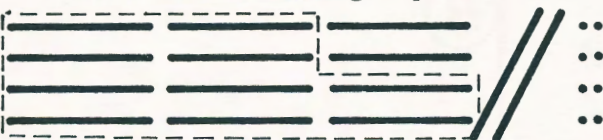
NEW TENS

$$\begin{array}{r} 2 \\ 37 \\ \times 4 \\ \hline 8 \end{array}$$

LOOSE BEANS LEFT

④

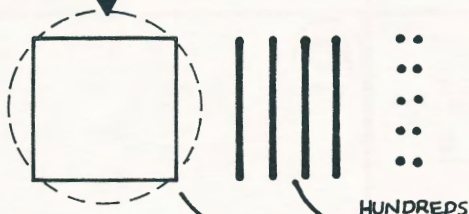
Combine sticks into groups of 10.



At this point we are multiplying 3 tens by 4 and adding the 2 tens that were carried.

⑤

Trade.



HUNDREDS AND TENS

$$\begin{array}{r} 2 \\ 37 \\ \times 4 \\ \hline 148 \end{array}$$

USE YOUR 10 x 10 RAFTS, 10-STICKS, AND LOOSE BEANS TO DO THESE PROBLEMS. Record your work.

$$\begin{array}{r} 46 \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{r} 63 \\ \times 2 \\ \hline \end{array}$$

$$\begin{array}{r} 29 \\ \times 4 \\ \hline \end{array}$$


$$\begin{array}{r} 78 \\ \times 3 \\ \hline \end{array}$$

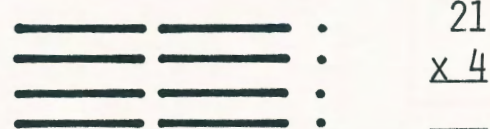
$$\begin{array}{r} 42 \\ \times 5 \\ \hline \end{array}$$


TYPE: Manipulative


ACTIVITY CARDS - BEANSTICKS - III (CONTINUED)

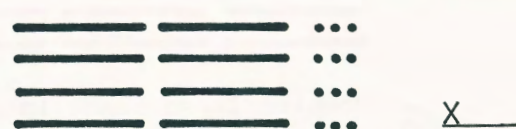
GIVE A MULTIPLICATION PROBLEM, COMBINE WHERE NECESSARY (SEE EXAMPLE) AND GIVE THE ANSWER FOR EACH.

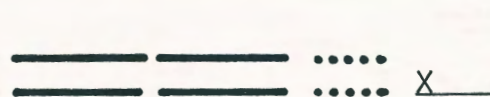
EXAMPLE:  $43 \times 6 \rightarrow$ $\begin{array}{r} 43 \\ \times 6 \\ \hline 258 \end{array}$

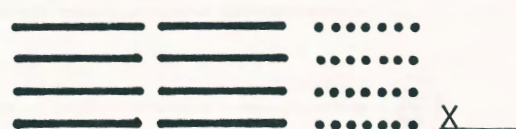
①  $\begin{array}{r} 21 \\ \times 4 \\ \hline \end{array}$

②  $\begin{array}{r} 2 \\ \times 2 \\ \hline \end{array}$

③  $\begin{array}{r} 3 \\ \times 3 \\ \hline \end{array}$

④  $\begin{array}{r} 4 \\ \times 4 \\ \hline \end{array}$

⑤  $\begin{array}{r} 5 \\ \times 5 \\ \hline \end{array}$

⑥  $\begin{array}{r} 6 \\ \times 6 \\ \hline \end{array}$

DRAW YOUR OWN SKETCH AND FIND THE ANSWER FOR THESE.

⑦ $\begin{array}{r} 12 \\ \times 2 \\ \hline \end{array}$

⑧ $\begin{array}{r} 46 \\ \times 5 \\ \hline \end{array}$

⑨ $\begin{array}{r} 37 \\ \times 4 \\ \hline \end{array}$

⑩ $\begin{array}{r} 45 \\ \times 2 \\ \hline \end{array}$

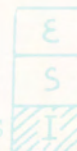
⑪ $\begin{array}{r} 17 \\ \times 5 \\ \hline \end{array}$

⑫ $\begin{array}{r} 29 \\ \times 6 \\ \hline \end{array}$



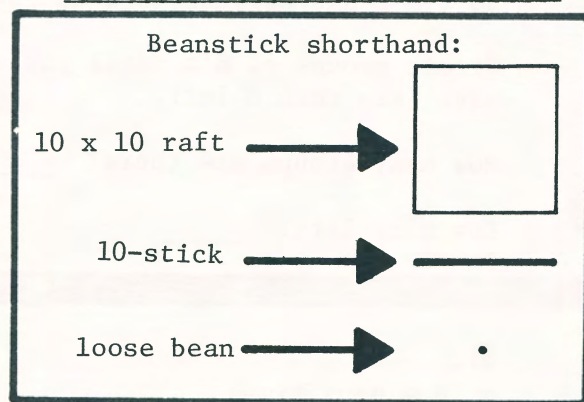
ACTIVITY CARDS - BEANSTICKS - IV

Model
Division
WHOLE NUMBERS



This activity may be performed using any multibase materials. For information on the construction, use and readiness for Beansticks or Multibase Blocks see the section on Lab Materials. You may need to guide your students through this to be sure they follow each step and get a feeling for the written method.

DIVISION IN BEANSTICK SHORTHAND



To DIVIDE 53 by 3

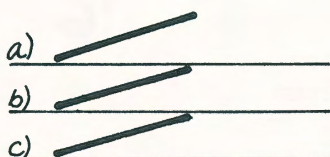
- ① Put 53 in a group to work with.



WRITE:

$$\begin{array}{r} 3 \overline{)53} \end{array}$$

- ② Distribute the 10-sticks into 3 equal groups.



REMAINING →

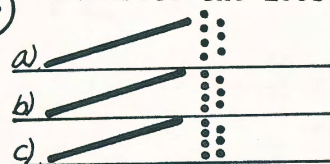


$$\begin{array}{r} 1 \overline{)3} \\ 3 \overline{)53} \end{array}$$

STICKS IN EACH GROUP
23 BEANS REMAINING

- ③ Exchange remaining 10-sticks.

- ④ Distribute the loose beans to the 3 equal groups.



REMAINING →

$$\begin{array}{r} 1(7)R(2) \\ 3 \overline{)53} \end{array}$$

BEANS IN EACH GROUP
BEANS REMAINING

USE YOUR 10-STICKS AND LOOSE BEANS TO DO THESE PROBLEMS.
RECORD YOUR WORK.

① $4 \overline{)57}$

② $6 \overline{)82}$

③ $5 \overline{)99}$

④ $3 \overline{)78}$

⑤ $2 \overline{)64}$

TYPE: Manipulative



Model
Division
WHOLE NUMBERS



HOW MANY GROUPS — HOW MANY LEFT ?

To solve a division problem using the repeated subtraction method, you want to think of your problem this way:

$30 \div 6$ means how many 6's in 30.



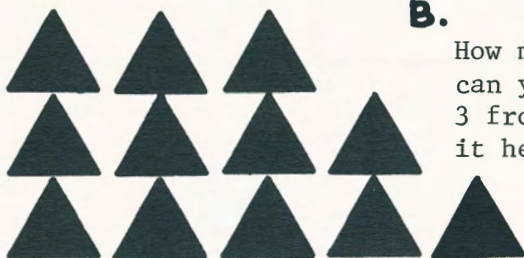
Circle groups of 6's until you have less than 6 left.

How many groups are there? _____

How many left? _____

1. A.

Find how many groups of 3 are in this pile by circling 3 at a time.



B.

12

How many times can you subtract 3 from 12? Do it here.

_____ \div _____ = _____ r _____

2. A. Find how many groups of 5 by circling.



B.

16

How many times can you subtract 5 from 16? Do it here.

_____ \div _____ = _____ r _____

3. A. Find how many groups of 11 by circling.



B.

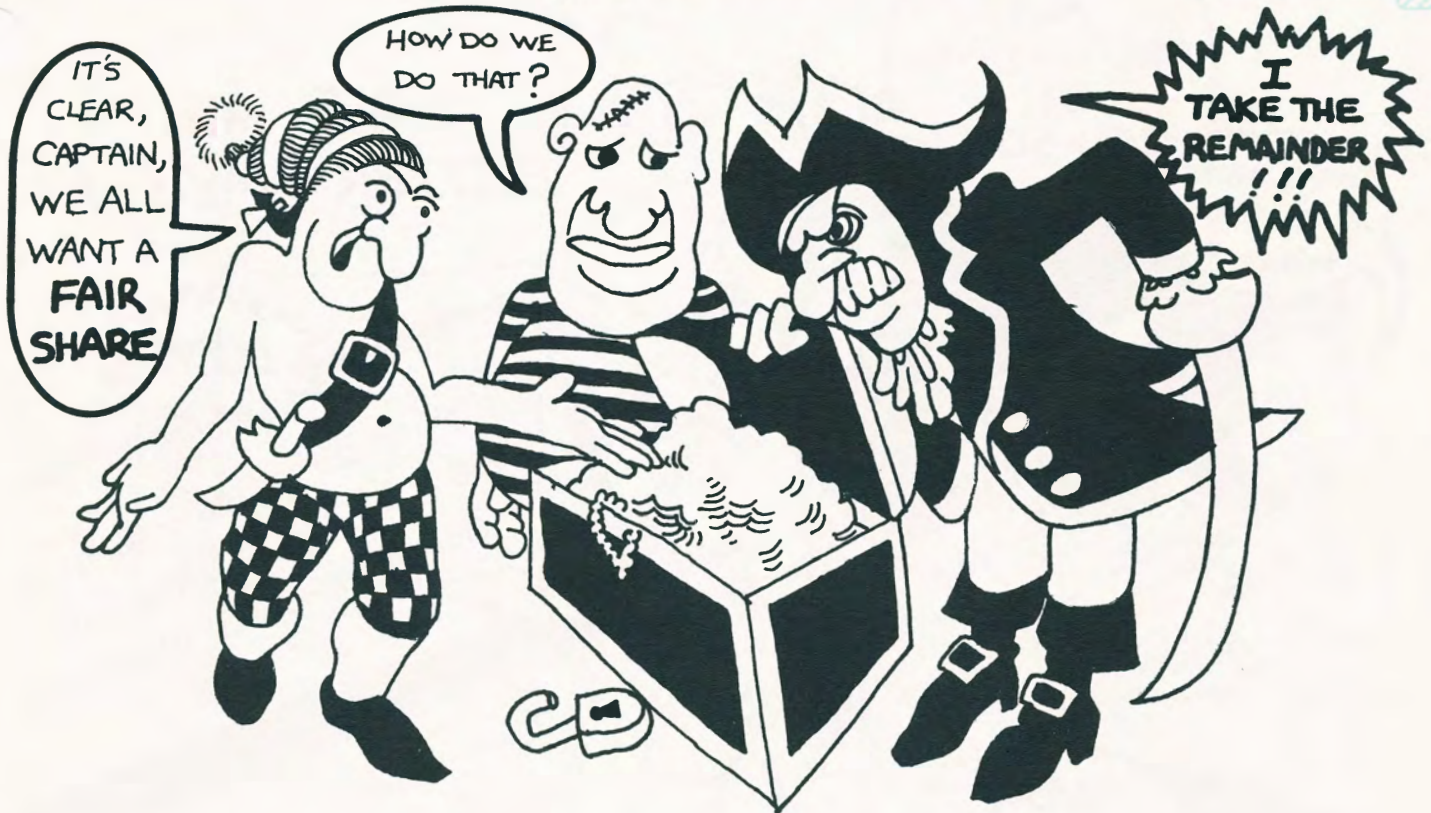
59

How many times can you subtract 11 from 59? Do it here.

_____ \div _____ = _____ r _____

FAIR SHARE

Model
Division
WHOLE NUMBERS



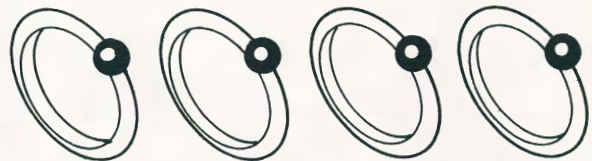
How would you divide each of these up into fair shares?

1



8 things 4 shares
Amount in each share _____

2



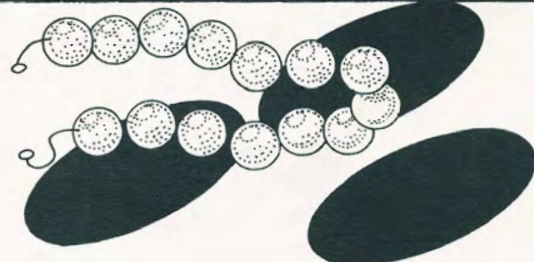
4 things, 3 shares
Amount in each share _____

3



3 things, 2 shares
Amount in each share _____

4



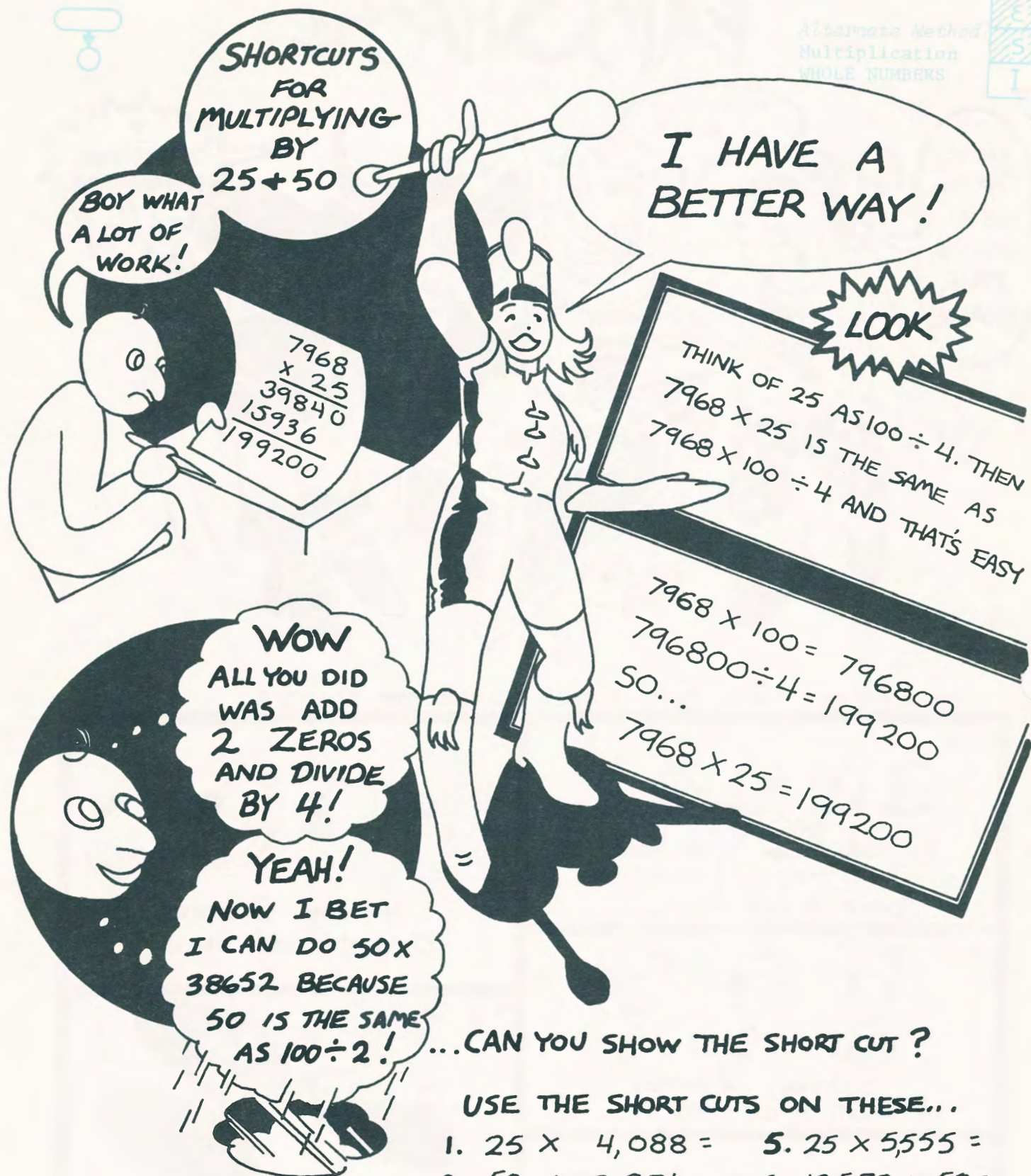
14 things, 4 shares
Amount in each share _____

5



5 things 10 shares
Amount in each shares _____

Make some problems of your own



USE THE SHORT CUTS ON THESE...

1. $25 \times 4,088 =$
2. $50 \times 3,871 =$
3. $8,076 \times 25 =$
4. $465 \times 50 =$
5. $25 \times 5,555 =$
6. $13,579 \times 50 =$
7. $681,372 \times 25 =$

8. $50 \times 987,654,321 =$



GRID MULTIPLICATION WITH INTERSECTIONS

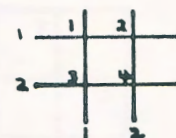
Basic Facts
Multiplication
WHOLE NUMBERS



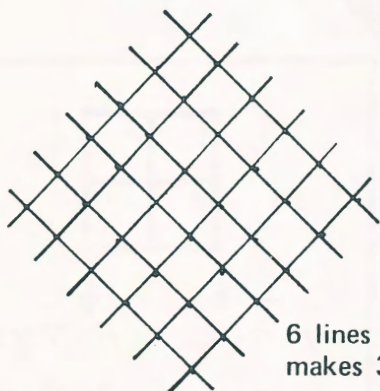
WHEN ONE LINE CROSSES ONE LINE, THEY MAKE ONE INTERSECTION.



WHEN TWO LINES CROSS TWO LINES, THEY MAKE FOUR INTERSECTIONS.



SINCE $1 \times 1 = 1$ AND $2 \times 2 = 4$ ARE MULTIPLICATION FACTS, WE CAN USE GRIDS TO FIND THE PRODUCTS OF TWO NUMBERS BY COUNTING THEIR INTERSECTIONS.



6 lines crossing 6 lines
makes 36 intersections

Look both ways
when crossing!



WHAT IS THE FEWEST NUMBER
OF LINES THAT WOULD MAKE:

- 12 INTERSECTIONS?
- 18 INTERSECTIONS?
- 24 INTERSECTIONS?
- 48 INTERSECTIONS?
- 56 INTERSECTIONS?
- 21 INTERSECTIONS?

SOLUTION TO GRID MULTIPLICATION USING INTERSECTIONS:

This is not the fastest
way of finding the
answers, but it works!



You could make a table.

Product	Fewest Lines	Most Lines (Without use of 1)	Other Ways
12	3×4	2×6	
24	4×6	2×12	3×8
18	3×6	2×9	
48	6×8	2×24	4×12
56	7×8	2×28	4×14
21	3×7		

Facts in Squares

Basic Facts
Multiplication
WHOLE NUMBERS



Fill in the
Products Across.
Use the up and
down products
for a
check.



1	3
2	

- ↓
1. 6×7 1. 5×9
2. 9×6 3. 3×8

1	3
2	

- ↓
1. 7×8 1. 5×11
2. 6×9 3. 8×8

1	3
2	

- ↓
1. 9×5 1. 7×6
2. 2×13 3. 8×7

1	3
2	

- ↓
1. 9×4 1. 7×5
2. 9×6 3. 8×8

1	3
2	

- ↓
1. 8×9 1. 2×37
2. 6×8 3. 4×7

1	3
2	

- ↓
1. 5×7 1. 8×4
2. 4×5 3. 5×10

1	3
2	

- ↓
1. 7×7 1. 8×6
2. 8×10 3. 9×10

1	3
2	

You
fill in
the blanks

- ↓
1. 8×6 1. 6×7
2. 7×3 3. _____

1	3
2	

Fill in
all three
blanks

- ↓
1. 9×7 1. _____
2. _____ 3. _____

1	3
2	

1. _____ 1. _____
2. _____ 3. _____

JIGSAW PUZZLES

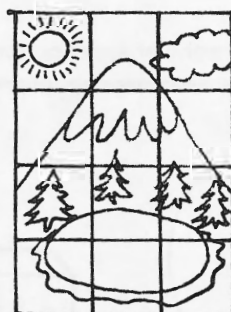
TEACHER IDEA

Basic Skills
Multiplication
WHOLE NUMBERS



1. Select an attractive picture about 9" x 12" to mount on an 11" x 14" sheet of poster board. (Calendars are a good source of colorful pictures.)
2. Use rubber cement to do the mounting, covering the entire back surface of the picture.
3. Cover the front of the picture and poster board with clear contact plastic. (optional)
4. Use a paper cutter to cut the borders away neatly in four strips, so that they can be mounted in place on a second sheet of poster board (also 11" x 14") to form a frame that the picture fits into easily. You might want to cut an additional 1/16" from the length and width of the picture before proceeding.
5. Draw lines on the back of the picture, dividing it into identical rectangles of any size. Write a problem in each rectangle, making certain that no two problems have the same answer.
6. Draw the same pattern of rectangles within the frame on the second piece of poster board. The answers to the problems are written in these rectangles, according to the scheme in the illustration.

Note match.



front of
picture

7x4	5x8	6x6
9x7	5x5	7x3
7x5	7x8	6x7
9x8	7x7	6x9

back of
picture

36	40	28
21	25	63
42	56	35
54	49	72

playing board

	40	28
21	25	63
42	56	35
54	49	72

picture being
formed

7. Cut the picture into rectangles. These are the playing cards. The framed poster board is the playing board.
8. The playing cards are held stacked in the hand problem side up. When the answer is located on the playing board, the card is laid on it picture side up.

This practice activity is self-correcting, as the picture reveals any misplaced cards. The instructions have been written in detail, so that your students may follow them to make puzzles for each other.

9. Can be extended to any operation or combination of operations.

TYPE: Puzzle



EVEN STEVEN

For information and readiness activities on Cuisenaire Rods see the section on Lab Materials.

Equipment: Box of Cuisenaire Rods
Squared centimetre paper--optional
Colored pencils or crayons--optional

Activity: Call a rod even if a train of red rods can be made equal to its length.

1. Make a train of red rods equal in length to the brown rod. Record by picture. Even? Yes
2. Can you make a train of red rods equal to the length of the blue rod? _____ Record by picture. Even? _____
3. Do this for every color of rod.
4. Arrange the even rods from longest to shortest. Record by picture.
5. If two even rods are put together will the rod formed be even? Try several.

I'm Even Steven. Can you give me a number name?



The name Cuisenaire and the color sequence of the rods are trademarks of the Cuisenaire Company of America, Inc

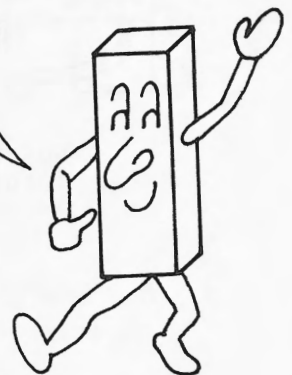
ODD ROD

Equipment: Box of Cuisenaire Rods
Squared centimetre paper--optional
Colored pencils or crayons--optional

Activity: Call a rod odd if a train of red rods cannot be made equal to its length.

1. Can you make a train of red rods equal to the length of an orange rod? _____ Odd? _____
2. What colors of rods are odd?
3. Arrange the odd rods from longest to shortest. Record by picture.
4. If a red rod is named 2 what are good number names for each of the even rods? What are good number names for each of the odd rods?
5. If two odd rods are put together will the rod formed be odd?
6. What if an odd rod and an even rod are put together?

I'm an Odd Rod. No Even Steven can make my length!



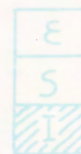
TYPE: Manipulative



ODD 'N EVEN I

(CONSTRUCTING MODELS)

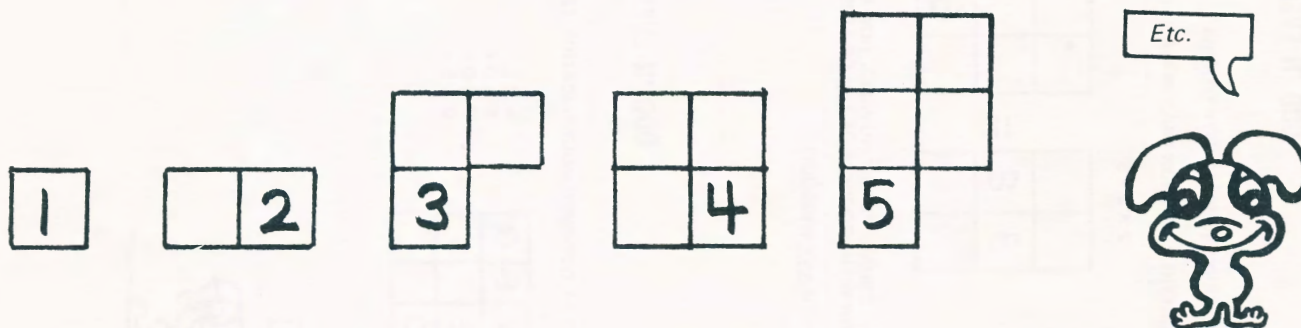
Odd and Even
Multiplication
WHOLE NUMBERS



Students will need some centimetre graph paper.

CUT OUT "GRID" NUMBERS FROM ONE TO TEN.
DO NOT HAVE MORE THAN TWO SQUARES IN A ROW.

NUMBER WITH ODD NUMERALS IN LOWEST LEFT SQUARE,
EVEN NUMERALS IN LOWEST RIGHT SQUARE.



ODD 'N EVEN II

(DISCOVERING PATTERNS)

6

PLACE YOUR "GRID" NUMBERS ON TOP OF EACH OTHER
WITH TEN ON THE BOTTOM AND ONE ON THE TOP.

DO ALL YOUR NUMERALS SHOW?

WHERE ARE THE ODD NUMERALS?
WHERE ARE THE EVEN?

Try it,
it's fun.



TYPE: Manipulative

SOURCE: *Activities with Squares for Well-Rounded Math*

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ODD 'N EVEN III

7

NOW FIT YOUR "GRID" NUMBERS TOGETHER TO SHOW ADDITION.

That 3 really is odd!

5	3
3	8

	8

$$\begin{array}{r} 5 \\ + 3 \\ \hline 8 \end{array} \quad \begin{array}{r} 3 \\ + 5 \\ \hline 8 \end{array} \quad \begin{array}{r} 8 \\ - 5 \\ \hline 3 \end{array}$$

$5 + 3 = 8$
 $3 + 5 = ?$

HOW MANY DIFFERENT "ADDITIONS" CAN YOU MAKE FROM YOUR TEN NUMBERS?

© Schreiner

ODD 'N EVEN V

9

SHARE YOUR "GRID" NUMBERS WITH OTHERS.

FIT THEM TOGETHER TO SHOW MULTIPLICATION.

EXAMPLES:

$2 \times 3 = 6$

$3 \times 2 = 6$



3	2

	6

	2
	2
	2

	6

IF FOUR PEOPLE SHARE "GRID" NUMBERS, HOW MANY DIFFERENT "MULTIPLICATIONS" CAN YOU SHOW WITH PRODUCTS OF TEN OR LESS?HOW MANY WITH ANY PRODUCT?

© Schreiner

ODD 'N EVEN IV

8

MAKE AN "ODD-EVEN" ADDITION TABLE.

+	O	E
O		
E		

$$\begin{array}{l} E + E = \\ E + O = \\ O + O = \\ O + E = \end{array}$$

$$\begin{array}{l} O - E = \\ E - E = \\ O - O = \\ E - O = \end{array}$$



I wonder if it's for odd?



-17-

© Schreiner

ODD 'N EVEN VI

10

TRY MAKING AN ODD-EVEN MULTIPLICATION TABLE.

X	E	O
E		
O		

$$\begin{array}{l} E \times E = \\ E \times O = \\ O \times O = \\ O \times E = \end{array}$$

$$\begin{array}{l} E \div E = \\ E \div O = \\ O \div O = \\ O \div E = \end{array}$$

I'm hungry!



If only the E's were eggs and the O's were olives!



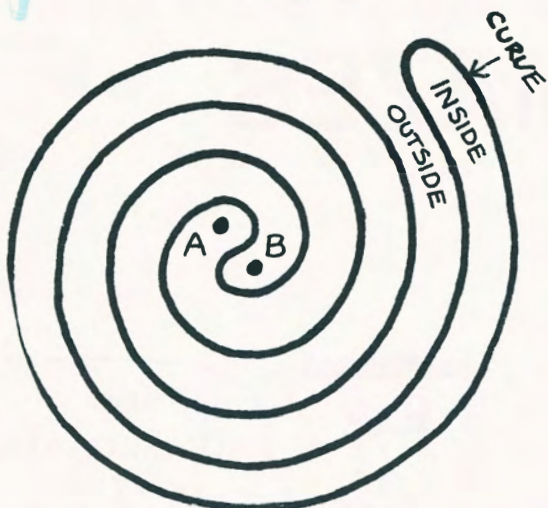
-19-

© Schreiner



INSIDE or OUTSIDE

Odd and Even
Multiplication
WHOLE NUMBERS



- Which point A or B is on the outside of the curve on the left?

To help you decide complete the table below by examining figure 1 and figure 2.

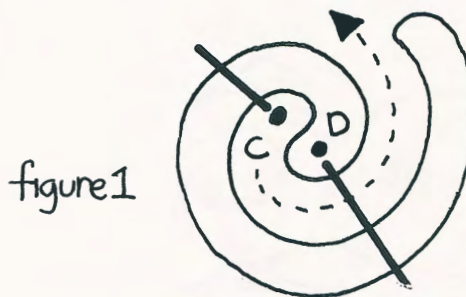
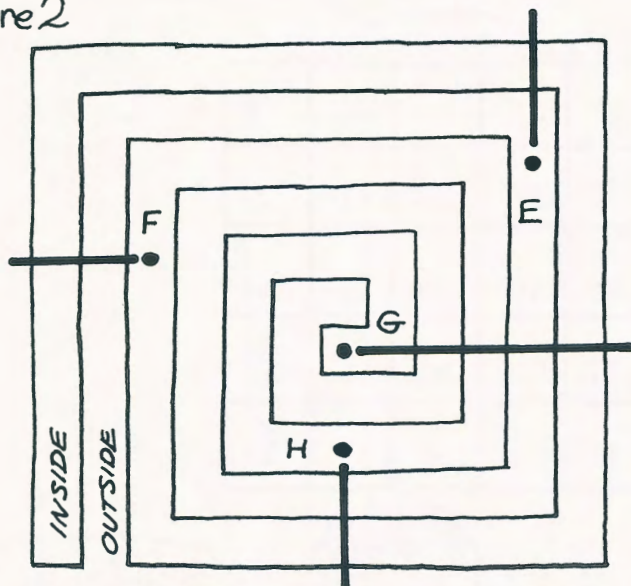


figure1

figure2



OUTSIDE	INSIDE	# OF TIMES THE HEAVY LINE CROSSES THE CURVE
C		
	D	
E		
	F	
	G	
	H	

If a straight line joining a point to the outside crosses the curve are EVEN number of times the point is OUTSIDE the curve.

- What kind of numbers are 2, 4, 6, 8, . . . ?
- Write a rule for using a line to tell if a point is inside or outside a curve.
- Which point is on the outside of the curve below?
- Make up your own curve. Try your rule.



MULTIPLE MAZES

Shade in
all multiples
of 6.



18	90	30	66	96
72	58	48	44	42
12	92	54	14	60
6	20	78	68	72
24	64	84	32	36

Shade in
all multiples
of 7.



28	63	35	84	56
70	33	23	110	49
42	98	77	7	91
77	93	48	65	56
14	58	95	55	21

Shade in
all multiples
of 8.



64	40	72	16	48
76	86	24	60	54
94	42	32	46	12
34	58	8	62	28
98	74	56	84	36

Shade in
all multiples
of 9.



18	58	32	96	63
54	83	29	66	45
90	9	45	54	27
81	43	74	23	99
36	56	98	84	72

On each maze move from Start to Finish using a pattern of multiples. (You may not move diagonally.)

Start

4	8	11	15	23	71	75	78
7	12	16	34	69	68	72	76
10	15	20	26	61	64	77	80
23	28	24	29	63	60	82	84
30	32	43	47	55	56	85	88
39	36	40	44	48	52	55	92
45	41	43	49	51	58	91	96
42	46	47	53	59	62	95	100

Finish

Start

9	17	26	35	43	52	60	69
18	28	37	46	55	216	231	242
27	26	35	54	72	204	223	224
36	45	54	63	71	207	216	225
44	53	62	72	90	198	219	234
108	99	90	81	89	189	190	243
117	107	98	89	90	180	188	252

Start

6	10	15	59	66	72	78	83
12	16	22	28	60	65	84	87
18	24	30	38	54	59	90	95
23	29	36	42	48	55	96	92
28	40	41	49	56	54	102	106
34	46	132	126	120	114	108	110
42	54	138	130	132	116	104	116
48	58	144	150	156	162	168	174

Finish

Start

7	14	21	35	56	56	63	70
13	42	28	35	42	49	55	77
20	35	37	41	28	119	89	84
27	34	41	119	112	105	98	91
34	176	125	126	118	175	91	97
29	170	139	133	141	148	168	175
35	161	148	140	147	154	161	182

MULTIPLE DESIGNS

Multiples
Multiplication
WHOLE NUMBERS



TO MAKE DESIGNS IN THE CIRCLES BELOW:

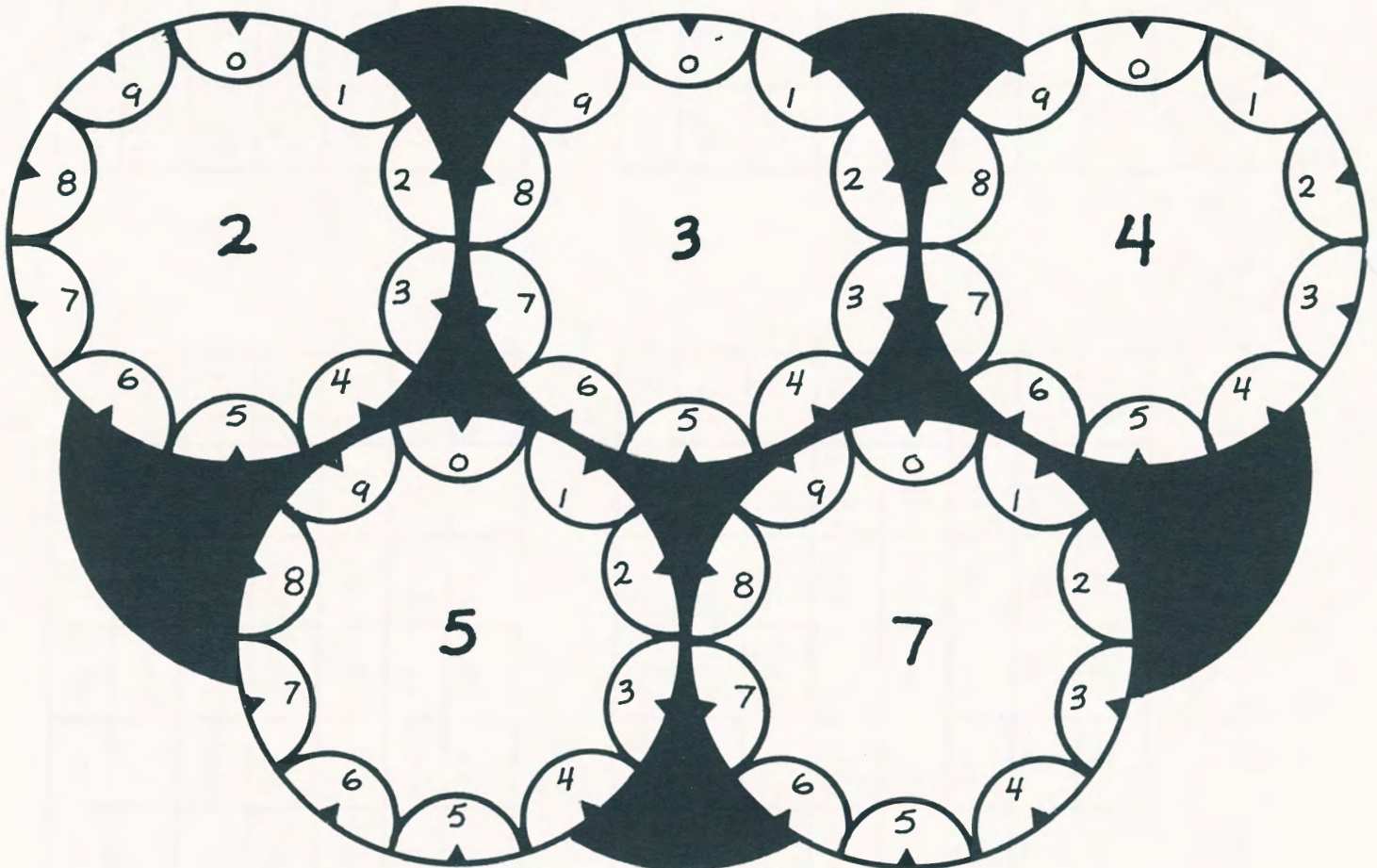
- 1) WRITE THE FIRST TEN MULTIPLES OF THE CENTER NUMBER OF A CIRCLE.
- 2) CONNECT THE UNITS DIGITS OF THESE MULTIPLES IN ORDER.

EXAMPLE: CIRCLE 6

MULTIPLES:

6 12 18 24 30 36 42 48 54 60

START
HERE



ON ANOTHER PIECE OF PAPER TRY MAKING STARS IN CIRCLES WITH CENTER NUMBERS: 8, 9, 10, 11, ETC.

TYPE: Paper & Pencil

IDEA FROM: *Aftermath*, Volume 3

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MISS FACTOR

Factors
Multiplication
WHOLE NUMBERS



Understandings Needed

Multiplication and division facts to the product of 81 — or — how to find the missing factor when given one factor and a product.

Players

Two to four

Equipment

20 product cards

81, 72, 64, 63, 56, 54, 49, 48, 45, 42, 40, 36, 35, 32, 28, 27, 24, 24, 21, 36

40 factor cards

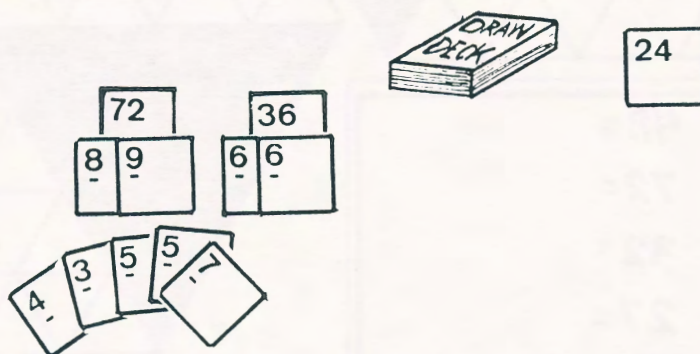
8 nines, 8 eights, 8 sevens, 6 sixes, 3 fives, 4 fours, and 3 threes

Rules

1. Draw card from product deck. Highest card is dealer and plays first.
2. Place "product" cards in center with one card turned up.
3. Deal seven cards to each player. As long as factor cards last, players replace any cards used, keeping seven in hand at all times.
4. Players make 'books' with two factors and one product card either by having the factors in hand or having one factor in hand and asking for and receiving other factor from another player.
5. Play moves when player can't make top product.
6. Players may only make one 'book' per turn.
7. If no player can make the turned up product, a new product is turned over and play continues.
8. Winner is player with most books after all cards are played.

SAMPLE GAME:

This player has already made combinations of factors and products for numbers 72 and 36. Number 24 is now up. The player may ask for a 6 to go with his 4 or an 8 to go with his 3, unless one of these combinations has already been made by another player.



TYPE: Game

IDEA FROM: *More Games and Aids for Teaching Math*

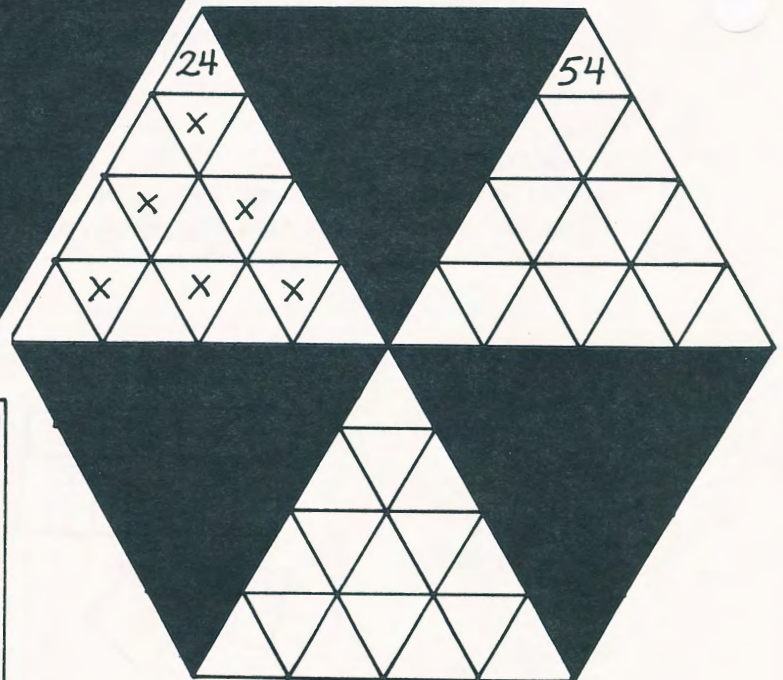
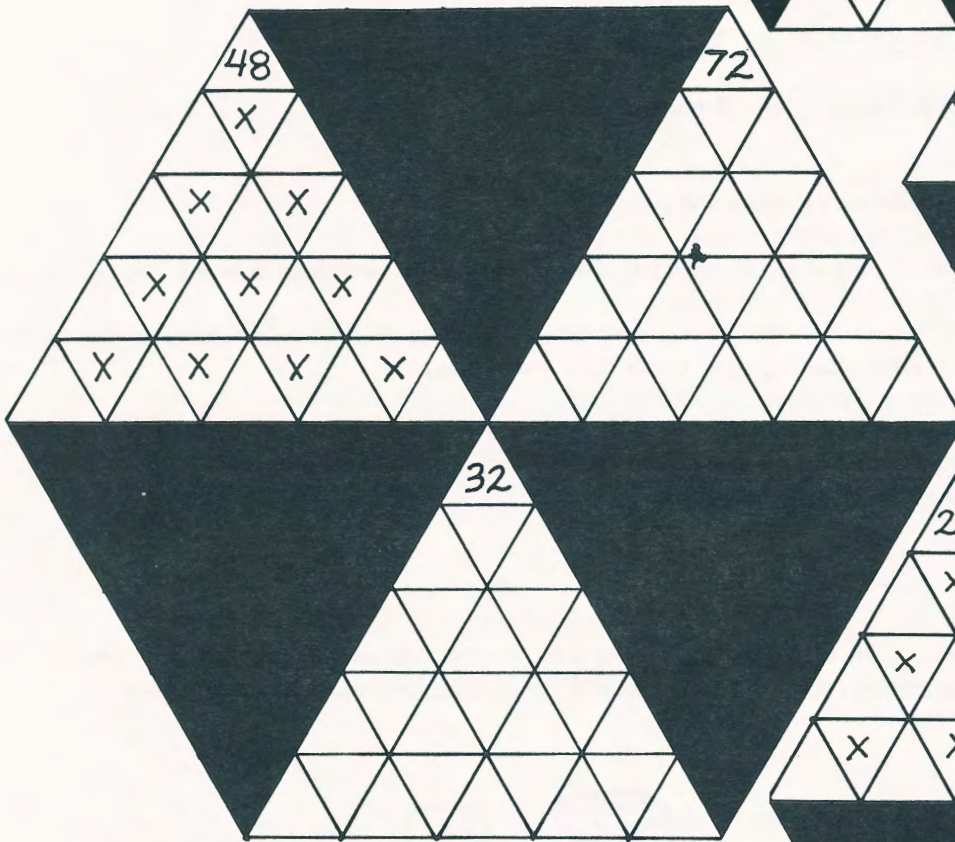
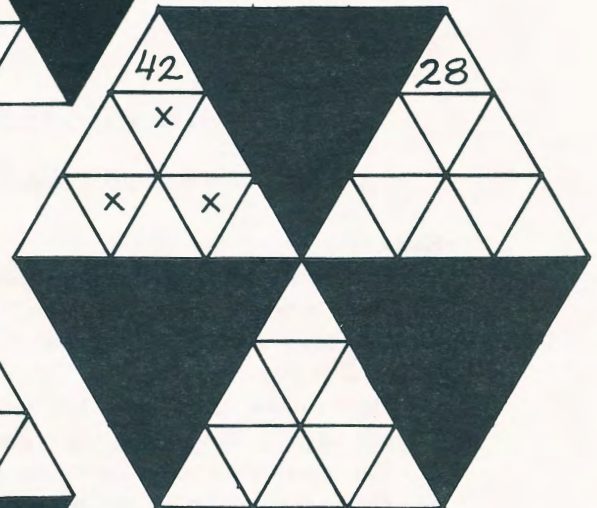
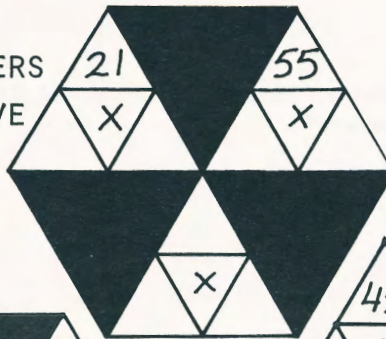
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FACTORING INTO PRIMES

Prime Factors
Multiplication
WHOLE NUMBERS



FIND THE PRIME FACTORIZATION OF THESE NUMBERS. RECORD YOUR ANSWERS IN THE TABLE BELOW. YOU WILL HAVE TO PUT SOME OF THE NUMBERS INTO HEXAGONS YOURSELF.



21 =	48 =
55 =	72 =
42 =	32 =
28 =	27 =
24 =	35 =
54 =	16 =

TYPE: Paper & Pencil

IDEA FROM: *Aftermath*, Volume 1 & 2

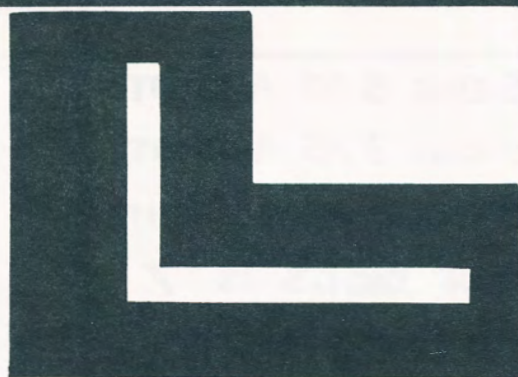
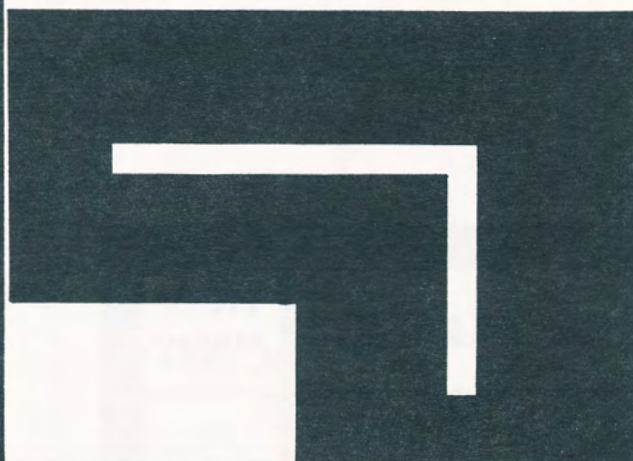
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WORD GAMES

Odd & Even
Multiples
Multiplication
WHOLE NUMBERS



THE ODD NUMBER NAMES, ONE THROUGH NINETEEN, ARE
HIDDEN IN THIS PUZZLE. THE WORDS GO IN THESE
DIRECTIONS: ➡ ↓



THE MULTIPLES OF TEN, TEN THROUGH ONE HUNDRED, ARE
HIDDEN IN THIS PUZZLE. THE WORDS GO IN THESE
DIRECTIONS: ➡ ↓

PE: Paper & Pencil/Puzzle

IDEA FROM: *Where's the Word? Book 1*

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NUMBER TYPES

Vocabulary
Multiplication
WHOLE NUMBERS



EQUIPMENT: HUNDREDS BOARD
NUMBER TILES 1 TO 100


ACTIVITY: PUT THE NUMBERS IN ORDER ON THE BOARD.
COMPLETE THESE CHARTS. FIND HOW MANY NUMBERS OF
EACH SET ARE ON THE HUNDREDS' BOARD.

This activity can be used
to introduce or review the
terms even, odd, multiple,
prime, digit and diagonal.



TYPE OF NUMBER	HOW MANY
EVENS	
ODDS	
MULTIPLES OF 10	
MULTIPLES OF 5	
MULTIPLES OF 3	
MULTIPLES OF 6	
PRIMES	
COMPOSITES	

MAKE UP
YOUR OWN PROBLEMS
AND PUT IN THE
BLANK
SPACES

TYPE OF NUMBER	HOW MANY
HAS ONE 5 AS A DIGIT	
HAS ONE 3 AS A DIGIT	
HAS ONE 0 AS A DIGIT	
SUM OF DIGITS IS 7	
ON A DIAGONAL 	

A NUMBER IS DIVISIBLE BY:

Divisibility
Division
WHOLE NUMBERS



TYPE: Transparency/Bulletin Board

IDEA FROM: *Aftermath*, Volume 3

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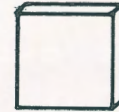
A SAREQ DEAL

Squares
Multiplication
WHOLE NUMBERS



MATERIALS: EACH STUDENT WILL NEED 100

- A) 1" CERAMIC TILES,
- OR B) CUBES (1", OR 1 CM, OR 2 CM),
- OR C) SMALL SQUARES CUT FROM TAGBOARD



USING THE TILES, MAKE THE
SMALLEST SQUARE YOU CAN.
HOW MANY TILES DID YOU USE?
MAKE THE NEXT SMALLEST SQUARE.
HOW MANY TILES ARE THERE ON
ONE SIDE OF THIS SQUARE?
HOW MANY TILES DID YOU USE?

THE INFORMATION ABOUT THESE TWO
SQUARES HAS BEEN PLACED IN THE
TABLE TO THE RIGHT.

FIND THE NEXT SMALLEST SQUARE YOU
CAN MAKE WITH YOUR TILES.
PLACE THE INFORMATION ABOUT THIS
SQUARE IN THE TABLE.

CONTINUE MAKING SQUARES AND FILLING IN THE TABLE UNTIL YOU FIND THE
TEN SMALLEST SQUARES.

THE NUMBERS IN THE RIGHT-HAND COLUMN OF YOUR TABLE ARE CALLED SQUARE
NUMBERS. WHY?

FIND THE NEXT TEN SQUARE NUMBERS.

FIND THE 25TH SQUARE NUMBER _____.

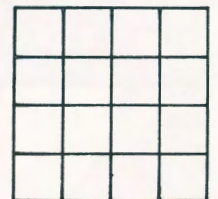
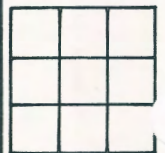
30TH SQUARE NUMBER _____.

50TH SQUARE NUMBER _____.

WHAT IS AN EASY WAY TO FIND A SQUARE NUMBER?

EXPLAIN HOW YOU WOULD FIND THE MILLIONTH SQUARE NUMBER.

SMALLEST SQUARES	
TILES ON ONE SIDE	TOTAL TILES USED
1	1
2	4



SQUARING 2-DIGIT NUMBERS

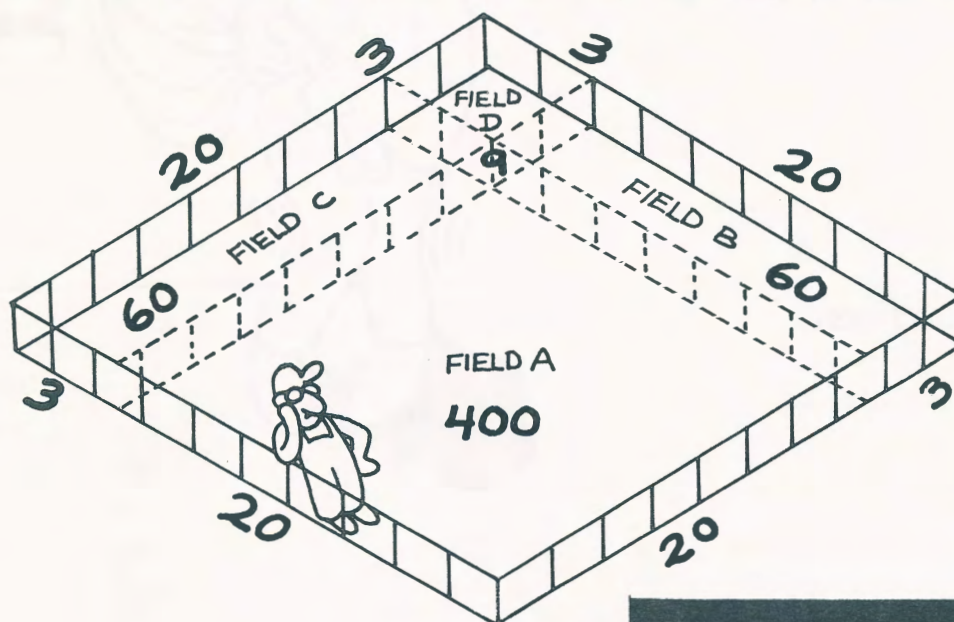
Squares
Multiplication
WHOLE NUMBERS



Can you compute 23^2 (or 23×23) in your head? Farmer Fred "squares" in his head by picturing fences in his field.

Here is how he does 23^2 .

He pictures fences as shown and adds up the areas of each field.



$$\begin{array}{r} 400 \\ 60 \\ 60 \\ + 9 \\ \hline 529 \end{array}$$

Here is a CHART of Farmer Fred's method:

X	20	3
20	400	60
3	60	9

$$23^2 = 400 + 60 + 60 + 9 = 529$$

Try Farmer Fred's method to find these "squares."

- | | |
|-----------|-----------|
| 1) 31^2 | 5) 48^2 |
| 2) 52^2 | 6) 67^2 |
| 3) 36^2 | 7) 81^2 |
| 4) 47^2 | 8) 99^2 |

Extension: Use Farmer Fred's method to find:

1) $27 \times 39 =$

X	20	7
30		
9		

TYPE: Paper & Pencil/Transparency

IDEA FROM: Number Power

Permission to use granted by Oakland County Mathematics Project



ATTENTION

Squares
Patterns
Multiplication
WHOLE NUMBERS



DO THESE MULTIPLICATIONS.

$$\begin{aligned} 15^2 &= 15 \times 15 = 225 \\ 25^2 &= 25 \times 25 = 625 \\ 35^2 &= 1225 \quad (3 \times 4 = 12, 5 \times 5 = 25) \\ 45^2 &= 2025 \\ 55^2 &= 3025 \end{aligned}$$

DO YOU SEE A PATTERN?

NOW DO THESE MULTIPLICATIONS.

$$\begin{aligned} 14 \times 16 &= 224 \\ 22 \times 28 &= 616 \\ 31 \times 39 &= 1809 \\ 43 \times 47 &= 2021 \quad (4 \times 5 = 20, 3 \times 7 = 21) \\ 55 \times 55 &= 3025 \end{aligned}$$

DO YOU SEE A PATTERN?

FOR WHICH OF THESE WILL THE PATTERN WORK?

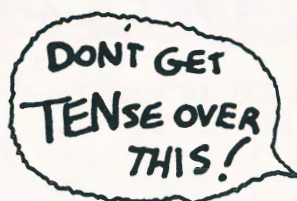
$42^2 =$	$31 \times 39 =$	$34 \times 31 =$
$89^2 =$	$32 \times 37 =$	$34 \times 36 =$
$75^2 =$	$33 \times 37 =$	$36 \times 33 =$

DESCRIBE THE PATTERN.

USE THE PATTERN TO DO THESE MULTIPLICATIONS.

$65^2 =$	$32 \times 38 =$	$45 \times 45 =$
$85^2 =$	$61 \times 69 =$	$77 \times 73 =$
$95^2 =$	$12 \times 18 =$	$58 \times 52 =$

For the pattern to work TENS' DIGITS must be the same, and the UNITS' DIGITS must add to 10.



MULTIPLE DIFFERENCES

Squares
Multiplication
WHOLE NUMBERS



$3 \times 3 =$ $2 \times 4 =$ DIFFERENCE =	$4 \times 4 =$ $3 \times 5 =$ DIFFERENCE =	$5 \times 5 =$ $4 \times 6 =$ DIFFERENCE =	$6 \times 6 =$ $5 \times 7 =$ DIFFERENCE =
--	--	--	--

IS THERE A PATTERN HERE?
WHICH IS GREATER? 7×7 OR 6×8 ? _____ BY HOW MUCH? _____

CAN
YOU
GUESS?

$60 \times 60 = 3600$
 $59 \times 61 =$
DIFFERENCE =

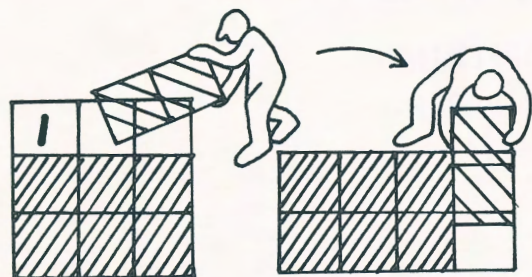
CAN YOU TELL A QUICK WAY
TO FIND 19×21 ?

$19 \times 21 =$
DIFFERENCE =

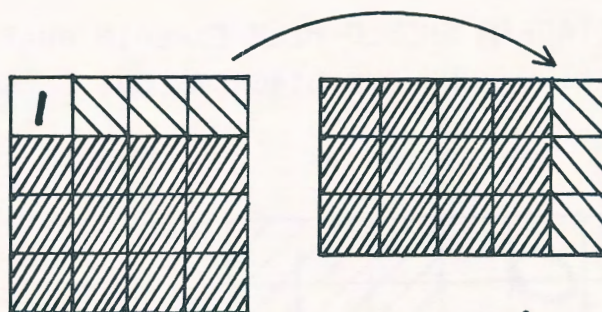
USE THIS IDEA TO FIND THE FOLLOWING ANSWERS.

1. 39×41 2. 79×81 3. 299×301 4. 8999×9001

THESE DIAGRAMS HELP EXPLAIN WHY THIS WORKS.



$$(3 \times 3) - (2 \times 4) = 1$$



$$(4 \times 4) - (3 \times 5) = 1$$

SHOW $(6 \times 6) - (5 \times 7) = 1$.

Here is the algebraic
justification for this method:

$$a \cdot a = a^2$$

$$(a - 1)(a + 1) = a^2 - 1$$

Difference = 1

TYPE: Paper & Pencil/Transparency

IDEA FROM: Mathex, Junior-Operations and Problem Solving

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MULTIPLE DIFFERENCES (CONTINUED)

$4 \times 4 =$ $3 \times 5 =$ DIFFERENCE =	$4 \times 4 =$ $2 \times 6 =$ DIFFERENCE =	$4 \times 4 =$ $1 \times 7 =$ DIFFERENCE =
$6 \times 6 =$ $5 \times 7 =$ DIFFERENCE =	$6 \times 6 =$ $4 \times 8 =$ DIFFERENCE =	$6 \times 6 =$ $3 \times 9 =$ DIFFERENCE =
$6 \times 6 =$ $2 \times 10 =$ DIFFERENCE =	$6 \times 6 =$ $1 \times 11 =$ DIFFERENCE =	

TRY THIS WITH 5×5 . ARE THE DIFFERENCES THE SAME? WHAT SEQUENCES DO THE DIFFERENCES MAKE? REPEAT FOR 7×7 , 8×8 , AND 9×9 .

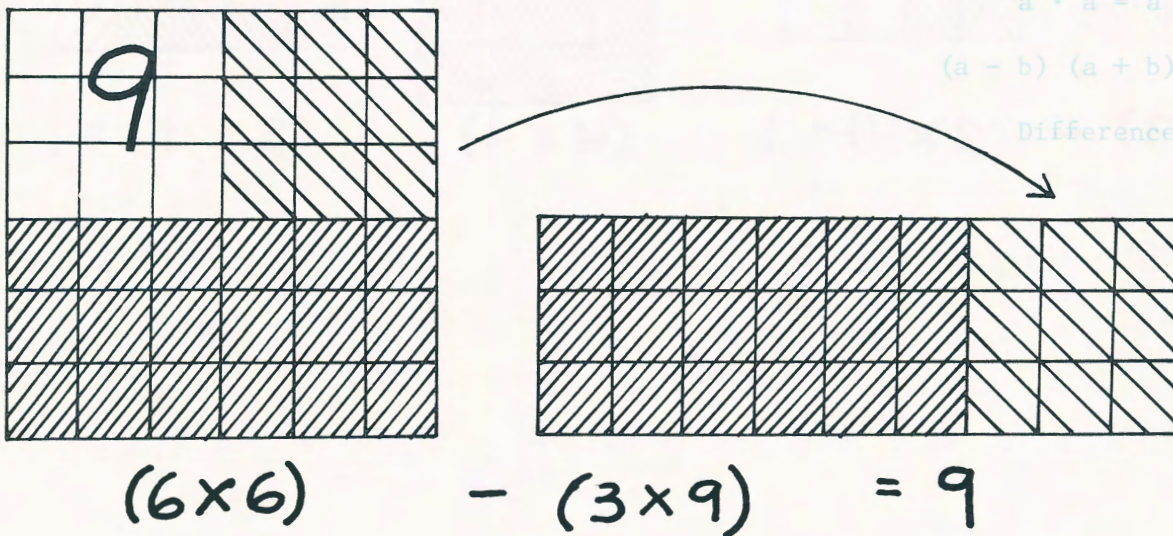
THIS DIAGRAM SHOULD HELP EXPLAIN WHAT IS HAPPENING.
TRY MAKING YOUR OWN DIAGRAMS OUT OF SQUARED PAPER.

Algebraic justification:

$$a \cdot a = a^2$$

$$(a - b)(a + b) = a^2 - b^2$$

$$\text{Difference} = b^2$$



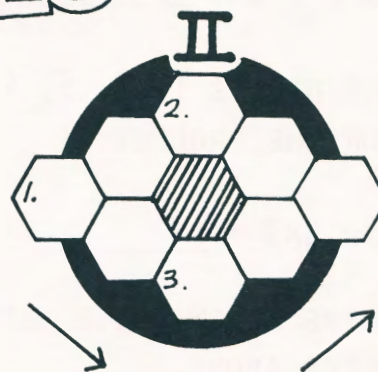
HEXED PRODUCTS

Multiplication
WHOLE NUMBERS



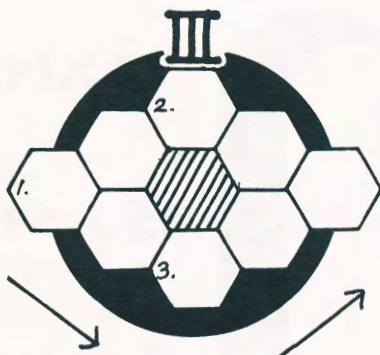
1. 9×22
2. 9×37

1. 9×17
3. 9×97



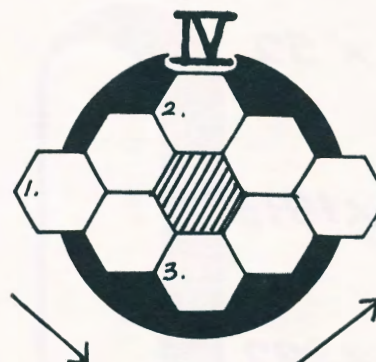
1. 11×33
2. 11×42

1. 11×34
3. 11×32



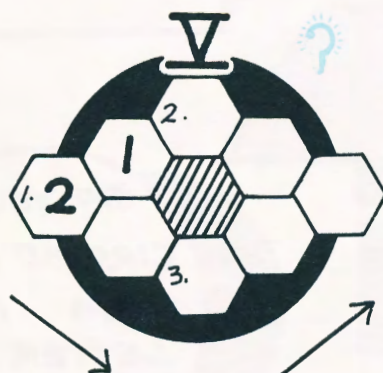
1. 12×16
2. 12×74

1. 12×14
3. 12×24



1. 15×19
2. 14×25

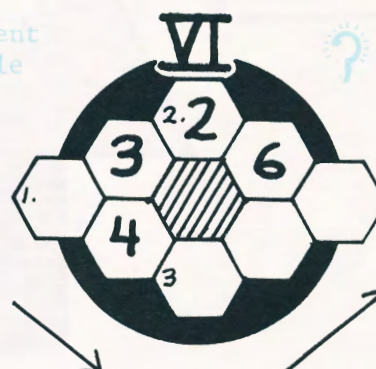
1. 13×21
3. 10×57



1. $7 \times$
2. $6 \times$

1. $5 \times$
3. 9×42

There are different
solutions possible
for puzzle V.



1. $9 \times$
2. $11 \times$

1. $13 \times$
3. $7 \times$

TYPE: Paper & Pencil/Puzzle

IDEA FROM: *Aftermath*, Volume 1

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Magic Digit Makers

Multiplication
WHOLE NUMBERS



COMPLETE: $43 \times 159 = \underline{\hspace{2cm}}$.

WHICH OF THE DIGITS 1, 2, 3, 4, 5, 6, 7, 8, 9 APPEAR IN THE FACTORS OR THE PRODUCT?

COMPUTE: $12 \times 483 = \underline{\hspace{2cm}}$. WHICH DIGITS APPEAR THIS TIME?

CIRCLE THE PAIRS BELOW WHICH GIVE ALL THE DIGITS LIKE THE TWO EXAMPLES ABOVE.

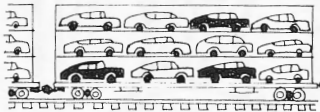
42×138	4×1963	27×198
318×52	18×297	72×198
4×1693	39×186	85×597
17×892	28×157	7×1234
71×453	4×1738	13×452



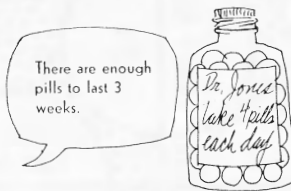
DO YOUR
CIRCLED PAIRS
FORM A
LETTER ?
THEY SHOULD
!!

TYPE: Paper & Pencil/Puzzle

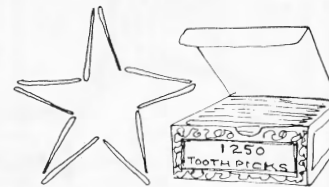
PICTURE PROBLEMS 2



There are 20 of these railroad cars in this train. How many autos are there altogether: _____



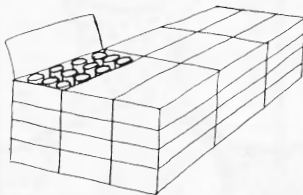
How many pills are in the bottle? _____



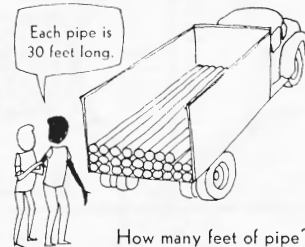
How many stars can be made from one box of toothpicks? _____



How much did each comic book cost? _____

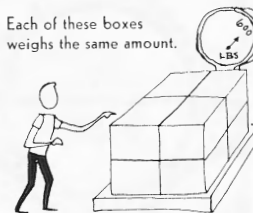


How many cans altogether: _____



How many feet of pipe? _____

Each of these boxes weighs the same amount.



How much does each box weigh? _____

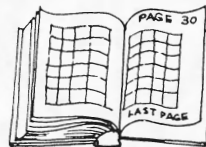


How many pounds should each boy carry? _____



In one day, there are 1440 minutes (24x60 = 1440)

How many minutes are there in a year (365 days) _____



How many stamps does it take to fill a book? _____

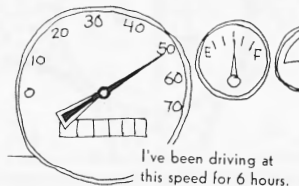


How much does the job pay per month? _____



How much will each boy have to pay? _____

Gotham City bought 16 police cars that cost \$2419 each. How much did the 16 cars cost altogether? _____



About how far has he driven? _____

START 220 MILE BOAT RACE

Exactly 5 hours later



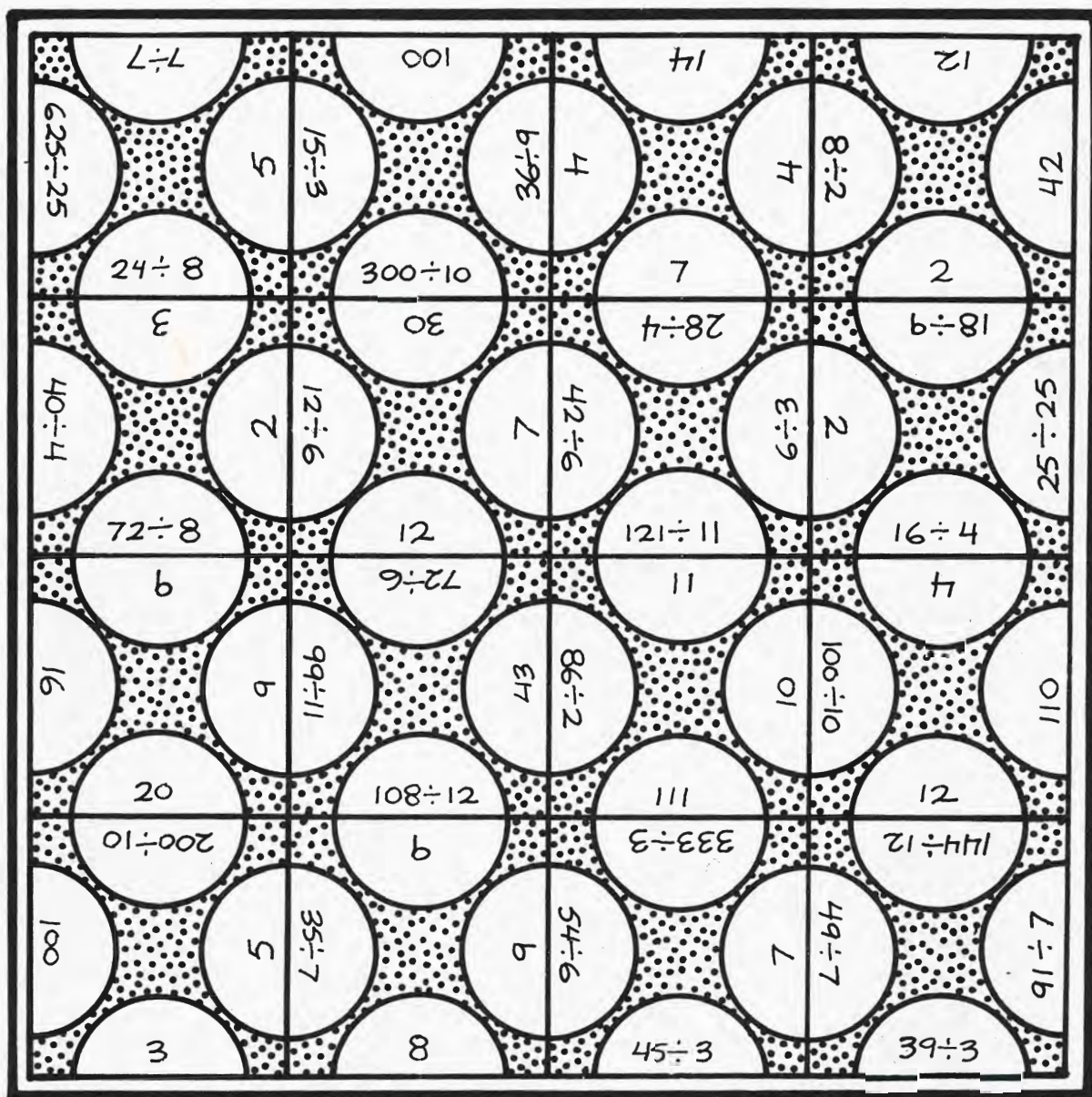
About how many miles did he travel each hour? _____

DIVISION JIGSAW



Cut out the pieces and give them to the students with directions to match each division problem to its answer. When completed, the puzzle will form a large 4 x 4 square.

Easier puzzles can be 3 x 3, harder ones 5 x 5. The puzzles can vary from basic operations--one kind, or mixed, to operations on fractions, decimals etc. The puzzle is more difficult if the outside border is cut off. Students can make their own puzzles, cut them out and give them to a friend to put together.



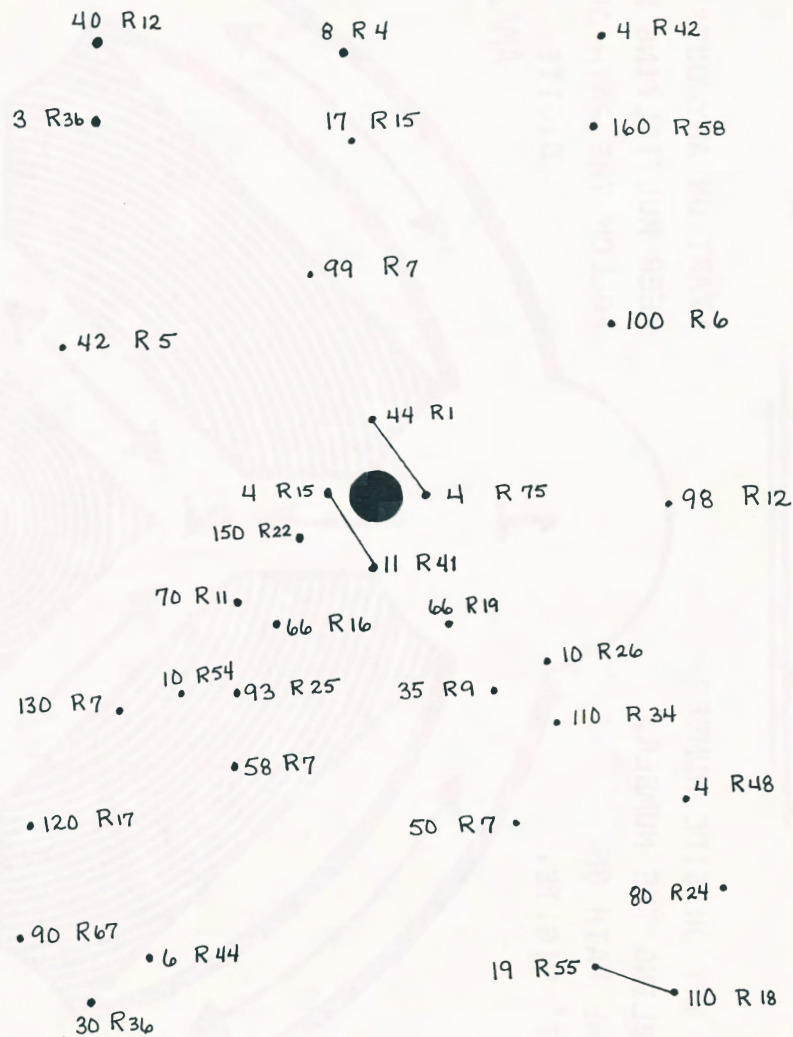
Clever Cut-up



Work each problem.
Connect your answers in order.



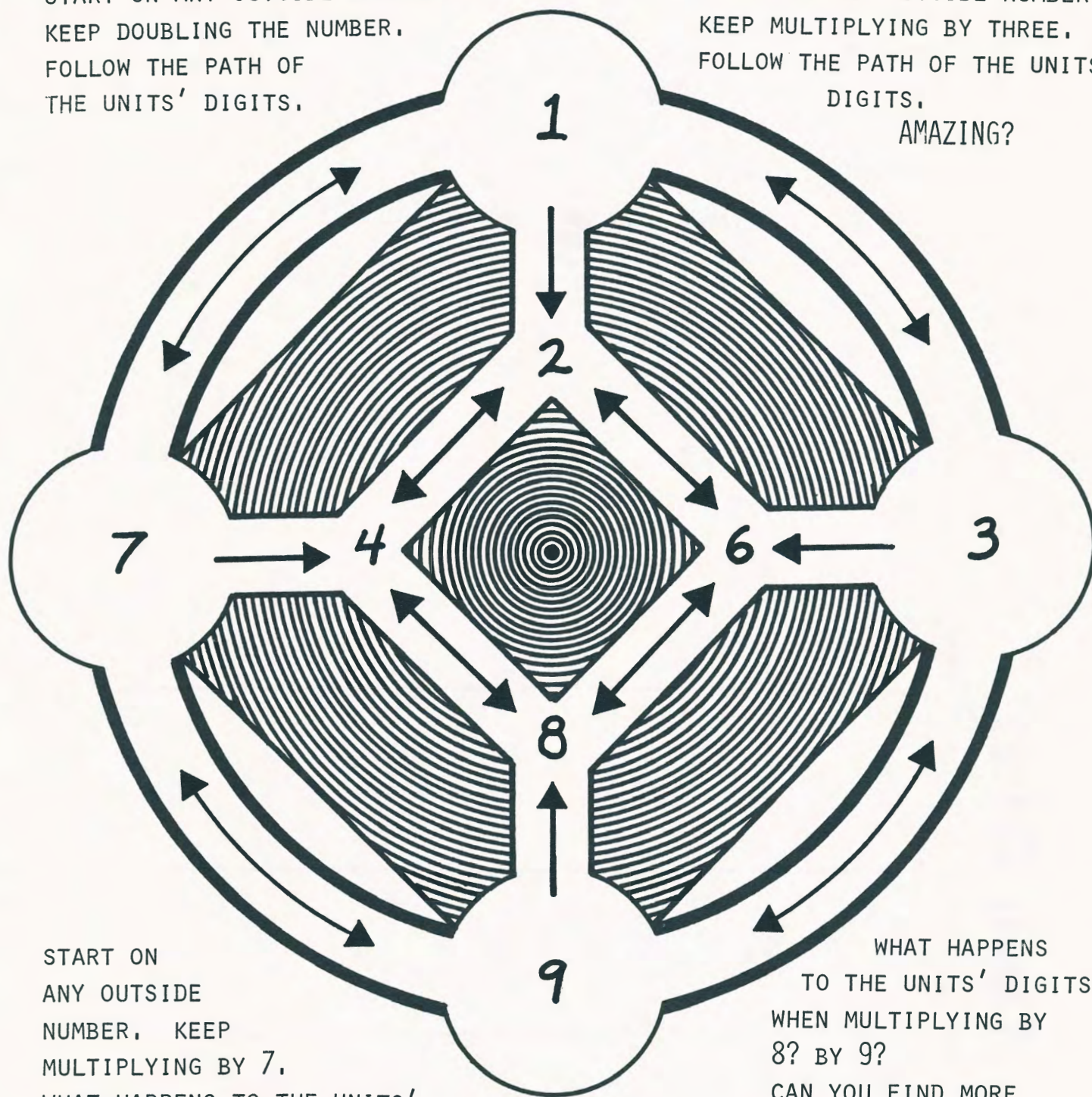
- | | | | |
|---------------------------|---------------------------|---------------------------|---------------------------|
| 1) $80 \overline{)8818}$ | 8) $54 \overline{)258}$ | 15) $62 \overline{)674}$ | 22) $56 \overline{)5233}$ |
| 2) $28 \overline{)2264}$ | 9) $31 \overline{)1365}$ | 16) $20 \overline{)2607}$ | 23) $29 \overline{)1930}$ |
| 3) $75 \overline{)348}$ | 10) $40 \overline{)1612}$ | 17) $35 \overline{)4217}$ | 24) $58 \overline{)679}$ |
| 4) $55 \overline{)6084}$ | 11) $51 \overline{)189}$ | 18) $76 \overline{)6907}$ | 25) $22 \overline{)1471}$ |
| 5) $73 \overline{)756}$ | 12) $39 \overline{)171}$ | 19) $63 \overline{)1926}$ | 26) $24 \overline{)849}$ |
| 6) $93 \overline{)447}$ | 13) $27 \overline{)4072}$ | 20) $99 \overline{)638}$ | 27) $10 \overline{)507}$ |
| 7) $67 \overline{)10778}$ | 14) $41 \overline{)2881}$ | 21) $75 \overline{)4357}$ | 28) $57 \overline{)1138}$ |



Amazing Units' Digit

START ON ANY OUTSIDE NUMBER.
KEEP DOUBLING THE NUMBER.
FOLLOW THE PATH OF
THE UNITS' DIGITS.

START ON ANY OUTSIDE NUMBER.
KEEP MULTIPLYING BY THREE.
FOLLOW THE PATH OF THE UNITS'
DIGITS.
AMAZING?



START ON
ANY OUTSIDE
NUMBER. KEEP
MULTIPLYING BY 7.
WHAT HAPPENS TO THE UNITS'
DIGITS?

WHAT HAPPENS
TO THE UNITS' DIGITS
WHEN MULTIPLYING BY
8? BY 9?
CAN YOU FIND MORE
PATTERNS?

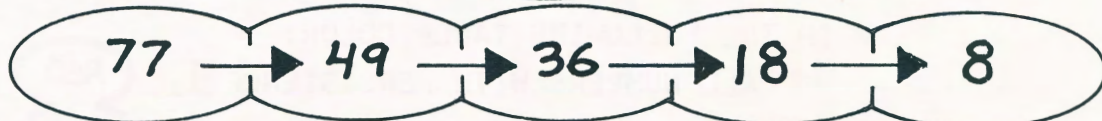
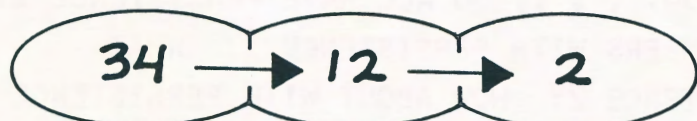


PERSISTENT NUMBERS

Multiplication
WHOLE NUMBERS



LOOK FOR A PATTERN IN THESE EXAMPLES:



IF YOU HAVE DISCOVERED THE RULE FOR THESE EXAMPLES, YOU SHOULD BE ABLE TO COMPLETE THE FOLLOWING SEQUENCES. (HINT: THINK MULTIPLICATION.)



THE ABOVE PATTERNS WERE COMPLETED BY COMPUTING PRODUCTS OF THE DIGITS OF ONE NUMBER TO FIND THE NUMBER TO ITS RIGHT. THE NUMBER OF ARROWS REQUIRED TO OBTAIN A SINGLE-DIGIT NUMBER IS CALLED THE PERSISTENCE OF THE NUMBER.



FIND THE PERSISTENCE OF EACH OF THE FOLLOWING NUMBERS ON A SEPARATE PIECE OF PAPER. (HINT: YOUR ANSWER SHOULD FORM TWO FAMOUS DATES.)

1. A) 91 B) 77 c) 26888999 D) 25
2. A) 321 B) 68889 c) 839844 D) 6788

TWO VERY PERSISTENT NUMBERS ARE 3,778,838,999 AND 277,777,788,888,999. ARE YOU PERSISTENT ENOUGH TO FIND THEIR PERSISTENCE? MAYBE YOU CAN FIND A HAND CALCULATOR TO HELP YOU.

TYPE: Paper & Pencil

IDEA FROM: "The Persistence of a Number," *Journal of Recreational Mathematics*, Spring, 1973

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PERSISTENT NUMBERS (CONTINUED)

THE TWO-DIGIT NUMBERS 10, 20, 30, . . . , 90 ALL HAVE PERSISTENCE 1. CAN YOU FIND ANY OTHER TWO-DIGIT NUMBERS WITH PERSISTENCE 1? WHAT ABOUT TWO-DIGIT NUMBERS WITH PERSISTENCE 2? HOW ABOUT WITH PERSISTENCE 3 OR 4?

IN THE FOLLOWING TABLE COLOR:

ALL NUMBERS WITH PERSISTENCE 1,

ALL NUMBERS WITH PERSISTENCE 2,

ALL NUMBERS WITH PERSISTENCE 3,

ALL NUMBERS WITH PERSISTENCE 4,



10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

WHAT ARE SOME INTERESTING PATTERNS IN THIS TABLE?

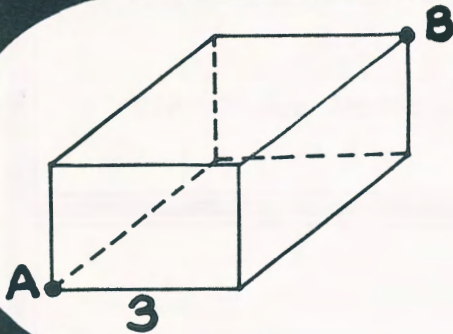
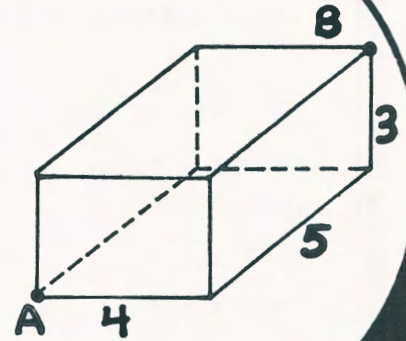


Path Products

Multiplication
WHOLE NUMBERS

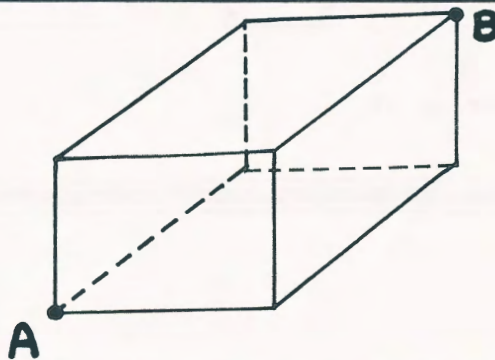


1. Can you find a path from A to B so that the product of the segments is 60?
2. Write a number sentence to show your path.
3. Find as many "60" paths as you can. Write a number sentence for each.
4. Can you find a path from A to B so that the product of the segments is greater than 60?
5. Can you find a path from A to B so that the product of the segments is less than 60?



Label the edges of this figure so that the product of one path from A to B is 30 and the product of another path from A to B is 120.

Write a sentence showing these two paths.



Label the edges of this figure so that the product of one path from A to B is 12 and the product of another path from A to B is 108. Write a sentence showing these 2 paths.

What is the product of the longest path from A to B in this figure without going over any segment twice?

TYPE: Paper & Pencil

This activity can be used to demonstrate the commutativity and associativity of multiplication.

IDEA FROM: *Computation and Structure*, Nuffield Mathematics Project

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FOUR INVESTIGATIONS

Patterns
Multiplication
WHOLE NUMBERS



1.

Look for a pattern as you do these multiplications.

$$1089 \times 1 = 1089$$

$$1089 \times 2 = 2178$$

$$1089 \times 3 = 3267$$

$$1089 \times 4 = 4356$$

$$1089 \times 5 = 5445$$

$$1089 \times 6 = 6534$$

$$1089 \times 7 =$$

$$1089 \times 8 =$$

$$1089 \times 9 =$$

2.

Look for a pattern as you do these multiplications.

$$10989 \times 9 = 98901$$

$$10989 \times 8 = 87912$$

$$10989 \times 7 = 76923$$

$$10989 \times 6 = 65934$$

$$10989 \times 5 = 54945$$

$$10989 \times 4 = 43956$$

$$10989 \times 3 =$$

$$10989 \times 2 =$$

$$10989 \times 1 =$$

Investigate 109989 and 1099989

3.

Look for a pattern.

$$\begin{array}{r} 12 \\ \times 9 \\ \hline \end{array}$$

$$108$$

$$\begin{array}{r} 123 \\ \times 9 \\ \hline \end{array}$$

$$1111104$$

$$\begin{array}{r} 1234 \\ \times 9 \\ \hline \end{array}$$

$$1107$$

$$\begin{array}{r} 9 \\ \times 9 \\ \hline \end{array}$$

$$11106$$

$$\begin{array}{r} 9 \\ \times 9 \\ \hline \end{array}$$

$$111105$$

Experiment with a multiplier of 99.

4.

$$12 \times 13 = 156$$

Subtract 5
from each
number

$$\begin{array}{c} 12 \\ \downarrow \\ 7 \\ \downarrow \\ 2 \end{array}$$

$$\begin{array}{c} 13 \\ \downarrow \\ 8 \\ \downarrow \\ 3 \end{array}$$

$$(7 + 8) = 15$$

$$(2 \times 3) = 6$$

$$\begin{array}{c} 15 \\ \times 6 \\ \hline \end{array}$$

Will this procedure work for other pairs of numbers?



REVERSALS LA SREVER

Calculators should be used here, especially if students are asked to find reversal pairs of their own.

$$\begin{array}{r} 34 \\ \times 86 \\ \hline \end{array}$$

$$\begin{array}{r} 43 \\ \times 68 \\ \hline \end{array}$$

$$\begin{array}{r} 64 \\ \times 23 \\ \hline \end{array}$$

$$\begin{array}{r} 46 \\ \times 32 \\ \hline \end{array}$$

$$\begin{array}{r} 28 \\ \times 41 \\ \hline \end{array}$$

$$\begin{array}{r} 82 \\ \times 14 \\ \hline \end{array}$$

Can you see why (34,86), (64,23) and (28,41) are called reversal pairs?

Which of the following are reversal pairs?

(83,36)

(57,32)

(53,23)

(42,36)

(93,26)

Find as many more reversal pairs as you can.

For two-digit numbers RS and rs, reversal pairs occur if $Rr = Ss$.
For example, (34,86), $3 \times 8 = 4 \times 6 = 24$.

For the very BRAVE: See if you can find all the reversal pairs below. (Hint: There are at least six!)



(336, 422)

(603, 204)

(693, 294)

(442, 224)

(366, 442)

(3636, 8064)

(264, 63)

(3756, 8064)

TYPE: Paper & Pencil

IDEA FROM: "Reversal Products," *Journal of Recreational Mathematics*, Spring, 1973

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For three-digit numbers RST and rst, reversal pairs occur if $Rr = Tt$ and if $Rs + Sr = St + Ts$.



REVERSAL-LIKE PAIRS

Multiplication
WHOLE NUMBERS



BEGIN WITH A REVERSAL
PAIR. (46,32)

LET'S
BUILD A
REVERSAL-LIKE
PAIR.

FIND:
$$\begin{array}{r} 4664 \\ \times 3223 \\ \hline \end{array}$$

Before this sheet, students
should have worked the
sheet called *Reversals*.

OK,
NOW LET'S TRY
 6446×2332 .
IS IT EQUAL TO
 4664×3223 ?

$$\begin{array}{r} 6446 \\ \times 2332 \\ \hline \end{array}$$

OH, I SEE,
WE CAN SAY THAT
(4664, 3223) IS A
REVERSAL-LIKE PAIR
SINCE 4664×3223
EQUALS $6446 \times$
 2332 .

TRY TO FIND A
REVERSAL-LIKE
PAIR USING
(28,41)

NEAT!
WE REVERSE
THE DIGITS
IN PAIRS.

CAN YOU FIND MORE REVERSAL-LIKE PAIRS? FOR
EXAMPLE, IS (84,36) A REVERSAL PAIR? IS
 $8448 \times 3663 = 4884 \times 6336$? FIND AT LEAST
FIVE MORE REVERSAL-LIKE PAIRS. BE SURE TO
CHECK THAT YOUR PAIRS ARE REVERSAL-LIKE.

TYPE: Paper & Pencil



28 ÷ 7 = 13

OR DO YOU BELIEVE THIS NONSENSE?

TEACHER DIRECTED ACTIVITY

Multiplication/Division
NUMBERS



DO YOU AGREE
WITH THIS
DIVISION?

$$\begin{array}{r} 1 \\ 7 \overline{)28} \\ \underline{7} \\ 21 \end{array}$$

$$\begin{array}{r} 13 \\ 7 \overline{)28} \\ \underline{7} \\ 21 \\ \underline{21} \end{array}$$

This activity should be done only after your students understand the correct algorithms.

SEVEN WON'T GO INTO
2, BUT IT WILL GO
INTO 8 ONE TIME.

EVERYONE KNOWS 7
GOES INTO 21 THREE
TIMES.

ONE WAY TO
CHECK A DIVISION
PROBLEM IS TO
MULTIPLY.

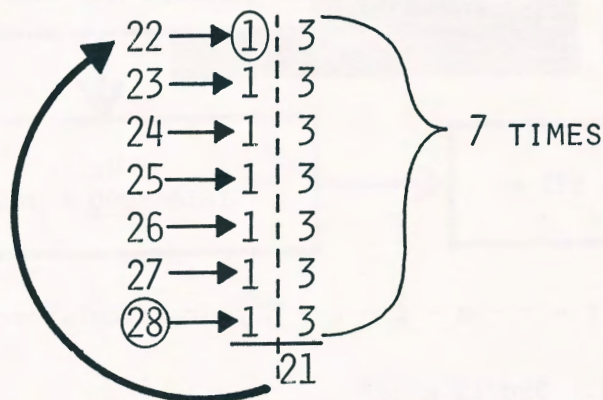
IF $28 \div 7 = 13$
THEN
 $7 \times 13 = 28$

MULTIPLICATION CHECK:

$$\begin{array}{r} 13 \\ \times 7 \\ \hline 21 \\ \underline{7} \\ 28 \end{array}$$

← 7 TIMES 3 = 21
← 7 TIMES 1 = 7

BUT MULTIPLICATION
IS JUST REPEATED
ADDITION.



CAN YOU FIND OTHER EXAMPLES
LIKE THIS?

There are 24 similar problems in all.
Here are a few more:

$$\begin{array}{l} 36 : 4 = 18 \\ 45 : 5 = 18 \\ 48 : 6 = 17 \\ 27 : 3 = 27 \end{array}$$



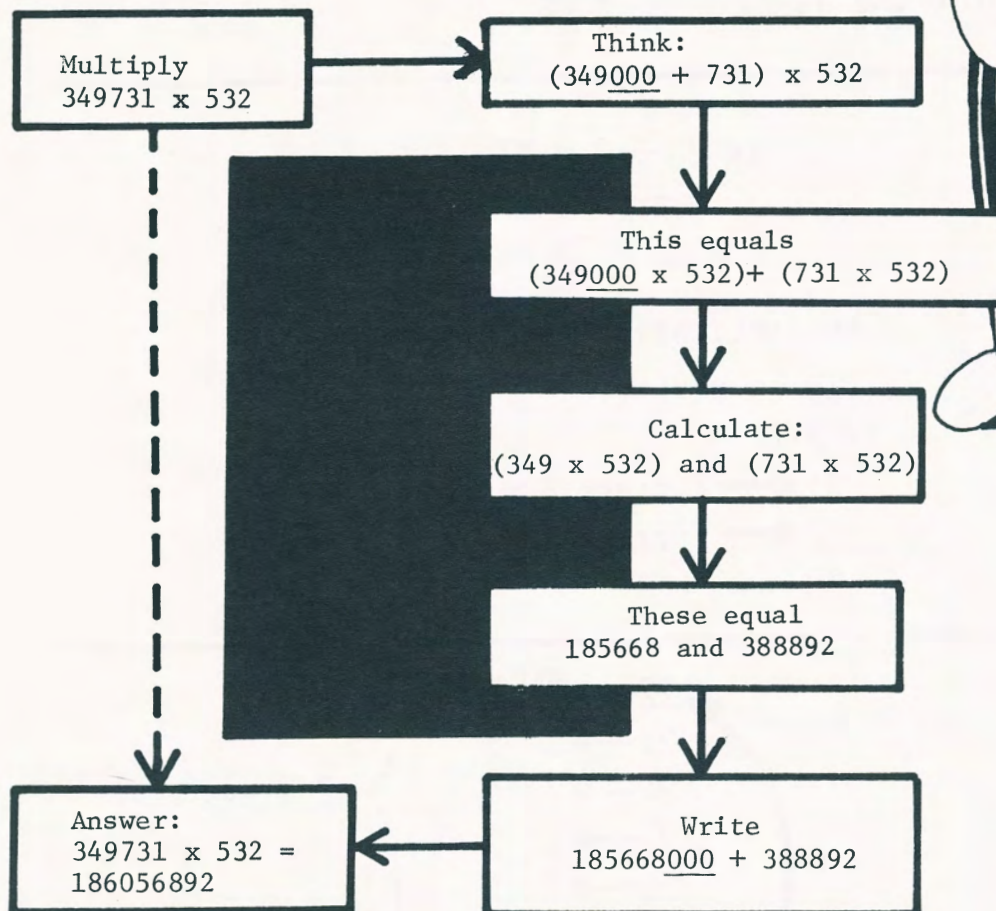
STRETCH

YOUR CALCULATOR

Multiplication
WHOLE NUMBERS



Can you find 349731×532 on your calculator? Atlas only has a 6 digit display on his calculator so he has to s - t - r - e - t - c - h his calculator. Here's how.



Can you s - t - r - e - t - c - h your calculator to find these products?

1. 596712×185
2. 617842×342
3. 487315×359
4. 564127×415

5. 47932×8916
6. 2345×98765
7. 4512×87654
8. 51326×62315

TYPE: Paper & Pencil



$$2 \times 3$$

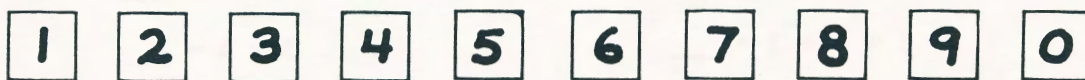
GOOD TIMES

Multiplication
WHOLE NUMBERS

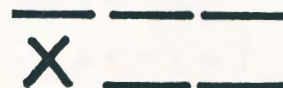


TEACHER DIRECTED ACTIVITY

MATERIALS: 10 small index cards--each with a different digit written on it.



OBJECT: To get the largest product.



- PROCEDURE:
1. Have each student copy the diagram at the right on a piece of paper.
 2. Shuffle the 10 digit cards and then show the student one at a time until 5 cards have been shown.
 3. As each number is shown student places it in the diagram. Once a number is placed in the diagram it may not be changed.
 4. When all 5 numbers are placed, students do the indicated multiplication.
 5. After the students have worked the problem you might ask, who got over 1000? . . . 2000? . . . 4000? until the winner is determined.
 6. Put the winning problem on the board for the class to see (and as a check).

EXAMPLE:

	Player A	Player B	2nd # shown	A	B
1st # shown	<div> <div>3</div> <div> <div>3</div> <div>X</div> <div></div> </div> </div>	<div> <div></div> <div> <div></div> <div>X</div> <div></div> </div> </div>		<div> <div>8 3</div> <div>X</div> <div></div> </div>	<div> <div></div> <div> <div></div> <div>X</div> <div>8</div> </div> </div>
3rd # shown	<div> <div>8 3 0</div> <div>X</div> <div></div> </div>			<div> <div>8 3 0</div> <div>X</div> <div>5</div> </div>	<div> <div>8 3 0</div> <div>X</div> <div>5 2</div> </div>
4th # shown		<div> <div></div> <div> <div></div> <div>X</div> <div>5</div> </div> </div>			
5th # shown					<div> <div>2 5 3</div> <div>X</div> <div>8 0</div> </div>

Player A wins.

- EXTENSIONS:
1. Have everyone use the 5 digits not shown and come up with the highest product.
 2. Find the lowest product.
 3. Find product closest to 10,000.
 4. Use a similar game with addition, subtraction or division.

TYPE: Activity



THE YOUNG GENIUS

Patterns
Multiplication
WHOLE NUMBERS



1

Calculate:

$$1 + 2 = \underline{\quad}$$

$$2 \times 3 = \underline{\quad}$$

$$1 + 2 + 3 = \underline{\quad}$$

$$3 \times 4 = \underline{\quad}$$

$$1 + 2 + 3 + 4 = \underline{\quad}$$

$$4 \times 5 = \underline{\quad}$$

$$1 + 2 + 3 + 4 + 5 = \underline{\quad}$$

$$5 \times 6 = \underline{\quad}$$

$$1 + 2 + 3 + 4 + 5 + 6 = \underline{\quad}$$

$$6 \times 7 = \underline{\quad}$$

Do you see a pattern? Explain.

Note: $1 + 2 + 3 + 4 + 5 + 6 = 21 = (6 \times 7) : 2$

2

When the world's greatest mathematician, C. F. Gauss, was a young boy, he was asked to add the first hundred numbers. Here's his surprising solution.

$$\begin{array}{r} 1 + 2 + 3 + \dots + 98 + 99 + 100 \\ + 100 + 99 + 98 + \dots + 3 + 2 + 1 \\ \hline 101 + 101 + 101 + \dots + 101 + 101 + 101 \end{array}$$

101 added 100 times or

$$100 \times 101$$

But I must divide this product by two.
(Why?) Therefore,

$$1 + 2 + 3 + \dots + 99 + 100 = \frac{100 \times 101}{2}$$

or 5050.



3

Predict the following sums using Gauss' method.

$$1 + 2 + 3 + \dots + 8 + 9 = \underline{\quad}$$

$$1 + 2 + 3 + \dots + 16 + 17 = \underline{\quad}$$

$$1 + 2 + 3 + \dots + 29 + 30 = \underline{\quad}$$

Add and check your answers. How were your predictions?

What is $1 + 2 + \dots + 49 + 50$ equal to?

4

CHALLENGE:

See if you can find sums using young Gauss' method.

$$4 + 5 + \dots + 49 + 50 = \underline{\quad}$$

$$7 + 9 + 11 + 13 + 15 + 17 = \underline{\quad}$$

$$1 + 5 + \dots + 97 + 101 = \underline{\quad}$$

$$1 + 2 + \dots + 199 + 200 = \underline{\quad}$$

$$5 + 7 + \dots + 15 + 17 = \underline{\quad}$$

$$76 + 78 + \dots + 272 + 274 = \underline{\quad}$$

See next page for discussion of using Gauss' method for these problems.

THE YOUNG GENIUS (CONTINUED)

TEACHER PAGE

Gauss' method works for any sequence of whole numbers which forms an arithmetic progression, i.e., where the interval between any two consecutive terms of the sequence is the same.

Example: $2 + 5 + 8 + 11 + 14$ --consecutive terms are 3 apart.

Below is an example of how to use the method to find sums of arithmetic progressions.

$$\begin{array}{r} 2 + 5 + 8 + 11 + 14 \\ + 14 + 11 + 8 + 5 + 2 \\ \hline 16 + 16 + 16 + 16 + 16 \end{array}$$

Observe:
 $2 + 14 = 16$
 (sum of first and last terms)

5 summands

$$5 \times 16 = 80; \quad 80 \div 2 = \underline{40} \quad (\text{sum of sequence})$$

How about $2 + 5 + 8 + \dots + 98$?

$$\begin{array}{r} 2 + 5 + 8 + \dots + 98 \\ 98 + 95 + 92 + \dots + 2 \\ \hline 100 + 100 + 100 + \dots + 100 \end{array}$$

How many summands?

To find how many terms are in the sequence:

Subtract first term from last term: $(98 - 2 = 96)$

Divide by the common interval: $(96 \div 3 = 32)$

Add 1: $(32 + 1 = 33)$

There are 33 terms in the last example, so

$$33 \times 100 = 3300; \quad 3300 \div 2 = 1650 \quad (\text{sum of sequences})$$

Here is another example:

$$3 + 8 + 13 + 18 + \dots + 118$$

$$3 + 118 = 121 \quad (\text{sum of first and last terms})$$

$$118 - 3 = 115; \quad 115 \div 5 = 23; \quad 23 + 1 = 24 \quad (\text{number of terms})$$

$$\frac{24 \times 121}{2} = 12 \times 121 = 1452 \quad (\text{sum of sequence})$$

If f is the first term, l is the last term and d is the difference between two consecutive terms (interval) then

$$S = \frac{(f + l) \times \left(\frac{l - f}{d} + 1\right)}{2}$$

For this example, $f = 3$,
 $l = 118$, $d = 5$

$$S = \frac{(3 + 118) \times \left(\frac{118 - 3}{5} + 1\right)}{2}$$

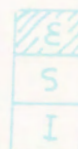
$$= \frac{121 \times 24}{2}$$

$$= 1452$$



SUM NUMBERS

Multiplication
WHOLE NUMBERS



Gauss' method of finding certain sums can be used to find what are called "sum numbers."

Compute:

$1 + 2 + \dots + 89 + 90 = 4095$	Add these	
$1 + 2 + \dots + 414 + 415 = 86320$	two numbers	$\rightarrow 90415$

What do you notice?

Some of the following are sum numbers. Can you find them?

- R $585910 = (1 + 2 + \dots + 584 + 585) = \underline{\hspace{2cm}}$
 $(1 + 2 + \dots + 909 + 910) = \underline{\hspace{2cm}}$ **ADD** \rightarrow $\underline{\hspace{2cm}}$
- T $1201545 = (1 + 2 + \dots + 119 + 120) = \underline{\hspace{2cm}}$
 $(1 + 2 + \dots + 1544 + 1545) = \underline{\hspace{2cm}}$ **ADD** \rightarrow $\underline{\hspace{2cm}}$
- S $758970 = (1 + 2 + \dots + 757 + 758) = \underline{\hspace{2cm}}$
 $(1 + 2 + \dots + 969 + 970) = \underline{\hspace{2cm}}$ **ADD** \rightarrow $\underline{\hspace{2cm}}$
- I $1999001 = (1 + 2 + \dots + 1998 + 1999) = \underline{\hspace{2cm}}$
 $1 = \underline{\hspace{2cm}}$ **ADD** \rightarrow $\underline{\hspace{2cm}}$
- A $39278 = (1 + 2 + \dots + 38 + 39) = \underline{\hspace{2cm}}$
 $(\underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \dots + \underline{\hspace{1cm}} + \underline{\hspace{1cm}}) = \underline{\hspace{2cm}}$ **ADD** \rightarrow $\underline{\hspace{2cm}}$
- G $1909415 = (1 + 2 + \dots + 1908 + 1909) = \underline{\hspace{2cm}}$
 $(\underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \dots + \underline{\hspace{1cm}} + \underline{\hspace{1cm}}) = \underline{\hspace{2cm}}$ **ADD** \rightarrow $\underline{\hspace{2cm}}$
- H $61549231 = (\underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \dots + \underline{\hspace{1cm}} + 6159) = \underline{\hspace{2cm}}$
 $(\underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \dots + \underline{\hspace{1cm}} + \underline{\hspace{1cm}}) = \underline{\hspace{2cm}}$ **ADD** \rightarrow $\underline{\hspace{2cm}}$

If you correctly unscramble the letters next to the sum numbers, you'll know that you're correct.

$$\begin{aligned} &\text{The sum of } 1 + \\ &2 + 3 + 4 + \dots \\ &+ n = \frac{n(n+1)}{2} \end{aligned}$$

TYPE: Paper & Pencil

This is a good exercise
for calculators.

NUMBER SENSE AND ARITHMETIC SKILLS

Placement Guide for Tabbed Dividers

Tab Label	Follows page:
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Whole Numbers Commentary: Mixed Operations (pages 401-408)

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Calculator Capers II	410	Using a calculator	Paper and pencil Manipulative
Mind Over Math	411	Mental Arithmetic	Activity
Backwards and Forwards	412	Computation	Game
Fill in the Wholes	413	Computation	Paper and pencil
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Rounded Results	415	Approximation	Paper and pencil Puzzle
Rounded Line Up	416	Approximation	Paper and pencil Puzzle
Don't Be Afraid to Guess	417	Approximation	Activity
A Slick Slip Stick	418	Approximation	Paper and pencil
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Operations with Whole Numbers	426	Computation	Paper and pencil Puzzle
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What's My Rule - I	430	Patterns	Paper and pencil Transparency
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Mom and Pop Steel Co.	445	Mixed operations	Activity
Change for a Quarter	447	Mixed operations	Activity

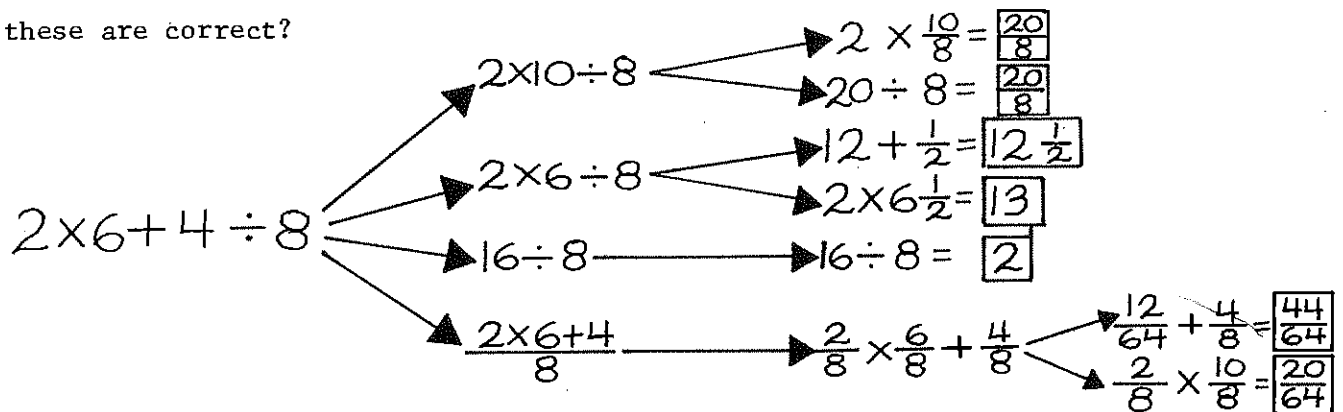
WHOLE NUMBERS:

MIXED OPERATIONS

Using each of the numbers 2, 4, 6 and 8 and any of the four operations (+, -, x, ÷) we can write the following numbers:

$$\begin{aligned} 1 &= [(4+6)-8] \div 2 \\ 2 &= [(2 \times 6)+4] \div 8 \\ 3 &= [(8-6)+4] \div 2 \\ 4 &= (2+6)-8+4 \\ 5 &= 4+[(8-6) \div 2] \end{aligned}$$

Is it possible to write all the whole numbers up to 50 in terms of these four numbers and four operations? Once you start you might not be able to stop until you get all the numbers--or are convinced that certain numbers can not be obtained. Students tend to approach this question with the same enthusiasm. They make up the problems and then solve them because there is a reason for doing so--a challenge. Look at all the practice they get; they are not only adding, subtracting, multiplying and dividing but also specifying the order in which they do the operations. Consider, for example, the expression $[(2 \times 6) + 4] \div 8$. We tend to take the groupings for granted. If the parentheses and brackets are dropped there are several ways the computations might be performed. Which, if any, of these are correct?



If a student is confused by parentheses he may be even more confused without them. In which of the above ways shall he interpret the problem? Multiplication and division take precedence over addition and subtraction, so without parentheses the correct answer to the above expression is $2 \times 6 + 4 \div 8 = 12 + \frac{1}{2} = 12\frac{1}{2}$. Confusion can also exist when only one operation is involved. $6 - 4 - 2 = 0$ when calculated from the left, but $6 - 4 - 2 = 4$ when calculated from the right. The use of parentheses makes the order of calculations clear. If a student writes $2 \times 6 + 4 \div 8$ and is interested in his answer being correctly interpreted as 2 then that student will be more inclined to use grouping notation correctly.

After the students have learned all four operations they need the opportunity to practice those skills in a meaningful way. One of the most important goals at this point is to apply the correct operation in a given situation. For example, if an automobile trip of 2842 km consumes 316 litres of gas what operation (or combination of operations) would you use to find the kilometres per litre (or litres per kilometre)?

The extension of skills in the four basic operations can be accomplished while students are engaged in problem solving and applications. In the process they can be improving their mental arithmetic and approximation skills and learning to make use of the calculator.




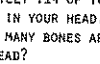






APPLICATIONS

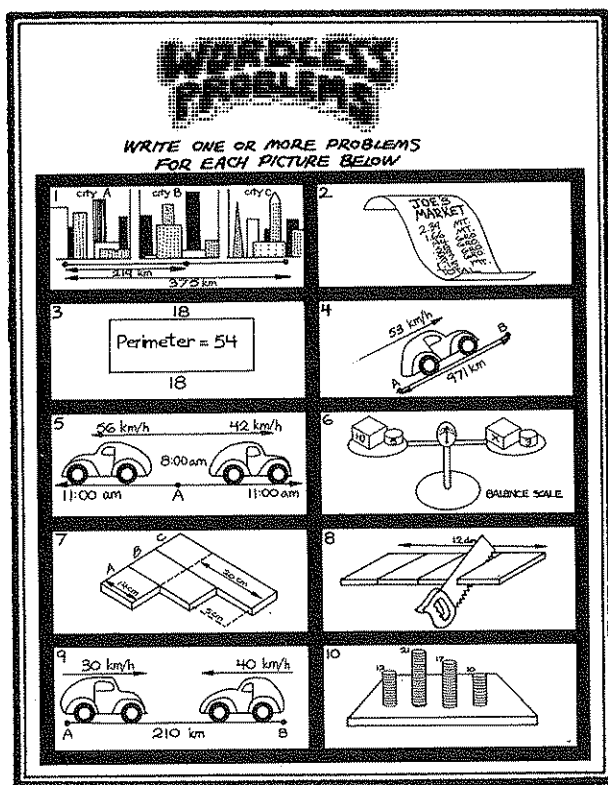
Once students have left school they will not be handed a page of division or subtraction problems to do, but they will be faced with situations which require them to apply the mathematics they have learned. We can have students apply their computation skills just like we have them apply their reading and spelling skills.

Most people think of "word problems" when they hear the word "applications." The image brought to mind is a list of typed problems filling a page. Word problems can be made more interesting with the addition of pictures coordinated with word problems.

Perhaps you can write a page of word problems for whole numbers using pictures like the student page shown at the right. The problems were chosen to give the students facts which might interest them. A page like this can be used as a transparency, a consumable paper/pencil page or as a laminated reusable card.

THAT'S JUST ABOUT THE SIZE OF IT!

1. MUSCLES MAKE UP ABOUT $\frac{1}{4}$ OF A PERSON'S BODY WEIGHT. ABOUT HOW MUCH DO YOUR MUSCLES WEIGH? 	2. YOUR INTESTINES (LARGE & SMALL) ARE ABOUT 7.5 M (25 FT.) LONG. HOW DOES THIS COMPARE WITH YOUR HEIGHT?
3. YOUR BODY CONTAINS 206 BONES. APPROXIMATELY $\frac{1}{14}$ OF YOUR BONES ARE IN YOUR HEAD. ABOUT HOW MANY BONES ARE IN YOUR HEAD? 	4. A PERSON'S BRAIN IS $\frac{1}{100}$ OF HIS BODY WEIGHT. HOW MUCH DOES YOUR BRAIN WEIGH? 
5. YOUR HEART BEATS ABOUT 80 TIMES A MINUTE. HOW MANY TIMES DOES IT BEAT IN A DAY? 	6. THE HUMAN BODY IS ABOUT $\frac{1}{6}$ WATER. HOW MANY POUNDS (KG) OF WATER ARE THERE IN YOU?
7. THE LARGEST GLAND IN YOUR BODY IS THE LIVER. IN A 160 LB (72.5 KG) PERSON THE LIVER IS ABOUT $\frac{1}{100}$ OF THE BODY WEIGHT. HOW MUCH DOES IT WEIGH? 	8. HUMAN BONES MAKE UP ABOUT $\frac{1}{4}$ OF A PERSON'S TOTAL BODY WEIGHT. HOW MUCH DO YOUR BONES WEIGH? 
9. THE EYE BLINKS ABOUT 25 TIMES EACH MINUTE. APPROXIMATELY HOW MANY TIMES DOES IT BLINK IN A DAY? 	10. THE BODY OF AN ADULT CONTAINS APPROXIMATELY 5 QUARTS (4.75 LITRES) OF BLOOD. A BLOOD DONOR USUALLY GIVES 1 PINT (.475 LITRES). WHAT FRACTION OF HIS BLOOD DOES A DONOR GIVE? 



IDEA FROM: *School Mathematics I*, by Robert E. Eicholtz, Phares, G. O'Daffer, Charles F. Brumfiel, Merrill E. Shanks, Charles R. Fiesnor. Copyright (c) 1971 by Addison-Wesley Publishing Company, Inc. All rights reserved. Reprinted by permission.

a) One teacher took a familiar age type problem, "Johnny is 12 years younger than his brother Tom, and Tom is twice as old as Johnny. How old is Tom?" and made up a similar problem about his own family. He told the class, "I can never remember my parents' ages, but I do remember that my mother is 4 years older than my father and that my father was 27 when my sister was born in 1952. How old are my mother and father?"

After working the problem the students were asked to write a similar "age" problem and bring it to class the next day. They were allowed to write age problems about themselves, relatives, pets or fictitious characters. The next day the problems were discussed and solved by the class. Some problems did not have enough information to solve; some problems had contradictory information.

For a change from word problems why not try some wordless problems? Pictures can be drawn which suggest problems. Students can then be asked to write a problem for each picture and then solve the problem. The graphics don't have to be fancy—just clear. Students in your class might like to draw wordless problems themselves. Problems like this could be shown on an overhead or given as a handout sheet. (The page at the left is included in the classroom materials.)

One of the drawbacks of word problems is that they seem so impersonal—they are always about some other people. How can we find applications that involve our students? Some ideas are given below.



The teacher tactfully made use of these problems to point out that in real life we often do not have enough information to solve a problem and that sometimes problems have no solutions. The problems were interesting to students because they involved people they knew and because they wrote the problems themselves.

b) Another teacher noticed she had a group of students interested in cars. They were forever looking at sports car magazines instead of doing their math problems. She decided to make use of their interest in cars and asked if she could borrow a couple of the magazines overnight. She found the magazines full of numbers which suggested problems for the students to solve. The next day the class was given problems based on information in the magazines. Here are three of the problems:

1. Read the ad at the right. How many cars did not finish the race? _____ What is the total number of miles that the 28 cars covered? _____ If a Porsche finished 2nd, 4th, 5th and 7th would you still conclude that the Renault is better?

Renault

The competition was fierce. But the Renault 17 Gordini factory team triumphed. Bearing the Fiat and Lancia factory teams as well as a Porsche Carrera and a Datsun 240 Z to win 1st, 3rd and 6th place in Gordini's first attempt in the Press On Regardless Rally (Oct. 30-Nov. 2). The only rally in America that's part of the world championship series.

Just as fierce was the course itself. 1,134 miles of treacherous timber trails and rough, rocky roads through the wilds of Michigan's Upper Peninsula. Out of 64 starters, only 28 finished.

ROAD TEST RESULTS

BRAKES

Minimum stopping distances, ft:
 From 60 mph 135
 From 80 mph 233
 Control in panic stop good
 Pedal effort for 0.5g stop, lb 35
 Fade: percent increase in pedal effort
 to maintain 0.5g deceleration in
 6 stops from 60 mph 43
 Parking: hold 30% grade? no
 Overall brake rating very good

2. Some of the road test results for a Honda CIVIC CVCC are given at the right. What is the difference of the stopping distance from 60 mph and 80 mph? _____ If a train was across the road 50 yds. ahead could this Honda be stopped in time at 60 mph? _____ 80 mph? _____ What do you think the minimum stopping distance for 70 mph is? _____ 50 mph? _____

Road Test Results from *Road and Track*, February 1975

Permission to use granted by CBS Consumer Publishing

The teacher did not assign the problems as a punishment for reading the magazine but rather as an acknowledgement of the students' interests. The students were allowed to share some of their information about cars with the rest of the class and plans were made to use the magazine again for decimal, ratio and percent problems.



PROBLEM SOLVING

To be effectively learned, problem-solving strategies (heuristics) must be regularly emphasized. There are many problem-solving strategies and they are more completely discussed in the overview book.

In the strict sense of the word asking students to add, subtract, multiply or divide whole numbers is not a problem-solving situation once they have learned the algorithms for those operations. On the other hand the resource page, *Fill in the Wholes*, asks students to use the algorithms in a way that is probably new to them. For example, if the sum of two numbers is 8, and their difference is 2, what are the numbers? Is there more than one answer? This new situation calls for a strategy. Some students may solve it by trial and error, while others may systematically list all pairs whose sum is eight and

FILL IN THE WHOLES

O AND Δ REPRESENT NUMBERS.
COMPLETE THE TABLES.

+	O	Δ	-
	4	2	
	6	3	
	12	9	
	5	2	
		6	3
12			4
	7	11	
7			7
		12	5
4	2		
	6	8	

+	-	O	Δ	\times	\div
		4	3		
		3		15	
				10	10
8	2				
		9	8		
15				56	
		11			11
		6	7		
12				36	
13				36	
24				144	
	0			121	

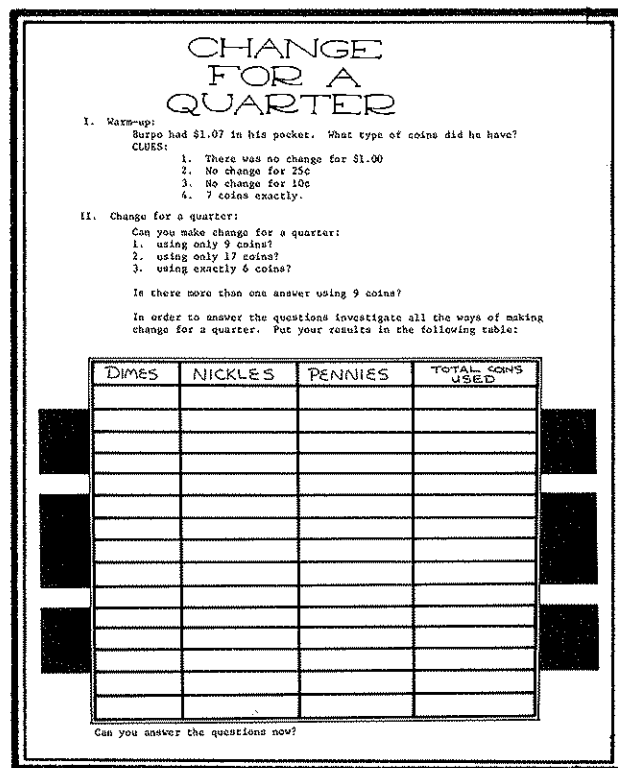
\times	O	Δ	\div
	6	2	
24	4		
	6		3
		2	4
5			5
3	1		
0		7	
		3	6
	9	8	
2		2	
	8	8	

MAKE YOUR OWN - STUMP A FRIEND.

+	-	O	Δ	\times	\div

choose the correct one. A class discussion focusing on different strategies used may divert the attention from answer seeking to methods or strategies for solution.

Another (teacher directed) activity that illustrates a different aspect of problem solving is the page *Change for a Quarter*. Here the strategy is organization of information. In other problems the organization might take the form of pictures or diagrams, but in this case it works nicely to use a table. When the table is completed a search can be made for patterns and relationships. Once again it is important that the solution of the problem, that is the answer, does not receive all the attention. The strategy, organization of information using a table, will probably be more important in the long run.



MENTAL ARITHMETIC AND APPROXIMATION

Properties of the decimal numeration system and properties of operations are often used when calculating an approximate or an exact result. The page *Mind Over Math* gives problems which can be solved quickly using number properties and mental arithmetic.

$11 + 12 + 13 + 17 + 18 + 19 = 30 + 30 + 30 = 90$

Finding exact answers by mental calculation requires different processes than finding sums by written algorithms. Once again, individual students may develop their own algorithms--and should be encouraged to do so. A standard way to mentally compute the sum $24 + 55$ is to think: $20 + 50$ is 70; $4 + 5$ is 9; therefore, $24 + 55$ is $70 + 9$ or 79.

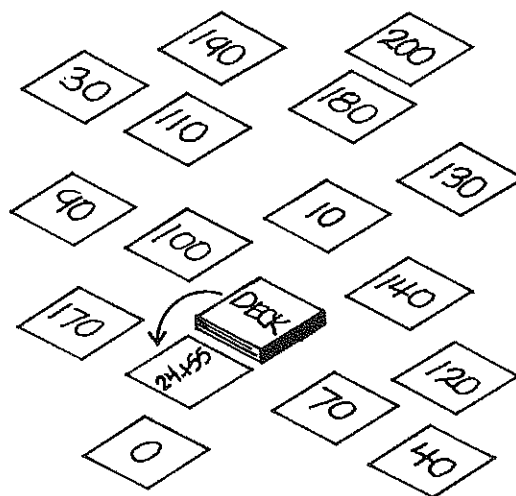
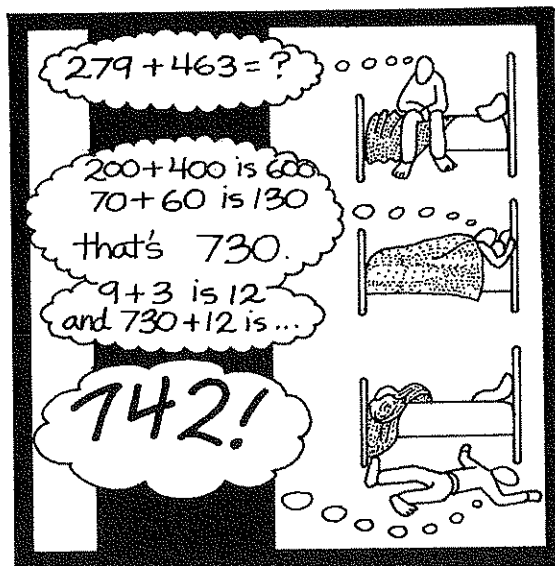
Notice that in the written algorithm we start with units, then add tens, etc., but mentally it seems easier to reverse the procedure. Haven't you noticed that when mentally totalling a restaurant check it is easier to first total the dollars, then the dimes and finally the pennies? That way even if you miscalculate, you at least get an approximation.

If we expect students to check the reasonableness of their answers and perform exact mental calculations, we must provide the opportunity for them to practice these skills periodically. Time devoted to such activities is certainly worthwhile in terms of number properties and the personal satisfaction it may give the individual.

The ability to approximate is important when checking for reasonable answers or when an approximate result is all that is necessary. We can encourage approximation skills with carefully chosen activities. The classroom pages, *Don't be Afraid to Guess*, *Rounded Line Up*, *Rounded Results*, *A Slick Slip Stick* and *About Right*, all emphasize approximation. In addition, you might adapt pages from fractions and decimals for whole number practice.

An adaptation of *The Almost Game* in FRACTIONS: Addition/Subtraction provides good practice for approximation. When a card showing the addition of two two-digit numbers is turned from the deck, the first person to point to the nearest multiple of ten for the sum gets a point.

The same approximation techniques can be used on three-digit sums, $279 + 463$. Rounding to the nearest hundred, our first approximation would be 800 ($300 + 500$). A more refined approximation could be obtained by rounding to the nearest tens to get $280 + 460 = 740$. Both approximations serve as good indicators of the vicinity of the correct sum.





CALCULATORS

Calculators can be used advantageously in many ways. Suggestions for introducing the calculator are given in *Calculator Capers*. With a calculator students are free to explore problems which would be drudgery with paper and pencil. The student page, *The Reverse Double Digit Magic Trick*, asks students to calculate $[(96 \times 23) - (69 \times 32)] \div 99$ and other strings of two-digit numbers. The objective of the page is to discover patterns and relationships. To compute the 12 long problems on the page by hand might not be much fun but with a calculator the computation is easy. The student's time is left free to consider questions like "Why do the first problems give zero? What other numbers work this way?"

It is easy to push the wrong button when using a calculator, so it is important to know when an answer is reasonable. Students must be able to approximate sums, differences, products and quotients. The page *A Slick Slip Stick* asks students to determine if answers found with a calculator are reasonable.

A calculator is useful for many of the activities in this resource. You can check the upper left hand corners of student pages for calculator keyed pages. The teaching emphases section in the appendix gives an overview and list of calculator pages.



CALCULATOR CAPERS I

Using a Calculator
Mixed Operations
WHOLE NUMBERS



MAKE A 2 APPEAR IN THE DISPLAY.
MAKE ALL ZEROS APPEAR IN THE DISPLAY.
WHAT DOES THE "C" BUTTON DO?
ADD 12 AND 72 ON THE CALCULATOR.
WHAT IS THE LARGEST NUMBER YOU CAN SHOW ON THE CALCULATOR?
ADD $792 + 792 + 792$. WHAT IS YOUR ANSWER?
DO $792 + 792 + 792$ AS A MULTIPLICATION PROBLEM.
DO $192 + 192 + 192 + 192 + 192$ AS A MULTIPLICATION PROBLEM.
FIND THE ANSWER TO 978×796 .
WORK THE FOLLOWING PROBLEM TWO DIFFERENT WAYS ON THE CALCULATOR.
 $(274 + 184) \times 492$
FIND THE ANSWER TO THE SUBTRACTION PROBLEM $3975 - 2886$.
HOW MANY TIMES CAN YOU SUBTRACT 28 FROM 168?
DIVIDE 45 BY 15.
SUBTRACT 15 FROM 45 AS MANY TIMES AS POSSIBLE. HOW MANY?
DID YOU GET THE SAME ANSWER FOR THE LAST TWO PROBLEMS?
DIVIDE 486 BY 54.
DIVIDE 9583 BY 63.
MAKE UP A DIVISION PROBLEM OF YOUR OWN AND WORK IT.
MAKE UP A MULTIPLICATION PROBLEM OF YOUR OWN AND WORK IT.
WORK THE FOLLOWING PROBLEM ON THE CALCULATOR. $(265 \times 75) + 3$.
MAKE UP SOME PROBLEMS AND WORK THEM.

TYPE: Paper & Pencil/Manipulative

IDEA FROM: *Mathematics Laboratory Handbook for the Junior High School*, San Diego City Schools

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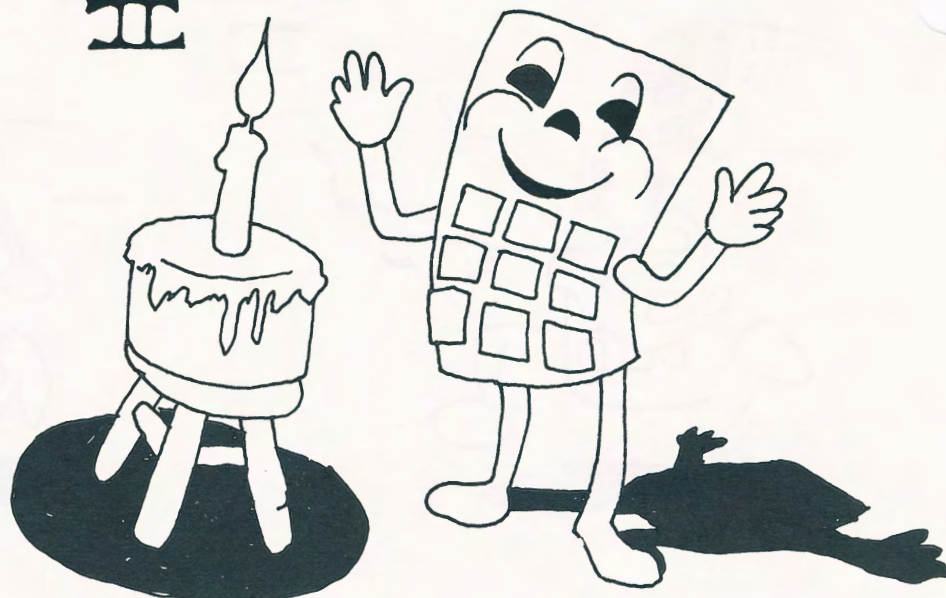


CALCULATOR CAPERS II

Using a Calculator
Mixed Operations
WHOLE NUMBERS



TAKE YOUR
AGE AT
YOUR LAST
BIRTHDAY
(IN YEARS)
AND FIGURE ON
THE CALCULATOR
THE FOLLOWING
INFORMATION.



1. THE NUMBER OF MONTHS YOU'VE BEEN ALIVE _____
2. THE NUMBER OF WEEKS YOU'VE BEEN ALIVE _____
3. THE NUMBER OF DAYS YOU'VE BEEN ALIVE _____
4. THE NUMBER OF HOURS YOU'VE BEEN ALIVE _____
5. THE NUMBER OF MINUTES YOU'VE BEEN ALIVE _____
6. THE NUMBER OF SECONDS YOU'VE BEEN ALIVE _____

IF YOU WERE 34 YEARS OLD AT YOUR LAST BIRTHDAY HOW MANY HOURS WOULD YOU HAVE LIVED? _____

IF YOU LIVE TO BE EXACTLY 90 YEARS OLD HOW MANY DAYS WILL YOU HAVE LIVED? _____

IF A DOG LIVES TO BE 16 YEARS OLD BY OUR CALENDAR AND EACH OF OUR YEARS IS EQUAL TO 7 DOG YEARS HOW MANY DOG HOURS WOULD THE DOG HAVE LIVED?

CAN YOU THINK OF SOME MORE INTERESTING IDEAS YOU'D LIKE TO EXPLORE ON THE CALCULATOR?

WRITE YOUR PROBLEMS AND SOLUTIONS HERE:

TYPE: Paper & Pencil/Manipulative

IDEA FROM: *Mathematics Laboratory Handbook for the Junior High School*, San Diego City Schools

MIND OVER MATH

Mental Arithmetic +, -, x, :
Mixed Operations
WHOLE NUMBERS



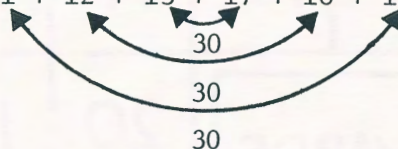
TEACHER DIRECTED ACTIVITY

- I. Answer Only Please - Place a series of problems on the board one at a time. Encourage students to look for a short way, then write only the answer to the problem.

Examples: $5 \times 22 = (5 \times 2) \times 11 = 10 \times 11 = 110$

$(53 \times 114) - (53 \times 104) = 53 \times 10 = 530$

$11 + 12 + 13 + 17 + 18 + 19 = 30 + 30 + 30 = 90$



More Problems:

$46 \times 5 =$

$24 \times 7 + 26 \times 7 =$

$(282 + 175) - (262 + 165) =$

$25 \times 8 \times 4 \times 7 =$

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 =$

$18 \times 53 \times 145 \times 119 \times 1347 \times 0 =$

$2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 =$

$(2 \times 7) + (3 \times 7) + (4 \times 7) + (5 \times 7) + (6 \times 7) =$

These are all problems which can be done mentally by using properties such as commutative, associative and distributive.

- II. How Quick Are You? - Place a series of sentences on the board one at a time. Instruct students to answer true or false to each statement as quickly as they can. They should do any necessary calculations mentally.

Examples:

$7 - 2 = 2 - 7$

$793 \times 1543 = 1543 \times 793$

$2 + 3 + 4 = 4 + 3 + 2$

$12 \times (4 \div 2) = (12 \times 4) \div 2$

$5 - (4 - 3) = (5 - 4) - 3$

$417 \times 24 = (417 \times 20) + (417 \times 5)$

$40 \div 80 = 80 \div 40$

$18 \times (2 - 1) = (18 \times 2) - 1$

$2 + (3 \times 4) = (2 + 3) \times 4$

$7 \times 0 \times 8 = 56$

$18 \times 3 \times 4 = 18 \times 12$

$385 \times 30 = (400 \times 30) - (15 \times 30)$

This activity may be viewed as asking students whether such properties as commutative, associative, and distributive are being used correctly.

TYPE: Activity



FILL IN THE WHOLES

Computation +, -, x, ÷
Mixed Operations
WHOLE NUMBERS



○ AND △ REPRESENT NUMBERS.
COMPLETE THE TABLES.

Some squares have no whole number answers. You may direct students to cross these squares out or encourage students to answer with fractions. Subtraction problems may also be answered with negative numbers.

+	○	△	-
6	4	2	2
	6	3	
	12	9	
5	2		
		6	3
12			4
	7	11	
7			7
		12	5
4	2		
	6	8	

x	○	△	÷
12	6	2	3
24	4		
	6		3
		2	4
5			5
3	1		
0		7	
		3	6
	9	8	
2		2	
	8	8	

+	-	○	△	x	÷
7	1	4	3	12	x
		3		15	
				10	10
8	2				
		9	8		
15				56	
		11			11
		6	7		
12				36	
13				36	
24				144	
	0			121	

MAKE YOUR OWN - STUMP A FRIEND.

+	-	○	△	x	÷

This type activity can also be used for fractions and decimals and integers.



RULE RUMMY

Computation $+$, $-$, \times , \div
Mixed Operations
WHOLE NUMBERS

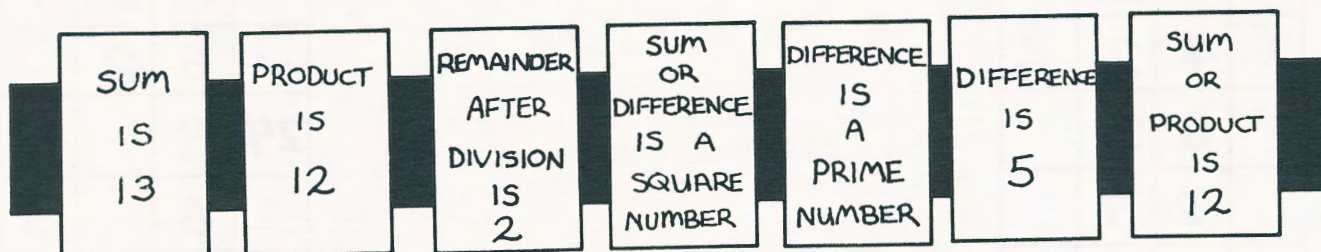


NEEDED: 2 TO 5 PLAYERS

DECK OF REGULAR CARDS - FACE CARDS REMOVED

DECK OF RULE CARDS.

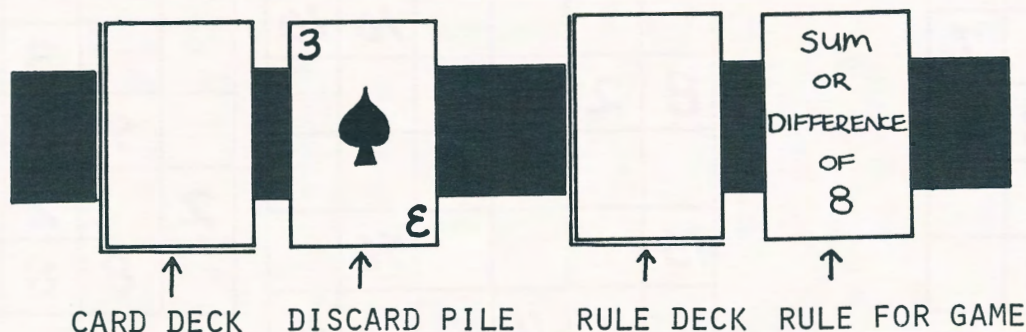
EXAMPLES:



RULES: DEAL 8 CARDS EACH FOR 2 PLAYERS

6 CARDS EACH 3-4 PLAYERS

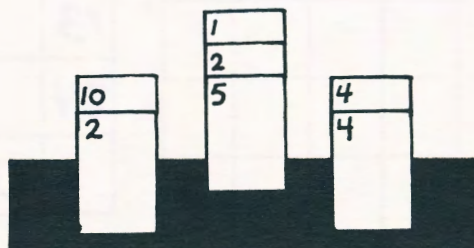
5 CARDS EACH 5 PLAYERS



ONE RULE CARD IS TURNED OVER AND USED FOR ENTIRE GAME.

EACH PLAYER IN TURN DRAWS A CARD FROM THE CARD DECK OR DISCARD PILE, MAKES A PLAY (IF POSSIBLE) AND DISCARDS AS IN RUMMY. PLAYS ARE MADE BY THE RULE.

EXAMPLE OF VARIOUS PLAYS
DIFFERENCE OR SUM OF 8



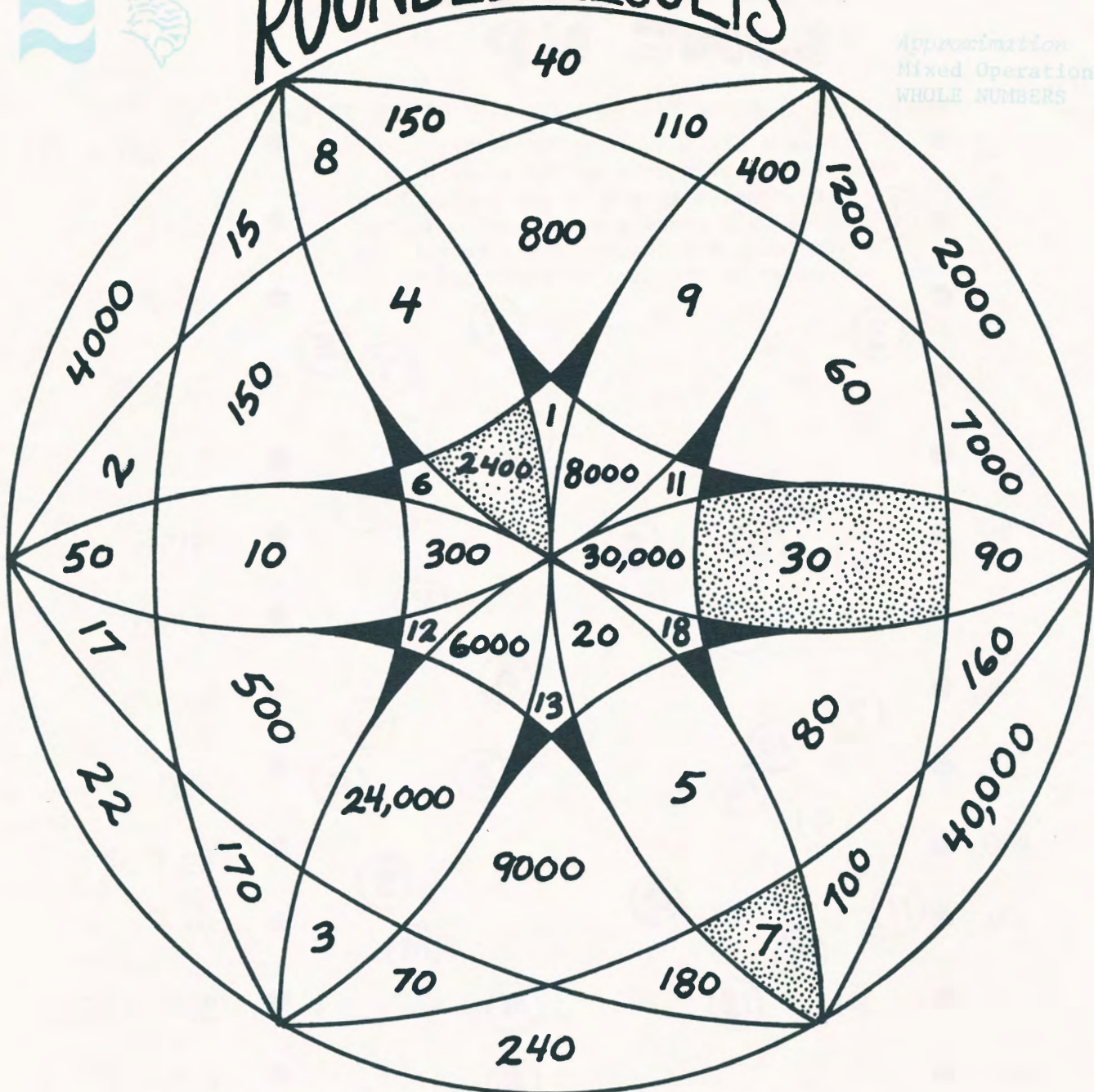
TO GO OUT: USE ALL CARDS AND DISCARD.

SCORING: TOTAL VALUE OF ALL CARDS PLAYED MINUS TOTAL VALUE IN HAND.

WINNER: PLAYER WITH GREATEST NUMBER OF POINTS AFTER FOUR HANDS.

TYPE: Game

IDEA FROM: *Mathex*, Junior-Operations and Problem Solving, Teacher's Resource Book No. 8



Find the **ROUNDED RESULT** (what the answer is close to) for each problem. Then shade in each area which contains one of your answers. The first three are done for you.

	THINK	ROUNDED RESULT
4×612	$\approx 4 \times 600 = 2400$	
$914 \div 31$	$\approx 900 \div 30 = 30$	
$\sqrt{50}$	$\approx 7 = 7$	
31×31	\approx _____	<u>900</u>
42×615	\approx _____	<u>25000</u>
$923 \div 3$	\approx _____	<u>300</u>
19×19	\approx _____	<u>400</u>
$158 \div 8$	\approx _____	<u>20</u>
$\sqrt{26}$	\approx _____	<u>5</u>

	THINK	ROUNDED RESULT
$999 \div 20 \approx$	<u> </u>	<u>50</u>
$305 \times 21 \approx$	<u> </u>	<u>6,000</u>
$149 \div 51 \approx$	<u> </u>	<u>3</u>
$\sqrt{65}$	<u> </u>	<u>8</u>
$192 \times 39 \approx$	<u> </u>	<u>8,000</u>
$296 \times 99 \approx$	<u> </u>	<u>30,000</u>
$53 \div 6 \approx$	<u> </u>	<u>9</u>
$245 \div 26 \approx$	<u> </u>	<u>10</u>
48×415	<u> </u>	<u>4</u>
5193	<u> </u>	<u> </u>



ROUNDED LINE UP

ROUNDED
RESULT

WHAT DOES A MATH TEACHER DO WHO LIKES TO EAT?

Draw a straight line connecting each problem or number on the right with its rounded result on the left. Each line will cross a number and a letter. The number tells you where to put the letter in the line of boxes below.

Approximation
Mixed Operations
WHOLE NUMBERS



10 ●		●	$37,621 \div 173$
20 ● (1)		●	2871
30 ●		●	23×32
40 ● (3)	(H)	● (T) (E)	$23 + 38$
50 ●	(A)	●	387
60 ●	(6) (8) (2)	● (I)	315×65
70 ● (5)	(4)	● (K)	37621
200 ●	(7)	● (A)	$40 - 9$
300 ●	(10) (12)	● (L) (G)	$4875 \div 9$
400 ●	(9)	●	39×54
500 ● (11)	(D)	● (S)	23
600 ●	(14) (13)	● (M)	$2852 + 1198$
2,000 ●		● (M) (I)	$495 - 215$
3,000 ●	(16) (15)	● (E)	$4340 \div 460$
4,000 ●		● (T)	39
20,000 ●	(17)	●	54
40,000 ●		●	2871×58
200,000 ●	(18) (E)	●	65

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
M	A	K	E	M	A	T	H	D	I	G	E	S	T	I	B	L	E

DON'T BE AFRAID TO GUESS

Approximation
Mixed Operations
WHOLE NUMBERS



- I. APPROXIMATION GAME** - Place problems on the board, one at a time. Encourage students to guess the answer without working the problem. Make a contest out of this. See whose guess comes closest to the correct answer. Use problems having numbers with two or more digits.

Examples: 217 4215
 391 3982
 682 1475 207 584
 395 8613 x 58 x 217
 702 9813

Estimate answer Estimate to
to nearest hundred . nearest thousand.

- II. THUMBS UP** - Write problems on the board one at a time. Have students put thumbs up if the answer is greater than N; thumbs down if the answer is less than N. All work should be done mentally.

Examples:

		Sums	Differences	Products	Quotients
Two Digit Numbers	Problems	24 + 17	75 - 37	37 x 72	98 ÷ 23
	N	100	50	2500	4
Three Digit Numbers	Problems	912 + 184	847 - 109	237 x 508	842 ÷ 196
	N	1000	500	250,000	5

Variations:

Write the problems on cards with the answers behind. Let a student hold the problem up.

Use larger numbers or mix 2, 3, and 4 digit numbers. Adjust N to a reasonable size.

Use 3 or more numbers in the problem.

Mix the operation: (37 + 82) ÷ 17.

- III. CLOSEST TO** - Write problems on the board one at a time. Use 2 digit numbers. Have the students indicate with a show of hands whether the answer is closer to zero, N, or M. All calculations should be done mentally.

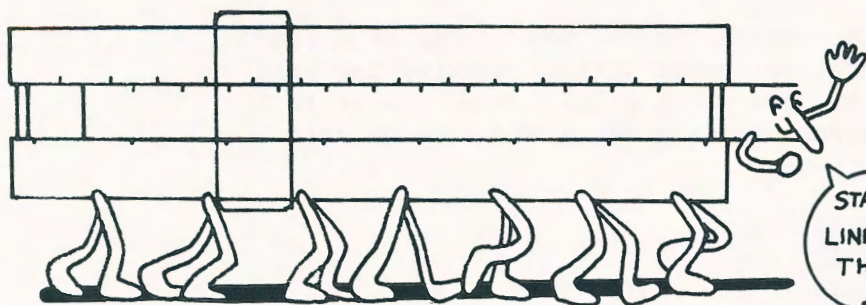
Examples:

	zero	N	M
Sums	zero	100	200
Differences	zero	50	100
Products	zero	2000	7000
Quotients	zero	5	10



A SLICK SLIP STICK

Approximation
Mixed Operations
WHOLE NUMBERS



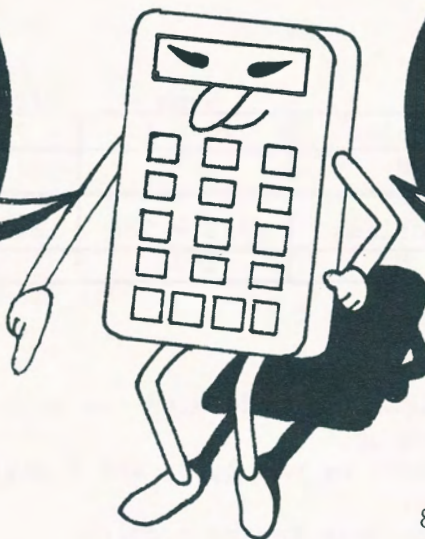
STAY IN
LINE BACK
THERE
!!

I am a slide rule.
I helped do these
calculations.
Approximate to
see if I was about
right.

1. $274 \times 452 = 124000$
2. $8742 \times 347 = 303000$
3. $643 \times 12 = 7700$
4. $257 \times 46 = 118000$
5. $371 \times 452 = 168000$

6. $437 \div 23 = 19$
7. $972 \div 36 = 27$
8. $97,444 \div 17 = 570$
9. $\frac{351 \times 161}{23 \times 13} = 190$

I am a hand calculator.
I was used to do these
problems. Approximate
to see if these answers are
reasonable.



