

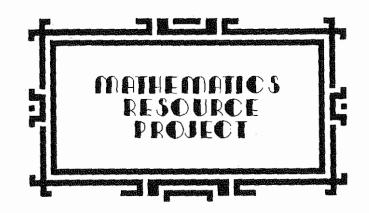
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ORGANIZATION

Placement Guide for Tabbed Dividers

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It is the intent of Congress that the resources developed by the Mathematics Resource Project will be made available within the school district using such material for inspection by parents or guardians of children engaged in educational programs or projects of that school district.

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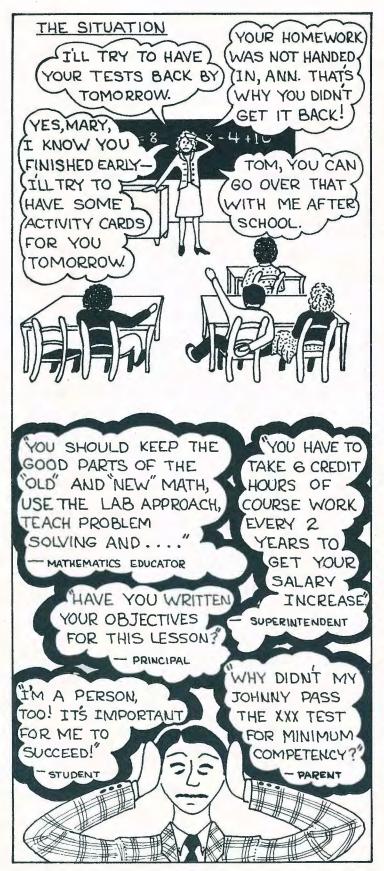
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The demands on teachers are heavy. The fifth or sixth grade teacher with 25 to 30 students is often responsible for covering many subjects besides mathematics. The seventh or eighth grade teacher may be teaching only mathematics but be working with 125 to 150 students each day. Within this assignment the teacher must find time for correcting homework, writing and grading tests, discussions with individual students, parent conferences, teacher meetings and lesson preparations. In addition, the teacher may be asked to sponsor a student group, be present at athletic events or open houses, or coach an athletic team.

Demands are made on the teacher from other sources. Students, parents and educators ask that the teacher be aware of students' feelings, self-images and rights. School districts ask teachers to enlarge their backgrounds in mathematical or educational areas. The state may impose a list of student objectives and require teachers to use these to evaluate each student. There are pressures from parents for students to perform well on standardized tests. Mathematicians and mathematics educators are asking teachers to retain the good parts of modern mathematics, use the laboratory approach, teach problem solving as well as to increase their knowledge of learning theories, teaching strategies, and diagnosis and evaluation.



There is a proliferation of textbooks and supplementary material available. Much of this is related to the demands on teachers discussed above. The teacher in small outlying areas has little chance to see much of this material, while the teacher close to workshop and resource centers often finds the amount of available material unorganized and overwhelming.

The Mathematics Resource Project was conceived to help with these concerns. The goal of this project is to draw from the vast amounts of material available to produce topical resources for teachers. These resources are intended to help teachers provide a more effective learning environment for their students. From the resources, teachers can select classroom materials emphasizing interesting drill and practice, concept-building, problem solving, laboratory approach, and so forth. When completed the resources will include readings in content, learning theories, diagnosis and evaluation as well as references to other sources. A list of the resources is given below. A resource devoted to measurement and another devoted to problem solving have been proposed.

NUMBER SENSE AND ARITHMETIC SKILLS (preliminary edition, 1977) RATIO, PROPORTION AND SCALING (preliminary edition, 1977) GEOMETRY AND VISUALIZATION (preliminary edition, 1977) MATHEMATICS IN SCIENCE AND SOCIETY (preliminary edition, 1977) STATISTICS AND INFORMATION ORGANIZATION (preliminary edition, 1977)



This resource is intended to help middle school and junior high school teachers by providing background materials and classroom ideas for use with students.

WHAT IS IN THIS RESOURCE?

The resource consists of the following components:

Content for Teachers Didactics Teaching Emphases Classroom Materials Glossary Annotated Bibliography Selected Answers

The Content for Teachers gives mathematical background in probability and statistics. Many examples and exercises are included. There are also 22 computer programs in the appendix to this section.

The Didactics papers give information on:

♥ Learning Theories

Teaching Techniques

 \sim Diagnosis and Evaluation

Goals and Objectives

The titles of the Didactics papers in this resource are:

Components of Instruction -- An Overview

Classroom Management

(W) Statistics and Probability Learning

the resources is given on page 11.

The Teaching Emphases are a collection of processes, approaches, and aids which are emphasized throughout the resource. These are:

G Critical Thinking

🏋 Decision Making

Problem Solving

Models and Simulations

Calculators and Computers

Laboratory Approaches

The Classroom Materials (Concepts and Skills and Applications) include:

- Worksheets (can be duplicated for whole class use)
- Activity cards (may be laminated and used as a nonconsumable card or used with other pages in a nonconsumable booklet)
- Transparency masters
- Games (individual, small group, and whole class)
- Teacher-directed activities
- Teacher ideas (ideas which teachers can develop into activities)
- Teacher pages (mathematical background for teachers for specific student pages)

The Teacher Commentaries which appear before the sections of the classroom materials intend to:

- Give a rationale for teaching a topic
- Suggest alternate ways to introduce or develop topics
- Suggest ways to involve students
- Highlight the classroom pages
- Give more ideas on the teaching emphases

The Glossary gives informal definitions for the statistics and probability used in this resource.

The Annotated Bibliography lists the A list of the Didactics papers for all of sources which were used to develop this resource. These sources contain many additional ideas which can be of help to teachers.

HOW ARE THE PARTS OF THE RESOURCE RELATED?

The classroom materials are keyed to each other, to the teaching emphases, to the commentaries, and to the didactics papers with symbols and teacher talk as shown on page 10.

The commentaries refer to specific classroom pages (cited in italics) and often a classroom page is shown reduced in size next to the discussion of the page. The commentaries relate the various teaching emphases to the mathematical topic of that subsection. When discussing methodology

and teaching strategies, the commentaries refer to the appropriate didactics papers (cited in italics).

Each teaching emphasis paper includes a rationale and examples from the classroom materials. The Applications pages are marked in the Concepts and Skills part of the Classroom Materials to point out the many applications in statistics and probability. References in italics are also made to the didactics papers.

The didactics papers refer to specific classroom pages (cited in italics) and occasionally a classroom page illustrating the topic of the paper is shown reduced in size.

HOW CAN THE RESOURCE BE USED?

The resource can be used by the teacher to provide a more successful, varied, and flexible mathematics curriculum and to obtain information about mathematics and didactics (teaching strategies, diagnosis and evaluation, learning theories and practices). The resource could also be used in in-service classes or workshops to emphasize critical thinking, problem solving, models and simulations, and so on.

More specifically, the resource can be used:

•As a source of ready-to-use activities

to supplement and vary the curriculum. Worksheets can be duplicated to provide a copy for each student; activity cards can be duplicated and laminated for repeated use; transparencies can be made from a page to form the basis of a teacher-led discussion; or a page can be the focal point of a bulletin board.

•To build basic skills.

For example, skills in reading graphs, in using vocabulary, and in computing averages are given attention in this resource.

•As a source of new or different ways to teach a topic.

A variety of teaching approaches can be found in the classroom materials. The commentaries discuss additional options, and the didactics papers in this resource give background in the teaching of statistics and probability. •To help students understand ideas in statistics and probability.

Concrete models are used in many of the classroom activities. Attention is given to concept building in the classroom materials and in the commentaries. Background for teachers is also provided through the teacher pages in the classroom materials, in the commentaries, and in the content for teachers.

•To gain some insight into difficulties

in learning probability and statistics. For example, some students find it difficult to use a systematic approach in listing permutations. This problem and related problems are discussed in the commentaries and in the didactics paper <u>Statistics and</u> Probability Learning.

•To improve attitudes.

Many activities can be used to help students have a feeling of success and accomplishment. Some students who do not succeed in arithmetic activities may be very successful in activities involving data gathering. Self-checking pages can help a student know when the activity is done correctly. Open-ended activities help students have confidence in their own methods. The many visual and hands-on materials can be used to capture student interest.

•To provide individual students or groups of students with material suitable for

their needs and interests. For example, classroom materials are included for students who are interested in applied problems, students who want to be challenged, and students who need much experience with concrete examples in experimental probability.

•To increase problem solving abilities. Many classroom pages can be used to give problem solving experiences. Each of these pages has a in the upper left-hand corner. Teacher hints and background for problem solving can be found in the Problem Solving teacher emphasis.

•As a springboard for developing activities, units, or curriculum.

The classroom pages can be used as models for teacher developed pages. An activity might have to be adapted to suit a specific teacher or class. For example, a page with too much reading might need to be rewritten as two pages. Activities can be developed from the pages of teacher ideas or from suggestions given in the commentaries or didactics papers. Units could be organized around a section, like experimental probability or a teaching emphasis, like problem solving.

•To obtain information about curriculum trends and research in mathematics education.

Many of the current trends in middle school mathematics are discussed in the teaching emphases section and keyed on the classroom pages. The didactics papers are an easy-to-read synopsis of the research related to teaching mathematics.

•To gain access to the many available sources for classroom ideas and teacher background.

The annotated bibliography can be used for selecting additional resources. Sources are also cited in the classroom pages, the commentaries, teaching emphases, and didactics papers.

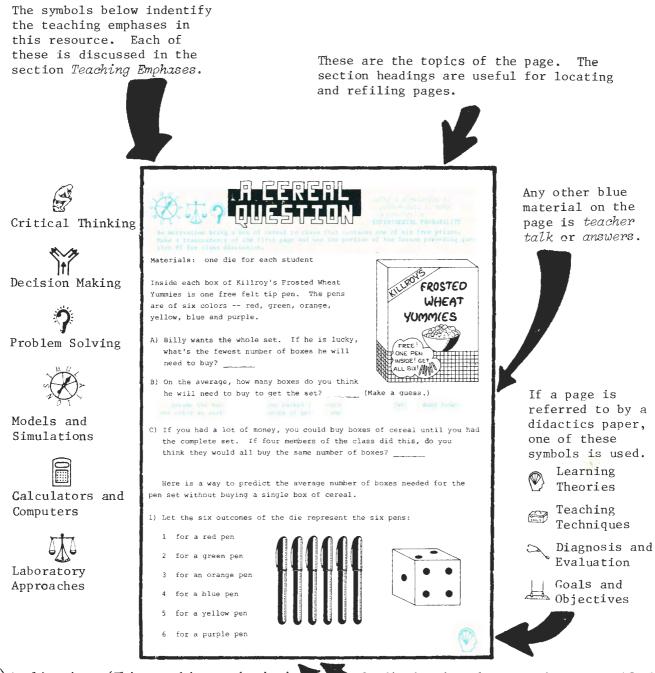
•To learn more about statistics and probability.

The <u>Content for Teachers</u> section has many ideas and useful background information for teachers who want to learn or review concepts in statistics and probability. There are several exercises included to help teachers improve or update their knowledge of the subject. Teachers will decide which material is appropriate for their students. Since the pages do not list the prerequisites needed for an activity, teachers need to carefully examine each page. In general, each subsection is arranged with the easier, introductory pages first. However, since there are several topics within each subsection, it is possible to find introductory pages throughout a subsection.

It is not expected that all of the information in this resource will be read or used by teachers in one year. Several stages of use are possible. One stage might be to use some of the classroom pages to supplement the curriculum. Another stage might be to organize a unit around a mathematical topic or teaching emphasis of the resource. A third stage could be to try a new approach to teaching (laboratory approach, problem solving, discovery lessons) as explained in the teaching emphases section and the various didactics papers. A fourth stage could be to use some of the sources cited throughout the resource and listed in the annotated bibliography. At any of these stages, teachers can add their own ideas and materials to the resource to personalize it and to keep it current.

FEATURES OF CLASSROOM PAGES

When a ditto master is made using the thermofax process, the material in blue will not reproduce. Thus, the student's copy will contain only the material printed in black. The corners are designed to describe the content on each page.

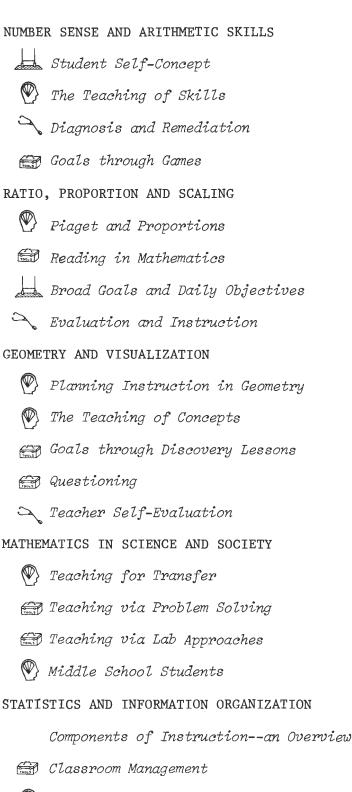


Applications (This teaching emphasis is discussed in the other project resources. In this resource there is an Applications part in the *Classroom Materials*, and in the Concepts and Skills part the symbol is used on appropriate classroom pages.)

Credit is given here to the *source* if the page is a direct copy. *Ideas from* other sources are also noted.

Here is the *type* of activity. This refers to the suggested use of the page.

LIST OF PAPERS ON THE LEARNING THEORY AND THE PLEASURABLE PRACTICE OF TEACHING



W Statistics and Probability Learning

NOTE: A complete collection of all the papers from each resource is available as a separate publication.



CONTENT FUR TEACHERS

Gathering Data Tables Graphs Scatter Diagrams Mean, Median, Mode Range & Deviation Sampling Experimental Probability Probability with Models Counting Techniques Probability with Counting Inferential Statistics Appendix--Computer Programs



Components of Instruction--An Overview

Classroom Management

🖗 Statistics and Probability Learning



- Critical Thinking Decision Making Problem Solving
- J Froblem Solving
- Models and Simulations
- Calculators and Computers
- Laboratory Approaches

CLASSROOM MATERIALS

SKILLS AND CONCEPTS

GATHERING DATA TABLES GRAPHS SCATTER DIAGRAMS MISLEADING STATISTICS MEAN, MEDIAN, MODE RANGE & DEVIATION SAMPLING EXPERIMENTAL PROBABILITY PROBABILITY WITH MODELS COUNTING TECHNIQUES

APPLICATIONS

BUSINESS & COMMERCE ENVIRONMENT HEALTH & MEDICINE PEOPLE & CULTURE RECREATION



ANNOTATED BIBLIOGRAPHY SELECTED ANSWERS

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STATISTICS AND PROBABILITY FOR THE MIDDLE SCHOOL YEARS

Statistics, the science concerned with collection, analysis, and interpretation of numerical information is important in the life of every citizen. It is needed for the proper evaluation of everyday matters such as advertising claims about gasoline mileage and relief from indigestion, public opinion polls and weather reports. It is indispensable for the solution of policy questions, from local affairs such as property assessment and predictions of school enrollments to national problems involving unemployment, crime, airplane safety and health. Even though numerical information is encountered everywhere, in newspapers and in magazines, on radio and on television, too few people have the training to accept such information critically and use it effectively.

> Overview and Analysis of School Mathematics, Grades K-12, 1975, Conference Board of the Mathematical Sciences, Washington, D. C. 1975, p.44

It is truth very certain that, when it is not in one's power to determine what is true, we ought to follow what is more probable.

--René Descartes

WHY INCLUDE STATISTICS AND PROBABILITY IN THE MIDDLE SCHOOL CURRICULUM?

Several justifications are given for teaching statistics and probability during the middle school years. Among these are:

- A great deal of useful information can be learned by reading tables and graphs that appear in newspapers, magazines, almanacs, surveys, and so forth. Students must know how to interpret the tables and graphs if they are to understand the information.
- Statistical statements in advertisements, political campaigns, and in daily conversations are often used to influence people. Students must think critically about these statements in order to judge whether the interpretations of statistical information are justified or not.
- Masses of data can be organized in various ways to give information not at first apparent. Sampling can give information about a population, and the reliability of this information must be judged. Simulations can help in understanding situations and making predictions. Students must know how to gather and organize information and how to use the information to make reasonable decisions or to provide evidence to support their position when there is a disagreement.
- Starting in elementary school, statistics and probability provide a natural link between mathematics and the physical and social sciences. Statistics and probability are powerful tools in studying these sciences, and the sciences provide motivation for learning about statistics and probability.

Students must learn that a correlation between two things does not necessarily mean there is a cause and effect.

- Statistics and probability can be centered in the students' world. Data about the class, school or community can be gathered, organized and interpreted. Probability can be related to games and sports. Students must have concrete and informal experience with statistics and probability in the middle school years in order to understand more formal methods in probability, counting and inferential statistics which may be studied later.
- Statistics and probability can be used to emphasize important processes such as active inquiry, discovery of relationships, the testing of conjectures, and critical thinking. Students must learn to use statistics and probability in problem solving to give reasonable answers when faced with uncertain situations.
- Students must realize that there are often no hard and fast answers. A high probability does not mean something will happen. Students must realize that everyone has to make decisions in spite of uncertain situations and that a knowledge of statistics and probability may enable them to make decisions with more confidence.

INTRODUCTION

Many problems we face in real life do not have hard and fast answers. They may involve decisions we must make even though we do not have all the necessary information. We may have to act in spite of uncertain and incomplete knowledge. For example:

- How does the manager of a shoe store decide what sizes of a particular shoe to stock?
- How many cartons of milk will a school cafeteria sell at each lunch period? What should be the ratio of chocolate milk to white milk, and how will this vary by the day of the week?
- How can I decide whom to vote for in the next election?
- Has the speed limit of 55 miles per hour resulted in less consumption of gasoline and/or fewer fatalities? Should it be retained?
- What does it mean for the weather forecaster to say, "The probability of rain tomorrow is 20%"?
- How many schools should be built or phased out in the next ten years in our district?
- How do we evaluate the claims of advertisers of competing products to determine what to buy?

Questions such as these have to be answered. All of us should understand how answers can be obtained so we can make good judgements on the answers proposed. The answers to these questions are based not only on arithmetic and geometry but also on statistics and probability. It is important that some ideas and methods of statistics be available to all people if they are to think critically and make intelligent decisions.

Some of the ideas in statistics are subtle, but many are relatively simple and can be introduced early in a child's education. The study of elementary statistics will give children many opportunities to review, apply and thus strengthen their skills in arithmetic and geometry. Furthermore, an early introduction and a regular follow-up will lead to a gradual familiarization with statistical ideas.

Knowledge of statistics enables us to more fully understand and challenge the statements of newspapers, politicians, advertisers, and researchers. Statistics also provides us with new tools to use in attacking problems that otherwise may seem insolvable.

How is this possible? Can problems involving uncertainty be tackled mathematically? What does it mean to solve a problem with an "answer" we are not sure of? We read that even though the Salk Vaccine was not a sure preventative against polio-

INTRODUCTION

myelitis, it was given to millions of children. Why? Experiments and large scale testing showed the chances of getting polio were much less if children had been given the vaccine. But what do we mean by the expressions "large scale," "experiment," "chances" and "much less"? Familiarity with these expressions are some of the ideas people can acquire in a study of statistics, not by formal definition but by actual experiences in trying to solve real problems. Such a problem might be a personal one for the student or the problem could be about a business or government agency in which students play the roles of adults making decisions.

A statistical approach to a problem often leads to the following questions:

- What kind of information (data) is necessary to approach this problem?
- How can the data be obtained? If by a survey, how is the survey to be organized? What are the problems in getting accurate data? Should the survey be of a sample or must it involve a census of the whole population? Is a census possible?
- Once the data is obtained, how can it be organized to get the necessary information?
- If the organization is in tables, how can these be best interpreted? If by graphs, what type of graph is most appropriate?
- How can graphs be drawn to be read easily and accurately? Are some graphs deceitful? How can we detect this and avoid such procedures ourselves?
- Once the data is at hand, can we find representative and summarizing numbers to help interpret the information easily?
- If we have information from a sample, can we use it to tell about the population? (To do this requires some knowledge of probability.)
- How can we determine the probability of an event?
- When we have made an inference from the sample to the population, how certain are we as to the accuracy of the conclusion? How can we judge this certainty?

Methods and ideas for answering these questions are developed in the following sections. This is just a beginning study of the methods of statistical analysis, but, hopefully, the methods will give some idea of the power and importance of the use of statistics in society.

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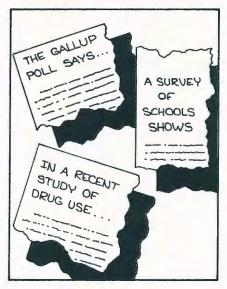
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INTRODUCTION

The first step in applying statistical methods to solving a problem, making a prediction, answering a question or making a decision is to gather the appropriate data. Books and public records may be consulted, polls and surveys undertaken to gather as much pertinent information as possible. The quality of the solution or decision relies heavily on the quality of the data collected. This quality is particularly important as it may not be possible to get all the data desired and decisions will have to be based on what is available.



APPROPRIATE DATA

When a problem is posed, the first thing to decide is what data is pertinent to the situation. Time and effort should not be wasted in trying to determine facts unless they bear on the question at hand. An example may help make this clear.

Suppose the question is "Should the candy and soft drink dispensing machines be removed from the school corridors?"

What facts are pertinent?

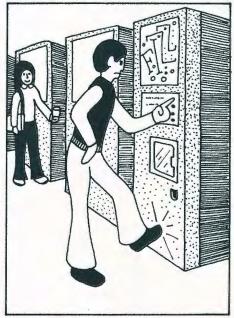
- The number of customers per day?
- The total value of sales?
- The profit per day?
- The kind of food and drink being sold?
- The health facts related to this food?

Whose opinions should be sought?

- Students, teachers, parents?
- Maintenance personnel, lunchroom personnel?
- Administrators, dentists, doctors?

Exercises (Answers given on p. 28)

- Consider the question: Should the Teachers' Union ask for a 20% raise this year? What data should be gathered to help determine an answer? What data could be gathered?
- 2. A committee of teachers is asked to study a recommendation that the current grading system be changed to a) a pass-fail system or b) a system with no grades but individual evaluation of each student. What data should be sought to help make a decision on this recommendation?



PRIMARY AND SECONDARY DATA

Once it has been decided what data to collect, the question as to how this is to be done depends on the choice made between primary and secondary data. Primary data is collected first hand by the person interested in the problem. Secondary data is collected by some other person but made available through publication or public record. Sometimes one or the other is not available so there is no choice. If both are possible the kind of data best suited depends on the problem. For example, if a dance committee wants to know the school's preference between Hard Rock and Rock-n-Roll music, they would probably try to collect the facts themselves. A class wanting to know how the school's choice compares to that of all students in the nation would also need secondary data found in the reports of record sales and requests for broadcasts on radio around the country.

Exercises

- 3. List two problems or questions for which primary data should be sought.
- 4. List two for which both primary and secondary would be worthwhile.
- 5. List two for which secondary data would be the only kind available.

GATHERING PRIMARY DATA

WHO SHOULD BE SURVEYED?

In order to collect primary data the first decision must be to choose the target population. If it is not too large, the data may be sought by a census of the whole group. But if data is sought about the students in a school of 1500, or in the whole state or nation, a census is impractical in terms of time, effort and expense. In this case, collection of data from only a sample is often made. In this case, care should be taken that the sample is truly representative of the



population. It is very easy to get results from a biased sample that does not at all represent those of the total population. The questions of the size of a sample, how it should be selected, how to predict results for the whole population from those of the sample and how accurate these predictions are will be taken up in *Sampling* and *Inferential Statistics* in the CONTENT FOR TEACHERS section.

HOW IS DATA OBTAINED?

Methods of gathering primary data are varied. Sometimes simple observation and counting is enough. Sometimes it is done through a survey. A <u>survey</u> consists of a questionnaire, i.e., a series of questions the answers to which will hopefully supply the required data. There are different ways of making a survey. It may be conducted by mail, by telephone or by personal interview. There are advantages and disadvantages to each.

The <u>mail survey</u> is relatively inexpensive and looks easy to do. However, the great danger is that not everyone will respond. This is true whether the survey is made of the whole population or only of a sample even if it is carefully selected to be truly representative. The trouble is the majority answering may be those with one point of view. If only 20% of those who get the questions in the mail send in answers, it may be impossible to predict what the whole population would have done. Advantages of a mail survey are that the questions are exactly the same for everybody and people have time to think about their answers before giving them.

A <u>survey by telephone</u> suffers from the fact that many people resent this kind of interruption, others do not take it seriously and give irrelevant answers. An advantage is that a call can be repeated until contact is made.

The <u>personal interview</u> is probably the best of the three methods of conducting a survey. If several interviewers are used, they need to be trained so that some uniformity is attained in attitude and approach. One of the advantages of interviews is the large percent of responses attained since return visits can be made if no one is available on the first attempt. Another advantage is the probability of a more serious attitude in the respondent since the interviewer



is physically present. Also an interview increases the probability of getting answers to most of the questions rather than only a selected few. Disadvantages are the possibility of the interviewer consciously or unconsciously influencing the respondent by actual suggestion or by attitude and behavior, and inaccuracy of recording responses, particularly if they are more than just a Yes or No.

QUESTIONS FOR A SURVEY

Any kind of survey involves making up the questions to be asked. This is always a tricky business because questions may be vague, biased, misleading, or their answers may not provide the information wanted. Many times a trial run is worthwhile to test the first attempt at writing questions. As George Gallup says, "Nothing is so difficult, nor so important, as the selection and wording of the questions.... No question, no matter how simple, must reach the interviewing stage without first having gone through a thorough pretesting procedure."*



Both of the above questions are biased. Perhaps a better one would be "Should America have gotten out of the Vietnamese war earlier than it did?" Of course students are not professional pollsters but they, or anyone taking a survey, should keep in mind the problem of making the questions fair, understandable, unambiguous and unbiased.

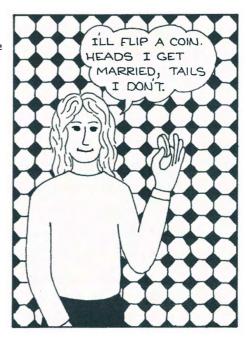
*George Gallup, The Sophisticated Poll Watchers Guide (Princeton: Princeton Opinion Press, 1972), p. 77.

Exercises

- 6. In each of the following pairs of questions, select the better one to put on a survey form.
 - a. "How many brothers and sisters do you have?" or "Are you a member of a large family?"
 - b. "Do you like that wonderful HI JINKS BAND?" or "Do you think the HI JINKS BAND is good?"
 - c. "Do you believe in freedom of the press?" or "Should pornographic magazines be banned from sale in public newsstands in this city?"
 - d. "Should the voting age be lowered to 18?" or "Should we let inexperienced kids of 18 vote?"

RESPONSES TO A QUESTION

Some questions allow a free answer but some require a choice from two to five responses. Here again care must be taken in writing the possible responses. Sometimes one response is blatantly more attractive than the others. Sometimes a sliding scale is offered that may be marked at any position. Some responses are not mutually exclusive. Sometimes a response does not really answer the question asked. Again students are not professional pollsters but if care is taken to select sensible responses, better results will follow.

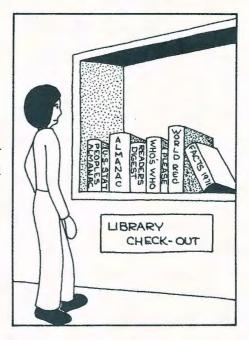


GATHERING SECONDARY DATA

If primary data is not needed, is not obtainable, or is not sufficient, it may be necessary to seek out secondary data. Where is such data to be found? The most likely sources are the various almanacs and fact books. Most of these are issued on a yearly basis and are available in school and public libraries. Typical ones

are The World Almanac, The Information Please Almanac, The Readers Digest Almanac, The U.S. Fact Book, and The Guinness Book of World Records. Less well known but valuable reference books are The People's Almanac, Historical Statistics of the United States and World Statistics in Brief.

These are not the only sources of data. Newspapers and magazines are full of tables, graphs and results of polls and surveys made by professionals and amateurs. Public records at municipal and state offices are frequently sources of easily obtained information. Public relations departments of businesses and business associations, of labor unions and professional associations are also possible sources.



Of course, the careful student considers the sources of data for bias and, if possible, checks against each other. Data on smoking from the U.S. Surgeon General's Office and from the Tabacco Growers Association may differ, as may unemployment figures from the Department of Labor and from the AFL-CIO. It should be noted that a word such as "unemployed" may be used with different meanings. Is a teacher "unemployed" during the summer or the Christmas vacation? Is a fledgling lawyer with

CONTENT FOR TEACHERS

GATHERING DATA

a newly opened office but as yet no clients "unemployed"? Is a coal-miner on strike for better working conditions and a living wage "unemployed"? Is a concert violinist "unemployed" between engagements? Different answers to such questions will lead to different statistics on unemployment. It is important that the student think about such possibilities and guard against misuse or misinterpretation of the data.

QUANTITATIVE DATA

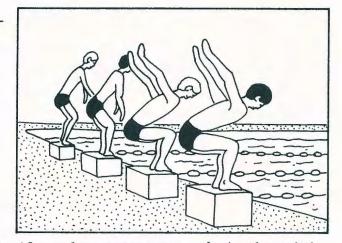
Whatever data is collected from primary or secondary sources will be either quantitative, i.e., expressed in numbers, or qualitative, i.e., expressed in words.

Questions that ask for height of the respondent or the number of children in a family are asking for quantitative data, but even here there are essential differences. Data that comes from counting as in the number of children is called <u>discrete</u>. All such numbers are positive integers. Sometimes we are interested in these exact numbers, sometimes approximations are more appropriate and easier to handle.

When to be exact and when to be approximate is a matter of judgement and experience. A bill of \$11.95 must be paid exactly but on the income tax forms \$11.25 may be rounded to \$11.00. The number of children in class today is exactly 34 but the number enrolled in all the schools is approximately 12,500. Even if it had been 12,473 on opening day there have undoubtedly been transfers in and out since then and the exact figure is no longer valid. Today's exact attendance could be found, if needed, by checking attendance records at all the schools.



Data that comes from measurement such as the height or weight of a person or the winning time of a race is called <u>continuous</u>. Such numbers are given to the nearest selected unit such as centimetres, kilograms or tenths of a second. All such numbers are approximate. They can never be exact in the way that counting numbers are. We can refine the measurement for greater precision but that is the best we can do. The precision desired for measurement numbers should be judged by the circumstances involved. Sometimes weight is measured to the nearest kilogram, sometimes to the nearest gram. Time may be measured to the nearest day, hour or millisecond, etc. Whatever the precision of the data gathered, it is important to record it accurately. No further calcula-



tions or implications will be worthwhile if careless errors are made in the original record. However, different individuals may measure or judge the same things and

get and record different results. Thus in a diving meet, judges give scores to the nearest half point. Five judges may give scores of 4.0, 5.0, 4.5, 4.5 and 4.0 to the same dive. These scores are recorded carefully and later on organized, summarized and evaluated.

In working with measurement numbers it should be noted that no derived numbers may be expressed with a precision greater than that of the least. precise original. If my desk is measured to be 165 cm long and 79.3 cm wide, the perimeter should be given as 489 cm and not 488.6 cm.

QUALITATIVE DATA

Questions that ask for the sex of a student

or about the kinds of milk (skim, regular or chocolate) the student prefers, are asking for qualitative data. But even here counting may take place. Each individual answers M or F in response to the question about sex but the interviewer counts the number of M's or F's to determine the ratio of boys to girls.

A survey might want to determine how the education of respondents correlates with their annual earned incomes. The qualitative data obtained in answer to a question about educational status could consist of a choice among "illiterate", "functionally literate" and "literate". Not only can these answers be distin-



CONTENT FOR TEACHERS

GATHERING DATA

guished but they can be ranked perhaps as 1, 2, 3 or 3, 2, 1 but not as 2, 1, 3. This rank ordering provides a set of data for further study. The choices offered might be greater than three with resulting finer ranking such as: no schooling, graduate of grammar school, graduate of high school, B.A. from college or equivalent, M.A., and PhD. with corresponding ranking from 1 to 6. Even if the data is quantitative, sometimes a rank is used. For instance, in a swimming race the order of finish is a ranking involving only positive integers.

Ranking of qualitative data is not always possible. If it can be done on the basis of some value judgement it converts the qualitative data into quantitative data for further analysis.



When making the original record of either quantitative or qualitative data, the student should constantly check the records for completeness, consistency and accuracy. It is surprisingly easy to make mistakes.

Exercises

- 7. A pollster made up a poll with the following questions. Which questions involve qualitative and which quantitative data? Which involve measurements? To which would you give exact answers? Are there some questions that do not involve data at all?
 - a. Do you watch TV?
 - b. If so, what stations do you watch?
 - c. How many stations do you get?
 - d. How many hours a day do you watch?
 - e. What kinds of programs do you like? Rank them in order of preference. Sports; drama; comedy; soap opera; cartoons; news; nature shows.
 - f. Do you have a color TV or black-and-white?
 - g. How many TV's do you have?
 - h. Are TV programs generally good or bad? Rate on a scale of 1 (poor) to 10 (excellent).
 - i. Add any other comments you would like to make.
- 8. Somebody opposed to increasing school taxes suggests that teachers don't work as hard as most people. This is a pretty general statement and therefore hard to refute off hand. To gather some data, you decide to conduct a poll. Should your questions involve qualitative or quantitative data? Suggest some questions you consider relevant. What kind of data do they involve? Are the numbers you expect to get exact or approximate? Whom should you poll?
- 9. Can you think of situations in which you would rather have qualitative than quantitative data?
- 10. Suppose you were at a dinner party and suddenly you began to have an appendicitis attack. What information about the people present would be helpful? What data might be irrelevant?

ANSWERS TO EXERCISES

 a) Some items might be: Current salary schedules for districts of the same size in this state Same in neighboring states Cost of living increases for past 10 or 20 years Local schedule increases for the same time Salary schedules for other municipal employees with comparable educational requirements and with less requirements Incomes of other professionals with similar experience
 b) Probably all but the last item would be matters of public record.

- 2. Opinions of parents, of students, of teachers with experience in such systems, of academic advisors, of college counselors. Results in schools that have adopted such systems, or the choice between such systems and a regular graded systems.
- 3. What is the average size family of students in this class? What is the maximum distance any student lives from school? What student walks the greatest distance to school?
- 4. How does the proportion of minority students in our school compare to the proportion of similar minorities in the population of our district? How does it compare to the proportion in the whole state?
- 5. What is the average age of Members of Congress? Who was the youngest and who the oldest man ever elected President of the United States?
- 6. a) First b) Second c) First
- 7. Quantitative: c), d), g), h) Qualitative: a), b), e), f) Measurements: d), h) Give exact answers: g) Could give exact: c), g) No data involved: i)
- 8. Questions should involve both, but mostly quantitative.
 - i) How many hours per week are you in the classroom?
 - ii) How many hours are spent in preparation?
 - iii) How many hours are spent correcting papers or tests?
 - iv) Do you feel emotionally drained after a day's work?
 - v) Is counseling kids after school a regular part of what you feel is your job? If so, how much time does it take?
 - vi) Do you feel that teaching is an 8-3 job?

These are suggestions. Similar questions might be asked of police officers, fire fighters, office workers, etc. in order to make comparisons.

- 9. In planning family dinners, preferences on various foods would be important. No use wasting money on liver if no one likes it, etc.
- 10. Are any of them doctors or nurses? Irrelevant information would be age and sex.

ORGANIZATION

Placement Guide for Tabbed Dividers

Tab Label	Follows page:
CONTENT FOR TEACHERS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
Probability with Counting	
Inferential Statistics	
Appendix-Computer Programs	
DIDACTICS	
Components of Instruction	
Classroom Management	
Statistics and Probability Learning	
TEACHING EMPHASES	
Critical Thinking	
Decision Making	
Problem Solving	
Models and Simulations	
Calculators and Computers	
Laboratory Approaches	
CLASSROOM MATERIALS	
CONCEPTS AND SKILLS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Misleading Statistics	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
APPLICATIONS	
Business & Commerce	
Environment	
Health & Medicine	
People & Culture	
Recreation	
GLOSSARY	
ANNOTATED BIBLIOGRAPHY	
SELECTED ANSWERS	



INTRODUCTION

Gathering data may be the first step in a statistical study but raw quantitative data is not much use until it has been organized in some fashion. The first such organization is usually into tables. Before we study how to do this it will be profitable to look at several tables prepared by others. Vast numbers of tables of data are published in books, magazines and newspapers. Trying to read and interpret the data in them will make us more conscious of the care needed in constructing our own tables.

In this section of CONTENT FOR TEACHERS we will consider how to construct and interpret tables correctly and accurately. We will also consider what makes a table inaccurate and biased. However, the classroom activities on misleading tables are in the MISLEADING STATISTICS section.

READING TABLES

Here are a couple of tables of figures reproduced exactly as printed, the first in a magazine, the second in a book. Can you read and understand them easily? Do they say something to you? Are they presented clearly or do you have to guess at the information they are trying to convey? One of the prime objectives of this section is to explain how to read and understand the data presented in tables and later on in graphs. Learning how to read tables will also help us learn how best to make effective presentations of our own data to others.

		Т	ABLE 1					
		HEIGHT -	WEIGHT	CHART				
Height	Small Frame Medium Frame Large Fr							
(in shoes)	Men	Women	Men	Women	Men	Women		
4'10"		95		101		111		
5' 0"		100		107		117		
5' 2"	116	106	(123)	113	133	123		
5' 4"	122	112	129	119	140	129		
5' 6"	128	119	136	127	147	137		
5' 8"	136	127	145	135	156	146		
5'10"	145	135	153	143	165	154		
6' 0"	153	143	162	151	174	163		
6' 2"	161		171		183			

TABLE 2										
BIRTH RATES FOR THE UNITED STATES, 1910 - 1973 (Rates are per 1000 Population)										
Year	Birthrate	Year	Birthrate							
1910	30.1	1945	20.4							
1915	29.5	1950	24.1							
1920	27.7	1955	25.0							
1925	25.1	1960	23.7							
1930	21.3	1965	19.4							
1935	18.7	1970	18.4							
1940	19.4	1973	14.9							

Data from Statistical Abstract of the United States, 1976, Table 68, p. 51.

In the first table, we assume the weights are in pounds although nothing is stated. What does the (123) in the third row, third column position of Table 1 mean? Is this the average weight of all men 5'2" tall? Mr. Lion is a man 5'8" tall and he weighs 178 lbs. What does the table tell him?

In the second table, what does "birthrate" mean? Should this information be in the table? Is it?

Any table should have a title specifying the content of the table. This will be either a headnote or a footnote. It is the first thing to look at and read carefully. Then the labels on the individual rows and columns should be examined. Do the numbers give actual values or percentages? What units are specified? Are the number approximate or exact?

Look again at Table 1. The units of weight should be specified as pounds. The label should indicate whether each weight is the actual average weight of individuals of that height and sex or the ideal weight as determined by a health expert.

Look at Table 2. Has the birthrate gone down steadily? No. What might have caused the fluctuations?

Here is Table 3. Let us try to read it carefully.

PUBLIC ELEMENTAR OF CL (Schools classified by t	ASSROOM	I TEAC	HERS 19	960 - 1	975			roup;			
elementary inclu		0			ondary	includ	les				
junior high school.) ITEM 1960 1965 1969 1970 1971 1972 1973 1974 1975											
Average Salary,											
All teachers \$1000	5.0	6.2	8.0	8.6	9.3	9.7	10.2	10.8	11.5		
Elementary \$1000	4.8	6.0	7.7	1	9.0	9.4	9.9	10.5	11.2		
INDEX $(1967 = 100)$	72.7	90.4	116.6	127.0	136.2	142.3	149.1	158.7	169.6		
Secondary \$1000	5.3	6.5	8.2	8.9	9.6	10.0	10.5	11.1	11.5		
INDEX $(1967 = 100)$	74.2	90.7	115.5	125.1	134.6	141.1	147.7	155.8	166.4		
Percent of Teachers											
Under \$7500	88.1	81.8	49.5	36.6	24.7	20.3	14.9	8.9	NA		
\$7500 - \$8499	11.9	10.1	18.1				16.8	14.6	NA		
\$8500 — \$9499		5.5	13.4				16.0	15.6			
\$9500 - \$11499		2.6	14.6					28.1			
\$11500 and over			4.3	10.3	19.0	23.1	27.7	32.7	NA		
Represents zero NA	Not Av	ailab	Le								

Data from Statistical Abstract of the United States, 1976, Table 220, p. 134.

Note both the headnotes and the footnotes. The source of the material is given so we can judge for ourselves whether we think the material is probably accurate or may be either intentionally or unintentionally biased to lead us to accept something perhaps untrue. The row labels indicate the units. Thus we read in the first row and second column the average salary of all school teachers in 1965 was \$6200 while the second entry in the same column indicates elementary school teachers received \$6000. The third and fifth rows labeled INDEX are not so clear. The note (1967 = 100) means these figures have something to do with the relation between that year and other years. The figures in the bottom half of the table are all percents. Those in any one column should add up to 100. A quick check shows that some columns only add up to 99.9 but this is because of rounding errors. The whole table gives a picture of rising salaries but without a table of the rising cost of living to compare with, we are not sure whether salaries have exceeded or dropped behind the results of inflation. It might be interesting to try to find a table that would help answer this question.

Exercises (Answers given on pp. 44-48)

- In Table 1, 145 appears in the 7th row, first column and in the 6th row, third column. What does this figure indicate? Why are there blanks in certain places in the table? About what should a large frame 6'4" man weigh according to this table? How about a small framed 5'5" woman?
- 2. In Table 2, the birthrate dropped slowly from 1910 to 1925 and then drastically in 1930. Can you give a reason? It slowly increased and then jumped in 1950. Why? There are two periods of steady decline. What are they?
- 3. What was the average salary of an elementary school teacher in 1960? In 1970? Of a high school teacher in 1975? For 1975 note the first figure 11.5 in the light of the 11.2 and 11.5 in the second and fourth rows of the column. Compare the corresponding three figures in all the other columns. Do you think the first 11.5 might be a mistake? Why? In 1974 a teacher earning \$9000 would have been better off than what percent? About how much would a teacher have to get in 1974 to be better off than 76.4% of all teachers? Use the given index figures to find the average salaries of elementary and high school teachers in 1967.

ORGANIZING DATA INTO TABLES

Secondary data is usually made available in tables like those we have just been reading. When primary data is collected it may be first recorded as a list in the order in which each item was found. Often it can be reorganized into some other order which will yield information more easily. Perhaps the most common way is to make a list in order of size. If many items are spread over a wide range, it may help to group them in intervals rather than try to record them individually. The number and size of the intervals is subject to some question, but a convenient number is one between 6 and 15. This number is chosen so the resulting intervals also will be of convenient size and will include all the data. If the number of different values in the data is small (less than 15), we may not need to use intervals.

Example 1

There are 30 classrooms in the Doherty School. The number of students per classroom is given in Table 4.

TABLE 4														
NUMBER OF STUDENTS PER CLASSROOM														
26,	33,	26,	29,	31,	17,	28,	29,	31,	27,	26,	32,	29,	31,	30,
31,	27,	28,	29,	32,	30,	33,	32,	29,	27,	31,	33,	31,	27,	29

TABLES

The first thing to notice is Room 6. The record shows only 17 students. Could this be an error in recording? Should the number really be 27? On checking, we find 17 is indeed correct. It might be worth investigating why this number is so small compared to the others. We should always examine the raw data to look for possible errors and inconsistencies.

TALLYING

Making a total list directly in order of size is difficult and involves checking and rechecking for errors and omissions. Various devices are available to help. The first is a simple tally. We make a list in order of the different values that appear in the table and next to each make a tally mark for each occurrence. The

fifth tally is made by crossing over the first four:### . This makes it easier to get the totals. Table 5 is a tally table made from Table 4. We do lose something in this process. From Table 5 we cannot tell which room has only 17 students or which two have 30 apiece. Since there are only nine different room sizes, no grouping is necessary.

TABLE 5						
STUDENTS PER CLASSROOM						
Students per Classroom	Frequency Tally	Number				
17	l	1				
26	111	3				
27	1111	4				
28	Ц	2				
29	++++ 1	6				
30	11	2				
31	14441	6				
32	111	3				
33	1(1	3				

Example 2

A grocer receives 43 boxes of apples and decides to weigh them as they come in the store to try and decide if he is being systematically short weighted. The weights in kilograms are given in Table 6.

		(TABLE 6 ES OF AF in kil	PLES	•		
16.3	16.2	16.3	17.1	15.0	15.6	16.3	16.8	16.1
15.9	17.1	15.3	16.4	16.3	16.7	16.0	16.1	17.0
16.1	15.7	16.3	16.3	15.9	16.2	16.3	16.4	16.2
16.1	15.6	16.3	17.0	16.3	15.7	16.2	16.1	16.3
16.7	16.3	15.5	15.7	16.1	16.0	16.3		

TABLES

The 43 items are scattered over the range from
15.0 to 17.1 covering 22 possible values. To reduce
this number we decide to group the data into sub-
intervals. 10 intervals of length two would do, as
would 8 of length three or 7 of length four. We
choose 8. The first interval could go from 15.0
to 15.2 or from 14.9 to 15.1 or from 14.8 to 15.0.
Again the choice is ours and we choose 14.8 to 15.0.
The reason is boxes are supposed to weigh not less
than 16 kg. With this choice one interval is 16.0
to 16.2 (see Table 7).

TABLE 7 BOXES OF APPLES (Weights in kilograms)					
Weight	Number of Boxes				
14.8-15.0	1				
15.1-15.3	1				
15.4-15.6	u				
15.7-15.9	++++				
16.0-16.2	11 ++++ ++++				
16.3-16.5	11111111111111				
16.6-16.8	111				
16.9-17.1	uu				

FREQUENCY DISTRIBUTIONS

If we count the number of items in each interval and record them, we have a new table called a frequency distribution. A frequency distribution table is one

of the most important tools of a statistical study. If we look at how the numbers cluster around a central value we are led to the various kinds of averages. The position of the numbers along the distribution leads to the ideas of quartiles, deciles and percentiles. These and the averages are considered in *Mean*, *Median and Mode* in the CONTENT FOR TEACHERS section. From the frequency distribution we construct the histograms and frequency polygons. These and the range and scatter of the numbers are discussed in *Range and Deviation* in the CONTENT FOR TEACHERS section.

TABLE 8 BOXES OF APPLES (Weights in kilograms)						
Weights Frequency						
14.8-15.0	1					
15.1-15.3	1					
15.4-15.6	3					
15.7-15.9	5					
16.0-16.2	12					
16.3-16.5	14					
16.6-16.8	3					
16.9-17.1	4					

STEM AND LEAF DIAGRAM

Another way of making a useful table is a stem-and-leaf diagram. We start by deciding on the intervals we want to use and listing them in a vertical column. In Table 9, the <u>stem</u> consists of the first two digits of the item and the <u>leaf</u> consists of the last digit. The stems are written down for each interval first. Then each item in the original list is tallied by entering its last digit as the leaf in the appropriate position. Table 9 displays the weights of the 43 boxes of apples as given in Table 6.

		TABLE 9 XES OF APPLES ts in kilograms)	
Interval	Stem	Leaf	Frequency
14.8-15.0	1 <u>4</u> 1 <u>5</u>	0	1
15.1-15.3	15	3	1
15.4-15.6	15	6,6,5	3
15.7-15.9	15	9,7,9,7,7	5
16.0-16.2	16	2,1,0,1,1,2,2,1,2,1,1,0	12
16.3-16.5	16	3,3,3,4,3,3,3,3,4,3,3,3,3,3	14
16.6-16.8	16	8,7,7	3
16.9-17.1	1 ⁶ ₇	1,1,0,0	4

The circled stem $\begin{pmatrix} 1\\5 \end{pmatrix}$ is written that way to indicate the digits in the stem may be either 14 or 15. In the third row the leaves 6,6,5 come from the table weights 15.6, 15.6 and 15.5.

From this table, the frequency distribution can be made. The stem-and-leaf diagram has an advantage over the tally table in that each item of the original data is still there. Therefore, mistakes in the recording can be easily spotted and corrected.

CUMULATIVE FREQUENCY

To determine the deciles or percentiles of a distribution, a table of cumulative frequencies may be constructed. This records the numbers of items at or below the uppermost value of each interval. Table 10 shows that 10 of the 43 boxes weigh 15.9 kilograms or less.

		TABLE 10 XES OF APPLES ts in kilograms)	
Interval	Frequency	Upper Most Value	Cumulative Frequency
14.8-15.0	1	15.0	1
15.1-15.3	1	15.3	2
15.4-15.6	3	15.6	5
15.7-15.9	5	15.9	10
16.0-16.2	12	16.2	22
16.3-16.5	14	16.5	36
16.6-16.8	3	16.8	39
16.9-17.1	4	17.1	43

Each number in the fourth column is the sum of the numbers in the second column up to that point. Care in adding is important, but an obvious check is that the last number in the cumulative frequency column must be the total number of the items being considered.

SUMMARY

To make a frequency table, remember to:

- 1. Decide on the number of intervals (usually 6-15).
- 2. Determine the end points of each interval so each item falls in just one interval.
- 3. Make the intervals equal in length unless there is good reason not to. For example, if the data is about ages, we might use 10-year intervals and the last one could go from 90 on up, since there are so few persons over 90.

A table should be clearly labeled and each row or column should have an appropriate heading.

Exercises

- 4. The height and weight of each number of a relatively small fifth grade class is given in the following table.
 - a) Make a tally of individual heights. Circle the tally marks for the girls. What do you notice?
 - b) Make a tally of individual weights. Circle the tally marks for the girls. What do you notice?
 - c) The heights vary from 144 cm to 165 cm, a range of 21 cm. If you want to group them in intervals, a convenient number would be 8 using intervals of length three units. Make two tallies, the first starting with the interval 143-145 and the second with 144-146. The tallies are slightly different but the general pattern is the same.
 - d) Do a similar grouped frequency tally for the weights.
 - e) Using intervals of length 5 units starting with 25-29 for weights and 140-144 for heights, make stem-and-leaf diagrams of the data.
 - f) Make frequency distribution tables and cumulative frequency tables from one of the grouped frequency tables in (c) and from the one in (d).

Н	EIGHTS AND WEIGH	TS OF A FIFTH GR.	ADE CLAS	S	
Sex	Height in cm	Weight in kg	Sex	Height in cm	Weight in kg
М	162	53	М	150	40
F	152	41	F	154	42
М	153	48	м	144	27
М	153	37	м	147	35
М	146	40	м	165	53
М	157	41	F	156	51
F	158	45	M	155	46
F	156	47	м	162	45

Г

- 5. For another fifth grade class, the heights and weights are given in the following table.
 - a) Make a grouped frequency tally and a frequency distribution table of the heights.
 - b) Do the same for the weights.
 - c) Make stem-and-leaf diagrams for weights and heights.
 - d) Make frequency distribution tables in each category for boys and girls separately. Do they differ markedly from each other?
 - e) Make cumulative frequency tables from all the tables.

	HEIGH	TS AND WEIGHTS OF	A FIFTH	GRADE CLASS	
Sex	Height in cm	Weight in kg	Sex	Height in cm	Weight in kg
М	158.5	43	F	158.5	51
F	155	43	М	152	62.5
М	149.5	43	М	135	27.5
F	152	50	М	149	37
F	160.5	42	F	148	38
М	168	50	М	161	59
М	156.5	43	F	156	44.5
М	159	45	М	151.5	40.5
F	152	40	М	159	44
М	146	42.5	F	161	46
М	156	44	м	144	39
М	163.5	52.5	F	153	44
F	152	39.5	F	157.5	40
М	151.5	37	F	161	59
М	163.5	50.5	М	141	31
F	149	46			

-

6. Look now at the tables you have made based on the data for Exercise 5. For what purpose would you use each table, i.e., what kinds of questions might each of them help you to answer?

INTERPRETING DATA IN TABLES

The reason for organizing data in tables is to give us a message. To get the message, we must interpret the data properly. This can be a very subtle and complicated problem depending heavily on the mathematical theory of probability. It is the core of inferential statistics. We are not going into such a study of probability at this time but by studying well-constructed tables we can almost always come to some valid and intelligent conclusions. As a result, we may be able to make more rational decisions for some of our problems than we could have before we looked at the data.

We recognize, of course, that there are many types of problems for which data collecting is inappropriate. This is true for moral dilemmas that involve value issues. President Lincoln's Cabinet voted unanimously against the issuance of the Emancipation Proclamation but this data did not affect his decision. We are concerned here not with those situations but with the many cases where a careful look at the data and its proper interpretation can help.

VARIABILITY AND BIAS OF THE DATA

Before we try to interpret data we must judge its accuracy and validity. Where did it come from? Who collected it? If the data in an advertisement seems to say that Heal Quick (HQ) painkilling drug is twice as effective as Old Reliable (OR), we should remember that twice zero is still zero so neither may be effective. Also if OR does stop my headache, why shift to another even if it's ten times stronger. What data supports the claim that HQ is better than OR? Did HQ's own lab try it out? Did they use people or animals? Or did 10 hospitals conduct controlled experiments? Even so, suppose 6 hospitals reported OR was better than HQ, 1 hospital said they could find no difference, and 3 said HQ was better than OR. Does the data show this? Probably not. The advertisement trumpets the fact that "Investigators at three different leading hospitals report HQ is twice as effective as OR. Shouldn't you switch to HQ?" No mention is made of the other seven hospitals and they hope you never find out about them. Obviously, the data reported is biased. We cannot always tell if data is biased but at least we should think about the possibility and in many such cases reserve judgment.

Consider the cereal box that claims "one cupful gives you 100% of the U.S. Recommended Daily Allowance of nine important vitamins and iron." What data supports this statement? Who tested it? The manufacturer's labs or U.S. Government labs? Do you suppose a high school student, a professional tennis player, a business executive and a retired school teacher all need the same amount of vitamins? Is the ad actually saying anything that you might be able to act on? Is the source of its information biased?

The Surgeon General of the United States reports that the National Cancer Institute's studies show a high correlation between cigarette smoking and the incidence of lung cancer. This says a greater proportion of the people who smoke cigarettes get lung cancer than those who do not smoke and the proportion increases with higher levels of smoking. A tobacco company may counter by saying "We don't believe your statistics. We think you are biased as our statistics show no such correlation. And even if you are correct, we don't believe smoking causes lung cancer. No one knows what causes lung cancer but we believe hyper-anxiety may be a cause of both lung cancer and heavy smoking. If you are not a hyper-anxious person, you might be able to smoke all you want and be no more susceptible to lung cancer than your non-smoking neighbor." Thus a high correlation may not mean a causal relationship. Even if there is a causal relationship it is possible that the direction of cause and effect is reversed. Thus lung cancer could conceivably cause hyper-anxiety and hyper-anxiety could cause smoking. In considering rival claims one must judge and interpret the data offered. Is there possible bias? Where does the bias lie? With whom do I agree, if either one? Do I stop smoking?

SUFFICIENCY OF THE DATA

Sometimes the question is "Do we have enough data?" For example, a student decides to toss a coin to decide whether to study or watch TV. "Heads I study, tails I watch TV." A toss gives a "head." "That's not a fair coin. It should have landed 'tails'." Question: Is there enough data to say the coin is not fair? Obviously not, at least for most disinterested bystanders. Suppose three tosses give three successive heads. Is it a biased coin? Suppose heads come up 5, 10, 20, 100 times in a row. Sooner or later, most people will say, "It's not a fair coin." At what point would you say that? How sure would you be?

TABLES

Most data is biased in some way and to some extent whether we know it or not. So conclusions drawn from it should be made with discretion and some caveats. But we may decide we have to buy a new car so we study the data the various dealers give us, recognize they are all biased one way or another and finally make a decision, our best decision in the light of all the uncertainties we recognize. We may get a "cream puff", or we may get a "lemon", but we do the best we can using all the evidence (data) we can get our hands on.

SUMMARY

- 1. Consider the source of the data for possible bias. If no source is given, be double wary.
- 2. Is the data really relevant?
- 3. Is there enough data to warrant drawing conclusions?
- 4. Can the conclusions be verified by collecting more data or by consulting another source?
- 5. Is the extra effort worth the time and money involved?

Exercises

- 7. In a city of about 1,000,000 people a proposal is made to build a limited access highway from the suburbs. <u>The Suburban News</u> reports that 67% of the people they polled are in favor. <u>The Inner City Daily</u> reports that their poll shows only 37% in favor. Your class wonders why the difference and interviews the two editors. <u>The Suburban News</u> conducted their poll at 6 P.M. to 7 P.M. at the bus stop. They talked to 125 men and women. <u>The Inner City Daily</u> polled 235 people between 2 and 3 P.M. on City Hall Plaza. Comment on possible bias and on the number of people interviewed. Your class wants to have a better poll. Can you suggest a better plan? (Comment: Planning a good poll is tough. Maybe you would rather wait until we have studied more about statistics before answering this question. It is enough at this time to recognize the possibility of bias.)
- 8. Consider the table below.
 - a) Determine the percent of male applicants who were hired at each school level.
 - b) Determine the percent of female applicants who were hired at each school level.
 - c) Were men or women applicants more likely to get teaching positions at each level?
 - d) Compare the total number of men hired and the total number of women.
 - e) Now find the total number of men and women applying for all the jobs available and the percent of each who were hired.
 - f) Were men or women applicants more likely to get teaching positions?
 - g) Discuss the possible bias of the data. What went wrong to give the contradictory results?

	Mal	e	Female		
	Applicants	Employed	Applicants	Employed	
University	65	31	10	5	
College	143	70	36	20	
High School	275	125	175	82	
Elementary School	46	17	360	152	

	TABLE 11				TABLE 12				
		UNNING A MAZ in Minutes)					ALL PER YEAR nt of Inches		
Minutes	Fre- quency	Cumulative Frequency	Relative Cumulative Frequency		Inches	Fre- quency	Cumulative Frequency	Relative Cumulative Frequency	
.5- 1.5	36	36	. 44		20-22	2	2	.04	
1.6-2.5	11	47	.53		23-25	1	3	.06	
2.6- 3.5	12	59	.72		26-28	4	7	.14	
3.6- 4.5	7	66	. 80		29-31	6	13	.26	
4.6-5.5	5	71	.87		32-34	11	24	.48	
5.6-6.5	2	73	. 89		35-37	10	34	.68	
6.6-7.5	4	77	.94		38-40	7	41	.82	
7.6- 8.5	3	80	.98		41-43	4	45	.90	
8.6-9.5	1	81	.99		44-46	3	48	.96	
9.6-10.5	1	82	1.00		47-49	2	50	1.00	

In Table 11, the frequency clusters in the first three intervals--72% fall there. Most of the rats can run the maze fairly quickly. There are two very slow ones out of the 82. In Table 12, the frequency clusters near the center rather than at one end as the previous one did. There are about as many items at a given distance above the center as below. We say the distribution is nearly symmetric. From the relative cumulative frequency we see that about a quarter of the items are below 31 and about 18% are above 40.

The thing that stands out in Table 13 is the lonely outlier, the one case of a hardness in the interval 85-89. Is that real or was a mistake made in recording the value? An outlier is usually a signal for further investigation. If it is a mistake, it is a reminder that care is needed in reading and recording data. If it is a correct value, what happened to make that particular piece of steel so different from all the others?

	TABL	E 13					
	HARDNESS (100 Sa						
Hardness	Frequency	Hardness	Frequency				
40-44	1	65-69	19				
45-49	4	70-74	0				
50-54	13	75-79	0				
55-59	26	80-84	0				
60-64							

Data from *Statistics: A New Approach* by W. A. Wallis and Harry V. Roberts, published by Free Press, p. 206.

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INTERPRETING FREQUENCY TABLES

Looking at data organized into a frequency table may enable us to pick out a general pattern. Here are two frequency tables with quite different patterns.

Exercises

9. Interpret the data in this table by answering questions such as:

- a) Is it a homogeneous group?

- b) Will any students probably need special attention?
 c) Are the IQ's scattered widely or fairly well concentrated?
 d) How would you answer questions (a), (b) and (c) if the top and bottom students were transferred to another room?

	1	1	1
I.Q.	Frequency	I.Q.	Frequency
80- 84	1	110-114	15
85- 89	0	115-119	6
90- 94	3	120-124	0
95- 99	7	125-129	0
100-104	9	130-134	1
105-109	8		

10. What do you see in the data below? Could you pick out which is a child's book and which is an adult's?

Number of letters in words	Frequ	ency B	Cumul Freq A	ative uency ^B	Relat Frequ A		Cumula Rela Freq A	
1- 2	14	11	14	11	.22	.22	.22	.22
3- 4	25	21	39	32	. 38	.43	.60	.65
5- 6	10	12	49	44	.15	.25	.75	.90
7- 8	8	5	57	49	.11	.10	.86	1.00
9-10	7	0	64	49	.12	.00	.98	
11-12	1	0	65	49	.02	.00	1.00	

11. Consider this table. Does the telephone company have any favorites? Would you get a similar distribution for the first digits?

LAS	T DIGITS	IN	A COLUMN	OF	TELEPHONE	NU	MBERS			
Last Digit	0	1	2	3	4	5	6	7	8	9
Frequency	10	8	8	7	8	8	8	4	8	12

12. Look at the table below. What is the largest possible count that might have happened? Are you surprised with the 12 as the first entry? Which interval did you expect to have the largest frequency?

(The 12 in the	POSITION OF THE FIRST ACE IN A WELL-SHUFFLED PLAYING DECK OF 52 CARDS (The 12 in the First Place Means that an Ace Appeared as Either the First, Second, or Third Card in 12 Deals out of 50.)							
Position	Frequency	Position	Frequency					
1- 3	12	16-18	4					
4- 6	5	19-21	2					
7- 9	11	22-24	2					
10-12	7	25-27	1					
13-15	6	28-30	0					

- 13. Look at Exercise 9. Some of the questions asked require clarification of meaning alone and then are factual. Other have embedded in them value judgments. Which parts of the exercise require understanding of value judgments before responding?
- 14. Look at Exercises 10 and 11. Can you think of any reason why data like this might be collected? Is anyone likely to make a decision on the basis of such data?

ANSWERS TO EXERCISES

- 1. The 145 in the 7th row, 1st column is what a small framed man 5 ft. 10 in. tall should weigh. The other 145 is the appropriate weight for a man with medium frame who is 5 ft. 8 in. tall. The blanks in the table indicate that there are not enough people at this height to arrive at a meaningful score. A large framed 6'4" man should weight about 192 pounds. Each additional two inches in height has meant an increase of about 9 pounds so we add 183 + 9 = 192. A small framed 5'5" woman might be expected to weigh about halfway between the weight of a 5'4" and a 5'6" woman or about 115 pounds.
- The drastic drop in birthrate in 1930 may reflect the effects of the great economic depression of 1929-30. The jump in 1950 is the baby boom after the 2nd World War. The steady decline periods were 1910-1935 and 1955-1973.
- 3. The average salary of an elementary school teacher in 1967 was \$6650. Use the proportion x/100 = 7300/109.9. For a high school teacher in 1975 the table gives \$11,500. The 11.5 given as the average salary of all teachers in 1975 is probably an error. In every other year the corresponding figure is between that in the 2nd and 4th row. The average salary of all teachers should always lie between that for elementary and secondary teachers and nearer that of elementary teachers as there are many more of them.

a)	Heights	Boys	Girls	Total	Weights	Boys	Girls	Total
b)	144	1		1	27	1		1
	146	1	-	1	35	1		1
	147	1		1	37	1		1
	150	1		1	40	11		11
	152		1		41	1	1	10
	153	11		11	42		1	0
	154		1	O	45	1	. 1	II)
	155	1		1	46	1		1
	156		11		47		1	0
	157			1	48	1		1
	158		1	()	51		1	
	162	()		11	53	11		11
	165	1		1				

The girls in the class seem to be somewhat nearer together in both height and weight than the boys. The four smallest individuals are boys as are the two largest.

4.

c) Heights

Interval	Tally	Frequency	Interval	Tally	Frequency	Cumulative Frequency
143-145	1	1	144-146	11	2	2
146-148	11	2	147-149	1	1	3
149-151	1	1	150-152	11	2	5
152-154	1111	4	153-155	101	4	9
155-157	1111	4	156-158	1111	4	13
158-160	1	1	159-161		0	13
161-163	11	2	162-164	11	2	15
164-166	1	1	165-167	1	1	16

d) Weights

Interval	Tally	Frequency	Cumulative Frequency
26-29	1	1	1
30-33		0	1
34-37	11	2	3
38-41	1111	4	7
42-45	111	3	10
46-49	111	3	13
50-54	111	3	16

e) Stem-and-leaf diagrams

	Weights			Heights	
Interval	Stem	Leaves	Interval	Stem	Leaves
25-29	2	7	140-144	14	4
30-34	3		145-149	14	6 7
35-39	3	7 5	150-154	15	2 3 3 0 4
40-44	4	10102	155-159	15	78665
45-49	4	85765	160-164	16	2 2
50-54	5	3 3 1	165-169	16	5

f) See the answers for (c) and (d).

5.	a)		Heights		b)		Weights	
	Interval	Tally	Frequency	Cumulative Frequency	Interval	Tally	Frequency	Cumulative Frequency
	135-137	1	1	1	26-29	1	1	1
	138-140		0	1	30-33	1	1	2
	141-143	1	1	2	34-37	11	2	4
	144-146	U	2	4	38-41	HHTI	6	10
	147-149	1411	4	8	42-45	HITTH	11	21
	150-152	4411	6	14	46-49	11	2	23
	153-155	11	2	16	50-53	HHT	5	28
	156-158	HTTI	6	22	54-57		0	28
	150 161	11141	C	20	58-61	11	2	30
	159-161	HHTI	6	28	62-65	1	1	31
	162-164	111	2	30		-	*	
	165-167		0	30				
	168-170	1	1	31]			

c)

Stem-and-Leaf for Heights

Interval	Stem	Leaves
135-139	13	5
140-144	14	4,1
145-149	14	9.5, 6, 9, 9, 8
150-154	15	2, 2, 2, 1.5, 2, 1.5, 3
155-159	15	8.5, 5, 6.5, 9, 6, 8.5, 6, 9, 7.5
160-164	16	0.5, 3.5, 3.5, 1, 1, 1
165-169	16	8

Stem-and-Leaf for Weights

Interval	Stem	Leaves
25-29	2	7.5
30-34	3	1
35-39	3	9.7, 7, 7, 8, 9
40-44	4	3, 3, 3, 2, 3, 0, 2.5, 4, 4.5, 0.5, 4, 4, 0
45-49	4	5,6,6
50-54	5	0, 0, 2.5, 0.5, 1
55-59	5	9,9
60-64	6	2.5

d)

Heights

			0			
		Boys			Girls	
Interval	Tally	Frequency	Cumulative Frequency	Tally	Frequency	Cumulative Frequency
135-139	1	1	1		0	0
140-144	11	2	3		0	0
145-149	11)	3	6	11	2	2
150-154	111	3	9	MH	4	6
155-159	HHT	5	14	1111	4	10
160-164	111	3	17	111	3	13
165-169	1	1	18		0	13

1.0	iork	to
We:	T 5 I	160
	- 0-	

		Boys			Girls	
Interval Tall		Frequency	Cumulative Frequency	Tally	Frequency	Cumulative Frequency
25-29	1	1	1		0	0
30-34	1	1	2		0	0
35-39	111	3	5	11	2	2
40-44	11111	7	12	14111	6	8
45-49	1	1	13	11	2	10
50-54	111	3	16	11	2	12
55-59	1	1	17	1	1	13
60-64	1	1	18			

e) See tables for (a), (b) and (d).

- 6. From tables (a) and (b), we can see patterns of clustering and spreading. Many children are near the center of the distribution and a few out at the ends. Tables in (d) show the boys differ among themselves much more widely than the girls. The stem-and-leaf diagrams in (c) give a vivid picture of heavy concentration at the center. The cumulative tables help to spot where the decile breaks are. All these ideas will be studied in the following sections on mean, median and mode and the one on range and deviation.
- 7. The Suburban News poll seemingly talked to commuters who would want better service to the city. The Inner City Daily might have talked to people whose homes might be razed to make way for the highway and would naturally be against it. The numbers are not so important as the place the poll was made. A better plan? Wait for later.
- 8. a) U: 47.7%; C: 49%; HS: 45.5%; ES: 37%
 - b) U: 50%; C: 55.6%; HS: 46.9%; ES: 42.2%
 - c) At each level women were more likely since a higher percent of applicants were hired.
 - d) 243 men were hired compared to 259 women.
 - e) Total number of men applying was 529 and 45.9% of them were hired. Total number of women applying was 581 and 44.6% of them were hired.
 - f) A higher percent of male applicants was hired than female so it was more likely for a male applicant to get a job than a female.
 - g) It seems curious that at each school level women applicants were more likely to be hired than men but in the total teaching profession men are more likely to be hired. The bias in the data is that so many more women applied for elementary school positions than men. Notice that the ratio of applicants, women to men, at the university level is $10/65 \approx .154$; at college is $36/143 \approx .252$; at high school is $175/275 \approx .636$ while at elementary school it is $360/46 \approx 7.83$.
- 9. a) No.
 - b) Yes, certainly the one with IQ at 82. Probably the one with high IQ needs special attention to or sheer boredom will mean no work and probably low grades.
 - c) Scattered widely.
 - d) Now the group is fairly homogeneous. The IQ's are reasonably concentrated. The bottom three students may need special help but the rest should get along fairly well.

11. O and 9 seem a little more common than the others but not very much so. First digits would probably have a similar distribution.

TABLES

- 12. Largest possible count is 49. Yes. I expected more in the 7-9 region.
- 13. (a) and (c) are essentially factual. (b) requires value judgments.
- 14. We might pick or discard a book for 3rd grade children by such a count as in Exercise 10. Exercise 11 is merely a matter of interest. No decisions rest on this data.

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ORGANIZATION

Placement Guide for Tabbed Dividers

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INTRODUCTION

When the data for a problem has been collected it may be organized into one or more tables. The impact of the numbers can usually be made more effective by a visual presentation. This is done by means of drawing graphs of various kinds. These may be the common <u>pictographs</u>, <u>circle graphs</u>, <u>bar graphs</u> and <u>line graphs</u>, or the less well known <u>scatter diagrams</u>, <u>histograms</u>, <u>frequency polygons</u> and <u>ogives</u>. We will consider the first four in this section and the others in later sections.

Graphs can be deceptive as well as meaningful. They may give as much misinformation as good information. It is important to know how to draw graphs properly. It is perhaps even more important to learn to read a graph correctly. We do not all have to draw graphs, but vast numbers of charts and graphs appear regularly in newspapers and magazines. Therefore, it behooves us and our students to know whether a graph has been drawn to give us a true message. Also it is important to know what makes a graph good or bad.

In this section of the CONTENT FOR TEACHERS, we will consider both the drawing and interpreting of poor and misleading graphs as well as of good graphs. However, the classroom activities on misleading graphs are in the MISLEADING STATISTICS section.

As the data for the study of different types of graphs let us consider the types of milk sold at lunch at the school cafeterias in the George Washington, Thomas Jefferson, and James Madison schools. They sell skim milk, regular milk, and choco-late milk. Some students collected the data for a given day and made the following table. The data has been rounded to the nearst 25 cartons.

TABLE 1 NUMBER OF CARTONS OF MILK SOLD ON SEPTEMBER 28, 1976											
George Washington Thomas Jefferson James Madison Tot											
Skim	50	75	100	225							
Regular	100	50	50	200							
Chocolate	150	200	225	575							
Total	300	325	375	1000							

PICTOGRAPHS

A pictograph uses little drawings of the objects to represent the number involved. In this case, milk cartons were used. We draw two pictographs to show the facts at one school. One is good and the other is misleading. Can you pick out the misleading graph?

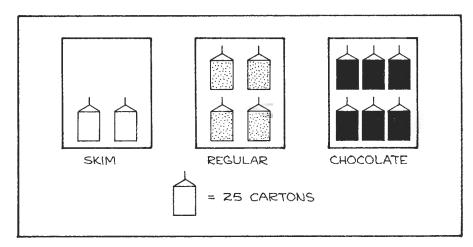


Fig. 1. Amount of Milk of Each Type Sold at George Washington School on September 28, 1976

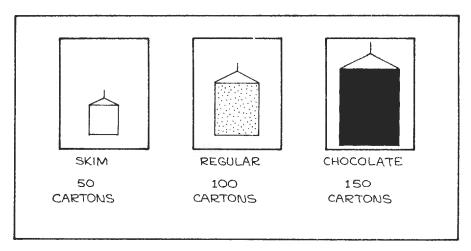


Fig. 2. Milk Sold at George Washington School

Figure 1 is the correct one. In it, each symbol represents 25 cartons sold and the figure shows clearly that twice as much regular milk was sold as skim milk and three times as much chocolate milk as skim milk. Note the "key." It indicates how many cartons each sketched symbol stands for. All the symbols are identical. The comparison is made by examining the number of symbols sketched for each category. Furthermore, the graph has a title indicating clearly what it represents.

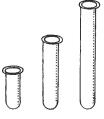
Figure 2 is misleading. The symbol representing regular milk is a carton drawn twice as tall as the one representing skim milk. But a similarly shaped carton twice as tall is also twice as wide and its area is four times as big thus

GRAPHS

giving the wrong impression of the relative sales. If the symbols were drawn in three dimensions, the sales of regular milk would seem to be 2^3 or 8 times as big as those of skim milk. Even though students may not know this exactly, they get an impression of relative size which is far too big. Pictographs should be drawn with the items all the same size and their number proportional to the corresponding number in the table.

Exercises (Answers given on pp. 64-68)

- 1. Draw pictographs showing the distribution of the kinds of milk sold that day at the other two schools and for the total at all three schools.
- 2. Draw a pictograph showing the distribution of the sale of chocolate milk at the three schools. Do the same for regular and for skim milk.
- 3. Look again at Figure 2. Suppose our symbol were a test tube instead of a bottle. Comment on the figure to the right to represent the milk sold at George Washington School. Does it still have the defects of Figure 2? Why or why not?



SKIM REG CHOC

CIRCLE GRAPHS

A circle graph is an excellent way to show the fractional parts into which a whole is divided. The circle is divided into sectors, one for each part. The central angles of the sectors are proportional to the fractional parts they represent. Thus to draw a circle graph of the data in the first column of Table 1, we determine that the skim milk is 50/300 or 1/6th part of the total sold. The regular is 100/300 or 1/3rd and the chocolate is 150/300 or 1/2 of the total. Doing this for each of the other schools we get Table 2.

	TABLE 2											
FRACTIONAL PARTS OF MILK SOLD												
	George Washington	Totals										
Skim	$50 \frac{50}{300} = \frac{1}{6} \approx .17$	75 $\frac{3}{13} \approx .23$	$100 \frac{4}{15} \approx .27$	$225 \frac{9}{40} \approx .23$								
Regular	$100 \frac{100}{300} = \frac{1}{3} \approx .33$	$50 \frac{2}{13} \approx .15$	50 $\frac{2}{15} \approx .13$	$200 \frac{8}{40} \approx .20$								
Chocolate	150 $\frac{150}{300} = \frac{1}{2} \approx .50$	$200 \frac{8}{13} \approx .62$	$225 \frac{9}{15} \approx .60$	$575 \frac{23}{40} \approx .57$								
Total	300	32.5	375	1000								

Since there is a total of 360° in the angles at the center of a circle we determine the angles of the sectors for Thomas Jefferson as $.23 \times 360^{\circ} = 83^{\circ}$, $.15 \times 360^{\circ} = 54^{\circ}$, and $.62 \times 360^{\circ} = 223^{\circ}$. The angles for the other graphs are computed in like manner. The result is Figure 3.

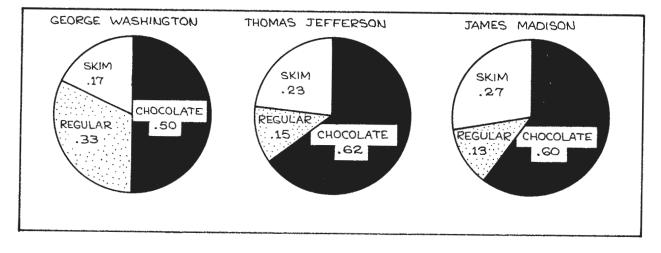


Fig. 3. Percent Sales of Different Kinds of Milk at Three Schools

Exercises

- Make a circle graph of the total sales of the different kinds of milk at the three schools.
- 5. Make a circle graph of the sales of skim milk at the three schools
- 6. Make a circle graph of the sales of chocolate milk at the three schools.

BAR GRAPHS

A circle graph enables us to compare the different percents for each school alone or to compare percents of, say chocolate milk, between schools. But we cannot make specific cross references very well. The reason is the circle graph does not give us the actual numbers of cartons of milk of different kinds or of the totals sold at the three schools. For instance, compare the record of chocolate milk sold at Jefferson School and at Madison. At Jefferson, a greater percent (62) was sold than at Madison (60). Yet a greater quantity (225 cartons) was sold at Madison than at Jefferson (200 cartons). How could we convey this information? In a circle graph, the circles are all the same size. If we try to make them of different size to represent the different totals we run into the same difficulty we had with pictographs. The area does not increase at the same rate as the radius and the area carries the message to our eye. We need a figure whose area does increase at the same rate as the length of one of its sides. This is true of a rectangle of constant width. A graph using such rectangles is called a <u>bar graph</u>. It is used to compare totals.

To draw a bar graph, draw a series of rectangles all of the same width. Any convenient width will do. The distances between adjacent rectangles should be the same in each group. The length of each bar is made to correspond to the proper number on an equally spaced scale on the appropriate axis. Some bar graphs have the bars vertical and some have them horizontal. Here is a vertical bar graph of the data in Table 1.

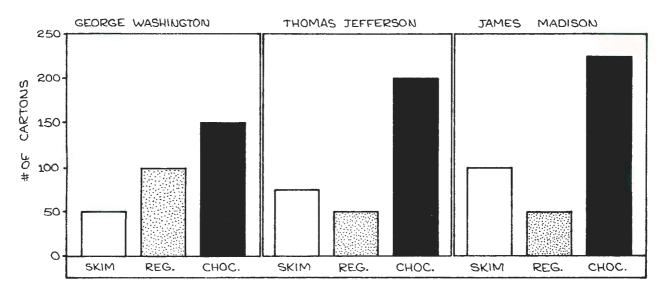
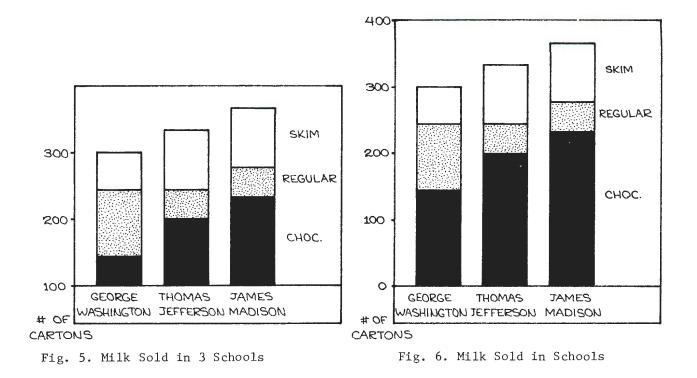


Fig. 4. Number of cartons of milk sold on September 28, 1976

For any graph, it is very important that it be clearly labeled. If the data is qualitative such as the types of milk sold, each bar should be identified. If numerical data is involved such as the number of cartons, the scale should be carefully and accurately marked on the appropriate axis. In this case it is the vertical one.

Another way of graphing this same data is shown in Figures 5 and 6. One of these is well-drawn, the other poorly. Do you see why?

GRAPHS



The poor one is Figure 5. In an effort to save space, the origin of the scale is omitted and the eye picks up a distorted image. Not only is the zero omitted, but the scale starts at 100. It looks as though the sales of chocolate milk in the Thomas Jefferson School were twice those in George Washington and about the same as those of skim milk in James Madison. Figure 6 shows the true relationship. If the eye is to get a quick picture of the data from the graph, we should be careful to insure it gets an accurate picture.

Always be careful to include the zero of the scale of a bar graph. We will find this is true for most other graphs. Always include the zero of the scale of any graph. Be suspicious of any graph in which the zero is not shown. It is likely to be misleading, probably intentionally, but perhaps just carelessly.

Exercises

- 7. From the School Board reports for your district or city, make a table showing at five year intervals from 1960-1975 the total expenditures for teachers' salaries, administrators' salaries, maintenance, books and supplies, and miscellaneous.
 - a) Make circle graphs for each year.
 - b) Make bar graphs for each year.

- 8. Look back at Exercise 7.
 - a) What kinds of questions are best analyzed with a circle graph?
 - b) What kinds of questions are best analyzed with a bar graph?
 - c) What questions that seem to be analyzable from the data are not well expressed by either graph?
 - d) For what questions does it not matter which graph you use?

9. Draw circle graphs for each of the four years in Table 3. The startling change might be made even more forcible by using different colors for the five parts of the circle. Can you be sure that the number of teachers getting \$11,500 and up in 1974 is actually greater than the number in 1970? What assumption would you have to make to be sure this is true? Do you believe the assumption is correct?

TABLE 3											
PERCENT OF TEACHERS AT EACH LEVEL											
SALARIES	1960	1965	1970	1974							
Under \$7,500 7,500 - 8,499 8,500 - 9,499 9,500 - 11,499 11,500 and up	88.1 11.9 	81.8 10.1 5.5 2.6 	36.6 19.7 14.4 19.1 10.3	8.9 14.6 15.6 28.1 32.7							

Data from Statistical Abstract of the United States, 1976, Table 220, p. 134.

10. Make graphs for the data in Table 4. You should make a circle graph for each row of the table and a bar graph for each column. Circle graphs are suitable for the rows but for the columns they would be almost useless. Why? What does the graph of the elementary school enrollment show? What does this indicate about the future of high school and college enrollments? Could you make any such predictions about future elementary school enrollments? In 1960 and 1974 the numbers in the columns do not add to give the totals. Why?

TABLE 4										
ENROLLMENT IN SCHOOLS AND COLLEGES (In Millions)										
	Total	Elementary	High School	College						
1950	30.3	21.4	6.7	2.2						
1960	46.3	32.4	10.2	3.6						
1970	58.9	36.8	14.7	7.4						
1974	58.3	34.0	15.4	8.8						

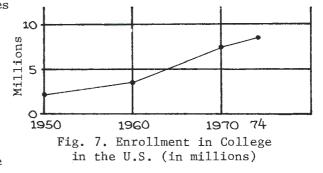
Data from Statistical Abstract of the United States, 1976, Table 189, p. 119.

GRAPHS

LINE GRAPHS

Line graphs usually represent the relationship between two variables. Frequently one of the variables is time and the graph shows how the other variable changes with time. Thus from the last column of Table 4, we can make a line graph (see

Fig. 7). The graph has a title and scales on both axes. First, we mark points for the data as given in the table. These points are connected by segments of straight lines. We do not know what the correct values are at such years as 1956 or 1972. Drawing a straight line means that lacking other evidence we assume the change was steady between the two points.



change was steady between the two points. If we had additional data we would put in the proper points and use them to draw a more accurate graph.

Graphing the data in the first row of Table 3 gives the result shown in Figure 8.

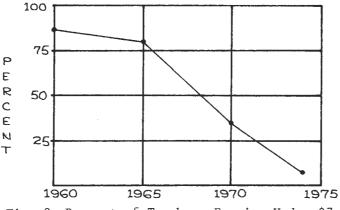
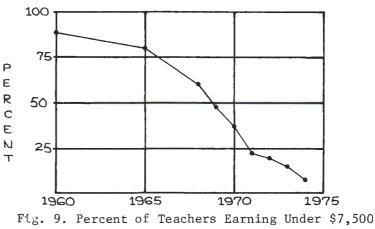


Fig. 8. Percent of Teachers Earning Under \$7,500

The additional data is in Table 5. Using it we get Figure 9. The additional information changes the picture a bit but not as drastically as it might have.

	TABLE 5											
	TEACHERS EARNING UNDER \$7,500											
	Year	Percent										
2	1960	88.1										
	1965	81.8										
	1968	61.1										
	1969	49.5										
	1970	36.6										
	1971	24.7										
	1972	20.3										
	1973	14.9										
	1974	8.9										
		1 1										

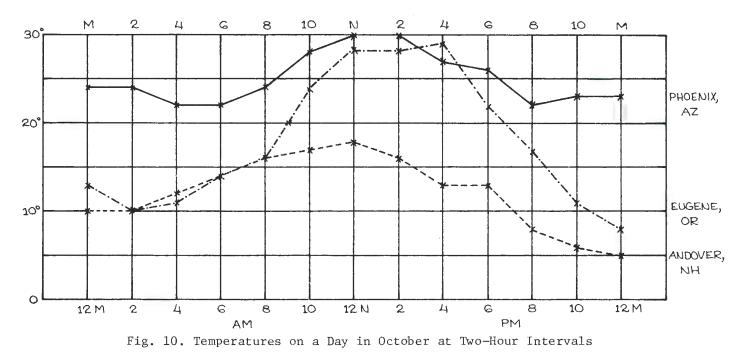


THREE GRAPHS IN ONE

The next graph drawn from the data in Table 6 shows how several line graphs on the same grid can yield additional information.

	TABLE 6														
TEMPERATURES ON A DAY IN OCTOBER AT TWO-HOUR INTERVALS (Temperature in Celsius)															
	12 Midnight	2	4	6 A.M.	8	10	12 Noon	2		6 Р.М.	8	10 M	12 idnight		
Eugene, OR	13	10	11	14	16	24	28	28	29	22	17	11	8		
Phoenix, AZ	24	24	22	22	24	28	30	30	27	26	22	23	23		
Andover, NH	10	10	12	14	16	17	18	16	13	13	8	6	5		

A scale for time is drawn on the horizontal axis and for temperature on the vertical axis. We mark points corresponding to the numbers for each city and connect those for Eugene with dots and line segments, those for Phoenix with solid line segments and those for Andover with broken line segments. The resulting graph is displayed in Figure 10.



Notice that the zero point on the vertical scale is carefully marked. The horizontal scales has a beginning point at 12 midnight but this is not really a zero.

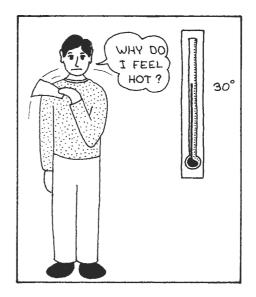
CONTENT FOR TEACHERS

GRAPHS

We have put all three graphs on one grid because we wanted to compare the temperatures in the three places at a given time. The graph also shows how the temperature changes in each locale as the day progresses. In all three, the change is from low to high and back to low. This is called a cyclic pattern since it would,

at least roughly, be repeated in succeeding days. Such repetitions, or cycles, could be noticed by reading the table carefully but for most people the graph makes the point more quickly and easily. Likewise, the graph makes obvious the fact that in Eugene, the temperature varied much more during this 24-hour period than at Phoenix or Andover.

From this graph we can estimate the temperature at times not given in the table. Thus at 3 P.M. the temperature at both Phoenix and Eugene is a little over 28° while at Andover it is about 14.5°. Estimating temperatures even at 2 A.M. on



the next day is much more uncertain. Probably that at Andover will be near 5° and that at Phoenix between 20° and 25°. More data over longer periods of time might help in looking ahead. Such forecasting is the business of many professional statisticians particularly in business and economic affairs as well as in weather. Even when they use great care, later events frequently prove that forecasters make mistakes.

Exercise

11. Consider Table 7 on the following page. In order to make good line graphs from this data the horizontal and vertical axes must be scaled properly. Equal intervals on the horizontal axis should correspond to equal time spans. Note that some of the dates are 20 years apart, some ten and some five or less. The vertical scale probably should have units of 100,000 and not of 1000 as given in the table. The six columns should give us six graphs. All six drawn on the same grid would be confusing. But to compare them, we might want two or three so drawn. Which ones? Maybe the first and fourth. Maybe the first, second and third. On three different sets of axes draw the six line graphs for this data putting together those you feel are most appropriate. As we pointed out before in connection with Figure 10, projections into the future are uncertain although sometimes very important. To indicate this uncertainty when a point is plotted for an estimated or projected value as for the last three in Table 7, the line segments connecting them to known points or to each other should be dotted.

	TABLE 7													
HIGH SCHOOL AND COLLEGE GRADUATES 1900-1984														
	11	School Gra 1 Thousand		41	ege Gradu Thousand									
	Total	Male	Female	Total	Male	Female								
1900 1920 1940 1950 1955 1960 1965	95 311 1221 1200 1351 1864 2665	38 124 579 571 648 898 1314	57 188 643 629 703 966 1351	27 49 187 432 286 392 501	22 32 110 329 183 254 289	5 17 77 103 103 138 213								
1970 1975 Estimated 1980 Projected 1984 Projected	2896 3139 3083 2728	1433 1554 1527 1351	1463 1585 1556 1377	792 944 NA NA	450 522 555 571	341 422 497 513								

Data from Statistical Abstract of the United States, 1976, Table 231, p. 140.

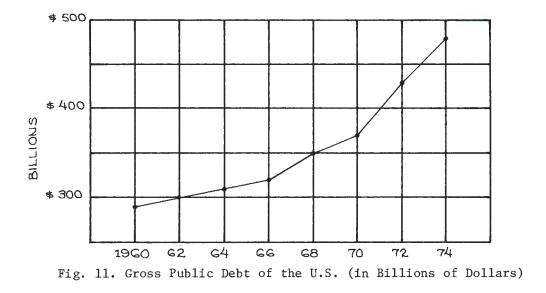
MISLEADING LINE GRAPHS

Consider the data in Table 8.

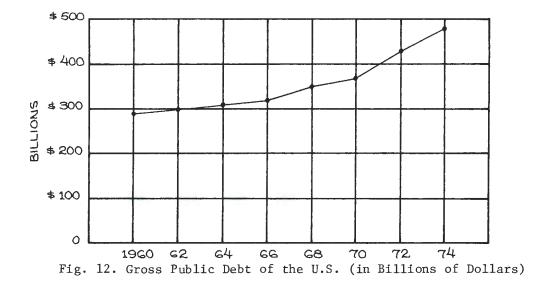
	TABLE 8											
P.	UBLIC DEBT OF THE (in B	1960 - 1974										
	Gross Debt	Interest Paid	Interest as a Percent of Federal Outlay									
1960 1962 1964 1966 1968 1970 1972 1974	\$284.1 295.4 308.1 316.1 345.4 370.1 426.4 474.2	\$ 9.2 9.1 10.7 12.0 14.6 19.3 21.8 29.3	$ 10.0 \\ 8.5 \\ 9.0 \\ 8.9 \\ 8.2 \\ 9.8 \\ 9.4 \\ 10.9 $									

Data from Statistical Abstract of the United States, 1976, Table 401, p. 246.

To present this data graphically a bar graph might be drawn. To indicate the historical trend probably a line graph is better. But a line graph can be misleading as well as the pictographs we saw earlier. Figure 11 is a line graph of the data in the first column of Table 8.



It is a nice graph but it deceives the eye. It looks as though the debt in 1974 were four times that of 1960. See what a different impression Figure 12 gives.

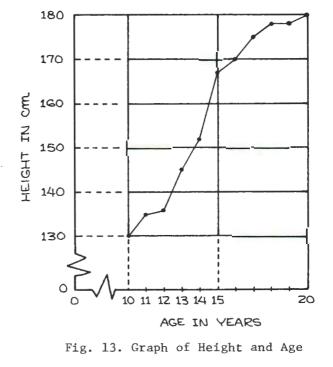


What makes the difference? In Figure 11, the starting point (0) of the vertical scale is missing, the first 250 units are missing, and the unit of the scale is twice as large as that in Figure 12. The combination makes the rise in debt seem even more drastic than it actually is. Goodness knows the facts are bad enough but at least the perspective is better. Sometimes the values we are plotting are all fairly far from zero. We do not want to have a wide empty space on the graph and yet we do not want to omit the zero point on the scale. We use a device called a <u>zero break</u> to overcome the difficulty at least partially. A zero break on the horizontal axes looks like this.

The - calls attention to the omission of part of the axis from 0 to 300. Table 9 gives the height of one boy from age 10 to 20.

TABLE 9											
HEIGHT OF SAM JONES FROM AGE 10 TO 20											
Age in Years	10	11	12	13	14	15	16	17	18	19	20
Height in Cm	130	135	136	145	152	167	170	175	178	178	180

The zero breaks on the two axes are clear signals of what has been done. The reader can adjust his interpretation to this fact.



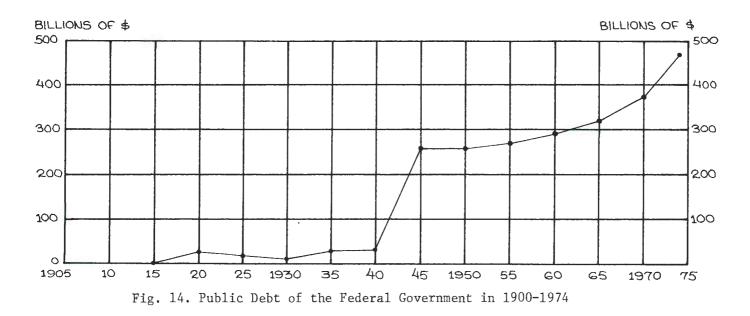
CONTENT FOR TEACHERS

Now consider the data in Table 10.

		TABLE	10						
PUBLIC DEBT OF THE FEDERAL GOVERNMENT (in Billions of Dollars)									
Date	Debt	Interest as Percent of Federal Outlay	Date	Debt	Interest as Percent of Federal Outlay				
1900	1.3	7.7	1940	43.0	11.5				
1905	1.1	4.3	1945	258.7	3.7				
1910	1.1	3.1	1950	256.1	14.5				
1915	1.2	3.0	1955	272.8	9.3				
1920	24.3	15.9	1960	284.1	10.0				
1925	20.5	28.8	1965	313.8	9.5				
1930	16.2	19.2	1970	370.1	9.8				
1935	28.7	12.6	1974	474.2	10.9				

Data from Statistical Abstract of the United States, 1976, Table 401, P. 246.

If a graph were drawn of the debt at 5 year intervals from 1900 as given in Table 10 another picture would emerge.



Of course, part of the increase represents the inflation of the economy. Perhaps the third column of Table 8 and the third and sixth column of Table 10 would make interesting graphs.

Exercises

- 12. Draw a line graph of the interest paid on the public debt from 1960 to 1974 using data from Table 8.
- 13. Draw a line graph of interest as a percentage of federal outlay. Use data from Table 10.
- 14. Compare the graphs in Exercises 12 and 13. What conclusions can you draw? Do you feel the graphs help to interpret the data better than the tables?
- 15. What kind of conjectures might be made about the past, present and future financial status of the United States based solely on the graph in Figure 14?
- 16. Answer the question asked in Exercise 15 if it were based solely on the graph you drew in Exercise 13.
- 17. Now answer the same question but base the answer on both graphs: the one in Figure 14 and the one in Exercise 13.

SUMMARY

To draw a good line graph from a given table

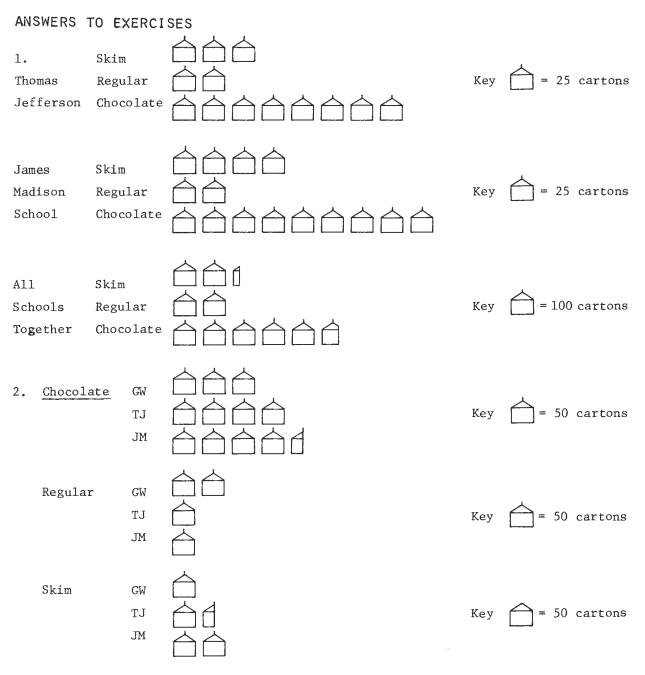
- 1. Select appropriate scales for both axes.
- 2. Be sure to include the zero of the vertical scale and if possible, of the the horizontal scale.
- 3. Mark the scale points on both axes.
- 4. Plot the points carefully and designate them clearly.
- 5. Draw the line segments connecting the points with care. Use a ruler. Use dotted lines to connect estimates to known values.

To read a graph

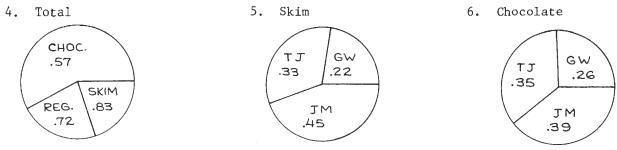
- 1. Be suspicious of a graph that does not show the zero of its scale.
- 2. Do not trust a graph in which the scale is not shown at all.
- 3. If the graph indicates that some points are the results of estimates, be grateful to the source but be wary. Estimates have a way of frequently not coming off.
- 4. The points plotted are as accurate as the data. But values on the line segments connecting the points are not necessarily accurate. Be careful of them.
- 5. Do not use a graph to estimate future values except with great caution and tentativeness. Figure 11 seems to show that the federal debt is going up steadily and at an increasing rate. Figure 14 shows that the rapid increases in the past were when the United States was involved in a war. With no war on in 1976, will the increase slow down or not? We simply do not know and the graph does not tell us.

CONTENT FOR TEACHERS

GRAPHS

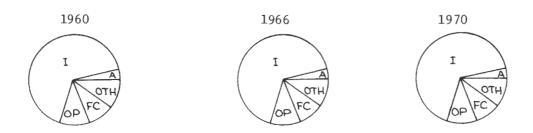


3. This figure does <u>not</u> have defects. Since the test tubes have the same diameter their heights do indicate the correct ratios of numbers of cartons sold.

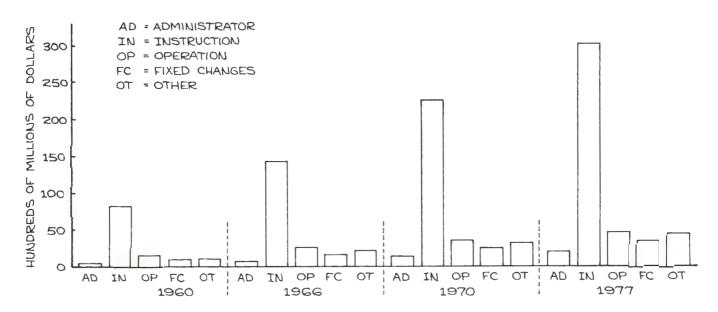


7. From <u>Statistical Abstract of the United States</u>, 1976, Table 227, p. 138. Totals given in millions.

	Administration	Instruction	Operation & Maintenance	Fixed Charges	Other	Total
1960	528	8,351	1,508	909	1,033	12,329
1966	938	14,445	2,387	1,701	1,583	21,053
1970	1,607	23,270	3,512	3,267	2,562	34,218



Note that in spite of drastic increases the percentages change hardly at all. The four circle graphs are practically not distinguishable.

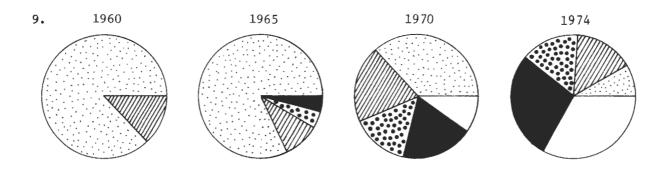


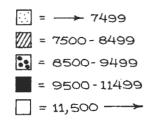
- 8. a) Circle graph is best in looking at percents of a total both in one year and from year to year.
 - b) Bar graphs compare totals in actual amounts rather than percents. They also show changes over a time period.
 - c) The rate of growth.
 - d) Reasons for the change. Comparisons to cost of living.

CONTENT FOR TEACHERS

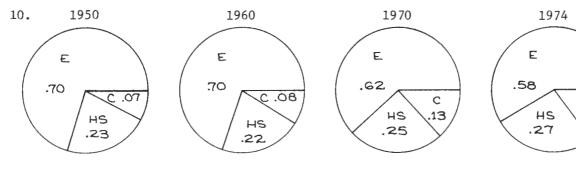
С

15

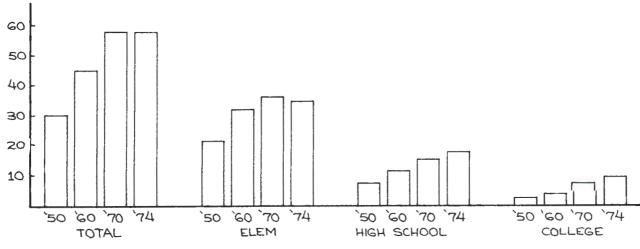




You cannot be sure but if there were more than a third as many teachers in 1974 as in 1970, then you are sure. For example, 10.3% of 15,000,000 = 1,540,000, while 32.7% of 5,000,000 = 1,620,000. This assumption is certainly correct so we are safe in making the statement.



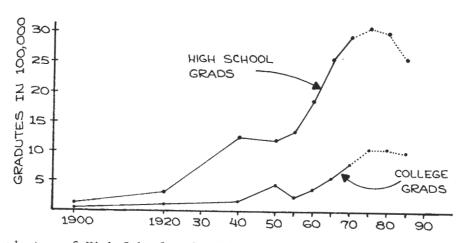
Enrollment in School and College



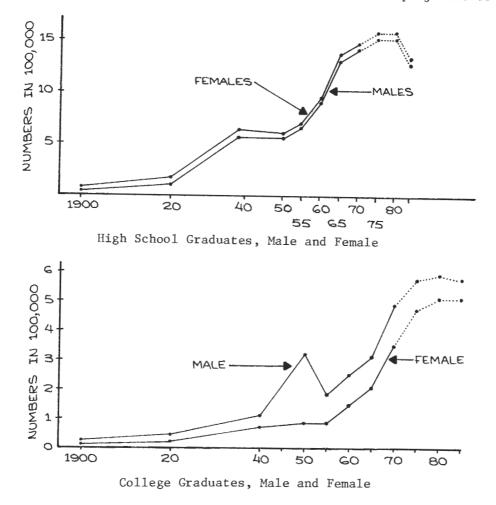
Enrollment in School and College

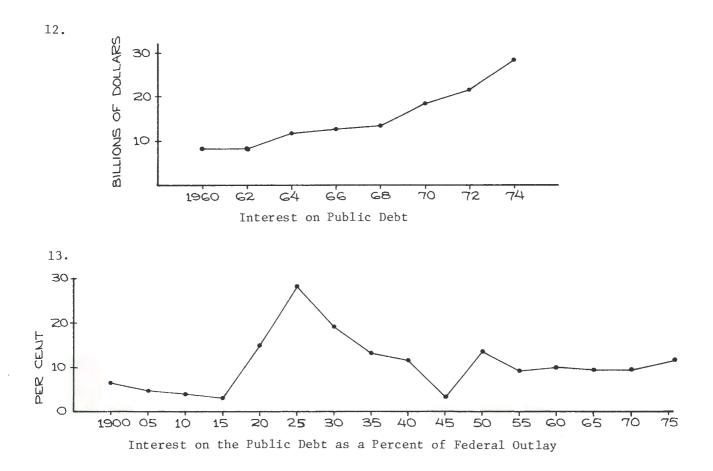
11.

Elementary school enrollments dropped in 1974 indicating a probably drop in high school enrollments in 1980 and in college enrollments in 1985. No predictions can be made from this data.



Graduates of High School and College from 1900 - 1970 projected to 1974





- 14. The only comparable parts are the part from 1960 on. Although the amount of interest paid is rising steeply the percentage of the total outlay is almost constant. This reflects inflationary increases in the total budget and rising deficits. The graphs certainly make more visible the facts.
- 15. This graph seems to indicate a long period of financial stability up to 1940 and then a dramatic increase in the public debt. The growth of the debt is sharp in the war period, 1940-1945, but even after that it seems to be increasing exponentially as we head for economic disaster.
- 16. In Exercise 13. It seems as though we have had a long stable period from 1945-1974. The cost of the debt remains at a constant percent of public spending. so what are we afraid of?
- 17. Combining both Figure 13 and the figure in Exercise 13, we see that the gross national outlay is increasing at roughly the same rate as the debt. Either inflation is taking a terrible toll or the whole economy is surging ahead or perhaps both are influencing the picture. But in either case the debt is no more to be worried about now than it was in 1940.

ORGANIZATION

Placement Guide for Tabbed Dividers

Tab Label	Follows page:
CONTENT FOR TEACHERS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
Probability with Counting	
Inferential Statistics	
Appendix-Computer Programs	
DIDACTICS	
Components of Instruction	
Classroom Management	
Statistics and Probability Learning	
TEACHING EMPHASES	
Critical Thinking	
Decision Making	
Problem Solving	
Models and Simulations	
Calculators and Computers	
Laboratory Approaches	
CLASSROOM MATERIALS	
CONCEPTS AND SKILLS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Misleading Statistics	
Mean, Median, Mode	
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Experimental Probability	604
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Counting Techniques	
APPLICATIONS	
Business & Commerce	
Environment	
Health & Medicine	
People & Culture	
Recreation	
GLOSSARY	
ANNOTATED BIBLIOGRAPHY	
SELECTED ANSWERS	



INTRODUCTION

Look back at the graphs in the previous section. Pictographs, circle graphs and bar graphs were essentially concerned with picturing "how many" of a certain item. We had pictographs of how many bottles of milk were sold. We had circle graphs and bar graphs of how many bottles of skim versus how many bottles of regular or chocolate milk were sold. Line graphs on the other hand represented the relationship between two different kinds of things, quantities or variables. Temperature changes with the time of day, the public debt changes with the year and so do the number of high school or college graduates.

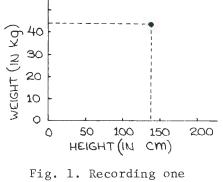
Now we turn to an extension of these ideas. Consider the following three cases. Suppose we want to explore the possible relationship between the height and weight of individuals or the income of a family and the appraised value of the home owned by the family or the weight of an automobile and the number of miles it goes per gallon of gas. Are these two variables related in any way? If one of them increases, does the other increase or perhaps decrease? If we have the data, can we record it in a graph in such a way as to help answer these and other questions?

A scatter diagram may do this very effectively.

DRAWING A SCATTER DIAGRAM

To make a scatter diagram, first select and mark a scale for one variable, say the height, on the horizontal axis and a scale for the other variable, say the weight, on the vertical axis. The scales will usually not be the same since the two items, in this case, height and weight, are measured in different units, cm and kg. The scales for different units do not have to be equal. The choice of which variable is recorded on the horizontal and which on the vertical axis is somewhat arbitrary. Usually we think of a person's weight as depending on the height. We mark weight on the vertical and height on the horizontal axis. Similarly in recording income and value of the home, we would put the value of the home on the vertical axis since we feel it depends on the income.

Next we plot a point on the diagram for each pair of numbers in the data. Thus if Tom's height is 143 cm and his weight is 44 kg, we go over on the horizontal scale to 143 and go up to a point opposite 44 on the vertical scale. A dot (\bullet) is made at this point to record the two numbers (see Figure 1).



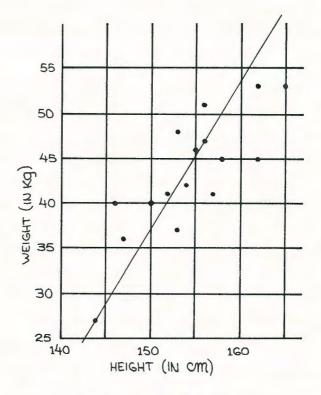
Pair of Values

		TABLI	8-1	and an and an	
WE	IGHTS AND HEIGHT	S OF A FIFTH GRAI	DE CLASS		
Sex	Height in cm	Weight in kg	Sex	Height in cm	Weight in kg
М	162	53	М	150	40
F	152	41	F	154	42
М	153	48	м	144	27
М	153	37	М	147	36
М	146	40	М	165	53
М	157	41	F	156	51
F	158	45	М	155	46
F	156	47	м	162	45

Using the data in Table 1, we make a scatter diagram of weight against height. Equal scales have been used even though they are not necessary. The result is Figure 2.

The purpose of a scatter diagram is to give some feel for the relationship between the two variables. We can see from Figure 2 that, in general, as height goes up so does weight. Sometimes a <u>trend line</u> for the data can be drawn. One easy way is to take a black thread and stretch it across the figure or use a transparent ruler. What you want is a line that seems to "fit" best. It should have about the

same number of points above it as below it. We have drawn one such line. Yours might differ somewhat but the trend would still be evident.





SCATTER DIAGRAMS

There are ways of determining the <u>line of best fit</u> exactly but we are content now with a line fitted by eye. Some students working with a scatter diagram for the first time will try to connect <u>all</u> the points by segments to get a line graph. It is sometimes hard to know when this is justifiable and when it is not. One good suggestion is that if the data comes from a survey or an experiment you should expect the graph to be a scatter diagram. Even here it may depend on how you are looking at the situation. If you want to see how the temperature at Eugene changes during the course of a day the line graph of Figure 10 in *Graphs* in the CONTENT FOR TEACH-ER section is appropriate. If you want to study the relationship between heights and weights of children, a trend line fitted to a scatter diagram as in Figure 2 above seems proper.

Look at Figure 2 again. There is no reason to have the same scales on both axes. In Figure 3 we change them. A real defect in Figure 2 is the lack of a zero point on both scales. This is poor technique for any graph as it fails to indicate the relative size of the change in the variables with respect to their starting values. The change in weight of a man from 50 to 55 kg is much less significant than the change in the weight of a baby from 5 to 10 kg. Figure 3 has zero points

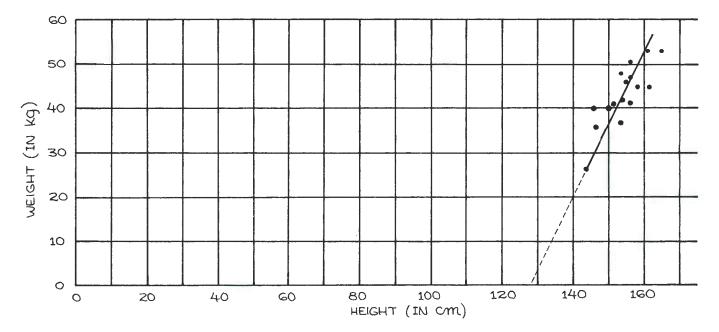


Fig. 3. Weight vs. Height in a Class of 5th Grade Children

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CONTENT FOR TEACHERS

SCATTER DIAGRAMS

but now the concentration of points in the upper right corner is so tight that the trend is harder to spot. Furthermore a large part of the graph is totally empty. We try again in Figure 4 with different scales and a zero break.

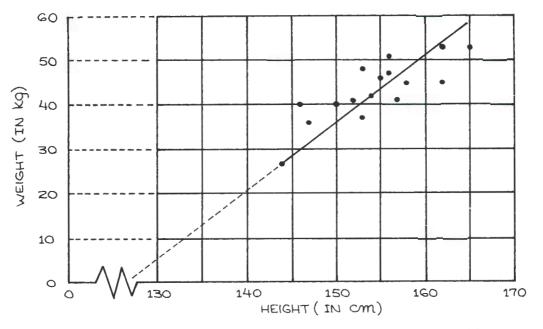


Fig. 4. Weight vs. Height in a Class of 5th Grade Children

The zero points on the scales are included. The squiggle on the horizontal axis calls attention to the fact we have omitted part of the graph. The scatter diagram now looks fine. We might have continued the zero break to 140. The reason for only going to 130 will be evident shortly.

CORRELATION

The trend line gives some idea of the relationship between the two variables, in this case the height and weight. If the points are all close to the line we say there is a good <u>correlation</u> between the variables. If the line goes upward to the right (as in Figure 4, where weight increases with height), then the correlation is positive. The highest correlation is determined if all the points lie exactly on

the trend line. In this case, the correlation is equal to 1. A negative correlation results if the trend line goes down to the right. The correlation is zero if no trend line can be drawn. Figure 5 illustrates various possibilities.

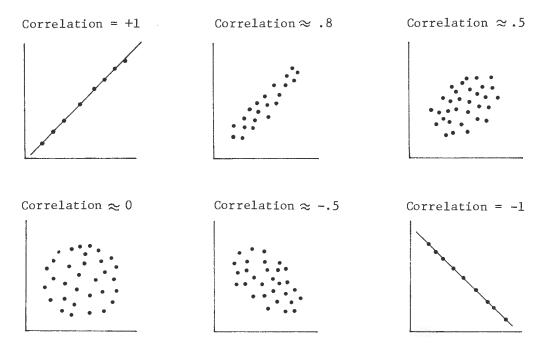


Fig. 5. Scatter Diagrams Showing Various Correlations

Usually the points will not be as evenly scattered as those in Figure 5 and the correlations may be hard to approximate without extensive calculations. We can still estimate whether the correlation is positive or negative and whether it is near -1, 0 or +1.

A trend line gives some idea of the relationship between the variables. Using it we may be able to predict from Figure 4 that a 5th grade child 151 cm tall will weigh about 39 kg. Could we use it to predict the weight of a child 130 cm tall? No. Why not?

Exercises (Answers given on pp. 81-82)

- Make a scatter diagram of weight vs. height for the data in the table on the following page. Put in a trend line. Make another diagram of height vs. weight. Which is correct? Which do you prefer? Why?
- 2. Make scatter diagrams of weights vs. heights for the boys and girls separately in the class you just worked with in Exercise 1. Put in the trend lines. Do either or both of them match the trend line in Exercise 1?

HEI	GHTS AND WEIGHT	S OF A FIFTH GRA	DE CLASS		
Sex	Height in cm	Weight in kg	Sex	Height in cm	Weight in kg
М	158.5	43	F	158.5	51
F	155	43	М	152	62.5
М	149.5	43	М	135	27.5
F	152	50	М	149	37
F	160.5	42	F	148	38
М	168	50	М	161	59
М	156.5	43	F	156	44.5
М	159	45	М	151.5	40.5
F	152	40	М	159	44
M	146	42.5	F	161	46
м	156	44	М	144	39
М	163.5	52.5	F	153	44
F	152	39.5	F	157.5	40
М	151.5	37	F	161	59
М	163.5	50.5	м	141	31
F	149	46			

- 3. A can with some holes in it was filled with water several times. A scatter diagram of the amount of water left at certain intervals was made in Figure 6. Put in a trend line. About how much water would be in the can at the end of 10 seconds? About when will the can be empty? What can you say about the correlation between the water in the can and the time since it was filled?
- 4. In one case in Figure 5, no trend line seemed possible. What significance does this have with respect to the two variables?
- 5. If a trend line is drawn, what reason would we have to extend it beyond the data we have? What dangers are involved if we do extend it? See Figures 3 and 4.

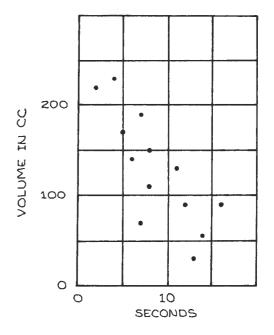


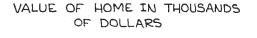
Fig. 6. Scatter Diagram of Volume of Water vs. Time

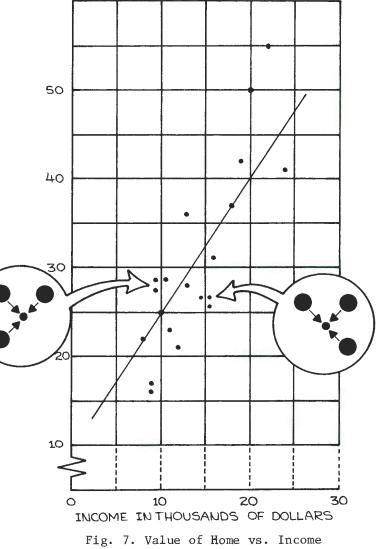
DUPLICATE VALUES

A survey of 20 families was made to determine the family income and the value of their home (in thousands of dollars). It was recorded in Table 2.

TABLE 2				
		UE OF HOM of Dollar		
Income	Value	Income	Value	
16	31	22	55	
8	22	15	26	
11	23	15	26	
10	28	20	50	
24	41	10	25	
10	28	13	36	
9	17	18	37	
13	28	9	16	
19	42	15	26	
10	28	12	21	

In Table 2, there are several duplicate sets of data. In order to represent them on the scatter diagram (see Figure 7), we have grouped them around the proper point with little arrows pointing to this spot. In this way, attention is drawn to them in Figure 7.



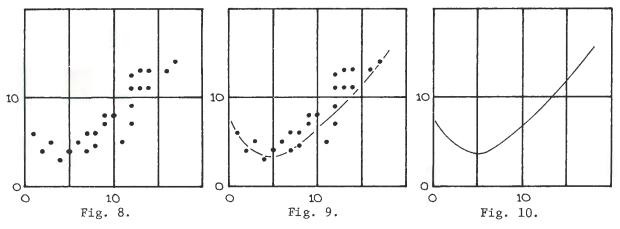


SCATTER DIAGRAMS

CONTENT FOR TEACHERS

CURVES OF BEST FIT

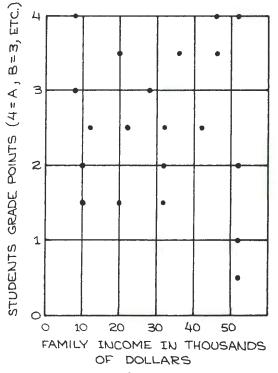
Sometimes the points of the scatter diagram will seem to fit better on a curve than on a straight line. If so, you may want to sketch in the curve. This may be fine but never erase the points of the scatter diagram and leave just the line or the curve. If Figure 8 is the actual scatter diagram and Figure 9 is your suggestion for a fitted curve, that is fine. Someone else may say "That's a poor fit" and may even try a straight line as his guess. But to give only Figure 10 is misleading.

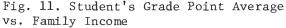


Scatter Diagram of Hypothetical Data

INTERPRETING SCATTER DIAGRAMS

What can we learn from a scatter diagram? A scatter diagram is used to display a comparison between two variables associated with a given set of individuals. If we draw such a diagram for the family income and a student's grade point average, we might get a scatter diagram like Figure 11.





SCATTER DIAGRAMS

The correlation here seems close to zero. We could not predict what a student's family income is if we know the grades. Neither could we predict the grade point average if we knew the family income.

On the other hand a diagram of a student's grade point average and the number of hours studied per week might look like Figure 12.

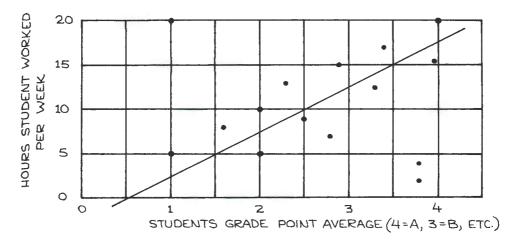


Fig. 12. Student's Grade Point Average vs. Hours Studied per Week

In this case, we (at least some of us) would see a trend line with a couple of outliers. We would be interested in the student who claims to study 20 hours a week and gets an average of 1, also the student who gets a 3.8 with only 2 hours a week of study. In general, we feel higher grades are the <u>result</u> of more studying but the graph certainly cannot tell us that. In <u>some</u> cases a reasonably high correlation is associated with a cause and effect relationship. If smoking causes lung cancer, there will be a high correlation between increased smoking and attacks of cancer. But it does not always work the other way. There may be a high correlation between smoking and lung cancer but there may be no causal relationship from either one to the other.

A scatter diagram with a trend line marked may be useful in other ways. If we are careful, we may use it to make predictions. For instance, looking at Figure 12, if a student studies 15 hours a week, he should get about a 3 grade point average. We must be careful, however. It would not be correct to <u>extend</u> the line and say studying 25 hours a week would result in a grade average of 5. Such an average doesn't exist, so no matter how hard the student studies, it can never be attained.

CONTENT FOR TEACHERS

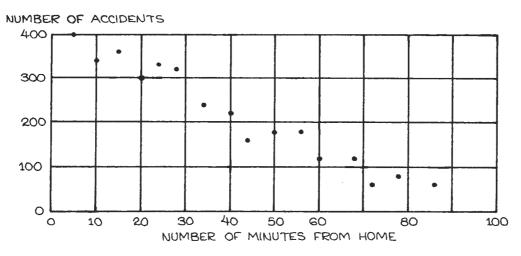


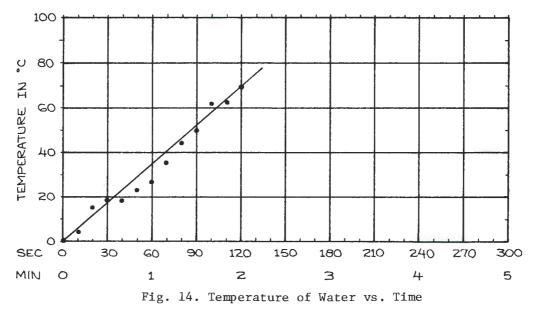
Fig. 13. Number of Accidents vs. Distance from Home

A study of automobile accidents revealed that the closer to home the driver was, the more accidents there were. Figure 13 shows this. A trend line is clearly visible. But does being nearer home cause a driver to get more careless and thus have more accidents? <u>No</u>. There are more accidents near home simply because more driving is done there. A correlation between two quantities as revealed by a trend line is no reason to assume causation from one to the other.

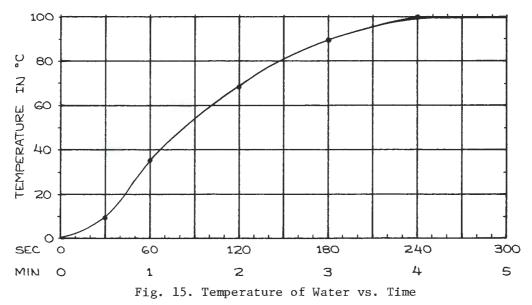
MAKING PREDICTIONS

When and how far may trend lines be extended to help in making predictions? This is a tricky and subtle question. Yet we do have to think about it because making reasonable predictions is one of the fundamental problems of statistics. In *Graphs* in the CONTENT FOR TEACHERS section, we considered a similar question briefly. In extending either the trend line of a scatter diagram or a line graph beyond the available data we must be sure we take into consideration all the information we have available. This is what we did at the end of the last paragraph knowing that the highest grade point average possible was 4. Extending the line in Figure 4 thoughtlessly might lead one to say that a student 130 cm tall would weigh about 4 kg, surely a foolish conclusion. Statisticians do use graphs about the public debt like Figure 14 in *Graphs* in the CONTENT FOR TEACHERS section to help in making predictions about the size of the debt in 1980 or 2000. We must realize they make these predictions, taking into account all the variables that are considered to have any effect. Even so they are at best only fallible estimates.

Perhaps the following illustration will help point up the problem. Suppose you put a tea kettle of ice water on your stove and turn the heat on. Now measure the temperature of the water every 10 seconds. The graph will look something like this:



What will the temperature be after 65 seconds? We would say about 37° C. What about after 3 minutes? Since it seems to be going up about 35° every minute the water should be about 105°. But! The temperature never goes over 100° unless the water is under pressure. So in an ordinary tea kettle, it would be impossible to get to 105°. Actually, the trend line is wrong. The points do not lie on a straight line but near a curve that looks somewhat like Figure 15.



CONTENT FOR TEACHERS

SCATTER DIAGRAMS

If we did not have the information about the boiling point of an unknown substance in the kettle, we might well look at a scatter diagram similar to Figure 14 and make a prediction which could be checked if further relevant data became available. Predictions about the future should be made in the light of all the data we have but inevitably some relevant data is missing. We must recognize this and remember that frequently we do have to act in the face of uncertainty.

Predicting beyond the known points is called extrapolation and must be handled very carefully. Some physics laws enable us to predict accurately what will happen tomorrow. The sun will rise. Water will boil if I apply enough heat. But will my pay check go up next year. Maybe yes. Maybe no. To say yes because it has gone up for three years in a row is to ignore many factors which may make a big difference.

To interpret the data in tables and graphs, two of the main objects we have considered are the amount and position of the clustering and the extent of the scattering. So far we have been content with a general impression of these ideas but to get a real handle on them we need to describe the distribution more exactly. This we will do in *Mean*, *Median and Mode* and *Range and Deviation* in the CONTENT FOR TEACHERS section.

Exercises

- 6. Make a scatter diagram of the EPA gas mpg City vs. the weight of a car as given in the table below.
 - a) Put in a trend line.
 - b) Is the correlation positive or negative?
 - c) Predict the EPA mpg City for a car weighing 3700 pounds.
 - d) Predict the EPA mpg City for a car weighing 1000 pounds.
 - e) Which prediction do you feel is more likely to be accurate?
 - f) Predict the mpg for cars weighing 6000 pounds, 500 pounds.

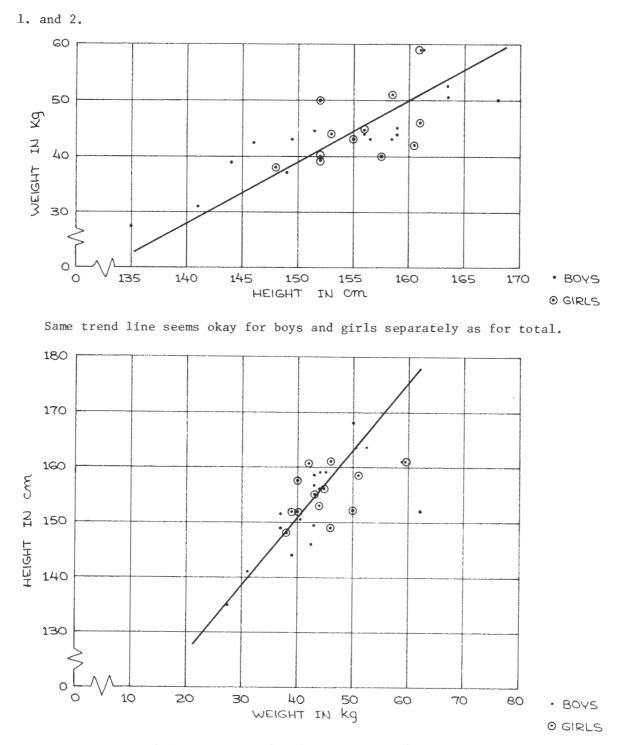
I	'HE EPA			F GAS Pound	•••••	PTION				
Weight of the car	2150	2650	3195	3440	2024	1762	1970	2692	4085	2160
EPA Gas mpg City	26	20	17	14	29	40	29	18	10	15
EPA Gas mpg Highway	39	29	21	23	45	52	41	27	14	26

7. Repeat Exercise 6 using highway mpg vs. weight.

8. Do you think curves would fit the data in Exercises 6 and 7 better than the trend lines? If so, draw them in. Use the curves to predict the mpg of cars weighing 3700 and 1000 pounds. Could you predict the mpg for cars weighing 6000 pounds? How about cars weighing 500 pounds? What precautions do you feel should be taken in making predictions from the table in Exercise 6?

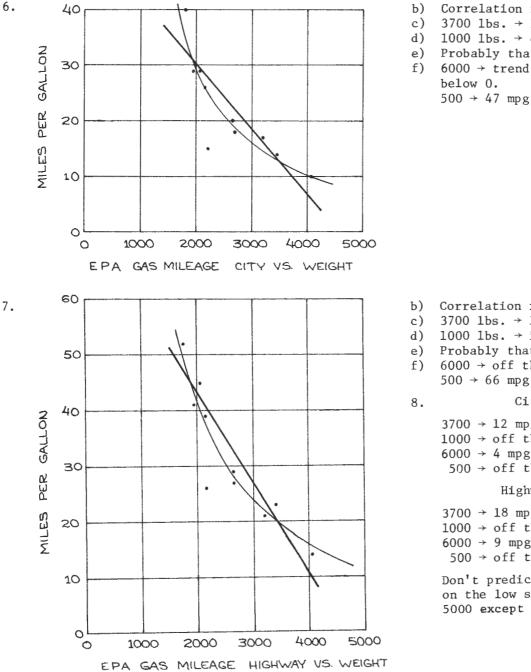
SCATTER DIAGRAMS

ANSWERS TO EXERCISES



Again same trend line seems to be okay. Neither diagram is any more correct than the other. Probably the first one is preferable as most people feel weight depends on height and usually we put the dependent variable on the vertical axis.

- 3. At the end of 10 seconds about 115 cc of water will be left. The can should be empty in about 16 seconds. The correlation is negative and closer to -1 than to 0.
- 4. If no trend line seems possible there is no correlation between the variables. We cannot predict one from the other.
- 5. One should be very wary of extending a trend line. It is often done in statistics and is one of the ways economists forecast the state of the country. But long range forecasts are much more hazardous than short range ones. Estimating from Figure 3 the weight of a boy whose height is 140 cm as 20 kg is a lot safer than saying a boy 130 cm tall would weigh 2 kg.



- Correlation is negative.
- 3700 lbs. → 10 mpg
- 1000 1bs. → 42 mpg
- Probably that for 3700.
- $6000 \rightarrow$ trend line goes below 0. $500 \rightarrow 47 \text{ mpg}$

- Correlation is negative.
 - 3700 lbs. \rightarrow 16 mpg
- 1000 lbs. → 59 mpg
- Probably that for 3700
- $6000 \rightarrow \text{off the map}$
- City
 - $3700 \rightarrow 12 \text{ mpg}$
 - $1000 \rightarrow \text{off the map}$
 - $6000 \rightarrow 4 \text{ mpg}$
 - $500 \rightarrow \text{off the map}$

Highway

 $3700 \rightarrow 18 \text{ mpg}$ $1000 \rightarrow \text{off the map}$ $6000 \rightarrow 9 \text{ mpg}$

 $500 \rightarrow off$ the map

Don't predict beyond 1500 on the low side and beyond 5000 except on the curve.

ORGANIZATION

Placement Guide for Tabbed Dividers

Tab Label	Follows page:
CONTENT FOR TEACHERS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
Probability with Counting	
Inferential Statistics	
Appendix-Computer Programs	
DIDACTICS	
Components of Instruction	
Classroom Management	
Statistics and Probability Learning	
TEACHING EMPHASES	
Critical Thinking	
Decision Making	
Problem Solving	
Models and Simulations	
Calculators and Computers	
Laboratory Approaches	
CLASSROOM MATERIALS	
CONCEPTS AND SKILLS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Misleading Statistics	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
APPLICATIONS	
Business & Commerce	
Environment	
Health & Medicine	
People & Culture	
Recreation	
GLOSSARY	
ANNOTATED BIBLIOGRAPHY	
SELECTED ANSWERS	



INTRODUCTION

A set of data obtained either from a primary or from a secondary source is usually organized into a frequency distribution. The set may be a very large one and we would like some way of summarizing the data. If the data is divided into categories such as the various kinds of milk sold at a certain school, we might try to determine such things as which kind is most popular or what is the average number of bottles sold. If the data is a set of numbers such as the weights of all the children in a given class we might be interested in determining: the least weight, the greatest weight, the most common weight, the average weight and the weight at the middle of the distribution. These numbers would be very different for the data from a third grade class and from a ninth grade class. We could use them to distinguish between two sets of data.

If we had to select one of these numbers to summarize or describe the class could we do it? Probably not, as the choice would depend on what aspect of the distribution we were interested in. Perhaps two or three of the numbers would do it. The least and greatest numbers determine the spread or <u>range</u> of the set. The other three each indicate something about the center of the distribution. Any one of them could be used to represent or locate the data but to describe it more completely we should also know the range or some other number to tell us something of how the data clusters closely or varies widely around the center.

In any set of numbers, the number that occurs most often is called the <u>mode</u>, the number at the middle is called the <u>median</u> and the average of all the numbers is called the <u>mean</u>. Usually there are many numbers in the data set but the methods for finding the mean, median and mode can be discussed just as well with data sets of relatively small size. In this section we will do this and save the study of the range and other measures of the variation of the set for the next section.

THE MEAN OF A DISTRIBUTION

Suppose there are n numbers in the distribution.

(The mean of any distribution is the sum of the numbers divided by n.

The mean, or the arithmetic mean as it is sometimes called, is what most of us have in mind when we use the word <u>average</u>. Thus suppose we want to find the mean grade of a student who has received grades of 68, 73, 65, 78, 72, 80, and 65. The sum of the seven grades is 501 so the mean is $501/7 = 71\frac{4}{7}$ or about 71.6.

We can think of the mean as a balance point. The mean of the numbers 1, 4, 9, 17, 19 is 10. If on a rod of length 20 units, where 20 is twice the mean 10, equal weights are hung at each of the five points as indicated in Figure 1 then 10 will be the balance point. If you put your finger under the 10, the rod and weights will balance.

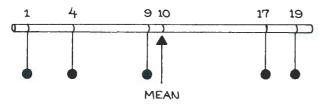


Fig. 1. The Mean as a Balance Point

Exercises (Answers given on p. 102)

- 1. Find the mean (average) of each of the following sets of numbers.
 - a) 6, 9, 2, 12, 7
 - b) 5, 5, 5, 5, 65
 - c) 73, 85, 92, 63, 46, 83
 - d) 73, 75, 77, 79, 81
 - Can you do this one without adding and dividing?
 - e) Make up a set of 5 different numbers for which you can get the mean with a minimum amount of calculation.

USING AN ASSUMED MEAN

Suppose we want to find the mean of the set 76, 78, 75, 68, 80, 71. Instead of adding up the fairly large numbers in this set, we can use smaller numbers if we proceed as follows. Inspect the numbers and guess a number that you think is near the mean. In this case, we might guess 75. We call this the <u>assumed mean</u>. Find the positive and negative differences between each number and the assumed mean. Write the positive and negative differences in separate columns. Add the numbers in



the two columns and then add the totals. The sum of all the differences is -2. Divide this sum by the count (6) getting $\frac{-2}{6}$ and add this correction to the assumed mean to get the exact mean. Thus the mean is

$$75 + (\frac{-2}{6}) = 75 - \frac{1}{3} = 74\frac{2}{3}$$
.
The reason this works is as fol-

lows. Call the mean we are looking for M, the assumed mean \overline{M} and the correction number C. By definition,

$$M = \frac{76 + 78 + 75 + 68 + 80 + 71}{6}$$

The sum of the differences is

 $(76 - \overline{M}) + (78 - \overline{M}) + (75 - \overline{M}) + (68 - \overline{M}) + (80 - \overline{M}) + (71 - \overline{M}).$ This equals $(76 + 78 + 75 + 68 + 80 + 71) - 6\overline{M}$. To get the correction number, divide this by 6.

Then C =
$$\frac{(76 + 78 + 75 + 68 + 80 + 71)}{6} - \frac{6\overline{M}}{6} = M - \overline{M}.$$

Therefore $C + \overline{M} = (M - \overline{M}) + \overline{M} = M$ or the exact mean equals the assumed mean plus the correction number.

Exercises

- 2. Use this method to find the mean of each of the following sets of numbers.
 - 76, 78, 80, 82, 86 a)
 - b) 91, 76, 83, 78, 72, 85 c) 3540, 3536, 3545, 3538
 - d) 151, 146, 153, 149, 157

A hand held electronic calculator makes finding a mean relatively easy. See Calculators and Computers in the TEACHING EMPHASES section. In the absence of a calculator, the above method is useful. With a little practice you can often do it in your head without even writing down the columns of differences as in Table 1. We want to find Tom's average if his grades are 63, 65, 58, 72, 68, 71, 60 and 68. There are eight grades in the set. Assume the average is 65. Note that the differences are successively -2, 0, -7, +7, +3, +6, -5 and +3. Keeping a running total of these in your head you would think

	TABLE 1				
	USING AN ASSUMED MEAN				
Numbers	Numbers Assumed Mean Differences				
76 78 75 68 80 71	75 75 75 75 75 75 75 75	1 3 0 5	-7 -4		
Sum = 9 + (-11) = -2					

Since there are eight numbers in the set, the correction number is 5/8 and the exact mean equals 65 + 5/8 or 65.625. Usually this would be rounded to 62.6.

Exercises

3. Find the mean of each of the following sets of numbers.

- a) 73, 68, 74, 65, 75
- b) 93, 86, 91, 87, 82, 95
- c) 57502, 57495, 57499, 57505, 57504
- d) 67, 72, 80, 75, 77

This method of solving a problem by assuming an answer and correcting it after a few calculations is the oldest recorded method of solving problems in all the history of mathematics. It is found in the Rhind papyrus of Egypt where it was described and used by Ahmese in 1500 B.C. It is still an excellent and sometimes an easy method that is not used as often as it might be.

THE MEAN IN A SET WITH REPETITIONS

If there are repetitions of numbers in the set, we may use multiplication as a substitute for repeated additions. Thus, if Mary's arithemtic tests are 93, 93, 97, 97, 93, 93 we can calculate her average either as

$$\frac{(93+93+97+97+93+93)}{6} = 94\frac{1}{3} \text{ or as}$$
$$\frac{(93 \times 4 + 97 \times 2)}{6} = 94\frac{1}{3} \text{ again.}$$

Note that the frequency distribution of the grades is given below:

Grade	Frequency	Product
93	4	372
97	2	194





To find the mean, multiply each grade by its frequency, add the products and divide by the sum of the frequencies which is the total number of grades. Setting it up in a table helps, particularly if there are many grades. Here is another example.

	TABLE 2	
THE MEAN OF A FREQUE	NCY DISTRIBUTIO	N OF GRADES IN A CLASS
Grade	Frequency	Product
65	3	195
70	5	350
75	6	450
80	11	880
85	7	595
90	2	180
95	1	95
Totals	35	2745
Me	an = 2745/35 =	78.4

Using a calculator to do the arithmetic may be a big help.

Exercises

- 4. A student got 70 on each of six weekly quizzes, 65 on three, 80 on five, and 76 on a final exam that was worth as much as four quizzes. Find the student's average.
- 5. In a large college course the distribution of grades is given in the adjoining table. Count A's as worth 95, B's as 85, C's as 75, D's as 65 and E's as 55. Determine the average grade in this class.

GRADES IN A	COLLEGE COURSE
Grade	Frequency
A	15
В	32
С	76
D	19
Е	5

This method is helpful if many numbers are in the set and there are many repetitions. In an experiment, a deck of cards was shuffled and cards dealt off and counted until an ace appeared for the first time. A tally was made of the number at which the ace appeared. The experiment was repeated fifty times and the results recorded in the first two columns of Table 3. To find the mean, find the product of each position number by its frequency, record it in the third column, add the results and divide by 50. The mean position of the appearance of the first ace is 9.54.

Even this takes fairly long and a good approximation can be found by grouping the results into intervals. This we will do in the next paragraph.

Exercises

6. Find the mean of each of the following sets of numbers.

a)			
Numbers	Frequency		
20	3		
25	4		
30	1		
35	2		
and the second sec			

APPEARA	NCE OF THE FI	RST ACE
Position	Frequency	Product
1	5	5
2	3	6
3	4	12
4	1	4
5	1	5
6	3	18
7	8	56
8	2	16
9	1	9
10	1	10
11	3	33
12	3 3 2 1 3	36
13	2	26
14	1	14
15	3	45
16	1	16
17	3	51
19	1	19
21	1	21
24	2	48
27	1	27
	Sum 50	477

b)						
Frequency						
1						
3						
4						
5						
1						

c) Numbers Frequency 1 19 2 7 3 2 4 1 5 1

d)					
Numbers	Frequency				
6,000	50				
12,000	5				
20,000	2				
100,000	1				

APPROXIMATE MEAN IN A SET GROUPED IN INTERVALS

If we group the results of the first ace experiment into intervals each of length 3, we get Table 4.

We can find an approximate mean by using the midpoint of each interval to represent all the numbers in that interval and proceeding as outlined in the last section.

The midpoints of the intervals here are (2,) 5, 8, etc. To find the approximate mean, we set the work up as follows: The numbers in the Product column of Table 5 are found by multiplying the value of the midpoint of the interval by the frequency of the numbers in that interval.

TABLE 4						
FREQUENCY OF THE POSITION OF THE FIRST ACE						
Interval	Frequency					
1- 3	12					
4- 6	5					
7-9	11					
10-12	7					
13-15	6					
16-18	4					
19-21	2					
22-24	2					
25-27	1					

	TAI	BLE 5	
APPROXIM	LATE MEAN OF THE	POSITION OF THE	FIRST ACE
Interval	Midpoint	Frequency	Product
1- 3	2	12	$2 \times 12 = 24$
4- 6	5	5	$5 \times 5 = 25$
7-9	8	11	88
10-12	11	7	77
13-15	14	6	84
16-18	17	4	68
19-21	20	2	40
22-24	23	2	46
25-27	26	1	26
		Sum 50	Sum 478

The approximate mean is then 478/50 = 9.56. Compare this with the true mean 9.54 that we got before. This mean is only approximate because not all the numbers in an interval are exactly equal to the one at the midpoint but the result is usually close enough to the true mean to serve us very well.

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Exercises

- 7. The weights of a class given in kg are: 39*, 48, 50, 39*, 41*, 33*, 50*, 46, 50, 29, 66, 45, 35, 46, 46, 41*, 35, 53, 43*, 55*, 33, 36, 49*, 35, 42, 38*, 44, 50, 60*, 48. The ones marked with a * were boys, the others girls. Group the weights in intervals of length 5 such as 28-32, 33-37, etc. The midpoints of these will then be 30, 35, etc. Follow through, using the method outlined above, to
 - a) Find the (approximate) mean weight of the class.
 - b) Find the (approximate) mean weight of the boys.
 - c) Find the (approximate) mean weight of the girls.



- The heights of the class in cm are: 149*, 157, 159, 153*, 159*, 149*, 165*, 154, 156, 145, 157, 155, 151, 151, 144, 152*, 150, 166, 156*, 166*, 144, 147, 154*, 146, 161, 150*, 156, 159, 174*, 153. Group the heights into intervals of convenient length. Determine the midpoints and carry on.
 - a) Find the mean (approximate) height of the class.
 - b) Find the mean (approximate) height of the boys.
 - c) Find the mean (approximate) height of the girls.

d) *Can you use the results of (b) and (c) to find (a)?

THE MODE OF A DISTRIBUTION

The mean gives us some information about the center of a distribution. But another number may represent the set better. Sometimes a distribution has repetitions. If there is a number which occurs more often than any other, it is called the mode.

In the set { 1, 3, 3, 3, 5, 165 }, the mean is 30 but most of the numbers in the set are a great deal smaller. The number 3 occurs more often than any other number. It is the mode. The mode of the set { 5, 9, 11, 11, 11, 11, 13 } is 11.

The distribution of weights of meat patties in hamburgers sold one day in a certain store is given in Table 6. The 100 g hamburger is the one sold most often. It is the mode of this distribution. It is perhaps most typical of the hamburgers sold in this store.

TABLE 6						
WEIGHT OF HAMBURGERS (In Grams)						
Weight Frequency						
99	5					
100	32					
101	15					
102	3					
103	1					
	And the second s					

The <u>mode</u> of any distribution is the number that occurs most often. Not every distribution has a mode. In the set { 2, 3, 3, 5, 7, 7, 9 } there is no mode since both 3 and 7 occur twice. Sometimes such a set is called bimodal indicating two modes. The set { 2, 2, 3, 3, 5, 5, 7, 7 } has no mode. If in Table 6 the 101 g hamburger also had a frequency of 32, we would say this distribution has no unique mode.

The mode is a second number (the first was the mean) for describing the distribution. It frequently is a good choice as a typical or representative number of the set.

Consider a teacher applying for a first job in a school district where the average salary is quoted as \$10,600. That sounds a lot better than the fact that most teachers, in fact 75% of them, got only \$9,000. The district employs 50 teachers at \$9,000, 10 department heads at \$12,000, five principals at \$20,000, and one superintendent at \$30,000. The distribution of pay is shown in Table 7.



TABLE 7						
SALARIES IN A	SCHOOL DISTRICT					
Salary	Frequency					
\$ 9,000 12,000 20,000 30,000	50 10 5 1					

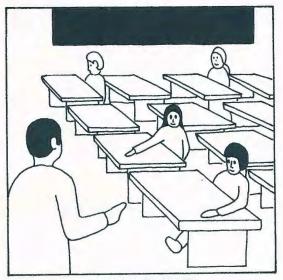
There are 66 employees whose total pay is \$700,000 making the mean salary just over \$10,600. But most teachers receive only \$9,000 and this figure, the mode, represents the district salary for teachers much better than the mean of \$10,600.



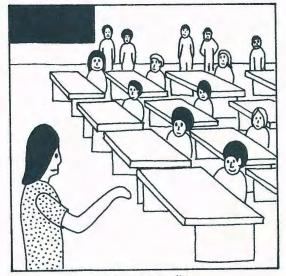
Another case where the mode of a distribution gives different and perhaps more valuable information than the mean occurs in the following example of the distribution of the ages of children and young people in two like cities, Jonesville and Smithtown.

	TAB	LE 8	
FRE	POPULATION UNDER	N OF THE AGES OF THE 21 IN TWO CITIES n Thousands)	
Jones	ville	Smith	ntown
Age Interval	Frequency	Age Interval	Frequency
0- 2	6	0-2	10
3-5	7	3-5	9
6-8	8	6-8	9
9-11	10	9-11	9
12-14	9	12-14	8
15-17	. 9	15-17	9
18-20	9	18-20	8

The mode will be that age group with the highest frequency. In Jonesville, the mode is the age group 9-11 while in Smithtown it is the group 0-2. The mean age in Jonesville is 10.7 and the mean in Smithtown is 9.7. Both of these are in the 9-11 interval. In Jonesville, the mean lies in the interval that also is the mode but in Smithtown it is far away. In Jonesville, the school board is faced with a declining school population, empty classrooms and an excess of tenured teachers. In Smithtown, the school population can be projected as fairly constant with perhaps a 10% increase in enrollment over the next ten years.



JONESVILLE



SMITHTOWN

Exercises

- 9. Check the mean of the distribution in Table 7. Is it \$10,600?
- 10. Check that in Table 8 the means are in fact 10.7 and 9.7 as stated.
- 11. Determine the modes in Tables 3, 4, and 8. Why are the modes in Tables 3 and 4 different?
- 12. A recent graduate is looking for a job in a small factory. He is told the average salary is \$8620. Having just had some statistics, he begins to wonder if that figure is representative of the salary structure of the factory. Further inquiry results in his getting the following distribution of salaries:

50 lathe and drill operators	3 (1	Ş6,000
5 foremen	Q	\$12,000
2 supervisors	Q	\$20,000
1 president and owner	a	\$100,000

Check that the average salary is indeed \$8620. What salary is the mode? What percent of employees get this salary? Does the mean or the mode represent this salary structure better? Why?

THE MEDIAN OF A DISTRIBUTION

Besides the mean and the mode there is a third number that helps to locate a distribution. It is the median. It lies at the middle of the distribution.

The median of any distribution is the number at the middle when the numbers are arranged in order of size.

Thus the median of the set $\{2, 3, 17, 19, 23\}$ is 17. The median of the set $\{26, 2, 37, 5, 7\}$ is 7 as can be seen by ordering the set to read $\{2, 5, 7, 26, 37\}$.

FINDING THE MEDIAN OF A SET

To obtain the median, first arrange the numbers in order of increasing size. If there is an odd number of elements in the set, say 7, pick the middle one. In this case it is the fourth element. More generally, if there are n numbers in the ordered set and n is an odd number, the median will be in the (n+1)/2 position. Thus if n = 7, then the median is the fourth number since (7 + 1)/2 = 4. If the number of elements is even then no number is in the middle position. In this case, the median is taken to be the number halfway between the two middle numbers. Thus the median of the set $\{2, 6, 9, 12, 15, 25\}$ is halfway between 9 and 12. It is the mean of 9 and 12, i.e., 10.5. In this case, the median is not a member of the set.

If there are repetitions in the set, the median may be one of the numbers that are repeated. For example, consider the set {12, 15, 15, 16, 19, 13}. Arranged in

order, the set is {12, 13, 15, 15, 16, 19}. There are six numbers in the set. The middle ones are the third and fourth. Both of these are 15 so the median is also 15. This time, the median does belong to the set.

Exercises

13. Find the median of each of these sets.

- a) { 22, 27, 17, 19, 31, 26 }
- b) {2, 2, 2, 2, 5, 7}
- c) {6, 9, 12, 15, 17, 19}
- d) {6, 9, 12, 15, 17, 482}
- e) {0, 1, 9, 12, 47, 165}
- f) If the median of the set {2, 3, 5, __, 8, 12} is 6, what is the fourth number?

FINDING THE MEDIAN OF A FREQUENCY DISTRIBUTION

In a frequency distribution there is a number greater than or equal to at least half the numbers in the set and a number less than or equal to at least half the numbers. If this is the same number, it is the median. If the numbers are different, the median is halfway between them. See the two examples following.

Find the median height of a class of 30 students whose heights are given in Table 9. Since there are 30 members of the class, the median height will be halfway between the heights of the 15th and 16th members. There are 14 members below 153 cm in height. There are of course 23 members less than or equal to 153 cm in height but the important fact is that there are at least 15 members less than or equal to 153 and at least 15 members greater than or equal to 153. The median height is then 153.

TABLE 9							
FREQUENCY DISTRIBUTION OF HEIGHTS OF A CLASS (Measured to the Nearest Centimetre)							
Height Frequency							
150 3 151 7							
152 4 153 9							
154 4 155 2							
170 1							

Find the median weight in a class of 40 students whose weights are given in Table 10. Of the 40 weights, 20 are less than or equal to 50 kg and 20 are greater than or equal to 52 kg. The median is the mean of these two or 51 kg.

TABLE 10						
FREQUENCY DISTRIBUTION OF WEIGHTS OF A CLASS OF 40 (Measured to the Nearest Even Kilogram)						
Weight Frequency						
46	3					
48	5					
50	12					
52 6						
54 9						
56	5					

Exercises

14. Find the medians of each of the following distributions.

a)	Weights in kilograms		51	5	2	53	5	54		55	56	5			
	Frequency		7		9	10)	8	3	3	2	2			
b)	Number of Letters in Words of 3 Sentences	the	1	2	3	4	•	5	6	7	8	9	10	11	12
	Frequency		4	13	10	3	3	3	4	3	4	0	2	1	2
c)	Ages of Children in a 5th Grade Class	10 ^y	r9 ^m	10	yr ₁₀	m	10	^{yr} 11	m	11 ^{yr}	113	r1 ^m	11	yr2"	
	Frequency		2		7			9		11		5		1	

15. Find the mean and mode for each of the distributions in Exercise 14 and compare them with the median.

16. Compute the mean and mode for the distribution of Table 9.

17. Compute the mean and the mode in Table 10.

FINDING THE MEDIAN OF A DISTRIBUTION OVER INTERVALS

If the frequency distribution consists of measures that are grouped into intervals, the median is determined as in the following example.

The expenditures per pupil for public schools in the 50 states are grouped in intervals in Table 11. The median expenditure lies halfway between that of the 25th state and that of the 26th state. Why?

TABLE 11						
AVERAGE PUBLIC SCHOOL EXPENDITURE PER STATE IN 1974-75 (Grouped in Intervals of \$200)						
Expenditures Frequency						
801 - 1000 1001 - 1200	13 17					
1201 - 1400 1401 - 1600	10 8					
1601 - 1800 1801 - 2000	1 0					
2001 - 2200	1					

Data adapted from *Statistical Abstract of the United States*, 1975, Table 220, p. 133.

The first interval contains the expenditures for 13 states and the second interval those for 17 states for a total of 30. Since the median is between the expenditures of the 25th and 26th states, it lies in this interval. Can you estimate roughly what the median is?

Although there is no state numbered 25.5, the median expenditure is defined to be at the 25.5 position. This is 12.5 places beyond the first 13. See Figure 2.

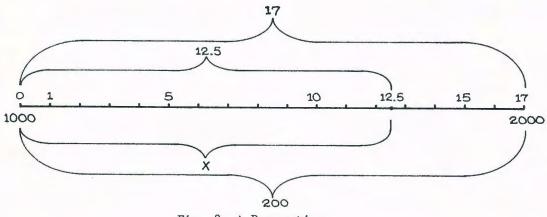


Fig. 2. A Proportion

If it is assumed that the 17 expenditures in the second interval are evenly spaced from 1000 to 2000 a proportion could be used to determine where along this interval the 12.5 position would be. Thus $\frac{12.5}{17} = \frac{x}{200}$ and x = 147.

From this an approximation to the median expenditure is determined as 1001 + 147 = 1148. Of course, this is only approximate but it is close enough to the true median (1140) to serve all practical purposes.

Exercises

18. Determine the median in each of the following distributions.

TEST SCORES	IN A MATH CLASS
Scores	Frequency
55 - 59	3
60 - 64	5
65 - 69	9
70 - 74	13
75 - 79	8
80 - 84	12
85 - 89	3
90 - 94	3
95 - 99	1

AVERAGE SCHOOL EXPEN POPULATION BY 50	
Expenditures	Frequency
151 - 200	3
201 - 250	14
251 - 300	15
301 - 350	13
351 - 400	4
401 - 450	0
451 - 500	0
501 - 550	1

- 19. Determine the approximate mean expenditure per state in Table 11.
- 20. Determine the mean and the mode of each of the distributions in Exercise 18.
- 21. You may be interested in the expenditures per state as given in the following table. Where does your state stand with respect to the median and the mean as computed above?

A	VERAGE PUE	SLIC SC	HOOL EXPEN	DITURE	PER PUPII	L 1974-	75 IN THE	50 STA	TES
AL AK AZ AR	\$ 871 1624 1176 896	HI ID IL IN	\$1384 910 1376 1074 1260	MA MI MN MS	\$1136 1547 14 2 3 834	NM NY NC ND	\$1052 2005 1052 1032	SD TN TX UT	\$ 973 903 894 942
CA CO CT DE FL GA	1201 1188 1507 1485 1147 869	IA KS KY LA ME MD	1240 1444 864 1034 918 1369	MO MT NB NV NH NJ	1078 1269 1211 1101 1095 1294	OH OK OR PA RI SC	1144 1009 1425 1446 1493 984	VT VA WA WV WI WY	1095 1054 1199 910 1323 1322

Data from Statistical Abstract of the United States, 1975, Table 220, p. 133.

MEAN, MEDIAN, MODE. WHICH?

The mean, median and mode can each be used to indicate some sort of average of a distribution. Each can be used as a representative of the set to describe or summarize the set. Which is the most useful? Is it always one of the three or does it depend on the problem? If the usefulness does depend on the problem, how can the best one be selected? Each is in some way an average. Is there a difference between the income of the average family in the United States and the average income of a family? In the presidential campaign in 1976, Mr. Carter apparently meant the median income when he talked about average income although most of the reporters interpreted it as the mean income. Maybe the modal income would have been a better choice than either the mean or the median.

If the distribution is of numbers, the mean, median and mode are all numbers. If the distribution is of categories such as kinds of milk or fruit sold there is no way to determine the mean. Sometimes the mode and the median can be found and sometimes only the mode. Here is an example in which the mode is the only one of the three that can be determined. In a grocery store selling fruit, the sales for one day in March are given in Table 12. There are more oranges sold than any other kind of fruit so the mode is "oranges." But it is meaningless to consider a mean or a median in this situation. It is impossible to add apples and oranges. There is no median fruit somewhere between a peach

TABLE 12	
SALE OF FRUITS FOR (Sales in Kilog	
Kinds of Fruit	Sales
Apples Bananas	200 425
Peaches	175
Oranges Grapefruit	525 275

and an orange. It is important to note that the distribution is concerned with fruit. It is not an average of the frequencies that is asked for but of the fruits. The trouble is that there is no way to order the different kinds of fruit or to find an average fruit.

Now consider a wholesale meat packer who sells four different grades of meat: prime, choice, good and commercial. His sales for one day in a certain week are given in Table 13. The mode is clearly the "commercial" grade. But this time, there is a definite order and therefore, the median grade can be determined. There were 3355 kg sold altogether. There were only 1485 kg sold of "prime" and "choice" so the median is in the next interval which is the "good."

TABLE 13		
SALE OF MEATS FOR (Sales in Kilog		
Grades of Meat	Sales	
Prime Choice	525 960	
Good Commercial	630 1240	

If the distribution is one of numbers such as the weights of the members of a class, or the sizes of shoes sold by a store, etc., all three measures, mean, median and mode can be determined. In the shoe store, the mode is most important because it tells the manager what size needs to be reordered first. The median is the best summary figure for a set if the set contains a few extreme values. The set {1, 2, 3, 4, 500} is better summarized by the median 3 than by the mean 102. If no really extreme values are in the set, the mean is usually the best summarizing figure. The representative you choose really does depend not only on the numbers themselves but on the situation in which they were determined and on the kinds of questions

being asked. Thus if the set of numbers { 1, 2, 3, 4, 500 } are the bills for the Christmas presents that Susie bought for her family, the mean of \$102 may be more representative of the month's bills than the median of \$3. Why? Because if \$3 is a representative bill in a set of five bills, Susie's family might expect the total to be \$15.00. But faced with an average bill of \$102.00 for five presents they know the total will be over \$500.00, in fact, \$510.00.

Another example may help illustrate the usefulness of the mode and the mean under different circumstances.

Slumberville is a fast growing suburb of the city Downtown. A construction company made a survey of family size in Slumberville to help decide what size houses to build. The company hoped that the distribution of family sizes in those families

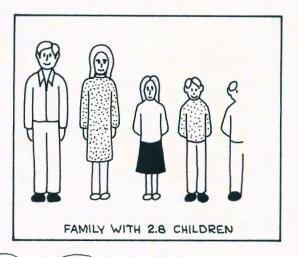
moving in would be about the same as those now living there. This hope may or may not be reasonable but the survey was the only information available so they had to rely on it. The distribution determined by the survey is given in Table 14.

The mean number of children per family is 2.8. Since no family exists with exactly 2.8 children, we might say the average or typical family has 3 children. However, more families have two children than any other number. Furthermore, over half of all families have only two, one, or no children so the median is two. Is the three child family the most representative of this

TABLE 14		
THE NUMBER OF CHILDREN PER FAMILY IN SLUMBERVILLE		
Number of Children	Frequency	
0	110	
1	110	
2	160	
3	125	
4	100	
5	60	
6	45	
7	20	
8	10	
9	5	
10	3	
11	2	
12	1	



group? Home builders who build most of their houses with the extra bedrooms needed for the "typical" three-child family may well price themselves out of the market. They would be better off building most of their houses for families with <u>less</u> than three children even though they will need some for the larger families with 3, 4, or even more children. The mode and the median of this distribution are both two.



The mean of a distribution may be strongly influenced by changes in the extremes but such changes usually have little effect on the mode or the median.

Changing any member changes the mean. The mean is sensitive. If the set {1, 2, 3, 4, 500} is changed to {1, 2, 3, 4, 5} the median is still 3 but the mean changes from 102 to 3. The set {1, 2, 3, 4, 15} again has median 3 but the mean is now 5. All things considered, the mean is probably the most useful measure unless there are particular reasons as noted above for using the mode or the median. The mean is easy to handle and it is sensitive. When samples of a population are studied later on, it is from the mean of the sample that a reasonable estimate can be made of the mean of the population. This is not true of either the median or the mode.

Exercises

- 22. A student's quiz grades are 10, 7, 8, 9, 10, 1. Find the mean, median, and mode. Which of these provides the best summary grade?
- 23. A pile of wooden dowels was examined and the length of each dowel recorded in cm: 2, 5, 7, 3, 6, 4, 5, 9, 36, 2, 5, 7. Find the mean, median, and mode. Which provides the best summary for the pile?
- 24. The sales in a certain store for one week were \$76, \$125, \$95, \$110, \$93, \$85, \$105. Find the mean and median. In another week, the sales were \$93, \$87, \$102, \$98, \$105, \$756, \$92. Find the mean and the median. Discuss the difference in the two weeks sales. Which measure detects the difference? Which measure probably indicates the normal sales of the store?
- 25. Look back at Exercises 22 and 23. Indicate circumstances under which you would select a different answer from the one you gave to the question of which summary measure is the best.

USING A COMPUTER TO FIND THE MEAN, MEDIAN AND MODE

A calculator can be a big help in computing the mean of a large set of numbers. If a computer is available, programs can be written once and stored for use as often as wanted. Such programs in the BASIC language will be found in the *APPENDIX* of the CONTENT FOR TEACHERS. Since there are many versions of BASIC used on different computers, this program may have to be modified to run on the computer available to you. But the changes should not be too numerous. There are programs given to find the mean, mode and median for sets of up to 50 numbers and for frequency distributions of numbers in up to 20 intervals.

Hand calculators, priced at less than \$25 in 1977, will give you the mean of a set of numbers at the push of one button after the numbers and their frequencies have been entered. Once the process of finding a mean by hand is understood, a calculator or a computer can make the computation quick and easy.

AN	SWERS TO EXERCISES
1.	a) 7.2 b) 17 c) $73\frac{2}{3}$ d) 77
2.	a) 80.4 b) $80\frac{5}{6}$ c) $.3539\frac{3}{4}$ d) 151.2
3.	a) 71 b) 89 c) 57,501 d) 74.2
4.	73.3
5.	77.2
6.	a) 26 b) 20.6 c) 1.6 d) 8620
7.	a) 44.5 b) 45 c) 44.2
8.	 a) 154.7 b) 157.7 c) 152.9 d) Yes, multiply the mean height of the boys by their frequency, multiply the mean height of the girls by their frequency, add the two results and divide the total number of boys and girls.
9.	\$10,606 to be exact
10.	Checks
11.	Table 3 mode = 7. Table 4 mode = 2. Table 8 mode = 10 in Jonesville, mode = 1 in Smithtown.
12.	Mode = 6,000, 86%, mode since the mean is displaced far to the high side by the \$100,000 salary of the president.
13.	a) 24 b) 2 c) 13.5 d) 13.5 e) 10.5 f) 7
14.	a) 53 b) 3 c) 10 ^{yr} 11 ^{mo}
15.	a) mean = 52.9; mode = 53 b) mean = 4.47; mode = 2 c) mean = $10^{yr} 11^{mo}$; mode = 11^{yr}
16.	mean = 152.9; mode = 153
17.	mean = 51.4; mode = 50
18.	a) median = $69 + \frac{12}{13} \ge 5 = 73.6$ b) $250 + \frac{8.5}{15} \ge 50 = 278.3$
19.	\$1184
20.	a) mode = 72; mean = 74.6 b) mode = \$275; mean = \$281
21.	Depends on state
22.	Mean = 7.5; median = 8.5; mode = 10. Probably the median. She had one day off, but 4 out of 6 were 80 or over.
23.	Mean = $\frac{91}{12} \approx 7.6$; median = 5; mode = 5. Median or mode of 5. Mean is thrown off by one extra long rod.
24.	First week mean = \$98, median = \$95. Second week mean = \$190.43, median = \$98. One very high day in the second week pushed the mean way up. Probably the median indicates normal sales.
25.	Mean is usually better if there are no outliers. The mode if most values are at one extreme. The median if there are one or two outliers.

ORGANIZATION

Placement Guide for Tabbed Dividers

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RAGE & DEVISION

INTRODUCTION

When trying to get information from a set of numbers, usually the first thing to do is to determine the frequency distribution of the set and study it. In *Mean*, *Median and Mode* of the CONTENT FOR TEACHERS section, we found that these three numbers each summarize in some way the distribution. Each tells us something about the tendency of the numbers in the distribution to cluster around some value. But two very different sets of numbers may have frequency distributions with the same mean, median and mode. In order to distinguish between them we also need to know how the numbers scatter along the distribution. Are all or most of them close together? Are some or most far away from each other? There are several numbers that will give us some of this information. Perhaps the simplest of these is the <u>range</u>.

RANGE

The range of a distribution is the difference between the smallest and largest numbers in the set. The best way to give the range is to give the actual interval in which the numbers occur.

Thus in a family of five, the range in the ages might be 35 - 4 = 31 years. In a family group of eight including grandparents, the range of ages might be 73 - 7 = 66 years. Suppose we have three groups of people. We consider the distribution of weights in each group. In the first group, weights go from 25 kg to 100 kg with a mean of 55 kg. In the second group, weights go from 50 kg to 65 kg, again with a mean of 55 kg. In the third group, the weights go from 80 kg to 95 kg with a mean of 90 kg. The range of the first group is 75 while that of both the second and third groups is 15. The means and the ranges with their intervals tell us that the groups are very different. Look at Figure 1.

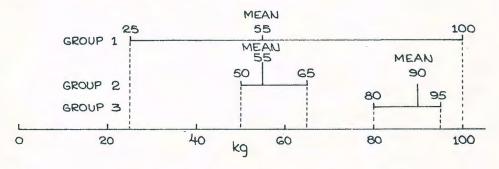


Fig. 1. Ranges and Means for Weights in Kilograms of Three Groups of People

The first and second groups have the same mean but different ranges. The second and third groups have the same range but different means. The wide range and fairly low mean of the first group suggests it is composed of both small children and adults. The second and third groups are each made up of people of about the same weight. The different means suggest the second might be a group of adolescents and the third a group of adults. The mean alone does not distinguish the second group from the first. The range alone does not distinguish the second group from the third. In order to describe the difference between pairs of groups it is necessary to examine both the ranges with their intervals and the means.

Sometimes the range is large but only because of one or two extreme values in the set while the rest of the numbers are close together. Such extreme values are called <u>outliers</u>. If a teacher weighing 100 kg joins the second group above, the range will jump to 50 but most of the weights are still close together. We might want to ignore the outlier in some calculations making a note of what we are doing and why. But we must be careful. An outlier is frequently an important signal. It may call attention to a significant fact that we ignore at our peril. Perhaps it was not due to a teacher being added to the group. Perhaps a mistake was made in recording one student's weight. Maybe a really obese student belongs to the class. Maybe an outsider has inadvertently been included.

The striking effect of an extreme outlier on the range indicates that another measure of scatter should be sought that might not be affected so much by an extreme. One such number is a trimmed range. This is obtained by dropping the extreme outliers on whichever end of the range they lie and at the same time dropping a corresponding number of items from the other end. A trimmed range is a measure of scatter not affected by extremes just as the median was an indicator of the center not affected by the extremes of the distribution.

However, there are other measures of the scatter even better than the ranges. These are the deviations.

Exercises (Answers given on pp. 126-129)

- 1. Determine the mean and range of each of the following sets.
 - a) $\{3, 1, 3, 2, 3, 4, 3, 5, 3\}$
 - b) {1, 2, 3, 1, 3, 5, 3, 4, 5}
 - c) { 3, 2, 1, 2, 3, 4, 5, 4, 3}
- 2. Determine the range and a trimmed range of each of the following sets. a) {3, 5, 7, 4, 1, 37}
 - b) { 37, 52, 41, 23, 29, 567, 1 }
 - c) { 32, 2, 1, 59, 36, 43, 55, 500 }

DEVIATIONS

Most sets of numbers studied statistically will be large, involving maybe many hundreds of numbers. The IQ scores in a school might involve over a thousand students with several hundred in each class and the individual scores ranging from 50 to 150. In spite of this, we use sets of a few small numbers for clarity in explaining the concepts of the deviations and the procedures for calculating them.

Consider the three sets of numbers:

a) 1, 3, 3, 3, 5 b) 1, 2, 3, 4, 5 c) 1, 1, 3, 5, 5

They have the same mean and the same range and yet present very different impressions as distributions. The members of the first set seem to cluster closer together around the mean than those in the second and those of the third seem even farther apart. A diagram called a dot diagram will make this even more striking. To make such a diagram, draw a horizontal line, put on it an appropriate scale and put a for each item above the scale at the proper position (see Figure 2).

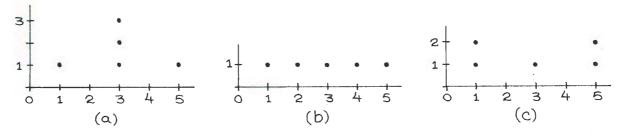


Fig. 2. Dot Diagrams of Three Distributions

The mean of each set is 3 and the range is 5 - 1 = 4. In these cases, even the median is 3. We might ask, "What is the difference, on the average, of the numbers in each set from the middle of the set?" We have the choice of two numbers to indicate the middle: the mean and the median. The mode is not even considered because not every distribution has a mode. Set (a) has 3 as its mode. Set (b) has no mode. Set (c) has no unique mode although it might be called bimodal at 1 and 5. In this case, the mean and the median are the same but often they would be different. We rather arbitrarily choose the mean.

THE MEAN DEVIATION

Our question now is, "What is the difference, on the average, of the numbers in the set from the mean of the set?" The mean is 3. Subtracting 3 from each number in the set we see that some differences or deviations as they are called are positive and some are negative. In set (a) we get 1-3 = -2, 3-3 = 0 and 5-3 = +2. The

sum of the deviations is (-2) + 0 + 0 + (+2) = 0. In set (b) we get (-2) + (-1) + 0 + (+1) + (+2) = 0 and in set (c), (-2) + (-2) + 0 + (+2) + (+2) = 0. In each case, the mean deviation is 0/5 = 0. The mean deviation of any set is always 0. To find some measure to help us, we have to reword our question.

THE MEAN ABSOLUTE DEVIATION

The only reason the deviations could add to 0 was because some of them were positive and some were negative. Suppose we consider only the size of the deviation of each number from the mean without regard to whether the number is to the right or left of the mean. Call this the distance from the mean and find the mean of these distances.

In set (a), 5 is two units away from the mean, 3, each of the 3's is zero units away and the 1 is two units away. So the sum of the distances away from the mean is 4. Since there are five distances altogether, the average distance is 4/5 = .8. In set (b), the sum of the distances from the mean is 2 + 1 + 0 + 1 + 2 = 6. The average distance from the mean is 6/5 = 1.2. In (c), the sum of the distances is 2 + 2 + 0 + 2 + 2 = 8 and the average distance from the mean is 8/5 = 1.6. The numbers .8, 1.2, and 1.6 do distinguish the three sets. They do increase from set (a) to set (c) as we intuitively want if they are to measure the increase in scatter from set to set.

The number thus obtained is called the <u>mean absolute deviation</u> of the set. "Deviation" because we measure the difference or deviation of each number from the mean; "absolute" because we always take the deviation as a positive distance; and "mean" because we took the average or mean of these distances.

To find the mean absolute deviation of even a small set of numbers it helps, though it is not necessary, to organize the work in a neat, systematic fashion. A table like Table 1 is convenient. Suppose we want the mean absolute deviation of the set { 25, 27, 31, 36, 20, 15 }. We find the mean in Columm 1, the deviations in Column 2 and the distances in Column 3. Usually decimal approximations are used. In this example we have given both the exact values and the decimal approximations for comparison.

		TABLE 1		
	FINDING	THE MEAN ABSOLUTE	DEVIATION, D	
	Devia	tion from Mean	Distar	ice from Mean
Numbers	Exact	Decimal Approximation	Exact	Decimal Approximation
25	$-\frac{2}{3}$ +1 $\frac{1}{3}$	7	$\begin{array}{c} \frac{2}{3}\\ 1\frac{1}{3} \end{array}$.7
27	$+1\frac{1}{3}$	+1.3	$1\frac{1}{3}$	1.3
31	$+5\frac{1}{3}$	+5.3	$5\frac{1}{3}$	5.3
36	$+10\frac{1}{3}$	+10.3	$10\frac{1}{3}$	10.3
20	$-5\frac{2}{3}$	-5.7	$5\frac{2}{3}$	5.7
15	$-10\frac{2}{3}$	-10.7	$10\frac{2}{3}$	10.7
Sum = 154	0	2	Sum = 34	34
Mean = $\frac{154}{6} = 25\frac{2}{3}$	Appr	oximately 0	Mean Absolute Deviation	$=\frac{34}{6}=5\frac{2}{3}\approx5.7$

Exercises

- 3. Find the mean, the range, and the mean absolute deviation in each of the following sets. (Hand calculators will help.)
 - a) 5, 9, 2, 4
- b) 7, 1, 11, 2, 3, 6

- a) 5, 9, 4, 7 c) 3, 2, 5, 7, 6 d) 5, 7, 0, 1, 7 f) 36, 39, 45, 52, 85, 90 h) 15.7, 16.3, 22.1, 54.3, 21.5 4. Can you find an example of two different sets with the same mean, median, mode and range?

The mean absolute deviation is a good measure of the scatter of a distribution. Unfortunately as more sophisticated methods are applied to the study of data it is found that this deviation is not convenient since it involves "absolute values" which are hard to work with. Fortunately there is another measure of scatter available. It is the standard deviation.

THE STANDARD DEVIATION

In determining the mean deviation, we found that the individual differences, considered as signed numbers, always added up to zero. To eliminate this problem, we used "absolute values" of the deviations, that is, considered each number as positive. Another way to eliminate the problem of the signs is to square each of the deviations and find their average. That is, find the mean of the squared deviations.

Let's try this in each of the three cases (a), (b) and (c) illustrated in Figure 2. In (a), the deviations are (1-3), (3-3), (3-3), (3-3) and (5-3). If we square and add we get $(-2)^2 + 0 + 0 + 0 + 2^2 = 8$. In (b) we get $(1-3)^2 + (2-3)^2 + (3-3)^2 + (4-3)^2 + (5-3)^2 =$ $(-2)^2 + (-1)^2 + (0)^2 + 1^2 + 2^2 = 4 + 1 + 1 + 4 = 10$. In (c) we get $(1-3)^2 + (1-3)^2 + (3-3)^2 + (5-3)^2 + (5-3)^2 =$ $(-2)^2 + (-2)^2 + 0^2 + 2^2 + 2^2 = 4 + 4 + 4 + 4 = 16$.

The means of the sum of the five squared deviations in each set are 8/5, 10/5 and 16/5 or 1.6, 2 and 3.2. These numbers do distinguish the sets. Like the mean absolute deviation they do increase from set (a) to set (b) to set (c) and so measure the increasing scatter of these sets. This mean of the squared deviations is called the <u>variance</u> of the set. To do it once more, let's find the variance of the lengths of a set of dowel rods. In cm, the lengths are 5, 7, 9, 12, 17. First find the mean, \bar{x} . $\bar{x} = (5 + 7 + 9 + 12 + 17)/5 = 50/5 = 10$. The deviations are 5-10 = -5, 7-10 = -3, 9-10 = -1, 12-10 = 2 and 17-10 = 7. The variance, v, is the mean of the squared deviations.

 $\mathbf{v} = [(-5)^2 + (-3)^2 + (-1)^2 + (2)^2 + (7)^2]/5$ = (25 + 9 + 1 + 4 + 49)/5 = 88/5 = 17.6.

The <u>variance</u> of a set is the mean of the squared deviations of each number in the set from the mean of the set.

Now since the original numbers in the above set are the measures of the lengths in cm of a set of dowel rods, the variance will be in cm² units. The mean is 10 cm and the variance 17.6 cm². We would like to have a measure of scatter in the same units as the original numbers in the set. We can do this if we simply take the square root of the variance. This square root is called the <u>standard deviation</u> of the distribution. The standard deviation of the lengths of the dowel rods is $\sqrt{17.6} \approx 4.2$ cm. The standard deviation of a set is the square root of the variance of the set.

In the sets (a), (b) and (c) above the standard deviations are $\sqrt{1.6}$, $\sqrt{2}$ and $\sqrt{3.2}$ respectively or approximately 1.3, 1.4 and 1.8. These numbers measure the scatter of the sets and they increase from (a) to (b) to (c) as we want.

The mean and standard deviation of a set are two of the most used numbers in statistics for describing a distribution.

COMPUTING THE STANDARD DEVIATION, s

The work in finding the standard deviation is easier if an organization similar to Table 1 is used. We omit the column of distances and add a column for the squared deviations. Table 2 is the work to find the mean, \bar{x} , and the standard deviation, s, of the numbers { 7, 9, 10, 15, 19, 25 }.

	TABLE 2	
	FINDING THE STANDARD DEVIATION,	S
Numbers	Deviations from the Mean	Squared Deviations
7 9 10 15 19 25	-7.2 -5.2 -4.2 .8 4.8 10.8	51.84 27.04 17.64 .64 23.04 116.64
Sum = 85 $\bar{x} = 85/6 \approx 14.2$		Sum = 236.84 v = 236.84/6 \approx 39.47 s $\approx \sqrt{39.47} \approx$ 6.28

Since the mean 14.2 in the first column is only an approximation to the actual value 85/6, all future computations will only be approximate.

To find the standard deviation of even a relatively small set of numbers by hand is lengthy. A hand calculator makes these computations much easier. A simple calculator can perform the necessary additions, subtractions, multiplications, divisions, and even the final square root rapidly and accurately.

Even knowing the correct sequence of steps may soon prove unnecessary as fairly inexpensive calculators are now on the market that will, at the push of a couple of buttons, automatically compute and display both the mean and the standard deviation of a set of numbers once the numbers have been entered correctly. But we should still understand what we are asking the calculator to do.

For larger sets of numbers a computer will be invaluable in saving much time. A BASIC program to find the variance and standard deviation is included in the *APPENDIX*. As was pointed out before, this program may have to be modified slightly to run on your computer as various versions of BASIC differ in details.

Exercises

5. Find x̄, the mean, and s, the standard deviation for the following sets of numbers.
a) 1, 5, 7, 7

- b) 9, 11, 7, 3, 10
- c) 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12
- d) 5, 5, 5, 5, 5
- e) 17, 103, 9, 36, 72

THE STANDARD DEVIATION, s, OF A FREQUENCY DISTRIBUTION

To find the standard deviation of a frequency distribution we have to multiply the squared deviation of each element of the set by its frequency just as we do for each element in finding the mean. A systematic method of doing this is a help. One such method is a table. Table 3 is the work to find the mean, \bar{x} , and the standard deviation of the weights in kg of a shipment of bags of grain. The original weights and frequencies are given in columns 1 and 2 of the table. A hand calculator is used to do the arithmetic.

		TABI	LE 3		
	FINDING	x AND s FOR A SH	HIPMENT OF BAGS OF G	GRAIN	
Weight (in Kg)	Number of Bags (Frequency)	Product Col.l x Col.2	Deviation from Mean $\overline{\mathbf{x}}$	Square of Deviation	Product Col.5 x Col.2
49.1 3 147.3 49.3 5 246.5 50.4 12 604.8 50.7 6 304.2 51.1 9 459.9 51.4 5 257.0		$ \begin{array}{c cccc} -1.4 & 1.96 \\ -1.2 & 1.44 \\1 & .01 \\ .2 & .04 \\ .6 & .36 \\ .9 & .81 \\ \end{array} $		5.88 7.20 .12 .24 3.24 4.05	
Total Number of Bags = 40 Total Weight = 2019.7				Sı	um = 20.73
	$\overline{\mathbf{x}} = \frac{2019.7}{40}$	≈ 50.5	$v \approx \frac{20.73}{40}$	≈.51825	
			$s \approx \sqrt{.5182}$	$\overline{5} \approx .72$	

If the numbers are grouped in intervals, simply use the midpoint of each interval as the representative for that interval.

Exercises

6. Find the mean \overline{x} and standard deviation s of the grades in a mathematics class as given in the first two columns in the following table. Some of the items have been filled in. Use a hand calculator and round to the nearest tenth.

Test Grades Interval	Frequency	Midpoint	Product Col.2 x Col.3	Deviation From Mean	Squared Deviation	Product Col.6 x Col.2
55-59 60-64	3 5 9	57 62	171			929.4
65-69 70-74	9 13		603	-7.6	57.8	
75-79	8	77				46.4
80- 4	12			7.4	54.8	657.6
85-89 90-94 95-99	3 3 1	92	276			
	Sum = f		Sum = t			Sum = w
	$\overline{\mathbf{x}} = \frac{\mathbf{t}}{\mathbf{f}} \approx$			$v \approx \frac{W}{f} \approx$ $s \approx \sqrt{v} \approx$		
				s≈√v≈		

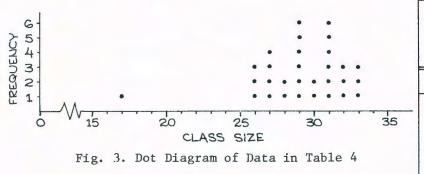
CAN X AND S DESCRIBE A DISTRIBUTION?

The standard deviation can serve to distinguish two different sets of numbers with the same mean and the same range. Furthermore the distribution that intuitively seems to have the greater scatter from the mean has the greater standard deviation. The standard deviations of the sets (a) = 1, 3, 3, 3, 5 (b) = 1, 2, 3, 4, 5 and (c) = 1, 1, 3, 5, 5 of Figure 2 turn out to be 1.3, 1.4 and 1.8 respectively. Can the standard deviation also be used to measure effectively the scatter in a single set of numbers? Yes. We noted earlier that if we know only the <u>mean</u> of a set of scores, we cannot make any prediction about how closely other scores cluster around the mean. In each case above the mean is 3. The standard deviation tells us something about how the scores do cluster around the mean. For example, if we have a set (d) with all the scores 3, what would the mean be? Obviously 3. What would the standard deviation be? 0. The question then becomes: How much can the standard deviation tell us about the closeness of other scores to the mean? The answer to this question and answers to other questions about a distribution can be clarified if we examine certain graphs of the data. These are the <u>histograms</u>, <u>frequency polygons</u> and <u>cumulative</u> <u>polygons</u> that are special cases of the bar and line graphs considered in *Graphs* of the CONTENT FOR TEACHERS section. We must spend some time now studying them.

