GCMP

GREATER CLEVELAND MATHEMATICS PROGRAM

TEACHER'S GUIDE
FOR SEQUENTIAL WRITE-IN TEXTS
LEVELS A THROUGH D

3

a b
c d

SRA

P8

SCOPE AND SEQUENCE

GREATER CLEVELAND MATHEMATICS PROGRAM

	K	1	2	3	4	5	6
Whole Numbers	•—	—	—	—	—	—	→
Development & Use of Models	✓	✓	✓	✓	✓	✓	
Investigation & Exploration	✓	✓	✓	✓	✓	✓	✓
Development of Computational Skill		✓	✓	✓	✓	✓	✓
Application		✓	✓	✓	✓	✓	✓
Fractional Numbers		•—	—	—	—	—	→
Development & Use of Models		✓	✓	✓	✓	✓	✓
Investigation & Exploration		✓	✓	✓	✓	✓	✓
Development of Computational Skill					✓	✓	✓
Application			✓	✓	✓	✓	✓
Integers					•—	—	→
Development & Use of Models					✓	✓	✓
Investigation & Exploration					✓	✓	✓
Application					✓	✓	✓
Rationals							•→
Development & Use of Models							✓
Investigation & Exploration							✓
Numeration		•—	—	—	—	—	→
Measurement	•—	—	—	—	—	—	→
Investigation & Exploration	✓	✓	✓	✓	✓	✓	✓
Development of Skills		✓	✓	✓	✓	✓	✓
Application			✓	✓	✓	✓	✓
Geometry	•—	—	—	—	—	—	→
Development & Use of Models	✓	✓	✓	✓	✓	✓	✓
Investigation & Exploration	✓	✓	✓	✓	✓	✓	✓
Problem Solving	•—	—	—	—	—	—	→

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GCMP

GREATER CLEVELAND MATHEMATICS PROGRAM

Teacher's Guide for Sequential Write-In Texts Levels A through D

3

Prepared by the staff of the Educational Research Council of America assisted by teachers from the participating Council Schools under the direction of George S. Cunningham.

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CONTENTS

Strengthening understanding of difference as a missing addend
Investigating sums and related differences associated with 10 tens, 10 hundreds, and 10 thousands
Using the idea of subtracting from 10 in computing differences of tens, hundreds, and thousands
Expressing a given number as a sum or difference in many ways
Using addition and subtraction in application situations

Pupil Pages B · 81–104
Guide Pages 95–122

UNIT 7 • ADDITION AND SUBTRACTION

Associating numbers with distances from 0 on the number line
Using algorisms in three-digit and four-digit addition and subtraction
Investigating a missing addend as a difference and a difference as a missing addend
Using guessing and testing as a means of discovery
Investigating a short addition algorism
Computing sums and differences of lengths
Using addition and subtraction in application situations involving money

Pupil Pages B · 105–124
Guide Pages 123–144

UNIT 8 • COMPUTATION OF BASIC PRODUCTS

Using partitioned arrays to investigate the products 1×6 through 10×6, 1×7 through 10×7, and 1×10 through 10×10
Expressing each factor of a product as a sum
Using partial products to compute products that have factors of 6, 7, or 10
Extending understanding of the distributive property
Organizing products
Computing products that have factors of 2, 3, 4, 5, 6, 7, or 10

Pupil Pages B · 125–132
Guide Pages 145–154

UNIT 9 • GEOMETRY

Measuring the surface of a rectangular figure by covering with square units
Recognizing that rectangular figures may have the same size but different shape
Recognizing that the smaller the unit square the more units needed to cover a rectangular figure
Folding paper to make square corners and perpendicular lines
Perceiving parallel lines
Recognizing perpendicular and parallel lines in familiar objects

Pupil Pages B · 133–152
Guide Pages 155–174

UNIT 10 • COMPUTATION OF BASIC PRODUCTS

Using partitioned arrays to investigate the products 1×8 through 10×8 and 1×9 through 10×9
Expressing each factor of a product as a sum
Using partial products to compute products that have factors of 8 or 9
Organizing products
Computing products that have factors of 0 through 10

Pupil Pages C · 153–172
Guide Pages 175–193

UNIT 11 • NUMERATION: THOUSANDS

Recognizing that a million is a thousand thousands
Interpreting numerals for large numbers
Using commas in numerals
Writing numerals for large numbers
Computing sums, and differences for thousands and millions
Computing products that are multiples of 10, 100, and 1000

INTRODUCTION

The young child has a natural curiosity about his world. He eagerly explores his environment and is able to make simple generalizations about the objects that surround him. As he progresses through the grades, this enthusiasm for learning tends to diminish unless it is carefully nurtured.

The teacher of the child in a primary grade is especially sensitive to the child's eagerness to understand the world around him, and makes every effort to create a learning environment that will stimulate and encourage this active curiosity. She knows that the young child is always at his best when he is in the process of discovery. She realizes that adhering rigidly to narrowly prescribed bounds and learning facts in neat categories may seem efficient, but this very efficiency may destroy enthusiasm for learning and stifle intellectual curiosity. She is acutely aware that her own intellectual curiosity has far-reaching effects in the learning environment. The zest for learning which the teacher evidences will be copied by the child. The teacher's use of questions will encourage the child to raise questions and investigate ideas.

Elementary school mathematics programs are beginning to take advantage of the child's curiosity by encouraging him to put a greater emphasis on the search for patterns, basic principles, and a logical structure of the mathematics he learns. There is a growing enthusiasm for skillful teaching procedures, well-planned activities, and the inclusion of new topics in the elementary school curriculum. The authors of the Greater Cleveland Mathematics Program are experienced classroom teachers, child psychologists, mathematics educators, curriculum specialists, and professional mathematicians. Each is aware of the trends in mathematics education and has utilized this awareness in the preparation of materials designed to provide variety and flexibility in meeting the needs of the child. The program is designed to help the child develop his power to reason, and security of understanding is provided in every phase.

GCMP GUIDELINES

The first concern of education is the learner. A good learning program provides an efficient and workable sequence of learning experiences to meet the challenge of providing better mathematics instruction for all pupils.

The concepts in the program may be as new to the teacher as to the pupil. Since instruction is the responsibility of the teacher, teacher-training films and KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER have been prepared to develop the understanding needed to teach the concepts effectively. This Teacher's Guide offers a rich variety of suggestions for effective teaching of the concepts and skills.

The following is a brief summary of the main features of GCMP.

The program meets two parallel objectives for mathematics education for the elementary school:

1. Development of a clear understanding of the number relationships needed in gaining proficiency in solving problems and in applying concepts.
   - Questions are emphasized more than answers. The effective application of mathematics requires the ability to define questions with precision.
   - Reasoning and understanding rather than memorizing rules are stressed.
   - Pupils are taught to look for powerful patterns and concepts that answer many questions of detail.
   - Every effort is made to develop an idea prior to the use of words that name the idea. Language is useless without ideas; the language of mathematics is learned as the pupil participates in developmental activities.
   - Review is built into the development and application of new concepts instead of being emphasized as an end in itself.
   - A variety of teaching techniques ranging from information-giving to guided discovery, is recommended in the activities used to develop concepts.
2. Development of computational skills.
   - Basic facts are learned by referring to a model or picture.
   - Models or pictures are used to develop algorisms for each operation.
   - Computational skills are developed and used in application situations.
   - More practice exercises than necessary for most classes are provided in the activities and on the pupil pages.
   - A variety of teaching techniques is recommended in the activities used to develop computational skills.

The program is articulated and sequential.
- No pupil's progress is handicapped.
- No important understanding is omitted from any pupil's experience.
- No pupil is burdened with busywork of dubious value.

The program is field tested.
- While the program demands serious and sustained effort from teachers, field testing has shown that this teacher effort results in better education for pupils.

The most important part of the program is the development of concepts and skills through a variety of experiences, many of which are described in detail in this Teacher's Guide.

–Using the pupil page is only one of many activities designed to introduce and develop basic concepts and skills.
–The pupil page may be used for discussion of ideas, practice, and to give a chance for the pupil to test what he has perceived.

Teaching the Program for Sequential Write-In Texts A, B, C, and D

Sequential Write-In Texts are textbooks rather than workbooks. They should be used by themselves rather than in conjunction with other books.

The activities used prior to work on the pupil pages are planned to stress single concepts. Some activities require the teacher to lead a group discussion, to ask carefully worded questions, and to guide the children to discover the underlying concept. Others indicate a direct, straightforward presentation. It seems to be true that the interaction of the children in a group is a necessary factor in the development of the child's reasoning ability.

The pupil pages marked *For Class Discussion* are used to develop significant ideas in the program. The discussion for these pages should include both pupil-pupil and teacher-pupil interaction. The pupil pages marked *reference page* contain important developmental ideas, discussion questions, generalizations, or basic facts. The teacher should plan to have pupils refer to these pages whenever they want to.

As the child participates in discovery situations, he perceives mathematics as a system of generalizations drawn from his experiences and begins to appreciate its simplicity. The skillful teacher realizes that it is better to gain insight into a principle than to memorize a great number of unrelated facts and procedures. The principle, when grasped, enables the child to discover the facts for himself and devise procedures whenever he needs them.

The teacher will notice the following features in the program contained in Sequential Write-In Texts A, B, C, and D.

1. Visual models are used extensively in deepening perception of number structure.
2. Many of the activities require the child to manipulate objects and make generalizations.
3. The concepts of sum and of difference are reviewed intensively from experience with sets of objects and number strips.
4. The concept of product is developed intensively from experience with arrays.
5. Place-value concepts are extended with the help of activities that use bundles of counters, number strips, wooden cubes, the Countingman, and the Bead Frame.
6. Placeholder addition, subtraction, and multiplication equations are used to show the number structure in given situations.
7. Perception of sums from 0 through 18, and related differences, is deepened and computation of these sums and differences is practiced extensively.
8. Algorisms for computing sums and differences are reviewed.
9. Partitioning of arrays into parts is used to show that a product is a sum of products.
10. Perception of products from 1×2 through 10×9 is deepened and computation of these products is practiced extensively. Some children will master all the basic facts. Most children will master the underlying concept of product.
11. Algorisms for computing products are introduced.
12. Measurement concepts are developed around the question "How much?" This question leads to review of the fractional numbers $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{3}$ and the introduction of the fractional numbers $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{10}$, $\frac{1}{20}$, and $\frac{1}{100}$. Rulers are used to develop an understanding of numbers as a measure of length. Gallon, quart, pint, and cup containers are used to develop an understanding of numbers as a measure of quantity.
13. A sense of the intervals of time that are designated as a second and a minute are developed.
14. The geometric concepts of perpendicular lines and parallel lines are explored. The geometric concept of area of rectangular figures is developed.
15. Addition, subtraction, and multiplication combinations are used to answer questions about money, time, linear measure, and liquid measure.

As the teacher guides the children in the program, she will see them making discoveries. As they make generalizations about their discoveries, she will want to lead them to new situations where their generalizations may be checked. She will encourage the children to search for new patterns, new ideas, and new relationships.

Pacing the Program

The teacher must assume responsibility for pacing the program to meet the needs of the individual child and of the group. The amount of time required for each unit depends on the purpose of the unit and on the ability of the group. Write-In Texts A, B, C, and D should be used in that sequence. No books should be omitted.

Sequential Write-In Text A

Units 1 and 2 introduce the concept of product as the number of objects in an array and investigate the properties of multiplication. These units offer the opportunity to move easily into the third-grade program since computation is not involved. Unit 5 uses partitioned arrays to investigate and compute some basic multiplication combinations. Pacing throughout these three units should allow time for a feeling of security as success is achieved.

In Unit 4, place value concepts are reviewed and extended through the thousands place. Investigations should proceed at a pace that allows for building a feeling of confidence.

In Unit 3, an intuitive approach to fractional numbers is utilized in work with $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{3}$. Since the major emphasis is on exploration rather than on mastery, pacing should proceed at a more rapid rate than in any other unit in this book.

Sequential Write-In Text B

Units 8 and 10 use partitioned arrays to investigate and compute the rest of the basic multiplication combinations. Pacing in these two units must provide for a feeling of security and success.

Unit 6 provides review of the concepts of sum and difference; review of computation of sums through 18, and related differences; and use of addition and subtraction combinations in determining sums and differences of tens, hundreds, and thousands. Unit 7 provides opportunity for maintaining skill with algorisms used in computing sums and differences through four-digit addends. Addition and subtraction skills are applied to determine sums and differences of money and length. Presentation of these two units should allow for building a feeling of confidence and computational skill while sustaining interest.

In Unit 9, provision is made for an exploration of geometric concepts. Since mastery is not intended, pacing should proceed at a more rapid rate than in any other unit in this book.

Sequential Write-In Text C

In Unit 11, place value concepts are extended through the millions place. In this unit, skill with the basic multiplication combinations is applied to computing products that involve multiples of 10, 100, and 1000. In Unit 12, a multiplication algorism for use in computing products of one-digit, two-digit, and three-digit factors is introduced. Pacing in these two units should allow for a feeling of security and development of computational skill.

Unit 13 provides for an exploration of measurement and geometric concepts while applying knowledge of multiplication. In Unit 14, an intuitive approach to fractional numbers is utilized in work with $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{10}$, $\frac{1}{20}$, and $\frac{1}{100}$. Since major emphasis is on exploration, pacing should proceed at a more rapid rate than in the other two units in this book.

Sequential Write-In Text D

In Unit 15, arrays are used to introduce the concept of missing factor. Unit 16 investigates several multiplication algorisms and provides practice in computation of products. The time allowed for these two units should provide for a feeling of success as well as development of computational skill.

Unit 19 provides practice in computing sums, differences, and products. Pacing should allow for building a feeling of confidence as well as sustaining interest.

In Unit 17, application of known number concepts to situations involving liquid measurement is provided. Unit 18 offers the opportunity to apply knowledge of multiplication to appropriate situations. Pacing in these two units should proceed at a more rapid rate than in the other units in this book.

Organization of the Teacher's Guide

Each unit has sections devoted to *Objectives*, *Key Ideas*, *Concepts*, *Scope*, *Fundamentals*, *Readiness for Understanding*, and *Developmental Experiences*. In most units there are *Pupil Work Page* suggestions and *Supplemental Experiences*. To help the teacher use the guide effectively, each of these sections is discussed here.

The *Objectives* section includes brief statements of the major goals of a unit; the *Key Ideas* and *Concepts* that are developed in the unit; and a reference to the chapter in KEY TOPICS IN MATHEMATICS FOR THE PRIMARY TEACHER that presents the relevant mathematical concepts in greater detail.

The *Key Ideas* are usually brief phrases that describe one or more of the major ideas or important insights for understanding.

The *Scope* section indicates specific concepts or skills to be developed in the activities that follow.

The *Fundamentals* section presents to the teacher a concise overview of the basic mathematical concepts contained in the material presented to the pupils and some pedagogical hints.

In the *Developmental Experiences* sections, the teacher will find a variety of activities used to introduce concepts, provide practice, and extend the pupil's perception. It also includes the materials list that indicates the devices and materials needed for the developmental experiences.

- The suggestions for *Pupil Work Pages* extend and implement the concepts presented on each page, and suggest additional activities and questions.
- *Supplemental Experiences* may be used for review work or to reinforce a pupil's understanding of a concept. No attempt has been made to classify a supplemental activity as "enrichment" or "remedial," since what serves as review for one pupil may bring initial understanding to another.

Different types of activities are marked as follows:

▶ Developmental activity

■ Supplemental activity

● Pupil Work Page suggestions

There is a materials list at the beginning of each *Developmental Experiences* section. Some teachers ask pupils to help prepare the materials, especially when numeral cards and tagboard strips must be cut. Teachers have also found it advisable for each pupil to have a small box to hold his own supply of numerals, placeholders, operation and comparison symbols (+, −, =, and >), and tagboard strips.

UNIT 1
THE PRODUCT
Pages 1 Through 10

OBJECTIVE
To recognize the array as a physical model of multiplication.

The child will arrange objects into arrays and name them as _____ by _____ arrays. He will begin to recognize common arrays such as a tiled ceiling or floor, panes of glass in the window, and lighting grids.

The child is helped to grasp the closure property of multiplication. He learns that the product of two whole numbers is itself a whole number.

See Key Topics in Mathematics for the Primary Teacher: Multiplication.

KEY IDEAS
2×3 is a number.
The whole is the sum of its parts.

CONCEPTS

array	multiplication
factor	product
	times sign ($\times$)

KEY IDEA
2×3 is a number.

Scope
To explore an array.

Fundamentals
A random arrangement of objects may make it difficult to determine the number of objects. An orderly arrangement often makes counting easier. Consider the array below:

● ● ● ● ● ●
● ● ● ● ● ●
● ● ● ● ● ●

In this array it is easy to count by ones, twos, threes, or sixes. Since the objects are arranged in an orderly form, the array may be described as a 3 by 6 or 6 by 3 array. The number of the array is the product 3×6 (or 6×3)—the number of rows times the number of rows. It is important for the child to observe many arrays and to be able to express the number of an array as a product (____ $\times$ ____).

In the set of whole numbers a product is always the number of an array. Consider the 2 by 3 array below.

● ● ●
● ● ●

The product is 2×3 or 3×2; the factors are 3 and 2. Each factor is the number of rows on one side or another of the array. The number of members in any array is properly expressed as a product—number of rows times number of rows. Note that the product is a number.

Since a 2 by 3 array can be rearranged to show a 6 by 1 array, we are able to say that 6×1 and 2×3 (or 3×2) are the same products.

Readiness for Understanding
Ability to count.

Developmental Experiences

for flannel board	*for each child*
disks	washers
array cards	sheet of 1″ squared paper
numerals	plastic numerals and symbols
symbols	

pocket chart
tagboard strips

▶ An understanding of products may be developed through the use of an array. Arrays can be created quickly and with ease on a bead frame, an abacus, the flannel board, or a pocket chart.

Present the array illustrated below.

● ● ●
● ● ●

Have the class suggest ways to name the number of this array. The responses may be many and varied: $2+2+2$, 6, $3+3$, $4+2$, $10-4$, and so forth. All appropriate answers are acceptable. Then call the children's attention to the arrangement of the objects. Say that the objects have been arranged in a 3 by 2 array or a 2 by 3 array. Explain that the product 3×2 or 2×3, is another way to express the number of members in this array. The product for this array is the number 3×2 or 2×3.

Write two forms for this product on the chalkboard. Read each to the class:

3×2 "3 times 2"
2×3 "2 times 3"

Show a 2 by 4 array.

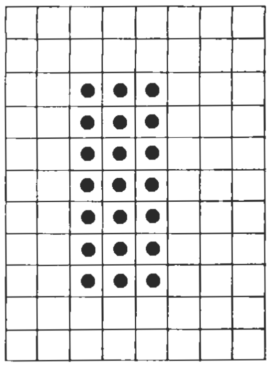
● ● ● ●
● ● ● ●

Ask a child to write the product for this array. He may write 2×4 or 4×2. Both products are correct.