If the answer
is not reasonable
what could have
happened?

1. $378 + 594 = 682$
2. $178 \times 342 = 60876$
3. $794 - 386 = 408$
4. $924 \times 12 = 11880$
5. $1468 \times 17 = 104228$
6. $5629 + 321 = 9000$
7. $21 \times 374 = 15414$

8. $9178 - 3904 = 5274$
9. $189 \div 20 = 9.45$
10. $4100 \div 50 = 820$
11. $6292 - 518 = 1074$
12. $38700 \div 300 = 12.9$
13. $2382 + 6894 = 12286$
14. $\frac{27 \times 43}{7 \times 8} = 1015.87$



ABOUT RIGHT

The people who wrote
this need to learn to
approximate better!



Approximation
Word Problems
Mixed Operations
WHOLE NUMBERS



Substitute sensible approximations for the ridiculous approximations
which are underlined.

- 1 The mad bicyclist can pedal at least 80 kilometres in one day. At this pace he should be able to travel 8000 kilometres in 10 days.
- 2 I make \$1.00 an hour babysitting. If I babysit 15 hours every week, in 12 weeks I will have earned about \$400.00.
- 3 hamburger monster ate the following number of hamburgers in 19 days. 13, 15, 15, 15, 13, 13, 15, 14, 13, 15, 14, 15, 15, 13, 13. He ate about 600 hamburgers.
- 4 The hamburger monster eats 13 to 15 hamburgers a day; at this rate it will take him about 500 days to eat 700 hamburgers.
- 5 If millionaire Randy Hart had \$37,452 in his pocket and gave \$26,987 away, he would have about \$1,000 left.
- 6 If a millipede with 2,474 legs was married to a millipede with 7,846 legs and their 3 offspring each had 1,986 legs, they would have to buy approximately 4000 pairs of boots to go hiking.
- 7 A supersonic jet flew across the United States (about 4200 kilometres) in 2 hours. This is about 8400 kilometres per hour.
- 8 I bought a motorcycle (\$698) with \$100 down and agreed to pay off the balance in two years with payments of \$75 a month.
- 9 I sleep for eight hours everyday, so in a year I sleep a total of about 50 days.
- 10 The basketball teams scored a total of 848 points in 21 games. That means they scored about 70 points a game.

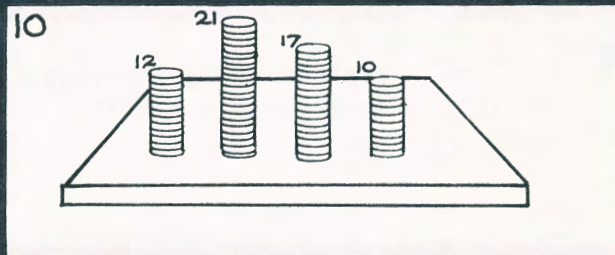
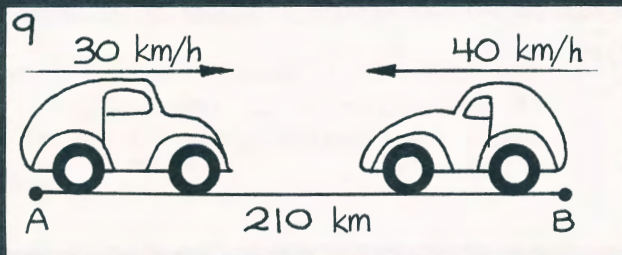
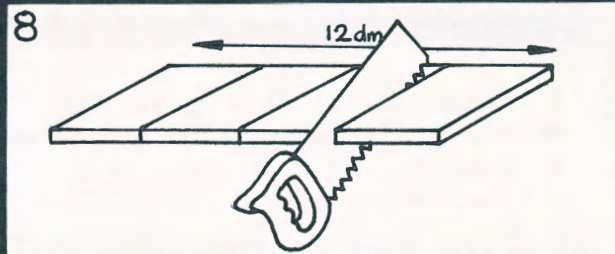
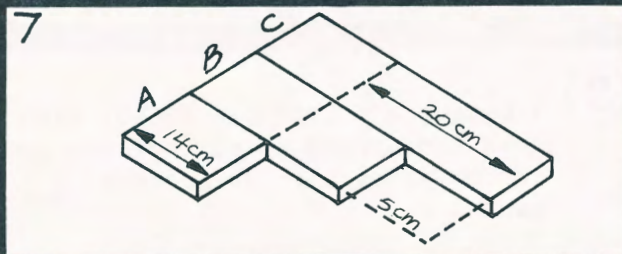
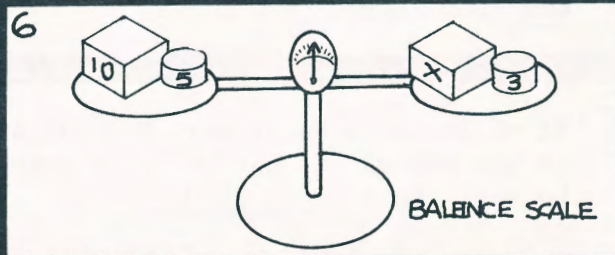
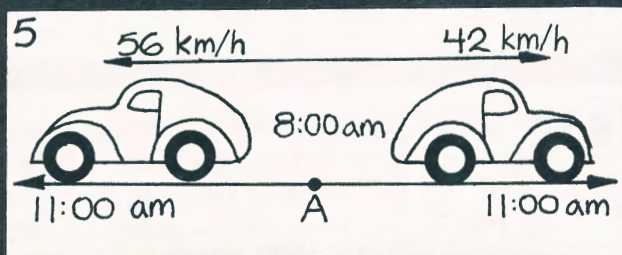
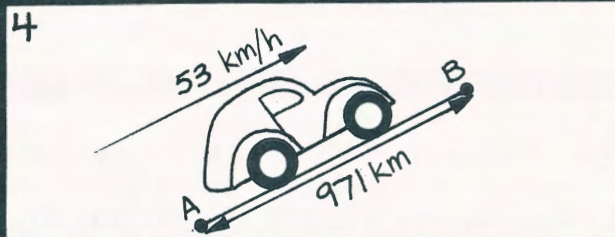
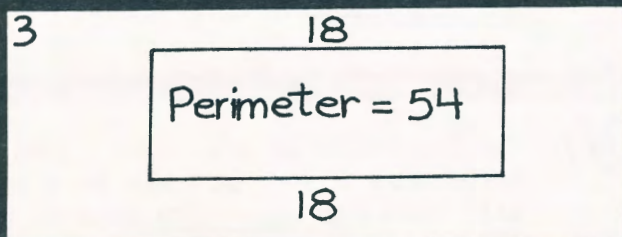
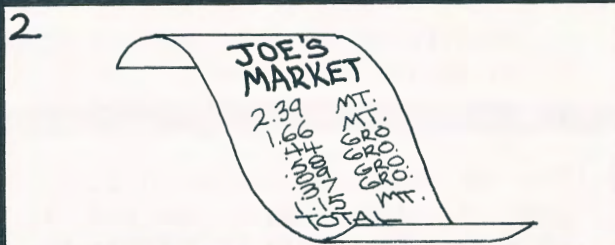
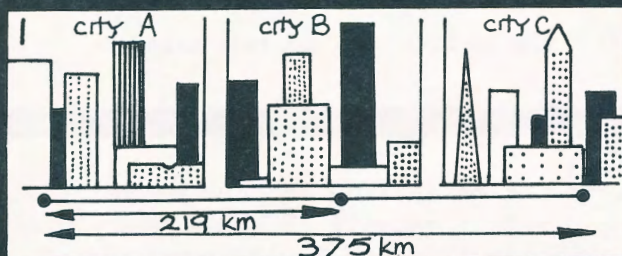


WORDLESS PROBLEMS

Word Problems
Mixed Operations
WHOLE NUMBERS



WRITE ONE OR MORE PROBLEMS
FOR EACH PICTURE BELOW



TYPE: Paper & Pencil

IDEA FROM: *School Mathematics I*, by Robert E. Eicholz, Phares, G. O'Daffer, Charles F. Brumfiel, Merrill E. Shanks, Charles R. Fleenor. Copyright (c) 1971 by Addison-Wesley Publishing Company, Inc. All rights reserved. Reprinted by permission.



ALTITUDES

Average
Mixed Operations
WHOLE NUMBERS

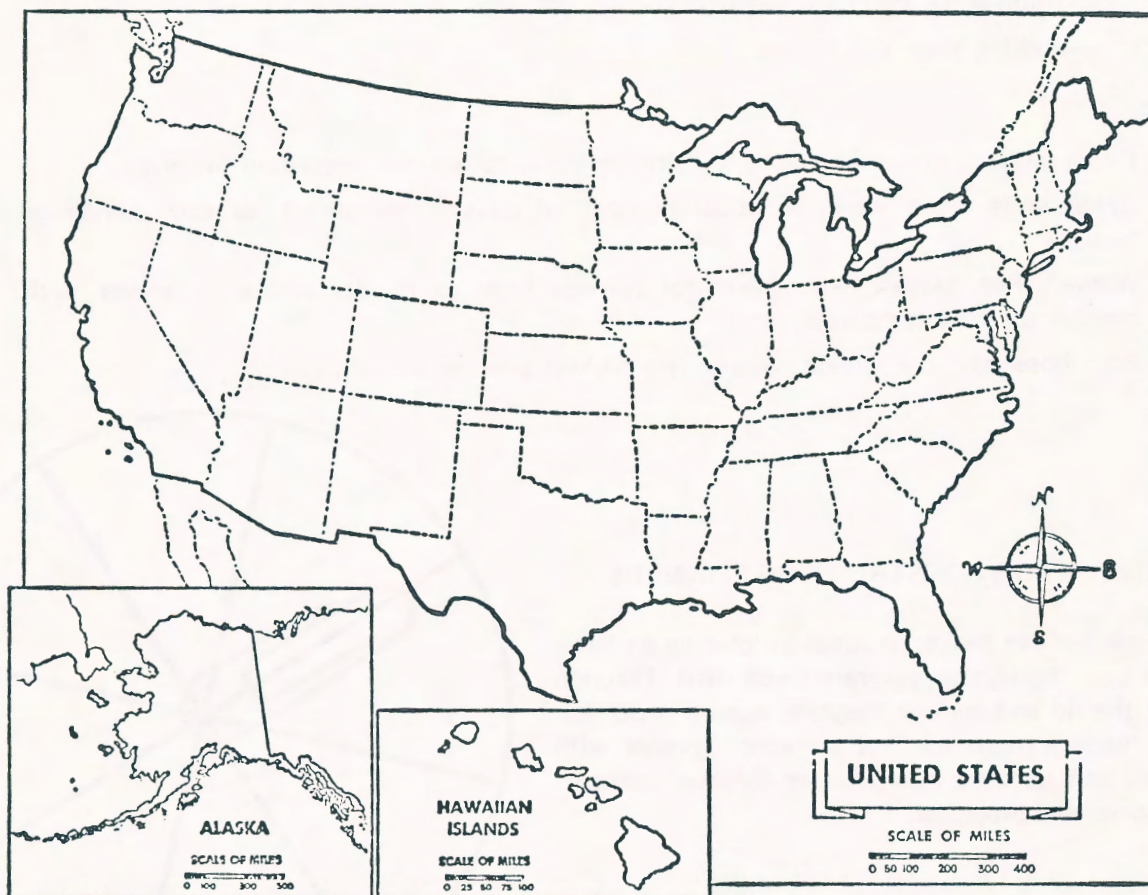


To do this project, you will need an INFORMATION PLEASE ALMANAC .

Use the index to locate the page for GEOGRAPHY (United States)
One of the charts lists Highest, Lowest, and Mean Altitudes in the U.S.
Your job is to study the chart for the mean altitudes of each state,
and then to color the map below according to this guide.

You may use
CRAYONS
or
PENCILS
or
FELT PENS

MEAN ALTITUDE	COLOR
0 —→ 1000	dark green
1001 —→ 2000	lt. green
2001 —→ 3000	blue
3001 —→ 4000	pink
4001 —→ 5000	orange
5001 —→ 6000	red
over 6000	brown



In a general way, the word MEAN is the same as AVERAGE. Using either the dictionary or your own memory, write a definition of AVERAGE :

WEATHER GRAPH GAME

Objectives: To give graphing practice.
To give opportunity for computing averages.
To give practice in column addition.

Construction:

1. Run off one graph-score sheet for each player.
2. Cover with acetate for permanent use and writing on.
3. Run off "degree" spinner on tag or poster board. (See "Suggestions for Constructing Spinners" in Introduction.) *See below.*

RULES FOR PLAYING WEATHER GRAPH GAME

Number of players: 2 — 4 or classroom teams.

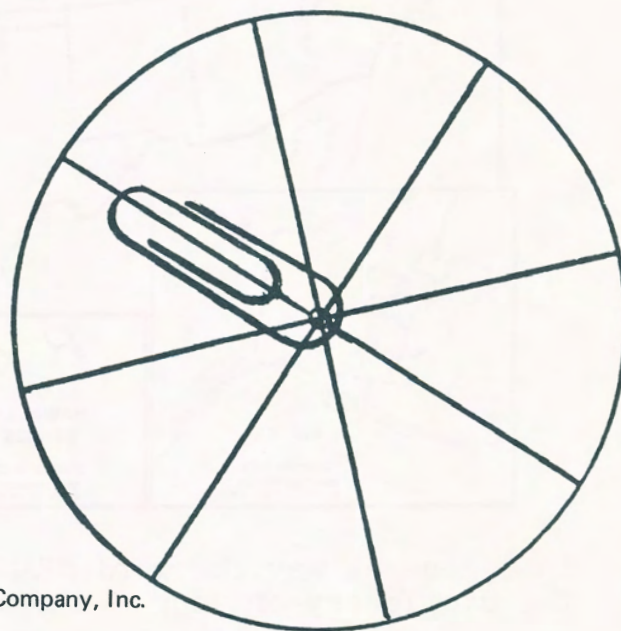
Equipment: Spinner
Graph-score sheet for each player
Marking pens or crayons

How to Play:

1. Either players or teams alternate spinning temperature and recording on graph.
2. Game ends when week or equal number of days is completed for each player or team.
3. Winner: For players who have not learned how to divide, winner is player with highest total of temperatures.
For those who can divide, winner has highest average temperature.

SUGGESTIONS FOR CONSTRUCTING SPINNERS

An excellent spinner can be constructed by placing a plastic coffee can lid over the spinner diagram to be used. Through the center of the lid and spinner diagram, push a paper fastener, which holds a paper clip for spinning. Spinner with plastic lid only can be used over spinner diagram transparencies on the overhead projector.

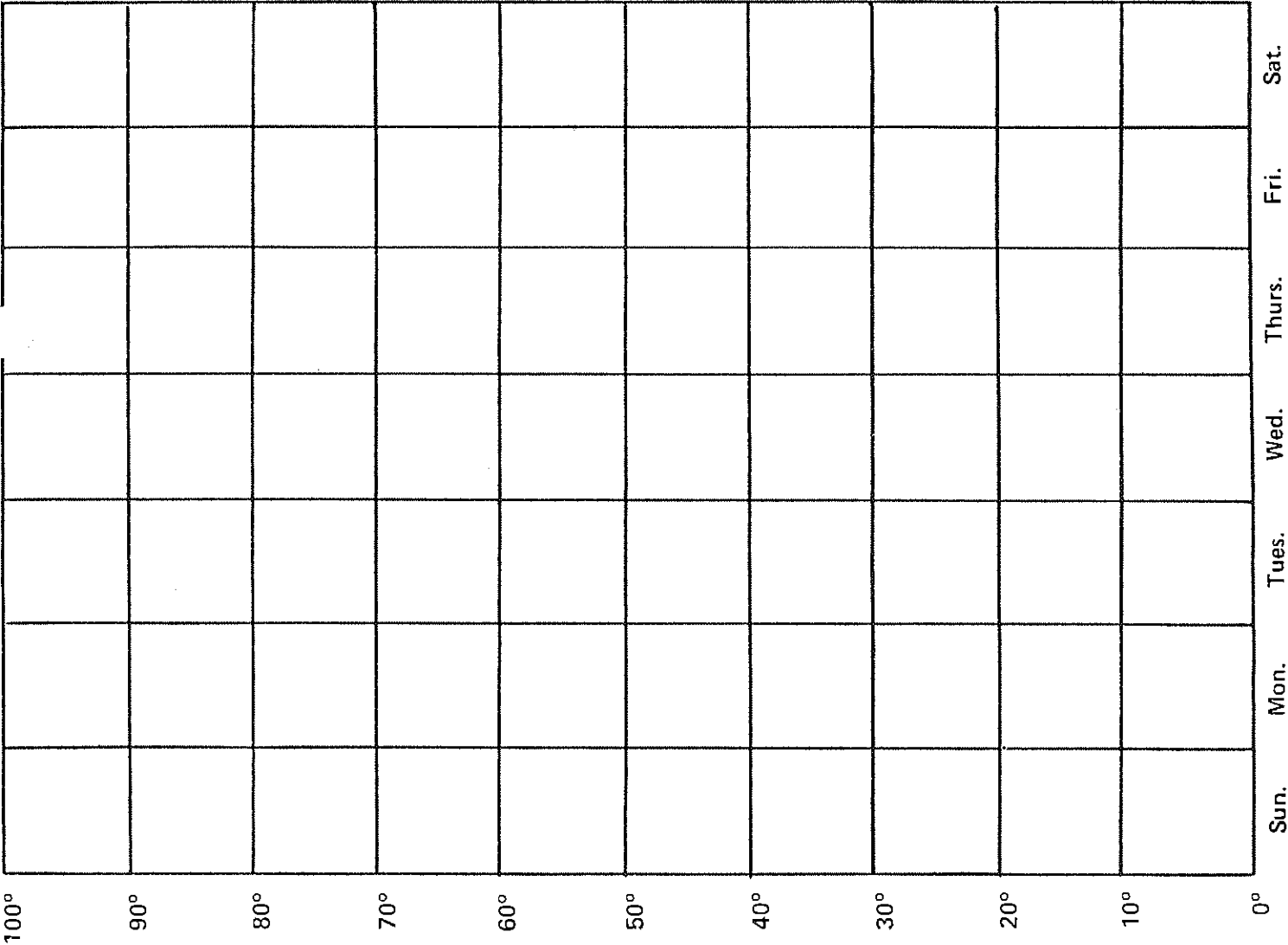


SOURCE: *Games and Aids for Teaching Math*

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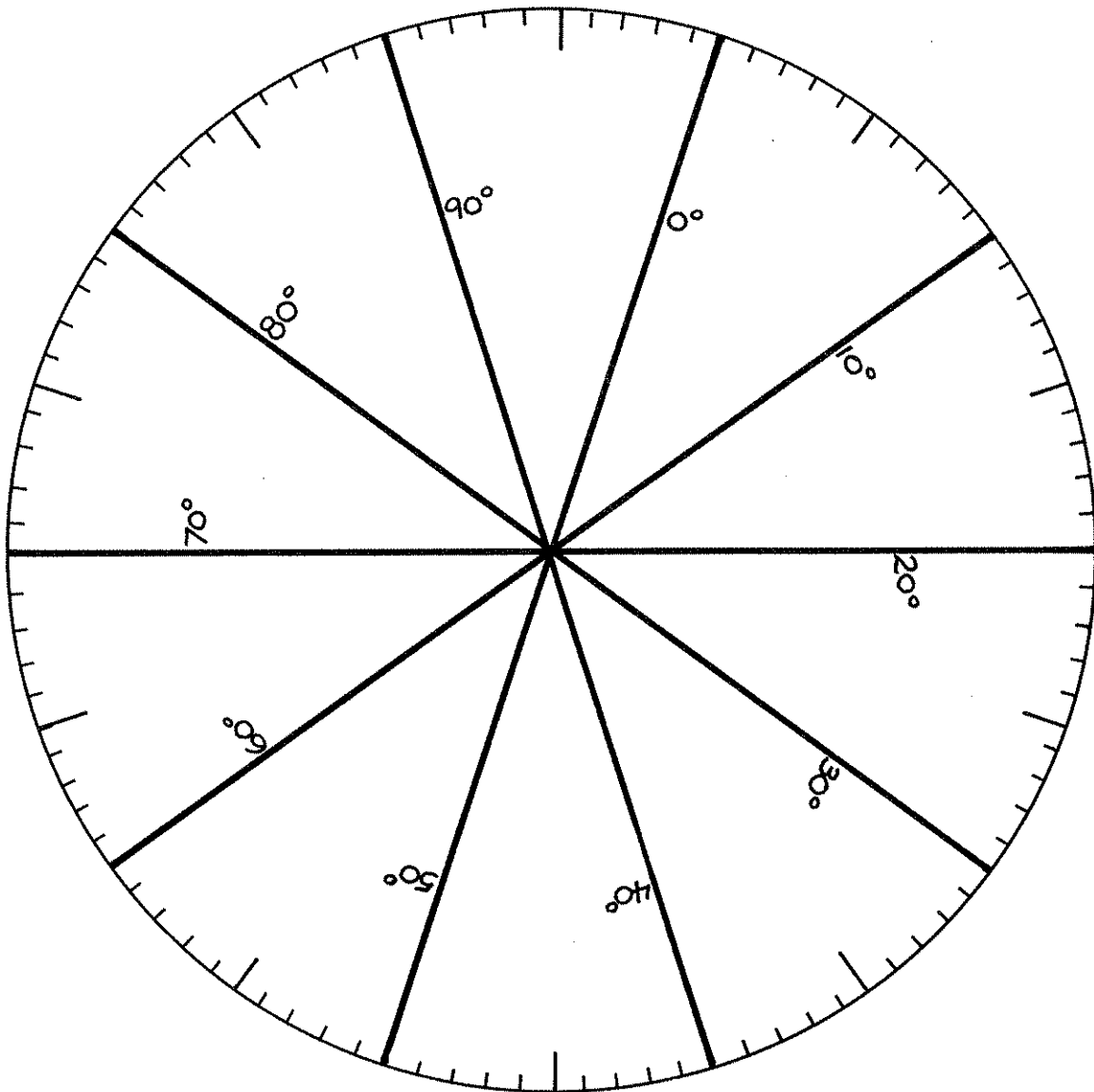
WEATHER GRAPH GAME (PAGE 2)

Record										
									Total	Average 7,



100° 90° 80° 70° 60° 50° 40° 30° 20° 10° 0°

WEATHER GRAPH GAME (PAGE 3)



SOURCE: *Games and Aids for Teaching Math*

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START

WIND UP

WRONG WAY

FINISH

1. $\begin{array}{r} 26 \\ + 17 \\ \hline \end{array}$

2. $\begin{array}{r} 86 \\ 75 \\ + 28 \\ \hline \end{array}$

3. $\begin{array}{r} 32 \\ \times 6 \\ \hline \end{array}$

4. $\begin{array}{r} 61 \\ \times 48 \\ \hline \end{array}$

5. $\begin{array}{r} 68 \\ - 39 \\ \hline \end{array}$

6. $\begin{array}{r} 171 \\ - 85 \\ \hline \end{array}$

7. $7 \overline{)161}$

8. $23 \overline{)943}$

9. $76 + \boxed{} = 38$

10. $(28 \times 25) + 336 =$

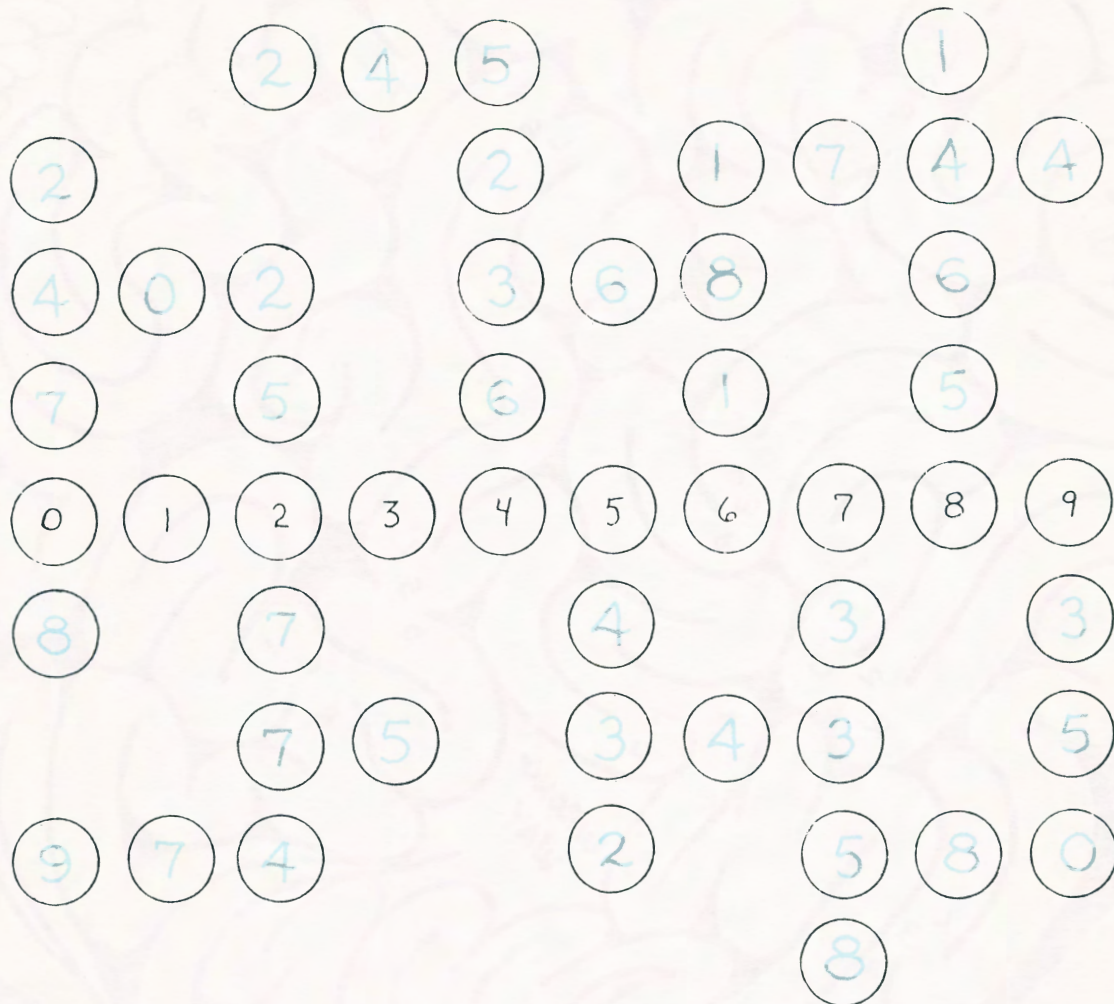
Left	Right
2	1
23	28
30	41
29	33
40	43
86	95
182	182
199	192
1036	2748
2928	17808

IDEA FROM: C.O.L.A.M.D.A.

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OPERATIONS WITH WHOLE NUMBERS

Computation +, -, x, ÷
Mixed Operations
WHOLE NUMBERS



Solve all the problems below. Then fit each answer into the circlegram.

Across

$$\begin{array}{r} 369 \\ + 605 \\ \hline \end{array}$$

$$\begin{array}{r} 892 \\ - 647 \\ \hline \end{array}$$

$$\begin{array}{r} 25 \\ \times 3 \\ \hline \end{array}$$

$$3 \overline{)1236}$$

$$6 \overline{)2208}$$

$$\begin{array}{r} 436 \\ \times 4 \\ \hline \end{array}$$

$$\begin{array}{r} 128 \\ + 235 \\ + 217 \\ \hline \end{array}$$

$$\begin{array}{r} 110 \\ + 106 \\ + 127 \\ \hline \end{array}$$

Down

$$\begin{array}{r} 265 \\ 384 \\ + 972 \\ + 195 \\ \hline \end{array}$$

$$\begin{array}{r} 55197 \\ - 40539 \\ \hline \end{array}$$

$$\begin{array}{r} 41147 \\ - 16439 \\ \hline \end{array}$$

$$\begin{array}{r} 8262 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 14043 \\ \times 18 \\ \hline \end{array}$$

$$\begin{array}{r} 13247 \\ - 3897 \\ \hline \end{array}$$

$$5 \overline{)27160}$$

$$7 \overline{)366548}$$

TYPE: Paper & Pencil/Puzzle



MAGIC TRICKS

Computation +, -, x, ÷
Mixed Operations
WHOLE NUMBERS



TEACHER DIRECTED ACTIVITY

	<u>EXAMPLE</u>	<u>GENERAL CASE</u>
Pick A Number	8	n
Add 7	15	$n + 7$
Multiply By 4	60	$4n + 28$
Subtract 12	48	$4n + 16$
Divide by 2	24	$2n + 8$
Add 4	28	$2n + 12$
Divide by 2	14	$n + 6$
Subtract 6	8 (Original Number)	n

Some students might be interested in seeing why these magic tricks work by seeing the general case. Later students could be encouraged to discover the general case on their own.

	<u>EXAMPLE</u>	<u>GENERAL CASE</u>
Pick A Number	11	n
Multiply By 3	33	$3n$
Add 9	42	$3n + 9$
Multiply By 2	84	$6n + 18$
Subtract 6	78	$6n + 12$
Divide By 3	26	$2n + 4$
Subtract 4	22	$2n$
Divide By 2	11 (Original Number)	n

	<u>Example</u>	<u>General Case</u>
Use Your House Number		H
Multiply by 2	684	$2H$
Add 5	1368	$2H + 5$
Multiply by 50	1373	$100H + 250$
Add your age (21)	68650	$100H + 250 + A$
Add 365	68671	$100H + 615 + A$
Subtract 615	69036	$100H + A$
	<u>684 21</u>	
	(House # Age)	

	<u>Example</u>	<u>General Case</u>
Pick A Number	10	n
Multiply by 2	20	$2n$
Add 5	25	$2n + 5$
Multiply by 50	1250	$100n + 250$
Add 1724	2974	$100n + 1974$
Subtract year of birth (1953)	<u>10 21</u>	$100n + 1974 - (1974 - \text{AGE})$
	(Original # Age)	$= 100n + \text{AGE}$

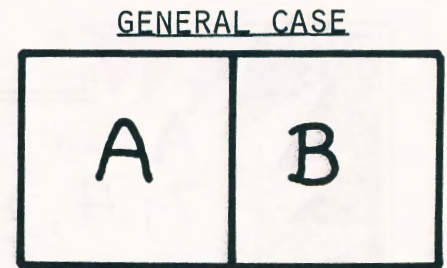
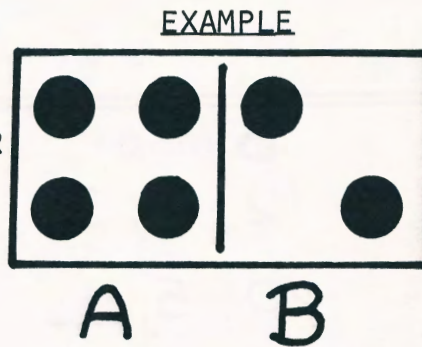
TYPE: Paper & Pencil/Activity

IDEA FROM: Yes, Math Can Be Fun and 4 the Math Wizard

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MAGIC TRICKS (CONTINUED)

MAKE A DOMINO ON YOUR PAPER
LABEL SIDES A & B



MULTIPLY SIDE A BY 5
ADD 8
MULTIPLY BY 2
ADD SIDE B
GIVE ME THE TOTAL
YOUR DOMINO IS:

$$\begin{array}{r} 4 \times 5 = 20 \\ 28 \\ 56 \\ 58 \\ 58 - 16 = 42 \end{array}$$

$$\begin{array}{r} A \times 5 = 5A \\ 5A + 8 \\ 10A + 16 \\ 10A + B + 16 \\ (\text{SUBTRACT } 16) \\ 10A + B \end{array}$$

AGE AND BIRTH DATE

ADD 5 TO YOUR AGE. DOUBLE IT. MULTIPLY BY 25. ADD YOUR BIRTHDAY OF THE MONTH. DOUBLE IT. SUBTRACT 500. THE FIRST TWO FIGURES ARE YOUR AGE; TAKE HALF OF THE LAST TWO FIGURES TO OBTAIN YOUR BIRTHDAY.

THREE GIVEN NUMBERS

PUT DOWN THREE NUMBERS, EACH LESS THAN TEN. MULTIPLY THE FIRST BY 2, ADD 5, AND MULTIPLY BY 5. ADD THE SECOND NUMBER AND MULTIPLY THE SUM BY TEN. ADD THE THIRD NUMBER. SUBTRACT 250. THE RESULT WILL BE THE THREE NUMBERS.

NUMBER MAGIC

HAVE A FRIEND CHOOSE A PRIME NUMBER GREATER THAN 3 (5, 7, 11, 13, ETC.) SQUARE IT. ADD 17. DIVIDE BY 12. THE REMAINDER WILL ALWAYS BE 6, REGARDLESS OF WHAT PRIME NUMBER IS CHOSEN.

IDEA FROM: *Yes, Math Can Be Fun* and *4 the Math Wizard*

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What's My Rule I

Patterns
Mixed Operations
WHOLE NUMBERS



TEACHER DIRECTED ACTIVITY

This page is intended to be used
as a transparency for class
discussion. A student page follows.

A Addition

7, 4, 11

5, 9, 14

3, 7, 10

4, 8, 12

B Division

12, 4, 3

20, 5, 4

14, 7, 2

21, 3, 7

C Addition

3c, 4c, 7c

5y, 3y, 8y

6d, 4d, 10d

5a, 3a, 8a

D Subtraction

9, 3, 6

8, 5, 3

13, 6, 7

15, 9, 6

E Multiplication

1, 0, 0

2, 1, 2

4, 8, 32

7, 6, 42

F Subtraction

8a, 4a, 4a

12y, 5y, 7a

17w, 3w, 14w

4c, 1c, 3c

G Multiplication

1, 1, 1

1, 2, 2

4, 0, 0

0, 9, 0

H Division

1, 1, 1

8, 4, 2

0, 7, 0

12, 6, 2

I Powers

3, 2, 9

2, 2, 4

5, 2, 25

1, 2, 1

What's My Rule I

CONTINUED

NAME: _____

1. 8, 2, 10

4, 14, _____

16, 7, _____

9, _____, 12

$\frac{2}{7}$, $\frac{3}{7}$, _____

3, 4, _____

2. 9, 4, 36

8, 8, _____

7, 9, _____

20, 3, _____

7, _____, 49

8, _____, 40

3. $\frac{2}{3}$, $\frac{4}{5}$, $\frac{8}{15}$

$\frac{1}{4}$, $\frac{2}{5}$, _____

$\frac{2}{9}$, $\frac{1}{3}$, _____

$\frac{1}{8}$, $\frac{3}{4}$, _____

$\frac{4}{7}$, $\frac{3}{4}$, _____

$\frac{4}{7}$, _____, $\frac{8}{35}$

$2a^3$, $4a^2$, $8a^5$

$8e^2$, $3e^5$, _____

$5d^3$, $6d^7$, _____

$10c^4$, $3c^8$, _____

$2w^3$, _____, $10w^7$

5. $4a^3$

$5a^4$

$7a^{11}$

$6a^8$

$5b^3$

$10c^9$

$22m^4$

$8w^1$

$12a^2$

$20a^3$

$77a^{10}$

6. Follow the examples

$$6a + 8b = (2 \cdot 3 \cdot a) + (2 \cdot 4 \cdot b) \\ = 2(3a + 4b)$$

$$12a + 16b = (4 \cdot 3 \cdot a) + (4 \cdot 4 \cdot b) \\ = 4(3a + 4b)$$

$10a + 25b =$ _____

$4a + 10b =$ _____

Make Your Own
What's My Rule and
Give To A Friend



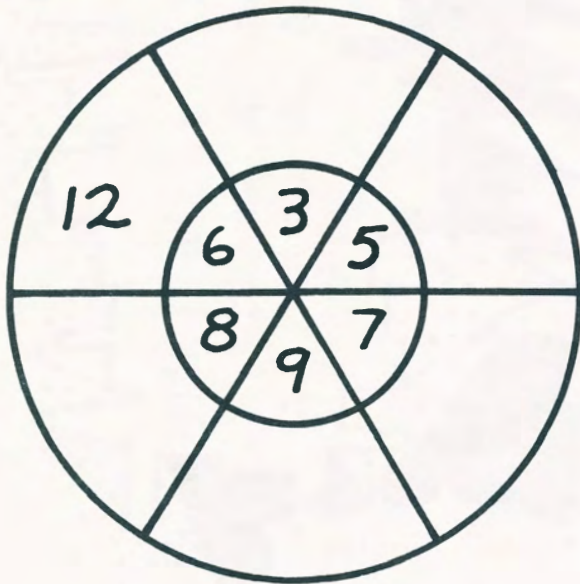
What's My Rule II

Patterns
Mixed Operations
Whole Numbers

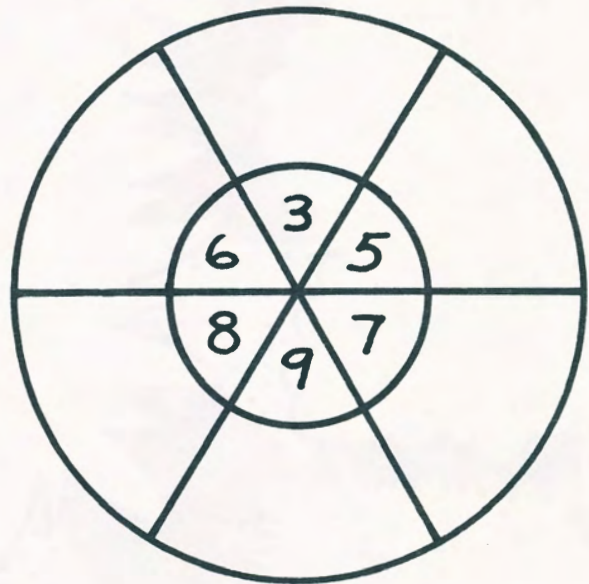


This page is intended to be used as a transparency for class discussion. A student page follows.

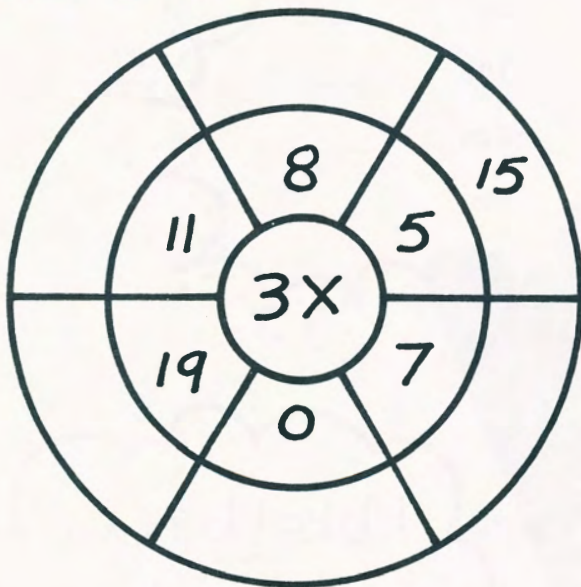
RULE: $N+6$ or $2 \times N$



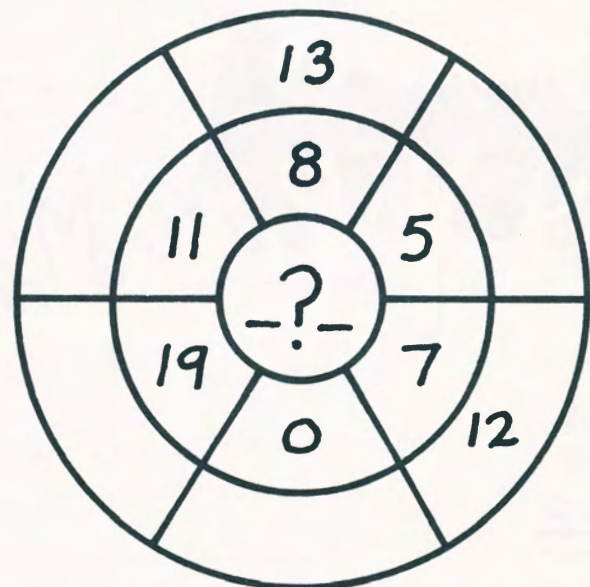
RULE: $(3 \times N) - 2$



RULE: $3 \times N$



RULE: $N+5$



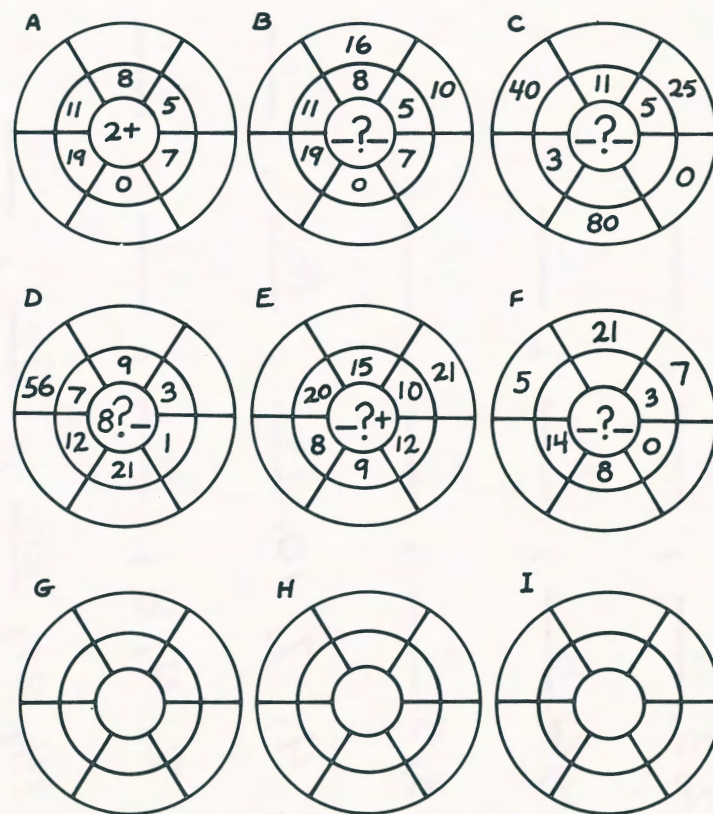
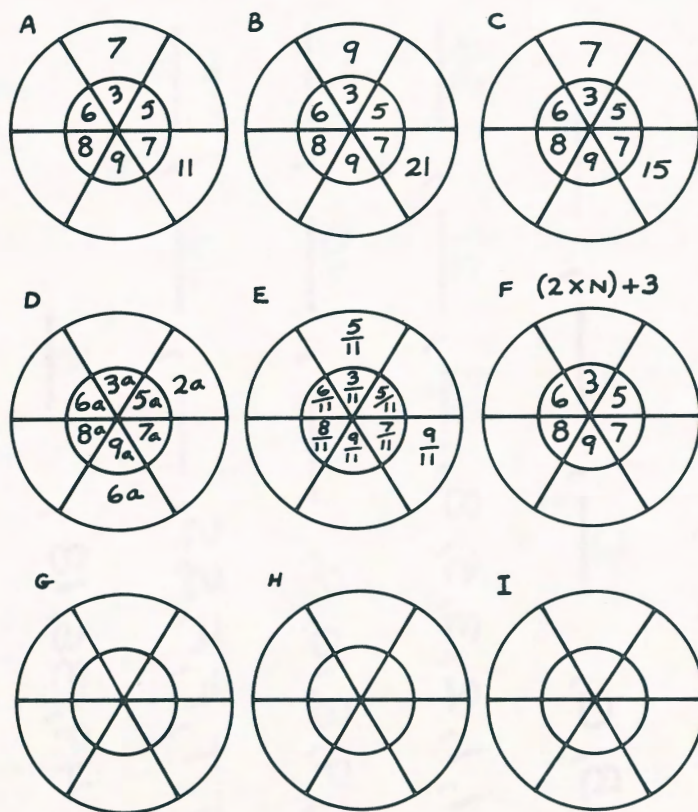
TYPE: Transparency/Paper & Pencil

IDEA FROM: *Daily Chores*

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What's My Rule II

(CONTINUED)



STUDENT PAGE

Use your own problems for G, H, I.



SEEING SEQUENCES

Patterns
Mixed Operations
WHOLE NUMBERS



This page is intended to be used as a transparency for class discussion. A student page follows.

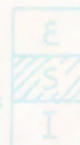
1. 1, 2, 3, 4 , 5 , 6
2. 2, 4, 6, 8 , 10 , 12
3. 1, 3, 5, 7 , 9 , 11
4. 1, 4, 7, 10, 13 , 16 , 19
5. 1, 2, 4, 8, 16 , 32 , 64
6. 7, 14, 21, 28 , 35 , 42
7. 1, 2, 4, 7, 11, 16, 22 , 29 , 37
8. A, B, C, D , E , F
9. 0, 1, 1, 2, 3, 5, 8, 13 , 21 , 34 , 55 (Fibonacci Sequence)
10. 8, 7, 9, 8, 10, 9, 11 , 10 , 12 , 11
(-1, +2, -1, +2, ...)
11. O, T, T, F, F, S, S, E , N , T , E
One, two, three
12. 77, 49, 36, 18 , 8
 7×7 4×9 3×6

SEEING SEQUENCES (CONTINUED)

STUDENT PAGE

Find the next terms in each of the following sequences. Find as many terms in each case as there are spaces.

1. 3, 6, 9, 12, __, __, __, __, __
2. 5, 10, 15, __, __, __, __, __
3. 1, 5, 9, 13, __, __, __
4. 9, 18, 27, 36, __, __, __
5. 27, 29, 31, 33, __, __, __
6. $\frac{3}{2}$, $\frac{4}{3}$, $\frac{5}{4}$, $\frac{6}{5}$, __, __, __
7. 1, 4, 8, 13, 19, __, __, __
8. a, c, e, g, __, __, __
9. 32, 16, 8, __, __, __, __
10. 72, 64, 56, 48, __, __, __, __, __
11. 3, 6, 12, 24, __, __, __, __
12. 1, 3, 2, 4, 3, __, __, __, __
13. 2, 9, 15, 20, __, __, __
14. $\frac{1}{3}$, $\frac{2}{3}$, 1, __, __, __, __
15. .3, .5, .7, .9, __, __, __
16. 1.6, .8, .4, .2, __, __, __
17. 1, 7, 14, 22, 31, 41, __, __, __
18. 4, 11, 9, 15, 12, 19, __, __, __
19. 3, 2, 6, 5, 15, __, __, __
20. IN THE SEQUENCE 2, 4, 6, 8, 10, . . ., what is the 10th term? the 100th term?
the 9,462,281st term? ____, ____, ____
21. Which of the following are in the sequence 3, 6, 9, 12, 15, . . .? Circle the
answers. 51, 65, 30, 83, 134, 276, 849
Why are they in the sequence? _____



This page is intended to be used as a transparency for class discussion. A student page follows.

...			...						
40	41	42	43	44	...				
30	31	32	33	34	35	36	37	38	39
20	21	22	23	24	25	26	27	28	29
10	11	12	13	14	15	16	17	18	19
	1	2	3	4	5	6	7	8	9

1. $23 \rightarrow = 24$
2. $12 \uparrow = 22$
3. $16 \nearrow = 27$
4. $44 \searrow \downarrow = 23$
5. $7 \rightarrow \uparrow \uparrow = 28$

6. $19 \rightarrow =$ _____
7. $43 \nearrow = 54$
8. $84 \leftarrow \uparrow \uparrow = 103$
9. $1972 \nearrow \nearrow = 1994$
10. $10 \searrow =$ _____

Both 20 and 10 are logical? Have students agree on "best" answer.

Have students agree on "best" answer.

ARROW MATH (CONTINUED)

STUDENT PAGE

USE THE FOLLOWING ARRAY TO ANSWER EACH OF THE PROBLEMS BELOW.



- | | |
|-------------------------|---------------|
| 1. 17 → = | 9. 24 ↗ ↑ = |
| 2. 19 ↘ = | 10. 35 → = |
| 3. 10 ↓ ↑ → = | 11. 54 → = |
| 4. 18 ↗ ↗ = | 12. 72 ← = |
| 5. 12 ← ← ← ← ↓ = | 13. 72 ↑ = |
| 6. 15 ← = | 14. 94 ↓ = |
| 7. 15 ↖ = | 15. 127 ↗ = |
| 8. 11 ↑ ↓ → ← ↑ ↑ ↓ ↓ = | 16. 700 → ↓ = |

IDEA FROM: *Aftermath*, Volumes 1-4

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CURIOSITIES

Patterns
Mixed Operations
WHOLE NUMBERS



COMPLETE EACH OF THE FOLLOWING "CURIOSITIES"

①

$$\begin{aligned} 1 \times 8 + 1 &= 9 \\ 12 \times 8 + 2 &= 98 \\ 123 \times 8 + 3 &= 987 \end{aligned}$$

=
=
=
=
=
=
=

$$123456789 \times 8 + 9 = 987654321$$

②

$$\begin{aligned} 1 \times 9 + 2 &= 11 \\ 12 \times 9 + 3 &= 111 \\ 123 \times 9 + 4 &= 1111 \end{aligned}$$

=
=
=
=
=
=
=

$$+10 = 1111111111$$

③

$$\begin{aligned} 9 \times 9 + 7 &= 88 \\ 98 \times 9 + 6 &= 888 \\ 987 \times 9 + 5 &= 8888 \end{aligned}$$

=
=
=
=
=
=
=

$$987654321 \times 9 - 1 = 8888888888$$

④

$$\begin{aligned} 2 \times 9 + 4 &= 22 \\ 23 \times 9 + 5 &= 212 \\ 234 \times 9 + 6 &= 2112 \end{aligned}$$

=
=
=
=
=
=

$$\begin{aligned} 23456789 \times 9 + 11 &= 211111112 \\ 234567900 \times 9 + 12 &= 2111111112 \\ &= \end{aligned}$$

⑤

$$\begin{aligned} 8 \times 9 + 5 &= 77 \\ 87 \times 9 + 4 &= 787 \\ 876 \times 9 + 3 &= 7887 \end{aligned}$$

=
=
=
=
=
=
=

$$- 3 = 788888887$$

Note what happens when you combine #2 and #3.

$$(1 \times 9) + 2 + (9 \times 9) + 7 =$$

$$(1 \times 9) + (9 \times 9) + 2 + 7 =$$

$$(1 \times 9) \times 9 + 9 = 90 + 9 = 99$$

(Using commutative, associative and distributive properties.)

ALSO

$$(12 \times 9) + 3 + (98 \times 9) + 6 =$$

$$(12 + 98) \times 9 + 9 = 990 + 9 = 999$$

Try the same idea with #4 and #5

Have students find more "curiosities."

Eck, Waiter!

Choosing Operation
Order of Operation
Mixed Operations
WHOLE NUMBERS



Apparatus: 68 (1 sq. in.) tiles,
felt pen, play board (1 sq. ft.)

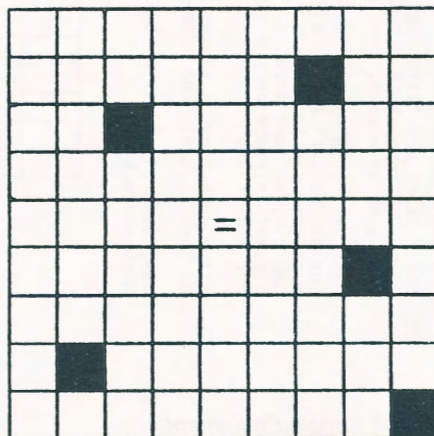
This is a Scrabble-type game involving equations. Although any number of players may play, this set handles up to four players effectively.

Preparation:

A. Mark tiles as follows:

Mark on tile:	Suggested number of copies:
2	5
3	5
4	5
5	3
6	4
7	1
8	5
9	2
10	4
11	1
12	5
+	4
-	4
X	4
\div	4
=	8
(blank)	4

B. Prepare the playing board as the diagram shows. Make the squares about 1 1/8-inch on an edge. Mark an '=' sign in the center square; black-in five squares as shown in the diagram:



EEK, WAITER! (CONTINUED)

The game:

- A. Place all tiles face down in a "draw pile."
- B. In Scrabble fashion, each player draws at random seven tiles. A player may look at his own tiles.
- C. First player plays a string of tiles in a (vertical or horizontal) line to make an equation, using the '=' sign printed on the board.
- D. Taking turns, each succeeding player may add tiles to make new equations, vertically or horizontally. At least one tile already on the board must be used in the new equation. No diagonal plays permitted.
- E. Blank tiles are "wild" and may be played as any number, 2 through 12, as an operation symbol (+, -, \times , \div), or an = sign, as specified by the player.
- F. Scoring: Each player keeps his cumulative score (or an "umpire" may keep all scores). In each turn, a player adds to his score the number of tiles he can play in that turn. If a player plays on a blacked-in space on the board, he doubles his score for that turn.
- G. After each play, a player replenishes his tile supply so that he again has seven unplayed tiles.
- H. Play proceeds until no one can make further plays. High score wins.

Example:

	3		6	+	5	=	11
	+		\div		-		+
	12	\div	6	=	2		2
	\div		+		=		-
	5		7		3		3
	=		=				=
	3	\square	8	-	1	=	10

Comments on Example:

1. Where grouping of symbols is ambiguous, player should specify the grouping he intends. In the Example, the equation

$$3 + 12 \div 5 = 3$$
 normally would require ()'s around the $3+12$. Since ()'s are not on the tiles, player verbally specifies the grouping when he plays his tiles.
2. The ' \square ' in the lower left corner of the Example represents a blank tile, used here as a '+',

Some modifications:

- Differentiate point values, so that a ':' tile played would get more points than a '+' tile, etc.
- Include new symbols, such as '>', '<'.
- Allow tiles to be used as multiple-digit numerals, as

2	1	-	1	7	=	4
---	---	---	---	---	---	---

- Make the playing board larger.



FRESNO

Choosing Operation
Order of Operation
Mixed Operations
WHOLE NUMBERS



MATERIALS: (1) A SPINNER WITH THE NUMBERS 0-9,
(2) A GAME SHEET FOR EACH PLAYER.

PLAYERS: ANY NUMBER MAY PLAY THIS GAME. IT MAY EVEN BE USED AS A SOLITAIRE.

OBJECT: TO FILL THE TARGET BOX WITH DIGITS TO FORM THE LARGEST NUMBER.
(SEE THE BOTTOM OF THE GAME SHEET ON THE NEXT PAGE.)

THE PLAY: THE SPINNER IS SPUN 4 TIMES. EACH PLAYER RECORDS THESE RESULTS IN THE 1ST GROUP OF 4 BOXES, MARKED "1." FOR EXAMPLE, IF THE NUMBERS SPUN WERE 1, 9, 4, 0, THEN THE FIRST BOX WOULD LOOK LIKE THIS:

1

1	9	4	0
---	---	---	---

IN THE SPACE AFTER YOUR NUMBERS, USE EACH NUMBER EXACTLY ONCE AND THE OPERATIONS YOUR TEACHER SELECTS, TO COMBINE THE NUMBERS TO GET 9 OR THE LARGEST POSSIBLE WHOLE NUMBER LESS THAN 9. (POSSIBLE OPERATIONS TO CHOOSE ARE ANY OF: ADDITION, SUBTRACTION, MULTIPLICATION, DIVISION, FACTORIALS, SQUARE ROOTS, OR POWERS.)

EXAMPLE: 1

1	9	4	0
---	---	---	---

 $(4+1) \times 0 + 9 = 9$

Have students discuss the different equations that they got.

TAKE THE DIGIT YOU HAVE MADE AND PLACE IT IN ONE OF THE 5 BOXES IN "A." HERE YOU ARE TRYING TO CREATE THE LARGEST 5 DIGIT NUMBER YOU CAN, SO IF YOU HAVE A LARGE DIGIT, PUT IT TO THE LEFT, SMALLER DIGITS SHOULD GO IN THE ONES COLUMN.

AFTER THIS PROCEDURE HAS BEEN REPEATED 5 TIMES, LINES 1-5 ARE COMPLETED, AND A IS FILLED. TAKE THE LARGEST 2 OR 3 DIGITS FROM A AND PLACE THEM IN THE TARGET BOX. IF THEY ARE LARGE DIGITS, PLACE THEM TOWARD THE LEFT. SMALLER DIGITS SHOULD GO TOWARD THE ONES COLUMN. THIS PROCESS IS REPEATED 5 MORE TIMES TO FILL 6-10 AND B, THEN 2 OR 3 MORE DIGITS ARE PLACED IN THE TARGET BOX. NOW YOU ARE READY TO COMPLETE 11-15 AND C. BE SURE YOU END UP WITH ALL 8 DIGITS OF THE TARGET BOX FILLED. THE PLAYER WITH THE LARGEST TARGET BOX NUMBER WINS.

VARIATIONS: (1) A shorter game would be to conclude after Box A is complete, (lines 1-5). The largest number in Box A wins.
(2) A second way to simplify would be to spin the spinner only three times for each number. The fourth box would remain empty, and the digit would be found by operating on only these three numbers.
(3) Numbers can be written in any order.

FRESNO

(CONTINUED)

1					
2					
3					
4					
5					

--	--	--	--	--

BOX A

6					
7					
8					
9					
10					

--	--	--	--	--

BOX B

11					
12					
13					
14					
15					

--	--	--	--	--

BOX C

--	--	--	--	--	--	--	--

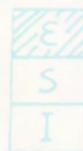
TARGET BOX



DIGIT IDEAS

TEACHER IDEAS

Choosing Operation
Order of Operation
Mixed Operations
WHOLE NUMBERS



NAME 4 BY USING

FOUR 1's... $1+1+1+1$

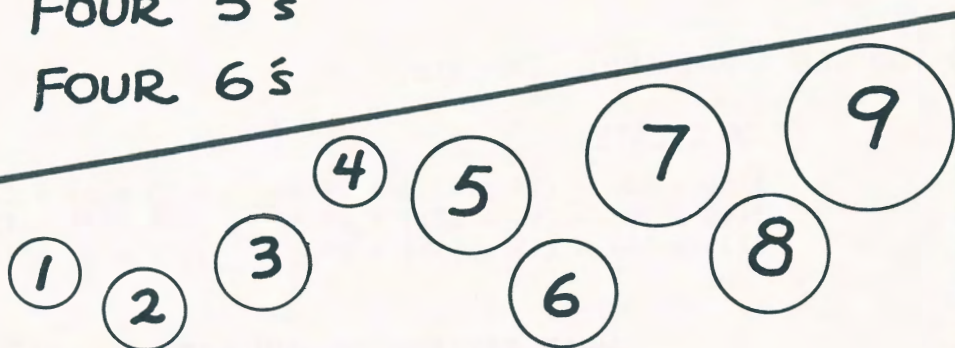
FOUR 2's... $2+2+2-2$

FOUR 3's

FOUR 4's

FOUR 5's

FOUR 6's



$$1+2+3+4+5+6+7+(8 \times 9) = 100$$

$$123-45-67+89 = 100$$

CAN YOU FIND OTHER WAYS
TO USE ALL NINE DIGITS
TO SHOW 100?

1975

75 MONTH 75									
		1	2	3	4	5			
6	7	8	9	10	11	12			
13	14	15	16	17	18	19			
20	21	22	23	24	25	26			
27	28								

$$1 = [(9+5) \div 7] - 1 \quad 6 =$$

$$2 = -1 - 9 + 7 + 5 \quad 7 =$$

$$3 = [(9+7) - 1] \div 5 \quad 8 =$$

$$4 = [9 - 1] \div (7 - 5) \quad 9 =$$

$$5 = ? \quad 10 =$$



THE REVERSE DOUBLE-DIGIT MAGIC TRICK - ACT 1

Mixed Operations
WHOLE NUMBERS



CALCULATE;

$$\begin{aligned} [(96 \times 23) - (69 \times 32)] \div 99 &= \underline{\quad}; (9 \times 2) - (6 \times 3) = \underline{\quad}. \\ [(84 \times 36) - (48 \times 63)] \div 99 &= \underline{\quad}; (8 \times 3) - (6 \times 4) = \underline{\quad}. \\ [(42 \times 12) - (24 \times 21)] \div 99 &= \underline{\quad}; (4 \times 1) - (2 \times 2) = \underline{\quad}. \end{aligned}$$

DO YOU SEE A PATTERN? EXPLAIN.

CALCULATE:

$$\begin{aligned} [(58 \times 53) - (85 \times 35)] \div 99 &= \underline{\quad}; (5 \times 5) - (8 \times 3) = \underline{\quad}. \\ [(65 \times 78) - (56 \times 87)] \div 99 &= \underline{\quad}; (6 \times 7) - (5 \times 8) = \underline{\quad}. \\ [(75 \times 56) - (57 \times 65)] \div 99 &= \underline{\quad}; (7 \times 5) - (5 \times 6) = \underline{\quad}. \end{aligned}$$

WHAT PATTERN DO YOU NOTICE HERE?

CAN YOU MAKE UP A SHORT-CUT BASED ON THESE PATTERNS SO THAT YOU CAN DO THE FOLLOWING CALCULATIONS MENTALLY? TRY IT! THEN CHECK YOUR ANSWERS.

$$\begin{aligned} (27 \times 31) - (72 \times 13) &= \underline{\quad} \\ (53 \times 57) - (35 \times 75) &= \underline{\quad} \end{aligned}$$

$$\begin{aligned} (47 \times 21) - (74 \times 12) &= \underline{\quad} \\ (97 \times 68) - (79 \times 86) &= \underline{\quad} \end{aligned}$$

$$\begin{aligned} (34 \times 95) - (43 \times 59) &= \underline{\quad} \\ (81 \times 17) - (18 \times 71) &= \underline{\quad} \end{aligned}$$

MAKE UP SOME MORE PROBLEMS LIKE THESE.

TYPE: Paper & Pencil

MOM & POP STEEL CO.

Mixed Operations
WHOLE NUMBERS



TEACHER DIRECTED ACTIVITY

MATERIALS: Page from phone book
 $\frac{1}{2}$ inch or cm. graph paper
Scissors (1 pair for every 2 students)
Ditto sheet for recording information (see next page)

PREPARATION: Form groups of 2 students each. Cut the graph paper into 12 x 1 strips. You will need 20 for every 2 students.

INSTRUCTIONS TO

- STUDENTS:**
- 1) Each group is a separate company. Each company sells steel rods, cut to specific lengths.
 - 2) As a business person you can purchase steel rods from a manufacturer in 12 foot lengths at a price of 50¢ a foot. Each company will begin by buying twenty 12' rods. What is the cost of these rods to your company?
 - 3) As a company, you will cut and fill orders for rods of lengths 1, 2, 3, 4, 5, 6, 7, 8, 9, or 10 feet. You sell these cut rods for \$1.00 a foot.
 - 4) Each cut you make costs \$1.00 because of labor and machine wear.
 - 5) If you have an order you can't fill, there is a \$10.00 cost for loss of future business.
 - 6) Each business will receive 10 orders of 4 rods per order.
 - 7) Any rods left over may be returned to the manufacturer according to the following:
 - 1' to 4' long - no money
 - 5' to 9' long - \$5.00 per rod
 - 10' to 12' long - \$10.00 per rod

- PROCEDURE:**
- 1) Supply each group with 20 strips of graph paper and a pair of scissors.
 - 2) Give the first order as follows: 3' rod, 5' rod, 7' rod and 9' rod. Check to see if students are able to follow the instructions.
 - 3) The rest of the orders are given out one at a time. Each group gets the same set of orders, but only after they finish an order do they receive the next order. Use the last 4 digits of phone numbers to generate random orders. (zero would be a 10' length.)
 - 4) Be sure each company records all the necessary information in all three tables as they do each order. (Order #1 and another sample are illustrated).
 - 5) Some students may not wish to cut strips of paper but prefer to keep track of inventories on paper only.

OBJECT: Make the most profit. As a company finishes order 10, have them calculate the total profit. Record it on the chalkboard. You will find that almost all the final answers will be different. This may lead into discussion.

CHALLENGE: What is the maximum amount of profit that could have been made from the 10 orders?

TYPE: Activity

MOM & POP STEEL CO. (CONTINUED)

SALES RECORDS

Order Number	Lengths Desired										
	1'	2'	3'	4'	5'	6'	7'	8'	9'	10'	
1			1		1		1		1		2 cuts needed
2		1			1			2			12 = 3 + 9
3											12 = 5 + 7
4											4 cuts needed
5											12 = 8 + 4
6											12 = 8 + 4
7											12 = 5 + 7
8											4 = 2 + 2
9											
10											

INVENTORY LIST

Order Number	After Lengths of Remaining Rods											
	12'	11'	10'	9'	8'	7'	6'	5'	4'	3'	2'	1'
0	20											
1	18											
2	15					1			1		1	
3												
4												
5												
6												
7												
8												
9												
10												

RUNNING EXPENSE AND INCOME

Order Number	\$ Lost on cuts	Money from Sale	Money Made
0	Spent \$120.00 to buy rods		
1	2	24	22
2	4	23	19
3			
4			
5			
6			
7			
8			
9			
10			

II. Change for a quarter:

Can you make change for a quarter:

1. using only 9 coins?
2. using only 17 coins?
3. using exactly 6 coins?

Is there more than one answer using 9 coins?

In order to answer the questions investigate all the ways of making change for a quarter. Put your results in the following table:

[illegible]

Can you answer the questions now?

CHANGE FOR A QUARTER (CONTINUED)

You may wish to put the completed chart on the overhead or board after sufficient time has passed. Here is the solution.

DIMES	NICKLES	PENNIES	TOTAL COINS
2	1	0	3
2	0	5	7
1	3	0	4
1	2	5	8
1	1	10	12
1	0	15	16
0	5	0	5
0	4	5	9
0	3	10	13
0	2	15	17
0	1	20	21
0	0	25	26

III. General Solution:

There is a general solution for finding the number of ways to make change for any amount greater than 10¢ using only dimes, nickels and pennies. The solution is valid for amounts less than or equal to 10¢ if a dime is included as a way of making change for 10¢, a nickel for 5¢ and a penny for 1¢.

1. If the amount ends in 0,1,2,3 or 4 the number of ways to make change is $(n + 1)^2$ ways where n is the tens' digit. For example: 70¢ (or 71¢, 72¢, 73¢, 74¢) has $(7 + 1)^2$ or 64 ways and \$1.22 has $(12 + 1)^2$ or 169 ways.
2. If the amount ends in 5,6,7,8 or 9 the number of ways to make change is $(n + 1)(n + 2)$ ways where n is the tens' digit. For example: 75¢ (or 76¢, 77¢, 78¢, 79¢) has $(7 + 1)(7 + 2)$ or 72 ways and \$1.48 has $(14 + 1)(14 + 2)$ or 240 ways.
3. Case 1 is the sum of the first $n + 1$ odd digits and case 2 is the sum of the first $n + 1$ even digits where n is the tens' digit.

NUMBER SENSE AND ARITHMETIC SKILLS

Placement Guide for Tabbed Dividers

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Fractions Commentary: Concepts (pages 452-460)

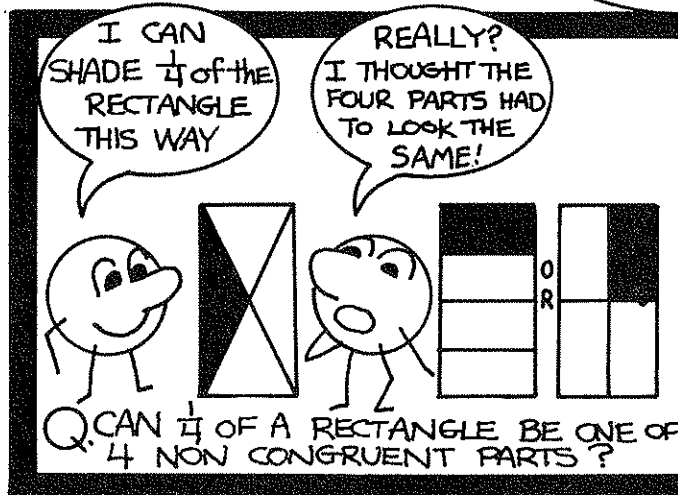
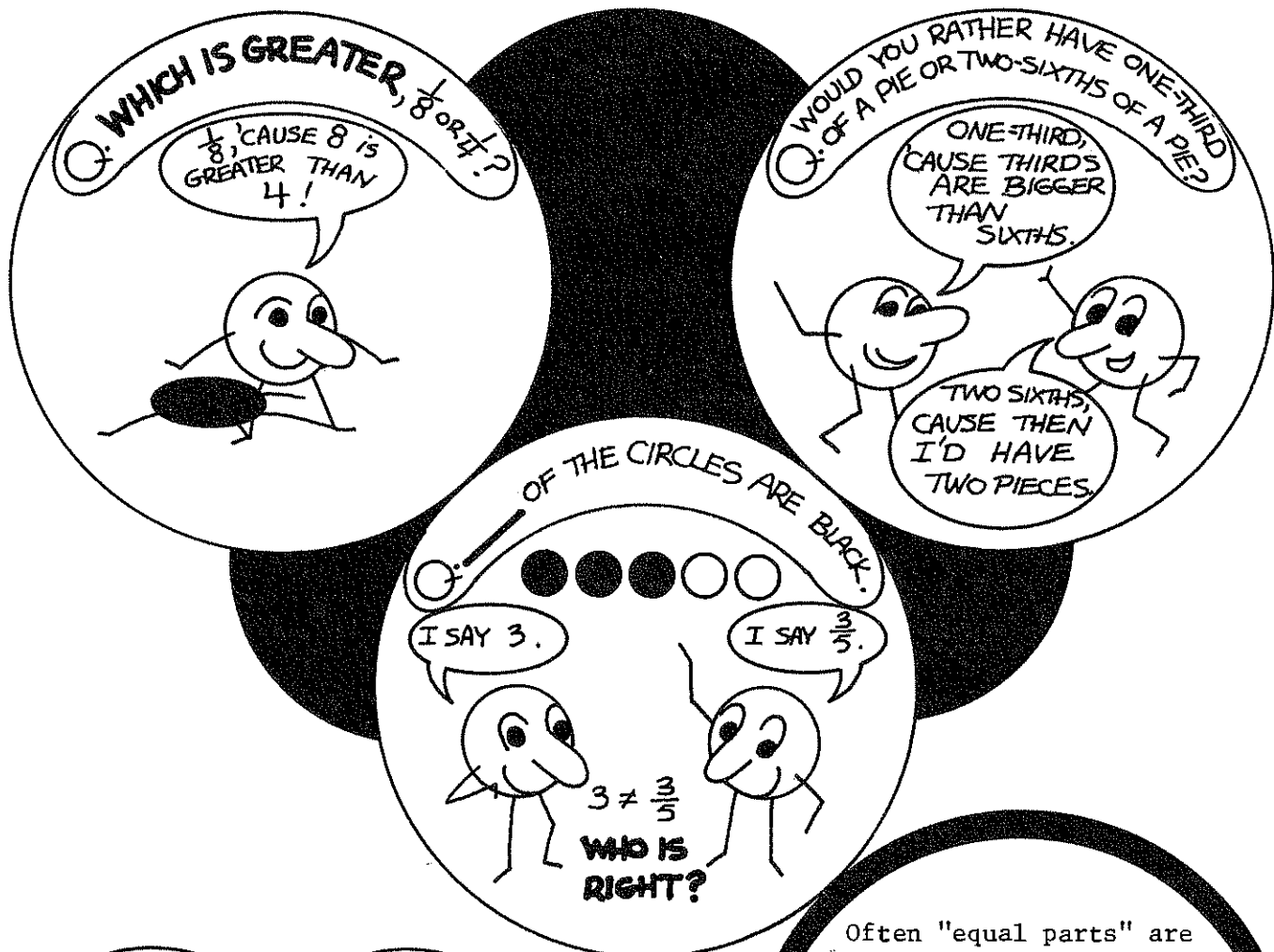
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Number Pairs/More Number Pairs	461	Definition	Paper and pencil Transparency
Activity Cards - Cuisenaire Rods - I	462	Whole model	Manipulative
Fraction Apple-cation	463	Whole model Equivalent	Activity Demonstration
Cooperation	464	Whole model	Game
Activity Cards - Circle Fractions - I	465	Whole model	Manipulative
Activity Cards - Geoboards - I	466	Whole model	Manipulative
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Color Clash	469	Whole model Estimation	Paper and pencil
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Fraction Dominoes	478	Fractional parts	Game
A Part of the Action	480	Fractional parts Measurement	Paper and pencil Transparency
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Activity Cards - Fraction Bars - I	482	Equivalent	Manipulative
Activity Cards - Circle Fractions - II	483	Equivalent Improper/mixed	Manipulative
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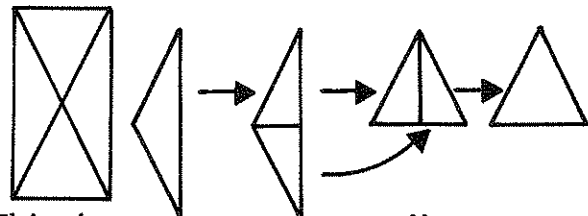
<u>TITLE</u>	<u>PAGE</u>	<u>TOPIC</u>	<u>TYPE</u>
Activity Cards - Clock Fractions - I	498	Improper/mixed	Manipulative
Activity Cards - Cuisenaire Rods - II	499	Improper/mixed	Manipulative
Name a Point at the Line Up/Find a Point!	500	Improper/mixed	Paper and pencil
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FRACTIONS: Concepts

So it's time to teach fractions again, is it? Have you ever encountered any of these situations?



Often "equal parts" are shown the same shape; however, it is the same measure (area, volume, length, etc.) that is necessary. This rectangle is divided into 4 parts. Do they have the same measure? Yes, since



This is not easy to see. Measurement and geometry ideas are needed.

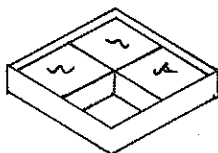
There are many conceptual problems that arise in learning fractions. In the following pages we will look at different approaches to fractions and the transition from concrete meanings to abstract symbolization.

MODELS FOR FRACTIONS

Students usually encounter fractions in everyday language before they study them more formally in school. "Half of an apple," "a quarter after six," and "one-third of a cup" are examples of common phrases. In elementary schools children are given a variety of experiences in which words and symbols for fractions are used to describe physical situations.



$\frac{1}{2}$ of an apple



$\frac{3}{4}$ of a cake

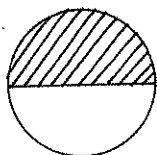


$\frac{1}{3}$ of the candy

These experiences with concrete objects become the basis for understanding and using fractions as numbers. Since fractions arise in different kinds of situations, there are several different approaches or views of fractions.

A Typical View of Fractions: The Whole Model

A typical treatment of fractions might begin with objects or shapes (usually called wholes). Fractions are then used to describe parts of these objects or shapes.

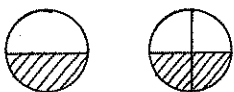


$\frac{1}{2}$ of the circle
is shaded



$\frac{2}{3}$ of the square is
not shaded

This basic 'picture' is referred to for equivalence, ordering and operations of fractions.



$$\frac{1}{2} = \frac{2}{4}$$



$$\frac{1}{2} < \frac{3}{4}$$



$$\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$$

When a 'picture' or manipulative is used as a basis for a number concept, we call the picture or manipulative a model. The view discussed above is called the "whole" model.

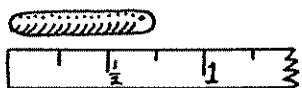
Other Views of Fractions

The whole model is useful for explaining fractions which describe "parts of wholes," but what about these fraction questions?



Q. What fraction of the boys are wearing caps? $\frac{2}{3}$

The fraction $\frac{2}{3}$ describes part of a set. Questions like this support the need for a SET model.



Q. What is the worm's length? $\frac{3}{4}$ unit

When we read a ruler, we use number line ideas. The NUMBER LINE can also become a model for fractions.



Q. If 3 steaks weigh 2 kilograms, how much does 1 steak weigh?

This question can be answered by dividing 2 by 3, and we would probably write $\frac{2}{3}$ kg ($\frac{2}{3} = 2 \div 3$). A fraction can be viewed as the QUOTIENT of two whole numbers.

Does $\frac{1}{2} = \frac{1}{3}$?

When fractions are used as numbers without a physical model in mind, we have moved from the concrete to a FORMAL stage. Since it would be inconvenient to always cut up apples or count out sets, it is eventually necessary to be able to work with fractions from a more abstract or FORMAL view.

There are, then, at least four interpretations from which we can abstract the concept of a fraction.*

- Fractions describe parts of wholes.
- Fractions describe parts of sets.
- Fractions name points on a number line.
- Fractions are quotients of whole numbers.



$\frac{1}{4}$

The chart on the following two pages shows how these interpretations explain the meaning, equivalence and ordering of fractions.

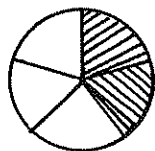
*Fractions can also be viewed as operators (see Stretchers and Shrinkers, Book 3, University of Illinois Committee on School Mathematics) or as ratios (see Ratio, Proportion and Scaling, Mathematics Resource Project).

FRACTION MODELS**WHOLE MODEL**

Fractions are used to describe parts of a set.

SET MODEL

Fractions are used to describe parts of a whole.

DEFINITION (EXAMPLE)

$\frac{2}{5}$ of the circle is shaded.

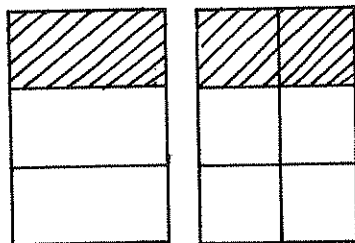


$\frac{2}{5}$ of the circles are black.

DEFINITION (GENERAL)

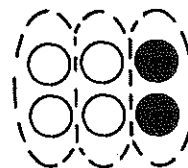
The whole or unit is divided into m equal parts and n of these parts is $\frac{n}{m}$ of the whole.

If a set is separated into m subsets of equal number, then n of the subsets is $\frac{n}{m}$ of the set.

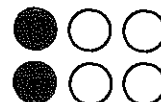
EQUIVALENCE**(EXAMPLE)**

$\frac{1}{3}$ and $\frac{2}{6}$ of the same sized rectangles are shaded. The areas of the shaded portions are equal.

This forms a basis for writing $\frac{1}{3} = \frac{2}{6}$, a number relationship.



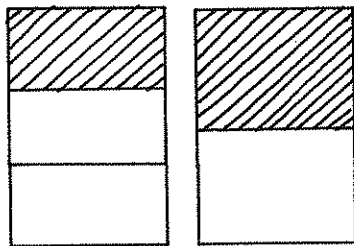
$\frac{1}{3}$ of the circles are black



$\frac{2}{6}$ of the circles are black

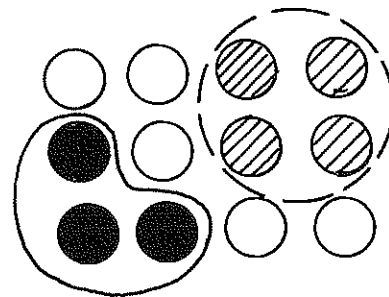
$\frac{1}{3}$ and $\frac{2}{6}$ of the same set are black. The number of black circles is 2 in both cases.

It is then logical to write $\frac{1}{3} = \frac{2}{6}$.

ORDERING**(EXAMPLE)**

Here $\frac{1}{3}$ and $\frac{1}{2}$ of the same sized rectangles are shaded, but the areas are not equal. A corresponding number relation is

$$\frac{1}{3} < \frac{1}{2}$$

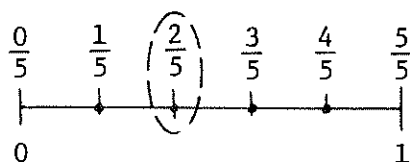


$\frac{4}{12}$ of the set has more circles than $\frac{3}{12}$ of the set. This is used to justify writing

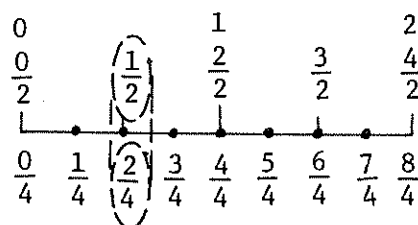
$$\frac{4}{12} > \frac{3}{12}$$

NUMBER LINE MODEL

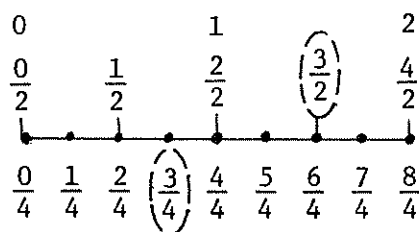
Fractions name points on a number line.



Divide the segment between 0 and 1 into m segments equal in length. The right endpoint of the n th segment has the name $\frac{n}{m}$.



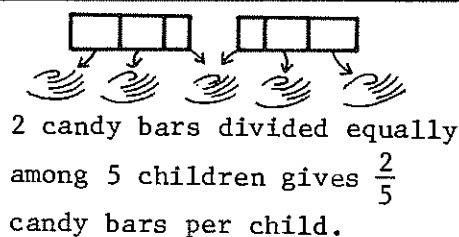
Two fractions are equivalent if they are names for the same point on the number line. $\frac{1}{2}$ and $\frac{2}{4}$ name the same point. We write $\frac{1}{2} = \frac{2}{4}$.



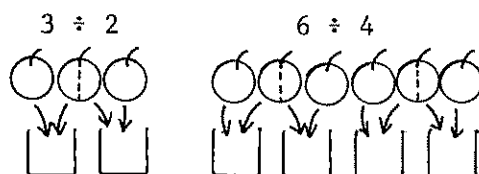
On the number line $\frac{3}{4}$ names a point to the left of the point named by $\frac{3}{2}$. We write $\frac{3}{4} < \frac{3}{2}$.

QUOTIENT MODEL

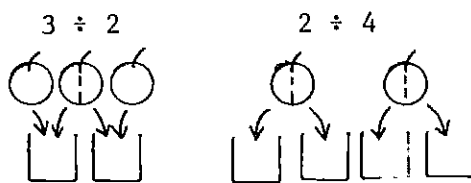
Fractions are the quotients of whole numbers.



The number $n \div m$ is the fraction $\frac{n}{m}$.



3 apples divided equally between 2 baskets gives the same amount of apples per basket as 6 apples divided equally among 4 baskets, so $\frac{3}{2} = \frac{6}{4}$.



3 apples divided equally between 2 baskets gives more apples per basket than 2 apples divided equally among 4 baskets. $\frac{3}{2} > \frac{2}{4}$

FORMAL MODEL

Fractions are number names.

$$\frac{2}{5}$$

A fraction is a pair of whole numbers $\frac{n}{m}$ with $m \neq 0$.

A fraction $\frac{n}{m}$ is equivalent to the fractions $\frac{2n}{2m}$, $\frac{3n}{3m}$, $\frac{4n}{4m}$, ...

Two fractions are equivalent if they are equivalent to the same fraction.

$$\frac{6}{8} = \frac{9}{12}, \text{ since } \frac{6}{8} = \frac{3 \times 6}{3 \times 8} = \frac{18}{24}, \text{ and } \frac{9}{12} = \frac{2 \times 9}{2 \times 12} = \frac{18}{24}.$$

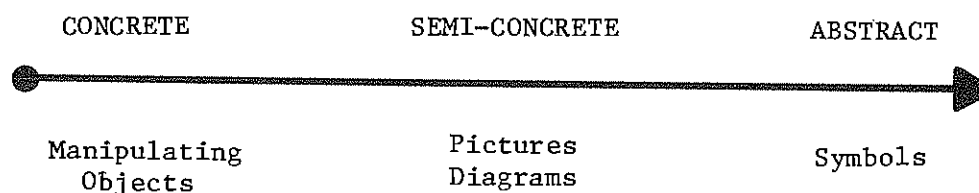
The greater of two fractions is determined by finding equivalent fractions with a common denominator and then comparing the numerators.

$$\frac{3}{4} < \frac{5}{6}, \text{ since } \frac{3}{4} = \frac{9}{12}, \frac{5}{6} = \frac{10}{12}, \text{ and } 9 < 10.$$

Which Model Should Be Used?

Which concrete setting should be used to give meaning to fractions? Should one model be chosen and maintained until equivalence, comparison and operations of fractions are covered? Would it be better to familiarize students with several models, so that they can select the model that seems most applicable in a given situation? We, as teachers, will have to answer these questions for ourselves and our students.

We can visualize the learning of fraction concepts as a continuum ranging from concrete introductory experiences to the formal symbolic model.



Ideally, students begin with concrete experiences and move toward abstract notation according to their individual development. Unfortunately, by the middle grades students are operating at all levels on this continuum. The burden is now on the teacher to discover each student's level and to provide the appropriate new experiences. Some students still need concrete experiences; a few will be operating abstractly; most will probably be in between. The use of different models may depend upon the individual student. Some students might become confused when confronted with several models. On the other hand, a different approach may be refreshing and provide new insights into fraction applications.

Regardless of how models are used in the understanding of fractions, it is helpful if we, as teachers, are aware of physical meanings underlying fraction questions. Being aware of the uses, limitations and problems in each model is also helpful. With this background a teacher is better able to diagnose, evaluate and remediate.

Using an Appropriate Model

Sometimes it is confusing to stay with one basic fraction model and attempt to extend it to meet all situations. When this happens, it is time to use another interpretation of fractions to solve the problems. If students say they can't see the "whole" in the question, "What fraction of the pencils are yellow?" then some work with sets of objects is in order. When students have trouble reading a ruler to the nearest quarter-inch or estimating fractional numbers on graphs, number line concepts can be emphasized.

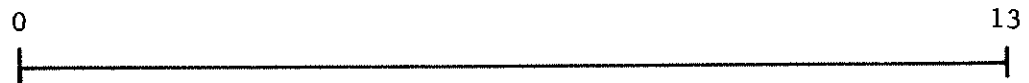
Treating fractions as quotients of whole numbers is expected in many situations.

Examples are:

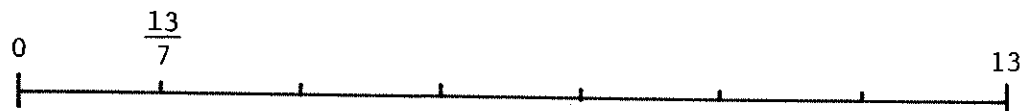
- a) Change $\frac{3}{4}$ to a decimal by dividing 3 by 4.
- b) Divide 12 by 5 and write the remainder in fraction form.
- c) To solve $3X = 5$ divide both sides of the equation by 3 to find $X = \frac{5}{3}$.

To make this quotient use of fractions more meaningful we could help students visualize the process. A class was given this problem:

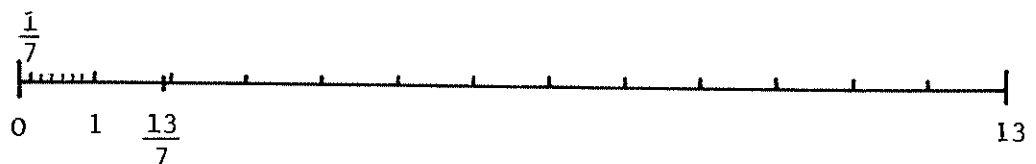
Find $\frac{13}{7}$ on this number line:



Students thinking of fractions as quotients divided the line segment in 7 equal parts and wrote $\frac{13}{7}$ as shown.



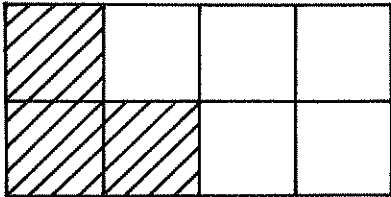
Students thinking of the whole model divided the segment into 13 equal parts to find 1, then they divided the unit between 0 and 1 into 7 equal parts and took 13 of these lengths to find $\frac{13}{7}$.



The quotient model gave the desired result much more directly, but most students and teachers would use the whole model attack.

Bringing It All Together

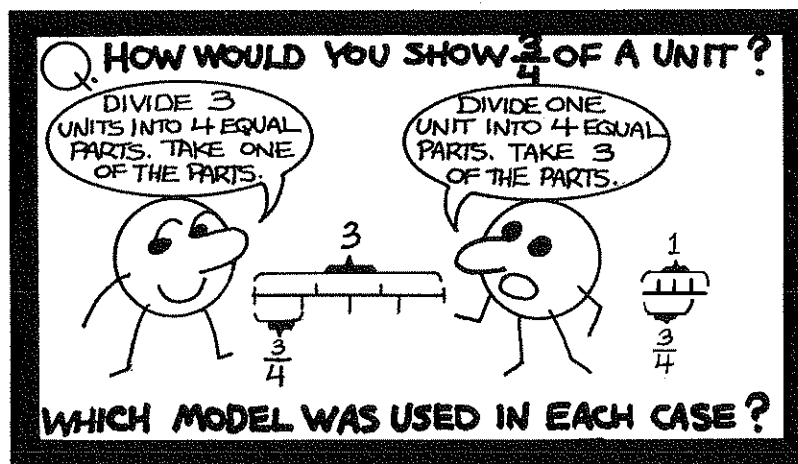
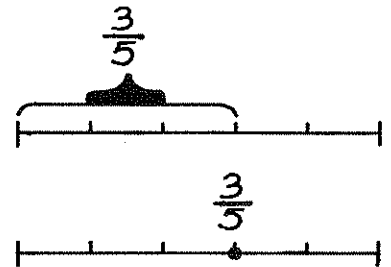
The various models give the same result in area, length, volume, etc., even though the steps involved are quite different.



This rectangle can be viewed as a whole or as a set of eight squares. The same picture suits both models. The picture could be described as $\frac{5}{8}$ of the rectangle is not shaded or $\frac{5}{8}$ of the squares are not shaded.

With the "whole" model $\frac{3}{5}$ of a unit might be shown like this:

In the number line model the symbol $\frac{3}{5}$ names the right endpoint of the $\frac{3}{5}$ unit.

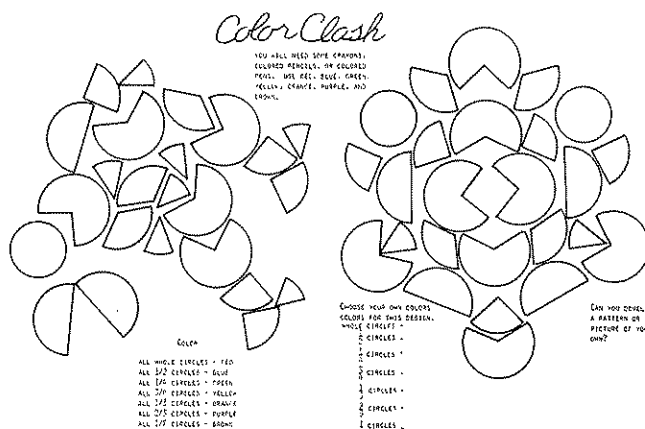


As we use concrete and semi-concrete experiences to form fraction concepts, there is a natural tendency to rush toward symbolism and the formal model. There are, however, many skill building activities and problem solving situations that can be posed before turning to the formal model. If the students can't understand these activities with a concrete model, then the formal model will probably not enhance their understanding of fractions. Some examples from the classroom materials of this resource are given on the next page.

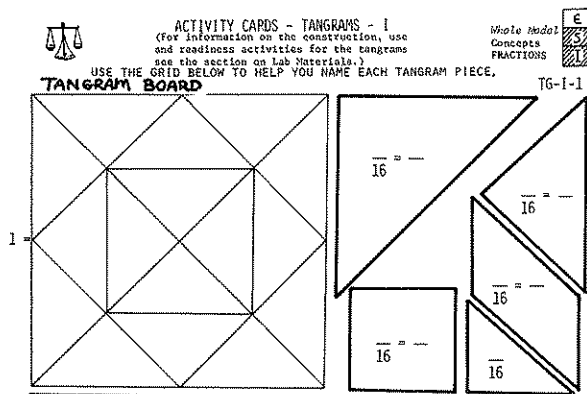
This skill building activity *Color Clash* shown at the right requires students to identify fractions of circles. Some students have difficulty assigning fractional numbers when divisional lines are not shown.

TYPE: Paper & Pencil

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Mathematics Resource Project



Whole Model
Concepts
Fractions



Tangrams are concrete manipulatives which can be used to reinforce understanding of fractions. This can lead to discovery and problem-solving activities. The first activity card shown at the left gives practice in assigning fractions to parts of a square.

Tangrams are discussed in general in the laboratory section of this resource. Construction hints, commercial sources and readiness activities are also given.

A good foundation in fraction concepts is essential before moving to fraction operations and algorithms. The development of addition, subtraction, multiplication and division of fractions will also be motivated by concrete and semi-concrete experiences.

Number Pairs and Fractions

Fill in the Blanks:

- of the children wear glasses.
 children in all.



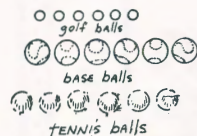
- of the children wear glasses.

1. piece of pie is missing
 pieces of pie at the beginning.



- of the pie is missing.

- baseballs.
 balls in all.



- of the balls are baseballs.

3. glasses are full.
 glasses in all.



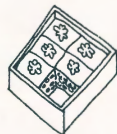
- of the glasses are full.

4. problems use addition.
 problems in all.

$$\begin{array}{r} 4 \\ + 3 \\ \hline 15 \end{array} \quad \begin{array}{r} 12 \\ - 5 \\ \hline 14 \end{array} \quad \begin{array}{r} 8 \\ + 7 \\ \hline - 8 \end{array}$$

- of the problems are addition.

5. pieces of cake left in the pan.
 pieces of cake in the pan at the beginning.



- of the cake is left in the pan.

More Number Pairs

part is shaded.
 parts in all.



of the region is shaded.



CAN YOU FILL IN THE CHART?



FIGURE	PART(S) SHADED	PARTS IN ALL	FRACTION SHADED

IDEA FROM: *Elementary School Mathematics*, Book 3, Second Edition by Robert E. Eicholz, Phares G. O'Daffer. Copyright (c) 1968 by Addison-Wesley Publishing Company, Inc. All rights reserved. Reprinted by permission.



ACTIVITY CARDS - CUISENAIRE RODS - I

For readiness activities for Cuisenaire
Rods see the section on Lab Materials.

Whole Model
Concepts
FRACTIONS



The name Cuisenaire and the color sequence of the rods
are trademarks of the Cuisenaire Company of America, Inc.

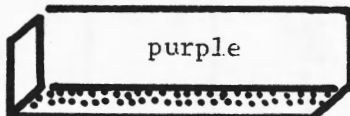
HALVES

1. What rod is half the length of a



rod? _____

2. What rod is half the length of a



rod? _____

3. What other rods can you find half of? _____

4. Make a train of orange and purple.

Can you find half of it? _____



Permission to use granted by Cuisenaire Company of America, Inc.

CR-I-1

CR-I-2

THIRDS

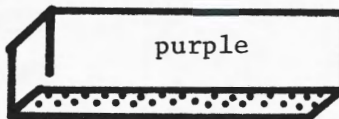
1. What rod is one third of a light green rod? _____

2. What rod is one third of a blue rod? _____

3. What other rod can you find $1/2$ of and $1/3$ of? _____

FOURTHS

4. What rod is one-fourth of a



rod? _____

5. What rod is one-fourth of a brown rod? _____

6. Try to find a rod that is three-fourths of a brown rod.

What color is it? _____

7. Find $1/4$ of the following trains: orange and red, two dark greens,
black and blue.



FRACTION APPLE-CATION

Whole Model
Equivalent
Concepts
FRACTIONS



TEACHER DEMONSTRATION

It is not always possible or practical for students to do experiments by themselves. A well-prepared teacher demonstration can fill this void. The materials needed for this demonstration are 1/2 dozen large apples, a sharp knife, a cutting board, some waxed paper, and a fraction worksheet for the students if you choose.

Students should gather around the demonstration table so they can see the parts as they are cut.



Pies or cupcakes
could be used for
this activity.

- A) Cut an apple into two parts.
 - 1) Each part is $1/2$ of the whole.
 - 2) Put the two halves together to show $2/2 = 1$.
 - B) Cut one of the halves into fourths.
 - 1) Show $2/4 = 1/2$ by comparing with the uncut half.
 - 2) Cut the other half into fourths.
 - 3) Put the fourths together to show $4/4 = 1$.
 - 4) Take $1/4$ out and show $1/4$ and $3/4$.
 - C) Cut one of the fourths into eighths.
 - 1) Show $2/8 = 1/4$ by comparing with an uncut fourth.
 - 2) Cut another. Show $4/8 = 2/4 = 1/2$.
 - 3) Cut another. Show $6/8 = 3/4$. (Students will need to visualize $3/4$ from the previous example, or a second apple can be cut to show $3/4$.)
 - 4) Cut the last fourth. Put pieces together to show $8/8 = 1$.
 - D) If the pieces are large enough, cut the eighths into sixteenths, and demonstrate the equivalent fractions.
- BE SURE TO HAVE ENOUGH PIECES AND LET THE STUDENTS EAT THE APPLE FRACTIONS.
- E) With another apple, cut out a $1/3$ slice.
(Tell the students that this cut is only a guess.)
 - 1) Show $1/3$ and $2/3$.
 - 2) Cut the $2/3$ to show $3/3 = 1$.
 - F) Cut one of the thirds to get sixths.
 - 1) Show $2/6 = 1/3$ by comparing with an uncut third.
 - 2) Cut another third into sixths.
 - 3) Show $4/6 = 2/3$ by visualizing $2/3$ or by cutting another apple.
 - 4) Cut the last third and then show $6/6 = 1$ by putting the pieces together.
 - G) If the pieces are large enough, cut the sixths into twelfths and demonstrate the equivalent fractions.

TYPE: Activity/Demonstration



COOPERATION

TEACHER IDEA

Whole Model
Concepts
FRACTIONS



THIS IS A GAME THAT CAN BE USED AS AN INTRODUCTORY ACTIVITY FOR FRACTIONS. IT REQUIRES NO COMPUTATION, ONLY COOPERATION AND TEAM WORK. IT MAY BE INTERESTING TO SIT BACK AND WATCH HOW YOUR STUDENTS INTERACT.

EACH TEAM SHOULD HAVE 5 PLAYERS.

EACH PLAYER RECEIVES AN ENVELOPE CONTAINING SOME FRACTION PIECES. THE ENVELOPES AND PIECES ARE LETTERED TO HELP GET THEM BACK IN THE PROPER ENVELOPE AT THE END OF THE GAME.

ENVELOPES



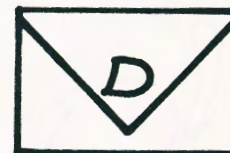
3 PIECES



4 PIECES



2 PIECES



2 PIECES



4 PIECES

THE OBJECT OF THE GAME IS TO MAKE 5 WHOLE SQUARES, (SHOWN BELOW)
EACH PLAYER MUST END UP WITH A SQUARE IN FRONT OF HIM.

RULES: NO TALKING

YOU MAY GIVE PIECES TO ANYONE ON YOUR TEAM.

YOU MAY NOT TAKE PIECES FROM ANYONE



TYPE: Game

Extension: Number the pieces 1-15. Have students estimate what fractional part of a whole square each piece is.



ACTIVITY CARDS - CIRCLE FRACTIONS - I

For information on the construction, use and readiness activities for Circle Fractions see the section on Lab Materials.

Whole Model
Concepts
FRACTIONS



Place the colored piece on the whole circle, and fill in the fraction name in the blank.

1. a) a green piece is _____ of the circle.
- b) a light blue piece is _____ of the circle.
- c) a yellow piece is _____ of the circle.
- d) a dark blue piece is _____ of the circle.

The student might estimate, cover the circle, or compare the size of the pieces.

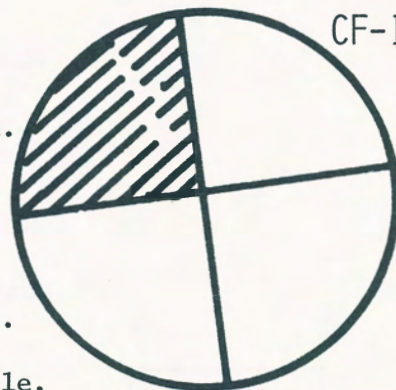
Write the Color.



2. a) a _____ piece is $\frac{1}{5}$ of the circle.
- b) a _____ piece is $\frac{1}{10}$ of the circle.
- c) a _____ piece is $\frac{1}{8}$ of the circle.
- d) a _____ piece is $\frac{1}{6}$ of the circle.

CF-I-1

3. a) 2 light blues are _____ of the circle.
- b) 3 light blues are _____ of the circle.
- c) 2 pinks are _____ of the circle.
- d) _____ greens are $\frac{2}{3}$ of the circle.
- e) _____ pinks are $\frac{5}{6}$ of the circle.
- f) _____ reds are $\frac{7}{10}$ of the circle.
- g) two _____ are $\frac{2}{5}$ of the circle.
- h) two _____ are the whole circle.
- i) three _____ are $\frac{3}{4}$ of the circle.



CF-I-2

Fill in
the
BLANKS





ACTIVITY CARDS - GEOBOARDS - I

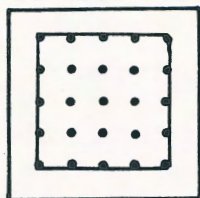
Whole Model
Concepts
FRACTIONS



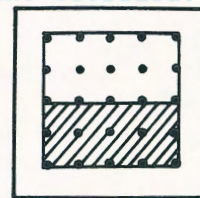
For information on the construction, use,
and readiness activities for the Geoboard,
see the section on Lab Materials.

Dot paper may be used for recording answers on instead of the Geoboard.

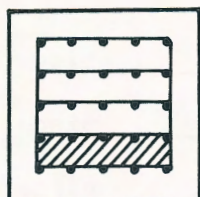
Make this square
on your board
with a rubber-
band.



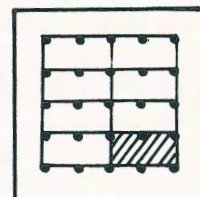
Use another rubberband
to make this picture.
What fraction of the
square is shaded?



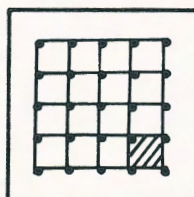
Make this
picture.
What fraction
of the square
is shaded?



Make this picture.
What fraction is
shaded?



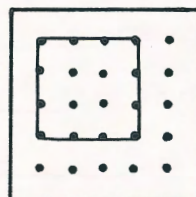
Make this picture.
What fraction is
shaded?



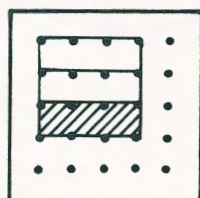
GB-I-1

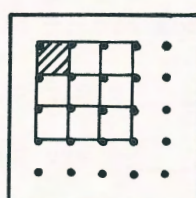
GB-I-2

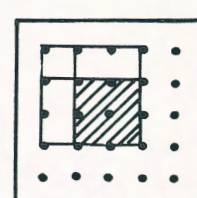
Make this square.



What fraction of the square is shaded?





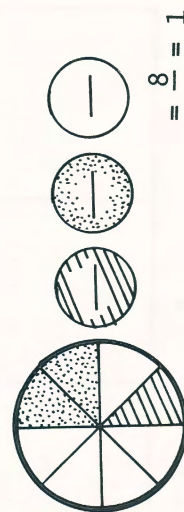
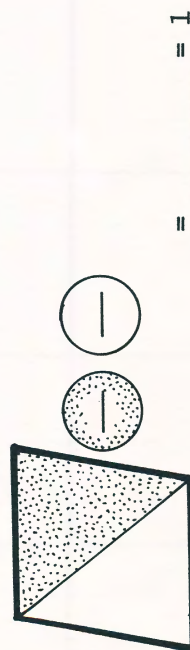
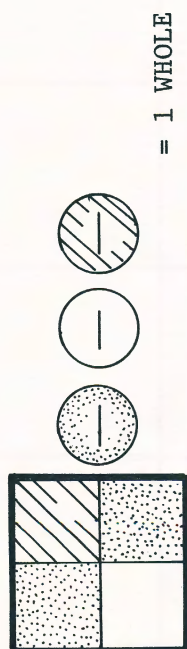
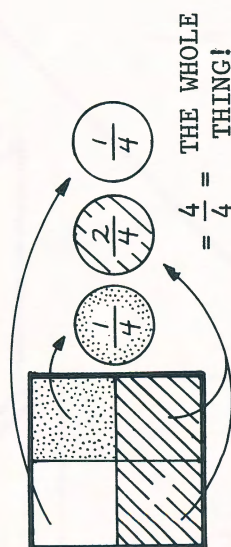
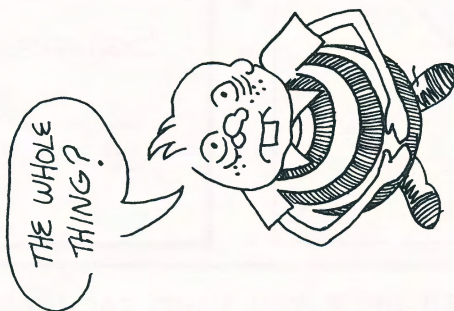
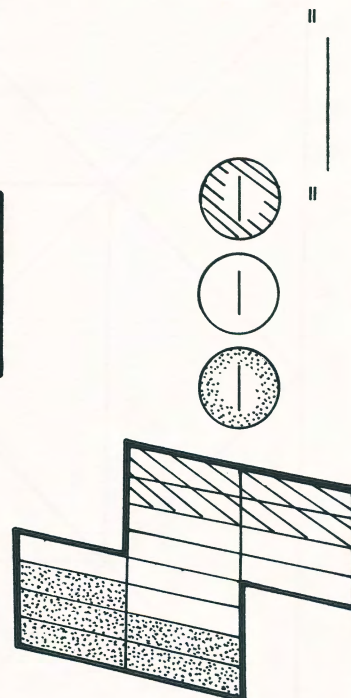
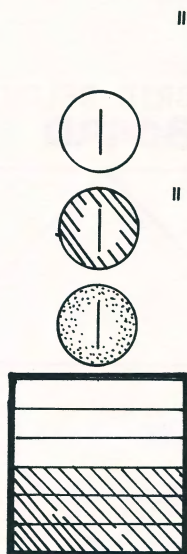
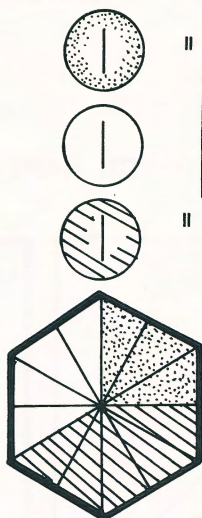
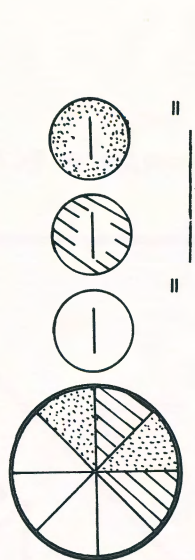


Find as many ways as you can to divide the square into halves, fourths, eighths.
Record your answers on dot paper.

TYPE: Manipulative

THE WHOLE THING

Whole Model
Concepts
FRACTIONS



TYPE: Paper & Pencil/Transparency

IDEA FROM: C.O.L.A.M.D.A.

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ACTIVITY CARDS - TANGRAMS - I



For information on the construction, use, and readiness activities for the Tangrams see the section on Lab Materials.

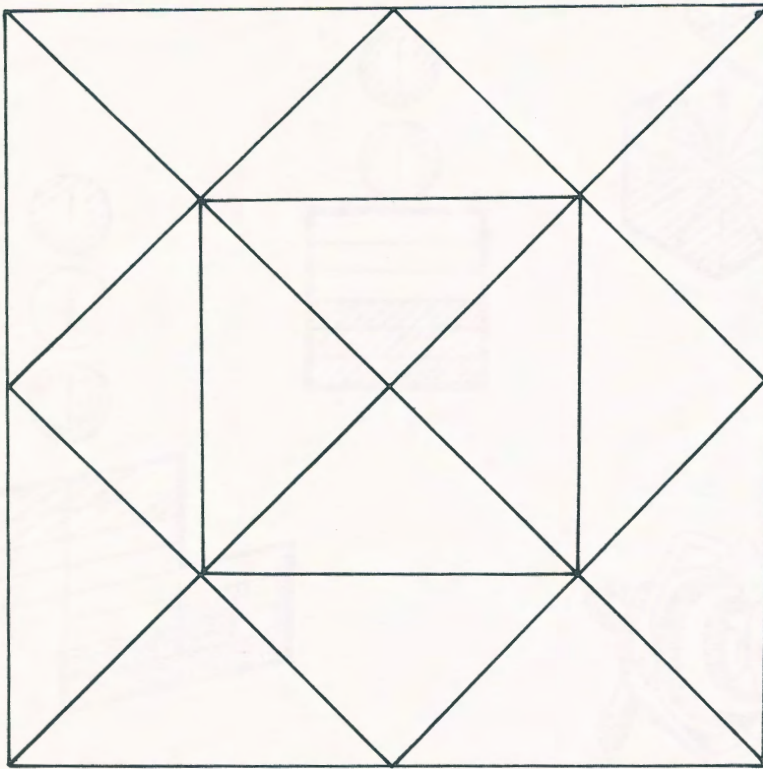
Whole Model
Concepts
FRACTIONS



USE THE GRID BELOW TO HELP YOU NAME EACH TANGRAM PIECE.

TG-I-1

TANGRAM BOARD



1 =

Each student will need a set of Tangrams.

$$\frac{\quad}{16} = \frac{\quad}{8} = \frac{\quad}{4}$$

LARGE
TRIANGLE

MEDIUM
TRIANGLE

$$\frac{\quad}{16} = \frac{\quad}{\quad}$$

SQUARE

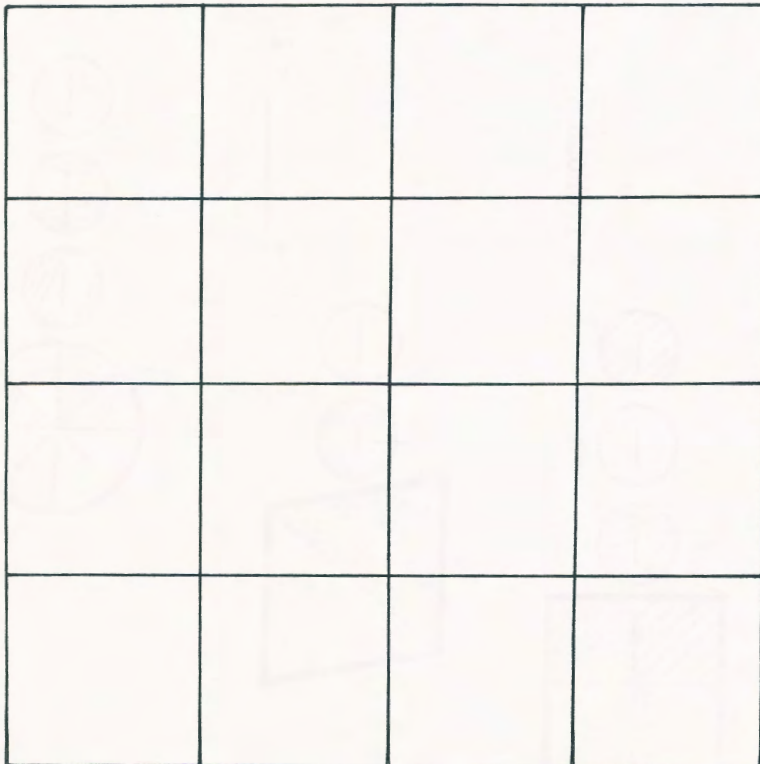
$$\frac{\quad}{16} = \frac{\quad}{\quad}$$

PARALLELO-
GRAM

SMALL TRIANGLE

USE THE GRID BELOW TO HELP YOU NAME EACH TANGRAM PIECE.

TG-I-2



1 =

$$\frac{\quad}{16} = \frac{\quad}{\quad}$$

$$\frac{\quad}{16} = \frac{\quad}{\quad}$$

$$\frac{\quad}{16} = \frac{\quad}{\quad}$$

$$\frac{\quad}{16} = \frac{\quad}{\quad}$$

$$\frac{\quad}{16}$$

CRAZY PEANUTS

Set Model
Concepts
FRACTIONS



I NEED SOME PEANUTS FOR WINTER.
WILL YOU HELP ME FIND SOME? TO MAKE THE
PEANUTS, JUST SHOW EACH FRACTION THIS WAY:
1. LOOK AT THE DENOMINATOR OF THE FRACTION,
DIVIDE THE DOTS INTO THAT MANY GROUPS, 2. LOOK
AT THE NUMERATOR OF THE FRACTION, AND SHADE THAT
MANY GROUPS. BE CAREFUL, THERE SHOULD BE THE
SAME NUMBER OF DOTS IN EACH GROUP.

LIKE THIS:

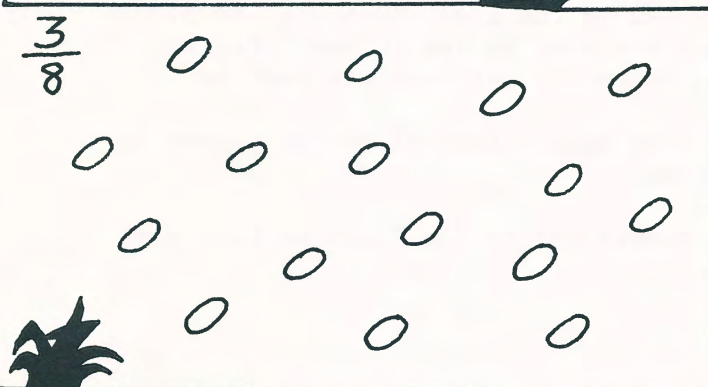
$\frac{1}{3} \rightarrow$ group shaded
groups in all



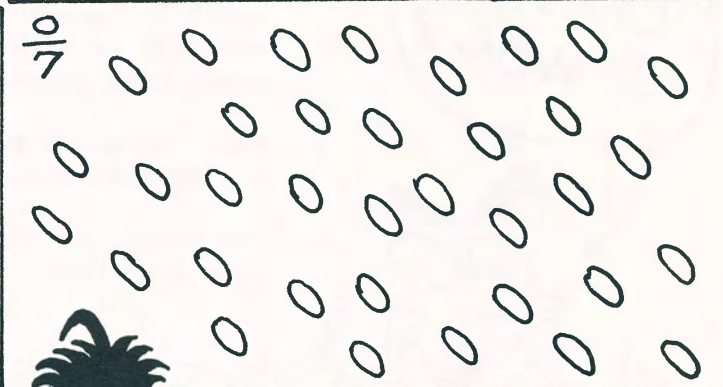
$\frac{1}{5}$



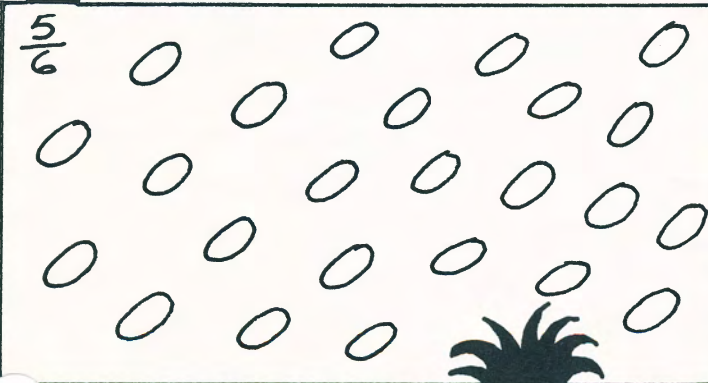
$\frac{3}{8}$



$\frac{6}{7}$



$\frac{5}{6}$



$\frac{3}{7}$



LESS THAN 5

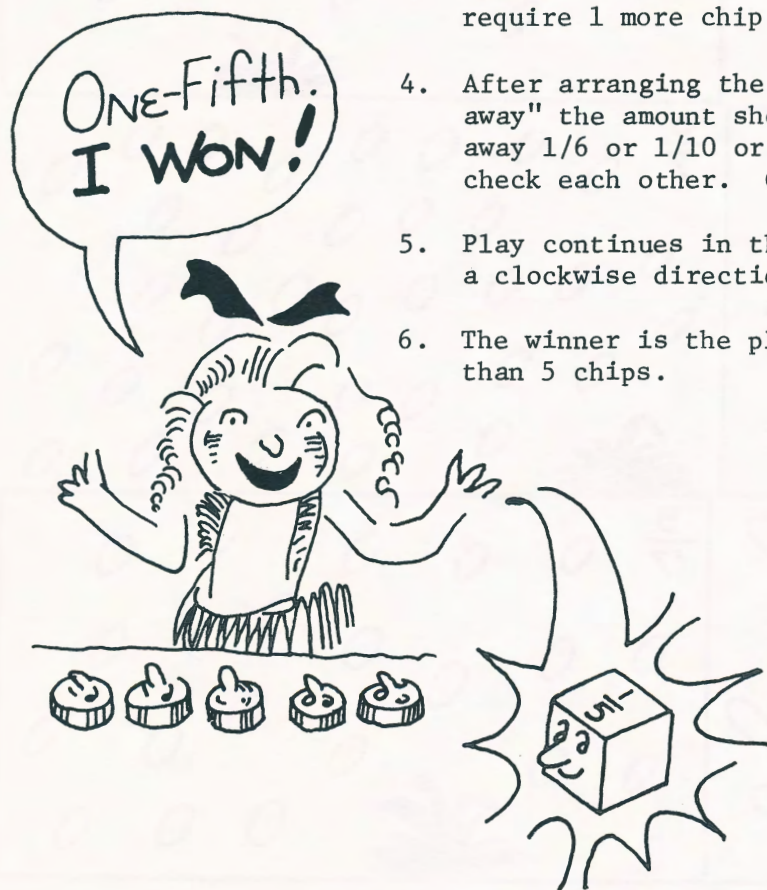
Set Model
Concepts
FRACTIONS



OBJECTIVE: Demonstrate fractions as part of a set.

MATERIALS: Chips or other small counting objects.
One die with these fractional numerals: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{10}$.
Paper plate, or box lid, or tray for each player.

- DIRECTIONS:
1. Each player counts out 60 chips from the bank and places them on his plate. Extra chips are kept in the bank in the center of the playing area.
 2. Each player rolls the die to determine who begins. The player rolling the largest fraction begins.
 3. The first player rolls the die and arranges his chips into as many equal subsets as the denominator of the fraction indicates. Example: If $\frac{1}{6}$ is rolled, the player arranges his chips into six equal subsets. If in some cases the set cannot be divided into equal subsets, the player must take chips from the bank to complete the set. Example: With 50 chips left if $\frac{1}{3}$ is rolled, the player would require 1 more chip.
 4. After arranging the chips into sets, the player "gives away" the amount shown on the die. Example: He gives away $\frac{1}{6}$ or $\frac{1}{10}$ or whatever he has rolled. Players check each other. Chips are put into the bank set.
 5. Play continues in this manner from player to player in a clockwise direction.
 6. The winner is the player who is the first to have less than 5 chips.



A simpler game uses 30 chips and one die with the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.

TYPE: Game

IDEA FROM: *Experiments in Fractions*



MAKE ME DIVIDE

Definition
Quotient Model
Concepts
FRACTIONS



TEACHER IDEA

Many students have difficulty understanding how the terms of a fraction relate to its physical representation. The following are three concrete examples of activities that relate the fractional name to concrete models through division.

USING MEASURING CUPS Idea for teacher demonstration.



Fill 2 measuring cups up to the 1 cup line.

Pour the contents into 3 measuring cups equally.

$$2 \div 3$$

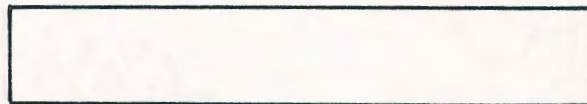
Each of the 3 cups will be $\frac{2}{3}$ full.

You could use rice, cornmeal, etc.

USING RULERS Ideas for student activities.

Cut a strip of paper 3" long.
Fold it into 4 equal parts.

$$3 \div 4$$



Use the ruler to measure each part.

Each will be about $\frac{3}{4}$ " long.



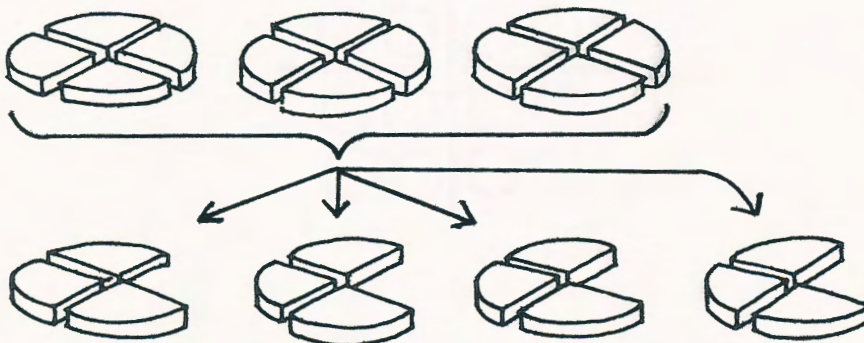
USING CIRCLE FRACTION PIECES

Your students may suggest other examples.

$$3 \div 4$$

Separate the 3 pies into 4 equal parts.

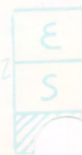
You have $\frac{3}{4}$ in each part.



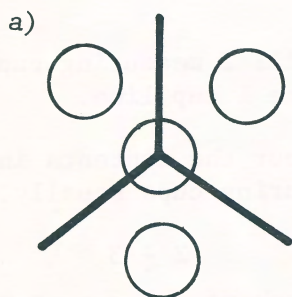
The quotient model is not usually emphasized, yet we expect students to use fractions as quotients. See the commentary to FRACTIONS: Concepts.

TYPE: Demonstration/Activity

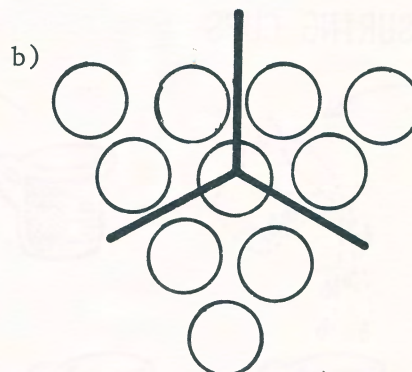
DIVIDING GROUPS



1. Each group of circles is split into equal sections. Divide the number of circles by the number of sections. Is this the size of each section?

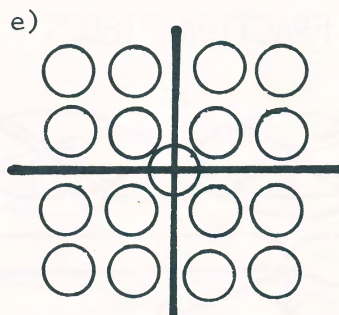
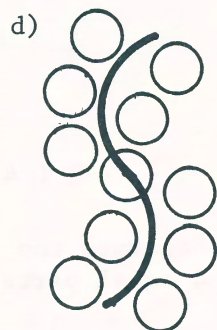
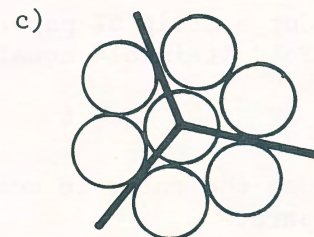
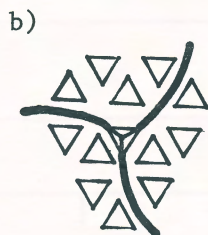
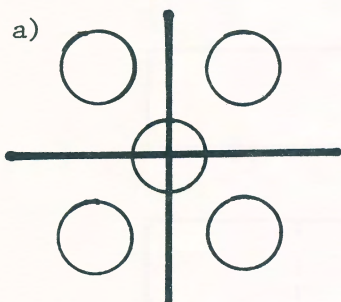


4 THINGS IN 3 SECTIONS



10 THINGS IN 3 SECTIONS

2. Give each example as a division of whole numbers. Then find the answer by looking at the size of each section.

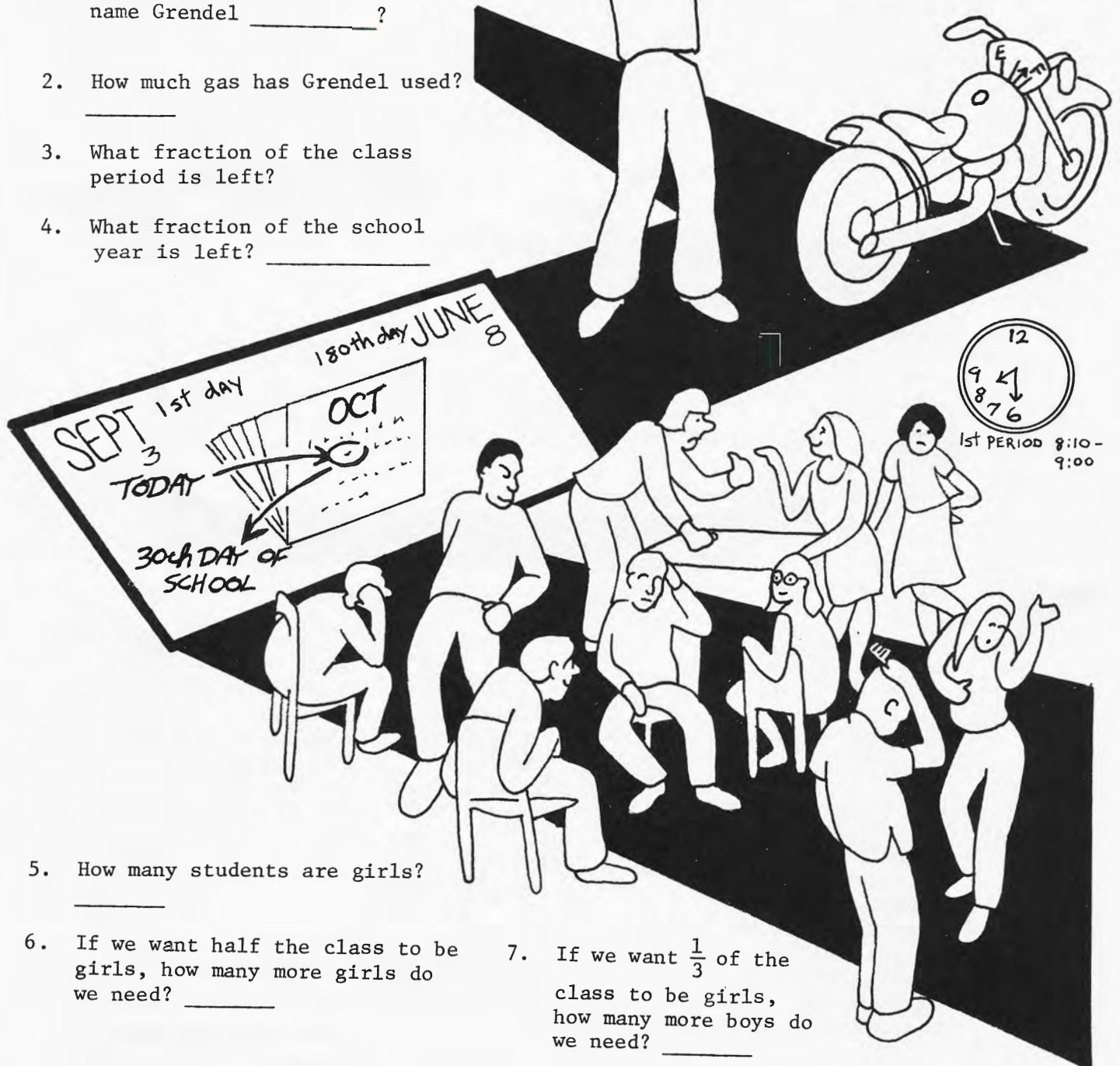


GRENDEL'S GAUGES

GIVE THE ANSWERS TO THE FOLLOWING QUESTIONS IN FRACTION FORM WHERE POSSIBLE.

1. What fractional part of the alphabet is used in the name Grendel _____?
2. How much gas has Grendel used?

3. What fraction of the class period is left?
4. What fraction of the school year is left? _____



5. How many students are girls?

6. If we want half the class to be girls, how many more girls do we need? _____
7. If we want $\frac{1}{3}$ of the class to be girls, how many more boys do we need? _____

FRACTION DOMINOES

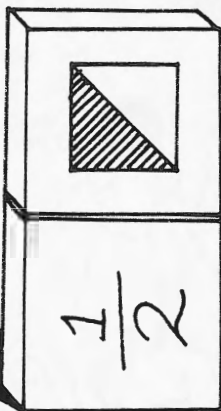
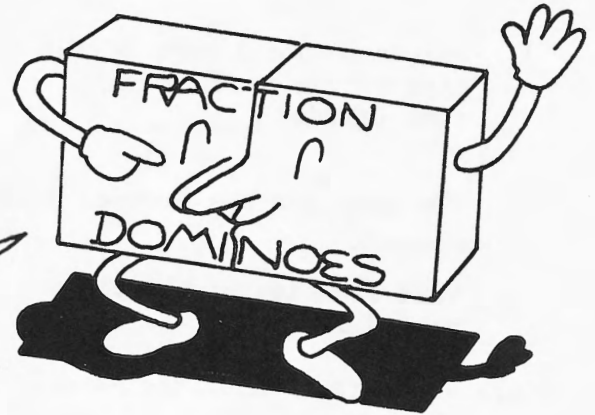
Want to make your own fraction dominoes?
Got some special fractions to work on?
Here's how!! . . .




Fractional Parts
Concepts
FRACTIONS

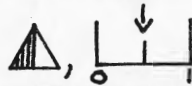


CHOOSE THE SET OF SIX
(6) FRACTIONS YOU WISH TO USE.
FOR EXAMPLE: $1/2$, $3/10$, $3/4$, $2/5$,
 $2/3$, $1/5$. PAIR THE $1/2$ WITH ALL 6
FRACTIONS, THE $3/10$ WITH THE LAST 5
FRACTIONS, THE $3/4$ WITH THE LAST FOUR,
ETC., TO MAKE 21 DOMINOES.

2 cm BY 4 cm IS A GOOD SIZE.

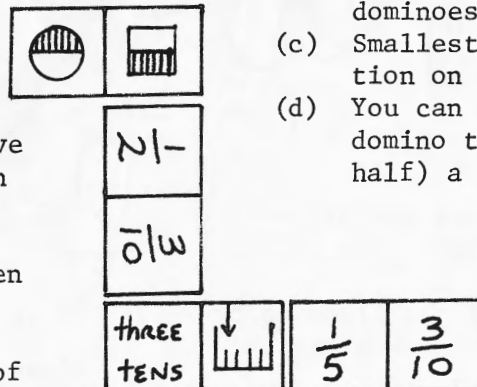


EACH FRACTION IS USED SEVEN TIMES. SUGGESTED
REPRESENTATIVES ARE $1/2$, ONE-HALF, , , ,



Remember the rules?

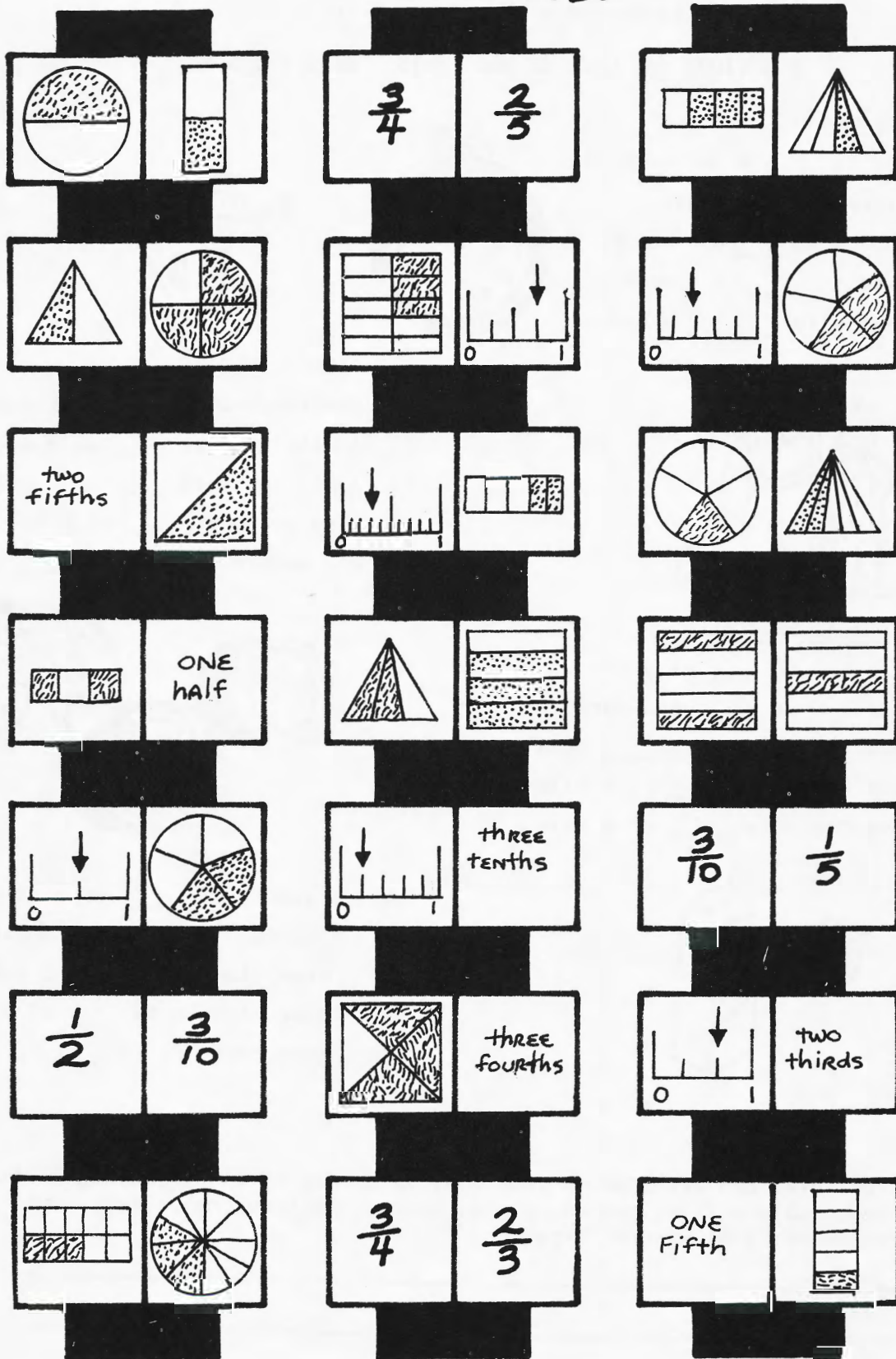
- (a) All dominoes placed face down.
- (b) Each player draws four dominoes.
- (c) Smallest double (same fraction on both ends) goes first.
- (d) You can play if you have a domino that matches (on one half) a domino already played.
- (e) If you can't play, you have to pull from the face down dominoes until you get a domino that plays.
- (f) When all dominoes have been pulled, a player loses a turn if he cannot play.
- (g) First player to play all of his dominoes is the winner.



TYPE: Game

FRACTION DOMINOES

CONTINUED



A PART OF THE ACTION

Fill in the blanks with reduced fractions.

FRACTIONS ARE USED IN MANY WAYS. HERE ARE SOME:

TIME

- 1) A minute is ____ of an hour.
- 2) An hour is ____ of a day.
- 3) Eight hours is ____ of a day.
- 4) Five days is ____ of a week.
- 5) Four months is ____ of a year.



WEIGHT

- 6) Twelve ounces is ____ of a pound.
- 7) A milligram is ____ of a gram.
- 8) A milligram is ____ of a kilogram.
- 9) Five grams is ____ of a kilogram.
- 10) One hundred pounds is ____ of a ton.



LINEAR MEASURE

- 11) Nine inches is ____ of a foot.
- 12) Nine inches is ____ of a yard.
- 13) Fifteen centimetres is ____ of a metre.
- 14) Fifteen metres is ____ of a kilometre.
- 15) Fifteen yards is ____ of a mile.



MONEY

- 16) A quarter is ____ of a dollar.
- 17) A nickel is ____ of a dollar.
- 18) Three dimes is ____ of a dollar.
- 19) Three nickels is ____ of a quarter.
- 20) Three quarters is ____ of a dollar.



VOLUME

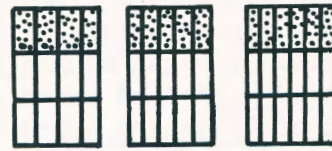
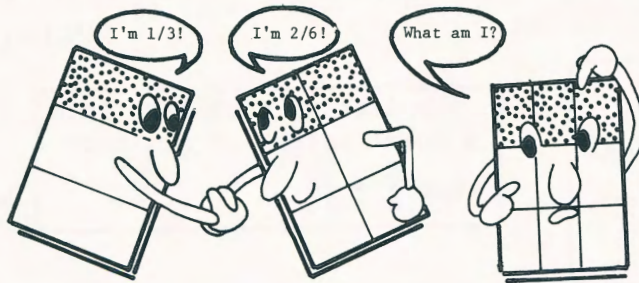
Make up and solve two problems of your own, using any of the following: litres, millilitres, cubic metres, ounces, pints, quarts, gallons, cubic feet, cubic yards, teaspoons, tablespoons, cups.

FAIR EXCHANGE

Equivalent
Concepts
FRACTIONS



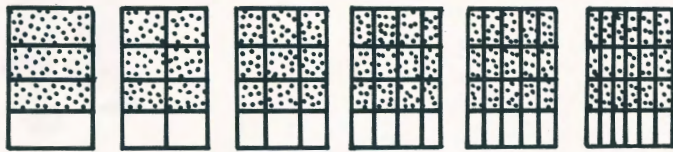
Find the equivalent fractions in each row and answer the questions.



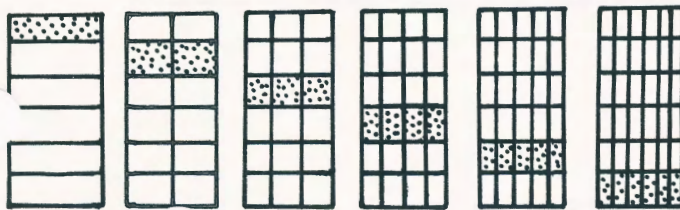
Name these.

Are the names equal to $\frac{1}{3}$?

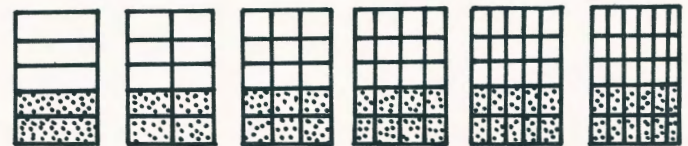
NAME THE FOLLOWING:



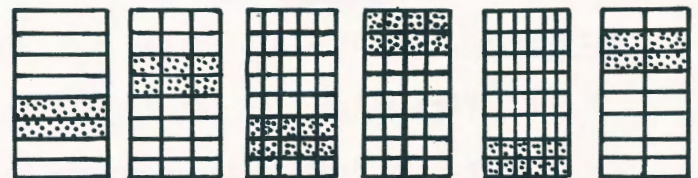
— = — = — = — = — = — = —



— = — = — = — = — = — = —



— = — = — = — = — = — = —



— = — = — = — = — = — = —

Does $\frac{1}{6} = \frac{3}{18}$? — Does $\frac{3}{18} = \frac{5}{30}$? — Does $\frac{1}{6} = \frac{5}{30}$? — Write a fraction that equals $\frac{12}{48}$. —

Write a fraction that equals $\frac{6}{24}$. —

Write a fraction that equals $\frac{2}{3}$. —

Write
fractions
for us.



Does $1 = \frac{8}{8}$? —

Does $\frac{3}{3} = \frac{5}{5}$? —

NOTE: Paper & Pencil/Transparency

IDEA FROM: C.O.L.A.M.D.A.

Permission to use granted by Northern Colorado Educational Board of Cooperative Services

ACTIVITY CARDS - FRACTION BARS - I



MAKING EQUAL FRACTIONS

For information on the construction, use and readiness activities for Fraction Bars see the section on Lab Materials.

Equivalent
Concepts
FRACTIONS

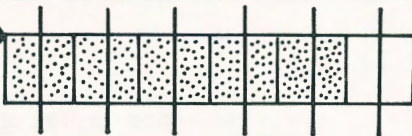


Materials Fraction bars

FB-I-1



Al! I've found a way to get equal fractions. Take a bar and split each part into 2 equal parts. I began with 5 out of 6 shaded parts and ended with 10 out of 12 shaded parts.



$$\frac{5}{6} = \frac{10}{12}$$

By splitting each part into 3 equal parts, we can show that $\frac{5}{6} = \frac{15}{18}$.



$$\frac{5}{6} = \frac{15}{18}$$

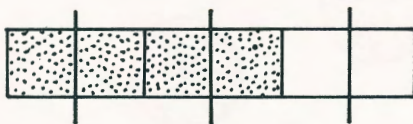


Drawing these lines does not increase or decrease the shaded amount.

Activities (1 or 2 students)

1. Pick a fraction bar for the following activities.
 - a. By drawing lines on the bar, split each part of the bar into 2 equal parts. Write the equation.
 - b. Erase the marks on the bar, and this time split each part of the bar into 3 equal parts. Write the equation.
 - c. Erase the marks on the bar. Now split each part of the bar into 4 equal parts. Write the equation.
2. Repeat these activities for a fraction bar of each color.

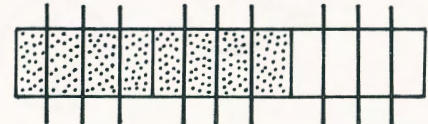
Examples



$$\frac{2}{3} = \frac{4}{6}$$



$$\frac{2}{3} = \frac{6}{9}$$



$$\frac{2}{3} = \frac{8}{12}$$

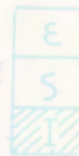
These examples show that $\frac{2}{3}$, $\frac{4}{6}$, $\frac{6}{9}$, and $\frac{8}{12}$ are fractions for the same amount.



ACTIVITY CARDS - CIRCLE FRACTIONS II

For information on the construction of Circle Fractions see the section on Lab Materials.

Equivalent
Improper - Mixed
Concepts
FRACTIONS



WRITE THE COLOR OR THE NUMBER OF PIECES NEEDED TO EXACTLY COVER THESE PIECES.

1. Two dark blue exactly cover 1 light blue.

$$\frac{2}{8} = \frac{1}{4}$$

2. Two pinks exactly cover _____ green.

$$\frac{2}{6} = \underline{\hspace{1cm}}$$

3. One light blue = 2 _____.

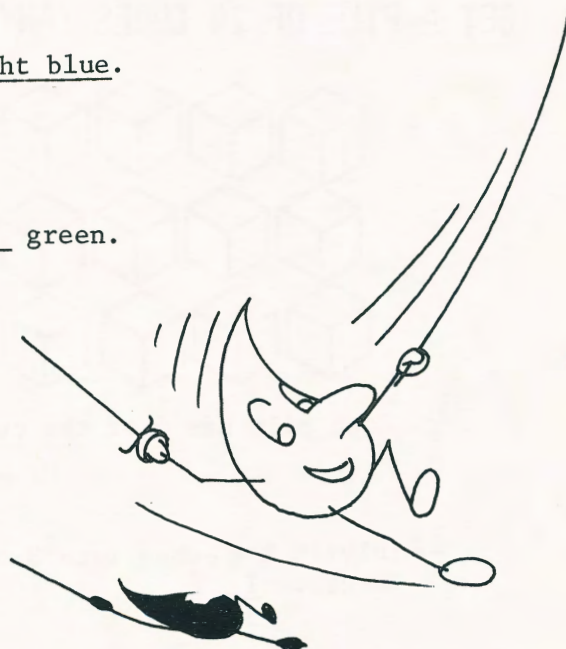
$$\underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

4. Three purple = _____ red.

$$\underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

5. Two yellow = 5 _____.

$$\underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$



CAN YOU FIND SOME MORE EQUIVALENT FRACTIONS USING THE CIRCLE FRACTIONS?

CF-II-1

CF-II-2

Here you need 2 white circles and 4 yellow pieces.

1. 1 yellow = $\frac{1}{??}$

2 yellow = $\frac{2}{2}$ or 1.

3 yellow = $\frac{3}{2}$ or $1\frac{1}{2}$.

4 yellow = _____ or 2.



Now use 2 white circles and 6 green pieces.

2. 1 green = $\frac{1}{?}$

2 green = _____

3 green = $\frac{?}{3}$ or _____

4 green = _____ or $1\frac{1}{3}$

5 green = $\frac{5}{3}$ or _____

6 green = _____ or _____

3. CAN YOU DO THE SAME THING WITH SOME PINK PIECES AND TWO WHITE CIRCLES?

TYPE: Manipulative



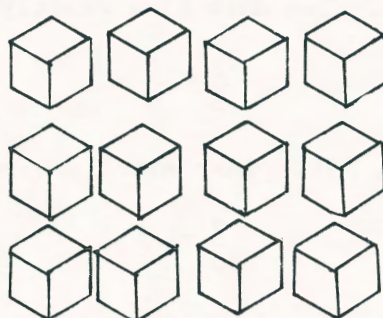
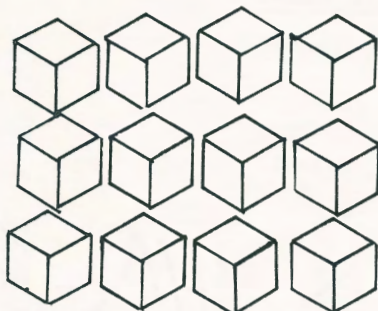
ACTIVITY CARDS - COLOR CUBES - II

For information on Color Cubes
see the section on Lab Materials.

Equivalent
Ordering
Concepts
FRACTIONS



GET A PILE OF 24 CUBES (ANY COLOR). DIVIDE THEM INTO 2 EQUAL PILES.



1. Each pile has $\frac{1}{2}$ of the cubes. How many cubes are in a pile? _____

We can say that $\frac{1}{2} = \frac{12}{24}$

2. Divide the cubes into 3 equal piles. Each pile has one-_____ of the cubes. $\frac{1}{3} = \frac{?}{24}$

3. Divide the cubes into 4 equal piles. Each pile has one-_____ of the cubes. $\frac{1}{4} = \frac{?}{24}$

CC-II-1

Continue dividing into 6, 8, 12, and 24 equal piles.

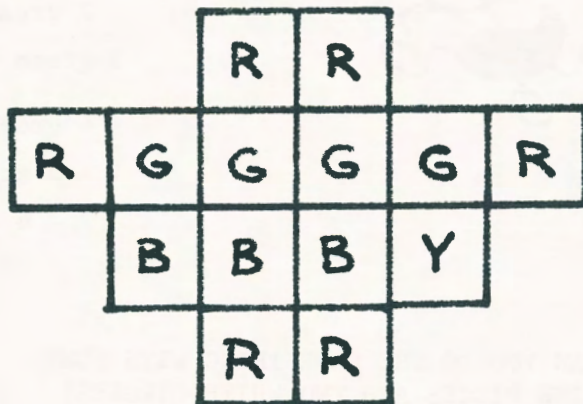
CC-II-2 MAKE YOUR OWN DESIGN USING AS MANY COLOR CUBES AS YOU LIKE.

USE MORE RED CUBES THAN GREEN CUBES,
MORE GREEN THAN BLUE,
MORE BLUE THAN YELLOW.

Write a fraction for each color, then arrange the fractions in order from largest to smallest.

_____ > _____ > _____ > _____

Example:



14 cubes

$$R = \frac{6}{14}, G = \frac{4}{14}, B = \frac{3}{14}, Y = \frac{1}{14}$$

$$\frac{6}{14} > \frac{4}{14} > \frac{3}{14} > \frac{1}{14}$$

You may wish to combine this
with Color Cubes I & III.

TYPE: Manipulative

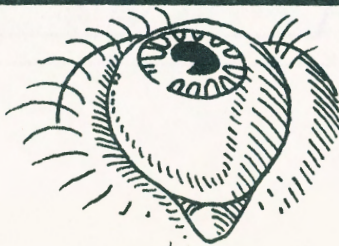


The Fraction Bar Sheet

Could your students help make a
bulletin board display of the
fraction bar sheet?
This sheet is for use with
Eyeballing the Fraction Bar Sheet.



1															
$\frac{1}{2}$								$\frac{1}{2}$							
$\frac{1}{3}$				$\frac{1}{3}$				$\frac{1}{3}$							
$\frac{1}{4}$			$\frac{1}{4}$			$\frac{1}{4}$			$\frac{1}{4}$						
$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$					
$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$					
$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$	
$\frac{1}{10}$		$\frac{1}{10}$		$\frac{1}{10}$		$\frac{1}{10}$		$\frac{1}{10}$		$\frac{1}{10}$		$\frac{1}{10}$		$\frac{1}{10}$	
$\frac{1}{12}$		$\frac{1}{12}$		$\frac{1}{12}$		$\frac{1}{12}$		$\frac{1}{12}$		$\frac{1}{12}$		$\frac{1}{12}$		$\frac{1}{12}$	
$\frac{1}{16}$		$\frac{1}{16}$		$\frac{1}{16}$		$\frac{1}{16}$		$\frac{1}{16}$		$\frac{1}{16}$		$\frac{1}{16}$		$\frac{1}{16}$	



Equivalent
Concepts
FRACTIONS

1	5	3
---	---	---

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THE FRACTION BAR SHEET

USE THE "EYEBALL" METHOD OR SOMETHING WITH A STRAIGHT EDGE AND THE FRACTION BAR SHEET TO FIND THESE EQUIVALENT FRACTIONS.

WRITE FIVE FRACTIONS
EQUIVALENT TO $\frac{1}{2}$.
____ = ____ = ____ = ____ = ____

WRITE THREE FRACTIONS
EQUIVALENT TO $\frac{3}{4}$.

$\frac{3}{4}$ = ____ = ____ = ____

COMPLETE THESE:

$\frac{1}{4}$ = ____ = ____

$\frac{3}{5}$ = ____ = ____

$\frac{2}{6}$ = ____ = ____

BE SURE TO KEEP THE STRAIGHT EDGE
PARALLEL TO THE SIDE OF THE PAPER.