GRAPHS OF A DISTRIBUTION

HISTOGRAMS

Table 4 is a frequency distribution of the number of class sizes in a certain school. We make a dot diagram of this distribution in Figure 3.



TABI	TABLE 4				
FREQUENCY DISTRIBUTION OF CLASS SIZES					
Class Size	Frequency				
17 26 27 28 29	1 3 4 2 6				
30 31 32 33	2 6 3 3				

Figure 3 makes the range very clear. It is 33 - 17 = 16. Also the outlier at 17 shows up vivid-

ly. The trimmed range of 33 - 26 = 7 gives a better idea of the scatter of most of the data. If instead of a \bullet to indicate each item, we use a small square \Box we get a new graph: a histogram.

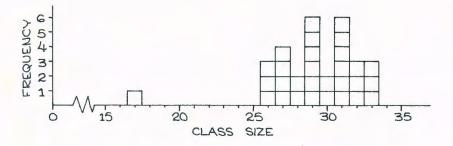


Fig. 4. Histogram of Data in Table 4

The squares are centered over the scale marks and are wide enough to have the vertical bars touch each other unless there are isolated values such as the 17 in Figure 4.

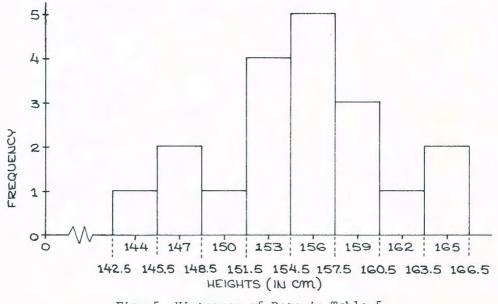
If the data is a set of measurements and is given over a series of intervals the problems of the intervals and their end points must be considered. Table 5 gives the heights of a class of students. They were originally given to the nearest

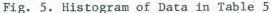
cm and are grouped in intervals such as 146-148 cm Since heights given as 146 cm or 148 cm might have been as low as 145.5 or as high as 148.5, we make the interval on the histogram extend from 145.5 to 148.5 and similarly for the other intervals. The interval is centered at the midpoint 147. Figure 5 is a histogram of the data in Table 5.

In a histogram based on data grouped into intervals, the vertical bars are drawn to a height that is proportional to the frequency. They are centered over the midpoint and extend from lower to upper interval boundary. The

TABLE 5	TABLE 5				
HEIGHTS OF A CLASS	OF STUDENTS				
Height (in centimetres)	Frequency				
143-145	1				
146-148 149-151	2				
152–154 155–157	4				
158–160 161–163	3				
164-166	2				

individual squares are omitted. For easier readability, the scales should be set so the maximum height of the histogram is no more than about 2/3 or 3/4 of its width.





The area above each interval in the histogram represents the number of items in that interval so it is important to have the intervals of equal length. If this is impossible because the data are not given that way, great care must be taken. In fact, at this stage: DO NOT TRY TO DRAW A HISTOGRAM FOR SUCH DATA. If you see a histogram with unequal or open intervals you should not try to interpret the data over those intervals.

Exercises

7. Make a histogram of the data in the following table. Label interval end points and midpoints accurately.

Height in Centimetres	148-150	151-153	154-156	157-159	160-162	163-165	166-168	169-171
Frequency	1	1	3	5	12	14	3	4

 Make a histogram of the following data. Group the data first into intervals such as 135-139, 140-144, etc.
 Heights of a class in centimetres: 158, 155, 149, 152, 160, 168, 156, 159, 152, 146, 156, 163, 152, 151, 163, 149, 158, 152, 135, 149, 148, 161, 156, 151, 159, 161, 144, 153, 157, 161, 141.

FREQUENCY POLYGONS

A histogram gives an accurate picture of the range of the distribution and of the mode if there is one. Both Figures 3 and 4 show the bimodal character of those distributions. Neither the mean nor the median is apparent. We get some idea of the scatter but we do not yet see just what the standard deviation can tell us.

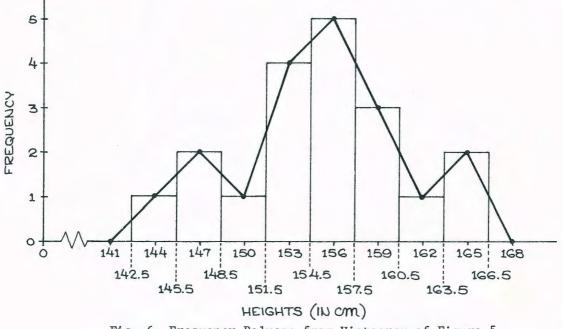


Fig. 6. Frequency Polygon from Histogram of Figure 5

Using the histogram we can draw a special line graph known as the <u>frequency</u> <u>polygon</u>. A frequency polygon is obtained from a histogram as follows. Locate the points at the center of the top of each bar of the histogram, add two zero points at the centers of the intervals above and below the last intervals you used. Connect consecutive points by a series of straight line segments (see Figure 6).

Exercises

9. Draw the frequency polygon for Exercise 7.

- 10. Draw the frequency polygon for Exercise 8.
- 11. Discuss what might be lost or gained in using Figure 6 as against Figure 5.

USING X, S AND A HISTOGRAM TO GET MORE INFORMATION

We come back now to the question: How can the mean and the standard deviation be used to describe the clustering and the scattering of a distribution? Primarily they are used in combination with the histogram. A histogram can take many different forms. If it looks like (a) in Figure 7, the mode and the mean are the same

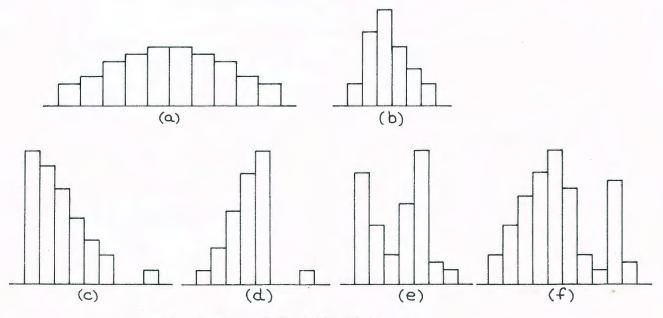
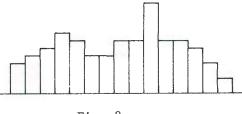


Fig. 7. Several Possible Histograms

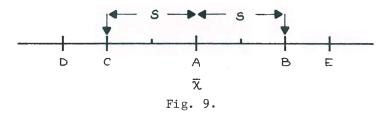
and the clustering is in the middle. So is it in (b) but there is far less scatter in (b) than in (a). What would be true about the standard deviations in (a) and (b)? In (c) and (d), there is clustering but it is at one end or the other. The mode is obvious but not the mean. (e) and (f) have still different appearance. Figure 8 shows a histogram of the weights of students at a university. You





might be justified in concluding that this was a coed university with more men enrolled than women. Why?

What can the standard deviation tell us about the distributions when the histograms are as different as those we have just drawn? The first thing is: no matter what the histogram looks like, if we know the mean \bar{x} and the standard deviation, s, we can say: <u>At least</u> 3/4 of all the items in the distribution will be found within a distance of 2s on either side of the mean. In Figure 9, if the mean of

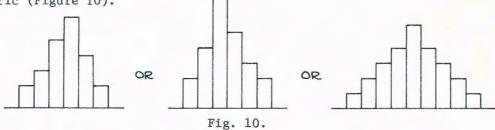


some distribution is at A and the standard deviation, s, is equal to \xrightarrow{S} then C is 2s units to the left of \overline{x} and B is 2s units to the right. We know that at least 3/4 of all items will be between C and B. Suppose D and E are 3s units on either side of A. What can we say about them? We know at least 8/9 of all the items lie between D and E. The fractions 3/4 and 8/9 are determined from the 2s and 3s distances we used. $\frac{3}{4} = 1 - \frac{1}{2^2}$ and $\frac{8}{9} = 1 - \frac{1}{3^2}$.

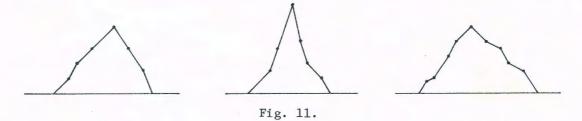
More generally, if the points P and Q are k standard deviations on either side of the mean \overline{x} , then we calculate $1 - \frac{1}{k^2}$ and we can say at least the fraction $1 - \frac{1}{k^2}$ of the distribution lies between P and Q. If $k \leq 1$, $1 - \frac{1}{1^2} \leq 0$ so we cannot say anything. Note well: Even though at least 3/4 or 75% of the measurements <u>must</u> be between $\overline{x} - 2s$ and $\overline{x} + 2s$ it might happen that 85% or 95% or even 100% of the measurements are within those limits. In fact, in most of the distributions we have met in actual practice, if we carry out the computations of \overline{x} and s it is evident that many more than 75% of all the items will be between points C and D. Can we say how many more? Yes, sometimes.

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Most of the distributions we encounter will have histograms that look more like the following diagrams than the last four in Figure 7. These are approximately symmetric (Figure 10).



We draw their frequency polygons in Figure 11. The polygons are also approximately symmetric and each has its highest point near the middle. If we draw smooth curves



to fit such frequency polygons as well as possible they will look something like those in Figure 12. Some of the curves may be narrower and higher than others but

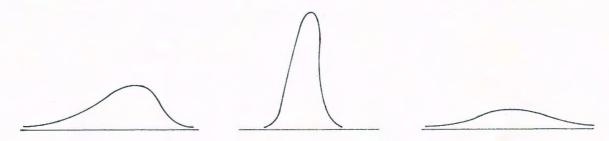


Fig. 12. Approximately Normal Curves

they are near what are called <u>normal curves</u>. Two characteristic properties of normal frequency curves are: they are symmetric around their mean; and if you move <u>one</u> standard deviation to either side of the mean, you include about 68% of the measurements. Also 95% of the data will be within 2 standard deviations and 99.7% within 3 standard deviations of the mean. Figure 13 may help to make this clearer.

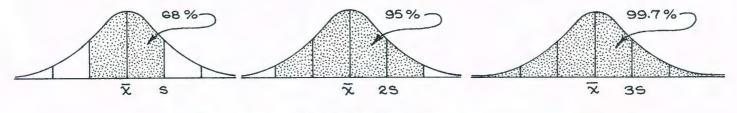


Fig. 13. Normal distributions with s = ---

Since most of our distributions approximate this situation we can say that these properties will hold for them.

A table may make these two theorems more vivid (see Table 6).

	TABLE 6	
x AND s DE	SCRIBE THE DISTRIBUTION	
	<u>All</u> Distributions	Approximately Normal Distributions
Interval	Contains at least	Contains about
$\overline{x} - s$ to $\overline{x} + s$	0%	68%
\overline{x} - 2s to \overline{x} + 2s	3/4 or 75%	95%
\overline{x} - 3s to \overline{x} + 3s	8/9 or 89%	99.7%

Suppose crates of peaches are being delivered to a store. Suppose we know the weights of the crates have a mean of 18 kg and a standard deviation of .5 kg, and we assume the weights are normally distributed. Then we know about 68% of the crates weigh between 18 - 1(.5) and 18 + 1(.5) kg or between 17.5 and 18.5 kg. Also about 95% are between 18 - 2(.5) and 18 + 2(.5) or 17 and 19 kg. Practically all of the crates are between 16.5 and 19.5 kg.

Exercises

In each exercise, assume a distribution that is approximately normal.

- 12. 1500 seventh graders at Sherman Junior High School have a mean IQ of 109. The standard deviation is 11.
 - a) Between what two IQ values will 95% of the student body lie?
 - b) About how many students may have an IQ greater than 142?
 - c) About how many students have IQ's between 98 and 120?

- 13. On a certain standardized test, the mean score is 500 and the standard deviation is 100.
 - a) What percentage of all students taking the test will have scores between 400 and 600? Out of a class of 140, how many students will have scores between 400 and 600? Between 300 and 700?
 - b) What percentage of all students will have scores between 500 and 600?
 - c) What percentage of all students will have scores greater than 700?
 - d) Lucy's score is 700. What is her percentile rank?
 - e) Tom's score is 600. What is his percentile rank?
- 14. A machine produces plastic covers for medicine bottles. When in good working order, it produces about 1000 covers per hour whose diameters have an approximately normal distribution with mean 1.50 cm and standard deviation .1 mm. About 95% of the covers will have diameters between two values A and B. What are these? Practically all covers will have diameters between two other values, C and D. What are they? One day the machine is out of order. The mean is still 1.50 cm but the standard deviation is now 1 mm. If the distribution is still at least roughly normal, what percent of the output is between 1.3 cm and 1.7 cm?

OGIVES OR CUMULATIVE POLYGONS

From Table 5, we construct the cumulative frequency table exhibited as Table 7, by adding the frequencies up to and including those at any given interval.

From this table we can see how many items are found <u>at or below</u> each upper interval value. We know there are 4 heights less than or equal to 151 cm. If we use the interval boundary, we know there are 4 items actually <u>below</u> the upper boundary, 151.5. To draw the cu-

TABLE 7 CUMULATIVE FREQUENCIES OF THE HEIGHTS OF A CLASS (in centimetres)						
Height (in centimeters) Frequency Cumulative Frequency						
143-145 146-148 149-151 152-154 155-157 158-160 161-163 164-166	1 2 1 4 5 3 1 2	1 3 4 8 13 16 17 19				

mulative polygon of this data as in Figure 14, plot 0 at the lower boundary of the first interval (142.5) and locate the proper point for each upper boundary. Thus at boundary point 151.5 we go up 4 units and mark the point, at 157.5 up 13 units, etc. We connect consecutive points by straight line segments. The resulting figure is usually shaped like an elongated or "lazy" S. It is the <u>cumulative frequency</u> polygon called for short an <u>ogive</u>. (The derivation of "ogive" is uncertain. It probably comes from a French word used to describe an arch. An ogive curve is shaped like half an arch.)

We draw the ogive corresponding to the frequency polygon in Figure 6. This gives us Figure 14.

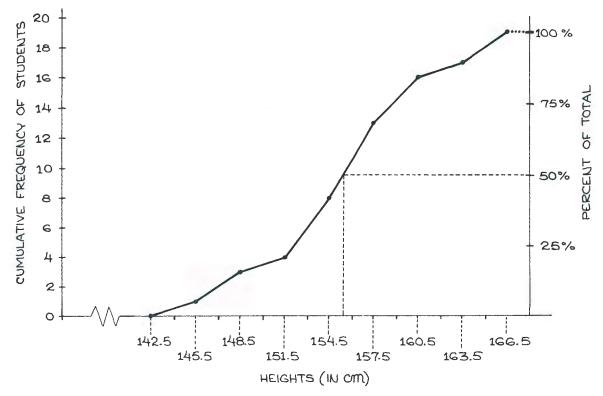


Fig. 14. Cumulative Polygon or Ogive for Heights from Table 6

The scale on the left side is the count of students less than a given height. The scale on the right side is a percent scale.

Exercises

- 15. Draw the ogives corresponding to the frequency polygons you drew for Exercises 9 and 10.
- 16. Use the ogives in Figure 14 and the ones you drew in Exercise 15 to determine the medians of the three distributions.

PERCENTILES, DECILES AND QUARTILES

An ogive may be used to determine not only the median of a distribution but also those other measures called the quartiles, deciles and percentiles. The median is that number such that half (50%) of the distribution is at or below it. In similar fashion, the quartiles divide the distribution into fourths, the deciles into tenths, and the percentiles into hundredths.

The first quartile is the number such that one-fourth or 25% of the distribution is at or below it, the second quartile, two-fourths, and the third quartile, three-fourths. Of course, the second quartile coincides with the median. The third decile, or the thirtieth percentile is the number such that three-tenths or 30% lie at or below it, the 95th percentile is at the point where 95% of the numbers lie at or below it.

The cumulative frequency polygon is usually drawn with the frequency scale marked on the left side of the diagram and the percentile scale on the right.

A histogram and its cumulative frequency polygon are shown below in Figures 15 and 16. From the cumulative polygon, the quartiles and percentiles can be read with reasonable accuracy. To find the 75th percentile, move up the right hand scale to the point marked 75, move over horizontally on a straight line to the point where it intersects the polygon, move down vertically to the intersection point with the horizontal axis and read the coordinate. The 75% or 3rd quartile is approximately \$1350. The 90% is about \$1580.

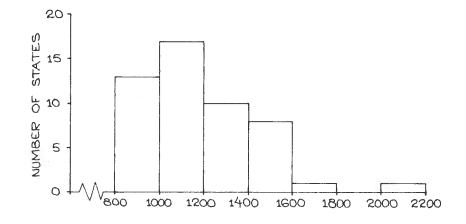


Fig. 15. Histogram of Average Expenditure Per Pupil in the 50 States, 1974-75

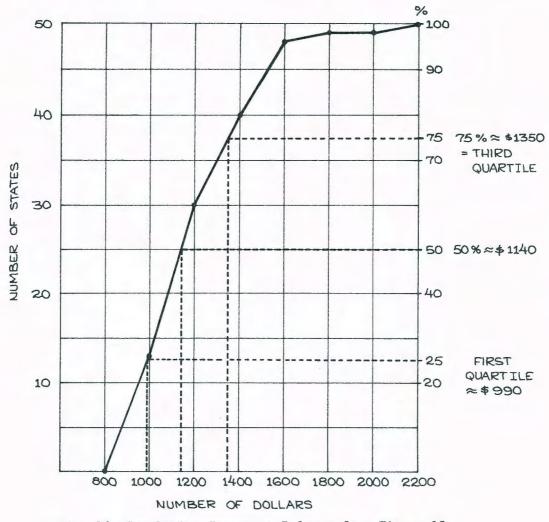
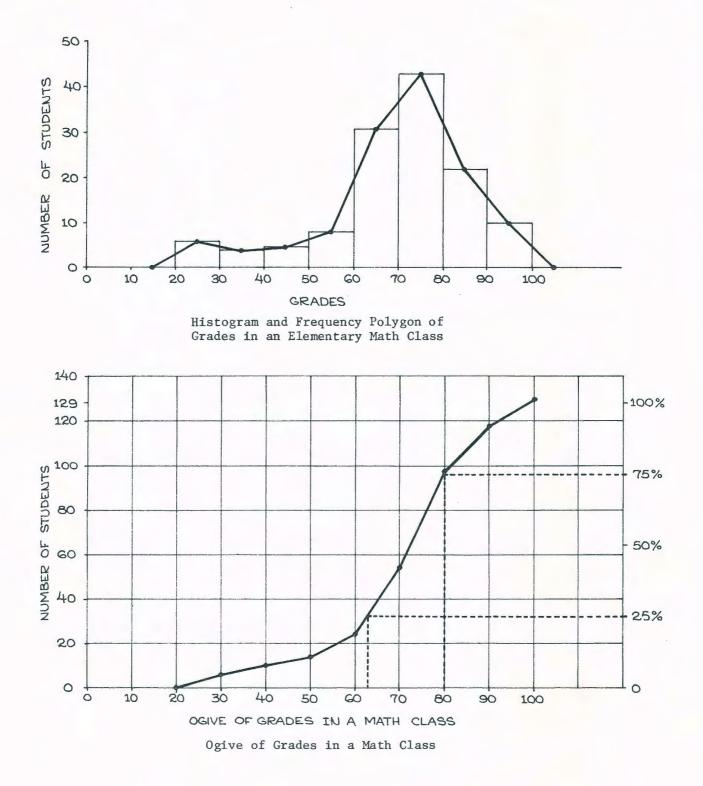


Fig. 16. Cumulative Frequency Polygon from Figure 15

Exercises

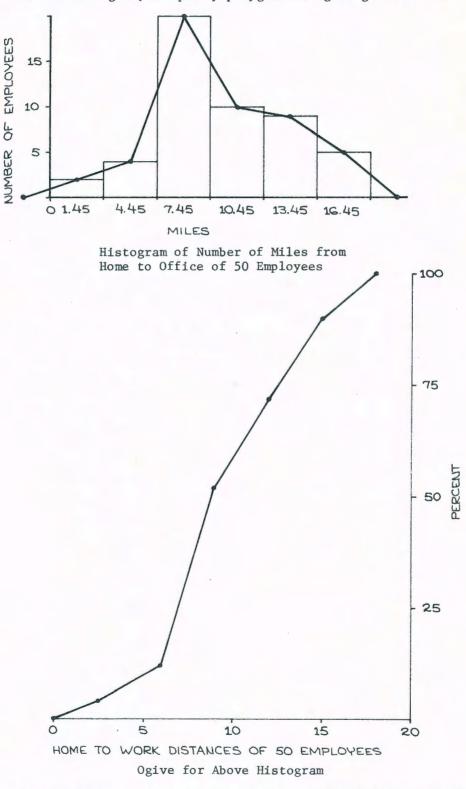
- 17. Determine from Figure 16 the 20, 35, 70, 80 and 95 percentile per pupil expenditures by states in 1974-75.
- 18. A histogram and its frequency polygon and ogive are given below.
 - a) Can you tell exactly the highest and lowest grades? Give an approximate range.
 - b) What grade is the mode?
 - c) How many students took the test?
 - d) What grade is at the 25%? What does this mean?
 - e) What percent of the students had grades better than 80?
 - f) What was the median grade?



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CONTENT FOR TEACHERS

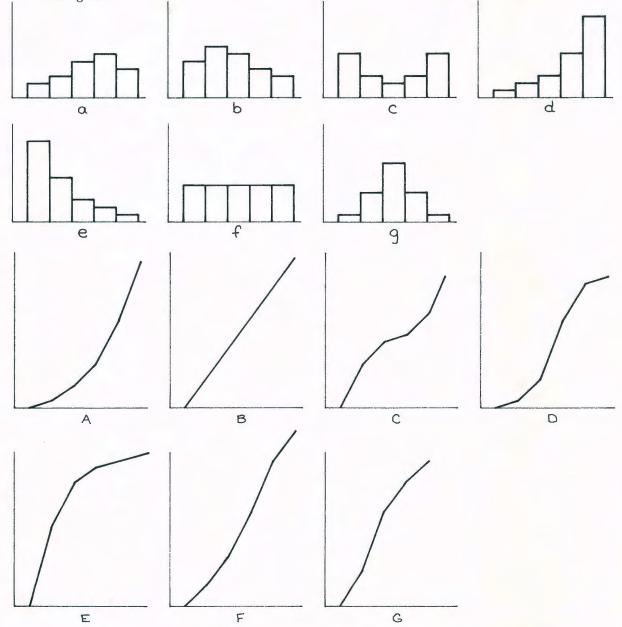




Graphs from *Statistics: A Conceptual Approach* by Sidney J. Armore, published by Charles Merrill, 1975, pp. 38 and 39. Permission to use granted by Charles E. Merrill Publishing Company

Use the graphs on page 22 to answer the following questions:

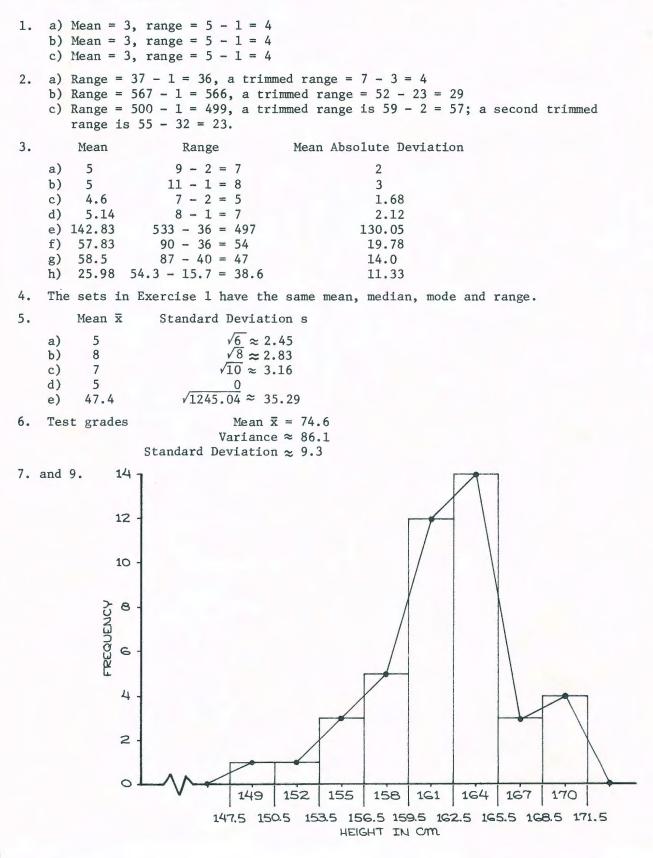
- a) What is the most common distance to work? What is the greatest distance anyone drives? What is the 25 percentile? What does this mean? What is the 50 percentile?
- b) If you wanted to be like most other workers, how far from work would you live in this community?
- c) If you wanted to "be different" where would you choose to live?
- 20. Here are some typical histograms and the corresponding ogives. Match them up. Thus f goes with B. What general interpretation can you give for each histogram?

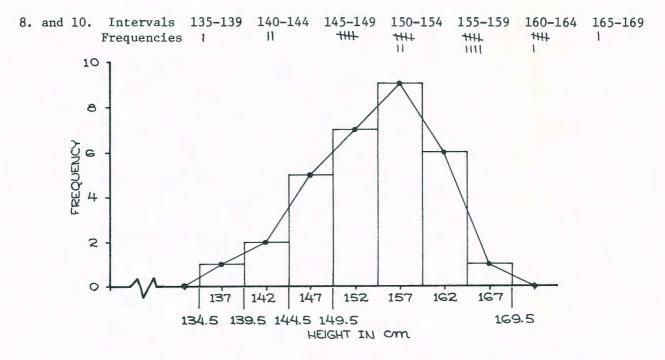


Histograms and ogives from *Introduction to Statistical Analysis and Inference* by Sidney J. Armore, published by John Wiley & Sons, Inc., 1966, pp. 69–71.

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ANSWERS TO EXERCISES

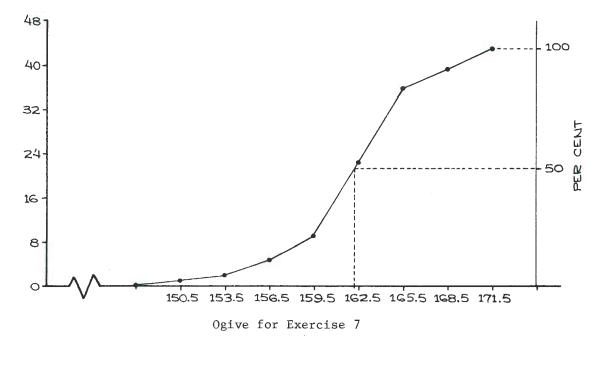


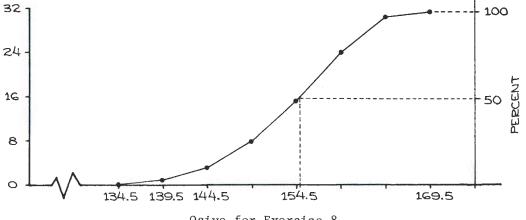


- 11. The frequency polygon gives perhaps a better idea of the way the heights are changing. In the histogram all the items in the interval are given one value, that at the midpoint of the interval. This may not be accurate as they could have had any value in the interval. The frequency polygon reflects this data.
- 12. a) 95% of the students are between 109 22 and 109 + 22 or 87 and 131.
 - b) .0015 x 1500 = 2.25 Say 2 students
 - c) .68 x 1500 = 1020 students will be between 98 and 120.
- 13. a) 68% will be between 400 and 600, 95 out of 140.
 95% will be between 300 and 700, 133 out of 140.
 b) About 34%, i.e., half of 68%
 - c) About 2.5% will be above 700.
 - d) 700 will give a percentile of 97.5.
 - e) 600 will give a percentile of 97.5
- 14. Mean = 1.50 cm, standard deviation = .1 mm = .01 cm 95% will lie between 1.50 - .02 and 1.50 + .02 or 1.48 to 1.52 cm.

Practically all, i.e., 99.7% will be between 1.47 and 1.53 cm. Now s = .1, therefore 95% are between 1.3 and 1.7 cm. If the distribution is no longer normal, still 75% will be in this range since it is $x - 2\overline{s}$ to $x + 2\overline{s}$.

15.	Data in Exercise 7 Upper Boundary Cumulative Frequency		Data in Exercise 8 Upper Boundary Cumulative Frequency		
	150.5	1	139.5	1	
	153.5	2	144.5	3	
	156.5	5	149.5	8	
	159.5	10	154.5	15	
	162.5	22	159.5	24	
	165.5	36	164.5	30	
	168.5	39	169.5	31	
	171.5	43			





Ogive for Exercise 8

- 16. Median in Exercise 7 ≈ 162 Median in Exercise 8 ≈ 155 Median in Figure 14 ≈ 155
 17. The percentiles are 20% ≈ \$960 80% ≈ \$1400 35% ≈ \$1050 95% ≈ \$1580 70% ≈ \$1300
 18. a) You cannot tell the exact lowest and highest grade. The approximate range is from 20 to 100.
 b) The mode is 75.
 c) 129 students took the test.
 d) The 25% is at 63. This means that about 32 students were at or below 63.
 - e) About 24% had grades better than 80.
 - f) The median grade was about 73.

19. a) Most common distance = mode = 7.5 miles Greatest distance is in interval 15-18, so 18. 25% is 7 miles, i.e., about 13 people drive 7 or less miles. 50% is 8.8 miles.

- b) Live from 6-12 miles from work
- c) Live either within 3 or more than 15 miles from work
- 20. a) F b) G c) C d) A e) E f) B g) D

ORGANIZATION

Placement Guide for Tabbed Dividers

Tab Label	Follows page:
CONTENT FOR TEACHERS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
Probability with Counting	
Inferential Statistics	
Appendix-Computer Programs	
DIDACTICS	
Components of Instruction	
Classroom Management	
Statistics and Probability Learning	
TEACHING EMPHASES	
Critical Thinking	
Decision Making	
Problem Solving	
Models and Simulations	
Calculators and Computers	
Laboratory Approaches	
CLASSROOM MATERIALS	
CONCEPTS AND SKILLS	
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APPLICATIONS	
Business & Commerce	
Environment	
Health & Medicine	
People & Culture	
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GLOSSARY	
ANNOTATED BIBLIOGRAPHY	
SELECTED ANSWERS	



INTRODUCTION

In previous sections we have discussed gathering data, organizing it into tables and graphs, summarizing it and describing it by measures of central tendency and scatter. One reason for this is the data may help in making a decision about a certain problem. The crucial words here are "may help." Some politicians may decide to vote on a current issue as a poll shows the majority of their constituents feel. Others may weigh the results of the poll but in the end let their own value judgment of the issue determine their vote. Data is important in making decisions but not all important. Many citizens filling out their income tax forms decide to be honest, not because a poll says whether a majority of fellow citizens would or would not cheat, but because their own values tell them to do the right thing.

If data can help, it behooves us to be as accurate and as complete as possible. But sometimes gathering data on the whole population is impractical or irrelevant. What should we do? Consider the following situations:

• A manufacturer of a new detergent, STOP DIRT, is considering a certain advertisement. Will it appeal to customers? Should a nationwide campaign be launched with this advertisement?

• An exporter of grain is buying carloads of wheat. It is suspected that some carloads are substandard. Hundreds of carloads are coming in from many different elevators. Should every carload be checked before being accepted or only one or two from each elevator?

• A school system buys light bulbs for its classrooms. It needs 500 cartons each with

twelve dozen bulbs. The purchasing agent is approached by a salesman from a new company offering bulbs at half the normal price. The salesman acknowledges that about 1% of his bulbs are defective and will burn out in less than normal time. How can the buyer check on the salesman's claims and be protected from buying an excessive number of defective bulbs?

• A suburban school systems feels pressure to integrate its classrooms. It could do it by busing in children from the city system. Data from a nearly similar suburb indicates that 20% of their integrated classes do less well in average scores on standardized achievement tests than before integration. In 40% they remain the same and in 40% there is improvement. Should the system integrate?



SAMPLING

In the first three cases, it is clear that decisions should be based on data about the members of the group involved. But it may be impractical or even impossi-

ble to obtain the data on the whole group. The advertiser cannot afford the chance of going nationwide immediately. The ad should be tried out in a small way first. The grain buyer cannot afford the time or money to inspect every carload but does look at a few. The school purchasing agent cannot test the length of life of all the bulbs for then there will be no fresh bulbs left to put in the classrooms. Only a few bulbs can be tested. In each of these cases the person responsible for the decision has to resort to selecting some sort of a sample, testing the items in the sample and then making the decision about



the whole population on the basis of the results from this sample.

In the last case, the sample is only of size one. Even so, is the data relevant to the decision? Suppose the percentages had been radically different, would it still be relevant? Is there a value judgment involved that should be subject to or should override the data whatever it is? These are tough questions that may have differing answers in differing times.

Exercises (Answers given on pp. 146-148)

- Buyers frequently inspect samples before making their decision.
 - a) List at least three situations where you feel this would be advantageous though perhaps not absolutely necessary.
 - b) List three where the population is so large that total inspection is impractical.
 - c) List three cases where the inspection is destructive as in the case of the light bulbs.



SAMPLING

CONTENT FOR TEACHERS

EVERYDAY SAMPLING

Not only is sampling important in commercial cases like those mentioned above, but it is frequently used in everyday living. If Mr. and Mrs. Brown buy a GOFAST car from the BUILDEM QUICK COMPANY and it turns out to be a lemon, they may feel this is a large enough sample to discourage them from buying any more GOFAST cars. Again suppose Mr. and Mrs. Brown are shopping in the GROW-RIGHT MARKET. At the lettuce bin, Mrs. Brown squeezes three or four heads of lettuce, finds them squishy and says to Mr. Brown, "Let's get out of here. Their produce is not good." She has used a very small sample to make her decision about the whole store. Meanwhile Mr. Brown looks at the tomatoes, picks up several and finds them good. He replies, "Hey, wait a minute, their tomatoes are good." Now they have sampled two different products with different results. Perhaps they decide they should look at the broccoli, spinach and other kinds of produce to get a larger sample before making a final decision. Maybe they decide not to buy there at all, maybe to buy only selectively after looking at each different kinds of produce. In any case, they are using samples to help make decisions.



Exercises

- Describe at least 3 situations where you have used sampling in the past year. It may be that in some cases only now, as you look back, do you realize you actually did use sampling.
- 3. Describe a situation where prejudices may be exhibited or caused by making decisions based on a small sample.
- 4. Describe some cases in which <u>one</u> bad item out of a large number would be enough to have you decide against the product. Under what circumstances would you be more lenient?
- 5. Sometimes we feel comfortable in generalizing from a sample of size 1; sometimes it is foolish to do so. Give examples of each situation.

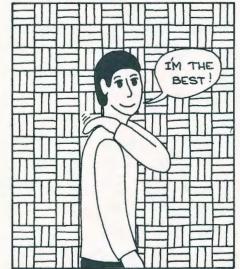
IMPORTANCE OF SAMPLING

The ideas and methods of sampling are important not only in problems like those above but in many others. Can enough preschool children learn to read in kindergarten to make reading a part of a kindergarten program? Is polio vaccine effective enough to be worth giving to everybody? Should the swine flu vaccine have been tried on a sample before the decision was made to produce enough for all adults? Suppose all of my sixth grade class reads at above the national sixth grade norm. Is this class a sample of all the sixth grade children in my district? Can I conclude they all or almost all read at this level or could it be my sixth grade class is exceptionally bright and not a typical sample?

In so many cases, inquirers or decision makers must gather data from a sample rather than from the population they are interested in.

For instance, in the problem discussed at the end of the last section concerning the weights of crates of peaches delivered to a store, we said, "If we know the mean and standard deviation of the weights of crates of peaches..." You may well say, "There are many crates being delivered yesterday, today, and on many tomorrows. How could we possibly know the mean and standard deviation of





crates that may not even be packed yet?" Of course, we do not know them exactly, and one of the main purposes of statistics is to learn how to make estimates of these numbers. To do it we would take a <u>sample</u> of say 20 crates. The population we are sampling is all the crates that might be delivered to that store this year. We find the mean and standard deviation of the weights of the samples and use them to estimate the mean and standard deviation of the population. Methods of making such predictions about the population from information obtained from a sample are important. To be useful in this way, the sample must be an appropriate one, that is, it must be representative of the population.

SAMPLING

It is therefore important to know how to decide the size and make-up of a representative sample and how to pick one. We need to study the whole question of samples, how to draw unbiased samples and how to make the estimates we want from the samples we have.

Sampling techniques have been and still are the subject of a great deal of study and research. Some spectacularly poor statements have been made based on poor

samples. Right up to election day in 1948 some newspapers predicted that Thomas Dewey would be elected president instead of Harry Truman. There is a remarkable picture of President Truman gleefully holding aloft a copy of the Chicago Tribune of the morning after the election, announcing that Mr. Dewey had been elected.

We should always be aware, if possible, what samples others have used to make claims to influence us.



Exercises

- Cite at least one other case of poor prediction due to poor sampling.
- 7. An advertisement says, "Doctors at three hospitals prescribe XYZ medicine more than any other pain killer." Discuss the sampling on which the ad might have been based.

SAMPLES AND POPULATIONS

What is a sample? It is a selection from the whole group we are considering. In statistics, the whole group is called the population and any smaller group selected from it is called a sample.

A pollster may interview 3000 people and ask them if they intend to vote for the Democratic or the Republican candidate in the next election. These 3000 people form a sample but it may be uncertain as to what the population is that is being sampled. If the survey were taken between 8 and 9 A.M. in the Grand Central Station in New York City, the population being sampled is certainly



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not that of the voters in New York City. It is a sample of the commuters who will stop to talk and who are willing to disclose their intentions, perhaps accurately, perhaps inaccurately. If the pollster is interested in voters in New York City, the sample is not representative of that population. It is called a <u>biased sample</u>.

Whenever we consider taking a sample of a population it is vital that we know what population is being aimed at so every effort can be made to draw a sample representative of that particular population. In the advertisement for detergent, the advertiser is interested in potential buyers. If the ad is tried out on people in an automobile salesroom, the sample may be biased in at least two ways. First, the proportion of men in the auto salesroom is probably higher than that of men among detergent buyers. Secondly, even though shoppers for automobiles may also be shoppers for detergents many people are in the market for detergents who are not now considering buying a car. A less biased sample would probably be the shoppers in a big supermarket.

As another illustration suppose a survey of the 6th graders in your school district were being considered to determine the interest in starting a new music program. You would not want to talk only to the members of the district sixth grade baseball team. Few of us would think that the ball team would be a representative sample of <u>all</u> the sixth graders. This problem of representativeness will be considered again a little later in this section.



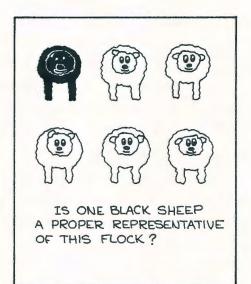
SELECTING A SAMPLE

We have said a sample should be <u>representative</u> of the population. What does representative mean? We considered the idea before when we felt the members of the baseball team were not really representative of all the sixth graders and the people in an auto showroom were not representative of the potential buyers of STOP DIRT detergent. Intuitively we feel a representative sample of sixth

SAMPLING

graders should include some boys, some girls, some athletes, some scholars, some musicians, etc., i.e., some of each different part of the population. But this may be impossible unless we make the sample so large as to be unwieldy. It is this fact that makes some people feel that sampling is a very suspect process. What can be done to meet this problem?

The best way to select a representative sample is to have some degree of randomness come into the selection process.



What is meant by randomness? What is a random sample? Perhaps the idea can best be approached through an example.

Suppose we want a random sample of three students from a class of 30 students. A sample is random if it is selected in such a way that each student has the same chance of being chosen as any other student. There are many ways of doing this. Some feel it could be done by simply having the teacher point randomly at these students. But many of us would suspect some bias in this method. A better way is to write the name of each student on separate but identical slips of paper, put all thirty slips in a basket, stir them up and draw out three slips. This is the usual way of drawing door prizes. The only trouble is that "stirring them up" is easier said than done. The lottery for the draft in 1970 was done this way and the results were far from being random. Why? Men with birthdays in December were much more likely to be called than those whose birthdays were in January. It was as though the slips for birthdays from January to December were put in a big drum in order, the drum turned over a couple of times supposedly to "stir them up" and the slips drawn out. The December slips were still largely on top, the November's next and so on down to the January's which were still largely grouped at the bottom of the drum. The chances of being drawn early were not equal for everybody. The drawing was not random.

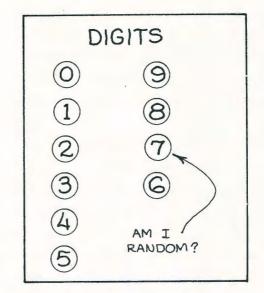
Situations like this demand better ways of assuring that a sample is truly random. We will describe such a process but before we do so we must remind you that even this does not assure us that any particular sample may not be very different

CONTENT FOR TEACHERS

from the population. A random selection of 9 sixth graders might be the members of the baseball team but the chance of this happening is very, very small. In a class of 30 it would be in the order of 6 in 100,000,000 but this is the same chance as that of selecting any other specified group of 9 students. We therefore decide that randomness in the selection process is desirable.

RANDOM NUMBERS AND THEIR USE

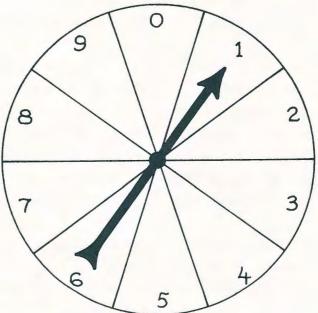
Perhaps the best way to pick a random sample is to use a table or list of so-called "random numbers" in a manner described below. What is a list ofrandom numbers? It is a list such that each digit from 0 to 9 has the same chance of appearing. The same is true of every two-digit number from 00 to 99 and also for numbers of three, four or more digits. Lists of up to 1,000,000 such random numbers are available. On page 10 is a short list of 2500 random numbers. If a table of



random numbers is not available we can find some device that will do reasonably well in generating a list of random numbers.

One such device is a carefully balanced spinner such as the one illustrated here. Simply give the arrow a hard snap, record the number on which it stops and repeat as many times as necessary. If the arrow stops exactly on a division line, ignore it and spin again. Other devices such as an icosahedral die are discussed in some of the student activities.

To use random numbers to select the sample of 3 children from a class



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of thirty proceed as follows. First number the children in any order, alphabetically would do as well as any other. If we are using the spinner, simply spin it twice and record the two digits in order. If the first spin gives a 4 and the second a 3, we record 43. Since the children's numbers only go up to 30 we ignore the 43 and try again. This time we get 0 and 5. We record 05 and note that the child numbered 5 has been chosen. We continue in this way until three different children are chosen for the sample.

To use the table of random numbers, we first have to decide where in the table to start. One way is to take a sharp pencil and while looking away from the page, put the pencil point down on the page and start at the digit the point touches. Read digits two at a time in order from the starting point, ignoring numbers above 30, until the three we need are obtained. If the successive numbers are 93, 88, <u>16</u>, <u>04</u>, 78, <u>29</u>, we would have chosen the children numbered 16, 4 and 29.

The same pattern could be followed if we wanted to get a random sample of 25 children from a school of about 950 students, this time of course using three digit numbers ignoring those over 950.

Exercises

- 8. In a class of 25 or more people ask each one to select a single digit at random and write it down. Do this three or four times and make a tally and a frequency distribution of the numbers. If the selections were truly random, what would the frequency distribution probably look like? Does your distribution look like this? Do you feel the selections were random or was there a tendency to choose certain numbers more than others?
- 9. Use a random number spinner to pick a random sample of three children out of a classroom of 10. Repeat twenty times (or have twenty different people each do it once). Make a frequency distribution of the number of times each child was chosen. Does the process seem random?
- 10. A school district employs 325 teachers. A committee of 10 is to be selected to represent the staff at a convention. Use a random number table to select this committee.
- In a certain city the telephone book has about 300 white pages of subscribers. We want to select three subscribers randomly. To do this, use a random number table in three stages.
 - a) Select three pages.
 - b) Select a column on each page chosen in (a).
 - c) Select a name from each column chosen in (b).
 - d) Repeat the process. Was any subscriber chosen twice?
- 12. A school has 925 students, 500 girls and 425 boys. Devise a method of using random numbers to select a sample that will tell you whether the student selected is a boy or a girl. Use this method to pick a sample of 10 students. How many girls are in your sample? Repeat the process. How many girls are in the second sample? Comment.

Table 2500 random digits

	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	2
)1)2)3)4)5	61 85 41 05 36	19 17 75	76 95 79	44 60 35	59 40 85	08 12 60	60 77 89	20 51 12	66 80 66	24 68 36 30 44	42 46 41	99 07 25	28 70 62	71 39 64	47 27 54	73 56 32	73 65 35	97 18 64	24 08 42	34 18 70 96 37	38 10 16	25 98 66	89 45 34	81 37 88 64 59	2 4 8
16 17 18 19 .0	73 94 28 38 49	78 15 37	81 00 11	49 92 05	27 41 75	50 16 43	50 37 72	97 07	46 99 36	27 37 66	27 90 43	04 76 43	75 87 91	38 87 05	38 83 44 71 73	97 34	69 76 59	20 45 50	30 94 95	87 53 83 94 25	19 96 37	54 07 64	35 46 64	70 62 21 61 14	2 9 1
1 2 3 4 5	85 07 82 83 19	55 67 92	13 78 56	99 13 64	88 04 46	48 83 18	59 85 94	19 53 30	75 97 33	51	96 11 77	24 63 71	56 50 38	41 67 98	26 81 19	43 97 31	14 08 52	85 14 23	11 96 35	61 47 86 38 59	07 82 38	46 89 55	03 66 63	92 44 78 29 22	1 0 6
- 6 - 7 - 8 - 9 20	90 90 61 41 58	92 28 25	84 77 77	99 99 73	45 36 71	45 11 53	79 00 3 0	20 00 05	51 83 60	94 76 58 05 38	67 83	05 67 30	60 15 20	68 23 59	84 69 30	10 59 54	08 60 79	27 02 23	22 12 59	60 49 94 75 52	55 56 78	93 25 30	13 84 07	41 14 29	3 9 3
21 22 23 24 25	99 88 19 53 40	61 28 61	29 81 62	18 63 34	05 24 47	30 04	29 96 37	08 56 40 74 70	94 11 97	33 59 09	58 36 77	88 16 36	68 01 92	42 02 80		08 36 99	11 16 26	15 26 28	96 83 24	84 06 53 24 07	15 41 54	51 09 63	48 50 43	85	9 0 5
6 7 8 9 0	65 95 75 92 43	79 26 45	58 86 48	84 16 29	86 42 84	65 56	04 03 60	73 22	69 43 64	94 68 07	89 87 71	12 68 46	93 70 35	84 09 31	88 29 18 52 16	72 92 21	62 85 80	79 94 61	66 60 25	71 98 32 30 07	65 97 31	17 44 99	54 95 58	94 69 82 07 62	5 7 0
1 2 3 4 5	14 79 61 71 76	93 37 00	51 36 51	89 14 72	07 84 62	25 94 59	25 14 18	29 96 87	18 55 82		50 05 74	48 34 04	21 47 46	47 88 24	74 62 66	61 57 39	37 73 82	03 75 30	51 02 87		87 49 41	97 18 10	63 83 53		4 4 2
6 7 8 9	26 58 45 27 57	54 33 26	81 94 48	74 97 62	22 70 10	32 96 83	45 27 63	46 26 03 45 38	40 89 30	88 63 92	30 37 48	91 57 32	46 66 46 96 79	86 16 67	52 18 26	71 78 95	42 55 90	99 78 65	36 54 07 50 93	75 98 46	12 03 09	94 46 95	11 57 58	47 09 47 67 42	8 3 2
+1 +2 +3 +4 +5	68 97 20 53 36	63 32 90	57 35 99	69 52 23	62 41 17	65 47 76	00 17 44	55 51 53 15 66	67 83 99	52 72 65	21 45 91	53 20 04	77 52 28 22 43	16 25 64	86 04 00	73 21 39	67 94 80	24 00 65	56 16 18 21 39	68 55 47	09 26 68	05 80 75	74 19 28	58 93 80 48 92	6 2 1
+6 +7 +8 +9 50	40 33 69 17 43	19 49 49	98 02 43	40 58 65	42 44 45	13 45 04	73 45 95	97 63 19 82 88	72 69 76	59 33 31	26 51 85	06 68 53		92 99 21	65	63 77 59	08 54 17	82 22 27	47 45 70 54 07	85 97 67	14 59 07	45 06 76	81 64 13	33 65 21 95 30	2 6 0

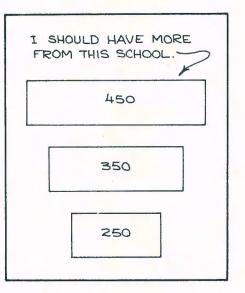
Table from STATISTICS BY EXAMPLE, Weighing Chances. Addison-Wesley Publishing Company, Inc.

140

STRATIFIED SAMPLES

Sometimes we want to be sure that certain different parts of the population are each represented in a sample. For instance, suppose we want a sample of thirty children from three different schools each of which has about 350 children. We might want the sample to have ten children from each school. We could do three random drawings of ten children, one for each of the three schools. On the other hand, to save time, we might draw just 10 random numbers choosing the child in each school corresponding to those numbers.

If the schools were of different sizes so one had about 450 children, the second about 350, and the third about 250, it might be wise to have each school represented in the sample in proportion to its size. To do this we would choose 13 from the first [because $\frac{450}{1050} \ge 30 = 12.85$], 10 from the second, and 7 from the third. This kind of a sample is called a <u>stratified sample</u>. It contains the same percent of each subgroup or "stratum" as the whole population.



Stratified samples are used extensively by the polls during elections. Pollsters want to be sure to have blue-collar workers, farmers,

blacks, Catholics, and other groups represented in their sample in proportion to their numbers in the general voting population.

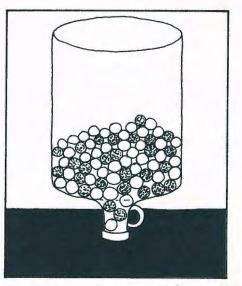
Exercises

- 13. In a school system employing 325 teachers, 150 are in district I, 100 are in district II, and 75 in district III. A committee of 10 is to be randomly selected.
 - a) Select the committee randomly from the whole population.
 - Select the committee as a stratified sample representing each district in proportion to its size.
 - c) Suppose teachers numbered 001 to 150 are in District I, those numbered 151 to 250 are in District II, and those from 251 to 325 are in District III. How are the different districts represented in the committee you selected in part (a)? Which method (a) or (b) do you prefer?

CONTENT FOR TEACHERS

THE PERILS OF SAMPLING

Is sampling a sure way of finding out the exact characteristics of the population? No, of course not. To see why, consider a rather artificial situation. Suppose we have a glass jar holding 1000 colored beads, some of them red and some of them blue. We shake the jar thoroughly (remember the difficulty of "stirring") and let 20 beads roll out through a funnel. If there were 500 red and 500 blue beads in the jar, it is possible we would get 20 red beads but the chance of this happening is very small, roughly one in a million.



Suppose we find that in the sample 12 are red and 8 blue, that is, 12/20 = 60% of the sample beads are red. Are 60% of the beads in the jar red? We are not sure so we put the beads back in the jar, shake it up again, and draw another sample. This time we get 15 red and 5 blue so 75% of the sample beads are red. If the beads are all identical except for color and if the shaking was thorough we might suspect that the jar actually has more red beads than blue ones but even so we would not be too sure of our conclusion. On the other hand, if we repeated the procedure 100 times and found that the mean number of red beads was 14.3 and the standard deviation was 2.1 we would be strongly tempted to say that about 14/20 or 70% of the beads in the jar were red. We know the result may not be exact but we can be reasonably sure we are near the mark. Furthermore, in about 95 out of the 100 cases, the number of red beads in the sample was between 10 and 18. In a few cases, we might have gone as low as 8 red ones or as high as 20.

Exercises

- 14. If colored beads or marbles and a sampling board are available, make up a population of 200 red and 100 blue marbles. Take 20 samples of 10 marbles each. Find the mean number of red marbles and the standard deviation of the number. How near is this to the 2/3 we know as the proportion in the population? Repeat using 10 samples of 20 marbles each. Repeat again using 10 samples of 25 marbles each. Compare the mean numbers obtained in the three experiments.
- 15. If no marbles are available, we can use a random number table to do a <u>simulation</u> of the experiment in Exercise 14. The ratio of red marbles to blue ones is 2 to 1. We read random numbers from the table and each time we read one of the digits 1, 2, 3, 4, 5, or 6 we say we have a red marble, while the digits 7, 8, and 9 stand for blue marbles. We ignore the 0 and continue until we have ten non-

zero digits for the first sample. Repeating this 9 times gives us the 10 samples. Draw 10 such samples 10, 20 and 25 marbles by random number simulation. Compare these results and those of Exercise 14.

- 16. How would you simulate the sampling of a population of colored beads with 80% red and 20% blue by using random numbers?
- 17. How would you simulate the sampling of a population that is 77% red and 23% blue?
- 18. If you have colored marbles, ask a friend of yours to put several cupfuls of red marbles and some different number of cupfuls of blue marbles in a bowl. Mix them thoroughly and draw 10 samples of 10. Find the mean number of red marbles. Estimate the proportion of red marbles in the population. Use samples of 20 and 25 again and again estimate! Now count the marbles in the actual situation. How close were your estimates? Which sample size gave the best results?

CHANCES IN SAMPLING

The question as to the chance that the mean of the sample is close to the mean of the population can only be answered after we have looked in the next section at the whole question of the chances of certain things happening. The concept of chances starts with relatively easy ones like the chances of getting a head when a coin is tossed, of getting a 6 when one die is thrown or getting an ace when one card is dealt from a well-shuffled deck of fifty-two cards.

Such chances are technically called probabilities. Their study is a major branch of modern mathematics. We will consider some elements of probability in the next two sections.

SAMPLE SIZE

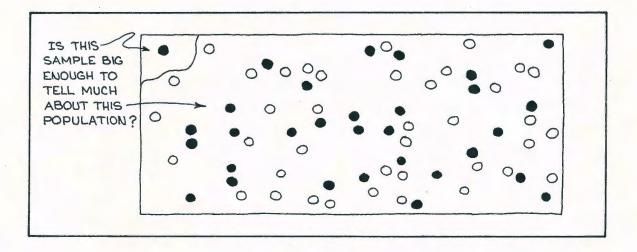
The purpose of investigating a sample is to determine something about the population without going to the expense or trouble of surveying the whole population. If we sample 3000 voters in the whole United States, can we predict the winner of an election? Would we do better with a sample of 3,000,000? If so, is the larger sample worth the added expense? In the problem about buying light bulbs (see page 1), how many bulbs should be tested out of 500 cartons?

Suppose the buyer takes the first carton of 144 bulbs, picks out 10 bulbs, and tests them by lighting them and leaving them on until they burn out. If 1000 hours is considered the minimum life for satisfactory bulbs and five of the ten bulbs burn out before that time, the buyer would surely feel that the bulbs had flunked the test and would probably reject the shipment. But suppose only one burned out? One

SAMPLING

bad bulb out of ten tested might imply the shipment is 10% defective. Perhaps the shipment should be rejected since the salesman had said only 1% were bad. But then the salesman would surely say the sample was too small. The buyer should have taken perhaps 20 cartons and tested 10 bulbs from each. Then if there were one or two bad bulbs, the buyer might decide that the shipment was living up the the guarantee and agree to accept all 400 cartons. The size of a sample is important. We will discuss it here and again in the section on Inferential Statistics.

Obviously a sample of small size does not tell us as much as a larger one. However, a representative small sample may well be better than a biased large sample. After a certain point increasing the size of a sample adds only insignificantly to our information about a large population. This fact has many times been compared to tasting soup. The cook in a small family judges the quality of a quart of soup by stirring it up and tasting a single spoonful. A hotel chef tests the quality of the soup in a ten gallon pot by stirring it up and tasting the same size spoonful.



The sample size needed to make accurate predictions is probably the least understood fact in sampling and the hardest to explain. Knowledge of it is the reason expert pollsters can make reasonably accurate predictions about the behavior or opinions of all adults in the United States by surveying what many people feel is much too small a sample. The Gallup poll regularly uses a stratified random sample of approximately 1500 people for surveys concerning either the population of one city or that of the whole country. If the result of such a poll is that 46% of the sample claim that they will vote for Jones, then it is almost certain that the ac-

SAMPLING

tual vote for Jones will be between 43% and 49%. Almost certain means if such a poll were taken with the same result, 46%, in 100 different elections, about 95 times the vote will fall in the 43% to 49% range and about five times it will fall outside that range. As the old saying goes "Nothing is certain except death and taxes" but statistical sampling does give good results with reasonable assurance.

SUMMARY

The main purpose of this section is to formulate the idea of a sample of a population, to consider representative samples and how to draw them using random numbers. We study a sample in order to draw conclusions about the population from which the sample came. How this is done and with what accuracy and reliability will be discussed in the section on inferential statistics. First we will have to look at some elementary probability.

ANSWERS TO EXERCISES

- 1. a) Sampling would be advantageous for a buyer of
 - i. sheets for a large hotel
 - ii. overshoes for the army
 - iii. microscope slides for a biology laboratory
 - b) Sampling where population is too large for a complete inspection
 - i. grass seed
 - ii. overshoes for the army
 - iii. new brand of ball point pens
 - c) Inspection is destructive in
 - i. Fourth of July firewords
 - ii. explosive caps for dynamite
 - iii. TV tubes
- Gas stations. Sampling of restrooms to determine future stops on a cross country trip. Grocery stores. In a new city to determine which store to patronize. Newspaper. Take a paper for a month to decide whether to subscribe.
- 3. One poor meal at a restaurant on a first visit makes you decide never to go there again.
- 4. If one of a dozen eggs is bad, I wouldn't want to depend on the others, but one bad apple in a barrel wouldn't make me throw the whole barrel away.
- 5. A first visit to a new family doctor is a sample of size 1. It is the basis on which many choose their doctor. Recovering successfully from a bad skid in a car may be a sample of what can happen but it would be foolish to generalize that one can recover from all bad skids.
- 6. Ford Motor Company's prediction that the Edsel would be a big success.
- 7. Maybe doctors at 10 hospitals did not do so. The ad says only what doctors at 3 hospitals said. The ad does not say how many doctors or what percent of the doctors favor XYZ. Maybe a sample of 1 doctor at each hospital is all that prescribe XYZ.
- 8. If the selections were truly random, each digit would appear about the same number of times. If the people you ask are like most people, they will have picked digits near 5 more often than those near 0 or 9.

9.	The position of the sy changed after ten spir the frequency of the the second ten and the given below. The spir random. The bias char tion of the spinner we	ns. Th first t e total nner is nges af	e tabi en spi of tw not i ter th	ins, fo venty : really	is		958 980 198 865 864 831 791 936 867 519			485 346 549 142 495 805 319 261 076 596	5 9 2 1 5 9 1 5
	Digits	0	1	2	3	4	5	6	7	8	9
	Frequency 1st 10	1	4	0	2	1	3	4	2	7	6
	Frequency 2nd 10	2	4	2	2	5	4	4	1	2	4
	Frequency Total 20	3	8	2	4	6	7	8	3	9	10

In the first 10 trials, 22 children numbered 5-9 were chosen and only 8 numbered 0-4. In the second trials, there were 15 in each set. Numbers 2 and 7 were low each time.

- 10. Read 3-digit numbers. If the number is in the series 001 to 325 the corresponding teacher is selected for the committee. Ignore all other numbers. Continue until 10 teachers are selected. One such series is 308 (395) (426) (542) (861) 225 (559) (490) (828) 221 290 (485) 141 (653) 299 (720) (994) 284 (336) 122 134 200.
- 11. Book used had 222 pages.
 - a) Select three pages. Read 3-digit numbers between 001 and 222. (651) (518) 135 (405) 136 (991) (518) (457) (528) (988) 125
 - b) Select one of 4 columns. Read single digits. Ignore 5, 6, 7, 8, 9, 0. (6) (0) (5) 1
 - c) There are about 100 names to a column. Read 2-digit numbers. <u>43</u>. The three selections are McAfer, McCormack and Leber.
 - d) Repeat the process. Page. 141 (620) (798) (774) (549) (670) 029 (478) (814) (927) (505) 073. Column. This time we pick columns for each page. 0 8 2 6 6 3 1. Name. Again we pick 3 numbers, one for each page: 20 08 26. We happened to get them in the first three pages. The three subscribers are: p. 141, c. 2, 1. 20, Millins; p. 029, c. 3, 1. 08, Barnes; p. 073, c. 1, 1. 26, City Hall, Public Works. No subscriber was chosen twice.
- 12. Use 3-digit random numbers. 001-500 for girls, 501-925 for boys. 051 369 915 184 575 289 881 252 033 (971) 194; 7 girls and 3 boys. Repeat: 892 419 234 606 016 865 282 390 281 090. Again 7 girls and 3 boys. Repeat: 166 609 762 008 266 631 638 141 620 798. This time 4 girls and 6 boys.
- 13. a) Select numbers 001-325. 082 663 163 814 162 079 877 454 967 002 947 881 492 750 507 346 272 704 753 883 976 920 305 319 543 562 282 815 009. Teachers 002, 009, 079, 082, 162, 163, 272, 282, 305 and 319 are selected.
 - b) 150/325 = .46, 100/325 = .31, 75/325 = .23. We select 5 teachers from District I, 3 from II and 2 from III. It seems as though III is over-represented. Probably a stratified sample would make the teachers in I and II happier.
- 14. Random number simulation of drawing marbles. $\frac{2}{3}$ red and $\frac{1}{3}$ blue. 1-6 red, 7-9 blue.

SAMPLES OF 10

6 R 8 R 5 R 5 R 7 R 7 R 7 R 7 R

604052,	872299
87124,	436036
140662,	470339
77399 6	33067
9180302	, 487029
117106,	13393
649058,	197049
44293,	832096
210926,	081784
902114,	24892

Mean number in 10	
trials is 6.5 out	
of 10, frequency	
65% ₂ red. Very close	Ð
to $\frac{2}{3}$.	

CONTENT FOR TEACHERS

SAMPLING

SAMPLES OF 20

9960505, 0601248, 0801886, 051733	13 R	
85447, 41674, 38421, 35129	16 R	
0438861, 2918, 053129, 56943	14 R	Mean number in 10
35888, 68421, 0081115, 960615	14 R	trials is 14.8 out
51483, 99619, 28816, 324309	13 R	of 20, frequency
640115, 93616, 01026036, 16268	18 R	74% red. Not as
35341, 09508501, 53616, 23447	17 R	close as before
043774, 970977, 36928, 045992	9 R	
62824, 24546, 34363, 5440034	19 R	
SAMPLES OF 25		
73566, 338401, 666097, 6200826, 63163	20 R	
81416, 207987, 74549, 6700294, 78814	13 R	Manager in 10
927505, 073462, 727047, 53883, 97692	14 R	Mean number in 10
0305319, 54356, 22828, 1500924, 16379	19 R	trials is 17.3 out
79937, 907687, 44347, 64594, 839607	12 R	of 25, frequency

 79937, 907687, 44347, 64594, 839607
 12 R
 of 25, frequency

 46219, 73837, 110575, 437207, 36664
 19 R
 69%. 2

 34391, 057131, 595095, 94376, 46461
 19 R
 to 2

 12498, 3040541, 497307, 47938, 12816
 16 R

 107387, 08071025, 013898, 14198, 4088309
 12 R

 5500266, 99442, 33899, 047369, 67522
 17 R

 36184, 89189, 279075, 51399, 88485
 12 R

16. Use digits 1, 2,...8 for red and 9, 0 for blue.

17. Use 2 digits numbers 01-77 for red and 78-00 for blue.

18. Depends on mixture.

ORGANIZATION

Placement Guide for Tabbed Dividers

Tab Label	Follows page:
CONTENT FOR TEACHERS	
Gathering Data	
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Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
Probability with Counting	
Inferential Statistics	
Appendix-Computer Programs	
DIDACTICS	
Components of Instruction	
Classroom Management	
Statistics and Probability Learning	
TEACHING EMPHASES	
Critical Thinking	
Decision Making	
Problem Solving	
Models and Simulations	
Calculators and Computers	
Laboratory Approaches	
CLASSROOM MATERIALS	
CONCEPTS AND SKILLS	
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Business & Commerce	
Environment	
Health & Medicine	
People & Culture	
Recreation	
GLOSSARY	
ANNOTATED BIBLIOGRAPHY	
SELECTED ANSWERS	

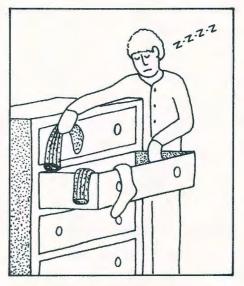


We talked about "chances" in the last section. Now we will investigate this idea a bit further. More formally, chances are called probabilities. What do we mean when we talk about the chance of getting two heads when two coins are tossed? How can we determine the probability that on the throw of two dice we get a sum of 9? These questions are put in terms of coins and dice because the study of probability began in response to inquiries from gamblers. It had been observed that the chances of getting two heads, one head or no heads seemed to be different. At least a bet on one head seemed to win more often than a bet on either of the other possibilities. Keeping track of the throws of two dice seemed to indicate a sum of 7 was more common than a sum of 9.

Perhaps keeping records of the way a certain event has happened in the past is a good way of estimating the chances of its happening the same way in the future. Suppose we have tossed two coins a thousand times and have gotten two heads 275 times. This is a little more than one-fourth of the time. Perhaps a good estimate of the chances of getting two heads when tossing two coins is about one-fourth. This is an example of estimating a chance empirically or experimentally. Sometimes such experimental results agree with our intuitive judgment and sometimes not. Sometimes we have no basis for any judgment except the experimental results.

Consider the following situation. When Joe Bright goes off to college, his mother buys him six pairs of red and six pairs of blue socks. After a couple of weeks, Joe can't be bothered pairing up his socks after laundering them and just throws them in the drawer. When he gets up he is sleepy and tired. He reaches in and grabs two socks without bothering to look at their color. How can we determine what the chances are that Joe gets a pair of the same color? If you guessed now what would your guess be?

Soon the blue socks begin to wear out faster than the red. There comes a day when the drawer



has only 10 red socks and 5 blue ones. How can we now determine what the chances are that Joe picks a pair? One way of estimating the chances in each case is just to watch Joe for a couple of weeks and see how he makes out. Does this seem reasonable? Would you get a better estimate if you watched for a week or if you watched for a month? The object of this section is to devise experiments to estimate probabilities such as these without having to watch Joe even for a week. To begin with, let's look at some examples where it is easier to see what happens.

EXPERIMENTS WITH SINGLE OUTCOMES

TOSSING A COIN

We toss a coin and use the results to estimate the chance of the coin falling head-up.

A coin has two sides, a head and a tail. When the coin is tossed it lands on the table either head-up or tail-up. It could stay on its edge but this happens rarely, if ever, and we ignore that possibility. The following results come from an actual series of trials.

First trial: A single toss. Result: T. That certainly does not tell us much. Try 10 tosses.

Second trial: 10 tosses. Result: HTTTT HTHTH

Look at the ratio of the number of heads to the number of tosses. Call this the relative frequency of heads and record it as f(H/N) = (number of heads)/(number of tosses)where H stands for the number of heads and N the number of tosses. This fraction is always some number between 0 and 1 inclusive since the number of heads must be greater than or equal to 0 and less than or equal to the number of tosses. In the first trial, the frequency of heads in one toss was 0 so f(H/1) = 0/1 = 0. In the second trial, f(H/10) = 4/10 = .4. This relative frequency is a preliminary estimate of the chance of getting a head on the toss of one coin. Is this estimate a good one? How could we make it better? Consider two possibilities:

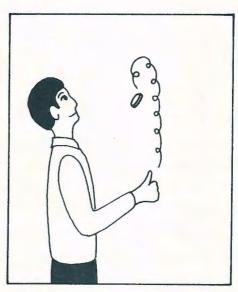


TABLE 1

100 TOSSES OF A COIN

HTHTH

THTTT

HTHHH

THHHH

TTHHT

HTHHH

TTHHH

THTHH

THHHH

HHHTH

HTTTT

HTHHH

HTTTH

TTHTH

TTHTT

THHTT

HHTHT

TTHHH

THTHH

First, make many more trials, say 50 or even 100 or 1000, and determine f(H/N). J.E. Kerrich tried this while he was interned during World War II. One of his results was 511 heads in 1000 tosses for f(H/1000) = .511.*

Second, make 100 trials and, instead of considering them all at once, consider them as a series of 10 repetitions of 10 tosses each.

The results for 100 trials are given in Table 1. Notice several interesting facts. The heads and tails do not alternate. Several runs of 3 heads and even two runs of 5 heads in a row occurred. The successive relative frequencies, f(H/10), are .4, .5, .6, .6, .3, .6, .5, .6, .7, .7. On the other hand, all the trials could be combined to give f(H/100) = .55. We could also compute the cumulative relative frequencies:

$f(H/10) = \frac{4}{10} = .4$	$f(H/50) = \frac{24}{50} = .48$	$f(H/80) = \frac{41}{80} = .5125$
$f(H/20) = \frac{9}{20} = .45$	$f(H/60) = \frac{30}{60} = .5$	$f(H/90) = \frac{48}{90} = .533$
$f(H/30) = \frac{15}{30} = .5$	$f(H/70) = \frac{35}{70} = .5$	$f(H/100) = \frac{55}{100} = .55$
$f(H/40) = \frac{21}{40} = .525$		

An important fact to notice is the variability of the relative frequency both in the individual series of 10 tosses and in the cumulative series. One might feel relative frequency is a poor way to estimate the chances of getting a head. We think there is a certain definite probability of the coin landing heads up. We may not know what it is, but we hope we can make an estimate of it by counting the relative frequency of heads in an experiment of tossing a coin many times. It is true probability is sometimes tough to estimate by experiments. However, at times experiments are the only way to get at an unknown probability. One of the most valuable lessons we can learn is that such estimates are not exact. Pollsters may use polls to estimate the probability of Mr. A being elected is .52. Many people forget that the estimate may be off by perhaps .03 and Mr. A may end up with only 49% of the vote and be defeated or with 55% and be elected by a "landslide."

^{*}Mosteller, Rourke and Thomas, *Probability with Statistical Applications*, 2nd Ed. (Reading, Mass.: Addison-Wesley Publishing Company, Inc., 1973), p. 17.

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Based on 100 tosses, our estimate of the chance of getting a head with this particular coin is .55. It may not be the exact chance and our estimate may change if we increase the number of tosses but for now we believe the chance is somewhere near .55.

TOSSING A DIE

Roll an ordinary die and check whether you get a five (.) or not. If you get a five, record the result with a l. If not, record a 0. These results were obtained on 6 series of 10 rolls each. The relative frequencies for each series vary from 0 to .3. The mean is .20. This is the same result as the relative frequen-

00000	00000	
10010	00001	
00000	00010	
00101	00000	
00110	01000	
10100	00100	
	and the second se	

cy in all 60 rolls since there were 12 successes in the 60 rolls and $\frac{12}{60} = .20$. It is the experimental estimate of the probability of rolling a five with this particular die. The estimate might change if the die were rolled 100 times or 1000 but .20 is the best estimate with these 60 rolls.

TOSSING A THUMBTACK

Many people feel intuitively the chance of a tossed coin landing head-up is or is near $\frac{1}{2} = .5$. Similarly we may feel the probability of rolling any particular score such as a five (\bigcirc) with a die is about $\frac{1}{6} \approx .17$. In the experiments above with a particular coin and a particular die, the relative frequencies and therefore the estimates of the probabilities came out near these values. For the coin it was .55 and for the die .20.

Now we look at a situation where we have no idea what the chances are. Consider a thumbtack. A thumbtack when tossed on a table can land in two positions: point down, D, α , or up, U, -. Can we estimate the probability it will land point down? We use the notation P(D) for this where P stands for probability and D for down. Using this notation the results of the previous two paragraphs could be written:

> For the coin $P(H) \approx .55$ For the die $P(Five) \approx .20$

We might toss one thumbtack 20 times or we can get similar results by tossing 20 of them at once and counting the number that fall in position D. We thus determine f(D/20) and repeat the experiment to get 25 results. We tally the 1

2

3

5

To

The

results, compute the 25 relative frequencies and finally determine their mean. You might do this and compare your results with ours. They probably will be quite different as the thumbtacks you use may not be the same as ours.

The results for 25 trials were as follows. The number of thumbtacks out of 20 that fell D were

8, 7, 5, 6, 6,	4, 3, 5, 6, 2,	7, 6, 2, 9, 3,
4, 4, 1, 6, 7,	4, 4, 5, 3, 6.	
find the relative frequencies,	divide by 20 getting:	
40, .35, .25, .30, .30, .20,	.15, .25, .30, .10, .35,	.30, .10, .45, .15
20, .20, .05, .30, .35, .20,	.20, .25, .15, .30.	
ese numbers can be grouped and t	allied as follows:	
.05 .10 .15 .	20 .25 .30 .35	.40 .45

As in the coin tossing experiment, note the great variability in the results. Nevertheless the mean of these numbers is .246 and we estimate $P(D) \approx .25$. A histogram of the relative frequencies is given in Figure 1. We will go into more detail about this kind of experiment in *Inferential Statistics* of the CONTENT FOR TEACHERS section.

3

6

3

1

1

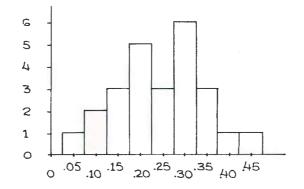


Fig. 1. Histogram of Relative Frequencies of D

This method of counting relative frequencies in an experiment has enabled us to make an estimate of a probability we had no other way to determine. We call the result an empirical or experimental probability.

CHOOSING A MARBLE

Suppose we have a bag of 5 marbles all alike except for color. There are 3 red and 2 blue ones in the bag. If we mix the marbles up thoroughly and pick one marble at random, what is the chance of picking a red marble? We did the experiment 100 times and picked a red marble 63 times. The relative frequency is 63/100 = .63 and is our first estimate of the chance of picking a red marble. Repeating the experiment we got 62, 52, 60 and 65 red ones. The mean of the resulting relative frequencies is .604. This is our estimate of the probability. Using this estimate would mean that if we drew 1000 times we would expect to get a red marble about 600 times.



Exercises (Answers given on p. 158)

- 1. Toss a nickel 10 times, count the number of heads and record the relative frequency, f(H/10). Do this 10 times. Determine f(H/10) each time. Find the mean of these 10 numbers. Is it the same as f(H/100)? What is your estimate of the probability of getting a head with your particular nickel?
- 2. Take a small piece of adhesive tape and fasten it over the head of the nickel you used in Exercise 1. Now repeat the experiment and give an estimate of the chance of getting a head with this weighted coin.
- 3. Put 3 red and 4 blue marbles in a bag. Draw a marble, note its color, put it back, shake the bag, draw again and continue until you have made 10 draws. What is the relative frequency of getting red? Do this ten times, find the mean and estimate the probability of drawing a red marble.

EXPERIMENTS WITH MORE THAN ONE OUTCOME

TOSSING TWO COINS

Toss two nickels. Count and record the number of heads. We can consider three possibilities. We get either two heads or one head or no heads at all. Would you expect the chances to be the same for each of these three? Let's look at a series of trials. Here is our record of an actual series of 70 tosses where 2 means



we get 2 heads, etc. On the first toss, we see no heads and record a 0, in the second toss one head and record a 1, etc. The three possibilities do not seem to occur with the same frequency. Most of us would say the chances of getting each of them are not the same. The relative frequency of two heads in the first series of 10 tosses was 1/10, in the second 4/10, and in the whole

01211	10110
02010	22120
21201	10122
20021	11111
01010	21111
01111	00111
10010	10120

series of 70 tosses 13/70. These results are not at all close together and we should be reluctant to say anything except that the chances seem to be less than 1/2. In fact, twice the relative frequencies were less than 1/5. What about the relative frequency of two tails? This is the same as no heads and it is 3/10, 4/10 and 22/70 in the three series of trials. Again all three are less than 1/2 and twice they are less than 1/3. Many more trials would have to be made to give us a clearer idea as to what the chances of two heads, one head or no heads really are. We will come back to this in *Probability With Models* of the CONTENT FOR TEACHERS section.

Consider a bag of 3 red and 2 blue marbles. This time after shaking the bag draw 2 marbles at once. There are three possible results. You could get 2 red marbles, 1 red marble or 0 red marbles. Are the chances for each the same? We do the experiment 100 times. We get 2 red marbles 34 times. Since the relative frequency is 34/100 our first estimate of the probability of getting 2 red marbles is 34/100 = .34. This is about 1/3. Could it be that the chances for 1 red marble and 0 red marbles are also about 1/3? In the 100 trials we got 1 red marble 49 times and 0 red marbles 17 times. The chances seem far from one third. We repeat the experiment with the results in Table 2.

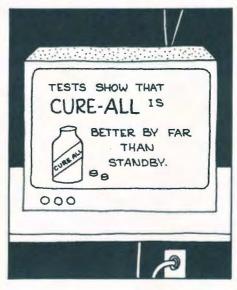
		TABLE 2		
		DRAWING 2 MARBLE G 3 RED AND 2 BL		Carlo Maria Para Second
	2 Red	1 Red	0 Red	Total
1st	34	49	17	100
2nd	34	53	13	100
3rd	44	41	15	100
4th	32	49	19	100
5th	30	59	11	100
Total	174	251	75	500

Again note the variability of the results from one series of 100 experiments to the next. For 2 red, the relative frequency varies from .30 to .44. For 500 trials it is $\frac{174}{500} = .348$. For 1 red, it is .502 and for 0 red, .15. Our estimates are that P(2 red) \approx .35, P(1 red) \approx .50 and P(0 red) \approx .15. We feel the real chances are somewhere near these figures, but probably do not equal them exactly. More trials would change them again. However, we do feel confident that P(0) < P(2) < P(1) rather than P(0) = P(1) = P(2) = $\frac{1}{3}$.

CONCLUSION

Estimating probability by experiments is time consuming. But it is worth doing because it helps us not only to know what to expect in general but also to realize how varied the results of an experiment may be. Sometimes the results of an experiment are the only way to estimate certain probabilities. Sometimes they may contradict our intuitive expectations. For example, suppose Al and Bob each roll an octahedral die with its faces marked with the digits from 1 to 8. A score is obtained by dividing the number Al rolls by the number Bob rolls and taking the first digit of the result. Thus if Al rolls a 7 and Bob a 5, the score is 1 since 7/5 = 1.4 and the first digit is 1. If Al rolls a 5 and Bob a 7, $5/7 \approx .71$ and the score is 7. Al wins if the score is 1, 2 or 3 and Bob wins if the score is 4, 5, 6, 7, 8 or 9. What are the chances that Al wins? Intuitively, we might feel that his chances are much less than Bob's but running the experiment a few times will soon convince anyone that the contrary is true.

Another reason an experimental approach to probability is important is that it may be the only way to estimate a probability vital in making an important decision. A drug manufacturer may claim a new wonder drug CURE-ALL is better than the old drug STANDBY. The Food and Drug Administration runs experiments to determine the probability that CURE-ALL is more effective than STANDBY. This is not a simple process but it is an application of experimental probability.



Exercises

- 4. We can tackle Joe Bright's problem not by watching him for a month but by making a model of the situation. Put 12 red and 12 blue marbles in a bag, draw two, note the color, and record a 1 if they are the same color, and a 0 if they are different. Repeat as many times as you want and note the relative frequency. It is your estimate. To find the chance of a pair in the second case, how many red and how many blue marbles would you put in the bag? Do this and find an estimate of the chance of getting a pair. Do your estimates agree with the guesses you made at the start of this section?
- 5. Just for fun: Instead of just picking the first two socks, Joe keeps pulling socks until he gets a pair. How many socks should he pull to be sure the chance of getting a pair is 1? In the second case above, how many socks must he pull out to be sure he gets a blue pair?
- 6. In rolling two dice the score consists of the sum of the two numbers rolled. There are several possible scores. We want to estimate the chances of rolling scores of 4, 7 and 10. Do you think the relative frequencies obtained in six trials would be enough to make good estimates? Do the experiment 20 times and estimate the chances requested. Would 100 trials give you better estimates? How many trials would be needed to find the probabilities exactly?
- 7. Have a friend put a few marbles (less than 20) in a bag, some red and some blue. We want to estimate the probability of drawing a red marble. Draw one marble, record the color, return the marble to the bag and shake the bag thoroughly. Repeat for ten trials. What is the relative frequency of drawing a red marble? Estimate P(R). Repeat for a second series of 10 trials. Repeat until you have a total of 100 trials. What is the final estimate of P(R)?
- 8. Use the same bag of marbles as in Exercise 7. This time draw two marbles and note whether you get 2, 1 or 0 red marbles. Return the marbles to the bag and repeat the experiment 100 times. Estimate P(2), P(1) and P(0) after each series of 10 trials. Are you more confident in your estimates of these probabilities after 10 trials or after 100? Why?

CONTENT FOR TEACHERS

ANSWERS TO EXERCISES

Since these exercises involve experiments, your results will probably differ from those of anyone else including the following.

- 1. f(H/10) = 3, 5, 3, 4, 4, 2, 6, 5, 5, 6 Mean = 4.3 Yes, it is = f(H/100) P(H) \approx .43
- 2. f(H/10) = 2, 2, 5, 4, 3, 6, 4, 4, 5, 4 = 39. P(H) = .4 not much change
- 3. We got 4, 3, 2, 4, 3, 7, 4, 3, 5, 3 with a mean of 3.9. Probability of getting a red marble is about 0.4.
- 4. a) Results are those of your own experiment.b) Put 10 red and 5 blue marbles in the bag and conduct your experiment.
- 5. a) 3 b) 12 (He might pull all the red ones first.)
- 6. You cannot find the exact probabilities by any number of trials. The best you can do is make a good estimate and it may take many trials to do even that.
- 7. Results depend on how many marbles of each color are put in the bag.
- 8. Results depend on how many marbles of each color are put in the bag.

ORGANIZATION

Placement Guide for Tabbed Dividers

Tab Label	Follows page:
CONTENT FOR TEACHERS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
Probability with Counting	
Inferential Statistics	
Appendix-Computer Programs	
DIDACTICS	
Components of Instruction	
Classroom Management	
Statistics and Probability Learning	
TEACHING EMPHASES	
Critical Thinking	
Decision Making	
Problem Solving	
Models and Simulations	
Calculators and Computers	
Laboratory Approaches	
CLASSROOM MATERIALS	
CONCEPTS AND SKILLS	
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GLOSSARY	
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SELECTED ANSWERS	



INTRODUCTION

In Experimental Probability of the CONTENT FOR TEACHERS section, we did some experiments to estimate the chances of certain simple events happening. Sometimes the results were about the same as our intuitive estimates. In this section, we investigate the circumstances in which this may happen. We examine how to combine intuition and experiment to find the chances involved in more complicated events built up as combinations of simple ones. Such

problems could be:

• What is the probability a professional baseball player whose batting average is .312 will get three or more hits in the five times he is at bat Friday?

• If the probability is .05 that a machine makes a defective car wheel, what is the probability at least one of the next five wheels is defective?

• If 20 fair coins are tossed, what is the probability they will fall with 8 or more heads? with from 8 to 11 heads? with exactly 8 heads?

Before we can tackle such problems we need

to think about simple chances or probabilities from a slightly different point of view than the experimental approach we considered before. We will make mathematical models and use them to study the real life situations.

FAIR COINS AND TRUE DICE

Consider the question of the chance of getting a head when tossing a coin. Intuitively, we feel the chance for a head with a fair coin is $\frac{1}{2}$. In the experiment in Experimental Probability of the CONTENT FOR TEACHERS section, the relative frequency in 100 trials was $\frac{55}{100}$ = .55. This was the estimate of the chance of getting a head on any one toss of this particular coin. Was this a fair coin? What is meant by saying a coin is "fair"? We will call a coin fair if when it is tossed many times it falls about as many times heads as tails. That is, in N tosses we would get about $\frac{1}{2}$ N heads. The expected relative frequency of heads is then $(\frac{1}{2}$ N) \div N which is $\frac{1}{2}$. For a fair coin the probability of getting a head on any one toss is $\frac{1}{2}$. By the same argument, the probability of getting a tail is also $\frac{1}{2}$.

Of course, a coin may be bent or worn as perhaps ours was since we got a few more heads than tails. But for a fair coin over the long run, we would expect about as many heads as tails.

What do we mean by a true die? A die is true if the chance of rolling any face such as a [] is the same as that of rolling any other face such as [] or []. Since there are 6 faces, a true die rolled 1200 times would result in about 200 of each face. We think the chances for each face then are about $\frac{200}{1200}$ or $\frac{1}{6}$. For a "true" die the probability is exactly $\frac{1}{6}$ but we recognize that in any experiment it would be very unlikely to get exactly 200 of each face in 1200 rolls. In the experiment in *Experimental Probability* in the CONTENT FOR TEACHERS section the estimate was .20, not too far from $\frac{1}{6} \approx .167$ but not as close as it might have been if we had rolled the die 1200 times instead of only 60 and if the die had been an expensive precision one instead of one from the local variety store.

In general, if a random experiment has n possible outcomes and we have no reason to expect any one outcome to occur more often than another, then in a large number of trials, say 1000, we would expect each one to occur $\frac{1000}{n}$ times. Therefore, the chance of any particular one would be $(\frac{1000}{n}) \div 1000$ or $\frac{1}{n}$. If the results of the experiment are far from this value we suspect that our intuition is wrong and we look for reasons. This might happen in an experiment of drawing 2 marbles from a bag containing 3 red and 2 blue marbles. There are three possibilities, either 2 red, 1 red or 0 red and because of the symmetry we might expect the chance of each to be $\frac{1}{3}$. But an experiment even with relatively few trials shows that the relative frequency of 1 red is much higher than that of either 2 red or 0 red.

Examples

- a) In tossing a fair coin, there are 2 equally likely cases and we assume $P(H) = P(T) = \frac{1}{2}$ where P(H) stands for the probability of getting a head.
- b) In rolling a true die there are 6 equally likely cases and $P(\bullet) = P(\bullet) = P$
- c) There are ten digits and in a random number table $P(0) = P(1) = ... = P(9) = \frac{1}{10}$ where P(n) is the probability that the digit selected is n.
- d) There are 100 two-digit numbers 00, 01, ..., 99 and in a random number table $P(\text{each one}) = \frac{1}{100} = .01.$

e) In rolling 2 dice there are 11 possible outcomes: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. Children may think these are equally likely. An experiment with only 30 rolls gave the following frequency distribution.

Roll	2	3	4	5	6	7	8	9	10	11	12
Frequency	0	2	2	4	5	7	3	3	3	0	1

The chances of rolling a 2 or a 12 seem much lower than those of rolling a number near 7.

SINGLE STAGE TREES

TREES FOR ROLLING A DIE

Example 1

Suppose on the roll of a die we are interested not in the exact number that shows up but simply whether the number is even. We could do an experiment 50 or 100 times to estimate the chance but let's use our intuition assisted by a model that turns out to be very helpful in many problems. We draw a "tree" to represent what can happen as the result of a single roll. Since the roll can have six different results, the tree has six "branches" each labeled with one of the results. See

Figure 1. For a true die we expect the rolls to be divided evenly among the six faces. The chance for each face would be $\frac{1}{6}$. This chance is associated with each branch and we label each of them with that number.

To identify any particular branch or branches, mark it or them heavily on the tree. The branch leading to 4 is so marked in Figure 2. Note that $p(4) = \frac{1}{6}$. If you rolled the die 1200 times, you would expect to roll a 4 about 200 times.

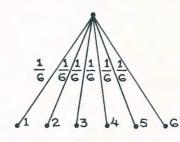


Fig. 1. Tree for One Roll of a Die

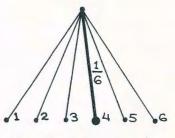


Fig. 2. Rolling a 4

CONTENT FOR TEACHERS

In this example, we are interested in the chance of rolling an even number. The even numbers that might be rolled are 2, 4 and 6. In 1200 rolls each should turn up about 200 times, so an even number will be rolled about 600 times. The chance of rolling an even number is about $600/1200 = \frac{1}{2}$. On the tree we mark the appropriate branches.

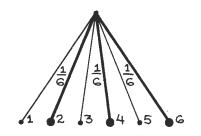


Fig. 3. Rolling an Even Number

Of course, on one roll of a die we cannot get more than one number. But the event of rolling an even number occurs if we get either a 2 or a 4 or a 6. We note that:

$$P(even) = \frac{1}{2}$$
.

How could we use the tree to determine this probability? We observe:

$$\frac{1}{2} = \frac{3}{6}$$
$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$$
$$= P(2) + P(4) + P(6)$$
$$= P(2 \text{ or } 4 \text{ or } 6)$$

If the event includes more than one branch, add the probabilities of each branch involved.

Example 2

In the same way the probability of rolling a number divisible by 3 is the probability of either a 3 or a 6. Experimentally, we think in a large number of rolls each one of these will happen about $\frac{1}{6}$ of the time and the chance for each is about $\frac{1}{6}$. The tree diagram has two branches marked with probability $\frac{1}{6}$. As before the probability of one or the other of these branches is the sum of the probability for each one.

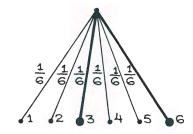


Fig. 4. Rolling 3 or 6

 $P(3 \text{ or } 6) = P(3) + P(6) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$.

TREES FOR DRAWING A MARBLE FROM A BAG

Example 3

Suppose a bag has 7 marbles in it, 5 red and 2 blue. If we mix the marbles up thoroughly and pick one at random each one will have the same chance of being drawn. In a large number of draws the relative frequencies of each will be about the same, $\frac{1}{7}$. We assume the probability of drawing each marble is $\frac{1}{7}$. Here is the tree: What is the chance of drawing a red marble? There are five branches that lead to a red marble and therefore

 $P(R) = \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} = \frac{5}{7}$.

What would P(B) be? There are two branches leading to blue.

$$P(B) = \frac{1}{7} + \frac{1}{7} = \frac{2}{7} .$$

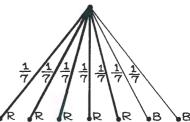


Fig. 5. Tree for 7 Marbles in a Bag

In general, if an experiment has n equally likely outcomes and the event E includes i of these outcomes, $P(E) = \frac{i}{n}$.

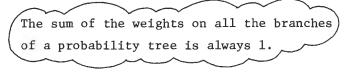
In the tree, n is the number of equally likely branches and i the number of branches that lead to E. So i is always less than or equal to n. This gives us the important result.

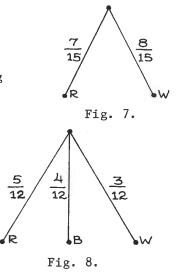
$$P(E) = \frac{i}{n} \leq 1$$
. The probability of any event is less than or equal to 1.

If we asked what is the probability the marble is either blue or red, the event would include <u>all</u> the branches of the tree. $P(B \text{ or } R) = P(B) + P(R) = \frac{2}{7} + \frac{5}{7} = 1$. This illustrates the idea that when anything is certain, we can say its probability is 1. If we draw a marble, it must be either blue or red since there are no other kind in this bag.

When n gets large, the tree of equally likely outcomes will have too many branches to draw easily. Since in this example we are interested only in whether the marble drawn is red or blue we could think of a new tree with just two branches marked R and B. But how do we label the branches? Think back to the first tree. In that example, $P(R) = \frac{5}{7}$ and $P(B) = \frac{2}{7}$. So this tree is labeled as in Figure 6. This is called weighting each branch of the tree with the probability belonging to that branch. R Fig. 6. Tree for Marbles It is nice to observe that one tree like this can be used for many different experiments if the branches are labeled properly. Thus if the bag has 15 marbles, 7 red and 8 white, the tree would be as in Figure 7. If the bag had 5 red, 4 blue, and 3 white, the tree would have three branches and be labeled as in Figure 8.

We now note the second important fact about probabilities.





TREES FOR A SPINNER

Example 4

Consider the three spinners in Figure 9. In each case, what is the chance of the pointer stopping in region 1?

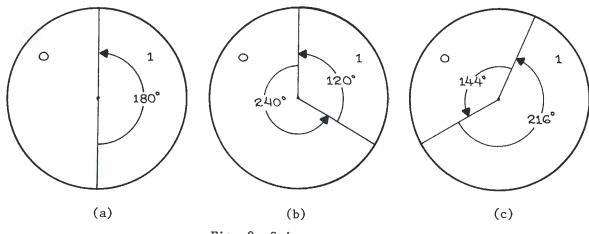


Fig. 9. Spinners

If each spinner is well balanced and spins freely, what would you expect to happen in each case? For (a) we would expect the pointer to stop in region 1 about $\frac{1}{2}$ of the time, for (b), about $\frac{120}{360}$ or $\frac{1}{3}$ of the time, and for (c), about $\frac{3}{5}$ of the time. We cannot draw a tree of n equally likely outcomes except for (a) when n = 2, but it does seem right to draw trees like those below. The branches would be weighted according to the chances we decided were appropriate. Here are the three diagrams:

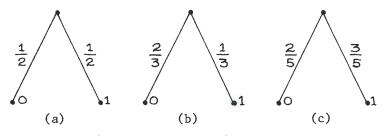


Fig. 10. Tree Diagrams for Spinners

MODELS USING RANDOM NUMBERS

Example 5

If we use a random number table to pick a single digit number, the probability of picking any given digit is $\frac{1}{10} = .1$. The tree of this experiment has 10 branches each labeled .1. Suppose we are interested in the event, E, that when we pick a digit randomly we get one of the set {1, 3, 5, 7}. Since each digit has probability .1 of being picked, in 1000 trials

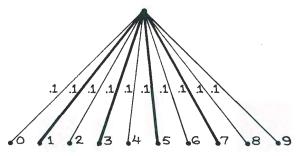


Fig. 11. Tree for Random Digits

we expect to get about 100 of each digit or 400 all together. The chance of getting one of the set is 400/1000 or .4.

$$P(E) = P(1 \text{ or } 3 \text{ or } 5 \text{ or } 7) = .4 = .1 + .1 + .1 + .1$$

= $P(1) + P(3) + P(5) + P(7)$.

If we mark the branches of the tree corresponding to the event E as in Figure 11, then P(E) equals the sum of the probabilities of all the branches that make up E.

Example 6

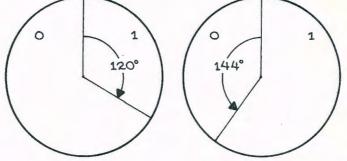
Sometimes a tree is not useful as a model. To model the selection of a single student at random from a class of 25 by a tree means using a tree of 25 branches. It is so big as to be useless. In *Sampling* in the CONTENT FOR TEACHERS section a random number table was used to select random samples. The model here consists in

assigning a number to each member of the class and using a random number table to make the selection of a single student from the class of 25. How can we use a random number table to do this? As we saw before, in the random number table, each twodigit number 01, ..., 99, 00, has the chance 1/100 of being chosen. How do we change these chances to 1/25 as we want? Here are two ways to do it. The first might be to assign each student 4 of the two-digit numbers. Thus student #1 would get 01, 02, 03 and 04 while #2 would get 05, 06, 07 and 08, etc. Each student would have the chance 4/100 = 1/25 of being chosen. This is fine since 25 goes evenly into 100 but suppose there had been 27 students in the class. A second method is to assign each student just one number. Thus #1 would get 01, #2 would get 02, etc. Only 27 two-digit numbers would be assigned. When we use the random number table to pick a number, if we pick one of these numbers, fine, that student is selected. But if we get any other number in the table we simply ignore it and pick again until we get one of the twenty-seven numbers that have been assigned. Since each of the twenty-seven assigned numbers has the same chance of being picked as any other, the chance of each one is 1/27 as we wanted. This method will work for a class of any size.

Exercises (Answers given on pp. 178-179)

- In rolling a die we win if we roll either a
 or a
- 2. In a class of 15 students there are 6 girls and 9 boys. Among the girls there are 4 red heads and among the boys 3 red heads.
 - a) Draw a 15-branch tree and label each branch appropriately with one of the labels GR, BR, GR, BR where GR stands for a red-headed girl, GR for a girl whose hair is not red and correspondingly for boys.
 - b) Draw a two-branched tree for girls and boys and another two-branched tree for red heads and those without red hair. Weight and label each branch properly.
 - c) Draw a four-branched tree with branches labeled GR, BR, GR, BR. Weight and label each branch properly.
 - d) If a class speaker is chosen at random, what is the chance:
 - i) the speaker is a girl?
 - ii) the speaker has red hair?
 - iii) the speaker is a red-haired girl?
 - e) If only redheads can be chosen as speakers and only children without red hair as singers, what is the chance the speaker will be a girl? What is the chance the singer will be a boy?

3. How would you use a random number table to simulate a spin of each of the spinners below?



- 4. A number is selected at random from the set {1, 2, 3, ..., 50}. How many branches are in the tree of equally likely outcomes? Don't draw the tree but determine how many branches lead to each of the following outcomes. Then determine the chance of drawing
 - a) an even number.
 - b) a number divisible by 3.
 - c) a perfect square.
 - d) a number divisible by 1.
 - e) a number divisible by 55.
 - f) a prime number.
- 5. A bag contains 23 marbles, 9 black, 7 white, 4 red and 3 green. A marble is drawn and its color determined. Draw the four-branched tree of outcomes and weight the branches according to the probability of each color. What is the chance the marble is
 - a) red?
 - b) not black?
 - c) red or green?
 - d) yellow?
 - e) round?
 - f) red and green?
 - g) not (red or black)?
 - h) not (red or black or green)?

MULTIPLE STAGE TREES

So far we have been looking at experiments that are completed in a single act: tossing a coin, rolling a die, choosing two marbles at once from a bag. We have considered the chance of one of the outcomes: the coin falls head-up; the die shows a 5; both marbles are red. We also considered other chances: the die shows a 3 or a 4; a single marble drawn is not red or black. Now we want to look at experiments that are made up of two or more acts done in succession and at experiments whose single act can be regarded as made up of a succession of simpler acts. Intuition seems to work better for simpler acts. We intuitively feel the chance of getting a head on the toss of a single fair coin is $\frac{1}{2}$. But the chance of getting one head when we toss three fair coins at once is not intuitively obvious. If we can break this down into a succession of simpler acts and learn how to combine their probabilities we can work out the probability.

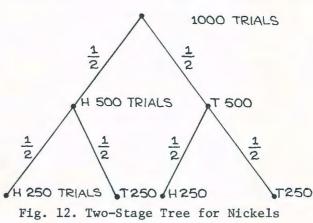
TOSSING TWO COINS

Let's start with tossing two nickels. What are the probabilities P(2 heads), P(1 head) and P(0 heads)? Instead of tossing the two nickels together think of tossing them one at a time in succession. A single trial now consists of tossing the first nickel, recording H or T and then tossing the second nickel and recording H or T.

We use our intuition and our previous results to work out the relative frequencies of various events and see how the known probabilities combine to give the answers we want. In 1000 trials, what results can be expected? The chance of getting a head the first time is 1/2 so we should get about 500 heads. Now consider only those trials that resulted in the 500 heads. What happens on the second toss? Again the chance of getting a head is 1/2 and so out of these 500 tosses, about 250 should be heads and about 250 tails. The relative frequency of 2 heads in succession is 250/1000 = .25 and the relative frequency of heads followed by tails is also 250/1000 = .25.

An experiment in which a trial consists of two things done in succession is called a two-stage experiment. A tree diagram for this is called a two-stage tree. To draw it, we draw the tree for the first stage and then at the end of each branch

draw the tree for the proper second stage. The diagram for tossing two nickels looks like Figure 12. The results at each stage are indicated. The chance of each result is written on the branch leading there. The left most branch leads to H at the first stage and H again at the second stage: the chance of following this branch is about 250/1000 or 1/4 which is the product of the chances at each stage. The event of getting 2 heads has probability branches lead to the event 1 head? Each



event of getting 2 heads has probability $\frac{1}{2} \ge \frac{1}{2} = \frac{1}{4} = .25$. What is P(1 head)? What branches lead to the event 1 head? Each of the two middle branches do since one leads to HT and the other to TH. Each of these branches is followed with probability

1/4. But when an event includes two or more branches, we find its probability by adding those of the branches. Therefore:

$$P(1 \text{ head}) = P(HT \text{ or } TH) = P(HT) + P(TH)$$

= $\frac{1}{4} + \frac{1}{4}$
= $\frac{1}{2} = .50.$

What is P(0 heads)? No heads means we got tails both times. P(0 heads) = P(TT) = $\frac{1}{4}$ = .25.

In an actual experiment two nickels were tossed 70 times. The relative frequencies resulting were:

> f(2 heads) = 13/70 = .19 f(1 head) = 35/70 = .50 f(0 heads) = 22/70 = .31

These vary a good deal from .25, .50 and .25, the probabilities we have just worked out by using our intuitive values of the chances at each of the two stages. If we had made 100 or 200 trials, the relative frequencies would probably have been closer.

CHOOSING TWO MARBLES

Example 7

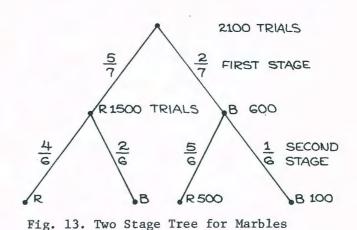
Suppose 2 marbles are drawn from a bag that has 5 red and 2 blue marbles. What are the chances that both marbles are red? We consider the single trial as made up of two successive events where one marble is drawn and then a second one without putting the first one back. What is the probability that the first marble is red? We saw before that this is $\frac{5}{7}$. Out of say 2100 trials about $\frac{5}{7} \times 2100 = 1500$ times we get a red marble. Now what is the situation in the marble bag? One red marble has been drawn so only 4 red and 2 blue ones remain. If we draw again the chance of getting a red one is $\frac{4}{6}$. Thus about $\frac{4}{6}$ of the 1500 times with a red marble on the first draw, we should get another red one on the second draw. The relative frequency of the event "two red marbles" turns out to be

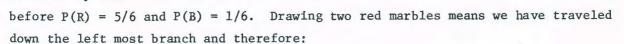
$$\left[\frac{4}{6} \times \left(\frac{5}{7} \times 2100\right)\right] \div 2100 \text{ or } \frac{4}{6} \times \frac{5}{7}.$$

Thus the chance of the outcome "two red marbles," considered as the results of two successive events, turns out to be the product of the chances of the two events.

CONTENT FOR TEACHERS

We draw the tree to illustrate the experiment. Note that this time, the probabilities at the second stage are different from those at the first stage. Where did we get 5/6 and 1/6 for the branches of the second stage tree on the right? To get to this position we had to draw a blue marble at the first stage. If this happens then 5 red and 1 blue marbles remain. By the methods we have used





$$P(2R) = \frac{5}{7} \times \frac{4}{6} = \frac{20}{42} = \frac{10}{21}$$

Drawing 1 red and 1 blue means we have gone down either of the two middle branches.

$$P(1R) = P(RB \text{ or } BR)$$

= $\frac{5}{7} \times \frac{2}{6} + \frac{2}{7} \times \frac{5}{6}$
= $\frac{10}{42} + \frac{10}{42} = \frac{10}{21}$

Finally, no reds means going down the right branch.

$$P(OR) = \frac{2}{7} \times \frac{1}{6} = \frac{1}{21}$$
.

Two important results come out of these examples.

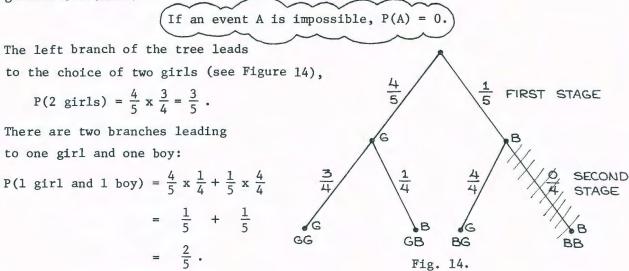
1. To get the probability of the event at the end of any branch, multiply the probabilities along the stages of that branch.

2. To get the probability of an event including several branches, add the probabilities computed for each of those branches.

So far we have seen only two-stage trees but the same principles hold for multiple-stage trees. These principles are the fundamental ideas underlying most applications of probability.

Example 8

Suppose we choose two children at random from a group of four girls and one boy. Draw the tree and find the chances of getting two girls; a boy and a girl; two boys. We quickly see that the third case is impossible. Since there are no cases, the relative frequency is $\frac{0}{4} = 0$ and the chance is 0. This illustrates the general statement.



IS IT A FAIR COIN?

Example 9

Suppose a friend of yours has a coin that she claims is fair, that is, on one toss the $P(H) = \frac{1}{2}$. To test the coin you toss it six times and it lands heads every time. Do you think it is fair? In Example 1 in *Experimental Probability* in the CON-TENT FOR TEACHERS section, when we tossed what we thought was a fair coin 100 times we did get a run of 5 heads. If we tossed this coin a large number of times, say 1000, we might get runs of 6 or even 10 or more heads in a row. But confronted with a strange coin and six successive heads, we might ask what is the chance of getting six heads in a row when tossing a fair coin? The tree has many branches



but we only have to follow along one and at each stage, for a fair coin, $P(H) = \frac{1}{2}$. Multiplying the probabilities along the branch for 6 successive heads we have $P(6 \text{ heads}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{64}$. This is such a small probability that most of us would be inclined to doubt that the coin is fair. If we got 10 successive heads the same argument would tell us that the probability of this event is 1/1024 which is less than .001. We would be almost sure the coin was not fair. It is a case of making a choice between two beliefs:

- The coin is fair and an extremely unlikely event, one with a chance of less than 1 in a thousand, has happened or
- 2. the coin is not fair.

This is an example of making a decision in the face of uncertainty. Help in making such decisions is one of the main results we hope to get out of studying statistics and probability. The situation here is that out of the many possible ways the 10 tosses could have fallen we have taken one sample consisting of 10 heads in a row. If the coin was fair, this was a very unusual sample; so unusual we might think the coin has heads on both sides. To test this, we look at both sides. If the coin has both a head and a tail we take another sample of 10 tosses. This time we get 9 heads and 1 tail. We are more than ever convinced that the coin is weighted heavily to show heads. We may, of course, be wrong but that is the decision we make in the light of the evidence we have.

THREE OR MORE COINS

Example 10

If we toss three pennies, what is the chance of getting three heads? This is the same as tossing one penny three times in succession. If we drew the whole three-stage tree it would have 8 branches. Do you see why? But three heads will result only if we get a head each times so we are interested in only one branch of the tree. We find the probability of the event at the end of that branch by multiplying the probabilities as we follow that branch from beginning to end. Thus

P(3 heads) = $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$.

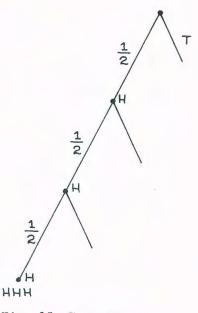


Fig. 15. Tree Branch for 3 Heads Can we do the same thing to find the probability of 2 heads? Yes, but we have to be more careful as there are more branches that lead to this result.

By drawing the whole tree and checking we see there are three such branches, the probability on each branch is $\frac{1}{8}$ and since any one of these branches gives the result we want, we find that P(2 heads) = $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$ (see Figure 16).

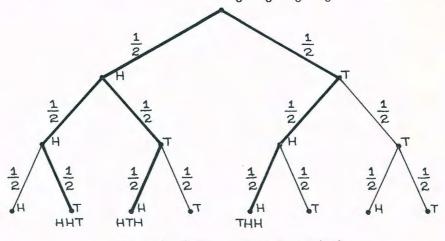
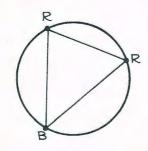


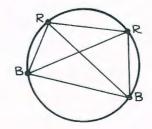
Fig. 16. 3-Stage Tree for P(2H)

OTHER MODELS

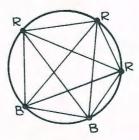
A tree is not the only model we can use in working out probabilities. We consider again the chances of drawing 2, 1 or 0 red marbles when choosing two at random from a bag containing red and blue marbles. The model consists of drawing a circle, placing on it a dot for each marble, labeling them R or B appropriately, and drawing lines connecting each pair of dots to represent possible choices. If the bag has 2 red and 1 blue marble we draw Figure 17a. Of the three choices of 2 marbles only



(a) 2 red and 1 blue



(b) 2 red and 2 blue Fig. 17. Circle Network Models



(c) 3 red and 2 blue

one has 2R, two have 1R and none has OR. The chance of 2R is $\frac{1}{3}$ and of 1R, $\frac{2}{3}$. If the bag has 2 red and 2 blue marbles, the model is Figure 17b and if it has 3 red and 2 blue the model is Figure 17c. In (b) we see there are six possible choices of 2 marbles, of these one has 2R, four have 1R and one has OR. The probabilities are

$$P(2R) = \frac{1}{6}$$
, $P(1R) = \frac{4}{6}$ and $P(0R) = \frac{1}{6}$

By counting in (c) we see in this case

$$P(2R) = \frac{3}{10}$$
, $P(1R) = \frac{6}{10}$ and $P(0R) = \frac{1}{10}$.

If the number of marbles in the bag is 6 or less, this model can be used but beyond that the network becomes so crowded, mistakes in counting are easy and the results become uncertain.

In the case of rolling two dice, the two-stage tree model has 36 branches, too many to consider easily. This time a model can be made by drawing a grid with

36 boxes in it as enclosed by the heavy lines in Figure 18. On the top and left side we indicate the possible throws of the two dice and in each square inside we mark the score resulting. Each square represents the result of following down one branch of the tree if we had drawn it. Intuitively we know the probability of each of the six branches in the first stage is $\frac{1}{6}$ and also of each branch in the second stage. The probability along any one branch of the whole

		SE	COND	D DI	E		
		1	2	3	4	5	6
	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
ШD	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
FIRST	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

Fig. 18. Throwing 2 Dice

tree is $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$. Since each square corresponds to a branch we can count up the squares that have a 5 in them to find P(5). There are 4 such squares and P(5) = $\frac{4}{36}$. Similarly P(10) = $\frac{3}{36}$ and so on for any possible roll of two dice.

What model is best? There is no answer. Different problems may lead to different models. The best one for you is the one that helps you the most to see the way to analyze the problem. It may not be the one your neighbor feels is the best. Different models may each contribute to better understanding for all of us.

Exercises

6. A bag has 3 red and 4 black marbles. You draw two marbles together at random. What is the chance both are black?

- 7. A bag has 3 red and 4 black marbles. You draw one marble, look at it, put it back and draw again.
 - a) What is the probability you draw two black marbles?
 - b) What is the probability the two marbles are the same color, i.e., both red or both black? Hint: Draw the tree and work out the probabilities along each branch.
 - c) What is the probability the marbles are of different color?
 - d) What is the sum of your answers to b) and c)? What significance does this have?
- 8. If a deck of 52 playing cards is well shuffled, the probability of drawing any card should be the same as that of drawing any other card.
 - a) How many branches does the tree for this experiment have?
 - b) What is the probability on any one branch?
 - c) How many branches end on spades? What is the probability of drawing a spade?
 - d) The K, Q, and J are known as face cards. How many face cards are in the deck? What is the probability of drawing a face card?
 - e) What is the probability of drawing an ace?
- 9. Two cards are dealt from a deck of 52 cards. To find the probability that both are aces, we think of two successive events. The first card is dealt. Four of the 52 branches give an ace. The probability is 4/52 or 1/13 that the first card is an ace. 51 cards are left in the deck of which only 3 are aces. What is the probability that the second card is an ace? What is the probability both cards are aces?
- 10. Two cards are dealt from a deck.
 - a) What is the probability of getting 2 face cards?
 - b) What is the probability of getting two cards of the same suit?
- 11. When we tossed a coin ten times and got ten heads in a row, we suspected the coin was biased. One possibility was that it had two heads. This is one hypothesis to explain what seemed an extremely unlikely event happened. We could verify or refute this hypothesis not by tossing the coin more times but by looking at it carefully. Can you think of some unlikely event that might occur for which such obvious verification or refutation of a hypothesis to explain it would be very difficult? Impossible?

SUMMARY

In working out problems involving probability there are several steps.

First: Try to count the number of outcomes of the experiment. If there are n of these and there is no reason why one is to be preferred to another, we assume the chance of each is 1/n. We can test this by performing the experiment many times and checking the relative frequencies. We diagram it by drawing a tree with n branches. Thus in drawing one marble from a bag with 13 identical marbles, the tree has 13 branches and the probability of drawing a certain marble in a random drawing is 1/13.

Second: If the event E consists of several branches, P(E) is the sum of the probabilities of the branches. If the marbles are numbered from 1 to 13, the

probability of getting a prime number is P(2 or 3 or 5 or 7 or 11 or 13) = 6/13. If there are 5 red and 8 black marbles, P(red) = 5/13.

Third: If the trial of the experiment is or can be broken down into successive events, the tree is no longer a single-stage one but rather has a number of stages corresponding to the number of successive events. The probability of the event at the end of any one branch is the product of all the probabilities along the branch leading to that event. Thus to find the probability of drawing two black marbles from the bag of 5 red and 8 black marbles, we follow along the right branch of the tree (see Figure 19) and find: $P(2B) = \frac{8}{13} \times \frac{7}{12} = \frac{56}{156}$.

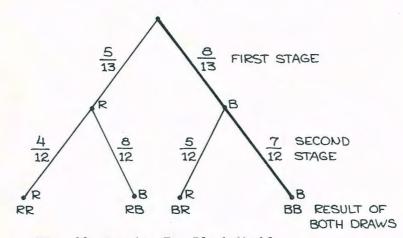


Fig. 19. Drawing Two Black Marbles

If more than one branch leads to an event we have to figure the probability along each such branch and add the results. To find P(1B), we note we end up with 1 black marble on each of the two middle branches so:

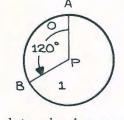
$$P(1B) = P(RB \text{ or } BR) = P(RB) + P(BR) = \frac{5}{13} \times \frac{8}{12} + \frac{8}{13} \times \frac{5}{12} = \frac{80}{156}$$

In this case, the probability of drawing a black marble changes in the second stage from that in the first. If we put the first marble back in the bag before drawing the second, the probabilities in the second stage would be the same as in the first. In this case: $P(2B) = \frac{8}{13} \times \frac{8}{13} = \frac{64}{169},$ slightly higher than the $\frac{56}{156}$ it was before.

Sometimes counting the branches of the tree that lead to the same event gets quite involved. The tree diagram for the outcomes of tossing a coin five times has 32 branches. Finding and counting the branches that lead to 3 heads and 2 tails is not easy. Looking at ways of counting is the subject of the next section.

Exercises

- 12. A bag of six marbles has 3 red, 2 blue, and 1 white. Two marbles are drawn. Set up the model of dots on a circle with chords joining two points.
 - a) What is the chance of getting the white marble?
 - b) What is the chance of getting two reds?
- 13. The marbles are as in Exercise 12. The marble drawn the first time is put back before the second drawing. What is the probability of getting 2 reds? of getting 2 whites? This time set up the two-stage tree as the model.
- 14. In this spinner, $\angle APB = 120^{\circ}$.
 - a) On one spin, what is P(1)?
 - b) What is P(not getting 1)?
 - c) On three spins, what is the probability of never getting a 1?
 - d) On three spins, what is the probability of getting 1 at least once?



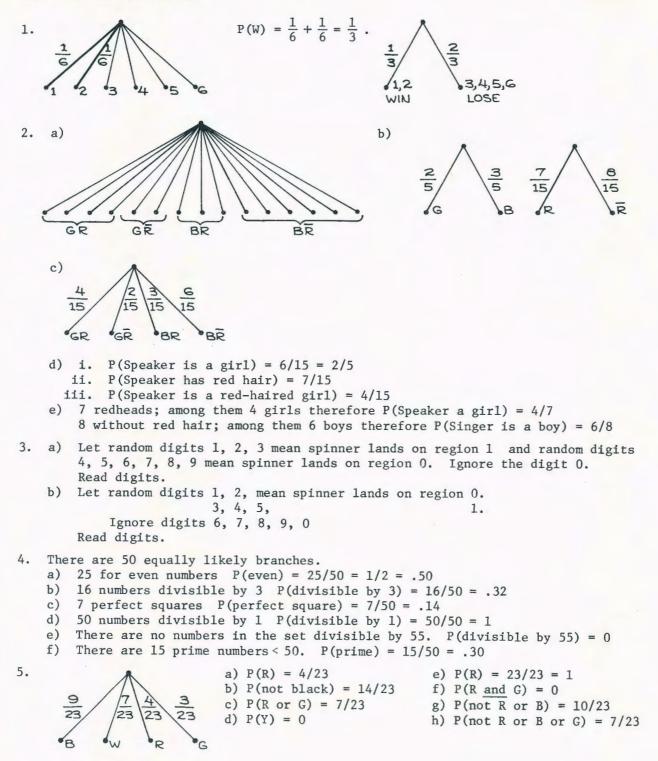
- 15. In rolling two dice, the score recorded is the sum of the dots showing on each die. Thus : scores as 8. Use the grid model illustrated in Figure 18.
 a) What is P(5) in rolling two dice?
 - b) What is P(11)? P(2)? P(7)?

16. 5% of the light bulbs produced in a certain factory are defective.a) If you pick a bulb at random off the shelf, what is the probability you

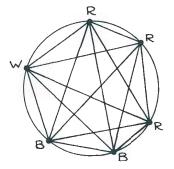
- get a defective one?
 b) If you take two, what is the probability both are defective? Set up a tree
 to show this.
- c) How could you use a random number table to simulate this experiment? Do it 50 times. What is the relative frequency of 2 defective bulbs?
- d) Suppose the experiment was to look at 5 bulbs and find the probability of getting two or more bad ones. Now a tree becomes almost too complicated to use. How could you use the random number tables to simulate this experiment? Do it 50 times. What is the relative frequency of two or more bad bulbs in five?

CONTENT FOR TEACHERS

ANSWERS TO EXERCISES



- 6. $4/7 \ge 3/6 = 2/7 \approx .286$
- 7. a) $4/7 \ge 4/7 = 16/49 \approx .327$
 - b) $9/49 + 16/49 = 25/49 \approx .51$
 - c) $3/7 \ge 4/7 + 4/7 \ge 3/7 = 24/49 \ge .49$
 - d) Sum is 1 because either the marbles are the same or they are different. These are all the possible outcomes.
- 8. a) 52
 - b) 1/52
 - c) 13, 13/52 = 1/4
 - d) 12, 12/52 = 3/13
 - e) 4/52 = 1/13
- 9. 3/51 or 1/17. $P(AA) = 1/13 \ge 1/17 = 1/221 \approx .0045$
- 10. a) $12/52 \ge 11/51 \approx .050$ b) $13/52 \ge 12/51 \approx .059$
- 11. If Sam gets 100 on 10 successive math tests, one hypothesis might be he was getting an advance look at the tests. No obvious verification is possible.
- 12.



a)	P(W) = 5/15 = 1/3
Ъ)	P(RR) = 3/15 = 1/5

- 13. P(RR) = 3/6 x 3/6 = 1/4 P(WW) = 1/6 x 1/6 = 1/36
- a) P(1) = 240/360 = 2/3
 b) P(not getting 1) = 1/3
 c) P(never getting a 1 on 3 spins) = 1/3 x 1/3 x 1/3 = 1/27
 d) P(getting 1 at least once) = 1 1/27 = 26/27
- 15. a) P(5) = 4/36 = 1/9b) P(11) = 2/36, P(2) = 1/36, P(7) = 6/36
- 16. a) P(defective) = 1/20
 - b) $1/20 \ge 1/20 = 1/400$
 - c) Consider two-digit numbers. Let 01, 02, 03, 04, 05 represent bad bulbs and all the others good ones. Read 2 consecutive 2-digit numbers 100 times and count the frequency of getting both numbers in the 01 to 05 group.
 - d) Look at 5 sets of 2 two-digit numbers and repeat 50 times. Count the number of times you get no bad bulbs or 1 bad one. Subtract from 50. This is the frequency of 2 or more bad ones. Dividing by 50 gives the relative frequency.

ORGANIZATION

Placement Guide for Tabbed Dividers

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Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
Probability with Counting	
Inferential Statistics	
Appendix-Computer Programs	
DIDACTICS	
Components of Instruction	
Classroom Management	
Statistics and Probability Learning	
TEACHING EMPHASES	
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Decision Making	
Problem Solving	
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Calculators and Computers	
Laboratory Approaches	
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INTRODUCTION

To determine the probabilities of the outcomes of an experiment we need to make counts of the results of the experiment. In the section on *Probability with Models* in CONTENT FOR TEACHERS we used different models to help in this counting: tree diagrams; networks; grids. The "grid" model for the rolling of two dice showed there were four cells, 1, 4, 2, 3, 3, 2 and 4, 1, out of 36 possibilities for the event "score of 5." Therefore $P(5) = \frac{4}{36}$. The "network" model for choosing two marbles from a bag of 3 red and 2 blue marbles showed there were 3 ways out of 10 for getting the event "2 reds." $P(2R) = \frac{3}{10}$. The "tree diagram" model for tossing a coin three times enabled us to count three branches out of 8 leading to the event "2 heads." $P(2H) = \frac{3}{8}$. In each experiment we had to count the total number of equally likely outcomes. We also had to count the number of outcomes in the part-icular event we were interested in.

Up to now, this counting has been relatively easy but it may be difficult if the numbers are large and the models more complicated. We want to develop some ideas to help us make such counts easier.

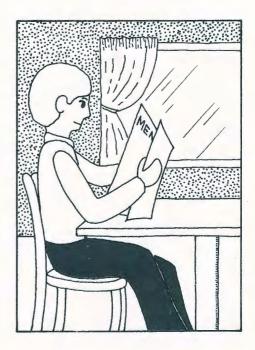
ALTERNATIVES

Suppose you can choose only one among several alternatives or combinations of alternatives. How many options do you have?

Example 1 - Single Courses

a) The BESTFOOD RESTAURANT offers five kinds of meat, three kinds of fish and four kinds of pasta as the entree in their regular dinner. Assuming John can select only one entree, in how many ways can he choose his main course? The answer is 12 since he can choose one of five meats or one of three kinds of fish or one of four pastas. The answer is the same whether he chooses directly or whether he first chooses his entree to be one of the meat dishes and then chooses one of the five in that group.

b) If seven air lines, three bus companies and one railroad run from New York to Chicago, you can make the trip in 7 + 3 + 1 = 11 ways.



In general, to find the number of ways to choose among alternatives simply add up the number available for each alternative.

Exercises (Answers given on pp. 197-198)

- 1. John and Joan have a date for Saturday night. They can go dancing or to the movies or bowling or stay home. There are three different bowling alleys they like, four dance halls and six movie houses. How many options do they have as to how to spend the evening?
- The university offers a number of courses at 8 AM: five in English, three in Mathematics, three in Economics and four in Science. If John has to take an 8 o'clock class, how many options does he have?

SUCCESSIVE CHOICES

Example 2 - Airline Routes

If seven airlines run from New York to Chicago and three airlines run from Chicago to Milwaukee, in how many ways can you go from New York to Milwaukee via Chicago. This time the answer is not the sum of 7 and 3. Rather it is the product since for each of the seven ways you choose for the first stage, there are three ways for the second

stage. A tree diagram may help. We have only drawn part of the tree as the whole tree has $7 \ge 3 = 21$ branches. Sometimes a simpler box diagram showing the number of choices at each stage is as good as the tree (see Figure 1).



Stage	1	st			2nd		Total
	New York	to Chicago		Chicago	to Milwaukee		
Number of choices		7	х		3	=	21

Fig. 1. Number of Routes from New York to Milwaukee

A slightly more complicated counting problem occurs if, for example, you might also have gone via Cleveland with the choice of 3 airlines from New York to Cleveland and 4 from Cleveland to Milwaukee. The number of routes via Cleveland would be 3 x 4. The total number of routes would be 33 altogether, 21 of them via Chicago plus 12 via Cleveland.



Example 3 - Dinner Menus

Besides 12 entrees which, you remember, consisted of five meats, three fish and four pastas, the BESTFOOD RESTAURANT offers the choice of 3 appetizers, 2 salads and 5 desserts. John and Joan go there for dinner one evening. In how many ways could John order dinner if for his entree he chooses some kind of meat and if he orders one of each of the other courses on the menu?

Joan decides to order some kind of fish but she feels that with fish only two of the appetizers and four of the desserts will go well, while either of the salads is okay. In how many ways can Joan order?

In each case, a tree might help answer the question. But a complete four stage tree would be very complicated. With a little care we can use the



box diagram suggested in Example 2. We would get the following diagram:

_	Stage			2nd Entree		3rd Salad		4th Dessert		Total Dinners	
For John	Number of Choices	3	x	5	x	2	x	5	=	150	
For Joan	Number of Choices	2	x	3	x	2	x	4	=	48	

Fig. 2. Box Diagram for Counting Possibilities

John can order dinner in 150 different ways and Joan in 48 for a total of 198. If we wanted to count the total number of different two dinner orders possible we would have to multiply 150 x 48 to get 7200.

This example illustrates the Fundamental Principle of Counting.

When successive choices are involved, the total number of possibilities is found by multiplying the number of options at each successive stage.

Exercises

- 3. Joan has four different colored shirts and five different pairs of jeans to choose from. In how many different ways can she dress?
- 4. John has 4 shirts, 3 pairs of pants and 3 pairs of shoes. In how many ways may he dress if he must wear shirt, pants and shoes? How many if he does not have to always wear a shirt?

- 5. At school Mary has to choose five courses, one each from 3 English, 2 Math, 3 Science, 4 Social Science and 2 Art classes. How many different programs are possible for Mary?
- 6. Christine, a teenager, belongs to high society. She feels wearing a different outfit each day of the year is necessary to keep her image. If she has 20 different sweaters, how many different skirts does she need to live up to her desires?

ARRANGEMENTS OR PERMUTATIONS

Example 4 - Flag Signals

A scout has four different flags. To send signals all four flags will be hoisted one above the other. How many signals can be sent? The scout chooses the first flag and ties it on the halyard. How many possibilities are there for this flag? With four different flags, there are exactly four. Now three flags are left. One of these is chosen and tied on below the first flag. Two possibilities remain for the next position. The last spot is filled automatically in one and only one way since only one flag is left in the flag locker. Thus there are $4 \times 3 \times 2 \times 1 = 24$ different signals. A box diagram similar to Figure 2 makes the

RED VELLOW GREEN BLUE

total count easy. But if the flags are colored red, yellow, green and blue, the different signals could be read off more easily by following the different branches

of the tree in Figure 3. Thus the second from the right gives us BGRY while the farthest to the left is RYGB. The tree illustrates the 24 different arrangements of the four colored flags. The model, box or tree, best for you to use depends on which model helps you more.

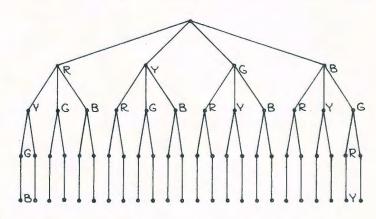


Fig. 3. Permutations of 4 Flags

An arrangement is also called a permutation.

A permutation of any set of objects is an arrangement of the objects in a definite order.

Thus there are 24 possible permutations of four different objects arranged in a row.

Exercises

- 7. If you have five different books, in how many ways can you arrange them in a row on your book shelf? You have five options for the one to go on the left end, 4 for the next position and so on. Your answer is the number of permutations of five different things.
- 8. How many permutations are there of six different flags? How many branches would there be in a tree corresponding to Figure 3?
- 9. Suppose the scout has six different flags but his halyard is so short he can only fly three at a time. How many signals can he make? Use a box diagram as in Figure 2. If you tried to draw a tree (don't) how many branches would it have in the third stage?
- 10. Most codes use "words" of five letters. How many different such "words" can be made using the letters A, B, C, D, E if each letter is used in each word?
- 11. How many five letter code "words" can be made from the letters of the word COLUMBIA if no letter is used more than once in any "word"?
- 12. In a certain state auto license plates use three letters followed by three digits. Thus TZZ 098. Note that repetitions are allowed. Use a six-stage box diagram. How many possibilities are there for the first stage? For the second and third? How many for the fourth, fifth and sixth stages? How many different license plates car this state issue? Your answer should be greater than 17.5 million.



13. When telephones were first invented people were called by name instead of number. Later as the list of subscribers got longer, <u>four</u> digit telephone numbers were assigned. What was the maximum number of subscribers for a given exchange?

Example 5 - Officers for a Group of Teachers

The twenty teachers in a certain school want to select a building president, vice-president and secretary-treasurer with no person holding two offices. If the selection is made at random, how many different sets of officers can be chosen? How many possibilities are there for president? Twenty. Once the president is chosen, how many teachers are left from whom the vice-president can be chosen? Nineteen. And then the secretary-treasurer can be chosen from the remaining eighteen.

?

1903

CONTENT FOR TEACHERS

Stages	lst President		2nd Vice-President		3rd Secretary-Treasurer		Total
Choices	20	x	19	x	18	-	6840

We draw a box diagram in Figure 4 similar to Figure 2.

Fig. 4. Box diagram for counting possibilities

Once again to find the total number of possibilities we multiply the number of choices at each stage getting 6840.

What is the probability that the officers will be: President - Jones; Vicepresident - Smith; Secretary-Treasurer - Brown? We assumed the selection was random. Therefore the probability of any given selection is equal to that of any other. As there are 6840 possible selections, the probability of Jones as President, Smith as Vice-President and Brown as Secretary-Treasurer is $\frac{1}{6840}$.

Exercises

- 14. A coin collector has 10 Indian-head pennies, each one dated with a different year from 1901 to 1910.
 - a) In how many different ways can three of the coins be selected and arranged in a line on the counter?
 - b) What is the probability that the arrangement is
- 15. A flag locker contains five different flags. A signal consists of three flags hoisted one above the other. How many signals can be sent?
- 16. A flag locker has nine different flags. Signals consist of four flags hoisted one above the other. How many signals can be sent?
- 17. A group of children consists of 3 boys and 3 girls. They are lined up for a picture.
 - a) In how many ways can this be done?
 - b) Suppose now the photographer wants to line them up with boys and girls alternating. In how many ways can this be done? Hint: Set up a box diagram similar to Figures 3 and 4.

Stages	lst		2nd		3rd		4th		5th		6th
Choices	6	x	3	x	?	x	?	x	?	x	?

He can choose any one of the 6 for the first position. Why are there only 3 choices for the second position? How many choices for the third position?

c) If the children are lined up at random, what is the probability that boys and girls alternate?

FACTORIALS

In the Examples and Exercises we have been counting permutations of different things. Sometimes these were permutations of all the things in a set such as the flags in a locker. Sometimes they were the permutations of only a limited number of the objects in the set as in the choice of officers for the teachers in a certain building. Counting permutations sometimes involves very large numbers and it would be nice to have some shorthand notation to use.

Example 6 - Lining Up for a Picture

Suppose a class of 20 students is to have a picture taken. The photographer wants to have the class lined up with 10 students in the first row and 10 in the second. In how many ways could this be done? We have seen that this can be thought of as a 20-stage process. How many choices are there for the front row, left end spot? 20. For the next? 19. And so on down. As each spot is filled there is one less in the group still to be chosen from until there is only one student left to fill the last spot. The total number of ways of lining up is 20 x 19 x 18 x 17 x 16 x 15 x 14 x 13 x 12 x 11 x 10 x 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1. This is a tremendously large number and we should

there are factors involved this number is read as "twenty factorial." The exclamation point may have been chosen to indicate our surprise at how rapidly this process generates a large number.

If the 6 teachers of the 5th grade wanted their picture taken they could line up in 6! (six factorial) or $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ ways.

If the five letters of the word PROUD are rearranged to form possible code "words" such as DROUP or UPROD, how many could be made? A five stage process is involved and the answer is 5! (five factorial) or 120 ways.



have some shorthand way to write it. The standard notation for it is 20!. Since



CONTENT FOR TEACHERS

COUNTING TECHNIQUES

The notation n! (read n factorial) is an abbreviation for the product of all the integers from n down to 1. $n! = n \times (n - 1) \times (n - 2) \times ... \times 3 \times 2 \times 1$

We see that the number of permutations of 20 people when we consider all of them is 20!. The number of signals we can fly using all five flags in the locker is

5!. In general:

The number of permutations of n objects taken all together is n!.

Could we use this factorial notation to express the answer to Example 5? In that example we wanted to count the permutations of 20 different people taken three at a time. The answer was 20 x 19 x 18.

But 20 x 19 x 18 =
$$\frac{20 \times 19 \times 18}{1}$$

= $\frac{20 \times 19 \times 18 \times 17 \times 16 \times ... \times 3 \times 2 \times 1}{17 \times 16 \times ... \times 3 \times 2 \times 1}$
= $\frac{20!}{17!}$.

This is the number of ways of choosing and arranging in a row three objects from twenty different ones. It is the number of permutations of 20 different objects taken three at a time.

Again, if you have 10 different books but only have room on your desk for six of them, in how many different ways could you arrange books in a row on your desk? We are asking for the number of permutations of 10 books taken 6 at a time. The answer is $10 \times 9 \times 8 \times 7 \times 6 \times 5$ since there are 10 choices for the first position, 9 for the second, etc. This could be written as $\frac{10!}{4!}$ using the same idea explained above.

Perhaps a more revealing notation would be $\frac{10!}{(10-6)!}$ where the 6 in the (10-6)! indicates the number of books you are using or the number of factors in the first form of the answer. In this form, the answer to Example 5 could be written as $\frac{20!}{(20-3)!}$.

In general:

The number of permutations of n things taken r at a time, where r < n, is $\frac{n!}{(n-r)!}$

Exercises

18.

Con	pute each of	the following:				
a)	<u>6!</u> 4!	b) $\frac{7!}{5! \times 2!}$	c)	$\frac{12!}{10!}$	d)	$\frac{12!}{9! \times 3!}$
e)	$\frac{20!}{16! \times 4!}$	f) $\frac{18!}{16!}$	g)	<u>31</u> 51	h)	$\frac{4! \times 3!}{7!}$

Express the answers to the following in factorial form but do not try to work them out.

- 19. In how many ways could 50 people be arranged in line?
- 20. How many permutations are there of 16 objects taken 4 at a time?
- 21. A boat has 10 different flags. Signals are made by hoisting flags one above the other.
 - a) How many signals can be made using 4 flags?
 - b) How many signals can be made using 6 flags?
 - c) How many signals can be made using 2 flags?
- 22. A boy scout has only four flags. He can make signals by flying either 4 or 3 or 2 or even 1 flag alone. How many signals can he make using all 4? Using 3? Using 2? Using 1? How many signals can he send in all? Now work out your answers. (The final answer should be 64.)

CHOICES OR COMBINATIONS

Suppose six people meet for dinner. After dinner a game of bridge is suggested. In how many ways can four of the six be chosen to play? From a group of 20 teachers, how many different committees of three can be chosen? In each of these cases we are not interested in the number of permutations of the four bridge players or the three committee members, but only in the number of different possible groups or committees. Such choices without regard to order are called <u>combinations</u>. The number of combinations of four objects from six is called "six choose four" and abbreviated by $_6 C_4$. Similarly we write $_{20} C_3$ for the number of committees of 3 that can be chosen from the 20 teachers. How do we calculate these numbers? Remember we are choosing without regard to order.

When the size of the group we are choosing from is small it is easy. But we want to build up a pattern that will enable us to find any such number we need. Suppose the group has only one object, A. We can choose one object from the group in only one way, i.e., $_1C_1 = 1$. Suppose the group has two objects, A and B.

We can choose two of them in only 1 way (AB) $_2C_2 = 1$ We can choose one of them in 2 ways (A)(B) $_2C_1 = 2$

In order to build up the patte	ern, we say in both cases there is	only one way to
choose no objects from the gro	Sup, i.e., ${}_{1}C_{0} = 1$ and ${}_{2}C_{0} = 1$.	
Suppose the original grou	up has three objects A, B, C.	
We can choose thre	ee objects in 1 way (ABC)	${}_{3}C_{3} = 1$
We can choose two	objects in 3 ways (AB)(AC)(BC)	$_{3}C_{2} = 3$
We can choose one	object in 3 ways (A)(B)(C)	${}_{3}C_{1} = 3$
Again we can choos	se no objects in 1 way ()	$_{3}C_{0} = 1$
For a group of four objec	cts the pattern goes:	
4 choose 4	(ABCD)	$_{4}C_{4} = 1$
4 choose 3	(ABC) (ABD) (ACD) (BCD)	${}_{4}C_{3} = 4$
4 choose 2	(AB) (AC) (AD) (BC) (BD) (CD)	$_{4}C_{2} = 6$
4 choose 1	(A) (B) (C) (D)	$_{4}C_{1} = 4$
4 choose 0	()	$_{4}C_{0} = 1$

Let's arrange these to make the pattern more visible adding a top line for a group of 0 objects.

0	objects						1					
1	object					1		1				
2	objects				1		2		1			
3	objects			1		3		3		1		
4	objects		1		4		6		4		1	
5	objects	?		?		?		?		?		?

The pattern can be continued indefinitely by noting:

- a) the first and last numbers in a row are always 1;
- b) the number in any position in a row is the sum of the two numbers in the row above lying to the right and left of its position.*

a) The first number in the next row is ${}_5C_5$, the number of ways you can choose all five objects. There is only one way ${}_5C_5 = 1$. The last number is ${}_5C_0$. We have assumed this is one. ${}_5C_0 = 1$.

b) We will show ${}_5C_3 = {}_4C_3 + {}_4C_2$. A set of 5 objects (ABCDE) is obtained from a set of 4 (ABCD) by including one different object E. Any combination of three from the five either includes E or not. If it includes E it is of the form (ABE), that is, it adds E to one of the ${}_4C_2$ groups. If it does not include E it is of the form (BCD) and is one of the ${}_4C_3$ groups. Therefore the number of combination ${}_5C_3$ is the sum of the numbers in these two groups, i.e., ${}_5C_3 = {}_4C_3 + {}_4C_2$.

^{*}For the interested reader, we offer proofs for certain special cases. These proofs can be generalized.

Figure 5 should help to make this procedure clear. This triangle of numbers is named after Blaise Pascal, a French mathematician who first called attention to it.

From the figure we see ${}_{5}C_{3} = 10$ and ${}_{6}C_{4} = 15$. It would be perfectly possible to determine ${}_{20}C_{3}$ by building up this figure but it would take a long time. We need a better way.

When we calculated in Example 5 the number of ways three class officers could be selected from twenty teachers, we were both selecting them and arranging them in

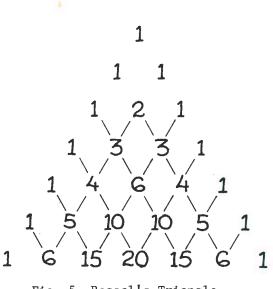


Fig. 5. Pascal's Triangle

order: President; Vice-president; Secretary-Treasurer. We found this could be done in 20 x 19 x 18 = $\frac{20!}{17!}$ ways. Now let's look at the problem in a slightly different way and consider it in two stages. At the first stage, we select the committee and after that we arrange them in order. The box diagram is drawn as Figure 6.

Stages	Choose the Committee		Arrange Them		Possible Arrangements
Numbers	20 ^C 3	x	3!	=	$\frac{20!}{17!}$

Fig. 6. Choosing and Arranging

If ${}_{20}C_3 \ge 3! = \frac{20!}{17!}$, we divide both sides by 3!. Then ${}_{20}C_3 = \frac{20!}{17!3!}$. (Note that 17 and 3 add to 20.) Finally

$$_{20}C_3 = \frac{20 \times 19 \times 18}{3 \times 2 \times 1} = 20 \times 19 \times 3 = 60 \times 19 = 1140.$$

In keeping with this pattern, let's compute 6 C4.

 $_6 C_4 = \frac{6!}{4!2!} = \frac{6 \times 5}{1 \times 2} = 15$ and this checks with what we had before.

If from a class of 12 children we choose a committee of 4, the number of different committees would be ${}_{12}C_4$. To compute this, we remember that the number of permutations or arrangements of 4 objects chosen from 12 is $\frac{12!}{(12-4)!} = \frac{12!}{8!}$.

CONTENT FOR TEACHERS

Counting by two stages, choosing first in ${}_{12}C_4$ ways and then arranging the four chosen ones in 4! ways, we see by our box diagram:

$${}_{12}C_4 \ge 4! = \frac{12!}{8!}$$
 and dividing by 4!
 ${}_{12}C_4 = \frac{12!}{8!4!} = \frac{12 \ge 11 \ge 10 \ge 9}{4 \ge 3 \ge 2 \ge 1} = 11 \ge 45 = 495.$

We repeat: The number of ways to choose 4 objects from 12 without regard to order is the number of combinations of 4 objects from 12.

It is important to remember:

Permutations involve selecting and ordering or arranging. Combinations involve selecting but not ordering.

In any problem of counting one of the first things to decide is whether ordering is involved or not.

These two ways of counting are mainly what we are trying to learn to do:

- (1) counting the ways of choosing things and arranging them in order.
- (2) counting the ways of choosing things without regard to order.

Exercises

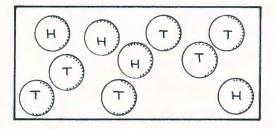
- 23. How many committees of 3 can be chosen from a group of 12 students?
- 24. A class has 20 students, 12 boys and 8 girls. How many different committees of 5 students can be chosen? How many committees will have boys as all the members? How many committees will consist only of girls?
- 25. The United States Senate has 100 members. A committee of 12 is to be chosen by lot. How many such committees are possible? (Leave answer in factorial notation.)
- 26. Seven people like to play bridge. How many different tables of 4 can be selected?
- 27. At a church fair, 500 numbered tickets are sold. Two different tickets are selected at random so that their holders may be given door prizes. How many different combinations of tickets could be selected?
- 28. On a ten question test, students are required to answer any 8 questions. How many students could be in the class if each one answered a different set of 8 questions?

CONTENT FOR TEACHERS

ARRANGEMENTS WITH REPETITIONS

Example 7 - Counting Heads

In tossing ten coins and trying to find the probability of getting four heads and six tails we looked at a tree with ten stages and a final total of 1024 branches. We needed to count the number of branches that led to the end result of 4 heads and 6 tails. Not just



HHHHTTTTTT but also perhaps THTTHHHTTT or TTHHTTTHTH or any other sequence of 4 heads and 6 tails. We are now ready to make this count but it still needs to be explained carefully. The four heads and six tails occur one by one at successive stages in the 10-stage tree. If we choose the four stages where H is to occur then T will automatically be at each of the other six stages and we will have one particular series of 4 H's and 6 T's. Thus the total number of such series is simply the number of ways we can choose 4 positions for H out of 10 possible ones, i.e., $_{10}$ C₄. This works out to be $\frac{10!}{4!6!} = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210$, and this is the number of branches of the tree that lead to the event 4 heads and 6 tails. Since the total number of branches is 1024, the probability of getting four heads is $\frac{210}{1024} \approx .205$.

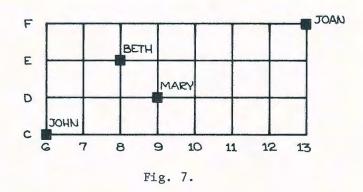
Similarly we can find the probability of getting five heads. We choose 5 positions for H in ${}_{10}C_5$ ways.

$${}_{10}C_5 = \frac{10!}{5!5!} = \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} = 252$$

Therefore, $P(5H) = \frac{252}{1024} \approx .246$.

Example 8 - Counting Paths

John lives on the corner of 6th Street and Avenue C. Joan lives at 13th Street and Avenue F. John has to walk at least ten blocks to visit Joan. He does not want to walk more than 10 blocks but he wants to vary his walk as much as possible. How many different routes can he take?



He has to go seven blocks East and three North. All we have to do is count the ways of arranging 7 E's and 3 N's. This is exactly the same as counting the ways of getting 7 H's and 3 T's in the toss of 10 coins. The answer is

$$10 C_7 = \frac{10!}{7!3!} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120.$$

In how many of these walks does John pass the corner where Mary lives at 9th and D? This is a two-stage event. In the first stage, he goes 4 blocks from his home to Mary's, 3 East and 1 North. This he can do in ${}_{4}C_{3} = \frac{4!}{3!1!} = 4$ ways. In the second stage he goes from Mary's house to Joan's, 6 blocks of which 4 are East and 2 North. He can do this in ${}_{6}C_{4} = \frac{6!}{4!2!} = 15$ ways. To get the total count in a two-stage case, we multiply the count at each stage so the answer is $4 \times 15 = 60$.

Suppose Beth lives at 8th and E. What is the probability John goes by Beth's house? We count the paths that go by Beth's house in 2 stages. The first stage has ${}_{4}C_{2} = \frac{4!}{2!2!} = 6$ paths and the second ${}_{6}C_{5} = \frac{6!}{5!6!} = 6$ paths. The total count is 6 x 6 = 36. Therefore the probability that John goes by Beth's house is 36/120 = 3/10.

Example 9 - More Code Words

How many code words can be made using all the letters of the word POOL? This differs from our previous code word problem since the letter 0 is repeated. Suppose we mark the two 0's so as to distinguish them by calling them 0_1 and 0_2 . Now we know there are 4! permutations of the 4 letters P, 0_1 , 0_2 , L. But these occur in pairs such as $L0_1PO_2$ and $L0_2PO_1$ and when we remove the distinguishing marks the two "words" become the same, LOPO. To find the number of words still different we have to divide 4! by 2. But 2 = 2! So our answer is $\frac{4!}{2!}$ or 12.

Again: how many different "words" can be made using all the letters of SASSY? This time there are three S's. If we mark them as S_1 , S_2 , and S_3 there are 5! different words possible. But the three S's can be arranged in 3! ways which are not distinguishable when the marks are removed. So the 5! words form groups of 3! or 6 indistinguishable words. Therefore to find the number of different words we have to divide 5! by 3! and the result is $\frac{5!}{3!} = 20$. On the other hand, if we want to follow the pattern of Example 7, we would choose the positions for the S's in ${}_{5}C_{3}$ or $\frac{5!}{3!2!} = 10$ ways and then in the second and third stages place the A and Y in 2 x 1 ways getting again a total of 20.

If more than one letter is repeated we can still proceed in either the pattern of this example or that of Example 7. Thus if we use the letters of PARALLEL there are 8 letters altogether with 3 L's and 2 A's. If the L's and A's have distinguishing marks the number of different words is 8!. But the three L's can be arranged in 3! ways. The two A's can be arranged in 2! ways.

Therefore to count the distinguishable words we have to divide by these numbers. The final count is $\frac{8!}{3!2!}$ = 3360 different permutations. A different approach would be to choose the positions for the L's in the first stage in ${}_{8}C_{3}$ ways. There are 5 positions remaining and in the second stage we choose two of these for the A's in ${}_{5}C_{2}$ ways. In the third stage, the P, R and E can be placed in the remaining positions in 3! ways. The final result is the product of these three numbers.

$${}_{8}C_{3} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56, \quad {}_{5}C_{2} = \frac{5 \times 4}{2 \times 1} = 10, \quad 3! = 6.$$

The answer is $56 \times 10 \times 6 = 3360$ agreeing with the former result.

How many different permutations are there of the letters of the word MISSISSIPPI? There are 11 letters altogether, 4 S's, 4 I's and 2 P's. Arguing in the same manner as before we get the final count as $\frac{11!}{4!4!2!} = 6930$ different permutations.

Exercises

- A. Successive Choices
- 29. The GOFORTOO HAMBURGER HOUSE offers four kinds of hamburgers, 5 kinds of soft drinks and 2 kinds of hot pies. If Tom orders a burger, a drink and a dessert, how many days can he go without duplicating his order?
- 30. In the country of SVOBODIA, license plates for cars have 5 digits. The first digit cannot be 0. How many different license plates are possible?
- 31. In SLOVENIA, license plates have two letters followed by 3 digits. No letters can be repeated. I and 0 look too much like digits and are not used as letters. No digits can be repeated. How many different license plates are possible?
- 32. There are 7 teachers in a certain school. One of them decides to invite one or more of the other six to dinner. In how many ways can this be done? Hint: think of a six-stage tree. At each stage, decide "invite" or "do not invite." How many branches are there in total? Should all the branches be counted?

B. Permutations

- 33. In how many ways may the four officers of a club be chosen from the 24 members?
- 34. Six men check their hats at a restaurant. When they leave, the hats are given out at random. In how many ways may this be done? What is the probability that every man gets his own hat?
- 35. How many different four letter code "words" can be made by arranging the letters of the word HOLD?

CONTENT FOR TEACHERS

- 36. How many different four letter code "words" can be made using only letters of the word GAMBLER?
- 37. A parking lot has seven spaces. If four shoppers park in the lot how many days could pass before they have to repeat their locations exactly?
- C. Combinations
- 38. From a class of 15 students a committee of 5 is to be chosen. In how many ways can this be done?
- 39. A class has 6 boys and 9 girls. A committee of 5 is to be chosen. How many committees can be chosen? How many committees will be all boys? How many will be all girls?
- 40. A class has 6 boys and 9 girls. A committee of 5 is to chosen. If the committee is to consist of 2 boys and 3 girls, how many different committees can be chosen? Hint: think of it as a two-stage event: first choose the boys and then the girls. What do you do with these two numbers? If the committee has 3 boys and 2 girls, how many different committees can be chosen?
- 41. A poker hand consists of 5 cards chosen at random from a standard deck of 52 cards.
 - a) How many different hands can be dealt? (Don't try to multiply out the
 - answer. Leave it in factorial form unless you have a hand calculator.)
 - b) How many hands will have only hearts?
- D. Miscellaneous
- 42. In the tree diagram for tossing a coin 8 times, how many branches will have 3 heads and 5 tails?
- 43. In rolling two dice how many ways are there of getting a score (sum of points showing) of 7? How many of 11? What is the probability of getting either a 7 of 11? Hint: how many cells are in the grid model?
- 44. How many different code "words" can be made using all the letters of the word SCHOOL?
- 45. How many different 4 letter words can be made using only the letters from the word SCHOOL? Hint: consider two cases:
 - a) Use both O's. First stage, choose two more letters from S, C, H, L. Second stage, arrange the 4 letters.
 - b) First, choose 4 letters from S, C, H, O, L. Second, arrange these 4 letters.
 - c) Since (a) and (b) represent alternatives, decide whether to multiply or add the results in (a) and (b).
- 46. How many poker hands (see #41) will have four cards of the same kind, i.e., 4 Aces or 4 Kings, etc.? (2 stages)
- 47. How many poker hands (see #41) will have exactly one pair, i.e., 2 cards of the same kind and the other three all different? (4 stages)

ANSWERS TO EXERCISES

```
1. 3 + 4 + 6 + 1 = 14
   2. 5 + 3 + 4 + 3 = 15
   3. 4 \times 5 = 20
  4. 4 \times 3 \times 3 = 36, 5 \times 3 \times 3 = 45
  5. 3 \times 2 \times 3 \times 4 \times 2 = 144
  6. 20 \times 18 = 360, 20 \times 19 = 380. She needs 19.
  7. 5 \times 4 \times 3 \times 2 \times 1 = 120
  8. 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720
  9. 6 \times 4 \times 4 = 120
10. 5 \times 4 \times 3 \times 2 \times 1 = 120
11. 8 \times 7 \times 6 \times 5 \times 4 = 6720
12. 26, 26, 26, 10, 10, 10, 26^3 \times 10^3 = 17,576,000
13. 10^4 = 10,000
14. 10 x 9 x 8 = 720, \frac{1}{720}
15. 5 \times 4 \times 3 = 60
16. 9 \times 8 \times 7 \times 6 = 3024
17. a) 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720 b) 6 \times 3 \times 2 \times 2 \times 1 \times 1 = 72 c) \frac{72}{720} = \frac{1}{10}
18. a) 30 b) 21 c) 132 d) 220 e) 4845 f) 306 g)\frac{1}{20} h) \frac{1}{35}
19. 50!
20. \frac{16!}{12!}
21. a) \frac{10!}{6!} b) \frac{10!}{4!} c) \frac{10!}{8!}
          16!
22. 4! \frac{4!}{1!}, \frac{4!}{2!}, \frac{4!}{3!}, 24 + 24 + 12 + 4 = 64
23. {}_{12}C_3 = \frac{12!}{9!3!} = \frac{12 \times 11 \times 10}{3 \times 2 \times 1} = 220

24. {}_{20}C_5 = \frac{20 \times 19 \times 18 \times 17 \times 16}{5 \times 4 \times 3 \times 2 \times 1} = 15504, {}_{12}C_5 = \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1} = 792,
         {}_{8}C_{5} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56
25. _{100} C_{12} = \frac{100!}{88!12!}
26. _{7}C_{4} = \frac{7!}{4!3!} = \frac{7 \times 6 \times 5}{3 \times 2 \times 1} = 35
27. _{500}C_2 = \frac{500 \times 499}{2 \times 1} = 124,750
28. _{10}C_8 = \frac{10!}{8!2!} = \frac{10 \times 9}{2 \times 1} = 45
29. 4 \times 5 \times 2 = 40
```

30. 9 x 10 x 10 x 10 x 10 = 90,000 31. $24 \times 23 \times 10 \times 9 \times 8 = 397,440$ 32. 63 33. $\frac{24!}{20!}$ 34. 6! = 720, $\frac{1}{720}$ 35. 4! = 2436. $\frac{7!}{3!} = 7 \times 6 \times 5 \times 4 = 840$ 37. $\frac{7!}{3!} = 840$ 38. ${}_{15}C_5 = \frac{15 \times 14 \times 13 \times 12 \times 11}{5 \times 4 \times 3 \times 2 \times 1} = 3003$ 39. ${}_{15}C_5 = 3003$, ${}_{6}C_5 = 6$, ${}_{9}C_5 = \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} = 126$ 40. ${}_{6}C_{2} \times {}_{9}C_{3} = 15 \times 84 = 1260, {}_{6}C_{3} \times {}_{9}C_{2} = 20 \times 36 = 720$ 41. $\frac{52!}{5!47!}$, $\frac{13!}{5!8!}$ 42. $_{8}C_{3} = \frac{8!}{5!3!}$, $\frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$ 43. 6, 2, $\frac{8}{36} = \frac{2}{9}$ 44. $\frac{6!}{2!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = 360$ 45. a) ${}_{4}C_{2} \ge \frac{4!}{2!} = \frac{4!}{2!2!} \ge \frac{4!}{2!} = 6 \ge 12 = 72$ b) ${}_{5}C_{4} \ge 4 = 5! = 120$ c) 120 + 72 = 19246. $13 \times 48 = 624$ 47. ${}_{4}C_{2} = 13 \times 48 \times 47 \times 46 = 1,349,088$

ORGANIZATION

Placement Guide for Tabbed Dividers

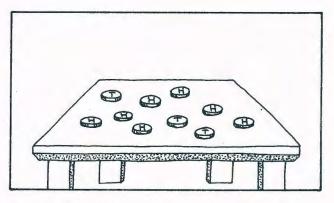
Tab Label	Follows page:
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INTRODUCTION

We have been basing our ideas of probability on the properties of the relative frequencies of the outcomes of an experiment. Since in the toss of a coin only two outcomes are possible, if heads comes up 503 times in 1000 tosses, tails must come up 497 times. The relative frequencies are $\frac{503}{1000}$ and $\frac{497}{1000}$ which add up to 1. The probability of heads on a single toss for this coin is about .5 and that of tails is about .5 and these also add up to 1.

Suppose you put a piece of adhesive tape on a nickel, toss the coin a large number of times and determine the relative frequency of heads for this particular biased coin. Assume the relative frequency turned out to be $\frac{73}{100} \approx .70$. On a further series of tosses we would expect heads to turn up about 70% of the time and we say



the experimental probability of heads is about .7. What is the probability of tails? If we expect heads about 70% of the time we would expect tails about 30% of the time and so the probability of tails is .3. The sum of these two probabilities is 1.

The same kind of argument tells us the probabilities of each of the various outcomes of a particular experiment will always add up to 1 so long as we are sure we have considered all of the possible outcomes.

Thus if we were to toss 5 coins and count the number of heads, we could get 6 different results since we can get 0, 1, 2, 3, 4 or 5 heads on a toss. If we could compute the probability of each of these events the sum would be 1.

The sum of the probabilities associated with all the outcomes of a particular experiment is always 1.

If there are only two possible outcomes of an experiment we sometimes rather arbitrarily call one of them a <u>success</u>, label it S and the other <u>not a success</u> and label it \overline{S} . Then by the principle above $P(S) + P(\overline{S}) = 1$.

In rolling a die we might consider the roll of a $[\bullet]^{\bullet}$ or a 3 to be a success. Then $P(S) = P(3) = \frac{1}{6}$ and $P(\overline{S}) = P(\text{not rolling a three}) = \frac{5}{6}$. For the biased nickel we might consider tossing a head to be a success. P(S) = P(Head) = P(H) = .7 and $P(\overline{S}) = P(\text{Tail}) = P(\text{T}) = .3$. Sometimes an event consists of several different outcomes of an experiment. We call the event E and try to determine P(E). We will present several examples to show how to find the answer to this kind of a question. The more difficult ones will involve careful counting as described in the section on *Counting Techniques* in CONTENT FOR TEACHERS.

COUNTING PROBABILITIES

Example 1 - Code "Words"

We make random code "words" by making all possible permutations of the letters of the word ROBIN. The event, E, consists of those words in which the consonants and vowels alternate. Thus, NIBOR would be considered a success for this event and BROIN a failure. What is P(E)? We count the number of permutations of the kind we want in E, and divide by the total number of permutations. This quotient is by definition P(E).

How many permutations are there of the letters in ROBIN? The answer, as we learned before is 5! = 120. Of these, how many have the characteristic of having consonants and vowels alternating? Of the five letters, three are consonants and two are vowels. Therefore consonants must be in the first, third and fifth position and vowels in the second and fourth. Since successive choices are to be made we multiply the number of possibilities at each stage to get:

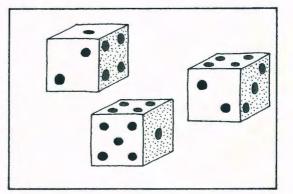
Stages	lst Consonant		2nd Vowel		3rd Consonant		4th Vowel		5th Consonant		
Choices	3	x	2	х	2	х	1	x	1	=	12

The product is 12. The probability of this event E is then $\frac{12}{120} = \frac{1}{10}$. Thus P(E) = .1. Since P(E) + P(E) = 1, P(E) = 1 - P(E) = 1 - .1 = .9.

Example 2 - Rolling Three Fair Dice

In rolling three true dice, suppose we consider the event a success if at least two dice show the same number. Thus the roll of (1,4,4), (1,1,6) or (2,2,2) would be considered a success. What is P(E)?

The counting here is a little tricky. In how many ways can the three dice fall?



Counting the branches of a three-stage tree shows there are 6 ways to choose at each stage so the answer is 6 x 6 x 6 = 216. If success is the event that at least two dice show the same number then failure means all three dice are different. This time it is easier to count the failures that the successes. For failure we have 6 options at the first stage, only 5 at the second since the second die cannot have the same number as the first die, and finally 4 at the third. There are therefore 6 x $6 \times 5 \times 4 = 120$ branches that lead to failure and $P(\overline{E}) = \frac{120}{216} = \frac{20}{36} = \frac{5}{9}$. Finally then $P(E) = 1 - \frac{5}{9} = \frac{4}{9}$. There are 216 - 120 or 96 branches that lead to E.

Could we count directly the number of branches that lead to E? Remember that E consists of all those rolls that have at least two dice showing the same number. The branches in E are of two kinds: first; those that have all three dice the same, and second; those that have two the same and one different. There are exactly 6 branches of the first kind. To count those of the second kind, remember that each branch has three stages. First choose the two stages that are to have the same number. This can be done in ${}_{3}C_{2} = 3$ ways. Then choose the number that appears twice. This can be done in 6 ways. The third choice is of the other number and can be done in 5 ways. Then the total number of branches of the second kind is $3 \ge 6 \ge 5 = 90$. Adding the counts we get 6 + 90 = 96 for the number of branches in E. P(E) is then equal to $\frac{96}{216} = \frac{4}{9}$ just as before.

Exercises (Answers given on p. 211)

- 1. If code "words" are made up from all the letters of the word MINUTE, what is the probability that the consonants and vowels alternate?
- 2. Four commuters drive into the city and park their cars. There are 4 different garages for them to park in and each garage is big enough to handle all the cars. If each driver chooses a garage at random what is the probability that the four cars are all in different garages? What is the probability they are all in the same garage?
- 3. On throwing two dice, what is the probability of scoring either a 7 or an 11?
- 4. A bag contains 4 marbles, each of a different color. One marble is drawn, its color recorded and the marble put back. This is done four times. What is the probability that
 - a) each color was drawn once?
 - b) each marble drawn had the same color?
 - c) three marbles were the same color and one a different color?
- 5. In a class of twenty students, there are three sisters. If a committee of three is chosen at random, what is the probability that exactly two of the three sisters are on the committee? (Hint: consider 2 stages. Choose the two sisters, then the third member.)

PROBABILITIES WITH MORE DIFFICULT COUNTING

Example 3 - Tossing a Biased Nickel with P(H) = .7

If we toss our biased nickel five times and consider getting either 4 or 5 heads a success for the event E, then getting 0, 1, 2, or 3 heads is a failure. What is P(E)?

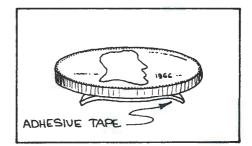
Consider a five-stage tree and determine how many branches lead to success, what the probability is for each branch and then add the results.

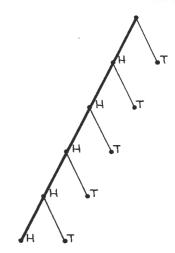
There is only one branch leading to 5 heads and at each successive stage

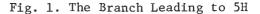
P(H) = .7.Multiplying the probabilities along this

branch we see that

There are several branches leading to 4 heads. Any such branch must have exactly one tail. We have seen how to count the number of such branches. We first count the number of ways to choose the positions for the 4 H's along the 5 stages. This is ${}_{5}C_{4} = \frac{5!}{4!1!} = 5$. What are the probabilities along each such branch? Each branch has 4 H's whose probabilities are each .7 and one T whose probability is .3. Therefore, multiplying the probabilities along this branch we find P(a branch with 4H and 1T) = (.7)(.7)(.7)(.7)(.3)= .07203.







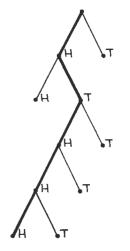


Fig. 2. One of the Branches Leading to 4H (HTHHH)

There are 5 such branches so P(4H) = 5 x .07203 = .36015. Finally, P(E) = P(5H) + P(4H) = .16807 + .36015

= .52822 ≈ .53

or just over a half. Of course, $P(\overline{E}) = 1 - .52822 = .47178 \approx .47$. A hand calculator will be a great help in determining such probabilities.

Note on notation. In the second part of this example we were looking for "the probability of getting exactly 4 heads when we know that we are tossing a coin 5 times with P(H) each time equal to .7." To save writing out such a long sentence, the following shorthand notation is frequently used:

P(4:5, .7)

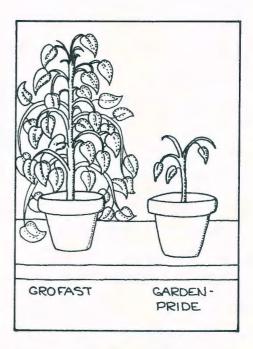
This is read "The probability of getting 4 successes (in this case 4 heads) in 5 trials of an experiment where the probability of success on one trial is .7."

What we have just found is $P(4:5, .7) \approx .36$.

This kind of a situation arises many times in questions about probability in everyday affairs as illustrated in the next two examples.

Example 4

A salesman claims the GROFAST plant food he sells is better than his rival's GARDENPRIDE. We ask how much better? He says at least 90% of the time, if two tomato plants are grown under the same conditions except for different food, the one given GROFAST will be in better shape at the end of a month. We decide to try out 10 pairs of tomato plants giving one in each pair GROFAST and the other GARDENPRIDE. At the end of a month only 8 of the plants given GROFAST are in better shape. Now the question comes up: Can we determine the probability of getting 8 successes in 10 trials with P(S) = .9 on each trial? This is simply P(8:10, .9). We look at a 10-stage tree and count the branches with 8 successes and 2 failures.



There are ${}_{10}C_8$ such branches. The probability on each branch is determined by multiplying the probabilities at each stage along the branch. This gives $(.9)^8 \times (.1)^2$. Thus the answer as worked out with a hand calculator* is $P(8:10, ..., 9) = 10Co(..., 9)^8(..., 1)^2$

$$(8:10, .9) = {}_{10}C_8(.9)^{\circ}(.1)^2$$
$$= {}_{10} \times {}_{9} \\ 2 \times 1 \\ (.4305)(.01)$$
$$\approx .1937 \approx .194$$

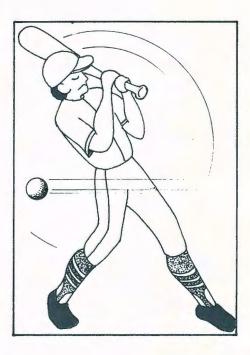
Is this so low that the salesman's claim should be rejected? Perhaps 10 was too small a number of trials. Further discussion of this kind of a problem will be found in the section on *Inferential Statistics* in the CONTENT FOR TEACHERS section.

Example 5

a11.

At a big league ballgame, your favorite player is Joe Hitemhard who has a batting average of .300. This means his relative frequency of hits out of times at bat is $\frac{300}{1000}$. So the probability of his getting a hit each time he comes up is estimated at .3. In this game, he does not get a single hit in his five times at bat. Some people would claim he is no .300 hitter. If Joe really is a .300 hitter, what is the probability that he gets no hits out of five times at bat? It is

 $P(0:5, .3) = ({}_{5}C_{0})(.3)^{0}(.7)^{5}$ $= (1)(1)(.16807) \approx .168 \approx \frac{1}{6}.$ Actually then, in about 1/6 of the games in which Joe comes to bat 5 times he will get no hits at



What is the probability that Joe will get at least one hit? This is a different question from asking what is the probability that he will get exactly one hit. The latter is $P(1:5, .3) = {}_{5}C_{1}(.3)^{1}(.7)^{4} \approx .360$. To find the answer to the first question we note that he either gets no hits at all or he gets at least one. Since this is all that can happen the two probabilities must add up to 1. We found before that $P(0:5, .3) \approx .168$. But

> P(no hits) + P(at least one hit) = 1. So .168 + P(at least one hit) = 1 P(at least one hit) = 1 - .168 \approx .832.

*An explanation on how to use a hand calculator is given at the end of this section.

A GENERAL CASE

To find a general formula for probabilities such as that of rolling exactly two sixes when rolling five dice, i.e., $P(2:5, \frac{1}{6})$, let's look again at one of the examples we worked out before. We said $P(8:10, .9) = {}_{10}C_8(.9)^8(.1)^2$.

We are interested in those branches of the 10-stage tree having 8 successes and 2 failures. How many such branches are there? One for each choice of the position of 8 successes along a 10-stage branch. The number of possible choices is $_{10}C_8$. The probability along each branch is found by multiplying together the probabilities of success, .9, on each of the 8 stages where we had a success and the probability of failure, .1, in each of the 2 stages where this happens. We have $_{10}C_8$ branches each with probability (.9)⁸(.1)². The final result is:

 $P(8:10, .9) = {}_{10}C_8(.9)^8(.1)^2.$

This formula could also be written as

$$P(8:10, .9) = {}_{10}C_8(.9)^8(1 - .9)^{10-8}$$

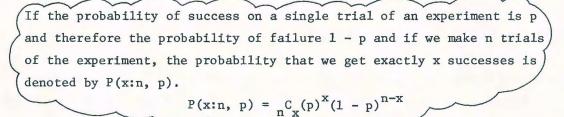
Thus for the dice

$$P(2:5, \frac{1}{6}) = {}_{5}C_{2}\left(\frac{1}{6}\right)^{2}\left(1 - \frac{1}{6}\right)^{5-2}$$
$$= \frac{5 \cdot 4}{2 \cdot 1}\left(\frac{1}{6}\right)^{2}\left(\frac{5}{6}\right)^{3}$$
$$= 10 \cdot \frac{1}{36} \cdot \frac{125}{216}$$
$$\approx .161$$

Following the pattern of these two examples

$$P(9:25, .3) = {}_{25}C_{9}(.3)^{9}(.7)^{16}$$

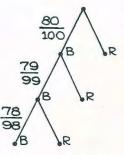
In general, we might be concerned with n trials of an experiment where the probability of success on each trial is p with 0 . The number of successes is calledx where <math>0 < x < n. The general statement is as follows:



If the number of marbles in a bag is very large compared to the number drawn in the sample, the probabilities involved in drawing with replacement and without replacement are very close. Probabilities with replacement are so much easier to work with; we use them in computing even when such a sample is drawn all at once instead of in successive stages with replacement. In the following exercises, you are asked to compute a few probabilities when n is fairly small. If n is larger than 5 or 6 the arithmetic becomes laborious unless you have a hand calculator or a computer. Also tables to use in these situations are available. In the next paragraph we will discuss their use as well as that of calculators and will suggest a program in BASIC to use if such a computer is available.

Exercises

- 6. A marksman has a probability of .8 of getting a bull's-eye. What is the probability he gets exactly 5 bull's-eyes out of his next six shots?
- 7. In the late afternoon poor light lowers the probability of getting a bull's-eye to .5. What is the probability he gets exactly 3 bull's-eyes out of his next 6 shots?
- 8. The light bulb machine produces bulbs with probability of a bad bulb of .1. In a sample of 10 bulbs, what is the probability of 0 or 1 bad bulbs?
- 9. A jar of colored marbles has 1/4 black and 3/4 red ones. A marble is drawn at random, its color recorded, the marble put back, and the experiment repeated until 4 marbles have been drawn. Find
 - a) P(no black marbles)
 - b) P(no red marbles)
 - c) P(1 black and 3 red marbles)
- 10. Suppose a bag of 100 marbles of which 80 are black and 20 red. If marbles are drawn in succession without replacement, what is the probability of getting 3 black marbles in succession? To do this, draw part of the threestage tree. Why do the probabilities change? Compare this result with what you would get if you replaced the marble after each draw.



11. One fifth of the marbles in a large bag are black and the rest are red. A sample of 5 is drawn all at once. Find P(3 black and 2 red).

DETERMINING P(X:N, P) MORE EASILY

USING TABLES

There are extensive tables of the probabilities P(x:n, p) for many values of n and p. Remember that x is the number of successes in n trials of an experiment and p is the probability of success on any one trial. At the end of this section, brief tables are given for n = 5, 10, 15, 20 and 25 and p = .1, .2, ..., .9. In each of these 5 tables the value of x is read on the left, the value of p on the top and

CONTENT FOR TEACHERS

the probabilities wanted at the intersection of the horizontal and vertical lines drawn through these values. Thus

P(8:10, .7) = .233 and P(12:15, .2) = 0+, i.e., less than .001 and P(19:25, .7) = .147.

Example 6

A fair coin is tossed 25 times. What is the probability of getting 14 heads? This is P(14:25, .5) and the table gives .133.

Example 7

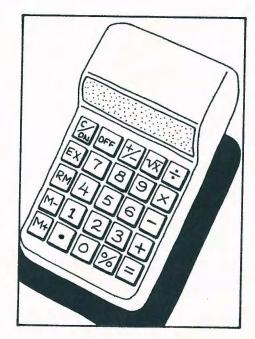
For a biased coin with P(H) = .7, how many heads are most likely to occur if we toss it 25 times? Looking in the table we see that P(18:25, .7) = .171 and this is larger than any other probability in the .7 column. The answer is that the number of heads is more likely to be 18 than any other number.

USING A HAND CALCULATOR

Suppose you want to compute P(7:10, .4) using a hand held calculator. If your calculator has an automatic constant this can be done as follows. First test the calculator for this feature. Turn the calculator on, enter 5, punch the x button, enter 3 and push the = button several times. If the readout is successively 15, 45, 135, your calculator is using 3 as a constant multiplier.

 $P(7:10, .4) = {}_{10}C_7(.4)^7(.6)^3$ $= \frac{10 \times 9 \times 8}{3 \times 2 \times 1}(.4)^7(.6)^3$

We work forwards right through the problem except that instead of multiplying 10 x 9 x 8 first we compute 10 \div 3 x 9 \div 2 x 8 so that numbers remain reasonably small. This would be particularly important in finding ₂₀ C₁₃ in the next example.



Enter 10, punch the \vdots button, enter 3, punch the x button, enter 9, etc. until the $_{10}C_7$ is computed. (It will read 199.999 instead of the exact value 120.) Then punch x and enter .4. The .4 is now in as a constant multiplier. Press the = button seven times, punch x, enter .6 and punch = three times. The answer, .04246731, is now displayed by the calculator. It can be checked in the tables below which, however, only give the answer to three places, .042. The following chart illustrates the procedure.

$$10 \div 3 \times 9 \div 2 \times 8 \times .4 = = = = = = \times .6 = = .04246731$$

Let's try P(13:20, .8) = ${}_{20}C_{13}(.8)^{13}(.2)^7$

$$\frac{20 \times 19 \times 18 \times 17 \times 16 \times 15 \times 14}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} (.8)^{13} (.2)^{7}$$

The sequence gives:

and the answer is .05454982 which to three places is .055, agreeing with the tables.

USING A BASIC COMPUTER PROGRAM

The BASIC language is perhaps the most common one available on time sharing computers in schools. A BASIC program to compute P(x:n, p) for any integral value of x from 1 to n-1 may be found as Program No. 16 in the *APPENDIX*. n must be a positive integer and p a positive number less than 1. If x = 0, the probability is $(1 - p)^n$ and if x = n, the probability is p^n , so all cases can be run off quickly. In fact, the program can be easily modified to let x run from 0 to n with the corresponding probabilities printed out in succession.

Exercises

12. A fair coin is tossed 20 times. What is the probability of more than 12 heads? This means 13 or 14 or any other number up to 20. To get the answer add up all the probabilities for all these numbers.

$$P(x > 12) = .074 + .037 + .015 + .005 + .001 + 0 + 0 + 0$$

= .132

What is the probability of either 10 or 11 or 12 heads?

13. If 60% of the voters in New York City are Democrats, what is the probability that in a random sample of 25 there are 15 Democrats?

14. Use a calculator to compute P(7:10, .8).

- 15. The probability of rolling a die to give a 💽 is 1/6. Use a calculator to compute the probability of exactly one 💽 in 10 rolls.
- 16. Read from the tables P(1:10, .1) and P(1:10, .2). Compare these with your answer in Exercise 15. Since 1/6 lies between .1 and .2, wouldn't you expect this to happen?

PROBABILITY WITH COUNTING

CONTENT FOR TEACHERS

ħ	x	.10	.20	. 30	.40	р .50	.60	.70	.80	.90	x
5	0	590	328	168	078	031	010	002	0+	0+	0
-	1	328	410	360	259	156	077	028	006	0+	1
	2	073	205	309	346	312	230	132	051	008	2
	3	008	051	132	230	312	346	309	205	073	3
	4	000	006	028	077	156	259	360	410	328	4
	4	01	000	020	077	100	237	500	410	520	
	5	0+	0+	002	010	031	078	168	328	590	5
10	0	349	107	028	006	001	0+	0+	0+	0+	0
	1	387	268	121	040	010	002	0+	0+	0+	1
	2	194	302	233	121	044	011	001	0+	0+	2
	3	057	201	267	215	117	042	009	001	0+	3
	4	011	088	200	251	205	111	037	006	0+	4
	5	001	026	103	201	246	201	103	026	001	5
	5			037	111	205	251	200	088	011	6
	6	0+	006						201	057	7
	7	0+	001	009	042	117	215	267			
	8	0+	0+	001	011	044	121	233	302	194	8
	9	0+	0+	0+	002	010	040	121	268	387	9
	10	0+	0+	0+	0+	001	006	028	107	349	10
15	0	206	035	005	0+	0+	0+	0+	0+	0+	0
	1	343	132	031	005	0+	0+	0+	0+	0+	1
	2	267	231	092	022	003	0+	0+	0+	0+	2
	3	129	250	170	063	014	002	0+	0+	0+	3
	4	043	188	219	127	042	007	001	0+	0+	4
	5	010	103	206	186	092	024	003	0+	0+	5
	5	010	043	147	207	153	061	012	001	0+	6
	6		043	081	177	196	118	035	001	0+	7
	7	0+				196	177	035	014	0+	8
	8	0+	003	035	118			147	014	002	9
	9	0+	001	012	061	153	207	14/	043	002	9
	10	0+	0+	003	024	092	186	206	103	010	10
	11	0+	0+	001	007	042	127	219	188	043	11
	12	0+	0+	0+	002	014	063	170	250	129	12
	13	0+	0+	0+	0+	003	022	092	231	267	13
	14	0+	0+	0+	0+	0+	005	031	132	343	14
	15	0+	0+	0+	0+	0+	0+	005	035	206	15
20	0	122	012	001	0+	0+	0+	0+	0+	0+	0
20	1	270	058	007	0+	0+	0+	0+	0+	0+	1
	2	285	137	028	003	0+	0+	0+	0+	0+	2
	23	190	205	028	012	001	0+	0+	0+	0+	3
	3		203	130	035	001	0+	0+	0+	0+	4
	4	090	210	130	055	005	01	01	0.	01	-

PROBABILITY T.	ABLES
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				PROI	BABILI'		LES (co	ont.)			
n	x	.10	.20	. 30	.40	р .50	.60	.70	.80	.90	x
20	5	032	175	179	075	015	001	0+	0+	0+	5
20	5 6	009	109	192	124	037	005	0+	0+	0+	6
	7	003	055	164	166	074	015	001	0+	0+	7
	/									0+	8
	8	0+	022	114	180	120	035	004	0+		0
	9	0+	007	065	160	160	071	012	0+	0+	9
	10	0+	002	031	117	176	117	031	002	0+	10
	11	0+	0+	012	071	160	160	065	007	0+	11
	12	0+	0+	004	035	120	180	114	022	0+	12
	13	0+	0+	001	015	074	166	164	055	002	13
	14	0+	0+	0+	005	037	124	192	109	009	14
	15	0+	0+	0+	001	015	075	179	175	032	15
		0+	0+	0+	0+	005	035	130	218	090	16
	16					001	012	072	205	190	17
	17	0+	0+	0+	0+						18
	18	0+	0+	0+	0+	0+	003	028	137	285	
	19	0+	0+	0+	0+	0+	0+	007	058	270	19
	20	0+	0+	0+	0+	0+	0+	001	012	122	20
25	0	072	004	0+	0+	0+	0+	0+	0+	0+	0
		199	024	001	0+	0+	0+	0+	0+	0+	1
	2	266	071	007	0+	0+	0+	0+	0+	0+	2
	3	226	136	024	002	0+	0+	0+	0+	0+	3
	1 2 3 4	138	187	057	007	0+	0+	0+	0+	0+	1 2 3 4
	5	065	196	103	020	002	0+	0+	0+	0+	5
	5 6	003	163	147	044	005	0+	0+	0+	0+	6
	0					014	001	0+	0+	0+	7
	7	007	111	171	080						78
	8	002	062	165	120	032	003	0+	0+	0+	0
	9	0+	029	134	151	061	009	0+	0+	0+	9
	10	0+	012	092	161	097	021	001	0+	0+	10
	11	0+	004	054	147	133	043	004	0+	0+	11
	12	0+	001	027	114	155	076	011	0+	0+	12
	13	0+	0+	011	076	155	114	027	001	0+	13
	14	0+	0+	004	043	133	147	054	004	0+	14
	15	0+	0+	001	021	097	161	092	012	0+	15
		0+	0+	001	009	061	151	134	029	0+	16
	16									002	17
	17	0+	0+	0+	003	032	120	165	062		
	18	0+	0+	0+	001	014	080	171	111	007	18
	19	0+	0+	0+	0+	005	044	147	163	024	19
	20	0+	0+	0+	0+	002	020	103	196	065	20
	21	0+	0+	0+	0+	0+	007	057	187	138	21
	22	0+	0+	0+	0+	0+	002	024	136	226	22
	23	0+	0+	0+	0+	0+	0+	007	071	266	23
	24	0+	0+	0+	0+	0+	0+	001	024	199	24
	25	0+	0+	0+	0+	0+	0+	0+	004	072	25

PROBABILITY TABLES (cont.)