Have several children show an array of their choice. Have each of these children select a volunteer to come to the board and write a product for the array.

▶ Give each child 26 washers and a sheet of 1-inch squared paper. Divide the class into 4 teams. List the names of each team's members on the chalkboard. Arrange the class so that members of the same team do not sit together.

Tell the children to use their washers to show an array of their choice on their sheets of squared paper. After the arrays have been built, tell the children to use their plastic symbols to indicate a product that is shown by the array.



3×7 (or 7×3)

Select a child from each team to record his team's score in the first round of the game. Have each child in the class name a product that is shown by his array. Have the team's recorder write that product on the chalkboard next to the child's name.

Give one point for every product named. Give a bonus point for each product that names a different array. Have the recorder of each team check products that already are given in the listing. For example, the following products may have been listed for Team I.

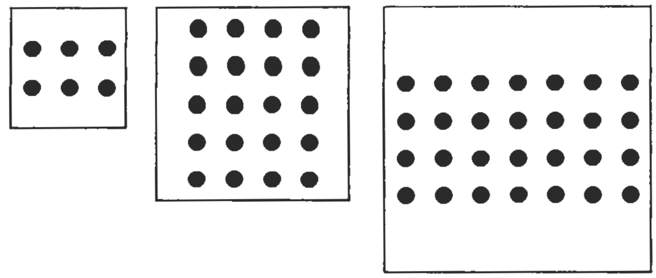
Team I	
Mary B.	3×2
Bob M.	4×6
Jim G.	7×3
Linda R.	2×3
Betty S.	2×10
Ben O.	6×4
Ralph T.	3×2

In this instance, Team I earns 7 points for the products it has named and a bonus of 4 points for the products 3×2, 4×6, 7×3, and 2×10. (2×3 and 6×4 are checked because they are listed above as 3×2 and 4×6; 3×2 is a repeated product.) After each team has totaled its points, have the recorders erase all products from the chalkboard.

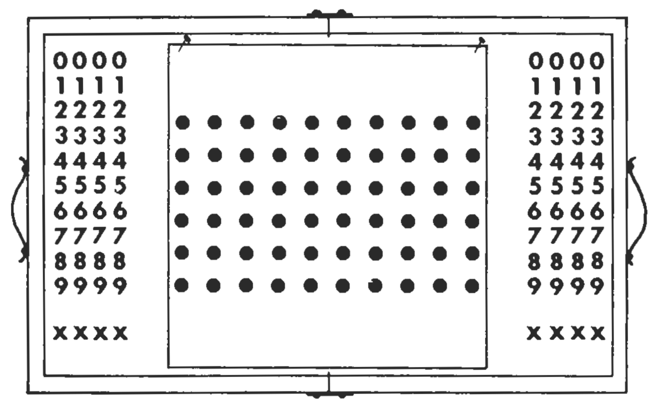
Begin another round of the game. Direct the children to remove the washers from their papers, but to leave the plastic numerals recording their first products on their desks. Then tell the children to construct a different array. The product shown on their desks will help them remember which array they used in the first round of the game. Again have a recorder list his team members' products and determine the points his team earns.

Continue the game for four or five rounds. Then have each team total its points and declare a winner. It is possible that somewhere during the game, an array with just one row may be used. The idea inherent in this model—the identity number for multiplication—will be treated in detail in the next unit.

▶ Design several array cards. Use square pieces of tagboard (from 4 inches by 4 inches through 12 inches by 12 inches) and $\frac{1}{2}$-inch gummed circle stickers to show 2 by 2 through 15 by 15 arrays.



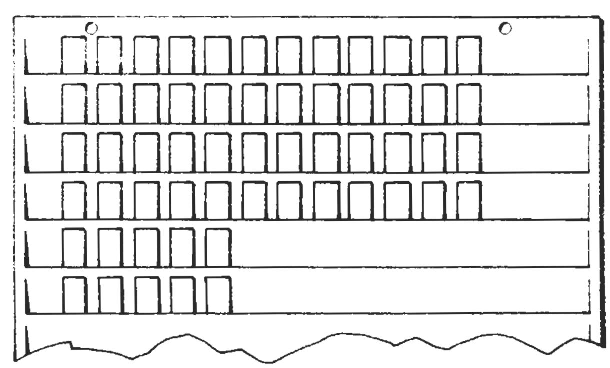
Arrange the flannel board as shown, with an array card in the center and sets of numerals and times symbols (×) on each side.



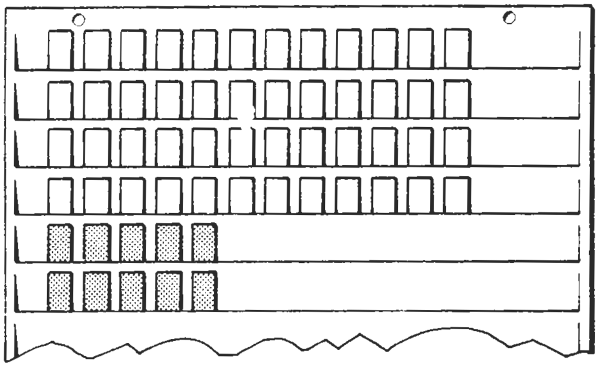
Assign a child to each side of the flannel board. Tell the children that at a given signal they are to come to the board and select symbols from their side to show a product that represents the array shown. Let the children see who can first complete the assignment. Then have each child tell how he decided upon the product he showed.

Continue to have other pairs of children show products that express given arrays.

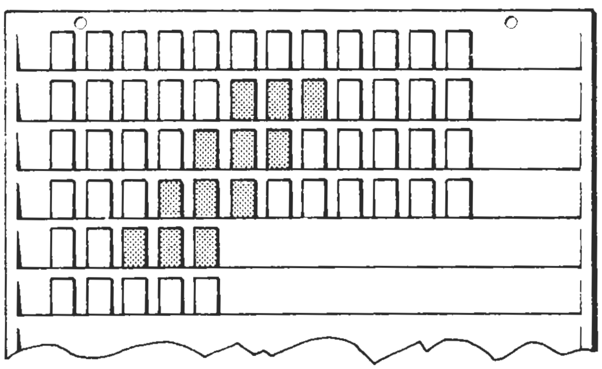
▶ Place several rows of tagboard strips in the pocket chart as illustrated.



The strips should be one color on one side and another color on the other side. Have the class look for models of products in these rows of strips. Give the children an opportunity to show their models by turning over the strips that helped them visualize the number of their product. For example, a child may show his model for 2×5 in the following way:



However, this is not the only way he could have shown it. Another child may see 3×4 in a less obvious way and show a model similar to the one in the following illustration.



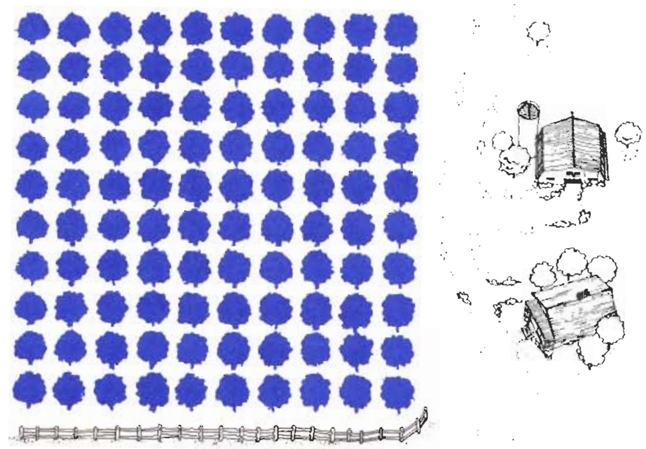
After a child has shown his model of a product, have him record his product on the chalkboard and turn the strips back to their original side. Have the children see how many different products they can show.

Name

UNIT 1 THE PRODUCT

For Class Discussion

Many years ago, Mr. Mason's father planted apple trees on the family farm. Last week Mr. Mason took his father for a ride in an airplane. As they flew over the apple orchard, they felt proud of the neat array of trees.



How many trees are in the orchard? *See pupil page suggestions.*

reference page

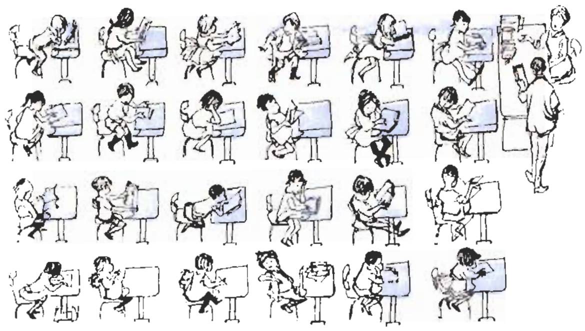
A-1

Pages 1 through 6

● Give the children time to examine the array on page 1. Help them read the paragraph. Then discuss the question of how many trees. As various responses are given, record them on the chalkboard and have each child tell how he decided upon the number he feels answers the question. For example, someone may say that he sees 10×10 trees; someone else may say that he sees 100 trees, or that he sees 5×20 trees; someone else may say that it is possible to count and find out how many trees there are; it is possible that another child may say that he added 2's and arrived at the answer to the question. There are other responses children may give and each should be considered.

The class should observe that all of the responses tell how many members belong to the array of trees. While the form of the responses may differ, all of them describe the number property of the array; the names in no way affect the number of trees in the array.

For Class Discussion



"Miss Jones, how many children are in your third-grade class?" asked Mr. South.

She looked at the class and answered, "Four times six."

"But you haven't told me the number of children!" exclaimed Mr. South.

"Oh yes I have," said Miss Jones.

____

Has Miss Jones told Mr. South the number of children in her class? *See pupil page suggestions.*

How do you know?

reference page

A-2

Complete the 5 by 5 array to show 5 × 5.

o o o o o
o o o o o
o o o o o
o o o o o
o o o o o

Complete the array for each product.

1. 3 × 2

o o o
o o o

2. 2 × 8

o o o o o o o o
o o o o o o o o

3. 7 × 3

o o o
o o o
o o o
o o o
o o o
o o o
o o o

4. 3 × 5

o o o
o o o
o o o
o o o
o o o

5. 6 × 4

o o o o o o
o o o o o o
o o o o o o
o o o o o o

6. 4 × 5

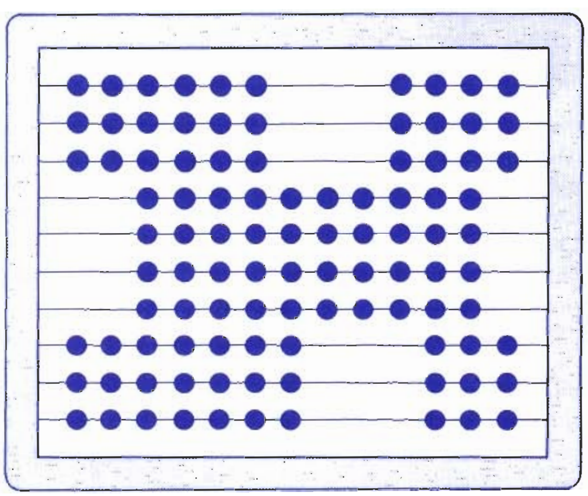
o o o o o
o o o o o
o o o o o
o o o o o

A-4

Name

For Class Discussion

The product △ × □ is the number of things in a △ by □ array.



What products do you see? *Any products through* 10 × 10.

reference page

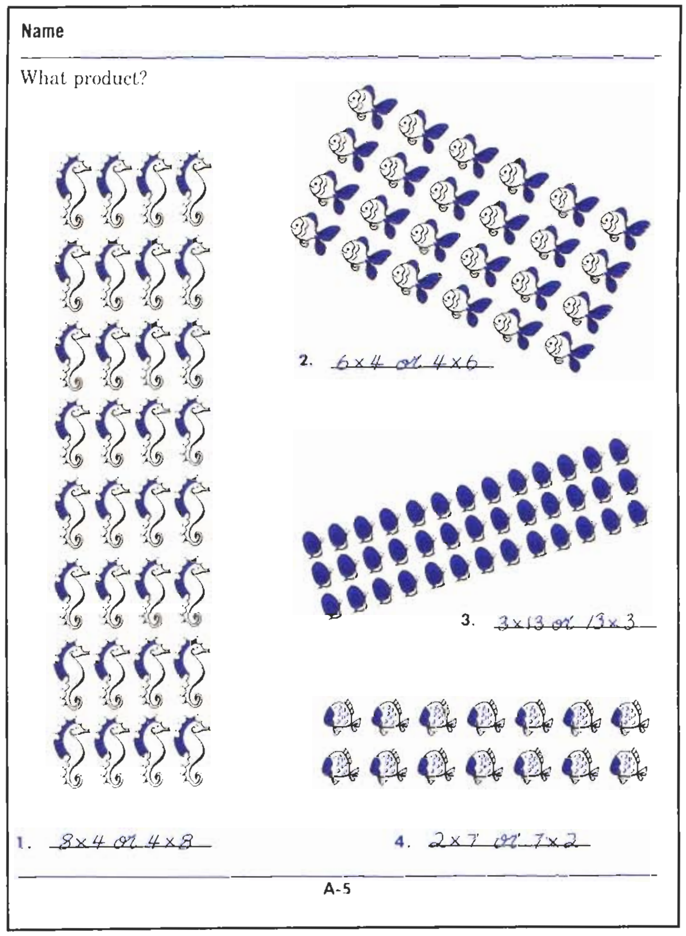
A-3

● After the illustration on page 2 has been examined and the story read, have the class discuss the question of 4×6 being a number. As children agree or disagree, have them explain their responses. For example, a child may not believe the number of children is known unless the count for this number is given. Help this child observe that the number property of the array stays the same whether or not the number is the product 4×6 or the sum 20+4. Help him observe that 4×6 is 20+4. Have this child show a 4×6 array on the flannel board. Then have him rearrange the array so that 20+4 objects are shown. Ask him to tell what he has concluded. Another child may say that 4×6 tells the number of children in the class because it gives him a picture of a particular number of children in an array.

● Have the children examine the beads in the illustration on page 3 and make a list of products they can see. Then discuss the products they listed.

If you draw a large model of the array's illustration on a sheet of chart paper and tape it to the chalkboard, the children may take turns showing the class how they saw the products they listed. They may then point out the models of their products. A record of the different products may be kept on the chalkboard.

● Use page 4 to give the children an opportunity to complete arrays to show given products. Work the example at the top of the page with the class. Assign exercises 1 through 6 as independent work.


Name

What product?

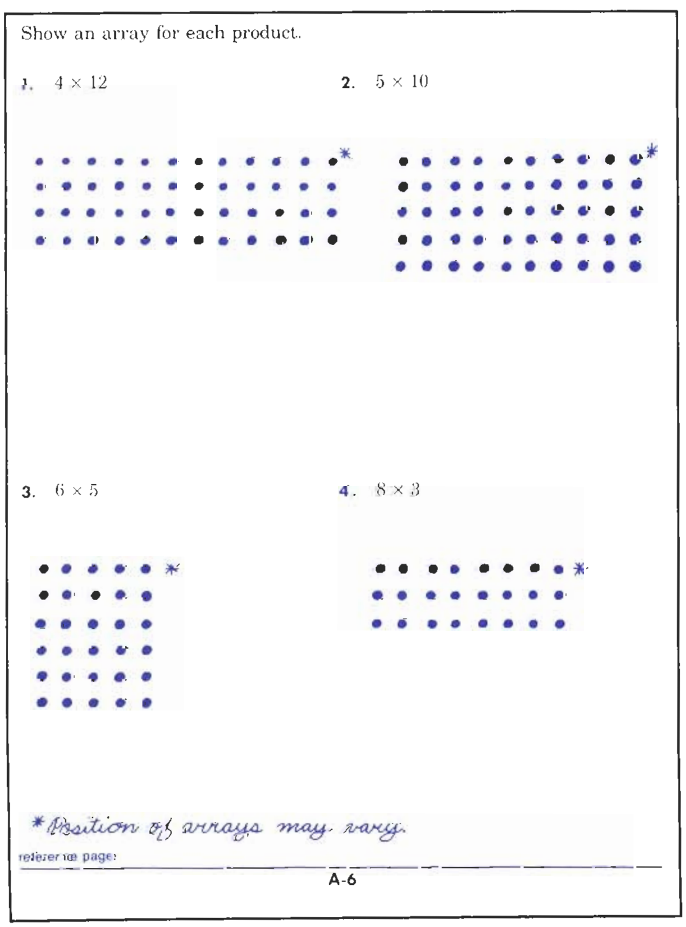
2. 6×4 or 4×6

3. 3×13 or 13×3

1. 8×4 or 4×8

4. 2×7 or 7×2

A-5


Show an array for each product.

1. 4 × 12

2. 5 × 10

3. 6 × 5

4. 8 × 3

*Position of arrays may vary.

reference page

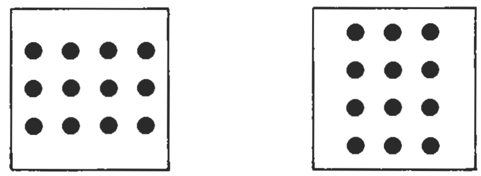
A-6

● Use page 5 to give the children more experience identifying products. For each array, two forms of the product represent the model; a child may list either one—both forms are correct. For the first array the product is 8 × 4 or 4 × 8.

● The exercises on page 6 provide children with experience in showing an array for a given product.

Supplemental Experience

■ Play "Simon Says" with cards that show arrays from 2×2 through 10×10. Design four cards for each product. For example, for 3×4 two sets of cards similar to the following could be used.



Have the children sit in a circle. Give each child two or three array cards. Tell the class to follow only the instructions that Simon says. Give commands such as the following:

Simon says, "Hold up a model for 4×8."
Simon says, "Put down a model for 5×7."
Hold up a model for 10×3.
Simon says, "Hold up a model for 6×2."
Put down a model for 9×7.

As the game continues, have the children exchange cards so that they stay alert and listen for different commands. Some of the commands may be made more involved if desired.

Simon says, "Hold up a model for 3×9 and a model for 5×8."
Simon says, "Hold up a model for 6×5."
Put down arrays of 4×7 members.
Put down arrays of 2×8 members and arrays of 9×8 members.
Simon says, "Put down arrays that have 7×3 members."

KEY IDEA

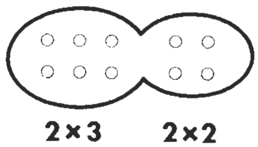
The whole is the sum of its parts.

Scope

To explore the partitioning of arrays.

Fundamentals

Partitioning an array into parts provides a model for the distributive property—a given product may be expressed as the sum of two or more products. Consider a 2×5 array partitioned into two parts: a 2 by 3 array and a 2 by 2 array.