WRITE A FRACTION EQUIVALENT
TO $\frac{2}{8}$? $\square = \frac{2}{8}$
WRITE ANOTHER ONE. $\triangle = \frac{2}{8}$

IS \square EQUIVALENT TO \triangle ?
YES OR NO

WRITE A FRACTION EQUIVALENT TO
 $\frac{8}{12}$. $\circ = \frac{8}{12}$.
ANOTHER ONE: $\frac{8}{12} = \text{hexagon}$

IS \circ EQUIVALENT TO hexagon ?
YES OR NO

IS
YOUR EYEBALL
STRAIGHT?

BE CAREFUL

USE YOUR FRACTION BAR SHEET
AND WRITE EITHER $<$, OR $=$, OR
BETWEEN THESE FRACTIONS.

A) $\frac{1}{2} \circ \frac{2}{3}$

C) $\frac{7}{16} \circ \frac{3}{8}$

B) $\frac{3}{4} \circ \frac{5}{8}$

D) $\frac{6}{10} \circ \frac{3}{5}$

G) IS $\frac{3}{10} < \frac{1}{2}$? (YES OR NO) IS $\frac{1}{2} < \frac{7}{10}$? (YES OR NO)
IS $\frac{3}{10} < \frac{7}{10}$? (YES OR NO)

H) IS $\frac{3}{4} > \frac{2}{3}$? IS $\frac{2}{3} > \frac{1}{2}$? IS $\frac{3}{4} > \frac{1}{2}$?

I) IS $\frac{1}{4} < \frac{1}{3}$? IS $\frac{1}{5} < \frac{1}{3}$? IS $\frac{1}{4} < \frac{1}{5}$?

J) IS $\frac{7}{8} > \frac{6}{10}$? IS $\frac{6}{10} > \frac{7}{12}$? IS $\frac{7}{8} > \frac{7}{12}$?

IF $\triangle < \square$
AND, IF $\square < \diamond$
IS $\triangle < \diamond$?

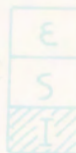
IF $\triangle < \square$,
AND, IF $\square < \diamond$,
IS $\triangle < \diamond$?





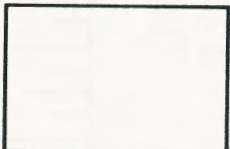
READING YOUR RULER

Equivalent
Number Line Model
Concepts
FRACTIONS



This activity provides practice in locating fractions on a number line. Number line concepts are then applied to reading the ruler. Below are suggestions for objects which can be measured when working the two student pages which follow. A sheet of rulers is also included (next page). Each student can have a copy of that page and can fold it to the proper ruler. The activity will take 1-2 days but could be done over several days and integrated with other materials. To begin the activity each student is given a copy of this page (or a similar teacher developed list), the page of rulers and the first student page.

Use your ruler to measure each of the following:

- Your pen or pencil.
 - Your shortest finger.
 - The width of your desk.
 - The length of this paper.
 - The width of this rectangle.
- 
- The widest part of this hexagon.
 - The narrowest part of this hexagon.
 - The width of your hand.
 - The length of the picture of the fish below.



The whole activity could be followed with problems like these:

Draw a segment that is $\frac{1}{2}$ inches long.

How many $\frac{1}{4}$ inches? _____

How many $\frac{1}{8}$ inches? _____

How many $\frac{1}{16}$ inches? _____

How many $\frac{1}{10}$ inches? _____

Draw a segment that is $\frac{7}{4}$ inches long.

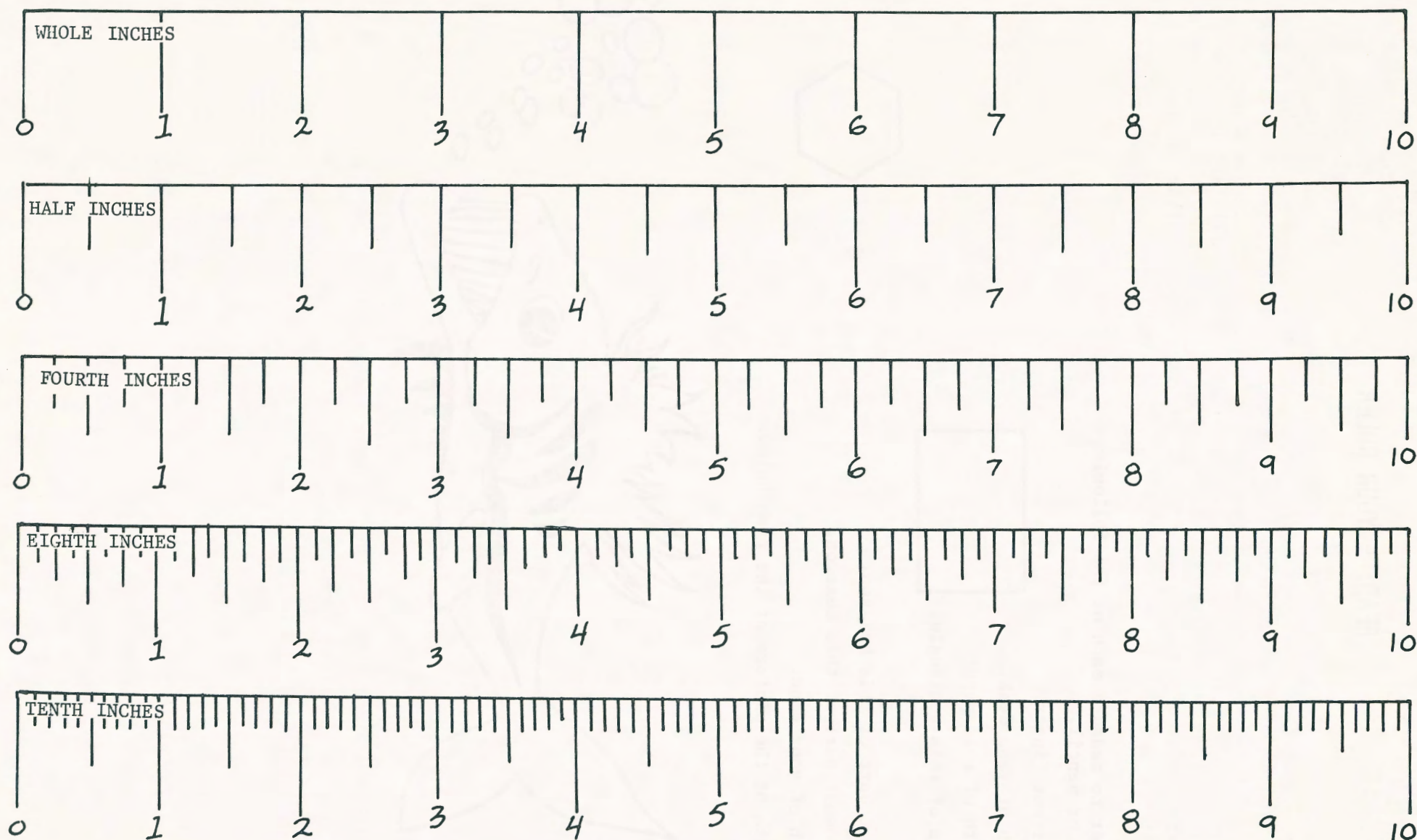
How many $\frac{1}{8}$ inches? _____

How many $\frac{1}{16}$ inches? _____

How many $\frac{1}{10}$ inches? _____

TYPE: Paper & Pencil/Manipulative

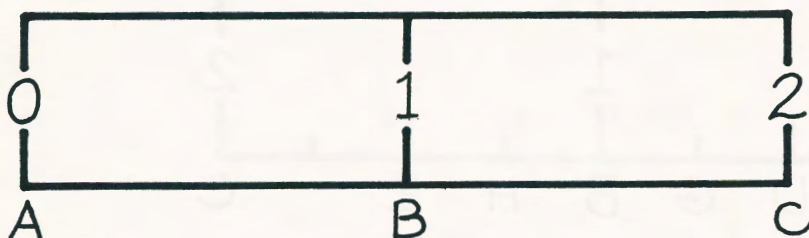
READING YOUR RULER (PAGE 2)



TO MEASURE SIMPLY FOLD THE PAPER SO THAT THE RULER YOU DESIRE IS
ALONG THE FOLDED EDGE. DON'T TRY TO APPROXIMATE IN-BETWEEN LINES.
JUST CHOOSE WHICHEVER LINE IT IS NEAREST TO.

STUDENT PAGE I

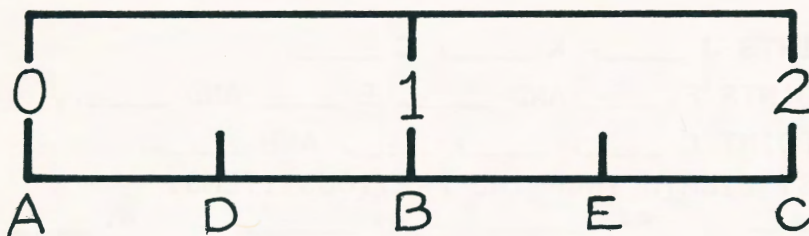
I. THE UNIT SCALE



1. ONE NAME FOR POINT A IS _____.
2. WHAT IS THE NAME FOR B _____ AND C _____?
3. MEASURE TO THE NEAREST INCH THE ITEMS DESCRIBED ON THE READING YOUR RULER PAGE. RECORD YOUR ANSWERS IN THE SPACE PROVIDED BELOW. (USE YOUR FOLDING RULER.)

- | | | | |
|----------|----------|----------|----------|
| a. _____ | b. _____ | c. _____ | d. _____ |
| e. _____ | f. _____ | g. _____ | h. _____ |
| i. _____ | j. _____ | | |

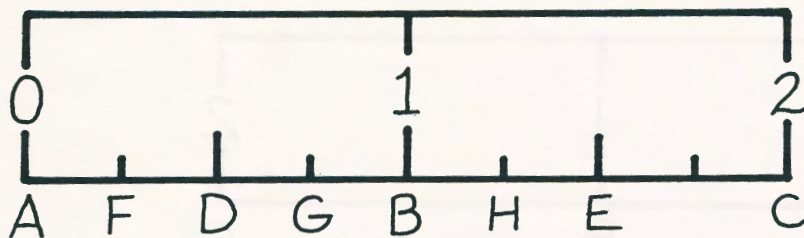
II. THE HALF-UNIT SCALE



1. ONE NAME FOR POINT A IS _____.
2. TWO NAMES FOR POINT B ARE _____ AND _____.
3. GIVE TWO NAMES FOR POINT C _____ AND _____.
4. NAME POINTS D AND E, _____ AND _____.
5. MEASURE TO THE NEAREST HALF-INCH THE ITEMS FROM THE READING YOUR RULER PAGE.

- | | | | | |
|----------|----------|----------|----------|----------|
| a. _____ | b. _____ | c. _____ | d. _____ | e. _____ |
| f. _____ | g. _____ | h. _____ | i. _____ | j. _____ |

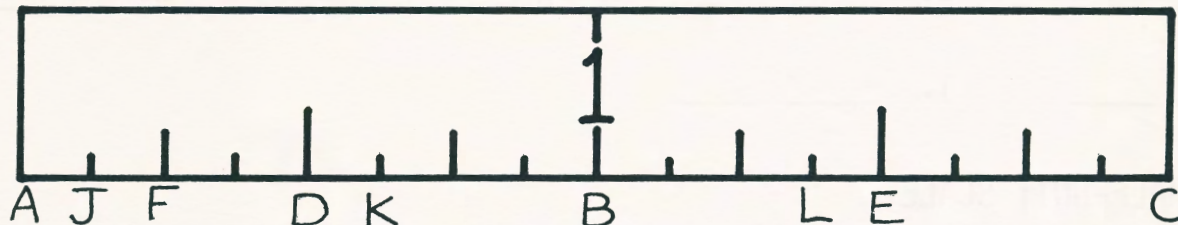
III. THE FOURTH-UNIT SCALE



1. GIVE ONE NAME FOR POINTS A _____, F _____, G _____, H _____.
2. GIVE TWO NAMES FOR POINTS D _____ AND _____, E _____ AND _____.
3. GIVE AS MANY NAMES AS YOU CAN FOR POINT B _____.
4. MEASURE TO THE NEAREST QUARTER-INCH THE PREVIOUS ITEMS,

a. _____ b. _____ c. _____ d. _____ e. _____
 f. _____ g. _____ h. _____ i. _____ f. _____

IV. THE EIGHTH-UNIT SCALE



1. GIVE ONE NAME FOR POINTS J _____, K _____, C _____.
2. GIVE TWO NAMES FOR POINTS F _____ AND _____, E _____ AND _____.
3. GIVE FOUR NAMES FOR POINT C _____, _____, _____, AND _____.
4. MEASURE TO THE NEAREST EIGHTH-INCH THE PREVIOUS ITEMS,

a. _____ b. _____ c. _____ d. _____ e. _____
 f. _____ g. _____ h. _____ i. _____ j. _____

V. THE TENTH-UNIT SCALE

1. DO YOU FIND TENTHS ON MOST RULERS? _____
2. HOW MANY TENTHS ARE IN ONE HALF? _____
3. MEASURE TO THE NEAREST TENTH-INCH THE PREVIOUS ITEMS,

a. _____ b. _____ c. _____ d. _____ e. _____
 f. _____ g. _____ h. _____ i. _____ j. _____

FRACTION TIC TAC TOE

Equivalent
Concepts
FRACTIONS



You will need to construct the
TIC TAC TOE mats shown here.

Players 2 - 4

Materials FRACTION TIC TAC TOE mat, fraction
dice, and markers.

Each player uses markers of one color.

On his turn, a player tosses the fraction dice and
places a marker on every fraction on the mat equal
to his fraction.

When a player has finished taking his turn, he pass-
es the fraction dice to the next player. If he has
missed any fractions, they may now be covered by the
other players.

A marker cannot be placed on a square which is
covered. If all the fractions equal to the frac-
tion a player tosses have already been covered, he
tosses again.

The first player to get 3 markers side by side in any row is the winner.

Variation

On his turn, a player tosses the fraction dice and places only 1 marker on the mat.
He may put this marker on any one of the fractions equal to the fraction tossed.

The first player to get 3 markers side by side in a row, column, or diagonal, is
the winner.

●	$\frac{1}{1}$	$\frac{2}{1}$	●	$\frac{4}{1}$	$\frac{5}{1}$	$\frac{6}{1}$	$\frac{7}{1}$	$\frac{8}{1}$	$\frac{9}{1}$	$\frac{10}{1}$	$\frac{11}{1}$
●	$\frac{1}{2}$	$\frac{2}{2}$	$\frac{3}{2}$	$\frac{4}{2}$	$\frac{5}{2}$	●	$\frac{7}{2}$	$\frac{8}{2}$	$\frac{9}{2}$	$\frac{10}{2}$	$\frac{11}{2}$
●	$\frac{1}{3}$	●	$\frac{3}{3}$	$\frac{4}{3}$	$\frac{5}{3}$	$\frac{6}{3}$	$\frac{7}{3}$	$\frac{8}{3}$	●	$\frac{10}{3}$	$\frac{11}{3}$
●	●	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{5}{4}$	$\frac{6}{4}$	$\frac{7}{4}$	$\frac{8}{4}$	$\frac{9}{4}$	$\frac{10}{4}$	$\frac{11}{4}$
●	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{5}{5}$	$\frac{6}{5}$	$\frac{7}{5}$	$\frac{8}{5}$	$\frac{9}{5}$	$\frac{10}{5}$	●
●	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	●	$\frac{5}{6}$	$\frac{6}{6}$	$\frac{7}{6}$	$\frac{8}{6}$	$\frac{9}{6}$	$\frac{10}{6}$	$\frac{11}{6}$
●	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{3}{7}$	$\frac{4}{7}$	$\frac{5}{7}$	$\frac{6}{7}$	$\frac{7}{7}$	$\frac{8}{7}$	$\frac{9}{7}$	$\frac{10}{7}$	$\frac{11}{7}$
●	$\frac{1}{8}$	●	$\frac{3}{8}$	$\frac{4}{8}$	$\frac{5}{8}$	$\frac{6}{8}$	$\frac{7}{8}$	$\frac{8}{8}$	●	$\frac{10}{8}$	$\frac{11}{8}$
●	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{4}{9}$	$\frac{5}{9}$	●	$\frac{7}{9}$	$\frac{8}{9}$	$\frac{9}{9}$	$\frac{10}{9}$	$\frac{11}{9}$
●	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	$\frac{10}{10}$	$\frac{11}{10}$
●	$\frac{1}{11}$	●	$\frac{3}{11}$	●	$\frac{5}{11}$	$\frac{6}{11}$	$\frac{7}{11}$	$\frac{8}{11}$	$\frac{9}{11}$	$\frac{10}{11}$	$\frac{11}{11}$
●	$\frac{1}{12}$	$\frac{2}{12}$	●	$\frac{4}{12}$	$\frac{5}{12}$	$\frac{6}{12}$	$\frac{7}{12}$	●	$\frac{9}{12}$	$\frac{10}{12}$	$\frac{11}{12}$

MIXED NUMBER TIC TAC TOE

Players 2 - 4

Materials MIXED NUMBER TIC TAC TOE mat, fraction
dice, and markers.

Played just like FRACTION TIC TAC TOE, only using
the Mixed Number mat.

Variation

One student holds the FRACTION TIC TAC TOE mat, the
other player holds the MIXED NUMBER TIC TAC TOE mat.
Pick one of the columns on the fraction mat, and
name all the fractions using lowest terms and mixed
number notation. Your classmate will tell you if
you have given the correct fraction or mixed number.
Take turns until all the rows on the FRACTION TIC TAC
TOE mat have been read and checked.

0	1	●	3	4	5	6	7	●	9	10	11
0	$\frac{1}{2}$	1	$1\frac{1}{2}$	●	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	$5\frac{1}{2}$
0	●	$\frac{2}{3}$	1	$1\frac{1}{3}$	$1\frac{2}{3}$	●	$2\frac{1}{3}$	$2\frac{2}{3}$	3	$3\frac{1}{3}$	$3\frac{2}{3}$
0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	●	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{3}{4}$
0	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	1	$1\frac{1}{5}$	$1\frac{2}{5}$	$1\frac{3}{5}$	$1\frac{4}{5}$	●	$2\frac{1}{5}$
0	$\frac{1}{6}$	●	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1	$1\frac{1}{6}$	$1\frac{1}{3}$	$1\frac{1}{2}$	$1\frac{2}{3}$	$1\frac{5}{6}$
0	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{3}{7}$	$\frac{4}{7}$	$\frac{5}{7}$	$\frac{6}{7}$	1	$1\frac{1}{7}$	$1\frac{2}{7}$	$1\frac{3}{7}$	$1\frac{4}{7}$
0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{3}{8}$
0	$\frac{1}{9}$	$\frac{2}{9}$	●	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{2}{3}$	$\frac{7}{9}$	$\frac{8}{9}$	1	$1\frac{1}{9}$	$1\frac{2}{9}$
0	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{3}{5}$	●	$\frac{4}{5}$	$\frac{9}{10}$	1	$1\frac{1}{10}$
0	$\frac{1}{11}$	$\frac{2}{11}$	$\frac{3}{11}$	$\frac{4}{11}$	$\frac{5}{11}$	$\frac{6}{11}$	$\frac{7}{11}$	$\frac{8}{11}$	$\frac{9}{11}$	$\frac{10}{11}$	1
0	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{4}$	●	$\frac{5}{12}$	$\frac{1}{2}$	$\frac{7}{12}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{11}{12}$

For information on the construction of dodecahedron
fraction dice see Dice in the Games and Puzzles
section in Lab Materials.

TYPE: Game

SOURCE: Fraction Bars, Card Set II

Permission to use granted
by Scott Resources, Inc.

TENTHS RUMMY

(For information on the construction of Rummy Cards see the section on Lab Materials.)

Needed: A deck of 10ths Rummy Cards and 2 to 4 players.

- Rules:
1. Each player is dealt seven (7) cards. The remaining cards are placed face down to form a stack and the top card of the stack is turned up to start a discard pile.
 2. Each turn a player draws one card from the stack or the top card from the discard pile. (If a player has two cards equivalent to the top card of the discard pile, he may pick up the entire pile.) A player must discard each turn.
 3. After drawing, a player may lay down any of these:
 - a. 3 or more equivalent cards.
 - b. 3 or more cards in consecutive order in tenths.
 - c. 1 or more cards that fit in with any cards already laid down by the player or an opponent.
 4. If all the stack gets used up before any player lays down all his cards, you should shuffle the discard pile and use it for a new stack.
 5. A hand is over when a player has laid down all his cards.
 6. Score 1 point for each card laid down, and 3 points if you are the first to lay down all your cards. Subtract 1 point for each card left in your hand.
 7. First player to get 30 points is the winner of the game.

Variations:

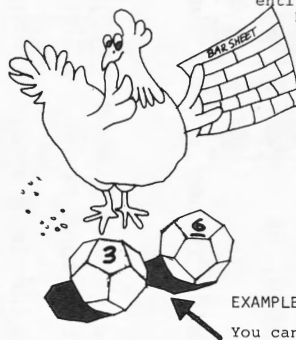
1. Play twelfths rummy with the twelfths deck. See Lab Materials section for construction information.
2. Play War with either deck.
3. Play In-Between
 - a. All cards are placed face down in the middle.
 - b. A player draws 2 cards and turns them face up.
 - c. The player now draws a third card. If it is between the first two, he gets to keep all three cards. If not, the cards are set aside.
 - d. Player with the most cards is the winner.



FRACTION BAR SHEET DICE GAME

(For pattern to construct a dodecahedral dice see the section on Lab Materials.)

- YOU NEED:
- A) 2 or more players.
 - B) 2 Dodecahedron dice each numbered 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 16.
 - C) A fraction bar sheet for each player. See page entitled *Fraction Bar Sheet*.
 - D) Some markers to cover the sheet.



HERE ARE THE RULES:

- ① Each player rolls the dice, small number is the numerator (top number), smallest fraction goes first.
- ② Doubles are rolled again.
- ③ Each player rolls the dice, names his fraction, and covers it or an equivalent fraction on the Fraction Bar Sheet.

EXAMPLE:

You can cover 1 of the halves or 2 of the fourths, or 3 of the sixths, or several other fractions. Can you name them?

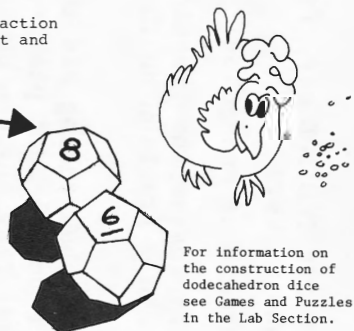
- ④ A player must use all of the fraction that is rolled. It may be split and used on more than one bar.

EXAMPLE:

You could cover 1 half and 1 fourth, or maybe .6 twelfths and 2 eighths.

⑤

The first player to get 3 of the bars covered is the winner.



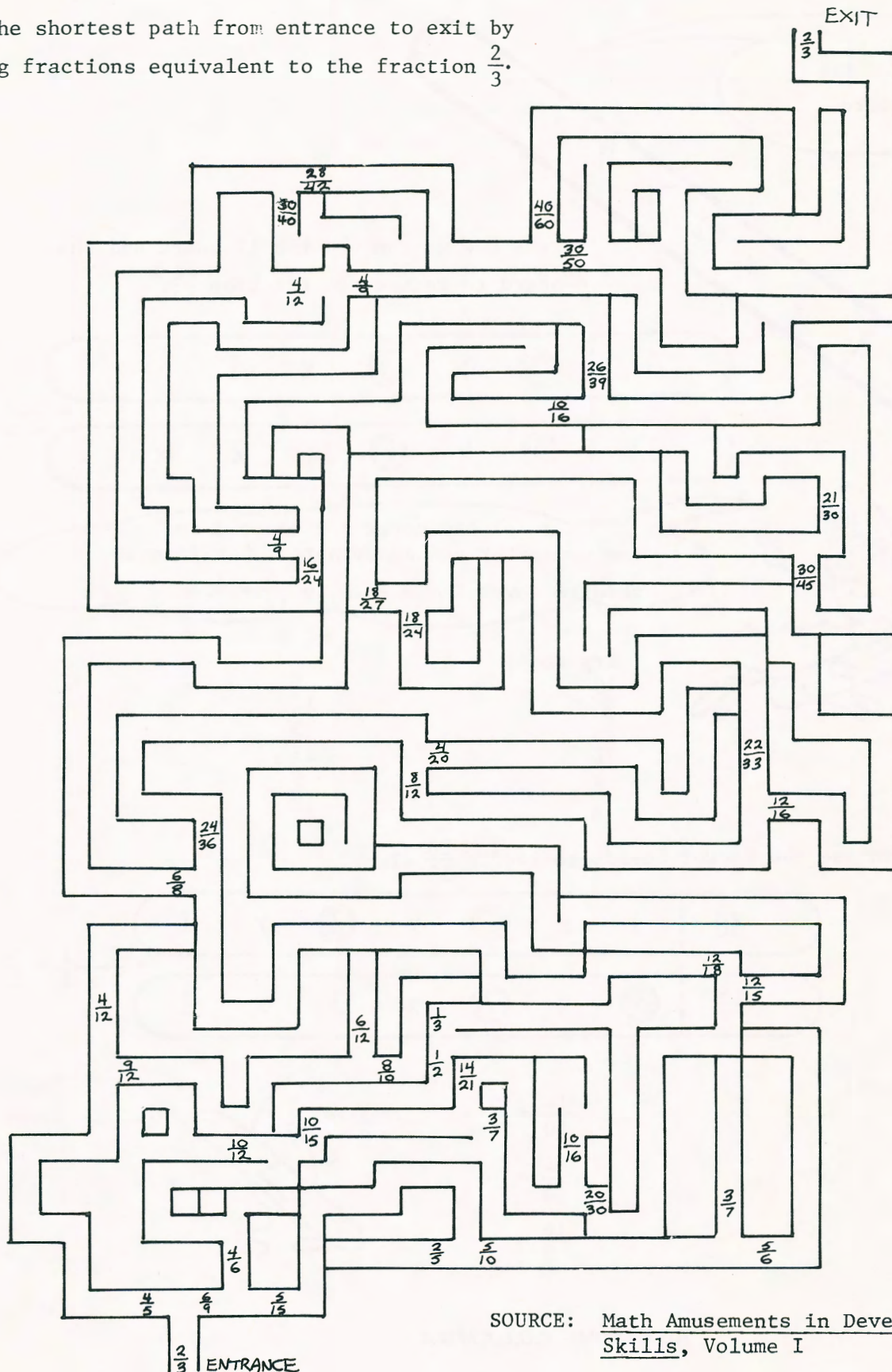
For information on the construction of dodecahedron dice see Games and Puzzles in the Lab Section.

EQUIVALENT FRACTIONS

Equivalent
Concepts
FRACTIONS



Find the shortest path from entrance to exit by finding fractions equivalent to the fraction $\frac{2}{3}$.



SOURCE: Math Amusements in Developing Skills, Volume I

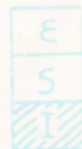
TYPE: Paper & Pencil/Puzzle

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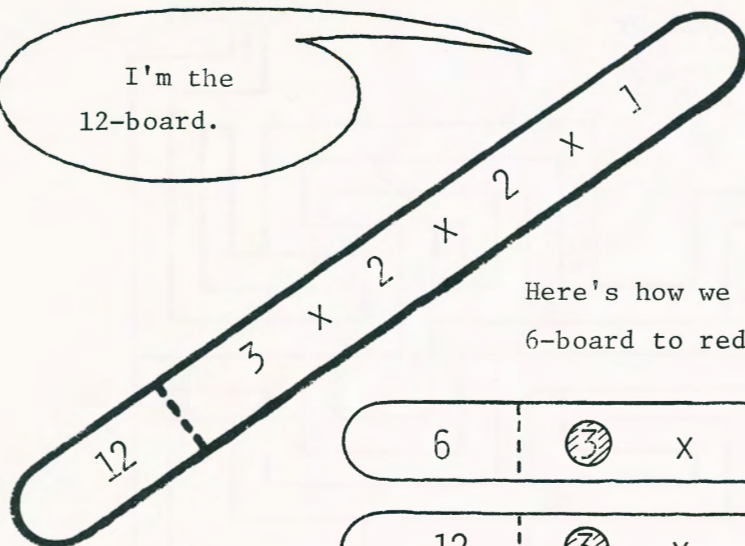


FACTOR BOARDS - I

Reducing
Concepts
FRACTIONS

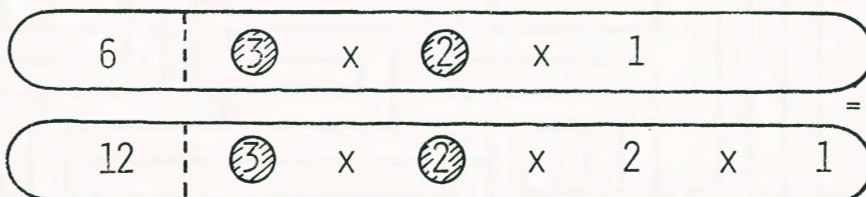


I'm the
12-board.



(For information on the
construction of Factor
Boards see the section
on Lab Materials.)

Here's how we can use the 12-board and the
6-board to reduce the fraction $\frac{6}{12}$.



I can cover a 2 and a 3 in
the numerator and denominator, but I never
need to cover the ones. So $\frac{6}{12} = \frac{1}{2}$.

Try some:

$$\frac{10}{20} =$$

$$\frac{6}{16} =$$

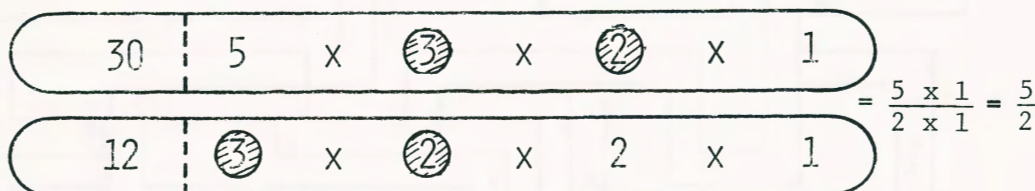
$$\frac{6}{8} =$$

$$\frac{8}{12} =$$

$$\frac{5}{15} =$$

$$\frac{9}{12} =$$

We can use the factor boards to reduce $\frac{30}{12}$ also.



Try these:

$$\frac{12}{9} =$$

$$\frac{6}{16} =$$

$$\frac{20}{5} =$$

$$\frac{14}{10} =$$

$$\frac{4}{3} =$$

$$\frac{16}{9} =$$



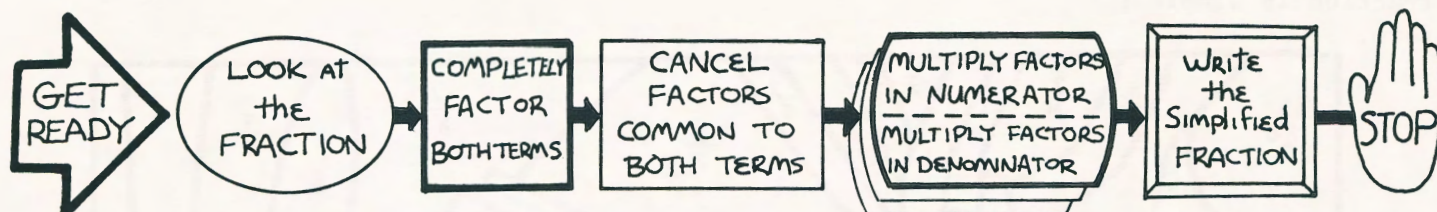
See me again for
multiplication
and division of
fractions.

TYPE: Manipulative

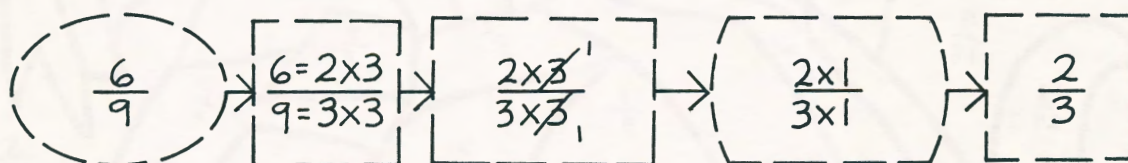
IDEA FROM: C.O.L.A.M.D.A.



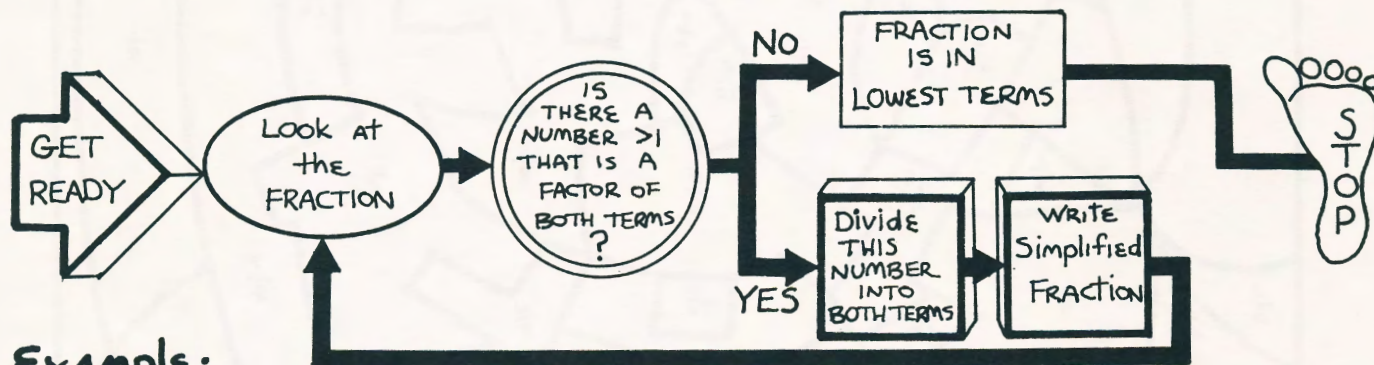
REDUCING FLOW CHARTS



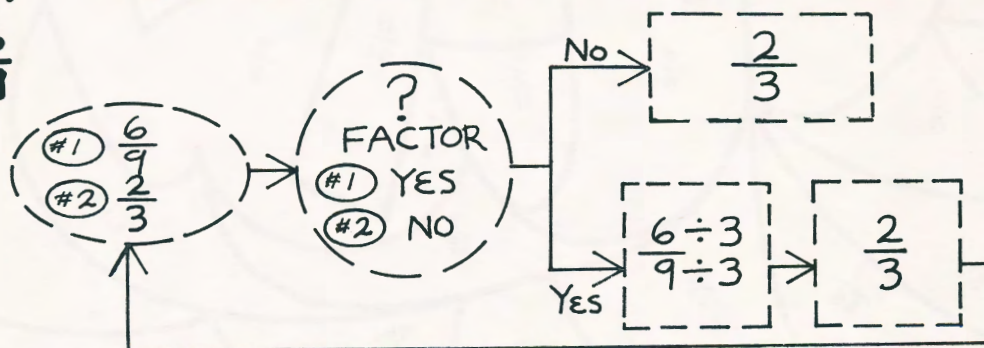
Example:
Reduce $\frac{6}{9}$



Students may not be familiar with all the vocabulary used here.



Example:
Reduce $\frac{6}{9}$



TYPE: Bulletin Board/Transparency



ACTIVITY CARDS - COLOR CUBES - III

For information on Color Cubes see
the section on Lab Materials.

Improper - Mixed
Concepts
FRACTIONS



CC-III-1

You will need some cubes (any color.)



= 1



= 1

Use eight cubes to cover
one whole completely.
What fraction is:

1 cube? $\frac{1}{8}$	5 cubes? _____
2 cubes? _____	6 cubes? _____
3 cubes? _____	7 cubes? _____
4 cubes? _____	8 cubes? _____

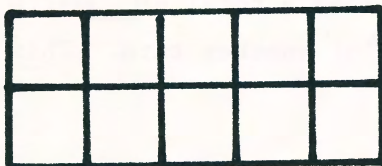
Using 9 cubes, you can
cover one rectangle
completely and place the
remaining cube on the
second rectangle.

Then 9 cubes = $\frac{9}{8}$ or $1\frac{1}{8}$

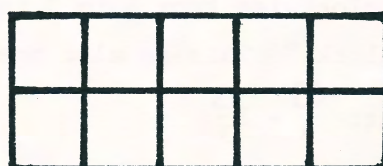
What about:

10 cubes? _____	13 cubes? _____
11 cubes? _____	16 cubes? _____

CC-III-2



= 1



= 1

Now 10 cubes are 1 whole.

Write as many fractions as you can for:

2 cubes: $\frac{2}{10}, \frac{1}{5}$	17 cubes: _____
5 cubes: _____	8 cubes: _____
12 cubes: _____	1 cube: _____
15 cubes: _____	10 cubes: _____

Students will need
grids to place their
cubes on.

TYPE: Manipulative



ACTIVITY CARDS - CLOCK FRACTIONS - I

For information on the construction and use of Clock Fractions, see the section on Lab Materials.

Improper - Mixed
Concepts
FRACTIONS



USING "CLOCK" A, ANSWER THE FOLLOWING QUESTIONS.

1. Rotating clockwise from 3 to 1 rotates the arrow across part of the "clock" or through $\frac{\square}{3}$ of the "clock."
2. Rotating clockwise from 3 to 2 rotates the arrow across parts of the "clock" or through $\frac{\triangle}{3}$ of the "clock."

3. Rotating from 3 to 3 rotates the arrow across parts of the "clock" or through $\frac{\bigcirc}{3}$ of the "clock." Did the arrow make one complete revolution?

We might say then, that $\frac{3}{3} = 1$.

4. Rotate the arrow from 3 to 3 to 3 to 3. Did you cross a total of 9 parts? Did you make 3 complete revolutions? Does the statement $\frac{9}{3} = 3$ seem logical?

CL-I-1

CL-I-2

STILL USING "CLOCK" A, CONSIDER THE FOLLOWING:

1. Rotating clockwise from 3 to 3 to 1 moves the arrow across a total of parts of the "clock." This is also one complete turn, and $\frac{\triangle}{3}$ of another turn. This shows that: $\frac{\square}{3} = 1\frac{\triangle}{3}$
2. Rotating from 3 to 3 to 3 to 2 moves the arrow across a total of parts. This is also complete turns and $\frac{\triangle}{3}$ of another turn.

CONCLUSION: $\frac{8}{3} = \text{hexagon} \frac{\triangle}{\bigcirc}$

3. Using "clock" C, give two ways of expressing the following: rotating the arrow from 6 to 6 to 6 to 6 to 5.

THEREFORE: $\frac{\triangle}{\square} = \underline{\hspace{2cm}}$

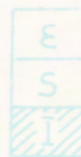
This activity is probably best with teacher direction.

TYPE: Manipulative

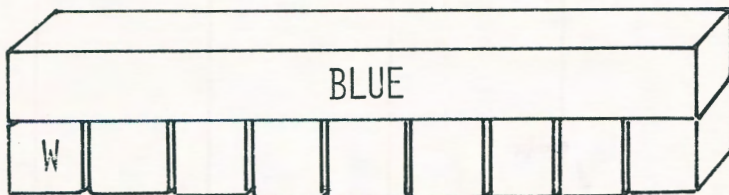


ACTIVITY CARDS - CUISENAIRE RODS - II

Improper - Mixed
Concepts
FRACTIONS



For readiness activities for Cuisenaire Rods
see the section on Lab Materials.



WHEN THE BLUE ROD IS 1 UNIT
A WHITE ROD IS $\frac{1}{9}$ OF THE UNIT.

GIVE FRACTIONS FOR THE OTHER RODS.

W $\frac{1}{9}$ R _____ G _____ K _____ D _____ P _____

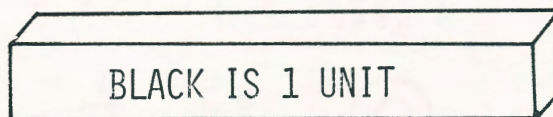
N _____ E $\frac{9}{9}$ OR 1 O _____ OR _____ Y _____

The name Cuisenaire and the color sequence of the rods are trademarks of the Cuisenaire Company of America, Inc.

CR-II-1

CR-II-2

GIVE FRACTIONS FOR THE OTHER RODS.



W _____ R _____ G _____ P _____ Y _____

D _____ OR _____ K _____ OR _____ N _____ OR _____ E _____ OR _____

O _____ OR _____

GIVE FRACTIONS FOR THE OTHER RODS IF YOU KNOW THAT THE WHITE ROD IS $\frac{1}{5}$.

W _____ R _____ G _____ P _____ Y _____

D _____ OR _____ K _____ OR _____ N _____ OR _____

E _____ OR _____ O _____ OR _____

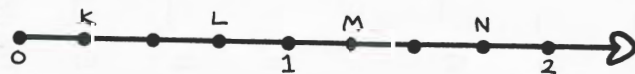
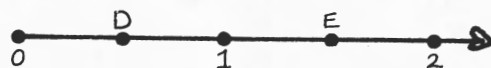
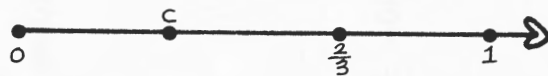
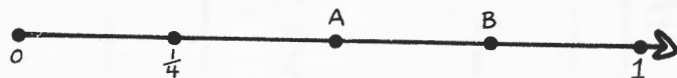
The name Cuisenaire and the color sequence of the rods are trademarks of the Cuisenaire Company of America, Inc.

Name _____
Point At _____
Line Up

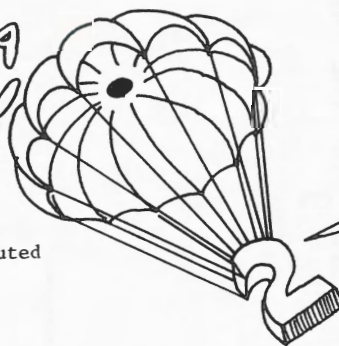
LINE UP

HOW GOOD A DETECTIVE ARE YOU?

SEVERAL LINES ARE DRAWN FOR YOU BELOW. LOOK AT HOW EACH ONE IS MARKED, THEN WRITE THE FRACTIONAL NAME FOR EACH POINT LABELED WITH A LETTER.



FIND A POINT!



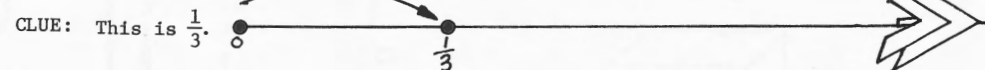
HELP!
WHERE AM I?

Six jet airplanes flew over an uncharted jungle and each had engine trouble. Six passenger numbers parachuted to safety.

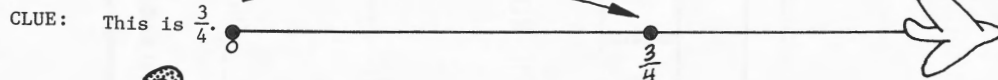
WHERE IS 1?



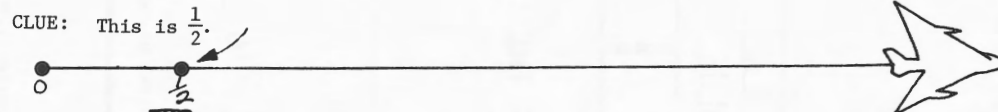
WHERE IS 1?



WHERE IS 1?



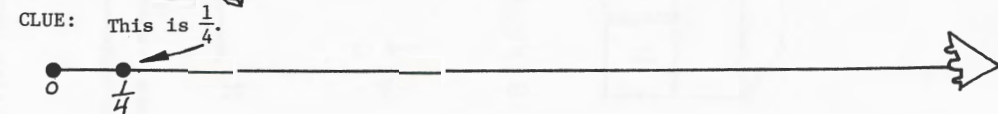
WHERE IS 2?



WHERE IS 3?



WHERE IS 2 1/4?



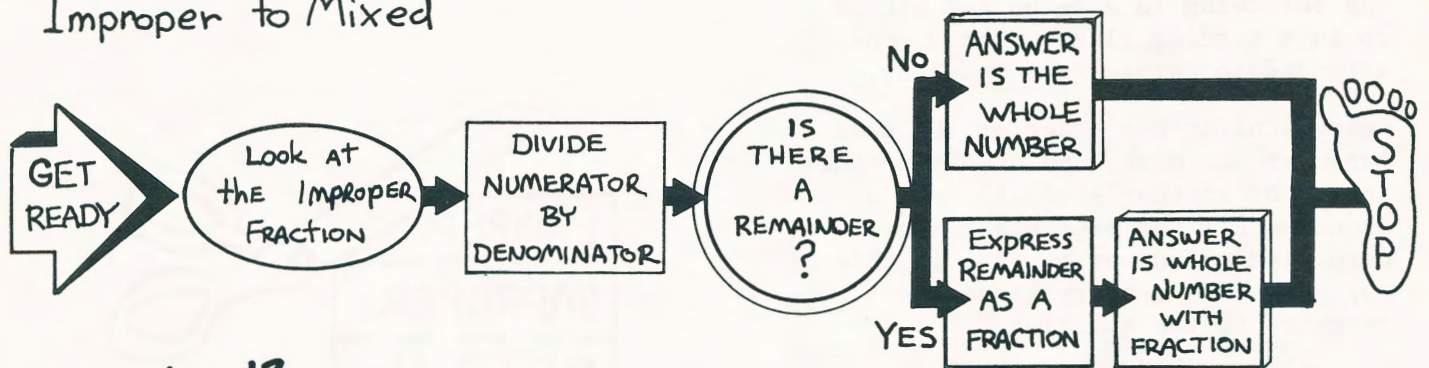


RENAMING FLOWCHARTS

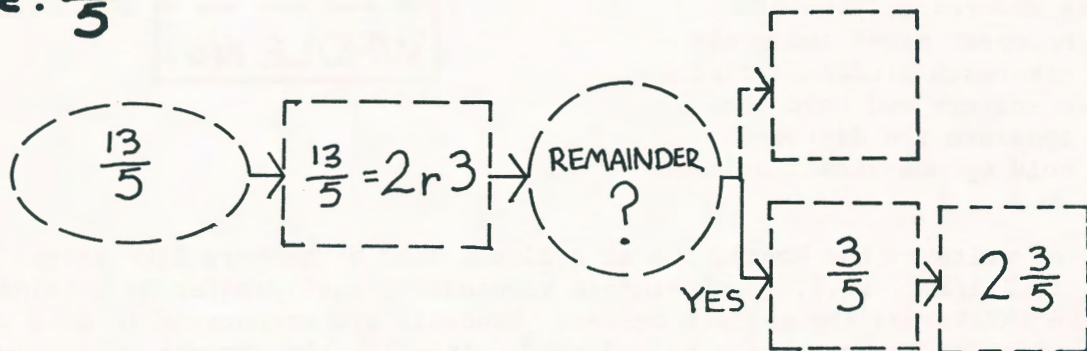
Improper - Mixed
Concepts
FRACTIONS



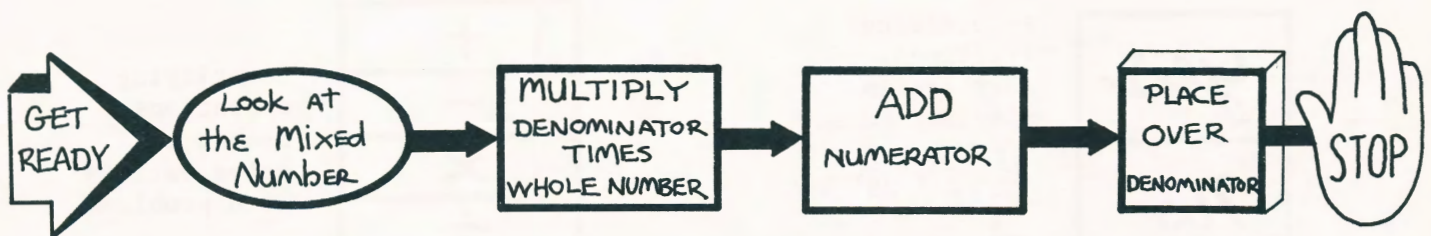
Improper to Mixed



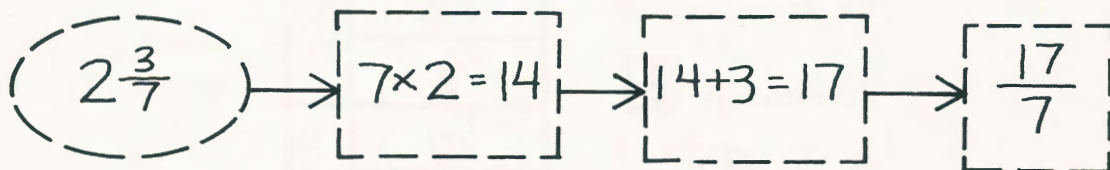
Example: $\frac{13}{5}$



Mixed to Improper



Example: $2\frac{3}{7}$



TYPE: Bulletin Board/Transparency



ESR - EVERY STUDENT RESPONDS

TEACHER IDEAS

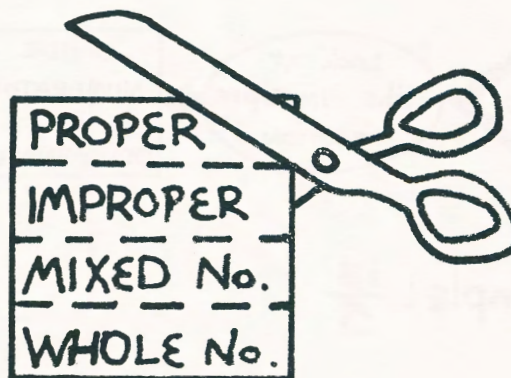
Improper - Mixed
Concepts
FRACTIONS



The following is a technique picked up in a reading class which can be extended to mathematics teaching.

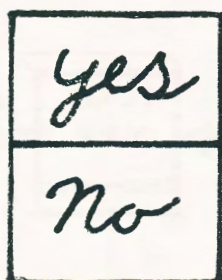
Every student has a set of the same cards at his desk, and as the teacher (aide, or another student) asks a question all the students raise a card with an answer on it. In this way you can see every student's response to the question.

Example: To reinforce fraction vocabulary and recognition of proper, improper, mixed and whole numbers, give each student a ditto with these answers and have the students separate the ditto, so they can hold up one answer at a time.

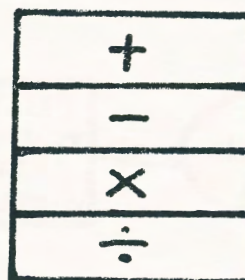


Call out, or write on the board, one at a time a list of numbers like these ($1/2$, $3/2$, $2/4$, 1 , $3 \frac{1}{2}$, . . .). Each student responds to each problem by holding up the card he thinks has the correct answer. Students are encouraged to make a decision without looking at anyone else's raised card. Identify the correct answer (or answers) after each problem. This technique can be useful as a review of material, as a means of evaluating learning, and as a diagnostic tool. Students enjoy this activity because of its novelty. However, if overused, the novelty could wear off.

It can be extended to:



recognizing fractions that are in simplest form.
($\frac{1}{2}$, $\frac{3}{21}$, $\frac{7}{28}$, $\frac{9}{16}$, $\frac{3}{51}$, . . .)

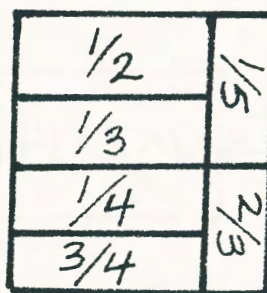


identifying operations needed to solve various word problems.

identifying names for fractions.

(two-thirds, ,

$1 \div 3$, , . . .)



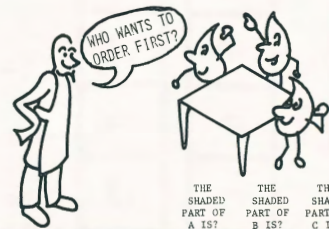
TYPE: Activity

This technique can be used for warm up, review or diagnosis.



A	B	THE SHADED PART OF A IS?	<, =, >	THE SHADED PART OF B IS?
		$\frac{1}{2}$	<	$\frac{3}{4}$
		$\frac{12}{16}$		
		$\frac{4}{5}$		$\frac{1}{5}$

A	B	THE SHADED PART OF A IS?	<, =, >	THE SHADED PART OF B IS?



THE SHADED PART OF A IS?	THE SHADED PART OF B IS?	THE SHADED PART OF C IS?	
			$\frac{2}{3}$ $\frac{1}{2}$ $\frac{1}{2} < \frac{2}{3} <$
			$\frac{1}{8}$ $\frac{1}{8} > \frac{1}{8}$
			$< <$
			$> >$
			BE CAREFUL
			$< <$

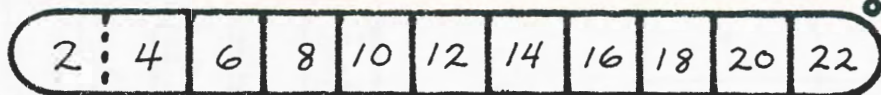


MULTIPLE BOARDS - I

Ordering
Concepts
FRACTIONS



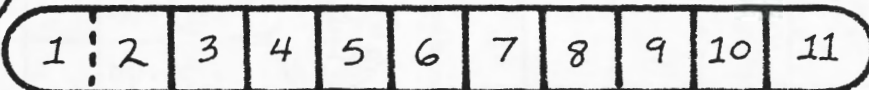
For information on the construction of Multiple Boards see the section on Lab Materials.



I'm the 2's board.

I'm the 1's board.

Use the 2's board and the 1's board to find equivalent fractions for $\frac{1}{2}$.



I'm the 2's board.



$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} = \frac{7}{14} = \frac{8}{16} = \frac{9}{18} = \frac{10}{20} = \frac{11}{22}$$

Try these:

I need to use the 1's board and the 3's board.

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \frac{6}{18} = \frac{7}{21} = \frac{8}{24} = \frac{9}{27} = \frac{10}{30} = \frac{11}{33} = \frac{12}{36}$$

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15} = \frac{12}{18} = \frac{14}{21} = \frac{16}{24} = \frac{18}{27} = \frac{20}{30} = \frac{22}{33} = \frac{24}{36}$$

$$\frac{4}{5} = \frac{8}{10} = \frac{12}{15} = \frac{16}{20} = \frac{20}{25} = \frac{24}{30} = \frac{28}{35} = \frac{32}{40} = \frac{36}{45} = \frac{40}{50} = \frac{44}{55} = \frac{48}{60}$$

$$\frac{3}{7} = \frac{6}{14} = \frac{9}{21} = \frac{12}{28} = \frac{15}{35} = \frac{18}{42} = \frac{21}{49} = \frac{24}{56} = \frac{27}{63} = \frac{30}{70} = \frac{33}{77} = \frac{36}{84}$$

$$\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20} = \frac{6}{24} = \frac{7}{28} = \frac{8}{32} = \frac{9}{36} = \frac{10}{40} = \frac{11}{44} = \frac{12}{48}$$

$$3\frac{1}{2} = \frac{7}{2} = \frac{14}{4} = \frac{21}{6} = \frac{28}{8} = \frac{35}{10} = \frac{42}{12} = \frac{49}{14} = \frac{56}{16} = \frac{63}{18} = \frac{70}{20} = \frac{77}{22}$$

$$4\frac{1}{3} = \frac{13}{3} = \frac{26}{6} = \frac{39}{9} = \frac{52}{12} = \frac{65}{15} = \frac{78}{18} = \frac{91}{21} = \frac{104}{24} = \frac{117}{27} = \frac{130}{30} = \frac{143}{33}$$

$$1\frac{1}{4} = \frac{5}{4} = \frac{10}{8} = \frac{15}{12} = \frac{20}{16} = \frac{25}{20} = \frac{30}{24} = \frac{35}{28} = \frac{40}{32} = \frac{45}{36} = \frac{50}{40} = \frac{55}{44}$$

$3\frac{1}{2} = \frac{7}{2}$. Use the 7's board and the 2's board.

TYPE: Manipulative

IDEA FROM: C.O.L.A.M.D.A.

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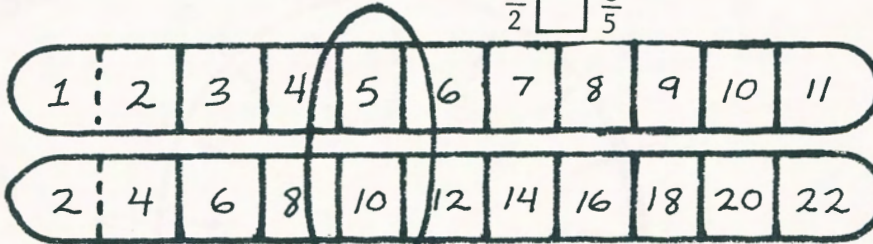
MULTIPLE BOARDS - I - CONTINUED

We can use the multiple boards to help us compare fractions.

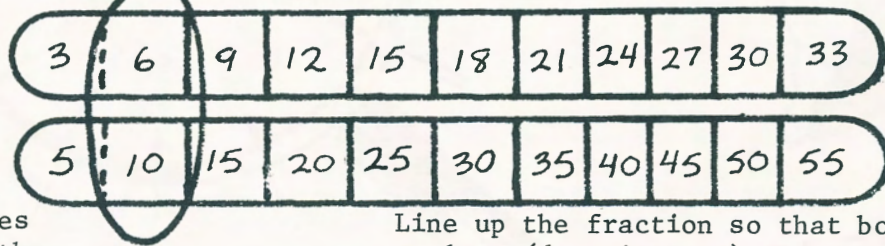
Compare $\frac{1}{2}$ and $\frac{3}{5}$. Is $\frac{1}{2}$ less than, equal to, or greater than $\frac{3}{5}$? (<, =, or >)

$$\frac{1}{2} \square \frac{3}{5}$$

Make $\frac{1}{2}$



Make $\frac{3}{5}$



There may be several places to match up the boards.

Line up the fraction so that bottom numbers (denominators) are the same.

Let's try another.

$$\frac{1}{3} \square \frac{2}{5}$$

$$\frac{1}{2} = \frac{5}{10}$$

$$\frac{3}{5} = \frac{6}{10}$$

$$\frac{1}{2}$$

<

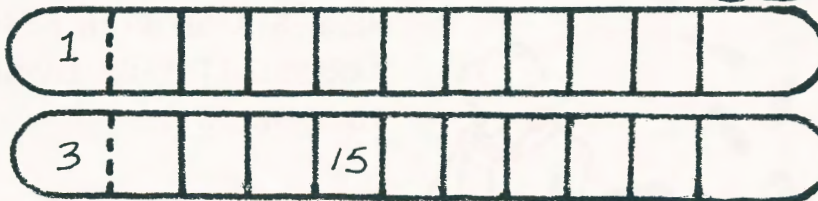
$$\frac{3}{5}$$

...

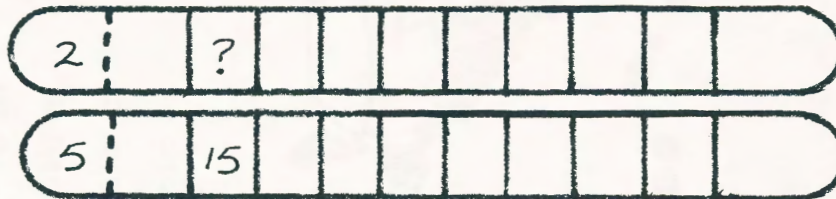
because

$$\frac{5}{10} < \frac{6}{10}$$

make $\frac{1}{3}$



make $\frac{2}{5}$



$$\frac{1}{3} = \frac{5}{15}$$

$$\frac{2}{5} = \frac{6}{15}$$

$$\frac{1}{3} \square \frac{2}{5}$$

because $\frac{5}{15}$ is less than $\frac{6}{15}$!

Try these: <, =, or >

$$\begin{array}{ll} 3/4 & 2/5 \\ 2/11 & 1/7 \\ 2/7 & 3/8 \end{array}$$

IDEA FROM: C.O.L.A.M.D.A.

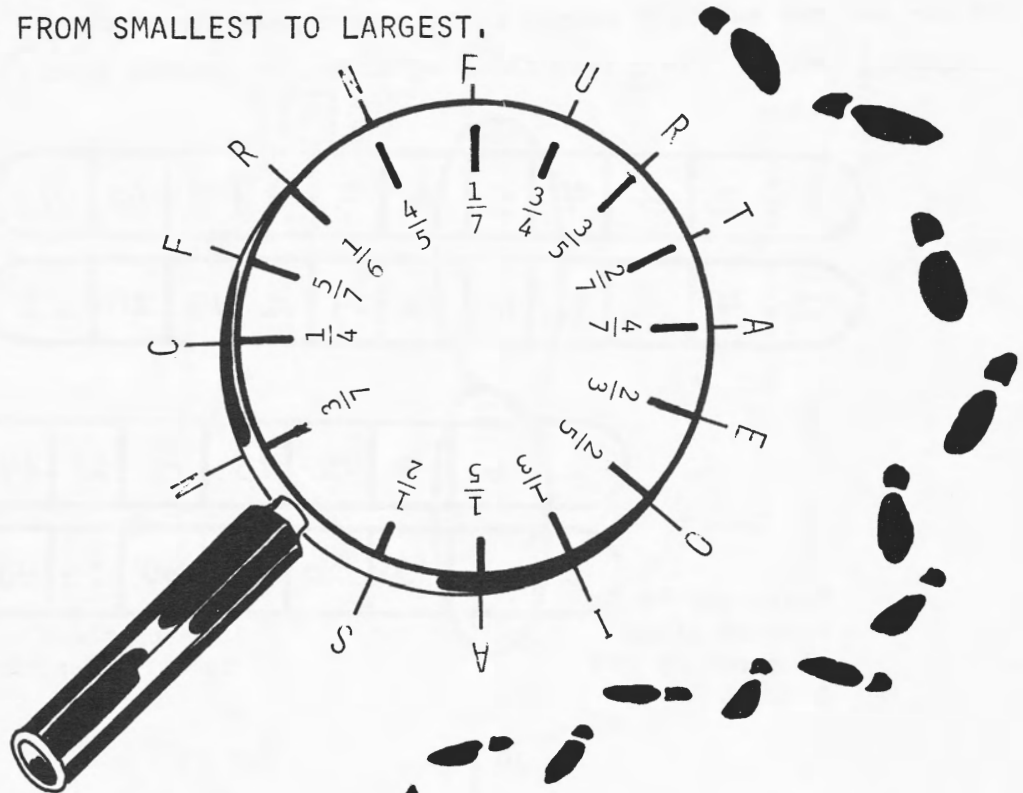
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WHAT'S THE MESSAGE?

Ordering
Concepts
FRACTIONS



DECODE THE MESSAGE BY ARRANGING THE FRACTIONS FROM SMALLEST TO LARGEST.



HERE'S THE
FIRST LETTER

F R A C T I O N S A R E F U N

These are very challenging in fractional form. This would be a good place to use a calculator or suggest the use of decimals. See *Calculated Codes* in DECIMALS.



HERE'S A SHORTER MESSAGE. ARRANGE IT FROM LARGEST TO SMALLEST.

Z	O	H	E	W	S
$\frac{8}{9}$	$\frac{4}{5}$	$\frac{5}{6}$	$\frac{9}{10}$	$\frac{7}{8}$	$\frac{11}{12}$

S E Z W B O

TYPE: Paper & Pencil/Puzzle

FRACTION DICE GAME

Ordering
Concepts
FRACTIONS



Players: 2 - 4

Materials: Dodecahedron fraction dice

Game 1

Kathy and Al took turns rolling the fraction dice until one of them got 5 fractions which were less than or equal to $\frac{1}{2}$. They recorded their fractions.



Kathy won.

Kathy $\frac{4}{7}$ $\frac{4}{1}$ $\frac{3}{3}$ $\frac{8}{7}$ $\frac{2}{1}$ $\left(\frac{2}{11}\right)$ $\frac{11}{8}$ $\frac{2}{3}$ $\frac{5}{6}$ $\left(\frac{1}{10}\right)$ $\frac{11}{5}$ $\frac{3}{5}$ $\frac{11}{4}$ $\left(\frac{4}{12}\right)$ $\frac{10}{8}$ $\left(\frac{1}{3}\right)$ $\left(\frac{1}{8}\right)$

Al $\left(\frac{2}{9}\right)$ $\left(\frac{5}{12}\right)$ $\frac{5}{3}$ $\frac{6}{2}$ $\frac{5}{7}$ $\frac{9}{9}$ $\frac{11}{8}$ $\frac{3}{4}$ $\left(\frac{5}{12}\right)$ $\frac{8}{5}$ $\frac{10}{1}$ $\left(\frac{4}{8}\right)$ $\frac{11}{1}$ $\frac{10}{8}$ $\frac{11}{9}$ $\frac{6}{9}$ $\frac{3}{3}$

Play this game with some of your classmates.

Game 2 Take turns rolling the fraction dice until someone gets 5 fractions which are greater than 1.

Game 3 Take turns rolling the fraction dice until someone gets 5 fractions which are greater than 1 but less than 2.

Game 4 Take turns rolling the fraction dice until someone gets 5 fractions in lowest terms.

Game 5 Take turns rolling the fraction dice until someone gets 5 fractions with even numerators and odd denominators.

BONUS TURNS

(for Games 1-5) When a player tosses one of the fractions he is after, he continues his turn by tossing again. When he tosses three such fractions on the same turn, he loses all three.

TYPE: Game

SOURCE: Fraction Bars, Card Set II

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For information on the construction of dodecahedon fraction dice see *Dice* in the Games and Puzzles section in Lab Materials.



INAROW

THIS GAME IS CALLED INAROW.
I NEED 2 OR MORE PLAYERS, AND
A DECK OF 12THS RUMMY CARDS.

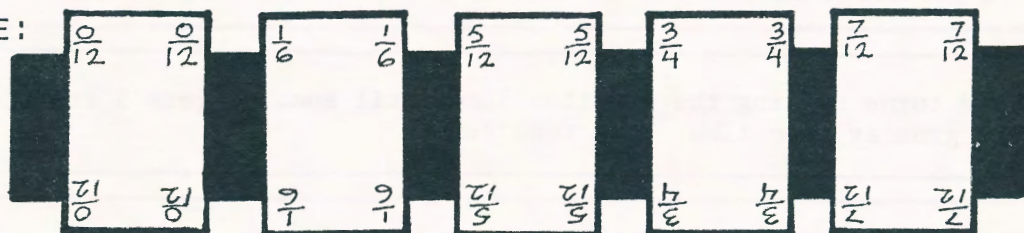
RULES:

- A) Dealer deals 5 cards face up to each player. Cards are placed in a row and may not be switched around.
- B) Cards not dealt are placed face down to form a stack.
- C) Each player is trying to get his cards ordered, smallest on the left to largest on the right.
- D) On his turn, a player draws a card from the stack and
 - 1) replaces any one of the cards in his row or
 - 2) discards the card because he can't use it.
- E) The first player to get his cards in a row wins.

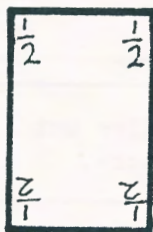


For information
on the
construction of
12ths rummy
cards see the
section on Lab
Materials.

EXAMPLE:



PLAYER DRAWS



AND REPLACES THE $\frac{3}{4}$ CARD TO
GET $\frac{0}{12}, \frac{1}{6}, \frac{5}{12}, \frac{1}{2}, \frac{7}{12}$ IN A
ROW AND WINS.

VARIATION:

AS A READINESS ACTIVITY FOR THE METRIC SYSTEM, THE GAME COULD
BE PLAYED WITH 10THS RUMMY CARDS.

TYPE: Game

IDEA FROM: *Fraction Bars*, Introductory Card Set

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More or Less Game

Math
Concepts
FRACTIONS



EQUIPMENT: Game board, markers,
2 dice

one marked:
two = faces,
two > (greater)
faces,
two < (less)
faces

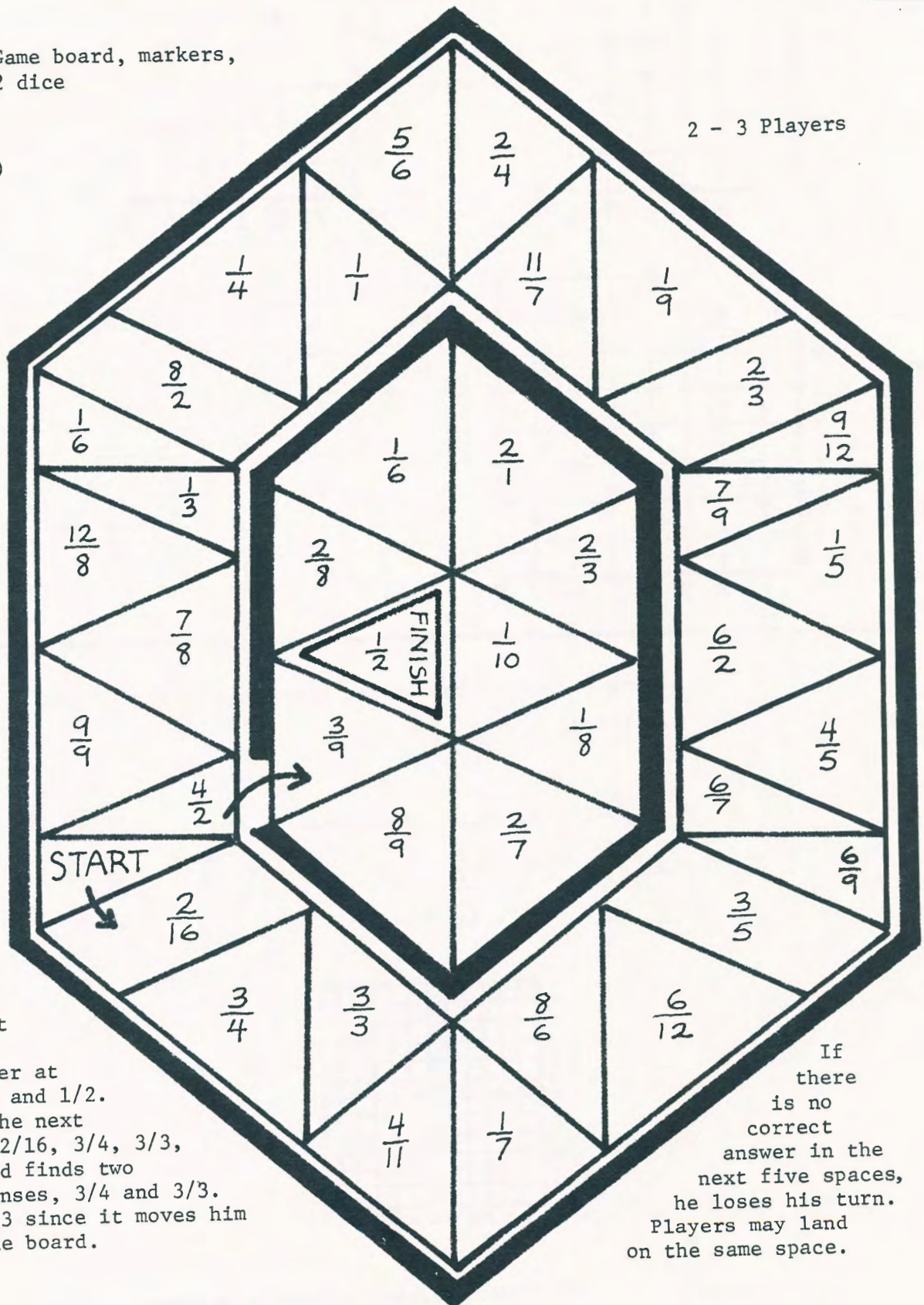


one marked:
 $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{3}$,
 $\frac{1}{2}$, $\frac{2}{3}$, 1



RULES: Roll
fraction die.
Player with
smallest frac-
tion goes
first.
All players
begin on
start.
First player
rolls both
dice. He can
place his
marker on
any one of
the next
five spaces
with a correct
answer. For
example, player at
start rolls > and $\frac{1}{2}$.
He looks at the next
five spaces ($\frac{2}{16}$, $\frac{3}{4}$, $\frac{3}{3}$,
 $\frac{4}{11}$, $\frac{1}{7}$) and finds two
correct responses, $\frac{3}{4}$ and $\frac{3}{3}$.
He chooses $\frac{3}{3}$ since it moves him
further on the board.

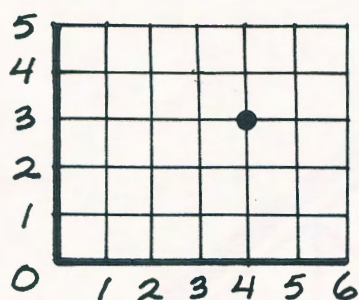
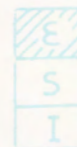
2 - 3 Players



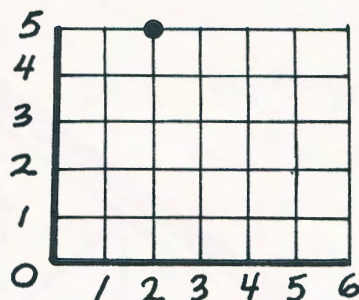
If
there
is no
correct
answer in the
next five spaces,
he loses his turn.
Players may land
on the same space.

ACTIVITY CARDS - GRAPHING FRACTIONS

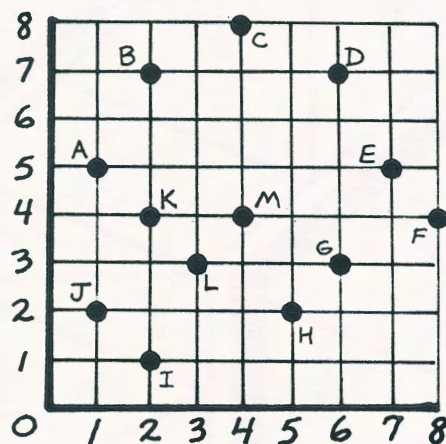
Ordering
Equivalent
Concepts
FRACTIONS



This dot represents $\frac{3}{4}$.



This dot represents $\frac{5}{2}$.



A = _____

E = _____

I = _____

B = _____

F = _____

J = _____

C = _____

G = _____

K = _____

D = _____

H = _____

L = _____

M = _____

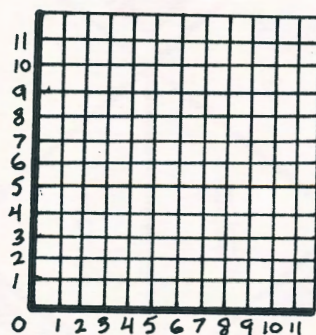
GF 1

GF 2

Using $\frac{1}{2}$ inch or centimeter graph paper, draw a grid as shown.

Graph the following fractions:

- | | | | | |
|------------------|-------------------|------------------|------------------|-------------------|
| a) $\frac{2}{3}$ | b) $\frac{6}{11}$ | c) $\frac{7}{3}$ | d) $\frac{8}{8}$ | e) $1\frac{2}{3}$ |
| f) $\frac{4}{6}$ | g) $\frac{3}{7}$ | h) $\frac{1}{4}$ | i) $\frac{2}{8}$ | j) $\frac{6}{9}$ |



TYPE: Paper & Pencil

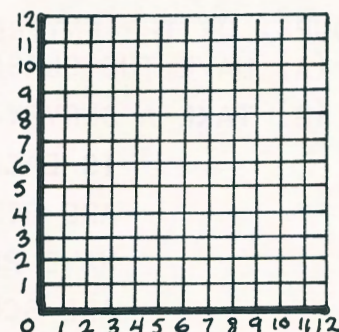
IDEA FROM: *The School Mathematics Project*, Book B

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ACTIVITY CARDS - GRAPHING FRACTIONS (CONTINUED)

Graph the following fractions:

- a) $\frac{1}{2}$ b) $\frac{2}{4}$ c) $\frac{3}{6}$
d) $\frac{4}{8}$ e) $\frac{5}{10}$ f) $\frac{6}{12}$



What do you notice about the graph of these fractions?

Look carefully at the fractions, are they all equivalent fractions?

On the graph the fractions should all have been on a straight line. Were they?

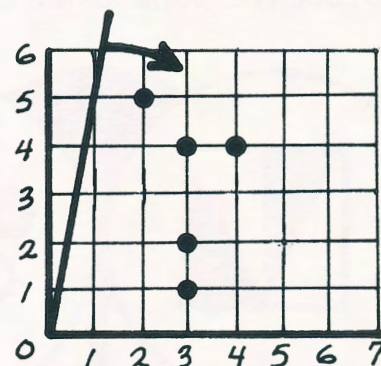
Use your graph paper to find 7 fractions equivalent to $\frac{2}{3}$.

GF 3

Here are graphs of the fractions: $\frac{1}{3}$, $\frac{2}{3}$, $\frac{4}{4}$, $\frac{4}{3}$, $\frac{5}{2}$,

Which is the smallest? Which is the largest?

A ruler can be used to order the fractions from largest to smallest. Can you see how?



GF 4

Use your graph paper and a rotating ruler to order the following sets of fractions from largest to smallest.

- 1) $\frac{1}{4}$, $\frac{1}{6}$, $\frac{2}{9}$, $\frac{2}{5}$, _____
2) $\frac{3}{5}$, $\frac{4}{7}$, $\frac{2}{3}$, $\frac{5}{8}$, _____
3) $\frac{12}{15}$, $\frac{7}{9}$, $\frac{3}{4}$, $\frac{5}{6}$, _____

MINIPROJECTS FOR FRACTIONS

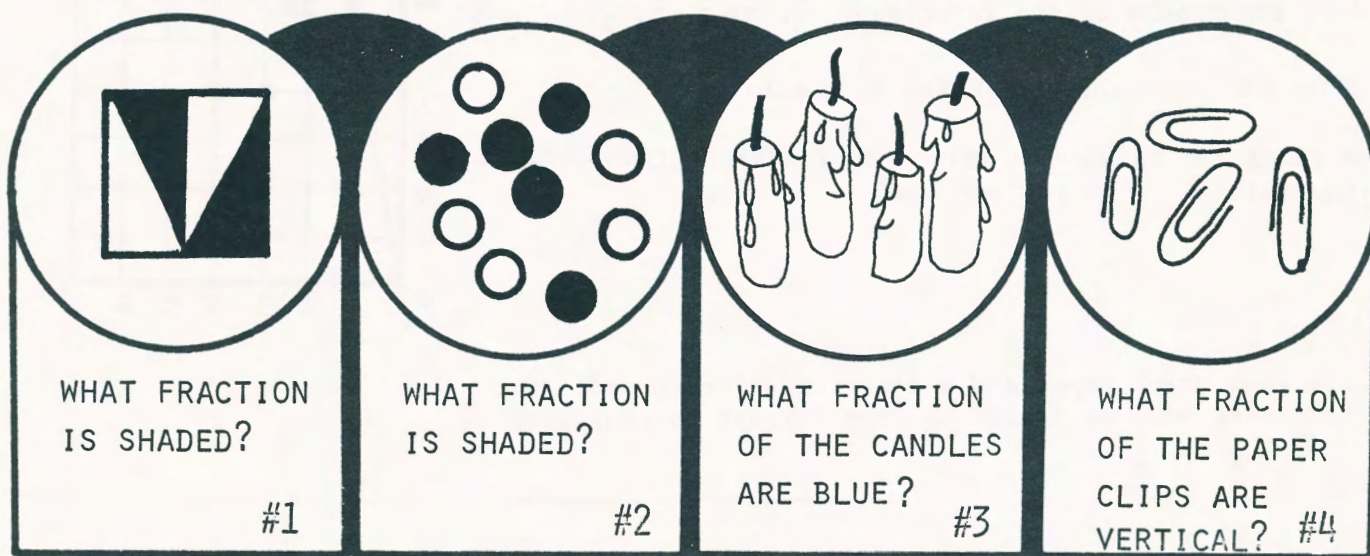
TEACHER IDEAS

Fractional Parts Concepts FRACTIONS



THE FOLLOWING ARE SAMPLES OF ACTIVITIES WHICH COULD BE COMPLETED IN ONE OR TWO DAYS. THEY COULD BE DONE BY INDIVIDUAL STUDENTS, SMALL GROUPS, OR THE ENTIRE CLASS.

1. (A) HAVE THE CLASS SEPARATE INTO HALVES, THIRDS, FOURTHS, ETC.
(B) TAKE A SURVEY. WHAT FRACTIONAL PART OF THE CLASS:
 - IS EACH STUDENT?
 - ARE GIRLS? BOYS?
 - HAVE PETS?
 - ARE SCORPIOS?
 - ETC.
2. FIND AND RECORD THINGS IN THE CLASSROOM THAT HAVE BEEN OR CAN BE DIVIDED INTO HALVES, THIRDS, FOURTHS, ETC.
3. (THIS ACTIVITY WOULD PROBABLY TAKE TWO DAYS TO COMPLETE.) ON THE FIRST DAY EACH STUDENT COULD MAKE A FRACTION PICTURE ON A PIECE OF CONSTRUCTION PAPER. EACH STUDENT WOULD MAKE AT LEAST ONE PICTURE. MAGAZINES, FELT PENS OR CRAYONS, EGG CARTONS, PAPER CLIPS, GLUE, AND A VARIETY OF OTHER MATERIALS COULD BE USED. IT MIGHT BE HELPFUL TO SHOW THE STUDENTS SOME EXAMPLES.



TEACHER NUMBERS THE PICTURES AFTER THEY ARE TURNED IN. THE FOLLOWING DAY EACH STUDENT NUMBERS ON A PIECE OF PAPER FROM 1-30 (OR HOWEVER MANY PICTURES YOU ARE USING). THEY EXCHANGE PICTURES WITH CLASSMATES, UNTIL ALL STUDENTS HAVE WRITTEN THE 30 ANSWERS ON HIS PAPER.

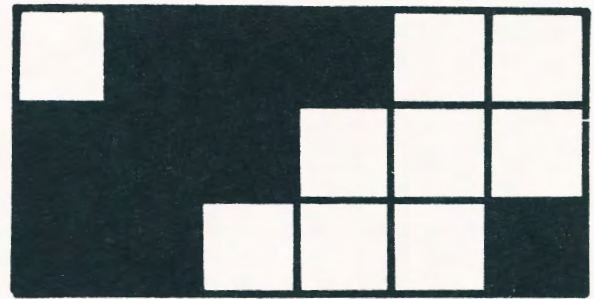
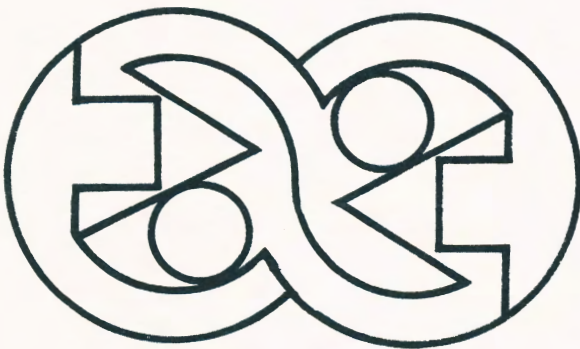
ART PROJECTS FOR FRACTIONS

TEACHER IDEAS

Fractional Parts
Concepts
FRACTIONS

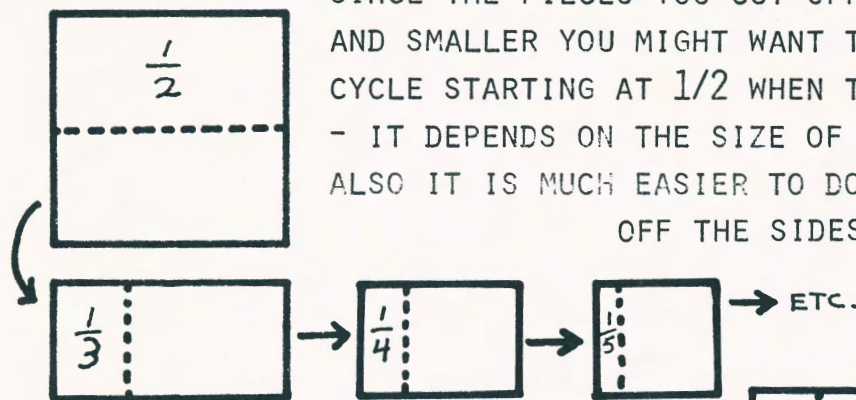


1. MAKE A SCRAPBOOK OR BULLETIN BOARD OF PICTURES OR DRAWINGS WITH DIFFERENT FRACTIONAL PIECES REMOVED.
2. DRAW PICTURES TO SHOW $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{8}$, AND $\frac{1}{10}$ OF AN OBJECT - STUDENTS MIGHT ENJOY PICKING THEIR OWN OBJECT TO DRAW. (HORSES, BASEBALL BATS, RECORDS, PHONE, ETC.)
3. MAKE A SYMMETRY MURAL TO ILLUSTRATE ONE-HALF. EXAMPLES - INKBLOTS.



4. CUT A PICTURE UP TO MAKE A JIGSAW PUZZLE. THE FIRST PIECE WOULD BE $\frac{1}{2}$ OF THE PICTURE. DIVIDE THE OTHER HALF INTO THIRDS, CUT OFF $\frac{1}{3}$. DIVIDE THE REMAINING PIECE INTO FOURTHS, CUT OFF $\frac{1}{4}$, ETC. YOU MIGHT WANT TO TRY THIS ACTIVITY YOURSELF BEFORE DOING IT WITH STUDENTS - SEVERAL INTERESTING THINGS HAPPEN.

EXAMPLE:



CUTS LIKE



OR



WILL LEAVE PIECES THAT ARE MORE DIFFICULT TO WORK WITH.

NUMBER SENSE AND ARITHMETIC SKILLS

Placement Guide for Tabbed Dividers

Tab Label	Follows page:
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Activity Cards - Cuisenaire Rods - III	528	Whole model Addition/subtraction	Manipulative
Activity Cards - Clock Fractions - II	529	Whole model Addition/subtraction	Manipulative
Activity Cards - Tangrams - III	531	Whole model Addition/subtraction	Manipulative
Activity Cards - Geoboards - II	533	Whole model Addition/subtraction	Manipulative
Fraction Subtraction	535	Whole model Subtraction	Paper and pencil
Number Line	536	Number line model Addition/subtraction	Manipulative
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A Masters Puzzle/Don't Get Clipped!	539	Like denominators Addition	Paper and pencil Puzzle
Inchworm Fractions	540	LCD Unlike denominators Addition	Paper and pencil
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LCD Flowchart I	542	LCD Addition/subtraction	Bulletin board Transparency
LCD Flowchart II	543	LCD Addition/subtraction	Bulletin board Transparency

<u>TITLE</u>	<u>PAGE</u>	<u>TOPIC</u>	<u>TYPE</u>
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Two-a-Part	551	Unlike denominators Addition	Game
World Track Recrods	552	Unlike denominators Addition/subtraction	Paper and pencil Transparency
The Greatest Game	553	Unlike denominators Addition/subtraction	Game
Fraction Magic Squares	554	Unlike denominators Addition/subtraction	Paper and pencil Puzzle
Can You Find a Path?	555	Unlike denominators Addition/subtraction	Paper and pencil
More Paths/Challenge Paths	556	Unlike denominators Addition/subtraction	Paper and pencil Puzzle
Almost Games	557	Approximation Addition/subtraction	Game

FRACTIONS:

ADDITION AND SUBTRACTION

Kids in school seem to use a fairly consistent strategy. Even good students use it much of the time; the bad students use it all the time; and everyone uses it when they feel under pressure. One way of describing this strategy is to say that it is answer-centered rather than problem-centered. (John Holt, How Children Fail)



A large group of seventh grade students were recently asked to work some arithmetic exercises. Here are two questions with some typical incorrect answers together with the strategy used to obtain those answers.* Do any of the answers look familiar to you?

Question: $\frac{2}{3} + \frac{1}{2} = \square$

Answer: $\frac{3}{5}$ "2 + 1 = 3 and 3 + 2 = 5"

Answer: $\frac{3}{6}$ "3 can't go into 2 and 2 can't go into 3, so I multiply them together (gave 6 for denominator). Then 2 + 1 = 3."

Answer: $\frac{1}{2}$ "Cancelled 2 into 2 for 1 and 1. Then 1 + 1 = 2 and 3 + 1 = 4 for $\frac{2}{4} = \frac{1}{2}$."

Question: $\frac{3}{4} - \frac{1}{2} = \square$

Answer: $\frac{2}{2}$ "3 - 1 = 2 and 4 - 2 = 2"

Answer: $\frac{5}{4}$ " $\frac{1}{2} = \frac{2}{4}$ and $\frac{3}{4} = \frac{3}{4}$, then 3 + 2 = 5. When you subtract, you don't subtract, you add the opposite."

Answer: $\frac{1}{1}$ "3 subtract 4 is 1. 1 subtract 2 leaves 1."

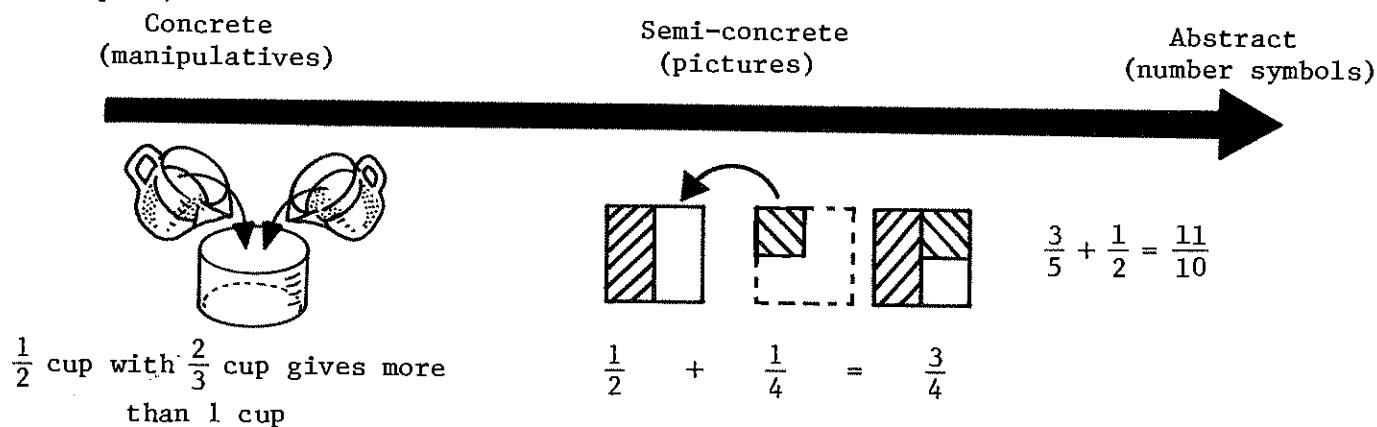
*Francis G. Lankford, Jr., Some Computational Strategies of Seventh Grade Pupils, Final Report (Charlottesville: University of Virginia, 1972).

Out of the total number of students who responded to the two questions almost 50% answered incorrectly. The unreasonableness of most of the answers is more striking than the fact that they are wrong. Any one of those students could probably have told you that one half-cup of water and two-thirds cup of water added together will total more than one cup. Yet their answers were all less than one--in fact, less than two-thirds. On the other hand, their answers to the subtraction exercise yielded answers equal to or greater than one. Somehow, many students become symbol pushers who believe that any answer is better than no answer at all, and the reasonableness of the answer is not even questioned.

MEANINGS OF ADDITION AND SUBTRACTION OF FRACTIONS

One important challenge for teachers of mathematics is to help students go beyond answer-seeking to understanding. What does the addition and subtraction of fractions mean in a physical setting? Is a sum that is obtained through symbol shuffling sensible when based on a concrete example? What is a logical sum for two fractions, and what strategy or algorithm can be used to arrive at that sum?

Here is where a sound understanding of fraction concepts can help. The meaning of addition and subtraction of fractions can be learned with the same models used to understand the meaning of fractions. Again we travel from the concrete world to the more abstract world of numbers. (See the commentary to Fractions: Concepts.)



Addition often arises from the physical act of combining; subtraction can often be related to a removing process. Concrete materials (like cups of water, parts of apples, Fraction Bars, etc.) can be used to demonstrate such situations. Activities with these materials can be used to promote verbalization of fraction situations. A student might say, "One-half cup with two-thirds cup gives more

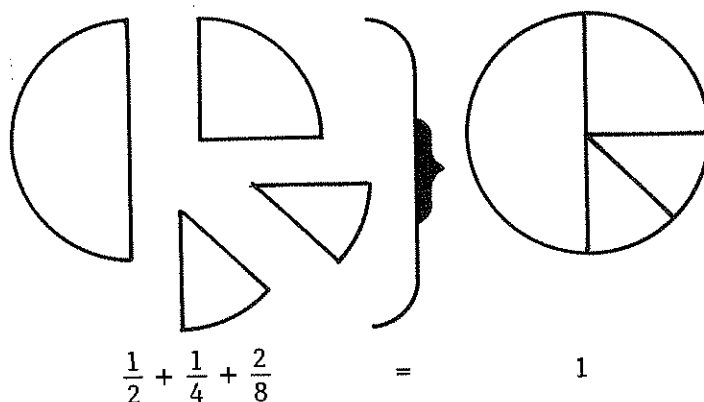
than one cup of water" or "If you have a cup and pour out one-fourth, you get three-fourths cup." Written symbolization is not necessary here, and it could obscure the concrete meaning.

Pictures or diagrams are the next step. Verbalization and written symbols can be added to bridge the gap between concrete models and abstract symbols. We can use concrete examples and abstract symbols as much as is feasible, taking care not to abstract too quickly and periodically relating work with symbols to pictures or concrete objects.

Whole Model

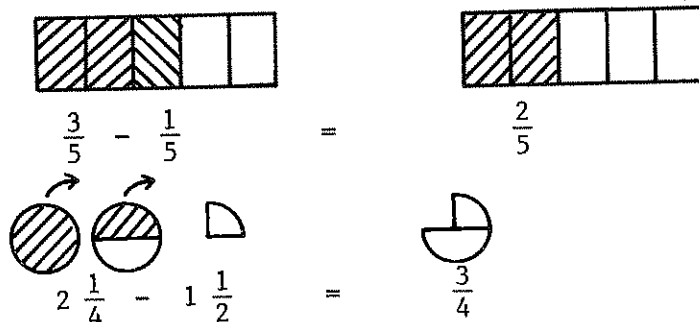
The whole model is especially useful in giving meaning to addition or subtraction of fractions. The diagram at the right shows that one half of a circle plus one fourth of a circle plus two eighths of a circle make one circle. The corresponding number sentence is:

$$\frac{1}{2} + \frac{1}{4} + \frac{2}{8} = 1$$



A student can write this number sentence based on the model without worrying about common denominators. The sum is based on physical meaning, not symbol shuffling. It is important to notice that the same whole is used throughout the problem. Questions like $\frac{1}{2}$ of + $\frac{1}{4}$ of = $\frac{?}{?}$ of can be asked, but they do not represent the number question, $\frac{1}{2} + \frac{1}{4} = ?$

Subtraction is a number operation often based on the physical act of removing or "taking away." This meaning of subtraction can be demonstrated with the whole model. Each of the diagrams here has a corresponding subtraction sentence. Notice that the differences can be obtained through a meaning of subtraction and a meaning of fractions. No symbol shuffling is necessary.



Set Model

Sets can be used to give meaning to addition and subtraction of fractions; however, they do not seem to be as versatile as the whole model or number lines. Figure A is a picture which could be used to justify the number sentence, $\frac{3}{10} + \frac{2}{10} = \frac{5}{10}$. Some conceptual problems might occur in trying to use sets to explain subtraction of fractions. For example, Figure B, which is intended to show $\frac{5}{7} - \frac{3}{7} = \frac{2}{7}$, might be interpreted as $\frac{5}{7} - \frac{3}{7} = \frac{2}{4}$ since three circles seem to leave the set. Figure C shows the circles sliding to the right but remaining in the set. We write, $\frac{5}{7} - \frac{3}{7} = \frac{2}{7}$.

Figure A:

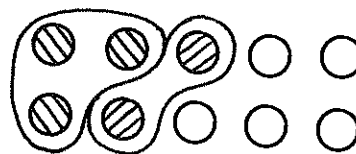


Figure B:

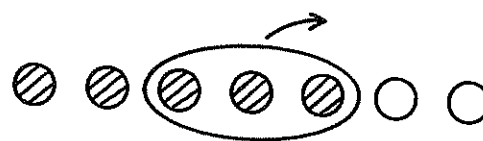
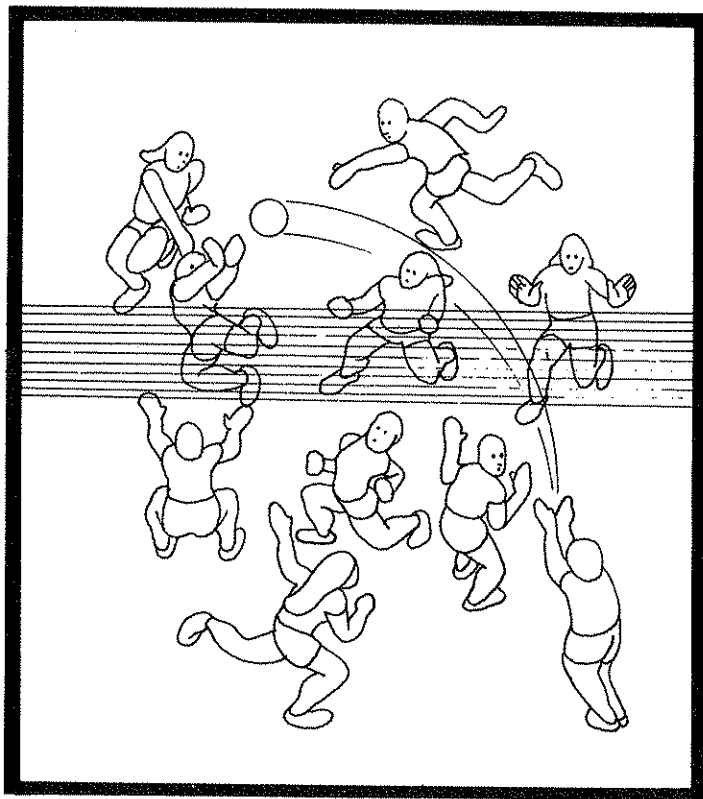


Figure C:



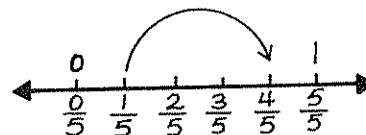
Keep an eye out for changing sets. Look at this problem: There are five boys and five girls. Three-fifths of the boys have long hair. One-fifth of the girls have long hair. What fraction of the students have long hair? A response is likely to be, $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$, but the answer is $\frac{4}{10}$. What happened? The sets changed, and the number sentence is really $\frac{3}{10} + \frac{1}{10} = \frac{4}{10}$. The answer $\frac{4}{10}$ supports the common error of adding numerators and denominators.

Number Line Model

The number line is a very useful device for motivating sums and differences of fractions. In addition, start on the point corresponding to the first fraction, then move forward by the value of the second fraction. (We are assuming that the fractions are non-negative.)

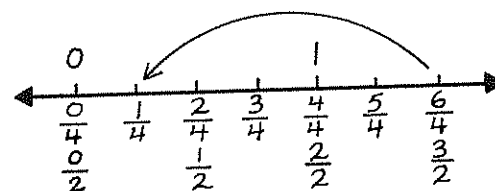
The sum is the fraction that names the stopping point. For subtraction, start at the point corresponding to the first fraction then move back the value of the second fraction. See the examples at the right. A slide rule can be used as a tool in computing sums and differences with this view. See *Fraction Slide Rule* in this section.

Addition Example:



$$\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$$

Subtraction Example:



$$\frac{3}{2} - \frac{5}{4} = \frac{1}{4}$$

Formalization

When students are learning about fractions via concrete and semi-concrete models, the foundation is being laid for the addition and subtraction algorithms for fractions. These algorithms are introduced as methods for finding sums and differences without relying on models or interpretations.

After working problems which involve models, some students will develop a method for adding and subtracting fractions with common denominators. Symbolically:

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \quad \text{and} \quad \frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$$

Other students will need to have this algorithm pointed out to them.

The next step is to consider fractions with unlike denominators. After appropriate activities students may see that fractions with unlike denominators can be written as equivalent fractions with common denominators before adding or subtracting them. See *Inchworm Fractions* in this section.

Symbolically: $\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad+bc}{bd}$

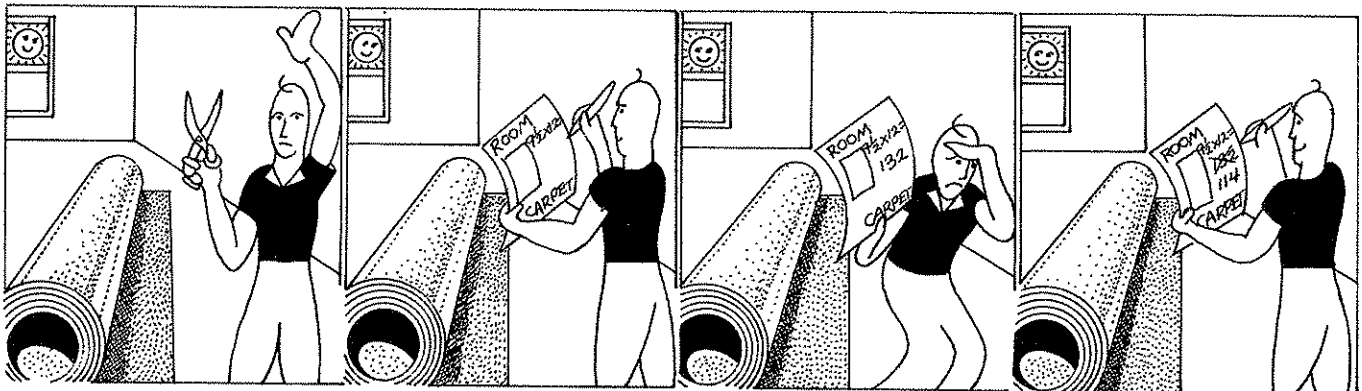
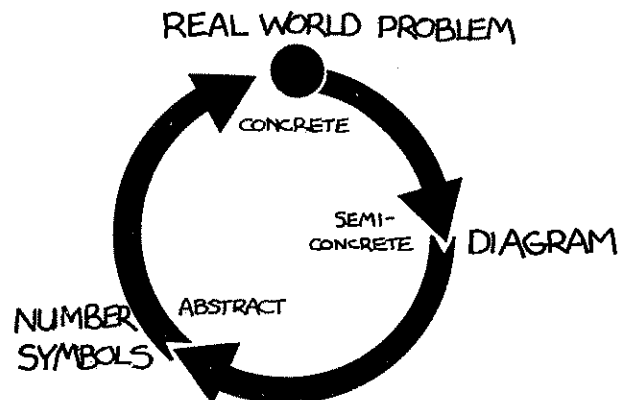
$$\frac{a}{b} - \frac{c}{d} = \frac{ad}{bd} - \frac{bc}{bd} = \frac{ad-bc}{bd}$$

These algorithms can and should be checked against previous work with models.

Why So Much Emphasis on Concrete Meanings?

It is important to solve many problems where the concrete meanings of fractions and operations are applied. Concrete examples form a solid reference for work with abstract symbols. "Reality checks" can be encouraged in exercises designed for skill building. In a "reality check" a student describes or sketches a concrete example to "prove" or support his sum or difference. We want students to be able to check the reasonableness of answers. They might have invented their own method or algorithm (correct or not) for a mathematical operation, and they should have some criteria for determining the validity of the method. Checking results against previously learned meanings is helpful throughout all of mathematics.

Goals of middle school mathematics include more than computing correctly with numbers. We also want students to be able to apply these numbers to everyday problems. When solving a "real world" problem such as ordering cement for a sidewalk or drawing a plan for a garden, we often make a sketch, do some figuring with numbers, then apply the results to the problem. The path for solving such a problem is usually circular as shown below. Concrete meanings help facilitate this path.



DIAGNOSING STUDENT ERRORS

As we saw at the beginning of this section, and as most teachers know, a large number of students never master addition and subtraction of fractions. However, in the process of studying fractions they do invent strategies for getting answers -- albeit wrong answers. Much can be learned by examining student solutions to exercises. Here are some questions to keep in mind: Are they making careless computational errors? Do they seem to be missing necessary prerequisite learning (e.g., an understanding of equivalent fractions)? Do they seem to consider the reasonableness of their answers? Have they learned an incorrect algorithm which they apply consistently? The answers to these and similar questions help us determine appropriate remediation. Look at the following examples and attempt to explain how each student obtained his/her answer before reading the accompanying explanation.

$$\frac{2}{3} + \frac{4}{5} = \frac{6}{8}$$

- This is the most common method of incorrectly adding fractions -- add the numerators and add the denominators.

$$\frac{7}{8} - \frac{1}{5} = \frac{6}{3}$$

- Subtracting numerators and denominators is the most common error in fraction subtraction. The order in which the fractions are written does not deter this strategy. In an exercise like $\frac{7}{8} - \frac{3}{10}$ the erroneous answer is most likely $\frac{4}{2}$ since $7 - 3 = 4$ and $10 - 8 = 2$.

$$\frac{1}{2} + \frac{2}{5} = \frac{2}{7} + \frac{10}{7} = \frac{12}{7}$$

- The strategy employed by this student is more difficult to diagnose. It appears as though a common denominator is obtained by adding the two denominators. The numerators appear to be obtained by multiplying denominators times numerators ($2 \times 1 = 2$; $5 \times 2 = 10$).

It is often times very difficult to determine an erroneous strategy by looking at written work alone. The only way to be sure that you have correctly interpreted a student's strategy is to have him/her relate that strategy orally. For the example below, two students obtained the same incorrect answer by different methods.

$$\begin{array}{r} 8\frac{2}{5} \\ -4\frac{3}{10} \\ \hline 3\frac{9}{5} \end{array}$$

Student I -- "10 will not go into 5, so change 8 to 7 and 2 (of $\frac{2}{5}$) to 12. Then I borrow one from 12 making it 11, and making 5 (of $\frac{2}{5}$) a 15. So $7\frac{11}{15} - 4\frac{3}{10} = 3\frac{9}{5}$. (4 from 7 = 3, 10 from 15 = 5, 3 from 11 = 9.)"*

Student II -- "Borrowed 1 from 8, made it 7 and 2 of $\frac{2}{5}$ into 12. Then $7\frac{12}{5} - 4\frac{3}{10} = 3\frac{9}{5}$. (4 take away 7 is 3, 3 take away 12 is 9, and 10 take away 5 is 5.)"*

We can help students learn correct procedures (algorithms) if we understand why they make the errors they do. When examining the type of errors and strategies that students use, we see that students are getting incorrect answers for many different reasons. Thus it will be necessary to employ different remedial treatments for different errors.

TEACHER STRATEGIES FOR THE REMEDIAL PROCESS

Developmental Work

When a student is employing an incorrect strategy, such as $\frac{1}{2} + \frac{2}{3} = \frac{3}{5}$, it will be necessary to probe more deeply to get to the heart of the problem. Does this student merely need more work with equivalent fractions? Can he/she correctly work exercises like $\frac{1}{6} + \frac{4}{6}$? If so, then the problem may center on equivalent fractions. If not, it may be necessary to go back to the meaning of fraction addition (or back even farther to the meaning of fraction).

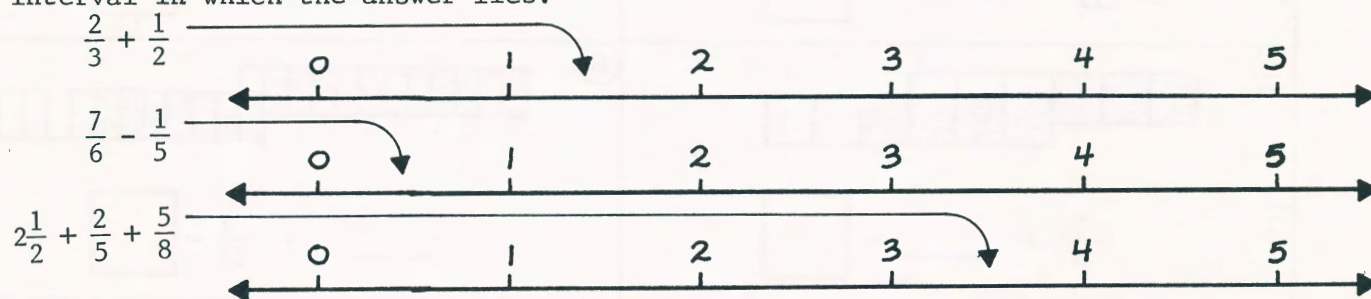
Can the student illustrate addition of fractions using diagrams or some concrete model? In the process of remediation it will often be necessary to revert to models to redevelop the meaning behind a particular concept. In some cases this may mean using diagrams, while in other cases it may mean employing manipulatives such as Circle Fractions or Fraction Bars. In any case, it is important that students can illustrate what addition or subtraction of fractions means in terms of some model or diagram before moving to symbol manipulation. Even when students can add or subtract fractions correctly, it is helpful to have them periodically illustrate the meaning of what they are doing in terms of some model or diagram. Some students

*Francis G. Lankford, Jr., Some Computational Strategies of Seventh Grade Pupils, Final Report (Charlottesville: University of Virginia, 1972).

are adept at learning rules (algorithms). Reference to a model or diagram may make their procedure more meaningful to them.

Approximate Calculations

One of the striking features of most incorrect answers is their unreasonableness. When students add and subtract fraction like $\frac{1}{2} + \frac{2}{5} = \frac{12}{7}$ or $\frac{7}{6} - \frac{1}{5} = \frac{6}{3}$, they should learn to give some attention to the relationship between the answer and the total exercise. Almost everyone would agree that one of our important goals is to encourage students to determine the reasonableness of their answers. Yet we tend to devote little time to the development of the skills necessary to make approximate calculations. One way to encourage and develop approximation abilities is to have students answer fraction addition and subtraction exercises by indicating the interval in which the answer lies.



Students having difficulties can be allowed to use a ruler as an aid. The patterns can be arranged by level of difficulty so that all students would be challenged. This activity would serve as a nice introduction to *The Almost Game* in this section. If we expect proficiency in testing reasonableness of answers we must devote time to developing the necessary skills.

Some General Strategies

Students who are still having computational difficulties as they enter middle grades usually have a history of failure (and maybe fear) in mathematics. Above all else, students need to believe they are valued people and are capable of attaining the needed skill. It is often advantageous, from the students' points of view, to have remedial instruction different from the methods with which they were previously taught (and experienced failure). These instructional techniques might include games, laboratory activities, drill and practice through problem solving, and manipulative devices, but it is important that students experience frequent encouragement and success.



ACTIVITY CARDS - FRACTION BARS - II

Whole Model
Addition/Subtraction
FRACTIONS



For information on the construction use and readiness activities for Fraction Bars see the section on Lab Materials.

FB-II-1

Write the missing fraction for each pair of bars, and then write the sum or difference of these fractions.

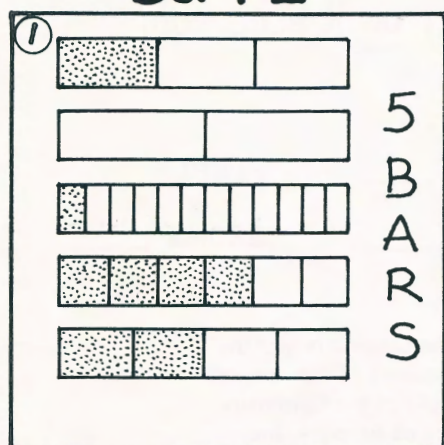
<p>①</p> $\frac{2}{6} + \underline{\hspace{2cm}} = \boxed{\hspace{2cm}}$	<p>②</p> $\underline{\hspace{2cm}} + \frac{2}{4} = \boxed{\hspace{2cm}}$
<p>③</p> $\frac{5}{12} + \underline{\hspace{2cm}} = \boxed{\hspace{2cm}}$	<p>④</p> $\frac{1}{6} + \underline{\hspace{2cm}} = \boxed{\hspace{2cm}}$
<p>⑤</p> $\frac{3}{6} + \underline{\hspace{2cm}} = \boxed{\hspace{2cm}}$	<p>⑥</p> $\underline{\hspace{2cm}} + \frac{7}{12} = \boxed{\hspace{2cm}}$
<p>⑦</p> $\frac{2}{3} + \underline{\hspace{2cm}} = \boxed{\hspace{2cm}}$	<p>⑧</p> $\underline{\hspace{2cm}} + \frac{9}{12} = \boxed{\hspace{2cm}}$
<p>⑨</p> $\begin{array}{r} \frac{10}{12} \\ - \frac{3}{12} \\ \hline \end{array}$	<p>⑩</p> $\begin{array}{r} \frac{3}{4} \\ - \frac{2}{4} \\ \hline \end{array}$
<p>⑪</p> $\frac{3}{6} - \frac{1}{6} = \boxed{\hspace{2cm}}$	<p>⑫</p> $\frac{5}{6} - \frac{4}{6} = \boxed{\hspace{2cm}}$

ACTIVITY CARDS - FRACTION BARS - II (CONTINUED)

Write the fraction for the bars below, change each to twelfths, and compute each sum of 5 fractions.

FB-II-2

SUM I



$$\frac{1}{3} = \frac{4}{12}$$

$$\frac{1}{2} = \frac{6}{12}$$

$$\frac{1}{12} = \frac{1}{12}$$

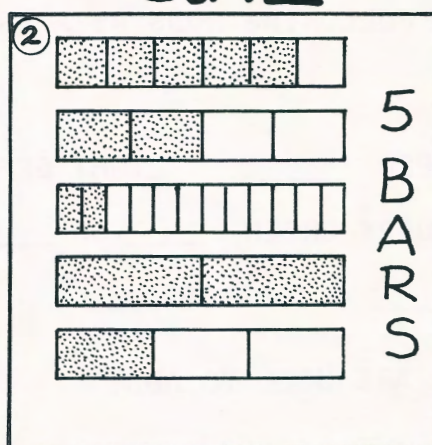
$$\frac{4}{6} = \frac{8}{12}$$

$$\frac{3}{4} = \frac{9}{12}$$

SUM

①

SUM II



$$\frac{5}{6} = \frac{10}{12}$$

$$\frac{1}{2} = \frac{6}{12}$$

$$\frac{1}{12} = \frac{1}{12}$$

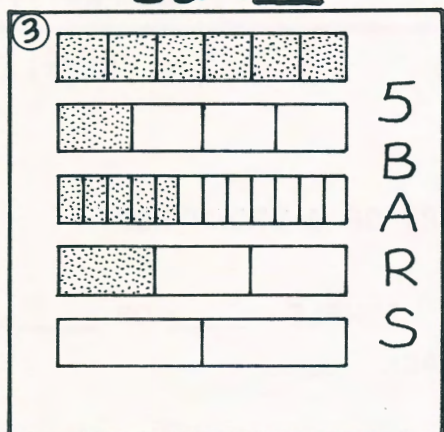
$$\frac{5}{6} = \frac{10}{12}$$

$$\frac{1}{2} = \frac{6}{12}$$

SUM

②

SUM III



$$\frac{5}{6} = \frac{10}{12}$$

$$\frac{1}{2} = \frac{6}{12}$$

$$\frac{1}{12} = \frac{1}{12}$$

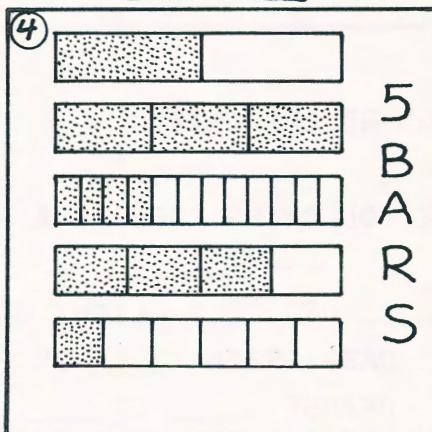
$$\frac{1}{2} = \frac{6}{12}$$

$$\frac{1}{2} = \frac{6}{12}$$

SUM

③

SUM IV



$$\frac{1}{2} = \frac{6}{12}$$

$$\frac{5}{6} = \frac{10}{12}$$

$$\frac{1}{12} = \frac{1}{12}$$

$$\frac{5}{6} = \frac{10}{12}$$

$$\frac{1}{2} = \frac{6}{12}$$

SUM

④

5. Which group of 5 fractions has the smallest sum? _____
6. Which group of 5 fractions has the greatest sum? _____
7. Which group of 5 fractions has the 2 fractions with the smallest sum? _____
8. Which group of 5 fractions has the 2 fractions with the greatest sum? _____
9. Which group of 5 has 4 fractions whose sum is the remaining fraction of the group? _____

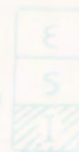
If the students use Fraction Bars allow them to pick 5 bars to add.



ACTIVITY CARDS - CUISENAIRE RODS-III

For readiness activities for Cuisenaire rods see the section on Lab Materials.

Addition/Subtraction FRACTIONS



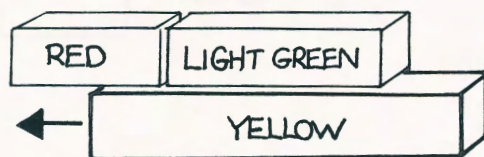
LET THE BLUE ROD BE ONE UNIT.

NAME EACH OF THE FOLLOWING RODS AS A FRACTIONAL PART OF A BLUE ROD.

WHITE _____ RED _____ LIGHT GREEN _____ OR _____ PURPLE _____
 YELLOW _____ DARK GREEN _____ OR _____ BLACK _____ BROWN _____
 BLUE _____ OR _____ ORANGE _____

SEE HOW THE RODS ARE USED TO ADD:

$$\frac{2}{9} + \frac{3}{9} = \frac{5}{9} \text{ (YELLOW)}$$



$$\begin{aligned} \frac{1}{9} + \frac{6}{9} &= \text{ () } \\ \frac{4}{9} + \frac{5}{9} &= \text{ () } \\ \frac{1}{3} + \frac{1}{3} &= \text{ () } \end{aligned}$$

$$\begin{aligned} \frac{1}{9} + \frac{2}{3} &= \text{ () } \\ \frac{5}{9} + \frac{1}{3} &= \text{ () } \\ \frac{2}{9} + \frac{1}{3} &= \text{ () } \end{aligned}$$

The name Cuisenaire and the color sequence of the rods are trademarks of the Cuisenaire Company of America, Inc.

CR-III-1

CR-III-2

LET THE BROWN ROD BE ONE UNIT.

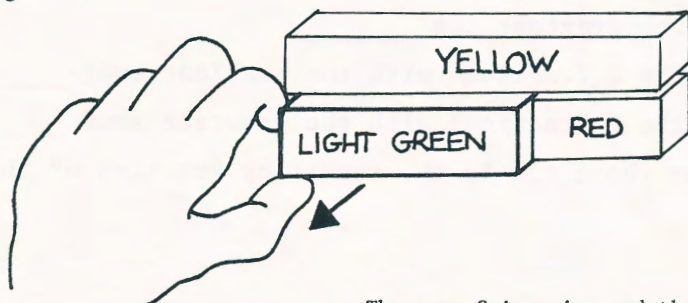
NAME EACH OF THE FOLLOWING RODS AS A FRACTIONAL PART OF A BROWN ROD.

WHITE _____ RED _____ OR _____ LIGHT GREEN _____ PURPLE _____ OR _____
 YELLOW _____ DARK GREEN _____ OR _____ BLACK _____
 BLUE _____ ORANGE _____ OR _____

SEE HOW THE RODS ARE USED TO SUBTRACT:

TRY THESE:

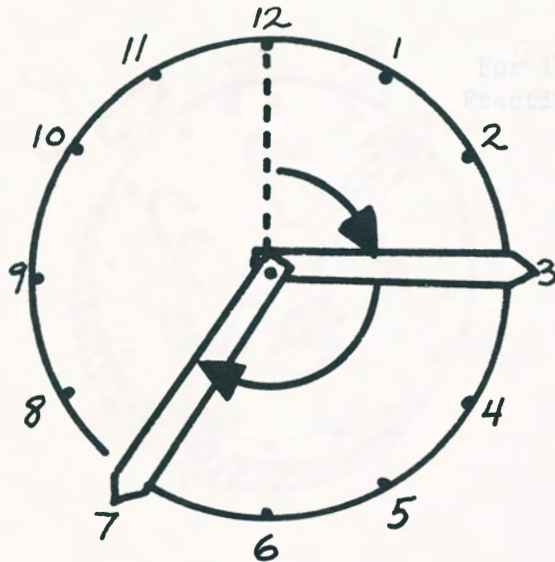
$$\frac{5}{8} - \frac{3}{8} = \text{ (RED)}$$



$$\begin{aligned} \frac{7}{8} - \frac{1}{8} &= \text{ () } \\ \frac{5}{8} - \frac{1}{2} &= \text{ () } \\ \frac{3}{4} - \frac{3}{8} &= \text{ () } \\ \frac{1}{2} - \frac{1}{4} &= \text{ () } \\ 1\frac{1}{8} - \frac{7}{8} &= \text{ () } \\ 1\frac{1}{4} - \frac{3}{4} &= \text{ () } \end{aligned}$$

The name Cuisenaire and the color sequence of the rods are trademarks of the Cuisenaire Company of America, Inc.

ACTIVITY CARDS - CLOCK FRACTIONS - II



Attach two arrows to "clock" D. Rotate both arrows from 12 to 3 (through $\frac{3}{12}$ of the "clock.") Holding the bottom arrow on 3, rotate the top arrow another $\frac{4}{12}$ of the "clock." The top arrow rotated through what part of the "clock?"

The example to the left should help you see that

$$\frac{3}{12} + \frac{4}{12} = \frac{7}{12}.$$

USE THE SAME PROCEDURES TO FIND:

A) $\frac{5}{12} + \frac{6}{12}$

B) $\frac{1}{12} + \frac{7}{12}$

C) $\frac{5}{12} + \frac{7}{12}$

D) $\frac{7}{12} + \frac{10}{12}$

CL-II-1

Using two arrows on "clock" D, rotate both arrows from 12 to 7. Holding the bottom arrow on 7, rotate the top arrow counter clockwise from 7 back across 3 parts of the "clock." Where is the top arrow now? Do you see that the example at the right shows that

$$\frac{7}{12} - \frac{3}{12} = \frac{4}{12}?$$

USE THE SAME PROCEDURE TO FIND:

A) $\frac{11}{12} - \frac{5}{12}$

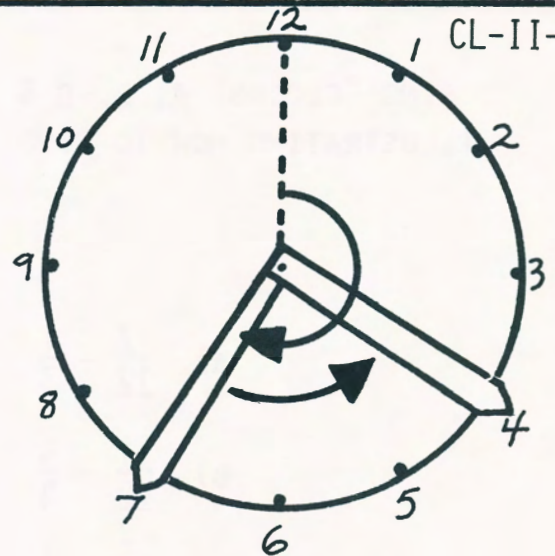
B) $\frac{9}{12} - \frac{8}{12}$

C) $\frac{12}{12} - \frac{9}{12}$

D) $1 - \frac{7}{12}$

E) $\frac{10}{12} - \frac{11}{12}$

F) $1\frac{1}{12} - \frac{5}{12}$



CL-II-2

THESE ARE TOUGH

TYPE: Manipulative

ACTIVITY CARDS - CLOCK FRACTIONS - II (CONTINUED)

USING "CLOCKS" A, B & D, TRY TO DISCOVER A METHOD FOR ILLUSTRATING HOW TO FIND ANSWERS TO PROBLEMS LIKE:



A) $\frac{1}{3} + \frac{1}{4}$

B) $\frac{2}{3} + \frac{3}{4}$

C) $\frac{2}{3} + \frac{5}{12}$

D) $\frac{3}{4} + \frac{7}{12}$

CL-II-3

CL-II-4

USING "CLOCKS" A, B, C & D, TRY TO DISCOVER A METHOD FOR ILLUSTRATING HOW TO FIND ANSWERS TO PROBLEMS LIKE:

A) $\frac{7}{12} - \frac{1}{3}$

B) $\frac{11}{12} - \frac{3}{4}$

C) $\frac{5}{12} - \frac{1}{6}$

D) $\frac{5}{5} - \frac{3}{4}$

E) $\frac{5}{6} - \frac{2}{3}$

F) $\frac{3}{4} - \frac{1}{3}$



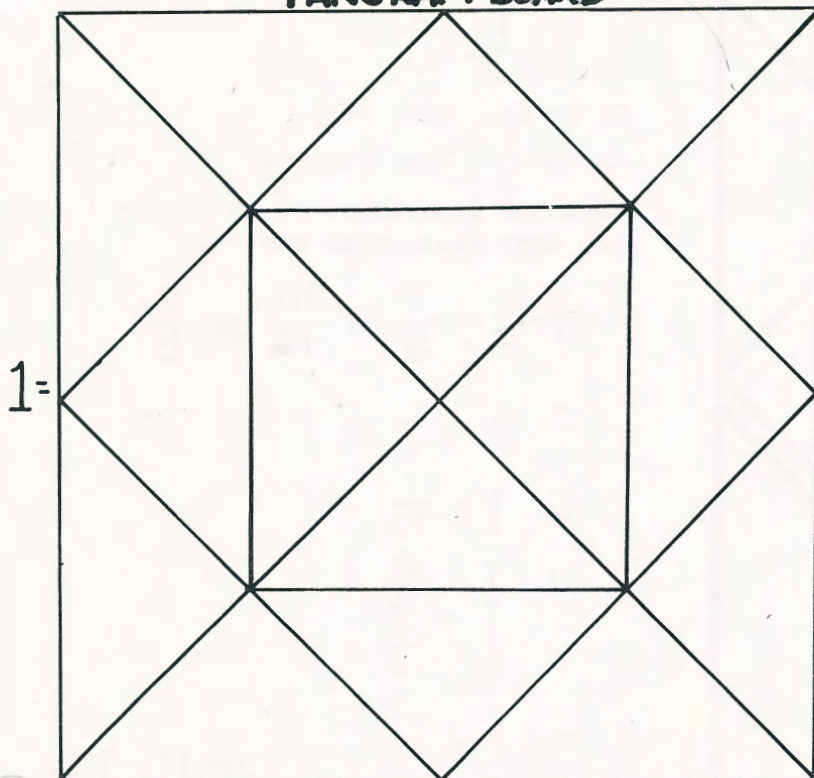
ACTIVITY CARDS - TANGRAMS - III

For information on the construction, use, and readiness activities for Tangrams, see the section on Lab Materials. Students should use a Tangram Board with these cards.

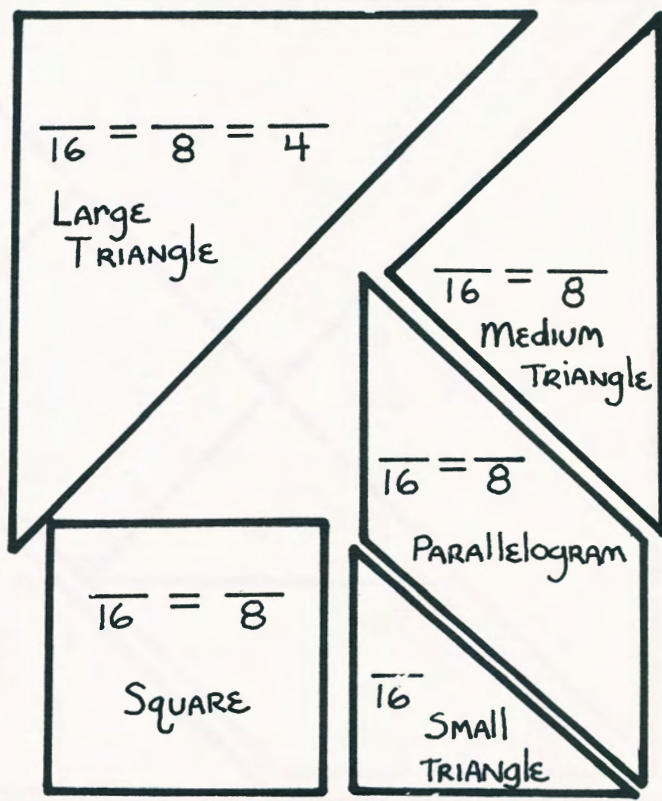
Whole Model
Addition/
Subtraction
FRACTIONS



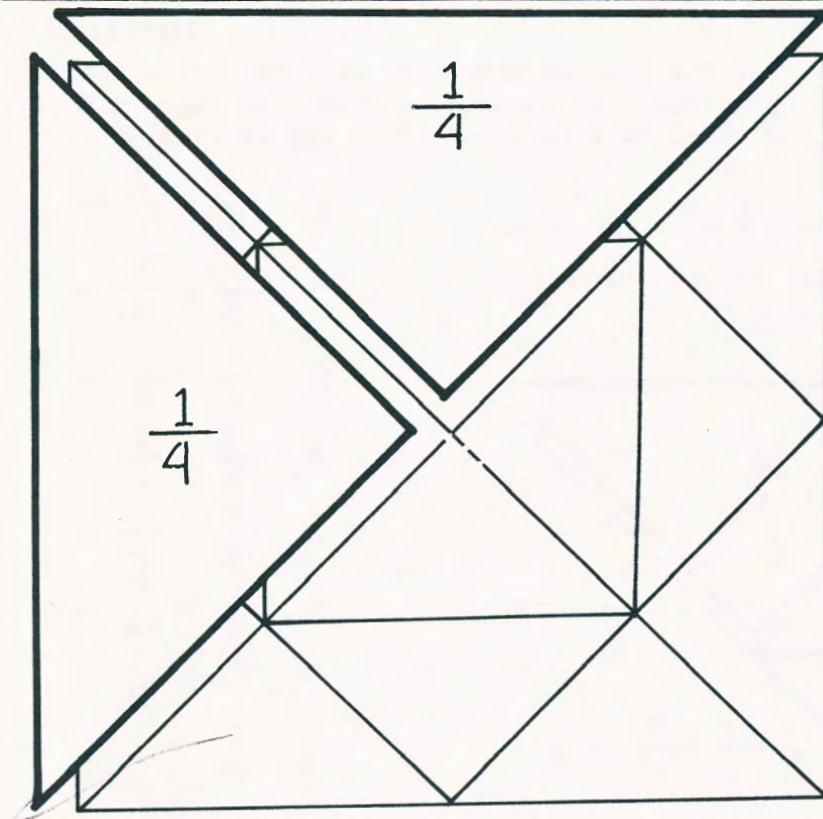
TANGRAM BOARD



TG-III-1



TG-III-2

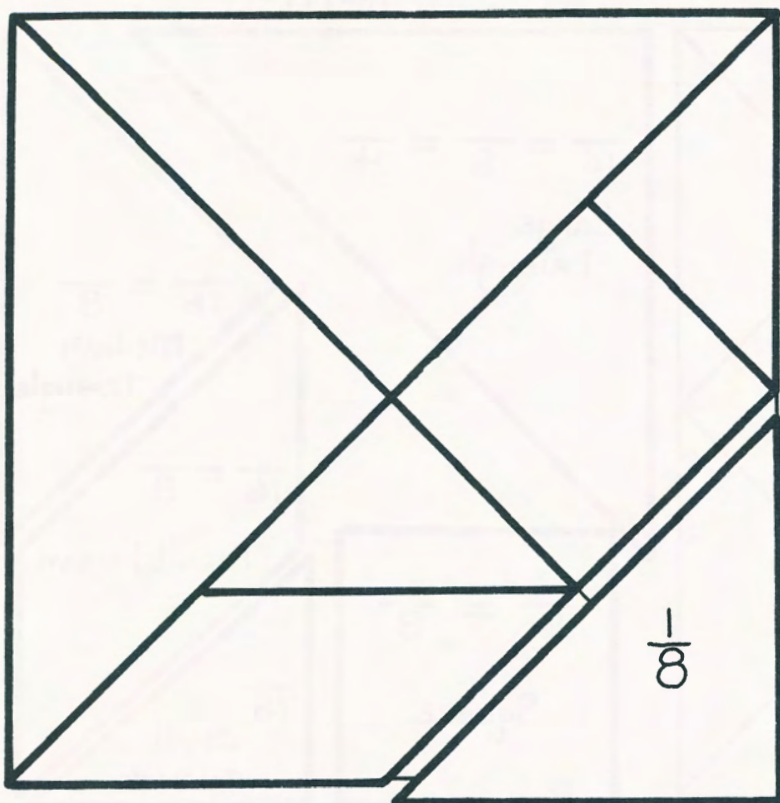


$$\frac{1}{4} + \frac{1}{4} = \frac{1}{2} \quad \text{or } \frac{2}{4} \text{ or } \frac{8}{16}$$

1. $\frac{1}{8} + \frac{1}{8} =$
2. $\frac{1}{16} + \frac{1}{16} =$
3. $\frac{2}{4} + \frac{2}{8} =$
4. $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} =$
5. $\frac{1}{8} + \frac{3}{16} =$
6. $\frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} =$
7. $\frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{4} =$

TYPE: Manipulative

ACTIVITY CARDS - TANGRAMS - III (CONTINUED)



$$1 - \frac{1}{8} =$$

Cover the board completely.

Remove $\frac{1}{8}$. What remains?

(Use the Tangram Board to figure how many sixteenths remain.)

$$1. \quad 1 - \frac{1}{8} = \frac{14}{16} \text{ or } \frac{7}{8}$$

$$2. \quad 1 - \frac{1}{4} =$$

$$3. \quad 1 - \frac{1}{16} =$$

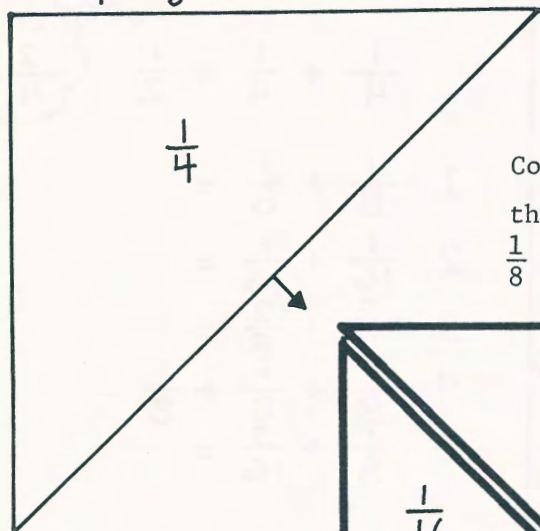
$$4. \quad 1 - \frac{5}{8} =$$

$$5. \quad 1 - \frac{7}{16} =$$

$$6. \quad 1 - \frac{3}{4} =$$

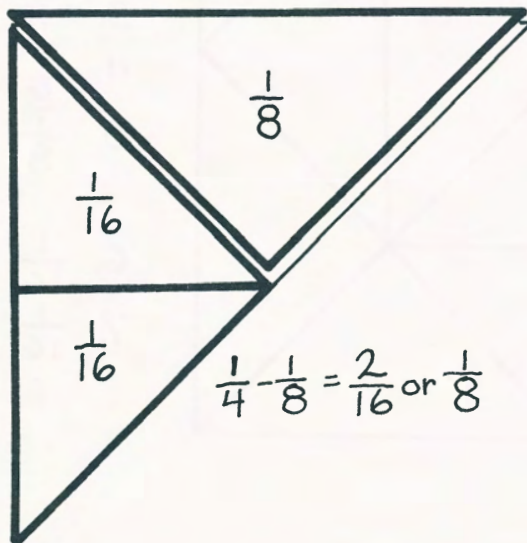
TG-III-3

$$\frac{1}{4} - \frac{1}{8} = ?$$



Use the tangram pieces to help you solve these problems. An example is shown but there may be several ways to demonstrate the problem.

Cover $\frac{1}{4}$ with pieces so that you can subtract $\frac{1}{8}$ from $\frac{1}{4}$.



$$\frac{1}{4} - \frac{1}{8} = \frac{2}{16} \text{ or } \frac{1}{8}$$

TG-III-4

$$1. \quad \frac{1}{4} - \frac{1}{16} =$$

$$2. \quad \frac{1}{8} - \frac{1}{16} =$$

$$3. \quad \frac{1}{2} - \frac{1}{8} =$$

$$4. \quad \frac{3}{8} - \frac{1}{8} =$$

$$5. \quad \frac{5}{16} - \frac{1}{8} =$$

$$6. \quad \frac{1}{4} - \frac{3}{16} =$$

$$7. \quad \frac{1}{2} - \frac{3}{8} =$$

$$8. \quad \frac{7}{8} - \frac{3}{8} =$$

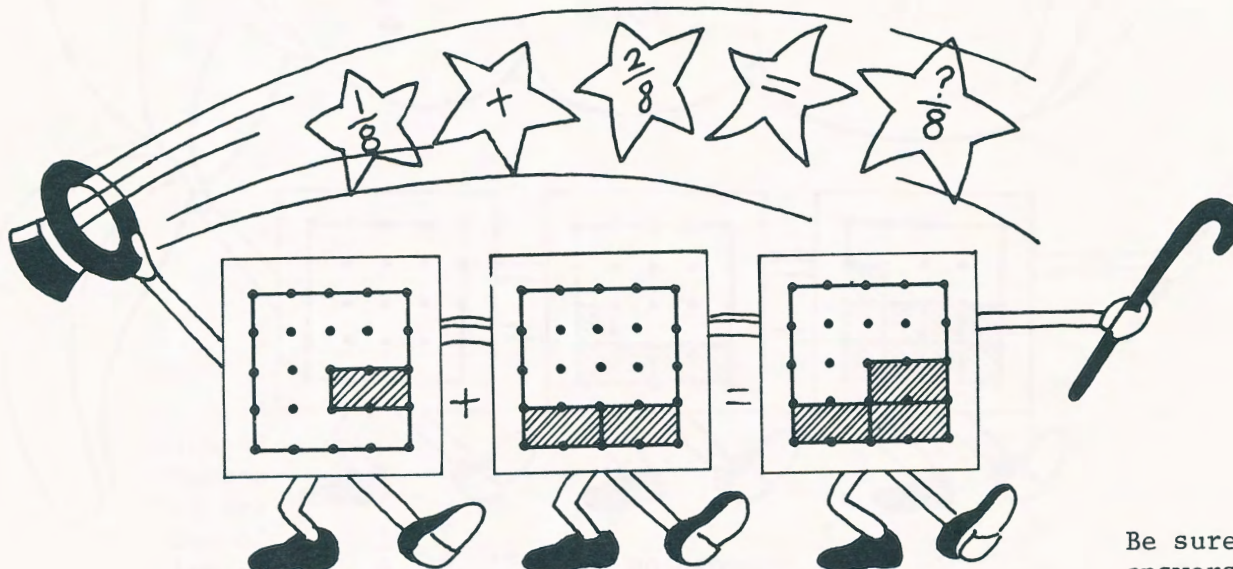


ACTIVITY CARDS - GEOBOARDS -II

For information on the construction, use and readiness activities for the Geoboard, see the section on Job Materials.

Grade 5
Addition/
Subtraction
FRACTIONS

Find a geoboard solution for each of the following problems.
Record your answers on dot paper.



1. $\frac{3}{16} + \frac{4}{16}$

3. $\frac{1}{4} + \frac{2}{4}$

5. $\frac{5}{16} + \frac{7}{16}$

2. $\frac{1}{2} + \frac{1}{2}$

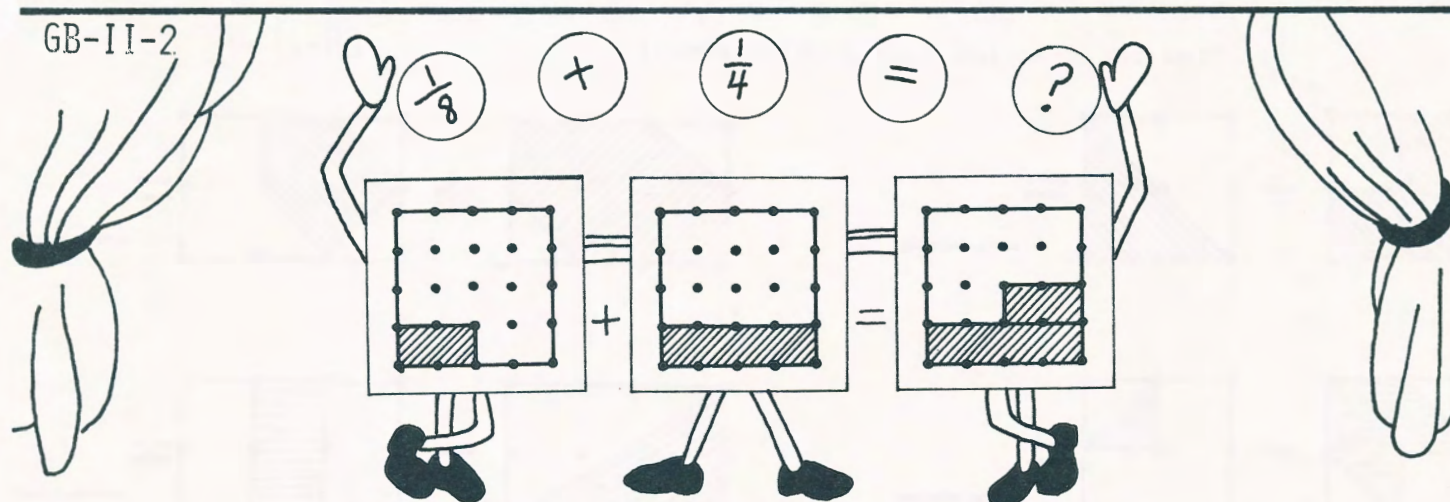
4. $\frac{3}{8} + \frac{3}{8}$

6. $\frac{5}{8} + \frac{3}{8}$

Be sure
answers
are in
lowest
terms!

GB-II-1

GB-II-2



Record on dot paper.

1. $\frac{3}{16} + \frac{1}{2}$

3. $\frac{1}{4} + \frac{5}{8}$

5. $\frac{1}{2} + \frac{3}{8}$

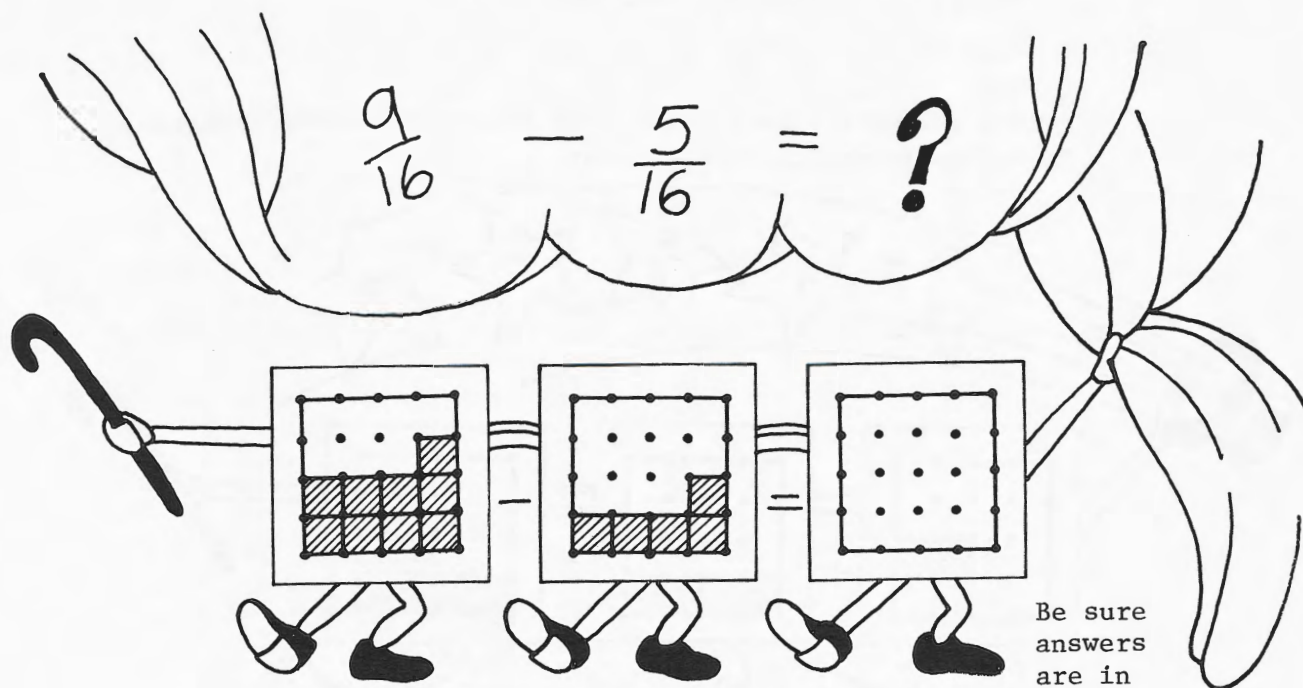
2. $\frac{3}{8} + \frac{5}{16}$

4. $\frac{3}{4} + \frac{1}{16}$

6. $\frac{5}{16} + \frac{5}{8}$

Be sure
answers
are in
lowest
terms!

ACTIVITY CARDS - GEOBOARDS - II (CONTINUED)



1. $\frac{6}{8} - \frac{2}{8}$

RECORD ON DOT
PAPER

4. $\frac{7}{8} - \frac{1}{4}$

2. $1 - \frac{1}{4}$

5. $\frac{11}{16} - \frac{1}{4}$

3. $\frac{3}{4} - \frac{1}{2}$

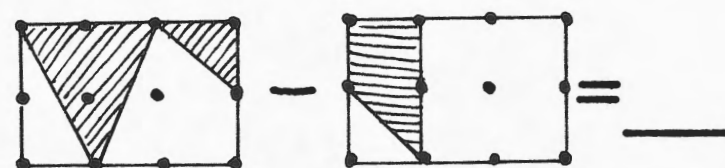
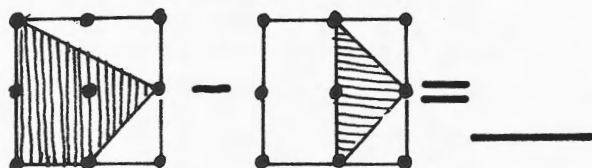
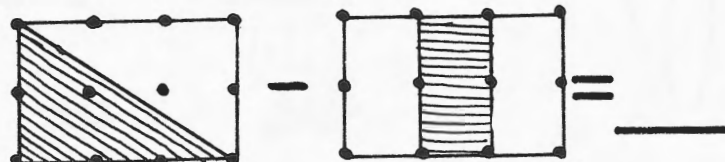
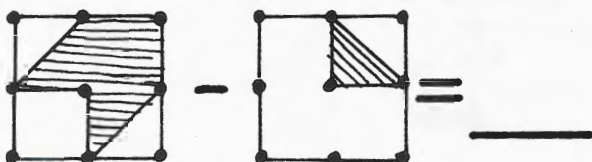
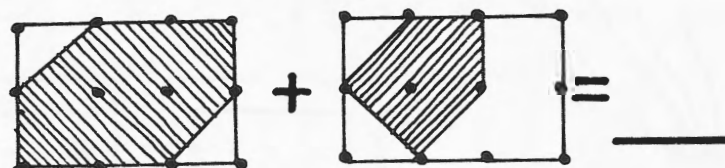
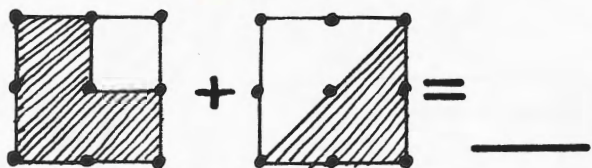
6. $\frac{15}{16} - \frac{7}{16}$

Be sure
answers
are in
lowest
terms!

GB-II-3

Find the fraction each problem names:

GB-II-4



For this card the fraction chosen will
depend on the unit assigned.

FRACTION SUBTRACTION

Whole Model
Subtraction
FRACTIONS



Find the answers to the following subtraction problems.
Use the diagrams to check your answers.

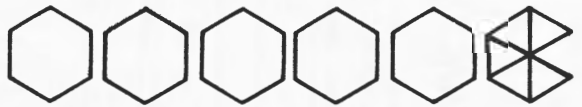
①

$$\begin{array}{r} 4\frac{3}{4} \\ - 2 \\ \hline \end{array}$$



③

$$\begin{array}{r} 5\frac{5}{6} \\ - 3\frac{1}{6} \\ \hline \end{array}$$



②

$$\begin{array}{r} 4 \\ - 1\frac{1}{4} \\ \hline \end{array}$$



④

$$\begin{array}{r} 6\frac{1}{4} \\ - 1\frac{3}{4} \\ \hline \end{array}$$



Make and solve a problem for these diagrams.

⑤



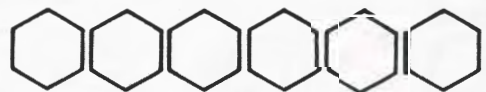
⑦



⑥



⑧



Make a diagram for these problems and solve them.

⑨

$$\begin{array}{r} 3\frac{1}{3} \\ - 1\frac{2}{3} \\ \hline \end{array}$$

⑪

$$\begin{array}{r} 8 \\ - 2\frac{2}{3} \\ \hline \end{array}$$

⑩

$$\begin{array}{r} 4\frac{5}{6} \\ - 3 \\ \hline \end{array}$$

⑫

$$\begin{array}{r} 6\frac{3}{8} \\ - 4\frac{7}{8} \\ \hline \end{array}$$

Choose some problems of your own, make diagrams, and solve them.

TYPE: Paper & Pencil

IDEA FROM: C.O.L.A.M.D.A.

Permission to use granted by Northern Colorado Educational Board of Cooperative Service

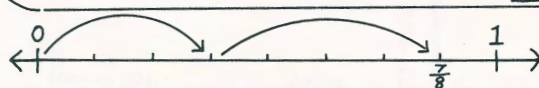
NUMBER LINE

ADDITION of FRACTIONS

When I add on the number line I start on 0 and go up as far as the first number says: $\frac{3}{8} + \frac{4}{8}$



...then I start where I left off and go up the amount of the second number: $\frac{3}{8} + \frac{4}{8}$



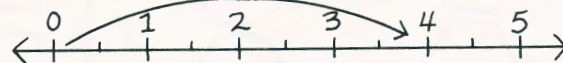
... I landed on $\frac{7}{8}$, so $\frac{3}{8} + \frac{4}{8} = \frac{7}{8}$!!!