ANSWERS TO EXERCISES

1. $\frac{6 \times 3 \times 2 \times 2 \times 1 \times 1}{6!} = \frac{\cancel{6} \times \cancel{2} \times \cancel{2} \times 2}{\cancel{6} \times 5 \times 4 \times \cancel{2} \times \cancel{2}} = \frac{1}{10}$ 2. P(different garages) = $\frac{4!}{4^4} = \frac{\cancel{4} \times 3 \times 2 \times 1}{\cancel{4} \times 4 \times 4 \times 4} = \frac{3}{32}$ P(all in same) = $\frac{4}{4^4} = \frac{1}{64}$ 3. P(7 or 11) = P(7) + P(11) = $\frac{6}{36} + \frac{2}{36} = \frac{8}{36} = \frac{2}{9}$ 4. P(all different) = $\frac{4!}{4^4} = \frac{3}{32}$ $P(all same color) = \frac{4}{4^4} = \frac{1}{64}$ P(3 same and one different) = $\frac{S}{44}$: To compute S, choose the repeating color in 4 ways, then choose the other color in 3 ways, then arrange 4 things three of which are alike in 4 ways. Therefore $S = 4 \times 3 \times 4 = 48$ and P(3 same and one different = $\frac{48}{256} = \frac{3}{16}$. 5. $\frac{{}_{3}C_{2} \times {}_{17}C_{1}}{{}_{20}C_{3}} = \frac{3 \times 17}{(20 \times 19 \times 18)/(3 \times 2 \times 1)} = \frac{17}{20 \times 19} = \frac{17}{380} \approx .045$ 6. P(5:6, .8) = ${}_{6}C_{5}(.8)^{5}(.2)^{1} = 6(.8)^{5}(.2) \approx .393$ 7. $P(3:6, .5) = {}_{6}C_{3}(.5)^{6} = 20(.5)^{6} \approx .313$ 8. $P(0:10, .1) + P(1:10, .1) = {}_{10}C_0(.1)^0(.9)^{10} + {}_{10}C_1(.1)^1(.9)^9$ = 1 x 1 x .314 + 10(.1)(.349) = .314 + .349 = .663 9. a) $P(0:4, .25) = {}_{4}C_{0}(.25)^{0}(.75)^{4} = (1)(1)(.316) = .316$ b) $P(4:4, .25) = {}_{4}C_{4}(.25)^{4}(.75)^{0} = 1(.004)(1) = .004$ c) $P(1:4, .25) = {}_{4}C_{1}(.25)^{1}(.75)^{3} = 4(.25)(.75)^{3} = .422$ 10. P(BBB) = $\frac{80}{100} \times \frac{79}{99} \times \frac{78}{98} \approx .508$. The probabilities change since if a black marble is drawn there remain only 79 black ones out of a total of 99. If the marbles are replaced each time P(B) = .8 and $P(BBB) = (.8)^3 = .512$, slightly higher. $P(3B) = P(3:5, .2) = {}_{5}C_{3}(.2)^{3}(.8)^{2} = 10(.008)(.64) \approx .0512$ 11. 12. P(10 or 11 or 12) = P(10:20, .5) + P(11:20, .5) + P(12:20, .5)+ .120 = .456 from tables ----.176 + .160 13. P(15:25, .6) from tables = .161 $P(7:10, .8) = {}_{10}C_7(.8)^7(.2)^3 = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} (.2097)(.008) \approx .201$ 14. 15. P(1:10, $\frac{1}{6}$) = ${}_{10}C_1(\frac{1}{6})^1(\frac{5}{6})^9 = 10 \times \frac{5^9}{610} \approx .322$ 16. P(1:10, .1) from tables = .387 P(1:10, .2) from tables = .268 1/6 = /167 which is 2/3 of the way from .1 to .2. 2/3 of the way from .387 to .268 = .387 - 2/3[.119] = .387 - .079 = .308, not too far from .323

ORGANIZATION

Placement Guide for Tabbed Dividers

Tab Label	Follows page:
CONTENT FOR TEACHERS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
Probability with Counting	
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Appendix-Computer Programs	
DIDACTICS	
Components of Instruction	
Classroom Management	
Statistics and Probability Learning	
TEACHING EMPHASES	
Critical Thinking	
Decision Making	
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Counting Techniques	
APPLICATIONS	
Business & Commerce	
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Health & Medicine	
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GLOSSARY	
ANNOTATED BIBLIOGRAPHY	
SELECTED ANSWERS	

ANNASSANNAL Strangs

INTRODUCTION

HOW CAN WE USE THE STATISTICAL IDEAS WE HAVE BEEN DEVELOPING TO HELP MAKE DECI-SIONS? We have posed several situations whose investigations involve such use.

• Does the mean, median or mode of the wages and salaries in a business or manufacturing concern give a person looking for a job the best guide for short term and/or long term salary prospects?

• Is a coin or a die fair? How can we find out?

• Should the buyer of ball point pens selling for 39¢ apiece test a sample of such pens? If so, how big should a sample be and what propor-

tion of poor pens can be tolerated? Would the decisions be different for a small local store than for a nationwide chain of 1250 such stores?

• Should I buy a new MONTEZUMA car? The salesman claims that 90% of former buyers feel it is the best car for the money. Is it worthwhile trying to find out on how many MONTEZUMA cars the claim is based?

• Are three tests in a semester a fair judge of a student's accomplishment?

• Is grade inflation a serious problem in my school? How can I even begin to tackle such a question?

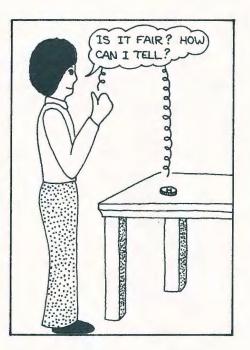
Looking more closely at the fair coin problem will enable us to see how statistical ideas can help.



WHEN IS A COIN FAIR?

We look at a coin. It has a head on one side and tails on the other. It looks like an ordinary coin and we assume it is a "fair" coin. The coin is tossed vigorously so that it spins many times in the air before falling on the table. If it comes up heads ten times in a row we may decide it is not a fair coin. Why? Of course, this decision involves knowing what we mean by a "fair coin." It also involves knowing what we mean by "decide." By a "fair coin" we usually mean a coin that in the long run comes up heads about as often as it comes up tails. That is, the ratio of the number of heads to the total number of tosses is close to 1/2. This is what we have called the experimental probability of heads and we have used it to write $P(H) \approx \frac{1}{2}$. Sometimes we say the ratio of heads to total tosses is one-half and write $p = \frac{1}{2}$.

Usually we "decide" if "we are reasonably sure." Of course, sometimes "we are positive" or "almost positive" but most people would not expect absolute certainty. Also "reasonably sure" may have different connotations under different circumstances.



Exercises (Answers given on pp. 237-241)

- To test the fairness of a new coin, Tom suggests that we toss it several times. He claims if it comes up heads more often than tails it is not fair and is biased towards heads. On the other hand, if it comes up tails more often than heads then it is biased towards tails. Is this test a good basis for deciding whether the coin is fair or not? What happens if the coin is tossed three times? How many times should the coin be tossed before a decision on fairness is made?
- 2. An advertisement of 9-volt Long Life batteries for a small calculator claims "the average life of these batteries is 100 hours." The Long Life battery in your calculator dies after 90 hours of service. You decide never to buy a Long Life battery again. Is your decision justified? Suppose the advertisement had read "the minimum life of these batteries is 100 hours." Would your decision be justified this time?
- 3. Suppose you roll a die three times and get a 6 each time. What is your decision as to the fairness of the die? How confident are you that you are right?
- 4. Three irate customers out of a hundred claim their 39¢ ball point pens are faulty. As store owner, would you refuse to stock this variety of pen?
- 5. Three irate customers out of 100 claim their \$390.00 HiFi systems are faulty. It costs \$100 apiece to fix up the systems. As store owner, would you refuse to stock that brand of HiFi systems again?



We assumed the coin came up heads 10 times in a row. If the coin is fair, $P(H) = \frac{1}{2}$ on each toss. The chance of getting ten heads in a row can be worked out by considering a tree of ten stages with only one branch leading to 10 heads. Multiplying along this branch we get $(\frac{1}{2})^{10} = \frac{1}{1024}$ thus P(10 heads in a row) = $\frac{1}{1024}$ or a little less than one in a thousand. Since this is so small we really feel quite confident that the coin is not fair. But of course it could have been. Just because the chance of tossing 10 heads in a row is less than one in a thousand does not mean it can <u>never</u> happen.

Suppose all 160,000,000 people in the United States whose age is 14 or more had each tossed a fair coin 10 times. About one person in a thousand would have tossed ten heads in a row. In other words, this would have happened in $\frac{160,000,000}{1,000}$ or 160,000 cases. Of course, we might have been one of these 160,000 cases but we are much more inclined to feel we would have been among the 159,840,000 people who did not get 10 heads in a row with a fair coin.

Since the 10 heads did show up we feel reasonably sure that the coin is not fair. We measure our certainty by saying if the chance of this happening with a fair coin is about .001 then, turning it around, we can say, when it does happen, the chance or probability that the coin is fair is only .001. Then the probability that the coin is biased (which is the only alternative) is 1 - .001 or .999. We can restate this to say "we are 99.9% sure that the coin is biased."

If we had only tossed the coin five times and gotten 5 heads in a row we would not have been so confident the coin was biased. The chance of this happening with a fair coin is $(\frac{1}{2})^5 = \frac{1}{32}$ or about .03. Again turning it around, if we get 5 heads in a row with a certain coin, we say the probability this coin is fair is .03 and the probability it is biased is 1 - .03 = .97.



Exercises

- 6. If the 1600 students in a certain school system each toss a fair penny six times, about how many should get six tails in a row?
- 7. Vital statistics of the country SLOVENIA show there are about 20,000 families with five living children. Of these, 700 families have five boys. If the probability of having a boy were 1/2, about how many families would have five boys? Do you think P(B) = 1/2 in SLOVENIA?
- 8. If a die is "fair," P(.) = 1/6. If 40,000 people each toss a fair die 4 times, about how many would get four . 's? If they tossed it five times about how many might get five ... 's?
- 9. In a rifle club, the award of GOODSHOT is given to anyone whose probability of getting a bull's-eye is .9. That is, over the season, a GOODSHOT averages 9 bull's-eyes out of 10 shots. John claims to be a GOODSHOT.



As you watch him shoot, you see he fails to make a bull's-eye in his first four shots. You decide he is lying. How confident are you that this decision is right? A SUPERSHOT award is given to those with P(Bull's-eye) = .95. Sue makes 12 bull's-eyes in a row. Is she really a GOODSHOT having an exceptional match or is she a SUPERSHOT? Can you decide?

THE DIVIDING LINE

Having tossed five heads in a row, we might be 97% sure the coin was biased but many of us would want to wait for further evidence.

When making a decision, do we want to be 90% sure we are right or 95% or 99% or, if possible, absolutely certain? It usually depends on the situation. If we have to make a decision on introducing a new medicine or a new procedure in medical treatment, we may want to be at least 99.9% sure it is not lethal. Tragic mistakes have been made because sufficient experiments were not made to insure this. Remember the introduction and use of Thalidomide some years ago before it was found to cause malformations in many of the babies of mothers who used it during pregnancy.

On the other hand, if the committee to select a band for a school dance are 95% sure or maybe even 80% sure that a majority of the students prefer a Hard Rock band to a Rock-n-Roll band, they might be perfectly happy about selecting such a band.

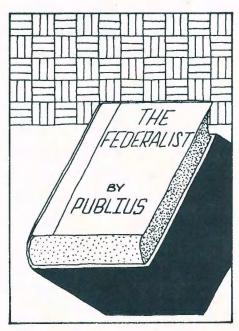
Exercises

- 10. What percent of certainty that you are correct would you want in making decisions in the following cases?
 - a) Breakfast cereal A is better than B.
 - b) Fluoridation of the water supply in your city is worthwhile.
 - c) As a doctor, a new local anesthetic for a minor operation is better than the old one.
 - d) As a doctor, a new anesthetic for major open-heart surgery is better than the old one.
 - e) To accuse a student of cheating on your test.
 - f) To bet on FLEETWING in the Kentucky Derby.
 - g) To buy a record in hopes it will please your mother.
 - h) To buy a book for yourself.
 - i) To quit teaching and try something new.
 - j) To introduce some statistics into your math class.

We assumed the coin was fair. We look at the result of the experiment, 10 heads in 10 tosses. The probability of this result is very low, one in 1024, or about .001. But it happened. We now turn the situation around and say, in the light of this evidence, the probability that the coin is fair is only about .001 and therefore the probability that it is biased is 1 - .001 or .999. Putting it as a percent, we say we are 99.9% sure that the assumption of fairness is not correct and the coin is actually biased. We have made a decision and if necessary we can act on it.

DECIDING AUTHORSHIP

The Federalist papers were written in 1787-88 to persuade the people of New York State to ratify the Constitution of the United States. Some of the papers were written by James Madison, some by Alexander Hamilton and some by John Jay but all were published under the name "Publius." Although the authors of most of the papers are well known, there are twelve for which the authorship has been disputed for many years. Recently Mosteller and Wallace* subjected the disputed papers to a careful analysis of style and word usage. They state the probability that Madison wrote the first of

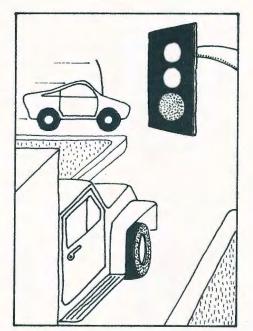


*Frederick Mosteller and David L. Wallace, "Deciding Authorship," in Statistics: A Guide to the Unknown, ed. by Judith M. Tanur (San Francisco: Holden-Day, Inc., 1972), p. 164. Permission to use granted by Holden-Day, Inc.

the disputed papers, #55, is .988, that he wrote the second one, #56, is .999 and that he wrote each of the other ten is even higher. In the light of this evidence if, of course, we accept it, most of us would be willing to credit the papers to Madison. Such high probability is not certainty but it is very convincing.

Exercises

- 11. Ms. Riley can take two different routes to get from her home to her school. The first route has three traffic lights that are not synchronized. The probabilities that the lights are green when Ms. Riley comes to each of the streets are .5, .6 and .6 respectively. The other route has 5 lights again not synchronized. The probability of hitting a green light at each of the first two lights is .6. Then she gets on a main road and for the next three lights the probabilities of green are .8, .9 and .8. Ms. Riley wants to take the route with the better chance of not having to stop even once for a red light. Which route should she take?
- 12. John Majic claims he has extrasensory perception. Jane is skeptical. She takes the A, K, Q of hearts and gives John the A, K, Q of spades. She arranges her cards face down in a line on the table and challenges John to arrange his in the same order without looking at hers. He does so and matches her arrange-



ment. She says, "You were just lucky. Let's see you do it with four cards." They each add the J and again after Jane rearranges her cards, John matches. What are the chances in each case that John was just lucky? (Hint: how many arrangements of three cards are there? of four?) What is the chance John was lucky both times? If John succeeds in matching five cards, what decision would you make as to his ability? Are you absolutely certain?

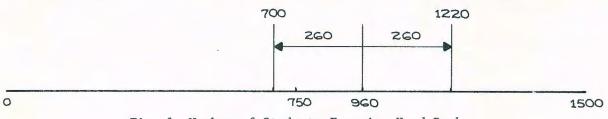
SELECTING A DANCE BAND

There are many kinds of problems to which statistical analysis may help us find answers. The answers are not likely to be certain as is the answer to "What is the sum of 5 and 3?". They may not even be exact but hopefully we can find an approximation to the answer and some degree of certainty as to the closeness of the approximation.

For instance, if a poll of 25 students in a school reports that 16 of them prefer a Hard Rock band to a Rock-n-Roll band for the next school dance, what can we say about the preference of all of the 1500 students in the school? In the sample,

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64% preferred Hard Rock. Would we be sure that 64% of 1500 or 960 students would prefer the Hard Rock band? No, not even if the 25 students polled had been a carefully selected random sample and the survey had been done with great care. All we could say would be that the number of students favoring Hard Rock would be somewhere "near" 960 and we are not even sure what the "near" is. Maybe, as a wild guess, it might mean within 260 on either side of the 960. The number favoring Hard Rock would then be somewhere in the interval from 700 to 1220. This is a very wide spread





going to even less than 750 which is 50% of the school. If the correct number is somewhere in the region from 700 to 1220, then the fraction of all the students fa-

voring Hard Rock is between $\frac{700}{1500}$ and $\frac{1220}{1500}$ or between 47% and 81%. Now 81% - 64% = 17% and 64% - 47% = 17% so we have a margin of error of 17% on either side of the 64% preference we found in our sample. We said before that this was a wild guess so we have little or no confidence in it.

Can our statistical methods help determine within what limits we can say the number or fraction of students favoring Hard Rock really lies and the confidence with which we can make this statement? <u>Yes</u>. We will begin to see how in the next paragraph and will keep working on it until the answer is determined on page 16.



Exercises

13. A pollster reports that his poll shows 52% of the voters in BIG CITY will vote for JOHN for mayor but the margin of error is 3%. He means that between (52-3)% and (52+3)% of the voters will actually vote for JOHN. If the vote cast is 236,522, JOHN's vote should be between what two values?

- 14. A poll of a sample of the students in a school of 2100 students claimed that Lucy would get 56% of the votes for Student Council, but the error in the predictions might be as high as 4%. In the election, Lucy actually got 1063 votes. Was the pollster right?
- 15. Tom claims that on the basis of his analysis of the buying habits of children at the school cafeteria between 350 and 400 of them will buy milk at lunch. There are 850 who go to lunch. What proportion is Tom predicting will buy milk and with what margin of error?
- 16. Poll A says Smith will get 52% of the votes in an election. They admit a possible error of 2%. Poll B says Jones will win with 51% with possible error in their results of 2%. In the election, 356,423 votes were cast and Smith got 179,832 votes. Which poll was correct?

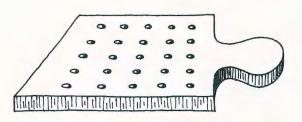
SIMULATION BY MARBLES

Perhaps it will help in considering the problem of the choice of a band to think of a simulation. We might think of a big bag of 1500 marbles, one for each student in the school with some of the marbles being red and some blue. The marbles represent the students, with the red ones representing those who favor the Hard Rock band and the blue those who favor the Rock-n-Roll. We do not know the number of each color nor the ratio of each kind to the whole. Taking a poll of a random sample of students is simulated by taking a random sample of marbles and counting the red and blue marbles. One quick and easy way to take such a sample is to shake up the marbles in the bag and then use a sampling board. This is a simple wooden paddle with 25 hemi-spherical holes.

The board can be thrust into the bag and brought out with 25 marbles, one in each hole (see Figure 2).

If such a sample showed 16 red marbles, it would simulate our poll in which 16 students favored the Hard Rock band. Suppose this happens.







What we want to do is use this result, 16 red marbles in a sample of 25, to determine as accurately as we can the ratio of red marbles to the total of red and blue and thus find the number of red marbles in the bag. Just because 64% of the marbles in the sample were red we cannot say 64% of all the marbles are red. What can we do?

SAMPLING FROM A POPULATION WITH KNOWN p

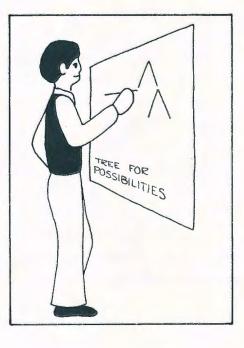
Have we seen any problem like this before? No, but in *Probability With Counting* in the CONTENT FOR TEACHERS section we looked at one which is the reverse of this. Maybe it will help us to study it again. In that problem we assumed we had a bag of 1500 marbles with say 70% red marbles. Then we figured out the probability that in a random sample of 25 marbles we would have 16 red ones. This was

$P(16:25, .7) = {}_{25}C_{16}(.7)^{16}(.3)^{9}$

We can calculate this or, fortunately, we can read it from the tables on page 12 of *Probability With Counting* of CONTENT FOR TEACHERS, as .134. This is not a very high probability. Also the situation here is not quite the same as the one required for the tables to give us an exact answer. We are drawing a sample of 25 marbles from the bag all at once. It is as though we were drawing them one after the other without replacement. In this case, the probability of getting a red marble on the second draw after drawing a red one the first time is not quite the same as the probability of getting a red one on the first draw. The tables assume the marbles are drawn with replacement and thus the probabilities are all the same. Are we justified in using the tables? Let's do some exercises to see how close the probabilities are when we draw with replacement and when we draw without replacement.

Exercises

- 17. A bag contains 3 red and 2 blue marbles. What is the probability of getting a red one on the first draw? After we record the color and put this marble back, we draw again. What is the probability of getting a red one? What is the probability that both the first and the second are red? If the marble is not put back, the situation in the bag depends on whether the first draw was a red or a blue. Find P(RR) in this case. (Hint: draw a tree and calculate the probabilities along the branches.)
- 18. A bag has 50 marbles: 35 red and 15 blue. Three marbles are drawn without replacement. What is the probability they are all red? If they are drawn with replacement, what is the probability they are all red?



19. A bag has 1500 marbles with 1050 red and 450 blue. Determine the two probabilities as in Exercise 18. Note how close the results are now.

In general, when the bag contains 50 or more marbles and the sample is 10% or less of those in the bag, the differences in the probabilities for drawing with replacement and drawing without replacement are so small the results are not significantly different. In these cases, we can use the table and using the table is much easier. Since we have 1500 marbles and samples of size 25, we are well inside the conditions stated and we use the tables.

To repeat: We found that when we know the percent, say 70%, of the whole school that favors Hard Rock we can determine the probability that in a sample of given size any specified number of students will favor Hard Rock bands. As a matter of fact, when the sample size is 25, we can read the probabilities from the table

as we did when we found that $P(16:25, .7) \approx .134$. Let's look at the whole column where p = .7. If we add up the probabilities for all the numbers from 0 - 13 inclusive, we get only .001 + .004 + .011 + .021 = .043. If we add all those from 22 - 25 we get .032 and the sum of these two is .043 + .032 = .075. Finally, adding up the probabilities for the middle group from 14 to 21 we get .923. Adding the probabilities for the three groups should give us 1 since we have included the probabilities for all possible outcomes in a sample of size 25. The fact that they add up to .998 instead of 1 is due to rounding errors. We can now say that if the percent favoring Hard Rock in the whole school is 70, then the probability is .923 that in our sample of 25 the number of children favoring Hard Rock will be somewhere between 14 and 21 (see Table 1).

What does this really mean? In sampling from the known population with 70% red could we never get all 25 red marbles or never get only 10? Certainly we could. What it does mean is if we took 100 random samples of size 25, about 92 of the samples would have from 14 to 21 red marbles and about 8 of them would have either less than 14 or more than 21. Thus a count of 10 or of 23 or of some other number not in the range 14 to 21 is possible but not probable, understanding that by not probable we mean that the probability for any or all of them is less than .075.

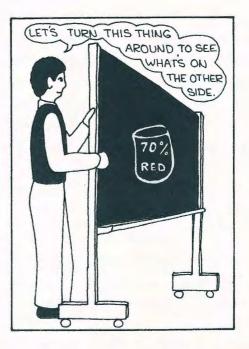
	TABLE	1
P (x:25,	.7)
x	Р	
0 1 2 3 4	0+ 0+ 0+ 0+ 0+	
5 6 7 8 9	0+ 0+ 0+ 0+ 0+	.043
10 11 12 13 14	.001 .004 .011 .027 .054	
15 16 17 18 19	.092 .134 .165 .171 .147	.923
20 21. 22 23 24	.103 .057 .024 .007 .001	.032
25	0+	J

Exercises

- 20. Look at the tables for probabilities in samples of size 25 on page 12 of *Probability with Counting* in the CONTENT FOR TEACHERS section. If the bag has 80% red marbles find the two numbers a and b corresponding to 14 and 21 above such that the probability is about .92 that the number of red marbles is between a and b inclusive.
- 21. Do the same if the bag has 40% red marbles and the probability is about .94 that n is between a and b.

SAMPLING FROM A POPULATION WITH UNKNOWN P

In the situation we are confronting, the whole thing is turned around. Instead of knowing the percent of red marbles in the bag and computing the probabilities of what can happen in a sample, we have a sample in front of us and are wondering about the population. We know that 16 of the 25 marbles or 64% of the sample are red. What percent of the population is red? Suppose we say 64%. How confident are we in this statement? All we really know is that at least 16 of the 1500 marbles are red and at least 9 are blue but we do not believe there are only 16 red or only 9 blue since both of these extremes are so unlikely. Rather we believe the percent of red marbles in the bag is somewhere near .64. But



what do we mean by "near"? The trouble is, of course, that if we put the first sample back in the bag, shake it up thoroughly and draw another sample we might get 14 or 17 or 19 or 12 or even 16 red marbles again.

How could we do better? We instinctively feel if we had a larger sample, say 250, we would be more confident in predicting the percent of red marbles in the population from that in the sample. We are right. But we may only use the formulas or tables for P(x:n, p) if the size of the population is more than about 50 and the sample less than about 10% of the population. Remember, the formulas and tables give us the probabilities when we draw a sample one marble at a time, replacing it and shaking up the bag before drawing the next marble. Drawing 25 marbles at a time from 1500 is drawing without replacement but it is a small enough sample so we can

TABLE 2										
NU	MBER (OF RED M	ARBLES	IN 10 S	AMPLES	OF SIZE	25			
Number	13	14	15	16	17	18	19	20		
Frequency	0	1	3	4	1	0	1	0		

safely approximate its probabilities by those from the formulas or tables. But a single sample of 250 drawn without replacement is too big to permit this. It is well over 10% of the population. How can we consider a sample of this size? We might think of drawing all 250 marbles one at a time without replacement and work out the actual probability by hand but this would be much too complicated and time consuming. A compromise would be to draw 10 samples of 25 each, replacing each sample before drawing the next. This is just what we have done and recorded in Table 2.

We could now say we have a sample of size 250 of which 159 or 63.6% are red. 159 is obtained simply by adding up the number of red marbles appearing in each of the ten samples.

The percent of red marbles in any sample is written as \bar{p} while the percent in the whole bag is written as p. What we have now is one sample is size 25 with $\bar{p} = .64$ and a larger sample with n = 250 and $\bar{p} = .636$. We are trying to use these values of \bar{p} to estimate the true but unknown value of p.

APPROXIMATING BY A NORMAL CURVE

Recall that in *Range and Deviation* in the CONTENT FOR TEACHERS section we stated that in most of the problems and experiments we are concerned with, the results fit reasonably well the familiar bell-shaped curve, the so-called normal curve. That is, if we took 100 samples of 25 marbles, recorded the percents \bar{p} of red marbles, drew a histogram of the results and then the frequency polygon, the graph would look very much like a normal curve (see Figure 3).

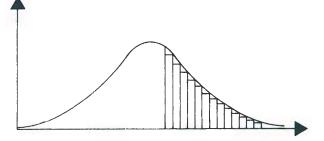
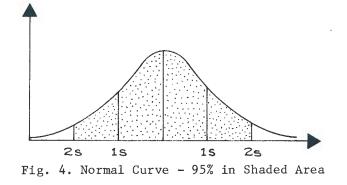


Fig. 3. Approximating by a Normal Curve

Now in a normal curve about 95% of the results will be within 2 standard deviations on either side of the central high point (see Figure 4).



That is, if we took many samples of size 25, 95% of the \bar{p} 's will be within two standard deviations of their mean. If we look at the ten samples we drew above we can work out \bar{p} for each and then the average of all 10 \bar{p} 's. It is .636 or as a percent it is 63.6%.

The statements we are making now are the result of a second approximation. First we approximated samples drawn without replacement by samples drawn with replacement. Now we are approximating the histogram in Figure 3 by the smooth normal curve in the same figure. Fortunately the approximations are close enough that the final answers are valid.

The following exercises are a review of the properties of the mean and standard deviation of a distribution that fits a normal curve.

Exercises

In each exercise, assume that the results fit approximately on the normal curve.

- 22. On a certain examination, the mean of all 600 scores was 73 and the standard deviation was 5. About how many scores were between 63 and 83?
- 23. On a standardized test, the mean is 500 and the standard deviation 100. About how many of the 7000 who took the test will get scores below 300 or above 700? If the distribution is symmetric, how many will be above 700?
- 24. 95% of the scores on an exam are between 65 and 95. What is the mean score and what is the standard deviation on that exam?
- 25. The mean height of male students at a university is 170 cm and the standard deviation is 8 cm. Out of 12,500 male students, 95% will have their heights between what two figures?
- 26. In 100 samples the relative frequency \bar{p} of red haired children was .09 and the standard deviation was .03. Between what two values will 95% of the \bar{p} 's lie?

THE STANDARD DEVIATION IN A SAMPLE OF SIZE n

In our problem about the students in a school or the simulation of this problem by marbles in a bag we have been concerned with the ratio of the students favoring Hard Rock to all students in the school. We have called this the percent or the relative frequency of students favoring Hard Rock and have denoted it by p. In the simulation p is the relative frequency of red marbles in the bag and \vec{p} the relative frequency, .64, in the sample. As a percent we may have written it as 64% but we should always remember that p and \vec{p} are both real numbers lying between 0 and 1 so that $0 and <math>0 < \vec{p} < 1$.

It has been proved that for any given n, the mean of all the possible values of \overline{p} in the samples is actually the value p we are looking for, the percent of red marbles in the bag. But we do not have all possible results. We just have one. How can we use this one sample percent \overline{p} to estimate p the unknown percent in the big bag of marbles? We know 95% of the \overline{p} 's will be within two standard deviations of their mean and we now know this mean is p. Fine!, if we know the standard deviation. Fortunately for us it has been proved that in situations like this the standard deviation deviation, s, of all possible \overline{p} 's can be found by a simple formula that depends only on p and on n, the size of the sample. We don't know p but hopefully our \overline{p} is near p and we use it as an approximation for p in the formula. The formula is

$$s = \sqrt{\frac{p(1-p)}{n}} \text{ which can be approximated by}$$
$$s = \sqrt{\frac{\overline{p}(1-\overline{p})}{n}}.$$

A hand calculator makes s easy to find when we know both \bar{p} and n.

Let's use this formula and see how it helps us in the two cases we have where n = 25 and n = 250. For n = 25, we have $\bar{p} = .64$, therefore $1 - \bar{p} = .36$ and

$$s \approx \sqrt{\frac{(.64)(.36)}{25}} = \sqrt{\frac{.2304}{25}} = \sqrt{.009216} \approx .096.$$

For n = 250 we have \overline{p} = .636, therefore 1 - \overline{p} = .364 and

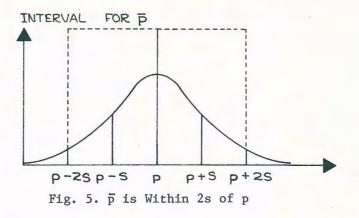
$$s \approx \sqrt{\frac{(.636)(.364)}{250}} \approx \sqrt{\frac{.2315}{250}} = \sqrt{.000926} \approx .03.$$

Exercises

27. Determine s in each of the following.

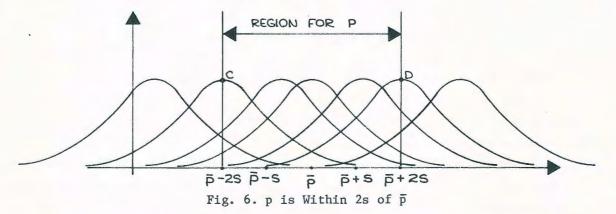
Dec					1011	01	cne	TOTTON	·							
a)	p =	1/2,	n	=	16					e)	$\overline{\mathbf{p}}$	52	.64,	n	=	400
b)	p =	.36,	n	=	25					f)	$\overline{\mathbf{p}}$	=	.5,	n	=	50
c)	p =	.49,	n	=	64					g)	p	=	.62,	n	=	50
d)	p =	.7,	n	=	100)				h)	$\tilde{\mathbf{p}}$	=	.75,	n		100

What we said before was that 95% of the time \bar{p} will be within two standard deviations of p. That is, \bar{p} will be in the interval in Figure 5.



ESTIMATING p FROM A KNOWN p

We now turn this around and say that if \overline{p} is within two standard deviations of p then p must be within two standard deviations of \overline{p} . This is similar to saying: If we know Tom lives within two blocks of Jim and we know where Tom lives we know where to hunt for Jim. But equally well, if we know where Jim lives we know where to hunt for Tom. This time we know \overline{p} and have a good approximation to s. Therefore in Figure 6 we do know the position of \overline{p} . Even if we do not know which of the normal curves we have drawn is the right one, nevertheless we know the right one is between the curve with p at C and the curve with p at D.

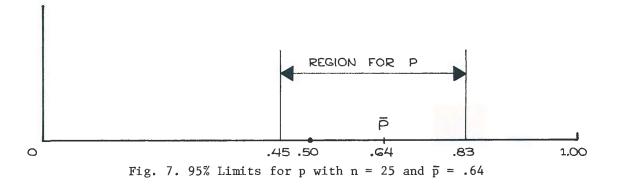


Let's see how this works for our two cases. When n = 25 and $\overline{p} = .64$, we have just found s = .096. At least 95% of the time the true percent p will lie between $\overline{p} - 2s$ and $\overline{p} + 2s$ or between .64 - 2(.096) and .64 + 2(.096). These bounds are .448 and .832. Our bounds on p are about .45 and .83 but these are much too far apart to

INFERENTIAL STATISTICS

CONTENT FOR TEACHERS

make us very happy. All we know is that the percent of red marbles in the bag is somewhere between 45% and 83% of the total. Even with 64% of the sample being red, we are not confident that as many as half of all the marbles are red (see Figure 7).



On the other hand if we use the sample of size 250 we have that \overline{p} = .636 since 159 out of 250 were red and we just computed s to be .030. Under these circumstances we can say we are 95% sure that p lies between .636 - 2(.030) = .576 and .636 + 2(.030) = .696 or roughly between .58 and .70. Now the boundaries on p are much closer and .50 is not included (see Figure 8).

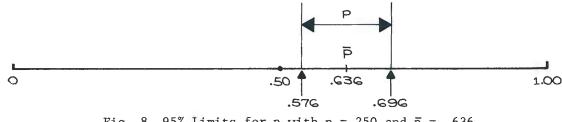


Fig. 8. 95% Limits for p with n = 250 and \overline{p} = .636

95% CONFIDENCE LIMITS FOR p

Going back to the children in the school, our simulation and our study of it has given us a good idea as to where we stand. When we took a random sample of 25 and found 64% of them preferred Hard Rock to Rock and Roll we cannot say with much confidence that even half the school had that preference. But with 10 random samples of size 25 considered as one random sample of size 250 we can say with a great deal of confidence if 63.6% of the sample prefer Hard Rock, at least 58% of the whole school will also.

The problem is finally solved.

We can make a decision on the basis of this evidence and hire the Hard Rock band feeling reasonably sure that most of the students will be pleased. Exercises

- 28. For each of the 8 cases in Exercise 27 determine the bounds on p for which we are 95% sure.
- 29. In the last few examples, we have discussed how to analyze data so as to be 95% satisfied that a population mean falls within certain ranges. We know one way to narrow the range but still maintain 95% confidence. What is it? Although you are 95% confident, you could be wrong. What would you have to do to be 100% confident? What would you say about the range if you were 100% confident in your statement?

The bounds we have been determining are called the 95% confidence limits for p.

The 95% confidence limits for p for a given n are those values A and B such that if we determine \bar{p} in a sample of size n, 95% of the time the true value of p in the population from which the sample was taken will lie between A and B. These limits are $A = \bar{p} - 2s$ and $B = \bar{p} + 2s$.

AN ELECTION POLL

In BIG CITY, a sample of 500 voters is randomly selected. 280 of them say they will vote for JOE PROMISALL for mayor. What are the 95% confidence limits on the proportion of voters who will vote for Joe? This time $\bar{p} = \frac{280}{500} = .56$.

 $1 - \overline{p} = 1 - .56 = .44$ and n = 500. Therefore in the formula

s
$$\approx \frac{\overline{p}(1-\overline{p})}{n}$$

we have s $\approx \frac{\sqrt{(.56)(.44)}}{500} = \sqrt{\frac{.2464}{500}} = \sqrt{.0004928} \approx .022.$

Thus we are 95% sure that p lies between .56 - 2(.022) = .56 - .044 = .516 and .56 + .044 = .604.

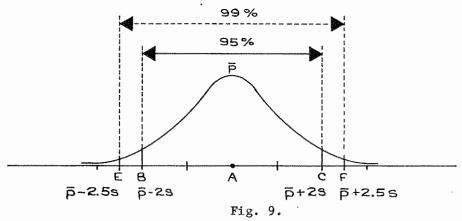
<u>Exercíses</u>

- 30. In a random sample of 453 teachers in the city of New Hampton, 215 said they were unhappy in their jobs and were thinking of leaving. Find the 95% confidence limits on the percent of all teachers in New Hampton who feel that way.
- 31. Out of 10 randomly chosen students, 7 said they disliked history. Are you 95% confident that even half of all students in the school really dislike history?
- 32. In an election in BIG CITY there are two referendum items A and B to be voted on. In a poll of 100 random voters, 73 said they would vote YES on item A but only 53 would vote YES on B. What are the 95% confidence limits on the percent of voters favoring passage of each item?
- 33. In Exercise 32, a new poll is to be taken to get a better idea as to whether B will pass. The new sample is of 500 people. Should A be included? 263 vote YES on B. What are the new 95% confidence limits of those in favor of B? Would you want a still larger sample?

CONTENT FOR TEACHERS

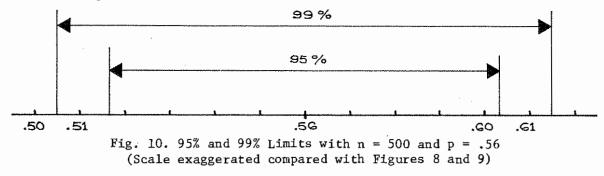
99% CONFIDENCE LIMITS

Sometimes when we want to have even greater confidence in our results we may ask for 99% limits. Now we are asking that 99 times out of 100 the true value of p will be inside the bounds we set around our sample value \bar{p} . In Figure 9, we represent the situation.



If \overline{p} is at point A and 95% of the values of p are in the region from B to C then to include 99% we will have to go even further away from A. In fact, if we go from E to F where E and F are each 2.5 standard deviations from A we will get 99%.

In the poll on the election for mayor we have a sample of size 500 with $\bar{p} = .56$. We found s = .022. To find the 99% confidence limits we must go 2.5 standard deviations or (2.5)(.022) on either side of .56. This gives us .56 - 2.5(.022) or .56 - .055 or .505 for the lower limit and .56 + .055 or .615 for the upper limit (see Figure 10).



We are 95% sure Joe will be elected with at least 51.6% of the voters and 99% sure he will get at least 50.5% of the votes. But there is an outside chance that he will get less than 50.5% of the votes and might well be defeated. There is also an outside chance he will get more than 61.5% but we are not terribly interested in that at the moment and so don't usually say anything about it.

Of course, all this assumes voters do not change their minds between the day the poll was taken and election day, the sample of voters was truly random, only two candidates were running, etc.



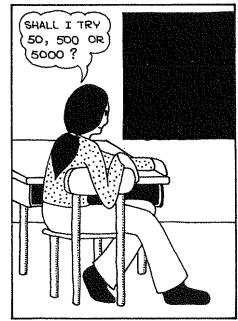
Exercises

- 34. Find the 99% confidence limits in Exercises 30, 32 and 33.
- 35. List three situations where you would be satisfied with 95% and three where you would want 99% or maybe even higher confidence in the results.
- 36. In Exercise 23, about how many students will score better than 750?
- 37. In Exercise 27, determine the 99% confidence limits on p.

SAMPLE SIZE

Up to this point, we have started with a sample whose size n and percent p are known and have asked what are the 95% or 99% confidence limits on p. In the last example, n = 500, \vec{p} = .56, $1 - \vec{p}$ = .44, s = .022 and we could state with 95% confidence that p might be as far away from \vec{p} as .044 or if we are using percentages, p - \vec{p} or \vec{p} - p < 4.4%. For 99% confidence we had to increase this to 5.5%.

Can we turn this around by asking how big n, the size of the sample, should be so that we can be 95% (or 99%) sure the difference between p and \bar{p} will be less than a specified amount say .02? The difference $p - \bar{p}$ or $\bar{p} - p$ may be written as an



absolute value $|p - \bar{p}|$ but if you are not familiar with this notation just call it D, for difference, and remember it is a positive number. For 95% confidence, as we said before, the outside limits of D should be equal to 2 standard deviations and for 99% it should be equal to 2.5 standard deviations. Thus for 95%, D = 2s, for 99%, D = 2.5s.

The trouble now is that since we have not taken the sample we have no idea what p or \overline{p} are and we need them because the formula for s is

$$s = \sqrt{\frac{p(1-p)}{n}}$$

But all is not lost!

We might <u>like</u> to know the smallest value of n that will give us our results with 95% confidence but what we are really interested in is a value of n small in relation to the size of the population that will give us this confidence whether it is the smallest or not. Thus if we determine that a sample of 1500 voters out of the voting population of 70,000,000 will give us 95% or better confidence in the limits we set, we do not really care

that the smallest sample to do this might be 1356. What is it we want? For 95% confidence limits on p we want D, the difference between p and \overline{p} to be less than 2s. Now s = $\sqrt{\frac{p(1-p)}{n}}$, and as n gets larger s, and therefore D, gets smaller. If we

want D< .02 and can show easily this will be true for n = 50, we are happy even though by much harder work we might be able to show n = 30 would also work. So we look again at the 95% confidence limits. These depend on D and D changes for different values of p but the largest possible value it can have for different values of p will be when p(1 - p) is as large as possible (see Figure 11).

Remember that p is between 0 and 1. Use various values of p to determine p(1 - p). The results are displayed in Table 3 and plotted as a graph in Figure 12.



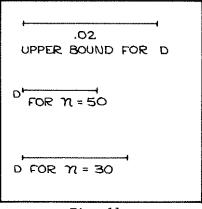
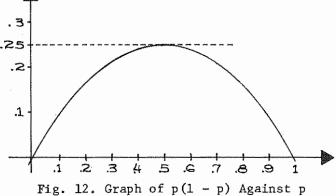


Fig. 11.

TABLE 3											
VALUES OF $p(1 - p)$ FOR $0 \le p \le 1$											
P	0	.1	.2	.3	.4	•5	.6	.7	.8	.9	1
p(1 - p)	0	.09	.16	.21	.24	.25	.24	.21	.10	.09	0
				• · · · · · · · · · · · · · · · · · · ·		A				L	



The largest value of p(1 - p) comes when $p = \frac{1}{2}$ and that value is $\frac{1}{4} = .25$. Therefore the largest value s can have is

$$s = \sqrt{\frac{1/4}{n}} = \sqrt{\frac{1}{4n}} = \frac{1}{2\sqrt{n}}$$

Then since D = 2s, the largest value D can have for any p is

$$D = 2\left(\frac{1}{2\sqrt{n}}\right) = \frac{1}{\sqrt{n}}$$

This equation is perhaps more easily worked with if we change its form slightly as follows.

$$D = \frac{1}{\sqrt{n}}$$
 gives $D^2 = \frac{1}{n}$ and finally $n = \frac{1}{D^2}$.

Thus if we want to be 95% sure p - $\overline{p} < .02$ we have D = .02 and therefore

$$n = \frac{1}{(.02)^2} = 50^2 = 2500.$$

2500 is the size of a sample that will do the trick for us. It may not be the smallest but it works.

If we only wanted D = .05 then n = $\frac{1}{(.05)^2} = 20^2 = 400$. If we want 99% confidence limits the only change is that D = 2.5s instead of D = 2s. Therefore we solve the equation D = 2.5 $(\frac{1}{2\sqrt{n}}) = \frac{5}{4\sqrt{n}}$.

From this we get $D^2 = \frac{25}{16n}$ and finally $n = \frac{25}{16D^2}$. Notice that this is $\frac{25}{16} \ge (\frac{1}{D^2})$ where $\frac{1}{D^2}$ is the value of n we found for 95% confidence. If we have already worked that out we can get the value for 99% simply by multiplying the first result by $\frac{25}{16}$ Thus for D = .05 we already have n = 400 for 95% confidence and so for 99% we have $n = \frac{25}{16} \ge 400 = 625$. For D = .02 and 99% confidence we can calculate directly

$$n \approx \frac{25}{16D^2} = \frac{25}{16(.02)^2} = \frac{25}{16(.0004)} = \frac{25}{.0064}$$
$$n \approx 3906.25$$

and we settle for a sample of size 3900.

Exercises

38. Determine the sample size that will insure the following.

	Confidence	Difference p -	٠Ē
a)	95%	.03	
b)	95%	.025	
c)	95%	.022	
d)	99%	.035	
e)	99%	.04	
f)	99%	.025	
g)	95%	.04	
h)	99%	.03	
i)	99%	.02	

The interesting point here is our computations for n, the sample size, do not depend on whether the sample is taken from the population of a city of 100,000, a state of 10 million or a country of 100 million voters. They depend only on the accuracy we demand and the degree of confidence we want to claim for that accuracy.

As a matter of fact, Gallup and Roper polls do not usually use very large samples in their opinion polls or voter polls. But they are very careful to get a random sample, usually a stratified random sample. Also they are careful to give the possible error. If in any poll you do not see a statement of the possible error you should be very wary of the announced results.

Exercises

- 39. A poll is taken of a random sample of 2000 people from a population of several million. If $\bar{p} = .52$, what are the 95% and 99% confidence limits on p?
- 40. Determine n if you want to be 95% sure your poll is within 5% of being right. Within 2.5%. 1%.
- Determine n if you want to be 99% sure your poll is within 5% of being right. Within 2.5%. 1%.

- 42. On a normal distribution about 68% are within one standard deviation and 87% within one and a half standard deviations of the mean. How big samples would you need to be 68% sure D = .05; to be 87% sure?
- 43. In a bowling league Dick kept a record and found that in the first twenty weeks of the season he bowled 622 frames and got 193 strikes. What are 95% confidence limits on the probability of his getting a strike?
- 44. Some parents were complaining that their 5th grade children could not pass a standard arithmetic test. A random sample of 50 children were given such a test and their average score was 72. What are the 95% confidence limits on the scores other children would make. Explain the result to a parent of a student in your class. If the passing score is 60, about what percent of students would you expect to fail?
- 45. In SLOVENIA, 700 out of a sample of 20,000 families with five living children had all boys. What is the relative frequency of 5 boys in this sample? If P(B) = 1/2 in SLOVENIA, what should be the relative frequency of 5-boy families? Is this within the 95% confidence limits on p?
- 46. In a rifle club, a person with a probability of .9 of getting a bull's-eye is awarded a GOODSHOT diploma. A probability of .95 for a bull's-eye earns a SUPERSHOT diploma. John claims to be a GOODSHOT but misses four times running. With what confidence can you say John is not a GOODSHOT? In a match, Sue shoots 20 targets of 5 shots each. She makes 90 bull's-eyes. In spite of this Sue claims to be a SUPERSHOT. Could this be true?
- 47. A professional baseball player has a current batting average of .310 based on 493 times at bat. Set 95% confidence limits on the probability of his getting a hit each time he is at bat in today's game. About how many hits should he get today if he bats five times?

A SAMPLING PLAN

Another kind of decision in which a statistical analysis of the problem can help is that confronting the buyer. The salesman claims that only 1% or less of the ball point pens he is selling are defective. The buyer says, "Let's take a random sample and see how many bad ones we get. Then I can decide." The crucial questions are:

- How big a sample should he take? and
- How many bad pens in the sample will make him reject the lot?

If the sample size is 25 and p = .01, he might decide to reject the lot if there are two or more bad ones and accept it if there are none or only one bad one. We need the probabilities P(0:25, .01) and P(1:25, .01). These are hard to calculate and unfortunately our tables do not give them. But larger tables do give them

CONTENT FOR TEACHERS

INFERENTIAL STATISTICS

as .778 and .196 respectively. Their sum is .974. This means that we will accept lots that are 1% or less defective about 97% of the time and reject such lots about 3% of the time. The salesman feels good but the buyer is still a bit unhappy for the following reason. Suppose the lot is not 1% defective but maybe 10%. Does this plan give him a good chance of catching those bad lots? Let's see what P(0:25, .10)and P(1:25, .10) are. They are in our tables and we find them to be .072 and .199 so their sum is .271 and so the buyer has a right to be unhappy. Why? Well, he has better than a 27% probability of accepting a really bad lot. What can be done? The answer is not immediately evident but by changing the size of the sample and the number of defectives allowed in the sample, we can try again to make both the salesman and the buyer happy that the test will pass a good lot and will catch a poor lot.

CONCLUSION

Statistics of course goes far beyond this. The point is not that each of us knows all the intricacies of how statistics works. But somehow we should get a feeling for the fact that statistical methods enable decisions to be made even when based on incomplete information. If the analysis of that information gives us a measurable and sufficiently high degree of confidence we are right, we go ahead. Statistical thinking suggests:

> When confronted with a problem, Decide what information is needed. Gather it by surveys, sampling, etc.; Organize it so you can Analyze the information to Determine results within the required Limits of confidence and then Make the decision in the light of all this evidence.

CONTENT FOR TEACHERS

ANSWERS TO EXERCISES

- 1. As stated the test is too vague. What does "several" mean? If it means a small number such as 3, then either heads must come up 2 or 3 times or tails must show 2 or 3 times and in both cases Tom would say the coin is biased. The test should involve a good many tosses. Again what does "good many" mean? And what ratio of heads to total tosses will convince us the coin is or is not biased? The answers to these questions are the main topic of this section.
- 2. If the claimed average life is 100 hours, a life of 90 hours may seem too small to make us feel confident that the claim is justified. It really depends on how many batteries were tested to find the average and what the standard deviation of the measures was. If 1000 batteries were tested and the standard deviation was 10 hours then we know that 95% of the batteries have lives between 80 and 120 hours. So batteries like ours, in fact batteries worse than ours are pretty common. But in this case the whole lot is so varied, we don't like the brand. If the standard deviation is 3.3 then about 2 batteries in a thousand would be as poor as ours. Again we wonder whether we were just unlucky or whether the average life was really down near 92 hours. In either case we are unhappy but maybe we would try one more Long Life battery. If the advertisement had said, "Minimum life is 100 hours," we would be definitely more justified in deciding not to try again.
- 3. Probably okay. The chances are $1/216 \approx .005$.
- Probably not. With pens so inexpensive, bad ones can be replaced without much cost and customers kept happy.
- 5. Probably yes. The cost of fixing 3 bad HiFi systems is high enough to make it doubtful that you would want to take that chance again. Of course, if you were making a profit of \$200 apiece on this brand and only \$100 apiece on a comparably priced other brand, you might decide to stick with the first brand.
- 6. $P(6T) = (\frac{1}{2})^6 = \frac{1}{64}$. Therefore about $\frac{1600}{64} = 25$ students should get six tails in a row.
- 7. $P(5B) = \frac{1}{32}$. $\frac{20000}{32} = 625$. 700 is close enough to 625 that $P(B) = \frac{1}{2}$ is not too unlikely and we tentatively say YES. But see Exercise 45.
- 8. $P(4 \text{ fives}) = (\frac{1}{6})^4 = \frac{1}{1296}$. In 40000 trees we would have about $\frac{1}{1296} \ge 40000 \approx 31$ people get 4 fives. In five tosses about 5 people might get 5 fives.
- 9. P(not getting bull's-eye) = .1 P(failure four times) = $(.1)^4$ = .0001 P(John is not a GOODSHOT) = 1 - .0001 = .9999. We are 99.9% sure he is lying. P(12:12, .9) = ${}_{12}C_{12}(.9)^{12}(.1)^0 \approx .28$ P(12:12, .95) = ${}_{12}C_{12}(.95)^{12}(.05)^0 \approx .54$ The chances are so much higher in the second case we conclude Sue is a SUPERSHOT. See also Exercise 46.

10.	This is a highly individual	matter. The answer given	here are one person's
	choice. Yours may be quite	different.	-
	a) 50% b) 75%	c) 75% d) 99.9%	e) 95%
	f) 55% g) 80%	h) 40% i) 80%	j) 75%
11	P(GGG) on first route = (5)	(6)(6) - 18	2

11. P(GGG) on first route = (.5)(.6)(.6) = .18P(GGGGGG) on second route = (.6)(.6)(.8)(.9)(.8) = .21Therefore the second route is better.

b) There are 4! = 24 permutations. $P(match) = 1/24 \approx .04$

c) Probability he was lucky both tosses is $1/6 \ge 1/24 = 1/144$.

d) There are 5! = 120 permutations. $P(match) = 1/120 \approx .008$ In the first case, not much credence should be given John's claim. In the second case, we might concede either that he does have some ESP or that he got a peek at Jane's cards as she arranged them. Just by luck, he could have matched up all five cards so we are not certain about his ESP. This might be the one time in 120 when a random arrangement by John matches the one Jane put down.

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14. (.52)(2100) = 1092, (.60)(2100) = 1260. 1063 is <u>not</u> between these limits. The pollster was wrong.

15.
$$\frac{575}{850} \approx .44$$
. $44\% \pm 3\%$ roughly.

16. A $\left\{\begin{array}{c} .50 \text{ x} (356,423) = 178,211 \\ .54 \text{ x} (356,423) = 192,468 \end{array}\right\}$ Smith B $\left\{\begin{array}{c} .49 \text{ x} (356,423) = 174,647 \\ .53 \text{ x} (356,423) = 188,904 \end{array}\right\}$ Jones

Smith's actual vote 179,832 is inside the interval for A. By Poll B, Smith should get between 167,519 and 181,776. Again the result is within these limits so both polls can claim to be correct.

- 17. 3/5, 3/5, 9/25 = P(RR) with replacement, 3/5 x 2/4 = 3/10 = P(RR) without replacement. 9/25 = .36 and 3/10 = .30.
- 18. P(RRR) without replacement = $\frac{35}{50} \times \frac{34}{49} \times \frac{33}{48} \approx .334$ P(RRR) with replacement = (.7)(.7)(.7) = .343
- 19. P(RRR) without replacement = $\frac{1050}{1500} \times \frac{1049}{1499} \times \frac{1048}{1498} \approx .3427 \approx .343$ P(RRR) with replacement = (.7)(.7)(.7) = .3430 = .343
- 20. 17 to 23 inclusive
- 21. 6 to 14 inclusive
- 22. 63 to 80 = \overline{x} 2s to \overline{x} + 2s therefore 95%(600) = 570
- 23. About 5% or 350, about 2.5% or 175 will be above 700
- 24. Mean is 80 and standard deviation is 7.5 assuming, of course, that the distribution is normal.

^{12.} a) There are 3! = 6 permutations. $P(match) = 1/6 \approx .17$

^{13. 115,896 ↔ 130,087}

CONTENT FOR TEACHERS

d)

() A

25. Between 170 - 2(8) and 170 + 2(8) or 154 to 186 cm.

 $.70 \pm .092 = .608, .792$

26. Between .09 - 2(.03) and .09 + 2(.03) or .03 to .15.

- 27. a) $\sqrt{\frac{1/2 \times 1/2}{16}} = \frac{1}{64} = \frac{1}{8}$ b) $\sqrt{\frac{(.36)(.64)}{25}} = \frac{(.6)(.8)}{5} = \frac{.48}{5} = .096$ f) $\sqrt{\frac{(.5)(.5)}{50}} \approx .024$ c) $\sqrt{\frac{(.49)(.51)}{64}} \approx .06$ d) $\sqrt{\frac{(.7)(.3)}{100}} \approx .046$ 28. a) $1/2 \pm 2(1/8) = 1/4, 3/4$ b) $.36 \pm 2(.096) = .168, .552$ c) $.49 \pm .12 = .37, .61$ e) $\sqrt{\frac{(.64)(.36)}{400}} \approx .024$ f) $\sqrt{\frac{(.62)(.38)}{50}} \approx .024$ g) $\sqrt{\frac{(.62)(.38)}{50}} \approx .069$ h) $\sqrt{\frac{(.75)(.25)}{100}} \approx .043$
- 29. One way to narrow the range and maintain 95% confidence is to increase the size of the sample. In situations like this you can <u>never</u> be 100% confident unless you increase the sample to include the whole population or widen the range to include all real numbers.

h)

 $.75 \pm .086 = .664, .836$

- 30. $\bar{p} = .475, 1 \bar{p} = .525, n = 453$ $s = \sqrt{\frac{(.475)(.525)}{453}} \approx .023$. Therefore we are 95% sure p lies between .475 - 2(.023) = .475 - .046 = .429 and .475 + 2(.023) = .475 + .046 = .521 We are 95% confident that the percent of unhappy teachers is between 42.9% and 52.1%.
- 31. $\vec{p} = .7, 1 \vec{p} = .3, n = 10$ $s = \sqrt{\frac{(.7)(.3)}{10}} = .145$ 95% confidence limits on p are .7 - .29 and .7 + .29 or .41 and .99. Therefore the answer is no since the percent who dislike history might be as low as 41%.
- 32. A: $\vec{p} = .73, 1 \vec{p} = .27,$ $s = \sqrt{\frac{(.73)(.27)}{100}} = .044$ B: $\vec{p} = .53, 1 - \vec{p} = .47$ $s = \sqrt{\frac{(.53)(.47)}{100}} = .050$ 95% confidence $.73 \pm 2(.044)$ or .642 and .818 95% confidence $.53 \pm 2(.05) = .43$ and .63
- 33. A does not need to be included since we are already 95% confident it will pass. $\overline{p} = \frac{263}{500} = .526$, $1 - \overline{p} = .464$, $s = \sqrt{\frac{(.526)(.474)}{500}} = .022$ Now 95% limits are $.526 \pm 2(.022) = .526 \pm .044 = .482$ to .570 A still larger sample might be no better. Even with a sample of 1000 and the same p, we still are not 95% sure that B will pass. The election is really too close to call.
- 34. 99% confidence limits are p + 2.5s. In Exercise 30, $\overline{p} \pm 2.5s = .475 \pm 2.5(.023) = .4175$ to .5325 In Exercise 32, A: $\overline{p} \pm 2.5s = .73 \pm 2.5(.044) = .62$ to .84 B: $\overline{p} \pm 2.5s = .53 \pm 2.5(.05) = .405$ to .655 In Exercise 33, $\overline{p} \pm 2.5s = .526 \pm 2.5(.022) = .471$ to .581

CONTENT FOR TEACHERS

35. 95% confidence in results would be satisfactory:

- a) In a poll to determine that a majority of students favored a Rock-n-Roll band to a Hard Rock band.
- b) In experiments to show CLEANWASH soap powder is better than STOPDIRT soap.
- c) To predict that tomorrow will be a sunny day for the annual picnic.
- 99% or higher confidence would be desirable
- a) For the safety of a new drug before its release.
- b) That a sampling plan will protect me as a buyer in a big chain store from buying more than 1 defective TV set in a hundred.
- c) In the safety of the space shuttle before I try it.
- 36. Scoring better than 750 means being above \overline{p} + 2.5s. But if 99% of the students are between \overline{p} 2.5s and \overline{p} + 2.5s then 1/2% will be above 750. Therefore (.005)(7000) = 35 students will score above 750.

37.		p	S	2.5s	p̄ — 2.5s	p̄ + 2.5s
	a) b)	.50 .36	.125	.313 .24	.187	.813 .60
	c)	.49	.06	.15	.34	.64
	d)	.7	.046	.115	.585	.815
	e)	.64	.024	.06	.58	.70
	f)	.50	.07	.175	.325	.675
	g)	.62	.069	.173	.447	.793
	h)	.75	.043	.108	.642	.858

38. For 95%,
$$n = \frac{1}{D^2}$$
, for 99%, $n = \frac{2J}{16} \times \frac{1}{D^2}$
a) $n = \frac{1}{(.03)^2} = (\frac{100}{3})^2 = 1111.1 \approx 1112$
b) $n = \frac{1}{(.025)^2} = 1600$
c) $n = \frac{1}{(.022)^2} \approx 2066$
d) $n = \frac{25}{16} \times \frac{1}{(.035)^2} \approx 1276$
e) $n = \frac{25}{16} \times \frac{1}{(.04)^2} \approx 977$
f) $n = \frac{25}{16} \times \frac{1}{(.025)^2} = 2500$
g) $n = \frac{1}{(.04)^2} = 625$
h) $n = \frac{25}{16} \times \frac{1}{(.03)^2} \approx 1737$
i) $n = \frac{25}{16} \times \frac{1}{(.02)^2} \approx 3906$
39. $n = 2000$, $\bar{p} = .52$, $1 - \bar{p} = .48$, $s = \sqrt{\frac{(.52)(.48)}{2000}}$

39. n = 2000, $\overline{p} = .52$, $1 - \overline{p} = .48$, $s = \sqrt{\frac{(.52)(.48)}{2000}} = .016$ 95% limits on p are $.52 \pm 2(.016) = .52 \pm .032 = .488$ and .55299% limits on p are $.52 \pm 2.5(.016) = .52 \pm .04 = .48$ and .56

40.
$$n = \frac{1}{(.05)^2}$$
, $\frac{1}{(.025)^2}$, $\frac{1}{(.01)^2}$
= 400 , 1600 , 10,000

INFERENTIAL STATISTICS

41.
$$n = \frac{25}{16} (\frac{1}{.05})^2$$
, $\frac{25}{16} (\frac{1}{.025})^2$, $\frac{25}{16} (\frac{1}{.01})^2$
 $= 625$, 2500, 15,625
42. For 68% confidence, $D = 1s = \frac{1}{2\sqrt{n}}$ or $n = \frac{1}{4b^2}$
For 8% confidence, $C = 1.5s = \frac{3}{2} \times \frac{1}{2\sqrt{n}}$ or $n = \frac{9}{16b^2}$
Therefore if $D = .05$ for 68%, $n = \frac{1}{4(\frac{1}{.05})^2} = 100$
for 8%, $n = \frac{1}{16(\frac{1}{.05})^2} = 225$
43. $\overline{p} = \frac{193}{622} \approx .31$, $1 = \overline{p} \approx .69$, $n = 622$, $s = \sqrt{\frac{(.32)}{622}} \approx .019$
957 limits on p are $.31 \pm 2(.019) = .31 \pm .038 = .272$ and $.348$
44. $\overline{p} = .72$, $1 - \overline{p} = .28$, $n = 50$, $s = \sqrt{\frac{(.72)}{50}} \approx .06$
95% limits on p are $.72 \pm 2(.06) = .72 \pm .12 = .60$ and $.82$
Therefore about 9% of the children will score between 60 and 82 with
 2.5% below 60 or failing and 2.5% above 82.
45. If $P(B) = 1/2$, $P(BB) = 1/32 = .03125$. With 700 out of 20,000 families in
SLOVENIA being all boys, p in this sample is 700/20,000 = .035.
 $s = \sqrt{\frac{(.2055)(.965)}{20,000}}$
95% confidence limits on p are $.035 \pm 2.(.0013) = .0324$ and $.0376$.
99% confidence limits on p are $.035 \pm 2.(.0013) = .0324$ and $.0376$.
99% confidence limits on p are $.035 \pm 2.5(.0013) = .03175$ and $.03825$.
Since $.03125$ is outside these limits we are 9% confident that P(B) in SLOVENIA
is greater than 1/2. But it is very close.
99. 7% confidence limits on p are $.035 \pm 2.5(.0013) = .0311$ and $.0389$ and these do
include 0.3125. So at this level we could not say P(B) is greater than 1/2.
46. If $P(B) = .9$, $P(\text{missing}) = .1$, $P(4 \text{ misses}) = (.1)^4 = .0001$.
This is so low we are better than 99% confidence that John is not a GOODSHOT.
In Sue's match, $\overline{p} = .9$ and $n = 100$.
 $s = \sqrt{\frac{(.92)(1.0)}{100}} = .03$. Therefore 95% confidence limits on
p are from .84 to .96. Since .95 is inside these limits we do not feel like
denying Sue's claim.
47. $\overline{p} = .310$, $1 - \overline{p} = .690$, $n = 493$, $s \approx \sqrt{\frac{(.331)(.69)}{493}} \approx .021$
95% limits on p are .310 $\pm 2(.021) = .310 \pm 0.422 = .268$ and .352.
He can be expected to get between 5(.268) a

ORGANIZATION

Placement Guide for Tabbed Dividers

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1 MEAN

100 REM THIS IS A PROGRAM TO FIND THE MEAN OF UP TO 50 NUMBERS 110 DIM X(50) 120 PRINT "HOW MANY NUMBERS DO YOU WISH TO INPUT? (UP TO 50)" 130 INPUT N 140 PRINT "ENTER A NUMBER AFTER EACH QUESTION MARK" 150 FOR I=1 TO N 160 INPUT X(I) 170 LET S=S+X(I) 180 NEXT I 190 LET M=S/N 200 PRINT 210 PRINT "THE MEAN IS ";M 220 END

2 MEDIAN

100 REM THIS IS A PROGRAM TO FIND THE MEDIAN OF UP TO 50 NUMBERS. 110 DIM X(50) 120 PRINT "HOW MANY NUMBERS DO YOU WISH TO INPUT? (UP TO 50)" 130 INPUT N 140 PRINT "ENTER A NUMBER AFTER EACH QUESTION MARK" 150 FOR I=1 TO N 160 INPUT X(I) 170 NEXT I 180 REM WE MUST NOW ARRANGE THE NUMBERS IN ORDER 190 FOR L=1 TO N 200 FOR J=1 TO N-L 210 IF X(J) <X(J+1) THEN 250 220 LET B = X(J+1)230 LET X(J+1) = X(J) 240 LET X(J) = B250 NEXT J 260 NEXT L 270 IF N/2=INT(N/2) THEN 300 280 LET K=(N+1)/2 290 GO TO 330 300 LET K=N/2 310 LET M=(X(K)+X(K+1))/2 320 GO TO 350 330 LET M=X(K) 340 PRINT 350 PRINT "THE MEDIAN IS"M 360 END

PHOGRAMS

COMPUTER

3 MODE

100 REM THIS IS A PROGRAM TO FIND THE MODE OF UP TO 50 NUMBERS 110 DIM X(50), T(50), S(50) 120 PRINT "HOW MANY NUMBERS DO YOU WISH TO INPUT? (UP TO 50)" 130 INPUT N 140 PRINT "ENTER A NUMBER AFTER EACH QUESTION MARK." 150 FOR I = 1 TO N160 INPUT X(I) 170 LET T(I) = 1180 NEXT I 190 LET T(N) = 0 200 FOR L = 1 TO N210 FOR J = 1 TO N - L220 IF X(J) < X(J+1) THEN 260 230 LET B = X(J+1)240 LET X(J+1) = X(J)250 LET X(J) = B 260 NEXT J 270 NEXT L 280 FOR I = 1 TO N-1290 IF X(I+1) = X(I) THEN 310 300 LET T(I) = 0310 NEXT I 320 LET S(1) = T(1)330 FOR I = 1 TO N-1340 IF T(I) <> T(I+1) THEN 380 350 LET S(I+1) = S(I) + 1360 NEXT I 370 GOTO 400 380 LET S(I+1) = T(I+1)390 GOTO 360 400 LET M = 1410 FOR I = 1 TO N420 IF M < S(I) THEN 440 430 GOTO 450 440 LET M = S(I)450 NEXT I 460 FOR I = 1 TO N-1470 IF M = S(I) THEN 500 480 NEXT I 490 GOTO 520 500 PRINT X(I+1) "IS A MODE OCCURING" M+1 "TIMES." 510 GOTO 480 520 END

PAGGAAMS

4 MEAN OF A FREQUENCY DISTRIBUTION (MEANFQ)

THIS IS A PROGRAM TO FIND THE MEAN OF A FREQUENCY DISTRIBUTION 100 REM THE NUMBERS TAKE ON N VALUES WITH N LESS THAN OR EQUAL TO 20 110 REM 120 REM 'THE VALUES ARE X(I). EACH X(I) OCCURS WITH FREQUENCY F(I). 130 DIM X(20) 140 PRINT "HOW MANY DIFFERENT VALUES DO YOU WANT? (UP TO 20)"; 150 INPUT N 160 PRINT "ENTER ONE VALUE AFTER EACH QUESTION MARK" 170 FOR I = 1 TO N180 INPUT X(I) 190 NEXT I 200 PRINT "ENTER A CORRESPONDING FREQUENCY AFTER EACH ?." 210 FOR I = 1 TO N220 INPUT F(I) 230 NEXT I 240 FOR I = 1 TO N 250 LET S = S + X(I) * F(I)260 LET F = F + F(I)270 NEXT I 280 PRINT 290 PRINT "THE MEAN IS" S/F 300 END

5 MEAN ABSOLUTE DEVIATION (ABSDEV)

100 REM THIS IS A PROGRAM TO COMPUTE THE MEAN ABSOLUTE DEVIATION OF UP TO 50 NUMBERS 110 REM 120 DIM X(50) 130 PRINT "HOW MANY NUMBERS DO YOU WISH TO ENTER? (UP TO 50)"; 140 INPUT N 150 PRINT "ENTER A NUMBER AFTER EACH QUESTION MARK." 160 FOR I=1 TO N 170 INPUT X(I) 180 LET S=S+X(I) 190 NEXT I 200 LET M=S/N 210 FOR J=1 TO N 220 LET D=D+ABS(X(J)-M) 230 NEXT J 240 LET D=D/N 250 PRINT 260 PRINT "THE MEAN ABSOLUTE DEVIATION IS ";D 270 END

PADGAAMS

MEAN, VARIANCE AND STANDARD DEVIATION (MVSDN) 6 100 REM THIS PROGRAM WILL COMPUTE THE MEAN, THE VARIANCE AND THE 110 REM STANDARD DEVIATION OF UP TO 50 NUMBERS. 120 DIM X(50) 130 PRINT "HOW MANY NUMBERS DO YOU WISH TO ENTER? (UP TO 50)" 140 INPUT N 150 PRINT "ENTER A NUMBER AFTER EACH QUESTION MARK." 160 FOR I=1 TO N 170 INPUT X(I) 180 LET S=S+X(I) 190 LET T=T+X(I)+2 200 NEXT I 210 LET M=S/N 220 LET V=T/N-(S/N) 2 230 LET D = SQR(V) 240 PRINT 250 PRINT "MEAN VARIANCE STANDARD DEVIATION" 260 PRINT 270 PRINT M,V,D 280 END MEAN, VARIANCE AND STANDARD DEVIATION OF A FREQUENCY DISTRIBUTION 7 100 REM THIS IS A PROGRAM TO COMPUTE THE MEAN, THE VARIANCE AND THE 110 REM STANDARD DEVIATION OF A FREQUENCY DISTRIBUTION. 120 REM THE NUMBERS TAKE ON N VALUES WITH N LESS THAN OR EQUAL TO 20. 130 REM THE VALUES ARE X(I). EACH X(I) OCCURS WITH FREQUENCEY F(I). 140 DIM X(20),F(20) 150 PRINT "HOW MANY DIFFERENT VALUES DO YOU WANT? (UP TO 20)" 160 INPUT N 170 PRINT "ENTER A VALUE FOR X(I) AFTER EACH QUESTION MARK." 180 FOR I = 1 TO N 190 INPUT X(I) 200 NEXT I 210 PRINT "ENTER THE CORRESPONDING VALUES OF F(I) AFTER EACH ?." 220 FOR I = 1 TO N230 INPUT F(I) 240 NEXT I 250 FOR I = 1 TO N260 LET F = F + F(I)270 LET S = S + X(I) * F(I)280 LET T = T + X(I) * X(I) * F(I)290 NEXT I 300 LET M = S/F310 LET V = T/F - M*M 320 LET D = SQR(V) 330 PRINT VARIAN CE 340 PRINT "MEAN STANDARD DEVIATION" 350 PRINT 360 PRINT M.V.D 370 END PANGAAMS

COMPUTER

PADGAAMS

8 RANDOM NUMBERS (RANDOM)

100 REM THIS PROGRAM GENERATES AND PRINTS A TABLE OF 110 REM 2500 RANDOM NUMBERS FROM 1 TO N FOR ANY N 120 REM BETWEEN 2 AND 12. IF YOU WANT TO 130 REM SIMULATE TOSSING A COIN INPUT N=2 AND 140 REM CHANGE STEPS 130 AND 140 TO READ 150 REM LET A\$(1)="H" AND LET A\$(2)="T". 160 REM TO GET A TABLE OF 10 RANDOM DIGITS INPUT N=10. 170 REM TO SIMULATE THE ROLL OF A DIE INPUT N=6. 180 REM TO SIMULATE THE ROLL OF A 4,8, OR 190 REM 12 SIDED DIE INPUT N=4,8,0R 12. IN THE 200 REM LAST CASE "E" WILL STAND FOR 11 AND "T" FOR 12. 210 DIM A\$(12) 220 PRINT "WHAT VALUE OF N DO YOU WANT"; 230 INPUT N 240 RANDOMIZE 250 LET A\$(1)="1" 260 LET A\$(2)="2" 270 LET A\$(3)="3" 280 LET AS(4)="4" 290 LET A\$(5)="5" 300 LET AS(6)="6" 310 LET A\$(7)="7" 320 LET A\$(8)="8" 330 LET A\$(9)="9" 340 LET A\$(10)="0" 350 LET A\$(11)="E" 360 LET A\$(12)="T" 370 FOR K=1 TO 50 380 FOR I=1 TO 10 390 FOR J=1 TO 5 400 LET R = INT(PND(X)*N+1) 410 PRINT AS(R); 420 NEXT J 430 PRINT " "; 440 NEXT I 450 PRINT 460 IF K/5<>INT(K/5) THEN 480 470 PRINT 480 NEXT K 490 END

COMPUTER

9 TOSSING ONE DIE (TOSS1D)

100 REM THIS IS A PROGRAM TO SIMULATE AND COUNT THE RESULTS OF TOSSING 110 REM A DIE N TIMES, WHERE N IS LESS THAN OR EQUAL TO 1000. 120 PRINT "WE WANT TO TOSS A DIE N TIMES (WHERE N IS LESS THAN OR EQUAL" 130 PRINT "TO 1000) AND PRINT THE RESULTS AFTER EVERY M TOSSES." 140 PRINT "THE RESULTS WILL BE TOTALLED AS WE GO ALONG." 150 PRINT "Ø<M<N" 160 PRINT "WHAT VALUES OF N AND M DO YOU WANT?" 170 INPUT NJM 180 IF N>1000 THEN 200 190 GO TO 220 200 PRINT "N MUST BE NO LARGER THAN 1000" 210 GO TO 160 220 PRINT 230 PRINT "RESULTS 1 2 3 4 5 6" 240 PRINT 250 RANDOMIZE 260 FOR I=1 TO N 270 LET X=INT(RND(X)*6+1) 280 ON X GOTO 390,370,350,330,310,290 290 LET N6=N6+1 300 GO TO 400 310 LET N5=N5+1 320 GO TO 400 330 LET N4=N4+1 340 GO TO 400 350 LETN3=N3+1 360 GO TO 400 370 LET N2=N2+1 380 GO TO 400 39Ø LET N1=N1+1 400 IF I/M<>INT(I/M) THEN 420 410 PRINT I, TAB(17); N1; TAB(22); N2; TAB(27); N3; TAB(32); N4; TAB(37); N5; TAB(4 2) / N 6 420 NEXT I 430 END



(

10 TOSSING TWO DICE (TOSS2D)

100 REM THIS IS A PROGRAM TO SIMULATE AND COUNT THE RESULTS OF TOSSING 110 REM TWO DICE UP TO 1000 TIMES. 120 RANDOMIZE 130 DIM N(12) 140 PRINT "HOW MANY TIMES DO YOU WISH TO TOSS THE DICE? (UP TO 1000)" 150 INPUT N 160 IF N>1000 THEN 180 170 GO TO 200 180 PRINT "N MUST BE NO LARGER THAN 1000" 190 GO TO 140 200 FOR I = 1 TO N 210 RANDOMIZE 220 LET R = INT(RND(X) * 6 + 1)230 LET S = INT(RND(X) \neq 6 + 1) 240 LET T = R + S250 FOR K = 2 TO 12 260 IF T = K THEN 300 270 NEXT K 280 NEXT I 290 GO TO 320 300 LET N(K) = N(K) + 1310 GO TO 280 320 PRINT 330 PRINT " 2 3 4 5 7 6 8 9 10 11 12 ** 340 PRINT 350 FOR I = 2 TO 12 360 PRINT TAB(1+5 - 2) JN(1) J 370 NEXT I 380 END

PADGAAMS

COMPUTER

<u>11</u> CHOOSING A COMMITTEE OF THREE (CHCOM1) 100 REM THIS IS A PROGRAM TO CHOOSE THREE CHILDREN OUT OF A CLASS OF 110 REM TEN, CHOOSING THEM AT RANDOM. THEN TO REPEAT THIS N TIMES (WHERE 120 REM N IS LESS THAN OR EQUAL TO 1000) AND PRINT THE DISTRIBUTION 130 REM OF THE CHOICES. 140 PRINT "HOW MANY TIMES DO YOU WISH TO REPEAT? (UP TO 1000)" 150 INPUT N 160 IF N>1000 THEN 180 170 GO TO 200 180 PRINT "YOU MAY REPEAT NO MORE THAN 1000 TIMES" 190 GO TO 140 200 FOR I = 1 TO N 210 RANDOMIZE 220 LET X(1) = INT(BND(X)*10 + 1) 230 LET X(2) = INT(RND(X) * 10 + 1)240 IF X(2) = X(1) THEN 230 250 LET X(3) = INT(RND(X)*10 + 1) 260 IF X(3) = X(1) THEN 250 270 IF X(3) = X(2) THEN 250 280 FOR J = 1 TO 3290 FOR K = 1 TO 10 300 IF X(J) = K THEN 330 310 NEXT K 320 GO TO 350 330 LET N(K) = N(K) + 1 340 GO TO 310 350 NEXT J 360 NEXT I 370 PRINT 380 PRINT " 1 З 4 5 2 6 7 8 9 10" 390 FOR L = 1 TO 10400 PRINT TAB(5*L - 3);N(L); 410 NEXT L 420 END

PANGAAM

12 CHOOSING A COMMITTEE OF TEN (CHCOM2)

100 REM THIS PROGRAM SIMULATES PICKING AT RANDOM A COMMITTEE OF 10 FROM 110 REM A SCHOOL OF 500 GIRLS AND 425 BOYS, REPEATING THIS N TIMES 120 REM (WHERE N IS LESS THAN OR EQUAL TO 20) AND FINDING THE AVERAGE 130 REM NUMBER OF GIRLS ON THE COMMITTEE. 140 PRINT "HOW MANY TIMES DO YOU WANT TO REPEAT? (UP TO 20)" 150 INPUT N 160 IF N>20 THEN 180 170 GO TO 200 180 PRINT "YOU MAY REPEAT NO MORE THAN 20 TIMES." 190 GO TO 140 200 FOR I = 1 TO N 210 RANDOMIZE 220 LET X(1) = INT(RND(X)*1000 + 1) 230 IF X(1) > 925 THEN 220 240 FOR J = 2 TO 10250 LET X(J) = INT(RND(X)*1000 + 1) 260 IF X(J) > 925 THEN 250 270 FOR K = 1 TO J - 1 280 IF X(J) = X(K) THEN 250 290 NEXT K 300 NEXT J 310 FOR L = 1 TO 10320 IF X(L) > 500 THEN 340 330 LET G = G + I340 NEXT L 350 PRINT "THE AVERAGE NUMBER OF GIRLS IN "I"SELECTIONS IS"G/I 360 NEXT I 370 END

PHOGRAMS

13 BATTER 100 REM THIS IS A PROGRAM TO SIMULATE THE NUMBER OF HITS A BATTER WITH A 110 REM BATTING AVERAGE OF P WILL GET WITH FIVE TIMES AT BAT 120 REM A GAME. IN N GAMES (WHERE N IS LESS THAN OR EQUAL TO 1000), HOW 130 REM MANY TIMES WILL THE BATTER GET 3 OR MORE HITS? 140 DIM H(1000) 150 PRINT "WHAT IS OUR PLAYER'S BATTING AVERAGE? (.0000<=P<=1.000)" 160 INPUT P 170 IF P<0 THEN 200 180 IF P>1 THEN 200 190 GO TO 220 200 PRINT "THE BATTING AVERAGE MUST BE BETWEEN Ø AND 1" 210 GO TO 150 220 PRINT "IN HOW MANY GAMES DOES OUR PLAYER PARTICIPATE? (UP TO 1000)" 230 INPUT N 240 IF N>1000 THEN 260 250 GO TO 280 260 PRINT "N MUST BE NO LARGER THAN 1000" 270 GO TO 220 280 FOR I = 1 TO N 290 RANDOMIZE 300 FOR J = 1 TO 5310 LET A = RND(X)320 IF A > P THEN 340 330 LET H(I) = H(I) + 1340 NEXT J 350 NEXT I 360 FOR I = 1 TO N 370 IF H(I) = 0 THEN 390380 GO TO 400 390 LET M = M + 1 400 FOR K = 1 TO 5 410 IF H(I) = K THEN 440420 NEXT K 430 GO TO 460 440 LET N(K) = N(K) + 1450 GO TO 420 460 NEXT I 470 PRINT "HITS GAMES" 480 PRINT " 0",M 490 FOR K = 1 TO 5500 PRINT K,N(K) 510 NEXT K 520 FOR K = 3 TO 5 530 LET H = H + N(K)540 NEXT K 550 PRINT "IN"N"GAMES THE BATTER GOT THREE OR MORE HITS"H"TIMES" 560 PRINT 570 LET S = 10*P(3*(1 - P)) + 5*P(4*(1 - P)) + P(5)580 PRINT "THE THEORETICAL PROBABILITY OF GETTING THREE OR MORE HITS IN" 590 PRINT "5 TIMES AT BAT FOR A PLAYER WITH BATTING AVERAGE"P"IS"S"." 600 PRINT "OUR EXPERIMENTAL EVALUATION OF THIS PROBABILITY IN"N"GAMES" 610 PRINT "IS"H/N 620 END

PANGAAMS

COMPUTER

14 AVERAGE NUMBER OF SUCCESSES (SUCCES)

100 REM THIS IS A PROGRAM FOR FINDING THE AVERAGE NUMBER OF SUCCESSES 110 REM IN ONE HUNDRED SAMPLES OF SIZE M DRAWN FROM A POPULATION OF 120 REM SIZE N WITH NI SUCCESSES. THE PROBABILITY P OF SUCCESS ON ONE 130 REM DRAW IS N1/N. NI<N. M SHOULD BE NO MORE THAN ABOUT 10% OF N. 140 REM M MUST BE LESS THAN OR EQUAL TO 100. 150 RANDOMIZE 160 DIM T(100),L(1000) 170 PRINT "THE SIZE OF THE POPULATION IS" 180 INPUT N 190 PRINT "THE NUMBER OF SUCCESSES IN THE POPULATION IS" 200 INPUT NI "THE SAMPLE SIZE IS (MUST BE NO MORE THAN 100)" 210 PRINT 220 INPUT M 230 LET P = N1/N 240 FOR K = 1 TO 100 250 LET T(K) = Ø 260 FOR I = 1 TO M 270 LET X = RND(X)280 IF X<=P THEN 310 290 NEXT I 300 GO TO 330 310 LET T(K) = T(K) + 1 320 GO TO 290 330 LET R = R + T(K)340 NEXT K 350 FOR I = 1 TO 100 360 IF T(I) = 0 THEN 380 37Ø GO TO 4ØØ 380 LET J = J + 1 390 GO TO 440 400 FOR K = 1 TO M 410 IF T(I) = K THEN 430 420 NEXT K 430 LET L(K) = L(K) + 1 440 NEXT I 450 PRINT "THE DISTRIBUTION OF SUCCESSES PER ONE HUNDRED SAMPLES IS" 460 PRINT "SUCCESSES", "FREQUENCY" 470 IF J = 0 THEN 490 480 PRINT " Ø", J 490 FOR K = 1 TO M 500 IF L(K) = 0 THEN 520510 PRINT K,L(K) 520 NEXT K 530 PRINT "IN ONE HUNDRED SAMPLES OF SIZE" M" THE AVERAGE NUMBER OF" 540 PRINT "SUCCESSES IS"R/100 550 END

PADGAAMS

COMPUTER

BINOMIAL COEFFICIENT (BINOM) 15 100 REM THIS IS A PROGRAM TO COMPUTE THE BINOMIAL COEFFICIENT NCR 110 REM N!/(R!*(N-R)!) FOR GIVEN N AND R WITH Ø<R<N<26 120 PRINT "REMEMBER, Ø<R<N<26" 130 PRINT "N AND R ARE EQUAL TO" 140 INPUT NJR 150 IF R<N-R THEN 170 160 LET S = N - R170 LET S = R 180 LET P = 1190 FOR I = 1 TO S 200 LET P = P*(N - I + 1)/I210 NEXT I 220 PRINT "THE BINOMIAL COEFFICIENT FOR N="N"AND R="R"IS"INT(P+.3) 230 END PROBABILITY P(X:N,P) (PROBXN) 16 100 REM THIS IS A PROGRAM TO COMPUTE P(X:N,P) WHERE X IS THE NUMBER OF 110 REM SUCCESSES IN AN EXPERIMENT OF N TRIALS WITH THE PROBABILITY OF 120 REM SUCCESS ON EACH TRIAL BEING P. N IS A POSITIVE INTEGER LESS THAN 130 REM OR EQUAL TO 50. P IS A POSITIVE NUMBER LESS THAN 1 AND 140 REM X TAKES ON INTEGRAL VALUES FROM 1 TO N - 1. 150 PRINT "WHAT ARE THE VALUES OF N, X AND P?" 160 INPUT N,X,P 170 IF N>50 THEN 190 180 GO TO 210 190 PRINT 'N MUST BE NO LARGER THAN 50" 200 GO TO 150 210 IF N - X < X THEN 240 220 LET Z = X230 GO TO 250 240 LET Z = N - X250 LET R = 1260 FOR I = 1 TO Z270 LET R = R*(N - I + 1)/I280 NEXT I 290 LET R = R*P*X*(1 - P)*(N - X) 300 PRINT "THE PROBABILITY REQUIRED IS"R 310 END PADGAAMS

CONFIDENCE LIMITS (CONLIM) 17 100 REM THIS IS A PROGRAM TO COMPUTE THE CONFIDENCE LIMITS ON THE 110 REM PERCENT OF SUCCESSES IN A POPULATION WHEN THE PERCENT OF 120 REM SUCCESSES IN A SAMPLE OF SIZE N IS KNOWN. 130 PRINT " THE NUMBER OF ELEMENTS IN THE SAMPLE IS" 140 INPUT N 150 PRINT "THE NUMBER OF SUCCESSES IN THE SAMPLE IS" 160 INPUT M 170 PRINT "THE PERCENT OF SUCCESSES IN THE SAMPLE IS"100*M/N 180 LET P = M/N 190 LET S = SQR(P*(1 - P)/N) 200 PRINT "THE CONFIDENCE LIMITS ON THE PERCENT OF SUCCESSES IN THE" 210 PRINT "POPULATION ARE:" 220 PRINT 230 PRINT "95%", 100*(P-2*S), 100*(P+2*S) 240 PRINT "99%", 100*(P-2.5*S), 100*(P+2.5*S) 250 PRINT "99.7%", 100*(P-3*S), 100*(P+3*S) 260 PRINT "IF THE SIZE OF THE POPULATION IS KNOWN TO BE T THEN INPUT" 270 PRINT "IT NOW" 280 INPUT T 290 PRINT "THE CONFIDENCE LIMITS THEN ARE:" 300 PRINT "95%", INT((P-2*S)*T), INT((P+2*S)*T + .999999) 310 PRINT "99%", INT((P-2.5*S)*T), INT((P+2.5*S)*T +.999999) 320 PRINT "9,9.7%", INT((P-3*S)*T), INT((P+3*S)*T + .999999) 330 END PROBABILITY OF SAME BIRTHDAYS (BIRDAY) 18 100 REM THIS IS A PROGRAM TO COMPUTE THE PROBABILITY THAT TWO OUT OF 110 REM N PEOPLE HAVE THE SAME BIRTHDAY. 120 PRINT "THE PROBABILITY THAT AT LEAST TWO OUT OF THE N PEOPLE" 130 PRINT "HAVE THE SAME BIRTHDAY:" 140 PRINT " N P" 150 FOR N = 10 TO 40 STEP 5 $160 \ LFT \ Q = 1$ 170 FOR I = 1 TO N - 1 $180 \text{ LET } Q = Q \times (365 - I) / 365$ 190 NEXT I 200 LET P = 1 - Q210 PRINT N,P 220 NEXT N 230 END