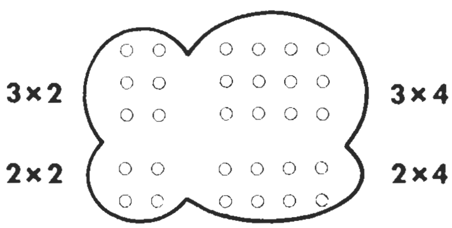


The partition introduces the sum (2×3) + (2×2) as the product 2×5. Thus the product of the array is the sum:

$$2\times5 = 2\times(3+2)$$
$$2\times5 = (2\times3) + (2\times2).$$

Although the meaning is the same with or without parentheses, (2×3) + (2×2) = 2×3 + 2×2, the use of parentheses by the teacher may help the child recognize that the whole product is the sum of partial products.

As another example, partition a 5 by 6 array into four parts.



We note that the sum (3×2) + (2×2) + (3×4) + (2×4) is the product 5×6.

Readiness for Understanding

Ability to describe an array as _____ by _____.

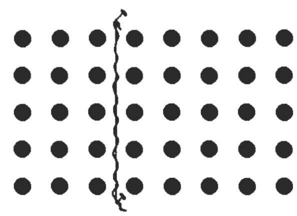
Developmental Experiences

for flannel board
disks
yarn

for each child
plastic numerals and symbols

bead frame
pocket chart
array cards
equation cards

▶ Place a 5 by 8 array on the flannel board. Have a child write the product on the chalkboard that represents the number of members in the array. Then partition the array with a piece of yarn as illustrated.

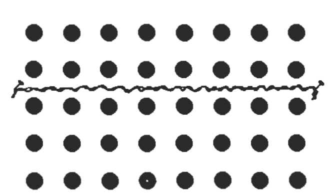


Let the children examine the two arrays that result. Ask them to tell the product for each array. Record their responses on the chalkboard beside 5×8. Show the relationship between the product 5×8 and the sum of the products 5×3 and 5×5 in an equation. There are many possible ways that the products could be recorded:

$$5\times8 = (5\times3) + (5\times5)$$
$$5\times8 = (5\times5) + (5\times3)$$
$$5\times8 = (3\times5) + (5\times5)$$
$$5\times8 = (5\times5) + (3\times5)$$
$$8\times5 = (5\times3) + (5\times5)$$
$$8\times5 = (5\times5) + (5\times3)$$
$$8\times5 = (3\times5) + (5\times5)$$
$$8\times5 = (5\times5) + (3\times5)$$

Ask the class what conclusions they have made from their observations of the partitioned array. One child may say that it is possible to make two small arrays out of the large array. Another may observe that a 5 by 8 array is made up of a 5 by 3 array and a 5 by 5 array. It is possible that a third child may say that when you add the products of the small arrays you get the product of the large array. Give the children ample time to tell about their observations.

Then ask the children whether this is the only way a 5 by 8 array could be partitioned into two smaller arrays. Call on a child to show another partitioning of this array. Perhaps he will choose to partition the array as:



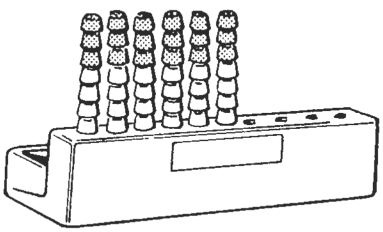
Have the class observe the two new arrays and name the product for each array. Help a child write an equation that shows the relationship between the product for the whole array and the product for the parts. He may write the following equation or one of its many possible forms:

$$5 \times 8 = (2 \times 8) + (3 \times 8)$$

Have other children partition the 5 by 8 array. In each instance, help a child write an equation that shows the relationship between the products for the whole array and the two parts.

Adapt this procedure to other arrays.

▶ On a bead frame, display an array such as the one illustrated.

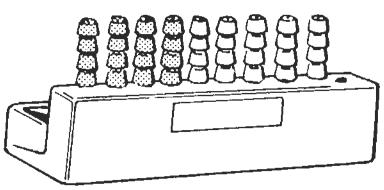


Have the children use their plastic symbols to build an equation on their desks that shows the relationship between the product for the whole array and the products for its parts. Tell the children that each part of the given array is indicated by a different color. The children may show the equation $7 \times 6 = (3 \times 6) + (4 \times 6)$ or any of its possible forms, for example:

$$7 \times 6 = (3 \times 6) + (4 \times 6)$$
$$6 \times 7 = (6 \times 3) + (6 \times 4)$$

If plastic parentheses are available, they may be used in building the equation. If parentheses are not available for this activity, then the children may do without them.

Place another array on the bead frame as illustrated.

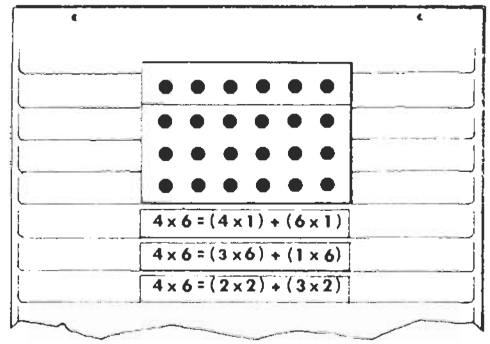


Again, have the children build an equation that shows the relationship between the product for the whole array and the products for the parts. The children may show the following equation or any one of its possible forms:

$$4 \times 9 = (4 \times 5) + (4 \times 4)$$

Continue to display other arrays on the bead frame. In each instance, have the children build an equation that shows the product for the given array as the sum of the products of its parts.

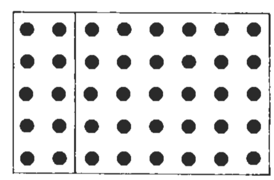
▶ Place a tagboard card in the pocket chart that shows an array partitioned into two parts. Place three equations below the array; only one equation should represent the illustrated array and its particular partitioning.



Have the children examine the equations and decide which one is illustrated by the partitioning of the given array. Ask a child to point out the equation he believes represents the illustration and to explain his choice. Have the class decide whether or not he is correct.

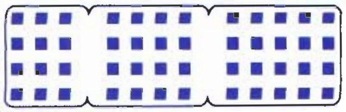
Continue in a similar way with other arrays. In some instances use only one equation that fits the illustration; in other instances use two equations that fit the illustration. For example, the following three equations could be used with a 5 by 8 array partitioned as shown. Only two of these equations fit the illustration.

$$5 \times 8 = (5 \times 2) + (5 \times 6)$$
$$8 \times 5 = (2 \times 5) + (5 \times 6)$$
$$5 \times 8 = (6 \times 2) + (6 \times 5)$$



As the children become more proficient, give them examples in which all three of the equations or none of the equations fit the illustration used.

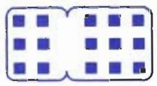
Name



$(4 \times 3) + (4 \times 4) + (4 \times 5) = 4 \times 12$

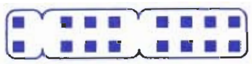
What product?

1.



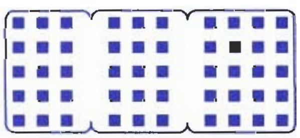
$(3 \times 2) + (3 \times 3) = 3 \times$ 5

2.



$(2 \times 1) + (2 \times 3) + (2 \times 4) = 2 \times$ 8

3.



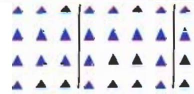
$(5 \times 3) + (5 \times 3) + (5 \times 4) =$ 5 $\times$ 10*

*Order of numbers may vary.

reference page

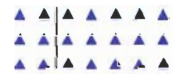
A-7

Name

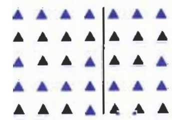


$4 \times 8 = (4 \times 3) + (4 \times 4) + (4 \times 1)$

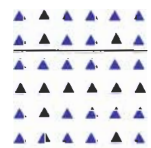
What product?



1. $3 \times 7 = (3 \times$ 2 $) + (3 \times$ 5 $)$*



2. $5 \times 7 = (5 \times$ 4 $) + (5 \times$ 3 $)$*



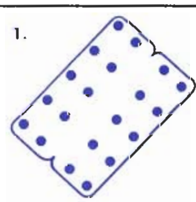
3. $6 \times 6 = ($ 2 $\times 6) + ($ 4 $\times 6)$*

*Order of numbers may vary.

A-9

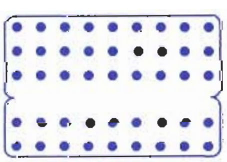
What product?

1.



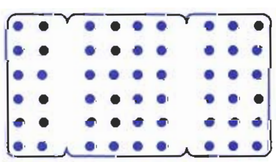
$(2 \times 5) + (2 \times 5) =$ 4 $\times 5$

2.



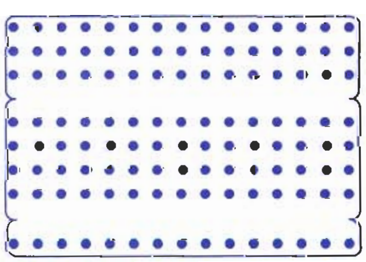
$(3 \times 9) + (2 \times 9) =$ 5 $\times$ 9*

3.



$(2 \times 6) + (4 \times 6) + (3 \times 6) =$ 9 $\times$ 6*

4.



$(3 \times 15) + (4 \times 15) + (1 \times 15) =$ 8 $\times$ 15*

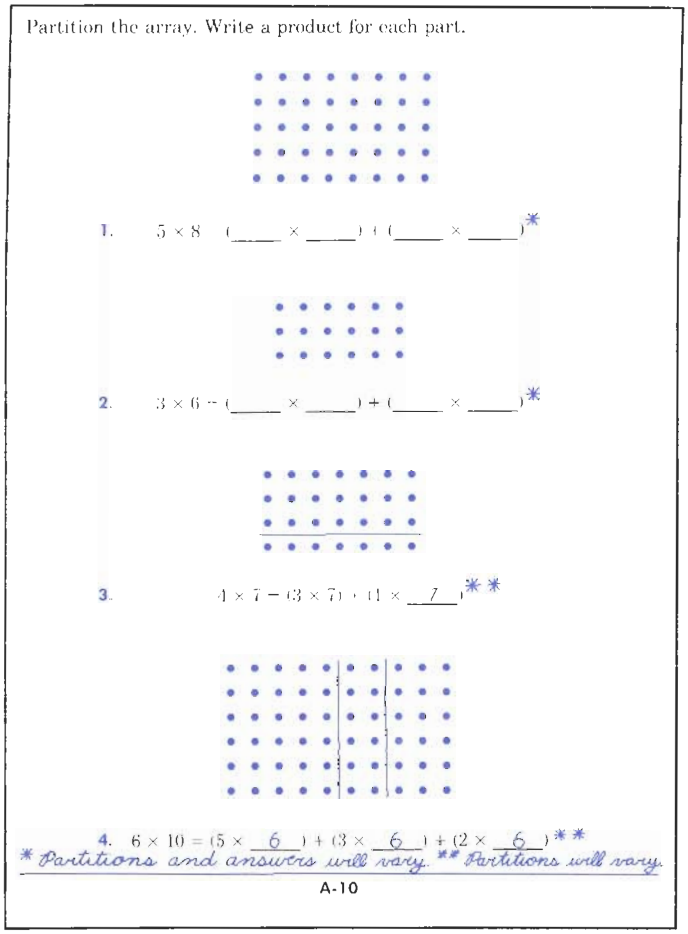
*Order of numbers may vary.

A-8

Pages 7 through 10

- Use pages 7 and 8 to provide experience using the concept that a product is a sum of products. Discuss the array at the top of page 7 with the class. Have them state the product for each of the three parts of the array and the product for the whole array. Have someone tell how these numbers are related. Tell the children that, on each page, they should observe each array carefully and complete the equation below each array. Work through one of the exercises on each page with the children before they work on their own.

- Pages 9 and 10 continue to develop the idea that a product is a sum of products. For these two pages, use the procedure suggested for pages 7 and 8.



Partition the array. Write a product for each part.

1. 5 × 8 = (____ × ____) + (____ × ____)*

2. 3 × 6 = (____ × ____) + (____ × ____)*

3. 4 × 7 = (3 × 7) + (1 × _7_)**

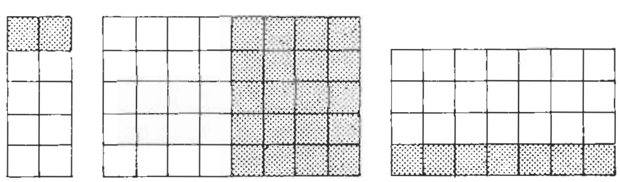
4. 6 × 10 = (5 × _6_) + (3 × _6_) + (2 × _6_)**

* Partitions and answers will vary. ** Partitions will vary.

A-10

Supplemental Experience

■ Cut several rectangles from sheets of tagboard. Construct several arrays such as those illustrated below.



Part of each array may be colored or shaded by using crayon or a felt-tip pen.

Fasten one array to the chalkboard. Separate the class into two teams. At a given signal a member from each team is to go to the chalkboard and write an equation that shows the relationship between the product for the whole array and the product for each of its parts. Give one point to the child who first completes the assignment correctly.

Continue with other arrays and other pairs of children until every child has had an opportunity to participate. Total each team's points to see which team won.

UNIT 2
PROPERTIES OF MULTIPLICATION

Pages 11 Through 20

OBJECTIVE

To extend the development of the properties of multiplication.

Through familiarity with the array, the child will develop an awareness of the properties of the Set of Whole Numbers with respect to multiplication. He has learned that 2×3 is a whole number (the closure property of multiplication), and knows intuitively that 2×3 is 3×2 (the commutative property of multiplication). The distributive property of multiplication over addition will become meaningful as the child recognizes that product plus product is product. The three-dimensional array will be used to develop understanding of the rearrangement and associative properties.

See Key Topics in Mathematics for the Primary Teacher: Multiplication.

KEY IDEAS

4×5 is 5×4.
Factors of products may be rearranged.
Multiplication is distributive over addition.
$1\times\square=\square$.
4×6 is $2\times6+2\times6$ or 2×12.

CONCEPTS

associative	identity number for multiplication (1)
commutative	rearrangement
distributive	

KEY IDEA

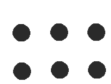
4×5 is 5×4.

Scope

To emphasize the number of an array as rows times rows.

Fundamentals

The child's ______ by ______ description of an array will not be limited by any one orientation of the array. When asked to describe an array such as the one below, some children will respond with 2 by 3, and others will respond with 3 by 2.



If the array were tilted, it would certainly evoke both responses, 3 by 2 and 2 by 3.

As the child uses the array as a model, he sees that the factors of a product may be rearranged without changing the product. When the child realizes that 2×3 is 3×2, he has developed an intuitive idea of the commutative property of multiplication.

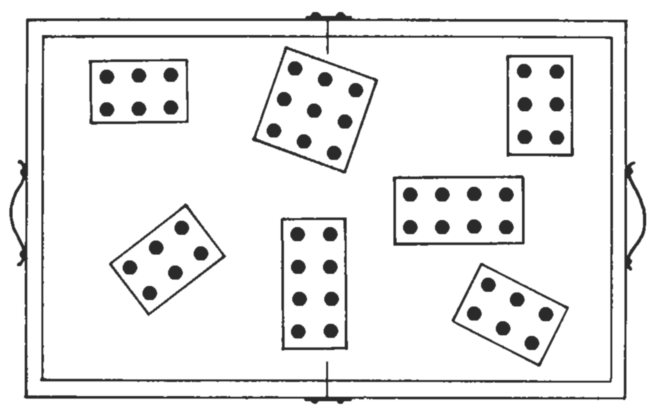
Readiness for Understanding

Ability to describe an array as ______ by ______.

Developmental Experiences

for flannel board	*for each child*
array cards	sheet of graph paper
	colored pencil
tagboard cards	

▶ Scatter several array cards on the flannel board. Three or four of the cards that show the same array should be arranged to show the array from different points of view.



Write a product on the chalkboard that represents the array that is shown more than once. Ask a child to remove all the array cards for the given product. Have him place these cards on the chalktray below the product. Have the class decide whether or not all of the appropriate arrays have been removed. If the child has not identified all of the arrays, let him choose someone to help find the others.

Continue to give other children the opportunity to find arrays for given products. Use arrays for the following products.

2×3	2×4	2×5
3×3	4×4	5×5
4×3	5×4	5×8
5×3		

▶ Write products from 2×2 through 9×9 on $\frac{3}{4}$ by 3-inch tagboard cards. Give each child a sheet of $\frac{1}{4}$-inch squared graph paper, a dark-colored pencil, and a product-card. Make sure that the commuted form for each product is among the cards given out.

Tell the children to make an array for their product. Show them how to make an array by drawing X marks or by coloring squares on the graph paper.

Have each child in turn hold up his product-card and his array so that all of the children are able to see them. Ask if any other child made the same array. For example, if one child displays an array for the product 4×6, a second child will have an array for the product 6×4. Have the two children display their arrays and product-cards side by side. Any orientation for the arrays is acceptable. Continue until each product and array has been shown with the commuted product and array.

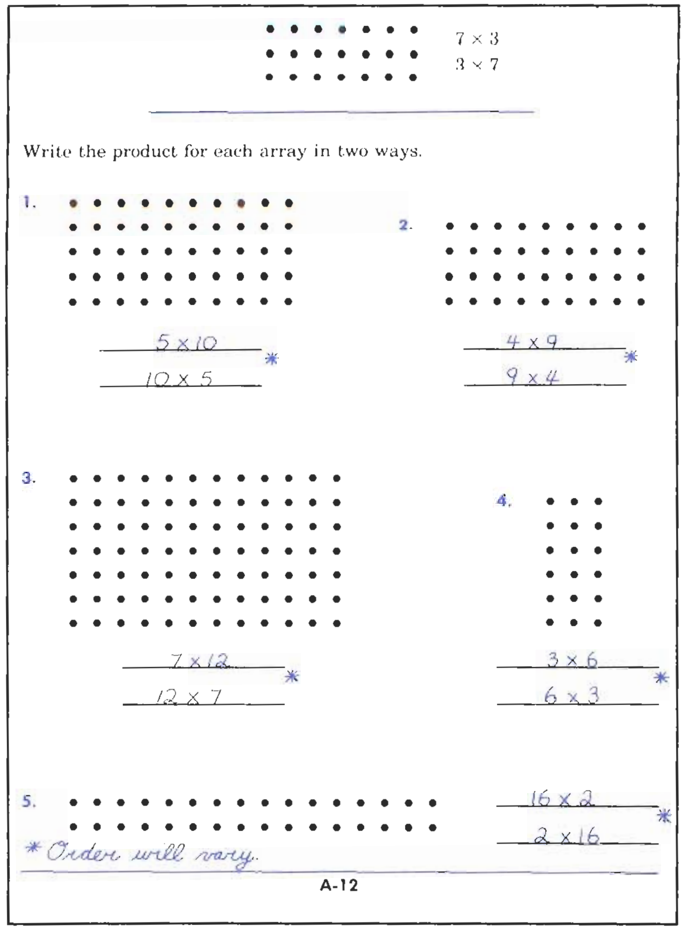
Repeat the activity; have the children exchange product-cards, construct arrays, and match arrays and products as before. Continue the activity until each child has made four to six arrays.