Now it's Your Turn...

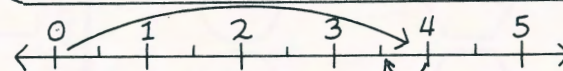
1. $\frac{2}{5} + \frac{4}{5} = \underline{\hspace{2cm}}$
2. $\frac{2}{3} + \frac{2}{3} = \underline{\hspace{2cm}}$ or $\underline{\hspace{2cm}}$
3. $\frac{3}{4} + \frac{1}{4} = \underline{\hspace{2cm}}$ or $\underline{\hspace{2cm}}$
4. $3 + \frac{2}{3} = \underline{\hspace{2cm}}$
5. $7 + \frac{1}{3} = \underline{\hspace{2cm}}$
6. $1\frac{1}{2} + 5\frac{1}{2} = \underline{\hspace{2cm}}$

SUBTRACTION of FRACTIONS

When I subtract on the number line I start on 0 and go up as far as the first number says: $4 - \frac{1}{2}$



...then I start where I left off and go back the amount of the second number: $4 - \frac{1}{2}$. I always go back UNDER the number line.



... I landed on $3\frac{1}{2}$, so $4 - \frac{1}{2} = 3\frac{1}{2}$!


Now it's Your Turn...

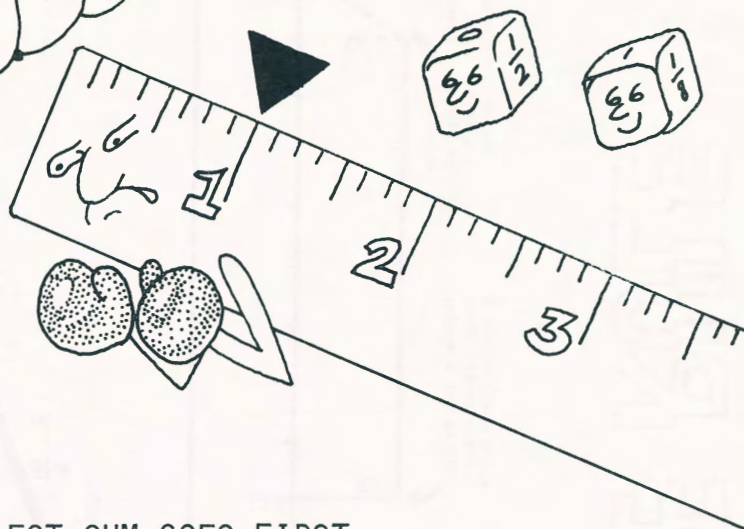
1. $3 - \frac{2}{3} = \underline{\hspace{2cm}}$
2. $2\frac{1}{2} - 1 = \underline{\hspace{2cm}}$
3. $4\frac{1}{2} - 2\frac{1}{2} = \underline{\hspace{2cm}}$
4. $\frac{5}{6} - \frac{3}{6} = \underline{\hspace{2cm}}$
5. $\frac{4}{5} - \frac{4}{5} = \underline{\hspace{2cm}}$

BEAT THE RULES

Number Line Model
Addition
FRACTIONS



NEEDED: 2 OR MORE PLAYERS
A 12" RULER FOR EACH
PLAYER
A MARKER FOR EACH
PLAYER 
A PAIR OF DICE, EACH
MARKED WITH 0, $\frac{1}{16}$,
 $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, 1



- RULES:
- 1) PLAYERS ROLL DICE, SMALLEST SUM GOES FIRST.
 - 2) EACH PLAYER ROLLS THE DICE AND, STARTING FROM ZERO, MOVES HIS MARKER A DISTANCE EQUAL TO THE SUM OF THE FRACTIONS ON THE DICE.
 - 3) IF DOUBLES ARE ROLLED, THE PLAYER GETS ANOTHER TURN.
 - 4) IF A PLAYER ADDS WRONG, HE LOSES THAT TURN, AND THE DISTANCE GOES TO THE PLAYER THAT FINDS THE MISTAKE.
 - 5) FIRST PLAYER TO MOVE HIS MARKER TO THE 12" MARK OR BEYOND IS THE WINNER.

Variations :

- 1) Same rules except the sum of the dice is subtracted from 12". First player to pass 0 wins.
- 2) Same rules except the winner is the first player to reach exactly 12. Player may choose to throw only one die for his turn. If a player's total moves his marker past 12, he loses that turn.
- 3) Rules of addition game are followed except if the sum has a numerator of 3 ($\frac{3}{4}$, $\frac{3}{8}$, $\frac{3}{16}$), the player loses his turn. (No lost turn if the sum is $\frac{6}{8}$ or $\frac{6}{16}$)
- 4) Use three dice. The player adds the fractions on any two dice and from that sum subtracts the fraction on the third die. Allow students to discover the best way to do this.



You could
GO METRIC
with this
activity
using a
centimeter
ruler and
by making
a different
unit die.

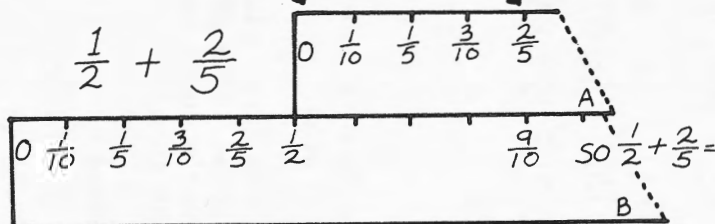
You may wish to make a
score sheet for students.

FRACTION SLIDE RULE

② LET'S TRY AN ADDITION PROBLEM

Place the 0 on the A scale directly above the $\frac{1}{2}$ on the B scale.

Find $\frac{2}{5}$ on the A scale, read directly down to find the answer on the B scale.



TRY THESE: $\frac{7}{10} + \frac{3}{5} =$ $1\frac{1}{5} + \frac{1}{2} =$ $\frac{1}{5} + 1\frac{3}{10} =$

Using the slide rule try to find a quick way to add three or more fractions.

$$\frac{3}{10} + \frac{1}{5} + \frac{9}{10} =$$

$$1 + \frac{2}{5} + \frac{1}{10} =$$

$$\frac{1}{5} + \frac{1}{10} + \frac{1}{2} + \frac{1}{10} =$$

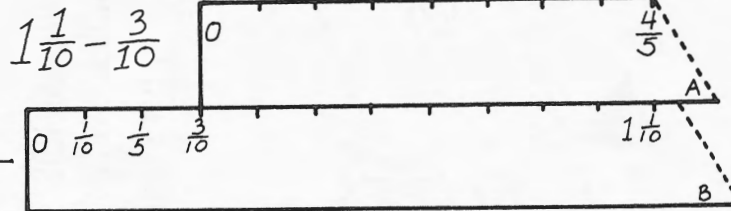
Hint; you shouldn't have to write anything down except your final answer.

$$\begin{array}{r} \frac{4}{5} \\ \frac{1}{10} \\ + \frac{1}{2} \end{array} \quad \begin{array}{r} \frac{3}{10} \\ \frac{7}{10} \\ + 1 \end{array} \quad \begin{array}{r} \frac{1}{10} \\ \frac{1}{5} \\ + \frac{2}{10} \end{array}$$

④ LET'S SUBTRACT

Place the 0 on the A scale directly above $\frac{3}{10}$ on the B scale.

Look on the A scale until you find $1\frac{1}{10}$. Read up to find your answer on the B scale.



⑤ TRY THESE:

$$\frac{9}{10} - \frac{1}{5} =$$

$$1\frac{1}{2} - \frac{7}{10} =$$

$$2 - \frac{4}{5} =$$

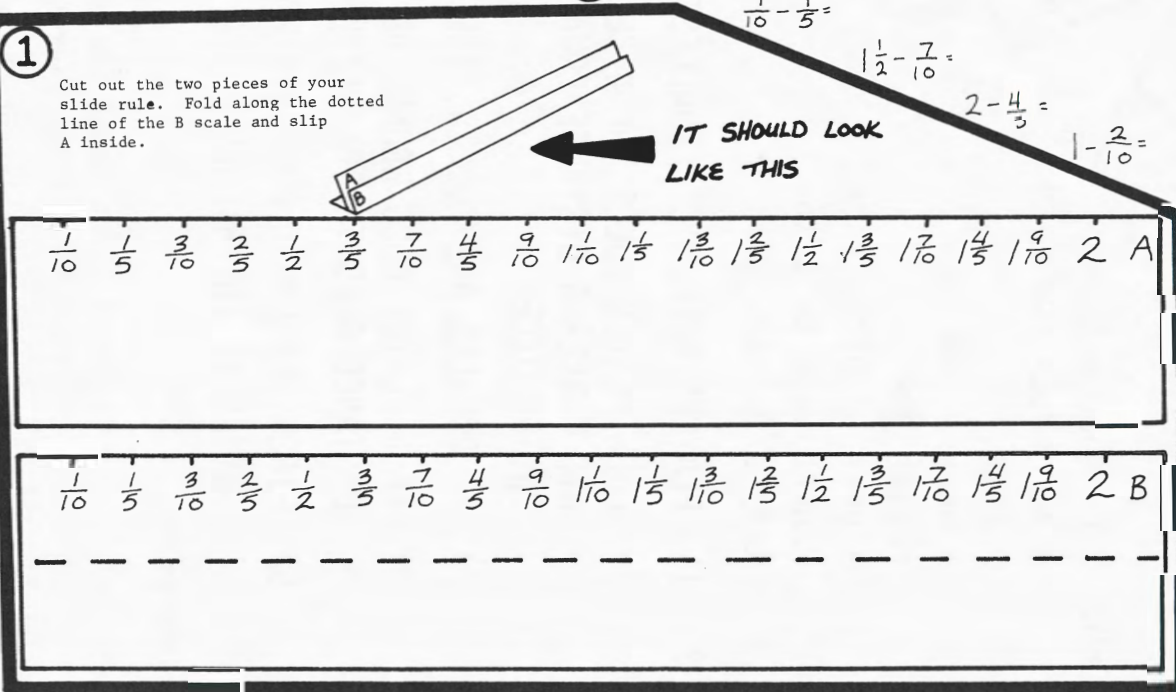
$$1 - \frac{2}{10} =$$

$$1\frac{3}{5} - \frac{9}{10} =$$

①

Cut out the two pieces of your slide rule. Fold along the dotted line of the B scale and slip A inside.

IT SHOULD LOOK LIKE THIS



A MASTERS PUZZLE

Fill the blanks in the puzzle below. SIMPLIFY your answer!



Sometimes you have to add up, down or from right to left

Try starting here

	=		+	$1\frac{11}{15}$	+	$\frac{5}{15}$		$1\frac{1}{7}$	+	$1\frac{6}{7}$	+		=	
+		=							+		=		=	
$3\frac{1}{3}$	=	$1\frac{21}{30}$	+	$\frac{7}{30}$	+	$1\frac{9}{30}$	+	$\frac{3}{30}$			=	$\frac{13}{14}$	+	$3\frac{7}{14}$
+		+							=		+		+	
$5\frac{2}{3}$			=	$\frac{6}{60}$	+	$\frac{9}{60}$	+	$\frac{19}{60}$			=	$2\frac{13}{14}$	+	$3\frac{5}{14}$
=								$2\frac{7}{10}$						
	=	$\frac{1}{9}$	+		+		+	$1\frac{2}{9}$	+	$\frac{2}{9}$	+	$2\frac{1}{9}$		
		+		=		=		$7\frac{6}{10}$						
			=	$2\frac{6}{18}$	+	$1\frac{8}{18}$		+					=	
		+		+		+			=	$2\frac{19}{20}$	+	$\frac{19}{20}$		
		$5\frac{5}{9}$		$1\frac{11}{18}$		$\frac{17}{18}$		=					+	
		=	$3\frac{1}{5}$	+	$2\frac{2}{5}$	+	$6\frac{4}{5}$		=	$9\frac{8}{20}$	+	$4\frac{16}{20}$		
				$1\frac{15}{18}$		$1\frac{9}{18}$								

Don't Get Clipped!

Add and simplify the following problems. In the puzzle below, connect your answers in order.

$3\frac{1}{2}$ $2\frac{5}{7}$ $4\frac{2}{7}$ $2\frac{1}{4}$
 $7\frac{2}{3}$ $5\frac{3}{4}$ $5\frac{3}{5}$ $4\frac{7}{8}$ $3\frac{3}{8}$
 $\frac{5}{9}$ $5\frac{3}{4}$ $7\frac{1}{8}$ $7\frac{1}{2}$ $8\frac{5}{8}$
 $8\frac{3}{4}$ $7\frac{2}{3}$ $7\frac{2}{3}$ $7\frac{1}{10}$ $4\frac{1}{2}$ $5\frac{1}{2}$
 $3\frac{2}{7}$ $6\frac{2}{3}$ $5\frac{1}{2}$ $5\frac{1}{2}$ $2\frac{7}{8}$

1) $4 + 3\frac{1}{2} =$

2) $5 + 2\frac{2}{3} =$

3) $\frac{1}{8} + \frac{5}{8} =$

4) $3\frac{3}{5} + 2 =$

5) $7 + \frac{1}{8} =$

6) $5\frac{1}{3} + 2\frac{1}{3} =$

7) $\frac{1}{5} + 4 =$

8) $5\frac{1}{5} + \frac{3}{5} =$

9) $2\frac{5}{8} + 6 =$

10) $\frac{1}{10} + 3\frac{3}{10} =$

11) $\frac{1}{4} + 2 =$

12) $2\frac{2}{8} + 1\frac{1}{8} =$

13) $4 + 3\frac{3}{5} =$

14) $\frac{1}{9} + \frac{4}{9} =$

15) $\frac{3}{4} + 8 =$

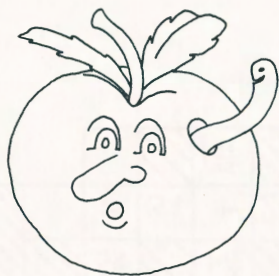
16) $\frac{2}{7} + 3\frac{3}{7} =$

17) $1 + 1\frac{7}{8} =$

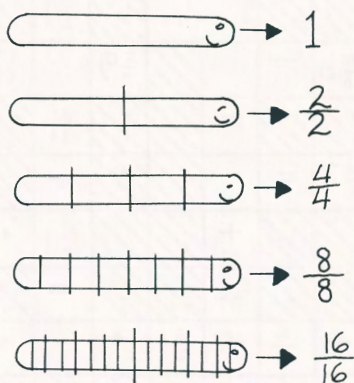
Like Denominators
Addition/Subtraction
FRACTIONS



INCH WORM FRACTIONS



Lines have been drawn to show how the inch worm is divided.



Use the inch worms to fill in these patterns.

$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16}$$

$$\frac{1}{8} = \frac{2}{16}$$

$$\frac{6}{8} = \frac{12}{16} = \frac{3}{4}$$

$$\frac{1}{4} = \frac{2}{8} = \frac{4}{16}$$

$$\frac{2}{8} = \frac{4}{16} = \frac{1}{4}$$

$$\frac{7}{8} = \frac{14}{16}$$

$$\frac{2}{4} = \frac{4}{8} = \frac{8}{16} = \frac{1}{2}$$

$$\frac{3}{8} = \frac{6}{16}$$

$$\frac{4}{8} = \frac{8}{16} = \frac{1}{2}$$

$$\frac{3}{4} = \frac{6}{8} = \frac{12}{16}$$

$$\frac{4}{8} = \frac{8}{16} = \frac{1}{2}$$

$$\frac{5}{8} = \frac{10}{16}$$

$$\frac{8}{8} = \frac{16}{16} = \frac{4}{4} = \frac{2}{2} = 1$$

THE INCH WORM CAN HELP YOU TO ADD LIKE FRACTIONS.

$$\frac{1}{8} + \frac{2}{8} = \text{worm with 3 parts shaded} = \frac{3}{8}$$

Shade in the parts to find these sums.

$$\frac{2}{4} + \frac{1}{4} = \text{worm with 3 parts shaded} = \frac{3}{4}$$

$$\frac{1}{2} + \frac{1}{2} = \text{worm with 2 parts shaded} = \frac{2}{2} \text{ OR } 1$$

$$\frac{1}{16} + \frac{7}{16} = \text{worm with 8 parts shaded} = \frac{8}{16} \text{ OR } \frac{1}{2}$$

$$\frac{4}{8} + \frac{3}{8} = \text{worm with 7 parts shaded} = \frac{7}{8}$$

$$\frac{5}{16} + \frac{9}{16} = \text{worm with 14 parts shaded} = \frac{14}{16} \text{ OR } \frac{7}{8}$$

$$\frac{1}{16} + \frac{2}{16} + \frac{3}{16} = \text{worm with 6 parts shaded} = \frac{6}{16} \text{ OR } \frac{3}{8}$$



NOW TRY SOME UNLIKE FRACTIONS.

$$\frac{1}{4} + \frac{1}{2} = \text{worm with 3 parts shaded} = \frac{3}{4}$$

$$\frac{1}{2} + \frac{1}{8} = \text{worm with 5 parts shaded} = \frac{5}{8}$$

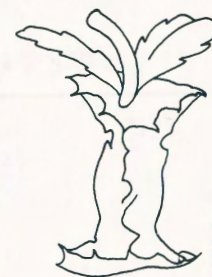
$$\frac{3}{4} + \frac{1}{8} = \text{worm with 7 parts shaded} = \frac{7}{8}$$

$$\frac{2}{16} + \frac{1}{4} = \text{worm with 5 parts shaded} = \frac{5}{16} \text{ OR } \frac{5}{16}$$

$$\frac{3}{8} + \frac{1}{4} = \text{worm with 5 parts shaded} = \frac{5}{8}$$

$$\frac{5}{8} + \frac{2}{16} = \text{worm with 7 parts shaded} = \frac{7}{8} \text{ OR } \frac{7}{8}$$

$$\frac{5}{16} + \frac{1}{2} = \text{worm with 11 parts shaded} = \frac{11}{16}$$



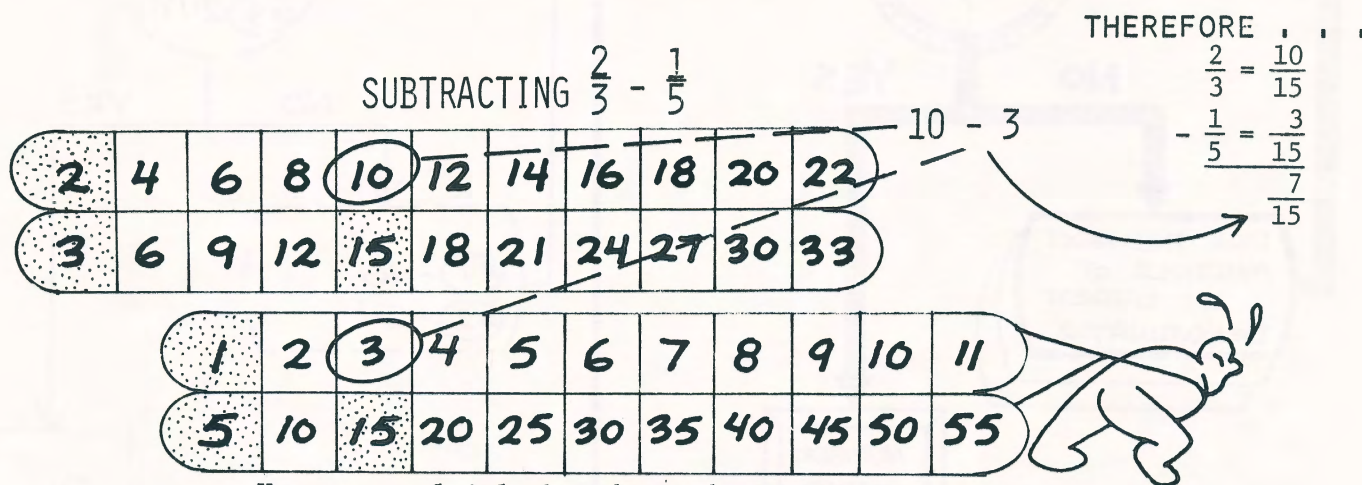
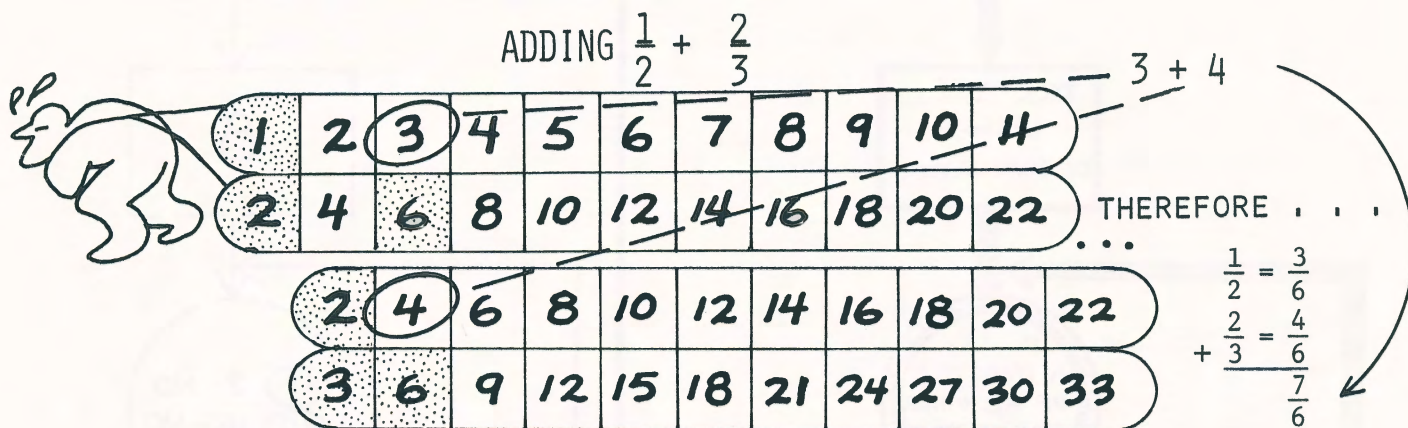
127
Addition
Fractions

5 3

MULTIPLE BOARDS - II

To add or subtract fractions:

- (1) Choose the multiple boards which begin with the same number as the terms of the fractions.
- (2) Move one pair of multiple boards until you have the same denominator lined up. (See the examples.)
- (3) Now you can add or subtract.



Use your multiple boards to do these problems.

- | | | |
|-----------------------------------|-----------------------------------|-------------------------------------|
| 1) $\frac{3}{4} + \frac{5}{6} =$ | 5) $\frac{2}{3} - \frac{4}{9} =$ | 9) $3\frac{1}{6} + 2\frac{3}{9} =$ |
| 2) $\frac{9}{10} - \frac{2}{3} =$ | 6) $\frac{7}{8} - \frac{5}{6} =$ | 10) $7\frac{2}{3} - 2\frac{3}{7} =$ |
| 3) $\frac{4}{7} + \frac{3}{8} =$ | 7) $\frac{2}{5} + \frac{2}{7} =$ | 11) $4\frac{5}{6} + 3\frac{4}{7} =$ |
| 4) $\frac{5}{8} - \frac{5}{12} =$ | 8) $\frac{3}{10} + \frac{2}{5} =$ | 12) $2\frac{1}{3} - 1\frac{3}{4} =$ |

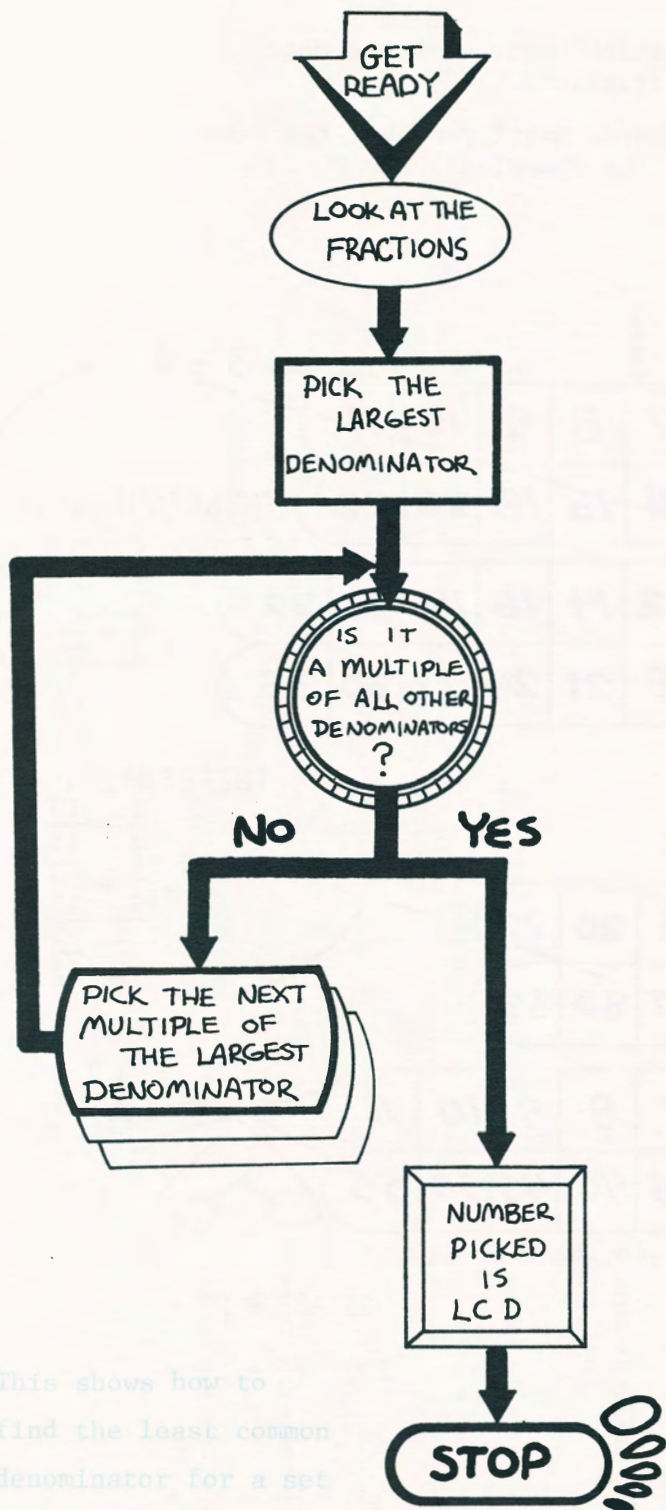


LCD FLOWCHART I

LCD
Addition/Subtraction
FRACTIONS

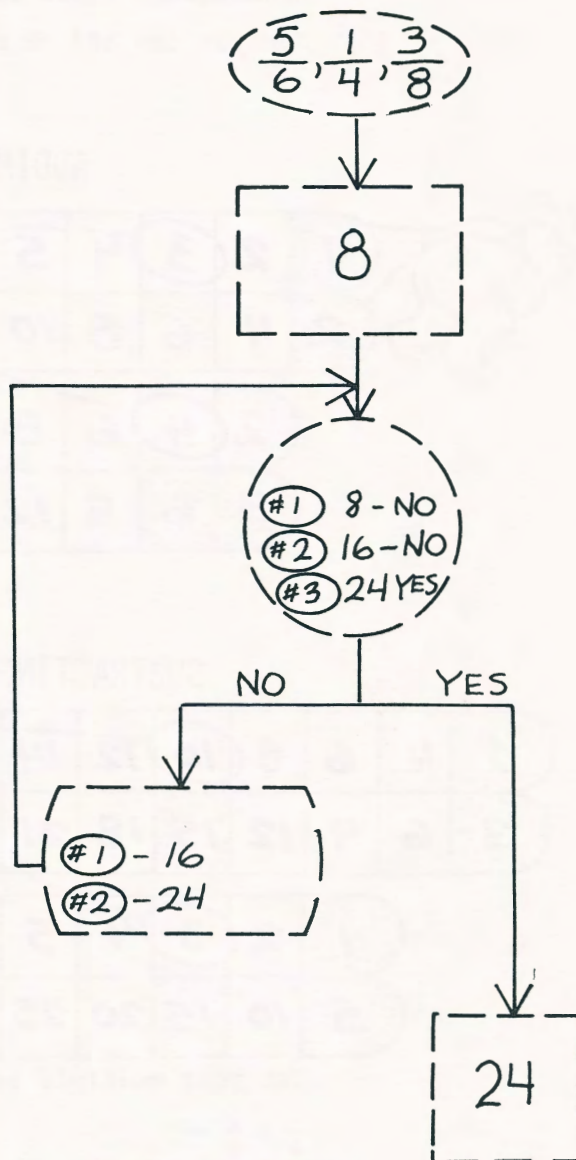


USING MULTIPLES



EXAMPLE :

$$\text{LCD } \left(\frac{5}{6}, \frac{1}{4}, \frac{3}{8} \right)$$



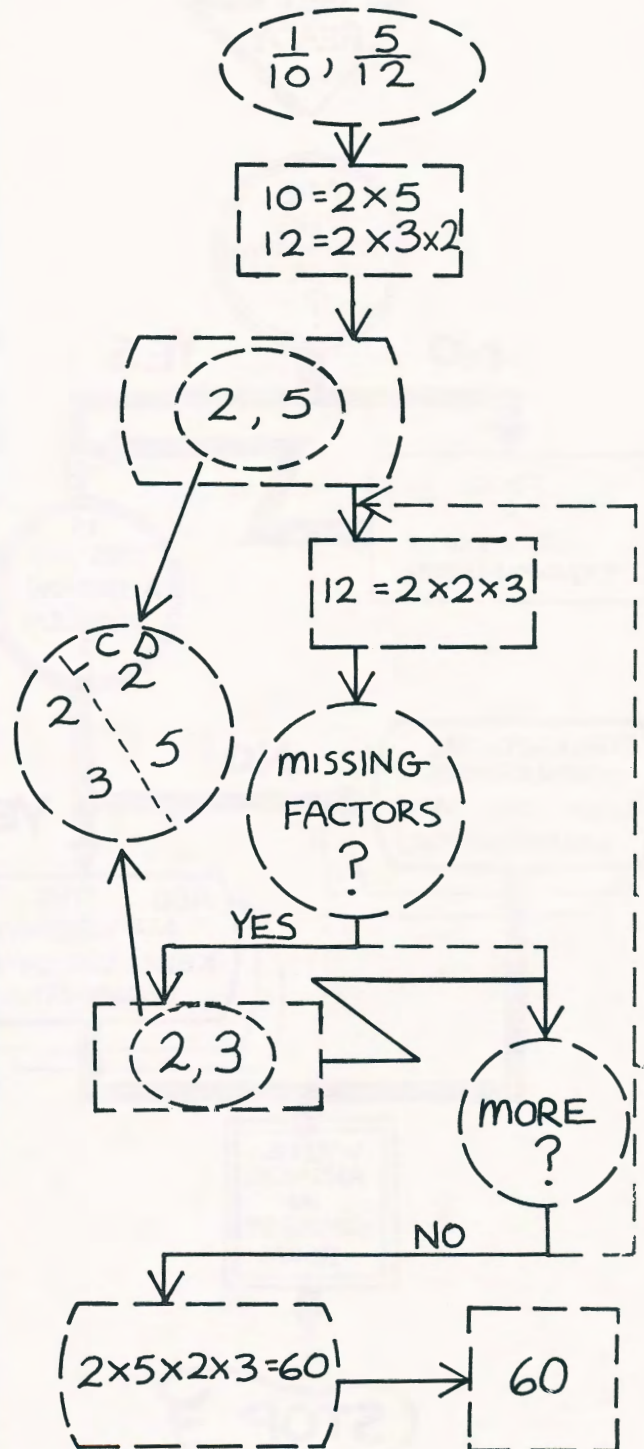
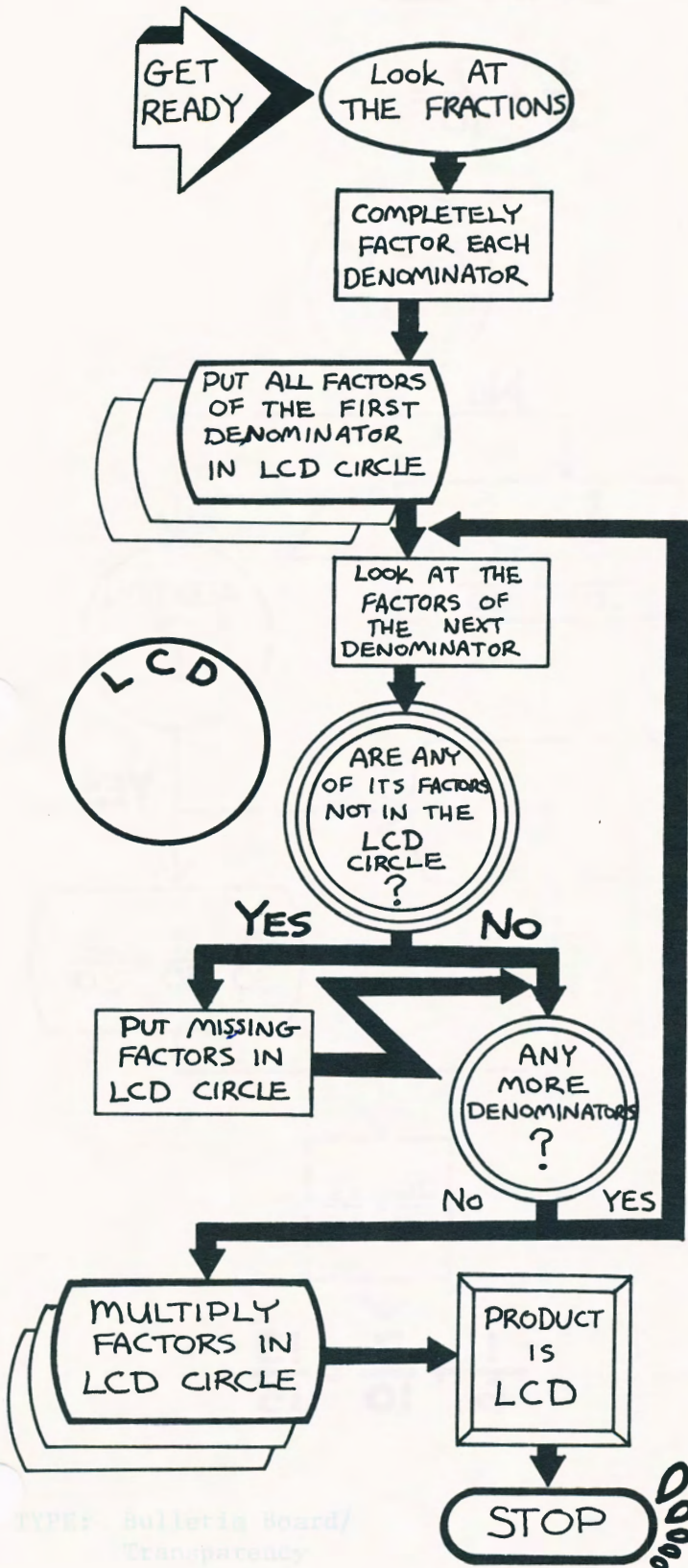
The least common denominator of the fractions $\frac{5}{6}$, $\frac{1}{4}$, and $\frac{3}{8}$ is 24.

LCD FLOWCHART II

LCD
Addition/Subtraction
FRACTIONS

USING PRIME FACTORS

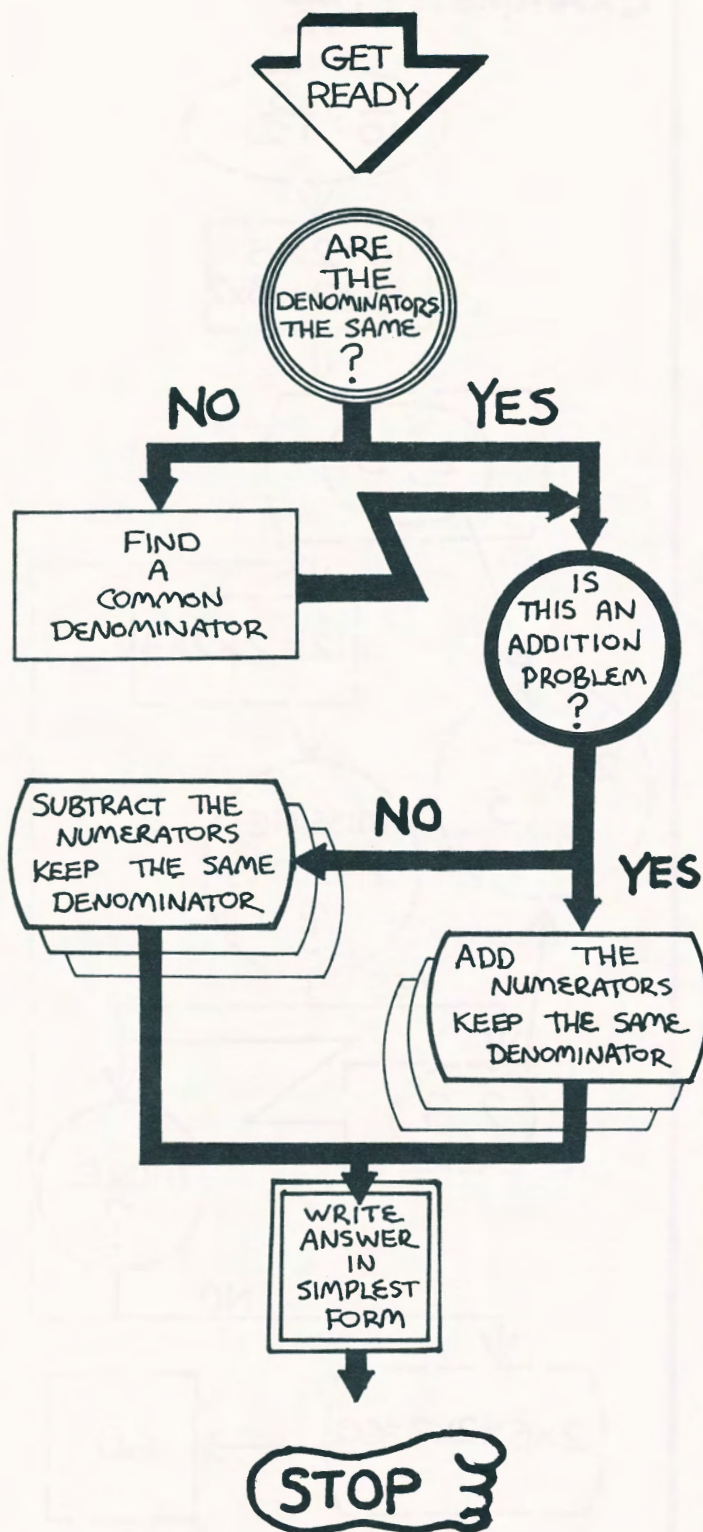
Example: $\frac{1}{10}, \frac{5}{12}$



FRACTION FLOWCHART

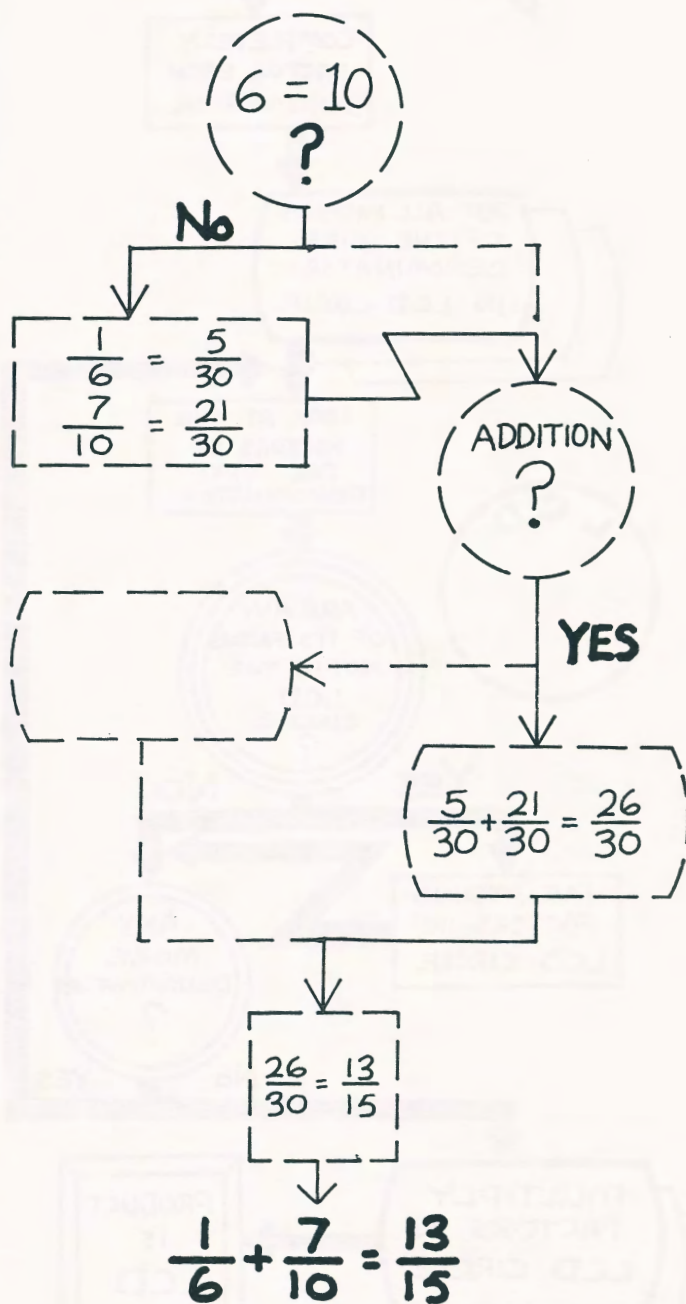
Unlike Denominators
Addition/Subtraction
FRACTIONS

TO ADD OR SUBTRACT FRACTIONS



EXAMPLE:

$$\frac{1}{6} + \frac{7}{10} =$$



Up-Down Approximation

I am a down arrow. ↓ What do I do?

$$3\frac{2}{3} \downarrow = 3$$

$$3\frac{1}{3} \downarrow = 3$$

$$3 \downarrow = 3$$

$$6\frac{9}{10} \downarrow = 6$$

$$6\frac{1}{2} \downarrow = 6$$

$$6\frac{1}{10} \downarrow = 6$$

$$\frac{2}{3} \downarrow = 0$$

$$\frac{1}{10} \downarrow = 0$$

$$\frac{17}{5} = 3\frac{2}{5} \downarrow = 3$$

Draw a ring around each true statement below.

$$2\frac{3}{4} \downarrow = 2$$

$$5\frac{1}{7} \downarrow = 5$$

$$8 \downarrow = 8$$

$$\frac{1}{3} \downarrow = 0$$

$$\frac{15}{4} \downarrow = 3\frac{3}{4} \downarrow = 3$$

$$4\frac{8}{9} \downarrow = 5$$

$$\frac{3}{4} \downarrow = 1$$

$$\frac{12}{5} \downarrow = 2\frac{2}{5} \downarrow = 2$$

$$\frac{16}{2} \downarrow = 8 \downarrow = 8$$

$$\frac{5}{6} \downarrow = 0$$

$$11\frac{1}{6} \downarrow = 11$$

$$\frac{1}{2} \downarrow = 0$$

Now complete these statements:

Example: $4\frac{1}{3} \downarrow = 4$

$$12\frac{5}{6} \downarrow = 12$$

$$7\frac{3}{4} \downarrow = 7$$

$$9\frac{1}{5} \downarrow = \underline{\hspace{2cm}}$$

$$4\frac{1}{3} \downarrow + 7\frac{3}{4} \downarrow = 4 + 7 = 11$$

$$12\frac{5}{6} \downarrow + 9\frac{1}{5} \downarrow = 12 + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$23\frac{1}{2} \downarrow = \underline{\hspace{2cm}}$$

$$15\frac{7}{8} \downarrow = \underline{\hspace{2cm}}$$

$$23\frac{1}{2} \downarrow - 15\frac{7}{8} \downarrow = \underline{\hspace{2cm}} - \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

Answer the following questions:

$$\text{Does } 18\frac{1}{4} \downarrow + 5 \downarrow = (18\frac{1}{4} + 5\frac{1}{4}) \downarrow ?$$

$$\text{Does } 23\frac{7}{8} \downarrow - 17\frac{5}{8} \downarrow = (23\frac{7}{8} - 17\frac{5}{8}) \downarrow ?$$

$$\text{Does } 13\frac{1}{3} \downarrow + 7\frac{1}{3} \downarrow = (13\frac{1}{3} + 7\frac{1}{3}) \downarrow ?$$

$$\text{Does } 17\frac{1}{5} \downarrow - 5\frac{4}{5} \downarrow = (17\frac{1}{5} - 5\frac{4}{5}) \downarrow ?$$

What are some numbers so that $\square \downarrow - \triangle \downarrow = (\square - \triangle) \downarrow$?

What do you think an up-arrow would mean? Do all the above problems using up-arrows instead of down-arrows.



Up-Down Approximation (Continued)

To introduce the \updownarrow (UP-DOWN ARROW) to your students, you might use the following examples:

You might use the following examples for introducing the \updownarrow (UP-DOWN ARROW)

$$3 \updownarrow = 3$$

$$2\frac{4}{5} \updownarrow = 3$$

$$2\frac{3}{5} \updownarrow = 3$$

$$2\frac{2}{5} \updownarrow = 2$$

$$2\frac{1}{5} \updownarrow = 2$$

$$2 \updownarrow = 2$$

$$3\frac{3}{4} \updownarrow = 4$$

$$3\frac{1}{2} \updownarrow = 3$$

$$3\frac{1}{4} \updownarrow = 3$$

Complete these statements:

$$7\frac{2}{3} \updownarrow = 8$$

$$8\frac{1}{3} \updownarrow = \underline{\hspace{2cm}}$$

$$8\frac{1}{5} \updownarrow = \underline{\hspace{2cm}}$$

$$4\frac{7}{9} \updownarrow = \underline{\hspace{2cm}}$$

$$7\frac{2}{3} \updownarrow + 8\frac{1}{5} \updownarrow = 8 + \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$$

$$8\frac{1}{3} \updownarrow - 4\frac{7}{9} \updownarrow = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$$

You might also have your students consider some or all of the following:

- Does $2\frac{1}{5} \updownarrow + 3\frac{2}{3} \updownarrow = 3\frac{2}{3} \updownarrow + 2\frac{1}{5} \updownarrow$?
- Does $17\frac{1}{2} \updownarrow - 8\frac{2}{3} \updownarrow = 8\frac{2}{3} \updownarrow - 17\frac{1}{2} \updownarrow$?
- Does $5\frac{1}{3} \updownarrow + 7\frac{1}{3} \updownarrow = 5\frac{1}{3} + 7\frac{1}{3}$?
- Does $5\frac{1}{3} \updownarrow + 7\frac{1}{3} \updownarrow = (5\frac{1}{3} + 7\frac{1}{3}) \updownarrow$?
- Does $10\frac{3}{4} \updownarrow - 8\frac{1}{4} \updownarrow = 10\frac{3}{4} - 8\frac{1}{4}$?
- Does $10\frac{3}{4} \updownarrow - 8\frac{1}{4} \updownarrow = (10\frac{3}{4} - 8\frac{1}{4}) \updownarrow$?

- Can you think of a problem that is like problem (c) and that will have a yes answer?
- Can you give a problem like (d) that will have a yes answer?
- Can you give a problem like (e) that will have a yes answer?
- Can you give a problem like (f) that will have a no answer?



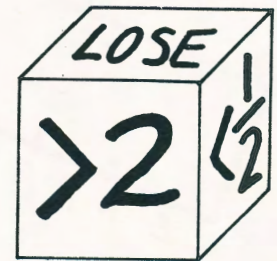
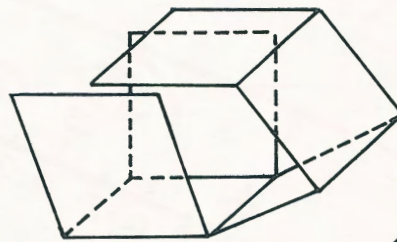
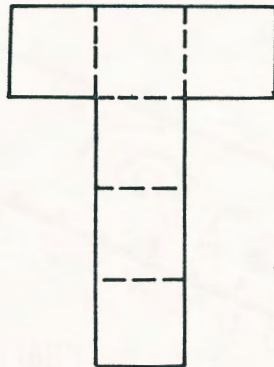
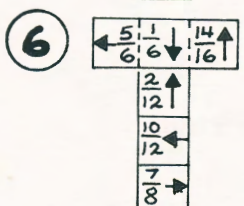
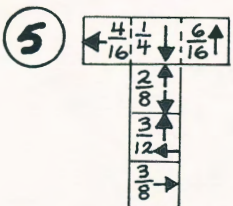
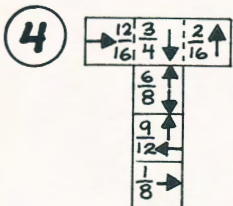
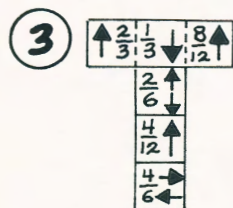
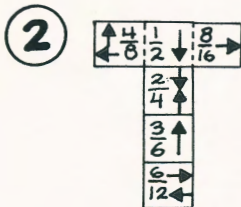
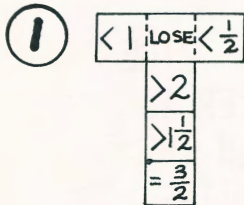
FRACTION SOLITAIRE

Unlike Denominators
Addition
FRACTIONS



MAKE THIS SET OF FRACTION CUBES FROM WOOD, FOAM RUBBER, STYROFOAM OR TAGBOARD. STICK-ON PAPER LABELS WORK WELL.

INSTRUCTION CUBE



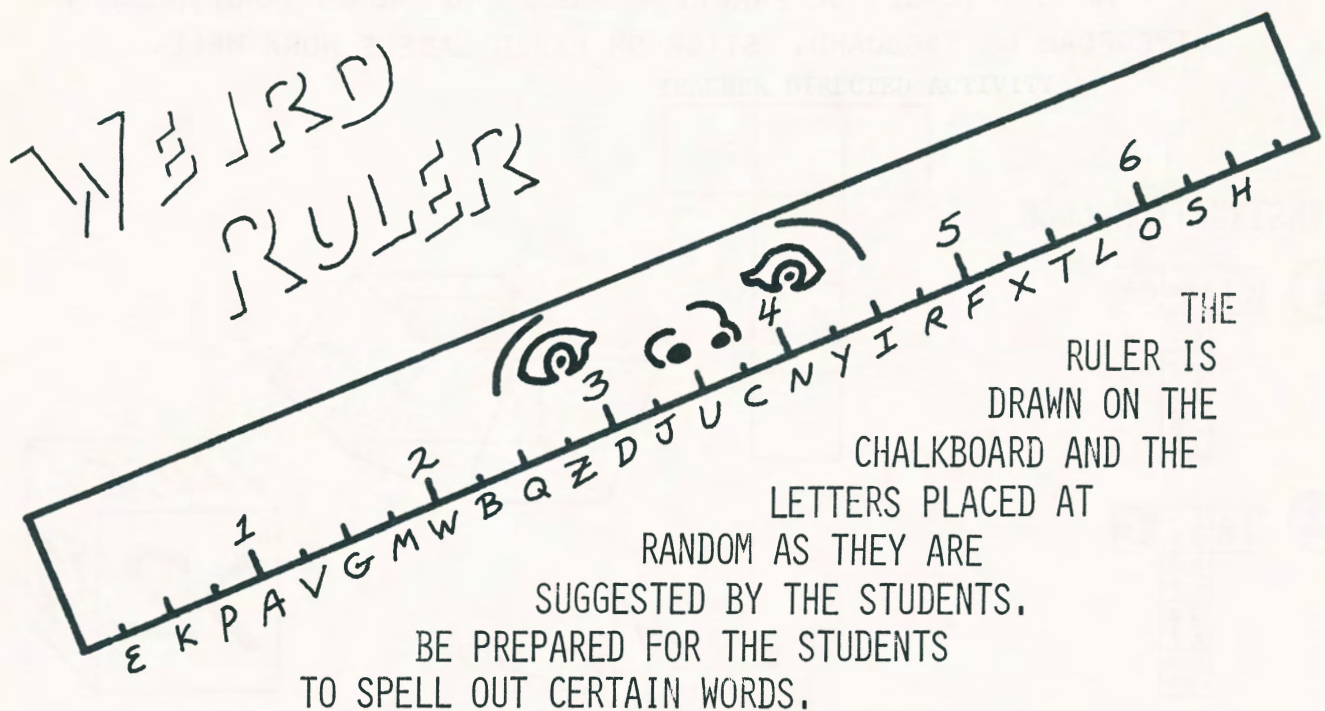
RULES:

1. ROLL THE CUBES.
2. ADD TWO OR MORE CUBES UNTIL YOU GET AN ANSWER ALLOWED BY THE INSTRUCTION CUBE OR UNTIL YOU SEE THAT IT IS IMPOSSIBLE.
3. THE CUBES MAY BE TURNED IN THE DIRECTION OF THE ARROWS TO GET EQUIVALENT FRACTIONS.
4. IF YOU CAN GET A CORRECT ANSWER, YOU GET A POINT. IF YOU CAN'T, YOU LOSE A POINT.
5. IN SOLITAIRE IF YOU GET 10 POINTS BEFORE YOU LOSE 10 POINTS, YOU WIN THE GAME.
6. IF YOU PLAY WITH OTHER STUDENTS, CHECK EACH OTHER'S ADDITION. THE FIRST ONE TO GET 10 POINTS IS THE WINNER OF THE GAME.

IDEA FROM: C.O.L.A.M.D.A.

TYPE: Game

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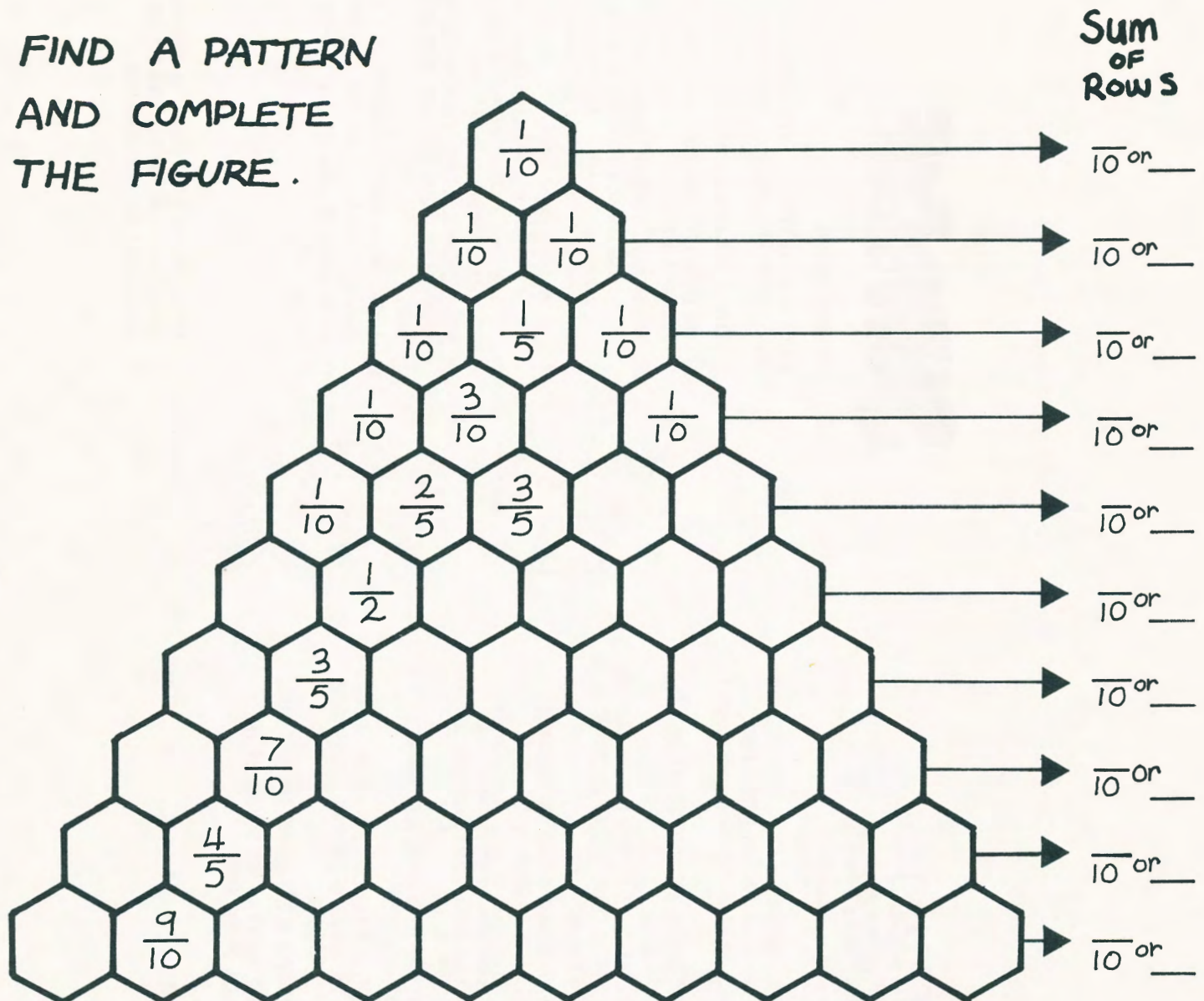


The following questions are examples which can be answered using the ruler.

1. How long is CAT? ($3\frac{3}{4} + 1 + 5\frac{1}{2} = 9\frac{1}{4}$)
2. Which student has the longest name?
3. Which is longer: forever, eternity, infinity?
4. Can you find a word exactly 3 units long?
5. Which is longer: MINUTE or HOUR?
6. What is the longest three letter word you can find?

PASCAL'S FRACTIONS

FIND A PATTERN
AND COMPLETE
THE FIGURE.



THIS WILL
BUILD YOUR TENTHS
SENSE!



FRACTION BLACK-JACK

For construction of Fraction Rummy
Cards see section on Lab Materials.

NEEDED: 2 OR MORE PLAYERS
A DECK OF 12THS FRACTION RUMMY CARDS.
(OR A DECK OF 10THS FRACTION RUMMY CARDS.)

- RULES:
- PLAYERS DRAW ONE CARD, LOWEST CARD IS PLAYER 1.
 - EACH PLAYER TRIES TO GET CARDS THAT ADD TO 2 OR AS CLOSE TO 2 AS HE CAN GET WITHOUT GOING ABOVE 2.
 - PLAYER 1 DRAWS A CARD AND ANNOUNCES WHETHER HE IS GOING TO "PLAY" OR "STAY,"
 - PLAY MEANS HE WILL DRAW ANOTHER CARD WHEN IT IS HIS NEXT TURN.
 - STAY MEANS HE DOES NOT WANT ANY MORE CARDS.
 - PLAY CONTINUES UNTIL ALL PLAYERS HAVE ANNOUNCED "STAY."
 - THE PLAYER CLOSEST TO 2 BUT NOT ABOVE 2 IS THE WINNER OF THE HAND. TIES COUNT 1 WIN FOR EACH.
 - RESHUFFLE ALL CARDS FOR THE NEXT HAND.
 - FIRST PLAYER TO WIN 5 HANDS IS THE WINNER OF THE GAME.



SUM-DIF

NEEDED: 2 OR MORE PLAYERS

A DECK OF 12THS FRACTION RUMMY CARDS
(OR A DECK OF 10THS FRACTION RUMMY CARDS)

- RULES:
- EACH PLAYER IS DEALT 3 CARDS FACE UP.
 - THE CARDS LEFT OVER ARE PLACED FACE DOWN TO FORM A STACK.
 - ON HIS TURN, A PLAYER DRAWS 2 CARDS.
 - IF THE SUM OR DIFFERENCE OF HIS 2 CARDS IS THE SAME AS THE FRACTION SHOWING ON ONE OF THE "UP" CARDS, HE KEEPS ALL 3 CARDS.
 - IF THE SUM OR DIFFERENCE IS NOT THE SAME, THE 2 CARDS ARE PLACED FACE-UP WITH THE OTHER FACE-UP CARDS.
 - WHEN THE STACK IS GONE, PLAYERS RECEIVE 1 POINT FOR EACH CARD THEY HAVE BEEN ABLE TO KEEP.
 - A BONUS OF 10 POINTS IS AWARDED IF A PLAYER IS ABLE TO USE ALL OF HIS FACE UP CARDS.
 - FIRST PLAYER TO SCORE 50 POINTS IS THE WINNER OF THE GAME.



VARIATION:

BEFORE DRAWING HIS 2 CARDS, AN OPPONENT COULD CALL EITHER "SUM" OR "DIFFERENCE," AND THE PLAYER WOULD USE THAT OPERATION.

Two-A-Part

Unlike Denominators
Addition
FRACTIONS



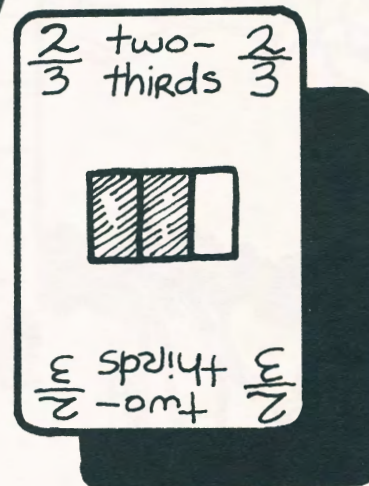
MAKE A SPECIAL DECK OF 52 CARDS.

THERE IS 1 EACH OF $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, $\frac{5}{5}$, $\frac{6}{6}$, $\frac{8}{8}$.

THERE ARE 2 EACH OF $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{1}{6}$. . .

$\frac{5}{6}$, $\frac{1}{8}$, . . . , $\frac{7}{8}$, $\frac{4}{3}$, $\frac{5}{3}$, $\frac{5}{4}$, $\frac{7}{6}$, $\frac{10}{8}$.

EACH CARD SHOULD LOOK LIKE THIS.



I HAVE
2 BOOKS

Rules:

1. 2-5 players, deal 7 cards each.
2. Put rest of the cards in a stack face down. Top card is turned face-up to start a discard pile.
3. Player to the left of the dealer draws top card (off stack or discard pile).
4. Players form "books" to lay down. Books are sets of cards that add up to 2.
5. Players discard 1 card unless it would give him less than 3 cards in his hand.
6. The game continues until one player "goes out" by laying down all his cards.
7. Scoring: 2-card book = 1 point, 3-card book = 2 points, 4 or more cards = 3 points, "going out" = 3 points. Subtract 1 point for each card not laid down.
8. 30 points win.
9. If the stack is used up and nobody has gone out, the discard pack is shuffled and used as a new stack.



IDEA FROM: C.O.L.A.M.D.A.

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WORLD TRACK RECORDS -FIELD EVENTS-

Math's Connections
Addition/Subtraction
FRACTIONS



WORLD TRACK RECORDS*		
	MEN	WOMEN
HIGH JUMP	7' 6 $\frac{5}{8}$ "	6' 4 $\frac{1}{2}$ "
POLE VAULT	18' 5 $\frac{3}{4}$ "	—
LONG JUMP	29' 2 $\frac{1}{2}$ "	22' 5 $\frac{1}{4}$ "
TRIPLE JUMP	57' 2 $\frac{3}{4}$ "	—
SHOT PUT	71' 7"	60' 0"
DISCUS	224' 5"	221' 9"
JAVELIN	308' 8"	213'

1. HOW MUCH HIGHER IS THE MEN'S POLE VAULT RECORD THAN THE MEN'S HIGH JUMP RECORD?
2. HOW MUCH FARTHER IS THE MEN'S JAVELIN RECORD THAN THE MEN'S DISCUS RECORD?
3. HOW MUCH FARTHER IS THE WOMEN'S DISCUS RECORD THAN THE WOMEN'S JAVELIN RECORD?
4. HOW MUCH HIGHER IS THE WOMEN'S LONG JUMP RECORD THAN THE WOMEN'S HIGH JUMP RECORD?
5. A PENTATHLON IS MADE UP OF FIVE EVENTS. SUPPOSE ONE WOMAN HELD ALL FIVE OF THE ABOVE LISTED WORLD RECORDS. WHAT WOULD BE THE TOTAL OF THIS "PENTATHLON"?
6. HOW DO YOU THINK THE WEIGHTS OF THE MEN'S AND WOMEN'S DISCUS COMPARE? WHICH ONE IS HEAVIER AND BY HOW MUCH?
7. WHY AREN'T THERE RECORDS FOR THE WOMEN'S POLE VAULT AND THE TRIPLE JUMP?

THE GREATEST GAME

NEEDED: 2 OR MORE PLAYERS
INDIVIDUAL GAME SHEETS
PENCILS
DIE

RULES: A LEADER ROLLS THE DIE, HE CALLS OUT THE NUMERAL SHOWN. EACH PLAYER DECIDES IN WHICH SHAPE TO PLACE THIS NUMERAL. THUS THE NUMERAL COULD BE WRITTEN IN ALL THE SQUARES, OR IN ALL THE CIRCLES, OR IN ALL THE TRIANGLES, OR IN THE DIAMOND WHICH INDICATES THE NUMERAL IS REJECTED.

THIS PROCESS CONTINUES THREE MORE TIMES. AFTER 4 NUMERALS HAVE BEEN ROLLED AND PLACED IN THE SPACES, EACH PLAYER COMPUTES THE SUM AND THE PLAYER WITH THE GREATEST TOTAL SUM IS THE WINNER.

GAME I

$$\frac{\bigcirc}{\square} = \underline{\hspace{2cm}}$$

$$\frac{\triangle}{\square} = \underline{\hspace{2cm}}$$

$$\frac{1}{\square} = \underline{\hspace{2cm}}$$

$$\frac{1}{\square} = \underline{\hspace{2cm}}$$

$$\frac{1}{\square} = \underline{\hspace{2cm}}$$

$$\frac{1}{\square} = \underline{\hspace{2cm}}$$

$$\frac{1}{\square} = \underline{\hspace{2cm}}$$

$$\frac{1}{\square} = \underline{\hspace{2cm}}$$

$$\frac{1}{\square} = \underline{\hspace{2cm}}$$

$$\frac{1}{\square} = \underline{\hspace{2cm}}$$

$$\frac{1}{\square} = \underline{\hspace{2cm}}$$

$$\frac{1}{\square} = \underline{\hspace{2cm}}$$

$$\frac{1}{\square} = \underline{\hspace{2cm}}$$

$$\frac{1}{\square} = \underline{\hspace{2cm}}$$

$$\frac{1}{\square} = \underline{\hspace{2cm}}$$

$$\frac{1}{\square} = \underline{\hspace{2cm}}$$

$$\frac{1}{\square} = \underline{\hspace{2cm}}$$

$$\frac{1}{\square} = \underline{\hspace{2cm}}$$

$$\frac{1}{\square} = \underline{\hspace{2cm}}$$

$$\frac{1}{\square} = \underline{\hspace{2cm}}$$

GAME II

$$\square + \frac{1}{\triangle} = \underline{\hspace{2cm}}$$

$$\bigcirc + \frac{1}{\triangle} = \underline{\hspace{2cm}}$$

$$\triangle - \frac{1}{\triangle} = \underline{\hspace{2cm}}$$

$$\diamond = \underline{\hspace{2cm}}$$

REJECT

TOTAL

GAME III

$$\frac{\bigcirc}{\triangle} + \frac{1}{2} = \underline{\hspace{2cm}}$$

$$\frac{\triangle}{\square} + \frac{1}{3} = \underline{\hspace{2cm}}$$

$$\diamond = \underline{\hspace{2cm}}$$

REJECT

TOTAL

FRACTION Magic SQUARES

Unlike Denominators
Addition/Subtraction
FRACTIONS



HERE ARE SOME
MAGIC SQUARES.
ALL ROWS, COLUMNS, AND
DIAGONALS SHOULD ADD
UP TO THE SAME
NUMBER.

MAGIC $1\frac{1}{4}$ SQUARE

$\frac{2}{3}$		$\frac{1}{2}$
		$\frac{7}{12}$

MAGIC 24 SQUARE

$5\frac{3}{5}$		$10\frac{2}{5}$
$6\frac{4}{5}$		

MAGIC $5\frac{2}{3}$ SQUARE

$2\frac{2}{3}$	$\frac{1}{3}$		
		$1\frac{2}{3}$	$\frac{1}{3}$
$\frac{3}{2}$	$\frac{7}{6}$		
	$\frac{7}{3}$		$\frac{1}{6}$

MAGIC $2\frac{1}{2}$ SQUARE



USE THESE NUMBERS
 $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, 1, 1\frac{1}{6}, 1\frac{1}{3}, 1\frac{1}{2}$.
BE CAREFUL.

MAGIC $8\frac{1}{8}$ SQUARE

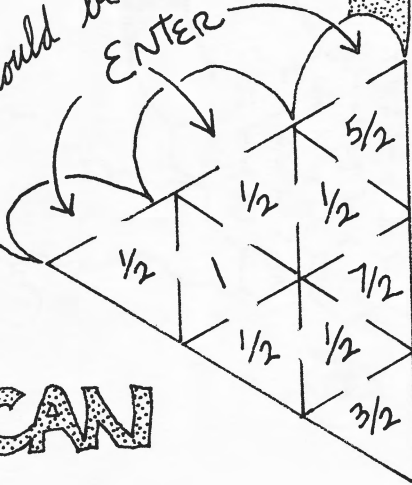
$2\frac{1}{8}$			1	
	$\frac{5}{8}$	$\frac{7}{8}$	$1\frac{3}{4}$	
		$1\frac{5}{8}$	$2\frac{1}{2}$	$2\frac{3}{4}$
$1\frac{1}{4}$		$2\frac{3}{8}$		$\frac{3}{8}$
$1\frac{3}{8}$	$2\frac{1}{4}$			$1\frac{1}{8}$

Have students explain
how they started filling
in the magic squares.



Here we half a maye.
the sum of your Path
should be seven.

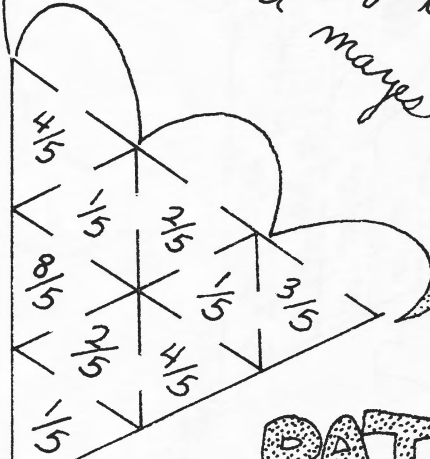
ENTER



CAN

Your Path
must add
up to the
number on
the balloon
basket.

Some people half
fifths trying to figure
out mayes.



PATH?

You

FINISH

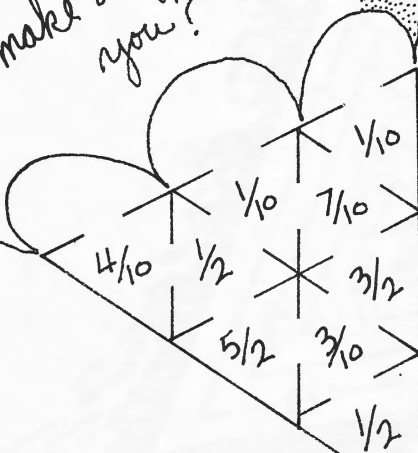
7

2

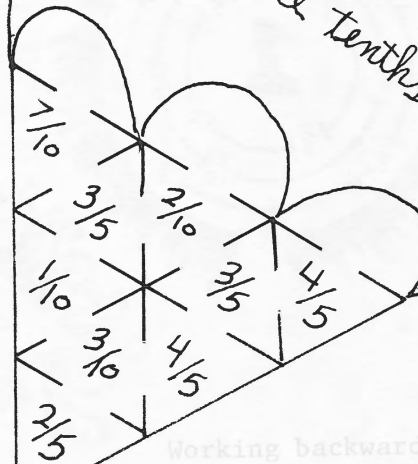
FIND

A

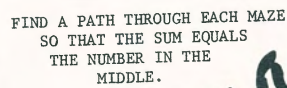
There are halves
in this maye. Does
this make any tenths to
you?



Would this be
easier if the fifths
were tenths?



Working backwards is a
good strategy for these
problems.



ALMOST GAMES

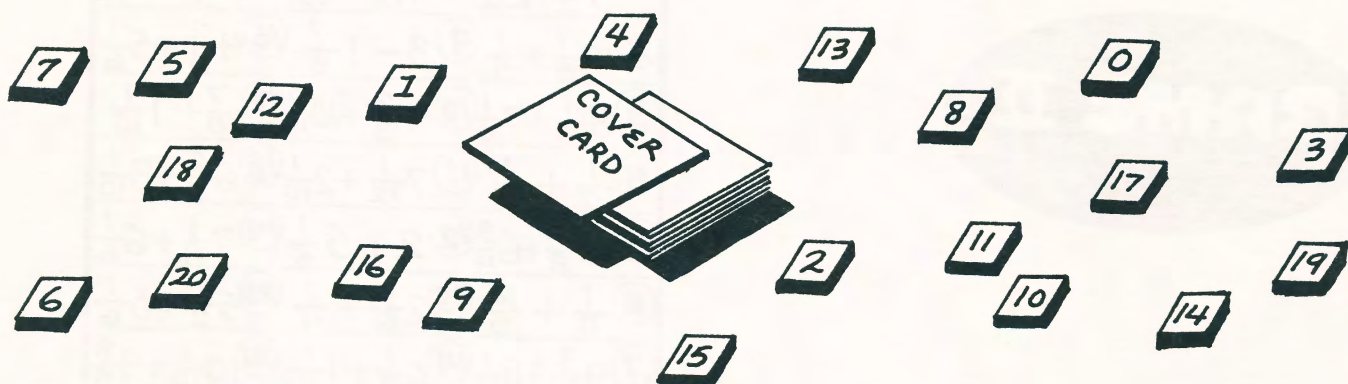
AN APPROXIMATION GAME FOR 2 TO 3 PLAYERS

WHAT YOU NEED:

- 21 TILES NUMBERED 0 THROUGH 20.
- DECK OF 21 GAME CARDS.
- 1 COVER CARD.

PLAYING:

PLACE THE GAME CARDS FACE UP IN A PILE IN THE CENTER OF THE DESK. PUT THE COVER CARD ON THE TOP OF THE DECK.



GAME I

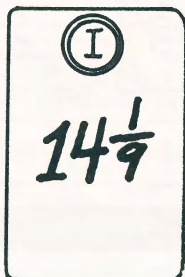
Remove the cover card. Look at the fraction for Game I. Decide which whole number it is closest to. Point to that whole number.

If you are the first player to point to the correct answer, you get to take the game card. Do not pick up the number tile. Continue playing until all the game cards are gone.

The winner is the player who ends up with the most game cards.

GAME CARDS

It helps to color code the games. Cards may be made on 3 x 5 index cards. You may decide to put each game on a separate set of cards, or make up problems of your own.



SAMPLE CARD I

21 CARDS					
① 14 $\frac{1}{9}$	⑧ 9 $\frac{2}{6}$	⑮ 16 $\frac{2}{8}$			
② 4 $\frac{1}{3}$	⑨ 10 $\frac{3}{10}$	⑯ 5 $\frac{9}{10}$			
③ 20 $\frac{3}{8}$	⑩ 18 $\frac{2}{3}$	⑰ 12 $\frac{4}{5}$			
④ 11 $\frac{5}{6}$	⑪ 1 $\frac{3}{9}$	⑱ 4 $\frac{5}{7}$			
⑤ 17 $\frac{4}{6}$	⑫ 7 $\frac{1}{2}$	⑲ $\frac{1}{8}$			
⑥ 1 $\frac{7}{8}$	⑬ 3 $\frac{1}{4}$	⑳ 7 $\frac{1}{5}$			
⑦ 16 $\frac{2}{4}$	⑭ 10 $\frac{3}{4}$	㉑ 15 $\frac{2}{7}$			

ALMOST GAMES

(CONTINUED)

Game II: Play the same as Game I only point to the whole number closest to the sum.

ADDITION

GAME II

21 CARDS

- | | | |
|----------------------------------|-----------------------------------|-----------------------------------|
| ① $3\frac{1}{2} + 2\frac{1}{2}$ | ⑧ $4\frac{1}{12} + \frac{2}{12}$ | ⑮ $5\frac{1}{3} + 14\frac{1}{12}$ |
| ② $12\frac{1}{5} + \frac{1}{5}$ | ⑨ $3 - 1\frac{7}{8}$ | ⑯ $4\frac{2}{3} + 5\frac{1}{6}$ |
| ③ $8\frac{1}{3} + 7\frac{1}{10}$ | ⑩ $4\frac{2}{3} + 10\frac{9}{10}$ | ⑰ $\frac{7}{10} + 1\frac{1}{10}$ |
| ④ $\frac{1}{2} + \frac{1}{4}$ | ⑪ $7\frac{1}{12} + 2\frac{1}{10}$ | ⑱ $6\frac{2}{5} + 8\frac{1}{10}$ |
| ⑤ $10\frac{3}{8} + 6\frac{3}{4}$ | ⑫ $2 + 5\frac{1}{5}$ | ⑲ $7\frac{1}{4} + 6\frac{1}{6}$ |
| ⑥ $\frac{1}{8} + \frac{2}{3}$ | ⑬ $2\frac{5}{6} + \frac{1}{4}$ | ⑳ $5\frac{1}{2} + 5\frac{1}{6}$ |
| ⑦ $9\frac{3}{4} + 8\frac{1}{4}$ | ⑭ $6\frac{1}{2} + 1\frac{1}{2}$ | ㉑ $10\frac{1}{10} + 9\frac{3}{5}$ |

SUBTRACTION

GAME III

Game III: Point to the whole number closest to the difference.

21 CARDS

- | | | |
|----------------------------------|------------------------------------|-----------------------------------|
| ① $17\frac{5}{6} - 5\frac{3}{8}$ | ⑧ $17\frac{6}{7} - 5\frac{1}{7}$ | ⑮ $21\frac{1}{3} - 5\frac{3}{5}$ |
| ② $19\frac{1}{6} - 7\frac{3}{8}$ | ⑨ $11\frac{8}{9} - 2\frac{4}{9}$ | ⑯ $35\frac{4}{5} - 18\frac{3}{5}$ |
| ③ $14\frac{1}{4} - 8\frac{3}{4}$ | ⑩ $9\frac{7}{9} - 5\frac{7}{10}$ | ⑰ $3\frac{3}{8} - \frac{1}{8}$ |
| ④ $16\frac{1}{2} - 6\frac{7}{8}$ | ⑪ $4\frac{1}{6} - 3\frac{5}{6}$ | ⑱ $15 - 8\frac{3}{10}$ |
| ⑤ $23\frac{1}{5} - 4\frac{2}{5}$ | ⑫ $20 - 5\frac{7}{9}$ | ⑲ $5 - 2\frac{6}{7}$ |
| ⑥ $17\frac{1}{3} - 9\frac{1}{4}$ | ⑬ $26\frac{9}{10} - 6\frac{7}{10}$ | ⑳ $24\frac{1}{2} - 6\frac{3}{4}$ |
| ⑦ $18\frac{4}{5} - 14$ | ⑭ $20\frac{1}{3} - 6$ | ㉑ $10\frac{1}{2} - 9\frac{1}{5}$ |

Variations:

- 1) If a player points to the wrong whole number, he gives one of his game cards to the player who finds the right answer.
- 2) A student may play by himself against a clock.
- 3) A student may use only the number tablets 1-5 and make up more game cards that round off to one of these numbers.

NUMBER SENSE AND ARITHMETIC SKILLS

Placement Guide for Tabbed Dividers

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Fractions Commentary: Multiplication/Division (pages 560-568)

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Activity Cards - Geoboards - III	572	Whole model Mult./Division	Manipulative
Rulers and Number Lines	573	Number line model Mult./Division	Paper and pencil Manipulative
Factor Boards - II	574	Mult./Division	Manipulative
Fraction Flowchart	576	Mult./Division	Bulletin board Transparency
Other Ways	577	Division	Paper and pencil Transparency
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FRACTIONS: MULTIPLICATION/DIVISION

The topic of fractions has been a troublesome one for students and teachers alike. Multiplication and division of fractions are especially complicated. Their algorithms are not simple to motivate from a concrete point of view, and the teaching of the algorithms themselves come under attack from all sides.

STUDENTS:

"MY DAD SAYS HE'S NEVER DIVIDED FRACTIONS SINCE HE LEARNED IT IN SCHOOL. WHY DO I HAVE TO LEARN IT?"

"WHY CAN'T I USE MY CALCULATOR? I'LL CHANGE ALL THE FRACTIONS TO DECIMALS WITH MY CALCULATOR, THEN USE IT TO MULTIPLY OR DIVIDE."

TEACHER:

"WE HAVE TOO MUCH TO TEACH. WHY DON'T WE LET THE JUNIOR HIGH COVER FRACTION OPERATIONS, AND WE CAN FIND MORE TIME FOR INFORMAL GEOMETRY?"

TEACHER:

"WHEN WE GO METRIC, WE WON'T NEED ALL THESE COMPLICATED FRACTIONS ANYWAY."

MATHEMATICS

EDUCATOR:

"WHEN AND HOW SHOULD THE FORMAL ALGORITHMS OF MULTIPLICATION AND DIVISION OF FRACTIONS BE INTRODUCED? IN SIXTH GRADE? IN ALGEBRA CLASSES? WITH CONCRETE MATERIALS?"

While these questions are being pursued in the teachers' lounge, at mathematics education conferences and in teacher-student discussions, the teaching of multiplication and division of fractions goes on. The metric system and calculators will surely affect our use and teaching of fractions, but there are good reasons for including these topics in the curriculum. If students can multiply and divide fractions, we can use this understanding to develop meaning for the multiplication and division of decimals. Operations with fractions form a basis for some of the topics in algebra. More immediately, an understanding of the operations of fractions can enhance students' number sense. Let's develop meaning and understanding of operation with simple fractions instead of focusing on pages of drill with fractions seldom used in the everyday world.



The algorithms for multiplication and division of fractions form an odd couple. The multiplication algorithm is quite easy to remember and seems so "natural"—the product of the numerators over the product of the denominators. (This same "natural" approach is the one that so many students apply to fraction addition $\frac{a}{b} + \frac{c}{d} = \frac{a + c}{b + d}$.) On the other hand, the division algorithm with its "invert and multiply" rule is unnatural, is harder to understand and causes much confusion. By the end of seventh grade a large number of students will not be able to correctly multiply fractions. Fewer will be able to divide fractions.

An examination of some common error patterns reveals the strategies that many students have adopted in order to produce answers to multiplication and division exercises. Look at each exercise and determine the probable strategy employed.

$$\frac{2}{3} \times \frac{1}{5} = \frac{10}{15} \times \frac{3}{15} = \frac{30}{15}$$

$$\frac{9}{10} \cdot \frac{3}{10} = \frac{3}{10}$$

$$\frac{2}{3} \times \frac{1}{5} = \frac{10}{3}$$

$$7 \frac{3}{4} \div \frac{3}{4} = 7 \frac{1}{1} = 8$$

$$3 \frac{1}{2} \times 4 = 12 \frac{1}{2}$$

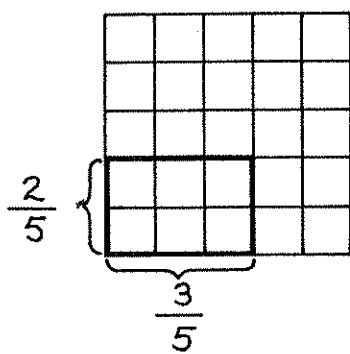
$$6 \frac{3}{4} \div 3 = 2 \frac{3}{4}$$

$$2 \frac{1}{4} \times \frac{2}{5} = 2 \frac{2}{20}$$

$$\frac{7}{8} \div \frac{2}{3} = \frac{21}{24} \div \frac{16}{24} = 1 \frac{5}{24}$$

Are these answers the result of symbol manipulation and devoid of any underlying meaning? Can these students relate the number statement, $\frac{2}{3} \times \frac{1}{5}$, to anything beyond symbols on a page? If there is no meaning behind the symbols, why aren't their answer seeking strategies (algorithms) as good as ours?

Why is each of the answer seeking strategies used on the previous page incorrect? Why should $\frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$ instead of $\frac{10}{15}$? There are several reasons. We want numbers and number operations to describe the world we live in. A rectangle with length $\frac{2}{5}$ metre and width $\frac{3}{5}$ metre will have area $\frac{6}{25}$ square metres.



$$\frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$$

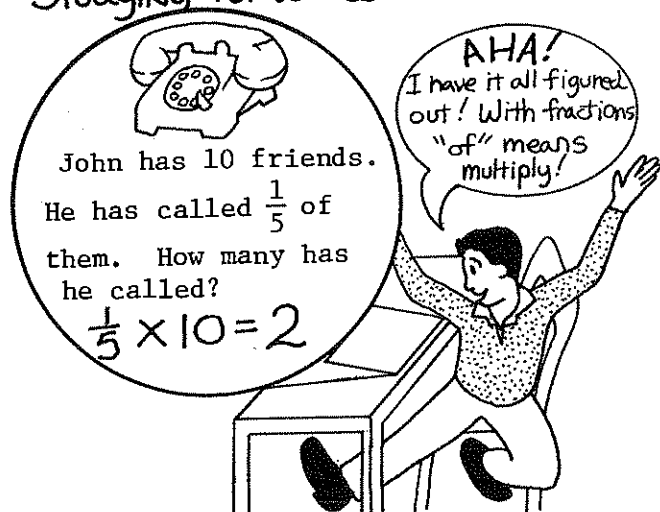


We also want to preserve relationships between whole numbers. The algorithm $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$ works for fractional forms of whole numbers. For example, $2 \times 3 = \frac{2}{1} \times \frac{3}{1} = \frac{2 \times 3}{1 \times 1} = \frac{6}{1} = 6$. Any algorithm which did not produce "6" would not be sensible. We can share these "secrets" with students so that our algorithms don't seem arbitrary.

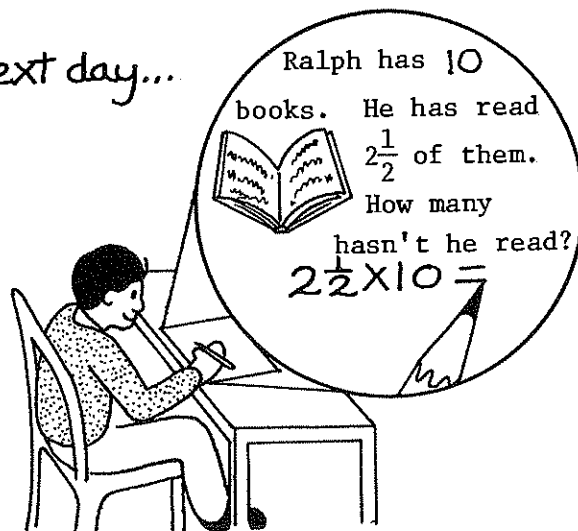
The unreasonableness of many of the results on the previous page indicates these students are not checking the reasonableness of answers or that they do not have any basis on which to do so. What would happen if these students, or for that matter students getting correct answers, were asked to show us what multiplication or division of fractions means to them in terms of diagrams, concrete objects or mathematical relationships? Both operations have concrete embodiments and mathematical rationales that motivate and provide a reasonable basis for their algorithms. Shouldn't students learning to multiply and divide fractions be able to explain what they are doing before using rote algorithms?

MEANING OF FRACTION MULTIPLICATION

Studying for a test...



Next day...



The search for meaningful ways to illustrate fraction multiplication reveals how complicated this operation is. When examining the subtleties of fraction multiplication, we begin to see why students have trouble with these algorithms. Let us examine the different multiplication ideas that can arise.

(Whole Number) \times (Fraction)

$3 \times \frac{1}{5}$

(Fraction) \times (Whole Number)

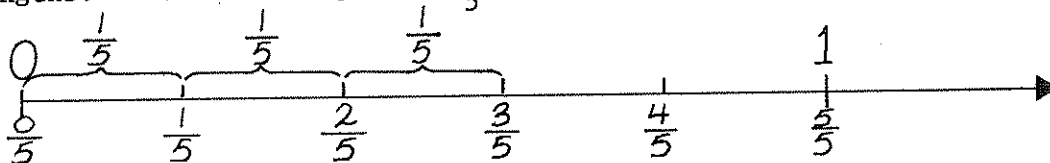
$\frac{2}{5} \times 3$

(Fraction) \times (Fraction)

$\frac{2}{3} \times \frac{4}{5}$

(Whole Number) \times (Fraction)

Because whole number multiplication is based on repeated addition, it is natural to think of $3 \times \frac{1}{5}$ as $\frac{1}{5} + \frac{1}{5} + \frac{1}{5}$ or $\frac{3}{5}$. This can be illustrated on a number line by marking off segments $\frac{1}{5}$ unit long. Starting at 0, mark off three $\frac{1}{5}$ unit lengths. The terminal point, $\frac{3}{5}$, is the product.



The same idea can be approached using a ruler. See the student page *Rulers and Number Lines*. For example, to find $5 \times \frac{3}{4}$ the student can draw five segments of length $\frac{3}{4}$ end-to-end and then measure the five segments. Of course, when using the ruler (or number line) the answer will probably be read as three and three-fourths. Unless students see this (and other answers) in the form $\frac{15}{4}$, they probably will not make the generalization that $a \times \frac{b}{c} = \frac{a \times b}{c}$.

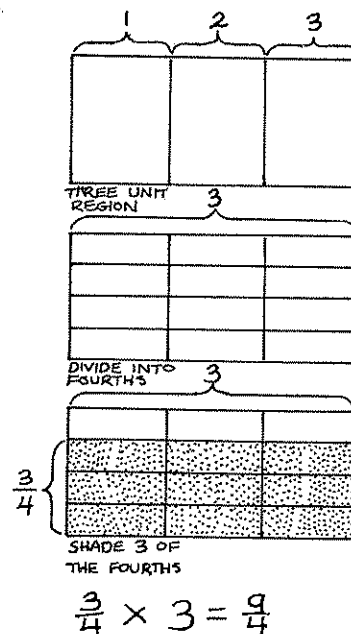
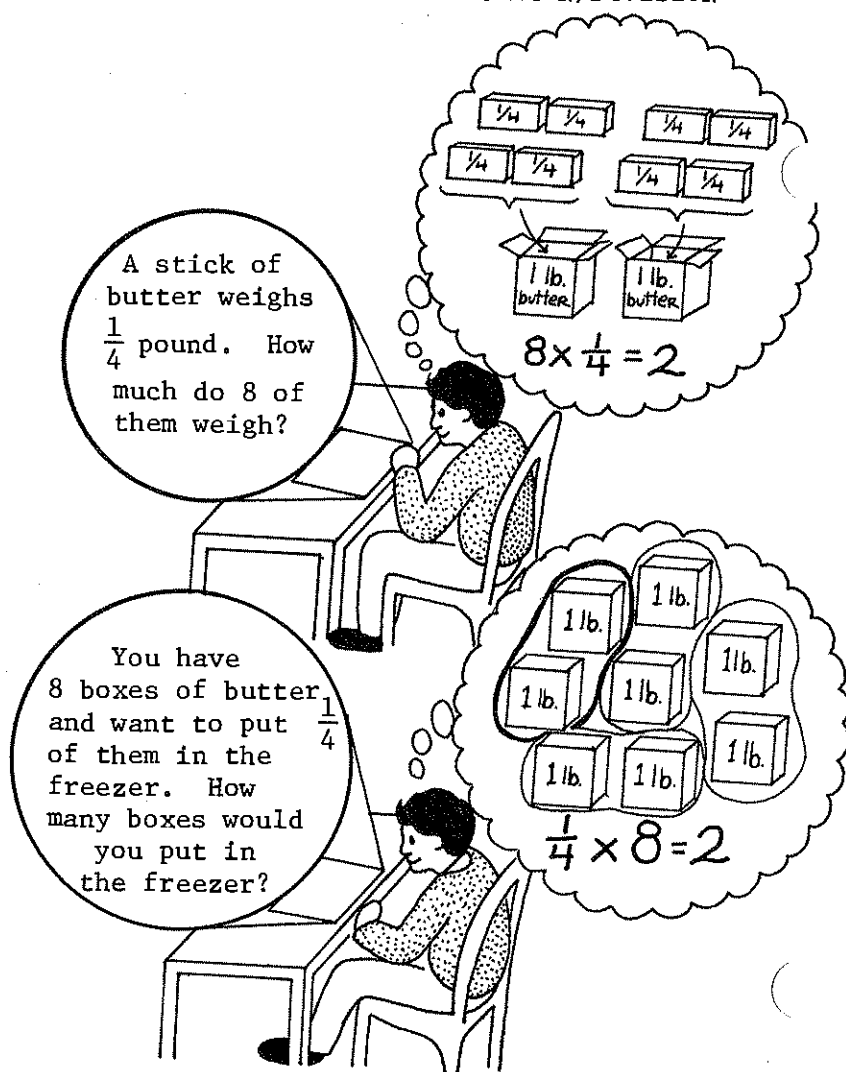
One of the important results of these activities is the realization that fractions like $\frac{3}{5}$ can be written as $3 \times \frac{1}{5}$.

(Fraction) x (Whole Number)

Multiplication of a fraction times a whole number, say $\frac{1}{4} \times 8$, requires a different interpretation. Repeated addition doesn't make sense—how can we add 8, $\frac{1}{4}$ times? Telling students that $\frac{1}{4} \times 8$ is the same as $8 \times \frac{1}{4}$ will also cause confusion when students confront a situation like the following: "There are 8 items, and you have $\frac{1}{4}$ of them." This sentence translates nicely into $\frac{1}{4}$ of 8 and then to $\frac{1}{4} \times 8$. The physical situation does not imply the addition of eight $\frac{1}{4}$'s ($8 \times \frac{1}{4}$).

Students who understand the fraction concept, " $\frac{1}{4}$ means one of four equal parts," may not have difficulty in seeing that $\frac{1}{4}$ of 24 means divide 24 into four equal parts and take one of them. (Notice that we tell students "of" means multiply, and $\frac{1}{4}$ of 24 is $\frac{1}{4} \times 24$. Yet, to compute $\frac{1}{4}$ of 24 you divide 24 by 4. No wonder there is confusion!) Likewise, $\frac{3}{4}$ of 24 means divide 24 into four equal parts and take three of them. However, finding $\frac{3}{4}$ of 3 poses a new problem because 3 doesn't split evenly into four whole number parts. (Some number sense can be encouraged here by having students approximate the product. Is the product more or less than 3? More or less than $\frac{3}{2}$?)

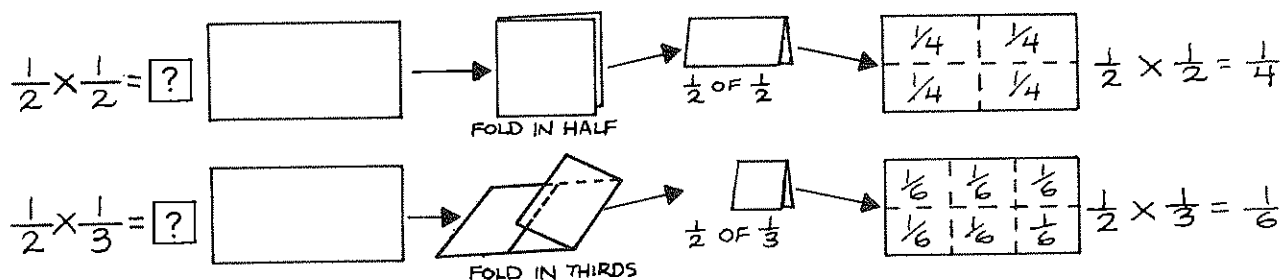
Let's examine this situation in terms of a diagram. Take three unit regions, divide the three regions into fourths and shade three of the fourths ($\frac{3}{4}$ of 3 is shaded). There are nine little shaded rectangles, each of which represents one-fourth of a unit. Therefore, $\frac{3}{4}$ of 3 is $\frac{9}{4}$. The tricky part of this demonstration is to understand that each of the little rectangles is $\frac{1}{4}$ of the basic unit region; the student who does not understand this might give $\frac{9}{12}$ as the product.



Experiences like these can lead students to see that $\frac{b}{c} \times a = \frac{b \times a}{c}$. At this point they might be led to see that $4 \times \frac{3}{7}$ and $\frac{3}{7} \times 4$ result in the same product. The generalization is that $a \times \frac{b}{c}$ and $\frac{b}{c} \times a$ are equal mathematically, even though they have different physical interpretations.

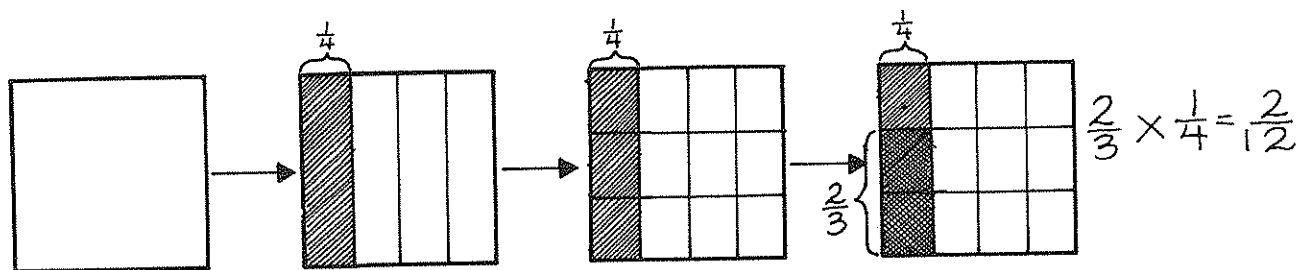
(Fraction) \times (Fraction)

Developing a meaningful interpretation for multiplying a fraction by a fraction is no more difficult than the other models. At the concrete level the activity of paper folding allows students to physically fold $\frac{1}{2}$ of $\frac{1}{2}$, $\frac{1}{2}$ of $\frac{1}{3}$, etc.

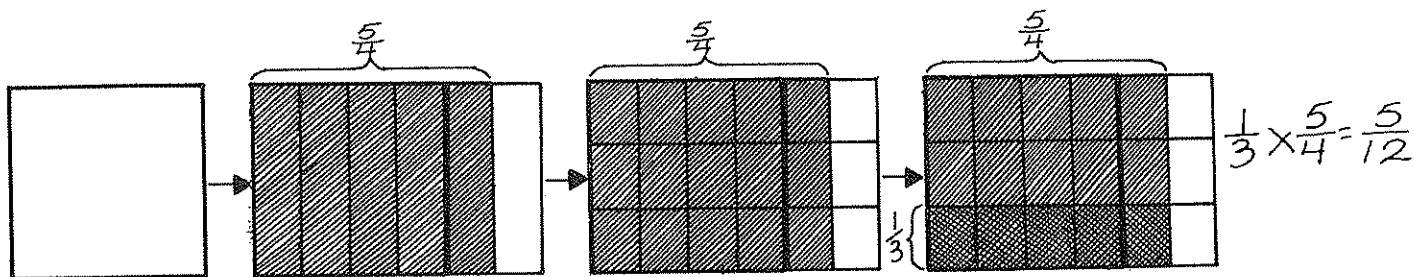


Students could be asked to create their own imaginative folds and name the result. (Take a piece of paper and try to show $\frac{1}{4}$ of $\frac{1}{3}$, $\frac{1}{3}$ of $\frac{1}{3}$ and $\frac{1}{6}$ of $\frac{1}{8}$ without using any measuring devices.)

A convenient way to show a fraction of a fraction at the pictorial level is to use a square region. Suppose, for example, we wish to show $\frac{2}{3}$ of $\frac{1}{4}$.



A similar method can be used to show $\frac{1}{3}$ of $\frac{5}{4}$. Students must be encouraged to relate the small rectangles back to the basic unit to avoid misinterpreting the last diagram.



This diagrammatic method can be used to show that $\frac{2}{3} \times \frac{1}{4}$ gives the same product as $\frac{1}{4} \times \frac{2}{3}$. After some experience students can be helped to see $\frac{1}{4} \times \frac{2}{3} = \frac{1 \times 2}{4 \times 3}$, that is, the product of the denominators determines the number of small rectangles into which the unit region is subdivided, and the product of the numerators yields the number of shaded regions. This is basically the algorithm, $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$.

MEANING OF FRACTION DIVISION

The topic of fraction division includes the following cases:

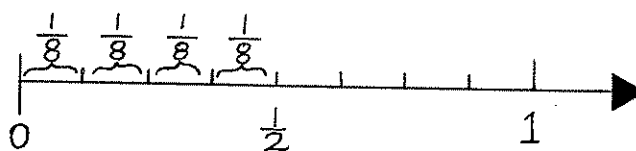
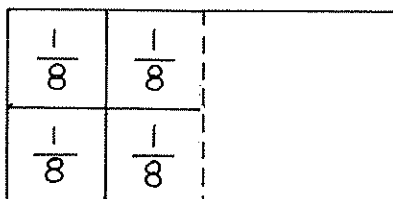
$$(\text{Fraction}) \div (\text{Whole number}) \qquad \frac{1}{2} \div 4$$

$$(\text{Whole number}) \div (\text{Fraction}) \qquad 4 \div \frac{1}{2}$$

$$(\text{Fraction}) \div (\text{Fraction}) \qquad \frac{1}{3} \div \frac{1}{2}$$

(Fraction) \div (Whole Number)

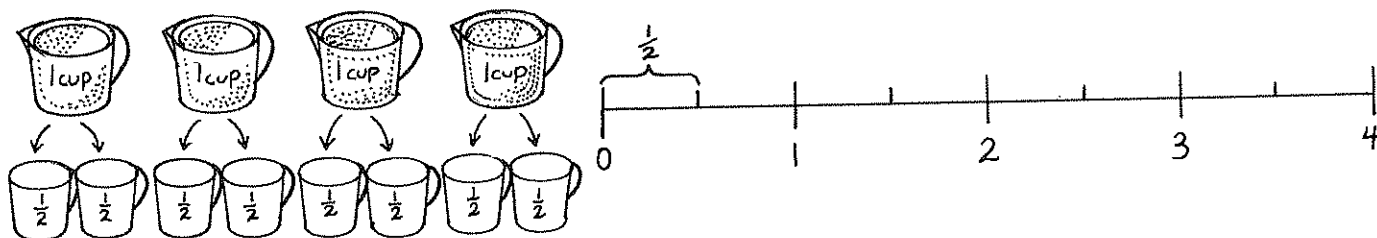
A statement like $\frac{1}{2} \div 4$ has a natural interpretation - dividing $\frac{1}{2}$ into four equal parts. At the concrete level this may mean folding or cutting $\frac{1}{2}$ sheet of paper into 4 equal parts or dividing $\frac{1}{2}$ of a candy bar into four equal parts. On the number line it can be interpreted as dividing $\frac{1}{2}$ unit into four equal parts. In the diagrams below we see $\frac{1}{2} \div 4 = \frac{1}{8}$.



When working with multiplication of fractions, $\frac{1}{4}$ of $\frac{1}{2}$ was interpreted as dividing $\frac{1}{2}$ into 4 equal parts or $\frac{1}{2} \div 4$. Thus, there is a close connection between $\frac{1}{2} \div 4$ and $\frac{1}{4} \times \frac{1}{2}$, but seeing this connection is not simple and requires maturity on the part of the learner.

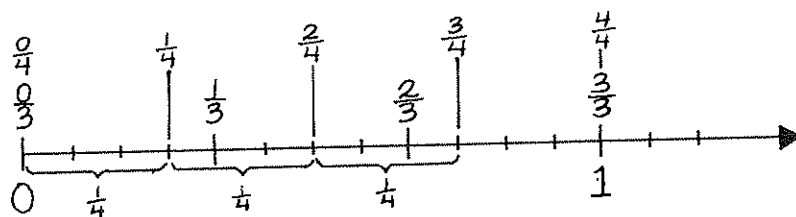
(Whole Number) \div (Fraction) and (Fraction) \div (Fraction)

While we can talk about and even illustrate dividing $\frac{1}{2}$ into 4 equal parts ($\frac{1}{2} \div 4$), it becomes meaningless to think of $4 \div \frac{1}{2}$ as dividing 4 into $\frac{1}{2}$ equal parts. It does make sense, however, to think of the number of $\frac{1}{2}$ units contained in 4 units. This notion can be represented concretely or pictorially.



At a higher level of abstraction we might rephrase the question "How many $\frac{1}{2}$'s in 4?" as "How many $\frac{1}{2}$'s in $\frac{8}{2}$?" Similarly, $6 \div \frac{3}{4}$ ("How many $\frac{3}{4}$ units are there in 6 units?") could be rephrased as $\frac{24}{4} \div \frac{3}{4}$ ("How many $\frac{3}{4}$ units are in $\frac{24}{4}$ units?")

Of course, not all fraction division comes out evenly. Consider, for example, $\frac{2}{3} \div \frac{1}{4}$. The number line can show that there are two $\frac{1}{4}$'s in $\frac{2}{3}$ with some part of the $\frac{2}{3}$ leftover. Perhaps some students can see that the leftover part



is $\frac{2}{3}$ of $\frac{1}{4}$ and conclude there are $2\frac{2}{3}$ one-fourths in $\frac{2}{3}$. It is more likely that the meaning is too complicated to pursue at this point except for establishing reasonable answers. The problem

$\frac{2}{3} \div \frac{1}{4}$ could be changed to the equivalent problem $\frac{8}{12} \div \frac{3}{12}$. Now, with a background in "nice" problems like $\frac{8}{9} \div \frac{2}{9} = 8 \div 2 = 4$, the uneven problem $\frac{8}{12} \div \frac{3}{12} = 8 \div 3$ or $2\frac{2}{3}$.

Building Toward the Algorithm

After students have dealt with fraction division concretely and geometrically, they can approach some situations in a more abstract manner. Appealing to previous work with whole numbers, we observe that

$$12 \div 3 = \boxed{4} \quad \text{because} \quad 12 = \boxed{4} \times 3$$

That is, for the problem $12 \div 3 = \boxed{}$ we can substitute the question "What can be multiplied times 3 to get 12?" This same idea can be applied to fractions.

$$\frac{3}{4} \div \frac{1}{8} = \frac{\triangle}{\square} \quad \longrightarrow \quad \frac{3}{4} = \frac{\triangle}{\square} \times \frac{1}{8}$$

At first the process can proceed by trial and error. In the later grades the formal algorithm for fraction division may be developed. A series of steps leading to the algorithm are outlined as follows:

$$\frac{3}{4} \div \frac{1}{8} = \frac{\triangle}{\square} \quad \text{implies} \quad \frac{3}{4} = \frac{\triangle}{\square} \times \frac{1}{8}$$

$$\frac{3}{4} \times \frac{8}{1} = \frac{\triangle}{\square} \times \frac{1}{8} \times \frac{8}{1}$$

$$\frac{3}{4} \times \frac{8}{1} = \frac{\triangle}{\square} \times 1$$

$$\frac{3}{4} \times \frac{8}{1} = \frac{\triangle}{\square}$$

$$\frac{24}{4} = \frac{\triangle}{\square}$$

What is needed to understand stages:

Defining division in terms of multiplication - with trial and error experience

Here we must know that every nonzero fraction has a multiplicative inverse (reciprocal) and the product of a fraction and its inverse is 1.

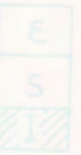
After going through this process several times, they may be able to generalize by looking at the second to the last step. That $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$.

The division algorithm for fractions is important - when students are ready to learn it. Most students are probably not ready to really understand its use or to remember the algorithm until the later junior high years. The idea of division of fractions, however, can be introduced much earlier and can be used when it arises in practical situations.



ACTIVITY CARDS - FRACTION BARS - III

Multiplication/Division
FRACTIONS

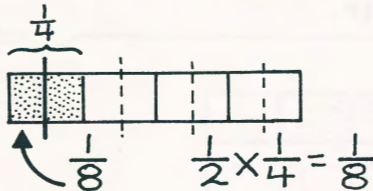


For information on the construction, use
and readiness activities for Fractions.
Bars see the section on Lab Materials.

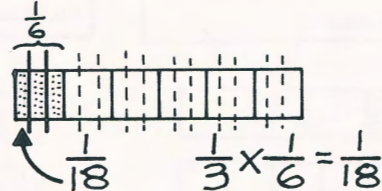
FB-III-1

To multiply a fraction times a fraction, you can split the shaded part of a fraction bar.

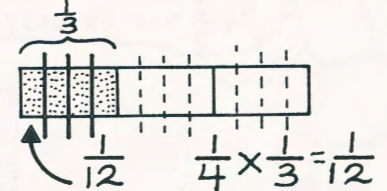
Split into 2 equal parts



Split into 3 equal parts



Split into 4 equal parts



Complete these equations.

Split each shaded
part into 2 equal parts

①
$$\frac{1}{2} \times \frac{1}{2} = \square$$

②
$$\frac{1}{2} \times \frac{1}{3} = \square$$

Split each shaded
part into 3 equal parts

③
$$\frac{1}{3} \times \frac{1}{2} = \square$$

④
$$\frac{1}{3} \times \frac{1}{3} = \square$$

Split each shaded
part into 4 equal parts

⑤
$$\frac{1}{4} \times \frac{1}{2} = \square$$

⑥
$$\frac{1}{4} \times \frac{1}{3} = \square$$

Now try these and decide how to split each shaded part.

⑦
$$\frac{1}{2} \times \frac{1}{6} = \square$$

⑧
$$\frac{1}{3} \times \frac{1}{4} = \square$$

⑨
$$\frac{1}{2} \times \frac{1}{4} = \square$$

⑩
$$\frac{1}{4} \times \frac{1}{4} = \square$$

⑪
$$\frac{1}{3} \times \frac{1}{6} = \square$$

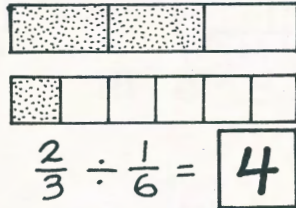
⑫
$$\frac{1}{4} \times \frac{1}{6} = \square$$

ACTIVITY CARDS- FRACTION BARS- III (PAGE 2)

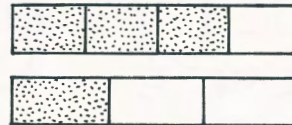
EXACT FITS



For the 2 bars I have taken, the shaded amount on the $\frac{1}{6}$ bar fits into the shaded amount on the $\frac{2}{3}$ bar exactly 4 times.



With the two bars I have, there is no exact fit. The shaded amount on the $\frac{1}{3}$ bar fits into the shaded amount on the $\frac{3}{4}$ bar twice with something leftover, so this is not an exact fit.



Activity (1 or 2 students)

Pick any 2 fraction bars which are not zero bars. Compare these bars to see how many times the shaded amount of one of your bars fits into the shaded amount of the other. Do this for 10 pairs of bars. See how many exact fits you can get.

Repeat this activity with a classmate. Take turns choosing pairs of bars. See who can be the first to get 5 exact fits.

FB-III-2

PRACTICING EXACT FITS

FB-III-3

Write the fraction for each bar and divide the greater fraction by the smaller. The first two equations have been started for you.

<p>①</p> $\frac{3}{6} \div \frac{1}{6} = \boxed{}$	<p>②</p> $\frac{7}{12} \div \frac{1}{12} = \boxed{}$	<p>③</p> $\text{---} \div \text{---} = \boxed{}$
<p>④</p> $\text{---} \div \text{---} = \boxed{}$	<p>⑤</p> $\text{---} \div \text{---} = \boxed{}$	<p>⑥</p> $\text{---} \div \text{---} = \boxed{}$

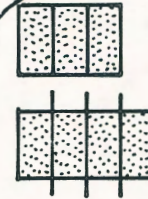
SOURCE: Fraction Bars, Introductory Card Set and Workbook I

FB-III-3

With these two bars, I can see that $\frac{1}{3}$ fits into $\frac{11}{12}$ 2 times with something leftover.

Compare what's leftover with the amount you are dividing by. This ratio is 3 to 4 or $\frac{3}{4}$. So, $\frac{1}{3}$ divides into $\frac{11}{12}$, 2 and $\frac{3}{4}$ times.

ratio of $\frac{3}{4}$



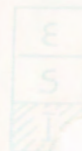
$$\frac{11}{12} \div \frac{1}{3} = 2\frac{3}{4}$$

Use the fraction bars to compute the quotients.

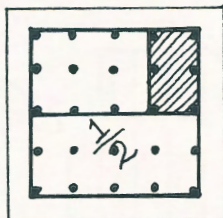
<p>①</p> <p>$\frac{7}{12} \div \frac{1}{6} = \square$</p>	<p>②</p> <p>$\frac{11}{12} \div \frac{1}{4} = \square$</p>	<p>③</p> <p>$\frac{11}{12} \div \frac{1}{3} = \square$</p>
<p>④</p> <p>$\frac{7}{12} \div \frac{1}{2} = \square$</p>	<p>⑤</p> <p>$\frac{4}{6} \div \frac{1}{4} = \square$</p>	<p>⑥</p> <p>$\frac{11}{12} \div \frac{1}{6} = \square$</p>
<p>⑦</p> <p>$\frac{5}{6} \div \frac{1}{3} = \square$</p>	<p>⑧</p> <p>$\frac{3}{4} \div \frac{1}{2} = \square$</p>	<p>⑨</p> <p>$\frac{5}{6} \div \frac{1}{4} = \square$</p>

ACTIVITY CARDS - GEOBOARDS - III

Whole Model
Multiplication/
Division
FRACTIONS



REMEMBER THAT $\frac{1}{4} \times \frac{1}{2}$ CAN BE WRITTEN $\frac{1}{4}$ OF $\frac{1}{2}$.



DO YOU SEE HOW THIS
PICTURE SHOWS THAT

$$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8} ?$$

$$\frac{1}{4} \text{ of } \frac{1}{2}$$

FIND A GEOBOARD SOLUTION FOR EACH OF THE
FOLLOWING PROBLEMS. RECORD YOUR ANSWERS
ON DOT PAPER.

1. $\frac{3}{4}$ of $\frac{1}{2}$

4. $\frac{1}{4}$ of $\frac{1}{4}$

2. $\frac{1}{2}$ of $\frac{1}{8}$

5. $\frac{5}{8}$ of $\frac{1}{2}$

3. $\frac{1}{4}$ of $\frac{3}{4}$

6. $\frac{1}{2}$ of $\frac{7}{8}$

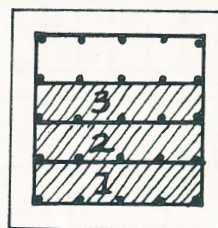
GB-III-1

$\frac{3}{4} \div \frac{1}{4}$ ASKS THE QUESTION: HOW MANY $\frac{1}{4}$'s IN $\frac{3}{4}$?

GB-III-2

DO YOU SEE THAT THIS PICTURE SHOWS

THAT THERE ARE THREE $\frac{1}{4}$ IN $\frac{3}{4}$?



FIND A GEOBOARD SOLUTION FOR EACH OF THE FOLLOWING.
RECORD YOUR ANSWERS ON DOT PAPER.

1. $\frac{1}{2} \div \frac{1}{4}$

4. $\frac{5}{8} \div \frac{1}{16}$

2. $\frac{1}{2} \div \frac{1}{8}$

5. $\frac{3}{4} \div \frac{1}{2}$ (BE CAREFUL)

3. $\frac{3}{4} \div \frac{1}{8}$

6. $\frac{7}{8} \div \frac{1}{4}$

RULERS AND NUMBER LINES

SAMPLES FOR STUDENT WORKSHEETS

I. NUMBER LINE Multiplication And Division

1. $5 \times \frac{2}{3} = 3\frac{1}{3}$

2. $4 \times \frac{3}{5} =$

3. $3 \times \frac{5}{8} =$

4. $3\frac{2}{3} \div \frac{1}{3} = 11$

5. $4\frac{3}{4} \div \frac{1}{4} =$

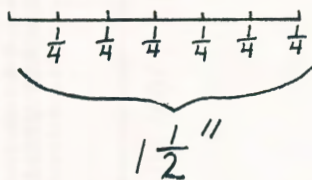
6. $2\frac{3}{4} \div \frac{1}{4} =$

II. OPERATIONS WITH A RULER

You can use your ruler to multiply and divide fractions.

Watch this!

MULTIPLICATION

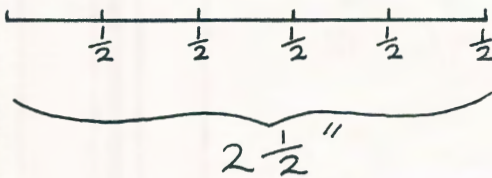


THIS SHOWS THAT $6 \times \frac{1}{4}$ is $1\frac{1}{2}$

- 1) Draw a line segment $\frac{1}{4}$ " long.
- 2) Extend this segment until there are six $\frac{1}{4}$ " segments together.
- 3) How long is the total line segment?

$$6 \times \frac{1}{4} = 1\frac{1}{2}$$

DIVISION



THIS SHOWS THAT $2\frac{1}{2} \div \frac{1}{2}$ is 5

- 1) Draw a line segment $2\frac{1}{2}$ " long.
- 2) On this segment mark off as many $\frac{1}{2}$ " pieces as you can.

- 3) How many could you mark off?

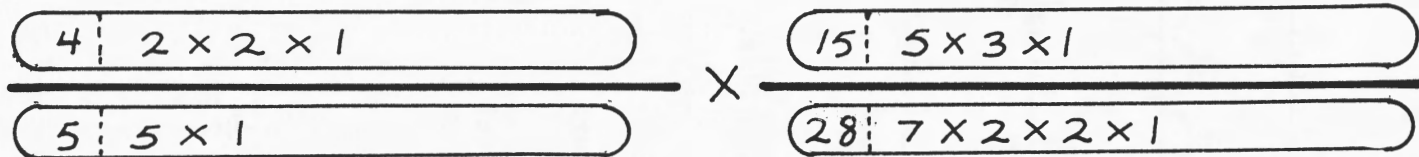
$$2\frac{1}{2} \div \frac{1}{2} = 5$$

IDEA FROM: C.O.L.A.M.D.A.

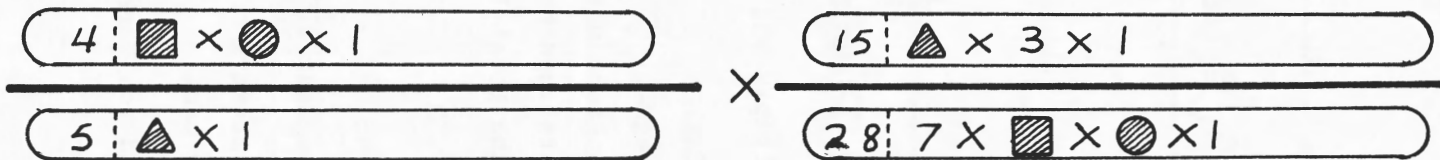
Permission to use granted by Northern Colorado Educational Board of Cooperative Services

TO MULTIPLY $\frac{4}{5} \times \frac{15}{28}$

MAKE EACH FRACTION USING THE BOARDS.



LOOK FOR COMMON FACTORS (OTHER THAN ONE) IN A NUMERATOR AND DENOMINATOR.
COVER UP EACH COMMON PAIR WITH A DISC OR SMALL PIECE OF PAPER.



MULTIPLY ALL THE REMAINING FACTORS IN THE NUMERATOR, $\frac{1 \times 3 \times 1}{1 \times 7 \times 1} = \frac{3}{7}$
MULTIPLY ALL THE REMAINING FACTORS IN THE DENOMINATOR,
TRY THESE:

$$\frac{2}{3} \times \frac{1}{8} =$$

$$\frac{5}{11} \times \frac{4}{15} =$$

$$\frac{7}{10} \times \frac{2}{3} =$$

$$\frac{5}{6} \times \frac{9}{14} =$$

$$\frac{7}{8} \times \frac{6}{14} =$$

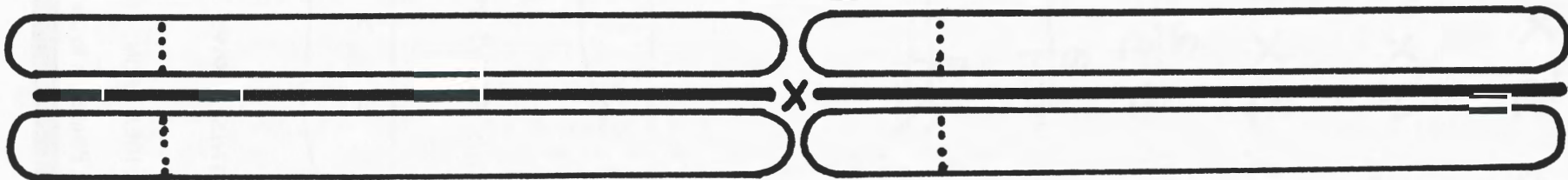
$$\frac{21}{35} \times \frac{42}{64} =$$

$$\frac{3}{4} \times \frac{16}{21} =$$

$$\frac{5}{3} \times 1\frac{2}{3} \times \frac{6}{15} =$$

$$2\frac{1}{2} \times 3\frac{3}{5} =$$

PUT YOUR FACTOR BOARDS ON THIS:

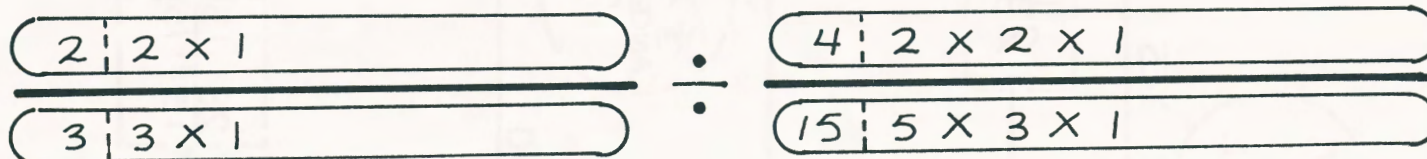


For information on the construction of Factor Boards, see the section on Lab Materials.

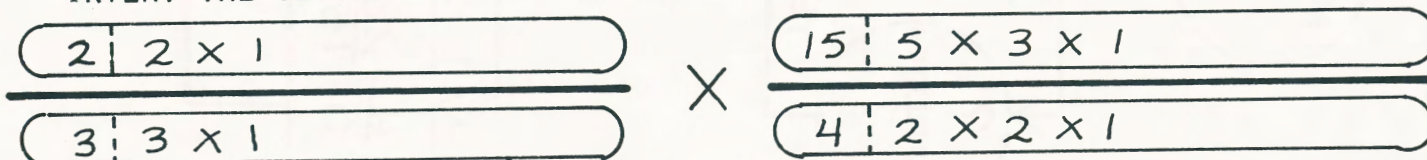
Multiplication/
Division
FRACTIONS

5/11

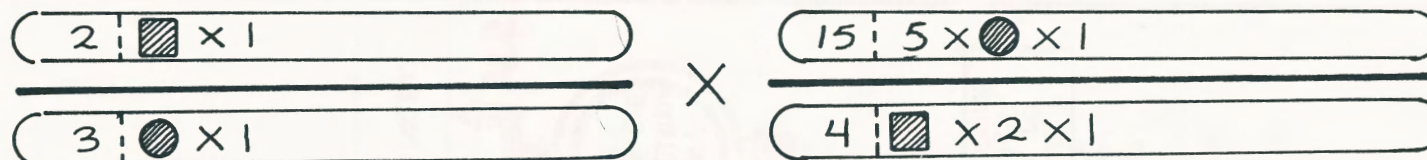
TO DIVIDE: $\frac{2}{3} \div \frac{4}{15}$ MAKE EACH FRACTION USING THE BOARDS.



INVERT THE SECOND FRACTION AND MULTIPLY,



LOOK FOR COMMON FACTORS (OTHER THAN ONE) IN A NUMERATOR AND A DENOMINATOR. COVER UP EACH COMMON PAIR WITH A DISC OR SMALL PIECE OF PAPER.



MULTIPLY ALL THE REMAINING FACTORS IN THE NUMERATORS.
MULTIPLY ALL THE REMAINING FACTORS IN THE DENOMINATORS.

$$\frac{1 \times 5 \times 1}{1 \times 2 \times 1} = \frac{5}{2} = 2\frac{1}{2}$$

TRY THESE:

$$\frac{5}{11} \div \frac{4}{11} =$$

$$\frac{7}{10} \div \frac{2}{3} =$$

$$1\frac{3}{8} \div \frac{5}{24} =$$

$$\frac{5}{4} \div \frac{9}{10} =$$

$$\frac{5}{6} \div \frac{5}{12} =$$

$$\frac{4}{9} \div \frac{7}{12} =$$

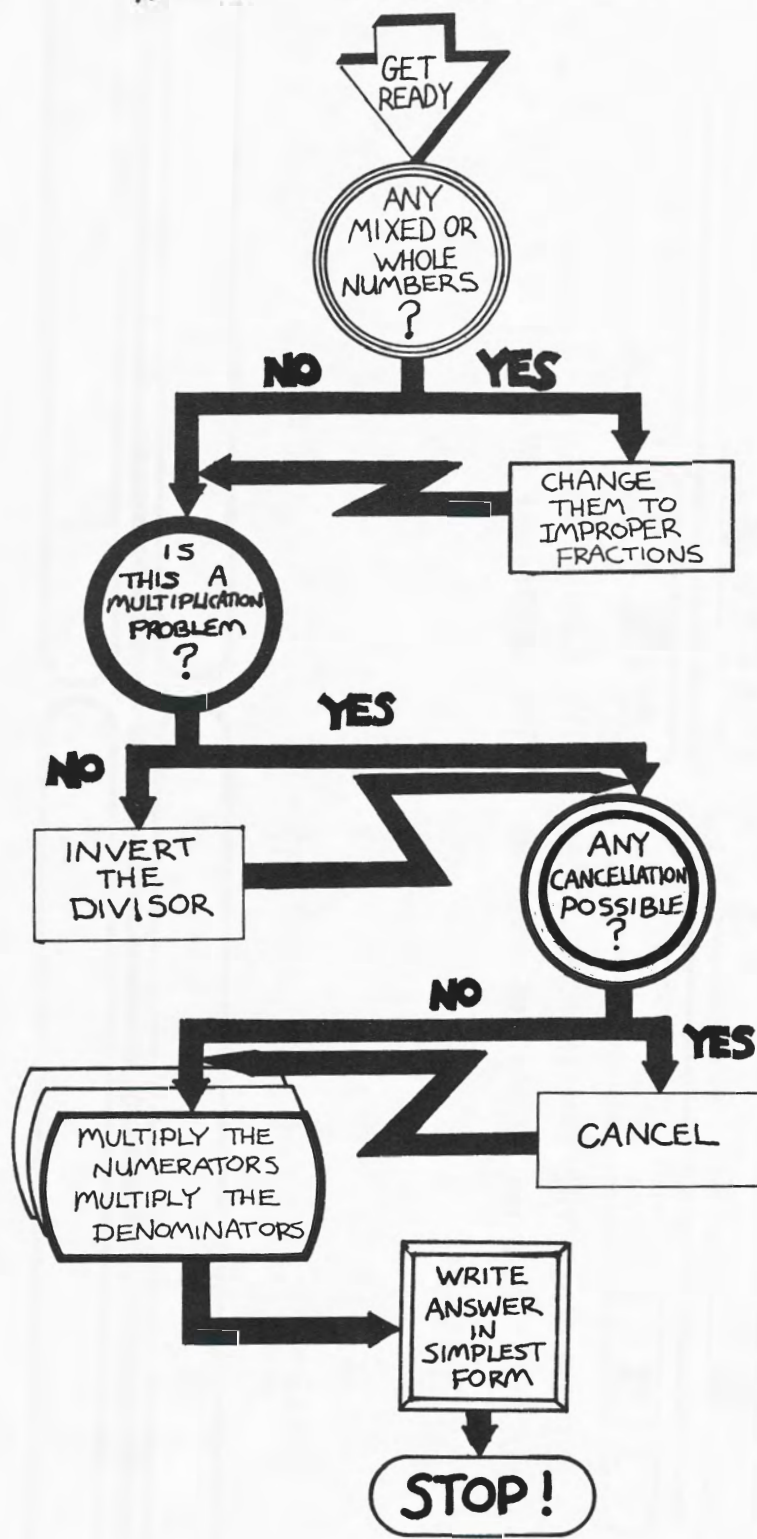
$$7\frac{1}{3} \div 2\frac{1}{5} =$$

$$\frac{6}{5} \div \frac{14}{15} =$$



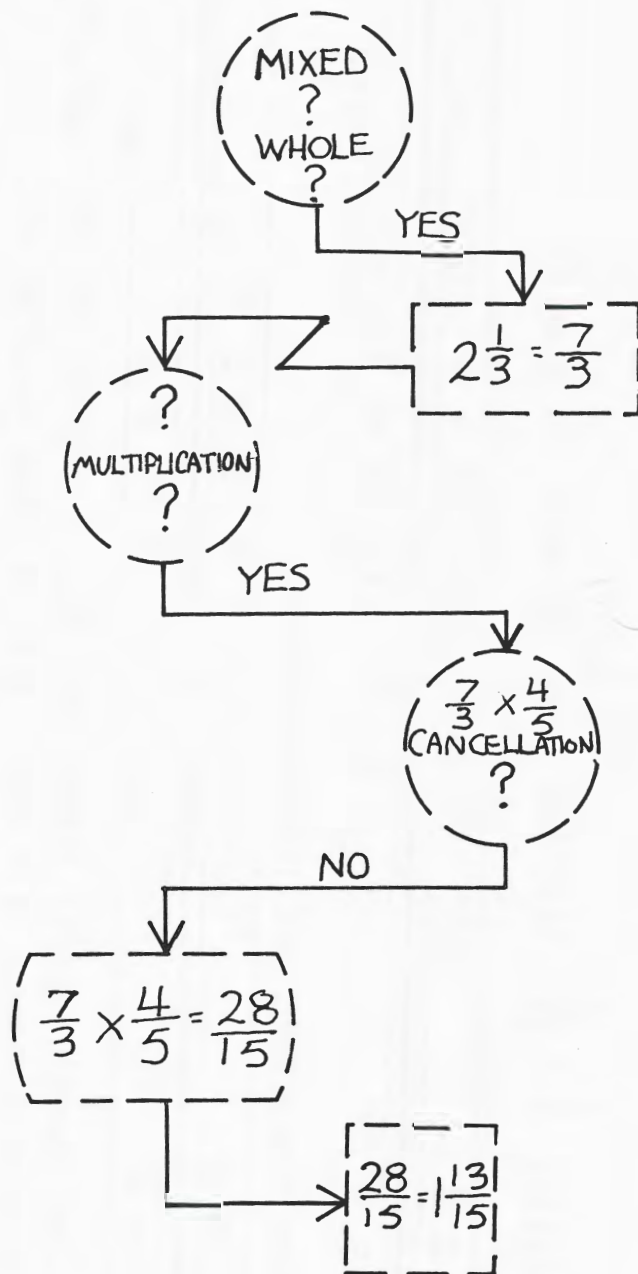
FRACTION FLOWCHART

TO MULTIPLY OR DIVIDE
FRACTIONS AND MIXED NUMBERS



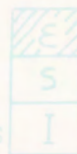
EXAMPLE :

$$2\frac{1}{3} \times \frac{4}{5} =$$



OTHER WAYS

Division
FRACTIONS



I'VE NEVER SEEN
A PROBLEM WORKED
LIKE THAT!!
I'LL TRY
ONE MYSELF

HOW WOULD
I DO ONE LIKE
THIS?...

$$\frac{9}{14} \div \frac{3}{7} = \frac{9 \div 3}{14 \div 7} = \frac{3}{2} = 1\frac{1}{2}$$

$$\frac{12}{27} \div \frac{4}{9} = \frac{12 \div 4}{27 \div 9} = \frac{3}{3} = 1$$

$$\frac{3}{16} \div \frac{2}{8} = \frac{3}{16} \times \frac{2}{2} \div \frac{2}{8} = \frac{3 \times 2 \div 2}{16 \times 2 \div 8} = \frac{3}{4}$$

TRY SOME YOURSELF

$$\frac{7}{8} \div \frac{3}{4} = \frac{7}{8} \times \frac{\boxed{}}{\boxed{}} \div \frac{3}{4} = \frac{\div 3}{\div 4} =$$

$$\frac{5}{9} \div \frac{2}{3} =$$

$$\frac{8}{11} \div \frac{1}{2} =$$

$$\frac{7}{9} \div \frac{2}{3} =$$

WELL, HOW ABOUT ONE LIKE THIS?

$$\frac{2}{7} \div \frac{3}{8} = \frac{2}{7} \times \frac{\boxed{}}{\boxed{}} \times \frac{\boxed{}}{\boxed{}} \div \frac{3}{8} = \frac{2 \times 3 \times 8 \div 3}{7 \times 3 \times 8 \div 8} = \frac{2 \times 8}{7 \times 3} = \frac{16}{21}$$

3 WON'T GO
INTO 2 AND 8
WON'T GO INTO
7

here is multiplication
by the reciprocal.

TRY THESE:

$$\frac{1}{2} \div \frac{3}{4} =$$

$$\frac{2}{3} \div \frac{5}{6} =$$

$$\frac{3}{8} \div \frac{2}{7} =$$

$$\frac{4}{10} \div \frac{3}{8} =$$

$$\frac{6}{7} \div \frac{5}{6} =$$

$$\frac{3}{5} \div \frac{1}{3} =$$

$$\frac{2}{3} \div \frac{5}{9} =$$

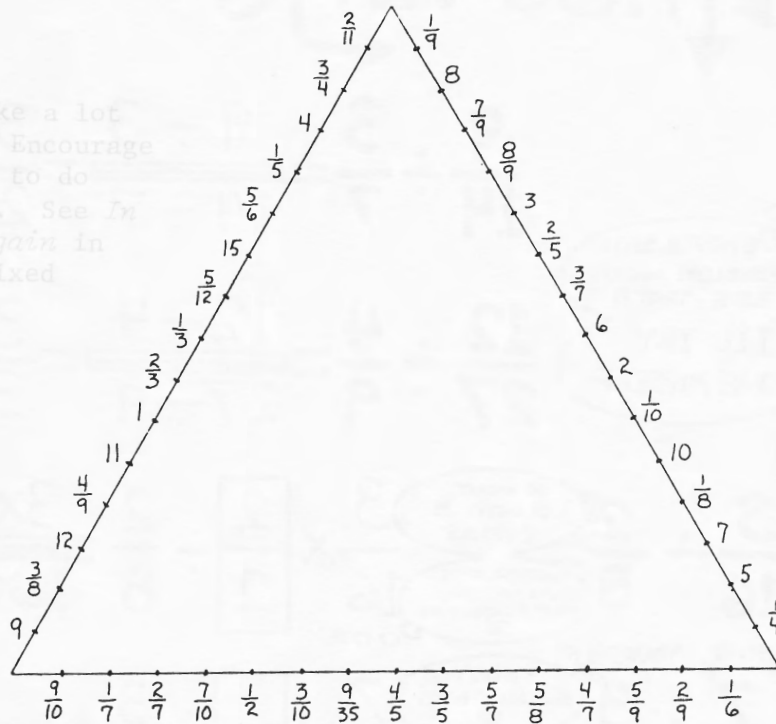
$$\frac{1}{7} \div \frac{5}{9} =$$

MULTIPLICATION OF FRACTIONS

Multiplication
FRACTIONS



This looks like a lot
of problems. Encourage
your students to do
them mentally. See *In
Your Head...Again* in
FRACTIONS: Mixed
Operations



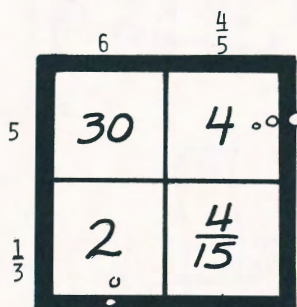
Connect "a" and "b" answers for each of the following problems.

- | | (a) | (b) | | (a) | (b) |
|--|-------|--|-------|--|-------|
| 1. $\frac{1}{2} \times \frac{4}{5} =$ | _____ | 6 $\times \frac{2}{3} =$ | _____ | 13. $\frac{3}{5} \times 10 =$ | _____ |
| 2. $\frac{2}{3} \times \frac{2}{3} =$ | _____ | $\frac{4}{7} \times \frac{7}{8} =$ | _____ | 14. $\frac{1}{12} \times 9 =$ | _____ |
| 3. $12 \times \frac{1}{6} =$ | _____ | $\frac{5}{9} \times \frac{6}{15} =$ | _____ | 15. $\frac{5}{8} \times \frac{8}{9} =$ | _____ |
| 4. $\frac{3}{10} \times \frac{6}{7} =$ | _____ | $\frac{15}{22} \times \frac{11}{20} =$ | _____ | 16. $\frac{3}{7} \times \frac{7}{15} =$ | _____ |
| 5. $\frac{1}{3} \times \frac{1}{3} =$ | _____ | $\frac{2}{5} \times \frac{5}{6} =$ | _____ | 17. $\frac{3}{5} \times \frac{20}{21} =$ | _____ |
| 6. $15 \times \frac{1}{3} =$ | _____ | $\frac{1}{5} \times 3 =$ | _____ | 18. $\frac{3}{4} \times \frac{5}{9} =$ | _____ |
| 7. $\frac{2}{13} \times \frac{13}{14} =$ | _____ | $\frac{8}{9} \times \frac{3}{4} =$ | _____ | 19. $\frac{2}{5} \times \frac{3}{4} =$ | _____ |
| 8. $\frac{2}{9} \times \frac{7}{2} =$ | _____ | $30 \times \frac{1}{2} =$ | _____ | 20. $9\frac{1}{3} \times \frac{3}{4} =$ | _____ |
| 9. $\frac{5}{6} \times \frac{3}{20} =$ | _____ | $\frac{5}{4} \times \frac{1}{2} =$ | _____ | 21. $\frac{4}{5} \times \frac{7}{8} =$ | _____ |
| 10. $16 \times \frac{1}{18} =$ | _____ | $\frac{10}{11} \times \frac{11}{12} =$ | _____ | 22. $1\frac{1}{3} \times \frac{3}{5} =$ | _____ |
| 11. $\frac{2}{3} \times 1\frac{1}{2} =$ | _____ | $\frac{4}{6} \times \frac{3}{7} =$ | _____ | 23. $\frac{3}{5} \times \frac{5}{9} =$ | _____ |
| 12. $\frac{6}{7} \times \frac{14}{15} =$ | _____ | $\frac{7}{10} \times \frac{5}{14} =$ | _____ | 24. $\frac{4}{5} \times \frac{5}{22} =$ | _____ |

Mixed Number Multiplication

FIRST, CAN YOU FIGURE OUT HOW WE GET THE NUMBERS IN THE LATTICE?

NOW TRY THESE PROBLEMS WITH LATTICES...



$$\frac{1}{3} \times 6\frac{4}{5} = 30 + 4 + 2 + \frac{4}{15} = 36\frac{4}{15}$$

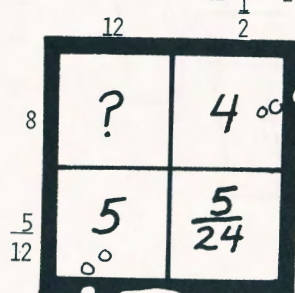
LET'S CHECK IT

$$\frac{1}{3} \times 6$$

$$5\frac{1}{3} \times 6\frac{4}{5}$$

$$\frac{16}{3} \times \frac{34}{5} = \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

HOW ABOUT $8\frac{5}{12} \times 12\frac{1}{2}$?



CHECK IT . . .

$$= \underline{\hspace{1cm}} + 4 + 5 + \frac{5}{24} = \underline{\hspace{1cm}}$$

$$8\frac{5}{12} \times 12\frac{1}{2}$$

$$\frac{101}{12} \times \frac{25}{2} = \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

$$\frac{60}{12}$$

$$8\frac{2}{3} \times 9\frac{1}{2}$$

$$10\frac{1}{3} \times 6\frac{1}{2}$$

$$5\frac{3}{8} \times 8\frac{1}{5}$$

$$5 \times 9\frac{5}{9}$$

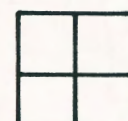
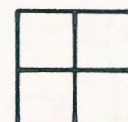
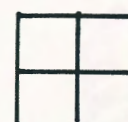
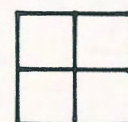
$$5\frac{1}{3} \times 5\frac{1}{3}$$

$$4\frac{1}{12} \times 3\frac{3}{4}$$

$$11 \times 3\frac{3}{16}$$

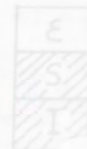
$$5\frac{1}{7} \times 8\frac{3}{4}$$

$$2\frac{2}{3} \times 10\frac{5}{8}$$



Mixed Numbers
Multiplication
FRACTIONS





CAN YOU GET Close Up

$24 \div 3\frac{1}{8}$

 $24 \div 3\frac{1}{8}$ is close to

$16 \div 3\frac{2}{3}$

 $16 \div 3\frac{2}{3}$ is close to

$10 \div 4\frac{3}{5}$

Handwritten notes: $3\frac{1}{8}$ is close to 3; $3\frac{2}{3}$ is close to 4

$18 \div 1\frac{3}{4}$

$8 \div 3\frac{1}{2}$

$21 \div 6\frac{1}{4}$

$12\frac{3}{4} \div 1\frac{5}{8}$

$6\frac{3}{4} \div \frac{11}{12}$

$8\frac{5}{8} \times 3\frac{1}{8}$

$10 \times 1\frac{1}{5}$

$12\frac{1}{4} \div 3\frac{1}{2}$

$2\frac{1}{8} \times 17\frac{9}{10}$

$4\frac{3}{10} \div 1\frac{2}{5}$

$5\frac{1}{7} \times 8\frac{3}{4}$

Approximate these and compare your answer with a friend.

$$6\frac{1}{5} \times 9\frac{3}{4} \approx \underline{\hspace{2cm}}$$

$$17\frac{9}{10} \div 3\frac{1}{8} \approx \underline{\hspace{2cm}}$$

$$5\frac{2}{5} \times 4\frac{3}{5} \approx \underline{\hspace{2cm}}$$

$$9\frac{7}{10} \div 3\frac{4}{7} \approx \underline{\hspace{2cm}}$$

You might want to see how close you came on some of these.



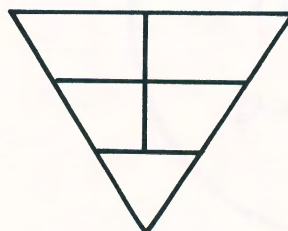
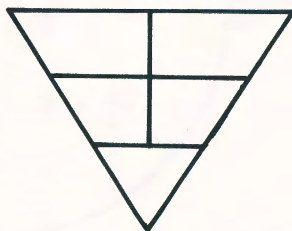
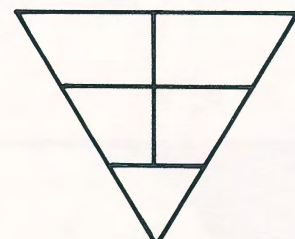
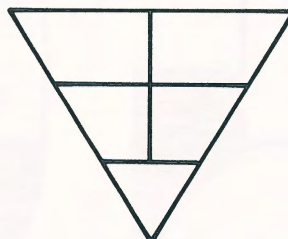
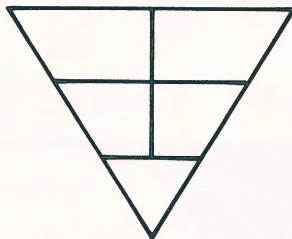
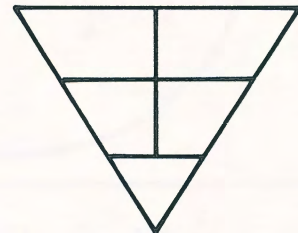
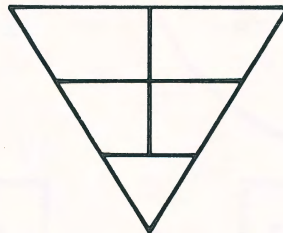
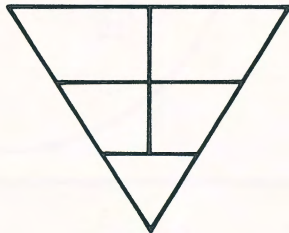
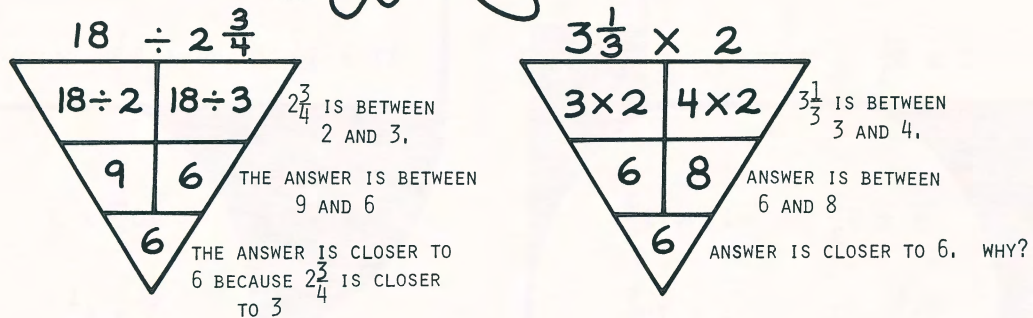
BETWEEN



Approximation
Mixed Numbers
Multiplication/Division
FRACTIONS



See the following page for
sample problem sets.



IN YOUR HEAD FIGURE OUT A
CLOSE ANSWER TO THESE
PROBLEMS. COMPARE YOUR
ANSWERS WITH A FRIEND.

You will need to put
some problems here.
See the next page
for suggestions.

TYPE: Paper & Pencil/Transparency

BETWEEN (CONTINUED)

PROBLEM SETS

I

$$24 \div 3\frac{5}{6}$$

$$6\frac{1}{4} \times 3$$

$$42 \div 6\frac{1}{7}$$

$$14 \times \frac{5}{6}$$

$$24 \div 1\frac{1}{8}$$

$$5\frac{3}{5} \times 4$$

$$6 \div 2\frac{2}{3}$$

$$2 \times 3\frac{1}{3}$$

$$9 \times 9\frac{2}{3}$$

$$12 \div 1\frac{7}{9}$$

$$3\frac{4}{7} \times 7$$

$$4 \div 1\frac{9}{10}$$

$$4 \times 8\frac{3}{4}$$

$$12 \div 3\frac{7}{8}$$

These problems are for the triangular grids.

II

$$4 \times 1\frac{1}{8}$$

$$12 \div 3\frac{1}{7}$$

$$2\frac{1}{8} \times 4$$

$$30 \div 6\frac{2}{5}$$

$$9 \times 3\frac{5}{6}$$

$$5 \div 5\frac{2}{3}$$

$$11 \times 7\frac{7}{8}$$

$$9 \div 3\frac{1}{3}$$

$$10\frac{4}{5} \times 3$$

$$12 \div 2\frac{1}{5}$$

$$5 \times 4\frac{1}{8}$$

$$15 \div 5\frac{2}{9}$$

$$3\frac{6}{10} \times 3$$

$$7\frac{1}{3} \div 1$$

These problems are to do mentally.

III

$$20 \div 4\frac{7}{9}$$

$$3\frac{1}{8} \times 4$$

$$12\frac{1}{3} \div 12$$

$$5\frac{3}{4} \times 7$$

$$10 \div 2\frac{1}{7}$$

$$10\frac{1}{3} \times 9$$

$$11 \div 6\frac{3}{8}$$

$$3\frac{3}{5} \times 6$$

$$25 \div 5\frac{2}{7}$$

$$1\frac{1}{2} \times 6$$

$$15 \div 3\frac{1}{4}$$

$$4\frac{5}{8} \times 5$$

$$16 \div 6\frac{5}{6}$$

$$12 \times 4\frac{1}{9}$$

IV

$$3\frac{3}{7} \times 3$$

$$9 \div 3\frac{1}{8}$$

$$7\frac{3}{4} \times 10$$

$$2\frac{7}{9} \div 10$$

$$8\frac{2}{5} \times 1$$

$$4\frac{2}{3} \div 4$$

$$10\frac{1}{2} \times 3$$

$$9 \div 2\frac{4}{7}$$

$$8 \div 2\frac{3}{5}$$

$$3\frac{3}{7} \times 9$$

$$3 \div 1\frac{1}{2}$$

$$4 \times 12\frac{1}{2}$$

$$4\frac{1}{4} \div 4$$

$$6 \times \frac{4}{11}$$



TRACK RECORDS IN SPACE

Mixed Numbers
Multiplication
FRACTIONS



YOU WILL NEED A 1973 (or newer) EDITION OF AN ALMANAC.

Each planet in our solar system pulls objects toward it with a different amount of force. In most cases, the smaller the planet, the smaller this force and the easier it would be to jump away from the surface. The idea of this activity is to find out what would happen if you tried 2 sports events while standing on other planets.

USE THE INDEX OF THE ALMANAC TO LOCATE THE OLYMPIC GAMES.

What is the 1972 Olympic High Jump record: _____

What is the 1972 Olympic Pole Vault record: _____



1. If you were on the Moon, you could jump 6 times higher.
 - a. Is the Moon's gravity stronger or weaker than Earth's? _____
 - b. How high would the record High Jump be on the Moon? _____
2. If you were on Mars, you could jump $2\frac{3}{5}$ times higher.
 - a. Is Mars' gravity stronger or weaker than Earth's? _____
 - b. How high would the record Pole Vault be on Mars? _____
3. If you were on Venus, you could jump $1\frac{1}{7}$ times higher.
 - a. Is Venus' gravity stronger or weaker than Earth's? _____
 - b. How high would the record High Jump be on Venus? _____
4. If you were on Neptune, you could jump only $\frac{5}{6}$ times as high.
 - a. Is Neptune's gravity stronger or weaker than Earth's? _____
 - b. How high would the record Pole Vault be on Neptune? _____
5. If you were on Jupiter, you could jump only $\frac{5}{12}$ times as high.
 - a. Is Jupiter's gravity stronger or weaker than Earth's? _____
 - b. How high would the record High Jump be on Jupiter? _____

It might be fun to have the students do a standing high jump in class and to figure how high they could jump on the various planets.