PADGAAMS

COMPUTER

19 ORDERING NUMBERS (ORDER)

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100 REM THIS IS A PROGRAM TO ORDER A SET OF UP TO 50 NUMBERS FROM
110 REM SMALLEST TO LARGEST.
120 DIM X(50)
130 PRINT "HOW MANY NUMBERS DO YOU WISH TO INPUT?"
140 INPUT N
150 PRINT "INPUT A NUMBER AFTER EACH QUESTION MARK."
160 \text{ FOR I} = 1 \text{ TO N}
170 INPUT X(I)
180 NEXT I
190 PRINT
200 FOR I = 1 TO N
210 \text{ FOR } J = 1 \text{ TO } N - I
220 IF X(J)<X(J+1) THEN 260
230 LET B = X(J+1)
240 \text{ LET } X(J+1) = X(J)
250 \text{ LET } X(J) = B
260 NEXT J
270 NEXT I
280 \text{ FOR I} = 1 \text{ TO N}
290 PRINT X(I);
300 NEXT I
310 END
```

20 SHUFFLING CARDS (SHUFLE)

100 REM THIS IS A PROGRAM TO SHUFFLE A DECK OF CARDS 110 PRINT 120 DIM A(52) 130 RANDOMIZE 140 FOR I = 1 TO 52150 LET A(I) = I 160 NEXT I 170 FOR I = 1 TO 52180 LET X = INT(RND(X) * 52 + 1)190 LET B = A(X)200 LET A(X) = A(I)210 LET A(I) = B 220 NEXT I 230 FOR I = 1 TO 52240 PRINT A(I); 250 NEXT I 262 END

PARGAM

21 ACES

100 REM THIS IS A PROGRAM TO SIMULATE THE WAITING TIME FOR THE FIRST 110 REM ACE WHEN A WELL SHUFFLED DECK OF CARDS IS DEALT ONE CARD AT A 120 REM TIME 130 RANDOMIZE 140 FOR K = 1 TO 1000 150 LET C = 52 160 FOR J = 1 TO 49 170 IF INT(RND(X)*C+1)<5 THEN 200 180 LET C = C - 1 190 NEXT J 200 LET S = S + J 210 NEXT K 220 PRINT "THE AVERAGE WAITING TIME FOR THE FIRST ACE IN 1000 DEALS IS" 230 PRINT S/1000 240 END

22 COLORED PENS (BALPEN)

100 REM THIS IS A PROGRAM TO SIMULATE HOW MANY PURCHASES YOU NEED TO 110 REM MAKE TO GET ALL THE PENS IF N DIFFERENTLY COLORED PENS ARE 120 REM PACKED ONE PEN TO A BOX OF CEREAL 130 REM THERE MAY BE UP TO 20 DIFFERENT XINDS 140 DIM A(20) 150 RANDOMIZE 160 PRINT "HOW MANY DIFFERENT PENS? (UP TO 20)" 170 INPUT N "HOW MANY DIFFERENT TRIALS DESIRED? (UP TO 100)" 180 PRINT 190 INPUT M 200 IF M>100 THEN 220 210 GO TO 240 220 PRINT "NO MORE THAN 100 TRIALS ARE ALLOWED" 230 GO TO 180 240 FOR I = 1 TO M 250 FOR J = 1 TO N 260 LET A(J) = Ø 270 NEXT J. 280 LET K = Ø 290 LET Z = INT(RND(X)*N + 1) 300 LET K = K + 1 310 LET A(Z) = 1320 FOR J = 1 TO N 330 IF A(J) = Ø THEN 290 340 NEXT J 350 LET S = S + K 360 NEXT I **370 PRINT** 380 PRINT "THE AVERAGE NUMBER OF PURCHASES IS"S/M"FOR"N"DIFFERENT PENS." 390 END

ORGANIZATION

Placement Guide for Tabbed Dividers

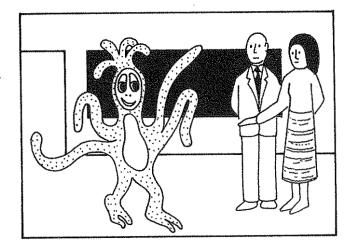
Tab Label	Follows page:
CONTENT FOR TEACHERS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
Probability with Counting	
Inferential Statistics	
Appendix-Computer Programs	
DIDACTICS	
Components of Instruction	
Classroom Management	
Statistics and Probability Learning	
TEACHING EMPHASES	
Critical Thinking	
Decision Making	
Problem Solving	
Models and Simulations	
Calculators and Computers	
Laboratory Approaches	
CLASSROOM MATERIALS	
CONCEPTS AND SKILLS	
Gathering Data	
Tables	
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Scatter Diagrams	
Misleading Statistics	
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APPLICATIONS	
Business & Commerce	
Environment	
Health & Medicine	
People & Culture	
Recreation	
GLOSSARY	
ANNOTATED BIBLIOGRAPHY	
SELECTED ANSWERS	



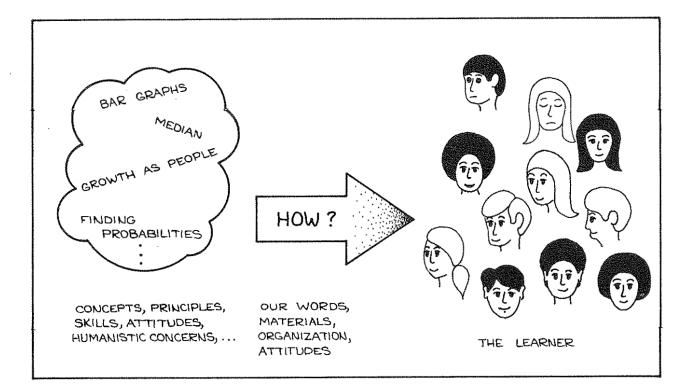
AN OVERVIEW

If a visitor from outer space visited you and asked, "What does a 'teacher' do?" how would you reply? No doubt part of your answer would mention transferring knowledge, thinking skills and attitudes to students. Explaining how this transfer is made might be difficult. Teaching is not yet a science (perhaps it should not be), although theory, research and experience do provide us with con-



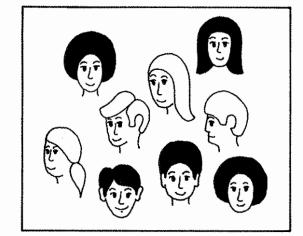


siderable guidance in our attempts to improve teaching. Several such facets as applied to the knowledge, skills and attitudes to be transferred, the nature of middle-school students, and some of the "how-to" have been touched on in the resources of the Mathematics Resource Project. This section presents an overview of many of these same facets.



THE MIDDLE SCHOOLER

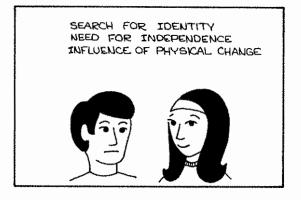
Let us start with the most important consideration in teaching--the learner. The most noticeable characteristic of middle schoolers is their variety of maturity levels. How can we possibly deal with a class made up of immature children, pre-adolescents, and a few relatively mature adolescents? Perhaps just keeping in mind that our students are <u>not</u> uniformly mature may help us. If our plans are



not going over well, we may have forgotten that many students need a greater variety, more changes of pace, or some less-verbal approaches than we may be providing.

Many of our students are likely to be early adolescents while they are in our

classes. There seem to be three outstanding features of adolescence: the search for an identity, the need for independence, and the impact of physical changes. These are likely to be interrelated; the physical growth toward an adult form is an obvious reminder to students that they <u>will</u> assume adult roles, which includes not being



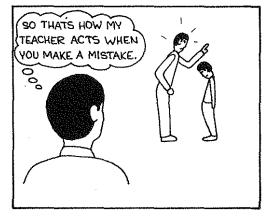
"treated like a kid" and <u>not</u> being bossed around. How can we help our students come to grips with the stresses and demands of adolescence? Perhaps by exposing the students to (their choices of?) many different situations in which they can assume adult roles (as in *What's on TV*? in the GATHERING DATA section or in the many activities in APPLICATIONS), we can allow them to sample situations which may aid them in seeing themselves as adults. Responding to their need for indepencence may be more difficult since some students seem to feel compelled to challenge even routine direction. Many teachers do manage to allow some student input into the choice of goals or activities. Finally, the students' growing awareness of their bodies and of themselves in relation to others means they are interested in data about themselves and others—and usually these data can be given graphical or statistical treatments. See, for example, *Data Sheet* (in the CATHERING DATA section), *What's*

In a Name? (in TABLES), My Favorite Color (in GRAPHS), Agree or Disagree (in SCATTER DIAGRAMS), To Each His Own, Muscle Fatigue, and What's Your Type? (all in HEALTH & MEDICINE).

People's self-concepts seem to be fragile things. And middle schoolers are people. Indeed, the best thing we can do for some middle schoolers is to build up their self-concepts. How? Well certainly not by reacting sarcastically to student responses. Certainly not by having obvious

favorites. Certainly <u>not</u> by mining obv favorites. Certainly <u>not</u> by ignoring the presence of particular students. Certainly <u>not</u> by...(you can likely name some practice which can detract from students' self-esteem). Enough <u>nots</u>. What are some positive things? Purkey [1970] gives some general areas.

RESPECT EACH STUDENT AS AN INDI-VIDUAL. Know names and if possible



something about the interests, strengths and weaknesses of individuals. PROVIDE EACH STUDENT WITH REASONABLE CHALLENGES--reasonable in the sense that

the student should be able to meet the challenge. In particular, watch that low expectations of usually weaker students do not betray you into communicating your expectations or into treating inferior work as praiseworthy. (It may be possible to give honest praise to effort, neatness, attendance, attitude, or the like when a student's work is subpar.)



BE SUPPORTIVE AND WARM TOWARD STUDENTS. Everyone likes a smile, a friendly greeting, or a genuine expression of interest. Students must realize that we are on their side and are not poised to "put them in their place" or to strike eagerly with the (perhaps) dreaded failing grade.

PROVIDE SOME SUCCESS EXPERIENCES. Doing this is so important but, at the same time, so difficult for the students who most need a taste of success. This need is perhaps one strong argument for including data-gathering work for graphing and statistics lessons or work with concrete materials in probability. In these sit-

uations even a weak student can make a contribution and enjoy some success.

Cognitive Development

The developmental psychologist Jean Piaget, after years of observation and experimentation, has come up with a description of mental development in terms of stages. (See the didactics paper *Piaget and Proportions* in <u>Ratio</u>, <u>Proportion and</u> <u>Scaling</u>, Mathematics Resource Project.) Most middle schoolers are at the *concrete*

operational stage or the formal operational stage or in transition between those two stages. Students at the concrete operational stage are not yet comfortable or systematic in dealing with the theoretically possible. For example, such a student might have difficulty in listing all the possible ways that two letters can be chosen from four letters and may have trouble with pages from the COUNTING TECHNIQUES section. Also,

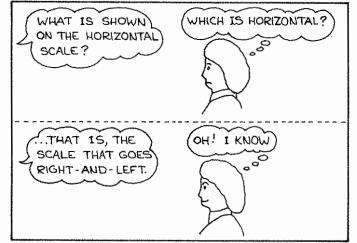
CHOOSE TWO IN ALL WAYS : A B C D CONCRETE OPERATIONAL AB, CD, uh, BD, AC, AD, uh, . FORMAL OPERATIONAL AB, AC, AD, BC, BD, CD. THAT'S ALL

concrete operational students are not usually adept at dealing with a relationship between two other relationships (e.g., equality between two ratios--that is, proportional thinking). Such students may be puzzled by an activity like *Take Your Choice* in the EXPERIMENTAL PROBABILITY section.

Other findings of the Piagetians which pertain directly to this resource are mentioned in a later didactics paper in this resource. Do keep in mind that the Piagetians have <u>not</u> usually tried to instruct the students they have worked with. They attempt only to determine the natural development of students' thinking patterns and capabilities. In most cases, how special schooling could alter these capabil-

ities has not been investigated.

You may have had the experience of having another student clear up some student's confusion, after your valiant efforts at explaining proved fruitless. We usually acknowledge the students' ability "to speak each other's language." This "speaking at the student's level" is touched on in *Planning Instruction in Geometry*



in <u>Geometry and Visualization</u>, although it is no doubt important in all areas of instruction. Briefly, investigators have hypothesized five levels of thinking in geometry and have pointed out that teacher talk may often be at level above that of the student. It makes sense to think that the same thing could happen with other topics.

SUMMARY

Let us remind ourselves frequently--particularly when faced with an exasperating student--that

- middle schoolers vary greatly in their maturity,
- adolescents feel the pressures of a search for an identity, a need for independence and a changing body,
- our influence on students' self-concepts may be one of our most important responsibilities, and
- students' thought and language patterns may not be as sophisticated as ours.

- Are there particular aspects of your students' behavior which might be explainable in terms of biological changes, a need for independence, or a search for identity?
- 2. Thumb through this resource and examine a few pages with these questions in mind: Could this activity be used in some fashion to make my students feel more independent? Could this activity contribute to some student's search for an identity?
- 3. Ms. Lead: "I've got students who have to be told every little thing to do. They can't even read the directions by themselves. I don't believe this 'need for independence' applies to my students." Your reply:
- 4. Students are usually interested in how they "stack up" as compared with other students.
 - a. How is this interest related to their search for identity?
 - b. This interest may lead to some stress. For example, a boy who is quite short or a girl who is quite tall may be sensitive about discussion of heights. How can a student page like Are You a Square or a Rectangle? (in the SCATTER DIAGRAMS section) make them less self-conscious about their heights?
 - c. Give some other topics that would require careful handling (e.g., weights, family income,...).
- 5. a. List some practices-to-be-avoided in dealing with students, with respect to damaging their self-concepts.
 - b. List some things which might enhance our students' self-concepts.
 - c. Think of a particular student who seems to have a low self-concept. Plan how you might work to bolster this student's self-concept.

For more background on

Early adolescence

Student self-concept

Piaget

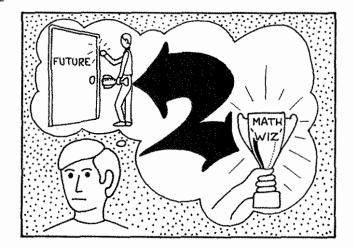
Levels of thinking

Broad goals. Faced with a district syllabus, a textbook-imposed sequence of topics or a local preoccupation with standardized test performance, we can easily lose sight of the broad goals of middle school education. As a result, some of our hardest efforts may be misdirected toward marginally important work. For example, should we replace some of the time on decimal division with time on critical reading of graphs (as in the INTERPRETING

see the Mathematics Resource Project's ...

- Middle School Students in Mathematics in Science and Society
- Student Self Concept in Number Sense and Arithmetic Skills
- Piaget and Proportions in Ratio, Proportion and Scaling
- Planning Instruction in Geometry in Geometry and Visualization

"WHERE ARE WE GOING?"... " "DID WE GET THERE?"



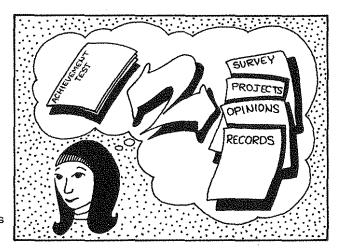
DATA section). Which of these, in view of the hand-held calculator, might better serve a broad goal like "preparation for everyday living"? Are we really working for "intelligent citizenship" if we stop with the calculation of mean, median and mode rather than continue to study when each of these is an appropriate summarizer of information (see, for example, The Average Family and What Do You Think? in the MEAN, MEDIAN, MODE section)? At regular intervals we should give a thoughtful reexamination of our mathematics curriculum's contribution to the school's broad goals.

Evaluation. If we claim to be working toward broad goals like, "to prepare for everyday living" or "to learn to work cooperatively" or "to possess a positive attitude toward learning," then we should try to find out, in some fashion, whether we are accomplishing those goals. What this means in most cases is that an evaluation of a program must go beyond measures of computational skills and scores from

COMPONENTS OF INSTRUCTION

DIDACTICS

standardized tests. Such "hard" data may be very important but at the same time give a very incomplete assessment of our achievements. Teacher opinions, anecdotal records, tallies of laboratory lessons or cross-disciplinary projects, requests (!) for project ideas, or student suggestions (!!) for school surveys may yield "soft" data, but at the same time data which give more valid measures of our contributions to broad goals than scores on skills.



SUMMARY

We must not lose sight of our broad goals, despite the immediate pressures of day-to-day teaching and the ready availability of measures of narrow skills. Not only must we keep these goals in mind, but we must also strive to evaluate, in as many ways as we can, our progress toward them.

??????

- 1. Read the broad goals for this resource (in the *Introduction*) to see where this resource might make unique contributions to your school's (or your own) broad goals.
- 2. How is the mathematics program at your school evaluated? Are there ways in which the evaluation could be supplemented?

For more background on...

see the Mathematics Resource Project's...

Broad goals

Broad Goals and Daily Objectives in <u>Ratio</u>, <u>Proportion and Scaling</u>

Evaluation

Evaluation and Instruction in <u>Ratio</u>, <u>Pro-</u>portion and Scaling

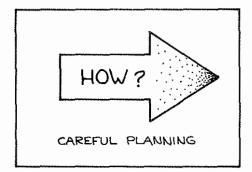
> THE TEACHING OF CONCEPTS

How do you proceed when you are teaching a new concept? (You may be in the habit of making a distinction between a concept and a principle; this distinction

is not made here since the guidelines for teaching both to middle schoolers are so similar.) There is some evidence that we should spend at least half of the class time on developing meaning and understanding through work with group discussions, concrete materials, teacher demonstrations, and laboratory activities. Here is a review of some of the other important elements in teaching concepts.

1. PREREQUISITES. It does not, of course, make any sense to resume teaching about a frequency distribution if our students don't remember what this is. Often we have a good idea from recent work how much review of prerequisites is needed; other times we may have to ask several questions or give a pretest.

2. MOTIVATION. It may be sufficient with some students to say only something like "Today we're going to study line graphs." Other classes may need considerably more than that: a line graph of phonograph record



TEACHING A CONCEPT
PREREQUISITES?
SOME INTERMINGLING OF
NAME FOR CONCEPT
DEFINITION
SEVERAL EXAMPLES
SOME NONEXAMPLES
FEEDBACK FROM STUDENTS
DIAGROSIS

sales, whale population, average weight of 12-year olds during this century, etc., may gain their attention better than words alone. Perhaps the key art of teaching is the ability to awaken interest in a topic. Teacher interest and enthusiasm, a "hot" topic, an interesting problem or challenge, a game, a response to some need-these things seem to work for many teachers. One of the aims of this resource is to supply you with ideas for introducing topics.

3. CONCEPT NAME. No one is likely to ignore the name for a concept completely, but perhaps we can give it more emphasis. Providing the name does signal that something name-worthy is coming up and does give a "peg" on which to put the idea. Writing the name on the board, commenting on its spelling, relating and contrasting it to other uses ("Who knows what a median in a highway is?") and pointing out its place in the lesson and its relationship to other concepts ("so this will give us three kinds of representative numbers: the mean, the median, and the mode)--all these are ways of giving attention to the concept name.

4. DEFINITION. Some concepts--e.g., stem-and-leaf display--would be awkward to

define carefully. However, for many concepts a definition can be given-and according to some researchers <u>should</u> be given when the language involved is not too complicated or lengthy. Two cautions are in order: (a) just because a definition has been given does <u>not</u> mean it has communicated and (b) just because a student can

MEDIAN - THE MIDDLE SCORE 1. PUT THE SCORES IN ORDER.

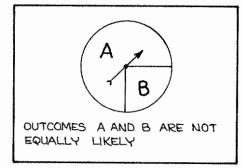
state a definition does <u>not</u> necessarily mean the concept has been mastered. *We can*not rely exclusively on words. Indeed, research indicates that a definition accompanied by examples and nonexamples gives better learning than a definition or example alone. [Klausmeier et al., 1974] It may also help to emphasize particular parts of a definition by using suitable examples and nonexamples, by underlining, or even by putting different parts of the definition on different lines.

5. EXAMPLES. The sentences above hint that we should have several, considered examples at hand. These examples should also vary the features irrelevant to the concept, or else the student may be misled into thinking that these are part of the concept or are *always* true about the concept. For example, we would want to make sure that our examples do not always result in the mean and median being equal, since the students may assume that they always are equal if they are in a few cases (the MEAN, MEDIAN, MODE page, *Median vs Mean*, might prevent such a misconception). Or since the mode is described as "the color that occurs most often" in *They Melt in Your Mouth* (in the GRAPHS section), a student might assume that the mode is a *color* if this is the *only* example ever given. The more examples we use, the less likely some irrelevant feature will occur in all of them.

6. NONEXAMPLES. Nonexamples enable us to emphasize particular parts of a definition by noting their absences in the nonexample: "This is not a random sample because each object was not equally likely to be picked." Nonexamples are most easily generated for concepts whose definitions have parts; make up a case which does not have that part and you have a nonexample. Usually you will use a nonexample

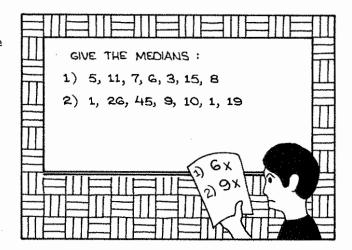
only after the students have seen some examples so that they will have a better chance of telling why the case is a nonexample. Finally, a nonexample may help to avoid overgeneralizing. After several sets of data have been fitted by straight lines,

a SCATTER DIAGRAMS page like Sounds Fishy to Me could prevent students from assuming that every set of data can be fitted by a straight line. Or after several experiences with equally-likely outcomes, a spinner like that in the figure could be used to show that sometimes outcomes are not equally likely.



7. FEEDBACK NEEDED. The warning was given above about relying too much on a student's statement of a definition as the only evidence of whether the student has learned a concept--that may indicate only that the student has learned a proper sequence of words. When we give the students several cases, they should be able to pick out which ones are examples and which are nonexamples. At some stage, they should be able to make up a new example. 8. DIAGNOSIS. Students often make careless errors, of course. But many times

a student's errors may reveal a misconception. The student in the figure might be miscounting--or might have the incomplete idea, "the median is the middle score," forgetting that the scores should be ordered first. A few seconds' thought or a question if the student is at hand ("Why do you say that?") can often reveal a misconception and allow us to do more than shake our heads and think, "Jay's not getting this."



The order in which to give attention to these items is still part of the art of teaching. Considerable study of, for example, whether to give the definition first or to precede the definition with some examples is being carried out. You are welcome to do your own experimenting!

SUMMARY

In teaching a concept, we should give attention to

- the prerequisites,
- thinking of some motivator,
- emphasizing the name of the concept,
- planning how to state and use a definition,
- developing several examples and nonexamples, and
- getting frequent feedback from as many students as possible.

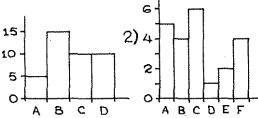
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- 1. Choose some concept you teach. What would be your responses for the various items given in the summary above?
- 2. Give some examples or nonexamples to clarify these misconceptions.
 - a. It is not necessary to label the axes on a bar graph.
 - b. The mode and the median are the same.
 - c. For any data, you can use either the mean, the median or the mode.
 - d. If there are six ways it can come out, each way has probability 1/6.
 - e. A graph that goes up as you go to the right is a graph of running totals.
- 3. Below are some exercises and Jay's answers. What is Jay's misconception?

1)

Exercises: Give the mode.

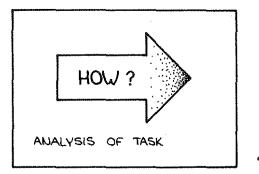
Jay's answers: 1) 15 2) 6



For more on concept teaching, see *The Teaching of Concepts* in <u>Geometry and</u> Visualization, Mathematics Resource Project.

THE TEACHING OF SKILLS

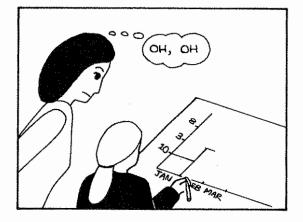
Making graphs, calculating means, listing outcomes of an experiment, determining probabilities--there are many skills involved in this resource. Let us survey several points in teaching a skill.



1. *PREREQUISITE SUBSKILLS*. To make a circle graph, for example, a student needs to know how to use ratios to figure out the part of the circle for

each category and how to use a protractor. If these subskills have been forgotten, they should be reviewed so their lack will not detract from your explanation. Some-: times it is easy to overlook prerequisites.

The simple matter (for us) of labelling and reading axes is not easy for many students. Many will need work with pages like *Completing Scales*, *Scale and Plot*, or *How Remarkable!* (all in the GRAPHS section). Or, before determining medians a review of ordering data may be a good idea. It is sometimes surprisingly difficult to analyze a task completely for subskills.



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2. CAREFULLY CHOOSE THE EXAMPLES YOU DEMONSTRATE. In demonstrating the making

ON TUESDAY THERE

WERE 110 ... WELL, AS YOU

CAN SEE

of a line graph, for example, one would avoid cases which have several categories, data over an extensive range, or data involving difficult plotting. For a first circle graph, one would likely choose a case for which the computations are not too complicated.

 DEMONSTRATE COMPLETELY to give students an idea of the whole process.

4. GO OVER THE STEPS, emphasizing the ones likely to be new or to cause trouble (e.g., the labelling of axes mentioned above).

5. BE PREPARED TO DEMONSTRATE ANOTHER CASE, perhaps having students parallel your work at their seats or come to the board to demonstrate particular steps.

6. GET THE STUDENTS DOING SOMETHING as quickly as possible. It is when the *learners* are trying the skill that you get feedback on pacing and points of confusion--and the learners are finding out where they get stuck. Try to plan so that in-class time is available for the students to try at least a few examples while you are on hand for additional information.

7. DIAGNOSE. A mis-plotted point could be only a slip-up in reading coordinates, but it might also result from reversing the coordinates. The rule of thumb is to ask yourself, "Might this error result from a misunderstanding?"

SUMMARY

- A key to the teaching of a skill is in the planning:
- analyze the skill for prerequisite subskills,
- plan your examples carefully,
- pace your demonstration carefully, emphasizing difficult points,
- involve the students as soon as possible, and
- seek feedback from all students, carefully examining errors for wrong ideas.

?????

- 1. List all the prerequisite subskills for making a circle graph. (Don't forget angles with measurements like 210°!)
- 2. What would be prerequisites for reading a pictograph?
- 3. List the prerequisite subskills for determining which of two situations gives the better chances of winning.
- 4. What sort of graphing might student work like Scale Sketches (in GRAPHS) profitably precede?

For more background on... Teaching skills

Diagnosis of errors

see the Mathematics Resource Project's ...

The Teaching of Skills in <u>Number Sense and</u> <u>Arithmetic Skills</u>

Diagnosis and Remediation (numerical examples) in the same resource.

TEACHING FOR TRANSFER Can our students apply what they have learned? Sometimes the answer is disappointing, especially when the situations are outside the mathematics classroom. Here are some suggestions for maximizing transfer: 1. IMITATE AS MUCH AS POSSIBLE

THE SETTINGS WHERE APPLICATIONS WILL LIKELY ARISE. Doing this is easy for work with reading graphs since graphs appear frequently in newspaper and magazines and can be

used for classwork. Situations in which data are to be collected and then graphed

can be virtually the same in classroom settings and "real" settings, particularly if questions of what data, how to collect them, the cost of collecting, and how best to report them are included on occasion.

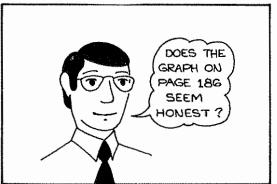
2. GIVE SUFFICIENT ATTENTION TO THE MAJOR POINTS. Rather than spending just one day on deceptive graphs and then



never mentioning them again, a teacher should frequently bring up the question of whether a new graph seems to give a fair presentation. Or as another example, re-

minders like, "When we say the probability of a win is 3/5, does that mean we'll win 3 times in 5 games?" may be necessary to avoid students' losing sight of the meaning of statements about probability.

3. USE A VARIETY OF SETTINGS. The richness of the topics of graphing,



statistics and probability invites many different applications. So many things can be graphed or summarized with statistics, and there are dice, cards, spinners, drawings, accident data, sales figures,...for probability. This guideline does not mean that *every* setting in which probability comes up should be discussed in a short period of time, but over the long term students should see the ideas in several different settings.

4. BE CAUTIOUS ABOUT NEGATIVE TRANSFER. Occasionally the learning of one topic interferes with the learning of another topic. Sometimes these situations can be predicted. For example, the similarity of the words "mean," "median," and "mode," and their uses with similar (or even the same) sets of data would signal that these ideas are likely to be mixed up by some students. When such situations are anticipated or discovered, we can warn against likely confusions and perhaps think up ways to enable the students to keep things straight.

5. EMPHASIZE APPLICATIONS. This resource contains many examples of reallife applications. See in particular the APPLICATIONS pages. Note, however, that applications within mathematics often occur, as in using graphing to study the outcomes from a probability experiment (e.g. Rolling Dice in the EXPERIMENTAL PROBABILITY section). Bulletin boards, talks by parents on careers, or news items can allow applications to come up. Devoting classtime to newspaper articles or topics brought in by students might help make students applicationconscious and rewards them for looking for applications.

SUMMARY

Transfer of learning is one of our most important goals. To help reach this goal, we might

- devise activities which resemble "real-world" situations,
- cover thoroughly the major ideas,
- use a variety of applications,
- watch out for negative transfer, and
- give lots of attention to applications.

PPPPP

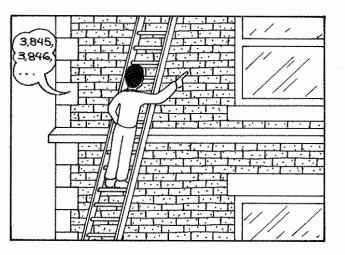
- Look through the APPLICATIONS pages to see...
 a. which pages you feel would be of value to the whole class.
 b. which pages you feel might have appeal to certain students.
- 2. Plan a "real" data-collecting and organizing activity with some group of your students. (See the commentary in the GATHERING DATA section.) Although you may prefer that the students choose their own theme, you might have a few in mind in case they have trouble coming up with an idea.
- 3. What do you feel are the major points to be made about graphs, statistics and probability?

For more background on transfer, see *The Teaching of Transfer* in <u>Mathematics in</u> Science and Society, Mathematics Resource Project.

PROBLEM SOLVING--GOAL AND TOOL

As is noted in *Problem Solving* in the TEACHING EMPHASES section, the word "problem" usually refers to situations which involve more than using only a wellknown procedure. Estimating (not just guessing) the cost of the bricks in a brick building, for example, would be a problem for most middle school students. On the

other hand, finding the cost of a <u>spec-</u> <u>ified</u> number of bricks, given the cost per brick, would for most students be only an exercise with the multiplication algorithm. Since we cannot possibly <u>know</u> all the mathematical problem situations our students will be faced with, we cannot hope to show all possible solutions. Yet we acknowledge the importance for our students of the goal of problem-solving ability. What <u>can</u>



we do to help our students grow in their ability to solve problems? One way which offers promise is to give them lots of problems and to emphasize general problem-solving steps like those of Polya mentioned in Problem Solving in the TEACHING EMPHASES section. Let us illustrate the steps with the problem above: Estimate the cost of the bricks in a specific brick building, given the cost of one brick.

1. UNDERSTANDING THE PROBLEM. Noting that only an <u>estimate</u> is called for should suggest that we do not need to know the exact number of bricks. (Many middle school students have not learned that exact answers, even when possible, are not always worth the extra work.) What is not clear from the problem is how close our estimate should be, so we would have to decide on that. The problem is oversimplified because the rate for a large number of bricks is likely less than the single-brick rate; we might wish to take this fact into account and seek information on bulk rates.

2. DEVISING A PLAN. This is the most difficult step. Problems--in the sense used here--usually involve determining what data are needed as well as how the data are related. Here, since the cost of each brick is given, we could estimate the value of the bricks if we could estimate how many bricks there are. Although a direct count of all bricks would be out of the question, perhaps the bricks in several small parts of the building (a sample) could be counted and, with (actual or estimated) measurements of the building, used to arrive at an estimate of the total number of bricks. Are the walls only one brick thick? Is there an air space? How should we pick our sample? Are there so many windows and doors that we would have to take them into account? How would we do this? How can we find the height of the building? If we can't perhaps we'll have to devise another plan--for example, counting the bricks in one row and estimating (or counting) the number of rows of bricks.

3. CARRYING OUT THE PLAN. In collecting the information, we may find deficiencies in our plan and have to revise it. For example, will our method of estimating take into account the mortar between bricks? It may happen, of course, that our plan can be carried through without modification.

4. LOOKING BACK. This step is often neglected but may be the most important since it involves an evaluation and review of our thinking. Is our estimate reasonable? Can we check it in some fashion? Is there another way to solve the problem---(e.g., calling the builders of the building and asking them!)? What was our method of solution? Are there other problems which could be solved by the same method? (This last question could be used to give middle schoolers some experience with thinking up problems, a valuable talent not promoted in most schools.)

If these steps are new to our students, we naturally will have to adjust our expectations of the students, particularly with respect to the devising-a-plan step. What we can do is to provide lots of problem-solving opportunities. We can be good problem-solving models for our students by following steps like Polya's. We can pursue false paths on occasion to show that such things <u>do</u> happen in solving problems.

Few would question growth in problem-solving ability as an important goal of mathematics teaching. Problems can also serve as a tool for instruction. For example, interesting problems like those

in *How Many Deer?* (in SAMPLING) can catch the imagination and <u>motivate</u> one to do some work willingly. Many teachers like to use a problem as a <u>lead-in</u> to a topic. For example, *A Cereal Question* in the EXPERIMENTAL PROBABILITY section provides an example of a simulation and would perhaps be a more attractive starting point for work with simulations than a lecture on simulations.

PROBLEMS CAN ... - AROUSE INTEREST - GIVE LEAD-INS - DISGUISE DRILL

The solutions of many problems may provide settings in which the student <u>practices</u> <u>other skills</u>, like finding averages, graphing, different forms of data collecting, critical thinking, basic computing, and so on.

SUMMARY

An important goal like problem-solving ability deserves all the attention we can give it. Providing (a) some problem-attack steps like those of Polya and (b) as many problem-solving opportunities as we can is one way to try for this goal. A problem-oriented approach has several bonus by-products: problems can motivate, give lead-ins to topics, and provide practice of subskills within the larger problem setting.

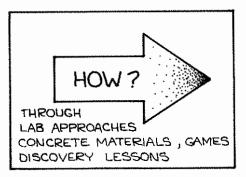
PPPPP

- 1. Rehearse how you might use Polya's problem-attack steps in attempting to solve a mathematical problem you know.
- Why would the following be regarded as exercises rather than problems?
 a. Calculating 87,984,923 x 6,412,385.
 - b. Making a bar graph for some given data (after much work with bar graphs).
 - c. Adding up all the grocery bills for a week.
- 3. Which one of the following uses of *How Many Deer?* (in the SAMPLING section) would better foster a problem-solving goal?
 - a. Pass out copies of the page and let students work on the page individually.
 - b. Ask the questions at the top of the sheet and invite discussion before passing the page out.
 - c. Use the page, but supply the data for the students.

(See the commentary in the SAMPLING section for other ideas on introducing this activity.)

For more background on problem solving, see *Teaching Via Problem Solving* in <u>Mathematics in Science and Society</u>, <u>Mathematics Resource Project</u>. Word problems are also discussed in *Reading In Mathematics* in the project's <u>Ratio</u>, <u>Proportion and</u> <u>Scaling</u> resource.

TEACHING WITHOUT TELLING Teaching without telling? You may have seen, or even tried, one of those striking demonstration lessons in which the teacher never says a word, but that is not what we have in mind here. Rather, these paragraphs treat two



teaching techniques which often overlap: laboratory approaches and discovery lessons. These take the teacher from center-stage and, it is hoped, lead to greater "audience participation."

Lab approaches. In these resources, a lab approach means any method which emphasizes *learning-by-doing*, especially as contrasted with learning-by-being-told. Lessons in which students gather their own data to graph (e.g., *Muscle Fatigue* in the HEALTH & MEDICINE section) or perform a dice-rolling experiment are sample lab lessons. What are the attractions of such lessons? Besides the active involvement of the students in the lesson, these five are usually mentioned:

1. LAB APPROACHES ALLOW ATTENTION TO THE PROCESSES OF LEARNING. Data gathering, data organizing, decision making, problem-solving skills—all these may be the student's responsibility in a lab lesson. In a teacher-centered lesson, the teacher may be the only one who practices most of those skills.

2. LAB APPROACHES ALLOW STUDENTS TO BE INDEPENDENT of the teacher in such activities as gathering and analyzing data. This does not mean the teacher plays no role in the lesson; the teacher becomes a resource person rather than the information giver.

The need for independence by some middle schoolers was mentioned earlier. Laboratory lessons in which the students can operate relatively independently of constant

teacher direction may help us meet that need. Laboratory lessons--particularly ones in graphing, sampling and simulations--can easily involve adult-like activities (see the APPLICATION pages) and can include small group work. Thus the lessons may also contribute to the search for identity by exposing the student to grown-up roles and by providing the opportunity for that student to note how others in the group react to him.

GOOD WORK! DO YOU THINK IT WOULD BE INTERESTING TO BE A MARKET RESEARCHER ? DIFFERENT COLORS

3. LAB APPROACHES FOSTER STUDENT-

STUDENT INTERACTION. At an age when the peer group is so important, this interaction may be a most valuable off-shoot. Students <u>can</u> learn from each other, and in the process may learn something about cooperation, working with others, and relating to others.

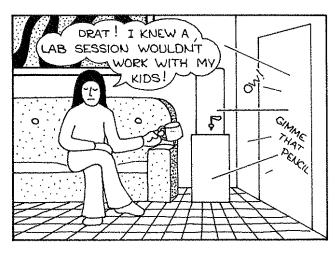
COMPONENTS OF INSTRUCTION

4. LAB APPROACHES MAY enable each student to contribute something and thus HELP THE STUDENT'S SELF-CONCEPT. In particular, with concrete materials at hand, the student can usually do <u>something</u> and perhaps check it with the materials before handing in an incorrect answer. If it is an experiment, almost anyone can throw dice or shuffle cards or draw marbles.

5. LAB APPROACHES OFTEN use concrete materials, which PROVIDE CONCRETE BASES

FOR ABSTRACT IDEAS. Concrete materials will be discussed in greater detail after a few sentences about the use of lab approaches.

The teacher role in a lab lesson changes from a whole-class focal point to a helper, guider, prodder, and reinforcer of individuals or small groups. Lab lessons require perhaps more preparation than teacher-centered instruction since guide-sheets must be readied and

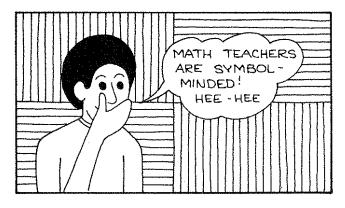


equipment arranged for. It is a good idea to start slow when the teacher or the students are not accustomed to lab lessons. *Laboratory Approaches* in the TEACHING EMPHASES section gives several suggestions for gradually easing into some lab lessons.

The use of concrete materials. During your college coursework did you ever grapple with an abstract idea with minimal success, and then suddenly "see" it when someone presented the idea in a more tangible form? For you as a college student that "tangible" form may have been a drawing or even something abstract like the set of integers. In that sense, pictures and familiar ideas might be considered "concrete materials" for college students. Although pictures and familiar abstractions may be "concrete" enough for many of our students, "concrete" will be used

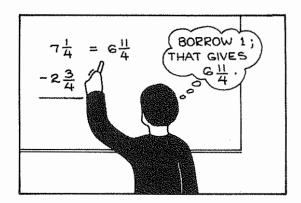
here to mean "physically real"-dice, matchsticks, stopwatches, slips of paper, graph paper, etc.

Perhaps the most compelling argument for concrete materials is that they can give a real-world basis for abstract ideas. So many mathematical entities--numbers, geometric figures, probabili-



ties, for example—are in fact abstractions which, properly speaking, exist only in our heads. But holding up three fingers or showing three blocks—concrete representation—does enable us to develop the abstract idea of three. It is easy—perhaps too easy—for mathematics instruction to become too abstract and bound to symbol

shuffling. Some students "study" perimeter without ever actually measuring anything! Some students "carry" and "borrow" in addition and subtraction exercises without knowing what justifies these procedures. Fortunately, the concepts and principles in this resource allow us to build many experiences with concrete materials into our instruction--see the list of student



pages at the end of Laboratory Approaches in the TEACHING EMPHASES section.

Some theorists advocate giving as many different *types* of concrete representation for an idea as possible. For example, the notion of randomness becomes richer through experiences with drawing slips of paper, tossing coins, throwing dice, flipping spinner arrows, etc.

Of course, all the types should not be given in a short time period; the idea is that over the years of students' school experiences, they should see as many concrete representations as practical. Note also that using a variety of materials may help us meet individual differences better. Only a little is known about learning styles, but it does seem likely that using many sorts of sensory input should help us reach more students. On the other hand, some slower or learning disabled students may not be able to digest several kinds of material and might profit most from work concentrated on one or two types.

<u>Games</u>. Instructional games have become increasingly popular in the last few years. They can put drill in an interesting setting, add variety to a class, provide enrichment, improve attitudes, and if play is observed carefully, enable the diagnosis of misconceptions. For example, *Crazy Quotients* (in the EXPERIMENTAL PROBABILITY section) could be used to provide division practice. *The Even/Odd Game* in the same section could help develop intuitions about probability. Discovery lessons. Discovery lessons could well be described as problem-solving lessons. In both, the students are confronted with a situation which does not immediately suggest an algorithm for a solution. Discovery lessons could also be considered laboratory lessons since they usually involve the same gather-data, organize-data, analyze-data, make-and-test-conjecture steps. They offer many of the same advantages as laboratory lessons: a student-centered atmosphere, emphasis on strategies of learning, peer-interaction, drill in a larger setting. For example, students could discover through their own efforts (and the guidance of *Rolling Dice* in EXPERIMENTAL PROBABILITY) that 7 is the most likely sum when tossing pairs of dice. Discovery lessons are particularily valuable if students have a misconception. For example, coin, die, and card experiences may lead to their thinking that each outcome in an experiment is equally likely. A two-outcome experiment like tossing a thumbtack (choose your thumbtacks carefully) can convince them otherwise.

SUMMARY

Laboratory approaches, games, and discovery lessons provide

- a learner-center climate,
- a student-generated rather than teacher-imposed solution, and
- student-student discussions of ideas.

Concrete materials can...

- give a firm foundation for abstract ideas and
- help to meet the individual needs due to differing abilities, differing learning styles, and the searches for independence and identity.

???????

- If you have had enough experience with laboratory approaches to give a fair evaluation, react to the five "attractions" listed for laboratory approaches.
- Pick out two of the list of classroom pages at the end of Laboratory Approaches (in the TEACHING EMPHASES section) and plan how you would use them to work toward the five "attractions" listed for laboratory approaches.
- 3. (Discussion) What sorts of things should a teacher do
 - a. during a lab lesson?
 - b. during a discovery lesson?

- 4. (Discussion) Do you use concrete materials in your teaching? Why or why not?
- 5. (Discussion) What has been your experience with instructional games? Do you have any testimonials or warnings about games to give?
- 6. D. Thomas: "I don't think I could try a lab lesson with small groups. My kids would spend all the time talking about ballgames and parties and what happened yesterday after school. They would get very little work done."

You:

- 7. Ms. Jackson: "Come on now! I've gone to college more than four years. I've thought very hard about how to explain things to my students. Are you telling me that all my background and experience should be ignored and the students left to figure things out themselves?" You:
- 8. Mr. Thomas: "I don't think my students need hands-on experience. Sure, I <u>talk</u> about concrete things, but if you use the actual materials, the kids won't exercise their imaginations." Your reaction to Mr. Thomas?

For more information on... Laboratory approaches

(and concrete materials)

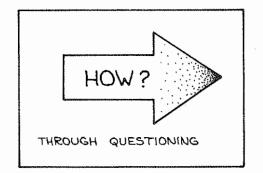
Games

Discovery lessons

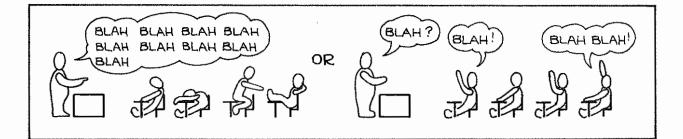
QUESTIONING

Experienced teachers know that when <u>they</u> do all the talking during a class session, students become inattentive, behavior problems may develop, and there is no feedback on what the students have learned. Our handiest tool for involving students--getting see the Mathematics Resource Project's

- Laboratory Approaches in the TEACHING EMPHASES section of this resource
- Teaching via Lab Approaches in <u>Mathematics</u> in <u>Science</u> and <u>Society</u>
- Goals Through Games in Number Sense and Arithmetic Skills
- Goals Through Discovery Lessons in Geometry and Visualization



them to do something or to react to something--is to use questions. Moreover, actively encouraging <u>students</u> to ask questions shows that you expect an <u>active</u> involvement from them, not just a <u>reactive</u> one.



Don't you use questions for these purposes?

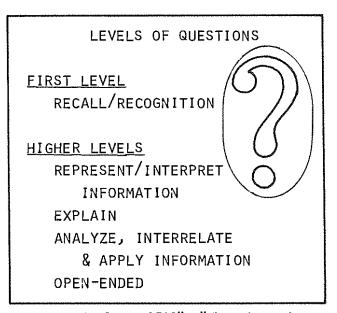
- -To motivate: "Would you rather have the mean income in the U.S. or the median income?"
- -To get students to evaluate or interact: "What do you think of group 1's method?"
- -To diagnose: "Then what did you do?" as well as

-To get general feedback: "Everyone calculate the probability of getting 11." (Let us include directions like "Find the median of these scores" as questions.)

This section touches on these things: asking some "higher level" questions, watching the pacing of our questions, and promoting the use of <u>student</u> questions. *QUESTIONS AT DIFFERENT LEVELS*

"First-level" questions are those which require only recall or recognition of specific facts, cases or routine procedures. Here are examples of "first-level" questions for middle schoolers: "Is the median greater than the mode?" "So 3 of the 4 outcomes have at least 1 head. What is the probability of at least one head?" "What is the middle score called ?" "What is the title of the graph?" "In which year was the kangaroo population greatest?" Such questions are, of course, very important in the classroom. They involve the students, giving them a chance to respond (usually successfully if they have been paying attention); they give us feedback; they enable us to emphasize particular facts or processes. It is when a teacher asks <u>only</u> first-level questions (or "zero-level" questions like "4 + 5 = 9.

Right, James?") that one becomes concerned. Questions requiring more than memory--"higher-level" questions--should also be asked. Otherwise we are not asking our students to exercise or develop cognitive skills other than memory. The outline in the box gives an indication of some categories of "higher-level" questions. Here are some samples, roughly categorized. (Assume that answers cannot be produced by mere recall or recognition of earlier classroom or text work.)

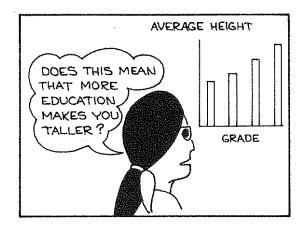


REPRESENT/INTERPRET INFORMATION. "What is the trend after 1970?" "What does the weather report mean when there's a 30% chance of rain?" "What does this graph seem

to imply?" "Make a table for these data."

EXPLAIN. "How do you decide what sort of graph to make?" "How do you figure out the probability of all heads?" "Why would a line graph be inappropriate here?"

ANALYZE, INTERRELATE AND APPLY INFOR-MATION. "Is the mean always greater than the median?" "How is this graph



misleading?" Word problems usually require analyzing and interrelating data--perhaps that's why they are so difficult.

OPEN-ENDED. "Find other ways to describe how scattered the numbers are." "How can we find out how many calories the kids in the whole school eat in a year?" "Which way is the easiest?" "How can you tell whether these dice are fair?" "How could you find out?"

If you would like to increase the number of higher-level questions in your lessons, it may be necessary--especially at first--to spend a few minutes of planning time thinking some up. As with most things, higher-level questions become easier to think up as we get more and more experience. Some work has shown a very high correlation between the level of a <u>teacher's</u> questions and the level of the the <u>student's</u> questions. (Two birds with one stone?)

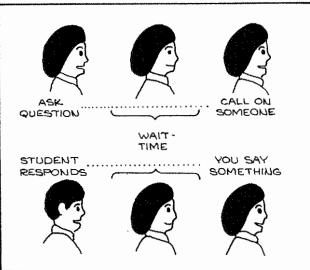
PACING OUR QUESTIONS

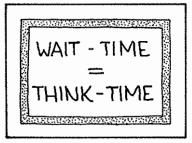
After you ask a question, how long do you wait before calling on a student for a response? Then after a student responds, how long do you wait until you comment on the response or ask another question? If your average "wait-time" is about one second, you

are like the teachers studied by Rowe [1969]. How much time does one second allow for *thinking*? Undoubtedly, one second is <u>not</u> enough time for many of our students to think of an answer or to evaluate and digest someone else's response. And it does help to wait longer. When Rowe's teachers extended their wait-time to 5 seconds,

fewer students said, "I don't know;" students gave longer answers and asked more questions; there was more studentstudent discussion of responses; the teachers asked a greater variety of questions; and they were able to treat students' responses more thoughtfully and flexibly. Truly something from "nothing"! QUESTIONS FROM STUDENTS--HOW?

Student questions promote student learning. However, student questions sometimes fall exclusively into the "first-level" ("How do you get the median?") or even a "zero-level" ("What page are we on?"). We all are thrilled when a student asks a question which shows imagination or insight: "Why don't we figure deviations from the median instead of from the mean?" What



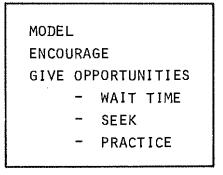




can we do to get more "higher-level" questions? Two things were mentioned in the earlier paragraphs: ask higher-level questions ourselves, and extend our postquestion and post-response wait-times. What else can we do? Certainly we can encourage good questions by reinforcing them when we get them, pointing out why you regard them as particularly good questions--"That question shows you have been looking for a pattern." (This statement is <u>not</u> meant to imply that low-level or routine

questions should be given condescending treatment. Common courtesy alone, if not a positive classroom climate, demands better than that!)

Statistics offers us many chances to get at student interests by having <u>them</u> ask questions about topics they would like to explore. We may need to give attention to question-asking. For example, the students could collect and



appraise a set of questions. Or they could prepare some poorly written ones and have other students improve them. Another way would be to present a graph, a table or student-collected data and have the students make up questions that can (and cannot) be answered from the given information.

SUMMARY

Questions can spur student reaction and participation. <u>Student</u> questions mean student involvement and learning. There may be great payoff in...

- striving to ask questions at a variety of "levels,"
- lengthening our "wait-times" after asking a question and after a student responds, and
- adopting as one of our goals an improved ability of students to ask questions.

PPPPP

- Think back over a very recent lesson you taught. Did you ask some "higher-level" questions? List some.
- 2. Tape a lesson and listen to a play-back. How are your "wait-times"?
- 3. Make a conscious effort to lengthen your "wait-times". Do there seem to be any

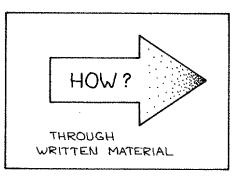
changes in the quality or quantity of the classroom interaction?

- 4. Should "wait-times" be greater for "higher-level" questions? Explain.
- 5. Visit a colleague's classroom just for the purpose of studying the colleague's questioning techniques and the quality of the student questions.
- 6. Are the following "higher-level" questions? Where would you classify them? Think of some similar questions.
 - a. Question 4 in Weather and Water Conditions, in the TABLES section.
 - b. Question 4 in It's Not Healthy to Drive Near Home, in SCATTER DIAGRAMS.
 - c. The questions at the bottom of What Do You Think?, in MEAN, MEDIAN, MODE.
 - d. Question #2 or #3 of Don't Lose Your Marbles, in SAMPLING.
- 7. How could you use the following to make points about question-asking? (The first three are from the GATHERING DATA section.)
 - a. A Traffic Problem
 - b. What's Your Bias?
 - c. Examining the Facts (in MISLEADING STATISTICS)
- 8. Johnson includes the following in his excellent article on questioning [1971]. What are your answers?
 - a. "Do the answers to your questions depend primarily on a good memory or a good understanding?"
 - b. "Does a student know whether his answer is right or wrong by your facial expression?"
 - c. "Do you spend as much time on preparing questions to ask students as you do on preparing the lecture material?"
 - d. "Are your questions usually directed at only a certain portion of the room (the action zone--center and front)?"
 - e. "Have you begun a permanent file on good questions for each concept that you teach during the year?"
- 9. (Discussion) What should you do when a student asks a question you can't answer?

For more information on questioning, see *Questioning* in <u>Geometry and Visualiza-</u> tion, Mathematics Resource Project.

READING IN MATHEMATICS

How can we take the difficulty out of reading mathematical material? The special symbols, the precise technical meanings of words and the compactness of mathematical expressions require careful attention, good background and



COMPONENTS OF INSTRUCTION

DIDACTICS

usually a slower reading rate than other material. It may be that we can help our students on two fronts: (a) making it less easy for the student to get by <u>without</u> reading, and (b) giving more instruction or greater emphasis to reading skills in mathematics.

Here are a few ways to "encourage" students to read:

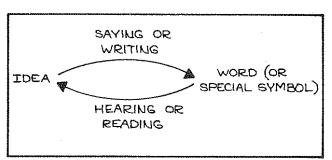
Give the reading part of the assignment first, tell the purpose of the reading, allow students time to read, and ask questions about the reading before giving individuals or the class the exercise assignment.

• In a list of word problems, choose problems which require a *variety* of computational approaches. If each problem in a list requires, say, finding the mean, a student will not have to read the words; he can just pick out numbers, add and divide. If you do not wish to mix up the types of computation in a list, you could include extraneous information or insufficient information in a problem. Alternatively, each problem could refer to a table or a graph and thus include a minimal amount of data itself.

Include a few questions in each assignment on the reading part of the assignment. If the reading part is long, some direction could be included in the questions: "According to paragraph 3..."

Particularly important to the work in this resource is the *reading of tables* and graphs. Since eye movements in ordinary reading are horizontal only, students often need instruction in looking for the title or caption for a table or graph, then looking for column headings, legends, labels for axes or scaling notes. Jumping back and forth between the text and the graph/table is often necessary. In addition, being alert to ways in which graphs can be misleading (see the INTERPRETING DATA section) is very important and deserves attention. For more on the reading of tables and graphs, see *Tables* in the CONTENT FOR TEACHERS section and the commentary in the GRAPHS section.

Vocabulary development is sometimes a key to improvement in a student's performance. There are, as is indicated to the right, two directions involved in vocabulary practice: (a) given the idea (usually through an example), produce the correct word, and (b) given the word, produce a definition or an



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example. Earle [1976] points out that practice at an even more primitive level may be necessary for some students: word (or other symbol) recognition. He suggests speed exercises like this one (adapted from 1976, p.11)*

> "Race against yourself. Look at the first word in each row. Then circle that word every time it appears in that row. When you finish, look at the chalkboard to see your time and record it in the blank." (The teacher points to 1, 2, 3, ... written on the chalkboard to keep track of the time.) 1. average average averaged averages average 2. mean meat clean near bean 3. median media median meridian mode 4. probability probably probility probability probity etc.

Another form of a word recognition exercise is in *Stats Search* in the RANGE & DEVIATION section. Earle also suggests "glossing" as a way to give attention to vocabulary:

"...some teachers have found that glossing--that is having students prepare their own glossary of terms--is a useful activity. In glossing, a small group of students is given a few mathematical symbols (or words) essential to the topic at hand. The students are asked to write their own definitions, using language that makes sense to them. The resulting definitions are not as elegant as dictionary definitions but prove considerably more useful. At the end of the activity (after students or the teacher has checked the definition)...the cards can be filed alphabetically for reference by other students. As with most such practices, however, the real benefit is derived by those who do the glossing." [1976, p.18]*

GUIDANCE in reading textual or problem material should help them to see the value in--and to profit from--their

reading.

• Tell the purpose of the assignment, perhaps by indicating what they will be able to do because of the reading. Doing this enables the student to look for, and focus on, what the lesson is about, rather than try to memorize everything.

• Indicate where this topic fits in "the big picture." Saying "Knowing about outcomes will help us figure out

*From Teaching Reading and Mathematics

Reprinted with permission of Richard A. Earle and the International Reading Association

THE PURPOSE OF THIS READING IS SO YOU WILL KNOW WHAT THE MEAN, MEDIAN, AND MODE ARE. P. 187 MEAN MEDIAN MODE

the chances of throwing two 6's with two dice" or "The last kind of graph we'll work with is called a circle graph" gives the students some notion of where the current work fits into a unit.

Review or explain important or unfamiliar words in the reading. Important new mathematical words could be written on the board to help the students recognize them in the reading and to remind them of their importance in the lesson.

For word problems especially, emphasize that more than one reading is necessary. Some authors recommend having students paraphrase word problems since that at least forces them to read for meaning.

Advise the students that to get the meaning from mathematical material it must be read more slowly than other kinds of material. READ A WORD PROBLEM:

- ONCE TO GET THE IDEA
- AGAIN TO SEEK RELATIONSHIPS
- AGAIN TO PICK OUT DATA
- AGAIN AT THE END TO SEE IF THE ANSWER MAKES SENSE,

SUMMARY

To help our students read mathematics we could

- give explicit attention to the reading of graphs and tables,
- emphasize vocabulary in whatever ways we can, and
- provide guidance by giving an overview for a reading assignment, reviewing and explaining terms and symbols, and pointing out that slower--and multiple-reading of mathematics is necessary.

PPPPP

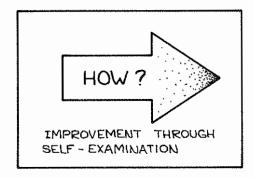
- How might Earle's word-recognition drill be used so as not to endanger a student's self-concept? (You might consider a "dry-run," repetitions, or competition with self rather than the rest of the class.)
- a. List the important vocabulary in one of the units you teach.
 b. Plan at least two ways (worksheet, game,...) you could give specific attention to those words.
- Suppose you plan to give a reading assignment using your textbook. Plan how you
 would prepare the students for the assignment.
- 4. Rehearse what you will say about reading tables or graphs, perhaps using *Studying* Some Sports (in the GATHERING DATA section) or Up, Up and Away (in the GRAPHS

section).

For more information on mathematical reading, see *Reading In Mathematics* in Ratio, Proportion and Scaling, Mathematics Resource Project.

WHAT MAKES A GOOD TEACHER?

What makes a good teacher? A more basic question is, good in what way? Student scores on standardized tests? Quiet classrooms? Student self-sufficiency as learners? Student performance in later mathematics courses? Student attitude toward learning? Student



curiosity?...Depending on your criterion for "good," you might find different teacher characteristics being more, or less, important than for someone else's criterion.

Many of the following characteristics, drawn largely from Rosenshine and Furst [1971, 1973], appear to be particularly related to student achievement as a criterion for "good." The Effortive Teacher's Manner

The Effective Teacher's Manner

One characteristic that seems to be important is the teacher's *enthusicasm* for the topic, for a student's idea, for a statistics project, for what just happened on a graph, for tomorrow,... This enthusiasm can be made obvious--and catching--by facial expressions, gestures, or voice patterns.

Effective teachers seem to be task-oriented, knowing what needs to be learned and having planned some means of achieving it. They encourage their students to work hard.

Students respect effective teachers

THE	EFFECTIVE TEACHER'S			
MANNER -	IS ENTHUSIASTIC			
-	IS ORGANIZED			
-	MERITS & GIVES RESPECT			
-	IS FAIR TO ALL			
LESSONS -	COMMUNICATE			
-	ARE WELL-STRUCTURED			
-	SHOW DIVERSITY			
	& FLEXIBILITY			
-	ALLOW AN OPPORTUNITY TO			
LEARN				
INTERACTION PATTERNS				
-	COMMUNICATE WITH STUDENTS			
-	USE MANY LEVELS OF			
QUESTIONS				
-	INVOLVE "PROBING" RESPONSES			
-	LACK HARSH CRITICISM			
	MANNER - - - - - - - - - - - - - - - - - - -			

and *are respected* by them. It is likely that this respect is earned by being effective rather than vice versa. We can communicate our respect for our students in various ways—courteous treatment of questions and answers, patience when a slow student is grasping for understanding, insistence on work of a caliber commensurate with the student's ability.

Fairness and uniformity in dealing with students also seem important and may just be sub-dimensions of treating students with respect. If you have ever laughed at a stage-whispered funny remark by a strong student but then a day or two later growled at a "troublemaker's" attempt at unsolicited humor, you realize that the last student's perception of your fairness might differ from your own.

THAT'S NOT FAIR! I HAD TO STAY AFTER SCHOOL WHEN I DID THAT!

The Effective Teacher's Lessons

One of the most noticeable features of effective teachers' lessons is that they are clear; they *communicate*. The presentation and activities seem to be at the right level for the students. The teacher's background and preparation are adequate to answer reasonable questions. Rather than give an unconvincing or muddled answer to a question for which she/he does not know an answer, the effective teacher is willing to acknowledge this lack and to present or seek a plan for looking for an answer.

Related to this clarity of lessons is the effective teacher's use of structuring remarks. For example, the teacher might provide an overview before a lesson ("We've learned about the mode, the median and the mean. Today we'll examine situations where the median is more appropriate than the mean"). Or, the teacher might summarize what has just happened. Even remarks like "Watch closely" or "These are important" or "Here is a key" add structure to a lesson. Such remarks, as well as mini-overviews or mini-summaries, could well appear several times during a lesson.

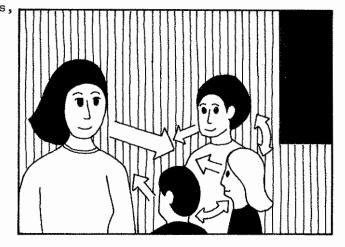
The effective teacher's *lessons vary*, both within a class period and over a sequence of class periods. Different activities, methods of evaluation, materials, and teaching devices seem more common in the effective teacher's classroom. Even within lessons, these teachers seemed able to shift in response to student needs,

responses and questions. This variety does <u>not</u> mean that every day should be a three-ring circus; effective teachers do have routines--but <u>every</u> day's lesson outline does not follow the same pattern.

Effective teachers provide students with *opportunities to learn*. Students need to collect and organize their own data, if they are to grow in data-collecting and organizing skills. They need experience with critical thinking exercises (e.g., as in the INTERPRETING DATA section), if they are to become critical thinkers. They

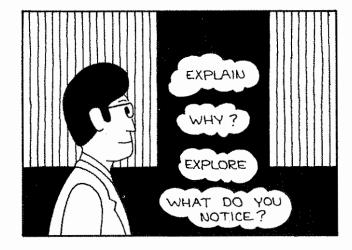
need practice at analyzing word problems, if they are to be able to do more than operation-designated calculations. The Effective Teacher's Interaction

Effective teachers communicate with, not just to, students. An atmosphere in which student questions, student reactions, and student-student evaluations or suggestions are encouraged can be fostered both by the teacher's attitude and by the teacher's



choice of activities. For example, small groups planning a survey or organizing the results of a poll will be talking with each other and learning from each other.

Effective teachers seem to ask questions at several levels, from the routine "What is...?" sort to more challenging "Why do you think that?" or "Which would be better? Why?" or "What happens when you look at more cases?" sort of questions. Such questions communicate to the students that they are responsible for—and expected to respond to—more than recitation of facts and skill with



computation, as well as giving them exposure to different sorts of thinking processes.

Related to this multi-level questioning is the *use of "probing."* When you ask a student to explain, to justify, to expand on, to clarify, to generalize, to evaluate, to apply---any sort of amplification of a response which requires more than

a surface understanding--you have "probed." Probes not only enable the teacher to determine the depth of understanding but also show the student that more than superficiality is expected.

Finally, effective teachers *do not* seem to *criticize students harshly*. (This does not mean they do not give minor reprimands or admonishments!) Positive student attitudes are conducive to learning, so it is wise to promote such attitudes rather than risk <u>negative</u> attitudes from severe criticism. A student may need just an excuse--"Teachers don't like me"--to stop trying or to follow less demanding pursuits (like skipping school).

SUMMARY

It may be helpful to take an occasional look in the mirror to compare ourselves with teachers who seem to be effective.

- Are we enthusiastic, organized, fair, respectful and respect-worthy?
- Are our lessons clear, well-structured, varied, and designed to cover all the things our students should learn?
- In working with students, do we avoid harsh criticism, foster student-teacher and student-student interactions, and ask questions at a variety of levels, including probing questions?

?????

- 1. (Discussion) Some teachers do not come across as being enthusiastic in class. Is there anything they can do to appear more enthusiastic?
- Think back over a week's lessons for a class that has gone well. What features did your lessons seem to have? If you have a class which has not been going too well, think about the lessons you have planned for it.
- 3. How much variety is there within a typical class of yours? Within a week's work for a class?
- 4. M. Lincoln: "Variety sounds good, but my third period class <u>needs</u> a stable routine, something they can count on. When I try something different, they become insecure, seem to feel threatened, aren't sure what their roles are. And once when I was introducing a lab lesson, a student said, 'Oh good, we don't have to do math today.' Is variety good for all classes?"

Your experiences? Your opinions?

5. Johnson [1975, pp.64-65] polled students in grades 5-9 about their views of the qualities of good teachers. Following are their most common answers. Do you

notice any surprises? Good teachers--nice, kind, helpful, etc. strict, firm, in control (less mentioned by younger children) interesting, makes learning fun patient not overly demanding knows subject fair, has no pets understanding good balance between control and freedom relates well to students

For more on examining one's teaching, see *Teacher Self-Evaluation* in the Mathematics Resource Project's <u>Geometry and Visualization</u>.

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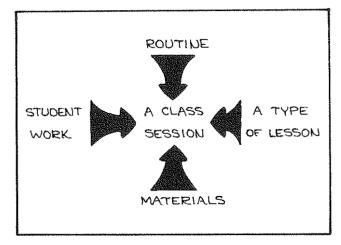
ORGANIZATION

Placement Guide for Tabbed Dividers

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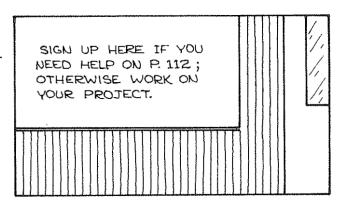
How long does it take your class to get "rolling"? Are you a little uneasy about trying a lab lesson? Are you spending hours checking homework? These questions relate to a potpourri of topics which might be loosely organized under a title like "classroom management" or even "little things mean a lot." You may be happy with all aspects of your classroom management. If not, there may be an idea or two for you in these pages.



MANAGING ROUTINE MATTERS

"As the twig is bent,..." The first days of school are very important in establishing routines. Time spent then in careful explanation of routines--and reasons for them--is time well spent.

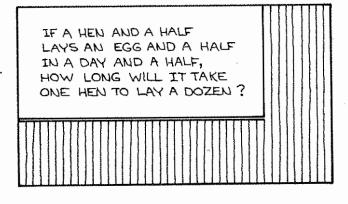
BEGINNING A CLASS. In some classrooms as much as 7-10 minutes may pass before anything related to formal learning takes place. In other classrooms the students seem to be involved in mathematics almost as soon as they enter the room. How do the teachers manage <u>that</u>? Some teachers prepare an overhead transparency with homework, a quiz or answers for an old quiz. They



have it showing when class starts; students understand that they are to check their homework. Other teachers, knowing that students are too answer-is-the-onlything oriented, intercept students at the door and have them put full solutions

to typical homework problems on the board or on overhead transparencies. Some teachers frequently start a class with a "warm-up" quiz for skill maintenance or for review of recent material. Still others write instructions on the board (the class routine calls for the students to read them) or write a puzzle or brain teaser, interesting question, or challenge on the board.

WHO'S WHERE? "What if I'm tardy?"



"Will this teacher let me go back to my locker if I forget something?" Such questions deserve explicit attention early in the year. Your answers should be uniformly enforced too, if you are to avoid charges of favoritism. (On the other hand, toilet calls might be handled on an individual basis, especially if you are confident of your ability to recognize distress.) Is taking roll something you can finish by the time the bell rings? Or can you delegate the task to students who refer to your seating chart? (In this case you would likely want two copies so you'll be reminded of who is absent.)

THE PAPER PARADE. Some systematic ways of collecting papers can save time in passing them back. If the papers are passed in in order, is it more efficient to leave them as they are and jump around in your gradebook, or to alphabetize them, record grades, and re-distribute the alphabetized papers? When do you collect homework? Toward the beginning or toward the end of class?



Most teachers use students--one for each row perhaps--to pass out whole class worksheets or tests. Some teachers pre-stack papers so that there is the correct number for each row. Many teachers put graded papers in a designated basket and students retrieve their own. Still others place papers in a set of large envelopes hooked to a bulletin board or chalk tray, each envelope containing papers for only



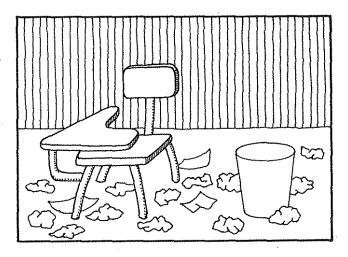
a part of the class.

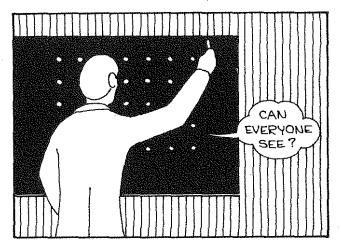
Having a clean-up routine is almost a must. A clean row might be a requirement for dismissal. Row monitors could have the responsibility of checking for a clean area, with the job being passed around or given to students "elected" because their desk area is usually littered.

BORED AT THE BOARD? Teachers use a chalkboard quite a bit during mathematics classes. Usually their writing

is large and legible enough -- and high enough on the board -- to be read by the whole

class. Experienced teachers leave what they've written on the board long enough to be studied or reviewed, and are careful to step aside to avoid shielding the writing from part of the class. Perhaps we could, however, make better use of the overhead projector. It is not just a substitute for a chalkboard. One advantage with an overhead is that material (graphs, answers, part of an explanation,...) can be prepared ahead



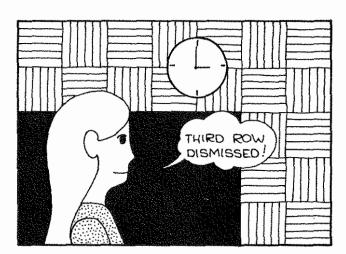


of time, with a substantial saving of class time. The lighted screen focuses the students' attention and since the teacher is facing the class, it is easy to main-tain good eye contact.

THE ASSIGNMENT. Although some might argue that giving the assignment orally helps teach listening skills, other prefer to teach listening skills some other way and do all they can to make certain the assignment has been noted. These teachers write it on the board, perhaps in a corner of the board where it would be saved until the next class meeting. Assignments should be given before the bell rings, of course. Students may find they are lost on the first exercise! In self-paced programs, several assignments may be posted or given to the student on paper.

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THEY'RE OFF! If you are still talking when the bell rings, the students should, out of courtesy, wait until you dismiss them. Even when the teacher is not talking, it does seem proper that the students should stay in their seats until they are dismissed. Students have a natural tendency to collect all their materials and "get in the starting blocks" several minutes before the bell rings, perhaps even wandering over to the wastebasket



(which happens to be close to the door). In recognition of this, some teachers deliberately plan some summarizing activity or some mental arithmetic drill to fill the last minutes of a class.

MANAGING A TYPE OF LESSON

TRYING SOMETHING DIFFERENT. We often are reluctant to try a different sort of lesson because we are not confident we can manage it. Suppose I try a game in small groups and they get too noisy? If I do try a lab lesson, won't Josh throw some-

thing? Will I be able to monitor everyone's work if we try projects?

Perhaps the best way to allay one's fear is to start on a small scale--for just a part of a period or with your smallest class. You will, or course, try to anticipate the problems and plan to avoid or minimize them. Anything new (a lab, a game, for example) should be rehearsed beforehand. Your prep-

EASE IN.
PREPARE YOURSELF,
PREPARE THE STUDENTS,

aration the first time or two will naturally be more extensive. Above all, you will want to prepare the students, if the lesson involves new and unfamiliar roles for them. Careful guidelines on behavior ("Talking to your partner is all right, but we'll have to keep it down so people can think") and procedures ("One person should check out the dice, the other person should bring them back at the end. Be sure to record your results carefully") should be communicated, as well as a positive atti-

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tude ("Won't it be interesting to find out whether adults are square?"--see Are You A Square or a Rectangle? in the SCATTER DIAGRAMS section).

Occasionally, if your idea is really different, you may also have to prepare the principal and the parents. For example, if a few students or the entire class will be polling neighborhoods, the principal should be informed about it and the parents should know--through a note, say--that this activity is indeed for mathematics class (in some cases your note should also make clear what the student will be learning from the activity). Again, the students should also be prepared: what the point of the work is, what they are to do, standards of behavior. Even sending a few students to the library (as for *Can You Guesstimate?, Information Please*, or *Getting to Know Your Almanacs*, all in the GATHERING DATA section) may require a few comments on route, speed and allocation of time once in the library.

ENRICHMENT. Projects and other forms of enrichment are attractive types of "lessons" for at least two reasons: the topics can be chosen to fit students' interests and backgrounds, and the students can exercise some choice in what they are doing. Barnard and Thornton [1977] report that frequent, short-term "projects" selected by the students are motivational and provide enrichment, application or practice of topics already covered. Their "projects" were in many cases one-page worksheets or puzzles.

If you would like to try more ambitious projects, how can you proceed? One way would be to search through these resources or other collections of activities for project ideas. For example, you might decide that experience in gathering and analyzing data would be good for your students. You might then read the commentary in the GATHERING DATA section and the *Gathering Data* pages in the CONTENT FOR TEACHERS section. After that, you could skim the student pages for ideas or activities that you could use with the whole class or, depending on their interests, with subsets of the class.

LABORATORY APPROACHES. Perhaps you have been attracted by some of these arguments for lab lessons: student involvement, work with concrete material, or interaction in small groups. After reading Laboratory Approaches in the TEACHING EMPHASES section and Teaching via Lab Approaches in <u>Mathematics in Science and Society</u>, you might have small groups all work on a single laboratory lesson--e.g., Roll That Cube from the EXPERIMENTAL PROBABILITY section. After an experience or two to familiarize students with lab procedures, you might try running two or three different lab activities at once. Your reason for using different lessons at the same time

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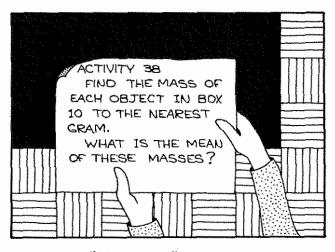
might be to meet individual differences better or perhaps to bypass the lack of enough equipment for whole-class work with a particular lesson.

MANAGING MATERIALS

As you make use of more and different materials, it will become increasingly important that you--and students--can find something you want without a lengthy search. Labeling boxes of equipment and file folders of activities is a must. Some sort of organization, indexing and cross-referencing--perhaps as in these resources--will enable you to track down an activity for a particular purpose. Great ideas which are misfiled or put in a "limbo-pile" may never get used.

Special precautions may be necessary with material like calculators or templates. It may help to have student "stock-keepers" to check material out and in.

A supply of scrap paper which is clean on one side and a few stubby pencils can help you side-step any "But I don't have ... " arguments. Some teachers keep pencils on hand and sell or demand collateral for the use of one. On a





more positive note, a collection of materials which early-finishing students can sample with a minimum of teacher attention can turn some otherwise-wasted time into profitable activity. In the same way, having available several mathematics-related books and magazines for use in class--or to check out--gives another release valve and a chance to catch someone's interest.

MANAGING STUDENT INVOLVEMENT

Students are the reason we are in the classroom. Their active involvement in

learning activities--not just passive presence--is one of our main concerns. But getting a willing involvement can be a challenge, as this quote from Johnson suggests:

If you asked a committee of fifth-graders to go out and count how many gravestones there were in the nearby cemetery...and how many of the people there died between 1800 and 1900, they'd rush right out, bright-eyed, pencil in hand to do the research...Ask some seventh-graders to do the same thing and they'd groan, object and, if necessary, procrastinate, but probably not bother to formulate the reasons why such an activity would be worthless. But ask ninth-graders to do it, and their first reaction would be (unless they thought you were joking) to inquire why the job should be done and what relation it might have to the purposes of the course. [Johnson, 1975, pp.59-60]*

Here is our usual repertoire of techniques for getting students to participate.

- seatwork and boardwork: Of great use when teaching skills, these methods can get everyone doing something. They also give us time to walk around, diagnosing, helping, encouraging—in general increasing the amount of one-person-to-oneperson interaction.
- questioning: The level of our questions and the way we pace them (see Questioning in the Geometry and Visualization resource) greatly affect the quality of this

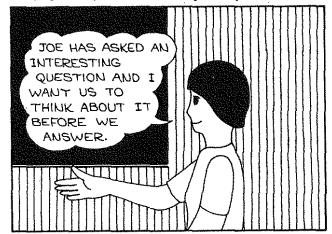
involvement. We should allow the students time to think and encourage them to ask questions. We can get some guidance from the students' facial expressions-thoughtful? Wanting to say something but not willing to raise a hand? Shy? Pre-occupied (about a sick parent, a fight with a friend, the game after school... things we have no idea about?)

student reports: As with student questions, these may be of greater interest than hearing the teacher talk all the time.

GENERATING EXCITEMENT. Experie

assignment is not exciting. How can we generate some "excitement"? Even if we are not "entertainers" we can exhibit *enthusicsm* ourselves. Perhaps we can use humor or exaggeration or surprise to add a little excitement to class.

Johnson [1975] noted that students



Experience indicates that merely rehashing a textbook



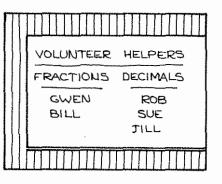
*From *How to Live Through Junior High School* by Eric W. Johnson. Copyright © 1959 by Eric W. Johnson. Reprinted by permission of J. B. Lippincott Company.

mentioned *encouraging* as a most helpful trait in their teachers. Perhaps many selfdoubting middle schoolers need only a little more encouragement to find that they <u>can</u> succeed and thus share in the excitement of learning. Questions can touch on student interests or present just the right amount of challenge (truly an art). Many teachers use discovery lessons to give the students the excitement of figuring something out for themselves. An occasional game is almost sure to generate some excitement, albeit not necessarily an excitement for learning. Perhaps we should examine each lesson we plan, hoping to see something exciting in it.



AN OPEN CLIMATE. No one hopes for a class atmosphere in which the students sit trembling, afraid to ask a question and fearing ridicule. We can hope that our students do feel free to ask questions, seek help, volunteer ideas, and contribute to discussions. How can we build this sort of climate? Perhaps the key is in *how we react to students*--do we brush them off, make sarcastic remarks, give abrupt answers and move on? Even adults, let alone sensitive middle schoolers, do not like such treatment. Shouldn't we instead treat students' questions as genuine concerns, give them careful treatment, and check with the questioners to see whether the questions <u>have</u> been answered? Student ideas, even ones not as fertile as we might hope for, should be followed up whenever practical. Allowing--even welcoming--

choices of some activities lends an air of freedom to a class. Fremont's idea [1977] of each student belonging to a small group of classmates and relying on them as a first "back-up" for instruction might provide an alternative form of involvement. Similarly, encouraging students to volunteer to help others can lead to more student-student work, the tutors learning through teach-

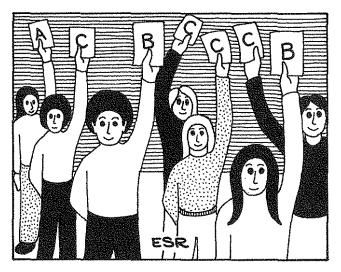




ing, and assistance for you.

FEEDBACK. If the students are actively involved, we can get feedback on how they are progressing. How else can we adjust our pacing? Questions, boardwork,

seatwork and short supervised study periods interspersed within a class session are commonly used for feedback. Feedback from <u>all</u> the students is important. Many teachers roam the room to observe and question many students, in particular the quiet and reticent ones. One idea, written up as *ESR--Every Student Responds* in the resource <u>Number Sense and Arithmetic Skills</u>, has each student with a set of cards labeled A, B, C or yes, no or the digits or

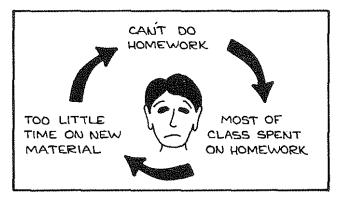


vocabulary words. The teacher asks a question for which the answer can be given with one of the cards and each student is to hold up the answer. Thus, every student is responding and the teacher can, at a glance, see how well the students are doing.

Homework, whether done in or out of class, is an important form of feedback. But homework presents problems, not all of which are easy to solve. Who corrects the homework? If the students do, will there be cheating? If teachers do, will their energy last for a whole school year? If homework is used for grading, will the students be tempted to copy mindlessly? Some teachers spot-check homework, looking at only one or two exercises for each student. Others have students check their homework most of the time but pick up the papers for one row or one class on occasion for a careful look. Other teachers give short quizzes made up of homework items.

In any case, if it is collected the homework should be reacted to in some way and returned quickly to the student.

How should homework be "gone over"? It is easy to become involved in the frustrating cycle in the figure to the right. The cumulative nature of many mathematics topics means we want to be



CLASSROOM MANAGEMENT

sure yesterday's work is understood before going on--thus homework most often is reviewed first in a class session. Johnson and Rising [1972, p.77] describe the following practice with an eye toward an efficient in-class treatment of homework: Students entering the classroom write on the board the number of any exercise they had trouble with. Other students look over these and put up solutions for any such exercise they know how to solve. All this takes place before the bell rings or a few minutes into the class.

Should middle schoolers be given answers before they have done the homework? Because of the abuses possible, most teachers would be reluctant to supply answers.

On the other hand, having answers at hand could help one avoid doing a whole assignment incorrectly. Perhaps a compromise--supply selected answers before class is over--would be best.

How much out-of-class work is reasonable? There is no profit in assigning so much work that many students will, out of self-defense, resort to copying answers. Do you have a school policy on homework? If not, do you talk to other teachers about how much homework



to other teachers about how much homework they are assigning, particularly when you are planning a big assignment?

MISBEHAVIOR. Some sorts of student "involvement" interfere with learning. Prevention is the best solution. Johnson and Rising advocate making standards of acceptable behavior clear from the beginning:

> ...classroom routines should be established and students should be made aware of what is expected of them and what they may expect of you...each teacher should set out to establish class regulations in a friendly, supportive atmosphere. The teacher should explain why these regulations are necessary, indicating his genuine and primary concern for the student and his progress. The teacher must be sincerely interested in students or such statements will sound foolish; students today are quick to react negatively to insincerity. [1972, pp. 68-69]*

Even with clear regulations, middle schoolers will test their teacher's patience, nervous systems, wills and alertness. Perhaps you can make frequent "adjustments" in where students sit. Or give a hyperactive student things to be passed out. Or,

*From Guidelines for Teaching Mathematics

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ever vigilant (!), move close to a location where an impending eruption threatens. You may have learned that there must be some "distance" between you and the students; too much "chumminess" and a controlled class interaction do not seem compatible.

> ...The (surveyed students in grades 8 and 9) rated firmness or being in control very high, while (the 5-6-7 graders) did not often mention it. It is not at all surprising that students in adolescent turmoil feel the need of being well controlled by teachers no matter how much they may complain about the control. However, if teachers who are prone to strictness take too much heart from this statistic, they find themselves exerting excessive external control, which looks good in the classroom and to the class but which can retard the development of self-control and maturity. [Johnson, 1975, p. 65]*

What do you do when someone <u>does</u> misbehave? There are no pat answers. The circumstances, the student, the nature of the offense--the variations possible in these make all-encompassing solutions impossible. There are some general guidelines which seem safe: Be fair; offense X cannot be completely ignored on one

occasion and harshly punished on another. <u>Over-reacting</u>, whatever that means, usually reflects an insecurity on our part rather than a calm assessment of the offense and the offender. It is probably counter-productive to force a confrontation in which the student must compound the offense to save face. Johnson offers these two do-nots: (1) Don't use a punishment that brings humiliation, and (2) don't spank or hit a junior high schooler. [1975, p.171]

Most productive in the long run is to look for the reason for the behavior. Sometimes it will be our fault--a long assignment, an unintended



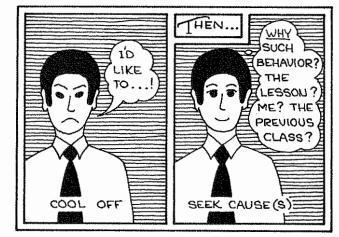
slight, missing the blow that provoked the one we saw. Or the student's fault-a lack of preparation, or a desire to be noticed, for example. But the causes may

*From How to Live Through Junior High School by Eric W. Johnson. Copyright © 1959 by Eric W. Johnson. Reprinted by permission of J. B. Lippincott Company.

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be outside the class--a remark from a "friend" in the hallway, parental pressure for good grades, lack of sleep or food,...A search for causes does enable us to take misbehavior less personally and, if we can identify the causes, to attempt to remove or avoid them.



PPPPP

- (Discussion) How do you get a class started? How do you handle routine matters like collecting papers, taking roll, dismissing the class?
- 2. Suppose you are planning a laboratory lesson which will involve groups of three students and lab equipment which will fit on a desk top. How should the furniture in your room be arranged? Who will arrange it that way? Who will restore it to the natural order?
- 3. (Discussion) What sorts of management problems have you noticed with laboratory lessons? With small-group work? With out-of-school projects? With games?
- 4. (Discussion) How do you run a typical mathematics class session?
- 5. How are materials and media (e.g., overhead projector, filmstrip projector,...) managed in your school? Do you know how to run the audio-visual equipment in your school?
- 6. (Discussion) How do you have your supplementary material organized? Could a student find something with a minimum of direction from you?
- 7. (Discussion) How do you handle quizzes and tests?a. Who corrects them?b. How quickly do you return them? Are students r
 - b. How quickly do you return them? Are students required to re-do the ones they have missed?
 - c. How often do you give quizzes? How long are they?
- 8. K. William: "Excitement!? That's exactly what my students DON'T need! I have enough trouble keeping them in their seats as it is." You:
- 9. (Discussion) Do you let students...
 - a. sit where they like?
 - b. help each other during seatwork? on homework?
 - c. sharpen their pencils anytime?



10. (Discussion) Hypothesize some possible causes of the following misbehaviors. What would be proper punishments for them?

- a. cheating on homeworkb. cheating on tests
- c. carving on a desk
- d. disfiguring school-owned books
- e. cursing you
- f. "casual" swearing
- g. provoking a fight
- 11. (Discussion) How do you handle note-passing? (Why do students pass notes?) Evaluate these methods.
 - a. "I read them out loud to the class."
 - b. "I pick them up and tear them up without comment."
 - c. "I pick them up and return them later, unread."
 - c. "I ignore them."
- 12. (Discussion) How do you handle the following? What might be their causes?a. a student who dominates a class discussionb. a student who never does any homework
 - c. a student who leaves his seat to talk with someone while you are talking
- 13. Think of a recent case of student misbehavior. What are some possible causes of the misbehavior? Can a repetition be avoided?
- 14. Johnson [1975] asked a large number of students, "What annoys you most about teachers?" The most common answer was, "Yelling." Why do teachers yell? What are alternatives?
- 15. Here are some student answers to "What are the most important characteristics of poor teachers?" Are there any surprises?

"Lets the kids do whatever they want"; "stops talking whenever there's the slightest noise"; "treats you like animals"; "doesn't explain things"; "intolerant of mistakes"; "covering up own mistakes"; "not willing to face up to the students' criticism"; "cries in class"; "grinding--too powerful, rules the kids like prisoners"; "always trying to find the bad in what kids do"; "too quick and rushy"; "he just wants the money." [Johnson, 1975, p. 66]

16. The Unified Science and Mathematics in Elementary School (USMES) project features "comprehensive" problem solving with challenges which can be real to the students---planning cafeteria lines, designing a soft drink, for example. One of their challenges deals with classroom management [1975]. The students are asked to help plan how to make the class run more smoothly (rules, noise, passing out papers,...) and to keep the room neat. Would you try this with your students if some aspect of classroom management needed improvement?



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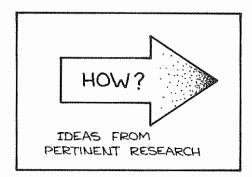
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Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
APPLICATIONS	
Business & Commerce	
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Health & Medicine	
People & Culture	
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GLOSSARY	
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In looking for studies on statistics and probability, one becomes acutely aware of how much research remains to be done. There seem to be very few studies dealing with statistics for middle schoolers, probably because it is a newer topic in the curriculum. There have been, however, several explorations devoted to topics in probability. All these are surveyed in the next few pages.

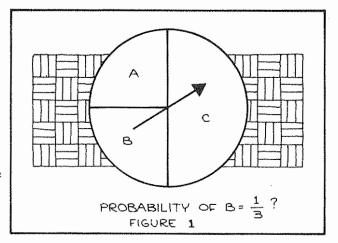


Can Middle Schoolers Handle Ideas from Statistics and Probability?

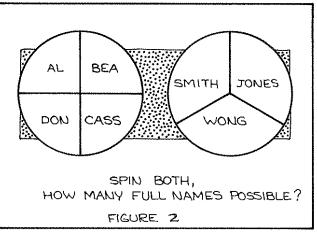
You may wonder whether your students can handle such ideas. Don't worry. With proper presentations, even students in the *primary* grades can work with simple experiments and ideas of more likely, less likely, and equal chances. [Jones, 1975; McLeod, 1971] Several workers have found that students from grades 4 through 9 can deal with such topics as: mean, median, mode; the probability of an event; and measures of scatter like the range. [Armstrong, 1972; Gipson, 1972; McClenahan, 1975; Moyer, 1975; Romberg and Shepler, 1973; Shulte, 1970; Smith, 1966; White, 1974; Wilkinson and Nelson, 1966] Moreover, middle school students, even without instruction, have some familiarity with the *ideas* (but not necessarily the technical language) of outcomes of an experiment, the relative likelihoods of simple events, and finding probabilities. [Doherty, 1966; Leake, 1962, 1965; Leffin, 1971]

TEACHING IS NEEDED. Despite this evidence, one argument for instruction in probability and statistics is that many students do have inaccurate or imprecise ideas which may handicap them in real-life situations. Some of these difficulties have to

do only with language; for example, Leffin [1971] found that many middle school students do not know the difference between "odds" (3 to 1) and "probability" (3/4). Other difficulties stem from the concepts. In Jones' work [1975] primary youngsters found that dealing with spinners like that in Figure 1 was more difficult than working with spinners with all the parts equal.



More complicated experiments not surprisingly cause greater difficulty. For example, Shepler [1969] found that most uninstructed students, in a situation like that in Figure 2, will think that there are 7 possible full names rather than 12--Al Smith, Al Jones, Al Wong, etc. (Similar difficulties should be anticipated with *The Un-Proverb* and *Aloha* in the COUNTING TECHNIQUES section.)



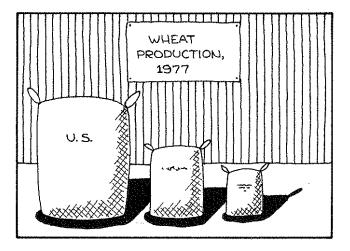
Alternate teaching methods have been developed for teaching some topics in probability and statistics. For example, the "tree-diagram" approach to probabilities (see the PROBABILITY WITH MODELS section) is a recent strategy. So is the method of using random numbers to solve problems (see *A Cereal Question* in the EXPERIMENTAL PROBABILITY section). Sanders [1971] has written about his work with these "simulation" techniques and junior high students.

Graphing

Line graphs are usually found to be more difficult for students to read than bar, circle or pictographs. [Culbertson & Powers, 1959; Peterson & Schramm, 1954] Bettis and Brown [1976] offer the opinion that no line graphs be taught before the sixth grade, on mental development grounds. What do you think? The difficulty with line graphs no doubt comes from having to find the scale readings on both axes. Other sorts of graphs are easier to read probably because of the more vivid size dif-

ferences in bars, parts of a circular region or rows of pictures.

As the page I Didn't Plant It That Way in the MISLEADING STATISTICS section illustrates, pictographs can easily be misleading with the figures failing to carry the correct message. Even when care is taken to maintain the proper proportions in pictographs (as in There's Music in the Air, in MISLEADING STATISTICS), children can still misinterpret them. For example,

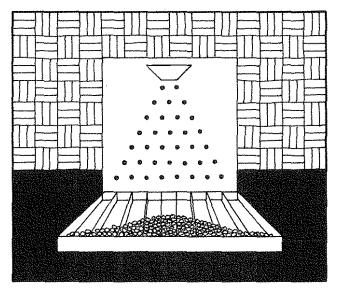


students have been found to believe that one country uses larger bags for wheat than does another country! Thus, students should learn that more units of the same size communicate better than a larger unit to show a difference in quantity. [Parker, in Whipple, 1933, p. 156] The students should also be alerted to the possibility of prepared graphs being drawn with units of different sizes. Probability

Because of the importance and frequency of probabilistic situations in life, several people have questioned students to find out their grasp of selected ideas. In most cases described below, the students have not been given instruction on the ideas.

RANDOMNESS. Piaget and Inhelder [1975] report that most children of ages 11-12 have a fairly clear idea of randomness. If one were to shake a box containing several blue marbles and several white marbles, pre-schoolers would not be surprised if all the blue marbles ended up on top. Older children would recognize that although such an occurrence is possible, some mix of the marbles would be more likely. You could ask similar questions about such situations to determine whether your students have some notion of randomness.

DISTRIBUTIONS. According to Piaget most middle schoolers will predict the rough symmetry that usually results when marbles are dropped through the slot at the top in the figure. (See Normal Distributions in RANGE AND DEVIATION for similar experiments.) Most of the marbles will end up in the center slots, with very few in the rightmost and leftmost slots and about the same number in symmetrically located slots. Students also have some "feel" for a uniform distribution, as evidenced by their pre-



dictions of roughly equal numbers of raindrops falling on each tile in a typical rain. Cohen and Hansel [1955a] also studied the development of the notion of distribution. From a large container of blue and yellow marbles (sometimes the ratio was given to the students, sometimes not), they drew several sets of four marbles. They asked students of ages 10-15 years to tell how many of the sets would have only

STATISTICS AND PROBABILITY LEARNING





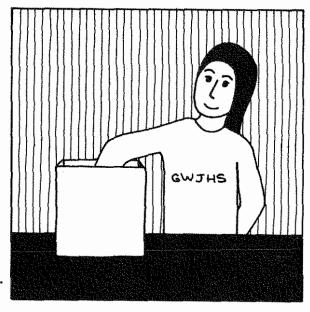
blue marbles (4,0); how many would have 3 blue and 1 yellow (3,1); and so on for the

other three possibilities: (2,2), (1,3), and (0,4). The students' responses led to the hypothesis of these four stages in the development of the idea of a distribution:

 By age 10 most students realize that the five possibilities--(4,0), (3,1), (2,2), (1,3), (0,4)--will not occur with equal frequencies.

 Later, there is a general tendency to believe that with a 50-50 mixture the (2,2) case will occur most often (although some regard it as <u>least</u> likely!).

3. Students at 124 years tend to

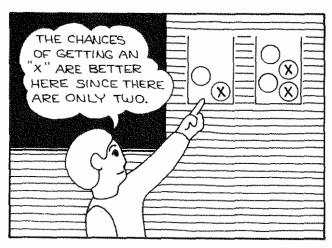


assign the same likelihood to symmetric outcomes like (3,1) and (1,3), with 50-50 mix-tures.

4. Even by 14 years, most students do not feel that (3,1) is more likely than (4,0). (The (3,1) outcome is more likely, when there are equal numbers of blue and yellow in the container.) This feeling develops later, to give the fourth stage.

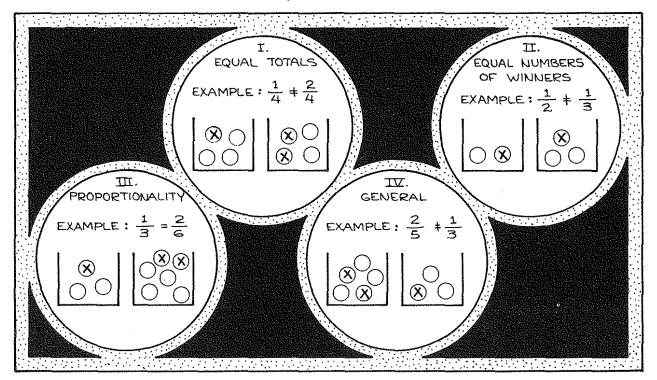
As in the Piagetian investigations, this one was carried out with students who had not been taught probability. Hence these stages can likely be accelerated. They do, however, suggest that activities should be undertaken to give our students chances to notice their misconceptions or to build up a background of experiences on which to base later ideas.

PROBABILITY IDEAS. The Piagetians report some fascinating conversations resulting from questions about which of two collections of counters (some marked with X's) gives the best chance of drawing a counter with an X. [Piaget and Inhelder, 1975, ch. 6] The child quoted to the right was 10 years, 7 months old. Many children around this age <u>can</u> handle situations involving simple proportions



like the 1:2 = 2:4 in the figure, which involves only a doubling. Some children will focus only on the "winners" and base their decisions solely on a comparison of these. In the setup pictured on the previous page, such a student would choose the box to the right. Cohen [1957] reports that some people are so entranced by the number of winners that they would prefer to draw from a box containing 100 cards with only 2 winning cards rather than to draw from a box containing 10 cards and 1 winning card. Sometimes students focus instead on the number of <u>losers</u> and would choose the box to the left in the preceding figure.

Here are some of the different types of setups considered by the Piagetians:

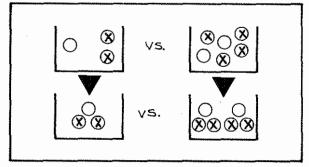


The tendencies mentioned earlier--focusing only on winners or on losers--may yield correct decisions some of the time. Many students seem to concentrate on the differences between the number of winners and the number of losers, another "sometimesit works" strategy. It is likely that many uninstructed middle schoolers will use such strategies. With this in mind we will want to question a student who is failing to give correct decisions in 2-box settings like those above (see *Take Your Choice* in the PROBABILITY WITH MODELS section). Even though such students may be able to give correct probabilities for simple situations (e.g., as in *South of the Border* or *Tree Diagram I* in the PROBABILITY WITH MODELS section), they may be misled by visual information in more complicated settings (e.g. *Be a Winner* in PROBABILITY WITH MODELS)

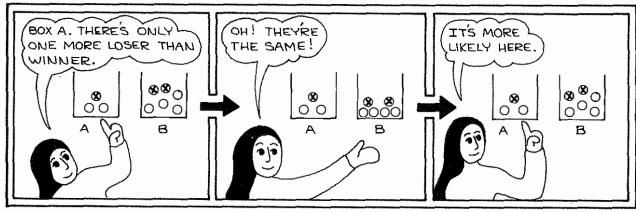
STATISTICS AND PROBABILITY LEARNING

and forget to use calculated probabilities.

Fischbein, et al., [1970] taught sixth graders to make groupings in such situations (as to the right). The Piagetians also tried this grouping method to see whether the students would then regard the two as having the same chances. Some students would, but when the original configuration was restored, they changed

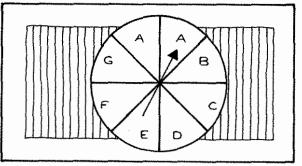


their minds! [P. get and Inhelder, 1975, pp. 154-156]



LARGE NUMBERS. Sometimes students can deal successfully with problems involving small numbers but cannot extend the reasoning to large numbers. For example, it may

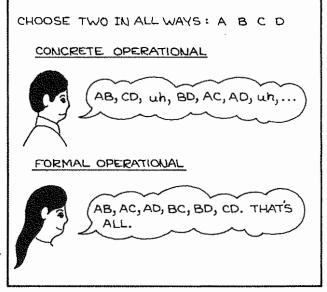
be easy to predict that on 16 spins of the spinner to the right, the pointer will point to A about 4 times and to B about 2 times. But some students will not agree that with 1600 spins, one could expect about 400 A's and 200 B's. Since it is difficult to visualize 1600, a



student who is only concrete operational may resort to differences (4 - 2 = 2, 400 - 200 = 200) and feel that the disparity in these differences must be too great for the 400 and 200 predictions to be correct. As students become mentally mature, they can reason proportionally and come to realize that the <u>ratio</u>, not the <u>difference</u>, is the important thing.

SYSTEMATIC COUNTING. Piaget uses the ability to list systematically all pos-

sible combinations as an earmark of formal operational thinking. Concrete operational students search for a system but they are only partly successful in finding one. Most of our students may profit from the concrete experiences suggested in the commentary in the COUNTING TECH-NIQUES section. In dealing with combinations, we will want to display for our students good thinking models and give them lots of guidance, as in the COUNTING TECHNIQUES section. It is likely that some of our students can already



generate all combinations, that others will pick up our systems rapidly, and that still others will not master systematic listing this year or will learn it only by rote. Piaget has indicated that without instruction combinations seem to be handled best after 11-12 years of age. The more difficult problem of systematically listing all <u>ordered</u> selections (see *Counting Techniques* in the CONTENT FOR TEACHERS section) without instruction, does not seem to be resolved until the ages of 14-15 or even later.

MORAL. Proceed cautiously. The activities in this resource were written with the above findings in mind, and many of the activities have been tried with students. But some ideas may not seem to register with <u>your</u> students. If this happens, you might choose to re-examine what went on in class, assess as best you can the stages of mental development of your students, and see if there might be mismatches in the two. In some cases, the work you do this year will be the foundation for later insights for your students.

?????

- 1. (Discussion) What are some common errors that you have observed when students have been reading or constructing graphs?
- With single students or with a small group, gather some evidence on their notions about some of the following:
 - a. randomness (see p. 3)
 - b. distributions (p. 3)
 - c. which of two boxes gives the better chances (see Take Your Choice and

STATISTICS AND PROBABILITY LEARNING



Be A Winner in the PROBABILITY WITH MODELS section)

- d. going from a small number of cases to a much greater one (see p. 6)
- e. systematic approaches to listing all choices (see p. 6)
- 3. Make a pictograph like that on page 2 and see how your students interpret it.
- 4. Give the situation in Figure 2 on page 2 and ask your students how many full names could result.
- 5. A problem not to be overlooked is the not-unusual conflict between a student's conviction and a prediction from theoretical probability. For example, after dutifully recording that the probability of a six on a toss of die is 1/6, a student may nonetheless *believe* the chances of a six are much less.
 - a. Ask some students what they believe, after they have completed *Finding Probabilities II* (in the PROBABILITY WITH MODELS section).
 - b. How would you handle students whose convictions differ from the "correct" probabilities?
 - c. (Research critique) Critique the Smock and Belovicz [1968] study.

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ORGANIZATION

Placement Guide for Tabbed Dividers

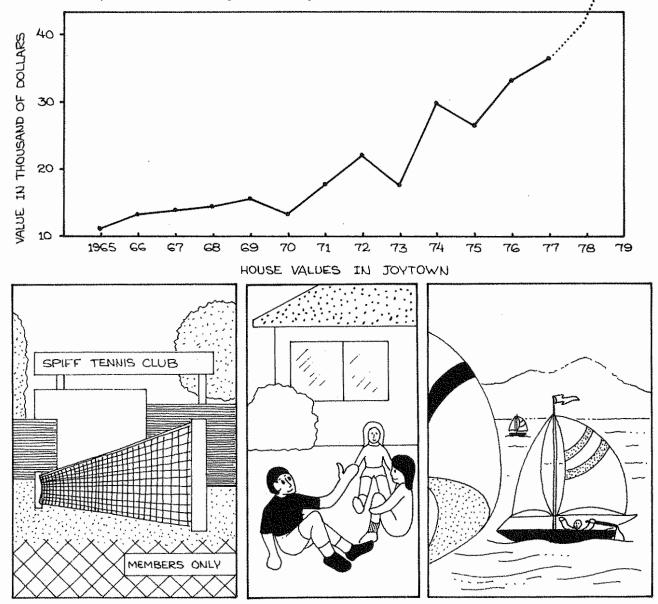
Tab Label	Follows page:
CONTENT FOR TEACHERS	
Gathering Data	
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Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
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Appendix-Computer Programs	
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TEACHING EMPHASES	
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CLASSROOM MATERIALS	
CONCEPTS AND SKILLS	
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CRITICAL THINKING

All of the statistical methods and ideas used throughout this resource should involve careful and critical thinking.

When we look at a table, a graph or the results of a poll, we either draw our own conclusions or consider those expressed by another. When we listen to or read an advertisement, we are asked to accept certain claims. The ability to think critically about methods of obtaining data and ways of presenting it can be of great importance.

A newspaper carries an advertisement for house lots in a new development in a local city JOYTOWN in the sate of ARIDA. There are graphs of rising values of lots bought in the last ten years, pictures of happy people on green lawns, of tennis clubs, of boats sailing peacefully on a smooth lake.



TEACHING EMPHASES

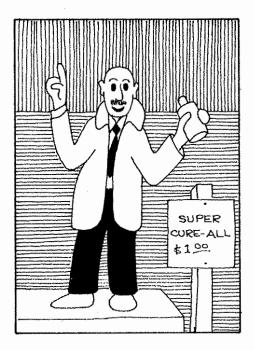
CRITICAL THINKING

A student who has been trained to examine claims critically would look at the graphs and pictures with a cautious eye, considering the following:

- Why does the graph start at \$10,000 rather than \$0?
- Is it to make the rise in value seem much greater that it was?
- Is the dotted extension into '78 and '79 justified or could there be a drop as in '72-'73?
- Are 15 year old homes better built and a better buy than new homes?
- Is the photo of houses on green lawns what a house will look like when it is bought at the advertised price or will additional thousands have to be spent for landscaping?
- How far away is the tennis club? How much extra does it cost to belong and to play?
- Is the lake in the picture part of the development or is it ten miles away?
- Are sewers and water lines laid?

This kind of critical thinking is vital for survival in the modern world. It is fostered and developed in many of the activities of this resource.

Critical thinking means to be skeptical and careful. Students should learn not to take statements of position pleaders, of proponents, or of promoters at their face value without careful examination. Of course, there are many honest people who advocate positions you or your students may question or disagree with. This honest disagreement is a far cry from deception. Honest disagreement may be clarified or resolved by questioning and challenging the information on which the arguments rest or the arguments themselves. But deliberate deception may be harder to spot and guard against.



- Is the graph of rising prices of homes in JOYTOWN an honest or a misleading graph?
- What makes a graph misleading?

- Does the graph show the resale value each year of a house built in 1965?
- Does it show the price of the lowest valued new home being built each year?
- If so, how do the structural details vary from year to year?
- Is the labeling of the graph correct?

In this resource, many activities are devoted to the ways in which a graph may be misleading.

Even an honest graph or table must be looked at critically. For instance, consider the presence of an outlier, a lonely individual result standing off by itself far from the crowd.

In a school attendance chart, most rooms have about 30 students.

- Why is Room 5 recorded with only 17?
- Is this an error?
- Should the number be 27 or is there some reason for this discrepancy?

Critical thinking should be involved whenever one listens to or reads someone else's interpretation of data.

- Is all the relevant data being presented?
- Is there misinformation as well as correct information?
- Is the misinformation intentional or accidental?
- Is the interpretation of the data logical and consistent?
- If misinformation is used intentionally or unintentionally, does the argument fail or is it supported by enough of the correct data to be valid?
 Another aspect of statistical methods that involves careful and critical think-

ing is the selection of an appropriate mathematical model for a given situation.

A boy has six shirts each of a different color. A girl has eleven different outfits. Each of them want to wear their clothes in a random order rather than a regular rotation. Their friends suggest rolling dice to determine the order, one die for the boy and two dice for the girl. Are there appropriate mathematical models for the two situations? Critical thinking might involve such questions as:

- Is rolling dice an experiment with random results?
- Are there the right number of outcomes in each case to match the requirements of the boy and the girl?
- Are the results of the roll of the die equally likely? Is this an important question for the boy?

- Does the girl have some favorite outfits she would like to wear more often?
- Are the results of the roll of 2 dice equally likely? Is this as important for the girl?

The critical thinking in these two examples involved asking many questions. Learning to think critically relies heavily on learning to ask questions.

- What data is needed?
- How can it be collected?
- Is it accurate?
- Is it pertinent?
- Is it complete?
- If not, is it sufficient?
- Is the graph honest?
- Is the interpretation justified?
- Is the model appropriate?
- Is the answer reasonable?
- What are the further implications?

Critical thinking deserves emphasis in all our teaching. It will help students from falling for fallacious arguments. It will also help them to understand better how to support their own positions with solid evidence, justifiable assumptions and sound arguments.

Developing the habit and attitude of critical thinking is important for all mathematics teachers and students. The study of statistics gives an excellent opportunity to develop such habits and attitudes.

EXAMPLES OF CRITICAL THINKING IN THE CLASSROOM MATERIALS

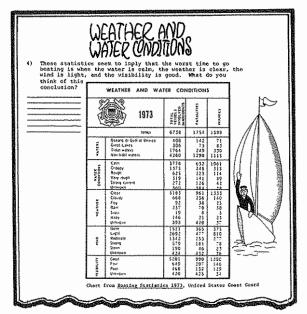
I. Thinking Critically about Tables

Sometimes incorrect conclusions are inferred from statistical data. Here students are asked to evaluate one possible conclusion from this table. They can probably think of a more valid conclusion.

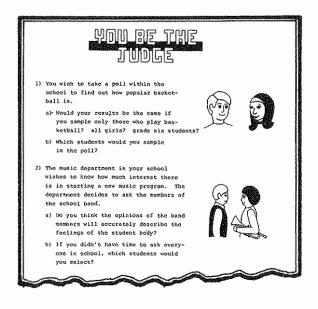
II. Thinking Cr	ritically about (Graphs
 1970 was about 200 million. In 1820, it was about 10 million. I. By reading pictograph A, you may think the people living in 1820 	200 Million	
were not as large as the people in 19701 Graph A is trying to show that the number of people living in the U.S. in 2970 was much greater than		
the <u>surper</u> in 1820. Let's examine pictograph A. 1) Draw a rectangle around each person but not the numbers. The		
 person dut not the numbers. Ine rectangle should be the smallest that will enclose the whole person. 2) Count the squares inside the scaller rectangle 		
 Count the squares inside the larger rectangle. The larger rectangle has 		
times as many squares as the smaller rectangle. So, the 1970 person is about times as large as the 1820 person.		
5) The 1970 population (200 mil- lion) is 20 times the 1820 po- pulation. Do the persons in Graph A correctly show thin?	1620 1970.	

III. Thinking Critically about Methods of Sampling

Students can evaluate methods of sampling and decide if a method does provide a representative sample. If not, they can suggest better ways to obtain representative samples.



Students can critically examine graphs to see if they are misleading. This activity compares a misleading graph to an honest graph of the same data.



CHOOSING THE BETTER QUESTION FOR A

DECIDING WHAT DATA TO COLLECT TO

COMPARING DATA FROM DIFFERENT

DISCUSSING BIASES OF SOURCES

DISCUSSING INTERVIEWS AND QUESTION-

QUESTIONNAIRE

ANSWER A QUESTION

ANALYZING ANSWERS

NAIRES

SOURCES

ROUNDING NUMBERS

CRITICAL THINKING FOUND IN CLASSROOM MATERIALS

GATHERING DATA

THAT'S A GOOD QUESTION

FACT OR OPINION?

A TRAFFIC PROBLEM

TAKING A SURVEY

ROUND UP ROUND DOWN

COMPARING DATA

WHAT'S YOUR BIAS?

TABLES

ARRESTS, BY TYPE OF OFFENSE	READING A TABLE
WEATHER AND WATER CONDITIONS	READING AND INTERPRETING A TABLE

GRAPHS

UP, UP AND AWAY	READING BAR GRAPHS
IT'S AN EMERGENCY, ACTIVITY V	READING AND MAKING PERCENT BAR GRAPHS
SCALE & PLOT	LABELING SCALES AND PLOTTING POINTS
A TREND FOR ALL SEASONS	EXAMINING TRENDS IN LINE GRAPHS
THINKING THROUGH PIE CHARTS	READING AND INTERPRETING A CIRCLE GRAPH
THE FAMILY CIRCLE	INTERPRETING CIRCLE GRAPHS
YOUR SHARE OF THE PIE	MAKING AND READING CIRCLE GRAPHS
WHAT KIND OF GRAPH?	CHOOSING AN APPROPRIATE TYPE OF GRAPH
HOW CAN WE DISPLAY THE DATA?	COMPARING DIFFERENT TYPES OF GRAPHS

E.

SCATTER DIAGRAMS

SCATTERED PATTERNS

COME RAIN OR COME SHINE

HABLA USTED EL ESPANOL?

IT'S NOT HEALTHY TO DRIVE NEAR

EXAMINING TRENDS IN SCATTER DIAGRAMS

MAKING A SCATTER DIAGRAM MAKING AND USING A TREND LINE

MAKING A SCATTER DIAGRAM THAT DOES NOT HAVE A TREND LINE

DISCUSSING CAUSE AND EFFECT FOR TWO RELATED VARIABLES

EXAMINING CLAIMS OF ADVERTISERS

EXAMINING A DECEPTIVE BAR GRAPH MAKING AN HONEST BAR GRAPH

EXAMINING A DECEPTIVE BAR GRAPH MAKING AN HONEST BAR GRAPH

EXAMINING DECEPTIVE BAR GRAPHS

SAME DATA

EXAMINING TWO LINE GRAPHS OF THE

EXAMINING DECEPTIVE PICTOGRAPHS

COMPARING TWO TYPES OF PICTOGRAPHS

MISLEADING STATISTICS

HOME

THIS DOESN'T "AD" UP EXAMINING THE CLAIMS OF ADVERTISEMENTS

EXAMINING THE FACTS

FIGURES NEVER LIE

BUTTER UP THE COOK

FOOD FOR THOUGHT

THE TUBE BOOM

I DIDN'T PLANT IT THAT WAY THERE'S MUSIC IN THE AIR

MEAN, MEDIAN, MODE

THE AVERAGE FAMILY

MEANS WITH M&M'S

WHAT DO YOU THINK?

CHOOSING MEAN OR MODE TO DESCRIBE A DISTRIBUTION

USING THE MEAN TO DESCRIBE A DISTRI-BUTION

CHOOSING MEAN, MEDIAN OR MODE TO DESCRIBE A DISTRIBUTION

CRITICAL THINKING



RANGE & DEVIATION

CLUSTERS AND SPREADS

LOOK, MOM, I'M GETTING BETTER

SAMPLING

ARE YOU SUCCESSFUL?

HOW MANY DOTS?

YOU BE THE JUDGE

EXPERIMENTAL PROBABILITY

THE GALLANT SIR LANCELOT

PROBABILITY WITH MODELS

MAYBE YES MAYBE NO

WHAT DO YOU EXPECT?

GHOSTS, GOBLINS AND "COINS THAT REMEMBER"

TRICKY STATEMENTS

BUSINESS & COMMERCE

MAKING DOLLARS AND SENSE

THE BUDGET DOLLAR

THE WALL STREET REPORT

CANADIAN WORKERS

IS FOOD EATING UP YOUR INCOME?

USING AVERAGE DISTANCE FROM THE MEAN TO DESCRIBE A DISTRIBUTION

EXAMINING A RUNNING TOTALS GRAPH OF TEST SCORES

SEEING HOW SAMPLES DESCRIBE A KNOWN POPULATION

USING SAMPLES TO ESTIMATE POPULATION SIZE

EVALUATING SAMPLING PLANS

MATCHING OUTCOMES TO SPINNERS

RELATING THE CHANCE OF AN EVENT TO A PROBABILITY SCALE

EXAMINING PROBABILITY SITUATIONS

EXPLORING IDEAS OF INDEPENDENCE AND THE "LAW OF AVERAGES"

EVALUATING STATEMENTS ABOUT THE LIKELIHOOD OF EVENTS

USING DECEPTIVE BAR GRAPHS TO INFLUENCE OPINIONS

READING A CIRCLE GRAPH ON THE NATIONAL BUDGET

USING PROBABILITIES TO PREDICT THE STOCK MARKET

STUDYING GRAPHS OF CANADIAN EMPLOYMENT

USING CIRCLE GRAPHS TO COMPARE PERCENT OF INCOME SPENT ON FOOD



ENVIRONMENT

GET A HORSE?

PULL FOR POOLING

TO SPRAY OR NOT TO SPRAY

CONTINENTAL FACTS

USING A TABLE TO COMPARE FUEL EFFICIENCY

GATHERING DATA ON THE NUMBER OF RIDERS IN CARS

READING A TABLE ON PESTICIDE USAGE

GRAPHING INFORMATION ABOUT CONTINENTS

HEALTH & MEDICINE

 FIFTY WAYS TO LOVE YOUR LIVER
 EXAMINING A DECEPTIVE BAR GRAPH OF NUTRIENTS

 UP IN SMOKE
 READING A BAR GRAPH ON HAZARDS OF SMOKING

PEOPLE & CULTURE

COUNTING EVERYBODY

HOW MANY CHILDREN?

GRAPHING THE WORLD'S POPULATION

IT'S NOT WHAT IT LOOKS LIKE

EVALUATING AN EVALUATION

WOW, I DIDN'T KNOW THAT

RECREATION

AGREE OR DISAGREE?

COMPLETING A TABLE OF CENSUS FIGURES

USING FAMILY SIZE TO PREDICT FUTURE POPULATION

MAKING AND USING A GRAPH TO PREDICT POPULATION TRENDS

READING A TABLE TO DISCOVER JOB DISCRIMINATION

ANALYZING A QUESTIONNAIRE USED IN TEACHER EVALUATION

ANALYZING AVERAGES

MAKING A SCATTER DIAGRAM TO COMPARE RATINGS

ORGANIZATION

Placement Guide for Tabbed Dividers

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DECISION MAKING

When Harry Truman was President of the United States, he kept a sign on his desk that said, "The Buck Stops Here." He had to make many fateful decisions including the one to drop the bomb on Hiroshima that ended World War II.

Most of us do not have to make such momentous decisions but we are confronted with minor ones every day and major ones often. Some of these are made glibly without thinking whether any information is available to help. A decision involves some future action. The possible results of that action determine the importance of the decision and the care and consideration that should be involved before making it.

What is meant by "making a decision glibly"? A family needs a new car. One member says, "The car dealer down the street has a beautiful red two-door. It's just the model and color I've always wanted. Let's get it." A decision made on this basis is obviously not made very thoughtfully.

For most families buying a car is a serious enough problem that the decision deserves more careful consideration of alternatives and consequences.

- Is a two door adequate and convenient?
- What kind of gas mileage does it give?
- Is the price within our reach?

What is the reputation of this make and model car for reliable performance? Each family will have other questions whose answers will be important in making the final decision with greater confidence. It may be to buy that "beautiful red two-door" but now it is based on relevant data, gathered and evaluated. This is typical of a statistical approach to decision making and the ideas and methods involved should be emphasized at every opportunity.

Decision making in the broad sense is what we do just before we act. The problem for most of us is how to make GOOD DECISIONS without too much agonizing and spending of time. It is important to know how to sort out the situations requiring

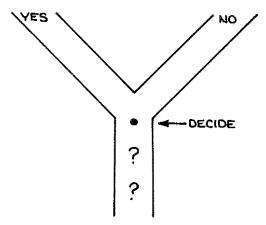


TEACHING EMPHASES

DECISION MAKING

decisions. Some are important enough to require careful, rational consideration of consequences. Others should be made routinely and quickly, perhaps even without conscious thought. But which is which? That in itself is a decision each one has to make on a personal basis.

Some people go shopping for groceries or clothes and buy on impulse. Others plan menus weekly and budget their clothes buying on a regular basis.



In the long run decisions are more likely to be satisfactory if some planning is done in advance. And all of us will have many decisions to make.

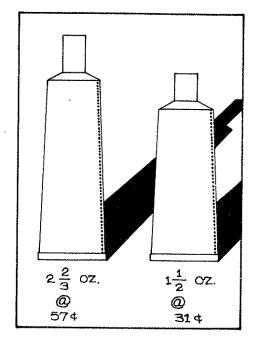
- Do I want to have my drinking water fluoridated?
- Should I buy a motorcycle or a small car?
- Should we invest money to install solar heat in our house?
- Should I elect another mathematics course?
- Should I go to college next year or get a job?
- Should I continue renting an apartment or should I buy a house?

The habit of looking at alternatives

and consequences in making decisions can be built up by practice. For example, selecting a brand of toothpaste might involve such questions as:

- Do we want a fluoridated paste?
- Is it smooth or gritty?
- How does the cost compare?
- Is it effective in preventing decay?
- Does it taste good?

Now if brands A and B are both satisfactory, final decisions may be made on availability, convenience, sale price, etc. without further thought.



DECISION MAKING

One important way to help students make good decisions is to stress the importance of their asking questions. This is true not only for a major decision but also for all those minor ones that occur during their consideration of the major one. A statistical investigation gives opportunity for both. For example:

Someone suggests the soft drink and candy machines should be eliminated from the school corridors.

- What data is necessary to make a good decision?
- Can it be gathered? If so, how?
 - Will the data be reliable?
 - Will the data be relevant?
- How should it be presented?

If by graphs, which kind? circle, bar, line, scatter diagrams? Are the graphs accurate or misleading?

- How can the data best be summarized?
- Is the decision based on the data or is the data used in a selected form to support a decision made on other grounds?
- Is there more or different data that would help make better decisions? If so, how can such data be gathered?

Many decisions made on impulse or personal prejudice are later supported by perhaps questionable "facts." This is unfortunate. Some knowledge of how statistical methods of analysis can be used in making rational decisions may help when the time comes to make an important decision.

Of course, some of the decisions we make on impulse are fine. But we should be careful to recognize what we are doing. For instance, grocery shopping is impulse buying for many people and displays are made to encourage this tendency. But setting up a shopping list of items needed, watching sales, and comparing prices per serving are examples of gathering, organizing and using information that are typical of a statistical approach to decision making.

If we are faced with the need to buy a new car, do we really look for pertinent information? How does the cost of a new car compare with the cost of a good second hand one or with the cost of getting the "old one" fixed up, repainted and reconditioned? How badly do we need it? Is good gas mileage important? How reliable are the advertised mileage statements?

Finally after all the facts are assembled, studied and evaluated, do we really use them in making the decision? Or do we say, "Oh, the heck with it. I'm tired

DECISION MAKING

of looking. Let's take this one. It's fancier than the one the Jones bought and we've got to keep ahead of them." That way can lead to disappointment if not disaster.

This resource should help students confronted with decisions to think about and compare the consequences of decisions made on impulse and those made after thoughtful inquiry and planning. They should learn to question and think before making up their minds.

A statistical approach to decision making is important because many decisions have to be made in the face of uncertainty and incomplete knowledge. In such a case, a statistical analysis of the data gives us the right to say there is a measurable degree of probability that the decision is correct. Surely this is better than a decision based on impulse, personal prejudice or hunch.

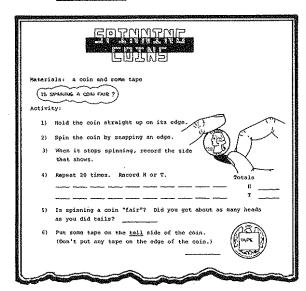


EXAMPLES OF DECISION MAKING IN THE CLASSROOM MATERIALS

I. <u>Deciding What Kind of Data to</u> Collect

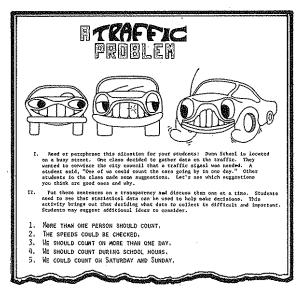
In this activity, students decide what data would be best to collect. The data could be used to help a traffic department decide whether or not to install a traffic light.

II. <u>Deciding If a Die, Coin or Game</u> Is "Fair"

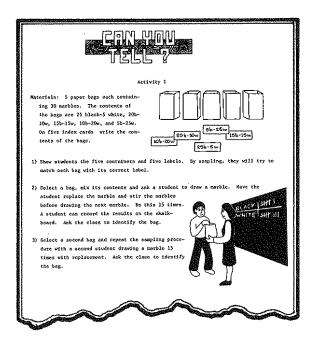


III. <u>Deciding the Make-up of a</u> Population

Students can take samples from a population and estimate the make-up of a population. In this activity students use sampling to help them decide what label best fits each population.



Students can gather data to help them decide if a die, coin or game is "fair." To be more confident of their decision, students may want to repeat the experiment several times or combine their data with that of other students.



DECISION MAKING

DECISION MAKING FOUND IN CLASSROOM MATERIALS

TABLES

THE LEXICOCHRONOGRAPHER

USING FREQUENCY TABLES TO ANALYZE WRITTEN MATERIAL

GRAPHS

FOREST FIRES ARE A REAL BURN HOW CAN WE DISPLAY THE DATA?

MEAN, MEDIAN, MODE

THE AVERAGE FAMILY

WHAT DO YOU THINK?

SAMPLING

DON'T LOSE YOUR MARBLES YOU BE THE JUDGE

EXPERIMENTAL PROBABILITY

THE EVEN-ODD GAME

PROBABILITY WITH MODELS

ONE \$M,ILL,ION

TAKE YOUR CHOICE

BE A WINNER

BUSINESS & COMMERCE

A NOISY PROJECT

COMPARING DIFFERENT TYPES OF GRAPHS

MAKING A PICTOGRAPH

CHOOSING MEAN OR MODE TO DESCRIBE A DISTRIBUTION

CHOOSING MEAN OR MODE TO DESCRIBE A DISTRIBUTION

USING SAMPLES TO ESTIMATE RATIOS

EVALUATING SAMPLING PLANS

PREDICTING OUTCOMES OF DICE

USING PROBABILITIES TO MAKE CHOICES USING PROBABILITIES TO MAKE CHOICES USING TREES TO ANALYZE GAMES

SURVEYING OPINIONS ON HOUSEHOLD TOOLS

DECISION MAKING



ENVIRONMENT

WHO NEEDS A BIKE PATH?

PULL FOR POOLING

PEOPLE & CULTURE

A SPORTY QUESTION

WHAT'S ON TV?

YOU TAKE A SURVEY

IS IT BEST TO BE GOLDEN?

GATHERING DATA ON BICYCLE AND CAR USAGE

GATHERING DATA ON THE NUMBER OF RIDERS IN CARS

GATHERING DATA TO INVESTIGATE DISCRIMINATION

COLLECTING DATA ON TYPES OF TELEVISION PROGRAMS

USING A SURVEY TO ANSWER QUESTIONS

COLLECTING DATA TO SEE IF GOLDEN RECTANGLES ARE THE MOST PLEASING SHAPE

ORGANIZATION

Placement Guide for Tabbed Dividers

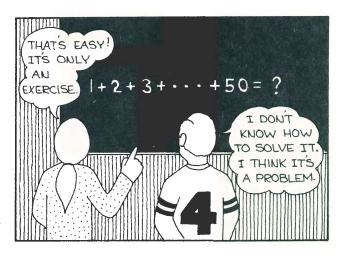
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One of the most important aspects of mathematics is posing and solving problems. Therefore in teaching mathematics at any level, problems and their solutions should be accorded a central position.

What is a "problem?" How does it differ from an "exercise?" What is a "solu-

tion?" A problem occurs when one feels the attainment of a certain goal is desirable but the route to that goal is not apparent. A solution is found when the route to the goal is determined. A problem becomes an exercise when the route reduces to the use of known techniques and formulas. A problem for Tom in the fifth grade may be only an exercise from him in the eighth grade. A problem for Ms. Porter in her first year of teaching may no longer be one after five years of experience. Exercises are



worthwhile in order to reinforce acquired techniques but not all assignments should be exercises. Practice with mathematical problems should develop habits of clear thinking, of careful consideration of alternatives, of planning and checking. Hopefully such habits will be of use when one is facing problems in later life.

Many statistical problems and their solutions are quite different from problems in arithmetic, algebra and geometry. A typical problem in statistics might be to determine the percent of students in a certain school or school district who are immune to measles. Its solution will require use of many statistical skills and methods. This problem may be only of intrinsic interest when posed. But the solution of the problem may become urgent if an epidemic of measles is feared and a decision has to be made whether to innoculate some or all of the children.

A second statistical problem is: What are the average weights of children at different age levels and how do children in this class compare to these averages? Others are: Is there a boy and a girl who are representative of the students in this class and, if so, how are the two to be selected? What is the probability a family of four children has two boys and two girls? The four aces from a deck of playing cards are dealt face down in some random order on the table. John identifies the four cards without looking at them. What is the probability he did this by pure chance and not by psychic power?

How do these problems differ from those encountered in arithmetic or geometry? Usually the data or information for a problem in arithmetic or the facts and conditions for a problem in geometry are given and the solutions



when found are definite. The answer is a set of numbers, a figure or a proof. The problem is how to work from what is given to what is wanted. If apples cost 11¢ apiece and oranges 12¢, how much money is needed to buy 5 oranges and 3 apples? A harder problem might be: How many apples and how many oranges can be bought for exactly \$1.00? In each case enough data is given. The answer to the first is 93¢. The answer to the second is one orange and eight apples. Both answers can be easily checked but the method of finding the second answer is not at all evident. The answer could be found by repeated guessing and checking. It should be noted that guessing and checking is a reputable method of solving problems that is too often overlooked. To find the general method of solving the second problem is in itself quite a difficult problem even in algebra.

Finding an exact answer is not usually possible for problems in statistics. Here the data must be gathered after it has been decided what data is relevant. The answers are likely to be couched in such terms as: We are 95% sure that only between 30 and 40 percent of the children are immune to measles. On the other hand the fundamental ideas and methods of thinking about problems are still much the same.

George Polya has written five books on problem solving.* Polya suggests four

^{*}In increasing order of difficulty these books are:

Polya, George. How to Solve It. 2nd Ed. Princeton: Princeton University Press, 1973.

_____ Mathematical Discovery, Vol. 1 & II. New York: John Wiley & Sons, Inc., 1962.

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essential phases or steps for solving problems. These steps are just as valid and valuable for problems in statistics as they are for problems in any field of mathematics or in any other field such as academic, business or human relations. Let us look at Polya's steps with particular emphasis on how they apply to statistical problems.

- UNDERSTANDING THE PROBLEM. Reformulate the problem. Determine as clearly as possible what is known. What limits are imposed? How exact an answer is wanted?
- DEVISING A PLAN. Determine what information is needed and where and how it is to be collected. Should it be done by sampling or otherwise? What model, if any, should be used? Is simulation desirable?
- 3. CARRYING OUT THE PLAN. Collect the data. Organize it into tables and graphs. Interpret the data to determine the solution.
- 4. LOOKING BACK. Was the plan or the model appropriate? Is the result reasonable? Does it solve the problem? If not, is it at least a step in the right direction? If yes, can we find a better or an easier method? Can we apply this method to solve another similar problem?

Let us apply these principles to the problem of determining the percent of students who are immune to measles.

UNDERSTANDING THE PROBLEM. The problem seems clear enough at first glance but a closer look reveals some uncertainties. Is the "measles" referred to regular measles or German measles? Are children immune if they were inoculated once before entering school or only if they were inoculated recently? What does "recently" mean?

DEVISING A PLAN. Once the problem itself is clarified, data must be gathered. Can it be obtained directly from school health records? If not, then questionnaires for the students or their parents will be needed to gather the necessary information. If the school district is large, then to save time and money probably only a sample group will be examined. Now statistical techniques and methods are directly involved.

CARRYING OUT THE PLAN. Decisions have to be made on such things as the questions to be asked, the size of the sample and how the poll is to be taken. How can it be assured that the sample is really representative of all the students? How are results to be presented in order to make the reports both fair and vivid?

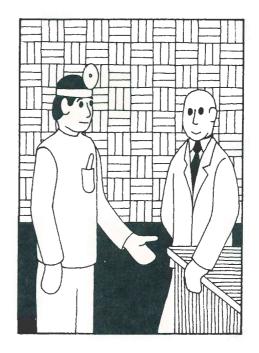
LOOKING BACK. How can projections from the sample to the whole student body be made and with what reliability? Are the results reasonable? Do they check with

SOURCE: How to Solve It, 2nd Ed.

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those from neighboring communities? Helping students see these as pertinent questions whose answers will help them solve other real problems is one of the purposes of this resource. One of the real difficulties may be to get students to accept an answer such as "We are 95% sure that only about one-third of the children are presently immune to regular measles." The answer is not a certainty as they may expect all answers to be. They must realize it is the best that can be done without unreasonable expenditures of time and money.

The second statistical problem



mentioned above was about the average weights of children of different ages and how the children in this class compare to these averages. Here the statistical concepts of average, range and deviation are important. Again, gathering, organizing and summarizing data are necessary before interpreting it to get the information necessary to solve the problem.

Frequently the solution of a statistical problem leads directly to a decision for action. Such a decision as choosing a class representative may be made arbitrarily but it is likely to be better with some statistical analysis. The original problem is really not very clear. As stated it is open to several interpretations depending on the answers to preliminary questions. Probably different interpretations will lead to different choices. Is the representative boy and/or girl to be chosen as the one nearest the average in height, in weight, in age, in academic achievement, in some form of athletic prowess or in a combination of any number of these or other items? Once this has been decided, a plan can be devised to make the selection. It can be carried out and the result checked.

Suppose the problem is clarified as follows. For a classroom competition, Room A is asked to select a boy for a baseball distance throw and a girl for accuracy in throwing a softball at a target. Problem: How to select the two representatives. Is the problem now clear? Yes, it seems so. Plan: Call for volunteers, have these



volunteers practice a few times and then conduct a preliminary competition to select the best ones. Four girls and three boys volunteer. Each one gets five throws. The results are tabulated.

TABLE	1
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	BOYS' BASEBALL DISTANCE THROW				
	(1	Distance	in metres)	
A1	46.2	47.3	48.8	49.2	45.5
Bob	47.3	39.2	49.6	54.1	41.7
Chris	51.1	50.2	48.6	42.3	42.4

TABLE	2
-------	---

GIRLS' SOFTBALL ACCURACY THROW (Score on target)

Donna	25	5	10	5	20
Evelyn	15	15	20	15	20
Frankie	0	10	25	25	25
Joan	15	20	20	5	25

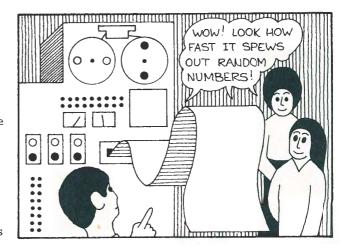
Now it seems the solution is not so clear. How is the interclass competition to be scored? By the average, by the first throw, by the best throw, by the average of the three best throws? This sort of information is necessary before the decision can be made. As work on the problem progresses, other information may be needed. Making a graph of the scores shows that Bob's throws vary most widely. The range of his throws is 15 m. He has the longest throw but the lowest average. Al has the largest average but Chris has the two best throws. Frankie got the most bull's-eyes but Donna had the best first throw. The final decision must be based on all the information about the contest and all the evidence about the individuals competing that can be gathered. Even so the chosen students may have a bad day and not do as well as we had hoped. Looking back: Was the plan a good one? Were the best prospects too reluctant to volunteer? Should the whole class have been tested? Were too many throws required so that fatigue entered in? Was too little practice allowed?

A method of problem solving widely used in statistics is that of using a model



to simulate the original problem. This is frequently done in studying the behavior of sampling, particularly when the sampling is destructive, as in studying the life of light bulbs or the quality of large shipments of eggs. The model may be built by simulating the sampling process by use of random numbers.

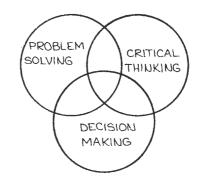
Random numbers for this purpose may be read from tables but generating them by computer or programmable calculator may be more practical since it is many, many times faster. The model and the method of simulation must be chosen with care so as to be appropriate to the problem at hand. Sampling problems are of great importance in business, industry, banking, manufacturing, medicine, law, etc. Methods for their solution will be valued by many students in later life and early acquaintance with these ideas will be helpful.



Problems in statistics frequently do not have hard and fast answers. But there are methods that lead to solutions. These solutions may not be exact or certain. However, there is usually a measurable degree of confidence in the result. On the basis of the solution "We are 95% sure that between 30 and 53 percent of the children in this district are immune to regular measles" the Board of Health has to decide whether to have all children inoculated to prevent a new epidemic.

The three ideas of problem solving, critical thinking and decision making are interrelated in many ways. The diagram to the right may help in thinking about them and their relationship to each other.

In some situations all three are inextricably involved, while in others the main emphasis is on only one or two



of the three. One might say that every attempt to solve a problem involves critical thinking. This is so whether it be to formulate it carefully, analyze it, create a

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plan for its solution, carry out the plan or to look back, reconsider, generalize and specialize the problem. So too, the solution of a problem may lead to making a decision. Indeed the problem may be: Should all the children be immunized against measles this fall before being permitted to start school? On the other hand, the problem "What is the probability of rolling a 5 with a fair die?" involves no decision at all.

Critical thinking may be involved in decision making. An advertisement says, "Buy a GASAVER CAR. The EPA mileage is 42 in the country and 34 in the city." Before we decide to buy this car we might ask: Is that with a car full of passengers or with just a driver? Is it with high test or regular gas? etc.

Sometimes critical thinking is important for our own satisfaction even if no problem is to be solved or immediate decision to be made. Is the School Committee's prediction of declining school enrollment next fall justified by the data they present or is it made to justify reducing the budget? Critical thinking should help a student understand better how to support a position with solid evidence and justifiable assumptions. It should help decide whether evidence is sufficient to persuade us to action. At this point, it becomes a vital adjunct to decision making.

Some decisions involve little critical thinking or problem solving. A teacher may decide what shoes to wear today simply by using a rotation system or by matching their color to the color of the clothes she decides to put on.

But in many cases and in many ways, decision making is an application to real world situations made on the basis of a solution to a problem. Critical thinking will be essential in deciding the mathematical model appropriate to the practical problem. The solution of the problem may then be the basis for making the decision for action. All three ideas are inextricably bound up in some situations. But the individual threads need to be studied separately to help us make the best use of our minds to think critically in solving our problems and making our decisions.

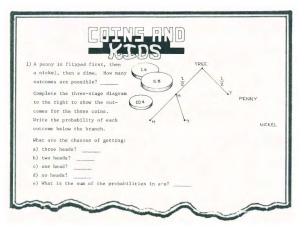


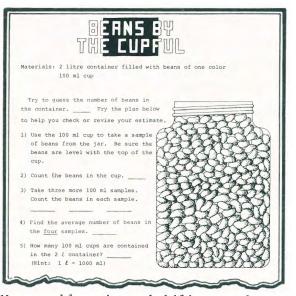
EXAMPLES OF PROBLEM SOLVING IN THE CLASSROOM MATERIALS

I. Using Sampling to Solve a Problem

Many problems in statistics involve making estimates or predictions about a population. Analyzing the data from samples can help in making estimates for the population.

II. <u>Drawing Probability Trees to Solve</u> Problems





Many problems in probability can be analyzed by drawing probability trees.



PROBLEM SOLVING FOUND IN CLASSROOM MATERIALS

GATHERING DATA

A TRAFFIC PROBLEM

DECIDING WHAT DATA TO COLLECT TO ANSWER A QUESTION

TABLES

AN AGE OLD PROBLEM

MAKING A STEM-AND-LEAF DISPLAY

GRAPHS

SCALE AND PLOT

LABELING SCALES AND PLOTTING POINTS

SAMPLING

CAN YOU TELL?	INCREASING THE NUMBER OF SAMPLES TO MAKE BETTER PREDICTIONS
ARE YOU SUCCESSFUL?	SEEING HOW SAMPLES DESCRIBE A KNOWN POPULATION
TWO SCOOPS OF RAISINS	USING SAMPLES TO CHECK FOR A UNIFORM DISTRIBUTION
HOW MANY DOTS?	USING SAMPLES TO ESTIMATE POPULATION SIZE
BEANS BY THE CUPFUL	USING SAMPLES TO ESTIMATE THE NUMBER OF BEANS IN A JAR
IT'S A PILE OF STONES!	USING SAMPLES TO PREDICT THE MASS OF STONES
HOW MANY DEER?	USING SAMPLES TO ESTIMATE POPULATION SIZE

EXPERIMENTAL PROBABILITY

SPINNING COINS

- PICKING MARBLES 2 AT A TIME FINDING THE RELATIVE FREQUENCY OF OUTCOMES
 - DETERMINING FAIRNESS OF SPINNING A COIN



I'LL FLIP YOU FOR IT RECORDING OUTCOMES OF FLIPS OF A COIN GET BACK ON THE TRACK USING COIN FLIPS TO SIMULATE A RANDOM WALK TOSSING PENNIES USING COIN FLIPS TO DETERMINE FAIRNESS OF A GAME A TREE SPINNER EXAMINING OUTCOMES OF A SPINNER THE HEXED HEXASPIN DETERMINING FAIRNESS OF A HEXASPIN ROLL THAT CUBE ESTIMATING THE FREQUENCY OF OUTCOMES OF DICE ROLL A DIE RECORDING THE OUTCOMES FOR ROLLS OF A DIE USING A SIMULATION TO GATHER DATA TO A CEREAL QUESTION MAKE A PREDICTION USING A COMPUTER TO FIND THE EXPECTED A COMPLETE SET NUMBER OF PURCHASES ROLLING DICE ESTIMATING THE FREQUENCY OF THE SUMS OF TWO DICE DETERMINING FAIRNESS OF A GAME CRAZY QUOTIENTS PROBABILITY WITH MODELS ONE \$M, ILL, ION USING PROBABILITIES TO MAKE CHOICES TAKE YOUR CHOICE USING PROBABILITIES TO MAKE CHOICES FINDING PROBABILITIES I FINDING PROBABILITIES FOR OUTCOMES PROBABILITY PROBLEMS TO PURSUE DRAWING ONE-STAGE TREES TO SOLVE WORD PROBLEMS



CARDS AND NUMBERS FINDING PROBABILITIES FOR OUTCOMES PICKY PROBLEMS FINDING PROBABILITIES FOR OUTCOMES WHAT DO YOU EXPECT? EXAMINING PROBABILITY SITUATIONS MARBLE PROBLEMS USING TWO-STAGE TREES TO SOLVE WORD PROBLEMS USING TREES TO SOLVE WORD PROBLEMS COINS AND KIDS SINK THAT FREE THROW! USING TREES TO SOLVE WORD PROBLEMS CHANCES ARE USING TREES TO SOLVE WORD PROBLEMS USING TREES TO SOLVE WORD PROBLEMS A MATCHED PAIR MORE MARBLE PROBLEMS USING TREES TO SOLVE WORD PROBLEMS PARTIAL TREES USING PARTIAL TREES TO SOLVE WORD PROBLEMS BE A WINNER USING TREES TO ANALYZE GAMES TO TREE OR NOT TO TREE USING TREES TO SOLVE WORD PROBLEMS ON THE AVERAGE STUDYING VARIABILITY IN THE TOSSES OF A COIN

COUNTING TECHNIQUES

WHAT'S NEW AT THE ZOO?	COUNTING ARRANGEMENTS
PASCAL'S PATTERNS	FINDING PATTERNS IN PASCAL'S TRIANGLE
PERPLEXING PROBABILITIES	USING COUNTING TECHNIQUES TO SOLVE PROBABILITY PROBLEMS

BUSINESS & COMMERCE

CHANCES AT THE SUPERMARKET	FIGURING THE CHANCES OF WINNING IN A SUPERMARKET GAME
JOIN THE QUEUE	USING RANDOM DIGITS TO INVESTIGATE CUSTOMER ARRIVAL
INSURANCE FOR A PREMIUM	USING A MORTALITY TABLE TO FIGURE THE PREMIUM FOR A LIFE INSURANCE POLICY

1



ENVIRONMENT

WHO NEEDS A BIKE PATH?

COUNTING CORNERS

DECAY AND HALF-LIFE

LET'S GO FISHING

GATHERING DATA ON BICYCLE AND CAR USAGE

ANALYZING DATA ON TRAFFIC AT INTERSECTIONS

USING DICE TO SIMULATE RADIOACTIVE DECAY

USING SAMPLING TO ESTIMATE THE MAKE-UP OF A POPULATION

PEOPLE & CULTURE

WHEN'S YOUR BIRTHDAY?

A SPORTY QUESTION

WHO'S #1?

AN ORDERLY ARRANGEMENT

USING A COMPUTER TO FIND PROBABILITIES

GATHERING DATA TO INVESTIGATE DISCRIMINATION

FINDING AND GRAPHING LETTER FREQUENCIES

COUNTING THE POSSIBLE PERMUTATIONS OF NOTES

ORGANIZATION

Placement Guide for Tabbed Dividers

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MODELS & SIMULATIONS

Models of real life situations are a frequent experience for most people. Model railroads and model planes are scaled down from actual size while in a science museum the model of the eye of a fly is greatly enlarged. To study changes in the design of an airplane wing, a model is made and subjected to wind tunnel tests. To get public reactions to plans for a new school, a model is built and displayed.

Models also occur frequently in the mathematics classroom. In many elementary classes, circle-fractions are models that give concrete realizations to the abstract ideas of fractions while manipulations with groups of counters are models of the notions of addition and subtraction.

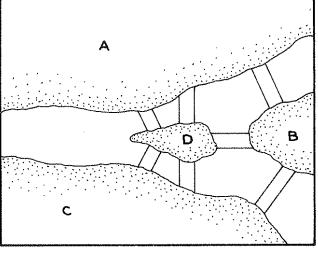
In later years the situation may sometimes be reversed. Instead of having concrete models of abstract mathematical ideas we have mathematical models to help study concrete real-life situations. As an illustration, consider the following example:

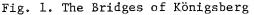
A famous problem in the early 1700's concerned the bridges in the city of Königsberg. There were seven such bridges, some across the river Pregel and some

giving access to an island in the river. The problem was: Could a family start from their home and take a walk during which they would cross each bridge once and only once! They might end up back at home or at a friend's home in another region. A rough map of the situation is drawn in Figure 1. No one had found a way to take such a walk but some people still thought it might be possible.

Finally in 1736, Leonard Euler, one of the great mathematicians of all time, solved the problem by making a mathematical model. The model consists of four points A, B, C and D representing the four regions, and seven arcs representing the bridges, drawn to co

A, B, C and D representing the four regions, and seven arcs representing the bridges, drawn to connect the points as the bridges connect the regions. The resulting figure is the graph in Figure 2, with four vertices and seven arcs. In this model, the bridge problem becomes: Can you start at some vertex and trace the whole graph without going over





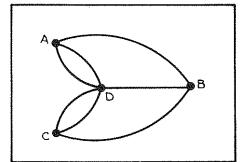


Fig. 2. Graph Model of the Bridges of Konigsberg

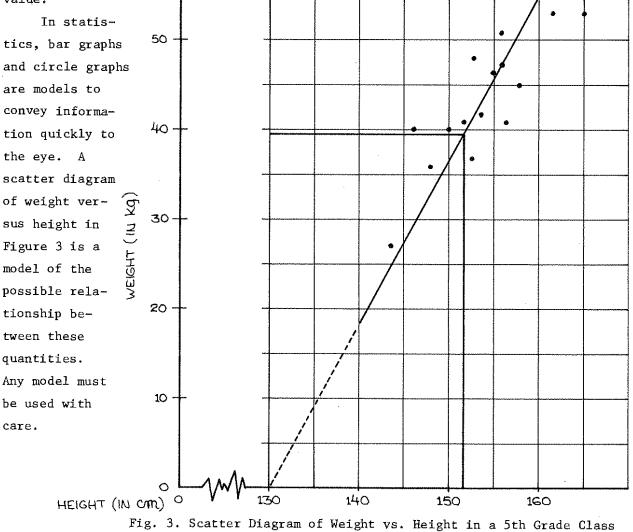
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MODELS AND SIMULATIONS

any arc twice? Euler proved the answer to this particular graph problem is NO. So the answer to the bridge problem is also NO. This type of model has proved very useful. It is used to solve many other problems involving communication networks such as roads and telephone lines.

We should note that the model ignores many features of the real life city such as the size of the island or the length of the bridges. A scale map would be a model of the city but it would not serve the purpose of analyzing the bridge problem as does Euler's graph model. He concentrates on the four regions and the seven bridges connecting the regions in certain pairs and ignores everything else.

Real situations are often complicated. The builder of mathematical models has to walk a fine line between oversimplifying the model so it serves no useful purpose or leaving it so complicated it cannot help solve the problem and again is of no value.



MODELS AND SIMULATIONS



For example, this model says a student whose height is 152 cm might weigh about 40 kg. But it would be foolish to say a student 130 cm in height should weigh about 0 kg as following the dotted portion of the trend line might indicate. Using a model beyond the appropriate region may suggest results that are absurd in the real situation. A Mercator projection map of the earth is a good model for regions near the equator but it makes Greenland as large as South America which a global map shows is not true.

Examples of models in this resource are the use of coin tossing or dice rolling to simulate a series of experiments. If the chances of success or failure in an experiment are even, then tossing coins may be an appropriate model. If the chance of success is 1/6 then rolling dice would be a better approach. Of course, not all experiments have their chance of success equal to either 1/2 or 1/6, so other models must be developed. The choice of an appropriate model is perhaps the most important concern for the model builder. Students will need practice and help in making such choices.

CHOOSING A MODEL

Example 1

In families of five children, what percent will have four of one sex and one of the other? Since the chances of a baby being a boy or girl are about the same, a model based on tossing coins would be appropriate.

Example 2

Sam has six shirts, each a different color. He wants to wear them each about the same number of times but doesn't like to wear them always in the same order. A good model for choosing his shirt might be to roll a die each morning using a die that has each face marked with the color of one of his shirts. Is this a good model? Yes, because the chance of each color showing on the die is 1/6 and in the long run each color will occur about the same number of times.

Example 3

Ella has eleven different outfits she wants to wear equally often but not always in the same order. Pete suggests she number her outfits from 2 to 12, roll two dice each morning and wear the outfit whose number comes up. Is this a good model? The answer is NO. Why? Although just eleven numbers can be rolled, the chances of

MODELS AND SIMULATIONS



getting a 7 are much higher than of getting a 2. Outfits 6, 7 and 8 would get much more wear than outfits 2 and 12.

What model could be designed to help Ella? Random digits can be used to design an appropriate model. This will be done near the end of this paper.

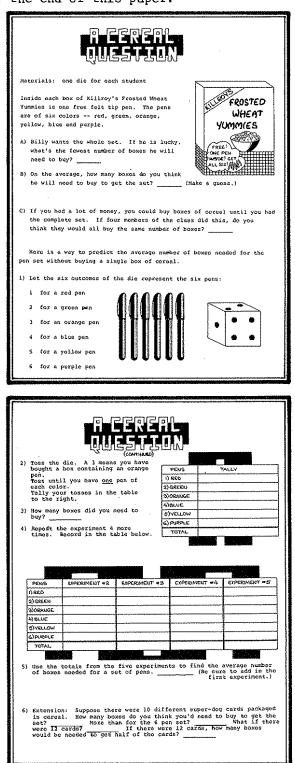
Example 4

In the activity A Cereal Question, the roll of a die does seem to be a good model for studying the problem of how many cereal boxes need to be bought to get at least one pen of each of the six different colors. Rolling the die simulates buying a cereal box, the result of the roll indicating the color of the pen in the box. But it can be noisy and time consuming.

Is there another model that will provide a simulation quieter, quicker and easier? Yes, the important quality of the model was that it had six equally likely outcomes to match the purchase of the six equally likely colored pens.

As suggested in the activity, a random digit table can be used to provide six equally likely outcomes. Thus a hundred simulations of the original purchase can be obtained quickly without buying a single box or rolling a single die. By use of a computer a thousand or ten thousand simulations can be done and the average number of purchases needed to get the six different pens easily figured.

Simulations by means of a model in which probability and random outcomes are used are called Monte Carlo methods. They are frequently used in statistical problems.





If the original experiment cannot be carried out, a simulation may be the only way to proceed. Such was the case in the original investigation into the feasibility of the atomic bomb. Simulation of the way the neutrons would behave as the chain reaction started was necessary both to determine the critical size of the bomb itself and to determine if, after such a chain reaction once started, it would stop before destroying the whole world.

Monte Carlo methods can involve cards, coins, dice, random numbers, spinners or any other devices that seem most appropriate. Cards are used in the activity *How Many Deer*? to simulate methods used to count wild animals in a park or fish in a lake.

SIMULATION: USING A MODEL

Tossing five coins was suggested as a good model for studying the composition of families of five children. To carry out the simulation we would record B for each head and G for each tail in a given toss to specify the number of boys and girls in that family. Doing this a hundred times and recording the number of families with four B's and one G or four G's and one B will give an approximation to the percent required. Since the chances of B or G are assumed to be 1/2, simulation by use of random numbers could be done by reading five successive digits for each family, recording B for every even digit and G for every odd one. This could be done even more rapidly by using a computer and the results obtained for a thousand or ten thousand families.

Random numbers could be used to help Sam pick his shirts just as they were used to simulate buying cereal boxes. We still need to design a model for Ella to use in selecting one outfit out of eleven with equal chances for each one. Consider a random number table and read two digits at a time. There are a hundred possibilities from 00, 01 to 99. These numbers can be divided into eleven groups with one number left over. Let it be 00. Group the remaining numbers from 01 to 99 in groups of 9 numbers.

01 t	o 09	group l	4	6 to	54	group	6
10 t	co 18	group 2	5	5 to	63	group	7
19 t	o 27	group 3	e	4 to	72	group	8
28 t	zo 36	group 4	7	3 to	81	group	9
37 t	co 45	group 5	5	2 to	90	group	10
			ç	l to	99	group	11

MODELS AND SIMULATIONS

If her outfits are numbered from 1 to 11, Ella can read a series of numbers from the random number table and pick the outfits for the week according to the groups the numbers fall in. She ignores the 00 whenever it occurs. If she reads 85, 40, 88, 30, 95, 00, 50, 02, 66, 99, 44 she would wear in succession outfits number 10, 5, 10, 4, 11, 6, 1, 8, 11 and 5.

Monte Carlo methods can be used to simulate sampling. Suppose a major league baseball player has a batting average of .353. What is his chance of getting three or more hits in five times at bat in today's game? These five times at bat can be considered as a sample of size five drawn from a large reservoir of times at bat where the probability of getting a hit each time is .353. Drawing a sample can be simulated by reading five successive three digit numbers from a random number table. Numbers from 001 to 353 stand for hits. Running a simulation for a hundred games and comparing the number of times three or more hits occur to a hundred gives an approximation to the chance we are looking for.

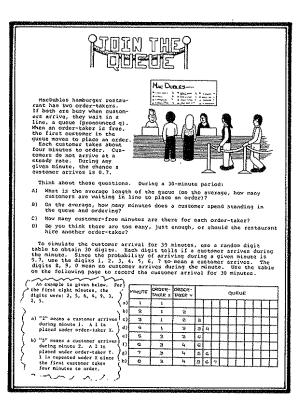
As another example, a salesman claims his light bulbs are guaranteed to be no more than 2% defective. Defective means they have a life of less than 1000 hours. How can tests be made to see if the claim is right? Testing bulbs destroys them so only samples can be tested. How big should a sample be and how many bad bulbs can be allowed in a sample before the whole shipment should be rejected? The salesman suggests taking a sample of 50 and passing the shipment if either no bad bulbs or only one is detected. This is a sampling plan with a sample size of 50 and a passing or acceptance mark of 1. Is this a fair test? How often will it reject a good shipment and how often will it pass a bad shipment in which 5 or 10% of the bulbs are bad? The probabilities involved could be computed or read from sufficiently large tables if they were available. What we want is a model we can modify to simulate many different sampling plans. By trying one plan after the other in simulation we would get an empirical basis for deciding which plan is best in that both buyer and seller agree it is fair. Then we use this plan for actually testing bulbs. The simulation could be done by reading sets of fifty three-digit numbers from a random number table. Defective bulbs would be represented by the numbers from 001 to 020 for the 2% case and by 001 to 050 or 100 in the 5 or 10% case. The other numbers would represent good bulbs. Many such samples could be run on a computer and a decision made as to whether the plan was good or not. If not, samples of different size and passing number could be tried until a satisfactory plan was found.

MODELS AND SIMULATIONS



In using models and simulations, it is important to remember what simplifying assumptions have been made about the original problem. Answers obtained from the model should be checked with the given conditions. Perhaps the model can be adjusted to take care of some conditions ignored the first time in making the assumptions. If so, progress is made toward better and more accurate solutions. Picking a suitable model may be very difficult. Practice in simple cases is important as is the discussion of the appropriateness of a particular model for a given problem.

EXAMPLES OF MODELS AND SIMULATIONS IN THE CLASSROOM MATERIALS

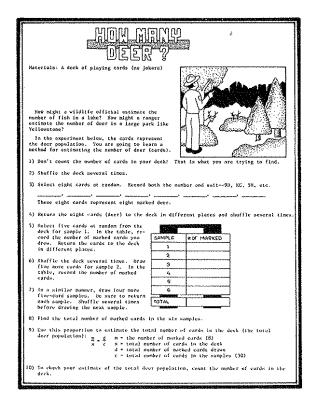


II. <u>Using Cards to Simulate a</u> Sampling Procedure

> Sampling is used to make estimates about populations, but sometimes the sampling procedures are difficult to understand. By simulating the sampling with cards or marbles, students learn how the sampling works. This activity simulates the marking of deer to determine the total number of deer in a population. Marbles could have been used in place of the cards.

I. <u>Using Simulations to Solve</u> Problems

Many problems can be solved through modeling and simulations. Using a model to simulate the problem situation provides data for solving the problem. Students can simulate the lines at a quick-order hamburger stand and obtain estimates for the waiting time and the length of the lines. This data could be helpful in making decisions about the number of clerks needed.



MODELS AND SIMULATIONS



MODELS AND SIMULATIONS FOUND IN CLASSROOM MATERIALS SAMPLING

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LET'S USE OUR RANUTAS

MAKING ESTIMATES

HOW MANY DEER?

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USING A RANDOM NUMBER TABLE FOR SAMPLING

USING SAMPLES TO ESTIMATE POPULATION SIZE

USING SAMPLES TO ESTIMATE POPULATION SIZE

EXPERIMENTAL PROBABILITY

A CEREAL QUESTION

A COMPLETE SET

USING A SIMULATION TO GATHER DATA TO MAKE A PREDICTION

USING A COMPUTER TO FIND THE EXPECTED NUMBER OF PURCHASES

BUSINESS & COMMERCE

JOIN THE QUEUE

ENVIRONMENT

DECAY AND HALF-LIFE

LET'S GO FISHING

RECREATION

YOU CAN'T GET A HIT ALL THE TIME

ONE DIE OR TWO

POKER WITH THE COMPUTER

USINC RANDOM DIGITS TO INVESTIGATE CUSTOMER ARRIVAL

USING DICE TO SIMULATE RADIOACTIVE DECAY

USING SAMPLING TO ESTIMATE THE MAKE-UP OF A POPULATION

USING A COMPUTER TO ANALYZE BATTING TRENDS

USING A COMPUTER TO COMPARE ROLLS OF A DIE WITH ROLLS OF TWO DICE

USING RANDOM DIGITS FROM A COMPUTER TO PLAY POKER

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ORGANIZATION

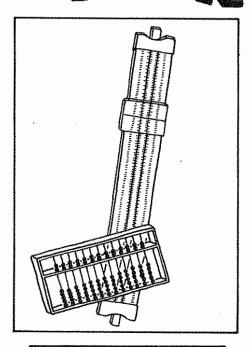
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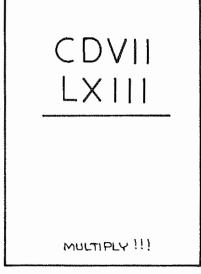
CALCULATORS

Since the use of numbers first began, people have devised and used calculators of various types to help and speed up the computations involved. Such calculators include fingers, small pebbles (Latin: calculi) lined up in rows, the abacus, the slide rule, the adding machine and now the hand-held electronic calculator and the computer. The computer is useful for handling large masses of numbers for repetitive and complex operations. For individual classroom use, the hand-held electronic calculator is, at the moment, more useful. The advent of this device marks a turning point . in the ease of mathematical calculations possibly as important as that of the shift from the Roman numeration system to our commonly used placevalue system.

The price of calculators over the last few years has been going steadily down but their power and sophistication has been going up. Simple four function calculators are now available for under \$10.00 and, if the trend continues, the cost may soon be something like \$1.98. At that price they could be provided for students as routinely as textbooks and pencils. Teachers need to decide <u>now</u> how to take advantage of calculators so they can train their students how to use this fascinating new tool to the greatest advantage.



<u>_OM</u>



The mathematics classroom no longer needs to be insulated from the numbers of the real world. When learning how to determine the perimeter and area of a rectangle, it is vital to learn why one adds in one case and multiplies in the other. Practice in using small integral values is needed while learning when to add and when to multiply. But in actual practice the numbers involved are more likely to be 23.4 and 32.3 than 20 and 30. A person using a calculator can compute the sum or product in each case with the same ease. In non-routine problems considerable time

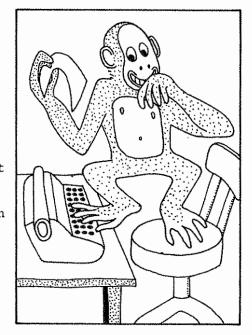
CALCULATORS AND COMPUTERS

must be spent on analysis of the problem and on selection of the proper algorithm to carry out the solution. It would be nice if less time had to be spent on the re-sulting calculations.

While a calculator allows a complicated series of arithmetic operations to be done by pushing a series of buttons, it does not eliminate the need for students to know the basic arithmetic skills. These skills are necessary for estimating and judging the reasonableness of answers obtained on the calculator as well as for routine calculations with small counting numbers.

An unreasonable answer to a problem should always be recognized by a student. An error with a calculator may be the result of carelessness in pushing the buttons or a fundamental mistake in the analysis of the problem. If the former, a mere repetition of the calculation should be enough. But if the latter, a complete review and reassessment of the analysis will be necessary. <u>The calculator does not replace</u> <u>thinking</u>. It is a tool to speed up calculations, to free us to think more carefully, and to analyze our problems more thoroughly.

An analogy to the value of a calculator in doing arithmetic computations might be that of a typewriter in writing a composition. Students need to learn when and how to add and multiply small whole numbers with or without pencil and paper just as they need to learn to write legibly and spell accurately. Long and involved computations are more easily done on a calculator once it has been mastered just as a long composition is more easily written on a typewriter once the touch system has been learned. Just as a typewriter produces gibberish if the keys are struck in the wrong order so will a calculator produce wrong answers if the + button is pushed when the x one is called for or 2.35 is entered instead of the correct 23.5.



In elementary statistics, an electronic calculator cannot be used to decide whether the mean, median or mode of a set of numbers is the most appropriate average to consider or the most appropriate representative number of the set. Nor will it serve to find the mode or median. Fortunately, the mean is most often desired and the calculator can be used to find it easily. This is as true if 53 numbers of four digits each are involved or only 10 numbers of one digit each. The process may be longer but for the calculator the difficulty is the same.

Other summarizing numbers used in statistics are the variance and the standard deviation. These are fairly messy to compute even with a small four function calculator. But by carefully recording the intermediate steps it can be done. The time and effort involved will be much less than in doing it by hand. Even this can be avoided with slightly more sophisticated calculators specially designed for statistical work that now sell for less than \$25.00. They will give both the mean and the standard deviation for any set of numbers as soon as the numbers are entered and the right buttons pushed. Such calculators will also give the correlation coefficient for two sets of numbers and the constants determining the equation of the line of best fit for the scatter diagram of the two sets. Without a calculator, determining these last items exactly is beyond the scope of the present project even though we have discussed the ideas in the CONTENT FOR TEACHERS sections.

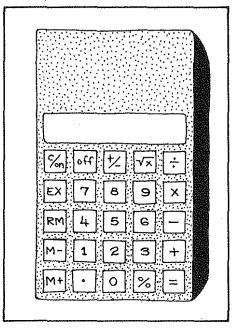
Probabilities can also be worked out on a calculator. Again it is important to remember that only a careful analysis of the problem will determine what probability is the right one to work out and the reasonableness of the result.

Most inexpensive calculators use a straight forward arithmetic sequence of operations performed as they are entered. Thus, punching in order 2, +, 5, x, 3, -, 9, = will display 12 as the answer. But if the problem is

$$\frac{(3+4) \times (6+5)}{7+2},$$

the intermediate answers, 7, 11 and 77 and 9 have to be stored either on scrap paper or in the calculator's memory for later recall and use. Some more expensive calculators have parentheses keys

and an algebraic logic so that the problem could be done as follows:



 $(, 3, +, 4,), x, (, 6, +, 5,), <math>\div, (, 7, +, 2,), =.$

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Other calculators use the Reverse Polish Notation (RPN) logic that may seem awkward at first. but eliminates the use of parentheses and is in fact quite easy to learn and use rapidly. On this kind of a calculator, the key sequence for the same problem would be:

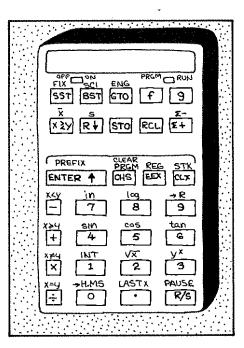
3,	Enter,	4, +,	6, E1	nter),	5, +,	
	x , 7	, Enter	r, 2,	+,[÷].	

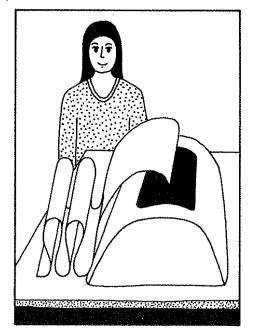
Some calculators use a so-called commercial logic much like the one used on old adding machines.

In addition to the different logics and methods of operation, each calculator has its own quirks and specialties to be learned from its instruction booklet. Someday manufacturers may standardize their machines.

A few of the late model calculators have the capacity to learn a program. They are called programmable calculators. On some of them the program has to be entered by hand; on others, it can be fed in from a previously recorded magnetic tape. What is a program? It is a series of pre-arranged steps that performs the same sequence of operations on whatever numbers are entered. Such a program is very useful if, for instance, one wants to compute the means of many different sets of numbers.

A computer is not yet a widely available classroom device even though many schools have access to large central computers through timesharing arrangements and local terminals. Some schools have their own mini-computer. More can be expected to have access to a computer in one way or another in the near future. The cost of a mini-computer is dropping rapidly and dramatically. Soon computers will be available to most students so teachers should know their capabilities and how to take advantage of this powerful new tool. Some students will gain experience with these computers by merely using programs already in the computer library or by inputting





programs prepared by someone else. Other students will want to write their own programs, an exciting endeavor to be encouraged for all students as it involves real skill in logical analyses and in careful and exact writing.

Some programs in the BASIC language have been provided in this resource to be copied and used or to serve as examples for students interested in writing their own. They will be found in the *APPENDIX* of the CONTENT FOR TEACHERS section. These programs will probably have to be modified slightly for use on your computer as each computer has its own idiosyncrasies and slight modifications of the BASIC language.

In the near future, the ability to use a computer will be just as important to a student as the ability to use a library. When that day comes the knowledge that a computer cannot do the analysis of a problem will be just as important as the knowledge that a computer can do the required calculations. We must see that our students are well prepared in both respects.

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EXAMPLES OF CALCULATORS AND COMPUTERS IN THE CLASSROOM MATERIALS

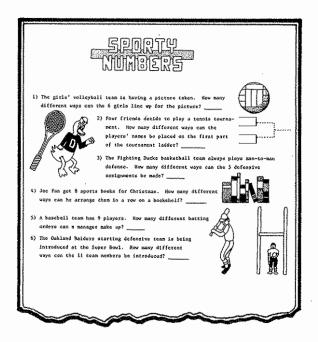
I. <u>Using Calculators to Compute</u> Running Totals

Calculators can be used to speed up the computation of running totals which lead into percentiles. Calculators are also useful for computing means, ranges and percents.

			NNYNG HANGS	
1)			Joan worked part- supermarket.	HONONY 4 3.75
			the right.	(TUTESONV + 4.25 ()
	has carned k	y the end of	f one day, two Those totals are	(TRUESDAY 44.50 FRIDAY 45.00
	called runn!	ing totals.		
		h table bolow in done for y Earnings		running totals. The first
	Monday Tuesday	\$3.75	\$3.75	75 + 54,25
	Wednosday Thursday Friday	\$4.00 \$4.50 \$5.00	_	App
2}		niow shows the		sents who ato at the school
	a) Maka a		al for the week. tudonts Rumning	Com.
	Monday	175	canon co adminini	LU III
	Tuesday	207		
	Wednesday	289		
	Thursday '	193		
	Friday	160		
	b) Row 1	many students	a wore served up	to and including Thursday?

II. <u>Using Calculators to Compute</u> Factorials

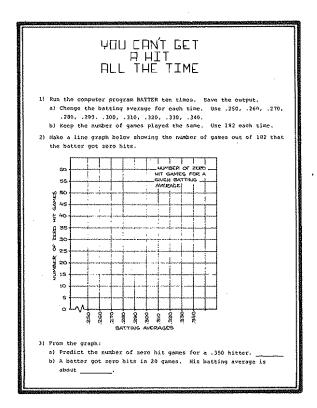
Formal methods for counting combinations and permutations involve factorials. Factorials can be computed quickly with a calculator.



i##

III. <u>Using a Computer to Simulate</u> Situations

Students can use the computer programs in the *APPENDIX* to the CONTENT FOR TEACHERS for many different kinds of simulations. This activity simulates the hits in one season (182 games) for players with different batting averages.



IV. Using a Computer to Determine
 Probabilities

When a sequence of probabilities are wanted, it is easier to use a computer. The probabilities from this activity are surprising, and students might want to gather data on birthdays before they believe that the results are reasonable.

CALCULATORS AND COMPUTERS

CALCULATORS AND COMPUTERS FOUND IN CLASSROOM MATERIALS GRAPHS

TO MAKE A CIRCLE GRAPH

RANGE & DEVIATION

RUNNING TOTALS

SAMPLING

ARE YOU SUCCESSFUL?

MAKING A CIRCLE GRAPH

FINDING RUNNING TOTALS

SEEING HOW SAMPLES DESCRIBE A KNOWN POPULATION

EXPERIMENTAL PROBABILITY

A COMPLETE SET

RANDOM NUMBERS VIA A CALCULATOR

CRAZY QUOTIENTS

COUNTING TECHNIQUES

SPORTY NUMBERS

WEBSTER'S DICTIONARY

BUSINESS & COMMERCE

CHANCES AT THE SUPERMARKET

CALLING ALL CARS

MAKING A MORTALITY TABLE

USING A COMPUTER TO FIND THE EXPECTED NUMBER OF PURCHASES

GENERATING RANDOM NUMBERS WITH A CALCULATOR

DETERMINING FAIRNESS OF A GAME

INTRODUCING AND USING FACTORIALS

COMPUTATION WITH FACTORIALS

IS FOOD EATING UP YOUR INCOME? USING CIRCLE GRAPHS TO COMPARE PER-CENT OF INCOME SPENT ON FOOD

> FIGURING THE CHANCES OF WINNING IN A SUPERMARKET GAME

COLLECTING AND STUDYING DATA ON FAMILY CARS

USING PROBABILITIES TO CONSTRUCT A MORTALITY TABLE

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CALCULATORS AND COMPUTERS

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ENVIRONMENT

PARTICULAR POLLUTANTS

WATER WASTES

HEALTH & MEDICINE

A BLOOD RELATIONSHIP

YOUR INTERNAL COMBUSTION MACHINE

READING A TABLE AND MAKING A GRAPH OF SOURCES OF AIR POLLUTION

READING A TABLE ON WATER POLLUTION

USING A CIRCLE GRAPH TO SHOW DISTRIBUTION OF BLOOD TYPES

COMPLETING A TABLE TO SHOW CALORIE USE

PEOPLE & CULTURE

POPULATION ESTIMATION

SIZING UP THE STATES

CANADA - NEIGHBOR TO THE NORTH

WHEN'S YOUR BIRTHDAY?

HOW DOES YOUR READING RATE?

RECREATION

YOU CAN'T GET A HIT ALL THE TIME

ONE DIE OR TWO

POKER WITH THE COMPUTER

READING A TABLE OF STATE POPULATIONS

USING A SOURCE BOOK TO GATHER

POPULATION DATA

READING A TABLE AND GRAPH ON CANADA'S POPULATION

USING A COMPUTER TO FIND PROBABILITIES

USING AVERAGES AND GRAPHS TO INCREASE READING RATE

USING A COMPUTER TO ANALYZE BATTING TRENDS

USING A COMPUTER TO COMPARE ROLLS OF A DIE WITH ROLLS OF TWO DICE

USING RANDOM DIGITS FROM A COMPUTER TO PLAY POKER

ORGANIZATION

Placement Guide for Tabbed Dividers

Tab Label	Follows page:
CONTENT FOR TEACHERS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
Probability with Counting	
Inferential Statistics	
Appendix-Computer Programs	
DIDACTICS	
Components of Instruction	
Classroom Management	
Statistics and Probability Learning	
TEACHING EMPHASES	
Critical Thinking	
Decision Making	
Problem Solving	
Models and Simulations	
Calculators and Computers	
Laboratory Approaches	
CLASSROOM MATERIALS	
CONCEPTS AND SKILLS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Misleading Statistics	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
APPLICATIONS	
Business & Commerce	
Environment	
Health & Medicine	
People & Culture	
Recreation	
GLOSSARY	
ANNOTATED BIBLIOGRAPHY	
SELECTED ANSWERS	

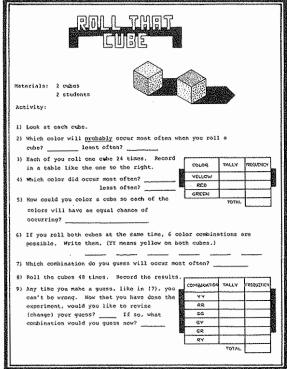
LABORATORY APPROACHES

Like "modern math" or "discovery learning" or "individualization," the phrase "laboratory approach" means different things to different people. Although the phrase may conjure up visions of manipulative materials, perhaps the most commonly agreed-on characteristic is that <u>in laboratory approaches the emphasis is on learning-by doing</u> as opposed to learning-by-listening. The important feature is that the student is an active participant rather than a passive receptor. Sometimes this involvement is accomplished through the use of a manipulative, but it could be accomplished through a pencil paper investigation, a class or individual project or even a teacher led discussion. This view of laboratory approaches ties in closely with problem solving and discovery learning.

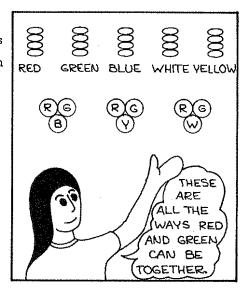
WHAT IS A LABORATORY ACTIVITY?

A laboratory activity is a task or mathematical exercise that emphasizes "learning by doing." Two examples are given below.

Jo and Lynn are working with the a) activity card to the right. Their teacher has given them plain cubes with colored dots on the faces. They each guessed green would occur most often when one die is rolled. Their experimental results supported this guess. They have agreed to guess that green-green (GG) will occur most often when two dice are rolled. They are rolling the dice to see what will happen. The tally starts to show more green-red (GR) pairs than GG. After 50 rolls they total their tallies and decide to revise their guess to GR. There is time left in the class period so



they roll the dice another 50 times. Again GR happens most often. Jo and Lynn discuss the activity and decide to ask their teacher if GR is "supposed" to happen more often. They return the activity card and cubes to a manila folder and hand in their papers. b) A class was challenged to find all the possible combinations of 3 different colors chosen from 5 colors. They were each given 35 centimetre cubes or chips, 7 of each of the 5 colors. Students started to arrange the chips and cubes on their desks. Some students started to write down what they found so they wouldn't duplicate. Students started to compare their findings. Someone said, "I've found six combinations." Someone else said, "I've found eight." One student asked if they could start writing the combinations on the



board so all the students could compare what they found. Combinations were written on the board until the students were convinced they had them all. The teacher saw they had found the ten possible combinations and asked the students to identify an organized way that could have been used to find all the combinations. For example, find the ways that red and green could be used, red and blue, red and white; then work on the next color, remembering to check for repetitions. The homework assignment was to try to find all the possible combinations of 3 different colors chosen from 6 colors.

The two lab activities above involved students in active learning. The teacher provided the activity or the challenge and then became a resource person. Students explored the activities at their own pace and in their own way. The activities provided opportunities for students to use problem solving processes--organizing information into a table, looking for patterns, making predictions and checking predictions. Cooperation among pairs or groups of students was encouraged and each student had a chance for success.

Lab activities can vary greatly in form from those described above. Active learning can be accomplished through a game, a demonstration, a paper and pencil exercise, a set of manipulatives with a task card, or an experiment using instruments to take measurements. An activity like *Tossing Pennies* from EXPERIMENTAL PRO-BABILITY has students play a game until they can decide if it is fair. The demonstration described in *Normal Distributions* in RANGE AND DEVIATIONS can help students understand normal curves. A lab activity could involve measuring to make a scatter



diagram, experimenting with spinners to approximate a probability, counting cars or bicycles to make recommendations on traffic regulations or taking surveys to determine opinions.

ORGANIZATION OF LABS

COLLECTING EQUIPMENT

A collection of lab materials is useful for some lab activity sessions. Items can be made, gathered or purchased. Below are three lists of suggested materials and manipulatives that have been used in various lab activities.

COMMON ITEMS

--Adhesives: tapes, glue

--Coloring Materials: pens, pencils, chalk, crayons, paint
--Fasteners or Binders: string, nails, pins, rubber bands, staples, wire, tacks
--Hard and Soft Wood: blocks, cork, boards, pegs, tagboard, toothpicks
--Paper and Write-on Materials: cardboard, construction paper, grid paper, letter stencils, plastic sheets, road maps, waxed paper, index cards, tracing paper
--Miscellaneous: measuring cups and spoons, modeling clay, needles, calendars, boxes, bottles, cans, cartons, cups, mirrors, catalogs, almanacs*, restaurant menus, phone books

TOOLS AND INSTRUMENTS

MANIPULATIVES

hammer	tape measures	Cuisenaire rods	dice
calculators	metre sticks	geoboards	spinners
paper cutter	metric weights	Soma cubes	sample boards
paper punch	balance scales	attribute blocks	marbles
paper stapler	stop watch	tangrams	blank cubes
scissors	thermometer	colored wooden cubes	coins
straightedges	T-square	linking cubes	polyhedral dice
protractors	templates	geoblocks	
compasses	clock	pattern blocks	



THE MATHEMATICS LABORATORY

The mathematics laboratory is an <u>environment</u> that provides for active learning and encourages active participation. In terms of physical organization, three basic kinds of mathematics laboratories are often discussed.

- 1) A decentralized laboratory--a self-contained set of lab materials stored in the teacher's classroom and readily available for the students to use.
- 2) A rolling or movable laboratory--a set of lab materials placed on a cart, stored in a central location, and wheeled from classroom to classroom as needed.
- 3) A centralized laboratory--a room especially designed (or adapted) and equipped for use as a permanent mathematics laboratory. Classes are usually brought into the lab room on a rotating schedule that allows each mathematics class to use the lab materials several times a week as needed.

For most schools, the decentralized laboratory is the most practical and desirable mathematics laboratory. Lab materials can be collected and organized at a modest rate as they are constructed, donated or purchased.

Eventually a set of lab materials will grow to a size large enough to be quite versatile. The classroom environment needs to be versatile as well. Flat tables, bookcases, movable carts and other furniture can be added to provide work areas for the students and storage space for the lab activities.

Lab materials may be packaged for student use in boxes, manila folders or envelopes. For example, a task card may be placed in a shoe box along with grid paper, scissors and several crayons that are required for the activity. Teachers may choose to package the materials with an activity card, or they may decide to have the students get needed materials for an activity from organized storage shelves. It is important that the students learn how to obtain the necessary lab items, how to read and follow the directions on a lab card, and how to cooperate and share their ideas.

ORGANIZATION OF THE STUDENTS

There are various ways of organizing the students for math lab participation. If a class is inexperienced in laboratory methods, whole class or large group activities might be easier to initiate. Other activities may be done in small groups where the students rotate among several lab stations. (See *Teaching Via Laboratory Approaches* in the resource <u>Mathematics in Science and Society</u>.) Often pairs of students work well; the teacher can carefully select the partners or, in some cases, allow the students to choose partners. Once the students understand the basic lab procedures, each pair can do a different lab activity. This cuts down the competition with classmates and allows the pair to make decisions and solve problems at their own rate.

CONTENT OF LAB ACTIVITIES

Lab activities can be organized around a mathematical topic, say experimental probability. The materials such as cubes, spinners and coins are collected for each activity. Once the materials have been selected, they can be organized into lab packages. The class can then be divided into pairs or groups for the lab sessions.

Another way is to organize a set of lab activities covering different topics or concepts. Each lab activity is unique and self-contained. For example, a series of ten lab activities could include the exploration of ten different topics, probability, length, mass, fractions, etc. One program which is organized this way is <u>Math-Lab-Junior High</u> [McFadden, et al., 1975].

THE ROLE OF THE TEACHER

The teacher's role often changes between one of disseminator of knowledge to resource person and moderator. The students do the lab activities; the teacher watches and encourages inquiry and independent thinking. Using a laboratory approach requires considerable preparation and forethought. The teacher needs to find, organize and store lab materials for easy use; tell students where lab materials are, what to do with them and how to schedule their use; carefully prepare task cards or directions for the lab activities; instruct students in problem-solving methods of attack and investigation; interact enthusiastically with students and share in their experiences; and evaluate each student's attitudes, work habits and accomplishments.

GETTING STARTED

There are many ways to incorporate laboratory approaches with present methods of teaching. The descriptions below provide several suggestions to consider when starting to use a laboratory approach.

Mr. Langford has a class of thirty seventh graders. He was not sure about using lab materials, so he decided to start small. He set up an "activity corner" in the room. Three lab cards with the necessary equipment (e.g. in this case, rulers, measuring tapes, grid paper) were set up in the "activity corner." Each day of one week a different group of six students was allowed to work in pairs using the lab materials. The rest of the class worked on related paper and pencil exercises. All week was spent on the study of mean, median and mode. All thirty students had a chance to do the lab activities, and the activities integrated well with the week's mathematics topic. Mr. Langford plans to collect or write task cards that mix well with his established curriculum. Later, he might try other ways of using the lab activity cards.

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LABORATORY APPROACHES

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Ms. Wilkins decided to assign each Friday as a "lab day" for her eighth-grade class of 28 students. She had watched several classes using a "lab day" once a week and decided to try it herself. She prepared two sets of seven lab cards covering seven different mathematical topics. Each student was assigned a partner, and the pair worked together for each of the seven "lab days." For seven weeks the students rotated to a new lab activity each Friday. They were asked to keep a record of their results and follow the planned rotation schedule. Ms. Wilkins found that this seven-week period with one "lab day" a week coincided well with the nine-week term. She developed a second set of lab materials for another seven weeks. This time lab was used twice a week and 14 task cards put into 14 manila envelopes along with manipulatives, paper, or other materials needed for each activity. Each card treated the topic of probability and contained various levels of abstraction and enrichment options for the students.

Mr. Jeffreys and Ms. Slone have adjoining sixth-grade rooms. They planned to team teach a number of units in mathematics and decided to try the lab approach for their unit on sampling. They made sample boards and gathered marbles, decks of cards and cubes. Mr. Jeffreys and Ms. Slone picked out five activities from the SAMPLING section. One activity was used each day for one week. Often the class compared the data in small groups or compiled the data to make better predictions.

The above are examples of teachers who were willing to support an active approach to learning. They prepared for using the lab approach by collecting and organizing materials and deciding on the content of lab activities.

Initially, when selecting materials and equipment to use in the math lab, find readily available materials in the school. As time goes on, you will be able to buy, make or scrounge other materials as they are needed for particular activities.

Pages from this resource that are marked with the lab symbol, $\frac{1}{42}$, the problemsolving symbol, $\frac{1}{2}$, or the modeling and simulations symbol, $\frac{1}{42}$, may include ideas for lab activities. The lab symbol itself mainly flags those pages which use a manipulative. Lab activities can also be found in the resources <u>Number Sense and Arithmetic Skills, Ratio, Proportion and Scaling, Geometry and Visualization</u>, and <u>Mathematics in Science and Society</u>. Ideas for laboratory activities can be found in any of the sources listed on page 8. Many periodicals (such as <u>The Arithmetic Teacher</u> or <u>The Mathematics Teacher</u>) include sections in each issue which contain ideas for activities that require a minimum of preparation and materials. Notice the interests of the students. Be creative and use your own ideas or their ideas as a source of lab activities. Discuss and exchange ideas about math labs with other teachers.

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Start small--in no way can most teachers and students survive a complete change of program. Students who have become passive learners need time to adapt to the role of active learners. The students need to develop inquisitive attitudes that motivate them to keep at a problem and not give up. They need supervision and guidance from the teacher as they learn to function in the lab environment. Eventually, the students should be able to select materials for each lab activity and return materials to the proper storage area when finished.

In the beginning it is a good idea to provide activities where each group member has a specific role. Have a specific objective in mind for each activity, and have a clear idea of its mathematical content. Go through the lab activity to find what background concepts or skills the students will need to tackle it. Check for any difficulties the students might encounter as they do the activity.

PLAN FOR EVALUATION

"Teacher evaluation of pupil progress should take two forms: (1) evaluation of written records, i.e., record papers, and (2) assessment of pupil competency based on observation and interaction with the youngster as he works. In both cases, the emphasis is on the progress of the individual in solving a given problem using his own particular talents and capabilities. Obviously, no evaluation is quite so valuable as that done first-hand. A laboratory approach offers unique opportunities to assess understandings and competencies through observation and discussion with the pupil as he completes his assigned task. In evaluating written records, provide positive reinforcement for carefully completed recordings. Encourage completeness of answers, keeping in mind that the record paper is primarily a communications device. If each pupil keeps a folder of his completed record papers, he can note his own improvement in recording throughout the school year." (Scott McFadden, et al., "Program Teacher Commentary On Using MathLab," published by Action Math Associates, 1976.)*

SUMMARY

The laboratory approach is a philosophy which emphasizes "learning by doing." "It is a system based on active learning and focuses on the learning process rather than on the teaching process." [Kidd, et al., 1970] At the level of their abilities and interests, the students discover relationships and study real-world problems which utilize specific mathematical skills. A laboratory approach can be integrated into the classroom and used along with, not in place of, many other equally valuable teaching strategies.

*Permission to use excerpt granted by Action Math Associates, Inc.

LABORATORY APPROACHES

SELECTED SOURCES FOR LABORATORY APPROACHES

- The Arithmetic Teacher. Reston, Virginia: The National Council of Teachers of Mathematics.
- Biggs, Edith E. and MacLean, James R. <u>Freedom to Learn</u>. Don Mills, Ontario: Addison-Wesley (Canada) Ltd., 1969.
- Kidd, Kenneth P.; Myers, Shirley S.; and Cilley, David M. <u>The Laboratory Approach</u> <u>to Mathematics</u>. Palo Alto, California: Science Research Associates, Inc., 1970.
- Krulik, Stephen. <u>A Mathematics Laboratory Handbook for Secondary Schools</u>. Philadelphia: W. B. Saunders Co., 1972.
- The Mathematics Teacher. Reston, Virginia: The National Council of Teachers of Mathematics.
- McFadden, Scott; Anderson, Keith; and Schaaf, Oscar. <u>MathLab--Junior High</u>. Eugene, Oregon: Action Math Associates, Inc., 1975.
- Reys, Robert F. and Post, Thomas R. <u>The Mathematics Laboratory</u>. Boston: Prindle, Weber & Schmidt, Inc., 1973.
- Sobel, Max and Maletsky, Evan. <u>Teaching Mathematics: A Sourcebook of Aids, Activi</u>ties, and Strategies. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1975.
- <u>Teacher-Made Aids for Elementary School Mathematics; Readings from the Arithmetic</u> <u>Teacher.</u> Reston, Virginia: The National Council of Teachers of Mathematics.

Laboratory and instructional materials can be obtained from the following publishing companies:

- Action Math Associates 1358 Dalton Drive Eugene, Oregon 97404
- Creative Publications, Inc. P.O. Box 10328 Palo Alto, California 94303
- Cuisenaire Company of America, Inc. 12 Church Street New Rochelle, New York 10805
- Educational Teaching Aids Division 159 West Kinzie Street Chicago, Illinois 60611
- Gamco Industries, Inc. Box 1911FG Big Spring, Texas 79720
- Ideal School Supply Company 11000 South Lavergne Avenue Oaklawn, Illinois 60453
- The Math Group, Inc. 396 East 79th Street Minneapolis, Minnesota 55420

- Midwest Publications Company, Inc. P.O. Box 307 Birmingham, Michigan 48012
- Mind/Matter Corporation P.O. Box 345 Danbury, Connecticut 06810
- Ohaus Scale Corporation 29 Hanover Road Florham Park, New Jersey 07932
- Scott Resources, Inc. 1900 E. Lincoln Box 2121 Fort Collins, Colorado 80521
- Selective Educational Equipment, Inc. 3 Bridge Street Newton, Massachusetts 02195
- Walker Educational Book Corporation 720 Fifth Avenue New York, New York 10019
- Webster Division McGraw-Hill Book Company 330 West 42nd Street New York, New York 10036



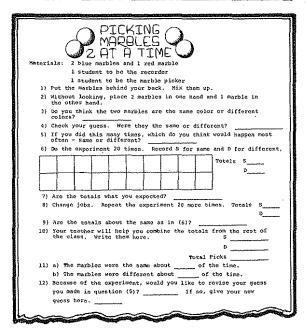
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EXAMPLES OF LABORATORY APPROACHES IN THE CLASSROOM MATERIALS

I. <u>Using Concrete Materials to</u> <u>Understand Concepts</u>

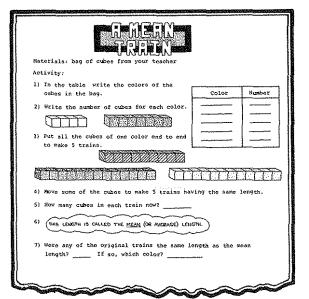
Sometimes a concrete representation will help students understand a concept. Here cubes are moved until the mean length of several trains is found.

II. Experimenting to Estimate Chances



III. Discovering Arrangements

Students can use colored chips to investigate the number of ways to arrange three colors in a row. They can then explore arrangements when colors are allowed to repeat.



Students can do experiments to gather data. The data can be compiled or averaged to draw conclusions about the chance of an outcome. In this activity students might be surprised that the chance is better for drawing 2 marbles of different colors than for drawing 2 marbles of the same color.

	REPERTING PERMUTATIONS
	Get 3 chips, each a different color.
1	Place a chip on each of these blanks.
2} (write the order of the colors. (Use an abbreviation)
	and the second second second
3):	Move the chips. Find difforent arrangements.
	Record each arrangement.
	THE THREE CHIPS CAN BE ARRANGED IN A
4)	Remove 1 chip. Get another chip with the
	same color as a chip you kept.
	QUESTION : CAN YOU STILL MAKE BIX DIFFERENT ARRANGEMENTS ?)
5)	Place a chip on each of these blanks,
6}	Write the order of the colors.

LABORATORY APPROACHES FOUND IN CLASSROOM MATERIALS

GRAPHS

THEY MELT IN YOUR MOUTH

SCATTER DIAGRAMS

ARE YOU A SQUARE OR A RECTANGLE?

A LENGTHY RELATIONSHIP

USING A GRAPH TO SHOW THE CONTENTS OF A PACKAGE OF M&M'S

GATHERING DATA TO MAKE A SCATTER DIAGRAM

MAKING SCATTER DIAGRAMS TO FIND RELATIONSHIPS

MEAN, MEDIAN, MODE

NUMBER PLEASE

A CLASSY MEDIAN

A MEAN TRAIN

A BALANCED ROD

A MEAN GUESS

MEDIAN VS. MEAN

RANGE & DEVIATION

ROLL THEM AGAIN

FILL UP THE GAPS

NORMAL DISTRIBUTIONS

FINDING THE MODAL LAST DIGIT IN A TELEPHONE NUMBER

DEMONSTRATING THE CONCEPTS OF MEDIAN AND MODE

FIND A MEAN LENGTH

RELATING THE MEAN TO A BALANCE POINT

DEMONSTRATING THE USE OF AN ASSUMED MEAN

SHOWING EFFECTS OF EXTREME VALUES ON MEDIAN AND MEAN

USING MEAN, MODE AND RANGE TO DESCRIBE A DISTRIBUTION

SHOWING SUMS OF DIFFERENCES ABOVE AND BELOW THE MEAN ARE EQUAL

INTRODUCING AND DEMONSTRATING NORMAL DISTRIBUTIONS

LABORATORY APPROACHES



SAMPLING

CAN YOU TELL?

AN AVERAGE SAMPLE, ACTIVITY I

AN AVERAGE SAMPLE, ACTIVITY II

TWO SCOOPS OF RAISINS

BEANS BY THE CUPFUL

IT'S A PILE OF STONES!

HOW MANY DEER?

EXPERIMENTAL PROBABILITY

PICKING MARBLES 2 AT A TIME

SPINNING COINS

I[†]LL FLIP YOU FOR IT

GET BACK ON THE TRACK

TOSSING PENNIES

A TREE SPINNER

- INCREASING THE NUMBER OF SAMPLES TO MAKE BETTER PREDICTIONS
- COMPARING A MEAN SAMPLE TO THE POPULATION
- COMPARING THE ACCURACY OF DIFFERENTLY SIZED SAMPLES
- USING SAMPLES TO CHECK FOR A UNIFORM DISTRIBUTION
- USING SAMPLES TO ESTIMATE THE NUMBER OF BEANS IN A JAR
- USING SAMPLES TO PREDICT THE MASS OF STONES
- USING SAMPLES TO ESTIMATE POPULATION SIZE
- FINDING THE RELATIVE FREQUENCY OF OUTCOMES
- DETERMINING FAIRNESS OF SPINNING A COIN
- RECORDING OUTCOMES OF FLIPS OF A COIN
- USING COIN FLIPS TO SIMULATE A RANDOM WALK

USING COIN FLIPS TO DETERMINE FAIRNESS OF A GAME

EXAMINING OUTCOMES OF A SPINNER

LABORATORY APPROACHES



THE HEXED HEXASPIN

ROLL THAT CUBE

ROLL A DIE

A CEREAL QUESTION

WILL THE SPIDER CATCH THE FLY?

ROLLING DICE

CRAZY QUOTIENTS

PROBABILITY WITH MODELS

TAKE YOUR CHOICE

CARDS IN A HAT

THE RED-BLUE GAME

ON THE AVERAGE

COUNTING TECHNIQUES

YES, THREE-LETTER WORDS, PLEASE

WHAT'S NEW AT THE ZOO? AND THE WINNER IS... REPEATING PERMUTATIONS DETERMINING FAIRNESS OF A HEXASPIN

- ESTIMATING THE FREQUENCY OF OUTCOMES OF DICE
- RECORDING THE OUTCOMES FOR ROLLS OF A DIE
- USING A SIMULATION TO GATHER DATA TO MAKE A PREDICTION
- USING DICE ROLLS TO SIMULATE A RANDOM WALK
- ESTIMATING THE FREQUENCY OF THE SUMS OF TWO DICE

DETERMINING FAIRNESS OF A GAME

USING PROBABILITIES TO MAKE CHOICES

USING SAMPLING TO PREDICT PROBABILITIES

- USING A TREE TO ANALYZE A GAME
- STUDYING VARIABILITY IN THE TOSSES OF A COIN
- USING TREES TO COUNT LETTER ARRANGE-MENTS COUNTING ARRANGEMENTS INTRODUCING COMBINATIONS

FINDING PERMUTATIONS WITH REPETITIONS



ENVIRONMENT

DECAY AND HALF-LIFE

LET'S GO FISHING

A CHILLING EXPERIENCE

IS 32° HOT OR COLD?

BALLOON BAROMETER

A SWINGING TIME

HANG TEN

HEALTH & MEDICINE

TO EACH HIS OWN

ARE YOU PHYSICALLY FIT?

YOU ARE WHAT YOU EAT

MUSCLE FATIGUE

PEOPLE & CULTURE

HOW'S YOUR POP?

YOU TAKE A SURVEY

A CHANCY COMPOSITION

- USING DICE TO SIMULATE RADIOACTIVE DECAY
- USING SAMPLING TO ESTIMATE THE MAKE-UP OF A POPULATION
- COLLECTING AND GRAPHING TEMPERATURE READINGS
- USING A GRAPH TO COMPARE FAHRENHEIT AND CELSIUS TEMPERATURES
- COLLECTING AND GRAPHING DATA FROM A HOMEMADE BAROMETER
- MAKING AND USING A GRAPH TO PREDICT THE PERIOD OF A PENDULUM
- COLLECTING AND GRAPHING DATA ABOUT ELASTICITY
- MAKING GRAPHS TO COMPARE BODY MEASUREMENTS
- COLLECTING DATA TO DETERMINE PHYSICAL FITNESS
- USING RATS TO OBSERVE THE EFFECTS OF DIET ON GROWTH

USING A LINE GRAPH TO SHOW MUSCLE FATIGUE

USING AN EXPERIMENT TO ANALYZE TASTE-TESTING ABILITIES

USING A SURVEY TO ANSWER QUESTIONS

USING CHANCE TO COMPOSE MUSIC

LABORATORY APPROACHES



RECREATION

CLASSROOM DECATHLON

DROPPING THE BALL

COLLECTING AND ORGANIZING DATA FROM A CLASS COMPETITION

MAKING A GRAPH TO STUDY THE BOUNCE OF A BALL

ORGANIZATION

Placement Guide for Tabbed Dividers

Tab Label	Follows page:
CONTENT FOR TEACHERS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
Probability with Counting	
Inferential Statistics	
Appendix-Computer Programs	
DIDACTICS	
Components of Instruction	
Classroom Management	
Statistics and Probability Learning	
TEACHING EMPHASES	
Critical Thinking	
Decision Making	
Problem Solving	
Models and Simulations	
Calculators and Computers	
Laboratory Approaches	
CLASSROOM MATERIALS	
CONCEPTS AND SKILLS	
Gathering Data	
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Experimental Probability	
Probability with Models	
Counting Techniques	
APPLICATIONS	
Business & Commerce	
Environment	
Health & Medicine	
People & Culture	
Recreation	
GLOSSARY	
ANNOTATED BIBLIOGRAPHY	
SELECTED ANSWERS	



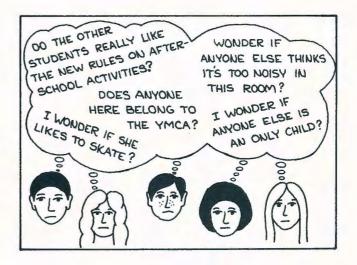
Commentary to GATHERING DATA (388-393)

TITLE	PAGE	TOPIC	TYPE
Statistics Around Us	394	Introducing statistics	Teacher idea
Stats Search	396	Introducing terms related to statistics	Worksheet Puzzle
Data Sheet	397	Collecting data about students	Teacher idea
A Traffic Problem	398	Deciding what data to collect to answer a question	Transparency Discussion
The Questionnaire	399	Learning about questionnaires	Worksheet
That's a Good Question	401	Choosing the better ques- tion for a questionnaire	Worksheet Puzzle
Fact or Opinion?	402	Analyzing answers	Worksheet Puzzle
Be Precise	403	Recognizing vague words	Worksheet Puzzle
Taking a Survey	404	Discussing interviews and questionnaires	Teacher directed activity Discussion
Getting to Know Your Almanacs	406	Using almanacs	Teacher directed activity
Can You Guesstimate?	407	Using a source book	Activity card Worksheet
Information Please	408	Using an almanac	Activity card Worksheet
Round Up Round Down	410	Rounding numbers	Worksheet
Comparing Data	412	Comparing data from different sources	Transparency Discussion
What's Your Bias?	414	Discussing possible biases of sources	Transparency Discussion
Almanacs and Other Sources of Data	415	Learning about sources of data	Teacher page

CATHERINC DATA

GATHERING DATA MEANINGFUL TO MIDDLE SCHOOL STUDENTS

Most students are very interested in knowing more about their classmates. Why not have students gather data related to their class? This can be done in many ways. The student pages What's In A Name? and How Much Are You Worth? use student names to provide data. Sisters and Brothers uses the number of sisters and brothers each student has. (These three activities are in the TABLES and GRAPHS sections because they also involve organizing the data.) Many other activities in this resource are similarly designed to



gather information about the class and the school.

A brief conversation can be the source of an activity on gathering and organizing data. The chance comment overheard by the teacher below can be the basis of an investigation by the student, a group of students, or the whole class. (See *A Sporty Question* in the PEOPLE AND CULTURE section for specific ideas on investigating sports discrimination.)

This activity could result in data to support a request for more girls' sports, or it could reveal reverse discrimination. Perhaps boys are not being allowed to participate in certain sports like volleyball. Students might even decide to carry out an attitude survey to see if girls and/or boys feel certain sports are only for one sex (A Sporty Question includes an example of such a survey).



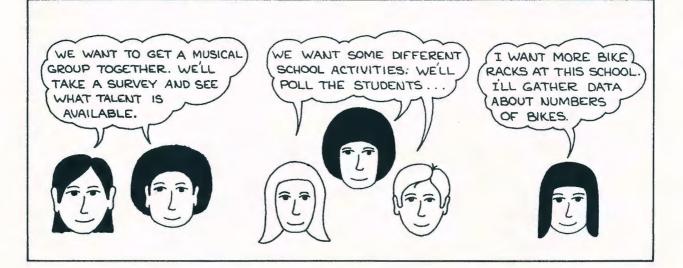
GATHERING DATA

Some students feel no one else could have the same feeling, fear or wish that they have. One junior high girl had aspirations to be a doctor, but she felt her classmates would think it was an odd profession for a woman. With the encouragement of her parents and a teacher, she surveyed the student body and discovered that many students thought it was appropriate for a woman to be a doctor. (Of course, results might not always be this positive.) Not only was she able to satisfy her own curiosity, but she also was involved in a meaningful application of statistical methods.



A school or local issue could also be the basis for an activity. If the budget has to be cut and either a sports program or a music program must go, a student opinion poll could be taken. (Be sure students are not led to believe their opinions will be considered in the decision unless this is true.) A poll could also be taken to see if students feel a new plan for lunch lines, absences, or hall passes is working. The poll could collect data on grade levels, age and sex so differences in student opinions by these categories could be determined.

No matter what activity is chosen, it is seldom guaranteed to interest all students. One suggestion for a class with diverse interests is to let some students work in small groups to decide what kind of information they would like to collect.



WHAT DATA SHOULD BE GATHERED?

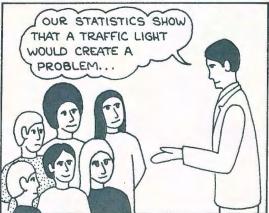
In order to decide what data to gather, it is often necessary to define the problem more narrowly. Suppose your class begins with the question: Should a traffic light be installed on the street by the school? Ask students:

- How can this question be answered? (One way is to collect data about pedestrian and motorist traffic on the street.)
- What data should be collected?
- Should we find the number of students crossing the street?
- Should we find the number of vehicles using the street?
- Should speeds be checked?
- Should a separate tally of children under ten years old be kept?
- Is it practical and possible to collect all the data we'd like to have?



The activity A Traffic Problem shown above can be used to stimulate more discussion. You can help your students see that some data is very important for answering the question but that there is no reason to collect more data than necessary.

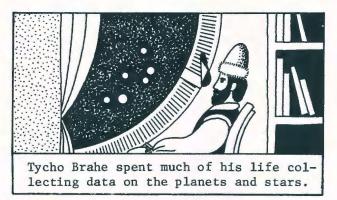
In the previous example, students might collect only that data supporting a bias toward installing the traffic light. If an unbiased answer to the question is to be made, it is necessary to collect data that might not support installing the light. The class might ask the city traffic control commission to send a representative to talk to the class about possible problems in overall traffic flow if a traffic light were installed. The initial question of installing a light might be discarded in favor of building a walk over the street or detouring pedestrians to another existing crosswalk.



In other activities, students will have to decide how precise the collected data should be. Have students decide whether to measure each student's height to the nearest centimetre or half-centimetre. They can also decide whether to record population figures from the almanac to the nearest million, thousand, or hundred.

Sometimes it is desirable to have data, but it is too expensive or time-consuming to collect all of it. Decisions will have to be made as to how complete the

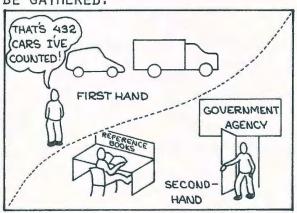
data should be. Sometimes a question must be left unanswered because the necessary data cannot be obtained within the limitations of the class. If this is discouraging to students, they might appreciate hearing about people like Tycho Brahe who spent a long time collecting data. (Years later, Kepler used the data to discover laws of planetary motion.)



You might want to provide more questions in which students can discuss and decide what data to gather. Examples are: Should the candy machine be removed from the school? Should a commercial company take over the hot lunch program? What day is best for a school party?

FROM WHAT SOURCES SHOULD THE DATA BE GATHERED?

Along with deciding what data should be collected, students will often need to decide what source to use. Certain problems may dictate the source, but sometimes several choices are available. Suppose the problem is to find the size of each student's family. Ask your class if it would be better to poll the students or



to obtain the information from the office. Which source is more accurate? Should data on a new ecology bill be obtained from the government agency that supported the bill or from the industries that lobbied against it or from both? Is there unbiased data available? Do all the sources agree? *What's Your Bias*? provides an opportunity for students to discuss possible biases of sources. More ideas on this are given in the MISLEADING STATISTICS section.

To decide what source or sources to use, students will need to be familiar with possible sources and to know how to obtain information from them. Activities like *Information Please*, *Can You Guesstimate*? and *Getting to Know Your Almanacs* will provide an introduction to reference books usually found in schools. A list of common almanacs and a description of each is given in *Almanacs and Other Sources of Data*. Hints on obtaining information from government agencies and corporations are given in *To Find Facts About Places*, *People*, *Things...*. Most of the classroom activities in this resource are accompanied by suggestions to students and teachers for choosing and using a source.

As the article below points out, two sources might use different methods to collect and report data.

West Berlin and Hungary have the highest suicide rates in the world, according to incomplete statistics of the World Health Organization. WHO reports that an international comparison of suicide rates is of "questionable value" so long as the reporting methods of specific countries vary widely. The report, however, does reveal some interesting trends. In 1970, for example, in West Berlin 67.5 men per 100,000 and 33.8 women per 100,000 took their own lives--15 and 30 times higher, respectively, than in Mexico. -But statisticians con-

sider Mexico's suicide statistics unreliable because of that country's religious constraints. Greece is the European country with the lowest suicide rate. Hungary, a country where many years ago a melancholy pop tune caused hundreds of people to take their lives, leads the suicide list with rates of 63.8 for men and 23.8 for women.* In an accompanying commentary, the WHO report adds that East Germany, Denmark, Hungary, Austria, Finland, Sweden, and Czechoslovakia are the European countries which

consistently have high

suicide rates.

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*These statistics are for countries; West Berlin's is for a city.

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The page *Comparing Data* asks students to look up facts in two different sources. The data is compared and students are asked to suggest possible reasons why the data is the same or different.

GATHERING DATA

HOW SHALL THE DATA BE GATHERED?

Deciding how to gather data can also be a problem. Students can discuss the following questions. Should a written questionnaire be used or a personal inter-

view? If a questionnaire is used, how should the questions be worded to avoid influencing the answers? If interviews are used, should the interviewer be allowed to explain what is meant by a question? Should information be gotten from an agency over the phone or should we ask to see written records?

The answers to these questions depend on the situation. Written records might be more exact in some



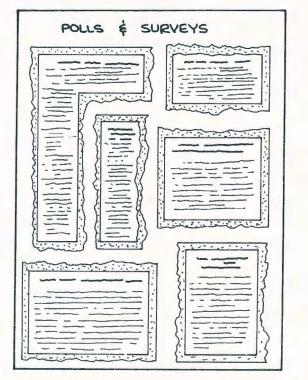
cases, but it might be more desirable to pursue verbal information over a phone in others. Designing a "fair" questionnaire or interview can be very difficult--Gathering Data of the CONTENT FOR TEACHERS section explains some of the concerns and problems in constructing questionnaires and interviews.

SOME EXTRA PAYOFFS IN GATHERING DATA

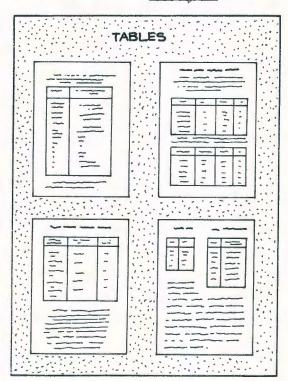
Having students use source books to gather data can have side benefits. Not only will they become more familiar with the sources, but they will probably broaden their general store of knowledge. While searching for data on one topic, students can often become absorbed in information given in tables about other topics. Gathering data firsthand can also result in more knowledge for students. Besides learning the statistical principles involved, students learn about the attitudes, opinions and preferences of their classmates. Here is a real opportunity to integrate mathematics with the social sciences and other subject areas.



To introduce the word statistics, ask students if they know what a statistic is. Write "statistics" on the chalkboard and carefully pronounce it. Ask students if they have ever seen a statistic. Prior to the class, prepare posters showing uses of statistics. Examples involving statistics (advertisements and articles) can be collected from newspapers, magazines and books and mounted on heavy paper. (Collect the examples several months before you begin the statistics unit.) One organization is shown below. Show the class each poster and briefly discuss the topic. Sample comments are suggested below for each poster. The presentation gives students a preview of ideas in statistics. After viewing the posters mention that <u>statistics</u> is the study of numerical data to obtain information. Each item in the data is called a <u>statistic</u>.



Statistics is used to help make decisions and answer questions. Often it is necessary to gather data first. Polls and surveys are two ways to get information. Refer to an example on the poster. Tell how the poll or survey was used to gather information to answer a question or make a prediction.



When numbers and facts are gathered they need to be put in order before they can be helpful. Tables are used to organize data. Tables help people communicate information in a concise form. It is important to read tables carefully so you understand the facts and don't reach incorrect conclusions. Refer to some facts in a table on the chart.



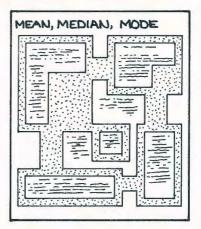
Graphs can also be used to help make sense out of data. Graphs are more eyecatching than tables. They also show comparisons and patterns of change more clearly. Refer to an example and show students they can notice trends without knowing the actual data. Again mention that graphs need to be read carefully so the reader is not misled.



Some data is organized into percentiles. Test results, for example the Stanford-Binet, are reported in percentiles. (Your local newspaper may have articles on how well the schools in the district scored on standardized tests.) If so, refer to these articles and point out that the students were sources of data for these reports.

PERCENTILES

Mean, median and mode are used to describe a collection of data. Students are probably familiar with the mean. Another name for mean is average. Ask students if they know how to find the average of three numbers. We often hear statements like the average temperature for Oklahoma City is 60°F. This does not mean that if you go there the temperature will be 60°F. Refer to an article that talks about an average.

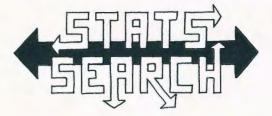


By taking a sample it is often possible to make an accurate prediction about a larger group. In polls a small group of people are questioned. Their answers are sometimes used to make a prediction about the entire U.S.

We see examples of probability when we read a weather report that says the chance of rain is 30%. Probability is used to describe if it is likely something will happen. Refer to an example. Point out that the "expected" often doesn't happen.



SAMPLING AND PROBABILITY



For example, CHART could be written as TRAHC.

pie chart	percentile	variance	simulate
interval	bias	decimal	scatter diagram
mean	range	survey	tally
deviation	questions	sample	pictograph
digit	bar graph	scale	histogram
horizontal poll statistics reliable predict table stem and leaf zero break distribution random frequency data mode truncate trend probability population outcome representative median	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	I D E R P E Q U E S T I B D S A A I O L C N L N T N E G L E U E R L Y F U E R L Y F B L P I C T G A A T L L G A A T L S A D E D E T A D E D E T A D E D E T A D E D E T A D E D E T A D E D N E C T I A N	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$



A sheet of paper (or a notecard) divided into 9, 12, 16 . . . parts can be used to collect data for use in several introductory statistics activities. The data is to be written down on cards and may be referred to by students. It is probably best not to include any questions that might be embarrassing to students such as weight, waist size, income of parents, etc. These types of items can best be handled in situations where the raw data is not identified with a particular student. A sample data sheet is shown below with a list of non-embarrassing questions that might be asked.

1 BAY CITY ROLLERS	 OLIVIA NEWTON - JOHN 	3 PINK PANTHER	
BROWN	5 BLUE	© Cat	
MATH	8 WELCOME BACK, KOTTER	9 PIZZA	

) Who is your favorite musical group?

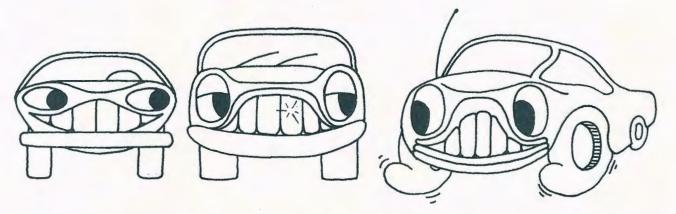
) Who is your favorite individual singer?

- 3) Who is your favorite cartoon character?
- 4) What is the color of your eyes?
-) What is your favorite color?
-) What kind of pet do you have?
- 7) What is your favorite school subject?
-) What is your favorite TV program?
-) What is your favorite food to eat?

Questions with numerical answers (How many sisters and brothers do you have?) or questions with a yes/no answer (Are you left-handed?) could be substituted for any of these. The data sheet could be used to provide data for several activities in the GRAPHS section. See Sisters and Brothers and My Favorite Color.

This idea could be used as an introduction to methods for collecting data: experiments, interviews, questionnaires, source documents, etc. You could also talk about firsthand data gathered by the individual or secondhand data collected from another person. Secondhand data can sometimes be obtained from written records like health records, birth certificates, driver's licenses, etc.





- I. Read or paraphrase this situation for your students: Dunn School is located on a busy street. One class decided to gather data on the traffic. They wanted to convince the city council that a traffic signal was needed. A student said, "One of us could count the cars going by in one day." Other students in the class made some suggestions. Let's see which suggestions you think are good ones and why.
- II. Put these sentences on a transparency and discuss them one at a time. Students need to see that statistical data can be used to help make decisions. This activity brings out that deciding what data to collect is difficult and important. Students may suggest additional ideas to consider.
- 1. MORE THAN ONE PERSON SHOULD COUNT.
- 2. THE SPEEDS COULD BE CHECKED.
- 3. WE SHOULD COUNT ON MORE THAN ONE DAY.
- 4. WE SHOULD COUNT DURING SCHOOL HOURS.
- 5. WE COULD COUNT ON SATURDAY AND SUNDAY.
- 6. LET'S COUNT PEOPLE CROSSING THE STREET.
- 7. "LET'S COUNT PEOPLE IN THE CARS.
- 3. WE DON'T NEED TO COUNT SMALL CARS LIKE VOLKSWAGENS AND HONDAS.
- 9. WE SHOULD START COUNTING AT 3:00.
- 10. WE SHOULD COUNT TRAFFIC ONLY IN ONE DIRECTION.
- 11. WE COULD COUNT FOR A MONTH.
- 12, WE SHOULD COUNT ON SUNNY DAYS AND RAINY DAYS.
- 13. WE SHOULD COUNT TRUCKS AND BIKES SEPARATELY.
- 14. WE SHOULD HAVE A TRIAL RUN.
- 15. A FORM SHOULD BE MADE FOR RECORDING.

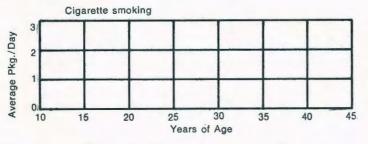
IDEA FROM: V. T. Thayer, *Mathematics in General Education:* A Report of the Committee on the Function of Mathematics in General Education for the Commission on Secondary School Curriculum, © 1940, p. 407. Reprinted by permission of Prentice-Hall, Inc., Englewood Cliffs, New Jersey.



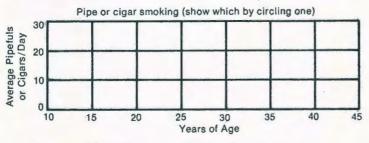
Some items on questionnaires or interviews ask for facts. Some ask for opinions. The questionnaire below is designed to obtain facts about adult smoking habits.

Smoking Experience Name		
City	County	State

Please draw a line from the age you started smoking to your present age or to the age you stopped smoking. Vary the height of your line according to how many packs of cigarettes you smoked per day.



At what age did you begin smoking cigarettes? _____ How many packages do you now smoke per day? _____ How long a cigarette butt do you usually leave? _____ $\frac{3}{4}$ of cig. ____ $\frac{1}{2}$ of cig. ____ $\frac{1}{3}$ of cig. ____ $\frac{1}{4}$ of cig. At what age did you quit smoking cigarettes (if you have)? _____



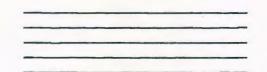
At what age did you begin smoking a pipe or cigar? _____ How many cigars or pipefuls do you now smoke per day?

How much do you now inhale?

____ Not at all ____ Slightly ____ Moderately ____ Deeply At what age did you quit smoking a pipe or cigar (if you have)? _____

If you have smoked only once in a while, please check here.

- 1. Study the questionnaire.
- 2. What are some advantages and disadvantages of not asking people to put their names on questionnaires?



3. Put an X to show

- a 20-year old who smokes one pack of cigarettes per day.
- b. a 42-year old who smokes
 6 cigars per day.
- 4. Smoker A checked "moderately" as the answer to "How much do you now inhale?". Smoker B checked "slightly". Is it possible Smoker A inhales less than Smoker B? Explain.
- Mister C has never smoked. Write a question to ask for this data.

Charts from *Well-Being*, Intermediate Science Curriculum Study, pp. 115-118. Permission to use granted by Silver Burdett Company and Florida State University



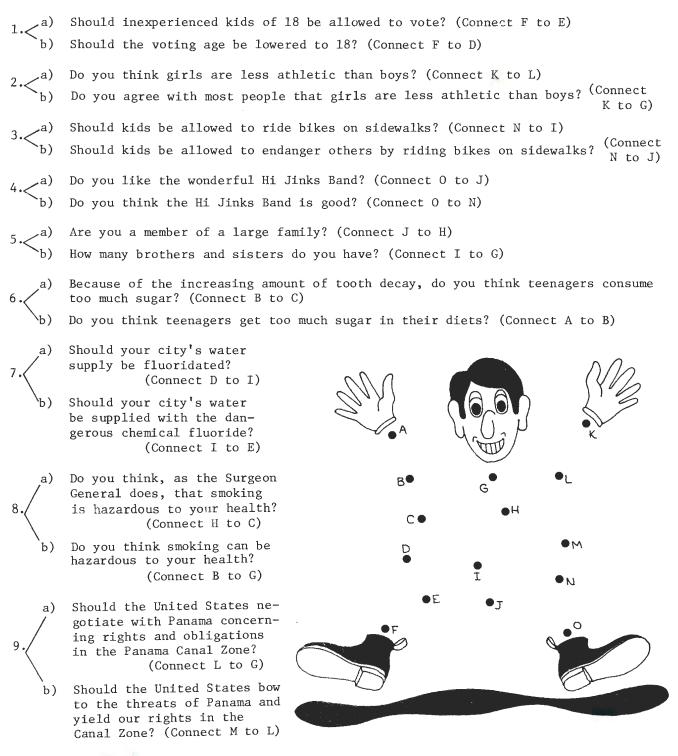
This portion of the questionnaire is designed to obtain opinions about adult smoking habits.

	Strongly agree	Mildly agree	Neither agree nor disagree	Mildly disagree	Strongly disagree	6.	On this po answered " for each s the answer	strongly tatement	disagr . Were	
A. Smoking costs more than the pleasure is worth.										
B. When I have children, I hope that they never smoke.				[]		7.	Show how y statements that best feeling ab five state	. Check describe out each	the bo s your	x
C. There is nothing wrong with smoking.		[***]								
D. Smoking is a dirty habit.					[]]	8.	A scale us 1 - 5 coul of the five statement.	d be used	d in pl	ace
E. There is nothing wrong with smoking as							TRONGLY GREE		STRO	NGLY SREE
long as a person doesn't smoke too						1	2	3	4	5
much.							Use the sca answer Stat			

Charts from *Well-Being*, Intermediate Science Curriculum Study, pp. 115-118. Permission to use granted by Silver Burdett Company and Florida State University



Good questions for questionnaires do not show biases or try to influence the person answering. Choose the better question in each pair below.

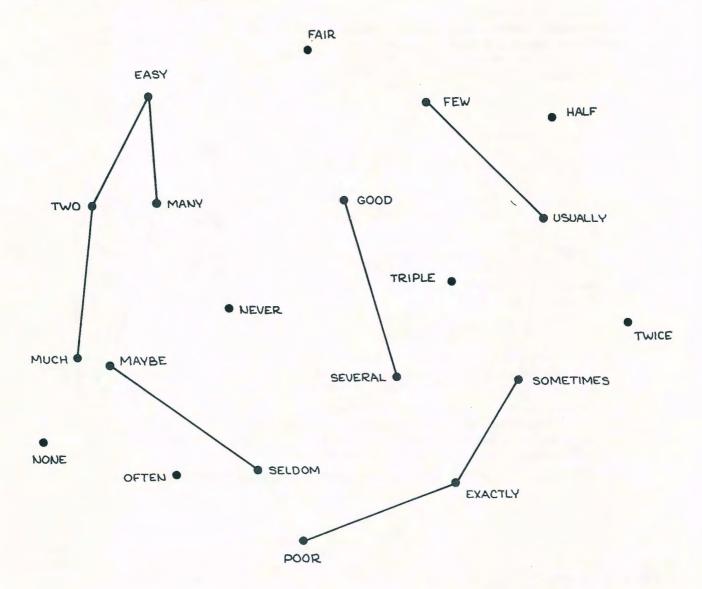




Are the answers to these questions fact (F) or opinion (0)? For each exercise, put the appropriate answer in the blanks below. What is your name? 1. Is the Democratic Party's approach to 2. urban problems better than the approach to If F, put t in blank 1. of the Republican Party? What is the best subject in school? < If F, put b in blank 8.If 0, put e in blank 11. 3. How much allowance should a l2-year old get each week? If F, put o in blank 4. 6. 7. Should the legal age for purchasing 8. liquor be 18 years? What is your present telephone number? < If F, put e in blank 4. If 0, put d in blank 15. 9. 10. Who is the best musical group? < If F, put o in blank 7.If 0, put l in blank 1. 11. 13. Should there be a traffic light at the intersection of 5th Avenue and Main Street? WHAT DID SNOW WHITE DISCOVER ABOUT PRINCE CHARMIN?

HE <u>1</u> <u>2</u> <u>3</u> <u>4</u> <u>5</u> TO <u>8</u> <u>E</u> <u>5</u> <u>0</u> <u>0</u> <u>E</u> <u>2</u> <u>E</u> <u>0</u> <u>11</u> <u>12</u> <u>13</u> <u>14</u> <u>15</u>

Vague words do not have definite meanings. Avoid using vague words whenever possible. Circle the words below that are vague. Connect the dots next to the vague words. Do it in alphabetical order. What geometric solid do you see?





Gallup, Roper, Neilson, Associated Press, United Press International -- all these organizations gather information by taking surveys. These surveys solicit facts and opinions on a wide range of topics.

An interview asking questions of people face-to-face is one type of survey. There are advantages and disadvantages to conducting an interview. Ask students to state reasons, both for or against conducting an interview. The following advantages and disadvantages can be brought out in a class discussion.

Advantages of an interview:

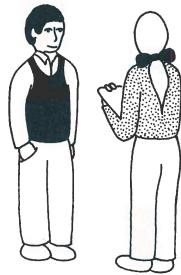
- Sometimes people will talk more than they will write.
- Movements, looks, or tone of voice may give other clues about what people really think.
- 3) In an interview, better data can be gathered by asking the respondent to make a point clear or to give more information.

Disadvantages of an interview:

- An interviewer usually talks to only one person at a time, so many interviews are necessary to obtain much information.
- If the results of several interviews are to be compared, the interviews must be as much alike as possible. Therefore, an interviewer should be highly trained and experienced.
- 3) The interview method can be very costly and time consuming.



Interviews are sometimes done over the telephone. This method may not give good information because people resent their telephones being used for this purpose and are reluctant to answer anything more than simple questions. Advantages of the telephone survey are that it is fast and costs less than the face-to-face interview.





A second type of survey used to obtain information is the written questionnaire. These are usually mailed to people. The people are asked to respond to the questions and then to mail back the answers. Like the interview method, the use of a questionnaire has both advantages and disadvantages. A class discussion can bring out these points.

Advantages of a questionnaire:

- 1) A questionnaire can reach more people than an interviewer can visit in person.
- 2) Using a questionnaire can save time.
- 3) Using a questionnaire is usually less expensive than interviewing people.

Disadvantages of a questionnaire:

- The responses people write will not be as complete as those given to an interviewer.
- Many mailed questionnaires are not returned so the information gathered is not as complete.
- There is no way to immediately check on the responses that are given.

The following questions could be posed for further student discussion:

- Which of the advantages (disadvantages) is most important?
- 2) When conducting an interview, would it be better to read the prepared questions or to memorize the questions?



- 3) Some people like to know that what they say is important enough to write down. Others get upset when they see an interviewer taking notes. Which is better: remembering the answers or writing them down? Do you think a tape recorder would help?
- 4) What percent of the written questionnaires do you think should be returned in order for the survey to be successful?
- 5) Which method, the interview or the written questionnaire, do you think would provide the most honest and frank answers?
- 6) Is the time of day an interview is conducted important?



Almanacs are good sources of statistical information, but many students have had little or no experience in using them. This activity provides an opportunity for students to become more familiar with these sources. Can You Guesstimate and Information Please also use almanacs.

- I. Gather as many almanacs or other statistical source books as you can. Use each almanac to make a list of four or five statements like those suggested below. Some statements are true. Others are false. Divide the class into small groups with one almanac and one set of statements based on that almanac.
- II. Tell students that many statements are made by people in everyday conversation, on T.V. and in newspapers. Often it is hard to know if a statement is true or not. One way to check is by using an almanac. Ask students to find and write down information from the almanac to show that each statement is either true or false. The almanacs and statement lists could be rotated so each group has a chance to use each different type of almanac.

Suggested Statements:

(Some information in almanacs changes each year so the statements below might not be accurate for the year of almanac you have available.)

Information Please Almanac

- a) The Baseball Writers Association chose Yogi Berra more often than
- any other individual as the most valuable player of the year.
- b) It is less than 2,000 miles from Chicago to San Francisco by airplane.
- c) More people were killed in the 1906 San Francisco earthquake than in any other earthquake in world history.

Guinness Book of World Records

- a) Pepsi-Cola is the world's top selling soft drink.
- b) The largest pyramid in the world is in Mexico.
- c) The largest eggs in the world are from whale-sharks.

The World Almanac

- THE WORD
- a) Fewer inches of rain and snow fall in Portland, Oregon than in Jacksonville, Florida in an average year.
 - b) Georgia has more nuclear power reactors than any other state.
 - c) The per capita income (average income per person) in North Dakota is less than the per capita income in New Jersey.

The U.S. Fact Book (The facts below are harder to find. A section from the table of contents is suggested for each.)

- a) In 1970, there were more American Indians in Arizona than in any other state. (See Population: race)
- b) There were almost 8,000 fewer prisoners in California jails in 1971 than in 1970. (See Law Enforcement: prisoners)
- c) The lumber industry produces more cedar for consumption than any other softwood. (See Forests and Forest Products: lumber production)



- I In the table write an estimate for each of the records.
- II Use the <u>Guinness Book of World</u> Records to check your estimates.



Estimate

Actual

THE WORLD'S

LARGEST PLAYABLE GUITAR IS

Record

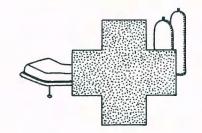
1)	Longest time for continuous balancing on one foot	hr.
2)	The length of the longest recorded fingernail	in.
3)	The greatest number of years lived by a cat	yr.
4)	The cost of the most expensive car	\$
5)	Height of the tallest monument	ft.
6)	The greatest speed attained by a dog	mph
7)	Most expensive pair of shoes	\$
8)	Diameter of the largest ball of string	ft.
9)	Number of words in the world's longest poem	
10)	Wing span of the largest butterfly	in.
11)	The greatest amount of snowfall in 24 hours	in.
12)	Value in dollars of the most valuable stamp	\$
13)	The fastest speed attained by a race car	mph
14)	The height of the tallest woman	ft. in.
15)	The length of the longest whale	ft.

INFORMATION PLEASE	
Use an Information Please Almanac to answer these questions. What year is your almanac?	
I. From the section on Sports	
 What is the highest attendance record for a professional double-header baseball game? 	
II. From the section on Aviation	
<pre>1. Who was the first licensed woman pilot?</pre>	
2. Who made the first round the world non-stop flight?	
3. What is the speed record for airplanes?	
<pre>4. What is the altitude record for helicopters? by</pre>	
5. What is the maximum speed of a commercial U.S. jet? by(type of plane).	



III. From the section on Disasters

- How many casualties resulted from the 1939 flood in China?
- 2. How many people were left homeless by the 1972 storm Agnes?
- 3. When did the Andrea Doria shipwreck take place?



- 4. How many people survived the 1972 Eastern Airlines crash?
- 5. What railroad train accident killed the greatest number of people?

IV.	From the section on Economy, American
	1. How many of these types of cars were made last year?
	a. Ford c. Plymouth
	b. Chevrolet d. AMC
x	2. How many American homes have these electrical appliances?
	a. colored TV sets c. refrigerators
	b. air conditioners d. electric blankets
	3. How many people were employed in these occupations?
	a. Telephone operators d. Physician
	b. Librariane. Meat cutter
	c. Carpenters f. Nurses, Registered
	4. What was the farm income received for these products?
	a. vegetables c. dairy products
	b. meat d. tobacco
	5. Which individual labor union had the largest number of members
	b. meat d. tobacco



Suppose there are 223,965,836 citizens of the United States alive. This is a complicated number with several digits. It is hard to remember. So we usually say

> The population of the U.S. is about 224,000,000 or 224 million.

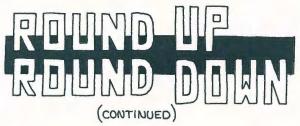
This is an example of rounding.

Also the population of the U.S. changes every minute. So we are usually only interested in a close approximation.



I. Fill in the blanks. Round 223,965,836 to the nearest

ten	
hundred	
thousand	
ten thousand	
hundred thousand	
million	
ten million	
hundred million	



II. When to Round

There are times when numbers should not be rounded. If your grocery bill is \$22.68, it is not wise to pay \$23 for the groceries. If the bill is \$22.24, the clerk will not accept \$22.

Is it reasonable to round numbers in these situations?

	yes	no
1. Report of the number of people in your state		
2. The number of people invited to supper		
3. A runner's time in a track meet		
4. The height of a mountain		
5. The altitude of an airplane		
6. A builder's measurement in constructing a house		

III. Where to Round

You must decide where to round numbers. You could say there are 732 or 730 or 700 students in your school. It depends on how accurate you want to be.

Decide on a good way to round these numbers.

- 1. There are 168,218,763 acres of federally owned land in Texas.
- 2. Indiana exported \$1,349,233,561 of goods in 1972.
- 3. In 1974 there were 34,253 million barrels of petroleum reserves.
- 4. Between 1960 and 1970, San Diego had a population increase of 169,259.
- 5. It cost \$652,747,359 to build one N.Y. World Trade Center Building.
- 6. People have bought 301,238,462 copies of Elvis Presley's records.

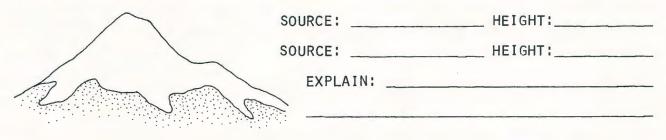


DIFFERENT SOURCES OFTEN GIVE DIFFERENT DATA, LOOK UP EACH OF THESE TOPICS IN TWO DIFFERENT SOURCES. EXPLAIN WHY THE DATA IS DIFFERENT OR THE SAME.

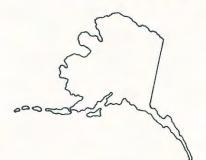
1. ESTIMATED WORLD POPULATION

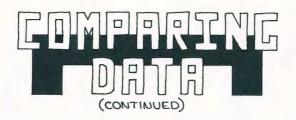
0	SOURCE:	POPULATION:
EST2	SOURCE:	POPULATION:
JAN Z	EXPLAIN:	

2. HEIGHT OF MT. MCKINLEY



3. NUMBER OF PEOPLE KILLED IN 1964 ALASKA EARTHQUAKE.

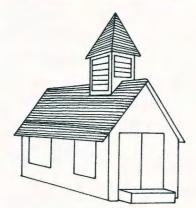




4. ESTIMATED NUMBER OF CHRISTIAN PEOPLE IN THE WORLD

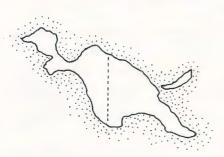
	SOURCE:	NUMBER:
$\langle \uparrow \rangle$	SOURCE:	NUMBER:
(H))	EXPLAIN:	
31,117		
<u></u>		

5. NUMBER OF TEACHERS IN PUBLIC SCHOOLS



SOURCE:	NUMBER:	
SOURCE:	NUMBER:	
EXPLAIN:		

6. AREA OF THE ISLANDS OF NEW GUINEA



SOURCE:	AREA:
SOURCE:	AREA:
EXPLAIN:	



GUM

1. The health aspects of gum chewing.

Your dentist

A gum commercial

 The most popular subject in your school.

A poll of the school band A poll of the whole school

3. The effect bottle deposits have on litter.

 Cola Bottling Co.
 54

State Highway Department

4. The health aspects of fluoridated water. A citizen's group against fluoridation

The city health department

5. Pros and cons of setting the legal drinking age at 18. A poll of 16 to 18 year olds A poll of parents

- 6. The unemployment in your city.A labor unionThe unemployment office
- 7. The amount of pollution caused by cars.

Environmental Protection Agency Auto Maker

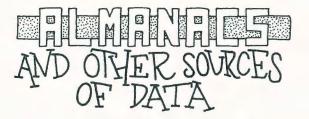
8. The effects of burning fields on air quality. A poll of people with asthma

An organization of farmers

9. The health aspects of cigarette smoking. The Surgeon General A tobacco company

10. The amount of accidents caused by drivers under 18. Accident reports compiled by

government agency Opinions expressed by teenagers



There are many reference books which are collections of data, tables and graphs. A selected list of these books is given below along with a summary of the information in each reference. Additional sources are listed in the bibliography.

The Associated Press. THE OFFICIAL ASSOCIATED PRESS SPORTS ALMANAC. \$2.25 paper.

This comprehensive almanac gives a brief description of many sports. Pictures and information about outstanding people in each sport are also included. Records, record holders and winners are given in easy-to-read tables. Besides the major sports like baseball and football, this book includes marbles, handball, bicycle racing, ice boating, cat shows, sled-dog racing, and many more.

Consumers Union of the United States. CONSUMER REPORTS, THE 1976 BUYING GUIDE ISSUE. \$1.95 paper.

Strengths and weaknesses of items on sale for U.S. families are described. There are numerous tables and descriptive information, including prices.

Dan Golen Paul Associates. INFORMATION PLEASE ALMANAC. \$2.95 paper.

Numerical data and descriptive text are combined in this almanac. There are written sections on how a president is elected, major events occurring on each day of the year, the defense system, the United Nations, world history, great disasters, astronomy, the space age, the Constitution, the presidents of the U.S., Nobel prize winners, celebrated persons, sports and so forth. Statistical tables accompany each of these topics.

McWhirter, Norris and McWhirter, Ross. GUINNESS BOOK OF WORLD RECORDS. \$1.75 paper.

This is an easy-to-read book giving records on what is the highest, lowest, biggest, smallest, fastest, slowest, oldest, newest, loudest, greatest, hottest, coldest, strongest in the world. Many pictures are included. Students generally find this book fascinating.

G. & C. Merriam Company. WEBSTER'S NEW COLLEGIATE DICTIONARY. Springfield, Massachusetts. \$10.95 hardback.

The biographical section lists several thousand people, their birthdates and facts about their lives. The geographical section lists several thousand places with statistics such as area, length, height, population, etc.

Newspaper Enterprise Association. THE WORLD ALMANAC AND BOOK OF FACTS. \$2.75 paper.

Along with a large section of statistics about the United States, this comprehensive almanac has information on mountains, bridges, cities, disasters, ... of the world. Sections are included on sports, memorable dates, laws and documents, the national park system, the states and cities of the U.S. and the nations of the world.

ALMANACS AND OTHER SOURCES OF DATA (page 2)

Reader's Digest Association, Inc. READER'S DIGEST 1976 ALMANAC AND YEARBOOK.\$4.95 hardback.

This is a comprehensive, illustrated source book that summarizes the major events of the previous year. Events from earlier years are also referenced. Numerical data and descriptive text are presented for many topics such as accidents and disasters, books, climate and weather, crime, history, language, medicine and health, religion and sports.

Rosenbloom, Joseph. CONSUMER COMPLAINT GUIDE 1977. Collier Macmillan Publishers. \$4.95 paper.

This book provides information for buyers on schemes that cheat consumers and describes laws that are intended to protect consumers. Extensive lists of agencies and organizations are included for those who wish to register a complaint or write for more information.

Showers, Victor. THE WORLD IN FIGURES. \$14.95 hardback.

Numerous tables of geographic and population data from around the world are included. There is also information on railroads, canals, dams, etc.

United Nations. WORLD STATISTICS IN BRIEF. New York, New York. \$3.95 paper.

This book is a compact, easy-to-read collection of important statistics for various countries. The first part of the book contains data on the population, agriculture, mining, manufacturing, consumption, communications, trade, transportation, education and culture for the world as a whole, selected regions of the world and major countries. The second part presents statistics on many of these categories for each of the 139 countries.

U.S. Bureau of the Census. HISTORICAL STATISTICS OF THE UNITED STATES, COLONIAL TIMES TO 1970, BICENTENNIAL EDITION. Washington, D.C. \$26.00 hardback.

This comprehensive two volume set of statistics and references is an update of two previous editions. Among the thirteen topics listed in the first volume are population, labor, agriculture and minerals. Energy, business enterprise and government are three of thirteen topics in the second volume.

U.S. Bureau of the Census. POCKET DATA BOOK. (For sale by Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402 or any U.S. Department of Commerce Field Office.) \$2.80 paper.

This book is a small, easy-to-read collection of tables and graphs about the United States. The book contains information on topics like population, government, prices, law enforcement, education, communication, and national defense. This book is less overwhelming and easier to use than the larger, more comprehensive almanacs.

ALMANACS AND OTHER SOURCES OF DATA (page 3)

U.S. Bureau of the Census. STATISTICAL ABSTRACT OF THE UNITED STATES. Washington, D.C. (Also published as THE U.S. FACT BOOK by Grosset and Dunlop, \$3.95 paper.)

This annual publication is the standard summary of statistics on the social, political, and economic organization of the United States. It is used as a reference and guide for many other statistical publications. The abstract is very comprehensive giving data on such specific topics as the school lunch program, characteristics of blind persons, credit card use, magazine advertising, fuel consumption, prices received by fishermen, automobile ownership, and volume of mail by classes. Many important primary sources of statistical information are referenced by subject area in an appendix.

Wallenchinsky, David and Wallace, Irving. THE PEOPLE'S ALMANAC. \$8.00 paper.

This huge collection of articles, facts and trivia claims to be the first reference book ever prepared to be read for pleasure. It contains no tables or graphs, but teachers and students alike might find it a fascinating source of facts and information. Here are a few facts you can learn from this book: Harvey Kennedy invented the shoelace and made \$2.5 million on his patent. Adolf Hitler owned 8,960 acres of land in Colorado. Mount Everest is 29,000 feet high. Surveyors worried that the public might consider this an estimate to the nearest 1000' so they falsely reported the height as 29,002 feet.

ORGANIZATION

Placement Guide for Tabbed Dividers

Tab Label	Follows page:
CONTENT FOR TEACHERS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
Probability with Counting	
Inferential Statistics	
Appendix-Computer Programs	
DIDACTICS	
Components of Instruction	
Classroom Management	
Statistics and Probability Learning	
TEACHING EMPHASES	
Critical Thinking	
Decision Making	
Problem Solving	
Models and Simulations	
Calculators and Computers	
Laboratory Approaches	
CLASSROOM MATERIALS	
CONCEPTS AND SKILLS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Misleading Statistics	
Mean, Median, Mode	
Range & Deviation	
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Experimental Probability	604
Probability with Models	634
Counting Techniques	
APPLICATIONS	
Business & Commerce	
Environment	
Health & Medicine	
People & Culture	
Recreation	
GLOSSARY	
ANNOTATED BIBLIOGRAPHY	
SELECTED ANSWERS	



Commentary to TABLES (420-423)

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TITLE	PAGE	TOPIC	TYPE
Arrests by Type of Offense	424	Reading a table	Transparency Discussion
Weather and Water Conditions	425	Reading and interpreting a table	Worksheet
Studying Some Sports	426	Reading a table	Worksheet
What's in a Name?	427	Making a frequency table	Activity card Worksheet
The Lexicochronographer	428	Using frequency tables to analyze written materials	Worksheet
How Much Are You Worth?	429	Making a frequency table	Teacher directed activity
Let's Pool the Data	430	Making a table	Worksheet
Branching Out	431	Introducing stem-and-leaf displays	Teacher directed activity
An Age Old Problem	432	Making a stem-and-leaf display	Worksheet
Cars for Sale	433	Making a stem-and-leaf display	Worksheet

TABLES

Tables are used in magazines, newspapers, advertisements, almanacs, cookbooks and other written material to organize data so it is easier to understand. Students must be able to <u>read the tables</u> which accompany newspaper and magazine articles if they are to fully understand the article. To make sense out of data that students collect, they need to know how to <u>organize the data into tables</u>.

READING TABLES

To help your students learn to read tables, you could enlarge an almanac table like that shown to the right for a transparency and ask questions like these:

- What does the circled number mean? (Students can use the row and column headings to determine that in 1971, 24.1% of the people arrested were under 18.)
- How many people were arrested for robbery in 1970?
 (You might have to remind them that the numbers are given to the nearest thousand so the answer is 74,000.)
- Find an offense showing fewer arrests in 1970 than in 1960.

What is the source of this data?

Based on reports representin	g 90 mil	lion of to	tal 1971 p	opulation.	x	Not appli	icable.	
	At	rrests (1,0	000)		rcent un 8 years o		Perc	
Offenses charged	1960	1970	1971	1960	1970	1971	1960	1971
Total	3, 307	4, 644	4, 439	14.4	23.8	(24.1)	11	15
Serious offenses	463	920	913	41.9	45.0	43.9	11	17
Criminal homicide	7	12	13	7.6	8.3	11.0	14	15
Forcible rape	7	11	11	17.4	18.2	21.1	X	X
Robbery	33	74	80	24.4	35.1	33.0	6	6
Aggravated assault	55	93	94	11.7	17.2	17.8	15	14
Burglary, breaking or								
entering.	115	200	200	46.3	51.0	49.1	3	5
Larceny, theft	193	432	424	47.7	50.2	49.3	17	28
Auto theft	54	97	93	60.5	54.6	50.8	4	6
Other offenses	2.845	3, 723	3, 527	9.9	18.6	19.3	11	14
Drunkenness	1,213	1,097	978	1.0	2.6	2.9	8	7
Disorderly conduct	394	437	426	11.8	19.9	20.3	13	15
Assault, other than above.	118	209	196	10.7	17.7	18.9	10	14
Driving while intoxicated	136	281	292	. 8	1.1	1.1	6	7
Liquor laws	79	137	125	20.5	32, 1	33.0	15	14
Vagrancy	133	82	58	5.7	11.0	11.3	8	26
	31	266	272	5.2	20.7	19.9	16	16
Narcotic drug laws			1,180	24.9	35. 3	34.7	16	21

(The skill of skimming a table quickly is one we might take for granted, but it might be hard for many students to pick out "vagrancy" or "drunkenness.")

• Are there any trends in the percent of arrests for those under 18 or for females? (It looks like females and persons under 18 are becoming a larger percent of those arrested. Of course, data for only two or three different years is not conclusive "proof." More recent data might give more or less support to that conclusion.)

(The table was printed in <u>Pocket Data Book USA 1973</u> but the source of the data is the U.S. Federal Bureau of Investigation. The data from two different sources might not agree because of the method of collecting it. One should also check the source to see if the data might be biased. More classroom ideas on biases are given in the MISLEADING STATISTICS section.)

The activities Weather and Water Conditions and Studying Some Sports can be used to help students learn to read tables. These activities explain that some numbers in tables can be read as they are, others must be related to a certain unit like thousands of people or millions of dollars. You could provide other tables suitable for fifth or sixth graders who have not had percents. There are more table reading activities in APPLICATIONS. To find these activities, check each section's Table of Contents. For more background in reading tables, see Tables in the CONTENT FOR TEACHERS section.

MAKING TABLES

The tables students make will generally be simpler than those found in almanacs or magazines. This does not mean it is easier to make tables than to read them. To make tables, students will need skills in tallying, in determining interval size,

Making a tally should not be difficult for most students. They can use the conventional tally method that groups by fives, or they could learn the tally method shown to the right that groups by tens. An example of a clever method for tallying--the stem-and-leaf display-- is shown to the lower right. This method is described in more detail on pages 6-7 in Tables of the CONTENT FOR TEACHERS section and in the activities Branching Out, An Age Old Problem, and Cars for Sale.

Most of the activities in this section give the row or column headings and intervals for the tables; however, there will be times when students will need to decide on table headings and the size of intervals. You could adapt It's An

1	2	3	4	5	6
1		111	1111	444	4441
	L .2	3 .	. 4.	5	6
	7 8	. 9			12
			\square		\square
	G1, 59 42, 56	, 47, 5 , 64	3, 65,	72, 59,)
	T	STEM	LEAF		
		4	7,2		
		5	9,3,	9,6	
		6	1,5,	4	
		٦	2		

and in labeling and organizing data.

TABLES

Emergency, Activity I in the GRAPHS section to give students some practice in choosing categories. Let's Pool the Data provides practice in making a table for given categories and age groupings. In Cars for Sale, students choose interval sizes for a stem-and-leaf display.

When organizing a table, students might have difficulties just keeping the information in straight rows or columns. You can encourage students to use the lines on their paper for straight rows and to draw vertical lines to keep the columns straight. Another way for students to "keep things straight" is to have them place a sheet of grid paper under the paper on which the table is to be made. The horizontal and vertical lines of the

I MADE A ALLOWANCE TABLE, BUT MY BROTHERSE BOB \$ 2.50 FRIEND SAYS SIGTERS SHE CAN'T READ JUAN 3 3.75 IT ! 2 SAM 2.00 000 3 GWEN 1 1.50 MATT 0 PENNY 3.00 4 2.25 2 ELSIE 1.00 0 RALPH CHIN 2.00 2 MARY 1.50

grid can be used as guidelines. Neatness counts! Students will probably need reminders to label the rows, columns and title of tables. A table is meaningless unless it has labels that describe the data. You might have students try to read each other's tables to be certain they are clearly labeled and organized.

A FINAL NOTE

You can help students see how very useful tables are by collecting and posting tables from magazines or newspapers. The following excerpt about tables is from pages 17-18 of Peter H. Selby's book <u>Interpreting Graphs and Tables</u> published by John Wiley. You might like to use some of his ideas to explain some of the advantages and disadvantages of tables to your students.

A good table is the product of careful thinking and hard work. It is not just a package of figures put into neat compartments and ruled to make it look more attractive. It contains carefully selected data put together with thought and ingenuity to serve a specific purpose.

Tables are, as mentioned before, the backbone of most statistical reports. They provide the basic substance and foundation on which conclusions can be based. They are considered valuable for the following reasons:

• Clarity - they present many items of data in an orderly and organized way.

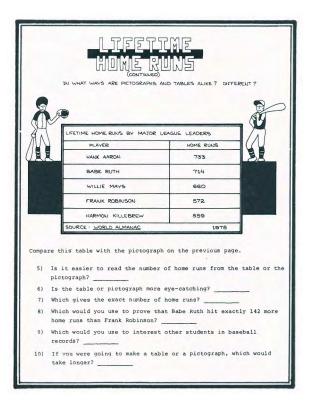
Interpreting Graphs and Tables. Peter H. Selby. John Wiley and Sons, Inc., 1976. Reprinted by permission of John Wiley and Sons, Inc.

- Comprehension they make it possible to compare many figures auickly.
- Explicitness they provide actual numbers which document data presented in accompanying text and charts.
- Economy they save space, and words.
- Convenience they offer easy and rapid access to desired items of information.

There are, of course, disadvantages of tables. Some of these include the following:

- Uninviting tables often look like difficult reading matter so many people ignore them.
- Undramatic significant relationships often are hard to find and comprehend.
- Too specific tabular data, because of their formal appearance, tend to make crude estimates and projections seem more precise than they actually are.

When graphs are studied, you can compare the advantages of tables to those of graphs. The first page of *Lifetime Home Runs* (in the GRAPHS section) shows a pictograph of home runs for some of the famous players in the Major League. The second page has the same information in a table. The questions below the table ask students to compare the two ways of presenting data. In some cases, the table is more useful than the graph. In others, the graph is better.



Interpreting Graphs and Tables. Peter H. Selby. John Wiley and Sons, Inc., 1976. Reprinted by permission of John Wiley and Sons, Inc.



BASED ON REPORTS REPRESENTING 90 MILLION OF TOTAL 1971 POPULATION

X NOT APPLICABLE

	1000	0 = 0 (1	000)		CENT UNI		PERCE	
	ARRE 1960	sтs (1, 1970	1971	1960	YEARS (1970	1971	FEMAI 1960	-E 1971
OFFENSES CHARGED	1900	1970	13/1	1900	1970	19/1	1300	19/1
TOTAL	3,307	4,644	4,439	14.4	23.8	24.1	11	15
SERIOUS OFFENSES	463	920	913	41.9	45.0	43.9	11	17
CRIMINAL HOMICIDE	7	12	13	7.6	8.3	11.0	14	15
FORCIBLE RAPE	7	11	11	17.4	18.2	21.1	X	Х
ROBBERY	33	74	80	24.4	35.1	33.0	6	6
AGGRAVATED ASSAULT	55	93	94	11.7	17.2	17.8	15	14
BURGLARY, BREAKING								
OR ENTERING	115	200	200	46.3	51,0	49.1	3	5
LARCENY, THEFT	193	432	424	47.7	50.2	49.3	17	28
AUTO THEFT	54	97	93	60.5	54.6	50.8	4	6
OTHER OFFENSES	2,845	3,723	3,527	9,9	18.6	19.3	11	14
DRUNKENNESS	1,213	1,097	978	1.0	2.6	2.9	8	7
DISORDERLY CONDUCT,.	394	437	426	11.8	19.9	20.3	13	15
ASSAULT, OTHER								
THAN ABOVE	118	209	196	10.7	17.7	18.9	10	14
DRIVING WHILE								
INTOXICATED	136	281	292	.8	1.1	1.1	6	7
LIQOUR LAWS	79	137	125	20.5	32.1	33.0	15	14
VAGRANCY	133	82	58	5.7	11.0	11.3	8	26
NARCOTIC DRUG LAWS.,	31	266	272	5.2	20.7	19.9	16	16
ALL OTHER (EXCEPT								
TRAFFIC)	740	1,214	1,180	24.9	35.3	34.7	16	21

SOURCE: U.S. FEDERAL BUREAU OF INVESTIGATION

Table from Pocket Data Book USA 1973, p. 122



- a. How many fatalities (deaths) occurred in non-tidal waters?
 - b. How many injuries occurred in very rough water conditions? _____
 - c. How many vessels were involved in accidents that occurred in strong winds?
- These numbers are on the chart. What does each number show?
 a. (70)
 - b. [129]
 - c. 1754

3) The largest number of vessels involved in accidents occurred when a. The water conditions were

- b. The weather was .
- c. The wind was
- d. The visibility was
- 4) These statistics seem to imply that the worst time to go boating is when the water is calm, the weather is clear, the wind is light, and the visibility is good. What do you think of this

conclusion?

	WEATHER AND WA	TER COM	DITION	S	
Contraction of the	1973	TOTAL VESSELS IN VOLVED IN ACCIDENTS	FATALITIES	INJURIES	
	TOTALS	6738	1754	1599	
WATERS	Oceans Gr Gulf of Mexico Great Lakes Tidal waters Non-tidal waters	408 306 1764 4260	142 73 249 1290	71 83 330 1115	
WATER	Calm Choppy Rough Very rough Strong current Unknown	3778 1375 625 319 272 369	632 248 223 141 126 384	1061 315 114 39 42 28	
WEATHER	Clear Cloudy Fog Rain Snow Hazy Unknown	5183 668 92 237 19 146 393	961 236 38 70 8 21 420	1333 140 25 38 3 23 37	
GNIM	None Light Moderate Strong Storm Unknown	1511 2692 1342 579 190 424	365 477 253 181 46 432	375 810 277 78 23 36	111
VISIBILITY	Good Fair Poor Unknown	5201 649 468 420	990 207 132 425	1290 146 129 34	





Use the table to answer these questions.

- 1) What four sports are shown?
- 2) How many different years are shown?
- 3) Look at the Unit column. What three things are shown?
 - a) Find 674 on the chart. There were 674 college football teams in 1950. Find 617. What does it mean?
 - b) Find (17,659). The unit is 1000. 17,659,000 people attended major league baseball in 1950. Find (30,467). What does it mean?
 - c) Find 3,800*. The unit is \$1000. This means \$3,800,000 in receipts. Find 11,847*. What does it mean?

4) Which year shows the fewest number of professional boxers?

- 5) Did attendance always increase in baseball games? ______ basketball? ______ football? ______
- 6) Did more people attend college or professional football games?
- 7) Why are the World Series attendance figures for 1950 and 1970 so much smaller?



RECREATION

Selected Recreational Activities: 1950 to 1973

Activity	Unit	1950	1960	1965	1970	1971	1972	1973
Baseball, major leagues:								
Attendance	1,000	17,659	20,261	22,806	29,000	29,544	27,330	30,467
Regular season	1,000	17,463	19,911	22,442	28,747	29,193	26,967	30,109
National league	1,000	8,321	10,685	13,581	16,662	17,324	15,529	16,675
American league	1,000	9,142	9,227	8,861	12,085	11,869	11,438	13,434
World series	1,000	196	350	364	253	351	363	358
Basketball, professional, attendance:								
National Basketball Assoc.	1,000		1,986	2,750	5,147	6,195	6,634	6,834
American Basketball Assoc.:	_,							
Regular season	1,000				1,753	2,230	2,437	2,400
Playoffs	1,000				213	299	360	364
Football:								
College, National Collegiate								
Athletic Association:								
Teams	Number	674	620	616	617	618	620	630
Attendance	1,000	18,962	20,403	24,683	29,466	30,455	30,829	31,283
Professional, Nat'l Football League:	2,000							
Attendance	1,000	2,115	4,153	6,546	9,991	10,560	10,929	11,257
		1,978	4,054	6,416	9,533	10,076	10,446	10,731
Regular season	1,000	1,978	99	130	458	484	483	526
Championship games	1,000	13/	99	130	430	404	405	520
Boxing, professional matches:								
Boxers	Number	3,940	2,920	2,202	5,071	5,783	6,136	7,384
Receipts, gross	\$1,000	3,800	5,902	8,264	10,642	10,237	11,847	12,634

Data from United States Statistical Abstract, 1974



How long is the longest (shortest) first name in your class? The longest (shortest) last name? What (is the most common length for a last name? How long is the longest complete name? (Use legal names, not nicknames.)

MY NAME IS JANE CASS. WY NAME IS JOHNATHAN THOMPSON.

To answer these questions:

- Collect the names of the students in your class. (Ask each student, have them sign a sheet or get a list from your teacher.)
- Find the number of letters in each first name and last name. Record the information in a table like the one to the right.
- 3) Count the number of students having one letter, two letters, three letters in their first, last and both names and record in a table.

	FREG	QUENCY F	OR
NUMBER OF LETTERS	FIRST NAME	LAST NAME	BOTH
1			
2			
3	3		
4	6	2	
~~~~~	the	the	1-

- 4) To get a better idea of the name lengths in your class, make a table like the one to the right. What does this table tell you?
- 5) Use the information from the table in #4 to make a graph.

			-	T.						
(0	5	~~~	~	A			5		~~~	-
STUDENTS	4			2	Wha N a s	t is the tudent's	most	frequent (first a	length nd last	1 Of E)?
STUD	3									
OF	2			1000000	Z					
NUMBER	1									
NUN	0			2.0		<u>s</u>				
		NL	MBER	OF LET	10-12 TERS					

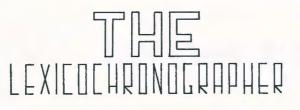
	LETTERS IN						
STUDENT	FIRST	LAST NAME	BOTH NAMES				
1) JIM CLARK	3	5	8				
2) PATTY WILSON	5	6	11				
3) GREG MUSSER	4	6	10				
m	1	h					

How long is the longest first name?

How long is the longest last name?

NUMBER OF LETTERS	FREQUENCY
1-3	
4-6	
7-9	
10-12	

of



 If a paragraph has a lot of long words, it will be harder to read. Count the number of letters in each word of the paragraph on the first page. Make a tally to record each word in the table to the right. In the frequency column, write the number of words with l, 2, 3, ... letters.

		CHAPPENS
NUMBER OF LETTERS	TALLY OF WORDS	FREQUENCY
1		
2		14
3	o the off the many o	20
4	matany and shift mat	25
5	ue enconnel:	
6	the state of the	12
7		
8		
9		
10		

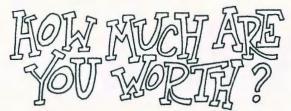
NUMBER O

2) Choose a page from a textbook, newspaper, encyclopedia, or magazine and repeat step 1 for the first one-hundred words.

- a) Which has more long words, your choice or the paragraph on the first page?
- b) Which seems harder to read?
- c) Does the word-length test seem to tell which is harder to read?
- d) What else might you test to tell how hard something is to read?

NUMBER OF LETTERS	TALLY OF WORDS	FREQUENCY
1		
Z		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

3) Choose another test to use on your selection and on the paragraph. (You could count the number of syllables in each word or the number of words in each sentence.) Organize the results in tables. Which test do you think is better?



Suppose letters were money as shown in the chart. Who has the most expensive name in class? Are more names worth over \$130 or under? Is there a most frequent value? (Use legal names, not nicknames.)

- 1) Ask your class the above questions.
- Display or have students make a copy of the table to the right.

G= \$7	M = \$ 13	5=\$19	
H = \$8	N = \$14	T = \$ 20	
I = \$9	0 = \$15	U = \$21	Y = \$ 25
J = \$10	P = \$16	V = \$22	Z = \$ 26
K = \$11	Q = \$17	W= \$ 23	
L = \$12	R = \$18	X = \$24	
	H = \$8 I = \$9 J = \$10 K = \$11	H = \$8 N = \$14 I = \$9 O = \$15 J = \$10 P = \$16 K = \$11 Q = \$17	I = \$9 $O = $15$ $U = $21$ $J = $10$ $P = $16$ $V = $22$ $K = $11$ $Q = $17$ $W = $23$

S

Α

N

W

0

N

G

\$ 19

\$21

\$ 19

\$ 14

\$23

\$ 15

\$ 14

133

1

3) Have students compute the value of their legal name. A handy organization is to write the letters of their name vertically. As they finish, have them sign their name on the chalkboard or on the overhead and list its value. Be sure to do your name.

- 4) Have students suggest ways to organize the results. Would any of these be more helpful than the list in (3)?
  - a) Arrange the names in alphabetical order. (Not more helpful for answering the questions at the top of the page.)
  - b) Group the boys and girls separately. (Not helpful unless the two groups are compared.)
  - c) List the names so the values are ordered from smallest to largest. (Yes.)
  - d) Group the results into intervals. (Yes, since few values will be the same, grouping lets you present the results more concisely.)
- 5) Have the students list the values from smallest to largest. Are there two students with names of the same value? More than two?
- 6) To display the results more concisely, have students choose a suitable interval size and make a frequency distribution of the data. Some students may need help. Where do most of the values lie?

For example, consider these values:

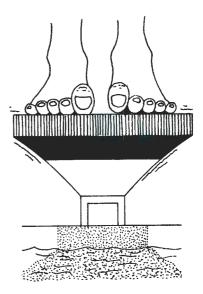
\$86, \$86, \$97, \$103, \$106, \$111, \$112, \$113, \$116, \$119, \$121, \$123, \$127, \$129, \$133, \$133, \$140, \$140, \$140, \$148, \$155, \$156, \$157, \$161, \$162, \$171, \$180. \$191, \$193.

Since they lie between \$80 and \$200, an interval of ten dollars would be convenient. Tables usually have 6-15 intervals. Remind students that the intervals must be the same size.

VALUE	TALLY	FREQUENCY
\$80 - \$89	38	2
\$ 90 - \$ 99	1	1
\$100 - \$109	11	2
\$110 - \$ 119	1111	5
\$120 - \$129	1111	4
\$130 - \$139	11	2
\$140-\$149	1115	4
\$150-\$159	111	3
\$160-\$169	11	2
\$170-\$179	1	1
\$180-\$189	1	1
\$190-\$199	11	2



I. Data has been collected on people's satisfaction with the swimming schedule for a city pool. The people surveyed range in age from 10 to 34. The data is to be arranged in intervals of five years and by these groupings: very satisfied, satisfied, neutral, dissatisfied, very dissatisfied. Draw a table for the data shown below. Your table should include columns and rows for age intervals and satisfaction groupings.





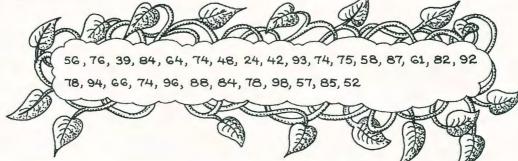
II. Here is the data from the survey. Enter it in the table you made in I.

III. What trends do you see in this table?



A stem-and-leaf display is a method of organizing information. It can be used for any frequency distribution, but is more convenient to use if the range of the data is not extreme. An advantage of the stem-and-leaf display is that individual scores can still be seen.

To introduce this method to the class, show them the scores from a math quiz.



- Ask the students to find the high and low scores. (24 and 98)
- 2) Since the scores are two digit numbers between 20 and 100, explain that the tens digits from 2 to 9 can be used as stems. List the stems in a column and draw a table as shown.
- 3) To complete the table, take each score and write the units digit in the leaf column opposite the correct stem. (The first eleven scores have been recorded in the table.) Tell students that the units digits (leaves) act as tallies. By evenly spacing the numbers, the finished table nicely displays the distribution.
- 4) When the display is finished, point to numbers in the leaf column and ask students to give the scores they represent.

STEM	LEAF
2	4
3	9
4	8,2
5	6
6	4
7	6,4,4
8	4
9	3

AFTER THE TALLY IS COMPLETED YOU COULD DRAW ANOTHER TABLE AND ORDER THE LEAF VALUES FROM SMALLEST TO LARGEST

company to set the set of the set

in second in the second second second in the second second

		ost common age of our country f		
when each was	firs		President	Age, lst Inauguration
(TAB	LE W	ACTS IN THE DERE FOUND IN MANAC.	George Washington John Adams Thomas Jefferson James Madison James Monroe	61 57 57
<ol> <li>To answer the question, make a stem-and-leaf display of the numbers.</li> </ol>			John Quincy Adams Andrew Jackson Martin Van Buren	57 61
USE THIS ST	EM	LEAF	William H. Harrison John Tyler	68 51
AGES FROM 40 000	4	3,3	James K. Polk Zachary Taylor	49
0000	+	9,8,6,9,7	Millard Fillmore	
(USE THIS STEM)	5	41.02.40411,41.2	Franklin Pierce	
	5	777876565556	James Buchanan Abraham Lincoln	
	6	LI,4,9,Z,t	Andrew Johnson	56
	6	8,5	Ulysses S. Grant Rutherford B. Hayes	
2) Order the left from smaller List them be	st t	-	James A. Garfield Chester A. Arthur Grover Cleveland Benjamin Harrison William McKinley	49 50 47 55
STEM		LEAF	Theodore Roosevelt William H. Taft	42
4	2.7		Woodrow Wilson	51 56

RN RGE ULD

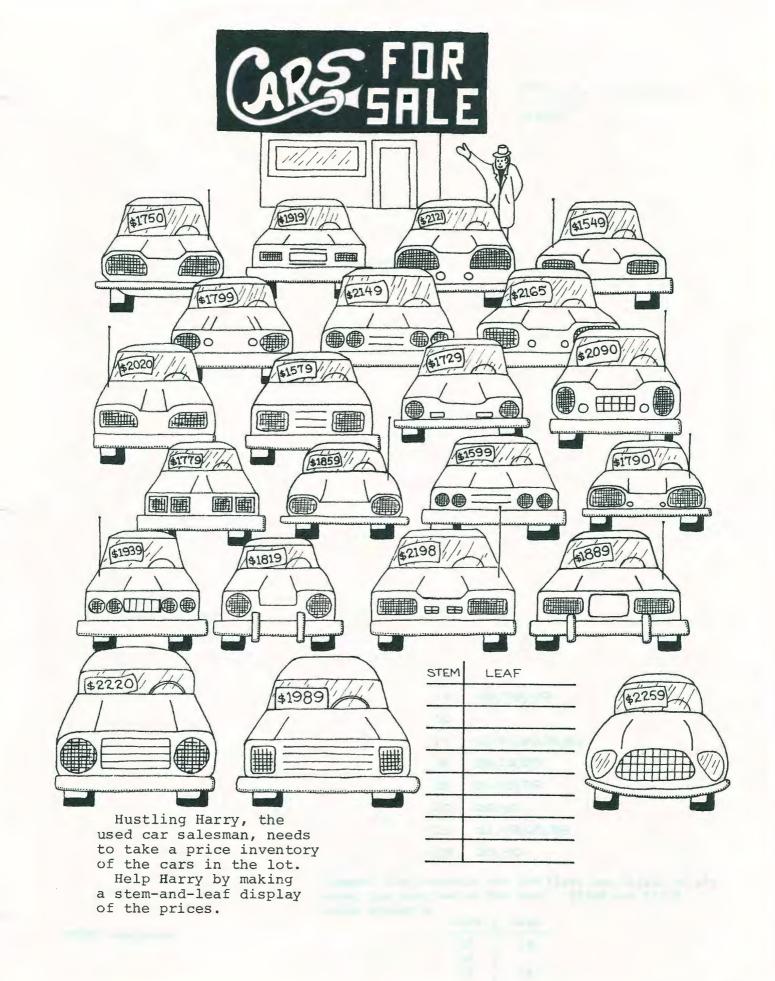
SIEM	LEAF
4	1,3
4	S, 7, 8, 9, 9
5	0,0,11,11,2,2,4444
5	655 65 6 5 6 7 T T T
6	0,1,1,1,2,4
6	5.0

- 3) What was the most common age?
- 4) How many presidents were older than sixty when inaugurated?
- 5) Were more presidents in their fifties or in their sixties?

52

Warren G. Harding ..... 55 Calvin Coolidge ..... 51 Herbert C. Hoover ..... 54 Franklin D. Roosevelt ..... 51 Harry S. Truman ..... 60 Dwight D. Eisenhower ..... 62 John F. Kennedy ..... 43 Lyndon B. Johnson ..... 55 Richard M. Nixon ..... 56

Gerald R. Ford ..... 61 James E. Carter .....



# ORGANIZATION

# Placement Guide for Tabbed Dividers

Tab Label	Follows page:
CONTENT FOR TEACHERS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
Probability with Counting	
Inferential Statistics	
Appendix-Computer Programs	
DIDACTICS	
Components of Instruction	
Classroom Management	
Statistics and Probability Learning	
TEACHING EMPHASES	
Critical Thinking	
Decision Making	
Problem Solving	
Models and Simulations	
Calculators and Computers	
Laboratory Approaches	
CLASSROOM MATERIALS	
CONCEPTS AND SKILLS	
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Health & Medicine	
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Recreation	
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SELECTED ANSWERS	

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# Commentary to GRAPHS (437-447)

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Make Mine M & M's	450	Making a bar graph	Teacher directed activity
Forest Fires are a Real Burn	451	Making a pictograph	Worksheet
Sisters and Brothers	452	Making a bar graph	Teacher directed activity
A Time to be Born	453	Making a bar graph	Worksheet
It's an Emergency, Activity I	454	Making a table	Worksheet
It's an Emergency, Activity II	456	Making a bar graph	Worksheet
It's an Emergency, Activity III	457	Making a bar graph Choosing a scale	Worksheet
It's an Emergency, Activity IV	458	Using bar graphs to compare wholes and parts of wholes	Worksheet
It's an Emergency, Activity V	459	Reading and making percent bar graphs	Worksheet
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Matching Scales	465	Matching data to appropriate scales	Worksheet Puzzle



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TITLE	PAGE	TOPIC	TYPE
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Charts	469	Reading and interpreting a circle graph	Worksheet Transparency
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What Kind of Graph?	475	Choosing an appropriate type	Transparency Discussion
How Can We Display the Data?	476	Comparing different types of graphs	Transparency Discussion

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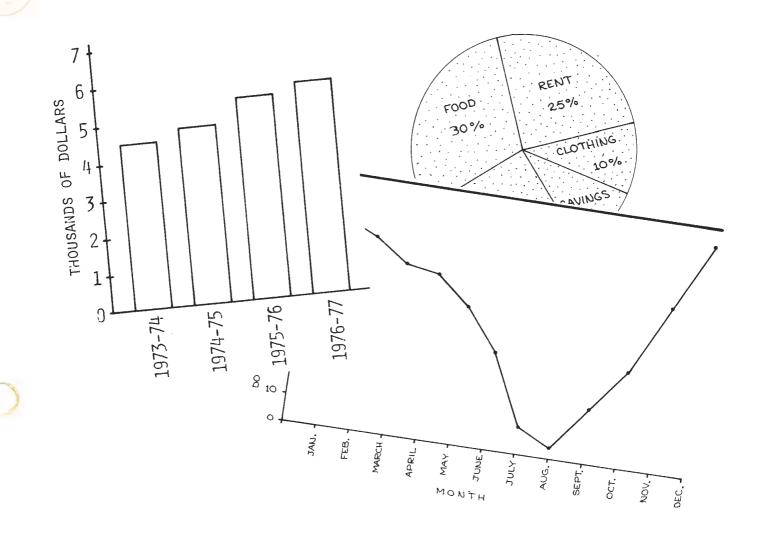
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C

436

# CRAPHS



We see graphs in almost every magazine or newspaper. Politicians, advertisers and news commentators use graphs to support positions, promote products, and report on happenings in the world. Graphs are used because they can quickly convey the general meaning of a mass of data. To be an intelligent voter, a wise consumer and an informed citizen, a person must be able to make sense out of the graphs presented. Students, too, can be very much influenced by graphs. Aren't the middle school years a good time to help people learn to <u>read graphs</u>?

Understanding graphs presented by others is important, but it is also useful to know how to <u>make graphs</u> to convey information, to see trends, to make comparisons or to support a statistical statement. The power of a graph to make sense out of a

GRAPHS

mass of data can and should be brought out in activities in which students are asked to make graphs. Other activities can show situations in which a table displays the data better than a graph (see *Lifetime Home Runs*). The Association of Teachers of Mathematics gives this warning about graphs:

The physical act of making a graph, the pleasure in its form, the appreciation of its compactness, are valuable aspects even if the math content of the feeling seems low. But these values are not cumulative: they do not increase with the number of times they are experienced. It seems wrong to suppose that every situation that <u>can</u> be graphed <u>should</u> be, or that because a kinds of situation has been graphed once it should always be graphed when it reappears. (The Association of Teachers of Mathematics, <u>Notes on Mathematics in Primary Schools</u>, [Cambridge: Cambridge University Press, 1969], p. 94.)*

As teachers we will have to decide how much time our students will spend learning to read and make graphs. You could coordinate your efforts with teachers who teach the grade ahead or behind yours. One good plan is to have students read and interpret pictographs before making them. The same idea of reading, then making can be used for bar graphs, line graphs and circle graphs. Xerox Education Publications publishes a sequence of four excellent, inexpensive pamphlets and a teacher's guide entitled <u>Weekly Reader Table and Graph Skills Series</u> which would be very helpful for planning in grades four through eight. (See the *ANNOTATED BIBLIOGRAPHY* for more information on this series and also the classroom materials for some example pages.)

#### PICTOGRAPHS

Most of your students probably made pictographs in the primary grades. They might have pasted a folded piece of paper after their names every time they read a book or perhaps a teacher put up a marker every time three books were read. The result was a pictograph (also called a picture graph or pictogram) in which each symbol stood for a certain number of books read.

COLBY	$\left[ \right]$	$\bigcirc \bigcirc $
KEY :	$\square$	MEANS ONE BOOK READ
DARREN	$\left[ \right]$	$\mathcal{O}$
KEY :	$\Box$	MEANS THREE BOOKS READ

*Permission to use excerpt granted by Cambridge University Press

When you have students make pictographs, keep the symbols simple. There is little value in spending hours copying a complicated symbol over and over. It is also helpful to have students use grids for making their pictographs. This will keep their pictures lined up vertically and evenly spaced. You can put the grids on their worksheet as shown to the right or have students place a grid under their pa-

BIKES AT THURSTON SCHOOL						
GRADE G	$\circledast$					
GRADE 7				Î	j.	
GRADE 8						
grade 9						
KEY: = 20 BIKES SOURCE: SCHOOL SURVEY COMMITTEE						

pers so they will have guidelines. Templates, stickers or rubber stamps could also be used to make pictographs more easily. Be sure students record the meaning for the symbol on their graphs.

Some pictographs have scales or numbers attached so they are easier to read. Such graphs combine the eye appeal of pictographs with the advantages of bar graphs. You might want to use such graphs as a transition from pictographs to bar graphs. (See Sisters and Brothers.)

## BAR GRAPHS

When your students learn to read bar graphs, be sure they examine the following things:

- The title and subtitles of the graph-do they help the reader understand what data is being presented?
- The scales on the graph--do they start at zero and are they evenly marked?

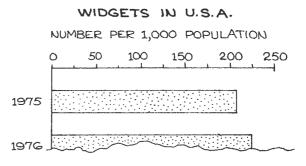
Rate per 1,000 Population 15 GAG GAG GAG GAG Births 1950 XXXXXXXXX Deaths XXXXXXXXXX 949 949 949 949 944 1960 1965 XXXXXXXXXX 1970 (prel.) 1972 (prel.) SOURCE: Pocket Data Book USA 1973, p. 57

**Birth and Death Rates** 

- If not, how does this change the looks of the graph? (See the MISLEADING STATISTICS section for more activities on this.)
- The labels of the horizontal and vertical axes--what is being shown on each?
- The source of data--is it likely to be reliable? Possibly biased?
- The bars--are they beside each other or is there a space between? (There usually is a space unless both axes have a number or time scale.)