If graph paper is not available, pegboards or 1-inch squared paper and adhesive-backed counters may be used for this activity.

Pages 11 and 12

● Use page 11 as a discussion guide. After the illustration has been examined and the story read, have the children express their ideas about the questions at the end of the story. If necessary, help the children reach the conclusion that both children are right because the array shows both 3×4 and 4×3. There are 3 rows one way and 4 rows the other way.

● Page 12 gives the children an opportunity to observe that the number of an array may be recorded in two ways. Changing the order of the factors in its product does not change the array. Discuss briefly the example at the top of the page with the class. Direct the children to examine each array on the page and to write a product for each array. Then for each product numeral they are to write the commuted form.


7 × 3
3 × 7

Write the product for each array in two ways.

1. 5 x 10 / 10 x 5 *

2. 4 x 9 / 9 x 4 *

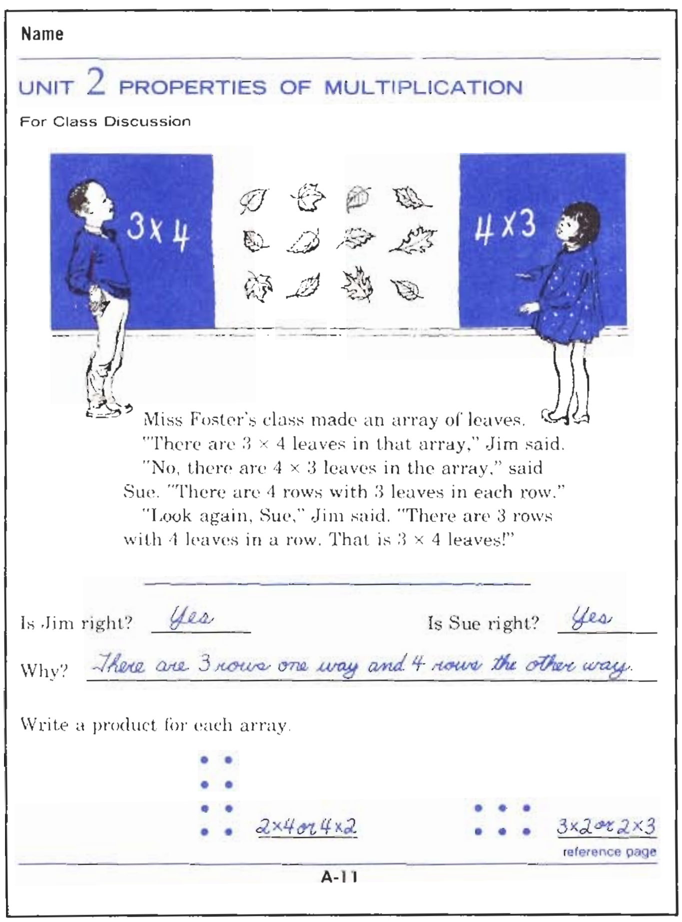
3. 7 x 12 / 12 x 7 *

4. 3 x 6 / 6 x 3 *

5. 16 x 2 / 2 x 16 *

* Order will vary.

A-12


Name

UNIT 2 PROPERTIES OF MULTIPLICATION

For Class Discussion

Miss Foster's class made an array of leaves.
"There are 3 × 4 leaves in that array," Jim said.
"No, there are 4 × 3 leaves in the array," said Sue. "There are 4 rows with 3 leaves in each row."
"Look again, Sue," Jim said. "There are 3 rows with 4 leaves in a row. That is 3 × 4 leaves!"

Is Jim right? Yes   Is Sue right? Yes

Why? There are 3 rows one way and 4 rows the other way.

Write a product for each array.

2x4 or 4x2   3x2 or 2x3

reference page

A-11

Supplemental Experience

■ Prepare squares of tagboard that show arrays of 2 by 2 through 9 by 9. Write the product of the array on the back of each card. As you show each array to the class call on a child to give the product for the array. Then have the child call on a second child to give the commuted form. Go through the pack of array cards several times, using this procedure.

KEY IDEA

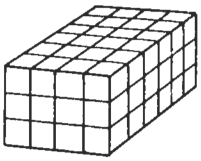
Factors of products may be rearranged.

Scope

To interpret a three-dimensional array as a model of multiplication.

Fundamentals

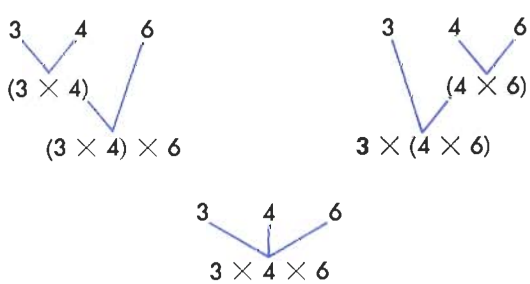
Just as a two-dimensional array is a model for multiplication, so is a three-dimensional array.



$\underline{3} \times \underline{4} \times \underline{6}$

As the three-dimensional array is viewed, the description of the array may have six variations, 3 by 4 by 6, 4 by 6 by 3, and so on. Since each of the six descriptions refers to the same array, we will have six different forms of the product; but in essence, there is only one product—rearranging the numerals for the factors does not change the product. In the example, $3\times4\times6$, the product is not affected by the arrangement of 3, 4, and 6 in the first, second, and third positions.

The above array also illustrates the associative property. The front face consists of 3×4 elements, and the array is 6 deep, so the product is $(3\times4)\times6$. When viewed from the top, the top face has 4×6 elements and there are 3 layers, so the product is $3\times(4\times6)$. Thus, $(3\times4)\times6 = 3\times(4\times6) = 3\times4\times6$. The way the factors are paired has no effect on the product; this is the associative property. The associative property is also illustrated in the following diagram.



Notice that it is not necessary to use parentheses.

Readiness for Understanding

Ability to count.

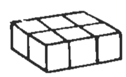
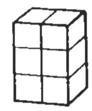
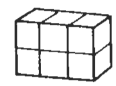
Ability to correctly interpret products represented by two-dimensional arrays.

Developmental Experiences

for each child
wooden cubes

sheets of tagboard
product-cards

▶ Have the children sit in a circle formation. Provide each child with 24 one-inch wooden cubes. Tell the children to use their cubes to show a 2 by 3 array on their desks. The children should be given complete freedom of choice in this situation. It is possible to show this array in the following ways.

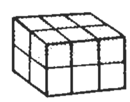
Ask the children to tell the product for the array. (3×2 or 2×3) Write an equation on the chalkboard to emphasize that the arrays shown by the children represent the same number.

$$3\times2 = 2\times3$$

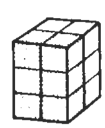
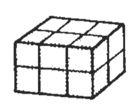
Each array has the same number of cubes even though some children may have expressed this number as 3×2 and others as 2×3. Remind the children that both 3×2 and 2×3 describe the same array.

Direct the children to build another array like their first one. Tell them where they are to place the cubes in this second array:

> Those children whose first array is just one row high are to place the cubes of their second array directly on top of the cubes in their first array.



> Those children whose first array is either two or three rows high are to place the cubes of their second array directly in front of the cubes in their first array.

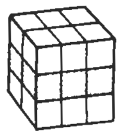



Tell the children that the array they now have on their desks has $3\times2\times2$ cubes. Write this product on the chalkboard. Ask if everyone sees the $3\times2\times2$ product in his array. Repeat the procedure with the products $2\times3\times2$ and $2\times2\times3$. Ask the class what conclusion they have drawn concerning the three products. The fact that the number of cubes in the array is all three products—$3\times2\times2$, $2\times3\times2$, and $2\times2\times3$—should become evident.

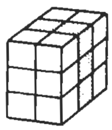
Direct the children to build a third array. Explain where the cubes in this array are to be placed:

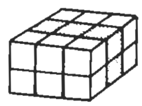
> Those children who built their first array just one row high are to place the cubes of their third

array directly on top of the other cubes on their desks.



The other children are to place the cubes of their third array directly in front of the other cubes on their desks.

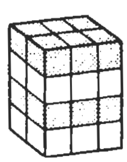




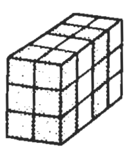
Have each child observe the arrangement of the cubes in his array and give the product for his array. Possible responses are 3×3×2 cubes, 3×2×3 cubes, and 2×3×3 cubes. Write the different responses on the chalkboard. Ask the class what conclusions they have reached concerning the products for their arrays.

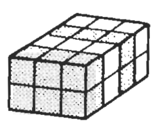
As a last step, direct the children to build a fourth array. Again explain where the cubes in the array are to be placed:

Children who have been building array on top of array should continue in this way.



Children who have been building arrays in front of arrays should continue in this way.





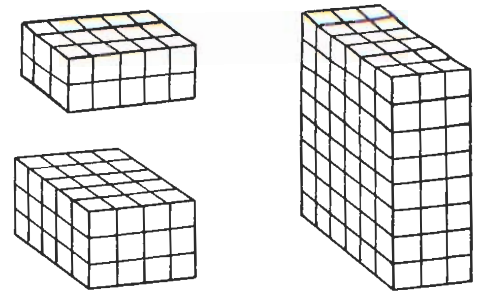
Have the children examine the arrangement of cubes in their array; ask them to tell the product for their array. The following responses are possible:

2×3×4
2×4×3
3×2×4
3×4×2
4×2×3
4×3×2

Record each different response on the chalkboard as it is given.

Now give the children time to discuss their arrays with each other. Encourage them to find out how the arrays are alike and how they are different. The children should be gaining some awareness of the fact that different arrangements of the members of a three-dimensional array do not change the number of objects in the array. They should have some intuitive idea that it is possible to change the order and the pairing of the factors of a product with no affect on the product.

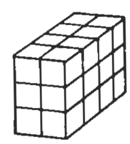
▶ Draw several three-dimensional arrays on sheets of 18 by 24 inch tagboard. Show only one array on each sheet. Some arrays that might be used are 2×5×3, 3×4×5, 8×3×6, 8×10×4, and 7×9×2.

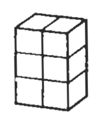


Show the class each array separately. Have the children tell what arrangements they see in each array and record their responses on the chalkboard. Six different arrangements are possible for each array. The children should be developing an intuitive feeling for the rearrangement property.

▶ Have the children build a 2 by 3 by 4 array on their desks with their cubes. Ask a child to name the product for his array and to record it on the chalkboard. He may use any one of the six possible forms of the product.

Ask the child to hold up the part of his array that the first two factors of his product represent. For example, if he used the product 2×3×4 he would show one of the 2×3 arrays.





(2 × 3)

Ask him where the 4 in the product came from. He may explain that there were four 2 by 3 arrays in the whole array. Record (2×3)×4 beside the product on the chalkboard. Explain that the parentheses show which numbers were paired.

2×3×4 (2×3)×4

Ask the children if there is another way to record this idea—four 2 by 3 arrays. Record their responses on the chalkboard.

2×3×4 (2×3)×4 4×(2×3)

Let other children tell the product for their arrays and demonstrate their perception of the array. You may want to record the pairings they develop in a chart such as the following:

Order of Factors	Pairings	
2×3×4	(2×3)×4	4×(2×3)
2×4×3	(2×4)×3	3×(2×4)
3×2×4	(3×2)×4	4×(3×2)
3×4×2	(3×4)×2	2×(3×4)
4×2×3	(4×2)×3	3×(4×2)
4×3×2	(4×3)×2	2×(4×3)

The children may observe that a particular pairing or association of the factors of a product does not affect the product. The children are intuitively developing the idea that multiplication is associative and commutative.

▶ Play "Commute Inside Parentheses." Make a set of 12 cards as illustrated.

1)	$(2\times5)\times3$	2)	$3\times(2\times5)$	3)	$(3\times5)\times2$
4)	$2\times(3\times5)$	5)	$(2\times3)\times5$	6)	$5\times(2\times3)$
7)	$(5\times2)\times3$	8)	$3\times(5\times2)$	9)	$(5\times3)\times2$
10)	$2\times(5\times3)$	11)	$(3\times2)\times5$	12)	$5\times(3\times2)$

Select two teams—six children on a team. Give cards 1 through 6 to one team and cards 7 through 12 to the other team. Ask a member of the first team to show his card. Tell the members of the other team to examine their cards carefully. Explain that they are looking for the card that shows commuting only within the parentheses. For example, if the first card shown is $(2\times5)\times3$ the card to be chosen by a member of the second team is $(5\times2)\times3$. The card that shows $3\times(5\times2)$ would not be a correct choice. It is an example of double commuting and will be used in the game "Double Commuting." A correct choice is worth one point; one point is subtracted for an incorrect choice.

Tell a member of the second team to show his card. Someone on the first team must show a card that properly illustrates the commutative property within the parentheses. Continue until six cards have been shown and matched. Total the scores and declare a winner.

▶ Play "Commute Outside Parentheses." Select two teams as before—six children on a team. Distribute the cards to each team as shown.

Team A	Team B
$(2\times5)\times3$	$3\times(2\times5)$
$(5\times2)\times3$	$3\times(5\times2)$
$(3\times5)\times2$	$2\times(3\times5)$
$(5\times3)\times2$	$2\times(5\times3)$
$(2\times3)\times5$	$5\times(2\times3)$
$(3\times2)\times5$	$5\times(3\times2)$

Follow the procedure of the preceding game. This time have the children match cards that illustrate commutativity outside the parentheses. For example, if the card $(2\times5)\times3$ is shown, the matching card is $3\times(2\times5)$.

Continue until six cards have been shown and matched.

▶ Play "Double Commute." Select two teams as before—six children on a team. Distribute the cards to each team as shown.

Team A	Team B
$(2\times5)\times3$	$3\times(5\times2)$
$3\times(2\times5)$	$(5\times2)\times3$
$(3\times5)\times2$	$2\times(5\times3)$
$2\times(3\times5)$	$(5\times3)\times2$
$(2\times3)\times5$	$5\times(3\times2)$
$5\times(2\times3)$	$(3\times2)\times5$

Follow the procedure of the preceding games. This time have the children match cards that illustrate both commutativity inside the parentheses and commutativity outside the parentheses. For example, if the card $(2\times5)\times3$ is shown, the matching card is $3\times(5\times2)$. Continue until six cards have been shown and matched.

▶ Play "Associate." Select two teams as before—six children on a team. Distribute the cards to each team as shown.

Team A	Team B
$(2\times5)\times3$	$2\times(5\times3)$
$3\times(2\times5)$	$(3\times2)\times5$
$(3\times5)\times2$	$3\times(5\times2)$
$2\times(3\times5)$	$(2\times3)\times5$
$(2\times3)\times5$	$2\times(3\times5)$
$5\times(2\times3)$	$(5\times2)\times3$

Follow the procedure of the preceding games. This time have the children match cards that illustrate associativity. For example, if the card $(2\times5)\times3$ is shown, the matching card is $2\times(5\times3)$. Continue until six cards have been matched.

Supplemental Experience

■ Play the game "Commute and Associate." This is a combination of the games "Commute Inside," "Commute Outside," "Double Commute," and "Associate."

Separate the class into groups with four or five children in each group. Each group of children will need twelve product-cards and four direction-cards.

Product-Cards

$(2\times5)\times3$	$2\times(5\times3)$	$(2\times3)\times5$
$(5\times2)\times3$	$2\times(3\times5)$	$(3\times2)\times5$
$3\times(5\times2)$	$(3\times5)\times2$	$5\times(3\times2)$
$3\times(2\times5)$	$(5\times3)\times2$	$5\times(2\times3)$

Direction-Cards

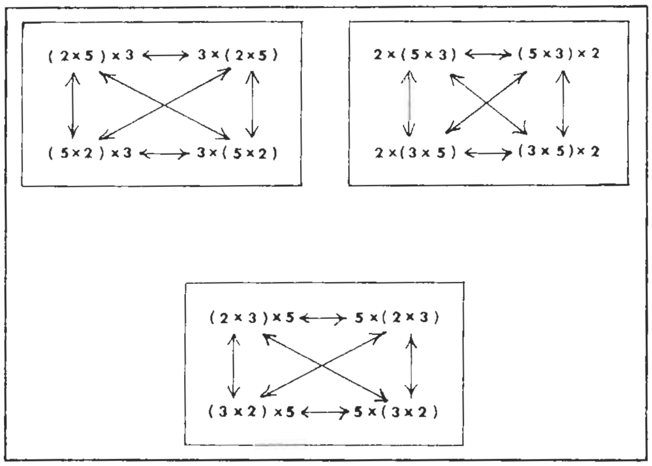
Commute Inside	Commute Outside
Double Commute	Associate

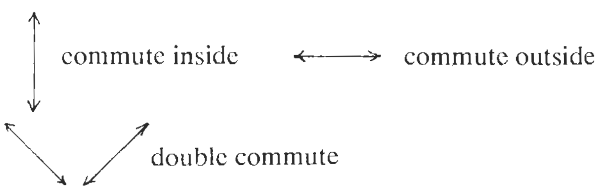
Tell each group of children to place their twelve product-cards face down on the table. Then have them place the direction-cards face down in another pile.

To play the game a child must turn over one product-card and one direction-card. He earns a point if he can change the product by doing what is indicated on the direction-card.

Chosen Cards	Correct Result
(5×3)×2	2×(3×5)
Double Commute	

The child must write his answer on a sheet of paper; then the other members of his group may check the result. You may wish to provide each group with an answer card.





Write the information about the arrows on the chalkboard and show how the arrows can be used. For example, if the cards (2×5)×3 and double commute were drawn, then follow the diagonal arrow from (2×5)×3 to 3×(5×2), the result of double commuting.

Although the arrows do not show associate, the answer card contains the necessary information. Tell the child that the associated form of a given product will not be found in the box containing the given product; it will be found in one of the other two boxes. For example, if the cards (5×2)×3 and associate were drawn, the child will find the associated form 5×(2×3) in the right-hand box or in the lower box.

Play the game until all 12 product-cards have been used. Tell the children to shuffle the cards before they begin the game again. If the children become skillful, then play a variation—choose two of the direction-cards instead of one. The same answer card may be used.

KEY IDEA

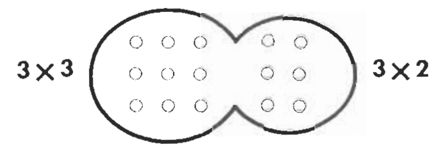
Multiplication is distributive over addition.