TYPE: Paper & Pencil

NUMBER SENSE AND ARITHMETIC SKILLS

Placement Guide for Tabbed Dividers

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Fractions: Mixed Operations

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PIZZA PUZZLE

Estimation
Mixed Operations
FRACTIONS



People will solve the "pizza puzzle" in different ways. Some have spent considerable time with complicated algebraic equations. Others have simply estimated how many times each part would fit into the circle and solved it in minutes. The most interesting solution involves relationships, as well as a reason for multiplying with fractions.

Ditto off a pizza puzzle for each student or group of students. Scissors should be available. Watch how your students go about solving the problem.

Some may estimate how many times each piece will fit into the whole circle.

Some may compare the pieces to each other after finding the value of 2 or 3 pieces. Examples of relationships used in this method are:

$$A + B = E, C + D = B, 3D = B, 2D = C$$

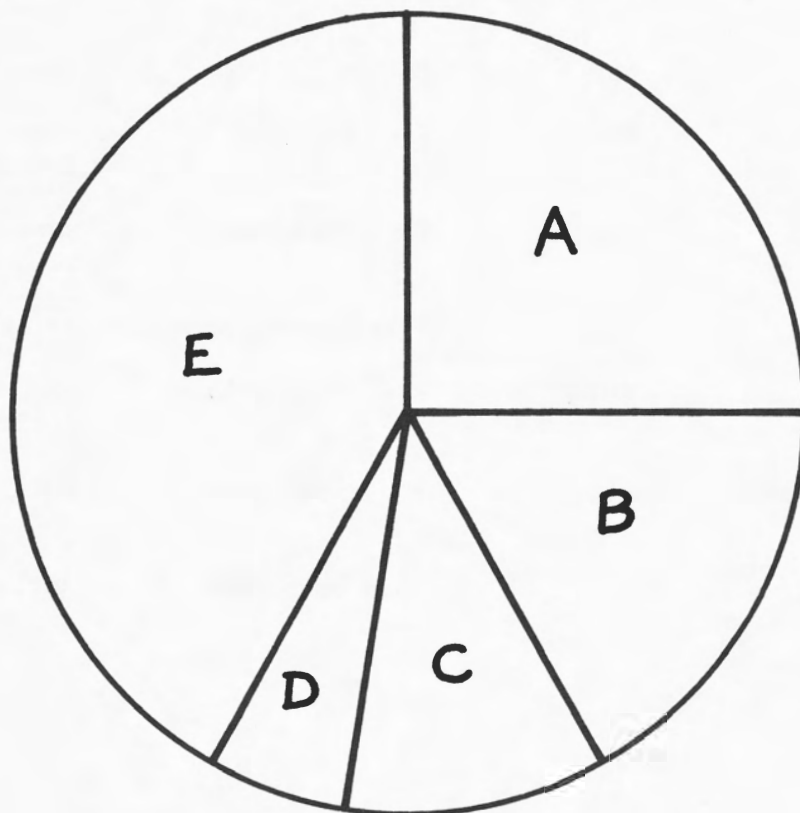
Some may see that $A = \frac{1}{4}$ and B is $\frac{2}{3}$ of A, so $B = \frac{2}{3} \times \frac{1}{4}$.

and that C is $\frac{2}{3}$ of B, so $C = \frac{2}{3} \times \frac{2}{3} \times \frac{1}{4}$.

that D is $\frac{1}{2}$ of C, so $D = \frac{1}{2} \times \frac{2}{3} \times \frac{2}{3} \times \frac{1}{4}$.

and finally that $E = 1 - (A + B + C + D)$.

Can you figure out what fraction of the pizza each piece is?



TYPE: Activity



WHERE'S YOUR HEAD AT?

Mental Arithmetic +,
Mixed Operations
FRACTIONS



I can do most any fraction problem
in my head. For example: $7 + 7\frac{1}{2}$ is easy!

Just add $7 + 7 = 14$
and $14 + \frac{1}{2} = 14\frac{1}{2}$

Try these:

$$8 + 7\frac{1}{2} =$$

$$9\frac{1}{3} + 2 =$$

$$4\frac{1}{5} + 10 =$$

$$6\frac{2}{3} + 4 =$$

$$3\frac{5}{8} + 5 =$$

$$11 + 7\frac{3}{4} =$$

Now for a couple quick subtractions, watch this . . .

$$40\frac{1}{2} - 20$$

$$40\frac{1}{2} \text{ means } 40 + \frac{1}{2}$$

$$\text{so } . . . 40 + \frac{1}{2} - 20 = 20 + \frac{1}{2} = 20\frac{1}{2}.$$

Try it! (Do these in your head.)

$$10\frac{1}{2} - 3 =$$

$$60\frac{1}{2} - 10 =$$

$$75\frac{1}{3} - 25 =$$

$$100\frac{7}{8} - 15 =$$

$$65\frac{2}{5} - 15 =$$

$$49\frac{6}{7} - 13 =$$

These are tricky. Can you figure out how to do these in

your head? Hmmm, $40 - 10\frac{1}{2}$ is the same as $40 - 10 - \frac{1}{2} \dots 30 - \frac{1}{2} = 29\frac{1}{2}$.

$$40\frac{1}{2} - 10\frac{1}{2} =$$

$$75 - 25\frac{1}{2} =$$

$$100 - 13\frac{1}{4} =$$

$$16 - 6\frac{3}{4} =$$

$$7 - 2\frac{3}{5} =$$

$$23 - 17\frac{6}{7} =$$

$$99 - 9\frac{6}{11} =$$

I think these will be very easy for you!

$$100\frac{1}{4} - 5\frac{1}{4} =$$

$$40\frac{2}{3} - 8\frac{2}{3} =$$

$$9\frac{1}{2} - 4\frac{1}{4} =$$

$$92\frac{5}{7} - 2\frac{3}{7} =$$

$$77\frac{3}{4} - 8\frac{1}{2} =$$

$$56\frac{5}{12} - 9\frac{1}{4} =$$

These look pretty difficult but use your head.

$$7\frac{1}{2} - 2\frac{1}{2} + 1\frac{1}{5} =$$

$$8\frac{3}{4} - 7\frac{3}{4} + 1\frac{6}{7} =$$

$$5\frac{1}{4} + 3\frac{1}{2} - 1\frac{1}{4} =$$

$$8\frac{1}{2} - 5\frac{1}{2} + 1\frac{1}{4} - \frac{1}{4} =$$

$$100 - 3\frac{1}{3} - 4\frac{1}{2} - 5 =$$

$$90 - 3\frac{1}{3} - 1\frac{1}{3} - 5\frac{1}{3} =$$

IDEA BASED ON: *Mental Computation* by Klaas Kramer, (c) 1965, Science Research Associates, Inc.

TYPE: Paper & Pencil/Transparency

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IN YOUR HEAD... AGAIN?

Mental Arithmetic or
Mixed Operations
FRACTIONS

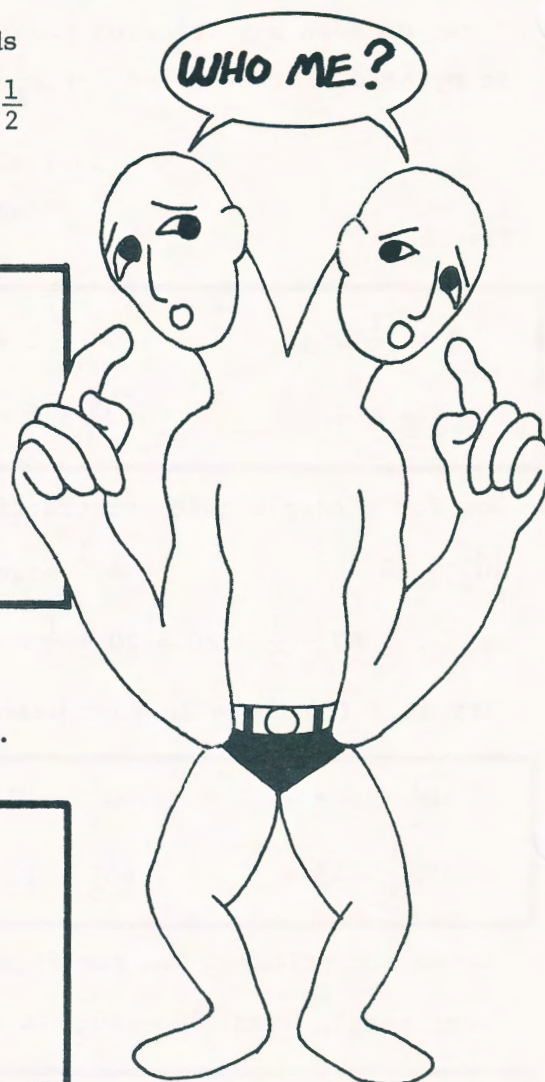


We can also do many multiplication problems in our heads if we give it a try.

For example $\frac{1}{2} \times 12$ we can think of as $\frac{1}{2}$ of 12 or $12 \div 2$, either way we get 6.

Try these:

$\frac{1}{2} \times 16 =$	$\frac{1}{2} \times 40 =$	$\frac{1}{2} \times 22 =$	$\frac{1}{2} \times 60 =$
$64 \times \frac{1}{2} =$	$70 \times \frac{1}{2} =$	$\frac{1}{4} \times 16 =$	$\frac{1}{5} \times 30 =$
$\frac{1}{3} \times 21 =$	$30 \times \frac{1}{3} =$	$\frac{1}{6} \times 54 =$	$28 \times \frac{1}{7} =$
$\frac{1}{2} \times 18 =$	$8 \times \frac{1}{8} =$	$\frac{1}{5} \times 35 =$	



You'll like these too! Use your head!

$\frac{2}{3} \times 12$ Find $\frac{1}{3}$ of 12, that's 4, then multiply by 2.
The answer is 8.

$\frac{2}{3} \times 9 =$	$\frac{3}{4} \times 8 =$	$\frac{2}{5} \times 25 =$	$\frac{4}{5} \times 10 =$
$12 \times \frac{3}{4} =$	$35 \times \frac{3}{5} =$	$\frac{5}{6} \times 42 =$	$\frac{2}{3} \times 3 =$
$\frac{9}{10} \times 100 =$	$81 \times \frac{7}{9} =$	$66 \times \frac{6}{11} =$	$\frac{3}{7} \times 7 =$

Here are problems that are m-i-n-d s-t-r-e-t-c-h-e-r-s!
Stick with it!

$\frac{1}{2} \times 14$ $1 \frac{1}{2}$ means $1 + \frac{1}{2}$,
 1×14 is 14 and $\frac{1}{2} \times 14$ is 7.
 $1 \frac{1}{2} \times 14$ is then $14 + 7$, or 21.

$\frac{1}{2} \times 18 =$	$2\frac{1}{2} \times 12 =$	$3\frac{1}{2} \times 8 =$	$4\frac{1}{2} \times 4 =$
$5\frac{1}{2} \times 12 =$	$16 \times 4\frac{1}{2} =$	$14 \times 2\frac{1}{2} =$	$5\frac{1}{3} \times 6 =$
$6\frac{1}{3} \times 3 =$	$24 \times 1\frac{1}{4} =$	$15 \times 2\frac{1}{5} =$	$42 \times 1\frac{1}{6} =$

BRAIN TEASERS

(no hints this time!)

$1\frac{2}{5} \times 15 =$	$2\frac{3}{4} \times 8 =$
$4\frac{2}{7} \times 7 =$	$2\frac{4}{5} \times 20 =$
$5 \times 2\frac{2}{5} =$	$3 \times 3\frac{2}{3} =$
$42 \times 1\frac{3}{7} =$	$100 \times 5\frac{3}{20} =$

TYPE: Paper & Pencil/Transparency

IDEA BASED ON: *Mental Computation* by Klaas Kramer, (c) 1965, Science Research Associates, Inc.

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GUIDED MAZE: FRACTIONS

Computation +, -, x, ÷
Mixed Operations
FRACTIONS



The answers to the following problems are divided into fractions, right and left, to indicate which direction you should go in the maze. Beginning with problem 1, work a problem and find the right answer in one of the columns. Then go to maze and trace a line until you reach the next problem to be worked. If you make one mistake, you will eventually end up in one of the boxes. If you make two mistakes, you will be stopped at a dead end.

1. $\frac{3}{8} + \frac{1}{4} =$ _____

4. $4\frac{1}{4} - 2\frac{1}{2} =$ _____

8. $\frac{1}{2} \times 1\frac{3}{4} =$ _____

2. $3\frac{1}{3} + 1\frac{5}{6} =$ _____

5. $\frac{2}{3} \times \frac{1}{4} =$ _____

9. $\frac{5}{6} - \frac{2}{3} =$ _____

3. $\frac{3}{4} - \frac{1}{2} =$ _____

6. $1\frac{1}{2} \times 2\frac{1}{3} =$ _____

10. $\frac{2}{3} \times 2 =$ _____

7. $\frac{4}{5} + \frac{1}{10} - \frac{3}{10} =$ _____

RIGHT

$\frac{1}{6} + \frac{2}{12}$ $\frac{3}{5} + \frac{6}{10}$ $3\frac{1}{2}$

$\frac{1}{4}$ $2\frac{4}{3}$ $2\frac{1}{8} + 2\frac{5}{8}$

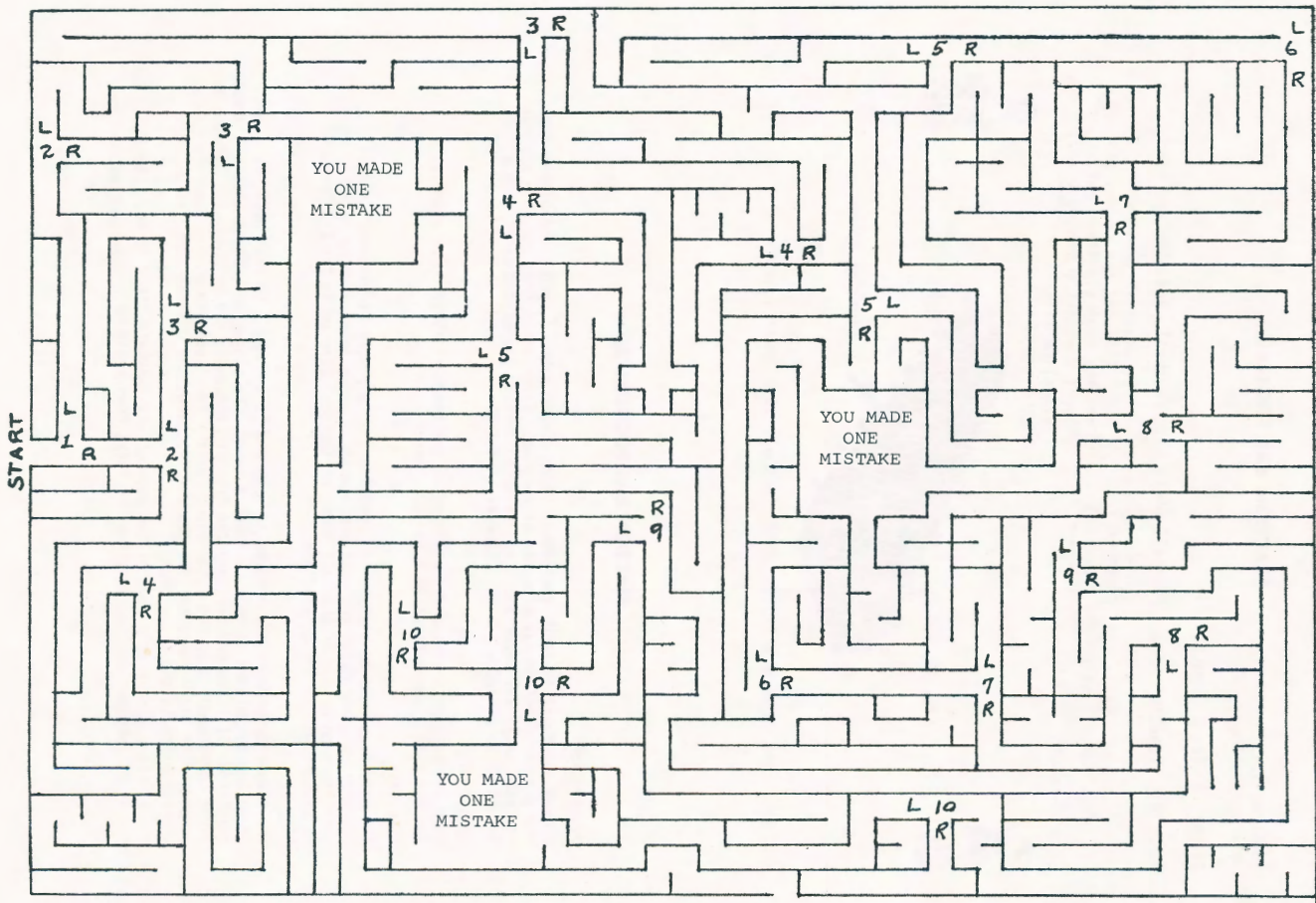
$\frac{1}{3} + \frac{4}{12}$ $11\frac{3}{8}$ $4\frac{6}{9} + 4\frac{2}{3}$

LEFT

$\frac{3}{8}$ $\frac{7}{8}$ $7\frac{3}{4}$

$\frac{2}{5}$ $2\frac{3}{2}$ $2\frac{1}{6}$

$\frac{5}{8}$ $\frac{4}{3} + 1\frac{1}{3}$ $4\frac{7}{6} + 5\frac{1}{6}$

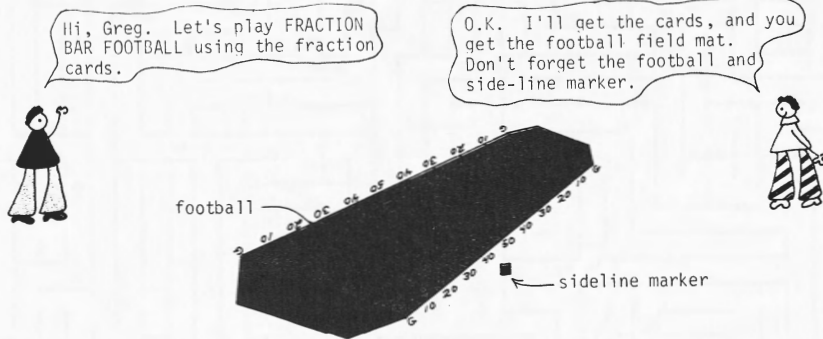


(subtraction, inequality,
and multiplication)

FRACTION BAR FOOTBALL

Players 2 (or 2 teams)

Materials Fraction cards, football field mat, football, and sideline marker



You will need to know a little about football to play this game.

FRACTION BAR FOOTBALL Rules

Preparation

Spread the fraction cards face down. Each player takes 1 card. The player with the greater fraction may kick-off or receive. Cards taken from the pile are not replaced until all cards are used. Playing through all of the cards once allows for about 25 plays, which is 1 quarter of the game.

Convention

Each one-twelfth corresponds to 1 yard on the football field. For example, the fraction $\frac{2}{3}$ corresponds to 8 yards, since $\frac{2}{3} = \frac{8}{12}$.

Kick-offs

The ball is placed on 1 of the 40 yardlines. The player kicking-off takes 1 fraction card and moves the ball the number of yards determined by multiplying that fraction by 6. If the ball lands on the Goal Line or in the End Zone, it is brought out to the 20 yardline.

Play

The player on offense has 4 turns to obtain a First Down. On each Down, he takes 2 cards and the player on defense takes 1 card. The player on offense uses his greater fraction. If it is greater than the fraction chosen by the Defense, the Offense gains yards corresponding to the difference. If the fractions are equal, there is no advance. If the Defense has the greater fraction, the Offense loses yards corresponding to the difference.

The sideline marker is placed on the side of the football field to indicate the number of the Down and the yardline the First Down began on. (Examples on next card)

FRACTION BAR FOOTBALL

Example 1



The Offense's greater fraction corresponds to 9 yards, and the Defense's fraction corresponds to 4 yards. So, the Offense gains 5 yards.

If the Offense had said, double, he would have gained 10 yards. (See Double Yardage, on back)

Example 2



The Offense's greater fraction corresponds to 8 yards, and the Defense's fraction corresponds to 10 yards. So the Offense loses 2 yards.

FRACTION BAR FOOTBALL Rules

Punts

The Offense takes 1 card and moves the ball the number of yards determined by multiplying that fraction by 6.

Points after Touchdown (Play starts from the 3 yardline)

Kicking for 1 point

One card is taken and that fraction is multiplied by 6. The ball must go at least 10 yards beyond the Goal Line.

Running for 2 points

The yardage is determined as on any Down. (Offense takes 2 cards and the Defense 1 card) The ball must go to the Goal Line or beyond for a score.

Field Goals (3 points)

The Offense takes 1 card and multiplies that fraction by 6. The ball must go at least 10 yards beyond the Goal Line for a field goal.

Double Yardage

The Offense may say, double, before taking his 2 bars. The Defense takes 1 card. If the yardage normally gained on the play is greater than 4 yards, the yardage is doubled. If it is less than or equal to 4 yards, the Offense loses his turn. As usual, it is still possible for the Offense to lose yardage.

Computation, or
Mixed Operations
FRACTIONS





Egyptian Fraction Tiles

Computation +, -, x
Mixed Operations
FRACTIONS



$$\boxed{\square} = \frac{1}{2}$$

$$\boxed{\bigcirc} = \frac{1}{6}$$

$$\boxed{\bigcirc} = \frac{2}{3}$$

$$\boxed{\bigcirc} = \frac{1}{3}$$

$$\boxed{\bigcirc} = \frac{1}{4}$$



FILL IN THE EGYPTIAN
FRACTION SYMBOLS TO
MAKE THESE STATEMENTS
CORRECT. A TILE CAN BE
USED ONLY ONCE FOR
EACH PROBLEM

$$\left(\frac{5}{12}\right) = \frac{3}{12} + \frac{2}{12} = \frac{1}{4} + \frac{1}{6} = \boxed{\bigcirc} + \boxed{\bigcirc}$$

A. $\frac{5}{6} =$

$$\boxed{\square} + \boxed{\bigcirc}$$

F. $\frac{7}{12} =$

$$\boxed{\square} + \boxed{\bigcirc} - \boxed{\bigcirc}$$

B. $\frac{7}{12} =$

$$\boxed{\bigcirc} + \boxed{\bigcirc}$$

G.

$$\boxed{\square} = \boxed{\bigcirc} - \boxed{\bigcirc}$$

C. $\frac{3}{4} =$

$$\boxed{\square} + \boxed{\bigcirc}$$

H.

$$\boxed{\bigcirc} = \boxed{\square} + \boxed{\bigcirc}$$

D. $\frac{5}{12} =$

$$\boxed{\bigcirc} + \boxed{\bigcirc}$$

I.

$$\boxed{\bigcirc} = \boxed{\square} \times \boxed{\bigcirc}$$

E. $\frac{3}{4} =$

$$\boxed{\bigcirc} + \boxed{\bigcirc} + \boxed{\bigcirc}$$

J.

$$\boxed{\bigcirc} = \boxed{\square} \times \boxed{\bigcirc}$$

TYPE: Paper & Pencil/Puzzle

IDEA FROM: *Aftermath*, Volume 3

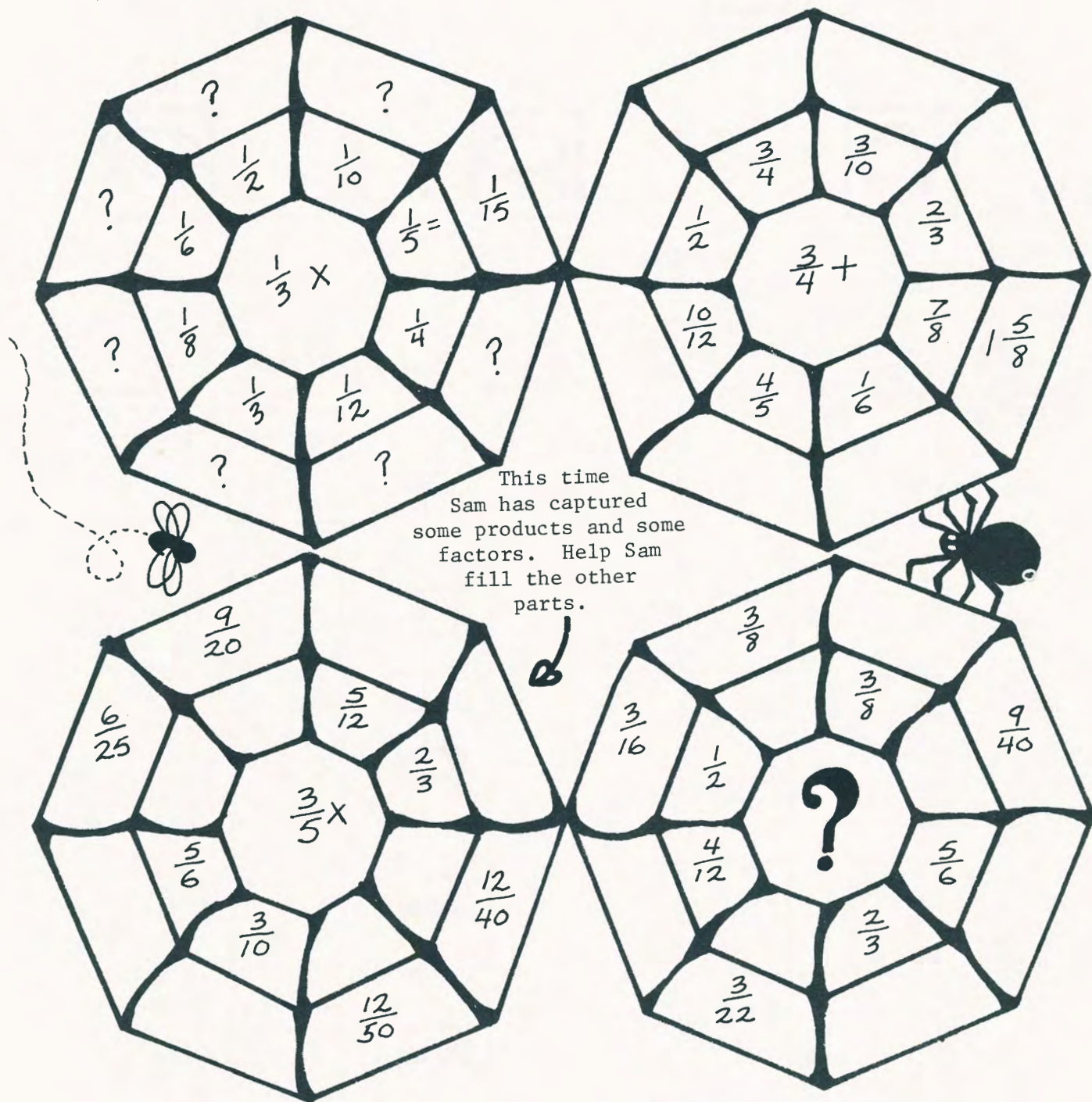
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SPIDER WEB FRACTIONS



Simple Sam, a spider, has spun a web and captured some fractions in parts of the web. Help Simple Sam find the fractions that go around the outside of the web. Write your fractions in simplest form.



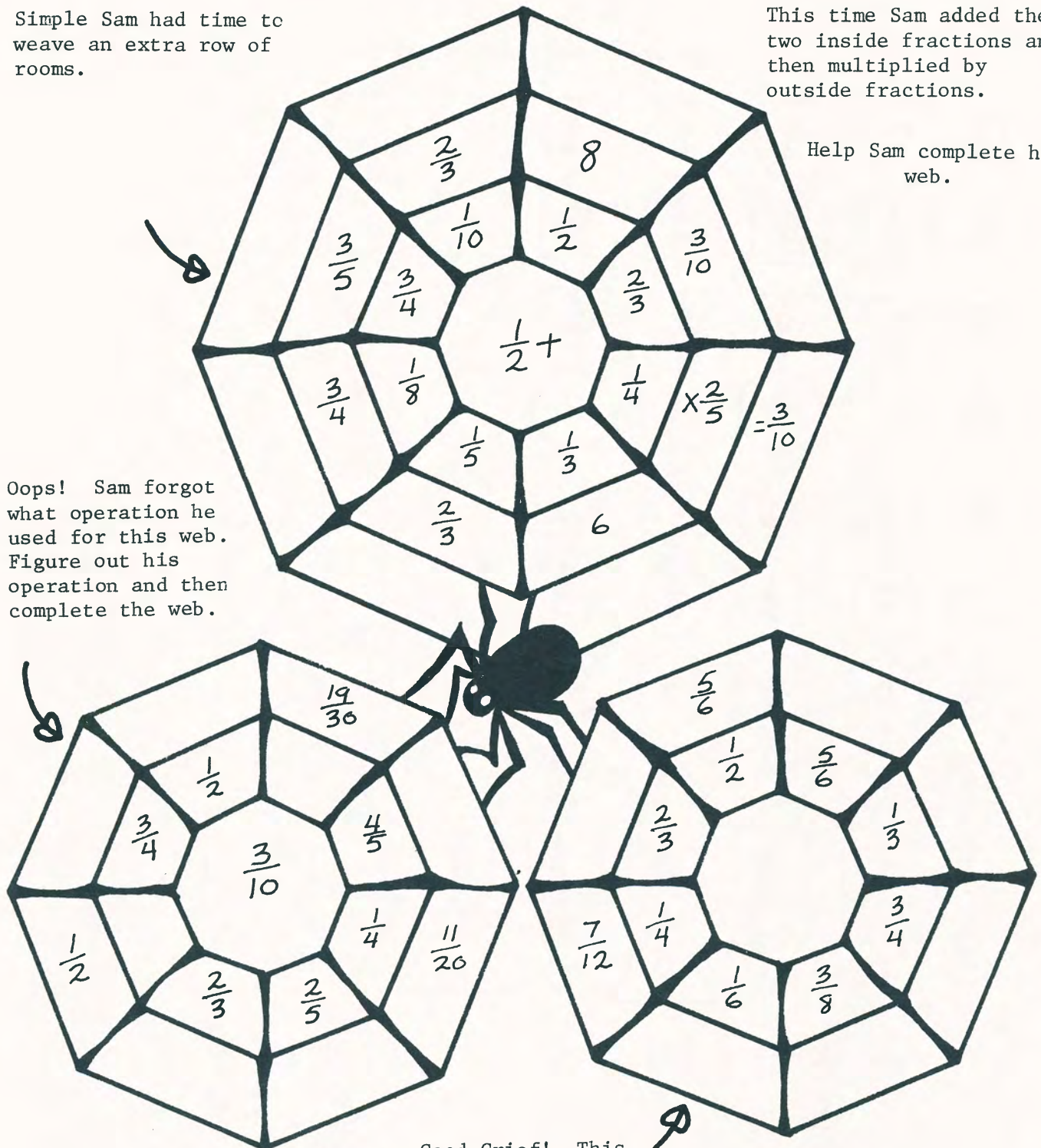
MORE WEB FRACTIONS

Simple Sam had time to weave an extra row of rooms.

This time Sam added the two inside fractions and then multiplied by outside fractions.

Help Sam complete his web.

Oops! Sam forgot what operation he used for this web. Figure out his operation and then complete the web.



Good Grief! This time he forgot the operation and the inside number. Can you find both of them?



FRACTIONS FOREVER

Patterns
Mixed Operations
FRACTIONS



Continue these fraction patterns.

A 0 $\frac{1}{2}$ 1 $1\frac{1}{2}$...

B $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{4}{5}$...

C 0 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$...

D 0 $\frac{2}{3}$ $1\frac{1}{3}$ 2 ...

E $\frac{1}{3}$ $\frac{1}{6}$ $\frac{1}{9}$ $\frac{1}{12}$...

F 0 $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$...

G 0 $\frac{3}{4}$ $1\frac{1}{2}$ $2\frac{1}{4}$...

H 0 $1\frac{1}{4}$ $2\frac{1}{2}$ $3\frac{3}{4}$...

I $\frac{1}{1}$ $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{7}$ $\frac{1}{11}$...

J 0 $2\frac{1}{2}$ 5 $7\frac{1}{2}$...

Extra: **K** $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots$

L $0, 2\frac{1}{3}, 4\frac{2}{3}, 7, \dots$

M $0, 1\frac{3}{4}, 3\frac{1}{2}, \dots$

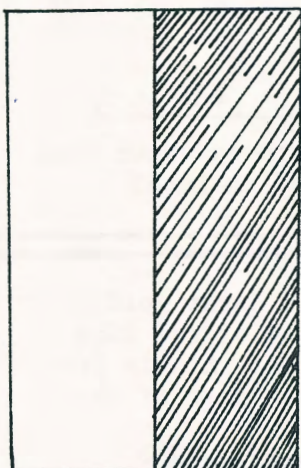
N $\frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$

O $\frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \dots$

P $\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \dots$

Q $\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{1}{20}, \frac{1}{30}, \frac{1}{42}, \dots$

R $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{1}{8}, \frac{1}{13}, \dots$

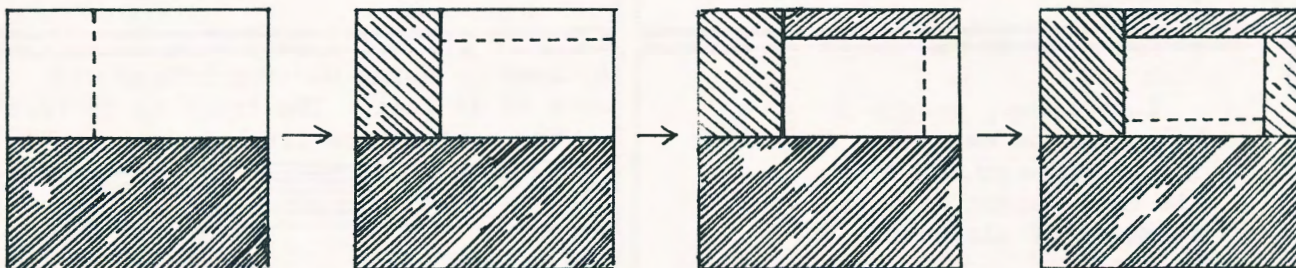


1. What fraction is shaded? _____
2. Shade $\frac{1}{2}$ Of What's Left. Now what fraction of the rectangle is shaded? _____
3. Shade $\frac{1}{2}$ OWL. Fraction shaded? _____
4. Shade $\frac{1}{2}$ OWL. Fraction shaded? _____
5. Shade $\frac{1}{2}$ OWL. Fraction shaded? _____
- ...
10. Shade $\frac{1}{2}$ OWL. Fraction shaded? _____

Another OWL

ONE HALF OF THE SQUARE HAS BEEN SHADED!

The students could do more OWL's for $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots$

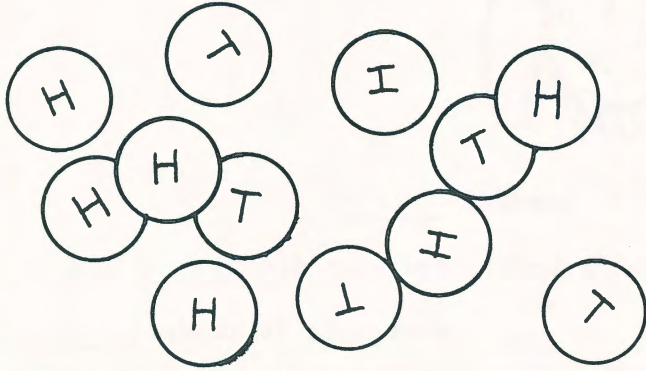


1. Now you shade $\frac{1}{3}$ Of What's Left. What fraction of the square is shaded now? Hint: $\frac{1}{2} + (\frac{1}{3} \cdot \frac{1}{2}) = \underline{\hspace{2cm}}$
2. Now shade $\frac{1}{4}$ Of What's Left. What fraction is shaded now?
3. Shade $\frac{1}{5}$ OWL. What fraction is shaded?
4. Shade $\frac{1}{6}$ OWL. What fraction is shaded? If you continue doing this, what fraction of the square would be shaded after you had shaded $\frac{1}{10}$ OWL?

Students could try an OWL with the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots$

PICTURE PROBLEMS

- 1 7 tosses are heads, 5 are tails.
What fraction are heads?
What fraction are tails?



2



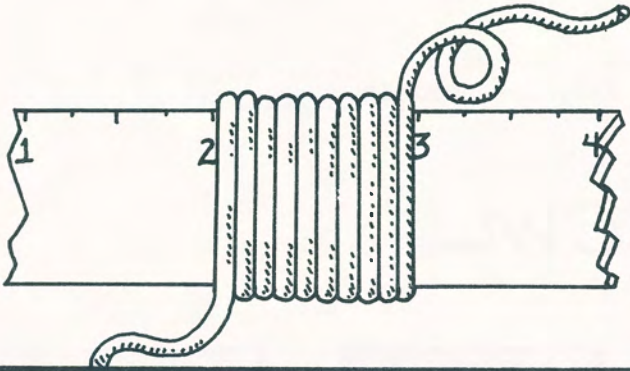
A kilometre is about
what fraction of a mile?



A centimetre is about what
fraction of an inch?

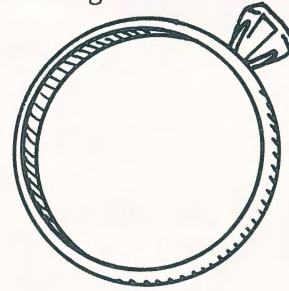
3

Here is a picture of a piece of
string wrapped around a ruler.
How thick is the string?



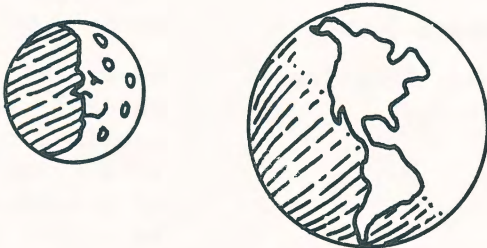
4

If a ring is 18 carat gold,
this means $\frac{18}{24}$ of the ring
is gold. Express this in low-
est terms. What part of the
ring is not gold?

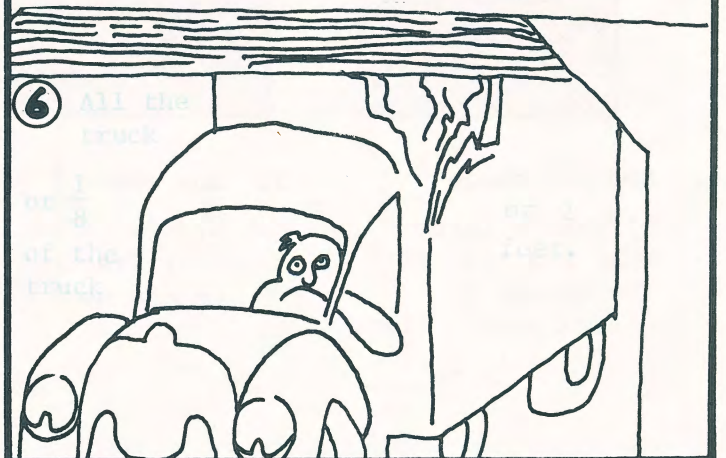


5

Luny Ludwig weighs 30
kilograms on earth but only
5 kilograms on the moon.
His moon weight is what
fraction of his earth
weight?

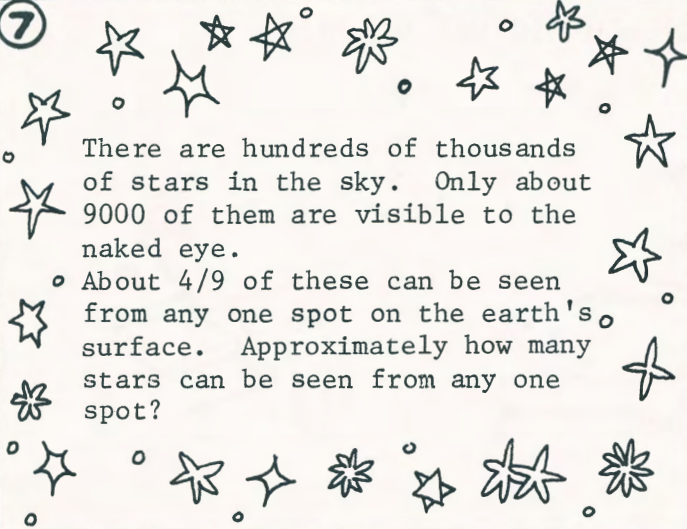


A truck is going under a bridge with clear-
ance of 14 feet. The truck is 16 feet high.
What part of the truck is in trouble?



PICTURE PROBLEMS (CONTINUED)

7



There are hundreds of thousands of stars in the sky. Only about 9000 of them are visible to the naked eye.

- About $\frac{4}{9}$ of these can be seen from any one spot on the earth's surface. Approximately how many stars can be seen from any one spot?

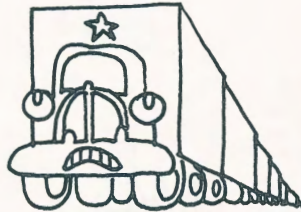
8

Human hair is about $\frac{1}{250}$ of an inch thick. Wool from a Merino sheep is the most expensive in the world and it is $\frac{1}{4}$ as thick as human hair. How thick is it?

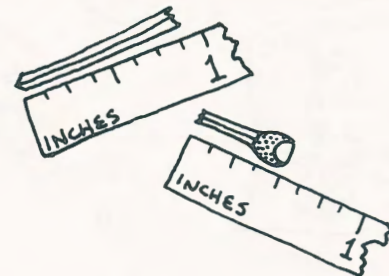


9

The longest vehicle in the world was over $\frac{1}{10}$ of a mile long, had 54 wheels, and a top speed of 20 m.p.h. It was the U.S. Army Overlord Train MK.II. About how many feet long was this truck?



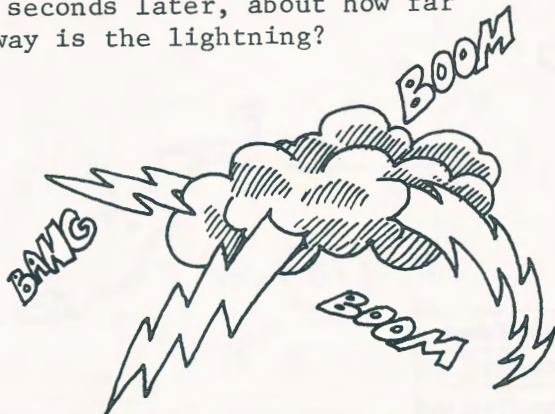
10



How long was the match before it was broken?

11

Sound travels about $\frac{1}{5}$ mile per second. If you see lightning and then hear the thunder 4 seconds later, about how far away is the lightning?



12

A boat uses $\frac{1}{5}$ gallon of gas per mile. If you want to plan a 25 mile trip up the river in your boat, how much gas should you get if you want to get back home?

You may get lots of "creative answers."





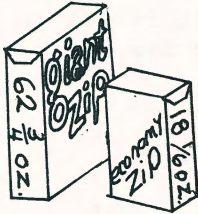
Wordless Problems

Word Problems
Mixed Operations
FRACTIONS

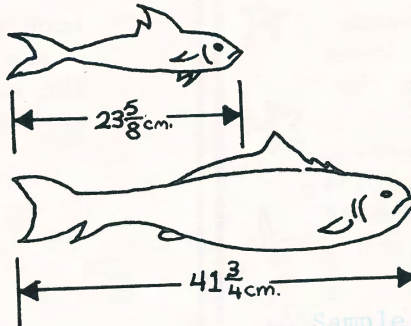


WRITE ONE OR MORE PROBLEMS FOR EACH PICTURE BELOW.

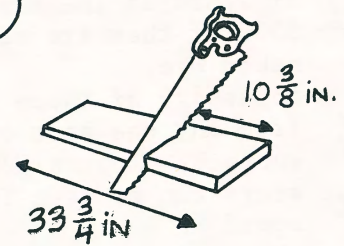
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2

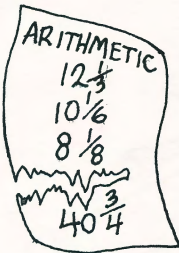


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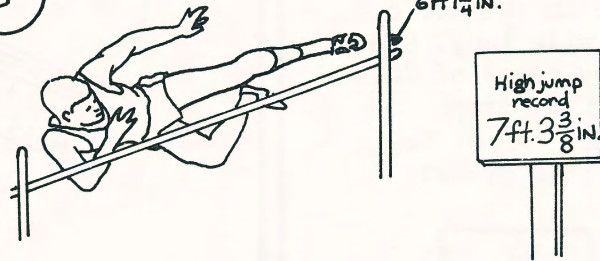


Sample problems:
How long is the other piece of the board?
How many $10\frac{3}{8}$ pieces can be cut from the board?

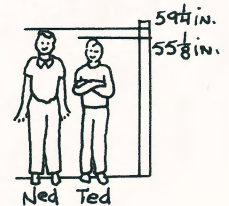
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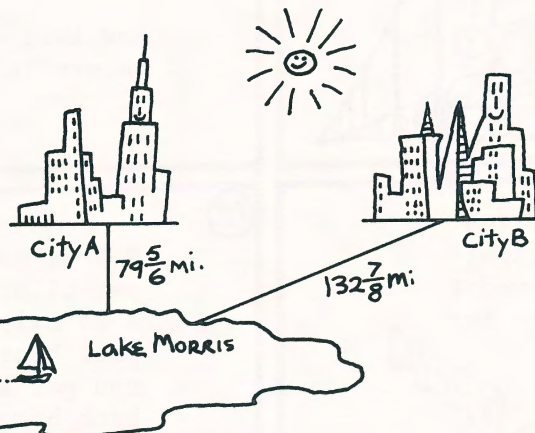
5



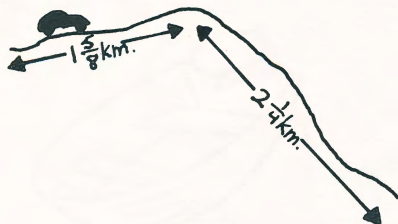
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7



8

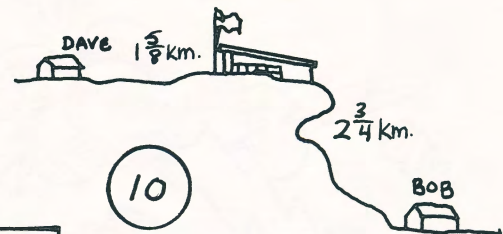


9



100 yd dash	
JONES	9 ⁹ / ₁₀ SEC
BROWN	10 ¹ / ₁₀ SEC
WHITE	10 ⁵ / ₁₀ SEC

10



Have students exchange and solve some of the problems.

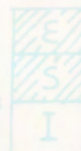
TYPE: Paper & Pencil



MAXIPROJECTS FOR FRACTIONS

TEACHER IDEAS

Mixed Operations
FRACTIONS



HOW FAR IS FAR ENOUGH?

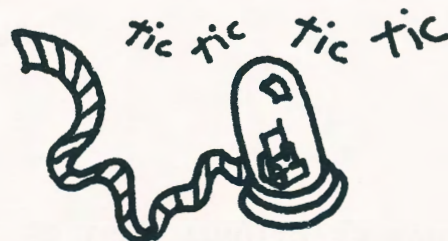
927430

ENCOURAGE STUDENTS IN THE CLASS TO CLOCK ON AN ODOMETER HOW FAR THEY LIVE FROM SCHOOL. GIVE THEM A COUPLE OF DAYS TO BRING IN THE RESULTS. STUDENTS WHO DON'T OR CAN'T GET THIS INFORMATION CAN ESTIMATE THE DISTANCE. THE MIXED NUMBERS AND FRACTIONS WILL HAVE DENOMINATORS OF 2, 5, OR 10.

THE CLASS COULD ORDER THE DISTANCES TO SEE WHO LIVES THE CLOSEST TO SCHOOL. STUDENTS CAN FIGURE HOW MUCH CLOSER THEY LIVE TO SCHOOL THAN THEIR FRIENDS AND HOW FAR THEY TRAVEL EACH DAY, EACH MONTH, EACH YEAR.

THE STOCK MARKET

STUDENTS CAN FOLLOW TRANSACTIONS ON THE STOCK MARKET BY LEARNING HOW TO READ THE STOCK MARKET REPORT AND HOW STOCK IS TRADED. AFTER STUDENTS HAVE BECOME FAMILIAR WITH THE STOCK MARKET REPORT, THEY DECIDE WHICH STOCK TO "BUY." FOR ABOUT 2 WEEKS EACH STUDENT KEEPS TRACK OF THE DAILY HIGH, LOW AND CLOSING PRICES FOR HIS STOCKS. ON A SPECIFIED DAY, ALL STUDENTS "SELL" THEIR STOCKS USING THE CLOSING PRICE FOR THAT DAY AND DETERMINE HOW MUCH THEY "MADE" OR "LOST."



STUDENTS CAN MAKE GRAPHS DEPICTING DAILY HIGHS, LOWS AND CLOSING PRICES. THIS PROJECT WILL USE HALVES, FOURTHS, EIGHTHS AND CONVERSIONS FROM FRACTIONS TO DECIMALS.

FOR MORE INFORMATION SEE: "A SEVENTH-GRADE MATHEMATICS CLASS TACKLES THE STOCK MARKET," FRIEDMANN, ESTHER K.,
ARITHMETIC TEACHER, JANUARY 1973, P. 45-47

TYPE: Activity

MAXIPROJECTS FOR FRACTIONS (CONTINUED)

HOW DO YOU SPEND YOUR TIME?

The following are samples of projects for a class or group that could involve a few days of research, examination and write up. They are meant to be open-ended. They are examples of how numbers can be used to describe and compare things people wish to investigate.

WHAT FRACTIONAL PART OF YOUR DAY DO YOU SPEND SLEEPING, PLAYING SPORTS, WATCHING TELEVISION, TALKING, ETC.?

MAKE A LIST OF THE THINGS YOU DO EACH DAY AND EITHER ESTIMATE HOW MUCH TIME YOU SPEND DOING THEM OR KEEP A LOGBOOK FOR A COUPLE OF DAYS. YOU WILL BE ABLE TO COMPARE WHAT YOU DO WITH YOUR FRIENDS. DISPLAY YOUR RESULTS.

IT WOULD BE VERY INTERESTING TO ESTIMATE WHAT PART OF A YEAR (OR YOUR LIFETIME) YOU SPEND DOING VARIOUS THINGS.

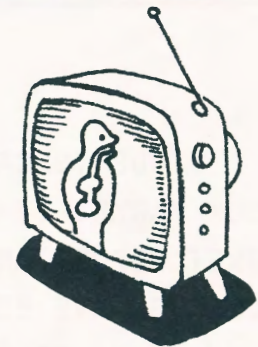


WHAT DO YOU REALLY SEE?

WHAT FRACTIONAL PART OF TELEVISION TIME IS SPENT ON ADVERTISEMENTS, NEWS, DIFFERENT TYPES OF PROGRAMS (COMEDIES, SOAP OPERAS, VARIETY SHOWS, ETC.)

YOU WILL HAVE TO DO A LITTLE RESEARCH HERE; DESIGN YOUR OWN METHODS OF RECORDING DIFFERENT TYPES OF PROGRAMS, AND MAKE ASSIGNMENTS FOR TIMES AND CHANNELS TO INVESTIGATE. IT MIGHT BE FUN TO MAKE A GUESS AS TO WHAT YOUR RESULTS WILL BE BEFORE YOU DO THE RESEARCH. DISPLAY YOUR RESULTS FOR THE REST OF THE CLASS.

YOU MIGHT ALSO COMPARE THE PROGRAMMING ON DIFFERENT CHANNELS, OR THE TYPES OF ADVERTISING THAT ARE MATCHED UP WITH DIFFERENT TYPES OF PROGRAMS.



IDEAS BASED ON: *Math Applications Kit* by Allen C. Friebe and Carolyn Kay Gingrich. © 1971, Science Research Associates, Inc.

FRACTION RECIPES

Mixed Operations
FRACTIONS



AN APPLICATION OF FRACTIONS

A THREE TO FIVE-DAY SUGGESTED OUTLINE

FIRST DAY: HAVE STUDENTS COMPLETE THE EXERCISE ENLARGING A RECIPE 10-15 MINUTES.

SECOND DAY: HAVE STUDENTS COMPLETE REDUCING A RECIPE 10-15 MINUTES.

THIRD DAY: STUDENTS ENLARGE THE SHORTBREAD COOKIE RECIPE.

FOURTH DAY: COOKIES ARE MADE IN CLASS, BAKED IN HOME-EC OVENS, AND EATEN. STUDENTS WORK ON WIENERBURGERS SHEET WHILE WAITING. (15-20 MINUTES)

FIFTH DAY: FOLLOW-UP OR EXTENSIONS.

ENLARGING A RECIPE

YOU ARE GOING TO A POTLUCK DINNER WHERE YOU ARE TAKING THE BEANS.
YOUR RECIPE FOR 8 HAS TO BE ENLARGED TO FEED 32 PEOPLE.

BARBECUE BEANS TO SERVE 8

1/2 CUP MOLASSES
1/4 CUP VINEGAR
1 1/2 TEASPOONS LEMON JUICE
1 TABLESPOON WORCHESTERSHIRE SAUCE
1/3 CUP WATER
1 TABLESPOON BUTTER
1/2 TEASPOON SALT
1/4 TEASPOON GARLIC SALT
1/2 TEASPOON DRY MUSTARD
2 TEASPOONS CHILI POWDER
1/2 TEASPOON RED PEPPER
1/2 TEASPOON PAPRIKA
3 1/2 CUPS CANNED PORK AND BEANS
1/2 CUP ONIONS CHOPPED
1/2 CUP COOKED BACON CRUMBS

BARBECUE BEANS TO SERVE 32

_____ MOLASSES
_____ VINEGAR
_____ LEMON JUICE
_____ WORCHESTERSHIRE SAUCE
_____ WATER
_____ BUTTER
_____ SALT
_____ GARLIC SALT
_____ DRY MUSTARD
_____ CHILI POWDER
_____ RED PEPPER
_____ PAPRIKA
_____ CANNED PORK AND BEANS
_____ ONIONS CHOPPED
_____ COOKED BACON CRUMBS

MIX MOLASSES, VINEGAR, LEMON JUICE, WORCHESTERSHIRE SAUCE, WATER, BUTTER, SALT, GARLIC SALT, DRY MUSTARD, CHILI POWDER, PAPRIKA, RED PEPPER, AND HEAT TO BOILING POINT. POUR OVER BEANS AND ONIONS AND MIX. BAKE AT 305° F. FOR 1 HOUR IN AN UNGREASED 2-QUART CASSEROLE. GARNISH LAST 10 MINUTES WITH BACON CRUMBS.

WILL THIS ENLARGED RECIPE FIT IN A 2-QUART CASSEROLE? WHAT SHOULD YOU USE?

REDUCING A RECIPE

A GUEST AT THE POTLUCK DINNER LIKED YOUR BAKED BEANS AND ASKED FOR THE RECIPE TO SERVE 4 PEOPLE.

BARBECUE BEANS TO SERVE 8

1/2 CUP MOLASSES
1/4 CUP VINEGAR
1 1/2 TEASPOONS LEMON JUICE
1 TABLESPOON WORCHESTERSHIRE SAUCE
1/3 CUP WATER
1 TABLESPOON BUTTER
1/2 TEASPOON SALT
1/4 TEASPOON GARLIC SALT
1/4 TEASPOON DRY MUSTARD
2 TEASPOONS CHILI POWDER
1/2 TEASPOON RED PEPPER
1/2 TEASPOON PAPRIKA
3 1/2 CUPS CANNED PORK AND BEANS
1/2 CUP ONIONS CHOPPED
1/2 CUP COOKED BACON CRUMBS

BARBECUE BEANS TO SERVE 4

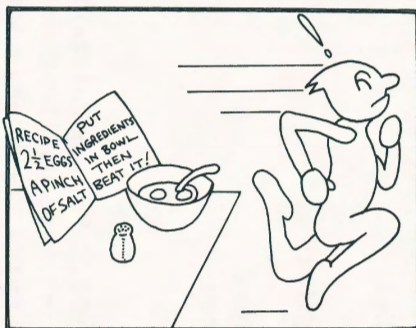
_____ MOLASSES
_____ VINEGAR
_____ LEMON JUICE
_____ WORCHESTERSHIRE SAUCE
_____ WATER
_____ BUTTER
_____ SALT
_____ GARLIC SALT
_____ DRY MUSTARD
_____ CHILI POWDER
_____ RED PEPPER
_____ PAPRIKA
_____ CANNED PORK AND BEANS
_____ ONIONS CHOPPED
_____ COOKED BACON CRUMBS

MIX MOLASSES, VINEGAR, LEMON JUICE, WORCHESTERSHIRE SAUCE, WATER, BUTTER, SALT, GARLIC SALT, DRY MUSTARD, CHILI POWDER, PAPRIKA, RED PEPPER, AND HEAT TO BOILING POINT. POUR OVER BEANS AND ONIONS AND MIX. BAKE AT 305° F. FOR ONE HOUR IN AN UNGREASED TWO-QUART CASSEROLE. GARNISH LAST 10 MINUTES WITH BACON CRUMBS.

CHANGE DIRECTIONS: BAKE IN _____-QUART CASSEROLE.

WIENERBURGERS

- 1 POUND ALL MEAT FRANKFURTERS, GROUND OR CHOPPED FINE
- 1/2 CUP SHREDDED SHARP CHEESE
- 2 HARD-COOKED EGGS, CHOPPED FINE
- 1/4 CUP KETCHUP OR CHILI SAUCE
- 2 TABLESPOONS PICKLE RELISH
- 1 TEASPOON PREPARED MUSTARD
- 1/2 TEASPOON GARLIC POWDER OR GARLIC SALT
- 8 SPLIT, BUTTERED BUNS, ANY BREAD WILL BE FINE



FILL BUNS WITH MIXTURE OF INGREDIENTS AND WRAP EACH IN FOIL. BAKE FOR 12 MINUTES AT 375° OR HEAT ON AN OUTDOOR GRILL.

HOW MANY BURGERS CAN YOU MAKE FROM THIS RECIPE? _____
CHANGE THE RECIPE SO YOU CAN MAKE THE FOLLOWING NUMBER OF BURGERS.

	16	24	12	2
FRANKFURTERS				
CHEESE				
EGGS				
KETCHUP				
RELISH				
MUSTARD				
GARLIC POWDER				
BUNS				

SHORTBREAD COOKIES

SUPPLIES: FLOUR, CUBED MARGARINE, SUGAR, LARGE MIXING BOWL AND SPOON, MEASURING CUPS, SHORTBREAD RECIPE, 2 OR MORE COOKIE SHEETS, USE OF THE OVENS IN HOME EC.

- 1/4 CUP BUTTER, SOFTENED OR MELTED
- 1/3 CUP SUGAR
- 1 3/4 CUP FLOUR

MIX COOKIES BY HAND IN MIXING BOWL, TAKE A GOLFBALL SIZE GLOB OF DOUGH, FLATTEN IN HANDS, PUT ON COOKIE SHEET. (DON'T GREASE THE COOKIE SHEET). IF COOKIES ARE TOO CRUMBLY - ADD MORE BUTTER. BAKE IN 350° OVEN FOR 10-12 MINUTES.

MAKES 2 DOZEN COOKIES.
PLACE THE RECIPE ON THE OVERHEAD AND HAVE STUDENTS DECIDE HOW MANY COOKIES THEY WANT; RESTRICTING CHOICES TO 1, 2 OR 3 EACH. CLASS THEN DECIDES TOTAL NUMBER OF COOKIES NEEDED, AND EACH STUDENT CHANGES THE RECIPE SO THAT THE DESIRED RESULT IS ATTAINED.

IF YOU HAVE ENOUGH TIME, YOU MAY WISH TO MAKE THE COOKIES ON THE SAME DAY YOU ENLARGE THE RECIPE. IF SO, THE NEXT STEP IS TO MAKE THE COOKIES. HAVE STUDENTS HELP WITH THE MIXING, AND LET EACH STUDENT MAKE 1, 2 OR 3 COOKIES AND PLACE THEM ON THE COOKIE SHEETS. RUSH THE SHEETS TO THE HOME-EC ROOM (STUDENTS COULD DO THIS TOO). WHILE YOU ARE WAITING, HAVE THE STUDENTS WORK THE WEINERBURGER ACTIVITY.

IF THERE IS A TIME PROBLEM, IT WOULD BE BETTER TO SPREAD THE ABOVE OVER TWO DAYS. AFTER THE COOKIES RETURN AND HAVE BEEN DIVIDED AMONG THE STUDENTS, THEY WILL WANT TO JUST EAT AND TALK.



FRACTION RECIPES (PAGE 4)

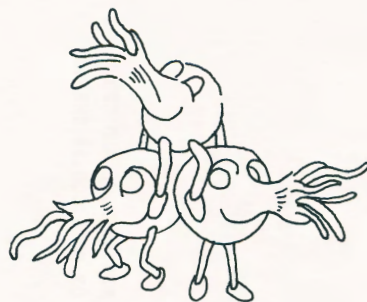
EXTENSIONS

Here is a recipe problem which adds another dimension.

The following recipe is for 9 servings. You have only one teaspoon of baking powder. How much of each of the other ingredients will you use and how many servings will you be able to make?

ONION PATCH PUDDING (9 servings)

2 cups chopped onions
1/4 cup butter or margarine
1 egg, well beaten
1 Tablespoon dried parsley flakes
1 teaspoon salt
1/8 teaspoon pepper
2 1/4 cups flour
3 teaspoons baking powder
1 teaspoon salt
1/2 cup butter or margarine
3/4 cup milk



and how about a METRIC RECIPE?

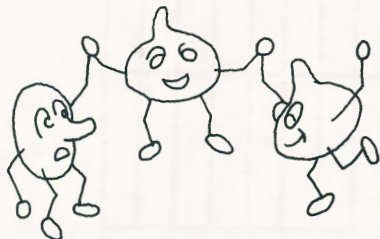
CHOCOLATE DESSERT (a Rich Munchie) (or Unbaked Metric Cake)

90 grams butter
110 grams chocolate chips
80 grams sugar
220 grams vanilla wafers
20 ml white karo syrup

Crush wafers into crumbs. Put chocolate in bowl and set this in bowl of hot water until it melts. Cream the butter, sugar and syrup together. Add the melted chocolate and wafer crumbs and mix thoroughly. Grease a pie tin with margarine. Put the mixture into the tin and smooth out the top. Place in a cold area for 3 or more hours.

Cut and serve.

Many other examples of NO BAKE recipes can be found in cookbooks. These can be used in class activities, or lab station activities involving one or two students.



IT'S A "FULLER" WORLD

This is a model of the 1967 edition of the DYMATION SKY-OCEAN WORLD, designed by Buckminster Fuller.

It is a continuous and almost distortionless map. This is not true of the "rectangular" or "orange peel" maps:



Also, the Dymation Air-Ocean World, unlike other maps can be made directly into a globe. The globe will be in the shape of an icosahedron, (a regular polyhedron, with 20 faces, each face an equilateral triangle).

This entire activity can be used as an interdisciplinary mathematics--social studies lesson.

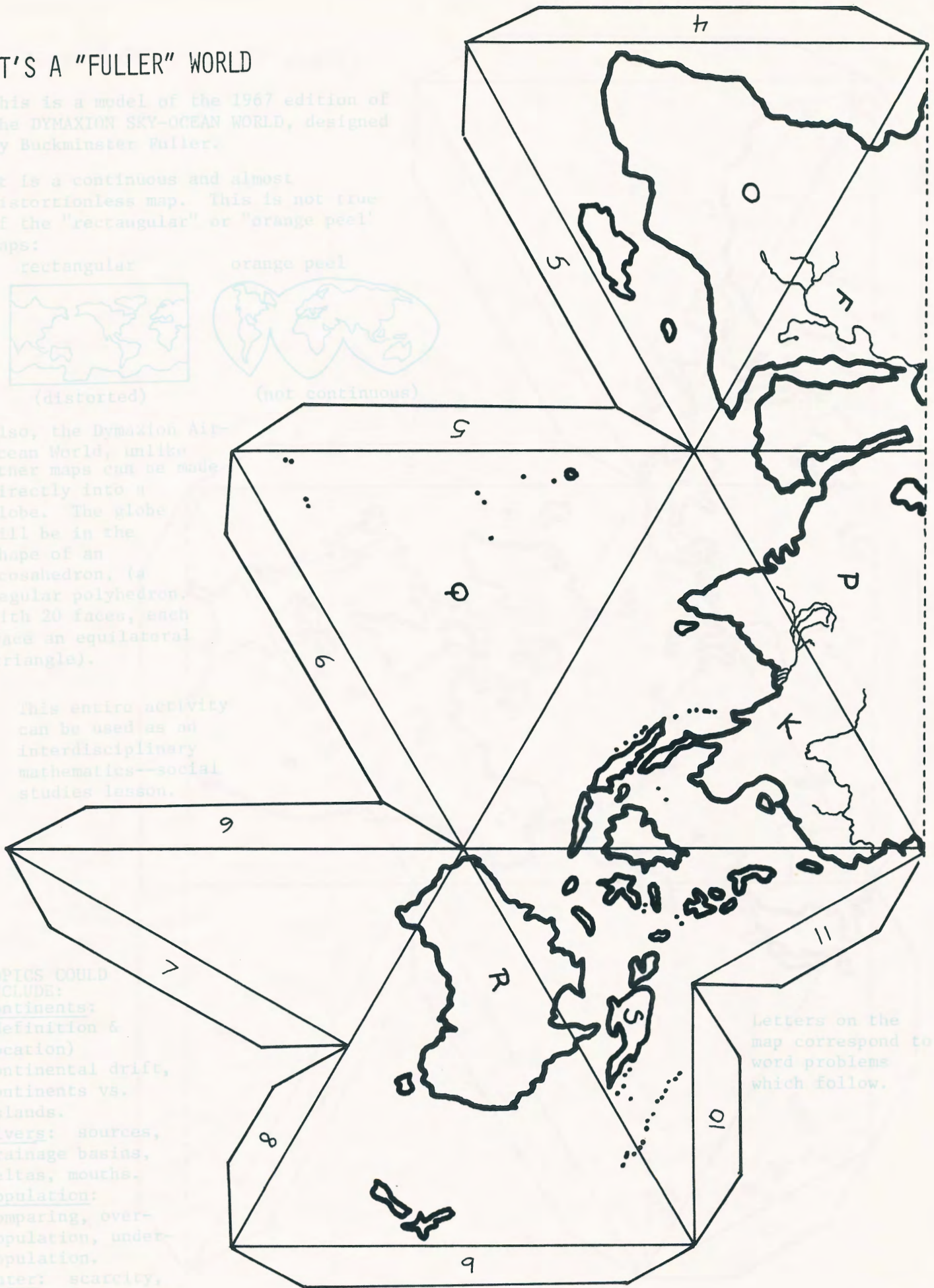
TOPICS COULD INCLUDE:

Continents:
(definition & location)
continental drift,
continents vs. islands.

Rivers: sources,
drainage basins,
deltas, mouths.

Population:
comparing, over-
population, under-
population.

Water: scarcity,
desalinization, etc.



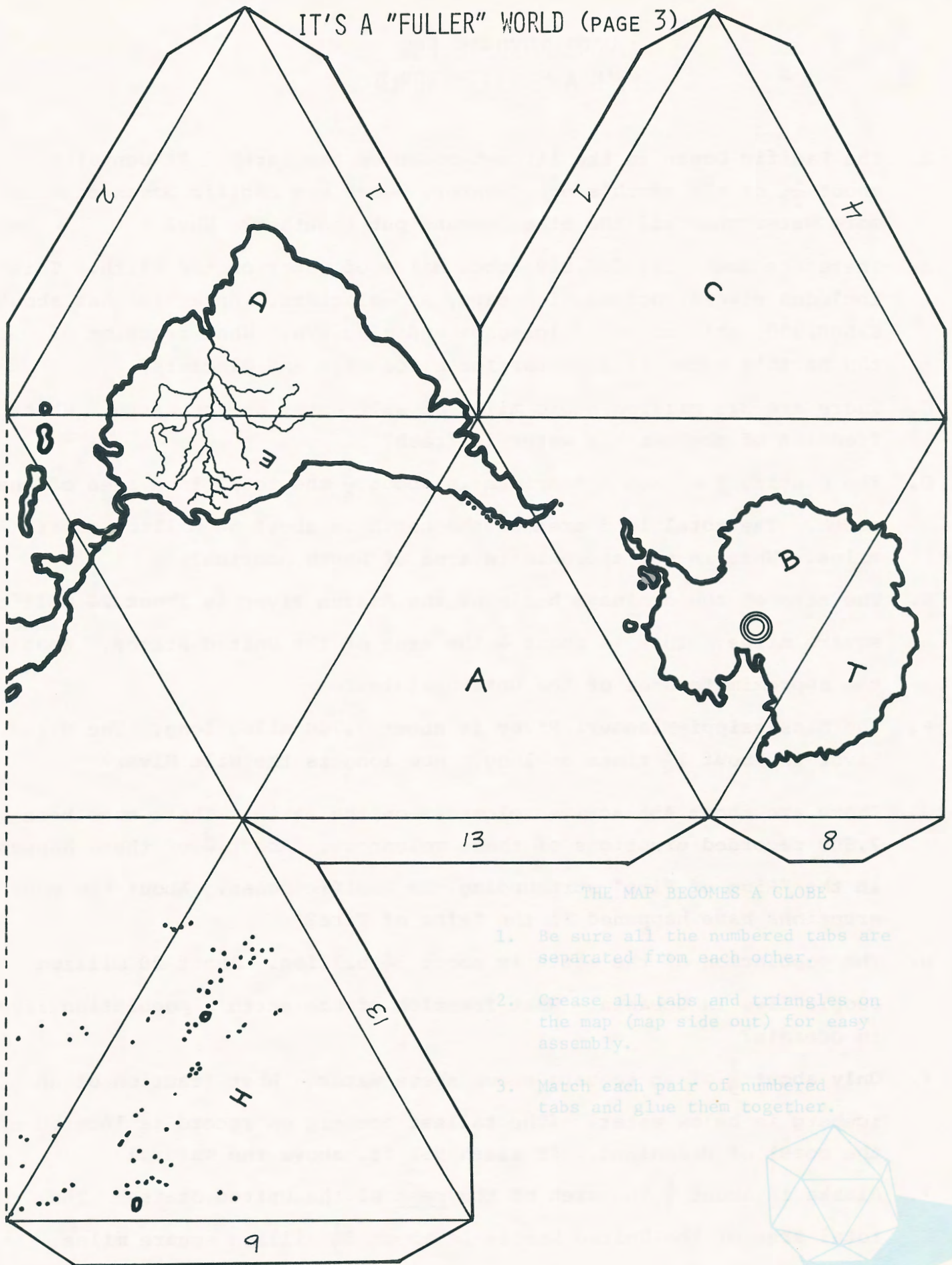
Letters on the map correspond to word problems which follow.

IT'S A "FULLER" WORLD (PAGE 2)

TO CONSTRUCT THE MAP
have your students:

1. Cut around the outer border of the map on each page.
2. Paste the map together in into one piece at the dotted lines. Be sure to keep the pages in order. Rubber cement works best.
3. Color the map. Maps may be colored to emphasize elevation, continents, etc.





THE MAP BECOMES A GLOBE

1. Be sure all the numbered tabs are separated from each other.
2. Crease all tabs and triangles on the map (map side out) for easy assembly.
3. Match each pair of numbered tabs and glue them together.





WORD PROBLEMS FOR IT'S A "FULLER" WORLD

Word Problems
Mixed Operations
FRACTIONS



Letters on the word problems correspond to letters on the map.

- A. The Pacific Ocean is the largest ocean on the earth. It contains about $\frac{9}{20}$ of the earth's ocean water. Does the Pacific Ocean contain more water than all the other oceans put together? Why?
- B. There are about 325,000,000 cubic miles of water on the earth. This includes rivers, oceans, ice caps, and glaciers. Antarctica has about 6,500,000 cubic miles of ice caps and glaciers. What fraction of the earth's water is in Antarctica's ice caps and glaciers?
- C. There are 315 million cubic miles of salt water on the earth. What fraction of the earth's water is fresh?
- D. The continent of South America is about $\frac{1}{8}$ the total land area of the earth. The total land area of the earth is about 56 million square miles. What is the approximate area of South America?
- E. The area of the drainage basin of the Amazon River is about $2\frac{1}{2}$ million square miles. This is about $\frac{2}{3}$ the area of the United States. What is the approximate area of the United States?
- F. The Mississippi-Missouri River is about 3,700 miles long. The Nile River is about $1\frac{1}{8}$ times as long. How long is the Nile River?
- G. There are about 455 active volcanoes on the earth. There have been 2,500 recorded eruptions of these volcanoes. About $\frac{4}{5}$ of these happened in the "Ring of Fire" surrounding the Pacific Ocean. About how many eruptions have happened in the "Ring of Fire?"
- H. The population of the earth is about $3\frac{1}{2}$ billion. About 20 million people live in Oceania. What fraction of the earth's population lives in Oceania?
- I. Only about $\frac{1}{9}$ of an iceberg shows above water. What fraction of an iceberg is below water? (The tallest iceberg on record is located off the coast of Greenland. It rises 550 ft. above the water.)
- J. Alaska is about $\frac{1}{5}$ the area of the rest of the United States. The total area of the United States is about $3\frac{1}{2}$ million square miles. What is the area of Alaska?

WORD PROBLEMS FOR
IT'S A "FULLER" WORLD (CONTINUED)

- K. In 1972 about 2 billion people were living in Asia. The population of the entire earth was about $3\frac{1}{2}$ billion. What fraction of the earth's population lived in Asia in 1972?
- L. The USSR is the largest country on the earth. The total land area of the earth is approximately 56 million square miles. The USSR contains $\frac{3}{20}$ of this land. What is the area of the USSR?
- M. Some of the most expensive land on the earth is in London. In 1972 it was worth about \$1,250 per square foot. How much is this per square inch?
- N. The Sahara Desert in North Africa is the largest desert on the earth. Its area is about $3\frac{1}{4}$ million square miles. What fraction is this of the total land area of the earth? (see problem L.)
- O. Of the 153 countries in the world, 43 are independent countries in Africa. Is this closer to $\frac{1}{3}$ or $\frac{1}{4}$ of the world's countries?
- P. The earth has 108 land peaks over 24,000 feet high. 96 of these peaks are in the Himalayan Range. What fraction of the 108 peaks are not in the Himalayan Range?
- Q. About 140 million square miles of the earth's surface is covered by water. Water covers about $\frac{7}{10}$ of the earth's surface. What is the total land area of the earth?
- R. Australia's population is concentrated in the East and South since $\frac{4}{5}$ of the country is too arid for people to live. What fraction of Australia is suitable for people?
- S. New Guinea is the second largest island on the earth; Greenland is the largest. New Guinea has $\frac{5}{12}$ the area of Greenland. The area of Greenland is about 840,000 square miles. What is the area of New Guinea?
- T. Antartica is almost completely covered by ice. Its area is $1\frac{1}{2}$ times the area of the United States. The area of the United States is about $3\frac{1}{2}$ million square miles. What is the area of Antartica?

NUMBER SENSE AND ARITHMETIC SKILLS

Placement Guide for Tabbed Dividers

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Cross-Number Puzzle	626	Motivation	Paper and pencil
Placing a Point	627	Estimation Place value	Paper and pencil Transparency
Dec.imal	628	Estimation Place value	Paper and pencil
Grid Decimals - I	629	Definition Place value	Paper and pencil
Activity Cards - Cuisenaire Rods - I	630	Definition Place value	Manipulative
Boxing It	631	Place value	Manipulative Paper and pencil
Activity Cards - Decimal Abacus - I	632	Definition Place value	Manipulative
Making Change	633	Place value	Paper and pencil
Making a Decimal Ruler	634	Number line Place value	Manipulative
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DECIMALS: CONCEPTS

HISTORICAL NOTES

The decimal point came into general usage in the early 1700's. It was preceded by various other notations for decimal fractions (those fractions whose denominators are powers of ten). In the sixteenth or seventeenth centuries the decimal numeral we now write as 28.75 might have been designated as $28\frac{75}{100}$, $28/75$ or $28\frac{75}{100}$, all of which fully utilize the principles of place value. Use of the decimal point is not universal even now. Many European countries use a comma to separate the

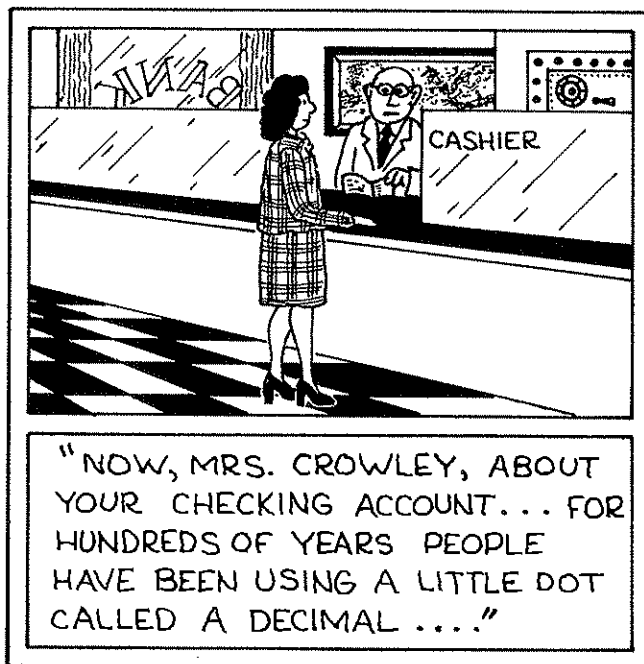
ones' place (often called units) from the tenths' place. Spaces are then used to separate thousands, millions, etc. These countries would write 13,425.67 as 13 425,67. Students in England would write three and five tenths as 3.5 and they would write three times five as 3.5; this reverses our own use of the point.

Why did this way of denoting numbers evolve and why do we want students to learn decimal notation? As societies and businesses became more complex demands grew for more efficient ways of writing numbers. Fractions with larger denominators were especially cumbersome so the efficient language of decimals replaced fractional notation in many business transactions. Today calculators and computers receive input and give output in decimal notation. Students must be able to handle decimal notation when dealing with money and making other transactions in the everyday world.

FRACTIONS VERSUS DECIMALS

Fractions are handy in many situations, but who wants to add $\frac{18}{37}$ and $\frac{25}{52}$? (These numbers might actually arise in business, science, etc.) It is also not simple to determine which of these two fractions is greater (something we might like to know in deciding a better buy). The fractions $\frac{21}{100}$ and $\frac{215}{1000}$ are much easier to add and compare.

Wouldn't life be easier if we could write every fraction with a denominator a power of ten? Then as a shorthand we could just write the numerator of the fraction and somehow note which power of ten is the denominator of the fraction.



Placement of a decimal point could designate which power of ten is in the denominator.

.21	the numerator is 21; denominator is 100 since there are two places after the decimal point
.005	numerator 5; denominator 1000
36.92	numerator 3692; denominator 100
25	numerator 25; denominator 1

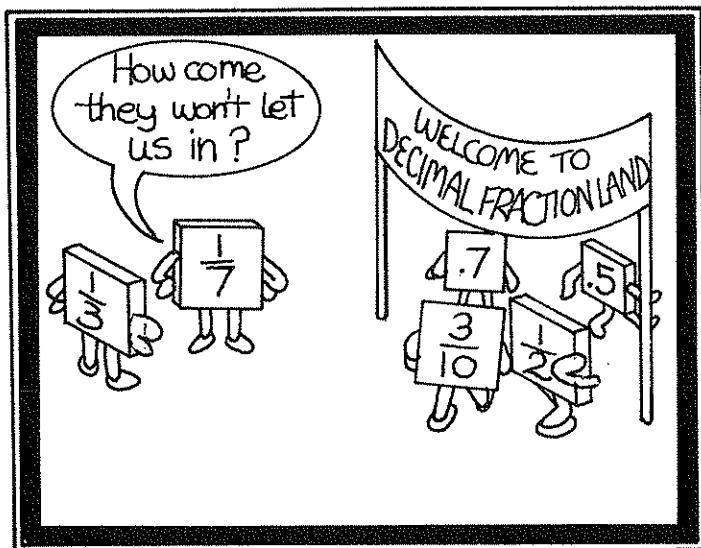
Decimal notation is efficient for writing fractions whose denominators are powers of ten, but what about fractions like $\frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{1}{6}$, etc.? Some of these can easily be changed to equivalent decimal fractions and then written in decimal notation.

$$\begin{aligned}\frac{1}{2} &= \frac{1 \times 5}{2 \times 5} = \frac{5}{10} = .5 \\ \frac{3}{4} &= \frac{3 \times 5 \times 5}{4 \times 5 \times 5} = \frac{75}{100} = .75 \\ 3\frac{2}{5} &= 3 + \frac{2 \times 2}{5 \times 2} = \frac{30}{10} + \frac{4}{10} = \frac{34}{10} = 3.4\end{aligned}$$

Other fractions are not so agreeable. All of the equivalent fractions for $\frac{1}{3}$ have a factor of 3 in their denominator, but no power of ten has three as a factor.

$$\underbrace{10 \times 10 \times 10 \times \dots \times 10}_{\text{power of ten}} = \underbrace{2 \times 5 \times 2 \times 5 \times 2 \times 5 \times \dots \times 2 \times 5}_{\text{prime factorization has only 2's and 5's}}$$

There is no whole number answer to $\frac{1}{3} = \frac{?}{10 \times 10 \dots \times 10}$



At first it seems decimal notation throws out all the fractions except special ones. It is not possible to write a terminating decimal (a decimal numeral with a finite number of digits after the decimal point) for fractions like $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{9}$ and $\frac{1}{7}$. This is because none of these fractions are equivalent to a fraction whose denominator is a power of ten. Each of these fractions can be written as

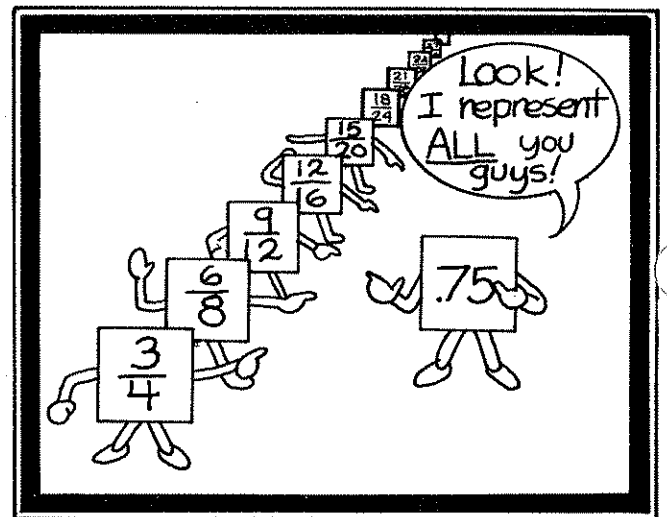
a repeating decimal (a decimal numeral with a repeating block of digits after the decimal point). Look at the examples below. The bar above a block of digits means the block repeats to the right. Notice that each fraction is really the sum of an infinite number of fractions whose denominators are powers of ten.

$$\frac{1}{3} = .33\overline{3} \quad \text{or} \quad \frac{1}{3} = \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \frac{3}{10,000} + \dots$$

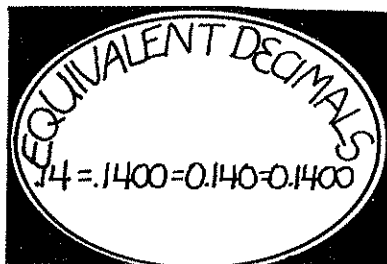
$$\frac{1}{6} = .16\overline{6} \quad \text{or} \quad \frac{1}{6} = \frac{1}{10} + \frac{6}{100} + \frac{6}{1000} + \dots$$

We can write a terminating decimal which approximates a fraction as closely as desired. $\frac{1}{3} \approx .33$ or $.33$ or $.333$ or \dots For many purposes the approximate decimal representation is close enough. Substituting approximate or exact decimal numerals for fractions allows us to use the algorithms for the operations of whole numbers with slight alterations for the decimal point.

There is an important difference between decimal and fraction notation. Let us look at the familiar fraction $\frac{3}{4}$. We have a clear understanding of the fraction $\frac{3}{4}$ and we experience a certain comfortableness using it. Is this same confidence felt with the fractions $\frac{51}{68}$ and $\frac{639}{852}$? These 3 equivalent fractions are just names for the same number. In decimal notation this number is written .75.



The decimal representation of a number is unique with these exceptions:



- a) Decimals can have zeros affixed to the right of the numeral (or in special cases to the left as shown).

$$.2 = 0.2 = .200 = 0.200$$

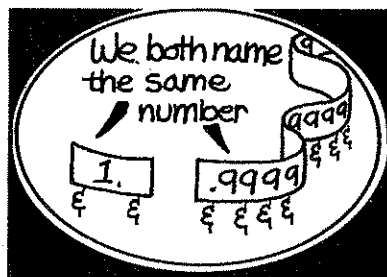
- b) Decimals can be written with repeating nines.

See *Incredible Equalities* in DECIMALS: Multiplication/Division.

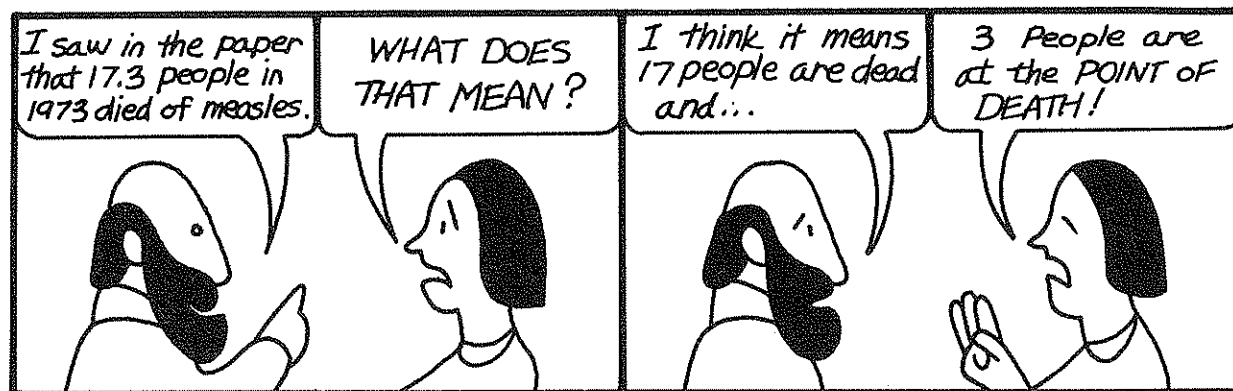
$$2.99\ldots \text{ can be written as } 3.0$$

$$3.499\ldots \text{ can be written as } 3.50$$

$$.999\ldots \text{ can be written as } 1$$

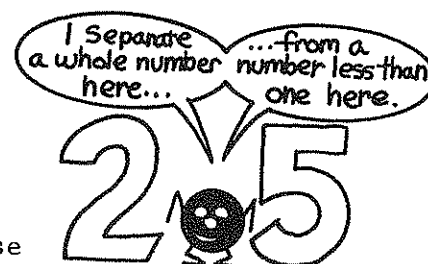


IMPORTANT DECIMAL IDEAS



Before students apply arithmetic operations to decimals they need to know the meaning of decimals and decimal notation. Some of the specific ideas about decimals are discussed below.

The decimal point separates a whole number from a number less than one. The decimal is the sum of these two numbers. Before relating decimals to place value or to equivalent fractions, activities can be used which rely on this understanding of the decimal point. See *Placing a Point* and *Dec.imal* in this section. Later these (or similar activities) can be used as a review or diagnostic exercise.

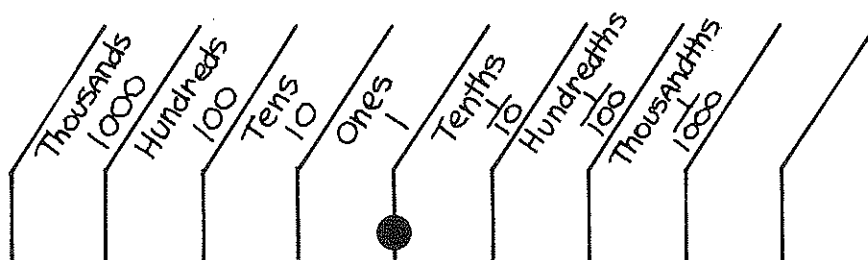


A fraction whose denominator is a power of ten corresponds to a terminating decimal, and vice versa. The position of the decimal point designates the power of ten.

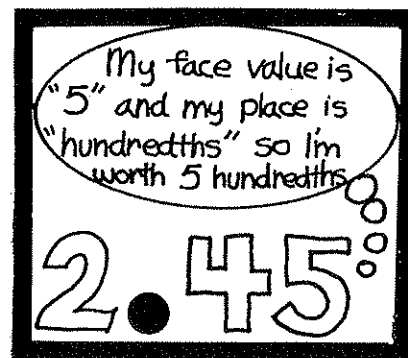
$$\begin{array}{r} \frac{4}{10} \\ \downarrow \\ .4 \end{array} \quad \begin{array}{r} \frac{8}{100} \\ \downarrow \\ .08 \end{array} \quad \begin{array}{r} \frac{215}{100} \\ \downarrow \\ 2.15 \end{array} \quad \text{or} \quad \begin{array}{r} 2\frac{15}{100} \\ \downarrow \\ 2.15 \end{array}$$

$$\begin{array}{r} .7 \\ \downarrow \\ \frac{7}{10} \end{array} \quad \begin{array}{r} .03 \\ \downarrow \\ \frac{3}{100} \end{array} \quad \begin{array}{r} 7.5 \\ \downarrow \\ \frac{75}{10} \end{array} \quad \text{or} \quad \begin{array}{r} 7.5 \\ \downarrow \\ 7\frac{5}{10} \end{array}$$

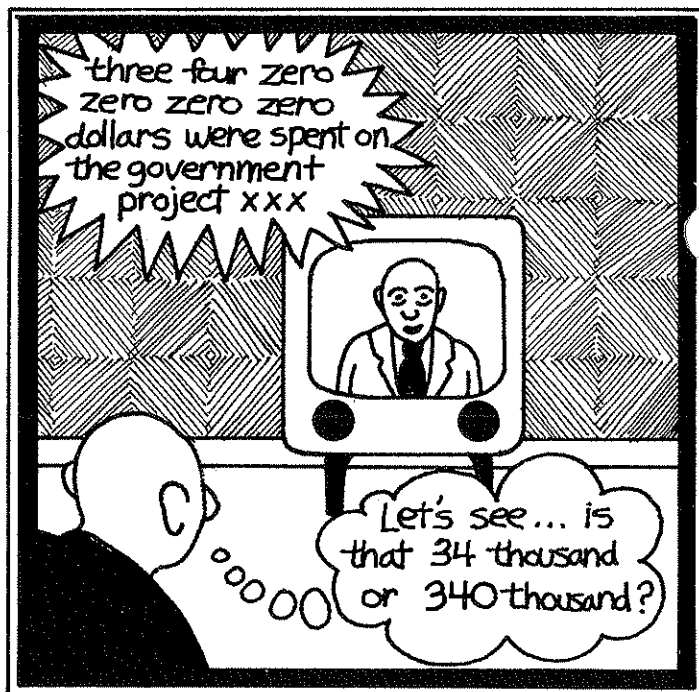
The ideas of place value used in whole number numeration are extended to decimals. A digit in a decimal numeral represents one-tenth of the number named by the same digit one place to the left. The place value names are symmetrical about the ones or units place, not around the decimal point.



The value of a digit in a numeral depends on its face value and its place value. In 38.715 ten is a place value and 3 is a face value, so the 3 represents 3 tens or 30; the 5 represents 5 thousandths or $\frac{5}{1000}$. See *Doing Your Thing* for a drill page on place value. Understanding of place value can help students understand why we "line up the decimal points" in addition and subtraction. This lining up keeps us from adding the number of tenths in one decimal to the number of hundredths in another.



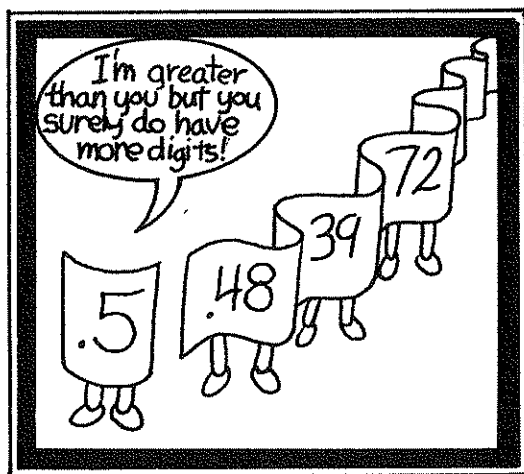
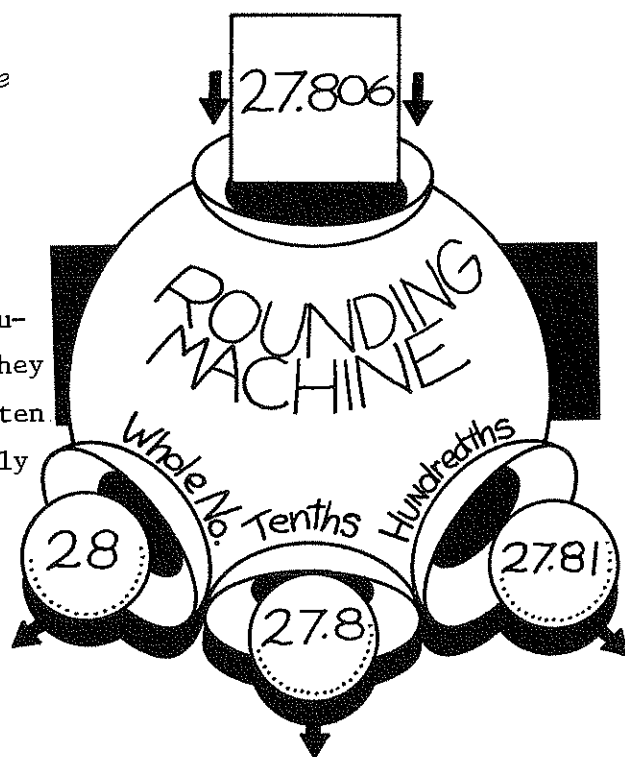
Reading and writing decimals depends on understanding place value and decimal notation. To read the numeral 617.014 as six hundred seventeen and fourteen thousandths, a student must read the whole number, replace the decimal point with "and," read the digits after the decimal point as if they represented a whole number and give the place value of the last digit. Writing a decimal numeral from the words requires that the process be reversed. In actual practice we might dictate the numeral above as six one seven point zero one four. A person can copy down this sequence of digits but there is little understanding of the magnitude or precision of this number when the place value words are removed. How would you react to the news announcer pictured at the right?



The standard way of reading 5.4 is "five and four tenths." Knowing that "and" replaces the decimal point makes the amount written on checks clear. It is helpful if students learn that 5.4 also represents fifty four tenths, five hundred forty hundredths (5.40), five and forty hundredths, etc. There are many correct readings for a numeral but only one is conventional. The unconventional readings help in understanding the meaning of the numeral. They are also useful when changing fractions to decimals and in the operations of decimals.

Affixing zeros to the right or left of a decimal numeral does not change its value. For example: $.2 = .20 = .200 = 0.2$. This is a hard concept. In $.2$ the focus is on "2"; in $.20$ students see the "20". Of course, affixing a zero to the right of some numerals necessitates supplying a decimal point: $30 = 30.0 = 30.00$. A student page giving drill in equivalent decimals is *What's the Point?* This idea of equivalent decimals is useful in the division algorithm. $.31 \overline{)4} \rightarrow .31 \overline{)4.00} \rightarrow 31 \overline{)400}$ Here 4 is viewed as 4 ones which is the same as 40 tenths or 400 hundredths. The advantage of learning different ways for reading a decimal becomes apparent in operations with decimals. See the commentaries for decimal operations.

Decimals can be rounded to any place--to the nearest whole number, nearest tenth, nearest hundred thousandth.... The rounding of decimals is helpful when approximating sums, differences, products and quotients of decimal numerals. 34.71×2.2 is approximately 35×2 or 70. Calculators round (truncate) out of necessity since they can display only a certain number of digits. Often we round a decimal so its magnitude is more easily understood. $.000213496$ might not be as handy a number as $.0002$, and if accuracy is not important then $.0002$ might be preferable. The table of contents gives several student pages providing practice and applications for rounding.



To compare or order decimals it is necessary to understand the meaning of the decimal numeral. Some students will write $.25 > .8$ since $25 > 8$, but those who understand decimals as fractions might reason $.25 = \frac{25}{100}$ and $.8 = \frac{8}{10} = \frac{80}{100}$ so $.25 < .8$. Others might think of place value and say there is less than 3 tenths in $.25$ but $.8$ has 8 tenths so $.25 < .8$. Decimals are compared to find a better buy, a higher rate of interest and longer lengths. Related student pages are keyed to "ordering" in the table of contents.

Fractions can be written in decimal notation.

Fractions whose denominators have only 2's and/or 5's as prime factors can be written easily as decimals. See *Decimal Double Plays*. Other fractions like $\frac{2}{3}$ and $\frac{1}{6}$ can be approximated in decimal notation with the aid of a manipulative before division of decimals is introduced. This is discussed under Decimals and Place Value Models below.

$$\frac{1}{2} \rightarrow \frac{5}{10} \rightarrow .5$$

$$\frac{2}{5} \rightarrow \frac{4}{10} \rightarrow .4$$

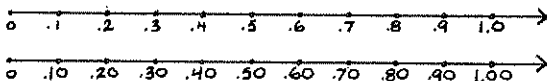
$$\frac{3}{20} \rightarrow \frac{15}{100} \rightarrow .15$$

$$20 = 2 \times 2 \times 5$$

MODELS FOR TEACHING DECIMALS

There are several good models for helping students understand decimals. Among these are the number line, place value models and money. Each of these is discussed below with suggestions for their use.

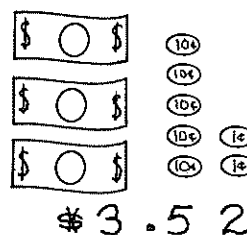
NUMBER LINES



PLACE VALUE MODELS



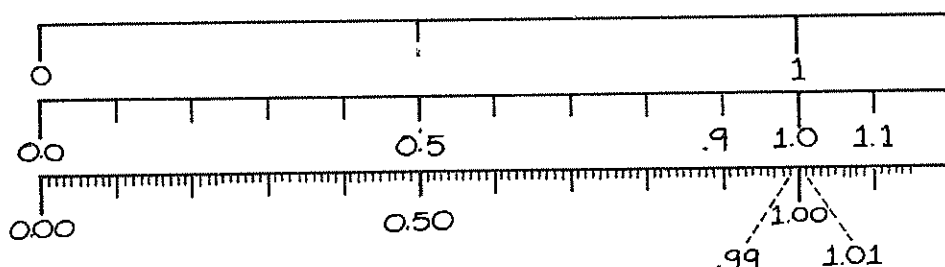
MONEY



Decimals and the Number Line

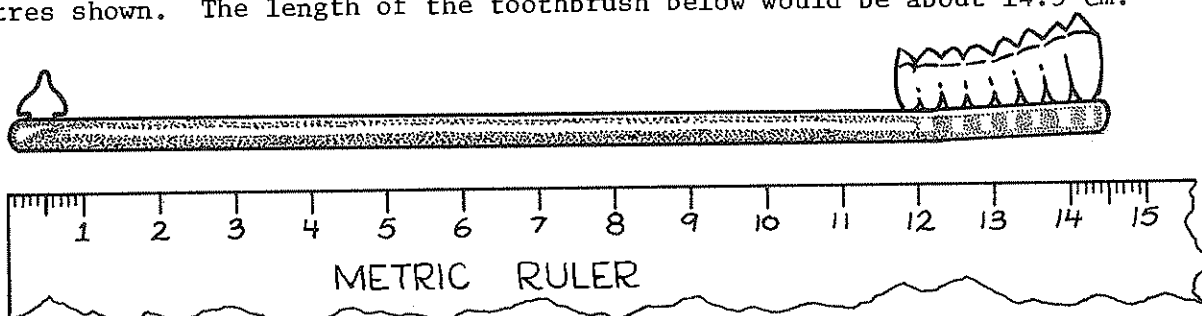
After students have represented whole numbers and fractions on a number line they can learn to represent decimals on a number line. A large number line can be made on a strip of paper tacked to the bulletin board or wall. Zero, one and two can be placed on the number line with at least a metre between zero and one. Tenths can be labeled and the divisions marks for hundredths added. As students are asked to locate the points corresponding to specific decimals they will soon stop counting the hundredth spaces and count by tenths. They eventually learn to find a point for .35 by finding .3 (3 tenths or 30 hundredths) and adding 5 more hundredth spaces. Thus, $.35 = .3 + .05$. The number line can provide the transition from a fraction understanding of decimals to a place value understanding.

The patterns used in labeling such number lines can help students realize that 1, 1.0 and 1.00 are names for the same mark on the number line.



Objects can now be measured and their lengths recorded to the nearest hundredth of a unit. Theoretically, this process of subdividing could be carried on indefinitely.

A metric ruler or metre stick is a good model for decimal notation. The ruler is usually marked in whole numbers of centimetres, with divisions of millimetres shown. The length of the toothbrush below would be about 14.5 cm.

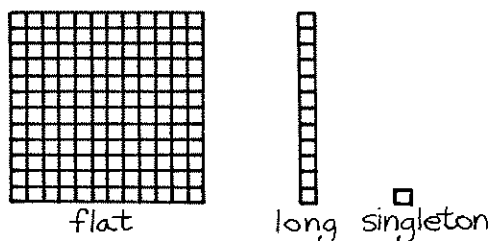


Of course, the length of the toothbrush is also 145 millimetres or 1.45 decimetres or .145 metres, but this transition is not easy. If decimal parts of a metre are desired adding machine tape could be used to make rulers marked in whole numbers of metres. Tenths and hundredths of a metre could be marked in decimal notation.

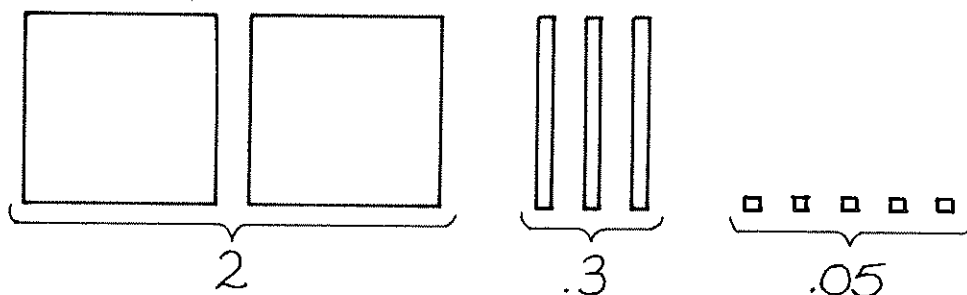
The number line is a versatile model and can be used to order decimals, to find equivalent decimals and to introduce place value. It can also be used to introduce addition and subtraction of tenths and hundredths.

Decimals and Place Value Models

Place value models such as multibase blocks, beansticks or the abacus are useful in learning decimal concepts. A set of paper place value models can easily be made from cover stock (file folder thickness) which has had centimetre grids dittoed on it. Flats (10 x 10 squares), longs (1 x 10 rectangles) and singletons (1 x 1 squares) can be cut from the paper grids.



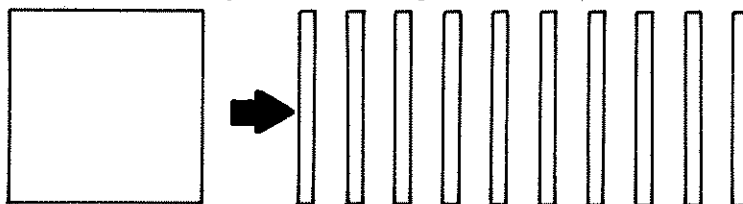
If a flat is chosen to represent 1, a long represents .1 and a singleton represents .01. Any decimal number in hundredths or tenths can be represented with this model. The decimal 2.35 is shown below.



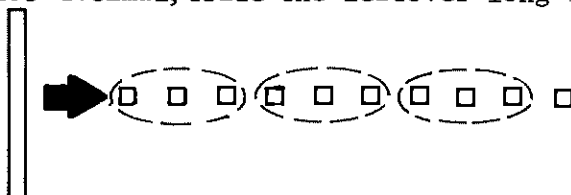
If students are not convinced that $2.35 = 2 + .3 + .05$ they can be encouraged to figure out the number of hundredths. If they have learned that $2.35 = \frac{235}{100}$ they will need to convince themselves that there really are 235 singletons represented; if they have learned that $2.35 = 2\frac{35}{100}$, they need only check that there are 35 singletons in addition to the 2 flats.

This model can be used to find an approximate decimal for a fraction.

Suppose we want to approximate $\frac{1}{3}$ with a decimal. $\frac{1}{3}$ means $1 \div 3$ so we could take a flat and divide it into 3 pieces, but we are not allowed to cut up the pieces--only to trade them in for other place value pieces. In base ten a flat is traded in for ten longs.

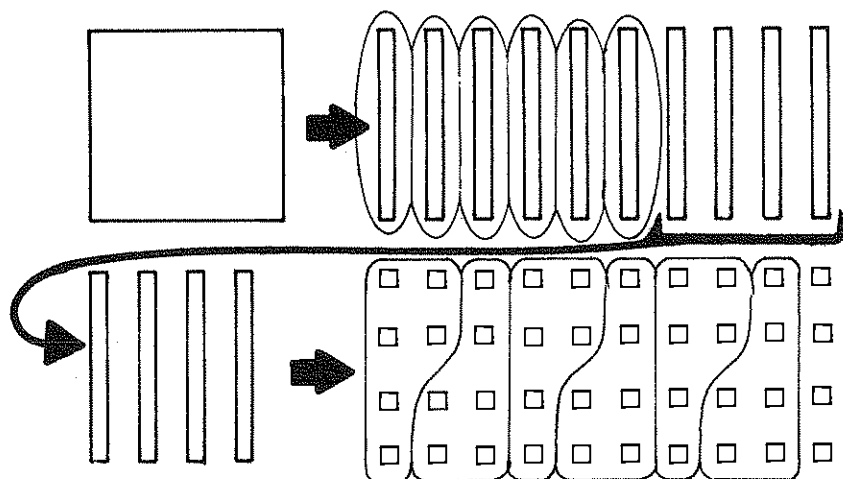


Now if the ten longs (ten tenths) are divided into 3 equal piles there are 3 longs per pile with one long leftover. Ignoring the leftover long, $\frac{1}{3} \approx .3$. If we want a more exact decimal, trade the leftover long for ten singletons.



Dividing by 3 gives 3 singletons (hundredths) per pile with one singleton left. $\frac{1}{3} \approx 3 \text{ tenths} + 3 \text{ hundredths} = .33$.

Here's another example: Find a decimal approximation for $\frac{1}{6}$.



$\frac{1}{6} \approx .1$ (This is not to the nearest tenth.)

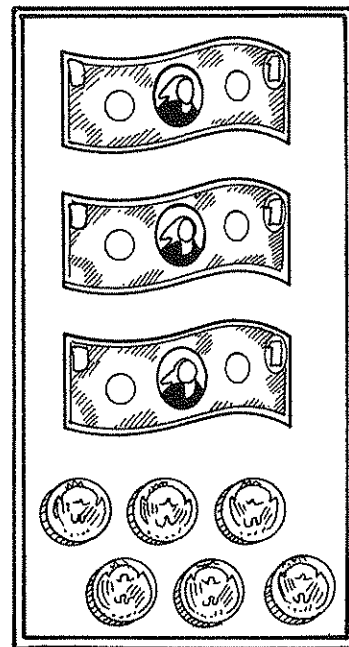
$$\frac{1}{6} \approx .1 + .06$$

so, $\frac{1}{6} \approx .16$

After a few such examples students will sometimes realize that the decimal will begin to repeat. In the example above 4 singletons are left just like 4 longs were left in the first step. The digit in the thousandths' place will again be 6.

Decimals and Money

For most students decimal points are first seen in connection with money notation. They see labels or advertisements giving prices of items. The decimal point separates the number of dollars from the number of cents, and we usually read \$3.98 as three dollars and ninety-eight cents rather than three and ninety-eight hundredths dollars. If exact payment were made for a pair of shoes priced at \$15.85 a customer might give the clerk a ten, a five, 3 quarters and a dime. It is not likely that the customer would follow place value and use 1 ten, 5 ones, 8 dimes and 5 pennies. Nevertheless, money notation is a good introduction to decimals and a good application of decimal notation.



Students learn that a decimal point is placed after a whole number of dollars. The digits after the decimal point represent some part of a dollar. A bill of \$12.35 means you need to pay 12 whole dollars plus some part of a dollar but less than 13 dollars.

If coins and bills are restricted to pennies, dimes, dollars, ten dollars, etc. then we can focus on place value in money notation. See *Making Change*. Ten pennies will buy as much as one dime and ten dimes will buy as much as one dollar.

A \$8.98 baseball bat can be purchased with 8 dollars, 9 dimes and 8 pennies (and that is the least number of pieces of money which could be used under this restriction) or 89 dimes and 8 pennies or 898 pennies. Although we usually write one dime as \$.10 it would be sensible to write one dime as \$.1 because a dime is one tenth of a dollar. A penny is one hundredth of a dollar and should be written \$.01. Ten pennies is ten hundredths of a dollar and is written \$.10, but ten pennies are also worth one dime so $.1 = .10$.

Rounding decimals can be practiced with money notation. \$349.53 can be rounded to \$350.00 (the nearest ten dollars or the nearest dollar) or to \$349.50 (the nearest tenth of a dollar). If the approximate cost of 3 yards of material at \$4.98 a yard were to be computed then the decimal \$4.98 could be rounded to \$5 giving an approximation of \$15.00. Comparing prices is also a good introduction to comparing decimals. Which costs more: a pair of jeans for \$8.48 or a pair for \$8.90? Which decimal is greater: 8.48 or 8.90? What about 8.48 or 8.9?

Students might benefit from inventing a new coin which is worth a tenth of a penny or \$.001. A name such as ten-penny could be suggested. Was there ever such a coin? The United States uses the word "mill" to designate $\frac{1}{1000}$ of a dollar. The word still occurs in discussing taxes as in mill levies, etc. Extending money notation to mills could be used as an introduction to thousandths.

Money notation, which students know something about, can be a good reference for learning decimal concepts. Later, money notation can be used to motivate the treatment of the decimal point in addition and subtraction.

DECIMALS WITH OR WITHOUT FRACTIONS?

The traditional approach is to study decimals after a thorough study of whole numbers and fractions; however, decimals can be approached directly from whole numbers. Our decimal money notation and the metric system, which groups by tens, both encourage the use of decimal notation early in the grades. Those students who have had much difficulty with fractions might benefit from an approach to decimals which, temporarily at least, by-passes fractional notation.

In an approach to decimals without fractions .47 would be viewed as naming a position on a number line; or as a convenient way of writing 47 of 100 equal parts of a whole; 47 out of 100 objects; $47 \div 100$. Much of the developmental work now done with fractions would have to be transferred to decimals. The relationship between commonly used fractions (like $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, etc.) would still be important.

Some educators speculate that fractions will become less popular as we move to the metric system. Surely the use of decimals will increase. The full impact of the metric system on our teaching of decimals is not yet known.



DEWEY DECIMALS

Motivation
Concepts
DECIMALS



In 1876, Melvin Dewey (an American librarian) devised a system for classifying non-fiction books.

000 → 99 includes: _____

100 → 199 includes: _____

200 → 299 includes: _____

Dewey divided all knowledge into 10 classes.

300 → 399 includes: _____

400 → 499 includes: _____

Go to the library and find out what each class means.

500 → 599 includes: _____

600 → 699 includes: _____

700 → 799 includes: _____

800 → 899 includes: _____

900 → 999 includes: _____

The Dewey Decimal Classification reference book in your library lists all subdivisions of the 10 main groups. Use this book to find the meanings of the numbers listed below.

BOOK NUMBER	MEANING OF EACH NUMBER	TITLE OF BOOK
973.3 means	9 →	History
	7 →	North America
	3 →	
	.3 →	
328.1 means	3	
	2	
	8	
	.1	
551.2 means	5	
	5	
	1	
	.2	
822.3 means	8	
	2	
	2	
	.3	
793.8 means	7	
	9	
	3	
	.8	



CROSS-NUMBER PUZZLE

Application
Concepts
DECIMALS



Use all of the numbers
to complete the puzzle.

3 Digits

3 6 2
2 0.4
2 3.8
6.3 6

4 Digits

9 9 0.1
5 1 4.2
2 8.8 4
1.9 3 0
1 8.7 5
9 4.3 6

5 Digits

7.6 5 3 2
6.5 2 3 4
2.8 3 0 4
9.3 1 4 2
3.8 7 7 5

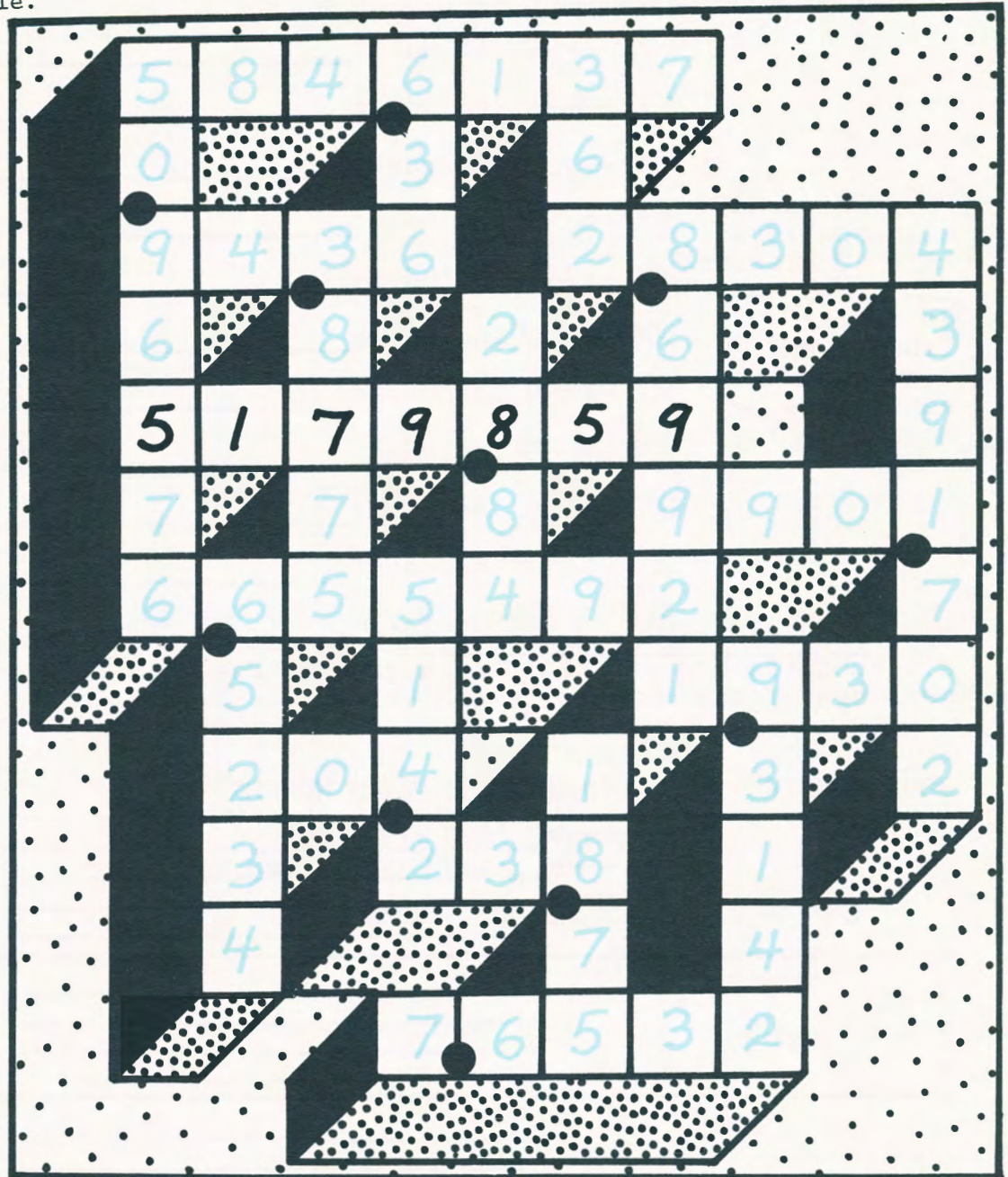
6 Digits

8.6 9 9 2 1

7 Digits

6.6 5 5 4 9 2
5 8 4.6 1 3 7
4 3 9 1.7 0 2
5 0.9 6 5 7 6

Have students
discuss their
strategies for
getting started.

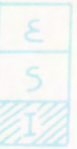


TYPE: Paper & Pencil/Puzzle



PLACING A POINT

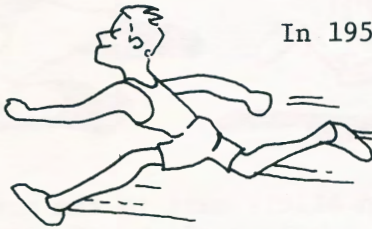
Estimation
Place Value
Concepts
DECIMALS



On January 20, 1954, Rogers Pass, Minnesota had a temperature of 697 degrees below zero.



The average amount of rainfall in San Francisco, California is 2501 inches per year.



In 1956 David Sime set a record when he ran 220 yards in 200 seconds.



Mary works as a babysitter on weekends. She is paid \$75 an hour.



On the average Mr. Brown's car travels 182 miles on 1 gallon of gasoline.



Sam traveled 40 miles on his bicycle. The trip took him 425 hours.



The population of the United States in 1950 was 1507 million.



A metre stick is a little longer than a yard. It is 3937 inches long.



The baby weighed 75 pounds at birth.



The patient's temperature in the morning was 1008 degrees.



The young man measured the palm of his hand and found that it was 325 inches wide.

TYPE: Paper & Pencil/Transparency

SOURCE: Project R-3

Permission to use granted by E.L. Hodges

Students could write similar problems of their own.



DECIMAL

Estimation
Place Value
Concepts
DECIMALS



Each of the following statements is incorrect because the decimal point is in the wrong place. Locate the mistake and make the correction by placing the decimal point in the right place. In some cases you may have to add a zero.



1. Frank Nelson, the fullback on the football team, weighs 19.5 pounds.
2. The Walkers' living room is 200 feet long and 1.2 feet wide.
3. In a close basketball game, the winning team scored 8500 points and the losing team scored 8.3 points.
4. Joan wrote a letter to her friend on a sheet of paper that was 1.05 inches long and 80 inches wide.
5. When Joan did some baby-sitting for the neighbors, they paid her \$12.50 an hour.
6. The ceiling in the Walkers' hotel room was 90 feet above the floor.
7. A major league baseball team played a season schedule of 1.82 games.
8. Mr. Johnson bought 10 gallons of gas for \$51.
9. Mr. Riley paid \$35.75 for his new car.
10. When Mr. Bates served tomato juice, he opened a can that contained 1800 ounces.
11. Bob bought 6 new baseballs for his team for \$.09.
12. When Alicia went to the movies she asked her father for \$225 to pay for her admission.
13. Mr. White took a business trip by plane. The plane traveled at a rate of 56 miles per hour.
14. East's last basketball game lasted 2500 hours.
15. For last Sunday's dinner, Mr. Hunter bought a turkey that weighed .12 pounds.
16. During their trip, the Stevens covered 16,000 miles in 4 hours of driving.
17. Mr. Larson bought a shirt and tie for \$.80.
18. The distance across the United States is about 300 miles.
19. During last winter, 47,000 inches of snow fell in Denver.
20. The distance between home plate and the pitcher's box is 6.05 feet.

TYPE: Paper & Pencil

IDEA FROM: C.O.L.A.M.D.A.

GRID DECIMALS - I

Definition
Place Value
Concepts
DECIMALS



A

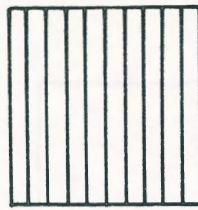


1 part

each part = 1

Shade in all of A.

B

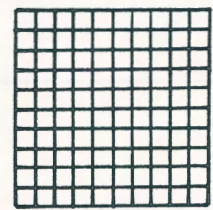


10 equal parts

each part = $\frac{1}{10} = .1$

Shade in any three columns of B.

C



100 equal parts

each part = = .01

Shade any 8 boxes of C.

What has been shaded?

Fraction
 $\frac{1}{1} + \frac{3}{10} + \frac{8}{100}$

Decimal
 $1 + .3 + .08 = 1.38$

What decimal has been shaded in each of the following?

<p>1. <u> </u></p>	<p>4. <u> </u></p>
<p>2. <u> </u></p>	<p>5. <u> </u></p>
<p>3. <u> </u></p>	<p>6. <u> </u></p>
<p>7. <u> </u></p>	

TYPE: Paper & Pencil

IDEA FROM: Project R-3

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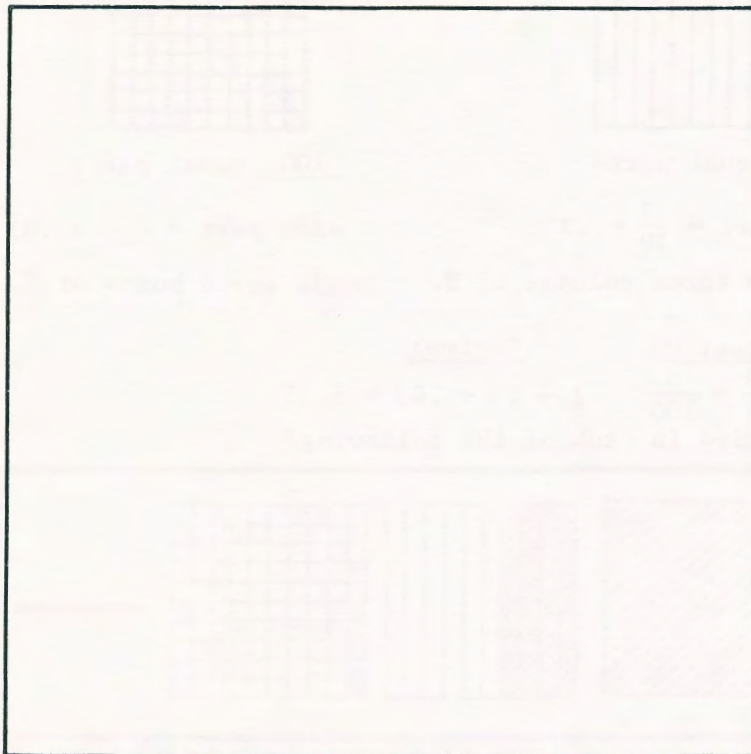
ACTIVITY CARDS - CUISENAIRE RODS - I

For readiness activities for Cuisenaire
Rods see the section on Lab Materials.

Definition
Place Value
Concepts
DECIMALS



The name Cuisenaire and the
color sequence of the rods are
trademarks of the Cuisenaire
Company of America, Inc.



1. For this activity you will need only the orange and white rods.
2. How many orange rods cover the square? _____
3. Does each orange rod cover $\frac{1}{10}$ of the square? _____ One tenth may also be expressed as .1, the decimal form.
4. How many white rods cover the square? _____
5. Each white rod covers how much of the square? _____

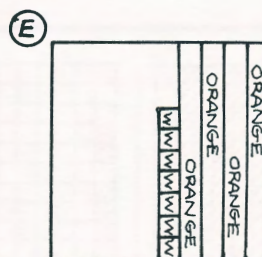
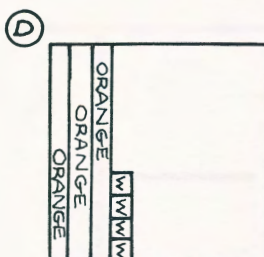
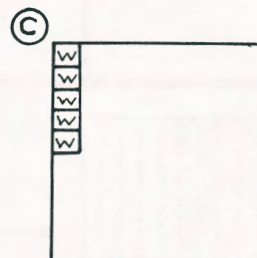
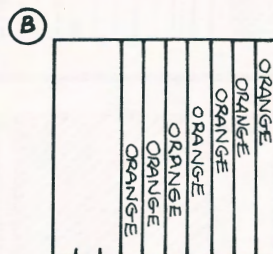
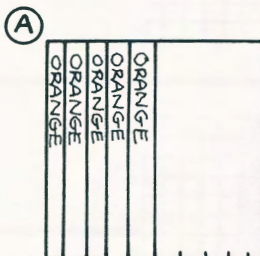
(Express your answer as both a fraction and as a decimal.)

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CR-I-1

CR-I-2

WHAT DECIMAL IS REPRESENTED BY EACH OF THE FOLLOWING?



- F
- .3 = ____ ORANGE + ____ WHITE
 .24 = ____ ORANGE + ____ WHITE
 .05 = ____ ORANGE + ____ WHITE
 .11 = ____ ORANGE + ____ WHITE

TYPE: Manipulative



BOXING IT

Place Value
Concepts
DECIMALS



WE ASSIGN THE NUMBER 1.00 TO THIS RECTANGLE
WE CAN THEN ASSIGN A DECIMAL NUMBER TO EACH
PIECE OF PAPER BELOW. CUT OUT THE PIECES AND
PLACE THEM ON THIS GRID TO GET THE ANSWERS.

PAPER #1 = _____

PAPER #5 = _____

PAPER #2 = _____

PAPER #6 = _____

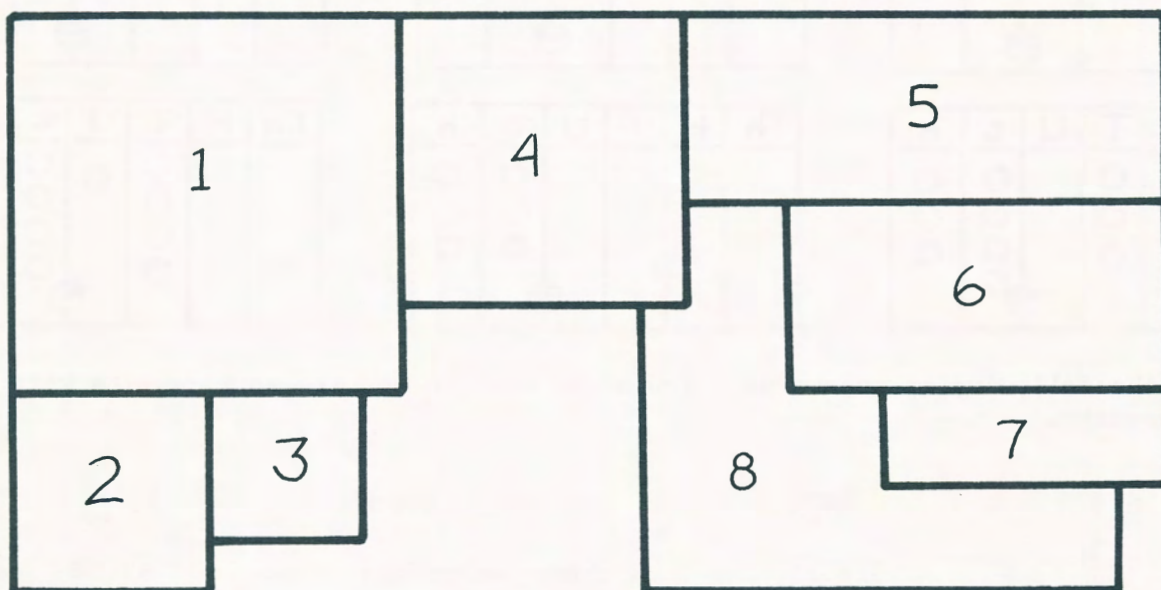
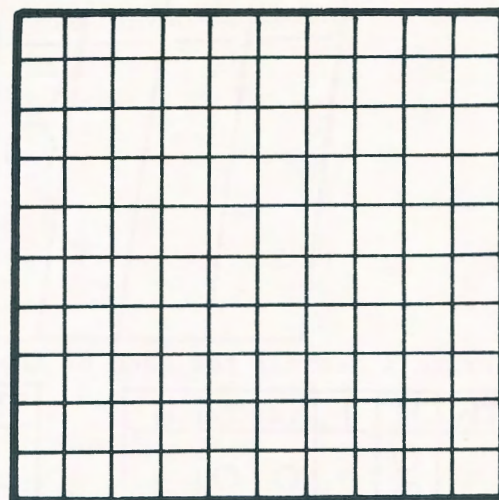
PAPER #3 = _____

PAPER #7 = _____

PAPER #4 = _____

PAPER #8 = _____

A transparent hundred grid could be placed
over the pieces instead of cutting them out.



ESTIMATE A DECIMAL NUMBER FOR EACH LETTER BELOW. THEN USE THE ABOVE GRID TO
FIND AN EXACT MEASURE (YOU MAY CUT AND RESHAPE THE FIGURES IF NECESSARY.)

AMORE

CAN YOU MAKE LETTERS FOR YOUR NAME AND FIGURE ITS DECIMAL SUM?

TYPE: Manipulative/Paper & Pencil

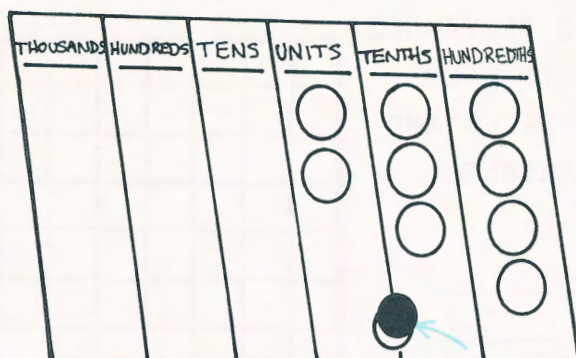
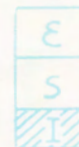
IDEA FROM: Project R-3

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ACTIVITY CARDS - DECIMAL ABACUS - I

Definition
Place Value
Concepts
DECIMALS



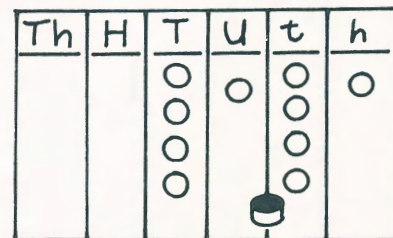
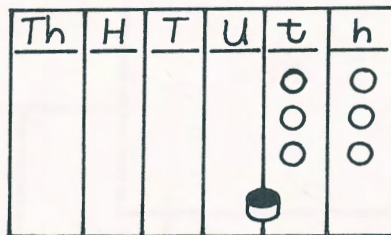
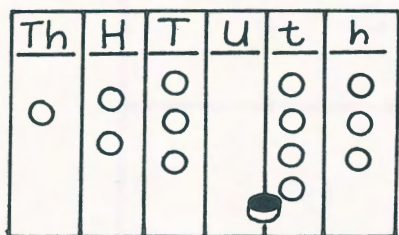
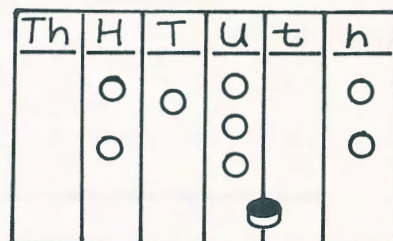
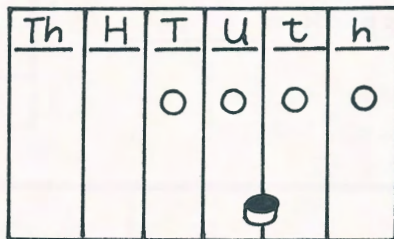
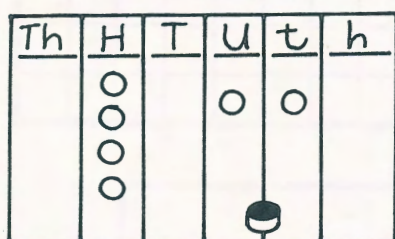
For information on the construction, use and readiness activities for the Abacus see the section on Lab Materials.

This abacus shows 2.34

2 units 3 tenths 4 hundredths.

1

Write a decimal for each of these. *Decimal Point*



2

Show the following on an abacus. You might want to compare your results with a classmate.

6.8

$24\frac{7}{10}$

two and one-tenth

$123\frac{4}{10}$

$7\frac{3}{10}$

314.25

three-hundredths

62.08

$13\frac{23}{100}$

$106\frac{37}{100}$

six and five-hundredths

.32

3

Make each of the following numbers on the abacus. What does the 4 stand for in the different numbers?

41.6 _____

1004 _____

6.4 _____

4321 _____

.04 _____

4.02 _____

4

How are the following numbers different? It may help to make each number on the abacus.

62.01 and 62.1

38 and 380

47.16 and 47.106

13 and 103

TYPE: Manipulative



MAKING CHANGE

Place Value
Concepts
DECIMALS



When Thomas Jefferson was president, he saw to it that our money system was changed over from the British to the decimal system.

Use a dictionary or an encyclopedia to find out when Jefferson was president of the U.S. _____

USE AS FEW COINS AS POSSIBLE TO MAKE CHANGE

PAY THIS AMOUNT	COINS TO USE		
	PENNIES	DIMES	DOLLARS
\$ 1.25			
\$ 3.74			
\$ 6.41			
60¢			
3¢			
12¢			
41¢			
\$ 11.39			
\$ 7.58			
87¢			

I wonder if Jefferson
had this in mind?



Since it's possible to make change using only pennies, dimes, dollars, why do we also have nickels, quarters, half dollars?

TYPE: Paper & Pencil

IDEA FROM: Project R-3

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MAKING A DECIMAL RULER

Place Value
Number Line
Concepts
DECIMALS



These can be prepared ahead of time and should be about 40 cm long.

Materials Needed: Strip of Tagboard

What To Do:

1. This segment represents $\frac{1}{10}$ or .1 of the length of the ruler you are to make. Use the tagboard and make a ruler one unit long. Divide it into tenths and label the marks using decimals.
2. Find 5 objects that are less than 1 unit long. Measure them to the nearest tenth. Record each measure as a decimal.
3. Carefully divide each of the tenths on your ruler into ten equal parts. Do this lightly first, in case you need to erase. Do not label them. How is your ruler divided up now?
4. Once again measure the 5 objects but now record each measure to the nearest hundredth of a unit. Record each measure as a decimal.
5. Measure each side of the seven-sided figure on this page. Measure to the nearest hundredth of a unit. Record each measure as a decimal and place it along the side measured.
6. What is the perimeter of the polygon surrounding these words?

CHALLENGE!

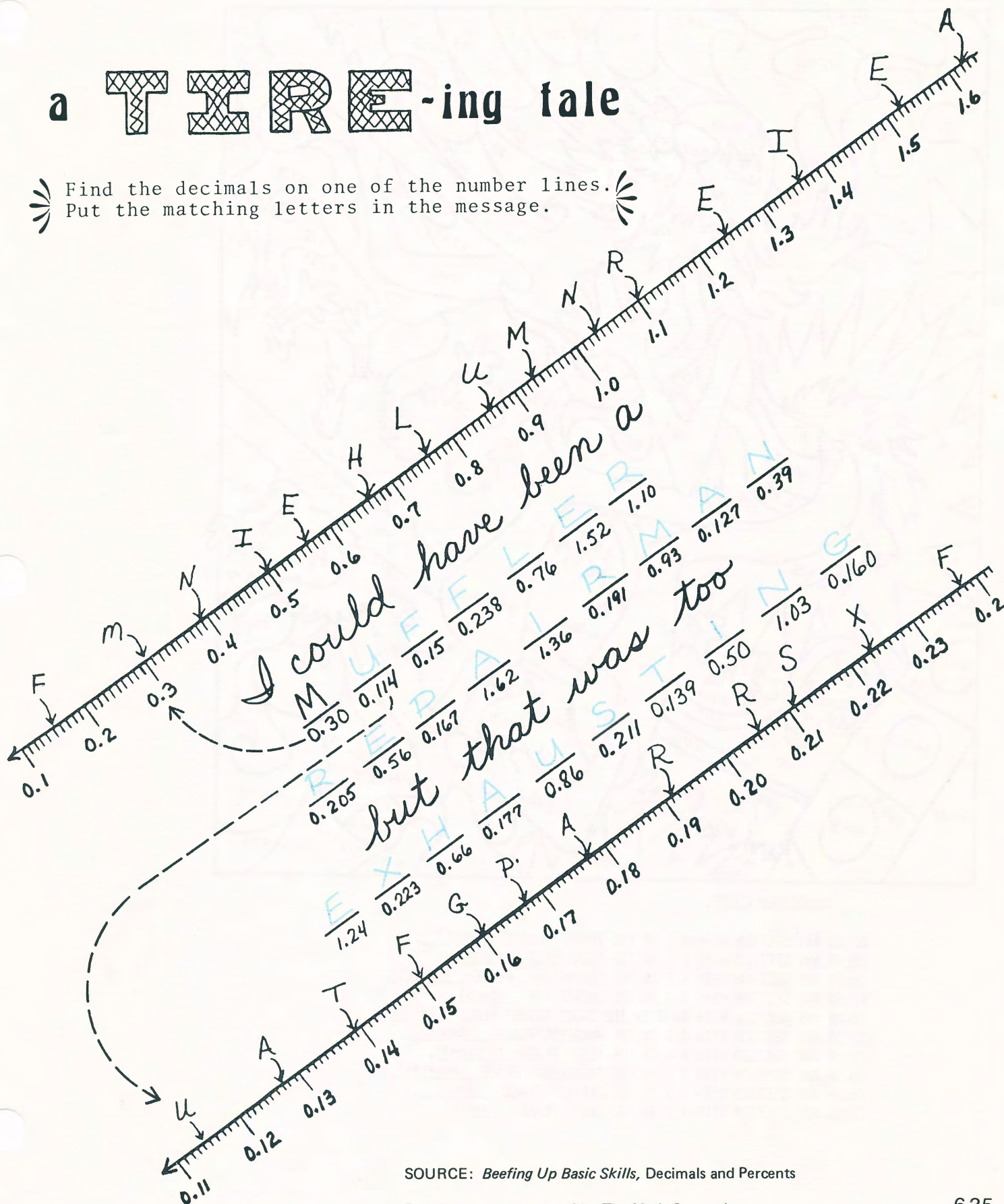
? Do you think a piece of paper is more or less than .001 unit thick? Develop a way to find out.



TYPE: Manipulative

IDEA FROM: MathLab, Level 18

Find the decimals on one of the number lines.
Put the matching letters in the message.



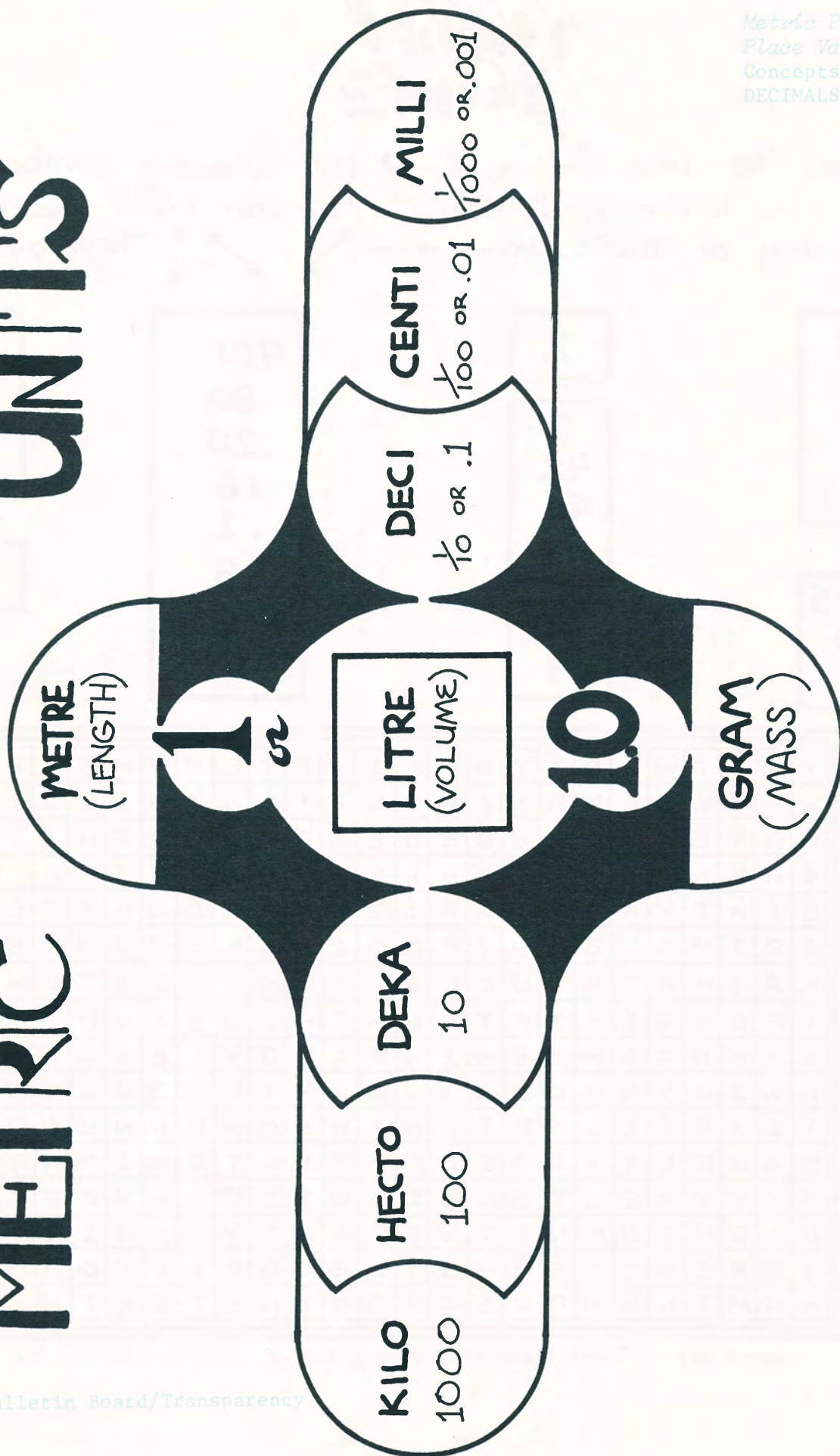
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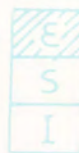
636

METRIC

UNITS



Metric Prefixes
Place Value
Concepts
DECIMALS



TYPE: Bulletin Board/Transparency

NAME GAME

Reading & Writing
Place Value
Concepts
DECIMALS



Find the name for each of the following numbers in the letter grid below. You can read the name in any of these ways \longleftrightarrow , \swarrow , \nearrow , \updownarrow . They're all there!

7.4
.5
76
80.1

.2

0
42
90
.15

4.01
80
.20
16
.1
.3
6
11

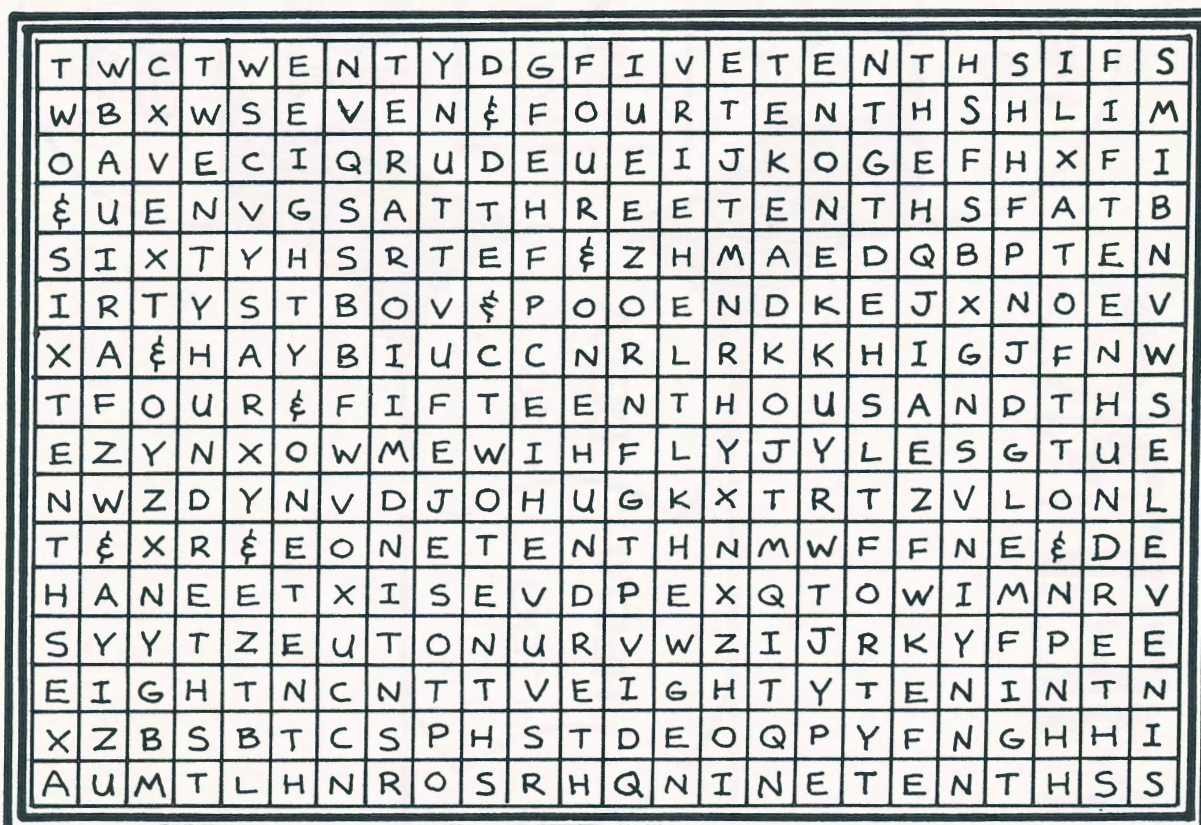
5
2.6
60

7

4.015
50
8

40
.9

& MEANS "AND"



There are at least three that aren't listed. Can you find them?

TYPE: Paper & Pencil/Puzzle

For more practice in reading and writing decimals see Mathimagination, Book E and Adventures with Arithmetic, Decimals.



ACTIVITY CARDS - CUISENAIRE RODS - II

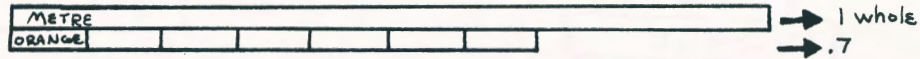
Rounding
Concepts
DECIMALS



For readiness activities for Cuisenaire Rods see the section on Lab Materials.

Each of the following decimals is between zero and one.
Round each decimal to the nearest whole number.
The metre stick is one whole.

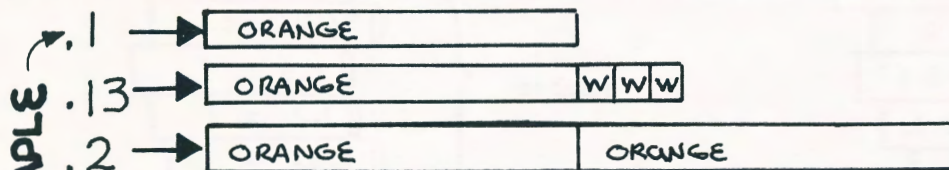
EXAMPLE



Decimal	More or less than half of metre stick	Nearest WHOLE No.
.7	MORE	ONE
.3		
.9		
.12		
.63		
.76		
.45		

The name Cuisenaire and the color sequence of the rods are trademarks of the Cuisenaire Company of America, Inc.

CR-II-1



EXAMPLE

Make	BETWEEN RODS	BETWEEN DECIMALS	Closer to
.13	1 ORANGE + 2 ORANGE	.1 and .2	.1
.37	— OR. — OR.		
.24	— —		
.08	0 OR. — OR.		
.49	— —		
.51	— —		

CR-II-2

Round each of the following to the nearest tenth. Do you need the rods?

.41 →
.08 →
.59 →
.37 →
.62 →
.75 →

Students will need to do Cuisenaire Rods - I in this section before doing this activity.

TYPE: Manipulative

The name Cuisenaire and the color sequence of the rods are trademarks of the Cuisenaire Company of America, Inc.



IT'S CLOSER THAN YOU THINK!!

(OR HOW DO YOU STACK UP?)

Rounding
Concepts
DECIMALS



Complete these patterns.

- 7.3 \approx 7
- 8.9 \approx 9
- 13.7 \approx 14
- 125.2 \approx 125
- .5 \approx 1
- 11.42 \approx 11
- 9.2 \approx
- 10.7 \approx
- 135.1 \approx
- 18.73 \approx
- 2.999 \approx
- 127.5 \approx

- .13 \approx .1
- .27 \approx .3
- 5.839 \approx 5.8
- 7.74 \approx 7.7
- 23.65 \approx 23.7
- .12 \approx
- .48 \approx
- 6.392 \approx
- 8.85 \approx
- 10.06 \approx
- 2.15 \approx
- 5.0505 \approx

How did you do them?

Explain your method.

#1

#2

- .206 \approx .21
- 5.472 \approx 5.47
- .093 \approx .09
- 3.666 \approx 3.67
- 2.108 \approx 2.11
- .1434 \approx .14
- .178 \approx
- .281 \approx
- .333 \approx
- 9.016 \approx
- 2.394 \approx
- .1685 \approx
- 4.815 \approx
- 74.888 \approx

ROUND TO THE NEAREST:

whole
number

- 92.02 \approx
- 1.6 \approx
- 3.2 \approx
- 7.85 \approx
- 4.25 \approx
- .9 \approx
- .49 \approx
- .31 \approx
- .089 \approx
- 1.39 \approx
- 23.797 \approx
- 1.709 \approx
- 50.095 \approx
- 4.163 \approx
- .278 \approx

tenth

hundredth

Good activity to be done as a class to encourage discussion of answers and methods. Maybe you could use an overhead projector.