This kind of attention to the details of a bar graph is brought out in *Count*ing All Cars. Also included in this activity are statements which can be completed by reading the graph closely.

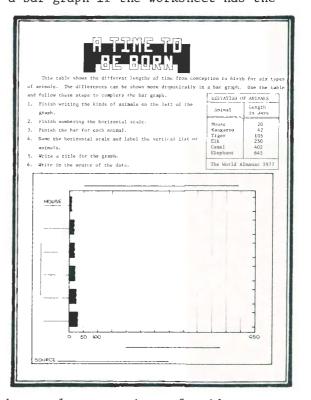
Some of your students might have difficulty reading a bar graph. If your students cannot approximate a reading for the bar to the right (about 210), you might want to have them read some number lines as given in *Mouth Control*. After they can give approximate numbers for points



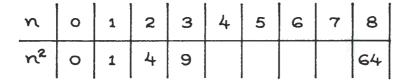
on number lines, you can help them transfer this skill to the reading of bar graphs. It will be easier for students to make a bar graph if the worksheet has the

scale already started. (See *Completing Scales* for a worksheet concentrating on finishing started scales.) The bars can also be started as shown in the activity to the right. Notice the marks at the right edge of the graph. Students can use these marks as guidelines for the bars. The solid vertical lines will help students draw the upper bars to the correct point on the graph. The blanks on the graph remind students to give a title, label the scales, record the source of the data, and give the year for the data.

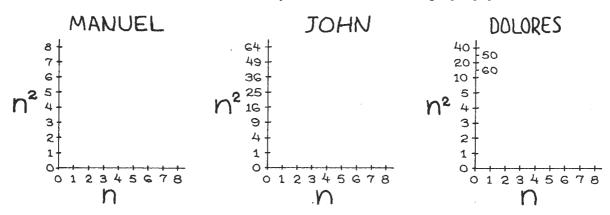
When asked to choose and label their own scales, students often have difficulty. This is not limited to middle school students. A class of older students was asked



to complete this table and make a graph of the results on a piece of grid paper.



GRAPHS



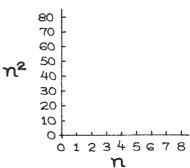
Here are some of the number lines they used to label the graph paper.

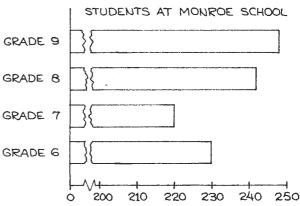
Manuel is labeling correctly, but he will not have room to show 9, 16, ..., 64 on his graph. John has used the square numbers in order and equally spaced. His first space shows a difference of 1, and the last space shows a difference of 15! His graph will be readable but will show a distorted "picture." Dolores saw she needed to place more numbers on her line and squashed them together. When she still did not have enough room, she tucked 50 and 60 in at random.

All three of these students need to develop skills in choosing and labeling a scale. They might be so used to number lines being labeled 0, 1, 2, 3, ... that they cannot conceive of them being labeled 0, 10,

20, ... or 0, 500, 1000, .... Scale and Plot and How Remarkable! were written to help students give different scales to number lines. Matching Scales asks them to match an appropriate scale to a set of data. After developing some of these skills, the students in the previous example could be asked to look at the largest square number (64) which must fit on the graph. They can then label the grid with multiples of 10 so all the data will fit.

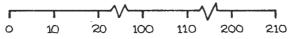
For some sets of data it is more convenient to use a scale with a zero break as shown to the right. (It is best to also show a break in the bars, but many graphs neglect to do this.) The data fits conveniently





GRAPHS

into a small space, but it is not as easy to compare one bar with another. The ninth grade does not have twice as many students as the seventh grade. Students might need help interpreting zero breaks in graphs. They should understand the false visual impressions such graphs create. Remind them to read the scale carefully and to compare the numbers from the scale--not the lengths of the bars. If you decide to have your students make graphs which use zero breaks, be sure they realize the breaks should be placed near zero on the scale. Some students will start to tuck them in at any convenient place. For example:



Unfortunately, some graphs in newspapers and magazines also do this, leaving the unwary reader with an incorrect interpretation.

In making graphs and labeling scales, students might be so meticulous that it takes forever. To increase their familiarity with scale labels other than 0, 1, 2, 3, ..., you might use the skill pages Scale Sketches, Completing Scales, Scale and Plot, How Remarkable! and Matching Scales.

## LINE GRAPHS

Many of the skills discussed previously for bar graphs are also necessary for reading and making line graphs. Students will need to read the scales on graphs. They will need to choose and label scales to fit the data they are planning to graph. Skills in locating points

that correspond to ordered pairs and in determining the coordinates for a specific point are also important. Some students, even at the eighth grade level, will need instruction in these skills before they can successfully read or make line graphs.

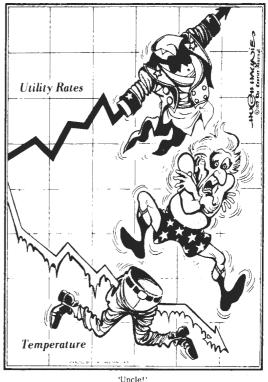
Line graphs are often used to show trends. It's not hard to see that the trend in the line graphs to the right is down--how much, we don't know because the

GRAPHS

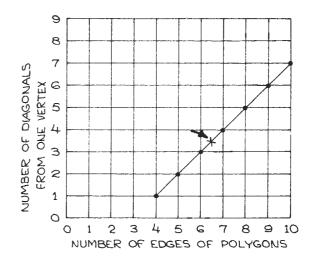
scales are noticeably missing. Have your students collect line graphs that show trends. You can display these graphs on a poster or make transparencies from them for discussion purposes.

Besides discussing the titles, labels, source and scales for a line graph, it is important to bring out these points:

- Line graphs are used to show trends, but we cannot usually be certain a trend will continue beyond the known data. The cartoon to the right indicates that the temperature is going down, while utility rates are going up. The arrows at the end of the lines encourage the reader to believe the trends will continue. The seasonal nature of temperature will, however, send the lower line graph up eventually. Warn your students: Do not assume a trend will continue! See A Trend for All Seasons for a student page on seasonal trends.
- It makes sense to read between the data points on some line graphs; it does not make sense to read between points on others. The point marked "x" on the graph to the right has coordinates (6¹/₂, 3¹/₂), but there is no polygon with 6¹/₂ edges. It does not make sense to read between the data points on this graph. (Some people say it is incorrect to join such graphs with line segments, but it is done so frequently that students should

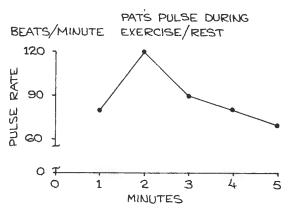


Cartoon from Courier Journal, 1977.



GRAPHS

learn to read such graphs critically.) It does make sense to read between the data points of this pulse graph. After  $2^{1}2$  minutes Pat's pulse rate was about 105 beats per minute.



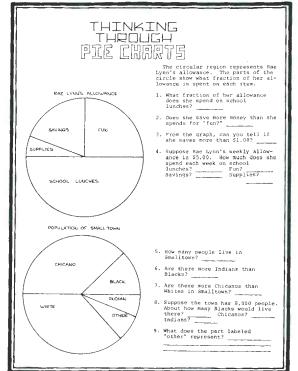
## CIRCLE GRAPHS

Background for circle graphs begins in the early grades when students use circle parts to learn about fractions. They learn to recognize one half, one third and one fourth of a circle, and they learn how these fractions of a circle compare in size.

With this background, students can read simple circle graphs like those shown in *Thinking Through Pie Charts*. They can compare a part to the whole circle as in question 1 and parts to each other as in question 2.

It is as important to know what information a graph does <u>not</u> give as what information it gives. Questions 3 and 5 ask about information that is not available from the graphs. Help your students become critical thinkers by discussing the questions on the activity thoroughly.

If an amount is given for the whole circle, students can estimate



the amount for the parts. (See Questions 4 and 8.) If an amount is given for one part, students can estimate the amount represented by the whole circle.

GRAPHS

If the data is planned carefully, students can make simple circle graphs using only their background in fractions. You might want to have your students do several exercises like those on Your Share of the Pie before introducing the idea of percents and angle measure for making circle graphs. (Of course, some intermediate classes might not cover percents at all.) Encourage your students to think about the graph as shown to the right.

When students understand percents, they will be more able to read the circle graphs used in newspapers and magazines. You might want to make transparencies from some of these "real world" graphs and discuss them with your stu-

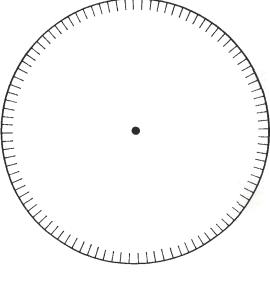


dents. Additional examples are included in *APPLICATIONS*. Encourage students to think critically as they read each graph. What conclusions can be made from the graph? What conclusions are not supported by the graph? Bring out the ideas in *The Family Circle* so students will realize circle graphs usually show how a whole amount is distributed, but circle graphs often do not give amounts for the parts.

Be certain your students are not misled by the size of the circle. Making a larger circle graph does not increase the amounts represented by the whole circle or its parts. You might explain this to students by showing two identical circle graphs, except for size, and ask students questions about the circle graphs. (See exercise 4 on Your Share of the Pie.)

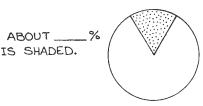
When you have students make circle graphs to represent data, point out that they must have "parts to whole" data. It makes no sense to represent the rainfall for five different years by using a circle graph. Another inappropriate use of circle graphs is pointed out in *How Can We Display the Data*?

You can avoid having students compute the number of degrees for the central angle of each part by giving them circles with 100 evenly spaced marks as shown to the right.



Students can compute the percents or you can give the percents. They then count the spaces between the marks to find each part. (Caution--be sure they don't confuse the 100 marks with the 360 degree marks on a protractor!)

Before having students construct circle graphs by using a protractor, try some of the ideas on Going Around In Circles with your students. Can they estimate the percent shaded of a circle? If they have a sound understanding of various percents of circles, they will be able to check the reasonableness of their calculations when they work with degrees.

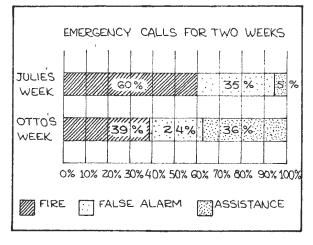


The page *To Make A Circle Graph* could be discussed and posted so students can quickly remind themselves of the steps in making circle graphs. You might also want to post a step-by-step flow chart for their calculators so they can accurately determine the degrees for each angle. An example is given below. Be sure it fits the calculators you have available before giving it to students.

TO DETERMINE THE ANGLE MEASURE FOR A PART

$$\begin{array}{c|c} \hline \text{Amount for part} \rightarrow \end{array} \div \\ \hline \Rightarrow \\ \hline \text{Amount for whole} \rightarrow \end{array} \Rightarrow \\ \hline x \rightarrow \\ \hline 360 \rightarrow \\ \hline = \end{array}$$

You might want to teach your students about other types of 100% graphs--often called area graphs. Some experience with these is given in *It's An Emergency, Activity V.* A long rectangle is used to represent the whole quantity and the rectangle is divided into parts. The rectangular 100% graph has the advantage of being easier for students to <u>make</u> because degree measure is not involved. The disadvantages are: they are not necessarily easier to <u>read</u>, they can be confused with bar graphs and they are not as commonly used as circle graphs.



#### GRAPHS

## DIFFERENT TYPES OF GRAPHS FOR DIFFERENT PURPOSES

In most activities, students will be asked to make a certain type of graph, but sometimes they will have to decide which type of graph to use. What Kind of Graph?, How Can We Display the Data? and It's An Emergency (Activities IV to VI) point out appropriate uses for each type of graph.

You can make a transparency of *What Kind of Graph?* and ask questions like this:

• Line graphs usually show trends. Do you see a trend in the first example? (The languages could be in other orders besides alphabetical. There is no trend.)

• Would a circle graph be better for this data? (No, it is not parts-to-whole data. There are lots of other languages.)

• Would a bar graph be better in Example I? (Yes. A bar graph is shown in blue on the page.)

• For the second example, would a line graph be appropriate? (No.) A circle graph? (Yes. The bar graph is all right in Example II but if it is desirable to LANGUAGE NUMBER SPEAKING MILLIONS 600 LANGUAGES OF THE WORLD ARABIC ENGLISH 150,000,000 350,000,000 500 HINDUSTANI 230,000,000 SN. MANDARIN (CHINA) 550,000,000 400 Heak 140,000,000 RUSSIAN 300 SPANISH 220,000,000 **JUMBE** 200 100 I NDIIS LANGUAGE BILLIONS MONEY SPENT ON DEFENSE BY 50. MONEY SPENT ON THE UNITED STATES IN 1965 DEFENSE BY U.S. IN 1055 11,552,000,000 13,339,000,000 18,146,000,000 ARMY 40 NAVY SPENT AIR FORCE 30 OTHER 4,061,000,000 I ARS 20 47,098,000,000 TOTAL 10 OTAL AVY AIR

convey a "parts-to-whole" idea, a circle graph would be better.)

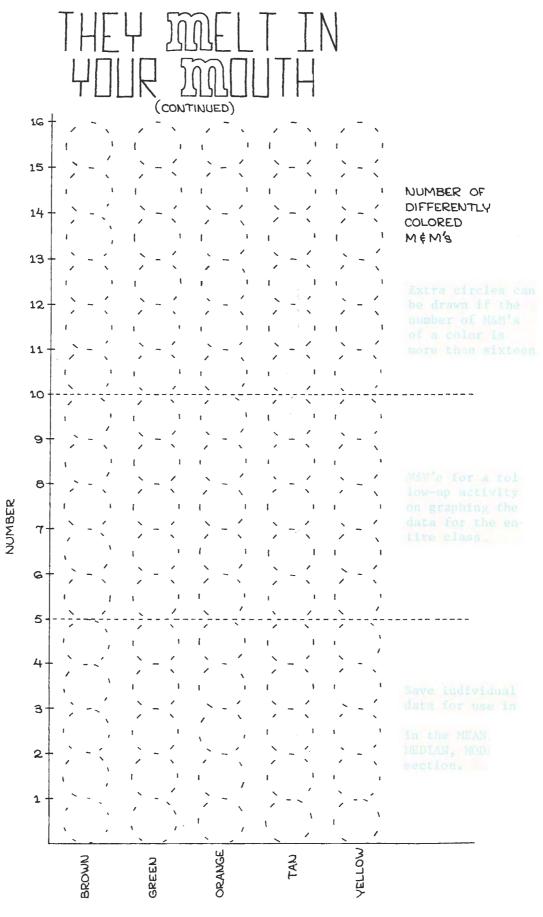


chin chronice of a products of months manine

### Materials: Package of M&M's

## Activity:

- 1) DON'T OPEN THE PACKAGE YET!
- 2) M&M's come in five colors: brown, green, orange, tan and yellow. Guess what color occurs most often in your package. Least often.
- dita, achier several translucaric chips of wrious solars on an atorhead. Suggest that it would be serier to gets interaction about the chips if they were extended. Now the chips into toleness of the same colur. Similate should find it worker to al chim the orders of each color by compare the accurate of the olor by compare the accurate of the olor hand within the orders of the olor hand within the orders of the olor
- 3) Open the package.
- 4) DON'T EAT ANY YET!
- 5) Arrange the pieces on the graph on the next page.
- 6) Record the total number and the number of each color. Total _____ brown ____ green ____ orange _____ tan ____ yellow _____
- 7) Which color occurs most often?
- 8) Is this the same as your neighbor's?
- 9) Which color occurs least often?
- 10) Is this the same as your neighbor's?
- 11) On the graph carefully mark the highest circle for each color.
- 12) Shade the circles below it.
- 13) NOW YOU CAN EAT THE M&M'S.



COLOR

To motivate this activity, discuss the differences in the student graphs from *They Melt in Your Mouth*. What do the graphs show usually happens in a package of M&M's? (Some graphs may show no yellows; others may show more than sixteen browns.) To get a better feeling for the relative amounts of each color in a package suggest that students pool their data and make a class graph.

- A) An easy way to collect the data for the class graph is described below.
  - Prepare five sheets, one for each color, as shown.
  - Students can file by the sheets and put a tally mark beside the appropriate number.
  - 3) The sheets can be assigned to five groups of students. Each group finds the frequency and total number of M&M's of their particular color.
  - Record the totals for each color on a transparency or the chalkboard so students can see the results.
- B) A discussion on finding an appropriate scale for a graph of the class data can follow.
  - 1) Students are to use page 2 of They Melt in Your Mouth.
  - 2) It is convenient to use a "nice" number for the scale: 1 circle represents 20 M&M's or 1 circle represents 25 M&M's.
  - Have each student make a graph by using the scale and shading the appropriate number of circles. Be sure students have labeled the scale correctly.

After making the bar graph displaying the data for the entire class, it could be compared to the individual bar graphs of the students. How are they alike (different)? How would a bar graph of a large 2-pound package of M&M's compare to this one? (Students might actually investigate this. Also a large package could be used to draw out samples of 10 or 20 to see if the samples can be used to predict the makeup of the entire package.)

GREEN MEMS

PACKAGE	TALLY	FREQUENCY	TOTAL
0			
1	1	1	1
2	ł	1	2
З	1111	4	12
4	-++++ 111	8	32
		$\sim$	$\sim$

	COLOR	TOTAL
	BROWN	
	GREEN	
	ORANGE	
	TAN	
	YELLOW	
CL	ASS TOTAL	





The table to the right gives the number of forest fires that occurred in one area last summer.

Use the data in the table to complete the pictograph below.

- Decide how many fires to let each symbol stand for. Write this number in the key.
- 2) For each month, draw the correct number of symbols. Be sure the symbols are equally spaced and lined up vertically. Try to make each symbol the same size and shape.

FOREST FIR	ES	
MONTH	FIRES	
MAY	20	
JUNE	14	
JULY	22	
AUGUST	18	
SEPTEMBER	45	1

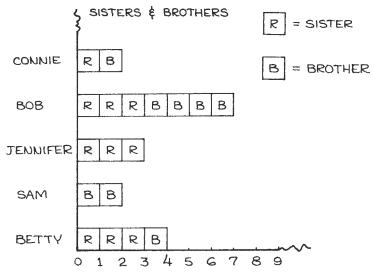
	FOREST	FIRES	
- MAY -			
- JUNE -			
- JULY -			
- AUGUST -			
- SEPTEMBER-			
KEY: M =	FIRES	NAME	



Making GRAPHS

Materials needed: red squares, blue squares, poster with axes and student names Activity:

- As they come into the room, have students get a red square for each of their sisters and a blue square for each of their brothers. Count half-sisters or half-brothers as sisters or brothers.
- 2) Have students place the squares next to their names. (Gummed labels work well.)



This activity can be used as a transition from pictographs to bar graphs.

The following points can be made to students:

- If each square is read as a separate symbol, the graph is a pictograph.
- If the squares are grouped together and the scale is used then the graph resembles a bar graph.

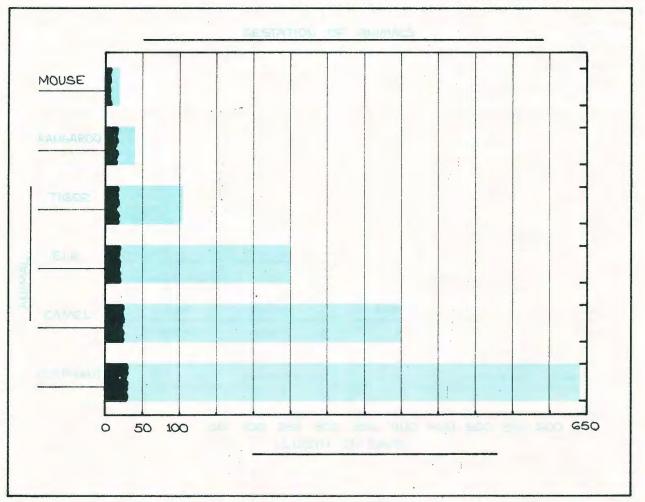
Another organization can be used to display the data. This technique can also be used for (a) number of girls or boys playing in various sports, (b) number of girls or boys interested in various occupations, (c) number of yes or no (true or false) answers given to a list of questions, (d) number of students that like or dislike various subjects, (e) number of girls and boys born in various months, etc. An example of this technique can often be found in an atlas. The highest and lowest points are often shown extending up or down from the center line which represents sea level.

The following questions could be asked: SISTERS & BROTHERS R = SISTER Suppose you want to know who has the BETTY В most sisters and brothers. B = BROTHER BB Which graph would be better, top or bottom? JENNIFER .... R R R Suppose you want to know who has BOB в в в the most brothers. CONNIE ····· R Which graph would be в better?



This table shows the different lengths of time from conception to birth for six types of animals. The differences can be shown more dramatically in a bar graph. Use the table and follow these steps to complete the bar graph.

- Finish writing the kinds of animals on the left of the graph.
- 2. Finish numbering the horizontal scale.
- 3. Finish the bar for each animal.
- 4. Name the horizontal scale and label the vertical list of animals.
- 5. Write a title for the graph.
- 6. Write in the source of the data.



the solution of the

GESTATION C	T ANIMALS
Animal	Length in Days
Mouse	20
Kangaroo	42
Tiger	105
Elk	250
Camel	402
Elephant	645

## ITS AN EMERGENEL ACTIVITY I

How many assistance or fire runs are made in your town each week? How much fire damage is reported in your town each month?

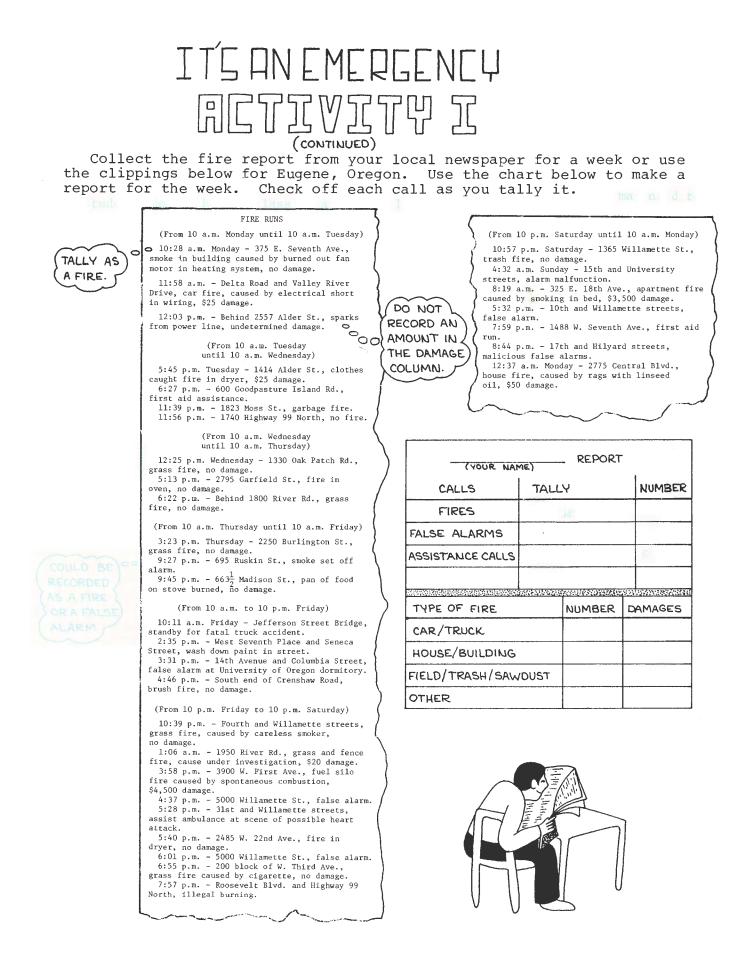
Four students in Mr. Johnson's math class decided to check the fire report in the local newspaper each night for a month and record the emergency calls. Each student selected one week of the month to report on. They agreed to list the types of fires.

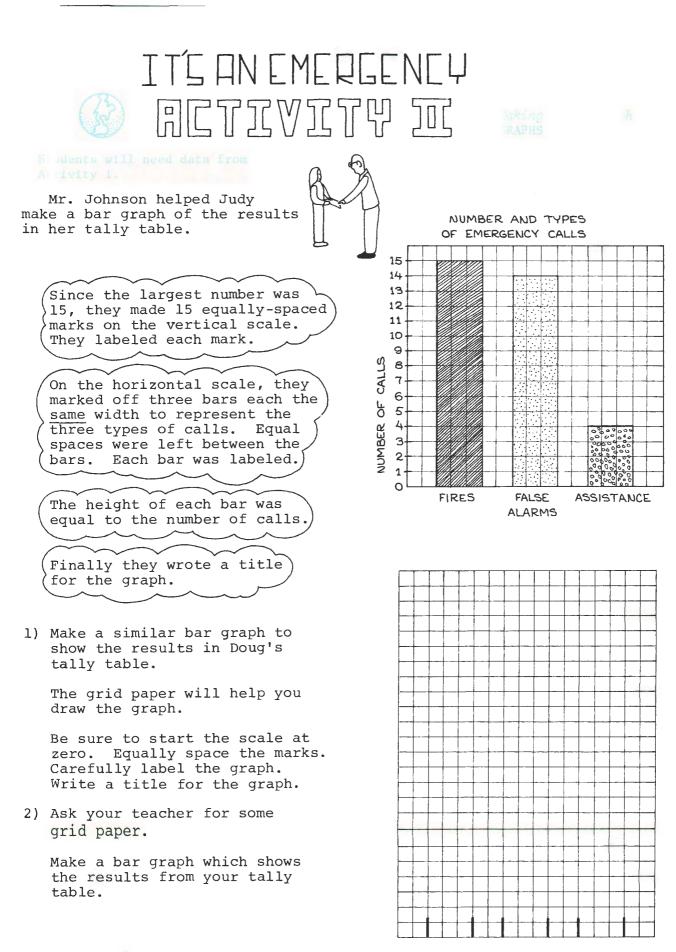
Ţ	UDY'S R	EPORT			
CALLS	TA		NUMBER		
FIRES	HIT HIT	HHT .	1	15	
FALSE ALARMS	1111 1111	. 111	14		
ASSISTANCE	un		4		
TYPE OF FIRE		NUMBER	T	AMAGES	
CAR/TRUCK		2	\$		
HOUSE/BUILDIN	G	5	\$	2300	
FIELD / TRASH /S	AWDUST	4	\$	0	
OTHER		4	\$	25	
TOT	AL	15	+		

DOUC	S'S RE	PORT			
CALLS	TA	LLY	NUMBE		
FIRES	++++ ++++	-++++++111		23	
FALSE ALARMS	++++ +++	t I		11	
ASSISTANCE	r 11	12			
			105150		
TYPE OF FIRE		NUMBER	DAMAGES		
CAR/TRUCK		4	\$ 3	3600	
HOUSE/BUILDING		6	\$	900	
FIELD/TRASH/SAW	DUST	5	\$	550	
OTHER		8	\$	30	
TOTA	4L	23			

(	otto's	REPOR	T				
CALLS	TALLY			NUMBER			
FIRES	HIT THE	r m		13			
FALSE ALARMS	HHT 11	1		8			
ASSISTANCE	HHT HH	tu		12			
TYPE OF FIRE	1676-912-02-02	NUMBE	-				
CAR/TRUCK		2		\$	90		
HOUSE/BUILDING		6		\$27,100			
FIELD/TRASH/SAWD	UST	2		\$	0		
OTHER		3		\$	0		
TOTA	L	13					

301	IE'S RI	EPORT				
CALLS	TAL	LY	NUMBER			
FIRES	+++++++	11 11		12		
FALSE ALARMS	HHT I			۲		
ASSISTANCE	l			1		
TYPE OF FIRE	1999-1999-1999 1999-1999-1999	NUMBE	-		AGES	
CAR/TRUCK		3		\$ '	007	
HOUSE/BUILDING		3		\$ 10,000		
FIELD/TRASH/SAL	5	:	\$	10		
OTHER	1	1	\$	0		
TOTAL	_	12				





# ITS AN EMERGENEY ACTIVITY DO

Otto suggested they make a bar graph to post on the hall bulletin board. The graph would compare the number of each type of emergency call for the month.

 Use the information in each student's report to complete the table below. Find the total of each row. Be sure to include Judy.

TYPE OF CALL	JUDY	DOUG	οττο	JULIE	TOTAL
FIRES	15				
FALSE ALARMS	14				
ASSISTANCE	4				

ON THE SCALE SHOULD EQUAL FOUR UNITS.

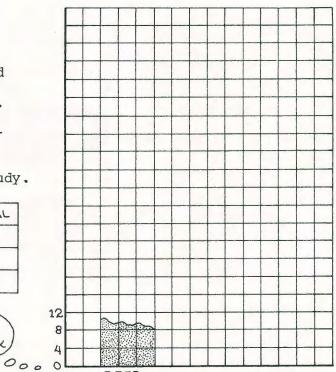
- Complete the bar graph above. Make each bar the same width. One bar is started for you. Finish labeling the scale and write a title for the graph.
- 3) Terry recorded the emergency calls for another week. She found: Fires - 16, False alarms - 3, Assistance - 4. Combine Terry's findings with the totals of the four students above. Complete the bar graph to the right to show the calls for five weeks.

#### Students sould use sould being from Actiwith a instand of Yosty's:

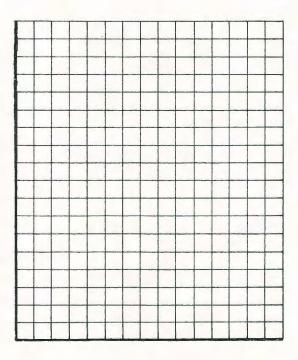
Pick a scale that lets you graph the largest number.

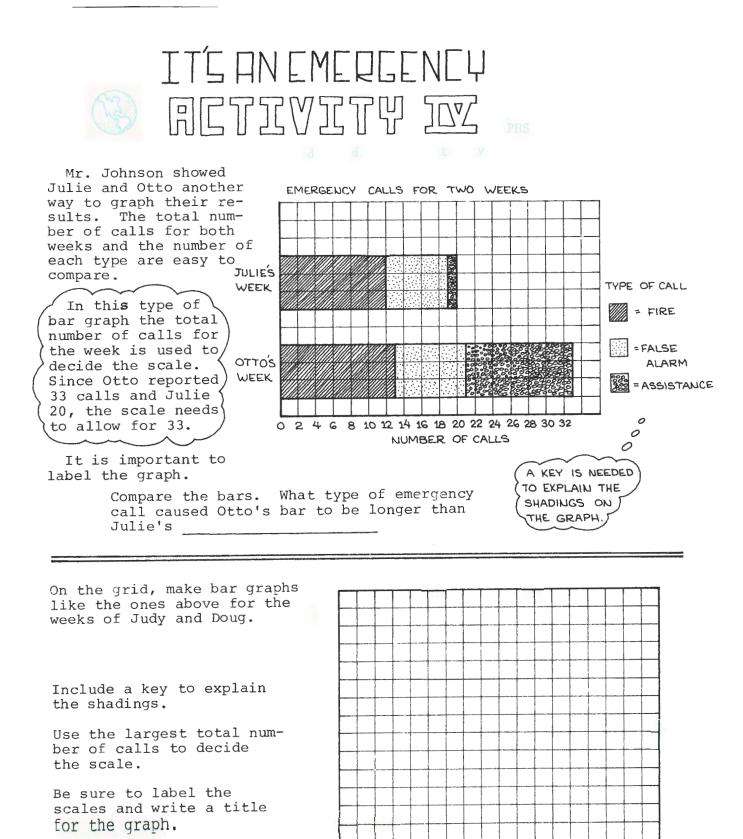
Start the scale at zero and equally space the markings. Label the scales and write a title for the graph.

TEPH: WORKSHOTL



FIRES



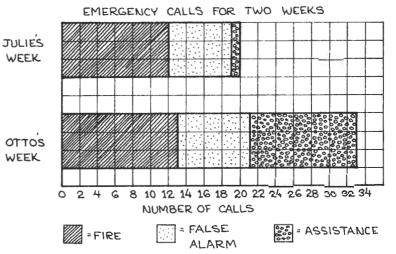


Which is more effective for showing differences in the types of calls for the students, the table or your graph?

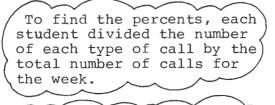
# ITS AN EMERGENEY

Julie and Otto made bar graphs to show the breakdown of emergency calls for two weeks. When they compared their bars, they noticed that during Otto's week there were more actual fire calls than during Julie's week.

Julie quickly pointed out that more than half of her total calls were fire calls: 12 out of 20 or 60% (12 ÷ 20). During Otto's week only 13 out of 33 or 39% were actual fire calls.



Their teacher, Mr. Johnson, complimented the students on their observations. He asked them to make percent bar graphs for their emergency calls.

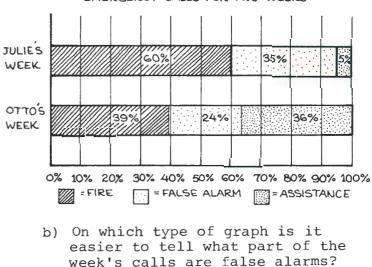


On grid paper the students drew bars 10 squares long. Each square shows 10% of the total.

They then marked off the percents on each bar and labeled them.

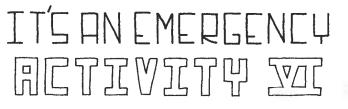
- Ask your teacher for some grid paper. Make percent bar graphs for the weeks of Doug and Judy.
- Look at the percent bar graphs and the bar graphs you drew in Activity IV.
  - a) Which type of graph tells you which week had the most actual fire calls?

	JULIE	OTTO
FIRE	12÷20 = 60%	13÷33 ≈ 39%
FALSE ALARM	7÷20 = 35%	8÷33 ≈ 24%
ASSISTANCE	1÷20 = 5%	12÷33 ≈ 36%



EMERGENCY CALLS FOR TWO WEEKS





Mr. Johnson grouped the students' fire calls together.



The data below is from Activity I.

	JU	YOUT			DOUG			0770				JULIE		
TYPE OF FIRE	NUMBER OF FIRES	1	DUNT OF MAGES											
CAR / TRUCK	2	\$	750	4	\$	3600	2	\$	90	3	\$	700		
HOUSE / APT / BUILDING	5	\$ 2	2300	ତ	\$	900	6	\$2	7,100	3	\$	10,000		
FIELD/TRASH/SAWDUST	4	\$	0	5	\$	550	2	\$	0	5	\$	10		
OTHER	4	\$	25	8	\$	30	3	\$	0	1	\$	0		

He showed Judy how to make a circle graph to show the types of fires for her week.

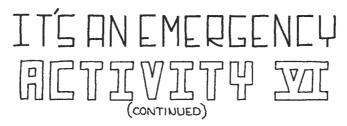
(First find the total number of fires ... 15)

You may wish to show students how to construct this circle graph

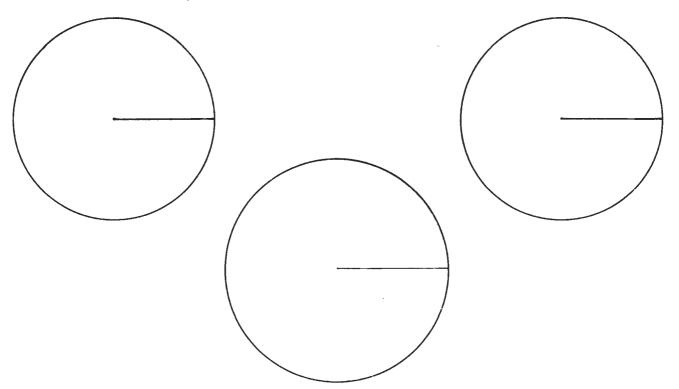
Write a ratio that compares the type  $\frac{2}{15}$ ,  $\frac{5}{15}$ ,  $\frac{4}{15}$ ,  $\frac{4}{15}$ 

HOUSE / APT / The entire circle represents the week's fires. BUILDING Use the ratios to find the part of the circle for each type of fire. Judy used a calculator to help her. CARS/ TRUCKS  $\frac{2}{15} \times 360^\circ = 48^\circ$   $\frac{5}{15} \times 360^\circ = 120^\circ$  $x 360^{\circ} = 96^{\circ}$ 15 FIELD/ TRASH/ SAWDUST OTHER Use a protractor to mark off angles of 48°, 120°, 96°, 96° to represent the number of TYPES OF FIRES each type of fire. DURING JUDY'S WEEK

TYPE: Worksheet



 Use the circles below to make circle graphs to show the types of fires for Doug's week, Otto's week and Julie's week.
 Label each graph.

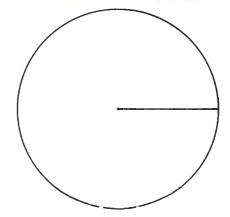


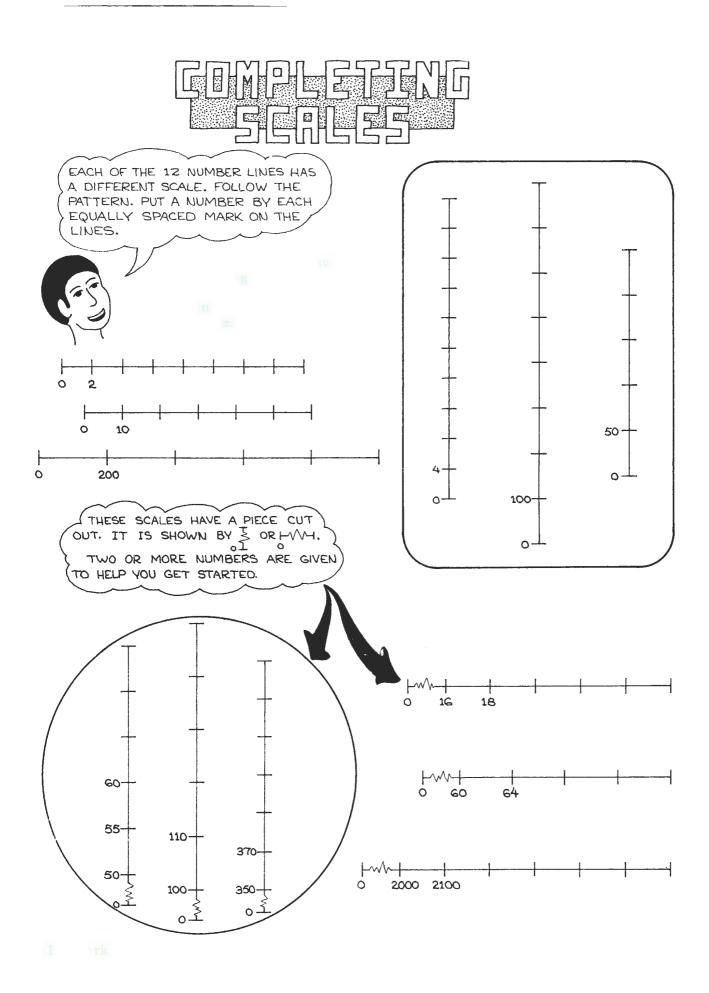
2) Do these graphs tell you which week had the most field/trash/sawdust fires?

Students should see that a circle graph is an effective way to show a comparison of parts of a whole. Bar graphs are useful to compare independent but related items.

3) Use a circle graph to show the part of the total damages each type of fire caused for the four weeks.

> Round the damages to the nearest thousand dollars before writing the ratios.







For each exercise, label the scales so the largest number shown will be near the top of the scale.

Put a dot on the scale for each number. Put the corresponding letter beside each dot.

2.

A) 95

50

75

30

16

B)

c)

D)

5.

A)

EXAMPLE 50 7 C A) 32 40-B) 24 30 - A c) 48 B 20 -D) 12 D 10 -0 1

1.

A)

B)

8

18

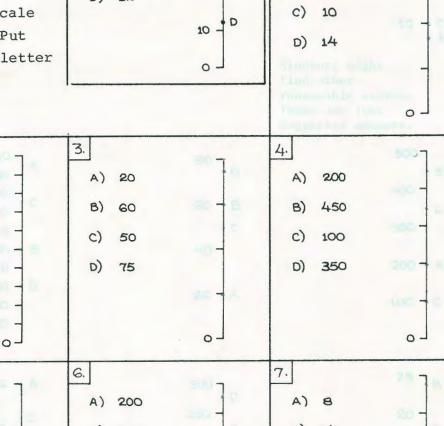
0 1

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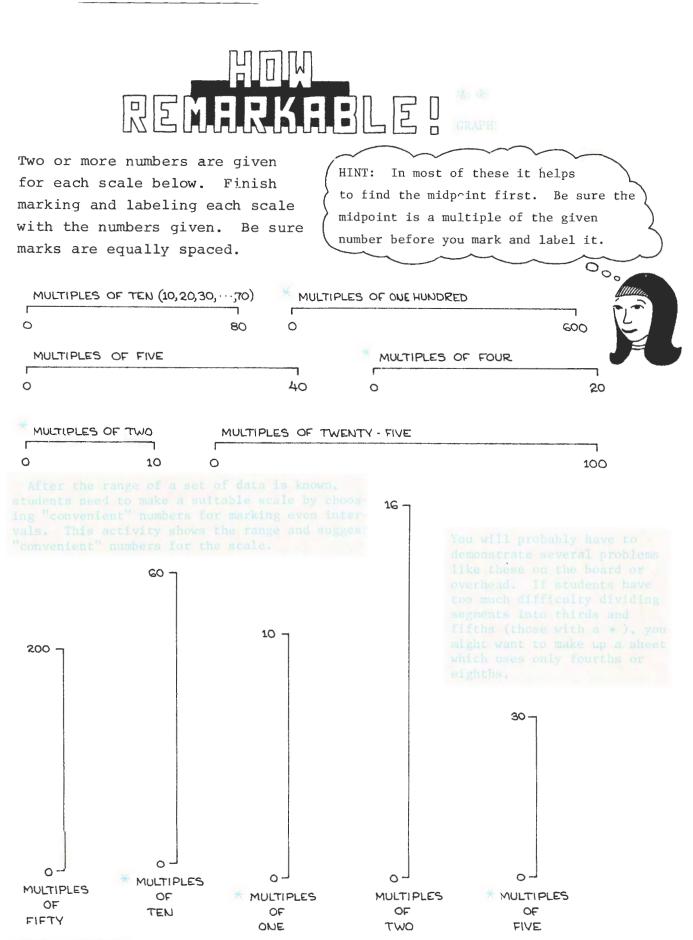
20 -

7

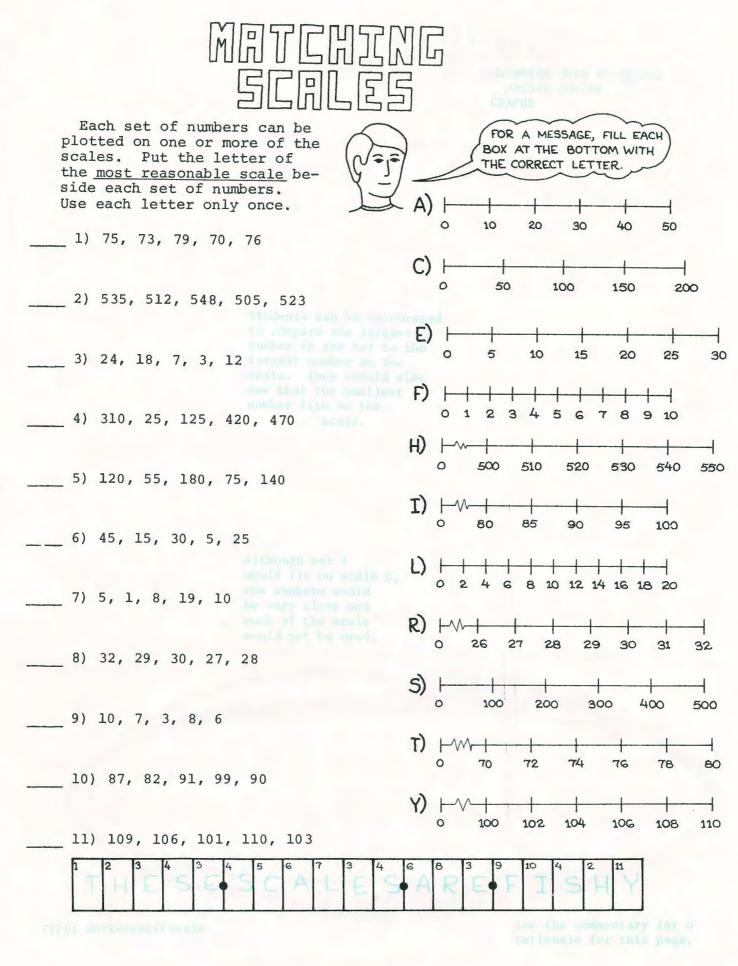
5



24 B) B) 10 B) 225 c) 13 C) 150 c) 18 100-12 D) 280 D) 10 D) 6 0. 0-0 -

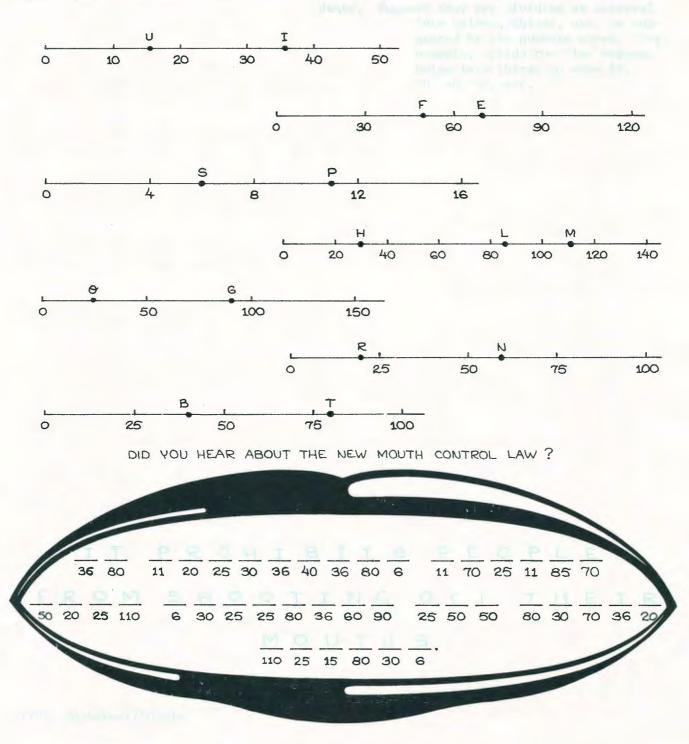


TYPE: Worksheet





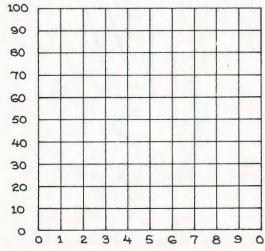
Look at each point named by a letter. Estimate its value. Write the value below the point. Find the value in the message below. Write the correct letter above it.



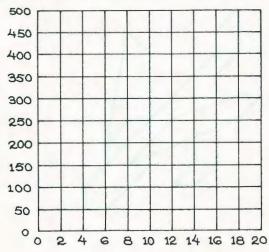


Connect each set of points with straight line segments. Do not connect points that have "Stop" between them. Cross off each ordered pair as you use it.

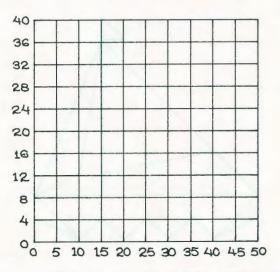
A. (5,60) (5,80) (6,80) (6,60) (5,60) Stop (4,0) (4,20) (2,40) (2,60) (4,60) (4,50) (3,50) (4,40) Stop (6,0) (6,20) (7,30) (7,70) (6,70) Stop (3,60) (3,70) (2,100) (3,100) (4,70) (4,100) (5,100) (5,80) Stop (6,60) (6,50) (7,50) Stop



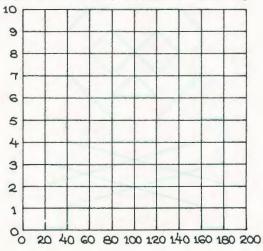
B. (10,250) (10,400) (8,450) (6,200) (6,150) Stop (14,200) (14,150) (12,100) (14,100) (16,150) (16,200) Stop (12,250) (18,300) (20,300) (18,250) (8,150) (12,250) Stop (10,250) (8,250) (6,200) Stop (10,150) (6,50) (4,50) (4,100) (16,400) (16,450) (18,500) (18,400) (16,350) (16,300) Stop

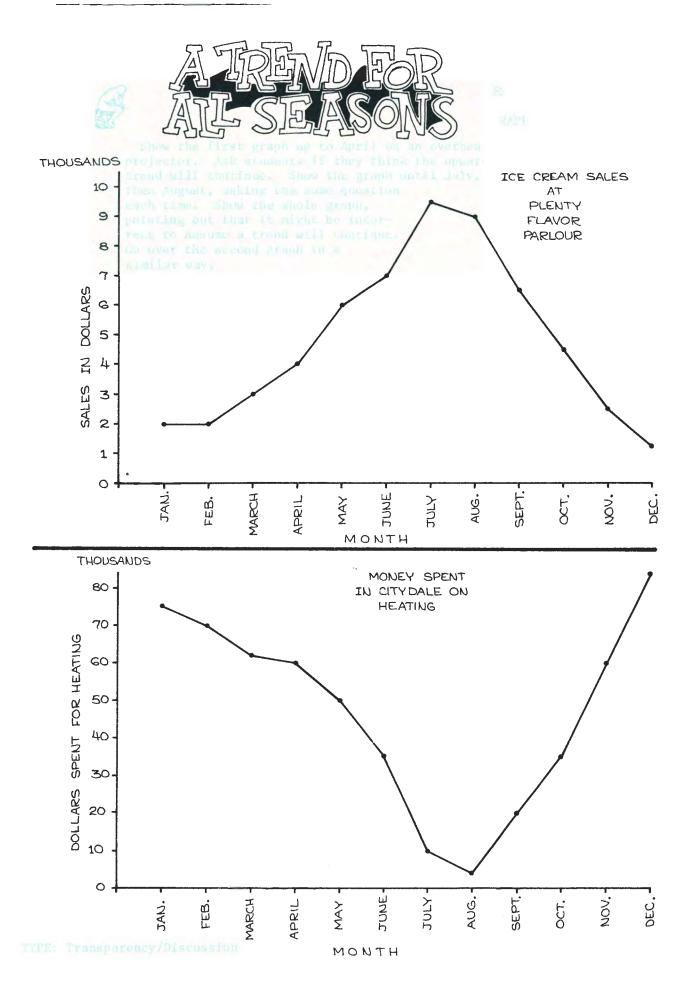


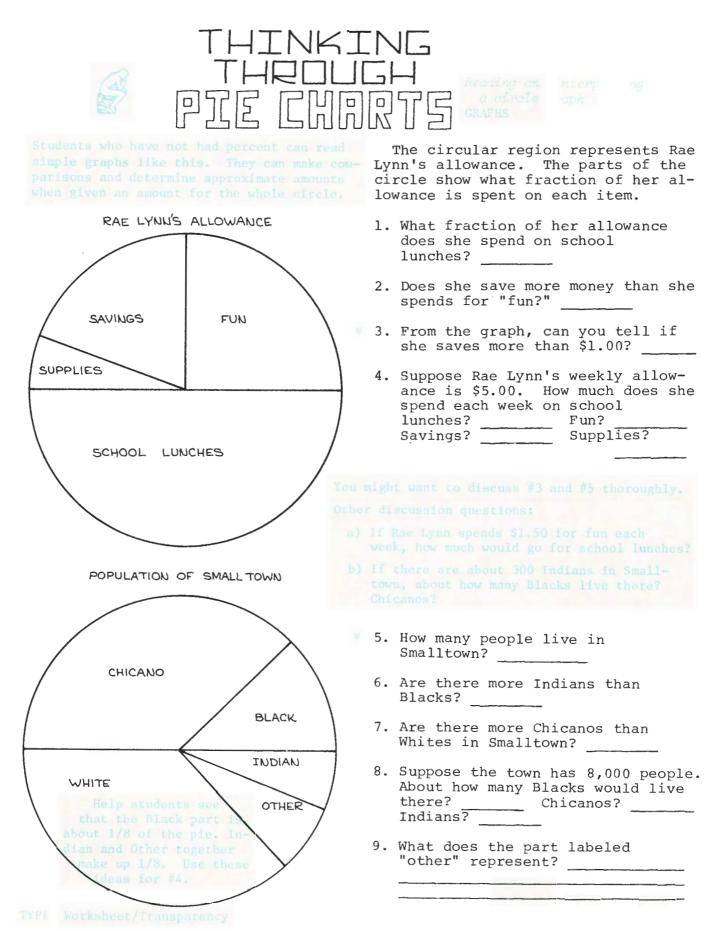
C. (15,8) (15,40) (20,36) (15,36) Stop (0,8) (15,36) (10,12) (0,8) Stop (10,0) (0,8) (50,8) (40,0) Stop (15,36) (50,8) Stop (15,12) (40,16) Stop (0,4) (10,0) (15,4) (20,0) (25,4) (30,0) (35,4) (40,0) (50,4) Stop



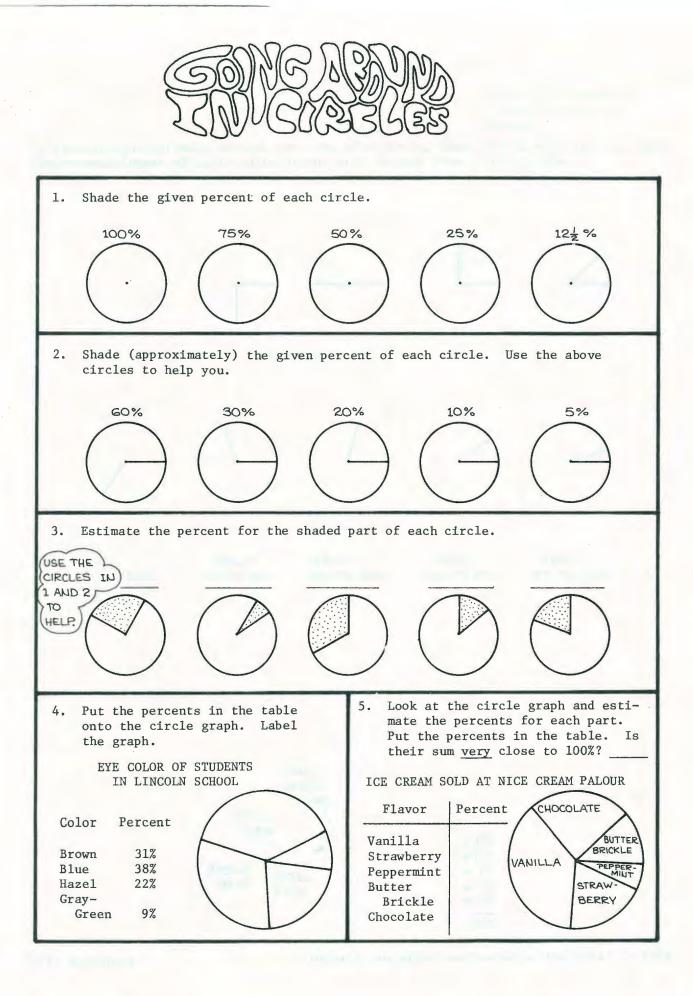
D. (20,5) (40,5) (40,4) (180,2) (180,0) (160,0) (160,1) (20,3) (20,5) Stop (60,10) (140,10) (160,9) (160,7) (120,5) (120,4) (80,4) (80,5) (40,7) (40,9) (60,10) Stop (60,8) (80,9) (100,8) (120,9) (140,8) (120,7) (100,8) (80,7) (60,8) Stop (80,6) (100,7) (120,6) (80,6) Stop (20,0) (20,2) (160,4) (160,5) (180,5) (180,3) (40,1) (40,0) (20,0) Stop







IDEA FROM: Teaching Modern Mathematics in the Elementary School, by Fehr & Phillips Permission to use granted by Addison-Wesley Publishing Company, Inc.





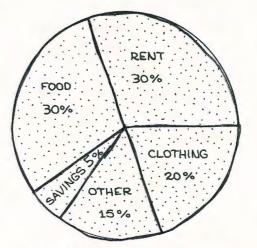
These graphs show how two families spend their monthly income.

 Do the graphs show the number of dollars each family spends on these items?

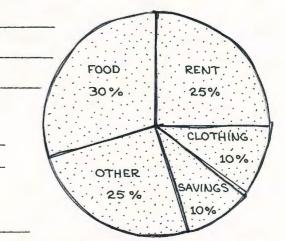
- 2) Can you tell from the graphs whether the two families have the same monthly income?
- 3) Which family spends a greater percent for rent? _____

Family A has a monthly income of \$500. Family B has a monthly income of \$800.

- 4) How much does family A spend for rent? _
- 5) How much does family B spend for rent? ____
- 6) Which family spends more money for rent? ____
- Compare your answers to exercises 3 and 6. Why do they differ?
- 8) Both families spend 30% of their monthly income for food. Which family spends more money for food? _____
- 9) In terms of percents, family B saves twice as much as family A. In terms of dollars, family B saves how many times as much as family A? _____

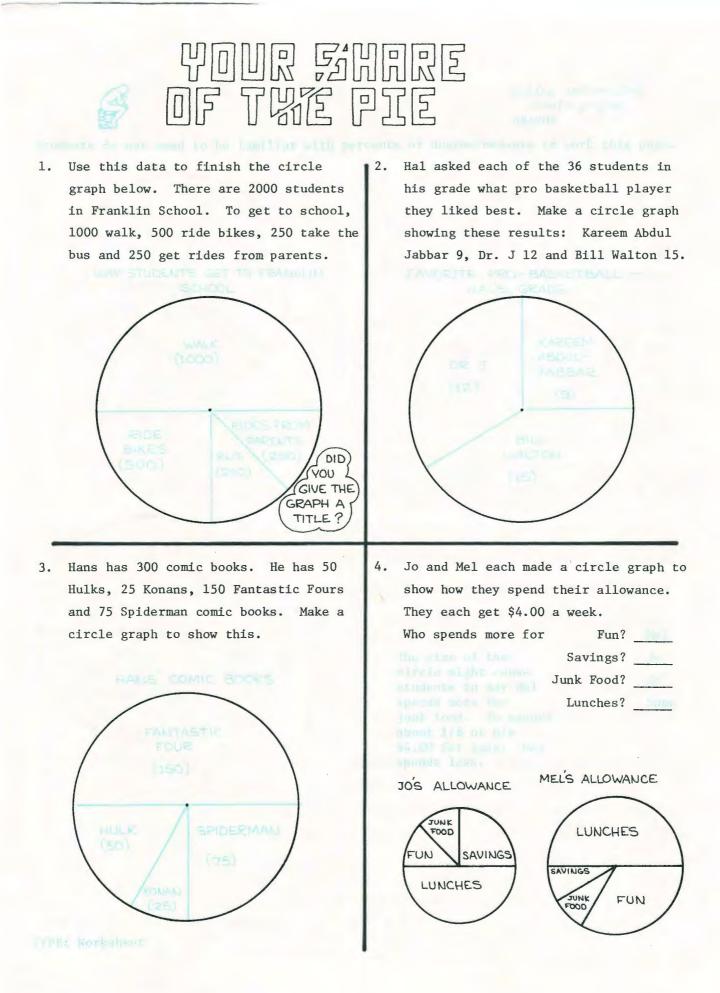


Monthly Budget for Family A



Monthly Budget for Family B

When graphs are used to compare percents, you should know the amounts on which the percents are based in order to interpret the graphs correctly.







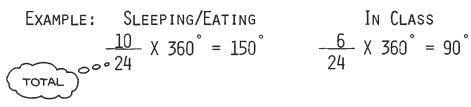
1. Be sure you have "parts to whole" data. Find the total of the data.

EXAMPLE: HOW I SPEND MY DAY

This page can be used to emphasize the steps in making a circle graph. SLEEPING/EATING10 hr.IN CLASS6 hr.AFTER SCHOOL SPORTS2 hr.HOME CHORES2 hr.SPARE TIME4 hr.TOTAL24 hr.



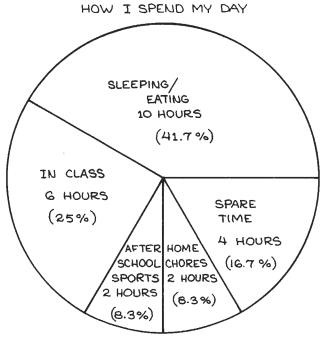
- 2. DRAW A CIRCLE OF ANY SIZE AND MARK ITS CENTER.
- 3. DETERMINE AN ANGLE MEASUREMENT FOR EACH ITEM.



- 4. DRAW A CENTRAL ANGLE FOR EACH ITEM. LABEL EACH PART OF THE CIRCLE.
- 5. WRITE A TITLE FOR THE GRAPH.

OPTIONAL: WRITE IN THE PERCENT FOR EACH ITEM. A CALCULATOR WILL HELP.

> EXAMPLE: SLEEPING/EATING  $\frac{10}{24} \approx .417 \text{ or } 41.7\%$ IN CLASS  $\frac{-6}{24} = .25 \text{ or } 25\%$





WHAT ARE THE FAVORITE COLORS IN YOUR CLASS ?

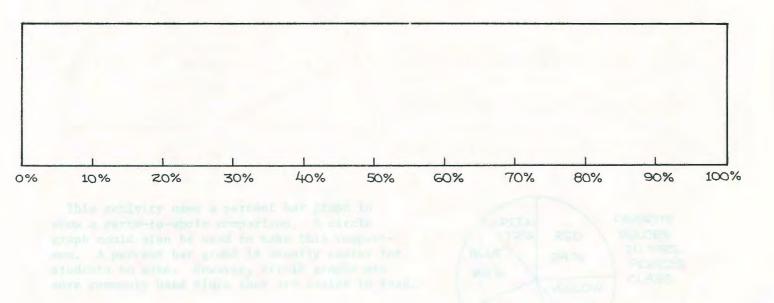
These are the favorite colors in Mrs. Perez's class.

This is a percent bar graph. It shows the percent of the class that prefers each color.

COLOR	TALLY	FREQUENCY		FAVORITE CON	LORS IN	MRS. PER	REZ'S CLASS	¥	PURPLE
RED	HTI	6							
GREEN	111	3		RED	GREEN	BLUE	ORANGE		YELLOW
BLUE	HHT	5		24%	12%	20%	24%	4	16%
ORANGE	HTTI	6						%	1010
PURPLE	1	1	°			•			
YELLOW	1111	4	000	6 OUT OF 25 IS 24%		0 5 OU (25 IS	\$ 1		OF 4%
	TOTA	NL 25	(	23 13 2470		(25 15	2070	10	470
l) Wri	te dow	n your favor	cite co	olor					
2) Put	your	tally mark o	on the	chart on	the ch	alkboar	d.		

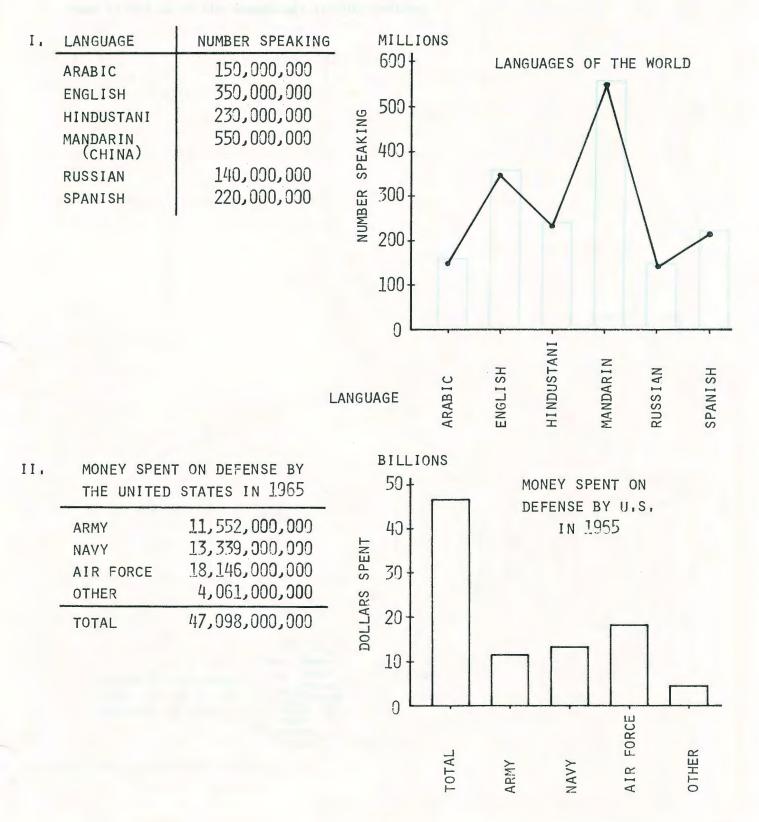
3) Make a bar graph of the favorite colors.

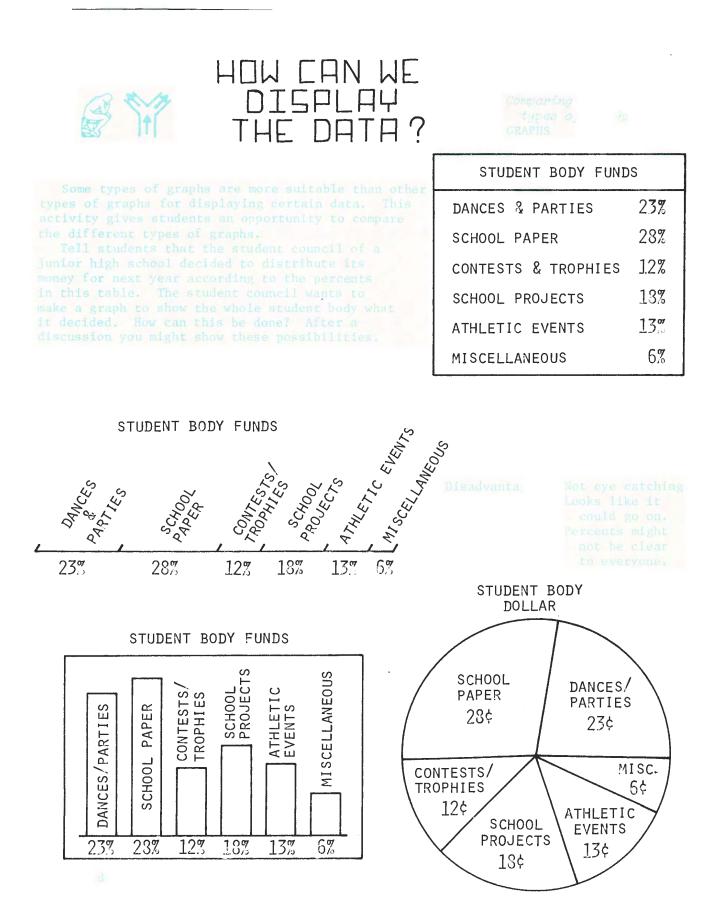
- 4) Make a percent bar graph for the favorite colors of the students in your class. Each mark shows 10%. Be sure to write a title for your graph and label each part.
- 5) What is the favorite color of the class? _____ What percent prefers this color?



25984 Mechanismi









Some students might think a circle graph is always best. Here is an example where a circle graph is not appropriate. Show students the table and graphs below. Which graph is most useful for showing a trend in amount of money in the student body funds?

YEAR	1973-74	1974-75	1975-76	1976-77	1977-78
TOTAL STUDENT BODY FUNDS	4,500	4,800	5,450	5,750	6,500
BODY	i-75	THOUSANDS		RING STUDENT DY FUNDS	
BODY	ING STUDENT FUNDS		1974-74 1974-75	1975-76	//-9/6T
THOUSANDS OF DOLLARS			_		
1973-74	1974-75 1975-76	1976-77 1977-78			

#### ORGANIZATION

#### Placement Guide for Tabbed Dividers

Tab Label	Follows page:
CONTENT FOR TEACHERS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
Probability with Counting	
Inferential Statistics	
Appendix-Computer Programs	
DIDACTICS	
Components of Instruction	
Classroom Management	
Statistics and Probability Learning	
TEACHING EMPHASES	
Critical Thinking	
Decision Making	
Problem Solving	
Models and Simulations	
Calculators and Computers	
Laboratory Approaches	
CLASSROOM MATERIALS	
CONCEPTS AND SKILLS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Misleading Statistics	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	604
Probability with Models	634
Counting Techniques	
APPLICATIONS	
Business & Commerce	
Environment	
Health & Medicine	
People & Culture	
Recreation	
GLOSSARY	
ANNOTATED BIBLIOGRAPHY	
SELECTED ANSWERS	



#### Commentary to SCATTER DIAGRAMS (480-482)

_.... __

TITLE	PAGE	TOPIC	TYPE
Scattered Patterns	483	Examining trends in scatter diagrams	Transparency
Are You A Square Or A Rectangle?	484	Gathering data to make a scatter diagram	Activity card Worksheet
Do They All Line Up?	486	Making a scatter diagram Drawing a trend line	Worksheet
It Sounds Like 20° C	488	Making a scatter diagram Using a trend line to predict	Worksheet
Come Rain Or Come Shine	490	Making a scatter diagram Making and using a trend line	Worksheet
¿Habla Usted, El Espanol?	491	Making a scatter diagram that does not have a trend line	Worksheet
A Lengthy Relationship	492	Making scatter diagrams to find relationships	Teacher directed activity
Lay It On The Line	494	Showing a relationship with perfect correlation	Worksheet
Sounds Fishy To Me	495	Making a scatter diagram with a curve for a trend line	Worksheet
It's Not Healthy To Drive Near Home	496	Discussing cause and effect for two related variables	Worksheet Discussion

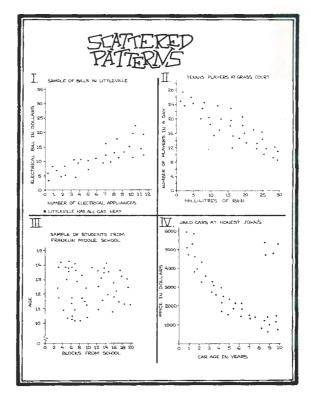
# SCATTER DIACRAMS

Students will need skills in reading scales, making scales and plotting points before they work with scatter diagrams. These skills are discussed in the commentary to GRAPHS. Activities in the GRAPHS section like *Scale Sketches*, *Mouth Control*, and *Scale and Plot* provide practice with scales and plotting points.

You can introduce or review scatter diagrams by making a transparency of the page *Scattered Patterns* and discussing the ideas below with your students.

• <u>Scatter diagrams are used to show</u> there is (or is not) a relation between <u>two variables</u>. Ask students what relation is shown by each scatter diagram to the right. (In Example I a person's; electrical bill tends to increase as more appliances are bought; in Example II the number of tennis players tends to decrease as the amount of rain increases; in III there doesn't seem to be a rela-tion between age and distance lived from school.)

• If points lie in a strip or oval, a trend line can be used to show an

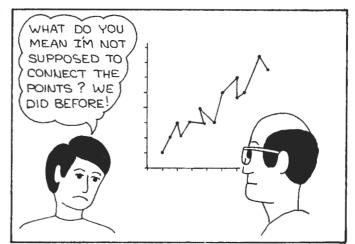


approximation of the relationship in a scatter diagram. (Use a transparent ruler

or piece of long uncooked spaghetti to fit a line on Graphs I and II.) Ask II this line would be useful for predicting one variable from a known value of the other. (Yes, <u>if</u> the dots are all fairly close to the line.) Can you be certain your prediction is correct? (No, unless it is known that the two variables always lie exactly on a straight line or curve.) Does it make sense to talk about a trend line for Graph III? (No.)

• <u>Sometimes a smooth curve can be used to show a trend in a scatter diagram</u>. In Example IV a smooth curve seems to be the best fit. Students will probably guess that the few outlying points are for special cars whose values tend not to decrease with age. COMMENTARY

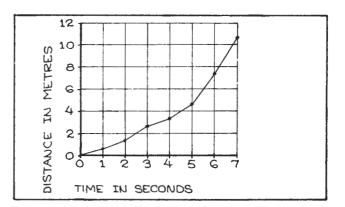
A frequent mistake by students is to treat a scatter diagram like a broken line graph. They will plot the points and then connect them just like they did for line graphs. They often find it hard to see a reason for connecting plotted points in one case and not connecting them in another. To help them understand, you might point to the several places where



dots are above each other. This does not occur in line graphs. Connecting all the points makes the graph look like the path of a maze! It is more useful to draw one straight line that best describes the trend of the points.

The activity Are You A Square or A Rectangle? asks students to make a scatter diagram showing heights and arm spans. There could be several different arm spans for one height. These points should not be connected with line segments.

A scatter diagram can be made of readings taken in an experiment. When <u>one</u> reading is taken after each second, these points <u>could</u> be connected with line segments. The sequence of line segments often approximates a line or curve of best fit. (Most data gathering in scientific experiments involves errors in measurement, so the data does not exactly fit the theoretical relationship.)



If the experiment is repeated and the new points are plotted, they will usually not coincide with the first points and the scatter diagram is more apparent. An example of this can be seen in *Decay and Half-Life* in the ENVIRONMENT section.

#### COMMENTARY

SCATTER DIAGRAMS

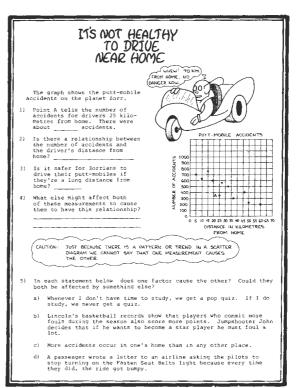
The activities in this section involve students in making scatter diagrams that show positive relations (Are You A Square or A Rectangle?, Do They All Line Up?, and It Sounds Like  $20^{\circ}C$ ), a negative relation (Come Rain or Come Shine) and no relation (*iHabla Usted*, El Español?). Other activities ask students to make scatter diagrams to see if a relationship is more likely to exist between one pair of variables than another (A Lengthy Relationship). Some activities show pairs of variables that fit exactly on a line or curve (Lay It On the Line and Sounds Fishy to Me).

Your students might think one variable is the "cause" and the other is the "effect." This is a common error in thinking. The scatter diagram below shows a

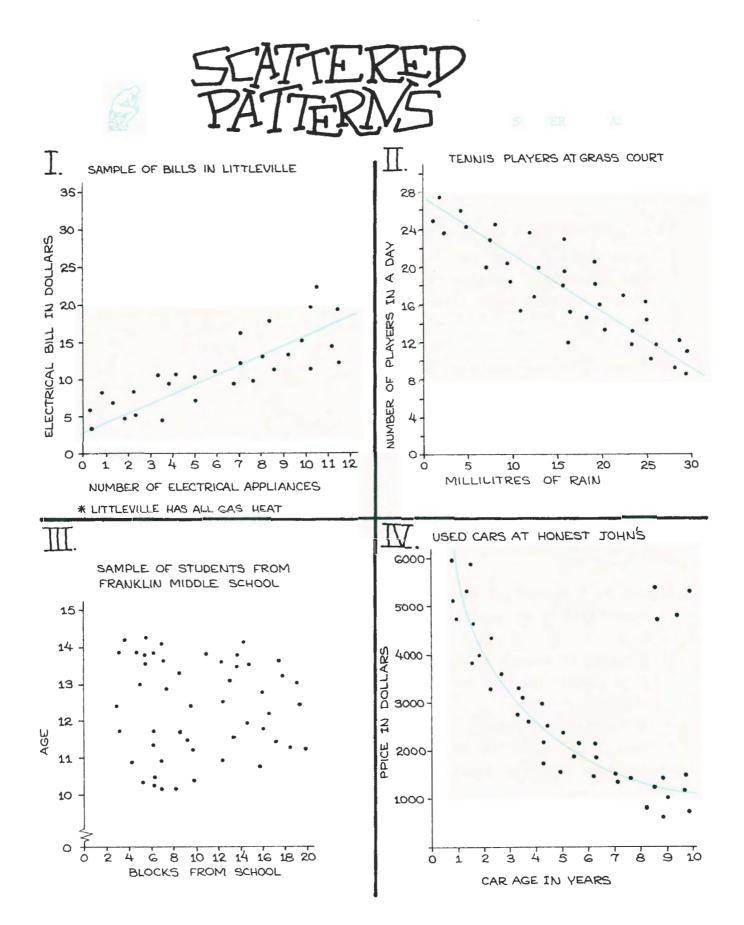
negative relationship between the distance from home and the number of accidents. Does this mean you are safer when you are driving in another state? (No.) Does the distance from home cause the number of accidents? (No, it is the greater number of miles driven close to home that accounts for the larger number of accidents happening near home.)

Help your students become critical thinkers by discussing with them the ideas in *It's Not Healthy To Drive Near Home*. They might be able to think of more examples of errors in cause and effect reasoning.

Although students will not see scatter diagrams as frequently as other



types of graphs, they can learn to appreciate their use in presenting statistical data. The activities in this section will give students experience in using scatter diagrams to look for relations.



### ARE YOU A SQUARE MOR A RECTANGLE ?

Materials: Two metric tape measures, two colored pencils

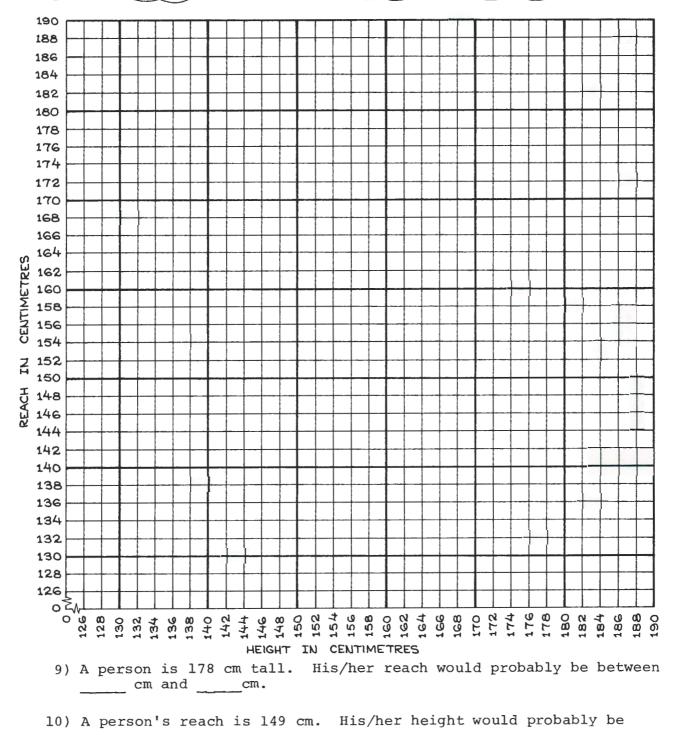
Activity:

- Attach one tape measure to the wall with the 0 mark at the floor. Attach the other tape measure to the wall about shoulder high with the 0 mark in a corner.
- 2) Stand with your back to the wall. Have a partner measure your height. cm
- 3) Stretch your arms out. Have a partner measure your reach (fingertip to fingertip). Be sure one fingertip touches the 0 mark in the corner ______ cm
- 4) How much longer or shorter is your reach than your height? _____ cm
- 5) A "square" person's reach is less than 3 cm longer or shorter than his/her height. A "rectangle" person's reach is more than 3 cm longer or shorter. Are you a square or a rectangle?

- 6) On the grid on the next page, graph a point to represent each person's height and reach. If the person is a square, use one color; if a rectangle use a different color.
- 7) Draw a loop around the point that represent square people.
- 8) What do you notice about the position of the loop on the grid?



This type of graph, called a scatter diagram, can be used to see relationships. In this case, you should see that as heights get taller, reaches get longer.



between _____ cm and _____ cm.

485



178-199 182-204

WOMEN MEN Desirable Weights for Ages 25 and Over Weight in Pounds (In Indoor Clothing) Desirable Weights for Ages 25 and Over Weight in Pounds (In Indoor Clothing) Height (with shoes on) 2-inch heels Ft In. Height (with shoes on) 1-inch heels Ft. In. Small Frame Medium Frame Large Frame Smali Frame Medium Frame Large Frame 4 10 4 11 5 0 5 1 126-141 129-144 92-98 94-101 118-129 121-133 96-107 104-119 112-120 115-123  $\begin{array}{c} 106\cdot 122\\ 109\cdot 125\\ 112\cdot 128\\ 115\cdot 131\\ 118\cdot 134\\ 121\cdot 138\\ 125\cdot 142\\ 129\cdot 146\\ 133\cdot 150\\ 137\cdot 154\\ 141\cdot 158\\ 145\cdot 163\\ 149\cdot 168\\ 153\cdot 173\\ \end{array}$ 4 11 5 0 5 1 2 3 5 5 5 4 5 5 5 5 5 6 5 5 5 7 5 9 5 10 98-110 101-113 124-136 127-139 130-143 134-147 132-148 135-152 96-104 4 5 7 8 9 118-126 118-126 121-129 124-133 128-137 132-141 136-145 99-107 102-110 105-113 108-116 101-113 104-116 107-119 110-122 113-126 135-152 138-156 142-161 147-166 151-170 155-174 159-179 164-184 168-189 173-194 134-147 138-152 142-156 146-160 150-165 154-170 158-175 162-180 167-185 172-190 113-126 116-130 120-135 124-139 128-143 132-147 136-151 111.119 111-119 114-123 118-127 122-131 126-135 130-140 136-145 140-150 144-154 148-158 152-162 156-167 160-171 164-175

Study the heightweight chart to the left.

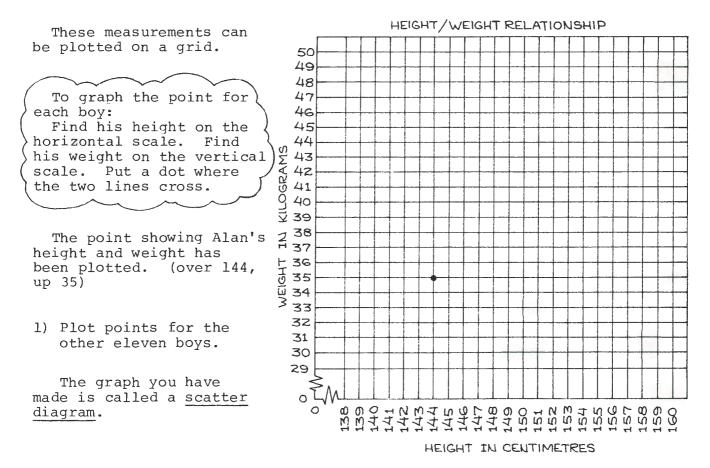
This chart, developed by the Metropolitan Life Insurance Company, suggests a relationship between height and weight.

To check this, twelve sixth grade boys were measured and weighed.

140.155

girls between 18 and 25, subtract 1 pound for each year under 25.

	P	LAN	NCK -	PUG	RANK		ILA S	OHN		ARK	NIKE	E A	OM
HEIGHT (CM)	144	144	160	147	151	150	141	157	154	140	142	160	
WEIGHT (Kg)	35	з4	45	34	41	43	37	49	45	29	32	50	



Permission to use chart granted by Metropolitan Life Insurance Company

For

134.144 138-148



Look at the scatter diagram on the first page. Notice that the points are not scattered all over the graph. They seem to lie in a narrow strip. It appears that as the boys' heights increase so do their weights.

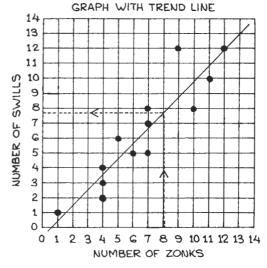
A line, called a <u>trend line</u>, can show this relationship. A trend line is drawn so it is close to each of the points.

To help you find a trend line:

- a) Use a black thread, a transparent ruler or a long piece of uncooked spaghetti.
- (b) Place the thread on the graph so about the same number of points are above the thread as below it.
- c) Draw in the line.

2) Draw a trend line for the graph on the first page.

A trend line can be used to make predictions. Suppose the number of zonks is 8. A good guess for the number of swills is about 7½. If the number of zonks is 3, the number of swills is about _____. If the number of swills is 9, the number of zonks is about _____.



- 3) Use your trend line on the first page to make these estimates.
  - a) A boy's height is 153 cm. A good guess for his weight is .
  - b) A boy's weight is 38 kg. A good guess for his height is



81.82.8

Students need to be familiar with trend lines. See Do They All Line Up? for an introductory activity.

Have you ever listened to a cricket chirp? Do they always chirp at the same rate?

A scientist listened to crickets.

He counted the number of chirps they made in one minute.

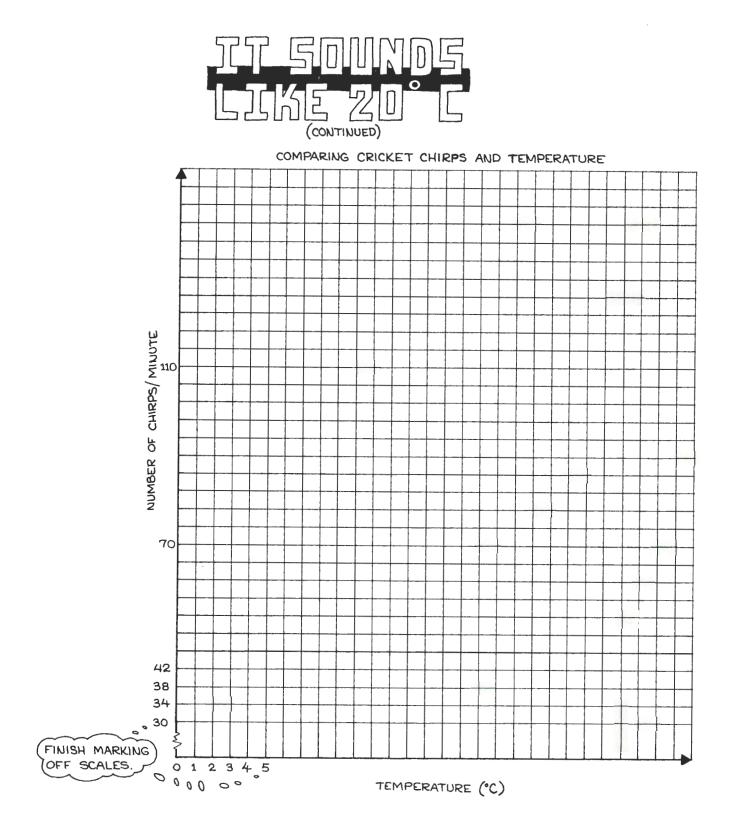
TEMPERATURE (C)	CHIRPS/MINUTE
පී	40
90	46
10°	52
11°	64
12°	ටට
13°	70
13°	רר
14°	80
15°	88
15°	90
1G°	90
16	୨୧
17	98

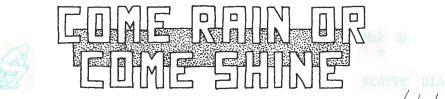
TEMPERATURE(C)	CHIRPS/MINUTE
17°	100
18°	101
18`	103
18°	108
19°	110
19°	115
20°	120
20°	126
21°	120
22°	130
22°	137
23°	143

Do you think there is a relationship between outdoor temperature and cricket chirps?

- 1) To check, make a scatter diagram of the data on the next page.
- 2) Do the points lie in a narrow strip? If so, draw a trend line.
- 3) Use the trend line to make some predictions.
  - a) A cricket chirps 56 times in a minute. What is the approximate temperature?
  - b) If the pattern continued, about how many chirps might a cricket make in one minute at 27° C?
  - c) At what temperature do crickets stop chirping?

*Discuss the dangers involved if a trend line is extended and use for predictions. Point out that the observer heard no cricket chirps a semperatures below 7°C. It may also be that crickets don't chirp at 27 Worksheet





I. A 6th grade class in Seattle has a car wash each year. They wanted to compare the amount of rain with the number of cars washed. By examining past records, the class found:

MILLIMETRES OF RAIN	20	3	20	16	0	10	4	22	4	12	12	10
NUMBER OF CARS	35	50	20	35	58	52	47	25	54	40	45	40

1) Do you think there is a relationship between the amount of rainfall and the number of cars?
2) Make a scatter diagram on the grid. Draw a trend line.
3) As the rainfall increases, does the number of cars washed increase or decrease?

This table gives the percent of field goals made from different distances by the teams in the Inter-Mountain Football Conference.

- 1) Is the percent of goals made related to the length of the field goal? _____
- Get some grid paper. Make a scatter diagram of the data. Find a trend line.
- 3) As the length of the field goals becomes shorter, what do you notice about the percent made?

PERCENT LENGTH OF FIELD OF FIELD GOAL (YDS.) GOALS MADE 15 75 40 20 35 35 30 35 10 95 20 80

5

65

45

25

CAR WASH DATA



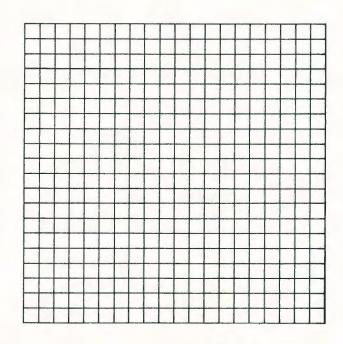
II.

# ¿HABLA USTED, EL ESPAÑOL?

In the table are test scores for ten students in Ms. Dias' class.

- Use the grid to make a scatter diagram of the test scores. Label the scales and write a title for the graph.
- Look for any relationship between the two test scores.
- 3) A student was absent for the geography test. She received a score of 45 on the Spanish test. Can you predict her score for the geography test?

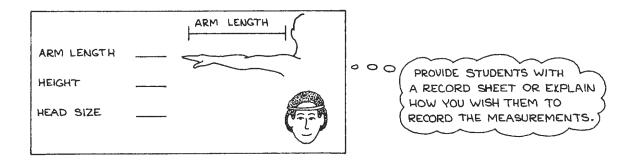
STUDENT	SPANISH TEST	GEOGRAPHY TEST
ALEX	90	40
CAROL	28	78
FRED	57	45
GAIL	76	30
INEZ	34	28
MARY	68	70
PENNY	86	92
RANDY	36	48
STEVE	84	60
WESLEY	50	90



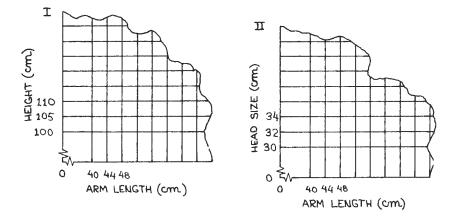


Some measurements are related (height-weight)--some are not (height-I.Q.). In this activity, scatter diagrams are used to investigate other possible relationships.

- 1) Group students in pairs. Students need a metre stick and string. Tell students they are to take some measurements. The class will then look at the results to see if the numbers are related.
- 2) Have students take and record (to the nearest centimetre) the measurements shown below.



3) In advance, prepare two transparencies (or posters) of these grids:



4) Place grid I on the overhead. As each student reports his/her arm length and head size, graph the ordered pair.



5) Have students look at the results. Do the points seem to lie in a narrow strip? If so, use a long piece of spaghetti to mark a trend line. Use the trend line to make predictions:

Lucy's arm is 60 cm long. How tall might she be?

Schroeder's height is 140 cm. How long would you expect his arm to be?

Point out that these predictions are only approximations. Also caution students against extending the line beyond the data to make predictions.

- 6) Place grid II on the overhead and repeat steps (4) and (5).
- 7) Tell students two measurements are closely related when the points lie close to the trend line.

Which two measurements are more closely related--arm length and height or arm length and head size?

Can students think of two other variables that might be closely related?

 Collect the students' data sheets. Compile two lists--one for boys, one for girls.

HEIGHT	ARM LENGTH	HEAD SIZE
h~~~		

9) On the following day, give each girl the set of girls' data and two grids. Do the same for the boys.

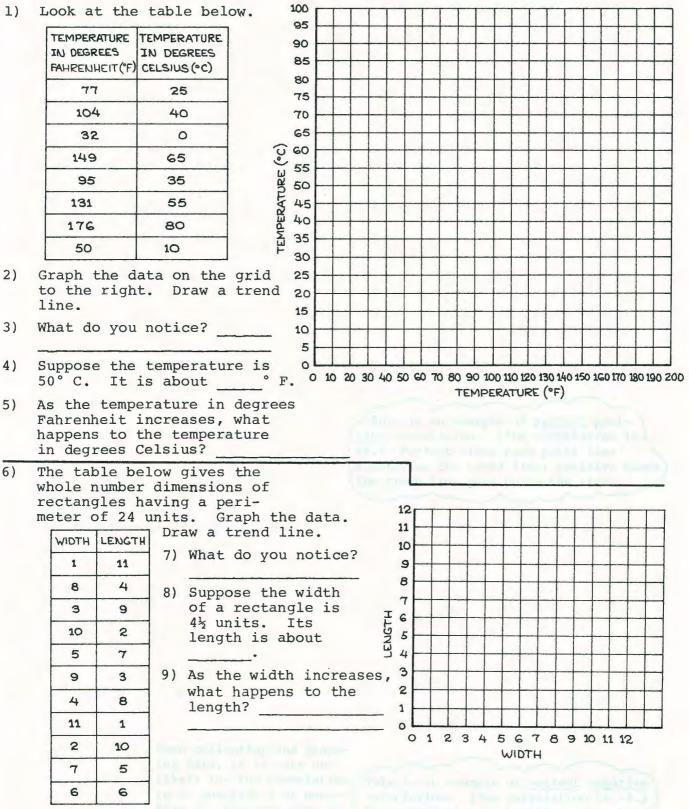
Have them draw scatter diagrams and mark trend lines.

10) Are height and arm length more closely related for either group (boys, girls) than they were for the whole class? Place grid I on the overhead for students to compare.

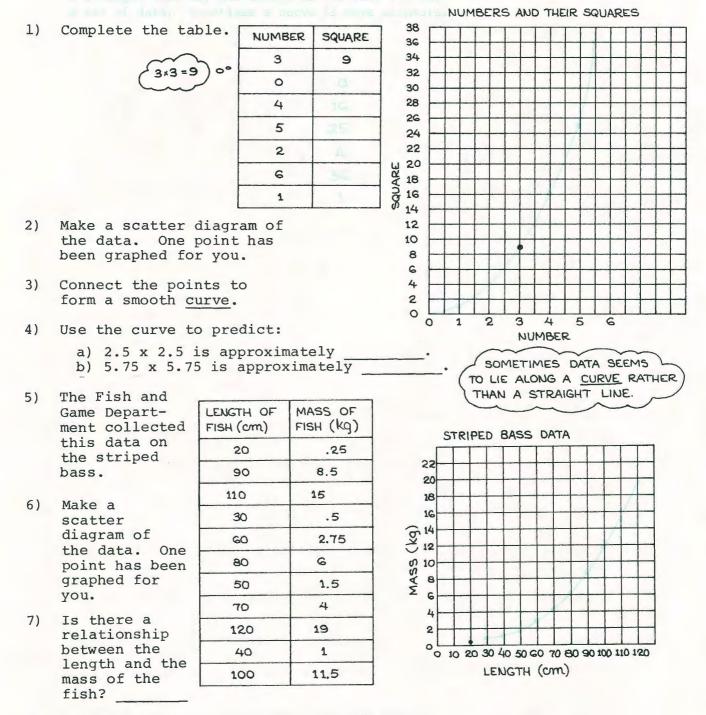
The answers may vary. Sometime: the measurement: for each group will be more closely related, sometimes not.

On some graphs, the points are so scattered that a pattern or trend line is not apparent. In this case, we say that the two measurements are not related For example, we would not expect any relationship between height and scores in math.

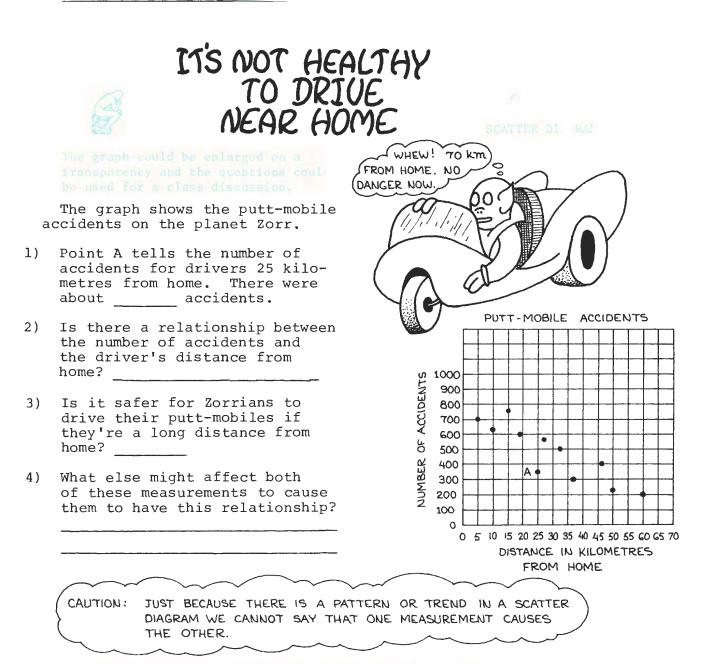








- 8) Connect the points to form a smooth curve.
- 9) Use the curve to predict: A fish is 75 cm long. Its mass is approximately A fish has a mass of 10 kg. Its length is approximately _____.



tatem its below buld be used for a lass di sur ion

- 5) In each statement below does one factor cause the other? Could they both be affected by something else?
  - a) Whenever I don't have time to study, we get a pop quiz. If I do study, we never get a quiz.
  - b) Lincoln's basketball records show that players who commit more fouls during the season also score more points. Jumpshooter John decides that if he wants to become a star player he must foul a lot.
  - c) More accidents occur in one's home than in any other place.
  - d) A passenger wrote a letter to an airline asking the pilots to stop turning on the Fasten Seat Belts light because every time they did, the ride got bumpy.

ks

## ORGANIZATION

### Placement Guide for Tabbed Dividers

Tab Label	Follows page:
CONTENT FOR TEACHERS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
Probability with Counting	
Inferential Statistics	
Appendix-Computer Programs	
DIDACTICS	
Components of Instruction	
Classroom Management	
Statistics and Probability Learning	
TEACHING EMPHASES	
Critical Thinking	
Decision Making	
Problem Solving	
Models and Simulations	
Calculators and Computers	
Laboratory Approaches	
CLASSROOM MATERIALS	
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Recreation	
GLOSSARY	
ANNOTATED BIBLIOGRAPHY	
SELECTED ANSWERS	



Commentary to MISLEADING STATISTICS (498-502)

TITLE	PAGE	TOPIC	TYPE
This Doesn't "Ad" Up	503	Examining the claims of advertisements	Teacher directed activity Transparency
Examining the Facts	505	Examining claims of advertisers	Transparency Discussion Worksheet
Figures Never Lie	507	Examining a deceptive bar graph Making an honest bar graph	Worksheet
Butter Up the Cook	508	Examining a deceptive bar graph Making an honest bar graph	Worksheet Transparency
Food for Thought	509	Examining deceptive bar graphs	Worksheet Transparency
I Didn't Plant It That Way	510	Examining deceptive pictographs	Worksheet Transparency
There's Music in the Air	512	Comparing two types of pictographs	Transparency Discussion

# MISLEADINC STATISTICS

Many people accept statistical arguments without considering that they might be misleading. One reason for including statistics in a general curriculum is to help people understand and carefully examine the statistical statements, tables and graphs used to influence them. Middle school students can begin to ask critical questions about statements made by advertisers, politicians and even their peers. They can also read tables and graphs critically. It is probably best to have students work with "honest" statistics before pointing out too many of the misuses of statistics. Concentrating too early on the misuses could make students distrustful of any reference to statistics.

#### STATISTICAL STATEMENTS

Help your students see that misleading statistical statements can be very subtle. During a much publicized trial, a television news commentator reported that the prosecution had spent three hundred thousand dollars while the defense had spent nearly a quarter of a million dollars. Some people in the audience probably thought the defense

DOLLARS PROSECUTING ANGELA DAVIS, WHILE NEARLY A QUARTER OF A MILLION DOLLARS HAS BEEN SPENT ON HER DEFENSE. WOW! A QUARTER OF A MILLION!

THE STATE OF CALIFORNIA HAS

had spent more because of the magical word "million." The figures might have been accurate, but the impression created could be false. Try reading the commentator's statement to your class. Are any of them misled?

Most students will need help initially in examining the statements of advertisers. The activity *This Doesn't "Ad" Up* helps students examine the claims in advertisements and political statements. (Some are misleading, others are not.) *Examining* 

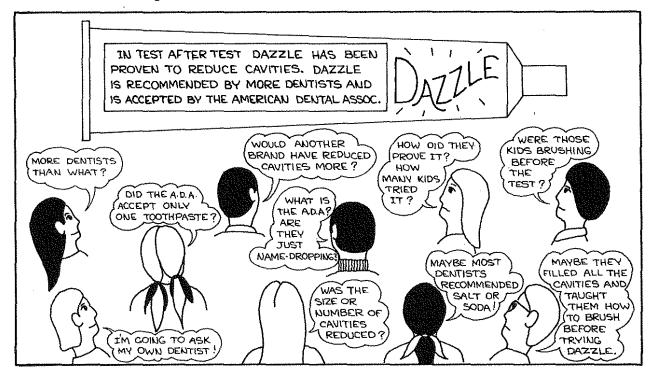
the Facts gives students additional experience in examining claims of advertisers. It is probably more interesting to bring actual advertisements or political statements to class. (Be sure to include advertisements which do not try to mislead statistically; students should not form the opinion that all advertising is false.)



498

#### COMMENTARY

Below is an ad for a hypothetical toothpaste and some possible questions and comments such an ad might stimulate.



Students in your class might want to complain about a particular advertisement. Specific information on filing an advertising complaint is contained in a bulletin called <u>If You Have a Complaint About Advertising...</u> published by the National Advertising Review Board (NARB), 845 Third Avenue, New York City, 10022. These points are summarized in the bulletin:

- Put your complaint in writing.
- 2. Be specific about where and when you saw or heard the advertisement. If it is a printed advertisement, send the original ad (or a copy) with your letter.
- Address your complaint to: NAD, 845 Third Avenue, New York City, 10022. (NAD is the National Advertising Division of the Council of Better Business Bureaus.)

(This refers to complaints about national advertising. File complaints on local advertising with your local Better Business Bureau listed in the phone book.) The bulletin says your complaint will be promptly acknowledged and investigated by NAD, and you will receive a report on the ultimate outcome of the case.

Other publications from NARB which might be of interest to students are the bulletin <u>Children's Advertising Guidelines</u> and the booklet <u>The National Advertising</u> <u>Review Board 1971-1975</u>. The booklet describes the complaints and decisions for that

four-year-period. An example is given below.

THE COCA COLA COMPANY, Foods Division (Hi-C Fruit Drink) Basis of inquiry - Television advertising claimed Hi-C to be "the favorite drink of kids and monsters everywhere." Notwithstanding the humorous reference to "monsters," it was the opinion of NAD's Children's Advertising Review Unit that the claim implied children's preference for Hi-C over all other drinks, and supportive data was requested. A consumer tracking study and market survey information was submitted which NAD did not find sufficiently related to the claim since children were not interviewed on brand preference, nor were any other drinks (sodas, instant drink mixes, etc.) other than ready-to-serve fruit drinks taken into consideration. When advised of NAD's position that the claim was not appropriately qualified to specify the ready-to-serve fruit drink category, the advertiser replied that it had discontinued use of the commercial and had replaced it with another stating Hi-C was "the favorite fruit drink of kids and monsters everywhere" (emphasis added). However, since the data did not address the powdered or concentrated fruit drinks, NAD held to its view that the material provided did not substantiate the revised claim. Although the advertiser disagreed with NAD's interpretation and considered the material fully supportive of the questioned statements it advised that, for its own reasons, the advertising would be discontinued.

News from NAD, for release on and after November 15, 1976.

#### TABLES

Tables can also be misleading. As skills in reading tables are taught, students can be encouraged to ask questions like, "Who made this table? Are they biased and trying to prejudice us? Where and how was the information gathered?" For example, suppose the table on the next page is being used to try to determine which cities are the most polluted. Bismarck, North Dakota, has a much higher reading than San Diego, Seattle, or Minneapolis/St. Paul. That seems strange. Bismarck has about 35,000 people and is located in a state with a low population and with few factories. What could have contributed to these unexpected results? Is it a misprint? Over what period of time were the readings taken? In what part of each city were the readings taken? Near the local smoke stack? Over a lake? Were the readings for Minneapolis taken during a rainy season while those for Bismarck were taken during plowing season when lots of dust was in the air? The table doesn't say. The source of the data is probably not biased; however, students should realize that the information cannot be used to conclude some cities are polluted until more is known about how the data was gathered. (Students should also realize that suspended particles are only one factor to consider in air pollution--see *City Circumstances* in the ENVIRONMENT section for a student activity on this.)

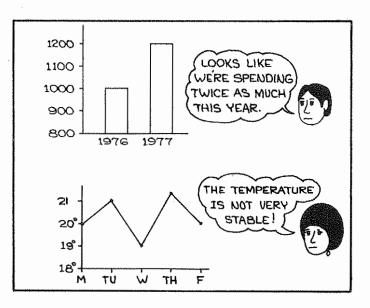
	(1.1.1	microg	rams per	cubic meter. ]			
Υ.	Min.	Max.	Geom. mean	STATION	Min.	Max,	Geom mean
hoenix akland n Diego n Francisco	60 21 26 29	283 100 140 121	144 57 39 60	New York Rochester	. 39	93 177 213	34 85 87
enver artford ew Haven	38 25 23	355 137 131	127 60 60	Columbus		207 135	87 78
iami Ilanta	32 39	132 203	62 82	City		194	67
onolulu bise hicago es Moines	25 20 39 34	70 254 336 202	40 69 97 85	Pennsylvania Pittsburgh Rhode Island Providence	- 69	259 129 157	88 135 69 63 95
ansas City	29	227	116	Nashville	54	249	10
ew Orleans altímore etroit inneapolis	43 44 41 31	119 166 236 130	73 90 111 64	Houston San Antonio	- 43 - 28	349 177 141	80 89 5-
. Louis (CAMP) ¹ maha eno	32 43 38	393 331 218	120 113 93	Virginia	. 9 - 24 - 33 - 41	116 109 181 197	93 55 41 98 93
	noenix	i Min. noenix	Min.         Max.           noenix	Min.         Max.         Geom. mean           ioenix	mean           incenix	Min.         Max.         Geom. mean         STATION         Min.           noenix	Min.         Max.         Geom. mean         STATION         Min.         Max.           noenix

Table from Statistical Abstract of the United States 1974, p. 180.

(Suspended Particulate Matter: Particles of smoke, dust, fumes and droplets in the air.)

#### GRAPHS

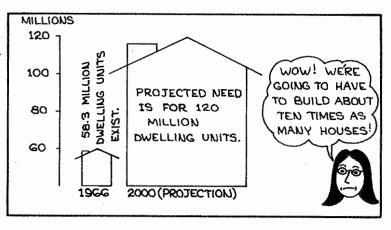
Most students probably take graphs at face value. They might look only at the different heights of a bar graph without noticing that the scale does not begin at zero. The sharp ups and downs of a line graph might be the only thing they notice. They might not realize that the temperature range of the line graph to the right is really very small.



COMMENTARY

MISLEADING STATISTICS

Pictographs can also be deceptive. The one to the right seems to show that even chimneys. on homes will need to be bigger! Not only does the scale not start at zero, but the area of the larger house is at least ten times the area of the smaller house, while the numbers to be represented (120 million versus 58.3 million) are in a ratio of about 2 to 1.



Misleading pictographs are examined in I Didn't Plant It That Way, and There's Music in the Air. The activities beginning with Figures Never Lie involve examining deceptive bar graphs, making "honest" graphs, and comparing the graphs.

You might want to make a list of things that might mislead. Students can help add to the list. Some suggestions are given below.

THINGS IN GRAPHS THAT MIGHT MISLEAD

- Does it have a title and labels?
- Is the source of the data given?
- If it's a pictograph is there a key?

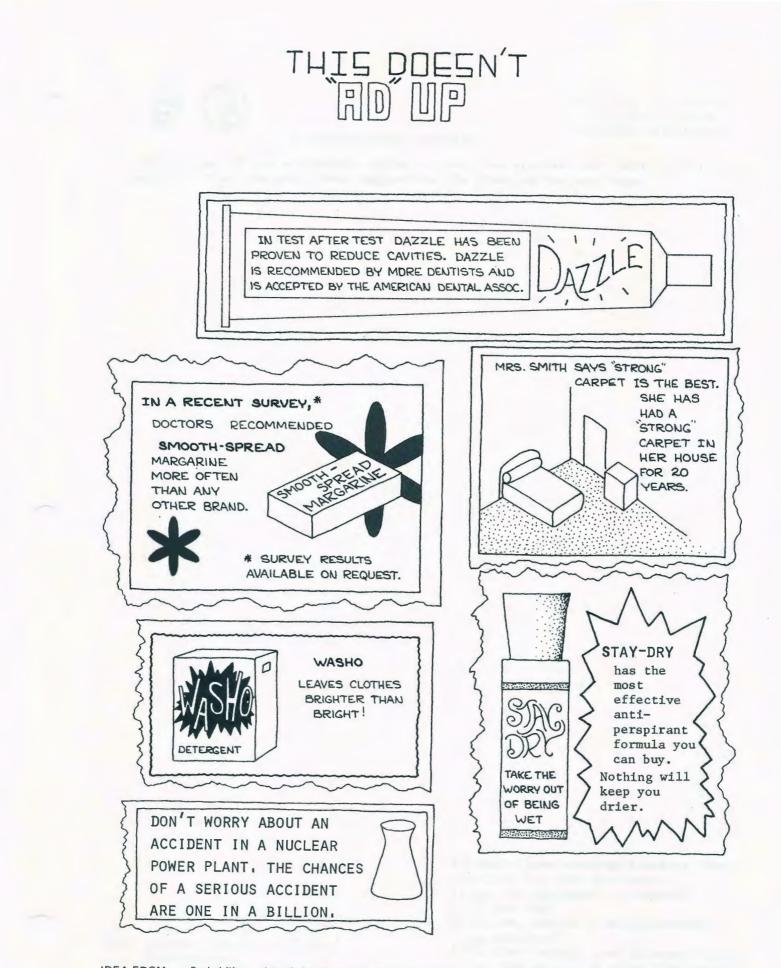
- are the symbols all the same size?

• If it has a scale - does it start with zero?

- if not, is a clear zero break shown?

- are the numbers on the scale evenly spaced?
- How might the graph change if the horizontal or vertical scales were stretched or shrunk?

Examples of misleading graphs from magazines or advertisements can make these activities more meaningful. Examples of honest graphs should be included so students can see that many graphs in written material are designed to convey data as clearly as possible.



IDEA FROM: Probability and Statistics, Nuffield Mathematics Project, John Wiley and Sons, Inc., 1969 Reprinted by permission of John Wiley and Sons, Inc.



#### Suggestions for the ads:

#### 1. Dazzle Toothpaste:

How many tests? Two? (See page 2 of the commentary for this section for additional questions on this ad.)

2. Smooth-Spread Magarine:

How many doctors were surveyed? How were they picked? Maybe doctors don't recommend margarine often. Was Brand B recommended 100 times and Smooth-Spread 101 times? One good point is that survey results are available on request. If students can find an ad like this, they might write for the results and try to make a better decision about the value of the product.

3. "Strong" Carpet:

Who is Mrs. Smith? Does she know anything about carpet? Has she used any other kind? In what room has she had "Strong" carpet? An unused guest room? A closet? Rolled up in an attic? Does the picture show "Strong" carpet or some other kind? In what kind of shape is the carpet?

4. Washo Detergent:

What is brighter than bright? Are the clothes bleached white? Does it change colors?

5. Stay-Dry Deodorant:

Does every anti-perspirant use the same formula? Will "nothing" keep you drier? Would it be good for a basketball player not to perspire?

6. Nuclear Power Plant:

Do you think it is known for certain what the chances are for a major nuclear accident? Can an event with very low chances occur? What damages would result if a serious nuclear accident did occur?

Note: This activity is more meaningful if actual ads are also discussed. You could make transparencies or slides of some and discuss them with the class. Others could be mounted for small group work. Groups could write questions that they would like answered about the product or about the claims on the ad. Have groups share this information with the class.

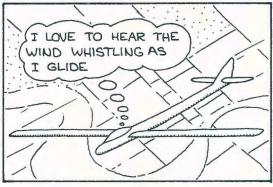
IDEA FROM: *Probability and Statistics,* Nuffeld Mathematics Project, John Wiley and Sons, Inc., 1969 Reprinted by permission of John Wiley and Sons, Inc.



The Federal Trade Commission is alert to incorrect or deceptive advertising. For this reason, manufacturers are careful that their claim is factually true. However, a factually true statement can be misleading if it doesn't mean anything or if the statement is true about all similar products. When reading an ad, ask yourself two questions. If the claim is true, is it really important? Is that claim true about all similar products?

 One car manufacturer claims its car is "more quiet than a glider." (Actually a glider is fairly noisy.) Does that tell you that the car is quiet?

Can you tell if the car is quieter than cars of the same size and price?_____



2. Another car company claims that more than 95% of all its cars sold in the U.S. in the past ll years are still on the road. What is this manufacturer trying to get you to believe?

Would you believe that these cars are better than all others if you knew that 95% of <u>all</u> cars sold in the past ll years are still on the road?

Would you be impressed if you knew the manufacturer sold very few cars 7, 8, 9, 10, and 11 years ago?_____

This same manufacturer claims that its sedan has a turning radius much shorter than that of a Cadillac Eldorado or Continental Mark IV. Does that convince you that this sedan is better when you realize that it is a small car, only 2/3 as long as the big cars mentioned above?_____ Do you know how the turning radius of the sedan compares to that of a V.W. bug?_____

Would you be surprised if the shorter car did <u>not</u> have a shorter radius?

3. One company claims its fruit juice has 10% more fruit solids than required by U.S. government standards. The government standard referred to requires 10% fruit solids. What is 10% more than 10%? Do you think 11% fruit solids is a lot better than 10%?



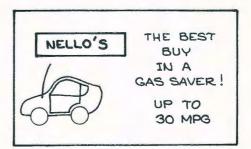
4. A certain home study school claims to have enrolled over 2 million people. From this claim, can you tell how many people graduated? (Actually only about 12% of all people enrolling in home study schools ever finish.)

Could you tell from the claim how many people got good jobs because of the home study training?_____

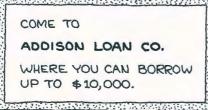
- 5. One pain reliever claims one pill was proven better than two aspirin in two hospital studies. Do you think two hospital studies are enough to be convincing? _____ Do you know on how many people the pain killers were tested? _____ Do you think these studies involved the type of pain that most people get? (Actually, they did not -- the studies involved types of pain that are relatively rare.)
- 6. A brand of pipe tobacco claims that it is 44% fresher. 44% fresher than what?_____
- 7. We are told that 4 out of 5 dentists recommend a brand of sugarless gum for their patients who chew gum. How many were surveyed?_______ Five?_____ Were the dentists picked in a random sample?_______ Were they employees of the gum company?

Do most	dentists	recommend c	or comment	on gun	chewing?
Can you	tell from	n the inform	mation abov	ve?	

- 8. A company claims its cranberry juice has more food energy than orange juice or tomato juice (food energy is measured in calories). Do you think most people want a drink with more calories?
- 9. Can you tell what gas mileage you will get from a car bought at Nello's?



10. Can you tell how much money you can borrow from Addison Loan Company?



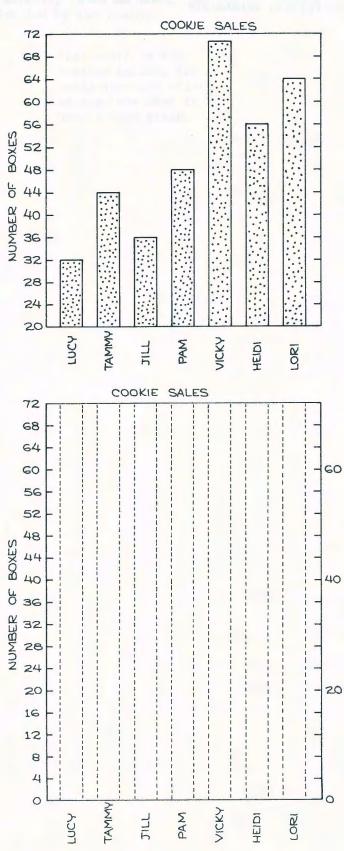


The bar graph shows the total boxes of Girl Scout cookies sold by each girl in the patrol.

- Look at the height of the bars for Lucy and Tammy. Tammy's bar appears times as high as Lucy's bar.
- 2) Tammy sold boxes. Lucy sold boxes. Has Tammy sold twice as many boxes as Lucy?
- 3) Look at Heidi's bar and Lucy's. Heidi's bar appears times as high as Lucy's.
- 4) Heidi sold boxes. Has Heidi sold three times as many boxes as Lucy?
- 5) Does the graph seem to mislead you even though the sales are reported correctly?
- 6) Use the information from the top graph to complete the bar graph to the right. Shade each bar to the appropriate height.
- 7) In the new bar graph, how does the height of Lori's bar compare to the height of Lucy's bar?

Has Lori sold twice as many boxes as Lucy?

 How does the scale on the bottom graph differ from the scale on the top graph.

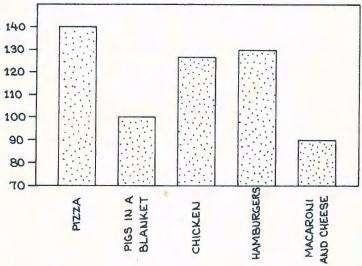




The students in Ms. Ne. class decided to survey the's school to find out which hot lunch meal was the favorite. The top five choices are listed to the right.

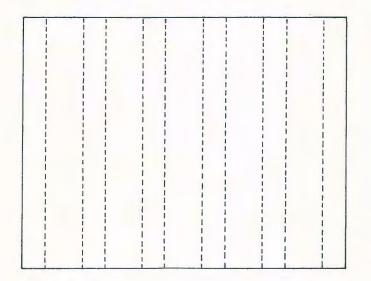
MEAL	NO. OF STUDENTS		
PIZZA	140		
PIGS IN A BLANKET	100		
CHICKEN	127		
HAMBURGERS	130		
MACARONI AND CHEESE	90		

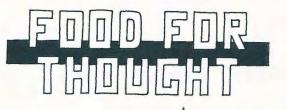
Alex hates macaroni and cheese. He drew the graph to the right to try to convince the cooks never to fix macaroni and cheese again.

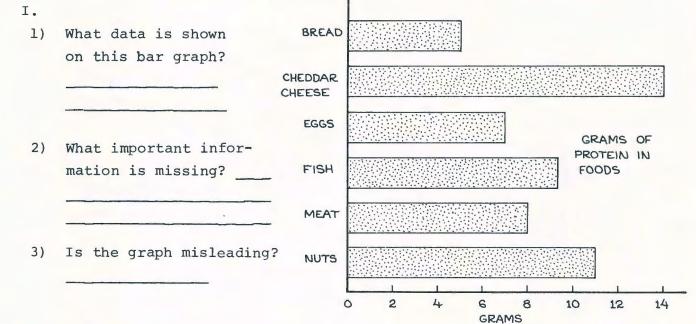


HOT - LUNCH FAVORITES

Lucy saw the graph. She told the cooks that Alex was trying to mislead them. Make an honest bar graph that Lucy could use to show them the true results.



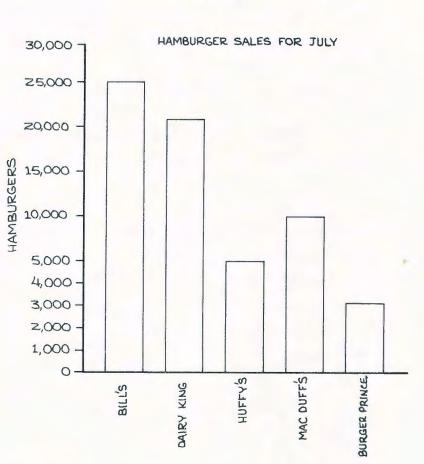




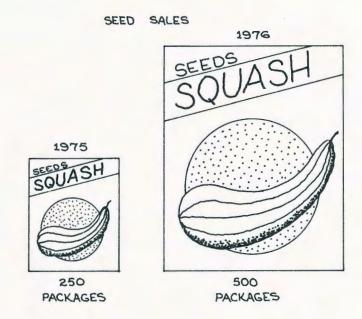
II.

1) How many hamburgers
 did each drive-in
 sell during July?
 Bill's
 Dairy King
 Huffy's
 MacDuff's
 Burger Queen ______

- 2) Is the graph misleading? _____ If so, how? _____
- Redraw the graph as you think it should be.



# I DIDN'T PLANT IT THAT WAY



A class at Brook's School sells packages of seeds each year to pay for a spring field trip.



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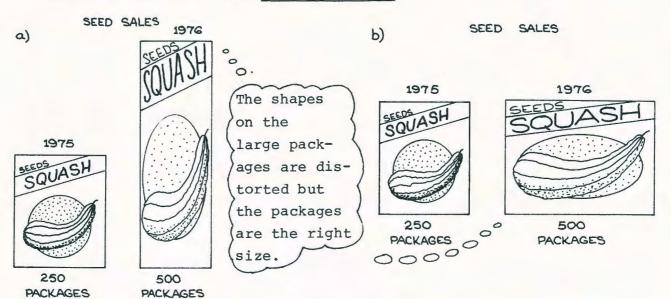
One of the students drew the pictograph above.

 Do the pictures show that the class of '76 sold twice as many packages as the class of '75?

To help you answer the question, trace the small package of seeds. Cut it out. Place it on the large package. How many small packages are needed to cover the large package? ______ The pictures imply the sales for 1976 are actually _________times as many as in 1975.

## I DIDN'T PLANT AT WAY (CONTINUED)

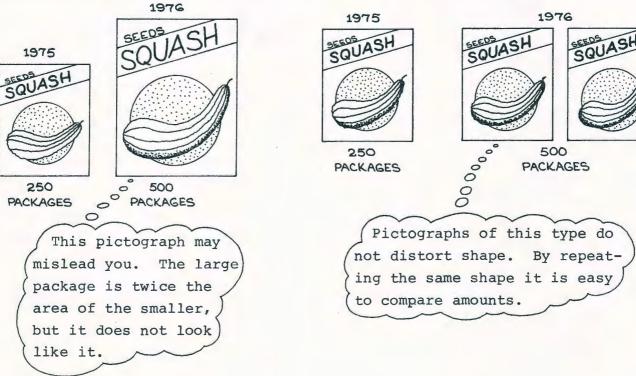
2) Look at the pictographs below. Each is drawn correctly. Which do you like best?



C) SEED SALES

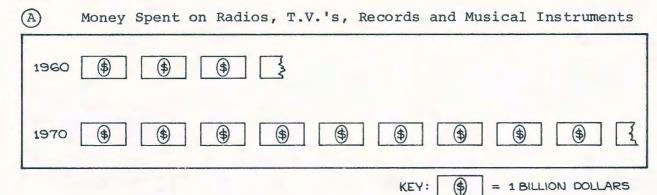
d)

SEED SALES

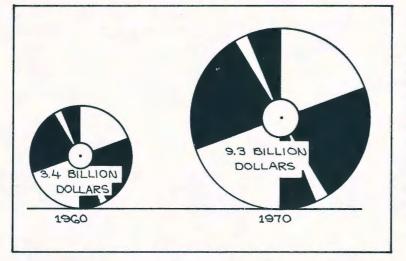




Look at the two pictographs below:



(B) Money Spent on Radios, T.V.'s, Records and Musical Instruments



Both graphs A and B are correctly drawn.

## ORGANIZATION

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Recreation	
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ANNOTATED BIBLIOGRAPHY	
SELECTED ANSWERS	

# CONTENTS

Commentary to MEAN, MEDIAN, MODE (515-521)

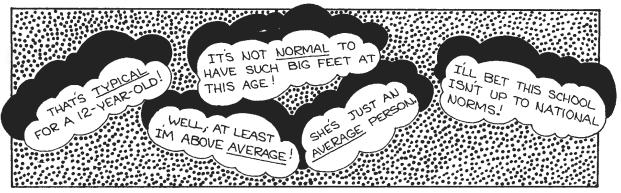
TITLE	PAGE	TOPIC	TYPE
Zappo the Zarian	522	Motivating mean, median, and mode	Discussion Transparency
Number Please	524	Finding the modal last digit in a telephone number	Activity card Worksheet
A la Mode	525	Finding the mode in a frequency graph	Worksheet
A Classy Median	526	Demonstrating the concepts of median and mode	Teacher directed activity
The Student in the Middle	527	Finding a median height	Worksheet
Get It Together	528	Demonstrating a mean area	Demonstration
A Mean Train	529	Finding a mean length	Activity card
A Balanced Rod	530	Relating the mean to a balance point	Activity card
A Mean Guess	531	Demonstrating the use of an assumed mean	Activity card Demonstration
A Shortcut to the Mean	532	Using an assumed mean	Worksheet
A Mean Method	533	Find the mean of a frequency distribution	Worksheet
Don't Quack Up!	534	Finding mean, median, or mode	Worksheet Puzzle
A Mean Location	535	Finding mean, median, or mode	Worksheet Puzzle
Median vs. Mean	536	Showing effects of extreme values on median and mean	Activity card
The Average Family	537	Choosing mean or mode to describe a distribution	Worksheet
Means with M & M's	538	Using the mean to describe a distribution	Worksheet Teacher directed activity
What Do You Think?	539	Choosing mean, median, or mode to describe a distribution	Worksheet

- - -----

_____

513

# MEAN, MEDIAN, MODE

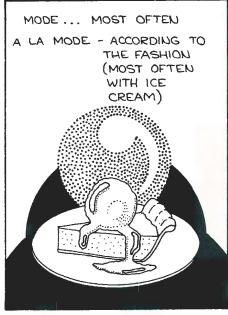


The everyday conversations of people show the tendency to generalize about large groups. Average, typical or normal people are discussed as though such people exist and are identifiable. The search is for standards (sometimes representative and sometimes not) against which everything else can be measured.

Adolescents, too, search for a "norm" with which to compare themselves. "Is it normal that I am this tall or short? That my feet are this big? That I feel this way?" This concern of middle school students for a standard of comparison can be used to motivate a unit on mean, median and mode. *Zappo the Zarian* is a story about a teenage being on another planet who is concerned about being "odd" and "not normal." He gathers data and discovers that he is unusual in some ways but is "normal" or "average" in other ways. You could have students read the story and then relate the concepts of mode, median and mean to the story.

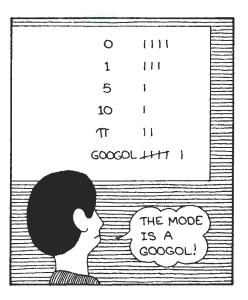
#### MODE

The idea of mode should be easy for most students to grasp, but they might have trouble keeping the word "mode" attached to the item that "occurs most often." You could point out to students that "mode" and "most often" begin with the same letters. The definition of a la mode might also help them remember the definition for mode. A la mode means according to the fashion or the <u>most popular</u> choice. A dessert of pie and ice cream is very popular, so it is called "pie a la mode."

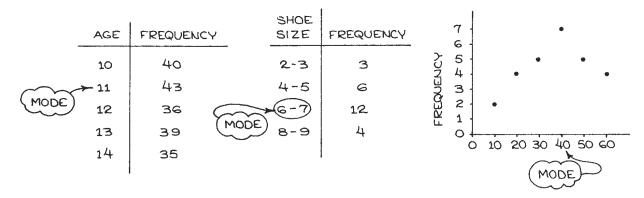


The mode can be introduced easily by asking students to write down a number. (You might want to restrict their choices to numbers between 1 and 10 or list possible choices on the board.) The number that students choose most often is the mode of the numbers picked.

By asking students their eye color, favorite athlete or type of pet, you can give students examples of modes that are not numbers, i.e., the mode (most frequent) eye color. Be sure to include some examples which do not have a mode such as { 2, 10, 6, 8, 5 } and examples that are bimodal like { 2, 2, 2, 3, 5, 7, 7, 7, 9 }.



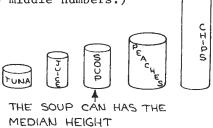
Demonstrate how easy it is to read the mode from a frequency table or a frequency graph. The item (or interval) with the greatest frequency is the mode. Have students work *A La Mode* to reinforce this idea.



#### MEDIAN

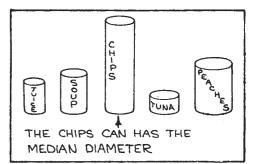
Students have probably heard about the median strip down the middle of two-way roads. Perhaps this can help them remember that median stands for the middle number or measurement (or the number midway between the two middle numbers.)

To demonstrate the concept of median, you could use the ideas in *A Classy Median*. Here is a similar demonstration. Bring five cans of various sizes. Arrange them on your desk in order of height. Measure the middle can to determine the median height.



#### COMMENTARY

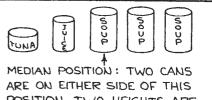
Ask a student to find and measure the median diameter of the cans. (The student measures the diameters and arranges the measurements in order or the cans are arranged in order as shown and the diameter of the middle can is measured.)



Add another can to the group and have a student reorder them by height. Explain that no can is in the middle, so the median height will be halfway between the two middle heights. Students can also find the median length of the six diameters. *The Student in the Middle* can be assigned to students to reinforce their skills in finding medians.

Students will need to put a set of data in order before they can determine the median of the set. After ordering the numbers, they can "tic" off the least and greatest numbers in pairs until the middle number is left. For an even number of data, students will need skills in finding the number halfway between the two middle values---add the two numbers and divide the sum by 2.

Many students will think that there are as many measurements greater than the median as measurements less than the median. To help them see this is not always true, show sets of cans where the median height is shared by several cans. In the set to the right, at least one half are less than or equal to the median height, and at least one half are greater than or equal to the median height.

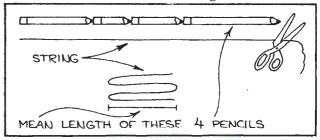


POSITION. TWO HEIGHTS ARE LESS THAN THE MEDIAN HEIGHT BUT NO HEIGHTS ARE GREATER THAN THE MEDIAN HEIGHT.

#### MEAN

Most students have computed an "average" for scores or measurements by adding the measurements and dividing by the number of measurements. You can give students

a concrete demonstration of a mean by using the ideas in *Does Everyone Have a Pencil?* or *A Mean Train*. You might also emphasize that the mean and the number of measurements can be used to find the sum of the measurements. Ask them to find the



total length of five pencils if the mean length is 12 cm.  $(5 \times 12 \text{ cm} = 60 \text{ cm})$  What is the total length of 20 pencils if the mean length is 15 cm?  $(20 \times 15 \text{ cm} = 300 \text{ cm})$ 

The mean is used more often than median or mode, so you will want to be certain students have skills in computing means by hand and with the aid of a calculator. These skills can be practiced by finding averages and handicaps related to baseball, basketball, golf, bowling or other sports (see the RECREATION section).

#### SOME STUDENT ERRORS IN FINDING MEAN, MEDIAN AND MODE

Here are some mistakes (misconceptions) that were observed in an eighth grade class.

• When students were asked to identify the mode of data in a frequency table as shown to the right, some students incorrectly gave 8, the highest frequency, instead of 25, the score with the greatest frequency.

Score	Frequency
20	7
22	5
25	8
27	4
30	2

- Sixteen different numbers were given and ordered. Some students thought the  $\frac{\text{median}}{2}$  (least number + greatest number). Some said, "The median is half of 16 or 8." Some thought the median was half of the sum of all 16 numbers. Others correctly found the average of the middle two numbers.
- When asked to find the <u>mean</u> of 16 numbers, most students correctly found the sum of the numbers and divided by 16. When they were asked to find the mean of data given in a frequency table, many students did not multiply the scores by their frequency even after instruction. Several incorrect methods were used. For the table shown above, they might have computed:  $\frac{1}{5}(20 + 22 + 25 + 27 + 30)$  or  $(20 + 22 + 25 + 27 + 30) \div (7 + 5 + 8 + 4 + 2)$  or  $\frac{1}{5}(7 + 5 + 8 + 4 + 2)$ .

#### CHOOSING AMONG MEAN, MEDIAN AND MODE

Students should be aware of advantages and disadvantages for each of the three common representative numbers: mean, median and mode. You might want to put a list like that given below on a poster for students.

THE MODE

#### Advantages

- Gives the item occurring most often
- Easy to read from a frequency table or graph

Disadvantages

- Might not be near center of the values
- Might be more than one mode
- Might change drastically with the addition of new scores
- Might not exist

#### THE MEDIAN

#### Advantages

- Gives the middle score (half are at or above the median; half are at or below the median)
- Not as easily influenced by extreme scores (see Median vs. Mean)

Disadvantages

- Does not give information about the total of the scores
- Requires that the scores be ordered
- Might not be a member of the set

#### THE MEAN

Advantages

- Is the most common meaning of the word "average"
- Can often be computed quickly by calculator
- Gives information about the total of the scores
- Is the most useful number in most statistical work

Disadvantages

Might be greatly influenced by extreme scores

Might not be a member of the set.

Activities like What Do You Think? and The Average Family ask students to choose among median, mean and mode to represent different situations. The ideas in these activities can be emphasized with class discussions. Students might offer logical reasons for using two different types of representative numbers.

#### COMMENTARY

#### THE CONFUSION OVER AVERAGE, MEAN, MEDIAN AND MODE

The word "average" is an everyday word that is usually synonymous with "mean." In this resource "average" has this common usage. You will want to warn your students that not everyone defines "average" as the mean.

This introductory paragraph to a height-weight chart involves several common ideas. The word "typical" is used--did the author intend the mode? The author refers to the 40th percentile; percentiles are related to medians because the median is the 50th percentile. Did the word "typical" stand for median? "Average" is used--is this the mean? We really don't know. (If the data was accumulated from a large sample or a large number of samples, the mean, median and mode might be very close.)

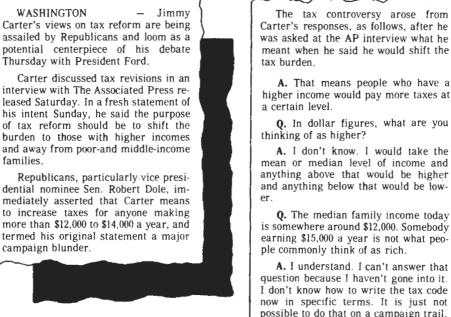
Warn students not to assume that all sources use the word "average" for the mean. You can reinforce this idea with the activitiy What Do You Think? You might also read students chapter 2, "The Well-Chosen Average" from Darrell Huff's <u>How to Lie With Statistics</u>. (Students can read the chapter themselves if they are about 9th or 10th grade reading level.)

# Where Do You Stand?

HE TYPICAL American man stands 5 feet 9 inches and weighs 172 pounds. The <u>typical</u> woman is 5 feet 3.6 inches tall and weighs 143 pounds. These are findings of a nationwide study of men and women aged 18 to 74 conducted from 1971 to 1974 by the Department of Health, Education, and Welfare.

The study just released shows that men generally weigh six pounds more and are 0.7 inches taller than in 1960-62. Women weigh three pounds more and are 0.5 inches taller. The chart below shows height and weight in terms of percentiles. If your weight equals the 40th percentile for your age group, then you're heavier than 40 per cent of Americans your age, and lighter than 60 per cent. But don't feel too satisfied. These aren't ideal weights, but <u>averages</u>, and the average American in your age group may be heavier than he ought to be.

In some cases it makes a lot of difference if the median is meant instead of the mean. You might read your class these parts of a news clipping from the 1976 presidential campaign.



#### Carter's responses, as follows, after he was asked at the AP interview what he meant when he said he would shift the

A. That means people who have a higher income would pay more taxes at

**0.** In dollar figures, what are you

A. I don't know. I would take the mean or median level of income and anything above that would be higher and anything below that would be low-

0. The median family income today is somewhere around \$12,000. Somebody earning \$15,000 a year is not what people commonly think of as rich.

A. I understand. I can't answer that question because I haven't gone into it. I don't know how to write the tax code now in specific terms. It is just not possible to do that on a campaign trail.

#### A FINAL NOTE

After all the discussion and computation of modes, medians and means, be sure you remind students of the main reason for studying these ideas. Point out that it is useful to have one or more numbers to represent or describe a set of data. These numbers can be used as rough standards for comparison; as standards for improvement; and as ways to get a quick (if very approximate) idea of the whole set of data.

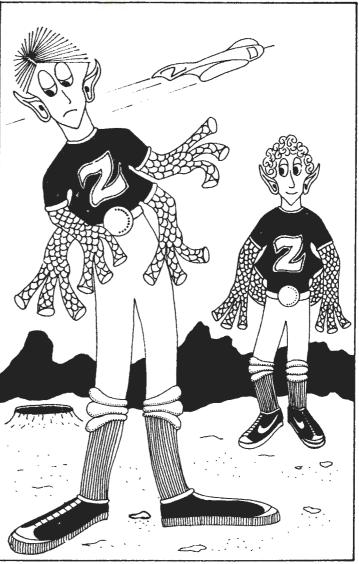


MEAN, MEDIAN MODE

Far away on the planet Zarabo lives a group of beings called Zarians. In one community, there is a Zarian named Zappo. Zappo has been sad for several months. His teacher Zoolinda had been trying to get Zappo to tell her why he was so sad.

Zappo finally blurted out, "I'm so odd! My feet are too long. My nose is too short. My hair is straight. My waist is higher than most teenage Zarians. I'll bet no one has arms as short as mine. Everybody has Nike tennis shoes but me and I'll bet I live farther from school than any one else. I'm just not normal!"

Zoolinda asked Zappo if he'd like to find out how "odd" he really is. Zappo said, "What do you mean?" His teacher explained they could gather data on the teenage Zarians in the class and see how he compares. Zappo ob-



jected "I don't want any one to know how odd I am!" Zoolinda assured him no one would have to write their names on the data card.

The next day Zoolinda gave the students measuring tape and had each student fill out a data card. Here is Zappo's card.

Length of right arm 50 cm	Distance lived from school <u>9</u> blocks
Length of right foot <u>30 cm</u>	Do you have tennis shoes? yes <u>x</u> no
Length of nose <u>2 cm</u>	If so, what brand? Brand x
Distance of waist from	Hair (check one) Curly
floor <u>100 cm</u>	Wavey
	Straight <u>x</u>



After school Zappo and Zoolinda looked at all the data cards. Zappo complained, "There are too many numbers, I still can't tell how odd I am." Zoolinda showed him how to make frequency tables for all the measurements. Here are some of the tables.

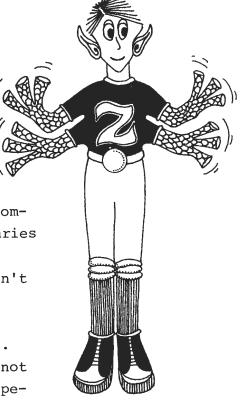
RIGHT ARM		RIGHT FOOT		NOSE			Tennis shoes?	
Length	Frequency	Length	Frequency	Lengt	ch .	Frequency	yes <u>18</u>	
48 cm	1	27 cm	6	1 0	cm	2	6	
49 cm	2	28 cm	5	1.5 0	zm	8	Hair:	
50 cm	2	29 cm	3	2 0	zm	4		
51 cm	5	30 cm	7	2.5 0	cm	7	curly <u>12</u>	
52 cm	7	31 cm	3	3 0	zm	3	wavey 11	
53 cm	6			L				
54 cm	1						straight <u>l</u>	

Zappo looked at the tables carefully. "Well," he said, "it looks like my arms are shorter than most Zarians in the class, but some have

even shorter arms. My feet are the most common length and my nose length is in the middle. I didn't know some of my classmates don't even have tennis shoes, but I am the only one with straight hair..."

Zoolinda pointed out some other things. "Although your nose length is in the middle, most of your classmates don't have the same nose length as you do--it is normal to have this much variation in nose length. Your foot length isn't in the middle but it is the most common length. At your age, the length of feet varies a lot. If we all had the same heights and body lengths, it would be a pretty boring world. Isn't it nice that each of us is average in some ways and special in others?"

Zappo brightened up and thanked his teacher. "I guess that's a good way to look at it. I'm not odd at all. I'm like others in some ways and special in others. After all, I'm the only one in the class with straight hair!"





What digit occurs most often as the <u>last</u> digit of a telephone number in your town? Guess _____

- 1) Get a page of telephone listings from your teacher.
- Select a column of telephone numbers. In the table below, tally the last digit of each number. Continue to the next column until you have tallied 100 numbers.



9?

	DIGIT	TALLY	FREQUENCY	Ŭ
*	0			181120
	1			-
	2			
	3			
	4			
	5			
	6			1.
	7			
	8			
	9			
		TOTA	L	

THE DIGIT OCCURRING MOST OFTEN IS CALLED THE MODE

- 3) In your sample, what digit is the mode?
- 4) What percent of your 100 telephone numbers ended in

3? _____ 5? ____

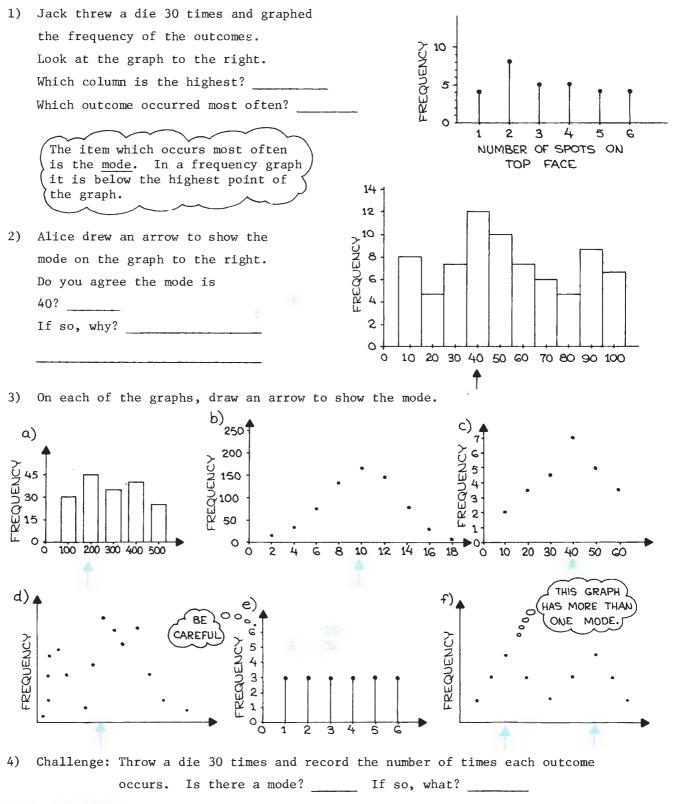
 Have each student support the mole for his/have marking insectified mides on the obsidement of overhead.
 What digit is most often a medel is this digit a componential shollow for prediction.

- the any Crequest lost digit of all place mediars to your most
- b) days each attraced report the Lock digit of sin/her phone number. Find the name, her days is consider to the edds in \$22

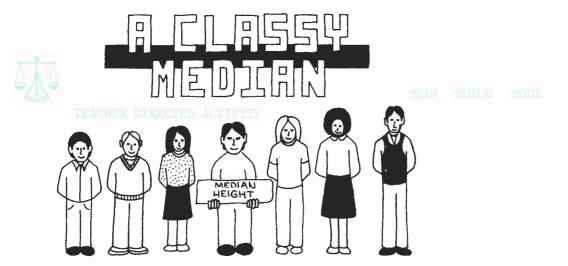
DEPT AND WERE DESCRIPTION

0?

# A LA MODE



TYPE Worksheet



The following demonstrates the concepts of median and mode.

Have the class (or just part of the class) line up from shortest to tallest. It is not necessary to know actual heights. Students can compare heights by standing back to back.

If the class has an odd number of members, the middle person can step forward. This student has the median height. Point out that the same number of students are on either side of this person.

If there is an even number of students, the median height is the average of the heights of the two students in the middle. Explain this by saying, "If a new student having a height between those of the middle two joined the class, this person would have the median height. Since there isn't, we say the median height is halfway between the heights of the middle two students." (If the students in the middle are the same height, each has the median height.)

Are there groups of two or more students having about the same height? If so, have these groups step forward. The group(s) with largest number of students has the modal height. If there is a mode(s), have only this group(s) remain forward. Is there more than one mode in the class?

Separate the students into two groups: boys and girls. Repeat the demonstration for both groups. Do both groups have a mode(s)? Is there a student with the median height in either group? If students sit in rows, the demonstration can be done with each row.

If the actual heights of the students are known, the mean height can be calculated. These questions could be investigated: Is there a student with the mean height? If not, which student(s) is closest to the mean height? Does the same person have the mean, median and mode height? Why? If a student 180 cm tall joined the class line up, where would the median and mode heights occur? Which changed the most, the position of the median or the position of the mean? Why? (The mean is more affected by extreme values.)

After the students have practiced lining up and identifying students with the median, mode and mean heights, the demonstration could be videotaped and viewed.

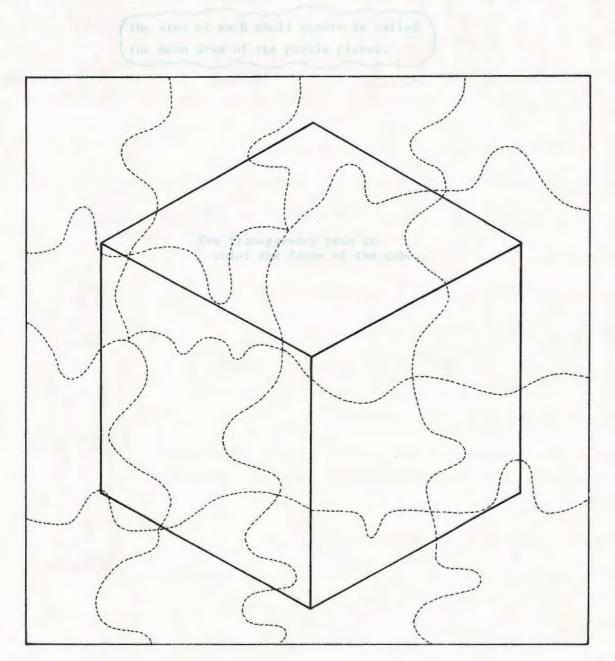
(PE r rected t t

THE STUDENT	
IN THE MIDDLE	
Materials: Centimetre ruler, scissors (optional)	
FRED E RUBY JON JON	JOAN
1) Find the height of each student above. Measure to the nearest	half centimetre.
Jon Ruby Fran Ed	
3) Circle the middle height in #2. WHEN NUMBERS ARE <u>ARRANGED</u> THE MIDDLE NUMBER IS THE M	
4) Which student has the median height?	
5) Randy wants to join the group. Find Randy's height	
6) What is the median height of the group now?	
SINCE THERE IS AN EVEN NUMBER OF HEIGHTS THE MEDIAN IS HALF WAY BETWEEN THE MIDDLE TWO HEIGHTS.	
7) Does any student have this median height?	
8) Wilbur walks over to the group. He is 6 cm tall. Draw Wilbur	RANDY
next to Joan. Find the median height of the group. (Include	
Randy.)	
9) Does any student have the median height?	
10) Cindy joins the group. She is 7 cm tall. Draw Cindy next to	
Bill. Find the median height of the group.	
11) Does any student have the median height?	

# GET IT TOGETHER

This oclivity deconstructed who content hi mus without using the traditional "has on the terms and storids by the moder of terms" approach.

- Make a transportance of this pape. Out out the parrie pieces. Place that an the neutheral, dak ecodemic has many partie pieces there are: Tet students: help conservable the partie on the sections.
- We a support transportancy draw a LE on a 15 on square. Place the brancparaney an the square covers the puzzle.
- 1) Divide the square into a grid of it small squares.
- 4) Tell students the 16 small squares have the same arms on the purfile.



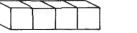
TPC: Demonstration



Materials: bag of cubes from your teacher

#### Activity:

- In the table write the colors of the cubes in the bag.
- 2) Write the number of cubes for each color.



- Put all the cubes of one color end to end to make 5 trains.



4) Move some of the cubes to make 5 trains having the same length.

5) How many cubes in each train now?

- 6) THIS LENGTH IS CALLED THE MEAN (OR AVERAGE) LENGTH.
- 7) Were any of the original trains the same length as the mean length? _____ If so, which color?

Students may need 11 cubes of each color

8) Get more cubes. Encourage students to start with 5 trains see with 6 cubes.

9) Make 5 trains so that

- a) the mean length is 6 cubes,
- b) one train has the same length as the mean length,

and c) each train is a different length.

Show your teacher.

10) Make 5 trains so that

a) the mean length is 7 cubes

and b) no train has the same length as the mean length.

Show your teacher.

TYPE: Activity Card

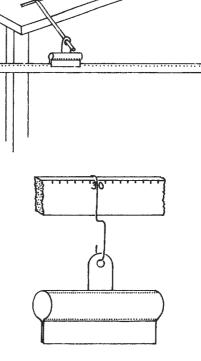
ME MEDIAN (OI

Each bag should contain 4 white green, 9 yellow, 12 blue and 13 cubes. Other colors may be substituted.

Color	Number



- Balance the metre stick as the picture to the right shows.
- The balance point of the metre stick should be at 50 cm.
- 3) Hang a clamp at 30 cm. Hang a clamp at 70 cm. Is the metre stick still balanced?
- 4) What is the mean of 30 cm and 70 cm?
- 5) Remove the two clamps. Now hang a clamp at 10 cm. Hang a clamp at 90 cm. Is the metre stick still balanced?
- 6) What is the mean of 10 cm and 90 cm?
- 7) Remove the clamps. Now hang a clamp at 25 cm. Where would you hang the other clamp to keep the stick balanced?



- 8) A rod is 80 cm long. The balance point is at 40 cm. Where would you put another clamp to keep the rod balanced if
  - a) one clamp is placed at 10 cm?
  - b) one clamp is placed at 55 cm?
- 9) A rod is 150 cm long.
  - a) Where is the balance point? _____ Where would you put another clamp to keep the rod balanced if
    - b) one clamp is placed at 110 cm?
    - c) one clamp is placed at 65 cm?

If it were possible to have a weightless rod, the balance point for several hanging weights will occur at the mean of the weights. For example, three clamps hanging at 10, 20 and 90 cm will have a balance point at 40 cm.

## JA MEAN METHOD

Jackie wanted to find the mean number of letters of the words in a paragraph in her math book. First she counted the letters in each word. The results are shown in the table to the right.

- I. To find the mean number of letters, Jackie needs to know the total number of letters and the total number of words.
- Multiply column 1 by column
   Write the products in column 3.

1	2	3
LETTERS IN A WORD	NUMBER OF WORDS	PRODUCT (COL. 1X COL. 2)
1	5	5
2	5	10
3	18	SA
4	12	48
5	10	50
6	2	12
7	4	28
8	3	24
9	1	9
TOTAL	60	240

- 2) Add the numbers in column 3 to find the total number of letters.
- 3) Add the numbers in column 2 to find the total number of words.
- 4) Find the mean number of letters by computing
- II. The students in Jackie's class counted the number of coins they had.

NUMBER OF COINS FOR	5	3	5	5	7	6	4
	5	2	6	6	4	6	1
EACH	2	1	5	0	3	3	6
STUDENT	4	7	2	6	0	5	9

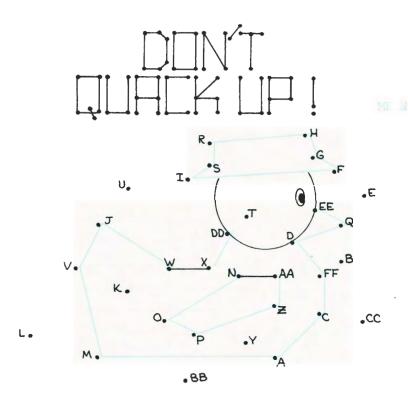
- Make a frequency distribution of the data.
- Use the method above to find the mean number of coins for the students in Jackie's class.

NUMBER OF COINS	FREQUENCY	PRODUCT (NUMBER X FREQUENCY)
0	2	Q.,
1	2	2
2	3	G
3	4	(2
4	3	12
5	G	30
6		36
7	No.	:4
TOTAL	BS	112

total number of letters total number of words

Mean = ____

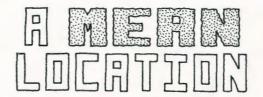
Will Arrende



Decide if each answer is True or False (T or F). Circle the answer and connect the dots.

1	) The mean of: 1,2,3,4, 5, is 3.	Conne T: R F: D	→H	11)	9 is the median of: 9,9,7,5,9.	т:	nect Q→EE B→Q
2	) The median of: 6,1,3, 7,4,3 is 3	T: AZ F: (		12)	The mean of: 16,5, 4,7,0,9,8 is 7.		CC→E X→DD
3	) 7 is the mode of: 2,6,9,7,5,6,1	T: U- F: J-		13)	4 is the mode of: 19,6,1,3,4,8		M≁O R≁S
4	) The median of: 2,9, 5,6,1,2,4 is 4	T: Q- F: G-		14)	The median of: 19, 6,1,3,4,8 is 5		O≁P K≁N
5	) The modes of: 6,6,7, 7,8,8,9,9,9 are 6,7,8.			15)	The mean of: 4,7,1, 6,5,8,2,9 is 5¼.		А→С А→Ү
6	) 6 is the mean of: 8,6,3,7,4,2.	T: X- F: D-		16)	The mode of: 5,19,5, 4,6 does not exist.		P→M I→S
7	) The mode of: 7,9, 13,2,7,5 is 7	T: M- F: DI	_	17)	The median of: 6,5, 4,7,1,3 is 4½.		C→FF B→CC
8)	The mean of: 2,8,4, 7,5,7,10,6,5 is 6.	T: H- F: B-		18)	The mean of: 9,9,9, 9,9 is 9.		J→K J→M
9)	The mode of: 4,4,4, 4,4 is 4.	T: V- F: L→		19)	The mode of: 4,2,9, 3,9,2,4,5,5 is 3.		B→EE I→F
10)	3 is the mode of: 4,4,5,4,4,3.	T: W→ F: AA		20)	The modes of: 1,7,2, 7,1,3 are 1 and 7.		G→F N→Z

TYPE: Works set/P :z



Solve each problem. Find the answer in the message at the bottom of the page. Above the answer write the letter of the problem.

- L) Find the mean of these values: 1, 2, 3, 4, 5, 6, 7, 8, 9.
- H) If a car travels at an average speed of 55 km/h for 3 hours, how far does it go?
- G) Find the median of: 4, 6, 8, 3, 1, 3, 2, 9, 4, 7.
- A) A dart player made these scores: 7, 10, 9, 4, 1, 11. Find the mean score.
- N) If the dart player above played again and scored 0, what would the mean score be then?
- F) Test scores in Ms. Romano's class are: 75, 80, 85, 80, 70, 90, 60, 70, 85, 85. Find the mode.
- B) The mean of five numbers is 8.6. What is the total of the five numbers?
- T) A car travels a distance of 315 km in 7 hours. What is the mean speed?
- U) Find the median value: VALUE FREQUENCY

FREQUENC
1
1
10
12

- R) A group of students received these test scores: 54, 60,73, 25, 62, 68. Find the mean score.
- The teacher decides to add four points to each of the six scores above. Find the mean score now.
- W) Here are the scores given to a group of students: 26, 14, 37, 17, 20, 32, 21, 41. Find the mean score.
- S) The teacher decides to double each of the eight scores above. Find the mean score now.
- E) The mean of four numbers is 9 and the mean of another six numbers is 14. Find the mean of the combined group of ten numbers.
- M) A train travels at 120 km/h. How many hours will it take to travel 300 km?

		H			N		E	AI	4	L	I	E	s	B	E	TA	NE	- E		1
	4	5 165	5 ]	12	2.	5	12	7	6	5	61	12	52	43	12	45	26	12 1	L2 6	5
														D		H	E	C		
45 1	.65	12	4		57	12	7	45	12	52	45	7	6		45	165	12			
	M																		R	S
52 2	2.5	7	5	5	1	2	52	45		85	4	5 165	12	6	9	2.5	43	12	57	52

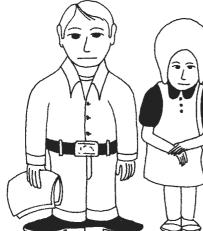
WHERE IS THE MEAN LOCATED ?

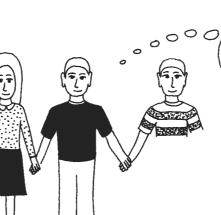
PPL: Knowle known A

	VI. AND AND MEAN NO. NO. NO. NO. NO. NO. NO. NO. NO. NO
Mat	erials: Cuisenaire Rods
1)	Use colored rods to make the train below.
2)	What color is the rod with the median length?
	What is this median length?
3)	Place two brown rods on the right end of the train. What color is the
	rod with the median length? What is this median length?
4)	Remove the brown rods. Replace them with two purple rods. What is the median length of the rods in the train?
5)	Remove the purple rods. Replace them with two orange rods. What is the median length of the rods in the train?
6)	Remove the two orange rods. Add one red and one white rod to the train. °°
7)	What is the median length of the rods in the train? (BE SURE THE RODS) Remove the red and white rods from #6. Can you add two rods SHORTEST TO LONGEST.
	so that the color of the rod with the median length will not be red?
8)	For each train, find the mean length of the rods.
	a) Train in #1 <u>14/9 or 1.56 cm</u> d) Train in #5
	b) Train in #3 c) Train in #6
	c) Train in #4
9)	When colored rods were added to the first train, which varied the most: the median or the mean?
10)	One week the daytime high temperatures were 7°C, 5°C, 8°C, 5°C, 6°C, 7°C, 11°C.
	Find the median temperature.
	Find the median temperature.
11)	On Monday the temperature jumped to 31°C.
	Find the median temperature for the 8 days.
	Find the mean temperature for the 8 days.
12)	Which measure was affected the most by the temperature jump,
	the median or the mean?
	Accivity The name Cuisenaire and the color sequence of the rods are trademarks of the Cuisinaire Company of America, Inc.

536







THE AVERAGE FAMILY IS MADE UP OF 1 ADULT MALE 1 ADULT FEMALE AND 2.35 CHILDREN

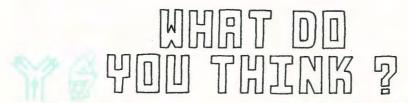
 A survey was conducted in a class of 25 students to find how many children were in each family. The results are shown in the table to the right.

/	$\sim \sim$	
	CHILDREN	NUMBER OF FAMILIES
	1	6
	2	8
$\langle \rangle$	3	5
5	4	3
	5	2
$\left  \right\rangle$	ç	1
(	$\sim$	$\sim$

- 2) Find the mean number of children
   for these families. Mean =
- 3) Are there any families with the mean number of children?
- 4) Find the mode of the number of children in the families.
- 5) You are to build 100 houses for a new housing development. Which would be more important for you to know, the mean or the mode of the number of children in the families? ______ Explain.
- 6) You are in charge of building a park for the new development. To help you decide the size of the playground, which would be more important to know, the mean or the mode of the number of children in the families? Explain.
- 7) Make a survey of your class to find the mean number of children in the families.
- 8) Would you expect your answer to #7 to be higher or lower than the average for all families in the city? _____ Explain.

MERNSWITH Mems
For this activity, each student will used the data from blo/hor package of MAN's. They will also need class results. Use the data from this Will Man Man's.
How many M&M's are in a 40 gram package? How many of each color are in a 40 gram package?
<pre>1) Find the mean number of M&amp;M's per package (to the nearest whole number)</pre>
2) Did you have the mean number of M&M's in your package?
3) Find the mean number of M&M's of each color in all the packages in the class. (to the nearest whole number) brown green orange
tan yellow
4) How many of the five rounded means are the same as the numbers in your package?
5) Find the sum of the five means in (3).
6) Is the sum close to your answer in (1)?
THE MEAN NUMBER IN (1) AND THE MEAN OF EACH COLOR IN (3) DESCRIBE WHAT IS IN A 40 GRAM PACKAGE OF M& M'S.
Stadants might wonder if the relation in (5) noise for the mode. Repart everyworst (3), (3) and (5) for mode: The sum of the modes up (5) probably does not equal the mode manner her partage. Notion could also be investigated. For payhopes of Mid's, the sum of the moderne in (6) will preferry be close to the modern number per partage. Moderner, this to not trad- her all note of data. Morelider seven hage contribute bins and objic marbles.
Total per bas 4, 4, 10, 14, 15, 17, 20

TOPE : Northalines / Twashers To see body Audio Live



Melcher Manufacturing Company boasts that they pay an average salary of \$10,000 to their employees.

Is the company telling the truth?

- 1) To help you decide, use the salary schedule to find mean salary: ______ median salary: ______ mode salary: ______
- 2) Which do you think is a more representative number for these salaries, the mean, median or mode?

-	E TOB	NUM	BER	SA	ADV	٦

TYPE OF JOB	EMPLOYED	SALARY
PRESIDENT	1	\$133,000
VICE - PRESIDENT	2	\$ 70,000
PLANT MANAGER	3	\$ 25,000
FOREMAN	12	\$ 8,000
SKILLED OPERATOR	21	\$ 6,000
UNSKILLED OPERATOR	36	\$ 5,000

Sometimes it is more helpful to know the median; sometimes the mean; other times the mode.

- If you wanted to find the total amount spent on junk food for a week by your class, would you want to know the mean, median or mode amount spent by the class?
- 2) If you wanted to know if you read more or less books per month than most people in the class, would you want to know the mean, median or mode?
- 3) The Big Wheel roller skating rink is ordering new skates. Which would be more useful to know, the mode, mean or median skate size?
- 4) The members of the Four Star riding club sold candy bars to finance a trip. The sales are shown in the table. Which best represents the number of bars sold by each member: the mode, mean or median?
- 5) You want to know which country has a large portion of people with low incomes. Which is most helpful to know for each country: the mean, mode or median income?

NUMBER OF BARS SOLD	NUMBER OF MEMBERS
5	1
15	8
20	5
30	4
40	2
200	1

### ORGANIZATION

### Placement Guide for Tabbed Dividers

Tab Label	Follows page:
CONTENT FOR TEACHERS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
Probability with Counting	
Inferential Statistics	
Appendix-Computer Programs	
DIDACTICS	
Components of Instruction	
Classroom Management	
Statistics and Probability Learning	
TEACHING EMPHASES	
Critical Thinking	
Decision Making	
Problem Solving	
Models and Simulations	
Calculators and Computers	
Laboratory Approaches	
CLASSROOM MATERIALS	
CONCEPTS AND SKILLS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Misleading Statistics	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	604
Probability with Models	634
Counting Techniques	
APPLICATIONS	
Business & Commerce	
Environment	
Health & Medicine	
People & Culture	
Recreation	
GLOSSARY	
ANNOTATED BIBLIOGRAPHY	
SELECTED ANSWERS	



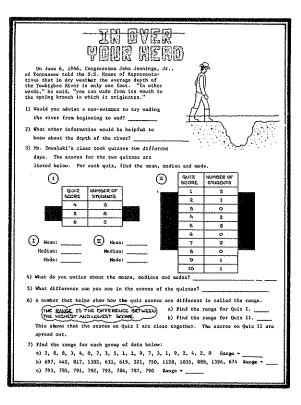
#### Commentary to RANGE & DEVIATION (542-550)

TITLE	PAGE	TOPIC	TYPE
In Over Your Head	551.	Using range to dis- tinguish two sets of data	Worksheet
Get Off the Stove, Grandpa	552	Finding the range	Worksheet Puzzle
Roll Them Again	553	Using mean, mode, and range to describe a distribution	Activity card Worksheet
Fill Up the Gaps	554	Showing sums of differences above and below the mean are equal	Worksheet
Differences from the Mean	555	Showing sums of differences above and below the mean are equal	Worksheet
Clusters and Spreads	556	Using average distance from the mean to describe a distribution	Teacher directed activity
Normal Distributions	558	Introducing and demonstrating normal distributions	Worksheet Teacher directed activities
Running Totals	562	Finding running totals	Worksheet
Stack Them Up	563	Making a graph showing running totals	Teacher demonstration Transparency
Miller's High Bars	564	Making a graph showing running totals	Worksheet
Look, Mom, I'm Getting Better	566	Examining running totals graphs of test scores	Worksheet
Sensitive Scores	567	Introducing percentiles on a running totals graph	Worksheet
Quartiles	568	Introducing quartiles on a running totals graph	Worksheet
Deciles and Percentiles	569	Reading deciles and percentiles from a running totals graph	Worksheet

# RANCE & DEVIATION

#### RANGE

Your students are probably familiar with the idea of range. They have had to look at the range of numbers to be graphed before picking a suitable scale. They might want to know the highest and lowest scores on a test so they can see how their scores compare. Students might also know the heights of the tallest and shortest basketball players on their favorite team. They probably do not find the difference between the two scores or heights (the range), but they realize that knowing the high and the low gives them valuable information about the scores or heights as a whole. To introduce a more formal study of range, you could use In Over Your Head shown to the right. In this activity, students see how ranges are used to compare two sets of data.



Most books define the range as the <u>difference</u> between the highest and lowest scores. Others define the range to be the highest and lowest scores. The difference can always be computed from these two scores, but the extreme scores cannot be determined from the difference. The advantage of knowing the two extreme scores can be explained to students through the following example.

Suppose the yearly incomes of ten people are being considered and you were told that the range for the incomes is \$20,000. Ask students if they think the person with the highest income has the possibility for much better living conditions than the person with the lower salary. (This depends on their incomes. If the low is \$2,000 and the high is \$22,000, there would be a lot of difference. If the low is \$1,000,000 and the high is



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#### RANGE AND DEVIATION

#### COMMENTARY

\$1,020,000, the \$20,000 wouldn't affect the living conditions very much.) You could relate this to the 1977 pay raise for the members of the U.S. Congress. Some people complained that a raise of \$13,000 was as much as their total salary; however, the pay raise is not as impressive relative to the 1976 Congressional salaries, the cost of living in Washington, D.C. and the salaries of persons in comparable roles.

#### DEVIATION FROM THE MEAN

Although the idea of range is familiar, it is less likely that students will have thought about the distance of scores from the mean. You might introduce the idea of deviation from the mean with the problem-solving activity below.



Tell students you are going to give them information about the lengths of a set of colored centimetre rods, and they are to guess what lengths the rods are. Have them give possibilities for the lengths after each new piece of information.

	Information	Possibilities
<b>5</b> 3	There are four rods and their mean length is 6 cm. What might the lengths be?	(6,6,6,6), (2,2,10,10), (1,5,8,10), Any combination of 4 whose sum is 24.
and	the rods range from 2 cm to 10 cm in length.	(2,6,6,10), (2,5,7,10) One is 2 cm, one is 10 cm, the other two add up to 12 cm.
and	on the average, the rods are 3 cm longer or shorter than the mean length.	$(2,6,6,10)  \frac{4+0+0+4}{4} = 2 \text{ (No)}$ $(2,4,8,10)  \frac{4+2+2+4}{4} = 3 \text{ (Yes!)}$

Point out to students how each new piece of information helped them know more about the set of rods. You might ask if it would have been as helpful to know the difference between the shortest and longest lengths as it was to know the shortest and longest lengths. (With a difference of 8 cm, the rods could be from 1 cm to 9 cm, 3 cm to 11 cm, ...) The average distance from the mean was needed to determine the lengths. Be sure students realize that larger sets cannot be determine <u>exactly</u> just by knowing the number of scores, the mean, the highest and lowest scores, and the average distance from the mean. If there were <u>six</u> rods with mean length 6 cm, ranging from 2 cm to 10 cm in length and on the average being 3 cm longer or shorter than the mean length, the set could be (2,3,4,8,9,10) or (2,2,5,7,10,10). The larger the set, the greater the number of possibilities.

You might have students compute the range (as a difference) and the average distance from the mean for sets of numbers having the same mean. Point out how both of these figures help distinguish the sets.

Example 1: 4 scores, mean = 6

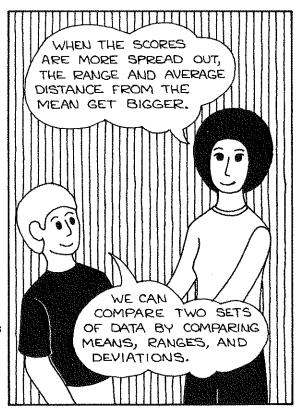
Example 2: 10 scores, mean = 8

Set	Range	Average Distance from Mean
(6,6,6,6)	0	0
(1,5,8,10)	9	3
(4,4,8,8)	4	2
(1,7,7,9)	8	2.5

-	•	
Set	Range	Average Distance from Mean
(5,6,6,8,8,8,8,8, 9,10,12)	7	1.4
(1,1,3,4,4,10, 12,14,15,16)	15	5.4

You might want to give students two sets of data and have them decide (without computing deviations) which set would have the greater deviation.

(2,5,10,13,20) or (21,23,24,24,26)? Do they see the first set is more spread out and will have a greater deviation?



Some students will have difficulty computing a deviation from the mean unless the data is set up in a table. You could give the problems in Example 2 from the previous page in the following form:

	Score	Mean	Distance fr Mean	om	Score	e Mean	Distance Mean	from
	5	8	3		1	8	7	
	6	8	2		1	8	7	
	6	8	2		3	8	5	
	8	8	0		4	8	4	
	8	8	0		4	8	4	
	8	8	· 0		10	8	2	
	8	8	0		12	8	4	
	9	8	1		14	8	6	
	10	8	2		15	8	7	
	12	8	4		16	8	8	
		Т	otal 14				Total 54	
Aver	age Dist	ance fro	m Mean = 14/	0 = 1.4	Average	Distance	from Mean =	54/10 = 5.4

(Students who have studied signed numbers might be asked to compute the signed difference for each score. The sum of these signed differences will always be zero so the distance without the sign is used. This is discussed more on pages 3 and 4 in *Range and Deviation* of the CONTENT FOR TEACHERS section. See also the student pages *Fill Up the Gaps and Differences from the Mean*.

The standard deviation is harder to motivate at the middle school grade levels. Standard deviations can also be a headache to compute if students do not have access to a calculator or computer. Here is a suggestion for introducing standard deviations with a minimum of computation and at least a little motivation.

Read this part of a newspaper article to your class. Students might want to discuss the purpose of the study and bring out critical points like, "Is 18 students a large sample?" or "Do volunteers drive like people who don't volunteer?" After discussion, focus on the fourth paragraph.



Drivers in late model cars without speedometers to nag them about speeding are most comfortable traveling about 70 miles an hour on the highway, according to student safety researchers.

The conclusion was reached in a research project carried out by students at Texas A & M University.

"The experiment clearly demonstrates that the average comfortable speed is well above the existing national speed limit of 55 miles per hour," said Dr. Ronald Morris of Texas A & M, when he presented a paper on the experiment at a meeting here of the SAFE Association, an organization of safety equipment researchers, manufacturers and users. "The analysis of our data resulted in an overall mean comfortable speed of 69.94 miles per hour with a standard deviation of 4.425 miles per hour. From this it is reasonable to conclude that the probability that the entire population's comfortable speed is 55 miles per hour is essentially zero."

Morris, also secretary of the association, said 18 volunteer students drove both ways over an isolated segment of Interstate 30 west of Texarkana, Tex., during daylight hours when weather was dry and sunny.

The students used a 1970 Datsun 240Z, a 1973 Ford Torino station wagon and a 1973 GMC Sports Van selected to represent the range of commercially available passenger vehicles.

You might want to read the fourth paragraph to students again. Write the values for the mean and standard deviation on the board. Explain that the standard deviation is a number used to describe how spread out scores are. Explain that in <u>any</u> distribution at least 75% of the scores are within 2 standard deviations from the mean. (See *Range and Deviation* of the CONTENT FOR TEACHERS section for more information on this.) At least 75% of the drivers tested had a comfortable driving speed between 69.94 -  $(2 \times 4.425)$  mph and 69.94 +  $(2 \times 4.425)$  mph. At least 75% were between 61.09 mph and 78.79 mph. Explain further that in any distribution at least 89% of the scores are within 3 standard deviations from the mean, in this case, between 56.665 mph and 83.215 mph. This study raises doubt about people being comfortable driving at 55 mph. (Of course, there are many questions to ask about the study and the conclusions. Was the sample representative of the population? Is a sample size of 18 large enough? Would you expect the <u>entire</u> population to agree on any one comfortable speed?)

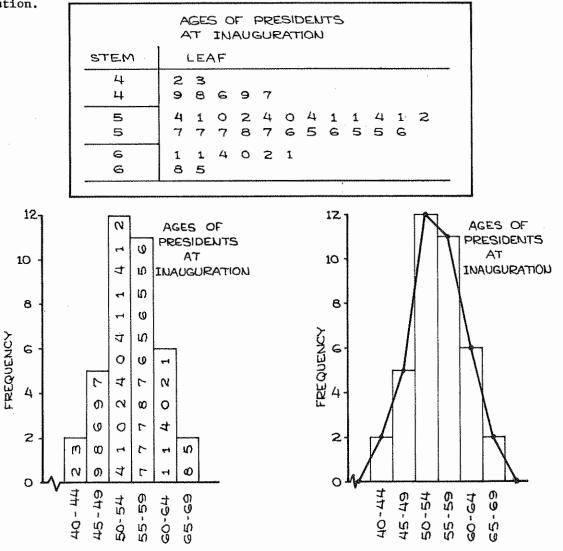
If students want to know how to compute a standard deviation you can show them the computation in a table like this:

First: Find the dis-	POINT	SCORED BY	THE DUNKERS IN	THEIR LAST Square of	1
tance of each score from the mean.	Player	Points	Distance from Mean	Distance from Mean	
<u>Second</u> : Square each distance.	Redhot Tree Slammer Floater Tip It	11 3 9 6 14	2 6 0 3 5	4 36 0 9 25	
Third: Average the squares.	Dribbler Assist Sky Hook Stealer	9 4 15 6	0 5 6 3	0 25 36 9	
<u>Fourth</u> : Take the square root of the	Fleetfoot	<u>13</u> 90	4 9 <b>∢</b> -The Mean	$\frac{16}{160}$	$\frac{160}{10} = 16$
average.			10 90		$\sqrt{16} = 4 \checkmark$ Standard Deviation
The standard devi-					

ation for this set is 4.

#### SHAPES OF DISTRIBUTIONS

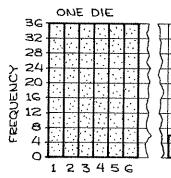
The stem-and-leaf displays from the activities Branching Out, An Age Old Problem and Cars for Sale in the TABLES section can be used to introduce students to the shapes of distributions. An Age Old Problem has students make a stem-and-leaf display of ages of the presidents at inauguration. If the "leaves" are lined up vertically, the display is like a frequency graph. Show students how the display can be turned and a broken line graph or curve added to show the shape of the distribution.

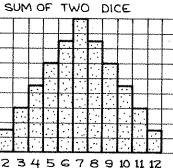


You can ask students questions about the shape of the distribution. For what age interval is the graph the highest? Does it make sense that the graph is low for 40-44 and 65-69? This graph is almost symmetrical about a center line. Do you think all frequency distributions would be shaped like this?

Students can look at the shapes of distributions from What's In A Name? and What Am I Worth? in the TABLES section. Are the distributions symmetrical? Students could graph and compare the distributions from rolling one, two or three dice. (Use the data from *Roll Them Again* in this section for three dice. See the EXPERI-MENTAL PROBABILITY section for activities with one or two dice.) The theoretical^{*} distributions for 216 rolls are given below. You could show these shapes on an overhead and let students compare their results with these.

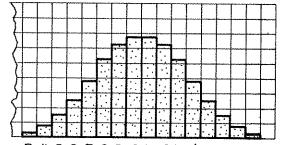
216 ROLLS



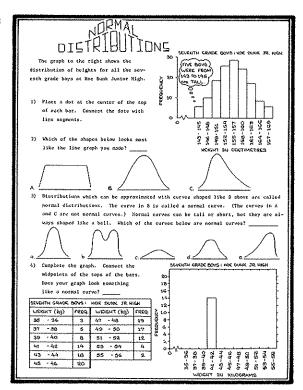


Some of the frequency graphs of students will approximate a normal curve. Normal Distributions can be used to introduce normal curves. The first page (shown to the right) is a worksheet for students. The next three pages of the activity contain suggestions for demonstrations or activities that lead to approximations of normal distributions. An inexpensive device called a Hextat that demonstrates an approximately normal distribution is available from Creative Publications. An example of a handmade "hextat" is given on page 371 of the 34th yearbook from the National Council of Teachers of Mathematics.

#### SUM OF THREE DICE

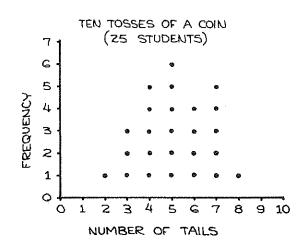


3456789101112131415161718



When you have students work activities from the EXPERIMENTAL PROBABILITY section, you can have them make a frequency graph of their results. In many cases students can observe an approximately normal distribution as shown to the right.

If you want students to relate standard deviation to normal curves, you could look at chapter 9 of <u>Mathematics A Human</u> <u>Endeavor</u> or chapter 8 of <u>Mathematics An</u> Everyday Experience.



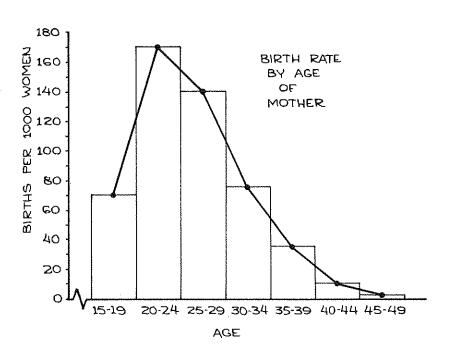
Some students might expect all distributions to be approximately normal. You might give students the table shown below and ask them to make a graph as shown to correspond to the table. Point out the "tail" on the right of the graph. Students should realize some distributions like this one are skewed to the right. Others are skewed to the left.

BIRTH RATE, BY AGE OF MOTHER			
Births per 1,000 Women in Each Group			
Age*	1968		
15-19 years 20-24 years 25-29 years 30-34 years	66.1 167.4 140.3 74.9		
35–39 years 40–44 years 45–49 years	35.6 9.6 .6		

SOURCE: Tables 35 and 36, U.S. Public Health Service

Table from Pocket Data Book USA 1973

*In 1968 the birth rate was 1.0 per 1000 women under age 15.



Could your students pick out the closest shape for each of these frequency tables? SCORE: FREQUENCIES DISTRIBUTION Α: в: C: D: Ε: F: G: H: Ι: J: I B <u>G</u> J FREQUENCY FREQUENCY FREQUENCY FREQUENCY FREQUENCY SCORE SCORE SCORE SCORE SCORE E F A <u>D</u> С FREQUENCY FREQUENCY FREQUENCY FREQUENCY FREQUENCY SCORE SCORE SCORE SCORE SCORE

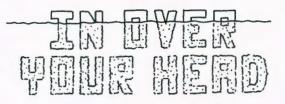
## You might have students match frequency tables with shapes of distributions.

#### PERCENTILES

Students might know their percentile scores from a Stanford Achievement Test or some other standardized test. The activities on running totals and percentiles in this section can help students understand their percentile scores and the percentiles that are frequently given in newspaper and magazine articles. The six activities beginning with Running Totals show a way to develop percentiles, deciles and quartiles.

#### A FINAL NOTE

Students can examine the distributions of data collected from their class, their school and their experiments in probability. They can compute the means and ranges and notice the shapes and variation of these distributions. All of this is an important part of understanding statistical data.



On June 6, 1946, Congressman John Jennings, Jr., of Tennessee told the U.S. House of Representatives that in dry weather the average depth of the Tombigbee River is only one foot. "In other words," he said, "you can wade from its mouth to the spring branch in which it originates."

- Would you advise a non-swimmer to try wading the river from beginning to end?
- 2) What other information would be helpful to know about the depth of the river?
- Ms. Zowalski's class took quizzes two different days. The scores for the two quizzes are listed below. For each quiz, find the mean, median and mode.

	QUIZ	NUMBER OF STUDENTS	
-	4	3	-
	5	8	
K.	6	3	
Mean: Median: Mode:		(II) Mean: Median: Mode:	

	QUIZ	NUMBER OF STUDENTS	
-	1	2	-
	2	1	
	3	0	
	4	2	
	5	5	
	6	0	
	7	2	
	8	0	
	9	1	
	10	1	

- 4) What do you notice about the means, medians and modes?
- 5) What difference can you see in the scores of the quizzes?
- 6) A number that helps show how the quiz scores are different is called the range.

THE RANGE IS THE DIFFERENCE BETWEEN THE HIGHEST AND LOWEST SCORE.

a) Find the range for Quiz I.b) Find the range for Quiz II.

This shows that the scores on Quiz I are close together. The scores on Quiz II are spread out.

π

- 7) Find the range for each group of data below:
  - a) 2, 8, 8, 3, 4, 0, 7, 3, 5, 1, 1, 9, 7, 3, 1, 9, 2, 4, 2, 0 Range = _____ b) 697, 442, 817, 1302, 631, 619, 321, 750, 1128, 1035, 888, 1396, 674 Range = ____ c) 793, 785, 791, 792, 793, 786, 787, 790 Range = ____

## GET OFF THE STOVE, GRANDPA

Find the range for each exercise. Place the letter above the correct answer below.