Scope

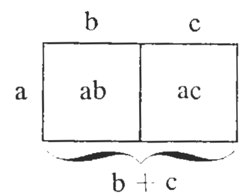
To explore the relationship between multiplication and addition.

Fundamentals

The observed relationship between the parts of an array and the whole array is the basis for the child's understanding of the distributive property. For example, consider the array shown below.

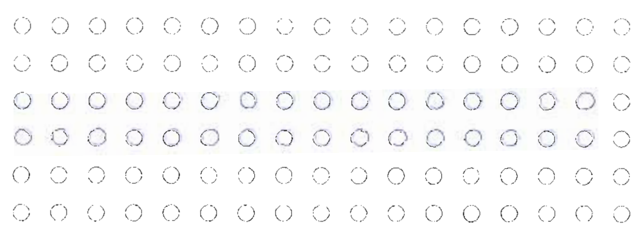


The partial products 3×3 and 3×2 make up the whole product 3×5 or 3×(3+2). Thus we observe that 3×5 = (3×3) + (3×2) or, in the general case, a×(b+c) = (a×b) + (a×c).



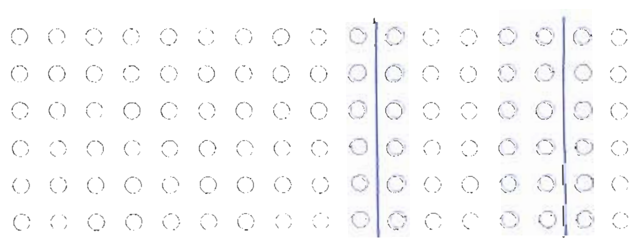
a(b+c) = ab + ac

Note that a product is denoted as _____ × _____ rather than with a standard numeral. It is important to use the _____ × _____ notation as an aid in understanding the meaning of product. Later, as the child learns the basic multiplication facts and begins more complicated computations, multiplication will be meaningful and its application will be more apparent in problem situations. By partitioning arrays, the child will be able to work easily with larger products.



6×17

In partitioning the 6×17 array, the number of arrays and the size of the arrays are a matter of choice. The three partial products—6×10, 6×5, and 6×2—might be selected.



6×10 **6×5** **6×2**

The product is the sum of the partial products.

$$6\times17 = 6\times(10+5+2)$$
$$= (6\times10) + (6\times5) + (6\times2)$$

The teacher will want to help the child thoroughly explore the relationship between multiplication and addition—the distributive property—in preparation for his later work with the multiplication algorism.

Readiness for Understanding

Understanding of the concept of product.
Understanding of the concept of sum.

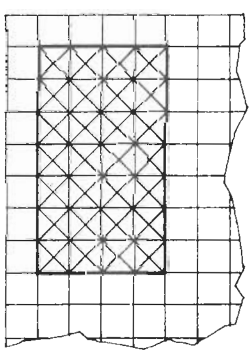
Developmental Experiences

for flannel board
tagboard strips
numerals and symbols
cutouts

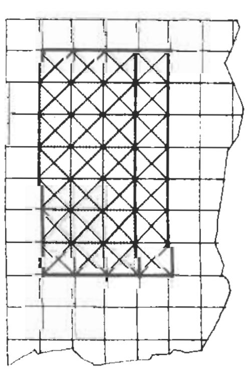
for each child
sheet of graph paper
colored pencils
plastic numerals and symbols

masking tape
felt-tip pen

▶ Give each child a sheet of $\frac{1}{4}$-inch squared paper and two colored pencils. Write the product 7×4 on the chalkboard and tell the children to draw an array that represents this product. Show them how to fill in the squares with X marks to make the array. Make certain they use just one color for this step.



Then direct the children to partition their arrays into 2 parts by shading in the squares in one part with their second pencil. When the children have finished, ask a child to show his partitioned 7 by 4 array. He may have drawn the following:



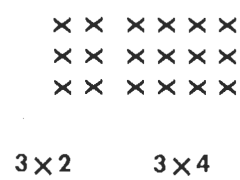
Have him tell the class how he interprets 7×4 as the sum of products represented by his partitioning. He may say that 7×4 is $(7\times3) + (7\times1)$. Have him show this relationship in an equation on the chalkboard. Ask if anyone else used the same partitioning; let these children show their arrays. Continue in this manner until every child has had the opportunity to show the partitioning of his array.

Keep a record of the sums of products for 7×4. After all of the children have shown their arrays, have the class examine the equations you've listed on the chalkboard. Discuss any of the children's conclusions. The children may observe that there are several ways a given array may be partitioned without changing the array, and that a given product is the sum of several different products.

Use the same procedure with products 4×4 through 9×9.

▶ Use strips of masking tape on the chalkboard to make a latticework frame with two openings, each about 12 inches square. Below one square write the product 3×2; below the other square write 3×4. Tell the children that 3×2 and 3×4 represent parts of an array and ask if anyone knows the product for the whole array. Record the children's responses on the chalkboard.

Then have two children illustrate the two products by making arrays of Xs in the frame. Tell the children to keep the Xs in their arrays as close to the center tape as possible, and to align the rows in both arrays.

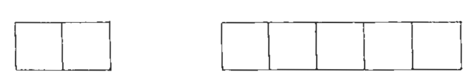


Have the class compare each of the listed responses to the whole array. Then have a child write an equation on the chalkboard that shows the relationship between the product for each part of the array and the product for the whole array.

$$(3\times2) + (3\times4) = 3\times6$$

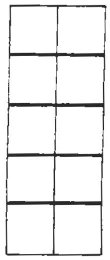
Adapt this procedure to products from 2×2 through 9×9.

▶ Use $1\frac{1}{2}$-inch wide tagboard strips that have been prepared for use on the flannel board. Cut out 81 strips so that you have 9 strips of each of these lengths: $1\frac{1}{2}$, 3, $4\frac{1}{2}$, 6, $7\frac{1}{2}$, 9, $10\frac{1}{2}$, 12, and $13\frac{1}{2}$ inches. Divide each strip into $1\frac{1}{2}$-inch squares and outline each square with a felt-tip pen.



a 3-inch long strip   a $7\frac{1}{2}$-inch long strip

Tell the children to close their eyes. Put five of the 3-inch strips on the flannel board as illustrated.

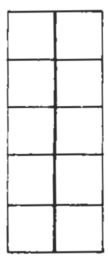


Tell the children to open their eyes. Have one child use felt numerals and symbols to show the product for this array. Then rearrange the strips in this manner.

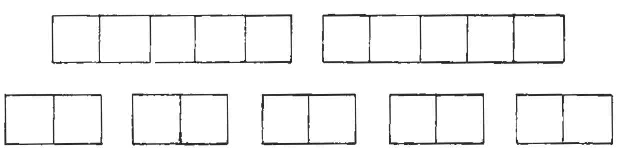
Ask what sum is described by the strips. (2 + 2 + 2 + 2 + 2) Have a child construct this sum beside the product 5×2 on the flannel board, and connect the two forms with an equal sign.

$$5 \times 2 = 2 + 2 + 2 + 2 + 2$$

Have the children close their eyes again. Construct a 5 by 2 array on the flannel board with two $7\frac{1}{2}$-inch strips.



Tell the children to open their eyes; ask a volunteer to use felt numerals and symbols to show the product for this array. Then rearrange the two $7\frac{1}{2}$-inch strips above the five 3-inch strips as shown.



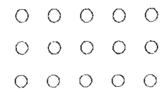
Ask what sum is described by the two top strips. (5 + 5) Have a child construct this sum beside the second 5×2 on the flannel board, and connect the two forms with an equal sign.

$$5 \times 2 = 2 + 2 + 2 + 2 + 2$$
$$5 \times 2 = 5 + 5$$

Encourage the class to describe and discuss any observations they have made about the relationship between 5×2 and the two sums. They may conclude that 5 × 2 is both 2 + 2 + 2 + 2 + 2 and 5 + 5.

Adapt this procedure to products such as 6×4, 3×8, and 7×9.

▶ Construct a 5 by 3 array on the flannel board with felt cutouts. Let a child show the product for this array with felt numerals and symbols.



5×3

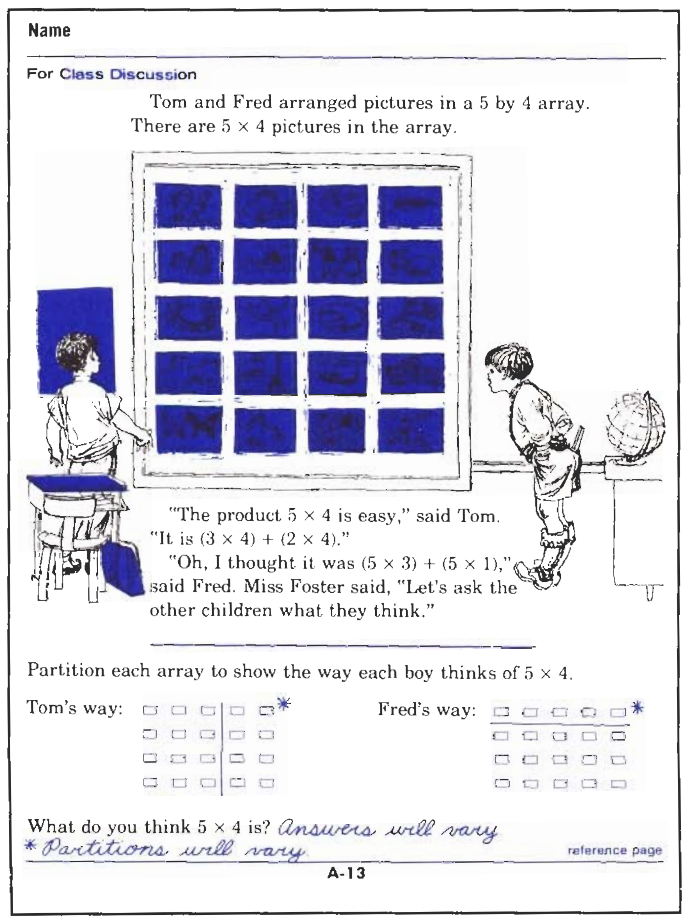
Tell the children to use their plastic numerals and operation symbols at their desks to show the sums indicated by the array and its product.

$$3 + 3 + 3 + 3 + 3$$
$$5 + 5 + 5$$

Adapt this procedure to demonstrate the products 6×9, 8×4, and 3×6.

Pages 13 through 18

● Have the children examine the illustration and read the story on page 13. As a result of the discussion the children should realize that 5×4 is the sum of several different products, the two sums given in the story, and others.


Name

For Class Discussion

Tom and Fred arranged pictures in a 5 by 4 array. There are 5 × 4 pictures in the array.

"The product 5 × 4 is easy," said Tom. "It is (3 × 4) + (2 × 4)."

"Oh, I thought it was (5 × 3) + (5 × 1)," said Fred. Miss Foster said, "Let's ask the other children what they think."

Partition each array to show the way each boy thinks of 5 × 4.

Tom's way: Fred's way:

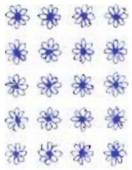
What do you think 5 × 4 is? *Answers will vary*

* *Partitions will vary*

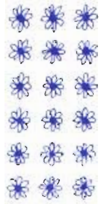
reference page

A-13

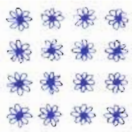
Partition each array into two parts. Write a product for each part.



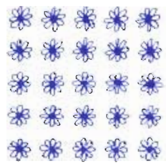
1. (___ × ___) + (___ × ___)*



2. (___ × ___) + (___ × ___)*

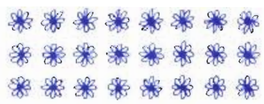


3. (___ × ___) + (___ × ___)*



4. (___ × ___) + (___ × ___)*

5. (___ × ___) + (___ × ___)*

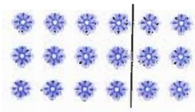


6. (___ × ___) + (___ × ___)*

* Partitions and products will vary.

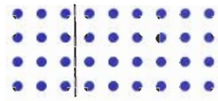
A-14

Name ______________________________

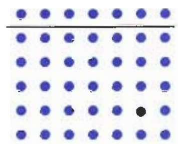


(3 × 4) + (3 × 2) = 3 × 6*

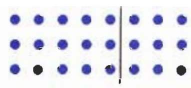
What products?



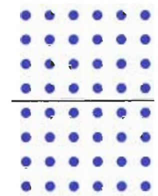
1. (4 × 3) + (4 × 6) = 4 × 9*



2. (7 × 5) + (7 × 1) = 7 × 6*



3. (5 × 3) + (3 × 3) = 8 × 3*



4. (4 × 6) + (4 × 6) = 8 × 6*

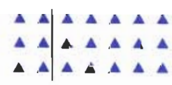
* Order may vary.

A-15

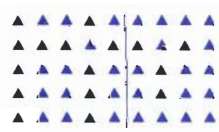
● Use page 14 to give the children an opportunity to partition arrays and to write products for the parts. You may wish to complete the first exercise with the class. The remaining exercises can be completed independently.

● Pages 15 and 16 provide an opportunity for the children to express a product as a sum of products. Discuss and work through the exercise at the top of each page with the class. Explain that the children are to work with one array at a time and complete the equation below the array. Explain that the partitions on page 15 will help them complete the equations. Tell the children that the incomplete equations on page 16 tell them how to partition the arrays.

Partition each array. Complete each sentence.

1. 

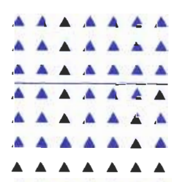
3 × 7 = (3 × 2) + (5 × 3)

2. 

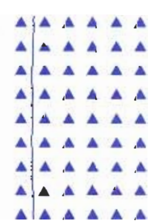
9 × 5 = (4 × 5) + (5 × 5)*

3. 

8 × 4 = (6 × 4) + (2 × 4)*

4. 

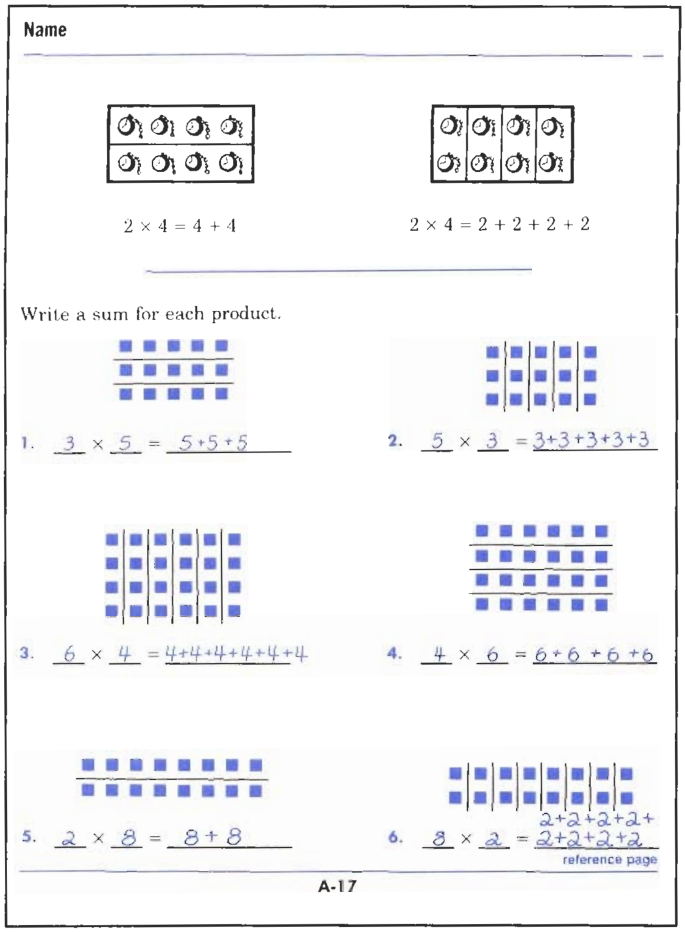
8 × 7 = (3 × 7) + (5 × 7)*

5. 

6 × 9 = (5 × 9) + (1 × 9)*

* Partition and order of numbers may vary.

A-16



Name

2 × 4 = 4 + 4

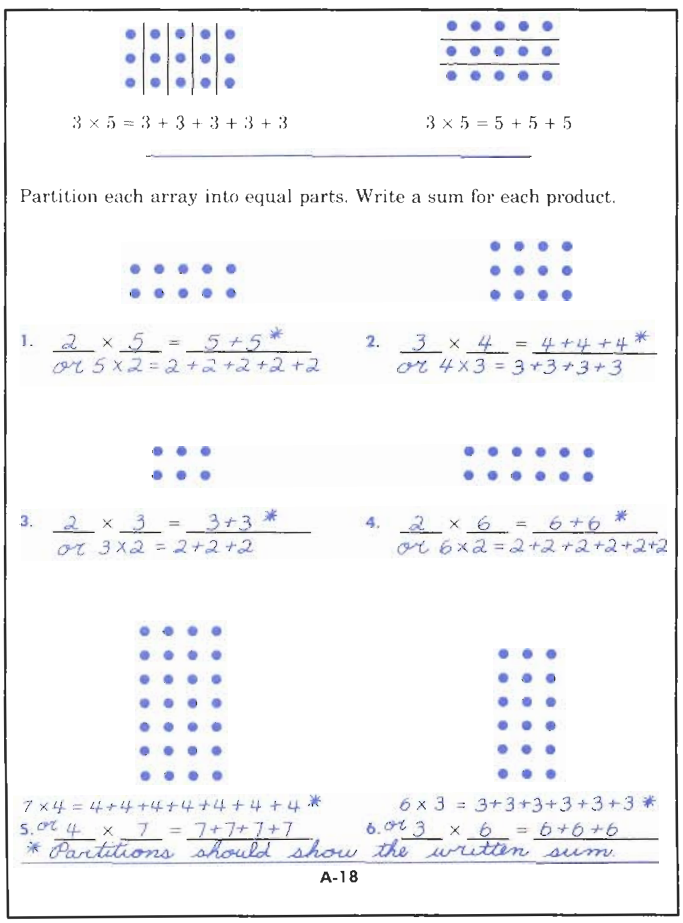
2 × 4 = 2 + 2 + 2 + 2

Write a sum for each product.

1. 3 × 5 = 5+5+5
2. 5 × 3 = 3+3+3+3+3
3. 6 × 4 = 4+4+4+4+4+4
4. 4 × 6 = 6+6+6+6
5. 2 × 8 = 8+8
6. 8 × 2 = 2+2+2+2+2+2+2+2

reference page

A-17



3 × 5 = 3 + 3 + 3 + 3 + 3

3 × 5 = 5 + 5 + 5

Partition each array into equal parts. Write a sum for each product.