What's happening?

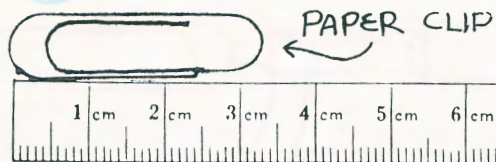
#3

TYPE: Paper & Pencil/Transparency



MATH TOOLS

Rounding
Concepts
DECIMALS



With this measuring device, the paper clip is about 3.3 centimeters.

Round off 3.3 to the nearest centimeter and it becomes : _____

ROUND OFF THESE NUMBERS TO THE NEAREST UNIT

1.9 becomes _____ because .9 rounds off to _____

4.3 becomes _____ because .3 rounds off to _____

3.5 becomes _____ because .5 rounds off to _____

0.8 becomes _____ because .8 rounds off to _____

2.2 becomes _____ because .2 rounds off to _____

this stuff is
getting scientific

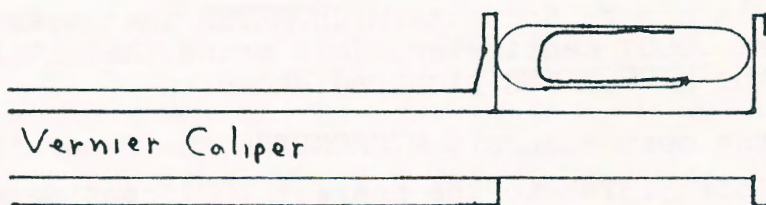


The ruler above can measure things to the nearest

1 centimeter
.1 centimeter
.01 centimeter

This device measures
things to the nearest
.01 centimeter.

It is more accurate
than the ruler above.



With this measuring device, the paper clip is now about 3.37 cm.

Round off 3.37 to the nearest .1 centimeter and it becomes : _____

ROUND OFF THESE NUMBERS TO THE NEAREST TENTH

2.87 becomes _____ because .87 rounds off to _____

5.43 becomes _____ because .43 rounds off to _____

1.09 becomes _____ because .09 rounds off to _____

3.75 becomes _____ because .75 rounds off to _____

4.24 becomes _____ because .24 rounds off to _____

TYPE: Paper & Pencil

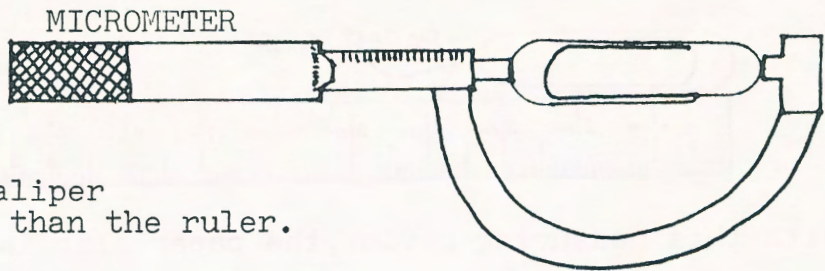
SOURCE: Project R-3

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MATH TOOLS CONT.

This device measures things to the nearest .001 centimeter.

It is 10 times more accurate than the vernier caliper and 100 times more accurate than the ruler.



With this measuring device, the paper clip is now about 3.374 cm.

Round off 3.374 to the nearest .01 centimeter and it becomes : _____

ROUND OFF THESE NUMBERS TO THE NEAREST HUNDREDTH

1.439 becomes _____ because .439 rounds off to _____

3.864 becomes _____ because .864 rounds off to _____

2.157 becomes _____ because .157 rounds off to _____

5.325 becomes _____ because .325 rounds off to _____

4.771 becomes _____ because .771 rounds off to _____

There is a more accurate MICROMETER that measures things to the nearest .0001 centimeter. This means that it is 10 times more accurate than the MICROMETER pictured above.

With the most accurate MICROMETER, the paper clip measures 3.3748 cm.

Round off 3.3748 to the nearest .001 centimeter and it becomes : _____

ROUND OFF THESE NUMBERS TO THE NEAREST THOUSANDTH

4.0963 becomes _____ because .0963 rounds off to _____

1.6548 becomes _____ because .6548 rounds off to _____

3.4725 becomes _____ because .4725 rounds off to _____

2.2631 becomes _____ because .2631 rounds off to _____

SOURCE: Project R-3



BASEBALL AUTO RACING RECORDS

Rounding
Concepts
DECIMALS



YOU WILL NEED AN ALMANAC.

USING THE INDEX, LOOK UP THE PAGE FOR BASEBALL--MAJOR LEAGUE LIFETIME RECORDS, LEADING BATTERS (OVER 2,000 HITS).

Name of batter	batting average	average rounded off to hundredths.....
Cobb , Ty		
Hornsby,Rogers		
Delehanty, Ed		
Williams, Ted		
Ruth , Babe		
Gehrig , Lou		
Sisler, George		
Musial , Stan		
DiMaggio, Joe		

Using the index, look up the page for AUTO RACING (Indy.500)

Year	Winner	Car	winning speed in miles/hour	m.p.h. rounded off to tenths.
1911				
1920				
1934				
1939				
1947				
1953				
1959				
1961				
1965				
1969				
1972				



DENSITY

Rounding
Ordering
Concepts
DECIMALS

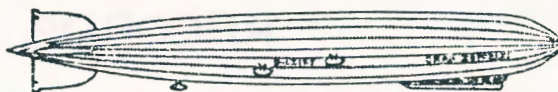


BALSA
WOOD



COPPER

A cubic foot of balsa wood weighs much less than a cubic foot of copper. We say that the density of copper is greater than the density of balsa wood.



This famous airship rose above the ground because it contained hydrogen gas. We say the density of hydrogen gas is less than the density of air.

Since oil rises to the top when mixed with water, it must be lighter. We say the density of oil is less than the density of water.



1. Which liquid floats on top?

Water
Gasoline

2. Which liquid floats on top?

Water
Sea water

3. (Use the dictionary.) Why is mercury so much heavier than the other liquids?

Type of liquid	Pounds per gallon	Pounds per gallon rounded to tenths
water	8.33	
gasoline	5.664	
mercury	113.288	
oil	7.497	
sea water	8.58	

4. Would a block of concrete float in melted glass? _____

5. Would a block of iron float in melted copper? _____

6. Would a block of gold float in melted copper? _____

Type of material	Pounds per cubic foot	Pounds per cu. ft. rounded to tenths
water	62.4	
balsa wood	8.11	
concrete	143.52	
copper	554.74	
glass	162.24	
gold	1204.3	
iron	486.72	

TYPE: Paper & Pencil/Transparency

SOURCE: Project R-3

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EYEBALLING DECIMALS

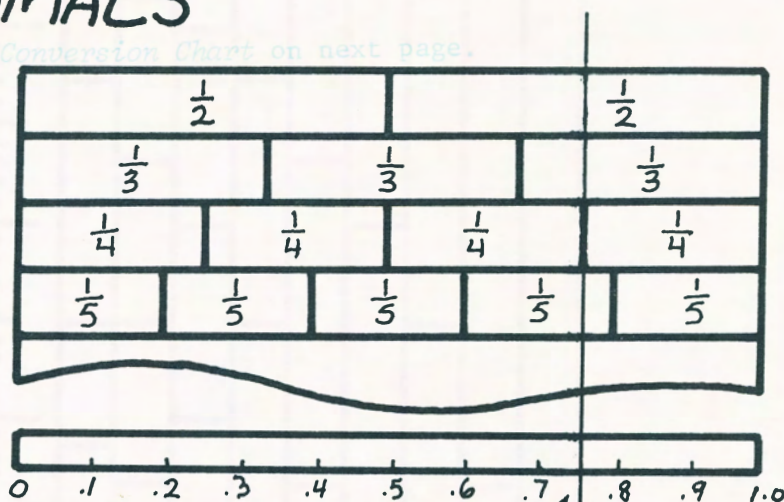
Relation to
Fractions
Concepts
DECIMALS



NOTE: Students need Conversion Chart on next page.

Use the Fraction - Decimal conversion chart and a straightedge to change the following fractions to decimals.

The example shows $\frac{3}{4} = .75$.



Be sure to keep your straightedge lined up with the edge of the paper.



- a) $\frac{1}{2} = \underline{\hspace{1cm}}$ b) $\frac{2}{3} = \underline{\hspace{1cm}}$ c) $\frac{2}{5} = \underline{\hspace{1cm}}$ d) $\frac{1}{6} = \underline{\hspace{1cm}}$
 e) $\frac{5}{8} = \underline{\hspace{1cm}}$ f) $\frac{9}{10} = \underline{\hspace{1cm}}$ g) $\frac{5}{12} = \underline{\hspace{1cm}}$ h) $\frac{13}{16} = \underline{\hspace{1cm}}$

Now try finding the decimals without using your straightedge or any other guide. Do it just by "eyeballing."

- a) $\frac{1}{8} = \underline{\hspace{1cm}}$ b) $\frac{5}{6} = \underline{\hspace{1cm}}$ c) $\frac{3}{16} = \underline{\hspace{1cm}}$ d) $\frac{7}{12} = \underline{\hspace{1cm}}$

Now check it with your straightedge.

Use your straightedge and find a fraction that is approximately equal to each of these decimals.

- a) .6 b) .67 c) .58
 d) .08 e) .92 f) .44

Find a decimal approximation for these fractions.

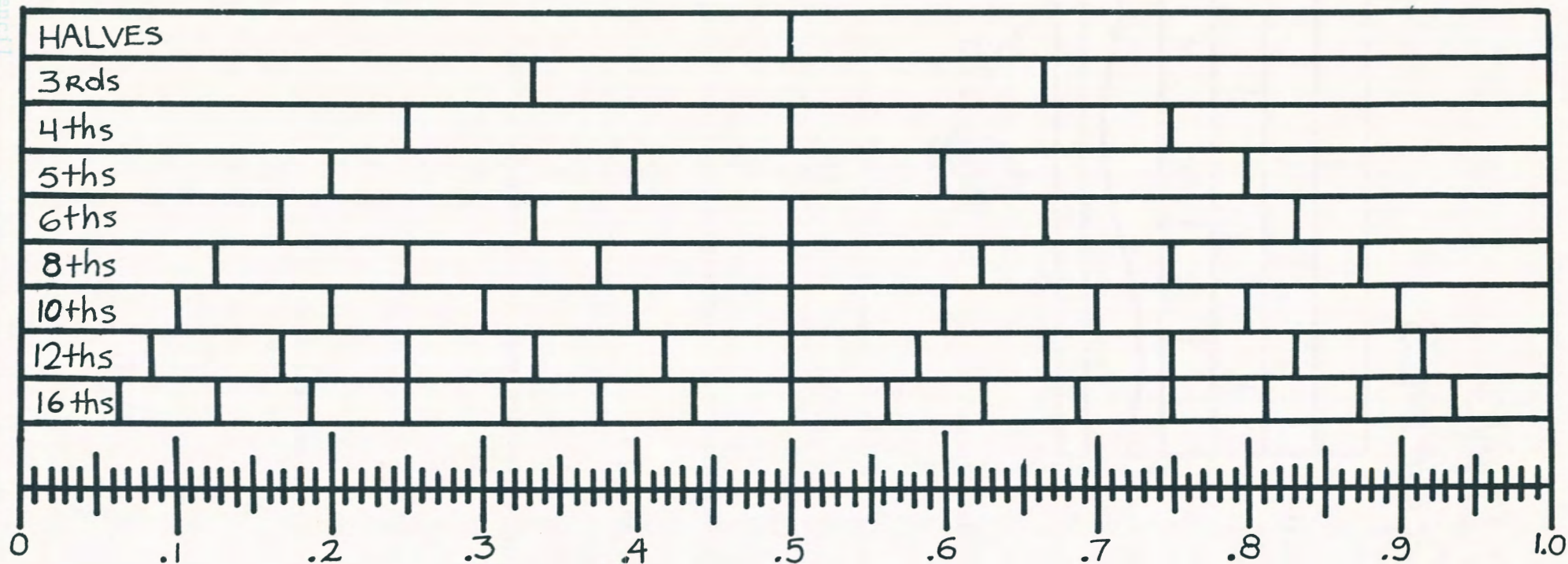
- a) $\frac{1}{7} \underline{\hspace{1cm}}$ b) $\frac{1}{9} \underline{\hspace{1cm}}$ c) $\frac{1}{14} \underline{\hspace{1cm}}$
 d) $\frac{5}{9} \underline{\hspace{1cm}}$ e) $\frac{20}{50} \underline{\hspace{1cm}}$ f) $\frac{9}{11} \underline{\hspace{1cm}}$

TYPE: Paper & Pencil

IDEA FROM: C.O.L.A.M.D.A.

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CONVERSION CHART



IDEA FROM: C.O.L.A.M.D.A.

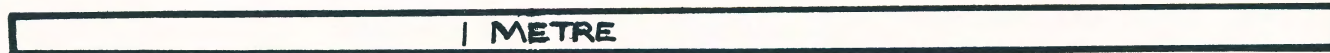
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ACTIVITY CARDS - CUISENAIRE RODS - III

For readiness activities for Cuisenaire Rods see the section on Lab Materials.

Relation to Fractions
Equivalent
Concepts
DECIMALS



→ orange

→ white

You will need a metre stick and some orange and white rods.

What fractional part of 1 metre is:

an orange rod? $\frac{1}{10}$ or $.1$
two orange rods? _____ or _____
three orange rods? _____ or _____
four _____ or _____
five _____ or _____
eight _____ or _____
ten _____ or _____

What fractional part of 1 metre is:

a white rod? $\frac{1}{100}$ or $.01$
two white rods? _____ or _____
three _____ or _____
five _____ or _____
seven _____ or _____
nine _____ or _____
thirty-eight _____ or _____

The name Cuisenaire and the color sequence of the rods are trademarks of the Cuisenaire Company of America, Inc.

CR-III-1

CR-III-2

A. USING ORANGE RODS B. USING WHITE RODS

REMEMBER !

*EACH ORANGE

ROD =

$\frac{1}{10}$ OR $.1$

*EACH WHITE

ROD =

$\frac{1}{100}$ OR $.01$

Fraction	# of orange Rods used	decimal	# of white used	decimal	
Find $\frac{1}{2}$ of the metre	5 →	.5	50 →	.50	.5 = .50
Find $\frac{1}{10}$ of the metre	1 →		→		
$\frac{3}{10}$		▶		▶	
$\frac{1}{5}$		▶		▶	
$\frac{2}{5}$		▶		▶	

TYPE: Manipulative

The name Cuisenaire and the color sequence of the rods are trademarks of the Cuisenaire Company of America, Inc.

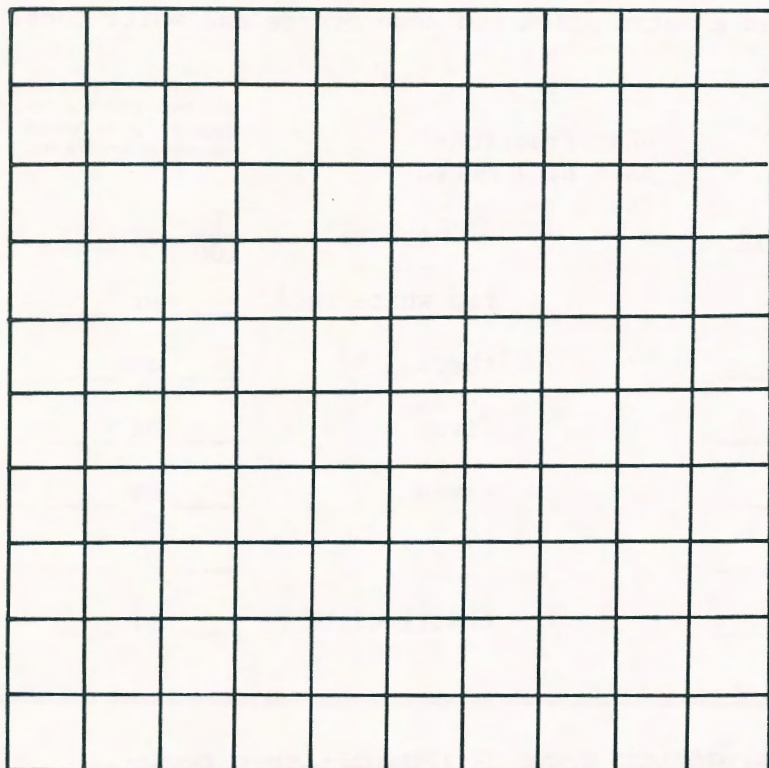


ACTIVITY CARDS - CUISENAIRE RODS - IV

Relation to Fractions
Concepts
DECIMALS



For readiness activities for Cuisenaire Rods see the section on Lab Materials.



→ One orange rod covers $\frac{1}{10}$ or . ____ of the square.

→ One white rod covers $\frac{1}{100}$ or . ____ of the square.

→ Cover $\frac{1}{10}$ of the square with white rods.
10 white rods = . ____

→ $\underbrace{1 \text{ orange}} = \underbrace{10 \text{ white}}$
. ____ = . ____

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CR-IV-1

CR-IV-2

Use the orange rods to find decimal equivalents for these.

Cover $\frac{1}{2}$ of the square, $\frac{1}{2} =$. ____ (____ tenths)

$\frac{1}{5} =$. ____ (____)

$\frac{3}{10} =$. ____ (____)

$\frac{7}{10} =$. ____ (____)

$\frac{2}{5} =$. ____ (____)

Use white rods or a combination of white and orange rods to find the following decimal equivalents.

$\frac{1}{4} =$. ____

$\frac{7}{20} =$. ____

$\frac{3}{4} =$. ____

$\frac{3}{10} =$. ____

$\frac{1}{20} =$. ____

$\frac{1}{50} =$. ____

TYPE: Manipulative



CHANGE UP

Relation to
Fractions
Concepts
DECIMALS



Fill in the blanks.

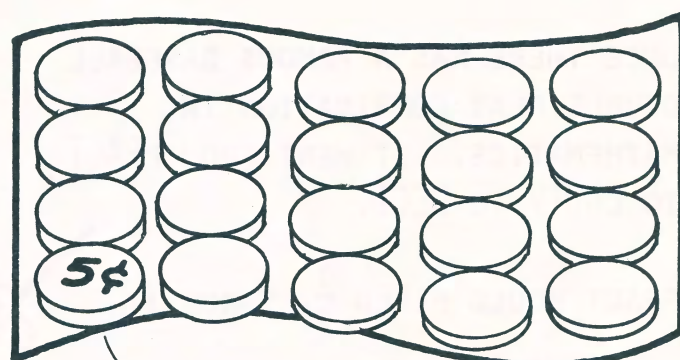
Example: 100 cents make up a dollar.

_____ nickels make up a dollar.

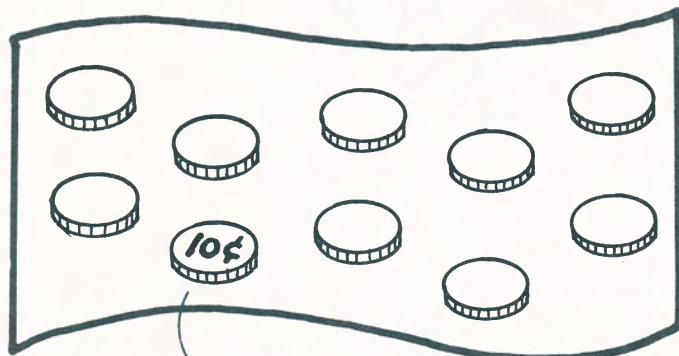


$$1\text{¢} = \frac{1}{100} \text{ of } \$1$$

or \$.__

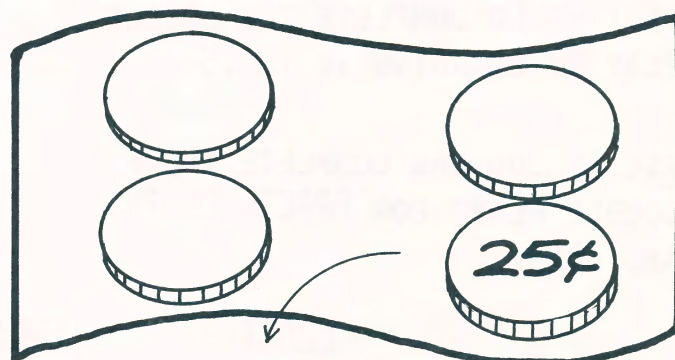


$\frac{1}{20}$ of \$1 or \$.__



$\frac{1}{10}$ of a \$1

or \$.__



$\frac{1}{4}$ of a \$1

or \$.__



$\frac{1}{2}$ of a \$1

or \$.__

Samples of additional questions . . .

1. Write a decimal for $\frac{2}{10}$ of a dollar.

For $\frac{7}{10}$, $\frac{3}{4}$, $\frac{9}{100}$, $\frac{4}{5}$, etc.

2. What coins would you use to show the following amounts?

\$.28 \$.03 \$.67 \$.16 \$.99

3. Write a decimal for:

a quarter plus a nickel.

3 dimes and 4 nickels.

6 pennies.

2 quarters, 4 dimes, 5 nickels
and 14 pennies.

TYPE: Paper & Pencil/Transparency

IDEA FROM: Basic Vocational Mathematics, Part I

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Curriculum Laboratory

DECIMAL Double Plays

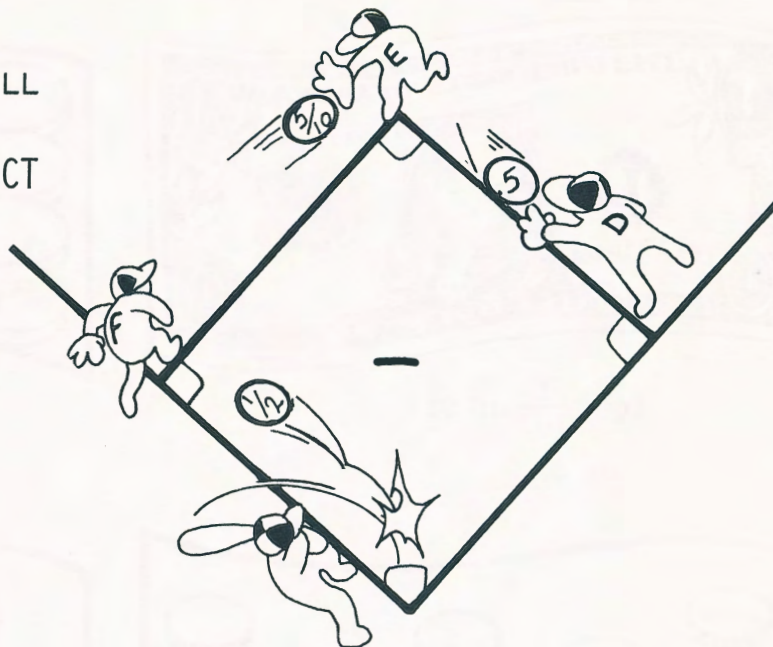
Relation to
Fractions
Concepts
DECIMALS



ONCE THERE WAS A FAMOUS BASEBALL
DOUBLE PLAY COMBINATION IN
MATHEMATICS. IT WENT FROM FRACT
TO EQUIV TO DECI.

FRACT WOULD FIELD $\frac{1}{2}$, THROW IT
TO EQUIV, WHO WOULD CHANGE IT
TO $\frac{5}{10}$ AND THROW IT TO DECI.

DECI WOULD COMPLETE THE DOUBLE
PLAY BY CHANGING IT TO .5



SEE IF YOU CAN COMPLETE THESE
DOUBLE PLAYS FOR FRACT, EQUIV,
AND DECI.

<u>FRACT</u>		<u>EQUIV</u>		<u>DECI</u>
$\frac{4}{5}$	→	$\frac{8}{10}$	→	.8
$\frac{3}{10}$	→	$\frac{3}{10}$	→	_____
$\frac{1}{10}$	→	_____	→	_____
_____	→	_____	→	.7
_____	→	$\frac{6}{10}$	→	_____
_____	→	_____	→	.4
$\frac{1}{2}$	→	_____	→	_____
_____	→	_____	→	.2

TYPE: Paper & Pencil/Transparency

DECIMAL Double Plays CONTINUED



In one of the tougher games of the year, Fract, Equiv and Deci made these double plays:

<u>Fract</u>		<u>Equiv</u>		<u>Deci</u>
$\frac{3}{4}$	→	$\frac{75}{100}$	→	.75
$\frac{7}{20}$	→	$\frac{35}{100}$	→	.35
$\frac{73}{100}$	→	$\frac{73}{100}$	→	.73

Complete these double plays:

$\frac{11}{20}$	→	$\frac{55}{100}$	→	—
$\frac{3}{50}$	→	$\frac{6}{100}$	→	—
—	→	—	→	.89
$\frac{1}{4}$	→	—	→	—
—	→	—	→	.16
—	→	$\frac{50}{100}$	→	—

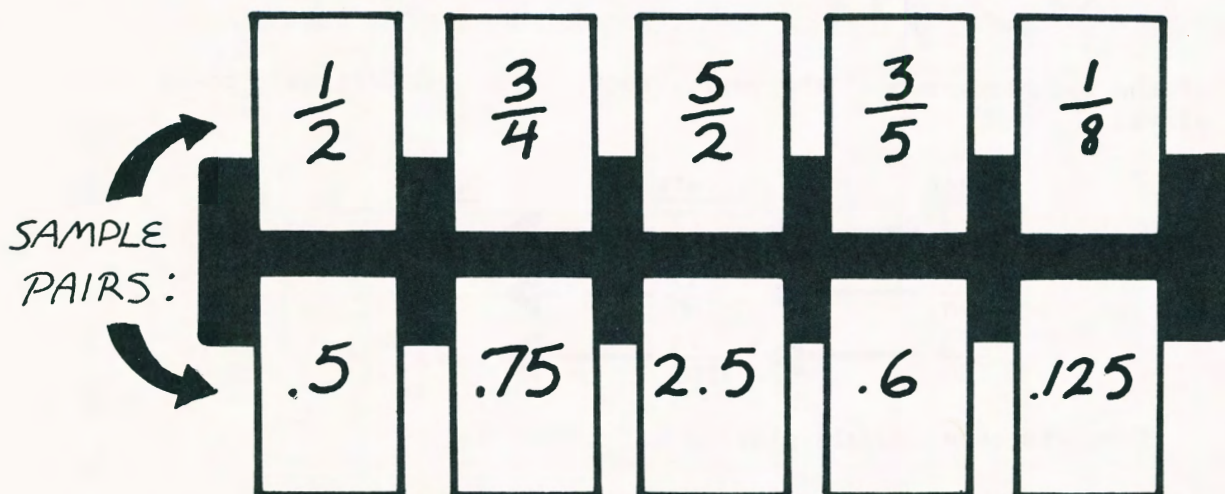
Fract got injured during one game and was replaced by MIX. The new double play combination was as good as the old one.

<u>Mix</u>		<u>Equiv</u>		<u>Deci</u>
$6\frac{2}{5}$	→	$6\frac{4}{10}$	→	—
$8\frac{4}{5}$	→	—	→	—
—	→	—	→	5.8
—	→	$3\frac{75}{100}$	→	—
$13\frac{1}{2}$	→	—	→	—
$5\frac{3}{20}$	→	—	→	—
—	→	—	→	2.65



FRACTIONS AND DECIMALS CONCENTRATION

For this game you will need 20 cards in pairs. Each pair will contain one fraction card and one decimal card.



The first time students play this game be sure to use only fractions that have terminating decimal forms.

Do not use fractions such as $\frac{1}{3}$ or $\frac{1}{6}$.

RULES: Place the cards facedown in to 4 rows of 5 cards each.

The first player turns any two cards faceup. If the two cards match (are equivalent), the player keeps them and turns over two more cards. If the two cards do not match, they are turned facedown again in the original dealt position.

Play proceeds to the left until all cards have been paired.

WINNER: The player with the most cards at the end of the game.

STRATEGY: Players should try to remember the placements of the cards as they are turned faceup.

Many variations of the game can be played:

- Equivalent fractions
- Equivalent decimals
- Equivalent fractions, decimals and percent
- More than 20 cards
- More difficult expressions

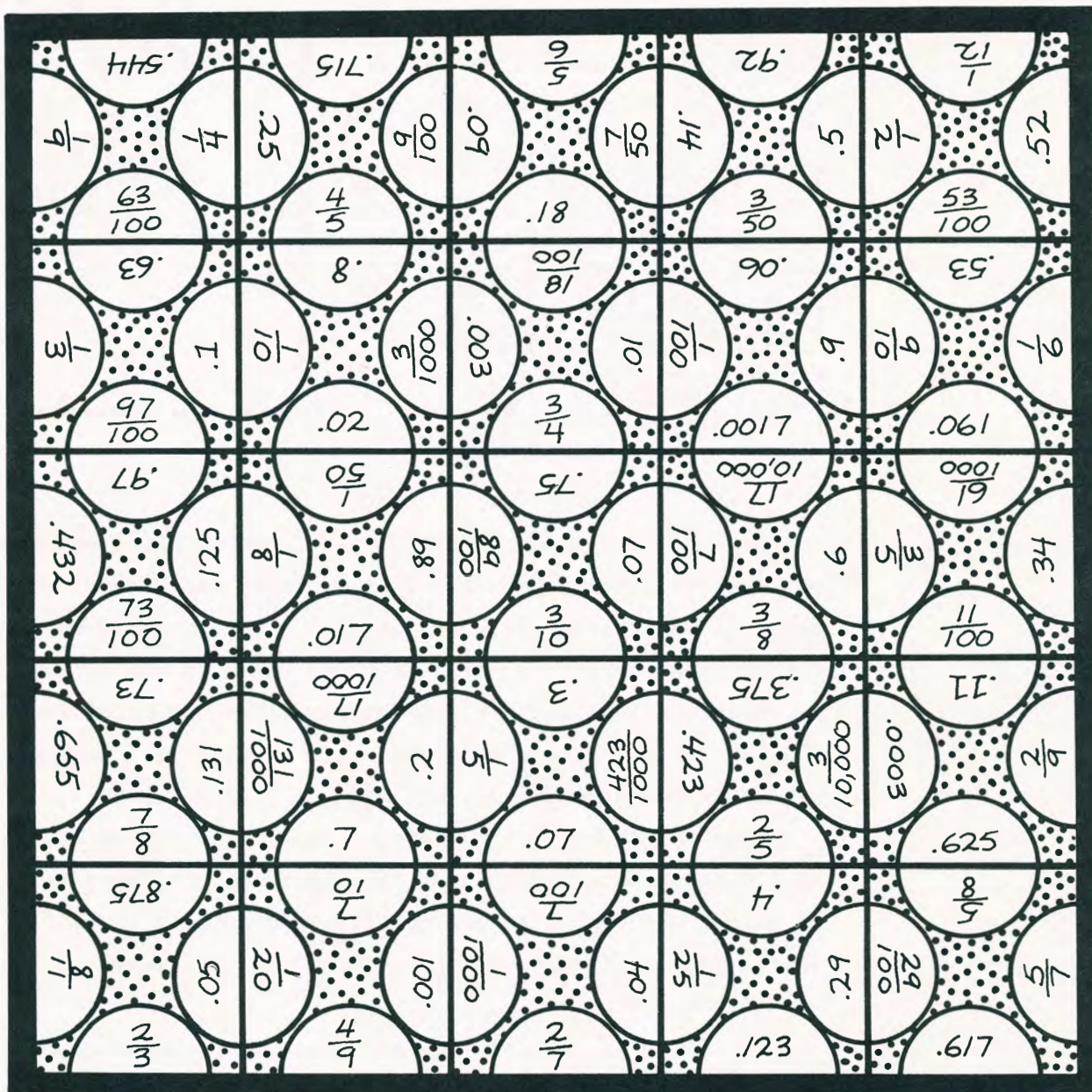
TYPE: Game

IDEA FROM: *Math Card Games*

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FRACTIONS AND DECIMALS PUZZLE

Mathematics
FRACTIONS
CONVERTING
FRACTIONS
TO DECIMALS



Cut up the puzzle. Give it to your students to put together. The puzzle is such
easy if you leave on the borders. For more information on border puzzles see
the discussion of Games and Puzzles in the section on Lab Materials.

MONSTER DECIMALS

Relation to Fractions
Equivalent
Concepts
DECIMALS



This game is played in much the same way as the card game Old Maid.

NUMBER OF PLAYERS: 2 to 6

MATERIALS:

49 cards - 12 groups of 4 cards each, and the monster card. Each group consists of 4 equivalent expressions.

Here are some sample groups:

$\frac{3}{4}$	SEVENTY FIVE HUNDRETHS	.75	.750
$\frac{5}{2}$	TWO AND FIVE TENTHS	2.5	TWENTY FIVE TENTHS
3	THIRTY TENTHS	3.	THREE HUNDRED HUNDRETHS



RULES:

Deal all the cards. Each player discards pairs of cards showing equivalent numbers. If the player has 3 cards that match he may discard only two of them. When this is completed each player has only unmatched cards in his hand.

Next, without looking, the dealer takes one card from the hand of the player on her right. If that card matches a card in her hand she may discard the pair faceup.

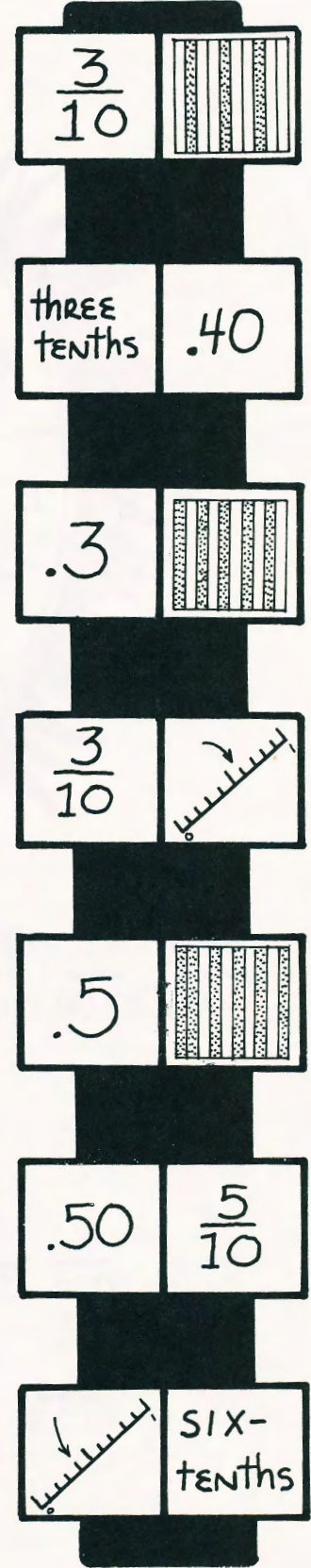
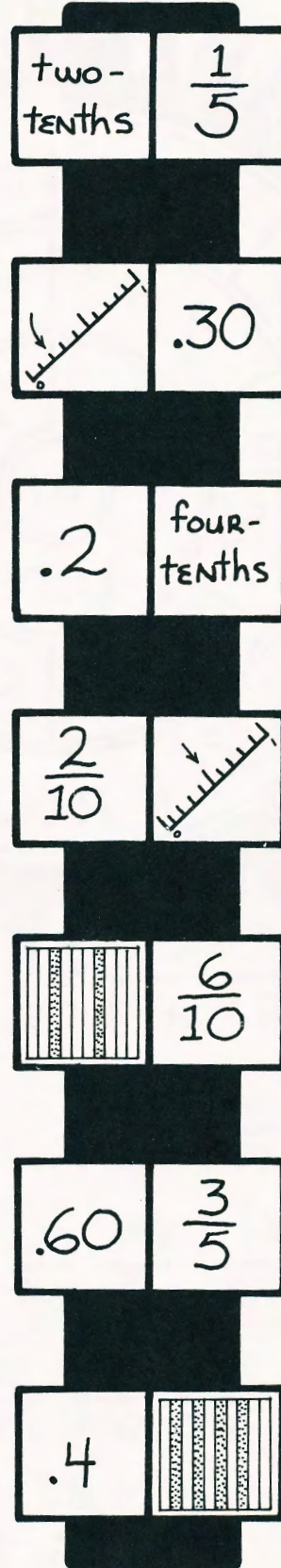
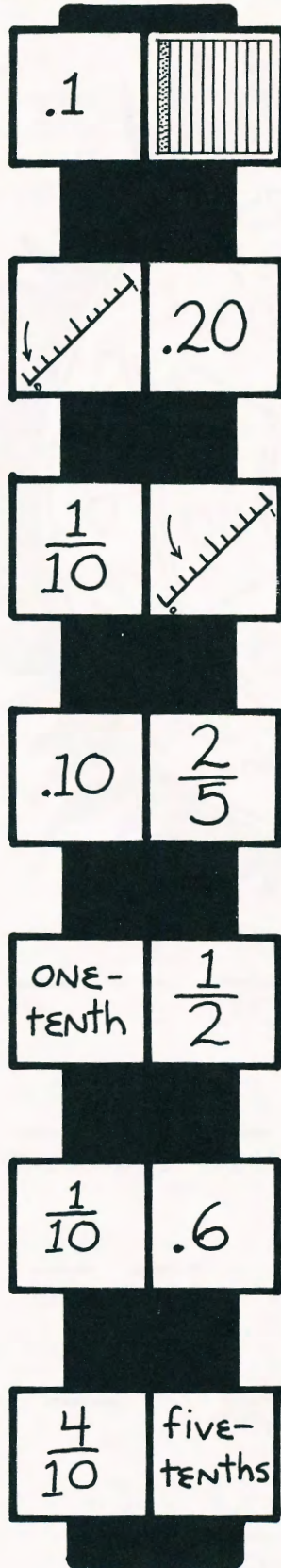
Play continues this way around the circle to the right. As the cards move from player to player they will eventually be paired and discarded - except for the monster card. When a player discards all his cards he is a winner and drops out of the play.

The loser is the one who last holds the monster card.

This game can be played with card decks using just equivalent fractions; just equivalent decimals; or with a combination of fractions, decimals and percents.

MORE DOMINOES

RULES:
PLACE ALL
DOMINOES FACE
DOWN. EACH
PLAYER DRAWS
SIX DOMINOES.
LARGEST
DOUBLE GOES
FIRST. YOU
CAN PLAY IF
YOU HAVE A
DOMINO
THAT MATCHES
ONE HALF A
DOMINO
ALREADY
PLAYED. IF YOU
CAN'T PLAY YOU
MUST PULL FROM
THE FACE-DOWN
DOMINOES UNTIL
YOU GET ONE
THAT CAN BE
PLAYED. WHEN
ALL THE
DOMINOES HAVE
BEEN PULLED,
A PLAYER LOSES
A TURN IF HE
CANNOT PLAY.
THE FIRST
PLAYER TO
PLAY ALL OF
HIS DOMINOES
IS THE WINNER.

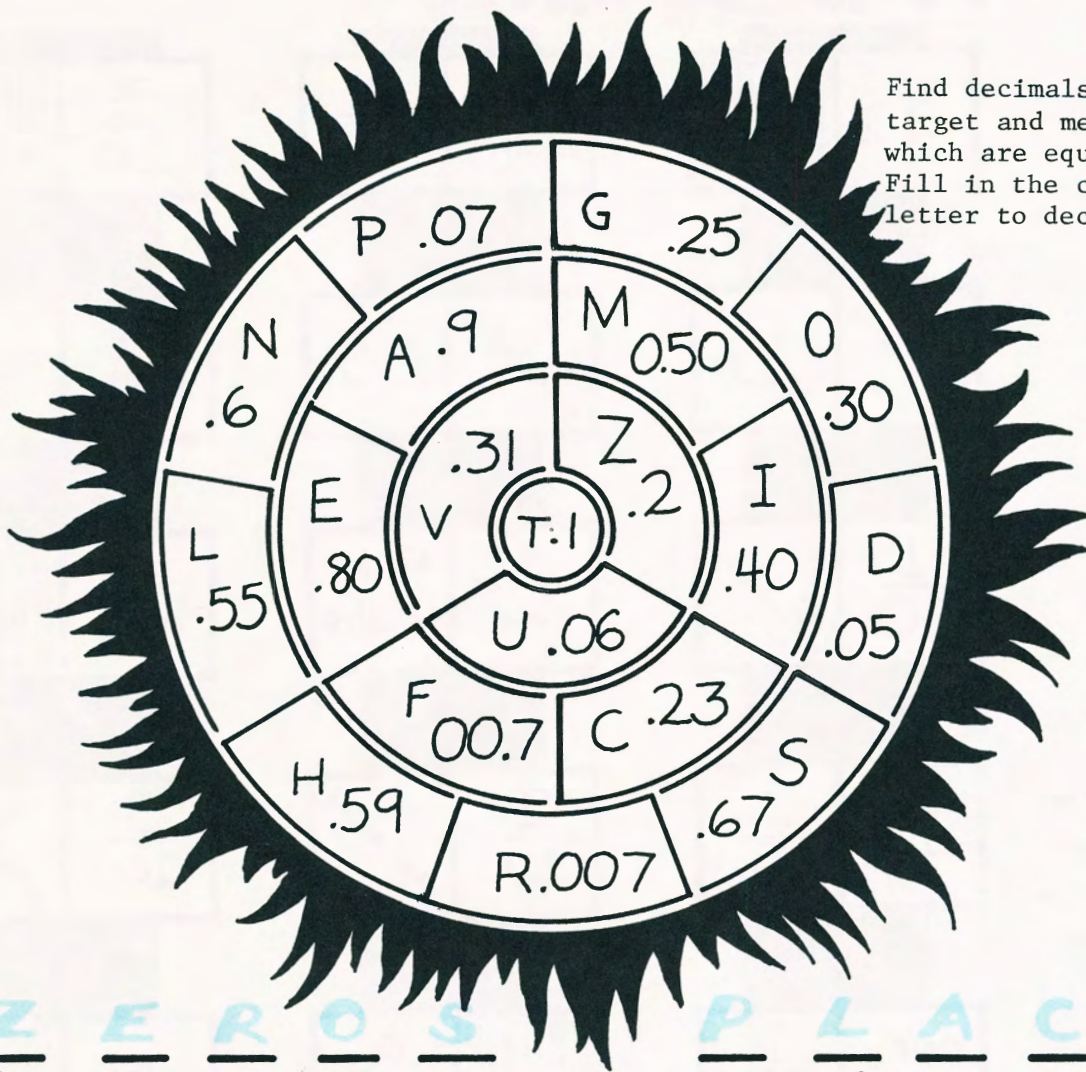


WHAT'S THE POINT?

Equivalent
Concepts
DECIMALS



Find decimals in the target and message which are equivalent. Fill in the corresponding letter to decode the message.



Z E R O S
.20 .80000 0.007 0.3 .670

P L A C E
.070 .550 0.9 .230 00.80 .050

O N T H E
.3000 0.60 .10 .5900 0.8

E N D O F A
0.80 .600 .0500 .3 .7 .9000

D E C I M A L
0.05 .8000 .2300 .400 .5 .900 0.55

D O N O T
.0500 .30 0.6 .300 .1000

C H A N G E
0.23 .590 0.90 .6000 .250 .800

I T S
.4 .100 .6700

V A L U E
.310 .90 .5500 .060 .8

TYPE: Paper & Pencil/Puzzle

METRIC BUILD A TRAIN

Equivalent
Metric Measure
Concepts
DECIMALS



MATERIALS: 1 metre stick per player.
1 box of Cuisenaire rods per player.
Set of game cards.*

See Cuisenaire Rods in the section on Lab Materials.

- PROCEDURE:**
1. Shuffle cards and place face down on the table.
 2. Players, in turn, draw one card from the deck and make the length shown on the card using the appropriate rod or rods. The rods are placed along each player's metre stick, beginning from one end.

Example: If the card says "2 cm," the player uses the rod that is 2 centimetres long, which is the red rod. If the card says ".1 metre," the player uses the orange rod which is 1 decimetre long or .1 of a metre.

3. The first person to reach the end of his metre stick wins the game.

*SAMPLE GAME CARDS

.16 m	.07 m	.1 dm	1.2 dm	2 cm
.18 m	.03 m	.2 dm	1.8 dm	3 cm
.14 m	.05 m	.3 dm	.4 dm	4 cm
.12 m	.08 m	.4 dm	1.3 dm	5 cm
.11 m	.01 m	.5 dm	1.6 dm	6 cm
.13 m	.02 m	.6 dm	2.1 dm	7 cm
.15 m	.04 m	.7 dm	1.5 dm	8 cm

To use this game for drill and practice in addition of decimals have students record their plays in a table and calculate the lengths of their "trains" after each turn.

TPE: Game



ACTIVITY CARDS - CUISENAIRE RODS - V

For readiness activities for Cuisenaire Rods
see the section on Lab Materials.

Ordering
Concepts
DECIMALS



CR-V-1

Make these: A. Using only white rods

B. Using as many orange rods as possible.

.23	<u>23</u> white rods	<u>2</u> orange + <u>3</u> white	$\frac{2}{10} + \frac{3}{100}$
.15	___ white rods	___ orange + ___ white	$\frac{1}{10} + \frac{5}{100}$
.37	___	___ + ___	$\frac{3}{10} + \frac{7}{100}$
.09	___	___ + ___	$\frac{0}{10} + \frac{9}{100}$
.52	___	___ + ___	$\frac{5}{10} + \frac{2}{100}$
.01	___	___ + ___	$\frac{0}{10} + \frac{1}{100}$
.44	___	___ + ___	$\frac{4}{10} + \frac{4}{100}$
.27	___	___ + ___	$\frac{2}{10} + \frac{7}{100}$

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A

B



COMPARE the lengths
A \lessgtr B

.21	<u>2</u> Or + <u>1</u> Wh	.22	<u>2</u> Or + <u>2</u> Wh	.21 < .22
.3	___ Or + ___ Wh	.4	___ Or + ___ Wh	.3 ___ .4
.2	___ Or + ___ Wh	.11	___ Or + ___ Wh	
.09	___ + ___	.2	___ + ___	
.37	___ + ___	.41	___ + ___	
.5	___ + ___	.57	___ + ___	
.2	___ + ___	.20	___ + ___	

CR-V-2

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For this activity the unit could be the metre stick or the hundreds grid.

"ORDER IN THE COURT"

Students may need some guidance in comparing relative values on the number line. (← less, → greater)

What does the bailiff say after he says, "Order in the court?"

Locate the numbers on the number line to help you find the correct answer for each question.

Write the letter of the correct answer above the question numbers at the bottom of this page.



- Example: (1) Which is less .1 or 1?
- (2) Which is less .48 or .5?
- (3) Which is greater .6 or .56?
- (4) Which is greater .86 or .09?
- (5) Which is less .6 or .56?
- (6) Which is greater .8 or .65?
- (7) Which number is between the other two? .60, .7, .89
- (8) Which number is between the other two? .3, .39, .08
- (9) Which is greater .14 or .41?
- (10) Which is less .76 or .27?
- (11) Which is less .2 or .48?
- (12) Which is greater .34 or 1?

H E R E C O M E S T H E J U D G E

1 12 3 12 11 5 8 12 7 6 1 12 10 9 4 2 12



POWERFUL PUZZLE

Read each sentence and circle your answer. On the next page, connect the two letters that go with your answer. The first one is done for you.

- 1) 14.7 has $\begin{cases} \text{two decimal places.} \\ \text{one decimal place.} \end{cases}$ CONNECT
K to Q
O to W
- 2) 0.7135 The 3 is in the $\begin{cases} \text{thousandths place.} \\ \text{hundredths place.} \end{cases}$ H to S
T to Y
- 3) Which is smallest $\begin{cases} 0.613 \\ 0.71 \\ 0.8 \end{cases}$ J to K
R to U
V to W
- 4) 2.68 has $\begin{cases} \text{three decimal places.} \\ \text{two decimal places.} \end{cases}$ M to H
G to D
- 5) 0.2459 The 2 is in the $\begin{cases} \text{ones place.} \\ \text{tenths place.} \end{cases}$ I to F
E to W
- 6) Circle the decimal fraction $\begin{cases} \frac{3}{4} \\ 0.5 \end{cases}$ P to Z
A to T
- 7) In 910.62 the whole number part is $\begin{cases} 910 \\ .62 \end{cases}$ V to S
M to N
- 8) Which is largest $\begin{cases} 0.511 \\ 0.55 \\ 0.5 \end{cases}$ Y to Z
C to L
T to U
- 9) 0.1864 The 8 is in the $\begin{cases} \text{thousandths place.} \\ \text{hundredths place.} \end{cases}$ E to I
G to R
- 10) 110.4406 has $\begin{cases} \text{four decimal places.} \\ \text{seven decimal places.} \end{cases}$ O to X
P to W

- 11) Which is smallest $\begin{cases} 0.302 \\ 0.4 \\ 0.3 \end{cases}$

M to R
J to U
K to L

- 12) In 3.605 the decimal fraction part is $\begin{cases} 3 \\ .605 \end{cases}$

V to Y
A to B

- 13) Circle the decimal fraction $\begin{cases} 8 \\ 0.31 \end{cases}$

O to F
D to H

- 14) $22 = 22.00$ $\begin{cases} \text{True} \\ \text{False} \end{cases}$

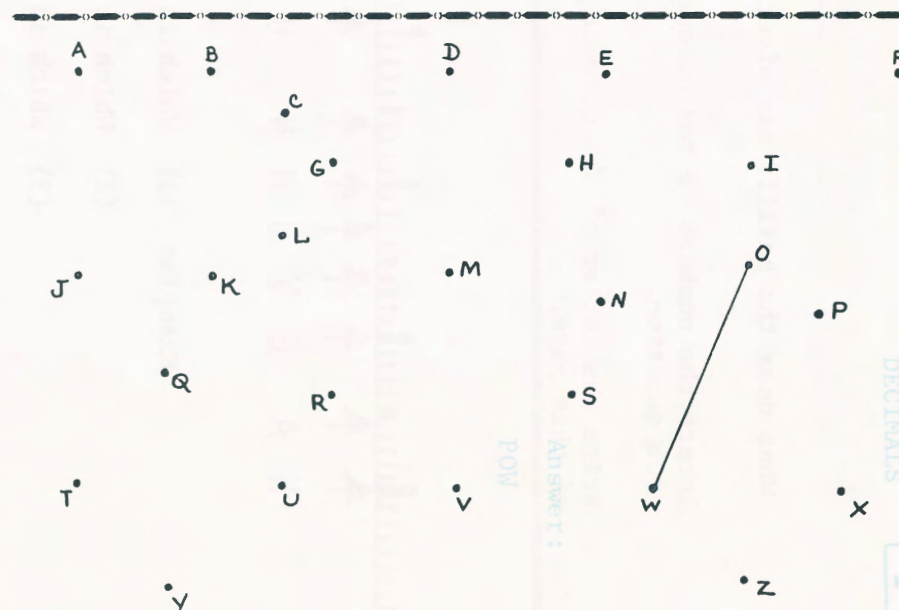
B to C
J to B

- 15) $0.9 = 0.090$ $\begin{cases} \text{True} \\ \text{False} \end{cases}$

X to Z
R to V

- 16) 0.3512 The 2 is in the $\begin{cases} \text{ten-thousandths place.} \\ \text{thousandths place.} \end{cases}$

F to X
N to I



Ordering
Place Value
Concepts
DECIMALS

DECIMAL SOLITAIRE

Ordering
Place Value
Concepts
DECIMALS



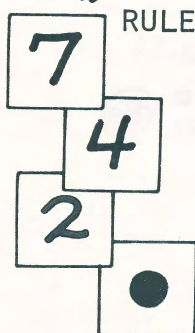
NEEDED: A SET OF DIGIT CARDS 0-9: 0 1 2 ... 9

2 DECIMAL POINTS: • •

PAPER AND PENCIL TO RECORD RESULTS.

1 GREATER THAN CARD: >

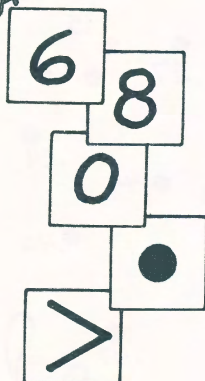
CARDS DRAWN



- RULES:
1. MIX UP THE DIGIT CARDS. PLACE THEM FACE DOWN IN A PILE.
 2. DRAW 3 DIGIT CARDS AND WITH A DECIMAL POINT MAKE A NUMBER WITH 2 DECIMAL PLACES.

EXAMPLE: 4 • 7 2

CARDS DRAWN



3. DRAW 3 MORE DIGIT CARDS, USE THE OTHER DECIMAL POINT CARD AND THE GREATER THAN CARD AND MAKE AS MANY NUMBERS AS YOU CAN THAT ARE LESS THAN THE FIRST NUMBER, RECORD YOUR ANSWERS.

EXAMPLE:

4	•	7	2	>	0	•	8	6
4	•	7	2	>	•	0	8	6
4	•	7	2	>	•	8	6	0

CAN YOU FIND MORE? TRY THE GAME A FEW MORE TIMES BY YOURSELF.
CAN YOU THINK OF A WAY TO PLAY WITH A FRIEND? MAKE UP YOUR OWN GAME -
AND TRY IT!!

TYPE: Game

SHARKY

Ordering
Concepts
DECIMALS



CONNECT THE DOTS IN ORDER FROM .01 TO 1.

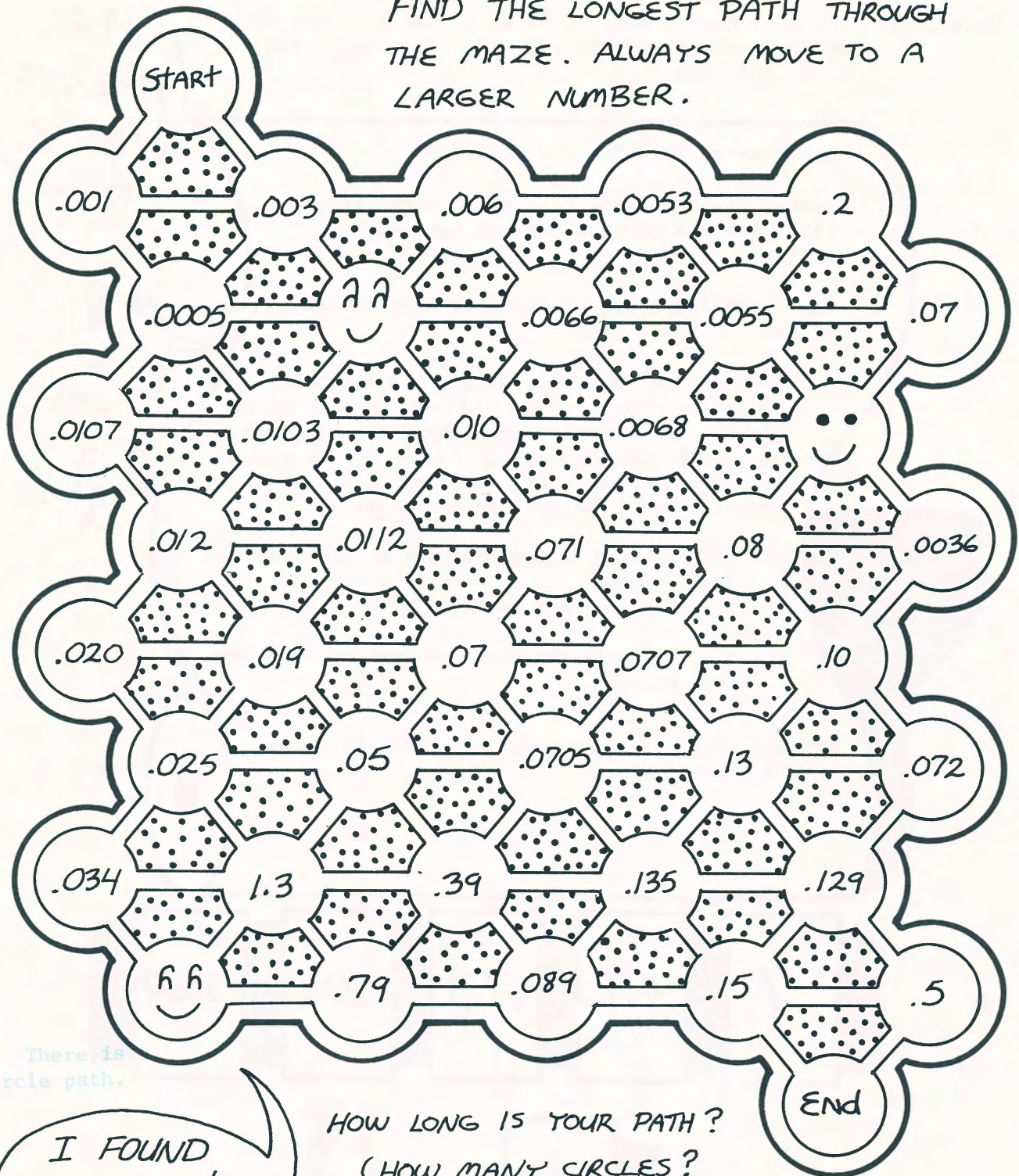


AMAZEMENT

Ordering
Concepts
DECIMALS



FIND THE LONGEST PATH THROUGH
THE MAZE. ALWAYS MOVE TO A
LARGER NUMBER.



Note: There is
23-circle path.

I FOUND
A 22-CIRCLE
PATH.

HOW LONG IS YOUR PATH?
(HOW MANY CIRCLES?
—DON'T COUNT "START"
AND "END")

TYPE: Paper & Pencil/Puzzle

IN-A-ROW DECIMALS

Ordering
Concepts
DECIMALS

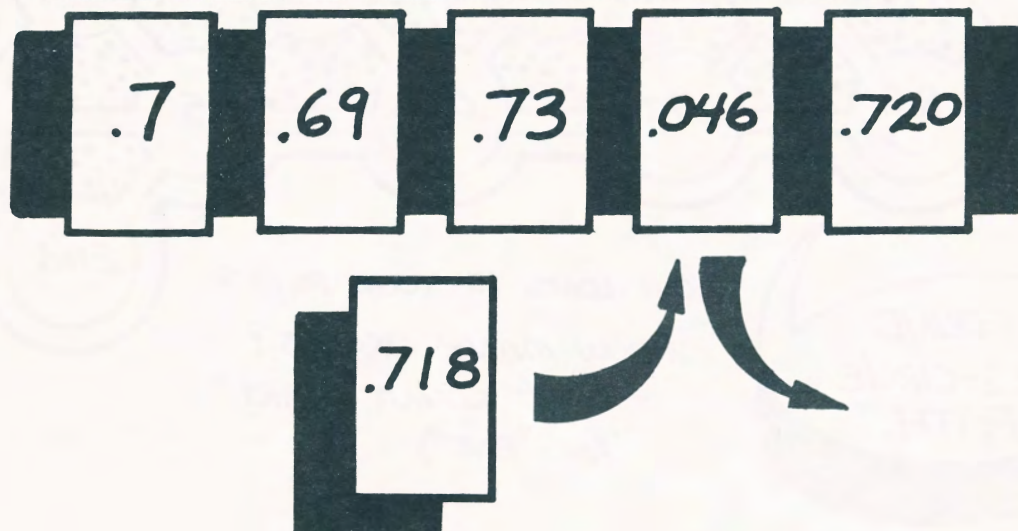


Materials: You will need a deck of 40 to 60 cards marked with decimal values such as .7, .69, .73, .046, .720, 5, 6.3, .005, etc.

**HERE
ARE THE
RULES.**

Rules same as for IN-A-ROW FRACTIONS

1. Dealer deals 5 cards face up to each player. Cards are placed in a row and may not be switched around.
2. Cards not dealt are placed face down to form a stack.
3. Each player is trying to get his cards in order; smallest on the left to largest on the right. (A variation could be for each player to decide before drawing a card if he wishes to place in order with smallest on the left, or instead, with smallest on the right.)
4. On his turn a player draws a card from the stack and:
 - a) replaces any one of the cards in his row, or
 - b) discards the card because he can't use it.
5. The first player to get his cards in a row is the winner.



TYPE: Game

NUMBER SENSE AND ARITHMETIC SKILLS

Placement Guide for Tabbed Dividers

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DIDACTICS	10
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The Teaching of Skills	24
Diagnosis and Remediation	42
Goals through Games	62
TEACHING EMPHASES	74
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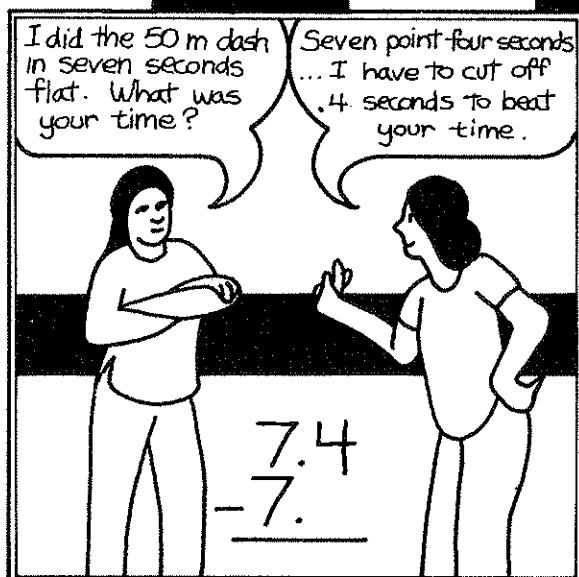
<u>TITLE</u>	<u>PAGE</u>	<u>TOPIC</u>	<u>TYPE</u>
Activity Cards - Cuisenaire Rods - VI	671	Model Addition/subtraction	Manipulative
Grid Decimals - II	672	Model Addition/subtraction	Paper and pencil Transparency
Activity Cards - Decimal Abacus - II	673	Model Addition/subtraction	Manipulative
Odometer Decimals	676	Model Addition	Paper and pencil
The Case of the Invisible Decimal Point	677	Lining up decimal point Addition/subtraction	Paper and pencil
Penny Slide	678	Addition/subtraction	Game
Money Exchange Rates	679	Addition/subtraction	Paper and pencil Transparency
4-Squares	680	Addition	Game
Lattice Mathematics I	682	Addition/subtraction	Paper and pencil
Decimal Add-a-Box	683	Addition	Paper and pencil Puzzle
Rounded Add-a-Box	684	Rounding Addition	Paper and pencil Puzzle
Decimal Trips	685	Measurement Addition	Paper and pencil

DECIMALS: ADDITION/ SUBTRACTION

"How come we're lining up decimal points now? It looks funny: You used to tell us to make the numbers line up on the right. I don't get it."

$$\begin{array}{r} 3.4 \\ .27 \\ + 4. \\ \hline \end{array}$$

Why do we have students add and subtract "ragged" decimals--those with different numbers of places after the decimal point? Some authors say that in practical applications these numbers would not occur since they are measurements to different degrees of accuracy, but there are everyday situations where decimals with different places occur.



We often solve such everyday situations in our heads--in fact, fourth and fifth graders on track teams seem to compare and subtract times (in tenths) in their heads before studying decimals formally. To write these problems on paper it is helpful to know seven seconds flat means 7.0 seconds and that \$2 can be written \$2.00. The addition of "ragged" decimals can increase number sense when approximation is used. What is a sensible answer for $4.5 + 7$? Would 5.2 be sensible? No, since 4.5 is more than 4 and $4 + 7 = 11$. The sum must be greater than 11.

A paper & pencil page could be centered around this idea before the addition algorithm for decimals is taught. A format like the student page *Never Hold a Grudge* in DECIMALS: Multiplication/Division could be used. Problems should be chosen carefully so students can use their decimal sense. No correct answers need to be given, only reasonable and unreasonable choices.

Examples:

Circle One				
3.61 + 3.9	IS CLOSEST TO	4	6	8
2.08 + .9	IS CLOSEST TO	2	3	4
1.8 + .12	IS CLOSEST TO	2	3	4

In the problems above students can round to the nearest whole before adding. Problems like $.08 + .012$ are more difficult to approximate and should not be included at this time. If students can see that $3.61 + 3.9$ is close to 8 then they can be helped to see that $\begin{array}{r} 3.61 \\ + 3.9 \\ \hline 4.00 \end{array}$ is not a reasonable way of working this problem.

Let's return to the original question, "Why do we line up the decimal points?" We usually think of place value when answering this--you have to add tenths to tenths and hundredths to hundredths.... This is a helpful idea and it is developed in the place value section of this commentary. Addition and subtraction of decimals can also be introduced through fractions.

ADDITION AND SUBTRACTION OF DECIMALS VIA FRACTIONS


Soon after learning the two notations for decimal fractions and the different ways of reading decimals students can add and subtract decimals in tenths.

NEVER

HOLD A GRUDGE...

FOR EACH OF THE SIXTEEN PROBLEMS CIRCLE THE MOST REASONABLE ANSWER. (THERE ARE NO CORRECT ANSWERS GIVEN, SO YOU SHOULDN'T HAVE TO WORK THE PROBLEMS.) PLACE THE LETTER IN THE CIRCLED BOXES IN THE APPROPRIATE SPACE BELOW.

1 $\begin{array}{r} 45 \\ \times 6 \\ \hline \end{array}$	280 P	2.7 U	24 T	11 $\begin{array}{r} 9.27 \\ \times 6 \\ \hline \end{array}$	556 J	5.6 R	.562 Y
2 $\begin{array}{r} 613.8 \\ \div 3 \\ \hline \end{array}$	24 C	2.2 H	.21 D	12 $\begin{array}{r} 339 \\ \div 9 \\ \hline \end{array}$.03 N	27 R	2.7 Z
3 $\begin{array}{r} 14 \\ \times 1.8 \\ \hline \end{array}$.32 A	3 E	35 G	13 $\begin{array}{r} 75.3 \\ \times 94 \\ \hline \end{array}$	707 D	7.078 C	71.5 I
4 $\begin{array}{r} 813.25 \\ \div 3 \\ \hline \end{array}$	47 H	.4 J	4.6 Y	14 $\begin{array}{r} 9813 \\ \div 9 \\ \hline \end{array}$.9 B	9 F	91 H
5 $\begin{array}{r} 293 \\ \times 4 \\ \hline \end{array}$	120 S	1002 K	9 T	15 $\begin{array}{r} 58.2 \\ \times 1.1 \\ \hline \end{array}$	57 M	62 L	640 S
6 $\begin{array}{r} 83386 \\ \div 3 \\ \hline \end{array}$	1083 Y	16 H	25 O	16 $\begin{array}{r} 24123 \\ \div 12 \\ \hline \end{array}$	201 P	2.1 Q	20 Y
7 $\begin{array}{r} 508 \\ \times 2.4 \\ \hline \end{array}$	13 E	121 B	1.4 C				
8 $\begin{array}{r} 7562 \\ \div 80 \\ \hline \end{array}$	80 P	.803 S	81 D				
9 $\begin{array}{r} 35.6 \\ \times 3.9 \\ \hline \end{array}$	14 X	1452 N	143.2 T				
10 $\begin{array}{r} 1751 \\ \div 51 \\ \hline \end{array}$	29 E	300 K	3 F				




1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
---	---	---	---	---	---	---	---	---	----	----	----	----	----	----	----

PROBLEM NUMBERS

$$\frac{7}{10} = .7, \quad \frac{18}{10} = 1.8, \quad \frac{120}{10} = 12.0$$

1.8 is read as one and eight tenths or as eighteen tenths.

They can write the decimals in fraction form, perform the addition or subtraction, then write the sum or difference as a decimal.

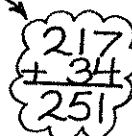


$$.3 + .5 = \frac{3}{10} + \frac{5}{10} = \frac{8}{10} = .8$$

$$1.8 + .2 = \frac{18}{10} + \frac{2}{10} = \frac{20}{10} = 2.0 \quad (\text{twenty tenths or two})$$

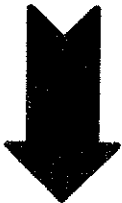
$$21.7 + 3.4 = \frac{217}{10} + \frac{34}{10} = \frac{251}{10} = 25.1$$

Computations like this might be done on scratch paper.



$$\begin{array}{r} 217 \\ + 34 \\ \hline 251 \end{array}$$

Students will often develop their own shortcuts by writing notes off to the side. They can be encouraged to verbalize any shortcut. "When adding tenths to tenths just add the number of tenths and put in the decimal point to remind you it's tenths." A similar pattern occurs in subtraction.



$$.8 - .2 = \frac{8}{10} - \frac{2}{10} = \frac{6}{10} = .6$$

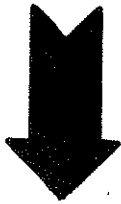
$$21.4 - 7.6 = \frac{214}{10} - \frac{76}{10} = \frac{138}{10} = 13.8$$

Scratch paper.




$$\begin{array}{r} 214 \\ - 76 \\ \hline 138 \end{array}$$

The same flow can be used for adding and subtracting hundredths.



$$.02 + .17 = \frac{2}{100} + \frac{17}{100} = \frac{19}{100} = .19$$

$$3.76 + 21.85 = \frac{376}{100} + \frac{2185}{100} = \frac{2561}{100} = 25.61$$



$$.18 - .05 = \frac{18}{100} - \frac{5}{100} = \frac{13}{100} = .13$$

$$41.72 - 3.98 = \frac{4172}{100} - \frac{398}{100} = \frac{3774}{100} = 37.74$$

Students will usually resort to vertical placement so they can easily apply the addition or subtraction algorithms. The teacher could ask, "Can anyone find a shortcut for finding the sum $3.17 + 4.22$?" Students might give some "shortcuts" that the teacher hasn't considered. A useful shortcut gives the correct answer (as found with the long method) and it applies to other problems. Students can be led to agree that a good shortcut to find $3.17 + 4.22$ is

$$\begin{array}{r} 3.17 \\ + 4.22 \\ \hline 7.39 \end{array}$$

What if we want to add tenths and hundredths?

$$.2 + .04 = \frac{2}{10} + \frac{4}{100} \quad \text{Now we need to change both to hundredths.}$$

$$= \frac{20}{100} + \frac{4}{100} = \frac{24}{100} = .24$$

What is a good shortcut for this problem? We know that $.2 + .04 = .24$ by the work above. How could this problem be written vertically?

$$\begin{array}{r} .2 \\ + .04 \\ \hline \end{array} \quad ? \quad \text{That doesn't help. What about } \begin{array}{r} .2 \\ + .04 \\ \hline \end{array} \quad ? \quad \text{or the equivalent } \begin{array}{r} .20 \\ + .04 \\ \hline \end{array} \quad ?$$

The last arrangements lead nicely to the sum .24 which is correct.

A good shortcut for this problem was to line up the decimal points (you can affix zeros to make the column on the right even). Does this shortcut work on other problems?

$$1.7 + .25 \quad \Rightarrow \quad \begin{array}{r} 1.7 \\ + .25 \\ \hline \end{array} \quad \text{or} \quad \begin{array}{r} 1.70 \\ + .25 \\ \hline 1.95 \end{array}$$

$$\text{Check: } 1.7 + .25 = \frac{17}{10} + \frac{25}{100} = \frac{170}{100} + \frac{25}{100} = \frac{195}{100} = 1.95 \quad \text{Yes!}$$

Involving the students in finding the algorithm can be more interesting for them. Here they are involved in discovering an algorithm which is based on understanding the decimal notation.

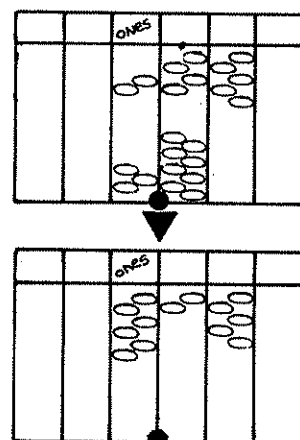
ADDITION AND SUBTRACTION OF DECIMALS VIA PLACE VALUE

Writing a problem like $2.45 + 3.8$ in expanded form as

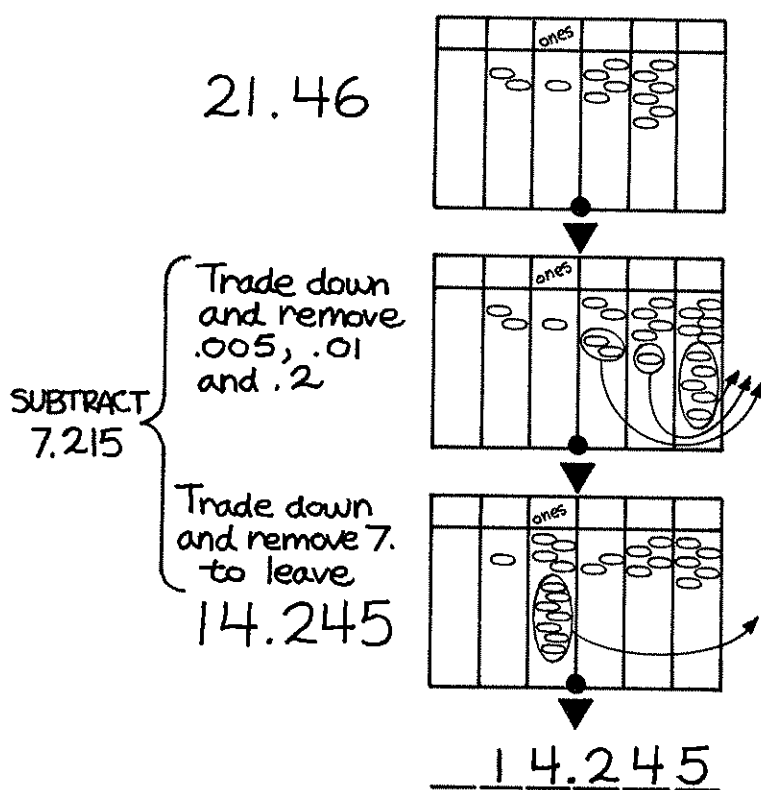
$$2 + (4 \times \frac{1}{10}) + (5 \times \frac{1}{100}) + 3 + (8 \times \frac{1}{10})$$

seems very complicated, but ideas of place value can be utilized very simply on an abacus. If students understand the ideas of trading ten for one and that numerals like 2.45 mean 2 ones + 4 tenths + 5 hundredths they can represent the numbers 2.45 and 3.8 on a chip abacus. See the diagram at the right. Trading ten tenths for a ones chip gives the sum 6.25.

$$\begin{array}{r} 2.45 \\ + 3.8 \\ \hline \end{array}$$



$$6.25$$



Subtraction is similar. If the problem is $21.46 - 7.215$ the number 21.46 can be shown. Trade downs can be made whenever chips can't be removed as required by the number being subtracted.

In working with the abacus the same flow of problems can be used--tenths (addition and subtraction) hundredths (+,-) mixing hundredths and tenths and then more complicated decimals.

The chip abacus is most useful for decimals when it has been used previously to emphasize whole number concepts and operations. It is used throughout the resource. Commercial sources and construction directions are given in the laboratory section of this resource.

Work with the abacus gives motivation for lining up the decimal points. If students represent the numbers correctly on the abacus the one, tenths, etc. are lined up so the corresponding numerals should be lined up by place value as well. Zeros can always be affixed to make the numbers line up on the right.



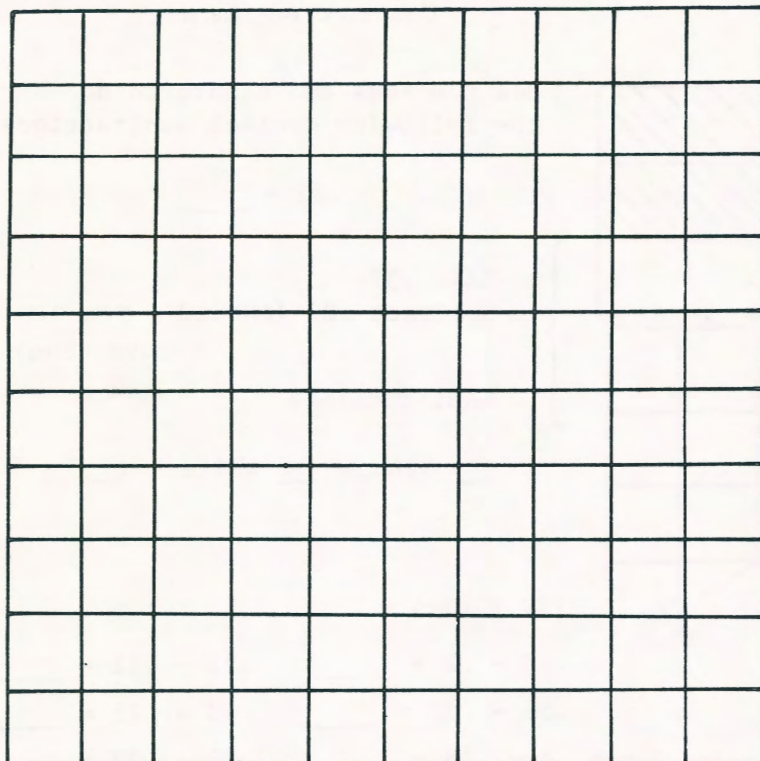
ACTIVITY CARDS - CUISENAIRE RODS - VI

The name Cuisenaire and the color sequence of the rods are trademarks of the Cuisenaire Company of America, Inc.

Model
Addition/Subtraction
DECIMALS



For readiness activities for Cuisenaire Rods see the section on Lab Materials.



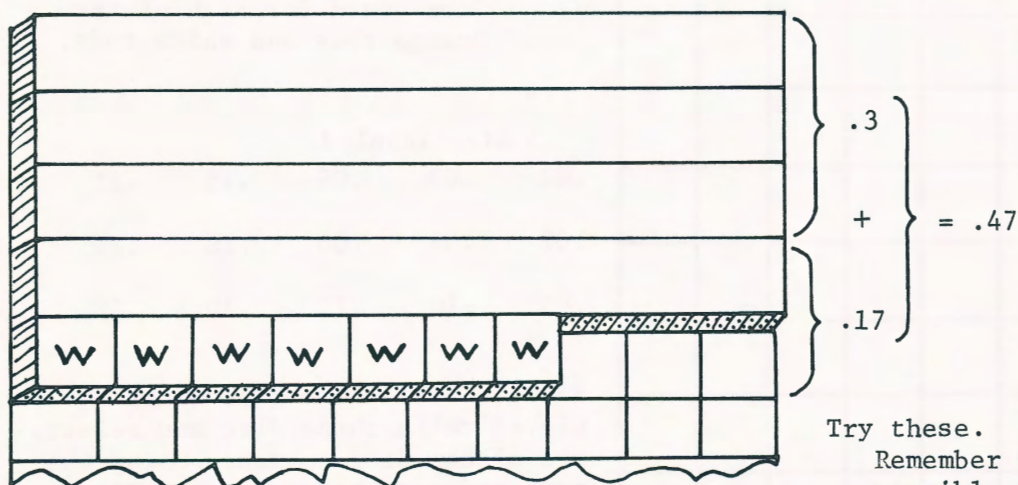
Use only orange and white rods. How many of each color do you need to cover the following parts of the square? Use as few rods as possible!

	ORANGE	WHITE
.03	none	3
.15		
.27		
.3		
.40		
.72		
.90		
.19		

CR-VI-1

CR-VI-2

Use the orange and white rods and the square to find an answer to this problem: $.3 + .17$



Try these.

Remember to use as few rods as possible in your final answer.

$.17 + .21 = \underline{\quad}$	$.40 + .07 = \underline{\quad}$	$.27 + .48 = \underline{\quad}$	$.35 + .25 = \underline{\quad}$
$.64 + .26 = \underline{\quad}$	$.50 + .25 = \underline{\quad}$	$.5 + .46 = \underline{\quad}$	$.19 + .38 = \underline{\quad}$

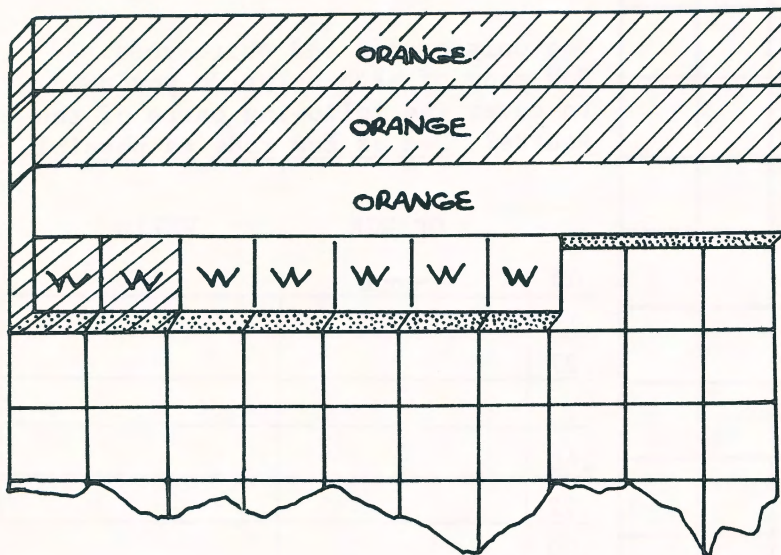
TYPE: Manipulative

The name Cuisenaire and the color sequence of the rods are trademarks of the Cuisenaire Company of America, Inc.

ACTIVITY CARDS - CUISENAIRE RODS - VI

(CONTINUED)

The name Cuisenaire and the color sequence of the rods are trademarks of the Cuisenaire Company of America, Inc.



Use the rods and square to do the following decimal subtractions.

$$.37 - .22 = \underline{\hspace{2cm}}$$

Make .37

Subtract .22 (shaded - you can remove them)

What remains?

 orange white = .

Try these.

$$.48 - .2 = \underline{\hspace{2cm}} \quad .33 - .11 = \underline{\hspace{2cm}}$$

$$.56 - .32 = \underline{\hspace{2cm}} \quad .43 - .25 = \underline{\hspace{2cm}}$$

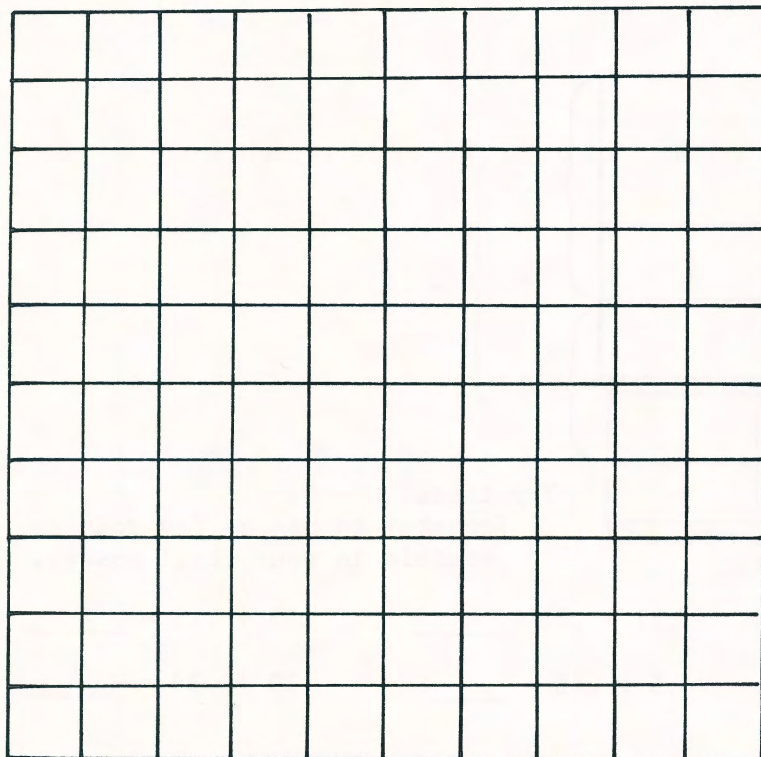
$$.4 - .18 = \underline{\hspace{2cm}} \quad .76 - .09 = \underline{\hspace{2cm}}$$

CR-VI-3

Be careful →

CR-VI-4

PLAY "THE COVER-UP"



GAME BOARD

Materials:

Game board for each player
Orange rods and white rods.

3 dice labeled

.01 .03 .09 .15 .21 .33

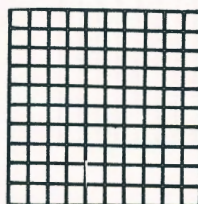
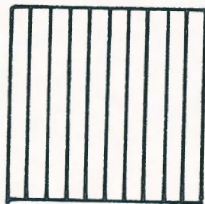
.02 .04 .08 .16 .24 .32

.05 .10 .15 .20 .25 .30

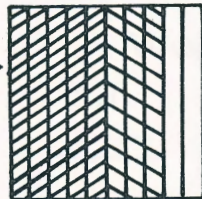
Rules: Decide who goes first. First player rolls three dice and selects one of the dice to use. Using the rods and game board, cover the amount shown on the chosen die. If all numbers on the dice are too large, player loses his turn. Winner is first player to cover his square exactly.



EACH OF THESE IS ALSO
1 UNIT



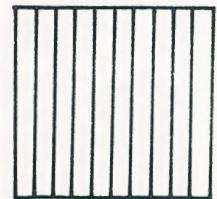
TO SHOW $\begin{array}{r} .5 \\ + .3 \\ \hline \end{array}$



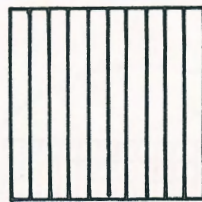
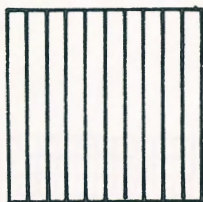
A. SHADE .5
B. SHADE ANOTHER .3

SHOW $\begin{array}{r} .4 \\ + .3 \\ \hline \end{array}$

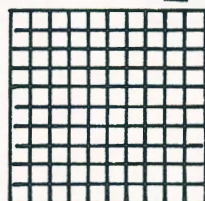
A. SHADE .4
B. SHADE .1



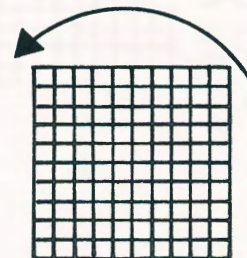
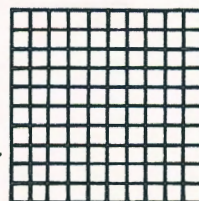
Ask students: What decimal sum is shown?



SHOW $\begin{array}{r} .8 \\ + .6 \\ \hline \end{array}$



SHOW $\begin{array}{r} .08 \\ + .05 \\ \hline \end{array}$



SHOW $\begin{array}{r} .8 \\ + .32 \\ \hline \end{array}$

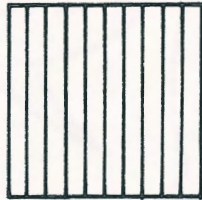
Get some graph paper and try some problems of your own.

Have students use graph paper and try similar problems.

TYPE: Paper & Pencil/Transparency

IDEA FROM: C.O.L.A.M.D.A.

GRID DECIMALS - II (CONTINUED)



TO SHOW $\begin{array}{r} .7 \\ - .3 \end{array}$

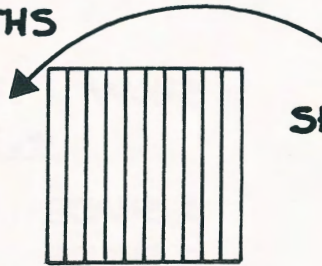
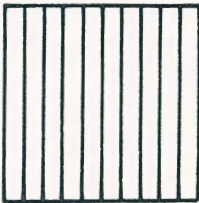
- A. LIGHTLY SHADE .7
- B. DARKEN 3 OF THE SHADED TENTHS.

What decimal is shown by the remaining lightly shaded part

SHOW

$\begin{array}{r} .9 \\ - .4 \end{array}$

- A. LIGHTLY SHADE .9
- B. DARKEN 4 OF THE SHADED TENTHS

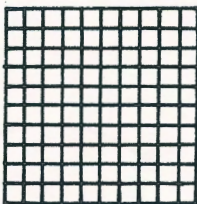


SHOW

$\begin{array}{r} 1.5 \\ - .7 \end{array}$

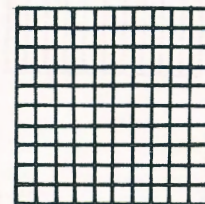
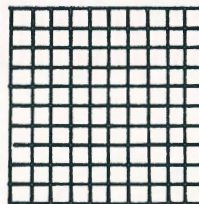
SHOW

$\begin{array}{r} .06 \\ - .03 \end{array}$



SHOW

$\begin{array}{r} 2 \\ - .43 \end{array}$



Get some graph paper and try some problems of your own.

Have students use graph paper and try similar problems.

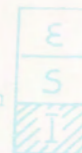
IDEA FROM: C.O.L.A.M.D.A.



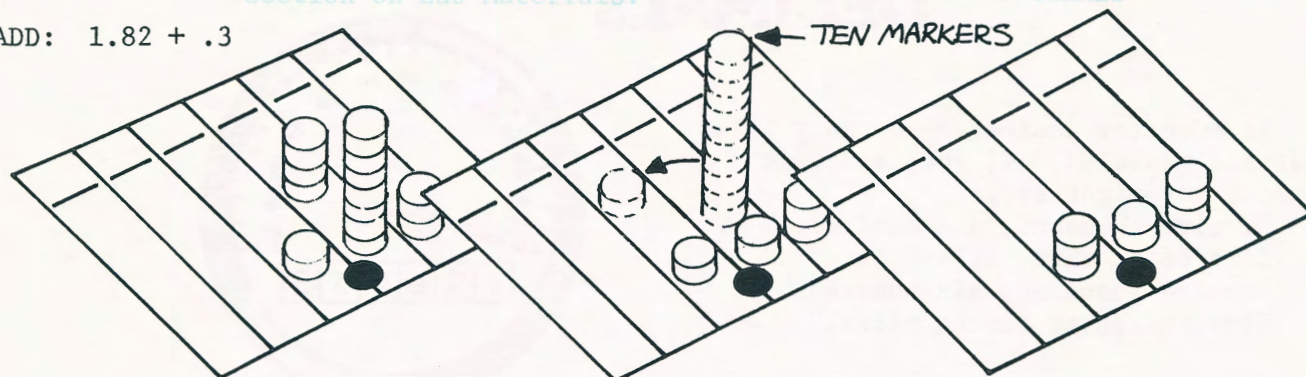
ACTIVITY CARDS - DECIMAL ABACUS - II

For information on the construction, use and readiness activities for the Abacus see the section on Lab Materials.

Model
Addition/Subtraction
DECIMALS



ADD: $1.82 + .3$



Trade if you have 10 or more markers in one place.

$$1.82 + .3 = 2.12$$

Try these on an abacus.

$$3.1 + 1.4 =$$

$$\begin{array}{r} 23.04 \\ + 8.19 \\ \hline \end{array}$$

$$\begin{array}{r} 104.3 \\ + 0.12 \\ \hline \end{array}$$

$$\begin{array}{r} 2.11 \\ + 1.95 \\ \hline \end{array}$$

$$36.1 + 27.3 =$$

$$4.91 + 3.68 =$$

$$\begin{array}{r} 16.8 \\ + 2.9 \\ \hline \end{array}$$

$$\begin{array}{r} 3.87 \\ + 0.08 \\ \hline \end{array}$$

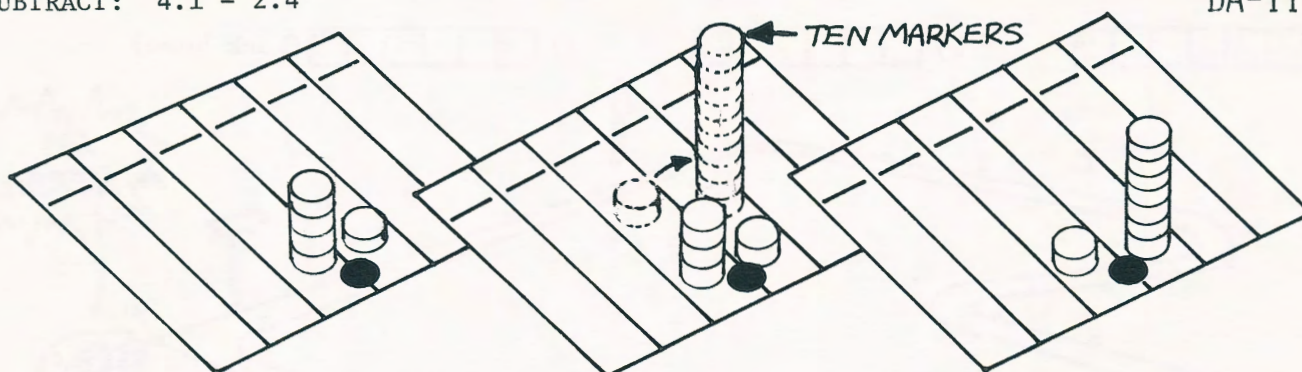
Try some problems of your own.

$$17.1 + 92 =$$

DA-II-1

SUBTRACT: $4.1 - 2.4$

DA-II-2



I need to subtract 4 markers from the tenths' place.

SUBTRACT: $4.1 - 2.4$

Try these on an abacus.

$$31.9 - 10.6 =$$

$$\begin{array}{r} 130 \\ - 2.4 \\ \hline \end{array}$$

$$\begin{array}{r} 3.72 \\ - .5 \\ \hline \end{array}$$

$$\begin{array}{r} 73.2 \\ - 8.06 \\ \hline \end{array}$$

$$8.21 - .17 =$$

$$15 - 7.3 =$$

$$6.34 - 1.75 =$$

$$\begin{array}{r} 9.1 \\ - 2.7 \\ \hline \end{array}$$

$$\begin{array}{r} 1.82 \\ - .42 \\ \hline \end{array}$$

Try some problems of your own.

TYPE: Manipulative



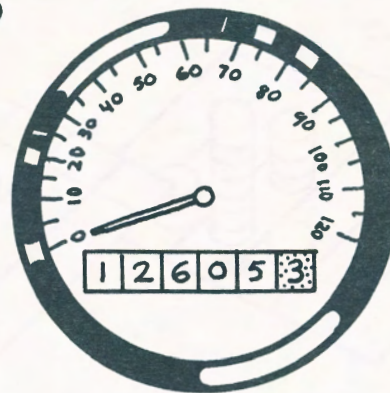
ODOMETER DECIMALS

Model
Addition
DECIMALS



An odometer indicates how many miles a car has traveled. If you read this odometer you might say,

"Twelve thousand, six hundred, five point three miles" or
"Twelve thousand, six hundred, five and three tenths miles."



On a piece of paper you would write:

12,605.3 or $12,605\frac{3}{10}$

Both mean the same thing. In 12,605.3 you used a point called a decimal point to separate the whole numbers and the fractional numbers.

Herbie's odometer reads **00598.6**. If each space on the road below is .1 of a mile, what will Herbie's odometer read at:

a)

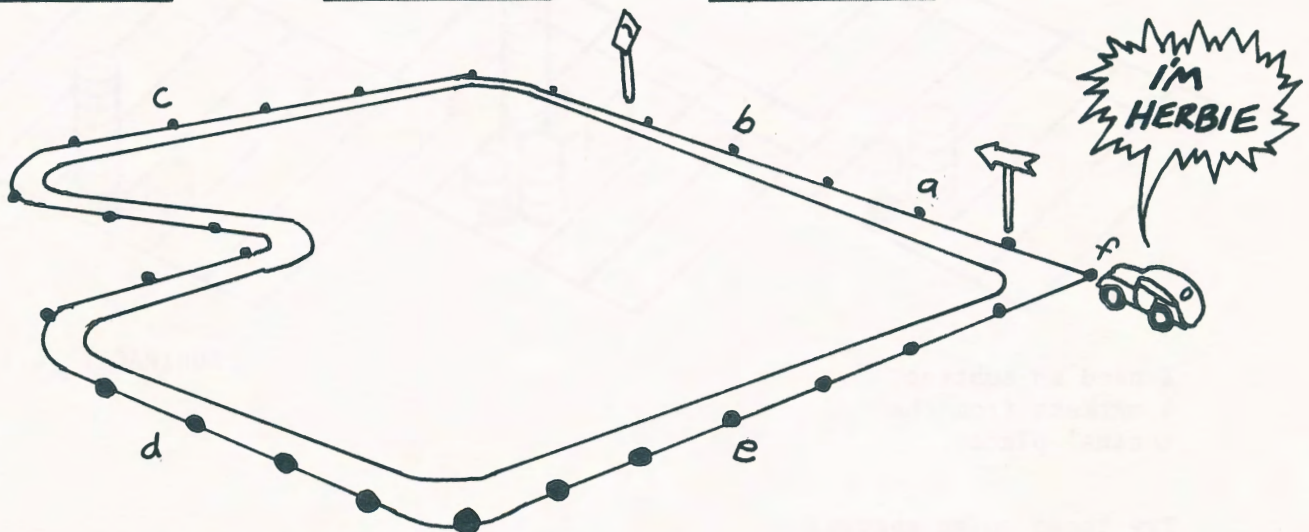
b)

c)

d)

e)

f) (back home)



If the odometer reads **99998.4** at the start what will the odometer read at each point?

a)

b)

c)

d)

e)

f)

TYPE: Paper & Pencil

THE CASE OF THE INVISIBLE DECIMAL POINT

HERLOCK HOLM AND DOCTOR WATSON NEED TO ADD UP THE NUMBERS IN EACH CLUE TO PREVENT A POSSIBLE **MURDER**. EVERYTHING ELSE ADDS UP—CAN YOU HELP?

HERLOCK, I FOLLOWED THEM EACH NIGHT JUST AS YOU SAID. HERE ARE THE DISTANCES. BUT HOW DOES IT ADD UP?

CLUE 1
 TUESDAY 11.6 KM
 WEDNESDAY 1.23 KM
 THURSDAY 3 KM

ELEMENTARY, MY DEAR WATS! WE'LL LINE THE NUMBERS UP.
 11.6
 1.23...
 ...BUT WHERE DOES THE **3** FIT IN?

HERE ARE THE BUTLER'S EXPENSES. AGAIN WE ARE MISSING A POINT!

CLUE 2
 PLANT \$2.98
 SHOVEL \$30.00
 POISON! \$9

TRUE, THERE IS A MISSING POINT BUT JUST COMPARE THIRTY DOLLARS, AND NINE DOLLARS, AND THE POINT BECOMES OBVIOUS!

LATER...

HERE ARE THE FLASKS. TWO CONTAIN ANTIDOTE, BUT ONE CONTAINS A LETHAL DOSE OF **POISON**!

CLUE 3...

87ml
 1.3ml
 2ml

THE POINT, WATS, IS TO ADD THEM TOGETHER TO COUNTERACT THE POISON

If the sum of your 3 clues is not 61.98, you need to check it out again. Locate any invisible decimal points to solve the following problems:

1) $15 + .43 =$ _____

2) $.17 + 8 =$ _____

3) $4.23 + 5 =$ _____

4) $17 - .23 =$ _____

5) $\$5.47 - \$3 =$ _____

6) $\$12 - \$.87 =$ _____

Penny Slide

Addition/Subtraction
DECIMALS



**IF YOU LAND IN THIS SECTION
SUBTRACT 1.0 FROM YOUR SCORE**

This playing board may be used by making a thermofax copy, then a ditto. Students will need a penny to slide from the start position. The game is played like shuffleboard with the winner determined when a certain total is reached or when time is up. 2-4 players work best.

IF YOU LAND IN THIS SECTION

SUBTRACT .5 FROM YOUR SCORE

1.6	1.5	1.4	1.3
.9	1.0	1.1	1.2
.8	.7	.6	.5
.1	.2	.3	.4

**IF YOU LAND IN THIS SECTION
SUBTRACT .5 FROM YOUR SCORE**

If the marker falls on one line, the player adds both numerals indicated. If it lands on the

intersection of four boxes, all four numerals are added to his score.

**IF YOU LAND
IN THIS SECTION**

START

**TAKE ANOTHER
TURN**

IDEA FROM: Fun With Numbers

TYPE: Game



Money Exchange Rates

Addition/Subtraction
DECIMALS



Only the Tuesday thru Saturday newspapers will contain the information you'll need. Many small town papers may not have the information; so you may have to use the library reference department.

Look thru the business section till you find a section called FOREIGN MONEY EXCHANGE RATES. If the paper is your own, cut out the section and paste it on the back of this page.

FOREIGN MONEY EXCHANGE RATES

COUNTRY	TODAY'S RATE	PREVIOUS DAY	CHANGE(indicate + or -)
Australia --- dollar			
Canada --- dollar			
France --- Franc			
W.Germany --- mark			
Britain --- pound			
Hong Kong --- dollar			
Israel --- pound			
Italy --- lira			
Japan --- yen			
Mexico --- peso			
Sweden --- krona			
Venezuela --- bolivar			

All the monies listed above are being compared to the U.S.

What is the purpose of listing these exchange rates: _____

Why do the exchange rates vary from day to day: _____

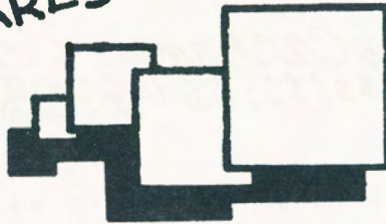
TYPE: Paper & Pencil/Transparency

SOURCE: Project R-3

Banks also have information
on daily money exchange rates.

Permission to use granted
by E. L. Hodges

4-SQUARES



MATERIALS: (See next page for construction hints)

1. Four sets of colored decimal pieces: red, blue, green and yellow
2. Four unit squares (5" x 5")
3. One spinner

OBJECT OF THE GAME: Upon completion of the game, the player with the highest total score is the winner.

NUMBER OF PLAYERS: Two - four

DIRECTIONS:

1. Put the unit squares on a table between the players. The number of unit cards should be the same as the number of players.
2. Each player takes an envelope containing colored decimal pieces. Each player should familiarize himself with the various shapes. Players should determine the value of each piece before beginning.
3. The spinner is placed between the players. Each player spins the spinner, and the one with the highest number plays first.
4. The first player spins the spinner. He puts a decimal piece, or pieces equal to the number he hit on the spinner on one or more of the unit squares.

EXAMPLE: George spins the spinner. The spinner comes to rest on .75. George could put a .50 piece and a .25 piece on a unit square, equaling .75. The player must use the largest units possible. For example, if George has a .50 piece and a .25 piece to equal .75, he cannot use three .25 pieces to make .75 or any other combination of smaller units to equal .75.

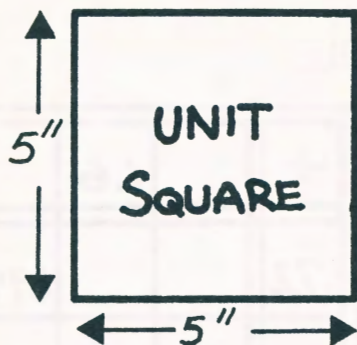
5. Players may put their decimal pieces on any of the unit squares, as long as there are open spaces on the squares.
- EXAMPLE:** Alice spins .35. She puts .25 on one card and .10 on another square, equaling .35.
6. If a number is spun that is larger than the total area of the uncovered parts remaining on the boards, the player loses his turn.
7. The game is completed when one of the following three events takes place:
 - a. All unit squares are completely covered.
 - b. One of the players has used up all his pieces.
 - c. No player is able to place a piece on any of the remaining spaces on the squares.
8. Upon completion of the game each player gathers up all the colored pieces he used on the squares and totals his scores. **EXAMPLE:** Jim used the following pieces:

		TOTAL
2	.50 pieces	1.00
1	.25 "	.25
3	.10 "	.30
5	.05 "	.25
		<hr/> 1.80

TYPE: Game

4-SQUARES (CONTINUED) CONSTRUCTION HINTS

SAMPLE UNIT SQUARE



You will need four unit squares or one for each player.

Each should be 5" on a side.

They can be made from heavy cardboard.

SAMPLE DECIMAL PIECES

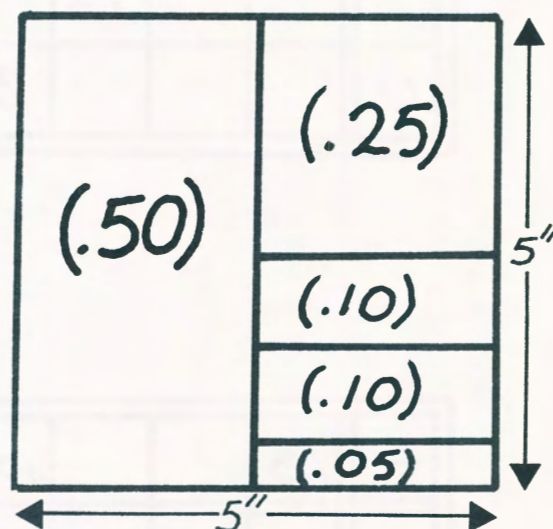
For each set of colored decimal pieces you will need:

- .50 - 2 pieces $(5" \times 2\frac{1}{2}")$
- .25 - 4 pieces $(2\frac{1}{2}" \times 2\frac{1}{2}")$
- .10 - 7 pieces $(2\frac{1}{2}" \times 1")$
- .05 - 6 pieces $(2\frac{1}{2}" \times \frac{1}{2}")$

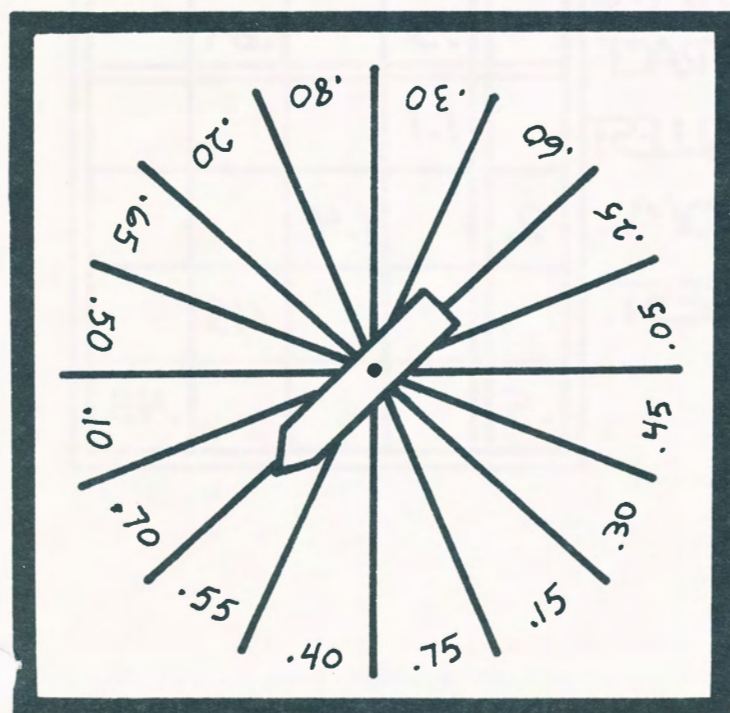
You will need a different colored set for each player.

Pieces may be made from tagboard.

Do not write the values on the pieces.



SAMPLE SPINNER



You will need one spinner. The dial face can be made from tagboard, placed on a 5" x 5" piece of cardboard and laminated. Attach the spinner to the face with a brass fastener, fold back the ends of the fastener and cover them with masking tape.

For more information on the construction of spinners see *Spinners* in the Games and Puzzles section of Lab Materials.



LATTICE MATHEMATICS - I

Addition/Subtraction
DECIMALS



COMPLETE EACH TABLE

+	.5		1	
	.7			
.9		1.3		
			2.3	
1.1				3

+		1.6		.007
.72			1.946	
2				
				.574
1.5	2.25			

Students may need help getting started.

-	1.3			.107
5	3.7			
.6			.174	
.87		.775		
				1.393

**ALWAYS
SUBTRACT
SMALLEST
FROM
LARGEST.**

-	.3		.64	
	1.1			
2		1.9		
			.43	
.5				.43

TYPE: Paper & Pencil

For a similar worksheet in
multiplication and division
see *Decimal Mathematics II*
in DECIMALS: Multiplication
and Division.

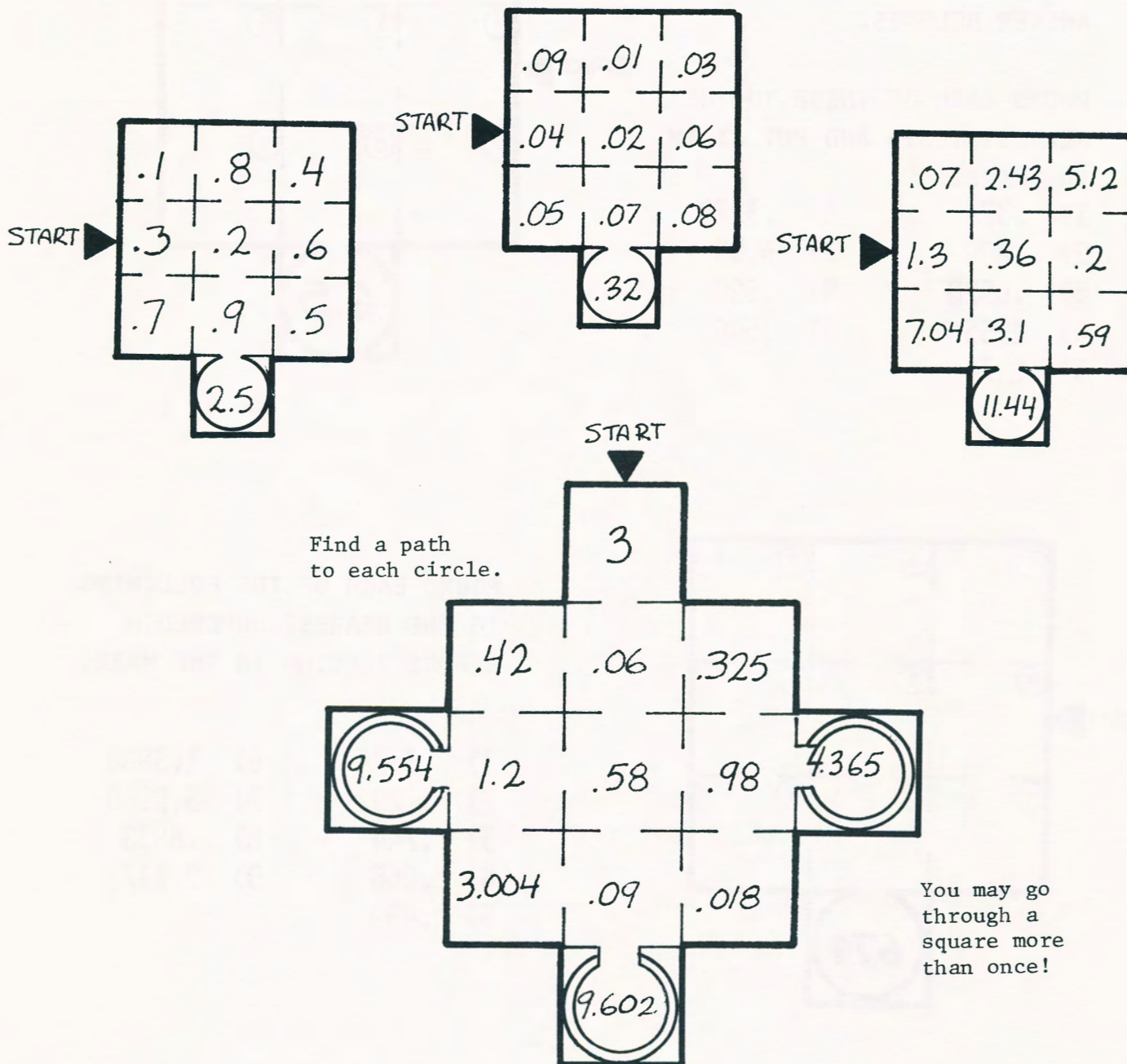


DECIMAL ADD + A + BOX

Addition
DECIMALS



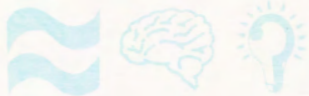
Draw a path to the answer in the bottom circle of each of the puzzles.
The sum of the numbers in the path must equal the bottom box.
The path cannot go through a corner.



TYPE: Paper & Pencil/Puzzle

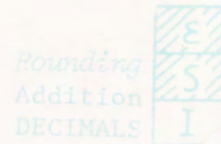
IDEA FROM: C.O.L.A.M.D.A.

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ROUNDED

ADD + A + BOX

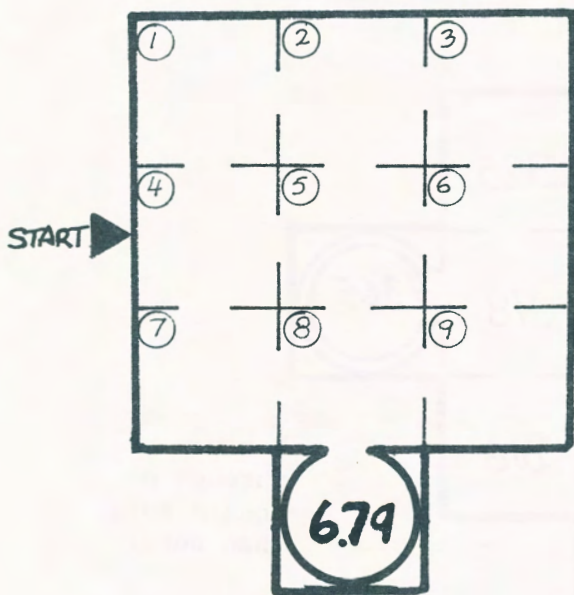
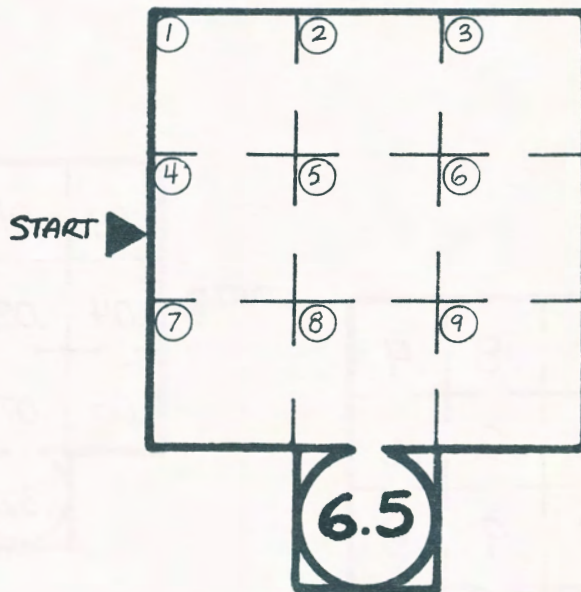


FIND A PATH THROUGH EACH MAZE THAT ADDS UP TO THE NUMBER IN THE CIRCLE AT THE BOTTOM.

EACH PROBLEM IS NUMBERED TO TELL YOU IN WHICH BOX THE ANSWER BELONGS.

ROUND EACH OF THESE TO THE NEAREST TENTH AND PUT IT IN THE MAZE.

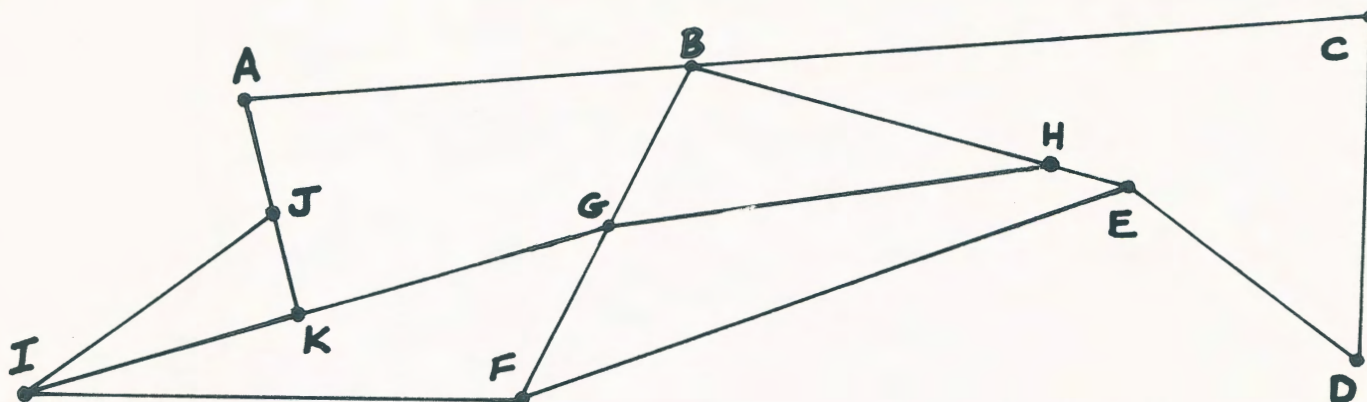
- | | |
|----------|----------|
| 1) .37 | 6) .3279 |
| 2) .809 | 7) 3.99 |
| 3) .0520 | 8) .890 |
| 4) 1.55 | 9) .546 |
| 5) 2.74 | |



ROUND EACH OF THE FOLLOWING TO THE NEAREST HUNDREDTH BEFORE PLACING IN THE MAZE.

- | | |
|-----------|-----------|
| 1) .337 | 6) 1.3888 |
| 2) 1.2929 | 7) 5.9960 |
| 3) .744 | 8) .6838 |
| 4) .066 | 9) 2.117 |
| 5) .499 | |

TYPE: Paper & Pencil/Puzzle



Suppose we decide to travel from H to J. How many different routes could we take if we do not wish to travel through any point twice? List the different routes you can find.

Place the two routes you think are the shortest in Charts I and II. Place the two routes you think are the longest in Charts III and IV.

I		II		III		IV	
FROM	CM	FROM	CM	FROM	CM	FROM	CM
H TO E		H TO		H TO		H TO	
E TO F							
F TO G							
G TO K							
K TO J							
TOTAL H - J		H TO J		H TO J		H TO J	

Which route is the LONGEST - I, II, III or IV? _____

Which route is the SHORTEST? _____

How many routes did you find? _____ Compare this answer with other students.

A MAP READING ACTIVITY WOULD BE A GOOD FOLLOW-UP.

NUMBER SENSE AND ARITHMETIC SKILLS

Placement Guide for Tabbed Dividers

Tab Label	Follows page:
DIDACTICS	10
Student Self-Concept	10
The Teaching of Skills	24
Diagnosis and Remediation	42
Goals through Games	62
TEACHING EMPHASES	74
Calculators	74
Applications	82
Problem Solving	92
Mental Arithmetic	104
Algorithms	112
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Mixed Operations	398
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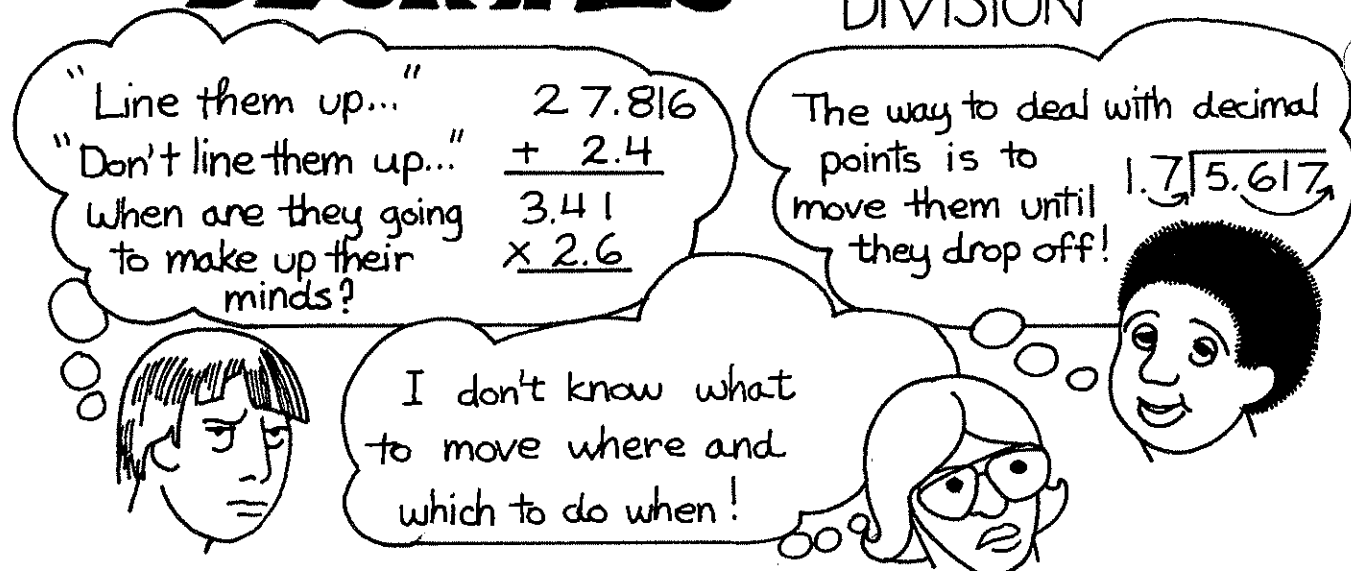
Decimals Commentary: Multiplication/Division (pages 690-697)

<u>TITLE</u>	<u>PAGE</u>	<u>TOPIC</u>	<u>TYPE</u>
Multiplication Paper Folding	698	Model Multiplication	Manipulative
Grid Decimals - III	699	Model Multiplication/division	Paper and pencil Transparency
Activity Cards - Decimal Abacus - III	701	Model Multiplication	Manipulative
Activity Cards - Decimal Abacus - IV	702	Model Powers of ten Multiplication/division	Manipulative
Dotman and Bobbin	703	Placing the decimal point Multiplication	Paper and pencil
Dotman's Decimal Notes	704	Placing the decimal point Multiplication	Paper and pencil
Keep Your Eye on the Dot	706	Placing the decimal point Division	Paper and pencil
Lots of Dots	707	Placing the decimal point Division	Paper and pencil
Lots of Dots Shortcut	708	Placing the decimal point Division	Paper and pencil
Division Distraction	709	Placing the decimal point Division	Paper and pencil Puzzle
Decimal Line Up	710	Placing the decimal point Division	Paper and pencil Puzzle
Seeing's Believing - I	711	Approximation Multiplication	Paper and pencil Transparency
Seeing's Believing - II	712	Approximation Division	Paper and pencil Transparency
Never Hold a Grudge	713	Approximation Multiplication/division	Paper and pencil Puzzle

<u>TITLE</u>	<u>PAGE</u>	<u>TOPIC</u>	<u>TYPE</u>
Nobody Knows but the Zeros	714	Approximation Multiplication	Paper and pencil Puzzle
The American Housewife	715	Multiplication	Paper and pencil Transparency
Giant Economy Size	716	Ordering Division	Paper and pencil Transparency
Speed and Sound	717	Multiplication/division	Paper and pencil Transparency
The Minimum Wage	718	Multiplication	Paper and pencil
Lattice Mathematics	719	Multiplication/division	Paper and pencil
Hands on Decimals	720	Model Relation to fractions Division	Manipulative
Decimal Niddy Griddy I	721	Relation to fractions Division	Puzzle
Decimal Niddy Griddy II	722	Relation to fractions Division	Puzzle
Calculated Codes	723	Relation to fractions Ordering Division	Paper and pencil Puzzle
Magic Square Decimals	724	Relation to fractions Division	Paper and pencil Puzzle
Stocks-Investments Funds/ Investment Funds	725	Relation to fractions Multiplication/division	Paper and pencil Activity
9-Time	726	Patterns Relation to fractions Division	Paper and pencil
Incredible Equalities	727	Relation to fractions Division	Activity Bulletin board
Repeating Decimals to Fractions	728	Relation to fractions Division	Paper and pencil

<u>TITLE</u>	<u>PAGE</u>	<u>TOPIC</u>	<u>TYPE</u>
Squared Off	729	Squares Multiplication	Paper and pencil Puzzle
Grass Roots	730	Square roots Multiplication	Paper and pencil Puzzle
Squares	731	Squares Multiplication	Paper and pencil Puzzle
Square Roots	732	Square roots Multiplication	Paper and pencil Puzzle
Square Root Game	733	Square roots Multiplication/division	Game

DECIMALS: MULTIPLICATION DIVISION



By the time multiplication and division of decimals is introduced students have been given many rules and algorithms for dealing with numbers. Sometimes these messages are arranged in a nice mental filing system so the right rule can be pulled out at an appropriate time, but often the rules and algorithms become jumbled. An algorithm can be more meaningful for students if they participate in its development. The algorithms for multiplication and division of decimals offer an opportunity for student involvement and discovery.

The algorithms for multiplication and division of terminating decimals are identical to the corresponding whole number algorithms with these exceptions: the treatment of decimal points and the adjoining of zeros in the dividend. These algorithms are often taught in a rote manner, "Here is how you multiply and divide decimals...." There are other ways to motivate the algorithms:

- Use approximation, patterns and reason.
- Use the students' understanding of decimals as fractions and their understanding of fraction operations.
- Use the understanding of place value.

When motivating algorithms with decimals, it is important to keep in mind what meanings have been given to decimal notation. If an activity uses the understanding of place value it is important that place value was emphasized during the time decimals were introduced. If a lesson uses the fraction approach (where 2.34 is just another name for $\frac{234}{100}$ as opposed to the place value view $2 + \frac{3}{10} + \frac{4}{100}$) then students should have had experience with decimals as fractions. Both of these meanings of decimals are important and can be useful in explaining the algorithms for multiplication and division.

APPROXIMATION, PATTERNS AND REASON

When students understand that 3.4 is a number between 3 and 4 and that 15.87 is between 15 and 16 they are ready to approximate the product or quotient of 3.4 and 15.87. Use can be made of the skills of rounding decimals to write

$$3.4 \times 15.87 \approx 3 \times 16 = 48 \quad \text{or} \quad 15.87 \div 3.4 \approx 16 \div 3 \approx 5$$

Students can be given approximation problems before the algorithms for multiplication and division are given. This puts their attention on the approximation process, not on the algorithm for multiplication or division. After the algorithms are introduced, the habit of checking the reasonableness of answers will help students find computational errors. See student pages *Seeing's Believing I and II* and *Never Hold a Grudge*.

If a student approximation exercise is given before teaching algorithms it is best to choose the numbers carefully. The decimals given above rounded nicely to whole numbers and then to whole number products or quotients. Problems like $.027 \div .14$ are not so nice. Both .027 and .14 round to the whole number 0, but $0 \div 0$ is not defined. After students have had practice in dividing decimals, they can write the approximations $.027 \div .14 \approx .028 \div .14 = .2$.

Number patterns and calculators could be used to discover the algorithms for multiplication and division of decimals. Students can find products and quotients of decimals with calculators. Can they discover how to find the products and quotients on pencil and paper?

Have students find and record these products one column at a time.

a) $1 \times 7 = 7$	b) $.1 \times 7 = .7$	c) $.01 \times 7 = .07$	d) $.1 \times .7 = .07$
$2 \times 7 = 14$	$.2 \times 7 = 1.4$	$.02 \times 7 = .14$	$.2 \times .7 = .14$
$3 \times 7 = 21$	$.3 \times 7 = 2.1$	$.03 \times 7 = .21$	$.3 \times .7 = .21$

What pattern do they see after column b)? column c)? d)?

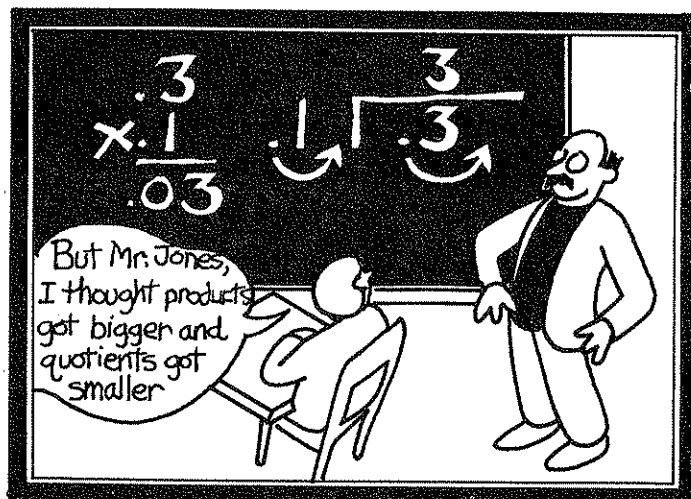
Can they find a rule to determine the product of 3 and .5? .3 and .5?

Can they find a general rule that works for all decimals?

To discover the division rule try the same approach starting with simple patterns.

a) $12 \div 1 = 12$	b) $12 \div .1 = 120$	c) $1.2 \div 1 = 1.2$	d) $1.2 \div .1 = 12$
$12 \div 3 = 4$	$12 \div .3 = 40$	$1.2 \div 3 = .4$	$1.2 \div .3 = 4$
$12 \div 6 = 2$	$12 \div .6 = 20$	$1.2 \div 6 = .2$	$1.2 \div .6 = 2$

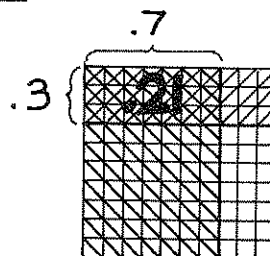
The product of two whole numbers greater than one is always greater than either of the numbers and the quotient is always less than or equal to the dividend. Decimals are peculiar. Sometimes products are bigger than both numbers, sometimes less than both and sometimes between the two numbers. The quotient of two decimals can be less than, greater than or equal to the dividend.



Students will be better able to check the reasonableness of answers if they understand that the product of two positive numbers less than 1 is a number less than either of the originals, the product of two numbers greater than 1 is greater than the two originals and that the product of a positive number less than 1 and a number greater than 1 is between the numbers. It is not realistic to expect all students to be able to generalize this way.

$.2 \times .81 = \square$	product is <u>less</u> than .2
$3.2 \times 7.5 = \square$	product is <u>greater</u> than 7.5
$3.2 \times .2 = \square$	product is <u>between</u> .2 and 3.2

To reinforce this idea products of decimals could be shown on a 100 grid using a dimension model. The cross-hatched portion (.3 x .7) is less than the .3 portion or the .7 portion. See *Multiplication Paper Folding*.




In division of decimals a quotient is less than the dividend when the divisor is greater than 1, a quotient is equal to the dividend when the divisor is 1 and a quotient is greater than the dividend when the divisor is less than 1.

$.8 \div 2.1 = \square$	quotient is <u>less</u> than .8
$.8 \div 1 = \square$	quotient is <u>equal</u> to .8
$.8 \div .43 = \square$	quotient is <u>greater</u> than .8

MULTIPLICATION OF DECIMALS VIA FRACTIONS

Multiplication of decimals can be introduced by writing the decimals as fractions, doing the computation and writing the resulting fraction in decimal notation. A lesson could be developed around each of the types of problems shown below. The computation should be kept fairly simple.

a) Products of one digit whole numbers and decimals in tenths.



$$2 \times .3 = 2 \times \frac{3}{10} = \frac{6}{10} = .6$$

$$3 \times .4 = 3 \times \frac{4}{10} = \frac{12}{10} = 1.2$$


$$5 \times 3.2 = 5 \times \frac{32}{10} = \frac{160}{10} = 16.0$$

Notice how helpful it is to have experience in naming 1.2 as "12 tenths" instead of just "1 and 2 tenths." The problems could also be worked vertically.

$$\begin{array}{r} .4 \\ \times 3 \\ \hline 12 \text{ tenths} \end{array} \rightarrow 1.2$$

The order of the factors can now be reversed and the same method applied.

b) Products of one digit whole numbers and decimals in hundredths.



$$3 \times .05 = 3 \times \frac{5}{100} = \frac{15}{100} = .15$$

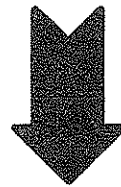
$$8 \times .25 = 8 \times \frac{25}{100} = \frac{200}{100} = 2.00$$

$$6 \times 3.05 = 6 \times \frac{305}{100} = \frac{1830}{100} = 18.30$$

Students can be reminded that 2.00 can be read as "200 hundredths" or as "2". The vertical form of this problem is below.

$$\begin{array}{r} .25 \\ \times 8 \\ \hline 200 \text{ hundredths} \end{array} \rightarrow 2.00$$

c) Products of powers of ten and decimals in tenths and hundredths.



$$10 \times .6 = 10 \times \frac{6}{10} = \frac{60}{10} = 6.0$$

$$100 \times 2.1 = 100 \times \frac{21}{10} = \frac{2100}{10} = 210.0$$

Some students might divide out (cancel) the tens. They would obtain the number 6 which is equivalent to 6.0.

Multiplying by 10 has the apparent effect of moving the decimal point one place to the right. Students might make similar observations when multiplying by 100, by 1000,....

d) Products of decimals in tenths and hundredths.

$$.3 \times .1 = \frac{3}{10} \times \frac{1}{10} = \frac{3}{100} = .03$$

$$2.4 \times .21 = \frac{24}{10} \times \frac{21}{100} \rightarrow \frac{504}{1000} = .504$$

(scratch work)

Some students will drop the horizontal writing of the problems on their own. Problems might be worked something like this:

$$.8 \times 3.5 =$$

Scratch work.

The computation on the scratch paper shows that this student has already decided he can temporarily ignore the denominators, use the whole number algorithm, multiply the denominators and then place the decimal point. A point of this can be made at the chalk board or overhead.

We could multiply the numbers as if they were whole numbers then take care of the placement of the decimal point.

It can be pointed out that the number of places after the decimal point in the product is the sum of the number of places after the decimal points in the original numbers; some students will find the pattern themselves. This transition to the multiplication algorithm gives some sense to what might be a rote memorization of a rule. With fractions to fall back on the student has a better chance of figuring out the "rule" if he happens to forget it. For related classroom material see *Dotman and Bobbin* and *Dotman's Decimal Notes*.

MULTIPLICATION OF DECIMALS VIA PLACE VALUE

A strict place value interpretation of decimals seems rather cumbersome in looking at multiplication of decimals. Consider this problem:

$$3.5 \times .27 = (3 + \frac{5}{10}) \times (\frac{2}{10} + \frac{7}{100})$$

$$\text{or } [3 + (5 \times \frac{1}{10})] \times [(2 \times \frac{1}{10}) + (7 \times \frac{1}{100})]$$

This seems only to complicate the situation. Often we resort to an abacus when we want to stress place value but there is no simple way to show $3.5 \times .27$ on an abacus.

It is in multiplication by whole numbers (especially powers of 10) that place value can be most useful. Suppose we want to emphasize that multiplication by 10 "moves" every digit in a decimal one place to the left (or more commonly put, "moves the decimal point one place to the right"). A problem is given, say $10 \times .023$. By the place value view $.023$

is interpreted to mean $\frac{2}{100} + \frac{3}{1000}$.

Using a chip abacus this number can be represented with 2 chips in the $\frac{1}{100}$'s column and 3 chips in the $\frac{1}{1000}$'s column.

Multiplying this number by 10 can be shown by placing ten such groups in the $\frac{1}{100}$'s and $\frac{1}{1000}$'s columns. After the trading is finished, 2 chips are left in the $\frac{1}{10}$'s place and 3 in the $\frac{1}{100}$'s place. Students can observe that the result was the same as moving the original chips one place to the left, or moving the decimal point one place to the right in the numeral. See *Decimal Abacus III and IV* for activity cards using a chip abacus.

100's	10's	ONES	$\frac{1}{10}$'s	$\frac{1}{100}$'s	$\frac{1}{1000}$'s
				00	000

.023

100's	10's	ONES	$\frac{1}{10}$'s	$\frac{1}{100}$'s	$\frac{1}{1000}$'s
				000000	0000000

10x.023

100's	10's	ONES	$\frac{1}{10}$'s	$\frac{1}{100}$'s	$\frac{1}{1000}$'s
			00	000	

.023

_____.230

DIVISION OF DECIMALS VIA FRACTIONS

Problems which involve division by a whole number and which "come out even" can be solved by changing the decimal notation to fraction notation. Even the division of a pair of decimals whose quotient is a whole number (like 3.2 and .8) make straightforward fraction problems.

Suppose we look at a possible lesson sequence.

a) Decimals divided by whole numbers.

$$.9 \div 3 = \frac{9}{10} \div 3 = \frac{3}{10} = .3$$

$$37.52 \div 4 = \frac{3752}{100} \div 4 = \frac{938}{100} = 9.38$$

Notice students will need to have the ability to work $\frac{a}{b} \div c = \frac{a \div c}{b}$. The problems could also be worked as below.

$$4 \overline{)39.52} \rightarrow 4 \overline{)3752} \xrightarrow{\text{938 hundredths}} 9.38$$

b) Decimals divided by decimals (quotient a whole number)

$$.1 \div .1 = \frac{1}{10} \div \frac{1}{10} = 1$$

$$1.2 \div .4 = \frac{12}{10} \div \frac{4}{10} = 3$$

$$9.36 \div .03 = \frac{936}{100} \div \frac{3}{100} = 312$$

The idea that $\frac{a}{b} \div \frac{c}{b} = a \div c$ is needed for these problems.

Alternately:

$$.4 \overline{)1.2} \xrightarrow{3} 4 \text{ tenths} \overline{)12 \text{ tenths}}$$

It might be tempting to write 3 tenths for the quotient. Students can check by multiplication.

$$4 \text{ tenths} \times 3 \text{ tenths} \neq 12 \text{ tenths}$$

c) Whole numbers divided by whole numbers (quotient not a whole number).

The situation is not so simple here; $5 \div 2$ is $\frac{5}{2}$ and $3 \div 7$ is $\frac{3}{7}$ but how should $\frac{2}{5}$ and $\frac{3}{7}$ be expressed as decimals? (See the commentary to DECIMALS: Concepts for a manipulative solution to this question.) This is really the problem of changing a fraction to a decimal. We can turn to the many names for a decimal number to help us.

Five can be read as 50 tenths, 500 hundredths, etc. How exact do we want our result to be? To the nearest hundredth? Then

$$5 \div 2 = 500 \text{ hundredths} \div 2 = 250 \text{ hundredths} = 2.50$$

$$\text{or } 2 \overline{)5} \longrightarrow 2 \overline{) \frac{250 \text{ hundredths}}{500 \text{ hundredths}}} \longrightarrow 2.50$$

Instead of writing out hundredths we could have used decimal points: $2 \overline{) \frac{2.50}{5.00}}$

If we want $3 \div 7$ to the nearest thousandth we can write

$$7 \overline{)3} \longrightarrow 7 \overline{) \frac{428 \frac{4}{7} \text{ thousandths}}{3000 \text{ thousandths}}} \approx 429 \text{ thousandths} = .429$$

Again using decimal points can save us writing thousandths: $7 \overline{) \frac{.428 \frac{4}{7}}{3.000}} \approx .429$

d) Decimals divided by whole numbers (results uneven).

$$5 \overline{)3.41} \longrightarrow 5 \overline{) \frac{68 \frac{1}{5} \text{ hundredths}}{341 \text{ hundredths}}} \approx 68 \text{ hundredths} = .68$$

$$12 \overline{)15.4} \longrightarrow 12 \overline{) \frac{12 \frac{10}{12} \text{ tenths}}{154 \text{ tenths}}} \approx 13 \text{ tenths} = 1.3$$

$$\begin{array}{r} 12 \\ 34 \\ \underline{24} \\ 10 \end{array}$$

e) General decimal division.

When the divisor in a problem is a whole number the problem is of the type discussed in c) or d). When the divisor is not a whole number we substitute an equivalent problem with a whole number divisor.

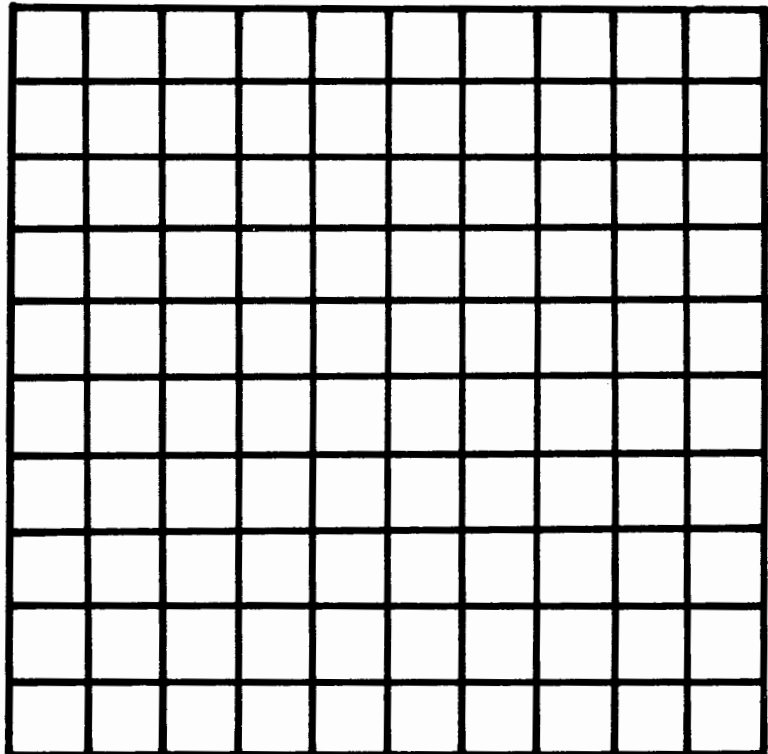
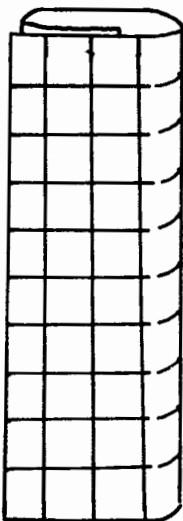
$$3.2 \overline{)14.75} \longrightarrow 32 \overline{)147.5}$$

MULTIPLICATION PAPER FOLDING

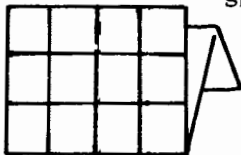
Materials needed: Each student needs several 10 x 10 grids. They may be dittoed from this sheet, or cut from cm or quarter inch graph paper.

Multiply $.3 \times .4$:

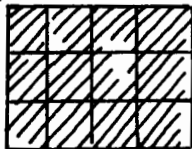
Fold grid so $.4$ of the paper is showing.



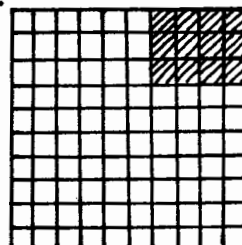
Now fold the paper to show $.3$ of this.



Color in the part showing.



Open the grid.



The part shaded is $.12$ of the whole.
So $.3 \times .4 = .12$

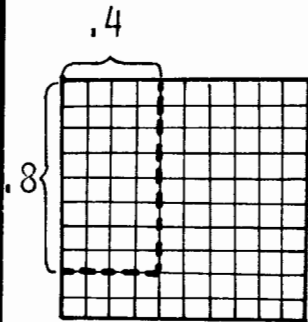
Fold your papers to find:

- 1) $.2 \times .8 = \underline{\quad}$ 2) $.5 \times .9 = \underline{\quad}$ 3) $.4 \times .7 = \underline{\quad}$

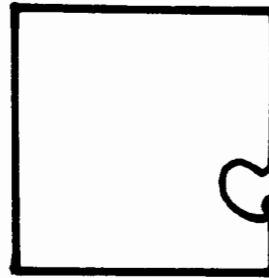
Can you see a pattern? Find answers for these problems, then check by folding.

- 4) $.3 \times .6$ 5) $.5 \times .8$ 6) $.6 \times .6$

GRID DECIMALS - III



$$.4 \times .8 = 32 \text{ HUNDREDTHS} \\ \text{OR } .32$$



USE A GRID TO SOLVE THE FOLLOWING PROBLEMS:

$$.2 \times .4 =$$

$$.8 \times .7 =$$

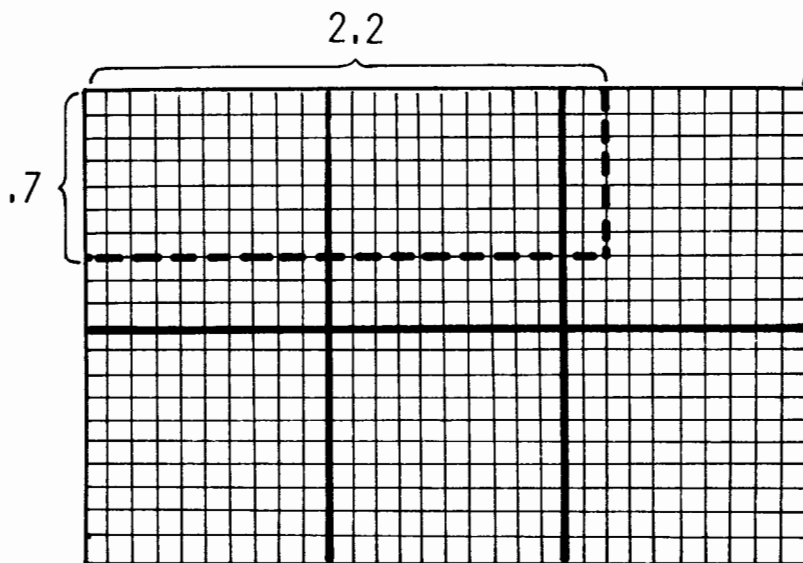
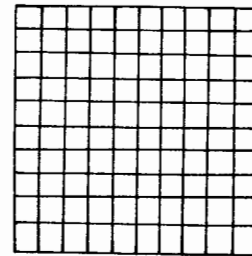
$$.3 \times .7 =$$

$$.9 \times .9 =$$

$$.1 \times .9 =$$

$$.15 \times .4$$

BE CAREFUL



EXAMPLE $.7 \times 2.2$

USE A GRID TO HELP YOU SOLVE THE FOLLOWING:

$$.7 \times 2.2 = 154 \text{ HUNDREDTHS} \\ \text{OR } 1.54$$

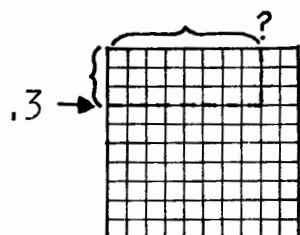
$$.5 \times 1.3 =$$

$$2.5 \times .3 =$$

$$1.1 \times 1.3 =$$

$$1.5 \times 2.4 =$$

GRID DECIMALS - III (CONTINUED)



$$.24 \div .3$$

I WILL MAKE A
RECTANGLE WITH
24 HUNDREDTHS,
ONE SIDE WILL
BE .3 LONG.

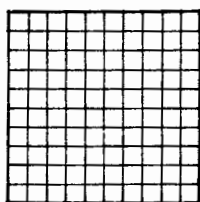
**NOW FOR
SOME
DIVISION!**



HOW LONG IS THE OTHER
SIDE OF THE RECTANGLE? _____

$$\text{so } .24 \div .3 = .8$$

TRY THESE USING A GRID:



$$.48 \div .6 =$$

$$.90 \div .9 =$$

$$.18 \div .3 =$$

$$.49 \div .7 =$$

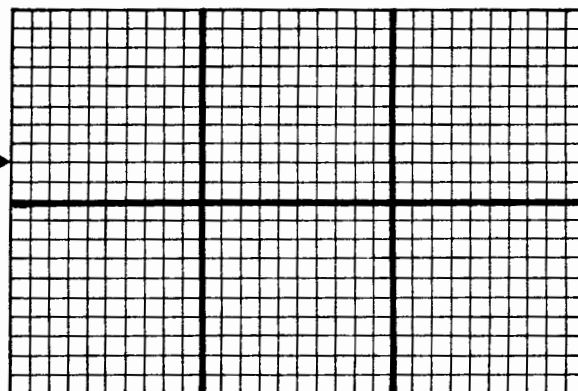
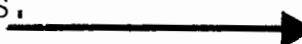
$$.72 \div .9 =$$

$$.25 \div .5 =$$

TRY THESE:

$$.48 \div 1.2 =$$

MAKE A RECTANGLE
WITH 48 HUNDREDTHS.
MAKE ONE SIDE 1.2



$$.36 \div .2 =$$

$$.65 \div 1.3 =$$

$$.36 \div 1.8 =$$

$$1.05 \div .7 =$$

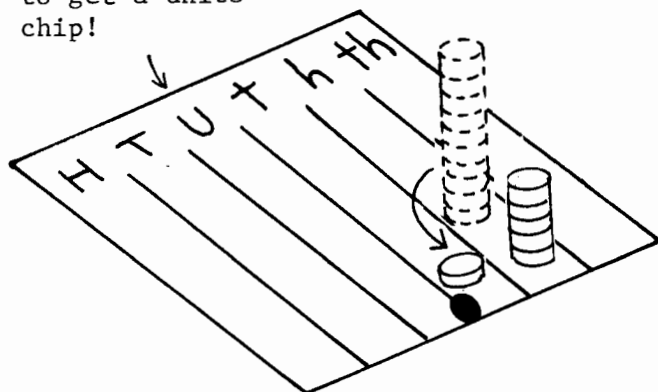
$$.65 \div .5 =$$

$$4.40 \div 2.2 =$$

ACTIVITY CARDS - DECIMAL ABACUS - III

Needed: 2-4 players
2 regular dice
Chips
Individual abacus boards

Be the first
to get a units'
chip!



Rules:

Each player rolls one die. Player with the largest number goes first.

First player rolls both dice and finds the product.

The product tells how many hundredths' chips can be placed on his board. Whenever ten or more chips are in one place, the player must trade them in for one chip in the next larger place.

Players take turns rolling the dice and adding chips to their boards.

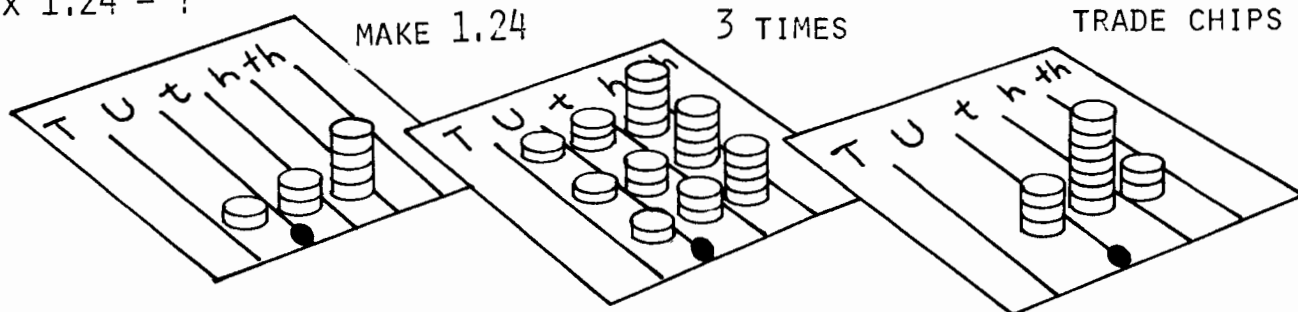
The first player to get a chip in the units' place is the winner.