(N) Cost of record albums in cents 279 389 498 559 639 789

 Eight highest waterfalls in the U.S. (in metres)

739 491 357 213 195 189 189 181

E Heights on an 8th grade basketball team (in centimetres)

 170
 183
 162
 173
 175
 173
 185
 170
 156
 165

A Eight quiz scores 76 85 92 96 83 72 83 87

(I) Number of floors in the ten tallest buildings in New York City

110 102 77 71 71 70 67 60 60 59

U Winning football score for Super Bowls I - IX 35 33 16 23 16 24 14 24 16

D Record long run Broadway plays G ^{Te}

G Ten longest vehicle tunnels in the United States (in metres) Ranimals (km/h)

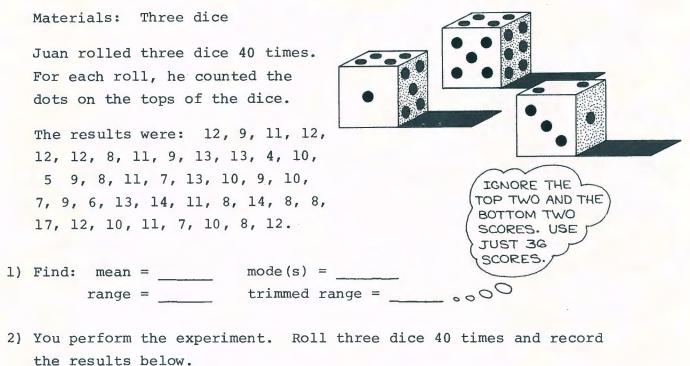
Abie's Irish Rose	2,327	Eisenhower Memorial (CO)	2,725	Antelope	98.2
Fiddler on the Roof	3,242	Copperfield (UT)	2,130	Cape hunting	•
Harvey	1,775	Allegheny (PA)	1,850		72.5
Hello, Dolly	2,844	Liberty Tubes (PA)	1,804	Cheetah	112.7
Life With Father	3,213	Zion National Park (UT)	1,757	Coyote	69.2
Man of La Mancha	2,328	East River Mountain (WV-VA)	1,725	Elk	72.5
My Fair Lady	2,717	Tuscarora (PA)	1,623	Gazelle	80.5
Oklahoma!	2,246	Kittatinny (PA)	1,441	Gray fox	67.7
South Pacific	1,925	Lehigh (PA)	1,335	Lion	80.5
Tobacco Road	3,182	Blue Mountain (PA)	1,323	Quarterhorse	76.5
topacco Road	5,102	Dide noticain (IR)	1,545	Wildebeest	80.5

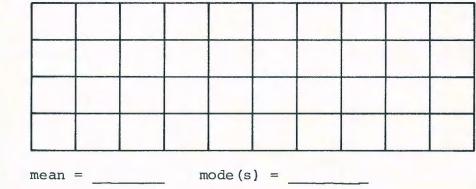
GET OFF THE STOVE, GRANDPA. YOU ARE TOO

TO 558 21 1467 45 51 1467 29 THE 45 24 510 1402 29



dat





3) Find: mean = ____ mode(s) = _____ range = ____ trimmed range = _____

- 4) Compare your results in #3 with Juan's in #1. Are any of the answers the same? _____ If so, which ones? _____
- 5) Which answers do you think would be more likely to be the same, the ranges or the trimmed ranges?
- 6) Compare your answers to #3 with a friend. Are any of the answers the same?

IDEA FROM: Centers and Spreads by Oakland County Mathematics Project Permission to use granted by Oakland County Mathematics Project

	FILL UP FMC COOC
Mat	erials needed: scissors
2)	Use the scale to give the height of each bar. Find the mean of the heights. Draw a horizontal line across the bars to show the mean height of the bars.
4)	On notebook paper, trace the bars. Cut off and save the part of the bars above the mean height line.
• 5)	On the figure above, can you fit the cut-off parts below the mean height line to fill up the gaps in the other bars?
7) 8)	Shade in the bars to       12         the right to show       11         these heights:       10         4, 11, 2,       9         6, 10, 9       8         Find the mean of the       7         heights.       6         Draw a horizontal       4         line across the bars       3         to show the mean       2         height of the bars.       1         On notebook paper       0         On notebook paper       0
10)	that lie above the mean height line. Can you fit the cut-off parts below the mean height line to fill up the gaps in the other bars? (Some may have to be cut.)
	THE TOTAL OF THE CUT-OFF PARTS ABOVE THE MEAN HEIGHT LINE WILL ALWAYS BE THE SAME AS THE TOTAL OF THE GAPS BELOW THE MEAN HEIGHT LINE.

## DIFFERENCES FROM THE MEAN

 Find the mean of the scores in the table.
 Mean =

 For each score find its difference from the mean. Write the differ-

ences in the table. Column 1 is for differences below the mean; Column 2 is for differences above the mean. The first score has been done for you.

SCORE	1	ABOVE (COL. 2)	
27			
29			
46			
18			
54			
ତେ			

3) Find the total of the differences below the mean (Col. 1):

above the mean (Col. 2):

What do you notice?

4) Find the mean of the scores in the table.

Mean =

- 5) For each score find its difference from the mean. Write the results in the table.
- 6) Find the total of the differences below the mean (Col. 1)

above the mean (Col. 2)

7) Alex says that the sum of the differences below the mean equals the sum of the differences above the mean. Do you agree?

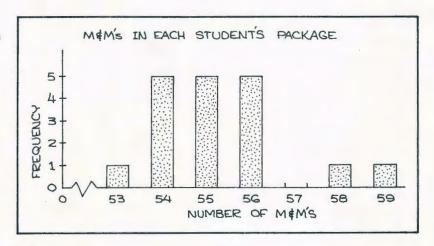
SCORE	DIFFERENCE FROM THE MEAN BELOW (COL. 1) ABOVE (COL. 2)	
 86		
42		
29		
58		
23		
80		
74		
64		

8) Check to see if Alex's conclusion is true for these scores: 59,52,60, 11,72,61,59,10,47,71,44,61,28,65. Make a table for the numbers. Find the mean. Find the differences above and below the mean.



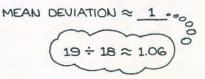
For this activity you will need data from *Make Mine M&M's* in the GRAPHS section.

- Display a graph like the one to the right showing the total number of M&M's in each student's package.
- 2) Ask students if the totals seem clustered together or spread out. (The totals should appear somewhat clustered.)
- Tell students they can compute a number that describes the amount of spread of the data. Organizing the data in a table simplifies the work.
  - a) First find the mean of the totals. [53 + (5 x 54) + (5 x 55) +
  - (5 x 56) + 58 + 59] ÷ 18 ≈ 55.3
    * If done without a calculator,
    round the mean to the nearest
    whole number (55) to simplify
    the calculations.
  - b) Find how far each total is from the mean. (53 is 2 away from 55.)
  - c) Multiply each distance from the mean by the frequency. (See Column 4 in the table.)
  - d) Find the distance total.
  - e) Divide the distance total by the frequency total. (19 ÷ 18 1.06)
     The average of the distances from the mean is a little more than 1. The totals for each package are tightly clustered.



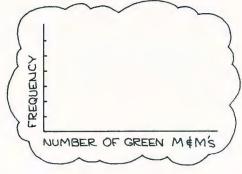
* MEAN = 55

TOTALS OF M&M'S	FREQUENCY	DISTANCE FROM MEAN	FREQUENCY x distance FROM MEAN
53	1	2	2
54	5	1	5
55	5	0	0
56	5	1	5
58	1	3	З
59	1	4	4
TOTAL	18	DISTANCE	19





- 4) Use the data from Make Mine M&M's. Divide the class into five groups and assign each group a color. Have each member of the group make a graph of the class frequencies for the group color.
- 5) Have the members of each group compare their graphs with the graph of the total number per package. Is the data on the students' graphs more or less spread out than the data on the graph of the total number per package?



- 6) Have each member compute the average distance from the mean for the group color. Provide students with a table similar to the one on the previous page to help them organize their work.
- 7) Have the members compare the average distance from the mean for their data to the average distance from the mean for the total number per package. If larger, point out that the spread of their data is greater.
- Have groups compare their average distance from the means. Use the group graphs to confirm the spread of the data.





The graph to the right shows the distribution of heights for all the seventh grade boys at Hoe Dunk Junior High.

- Place a dot at the center of the top of each bar. Connect the dots with line segments.
- 2) Which of the shapes below looks most like the line graph you made? _____

B.

30 FIVE BOYS WERE FROM 143 TO 145 20 FREQUENCY om TALL O 0 10 0 - 166 169 143 - 145 152-154 146 - 148 158-160 161-163 155-157 149-151 1 164 167 CENTIMETRES HEIGHT IN

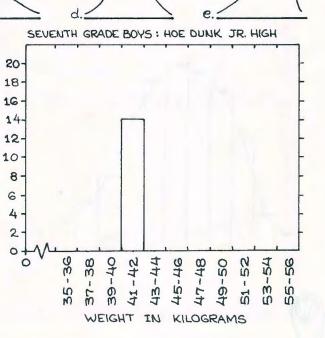
3) Distributions which can be approximated with curves shaped like B above are called normal distributions. The curve in B is called a normal curve. (The curves in A and C are not normal curves.) Normal curves can be tall or short, but they are always shaped like a bell. Which of the curves below are normal curves? ______

a. _____

A.

4) Complete the graph. Connect the midpoints of the tops of the bars. Does your graph look something like a normal curve?

SEVENTH GRADE	BOYS :	HOE DUNK JR. H	IGH	
WEIGHT (kg)	FREQ.	WEIGHT (kg)	FREQ.	3
35 - 36	3	47 - 48	19	FREQUENCY
37 - 38	5	49 - 50	17	REQ
39 - 40	8	51 - 52	12	L L
41 - 42 .	14	53 - 54	4	
43 - 44	18	55 ~ 56	2	
45 - 46	20			



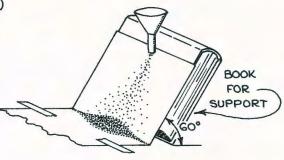


Several demonstrations and activities can be used to show normal distributions. A collection of these is given below.

I. A Normal Distribution of Salt. (Idea from: <u>Goals for the Correlation of Elementary</u> <u>Science and Mathematics</u> -- The Report of the Cambridge Conference on the Correlation of Science and Mathematics in the Schools.)

Crease a piece of graph paper in the middle. Tape one side onto a flat surface. Support the other side with a book at about a 60° angle. (You can experiment with different angles.)

Hold your finger over the end of a small funnel and fill it with about

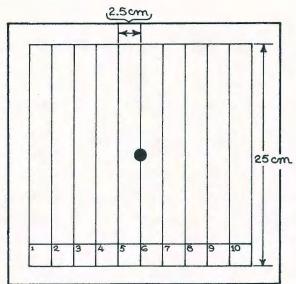


one-half cup of salt. Let the salt come out of the funnel in a smooth stream and slide down the side of the paper to the fold. Have students look at the outline of the top of the salt against the paper. It should look like a normal curve. You can spray the salt and paper with paint, remove the salt, and see the curve more clearly. An alternative is to carefully draw a curve along the top of the salt mound.

II. A Distribution of Rice (Idea from: <u>A Handbook of Aids for Teaching Junior-Senior</u> High School Mathematics by S. Krulik)

Enlarge the pattern to the right on a 30 cm by 30 cm sheet of paper. The columns are 2.5 cm wide and 25 cm long. The columns are numbered and a space is included at the bottom to help in recording.

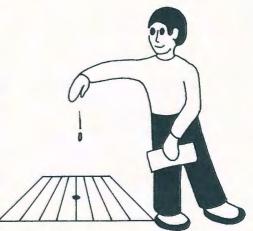
Note: In this activity, the rice does not become arranged in a normal distribution like the salt, but the <u>graph</u> of the number of grains in each column will approximate a normal curve.



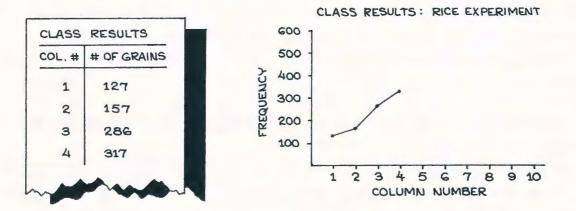
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Have students work in groups of two or three. Give each group a copy of the pattern and an envelope containing at least 50 grains of rice. Have one student in each group drop a few grains of rice at a time directly over the large dot from a height of about 30 cm above the paper. (The paper can be placed on the floor so it is flat and the rice won't spill all over.) Any grain of rice that rolls out of the ten columns should be dropped again. Continue until 50 grains are on the paper.



Have students count the grains in each column and record the total for each column. The class results can then be combined and graphed as shown. The graph should approximate the normal curve.



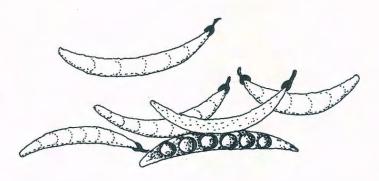
Note: A variation of this experiment is described by Edward L. Spitznagel, Jr., in "An Experimental Approach in the Teaching of Probability" in <u>The Mathematics Teacher</u>, October, 1968. Spitznagel describes a paper ruled into 20 columns covered with carbon paper and placed on the floor. A marble is then rolled down the groove of a ruler (inclined 1 in.) and four feet across a table to drop onto the carbon paper. This is done 400 times and a frequency graph similar to that shown above is drawn. The graph will usually approximate a normal curve. (The article has many specific suggestions for doing this experiment with success.)

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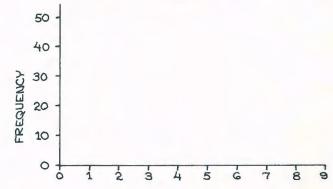
#### III. A Distribution of Peas

Bring enough pea pods so each student has three pods. (If you do this activity with a small group, you might give each student more pods.) Have each student count the mature peas in each pod. Accumulate the data in a frequency table and graph.



PEAS IN ONE HUNDRED PODS		
# OF TALLY		FREQUENCY
0		
1		
2		
3 4		
4		
5		
G		
7		
8		
9		







T)	During spring	vacation Joan	n worked part-	$\sim$	$\sim$
	time as a bag	ger at the sup	permarket.	MONDAY	\$ 3.75
	Her earnings	are shown to t	the right.	TUESDAY	\$ 4.25
	Joan wants to	know the tota	al amount she	WEDNESDAY	\$ 4.00
	has earned by	the end of or	ne day, two (	THURSDAY	\$ 4.50
	days, three d	ays, etc. The	ese totals are	FRIDAY	\$ 5.00
	called runnin	g totals.			
	Complete the	table below to	o show Joan's runn	ing totals	. The first
	two have been	done for you.	•		
		Earnings 2	Running Totals		2,2
	Monday	\$3.75	\$3.75	$\sim$	-
	Tuesday	\$4.25	\$8.00 -0 (\$3.75 +	\$4.25	
	Wednesday	\$4.00	- ~	~	
	Thursday	\$4.50		(	4
	Friday	\$5.00			
					$\sim$

2) The chart below shows the number of students who ate at the school cafeteria each day for one week.

a) Make a running total for the week. Number of students Running Totals onday 175

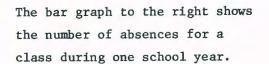
(D)

Monday	175	
Tuesday	207	
Wednesday	189	
Thursday	193	
Friday	160	

b) How many students were served up to and including Thursday?
3) The advance ticket sales for a school play were as follows:

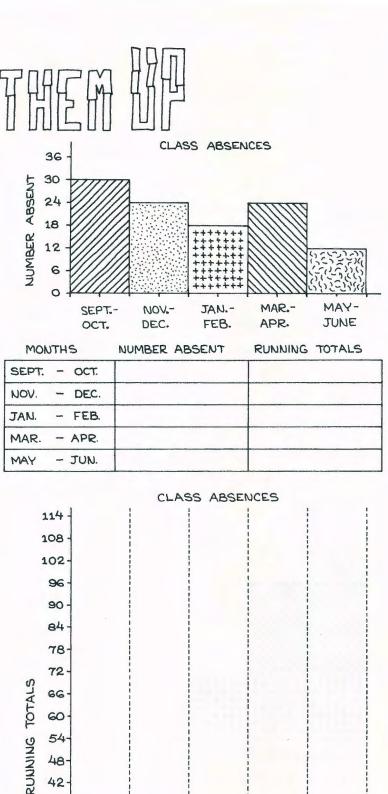
	ſ	Tickets Sold	Running Totals	
	Wednesday	250		a) When had one-fourth
Week 1	Thursday	147		of the advance tick-
	Friday	100		b) When had one-half of
	Monday	75		the advance tickets
Week 2	Tuesday	197		been sold? []]
	Wednesday	200		
	Thursday	250		

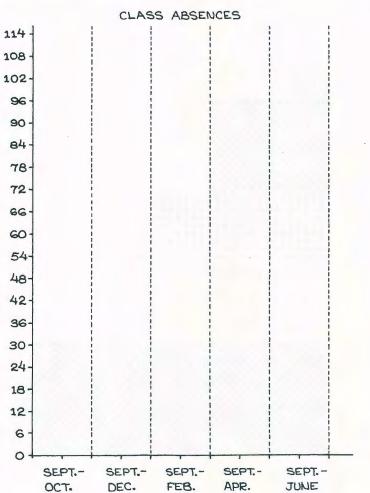




- 1) Use the graph to complete the running totals table.
- 2) How many absences in September-October?
- 3) How many absences in May-June?
- 4) When did the absences reach 50? 100?

The bars from the top graph can be used to draw a running totals graph.





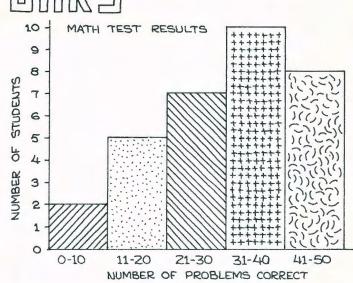


Ms. Miller gave her class a 50 question math test.

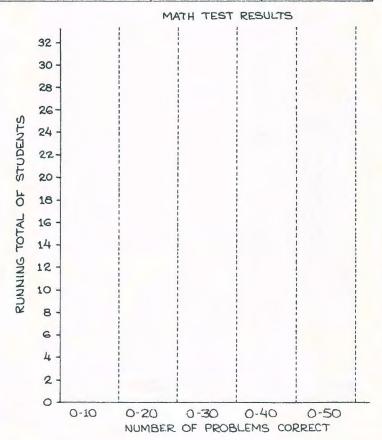
The results are shown in the graph. The first bar shows that 2 students each got 10 or fewer problems correct.

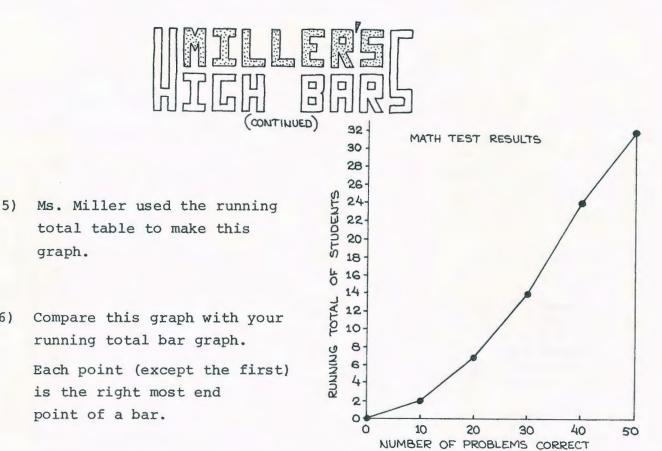
The second bar shows that 5 students each got more than 10 but not more than 20 correct.

- Use the top graph to compare the running total table.
- Use the table to make a running total bar graph.
- 3) How many students each got 40 or fewer problems correct?
- 4) How many students each got 20 or fewer problems correct?



NO. PROBLEMS CORRECT	NO.	STUDENTS	RUNNING TOTALS
0 - 10			
11 - 20			
21 - 30			
31 - 40			
41 - 50			





NUMBER OF GREATEST

41 - 50

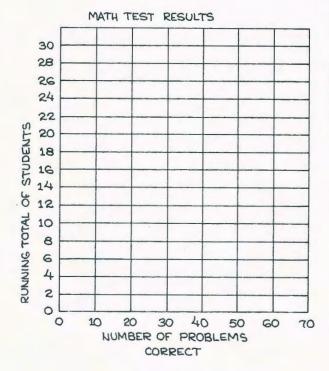
51 - 60

61 - 70

7) Complete the table to the right. Use it to make a graph like the one above.

graph.

6)



PROBLEMS NUMBER IN NUMBER OF RUNNING CORRECT INTERVAL STUDENTS TOTALS 0 - 10 10 2 11 - 20 20 2 21 - 30 4 31 - 40 7

> 8) How many students each have 60 or fewer problems correct?

5

7

3

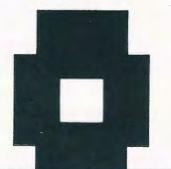
9) Challenge: Half of the students each have or fewer problems correct.



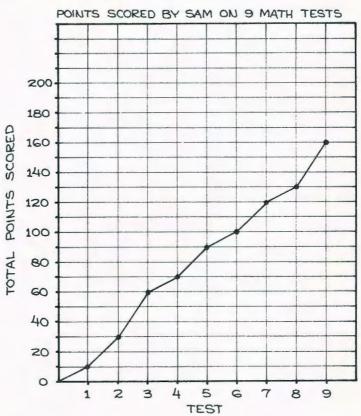
Look at this running totals graph of Sam's scores of 9 math tests. Each test had 40 points.

- It looks like
  - Sam is getting better in math class.
  - He is progressing at a steady rate.

Use the graph to fill in the table below.



TOTAL POINTS AFTER TEST			S SAM
9		9	
8		8	
7		7	
6		6	
5		5	
4		4	
З		3	
2		2	
1		1	



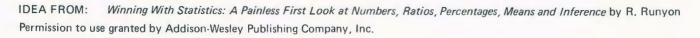
3) Is Sam getting better in math class?

4) Is his rate of progress steady?

MORAL: Beware of a running totals graph. (a) A running totals graph always goes up or at least stays the same. An upward trend

b) By adding <u>all</u> previous test scores, the graph looks smooth. A smooth graph does not imply consistent test scores.

does not imply improvement in individual



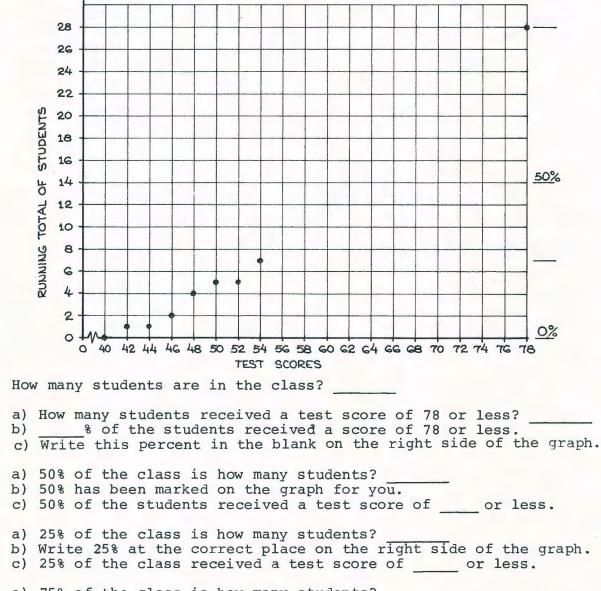
scores.



1) The students in Mr. Frank's class received these scores on a social science test:

TEST SCORES	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68	70	72	74	76	78
FREQUENCY	0	1	0	1	2	1	0	2	3	0	2	2	4	2	1	3	0	1	2	1

 Use the table to complete the running total graph below. Connect the dots with line segments. Use the graph to help you answer the following questions.



SOCIAL SCIENCE TEST SCORES

7) a) 75% of the class is how many students?
b) Write 75% at the correct place on the graph.
c) 75% of the class received a test score of _____ or less.

3)

4)

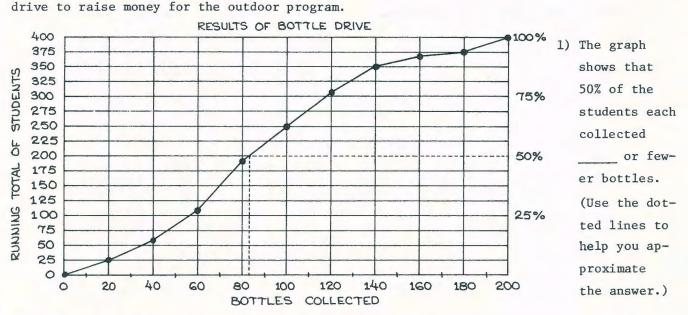
5)

6)



The students at Putnam Junior High had a bottle

Ι

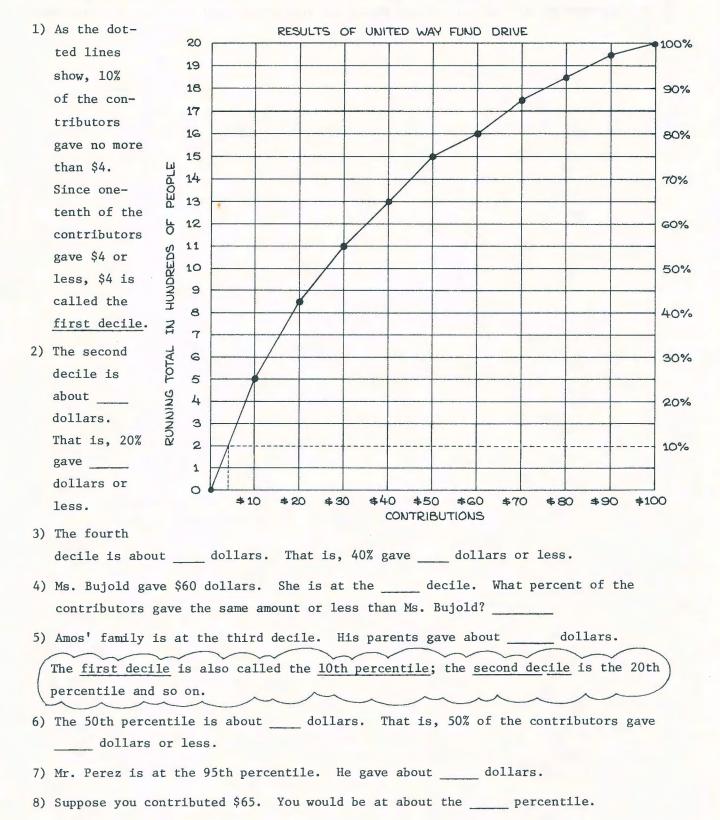


- 2) 25% of the students each collected 55 or fewer bottles. Since one-quarter of the students each collected 55 or fewer bottles, 55 is called the <u>first quartile</u> of the number of bottles collected.
- 3) 75% of the students each collected about _____ or fewer bottles. Since threequarters of the students each collected this number or fewer bottles, the third quartile is _____ bottles.
- II. The graph shows the running totals of the under 21 population of Jonesville.

JONESVILLE POPULATION UNDER 21 1) The first quartile age is 50 45 100% about _____ years. That is, 25% of the under 21 5 40 75% SGINSANDS population is _____ years or less. 50% 2) The second quartile age is 3 20 about _____ years. That TOTAL 15 is, 50% of the under 21 25% population is _____ years 10 RUNNING 5 or less. 0 3) The third quartile age is 15 18 21 0 3 6 9 12 AGE IN YEARS about _____ years.



I. The graph shows contributions to the United Way fund drive.



## ORGANIZATION

### Placement Guide for Tabbed Dividers

Tab Label	Follows page:
CONTENT FOR TEACHERS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
Probability with Counting	
Inferential Statistics	
Appendix-Computer Programs	
DIDACTICS	
Components of Instruction	
Classroom Management	
Statistics and Probability Learning	
TEACHING EMPHASES	
Critical Thinking	
Decision Making	
Problem Solving	
Models and Simulations	
Calculators and Computers	
Laboratory Approaches	
CLASSROOM MATERIALS	
CONCEPTS AND SKILLS	
Gathering Data	
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Graphs	
Scatter Diagrams	
Misleading Statistics	
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Business & Commerce	
Environment	
Health & Medicine	
People & Culture	
Recreation	
GLOSSARY	
ANNOTATED BIBLIOGRAPHY	
SELECTED ANSWERS	

## CONTENTS

Commentary to SAMPLING (573-577)

TITLE	PAGE	TOPIC	TYPE
Don't Lose Your Marbles	578	Using samples to estimate ratios	Teacher directed activity
Can You Tell?	580	Increasing the number of samples to make better predictions	Teacher directed activity Activity card
An Average Sample, Activity I	583	Comparing a mean sample to the population	Activity card Worksheet
An Ample Sample	584	Making a sample board	Teacher idea
An Average Sample, Activity II	585	Comparing the accuracy of differently sized samples	Activity card Worksheet
Let's Use Our Ranutas	586	Using a random number table for sampling	Activity card Worksheet
Are You Successful?	589	Seeing how samples describe a known population	Worksheet
Two Scoops of Raisins	591	Using samples to check for a uniform distribution	Activity card
Making Estimates	592	Using samples to estimate population size	Worksheet
How Many Dots?	593	Using samples to estimate population sizes	Transparency Activity card
Beans by the Cupful	594	Using samples to estimate the number of beans in a jar	Activity card
It's a Pile of Stones!	595	Using samples to predict the mass of stones	Teacher directed activity
How Many Deer?	597	Using samples to estimate population size	Activity card Teacher directed activity

•

TITLE	PAGE	TOPIC	TYPE
You Be the Judge	598	Evaluating sampling plans	Transparency Discussion
At Random	601	Introducing randomness	Transparency Discussion
Area By Random Dots	602	Using randomly arranged dots to estimate area	Worksheet

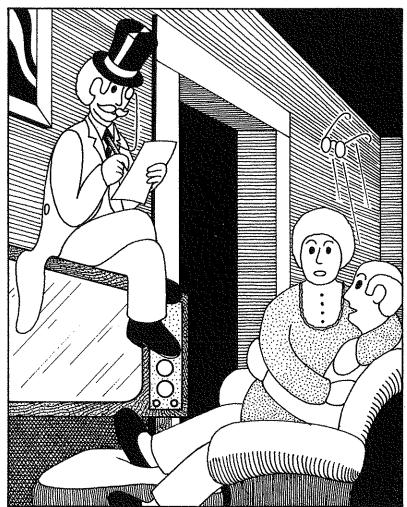
# SAMPLING

#### WAYS TO INTRODUCE SAMPLING

The need for sampling can be introduced with a class discussion. Ask students if they have ideas on how to determine the nation's most popular television show at 8:00 on Wednesday night. Students might suggest that everyone be asked. Discussion could include questions like: Is it worthwhile (or even possible) to spend time and money polling everyone in the country? Do you think it is reasonable to ask a sample of people? If so, how many people do you think should be in the sample? Have any of your favorite shows been cancelled because of a poor rating? Were you

asked your opinion? How are the Nielsen (television) ratings actually made?

Some students might know that data for television and radio ratings are collected in several ways. One way is to attach an automatic recorder to television sets. This recorder keeps track of the station to which the television is tuned at different times of the day. Another way is to have people write down the shows they watched in a certain time period. In each case, a sample of people is used to predict the preferences of the entire population. Have students discuss some possible errors in each method. Here are two ideas that might come up: The automatic recorder



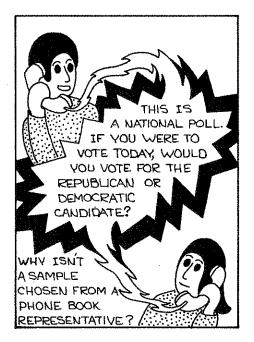
has no way of knowing how many people, if any, are watching. The people recording their choices might not remember what they watched or they might be unwilling to answer honestly.

#### COMMENTARY

SAMPLING

If the class has collected newspaper articles referring to polls, you might go over these and point out the ones that polled a sample. Students might be surprised to learn only about 1500 people are questioned in a Gallup poll to predict national votes.

Students may point out that a sample might not reflect the total population at all. This is a perfect chance for you to ask students how a sample should be chosen so the sample would have a reasonable chance of reflecting the population's opinions on television shows, political candidates or ecological issues. Discussion should bring out



the importance of including people from all age groups, from all parts of the United States, from all political parties, from different religions, races, etc. Ideas of random sampling (when each member of a population has the same chance of being chosen) can also be brought out at this time. The suggestions on You Be the Judge could be discussed.

Here is an interesting sampling situation to present to your class. A group of ten girls decided they didn't believe there are as many boys in families as there are girls. They each wrote down the number of boys and number of girls in their families. The totals showed 16 girls and 10 boys. They decided there probably are more girls than boys in most families. What do you think of this conclu-

sion? Students might say the sample of ten families is not large enough--if so, ask them what they would think about polling the families of all the girls in the school or 10,000 girls picked at random from the country. See if someone in the class will realize the samples are very biased because there is no chance for families with no girls to be included. You could have the girls and boys in your class separately compile data about their families and compare the totals.



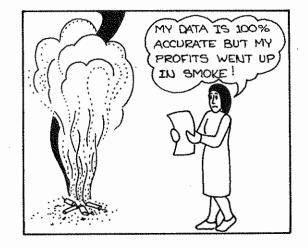
#### SAMPLING

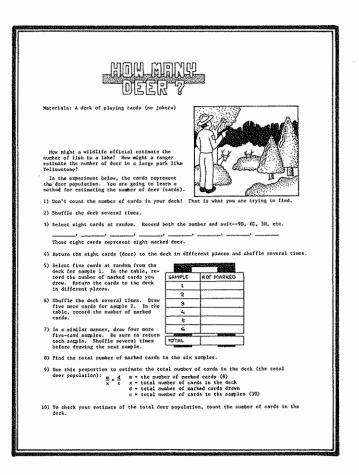
#### COMMENTARY

Another way to motivate sampling is to pose situations like this: A manufacturer of matches (or balloons, firecrackers, light bulbs, bike tire tubes, etc.) wants to make sure the product is of good quality, e.g., the matches will burn properly. How can the manufacturer by reasonably sure of the quality without burning all the matches? Students might suggest randomly sampling one match out of each box or choosing at random, one box out of every 1000 boxes.

#### MAKING A SETTING FOR AN ACTIVITY

Many activities in sampling will be more meaningful if a discussion is held beforehand. The activity below should be more meaningful if you pose some of these questions for discussion. "How is it possible to estimate the number of blue whales? The number of elk in Alaska? The number of trout in a lake?" Students might suggest you could count them all--not very practical because it is very difficult to keep the counted ones from mixing with the uncounted ones and also impossible to be sure you have them all. Someone might suggest marking the animals as described on the television shows "Wild Kingdom" and "The Undersea World of Jacques Cousteau." Just how marking a sample of animals will help determine





#### COMMENTARY

the total number is usually not clear to students. If you are lucky, someone might suggest putting the marked animals back into the population, letting them mix, and looking at the percent of marked animals in a new sample. This idea can be explained to the class before it is simulated as suggested in *How Many Deer*? By explaining that the animals are often caught, marked, and let loose in different parts of the park, lake or ocean, you can set the pattern for students to choose cards from (and return them to) different parts of the deck.

The accuracy of the estimate obtained by using cards will vary according to how well the cards are mixed after each sample is taken. One class pooled their data and obtained an estimate of 46.7 cards for a true population of 49. Another class estimated 58.3 cards--not nearly as close. You might want to point out that marbles instead of cards could have been used to simulate this sampling problem.

#### BRINGING OUT THE MAIN IDEAS IN SAMPLING

The main purpose of the activities in this section is to show students <u>it is</u> <u>possible to make fairly accurate estimates about a population by using well-planned</u> <u>sampling</u>. The activities Don't Lose Your Marbles, An Average Sample, Let's Use Our Ranutas, Can You Tell? and in the ENVIRONMENT section Let's Go Fishing use sampling to estimate the <u>composition</u> of a population--what part of the marbles are red, what part of the fish are trout, etc. How Many Dots, Making Estimates, Beans by the

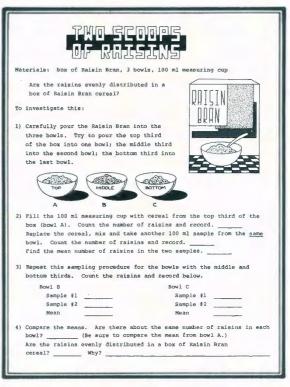
Cupful, and How Many Deer? use sampling techniques to estimate the <u>total number</u> in a population. You can bring out the main purpose of this section by summing up each activity with statements like, "We made a pretty good estimate, didn't we? Not exact, but pretty close." (Of course, the samples won't <u>always</u> give good estimates, but that is an important point to bring out, too!) "Do you see how useful this method would be to approximate national votes or opinions?"



#### COMMENTARY

SAMPLING

You will want to emphasize that sampling must be well planned. Ask questions like, "Do you see how far off the estimate could be if we didn't thoroughly mix the two colors of marbles (or mix the cards, choose samples from different parts of the population, ...)?" Two Scoops of Raisins points out that raisins are not randomly distributed in Raisin Bran-they tend to settle to the bottom. Ask students how they might take samples to estimate the number of raisins in one box. They might suggest taking three 250 ml samples--one each from the top, middle and bottom, averaging and multiplying by the number of 250 ml containers-full in the package. Another suggestion might be to pour it out, stir it up and take a sample.



The activities Don't Lose Your Marbles and An Average Sample Activity II emphasize that a larger sample will usually give a better estimate of a population. Students should realize that even a large sample must be chosen so it will reflect the population. It also can be pointed out that taking a larger sample than necessary is a waste of time and effort. (See Sampling of the CONTENT FOR TEACHERS section for more information on the size of a sample for various populations.) The number of samples taken can also affect the accuracy of an estimate. Usually, <u>if</u> more samples are taken and the results are averaged, the estimate of the population will be more accurate. Of course, students should take samples of the same size if they want to average the data from the samples.

#### A FINAL NOTE

Most of the sampling activities in this section involve students in actively gathering, organizing and interpreting data. This can be tedious if it takes too

much time to collect or organize the data. An Ample Sample describes a laboratory aid that can be used to obtain 5, 10 or 25 samples of marbles quickly. You might think of other ways to gather data efficiently. Having students pool their data as suggested in most activities also shortens the data collecting process.

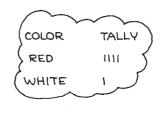
0	0	0	0	0	1	
0	0	0		0	1	_
0	•	0	0	0		
•	•	•	0	0		-
0	0	0	0	0	/	-



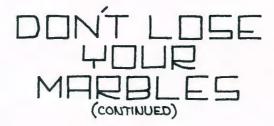
Materials: Prepare a collection of 300 marbles: 200 white, 100 red. Other colors and objects could be used such as centimetre cubes, poker chips or beans. Place the collection in a paper bag.



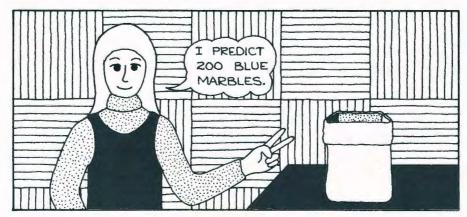
- 1) Tell the class that you have a collection of 300 marbles of different colors but all the same size.
- Ask, "How could we tell how many marbles there are of each color?" Suggestions should include looking at the contents of the bag and counting the number of marbles of each color.
- 3) Say, "Suppose we don't need to know the number of each color exactly and we don't wish to take the time to count the marbles?" Discussion can lead to ideas of taking a sample and using it to predict the contents of the bag.
- 4) Ask a student to draw a sample of one marble. Record the color on the chalk-board. Ask students what this sample tells about the contents of the bag. (There is at least one marble of that color in the bag. Some students might guess they are all that color.)
- 5) Have four more students each draw a marble. A student can record the results on the chalkboard. Ask what the sample of five tells about the contents of the bag. If the sample is 4 red and 1 white, students could say there probably are more red than white. However, it should be brought out that they don't know for sure.



6) Have five more students each draw a marble. Record the results. Ask students to use the sample of 10 to predict the contents of the bag. If the sample is 6 white, 4 red, students could say there probably are more white than red or there are about the same number of each color. Again students don't know for certain.



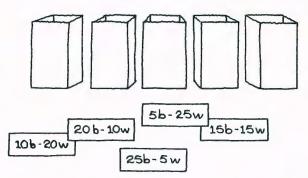
- 7) Have a student draw 10 more marbles. Record. Does the 20 marble sample change their feelings about the make-up of the bag? (If the make-up of the sample changes drastically, so should their feelings about the make-up of the bag.)
- 8) Have three more students each draw 10 marbles. Record. Use the sample of 50 to predict the make-up of the bag. Proportions could be used to estimate the number of each color.
- 9) Ask students which of the samples they think gives the best information for estimating the make-up of the bag. A sample of 5, 10, 20 or 50 marbles? (Even though a small sample might accurately describe the make-up of the collection, a larger sample is more likely to be accurate.).)
- 10) On the basis of the sampling so far, ask students if they would be willing to bet money that they could predict <u>exactly</u> the number of red marbles in the bag. Would it be safer to make a bet on an interval for the number of red marbles? Would they rather have a sample of 100 to work with? What size of a sample would they like in order to be willing to bet on the make-up of the bag? (Samples can be used to give accurate estimates. It would be much safer to bet on an interval.)
- 11) Extensions:
  - a) Prepare a collection of 400 marbles: 200 white, 150 red, 50 blue. Repeat steps 1 through 10.
  - b) Don't tell students the total number of marbles in the bag. By sampling, the students are to determine an estimate of the ratio of marbles of each color. After ratios are established, questions such as "If there are 500 marbles, how many are blue?" could be asked.





Activity I

Materials: 5 paper bags each containing 30 marbles. The contents of the bags are 25 black-5 white, 20b-10w, 15b-15w, 10b-20w, and 5b-25w. On five index cards write the contents of the bags.



- 1) Show students the five containers and five labels. By sampling, they will try to match each bag with its correct label.
- 2) Select a bag, mix its contents and ask a student to draw a marble. Have the student replace the marble and stir the marbles before drawing the next marble. Do this 15 times. A student can record the results on the chalk-board. Ask the class to identify the bag.
- Select a second bag and repeat the sampling procedure with a second student drawing a marble 15 times with replacement. Ask the class to identify the bag.



4) Repeat the process for the other three bags. Do the students feel they have matched each bag with the correct label? If not, which ones would they like to switch? Would they like to take more samples from any of the bags to help them feel more certain? Which bags seem the easiest to identify?



Activity II

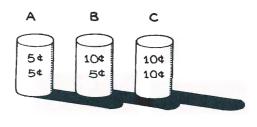
Can fewer than 15 samples of one marble be taken to identify the contents of the bags? Are more samples than 15 necessary?

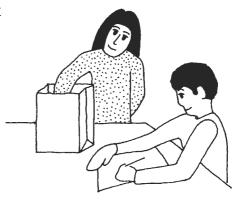
To investigate this:

- a) Organize the class into two-person teams.
- b) Give each team two previously prepared bags containing 25b-5w and 20b-10w or 20b-10w and 15b-15w or 15b-15w and 10b-20w or 10b-20w and 5b-25w.
- c) Also give each team all five label cards.
- d) Supply each team with the following student page.
- e) Discuss the results.

#### Activity III

Challenge: One can contains two dimes, one can contains two nickels and one can contains one dime and one nickel. Each can is labeled with an incorrect label. What is the fewest number of coins you need to draw (with replacement) to know the contents of all three cans? Erom which can(s) are the coins drawn?



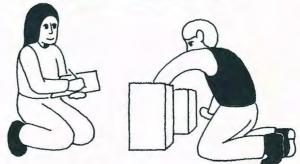


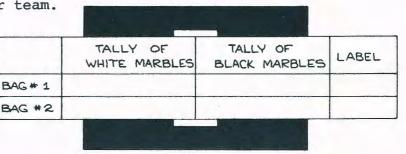


Materials: Two paper bags each containing 30 marbles Five labels, two of which describe the contents of the bags.

- 1) Without looking inside, draw a marble ten times from one bag. Replace each marble and stir the marbles before drawing the next marble. Have your partner tally the results. Use the samples to select the label that best describes the contents of the bag. Record the label in the table.
- Switch roles with your partner. Repeat the above sampling plan for bag #2. Try to identify the contents of the bag.
- 3) Do you feel as certain about the contents of the bags from the 10 samples as you did with the 15 samples used by the class?
- 4) Check to see if your predictions are correct.
- 5) Exchange your bags with another team.
- 6) Repeat steps 1 and 2 but draw a marble 30 times with replacement. Record the samples in the table. Try to identify the contents of the bags.
- 7) To identify the contents of the bags, which would you feel most certain using: 30 one marble samples, 15 one marble samples or 10 one marble samples?

	TALLY OF WHITE MARBLES	TALLY OF BLACK MARBLES	LABEL
#1			
#2			





Record the label.



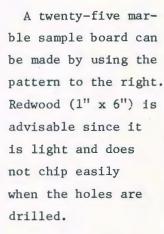
Materials: Collection of 200 marbles -- 20 red, 40 white, 140 blue

- A) If you draw two 10-marble samples, would you expect to draw the same number of red marbles in each sample? _____ (guess)
- B) If you took a random sample of ten marbles, on the average how many red, white and blue would you expect to get? _____ (guess)
  - Draw a random sample (every marble has the same chance of being drawn) of ten marbles. Count the marbles of each color. Record the numbers in the table. Put the marbles back and mix them up. Draw nine more samples of ten marbles. Record.

SAMPLE NUMBER	RED	WHITE	BLUE	
1				
2				
3				
4				
5				
6				
7				
8				2) Look at the table. Do the sample contain the same number of each
9				color?
10				3) For the samples find the average
TOTALS	97697999999999999999999999999999999999	1. 34 (You) 20() (You) 20()		number of: red marbles white marbles
				blue marbles

- 4) Compare the numbers in #3 with the make-up of the total collection. Could these averages be used to describe the make-up of the total collection?
- 5) Compare the numbers in #3 with your guess to question B. You may revise your guess here.
- 6) Challenge: You have a collection of 1500 marbles. On the average, a sample of 10 marbles has this make-up: 4 white, 1 black, 5 yellow. How many marbles of these colors would you expect in the 1500 marble collection?

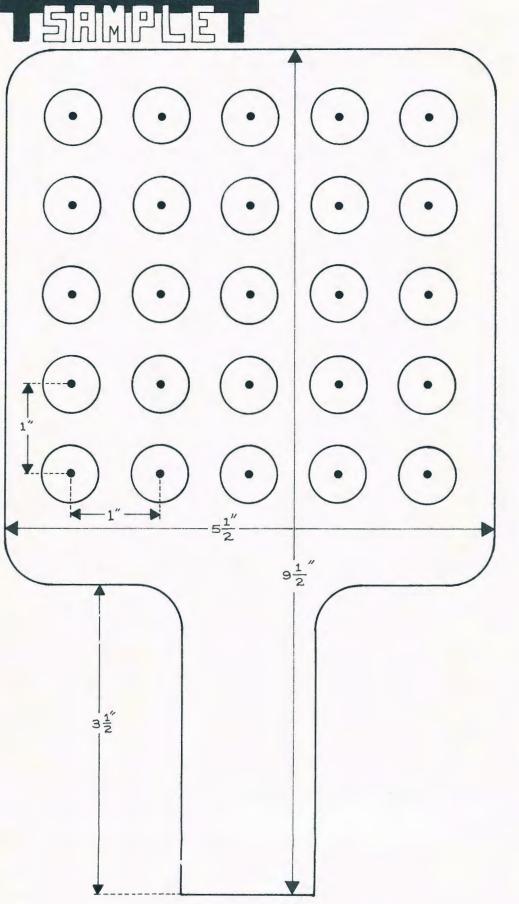
white _____, black _____, yellow _____.



0

Cut the board and transfer the markings for the holes. A 5/8" drill bit makes appropriately sized holes to fit marbles from a Chinese Checkers game. You may need to use a different size for your marbles.

Drill the holes almost through the board. A rotary cutter bit can then be used to smooth out the holes.





Materials: collection of 200 marbles -- 50 red, 150 white o o o If you took a random sample of 10 marbles from the collection, on the average would you expect to get 3 red marbles and 7 white marbles?

Test your answer by drawing a random sample of 10 marbles. Record the number of each color in the table. Replace the sample. Stir the collection. Draw nine more samples with replacement.

 For the samples find the average number of:

red marbles

white marbles

- 2) Repeat the above experiment but draw 10 random samples of <u>20</u> marbles. Replace each sample before drawing the next. Stir the collection each time. For each sample, record the number of each color in the table to the right.
- 3) For the samples, find the average number of:

red marbles _____ white marbles

4) On the average, which sample size better represents the make-up of the original collection:

10 marbles or 20 marbles

SAMPLE NUMBER	RED	WHITE
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
TOTALS		

0 0 0

Long - Salt - Sa		and the second second
SAMPLE NUMBER	RED	WHITE
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
TOTALS		
	1.	

LET'S USE DUR RANDTAS

Drawing samples of marbles may be slow and requires many materials. You can use your RAndom NUmber TAbleS to simulate drawing samples of marbles. Get a list of random digits from your teacher. Start at the arrow marked on your table. Read the digits in the direction of the arrow. When you come to the end of a row or column, start at the beginning of the next one.

 Use a random number table to simulate drawing a sample of 25 marbles from a bag containing 200 white and 100 red marbles. Twice as many are white so twice as many digits should stand for white. Use 1, 2, 3, 4, 5, 6 for white marbles. Use 7, 8, 9 for red. Ignore the 0 if you happen to read it. Read 25 non-zero digits for your sample. As you read, tally the results.

(For example, if the digits a	are	$\sim$ $\sim$	
1		03 39 71 19 48 20	9
The tally and totals are	RED	++++ ++++ )	(11)
	WHITE	++++ ++++ 1111	(14)
1		~~ ~ ~ ~ ~	

Repeat until you have 10 samples. Record the tallys and totals here.

	SAMPLE 1	SAMPLE 2	SAMPLE 3	SAMPLE 4
RED				
WHITE	-			

	SAMPLE 5	SAMPLE 6	SAMPLE 7	SAMPLE 8
RED				
WHITE				

SAMPLE 9

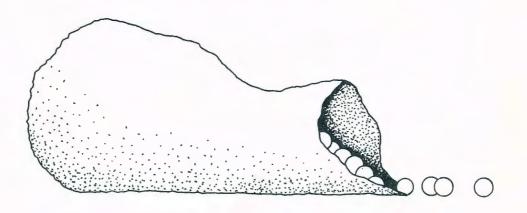
SAMPLE 10 MEAN OF SAMPLES

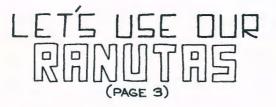
RED	MEAN OF RED =
WHITE	MEAN OF WHITE = "

Write the fraction: mean of reds/mean of whites. Is this fraction close to 1/2? _____ Do you think it should be close to 1/2? ____ Why or why not? _____



- 2. Suppose a bag contains 200 red and 300 black marbles. You want to simulate drawing samples by reading digits from a random number table. How many of the 10 digits should stand for red marbles? ______ How many for black? ______ Choose the digits for red. ______ For black. ______
- 3. Do the same if the bag has 200 red and 500 black marbles.
- 4. Suppose the bag has 350 red and 650 black marbles. Use the table to read two-digit numbers such as 21 or 36 or 99 or 00. Let the thirty-five numbers from 01 to 35 stand for red. Which ones will stand for black? How many should there be? _____ Can any be ignored? _____
- 5. Suppose the bag has 370 red and 470 black. Which two-digit numbers could stand for red? ______ Which for black? ______
  Which should be ignored? ______





6. Suppose a bag has 200 green, 150 black and 50 white marbles. There are four times as many green as white marbles and three times as many black as white.

When you simulate drawing samples from this bag by reading random digits, let one digit stand for a white marble.

Choose a digit to stand for a white marble. Write it here. _____ How many digits should stand for black marbles? _____ Choose the digits to stand for black marbles. Write them here. _____ How many digits should stand for green marbles? _____ Choose the digits to stand for green marbles. Write them here. _____ How many digits should you ignore? _____ Which digits will you ignore? _____

Simulate 10 samples of size 25. Record the results here.

	SAMPLE 1	SAMPLE 2	SAMPLE 3	SAMPLE 4
GREEN				
BLACK				
WHITE				

	SAMPLE 5	SAMPLE G	SAMPLE 7	SAMPLE 8
GREEN				
BLACK				
WHITE				

 SAMPLE 9
 SAMPLE 10
 MEAN OF SAMPLES

 GREEN
 MEAN OF GREEN =
 MEAN OF BLACK =

 BLACK
 MEAN OF BLACK =
 MEAN OF WHITE =

What is the ratio of the mean number of blacks to whites? ______ What is the ratio of the mean number of greens to whites? ______ What numbers do you think these ratios should be close to? _______ Why?





II. To check your guesses, run the computer program SUCCES Let N = 2000 N1 = 20 (the red marbles are called successes)

M = 50

The computer will draw one hundred 50-marble samples.

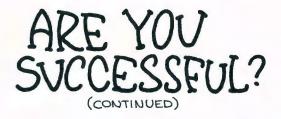
III. In the table record the distribution of successes (red marbles) for the one hundred samples.

Compare the results with your guesses. Were you close?

From the printout what is the average number of successes (red marbles)?

This number is _____ percent of the 50 marbles.

NUMBER OF SUCCESSES (RED MARBLES)	FREQUENCY



- IV. Make a graph of the distribution of red marbles (successes) in the one hundred samples. Which column(s) are high?
- V. Run the program again. Make a graph of the new distribution. Are the graphs similar?
- VI. Suppose 40 of the 2000 marbles were red (2% of 2000). In a sample of 50 marbles how many would you expect to be red? ______ For this 2% mixture do you think a graph of the distribution of red marbles in one hundred 50-marble samples would differ from the ones you drew in IV and V? _____

To see, run the program SUCCES.

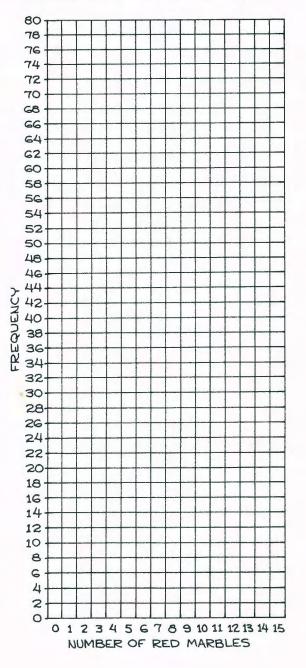
- Let N = 2000
  - N1 = 40
  - M = 50

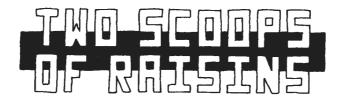
Make a graph of the distribution. Which column(s) are high?

From the printout what is the average number of successes (red marbles) for this 2% mixture?

This number is _____ percent of the 50 marbles.

#### DISTRIBUTION OF RED MARBLES (SUCCESSES) IN ONE HUNDRED SAMPLES FROM A POPULATION WITH 1% RED MARBLES

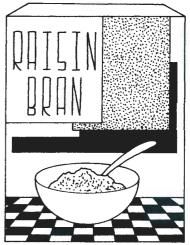


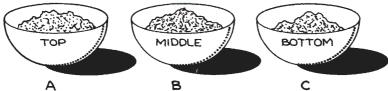


Materials: box of Raisin Bran, 3 bowls, 100 ml measuring cup

Are the raisins evenly distributed in a box of Raisin Bran cereal?

- To investigate this:
- Carefully pour the Raisin Bran into the three bowls. Try to pour the top third of the box into one bowl; the middle third into the second bowl; the bottom third into the last bowl.





- 3) Repeat this sampling procedure for the bowls with the middle and bottom thirds. Count the raisins and record below.

 Bowl B
 Bowl C

 Sample #1
 Sample #1

 Sample #2
 Sample #2

 Mean
 Mean

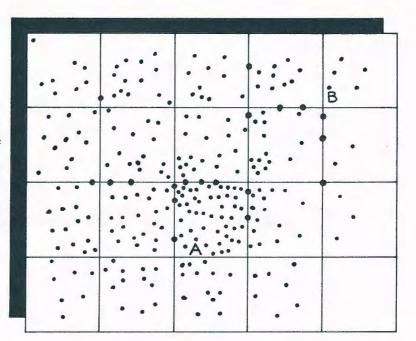
4) Compare the means. Are there about the same number of raisins in each bowl? ______ (Be sure to compare the mean from bowl A.) Are the raisins evenly distributed in a box of Raisin Bran cereal? _____ Why? _____



One year in California, aerial pictures were used to count 4.5 million ducks and 731,000 geese. Are these numbers exact or approximate? How do photo interpreters make these estimates?

Suppose each dot in the picture represents a duck. Can you guess the number of ducks?

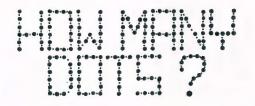
- Find section A. Count the dots. If a dot is on a line, count it.
- 2) How many sections in this picture?
- To get a total estimate, multiply your answer to #1 by your answer to #2.



4) Is this a good estimate of the total number of dots? _____ Explain.

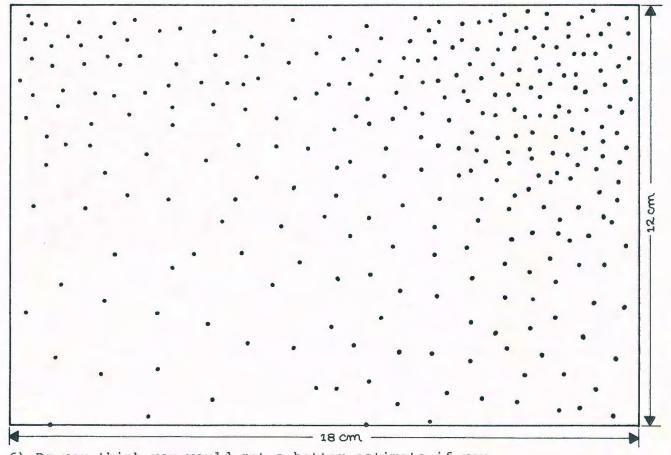
5) Find section B. Count the dots. ____ Find the total estimate.____

- 6) Are your two estimates very different? _____ If so, explain why._____
- 7) Select a section you feel will give you a good estimate. Count the dots. _____ Find the total estimate. _____
- 8) Compute the average number of dots in sections A, B, and your section.
  Use the average to make a total estimate.
- 9) Compare the estimates in #7 and #8 with those in #3 and #5. Which estimates seem most accurate?



#### HOW MANY DOTS ARE IN THE RECTANGLE?

- To get an estimate, use the 3 cm by 3 cm square provided by your teacher. Place the square inside the rectangle. Count the dots in the square and on the edges of the square. Record the number of dots.
- 2) Do this two more times. Each time place the square in a different part of the rectangle. Record.
- 3) Find the mean number of dots in these three samples.
- 4) If placed side by side, how many 3 cm by 3 cm squares would cover the rectangle?
- 5) To estimate the number of dots in the rectangle, multiply your answer to #3 by your answer to #4.



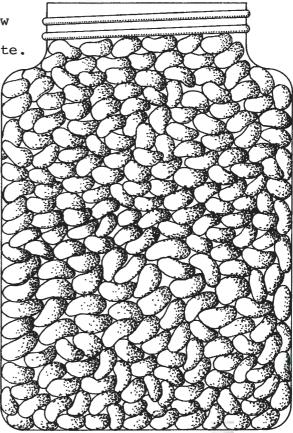
- 6) Do you think you would get a better estimate if you
  - a) used a 2 cm by 2 cm square or a 4 cm by 4 cm square?b) took one sample where the dots are crowded, one where there are few dots and one where there is a middle amount?
  - c) took 5 samples?
- Try the suggestions in #6. Write down your estimate for each suggestion.



Materials: 2 litre container filled with beans of one color 100 ml cup

Try to guess the number of beans in the container. ____ Try the plan below to help you check or revise your estimate.

- Use the 100 ml cup to take a sample of beans from the jar. Be sure the beans are level with the top of the cup.
- 2) Count the beans in the cup.
- Take three more 100 ml samples.
   Count the beans in each sample.
- 4) Find the average number of beans in the four samples.
- 5) How many 100 ml cups are contained in the 2 l container? ______ (Hint: 1 l = 1000 ml)



- 6) Use your answers from #4 and #5 to estimate the number of beans in the 2 l container.
- 7) Suppose the 2 & container was filled with flibbers. When four 100 ml samples were taken, the counts were 176, 187, 194, 167. Find the average number of flibbers in the four samples.

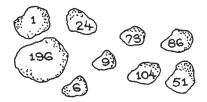


8) Estimate the number of flibbers in the 2  $\ell$  container.



The following activity can be used to help students see that a random sample can give a good estimate for a population (perhaps a better estimate than a sample which they carefully choose).

Preparation of materials:



	Ę
1) 22	51)46.0 }
2) 86.5	52) 41.3
3) 28	53)40
4) 75.5	54) 28.6 3
hann	m

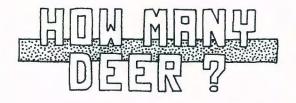
- a) Collect 200 small stones. The stones should vary in mass from about 20 grams to 100 grams. You could have the students or an aide gather the stones.
- b) Number the stones from 1 to 200. (Do not number them in order of increasing size.)
- c) Find the mass of each stone to the nearest gram or to the nearest tenth of a gram.
- d) Prepare a record sheet that lists the stones in order from 1 to 200 with their masses.
- e) Find the total mass of the stones and divide by 200 to find the mean mass of the stones.

#### Experiment:

- Place the stones on a table with the numbered sides facing up. Tell students they will each try to estimate the total mass of the stones by taking a representative sample.
- 2) Have each student inspect the stones and record the numbers of ten stones that he/she feels are representative of the collection. Some students will select five large and five small, others will try to pick ten that seem about average and others will try to use no pattern. (Don't suggest any of these plans in advance to the students.) The activity works smoother if about five students are at the table at one time.



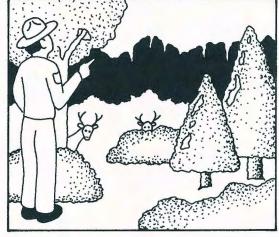
- 3) As each student finishes selecting the sample, give him/her a copy of the record sheet. (Alternatively, several could be posted in the room.) Tell the student to find the total mass of his/her sample and the mean of the sample. Then each student multiplies by 200 to approximate the mass of the collection. Have each student record his/her approximation on the chalkboard.
- 4) After the class has finished sampling, place a transparency of random digits on the overhead and select ten numbers from 1 to 200 inclusively. Use the record sheet to find the masses for the stones with these numbers. Find the mean of this sample and multiply by 200 to approximate the mass of the collection. Record the results on the chalkboard.
- 5) Tell the class the total mass of the stones based on the weighings.
- 6) Have each student find the difference between this amount and his/ her approximation. You do the same for the random-digit approximation.
- 7) Compare your difference with the students' differences. Find how many students got closer approximations than you did.
- 8) The random digits can be used to select another sample. Again, compare the results with the students' approximations.
- 9) Discuss the students' methods for selecting the stones in their samples. At least some of the students should feel that the random-digit method of selecting a representative sample is as good or better than any of their plans.



Materials: A deck of playing cards (no jokers)

How might a wildlife official estimate the number of fish in a lake? How might a ranger estimate the number of deer in a large park like Yellowstone?

In the experiment below, the cards represent the deer population. You are going to learn a method for estimating the number of deer (cards).



- 1) Don't count the number of cards in your deck! That is what you are trying to find.
- 2) Shuffle the deck several times.
- 3) Select eight cards at random. Record both the number and suit--9D, KC, 5H, etc.

These eight cards represent eight marked deer.

- 4) Return the eight cards (deer) to the deck in different places and shuffle several times.
- 5) Select five cards at random from the deck for sample 1. In the table, record the number of marked cards you drew. Return the cards to the deck in different places.
- Shuffle the deck several times. Draw five more cards for sample 2. In the table, record the number of marked cards.
- 7) In a similar manner, draw four more five-card samples. Be sure to return each sample. Shuffle several times before drawing the next sample.

	No. of Concession, Name
SAMPLE	#OF MARKED
1	
2	
3	
4	
5	
6	
TOTAL	

- 8) Find the total number of marked cards in the six samples.
- 9) Use this proportion to estimate the total number of cards in the deck (the total deer population):  $\frac{m}{x} = \frac{d}{c} \qquad m = \text{ the number of marked cards (8)}$  x = total number of cards in the deck d = total number of marked cards drawn

c = total number of cards in the samples (30)

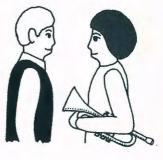
10) To check your estimate of the total deer population, count the number of cards in the deck.

IDEA FROM: STATISTICS BY EXAMPLE, Exploring Data. F. Mosteller, et al. Addison-Wesley Publishing Company, Inc.

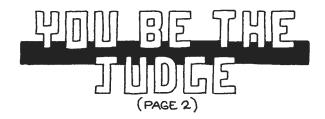


- You wish to take a poll within the school to find out how popular basketball is.
  - a) Would your results be the same if you sample only those who play basketball? all girls? grade six students?
  - b) Which students would you sample in the poll?
- The music department in your school wishes to know how much interest there is in starting a new music program. The department decides to ask the members of the school band.
  - a) Do you think the opinions of the band members will accurately describe the feelings of the student body?
  - b) If you didn't have time to ask everyone in school, which students would you select?
- 3) Goldilocks was hungry and saw the bears' porridge sitting on the table. She carefully skimmed a spoonful off the top of each bowl to see if it was the right temperature to eat.
  - a) Will Goldilocks' method of sampling mislead her?
  - b) If so, how should she sample the porridge?

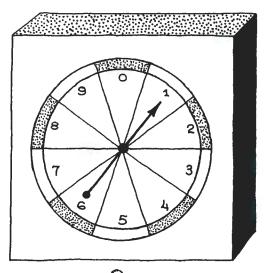




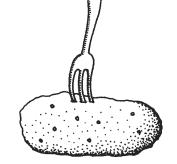




- 4) A math teacher wishes to pick three students to collect homework. To avoid showing favoritism, the teacher numbers the students from 1 to 30. The teacher then spins a spinner twice to pick a student. (Spins of 2 and 3 mean that student #23 is selected.)
  - a) Do you think this is a fair way to choose students?
  - b) Can you think of another way so everyone has an equal chance?



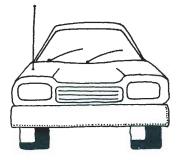
5) Do you think the following sampling methods would give accurate information? If not, suggest a better plan.



a) Deciding whether potatoes are cooked by piercing one potato with a fork.

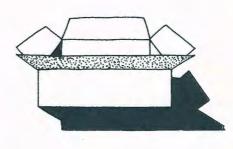


- b) Determining the most popular brand of cigarette by questioning customers in the non-smoking section of a restaurant.
- c) Determining the most popular make of car by counting
   500 cars in a large shopping center parking lot.





- 6) BUY FROM US mail-order company opens some of its outgoing packages to see if the orders are being filled correctly. The inspectors check packages from 9:00 A.M. to 10:00 A.M. on Wednesdays. The inspectors have found few mistakes, but many customers are complaining.
  - a) What is wrong with the inspectors' method?

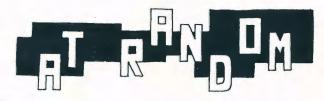


- b) Inspector Ike suggests a new plan: Check packages every day, but only those heavier than 20 kg. Do you think Ike's plan will work? If not, what would you suggest?
- 7) In 1936, the Literary Digest magazine conducted a poll to predict the next president of the United States. The magazine mailed 10 million sample ballots to persons on telephone and automobile owner lists. 2.3 million ballots were returned. 57% voted for Landon; 43% for Roosevelt. However, Roosevelt won the election getting 63% of the vote! What might have caused the error in the Literary Digest's poll?

ROO	SEVEL	T THE
	WINN	ER
DA	ILY ST	R-NEWS

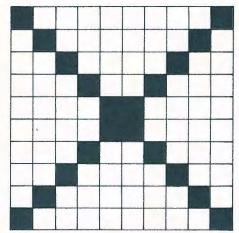
8) Today the Gallup Poll conducts surveys to predict the feelings and opinions of Americans. About how many people do you think Gallup polls in its nationwide surveys?

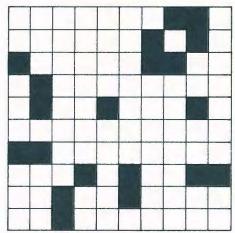




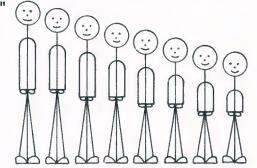
### WHAT DOES "AT RANDOM" MEAN?

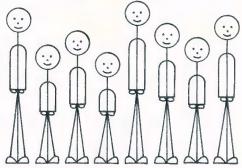
I. Twenty squares are shaded in each grid below. The first grid was shaded according to a planned pattern. The second grid was shaded "at random."





II. Eight students are standing in a line. The first time they are arranged in order of height. The second time they are arranged "at random." (...)





III. The first list shows ten words arranged in alphabetical order. The second list shows the same words arranged "at random."

percentile
population
range
sample
statistics

the second se	
sample	median
mode	range
average	population
mean	statistics
percentile	graph

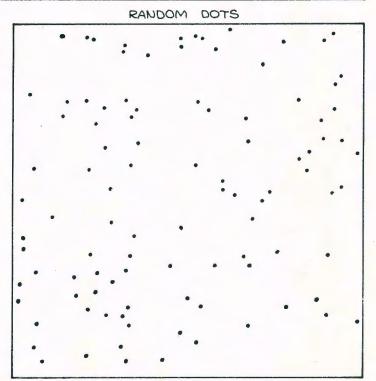
IDEA FROM: Making Mathematics, A Secondary Course, Book 4, by D. Paling, et al.

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## AREA BY RANDOM DOTS

	GRID OF DOTS										
	•	•	•	•	•	•	•	•	•	•	
Materials: 5 cm by 5 cm square of transparency Irregular shapes of		•	•	•	•	•	•	•	•	•	
transparencies (see second page)	•	•	•	•	•	•	•	•	•	•	
1. What is the area of the square to the right? How many	•	•	•	•	•	•	•	•	•	•	
dots are in the square?	•	•	•	•	•	•	•	•	•	•	
<ol> <li>What is the area of the 5 cm by 5 cm square of transparency?</li> </ol>	•	•	•	•	•	•	•	•	•	•	-10cm
Place it inside the large square. How many dots does it cover? (If dots fall on the edge, count	•	•	•	•	•	•	•	•	•	•	
half of them.)	•	•	٠	•	•	•	.•	•	•	•	
3. Place the transparent square any- where inside the square four more	•	•	•	•	•	•	•	•	٠	•	
times. Each time count the number of dots covered. Find the mean	•	•	•	•	•	•	•	•	•	•	
number of dots covered for five trials.	-				10	)cm -				-	

- 4. On the average, what fraction of the total dots are covered by the transparent square each time?
- 5. There are 100 dots arranged at random on the square to the right. Place the transparent square inside the square. How many dots does it cover?
- Do this four more times. Find the mean number of dots covered for the five trials.
- 7. On the average, what fraction of the total dots are covered by the transparent square each time?
- The tranparent square covers about 1/4 of the dots because its area is the area of the large square.



IDEA FROM: Making Mathematics, A Secondary Course, Book 4, by D. Paling, et al.



Note: On the previous page, you showed that a shape with 1/4 of the area of the square should cover about 1/4 of the dots. It is also true that if a shape covers 1/4 of the dots, it should have about 1/4 of the area of the square. A simi-lar relationship holds for other shapes and fractions.

- 9. Trace these shapes onto a transparency and cut them out. 10. Place each of them inside the RANDOM DOTS square five times. Find the mean number of dots covered by each shape. parallelogram: ( ____ + ___ + ___ + ___ + ___ ) ÷ 5 = ___ ( _____ + ____ + ____ + ____ ) ÷ 5 = _____ triangle: 11. What would be a good estimate for the area of the parallelogram? For the area of the triangle? 12. Trace these shapes onto a transparency and cut them out. 13. Place each of them in the RANDOM GLOB #1 DOTS square five times. Find the mean number of dots GLOB #2 covered by each glob. glob #1 _____ glob #2 14. What would be a good estimate for the area of glob #1? glob #2? 15. Suppose the RANDOM DOTS square on the
- previous page had <u>200</u> dots. Estimate the area of each shape given in the table below.

Shape	Mean number of dots covered	Approximate area of shape
	100	
A	100	
В	50	
С	20	
D	45	
E	36	

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## ORGANIZATION

## Placement Guide for Tabbed Dividers

Tab Label	Follows page:
CONTENT FOR TEACHERS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
Probability with Counting	
Inferential Statistics	
Appendix-Computer Programs	
DIDACTICS	
Components of Instruction	
Classroom Management	
Statistics and Probability Learning	
TEACHING EMPHASES	
Critical Thinking	
Decision Making	
Problem Solving	
Models and Simulations	
Calculators and Computers	
Laboratory Approaches	
CLASSROOM MATERIALS	
CONCEPTS AND SKILLS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Misleading Statistics	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	604
Probability with Models	634
Counting Techniques	
APPLICATIONS	
Business & Commerce	
Environment	
Health & Medicine	
People & Culture	
Recreation	
GLOSSARY	
ANNOTATED BIBLIOGRAPHY	
SELECTED ANSWERS	



### Commentary to EXPERIMENTAL PROBABILITY (607-614)

TITLE	PAGE	TOPIC	TYPE
Picking Marbles 2 at a Time	615	Finding the relative frequency of outcomes	Activity card Worksheet
Spinning Coins	616	Determining fairness of spinning a coin	Activity card Worksheet
I'll Flip You For It	617	Recording outcomes of flips of a coin	Activity card Worksheet
Get Back on the Track	618	Using coin flips to simulate a random walk	Activity card Worksheet
Tossing Pennies	619	Using coin flips to determine fairness of a game	Activity card Worksheet
A Tree Spinner	620	Examining outcomes of a spinner	Activity card Worksheet
The Gallant Sir Lancelot	621	Matching outcomes to spinners	Worksheet Puzzle
The Hexed Hexaspin	622	Determining fairness of a hexaspin	Activity card Worksheet
Roll That Cube	623	Estimating the frequency of outcomes of dice	Activity card Worksheet
Roll a Die	624	Recording the outcomes for rolls of a die	Activity card Worksheet
A Cereal Question	625	Using a simulation to gather data to make a prediction	Activity card Worksheet
A Complete Set	627	Using a computer to find the expected number of purchases	Worksheet
700 Random Tosses of a 6-Sided Die	628	Using a random number table	Worksheet

605

TITLE	PAGE	TOPIC	TYPE
Random Numbers via a Calculator	629	Generating random numbers with a calculator	Teacher idea Transparency
Will the Spider Catch the Fly?	630	Using dice rolls to simulate a random walk	Activity card Game
Rolling Dice	631	Estimating the frequency of the sums of two dice	Activity card Worksheet
Crazy Quotients	632	Determining fairness of a game	Game
The Even/Odd Game	633	Predicting outcomes of dice	Game

(

## EXPERIMENTHL PROBRELITY

Words related to probability are already incorporated in the vocabulary of students. They know that the exact outcomes of many games, sporting events and everyday happenings cannot be predicted, so their conversations include phrases like those to the right.

We all use our experiences to help us predict what will happen in the future. Here is an example you can use to relate experimental probability to



everyday life. Sid has had five meetings with Fred. Fred has been late four of these times. From these experiences, would you guess Fred will be late or on time for the next meeting with Sid? (Probably late. Based on this small amount of data, there are 4 chances out of 5 that Fred will be late and 1 chance out of 5 that he will be on time. Of course, one would want to collect more information on Fred before assuming he is late 4/5 of the time.)

The experiments in this section provide an opportunity for students to collect, organize and interpret data to gain a better general understanding of the likelihood of certain events happening. Here is the general format for the activities.

I. A question or problem is posed.



"If you flipped a coin 100 times, would there be more heads or tails?" (See I'll Flip You For It.)

"Is it easier to roll a 4 than a 6 with one die?" (See Roll A Die.)

"If you drop a thumbtack 100 times, what part of the time will it land point up?"

II. Students are asked to make a guess. (Their guess can tell you a little about their subjective ideas of probability. One student might think heads is lucky or 6's very hard to get, and so forth.) III. An experiment is outlined for the student to do.

A coin is flipped 100 times.

A die is rolled 60 times.

Ten thumbtacks are dropped 10 times.

IV. The data from each student is recorded. The data from different students is compared and/or compiled. Patterns or trends are discussed. Students might revise their guesses.

"There were more heads for 11 students and more tails for 14 students. The totals showed 1253 heads out of 2500 flips. Sometimes tails comes up more. Sometimes heads."

"4s came up more often than 6s for 8 students, but 6 happened more often for 12 students and they tied for 5 students. Sometimes 4s come up more and sometimes 6s.

"The fraction of thumbtacks landing point up ranged from 54/100 to 67/100. Seems like they land point up over half of the time."

#### THE UNEXPECTED CAN HAPPEN

Experiments will not necessarily convince a student that certain events have the same chance of occurring. Two examples are given below.

A teacher decided to question an eleven year old sixth grade boy to see if he thought heads and tails had the same chance of occurring in a coin flip. The following conversation resulted.

Teacher: If you flipped a coin 100 times, how many times would it come up heads and how many times tails?

Student:	I'd gu	iess	about	32	heads
	and 68	3 ta:	ils.		
Teacher:	You th	nink	tails	is	more

likely than heads?

Student: Yep.

Teacher: Have you ever tried it 100 times?

Student: No, but my dad did and that's what he got. Want to try it?



#### COMMENTARY

Teacher: Sure.

Student:	(100 flips later) 53 tails! I knew tails would win!
Teacher:	Do you think that would happen if we did it again?
Student:	Well, maybewant to go best two out of three?
Teacher:	Sure.
Student:	Come on tails! (100 flips later) 57 tails! See!
Teacher:	Do you think tails would always win?
Student:	Well, I guess heads could win, but tails is lucky for me.

The teacher decided not to force the idea that heads and tails have the same chance with a fair coin. The student now had three experiments on which to base his judgements--all of which favored tails, but not so radically as the one experiment he had heard his father describe. In class, the student would be able to observe the results of other students and see (most likely) that tails doesn't always win. This data could be accumulated and the relative frequency for both heads and tails should settle <u>around</u> .5.

The idea that tails (or heads) is lucky for some people is quite common even among people who know that the chances for each are the same. Many adults have a similar feeling about luck. One college basketball coach was distressed when a bag containing his favorite suit was misplaced during a plane flight. When the suit was found in time for the game and his team won, he said, "I knew we couldn't lose; not when I'm wearing my lucky suit!"

A teacher of an eighth grade general mathematics class introduced the ideas on *Roll A Die* by asking the students if they thought any number was harder to roll than the others. Most students thought 1s and 6s were harder to get. (Possibly because 1s and 6s are needed for many games.) A die was passed around the room. Each student rolled twice and recorded the rolls by placing a cube on the front desk over the correct number. After twelve rolls were completed, the cubes showed four 6s, two 5s, one 4, three 3s, two 2s and no 1s. The students were saying, "It can't happen!" or "It never happens this way!"



#### COMMENTARY

The teacher asked the students to predict the next roll. Some thought it was time for a 1 or a 4. Others thought the best guess was 6 because it had occurred most often so far. After the sixteen students had rolled twice, the results were this:

Number on Die	1	2	3	4	5	6	
Frequency	4	4	7	4	4	9	

Comments from students included, "But tomorrow we'd get something different;" "This doesn't prove anything!" and "I still think is and 6s are harder to get--the is didn't show up much."

The teacher passed out copies of Roll A Die and had the students work in pairs to gather more data. She was <u>hoping</u> 1, 2, 3, 4, 5 and 6 would occur about the same number of times so students would see they are equally likely. After each pair had rolled 60 times, the results given to the right were recorded on the board.

Some students were reinforced in their belief that 1s are harder to get because of the low total in that column.

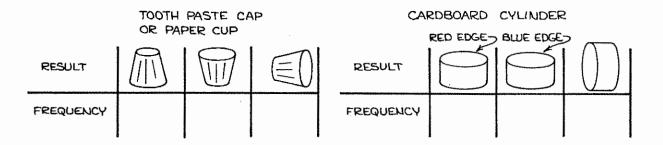
KEDULID FROM ROLLAVIE								
	1	2	З	4	5	ရ	TOTALS	
PAIR 1	6	10	10	11	11	12	ଦେ	
PAIR 2	8	8	13	12	12	7	60	
PAIR 3	9	10	10	12	4	15	60	
PAIR 4	G	Э	9	11	12	13	ଚେ	
PAIR 5	9	11	12	10	B	10	ଦେ	
PAIR G	11	7	9	େ	13	14	60	
PAIR 7	9	13	8	17	е	4	0 N	
PAIR 8	10	7	6	16	11	10	60	
TOTALS	68	75	77	95	80	85	(480)	
	-			-			ROLLS	

They were surprised that 6s had the second highest total. Because more variation had occurred in the rolls that teacher has expected, she did not try to convince students the numbers were equally likely. She had the students discuss the variability of the numbers which ranged from four 6s to seventeen 4s. The students were interested in the frequency of the numbers in the table and the teached helped them see that 10 was most frequent. (This will not always be true.) They knew it was unlikely for this exact distribution to happen again.

Students were asked to make predictions, "If you throw the die 6000 times, about how many 1s, 2s, ... 6s would you expect?" They said, "You can't tell until you do it." or "It'll be different every time." Some students multiplied their experimental results by 100. The teacher realized this was a sensible use of their experiment and that the "expected value" of 1000 for each number was an idea better saved until students were more familiar with equally likely outcomes. These two examples show that experimental probability will not always give the hoped-for results; students might not become convinced that certain events are equally likely.

#### SOME FIRST EXPERIMENTS IN PROBABILITY

Because students have had experiences with coins and dice, they may have formed ideas about tails happening more often, 6s being hard to get and so on. These sometimes poorly founded ideas can interfere with logical interpretations of experiment with objects that do not have commonly accepted probabilities such as thumbtacks to estimate the number of times a thumbtack will land point up. You could also use toothpaste or shampoo caps. Students could also investigate cardboard cylinders or different lengths cut from the center of paper towel rolls, or small blocks of wood or foam that are <u>not</u> cubes.



When students are tossing bottle caps or cardboard cylinders, be sure they give the cap or cylinder plenty of "flip" for a fair toss. You can ask if they are willing to predict the <u>approximate</u> results of 1000 tosses based on the results of one toss. Ten tosses? One hundred tosses? They should realize that the more tosses, the more likely they can make a reasonable approximation. If you have students compile their data (this provides a large number of trials without boring one pair of students), be certain their bottle caps or other models are as similar as possible. A toothpaste cap with a hunk of dried toothpaste will not have the same results as a very clean one.

Students should relate the results of their experiments to the design of the physical objects they are tossing or spinning. This might be easier to do when the possible results are not "equally likely." (That is, a student might flip 53 tails out of 100 and feel there is definitely a better chance for tails. It is unlikely

1. Salar

#### EXPERIMENTAL PROBABILITY

#### COMMENTARY

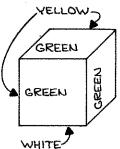
to have exactly 50 tails occur.) You can have students experiment with objects like those below and ask them to explain their results in terms of the model. These experiments also give you a good chance to observe their intuitive ideas of probability.

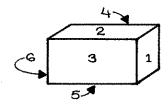
• If a student rolls this cube sixty times, the results will probably show more greens than yellows and more yellows than whites. The student can explain this by saying there are more green faces than yellow and more yellow than white.

• If a student rolls this oblong block of foam sixty times, the 6 and the 1 will most likely occur much less frequently than the 2, 3, 4 or 5. Students can explain this in terms of the shape of the block. "There is less room for the 1 and 6 end to land on" or "It is more stable when it is lying down." Without the experiment, <u>some</u> students might think the numbers have the same chance of occurring.

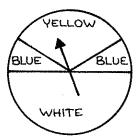
• If a marble is drawn from the bag, its color recorded, the marble returned to the bag and the procedure repeated 20 times, probably more blues will be recorded than reds. The results can be explained in terms of the greater number of blue marbles in the bag. (Young students might feel red is more likely because it is their favorite color or because there is only one red--see pages 3-4 in *Statistics and Probability Learning*.)

• For this spinner, students might think blue has a better chance because it shows up in two places. An experiment can show that white occurs more often. Students can relate these results to the greater area of the white section than the total area of blue. Note: Unbiased spinners are difficult to make. The way they are spun can also influence results.









#### COMMENTARY

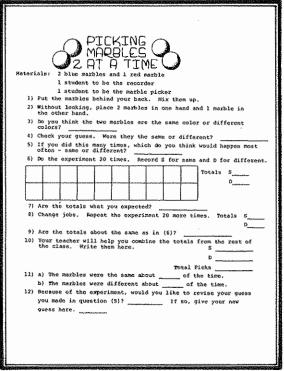
#### BRINGING OUT THE IDEA OF EQUALLY LIKELY

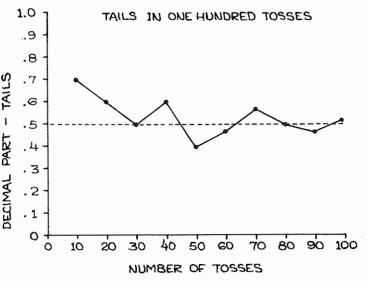
It is hard to convince some students that heads and tails are equally likely--that they have the same probability of happening--by flipping a coin a predetermined number of times. Even after 1000 flips, some students might believe tails is more likely if tails occurred 505 times. They focus on the greater number of tails rather than the ratio of tails to total number of tosses. There are several ways to avoid this.

I

Have students work first with models where the outcomes are not equally likely. (Some simple examples were given in the previous section.) The activity Picking Marbles 2 At A Time has students collect data which will help them see it is twice as likely to draw 2 marbles of the same color as it is to draw 2 marbles of different colors. The activity can then be extended to choosing 2 marbles out of 3 blues and 1 red. In this second experiment, the number of differently colored pairs should be much closer to the number of same colored pairs. The contrast in the data. for the two experiments helps students be more willing to accept the idea that "same" and "different" are equally likely in the second experiment.

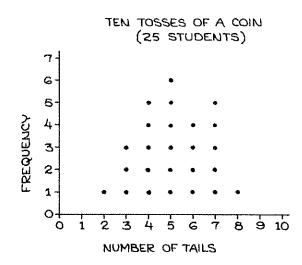
Build the idea of ratios as a decimal into the activities. Instead of recording only 27 tails out of 50 flips, have students write this as 27/50 or .54. This number approximates the probability of getting a tail. Students could calculate decimals after 10, 20, 30, ..., 100 tosses and see that it stabilizes (usually) around .5. Data from the whole class could also be compiled and a decimal ratio plotted on a graph.





#### COMMENTARY

• Look at a distribution of the results from several repeats of the experiment. Have each student in, the class flip a penny ten times. Make a graph of the individual results. If your class has 25 students the graph might look like the one shown here. You can point out how the results cluster around 5--half heads and half tails. Much variability is possible, but this is fairly evenly distributed around the 5.



Once the idea of equally likely is accepted as reasonable, coins, dice, spinners and random digits can be used in simulations. A Cereal Question and A Tree Spinner use these models to simulate situations. In these activities, a question is posed, data gathered from the simulation and a generalization is made from the data. (You might want students to consider why a particular model is used in a situation. See Models and Simulations in the TEACHING EMPHASES section.)

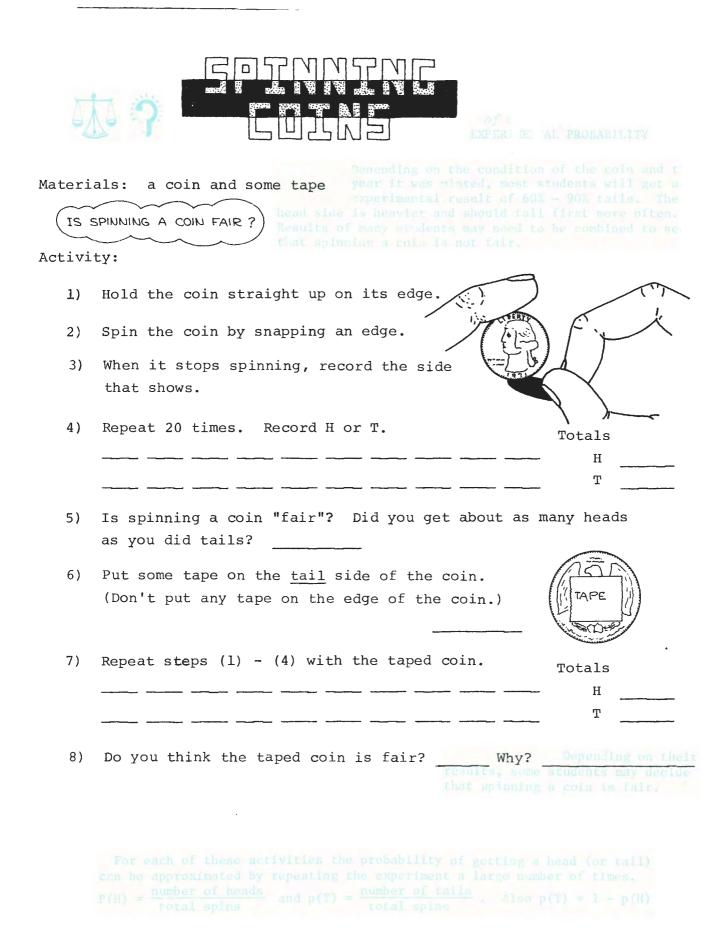
J. Ý	PICKING MARBLES ZATATIME	Finding the relative frequency of outcomes EXPERIMENTAL PROBABILITY
Materials:	2 blue marbles and 1 red marble	

- l student to be the recorder
- 1 student to be the marble picker
- 1) Put the marbles behind your back. Mix them up.
- 2) Without looking, place 2 marbles in one hand and 1 marble in the other hand.
- 3) Do you think the two marbles are the same color or different colors?
- 4) Check your guess. Were they the same or different?
- 5) If you did this many times, which do you think would happen most often same or different?

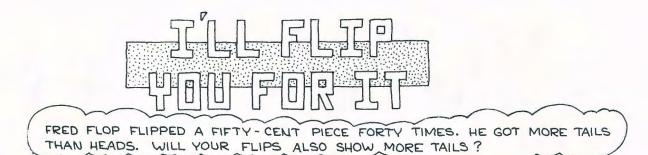
6) Do the experiment 20 times. Record S for same and D for different.

		~	Totals S D
<ol> <li>7) Are the total</li> <li>8) Change jobs.</li> </ol>			pre times. Totals SD
		ombine the to	
12) Because of th	s were differen e experiment, w	about t about ould you like	otal Picks of the time.
guess here.	ues cron (5):	<u> </u>	SO, give your new
could be good of the colors. The activity can be commarble. Encourage stuthey will guess that the ferent colors. As the three of the six picks colors. To emphasize in each hand. One han and the other hand mus TYPE: Activity Card/Wo	picked. In two of In the long run th e of getting diffe ntinued using thre dents to guess as he picked marbles diagram shows, th give the same col the equal chances, d must show two ma t show two marbles rksheet	the three picks e chance of gett rent colors is 2 e blue marbles a in (4) and (5). will more often e chances are ec or and three giv put two of the rbles with the s with different	and one red Usually have dif- qual since ve different marbles same color colors.
DEA FROM: the Probability ar for the Intermedi	nd Statistics strand of the p	reliminary experimenta	al version of CSMP Mathematics

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tivit urd, rk



Materials: a coin (no two-headed coins allowed) Activity:

- 1) Flip your coin twenty times. Record H for heads or T for tails.
- Record the number of heads and the number of tails from your twenty flips.

	FREQUENCY	
н		
т		

- 3) Which occurred more for you heads or tails?
- 4) Check with other students. Which occurred more?
- 5) Pretend you flipped ten heads in a row.
  - a) What do you think the next flip will be?
  - b) Is the llth flip affected by the results of the first ten flips?



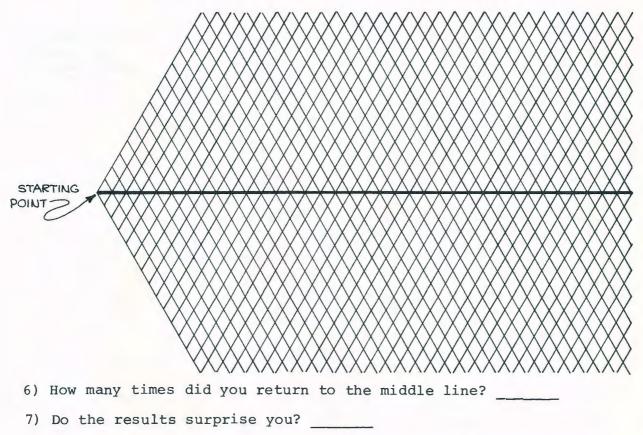


Materials: a coin

#### Activity:

- 1) Flip the coin.
- 2) Heads (H) means move one space up.
- 3) Tails (T) means move one space down 🔪
- 4) In 50 flips how many times do you expect to return to the middle line?
- 5) Do the experiment. Flip 50 times. Record your results.





8) In the 50 flips, how many heads _____ and how many tails _____ did you get?



Gary, the gambler, is setting up a friendly game. He suggests to Tony, "Let's toss pennies. I'll toss first. If it comes up heads, I win. If it comes up tails, you toss. If your penny comes up heads, I win; if it's tails, you win."

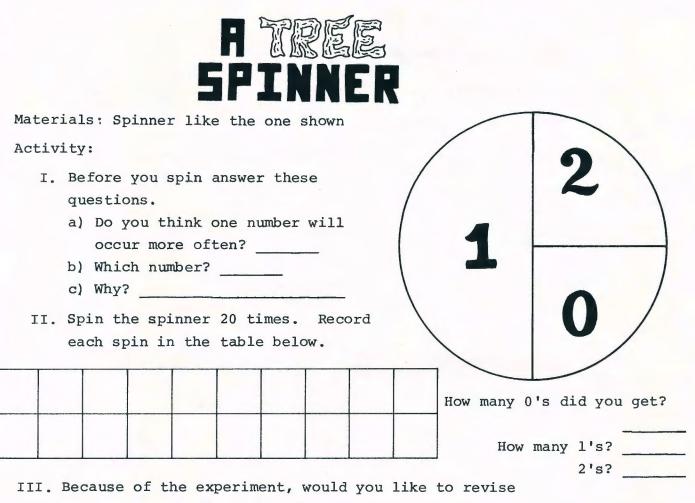
Tony says, "That's not fair! You'll win twice as often." Gary: "O.K., then! I get one penny if I win -- you get two if you win." Tony: "That's better! Let's play!"

Play the game 20 times. How much did Gary win? How much did Tony win?

Based on your data in the table, is the game fair? ______ Give reasons.

TURN	GARY'S TOSS	TONY'S TOSS	WINNER	GARY'S WINNINGS	TONY'S WINNINGS
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					
13					
14					
15					
16					
17					
18					
19					
20					

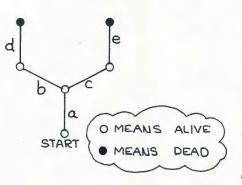
SOURCE: *Taking Chances,* Oakland County Mathematics Project Permission to use granted by Oakland County Mathematics Project



your answer to (I-a) above?

IV. Were your results similar to your neighbor's results?

V. A plant grows by producing 1 or 2 new branches and dies when 0 new branches are produced. For example, the tree below was grown with these six spins: 1, 2, 1, 1, 0, 0

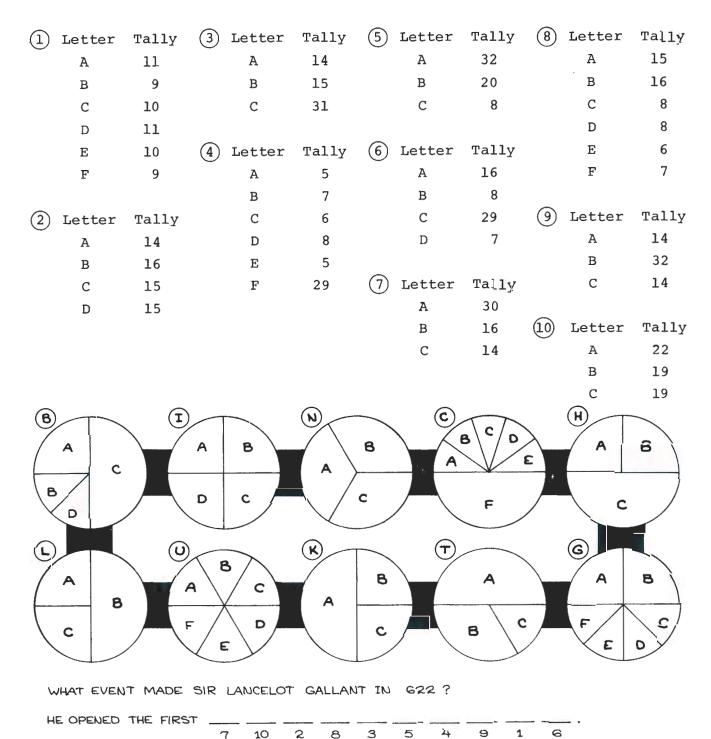


- 1) The first spin grows one new branch (a).
- 2) The second spin grows two new branches (b) and (c).
- 3) The third spin grows 1 new branch (d) on branch (b). (Start at the left.)
- 4) The fourth spin grows 1 new branch (e) on branch (c). (Move right to the next live branch on the same level.)
- 5) The fifth spin causes branch (d) to die. (Start again at the left since there are no more live branches to the right.) 6) The sixth spin causes branch (e) to die.
- VI. Grow 5 plants. For each plant spin no more than 10 times or until all branches are dead.

a) After 10 spins, how many of the 5 plants are still growing?

# THE GRULENT SIR LANCELOT

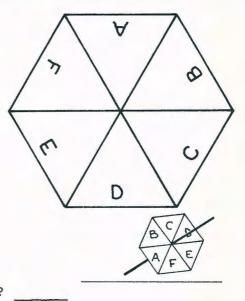
Match each spinner with the tally most likely to occur from 60 spins. Put the correct letter in the blank to answer the riddle at the bottom.



621



- I. To make a hexaspin,
  - a) Carefully cut out this hexagon.-->
  - b) Poke a round toothpick exactly through the center of the hexagon.



TALLY OF LANDINGS FREQUENCY

6

4

- II. Make some guesses about the hexaspin.
  - a) If you spin it several times do you think it will stop on one letter more often than any other letter? _____ If so, which letter?
  - b) Guess how many times it will stop on a D in 30 spins.

LETTERS

ON HEXASPIN

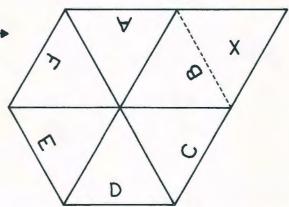
A

B

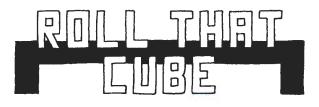
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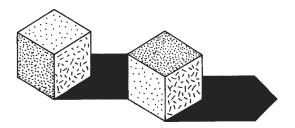
- III. Spin the hexaspin exactly 30 times. Record in a
  - table like this.
  - a) Were your results what you expected?
  - b) Compare your table with your classmates' tables. Are their results similar to yours?
- IV. To make a hexed hexaspin,
  - a) Carefully cut out this shape.-->
  - b) Fold on the dotted line and glue X to B.
  - c) Poke a round toothpick through the center of the hexagon.
- V. Repeat II and III for this spinner. Why do you think it is called "hexed"?



IDEA FROM: *Making Mathematics*, A Secondary Course, Book 2, by D. Paling, et al. Permission to use granted by Oxford University Press



Materials:	2 cubes	
	2 stude	nts



Activity:

- 1) Look at each cube.
- 2) Which color will <u>probably</u> occur most often when you roll a cube? _____ least often? _____
- Each of you roll one cube 24 times. Record in a table like the one to the right.
- 4) Which color did occur most often? ______ least often? _____
- 5) How could you color a cube so each of the colors will have an equal chance of occurring?

COLOR	TALLY	FREQUENCY
YELLOW		
RED		
GREEN		
	TOTAL	

- 6) If you roll both cubes at the same time, 6 color combinations are possible. Write them. (YY means yellow on both cubes.)
- 7) Which combination do you guess will occur most often?
- 8) Roll the cubes 48 times. Record the results.
- 9) Any time you make a guess, like in (7), you can't be wrong. Now that you have done the experiment, would you like to revise (change) your guess? _____ If so, what combination would you guess now? _____

		1 1
COMBINATION	TALLY	FREQUENCY
YY		
RR		
GG		
GY		
GR		
RY		
	TOTAL	



Materials: One die Two person team

Millard Morse says that with one die it is easier to roll a 4 than a 6. Do the following experiment to check out Millard's claim.

1) Roll the die 30 times. Have your partner record the rolls below.

 	 -	 	 	 

_ ___

----

 Have your partner roll the die 30 times. You record the rolls below.

_

3) How many times did a 4 come up? a 6? _____ Complete the table to summarize the results of the tosses.

4) Do you agree with Millard's claim?



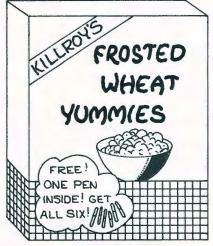
	FACE OF DIE	FREQUENCY	
-	1		
	Z		
	3		
	4		
	5		
	6		
	TOTAL		



Materials: one die for each student

Inside each box of Killroy's Frosted Wheat Yummies is one free felt tip pen. The pens are of six colors -- red, green, orange, yellow, blue and purple.

A) Billy wants the whole set. If he is lucky, what's the fewest number of boxes he will need to buy?

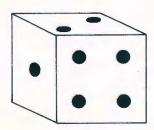


- B) On the average, how many boxes do you think he will need to buy to get the set? _____ (Make a guess.)
- C) If you had a lot of money, you could buy boxes of cereal until you had the complete set. If four members of the class did this, do you think they would all buy the same number of boxes?

Here is a way to predict the average number of boxes needed for the pen set without buying a single box of cereal.

- 1) Let the six outcomes of the die represent the six pens:
  - for a red pen
     for a green pen
     for an orange pen
     for a blue pen
     for a yellow pen
     for a purple pen





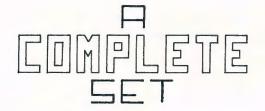


- 2) Toss the die. A 3 means you have bought a box containing an orange pen. Toss until you have <u>one</u> pen of each color. Tally your tosses in the table to the right.
- 3) How many boxes did you need to buy?
- 4) Repeat the experiment 4 more times. Record in the table below.

PENS	TALLY	
I) RED		
2) GREEN		
3) ORANGE		
4) BLUE		
5) YELLOW		
6) PURPLE		
TOTAL	<u> </u>	

PENS	EXPERIMENT #2	EXPERIMENT #3	EXPERIMENT #4	EXPERIMENT #5
I) RED				
2) GREEN				
3)ORANGE				
4) BLUE				
5) YELLOW				-
G)PURPLE				
TOTAL				

- 5) Use the totals from the five experiments to find the average number of boxes needed for a set of pens. _____ (Be sure to add in the first experiment.)
- 6) Extension: Suppose there were 10 different super-dog cards packaged in cereal. How many boxes do you think you'd need to buy to get the set? More than for the 6 pen set? What if there were 12 cards? If there were 12 cards, how many boxes would be needed to get half of the cards?

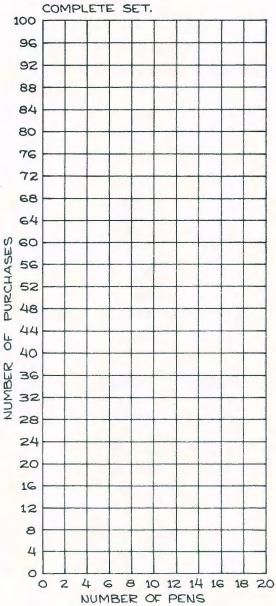


- I. Inside each box of cereal is packed one free pen.
  - a) Suppose there are two differently colored pens. Predict the number of boxes you would expect to buy to get a complete set (a pen of each color).
    b) Suppose there are six differently colored pens. Predict the number of boxes
  - you would expect to buy to get a complete set.
  - c) Ten differently colored pens.
  - d) Fourteen differently colored pens.
- II. Run the computer program BALPEN four times. Use N = 2, then 6, then 10, then 14. Use M = 10 each time.
- III. Round each average number of purchases from the computer print out to the nearest whole number. Write the results in the table below.

Number of Pens	2	6	10	14
Number of Purchases				

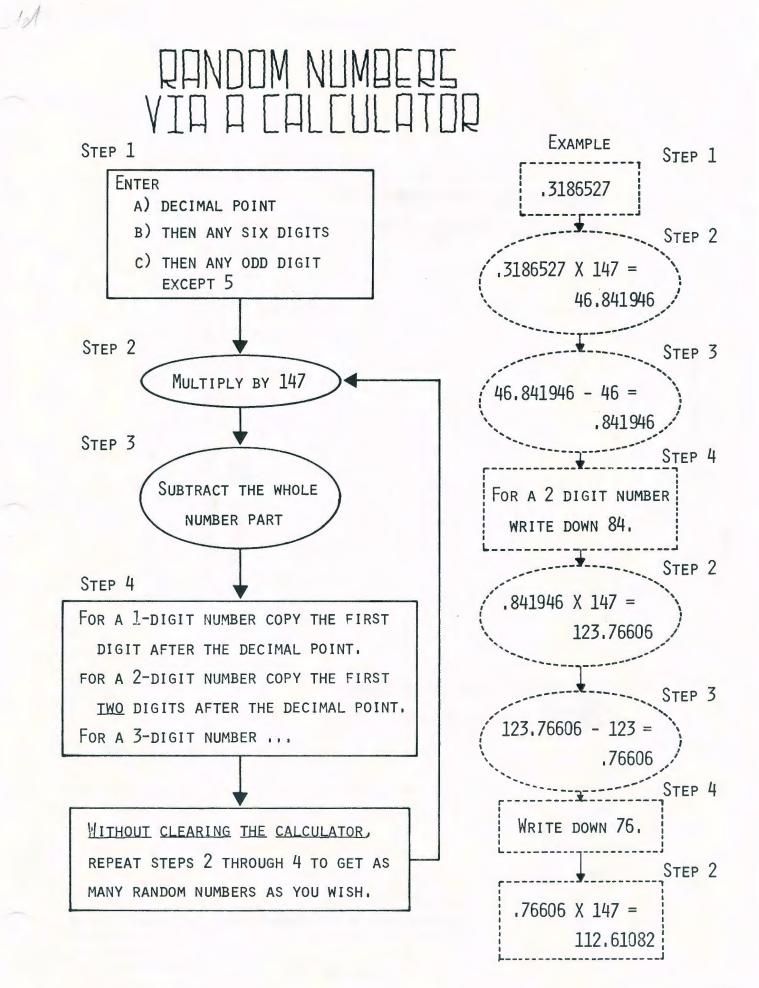
- IV. Compare the numbers in the table with your predictions. Would you like to revise your guesses?
- V. Graph the data in the table on the grid to the right. Join the dots with line segments.
- VI. Use the graph to predict the number of purchases needed if there are twelve pens.
- VII. Use the graph to predict the number
   of purchases needed if there are
   eighteen pens. _______(You will need
   to extend the graph.)
- VIII. Let N = 12, 18, 20. Use M = 10. Run the computer program BALPEN three times to check your predictions in VII.

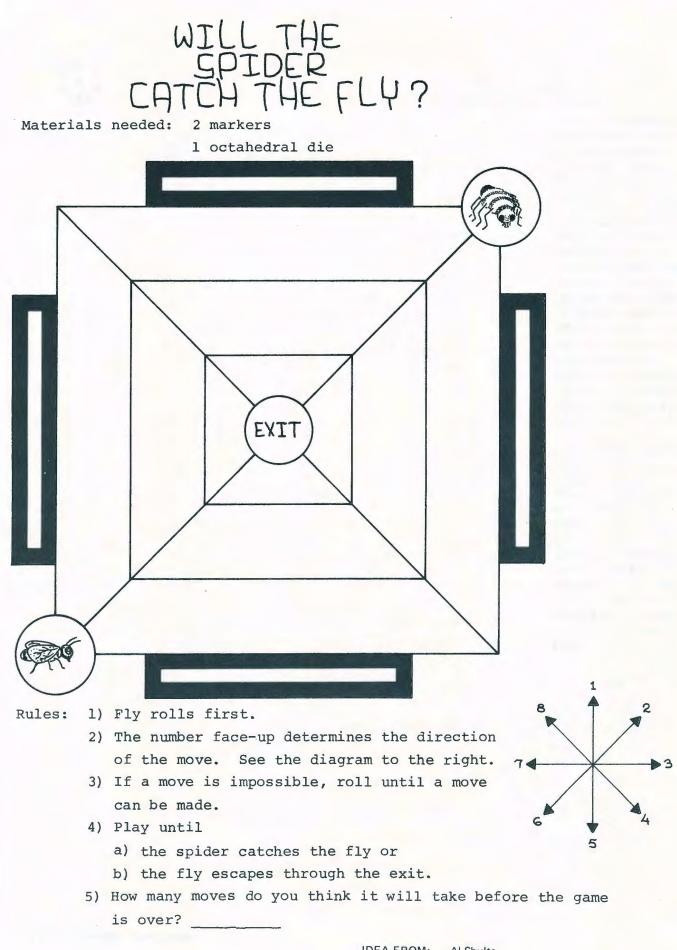
THE EXPECTED NUMBER OF PURCHASES TO GET A





66534	54365	21644	32524	64631	42523	33652
54415	16611	25655	32122	33365	45165	26556
46353	51643	51246	54655	52241	26346	36345
33623	32364	35414	21451	65133	34143	45163
14361	66264	61164	13531	43255	34166	54343
14215	34124	45654	25316	15646	14342	62535
44311	63511	44541	62515	36455	12123	25542
63361	22633	44561	56411	22445	51443	24222
61332	16655	44141	56222	26633	14256	36315
15354	46644	16246	56541	44555	35516	24334
12563	33521	16361	22116	11226	54361	32531
13513	13542	52151	24613	65261	56554	11441
	_					
45266	11611	13121	63655	43212	61646	13414
14233	45665	31154	45122	26424	24354	45552
12533	43643	25523	44413	66563	11535	12366
52122	11131	22423	55442	66263	34625	42533
44153	13311	26145	61541	44413	24515	52224
36654	24242	25654	31122	64536	66244	32345
			6 6 6 1 1	0.000	1.600.6	
61113	55635	53235	66211	26664	16236	65461
23565	36121	56465	15541	52631	42331	13422



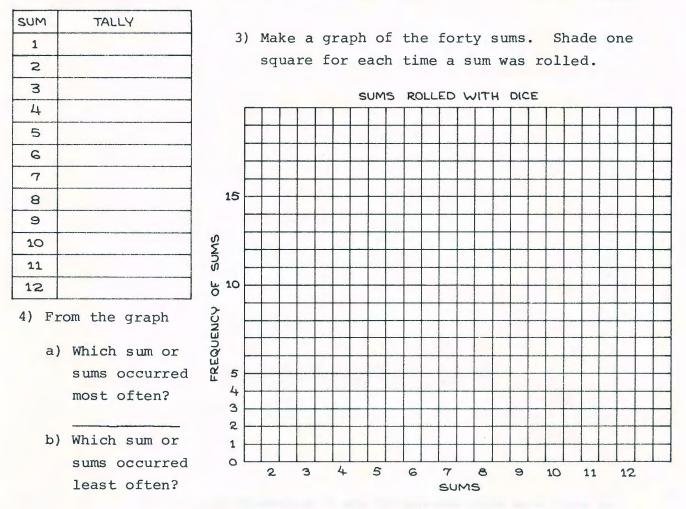


IDEA FROM: Al Shulte Permission to use granted by Oakland County Mathematics Project



Materials: Pair of dice for every two students Activity:

- When the dice are rolled a large number of times, what sum do you think will occur most often?
- 2) You and your partner each roll the dice 20 times. Tally the sums.



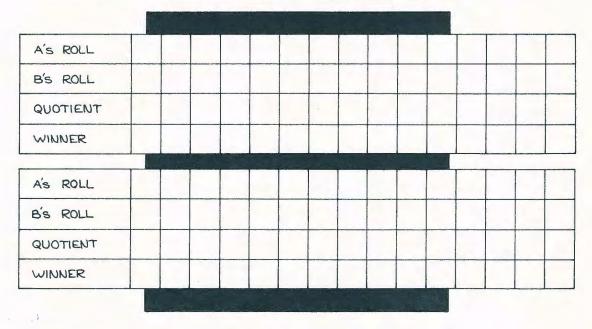
- 5) Your teacher will help you combine your results with the results of the rest of the class. For your class:
  - a) Which sum or sums occurred most often?
  - b) Which sum or sums occurred least often?
- 6) A <u>guess</u> like you made in question 1 can't be wrong. Now what sum do you think will occur most often?



Materials: 2 players each with an octahedral die

Rules:

- 1) Each player rolls the die.
- 2) Higher number is player A. Lower number is player B.
- 3) Player A rolls. Player B rolls. Divide A's number by B's number.
  - a) If the 1st digit of the quotient is 1, 2 or 3, A wins.
  - b) If the 1st digit of the quotient is 4, 5, 6, 7 or 8, B wins.
  - c) Example: A rolls 4; B rolls 2.  $4 \div 2 = 2$  A wins.
    - : A rolls 3; B rolls 6. 3 ÷ 6 = .5 B wins. : A rolls 5; B rolls 8. 5 ÷ 8 = .625 B wins.
      - : A rolls 5; B rolls 3. 5 ÷ 3 = 1.666... A wins.
- 4) Is this a "fair" game? Do both players have the same chance of winning?
- 5) Play the game 30 times. Record in the table below. You can do the division on a calculator to speed up the game.



IDEA FROM: *Probability for Intermediate Grades,* Teacher Commentary, School Mathematics Study Group Permission to use granted by the School Mathematics Study Group



Materials: Score sheet for each player, 4 dice

- 1) To score points you roll 1, 2, 3 or 4 dice and try to get all even numbers.
- 2) For your turn, you choose how many dice to roll.
- 3) Points are scored like this:

a) If you decide to	roll and you	get score
	l die l even	number 2 points
	2 dice 2 even	numbers 4 points
	3 dice 3 even	numbers 8 points
	4 dice 4 even	numbers16 points
b) If you roll any	odd numbers, score 0	points for that turn.

# OF

DICE

1

2

3

4

SCORE

VALUE

2

4

8

16

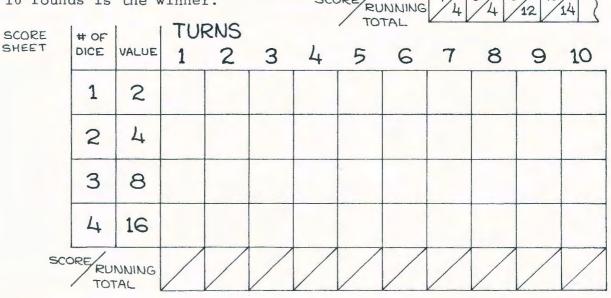
1

Х

4

0

- 4) To play:
  - a) On the score sheet mark your choice with an X.
  - b) Roll the number of dice you chose.
  - c) Mark your score.
  - d) Find your running total. The sample score sheet to the right shows a player's results for four turns.
- 5) Each player takes a turn.
- Player with highest total after 10 rounds is the winner.



TURNS

3

Х

8

2

Х

4

Х

2

## ORGANIZATION

## Placement Guide for Tabbed Dividers

Tab Label	Follows page:
CONTENT FOR TEACHERS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
Probability with Counting	
Inferential Statistics	
Appendix-Computer Programs	
DIDACTICS	
Components of Instruction	
Classroom Management	
Statistics and Probability Learning	
TEACHING EMPHASES	
Critical Thinking	
Decision Making	
Problem Solving	
Models and Simulations	
Calculators and Computers	
Laboratory Approaches	
CLASSROOM MATERIALS	
CONCEPTS AND SKILLS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Misleading Statistics	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
APPLICATIONS	
Business & Commerce	
Environment	
Health & Medicine	
People & Culture	
Recreation	
GLOSSARY	
ANNOTATED BIBLIOGRAPHY	
SELECTED ANSWERS	



Commentary to PROBABILITY WITH MODELS (638-646)

TITLE	PAGE	TOPIC	TYPE
Maybe Yes Maybe No	647	Relating the chance of an event to a probability scale	Transparency Worksheet
Tree Diagram I	649	Drawing one-stage trees	Worksheet
Tree Diagram II	650	Drawing one-stage trees for spinners	Worksheet
Tree Diagram III	651	Drawing one-stage trees for outcomes not equally likely	Worksheet
Tree Diagram IV	652	Drawing one-stage trees for outcomes not equally likely	Worksheet
Tree Diagram V	653	Drawing one-stage trees for outcomes not equally likely	Worksheet
South of the Border	654	Matching trees to spinners and containers	Worksheet Puzzle
One Million	655	Using probabilities to make choices	Worksheet
Take Your Choice	656	Using probabilities to make choices	Activity card Worksheet
Cards in a Hat	657	Using sampling to predict probabilities	Activity card
Finding Probabilities I	658	Finding probabilities for outcomes	Worksheet
Finding Probabilities II	659	Finding probabilities for outcomes	Worksheet
Probability Problems to Pursue	660	Drawing one-stage trees to solve word problems	Worksheet
Cards and Numbers	661	Finding probabilities for outcomes	Worksheet

TITLE	PAGE	TOPIC	TYPE
Picky Problems	662	Finding probabilities for outcomes	Worksheet
What Do You Expect?	663	Examining probability situations	Worksheet
Two-Stage Trees I	664	Drawing two-stage trees	Teacher directed activity Worksheet
Two-Stage Trees II	666	Drawing two-stage trees	Worksheet
Two-Stage Trees III	667	Finding probabilities for outcomes	Worksheet
Marble Problems	668	Using two-stage trees to solve word problems	Worksheet
Coins and Kids	669	Using trees to solve word problems	Worksheet
Sink That Free Throw!	670	Using trees to solve word problems	Worksheet
Tossing Pennies Revisited	671	Using a tree to analyze games	Teacher idea
Chances Are	672	Using trees to solve word problems	Worksheet
A Matched Pair	673	Using trees to solve word problems	Worksheet
More Marble Problems	675	Using trees to solve word problems	Worksheet
The Red-Blue Game	676	Using a tree to analyze a game	Game Activity card
Partial Trees	677	Using partial trees to solve word problems	Worksheet
Be a Winner	678	Using trees to analyze games	Worksheet
To Tree or Not to Tree	680	Using trees to solve word problems	Worksheet
And the Sum is	681	Finding probabilities of sums of dice	Worksheet

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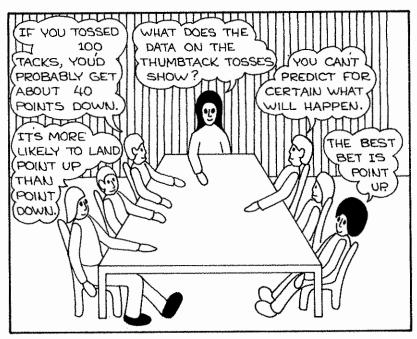
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TITLE	PAGE	TOPIC	TYPE
Roll That Cube Revisited	683	Using trees and grids to analyze probabilities	Teacher idea
Which Die is Better? Revisited	684	Using grids and trees to analyze probabilities	Teacher idea
Crazy Quotients Revisited	685	Using a grid to analyze a game	Teacher idea
A Sensitive Question	686	Using probability to find an answer to a sensitive question	Teacher directed activity
Ghosts, Goblins, and "Coins that Remember"	688	Exploring ideas of independence and the "law of averages"	Teacher idea
Tricky Statements	691	Evaluating statements about the likelihood of events	Transparency
On the Average	692	Studying variability in the tosses of a coin	Activity card Worksheet

# PROBABILITY WITH MODELS (

# ASSIGNING NUMBERS FOR PROBABILITIES

The activities in the EXPERIMENTAL PROBABILITY section involve students in gathering data to compare the frequency of different outcomes. This will usually lead students to conclusions like, "It is more likely to be two marbles of different colors than two marbles the same," "It's not very likely that a sum of 12 is rolled with 2 dice" or "Heads and tails have about the same chance." These conclusions



are fine but they could be more informative if students used numbers to describe the "likelihood" of each outcome. Here are some suggestions for introducing the assignment of numbers for probabilities.

# Use Percents

Even before studying percents in school, many students will know that a 90% chance means pretty likely and a 5% chance means unlikely. If your students have this general understanding, they are already associating numbers with probabilities. You might start a class discussion with questions about percents and chance. "Which is more likely to happen, something with a 40% chance or something with a 60% chance?; What percent would be used for the chance of an outcome that is certain to happen?; If there is a 40% chance of rain, what is the percent chance of no rain?; If there is a 50% chance of rain, is it more likely to rain or not?"

If your students can answer the above questions, show them how to translate the "percent chance" statements to probability statements. "100% is another name for 1. Something with a 100% chance of happening has the probability of 1 of happening. What is the probability that the sun will rise tomorrow? That this pencil will fall if I let go of it?" Discussions like this can help students see that the probability of something certain to happen is 1, the probability of some-

### PROBABILITY WITH MODELS

### COMMENTARY

thing just as likely to happen as not is  $\frac{1}{2}$  and the probability of an outcome happening plus the probability of the outcome not happening equals 1. (Maybe Yes Maybe No gives a development of this without percents.)

# Use Relative Frequencies from Experiments

Students can use the data gathered in activities from the EXPERIMENTAL PROBA-BILITY section to approximate probabilities. You could define

The Approximate Probability of an Outcome = Number of Times Outcome Occurred (The degree of certainty)

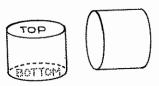
Point out to students that this fraction is always between 0 and 1, inclusive. In drawing one marble with replacement from a bag of all red marbles, students should see that the probability for a red marble is 1 because the numerator and denominator are always equal. The probability of drawing a blue marble is 0 because blue will occur 0 times. Students can use data from thumbtack tosses, coin flips or marble draws to approximate the probability for an outcome. These approximations can make it seem reasonable to conclude that the probability of a head on one flip of a coin is  $\frac{1}{2}$ , the probability of rolling a five in one roll of a die is  $\frac{1}{6}$ , and so on.

Point out situations where experimental data is the <u>only</u> way to approximate a probability. Batting averages and free throw percentages are determined by the data available on players. These averages and percents are a good approximation for the likelihood of a player getting a hit or sinking a free throw.

# Use the Symmetry of the Object

You can try to convince students of a probability by having them examine the spinner, marbles, die or coin being used. If a spinner is half red and half blue it seems reasonable that a spin of red should have the same probability as a spin of blue. Each color should occur about  $\frac{1}{2}$  of the time so the probabilities are both  $\frac{1}{2}$ . A die is a cube and, because of its symmetry, should not favor any of the six outcomes. Each result should happen about  $\frac{1}{6}$  of the time so it is reasonable to assign a probability of  $\frac{1}{6}$  for a  $\boxed{\bullet^{\bullet}}$  on one roll.

Sometimes students will be able to use the symmetry of an object to reason that two probabilities are equal, but they will need to use experimental data to estimate all the probabilities involved. For example, in tossing a short cylinder, the probability it lands on its top should equal the probability it lands on its bottom.



### COMMENTARY

### PROBABILITY WITH MODELS

Students will need to toss the cylinder to get an estimate of the probability it will land on the curved surface.

When you have students determine probabilities in this "reasonable" way, watch for possible misconceptions they might have. Some students might ignore the physical make-up of the object and decide that their favorite color is most likely to be spun or the number they need most for a game is least likely to be rolled with a die. You might want to give some of the diagnostic exercises suggested below so you will have a better idea if your students pay attention to the structure of the object being tossed or spun. The paper *Statistics and Probability Learning* in this resource gives more background on student misconceptions about probability.

# SOME DIAGNOSTIC EXERCISES

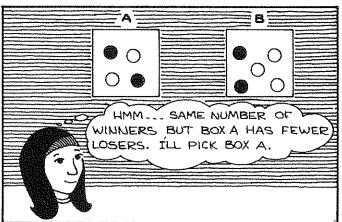
If a student answers red for both Exercises 1 and 2, red might be the student's favorite color. This student is not looking at the make-up of the spinner. Exercise 3 might be answered "Blue" because blue occurs in two places. Show the spinner divided in 8 congruent pie shapes so students can see that green covers as much area as blue. Some may still feel blue has a greater chance than green. Students who have these misconceptions may need extra help in understanding probability. Students will probably answer Exercise 4 by counting yellows versus blues. Exercises 5 and 6 will give some information about how students relate chance to the contents of a bag of marbles.

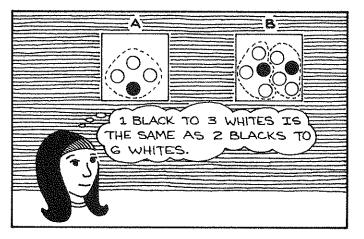
RED 1. What is more likely 4. What is more likely for this spinner, to be drawn from red or green? GREEN this bag, a yellow marble or a blue marb1e? RED 2. What is more likely 5. If you want an equal for this spinner, chance of drawing red or green?_ red or blue, you GREEN (B) would put in (8) **B**) red marbles. GREEN 3. What is more likely 6. If you wanted to draw a BLUE RI DË for this spinner, red marble twice as blue or green? often as a blue marble RED you would put in (B) red marbles. **(B**)

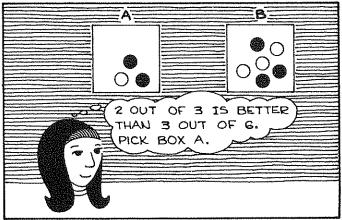
# COMMENTARY

The student page Take Your Choice has additional ideas for diagnostic questions. On the page are pairs of boxes containing black marbles and white marbles. Students are to choose the box in each pair from which there is a better chance of drawing a black marble. Students usually use several strategies, both correct and incorrect, for making these choices. Some of the incorrect strategies are discussed in the paper Statistics and Probability Learning. Some correct strategies are given below.

- If the boxes have the same number of winners (black marbles) students can compare the number of losers (white marbles).
   If the boxes have the same number of losers, the box with more winners has the better chance.
- If the marbles can be grouped in the same ratio, the boxes have the same chance. (This physical grouping is probably hard for most students and it might be easier to write ratios or fractions.)
- Other arrangements are even more difficult and many students will not be able to give good reasons for their choices without writing and comparing fractions or ratios to represent the chances.







RED

# COMMENTARY

# USING MODELS TO FIND PROBABILITIES

Several models have been developed to help students see how to find probabilities. Some of these models help in counting equally likely outcomes.

A grid can be used to count the possible totals when rolling a red die and a green die as shown in And The Sum Is... Students can find the probability of rolling a 7 by dividing the number of the 7's by the total number of possibilities. Grids are used as models in Roll That Cube Revisited, Which Die Is Better? Revisited and Crazy Quotients Revisited. Another counting model, that of drawing chords on a circle, is shown in Picking Marbles 2 At A Time in the EXPERI-MENTAL PROBABILITY section.

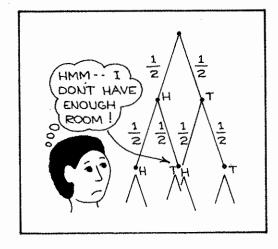
The most versatile model seems to be the tree diagram. Students can use tree

۰. ٠. • ... 4 5 G 7 2 з • 3 4 5 6 7 8 ٠. 7 4 5 6 8 9 GREEN •• 7 5 6 8 9 10 10 11 6 7 8 9 4 4 4 7 12 8 9 10 11 ...

diagrams to represent equally or unequally likely events or to find the probability of several things happening in succession. Because of this, *Probability With Models* in the CONTENT FOR TEACHERS section and the classroom materials in this section rely mainly on tree diagrams for determining probabilities. You might want to read the content paper before using probability trees with your students.

Students may have difficulties with trees. Here are some ideas that might help.

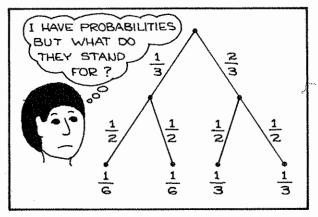
 Encourage your students to draw "fat" trees. Students may not leave enough space for labeling or for the second or third stages.



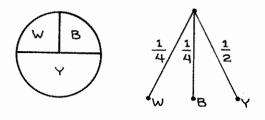
# PROBABILITY WITH MODELS

# COMMENTARY

 Have students label the tree with outcomes and probabilities at each stage of a 2 or 3-stage tree.
 If they don't label they might forget what each branch represents.



Be sure students relate the physical makeup of the spinner, die or coin to the probabilities placed on the tree. You can use a page like South of the Border to see if students can match spinners and containers with appropriate trees.



Some students will not need to use trees on some of the activities in this section. They will be able to determine the probabilities using counting methods. Remind students that they can return to using trees when they "get stuck" on a problem or when a problem involves more than one step (like rolling a die then flipping a coin).

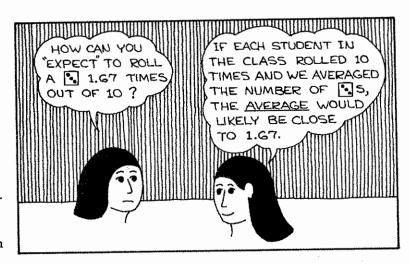
# DETERMINING EXPECTED NUMBERS

Many of the activities in the EXPERIMENTAL PROBABILITY section have students guess how many times an outcome would occur in a certain number of trials. They guess how many times a coin might land tails up in 40 flips and how many 's 's might be rolled in 60 rolls of a die. After the experiments are finished, the data from different students are averaged. These averages are estimates for the expected number of outcomes just like the relative frequency of number of successes to number of trials is an estimate for the probability. After students assign probabilities to outcomes, they can calculate expected numbers by multiplying the number of trials by the probability for the outcome. Here are some problems you can have them solve:

# PROBABILITY WITH MODELS

# COMMENTARY

What is the expected number of s in 60 rolls of a die? (1/6 of 60 = 10) 300 rolls of a die? (1/6 of 300 = 50) 10 rolls of a die? (1/6 of 10 ≈ 1.67) The last problem will probably create all sorts of questions--see the cartoon to the right. If your



students have difficulty with the words "expected number" try using "on the average, how many is would you expect to be rolled?"

- In 100 rolls of a die how many times would you expect to roll an even number? (P(even number) = 1/2 so 1/2 x 100 = 50) an odd number? (1/2 x 100 = 50) a prime number? (2,3 and 5 are prime. 1/2 x 100 = 50)
- During the basketball season, Sinker Sue has made 21 out of 35 attempted free throws. Estimate the probability of Sue making her next free throw.  $(\frac{21}{35} = .6)$  Using the probability .6, how many free throws would you expect Sinker Sue to make out of her next 10 attempts? (.6 x 10 = 6) This idea could be extended to an activity where students gather data on a basketball player and use it to approximate a probability. They can then compare the number of free throws actually made out of those attempted in a recent game with the expected number. Students might want to discuss free throw percentages of players from their school or from a favorite college or professional team. They might also relate these percentages to choices of people to foul in the last minutes of a game.

# COMMENTARY

# FIGURING ODDS

Odds and probabilities are commonly used in newspapers and everyday speech, but many people confuse the two ideas. Students should know the difference. Perhaps your students already know that "even" odds, a 50-50 chance and a probability of 1/2 mean the same thing. То help students discover the general relationship between odds and probabilities, you could have them complete and examine patterns in a table like that shown below. To fill in this table students will need to know how to assign probabilities and that the odds for something happening is the ratio of the number of chances it will happen to the number of chances it will not happen.



Loretta McKay of Rainier, Or., gave birth to her third set of twins in six years. The odds against such a thing happening were given as 512,000 to 1, but other odds-makers, their pocket calculators flashing crazily, said it was more like winning the Irish Sweepstakes every day for a year.

111/21 to 1babilities. Point out that a probability of 1 means would be certain and it is not.121/31 to 2lity of 1 means would be certain and it is not.232/52 to 3model is impossible.	nbi- ( red / A	1

# COMMENTARY

Ask your students if they can find the odds for drawing a red marble if the probability of drawing red is 3/4. Can they use the patterns in the table to see the odds is the ratio of the numerator of the probability fraction to the difference of the denominator and numerator? You might want to give them some odds and have them compute corresponding probabilities.

# COINS AND DICE THAT "REMEMBER"

Most of us know a coin can't remember but many people saythings to the contrary. Do your students believe there is a better chance for flipping a tail after three heads have been flipped? Lots of adults believe this but, of course, the coin cannot remember. No matter how many heads have been flipped, there is still a 50-50 chance for getting a head on the next try. You can use the ideas in *Ghosts*, *Goblins* and "Coins that Remember," Tricky Statements and On the Average to promote discussion about such ideas. You could have students flip coins until they flip three heads in a row. Have them record the next flip. What fraction of the students got a head on the fourth flip? Don't be too disappointed if your students still believe that after a sequence of heads they are "due" for a tail. Sometimes even data to the contrary cannot change people's minds on this subject!

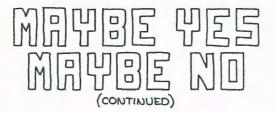


WHAT IS THE CHANCE OF EACH OF THESE EVENTS OCCURRING? WRITE ONE OF THESE FIVE ANSWERS.

(	NEVER PROBABLY NOT (MAYBE YES - MAYBE NO) (PROBABLY) (ALWAYS)
1)	YOU WILL GROW ANOTHER HEAD.
2)	YOU WILL GET A "TAIL" WHEN YOU FLIP A COIN.
3)	YOU HAVE A HEART.
4)	YOU WILL TAKE A SPACE TRIP TO THE MOON.
5)	YOU WILL EAT SUPPER TONIGHT.
6)	YOU WILL GET A "HEAD" WHEN YOU FLIP A COIN.
7)	YOU WILL GET A "TWO" WHEN YOU ROLL A DIE.
8)	YOU WILL GET OLDER.
9)	YOU WILL GET A NUMBER GREATER THAN "TWO" WHEN YOU ROLL A DIE.
10)	YOU WILL SEE A LIVE DINOSAUR AT THE ZOO.

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Mathematicians use a probability line to show the chance of an event. In the appropriate places write the numbers of the exercises on the previous page along this probability line.

NEVER

ALWAYS

An event that can never happen has a probability of 0.

An event that has the same chance of happening or not happening has a probability of  $\frac{1}{2}$ .

- 11) What do you think is the probability of an event that
  - a) always happens?
  - b) probably happens?

c) probably does not happen?

Write the number of each exercise below in the appropriate place on the probability line.

12) You will live to be 100 years old.

13) You will draw the Ace of clubs from a regular deck of cards.

14) You will get an A in math class.

15) You will draw a red card from a regular deck of cards.

16) You will watch television today.

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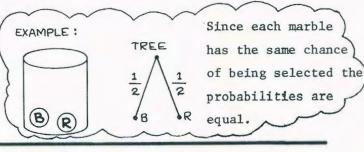
# TREE DIAGRAM

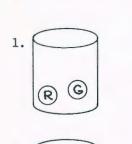
In problems #1-10, sketch a tree diagram to show the probability of drawing <u>each marble</u> from the container. Write the probability on each branch.

TREE

TREE

TREE



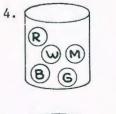


2. G B B

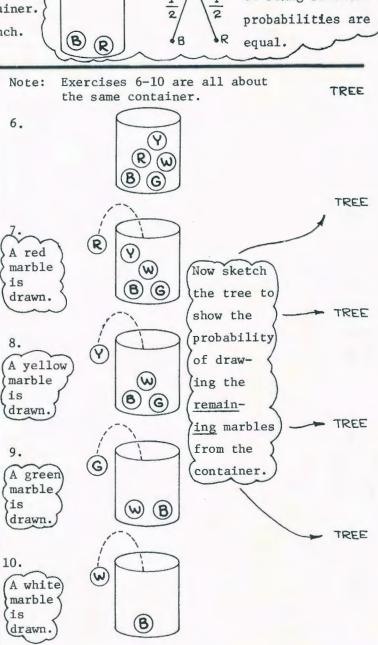
3. R B

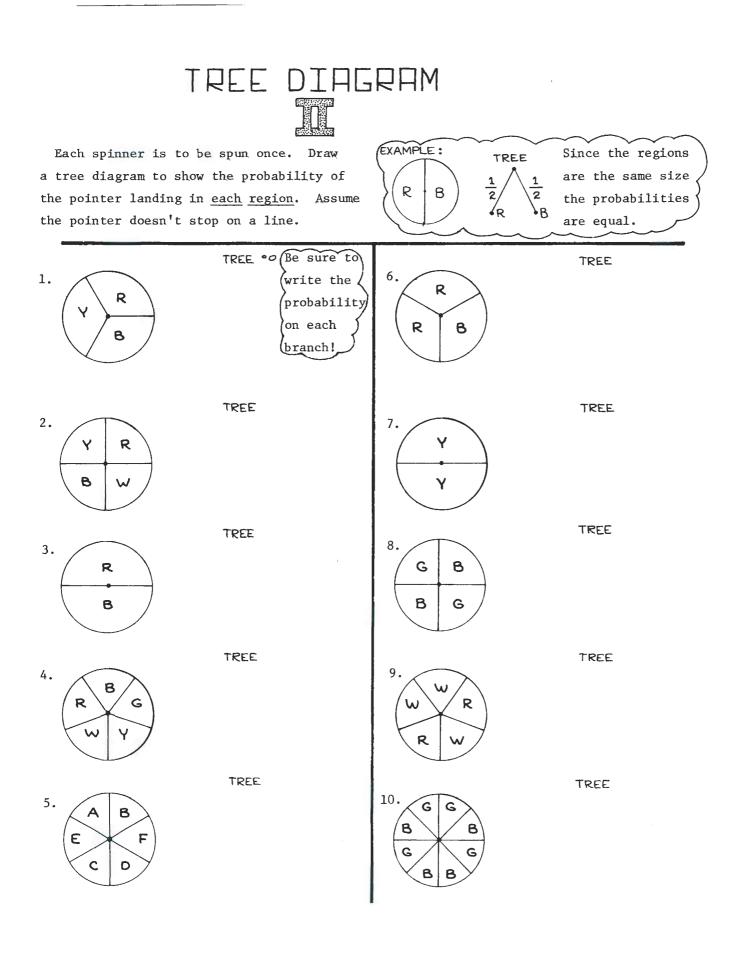
TREE

TREE



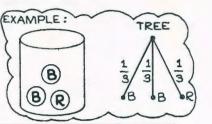
5.



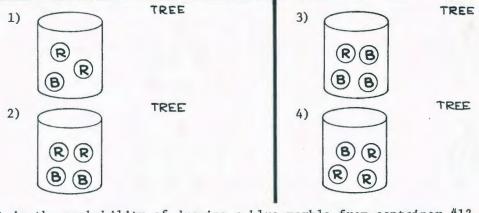




In exercies #1-4, sketch a tree diagram to show the probability of drawing <u>each marble</u> from a container. Be sure to write the probability on each branch.



. . .



5a) What is the probability of drawing a blue marble from container #1? ____

b) What is the probability of drawing a red marble from container #1?

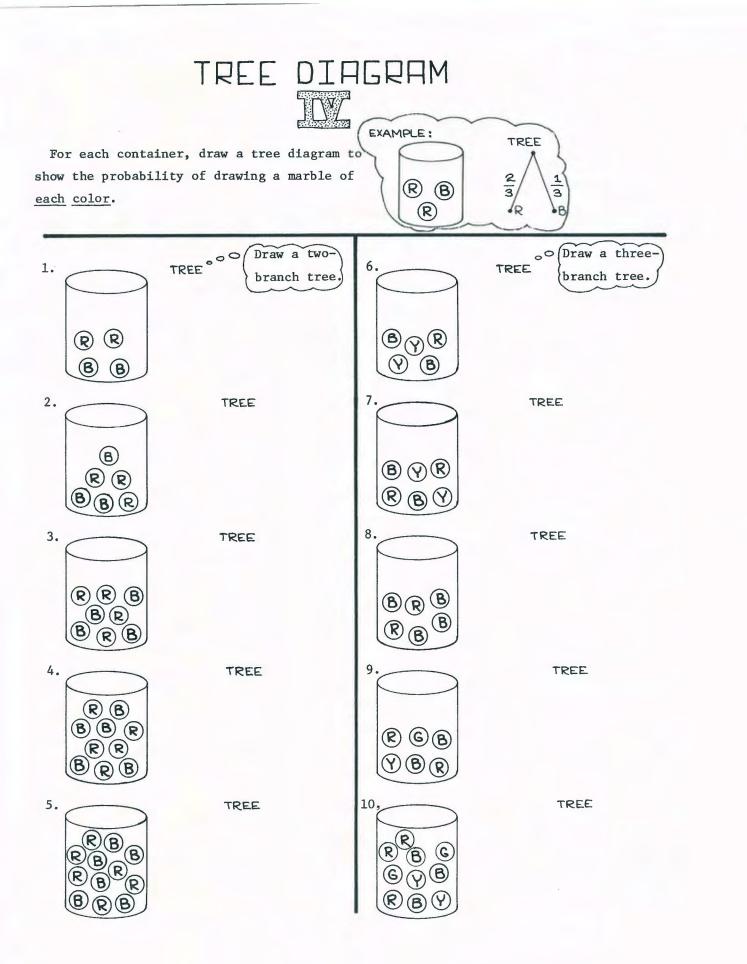
THE PROBABILITY OF DRAWING EACH MARBLE IS 1/3. THE PROBABILITY OF DRAWING A RED MARBLE IS 1/3 + 1/3 = 2/3.

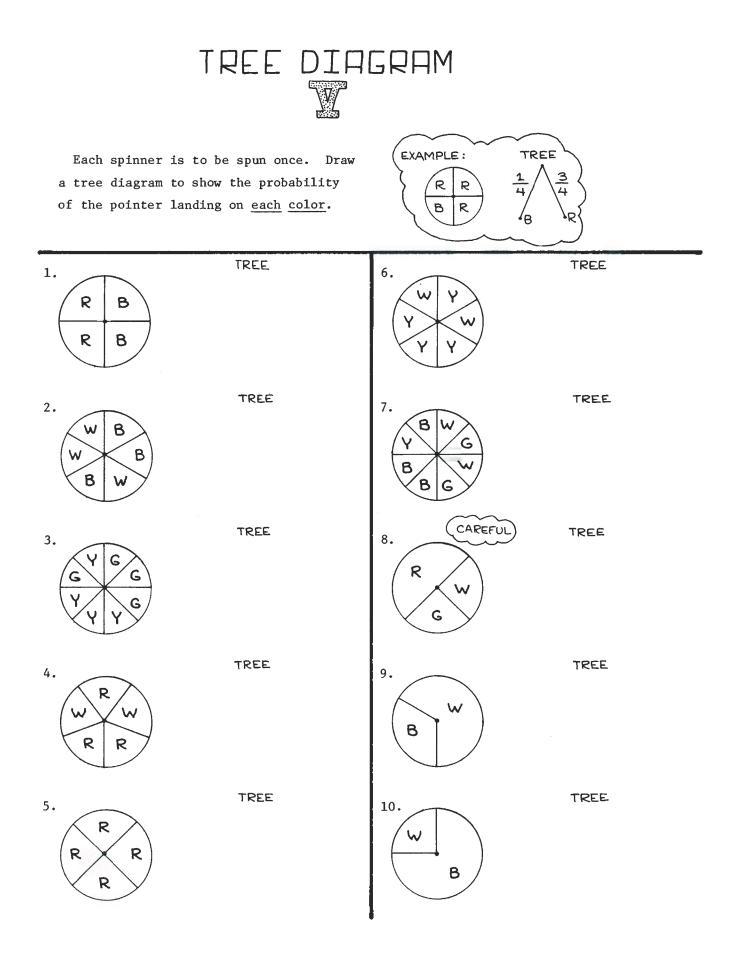
- 6) Write the probabilities for these drawings:
  - a) A blue marble from container #2 _____ d) A red marble from container #3 _____
  - b) A red marble from container #2 _____ e) A blue marble from container #4 _____
  - c) A blue marble from container #3 f) A red marble from container #4
- 7) Because there are two colors in each container a <u>two</u>-branch tree can be used to show the probability of drawing a red or a blue marble from the containers. For container #1, the tree is:
  2/3
  2/3
  R
  1/3
  Container #1
  <pContainer #1</p>
  Container #1
  <pContainer #1</p>
  Cont

Container #2:

Container #3:

Container #4:

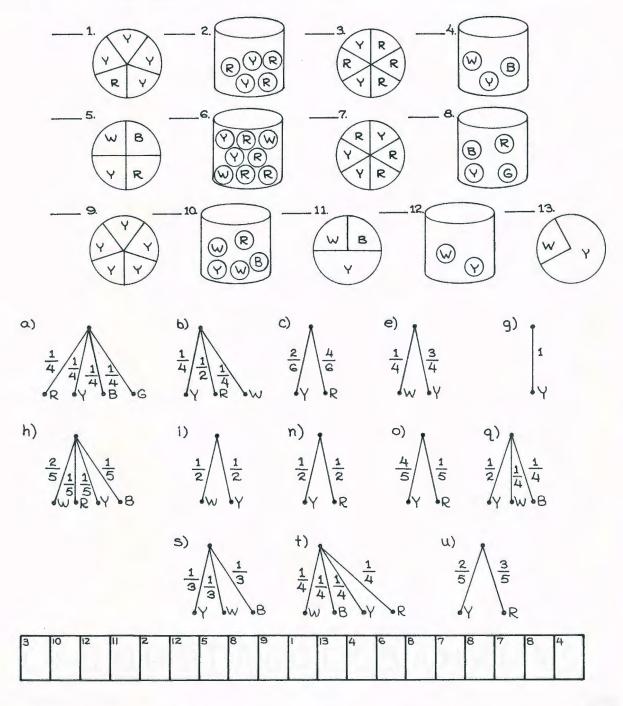


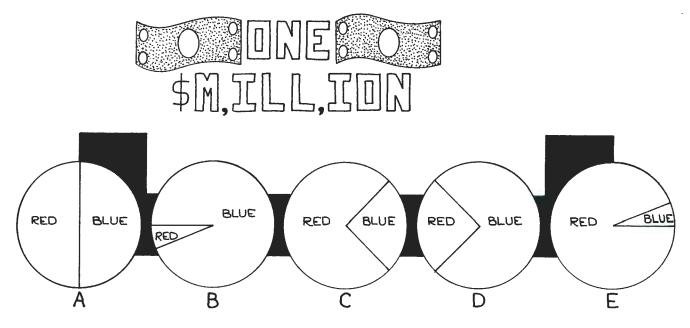




Each spinner or container can be matched with a tree diagram. Some diagrams show the probability of the pointer landing on <u>each color</u> for one spin of a spinner. Others show the probability of drawing a marble of <u>each color</u> for a one-marble draw.

Write the letter of the correct tree diagram beside the spinner or container. Then fill in the answer blanks at the bottom of the page to find out what happened on a South American plantation on November 21, 1977.





In each pair below circle the letter of the spinner that will more likely show red than blue.

1)	в	or	Е	4)	A	or	D
2)	С	or	A	5)	В	or	С
3)	Е	or	С	6)	в	or	D

Suppose a rich person (Money Bags) will give you \$1,000,000 if you can get red on one spin of a spinner.

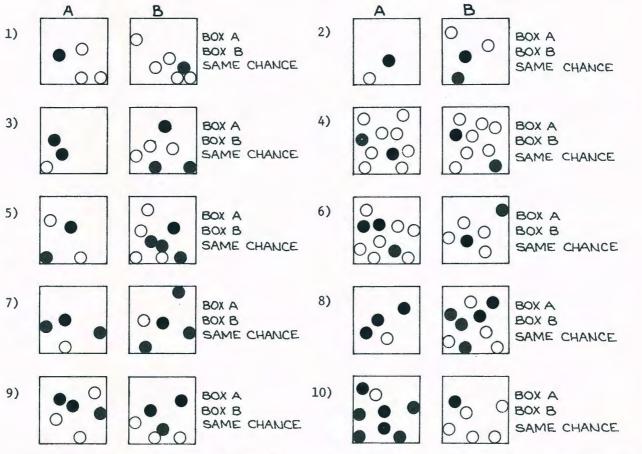
- 7) Which spinner would you choose?
- 8) Which spinner would Money Bags choose for you to use if she didn't want you to win?
- 9) If you could make your own spinner, how would you color it?
- 10) If Money Bags could make a spinner for you and didn't want you to win, how would she color the spinner?



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Each box has some black and some white marbles. A person chooses one box from each pair and draws one marble. The person wins if the marble is black. Which box in each pair gives the best chance of winning? Circle your choice.



11) Get two boxes. Pick one of experiments 3, 4, 6 or 7. Put the correct marbles in each box.

a) Without looking pick a marble from box A. Record the color. Put the marble back in the box. Shake the box.

BOX

A

B

BLACK WHITE

- b) Repeat 24 more times.
- c) Without looking pick a marble from box B. Record the color.
- d) Repeat 24 more times.
- e) Does the experiment agree with your choice?
- f) Would you like to change your choice?

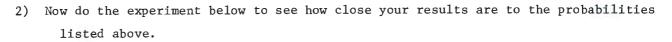
IDEA FROM: A Study of Parts of the Development of A Unit by J. Shepler

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Materials: Package of file cards Hat (Can)

1) Count out 10 cards.
Letter 2 of them with an X. Letter 5 of them with a Y.
Letter 3 of them with a Z. Mix them up and put them in the hat.
Think about drawing a single card from the hat.
The probability that this card would be an X is 2/10.
What is the probability that this card would be:
a) y?
b) z?



- a) Select a card. Record its letter. Put it back in the hat. Shake the hat.
- b) Perform the experiment a total of 40 times.
- c) Is the ratio of  $\chi$  's to 40 close to 2/10?
- d) Write about your results. Do you think you took a large enough sample? Discuss this with your teacher.
- 3) Remove the cards from the hat. Secretly change some of the letters so that the probabilities are different. Don't introduce any new letters. Keep only x's, y's and z's.

Now see if your partner can determine the new probabilities.

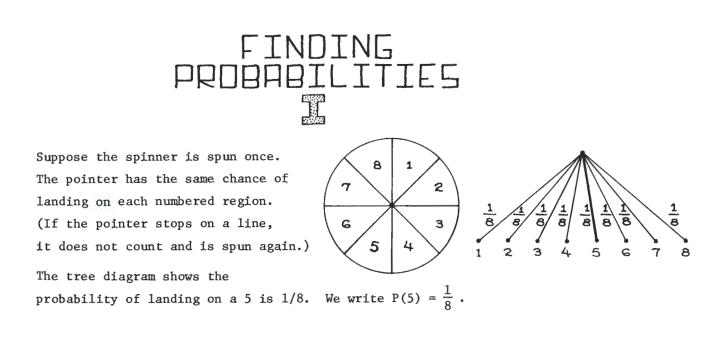
 Repeat the steps given in Exercise 3.
 This time, you draw cards out of the hat after your partner secretly changes some of the letters.

EXTENSION: Put 5 cards in the hat: 2  $\times$  's and 3  $\gamma$  's. Guess the probability of selecting 2 cards and getting

a) both x's b) both y's c) one of each.

Now conduct an experiment to check your guesses. Write about your experiment.

SOURCE: *Mathlab – Junior High* by S. McFadden, et al. Permission to use granted by Action Math Associates, Inc.



1. Fill in the probabilities below for one spin of the spinner.

			P(even number) =
b	) P(8) =	g)	P(number is greater than 4) = P(number is less than 4) =
с	) $P(2 \text{ or } 8) = \o \circ (P(2) + P(8))$	h)	P(number is less than 4) =
d	) P(7 or 6) =	i)	P(number is greater than 7) =
е	) P(odd number) =	j)	P(1 or 2 or 3 or 4 or 5 or 6 or 7 or 8) =

2. Use the tree diagram above to fill in the probabilities for one spin of the spinner.

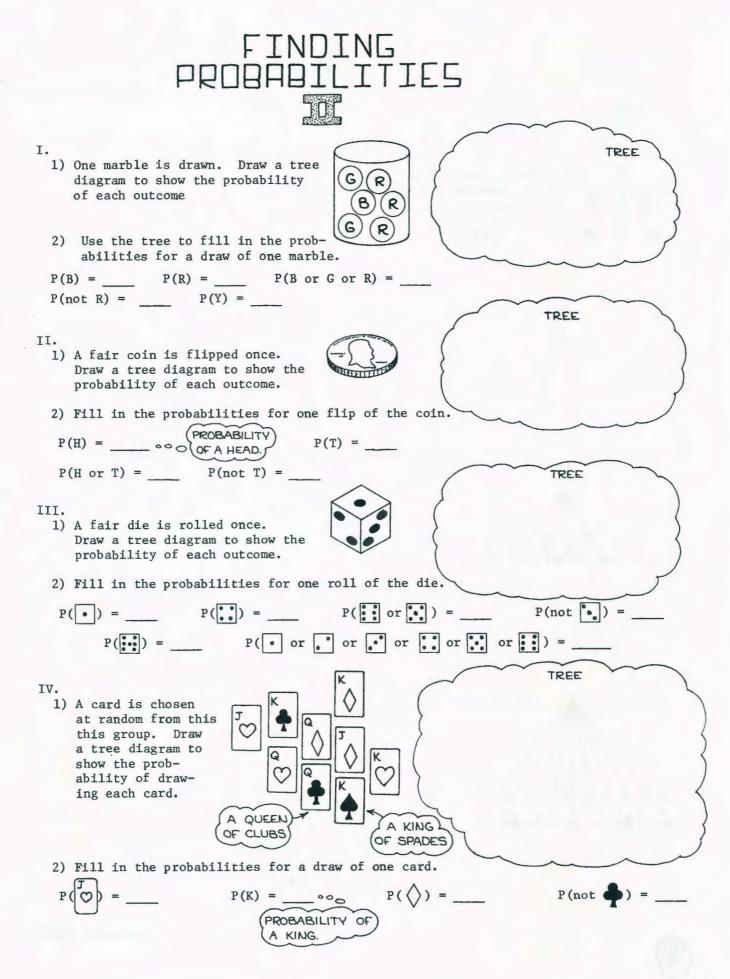
a) P(8) = _____
b) P(not 8) = _____
c) P(13) = _____
c) P(13) = _____

- c) P(4) = _____
- d) P(not 4) = _____

 $\begin{array}{c} P(13) = \underline{\qquad} \\ p(13) = \underline{\qquad} \\$ 

3. The probabilities are for one spin of the above spinner. Use the tree diagram to select an outcome that has each probability. e.g.,  $P(\underline{7 \text{ or } 6 \text{ or } 5}) = \frac{3}{8}$ 

a)  $P(___] = \frac{1}{8}$ b)  $P(__] = \frac{2}{8} \text{ or } \frac{1}{4}$ c)  $P(__] = \frac{1}{2}$ d)  $P(__] = \frac{7}{8}$ e)  $P(__] = 1$ f)  $P(__] = 0$ 





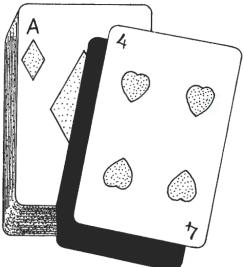
For each situation, <u>draw a tree diagram</u> to show the probability of each outcome. Use the tree diagram to answer the question.

1)	A bag contains 4 red, 3 green and 2 yellow	TREE
	marbles. A marble is picked at random. What	
	is the probability of picking	
	a) a green marble?	
	b) a green or red marble?	
	c) neither a green nor a red marble?	
2)	There are 10 slips of paper in a hat. The slips	TREE
	are numbered from 1 through 10. One slip is	
	drawn at random from the hat. What is the	
	probability that the slip has	
	a) a 5 on it?	
	b) a number greater than 6?	
	c) an even number on it?	
	d) a number evenly divisible by 3?	
3)	Mr. Jacobs has 14 good eggs and 4 rotten eggs	TREE
	lying on the counter. His wife puts the eggs	
	together and places them in the refrigerator.	
	Mr. Jacobs chooses one at random. What is	
	the probability he picks a good egg?	
	One latter of the second "statistication" is colored	TREE
4)	One letter of the word "statistician" is selected	TREE
	at random. What is the probability the letter is	
	a) "t"?	
	b) "a"?	
	c) a vowel?	
	d) a consonant?	
5)	In Olivia's class, 2 students have red hair,	TREE
	4 have black hair, 6 have blond hair and 12 stu-	
	dents have brown hair. One student is chosen at	
	random. What is the probability the student has	
	a) blonde hair?	
	b) not red hair?	
	-,	



A deck of playing cards has 52 cards. There are four suits: clubs ╇ , diamonds 💧 , hearts 🖤 , and spades 🌩 . Each suit has thirteen cards: ace (A), 2, 3, 4, 5, 6, 7, 8, 9, 10, and three face cards - Jack (J), Queen (Q), King (K).

If a tree diagram were drawn to show the probability of drawing a single card at random, how many branches would it have?



- 1) Think about how many branches lead to each outcome below. Write the probability for each outcome.
  - P(the A of spades) = _____ P(the J of clubs) = _____ P(not the J of clubs) = P(the 2 or the 3 of diamonds) = P(a King) = _____ P(an ace) = _____

P(a heart) =_____ P(not a spade) = P(a K or a Q) = _____ P(not a 10) = _____ P(the 14 of hearts) = P(a face card) = ____

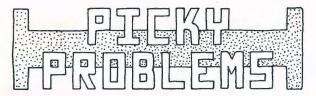
If a tree diagram were drawn to show the probability of choosing a number at random from this set, how many branches would it have?

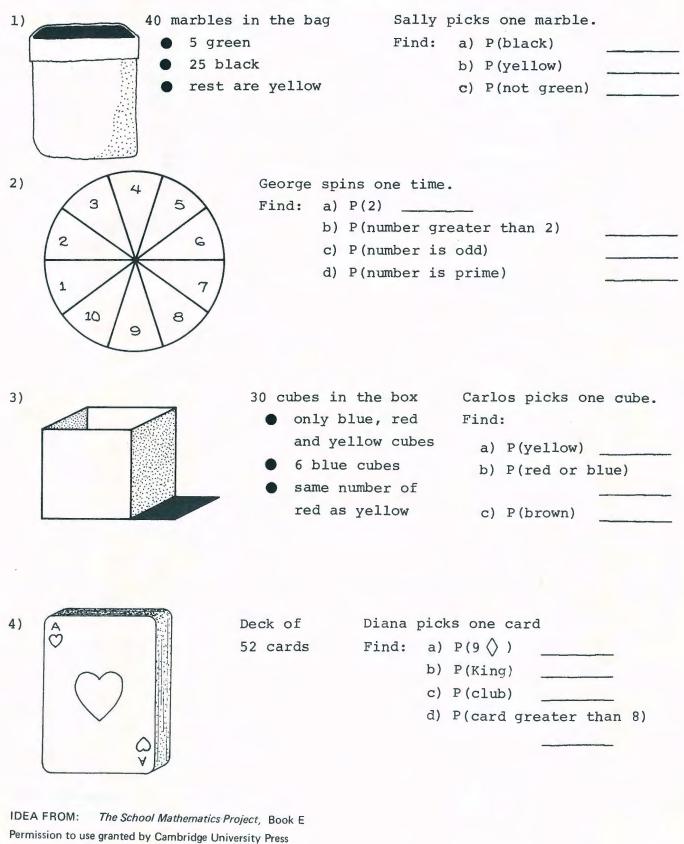
2) Think about how many branches lead to each outcome below. Write the probability for each outcome.

 $P(2) = ____ P(2 \text{ or } 3) = ____$ P(a number greater than 20) = _____ P(not 17) =_____ P(an even number) =

$\frown$	$\sim$	$\sim$	$\sim$	$\sim$
	2	3	4	5
6	7	8	9	10 )
/ 11	12	13	14	15 🖉
2 16	17	18	19	20)
21	22	23	24	25
26	27	28	29	30)
		$\sim$	$\sim$	$\sim$

P(31) =P(a perfect square) = P(a number divisible by 1) = P(a number evenly divisible by 3) = ____ P(a prime number) = ____







Answer T (true) or F (false) for statements 1-10.

- If a tossed coin does not stand on its edge, it is certain to be either heads or tails.
- 2) If you toss a fair coin once, you are as likely to get a head as a tail.
- 3) If you toss a fair coin 100 times, it may be heads 0 times or 100 times or any whole number in between.
- 4) If you toss a fair coin 1000 times, it is very unlikely that you will get 900 tails.
- 5) Whether you get heads or tails when you toss a fair coin is a matter of chance.
- 6) You might toss a fair coin 50 times without getting a head. ____
- 7) A box contains two blue marbles and one red marble. You pick one marble without looking. The chances are 1 out of 3 it will be blue.
- 8) In statement 7, your chances of picking a red marble are two out of three.
- 9) In statement 7, your chance of picking a green marble is zero.
- 10) Joe is eight years old. It is more likely he is four feet tall than 10 feet tall.

Statements 11 - 15 refer to the spinner. Answer yes or no.

- 11) If you spin the spinner 10 times, are you likely to get the same number of reds as whites?
- 12) Are you likely to get more whites than reds? _____
- 13) If the chances of getting red are 3 out of 4, are the chances of getting white 1 out of 4?
- 14) Can you be certain of getting at least one red in 10 spins?
- 15) Is it very likely that you will get no whites in 10 spins?

Read the following statement. Then answer statements 16 - 20.

James has three green marbles and two blue marbles in his pocket.

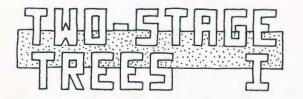
What is the least number of marbles he must remove to be sure of getting

- 16) a blue marble? _____
- 17) both blue marbles? _____
- 18) both colors?
- 19) a green one?
- 20) If James removes one marble, there are three chances out of ______ it will be a green one.

SOURCE: *Probability for Intermediate Grades,* Teacher's Commentary, by School Mathematics Study Group Permission to use granted by the School Mathematics Study Group

RED

WHITE



Suppose you have the two spinners to the right. Each is spun once. What is the probability of first getting Blue on Spinner I and then getting True on Spinner II? A tree diagram can help you answer this question.

- Draw a tree diagram for Spinner I. Since each color has the same chance of occurring the probability for each branch is 1/3.
- Suppose the pointer of Spinner I lands on <u>Red</u>. When Spinner II is spun, True or False have the same chance of occurring. The probability of each is 1/2. Draw two brances at the end of the R-branch to show this.
- 3) Suppose the pointer of Spinner I lands on <u>White</u>. Again the pointer of Spinner II has the same chance of landing on True or False. Draw two branches with probabilities 1/2 at the end of the W-branch.
- 4) Suppose the pointer of Spinner I lands on <u>Blue</u>. Both outcomes of Spinner II have the same chance of occurring. Draw two branches at the end of the B-branch with probabilities 1/2.
- 5) The two-stage tree has six outcomes: (R,T), (R,F), (W,T), (W,F), (B,T), (B,F). <u>Each has the same chance of occurring</u> so the probability of each is 1/6.

SPINNER I SPINNER II 3 13 3 B 1 1ST SPIN 3 3 3 2ND SPIN 2 1 3 3 1 1 2 2 (R,T)(R,F) OUTCOME : (W,T)(W,F) (B,T) (B,F) PROBABILITY : 1 6 16 16 16 6 (R,T) means the pointer of Spinner I is on Red and the pointer of Spinner II is on True.

BLUE

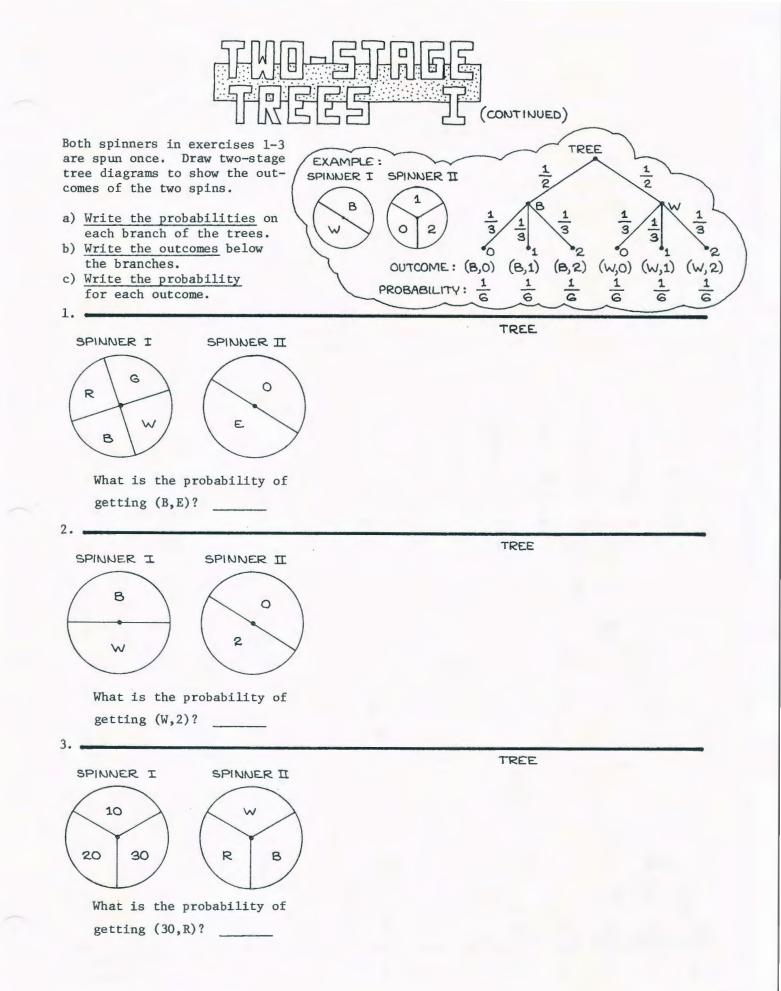
RED

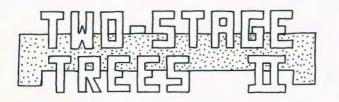
WHITE

TRUE

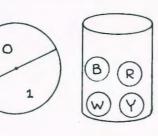
FALSE

If each spinner is spun once, the probability of getting Blue on Spinner I and True on Spinner II is 1/6.





 The spinner is spun once and then a marble is drawn from the bag. Draw a twostage tree to show the outcomes. What is the probability of getting a zero on the spinner and a red marble from the bag? _____



2) A coin is flipped once and a die is rolled once. Draw a two-stage tree to show the outcomes. What is the probability of getting a tail and a six?

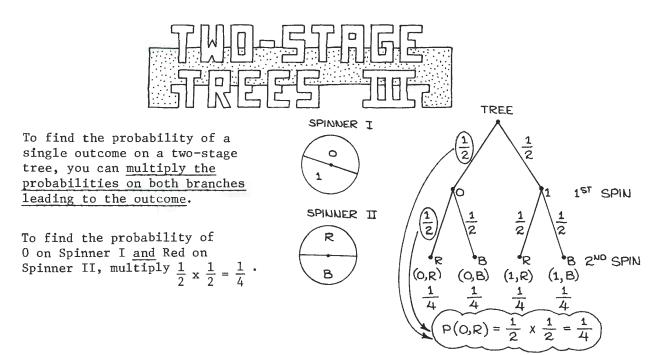
3) Each coin is flipped once. Draw a two-stage tree to show the outcomes. What is the probability of getting two heads? What is the probability of getting one head and one tail?



4) One coin is flipped twice. Do you see how your tree diagram in #3 can be used to show the outcomes? What is the probability of getting two tails? ____ TREE

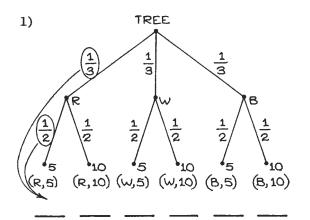
TREE

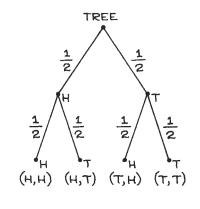
TREE

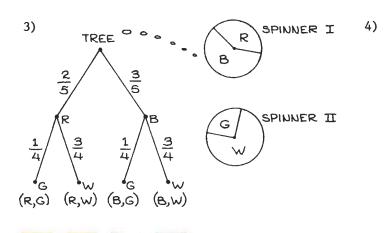


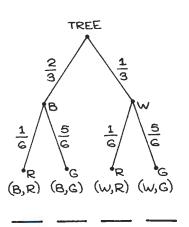
Study each diagram below. Multiply the probabilities on both branches to find the probability of a single outcome. Find and write the probability below each outcome.

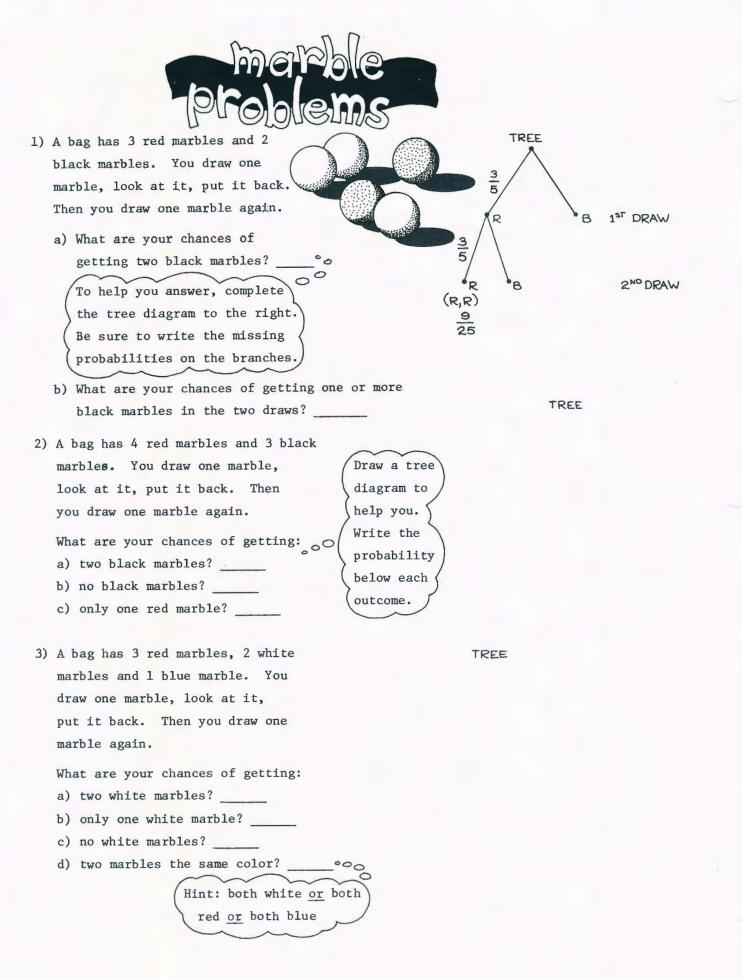
2)

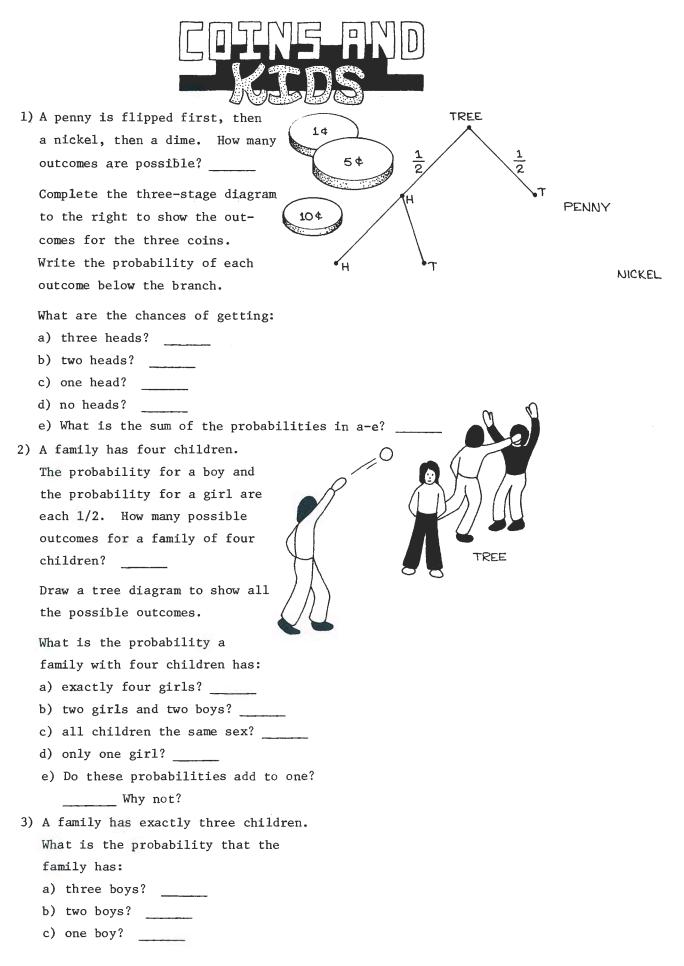














- I. North Salem and South Salem play each other four times during the regular basketball season. For any one game North's probability of winning is 2/3; South's probability of winning is 1/3.
  - a) Who do you think is more likely to win the most games?
  - b) For any one game what is North's probability of losing? _____
  - c) Complete the tree diagram to show all the possible outcomes (Wins and Losses) for North for the four games.

For game #1, North wins with probability 2/3; loses with probability 1/3.

Below each final outcome in the diagram write the probability.

- d) What is the probability
  - North will win all four games? _____
  - 2) each school wins two games?
  - 3) South wins three games? _____
- II. North and South Salem are to play each other three times during a holiday tournament. One of North's key players is sick and can't play. So for any one game North's probability of winning is 3/5; South's probability of winning is 2/5. What is the probability a) North loses all three games? ______ b) South wins two of the three games? _______ c) North wins two of the three games?

GAME # 1

TREE

TREE

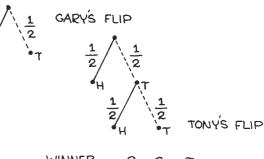
3



On the page *Tossing Pennies* in the Experimental Probability section, Gary offers to play the following game with Tony. Gary flips a coin. If it is heads, Gary wins. If it is tails, Tony flips a coin. If Tony's coin is a head, Gary wins. If it is a tail, Tony wins. Tony says the game is unfair. Gary then offers to accept one penny when he wins and to pay Tony two pennies for a loss. A tree diagram can be used to analyze the game.

At stage 1 (Gary's flip), the chances of H or T are equally likely. Gary will win half of the time.

At stage 2 (Tony's flip if Gary got a tail), the chances of H or T are equally likely. At stage 2, Gary wins half of the time. Tony wins only on the dotted branch.



WINNER G G T PROBABILITY  $\frac{1}{2}$   $\frac{1}{4}$   $\frac{1}{4}$ 

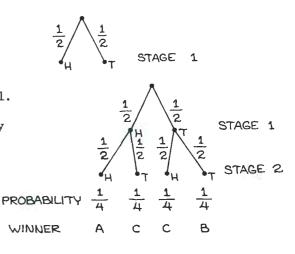
The tree shows Gary's chances of winning a game are 3/4. If 4 games are played and Gary won 3, he would collect 3¢ while Tony would only win 2¢. A fairer bet would be for Gary to pay Tony 3¢ when Tony wins. Then in the 4 games, if Gary won 3, each would win 3¢.

A second activity mentioned on the page *Tossing Pennies* involves flipping two coins. The class is divided into two teams A and B and the teacher is team C. Each student flips two coins. If both are heads, team A wins the two coins. If both are tails, team B wins the two coins. If one is a tail and one is a head, team C wins the two coins. Below is an analysis of the game using a tree diagram.

At stage 1, the chances of a head or a tail are equally likely.

At stage 2, the chances of a head or a tail are equally likely for each outcome of stage 1.

The completed tree shows four equally likely outcomes, two of which are winners for C. This game may dispell the misconception of three equally likely outcomes for flipping two coins -- two heads, two tails or one of each.





- Pete has two bags. One contains 3 black marbles and 1 white marble. The other contains 3 black and 2 white. Pete shuffles the bags so Dick can't tell their contents. Dick chooses a bag and draws one marble. What are the chances he draws a white marble?
  - a) To answer the question, imagine Dick spins a spinner to choose the bag. (Each bag has the same chance of being selected.)
  - b) Dick then draws a marble from the bag.
     Complete the tree to show the outcomes of drawing a marble. Be sure to write the probability below each outcome.

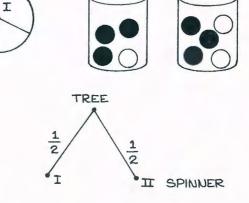
Add the probabilities of drawing

a white from Bag I or a white from Bag II.

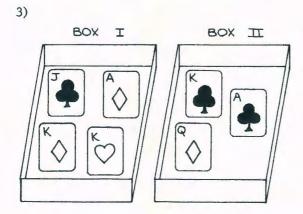
c) Find the probability Dick draws a white marble.



Π



2) Penelope has two bags. One contains 4 black marbles and 3 white marbles. The other contains 2 black and 4 white. Sue chooses a bag and draws one marble. What are the chances she draws a black marble?



A coin is tossed to choose Box I or Box II. Without looking, Alan draws a card from the box. What are the chances that Alan draws a) a Queen (Q)? _____ b) an Ace (A)? _____ c) a club ( )? _____ d) a King (K)? _____



Haphazard Hal has 6 identical pairs of socks, except 3 pairs are brown and 3 pairs are black.

Hal never bothers to pair up his socks. He just tosses them into the sock drawer. When he gets up in the morning he is sleepy. He reaches into the drawer and grabs two socks without looking at their color.

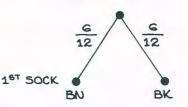
What are the chances Hal gets a pair the same color? (Assume the sock drawer has 6 brown and 6 black socks each morning.)

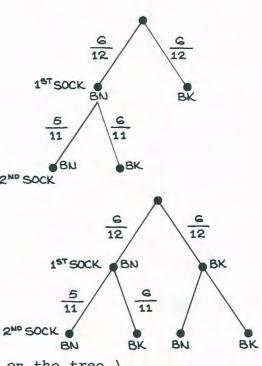
A tree diagram can help you find the chances. When Hal draws the first sock, the chances of getting a brown sock are  $\frac{6}{12}$ . The chances of getting a black sock for the first sock are also  $\frac{6}{12}$ .

To figure the chances for the <u>second</u> sock we need to know the color of the first sock.

- 1) Suppose the first sock was brown. There are 11 socks left in the drawer: 5 brown, 6 black. The chances of the second sock being brown are  $\frac{5}{11}$ . The chances of the second sock being black are  $\frac{6}{11}$ . These probabilities are shown on the tree.
- 2) Suppose the first sock was black. There
  are 11 socks left: 6 brown, 5 black.
  The chances of the second sock being
  bility on the tree.)
  The chances of the second sock being
  bility on the tree.)
  The chances of the second sock being
  black are _____. (Write this probability on the tree.)









*To find the chances of getting a brown and then a brown, multiply  $\frac{6}{12} \times \frac{5}{11} = \frac{30}{132}$ .

The chances of getting a brown and then a black,  $\frac{6}{12} \times \frac{6}{11} = \frac{36}{132}$ .

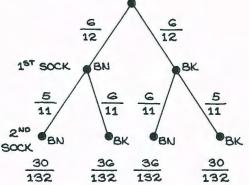
The chances of getting a black and then a brown,  $\frac{6}{12} \times \frac{6}{11} = \frac{36}{132}$ .

*The chances of getting a black and then a black,  $\frac{6}{12} \times \frac{5}{11} = \frac{30}{132}$ .

To find the chances that Hal gets a matched pair add*  $\frac{30}{132} + \frac{30}{132} = \frac{60}{132}$ .

ARE LESS THAN

2) Now suppose Hal has 4 brown socks and 6 black socks. What are the chances he gets a matched pair? ______ Are his chances better than in #1? ______ What are his chances of not getting a matched pair? ______





- A bag has 3 red marbles and 2 black marbles. You draw one marble, look at it, and do not put it back. Then you draw another marble.
  - a) What are your chances of drawing two black marbles? Study the tree carefully. There are only four marbles left after the first draw. Write the missing probability on the branch to complete the diagram.
  - b) What are your chances of getting one or more black marbles in the two draws?

2

4

(R,R)

ေ

20

Draw a

diagram to help

tree

you.

R

(R,B)

ရေ

20

TREE

5

3

4

(B,R)

6

20

R

В

(B,B)

2) A bag has 4 red marbles and 3 black marbles. You draw one marble, look at it and do <u>not</u> put it back. Then you draw another marble.

3) A bag has 3 red marbles, 2 white marbles and 1 blue marble. You draw one marble, look at it and do not put it back. Then you draw another marble.

What are your chances of getting:

- a) two white marbles?
- b) only one white marble? _____
- c) no white marbles?
- d) two blue marbles?
- e) two marbles the same color?

TREE

1ST DRAW

2ND DRAW

TREE



Materials: pencil, paper clip or hair pin Activity:

- Bend one edge of the paper clip until it is straight.
- Arrange the paper clip and pencil on the circle like the diagram shows.
- Snap the end of the paper clip so it spins. Practice a few times.

#### Game for two players:

1) Red counts 3 points -- blue counts 2 points. First player to 30 points is the winner.