1. 2 × 5 = 5+5* or 5×2 = 2+2+2+2+2
2. 3 × 4 = 4+4+4* or 4×3 = 3+3+3+3
3. 2 × 3 = 3+3* or 3×2 = 2+2+2
4. 2 × 6 = 6+6* or 6×2 = 2+2+2+2+2+2
5. 7×4 = 4+4+4+4+4+4+4* or 4 × 7 = 7+7+7+7
6. 6×3 = 3+3+3+3+3+3* or 3 × 6 = 6+6+6

* Partitions should show the written sum.

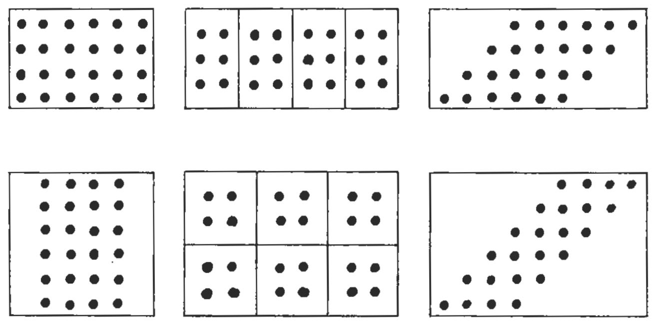
A-18

● Page 17 shows arrays that have been partitioned into equivalent arrays. The children are to interpret a product in terms of a sum of equal addends. Discuss the example at the top of the page with the class. Then have the children write equations that link the product and sum shown by the partitioning of each array.

● Use page 18 to give children experience in partitioning an array into equivalent parts and in interpreting the product in terms of a sum of equal addends. You may wish to complete the first exercise with the class. The children should understand that there is more than one way to interpret each array.

Supplemental Experience

■ Make several array-cards for each of the products 2×3 through 9×8. For example, arrays such as the following could be shown for the product 6×4.



Place a product-card, such as 3×4, in the pocket chart. Below the product-card, place several array-cards; some arrays should represent the given product, some should not.

Separate the class into two teams. Tell the first member of one team to remove all of the arrays that match the product shown. If the child finds all of the cards, he earns a point for his team. If he does not, allow a member of the other team to find the other arrays and earn a point for his team.

Proceed in this way until all of the children have had an opportunity to participate in the activity. Then total the points and declare a winner.

KEY IDEA

$1 \times \square = \square$.

Scope

To recognize the identity number for multiplication (1).

Fundamentals

The child's perception of product has been developed through experiences with arrays. The 1 by n array,

o o o . . .
1 × n

is the simplest array and the product 1 × n can easily be seen to be n. For example,

o o o o o

1 × 5 = 5

Readiness for Understanding

Understanding of the concept of product.

Developmental Experiences

for flannel board	*for each child*
tagboard strips	counters
product-cards	plastic numerals
felt-tip pen	and symbols

▶ Have the children show a 1 by 5 array by using counters on their desks. Direct them to use their plastic numerals and operation signs to show the product beside their array.

● ● ● ● ● **1 × 5 (or 5 × 1)**

Then have the children construct the following arrays and the product for each below their first array.

6 by 1
10 by 1
1 by 3
1 by 15

Have the class examine the arrays and products and discuss what they see. The children may make the following observations:

There are arrays that have just 1 row.

The number 1 is a factor in each of the products; it is either the number of rows or the number of members in each row.

When 1 is a factor of a product, the number of members belonging to the array is the same as the other factor of the product.

▶ On the chalkboard, make a list of products that have 1 as a factor, from 1×1 through 20×1. Have the children use counters at their desks and build a 1 by 1 array. Ask whether or not it is possible to rearrange the array for the product 1×1 to show a different array. (no) Ask if it is possible to rearrange the array for the product 4×1 to show a different array. (yes, a 2 by 2 array) Note that 2×2 and 4×1 are the same number; they are the same product.

Follow this procedure with the other products from 2×1 through 20×1. Whenever an array may be rearranged to form a different array, list the product for the new array below the original product. (2 × 2 is listed below 4×1.)

1×1	2×1	3×1	4×1	5×1	6×1	7×1
			2×2		3×2	

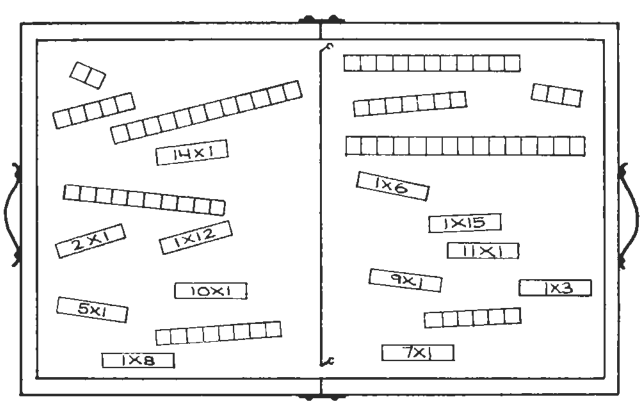
8×1	9×1	10×1	11×1	12×1	13×1	14×1
4×2	3×3	5×2		3×4		7×2
				6×2		

15×1	16×1	17×1	18×1	19×1	20×1
5×3	4×4		9×2		4×5
	8×2		3×6		2×10

Let the children discuss what they observe about the products listed in the chart on the chalkboard. The children may observe that there are some products that have only one form—the form that has 1 as one of the factors.

▶ To represent the products 1×1 through 20×1, cut the following lengths from $1\frac{1}{2}$-inch-wide tagboard strips for use on the flannel board: 15, $16\frac{1}{2}$, 18, $19\frac{1}{2}$, 21, $22\frac{1}{2}$, 24, $25\frac{1}{2}$, 27, $28\frac{1}{2}$, and 30 inches. Use a felt-tip pen to outline $1\frac{1}{2}$-inch squares on each strip. Combine these strips with one group of the strips used in an earlier activity.

Set up the flannel board as shown. Note that one product-card in each section of the board does not match any of the arrays displayed.



Have the children form two teams; assign a section of the board to each team. Explain that at a given signal a representative from each team is to match the arrays and products within their section. Any card that cannot be matched should be removed from the board. The first child to finish correctly earns a point for his team.

Place different cards on the board for each new pair of competitors. Continue the game until all of the children have participated. Then total the points for each team to determine a winner.

Supplemental Experience

■ Play "Train" with array-cards for 1×1 through 10×10. Select one child to be the conductor and have him stand by the desk of another child. Hold up an array-card and have both children try to name the product for the array. If the conductor names the product first, he may move on to the next child. If the other child responds with the correct product first, he becomes the new conductor. Whoever is conductor continues around the room until he misses. Change the array each time the product is named.

Children who give an incorrect response may be switched to a side track to carefully examine the array whose product they named incorrectly. They may rejoin the train by giving the correct product when another child misses.

KEY IDEA

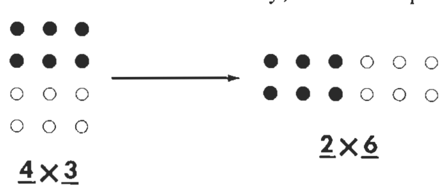
4×6 is 2×6 + 2×6 or 2×12.

Scope

To introduce equivalent products.

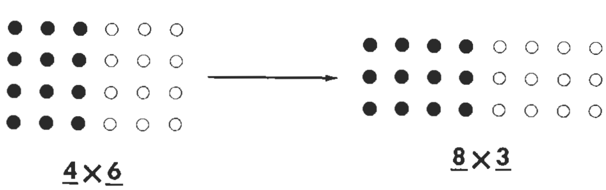
Fundamentals

A product is not changed as parts of an array are rearranged to form a new array, for example:



4×3=2×6

It may be seen that the new array has been formed by rearranging parts of the original array. The following example shows that 4×6 = 8×3.



The above rearrangement of parts of an array has the effect of halving one dimension while doubling the other dimension. This use of the principle of compensation will help the child understand the more sophisticated computation techniques that he will learn in later grades.

Readiness for Understanding

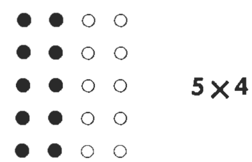
Ability to partition an array.

Developmental Experiences

for flannel board	*for each child*
disks	tagboard disks or squares
numerals and symbols	plastic numerals and symbols

pocket chart
array-cards

▶ Use felt disks that are a different color on each side to construct a 5×4 array on the flannel board. This array should be one color. Have a child show the product for this array with felt numerals and symbols. Ask if anyone can show $\frac{1}{2}$ of this array. Let a volunteer turn over disks that represent $\frac{1}{2}$ of the total array. Have the children check his work and explain why they do or do not agree with the solution. Their reasoning should be based on whether or not there are the same number of members in both parts of the array.

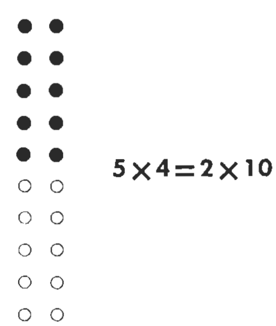


Ask if anyone can rearrange the two parts of this array to show a different product. The most likely response is a 2×10 array. After a child has moved the disks to show the new arrangement, have him place symbols for 2×10 beside the 5×4.

Have the class discuss the relationship between the two products. The following facts should be examined:

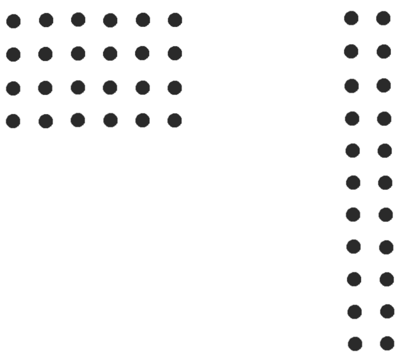
> 5×4 is 2×10.
>
> Both 5×4 and 2×10 tell how many members belong to the 5 by 4 array and the 2 by 10 array.
>
> No new members were added to the 5 by 4 array and none were removed; the array was rearranged.

Have a child place an equal sign between 5×4 and 2×10 to summarize and reinforce the fact that 5×4 is 2×10.



Continue the activity with products such as 7×6, 4×8, 6×5, and 3×12.

▶ Give each child 100 tagboard disks (or squares) about $\frac{3}{4}$ inch in diameter. Direct the children to show a 4 by 6 array on their desks. Beside this array, have them build another that has the same number of members in a different arrangement. Then tell them to use their plastic numerals and symbols to make an equation using the number of their first array and the number of their second array.

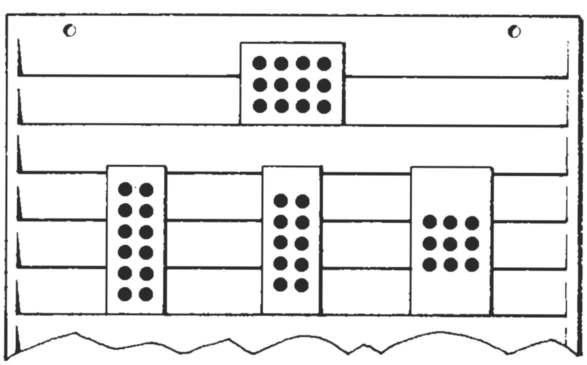


$4 \times 6 = 2 \times 12$

Call on a child to tell about the two arrays he built. He may say that he has a 4 by 6 array and a 2×12 array. Have him read his equation. Ask if anyone made a different array. It is possible that some children may have a 4 by 6 array and a 3 by 8 array. Have the equations read.

Continue the activity with the products 4×7, 2×8, 8×6, and 6×6.

▶ Place a card in the pocket chart that shows an array whose members can be rearranged to form an equivalent array. For example, the members of a 3 by 4 array can be rearranged to form a 6 by 2 array. Place three other array-cards below the first. One of the cards should show a different arrangement of the original array.



Ask a child to choose an array from the lower section of the pocket chart that has the same number as the given array. Have him explain his selection. He may point out the 3×4 members in the 6 by 2 array, or the 2×6 members in the 3 by 4 array. Have another child show the relationship between the products of the two arrays by writing an appropriate equation on the chalkboard.

$$3 \times 4 = 6 \times 2$$

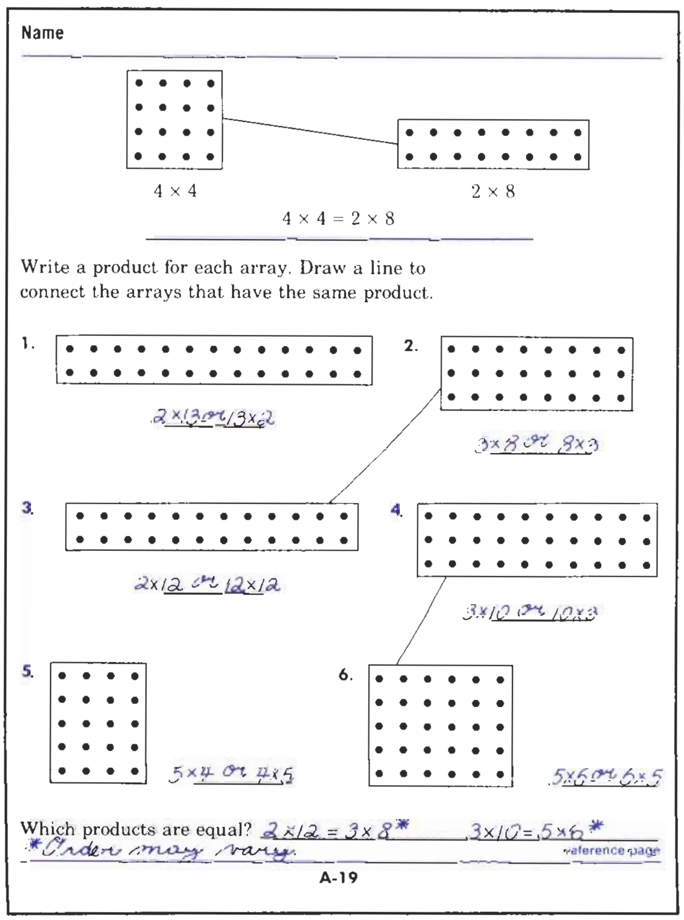
Adapt this procedure to several other pairs of products such as the following:

4×4 and 2×8
2×10 and 4×5
6×6 and 3×12
8×5 and 4×10
9×6 and 18×3

Pages 19 and 20

These pages give the children practice in naming the product for an array in different ways. Discuss the example at the top of each page with the class.

● Have the children write a product for each array on page 19. Then ask the pupils to draw lines to connect arrays whose products are equal. Have them write an equation for each pair of equal products. When the page is completed, ask individual children to tell the products they chose for a particular array. Have the child who is discussing the array construct it on the flannel board and then rearrange the array to represent his other product.


Name

4 × 4 2 × 8

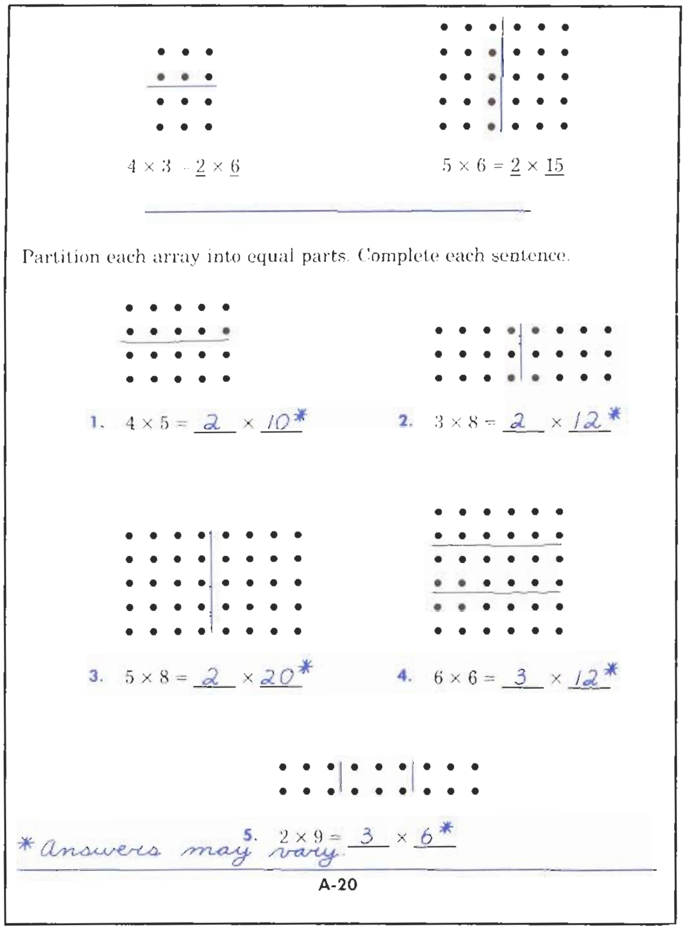
4 × 4 = 2 × 8

Write a product for each array. Draw a line to connect the arrays that have the same product.

1. 2×13 or 13×2
2. 3×8 or 8×3
3. 2×12 or 12×2
4. 3×10 or 10×3
5. 5×4 or 4×5
6. 5×6 or 6×5

Which products are equal? 2×12 = 3×8* 3×10 = 5×6*
*Order may vary. reference page

A-19



4 × 3 = 2 × 6 5 × 6 = 2 × 15

Partition each array into equal parts. Complete each sentence.

1. 4 × 5 = 2 × 10*
2. 3 × 8 = 2 × 12*
3. 5 × 8 = 2 × 20*
4. 6 × 6 = 3 × 12*
5. 2 × 9 = 3 × 6*

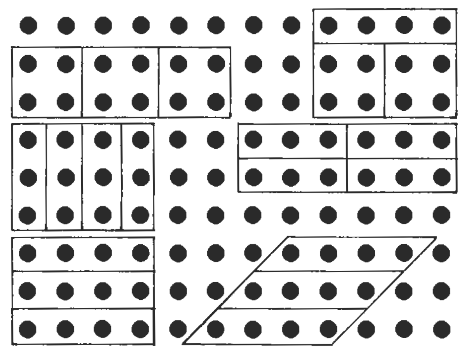
*answers may vary.

A-20

● Tell the class to examine the example at the top of page 20 and to note that the product for the given array can be thought of in other ways. Have the children partition each array into equal parts and then complete the sentence below each array.

Supplemental Experience

■ Draw an array of dots, 9×12 or larger, on the chalkboard. Have a child outline a set of dots that has 3×4 members. Let other children show different sets of 3×4 members. Allow the children complete freedom in their choice of dots for a 3×4 array. There are several ways to illustrate this number. Do not insist that all of them be found. Have each child that made a 3×4 array indicate how he sees 3×4 by partitioning his array with lines of a second color.



Adapt this procedure to other products such as 4×4, 6×3, 4×6, and 5×4.

UNIT 3
TIME

Pages 21 Through 24

OBJECTIVE

To develop an understanding of the time duration of *a second* and *a minute*.