Example: Player rolls
3 and 5. $3 \times 5 = 15$
This means 15 chips can be put in the hundredths' place or 1 tenth and 5 hundredths.

DA-III-1

DA-III-2

$$3 \times 1.24 = ?$$



TRY THESE:

$$6 \times .2 =$$

$$7 \times .3 =$$

$$3 \times .05 =$$

$$5 \times 1.34 =$$

$$3 \times 38.1 =$$

$$4 \times .18 =$$

$$6 \times .28 =$$

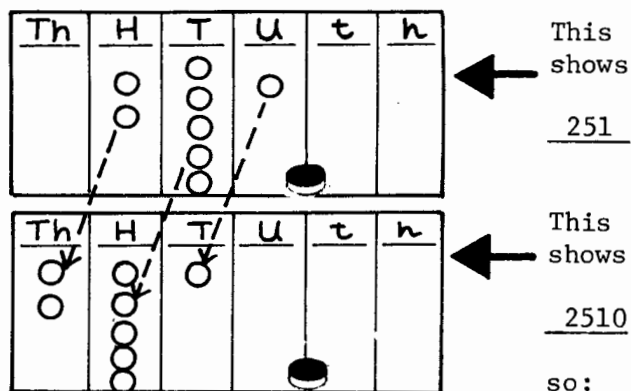
$$9 \times .092 =$$

$$10 \times 6.18 =$$

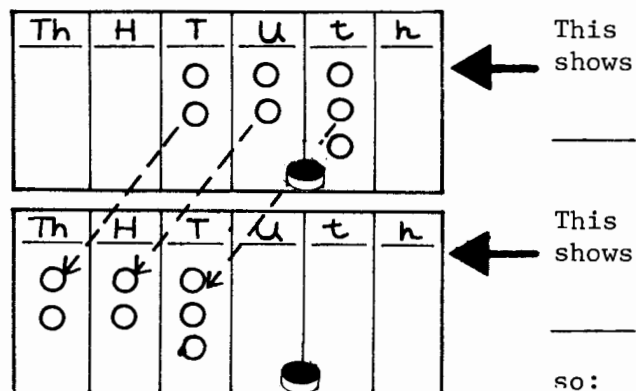
TRY MULTIPLYING SOME MORE DECIMALS BY TEN.

WHAT HAPPENS?

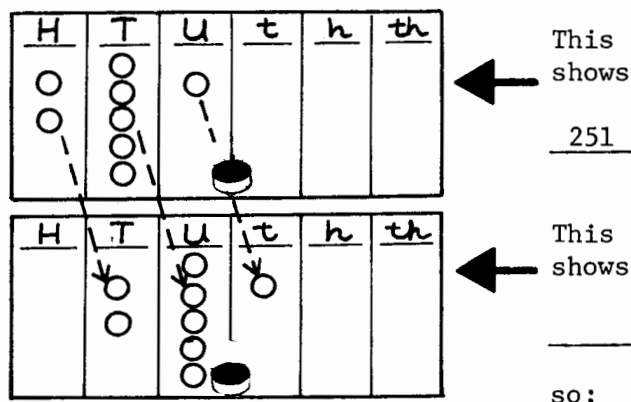
ACTIVITY CARDS - DECIMAL ABACUS - IV



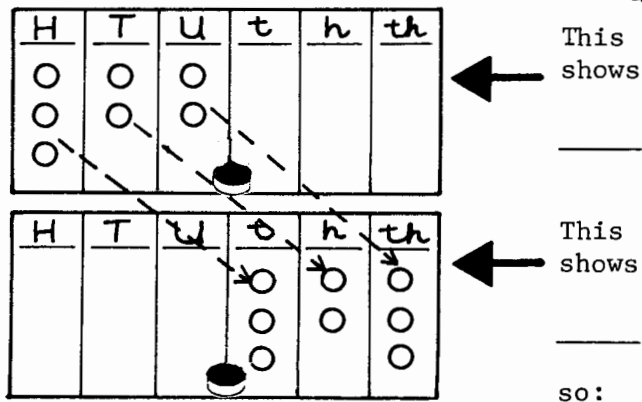
$$251 \times 10 = 2510$$



$$\text{_____} \times 100 = \text{_____}$$



$$251 \div 10 = \text{_____}$$



$$\text{_____} \div 1000 = \text{_____}$$

Complete the sentences and check using two abaci.

$$332 \times 10 = \text{_____}$$

$$41 \times 1000 = \text{_____}$$

$$910 \times 100 = \text{_____}$$

$$4.8 \times 100 = \text{_____}$$

$$50.6 \times 10 = \text{_____}$$

$$.08 \times 10 = \text{_____}$$

$$670 \div 10 = \text{_____}$$

$$842 \div 100 = \text{_____}$$

$$72.1 \div 10 = \text{_____}$$

$$1.78 \div 10 = \text{_____}$$

$$56 \div 100 = \text{_____}$$

$$.003 \times 100 = \text{_____}$$

Use two abaci to compare the numbers in each problem. Complete each sentence.

$$93 \times \text{_____} = 930$$

$$.046 \times \text{_____} = 460$$

$$.801 \div \text{_____} = .0801$$

$$4800 \div \text{_____} = 480$$

$$5100 \div \text{_____} = 510$$

$$.076 \times \text{_____} = 76$$

$$43.1 \div \text{_____} = .431$$

$$7.48 \times \text{_____} = 74.8$$

$$.86 \times \text{_____} = 860$$

DOTMAN, MY TEACHER **THE JOKER**
HAS ME ALL TIED UP WITH
**MULTIPLYING
DECIMALS....**

DOTMAN AND BOBBIN

FAR BENEATH GOTHIC CITY...

**MAYBE
WE CAN FIND AN
EASY WAY TO DO IT.**
LET'S LOOK AT MY OLD
NOTES. IT SAYS HERE THAT
ALL DECIMAL PROBLEMS CAN
BE MULTIPLIED BY CHANGING
THE DECIMALS TO EQUIVALENT
FRACTIONS. BUT LET'S EXAMINE
THESE PROBLEMS CLOSELY.
MAYBE WE WILL BE
ABLE TO SEE
SOMETHING
ELSE.

$$.4 \times .7 = \frac{4}{10} \times \frac{7}{10} = \frac{28}{100} = .28$$

THERE ARE
SOME DIGITS
UNDERLINED.

**LOOK AT
THIS!**

RIGHT!

**2 DIGITS ARE UNDERLINED IN THE
QUESTION.**

**2 DIGITS ARE UNDERLINED IN THE
ANSWER.**

**ALL THE UNDERLINED DIGITS ARE TO
THE RIGHT OF THE DECIMAL
POINT. $4 \times 7 = 28$**

**I SEE A
PATTERN.
DO YOU?**

LOOK AT THIS:

$$1.\underline{3} \times \underline{1}1 = \frac{13}{10} \times \frac{11}{100} = \frac{143}{1000} = .\underline{1}\underline{4}\underline{3}$$

HOW MANY DIGITS ARE UNDERLINED IN
THE QUESTION? ____ . HOW MANY

DIGITS ARE UNDERLINED IN THE
ANSWER? ____ . ON WHICH

SIDE OF THE DECIMAL POINT ARE
ALL THE UNDERLINED DIGITS? ____

HOW MUCH IS 13×11 ? ____ .

NOW YOU
ARE READY TO
LOOK AT MY
OLD DOT MATH
NOTES...

DOTMAN'S DECIMAL NOTES

In the following problems the digits have been written in disappearing ink. All digits left and right of the decimal point have been underlined.

Can you put the decimal points in the correct position, even though the numbers have disappeared? (Remember to count the digits to the RIGHT of the decimal point.)

EXAMPLES:

A) $\underline{.2} \times \underline{.6} = \underline{\quad} \underline{\quad}$

B) $\underline{3} \underline{7} \times \underline{.4} = \underline{\quad} \underline{\quad} \underline{\quad}$

C) $\underline{\quad} \underline{\quad} \times \underline{\quad} = \underline{\quad} \underline{\quad} \underline{\quad}$

Now, place the missing decimal point for these:

1) $\underline{\quad} \times \underline{\quad} = \underline{\quad} \underline{\quad}$

2) $\underline{\quad} \underline{\quad} \times \underline{\quad} = \underline{\quad} \underline{\quad} \underline{\quad}$

3) $\underline{\quad} \times \underline{\quad} = \underline{\quad} \underline{\quad}$

4) $\underline{\quad} \times \underline{\quad} = \underline{\quad} \underline{\quad}$

5) $\underline{\quad} \times \underline{\quad} = \underline{\quad} \underline{\quad}$

Try some of these. Then check by changing the decimals to fractions.

EXAMPLE: $\underline{.7} \times \underline{.5} = \underline{.35}$ CHECK: $\frac{7}{10} \times \frac{5}{10} = \frac{35}{100} = \underline{.35}$

6) $\underline{.17} \times \underline{.4} =$

7) $\underline{7.3} \times \underline{.2} =$

8) $\underline{.001} \times \underline{.02} =$

9) $\underline{.05} \times \underline{.02} =$

10) $\underline{3.1} \times \underline{1.2} =$

In the next problems you are to also find the missing decimal point, but it could be in any of the 3 parts of the problem.

Find all the missing decimal points.

11) $\underline{\quad} \times \underline{\quad} = \underline{\quad} \underline{\quad}$

15) $\underline{\quad} \times \underline{\quad} = \underline{\quad} \underline{\quad}$

12) $\underline{\quad} \times \underline{\quad} = \underline{\quad} \underline{\quad}$

16) $\underline{\quad} \times \underline{\quad} = \underline{\quad} \underline{\quad}$

13) $\underline{\quad} \times \underline{\quad} = \underline{\quad} \underline{\quad}$

17) $\underline{\quad} \times \underline{\quad} = \underline{\quad} \underline{\quad}$

14) $\underline{\quad} \times \underline{\quad} = \underline{\quad} \underline{\quad}$

DOTMAN'S DECIMAL NOTES (CON-TINUED)

PLACE THE DECIMAL POINT IN THE ANSWER TO EACH PROBLEM.

1) $.3 \times .7 = 21$ 2) $1.5 \times .7 = 105$

3) $.17 \times .8 = 136$ 4) $.11 \times 8. = 88$

5) $1.4 \times 3.8 = 532$ 6) $.2 \times .4 = 08$

7) $.3 \times .2 = 6$ 8) $.003 \times .04 = 12$

9) $12 \times .7 = 84$ 10) $21.6 \times .003 = 648$

$$\begin{array}{r} 11) \quad 3.7 \\ \quad .4 \\ \hline 148 \end{array}$$

$$\begin{array}{r} 12) \quad 1.56 \\ \quad \times 7.3 \\ \hline 468 \\ 1092 \\ \hline 11388 \end{array}$$

$$\begin{array}{r} 13) \quad .015 \\ \quad \times 1.3 \\ \hline 45 \\ 15 \\ \hline 195 \end{array}$$

MULTIPLY AND PLACE THE POINT IN THE ANSWER.

$$\begin{array}{r} 14) \quad 7.3 \\ \quad .07 \\ \hline \end{array}$$

$$\begin{array}{r} 15) \quad .039 \\ \quad \times 3.5 \\ \hline \end{array}$$

$$\begin{array}{r} 16) \quad 1.054 \\ \quad \times .003 \\ \hline \end{array}$$

17) $.15 \times .7 =$

18) $.0134 \times .002 =$

KEEP YOUR EYE ON THE DOT

EXAMINE THE FOLLOWING:

$$2 \overline{) .8} = 2 \overline{) 8 \text{ tenths}} = 4 \text{ tenths} = .4$$

$$3 \overline{) .15} = 3 \overline{) 15 \text{ hundredths}} = 5 \text{ hundredths} = .05$$

NOW LOOK AT IT THIS WAY:

$$\begin{array}{r} .4 \\ 2 \overline{) .8} \end{array}$$

What can you say about the position of the decimal point?

$$\begin{array}{r} .05 \\ 3 \overline{) .15} \end{array}$$

What can you say about the numbers in the division problems, when you ignore the decimal points?

What is $2 \overline{) 8}$? _____

What is $3 \overline{) 15}$? _____

TRY THESE:

(1) $2 \overline{) .6}$

(2) $3 \overline{) .12}$

(3) $6 \overline{) .336}$

(4) $7 \overline{) 1.4}$

(5) $8 \overline{) .4}$ (Hint: 4 tenths = 40 hundredths)

(6) $4 \overline{) .02}$

(7) $12 \overline{) .24}$

(8) $15 \overline{) 1.35}$

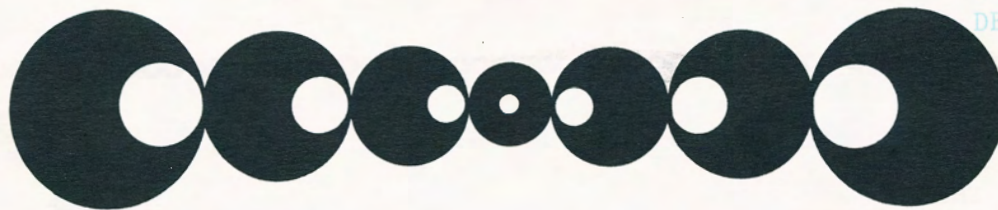
(9) $25 \overline{) 122.5}$

(10) $11 \overline{) .0121}$

How do you locate the decimal point in the answer when the divisor is a whole number?

LOTS OF DOTS

Placing the
Decimal Point
Division
DECIMALS



Is $6 \overline{)12}$ equivalent to $60 \overline{)120}$? *Yes, the quotients are the same.*

CIRCLE OTHER PROBLEMS THAT ARE EQUIVALENT TO $6 \overline{)12}$.

- (A) $600 \overline{)1200}$ (B) $6000 \overline{)12000}$ (C) $.6 \overline{)1.2}$
 (D) $.06 \overline{)1.2}$ (E) $.006 \overline{)0.12}$ (F) $.0006 \overline{)0.012}$

Did you circle every problem? Can you make more problems of your own that are equivalent to $6 \overline{)12}$?

WRITE 3 PROBLEMS WHICH ARE EQUIVALENT TO EACH OF THESE:

- (1) $3 \overline{)15}$ _____, _____, _____
 (2) $5 \overline{)110}$ _____, _____, _____
 (3) $.6 \overline{)1.2}$ _____, _____, _____
 (4) $.04 \overline{)24}$ _____, _____, _____

CHANGE EACH OF THE FOLLOWING PROBLEMS TO AN EQUIVALENT PROBLEM WITH A WHOLE NUMBER DIVISOR, THEN DIVIDE.

- (5) $.7 \overline{)14} = 7 \overline{)14} = .2$
 (6) $.03 \overline{)156} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
 (7) $.11 \overline{)1.21} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
 (8) $.08 \overline{)7.2} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
 (9) $.2 \overline{)4.0} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
 (10) $.006 \overline{)12.6} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
 (11) $1.2 \overline{)2.4} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
 (12) $1.5 \overline{)30.} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
 (13) $1.1 \overline{)0.132} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
 (14) $.012 \overline{)36.} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
 (15) $.09 \overline{)18} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$



TYPE: Paper & Pencil



LOOK AT THESE:

$\cdot 7$ $\sqrt{.35}$
 3 5

The decimal point moved ____ place(s) to the ____ in both the divisor and dividend. We now divide $7 \overline{)3.5}$.

HERE IS ANOTHER EXAMPLE:

$$\begin{array}{r} 540. \\ 16 \overline{) 8640.00} \end{array}$$

The decimal point moved ____ place(s) to the ____ in both the divisor and dividend. We now divide $3 \overline{1620}$.

THIS EXAMPLE HAS EVERYTHING BUT THE NUMBERS:

Was the decimal point in the answer placed correctly?

PLACE THE DECIMAL POINT IN THE ANSWER FOR EACH PROBLEM:

(1) \cdot \cdot

(11)

(2) .

(12) _____ . _____ | . _____

(3) \cdot \cdot

\cdot <u> </u>

(13) _____ • _____

(4) \cdot \cdot

(14) $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

(5) $\frac{\underline{\quad} \cdot \underline{\quad}}{\underline{\quad} \quad \underline{\quad} \cdot}$

(15) .

(6) . _____

(7)

(8) $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

(9) .

(10)

SOURCE: *Beefing Up Basic Skills, Decimals and Percents*

TYPE: Paper & Pencil/Puzzle



DIVISION



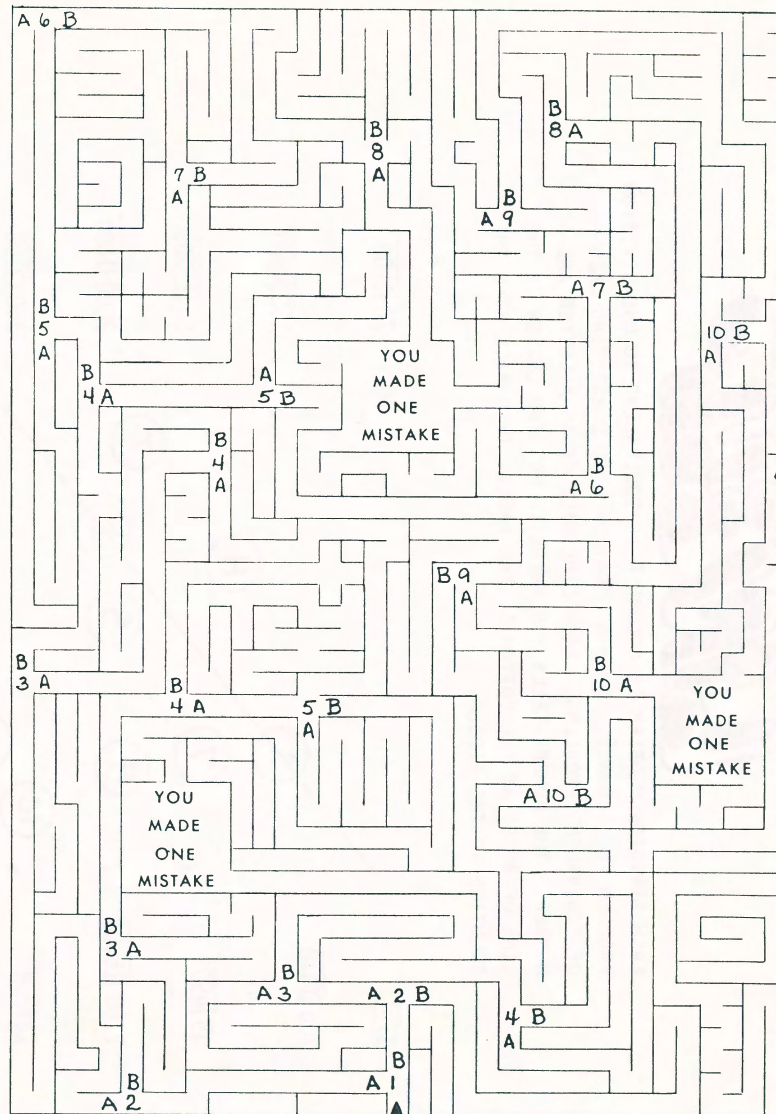
ISTRACTION

Enter the maze on the next page and draw a line to the number 1. Then work problem 1. If the correct answer is A, follow the path labeled A until you get to number 2. If the correct answer is B, follow the path labeled B until you get to number 2. Then work problem 2 and so on. If you come to a dead end you have made 2 mistakes.



Do not work these problems! Just choose the answer which is set up correctly for division.

	A	B
1) $1.5 \overline{) 19.5}$	$1.5 \overline{) 19.5}$	$1.5 \overline{) 19.50}$
2) $2.3 \overline{) 2.16}$	$2.3 \overline{) 2.16}$	$2.3 \overline{) 2.16}$
3) $0.12 \overline{) 0.0168}$	$0.12 \overline{) 0.0168}$	$0.12 \overline{) 0.0168}$
4) $0.015 \overline{) 0.255}$	$0.015 \overline{) 0.255}$	$0.015 \overline{) 0.255}$
5) $19 \overline{) 1.71}$	$19 \overline{) 1.71}$	$19 \overline{) 1.71}$
6) $0.45 \overline{) 202.5}$	$0.45 \overline{) 202.5}$	$0.45 \overline{) 202.50}$
7) $0.015 \overline{) 19.5}$	$0.015 \overline{) 19.5}$	$0.015 \overline{) 19.500}$
8) $2.7 \overline{) 864}$	$2.7 \overline{) 864.0}$	$27 \overline{) 864}$
9) $0.31 \overline{) 995.1}$	$0.31 \overline{) 995.1}$	$0.31 \overline{) 995.10}$
10) $0.05 \overline{) 2715}$	$0.05 \overline{) 2715.00}$	$0.05 \overline{) 2715.0}$



ENTER

END

Placing the
Decimal Point
Division
DECIMALS



DECIMAL LINEUP

Placing the
Decimal Point
Division
DECIMALS



EACH DIVISION PROBLEM IN COLUMN A IS EQUIVALENT TO ONE DIVISION PROBLEM IN COLUMN B. DRAW A STRAIGHT LINE CONNECTING EACH PAIR OF EQUIVALENT PROBLEMS. EACH LINE WILL CROSS A LETTER AND A NUMBER. THE NUMBER TELLS YOU WHERE TO PUT THE LETTER IN THE LINE OF BOXES AT THE BOTTOM OF THE PAGE. THIS WILL GIVE THE SOLUTION TO THE RIDDLE.

A		B
$3 \overline{)21}$	•	$70 \overline{)1400}$
$7.0 \overline{)14.0}$	•	$150 \overline{)4500}$
$15 \overline{)450}$	•	$6. \overline{)1200.}$
$2.00 \overline{)16.00}$	•	$2000 \overline{)1600}$
$12 \overline{)2.4}$	•	$300 \overline{)2100}$
$30 \overline{)21.0}$	•	$4. \overline{)24.4}$
$7 \overline{)140}$	•	$11. \overline{)6.6}$
$15 \overline{)45}$	•	$1500 \overline{)4500}$
$20 \overline{)16.0}$	•	$120 \overline{)24.}$
$12 \overline{)24}$	•	$120 \overline{)240}$
$.4 \overline{)2.44}$	•	$70 \overline{)140}$
$.06 \overline{)12.00}$	•	$200 \overline{)1600}$
$1.1 \overline{)66}$	•	$300 \overline{)210}$

Handwritten connections and letters/numbers:

- Line from $3 \overline{)21}$ to $70 \overline{)1400}$ (crosses 1)
- Line from $7.0 \overline{)14.0}$ to $150 \overline{)4500}$ (crosses 7)
- Line from $15 \overline{)450}$ to $6. \overline{)1200.}$ (crosses 4)
- Line from $2.00 \overline{)16.00}$ to $2000 \overline{)1600}$ (crosses 10)
- Line from $12 \overline{)2.4}$ to $300 \overline{)2100}$ (crosses 2)
- Line from $30 \overline{)21.0}$ to $4. \overline{)24.4}$ (crosses 13)
- Line from $7 \overline{)140}$ to $11. \overline{)6.6}$ (crosses 5)
- Line from $15 \overline{)45}$ to $1500 \overline{)4500}$ (crosses 12)
- Line from $20 \overline{)16.0}$ to $120 \overline{)24.}$ (crosses 9)
- Line from $12 \overline{)24}$ to $120 \overline{)240}$ (crosses 3)
- Line from $.4 \overline{)2.44}$ to $70 \overline{)140}$ (crosses 6)
- Line from $.06 \overline{)12.00}$ to $200 \overline{)1600}$ (crosses 8)
- Line from $1.1 \overline{)66}$ to $300 \overline{)210}$ (crosses 11)

Letters and numbers scattered in the center:

- Letters: H, S, I, T, N, E, M, K, A, E
- Numbers: 1, 7, 4, 10, 2, 13, 5, 12, 9, 11, 3, 6, 8

WHAT DID THE DECIMAL POINT SAY TO THE DOLLAR SIGN?

1	2	3	4	5	6	7	8	9	10	11	12	13
I	M	A	K	E	T	H	E	C	E	N	T	S

TYPE: Paper & Pencil/Puzzle



SEEING'S BELIEVING - I OR ARE YOU REASONABLE?

Approximation
Multiplication
DECIMALS



TRY THESE:

$$\begin{array}{r} 2.3 \\ \times 1 \\ \hline \end{array}$$

$$\begin{array}{r} .7 \\ \times 1 \\ \hline \end{array}$$

$$\begin{array}{r} 86.7 \\ \times 1 \\ \hline \end{array}$$

When we multiply by one, the product is _____ to the original number.

$$\begin{array}{r} 2.3 \\ \times 1.9 \\ \hline \end{array}$$

$$\begin{array}{r} .7 \\ \times 3 \\ \hline \end{array}$$

$$\begin{array}{r} 86.7 \\ \times 2.1 \\ \hline \end{array}$$

When we multiply by a number greater than one, the product is _____ than the original number.

$$\begin{array}{r} 2.3 \\ \times .5 \\ \hline \end{array}$$

$$\begin{array}{r} .7 \\ \times .32 \\ \hline \end{array}$$

$$\begin{array}{r} 86.7 \\ \times .9 \\ \hline \end{array}$$

When we multiply by a number less than one, the product is _____ than the original number.

Which of these answers are reasonable? Circle them.

$$\begin{array}{r} 12.7 \\ \times 2 \\ \hline 25.4 \end{array}$$

$$\begin{array}{r} .8 \\ \times .4 \\ \hline 1.02 \end{array}$$

$$\begin{array}{r} 3.71 \\ \times .5 \\ \hline 1.855 \end{array}$$

$$\begin{array}{r} 3.65 \\ \times 4 \\ \hline 14.60 \end{array}$$

$$\begin{array}{r} 1.6 \\ \times .7 \\ \hline .112 \end{array}$$

$$\begin{array}{r} 4.37 \\ \times 1.8 \\ \hline 7.866 \end{array}$$

GETTING CLOSER:

$$\begin{array}{r} 4.7 \\ \times 1.9 \sim (2) \\ \hline \end{array}$$

Is your answer about twice as big as 4.7?

$$\begin{array}{r} 3.6 \\ \times 3.1 \sim (3) \\ \hline \end{array}$$

Is your answer about three times as big as 3.6?

$$\begin{array}{r} 15.093 \sim (15) \\ \times 3 \\ \hline \end{array}$$

Is your answer about 3 x 15?

$$\begin{array}{r} 4.7 \\ \times .5 \sim (\frac{1}{2}) \\ \hline \end{array}$$

Is your answer about half as big as 4.7?

$$\begin{array}{r} 3.6 \\ \times .33 \sim (\frac{1}{3}) \\ \hline \end{array}$$

Is your answer about one-third as big as 3.6?

$$\begin{array}{r} 86.7 \\ \times .9 \sim (1) \\ \hline \end{array}$$

Is your answer close to 86.7?

Which of these answers are reasonable? Circle them.

$$\begin{array}{r} .62 \\ \times .8 \\ \hline 4.96 \end{array}$$

$$\begin{array}{r} 8.09 \\ \times .7 \\ \hline 5.663 \end{array}$$

$$\begin{array}{r} 8.04 \\ \times 3.2 \\ \hline 2.5728 \end{array}$$

$$\begin{array}{r} 4.81 \\ \times 9.2 \\ \hline 44.252 \end{array}$$

$$\begin{array}{r} 6.02 \\ \times .1 \\ \hline 6.02 \end{array}$$

$$\begin{array}{r} 5.37 \\ \times .2 \\ \hline 1.074 \end{array}$$



SEEING'S BELIEVING - II

Approximation
Division
DECIMALS



TRY THESE:

$$1 \overline{)2.3}$$

$$1 \overline{).7}$$

$$1 \overline{)78.4}$$

When we divide by one,
the quotient is
_____ to the
original number.

$$.5 \overline{)2.5}$$

$$.7 \overline{).7}$$

$$.32 \overline{)78.4}$$

When we divide by a
number less than one,
the quotient is
_____ than the
original number.

$$5 \overline{)2.5}$$

$$1.4 \overline{).7}$$

$$3.5 \overline{)78.4}$$

When we divide by a
number greater than
one, the quotient is
_____ than the
original number.

Which of these answers are reasonable? Circle them.

$$\begin{array}{r} 74 \\ .2 \overline{)14.8} \end{array}$$

$$\begin{array}{r} 1.1 \\ 6 \overline{)0.66} \end{array}$$

$$\begin{array}{r} .312 \\ 2 \overline{)6.24} \end{array}$$

$$\begin{array}{r} 1.25 \\ 6.2 \overline{)7.75} \end{array}$$

$$\begin{array}{r} 66.9 \\ .5 \overline{)33.45} \end{array}$$

GETTING CLOSER:

Is your answer
about three
times as
big as
61.5?

$$\begin{array}{r} .3 \overline{)61.5} \\ \downarrow \left(\frac{1}{3} \right) \end{array}$$

Is your answer
about
twice as
big as
12.8?

$$\begin{array}{r} .5 \overline{)12.8} \\ \downarrow \left(\frac{1}{2} \right) \end{array}$$

Is your answer
about four
times as
big as
18.5?

$$\begin{array}{r} .25 \overline{)18.5} \\ \downarrow \left(\frac{1}{4} \right) \end{array}$$

Is your
answer
about
 $\frac{31}{2}$?

$$\begin{array}{r} 2 \overline{)62.4} \\ (62) \end{array}$$

Is your
answer
about
 $\frac{.05}{7}$?

$$\begin{array}{r} 6.9 \overline{).354} \\ \downarrow (7) \quad \downarrow (.35) \end{array}$$

Is your
answer
about
 $\frac{4}{10}$?

$$\begin{array}{r} 9.5 \overline{)40.28} \end{array}$$

Which of these answers are reasonable? Circle them.

$$\begin{array}{r} 4.2 \\ .4 \overline{)16.8} \end{array}$$

$$\begin{array}{r} 4.37 \\ 25 \overline{)10.925} \end{array}$$

$$\begin{array}{r} .327 \\ 4.1 \overline{)13.407} \end{array}$$

$$\begin{array}{r} 72.9 \\ .6 \overline{)43.74} \end{array}$$



NEVER

HOLD A GRUDGE...

Approximation
Multiplication/
Division
DECIMALS



FOR EACH OF THE SIXTEEN PROBLEMS CIRCLE THE MOST REASONABLE ANSWER, (THERE ARE NO CORRECT ANSWERS GIVEN, SO YOU SHOULDN'T HAVE TO WORK THE PROBLEMS.) PLACE THE LETTER IN THE CIRCLED BOXES IN THE APPROPRIATE SPACE BELOW.

1	$\begin{array}{r} 45 \\ \times 6 \\ \hline \end{array}$	$\rightarrow 280_P$	2.7_U	24_T
2	$6 \overline{)13.8}$	$\rightarrow 24_C$	2.2_H	$.21_D$
3	$\begin{array}{r} 1.4 \\ \times 1.8 \\ \hline \end{array}$	$\rightarrow .32_A$	3_E	35_G
4	$8 \overline{)3.25}$	$\rightarrow 47_H$	$.4_J$	4.6_Y
5	$\begin{array}{r} 29.3 \\ \times 4 \\ \hline \end{array}$	$\rightarrow 120_S$	100.2_K	9_T
6	$83 \overline{)138.6}$	$\rightarrow 1083_Y$	16_H	25_O
7	$\begin{array}{r} 5.08 \\ \times 2.4 \\ \hline \end{array}$	$\rightarrow 13_E$	121_B	1.4_C
8	$7 \overline{)56.2}$	$\rightarrow 80_P$	$.803_S$	8.1_D
9	$\begin{array}{r} 35.6 \\ \times 3.9 \\ \hline \end{array}$	$\rightarrow 14_X$	1452_N	143.2_T
10	$1.7 \overline{)51}$	$\rightarrow 29_E$	300_K	3_F

11	$\begin{array}{r} 9.27 \\ \times 6 \\ \hline \end{array}$	$\rightarrow 55.6_J$	5.6_R	$.562_Y$
12	$.33 \overline{)9}$	$\rightarrow .03_N$	27_R	2.7_Z
13	$\begin{array}{r} 75.3 \\ \times .94 \\ \hline \end{array}$	$\rightarrow 707_D$	7.078_C	71.5_I
14	$9 \overline{)8.13}$	$\rightarrow .9_B$	9_F	91_H
15	$\begin{array}{r} 58.2 \\ \times 1.1 \\ \hline \end{array}$	$\rightarrow 57_M$	62_L	640_S
16	$2.4 \overline{)42.3}$	$\rightarrow 201_P$	2.1_Q	20_Y



T H E Y
1 2 3 4

S H E D
5 6 7 8

T E R R I B L Y
9 10 11 12 13 14 15 16

PROBLEM NUMBERS





THE AMERICAN HOUSEWIFE

Multiplication
DECIMALS



A survey conducted by economists for a large bank revealed the worth of an average American housewife. Here are 12 of the tasks a housewife is called upon to perform daily. Calculate her earnings per week for each job. ROUND OFF ANSWERS TO THE NEAREST WHOLE CENT.

JOB	HOURS PER WEEK	RATE PER HOUR	VALUE PER WEEK
Nursemaid	44.5	\$2.00	
Dietitian	1.2	\$3.60	
Food Buyer	3.3	\$2.58	
Cook	13.1	\$3.48	
Dishwasher	6.2	\$2.00	
Housekeeper	17.5	\$3.30	
Laundress	5.9	\$2.88	
Seamstress	1.3	\$3.24	
Practical Nurse	.6	\$3.00	
Maintenance Man	1.7	\$3.42	
Gardener	2.3	\$2.58	
Chauffeur	2.0	\$3.12	
WHAT IS THE TOTAL EARNED IN ONE WEEK →			
WHAT WOULD BE HER YEARLY SALARY (52weeks=1year) →			

TYPE: Paper & Pencil/Transparency

SOURCE: Project R-3

Permission to use granted
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GIANT

ECONOMY SIZE

IS THE **GIANT ECONOMY SIZE** MORE ECONOMICAL?

Ordering
Division
J



COMPLETING THE TABLE BELOW WILL HELP YOU ANSWER THE QUESTION.

The students can collect the data at a local supermarket or the data may be provided for an in-class activity. You may wish to have a class poll their favorite cereals. Choose metric or standard units.

BREAKFAST CEREALS			
BRAND	SIZE OF PACKAGE	COST OF PACKAGE	
CORN FLAKES	18 oz. 510 grams	69¢	18 5.10
CORN FLAKES	12 oz. 340 grams	49¢	
CORN FLAKES	8 oz. 226 grams	38¢	
RICE KRISPIES	13 oz. 368 grams	79¢	
RICE KRISPIES	10 oz. 283 grams	64¢	
Cheerios	15 oz. 425 grams	85¢	
Cheerios	10 oz. 283 grams	61¢	
Wheaties	18 oz. 510 grams	85¢	
Wheaties	12 oz. 340 grams	61¢	
Special K	15 oz. 425 grams	97¢	
Special K	11 oz. 312 grams	84¢	
Shredded wheat	15 oz. 425 grams	66¢	
Shredded Wheat	10 oz. 284 grams	47¢	

WAS THE LARGER SIZE ALWAYS MORE ECONOMICAL? _____

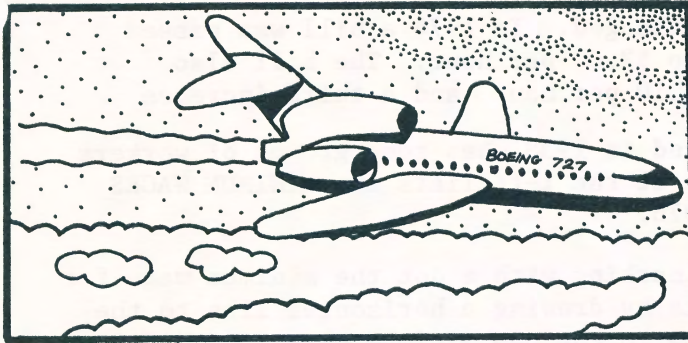
WHICH CEREAL IS THE BEST BUY? _____

CAN YOU FIND AN ITEM WHERE THE GIANT SIZE
IS NOT MORE ECONOMICAL? _____



SPEED & SOUND

Multiplication/Division
DECIMALS



Aircraft	Speed
Small plane	Mach .21
Propeller airliner	Mach .52
Jet airliner	Mach .9
Jet fighter	Mach 1.9
Supersonic transport	Mach 2.2
Rocket	Mach 5.75

At sea level (32°F temperature) sound travels at a speed of about 742 miles per hour. At higher altitudes, where the air is thinner and cooler, sound travels more slowly. The speeds of aircraft are often given according to the speed of sound. A speed of Mach 1 is the speed of sound, Mach .5 is half the speed of sound, Mach 2 is twice the speed of sound, Mach 2.2 is 2.2 times the speed of sound, and so on. Mach is the name of the scientist who made some important discoveries about sound.

For the chart below, use 670 m.p.h. for the speed of sound (this assumes that all aircraft will be flying at the same cruising altitude which is about 20,000 feet.)

AIRCRAFT	SPEED IN MILES PER HOUR
1. Small plane	
2. Propeller airliner	
3. Jet airliner	
4. Jet fighter	
5. Supersonic transport	
6. Rocket	

To find the MACH number for any aircraft speed, $\frac{\text{aircraft speed}}{\text{speed of sound}}$
For these problems, use 742 m.p.h. as speed-of-sound.

7. 1340 mph = MACH _____

8. 2010 mph = MACH _____

9. 1600 mph = MACH _____

10. 500 mph = MACH _____

TYPE: Paper & Pencil/Transparency

SOURCE: Project R-3

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The Minimum Wage

Multiplication
DECIMALS



The minimum wage law is frequently changed. In 1974 a bill was passed increasing the minimum wage from \$1.60 to \$2.00 per hour. The bill also provided a second increase in 1975 to \$2.10 per hour and a third increase to \$2.30 per hour in 1976.

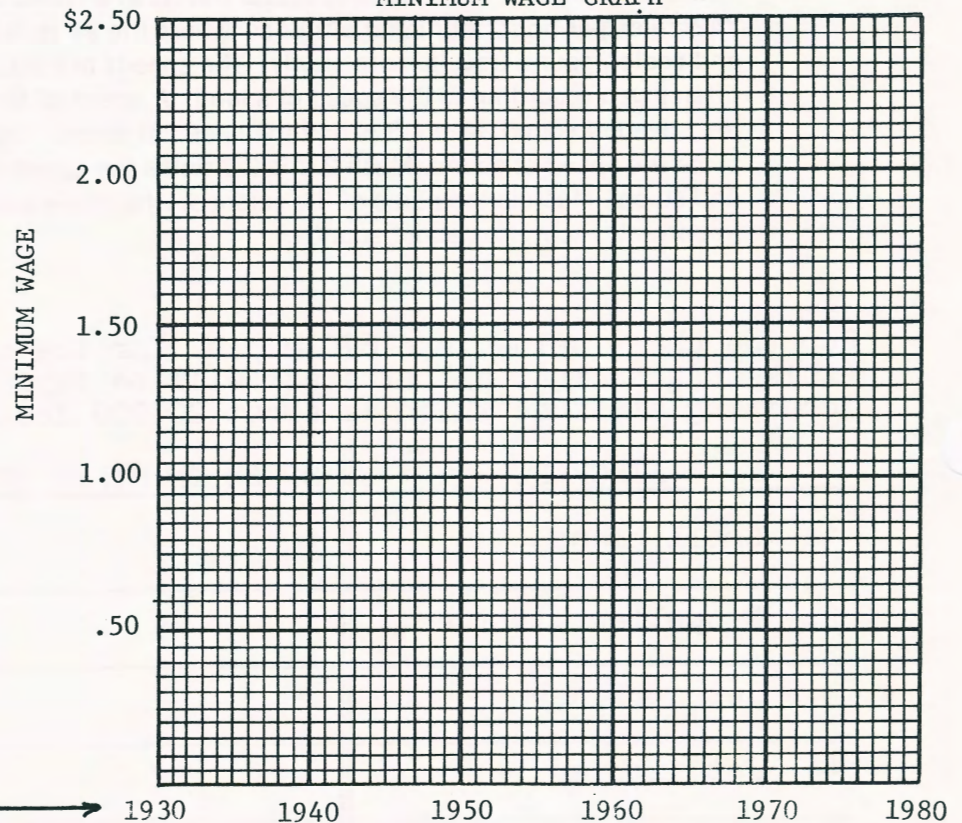
The first minimum wage law was passed in 1938 when some groups of workers were guaranteed 25¢ per hour. The table at the left lists the MINIMUM WAGES AS IT HAS INCREASED FROM 1938 THROUGH 1976.

Complete the graph on the right by marking with a dot the minimum wage for each year in the table. Connect the dots by drawing a horizontal line to the next year indicated, then a vertical line to the new minimum wage.

MINIMUM WAGE TABLE

YEAR	MINIMUM WAGE
1938	\$.25
1944	.40
1950	.75
1956	1.00
1961	1.15
1963	1.25
1967	1.40
1974	2.00
1975	2.10
1976	2.30

MINIMUM WAGE GRAPH



How much money would a minimum wage worker receive in:

(1) an eight-hour day

(2) a five-day week

(3) a 52-week year

1938 \$2.00

\$10.00

\$520

1944 a. _____

a. _____

a. _____

1950 b. _____

b. _____

b. _____

1956 c. _____

c. _____

c. _____

1961 d. _____

d. _____

d. _____

1963 e. _____

e. _____

e. _____

1967 f. _____

f. _____

f. _____

1974 g. _____

g. _____

g. _____

1975 h. _____

h. _____

h. _____

1976 i. _____

i. _____

i. _____



COMPLETE EACH TABLE

X	.5		1.1	
	.35			
.3		.9		
			.66	
2				4.6

X	.09			3
.61			.244	
8		7.2		
				.006
	1.35			

 ALWAYS DIVIDE BY THE TOP NUMBER.

÷	.5		1	
.9	1.8			
.3				.5
			60	
3		1.5		

÷	1.2	.06	4	
	5			2.5
7.2				
.96			.24	
	1.8			

EXAMPLE: $.9 \div .5 = 1.8$

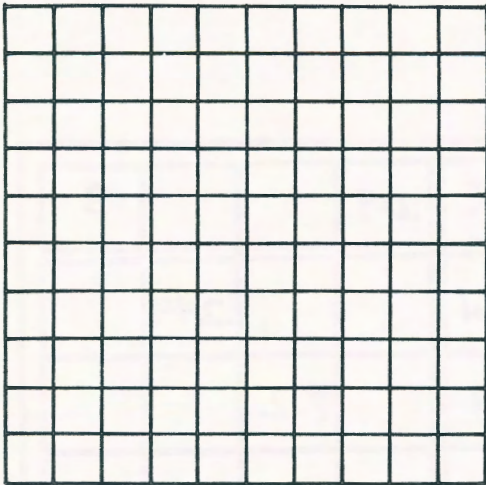


HANDS ON DECIMALS

Model
Relation to
Fractions
Division
DECIMALS



MATERIALS: 100 wooden cubes, tiles or similar objects.



Using the cubes, make a 10 X 10 square. Consider this one unit. Each single cube then represents

.01

Separate the cubes into 4 equal groups. How many cubes are in one of the groups? _____

Is this $\frac{1}{4}$ of the UNIT?

Do you see that $\frac{1}{4} = .25$?

Separate the cubes into 3 equal groups. How many are in each group? _____

Are there any cubes left over? _____

We can't find $\frac{1}{3}$ of the unit exactly like we could $\frac{1}{4}$, but we can say that $\frac{1}{3}$ is approximately equal to .33 or that $\frac{1}{3} \approx .33$.

Find an approximate decimal for each of the following: $\frac{1}{8} \approx$ _____ $\frac{1}{6} \approx$ _____ $\frac{1}{12} \approx$ _____

$\frac{1}{11} \approx$ _____ $\frac{7}{9} \approx$ _____ $\frac{5}{11} \approx$ _____

You may need to remind students that $\frac{1}{3}$ is one of three equal parts, so they should divide the squares into 3 equal parts and then cover one part.

In the last exercise you approximated some decimal equivalents for fractions like $\frac{1}{8}$.

Now try this:

DIVIDE 1 by 8, placing a decimal after the 1 and putting three zeros to the right of the decimal point.

$$8 \overline{) 1.000} = \underline{\hspace{2cm}}$$

How does this answer compare with your answer for $\frac{1}{8}$ in the last exercise?

Try some more - compare your answers with the approximations.

$$\frac{1}{12}; \quad 12 \overline{) 1.000}$$

$$\frac{7}{9}; \quad 9 \overline{) 7.000}$$

Change each of the following fractions to decimal approximations by dividing.

$$\frac{2}{3} \approx \underline{\hspace{2cm}} \quad \frac{5}{6} \approx \underline{\hspace{2cm}} \quad \frac{1}{9} \approx \underline{\hspace{2cm}}$$

$$\frac{3}{7} \approx \underline{\hspace{2cm}} \quad \frac{7}{8} \approx \underline{\hspace{2cm}} \quad \frac{7}{12} \approx \underline{\hspace{2cm}}$$

TYPE: Manipulative

DECIMAL NIDDY GRIDDY I

Relation to
Fractions
Division
DECIMALS

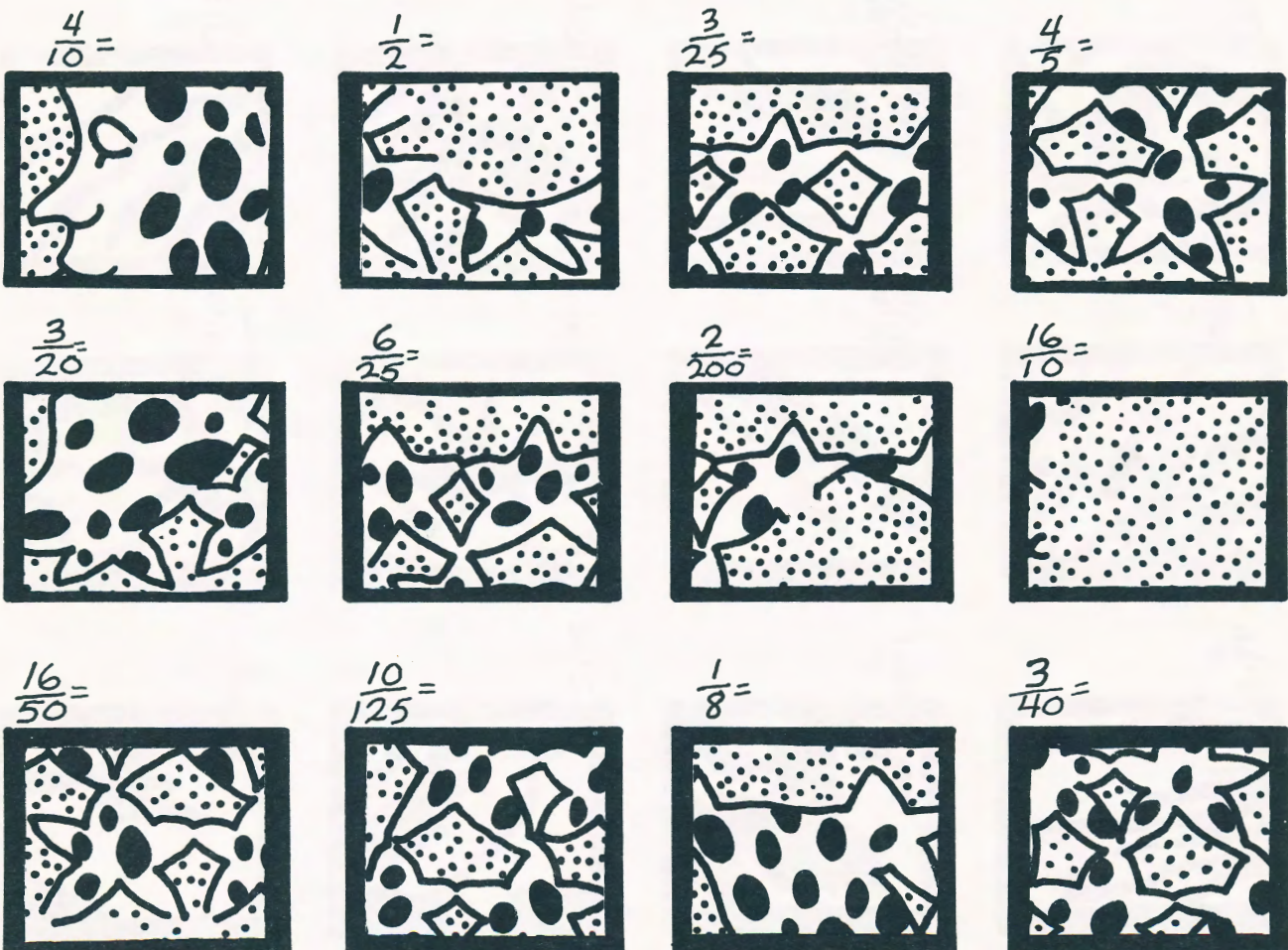


Solve each problem.

Cut out one piece at a time and paste it in the box with the correct answer.

Cut inside the border of each piece.

.24	.01	.125	.12
.08	1.6	.40	.075
.32	.5	.15	.8



TYPE: Puzzle

For a more permanent puzzle laminate each piece, and be sure to record the fraction problem on the back.

DECIMAL NIDDY GRIDDY II

Relation to
Fractions
Division
DECIMALS



Solve each problem.

Cut out one piece at a time and paste it in the box with the correct answer. Cut inside the border of each piece.

.72	1.6	.583	1.06
.6	.20	1.375	.83
.165	.875	.3	.8

$$\frac{3}{15} =$$



$$\frac{53}{50} =$$



$$\frac{1}{3} =$$



$$\frac{24}{30} =$$



$$1\frac{3}{5} =$$



$$\frac{33}{200} =$$



$$\frac{7}{12} =$$



$$1\frac{3}{8} =$$



$$\frac{5}{6} =$$



$$\frac{7}{8} =$$



$$\frac{8}{11} =$$



$$\frac{2}{3} =$$



TYPE: Puzzle



CALCULATED CODES

Relation To
Fractions
Ordering
Division
DECIMALS

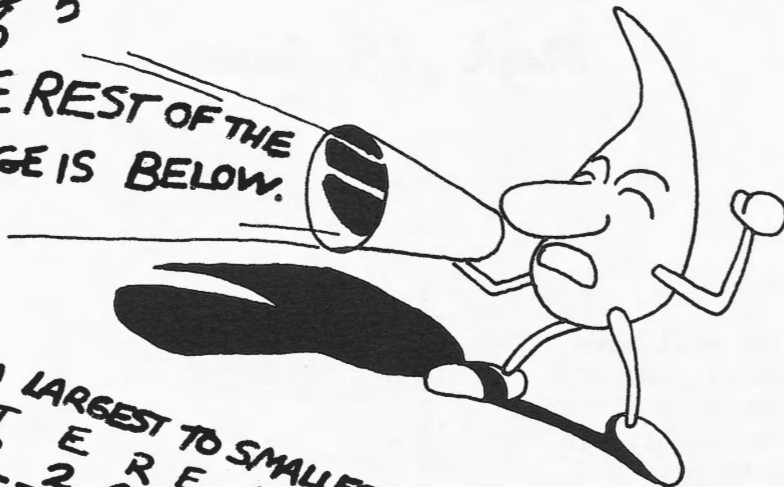


USE ME TO FIND THE DECIMAL VALUE OF EACH OF THE FRACTIONS, THEN CRACK THE CODE BY ORDERING FROM THE SMALLEST TO THE LARGEST, FROM LEFT TO RIGHT.



THE REST OF THE MESSAGE IS BELOW.

Y $\frac{4}{11}$ D $\frac{4}{13}$ A $\frac{2}{5}$ T $\frac{4}{5}$ E $\frac{5}{11}$ H $\frac{6}{11}$ G $\frac{5}{12}$ O $\frac{3}{8}$ A $\frac{4}{7}$ E $\frac{6}{13}$ U $\frac{5}{13}$



ORDER THESE FROM LARGEST TO SMALLEST, LEFT TO RIGHT.

L $\frac{2}{7}$ C $\frac{5}{16}$ C $\frac{3}{8}$ L $\frac{1}{9}$ R $\frac{3}{35}$ L $\frac{1}{3}$ A $\frac{5}{18}$ T $\frac{3}{16}$ E $\frac{2}{17}$ R $\frac{3}{17}$ E $\frac{1}{12}$ A $\frac{4}{11}$ H $\frac{1}{8}$ E $\frac{3}{34}$ H $\frac{1}{11}$ U $\frac{5}{17}$ O $\frac{2}{11}$ S $\frac{1}{6}$ P $\frac{3}{42}$

This could be done as a two-person activity. For the top part of the message one person finds the equivalent decimal using a calculator while the second person records. For the bottom part switch jobs.

MESSAGE: Do you agree that calculators help here?

TYPE: Paper & Pencil/Puzzle

MAGIC SQUARE DECIMALS

Relation to
Fractions
Division
DECIMALS



Change each fraction to its decimal equivalent. Place the answers in the grid. If your answers are correct each row, column and diagonal adds to the same number.

① .8	④ .1	⑦ .6
② .3	⑤ .5	⑧ .7
③ .4	⑥ .9	⑨ .2

Magic 1.5 SQUARE

- ① $\frac{8}{10} = \underline{\hspace{1cm}} .8$
- ② $\frac{3}{10} = \underline{\hspace{1cm}}$
- ③ $\frac{2}{5} = \underline{\hspace{1cm}}$
- ④ $\frac{1}{10} = \underline{\hspace{1cm}}$
- ⑤ $\frac{1}{2} = \underline{\hspace{1cm}}$
- ⑥ $\frac{9}{10} = \underline{\hspace{1cm}}$
- ⑦ $\frac{3}{5} = \underline{\hspace{1cm}}$
- ⑧ $\frac{7}{10} = \underline{\hspace{1cm}}$
- ⑨ $\frac{1}{5} = \underline{\hspace{1cm}}$

You will have to figure out where to place the numbers in this grid. First change each fraction to its decimal equivalent.

- $\frac{3}{20} = \underline{\hspace{1cm}}$
- $\frac{3}{4} = \underline{\hspace{1cm}}$
- $\frac{13}{20} = \underline{\hspace{1cm}}$
- $\frac{1}{20} = \underline{\hspace{1cm}}$
- $\frac{7}{20} = \underline{\hspace{1cm}}$
- $\frac{11}{20} = \underline{\hspace{1cm}}$
- $\frac{1}{4} = \underline{\hspace{1cm}}$
- $\frac{17}{20} = \underline{\hspace{1cm}}$
- $\frac{9}{20} = \underline{\hspace{1cm}}$

.75	.05	.55
.25	.45	.65
.35	.85	.15

Magic 1.35 SQUARE

When making similar magic squares, be sure the decimal equivalents form an arithmetic progression.

STOCKS - INVESTMENTS FUNDS

You will need the financial section of the Tuesday thru Saturday edition of a big city newspaper to do this assignment.

Scientists do their measurements almost exclusively in the decimal system. Business and industry, however, use both the fraction and the decimal systems. Therefore, the need for interchanging these number systems is greatest in these two fields. The most obvious everyday use of both fractions and decimals is in the stock market.

Find the page that lists the NEW YORK STOCK EXCHANGE.
How are the transactions listed: as decimals or as fractions _____

Find the page that lists the INVESTMENT FUNDS.
How are these transactions listed: as decimals or as fractions _____

NEW YORK STOCK EXCHANGE listing of companies	CLOSE PRICE as a fraction	CLOSE PRICE converted to a decimal with 2 place accuracy.
Chrysler		
Lucky Stores		
Pacific Gas + Electric		
Xerox		

INVESTMENT FUNDS listing of companies	ASK PRICE as a decimal	ASK PRICE rounded off to tenths..	ASK PRICE converted to a reduced fraction
Franklin Group (Growth)			
Mass. Financial (M.I.T.)			
Dreyfus Group (dreyfus)			
Value Line Fund (Income)			

RESEARCH QUESTION: Why are New York Stock Exchange listings given as fractions, while the Investment Funds are given as decimals?

INVESTMENT FUNDS

Private companies need lots of money to compete and grow. When they need more than they can borrow from banks, the companies sell SHARES to the public. People who want to buy these shares can do it in two main ways: (1) buy them on their own at a stock exchange (2) have an investment company buy them for you. Your job is to find out a few things about INVESTMENT COMPANIES. Use a newspaper for this activity. Turn to the business section of the paper and locate

INVESTMENT FUNDS

Date of paper: _____ Name of paper: _____

You buy 100 shares of Allstate.

According to the paper, the ask price (buy) is _____ per share.
Therefore, the total price for 100 shares is _____

You already own 100 shares of American Express (Invest.)

According to the paper, the bid price (sell) is _____ per share.
If you sell, how much money will you have: _____

You buy 25 shares of Dreyfus (Dreyf)

The bid price is _____ per share.
The total buy price is _____

You already own 75 shares of Fidelity (Trend).

The ask price is _____ per share.
If you sell, how much money will you have: _____

You buy 150 shares of Keystone Funds (Cus B4)

The bid price is _____ per share.
The total buy price is _____

From any source that will help you (parent, librarian, teacher) get help to answer these questions.

---What does an Investment Fund do with your money after you buy their shares: _____

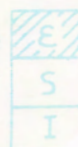
---When you buy shares in an Investment Fund, are you taking more of a risk than when you buy shares in a private company: _____





9-time

Patterns
Relation to
Fractions
Division
DECIMALS



To avoid overwhelming the students introduce this activity at the chalkboard.

MULTIPLY:

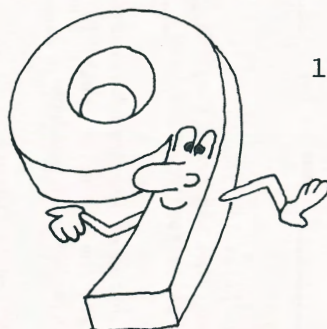
$$\begin{array}{r} 12,345,679 \\ \times 81 \\ \hline 999,999,999 \end{array}$$

$$\begin{array}{r} 76,923 \\ \times 13 \\ \hline 999,999 \end{array}$$

$$\begin{array}{r} 588,235,294,117,647 \\ \times 17 \\ \hline 9,999,999,999,999,999 \end{array}$$

$$\begin{array}{r} 142,857 \\ \times 7 \\ \hline 999,999 \end{array}$$

$$\begin{array}{l} 142,857 \times 6 = 857,142 \\ \times 5 = 714,285 \\ \times 4 = 571,428 \\ \times 3 = 428,571 \\ \times 2 = 285,714 \\ \times 8 = 1,142,856 \\ \times 9 = 1,285,713 \end{array}$$



$$\begin{array}{l} 12,345,679 \times 9 = 111,111,111 \\ \times 18 = 222,222,222 \\ \times 27 = \\ \times 36 = \\ \cdot \\ \cdot \\ \cdot \\ \times 81 = \end{array}$$

$$\begin{array}{l} 76,923 \times 3 = 230,769 \\ \times 9 = 692,307 \\ \times 12 = 923,076 \end{array}$$

CHANGE TO A DECIMAL

$$\frac{1}{7} = .142857$$

$$\frac{2}{7} = .285714$$

$$\frac{3}{7} = .428571$$

$$\frac{4}{7} =$$

$$\frac{5}{7} =$$

$$\frac{6}{7} =$$

$$\frac{8}{7} =$$

$$\frac{9}{7} =$$

$$\frac{1}{9} = .111$$

$$\frac{2}{9} = .222$$

$$\frac{3}{9} = .333$$

$$\frac{4}{9} =$$

$$\frac{5}{9} =$$

$$\frac{6}{9} =$$

$$\frac{7}{9} =$$

$$\frac{8}{9} =$$

$$\frac{1}{13} = .076923$$

$$\frac{3}{13} = .230769$$

$$\frac{9}{13} =$$

$$\frac{12}{13} =$$

$$\frac{1}{81} = .012345679$$

And finally: $\frac{1}{17} =$

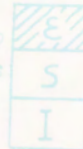
Do you see any connections between the top and bottom of the page?

Use a calculator to help you explore your ideas.

INCREDIBLE EQUALITIES

TEACHER DIRECTED ACTIVITY

Relation to
Fractions
Division
DECIMALS



Have the following statement written on the board when the students come to class.

.9999... is equal to 1

The activity could be reversed allowing the students to discover that .9999... = 1.

Allow student to debate the issue, then show them each of the following:

<p style="text-align: right;">ADDITION</p> $\begin{array}{r} \frac{1}{3} = .3333\ldots \\ \frac{1}{3} = .3333\ldots \\ + \frac{1}{3} = .3333\ldots \\ \hline \frac{3}{3} = .9999\ldots \end{array}$ <p style="text-align: right;">But $\frac{3}{3} = 1$</p>	<p style="text-align: right;">MULTIPLICATION</p> $\begin{array}{l} \frac{1}{9} = .1111\ldots \\ 9 \times \frac{1}{9} = \frac{9}{9} \\ 9 \times .1111\ldots = .9999\ldots \\ \text{But } \frac{9}{9} = 1 \end{array}$
<p style="text-align: right;">PATTERN</p> $\begin{array}{l} \frac{1}{9} = .1111\ldots \\ \frac{2}{9} = .2222\ldots \\ \frac{3}{9} = .3333\ldots \\ \frac{4}{9} = \\ \frac{5}{9} = \\ \frac{6}{9} = \\ \frac{7}{9} = \\ \frac{8}{9} = \\ \frac{9}{9} = \end{array}$ <p style="text-align: right;">Have students continue the pattern.</p> <p style="text-align: right;">But $\frac{9}{9} = 1$</p>	<p style="text-align: right;">DIVISION</p> $\begin{array}{l} \frac{9}{9} = 1, \\ \text{and } \frac{9}{9} \text{ means} \\ 9 \div 9 \end{array}$ $\begin{array}{r} .9999 \\ 9 \overline{) 9.0000} \\ \underline{81} \\ 90 \\ \underline{81} \\ 90 \\ \underline{81} \\ 90 \end{array}$ <p style="text-align: right;">Before trying this, be sure students will accept remainders equal to the divisor.</p>

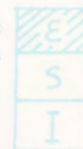
TYPE: Activity/Bulletin Board

This page can be used as motivation before students are presented the formal algorithms for changing repeating decimals to fractions. See *Repeating Decimals to Fractions*.

REPEATING DECIMALS TO FRACTIONS

TEACHER PAGE

Relation to
Fractions
Division
DECIMALS



Below are given several examples illustrating the algorithm for changing repeating decimals to fractions.

.9999...
Let $N = .9999\ldots$
THEN



(Multiply by 10 because 1 digit repeats)

$$\begin{array}{r} 10N = 9.9999\ldots \\ - N = .9999\ldots \\ \hline 9N = 9 \end{array} \quad \text{(subtract)}$$

$$N = \frac{9}{9} = 1$$

.4747
Let $N = .4747$
THEN



(Multiply by 100 because 2 digits repeat)

$$\begin{array}{r} 100N = 47.4747 \\ - N = .4747 \\ \hline 99N = 47 \end{array}$$

$$N = \frac{47}{99}$$

.12525
Let $N = .12525$
THEN $10N = 1.2525$
(Repeating part now
next to decimal point)
AND



(Multiply by 100 - 2 digits repeat)

$$\begin{array}{r} 1000N = 125.2525 \\ - 10N = 1.2525 \\ \hline 990N = \end{array}$$

$$N = \frac{124}{990} = \frac{62}{495}$$

.27345345
Let $N = .27345345$
THEN $100N = 27.345345$



(Multiply by 1000 - 3 digits repeat)

$$\begin{array}{r} 100000N = 27345.345 \\ 100N = 27.345 \\ \hline 99900N = 27318 \end{array}$$

$$N = \frac{27318}{99900} = \frac{4553}{16650}$$

TYPE: Paper & Pencil



SQUARED OFF

Squares
multiplication
DECIMALS

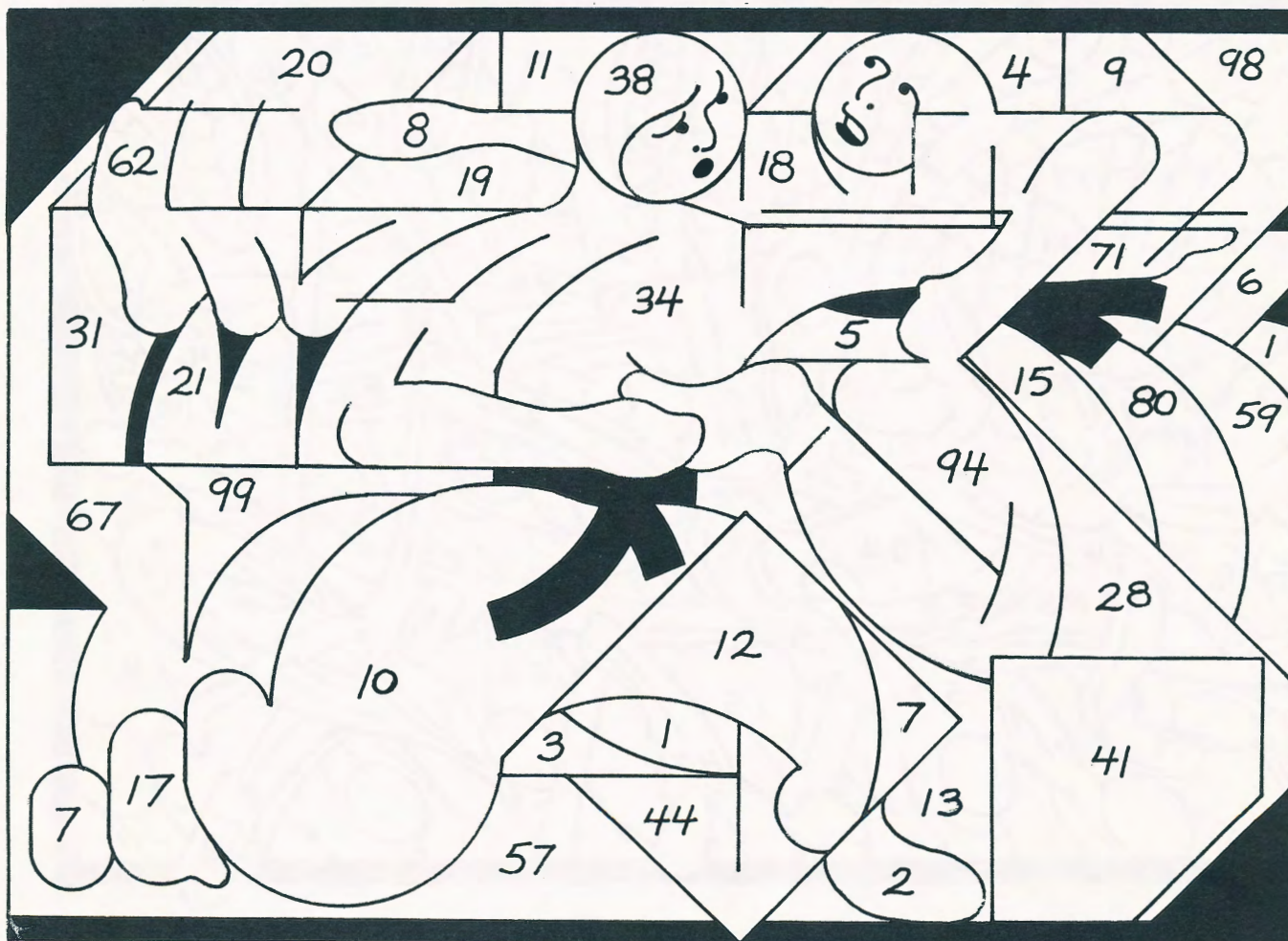
ϵ
5
2
1

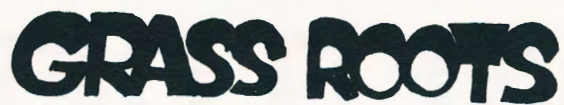
Circle the most reasonable answer for each of the twelve problems below.

$4.3^2 \approx$	18	21	22
$2.3^2 \approx$	3	5	7
$5.8^2 \approx$	31	34	28
$8.4^2 \approx$	67	80	71
$1.5^2 \approx$	1	2	4
$7.9^2 \approx$	62	57	59
$6.2^2 \approx$	38	41	44
$9.7^2 \approx$	94	98	99
$2.9^2 \approx$	6	7	8
$3.5^2 \approx$	11	12	15
$3.2^2 \approx$	9	10	13
$4.1^2 \approx$	20	19	17

n	n^2
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100

Shade in the areas containing your circled answers in the picture below.





$\sqrt{19} \approx 4.1$	5.1	4.4
$\sqrt{39} \approx 6.6$	6.8	6.2
$\sqrt{62} \approx 7.5$	7.9	8.2
$\sqrt{70} \approx 8.1$	8.4	8.7
$\sqrt{27} \approx 5.2$	5.0	4.8
$\sqrt{83} \approx 9.3$	9.6	9.1

$\sqrt{78} \approx 8.3$	8.5	8.8
$\sqrt{13} \approx 3.6$	3.9	3.3
$\sqrt{90} \approx 9.5$	9.2	9.7
$\sqrt{48} \approx 6.5$	6.9	6.7
$\sqrt{97} \approx 9.3$	9.4	9.8
$\sqrt{55} \approx 7.1$	7.4	7.7



SQUARE ROOTS OR GETTING THE MESSAGE

Square Roots
Multiplication
DECIMALS



OBSERVE: Students will need a Square Roots Table.

$$\sqrt{3700} \approx 60.83 \quad \sqrt{370} \approx 19.24 \quad \sqrt{37} \approx 6.083 \quad \sqrt{3.7} \approx 1.924 \quad \sqrt{.37} \approx .6083$$

How could you have used the table to do these exercises?

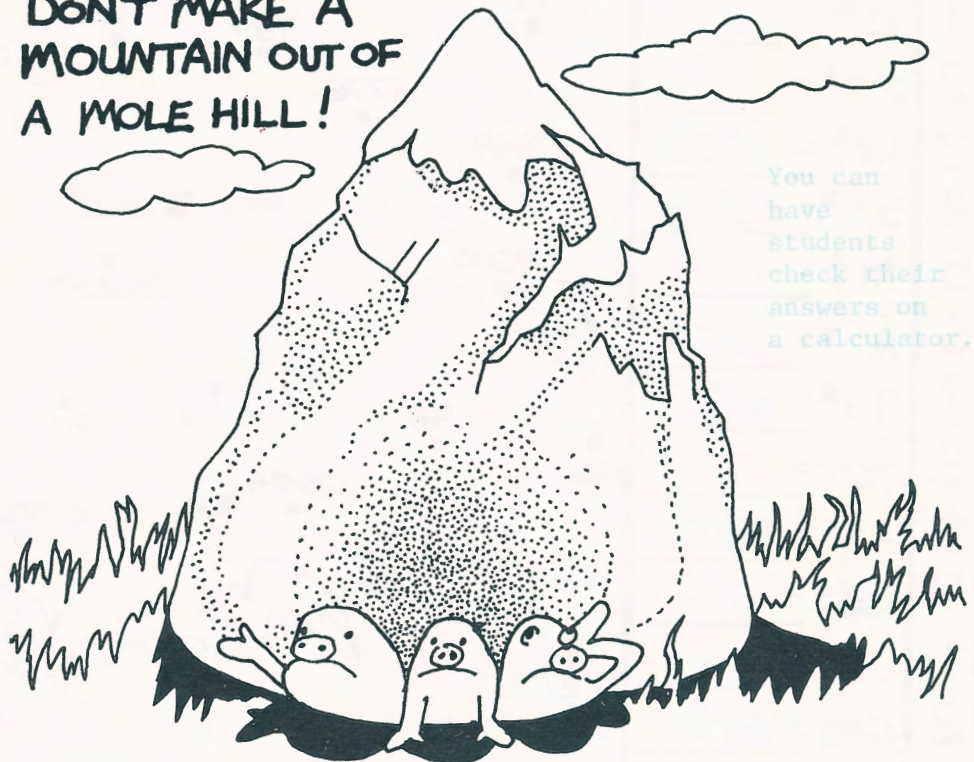
Use the table to find approximate answers for:

$$\begin{array}{lllll} \sqrt{7900} \approx \underline{\hspace{1cm}} & \sqrt{790} \approx \underline{\hspace{1cm}} & \sqrt{79} \approx \underline{\hspace{1cm}} & \sqrt{7.9} \approx \underline{\hspace{1cm}} & \sqrt{.79} \approx \underline{\hspace{1cm}} \\ \sqrt{500} \approx \underline{\hspace{1cm}} & \sqrt{50} \approx \underline{\hspace{1cm}} & \sqrt{5} \approx \underline{\hspace{1cm}} & \sqrt{.5} \approx \underline{\hspace{1cm}} & \sqrt{.05} \approx \underline{\hspace{1cm}} \end{array}$$

Use the table to help you solve the fifteen problems below. Place the letter next to each problem in the appropriate spaces below to complete the message.

A	$\sqrt{2.8} \approx 1.673$
E	$\sqrt{28} \approx 5.292$
F	$\sqrt{280} \approx 16.73$
H	$\sqrt{47} \approx 6.856$
I	$\sqrt{4.7} \approx 2.168$
L	$\sqrt{4700} \approx 68.56$
M	$\sqrt{.36} = .6$
O	$\sqrt{3.6} \approx 1.897$
R	$\sqrt{360} \approx 18.97$
S	$\sqrt{810} \approx 28.46$
T	$\sqrt{8.1} \approx 2.846$
U	$\sqrt{81} = 9$
V	$\sqrt{.6} \approx .7746$
W	$\sqrt{6} \approx 2.449$
Y	$\sqrt{60} \approx 7.746$

**DON'T MAKE A
MOUNTAIN OUT OF
A MOLE HILL!**



You can
have
students
check their
answers on
a calculator.

THE MOLES WILL
HATE YOU FOREVER!

TYPE: Paper & Pencil/Puzzle



SQUARE ROOT GAME

TEACHER IDEA

Square Roots
Multiplication/Division
DECIMALS



The square root game can be used to give students a more comfortable feeling about square roots and can also greatly extend their ability to use the hand calculator.

PLAYERS: The class may play as individuals, or small groups may work together in teams.

READINESS: Demonstrate the "Divide and Average" method of finding a square root as outlined on the game sheet. IT IS SUGGESTED THAT ALL APPROXIMATIONS NOT EXCEED 3 DECIMAL PLACES OF ACCURACY. Students will probably need to see two or three examples. You may also wish to review the definitions of square and square root.

PROCEDURE: For each round students are asked to find the square root of a number between zero and one hundred.

Students try to guess a whole number which is close to the square root of the number, following the outlined procedure on the game sheet. (Use a calculator for the computations.) The work should be recorded on the game sheet.

The students now find the approximate square root of the number, following the outlined procedure on the game sheet. (Use a calculator for the computations.) The work should be recorded on the game sheet.

Approximations continue until the new approximation is the same as the previous approximation. (Be sure not to exceed 3 decimal places of accuracy.) This last approximation is the desired answer.

This ends the round. Score the round and begin again, using a new square root problem.

SCORING: The score for any round is the step number of the last approximation. The game consists of five to ten rounds.

WINNING: The winner is the player with the lowest score at the end of the game.

SUGGESTION: Place game sheets under clear plastic or laminate. Students record answers with felt tip pens - erase with damp cloth for next round.

EXTENSION: After students are familiar with the game, you may wish to let them use decimals for their first guess.

SQUARE ROOT GAME (CONTINUED)

SCORE SHEET

FIND THE SQUARE ROOT OF
the NUMBER: 5

WHOLE NUMBER
GUESS: 2

STEP

1

$[(\text{Number} \div \text{Guess}) + \text{Guess}] \div 2 = \text{First Approximation}$

$$[(\underline{5} \div \underline{2}) + \underline{2}] \div 2$$

$$[\underline{2.5} + \underline{2}] \div 2$$

$$[\underline{4.5}] \div 2 =$$

2.25

STEP

2

$[(\text{Number} \div \text{First A}) + \text{First A}] \div 2 = \text{Second Approximation}$

$$[(\underline{5} \div \underline{2.25}) + \underline{2.25}] \div 2 =$$

$$[\underline{2.222} + \underline{2.25}] \div 2 =$$

$$[\underline{4.472}] \div 2 =$$

2.236

STEP

3

$[(\text{N} \div \text{Second A}) + \text{Second A}] \div 2 = \text{Third Approximation}$

$$[(\underline{5} \div \underline{2.236}) + \underline{2.236}] \div 2 =$$

$$[\underline{2.236} + \underline{2.236}] \div 2 =$$

$$[\underline{4.472}] \div 2 =$$

2.236

STEP

4

$[(\text{N} \div \text{Third A}) + \text{Third A}] \div 2 = \text{Fourth Approximation}$

$$[(\underline{\quad} \div \underline{\quad}) + \underline{\quad}] \div 2$$

$$[\underline{\quad} + \underline{\quad}] \div 2$$

$$\div 2 =$$

STEP

5

$[(\text{N} \div \text{Fourth A}) + \text{Fourth A}] \div 2 = \text{Fifth Approximation}$

$$[(\underline{\quad} \div \underline{\quad}) + \underline{\quad}] \div 2$$

$$[\underline{\quad} + \underline{\quad}] \div 2$$

$$\div 2 =$$

STEP

6

$[(\text{N} \div \text{Fifth A}) + \text{Fifth A}] \div 2 = \text{Sixth Approximation}$

$$[(\underline{\quad} \div \underline{\quad}) + \underline{\quad}] \div 2$$

$$[\underline{\quad} + \underline{\quad}] \div 2$$

$$\div 2 =$$

ROUND	SCORE
1	3
2	
3	
4	
5	
6	
7	
8	
9	
10	

EXTENSION: After playing the game have the students make their own square root table.

NUMBER SENSE AND ARITHMETIC SKILLS

Placement Guide for Tabbed Dividers

Tab Label	Follows page:
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CONTENTS

Decimals Commentary: Mixed Operations (pages 736-741)

<u>TITLE</u>	<u>PAGE</u>	<u>TOPIC</u>	<u>TYPE</u>
The Diamond Game	742	Computation	Paper and pencil Game
World Track Records	743	Computation	Paper and pencil Transparency
"The Answer"	744	Computation	Paper and pencil
Nothing is Forever	745	Patterns	Paper and pencil Transparency
Approx-Appraisals	746	Approximation Word problems	Paper and pencil Transparency
Operation Please!	747	Word problems	Paper and pencil Transparency
Wordless Problems	748	Word problems	Paper and pencil Transparency
Simplify the Numbers	749	Simplifying problems Word problems	Paper and pencil
That's Just About the Size of It!	750	Word problems	Paper and pencil
Getting 'Round to Calculating	751	Approximation	Paper and pencil
Up and Down with the Calculator	752	Rounding	Activity
One-liners	754	Mixed operations	Paper and pencil
Shopping with a Newspaper	755	Mixed operations	Activity

DECIMALS: MIXED OPERATIONS

Once the basic understanding and skills of the four arithmetic operations of decimals are covered, students are ready for extended lessons involving mixed operations. More activities stressing applications, problem solving, use of calculators, approximation and verbalization can be developed. These activities can build skills in computation with decimals while fulfilling their important tasks in the areas mentioned.

Applications, estimation/approximation, problem solving and calculators are teaching emphases throughout the resources. A rationale and table of contents for each of these emphases is found in the appendices. Verbalization can help students understand and remember mathematical ideas and concepts. Some additional remarks about applications, problem solving and verbalization are given below.



APPLICATIONS

Decimal notation is used extensively in our society. We must be able to interpret the decimals we see in newspapers, business forms, or prices. With hand calculators readily available, computation with decimals can be accomplished quickly and accurately. Being free from cumbersome calculations, more time and emphasis can be placed on interpreting numbers and determining what operations to use in finding the solutions to problems. The classroom materials in this resource are keyed to applications (see their upper lefthand corners for the symbol of a globe). In addition you can search out ideas of your own.

Keep your eyes open for situations in your life or in the student's world which could make good problems for the classroom. A problem that is based on a real situation encountered by a teacher or a student will usually generate interest.

Starting Points

a) One creative teacher brought a menu from a restaurant. He said, "I have a problem. When I take my family out to eat, it costs too much." He told the class what each member in his family had ordered the last time they went to the restaurant.

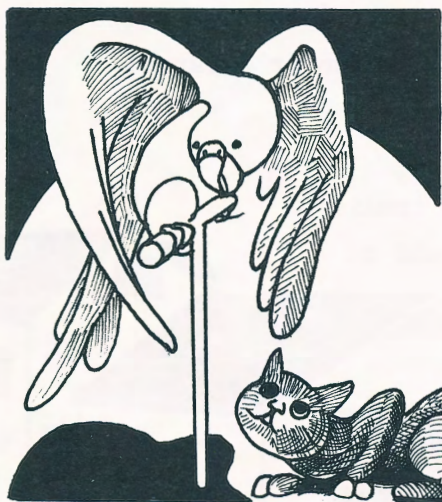
Chez Crepe			
steak		4	98
shrimp		3	75
hamburger plate		2	50
chicken		2	75
milk	2		60
tea	2		50
TAX			
TOTAL			

The class was asked to determine the bill, tax and tip.* The teacher then asked the class to break into groups and determine how the family could eat at the restaurant and keep the total cost under \$12.00. They then decided to get sample menus from several places in the area and determine which eating place was the most economical. Questions of quality versus price were discussed. What about distance from the teacher's home? Atmosphere? Variety of foods? Was it less expensive to leave the children home and pay a babysitter? The above activity was a practical application, and it was interesting for the students since it involved a real problem for their teacher.



b) Are your students interested in track? What's the time of the fastest runner in your school for the 50-yd. dash? The 660? The mile? How do those times compare with the U.S. records? What is the difference in time for each record? If 50 yds. are run in 6.8 seconds, what would be a reasonable time for 50 metres?

c) Do your students plan social activities? How much would it cost to have a class party including snacks and prizes? What would the cost be for a birthday party including roller skating and treats? Would it be cheaper to go bowling? Have a home party? A kite flying contest? A bike hike? How can money be raised for such a social event?



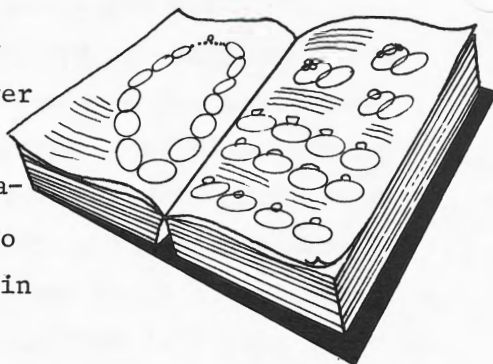
d) Do your students like pets? What is the cost of owning a pet? Purchase price, shots, food, veterinary, neutering, cages and miscellaneous costs can be considered. What pet would be the cheapest to care for? A turtle? Snake? Gerbil? Goldfish?



*These activities involve percent. If percent is not understood by the class, tax tables or tip tables could be used.

e) Do your students like to look at catalogues?

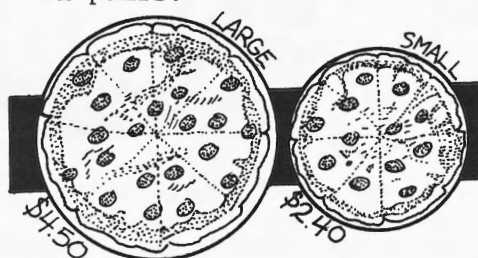
A collection of catalogues from sports stores, department stores, jewelry stores and bike stores should cover a variety of interests. Students can make out orders, determine shipping charges, compute the tax, if applicable, and fill out a check with the correct payment. To make this more interesting the assignment can be made in story form.



"You have just broken the city record for the 600-m run. The prize is a trophy and \$100 of merchandise from XXX Sports Company. Decide what you want and fill out this order form."

"You have just won a 2-week trip to Hawaii and all that you will need for the trip. You may select luggage, clothing and miscellaneous items for your trip from XXX Catalogue. Because you will be traveling by plane, your total packed luggage cannot exceed 30 kg (about 66 pounds).

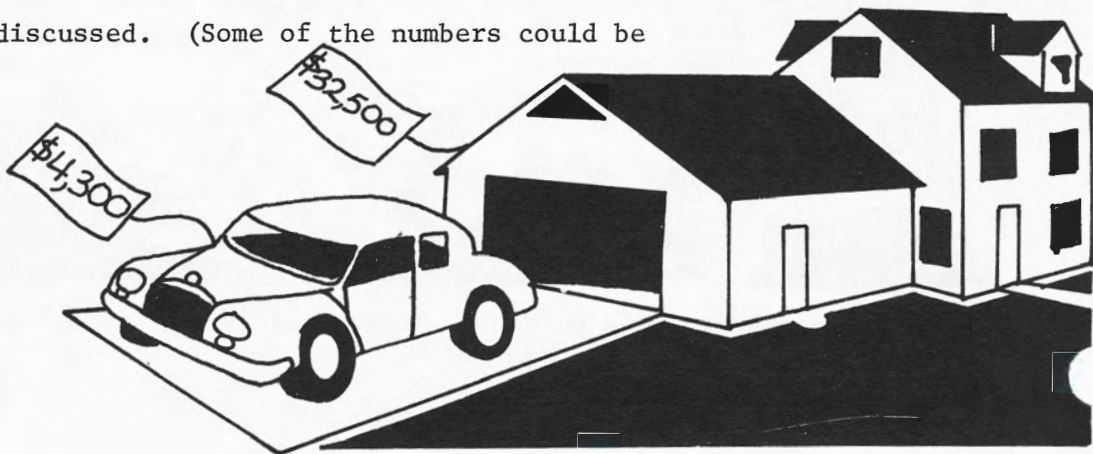
If you know your students well, story problems can be related to their interests. Four problems like those given above could be listed on the board and the students allowed to choose a problem from the list. The problems could be worked in pairs.



f) Are your students fond of pizza? How do the areas of different sized pizzas compare to their prices? Which size pizza gives more pizza per dollar? If a group of students goes to a pizza parlor, is it better to order 3 medium or 2 large?

g) Are you thinking of buying a new car or house?

Students might be interested in helping you decide if you can afford it. Insurance, taxes, interest and credit ratings can be discussed. (Some of the numbers could be fictitious.)



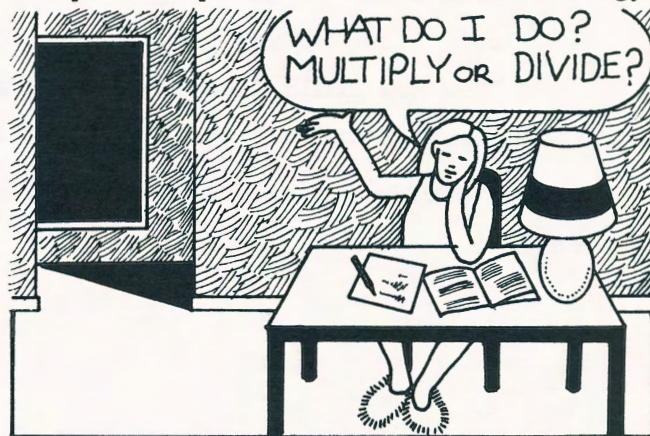


PROBLEM SOLVING

Problem solving is an important part of mathematics and of many other areas of life. Because of its importance it is a teaching emphasis throughout the resources. A general discussion of problem solving is contained in the overview book for the resources, and the light bulb symbol is used on the classroom materials to signal places where problem solving might be used.

There are many useful problem-solving strategies and heuristics. These strategies are discussed in the overview book. One of the most helpful strategies is SIMPLIFY THE PROBLEM. One way to simplify a problem is to substitute simpler numbers. Word problems involving decimals are a perfect place to teach this strategy.

When given a word problem with decimals, a student often "draws a blank." A common question is "Do I multiply or divide?" Substituting familiar whole numbers for decimals (simplifying the problem) can help students choose an appropriate operation. Students can be taught to try this strategy. Research



has suggested that problem-solving strategies are better learned when the focus is on the strategy, not on the correct answer to the problem. The student page *Simplify the Numbers* was written to emphasize a solution strategy, not the numerical answer. On this sheet students are asked to substitute familiar whole numbers (like 4 and 8 or 5 and 10) for the decimals and solve the whole number problem. They then use the corresponding decimals and the operation chosen for the whole number solution to set up (but not solve) the problem.

④ ← Substitutions → ⑧

Example: A rock weighs 24.712 kg. Its volume is 3610.9 cm³. How many kg does one cm³ of this rock weigh?

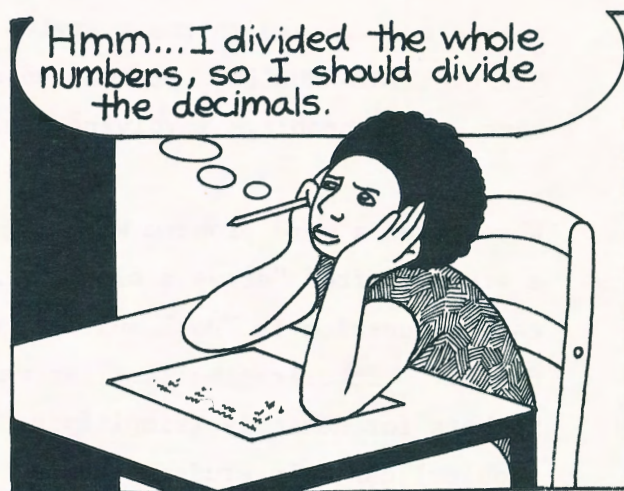
$$\begin{array}{l} \text{Whole} \\ \text{Number} \\ \text{Solution} \end{array} \left\{ \begin{array}{l} 4 \div 8 = \frac{1}{2} \end{array} \right.$$

$$\begin{array}{l} \text{Decimal} \\ \text{Set up} \end{array} \left\{ \begin{array}{l} 24.712 \div 3610.9 \\ \text{or} \\ 3610.9 \overline{)24.712} \end{array} \right.$$

Notice that the whole numbers substituted for the decimals need not be approximations of the decimals. Approximation as a helpful technique can be stressed at another time. The point here is to determine a useful operation using the two numbers. The approximations: 25 and 3611 might not help in determining the operation. After substituting the familiar numbers 4 and 8, a student might reason as follows:

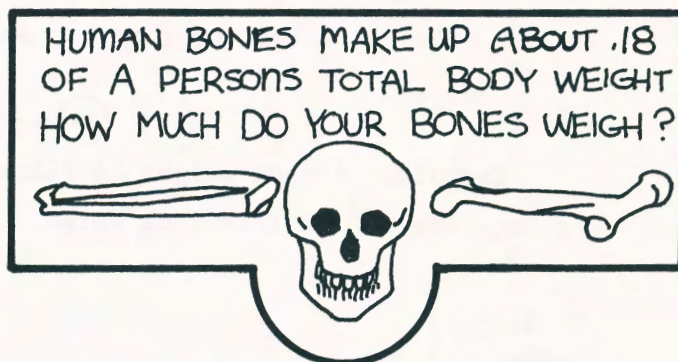
"If 8 cm^3 weigh 4 kg,
 then 2 cm^3 weigh 1 kg
 and 1 cm^3 weighs $\frac{1}{2}$ kg."

"Now how could I have gotten $\frac{1}{2}$ with the numbers 8 and 4? $4 \div 8 = \frac{1}{2}$, so I should divide the decimals in the same way."



It would be best to substitute simple fractions for most of the problems in *That's Just About the Size of It*. You might examine the problems on these pages to see why. When simpler numbers are substituted, it is important that the operation with the simpler numbers be the same as the operation with the original numbers.

Try substituting a whole number for the decimal in the problem at the right. Does the problem still convey the same meaning?



VERBALIZATION

The student page *Wordless Problems--Decimals* shows picture situations, and the students are asked to create and solve a problem for each picture. This encourages them to see relationships and to organize questions. The ability to write such a question is worthwhile. It is also helpful to have students verbalize the relationships they see and to give the question in words. Students could be asked to explain how they solved their problems and why that method makes sense. Students will write different problems; there is no one correct problem for these.

The student page *Operation Please* asks the student to identify the operation used to solve the problem. There is a danger here. As with whole numbers and fractions, decimal problems can be solved in several "correct" ways. Problem 2 at the right can be solved by counting tenths instead of adding. Problem 7 could be solved by multiplying 12.5 by various numbers instead of dividing 5262.5 by 12.5.

$$421 \left\{ \begin{array}{l} 100 \times 12.5 = 1250 \\ 400 \times 12.5 = 5000 \\ 20 \times 12.5 = 250 \\ 1 \times 12.5 = 12.5 \end{array} \right\} 5262.5$$

After students have solved a problem logically and correctly, it is helpful if they identify what was done. We must be careful not to expect them to solve the problems the way we would. The goal should be: "Decide how to solve the problem, then solve it," not "Decide which operation the teacher will accept, then solve it." Perhaps verbalization would help here. Students could describe how they solved the problem. They could be encouraged to use their own way.

Problems from *Operation Please*

2. The odometer on an automobile read 9999.9 miles. What would it read .2 miles later?

This problem is solved by +, -, x, ÷.

The answer is: _____

7. Boxes of canned tomatoes were sent from Sacramento to Los Angeles. The total weight was 5262.5 kg. If each box weighed 12.5 kg, how many boxes were shipped?

This problem is solved by +, -, x, ÷.

The answer is: _____



THE DIAMOND GAME

Computation +, -, \times , \div
Mixed Operations
DECIMALS



$$\begin{array}{r} \bullet \\ \bullet \quad .4 \\ \bullet \quad \times .8 \quad \bullet \end{array}$$

$$\bullet \quad 9 \overline{) 11.8} \quad \bullet \quad 2 \overline{) .6} \quad \bullet$$

$$\bullet \quad \begin{array}{r} .2 \\ + .1 \\ \hline \end{array} \quad \bullet \quad \begin{array}{r} .9 \\ .4 \\ \hline \end{array} \quad \bullet \quad 4 \overline{) 5.2} \quad \bullet$$

$$\bullet \quad .7 \times .6 = \bullet \quad .3 + .5 = \bullet \quad 2.4 \div 6 = \bullet \quad .7 - .2 = \bullet$$

$$\bullet \quad 6 \overline{) 4.8} \quad \bullet \quad \begin{array}{r} .5 \\ \times .1 \\ \hline \end{array} \quad \bullet \quad \begin{array}{r} .8 \\ + .7 \\ \hline \end{array} \quad \bullet \quad 3 \overline{) .09} \quad \bullet \quad \begin{array}{r} .12 \\ - .07 \\ \hline \end{array} \quad \bullet$$

$$\bullet \quad 7 \overline{) .56} \quad \bullet \quad 5.5 \div 11 = \bullet \quad 3.2 + .7 = \bullet \quad 9.4 - 2.6 = \bullet \quad \begin{array}{r} .6 \\ \times .7 \\ \hline \end{array} \quad \bullet \quad 8 \overline{) .64} \quad \bullet$$

$$\bullet \quad \begin{array}{r} .06 \\ + .6 \\ \hline \end{array} \quad \bullet \quad \begin{array}{r} .32 \\ .25 \\ \hline \end{array} \quad \bullet \quad \begin{array}{r} .03 \\ \times .7 \\ \hline \end{array} \quad \bullet \quad 4 \overline{) .016} \quad \bullet \quad 4.23 \times 0 = \bullet$$

$$\bullet \quad .3 \times .3 = \bullet \quad 8 \div 8 = \bullet \quad .9 + .1 + .2 = \bullet \quad 5 \times .1 = \bullet$$

$$\bullet \quad 24.1 + .36 = \bullet \quad 15 \times .2 = \bullet \quad .75 - .2 = \bullet$$

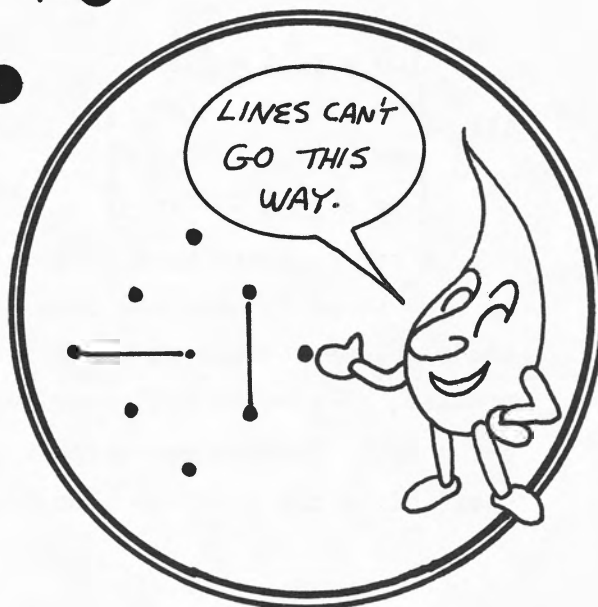
RULES:

2 OR MORE PLAYERS

- PLAYERS TAKE TURNS DRAWING LINES TO CONNECT TWO DOTS. LINES MAY NOT GO VERTICALLY OR HORIZONTALLY.
- THE PLAYER WHO DRAWS A LINE THAT COMPLETES A DIAMOND DOES THE PROBLEM IN THE DIAMOND. IF CORRECT, THE PLAYER PLACES HIS INITIAL IN THE DIAMOND. IF WRONG, THE NEXT PLAYER TRIES TO GET THE CORRECT ANSWER AND THE DIAMOND.
- THE PLAYER WITH THE MOST DIAMONDS AT THE END OF THE GAME WINS.

$$\bullet \quad .7 \div 1 = \bullet \quad .14 - .07 = \bullet$$

$$\bullet \quad .09 - .03 = \bullet$$



IDEA FROM: WYMOLAMP, Elementary Materials

Permission to use granted by Fremont County Unified School District #25



WORLD TRACK RECORDS

-RUNNING EVENTS-

Computation +, -, ×, ÷
Mixed Operations
DECIMALS



For world records in
the field events see
World Track Records.
FRACTIONS: Addition/
Subtraction.

WORLD TRACK RECORDS*		
	MEN	WOMEN
100-YD. DASH	9.1 SEC.	10.0 SEC.
220	20.0 (TURN)	22.6
440(QUARTER MILE)	44.5	52.2
880 (HALF MILE)	1 MIN. 44.6 SEC.	2:02.0
MILE	3:51.1	4:29.5
6-MILE	26:47.0	



* Sports Almanac 1974

1. FOR THE MEN, HOW MUCH LONGER IS THE TIME:
FOR THE 440 THAN FOR THE 220?
FOR THE 220 THAN FOR THE 100?
FOR THE 880 THAN FOR THE 440?
FOR THE 6-MILE THAN FOR THE MILE?
2. FOR THE WOMEN, SUPPOSE YOU COULD RUN A MILE AT THE RECORD PACE
FOR THE QUARTER MILE.
A) HOW FAST COULD YOU RUN THE MILE?
B) WHAT IS THE DIFFERENCE BETWEEN THIS AND THE WORLD RECORD FOR
THE MILE?
3. SUPPOSE A MAN RAN A MILE AT THE RECORD PACE FOR THE 6-MILE,
(26 MINUTES, 47 SECONDS).
A) HOW FAST WOULD HE RUN THE MILE?
B) WHAT IS THE DIFFERENCE BETWEEN THIS AND THE WORLD RECORD
FOR THE MILE?
4. THE BOSTON MARATHON IS 26 MILES, 385 YARDS. ESTIMATE THE RECORD
TIME FOR THE RACE. CHECK YOUR ESTIMATE IN THE LATEST ALMANAC.

IDEA FROM: Project R-3

Permission to use granted by E.L. Hodges

TYPE: Paper & Pencil/Transparency

"THE ANSWER"

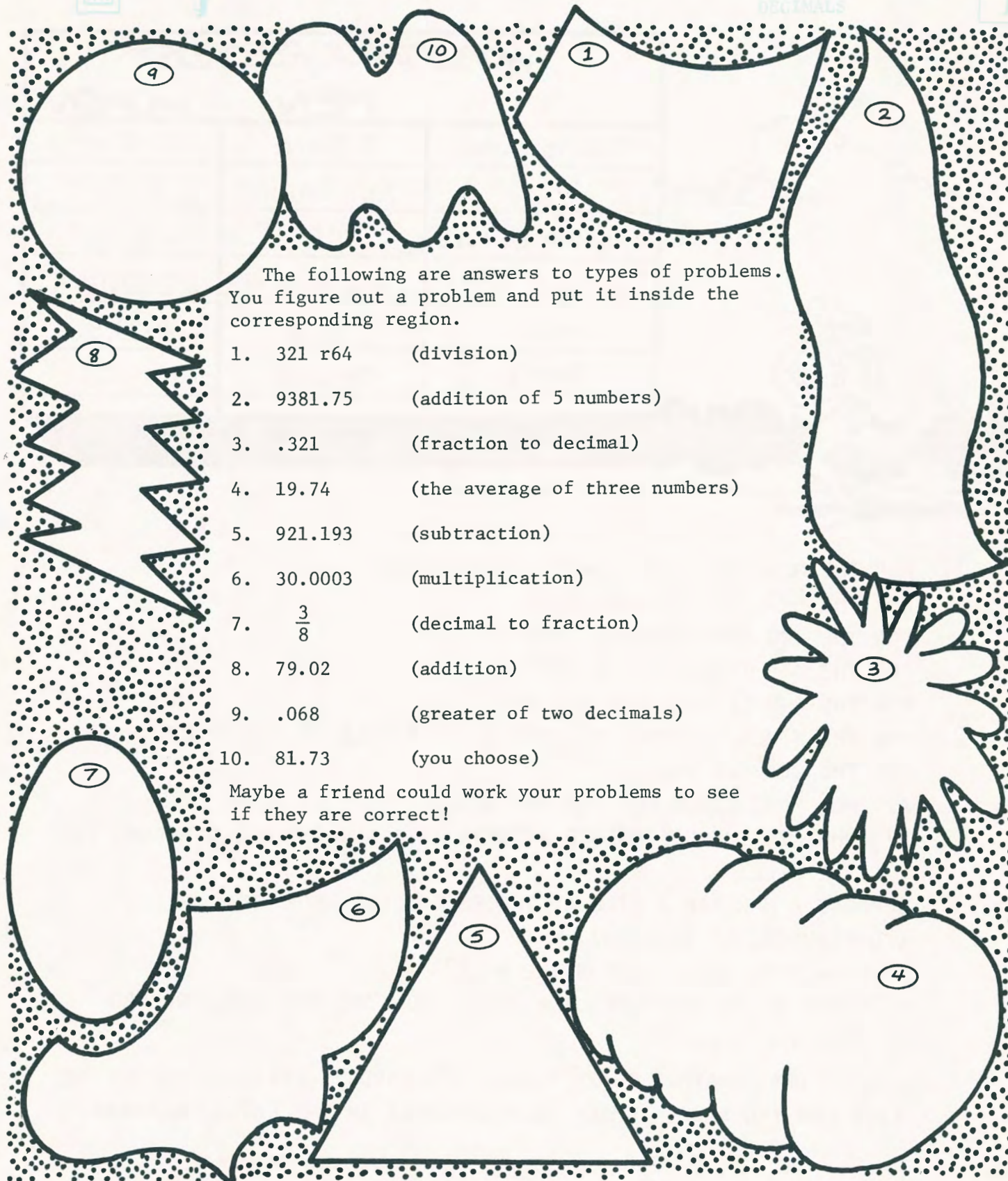
Computation 4, -3, 5
Mixed Operations
DECIMALS



The following are answers to types of problems.
You figure out a problem and put it inside the
corresponding region.

1. 321 r64 (division)
2. 9381.75 (addition of 5 numbers)
3. .321 (fraction to decimal)
4. 19.74 (the average of three numbers)
5. 921.193 (subtraction)
6. 30.0003 (multiplication)
7. $\frac{3}{8}$ (decimal to fraction)
8. 79.02 (addition)
9. .068 (greater of two decimals)
10. 81.73 (you choose)

Maybe a friend could work your problems to see
if they are correct!





NOTHING IS FOREVER

Patterns
Mixed Operations
DECIMALS



CONTINUE THESE DECIMAL PATTERNS

1. ...
2. ...
3. ...
4. ...
5. ...
6. ... *Add 1.11*
7. ... *Add .7; 1.7; 2.7; etc.*
8. ...
9. ... *$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \text{ etc.}$*
10. ... *Multiply by 1.2*
11. ... *$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$
or multiply by .5*
12. ... *$\frac{0}{1}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \text{ etc.}$*

TYPE: Paper & Pencil



Approx-Appraisals

Approximation
Word Problems
Mixed Operations
DECIMALS



At the end of each problem there are three numbers.
CIRCLE the number you think is nearest to the right answer.

DO IT IN YOUR HEAD. Don't forget to use rounded figures to find the approximate answer.

1. Last summer a salesman drove his car 5,932 kilometres in 10 weeks. Which answer gives the approximate average number of kilometres he drove his car each week?
400 kilometres 500 kilometres 600 kilometres
2. At the grocery store a housewife spent \$2.89 for fresh fruit and vegetables, \$14.22 for meat, and \$6.75 for other items. About how much did she spend altogether?
\$20.00 \$25.00 \$30.00
3. A boy sold 29 dozen eggs at 51¢ a dozen. Which answer shows approximately how much he got for the eggs?
\$1.50 \$15.00 \$150.00
4. Mary bought 6 metres of material at \$1.49 a metre. About how much did the material cost?
\$3.00 \$6.00 \$10.00
5. A train goes 629 kilometres in 9 hours. Which answer approximately gives the average speed of the train?
70 km/h 75 km/h 80 km/h
6. About how much change is left from two \$20 bills if you pay \$19.89 for a tennis racket and \$3.49 for tennis balls?
\$12.00 \$16.00 \$20.00
7. Bill bought a pen and pencil set for \$8.95 and a small package of paper for \$.49. About how much did he pay in all?
\$9.50 \$10.00 \$10.50
8. If an auditorium contains 28 rows of 32 seats in a row, approximately how many people will the auditorium seat?
900 1,000 800
9. If a girl earns \$1.95 an hour, about how many hours will she have to work to earn \$50.00?
30 25 20

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Students learn to do these problems quickly, the choices can be refined. For example, Problem #2 could use \$22.00, \$23.00, or \$24.00.

IDEA FROM: C.O.L.A.M.D.A.



OPERATION PLEASE!

Word Problems
Mixed Operations
DECIMALS



Show all your calculations on this page and tell if the problem is +, -, x, ÷. (Circle one)

1. Stan ran the 880 event at the track meet in 2 minutes, 15.5 seconds. Lydia ran the same event in 2 min., 22.8 seconds. How much faster did Stan run?

This problem is solved by +, -, x, ÷.

The answer is: _____

2. The odometer on an automobile read 9999.9 miles. What would it read .2 miles later?

This problem is solved by +, -, x, ÷.

The answer is: _____

09999.9

3. If gasoline costs \$.64 per gallon, what is the total cost of 10.5 gal.?

This problem is solved by +, -, x, ÷.



The answer is: _____

4. One day Antonio rode his bike to and from the park 4 times. If his odometer showed 6 kilometers, how far was one round trip?

This problem is solved by +, -, x, ÷.

The answer is: _____

5. There are 23 charities. A man gave \$15.50 to each. How much did he give altogether?

This problem is solved by +, -, x, ÷.

The answer is: _____

6. In 1955 Col. Talbott won the Bendix Air Trophy with an average speed of 982.841 km/h. On the following day someone else flew 340.220 km/h faster. What was the new record?

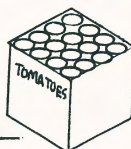
This problem is solved by +, -, x, ÷.

The answer is: _____

7. Boxes of canned tomatoes were sent from Sacramento to Los Angeles. The total weight was 5262.5 kg. If each box weighed 12.5 kg, how many boxes were shipped?

This problem is solved by +, -, x, ÷.

The answer is: _____



8. At the Olympics in 1960 a woman ran the 880 in 2 min., 4.3 seconds. In 1964 another woman ran the same race in 2.2 seconds less time. What was the new record?

This problem is solved by +, -, x, ÷.

The answer is: _____

TYPE: Paper & Pencil/Transparency

IDEA FROM: Project R-3

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Students need not find the answers. You may use this page to stress which operation is needed.



WORDLESS PROBLEMS

Word Problems
Mixed Operations
DECIMALS



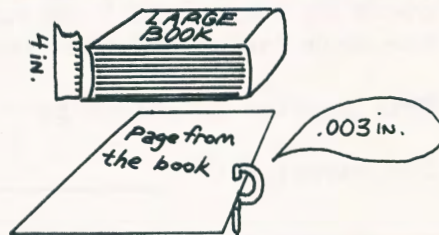
WRITE ONE OR MORE PROBLEMS FOR EACH PICTURE BELOW.

1.

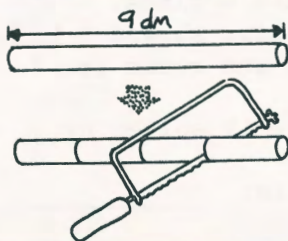


PERIMETER IS
10.96 cm

2.



3.



4.

WEEK	MILES	GALLONS USED
1	562.3	41.2
2	467.5	34.6
3	227	16.2
4	389.3	28.7

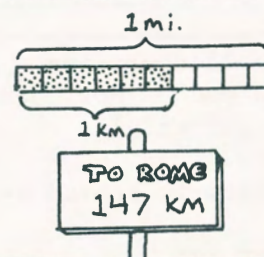
5.

APPLES



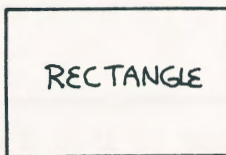
29.6 KG

6.



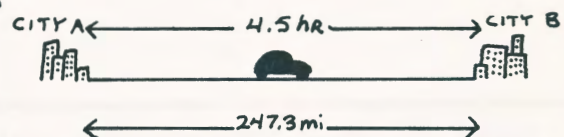
7.

WIDTH
2.39 cm

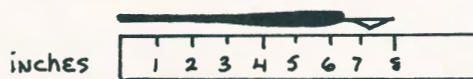
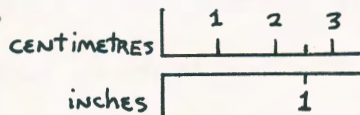


PERIMETER IS
10.14 cm

8.



9.



10.



TYPE: Paper & Pencil/Transparency

IDEA FROM: *School Mathematics I*, by Robert E. Eicholz, Phares, G. O'Daffer, Charles F. Brumfiel, Merrill E. Shanks, Charles R. Fleenor. Copyright (c) 1971 by Addison-Wesley Publishing Company, Inc. All rights reserved. Reprinted by permission.

SIMPLIFY THE NUMBERS

Simplifying Problems
Word Problems
Mixed Operations
DECIMALS



Before using this page it would be helpful to read the commentary to
DECIMALS: Mixed Operations under Problem Solving.

<p>1. A board is 325.6 cm long. How many shelves can be cut from the board if each shelf is 60.7 cm long?</p>	<p>Have your students:</p> <p>a) Read the problem. b) Make the problem easier by writing a simple whole number above the decimal.</p>
<p>2. A 2.7 kg roast costs \$8.40. How much would a 1 kg roast cost?</p>	<p>c) Work the problem using the whole numbers. d) Set up the decimal problem by replacing the whole numbers with the corresponding decimals.</p>
<p>3. A plane was traveling 672.81 km each hour. After traveling for 2.6 hours, how far had the plane gone?</p>	<p>Some examples are shown below. Your students might pick different numbers.</p>
<p>4. A rock weighs 24.712 kg. Its volume is 3610.9 cm³. How many kg does one cm³ of this rock weigh?</p>	<p>$5 \div 10 = \frac{1}{2}$ OR $10 \overline{) 5.0}$ 50</p> <p>$3610.9 \overline{) 24.712}$</p>
<p>5. Peppermint candy costs \$1.85 a pound. How much would .75 pounds cost?</p>	<p>$2 \times 6 = 12$</p> <p>$1.85 \times .75$ OR $\begin{array}{r} 1.85 \\ \times .75 \\ \hline \end{array}$</p>
<p>6. A tack weighs about .75 g. How many tacks would it take to weigh 15.75 g?</p>	<p>$8 \div 2 = 4$</p> <p>$15.75 \div .75$ OR $.75 \overline{) 15.75}$</p>



THAT'S JUST ABOUT THE SIZE OF IT!

Body weights can be in metric or standard units.

Word Problems
Mixed Operations
DECIMALS



1. MUSCLES MAKE UP ABOUT $.4$ OF A PERSON'S BODY WEIGHT. ABOUT HOW MUCH DO YOUR MUSCLES WEIGH?



2. YOUR INTESTINES (LARGE & SMALL) ARE ABOUT 7.5 M (25 FT.) LONG. HOW DOES THIS COMPARE WITH YOUR HEIGHT?

3. YOUR BODY CONTAINS 206 BONES. APPROXIMATELY $.14$ OF YOUR BONES ARE IN YOUR HEAD. ABOUT HOW MANY BONES ARE IN YOUR HEAD?

4. A PERSON'S BRAIN IS $.02$ OF HIS BODY WEIGHT. HOW MUCH DOES YOUR BRAIN WEIGH?



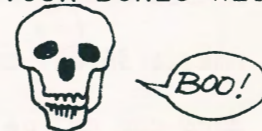
5. YOUR HEART BEATS ABOUT 80 TIMES A MINUTE. HOW MANY TIMES DOES IT BEAT IN A DAY?



6. THE HUMAN BODY IS ABOUT $.6$ WATER. HOW MANY POUNDS (KG) OF WATER ARE THERE IN YOU?

7. THE LARGEST GLAND IN YOUR BODY IS THE LIVER. IN A 160 LB (72.5 KG) PERSON THE LIVER IS ABOUT $.03$ OF THE BODY WEIGHT. HOW MUCH DOES IT WEIGH?

8. HUMAN BONES MAKE UP ABOUT $.18$ OF A PERSON'S TOTAL BODY WEIGHT. HOW MUCH DO YOUR BONES WEIGH?



9. THE EYE BLINKS ABOUT 25 TIMES EACH MINUTE. APPROXIMATELY HOW MANY TIMES DOES IT BLINK IN A DAY?



10. THE BODY OF AN ADULT CONTAINS APPROXIMATELY 5 QUARTS (4.75 LITRES) OF BLOOD. A BLOOD DONOR USUALLY GIVES 1 PINT (.475 LITRES). WHAT FRACTION OF HIS BLOOD DOES A DONOR GIVE?

TYPE: Paper & Pencil

Answers will vary. Students might compare answers with a friend who is about the same size.



GETTING 'ROUND TO CALCULATING

TEACHER IDEAS

Approximation
Mixed Operations
DECIMALS



Here is a fast way to find approximate answers. Can you tell how it is done?

$$1) \frac{31.18 \times 42.7}{161} \approx \frac{30 \times 40}{160} = \frac{30}{4} \approx 7$$

Finish these problems by filling in the rectangles:

$$2) 19.7 \times 12.3 \approx \boxed{} \times 10 = \boxed{}$$

$$3) \frac{28.3 \times 17.2}{12.4} \approx \frac{\boxed{} \times \boxed{}}{\boxed{}} = \boxed{}$$

These problems are especially useful for developing the number sense needed for the effective use of hand calculators. Each section can be expanded by adding similar problems. Answers will vary.

See if you can tell how these problems were done:

$$4) 19.7 \times .123 \approx 20 \times .1 = 2$$

$$5) \frac{.826}{.174} = \frac{826}{174} \approx \frac{800}{200} = 4$$

Now finish these problems by filling in the rectangles:

$$6) 57.7 \times .081 \approx \boxed{} \times \boxed{} = \boxed{}$$

$$7) 287.3 + 9.6 + 74.206 \approx \boxed{} + \boxed{} + \boxed{} = \boxed{}$$

$$8) \frac{1.84}{.29} = \frac{184}{\boxed{}} \approx \frac{\boxed{}}{\boxed{}} \approx \boxed{}$$

Some of these answers below are obviously wrong. Find the approximate answer for each of these.

Circle the answers that must be wrong.

$$9) 27.30 \times 41.67 = 2970$$

$$12) \frac{17.0}{.989} = 17.2$$

$$10) 243.3 + 71.1 + 105.6 = 428.0$$

$$13) \frac{24.3}{5.4} = .45$$

$$11) 28.82 \div .93 = 3.1$$

The answers to the problems below are correct, but the decimal points have been left out. Find the approximate answer for each and place the decimal point where it belongs in each answer.

$$14) 5.03 \times 17.6 = 886$$

$$19) \frac{68.64}{4.4} = 156$$

$$15) 6.23 \times 17.91 \times .131 = 144$$

$$20) 16.55 \times .008 = 488$$

$$16) \frac{.726}{.154} = 471$$

$$21) \frac{400.14}{85.5} = 488$$

$$17) \frac{.198}{.090} = 22$$

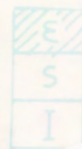
$$22) \frac{.7265}{.054} = 1344$$

$$18) 5.824 \times .36 = 209664$$



UP↑ AND DOWN↓ WITH THE CALCULATOR

Rounding
Mixed Operations
DECIMALS



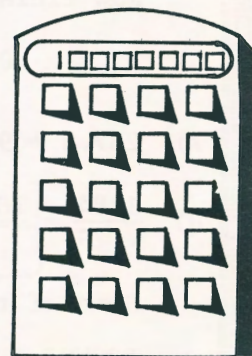
$$\begin{aligned} 26.75 \uparrow &= 27 && \text{(NEAREST WHOLE NUMBER GREATER THAN 26.75)} \\ 5.6 \downarrow &= 5 && \text{(NEAREST WHOLE NUMBER LESS THAN 5.6)} \\ 17.31 \updownarrow &= 17 && \text{(ROUNDED TO NEAREST WHOLE NUMBER)} \end{aligned}$$

THE NUMBERS $26.75 \uparrow$, $5.6 \downarrow$, AND $17.31 \updownarrow$ CAN ALSO BE FOUND USING A CALCULATOR.

IN THIS ACTIVITY THE SYMBOL 1D WILL MEAN TO ENTER 1 IN YOUR CALCULATOR AND FILL THE REMAINDER OF THE DISPLAY WITH ZEROS. FOR EXAMPLE, IN AN 8-DIGIT DISPLAY 1D MEANS 10,000,000 (1 FOLLOWED BY 7 ZEROS).

TO FIND $26.75 \uparrow$: (USING 8-DIGIT DISPLAY)

- DISPLAY 26.75 ON YOUR CALCULATOR.
- ADD 1 (DISPLAY SHOULD NOW SHOW 27.75).
- DIVIDE BY 1D (DISPLAY SHOULD NOW SHOW .00000027)
- MULTIPLY BY 1D (DISPLAY SHOULD NOW SHOW 27)



TO FIND $N \uparrow$ FOR ANY DECIMAL N, FOLLOW THESE DIRECTIONS:

$$(N + 1) \quad 1D \times 1D = \boxed{N \uparrow}$$

USE THE CALCULATOR TO FIND THE FOLLOWING:

$$2.3 \uparrow$$

$$3.41 \uparrow$$

$$0.7 \uparrow$$

$$273.365 \uparrow$$

DID YOU GET 3, 4, 1, AND 274?

TRY TO EXPLAIN WHY THE PROCESS FOR FINDING UP-ARROWS WORKS.



TYPE: Activity

UP AND DOWN WITH THE CALCULATOR (CONTINUED)

17.31 \updownarrow (USING 8 DIGIT DISPLAY)

- DISPLAY 17.31 ON YOUR CALCULATOR.
- ADD .5 (DISPLAY SHOULD NOW SHOW 17.81)
- DIVIDE BY 10 (DISPLAY SHOWS .00000017)
- MULTIPLY BY 10 (DISPLAY SHOWS 17)

THE FOLLOWING PROCEDURE CALCULATES N:

$$(N + .5) \div 10 \times 10 = \boxed{N \updownarrow}$$

USE THE CALCULATOR TO FIND THE FOLLOWING:

3.5 \updownarrow

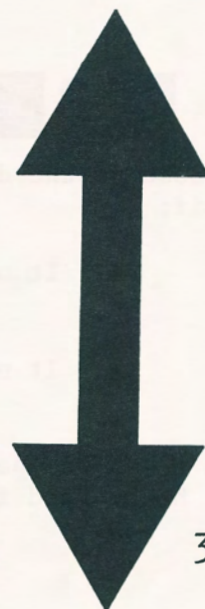
14.7 \updownarrow

43.3 \updownarrow

127.499 \updownarrow

36.99 \updownarrow

(DID YOU GET 4, 15, 43, 127 AND 37?)



COMPLETE THE FOLLOWING DIRECTIONS FOR FINDING DOWN-ARROWS.

$$N = \boxed{N \downarrow}$$

USE YOUR DIRECTIONS AND THE CALCULATOR ON THE FOLLOWING NUMBERS?

37.2 \downarrow

437.6 \downarrow

51.73 \downarrow

312.312 \downarrow

(DID YOU GET 37, 437, AND 312?)

HERE ARE SOME MORE PRACTICE PROBLEMS. BE SURE TO CHECK YOUR ANSWERS MENTALLY.

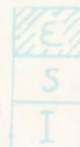
18.3 \uparrow	+	4.6 \downarrow	=	<u>19</u>	+	<u>4</u>	=	<u>23</u>
37.2 \updownarrow	-	14.7 \uparrow	=	<u>37</u>	-	_____	=	_____
8.75 \downarrow	-	5.75 \updownarrow	=	_____	-	<u>6</u>	=	_____
16.25 \updownarrow	+	3.6 \updownarrow	=	_____	+	_____	=	<u>20</u>
13.4 \uparrow	-	13.4 \downarrow	=	_____	-	_____	=	_____



one-liners

TEACHER IDEAS

Mixed Operations
DECIMALS



1.

■ 3 ■ ■ 8. ■ ■ 4 ■

Some of the digits in the above decimal are covered. What is the decimal if:

A. It names the largest number possible and has no two digits alike?

93,768.5241

B. It names the smallest number possible and has no two digits alike?

03,128.5647 or 13,028.5647

2. A man was paid \$2.00 for sawing a board into 2 pieces. How much should he be paid for sawing the board into 4 pieces? \$6.00



3. Which do you prefer - a truck load of nickels or half-a-truck load of dimes?

$\frac{1}{2}$ truck load of dimes

4. What is the greatest amount of money you could have (using only pennies, nickels, dimes and quarters) and still not be able to give someone change for:

half a dollar? 69c

a dollar? 119c

5. Suppose the metric system was used to measure time and money.

A. How much money would you have if you had:

a "kilobuck?"



a "centibuck?"



B. Would you like your class period better if it were a "deciday" long? one deciday = 2.4 hours

C. Which would be longer, a "milliday" or one minute? milliday

D. What would be meant by a "decimilliday?" $\frac{1}{10,000}$ of a day or 8.64 sec.

E. What changes might be made on the clock if time were measured in the metric system?

Suggestions: 10 hours = 1 day, so 1-10 on the face
(1 hour = a deciday)

100 min. = 1 hour (1 min. = a centihour)
100 sec. = 1 min.

TYPE: Paper & Pencil

IDEA FROM: *Elementary School Mathematics*, Book 6, Second Edition by Robert E. Eicholz and Phares G. O'Daffer: Copyright (c) 1968 by Addison-Wesley Publishing Company, Inc. All rights reserved. Reprinted by permission.



SHOPPING WITH A NEWSPAPER

TEACHER DIRECTED ACTIVITY

Mixed Operations
DECIMALS



MATERIALS: DAILY NEWSPAPERS FROM LOCAL AREA

PREPARATION: PLACE THE FULL PAGE ADVERTISEMENTS FROM LOCAL GROCERY STORES AROUND THE ROOM. USE ADS FROM 3 TO 5 DIFFERENT STORES. (WITH A LARGE CLASS YOU MAY WISH TO HAVE MORE THAN ONE COPY OF EACH AD.)

**INSTRUCTIONS
TO STUDENTS:**



1. PLAN A PARTY FOR 10 OR 20 OF YOUR FELLOW STUDENTS. YOU MAY USE ONLY THE ITEMS LISTED IN ANY OF THE ADS. YOU MAY SHOP AT MORE THAN ONE STORE.
2. WRITE THE MENU FOR YOUR PARTY.
3. MAKE A LIST OF EVERYTHING YOU WILL NEED AND HOW MUCH OR MANY OF EACH ITEM.
4. FIND THE COST OF YOUR PARTY - BE SURE TO CHECK THE ADS AND GET THE BEST PRICE ON EACH ITEM.

- FOLLOW-UP:**
1. PREPARE A LIST OF 5 TO 10 ITEMS TO BE PURCHASED BY EVERYONE. BE SURE EACH ITEM IS ADVERTISED BY AT LEAST 3 STORES.
 2. HAVE STUDENTS FIND THE "BEST BUY" FOR EACH ITEM ON THE LIST.
 3. HAVE STUDENTS FIND THE TOTAL COST OF THE LIST (LOWEST POSSIBLE).
 4. YOU MAY WISH TO PREPARE A CHART SIMILAR TO THE ONE SHOWN BELOW.

EXAMPLE :

ITEM	STORE A	STORE B	STORE C	STORE D	STORE E	BEST BUY
2 POUNDS GROUND BOUND						
3 POUNDS HOT DOGS						
6 SMALL CANS FROZEN ORANGE JUICE						
3 POUNDS COFFEE						
1 CAN TOMATO SOUP						

TYPE: Activity

TOTAL

NUMBER SENSE AND ARITHMETIC SKILLS

Placement Guide for Tabbed Dividers

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LARGE AND SMALL NUMBERS: AWARENESS

We live in an era of large numbers. It is almost impossible to find any current newspaper or magazine that doesn't include "millions," "billions" and occasionally even "trillions." These large numbers occur not only in the news but in the cartoons and advertisements. The Federal Government informs us that our new budget will be around 300 billion dollars; environmentalists say that we throw away 50 billion cans each year; and our local spaghetti house tells us that they served 250 million feet of noodles last year.

The department had announced it expected Americans to spend about \$135 billion on food this year, compared to \$125 billion last year.

But a 20 per cent increase would take it closer to \$150 billion.

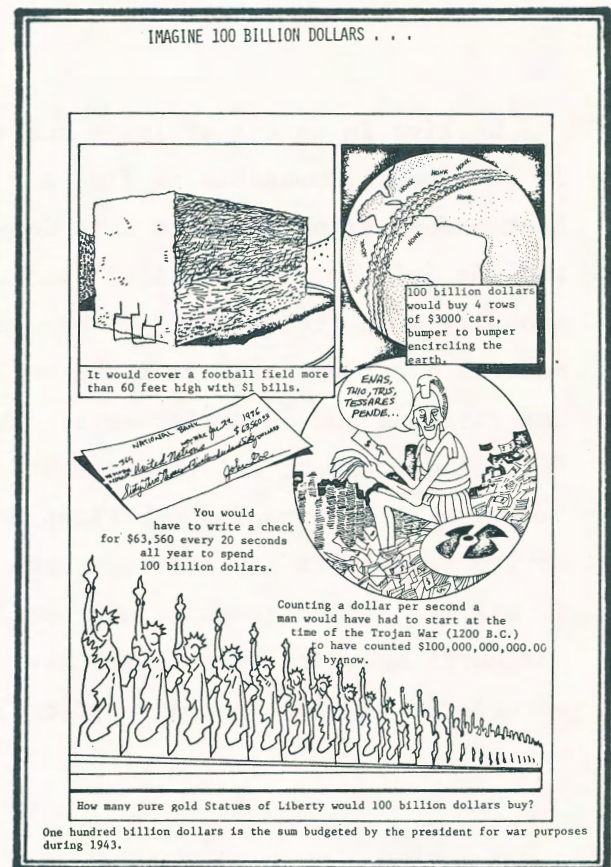
WASHINGTON

Personal income in the United States rose \$6.7 billion in March and for the first time exceeded \$1 trillion at an adjusted annual rate, the Commerce Department reported today.

The new \$344 million cost increase brings the present estimate for the B1 to \$13.7 billion, 2.5 billion over the original estimate of \$11.2 billion.

The biggest cost increases listed in the Pentagon report after the F111 jetfighter and the B1 bomber are \$2 billion for the Safeguard antimissile system, \$1.7 billion for the C5A supercargo jet, \$1.6 billion for the Minuteman Missile III, \$1.2 billion for the Navy's new F14 swingwing jetfighter and \$1 billion for the Navy's A7E jetfighter.

Every so often a journalist, observing a particularly large newsworthy number, tries to make the public aware of its significance. The 1943 budgeting of 100 million dollars for war purposes was compared to a stack of \$1 bills covering a football field to a height of 60 feet. The 1974 federal budget of 300 billion was compared to spending \$10,000 a minute from the moment of birth until age 57. The recent trillion dollar high for personal income prompted one writer to stack \$1000 bills. A million dollars in crisp new \$1000 bills makes a pile 8 inches high. A billion such bills (one trillion dollars) stacks 125 miles high.



While such comparisons are surprising and fun to read, do they really help us understand how large such numbers are? Can we learn to understand the significance of such numbers? If we read that there is a 21 billion dollar cost overrun in a federal project what should that mean to us? Buckminster Fuller has said, "Like parrots, we learn to recite numbers without any sensorial appreciation of their significance. We have yielded so completely to specialization that we disregard the comprehensive significance of information."

A LITTLE HISTORY

Sometime between 200 B.C. and 300 B.C. Archimedes calculated the number of grains of sand needed to fill the universe. In order to carry out his calculations he had to extend the Greek numeration system and invent a new system for expressing large numbers. In general, however, ancient arithmetic rarely had a need for large numbers. Even in the 800 year history of our Hindu-Arabic numeration system the need for large numbers has been very recent. The word "million" meaning "great thousand" was not known before the thirteenth century and it wasn't until the seventeenth century that people used "million" instead of "a thousand thousand." The word "billion" first appeared in the fifteenth century as the name for 10^{12} (as the British still use it*) but the French attached "billion" to 10^9 or 1,000,000,000 and probably influenced American usage after the Revolutionary War. The terms "million" and "billion" became popularly used during the 20th century in connection with wars and budgets in a technological society.

Power of Ten	Number of Zeros	American System
10^3	3	thousand
10^6	6	million
10^9	9	billion
10^{12}	12	trillion
10^{15}	15	quadrillion
10^{18}	18	quintillion
10^{21}	21	sextillion
10^{24}	24	septillion
10^{27}	27	octillion
10^{30}	30	nonillion
10^{33}	33	decillion
10^{36}	36	undecillion
10^{39}	39	duodecillion

In the earliest American arithmetic books large number names were in fashion and students sometimes learned number names to "duodecillions" before taking up addition. Currently we do not place emphasis on learning the technical names of large numbers. Because of our convenient exponential notation we can write 10^{47} and do not have to memorize the name "one hundred quattuordecillion." However, writing a symbol or knowing a name do not make numbers meaningful.

*In the British system the names million, billion, trillion, etc. represent 10^6 , 10^{12} , 10^{18} , respectively.

DEVELOPING LARGE AND SMALL NUMBER SENSE

One of the basic ingredients in any learning situation is motivation. Many people are interested in large and small numbers--at least with the astounding facts using large and small numbers. This is why many newspapers and magazines use large and small number facts as "fillers." Facts like: One inch of rain falling on one acre weighs 200,000 pounds (100 tons); The smallest animal, *Mycoplasma laidlawii*, is so small that 400,000,000 of them will fit on the head of a pin; A "googol" is the name for the number written as a one followed by one hundred zeros. Some people even use their large and small number facts incorrectly--but very few people notice.

By occasionally introducing interesting facts in our classes and having students collect facts that are interesting to them we have a starting point for developing large and small number sense.

There are at least two important components to developing large number sense. The first deals with orders of magnitude. If we think of three numbers like 10, 100 and 1000 we can imagine, for example, cities that are 10, 100 and 1000 miles away. As we multiply by ten the distance away grows rapidly. If 1000 miles is multiplied by ten to get 10,000 miles we are half way around the earth. Thus, as we increase numbers by a factor of 10 they rapidly grow large. Most people think of a million as large, a billion as larger and a trillion as still larger but do not comprehend the relative sizes as a million, a billion (thousand million) and a trillion (thousand thousand million). One way to illustrate these orders of magnitude is to think of seconds or heartbeats--since a heart beats about once a second. A million seconds is slightly more than 11 days, a billion seconds is over 30 years and a trillion seconds is over 30,000 years. While it is difficult to imagine 30,000 years, it is easy to comprehend 11 days and 30 years. Relating

these "illions" to seconds, days and years (something personal and understandable) increases the comprehension of multiplying by a factor of 1000.

The second prerequisite for understanding large and small numbers is the ability to devise meaningful comparisons. The nature of these comparisons may vary from individual to individual. Consider, for example, the recent audit of the U.S. supply of gold stored at Fort Knox. About 6 billion dollars worth of gold rests in the vaults in the form of gold bricks, each weighing about 27 pounds and worth \$64,000 on the average. Is there any way to comprehend or reify this amount? Using simple arithmetic we can devise many different comparisons.

- a. \$30 for every man, woman and child in the United States
- b. $\frac{1}{50}$ or 2% of the federal budget for a single year
- c. \$60,000 for every person in attendance at the Rosebowl football game (a crowd that can be seen on T.V.)
- d. 1 gold brick for each person at the Rosebowl game
- e. \$600,000 for every person in a town of 10,000
- f. 1 pound of gold for every person in the State of Oregon
- g. \$1.50 for every person in the world

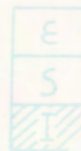
The list of comparisons could become quite lengthy and, undoubtedly, some of the comparisons may be just as meaningless as the original to some students. Students should devise their own ingenious comparisons. Hopefully, we will some day reach the point where informed citizens reading about a cost overrun of 21 billion dollars will be able to comprehend the significance of the amount by thinking, "Hey, that amounts to about \$100 a person or \$400 a family!"

Very small numbers are not nearly as common as large numbers. However, when they are encountered they may be treated much like large numbers. For example, the smallest virus is .000020 mm in diameter. Thus, 1 million such viruses side-by-side would stretch 20 mm or 2.0 cm. If an understanding of orders of magnitude is developed it can be applied to very small objects by determining how many of these objects combined make a comprehensible amount. These ideas can be developed from resource pages that deal with small numbers.



BULLETIN BOARD IDEA

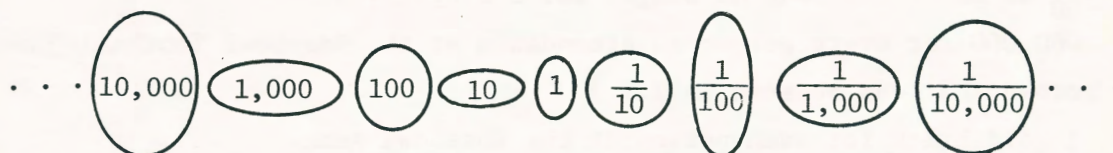
Number Sense-Powers of Ten
Awareness
LARGE & SMALL NUMBERS



TEACHER IDEA

Students can participate in creating a bulletin board designed to improve their understanding of large and small numbers. This bulletin board will grow as it is used and eventually might cover one wall of your room.

The board is started by evenly spacing columns across the wall. The headings consist of powers of 10. Start with 1 in the center of the wall, then in a horizontal line write other powers of 10 in sequence on either side of one:




As the students become familiar with large and small numbers, they will be able to supply information about these numbers such as:

- (1) Number name ONE HUNDREDTH
- (2) Number, as a power $\frac{1}{10^2}$
- (3) Examples of how the numbers are used.
 - a) News clippings . . .
 - b) Almanac facts . . .
 - c) Photos or drawings.


SAMPLE COLUMN → $\frac{1}{100}$

(1) Number name ONE HUNDREDTH

(2) Number, as a power $\frac{1}{10^2}$



The Louse Larure is $\frac{1}{100}$ mm in diameter.



1 CENTILITRE → $\frac{1}{100}$ of a litre is a centilitre.

Each of these items are written or placed column fashion below the appropriate number on the wall.

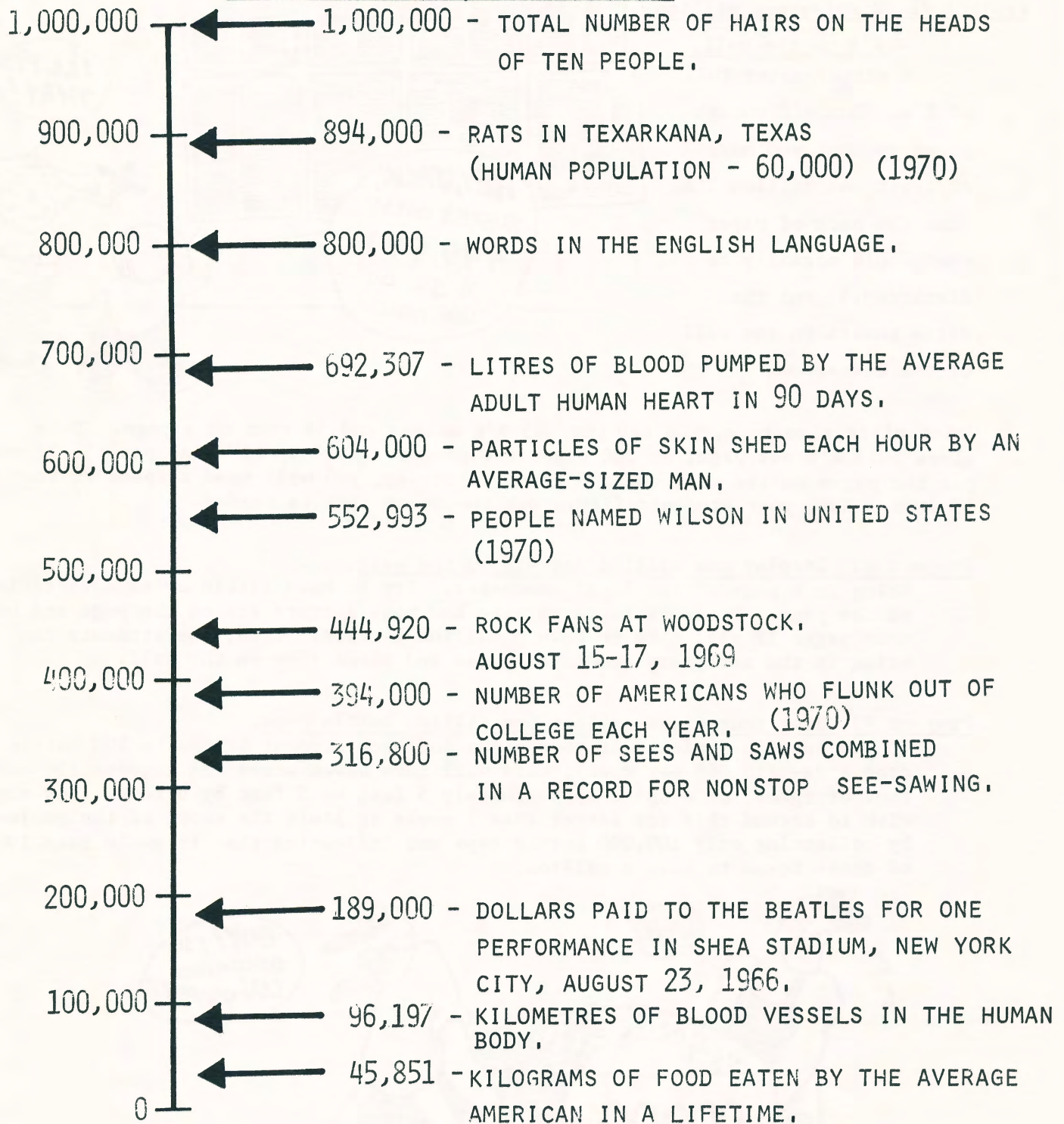
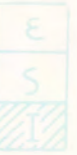
FOLLOW-UP:

If the pictures are good, take slides of them; then, a month or so later, flash the pictures to the class and ask them to give the number that comes to their minds.

TYPE: Bulletin Board

SCALE OF ONE MILLION

Number Sense-Large
Awareness
LARGE & SMALL NUMBERS



TYPE: Bulletin Board/Transparency

If you use this as a bulletin board make the scale large enough so students can add other interesting facts.

HOW BIG IS A MILLION - 3 PROJECTS

Number Sense-One Million
Awareness
LARGE & SMALL NUMBERS



TEACHER IDEAS

Project #1 Display one million

X's on the wall.

Type a ditto master full of X's. Run off enough pages so that you have at least one million X's. (Use the back of paper that would normally be discarded.) Put the ditto sheets on the wall of the classroom.



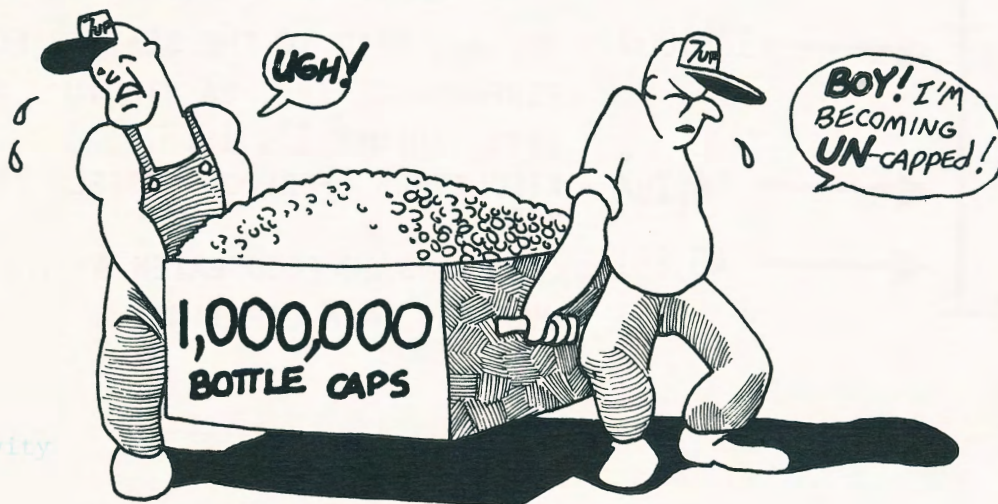
Using elite size type, you can put 85 X's across and 59 rows on a page. This gives 5015 X's per page, so 200 pages are needed. ($8\frac{1}{2}$ " by 11" size paper). If you put the pages on the wall using 8 rows, 25 across, you will need a space $7\frac{1}{2}$ feet by 18 feet. Have your students figure out the space that is needed.

Project #2 Display one million letters on the wall.

Bring in a page of the local newspaper. Try to have little or no advertising on the page. Students can determine how many letters are on the page and how many pages it will take to have a million letters. Then, the students can bring in the necessary number of pages and place them on the wall.

Project #3 Have your class collect one million bottle caps.

If you have 30 students in your class and each student brings in 100 bottle caps a day (15,000 per week), this will take seven weeks and require 150 cubic feet of space, or a space approximately 5 feet by 5 feet by 6 feet. You may wish to extend this for longer than 7 weeks or limit the scope of the project by collecting only 100,000 bottle caps and indicating that it would take 10 of these boxes to make a million.



TYPE: Activity



ONE MILLION CUBES

Number Sense-One Million
Awareness
LARGE & SMALL NUMBERS

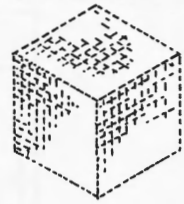


You may want to use centimetre cubes and metre sticks for this activity.

Suppose we took 1,000,000 centimetre cubes and made a large cube.

The volume of this cube is _____ cubic cm.

Notice that 1 cubic metre = 1 m x 1 m x 1 m = 100 cm x _____ cm
x _____ cm = _____ cubic cm.



Therefore, the volume of this cube is _____ cubic metre.

The dimensions of the cube could be illustrated with three metre sticks.

The volume of the large cube is approximately the volume of:
(circle one)

a textbook

the teacher's desk

the classroom

Suppose we took these 1,000,000 cm cubes and made a square-shaped figure one centimetre thick.

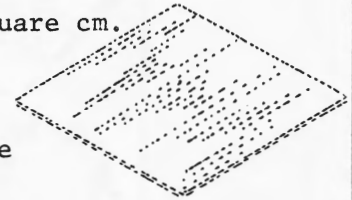
The area of the top of this figure is _____ square cm.

Notice that 1 square metre = 1 m x 1 m =

100 cm x 100 cm = _____ square cm.

Therefore, the area of the top of this figure is _____ square metres.

How long would the figure be on each side? _____



The area of the top of the square figure is approximately the area of:
(circle one)

a ping pong table

a volleyball court

a soccer field

Now imagine those 1,000,000 cm cubes stacked up in a column.

The height of this column of cubes is _____ cm.

Notice that 1 metre = _____ centimetres

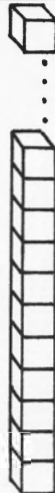
Therefore, the height of this column of cubes is _____ metres.

The height of this column of cubes is approximately the height of:
(circle one)

a flag pole

the Statue of Liberty

Mt. Everest



IMAGINE 100 BILLION DOLLARS . . .

Number Sense - 100 Billion
Awareness
LARGE & SMALL NUMBERS



It would cover a football field more than 60 feet high with \$1 bills.

100 billion dollars would buy 4 rows of \$3000 cars, bumper to bumper encircling the earth.

You would have to write a check for \$63,560 every 20 seconds all year to spend 100 billion dollars.

Counting a dollar per second a man would have had to start at the time of the Trojan War (1200 B.C.) to have counted \$100,000,000,000.00 by now.

How many pure gold Statues of Liberty would 100 billion dollars buy?

One hundred billion dollars is the sum budgeted by the president for war purposes during 1943.

Have students consider questions like these:

1. How much is gold worth today?
2. How heavy is the Statue of Liberty?
3. How much would 100 billion dollars in pennies weigh?
4. How high would 100 billion dollar bills be if piled up?
5. How far would 100 billion dollar bills reach if layed end to end?
6. How many people would have to give you \$500.00 for you to have \$100,000,000,000.00?

TYPE: Bulletin Board/Transparency



BEANS IN A JUG

AND OTHER ESTIMATING ACTIVITIES

Suggested as a small group project for four students.

Estimation
Computation-Logic
Awareness
LARGE & SMALL NUMBERS



MATERIALS: A sealed gallon jug filled with beans.

$\frac{1}{2}$ gallon milk carton about $\frac{2}{3}$ filled with beans.

1 quart milk carton.

3 measuring cups (1 cup size).

4-6 measuring cups ($\frac{1}{4}$ cup size).

PROCEDURE: Guess the number of beans in the jug. Each person in the group writes a guess on a piece of paper, folds the paper and places it in the middle of the table or work space. After everyone has made a guess, unfold the papers and agree as a group on one of the guesses. This is the group guess. Record all the guesses in the table below. (A group may not wish to accept any of the guesses for the group guess. If this is the case, select another number as the group guess.)

NAME:	GUESS:	
_____	_____	<i>Group Guess</i> _____
_____	_____	
_____	_____	

To find a "good" estimate of the actual number of beans in the jug use the beans in the $\frac{1}{2}$ gallon carton (do not open the jug) and follow the suggestions below.



Fill 4 of the measuring cups ($\frac{1}{4}$ cup size) full of beans so that the beans in each cup are level with the top of the cup. Count and record below the actual number of beans in each $\frac{1}{4}$ cup.

Cup # 1 _____

Cup # 2 _____

Cup # 3 _____

Cup # 4 _____

The average number of beans in
 $\frac{1}{4}$ cup is _____

How many $\frac{1}{4}$ cups does it take to fill the one-cup size measuring cup? _____

Give an estimate for the number of beans in one cup. _____

How many cups in a quart? _____

Estimate the number of beans in one quart. _____

How many quarts in a gallon? _____

How many beans do you now think are in the gallon jug? _____

How does this compare with the original guesses?

How far off were the original guesses?

BEANS IN A JUG (CONTINUED)

TEACHER IDEAS

Here are several suggestions for more class activities. They could be left open-ended or developed into structured activities similar in format to *Beans in a Jug*.

1. How many raisins are in a box of raisin cookies?
(The data may be eaten when the activity is finished.)
2. How many watched the last (basketball) game at your school?
Hints: a. How many from your class?
b. How much of the seating was filled with students?
3. How many telephones are there in your area?
Hints: a. How many phone numbers are listed on one of the white pages in your phone book?
b. How many white pages are there?
4. How many people in the United States are named Jones?
Hints: a. How many people in your area are named Jones?
b. How many people are there in your area?
c. How many people are there in the United States?
5. How many people in the country have the same last name as yours?
6. How many words in your dictionary?
7. How many letters are there in all the words in the Sunday paper?
8. How many grains of sugar are there in a 5-pound bag?
9. How many bricks (floor or ceiling tiles) are there in your school?
10. How many blades of grass are on your school ground?
Hints: a. How many are in one square inch (square centimetre)?
b. What is the total area of the part of your school ground covered by grass?
11. How much space is needed for a lawn containing: one million blades of grass?
one billion blades of grass?
one trillion blades of grass?

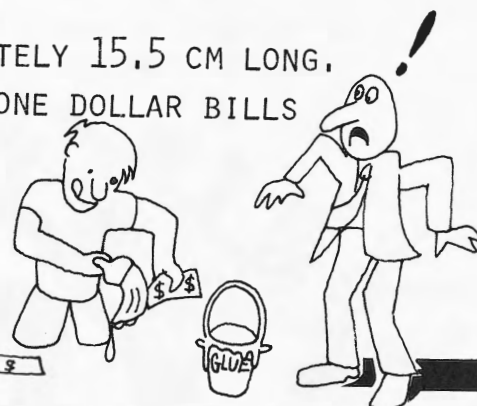
ADDITIONAL SUGGESTIONS:

- A. For beans in a jug try using candy eggs or jelly beans, especially at Easter-time. Have students guess each day for a week and do the activity at the end of the week. You could use varying sizes of eggs in the jar. Give a prize for the best estimate and give away the candy when done with the activity.
- B. Encourage students to come up with their own estimating activities.

STRETCHING YOUR DOLLARS



A ONE DOLLAR BILL IS APPROXIMATELY 15.5 CM LONG.
ABOUT HOW FAR WILL 10,000,000 ONE DOLLAR BILLS
REACH IF LAID END TO END?