7

3 spins

4 spins

1ST TURN

2ND TURN

3RO TURN

4TH TURN

5TH TURN

BB

R

- Make a table for each player as shown to the right.
- 3) On your turn, spin the spinner. If you get blue, continue to spin. Stop when you

a) have spun 4 times

- or b) get red.
- 4) Record your spins and points.

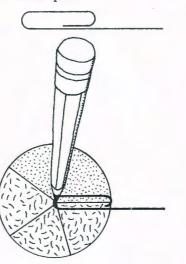
#### After the game:

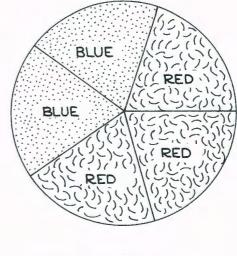
1) How many turns did the two players make? How many turns were 1 spin? ______ 2 spins?

2) This tree diagram shows all the possible outcomes for one turn. In each circle, write the score earned if the turn ended on that spin.

- 3) Find the probability of a turn ending with
  - 1 spin. _____ 4 spins with red on the 4th spin. _____
  - 2 spins. _____ 4 spins with blue on the 4th spin. _____
  - 3 spins.







POINTS

TOTAL

1ST SPIN

2ND SPIN

3RD SPIN

4TH SPIN

NAME

R

SPINS



To solve some probability problems, it is not necessary to draw the whole tree diagram.

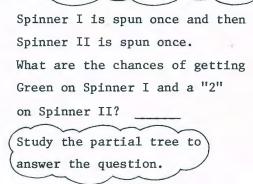
SPINNER I

1

SPINNER II

2

1)



- 2) A red die is rolled and then a green die is rolled. What are the chances of getting a six on the red die and an even number on the green die?
- 3) A card is drawn from a wellshuffled deck of 52 playing cards. The card is replaced, the deck reshuffled and another card is drawn. What are the chances that both cards are aces? _____
- 4) A card is drawn from a well-shuffled deck of 52 playing cards. The card is not replaced. A second card is drawn. What are the chances that the first card is a club and the second card is a heart?

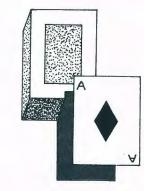


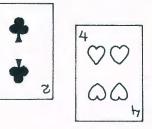
TREE

2

G

(G,2)





677



R

In each problem below, write an X in the blank beside the game which gives you the best chance of winning. If the chances are the same, write an X in the blank beside "Same chance of winning." To help you decide, draw tree diagrams to find the probabilities of winning for each game.

TREES

TREES

1)

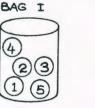
SPINNER II

SPINNER I

G

Game I: You spin Spinner I once. You win if you get white. Game II: You spin Spinner II twice. You win if you get Red on the first spin and White on the second spin. Same chance of winning.

2)

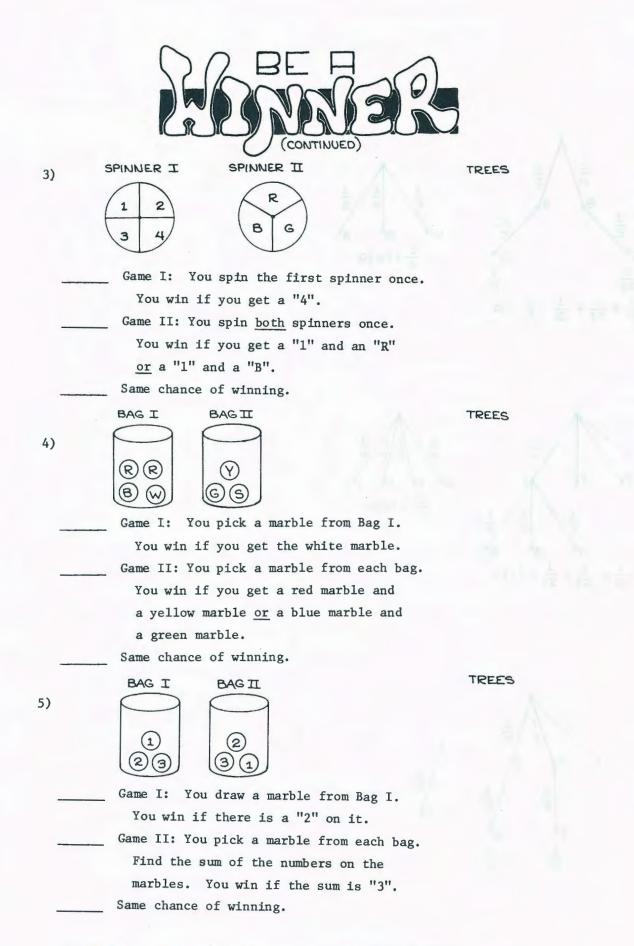


BAG II

Game I: You draw a marble from Bag I. You win if there is a "3" on it. Game II: You draw a marble from Bag II. (Don't put the marble back.) You draw another marble. You win if you get a "1" on the first marble and a "3" on the second marble.

Same chance of winning.

**IDEA FROM:** A Study of Parts of the Development of a Unit by J. Shepler Permission to use granted by Dr. Jack Shepler



IDEA FROM: A Study of Parts of the Development of a Unit by J. Shepler Permission to use granted by Dr. Jack Shepler



- 1) A dozen eggs from a farm contains three eggs with spotted yolks.
  - a) If you pick one egg from this dozen, what is the probability the yolk is not spotted?
  - b) If you pick one spotted egg, what is the probability a second egg picked will be spotted?
  - c) Draw a two-stage tree diagram to find the probability of picking two eggs with spotted yolks ______ two eggs with good yolks ______ one good and one spotted
- 2) A radio signal has probability 9/10 of being correctly interpreted. To improve the chances of receiving a correct message, each symbol is transmitted three times. A symbol is correctly received if it is correctly interpreted two of the three times or all three times. Use the tree diagram to find the probability of a symbol being correctly received.

Symbol received

Probability of correct reception

- C means correctly interpreted. I means incorrectly interpreted.
- 3) The Community Club has a cake raffle each month. Each member buys one ticket. John has never won.
  - a) If the club has 15 members, what is the probability John will win this month if everyone has an equal chance of winning?
  - b) Sam decides to help John by putting 5 extra tickets for John in the ticket box. Now what is John's probability of winning?
  - c) What is the probability that Sam will win?
  - d) What is the probability that another member will win?
  - e) At the last minute Juanita brings an extra cake. The club decides to draw two tickets without replacing the first ticket. What is the probability John will win both cakes? _____
  - f) What is the probability John still will not win either cake?

## AND THE SUM IS . .

Louise was playing Monopoly. She wondered what sum occurred most often when the dice were rolled. What do you think?

٠.

• •

••••

•••

DIE

- Fill in the table to find the sums for rolling two die.
- 2) Which sum occurs most often? _____
- 3) a) How many of the sums are 5?
  - b) List them with the green die number listed first.
  - c) The probability of rolling a sum of 5 is _____ out of ____.

This can be written  $P(sum = 5) = \frac{4}{36}$ .

- 4) Find P(sum = 10).
- 6) Find P(sum = 7).
- 8) Find P(sum = 7 or 11).
- 10) Find P(sum < 4).
- 12) Find P(sum is prime).

- 5) Find P(sum = 2).
- 7) Find P(sum = 11).

RED

5

DIE

9

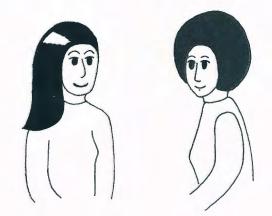
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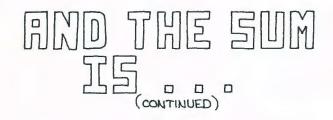
•••

- 9) Find P(sum > 6).
- 11) Find P(sum is even).

Louise asked her teacher what would happen if the dice were rolled 72 times. Ms. Garcia answered, "On the average you could expect to get the sum of 5 eight times out of 72 rolls. This can be shown by multiplying  $\frac{4}{36} \ge 72$ ."

For practice, Ms. Garcia asked Louise to find these expected results.





12)	Sum	Number of rolls	Louise thought	Answer
a)	6	72	$\frac{5}{36}$ x 72	
b)	9	180	$\frac{4}{36}$ x —	
c)	11	144	x 144	
d)	7	3600	x	

In the library, Louise found a book called <u>72 Hours at the Crap Table</u> by B. Mickelson. Craps is a game played by rolling a pair of dice and betting on the sum that comes up. The table below shows what actually happened on 14,976 rolls of the dice.

SUM	2	3	4	5	6	7	8	9	10	11	12
ACTUAL	402	854	1237	1669	2009	2575	2082	1602	1254	875	417
EXPECTED	-	0.92	1248	1054		24146	20110	1000	1741	632	-015

13) Use the probability for getting each sum to find the expected number of sums for the 14,976 rolls. Write your answers in the table. A calculator will make the calculation easier.

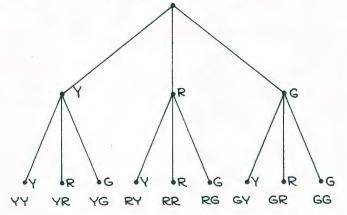
$\sim$	$\sim$
2 × 14,976	ON THE
CALCULATOR IS	2 ÷ 36 × 14,976.
Can	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

# REVISITED

Depending on the level of your class, students could compute the probabilities for each of the events described in *Roll That Cube* in EXPERIMENTAL PROBABILITY.

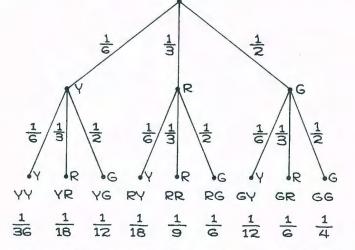
When rolling one cube there are six possible outcomes. Of the six, yellow occurs 1 time, red 2 times and green 3 times. The respective probabilities are yellow  $\frac{1}{6}$ , red  $\frac{1}{3}$  ( $\frac{2}{6}$ ) and green  $\frac{1}{2}$  ( $\frac{3}{6}$ ). Exercise 3 in *Roll That Cube* could be answered by saying yellow should occur about 4 times, red about 8 times and green about 12 times.

When rolling two cubes there are nine possible outcomes. These can be illustrated with a tree diagram or a matrix.



	Y	R	G
Y	YY	YR	YG
R	RY	RR	RG
G	GY	GR	GG

The probabilities associated with each of the nine outcomes can also be illustrated.



		Y		R		G	
			-10				え
Y		YY		YR		YG	
	- 6		136		18		12
R		RY		RR		RG	
	13		18		19		6
G		GY		GR		GG	
	12		12		-6		+

GR and RG represent the same color combination since order is not important. The probability for rolling GR is the sum of the probabilities for rolling GR and RG:  $(\frac{1}{6} + \frac{1}{6} = \frac{1}{3})$ . The "correct" guess for exercise 6 in *Roll That Cube* is GR. Many students may guess that GG will occur most often.



This activity analyzes the situations shown in *Which Die Is Better*? in EXPERIMENTAL PROBABILITY. In the long run, A will win over B, B will win over C, C will win over D and D will win over A. In each case, of the 36 possible combinations, the one die will win 24 times. The person choosing second can <u>always</u> choose a die that will have a probability of winning of  $\frac{2}{3}$ . This can be illustrated by the use of a grid or a tree diagram.

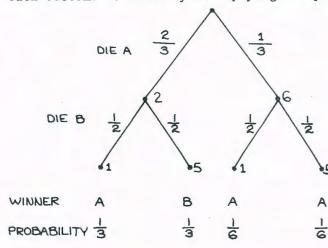
Using Die A and Die B, the grid would look like the one below.

	В					
	1	1	1	5	5	5
2	Α	A	A	в	в	в
2	A	A	A	в	B	в
2	A	A	A	в	в	в
2	A	A	A	в	B	в
6	A	A	A	A	A	A
G	Α	A	A	Α	A	A
	Q Q Q Q	2 A 2 A 2 A 2 A 6 A	2 A A 2 A A 2 A A 2 A A 2 A A 6 A A	1       1       1         2       A       A       A         2       A       A       A         2       A       A       A         2       A       A       A         2       A       A       A         2       A       A       A         2       A       A       A         6       A       A       A	1       1       1       5         2       A       A       A       B         2       A       A       A       B         2       A       A       A       B         2       A       A       A       B         2       A       A       A       B         2       A       A       A       B         2       A       A       A       B         2       A       A       A       B         6       A       A       A       A	1       1       1       5       5         2       A       A       A       B       B         2       A       A       A       B       B         2       A       A       A       B       B         2       A       A       A       B       B         2       A       A       A       B       B         2       A       A       A       B       B         2       A       A       A       A       B       B         6       A       A       A       A       A       A

A is the winner in 24 of the 36 possibilities.

The probability of A winning is  $\frac{2}{3}$ .

A tree diagram of the above example shows the probabilities of rolling each of the numbers. The winner and the probability of winning are shown. The probability of each outcome is found by multiplying the probabilities along the branches.



Probability of A winning is  $\frac{1}{3} + \frac{1}{6} + \frac{1}{6} = \frac{2}{3}$ . Probability of B winning is  $\frac{1}{3}$ .

You may want to check out each of the other possibilities. If Die A and Die C are used, the probability of A winning is  $\frac{5}{9}$ . If Die B and Die D are used, Die B should win  $\frac{1}{2}$  of the time.



In the game *Crasy Quotients* in EXPERIMENTAL PROBABILITY, what appears to be an unfair game for player A actually turns out to be an unfair game for player B. An examination of the table of quotients (only the first digit is shown) shows that 1, 2 and 3 occur more frequently.

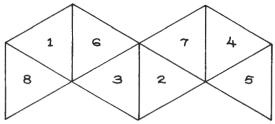
	PLAYER B								
		1	2	з	4	5	6	7	8
	1	1	.5	.3	.2	.2	.1	.1	.1
	2	2	1	.6	.5	.4	.3	.2	.2
	3	3	1.	1	.7	.6	.5	.4	.З
RA	4	4	2	1.	1	.8	.6	.5	.5
	5	5	2.	1.	1.	1	.8	.7	.6
	6	6	з	2	1.	1.	1	.8	.7
	7	7	З.	2.	1.	1.	1.	1	.8
	8	8	4	2.	2	1.	1.	1.	1

Of the 64 possible outcomes, a 1, 2 or 3 occurs as the first digit 40 times. Thus the probability of A winning is  $\frac{40}{64}$ . P(A wins) =  $\frac{40}{64}$ . The probability of B winning is  $\frac{24}{64}$ . P(B wins) =  $\frac{24}{64}$ . In reduced form and decimal form, the respective probabilities are  $\frac{5}{8}$  and  $\frac{3}{8}$  or .625 and .375.

PLAYER A

The game could be played with normal dice using the rule that A wins on 1, 2 and 3 and B wins on 4, 5 and 6. The game now looks reasonably fair. But A's probability of winning is  $\frac{23}{36} \approx .64$  and B's probability of winning is  $\frac{13}{36} \approx .36$ .

The net below can be used to make an octahedral die. A set containing 4-sided, 6-sided, 8-sided, 12-sided and 20-sided dice can be purchased from Creative Publications.





Have you ever wanted to find the answer to a sensitive question like "How many kids smoke marijuana?". In many groups this question would be answered with the socially acceptable answer of none. This activity illustrates a method where a group of respondents can answer truthfully but answers of individuals are not identified. The technique involves the use of randomly occurring events.

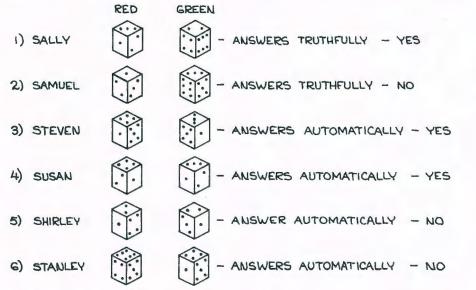
To explain the technique to your class use a non-embarassing question that can easily be checked. Let the random event be the rolling of a red die and a green die. The question is "Are you a girl?".

Give these directions to the class: Each student rolls the dice.

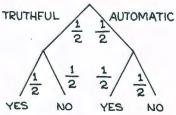
- 1) If an odd number shows on the upper face of the <u>red</u> die, answer the question truthfully.
- 2) If an even number shows on the red die, look at the green die.
  - a) If an odd number shows, automatically answer yes.

b) If an even number shows, automatically answer no.

The following show possible responses.



Since rolling dice is a random event, we would expect the red die to be odd half the time and even half the time. So we would expect half the students to answer truthfully and half to answer automatically. For those answering automatically we would expect half (1/4 of the total) to answer yes and half to answer no.





To determine the portion of truthful responses to the question:

- 1) Find the total number of yes responses and the total number of no responses.
- 2) Subtract from each total 1/4 of the total number of respondents.
- 3) Divide the remaining numbers by 1/2 of the total number of respondents to get the fraction or decimal for each response. (You will probably want to convert these to percents.)

For example, using the above technique with a class of 32 students, 17 yes answers and 15 no answers occurred to the question "Are you a girl?". Subtracting 8 (we would expect 1/4 of the total to automatically answer yes) from the yes answers and 8 (we would expect 1/4 of the total to automatically answer no) from the no answers gives 9 yes answers and 7 no answers. 9 yes answers out of 16 answers implies 56% of the students are girls and 7 no answers out of 16 answers implies 44% of the students are boys. The actual class had 16 girls (50%) and 16 boys (50%).

The technique was illustrated using the question "Are you a girl?" so students could immediately verify the results. After they are convinced (?) the technique is a reasonable one, sensitive and/or embarassing questions can be used.

Three words of caution: (1) Each student must keep the results of his/her random event secret. (2) As with any statistical event, particularly with a sample size as small as a class, there is a chance that the results will not be as expected. The results of one class with 21 boys and 11 girls (66% and 34%) indicated 31% boys and 69% girls. (3) The technique determines only the responses of the group as a whole and in no way does it describe an individual's response.

Other random events that could be used are flipping two coins (head or tail), spinning a spinner twice (two different colors or odd or even number), drawing two cards from a deck (red or black), using the middle two digits from the last four digits of a telephone number (odd or even), using the last two digits of the serial number of a dollar bill (odd or even) or using a table of random digits (odd or even).

The technique could be used for questions with three responses like "How often do you smoke pot? -- never, once a week or less, more than once a week." For those answering automatically, a 1 or 2 on the green die means to answer "never"; a 3 or 4 means to answer "sometimes"; and a 5 or 6 means to answer "often".

### GHOSTS, GOBLINS & "COINS THAT REMEMBER"

- 1. Do you believe that there are ghosts?
- 2. Do you believe that there are goblins?
- 3. Do you believe that a coin can remember?

You probably answered "No" to all these questions. Yet often we hear people talk as if they believe that coins can think and remember. They really do not understand the ideas in the <u>law of large numbers</u>. Most people call it the law of averages, and they often draw wrong conclusions from it.

You have heard people say:

A. "I have tossed an honest coin four times. Each time it came up heads. The law of averages says that the next toss will be tails."

Do you believe that the next toss is more likely to be tails than heads?

B. "My teacher uses a spinner to assign positions for the baseball game of "work up". I haven't been assigned as a pitcher yet this year. Therefore, by the law of averages, I'm sure to be assigned as pitcher today."

Do you think that this pupil is more likely than not to be chosen as a pitcher?

C. "I have been tossing an honest die. In 23 tosses, the face with one dot on it has never been up. By the law of averages, it is very likely that it will come up on the next toss."

Do you think the face with one dot is more likely than any other face?

Let's look at each of these examples of a misunderstanding of the "law of averages". Look back at statement A .

A. A coin does not have a memory. It cannot "remember" that it has been heads on the last four tosses. There is an equal chance for heads or for tails on the next toss.

We can use mathematics to prove that it is "unusual" to have a coin show four heads in four tosses. We can draw a tree diagram, make a table, or look at the fourth row in the Pascal Triangle. How many different outcomes are there when 4 coins are tossed or when one coin is tossed 4 times? How many of these outcomes consist of four heads? So, P(4 heads) =  $\frac{1}{16}$ .

However, this also means that we expect 4 heads in a row, once every 16 times that we toss 4 coins. The coin while flying through the air on the fifth toss cannot say to itself, "Well, that's 4 heads in a row; I better twist a bit more and be sure to land tails or I'll mess up the 'law of averages'." The probability of heads on the next toss is of course  $\frac{1}{2}$ , the same as any other individual toss. Some people who misunderstand the law of averages think the probability of tails is much greater than  $\frac{1}{2}$  after a coin has been heads several times in a row. Do you know people like this? They have forgotten that what happens on one toss has NOTHING to do with what will happen on the next toss.

## GHOSTS, GOBLINS & "COINS THAT REMEMBER" (PAGE 2)

Refer to statements B and C.

- B. If there are 9 positions on the baseball field, then the probability of getting any one position is 1 out of 9. The fact that this pupil has not been a pitcher yet does not cause the spinner to favor one position for him over the others. He still has only 1 chance in 9 of being a pitcher today.
- C. This person is overlooking one simple fact about a die -it cannot think! It cannot say, "Let's see now. I know the probability of any face is  $\frac{1}{6}$ . My face with one dot on it has not been up in 23 tosses, so on the next toss I'll land so that the face with the one dot is on the top."

This person is thinking, "One face hasn't been up for a long time, so that face is more likely to come up than any of the others." This is a mistake about the law of averages that people often make. He doesn't really believe that dice can think, yet he is acting as if they could. Each face on a die has just as much chance to be up as any other face. If the face with one dot has not been up in 100 tosses, it <u>still</u> has no more chance than any other face to be up on the next toss. In fact, it has just one chance out of six. Can you think of other correct or incorrect statements that you have heard about the law of averages? List some of them.

Why do so many people misunderstand the law of averages? It is too bad, but we all believe things and arrive at conclusions which just aren't true.

Which of these statements are false?

- 1. Lightning never strikes twice in the same place.
- 2. If you handle a frog, you'll get warts.
- The end of the Panama Canal on the Pacific Ocean side is farther west than the end on the Atlantic Ocean side.
- Horses are smarter than pigs.
- 5. George Washington threw a dollar across the Potomac River.
- 6. Columbus discovered America.

Many people believe some of these statements. Did you believe any of them? If you did, it isn't at all surprising. However, the six statements are all false. Most of us believe some things which really aren't true. Why is this so? There are many reasons. Among them are:

- We are told or we have read something which is not true, but we remember it.
- 2. We did not understand what we were told or what we have read.
- 3. We reasoned incorrectly.
- 4. Our experience caused us to believe something that wasn't true.

### GHOSTS, GOBLINS & "COINS THAT REMEMBER"

(PAGE 3)

5. We jumped to a conclusion without knowing enough facts.

6. We failed to check our belief against the facts.

This list could go on and on. There may be other reasons that you can think of. People arrive at false ideas about the law of averages for many of these same reasons. We can be fooled unless we are very careful. We might believe that an outcome, such as heads on a toss of a coin, is bound to happen if it hasn't happened for many tosses. It is easy to understand how our brain fools us in this case. It tells us that for a large number of tosses of a coin, heads will occur about half of the time -- and this is true. This is an example of the law of large numbers. Then we observe that heads hasn't occurred for several tosses and we make the mistake of thinking that heads must now start occurring more often to "catch up" with the number of tails. This is not true. Remember, a coin can't think. On each toss, there is just as much chance for heads to turn up as for tails.

By using mathematics, we can learn many interesting things. For example, from 15 children in your room, there are 6, 435 different ways you can have 7 children on a committee. If you choose a 7-member committee from 30 students, you have a choice of 2, 035, 800 different committees. Another example is if a coin has been tossed and heads have occurred 7 out of 10 times, chances are less than  $\frac{1}{2}$  that tails will "catch up" in 100 tosses. The mathematician can tell what will probably happen in cases such as this.

The next time that you hear some statement about the "law of averages", listen carefully. Try to find what the person believes and see if he is using it correctly.

#### ASSIGNMENT:

- 1. Complete the page *Tricky Statements*. Discuss your answers with your teacher.
- Do the activity On the Average. Discuss the results with your teacher.

16

Perhaps now you can see why some people misunderstand the law of averages. With many tosses of a coin, we do see that about half of the tosses are heads. That is, it takes 2 tosses, <u>on the average</u>, to get heads. But, when you tossed a coin, you found that sometimes you tossed a head on only 1 toss. Other times, you had to toss the coin several times to get a head. This should help you understand that these people fail to see that the "average" is made from numbers that differ quite widely and that there is NOT a "law" which says that you <u>must</u> get a head after tossing 5 tails, for example.



Mark these TRUE or FALSE.

- 1. You have been spinning a spinner that has a dial which is  $\frac{1}{2}$  black and  $\frac{1}{2}$  red. The last four spins have landed on black. It is more likely that the spinner will show red on the next spin than black.
- 2. The last five new pupils who came to our school were boys. The chances are better than equal that the next new pupil will be a girl.
- 3. The hospital reported that the last seven babies born there were girls. It is more likely that the next baby born there will be a boy than that it will be a girl.
- 4. The weatherman says that on the average it rains 4 days during the month of July. Today is the 27th of July and it has not rained all month. Therefore, it will rain tomorrow.
- 5. An auto dealer has 250 new cars and he knows that one out of every five new cars he sells is colored black. This week he has sold a blue, a white, a green, and a grey car. It is more likely than not that the next car he sells will be a black one.

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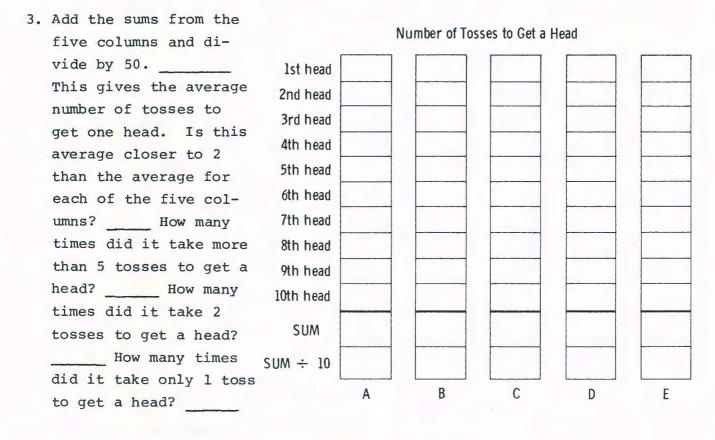


This experiment may help you to see why some people draw wrong conclusions from the law of averages.

Problem: How many times, <u>on the average</u>, do you think that you would have to toss a coin before it comes up heads? Use the chart below to answer the question.

Procedure:

- Toss a coin. Count the number of tosses until you get a head. For example: If you get a head on the first toss, write 1 in the column just to the right of "1st head." Start over. If you do not get a head until the fourth toss, write a 4 just to the right of "2nd head." Continue until you have completed Column A. Repeat for Columns B through E. Each column provides space to record the tosses for 10 heads.
- 2. After you have tossed 50 heads, add the number of tosses to get each group of ten heads. Divide each of these sums by 10 to find the <u>average</u> number of tosses needed to get one head for that column.



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## ORGANIZATION

## Placement Guide for Tabbed Dividers

Tab Label	Follows page:
CONTENT FOR TEACHERS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
Probability with Counting	
Inferential Statistics	
Appendix-Computer Programs	
DIDACTICS	
Components of Instruction	
Classroom Management	
Statistics and Probability Learning	
TEACHING EMPHASES	
Critical Thinking	
Decision Making	
Problem Solving	
Models and Simulations	
Calculators and Computers	
Laboratory Approaches	
CLASSROOM MATERIALS	
CONCEPTS AND SKILLS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Misleading Statistics	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
APPLICATIONS	
Business & Commerce	
Environment	
Health & Medicine	
People & Culture	
Recreation	
GLOSSARY	
ANNOTATED BIBLIOGRAPHY	
SELECTED ANSWERS	

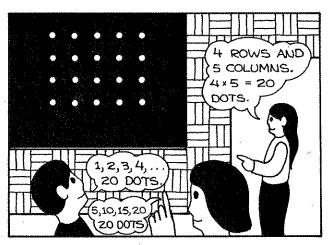


Commentary to COUNTING TECHNIQUES (694-703)

TITLE	PAGE	TOPIC	TYPE
Math in Many Ways	704	Counting paths to spell words	Worksheet
Yes, Three Letter Words, Please	705	Using trees to count letter arrangements	Teacher directed activity
What's New at the Zoo?	707	Counting the number of arrangements	Teacher demonstration
5 Coins in the Fountain	708	Using the fundamental counting principle	Worksheet
The Un-Proverb	709	Using the fundamental counting principle	Worksheet Transparency
Aloha	710	Using the fundamental counting principle	Worksheet
Numbers and Letters	711	Using the fundamental counting principle	Worksheet
Sporty Numbers	712	Introducing and using factorials	Worksheet
Webster's Dictionary	713	Computing with factorials	Worksheet Puzzle
And the Winner is	714	Introducing combinations	Worksheet
Choosing Points on a Circle	715	Finding combinations	Teacher directed activity
Repeating Permutations	718	Finding permutations with repetitions	Activity card Worksheet
That Rascal Pascal	721	Seeing relationships in Pascal's triangle	Worksheet
Shading Pascal's Triangle	723	Shading patterns in Pascal's triangle	Teacher idea
Pascal's Patterns	724	Finding patterns in Pascal's Triangle	Worksheet
Pascal and Probability	726	Using Pascal's triangle to find probabilities	Worksheet
Perplexing Probabilities	728	Using counting techniques to solve probability problems	Worksheet

## COUNTING TECHNIQUES

Counting begins during pre-school when students count objects one at a time. In school they soon learn to count by 2's, 5's or 10's and to find the products of whole numbers. Both of these techniques can be used to find "how many" without counting objects one at a time. The formal counting techniques associated with probability have this same purpose--to find out "how many" without having to count one at a time.



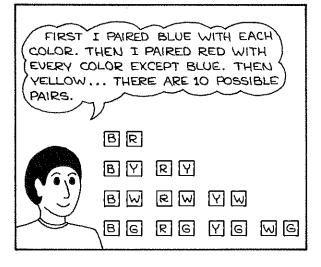
#### SOME CONCRETE EXPERIENCES WITH COMBINATIONS

As in learning many mathematical concepts and skills, it might be helpful to your students to solve some problems with concrete objects before attempting to use abstract methods of counting. (Depending on the level of your students, you might decide to give only these concrete experiences and leave the formal methods for counting to later grades.) Here are some informal activities to try with your class.

• Give each student an envelope containing 50 markers or slips of paper, 10 each of blue, red, yellow, white and green. Ask them to use the markers to show all the possible pairs of 2 different colors. Here the pair blue-yellow is the same as yellow-blue. These are called <u>combinations</u> of colors. Watch how students solve the problem. Do they guess at pairs and then look to see if they have duplicated a pair

or are they using a system? If they are just guessing, encourage them to develop a system of pairing the markers so they won't miss any.

After everyone has had a chance to work on the problem, some of the students can share their methods with the class. If students notice the number pattern 4 + 3 + 2 + 1, point out that the method works only for choosing <u>pairs</u> of colors. They will be learning more general number patterns for counting combinations later.



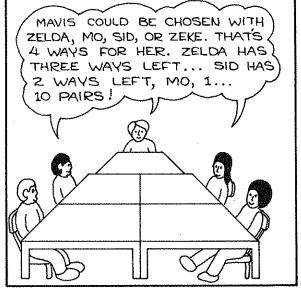
#### COUNTING TECHNIQUES

#### COMMENTARY

Change the problem by adding another color. (With 6 colors there are 15 pairs.) Change the problem again by allowing pairs to be the same color--that is, allow red-red, blue-blue, etc. (For 5 colors there are 15 possible pairs, for 6

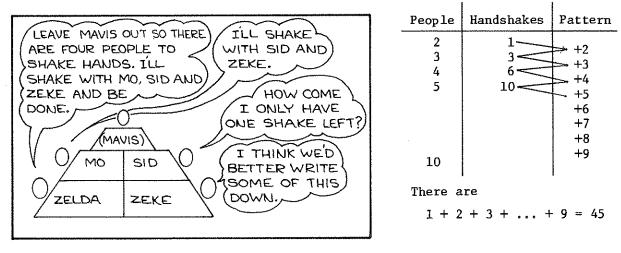
• Organize the class into groups of five or six. Ask students to find all possible pairs that could be chosen out of their group. They probably will not recognize this as the same problem (mathematically) as choosing pairs of colors from 5 or 6 colors; only the things to be selected are different. You might want to pose the problem again in another disguise: Andrea wants to take 2 books to read on her trip. She has 5 books to choose from. How many ways can she choose 2 books out of 5? Do your students try to list the pairs or d

colors, there are 21 pairs.)



Do your students try to list the pairs or do they realize the answer is 10 again? For 6 books, do they answer 15 pairs?

• While the class is in groups, ask them to figure out how many handshakes would happen if everyone shook hands with everyone else exactly once. (This is the "same" problem again---some might recognize it.) This time you might want to build a pattern for determining the number of pairs chosen out of 10 people, objects, ... Ask students to find the number of handshakes for 2 people, 3 people, 4 people and 5 people. They can then look for patterns to find the number of handshakes for 10 people.



#### COMMENTARY

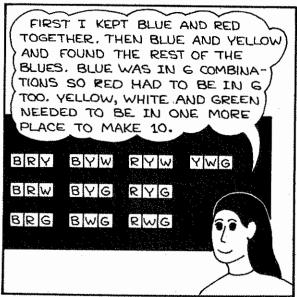
REMIND STUDENTS: The number pattern on the previous page works for choosing <u>pairs</u>. The activity *Choosing Points on a Circle* shows number patterns for choosing combinations of 3 or more out of a set.

Ask students to show all the possible combinations of three colors chosen out of four colors. Again, challenge them to find a system and later to explain it to the class. (Some students might be able to see that choosing 3 out of 4 is like choosing 1 out of 4 to leave out.) Using 4 colors, they should find that 4 combina-

tions of 3 colors are possible. Extend the problem to choosing combinations of 4 colors out of 5, 5 colors out of 6, ...

• Change the problem by asking for the number of possible combinations of 3 colors chosen from 5 colors. (Some students might relate this to the ways of choosing 2 colors to leave out.)

The last activity given above is more difficult. It is hard to keep track of the colors used and to decide how to proceed next. Encourage students to write statements down to keep things straight--



"There can't be any more blue with red; I've found all the combinations with blue; Each color must show up six times;" etc.

These activities all involve combinations of objects; the arrangement or order of the objects does not matter. Students are often able to solve these simple combination problems by systematically listing the combinations. To solve more complicated questions about combinations, students will need to generalize. This is discussed in the last part of this commentary and in *Counting Techniques* of the CONTENT FOR TEACHERS section.

#### SOME CONCRETE EXPERIENCES WITH PERMUTATIONS

The order in which objects are picked does not matter for combinations. The handshakes were combinations because the result is the same if Zelda shakes hands with Sid or Sid shakes hands with Zelda. In some situations order does make a difference. Suppose the problem is to find the number of valentines sent if each person in a group sends one valentine to every other person. Tasha sending Igor a valentine is different from Igor sending Tasha a valentine. The two orderings of Tasha

#### COUNTING TECHNIQUES

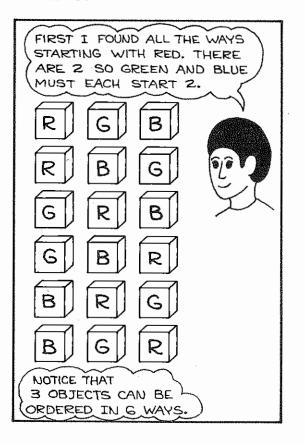
#### COMMENTARY

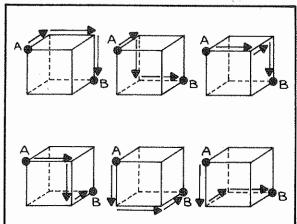
and Igor means two different valentines. The word "permute" is defined as "to change the order." Sets whose order is important are called permutations. Here are some activities for your class in counting permutations.

Give each student cubes or counters of 3 different colors. Ask them to find and record all the ways the three colors can be arranged in a row so they are in a different order each time. You might restate the problem pointing out that each student has one set of colors. All three colors will be used in each arrangement. Encourage students to use a system such as the student is describing in the illustration.

Here is the same problem in disguise. Ask students how many mathematics problems can be made by putting the numbers 4, 5 and 7 in the blanks. (_____) ÷ ____ = ? Have them write out the problems and solve them. (Some of the problems will involve negative numbers. For example, (4 - 7) ÷ 5 = -.6.

Make a worksheet with nine identically drawn cubes with vertices labeled A and B. Have students draw arrows showing all the possible paths 3 units long on the edges of the cube from A to B. (To get from A to B, a point must move one unit in each direction: back, right and down. There are six ways to order these three words so there are six paths.)





COMMENTARY

• Another geometric variation of arranging three things in order is to find all the ways to connect 3 points <u>in order</u>. Notice that  $A \circ \longrightarrow \circ B$ 

is not allowed because the points aren't connected in order. This could be extended to connecting 4 points in order.

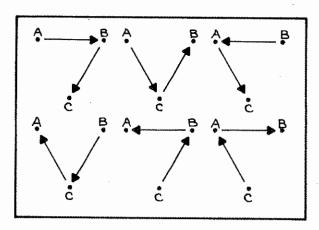
• Vary the first permutation problem above by adding a cube of a new color. Challenge students to find a system to determine how many orderings (permutations) of the four cubes are possible. Some students will experiment with orderings and check to see if they have duplicates. Others will find all the ways to start with one color and multiply by the number of colors as shown to the right. Another way is to look at the arrangements of three colors and realize for each arrangement the new color can go in any one of 4 available positions:

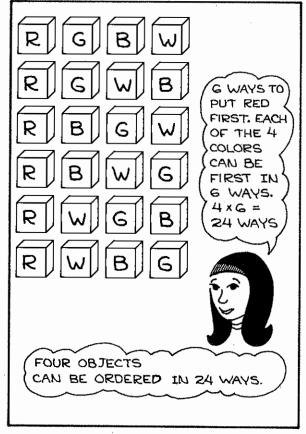


There are 6 ways to arrange three cubes in a row so there must be  $4 \ge 6$  or 24 ways to arrange four cubes in a row. (See What's New at the Zoo? and the last part of this commentary for more ideas on this.)

#### SPECIAL COUNTING PROBLEMS

Sometimes there are special conditions on a counting problem. Suppose the question is to find the number of ways to arrange 4 suits of cards in your hand if no suit can be beside another suit of the same color. Another example is to find the number of ways to get 2 heads and 2 tails out of 4 flips of a coin. These problems and the other activities discussed below can be given to students before generalizations or counting formulas are studied.

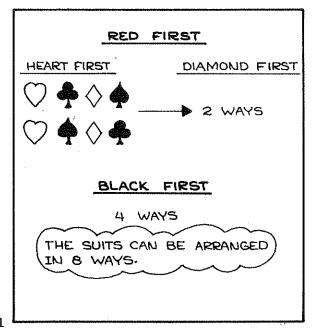




#### COMMENTARY

• Students can solve the card suit problem by listing all the possibilities. Better yet, some students will realize they can figure out the ways to start with one suit, say hearts, and multiply by 4 to find all the ways. Some students might organize their information as shown to the right.

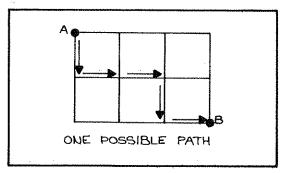
• Listing the ways to get 2 heads and 2 tails out of 4 flips is easier if a system is followed. Encourage students to follow a pattern. Here is one possible pattern: First find all the ways the 2 heads can be in a row: HHTT, THHT, TTHH. Now find the ways the heads can have 1 tail



between them: HTHT, THTH. Last, find the ways the heads can have 2 tails between them: HTTH. There are 6 ways to get 2 heads and 2 tails in 4 flips. (You might want to pose the problem so students can relate it to choosing a combination of pairs: How many ways can you choose 2 positions for heads in 4 flips of a coin?)

Some students might think the card suit problem with 2 reds and 2 blacks is the same as the problem with 2 heads and 2 tails. The card problem has 4 distinct suits, but their arrangement is restricted to RBRB or BRBR. The coin problem does not restrict the arrangement and there are not 4 distinct things. Heads and tails are both repeated. Arrangements where one thing can be used more than once are called permutations with repetition. *Repeating Permutations* gives several other related problems. Some additional background for permutations with repetition is also given on page 3 of *Repeating Permutations*.

Make a worksheet of fifteen or so 2 by 3 rectangles about the size shown here. Ask students to find the number of 5 unit paths from A to B. (So as not to spoil your fun, the paths are given on page 10 of this commentary.) Students can find the paths by trial and error--pooling their paths and eliminating duplicates.



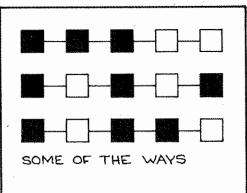
COUNTING TECHNIQUES

#### COMMENTARY

Some might attack the problem systematically. <u>One</u> system is shown on page 10 of this commentary.

• Make a worksheet of about 15 sets of 5 squares in a row as shown. Ask students to find the number of ways 3 of the squares can be shaded. The graphic to the right shows three of the ways, but no organized way of searching is apparent. With a system, such as finding all the ways with 3 shaded in a row, then 2 in a row, then no 2 beside each other, all the ways can be found.

(See the solution and further comments on page 10.



You might prefer to have your students view this problem as a combination problem: How many ways can 3 positions for the shaded squares be chosen out of five

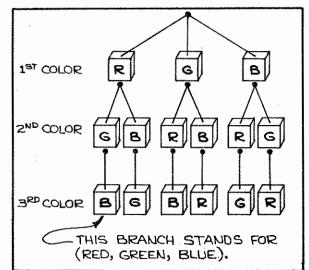
positions?)

The activities suggested so far have posed problems which can be solved by moving cubes or counters around and/or listing the different possibilities. Such experiences have value as a background for more formal study of permutations and combinations. More importantly, they allow students to explore, see patterns, and develop their own systems for solving problems.

#### USING A SYSTEMATIC APPROACH

Much was said in the previous pages about encouraging students to develop a systematic method for finding combinations or permutations. There is usually a lot of satisfaction in knowing that all the possibilities have been found. How will students know this unless they are systematic? (Some students will answer, "I've found them all because there aren't any more" or "I don't want to look any more.")

You might want to show your students how to use "trees" to find permutations. The graphic to the right shows all the possible orderings of a red cube, a green cube and a blue cube. Some students might become confused trying to change a vertical ordering into a horizontal listing. You might want to show the tree in a horizontal way. The activity Yes, Three Letter Words, Please shows more examples with trees. In Counting Techniques of the CONTENT FOR



TEACHERS section there are also examples using trees. *Probability With Models* of the CONTENT FOR TEACHERS section and PROBABILITY WITH MODELS in the student materials both employ trees. Trees are a very useful organizational model in both counting and probability.

#### WHICH SHOULD BE TAUGHT FIRST, PERMUTATIONS OR COMBINATIONS?

Not everyone agrees what should be taught first, permutations or combinations. Piaget in his book <u>The Origins of the Idea of Chance in Children</u> notes that with uninstructed students, the ability to list systematically all permutations occurs later than the ability to list combinations. It appears that students should be given many concrete experiences with permutations and combinations before too much formalizing of methods of counting.

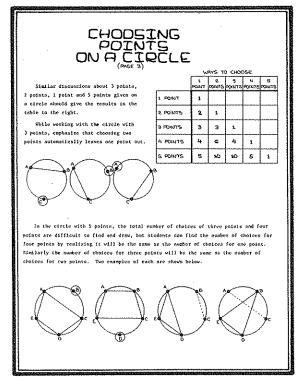
#### DEVELOPING A METHOD FOR COUNTING WITHOUT LISTING

As the number of things to be combined or ordered (permuted) increases, it becomes more difficult and time-consuming to list all the possible ways. If you have your students work with more complicated problems, they will need to have a generalized method for finding the number of possible combinations or permutations. There are several ways you could introduce these generalizations.

#### Through Number Patterns

For combinations use the number patterns shown in *Choosing Points on a Circle*. You could introduce Pascal's triangle and how it relates to combinations. (Background information on this is given in *That Rascal Pascal* and on page 11 of *Counting Techniques* of the CONTENT FOR TEACHERS section.

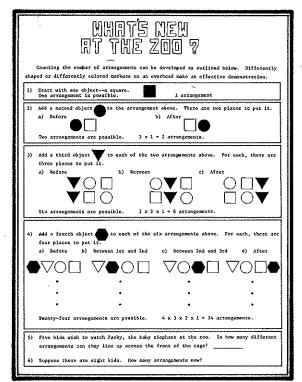
For permutations of n objects with no repetitions follow the development in What's New at the Zoo shown on the next page. You could use small, differently shaped objects on an overhead projector to help students see the arrangements. After they see the 2



#### COUNTING TECHNIQUES

#### COMMENTARY

ways of ordering two objects, show them the 3 places a new object could go in each of the 2 arrangements. Help students see that there are three times as many ways to order three objects as two objects or  $3 \times 2 \times 1$  ways. A fourth object can go in any one of four places in each of the six arrangements as before or  $4 \times 3 \times 2 \times 1$ ways. You might want to record this in a table to show what is happening, as shown below. The number pattern in the table should help students guess that <u>5</u> objects can be ordered in <u>5</u>  $\times 4 \times 3 \times 2 \times 1$  ways, 6 objects in <u>6</u>  $\times 5 \times 4 \times 3 \times 2 \times 1$  ways and so on.

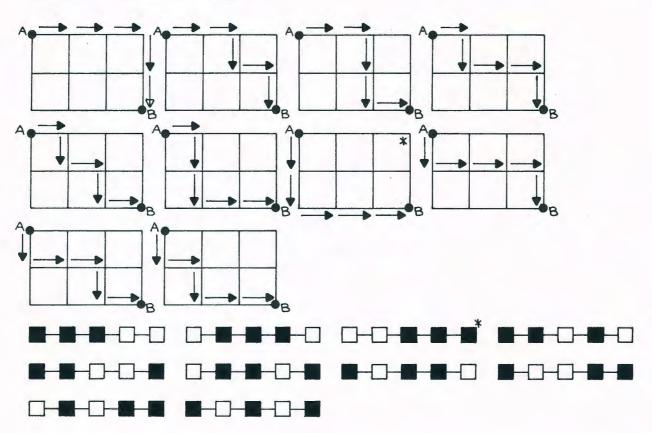


ONE	TWO	THREE	FOUR
OBJECT	OBJECTS	OBJECTS	OBJECTS
1	2×1	3×2×1	4×3×2×1

#### Through The Fundamental Counting Principle

A second way (the most traditional and generalizable way) is to teach students the Fundamental Counting Principle. The Fundamental Counting Principle is discussed thoroughly in *Counting Techniques* of the CONTENT FOR TEACHERS section. The activities 5 *Coins in the Fountain, The Un-Proverb* and *Aloha* suggest ways for introducing and using this principle. It is applied to counting permutations in *Numbers and Letters* and *Sporty Numbers*.

#### COMMENTARY



Solutions for problems on page 7:

The two problems above are similar. The first is the problem of arranging 3 R's (or 3 rightward movements) and 2 D's (or 2 downward movements) in order. The second is to arrange 3 B's (black squares) and 2 W's (white squares) in order. The rectangle with a * matches the row of squares with a *. You might have students "match" all ten of their solutions.



Use the letters to the right. Start at M. Trace a path to H to spell MATH.

A PATH CAN ONLY GO TO THE LETTERS JUST BELOW A LETTER.

How many ways do you think it can be done? These steps will help you to check your guess.

1) How many paths from the M to the left A? ______ the right A? _____

Write your answers in the blanks.

2) How many paths from the M to the left T? _____ the right T? _____

(careful) the middle T?

DID YOU SAY 2 PATHS TO THE MIDDLE T? TRACE THE 2 PATHS WITH YOUR FINGER.

Write your answers in the blanks.

3) How many paths from the M to the left H? ______ the right H? ______ (careful) the left middle H? ______

(careful) the right middle H?

DID YOU SAY 3 PATHS TO THE RIGHT MIDDLE H ? TRACE THE 3 PATHS WITH YOUR FINGER.

Write your answers in the blanks.

- 4) Now, how many paths can you trace to spell MATH?
- C A A N N N D D D D Y Y Y Y

M

нннн

M

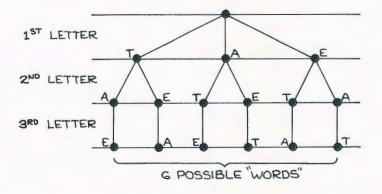
M

M

- 5) Starting at C, how many paths can you trace to spell CANDY?
- Make up a word triangle of your own. You could use your name.

## YES, THREE LETTER WORDS, PLEASE

- 1) On the board, draw and number three blanks.
- 2) With your own chips and container demonstrate how to mix the chips; pick one chip; fill in the first blank with the letter on the chip; pick a second chip; fill in the second blank; pick the third chip; and fill in the third blank. A chip is not to be replaced after it is picked.
- 3) Have each student draw two sets of blanks and do the experiment twice.
- 4) On the board, record the results. If the experiment is done many times, the six "words" should occur about the same number of times. TAE, TEA, ATE, AET, EAT, ETA Interestingly, only TAE is not listed in <u>Webster's New Collegiate Dictionary</u>. AET is an abbreviation meaning aged. ETA is the seventh letter of the Greek alphabet.
- 5) Ask if any more words can be formed. Students will probably think they have all the words. Use a tree diagram to show the six possible words. Build the tree one step at a time. Trace along a branch. Have students identify the word.

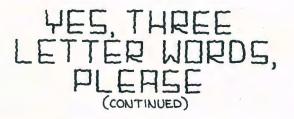


3

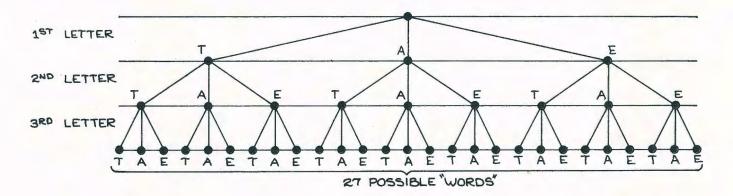
With only one example, it is best <u>not</u> to have students relate the total number of words to the number of letters. Later the fundamental counting principle can be introduced.

To find the number of ways of making several decisions in succession, multiply the number of choices that can be made in each decision.

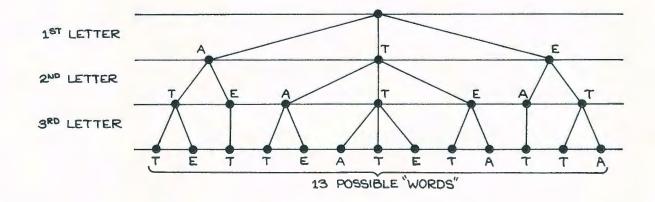
In the example above, the number 6 is obtained by multiplying  $3 \ge 2 \ge 1$ , but 3 + 2 + 1 is also 6. Students might think of adding instead of multiplying. With 4 chips,  $4 \ge 3 \ge 2 \ge 1$  words are possible but not 4 + 3 + 2 + 1 words.



The experiment can be repeated, but this time replace the chip before making the next pick. When the results of two trials per pair of students are recorded, all possible words probably won't be listed. Again ask students if other words are possible. Suggest making a tree diagram to show possible words.

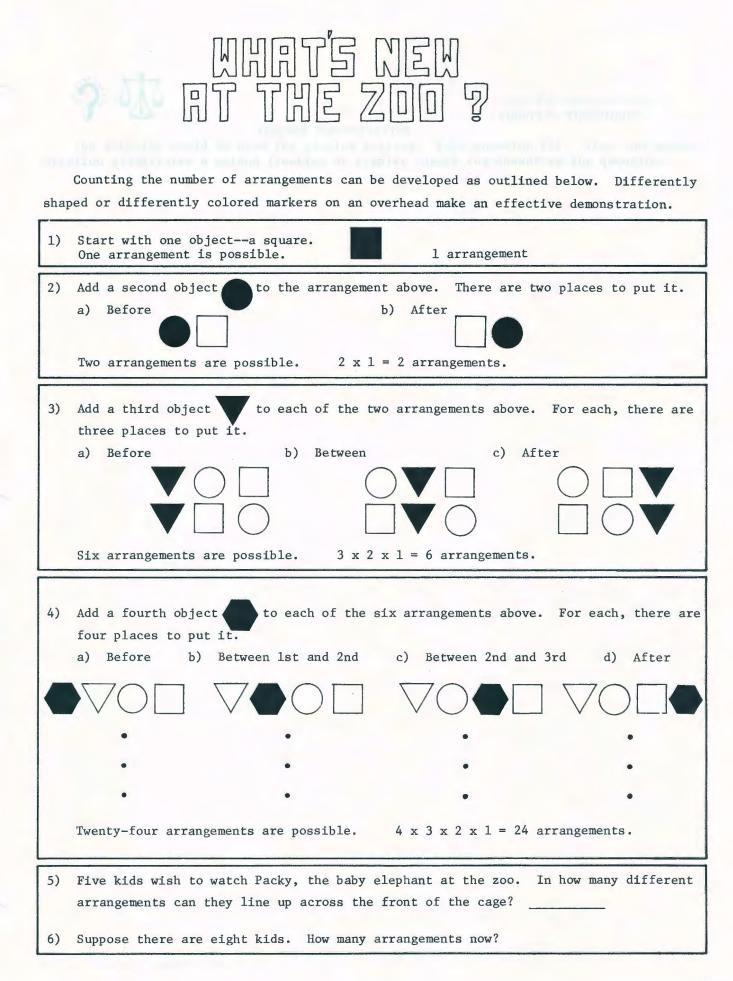


A more difficult experiment uses partial replacement. If T is picked, replace. If either A or E is picked, do not replace. Follow the above procedures for recording results, drawing the tree, etc. The fundamental counting principle can not be used for this experiment because a consistent number of ways for choosing the 2nd and 3rd letter can't be made. The tree diagram is an efficient way to show the results.



IDEA FROM: The preliminary experimental version of CSMP Mathematics for the Intermediate Grades, Entry Module I, and the preliminary experimental version of CSMP Mathematics for the Upper Primary Grades, Part II.

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The shopping center has a fountain. People throw money into the water. The money is usually given to charity. The merchants wanted to advertise a special sale. They offered all the money in the fountain to any 5th, 6th, 7th or 8th grader who could figure out how many different ways 5 coins could be arranged in a row. The coins are a penny, a nickel, a dime, a quarter and a half dollar.

1) What is your guess?

Tammy tried to solve the problem this way. She drew 5 blanks. She thought, "Any of the 5 coins could be in the first blank. I will put a 5 in the 1st blank."