The use of a visual device such as a seconds pendulum that demonstrates the duration of a second—one beat per second—will enable the child to develop an understanding of the time that elapses in a second and in a minute. The child has learned to tell time to the hour, half-hour, quarter-hour, and minute. He now develops a sense of the time intervals second and minute. He will learn how much can be accomplished within a second and a minute. He will relate seconds to parts of a minute.

See Key Topics in Mathematics for the Primary Teacher: Measurement.

KEY IDEAS

A second is a long time.
The minute is sixty seconds.
Thirty seconds is a half-minute.

CONCEPTS

minute second

KEY IDEA

A second is a long time.

Scope

To explore the duration of a second through the use of a seconds pendulum.

Fundamentals

The seconds pendulum is a free-swinging string, 39 inches long, with weights fastened to the end of it. A string shorter than 39 inches swings too rapidly; a longer string swings too slowly.

Each swing of a 39-inch pendulum is one second in duration no matter how much force is used to start it. The pendulum will describe a greater arc, but the time of each swing will remain the same.

Readiness for Understanding

Ability to count.

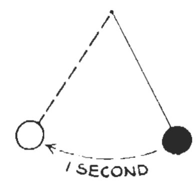
Developmental Experiences

one 45″ string weights

▶ Construct a pendulum from any type of string and a weight. Metal washers or lead sinkers may be used as weights; however, any weight will do. Tie the weight to one end of the string and attach the other end to a hook suspended from the ceiling or a light fixture. The completed pendulum (string and weight) should be 39 inches long. It should be placed where it can swing without interference, and where each swing can be seen by the class.

Ask several children to demonstrate how long they think a second is by clapping their hands to indicate the beginning and the end of one second. Many children may think that the second is almost too short a time to clap twice.

Tell the children that the string with a weight on the end is a seconds pendulum. Start the pendulum. As the boys and girls watch the swinging pendulum, explain that each time the pendulum completes a swing from one side to the other, 1 second has passed.



Ask the class to tilt their heads from side to side and follow the rhythmic movement of the pendulum for 10 seconds. Remind them that each swing of the pendulum represents 1 second.

Have several children take turns being the timekeeper. Tell the timekeeper to silently count the pendulum swings and clap his hands when 10 seconds have passed. If this is too difficult for him, suggest that he count aloud. "One" as the pendulum completes its first swing and "two" when the pendulum returns to its starting point.

Ask the children to tilt their heads from side to side in time with the pendulum while the timekeeper counts out 10 seconds. After the class gets the swing of it, explain that by moving their heads with each swing they will have an idea of how long a second is.

Finally, ask the children to clap their hands each second as the pendulum swings 15 times.

▶ Have the children demonstrate the length of a second by using their arms as pendulums. Suggest that they swing one arm freely in front of their bodies. Then ask the children to watch the classroom pendulum swings and to count 15 seconds silently while they swing one arm in time with its rhythm.

Let the children suggest other physical activities that might be done to the rhythm of the seconds pendulum. Remind them that each swing of the pendulum takes one second.

▶ Start the pendulum and have the class count 20 seconds aloud. Then appoint a timekeeper. Tell the class that the timekeeper will choose a number of seconds less than 11 seconds. He will clap his hands when he begins timing, and clap again when time is up. Have half of the children turn their backs to the pendulum. Ask them to judge how many seconds have passed between the timekeeper's claps. Suggest that the children who are still facing the pendulum silently count the number of seconds that pass. Ask the boys and girls in the guessing group to give their estimates of the number of seconds the timekeeper chose. Then ask the timekeeper to announce the number of seconds he counted so that the children in the guessing group can check the accuracy of their estimates.

Change timekeepers. Let the other half of the class turn their backs to the pendulum and estimate the number of seconds chosen by the timekeeper. After the timekeeper has revealed the count, ask the children in the guessing group who judged the time exactly, to explain how they arrived at their answers. (No doubt they counted.)

Explain to the children that a way in which some people estimate a number of seconds is to count: one thousand one, one thousand two, one thousand three, and so on. Start the pendulum and have the children count 10 seconds out loud using the one-thousand method.

Let the children walk around the room and count in this manner. Select one child to watch the pendulum and lead the counting for 20 seconds by saying, "One thousand one, one thousand two . . . one thousand twenty." Tell the others that each step should be completed as they hear a count. Continue the activity, changing the counting leader frequently, until most of the children have developed the rhythm of walking one step per second.

Now vary the activity by having a child not look at the pendulum and take one step each second for 8 seconds. Ask the rest of the class to watch the pendulum and count the seconds silently. Tell the walker to say "Eight" at the end of eight steps. Let the class tell him whether he walked too fast or too slow. Let others volunteer and continue the activity. Vary the number of seconds assigned to the walkers.

The game can be varied by having the children walk two steps per second.

Play "Stand up, Sit down" to help the children develop a feeling for the length of one second. Start the pendulum and tell the children to stand up and count aloud "One" at the end of the first pendulum swing. When the pendulum returns to its starting point, they should sit down and count aloud "Two." Continue in this manner for 10 seconds.

As a variation, tell half of the class to stand at the end of the first complete swing and count "One." Let the rest of the class start the game standing up. They must sit down on the "one" count. Have them continue the activity through a count of 6.

Many words, phrases, and poems have a definite meter. Peter Piper picked a peck of pickled peppers may be chanted to the swing of the pendulum. The chant has twelve beats—the first, third, fifth, seventh, ninth, and eleventh beats are stressed; the even syllables are not stressed.

Pe	ter	Pi	per	picked	a	peck	of
1	2	3	4	5	6	7	8
pick	led	pep	pers.				
9	10	11	12				

Ask the pupils to say the chant 2 beats per second. While the pendulum swings once, they must say the first 2 syllables, emphasizing the first syllable:

Pe	ter
1	2

As the pendulum returns to its starting point, they must chant the third and fourth syllables, stressing the third syllable:

Pi	per
3	4

It takes 6 seconds to say the entire chant using this tempo.

Continue the activity. Use songs, poems, jump-rope chants, phrases, or words that may be chanted in a stressed and unstressed pattern per second.

To further help the children estimate seconds, ask half of the class to turn their backs to the pendulum and estimate the number of seconds that pass during an activity. Direct the rest of the class to silently count the pendulum swings to establish the number of seconds that pass. Tell the class that they are to estimate how many seconds the lights stay on in the room. Select a child who does not yet have an accurate sense of one second to be the timekeeper. Secretly tell the timekeeper to turn the lights on, allow them to stay on for 7 seconds, and then turn them off.

When the time is up, ask the estimating group to give their estimates and record these on the chalkboard. Then let the timekeeper tell the number of seconds the lights were on.

Choose a new timekeeper and quietly tell him to play a record for 13 seconds. Let the same part of the class judge the number of seconds the recording plays, while the others silently count the seconds. When the record has stopped, ask the pupils for their estimates and record their answers on the chalkboard. Compare them with the number of seconds that the timekeeper let the record play.

Select another timekeeper and ask him to ring a hand bell for 4 seconds. The same group of children should estimate the number of seconds that the bell rings while the others silently count the seconds that pass. Record the estimates on the chalkboard and compare them to the actual amount of time.

Repeat the three activities; have the other half of the class estimate their duration. This time have lights turned on for 13 seconds, the record played for 9 seconds, and the bell rung for 2 seconds.

After these activities have been completed, let several children describe:

the method used to estimate the number of seconds,

changes in their ideas of a second,

the type of activity that was hardest to estimate in seconds.

Have the class investigate the number of seconds it takes to carry out some ordinary activities at a normal rate of speed. Construct the following chart on the chalkboard.

ACTIVITY	NUMBER OF SECONDS
1. Comb hair	
2. Sharpen pencil	
3. Put on boots	
4. Take off boots	
5. Pledge to the flag	
6. Go to the door	
7. Get a drink	
8. Put on coat and button it	

The children may suggest other activities such as finding a given page in a book, writing their names, and so forth.

Explain that this is not a contest to determine who can complete the activities the fastest. Choose one pupil to perform each activity. Ask the class to determine the duration of each activity by counting the swings of the seconds pendulum. Select a child to record the number of seconds used in each activity. Then let other children perform the same activities and compare the number of seconds they required.

▶ When the children have an idea of the length of a second, the development of a rhythm code can be an interesting exercise. Divide the class into two groups; call one group A and the other B. Explain that Group A should silently count 6 seconds after you nod your head. Then they should tap their desks with a pencil at the beginning of each of the next 12 seconds. Let them practice once or twice. Tell Group B to silently count 12 seconds after you nod your head. On the count of 12 they should clap their hands once at the beginning of each second for 6 seconds. Let this group practice once or twice; then have both groups perform together.

It may take several trials before each group can act rhythmically as a group. It may be necessary to let them watch the pendulum for the first few times the groups try the activity. However, it is very satisfying to the children when their group can perform a simple rhythm without visual or audible clues.

Supplemental Experience

■ If there is a clock with a pendulum available (in the office or on a field trip), arrange for pupils to see it. As they observe the swing of the pendulum, let them decide whether or not it is a seconds pendulum. In many cases, the time it takes the pendulum on a clock to swing from one side to the other is less than a second.

KEY IDEA

The minute is sixty seconds.

Scope

To become familiar with the duration of a minute through activities that can be accomplished in minutes and seconds.

Fundamentals

The minute is introduced as 60 seconds. After they participate in the following activities, the children will know a minute in a practical, functional way—they will have an idea of the amount of work they can accomplish in one minute.

Readiness for Understanding

Ability to count by twos.
Understanding of the seconds pendulum.

Developmental Experiences

- seconds pendulum
- record player
- two musical records (one relaxing and one animated)
- 2 sets of computation exercises (one-digit and two-digit sums)
- large ball
- stopwatch

▶ Tell the children that 60 swings of the seconds pendulum is 1 minute. Start the pendulum and have the class count 60 swings of the pendulum aloud.

Ask the children how many seconds it takes the pendulum to swing across and back once. (2 seconds) After the children respond, suggest that they watch only the starting position of the pendulum and count 2 seconds every time the pendulum returns to that position—2, 4, 6, 8, ... Have the class count aloud in this manner for 60 seconds.

Assign three children to watch the pendulum and count aloud while the rest turn away from the pendulum. Tell the class that for 20 seconds the three children will count aloud while the rest count silently. Then everyone will continue counting silently until they reach 60 seconds. Tell the three timekeepers to say "60" when the end of the 60th second is reached. At the end of the activity, ask, "Who reached the count before the timekeeper did?" "Who reached 60 at the same time?" and "Who had not counted to 60 when time was called?" Continue the activity until most of the children get the rhythm for counting 60 seconds.

▶ Tell the children that they will be listening to a record for 1 minute. Use a relaxing selection. Start the pendulum; then start the record and let it play for 60 seconds.

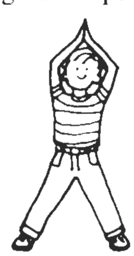
Have the children turn away from the pendulum. Tell them that you will play the same music and that they should stand up when they think one minute has passed. Select three timekeepers. Tell one of the timekeepers to count the children who stand after 55 seconds of music; tell another to count those who stand exactly 60 seconds after the music has started; tell the third to count the children who stand after 60 seconds have elapsed. Stop the music after 90 seconds. Let the timekeepers tell how many children were within 5 seconds of estimating 1 minute and how many stood exactly at the count of 60 seconds. The timekeepers may give the names of these children if desired.

Now play the same game with animated music such as a march. Ask the children to count a period of 60 seconds and raise their hands when they believe 1 minute has elapsed. Let three timekeepers record how many children come within five seconds (either side of sixty) of estimating correctly. Discuss whether it is more difficult to estimate one minute with slow music or with fast music.

▶ Introduce the game "Statue." Tell the children to form a circle with an arm's distance between each child. Ask the children to turn to the right and face the back of the person to his right. Tell them to move counterclockwise around the room while the music is playing. When the music stops they are to freeze in statue positions (that is, no movement) until they believe 1 minute has passed. Then they should raise their hands and move to a normal standing position.

Play the game several times to determine the winners—the children who most often closely estimate one minute. Choose three timekeepers to start the pendulum when the music stops and to record the names of the children who stay in statue positions for more than 55 seconds but less than 65 seconds. These children will compete in the next game. Play at least three games to determine the winners. The children who are least accurate in estimating a minute should be selected as timekeepers for the next round of games.

▶ Explain the Jumping Jack calisthenic. Tell the children to stand erect with their arms at their sides and their feet together. Then tell them to jump at their places, spread their feet apart, and clap their hands together over their heads. These two positions are repeated. Allow the boys and girls to practice for a short time.



Select eight children to stand an arm's length apart in a straight line. Ask two timekeepers to start the pendulum, silently count 60 seconds, and announce the finish of the activity. Explain that the eight children are to do Jumping Jacks rhythmically when the timekeeper tells them to begin. Ask the rest of the class to silently count the number of calisthenics completed in one minute. Ask how many Jumping Jacks were done in 1 minute.

Ask the next group of children to do 1 Jumping Jack (two motions) per second. Ask the timekeeper to count aloud as the pendulum swings for the first 10 seconds. Then the jumpers should continue to count the seconds silently while the class counts the number of Jumping Jacks completed.

Vary the exercise by asking the next group of children to do 2 Jumping Jacks per second during 1 minute.

▶ Efficiency and accuracy in a timed test result when the children work in a rhythmic manner. The following exercises will help to develop a rhythm in computing. Prepare basic addition fact exercises in three rows, 10 to each row. The 10 exercises in the first row should result in a one-digit sum, for example, $2 + 3 = 5$. The 10 exercises in the second row may result in some one-digit sums as well as two-digit sums, for example, $9 + 8 = 17$. All the exercises in the third row should result in two-digit sums. Duplicate the exercises and give each child a copy.

Tell the children that they are going to try to develop a rhythm in computing and writing sums. They are not to move ahead or spend more than the allotted time on one exercise.

Call attention to the first row. Explain that they will be allowed 6 seconds for each of the 10 exercises. Tell the children that when you say "Write" they are to think and then write the first sum. When you say "Write" the next time, they are to think and then write the second sum, and so on. Use a stopwatch for timing; give the first direction as the watch is started and repeat the direction every 6 seconds.

At the end of 60 seconds tell the children to stop. Have the children check their papers. Discuss how the class felt about the length of time allowed for each exercise.

Ask how many of the children felt an easy rhythm in the work they just completed. Tell them that working in a rhythmical pattern makes it possible to accomplish more work than when no definite rhythm or pattern is established. Then call attention to the second row of addition exercises. Tell the class that they have 4 seconds to think and write each sum. Again use the stopwatch and give a direction every 4 seconds. At the end of 40 seconds tell the children to stop. Discuss how the class felt about the length of time allowed for each exercise. Have the children check the accuracy of their work.

Let the class try the third row of 10 exercises at the rate of 2 seconds each. Ask them to try not to break their rhythm by getting excited about one exercise, but to move ahead whenever they hear you say "Write." It is quite possible that several children will not be able to function because they have broken their rhythm. Tell them that they will have other chances to try this type of rhythm work.

On the following day give the class 30 basic addition exercises to complete in 60 seconds. Ask the children to set their own rhythm and to try to keep it throughout the page. Walk around the room and note the rhythm (or lack of it) in each child's work. Time the minute with a stopwatch. At the end of that time, let the children tell how many exercises they completed in a minute. Then have them check the accuracy of their work.

▶ Let the children sing familiar songs at a normal tempo to see how much of a given song is sung in one minute. For example, have the class sing "America the Beautiful" while two timekeepers start the seconds pendulum at the beginning of the song and silently count 60 seconds while watching the pendulum. At the end of 60 seconds they should tell the class to stop. In a similar manner investigate several other songs, such as "Yankee Doodle."

Then ask the children to investigate how many words they read in a minute when they are reading at a normal rate. Have each child select a book and turn to the beginning of any story or chapter he wishes. Let two timekeepers start the pendulum as they tell the class to begin. At the count of 60, the timekeepers should tell the class to stop. Then let each child count and record the number of words he read in 1 minute. Repeat this activity two more times, each time asking the children to start at the beginning of a new story or chapter. Remind the children to read at a comfortable rate. Discuss whether or not the number of words per minute was about the same each time, and why.

▶ Introduce a rhythm activity for the children to learn. Have eight pairs of children sit on chairs facing each other. Give them directions for each of the five steps in the rhythm and let them practice each step:

slap your thighs,
clap your hands,
clap your partner's hands with palms meeting,
touch your shoulders,
nod your head.

Let each pair of children practice the routine several times to get the feeling of repeating the five steps.

Tell the children that the five steps make one complete exercise. Each set of partners will do as many complete exercises in one minute as possible without breaking the rhythm or making an error in the pattern. Assign a third child to each pair to count the number of exercises completed in a minute. Select two other timekeepers to give a signal to begin the exercise when they start the pendulum, count 60 seconds, and give a signal to stop.

Continue the activity with other pairs of children. If desired, let the children change partners to try to complete more exercises in one minute.

Supplemental Experiences

■ Have some pupils investigate how many times they can bounce a ball in one minute. If possible, have the children use balls of different sizes.

■ Some children may wish to investigate the number of words they can read aloud, with clarity and expression, in one minute. Emphasize that this is not a contest to determine how fast one can read aloud.

KEY IDEA

Thirty seconds is a half-minute.

Scope

To explore the fractional parts ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$) of a minute and to express these parts as seconds.

Fundamentals

The children have learned to use the fractional numbers $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ to represent parts of a unit. The teacher will want to emphasize that three-thirds and four-fourths are each one unit. The child will discover, by guessing and testing, the number that is one-half of 60. That is, he will guess the number which, if added to itself, is 60. He can prove that the guess is correct by adding.

Readiness for Understanding

Understanding of the fractional numbers $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

Understanding of the time duration of a second and a minute.

Developmental Experiences

seconds pendulum
demonstration clock
10 pieces of paper

▶ You will need 6 pieces of paper that are 4 by 18 inches for this activity. Write 1 minute = ___ seconds on the chalkboard. Then have a child complete the sentence. Now hold up one piece of paper and tell the children that it represents one minute. Ask a child to write 1 minute on the paper. Hold up another piece of paper and ask some child to compare the two and see if the second piece also represents one minute. When it is established that both pieces are the same size, ask a child to fold the second piece of paper to show $\frac{1}{2}$ minute. Then have him open the paper, draw a line down the fold, and label each part $\frac{1}{2}$ minute. Ask how many of the $\frac{1}{2}$ parts it takes to make 1 minute. (2)

Display another piece of paper. Ask a child to compare it to the piece labeled 1 minute. When it is established that the two pieces are the same size, ask a child how many seconds the blank piece of paper represents. Have the piece labeled 60 seconds. Then hold up another sheet of paper. Demonstrate that it is the same size as the piece for 60 seconds and fold the paper to show $\frac{1}{2}$ of 60 seconds. Ask the children how many of the $\frac{1}{2}$ parts it takes to equal 60 seconds.