HOW MANY DOLLAR BILLS WOULD YOU NEED TO STRETCH FROM:

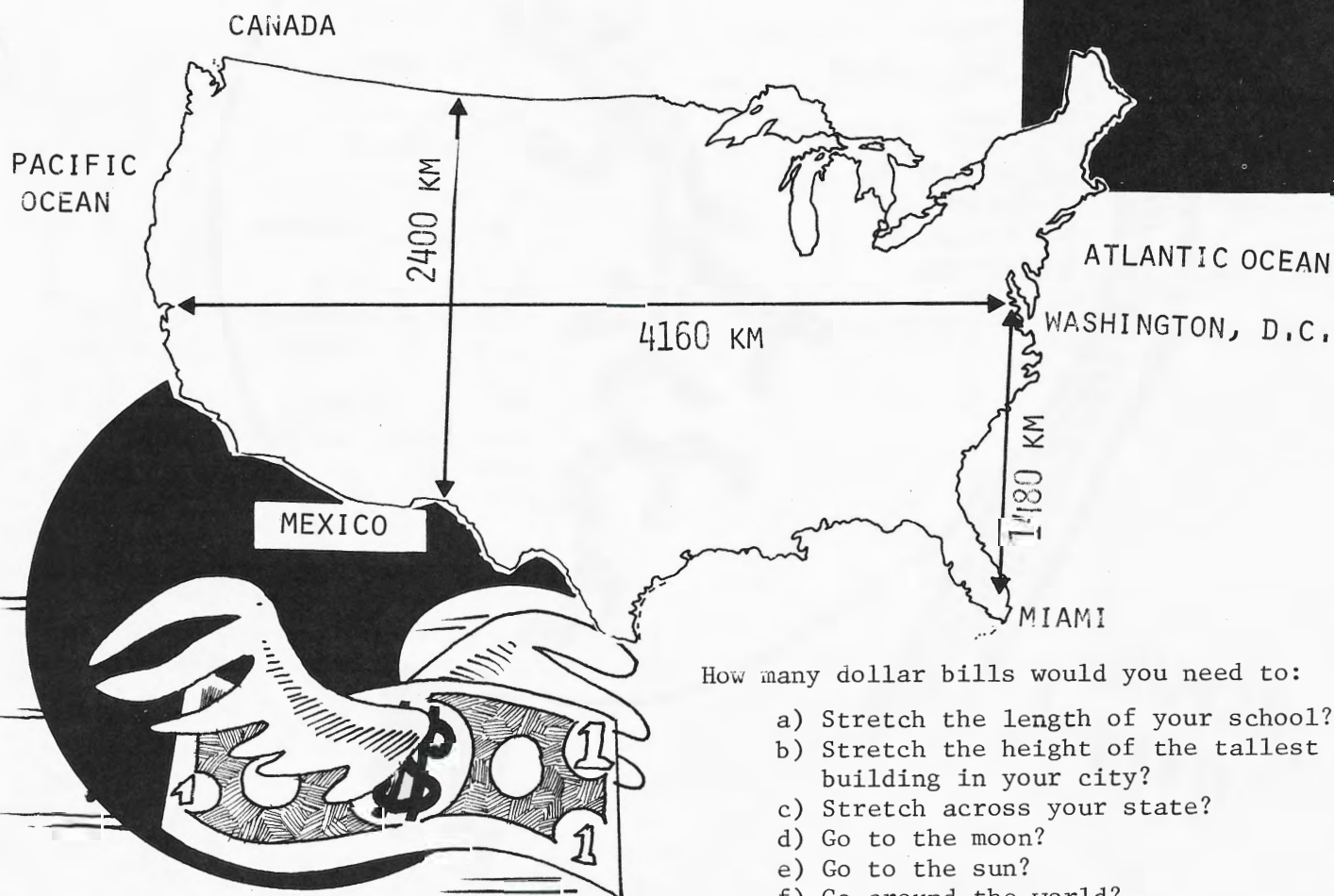
_____ CANADA TO MEXICO?

OR

_____ PACIFIC TO ATLANTIC?

OR

_____ MIAMI TO WASHINGTON?



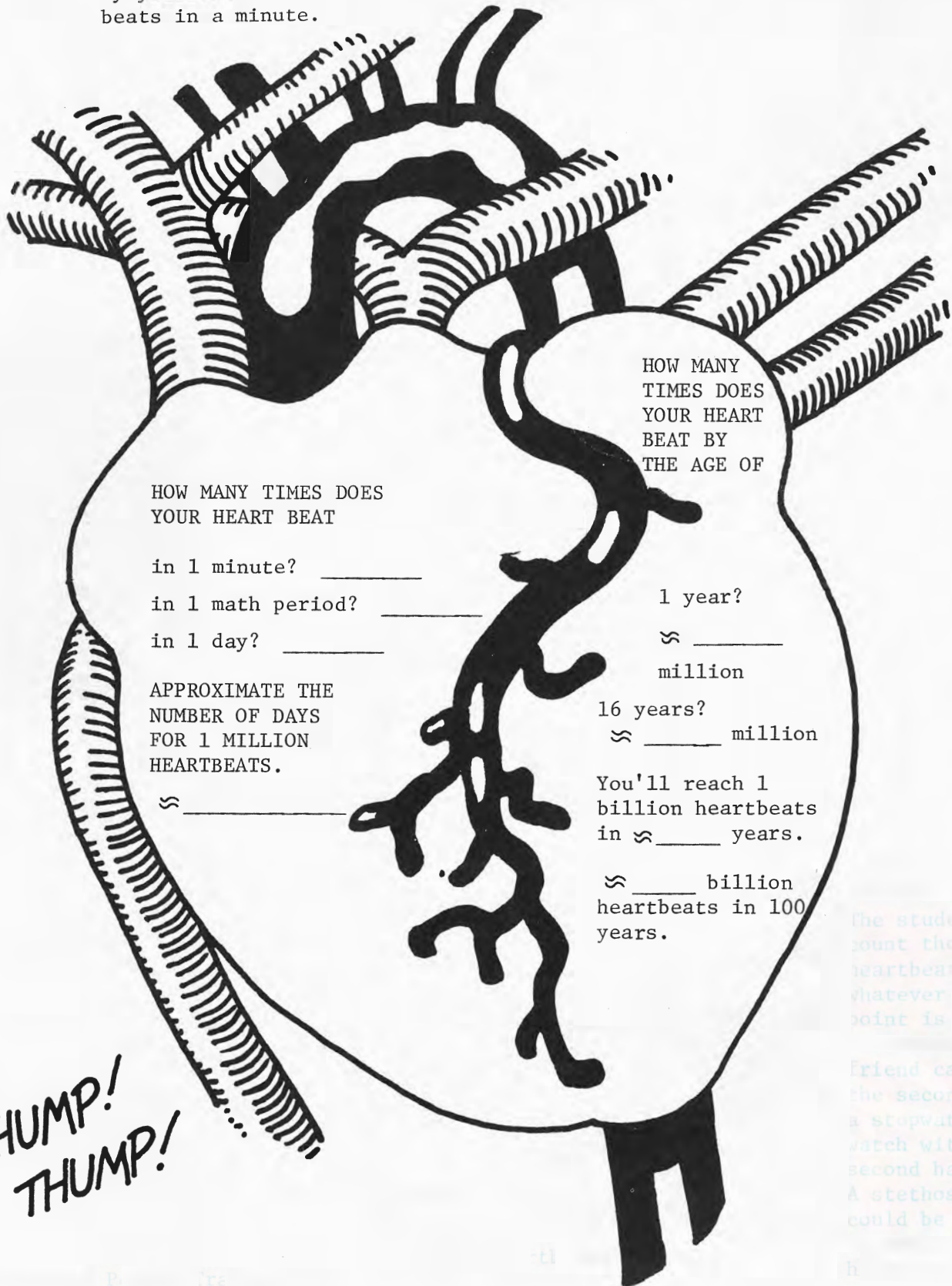
How many dollar bills would you need to:

- Stretch the length of your school?
- Stretch the height of the tallest building in your city?
- Stretch across your state?
- Go to the moon?
- Go to the sun?
- Go around the world?



your heartbeats

By yourself or with a friend count the number of times your heart beats in a minute.



HOW MANY TIMES DOES
YOUR HEART BEAT

in 1 minute? _____

in 1 math period? _____

in 1 day? _____

APPROXIMATE THE
NUMBER OF DAYS
FOR 1 MILLION
HEARTBEATS.

≈ _____

HOW MANY
TIMES DOES
YOUR HEART
BEAT BY
THE AGE OF

1 year?

≈ _____

million

16 years?

≈ _____ million

You'll reach 1
billion heartbeats
in ≈ _____ years.

≈ _____ billion
heartbeats in 100
years.

THUMP!
THUMP!

The student
count the
heartbeats
whatever
point is

Friend can tell
the seconds
a stopwatch
watch with a
second hand.
A stethoscope
could be used



SECONDS...TIMELINE

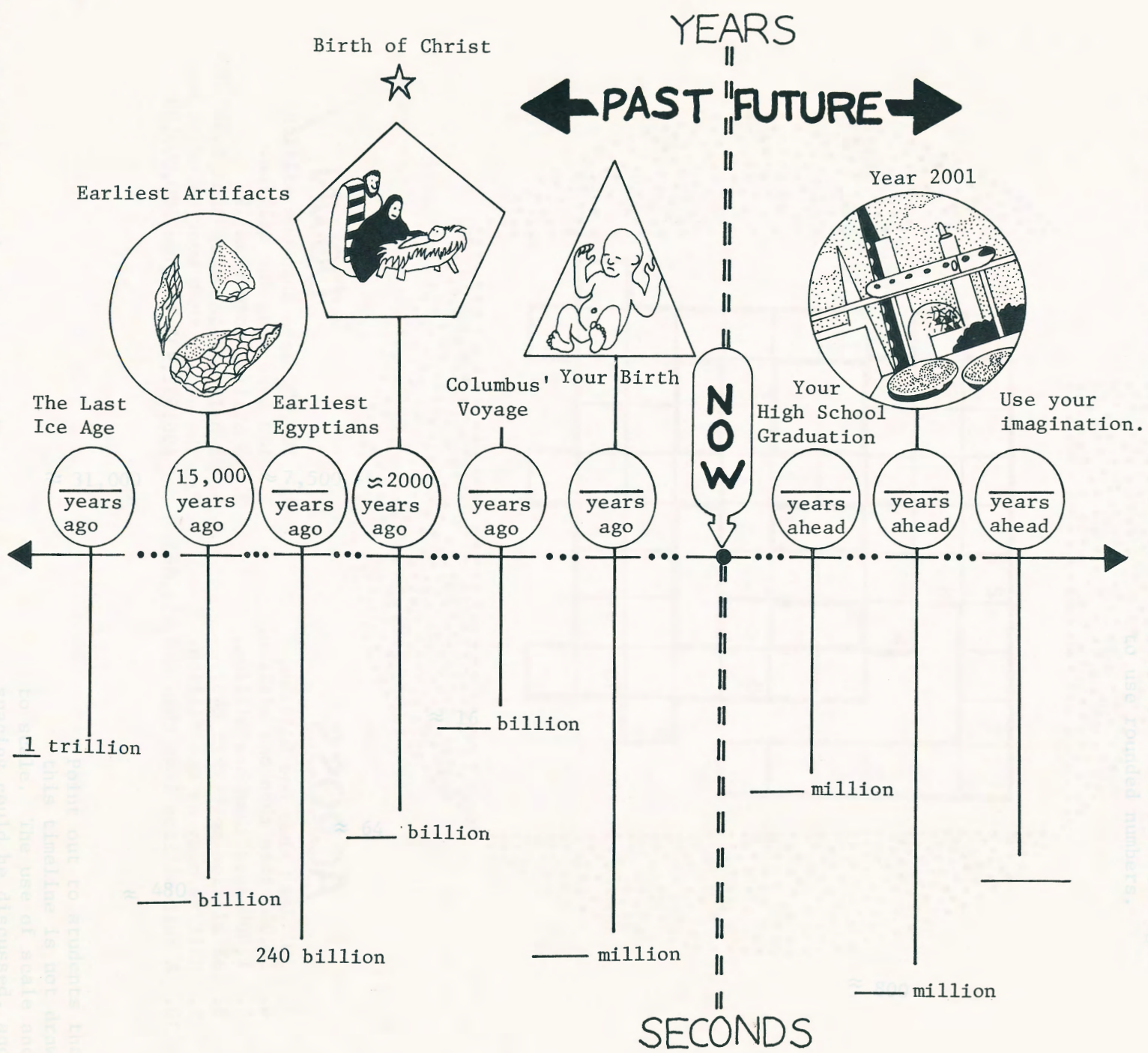
Longer than...
Awareness
LARGE & SMALL NUMBERS



FACTS

1 minute = _____ seconds
1 hour = _____ seconds

1 day = _____ seconds
1 year = _____ seconds



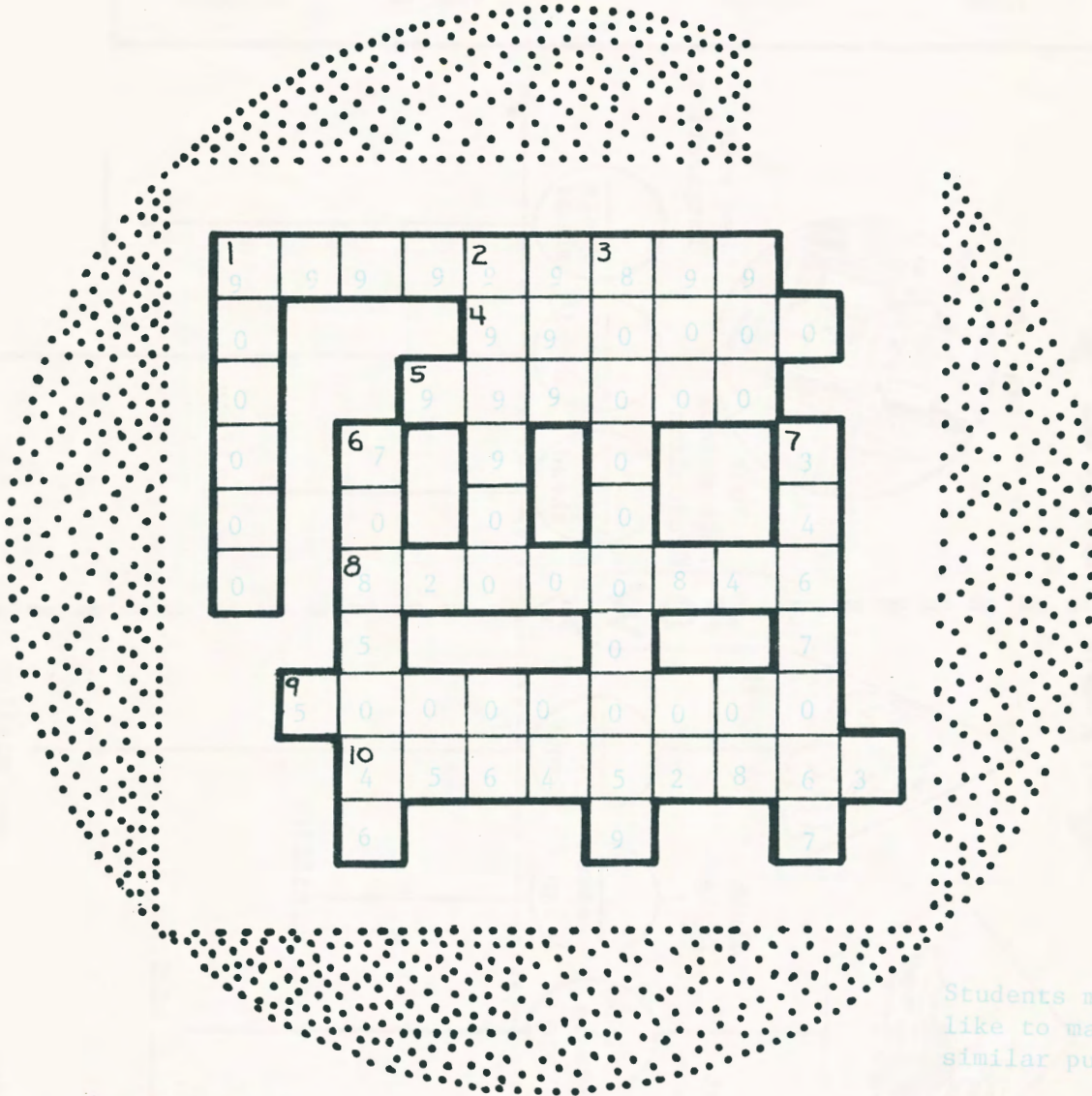


GOING BIG TIME

Computation—Large
Awareness
LARGE & SMALL NUMBERS



Complete the puzzle by writing the answer for each problem in the appropriate squares.



Students might like to make up similar puzzles.

ACROSS

1. 101 less than one billion.
4. 10,000 less than one million.
5. 1,000 less than one million.
8. 82 million more than 846.
9. Half as many as one billion.
10. A half-million less than 456,952,863

DOWN

1. 100,000 less than one million.
2. 100 less than one million.
3. 8 billion more than 59.
6. A half-million more than 6,585,046
7. The amount that must be added to 496,532,933 to give 500,000,000.

A FREE SUNDAE

TEACHER IDEA

The following is a problem for students that uses the problem-solving strategies of simplifying the problem, building up, and looking for a pattern.

After reading the story let students experiment with the problem. Some may develop their own methods of solution. After some discussion, hand out the student page.

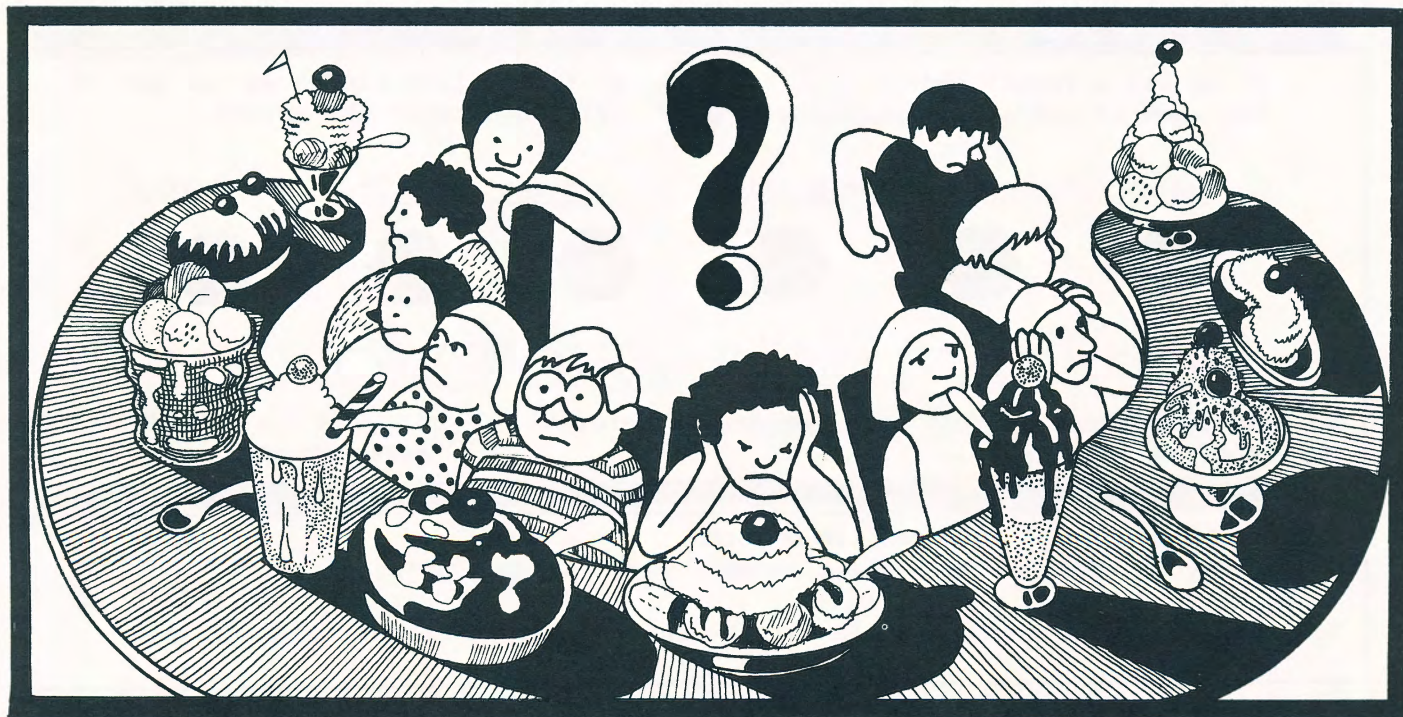
You might follow up the activity by having students consider how many ways they could sit in their classroom. Would the arrangement of the chairs have an effect on the solution?

Ten kids decided to celebrate their graduation from junior high school with sundaes at Farrell's. When they got together, they started arguing as to which seat each should occupy at the soda counter. Some of them wanted to sit alphabetically, others according to age, still others according to height, etc. The argument went on and on, and none would sit down. The problem was solved by the waiter.

He said, "Stop arguing. Sit down where you are and listen to what I have to say. One of you write down the order in which you are now sitting. Return here tomorrow and sit down in a different order. Keep doing this until you have tried all the ways. When the time again comes for you to sit down at the places where you are now sitting, I promise to serve you free of charge anything that you want."

The suggestion was tempting, and the kids decided to meet at Farrell's every day and to try every possible way of sitting at the counter.

That day of the free sundae, however, never came, and not because the waiter failed to keep his word, but because there are too many different ways for ten kids to sit at a counter--in fact, 3,628,800 of them. And to try all of them, it would take almost 10,000 years.




A FREE SUNDAE (CONTINUED)

STUDENT PAGE

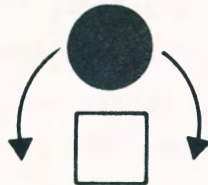
Let's look at some simpler problems:
There is only one way to arrange one object.



If we add a second object  there are two places we can put it.


1. Before the square

2. Behind the square



We have twice as many arrangements as before or

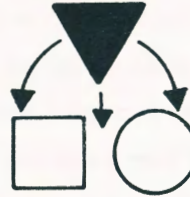
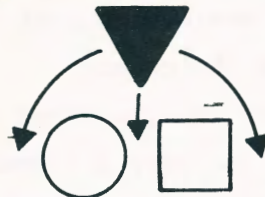
$$1 \times 2 = 2 \text{ arrangements}$$

If we add a third object  there are three places we can put it.

1. Before

2. Between

3. Behind



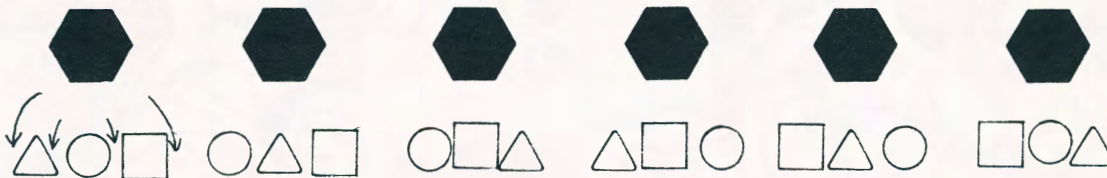
We have three times as many arrangements as before.

$$1 \times 2 \times 3 = 6 \text{ arrangements}$$

If we had a fourth object for each of our six arrangements.

there are four places we can put it
Finish drawing the arrows.

1. Before 2. Between 1st & 2nd 3. Between 2nd & 3rd 4. Behind



We have four times as many arrangements as before.

We can write this as:

$$1 \times 2 \times 3 \times 4 = 24 \text{ arrangements}$$

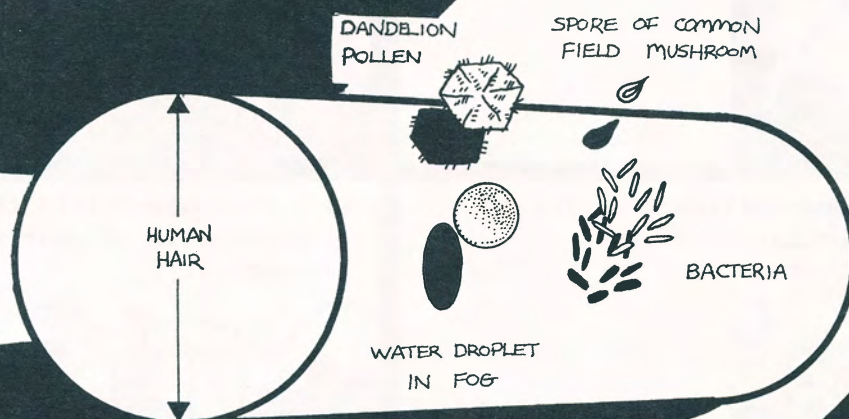
Explain how we could arrange five objects.

Could they be arranged in 5 times as many ways as the 4 objects?

Now, how many ways could ten kids sit at the counter in Farrell's?

LET'S ~~SPLIT~~ HAIRS

THIS IS A THICKNESS OF HAIR MAGNIFIED 600 TIMES.
THE OTHER ITEMS ON THE HAIR ARE DRAWN TO SCALE.



GET A SCIENCE BOOK OR ENCYCLOPEDIA TO FIND THE SIZE AND SHAPE OF OTHER THINGS. DRAW PICTURES MAGNIFYING YOUR THINGS IN PROPORTION TO THIS HAIR.

HERE ARE SOME SUGGESTIONS:

- 1) RED CORPUSCLES
- 2) VARIOUS SMALL INSECTS (LOUSE, FLEA, ETC.)
- 3) GRAIN OF SAND (OR ANY OTHER CRYSTALS)
- 4) ALGAE (VARIOUS DIFFERENT TYPES)
- 5) FUNGI (MUSHROOMS, SMALLER FUNGI)
- 6) DIATOMS
- 7) CELLS (COMPARE HUMAN OR ANIMAL CELLS TO PLANT CELLS)
- 8) MOLECULES
- 9) PROTOZOA
- 10) VIRUSES

MAKE A BULLETIN BOARD OF SMALL THINGS.

TYPE: Bulletin Board/Activity

IDEA FROM: *Arithmetical Excursions: An Enrichment of Elementary Mathematics*

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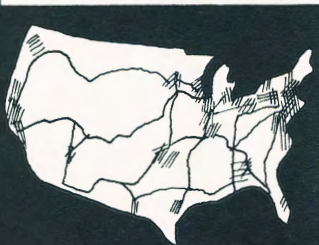


ONE-MILLIONTH ONE-MILLIONTH

Computation
Awareness
LARGE & SMALL NUMBERS



- 1) Find one-millionth of the total length of roads in the United States.



(1961)
3,710,299 miles
One-millionth is
 ≈ 3.7 miles.
5,971,114 km
One-millionth is
 ≈ 6 km.

- 2) Find one-millionth of the total length of the coastline of the United States.



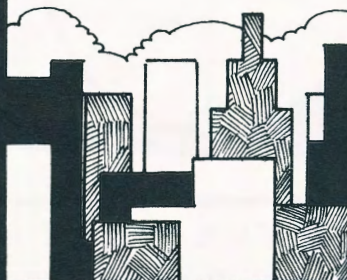
(1961)
12,400 miles
One-millionth is
 ≈ 65.5 ft.
19,960 km
One-millionth is
 ≈ 20 m.

- 3) Find one-millionth of the total population of the United States.



204,816,000 people
One-millionth is
 ≈ 204 people.

- 4) Find one-millionth of the population of your state; your city.



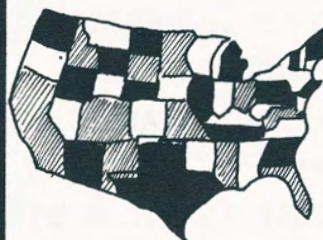
Students will need information on the population of your state and city.

- 5) Find one-millionth of the total land and water area of the United States.



3,676,000 sq. miles
One-millionth is
 ≈ 3.7 sq. miles.
9,520,000 sq. km
One-millionth is
 ≈ 9.5 sq. km.

- 6) Find one-millionth of the total area of your state; your city.



Students will need information on the area of your state and city.

- 7) Find one-millionth of the distance across the United States.



From San Francisco to New York
2,570 miles
One-millionth is
 ≈ 13.5 ft.
4170 km
One-millionth is
 ≈ 4.2 m.

- 8) Find one-millionth of the distance between _____ and _____.



Fill in two places your students would be familiar with.



MORE INVESTIGATIONS

Computation
Awareness
LARGE & SMALL NUMBERS



TEACHER IDEAS

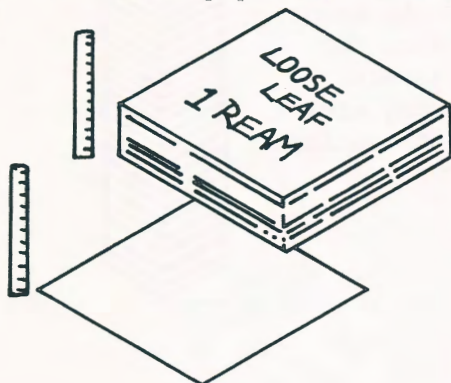
1

Island X has a population of 1,000 people. Every 30 years the population of the island doubles. What will the population be in 30 years, 60 years,..., 300 years? When will the population be over a million?
over a billion?



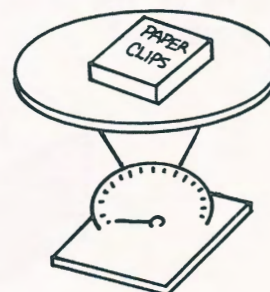
2

How thick is a sheet of paper?
Hint: First measure the thickness of a ream of paper. How many sheets of paper are in a ream?



3

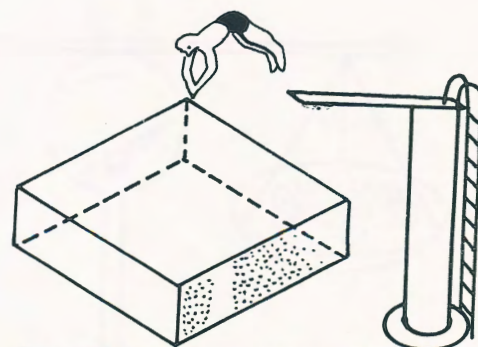
How heavy is a paper clip?
Hint: First weigh a new box of clips. How many clips are in the box? How about the weight of the box?



4

Do you drink enough water in one year to fill a swimming pool?

Most of these investigations are designed to be open-ended. Students will need to determine their own method for finding answers, and there will probably be no one "right answer." The investigations can be used as start questions for in-depth class activities; as interest-getters for the beginning of a class period; or as ideas for extra credit.



Be selective - some of the investigations are much more difficult than others.

TYPE: Paper & Pencil/Activity

IDEA FROM: *Aftermath*, Volume 1

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MORE INVESTIGATIONS

(CONTINUED)

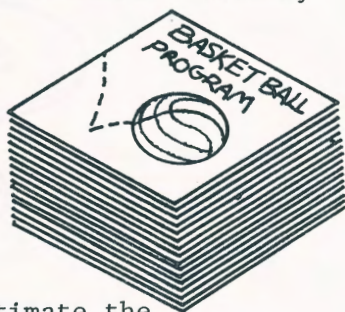
- 5 Suppose we have a line of people stretching from New York City to Los Angeles. They are standing side-ways and holding hands. The first person in line (in New York City) squeezes the hand of the 2nd person in line, who in turn squeezes the hand of the 3rd person, and so on. How long would it take before the last person in Los Angeles had his hand squeezed in this "chain" reaction?



- 6 How would you estimate the amount of blood that should be stored in blood banks at the hospital for emergencies? Find some information about blood banks from your nearest hospital.



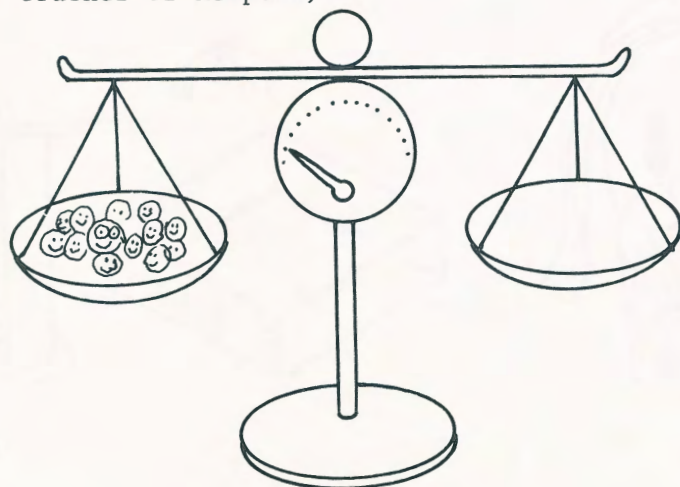
- 7 You need to order programs to be sold at the next home basketball game. How would you estimate the number for the printer? Some facts you need to find out first: 1. How many are expected? (Past ticket sales, will the opposing team help draw a crowd?) 2. What fraction of the attendance usually buys programs?



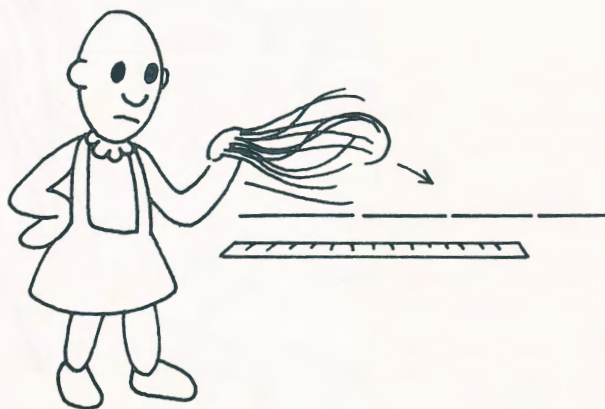
- 8 Estimate how much it would cost to buy a mathematics textbook for every student in your school. Estimate how much money is spent on paper for worksheets, quizzes, etc. in mathematics classes.



- 9 If $1 \text{ kg} \approx 2.2 \text{ lbs.}$, estimate the total weight in kilograms of everyone in your class (including the teacher or helpers).



- 10 Estimate in metres the total length of hair on your head. If you were to have it cut, how much total length would be taken off?



NUMBER SENSE AND ARITHMETIC SKILLS

Placement Guide for Tabbed Dividers

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Large Numbers on the Abacus - I	802	Powers of ten	Paper and pencil Manipulative

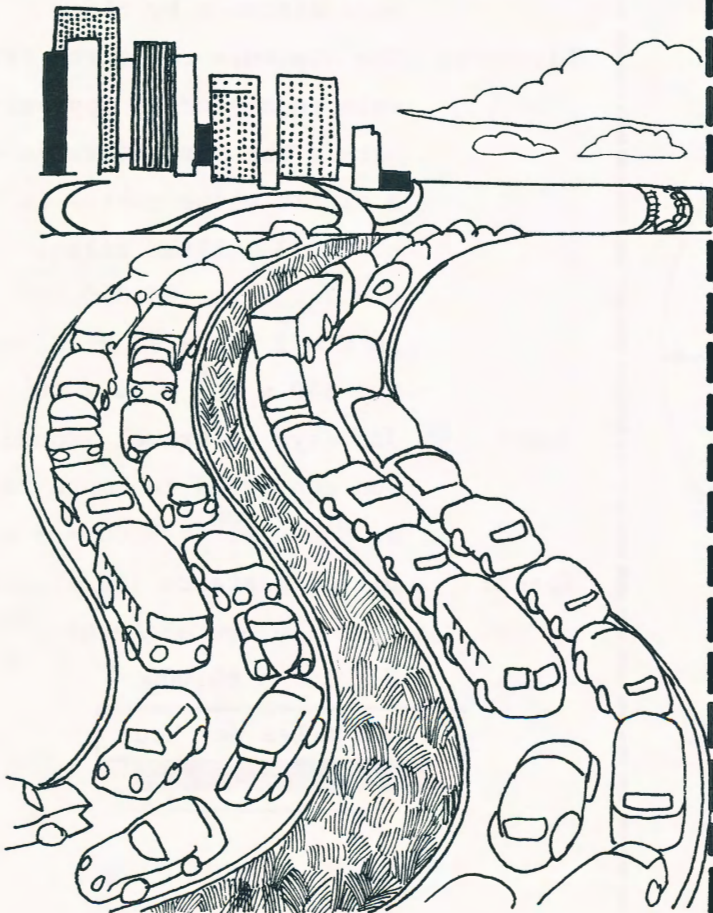
<u>TITLE</u>	<u>PAGE</u>	<u>TOPIC</u>	<u>TYPE</u>
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Treasure Hunt	810	Scientific notation	Paper and pencil Puzzle
I Didn't Planet That Way	811	Scientific notation	Paper and pencil Transparency
Scientific Facts	812	Scientific notation	Paper and pencil
Grains of Sand and Turning Wheels	813	Scientific notation	Paper and pencil

LARGE AND SMALL NUMBERS: EXPONENTIAL NOTATION

[illegible]

There are reasons to study exponents other than as a readiness activity for algebra. When a very basic understanding of exponents is combined with approximate calculations and mental arithmetic we have the power to perform calculations which ordinarily are considered difficult. Let's look at a few situations where exponents can facilitate our calculations.

A 1973 report indicated that 23 of 25 busiest highway sections in the U.S. are in Chicago. In fact, one half-mile stretch of the Dan Ryan Expressway carries about 254,700 vehicles a day.



Q. About how many vehicles does this section carry in one year?

Think: (vehicles per day) x (days per year)

$$254,700 \times 365 =$$

Approximation: $250,000 \times 400 =$

Exponentiation: $25 \times 10^4 \times 4 \times 10^2 =$
 $100 \times 10^6 = 10^8 =$
 $100,000,000 \text{ a year}$

Q. About how many vehicles does this section carry in a minute?

Think: $(\text{vehicles per day}) \div (\text{minutes per day})$

$$254,700 \div (24 \times 60)$$

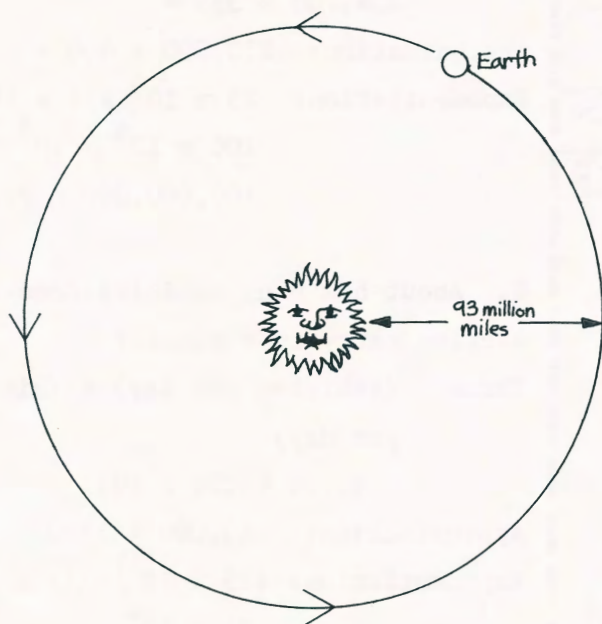
Approximation: $250,000 \div 1500$

Exponentiation: $(25 \times 10^4) \div (15 \times 10^2) =$
 $\frac{25 \times 10^4}{15 \times 10^2} \approx 2 \times 10^2 = 200$
 200 a minute

An environmental magazine claims that each year over 50 billion cans are discarded in the U.S.



The earth is approximately 93 million miles from the sun.



Q. Is this a reasonable claim?

There are over 200 million (200×10^6) people in the U.S. Figuring about 4 people per family gives 50 million (50×10^6) families.

Think: 50 billion cans distributed among 50 million

$$50 \text{ billion} \div 50 \text{ million}$$

$$\text{Exponentiation: } 50 \times 10^9 \div 50 \times 10^6 = \frac{50 \times 10^9}{50 \times 10^6} = 10^3$$

1000 cans per family
per year
or
about 3 cans a day

Q. About how fast is the earth moving about the sun?

Think: To compute the speed we divide distance by time.

Distance: The distance the earth travels in a year is approximately the circumference of a circle whose radius is about 93 million miles.

$$C = 2\pi r = 2\pi (93,000,000)$$

$$\text{so } C \approx 2 \times 3 \times 90 \times 10^6 \text{ or}$$

$$C \approx 540 \times 10^6 = 54 \times 10^7$$

Time: 365 days to travel around the sun converted to hours is 365×24 or $360 \times \frac{100}{4} = 9000 = 9 \times 10^3$

Speed: Divide distance (in miles) by time (in hours) to get $\frac{54 \times 10^7}{9 \times 10^3} = 6 \times 10^4 = 60,000$

60,000 miles per hour

The value of these kinds of problems or situations is that they utilize the four operations (+, -, ÷, x), large and small numbers, exponents, approximate calculations and the logic needed to decide what operation to use in a given situation. What is even more important is that there are problems available for many student interests. Newspapers and magazines will provide sufficient material. Students can bring articles of interest to school and explore the mathematical significance of those articles.

If we decide that the problem situations above are a worthy goal, what knowledge about exponents must be acquired? A general introduction to exponents will usually have varying exponents (e.g., 2^5 , 4^3 , 3^2 , etc.). Once the basic definition has been mastered it may be appropriate to concentrate on powers of ten. Most applications at the beginning level will use powers of ten because they are so closely related to our numeration system. When working with applications that use powers of ten there are several ideas that are needed.

- a. 10^n means n factors of 10, and the product of n factors of 10 can be written as 1 followed by n zeros. The names thousand, million, billion and trillion are associated with 10^3 , 10^6 , 10^9 and 10^{12} respectively.
- b. Any number can be expressed in many different ways. A number like 69730 can be expressed as 6973×10 , 69.730×10^3 , $.69730 \times 10^5$, etc.
- c. Products and quotients of powers of 10 can be computed.

$$10^5 \times 10^3 = \underbrace{10 \times 10 \times 10 \times 10 \times 10}_5 \times \underbrace{10 \times 10 \times 10}_3 = 10^8$$

$$\frac{10^5}{10^3} = \frac{10 \times 10 \times 10 \times 10 \times 10}{10 \times 10 \times 10} = 10 \times 10 = 10^2$$

$$\frac{10^3}{10^5} = \frac{10 \times 10 \times 10}{10 \times 10 \times 10 \times 10 \times 10} = \frac{1}{10 \times 10} = \frac{1}{10^2}$$

d. There are many other minor, but important, ideas that will need to be reviewed while working problems and applications. For example,

i) $\frac{32 \times 10^2}{4}$ is $\frac{32}{4} \times 10^2$ and not $\frac{32}{4} \times \frac{10^2}{4}$

ii) $7 \times (3 \times 10^3)$ is $(7 \times 3) \times 10^3$ and not $7 \times 3 \times 7 \times 10^3$

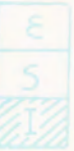
iii) $\frac{6 \times 10^4}{15}$ can be easily handled by renaming it as $\frac{60 \times 10^3}{15} = 4 \times 10^3$

iv) $(3 \times 10^4) \times (4 \times 10^3)$ can be rewritten as $3 \times 4 \times 10^4 \times 10^3$ or 12×10^7

Up to this point we have considered only positive powers of 10. Some students may be ready to move on to zero and negative powers of ten. These notations become much more complicated and need not be rushed. The definition $10^0 = 1$ seems strange to some students and using negative exponents too early usually causes confusion.

STATEMENT

Motivation
Exponential Notation
LARGE & SMALL NUMBERS



DISCOVER THE PATTERN.

MISSISSIPPI	→	M ¹ I ³ S ⁴ P ²	→	I ³ M ¹ P ² S ⁴
ILLINOIS	→	I ³ L ² N ¹ O ² S ¹	→	I ³ L ² N ¹ O ² S ¹
CONNECTICUT	→	C ² O ¹ N ² E ² T ¹ I ¹ U ¹	→	C ² E ¹ I ² N ² O ² T ¹ U ¹

FILL IN THE BLANKS.

ARKANSAS	→	_____	→	_____
COLORADO	→	_____	→	_____
_____	→	T ¹ E ⁴ N ² S ²	→	_____
_____	→	H ² A ¹ W ¹ I ²	→	_____
_____	→	_____	→	A ² C ¹ E ¹ H ¹ M ¹ S ⁴ T ² U ¹
_____	→	_____	→	A ² D ¹ I ² N ²

USE THE PATTERN TO
FINISH DECODING THE
FOLLOWING QUESTION.

A³C¹E²F²I²N²O³R¹S²T²U²Y¹

AAACEEFF UUY

A R E Y O U I N A
S T A T E O F
C O N F U S I O N ?

TYPE: Paper & Pencil/Puzzle

IDEA FROM: *Fun and Games with Mathematics*

787

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AN EXPONENT EXPERIENCE

Exponential notation is a NON-SECRET CODE used to tell how often a number is used as a factor.

3^4 means $3 \times 3 \times 3 \times 3$.
How often is 3 used as a factor? _____

$3^4 = 3 \times 3 \times 3 \times 3 =$ _____ (multiply and find the answer.)

base $\rightarrow 3^4 \leftarrow$ exponent

We sometimes say that 3^4 or 81 is the fourth power of 3.

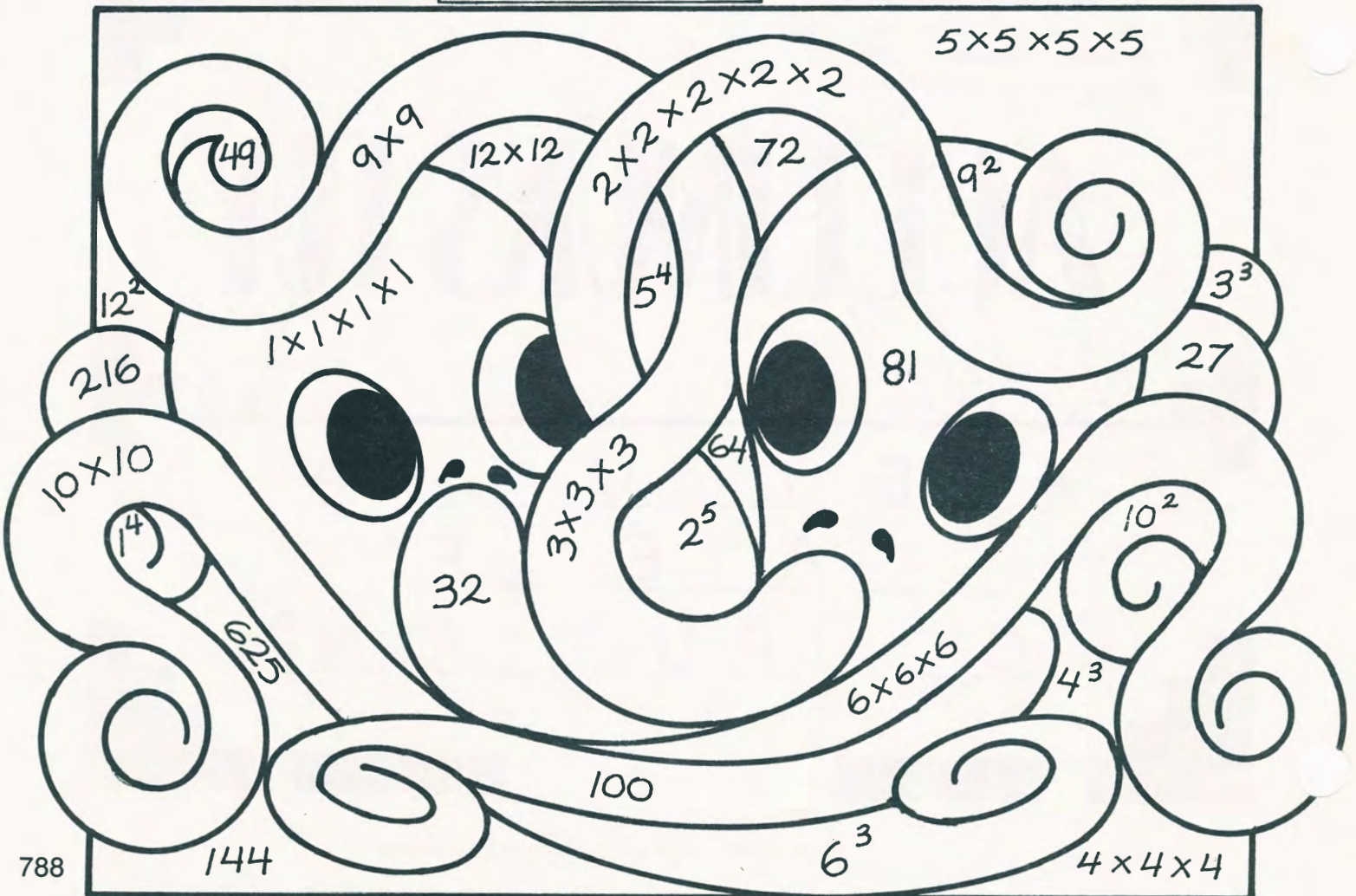
What is the fifth power of 3? _____ or _____
the fourth power of 2? _____ or _____

Fill in the blanks in the table and then follow the color code to color the picture below.

RED
$9^2 = 9 \times 9 = 81$
$10^2 = \quad \times \quad =$
$\quad = \quad = 27$

YELLOW
$\quad = \quad = 144$
$\quad = 4 \times 4 \times 4 =$
$5^4 = \quad =$
$\quad = \quad = 49$

BLUE
$1^4 = \quad =$
$\quad = 6 \times 6 \times 6 =$
$\quad = 2 \times 2 \times 2 \times 2 \times 2 =$



A POWERful PATTERN

This is an exercise designed to show that $n^1 = n$ and $n^0 = 1$.

Exponential Notation
LARGE & SMALL NUMBERS



Note the pattern and fill in the missing numbers.

$$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$$

$$3^4 = 3 \times 3 \times 3 \times 3 = 81$$

$$3^3 = 3 \times 3 \times 3 = 27$$

$$3^2 = 3 \times 3 = 9$$

$$3^1 = 3 = 3$$

$$3^0 = \text{~~~~~} = \text{Hexagon}$$

CLUE:

$$(81 = 243 \div 3)$$

$$(27 = 81 \div 3)$$

$$(9 = 27 \div 3)$$

$$(3 = 9 \div 3)$$

$$(\text{Hexagon} = \text{~~~} \div 3)$$

What is the pattern?

Did you put "1" in the hexagons?

Complete the following. Find a pattern for each.

$$2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$$

CLUE:

$$2^5 = \text{~~~~~} = \text{Circle}$$

$$(\text{Circle} = 64 \div 2)$$

$$2^4 = \text{~~~~~} = \text{~~~~~}$$

$$= \div$$

$$2^3 = \text{~~~~~} = \text{~~~~~}$$

$$= \div$$

$$2^2 = \text{~~~~~} = \text{~~~~~}$$

$$= \div$$

$$2^1 = \text{~~~~~} = \text{~~~~~}$$

$$= \div$$

$$2^0 = \text{~~~~~} = \text{~~~~~}$$

$$= \div$$

$$10^5 = \text{~~~~~} = 100,000$$

$$10^4 = \text{~~~~~} = \text{~~~~~}$$

$$10^3 = \text{~~~~~} = \text{~~~~~}$$

$$10^2 = \text{~~~~~} = \text{~~~~~}$$

$$10^1 = \text{~~~~~} = \text{~~~~~}$$

$$10^0 = \text{~~~~~} = \text{~~~~~}$$

$$4^4 = 256$$

$$4^3 = 64$$

$$4^2 = \text{~~~~~}$$

$$4^1 = \text{~~~~~}$$

$$4^0 = \text{~~~~~}$$

PREDICTIONS:

$$23^0 = \text{~~~~~}$$

$$17^1 = \text{~~~~~}$$

$$194^0 = \text{~~~~~}$$

$$942^1 = \text{~~~~~}$$

$$17215^0 = \text{~~~~~}$$

$$297,213^1 = \text{~~~~~}$$

This works well as a teacher directed activity using the chalkboard or the overhead.

Power Paths

Computation
Exponential Notation
LARGE & SMALL NUMBERS



Complete the tables for powers of 2, 3, 4, and 5. Check your answers by shading a path of the powers of each number from the start to the center.

$2^0 = 1$	$2^4 = 16$
$2^1 = 2$	$2^5 = 32$
$2^2 = 4$	$2^6 = 64$
$2^3 = 8$	$2^7 = 128$
$2^8 = 256$	

$3^0 = 1$	$3^4 = 81$
$3^1 = 3$	$3^5 = 243$
$3^2 = 9$	$3^6 = 729$
$3^3 = 27$	$3^7 = 2187$
$3^8 = 6561$	

START

2

START

3

START

5

START

4

$5^0 = 1$	$5^4 = 625$
$5^1 = 5$	$5^5 = 3125$
$5^2 = 25$	$5^6 = 15625$
$5^3 = 125$	$5^7 = 78125$
$5^8 = 390625$	

$4^0 = 1$	$4^4 = 256$
$4^1 = 4$	$4^5 = 1024$
$4^2 = 16$	$4^6 = 4096$
$4^3 = 64$	$4^7 = 16384$
$4^8 = 65536$	

Do some of the shaded numbers belong to more than one group? Why?
Must some of the shaded numbers belong to only one of the groups? Why?

TYPE: Paper & Pencil/Puzzle



Computation
Exponential Notation
LARGE & SMALL NUMBERS



WHAT IS AN OCTOPUS?

To find out connect each number in column A to its equivalent in column B, then connect that expression in column B to its equivalent in column C. Passing from A to B to C, your path will cross a letter and a number. The number tells you where to put the letter in the line of boxes.

(A)		(B)		(C)
$.4^3$		7×7		81
5^2	(N)	5×3	(5)	12
$(\frac{1}{3})^3$	(I)	$3 \times 3 \times 3 \times 3$.064
7^2	(D)	5×5	(2)	49
$(\frac{1}{2})^4$		$.4 \times .4 \times .4$	(4)	32
3^4	(T)	$\frac{1}{10} \times \frac{1}{10}$	(10)	25
5^3		$\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$		125
$.2^2$	(A)	$2 \times 2 \times 2 \times 2$	(9)	$\frac{1}{27}$
$(\frac{1}{10})^2$		$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$	(1)	15
2^5	(C)	$5 \times 5 \times 5$	(7)	.04
	(E)	3×4	(3)	$\frac{1}{100}$
		$.2 \times .2$		$\frac{1}{16}$
				.4

1	2
A	N

3	4	5	6	7
E	I	G	H	T

8	4	9	3	9
S	I	D	E	D

10	1	7
C	A	T

EXPONENT MAGIC SQUARE

Computation
Exponential Notation
LARGE & SMALL NUMBERS



Place the answers to each of the following sixteen problems in the corresponding boxes of the bottom square.

MAKING YOUR OWN EXPONENT MAGIC SQUARES

Magic squares made from multiples of squares and cubic numbers will generate a variety of similar problems using lots of exponents.

Example: (Using consecutive multiples of four)

32	4	24
12	20	28
16	36	8

will generate

2^5	2^2	$2^3 \times 3$
3×2^3	5×2^2	7×2^4
4^2	3×2^3	2^3

1 $2^2 \times 13'$	2 $2^2 \times 3'$	3 2^3	4 2^6
5 2^5	6 $2^2 \times 10'$	7 $2^2 \times 11'$	8 $2^2 \times 5'$
9 $4^2 \times 3'$	10 $2^3 \times 3'$	11 $2^2 \times 7'$	12 6^2
13 2^2	14 $2^2 \times 15'$	15 $2^3 \times 7'$	16 2^4

For information on constructing magic squares see Magic Squares in WHOLE NUMBERS: Addition/Subtraction.



If your answers are correct, you have formed a 4 x 4 magic square. What is the magic sum?

136

1 52	2 12	3 8	4 64
5 32	6 40	7 44	8 20
9 48	10 24	11 28	12 36
13 4	14 60	15 56	16 16

TYPE: Paper & Pencil/Puzzle

CHEERY SEQUENCES

TEACHER DIRECTED ACTIVITY

Start with any counting number. Square the digits and sum the squares. Continue the process and see what happens.

Consider 29

$$\begin{aligned} 29 &\rightarrow 2^2 + 9^2 = 85 \\ 85 &\rightarrow 8^2 + 5^2 = 89 \\ 89 &\rightarrow 8^2 + 9^2 = 145 \\ 145 &\rightarrow 1^2 + 4^2 + 5^2 = 42 \\ 42 &\rightarrow 4^2 + 2^2 = 20 \\ 20 &\rightarrow 2^2 + 0^2 = 4 \\ 4 &\rightarrow 4^2 = 16 \\ 16 &\rightarrow 1^2 + 6^2 = 37 \\ 37 &\rightarrow 3^2 + 7^2 = 58 \\ 58 &\rightarrow 5^2 + 8^2 = 89 \end{aligned}$$

(PROCESS CONTINUES FOREVER)

Consider 121

$$\begin{aligned} 121 &\rightarrow 1^2 + 2^2 + 1^2 = 6 \\ 6 &\rightarrow 6^2 = 36 \\ 36 &\rightarrow 3^2 + 6^2 = 45 \\ 45 &\rightarrow 4^2 + 5^2 = 41 \\ 41 &\rightarrow 4^2 + 1^2 = 17 \\ 17 &\rightarrow 1^2 + 7^2 = 50 \\ 50 &\rightarrow 5^2 + 0^2 = 25 \\ 25 &\rightarrow 2^2 + 5^2 = 29 \\ 29 &\rightarrow 85 \rightarrow 89 \rightarrow \dots \rightarrow 37 \end{aligned}$$

Consider 23

$$\begin{aligned} 23 &\rightarrow 2^2 + 3^2 = 13 \\ 13 &\rightarrow 1^2 + 3^2 = 10 \\ 10 &\rightarrow 1^2 + 0^2 = 1 \\ 1 &\rightarrow 1^2 = 1 \\ &\text{(FOREVER)} \end{aligned}$$

For the above examples, we say that the Cheery Sequences for 29, 121, and 23 are:

For 29: 29, 85, 89, 145, 42, 20, 4, 16, 37, 58, 89, . . .
For 121: 121, 6, 36, 45, 41, 17, 50, 25, 29, 85, . . ., 16, 37, . . .
For 23: 23, 13, 10, 1, 1, 1, . . .

Any number which generates a 1 (one) in its Cheery Sequence is said to be HAPPY. 23 is a happy number.

Any number which generates a 37 in its Cheery Sequence is said to be SAD or LONELY. 29 and 121 are sad numbers.

If calculators are available, have students work in pairs, one using the calculator, the other recording results. Investigate some of the following questions:

1. Do all counting numbers have a Cheery Sequence which generates either a 1 or a 37?
2. Are there any consecutive counting numbers, both of which are HAPPY?
3. Is there an infinite number of HAPPY numbers?
4. Are there strings of more than two consecutive HAPPY numbers?

Challenge: Does Happiness depend on the base in which a number is expressed?

$$23_{10} = 43_{\text{five}}$$

$$43_{\text{five}} \rightarrow 4^2 + 3^2 = 31_{\text{five}} + 14_{\text{five}} = 100_{\text{five}}$$

$$100_{\text{five}} \rightarrow 1^2 + 0^2 + 0^2 = 1$$

TYPE: Paper & Pencil/Activity

IDEA FROM: "Happy Integers," *The Mathematics Teacher*, Nov. 1972

Permission to use granted by the National Council of Teachers of Mathematics

CHEERY SEQUENCES

CONTINUED

A. Consider the word EASTER. If we assign a value to each letter of the alphabet (A = 1, B = 2, C = 3, . . . , Z = 26), we find that EASTER = 68, a HAPPY number.

1. Find some HAPPY words.
2. Is your name a HAPPY or SAD name?
3. Many other questions may come to mind.

B. Investigate what happens when you cube the digits.

$$37 \rightarrow 3^3 + 7^3 = 370$$

$$370 \rightarrow 3^3 + 7^3 + 0^3 = 370$$

Do all numbers generate one of the following numbers?
370, 371, 153 or 407

Examine: $97 \rightarrow 9^3 + 7^3 = \underline{\hspace{2cm}}$
 $\rightarrow \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

C. How about the fourth power of the digits? Fifth power? . . .

Here are some interesting results.

$$1^3 + 5^3 + 3^3 = 153$$

$$4^5 + 1^5 + 5^5 + 1^5 = 4151$$

$$3^3 + 7^3 + 1^3 = 371$$

$$8^4 + 2^4 + 0^4 + 8^4 = 8208$$

D. $(1 + 6 + 9)^2 = 256$

$$(2 + 5 + 6)^2 = 169$$

E. Investigate this result:

$$41 \rightarrow (4 + 1)^3 = 5^3 = 125$$

$$125 \rightarrow (1 + 2 + 5)^3 = 8^3 = 512$$

$$512 \rightarrow (5 + 1 + 2)^3 = 8^3 = 512$$

NOTE: Most of these investigations will be much more rewarding for students if calculators are used.



Power of Two
Exponential Notation
LARGE & SMALL NUMBERS



GREAT-GREAT-GREAT-...GRANDPARENTS

Have you ever wondered about the number of ancestors you have had over the centuries? Find out how many ancestors you had 7 centuries ago.

To solve this problem you need an estimate of how many generations of parents and grandparents live each century. Let's say 3 generations in each century.

One way to solve this would be to make a table of your "greats" in each century.

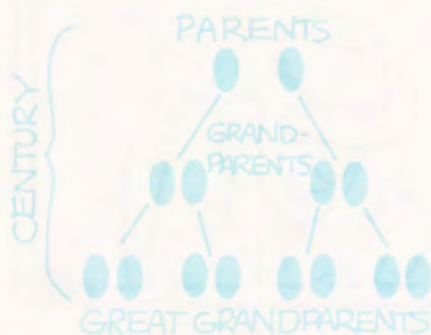
Clue for filling in your table:

First, you had two parents. Each of your parents had 2 parents, so you had 4 or $(2 \times 2 \text{ or } 2^2)$ grandparents. Each of your 4 grandparents had 2 parents, so you had 8 or $(4 \times 2 \text{ or } 2 \times 2 \times 2 \text{ or } 2^3)$ great-grandparents.

Complete the rest of your table.

Decide how much, if any, of this table to supply for your students.

You may wish to introduce the problem with a tree diagram, and then use the table on this page to summarize the results.



CENTURIES	GENERATIONS	RELATIVES	NUMBER OF RELATIVES
1	1	PARENTS	2
	2	GRAND-PARENTS	2 ²
	3	GREAT-GRANDPARENTS	2 ³
2	4	Great-great-grandparents	2 ⁴ or 16
	5	(2) Great-grandparents	
	6	(4) Great-grandparents	

For a more challenging problem give the students the problem and let them develop tactics to solve it.

IDEA FROM: *Arithmetical Excursions: An Enrichment of Elementary Mathematics*

TYPE: Paper & Pencil

Permission to use granted by Dover Publications, Inc.



SIMPLE SUMS

Powers of Two
Exponential Notation
LARGE & SMALL NUMBERS



Calculate: $1 + 2 = \underline{3}$

$2^2 = 2 \times 2 = \underline{4}$

$1 + 2 + 4 = \underline{7}$

$2^3 = 2 \times 2 \times 2 = \underline{8}$

$1 + 2 + 4 + 8 = \underline{15}$

$2^4 = \underline{16}$

$1 + 2 + 4 + 8 + 16 = \underline{31}$

$2^5 = \underline{32}$

Do you see a pattern? Explain.

Use the pattern to do these.

$1 + 2 + 4 + 8 + 16 + 32 = \underline{63}$ (6 terms)

$2^6 = \underline{64}$

$1 + 2 + 4 + \dots + 64 = \underline{127}$ (7 terms)

$2^7 = \underline{128}$

$1 + 2 + 4 + \dots + 128 = \underline{255}$ (8 terms)

$2^8 = \underline{256}$

Continue using the pattern.

$1 + 2 + 4 + \dots + 256 = \underline{511}$ (9 terms) $2^9 = \underline{512}$

$1 + 2 + 4 + \dots + 512 = \underline{1023}$ (10 terms) $2^{10} = \underline{1024}$

What is $1 + 2 + 4 + \dots + 1024$?

Can you do the problem in your head?

Double Agent Dan is sent on a dangerous 20-day mission. The bad guys offer him \$1,000.00 to work for them. The good guys offer him 1¢ the first day, 2¢ the second, 4¢ the third, and they keep doubling the amount each day. Who should he work for and why?



He would make \$10,485.75 working for the good guys.
 $.01 + .02 + .04 + .08 + \dots + 5242.88$ (20 terms) $= 2^{20} - 1$.

GENERAL SOLUTION:

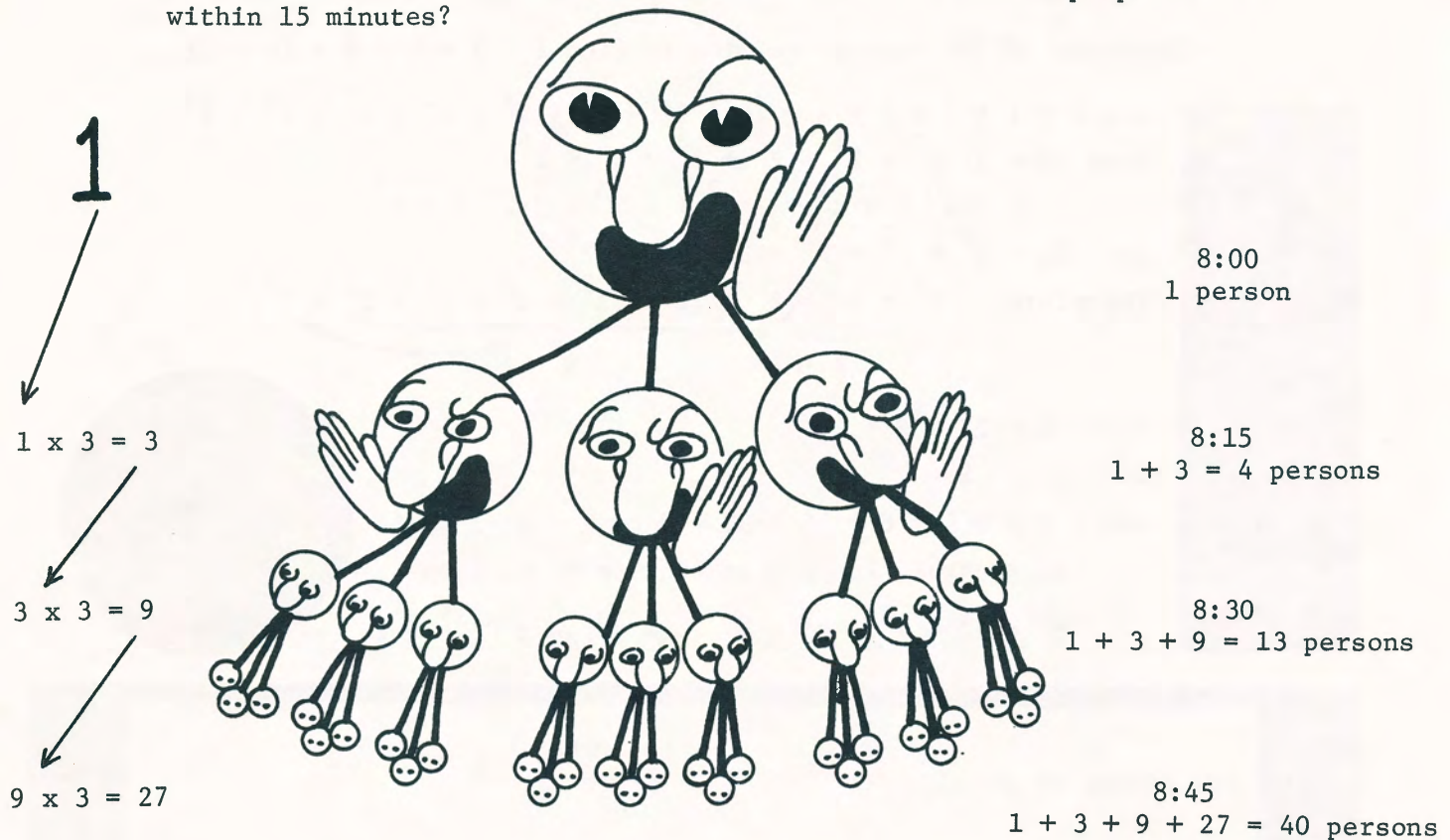
$1 + 2 + 4 + \dots + 2^n = 2^{(n+1)} - 1$

See Jan's *Handwriting* which follows *Math* for a more detailed explanation.

RUMORS

OR I HEARD IT THROUGH THE GRAPE VINE

How long would it take to spread a rumor in a town of 80,000 people if each person who hears the rumor tells it to three new people within 15 minutes?



TIME	NEW PEOPLE	TOTAL PEOPLE	
8:00	$3^0 = 1$	1	= 1
8:15	$3^1 = 3$	$1 + 3$	= 4
8:30	$3^2 = 3 \times 3 = 9$	$1 + 3 + 9$	= 13
8:45	$3^3 = 3 \times 3 \times 3 = 27$	$1 + 3 + 9 + 27$	= 40
9:00	$3^4 = 3 \times 3 \times 3 \times 3 = ?$	$1 + 3 + 9 + 27 + ?$	= _____
9:15			
9:30			
9:45			
10:00			
10:15			
10:30			
10:45			
11:00			

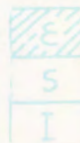
For sake of calculations, students can be shown the short cut suggested by the 3rd power, which follows.

Students could consider how long it would take if each person could tell 5 new people within 15 minutes; 10 new people, etc.

Personalize the activity by having the students find out how long it would take to spread a rumor through the entire school.

LET'S GENERALIZE

Power Series
Exponential Notation
LARGE & SMALL NUMBERS



TEACHER PAGE

Here is a general method for finding the sum of a sequence of powers for any counting number.

FOR POWERS OF 2: Suppose we wish to find $1 + 2 + 4 + 8 + 16 + 32$.

$$\text{Let } S = 1 + 2 + 4 + 8 + 16 + 32 = 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5$$

$$\text{Then } 2S = 2 \cdot (2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5)$$

$$= 2 \cdot 2^0 + 2 \cdot 2^1 + 2 \cdot 2^2 + 2 \cdot 2^3 + 2 \cdot 2^4 + 2 \cdot 2^5$$

$$\text{Or } 2S = 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6$$

$$\text{Therefore } 2^0 + 2S = \underbrace{2^0 + (2^1 + 2^2 + 2^3 + 2^4 + 2^5)}_{S + 2^6} + 2^6$$

$$\text{So } 1 + 2S = S + 2^6$$

$$\text{Or } S = 2^6 - 1$$

$$\text{and } 1 + 2 + 4 + 8 + 16 + 32 = 2^6 - 1 = 64 - 1 = 63$$

In general, for powers of 2 only, we have

$$2^0 + 2^1 + 2^2 + 2^3 + \dots + 2^n = 2^{(n+1)} - 1$$

FOR POWERS OF 3:

$$S = \frac{3^{(n+1)} - 1}{2}$$

To find $1 + 3 + 9 + 27 + 81 + 243 + 729 + 2187 + 6561 + 19683 + 59049$

$$(3^0 + 3^1 + 3^2 + 3^3 + 3^4 + 3^5 + 3^6 + 3^7 + 3^8 + 3^9 + 3^{10})$$

we need only calculate 3^{11} or $(3^{10} \times 3)$, subtract 1 and divide by 2.

$$\frac{(59049 \times 3) - 1}{2} = \frac{177146}{2} = 88,573$$

NOTE: By 10:30, 88,573 people would have heard the rumor, that is, in $2\frac{1}{2}$ hours.

FOR POWERS OF 4:

$$1 + 4^1 + 4^2 + 4^3 + \dots + 4^n = S$$

$$\text{can you show that } S = \frac{4^{(n+1)} - 1}{3}?$$

In general,

for any counting number, $a > 1$
 $a^0 + a^1 + a^2 + a^3 + \dots + a^n =$

$$\frac{a^{(n+1)} - 1}{a - 1}$$



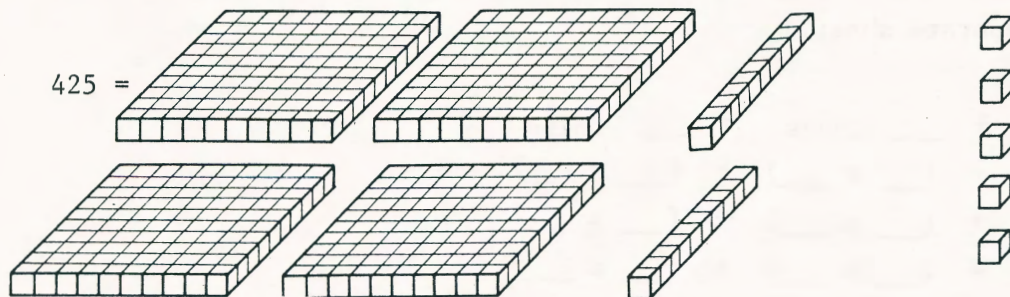
MULTIBASE BLOCKS - I

Expanded Notation
Base Ten
Exponential Notation
LARGE & SMALL NUMBERS



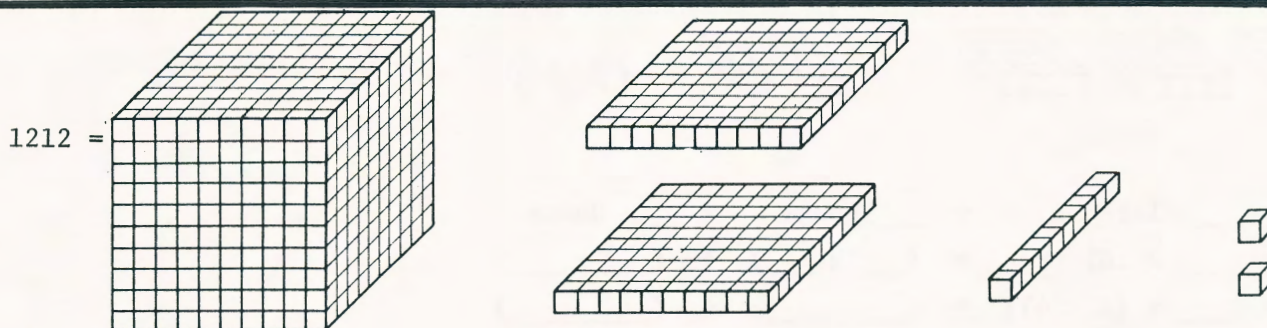
For information on the construction, use and readiness activities for Multibase Blocks see the section on Lab Materials.

Find the pattern. Fill in any missing blanks.



$$\begin{aligned} 425 &= 4 \text{ Flats} & + & 2 \text{ Longs} & + & 5 \text{ Units} \\ 425 &= (4 \times 100) & + & (2 \times 10) & + & (5 \times 1) \\ 425 &= [4 \times (10 \times 10)] & + & (2 \times 10) & + & (5 \times 1) \\ 425 &= (4 \times 10^2) & + & (2 \times 10^1) & + & (5 \times 10^0) \end{aligned}$$

This is called the
EXPONENTIAL FORM of
expanded notation.



$$\begin{aligned} 1212 &= \underline{\quad} \text{ Blocks} & + & \underline{\quad} \text{ Flats} & + & \underline{\quad} \text{ Longs} & + & \underline{\quad} \text{ Units} \\ 1212 &= (\underline{\quad} \times 1000) & + & (\underline{\quad} \times \underline{\quad}) & + & (\underline{\quad} \times \underline{\quad}) & + & (\underline{\quad} \times \underline{\quad}) \\ 1212 &= [\underline{\quad} \times (10 \times 10 \times 10)] & + & [\underline{\quad} \times (\underline{\quad} \times \underline{\quad})] & + & (\underline{\quad} \times \underline{\quad}) & + & (\underline{\quad} \times \underline{\quad}) \\ 1212 &= (\underline{\quad} \times 10^3) & + & (\underline{\quad} \times \underline{\quad}) & + & (\underline{\quad} \times \underline{\quad}) & + & (\underline{\quad} \times \underline{\quad}) \end{aligned}$$

2136 = (Cut out your own pieces from the cutout sheet and glue them in place here.)

(You will find a cutout sheet for these two pages following the next page.)

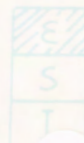
$$\begin{aligned} 2136 &= \underline{\quad} \text{ Blocks} & + & \underline{\quad} \text{ Flats} & + & \underline{\quad} \text{ Longs} & + & \underline{\quad} \text{ Units} \\ 2136 &= (\underline{\quad} \times 1000) & + & (\underline{\quad} \times \underline{\quad}) & + & (\underline{\quad} \times \underline{\quad}) & + & (\underline{\quad} \times \underline{\quad}) \\ 2136 &= [\underline{\quad} \times (\underline{\quad})] & + & [\underline{\quad} \times (\underline{\quad})] & + & (\underline{\quad} \times \underline{\quad}) & + & (\underline{\quad} \times \underline{\quad}) \\ 2136 &= (\underline{\quad} \times 10^3) & + & (\underline{\quad} \times \underline{\quad}) & + & (\underline{\quad} \times \underline{\quad}) & + & (\underline{\quad} \times \underline{\quad}) \end{aligned}$$

EXTENSIONS: Show larger
numbers using long blocks,
flat blocks, and block blocks.



MULTIBASE BLOCKS - II

Expanded Notation
Other Bases
Exponential Notation
LARGE & SMALL NUMBERS



Find the pattern. Fill in any missing blanks.

345 = (Glue pieces on a separate sheet.)

$$\begin{aligned} 345 &= \text{___ Flats} & + & \text{___ Longs} & + & \text{___ Units} \\ 345 &= (\text{___} \times \text{___}) & + & (\text{___} \times \text{___}) & + & (\text{___} \times \text{___}) \\ 345 &= [\text{___} \times (\text{___} \times \text{___})] & + & (\text{___} \times \text{___}) & + & (\text{___} \times \text{___}) \\ 345 &= (\text{___} \times \text{___}) & + & (\text{___} \times \text{___}) & + & (\text{___} \times \text{___}) \end{aligned}$$

You will find a cutout sheet for the multibase pieces on the next page.



$$\begin{aligned} 323_4 &= \text{___ Flats} & + & \text{___ Longs} & + & \text{___ Units} \\ 323_4 &= (\text{___} \times 16) & + & (\text{___} \times \text{___}) & + & (\text{___} \times \text{___}) \\ 323_4 &= [\text{___} \times (4 \times 4)] & + & (\text{___}) & + & (\text{___}) \\ 323_4 &= (\text{___} \times 4^2) & + & (\text{___}) & + & (\text{___}) \end{aligned}$$

345₇ = (Glue pieces on a separate sheet.)

$$\begin{aligned} 345_7 &= \text{___ Flats} & + & \text{___ Longs} & + & \text{___ Units} \\ 345_7 &= (\text{___} \times 7^2) & + & \text{___} & + & \text{___} \end{aligned}$$

1231 = (Glue pieces on a separate sheet.)

Remind students that no subscript implies base 10.

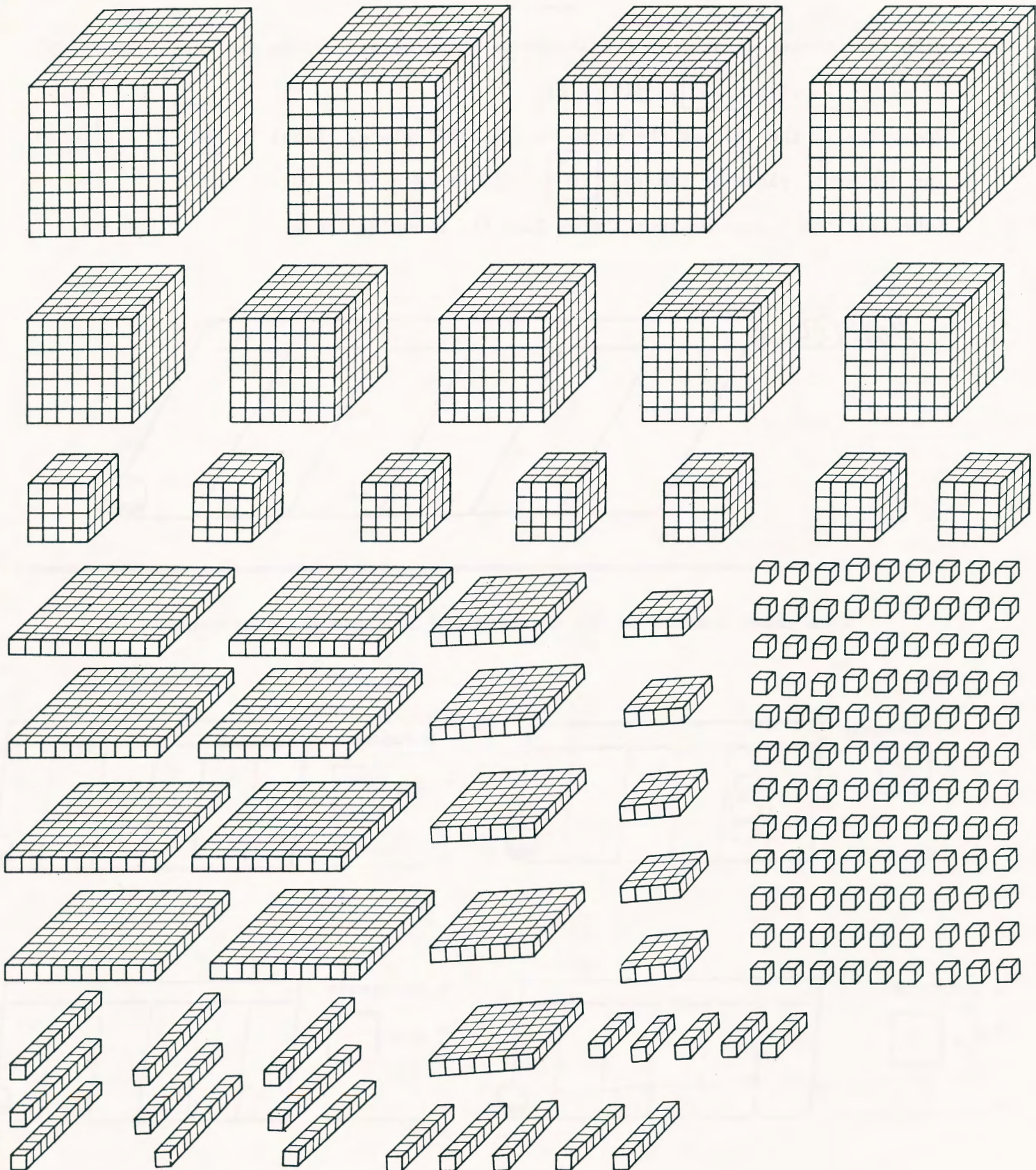
$$\begin{aligned} 1231 &= \text{___ Blocks} & + & \text{___ Flats} & + & \text{___ Longs} & + & \text{___ Units} \\ 1231 &= (\text{___} \times 10^3) & + & \text{___} & + & \text{___} & + & \text{___} \end{aligned}$$

1231₄ = (Glue pieces on a separate sheet.)

$$\begin{aligned} 1231_4 &= \text{___ Blocks} & + & \text{___ Flats} & + & \text{___ Longs} & + & \text{___ Units} \\ 1231_4 &= (\text{___} \times 4^3) & + & \text{___} & + & \text{___} & + & \text{___} \end{aligned}$$

Try some more of your own.

MULTIBASE BLOCK CUTOUTS (BASES 10, 7 AND 4)



LARGE NUMBERS ON THE ABACUS - I

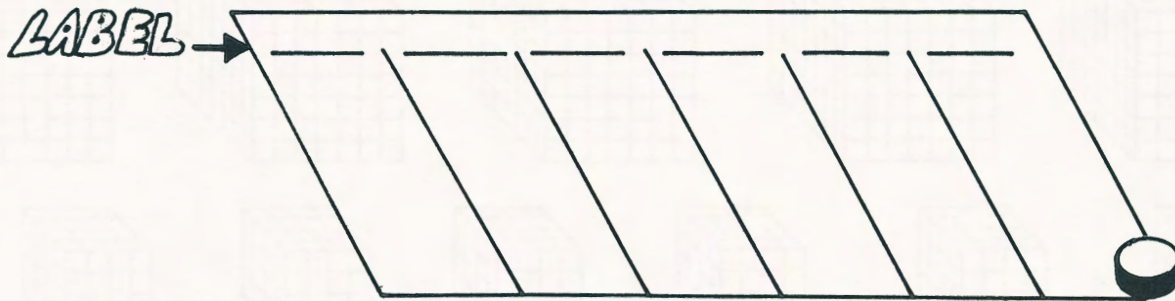
With the checker located on the abacus as shown below, locate and label the UNITS' place.

Label it 10^0 . ($10^0 = 1$)

One place to the left of the units is the TENS' place. Label it 10^1 . ($10^1 = 10$)

The HUNDREDS' place is labeled 10^2 . ($10^2 = 10 \times 10 = 100$)

Label all the places on your abacus like the examples above.



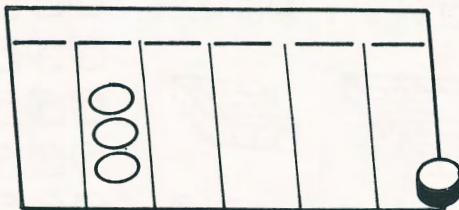
Show these numbers on the abacuses and complete the statements.

Example:

3 ten-thousands

$$= 3 \times 10^4$$

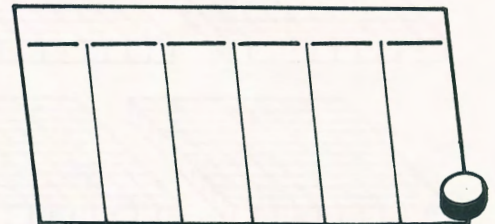
$$= 30,000$$



5 hundred-thousands

$$= 5 \times \square$$

$$= \underline{\hspace{2cm}}$$



2 hundreds

$$= 2 \times \square$$

$$= \underline{\hspace{2cm}}$$



4 thousands

$$= 4 \times \square$$

$$= \underline{\hspace{2cm}}$$



SMALL NUMBERS ON THE ABACUS - I

Exponential Notation
LARGE & SMALL NUMBERS

For information on the construction, use, and readiness activities for the Abacus see the Lab Materials section. Students can actually use an abacus at their seats and use the paper to record the results, or they can do the problems on the paper abacus provided on this page by drawing lines for wires.

With the checker located on the abacus as shown below, locate and label the UNITS' place.

Label it 10^0 . ($10^0 = 1$)

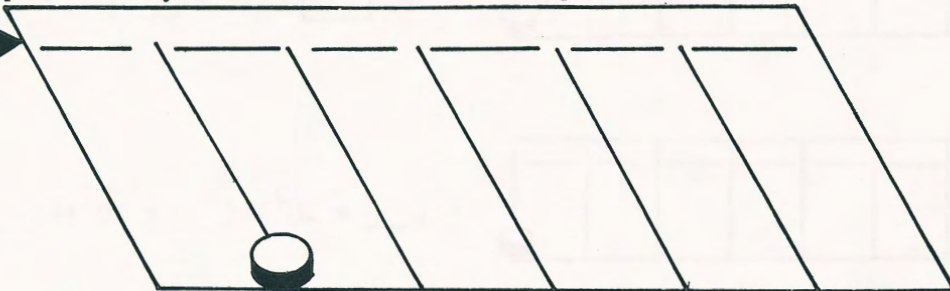
One place to the right of the units' place is the TENTHS' place.

Label it $\frac{1}{10^1}$. ($\frac{1}{10^1} = \frac{1}{10}$)

The HUNDREDTHS' place is labeled $\frac{1}{10^2}$. ($\frac{1}{10^2} = \frac{1}{10 \times 10} = \frac{1}{100}$)

Label all the places on your abacus like the examples shown.

LABEL →

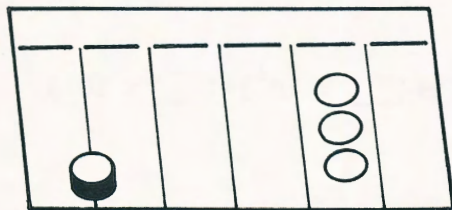


Show these numbers on the abacuses and complete the statements.

3 ten-thousandths

$$= 3 \times \frac{1}{10^4}$$

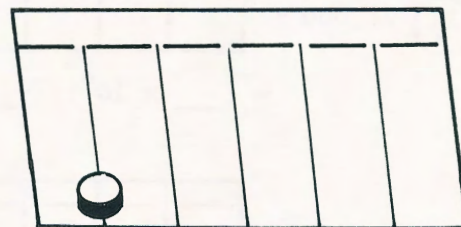
$$= \underline{.0003}$$



5 hundred-thousandths

$$= 5 \times \boxed{}$$

$$= \underline{}$$



2 hundredths

$$= 2 \times \boxed{}$$

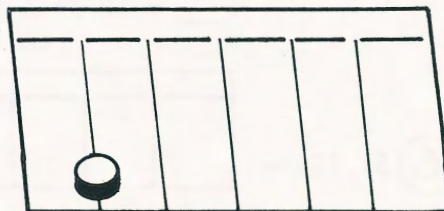
$$= \underline{}$$



4 thousandths

$$= 4 \times \boxed{}$$

$$= \underline{}$$

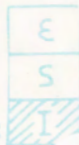




LARGE NUMBERS ON THE ABACUS - II

For information on the construction, use and readiness activities for the Abacus see the section on Lab Materials.

Expanded Notation
Base 10
Exponential Notation
LARGE & SMALL NUMBERS



Use the abacus to show these numbers. Fill in the blanks.

Example:

$$43 = \begin{array}{|c|c|c|c|c|c|} \hline 10^5 & 10^4 & 10^3 & 10^2 & 10^1 & 10^0 \\ \hline & & & & \text{4 beads} & \text{3 beads} \\ \hline \end{array} = (4 \times 10^1) + (3 \times 10^0)$$

① $25 = \begin{array}{|c|c|c|c|c|c|} \hline & & & & & \\ \hline \end{array} = (2 \times \boxed{}) + (5 \times \boxed{})$

② $312 = \begin{array}{|c|c|c|c|c|c|} \hline & & & & & \\ \hline \end{array} = (3 \times \boxed{}) + (1 \times \boxed{}) + (2 \times \boxed{})$

③ $503 = \begin{array}{|c|c|c|c|c|c|} \hline & & & & & \\ \hline \end{array} = (__ \times 10^2) + (__ \times 10^1) + (__ \times 10^0)$

④ $24,000 = \begin{array}{|c|c|c|c|c|c|} \hline & & & & & \\ \hline \end{array} = (__ \times 10^4) + (__ \times 10^3) + (__ \times 10^2) + (__ \times 10^1) + (__ \times 10^0)$

⑤ $2,430 = \begin{array}{|c|c|c|c|c|c|} \hline & & & & & \\ \hline \end{array} = (__ \times 10^3) + (4 \times \boxed{}) + (__ \times \boxed{}) + (__ \times \boxed{})$

⑥ $51,234 = \begin{array}{|c|c|c|c|c|c|} \hline & & & & & \\ \hline \end{array} = (__ \times \boxed{}) + (__ \times \boxed{}) + (__ \times \boxed{}) + (__ \times \boxed{}) + (__ \times \boxed{})$

TYPE: Paper & Pencil/Manipulative



SMALL NUMBERS ON THE ABACUS - II

For information on the construction, use, and readiness activities for the Abacus see the section on Lab Materials.

Expanded Notation

Base Ten

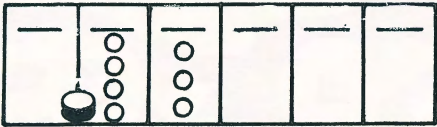
Exponential Notation

LARGE & SMALL NUMBERS




Use the abacus to show these numbers. Fill in the blanks.

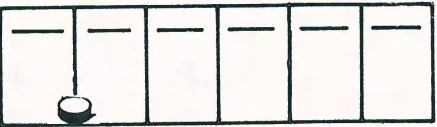
Example:


43 hundredths =  = $(4 \times \frac{1}{10^1}) + (3 \times \frac{1}{10^2})$


① 2 and 5 tenths =  = $(2 \times \boxed{}) + (5 \times \boxed{})$

② 32 hundredths =  = $(3 \times \boxed{}) + (2 \times \boxed{})$

③ 503 thousandths =  = $(\boxed{} \times \frac{1}{10^1}) + (\boxed{} \times \frac{1}{10^2}) + (\boxed{} \times \frac{1}{10^3})$

④ 24 hundred-thousandths =  = $(\boxed{} \times \frac{1}{10^4}) + (\boxed{} \times \frac{1}{10^5})$

⑤ 243 ten-thousandths =  = $(\boxed{} \times \frac{1}{10^2}) + (4 \times \boxed{}) + (\boxed{} \times \boxed{})$

⑥ 51 hundred-thousandths =  = $(\boxed{} \times \boxed{}) + (\boxed{} \times \boxed{})$

TYPE: Paper & Pencil/Manipulative

Directions: First, find the answer to a problem from the Problems and Answers Table.

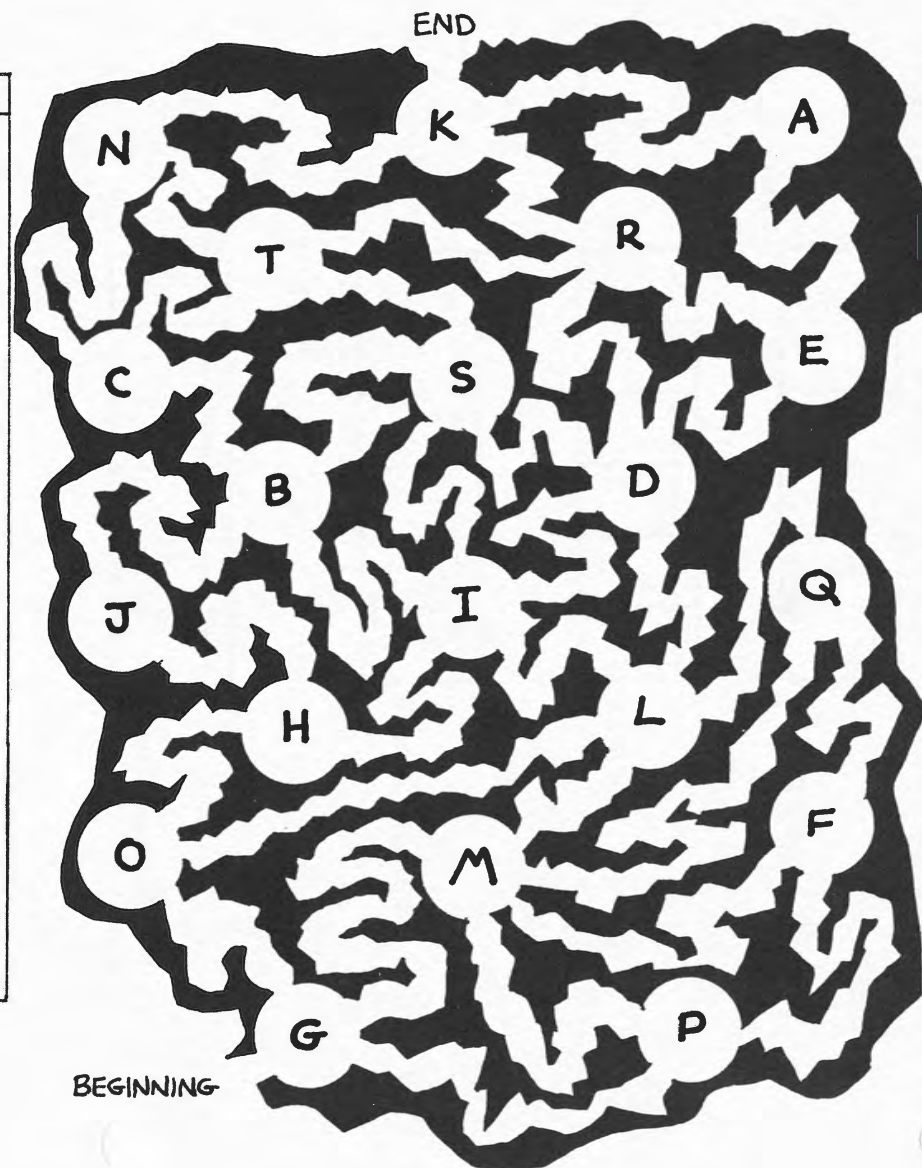
Second, in the Cave-Maze shade the path connecting the letter of the problem to the letter of the answer.

When you have finished, you should have a complete path from the BEGINNING to the END which is shaded. These letters will spell out what you found in the Cave-Maze.

PROBLEMS AND ANSWERS TABLE

ANSWERS		PROBLEMS
40,000.67	R	A $(5 \times 10^3) + (9 \times 10^1) =$
5900	P	G $(5 \times 10^3) + (9 \times 10^2) =$
190.706	G	R $(5 \times 10^3) + (9 \times 10^0) =$
203,084	C	D $(4 \times 10^0) + (6 \times \frac{1}{10^2}) + (7 \times \frac{1}{10^3}) =$
3,040,008	S	Q $(4 \times 10^4) + (6 \times 10^0) + (7 \times \frac{1}{10^2}) =$
5090	E	E $(4 \times 10^4) + (6 \times \frac{1}{10^1}) + (7 \times \frac{1}{10^2}) =$
109.076	O	B $(2 \times 10^3) + (3 \times 10^0) + (8 \times \frac{1}{10^3}) + (4 \times \frac{1}{10^5}) =$
230,840	D	S $(2 \times 10^5) + (3 \times 10^4) + (8 \times 10^2) + (4 \times 10^1) =$
5009	T	N $(2 \times 10^5) + (3 \times 10^3) + (8 \times 10^1) + (4 \times 10^0) =$
3,400,800	J	P $(1 \times 10^1) + (9 \times 10^0) + (7 \times \frac{1}{10^1}) + (6 \times \frac{1}{10^2}) =$
40,006.07	M	L $(1 \times 10^2) + (9 \times 10^0) + (7 \times \frac{1}{10^2}) + (6 \times \frac{1}{10^3}) =$
3,000,000.048	A	O $(1 \times 10^2) + (9 \times 10^1) + (7 \times \frac{1}{10^1}) + (6 \times \frac{1}{10^3}) =$
19.76	F	H $(3 \times 10^6) + (4 \times 10^5) + (8 \times 10^2) =$
4.067	L	T $(3 \times 10^6) + (4 \times 10^4) + (8 \times 10^0) =$
2003.00804	I	K $(3 \times 10^6) + (4 \times \frac{1}{10^2}) + (8 \times \frac{1}{10^3}) =$

CAVE-MAZE



GOLD DIGGER

LET'S GET SCIENTIFIC

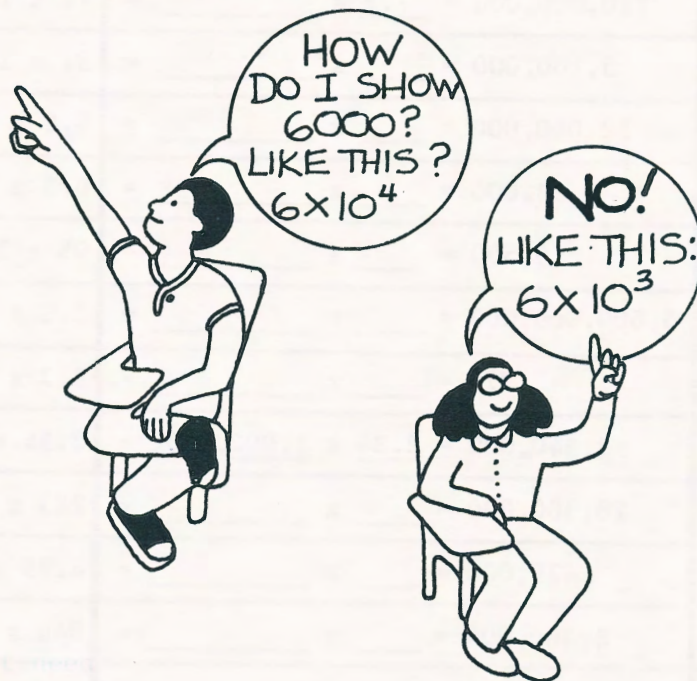
Study these sets: $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
 $B = \{10^0, 10^1, 10^2, 10^3, 10^4, 10^5, 10^6, 10^7, 10^8, 10^9, 10^{10}\}$

Do you know the numerical value of the members of set B?

$10^0 = \underline{\hspace{1cm}}$, $10^1 = \underline{\hspace{1cm}}$, $10^2 = \underline{\hspace{1cm}}$, $10^3 = \underline{\hspace{1cm}}$, $10^9 = \underline{\hspace{1cm}}$

Can you express each of the numbers below as a product of two numbers such that one number is from set A and the other from set B?

1. 6,000 =
2. 9,000,000 =
3. 200,000 =
4. 70,000 =
5. 500 =
6. 40,000,000 =
7. 30 =
8. 800,000,000 =
9. 6 =
10. 20,000,000,000 =



Your answers to problems 1 - 10 are written in scientific notation.

Definition: Scientific notation is a symbol that expresses any number as a power of ten multiplied by some number between 1 and 10 (including 1).

GETTING MORE SCIENTIFIC...

Scientific Notation
Exponential Notation
LARGE & SMALL NUMBERS



A number is in
scientific notation
when it is written as



a number which
is at least one
but less than ten



a power
of ten

Examples:

$$\begin{array}{llllll} 36,000 = 3.6 & \times & 10,000 = & 3.6 & \times & 10^4 \\ 2,820 = 2.82 & \times & 1,000 = & 2.82 & \times & 10^3 \\ 1,987,000 = 1.987 & \times & 1,000,000 = & 1.987 & \times & 10^6 \end{array}$$

For each of the fifteen problems below, circle the correct answer that is in scientific notation. Place the letter in the circled boxes in the appropriate spaces below to find the rest of the message.

THERE IS NO FOOL LIKE AN OLD FOOL...

1	480,000 = <u>4.8</u> x <u>100,000</u> =	4.8 x 10 ²	A	4.8 x 10 ⁷	T	4.8 x 10 ⁵	Y
2	720,000,000 = <u>7.2</u> x _____ =	72 x 10 ⁸	H	7.2 x 10 ⁸	O	72 x 10 ⁷	R
3	3,100,000 = _____ x _____ =	31 x 10 ⁵	E	3.1 x 10 ⁵	L	3.1 x 10 ⁶	U
4	52,000,000 = _____ x _____ =	5.2 x 10 ⁷	J	5.2 x 10 ⁸	C	5.2 x 10 ⁹	R
5	83,000 = _____ x _____ =	8.3 x 10 ³	K	83 x 10 ⁴	O	8.3 x 10 ⁴	S
6	9,500 = _____ x _____ =	95 x 10 ⁴	L	9.5 x 10 ³	T	95 x 10 ²	E
7	5,600,000,000 = _____ x _____ =	5.6 x 10 ⁸	N	5.6 x 10 ⁹	C	5.6 x 10 ⁷	O
8	370 = _____ x _____ =	3.7 x 10 ²	A	370	E	37 x 10 ¹	I
9	2,340,000 = <u>2.34</u> x <u>1,000,000</u> =	2.34 x 10 ⁶	N	23.4 x 10 ⁵	T	2.34 x 10 ⁵	B
10	28,300,000 = _____ x _____ =	283 x 10 ⁵	S	2.83 x 10 ⁷	B	28.3 x 10 ⁶	M
11	479,000 = _____ x _____ =	4.79 x 10 ³	G	4.79 x 10 ⁴	F	4.79 x 10 ⁵	E
12	8,460,000 = _____ x _____ =	846 x 10 ⁴	Z	8.46 x 10 ⁴	N	8.46 x 10 ⁶	X
13	537 = _____ x _____ =	5.37 x 10 ²	P	53.7 x 10 ²	L	5.37 x 10 ⁰	E
14	62,100 = _____ x _____ =	6.21 x 10 ²	A	6.21 x 10 ³	S	6.21 x 10 ⁴	R
15	958,000,000 = _____ x _____ =	95.8 x 10 ⁸	S	9.58 x 10 ⁸	I	95.8 x 10 ⁶	O

Y O U J U S T C A N T
1 2 3 4 3 5 6 7 8 9 6

R E A T E X P E R I E N C E
10 11 8 6 11 12 13 11 14 15 11 9 7 11

EVEN MORE SCIENTIFIC

Scientific Notation
Exponential Notation
LARGE & SMALL NUMBERS



How do you represent .005 in scientific notation?

Recall that $.005 = 5 \times \frac{1}{1000} = 5 \times \frac{1}{10^3}$ in expanded notation.

Is $5 \times \frac{1}{10^3}$ also in scientific notation? Check your definition.

Here are some more examples:

$$.07 = 7 \times \frac{1}{100} = 7 \times \frac{1}{10^2}$$

$$.0054 = 5.4 \times \frac{1}{1000} = 5.4 \times \frac{1}{10^3}$$

For each of the thirteen problems circle the correct answer that is in scientific notation. Place the letter in the circled boxes in the appropriate spaces below to find the rest of the message.

DON'T BE YOUR OWN WORST ENEMY....

1	$.00008 = \underline{\quad 8 \quad} \times \frac{1}{100000} =$	$.8 \times \frac{1}{10^5}$ Y	$8 \times \frac{1}{10^5}$ G	$8 \times \frac{1}{10^4}$ T
2	$.0004 = \underline{\quad 4 \quad} \times \underline{\hspace{1cm}} =$	$4 \times \frac{1}{10^3}$ H	$.4 \times \frac{1}{10^3}$ O	$4 \times \frac{1}{10^4}$ I
3	$.0000007 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} =$	$7 \times \frac{1}{10^7}$ V	$7 \times \frac{1}{10^6}$ U	$7 \times \frac{1}{10^5}$ E
4	$.002 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} =$	$2 \times \frac{1}{10^3}$ E	$.2 \times \frac{1}{10^2}$ N	$.2 \times \frac{1}{10^3}$ R
5	$.5 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} =$	$.5 \times 10^0$ M	$.5$ F	$5 \times \frac{1}{10^1}$ S
6	$.00009 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} =$	$.9 \times \frac{1}{10^4}$ A	$9 \times \frac{1}{10^5}$ O	$90 \times \frac{1}{10^6}$ E
7	$.38 = \underline{\hspace{1cm}} \times \frac{1}{10} =$	$38 \times \frac{1}{10^1}$ T	$3.8 \times \frac{1}{10^0}$ E	$3.8 \times \frac{1}{10^1}$ M
8	$.000047 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} =$	$4.7 \times \frac{1}{10^4}$ H	$4.7 \times \frac{1}{10^5}$ N	$4.7 \times \frac{1}{10^6}$ R
9	$.039 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} =$	$3.9 \times \frac{1}{10^1}$ P	$3.9 \times \frac{1}{10^2}$ L	$39 \times \frac{1}{10^3}$ E
10	$.0000021 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} =$	$2.1 \times \frac{1}{10^6}$ S	$2.1 \times \frac{1}{10^7}$ R	$21 \times \frac{1}{10^7}$ B
11	$.00284 = \underline{\hspace{1cm}} \times \frac{1}{1000} =$	$28.4 \times \frac{1}{10^3}$ F	$284 \times \frac{1}{10^3}$ S	$2.84 \times \frac{1}{10^3}$ A
12	$.000397 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} =$	$3.97 \times \frac{1}{10^4}$ C	$3.97 \times \frac{1}{10^3}$ O	$3.97 \times \frac{1}{10^6}$ D
13	$.00526 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} =$	$5.26 \times \frac{1}{10^5}$ R	$5.26 \times \frac{1}{10^3}$ H	$52.6 \times \frac{1}{10^4}$ L

G I V E S O M E O N E E L S E
1 2 3 4 5 6 7 4 6 8 4 4 9 10 4
A C H A N C E !
11 12 13 11 8 12 4

TYPE: Paper & Pencil/Puzzle

TREASURE HUNT

Each of the rooms in the maze contains a number. Only twelve of these numbers are the correct answers to the problems below.

Work each problem. Find the answers in the maze and circle them.

When you have finished connect some of the circled answers, so that you have a path from ENTER to the TREASURE that only goes through circled answers.



1) 7×10^3

5) 5.2×10^2

9) $4.8 \times \frac{1}{10^1}$

2) 4×10^7

6) 7.4×10^6

10) $7.4 \times \frac{1}{10^3}$

3) 2×10^5

7) 6.9×10^5

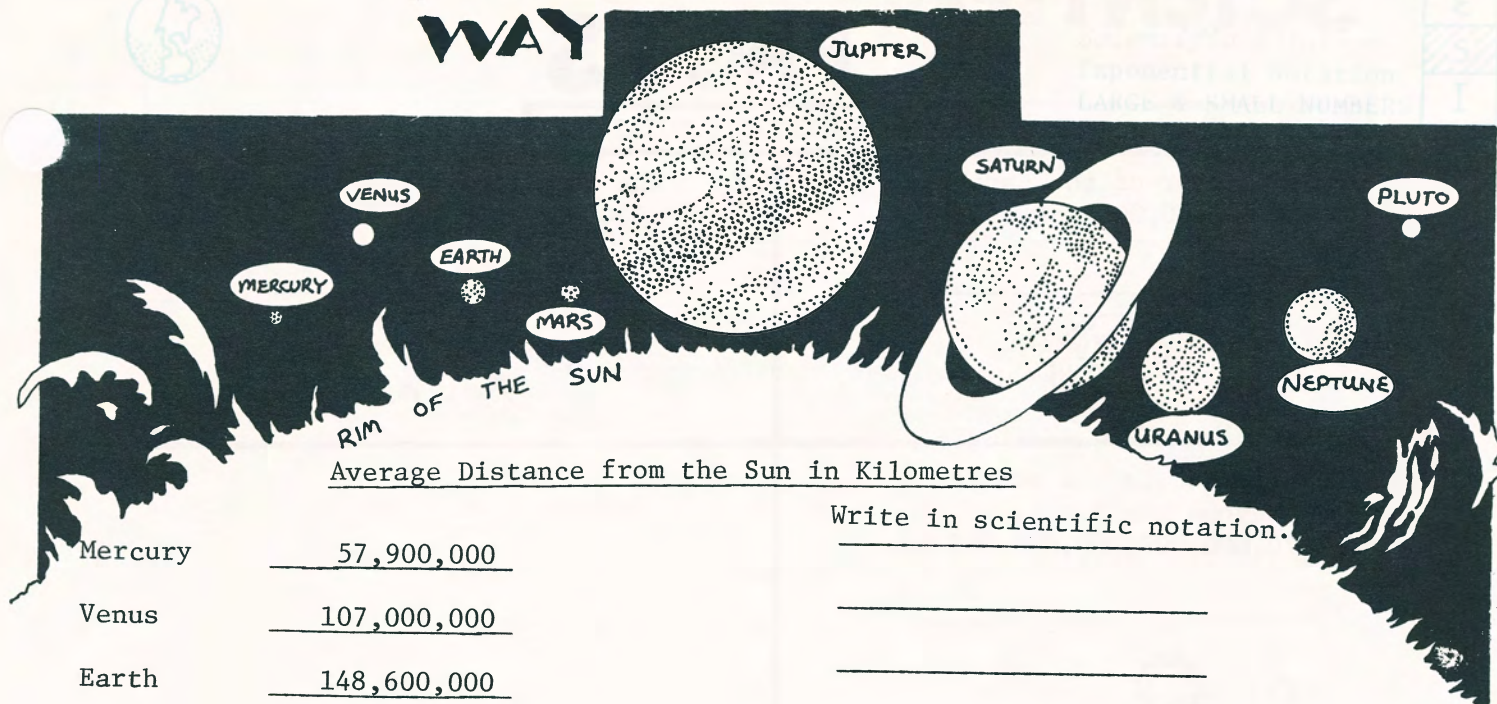
11) $5.2 \times \frac{1}{10^6}$

4) 5×10^4

8) 4.8×10^7

12) $6.9 \times \frac{1}{10^2}$

I DIDN'T PLANET THAT WAY



Average Distance from the Sun in Kilometres

Mercury	<u>57,900,000</u>
Venus	<u>107,000,000</u>
Earth	<u>148,600,000</u>
Mars	<u>225,900,000</u>
Jupiter	<u>772,800,000</u>
Saturn	<u>1,412,200,000</u>
Uranus	<u>2,853,900,000</u>
Neptune	<u>4,459,200,000</u>
Pluto	<u>5,797,000,000</u>

Write in scientific notation.

Are these numbers easier to compare in regular or scientific notation?

AGES

Scientists think that:

The Sun has existed approximately 5×10^{12} years.

The Earth has existed approximately 3×10^9 years.

Life has existed on earth approximately 1×10^9 years.

Humans have existed on earth approximately 5×10^5 years.

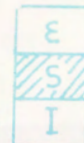
Write these numbers in regular notation, and with word phrases.

Sun _____ or five _____ years
 Earth _____ or three _____ years
 Life _____ or one _____ years
 Humans _____ or five hundred- _____ years



SCIENTIFIC FACTS

Scientific Notation
Exponential Notation
LARGE & SMALL NUMBERS



Write each of the following in SCIENTIFIC NOTATION.

- ① The diameter of the Earth is about 12,700,000 metres.



- ② The length of common viruses is .0000009 cm.



- ③ The distance to the nearest star beyond the Sun is 40,070,000,000,000,000,000 cm.



Write each of the following as a DECIMAL.

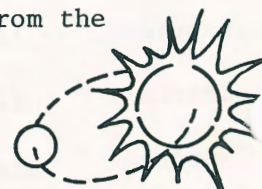
- ⑤ The diameter of a hydrogen atom is



$$1.35 \times \frac{1}{10^8} \text{ cm.}$$

- ⑥ The average distance from the Earth to the Sun is

$$1.5 \times 10^8 \text{ km.}$$



- ⑦ The time required for one addition problem on the IBM 7090 computer is

$$4.4 \times \frac{1}{10^6} \text{ seconds.}$$

- ⑧ The total number of ways of arranging 52 cards in a deck of cards is about

$$8.0658 \times 10^{67}.$$

After working this problem, students should see the advantages of writing numbers in scientific notation.

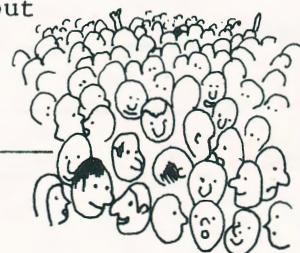
- ⑨ The probability of being dealt all 13 cards of one suit when dealing a whole deck of 52 cards to four people is

$$6.3 \times \frac{1}{10^{12}}.$$



- ⑩ The total number of people who have ever lived is about

$$6.4 \times 10^{10}.$$

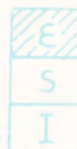




GRAINS OF SAND AND TURNING WHEELS

TEACHER IDEAS

Scientific Notation
Exponential Notation
LARGE & SMALL NUMBERS



Here are two examples of using approximation techniques to obtain estimates of large quantities.

I. HOW MANY REVOLUTIONS DOES AN AUTOMOBILE WHEEL MAKE ON A TRIP FROM NEW YORK CITY TO LOS ANGELES?

The diameter of an automobile wheel is about 76.2 cm, so the circumference is about 2.4 metres.

It is about 4800 km from New York City to Los Angeles.

SOLUTION:



Find the distance from N.Y. to L.A. in metres and divide this by the circumference of an automobile wheel in metres.

$$\frac{4800 \text{ km} \times 1000 \frac{\text{m}}{\text{km}}}{2.4 \text{ m}} = \frac{48,000,000}{24} = \boxed{2,000,000 \text{ revolutions}}$$

II. HOW MANY GRAINS OF SAND ARE NEEDED TO FILL A SPHERE THE SIZE OF THE EARTH?

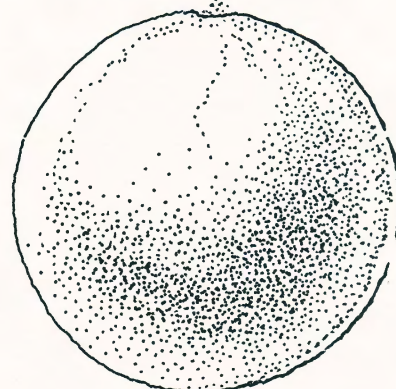
a) The average grain of sand is about $\frac{1}{400,000}$ of a cm^3 (one four hundred-thousandth of a cubic centimetre) in size. So, 400,000 grains of sand are needed to fill one cubic centimetre.

b) The radius of the earth is about 6400 km.

SOLUTION: Find the volume of the earth in cubic centimetres, then multiply by 400,000.

$$\begin{aligned} \text{Volume} &= \frac{4}{3} \times \pi \times (\text{radius})^3 \\ &= \frac{4}{3} \times 3.1416 \times (6400 \text{ km} \times 1000 \frac{\text{m}}{\text{km}} \times 100 \frac{\text{cm}}{\text{m}})^3 \\ &\approx \frac{4}{3} \times 3 \times (64 \times 10^7)^3 \text{ cm}^3 \\ &\approx \frac{4}{3} \times 3 \times (250,000 \times 10^{21}) \text{ cm}^3 \\ &= 4 \times (250,000 \times 10^{21}) \text{ cm}^3 \\ &= 1,000,000 \times 10^{21} \text{ cm}^3 \\ &= 1 \times 10^{27} \text{ cm}^3 \end{aligned}$$

changing radius
to centimetres



So, grains of sand $\approx 400,000 \times (1 \times 10^{27})$

$$\boxed{= 4 \times 10^{32}}$$

The greatest distance that one can see with the 200-inch Mt. Palomar Telescope is 7 billion light years (about 66×10^{21} km). A sphere whose radius is 7 billion light years can be filled with about 10^{90} grains of sand.

Now, does the Googol (10^{100}) have a little more meaning?

How about a Googolplex? ($10^{\text{googol}} = 10^{10^{100}}$).

NUMBER SENSE AND ARITHMETIC SKILLS

Placement Guide for Tabbed Dividers

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ANNOTATED BIBLIOGRAPHY

The following is a list of sources used in the development of this resource. It is not a comprehensive listing of materials available. In some cases, good sources have not been included simply because the project did not receive permission to use the publisher's materials or a fee requirement prohibited its use by the project.

ACCENT ON ALGEBRA. Patrick J. Boyle and William J. Juarez. Palo Alto, California: Creative Publications, Inc., 1972. (Creative Publications, Inc., P.O. Box 10328, Palo Alto, CA 94303)

126 pp; cloth; workbook; b/w; medium reading level

A collection of supplementary activities in the form of cross-word and cross number puzzles, alphametics and graphs on a variety of algebraic topics that can be reproduced for classroom use. The book can be used with students from upper elementary to junior college levels.

ACTIVITIES IN MATHEMATICS, Second Course. Donovan A. Johnson, Viggo P. Hanson, Wayne H. Peterson, Jesse A. Rudnick, Ray Cleveland, and L. Carey Bolster. Glenview, Illinois: Scott, Foresman and Company, 1973. (Scott, Foresman and Company, 1900 E. Lake Ave., Glenview, IL 60025)

432 pp; cloth; teacher's guide; color

A collection of 85 student activities covering four topics: graphs, statistics, proportions and geometry are contained in this book. Each of the four sections can be purchased separately in paperback format for student use.

ACTIVITIES WITH SQUARES FOR WELL-ROUNDED MATH. Nikki Bryson Schreiner. Palos Verdes Estates, California: Touch and See Educational Resources, 1973. (Touch and See Educational Resources, P.O. Box 794, Palos Verdes Estates, CA 90274)

119 pp; paper; workbook; b/w; low reading level

The book is a collection of task cards on several topics such as numeration, measurement, sets, numbers and operations. The cards can be reproduced for classroom use.

ADVENTURES WITH ARITHMETIC, Decimal. Lawrence N. Swienciki. Palo Alto, California: Creative Publications, Inc., 1974. (Creative Publications, Inc., P.O. Box 10328, Palo Alto, CA 94303)

48 pp; paper; workbook; b/w; medium reading level

This book contains twenty disguised drill activities on decimals which involve a story and related puzzle. Similar books are available on these topics: fractions, percent and algebra.

AFTERMATH, Volumes 1-4. Dale Seymour, Mary Laycock, Bob Larsen, Ruth Heller and Verda Holmberg. Palo Alto, California: Creative Publications, Inc., 1971. (Creative Publications, Inc., P.O. Box 10328, Palo Alto, CA 94303)

120 pp for each book; paper; workbook; b/w; low reading level

A collection of four booklets each including reproducible student worksheets on a variety of topics.

ARITHMETICAL EXCURSIONS: AN ENRICHMENT OF ELEMENTARY MATHEMATICS. Henry Bowers and Joan E. Bowers. New York: Dover Publications, Inc., 1961. (Dover Publications, Inc., 180 Varick St., New York, NY 10014)

320 pp; paper; b/w; teacher reference

This is a book of enrichment topics in mathematics ranging from counting through the arithmetic operations to figurate, perfect and amicable numbers and then on to mysteries and folklore of numbers. Exercises and answers are provided for each of the 27 topics.

THE ARITHMETIC TEACHER. Reston, Virginia: The National Council of Teachers of Mathematics. (National Council of Teachers of Mathematics, 1906 Association Dr., Reston, VA 22091)

This magazine, published eight times a year, January through May and October through December, contains a wealth of ideas and activities for the elementary and middle school mathematics teacher.

THE BALANCE BOOK. Lee Jenkins. Hayward, California: Activity Resources Company, Inc., 1974. (Activity Resources Company, Inc., P.O. Box 4875, Hayward, CA 94540)

62 pp; paper; workbook; b/w; low reading level

The booklet contains a group of activity cards that involve using combinations of weights on a math balance to explore the four basic operations with whole numbers. The cards can be duplicated for use with students.

BASIC SECONDARY MATHEMATICS PROGRAMS. Seattle Public Schools. Seattle: Seattle Public Schools, 1972. (Seattle Public Schools, Basic Skills Department, Mathematics Section, 515 W. Galer St., Seattle, WA 98119)

277 pp; paper; teacher resource; b/w

The book is an outline of the pre-algebra mathematics curriculum used in the Seattle School District middle and junior high schools. Included are objectives, time allotments, activities and tests.

BASIC VOCATIONAL MATHEMATICS, Part 1. Vocational-Technical Curriculum Laboratory. New Brunswick, New Jersey: Rutgers--The State University, 1971. (Vocational-Technical Curriculum Laboratory, Bldg. 4103--Kilmer Campus, Rutgers University, New Brunswick, NJ 08903)

221 pp; paper; student references; b/w; medium reading level

The book contains pencil-paper activities to develop and drill various topics in basic mathematics. Most word problems involve a vocational setting and reprints of newspaper articles about successful tradespersons are included.

BEEFING UP BASIC SKILLS. Minneapolis: The Math Group, Inc., 1974. (The Math Group, Inc., 396 East 79th St., Minneapolis, MN 55420)

40 pp each booklet; paper; workbook; b/w; low reading level

The series includes three booklets: Whole Numbers, Fractions and Decimals and Percents. Each booklet contains puzzle pages for drilling basic skills.

CHIP TRADING ACTIVITIES. Patricia Davidson, Grace Galton, and Arlene Fair. Fort Collins, Colorado: Scott Resources, Inc., 1972. (Scott Resources, Inc., P.O. Box 2121, Fort Collins, CO 80522)

learning kit

The kit contains chipboards, abacus boards, chip tills, cards, chips, dice and four activity books. The activities in the set of books is a program which uses the trading of colored chips for teaching the following concepts and skills: place value; regrouping in addition and subtraction; multiplication and division processes; patterns and relationships in numeration; decimal notation; numeration systems other than base ten.

C.O.L.A.M.D.A. Longmont, Colorado: Personalized Instruction Center, Northern Colorado Educational Board of Cooperative Services, n.d. (Northern Colorado Educational Board of Cooperative Services, 830 South Lincoln, Longmont, CO, 80501)

374 activities; workbook; b/w; unbound; medium reading level

The collection of materials provides instruction for reluctant learners in grades 7-12 through a laboratory approach. Included are games, disguised drill worksheets, activity cards and a guide for recommended use.

COMPUTATION AND STRUCTURE. Nuffield Mathematics Project. New York: John Wiley and Sons, Inc., 1967. (John Wiley and Sons, Inc., 605 Third Ave., New York, NY, 10016)

103 pp; paper; teacher's guide; color

This book belongs to a collection of teacher's guides designed to teach mathematics to children ages 5 to 13. The guides use a spiral development and each book contains teaching suggestions, examples of non-traditional topics that can be used to develop mathematical awareness, examples of student's work and suggestions for class discussion. This book deals with the concepts of counting, place value, time, money and addition.

DAILY CHORES. Chuck Allen. Palo Alto, California: Creative Publications, Inc., 1970. (Creative Publications, Inc., P.O. Box 10328, Palo Alto, CA 94303)

111 pp; paper; workbook; b/w; low reading level

Includes pages for making transparencies and student worksheets. The activities can be used for motivation, introducing new concepts or drilling students on previously taught concepts.

"Diffy." Herbert Willis. *THE ARITHMETIC TEACHER*, Vol. 18, No. 6 (October, 1971), pp. 402-405. (National Council of Teachers of Mathematics, 1906 Association Dr., Reston, VA 22091)

"A discovery approach with ancient numeration systems." Robert W. Keller. *THE ARITHMETIC TEACHER*, Vol. 19, No. 7 (November, 1972), pp. 543-544. (National Council of Teachers of Mathematics, 1906 Association Dr., Reston, VA 22091)

"Discovery with cubes." Robert E. Reyes. *THE MATHEMATICS TEACHER*, Vol. 67, No. 1 (January, 1974), pp. 47-50. (National Council of Teachers of Mathematics, 1906 Association Dr., Reston, VA 22091)

DRILL AND PRACTICE AT THE PROBLEM SOLVING LEVEL, Activity Pages. Robert W. Wirtz. Washington, D.C.: Curriculum Development Associates, Inc., 1973. (Curriculum Development Associates, Inc., Suite 414, 1221 Connecticut Ave. N.W., Washington, D.C. 20036)

215 pp; unbound; workbook; b/w; low reading level

The resource contains activities, reproducible for class use, at three levels: manipulative, representational and abstract for teaching whole number concepts to elementary school children in a problem solving setting.

DRILLING 4 FUN. John Geer and Charles Geer. Burlingame, California: Modern Math Materials, 1976. (Modern Math Materials, 1658 Albermarle Way, Burlingame, CA 94010)

97 pp; paper; workbook; b/w; low reading level

The book contains enrichment activities in the form of games and puzzle pages on whole number operations.

ELEMENTARY MATHEMATICS FOR TEACHERS. Charles F. Brumfiel and Eugene F. Krause. Reading, Massachusetts: Addison-Wesley Publishing Company, Inc., 1969. (Addison-Wesley Publishing Company, Inc., 2725 Sand Hill Rd., Menlo Park, CA 94025)

470 pp; cloth; teacher reference; b/w

The book is written for prospective elementary teachers and explores the practical and theoretical aspects of elementary mathematics.

ELEMENTARY SCHOOL MATHEMATICS, 2nd ed. Robert E. Eicholz and Phares G. O'Daffer. Menlo Park, California: Addison-Wesley Publishing Company, Inc., 1968. (Addison-Wesley Publishing Company, Inc., 2725 Sand Hill Rd., Menlo Park, CA 94025)

cloth; textbook series; color

This textbook series covers grades K-8. The usual topics of an elementary curriculum are covered. The series was written to provide students with many opportunities to think mathematically.

EXPERIENCES IN MATHEMATICAL IDEAS, Volumes I & II. Reston, Virginia: National Council of Teachers of Mathematics, 1970. (National Council of Teachers of Mathematics, 1906 Association Dr., Reston, VA 22091)

285 pp; unbound; worksheets; b/w

The topics of place value, multiplication, measurement, fractions, statistics and probability are covered through a laboratory-activity approach.

EXPERIMENTS IN FRACTIONS. Donald Buckeye. Troy, Michigan: Midwest Publications Company, Inc., 1972. (Midwest Publications Company Inc., P.O. Box 129, Troy, MI 48099)

48 pp; paper; workbook; b/w; low reading level

The book contains twenty-five task cards and three games to help develop and extend the student's concept of fractions.

"A fifth grade's revision of our number system." Morris Pincus, *THE ARITHMETIC TEACHER*, Vol. 19, No. 3 (March, 1972), pp. 197-199. (National Council of Teachers of Mathematics, 1906 Association Dr., Reston, VA 22091)

FIGURES FOR FUN. Ya Perelman. Moscow, U.S.S.R.: Mir Publishers, 1973. (Mir Publishers, Moscow, U.S.S.R., c/o Imported Publications, Inc., 320 West Ohio, Chicago, IL 60610)

184 pp; paper; teacher reference; b/w

This is an interesting collection of 123 brain-teasers. Answers are provided for each puzzle. The book is available through Imported Publications, Inc., Chicago, Illinois.

FINITE DIFFERENCES. Dale Seymour and Margaret Shedd. Palo Alto, California: Creative Publications, Inc., 1973. (Creative Publications Inc., P.O. Box 10328, Palo Alto, CA 94303)

116 pp; paper; workbook; b/w; medium reading level

The book develops the problem solving technique of finite differences. Many problems, including patterns and number sequences, are included that can easily be solved with this technique.

4 *THE MATH WIZARD.* Louis Grant Brandes. Portland, Maine: J. Weston Walch, Publisher, Inc., 1962. (J. Weston Walch, Publisher, Inc., Box 658, Portland, ME 04104)

252 pp; paper; b/w; workbook; medium reading level

The book contains a collection of ideas and activities for enrichment purposes and practical suggestions for implementing them.

FRACTION BARS. Albert Bennett and Patricia Davidson. Fort Collins, Colorado: Scott Resources, Inc., 1973. (Scott Resources, Inc., P.O. Box 2121, Fort Collins, CO 80522)

activity kit

This kit contains a program of games, activities, manipulative materials, objectives, workbooks, and tests for teaching fractions. A model for fractions with denominators of 2, 3, 4, 6 and 12 is used throughout this excellent kit.

FUN AND GAMES WITH MATHEMATICS. James Haugaard and David Horlock. Los Gatos, California: Contemporary Ideas, 1972. (Prentice-Hall Learning Systems, Inc., P.O. Box 47X, Englewood Cliffs, NJ 07632)

302 pp; paper; workbook; b/w; low reading level

This is a book of enrichment activities for elementary, junior high and senior high students. Included are games, puzzles, and patterns. The book can be used for making transparencies and worksheets for classroom use.

FUN WITH NUMBERS. San Francisco Unified School District. San Francisco, California: n.d. (San Francisco Unified School District, 135 Van Ness Ave., San Francisco, CA 94102)

GAMES AND AIDS FOR TEACHING MATH. Nikki Bryson Schreiner. Palos Verdes Estates, California: Touch and See Educational Resources, 1973. (Activity Resources Company, Inc., P.O. Box 4875, Hayward, CA 94540)

74 pp; paper; workbook; b/w; low reading level

The book contains instructions for making and the rules for playing games that drill whole number, fraction and measurement concepts. In addition, suggestions for the use of games and the management of a math game lab are given.

"Happy integers." Donald C. Duncan. *THE MATHEMATICS TEACHER*, Vol. 65, No. 7 (November, 1972), pp. 627-629. (National Council of Teachers of Mathematics, 1906 Association Dr., Reston, VA 22091)

HOW TO TAKE A CHANCE. Darrell Huff and Irving Geis. New York: W.W. Norton and Company, Inc., 1959. (W.W. Norton and Company, Inc., 500 Fifth Ave., New York, NY 10036)

173 pp; paperback; teacher reference

In an entertaining manner, the book describes how the ideas of probability influence our lives and affect our decisions. Also included are probability problems and puzzles.

INGENUITY IN MATHEMATICS. Ross Hornsberger. New York: Random House, Inc., 1970. (Random House, Inc., 201 East 50th St., New York NY 10022)

204 pp; paper; teacher reference; b/w

This book is from the New Mathematical Library collection written by professional mathematicians and is intended for high school students and adults. The nineteen essays in the book deal with the topics of number theory, geometry, logic and probability.

INVESTIGATING SCHOOL MATHEMATICS. Charles R. Fleenor, Robert E. Eicholz, and Phares O'Daffer. Menlo Park, California: Addison-Wesley Publishing Company, Inc., 1974. (Addison-Wesley Publishing Company, Inc., 2725 Sand Hill Rd., Menlo Park, CA 94025)

cloth; textbook series; color

This is a textbook series for grades 1-6. The books are colorfully illustrated and include the usual topics in mathematics. A spiral approach is used for most topics and many thought-provoking extensions are included.

IT'S A TANGRAM WORLD. Lee Jenkins and Peggy McLean. San Leandro, California: Educational Science Consultants, 1968. (Educational Science Consultants, P.O. Box 1647, San Leandro, CA 94577)

92 pp; paper; workbook; b/w

The book contains a series of activities designed for use with tangrams to help students learn about part-whole relationships through experience.

JOURNAL OF RECREATIONAL MATHEMATICS. Farmingdale, New York: Baywood Publishing Company, Inc. (Baywood Publishing Company, Inc., 120 Marine St., Farmingdale, NY 11735)

This journal is published four times a year. Each issue contains a variety of problems, puzzles, book reviews and articles dealing with topics in mathematics and mathematics teaching. The journals are an excellent source of enrichment topics for a classroom.

MAN AND NUMBER. Donald Smeltzer. London: A. and C. Black, Ltd., 1970. (A. and C. Black, Ltd., 35 Bedford Row, London, WC1R 4JH, England)

115 pp; paper; teacher reference; b/w

The book describes the development of our number system and methods of doing calculation. Included are illustrations, photographs and a bibliography.

MASTER BOOK OF MATHEMATICAL RECREATIONS. Fred Schuk. New York: Dover Publications, Inc., 1968. (Dover Publications, Inc., 180 Varick St., New York, NY 10014)

430 pp; paper; teacher reference; b/w

This book contains a collection of problems, games and puzzles and the mathematics behind the strategies involved in their solutions.

MATH ACTIVITIES WITH SIMPLE EQUIPMENT. John Ginther. Troy, Michigan: Midwest Publications Company, Inc., 1972. (Midwest Publications Company, Inc., P.O. Box 129, Troy, MI 48099)

121 pp; paper; workbook; b/w; low reading level

The book contains a collection of enrichment activities involving whole numbers, fractions and geometry. Many require manipulatives. Pages may be reproduced for classroom use.

MATH ACTIVITY WORKSHEET MASTERS. rev. ed. William T. Stokes and Mary Laycock. Palo Alto, California: Creative Publications, Inc., 1974. (Creative Publications, Inc., P.O. Box 10328, Palo Alto, CA 94303)

MATH AMUSEMENTS IN DEVELOPING SKILLS, Vol. I. Alice A. Clack and Carol H. Leitch. Troy, Michigan: Midwest Publications Company, Inc., 1972. (Midwest Publications Company, Inc., P.O. Box 129, Troy, MI 48099)

87 pp; paper; workbook; b/w

The book contains fifty-seven puzzles for disguised drill in whole number, fraction and decimal operations. The pages can be reproduced for classroom use.

MATH APPLICATIONS KIT. Allen C. Friebe and Carolyn Kay Gingrich. Chicago: Science Research Associates, 1971. (Science Research Associates, 259 E. Erie St., Chicago, IL 60611)

270 cards; color; activity cards; medium reading level

This is an excellent collection of 270 activity cards that presents a large number of problems and activities in science, sports and games, occupations, social studies and everyday things that students, using elementary school mathematics, can explore. Also included are 10 reference cards and student and teacher handbooks.

MATH CARD GAMES. Walter Rucker and Clyde Dilley. Palo Alto, California: Creative Publications, Inc., 1974. (Creative Publications, Inc., P.O. Box 10328, Palo Alto, CA 94303)

86 pp; paper; teacher reference; b/w

The book contains the instructions for making and the rules for playing card games that drill whole number facts and operations.

MATHEMATICS. David Bergamini. (Life Science Library Series) New York: Time, Inc., 1967. (Time Life Books, Time and Life Bldg., Rockefeller Center, New York, NY 10020)

200 pp; cloth; student reference; color; medium reading level

The colored and black and white photographs and graphics in this historical survey of mathematics makes this an excellent resource for student and teacher alike. The book covers the development of mathematics from counting to computers, probability and modern geometries.

MATHEMATICS A HUMAN ENDEAVOR. Harold R. Jacobs. San Francisco: W.H. Freeman and Company, Publishers, 1970. (W.H. Freeman and Company, Publishers, 660 Market St., San Francisco, CA 94104)

529 pp; cloth; textbook; b/w; high reading level

This text for a liberal arts course is an excellent resource book. Many of the ideas are suitable for or could be adapted for middle school students.

MATHEMATICS INVOLVEMENT PROGRAM. David Gladstone and Joseph Gladstone. Chicago: Science Research Associates, 1971. (Science Research Associates, 259 E. Erie St., Chicago, IL 60611)

cards; color; activity cards; medium reading level

This is a kit of manipulative materials and activity cards designed to cover many of the concepts of an elementary mathematics program, grades K-6.

MATHEMATICS LABORATORY HANDBOOK FOR THE JUNIOR HIGH SCHOOL. San Diego City Schools. San Diego, California, n.d. (San Diego City Schools, Materials Development Office, Rm. 2002, Education Center, 4100 Normal St., San Diego, CA 92013)

THE MATHEMATICS TEACHER. Reston, Virginia: The National Council of Teachers of Mathematics. (National Council of Teachers of Mathematics, 1906 Association Dr., Reston, VA 22091)

This magazine, published eight times a year, January through May and October through December, contains a wealth of ideas and activities for the middle school and secondary mathematics teacher.

MATHEX. Montreal, Canada: Encyclopaedia Britannica Publications, Ltd., 1970. (Encyclopaedia Britannica Publications, Ltd., 2 Bloor St. W., Suite 1100, Toronto, Ontario, Canada M4W 3J1)

approx. 45 pp in each; paper; workbook; b/w; medium reading level

MATHEX is a series of ten student workbooks with corresponding Teacher Resource books. Books 1-5 are designed for primary grades and books 6-10 are designed for grades 4-6. The topics covered in books 1-5 are matching and graphing, numeration, operations, geometry, measurement and estimation; in books 6-10 are graphing and probability, numeration, operations and problem solving, geometry and measurement.

MATHIMAGINATION, Books A-F. Steve Marcy and Janis Marcy. Palo Alto, California: Creative Publications, Inc., 1973. (Creative Publications, Inc., P.O. Box 10328, Palo Alto, CA 94303)

48 pp in each; paper; workbooks; b/w

Each book provides drill in the form of a puzzle on basic concepts and skills of a mathematics topic; Book A: beginning multiplication and division; Book B: operations with whole numbers; Book C: number theory; Book D: fractions; Book E: decimals and percents; Book F: geometry, measurement and cartesian coordinates.

MATHLAB. Scott McFadden, Judy Johnson, Mary Ann Todd, and Oscar Schaaf. Eugene, Oregon: Action Math Associates, Inc., 1974. (Action Math Associates, 1358 Dalton Dr., Eugene, OR 97404)

series; activity cards; b/w; medium reading level

The program is a collection of activity cards that are interdisciplinary in nature and involve students in "hands on" problem solving investigations. In the first edition each activity is printed on cardstock; in the new metric edition groups of activities are packaged in book form. The groupings are by grade level: K, 1-2, 3-4, 5-6 and junior high.

MENTAL COMPUTATION. Klass Kramer. Chicago: Science Research Associates, 1965. (Science Research Associates, 259 E. Erie St., Chicago, IL 60611)
paper; b/w; workbooks and teacher guides

This is a series of six student workbooks and teacher guides designed as a program to teach elementary students how to perform mathematical operations without paper and pencil.

MINNEMAST, Unit 22: PARTS AND PIECES. Minnesota Mathematics and Science Teaching Project. Minneapolis: University of Minnesota, 1970. (Minnesota Mathematics and Science Teaching Project, 720 Washington Ave., Minneapolis, MN 55414)

132 pp; paper; b/w; teacher's guide

This title is one of 29 units for grades K-3. The unit is designed to teach fraction concepts and operations through concrete materials. Emphasis is placed on understanding rather than on drill and practice.

MORE GAMES AND AIDS FOR TEACHING MATH. Nikki Bryson Schreiner. Palos Verdes Estates, California: Touch and See Educational Resources, 1973. (Touch and See Educational Resources, P.O. Box 794, Palos Verdes Estates, CA 90274)

74 pp; paper; workbook; teacher reference; b/w

The book contains instructions for making and the rules for playing games that drill whole number, fraction and measurement concepts. In addition, suggestions for the use of games and the management of a math game lab are given.

MULTIBASE ACTIVITIES, Base 10. Robert E. Willcott, Mark A. Spikell and Carole E. Greenes. Palo Alto, California: Creative Publications, Inc., 1974. (Creative Publications, Inc., P.O. Box 10328, Palo Alto, CA 94303)

145 pp; paper; workbook; b/w; low reading level

This book is one of three multibase activity books (the other two are for bases 4 and 5) designed for students in grades 4-7 that uses multibase wood materials to develop the ideas of counting, place value and the four basic operations with whole numbers. The pages can be reproduced for classroom use.

"Multi-bet." Arthur K. Miki. *THE ARITHMETIC TEACHER*, Vol. 20, No. 8 (December, 1973), pp. 694-695. (National Council of Teachers of Mathematics, 1906 Association Dr., Reston, VA 22091)

NUMBER POWER. Oakland County Mathematics Project. Pontiac, Michigan: Oakland Schools, 1970. (Oakland County Mathematics Project, Oakland Schools, 2100 Pontiac Lake Rd., Pontiac, MI 48054)

paper; b/w; workbooks; medium reading level

This is one of a good series of 23 topical workbooks covering such general areas as geometry, statistics, probability, measurement, ratio and proportion.

NUMBER WORDS AND NUMBER SYMBOLS. Karl Menninger. Cambridge, Massachusetts: Massachusetts Institute of Technology Press, 1969. (Massachusetts Institute of Technology Press, 28 Carleton St., Cambridge, MA 02142)

480 pp; teacher resource; b/w; cloth

The book is a cultural history of the development of numbers. Included are about 300 photographs and drawings. Some of the topics covered are number systems in actual use, numeration systems of primitive tribes and various counting and computing devices.

NUMBERS IN THE NEWS. Houghton Lake, Michigan: Christopher Lee Publications, n.d. (Christopher Lee Publications, Route 1, Box 582 A, Houghton Lake, MI 48629)

activity from subscription service

This subscription service provides current miscellaneous raw number data such as baseball statistics and related student questions in ditto master format.

"Numerology." Karen Billings. *OCTM MONOGRAPH*, (November, 1974), p. 1. (Oregon Council of Teachers of Mathematics, 4015 S.W. Canyon Rd., Portland, OR 97221)

OCTM MONOGRAPH. Eugene, Oregon: Oregon Council of Teachers of Mathematics. (Oregon Council of Teachers of Mathematics, 4015 S.W. Canyon Rd., Portland, OR 97221)

The OCTM MONOGRAPH is now part of the OREGON MATHEMATICS TEACHER, a magazine published eight times a school year that contains many useful ideas for teachers of mathematics at all levels.

"The original counting systems of Papua and New Guinea." Edward P. Wolfers. *THE ARITHMETIC TEACHER*, Vol. 18, No. 2 (February, 1971), pp. 77-83. (National Council of Teachers of Mathematics, 1906 Association Dr., Reston, VA 22091)

"Perimeter-magic polygon." Terrell Trotter, Jr. *JOURNAL OF RECREATIONAL MATHEMATICS*, Vol. 7, No. 1 (Winter, 1974), pp. 14-20. (Baywood Publishing Company, Inc., 120 Marine St., Farmingdale, NY 11735)

"The persistence of a number." N.J.A. Sloane. *JOURNAL OF RECREATIONAL MATHEMATICS*, Vol. 6, No. 2 (Spring, 1973), pp. 97-98. (Baywood Publishing Company, Inc., 120 Marine St., Farmingdale, NY 11735)

"Pica-Centro--a game of logic." Douglas B. Aichele. *THE ARITHMETIC TEACHER*, Vol. 19, No. 5 (May, 1972), pp. 359-361. (National Council of Teachers of Mathematics, 1906 Association Dr., Reston, VA 22091)

PROBLEMS--GREEN SET. Nuffield Mathematics Project. New York: John Wiley and Sons, Inc., 1969. (John Wiley and Sons, Inc., 605 Third Ave., New York, NY 10016)

52 pp; paper; teacher's guide; b/w

The booklet contains fifty-two problems designed to motivate and challenge elementary school children. Solutions and extensions are provided. The problems can be purchased on cards for student use. Two other problem sets--Red Set and Purple Set are also available.

PROJECT R-3. E.L. Hodges, ed. San Jose, California: T.M.T.T., 1974. (E.L. Hodges, 990 Asbury, San Jose, CA 95126)

looseleaf; worksheets; b/w

Four packets of student pages and a packet of answer sheets and suggested forms for grading, record keeping, etc. make up the materials. The four packets cover whole numbers, fractions, decimals and percents.

RETRIEV-O-MATH. Eugene Smith and Carlos Wilhite. Birmingham, Michigan: Midwest Publications Company, Inc., 1968. (Midwest Publications Company, Inc., P.O. Box 129, Troy, MI 48099)

76 cards; b/w

On each of seventy-three cards is a student activity. Some of the topics included are arithmetic geometry, magic, number theory, algebra and logic. The cards could be used for enrichment and motivation. Three cards show sample bulletin board displays.

"Reversal products." Jean Meeus. *JOURNAL OF RECREATIONAL MATHEMATICS*, Vol. 6, No. 2 (Spring, 1973), pp. 99-100. (Baywood Publishing Company, Inc., 120 Marine St., Farmingdale, NY 11735)

ROAD AND TRACK. Newport Beach, California: CBS Consumer Publishing. (CBS Consumer Publishing, 1499 Monrovia Ave., Newport Beach, CA 92663)

SCHOOL MATHEMATICS I & II. Robert E. Eicholz, Phares O'Daffer, Charles F. Brumfiel, Merrill E. Shanks and Charles R. Fleenor. Menlo Park, California: Addison-Wesley Publishing Company, Inc., 1971. (Addison-Wesley Publishing Company, Inc., 2725 Sand Hill Rd., Menlo Park, CA 94025)

cloth; textbook series; b/w; medium reading level

SCHOOL MATHEMATICS I and SCHOOL MATHEMATICS II are the 7th and 8th grade texts that complement the ELEMENTARY SCHOOL MATHEMATICS series from the same publishers.

THE SCHOOL MATHEMATICS PROJECT, Books A-H. The School Mathematics Project. Great Britain: Cambridge University Press, 1969. (Cambridge University Press, 32 East 57th St., New York, NY 10022)

approx. 300 pp in each; paper; textbook; b/w; medium reading level

The series is intended for students in grades 7-10. The books present a spiral activity-oriented approach that interweaves arithmetic, algebra and geometry with an attempt to emphasize the practical application of each topic.

THE SCHOOL MATHEMATICS PROJECT, Books 1-5 [Metric]. The School Mathematics Project. Great Britain: Cambridge University Press, 1969. (Cambridge University Press, 32 East 57th St., New York, NY 10022)

approx. 300 pp in each; cloth; textbook; b/w; high reading level

This is the first series of textbooks produced by the School Mathematics Project. Like Books A-H, it presents a spiral activity-oriented approach but the coverage of topics is more comprehensive and the reading level is higher.

SOME COMPUTATIONAL STRATEGIES OF SEVENTH GRADE PUPILS. Final Report. Francis G. Lankford, Jr. Charlottesville: University of Virginia, 1972. (Francis G. Lankford, Jr., 102 Cavalier Dr., Colthurst Farm, Charlottesville, VA 22901)

STUDENT ACTIVITY CARDS FOR CUISENAIRE RODS. Patricia Davidson, Grace Galton and Arlene Fair. New Rochelle, New York: Cuisenaire Company of America, Inc., 1971. (Cuisenaire Company of America, Inc., 12 Church St., New Rochelle, NY 10805)

130 cards; b/w

The kit contains 130 cards in ten sets with an accompanying teacher's manual. The cards are to be used with Cuisenaire rods and include games and problems which develop mathematics concepts in grades K-6.

THE STUDY OF POWERS, ROOTS AND LOGARITHMS. Z.P. Dienes. (Children's Instruction Cards adapted by E.W. Golding.) New York: Herder and Herder, Inc., 1968. (McGraw-Hill Book Company, Webster Division, 1221 Avenue of the Americas, New York, NY 10020)

107 pp: paper: b/w

The book contains forty-one instruction cards for students that utilize multibase blocks and a story setting to develop the concepts of powers, root and logarithms. Background materials are provided for the teacher.

"Sum numbers." Charles W. Trigg. *JOURNAL OF RECREATIONAL MATHEMATICS*, Vol. 6, No. 3 (Summer, 1973), pp. 224-227. (Baywood Publishing Company, Inc., 120 Marine St., Farmingdale, NY 11735)

WAYS TO FIND HOW MANY. David A. Page. Newton, Massachusetts: The Arithmetic Project, Education Development Center, Inc., 1965. (Education Development Center, Inc., 39 Chapel St., Newton, MA 02160)

35 pp; paper; workbook; b/w; low reading level

The booklet contains activities to develop skills in estimating length and the number of squares in a drawing. In addition, there are activities to increase student's ability to count cubes, visualize the arrangement and determine the number of cubes from a drawing and construct shapes from cubes given a drawing.

WHERE'S THE WORD?, Book 1. Diana Hestwood and Jackie McClees. Minneapolis: The Math Group, Inc., 1974. (The Math Group, Inc., 396 East 79th St., Minneapolis, MN 55420)

28 pp; paper; workbook; b/w; low reading level

Each puzzle relates to a topic and consists of a letter maze with corresponding word list. The pages can be used to make ditto or mimeograph masters.

WYMOLAMP, Secondary Materials. Fremont County Unified School District No. 25. Riverton, Wyoming: Fremont County Unified School District No. 25, 1972. (Fremont County Unified School District No. 25, 121 North Fifth St. W., Riverton, WY 82501)

approx. 350 activities; workbook; b/w; unbound; medium reading level

The collection of materials provides instruction for reluctant learners in grades 7-12 through a laboratory approach. Included are games, disguised drill worksheets, activity cards and a guide for recommended use.

YES, MATH CAN BE FUN! Louis Grant Brandes. Portland, Maine: J. Weston Walch, Publisher, Inc., 1960. (J. Weston Walch, Publisher, Inc., Box 658, Portland, ME 04104)

263 pp; paper; workbook; b/w; medium reading level

The book is a collection of recreational activities in the form of puzzles, games, number oddities, illusions, problems and projects to stimulate students.