5		(0)	$\sim$	
			~~~	$\sim$
"Once I put	a coin in the first bla	nk, 4 coins are left to cho	oose from for th	e second
blank."	4			
Finish Tammy	's work. How many coin	s are left to choose from :	for the third bl	lank; then
for the four	th blank; then for the	fifth blank?		
2) How m	any different ways can	the 5 coins be arranged?		
3) Is yo	our answer to (2) close	to your guess in (1)?		

4) How many ways can 4 people sit in 4 seats in a row at a theater?

5) How many ways can 6 books be arranged in order on a shelf?

6) How many ways can 7 bottles of pop be placed in a row on a grocer's shelf?



A		D	
l A rolling stone	 -	gathers no moss.	1
2 A bird in the hand	 -	is worth two in the bush.	2
3 A penny saved	 -	is a penny earned.	3
4 A stitch in time	 	saves nine.	4
5 The early bird	 -	gets the worm.	5
6 Every cloud	 -	has a silver lining.	6

The proverbs above are divided into two parts, A and B Create your own un-proverb.

- 1) Roll a die to get a first part.
- 2) Roll a die to get a second part.
- 3) Write your proverb or un-proverb here.
- 4) a) How many ways can the parts above be paired together?
 - b) How many pairings would be un-proverbs?
- 5) If you had eight proverbs, how many pairings would be un-proverbs?
- 6) If you had twelve proverbs, how many pairings would be un-proverbs?
- 7) What un-proverb is shown in the drawing to the right?
 8) JACK AND JILL WENT UP THE HILL TO FETCH A PAIL OF WATER. JACK FELL DOWN AND BROKE HIS CROWN AND JILL CAME TUMBLING AFTER.

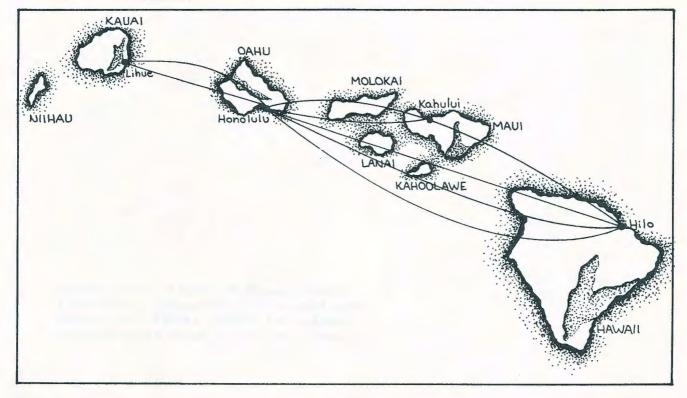
IDEA FROM: "Mathematical Games" by M. Gardner, *Scientific American*, February 1977 Permission to use granted by *Scientific American* and Martin Gardner



1) A hale aina (restaurant) in Hawaii has these items on the menu.

Meat	Vegetable	Beverage
Iwi aoao (spare ribs)	Ipu kalua (baked squash)	Wai (juice)
Pipi ku (roast beef)	Lau ai ai (salad)	Waiu (milk)
Pelehu oma (roast turkey)	Limu (sea weed)	Koka (soda pop)
Opihi (shellfish)	Poi (taro root)	Ki (tea)
Lau lau (butterfish)	Ula paa (breadfruit)	Kope (coffee)
Kalua paa (roast pig)	Uala (sweet potato)	
A ku (tuna)		
Kamano (salmon)		

- a) How many ways can you choose from the meat column?
- b) How many different pairs can you order if you take one from the meat column and one from the vegetable column?
- c) Order one item from each of the three columns. How many different ways could you do it? _____



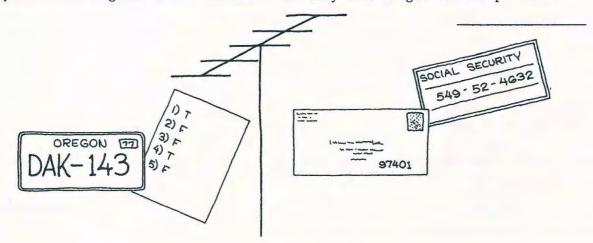
- 2) The Island Travel Agency offers trips by plane between cities.
 - a) How many direct flights from Honolulu to Lihue?
 - b) How many direct flights from Honolulu to Hilo?
 - c) How many flights from Lihue to Hilo by going only through Honolulu?
 - d) How many direct flights from Hilo to Kahului to Honolulu to Lihue to Honolulu to Hilo?
- 3) The Hawaiian alphabet has only 12 letters 5 vowels a, e, i, o, u and 7 consonants h, k, l, m, n, p and w. How many different three letter words can be made if the word has 2 vowels with a consonant in the middle? The vowels may be the same.

NUMBERS AND LETTERS

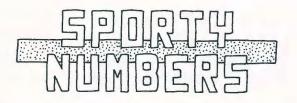
- A zip code has 5 digits. How many zip codes are possible if each digit can be 0 - 9?
- 2) A social security number is a 9 digit number written in three parts. 2 9 8 - 8 4 - 8 0 5 9

Everyone's social security number is different.

- a) If all digits 0 9 can be used, how many social security numbers are possible?
- b) Are there enough numbers so <u>each</u> citizen in the United States can have a different number?
- 3) License plates in Oregon have 3 letters and 3 digits. I's and O's are not used as letters. The first letter shows the month the license plate was issued: A for January, B for February, etc. (remember the letter I is not used).
 - a) What letter represents licenses issued in November?
 - b) How many license plates are possible?
 - c) In 1974, 1,579,736 cars, trucks and buses were licensed in Oregon. How many possible license plates were not being used?
- 4) TV and radio stations have station codes starting with K or W (KWAX or WJW). Some stations use 3 letters. Other stations use 4 letters. Letters may be repeated.
 - a) How many station codes with 3 letters are possible?
 - b) How many station codes with 4 letters are possible?
 - c) An almanac lists 4,357 AM, 2,448 FM and 721 TV stations in the United States in 1974. If each station <u>did</u> have a different code, how many station codes are not used?
- 5) You are going to take a True False test with 10 questions. You forgot to study. So you decide to guess at the answers. How many sets of guesses are possible?



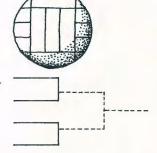
LETTERS AND NUMBERS



 The girls' volleyball team is having a picture taken. How many different ways can the 6 girls line up for the picture?



2) Four friends decide to play a tennis tournament. How many different ways can the players' names be placed on the first part of the tournament ladder? _____



- 3) The Fighting Ducks basketball team always plays man-to-man defense. How many different ways can the 5 defensive assignments be made?
- 4) Joe Fan got 8 sports books for Christmas. How many different ways can he arrange them in a row on a bookshelf?
- 5) A baseball team has 9 players. How many different batting orders can a manager make up?
- 6) The Oakland Raiders starting defensive team is being introduced at the Super Bowl. How many different ways can the 11 team members be introduced?

Products like the ones above are called factorials. You start with a number and keep multiplying by the next smaller number until you get to 1. An exclamation mark (!) is used to show a factorial. $6 \times 5 \times 4 \times 3 \times 2 \times 1$ can be written as 6! (6! is read "six factorial.")

7) Write the factors for 7!. (7! is read "seven factorial.")

- 8) Find the value for 7!
- 9) What is the largest factorial your calculator can display without indicating an overflow?

IDEA FROM: Counting and Choosing, Oakland County Mathematics Project Permission to use granted by Oakland County Mathematics Project



Work each exercise. Find the correct answer in the riddle below. Put the appropriate letter in each blank.

E	How many different ways can you arrange the letters in MIX?
(A)	What is the value of 6 X 5!?
0	How many different telephone numbers can be made by rearranging the digits in 348-5927 if no digits are repeated?
R	What is the value of 6! + 3!?
F	5! is how much smaller than 5 ³ ?
\bigcirc	What is the value of 7! ÷ 4!?
S	How many different ways can you arrange the letters in SCRAMBLE?
B	What is the value of 7! - 6!?
C	How many different zip codes can be made by rearranging the digits in 97402 if no digits are repeated?
H)	What is the value of 3! + 5!? O O O O O O O O O O O O O O O O
120 72	0 120 210 4320 6 5 5040 120 6
	126 5040 120 40,320 G

AND THE WINNER IS
Materials: 4 pennies - each with a different date
Activity: 1) Place the pennies in the circles. Write the date below each penny. A B C D
A contest is being held to pick
two pennies. I will choose two dates but won't tell you.
You will win the pennies if you choose the correct pair. (YOU WON'T REALLY 2) What are the choices you could make? (WIN TWO PENNIES. THIS)
a) If you choose A first, the pairs could be
A, A, or A
b) Choosing B first, B, B, or B
c) Choosing C first, C, C, or C
d) Choosing D first, D, D, or D
Look closely. You win by choosing the correct pair. Penny A and Penny B is the same pair as Penny B and Penny A.

- 3) In (2), circle a pair. Cross out any pair that is the same. Continue circling and crossing out until no repeats are left. Now how many different pairs can you choose from?
- 4) Suppose another penny is added; label it E.a) Write all the pairs.

A	 3	A	 ,	A	 5	A	
В	 3		 5		 ,	-	
	 ,		 ,		 ,		

b) Circle a pair. Cross out any pair that is the same. How many different pairs are there?



CHOOSING POINTS ON A CIRCLE

This activity shows how to form combinations using points on a circle. Segments, triangles or quadrilaterals help to illustrate the different ways to make choices. Differently colored pens or chalk may help to emphasize choices.

Write the table below on the board or a transparency. Fill it in as the different ways are found. A circle with four points is used for this demonstration.

	WAYS TO CHOOSE									
	1 POINT	2 POINTS	3 POINTS	4 POINTS						
CIRCLE WITH 4 POINTS										

- a) The circle has four points marked. How many ways can you choose one point?
 - You can choose A (point to A).
 - You can choose B (point to B).
 - You can choose C (point to C).
 - You can choose D (point to D).

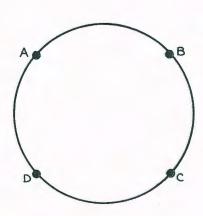
You can choose one point in 4 ways (record 4 in the table).

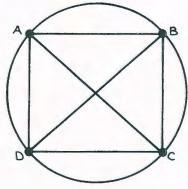
b) How many ways can you choose two points?

Show your choice of two points by connecting the points with a line segment.

You can choose A and B (draw the segment). You can choose A and C (draw the segment). (Continue until all 6 choices are shown. Emphasize that the choice of A and B is the same as the choice B and A. Only one segment is drawn between A and B. The order that the two points were chosen doesn't matter.)

You can choose two points in 6 ways (record 6 in the table).





CHOOSING POINTS ON A CIRCLE

c) How many ways can you choose three points? Show your choice by drawing a triangle to connect the three points.

You can choose A, B and C (draw the triangle).

You can choose A, B and D (draw the triangle).

You can choose A, C and D (draw the triangle).

You can choose B, C and D (draw the triangle).

You can choose three points in 4 ways (record 4 in the table).

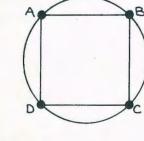
(Using the four graphics have students see that a choice of three points together is like leaving one point out. Choosing A, B and C together leaves D out. The number of ways of choosing three points out of four is the same as the number of ways of choosing one point out of four. Refer to the table to show the numbers are the same.)

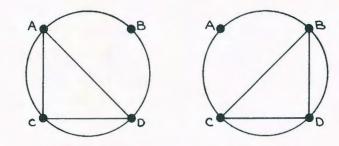
 d) How many ways can you choose four points? Show your choice by drawing a quadrilateral to connect the four points.

You can choose A, B, C and D (draw the quadrilateral).

You can choose four points in 1 way (record 1 in the table).

(Emphasize that the choices of B, C, D and A or C, B, D, and A will give the same quadrilateral. If a student suggests the points A, C, D and B you will need to emphasize that these are the same four points even though the figure looks different.)





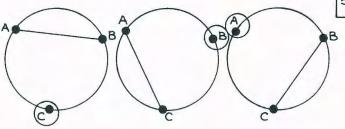
B

CHODSING POINTS ON A CIRCLE

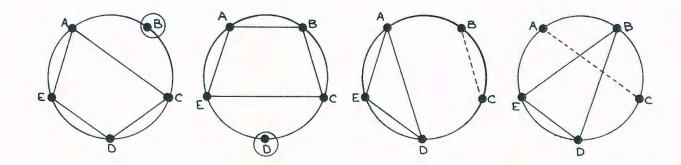
Similar discussions about 3 points, 2 points, 1 point and 5 points given on a circle should give the results in the table to the right.

While working with the circle with 3 points, emphasize that choosing two points automatically leaves one point out.

	WAYS TO CHOOSE									
	1 POINT	1 2 3 4 POINT POINTS POINTS POINT								
1 POINT	1									
2 POINTS	2	1								
3 POINTS	3	3	1.							
4 POINTS	4	G	4	1						
5 POINTS	5	10	10	5	1					



In the circle with 5 points, the total number of choices of three points and four points are difficult to find and draw, but students can find the number of choices for four points by realizing it will be the same as the number of choices for one point. Similarly the number of choices for three points will be the same as the number of choices for two points. Two examples of each are shown below.



	PEPERTING
1)	Get 3 chips, each a different color. Place a chip on each of these blanks.
2)	Write the order of the colors. (Use an abbreviation)
3)	Move the chips. Find different arrangements.
	THE THREE CHIPS CAN BE ARRANGED IN A ROW SIX DIFFERENT WAYS.
4)	Remove 1 chip. Get another chip with the same color as a chip you kept.
	QUESTION : CAN YOU STILL MAKE SIX DIFFERENT ARRANGEMENTS ?
5)	Place a chip on each of these blanks.
6)	Write the order of the colors.
7)	Take the 2 chips with the same color and trade places. Write the order of the colors.
8)	Is there a difference in the answers to (6) and (7)?
9)	Move the chips to make as many different arrangements as you can. Write each new arrangement of the colors.
10)	Including the arrangement in (6), how many did you find?
11)	Explain why there are fewer arrangements when a color is repeated.

TELL ADDAULTY CATRONIC MANAGE



The Beebe family, Mom, Dad, Otto and Barbara, are interested in how many different arrangements of the letters in each name can be made.

Help them out by writing the letters of each name on separate slips of paper. (Don't do Barbara's name.) Arrange the letters in as many different ways as you can. Record each different arrangement.

How many different arrangements did you find?

b) DAD Place and rearrange the letters here: Record here.

How many different arrangements?

How many different arrangements?

d) BEEBE ______ ____

How many different arrangements?

e) BARBARA Guess how many different arrangements there are. ______ Check with your teacher for the answer.



How many different arrangements of the letters in ALASKA? If each letter were different, $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6!$ arrangements would be possible. But switching the A's does not distinguish one arrangement from another. For example, ALASKA - ALASKA. Can you tell the first A and the last A were switched in the second spelling? In ALASKA there are $3 \times 2 \times 1 = 3!$ different ways to arrange the A's and not change the spelling. So the different arrangements of the letters are $\frac{6!}{3!}$ or $\frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} = 120$. On the first page, the different arrangements of the chips (two with the same color) is $\frac{3!}{2!}$ or $\frac{3 \times 2 \times 1}{2 \times 1} = 3$.

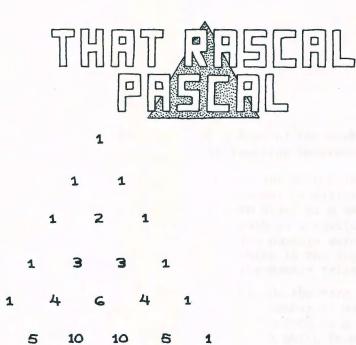
The different arrangements of the letters in TENNESSEE would be

 $\frac{8!}{4!2!2!} \text{ or } \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1 \times 2 \times 1 \times 2 \times 1} = 420.$

If you have developed this concept with your class, you can ask them to find the different arrangements of letters in words such as states: ALASKA, TENNESSEE, MISSISSIPPI, (your state, city or town), animal names: ELEPHANT, KANGAROO, ANACONDA, fruit: TANGARINE, BANANA, APPLE, etc.

Palindromic words, spelled the same backwards as forwards, could be used to investigate permutations with repetition. RADAR, TOOT, ZOONOOZ (a publication of the San Diego Zoo), MADAM, I'M ADAM are just a few examples.

Students may enjoy finding the number of permutations in the letters of their names; first, last or both.



1

a) \9

36/

This number triangle is called Pascal's triangle. Blaise Pascal was a French priest who was very interested in mathematics. The patterns in the triangle can be extended indefinitely.

1) Find these four groups in Pascal's triangle.

120/

- 2) Do you see a relationship among the three numbers in each group? ____
- 3) Use the relationship you found to write three more rows on the triangle above.

4) These groups occur later in the triangle. Find each missing number.

c) \210

462

d)' ?

120

?

330

45

IDEA FROM: *Counting and Choosing,* Oakland County Mathematics Project Permission to use granted by Oakland County Mathematics Project

b) ??



The pages Math in Many Ways and Choosing Points on a Circle show Pascal's triangle can be helpful in finding the number of different ways of selecting objects if order is not important.

The following points are needed if students are to read and use the triangle.

1)	If you are picking from 3 objects, 1	ook						1						
	at the row 1, <u>3</u> , 3, 1. If you are p	ick-												
	ing from 5 objects, look at the row						1		1					
	1, <u>5</u> , 10, 10, 5, 1, etc.													
2)	The 1 in the first position of					1		2		1				
	each row tells the number of ways						-							
	O objects can be chosen. For				1		3		3		1			
	example, choosing 0 objects out					1.				1.				
	of 3 can be done in 1 way.			1		4		6		4		1		
3)	The 1 in the last position tells				~						-			
	the number of ways all the objects		1		5		10		10		5		1	
	can be chosen. For example,			~				~~		.~		-		
	choosing 3 objects out of 3 can	1		6		15		20		15		6		1
	be done in 1 way.													

If there are 5 objects, 0 objects can be chosen in 1 way, 1 object in 5 ways, 2 objects in 10 ways, 3 objects in 10 ways, 4 objects in 5 ways and 5 objects in 1 way.

- 4) Choosing a number of objects automatically forms another set of objects that was not chosen. Pascal's triangle shows this by its symmetrical nature. Choosing 2 objects from 3 can be done in 3 ways; choosing 1 object from 3 can also be done in 3 ways.
- 5) The usual notation used for combinations is $\binom{n}{r}$ indicating r objects chosen from n objects without regard for the order of the objects.
 - a) $\binom{n}{1} = n$ See (1) above. b) $\binom{n}{0} = 1$ See (2). c) $\binom{n}{n} = 1$ See (3). d) $\binom{n}{r} = \binom{n}{n-r}$ See (4). e) $\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$ Remember the rule for generating Pascal's triangle.
 - f) $\begin{pmatrix} 0\\ 0 \end{pmatrix} = 1$ This allows the triangle to be complete.



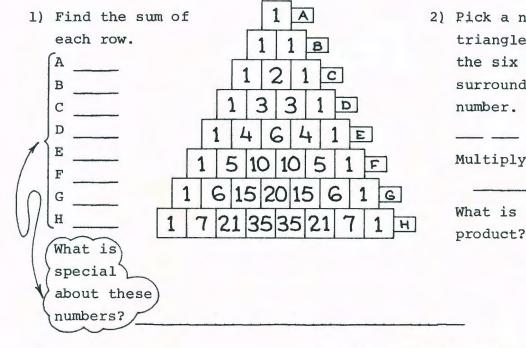
Many interesting patterns emerge when parts of Pascal's triangle are shaded. Some suggestions are given below. Students might like to draw Pascal's triangle on a large poster and add more rows to it.

Examples of patterns that could be shaded in different colors include:

- 1) even numbers
- 2) odd numbers
- 3) multiples of 3
- 4) multiples of 5
- 5) multiples of 10
- 6) multiples of a prime

G G





2) Pick a number on the triangle (not 1). Write the six numbers that surround the picked number

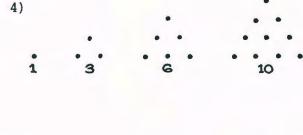
Multiply the six numbers.

What is special about this product?

3)

5)

These are the first four counting numbers. With your 1,2,3,4 finger, trace the diagonal in Pascal's triangle where these numbers are found. Write the next two counting numbers.



These are the first four triangular numbers. With your finger, trace the diagonal in Pascal's triangle where these numbers are found. Write the next two triangular numbers.

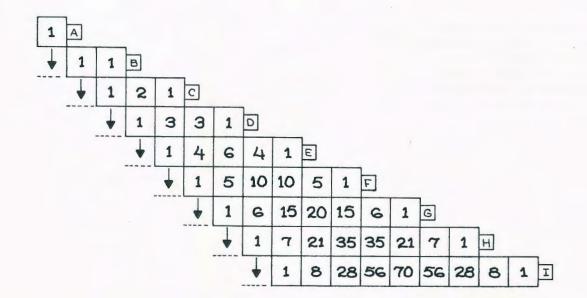
These are the first three pyramidal numbers. With your finger, trace the diagonal in Pascal's triangle where these numbers are found. Write the next two pyramidal numbers.

IDEA FROM: *Making Mathematics*, A Secondary Course, Book 4, by D. Paling, et al. Permission to use granted by Oxford University Press



6)	Cut out the strips of the triangle above.	A
	Arrange strips A, B, C and D like this.	
	Find the sum of each column.	В
	Compare the sums with strip E. What do	C
	you notice?	
	Arrange strips A - H to find the numbers	
	that would go on strip I.	
		I

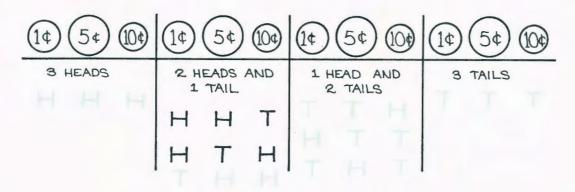
7) Arrange the strips like the diagram below.



Find the sum of the columns marked with an arrow. Write the sums.



What is the probability of getting 2 heads and 1 tail when you flip 3 coins -a penny, a nickle and a dime? Use the table below to write all the outcomes of heads and tails. Two outcomes are done for you.



1) How many different outcomes in all?

2) How many of the outcomes show 2 heads and 1 tail?

3) What is the probability of flipping 3 coins and getting 2 heads and 1 tail?

Pascal's triangle can also be used. Look at the row with the numbers 1, 3, 3, 1. These are the numbers of outcomes you got for each column in the table.

There is (are) 1 way to get 3 heads and 0 tails, 3 ways to get 2 heads and 1 tail, 3 ways to get 1 head and 2 tails, 1 way to get 0 heads and 3 tails. 1 1 1 1 2 1 1 3 3 1 1 4 6 4 1 1 5 10 10 5 1 1 6 15 20 15 6 1 1 7 21 35 35 21 7 1 1 8 28 56 70 56 28 8 1 1 9 36 64 126 126 84 36 9 1 1 10 45 120 210 252 210 120 45 10 1

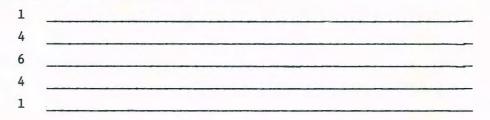


4) Look at the row with the numbers 1, 5, 10, 10, 5, 1.

There is (are) _____ way to get 5 heads and 0 tails, 5 ways to get 4 heads and _____ tail, 10 ways to get _____ heads and 2 tails, _____ ways to get 2 heads and _____ tails, _____ ways to get _____ heads and _____ tails, _____ way to get _____ heads and _____ tails,

What is the probability of flipping 5 coins and getting 1 head and 4 tails?

5) Look at the row with the numbers 1, 4, 6, 4, 1. These numbers count the outcomes for flipping 4 coins. Write what each number means.



What is the probability of flipping 4 coins and getting 2 heads and 2 tails?

6) Use Pascal's triangle to find these probabilities.

a)	Flipping	5	coins	and	getting	3	heads	and	2	tails	
----	----------	---	-------	-----	---------	---	-------	-----	---	-------	--

- b) Flipping 5 coins and getting 4 heads and 1 tail
- c) Having 8 children and getting 3 girls and 5 boys
- d) Having 6 children and getting 6 girls and 0 boys

e) Spinning this spinner (0+1) 4 times and getting 2 0's and 2 1's

f) Choosing 5 numbers between 1 and 100 at random and getting 5 even numbers

- g) Choosing 7 numbers between 1 and 100 at random and getting 7 even numbers
- h) Answering a 10 question true-false test at random and getting 10 true answers

PEPPLEXING

- Television and radio stations have station codes starting with K or W (KAAW or WJW). Some stations use 3 call letters. Other stations use 4 letters. Letters may be repeated.
 - a) How many station codes with 3 letters are possible?
 - b) How many 3-letter station codes have all 3 letters the same?
 - c) What is the probability that a station with a 3-letter code has all 3 letters the same?
 - d) How many 3-letter station codes have the first and last letter the same?
 - e) What is the probability that a station with a 3-letter code has the first and last letters the same?
 - f) How many station codes with four letters are possible?
 - g) What is the probability that a station with a 4-letter code has all 4 letters the same?
 - h) What is the probability that a station with a 4-letter code has all
 4 letters different?
- 2) A zip code has 5 digits. Digits may be repeated.
 - a) How many zip codes are possible?
 - b) If all zip codes were possible, what is the probability that a zip code has the same first and last digit? ______
 - c) If all zip codes were possible, what is the probability that a zip code has all odd digits?
 - d) What is the probability of a zip code reading the same backwards as forwards (e.g., 12321)?
- 3) You are going to take a true-false test with 10 questions.
 - a) How many different sets of answers are possible?
 - b) What is the probability that all 10 answers are false?
 - c) What is the probability that a set of answers has 3 true and 7 false answers?
 - d) What is the probability of having exactly 6 true answers arranged one after the other? ______





ORGANIZATION

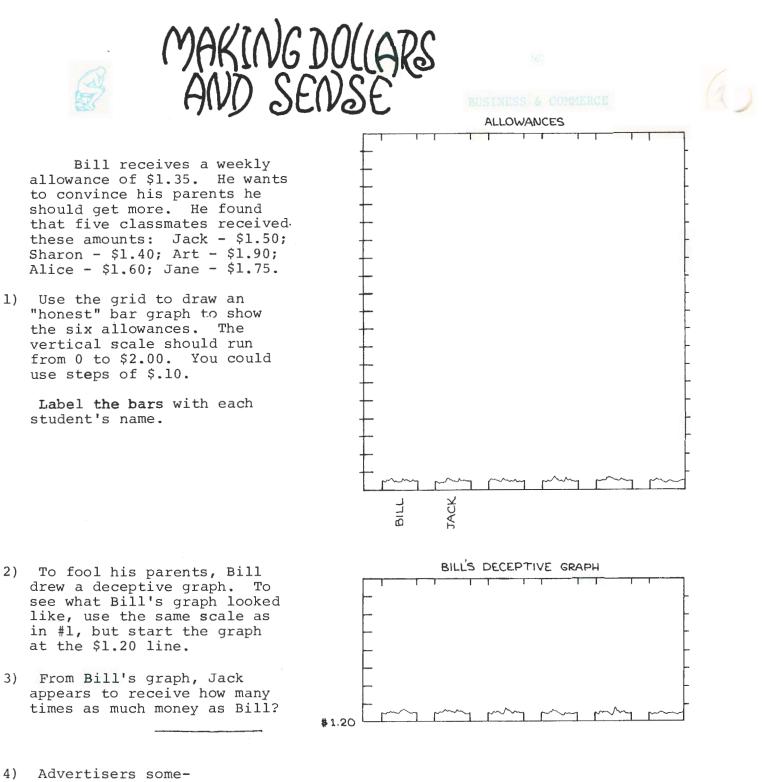
Placement Guide for Tabbed Dividers

Tab Label	Follows page:
CONTENT FOR TEACHERS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
Probability with Counting	
Inferential Statistics	
Appendix-Computer Programs	
DIDACTICS	
Components of Instruction	
Classroom Management	
Statistics and Probability Learning	
TEACHING EMPHASES	
Critical Thinking	
Decision Making	
Problem Solving	
Models and Simulations	
Calculators and Computers	
Laboratory Approaches	
CLASSROOM MATERIALS	
CONCEPTS AND SKILLS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Misleading Statistics	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
APPLICATIONS	
Business & Commerce	
Environment	
Health & Medicine	
People & Culture	
Recreation	
GLOSSARY	
ANNOTATED BIBLIOGRAPHY	
SELECTED ANSWERS	



BUSINESS & COMMERCE

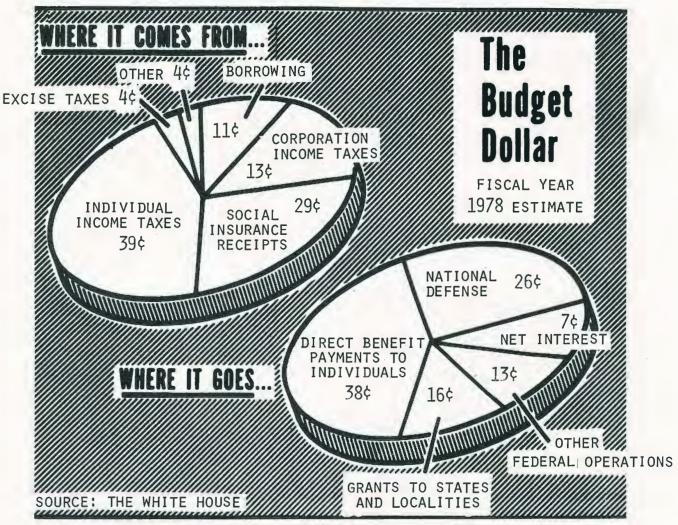
TITLE	PAGE	TOPIC	TYPE
Making Dollars and Sense	730	Using deceptive bar graphs to influence opinions	Worksheet
The Budget Dollar	731	Reading a circle graph on the national budget	Transparency Worksheet
The Wall Street Report	733	Using probabilities to predict the stock market	Activity card
Canadian Workers	734	Studying graphs of Canadian employment	Worksheet
Canadian Imports and Exports	736	Studying a graph of Canadian imports and exports	Worksheet
Chances at the Supermarket	737	Figuring the chances of winning in a supermarket game	Teacher idea
A Noisy Project	738	Surveying opinions on household tools	Activity card
Calling All Cars	739	Collecting and studying data on family cars	Worksheet
Join the Queue	740	Using random digits to investigate customer arrival	Worksheet
Making a Mortality Table	742	Using probabilities to construct a mortality table	Worksheet
Insurance for a Premium	744	Using a mortality table to figure the premium for a life insurance policy	Worksheet



 Advertisers sometimes use Bill's graphing to fool the readers.

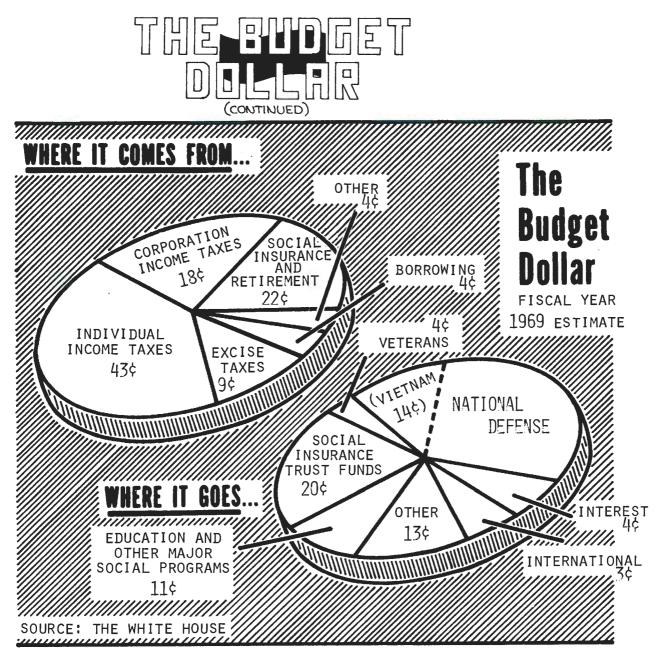
> Study the graph. Does it accurately show the increased costs?____

THE BUDGET DOLLAR



Where the 1978 budget dollar will come from, where it will go

- I. USE THE GRAPH TO ANSWER THESE QUESTIONS:
 - What source provides the greatest part of the budget dollar?
 - 2. Where is the greatest part of the dollar budgeted to go?
 - 3. National defense gets about what fraction of the budget dollar?
 - 4. Social insurance receipts provide about what fraction of the budget dollar? _____
 - 5. Does the graph give the total amount in the budget?
 - 6. Can you tell from the graph if the budget is balanced? That is, can you tell if as many dollars are taken in as are spent?



- II. COMPARE THE GRAPHS FOR THE 1969 BUDGET ABOVE TO THE GRAPHS FOR THE 1978 BUDGET. ANSWER THESE QUESTIONS:
 - 1. Did the corporation income taxes provide a greater part of the budget dollar in 1969 than in 1978? _____ Did corporation income taxes provide more dollars for the total budget in 1969 than in 1978? _____
 - 2. In 1969 how many cents out of each dollar were budgeted to national defense? _____ Can you tell from the graphs if more total dollars were budgeted for national defense in 1969 than in 1978?____
 - 3. The graphs for the budget dollar coming in in 1969 and 1978 are very much the same. Why are the graphs for the budget dollar being spent in 1969 and 1978 so different?



Materials: Five daily New York Stock Exchange reports (Your local newspaper probably has this report.)

Activity:

- 1) Use one column of the stock report.
- 2) How many stocks increased in value? (Look for a +)
- 3) How many stocks decreased in value?_____ (Look for a -)
- 4) If a stock is picked at random from your column, what is the probability it increased in value?
- 5) Keep a record for each of the five days for the stocks in your column.

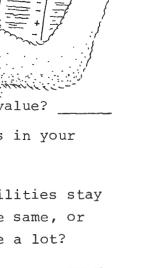
DAY	PROBABILITY STOCKS WENT UP
1	
2	
З	
4	
5	

6) Did the probabilities stay pretty much the same, or did they change a lot?

Extensions:

- 8) Read about the Dow-Jones average. Write a short report.
- 9) Read about a "bear" market; a "bull" market. Write a short report.

SOURCE: *What Are My Chances?, Book A,* by A. Shulte and S. Choate Permission to use granted by Creative Publications, Inc.



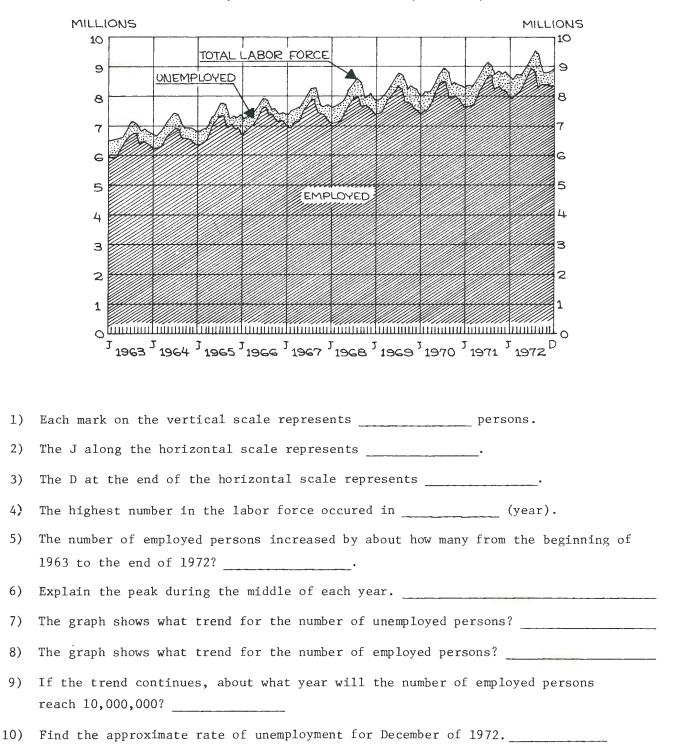
Exchance

-B



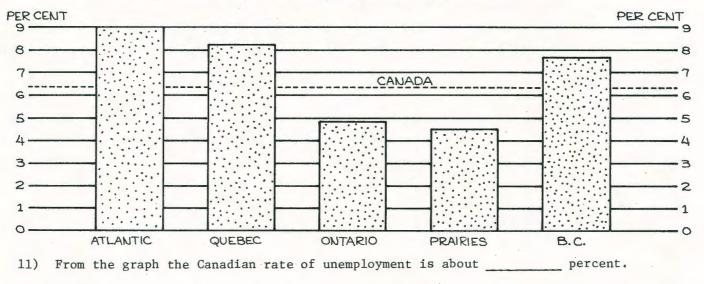
BUST ISS & COMMERCE







UNEMPLOYMENT RATE, BY REGION, 1972 ANNUAL AVERAGES

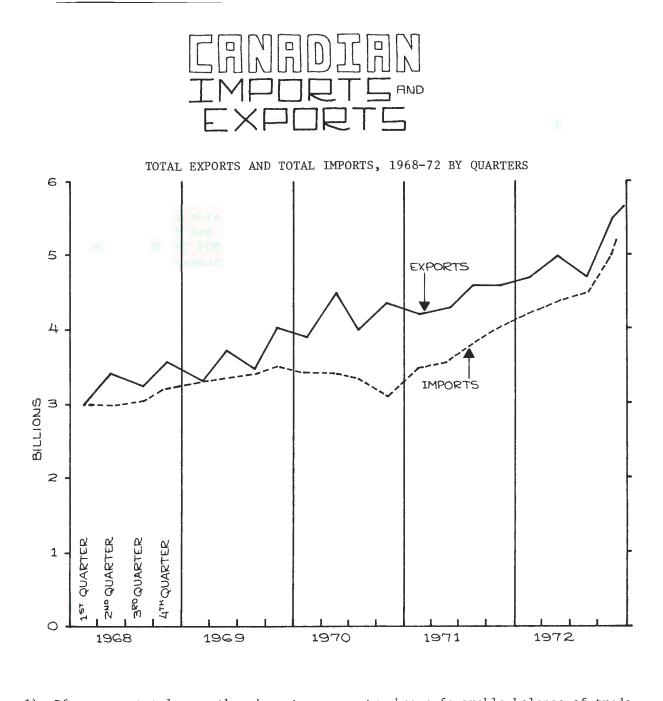


- 12) Which regions had rates higher than the Canadian rate of unemployment?
- 13) If you cut the strips above the Canadian rate of unemployment, would they exactly fill up the strips below the Canadian rate? _____ Explain. _____

Age Group		Men	Women					
	Number	Participation Rate	Number	Participation Rate %				
	1000	%	1000					
14-19 years	545	40.8	407	32.0				
20-24 years	827	84.0	580	60.5				
25-44 years	2,636	96.8	1,171	42.8				
45-64 years	1,791	89.2	756	36.3				
65 years and over	139	18.7	39	4.3				

THE LABOR FORCE, 1972, ANNUAL AVERAGES

The chart says there are 545,000 employed men in the 14-19 years age group. This represents 40.8% of all 14-19 year old men.



 If exports total more than imports, a country has a <u>favorable balance of trade</u>. Does Canada have a favorable balance of trade? ______
 In what quarter of what year did the exports exceed \$4 billion? ______
 In what quarter of what year did the imports exceed \$4 billion? ______
 In what quarter of what year was the difference between exports and imports the greatest? _______ the smallest? _______
 Before 1968, do you think Canada had an <u>unfavorable</u> balance of trade? _______

6) When do you think Canada's exports will reach \$6 billion?



BUSI

Sometimes supermarkets have games that can be used as motivation for an investigation in proability. Here is a table from an advertisement used to promote a game for a supermarket chain in the Northwest.

Enlarge the table on a transparency and show it to your class. Pose these questions:

- What are the chances for winning \$1000 in 5 visits?
- The game ends officially when all the game cards are distributed. How many game cards are there?

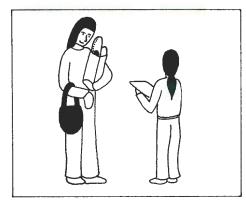
	-		
Cł	HANCES T	O WIN TABLE	:
PRIZES	NO. OF PRIZES	CHANCES FOR 1 VISIT	13 VISITS
\$1000	9	1 IN 188,889	1 IN 14,530
1000	45	1 IN 37,778	1 IN 2,906
20	170	1 IN 10,000	1 IN 769
10	340	1 IN 5,000	1 IN 385
5	680	1 IN 2,500	1 IN <u>192</u>
1	13,600	1 IN 125	1 IN 10
			ſ

The table does not answer the questions but, by examining patterns in the table, it is possible to compute the answers. Have your students look for patterns. By using a calculator, they can discover that dividing each of the numbers under the "1 visit" column by 13 gives the numbers under the "13 visit" column. A way to find the chances of winning \$1000 in 5 visits is to divide 188,889 by 5. There is about 1 chance in 37,779 to win \$1000 in 5 visits.

A second pattern can be discovered by multiplying the number of prizes in each category by the corresponding number under the "l visit" column (9 x 188,889). In each case the product is about 1,700,000. Could it be there are 1,700,000 game cards? There are nine \$1000 prizes. 9 chances out of 1,700,000 \approx 1 chance out of 188,889. Your students might ask other questions and discover other patterns in the table. (What are the chances of winning a prize of any size in 1 visit? 14,844 out of 1,700,000 \approx 1 out of 100. Did each store receive an equal number of cards? Can't tell.) You can have students bring in other promotional games for the class to analyze.



Surveying opinions (household tools BUSINESS & COMMERCE



Make plans to take a poll of 50 people (or if more convenient two homes per pupil and plan so that there will not be duplication of households). Use these questions or develop a set of your own that might be more relevant to your community.

- Would you be willing to pay 8% more for an air conditioner if it made less noise? More specifically, would you pay \$310 as compared to \$288?
- 2) Would you be willing to pay \$99 for a less noisy vacuum cleaner if the same model but noisier was available for \$90?
- 3) Would you be willing to pay \$1.25 more for a less noisy hair dryer?
- 4) Would you be willing to pay \$5.00 more for a less noisy lawn mower?

	"YES"		"N	0"	"UNDECIDED"				
ITEM	NO.	90	NO.	8	NO.	9			
Air Conditioner									
Vacuum Cleaner									
Hair Dryer									
Lawn Mower									

Tabulate your results on a chart similar to the one shown here.

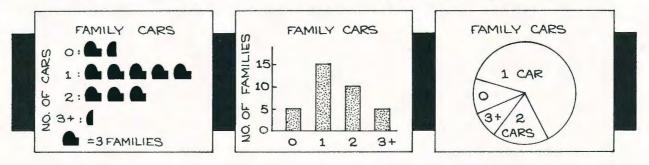
If you can come to any conclusions that might influence a manufacturer of one of these items, why not write to such a manufacturer explaining what you have done and offer some concrete suggestions to him for future production plans. Wouldn't it be interesting to find out if the company would respond to your suggestions?

TYPE: Activity Card

SOURCE: *Pollution: Problems, Projects and Mathematics Exercises,* Wisconsin Department of Public Instruction Permission to use granted by Wisconsin Department of Public Instruction

CALLING ALL CARS

- 1) Take a poll among the students in your class. Find out how many families have
 - a) no cars c) 2 cars
 - b) 1 car d) 3 or more cars
- 2) Make some kind of graph for the information. Here are some samples.



- 3) In your poll, what percent of the families have one car? _____ Two or more? _____
- 4) How do these percents compare with the percents for 1970 in this table?

AUTOMOBILE OWNERSHIP

	1950	1955	1960	1965	1970
Millions of families	45.2	49.1	53.4	58.5	63.7
% of families owning autos	59	70	77	79	80
a) one auto (%)	52	60	62	55	50
b) two or more (%)	7	10	15	24	30

Table from Environmental Science, Intermediate Science Curriculum Study

Permission to use granted by Silver Burdett Company and Florida State University

- 5) For each year in the table, what percent of the families did not own an automobile?
- 6) How many millions of families owned automobiles for each of the years in the table?
- 7) Which is growing at a faster rate: the number of families, or the number of families owning automobiles?
- 8) Predict figures for 1975 and 1980.

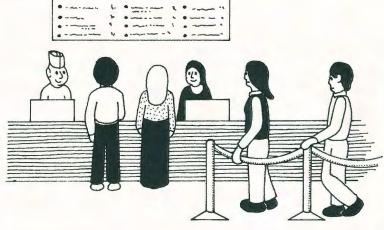
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MAC DUBLES

MacDubles hamburger restaurant has two order-takers. If both are busy when customers arrive, they wait in a line, a queue (pronounced q). When an order-taker is free, the first customer in the queue moves to place an order. Each customer takes about four minutes to order. Customers do not arrive at a steady rate. During any given minute, the chance a customer arrives is 0.7.

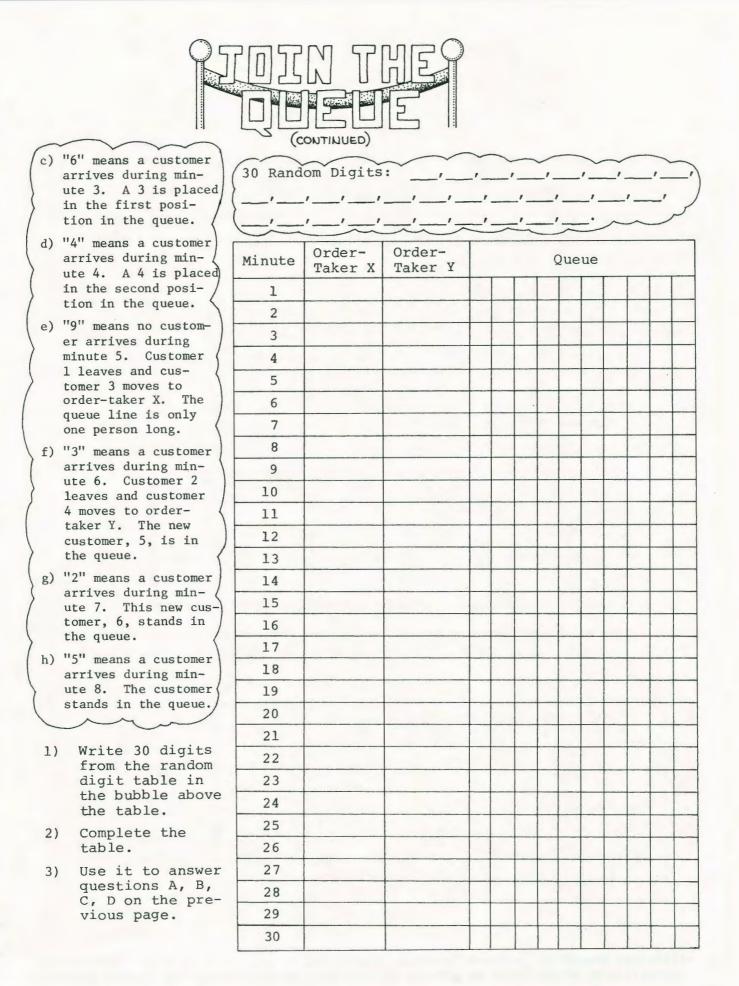


Think about these questions. During a 30-minute period:

- A) What is the average length of the queue (on the average, how many customers are waiting in line to place an order)?
- B) On the average, how many minutes does a customer spend standing in the queue and ordering?
- C) How many customer-free minutes are there for each order-taker?
- D) Do you think there are too many, just enough, or should the restaurant hire another order-taker?

To simulate the customer arrival for 30 minutes, use a random digit table to obtain 30 digits. Each digit tells if a customer arrives during the minute. Since the probability of arriving during a given minute is 0.7, use the digits 1, 2, 3, 4, 5, 6, 7 to mean a customer arrives. The digits 8, 9, 0 mean no customer arrives during the minute. Use the table on the following page to record the customer arrival for 30 minutes.

(An example is given below. For) the first eight minutes, the digits were: 2, 5, 6, 4, 9, 3,)	MINUTE	ORDER- TAKER X	ORDER- TAKER Y				(QUE	EUE	:	 _	
2, 5.	a) 1	1										
) 2	1	2									
	:) 3	1	2	З								
during minute 1. A 1 is placed under order-taker X.	1) 4	1	2	з	4							
	2) 5	3	2	4							T	
b) "5" means a customer arrives during minute 2. A 2 is) 6	З	4	5								
placed under order-taker Y. /g	g) 7	3	4	5	6							
the first customer takes	a) 8	З	4	5	6	7						
four minutes to order.												



ES N



How is a mortality table made?

Life insurance companies keep careful records of the lives and deaths of their policy holders. Data is collected on the ages of those being insured and the number that die during a year for each age group. From these facts, the number of deaths per 1000 people is found.

Finding number of deaths per 1000 people

Several insurance companies report this data: At the beginning of a year, 4280 thirteen-year-olds are insured. By the end of the year, 6 have died.

The probability of dying is $\frac{6}{4280}$. To find the deaths per 1000 people, solve this proportion: $\frac{6}{4280} = \frac{x}{1000}$. x = 1.40. Based on past data, the expected

number of deaths per 1000 thirteen-year olds is estimated at 1.40.

I. Use the infor	mation below	to find	the number	of deaths	per 1000	people for	each age.
------------------	--------------	---------	------------	-----------	----------	------------	-----------

AGE	NUMBER INSURED AT BEGINNING OF YEAR	NUMBER OF DEATHS	NUMBER OF DEATHS PER 1000 PERSONS
14	4500	7	G
15	5000	8	1.60
16	4750	8	1.68
17	5800	10	1.72

II. For an age group, the actual number of deaths each year varies. One insurance company found this information over a five year span for its thirteen-year-old policy holders.

			S
YEAR	NUMBER INSURED AT BEGINNING OF YEAR	NUMBER OF DEATHS	NUMBER OF DEATHS PER 1000 PERSONS
1971	5550	8	1.44
1972	4800	9	1.88
1973	6050	12	1.98
1974	5700	11	1.93
1975	5060	7	1.38

Insurance companies use many years of data to establish a reliable death rate for each age.

To obtain a reliable number for predicting future deaths for thirteen-year-olds, find the average of the numbers in the last column.

TYP Worksheet



Making the mortality table

Edmund Halley (1656-1742) published the first complete mortality table.

Once a reliable death rate per 1000 people is known for each age group the mortality table can be constructed.

From past records, an insurance company reports these death rates per 1000 people:

С

AGE	DEATH PER 1000 PEOPLE	
13	1.40	
14	1.47	
15	1.52	
16	1.62	
17	1.70	

The company wishes to construct a mortality table starting with 40,000 thirteen year olds. The steps are shown below the table.

A B

AGE	NUMBER LIVING	DEATHS DURING YEAR	DEATHS PER 1000 PEOPLE	
13	40,000	56	1.40	
14	39,944	59	1.47	
15	39,885	61	1.52	
16	39,824	65	1.62	
17	39,759	GB	1.72	

Students can use a calculator to complete the table.

To complete the table:

1) Use the death rate from column C to find the number of deaths for age 13.

$\frac{1.40}{1000}$	x	40,000	=	56
---------------------	---	--------	---	----

Round to the nearest whole number.

- 2) Write your answer in column B.
- 3) Subtract your answer from 40,000 to find the number living at age 14. Write the number in the table.
- 4) To find the deaths during the year for 14 year olds, multiply the number living by the death rate $(\frac{1.47}{1000})$. Round the answer to the nearest whole number and write it in column B.
- 5) To find the number living for age 15, subtract the number in column B from column A for age 14. Write the answer in column A.
- 6) Continue the calculations to complete the table.

Volume Three of The World of Mathematics by James E. Newman contains an account written by Edmund Halley of his development of the first life insurance tables.



When you purchase a life insurance policy, you agree to pay the company a fixed amount of money annually called the premium. Insurance companies use mortality tables to help them figure the amount of the premium for each policy holder.

Mr. Phillips wishes to start a life insurance company. He plans to insure 2000 thirteen-year olds for \$5000 each for one year. How much should he collect from each person in order to pay the death claims for those who may die during the year? (Mr. Phillips must pay \$5000 for each person who dies.)

The deaths per 1000 for thirteen-year olds in the mortality table is 1.32. Since there are 2000 to insure, the expected number of deaths is 2.64 or about 3. Mr. Phillips can expect to pay claims equal to 3 x \$5000 or \$15,000. (The actual number of deaths during the year could be greater than or less than 3. Over many years, the number of deaths per year for 2000 thirteen-year olds would average about 3.)



How much money should Mr. Phillips collect from each of the 2000 persons to pay the death claims of \$15,000?

$$15,000 \div 2000 = $7.50.$

(This amount pays only the death claims. A higher fee is needed to cover Mr. Phillips' operational costs.)

- Mr. Phillips plans to insure 4000 fifteen-year olds for \$3000 for one year. He wishes to collect enough money to pay the expected death claims for this group. How much should the premium be for each person?
- 2. The Stinnet Insurance Company has 7480 policy holders of age 21. The company insures each person for \$10,000 for one year. In order to pay the expected death claims for this group, how much should the premium be for each person?
- 3. You own an insurance company. 4250 thirty-year olds wish to be insured for \$15,000 for one year. In order to pay the expected death claims for this group, what premium should you charge each person?

ORGANIZATION

Placement Guide for Tabbed Dividers

Tab Label	Follows page:
CONTENT FOR TEACHERS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
Probability with Counting	
Inferential Statistics	
Appendix-Computer Programs	
DIDACTICS	
Components of Instruction	
Classroom Management	
Statistics and Probability Learning	
TEACHING EMPHASES	
Critical Thinking	
Decision Making	
Problem Solving	
Models and Simulations	
Calculators and Computers	
Laboratory Approaches	
CLASSROOM MATERIALS	
CONCEPTS AND SKILLS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Misleading Statistics	
Mean, Median, Mode	
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Sampling	
Experimental Probability	604
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Counting Techniques	
APPLICATIONS	
Business & Commerce	
Environment	
Health & Medicine	
People & Culture	
Recreation	
GLOSSARY	
ANNOTATED BIBLIOGRAPHY	
SELECTED ANSWERS	



ENVIRONMENT

TITLE	PAGE	TOPIC	TYPE
Get a Horse?	747	Using a table to compare fuel efficiency	Worksheet
Air Pollution	748	Reading a table Making a graph on air pollution	Teacher idea
City Circumstances	749	Reading a table on air pollution	Worksheet
Particular Pollutants	750	Reading a table and making a graph of sources of air pollution	Worksheet
Who Needs a Bike Path?	751	Gathering data on bicycle and car usage	Activity card
Counting Corners	752	Analyzing data on traffic at intersections	Teacher idea
Pull for Pooling	754	Gathering data on the number of riders in cars	Activity card
To Spray or Not to Spray	755	Reading a table on pesticide usage	Transparency
Water Wastes	756	Reading a table on water pollution	Worksheet
Decay and Half-Life	757	Using dice to simulate radioactive decay	Worksheet
Rocky Mountain High	760	Making a stem-and-leaf display of each state's highest point	Worksheet
Continental Facts	761	Graphing information about continents	Worksheet
Highs and Lows	763	Using a graph to compare continental highs and lows	Teacher demonstration Bulletin board
Let's Go Fishing	764	Using sampling to estimate the makeup of a population	Worksheet Activity card

TITLE	PAGE	TOPIC	TYPE
A Chilling Experience	765	Collecting and graphing temperature readings	Activity card Worksheet
Is 32 Degrees Hot or Cold?	766.	Using a graph to compare degrees Celsius to degrees Fahrenheit	Worksheet
Balloon Barometer	767	Collecting and graphing data from a homemade barometer	Activity card
A Swinging Time	768	Making and using a graph to predict the period of a pendulum	Activity card Worksheet
Hang Ten	770	Collecting and graphing data about elasticity	Activity card Worksheet

_



It is more efficient on a trip for a car to have 4 people in it than just 1. That way more people are moved many miles for about the same amount of gasoline. When two people go 10 miles each, you can say that they have traveled 20 people-miles. The number of people-miles per gallon gives one way to compare different kinds of vehicles.

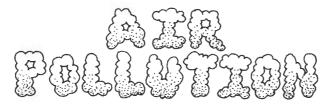
Vehicle	Average number of miles per gallon	People in vehicle	People-miles per gallon
Car	13.3	. 1	13.3
		1.3*	17.3
		2	26.6
		5	66.5
Bus	5.9	5	29.5
		50	295.0
Jet aircraft	0.24	73	17.5
Electric train ^a	1.88	600	1130 (level ground)

*Estimated average

^aElectricity used is expressed in terms of fuel.

Data updated from *Principles of Environmental Science* by Kenneth Watt. Copyright 1973, McGraw-Hill Book Company. Used with permission of McGraw-Hill Book Company.

- According to the table, what type of vehicle gives the most people-miles per gallon? _____ The least? _____
- 2) Since jets do not give a very high people-miles per gallon figure, why do people going on long trips like to fly?
- 3) Since cars have a low people-miles per gallon figure, why do so many people drive instead of taking a bus?
- 4) What does the 1.3 in the "people in vehicle" column mean?
- 5) How was the last column figured out?
- 6) Some cars get 35 miles per gallon. With 2 people, what would the people-miles per gallon be for such a car?
- 7) Suppose a jet had 150 people (including the crew) in it and got the same number of miles per gallon as in the table. How many people-miles per gallon would that give? ______ Would this be more efficient than 3 buses?
- 8) Besides fuel efficiency, why would one bus with 50 people be better than 10 cars, each with 5 people?



This is an example of a table showing air pollution data. Local information is available from the nearest air pollution authority. You can find a number in the governmental section of the telephone book. Ask for data in table or graph form relating to air pollution in your area.

SUMMARY

SUSPENDED PARTICULATE

1972

0 - 60 Light

Pollution Index

61 - 100 Moderate 101 - 260 Heavy 261 - Very Heavy

City Station Location	Station Location	Field Station Number	Suspended Particulate $(\mu g/m^3)$ * Monthly Average								Yearly Average				
		Jan	Feb	Mar	Apr	May	June	July	Aug	Sep	0ct	Nov	Dec		
Eugene	City Hall	20-18-32	88	108	72	78	103	92	164	135	159	107	64	96	106
	Commerce Bldg.	20-18-35	-	79	58	77	84	91	75	97	92	98	50	88	81
Springfield City Library	City Library	20-33-37	114	138	92	89	93	59	88	107	109	142	75	95	100
	City Shops	20-33-35	80	99	88	75	111	119	122	138	114	131	80	92	104
Junction City	City Offices	20-24-04	68	75	58	49	68	60	68	75	91	84	54	44	66
Cottage Crove	City Hall	20-09-01	-	-	47	39	46	38	45	51	47	59	45	53	47
Oakridge	Fire Station	20-30-01	-	-	85	56	89	78	79	105	83	106	73	69	82
Rural	LRAPA, Airport	20-00-33	25	36	25	23	38	48	42	50	53	72	30	32	40

*micrograms per cubic metre

- 1) Students can select a color for each category in the pollution index and color the monthly averages in the table.
- 2) Students can answer questions based on the table. For example,
 - a) What is the highest monthly average in the table?
 - b) For Oakridge, how many months show moderate pollution?
 - c) Which field station reported the most months with heavy pollution?
 - d) Which month shows more readings in the light category?
- 3) Students can compute the yearly averages for each field station. An average of the readings for each month can also be figured.
- 4) For each field station, students can make a line graph of the monthly readings to check for trends. By comparing graphs, students can identify the stations with the most (least) variation in readings. Similiarities or differences in trends for the stations can be investigated.

CITY CIRCUMSTANCES

STATION		Min. Max.		Geom. Mean	STATION	Min.	Max.	Geom. Mean
Arizona	Phoenix	60	283	144	New			
California	Oakland	21	100	57	Hampshire Concord	20	93	38
	San Diego. San	26	140	59	New York Rochester.	39	177	88
	Francisco	29	121	60	North Dakota. Bismarck	24	213	87
Colorado	Denver	38	355	127	Ohio Cincinnati	38	207	87
Connecticut.	Hartford	25	137	60	Columbus	28	135	78
	New Haven.	23	131	60	Oklahoma Oklahoma			
Florida	Miami	32	132	62	City	28	194	67
Georgia	Atlanta	39	203	82	Oregon Portland	29	255	86
lawaii		25	70	40	Pennsylvania. Pittsburgh	69	259	135
[daho	Boise	20	254	69	Rhode Island. Providence	38	129	69
Illinois	Chicago	39	336	97	South			
Lowa	Des Moines	34	202	85	Carolina Columbia	23	157	63
Kansas	Kansas				Tennessee Memphis	42	395	92
	City	29	227	116	Nashville.	54	249	106
Louisiana	New				Texas Dallas	29	349	86
	Orleans	43	119	73	Houston	43	177	89
Maryland		44	166	90	Utah Salt Lake			
fichigan	Detroit	41	236	111	City	51	211	95
Minnesota	Minnea-				Virginia Norfolk	9	116	55
Missouri	polis St. Louis	31	130	64	Washington Seattle West	24	109	47
	$(CAMP)^1$	32	393	120	Virginia Charleston	33	181	98
Nebraska	Omaha	43	331	113	Wisconsin Milwaukee.	41	197	92
Nevada	Reno	38	218	93	Wyoming Cheyenne	7	148	30

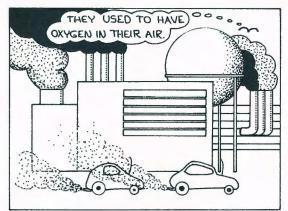
SOURCE: U.S. Environmental Protection Agency, Monitoring and Air Quality Trends Report, 1972.

Table from Statistical Abstract of the United States 1974

(Suspended Particulate Matter: Particles of smoke, dust, fumes and droplets in the air.)

- Using a standard of 75 microgram/m³ (annual geometric mean), what cities had too much suspended particulate matter?
- 2) Which city had the highest geometric mean? the lowest?
- 3) What city had the highest maximum amount of suspended particulate matter? ____
- 4) These are selected cities, not the only cities with air problems. How many states are not represented? ______
- 5) What are some cities with obvious air pollution problems that are not mentioned here?
- 6) Suspended particulate matter does not include carbon monoxide, ozone, sulfur oxides or nitrogen oxides. What information do you need before you decide whether the cities in the table have polluted air? ______

PARTICULAR POLLUTANTS



SOURCES OF U.S. AIR POLLUTION (1974, est.)

(millions of metric tonnes)

Kind of Pollutant

Source	Carbon monoxide	Sulfur oxides	Hydro- carbons	Nitrogen oxides	Particu- lates	Total for source
Transportation	73.5	.8	12.8	10.7	1.3	
Heating	.6	5.6	1.6	4.0	2.6	
Power plants	.3	18.7	0.1	7.0	3.3	10.4
Industry	12.7	6.2	3.1	0.6	11.0	15.8
Burning waste	2.4	0	0.6	0.1	0.5	1.1
Uncontrolled and miscellaneous	5.1	0.1	12.2	0.1	0.8	10.
Totals			19-4	55.0		109.4

Data from Statistical Abstract of the United States 1975

1) Find the totals for each source and for each kind of pollutant.

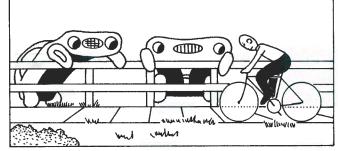
2) What was the grand total of air pollution in 1974?

3) About what percent of the total pollution does transportation put in the air?

4) Make a circle graph for either the sources (use the totals column) or the kinds of pollutants (use the bottom row). (Round the grand total to 200 million and each of the other totals to the nearest million.)



Do a lot of cars and bikes travel near your school each day? Many cities are establishing bike paths so that both types of vehicles won't have to travel in the same lanes.



COCATION . 1200 ALDER rime:

BIKES

3 44++ +4++

Find several locations to take bike and car counts, and collect the following information CARSY for a period of one hour.

				Approximate bike
Street	Time	Bikes/hour	Cars/hour	to car ratio
13 [™] € ALDER	8-9 AM	64	252	1:4
				:
				:
	·			

Do you think a separate bike path is needed? If so, who should have the information so that a path could be established?

Do you think the number of bicycles would increase if a path were established?

Why should bicycles be encouraged?

Compare the motor vehicle registration and bicycle registration for your city. Look at each for the last 5 years to see if there are any general patterns.

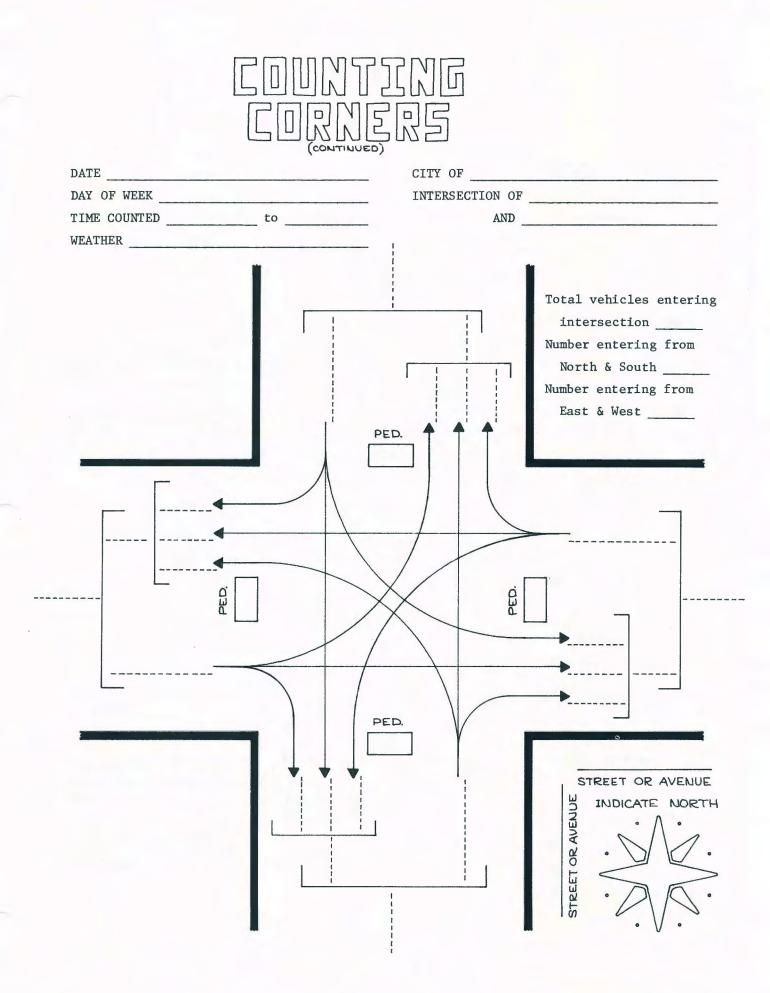


The traffic divisions of cities gather data on the amount and type of traffic at busy intersections. Sometimes a cable is stretched over one lane of a street. Every 15 seconds a mechanical device records on a paper tape the number of cars crossing the cable. Often the traffic at an intersection must be counted by observation. The person counting might use a large board with buttons to tally the count. The data is stored in a computer and it is used to help make decisions about traffic signs and lights, turn lanes, bike lanes, widening streets or creating one-way streets.

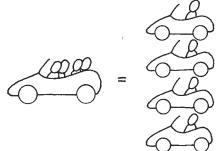


To show what is happening at a particular intersection the data is recorded on a diagram as shown on the next page. Here are some ideas for using this diagram:

- I. Have students take a count at a busy intersection near school and record it on the diagram. You could form 8 teams, (a) through (h), and have each team do the counts as labeled in blue on the next page. You might want to simplify the diagram and have students count only the traffic entering and leaving on each street. You might want them to count bike traffic separately. A discussion of the data could include questions like, "Should there be a different type of traffic control--lights or signs? Should there be a bike lane, left turn lane or a better pedestrian crossing?"
- II. Show the diagram on an overhead. Add hypothetical data like that shown in blue on the diagram. Ask questions like those in I above.
- III. Show the diagram and give only the numbers at each of the arrowheads (16 numbers). Have students figure out the numbers for the rest of the blanks.
- IV. Give students the 4 outside numbers and have them figure out possible numbers for the rest of the blanks. (Traffic engineers sometimes have to do this. If they have the data from the surrounding intersections, they can figure out a logical flow of traffic for an intersection without counting.)
- V. Have someone from the traffic division in your city visit your class and describe how they collect and use data on traffic.







Make plans to take a count of cars traveling certain routes, at certain hours, on various days in your community.

Suggestions:

Go to the location you have chosen. Count the cars traveling in one direction and the number of passengers in each car. Do this for $\frac{1}{2}$ hour during a morning rush hour, $\frac{1}{2}$ hour during an evening rush hour, and $\frac{1}{2}$ hour during a midday hour.

Determine how many days you will do this and vary your days so that some are weekdays and some on weekends.

When you arrive at what you feel is a fair sampling determine how many fewer cars would have been needed if each car would have carried 3 passengers.

Determine what per cent of the cars carried only 1 person; 2 persons; 3 persons; more than 3 persons.

What conclusions can you form as an individual or as a group carrying out this project? Can you use these conclusions to make some recommendations to your own family (families)? To the staff of your school? To the members of your community? To your traffic department?

SOURCE: *Pollution: Problems, Projects and Mathematics Exercises*, 6-9, Wisconsin Department of Public Instruction Permission to use granted by Wisconsin Department of Public Instruction

TO SPRRY OR NOT TO SPRRY

PESTICIDE USAGE AND AGRICULTURAL USAGE

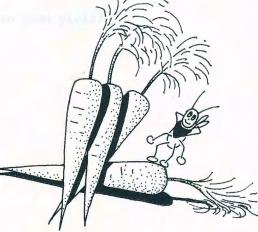
	Pesticide	e Use	Yield	
Area or Nation	Grams per hectare	Rank	Kilograms per hectare	Rank
Japan	10,790	1	5,480	1
Europe	1,870	2	3,430	2
United States	1,490	3	2,600	3
Latin America	220	4	1,970	4
Australia and Pacific Islands	198	5	1,570	5
India	149	6	820	7
Africa	127	7	1,210	6

Table from Patterns and Perspectives in Environmental Science, National Science Board. A hectare is 10,000 square metres.

- And produces building and and produces a first hits and the second second the second second second second second
- many class in hand particult on Thefenitical Vivid pre Sacting as sicked in many har
- 3) Dress said practities diverse better adricultural chains (Generality citche lenses) of posticide usees increases, develoc, as saris conclude from the labor of actil higher field will be estimated by a higher usersed

4] One: after Another the part of particular in related to part y BELITHER, while supply, while Lyps, machinelexibles of appropriate group of provide bland, size 1

Permission to use table granted by National Science Board





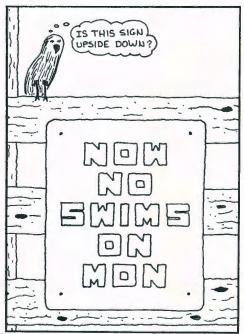
WATER POLLUTION OF MAJOR DRAINAGE AREAS (REGIONS DRAINED BY INTERRELATED STREAMS)

	Total length of streams in	Pollut	ed km	1971 duration-		
Name of area	the area (km)	1970	1971	intensity factor*		
Ohio	46,400	15,800	38,500	.42		
Southeast	18,800	5,000	7,200	.74		
Great Lakes	34,200	10,500	14,000	.45		
Northeast	51,900	19,000	9,300	.61		
Middle Atlantic	51,100	7,400	9,000	.47		
California	45,200	8,600	13,500	.27		
Gulf of Mexico	103,600	26,600	18,600	.35		
Missouri	16,700	6,800	2,900	.31		
Columbia	48,700	11,900	9,100	.12		
United States .	416,600	111,600	122,100	.41		

*This indicates how badly the stream was polluted and for how long during the year. The greater the decimal, the worse the pollution.

Table adapted from Statistical Abstract of the United States 1975

- 1) Which area within the U.S. has the greatest number of kilometres of stream length?
- 2) Which areas had fewer polluted kilometres in 1971 than in 1970?
- 3) Which area had the highest duration-intensity factor in 1971? The lowest?
- 4) For each area, find the percent of the total stream length polluted in 1971. Arrange the areas in order from greatest to least percent.
- 5) What are some streams in your drainage area? Check with a pollution agency to see if the streams are polluted.





Materials Needed:

Graph paper, set of 20 cubes with one face of each cube marked with a ${\rm D}$



Each die in the set represents a radioactive particle. If the side marked D turns up, a particle has decayed. This die is removed from the pile before the next roll.

AD

I Begin by rolling all 20 particles. Remove any particles that decay (the side marked D turns up). Record your results in the table to the right.

> Continue rolling until all active particles have decayed. You may need to extend the table.

II Do you think the results would be exactly the same if you rolled the 20 radioactive particles again? _____

	ROLL	PARTICLES	PARTICLES REMAINING
\sim	START	0	20
AFTER THE	1		
ONLY ROLL THOSE	2		
PARTICLES THAT	3		
(RADIOACTIVE.)	4		
	5		
	6		
	7		
	8		
	9		
	tim	han	turn

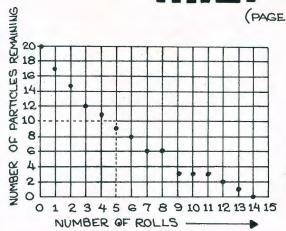
I

III Make at least five more trials. Each time start with 20 radioactive particles. As in part I, roll until all radioactive particles have decayed. Make tables like those below and record your results.

		ON GILG I	Lecord J.		1.00.						
4	Providence -	1 / /.:	DECONCLES	Renau ES	PARTICLES	PARTICLES REDUCILES	PARTICLE'S	Remarkes	PRETICIES	PERMALLES	Snin
1	1	[]				<u></u>				(
2											
3											
4								-			
5											
6											
7											
8											
2	Lan	L	Im	Lon	Im	In	han	hand	han	Land	

IDEA FROM: Laboratory Activities for Teachers of Secondary Mathematics by G. Kulm Permission to use granted by Prindle, Weber and Schmidt, Inc

DECRY AND HALF-LIFE (PAGE 2)



Graph each of your six trials on the grid below. The graph of a trial should look something like the one to the left. Draw a smooth curve which best fits your points.



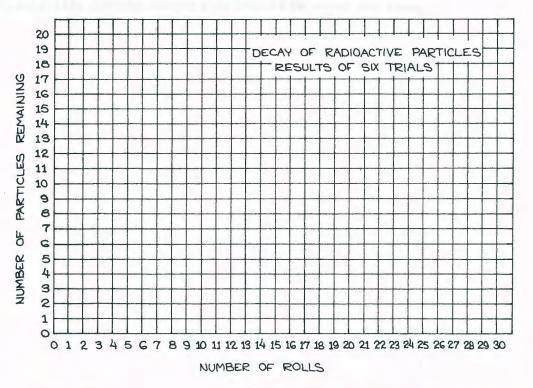
From the graph above I can see it took about 5 rolls before half the particles decayed. So the half-life of the cubes is 5 rolls.

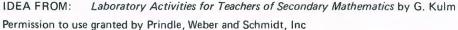
V The half-life of a radioactive substance is the time it takes for half of the particles to decay.

Use your graph from part VI below. On the average, the half-life of a sample of 20 radioactive particles would be _____ rolls.

VI What would happen to the half-life if your starting pile was twice as large? Make a prediction for the half-life. _____ rolls

Perform several trials to check your prediction.

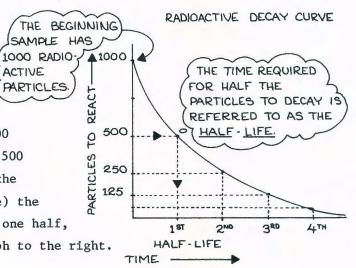






Nuclear reactors are becoming more common because of the energy crises. A byproduct of the reactors is radioactive chemical waste which must be safely stored. This is causing some problems because of the long half-lives of some of the radioactive isotopes.

The half-life of a radioactive substance is the time it takes for half of the substance to lose its radioactivity, that is, to decay. For example, let's suppose, there are 1000 ratioactive particles. In one half-life 500 particles (half of the 1000) decay. In the second half-life (the same length of time) the remaining particles are again reduced by one half, that is half of 500 or 250. See the graph to the right.



Decay for any particular particle is completely random and can't be measured, but the concept of half-life allows scientists to quite accurately predict the decay of an entire sample.

The half-lives of nuclear particles wary from a tiny fraction of a second to more than a billion years. For strontium-90 and cesium-137, waste products in nuclear reactors, an isolation time of 600 years is required before it is safe to release them into the environment.



CARBON-14 DATING:

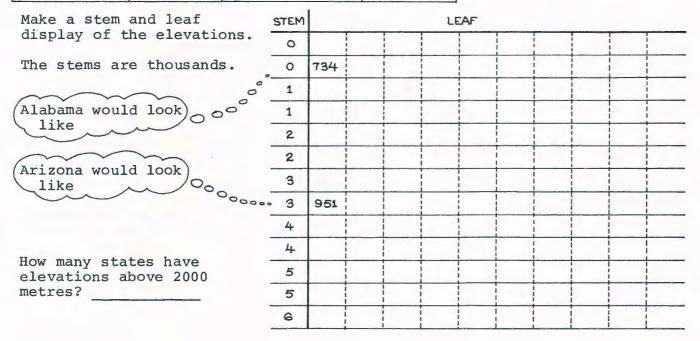
All living material has a certain percent of carbon in its chemical makeup. Most of this is stable carbon-12, but there is a small amount of carbon-14 which has a half-life of slightly more than 5000 years. Scientists can analyze artifacts and determine the percent of carbon-14 left to establish its age. This method is effective for organic material 1,000 to 50,000 years old.

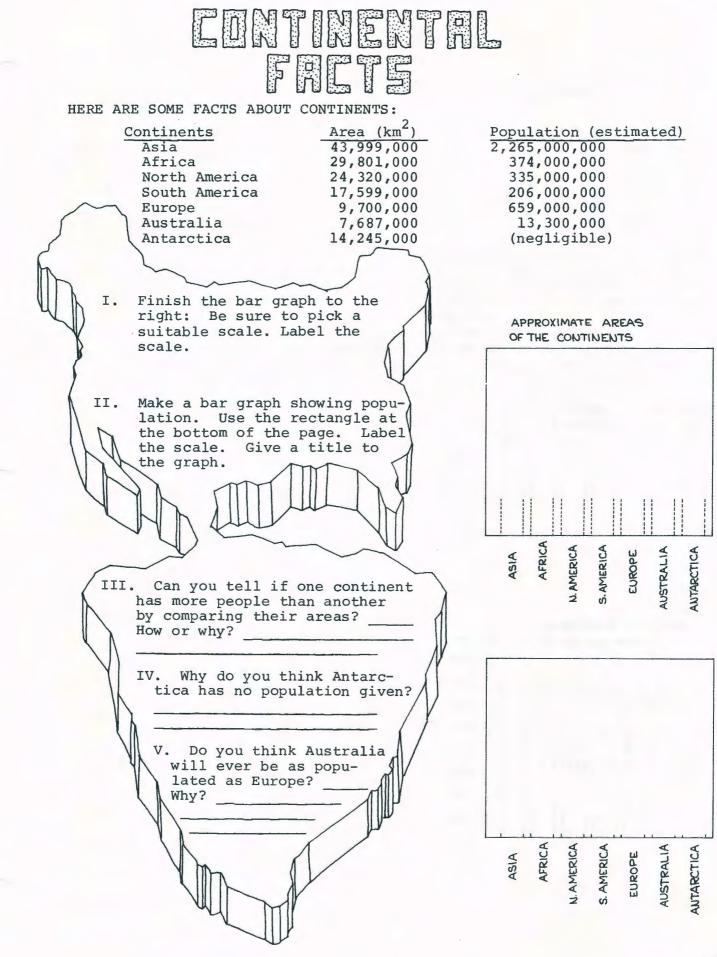
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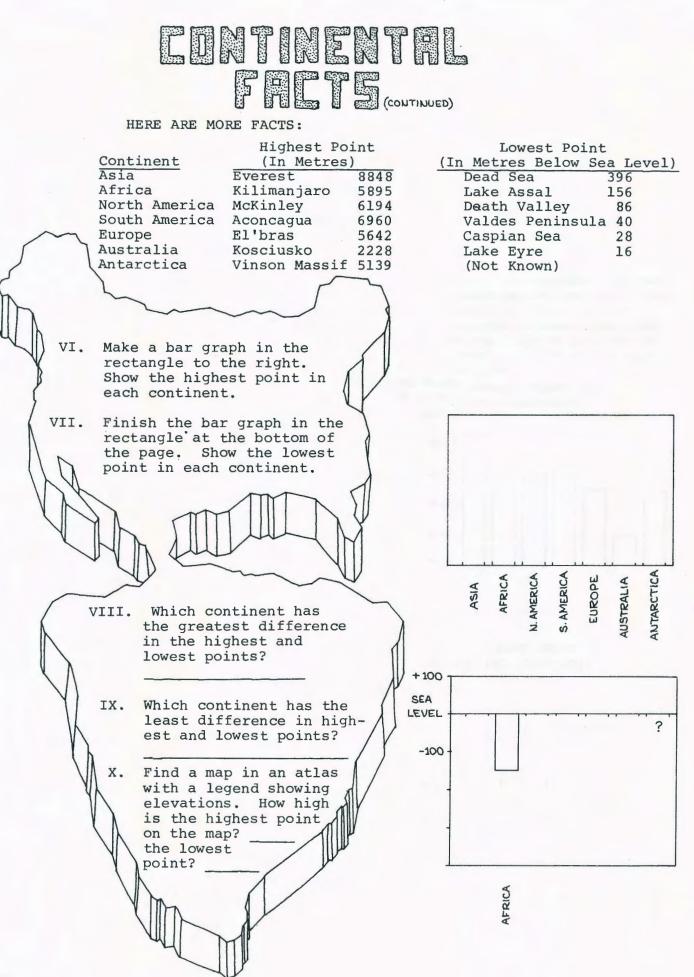


Peak-Scaler Pike collected the elevations of the highest points in each of the states.

State	Elevation (metres)	State	Elevation (metres)	State	Elevation (metres)
Alabama	734	Louisiana	163	Ohio	472
Alaska	6194	Maine	1656	Oklahoma	1516
Arizona	3951	Maryland	1024	Oregon	3424
Arkansas	839	Massachusetts	1064	Pennsylvania	979
California	4418	Michigan	604	Rhode Island	247
Colorado	4399	Minnesota	701	South Carolina	1085
Connecticut	725	Mississippi	246	South Dakota	220.7
Delaware	135	Missouri	540	Tennessee	2025
Elorida	105	Montana	3901	Texas	2667
Georgia	1458	Nebraska	1654	Utah	4123
Hawaii	4205	Nevada	4005	Vermont	1339
Idaho	3859	New Hampshire	1917	Virginia	1746
Illinois	376	New Jersey	550	Washington	4392
Indiana	383	New Mexico	4011	West Virginia	1482
Iowa	509	New York	1629	Wisconsin	595
Kansas	1231	North Carolina	2037	Wyoming	4207
Kentucky	1263	North Dakota	1069		







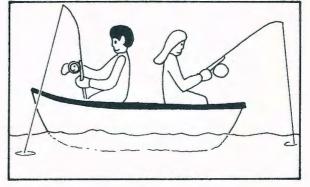
HIGHS AND LOWS

HIGHEST AND LOWEST POINTS IN EACH CONTINENT 9000 The second page of Continental Facts has students graph the highest and lowest points on each continent. Because the two sets of data have such different 8000 ranges, students probably picked two very different scales. A better comparison of the highs and lows can be made if they 7000 are both placed on one graph with one scale. You could prepare this graph and 6000 use it as a contrast to the two graphs on Continental Facts or making the graph could be a student project. 5000 Mark a 1 metre strip of poster board in centimetres. • Mark "SEA LEVEL" ten centimetres 4000 from the bottom. • Label the scale using 1 cm : 100 m. (The graph to the right has a scale 3000 of 1 cm : 500 m so it will fit the page.) • Complete the bar graph. Strips of 2000 . tape or paper could be used for the bars. Have students compare this graph to 1000 those on Continental Facts. 500 Have students answer questions about the graph: which has a greater SEA LEVEL range, the highs or lows? ... ? - 500 ANTARCTICA AMERICA AUSTRALIA EUROPE AMERICA AFRICA ASIA



Goal: To estimate the percent of each type in a mixture.

Needed: Bag with 3 kinds of items



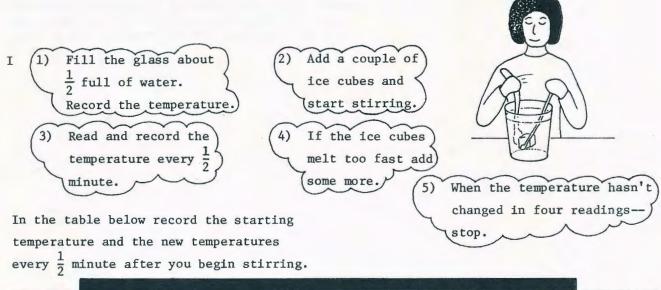
Sometimes wildlife specialists want to know what percent of the fish in a body of water are game fish like trout, bass or salmon. Here is one way to estimate the percents.

- I Mix up the contents of the bag.
- II Without looking, reach in the bag and grab a handful. Count the number of each type. Record in a table like the one below. Put the handful back in the bag.
- TYPE III In the same way, repeat steps I B C and II to get two more samples. SAMPLE 1 SAMPLE 2 GRAND IV Find the totals for each type. TOTAL SAMPLE 3 Add these totals together to find the grand total of the TOTALS samples. % OF GRAND TOTAL V Each type is what percent of the grand total? Record your answers in the table above. VI Count the items in the bag and B C TOTAL record in the table to the right. A Each type is what percent of the NUMBER total? How close were your answers in V? PERCENT

Many lakes have report forms people can use to list the different types of fish they have caught. By combining their results like you did in steps IV and V, wildlife specialists can estimate the percent of each type of fish in the lake.

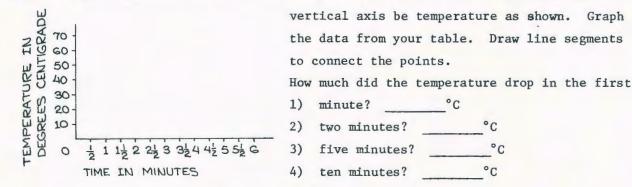


Materials Needed: Ice cubes, warm water, Celsius thermometer, stirrer, watch and a glass or cup

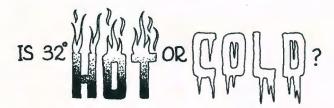


LENGTH OF TIME STIRRED IN MINUTES	START	12	1	112	2	21/2	3	312	4	412	5	$5\frac{1}{2}$	5
TEMPERATURE IN DEGREES CELSIUS													3

II Make a large graph like the one below. Let the horizontal axis be time and the



III You be the scientist. Can you explain why the water and ice cube mixture didn't get colder than 0°C?



Materials Needed: One Celsius thermometer, one Fahrenheit thermometer, ice cubes, hot water and salt

> HOW CAN IT BE SO HOT, I THOUGHT WATER

FROZE AT 32° ?

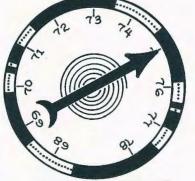
Measure and record the temperature of each item with the Celsius and Fahrenheit thermometers.

Crasseou REFRICE OUTSIDE Pers LXIL. 2CE CELSIUS FAHRENHEIT

Plot these pairs of temperatures on the grid below. Place the Celsius readings on the horizontal axis and the Fahrenheit readings on the vertical. Draw a line of best fit for the points.

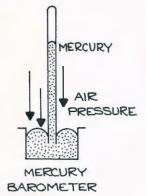
Usi	ng the graph det	ermine	2	-2												-
the	temperatures in	Celsi	us.	-16	30-											
A)	Normal body tem	peratu	ire		+	+										+
	°C			15	50											
B)	A warm day	°C		117	20-											-
C)	A cold day	°C		FAHRENHE	0											
D)	Water freezes _		°C	1 1	-0											
					30											
		-30	-20	-10	0	10	20	30	40	50	60	70	80	90	100	+
					30				CEL	STUS	5					
					60											





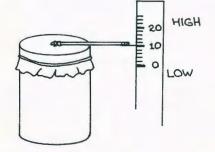
Instruments used to measure air pressure are called barometers.

Meteorologists use barometers to detect changes in the atmospheric pressure. This helps them forecast the weather.



ANEROID BAROMETER

Here is a method of making a barometer using an empty large wide mouth jar (a coffee can also works), a large round balloon, a strong rubber band, a soda straw and a needle.



I Cut a balloon so it may be stretched over the mouth of the jar. Secure the balloon to the jar with the rubber band. It is important that the balloon be as tight as possible. To one end of the straw attach the needle for a pointer. Tape the other end to the middle of the balloon.

NOON

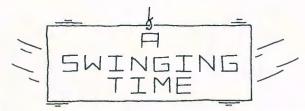
TIME

When finished, set the barometer and scale in a protected area. It works best when kept away from drafts and large temperature changes.

II Make a millimetre scale as shown. Support it so the needle points at 10.

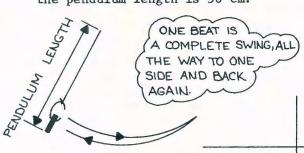
	TIME : MARCH 7	8:00 A.M.	9:00	10:	III	Make a daily recording table similar to the one shown to the left and record the barometer				
	READING	11	12	12		reading every hour. Record any				
	WEATHER CONDITION	CLOUDY	CLEARIN	IG }		changes in the weather condition.				
IV	Make a line g the one at th day you recor meter. Indic changes on th appropriate t	e right for d with the ate any wea e graph at	r each baro- ather	READING ON BAROMETE		ERRING 9 AM. 10 AM 11A.M. 12 1.P.M. 2.P.M. 3.P.M.				

IDEA FROM: Build-It-Yourself Science Laboratory by R. Barrett Permission to use granted by Doubleday and Company

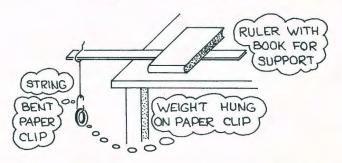


Materials needed: Metre stick, about a metre of string, paper clip, identical weights (fishing sinkers or metal washers), watch with a second hand.

I Set up a pendulum as shown to the right. Tie the string so the pendulum length is 50 cm.



A simple pendulum can be made by hanging weights on a string.



Pull the weight back about 10 cm and release it. Measure the amount of time for 10 beats. Make three separate trials and enter the results below.

		Length of Pendulum	Time for 10 beats
Trial	1	50 cm	
Trial	2	50 cm	
Trial	3	50 cm	

AVERAGE TIME FOR THE THREE TRIALS

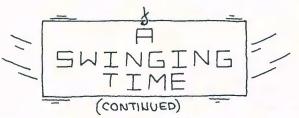
DONÍT HANG THE WEIGHTS LIKE THIS: DONÍT HANG THE WEIGHTS LIKE THIS: DOUBLE THE NUMBER OF WEIGHTS HUNG ON THE PENDULUM O O O O O TTI

What effect do you think additional weights will have on the time required for ten beats?

Try it.

	Length of Pendulum	Time for 10 beats	Time for 1 beat	
ial 4	50 cm			•

Predict the time required for 10 beats if the number of weights were tripled. Check to see if you are correct.



III Use the single weight as in trial 1. For each trial make the pendulum length the same as shown in the table below. What effect do you think this will have on the time for 10 beats?

Try it.

Record your results in the table below.

		Pendulum Length	Time for 10 Beats	Time for 1 Beat
Trial	5	40 cm		
Trial	6	30 cm		
Trial	7	20 cm		
Trial	8	60 cm		
Trial	9	70 cm		
Trial	10	Your own choice:		

40

30

50

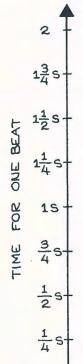
LENGTH OF PENDULUM IN CM

60

70

80

IV Make a graph of trials 5 through 10.



10

20

CHANGE THE LENGTH OF THE PENDULUM A LITTLE

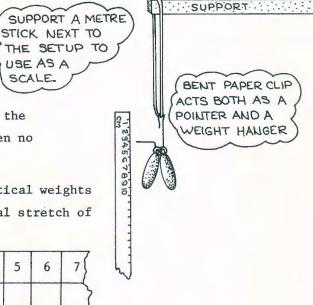
A seconds pendulum has one beat in exactly one second. Using your graph, what would be the length of a seconds pendulum? _____ cm



Materials Needed: Two identical rubber bands, about 20 identical weights (such as fishing sinkers, machine washers or nuts), metre stick, paper clip and graph paper

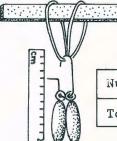
Assemble the materials as shown. Adjust the Ι metre stick so the pointer is at zero when no weights are attached.

After the setup is complete add the identical weights one at a time. Each time record the total stretch of the rubber bands.



Number of weights	0	1	2	3	4	5	6	7
Total stretch (cm)	0							

II Hang two identical rubber bands over the support with the paper clip suspended



from both of them as shown. Adjust the metre stick so the pointer is at zero. As in part I, add the weights one by one. Record the total stretch of the rubber bands.

Number of weights	0	1	2	3	4	5	6	7	8	9}
Total stretch (cm)										\mathbb{Z}

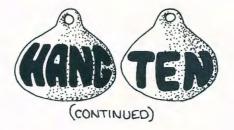
USE AS A

SCALE.

III Change the two rubber bands so they are tied together, hanging in tandem. Adjust the metre stick so the pointer is at zero. Will this setup produce a different stretch than both parts I & II?

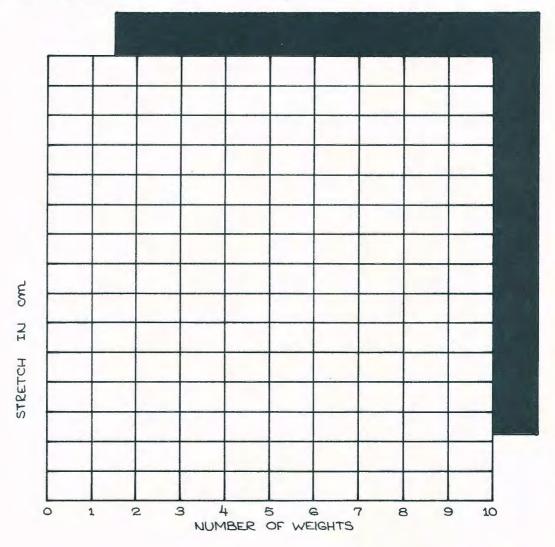
Add weights one by one to find out.

Number of weights	0	1	2	3	4	5	6	7	8	9	10
Total stretch (cm)											



IV Graph the results.

- 1) Choose and label the vertical scale.
- 2) Plot the results of part I. Connect the points.
- Use the same axes to plot the results of part II. Connect the points.
- Plot the results of part III on the same axes. Connect the points.
- 5) Do you see any similarity between the three graphs?



ORGANIZATION

Placement Guide for Tabbed Dividers

Tab Label	Follows page:
CONTENT FOR TEACHERS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
Probability with Counting	
Inferential Statistics	
Appendix-Computer Programs	
DIDACTICS	
Components of Instruction	
Classroom Management	
Statistics and Probability Learning	
TEACHING EMPHASES	
Critical Thinking	
Decision Making	
Problem Solving	
Models and Simulations	
Calculators and Computers	
Laboratory Approaches	
CLASSROOM MATERIALS	
CONCEPTS AND SKILLS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Misleading Statistics	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
APPLICATIONS	
Business & Commerce	
Environment	
Health & Medicine	
People & Culture	
Recreation	
GLOSSARY	
ANNOTATED BIBLIOGRAPHY	
SELECTED ANSWERS	

CONTENTS

HEALTH & MEDICINE

TITLE	PAGE	TOPIC	TYPE
What's Your Type?	774	Reading a table of body types	Worksheet
Stretch Smith	775	Using graphs to study growth rates	Worksheet Transparency
Graphs of Growth	777	Using graphs to understand a person's growth rate	Transparency Bulletin board
To Each His Own	779	Making graphs to compare body measurements	Activity card Worksheet
Are You Physically Fit?	780	Collecting data to deter- mine physical fitness	Teacher idea
Your Internal Combustion Machine	784	Completing a table to show calorie use	Worksheet
Fifty Ways to Love Your Liver	785	Examining a deceptive bar graph of nutrients	Worksheet Transparency
You Are What You Eat	786	Using rats to observe the effects of diet on growth	Teacher idea
A Chilling Factor	787	Studying a table of the effects of wind and temperature	Worksheet
Football Injuries	788	Reading a table about football injuries	Worksheet
Up in Smoke	789	Reading a bar graph on the hazards of smoking	Worksheet
Pulse and Temperature	790	Reading temperature and pulse from a hospital chart	Worksheet
Muscle Fatigue	791	Using a line graph to show muscle fatigue	Teacher idea
A Blood Relationship	792	Using a circle graph to show distribution of blood types	Worksheet

WURTS YOUR TYPE ?

- 1) Use a metric tape measure to find your height in centimetres.
- 2) Use a metric scale to find your weight in kilograms. kg
- 3) From the chart, what is your body type?

NOTE: HEAVY IS NOT THE SAME AS FAT. LIGHT IS NOT THE SAME AS SKINNY. A 170 cm, 110 kg FOOTBALL PLAYER WOULD BE TALL, HEAVY BUT HE CERTAINLY WOULD NOT BE FAT. A 142 cm, 40 kg GYMNAST WOULD BE SHORT, LIGHT BUT SHE WOULD NOT BE SKINNY.

N	Anna in 1	(ile steame								
- Mass in kilograms -Height in centimetres				GROWTH CHART FOR GIRLS						
		10 Yrs.	11 Yrs.	12 Yrs.	13 Yrs	14 Yrs.	15 Yrs.			
	Tall	143-155	153-163	157-168	162-170	162-173	164-173			
KA	verage	134-142	140-152	147-156	152-161	154-161	156-163			
	Short	125-133	130-139	135-146	140-151	146-153	147-155			
H	leavy	40-52	45-59	49-63	55-68	57-71	60-72			
4 A	verage	29-39	33-44	36-48	41-54	45-56	47-59			
1	ight	23-28	25-32	28-35	31-40	36-44	39-46			
- M	ass in k	kilograms entimetres]	GROWTH	CHART FO	OR BOYS				
- M	ass in k	kilograms]		CHART FO	DR BOYS	15 Yrs.			
- He	ass in k	kilograms entimetres		GROWTH		[15 Yrs. 169-185			
	ass in k ight in Ce	kilograms entimetres 10 Yrs.	11 Yrs.	GROWTH 12 Yrs.	13 Yrs.	14 Yrs.				
	ass in k ight in ce Tall	kilograms entimetres 10 Yrs. 149-155	11 Yrs. 149-163	GROWTH 12 Yrs. 157–168	13 Yrs. 162-178	14 Yrs. 169-183	169-185			
	ass in k ight in ce Tall verage	(ilograms entimetres 10 Yrs. 149-155 134-148	11 Yrs. 149-163 139-148	GROWTH 12 Yrs. 157-168 142-156	13 Yrs. 162-178 149-161	14 Yrs. 169-183 154-168	169-185 159-168			

29-37

34-44

31-38

40-48

4) Sally is 15 years old, is 170 centimetres tall, and weighs 58 kilograms. What is her body type? _____

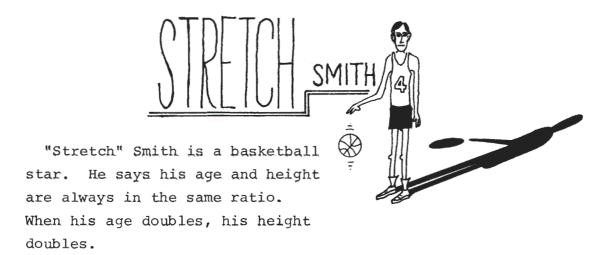
- 5) John is ll years old, is 135 centimetres tall, and weighs 30 kilograms. What is his body type?
- 6) Which body type would be best for these sports?

27-32

23-29

iaht

- a) Gymnastics b) Basketball c) Archery
- d) Horse racing e) Wrestling f) Shot putting

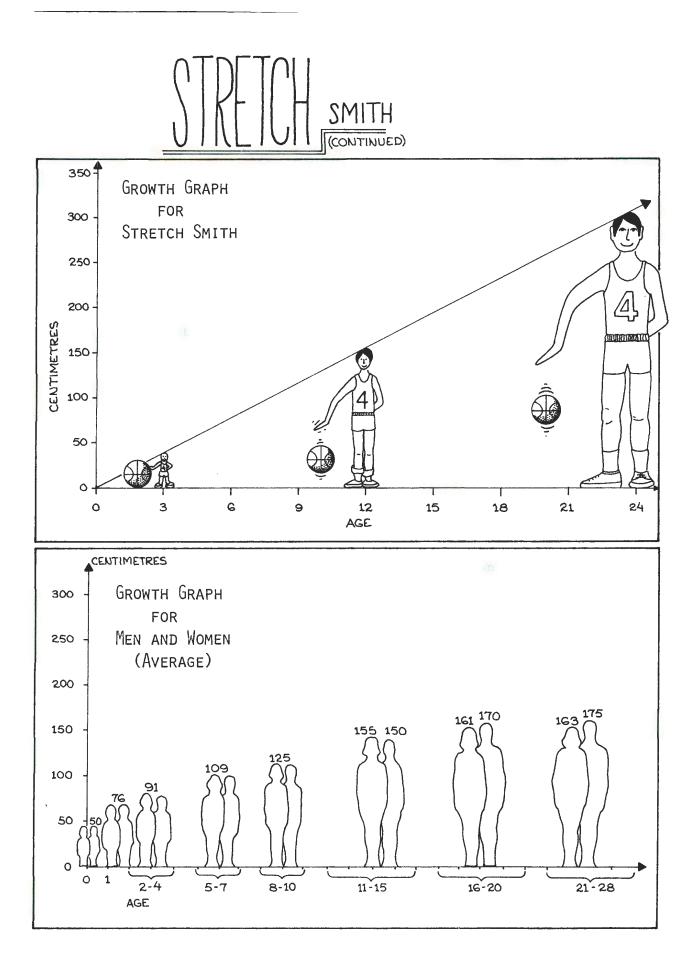


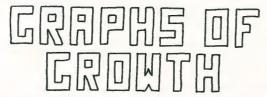
1) Assume "Stretch" is right. Fill in the tables below.

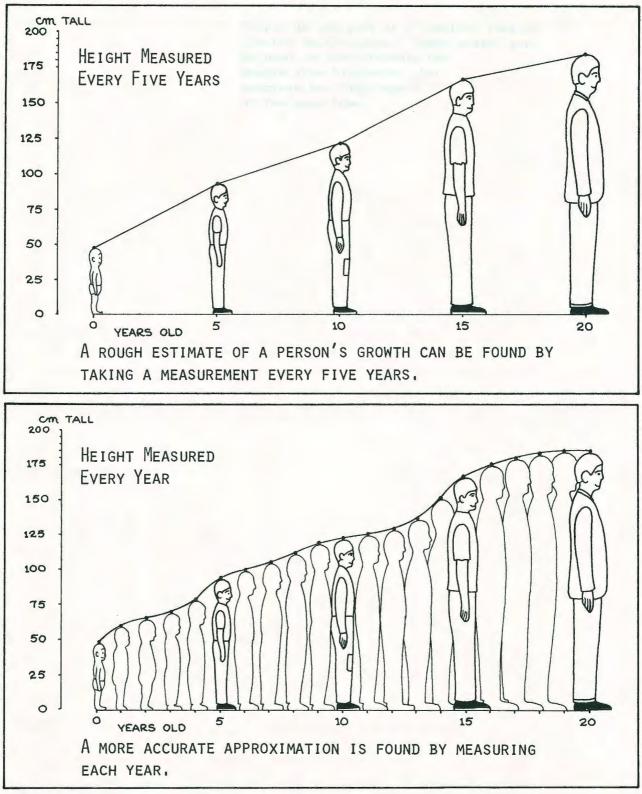
How Old Was Stretch?									
	A	н							
	12 Yrs.	160 cm							
	Yrs.	80 cm							
	Yrs.	40 cm							
	Yrs.	20 cm							

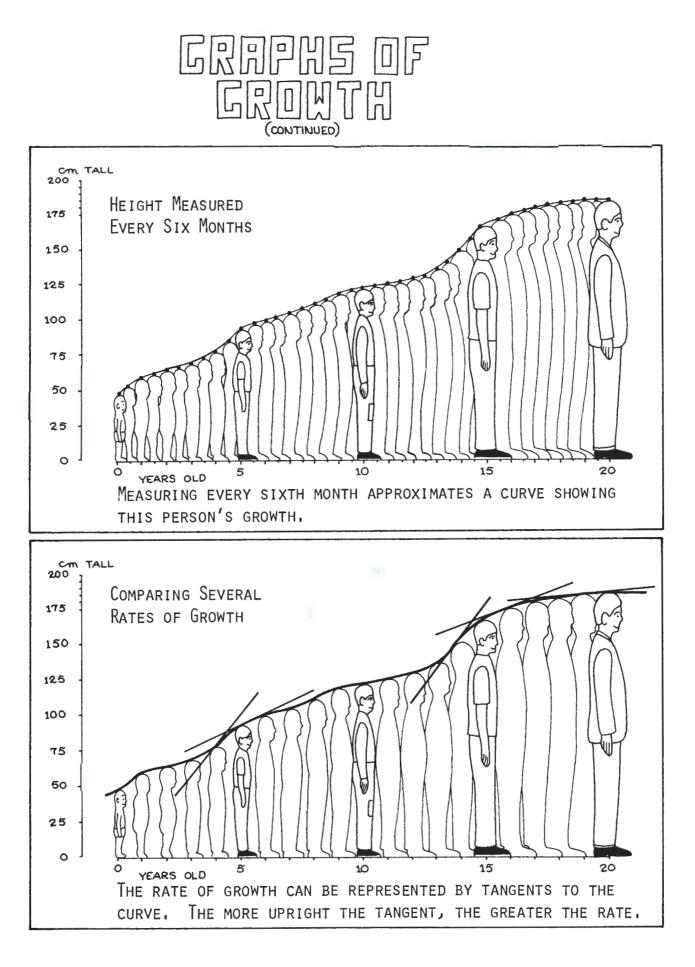
Ho	w Tall Wil	l'Stretch	Be?
	A	H	
	12 Yrs.		
	24 Yrs.		
	36 Yrs.		
			-

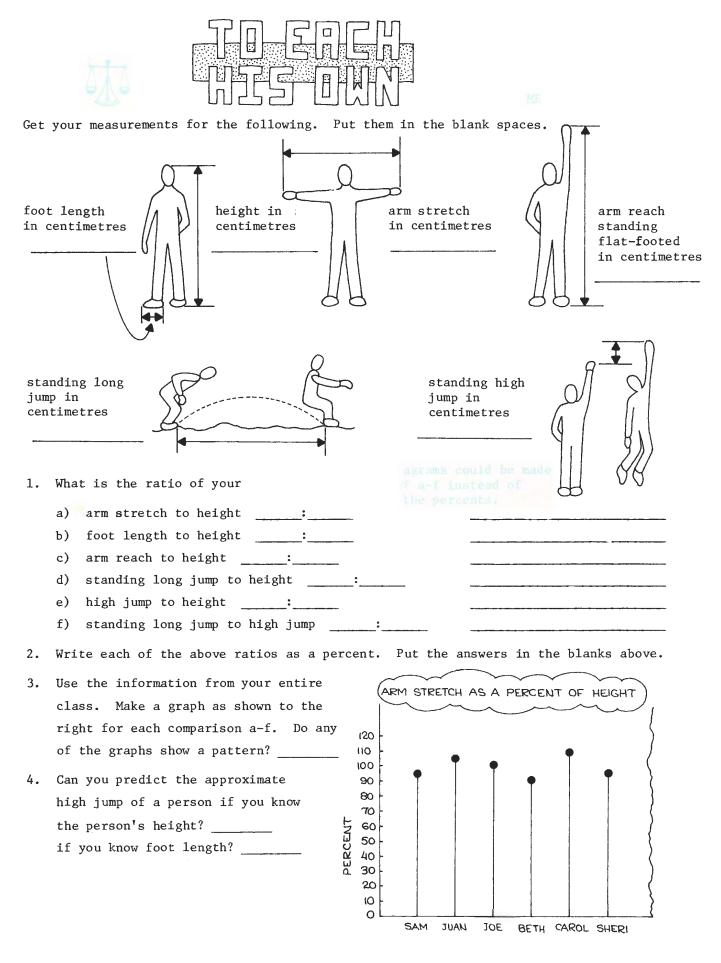
2) Graph the information in the tables. The graph is started for you. 550 500 Join the points with a height in centimetres 450 straight line. 400 350 3) Use the graph to approxi-300 mate these heights and ages. 250 A = 30 yrs., H = ____ 200 A = 0 yrs., H =150 A =____, H = 250 cm100 A = , H = 100 cm50 4) Does "Stretch" know what 0 3 6 9 12 15 18 21 24 27 30 33 he is talking about? Explain. age in years













An interesting and fun activity (perhaps in cooperation with the physical education teacher) would be to test the physical fitness of your students. The President's Council on Physical Fitness has suggested the following seven tests: pullups, situps, standing long jump, shuttle run, fifty-yard dash, softball throw for distance and six hundred-yard run/walk.

Standards of performance for boys and girls, ages 10-17, are shown below. Notice that the charts also include standards for qualifying for the Presidential Physical Fitness Award. Official application forms and complete information can be obtained by writing to Presidential Physical Fitness Awards, 1201 Sixteenth Street N.W., Washington, D.C. 20036.

The data collected during testing can be used for motivating several concepts of descriptive statistics such as mean, median, mode, range, etc. The data can be used to make several kinds of graphs, e.g., a bar graph comparing performances of students in a class or a line graph to record progress of an individual student. Students will probably be very interested in comparing results from earlier tests.

PULLUPS

A boy grasps the bar with palms forward and hangs from the bar so his feet don't touch the floor. A successful pullup is completed when the boy pulls his body up until his chin is over the bar and then lowers his body until his elbows are fully extended.

FLEXED ARM HANG

Two spotters help a girl to raise her body so her chin is above the bar, palms forward, elbows flexed and feet off the floor. A timer records the time, in seconds, that the girl can hold this position.

IDEA FROM: Youth Physical Fitness

BOYS

[Number	of	pu.	LLυ	ıps	1

	Age										
Rating	10	11	12	13	14	15	16	17			
Excellent	8	8	9	10	12	13	14	16			
Presidential Award	6	6	6	8	10	10	12	12			
Good	5	5	5	7	8	10	11	12			
Satisfactory	3	3	3	4	6	7	9	9			
Poor	1	1	1	2	4	5	6	7			

GIRLS

[In	seconds	l
-----	---------	---

	Age										
Rating	10	11	12	13	14	15	16	17			
Excellent	31	35	30	30	30	33	37	31			
Presidential Award	21	20	19	18	19	18	19	19			
Good	18	17	15	15	16	16	16	16			
Satisfactory	10	10	8	9	9	10	9	10			
Poor	6	5	5	5	5	6	5	6			



SITUPS

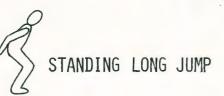


A pupil lies on his back and locks his fingers behind his head while a partner holds his ankles to keep his heels on the floor. Two situps are completed when the pupil (1) sits up, (2) touches right elbow to left knee, (3) lies back down, (4) sits up, (5) touches left elbow to right knee and (6) lies back down.

BOYS

[Number of situps]

	Age												
Rating	10	11	12	13	14	15	16	17					
Excellent Presidential Award Good Satisfactory Poor	100 100 76 50 34	100 100 89 50 35	100 100 100 59 42	100 100 100 75 50	100 100 100 99 60	100 100 100 99 61	100 100 100 99 63	100 100 100 85 57					
		GIR	LS										
Excellent Presidential Award Good Satisfactory Poor	50 50 50 39 26	50 50 50 37 26	50 50 50 39 26	50 50 50 38 27	50 50 49 34 25	50 50 42 30 24	50 50 41 30 24	50 50 45 30 23					



		Age														
Rating	1	0	1	1	1	2	1	3	14		15		16		17	
	ft.	in.	ft.	in.	ft.	in.	ft.	in.	ft.	in.	ft.	in.	ft.	in.	ft.	in.
Excellent Presidential	6	1	6	3	6	6	7	2	7	9	8	0	8	5	8	6
Award	5	8	5	10	6	2	6	9	7	3	7	6	7	11	8	1
Good	5	7	5	9	6	1	6	9 7	7	0	7	6	7	9	8	0
Satisfactory	5	2	5	4	5	8	6	0	6	7	7	0	7	4	7	6
Poor	4	10	5	0	5	4	5	7	6	1	6	6	6	11	7	0
						GI	RLS									
Excellent Presidential	5	8	6	2	6	3	6	3	6	4	6	6	6	7	6	8
Award	5	4	5	8	5	9	5	10	6	0	6	1	6	2	6	2
Good	5	2	5	6	5	8	5	8	5	10	6	0	6	0	6	0
Satisfactory	4	10	5	0	5	2	5	3	5	5	5	6	5	6	5	7
Poor	4	5	4	8	4	9	4	10	5	0	5	1	5	2	5	2

BOYS

A pupil stands comfortably with knees flexed and then jumps forward as far as possible. The distance is measured from the starting line to the point where the heels first touch the floor.



SHUTTLE RUN

Two blocks of wood, 2" x 2" x 4", are placed 30 feet from a starting line. A pupil runs to the blocks, picks up one block, runs back to the starting line and places the block behind it. This action is repeated to place the second block behind the starting line. The time needed to accomplish the task is recorded to the nearest 10th of a second.



[In seconds to nearest 10th]												
Rating	Age											
Kating	10	11	12	13	14	15	16	17				
Excellent Presidential	10.0	10.0	9.8	9.5	9.3	9.1	9.0	8.9				
Award	10.4	10.3	10.0	9.9	9.6	9.4	9.2	9.1				
Good	10.5	10.4	10.2	10.0	9.8	9.5	9.3	9.2				
Satisfactory	11.0	10.9	10.7	10.4	10.0	9.8	9.7	9.6				
Poor	11.5	11.3	11.1	10.9	10.5	10.1	10.0	10.0				
			GIR	LS	<u></u>		deu					
Excellent	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0				
Presidential												
Award	10.8	10.6	10.5	10.5	10.4	10.5	10.4	10.4				
Good	11.0	10.9	10.8	10.6	10.5	10.7	10.6	10.5				
Satisfactory	11.5	11.4	11.3	11.1	11.0	11.1	11.0	11.0				
Poor	12.0	12.0	11.9	11.8	11.5	11.6	11.5	11.5				

BOYS

50-YARD DASH

The time needed for a pupil to run 50 yards is recorded to the nearest 10th of a second.



[In seconds to nearest 10th]

Datina			,	Age	2			
Rating	10	11	12	13	14	15	16	17
Excellent Presidential	7.0	7.0	6.8	6.5	6.3	6.1	6.0	6.0
Award	7.4	7.4	7.0	6.9	6.6	6.4	6.2	6.1
Good	7.5	7.5	7.2	7.0	6.7	6.5	6.3	6.2
Satisfactory	8.0	7.8	7.6	7.3	7.0	6.7	6.5	6.5
Poor	8.5	8.1	8.0	7.6	7.2	7.0	6.8	6.7
		G	IRLS					
Excellent	7.0	7.0	7.0	7.0	7.0	7.1	7.0	7.1
Presidential								
Award	7.5	7.6	7.5	7.5	7.4	7.5	7.5	7.5
Good	7.7	7.7	7.6	7.6	7.5	7.6	7.5	7.6
Satisfactory	8.2	8.1	8.0	8.0	7.9	8.0	8.0	8.0
Poor	8.8	8.5	8.4	8.4	8.3	8.3	8.5	8.5



SOFTBALL THROW

A softball is thrown as far as possible with an overhand motion. The best of three throws is recorded by measuring the distance from the starting line to the point where the softball first touches the ground.



BOYS [In feet]

Rating		Age											
Kating	10	11	12	13	14	15	16	17					
Excellent	138	151	165	195	208	221	238	249					
Presidential													
Award	122	136	150	175	187	204	213	226					
Good	11.8	129	145	168	181	198	207	218					
Satisfactory	102	115	129	147	165	180	189	198					
Poor	91	105	115	131	146	165	172	180					
		(GIRLS	•		,	1	•					
Excellent	84	95	103	111	114	120	123	120					
Presidential													
Award	71	81	90	94	100	105	104	102					
Good	69	77	85	90	95	100	98	98					
Satisfactory	54	64	70	75	80	84	81	82					
Poor	46	55	59	65	70	73	71	71					

600-YARD RUN/WALK

The time needed for a pupil to run 600 yards is recorded in minutes and seconds. Walking is permitted, but the objective is to cover the 600 yards in the shortest possible time.



Detine				Age				
Rating	10	11	12	13	14	15	16	17
Excellent Presidential	1:58	1:59	1:52	1:46	1:37	1:34	1:32	1:31
Award	2:12	2:08	2:02	1:53	1:46	1:40	1:37	1:36
Good	2:15	2:11	2:05	1:55	1:48	1:42	1:39	1:38
Satisfactory	2:26	2:21	2:15	2:05	1:57	1:49	1:47	1:45
Poor	2:40	2:33	2:26	2:15	2:05	1:58	1:56	1:54
			GIRLS					
Excellent Presidential	2:05	2:13	2:14	2:12	2:09	2:09	2:10	2:11
Award	2:20	2:24	2:24	2:25	2:22	2:23	2:23	2:27
Good	2:26	2:28	2:27	2:29	2:25	2:26	2:26	2:31
Satisfactory	2:41	2:43	2:42	2:44	2:41	2:40	2:42	2:46
Poor	2:55	2:59	2:58	3:00	2:55	2:52	2:56	3:00

BOYS

[In minutes and seconds]





The food you eat is used for growth, repair and energy. When you eat more Calories than your body can use, the extra Calories are stored as fat. 1600 extra Calories convert to about 1 kilogram of body mass. By increasing your activity you can burn more Calories.

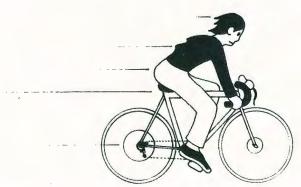
The chart below shows the average number of Calories used per minute for each type of activity. (These amounts can vary with individuals.)

Activity	Walking	Biking	Swimming	Running	Reclining
Calories Used Per Minute	5.2	8.2	11.2	19.4	1.3



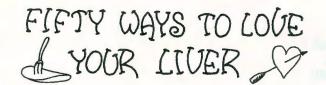
The table below lists some foods and the approximate number of Calories they contain. Complete the table to show the approximate number of minutes of each activity it would take to burn up the food. Use a calculator to help you.

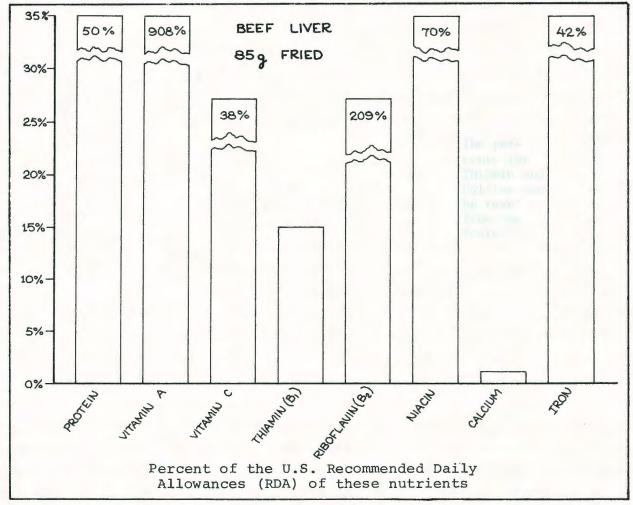
Dec 1	Number of	Number of Minutes										
Food	Calories	Walking	Biking	Swimming	Running	Reclining						
Milk, 1 glass	166											
Cookie, Chocolate Chip	51											
Spaghetti and Sauce, 360 ml	396											
Cheeseburger	425											
Pancake with Syrup	124											
Chocolate Bar, 60 gm	294											
Beans, Green, 240 ml	27											



Do you wish to lose weight? Boys your age need about 2800 Calories per day; girls about 2400 Calories.

Get a Calorie chart and keep track of the Calories you eat each day. If you wish to lose weight, you should eat fewer Calories and/or exercise more.

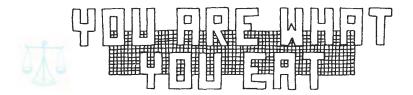




Graph adapted from Comparison Cards, Card 9, National Dairy Council

Courtesy of National Dairy Council

- The vertical scale on the left of the graph is drawn between what two percents?
- 2) The bars for Protein, Vitamin A, Niacin, and Iron are all the same height. Do they represent the same percents?
- List each nutrient and its percent that 85 g of fried beef liver supplies in order from lowest to highest.
- 4) Suppose the graph had bars all in one piece and the scale still started at zero. The Vitamin A bar would be ______times as long as the Protein bar.
- 5) Would it be practical to draw the graph with the bars all one piece? _____



Often students don't realize the need for a well-balanced diet. They don't directly observe the effects of eating only junk food or of eliminating an important food from their diets. Classroom experiments where young rats are fed contrasting diets will show how food affects growth and sometimes behavior patterns.



An excellent publication <u>Animal Feeding Demonstration...</u> <u>for the Classroom</u> can be ordered from the National Dairy Council, 6300 N. River Road, Rosemont, Illinois 60018. (You may wish to check with your local dairy council for this or similar information.) The publication thoroughly explains

the experiments. Sources are listed for obtaining rats, instructions are given for constructing cages and for proper care of the rats and various diets are furnished. Ample teacher information is provided to help explain and facilitate the activities.

Applications to Statistics: Much information needs to be organized and recorded accurately. Each rat needs to be labeled. Once a diet has been selected, food has to be weighed or measured and the amounts recorded. This is a good opportunity to teach and give students practice using the metric system. Weekly, the rats are weighed and their appearance and behavior are noted. Line graphs are an effective means for showing the growth rate. If the data for both the test and control rats are plotted on the same graph, comparisons can easily be made. Near the end of the experiment, both rats can be fed the nutritious diet and students can observe the effects on the rat previously fed the poor diet. After the experiment is completed, students can study the data, summarize the results and relate the conclusions to their own eating patterns. Other questions can be discussed: Do students think the outcomes will be nearly the same if the same diets are used with new rats? Other than diet, what could affect growth rate? Will the results be more believable if several rats exhibit similar growth patterns with the same diets?



WIND-CHILL CHART

WIND		THERMOMETER READING DEGREES CELSIUS													
SPEED	4	2	-1	-4	-7	-9	-12	-15	-18	-21	-23	-26	-29	-32	-34
CALM	4	2	-1	-4	-7	-9	-12	-15	-18	-21	-23	-26	-29	-32	-34
8 km/h	З	1	-3	-6	-9	-11	-14	-17	-21	-24	-26	-29	-32	-36	-37
16 km/h	-2	-6	-9	-13	-17	-19	-23	-26	-30	-33	-36	-39	-43	-47	-50
24 km/h	-6	-9	-12	-17	-21	-24	-28	-32	-36	-40	-43	-46	-51	-54	-57
32 km/h	-8	-11	-16		-23	-27	-31	-36	-40	-43	-47	-51	-56	-60	-63
40 km/h	-9	-14	-18	-22	-26		-34	-38	-43	-47		-55	-59	-64	-67
48 km/h	-11	-15	-19	-24	-28	-32	-36	-41		-49		-57	-61	-66	-70
56km/h	-12	-16	-20	-25		-33	-37	-42	-47	-51	-55	-58	-64	-68	-72
64 km/h	-13	-17	-20	-26	-30	-34	-38	-43	-48	-52	-56	-60	-66	-70	-74

Table adapted from <u>Emergency Care and Transportation of the Sick and Injured</u> by American Academy of Orthopaedic Surgeons, 1971.

When you're outdoors and the wind comes up, the air feels colder than it actually is. The chart above shows the wind chill (how cold it feels) for different actual temperatures and different wind speeds. For example, if the temperature is $-1^{\circ}C$ and the wind speed is 24 km/h, it feels the same as $-12^{\circ}C$ and no wind.

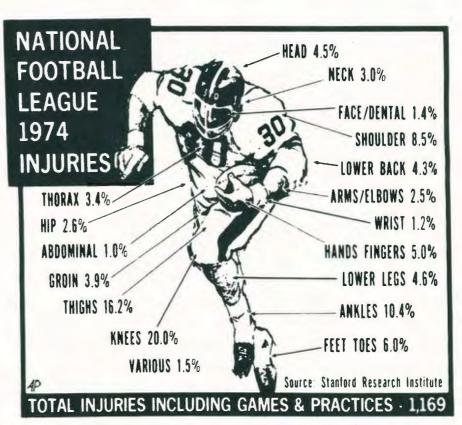


- If the wind is 40 km/h and the temperature is -7°C, it feels the same as _____
 with no wind.
- 2) Find the wind chill if the temperature is -15°C and the wind is 24 km/h. Is this chill factor classified as very cold, bitter cold or extreme cold?
- 3) If the temperature reading is -21°C and the wind chill is -49°C, then the wind speed is approximately?
- 4) Does it feel colder at 2°C with a wind blowing at 56 km/h or at -12°C with a wind at 8 km/h?
- 5) If the chill factor is classified as extreme cold, exposed flesh on a person's body will freeze in less than an hour. Jane is skiing. The temperature is -21°C and the wind speed is 40 km/h. She plans to make four runs that could take more than an hour. What advice would you give her? ______

Permission to use table granted by the American Academy of Orthopaedic Surgeons

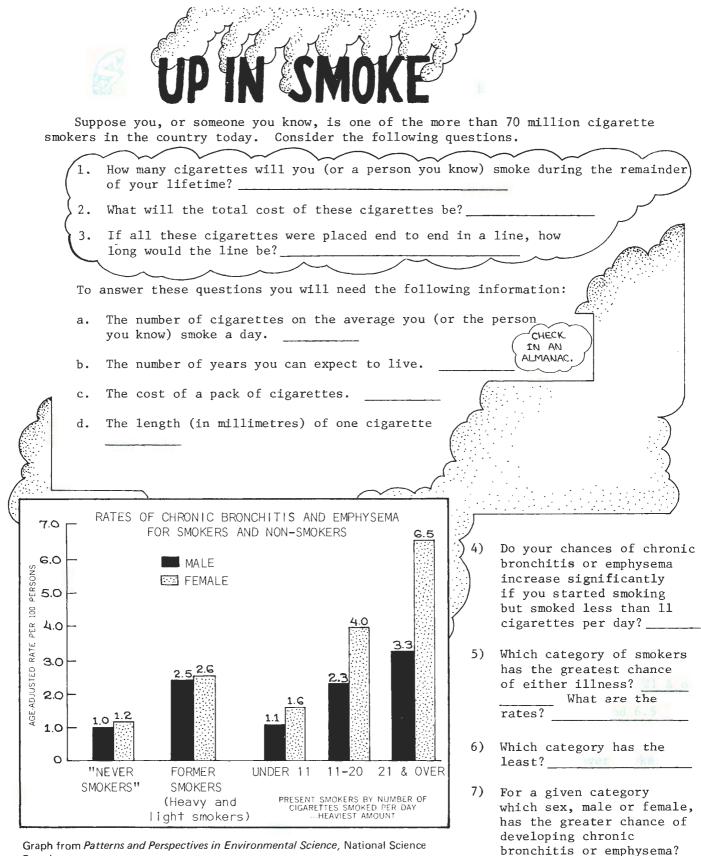


- A. What part of the body sustained more injuries than any other part? ______
 - B. What percent of the total was it? _____
 - C. Of the 1,169 injuries about how many were knee injuries?
- 2. The leg accounted for what percent of the total NFL injuries in 1974? ______ Is this more or less than half?



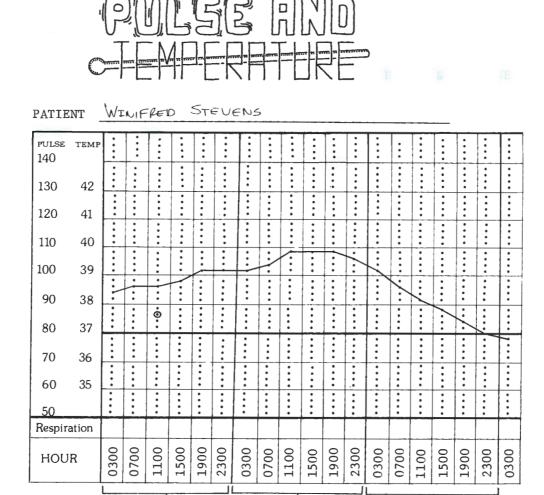
Permission to use granted by Stanford Research Institute

- 3. According to this study, of all injuries counted in 1974, 15.4% were reoccurrences of previous injuries. What percent of the total were first time football injuries?
- 5. The study showed younger players received more injuries than their older teammates. The eight teams at the top of the league received fewer injuries than the bottom eight teams. Can you explain?



Board

Permission to use granted by National Science Board



This is a real hospital chart. Nurses take and record the pulse and temperature of patients. Nurses do this with two differently colored pencils.

Day 1

Day 2

Day 3

1)	Look at the chart. The heavy straight line shows the normal tempera- ture in degrees Celsius. What is this temperature?
2)	Each small dot shows an increase of a) degrees in temperature. b) beats in pulse.
3)	Look at the circled dot. a) What temperature is shown? b) What pulse is shown? c) What time is it?
4)	Look at the hours. If 0300 is 3:00 A.M. (in the morning), what is 0700 1100 1500 1900 2300
5)	Look at the line graph on the chart. a) What day and time did the temperature start to fall?
	b) What day and time did it reach normal?

PE: Workshe t



Many sporting, musical or recreational activities involve an extensive use of muscles. Usually those who have conditioned their muscles through practice perform better and have less muscle fatigue. In this activity students will measure muscle fatigue.

Three students are needed per group; one as the "muscle" person, one as the timer, and one as the counter.

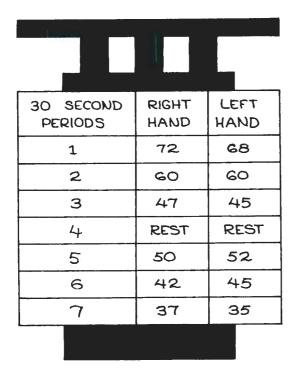
The "muscle" person places the right forearm flat on a table so the back of the fingertips are flat on the table top. He/she closes and opens the right hand as fast as possible until the timer says stop, being sure the fingertips touch the palm when closed and the fingertips touch the table when open.

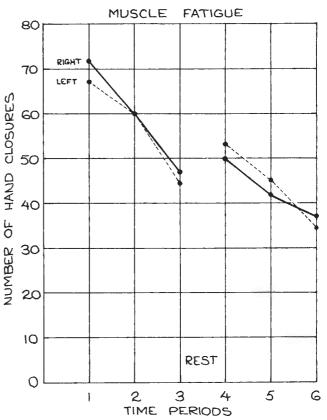
The timer times each trial for 30 seconds and records the count on the table. Between the 3rd and 4th trials the "muscle" person is given a 30-second rest period.

The counter counts the number of times the fingertips touch the table, relays the count to the timer, and begins the count over at 1 for each trial.

After the activity has been completed for the right hand, repeat the steps for the left hand.

A table with sample data and a graph of that data are shown below.





Squeezing a tennis ball or a small rubber ball is an exercise used to increase strength in the hand and wrist muscles. Students could practice this exercise for 15 minutes each day for two weeks. Then repeat this muscle fatigue activity and compare the results.

791



Blood Bank

De	one	DES	f	DT	W	ed	ne	sda	ay		4	
A Positive						¢						16
A Negative												1
O Positive	•											4
O Negative												1
B Positive												2
AB Positive	е											1

The Lane Memorial Blood Bank, 740 E. 13th Ave., Eugene, will be open for appointments from 10:30 a.m. to 1 p.m. and from 2 to 4:30 p.m.

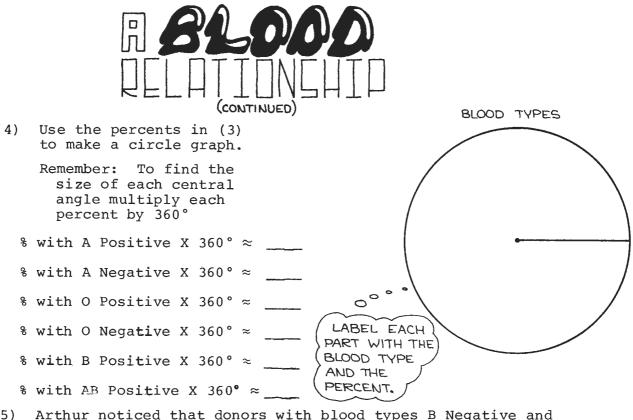
Arthur saw this ad in the newspaper.

His father explained that each person has one of eight types of blood (A positive, A negative, B positive, B negative, O positive, O negative, AB positive, AB negative). A person's blood type must be considered when he/she receives a transfusion.

Arthur decided to keep a record of the requests. He wanted to see which blood types were needed the most.

	TYPE					NUM	BERS	S OF	DON	ORS	NEE	DED					
A	POSITIVE	7	8	5	6	7	6	8	10	16	8	10	6	8	15	5	6
A	NEGATIVE	4	2	1	1	1	1	1	1	1	3	0	1	0	1	2	1
O POSITIVE 12 12 6 5 10 8 12 10 4 10 10 6 12 15 10 6									6								
ON	IEGATIVE	0	1	0	0	3	1	1	4	1	4	2	1	1	7	2	1
B POSITIVE 4 0 2 2 1 1 2 1 2						1	2	1	0	1	1	1					
AB	POSITIVE	1	1	2	1	1	1	1	2	1	1	1	2	1	3	1	2
_)	Find the A Posit O Negat	ive ive			1	A Ne B Pc	egat osit	ive ive			AI) Po	sit	ive			
2) Find the total number of donors needed.								ors	nee	ded	•						
	3) Find the percent of the total for each blood type.																

NUMBER OF A POSITIVE NEEDED X 100 TOTAL NUMBER OF DONORS NEEDED 0000 % with A Positive S % with O Negative % % with A Negative 8 % with B Positive % % with O Positive 8 % with AB Positive %



- 5) Arthur noticed that donors with blood types B Negative and AB Negative were not listed in the newspaper. What might be the reason?
- 6) To find the actual percent of people having each blood type, Arthur visited the blood bank.

DONOR INFORMATION

WHO CAN DONATE BLOOD?

Healthy persons between 18 and 66 years and who weigh 102 pounds or more. "Donors between 17 and 18 years must have written permission from a parent or guardian before they can donate."

Donors may donate every three months.

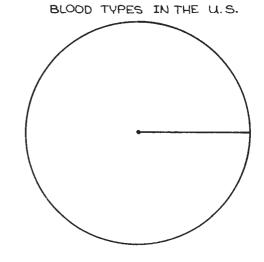
Who cannot donate blood?

Persons with a cold, sore throat or an infection. Persons with a history of jaundice, hepatitis, or heart disease. All donors are carefully screened each time they donate blood.

BLOOD GROUPS APPEAR IN THE UNITED STATES AS FOLLOWS

Blood Group and Rh	How many have	it quency
O (Rh positive)	1 person in	3 37.4%
O (Rh negative)	1 person in	15 6.6%
A (Rh positive)	1 person in	3 35.7%
A (Rh negative)	1 person in	16 6.3%
B (Rh positive	1 person in	12 8.5%
B (Rh negative)	1 person in	67 1.5%
AB (Rh positive)	1 person in	29 3.4%
AB (Rh negative)	1 person in	167 .6%

(USE THE PERCENTS IN THE TABLE TO MAKE A CIRCLE GRAPH.



7) Use the table in (6) to predict the number of students in your class with each type of blood.

ORGANIZATION

Placement Guide for Tabbed Dividers

Tab Label	Follows page:
CONTENT FOR TEACHERS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
Probability with Counting	
Inferential Statistics	
Appendix-Computer Programs	
DIDACTICS	
Components of Instruction	
Classroom Management	
Statistics and Probability Learning	
TEACHING EMPHASES	
Critical Thinking	
Decision Making	
Problem Solving	
Models and Simulations	
Calculators and Computers	
Laboratory Approaches	
CLASSROOM MATERIALS	
CONCEPTS AND SKILLS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Misleading Statistics	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	604
Probability with Models	634
Counting Techniques	
APPLICATIONS	
Business & Commerce	
Environment	
Health & Medicine	
People & Culture	
Recreation	
GLOSSARY	
ANNOTATED BIBLIOGRAPHY	
SELECTED ANSWERS	

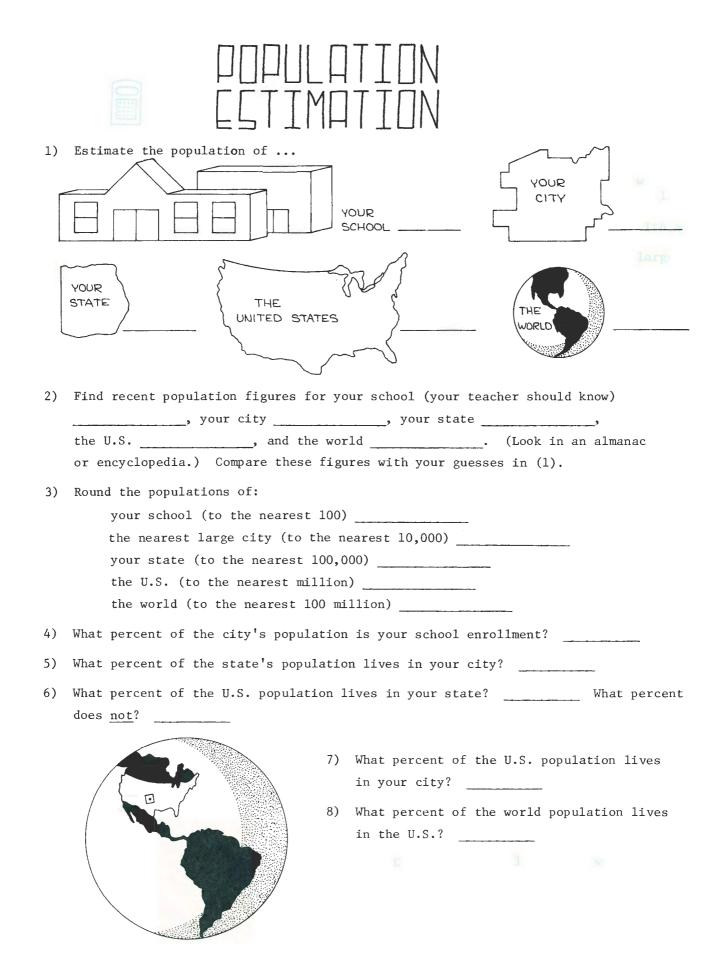
CONTENTS

PEOPLE & CULTURE

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TITLE	PAGE	TOPIC	TYPE
Population Estimation	797	Using a source book to gather population data	Worksheet
Sizing Up the States	798	Reading a table of state populations	Worksheet
Counting Everybody	800	Completing a table of census figures	Teacher idea Worksheet
Canada, Neighbor to the North	802	Reading a table and graph on Canada's population	Worksheet
How Many Children?	804	Using family size to predict future population	Worksheet
Graphing the World's Population	805	Making and using a graph to predict population trends	Worksheet
Population Projects	8,06	Gathering and interpreting population data	Teacher ideas
When's Your Birthday?	807	Using a computer to find probabilities	Worksheet
How's Your Pop?	808	Using an experiment to analyze taste-testing abilities	Activity card
A Sporty Question	809	Gathering data to investi- gate discrimination	Teacher directed activity
It's Not What it Looks Like	811	Reading a table to discover job discrimination	Worksheet
Evaluating an Evaluation	812	Analyzing a questionnaire used in teacher evaluation	Worksheet Discussion
Wow, I Didn't Know That	814	Analyzing averages	Bulletin board Transparency
What's on TV?	815	Collecting data on types of television programs	Activity card

TITLE	PAGE	TOPIC	TYPE
You Take a Survey	817	Using a survey to answer questions	Activity card
The Ten Most Wanted List	818	Using sampling to make identifications	Worksheet
Is it Best to be Golden?	821	Collecting data to see if golden rectangles are the most pleasing shape	Teacher directed activity
How Does Your Reading Rate?	824	Using averages and graphs to increase reading rate	Activity card Worksheet
The Fry Test	825	Using the Fry test to determine reading level	Teacher page
Who's #1?	827	Finding and graphing letter frequencies	Teacher directed activity
Will the Real "A"			
Please Stand Up?	830	Analyzing a table of musical frequencies	Worksheet
Can You Hit High "C"?	831	Reading a table of instru- ments and voice ranges	Worksheet
Beethoven Bar Graphed	832	Using a bar graph to analyze music	Teacher idea
A Chancy Composition	836	Using chance to compose music	Activity card
An Orderly Arrangement	837	Counting the possible permutations of notes	Worksheet





- 1) These population figures have been rounded to the nearest .
- Shade each state on the map on the following 2) page according to its population. Choose your own colors.

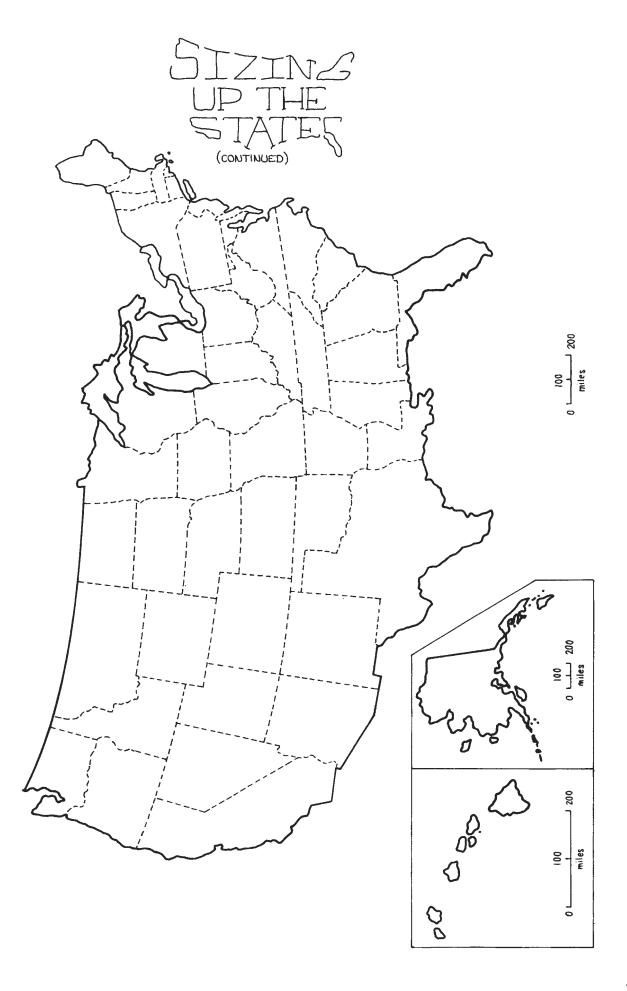
Color used

0	-	1	million	
1	-	2	million	
2	-	5	million	
5	-	10	million	
10	~	25	million	

If you do not have a calculator, round the population figures to the nearest million for the percent problems that follow.

- 3) The state with the largest population is _____. Its population is what percent of the U.S. total population?
- 4) What is the fewest number of states required to get $\frac{1}{3}$ or 33% of the total U.S. population? List them.
- What is the fewest number of states that con-5) tains $\frac{1}{2}$ or 50% of the U.S. population? List them.
- 6) Study the map you shaded. Why are some areas much more populated than others?
- 7) Use the population figures in the table. What is the largest number of state populations you can add and still be less than California's population?

	July 1, 1974 Population Estimates*
ALABAMA	3,577,000
ALASKA	337,000
ARIZONA	2,153,000
ARKANSAS	2,062,000
CALIFORNIA	20,907,000
COLORADO	2,496,000
CONNECTICUT	3,088,000
DELAWARE	573,000
FLORIDA	8,090,000
GEORGIA	4,882,000
HAWAII	847,000
IDAHO	799,000
ILLINOIS	11,131,000
INDIANA	5,330,000
IOWA	2,855,000
KANSAS	2,270,000
KENTUCKY	3,357,000
LOUISIANA	3,764,000
MAINE	1,047,000
MARYLAND	4,094,000
MASSACHUSETTS	5,800,000
MICHIGAN	9,098,000
MINNESOTA	3,917,000
MISSISSIPPI	2,324,000
MISSOURI	4,777,000
MONTANA	735,000
NEBRASKA	1,543,000
NEVADA	573,000
NEW HAMSHIRE	808,000
NEW JERSEY	7,330,000
NEW MEXICO	1,122,000
NEW YORK	18,111,000
NORTH CAROLINA	5,363,000
NORTH DAKOTA	637,000
OHIO	10,737,000
OKLAHOMA	2,709,000
OREGON	2,266,000
PENNSYLVANIA	11,835,000
RHODE ISLAND	937,000
SOUTH CAROLINA	2,784,000
SOUTH DAKOTA	682,000
TENNESSEE	4,129,000
TEXAS	12,050,000
UTAH	1,173,000
VERMONT	470,000
VIRGINIA	4,908,000
WASHINGTON	3,476,000
WEST VIRGINIA	1,791,000
WISCONSIN	4,566,000
WYOMING	359,000
TOTAL	210,669,000





		LATION DED TO EEST ON (B)	CHANGE FROM PRIOR CENSUS	PERCENT (D) CHANGE FROM PRIOR CENSUS
CENSUS YEAR	POPULATION	POPUI	CHAN FRON PRIO CENS	PERC CHAN(PRIOF
1790	3,929, 214	4,000,000		
1800	5,308,483	5,000,000	1,000,000	25.0%
	- (k. dili - 1. i k			
1970				

After the students have completed the table on the following page, the information could be graphed.

Students can discuss:

- what information is appropriate to graph: actual; increase of; or the percent of increase of population.
- what type of graph is appropriate (line, bar, circle, picture).

Use the table and the same graphing procedures for population figures for your state and city. Each state census is available in an almanac. The city census is available in each state's governmental handbook at the public library or from the Chamber of Commerce.

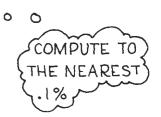
Compare the graphs of the actual population for the U.S., your state and city. The population of foreign countries could also be used and compared in a similar way.

COUNTING EVERY BODY

		z 2 (b)		d y s
		PODULATION ROUNDED TO NEAREST	C	ISU S
		ION LA	US (
CENSUS		L'AT DU	NS	NG E
YEAR	POPULATION	PODULATION ROUNDED TO NEAREST	CHANGE FROM PRIOR CENSUS	PERCENT CHANGE FROM PRIOR CENSUS
1790	3,929,214	4,000,000		
1800	5,308,483	5,000,000	1,000,000	25.0%
-				
1970				

You will need a 1971 or newer almanac. Use the index to find the page location for United States population.

The national census was first taken in 1790.



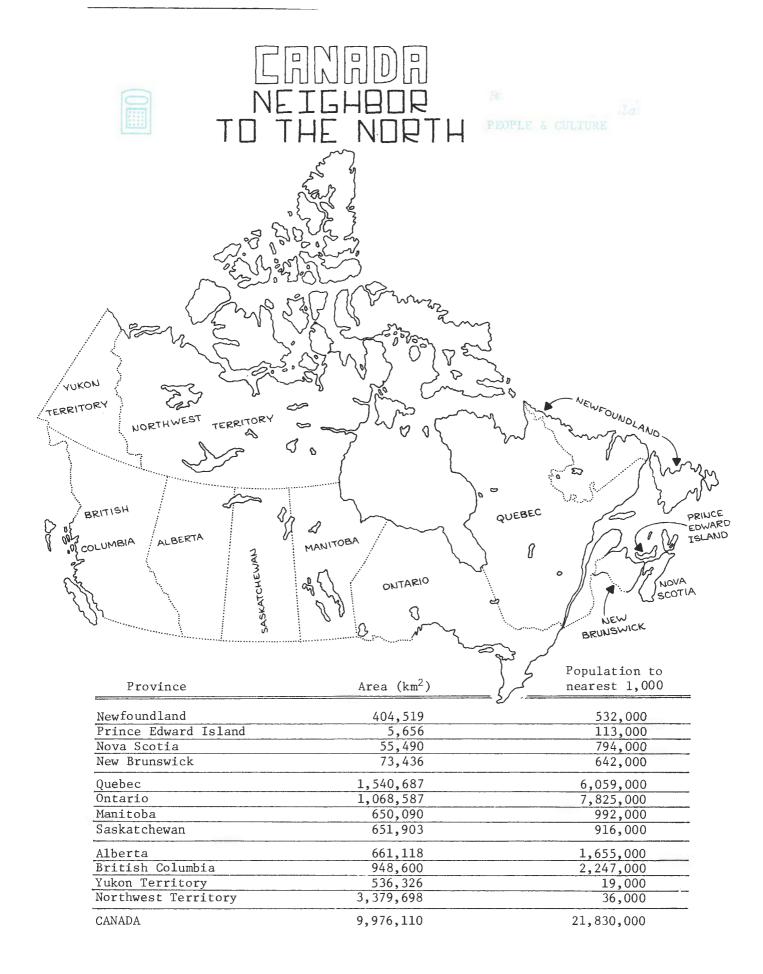
Since then a census has been taken every 10 years.

a) From the almanac copy the population for each ten-year period from 1790 to 1970. b) Round each figure to the nearest million. c) Find the change in population for each period--use the numbers in Column (b) to get your answer. d) Find the percent change for each period by dividing the change--Column (c)-by the rounded population of the previous census year--

Р	Ŧ.	<u> </u>		<u> </u>	0	u.s	<u> </u>
С	0	1	11	m	n	(5).	

Which column gives you more information--Column (c) or Column (d?

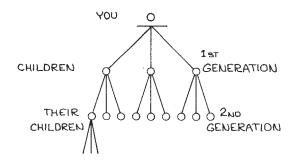
What major trend do you see?



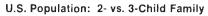
		(CONTINUED)		
1)	How many provinces are in Cana			
2)	Which province has the smalles	st area? la	argest?	
3)	Which province has the smalles	st population?	largest?	
4)	The four provinces of Quebec,			
-	percent of Canada's total popu			
5)	The Northwest Territory has ab	oout what percent of Canada	a's area?	
-	Canada's population?			
6)	If the land in the province we	ere divided evenly, how muc	ch land would each person in	
	these provinces have?			
	a) The Northwest Territory _	· · · · · · · · · · · · · · · · · · ·		
	b) Ontario			
	c) Prince Edward Island			
	d) All of Canada			
7)	If the land were divided even	ly, how much would each per	cson in your state have?	
1	MALES	FEMALES	The graph shows the po	р-
	80 +		ulation of Canada both by	-
	975 - 75		age and by sex.	
	70-74		(8) About how many males	
	65-69		are in the 0-4 age	
	60-64 65-59		group?	
	50-54		(9) About how many fe-	
	45-49		males are in the	
	40-44		10-14 age group?	
	35-39			
	30-34		10) In the 80+ age group,	
	25-29		are there more males	
	20-24		or females?	
	10-14		11) In 10 years, will	
	5-9		Canada need more or	
	0-4		fewer public school	
1200	1 1 1 1 1 1 1 1 1 1	0 200 400 600 800 1000 120	teachers?	
	THOUSANDS	THOUSANDS		

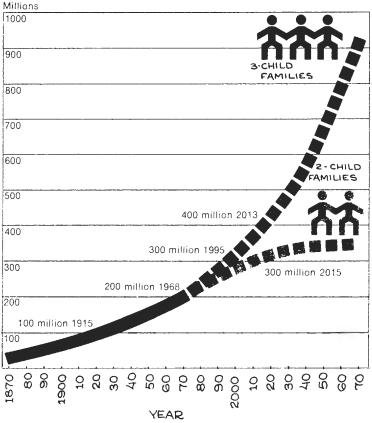


- Some people don't have children. Some have 1 or 2 or 3 or more. How many children do you think you will have when you grow up?



- 3) Suppose that on the average each person has 2 children in each generation. How many children will be in the 4th generation? (Hint: Make a diagram like the one above.)
- 4) How many children will be in the 4th generation using your number from exercise 1?
- 5) Would a 4-child average give twice as many children in each generation as a 2-child average?
- 6) According to the graph, about when was the population 200 million?
 - 100 million?
 - 50 million?
- 7) What could decide whether the population is greater or less than 300 million by the year 2000?
- SOURCE: Commission of Population Growth and the American Future (1972)



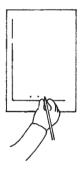


Graph from *Options; A Study Guide to Population and the American Future,* Produced by The Population Reference Bureau, Inc.

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graphing the worlds population



This table gives the population of the world from 4000 B.C.

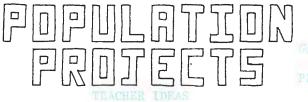
1) How many years did it take the population to double, from 250 million to 500 million? ______ 500 million to 1 billion? ______ 1 billion to 2 billion? ______ 2 billion to 4 billion? ______ Estimate when the population will be double the 1960 population.

Wo	Norld Population (estimated)				
Date		Populat	tion	(millions)	
4000 B	.C.	75	5		
2000 в	.C.	150	D		
0		250	C		
1650 A	.D.	500)		
1830 A	.D.	1000)		
1930 A	.D.	2000)		
1960 A	.D.	3000)		
1970 A	.D.	3600)		
1975 A	.D.	4000)		

 Use a whole piece of graph paper and graph the ordered pairs in the table.
 Draw a smooth curve through the points.

- 3) From the graph and exercise 1, what would be a reasonable estimate of the world population in the year 2000?
- 4) Why did the population increase so slowly for so many years?

IDEA FROM: *Environmental Science*, Intermediate Science Curriculum Study Permission to use granted by Silver Burdett Company and Florida State University



Gathering and interpreing population PEOPLE & CULTURE

- 1. Find which city has the fastest growth rate in your county. The office which has this information will vary from state to state. Try the records office or the county information number. The local Chamber of Commerce might have figures for a few years. List some cities and their population changes in the past few years. Which city had the greatest increase in numbers? What is the growth rate of the closest large city? The greatest percent of increase? Look in an almanac and find the state that is growing at the greatest percent of increase.
- 2. Find out how many classmates have pets. How would you estimate the number of dogs and cats in your city?
- 3. Look at a map of your state. Is one county of the state more populous than another? Figure the population density (ratio of number of people to number of square kilometres) of each county. Which is the most dense? Make up a color code and color the counties of the state on a map according to their population densities.
- 4. Look in an almanac to find which state's population is most dense. Which European country's population is most dense?
- 5. Use an almanac to find the country from which the greatest number of immigrants came to the U.S. in a recent year (not in all almanacs). Or, graph the number of immigrants to the U.S. for a large number of years and try to explain any changes.
- 6. Graph your school district's enrollment for the years that information is available. (Call the superintendent's office.) Is the enrollment growth proportional to the city's population growth?
- 7. Find information from state tourist bureaus or your local Chamber of Commerce about the number of tourists visiting your state or your city. What are the advantages and disadvantages of TOURIST tourism? INFORMATION 8. Contact the motor vehicle department and ask for the outof-state registrations turned in last year. These figures will give you an STATE approximate number of families MAP moving into your state. Compare the number from each state. Are the majority coming from one state? Graph the results. ALMANAC





Using a computer to find probabilities PEOPLE AND CULTURE

This activity uses the computer program BIRDAY, located in the appendix to the CONTENT FOR TEACHERS section, to find the probability that at least two people in a group have the same birthday.

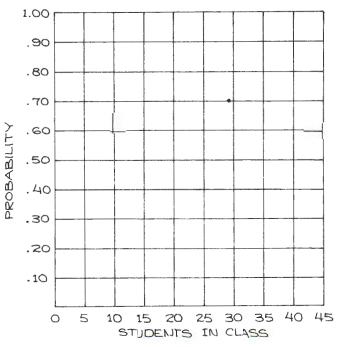
Do you think two or more students in your class have the same birthday (same day, same month)? How many students do you think it would take for the probability of this happening to be .50? Make a guess.

- I. To find out, use the computer. Your teacher will tell you how to run the program BIRDAY.
- II. Write the probabilities from the computer printout in Column 2 in the table to the right.
- III. Round each probability to the nearest one-hundredth and record in Column 3.
- IV. Graph the rounded probabilities below. Connect the points to make a line graph.
- VI. Use the graph to find the probability that at least two students in your class have the same birthday. _____
- VII. Take a survey of your class to see if any students have the same birthday.

Fo calculate the probabilities for each number of people from 15 to 30, change statement 150 to read 150 for N = 15 to 30

	2	З	
NUMBER IN CLASS	PROBABILITY ROUNDED PROBABILIT		
10			
15	0.252901	0.25	
20	0.411438	0.41	
25	0.5687	0.57	
30	0.70631	0.71	
35			
40			

THE PROBABILITY THAT TWO OR MORE STUDENTS HAVE THE SAME BIRTHDAY





Suppose three cups of cola drink are on the table. One is Coke, one is Pepsi, and one is RC Cola. The cups have labels A, B, and C only. You are to taste from each cup and tell which cup contains which cola.

 Make a list of all of the possible ways the colas could have been put into the three cups.

e cups.			
A	B	C	
Coke	Pepsi	RC Cola	
		·····	

- If you guessed without tasting, you would have 1 out of _____ chances of being correct.
- 3. If 24 students guessed without tasting, how many would you expect to guess all three colas correctly?
- 4. If all of the students in your class guessed without tasting, how many would you expect to guess all three colas correctly?

TRY THE EXPERIMENT WITH YOUR CLASS.

Ask your teacher to prepare 3 identical cups labeled A, B, and C: one with Coke, one with Pepsi, and one with RC Cola for each student. Taste a small amount of cola from each cup.

- 5. Write down which cola comes from which bottle. According to your taste, A is _____, B is _____, and C is _____.
- 6. How many students in your class got all three colas right?
- 7. Is this more or less than you expected if they were just guessing?

MATERIALS: paper cups (3 for each student), colas (Coke, Pepsi, RC)*, pencil.

*This activity can also be done using 3 varieties of chewing gum in place of the colas.



Many people are concerned about discrimination in our society. There have been movements to prevent discrimination against blacks, Indians, women, children, the aged... Your students might be wondering if their school, city, state or country discriminates against certain groups. This activity suggests how information can be found to answer a specific discrimination question.

PROBLEM: Do school and community sports programs discriminate against girls?

CLARIFYING THE PROBLEM:

During a discussion of this problem students might question what it means to be discriminated against. They could decide to find out what portion of the sports budget is spent for each of girls' and boys' sports, which sports are limited to only girls or only boys, how many different types of sports are available to girls and how many to boys, if athletic facilities are available to boys more often than to girls or if girls' teams are allowed to compete away from home as often as boys' teams.

Other information will also be helpful. How many girls and how many boys go out for each type of sport? (30% of the sports budget might be spent for girls' sports, 20% for coed sports and 50% for boys' sports, but 35 girls and 100 boys might go out for the program.) Are there girls who would like to play baseball or boys who would like to play volleyball but are not allowed to? Are some sports viewed by students, faculty or parents as being strictly for girls or only for boys?

GATHERING THE INFORMATION:

Most of the information can be obtained from the school office or community group in charge of the sports program. A questionnaire similar to that shown on the following page could be used to obtain information about student attitudes towards sports.

ORGANIZING THE INFORMATION:

The data from the questionnaire will be hardest to organize. Students will probably want to tally the responses from girls and boys separately. They might also tally the responses by grade level to compare differences and see trends.

INTERPRETING THE INFORMATION:

Students might suggest different interpretations or you might have some suggestions ready for them to consider. The data could be shown to someone in charge of the sports program for another interpretation.



STUDENT QUESTIONNAIRE

I.	GRADE	LEVEL:	Fifth Sixth Seventh Eighth	 II.	SEX:	Male Female	
			EIGULU				

III. WHAT ORGANIZED SPORTS DID YOU PLAY IN THE LAST YEAR? (DON'T COUNT THE SPORTS IN P.E. CLASSES.)

baseball	ice hockey		track	
basketball	karate or judo		volleyball	
cross-country	softball		wrestling	
field hockey	soccer		other	
football gymnastics	swimming tennis	النصبيب مي		
gy milds c2 c5	cenniis			

IV. WHICH OF THE ABOVE SPORTS WOULD YOU PLAY IN IF THEY WERE OFFERED?

field hockey	soccer swimming	<u></u>	other	<u> </u>
football	swimming			
gymnastics	tennis			

V. AFTER EACH SPORT BELOW PUT G-B IF APPROPRIATE FOR BOTH GIRLS AND BOYS G IF APPROPRIATE FOR GIRLS ONLY B IF APPROPRIATE FOR BOYS ONLY

baseball	ice hockey	track	
basketball	karate or judo	 volleyball	
cross-country	softball	wrestling	
field hockey	soccer	 other	
football	swimming		
gymnastics	tennis		

VI. PUT AN E AFTER EACH SPORT AT LEAST ONE PARENT OR GUARDIAN HAS ENCOUR-AGED YOU TO PLAY. PUT A D AFTER EACH SPORT AT LEAST ONE PARENT OR GUARDIAN HAS DISCOUR-AGED YOU FROM PLAYING.

baseball	 ice hockey	track	
basketball	karate or judo	 volleyball	
cross-country	softball	wrestling	
field hockey	 soccer	other	
football	 swimming		
gymnastics	 tennis		
			the second se



d ib

PEOPLE & CULTURE

Statistics can support many positions. The individual categories in the table seem to show no discrimination against females, but the totals show that even though fewer males applied, more were accepted for employment. The paradox of this example is explained in grades K-8. An overwhelming number of women applied for jobs in this category where the number of positions is lowest with respect to the number of applicants.

HYPOTHETICAL DATA SHOWING THE NUMBER OF MALE AND FEMALE APPLICANTS FOR VARIOUS TEACHING CATEGORIES, THE NUMBER ACCEPTED, AND THE PERCENT ACCEPTED						
Job Category Teacher	Number Applied	Male Number Accepted	% Accepted	Number Applied	Female Number Accepted	% Accepted
Grades 13-14 (2-year College)	150	30	20	40	16	40
Grades 11-12	200	70	35	50	35	70
Grades 9-10	100	15	15	50	15	30
Grades K-8	50	2	4	600	48	8
Total	500	117	23	740	114	15

1) Look at the table. What percent of female and male applicants were accepted for employment in:

	Female %	Male %
13-14	40	20
11-12	70	35
9-10	30	15
K-8	8	4
	11-12 9-10	13-14 40 11-12 70 9-10 30

2) For each individual category the <u>percent</u> of female applicants accepted is how many times as much as for male applicants?

3) For each individual category is the <u>number</u> of female applicants accepted twice as much as male applicants?

4) Now examine the totals in the table.

a) What percent of females were accepted? 15 males? 23

b) How many females were accepted? _____ males? _____

- 5) a) Do the individual categories seem to show discrimination against females?
 - b) Do the totals seem to show discrimination against females?

TYPE: Worksheet

Г

IDEA FROM: Winning With Statistics: A Painless First Look at Numbers, Ratios, Percentages, Means and Inference by R. Runyon Permission to use granted by Addison-Wesley Publishing Company, Inc.





The teachers of a junior high school wanted more feedback from students on how they could improve their teaching. They asked the student council to help. The student council made up a questionnaire somewhat like the one on the next page. Read the questionnaire and answer these questions.

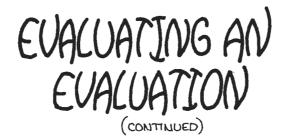
 Are there items you would reword? _____ If so, circle the number and reword it on another sheet of paper.

2. Are there items you would omit? _____ Which ones? _____

- Are there other items you would like included if you were to evaluate teachers? ______
 List them.
- 4. Suppose each of 30 students in 5 classes evaluated one teachers. How would you suggest organizing the data from the 150 questionnaires? ______
- 5. Who should tabulate the data? the teacher? _____ the principal? _____ a student or group of students? _____ Other suggestions ______

6. Who should see the results? Each teacher should see his/her own. _____ Any student, teacher or parent should be allowed to see the results. _____ The principal should see them all. _____ They should be shown to the local editor of a paper. _____

- 7. Suppose the questionnaire were used in a school. In each pair, check what you think is most likely to happen.
 - _____ a. Teachers would not change.
 - b. Teachers would make use of the evaluation and improve their teaching.
 - a. Students would feel their opinions counted more.
 - b. Students wouldn't take the time to fill out the questionnaire.
 - a. Students would honestly try to help teachers.
 - b. Students would be very critical of hard teachers.



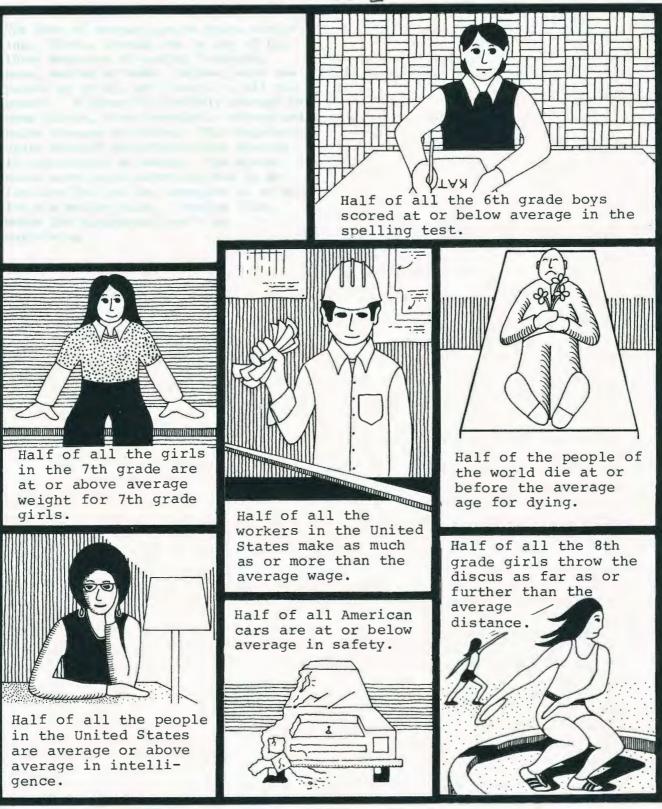
This teacher evaluation is your way to help teachers improve their teaching.

Look at the question, read the examples, then rate the teacher poor, fair, average, good or excellent compared to your ideal teacher.

This teacher evaluation is for your own benefit. By filling this out honestly and seriously you can help the teachers improve their teaching skills.

1.	ATTITUDE TOWARDS STUDENT IDEAS: (Does the teacher have respect for the things you say in class?)	1 poor	2 fair 	3 avg 	4 good 	5 exc
2.	FAIRNESS: (Is the teacher fair and impartial in his/her treatment of all students in the class?)	poor	fair 	avg 	good 	exc
3.	ENTHUSIASM: (Does he/she show interest and enthusiasm for the subject? Does the teacher appear to enjoy teaching the subject?)	poor 	fair 	avg 	good 	exc
4.	PATIENCE: (Does the teacher know how to deal with someone who doesn't understand?)	poor	fair 	avg	good 	exc
5.	KNOWLEDGE OF SUBJECT: (Does the teacher have a thorough knowledge and understanding of his/her teaching field?)	poor	fair 	avg	good 	exc
6.	CLARITY OF PRESENTATION: (Are ideas presented at a level you can understand?)	poor	fair 	avg	good	exc
7.	ENCOURAGEMENT OF STUDENT PARTICIPATION: (Does the teacher encourage you to raise questions in class?)	poor 	fair 	avg 	good 	exc
8.	CLASS PREPARATION: (Is the teacher prepared for class?)	poor	fair 	avg	good 	exc





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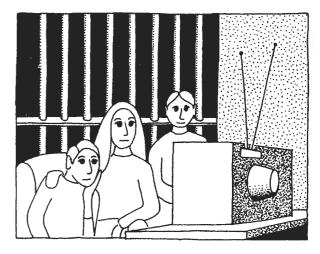




Collecting data (types of television (grams PEOPLE & CULTURE

Some television networks concentrate on certain types of programs, like Westerns. Is there a trend this season? Study the types of programs shown on TV for a week.

Use either the TV listing in your daily newspaper or a weekly TV guide. Record the programs from 3 p.m. to 11 p.m. Put the total amount of time for each type of show. Use .5 for 30 minute programs; 1 for hour programs; 1.5 for hour and one-half programs, etc.

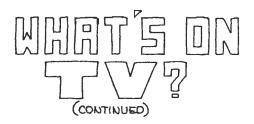


	Comedy	Western	Crime	News	Culture Program	Science Fiction	Musical, Variety
Mon.							
Tues.							
Wed.							
Thurs.							
Fri.							
Sat.							
Sun.							
Totals	·····						

What do you think? Is one type of program more popular this season?

Help students decide on what programs fall into the categories: crime programs usually involve confrontations with criminals, culture programs include ballets, sym phonies, etc. and could include educational programs like National Geographic. Students might like to make up their own classification of programs and their own table.

TYP Activity Card



The student page What's on TV? could be followed with other activities like these:

- <u>Violence</u>: How much violence is there in television programs? Is there more violence later or earlier in the day? Students could decide by surveying different programs and recording each time there are verbal threats, physical force, weapon usage and so forth.
- 2) <u>Time for Commercials</u>: Measure the amount of time devoted to commercials during a one or two hour period. What percent of time is "usually" devoted to commercials? Graphs could be made to study whether certain hours have more commercials.
- 3) <u>Types of Commercials</u>: Discuss the different ways commercials try to influence: uses of a well-known person, use of an authority figure (doctor type), use of ordinary people, use of humor, use of fear of failure (bad breath, acne), etc. Develop a list with your class. Have students watch a program and classify each commercial according to the developed list. New types may need to be added to the list.
- Favorite Programs: Ask students what their favorite programs are. You or the students could make up a questionnaire to study viewing patterns of students in the class; in the school.
- 5) <u>Show Dropping</u>: Why is your favorite show of last year no longer on TV? How are decisions to drop or retain programs made? By whom? Discuss the Nielsen ratings.
- 6) <u>Saturday Morning</u>: Some of the old favorites are now animated for "Saturday morning specials." What types of shows are on Saturday? What percent of crime? What percent of time for commercials? What types of commercials?
- 7) <u>Future Programs</u>: What kind of programs would the students like to see on TV? They might like to write descriptions of new programs not yet on TV. Students could survey the school to see what others think about their proposed shows.

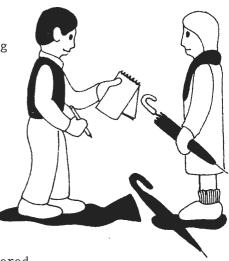




Materials: Whatever is necessary for your survey

Here are some questions that could be answered by taking a survey:

- What is the most popular TV program?
- What percent of the people are left-handed?
- What is the most popular song?
- What are the chances that a person drives with a seat belt on?
- What is the most popular automobile color?



- Write down three other questions that could be answered by taking a survey.
- 2. Select a question--either one of your own or one of those listed above. Decide how to conduct the survey. Before starting, talk with your teacher about your plan.
- 3. Display your results so that others can easily understand them.
- 4. Do you feel that your results would apply to
 - the rest of the school?
 - the community?
 - your state?
 - the entire nation?
 - Explain your answers.
- EXTENSION: Check to see if your teacher or school library has the book, <u>How to Lie</u> <u>with Statistics</u>, by Darrell Huff. Perhaps other books on probability and statistics are available for you to read.

SOURCE: *MathLab - Junior High* by S. McFadden, et al. Permission to use granted by Action Math Associates, Inc.

THE TEN MOST WANTED LIST

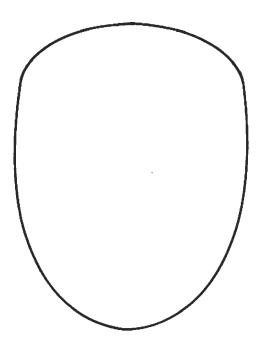
Using sampling to make identifications PEOPLE & CULTURE

Many police departments have artists that draw sketches of suspected criminals. Some departments even have books containing many drawings of differently shaped noses, mouths, etc. From these books a victim or witness can pick features and the artist can make a composite sketch.

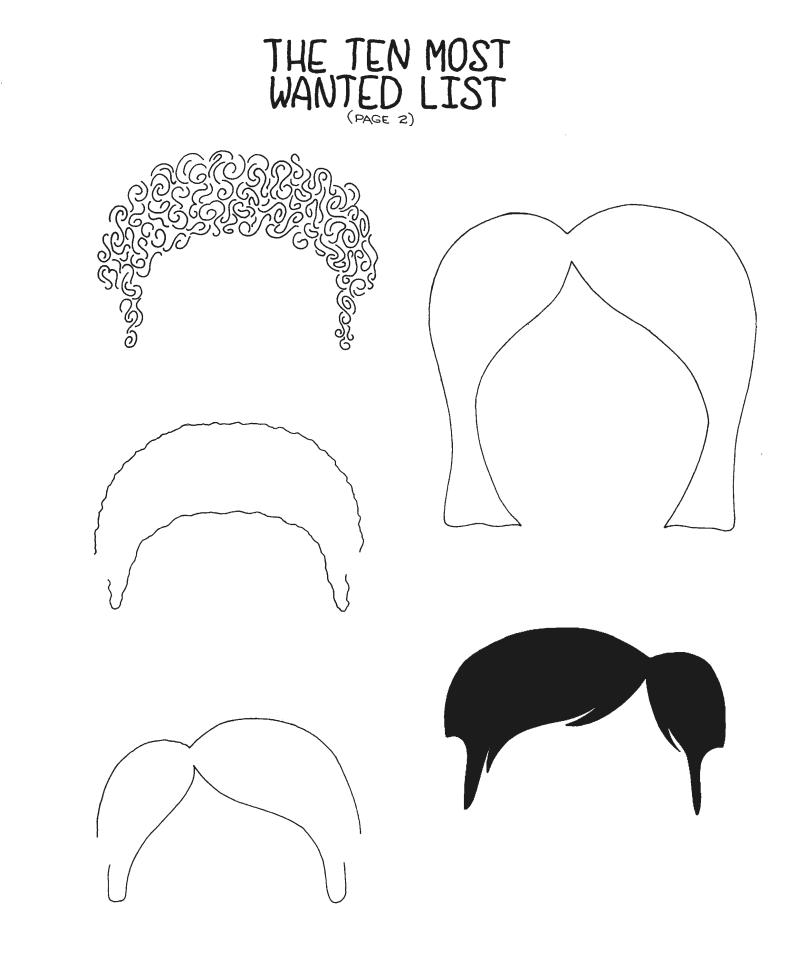
- 1) Look at the features on the next two pages.
- 2) Cut out your favorite feature from each of the groups.
- 3) Use glue or transparent tape to attach the features to the face outline below.
- 4) How many different faces could be made using the features on the next page? 3125

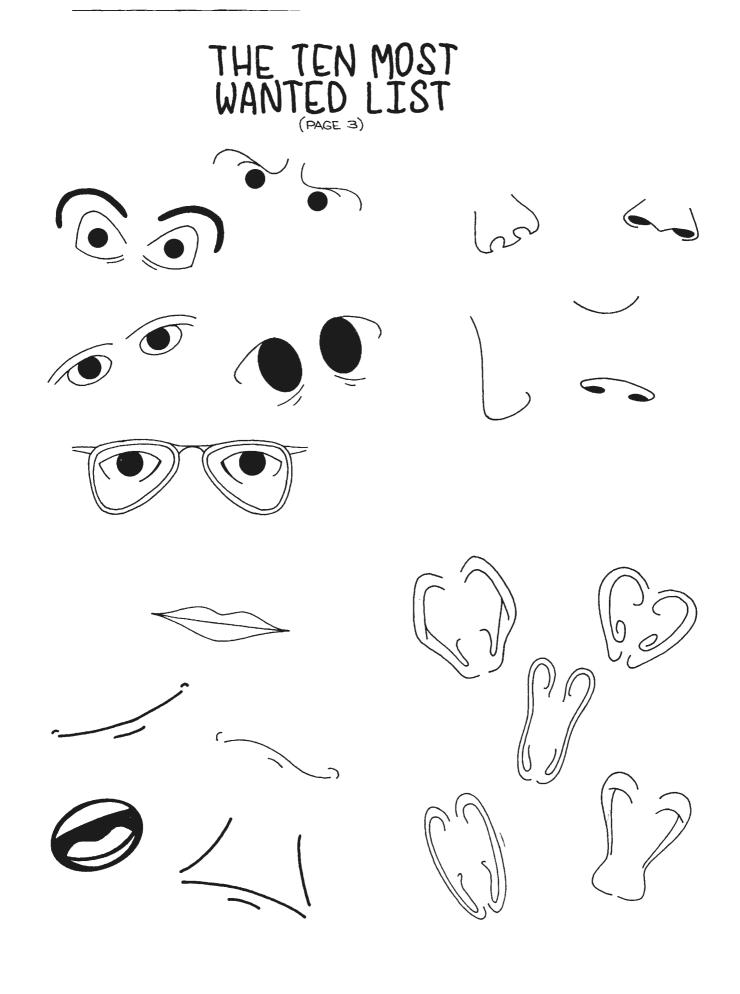
5	x	5	x	5	x	5	х	5
hair		nose		eyes		mouth		ears

- 5) What is the probability that another student, picking at random, makes the same face as you? 3125
- 6) Survey the class to find the features favored by most of the students. Make a sketch using these features.









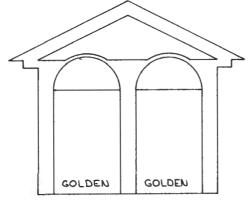


- I. Show the class several examples of golden rectangles in art and architecture. Use examples as shown on pages 94 and 95 in <u>Mathematics</u> from the Life Science Library or use the examples on the top of the following page. While pointing out the golden rectangles, explain that many people from the time of the Greeks have believed golden rectangles are the most pleasing rectangular shapes.
- II. Present the problem: Are there any golden rectangles in our room? Students might be able to eliminate a blackboard for being "too long" and skinny or a floor tile for being too "square", but be puzzled as to how they should decide if some rectangular shapes are really golden. A worksheet of carefully drawn golden rectangles (see bottom of next page) could be given to students. They can then measure the rectangles and discover that the quotient of each pair of lengths and widths is about 1.6. Now the "about 1.6" criteria can be used to determine golden rectangles.
- III. Some students might have questioned whether golden rectangles are the most pleasing. "I like squares best!" can be a lead into a new problem: Is it true that golden rectangles are the most pleasing shapes? A survey of the class can be taken where students pick their favorite shape from a collection of rectangles. (See the third page of this activity for such a collection.) Was the golden rectangle chosen most often?
- IV. Perhaps the results were biased by so much previous discussion of the golden rectangle. Ask students what they think about this. Students might also wonder if the arrangement of the rectangles on the page influenced the results. Size of the golden rectangle might also effect results.
- V. Have a committee of students survey several other classes and compare the results to their class' survey. Students could take home page 3 and have family members vote. Does the data support the idea that the golden rectangle is most pleasing?

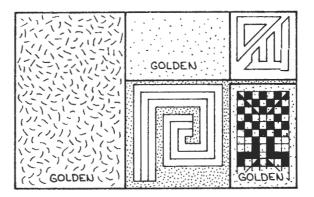
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Some people think golden rectangles are the most pleasing to look at. Golden rectangles are often used in art and architecture. Here are some examples.

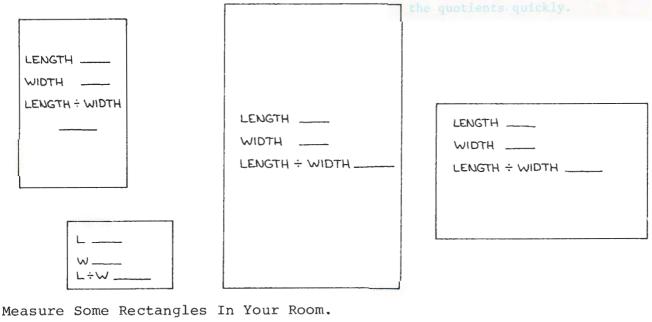


Many buildings have golden rectangles.



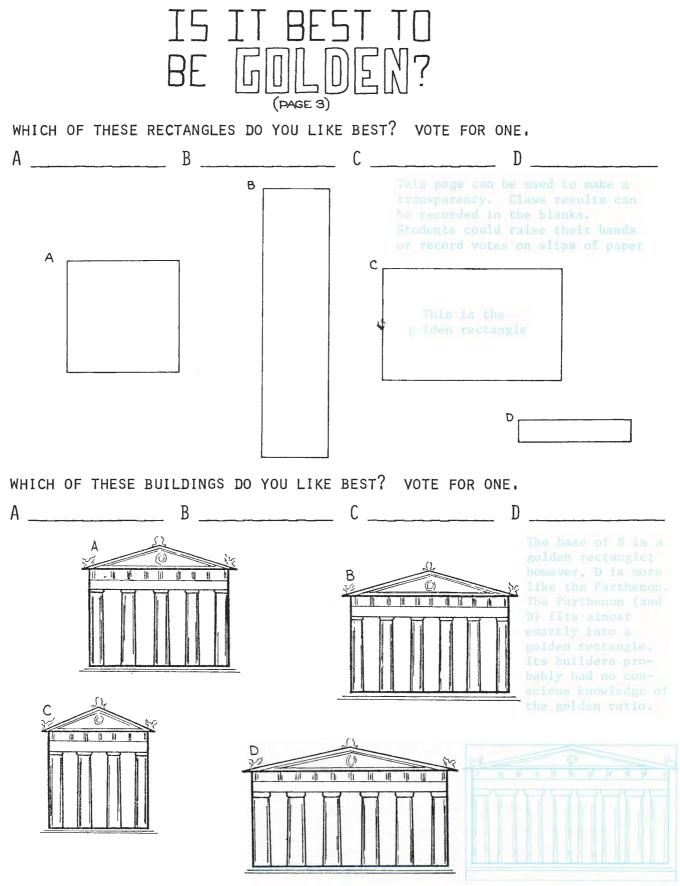
Pop artists sometimes use golden rectangles. (The whole picture is also a golden rectangle.)

What Makes A Rectangle Golden? Measure Each of These Golden Rectangles To Find Out.



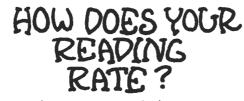
Are some books golden? _____ Are some windows golden? _____

List four things in your room that have the shape of a golden rectangle.



Greek temples drawn by Nancy Linn.

From the book *The Golden Mean* by Charles F. Linn Permission to use granted by Doubleday and Company



Keeping track of your reading rate can help you see if your rate is improving. Here's how to find your reading rate:

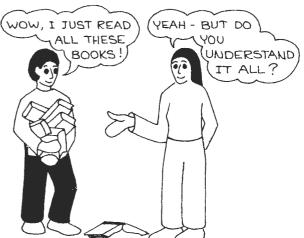
- Choose a book that is not difficult for you to read.
- Choose a page in the book that is all words-no pictures or illustrations.



- 3. Find the approximate number of words on this page by doing these steps:
 - a. Choose ten lines at random from the page.
 - b. Find the average number of words in the ten lines. (You'll have to count the words in each line, add, then divide the total by ten.)
 - c. Count the lines on the page. _____ Multiply this number by the result in (b). _____
- 4. Find your reading rate by doing these steps:
 - a. Read the page. Have someone time you and record the number of seconds it takes to read the page.
 - b. Divide the number of words on the page (the second number from 3(c) above) by the number of seconds from 4(a) above. _____ This is your average number of words per second.
 - c. Multiply the number in 4(b) by 60 to find your reading rate--the average number of words per minute.

Do this two or three times a week. Use books at about the same level of reading. Choose different pages. Try to increase your rate of reading. Keep a record of your reading rate and after several weeks, graph the rates. Is there a trend to show your reading rate is increasing?

> CAUTION: "Reading" means <u>understanding</u> the words, too. Don't try to move your eyes over the words so fast that you don't understand. Some material isn't meant to be read quickly (mathematics, for instance!).





The reading level of written material is of concern to editors, writers, and teachers. To help establish the level of reading material several tests have been designed. These tests use measures of sentence length and number of syllables in words.

Edward Fry, a professor at Rutgers University Reading Center, has developed a procedure that is quick and easy to use. His method is described below.

- 1) Choose three one-hundred-word passages, one each from near the beginning, middle, and end of the book. Skip all proper nouns and numerals.
- 2) Count the total number of sentences in each hundred-word passage. (Estimate to the nearest tenth of a sentence.) Average these three numbers.*
- 3) Count the total number of syllables in each hundred-word sample. (You could count every syllable over one in each word and then add 100.) Average the total number of syllables for the three samples.*
- 4) On the graph on the following page, plot the average number of sentences and the average number of syllables per hundred words to determine the area of readability level. Most plot points will fall near the heavy curved line.

Example

\sim	SENT	NUMBER OF	NUMBER O WORDS SYLLABLES PER 1	
(100- WORD SAMPLE	PAGE 5	9.1	122	r
(100- WORD SAMPLE	PAGE 89	8.5	144	
(100- WORD SAMPLE	PAGE 160	7.0	129	2
Z		3)24.6	3)395	
\$	AVERAGE	8.2	132	$\langle \rangle$
			GRAPH, FALL IN THE 5TH	
		\sim	m	

If great variability occurs either in sentence length or in syllable count for the three selections, select several more passages and average before plotting.

IDEA FROM: "Readability Formula that Saves Time," by E. Fry, *Journal of Reading*, April, 1968 "Updating the Fry Readability Formula," by J. Kretschmer, *The Reading Teacher*, March 1976

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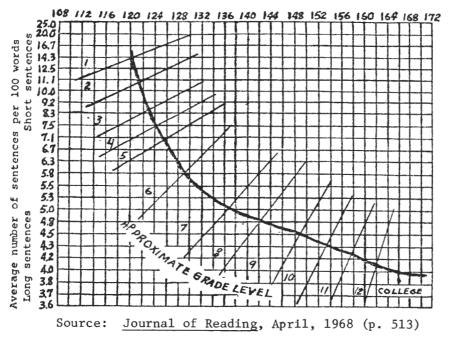


Graph for Estimating Readability

by Edward Fry, Rutgers University Reading Center Average number of syllables per 100 words

Short words

Long words



How accurate is the score?

Fry claims his method is accurate to within a grade level. However, there are no rigorous standards of just what constitutes a particular grade level of difficulty. The standards set so far have been based on subjective agreements between publishers and educators, relative ranking (comparing the orders in which a group of books is ranked by several formulas or tests), and comprehension tests.

When compared to the other more involved readability tests, Fry's formula does seem to rank the readability level of printed material higher. To correct this, Joseph Kretchmer has updated Fry's method by identifying a group of basic words of two or three syllables to be counted as one syllable words. The group of words is listed below. Using this list as a reference, apply the Fry

about after again ago almost along also always another any anything	around away because before better city ever every family father funny going	happy into little many mother morning money myself never o'clock only	open other over people pretty second seven something today under upon	until very woman yellow yesterda
---	--	---	---	--

formula in the usual manner but count these words as only one syllable when obtaining the syllable count. With this correction, the accuracy of the Fry erday method should be increased.

IDEA FROM: "Readability Formula that Saves Time," by E. Fry, *Journal of Reading*, April, 1968, and "Updating the Fry Readability Formula," by J. Kretschmer, *The Reading Teacher*, March 1976

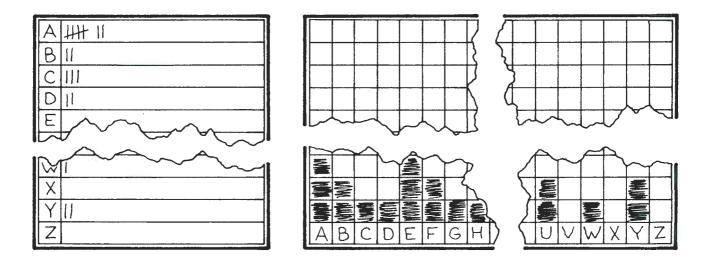


TEACHER DIRECTED ACTIVITY

Which letter of the alphabet occurs most frequently in printed material?

(The following is written as an individual activity but can be done in a two-person group.)

- 1) Each student chooses a book.
- 2) Each student then selects <u>five</u> lines of print and keeps a tally to count how many times each letter occurs. If graph paper is used, and a square shaded for each occurence of a letter, a bar graph will be constructed.



- 3) To compile the results, several methods can be used.
 - a) Students can report their most frequent letter to the class, and a tally on the blackboard can be kept.
 - b) A large bar graph can be constructed on the bulletin board, and the students can shade in their results. The large bar graph may need to be scaled to make the size manageable. For each letter a ratio that compares the number of occurrences for the letter to the total number of occurrences of all letters tallied by the class can be written. A calculator can be used to convert the ratio to a percent.
 - c) Students can examine their own tally for each letter and write the ratio, number of times the letter occurred : total number of letters counted in 5 lines of print. The ratio can also be converted to a percent (use the calculator). It may be desirable to have students construct a bar graph showing the percents.

IDEA FROM: Readings in Mathematics, Book 2, and Mathematics A Human Endeavor

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- 4) Suggestions for analyzing the results.
 - a) Students can compare their tallies with their neighbor's.
 - b) If students have expressed their tallies with percents, the percents of each student can be added to see how close the sum is to 100%.
 (Rounding error might make the sum not equal to 100%.)
 - c) The individual percents can be compared with the percents calculated from the class bar graph. Which set of data is more valid and why? Do the percents for the class bar graph add to 100%?

Extensions:

I. Morse Code

Morse Code is used to send messages rapidly. Letters are formed by a combination of no more than four dots and/or dashes; digits by a combination of five dots and/or dashes as shown in the tables. A dash is formed by depressing the telegraph key for a time unit three times as long as for a dot. The space between dots and dashes in the same letter has the same time unit as the dot. For example,

L'in Morse code is ". - . . ". "L"has a time unit length of 9, Number Morse Character 1 2 The cost of sending a mes-3 sage depends on the number of time 4 units in the length of the message. This is dependent on the number of 5 times each letter of the alphabet 6 occurs in the message. To devise a code that is the most economical, 7 those letters that occur most fre-8 quently should be represented by code characters that have the 9 shortest time unit lengths.

$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	English	Morse	Time	Frequency in	Time unit length
	Letter	Character	Units	100 letters	in 100 letters
LIATAL THE O LIMIT LONGTO IL 1/	Z		11	4	5 ¹ / ₂ 26 2 ³ / ₄

IDEA FROM: *Readings in Mathematics*, Book 2, *Mathematics A Human Endeavor*, and "Hamming it up mathematically"

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Have students create their own "Morse code" based on the percent frequency from their individual tallies. For a more efficient code, the percent frequency from the large bar graph could be used by the entire class.

The table lists the Morse code character for each letter of the alphabet and gives the total time unit length of the average 100-letter message. Students may wish to compare their codes with the Morse code. Are their codes more efficient than Morse's?

Is the Morse code the most efficient code in terms of economy?

Research projects:

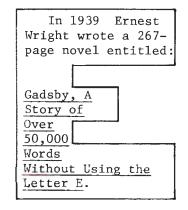
- 1) Is the keyboard arrangement of the typewriter efficient? Were percent frequencies of letters considered in assigning letters to the keys? (For information, see The Codebreakers by David Kahn.
- 2) Check the letter frequencies in a Scrabble game. Find the percent frequency for each letter.

NOTE: In creating the Morse code, Samuel F. B. Morse in 1838 counted the letters in a Philadelphia newspaper's typecase to help him assign the characters. Had he assigned the symbols haphazardly, the average message would have cost 25% more.

- II. 1) Have students use the percent frequencies of letters based on the large bar graph to estimate the number of letters in each of the following:
 - a) E's in 300 letters
 - b) M's in 500 letters
 - c) Y's in 3000 letters
 - 2) Consider the sentence: "Pack my box with five dozen liquor jugs."
 - a) Find the percent frequency of the letter E in this sentence.
 - b) How does this percent compare with the percent frequency for the letter E from the large bar graph?
 - c) Check the percent frequencies of other letters in the sentence and compare them with the percent frequencies from the large bar graph.
 - 3) "This group of words is unusual. If you look at it analytically you may find out why. Do not quit. By staying with it you will not fail.

Have students try to compose a paragraph or sentence without using the letter E!

Suggested Reading: <u>The Codebreakers</u> by David Kahn, Macmillan, 1967. <u>Famous Stories of Code and Cipher</u> edited by Raymond T. Bond, Collier Books, 1965 <u>Have</u> His Carcase by Dorothy Sayers, Avon, 1974.



IDEA FROM: Readings in Mathematics, Book 2, and Mathematics A Human Endeavor Permission to use granted by W. H. Freeman and Company, Publishers, and Ginn and Company (Xerox Corporation)

Analyzing a table f misical frequenc es PEOPLE & CULTURE

People haven't always agreed on the pitch of a note. The table below gives many different frequencies (and many different pitches) that have been used for the first A above middle C on the piano. If all these "A's" were played together the result would be noise not music.

1. The table lists the frequencies least to greatest and gives the year that frequency was used. Find the median frequency for the table.

2. What is the range of the frequencies in the table?

Frequencies for the first A above middle C

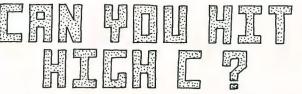
Frequency for A	Date	Place
374.2	1700	Lille, organ of L'Hospice Comptesse
376.6	1766	Paris, from model after Bedos
393.2	1713	Great organ in Strassburg cathedral
395.2	1759	Organ at Trinity College, Cambridge, England
398.7	1854	Lille, restored organ of La Madeleine
402.9	1648	Paris, Mersenne's spinet
409.	1783	Paris, court clavecins
414.4	1776	Breslau, clavichords
415.	1754	Presden, Silbermann organ
421.6	1780	Vienna, Stein pianos, used by Mozart
422.5	1751	England, Handel's tuning fork
423.2	1815	Band of Dresden opera, under von Weber
427.	1811	Paris, Grand Opera
427.8	1788	England, St. George's Chapel, Windsor
433.	1820	London Philharmonic
4 35.4	1859	Paris, Diapason normal of Conservatoire
440.2	1834	Scheibler's "Stuttgart Standard"
441.7	1690	Organ at Hampton Court Palace, England
444.2	1880	United States "low organ pitch"
445.6	1879	London, Covent Garden Opera
448.4	1857	Berlin Opera
451.7	1880	United States, Chickering's standard fork
458.	1880	United States, Steinway's pitch
474.1	1708	London, Chapel Royal, St. James
484.2	1688	Hamburg, St. Jacobi Kirche, approved by Bach
503.7	1636	Paris, Mersenne's ton de chapelle
563.1	1636	Paris, Mersenne's chamber pitch
567.3	1619	North German church pitch

Table from The Physics of Musical Sound by Jess J. Josephs, pp. 64-65.

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In 1939, an International Pitch Conference was held in London. This conference recommended a frequency of 440 cycles per second for the standard of the first A above middle C. The U.S. National Bureau of Standards broadcasts this frequency so people can standardize their "A's" by radio.

lorksheet



The table below gives the range in pitch for voices and instruments. The frequency column tells how many vibrations per second are necessary to create a tone with the given pitch. Look at the table and answer the questions below.

В G F GA 8 Ε GAB C DE G CDE G DE C Soprano c Alto E G Tenor A Bass E Violin G Vicla E Cello A C Bass D Bb Piccolo P Flute Ç C Bb Oboe G English Hor A Clarinet RE D Bass Clarinet F Db Alto Saxophone Ab Tenor Saxophone EK Baritone Saxophone At Bb Bassoon Eb Bb Contra Bassoon Ab Bb Trumpet Ē Cornet BE E French Horn B F Trombone E D Bass Trombone C F Tuba D Harp G# Glockenspiel C G Celesta С C Xylophone [] C C C Tubular Bells F Timpani

- 1. Which instrument (not including the piano) on the table above has the greatest range in pitch? ______ Give the frequency of this instrument's lowest pitch. The frequency of its highest pitch.
- 2. Which instrument has the least range in pitch? _____ Give the frequency of this instrument's lowest pitch. _____ The frequency of its highest pitch.
- 3. If you play an instrument see if it is on the table (if not, pick any instrument on the table). How does its range of pitch compare to a trumpet? (greater, equal to, less than) To a clarinet? (greater, equal to, less than)
- 4. Which type of voice has the greatest range in pitch?

Table from *Listeners Guide to Musical Understanding* published by William C. Brown Company Publishers, page 56, Fourth Edition.

Permission to use table granted by William C. Brown Company Publishers



Many people recognize a composer or singer when they hear the music played or sung. There's something familiar about the melody or its rhythm that makes it identifiable. Can composers be identified by some mathematical structure in their music?



Some components of music that are easy to represent with numbers or number sequences are the placement of notes in the scale and interval jumps.

Most students could easily analyze a piece of music by studying the frequency of interval jumps. Bar graphs can be used to display the information obtained.

Activity for Students:

Assign the numbers 1 to 88 in ascending order to the 88 notes on the piano. Each half-step interval is given a new number in this manner:



The melody line of some piece of music is then changed to a sequence of numbers. Beethoven's "Minuet in G" is numbered here as an example:





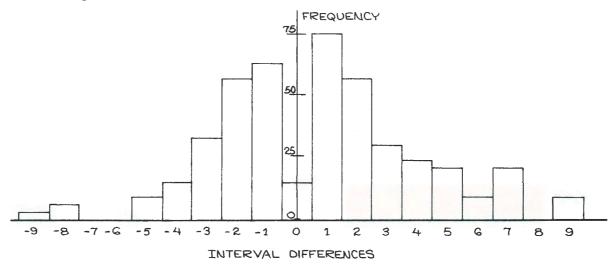




The next step in the process is most quickly done by two people. The first person looks at the numbers representing two consecutive notes, and reads the difference to a second person who records the difference. The interval jump is considered positive when the second note goes up and negative when the second note goes down in pitch. The first interval of the "Minuet," 51 to 52, would read as a positive 1. The next interval read is 52 to 54 or positive 2. Students could record the number of each kind of interval jump in a frequency table like this:

INTERVAL DIFFERENCE	-9	-8	-7	-6	~5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9
FREQUENCY															1				

The data is much more interesting when put into a bar graph. Your bar graph might look something like this:



Questions about the bar graph could be asked: Are there any patterns in the graph? Do the patterns occur in another piece by Beethoven?

Other graphs could show the number of occurances of each given note on the scale.

Extension of Activity:

More graphs could be done showing the same data about another piece by the same composer and compared. Other comparisons could be done on different composers, different styles of music, or different times in history.

Question: Can we get a computer to help in the process? See the advertisement on the next page.



Solving Musical Mysteries by Computer

Plagiarism didn't concern sixteenthcentury Italian song writers. They didn't think twice about borrowing each other's best tunes. And nobody minded.

That's the way it was back then. But today, musicologists want to trace precisely who borrowed what from whom. And the computer is helping them do this, just as it is helping them in many other kinds of musical research.

The two scores below show how an obscure musician, Nicola Broca, borrowed a melody from a better-known composer, Josquin des Prés. Nicola even went so far as to twist Josquin's words, and turn a sacred song into a ditty of disappointed love. Out of some 40,000 different tunes, the IBM System 370 Model 158 at the State University of New York in Binghamton selected these two, because they had such similar melodic form. Dr. Harry Lincoln. Chairman of the Department of Music, was then able to compare the printouts of the opening themes, scrutinize publication dates and trace the borrowing.

Musicologists like Dr. Lincoln have to cope with such a vast repertory they just couldn't tackle much research of this kind without computer help. Of course, they must have a way to put a musical score into computer-readable form. And that's why so many musicologists today are using a coding system called DARMS.

DARMS, Digital Alternate Representation of Musical Scores, was developed by Stefan Bauer-Mengelberg, a visiting professor at Binghamton who is also a staff member at the IBM Systems Research Institute. He says, "Now a musicologist can take any piece of music in standard notation and transcribe it into a code for entry into a computer."

ETHOVE

(PAGE 3)

Since musicologists have a way to tell the computer precisely what a composer has scored, they can now process a formidable volume of data. In fact, in many universities today music departments are among the biggest computer users.

With DARMS as their tool, musicologists can develop programs to analyze a composer's use of harmony, rhythm and counterpoint. With this knowledge they can develop a theory about his style, and study how it evolved. They can even attempt to determine when Bach, for example, composed a particular work.

"Once you know enough about composers' stylistic techniques," says Bauer-Mengelberg, "much music that was once dubbed 'anonymous', or was wrongly attributed, can be ascribed to the right composer. This is especially important in early music, where title pages from folios are often lost."

Looking to the future, Bauer-Mengelberg speaks of how the computer could be used to print musical scores: "Now that a way has been found to make music machine-readable, we hope the day is not far off when we will be able to use the computer in the preparation of master plates for music printing." Information in advertisements can be used as resource material and as ideas for projects.

This advertisement would be interesting after students have analyzed some music. Their work was probably timeconsuming and they will see the advantages of using the computer.

Ask students to look for advertisements or articles relating the computer and music.

If there is a computer terminal available, check on programs or simulations involving music. Students do not need to know a computer language to use the prepared programs.

Advertisement from *Scientific American*, June 1975, p. 46.

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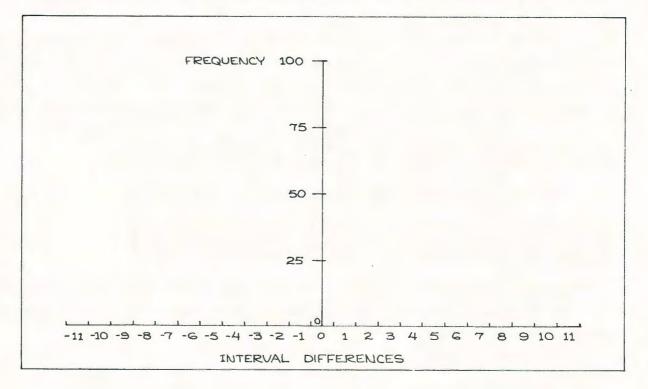
Two songs. Two composers. Similar melodies. The computer helps solve the mystery: who borrowed from whom.

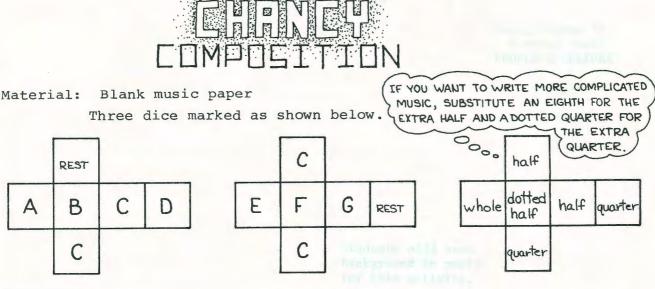


TITLE OF MUSIC

INTERVAL DIFFERENCE	TALLY	INTERVAL DIFFERENCE	TALLY
-11		0	
-10		1	
- 9		2	
- 8		З	
- 7		4	
- 6		5	
- 5		6	
- 4		7	
- 3		8	
- 2		9	
- 1		10	
		11	

INTERVAL DIFFERENCE	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	з	4	5	6	7	8	9	10	11
FREQUENCY																							





Procedure:

--Place the treble clef sign and the 4 time signature on the top staff of the music paper. (See the example below.)

--Mark off the number of measures you want to compose or spin a spinner to determine the number of measures.

--Roll one of the lettered die (your choice) and the time die to determine the pitch and length of the first note. Record this on your staff. --Continue this process until the measures are full.

Special Rules:

--If a C is rolled, the note can be placed in either C position: _____ or _____ The same is true for other notes.

--If a time is rolled that will not fit into a measure, roll a new time. If a quarter note or rest is necessary to fill the measure, the time die need not be rolled. Whole rests are not allowed.

--A composer may decide to begin and end each composition with a C.



It looks like music but what does it sound like? Play your composition or have a musically inclined friend play it for you. How do you like your "composition by chance?"



Question: Suppose you want to write a melody using exactly 5 quarter notes and choosing from 7 different pitches: A, B, C, D, E, F, G. How many different melodies could you write? (Hints are given below.)

- I. Let's simplify the problem:
 - a) What if you wanted to use 2 quarter notes and choose from 2 pitches,A and B? There are 4 possible "melodies." Can you write them below?

<u>A</u> <u>A</u>, ____, and ____

b) Write the melody arrangements for 2 quarter notes and 3 pitches,

		A, B and C/////
		/ / / / /
		How many are there? or x
	c)	How many melody arrangements for
		2 quarter notes and 4 pitches? or x
		2 quarter notes and 5 pitches? or x
		2 quarter notes and 6 pitches? or x
-	1	2 quarter notes and 7 pitches? or x
	l I	2 quarter notes and 12 pitches? or x
II.	Try	the same reasoning for 3 quarter notes.
	a)	3 quarter notes and 2 pitches. The melody arrangements are
		//////////
		/////////
		There are or x arrangements.
	b)	How many melody arrangements for
		3 quarter notes and 3 pitches? or x x ° (you can
		3 quarter notes and 4 pitches? or x x of USE
		3 quarter notes and 5 pitches? or x (EXPONENTIAL)
		3 quarter notes and 6 pitches? or x x (3, to)
		3 quarter notes and 7 pitches? or x x SAVE
		3 quarter notes and 12 pitches? or x x (TIME.)
III.	Try here	to answer the question at the top of the page. Write your answer

T ?E Worksheet

ORGANIZATION

Placement Guide for Tabbed Dividers

Tab Label	Follows page:
CONTENT FOR TEACHERS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
Probability with Counting	
Inferential Statistics	
Appendix-Computer Programs	
DIDACTICS	
Components of Instruction	
Classroom Management	
Statistics and Probability Learning	
TEACHING EMPHASES	
Critical Thinking	
Decision Making	
Problem Solving	
Models and Simulations	
Calculators and Computers	
Laboratory Approaches	
CLASSROOM MATERIALS	
CONCEPTS AND SKILLS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Misleading Statistics	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	604
Probability with Models	634
Counting Techniques	
APPLICATIONS	
Business & Commerce	
Environment	
Health & Medicine	
People & Culture	
Recreation	
GLOSSARY	
ANNOTATED BIBLIOGRAPHY	
SELECTED ANSWERS	



RECREATION

TITLE	PAGE	TOPIC	TYPE
Sports in the News	840	Using newspapers and magazines as sources of data	Teacher idea
Classroom Decathlon	842	Collecting and organizing data from a class competition	Activity card Worksheet
Analyzing Activities	845	Reading a table of recreation activities	Worksheet
Out of the Ball Park	846	Using stem-and-leaf displays to study distributions of home runs	Worksheet
Dropping the Ball	847	Making a graph to study the bounce of a ball	Activity card Worksheet
You Can't Get a Hit All the Time	848	Using a computer to analyze batting trends	Worksheet
One Die or Two	849	Using a computer to compare rolls of a die with rolls of two dice	Worksheet
Agree or Disagree?	850	Making a scatter diagram to compare ratings	Worksheet
Judging Gymnastics in the Olympics	851	Finding the mean to score a gymnastics event	Worksheet Transparency
Gymnastics in High School	852	Using mean and median to score a gymnastics event	Worksheet Transparency
Statistics: Baseball	853	Finding batting averages	Worksheet
Slugger	854	Finding slugging averages	Worksheet
Football Scores	855	Analyzing trends in football scores	Teacher idea
Diving Competition	856	Using the range to find diving scores	Worksheet
Poker with the Computer	857	Using random digits from a computer to play poker	Game



Save clippings from the newspapers and magazines that include sports records, statistics about players, advertisements about equipment and events, and pictures for the bulletin board. The sports section of a newspaper is the easiest place to find facts and figures, but you might also find sports information in magazines and other periodicals. The statistics from the preliminaries below could be used to have students determine who should be in the finals.

As much as possible relate any statistics to those of your students and/or school records. Students could collect articles, write questions about the articles and pass them around the class.

World swim championships

Men (PRELIMINARIES)

,

200-meters Backstroke

FIRST HEAT --- 1, Lutz Wanja, East Germany, 2:11.62, 2, Bab Tierney, United States, 2:12.77, 3, Reinhold Becker, West Germany, 2:15.47, 4, Gory Abrahams, Great Britain, 2:15.62, 5, Igor Omel-chenka, Saviet Union, 2:16.80, 6, Larenza de la Torre, Calombia, 2:29.60.

SECOND HEAT — 1, Paul Hove, United States, 2:08.73. 2, Abdul Karin Ressang, The Netherlands, 2:09.97. 3, Steve Pickell, Canada, 2:11.120. 4; Krasi-mir Petkav, Bulgaria, 2:11.179, S, Conrado Porta, Argentina, 2:12.78. 6, Enrique Ledesma, Ecuador, 2:22.90.

THIRD HEAT — 1, Raland Matthes, East Ger-many, 2:08.73. 2, Soltan Rudolf, Hungary, 2:08.92. 3, James Carter, Great Britain, 2:09.77. 4, Fran-cisca Santos, Spain, 2:11.94. 5, Lopo Cianchi, 2:13.80.

FOURTH HEAT -- 1, Mark Tanelli, Australia, 2:06.08.•2, Zaltan Verroszto, Hungory, 2:07.91. 3, Santiago Esteva, Spain, 2:08.94. 4, Butch Batche-ior, Canada, 2:13.12. 5, Bodo Schlag, West Germa-ny, 2:16.48. 6, Genar Otero, Calombia, 2:19.02. 7, Jose Luis Lopez, Ecuador, 2:28,15.

400-meter Freestvle

FIRST HEAT — 1, Gordon Downie, Great Brit-ain, 4:05.60. 2, Peter Pettersson, Sweden, 4:06.22, 3, Alexandr Samsanov, Soviet Union, 4:11.67. 4, Michael Ker, Conado, 4:13.79. 5, Juan Alfredo Uribe, Colombia, 4:31.56.

SECOND HEAT — 1, Frank Pfutze, East Ger-many, 4:04.25, 2, Rainer Strohbock, East Germa-ny, 4:05.02, 3, Bengt Gingsjo, Sweden, 4:09.72, 4, Jim Green, Canada, 4:15.47, 5, Krasimir Enchev, Bulgaria, 4:20.27, 6, Gullermo Pacheca, Peru, 4:20.09

THIRD HEAT — 1, Bruce Furniss, United States, 4:03.31. 2, Werner Lape, West Germany, 4:07.37. 3, Max Metzker, Austrolia, 4:08.53. A Brett Taylor, New Zealand, 4:09.24. 5, Henk Etzer-man, The Netherlands, 4:12.52. 6, Tomas Becerra, Calarbia (2020) Colombia, 4:22.00

FOURTH HEAT — 1, Tim Shaw, United States, 4:04.23. 2, Graham Windeatt, Australia, 4:04.76. 3, Andrei Krylov, Saviet Union, 4:05.62. 4, Marc Laz-zard, France, 4:07.70. 5, Josy Wilwert, Luxem-bourg, 4:28.91.

Women (PRELIMINARIES)

100-meter Butterfly

FIRST HEAT — 1, Jill Symans, United States, 1:03.56. 2, Gunilla Andersson, Sweden, 1:05.28. 3, Natalia Popova, Saviet Unian, 1:05.41. 4, Flavia Nadaluti, Romania, 1:05.90. 5, Eva Lyndanvi, Hungary, 1:06.50. 6, Mantserrat Maja, Spain, 1:07.41 1:07.41.

1:U.41. SECOND HEAT 1, Comilie Wright, United States, 1:03.74. 2, Maria de Milagros Paris, Costa Rica, 1:04.48. 3, Tamara Shelafastava, Saviet Unian, 1:04.48. 3, Tamara Shelafastava, Saviet Unian, 1:04.97. 4, Kuniko Banno, Japan, 1:05.09. 5, Rosemarie Ribeiro, 1:05.41. 6, Schiavon Donatella, Italy, 1:06.15. 7, Beate Jasch, West Germany, 1:07.09. 8, Maria Teresa Carrera, Ecuador, 1:14.05.

1:14.05. THIRD HEAT — 1, Rosemarie Kother, East Germany, 1:02.83, 2, Gudrun Beman, West Ger-many, 1:04.23, 3, Lynda Hanel, Australia, 1:04.62, 4, Wendy Quirk, Canada, 1:04.73, 5, Lynne Rawe, New Zealand, 1:05.23, 6, Joanne Atkinson, Great Britain; 1:06.36, 7, Cinzia Rampazza, Italy, 1:06.47, 8, Maria Matilde Menocal, Colombia, 1:10.43."

o, muria maniae Menocal, Colombia, 1:10.43. FOURTH HEAT — 1, Kornella Ender, East Germany, 1:02.86. 2, Barbara Clark, Conada, 1:04.85. 4, Mina Stove, Australia, 1:04.85. 4, Mon-ique Rodahi, New Zealand, 1:05.27. 5, Yasue Hathuda, Japan, 1:05.88. 6, Anne Adams, Great Britain, 1:05.97. 7, Jose Damen, The Netherlands, 1:06.45.

400-meter Individual Medley

FIRST HEAT — 1, Kathy Heddy, United Stotes, 5:04.25. 2, Judy Hudson, Australia, 5:05.47. 3, Liz McKinnon, Canada, 5:05.71. 4, Deborah Simpson, Great Britain, 5:17.25.

SECOND HEAT — 1, Jenni Franks, United States, and Karlo Linke. East Germany, tied for first, 5:04.70. 3, Susan Richardsson, Great Britain, 5:10.54. 4, Marina Moliutina, Soviet Union, 5:14.40. 5, Colette Grabbe. Belgium, 5:21.42. 6, Birgit New-man, West Germany, 5:22.01.

THIRD HEAT — 1, Ulrike Tauber, East Ger-many, 5:00.55. 2, Cheryl Gibson, Canoda, 5:04.39. 3, Susan Hunter, New Zealand, 5:07.53, 4, Paola Marozzi, Italy, 5:32.39.

'FINALS'

200-meter Backstroke

1, Zoltan Verrasito, Hungary, 2:05.05. 2, Mark Tonnelli, Australia, 2:05.78. 3, Paul Hove, USA, 2:06.49. 4, Roland Matthes, E. Germany, 2:07.09. 5, Soltan Rudalf, Hungary, 2:07.15. 6, Santiaga Esteva, Spain, 2:09.21. 7, Abdul Karin Ressana, Holland, 2:09.78. 8, James Carter, G. Britain, 2:11.67. 2:31.67.

400-meter Freestyle

Tim Shaw, USA, 3:54.88 (Betters meet rec-ard of 3:58.18, set by Rick Demont, USA, 1973). 2, Bruce Furniss, USA, 3:57.71. 3, Frank Pfutze; E. Germany, 4:01.10. 4, Graham Windeatt, Australia, 4:02.72. 5, Gardon Downle, G. Britain, 4:02.88.6, Rainer Strahbach, E. Germany, 4:05.00.

(FINALS)

100-meter Butterfly

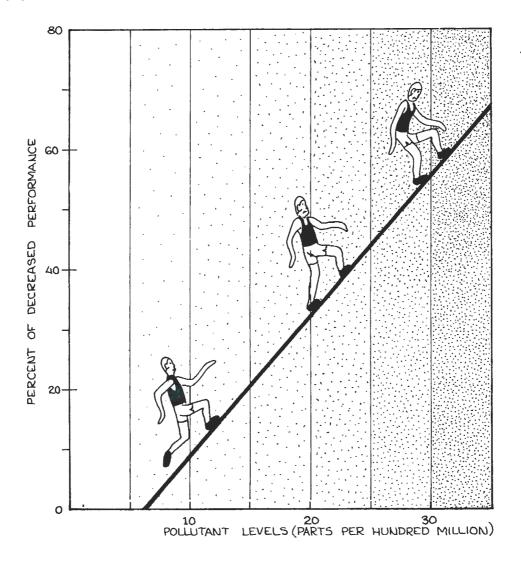
1, Kornelia Ender, E. Germany, 1:01.24 (Betters world record of 1:01.33, set by Korenila Ender, E. Germany, 1975). 2, Rosemarie Kother, E. Germany, 1:01.80. 3, Camille Wright, USA, 1:02.79. 4, Jill Symons, USA, 1:03.51. 5, Maria Dei Milagro Paris, Costa Rica, 1:03.86. 6, Barbard Clark, Canada, 1:04.06. 7, Gudrun Beckmann, W. Germany, 1:04.29. 8, Lynda Hanel, Australia, 1:04.33.

400-meter Individual Medley

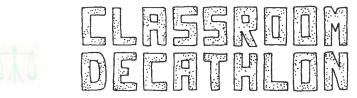
1. Ulrike Tauber, E. Germany, 4:52,76 (Betters meet recard of 4:57,51, set by Gudrun Wegner, E. Germany, 1973). 2. Karlä Linke, E. Germany, 4:57,83. 3. Kathy Heddy, USA, 5:00.46. 4, Jenny Franke, USA, 5:03.15. 5, Liz Mackinnon, Canada, 5:03.81. 6, Susan Hunter, N. Zealand, 5:03.85. 7, Cheryl Gibson, Canada; 5:04.04. 8, Judy Hudson, Australia, 5:04.99.



Showing the relationship of sports to other problems in society can be interesting. See the graph below. Questions could be made from the chart for use by you or your students.



This chart shows the effects of certain air pollutants on the performance of crosscountry track teams as pollution in an area became worse. Pollution levels were recorded one hour before each meet.



This activity consists of ten events for you to perform. The best performance in each event will be scored 100 points. The remaining performances are scored as a percent of the best performance.

The ten events of this decathlon are:

- Cotton ball throw
 Throw a ball of cotton as far as you can. Measure
 the distance from the starting line to the point
 where the cotton ball first touches the floor.
- 2. Basketball put

In the gym or outside throw a basketball as far as you can. Use a motion like a shot putter. Measure the distance from the starting line to the point the basketball first touches the floor.

- 3. Standing long jump Jump as far as you can from a standing start. Measure the distance from the starting line to the back of your heels where you jump.
- 4. Tiddley-wink snap Snap a tiddley-wink as far as possible. Measure the distance from the starting line to the point where the tiddley-wink first touches the floor.

0 0

- 5. Twenty-metre, one-legged dash In the gym or outside record the time it takes to hop 20 metres while holding one leg up off the floor.
- 6. Plastic straw throw Throw a plastic straw as far as possible. Measure the distance from the starting line to the point where the straw first touches the floor.
- Fifty-metre backwards run Outside, record the time it takes to run 50 metres backwards.
- 8. Airplane fly In the gym or outside throw a paper airplane as far as you can. Measure the distance from the starting line to the point where the plane first touches the floor.
- 9. Rubber band snap Snap a rubber band as far as you can. Measure the distance from the starting line to the point where the rubber band first touches the floor.
- 10. Paper toss score

From a distance of 4 metres record how many wads of paper you can throw, 1 at a time, into a wastepaper basket in 30 seconds.

Pick a partner. For each of the ten events, as one person is performing, the partner will be either timing or measuring distances. Each person will perform three separate trials in each event. Label and record the trials on your individual record sheet.



PERSONAL RECORDS OF CLASSROOM DECATHLON

NAME:

PARTNER:

	Trial l	Trial 2	Trial 3
EVENT:			

Circle your best performance for each event.

843



PERSONAL RECORD OF CLASSROOM DECATHLON

Final Summary Card

NAME:			
PARTNER:			
EVENT:	BEST:	SCORE:	
		TOTAL SCORE:	

The best performance in the class in each event is assigned a value of 100 points. Each remaining performance is compared to the best performance to determine a score. For example, if the best distance in the cotton ball throw is 8.0 metres, it would be worth 100 points. The score of a 6.5 metre throw would be as follows:

 $\frac{6.5}{8.0} = \frac{\chi}{100} \implies 8.0\chi = 650 \implies \chi = 81.2 \text{ OR B1 POINTS}$

- Stadente can find the range of the total scores to check the synamic. If there are extremes, the arised range could be used.
- toral scenes.
- 3) To display the results of individual counts clobares day rade a bar graph. The data should be grouped by intervalue.
- 4) The distribution of spaces for an even public so mailed of flating the composided deviation from the mean. See comp Sector will a the propheter IDS TEACHERS. Method.



Charts or tables enable a person to find facts rapidly because the information is well-organized and kept as brief as possible. Use the information in this chart to answer these questions.

PARTICIPATION IN SELECTED OUTDOOR RECREATION ACTIVITIES: 1972

ACTIVITY	Participants (millions)	Participants as percentage of total population	
1) Camping in			
remote or wilderness areas	7.7	5	IFN
Camping in developed camp grounds	17.5	11	10 0107
Hunting	22.2	14	
0	38.0	24	17-1
Fishing			
Riding motorcycles off the road	7.4	5	A A
2) Hiking with a pack,		_	
mountain/rock climbing	8.6	5	
Nature walks	26.7	17	
Walking for pleasure	54.2	34	
Bicycling	16.7	10	
Horseback riding	8.7	5	
·	8.5	5	
3) Water skiing Sailing	4.1	3	J UE
•	23.3	15	
Other boating		18	
Outdoor pool swimming	28.5		
Other swimming	53.8	34	
4) Golf	7.7	5	
Tennis	8.6	5	8
Playing other			
outdoor games or sports	35.0	22	S
5) Going to outdoor sports events Visiting zoos, fairs,	. 18.9	12	井1 1
amusement parks	38.7	24	
	59.8	37	
Sightseeing	74.4	47	- [A] - E
Picnicking	54.5	34	
Driving for pleasure			6, 10
6) Snow skiing	7.2	5	T B
Snowmobiling	7.2	5	
Other winter sports	24.8	16	363
Data from <u>United States Statistical Abstr</u> a) What is the most popular activity			
b) The second most popular?			
	ten una signi		
c) What fraction of the U.S. populat fishing as an activity?	tion uses pichi	.CKing as an activity:	
d) Do more people camp in campground	ls or wildernes	s areas?	
e) Do more people hunt or fish?	Snow	ski or water ski?	
f) What fraction of the population r	ides bicycles?		
g) Do twice as many people bicycle a	is play golf?		
h) Are golf, tennis and fishing all			
i) How many millions of people parti			
j) The population of the United Stat for the number of participants to	es is about 22 be larger that	20,000,000 people. Why is an the U.S. population?	it possible



Ralph wanted to organize the number of home runs his heroes made. He made this stem-and-leaf display. It shows the total home runs hit by the champion home run hitters in the American League, 1920-1974.

HOME RUN LEADERS

	National League
Year	
1920	C. Williams15
1921	G. Kelly23
1922	R. Hornsby42
1923	C. Williams41
1924	J. Fournier27
1925	R. Hornsby
1926	H. Wilson21
1927	H. Wilson, C. Williams 30
1928	H. Wilson, J. Bottomley31
1929	C. Klein43
1930	H. Wilson56
1931	C. Klein
1932	C. Klein, M. Ott
1933	C. Klein28
1934	Collins, M. Ott35
1935	W. Berger
1936	M. Ott
1937	M. Ott, J. Medwick31
1938	M. Ott
1939	J. Mize28
1940	J. Mize43
1941	D. Camilli
1942	M. Ott
1943	B. Nicholson29
1944	B. Nicholson33
1945	T. Holmes
1946	R. Kiner23
1947	R. Kiner, J. Mize51
1948	R. Kiner, J. Mize40
1949	R. Kiner
1950	R. Kiner
1951	R. Kiner
1952	R. Kiner, H. Sauer37
1953	E. Mathews47
1954	T. Kluszewski49
1955	W. Mays51
1956	D. Snider43
1957	H. Aaron
1958	E. Banks
1959	E. Mathews46
1960	E. Banks
1961	0. Capeda
1962	W. Mays
1963	H. Aaron, W. McCovey44
1964	W. Mays
1965	W. Mays
1966	H. Aaron, W. McCovey44
1967	H. Aaron
1968	W. McCovey
1969	W. McCovey45
1970	J. Bench
1971	W. Stargell
1972	J. Bench
1973	W. Stargell44
1974	M. Schmidt36

STEM	LEAF
2	2,4
2	
З	3, 4, 2, 3, 2, 2, 2, 3, 2, 2,
3	9, 6, 5, 7, 6, 9, 7, 7, 7
	1, 1, 4, 3, 3, 2, 2, 2, 0, 4, 4, 4
	6, 7, 6, 9, 6, 8, 9, 6, 6, 8, 5, 9, 9, 9
5	4,4,2
5	9, 8, 8
6	0, 1

1) Make a stem-and-leaf display for the home run totals of the National League.

STEM	LEAF	
1		
<u></u>		

- 2) Is your display for the National League similar to Ralph's for the American League?
- 3) Were the home run totals higher for the National League?





100cm

WHICH BALL BOUNCES THE HIGHEST

Materials needed: Any of these: soccer ball, table tennis (ping pong) ball, basketball, tennis ball, volleyball, softball, golf ball.

HEIGHT

AT

FIRST

BOUNCE

metre stick masking tape

I. Select one of the balls for the first experiment. Attach the metre stick to a table or desk, so the stick stands vertically.

II. Drop the ball from the 100 cm mark. Have your partner measure how high it bounces. Read the mark to the nearest cm at the bottom of the ball. Record in the table to the right. Drop the ball from the

other heights listed and record the bounces. ocm BALL 100

BOUNCED (CM)

BALL

HEIGHT 20

90

80

70

60

50

40

30

10

0

III. Plot the points of your table on this graph.

IV. The points probably do not lie in a straight line, but they do follow a pattern. Connect the points.

V. Use the graph to predict the bounce of a ball dropped from 90 cm, from 120 cm.

VI. Try the same experiment with another ball. Record and then graph the height of the bounces. Do both balls follow the same kind of pattern?

VII. For a given height, which ball has the highest bounce?

0 10 20 30 40 50 60 70 80 90 100 110 120 HEIGHT BALL DROPPED (cm)

BALL

HEIGHT

BOUNCED

BALL

(Cm)

HEIGHT

BALL

DROPPED

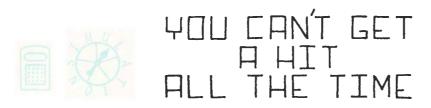
(cm)

100

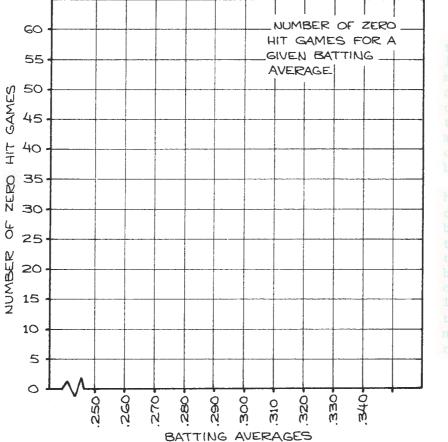
80

60

50 40 20



- 1) Run the computer program BATTER ten times. Save the output.
 - a) Change the batting average for each time. Use .250, .260, .270, .280, .290. .300, .310, .320, .330, .340.
 - b) Keep the number of games played the same. Use 182 each time.
- 2) Make a line graph below showing the number of games out of 182 that the batter got zero hits.



tical scale, similar line graphs could be made for 1, 2, 3, 4 and 5 hit games. Stu dents could analyze t ten outputs and deter mine the mode number of hits for each given batting average. A

hit games for a student's favorite baseball player to maintain his present batting average could be determined by the computer. Simply input the present batting average and the number of games remaining in the seaso

- 3) From the graph:
 - a) Predict the number of zero hit games for a .350 hitter.

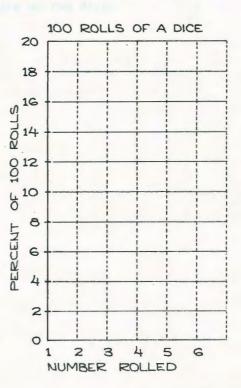
1

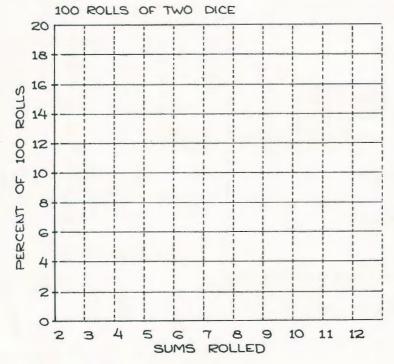
 b) A batter got zero hits in 20 games. His batting average is about

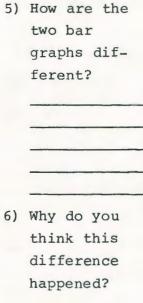
TYI Work weet



- Run the computer program called TOSSID. Let N and M be 100.
- 2) Make a bar graph to the right showing the percent of the 100 rolls that were a 1, 2, 3, 4, 5 or 6.
- Run the computer program called TOSS2D. Toss the die 100 times.
- Make a bar graph below showing the percent of the sums that were 2, 3, 4, ..., 11 or 12.









Sue and Hal were asked to judge a dance contest. The best dancer was given a 1. The second best dancer was given a 2 and so on.

DANCERS	A	в	с	D	ε	न	G	н	I	J
SUE	6	з	1	7	9	5	2	8	10	4
HAL	5	4	1	6	10	8	з	7	9	2



- Which dancer(s) did Sue and Hal exactly agree about?
- 2) For which dancer(s) are the scores farthest apart?
- 3) On the grid plot the scores of each dancer. Be sure to number the scales on the graph.

HAL'S SCORES

 Draw a trend line which best shows the relationship between the ratings.

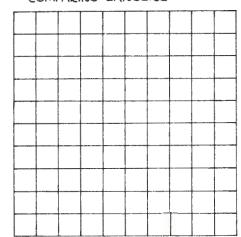
Extension:

1) Choose ten pop recording artists or rock groups. Rate them from 1 to 10, giving the one you like the best a mark of 1; the second best a 2; and so on.

2) Get a partner to do the same thing with the same artists.

- 3) Make a table of the results.
- 4) Draw a scatter diagram.
- 5) See if it is possible to find a trend line.
- 6) Do you and your partner agree or disagree on the groups?

COMPARING DANCERS

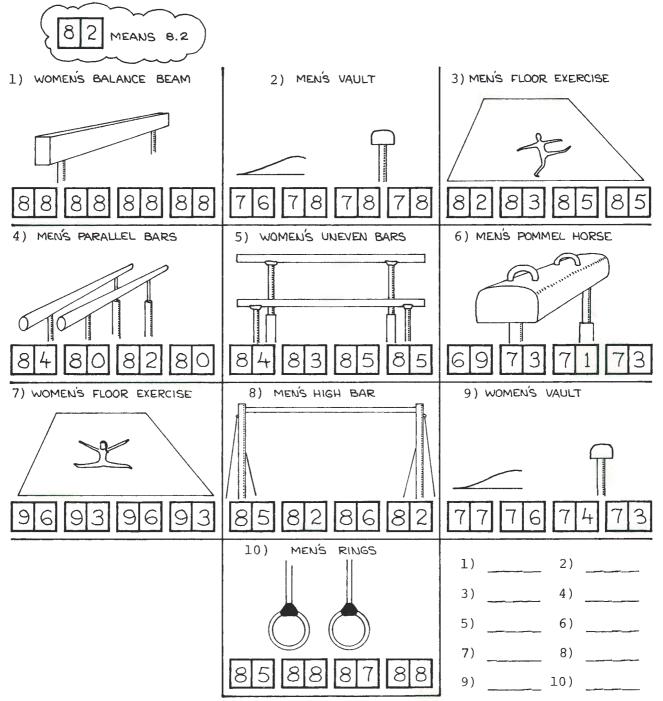


SUE'S SCORES

Wa

JUDGING GYMNASTICS IN THE OLYMPICS

In the Olympics the scores of four judges are used to rate a gymnast. The high score and the low score are dropped. The other two scores are averaged. Find the score for each of these events. Write the scores at the bottom of the page.



GYMNASTICS IN HIGH SCHOOL

In high school meets, the ratings of three judges are used to score a gymnast. Two methods are used.

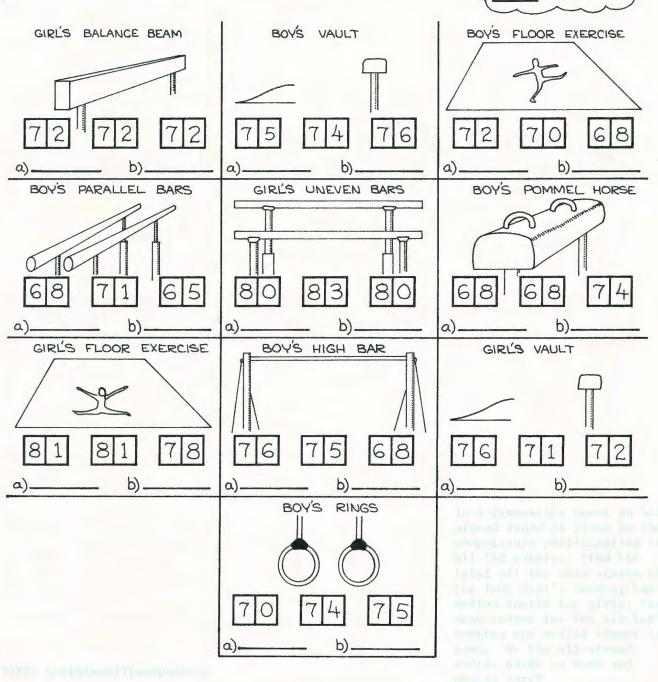
a) All three scores are averaged. The mean score is used.

2

MEANS 7.2

b) Only the median score is used.

Use both methods to find the score for each of these events.



STATISTICS: ORSEORLL

To calculate a player's batting average divide the number of hits (H) by the number of times at bat (AB). Times at bat does not include walks, hit-by-pitcher or sacrifices.

Find the batting average for each of the following, accurate to three decimal places.

Player and Team	(G) Games	(AB) At Bat	(H) Hits	Batting Average
Willie Stargell, Pittsburgh	148	522	156	,299
Cesar Cedeno, Houston	139	525	168	- 320
Dave Roberts, San Diego	127	479	137	. 286
Pete Rose, Cincinnati	160	680	230	-338
Lou Brock, St. Louis	160	650	193	1297
Dusty Baker, Atlanta	159	604	174	. "89



The following six baseball clubs make up the eastern division of the National Baseball League. Compute the percent of games won for each team. Who was the eastern division winner?

		Won	Lost	Percent Won	
	Montreal	79	83		\bigcap
	Pittsburgh	80	82	- 19-	
	Philadelphia	71	91	418	
	New York	82	79	-309	
	St. Louis	81	81		
B	Chicago	77	84	678-	8



457

During the 1921 season, Babe Ruth got 85 singles, 44 doubles, 16 triples, and 59 home runs. A single is worth one base, a double two bases, a triple three bases and a home run is worth four bases.

HIT	FREQUENCY	TOTAL BASES
Singles	85	85
Doubles	44	88
Triples	16	48
Home Runs	59	236

RUTH'S 457 TOTAL BASES STILL STANDS AS A RECORD FOR THE MOST TOTAL BASES IN A SINGLE SEASON.



To find a baseball player's slugging average divide the "times at bat" into the total bases the player has. Calculate the slugging average for the following baseball players.

PLAYER	AT BATS	SINGLES	DOUBLES	TRIPLES	HOME RUNS	TOTAL BASES	SLUGGING AVERAGE
Gonzales	352	70	23	11	26		. 1
Jones	292	37	16	5	12		
Marsh	470	58	21	13	22		
Segovia	468	63	31	18	27		
Loren	204	28	19	6	6		1.000

BABE RUTH HAS THE HIGHEST LIFETIME SLUGGING AVERAGE OF .690 FOR 21 YEARS OF PLAYING IN THE MAJOR LEAGUES.



Each week during the football season, collect the scores involving the local high school team. You may wish to use professional or college scores. Put them together at the end of the season (see sample below) and have students do the following.

1. Place the winning scores in order--largest to smallest.

2. Write the highest winning score.

Write the lowest winning score.

Find how far apart the highest and lowest winning scores are.

(RANGE)

- Determine the most frequent winning score. (MODE) 3.
- Determine the median winning score. (MEDIAN) 4.
- 5. Determine the mean winning score. (MEAN)
- Arrange the LOSING scores in order--largest to smallest. 6.
- Answer 2, 3, 4, and 5 for the losing scores. 7.
- Were there any games where the winning and losing scores were the 8. same as the median winning and losing scores?

District 5AAA Football Scores Week 2

Week 3 North Eugene 14, Cottage Grove 7

North Eugene 14, Cottage Gro Sheldon 22, South Eugene 21

Thruston 22, Willamette 8

Marshfield 26, Churchill 6

Thurston 28, Sheldon 8 Marshfield 21, Cottage Grove 12 North Bend 34, Churchill 18 South Eugene 26, Willamette 8 Springfield 12, North Eugene 7

Week 1

North Bend 22, Sheldon 20 Churchill 21, South Eugene 20 Willamette 29, North Eugene 8 Cottage Grove 40, Springfield 18 Marshfield 59, Thurston 28

Week 5

North Bend 24, Springfield 0 Week 6 Willamette 27, Sheldon 22 North Eugene 28, Churchill 6

Churchill 25, Springfield 0 Marshfield 13, Sheldon 12 South Eugene 34, Cottage Grove 26 Cottage Grove 7, Thurston 0 Thurston 16, North Eugene 14 North Bend 40, Willamette 26

Week 4

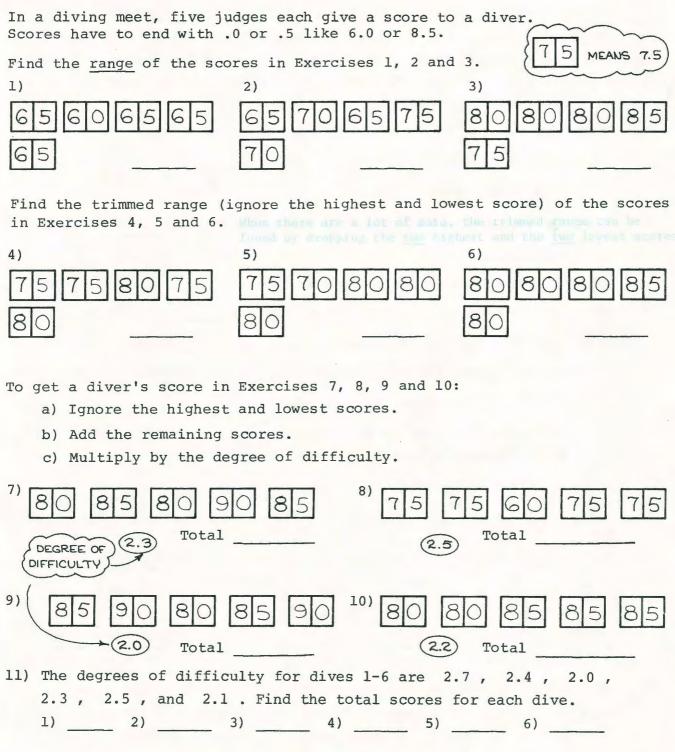
North Bend 42, North Eugene 20 South Eugene 36, Marshfield 15 Sheldon 12, Springfield 7 Churchill 35, Willamette 21

South Eugene 28, Thurston 21 Marshfield 20, Springfield 0 North Bend 34, Cottage Grove 21

- Use the scores to make league standings for each of the weeks of the 9. season. Include the number of games won, number of games lost and the percent of games won expressed as a 3-place decimal like .750.
- Were the scores of the first-place team all above the mode of the 10. winning scores? median? mean?

)





12) The odd-numbered dives (1,3,5,7,9) were done by diver A. The evennumbered dives (2,4,6,8,10) were done by diver B. Which diver won the diving competition?



Read all the instructions before you go to the computer.

To play poker with several friends (include the computer as a player):

1) Run the computer program SHUFLE.

The output lists the numbers from 1 to 52 in a random arrangement. A sample is shown below:

38 48 19 42 50 37 43 35 39 7 51 23 29 41 6 21 52 5 16 30 27 22 3 26 34 17 49 33 46 24 18 32 40 2 4 14 31 13 15 44 8 10 12 36 9 25 28 20 47 45 11 1 The numbers 1-13 are the Ace of Spades, 2S, ..., JS, QS, and King of Spades; 14-26 are the Ace of Hearts, 2H, ..., JH, QH, and King of Hearts; 27-39 are the Diamonds; and 40-52 are the Clubs.

- 2) From the computer printout, pick your numbers just like you were dealing. For example, if there are three players, player 1 gets the 1st (38), 4th, 7th, 10th and 13th numbers. Player 2 gets the 2nd (48), 5th, 8th, 11th and 14th numbers. Player 3 gets the 3rd (19), 6th, 9th, 12th and 15th numbers.
- 3) Write your numbers and actual cards in tables like the ones shown. A three-person game is using the output listed above.

Card Number	Actual Card
38	QD
42	3C
43	4C
7	75
29	3D

Computor

rlay	erz
Card Number	Actual Card
48	9C
50	JC
35	9D
51	QC
41	2C

Player 3				
Card Number	Actual Card			
19	6н			
37	JD			
39	KD			
23	10 H			
6	<u>65</u>			

- 4) Figure out the best hand. Computer has a pair of 3's. Player 2 has a pair of 9's. Player 3 has a pair of 6's. Player 2 wins.
- 5) The ranking of winning hands in poker is shown to the right.
- 6) Overall, do you think the computer has the highest probability of winning? Keep a tally of games won by each player to find out.

Royal flush Straight flush 4 of a Kind Full House Flush Straight 3 of a Kind 2 Pairs 1 Pair High Card ridge player, you ign want to as the computer to run off a orige of bridge hands dier players if they they players if they high the hands were a stard a sum of the and usually dealt be be players the monthly

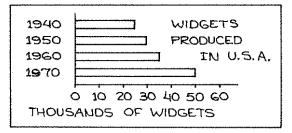
ORGANIZATION

Placement Guide for Tabbed Dividers

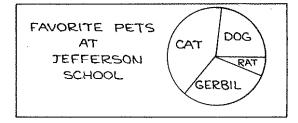
Tab Label	Follows page:
CONTENT FOR TEACHERS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
Probability with Counting	
Inferential Statistics	
Appendix-Computer Programs	
DIDACTICS	
Components of Instruction	
Classroom Management	
Statistics and Probability Learning	
TEACHING EMPHASES	
Critical Thinking	
Decision Making	
Problem Solving	
Models and Simulations	
Calculators and Computers	
Laboratory Approaches	
CLASSROOM MATERIALS	
CONCEPTS AND SKILLS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Misleading Statistics	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	604
Probability with Models	634
Counting Techniques	
APPLICATIONS	
Business & Commerce	
Environment	
Health & Medicine	
People & Culture	
Recreation	
GLOSSARY	
ANNOTATED BIBLIOGRAPHY	
SELECTED ANSWERS	



- almanac. A source book, usually published annually, that contains statistical and general information.
- arithmetic average. The mean of a set of data.
- assumed mean. An estimate of the mean of a set of data. It is used to calculate the true mean.
- average. 1. noun: Usually the mean of a set of data. Sometimes used for median or mode. 2. verb: To calculate the mean of a set of data.
- bar graph. A graph that uses rectangular bars to represent data. The bars may be vertical or horizontal.



- central tendency. The tendency of a distribution to cluster about some "middle" or "average" value. Measures of central tendency are the mean, median and mode.
- chance. 1. An everyday word for probability as in "There is 30% chance of rain." 2. Opportunity or possibility. "There are 3 chances to draw a red marble."
- circle graph. A graph in the shape of a circle that shows "parts to whole" data.

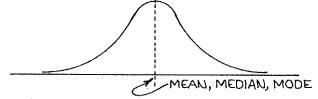


combination. A group of items chosen from a set. The items in a combination have no particular order.

- correlation. A measure expressing the extent to which two variables are related, for example, height and weight.
- data. Facts (often numerical) used to understand situations, make decisions or support positions. Used commonly as both singular and plural.
- *decile.* One of the numbers dividing an ordered distribution into ten parts of equal frequency. The fifth decile is the median.
- deviation. See mean absolute deviation and standard deviation.
- equally likely. Having the same chance or probability of occurring.
- event. A set of one or more possible outcomes of an experiment.
- experiment. A situation involving chance or probability. For example, a die is rolled once, a coin is tossed three times or a marble is drawn from a jar.
- fair coin. A coin where heads and tails have the same chance of occurring.
- fair die. A die where each face has the same chance of showing.
- fair game. A game is fair if each player has the same chance of winning (assuming both players have the same ability in playing the game).
- firsthand data. Data collected by the individual by observation or by surveying.
- frequency. The number of items in a category.
- frequency table. An organization of data showing the frequency for each value or category.

alm-fre 859

- fundamental counting principle. A rule for counting: The total number of ways of making several decisions in order is the product of the number of ways to make each decision.
- mean. The average of a set of numerical items. The mean is computed by finding the sum of the items and dividing by the number of items.
- mean absolute deviation. The average distance of the scores from the mean. It describes the amount of spread in a distribution.
- *median.* The middle item in an ordered set of data. If the data has an even number of items the median is the average of the two middle items.
- *mode.* The item occurring most often in a distribution.
- normal curve. A bell shaped curve as shown.



- normal distribution. A distribution whose graph is a normal curve. The mean, median and mode are equal for a normal distribution.
- odds. For equally likely outcomes, the ratio of the number of favorable outcomes to the number of unfavorable outcomes.
- outcome. One of the possible results of an experiment.
- percent bar graph. A graph where the entire rectangular bar represents 100%. Also known as an area bar graph.

MADISON	SCHOOL'S I	AVORITE	SINGER
BARBRA	лног	MOTT	
STREISAND	DENVER	THE	OTHER
25%	, 30%	HOOPLE 20%	25%

- percentile. One of the numbers dividing an ordered distribution into 100 parts of equal frequency. The 50th percentile is the median.
- permutation. An ordered arrangement of items chosen from a set.

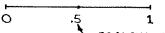
permute. To change the order.

pictograph. A graph that uses pictures to represent the data.

to to represent the data.
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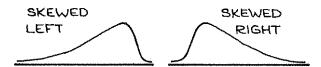
pie chart. A circle graph.

- *poll.* A survey of selected people to obtain information.
- population. The set of all items being considered for an investigation.
- probability. 1. A number assigned to an event to measure its likelihood of occurring. The probability of an event is always between 0 and 1 inclusive. 2. The branch of mathematics that studies probabilities.
- probability scale. A scale extending from 0 to 1 on which the probability of an event can be represented.

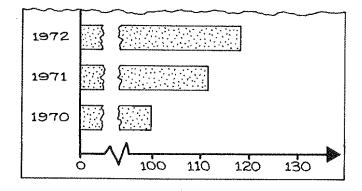


- 50% CHANCE OF RAIN quartile. One of the numbers dividing an ordered distribution into 4 parts of equal frequency. The 2nd quartile is the median.
- random. Items are arranged at random if there is no predetermined pattern for the arrangement and each item has the same chance of occurring in any position.

- random number table. A list of digits where each digit has an equal chance of appearing in any position of the table.
- range. The difference between the highest and lowest scores in a set of data. Sometimes the range is given as the highest and lowest scores.
- relative frequency. The ratio of the number of occurrences of an event to the total number of trials.
- running total. In a set of data, the sum of the items up to and including a given item.
- sample. A set of items selected from a population. Often used to make a prediction about the population.
- sample space. The set of possible outcomes in an experiment.
- sampling. The process of taking a sample (to predict a population).
- scatter diagram. A graph showing the relationship (or lack of) between two variables.
- secondhand data. Data collected by another person. Secondhand data is found in written records as health records, birth certificates and source books such as almanacs.
- simulation. The process of representing one situation with another to gather information about the original situation.
- skewed. Not symmetrical. Extending to one side.



- standard deviation. A number which describes the amount of spread in a distribution. Its calculation involves squares and square roots.
- statistics. 1. The study of numerical data to obtain information. 2. Another word for data.
- stem-and-leaf display. A frequency
 table from which individual data can
 be read.
- survey. The process or product of gathering information from a group of people about themselves or about a topic. Interviews and questionnaires are types of surveys.
- trend line. A line on a scatter diagram that best shows the relationship between two variables.
- trial. Doing an experiment once. For example, flipping a coin once or rolling a die once.
- trimmed range. The range of a set of data after one or more of the highest and the same number of the lowest scores have been omitted.
- zero break. The break in a number line to show a piece of the number line has been omitted. Used often in graphs.



ORGANIZATION

Placement Guide for Tabbed Dividers

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The following is a list of sources used in the development of this resource. It is not a comprehensive listing of materials available. In some cases, good sources have not been included simply because the project did not receive permission to use the publisher's material or a fee requirement prohibited its use by the project.

ANNOTATED BIBLIOGRAPHY

ACTIVITIES IN MATHEMATICS, Second Course. Donovan A. Johnson, Viggo P. Hanson, Wayne H. Peterson, Jesse A. Rudnick, Ray Cleveland, and L. Carey Bolster. Glenview, Illinois: Scott, Foresman and Company, 1971. (Scott, Foresman and Company, 1900 East Lake Ave., Glenview, IL 60025)

375 pp; hardbound; color; teacher reference; medium reading level

ACTIVITIES IN MATHEMATICS consists of four units. The graphing unit includes plotting points and drawing line graphs of data. Gathering data, making bar graphs and investigating counting techniques are covered in the statistics unit. The four units are consolidated in hardbound and separated in paperbound.

APPLICATIONS IN MATHEMATICS, Course A, Sampling and Statistics. Donovan A. Johnson, Viggo P. Hanson, Wayne H. Peterson, Jesse A. Rudnick, Ray Cleveland, and L. Carey Bolster. Glenview, Illinois: Scott, Foresman and Company, 1972. (Scott, Foresman and Company, 1900 East Lake Ave., Glenview, IL 60025)

62 pp; paper; color; textbook; high reading level

This booklet offers an activity-oriented approach to sampling and statistics. A survey sheet is used to gather data on the class; this information is then used to study graphing and measures of central tendency.

APPLICATIONS IN MATHEMATICS, Course B, Decision Making. Donovan A. Johnson, Viggo P. Hanson, Wayne H. Peterson, Jesse A. Rudnick, Ray Cleveland, and L. Carey Bolster. Glenview, Illinois: Scott, Foresman and Company, 1972. (Scott, Foresman and Company, 1900 East Lake Ave., Glenview, IL 60025)

62 pp; paper; color; textbook; high reading level

This booklet teaches many decision-making skills (estimation, sampling, making drawings, using statistics, taking surveys, etc.) and how to use them to solve problems.

AVERAGES. Jane Jonas Srivastava. New York: Thomas Y. Crowell Company, 1975. (Copyright now held by Harper and Row, Publishers, Inc., 10 E. 53rd St., New York, NY 10022)

33 pp; hardbound; color; student references; low reading level

Mean, Median and Mode are explained in a simple but delightful way.

BUILD-IT-YOURSELF SCIENCE LABORATORY. Raymond Barrett. Garden City, New York: Doubleday and Company, Inc., 1963. (Doubleday and Company, Inc., 245 Park Ave., New York, NY 10017)

340 pp; paper; teacher reference; b/w

This book contains a collection of science activities that can be set up and performed using homemade, inexpensive equipment.

CANADA 1974, English Edition. Constance McFarland and Margaret Johnstone, ed. Ottawa, Canada: under the authority of the Minister of Industry, Trade and Commerce, 1973. (Ministry of Industry, Trade and Commerce, Ottawa, Ontario KlA OH5, Canada)

348 pp; paper; color; teacher reference; medium reading level

The book contains information on Canada's geography, environment, history, religion and economics. Much statistical material in the form of tables and graphs is included.

CENTERS AND SPREADS. Oakland County Mathematics Project. Pontiac, Michigan: Oakland County Mathematics Project, 1971. (Oakland Schools, 2100 Pontiac Lake Rd., Pontiac, MI 48054)

57 pp; paper; b/w; textbook; high reading level

This is an excellent source of activities on measures of central tendency, range, mean deviation and standard deviation.

CHANGING TIMES, The Kiplinger Magazine. Editors Park, Maryland: The Kiplinger-Washington Editors, Inc. (The Kiplinger- Washington Editors, Inc., 1729 H. St., N.W., Washington, D.C. 20006)

Suggestions and cautions to consumers form the basis for this magazine. Many tables, graphs and statistics that can be used in a mathematics classroom are included in each issue.

COLLECTING, ORGANIZING AND INTERPRETING DATA. (Topics in Mathematics, Booklet 16) Reston, Virginia: National Council of Teachers of Mathematics, 1969. (National Council of Teachers of Mathematics, 1906 Association Dr., Reston, VA 22091)

42 pp; paper; teacher reference

This is one of a series of topical booklets in mathematics for elementary school teachers. It describes methods of gathering, organizing and interpreting data and gives ideas for classroom activities and discussions.

COMPARISON CARDS. See U.S. RDA Comparison Cards and Guide for Teachers and Other Readers. COUNTING AND CHOOSING. Oakland County Mathematics Project. Pontiac, Michigan: Oakland County Mathematics Project, 1972. (Oakland Schools, 2100 Pontiac Lake Rd., Pontiac, MI 48054)

34 pp; paper; b/w; textbook; medium reading level

Simple ideas about permutations and combinations using the multiplication principle and Pascal's triangle are included in this book.

- "Deciding Authorship." Frederick Mosteller and David L. Wallace. STATISTICS: A GUIDE TO THE UNKNOWN. Judith M. Tanur, editor. San Francisco: Holden-Day, Inc., 1972. (Holden-Day, Inc., 500 Sansome St., San Francisco, CA 94111)
- EMERGENCY CARE AND TRANSPORTATION OF THE SICK AND INJURED. Chicago: American Academy of Orthopaedic Surgeons, 1971. (American Academy of Orthopaedic Surgeons, 430 North Michigan St., Chicago, IL 60611)
- ENVIRONMENTAL SCIENCE. (Probing the Natural World/ Level III) Intermediate Science Curriculum Study. Morristown, New Jersey: Silver Burdett General Learning Corporation, 1972. (Silver Burdett Company, 250 James St., Morristown, NJ 07960)

146 pp; paper; color; workbook; medium reading level

One of a series of eight student workbooks, this book explores plagues, pollution, and population problems.

- ESTIMATES OF SCHOOL STATISTICS, 1974- 1975. Washington, D.C.: National Education Association, 1975. (National Education Association, Research Division, 1201 16th St., N.W., Washington, D.C. 20036)
- "Estimating the Size of Wildlife Populations." Samprit Chatterjee. Exploring Data, from the program STATISTICS BY EXAMPLE. Edited by F. Mosteller, W. Kruskal, R. Link, R. Pieters, and G. Rising. Copyright (C) 1973 by Addison-Wesley Publishing Company, Inc. (Addison-Wesley Publishing Company, Inc., Sand Hill Rd., Menlo Park, CA 94025)
- EXPLORATIONS IN MATHEMATICS. J. Arthur Wiebe and James W. Goodfellow. New York: Holt, Rinehart and Winston, Publishers, 1970. (Holt, Rinehart and Winston, Publishers, 383 Madison Ave., New York, NY 10017)

504 pp; hardbound; color; textbook; high reading level

This textbook for a general mathematics class contains chapters on graphing, chance, statistics and permutations. Emphasis is placed on gathering, organizing and analyzing data.

THE FIGURE FINAGLERS. Robert S. Reichard. New York: McGraw-Hill Book Company, 1974. (McGraw-Hill Book Company, 1221 Avenue of the Americas, New York, NY 10020)

274 pp; paper; b/w; teacher reference; high reading level

This book attempts to classify, document and analyze the many ways statistics can be used to confuse and mislead.

FLAWS AND FALLACIES IN STATISTICAL THINKING. Stephen K. Campbell. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1974. (Prentice-Hall, Inc., Englewood Cliffs, NJ 07632)

200 pp; paper; teacher reference; b/w

This book exposes the fallacies and wrong methods used in statistical thinking, and guides the layman into distinguishing between valid and faulty statistical reasoning.

"A fun way to introduce probability." Rich Billstein. THE ARITHMETIC TEACHER, Vol. 24, No. 1 (January, 1977) pp. 39-42. Reston, Virginia: National Council of Teachers of Mathematics. (National Council of Teachers of Mathematics, 1906 Association Dr., Reston, VA 22091)

The author shows how both the traditional coin-flipping and dice-rolling approaches to probability can be extended into game situations that have surprising results.

GOALS FOR THE CORRELATION OF ELEMENTARY SCIENCE AND MATHEMATICS, Report of the Cambridge Conference on the Correlation of Science and Mathematics in the Schools. Boston: Houghton-Mifflin Company, 1969. (Houghton-Mifflin Company, Educational Division, 1 Beacon St., Boston, MA 02107)

208 pp; paper; teacher reference

This is a report of a workshop-style conference held in 1967 that includes sections entitled Goals, Implications for the Mathematics-Science Curriculum, Curricula and Teacher Training, and Recommendations for Immediate Implementation. Twenty-five appendices include many specific suggestions for the classroom.

- GOLDEN MEAN: MATHEMATICS IN THE FINE ARTS. Charles Linn. New York: Doubleday and Company, Inc., 1974. (Doubleday and Company, Inc., 245 Park Ave., New York, NY 10017)
- GUIDELINES FOR TEACHING MATHEMATICS, 2nd ed. Donovan Johnson and Gerald Rising. Belmont, California: Wadsworth Publishing Company, Inc., 1972. (Wadsworth Publishing Company, Inc., 10 Davis Dr., Belmont, CA 94002)
- "Hamming it up mathematically." Dale M. Shafer. SCHOOL SCIENCE AND MATHEMATICS. Indiana, Pennsylvania: School Science and Mathematics Association, Inc.(School Science and Mathematics Association, Inc., Indiana University of Pennsylvania, Box 1614, Indiana, PA 15701)
- A HANDBOOK OF AIDS FOR TEACHING JUNIOR-SENIOR HIGH SCHOOL MATHEMATICS. Stephen Krulik. Philadelphia: W.B. Saunders Company, 1971. (W.B. Saunders Company, 218 W. Washington Square, Philadelphia, PA 19105)

120 pp; paper; b/w; teacher reference

The manual contains many aids that can be used to teach mathematics. Included is a probability board and experiment that generates a normal curve of distribution.

HOW TO LIE WITH STATISTICS. Darrell Huff. New York: W. W. Norton and Company, Inc., 1954. (W. W. Norton and Company, Inc., 500 Fifth Ave., New York, NY 10036)

142 pp; paper; b/w; teacher reference; student reference; medium reading level

This is a delightful book that illustrates how statistics can be used to demonstrate any point of view. Although published in 1954, the examples of distortion and suggestions for critical examination are still relevant.

- HOW TO LIVE THROUGH JUNIOR HIGH SCHOOL. Eric Johnson. Philadelphia: J. B. Lippincott Company, 1975. (J. B. Lippincott Company, 521 Fifth Ave., New York, NY 10017)
- HOW TO SOLVE IT: A NEW ASPECT OF MATHEMATICAL METHOD. G. Polya. Princeton: Princeton University Press, 1973. (Princeton University Press, Princeton, NJ 08540)
- INTERPRETING GRAPHS AND TABLES. Peter Selby. New York: John Wiley and Sons, Inc., 1976. (John Wiley and Sons, Inc., 605 Third Ave., New York, NY 10016)
- AN INTRODUCTION TO PROBABILITY AND STATISTICS. Boston: Houghton-Mifflin Company, 1964. (Houghton-Mifflin Company, Educational Division, 1 Beacon St., Boston, MA 02107)

103 pp; paper; b/w; teacher reference

This booklet introduces the basic ideas of statistics and probability. Among the topics discussed in some detail are collection of data, reading and interpreting graphs, probability, distributions, regression lines and correlation.

- INTRODUCTION TO STATISTICAL ANALYSIS AND INFERENCE. Sidney J. Armore. New York: John Wiley and Sons, Inc., 1966. (John Wiley and Sons, Inc., 605 Third Ave., New York, NY 10016)
- JOURNAL OF READING. Newark, Delaware: International Reading Association. (International Reading Association, 800 Barksdale Rd., Newark, DE 19711)
- THE KENT MATHEMATICS PROJECT. Turnbridge Wells, England: Kent Mathematics Project (not yet published).

paper; b/w

This is a materials bank from which teachers can select individual courses of study for students from 9-16 years of age. The expected date of availability of the materials is September 1977.

LABORATORY ACTIVITIES FOR TEACHERS OF SECONDARY MATHEMATICS. Gerald Kulm. Boston: Prindle, Weber and Schmidt, Inc., 1976. (Prindle, Weber and Schmidt, Inc., 20 Newbury St., Boston, MA 02116)

paper; b/w; teacher reference

This is a collection of laboratory activities designed for secondary students.

LADY LUCK: THE THEORY OF PROBABILITY. Warren Weaver. Garden City, New York: Anchor Books, 1963. (Anchor Books, imprint of Doubleday and Company, Inc., 245 Park Ave., New York, NY 10017)

392 pp; paper; b/w; teacher reference; high reading level

Through interesting examples, this highly readable book explains many of the concepts of statistics and probability. Some of the examples are historical, some of the examples are paradoxical; but all of the examples are understandable.

LEARNING TO TEACH SECONDARY SCHOOL MATHEMATICS. Otto C. Bassler and John R. Kolb. Scranton: International Textbook Company, 1971. (International Textbook Company, INTEXT, Oak and Pawnee Streets, Scranton, PA 18508)

434 pp; paper; teacher reference

This is a textbook on methods of teaching secondary mathematics. The appendices of this book contain a list of goals for a unit on statistics, a list of objectives of a unit on statistics, and a sample lesson plan for statistics.

- LISTENER'S GUIDE TO MUSICAL UNDERSTANDING, Fourth Edition. Leon Dallin. Dubuque, Iowa: William C. Brown Company Publishers, 1972. (William C. Brown Company Publishers, 2460 Kerper Blvd., Dubuque, IA 52001)
- MAKING MATHEMATICS, A Secondary Course, Books 1-4. D. Paling, C. S. Banwell, and K. D. Saunders. Ely House: Oxford University Press, 1972. (Oxford University Press, 200 Madison Ave., New York, NY 10016)

125 pp. each; booklets; medium reading level

These are a series of booklets that are suitable for grades 5-8. The booklets provide class, group and individual activities in various mathematical topics. Graphs, statistics and random numbers are included as topics.

"Mathematical Games." Martin Gardner. SCIENTIFIC AMERICAN, Vol. 236, No. 2 (February, 1977), pp. 121-125. New York: Scientific American, Inc. (W. H. Freeman and Company, Publishers, 660 Market St., San Francisco, CA 94104)

This popular feature of SCIENTIFIC AMERICAN describes several interesting and amusing activities involving wordplay and mathematics.

MATHEMATICS. (Life Science Library Series) David Bergamini. Morristown, New Jersey: Time-Life Books, Inc., 1970. (Time-Life Books, Inc., Alexandria, VA 22314)

196 pp; hardbound; student reference; color; medium reading level

The colored, and black and white photographs and graphics in this historical survey of mathematics make this an excellent resource for student and teacher alike. The book covers the development of mathematics from counting to computers, probability and modern geometries. MATHEMATICS A HUMAN ENDEAVOR. Harold Jacobs. San Francisco: W. H. Freeman and Company, Publishers, 1970. (W. H. Freeman and Company, Publishers, 660 Market St., San Francisco, CA 94104)

529 pp; hardbound; b/w; textbook; high reading level

This liberal arts course text is an excellent resource book. Many of the ideas are suitable for or could be adapted for middle school students.

MATHEMATICS AND LIVING THINGS, Teacher's Commentary. School Mathematics Study Group. Palo Alto, California: The Board of Trustees of the Leland Stanford Junior University, 1965. (Distributed by A. C. Vroman, 2085 East Foothill Blvd., Pasadena, CA 91109)

170 pp; hardbound; b/w; teacher's guide

This is a teacher reference book that uses exercises in biological sciences to obtain data for introducing mathematical concepts and principles.

MATHEMATICS, AN EVERYDAY EXPERIENCE. Vern E. Heeven and Charles D. Miller. Glenview, Illinois: Scott, Foresman and Company, 1976. (Scott, Foresman and Company, 1900 East Lake Ave., Glenview, IL 60025)

519 pp; hardbound; b/w; textbook; high reading level

This is a general math book with an emphasis on applications. The book has many beautiful photographs relating the everyday world to mathematics. About 150 pages are devoted to probability and statistics. The book would be useful in general mathematics, liberal arts mathematics or mathematics for elementary teachers.

MATHEMATICS FOR THE INTERMEDIATE GRADES, Part I. Comprehensive School Mathematics Program. St. Louis: CEMREL, Inc., 1976. (CSMP, CEMREL, Inc, 3120 59th St., St. Louis, MO 63139)

The "Comprehensive School Mathematics Program" is a curriculum development project of CEMREL, Inc., a national educational laboratory with offices in St. Louis, Missouri.

MATHEMATICS FOR TODAY, Level Blue. Ed Williams, Sawl Katz and Patricia Flagholz. New York: Oxford Book Company, Inc., 1976. (Oxford Book Company, Inc., 11 Park Place, New York, NY 10007)

256 pp; hardbound; b/w; teacher's guide; medium reading level

The book provides drill practice while focusing on mathematical skills required in everyday life.

MATHEMATICS FOR UPPER PRIMARY GRADES, Part II. Comprehensive School Mathematics Program. St. Louis: CEMREL, Inc., 1976. (CSMP, CEMREL, Inc., 3120 59th St., St. Louis, MO 63139)

The "Comprehensive School Mathematics Program" is a curriculum development project of CEMREL, Inc., a national educational laboratory with offices in St. Louis, Missouri.

MATHEMATICS IN GENERAL EDUCATION. V. T. Thayer. New York: Appleton-Century-Crofts, Inc., 1940. (Copyright now held by Prentice-Hall, Inc., Englewood Cliffs, NJ 07632)

423 pp; hardbound; teacher reference

This is a report of the Committee on the Function of Mathematics in General Education for the Commission on Secondary School Curriculum of the Progressive Education Association.

 A MATHEMATICS LABORATORY HANDBOOK FOR SECONDARY SCHOOLS. Stephen Krulik. Philadelphia:
 W. B. Saunders Company, 1972. (W. B. Saunders Company, 218 W. Washington Square, Philadelphia, PA 19105)

107 pp; paper; b/w; teacher reference

The booklet includes a description of the mathematics laboratory approach to teaching, its place in the curriculum, and activities designed for the mathematics laboratory (including five activities on statistics and probability).

MATHLAB-- Junior High. Scott McFadden, Keith Anderson and Oscar Schaaf. Eugene, Oregon: Action Math Associates, Inc., 1975. (Action Math Associates, Inc., 1358 Dalton Dr., Eugene, OR 97404)

80 pp; paper; b/w; teacher's guide; student reference; low reading level

This book is a collection of activity cards designed to provide students with problem-solving situations. Eight activities dealing with probability are included.

THE NATIONAL ADVERTISING REVIEW BOARD, 1971-1976. New York: National Advertising Division of Better Business Bureaus. (National Advertising Division of Better Business Bureaus, 845 Third Ave., New York, NY 10022)

This booklet describes complaints handled by NARB (National Advertising Review Board) during the years 1971 through 1976, and the decisions that were made by the board. This booklet can be obtained at a cost of \$1.00 plus 68¢ for mailing.

THE NEWS MATH. Betty Buzitis and Jeanette Keller. Seattle: Seattle Times Company, 1974.

117 pp; paper; teacher reference; b/w

The booklet sensitizes students to the newspaper as a source of data. Sample activities and suggestions for lessons that involve the newspaper are given. These activities apply many mathematics concepts and skills: graphs and charts, averages, estimation, fractions, classification, etc.

- NOTES ON MATHEMATICS IN PRIMARY SCHOOLS. The Association of Teachers of Mathematics. New York: Cambridge University Press, 1969. (Cambridge University Press, 32 East 57 St., New York, NY 10022)
- OPTIONS; A STUDY GUIDE TO POPULATION AND THE AMERICAN FUTURE. Kathryn Horsley. Washington, D.C.: Population Reference Bureau. (Population Reference Bureau, 1337 Connecticut Ave., N. W., Washington, D.C. 20036)
- OVERVIEW AND ANALYSIS OF SCHOOL MATHEMATICS, GRADES K-12. Washington, D.C.: Conference Board of the Mathematical Sciences, 1975. (Conference Board of the Mathematical Sciences, 832 Joseph Henry Building, 2100 Pennsylvania Ave., N.W., Washington, D.C. 20037)
- PATHWAYS TO PROBABILITY. Amy C. King and Cecil B. Read. New York: Holt, Rinehart and Winston, Publishers, 1963. (Holt, Rinehart and Winston, Publishers, 383 Madison Ave., New York, NY 10017)

139 pp; paper; b/w; teacher reference; student reference; medium reading level

The book gives a historical development of probability. Included are brief sketches of the lives, accomplishments and problems of some of the persons involved.

- PATTERNS AND PERSPECTIVES IN ENVIRONMENTAL SCIENCE. Washington, D.C.: National Science Board, n.d. (National Science Board, 1800 G. St., Washington, D.C. 20550)
- A PHILOSOPHICAL ESSAY ON PROBABILITIES. Marquis de Laplace. New York: John Wiley and Sons, Inc., 1952. (John Wiley and Sons, Inc., 605 Third Ave., New York, NY 10016)

196 pp; paper

Mostly of historical interest, this is an essay written in 1795 to explain the principles and general results of the theory of probability and how they apply to "important questions of life."

- THE PHYSICS OF MUSICAL SOUND. Jess Josephs. New York: Crane, Russak and Company, Inc., 1965. (Crane, Russak and Company, Inc., 347 Madison Ave., New York, NY 10017)
- POCKET DATA BOOK USA 1973. Washington, D.C.: United States Government Printing Office, 1973. (United States Government Printing Office, Division of Public Documents, Washington, D.C. 20402)

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POLLUTION PROBLEMS, PROJECTS AND MATHEMATICS EXERCISES, 6-9. (Bulletin No. 1082) Madison, Wisconsin: Wisconsin Department of Public Instruction, n.d. (Wisconsin Department of Public Instruction, 126 Langdon St., Madison WI 53702)

84 pp; paper

This is a teacher-written collection of problems and projects. Exercises are grouped under whole numbers, rational numbers, real numbers, percent and proportion, measurement and statistical measures and graphs.

- PRINCIPLES OF ENVIRONMENTAL SCIENCE. Kenneth E. Watts. New York: McGraw-Hill Book Company, 1973. (McGraw-Hill Book Company, 1221 Avenue of the Americas, New York NY 10020)
- "Probability." David A. Page. THE GROWTH OF MATHEMATICAL IDEAS, Grades K-12. Reston, Virginia: National Council of Teachers of Mathematics, 1959. (National Council of Teachers of Mathematics, 1906 Association Dr., Reston, VA 22091)

"Probability" gives some background for elementary probability and suggestions for sprinkling probability ideas throughout the K-12 years.

PROBABILITY AND STATISTICS. Nuffield Mathematics Project. New York: John Wiley and Sons, Inc., 1969. (John Wiley and Sons, Inc., 605 Third Ave., New York, NY 10016)

53 pp; paper; b/w; teacher reference; classroom ideas

One of a series from the Nuffield Mathematics Project, this booklet is full of interesting classroom ideas for probability and statistics on a middle school level. Included are examples of student work.

PROBABILITY AND STATISTICS--AN INTRODUCTION THROUGH EXPERIMENTS. Edmund C. Berkeley. Newtonville, Massachusetts: Berkeley Enterprises, Inc., 1961.

121 pp; paper; b/w; teacher reference

The book provides an introduction to probability and statistics by means of actual physical experiments.

PROBABILITY FOR INTERMEDIATE GRADES, Teacher's Commentary. Revised Edition. School Mathematics Study Group. Palo Alto, California: The Board of Trustees of the Leland Stanford Junior University, 1965. (Distributed by A. C. Vroman, 2085 East Foothill Blvd., Pasadena, CA 91109)

192 pp; paper; b/w; teacher's guide; medium reading level

This book is a followup to a book written for primary grades. It uses dice, coins, colored cubes and spinners as models for probability experiments. Student texts are available. PROBABILITY FOR PRIMARY GRADES, Teacher's Commentary, Revised Edition. School Mathematics Study Group. Palo Alto, California: The Board of Trustees of the Leland Stanford Junior University, 1965. (Distributed by A.C. Vroman, 2085 East Foothill Blvd., Pasadena, CA 01109)

135 pp; paper; b/w; teacher's guide; medium reading level

Simple probability ideas are explored using dice, coins, colored cubes and spinners as models. Student texts are available.

Probability, Statistics and Combinatories. MATHEMATICS FOR THE INTERMEDIATE GRADES, Entry Module 1. Comprehensive School Mathematics Program. St. Louis: CEMREL, Inc., 1976. (CSMP, CEMREL, Inc., 3120 59th St., St. Louis, MO 63139)

96 pp; paper; b/w; teacher reference

One of a series, this book describes introductory lessons for probability, statistics and counting techniques.

PROBABILITY: THE SCIENCE OF CHANCE, Teacher's Manual. Margaret F. Willerding. Chicago: Franklin Publications, Inc., 1970. (Distributed by Lyons and Carnahan, Educational Division of Meredith Corporation, 407 E. 25th St., Chicago, IL 60616)

hardbound; color; teacher's guide; medium reading level

Written at the junior high level, the book treats the following topics in probability theory: probabilities with tossing coins and dice, meaning of outcome, experiment, sample space and event, statistical probability, mathematical expectation, probability in business and conditional probability. The book endeavors to create a desire for further study of the subject.

- PROBABILITY WITH STATISTICAL APPLICATIONS. Rourke Mosteller and Thomas Mosteller. Reading, Massachusetts: Addison-Wesley Publishing Company, Inc., 1973. (Addison-Wesley Publishing Company, Inc., South St., Reading, MA 01867)
- "Random Digits and Some of Their Uses." Roger Carlson. *Weighing Chances* from the program *STATISTICS BY EXAMPLE*. Edited by F. Mosteller, W. Kruskal, R. Link, R. Pieters, and G. Rising. Copyright (C) 1973 by Addison-Wesley Publishing Company, Inc. (Addison-Wesley Publishing Company, Inc., Sand Hill Road, Menlo Park, CA 94025)
- "Random models in the classroom." Keith Selkirk. MATHEMATICS IN SCHOOLS, Vol. 3
 No. 2(March, 1974), pp. 15-17. Essex: Longman Group Limited.(Longman Group
 Limited, 5 Bentinck St., London, England WIM 5RN)

This article describes eight ideas for using random digits as models for probabilistic situations.

"A readability formula that saves time." Edward Fry. JOURNAL OF READING, Vol. 11, No. 7 (April, 1968), pp. 513- 516. Newark, Delaware: International Reading Association. (International Reading Association, 800 Barksdale Rd., Newark, DE 19711)

This article describes a test developed by Edward Fry that is used to determine the readability level of written materials.

READINGS IN MATHEMATICS: BOOK 2, edited by Irving Adler, (C) Copyright, 1972, by Ginn and Company (Xerox Corporation). Used with permission. (Ginn and Company, Xerox Corporation, 191 Spring St., Lexington, MA 02173)

188 pp; paper; b/w; teacher reference

Many selections written by both contemporary authors and by great scientists and mathematicians of the past are included in this book. The selections relate mathematics to everyday life, science and literature. The book is interesting reading for the lay person as well as the mathematician.

THE SCHOOL MATHEMATICS PROJECT, Books A-H. School Mathematics Project. London: Cambridge University Press, 1972. (Distributed by Cuisenaire Company of America, Inc., 12 Church St., New Rochelle, NY 10805)

approx. 300 pp. each book; paper; b/w; textbook; medium reading level

Each of these books contains some statistics and/or probability. The project uses a spiral, activity-oriented approach, with the simpler concepts in the earlier books.

- SCIENTIFIC AMERICAN. San Francisco: W.H. Freeman and Company, Publishers. (W.H. Freeman and Company, Publishers, 660 Market St., San Francisco, CA 94104)
- SECONDARY SCHOOL MATHEMATICS. School Mathematics Study Group. Palo Alto, California: The Board of Trustees of the Leland Stanford Junior University, 1971. (Distributed by A. C. Vroman, 2085 East Foothill Blvd., Pasadena, CA 91109)

96 pp; paper; textbook; high reading level

Chapters 23 (Quadratic Functions) and 24 (Statistics) are published in paperbound books. The chapter on statistics is intended for high school students. It includes coverage of normal curves and standard deviation.

SETS, PROBABILITY AND STATISTICS: THE MATHEMATICS OF LIFE INSURANCE. Paul C. Clifford, Mildred Keiffer and Max A. Sobel. New York: Institute of Life Insurance, Education Division, 1972. (American Council of Life Insurance, 8150 K. St., N.W., Washington, D.C. 20006)

35 pp; paper; workbook; color; medium reading level

The booklet describes (in student lesson form) sets, elementary probability and the mathematics used in the life insurance business. "A seventh-grade mathematics class tackles the stock market." Esther K. Friedmann. *THE ARITHMETIC TEACHER*. Vol. 20, No. 1 (January, 1973), pp. 45-47. Reston, Virginia: National Council of Teachers of Mathematics. (National Council of Teachers of Mathematics, 1906 Association Dr., Reston, VA 22091)

This article describes how a class used the stock market as an application of mathematics.

SOME USES OF MATHEMATICS: A SOURCE BOOK FOR TEACHERS AND STUDENTS OF SCHOOL MATHEMATICS. (Studies in Mathematics, Vol. XVI) Max S. Bell, ed. Palo Alto, California: School Mathematics Study Group, 1967. (Distributed by A. C. Vroman, 2085 East Foothill Blvd., Pasadena, CA 91109)

252 pp; paper; teacher reference; b/w; high reading level

Twenty-four articles dealing with applications of mathematics and mathematical models make up this book.

SOURCES OF STATISTICS. Joan Harvey. Hamden, Connecticut: Linnet Books, 1971.

125 pp; hardbound; teacher reference

A list and descriptions of the main statistical publications of the United Kingdom and the more important publications of the United States and of various international organizations are included. These are grouped into chapters such as social problems, education, tourism and advertising.

STARTING STATISTICS. E. L. Hanson and G. A. Brown. Amersham, Bucks, Great Britain: Hulton Educational Publications, Ltd., 1975.

96 pp; paper; color; textbook; high reading level

The basic ideas of statistics and probability are illustrated with many charts and examples. The last chapter on two dimensional statistics is appropriate for students with some knowledge of algebra.

STARTING STATISTICS. K. Lewis and H. Ward. London: Longman Group Limited, 1969. (Longman Group Limited, 5 Bentinck St., London, England WIM 5RN)

138 pp; paper; b/w; teacher reference; medium reading level

In an easy reading style, this book explores many statistical ideas and presents class activities on: collecting and displacing data, averages, correlation, frequency distributions, running totals, probability, sampling spread, and continuity. The authors have stressed the practical aspects of the subject and have encouraged students to get their own facts and to discuss, analyze and justify any conclusions they may deduce from the facts.

STATISTICAL ABSTRACT OF THE UNITED STATES, 1976. Washington, D.C.: U.S. Bureau of the Census. (Commerce Department, Bureau of the Census, Washington, D.C. 20233)

STATISTICAL THINKING: A STRUCTURAL APPROACH. John L. Phillips, Jr. San Francisco: W.H. Freeman and Company, Publishers, 1973. (W.H. Freeman and Company, Publishers, 660 Market St., San Francisco, CA 94104)

124 pp; paper; graphics; teacher reference; high reading level

Persons beginning a study of statistics can use this as a resource book that explains many statistical terms and tests using uncomplicated examples.

STATISTICS. Irene Campbell. (Mathematics in the Making 12) Boston: Houghton-Mifflin Company, 1970. (Houghton-Mifflin Company, Educational Division, 1 Beacon St., Boston, MA 02107)

32 pp; paper; student reference; teacher reference; color

The book describes some uses of statistics and outlines introductory activities for students to investigate.

STATISTICS. (Topics from Mathematics) David S. Fielber. New York: The Syndics of the Cambridge University Press, 1967. (Cambridge University Press, 32 East 57 St., New York, NY 10022)

31 pp; paper; color; teacher reference; student reference.

The booklet introduces statistics and discusses distributions, samples, and graphs.

"Statistics." Richard S. Pieters and John J. Kinsella. THE GROWTH OF MATHEMATICAL IDEAS, Grades K-12. Reston, Virginia: National Council of Teachers of Mathematics, 1959. (National Council of Teachers of Mathematics, 1906 Association Dr., Reston, VA 22091)

"Statistics" gives an overview of statistics at the pre-college level and makes recommendations for the teaching of statistics in grades K-12.

STATISTICS. Jane Jonas Srivastava. New York: Thomas Y. Crowell Company, 1973. (Copyright now held by Harper and Row, Publishers, Inc., 10 E. 53rd St., New York, NY 10022)

34 pp; hardbound; color; student reference; medium reading level

This juvenile literature book is a simple introduction to the concept and uses of statistics.

STATISTICS: A CONCEPTUAL APPROACH. Sidney J. Armore. Columbus, Ohio: Charles
E. Merrill Publishing Company, 1975. (Charles E. Merrill Publishing Company,
1300 Alum Creek Dr., Columbus, OH 43216)

STATISTICS: A GUIDE TO THE UNKNOWN. Judith M. Tanur, ed. San Francisco: Holden-Day Inc., 1972. (Holden-Day, Inc., 500 Sansome St., San Francisco, CA 94111)

430 pp; paper; teacher reference; high reading level

This is an excellent collection of essays describing the use of statistics for applications in diverse fields, from deciding whether James Madison or Alexander Hamilton wrote several of the Federalist papers to deciding whether bunting in baseball is a helpful strategy.

STATISTICS AND PROBABILITY. (The School Mathematics Project) J. H. Durran. London: Cambridge University Press, 1970. (Cambridge University Press, 32 East 57 St., New York, NY 10022)

488 pp; hardbound; b/w; teacher reference; high reading level

This book is a sophisticated coverage of probability and statistics for teachers and high-ability students.

- STATISTICS; A NEW APPROACH. W. A. Wallis and H. V. Roberts. New York: Free Press, 1956. (Distributed by Macmillan Publishing Company, Inc., 866 Third Ave., New York, NY 10022)
- STATISTICS BY EXAMPLE. Frederick Mosteller, William H. Kruskal, Richard F. Link, Richard S. Pieters, and Gerald R. Rising, ed. Menlo Park, California: Addison-Wesley, Publishing Company, Inc., 1973. (Addison-Wesley Publishing Company, Inc., Sand Hill Rd., Menlo Park, CA 94025)

150 pp. each; paper; b/w; teacher reference; high reading level

The four books in the series, Exploring Data, Weighing Chances, Detecting Patterns and Finding Models, illustrate many situations that explain real-life problems in terms of statistics. A knowledge of intermediate algebra is necessary to understand the mathematics involved.

"Statistics, choice and chance." Edith Biggs. MATHEMATICS FOR OLDER CHILDREN. New York: Citation Press, 1972, pp. 53-62. (Citation Press, imprint of Scholastic Book Services, 50 W. 44th St., New York, NY 10036)

117 pp; paper; teacher reference; b/w; medium reading level

This chapter shows examples of work done by students in a discovery-type environment.

STATISTICS FOR PROBLEM SOLVING. C. Mitchell Dayton and Clayton L. Stankard. New York: McGraw-Hill Book Company, 1971. (McGraw-Hill Book Company, 1221 Avenue of the Amercias, New York, NY 10020)

290 pp; hard; b/w; teacher reference; high reading level

The book is designed to serve as a textbook for introductory applied statistics for students in education, educational psychology and related behavioral science fields.

STATISTICS FOR TEACHERS. Estelle S. Gellman. New York: Harper and Row, Publishers, Inc., 1973. (Harper and Row, Publishers, Inc., 10 E. 53rd St., New York, NY 10022)

238 pp; paper; b/w; teacher reference; high reading level

Emphasis is on the application of statistics in describing classroom performance and in testing hypotheses relevant to the classroom situation. A knowledge of elementary algebra is required.

A STUDY OF PARTS OF THE DEVELOPMENT OF A UNIT IN PROBABILITY AND STATISTICS FOR THE ELEMENTARY SCHOOL. Jack Shepler. Unpublished Ph.D. dissertation, University of Wisconsin, 1969. (Available from University Microfilms International, Ann Arbor, MI 48104)

This disseration is an excellent source of ideas and activities for both experimental and formal probability, as well as a discussion of how students perform on the activities.

- "Suicide." Lloyd Shearer. PARADE MAGAZINE, April 1976. New York: Parade Publications, Inc. (Parade Publications, Inc., 733 Third Ave., New York, NY 10017)
- TAKING CHANCES. Oakland County Mathematics Project. Pontiac, Michigan: Oakland County Mathematics Project, 1971. (Oakland Schools, 2100 Pontiac Lake Rd., Pontiac, MI 48054)

70 pp; paper; b/w; textbook; medium reading level

Simple probability ideas are presented as a series of activities using dice, coins, cards and spinners.

TEACHING MODERN MATHEMATICS IN THE ELEMENTARY SCHOOL. H. W. Fehr and J. Mc. Phillips. Addison-Wesley Publishing Company, Inc., 1972. (Addison-Wesley Publishing Company, Inc., South St., Reading, MA 01867)

515 pp; hardbound; teacher reference

A book on the curriculum and pedagogy of contemporary elementary school mathematics. The book is intended primarily for a methods course for preservice elementary school teachers. It is a very useful reference book.

THE TEACHING OF PROBABILITY AND STATISTICS. Lennard Rode, ed. New York: John Wiley and and Sons, Inc., 1970. (John Wiley and Sons, Inc., 605 Third Ave., New York, NY 10016)

373 pp; hardbound; teacher reference; high reading level

This is a collection of the papers presented at the first CSMP International Conference on the teaching of probability and statistics. Although some topics are advanced, there are many ideas for classroom activities.

TEACHING READING AND MATHEMATICS. Richard A. Earle. Newark, Delaware: International Reading Association, 1976. (International Reading Association, 800 Barksdale Rd., Newark, DE 19711)

"To find facts about places, people, things...." CHANGING TIMES, The Kiplinger Magazine, Vol. 29, No. 11 (November, 1975), p. 18. Editors Park, Maryland: The Kiplinger-Washington Editors, Inc. (The Kiplinger-Washington Editors, Inc., 1729 H. St., N.W., Washington, D.C. 20006)

The article describes many sources of information.

"Updating the Fry readability formula." Joseph C. Kretschmer. THE READING TEACHER, Vol. 29, No. 6 (March, 1976), pp. 555-558. Newark, Delaware: International Reading Association. (International Reading Association, 800 Barksdale Rd., Newark, DE 19711)

This article describes Kretschmer's effort to make Fry's readability formula more accurate.

- U.S. NEWS AND WORLD REPORT. Washington, D.C.: U.S. News and World Report, Inc. (U.S. News and World Report, Inc., 2500 N. St., N.W., Washington, D.C. 20037)
- U.S. RDA COMPARISON CARDS AND GUIDE FOR TEACHERS AND OTHER READERS. Chicago: National Dairy Council, 1974. (National Dairy Council, 6300 N. River Rd., Rosemont, IL 60018)

color; teacher reference

This set contains multi-colored bar graphs of the main nutrients found in 57 foods (each graph is drawn on card stock), spirit duplicating masters for class activities, and a teacher/leader guide.

WEEKLY READER TABLE AND GRAPH SKILLS SERIES. Beth Atwood, George Brown, Morton Malkofsky and Carolyn Paine. Columbus, Ohio: Xerox Corporation, 1968. (Xerox Corporation, 245 Long Hill Road, Middletown, CT 06457)

47 pp. each; pamphlets; workbook; teacher reference; b/w

The series, comprised of four pamphlets, develops the necessary skills for reading and interpreting tables and graphs. Experience is also provided in collecting data, organizing data in tables and constructing graphs. In each pamphlet a skill is introduced, extended, reinforced and then reviewed. Although the pamphlets are written at a 3-6 grade reading level, pamphlets C and D include mathematical content appropriate for developing units at the junior high and senior high general math levels. An accompanying teacher's guide explains how to use each pamphlet.

WELL-BEING. (Probing the Natural World/ Level III) Intermediate Science Curriculum Study. Morristown, New Jersey: Silver Burdett General Learning Corporation, 1972. (Silver Burdett Company, 250 James St., Morristown, NJ 07960)

143 pp; paper; color; workbook; medium reading level

One of a series of eight student workbooks, this book has activityoriented lessons dealing with the senses and functions of the body, and how these are affected by food, smoking, alcohol, and other drugs.

WHAT ARE MY CHANCES, Book A. Albert P. Shulte and Stuart A. Choate. Palo Alto, California: Creative Publications, Inc., 1977. (Creative Publications, Inc., Box 10328, Palo Alto, CA 94303)

97 pp; paper; b/w; teacher reference; medium reading level

Seventy-three activities are used to develop an intuitive feeling about probability. Extensive teacher notes are included.

WINNING WITH STATISTICS: A PAINLESS FIRST LOOK AT NUMBERS, RATIOS, PERCENTAGES, MEANS AND INFERENCE. Richard P. Runyon. Reading, Massachusetts: Addison-Wesley Publishing Company, Inc., 1977 . (Addison-Wesley Publishing Company, Inc., South St., Reading, MA 01867)

210 pp; paper; b/w; teacher reference

The book is a humorous introduction to statistics, its many measures, and the many ways it can be used to mislead.

THE WORLD OF PHYSICS. Robert Hulsizer and David Lazarus. Reading, Massachusetts: Addison-Wesley Publishing Company, Inc., 1972. (Addison-Wesley Publishing Company, Inc., South St., Reading, MA 01867)

518 pp; hardbound; b/w; textbook

This book, written as a high school physics text, is full of hand-drawn graphics that are easy to understand.

YOUTH PHYSICAL FITNESS: SUGGESTED ELEMENTS OF A SCHOOL-CENTERED PROGRAM. Washington, D.C.: United States Government Printing Office, 1967. (United States Government Printing Office, Division of Public Documents, Washington, D. C. 20402)

109 pp; paper; b/w; teacher reference

This book has exercises to evaluate and promote physical fitness of students.

ORGANIZATION

Placement Guide for Tabbed Dividers

Tab Label	Follows page:
CONTENT FOR TEACHERS	
Gathering Data	
Tables	
Graphs	
Scatter Diagrams	
Mean, Median, Mode	
Range & Deviation	
Sampling	
Experimental Probability	
Probability with Models	
Counting Techniques	
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Inferential Statistics	
Appendix-Computer Programs	
DIDACTICS	
Components of Instruction	
Classroom Management	
Statistics and Probability Learning	
TEACHING EMPHASES	
Critical Thinking	
Decision Making	
Problem Solving	
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Counting Techniques	
APPLICATIONS	
Business & Commerce	
Environment	
Health & Medicine	
People & Culture	
Recreation	
GLOSSARY	
ANNOTATED BIBLIOGRAPHY	
SELECTED ANSWERS	



TABLES

- 425 (1a) 1290 (1b) 39 (1c) 579 (2a) number of fatalities in foggy weather (2b) number of injuries in poor visibility (2c) total reported number of fatalities in 1973 due to boating accidents (3a) calm (3b) clear (3c) light (3d) good
- 426 (1) baseball; basketball; football; boxing (2) 7 (3) 1000; number; \$1000 (3a) 617 college football teams in 1970 (3b) 30,467,000 attendance in 1973 (3c) \$11,847,000 in receipts in 1972 (4) 1965 (5) No; yes; yes (6) college (7) The number of games played was fewer
- 432 51, 54 or 57 (4) 7 (3) (5) 23

GRAPHS

- 459 (2a) bar graph (2b) percent bar graph
- 465 (1)Т (2) н (3) E (4) S (5) C (6) A (7) L (8) R (9) F (10) I (11) Y
- 471 (1)No (2) No (3) Family A (4) 150 (5) 200 (6) Family B Even though 30% is greater than 25%, 25% of \$800 is greater than (7)30% of \$500. (8) Family B (9) More than 3 times as much . .

SCATTER DIAGRAMS

487 (2) 8; 75

PAGE NUMBER

)

)

- 10° or 11° 488 (3a) (3b) maybe 160
- 490 I. (3) decrease (3) They increase. II.

(3)The points lie in a straight line.

125° (4)

> (5) It increases.

- (7)The points lie in a straight line.
- (8) 7½ units

6

350

- (9) It decreases.
- 495 (4a) (4b) 32
 - (7) Yes

(1)

- (9) 5;95
- 496

494

- (2)Yes
- (3) Not necessarily, even though it appears so from the graph.
- (4) Most driving is done close to home so more accidents happen
 - close to home.

881

A Second Sec

PAGE NUMBER

MISLEADING STATISTICS

505	(1) (2)	No; no. That they make sturdy, reliable cars; not necessarily; no; no;
506	(3) (4) (5) (6) (7) (8) (9) (10)	no; yes 11%; not much No; no Probably not; no; no Can't tell Can't tell; can't tell; don't know; don't know; not sure; no No No No
507	 (1) (2) (3) (4) (5) (7) (8) 	2 44; 32; no 3 56; no Yes Yes The scale starts at zero. It is not cut off at the bottom.
50 9	(1) (2)	25,000; 20,000; 5,000; 10,000; 3,000 Yes; the scale does not show equal intervals.
		MEAN, MEDIAN, MODE
527	(1)	Jon - 4.5 cm; Ruby - 5 cm; Fran - 7 cm; Ed - 5 cm; Fred - 6 cm; Bill - 6 cm; Joan - 5.5 cm
529	(5) (9) (10)	9 (7) Yes; yellow Possible lengths are 2, 4, 6, 8, 10 or 4, 5, 6, 7, 8. Possible lengths are 4, 5, 6, 9, 11.
530	(3) (8a)	Yes (4) 50 cm (5) Yes (6) 50 cm (7) 75 cm 70 cm (8b) 25 cm (9a) 75 cm (9b) 40 cm (9c) 85 cm
532	(a)	mean = 37 (b) mean = 7 (c) mean = 34
537	(2) (5) (6) (8)	3.1 (3) No (4) 2 Mode. In the table 14 of 21 families have 2 or less children. More smaller homes are needed. Mean. The park should accomodate the total number of children. This can be found by knowing the mean number of children per family. Probably higher, since each family in the class has at least one child.
539 Bi	Top ottom	

PAGE NUMBER

RANGE AND DEVIATION

551	(4) (5) (6a)	I. 5; 5; 5 II. 5; 5; 5 All the same More spread out 2 (6b) 9 9 (7b) 1075 (7c) 8					
553	(1) (5)	10.325; 12, 8; 13; 7 Students should observe less variability in the trimmed ranges.					
554	(1)	7; 2; 6; 4; 6 (2) 5 (5) Yes (7) 7 (10) Yes					
555	(1) (4) (8)	39 (3) 43; 43; The numbers are the same 57 (6) 77; 77 (7) Yes Mean = 50; sum above = sum below = 110					
558	(2)	B (3) a, c, d (4) Yes					
562	(1) (2) (3)	12.00; 16.50; 21.50 175; 382; 571; 764; 924 250; 397; 497; 572; 769; 969; 1219					
563	(1)	Number absent - 30, 24, 18, 24, 12; Running totals - 30, 54, 72,					
	(2)	98, 108 32 (3) 12 (4) Sometime in NovDec.; Sometime in May-June					
564	(1) (3)	Number of students - 2, 5, 7, 10, 8; Running totals - 2, 7, 14, 24, 32 24 (4) 7					
567	(3) (7a)	28 (4a) 28 (4b) 100 (5a) 14 (5c) 62 (6a) 7 (6c) 54 21 (7c) 68					
5681. II.	(1) (1)	82 (3) 118; 118 6; 6 (2) 10; 10 (3) 14					
569I.	(2) (7)	8; 8 (3) 18; 18 (4) 8th; 80 (5) 13 (6) 26; 26 85 (8) 83rd					
SAMPLING							
583	(6)	600; 150; 750					
585	(4)	20 marbles					
586 587		Yes; the fraction of reds to whites in the population is 1/2. 4; 6; 1, 2, 3, 4; 5, 6, 7, 8, 9, 0 Digits for red: 1, 2. Ignore 8, 9, 0; Digits for black: 3, 4, 5, 6, 7					
	(4) (5)	36 to 00; 65; no 01 to 37; 38 to 84; 85 to 00					

PAGE NUMBER		SAMPLING (continued)								
588	(6)	(6) 3/1; 4/1; the fractions of blacks to whites and greens to whites are 3/1 and 4/1 respectively in the population.								
591	(4)	(4) Probably not; Probably not. The weight of the raisins will cause them to settle to the bottom of the box.								
592	(1) (5)									
594	(7)	181								
602	(1) (6)									
603	(10) ≈ 16 ; ≈ 8 (11) $\approx 16 \text{ cm}^2$; $\approx 8 \text{ cm}^2$ (13) ≈ 26 ; ≈ 10 (15) A - (100/200) x 100 cm ² = 50 cm ² B - (50/200) x 100 cm ² = 25 cm ² C - 10 cm ² D - 22.5 cm ² E - 18 cm ²									
		EXPERIMENTAL PROBABILITY								
617	(5b)	No								
623	(2) (6)	Green; yellow (5) Use each color on two faces YY; RR; GG; GR; GY; RY								
		PROBABILITY WITH MODELS								
647	(5)	Never (2) Maybe Yes - Maybe No (3) Always (4) Probably not Probably (6) Maybe Yes - Maybe No (7) Probably Not Always (9) Probably (10) Never								
648	(lla)	1 (11b) Between $1/2$ and 1 (11c) Between 0 and $1/2$								

- 655 (1) E (2) C (3) E (4) A (5) C (6) D (7) E (8) B (9) All red (10) All blue
- 658 (1a) 1/8 (1b) 1/8 (1c) 2/8 or 1/4 (1d) 2/8 or 1/4 (1e) 4/8 or 1/2 (1f) 4/8 or 1/2 (1g) 4/8 or 1/2 (1h) 3/8 (1i) 1/8 (1j) 8/8 or 1 (2a) 1/8 (2b) 7/8 (2c) 1/8 (2d) 7/8 (2e) 8/8 or 1 (2f) 0/8 or 0 (2g) 4/8 or 1/2 (2h) 8/8 or 1 (3) Answers may vary.
- 659 I.(2) 1/6; 1/2; 1; 1/2; 0 II.(2) 1/2; 1/2; 1; 1/2 III.(2) 1/6; 1/6; 1/3; 5/6; 0; 1

IV.(2) 1/9; 4/9; 1/3; 7/9

PAGE NUMBER	PROBABILITY WITH MODELS (continued)						
(2a) (3)	1/3 (1b) 7/9 (1c) 2/9 1/10 (2b) 1/3 (2c) 1/2 (2d) 3/10 7/9 1/4 (4b) 1/6 (4c) 5/12 (4d) 7/12						
	1/4 (4b) $1/0$ (4c) $5/12$ (4d) $7/120 1/4 (5b) 11/12$						
661 (1)	52; 1/52; 1/52; 51/52; 2/52 = 1/26; 4/52 = 1/13; 4/52 = 1/13; 13/52 = 1/4; 39/52 = 3/4; 8/52 = 2/13; 48/52 = 12/13; 0/52 = 0; 12/52 = 3/13; 30						
(2)							
(2a)	 25/40 (1b) 10/40 (1c) 35/40 1/10 (2b) 8/10 (2c) 5/10 (2d) 4/10 12/30 (3b) 18/30 (3c) 0/30 1/52 (4b) 4/52 (4c) 13/52 (4d) 20/52 or 24/52 depending on whether aces are 1 or 13. 						
663 (1)	T (2) T (3) T (4) T (5) T (6) T (7) F (8) F (9) T						
(10)	T (11) No (12) No (13) Yes (14) No (15) No (16) 4 5 (18) 4 (19) 3 (20) Five						
665 (1)	1/8 (2) 1/4 (3) 1/9						
666 (1)	1/8 (2) 1/12 (3) 1/4; 1/2 (4) 1/4						
667 (1) (2) (3) (4)	1/4; 1/4; 1/4; 1/4 2/20 = 1/10; 6/20 = 3/10; 3/20; 0/20						
	4/25 (lb) 16/25 9/49 (2b) 16/49 (2c) 24/49 1/9 (3b) 4/9 (3c) 4/9 (3d) 7/18						
669 (1) (2) (2d) (3a)	16 (2a) 1/16 (2b) 6/16 or 3/8 (2c) 2/16 or 1/8						
670 I.(a) II.(a)							
672 (lc)	13/40 (2) 19/42 (3a) 1/6 (3b) 7/24 (3c) 11/24 (3d) 5/12						
673 (2)	6/11; 5/11						
674 (1)	3/7; 4/7 (2) 7/15; yes; 8/15						
	1/10 (1b) 14/20 (2a) 1/7 (2b) 2/7 (2c) 4/7 1/15 (3b) 8/15 (3c) 2/5 (3d) 0 (3e) 4/15						

PAGE NUM	BER PROBABILITY WITH MODELS (continued)
676	(3) 3/5; 6/25; 12/125; 28/625; 16/625
677	(1) 1/6 (2) 1/12 (3) 1/169 (4) 13/204
678	(1) Same chance of winning (2) Game I
679	(3) Game I (4) Same chance of winning (5) Game I
680	(la) 9/12 (lb) 2/11 (lc) 6/132; 72/132; 54/132 (3a) 1/15 (3b) 6/20 (3c) 1/20 (3d) 13/20 (3e) 30/380 (3f) 182/380
681	(2) 7 (3a) 4 (3b) 4,1; 3,2; 2,3; 1,4 (3c) 4; 36 (4) 3/36 (5) 1/36 (6) 6/36 (7) 2/36 (8) 8/36 (9) 21/36 (10) 6/36 (11) 18/36 (12) 15/36
682	(12a) 10 (12b) 180; 20 (12c) 2/36; 8 (12d) 6/36; 3600; 600
	COUNTING TECHNIQUES
707	(5) 120 (6) 40,320
708	 (2) 120 (4) 4 x 3 x 2 x 1 = 24 (5) 6 x 5 x 4 x 3 x 2 x 1 = 720 (6) 7 x 6 x 5 x 4 x 3 x 2 x 1 = 5040
709	(4a) $6 \ge 6 = 36$ (4b) $6 \ge 5 = 30$ (5) $8 \ge 7 = 56$ (6) $12 \ge 11 = 132$ (8) $4 \ge 3 \ge 2 \ge 1 = 24$ but one is the poem as is. There are 23 unpoems.
710	(1a) 8 (1b) $8 \times 6 = 48$ (1c) $8 \times 6 \times 5 = 240$ (2a) 2 (2b) 3 (2c) $2 \times 3 = 6$ (2d) $1 \times 2 \times 2 \times 2 \times 3 = 24$ (3) $5 \times 7 \times 5 = 175$
711	<pre>(1) 100,000 (2a) 1,000,000,000 (2b) Yes, U.S. population is about 220 million (3a) L (3b) 6,912,000 (3c) 5,332,264 (4a) 1352 (4b) 35,152 (4c) 28,978 (5) 1024</pre>
712	(1) 720 (2) 24 (3) 120 (4) 40,320 (5) 362,880 (6) 39,916,800 (7) 7 x 6 x 5 x 4 x 3 x 2 x 1 (8) 5040 (9) Varies
714	(2a) B; C; D (2b) A; C; D (2c) A; B; D (2d) A; B; C (2e) 12 (4b) 10
718	(8) No (10) 3 (11) An exchange between two repeated letters does not distinguish one arrangement from another.
719	 (a) MOM; MMO; OMM; 3 (b) DAD; ADD; DDA; 3 (c) OTTO; OTOT; OOTT; TOTO; TOOT; TTOO; 6 (d) BEEEB; BEEBEE; BEBEEE; BBEEEE; EBEEBE; EBEEBE; EEBEEB; EEBEBE; EEBEB; 10
	(e) 210
721	(2) Yes; $1 + 2 = 3$ or $3 - 1 = 2$ or $3 - 2 = 1$; $6 + 4 = 10$; $3 + 3 = 6$; 4 + 1 = 5 (5a) 45 (5b) 45 (5c) 252 (5d) 210; 165; 495

(

COUNTING TECHNIQUES (continued)

- 724 (1) A 1; B 2; C 4; D 8; E 16; F 32; G 64; H 128; Powers of 2, all possible combinations of n objects. For n = 4, <u>2 X 2 X 2 X 2 = 16</u> combinations. There are two choices for each object, either it is counted or it is not counted.
 - (2) Always a perfect square (3) 5; 6 (4) 15; 21 (5) 20; 35
 (6) Sums are the numbers (except the first) on strip E; 4, 6, 4, 1;
 - 1, 8, 28, 56, 70, 56, 28, 8, 1
- 726 (1) 8 (2) 3 (3) 3/8

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- (4) 1; 1; 3; 10; 3; 5; 1; 4; 1; 0; 5; 5/32
- 727 (5) 1 way to get 4 heads and 0 tails; 4 ways to get 3 heads and 1 tail; 6 ways to get 2 heads and 2 tails; 4 ways to get 1 head and 3 tails; 1 way to get 0 heads and 4 tails; 6/16
 - (6a) 10/32 (6b) 5/32 (6c) 56/256 (6d) 1/64 (6e) 6/16
 - (6f) 1/32 (6g) 1/128 (6h) 1/1024

BUSINESS AND COMMERCE

- 734 (1) 1,000,000 (2) January (3) December (4) 1972
 (5) about 2,400,000 (6) Summertime jobs for students and agriculture workers (7) Number is about the same (8) Number is increasing
 (9) 1980-1982 Answers will vary. (10) about 6%
 - 735 (11) 6.3% (12) Atlantic, Quebec, B.C. (13) No; The regions have different populations. The base figure is not the same for each region. (14) 25-44 (15) 20-24 (16) 756,000 (17) 2,080,000 (18) 72%
 - 736 (1) Yes (2) 1969 (3) 1970 (4) late 1970; early 1969
 (5) Can't tell. Students may answer yes. (6) Probably 1973. This could be checked in an almanac.
 - 744 (1) \$4.50 (2) \$18.72 (3) \$35.30

HEALTH AND MEDICINE

- 774 (4) Tall, average (5) Short, light (6) Answers may vary.
- 785 (1) 0% and 35% (2) No (3) Calcium 1%, Thiamin (B₁) 15%, Vitamin C 38%, Iron 42%, Protein 50%, Riboflavin (B₂) 209%, Vitamin A 908% (4) 18
- 787 (1) -26°C (2) -32°C; Bitter cold (3) 48 km/h (4) 2°C with a wind at 56 km/h (5a) 2°C; 16 km/h (5b) 4°C; 24 km/h
 (6) She shouldn't make four runs. Exposed flesh will freeze.
- 790 (1) 37° (2a) 37.6°C (2b) 86 (2c) 11 o'clock (3a) .2 (3b) 2
 (4) 7:00 A.M.; 11:00 A.M.; 3:00 P.M.; 7:00 P.M.; 11:00 P.M.
 (5a) Day 2, 1900 (5b) Day 3, 2300
- 792
 (1)
 131; 21; 149; 30; 20; 22
 (2)
 373
 (3)
 35; 40; 6; 5; 8; 6

 (4)
 126°; 144°; 22°; 18°; 29°; 22°
- 793 (5) Few people have these blood types. Few donors are needed.

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PEOPLE AND CULTURE

- 803 (1) 12 (2) Prince Edward Island; Northwest Territory
 (3) Yukon Territory; Ontario (4) 80% (5) 33%; .2%
 (6a) 94 km² (6b) .14 km² (6c) .05 km² (6d) .46 km²
 (8) 900,000 (9) 1,100,000 (10) Females (11) Less
- (Ia) AB, BA, BB (Ib) AA, AB, AC, BB, BA, BC, CC, CA, CB; 9 or 3 x 3
 (Ic) 16 or 4 x 4; 25 or 5 x 5; 36 or 6 x 6; 49 or 7 x 7; 144 or 12 x 12
 (IIa) AAA, AAB, ABA, BBA, BAA, BAB, ABB, BBB; 8 or 2 x 2 x 2
 (IIb) 3³ or 3 x 3 x 3; 4³ or 4 x 4 x 4; 5³ or 5 x 5 x 5; 6³ or 6 x 6 x 6; 7³ or 7 x 7 x 7; 12³ or 12 x 12 x 12 (III) 7⁵ or 7 x 7 x 7 x 7 x 7

RECREATION

- 849 (5) The numbers in the top graph occur about the same number of times while the middle sums occur more.
 - (6) With one die the chance of rolling any number is about the same. With two dice the middle sums have more chances to occur.
- 851 8.8; 7.8; 8.4; 8.1; 8.45; 7.2; 9.45; 8.35; 7.5; 8.75

852	(a)	7.2	(b)	7.2;	(a)	7.5	(Ъ)	7.5; (a)	7.0	(Ъ)	7.0;
	(a)	6.8	(b)	6.8;	(a)	8.1	(Ъ)	8.0; (a)	7.0	(b)	6.8;
	(a)	8.0	(Ъ)	8.1	(a)	7.3	(Ъ)	7.5; (a)	7.3	(b)	7.2;
	(a)	7.3	(b)	7.4							

856 (1) .5 (2) 1.0 (3) 1.0 (4) .5 (5) .5 (6) 0 (7) 57.5 (8) 56.25 (9) 52.0 (10) 55.0 (11) 52.65; 49.2; 48.0; 52.9; 58.75; 50.4 (12) Diver A wins; 268.9 to 263.75