Write ___ + ___ = 60 on the chalkboard. Then ask how many seconds there are in $\frac{1}{2}$ of 60 seconds. Let the children suggest numbers; record each number in the equation. Ask the children why they agree or disagree with each suggestion. The children should agree from this exploration that each of the two $\frac{1}{2}$ parts will be represented by the same number. They should also agree that the sum of the parts should be 60. Through trial and error they will arrive at $30 + 30 = 60$ and decide that $\frac{1}{2}$ of 60 seconds is 30 seconds.

Continue the activity by folding a fourth piece of paper to show $\frac{1}{4}$ of a minute. Have still another piece folded to show $\frac{1}{4}$ of 60 seconds. Ask how many of the $\frac{1}{4}$ parts it takes to equal 60 seconds.

$$\_\_\_ + \_\_\_ + \_\_\_ + \_\_\_ = 60$$

Again let the children suggest how many seconds $\frac{1}{4}$ of 60 seconds would be. Have the children experiment with numbers until they find the number that it takes 4 of to equal 60. Give them the chance to discover the fact that one-fourth of 60 seconds is 15 seconds.

▶ Ask the class how many seconds there are in $\frac{1}{2}$ of 60 seconds. Then ask several of the children how they could use the pendulum to count two $\frac{1}{2}$-minutes. Some child may decide to count each swing until he has counted 30 seconds and then start over and count the next 30 seconds. Another child may suggest counting every 2 swings of the pendulum in a similar manner. Let a child start the pendulum and count aloud 2 intervals of 30 seconds each, while the rest of the class silently counts 60 seconds. Discuss whether or not the child counted for the same length of time as the class.

Ask a volunteer how many seconds there are in $\frac{1}{4}$ of 60 seconds. Then ask several children how they could count four $\frac{1}{4}$-minutes by using the pendulum. A child may decide to count each swing until he has counted 15 seconds and then repeat the procedure three more times. Let one child start the pendulum and count aloud 4 intervals of 15 seconds each, while the rest of the class silently counts 60 seconds. Discuss whether or not the child counted for the same length of time as the class.

▶ Have 4 pieces of paper, 4 by 18 inches, on hand for this exercise. Write 1 minute = ___ seconds on the chalkboard and let a child complete the sentence. Display the 4 by 18-inch paper that was labeled 1 minute in the preceding activity. Hold up a second sheet of paper and ask a child to compare the two pieces. When it has been established that both pieces are the same size, fold the second piece of paper to show $\frac{1}{3}$-minute, open the paper, and draw lines on each fold. Ask a child what part of a minute each part represents. Have each part labeled $\frac{1}{3}$-minute. Ask a child how many of the $\frac{1}{3}$ parts it takes to make 1 minute.

Display the 4 by 18-inch paper that was labeled 60 seconds in the previous activity. Take a blank sheet and show the class that it is the same size as the paper for 60 seconds. Fold the paper to show $\frac{1}{3}$ of 60 seconds. When the paper is opened, draw lines on the folds. Ask a child what part of 60 seconds each part represents. Let the class tell how many of the $\frac{1}{3}$ parts it takes to equal 60 seconds. Write ___ + ___ + ___ = 60 on the chalkboard. Then ask how many seconds there are in $\frac{1}{3}$ of 60 seconds. Let the children suggest numbers and then record each number in the equation. Ask the children why they agree or disagree with each suggestion. The children should agree from this exploration that each of the three $\frac{1}{3}$ parts will be represented by the same number. They should also agree that the sum of the parts is 60. Through trial and error they will arrive at $20 + 20 + 20 = 60$ and decide that $\frac{1}{3}$ of 60 seconds is 20 seconds.

Continue the activity by folding one 4 by 18-inch paper to show $\frac{1}{6}$ of a minute, and fold another piece to show $\frac{1}{6}$ of 60 seconds. Ask how many of the $\frac{1}{6}$ parts it takes to equal 60 seconds.

$$\_\_\_ + \_\_\_ + \_\_\_ + \_\_\_ + \_\_\_ + \_\_\_ = 60$$

Again let the children suggest how many seconds $\frac{1}{6}$ of 60 seconds would be. Let the children experiment with numbers until they find the number that it takes 6 of to equal 60, or that one-sixth of sixty seconds is 10 seconds.

▶ Write the following incomplete sentences on the chalkboard.

___ + ___ = 60

___ + ___ + ___ = 60

___ + ___ + ___ + ___ = 60

___ + ___ + ___ + ___ + ___ + ___ = 60

Ask what number it takes 2 of to equal 60. Have a child complete the first sentence. Continue with similar questions and have the rest of the sentences completed. Then ask the children the number of seconds in $\frac{1}{2}$ of 60 seconds, in $\frac{1}{3}$ of 60 seconds, in $\frac{1}{4}$ of 60 seconds, and in $\frac{1}{6}$ of 60 seconds.

Let one child use the seconds pendulum to demonstrate the count for three $\frac{1}{3}$-minutes by counting 20 swings of the pendulum three times in succession. In a similar manner, have all of the children count four $\frac{1}{4}$-minutes, two $\frac{1}{2}$-minutes, and six $\frac{1}{6}$-minutes.

Write on the chalkboard:

$\frac{1}{6}$-minute
$\frac{1}{3}$-minute
$\frac{1}{2}$-minute
$\frac{1}{4}$-minute

Ask the children which period of time is the longest, next longest, and so forth. Ask the children to explain how they know. Some children may say that if a minute is divided into only 2 parts, each part is longer than the part of a minute that is divided into more than 2 parts. Other children may explain in terms of seconds—one-half minute is 30 seconds, or that each of the other parts of a minute listed is less than 30 seconds.

Pages 21 through 24

● Page 21 lets the children identify parts of a minute and parts of 60 seconds. Discuss the model at the top of the page. Ask the children what the two strips at the top of the page tell them. Someone will volunteer that 1 minute is equal to 60 seconds.

Call attention to exercises 1, 2, and 3. Remind the class that the pictures beside the exercises demonstrate parts of a minute. Write $\frac{\triangle}{\triangledown}$ on the chalkboard. Ask the children why they think the symbol $\frac{\triangle}{\triangledown}$ appears on each part of the picture in the first exercise. Someone may suggest that each number is a fractional number or a number like $\frac{1}{2}$.

Encourage the pupils to look over the rest of the illustrations and exercises on the page and to ask questions about anything they do not understand. Since the developmental experiences have provided a background for this page, most of the children should be able to proceed with little difficulty.

● Use page 22 to help determine each pupil's ability to tell time. Before the page is introduced, display a demonstration clock that shows 8 o'clock. Ask the children to write the time shown. It may be written as 8 o'clock or as 8:00. Set the clock at 3:35 and let the children read and write the time. Review the idea that it is 35 minutes after 3, 25 minutes to 4, or 3:35. Then introduce page 22 and let the children proceed independently. They need to give only one reading for each time.

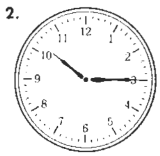
● To introduce page 23, call attention to the strips shown at the top of the page. Let individual children tell what each strip represents and what they observe about the size of the two strips. Many in the class may already know that 1 hour is 60 minutes. Remind the class that the illustrations beside exercises 1, 2, and 3 show parts of an hour and that the illustrations beside the other exercises show parts of 60 minutes. Ask the children to complete as many of the exercises on the page as possible. When most of the children have finished, discuss their answers and the reasons for them. Using a demonstration clock, let the children move the minute hand through $\frac{1}{2}$ hour, $\frac{1}{3}$ hour, $\frac{1}{4}$ hour, and $\frac{1}{6}$ hour. Let others demonstrate that 2 halves of 60 minutes are 60 minutes, and so forth.

15 minutes past 6
6:15
quarter past 6

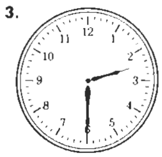
What time is it? *Answers will vary.*

1.


12:00

2.


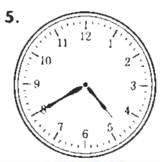
10:15
15 minutes past 10
quarter past 10

3.


2:30
30 minutes past 2
half past 2
30 minutes to 3

4.


2:45
15 minutes to 3
quarter to 3

5.


4:40
40 minutes past 4
20 minutes to 5

6.


7:50
50 minutes past 7
10 minutes to 8

A-22

Name

UNIT 3 TIME

For Class Discussion

1 minute ⇔ 60 seconds

How much time?

1 minute

1. △ is $\frac{1}{2}$ minute.

2. □ is $\frac{1}{3}$ minute.

3. ○ is $\frac{1}{4}$ minute.

60 seconds

4. △30 + △30 = 60

5. □20 + □20 + □20 = 60

6. ○15 + ○15 + ○15 + ○15 = 60

7. How many seconds is $\frac{1}{3}$ of 60 seconds? *20 seconds*

8. How many seconds is $\frac{1}{2}$ of 60 seconds? *30 seconds*

9. How many seconds is $\frac{1}{4}$ of 60 seconds? *15 seconds*

reference page

A-21

Name

For Class Discussion

60 minutes ⇔ 1 hour

1 hour

How much time?

1. △ is $\frac{1}{2}$ hour.

2. □ is $\frac{1}{3}$ hour.

3. ○ is $\frac{1}{4}$ hour.

60 minutes

4. △30 + △30 = 60

5. □20 + □20 + □20 = 60

6. ○15 + ○15 + ○15 + ○15 = 60

7. How many minutes is $\frac{1}{2}$ of 60 minutes? *30 minutes*

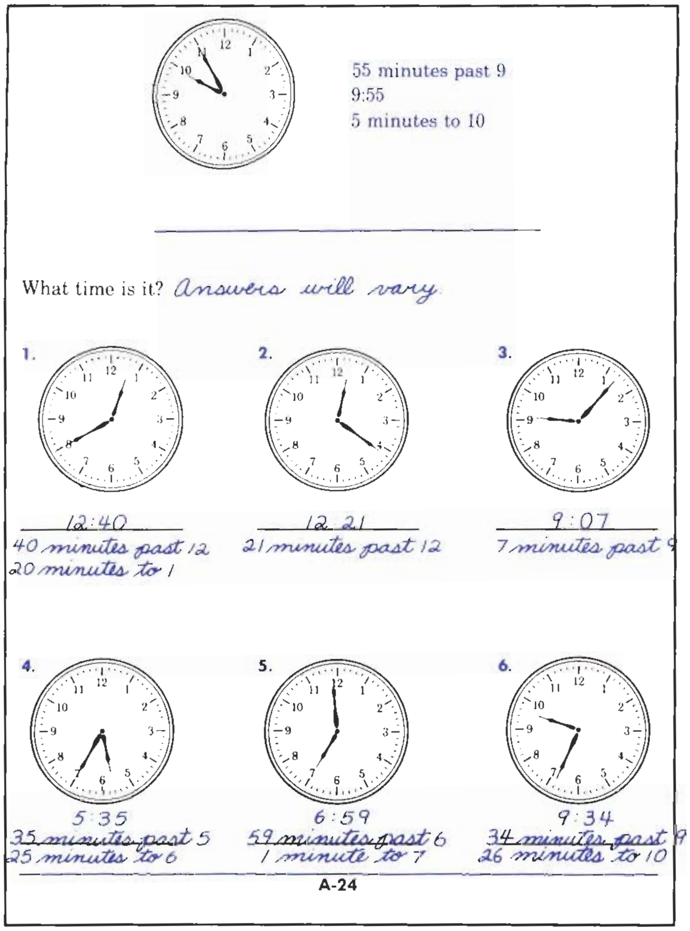
8. How many minutes is $\frac{1}{4}$ of 60 minutes? *15 minutes*

9. How many minutes is $\frac{1}{3}$ of 60 minutes? *20 minutes*

10. How many minutes is $\frac{1}{6}$ of 60 minutes? *10 minutes*

reference page

A-23



● Page 24 provides a further opportunity for reading a clockface. Set a demonstration clock at 5:15. Ask the children to read the time in several ways. They may suggest quarter after 5, 15 minutes past 5, or 45 minutes before 6. Repeat the activity with the clock set at 5:17. Assign the page so that the children can find out how accurately they can read time from a clock. They need to give only one reading for each clock.

Supplemental Experiences

■ This activity and the others that follow may be used with children who have trouble reading time from a clock.

Ask the children to watch the hands on a real clock (use a clock with a large dial or a geared model clock). Turn the hands. Ask questions such as:

What do you see on the clock?
Are the hands the same length?
Do both hands move at the same time?
Which hand moves faster?

Set the long hand of the clock on 12 and the short hand on 3; review the fact that at the hour the long hand points to 12 and the short hand indicates the hour. Ask a child to tell the time shown by the clock. (3 o'clock) Continue to set the clock for times on the hour, and let the children read the time aloud.

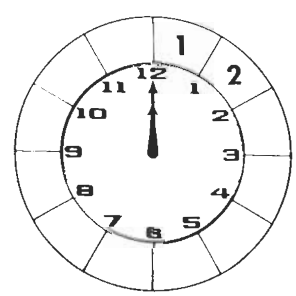
Draw a large clockface that shows 9 o'clock on the chalkboard. Beside the clock write ___o'clock and ___:00.

____ O'CLOCK

____ :00

Have a child write the correct numeral on each line. Continue this activity by setting different hours on the clock and asking the children to write the time shown.

■ Draw a large clockface on the chalkboard. Draw a border about 3 inches wide around the clockface and divide it into 12 equal parts. Draw the hands to show 12 o'clock and ask the children to tell the time shown on the clock. Then write 1 and 2 in the border as shown.



Explain that the time it takes the hour hand to go from 12 to 1 on the clock is 1 hour. Tell the children that you started to number the hours after 12 o'clock. Ask them to number the rest of the hours after 12 o'clock.

Point to the line above 12. Say that the first hour after 12 o'clock begins at 12 o'clock. Ask a child to find the line that shows the end of the first hour after 12 o'clock. Erase the hour hand of the clock and redraw it to point to the line the child indicated. Then ask someone to tell the time shown by the clock at the end of the first hour.

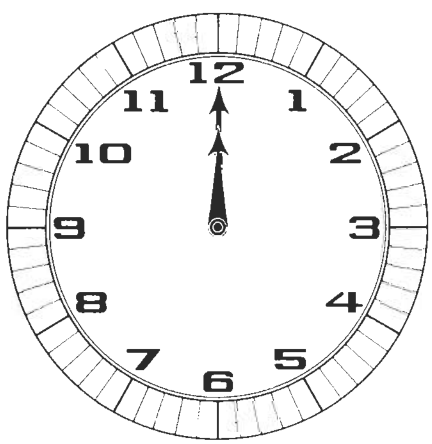
Move your finger across the part of the border that shows the second hour after 12 o'clock. Ask a child to point to the line that shows where the second hour begins and the line that shows where it ends. Let a volunteer read the time that the second hour after 12 o'clock begins. Erase the hour hand and redraw it so it points to 2. Have the time at the end of the second hour read.

Let the children list on the chalkboard the positions of the hour hand for each hour after 12 o'clock. Review the times at the beginning and at the end of the first hour as you write them on the board. Have a volunteer write the time shown by the hour hand at the beginning and at the end of the second hour.

After 12:00	Begins at	Ends at
1st hour	12:00	1:00
2nd hour	1:00	2:00
3rd hour		
4th hour		
5th hour		
6th hour		
7th hour		
8th hour		
9th hour		
10th hour		
11th hour		
12th hour		

Have other volunteers suggest the beginning and end of the third through the twelfth hours and list them on the chart.

■ On a tagboard circle 16 inches in diameter, draw a clock dial about 12 inches in diameter and mark the border to show minutes.



It may be helpful to make every 5th line in the border a little thicker than the other lines to show the intervals of 5 minutes between two numerals on the clock. Use a brad to attach the hands.

Ask a child to set the clock to show 12 o'clock. Explain that each part of the border shows one minute. Let the children number the minutes after 12:00.

Ask some child to show where the 1st minute after 12:00 begins and where it ends and record the information in chart-form on the chalkboard.

After 12:00	Begins at	Ends at
1st minute	12:00	12:01
2nd minute		
3rd minute		

Ask the child to read the time that begins at the 1st minute after 12:00. Tell the children that the time at the end of the 1st minute is read, "One minute after twelve." Ask another child to move the minute hand to point to the time that would be shown at the end of the 1st minute. Continue to have the children point out and record in the table the time at the beginning and end of successive minutes after 12:00. After the 10th minute after 12:00 has been recorded, let the children choose any minute after 12:00 and tell the time at the beginning and end of that minute.

■ Use a geared demonstration clock or a real clock and set it at 12:00. Ask the children to watch both hands of the clock as you turn them slowly. Pause briefly at 12:15, 12:30, and at 12:45 and discuss the children's observations. They may notice that the hour hand is moving slowly toward 1 o'clock. When the clockface reads 12:59, ask a child to write the time at the beginning of the 60th minute after 12:00. Move the hands 1 minute more and ask a child to write the time that would be shown at the end of the 60th minute after 12:00. The children may observe that the time shown is 1:00 and it is the end of the 1st hour after 12:00.

■ Display the tagboard demonstration clock with the 60-minute intervals numbered. Have the children point out the end of the 5th minute, 10th minute, and the 15th minute after 12:00. Ask if there is an easy way to recognize the location of the minute hand at these times. The children may observe that the minute hand points directly to a numeral on the clock. Ask for volunteers to set the clock to show other times when the minute hand points directly to a numeral. Have them read the time they have shown.

■ Let a child set the tagboard clock to show 12 o'clock. Ask what length of time each part of the border shows. (1 minute) Ask a child to show where the 1st minute after 12:00 ends by moving the minute hand to that position. Continue the activity by having the children set the hands to show such times as 12:10, 12:11, 12:25, 12:40, and 12:59. The children may point to the spaces in the border as they count, if they wish.

Set the clock to show 12:30. Ask one child to count the number of minutes after 12, while another child counts the number of minutes it will take before the clock shows 1 o'clock. Write these incomplete statements on the chalkboard:

____ minutes after 12
____ minutes before 1

Let each child record his count. Continue the activity in a similar manner for such times as 12:40, 12:50, 12:55, 12:10, and 12:22.

■ Display the tagboard demonstration clock with the 60-minute intervals numbered. Set the minute hand at the end of the 15th minute. Ask a child how many minutes after the hour is shown on this dial. The child may notice that the minutes are numbered and that the end of the 15th minute means that 15 minutes have passed. Let another child find the number of minutes before the next hour. He may count if he knows no other way to find the answer.

Then have a child write the number of minutes after the hour and the number of minutes before the next hour on the board and add them.

$$\begin{array}{r} 15 \text{ minutes} \\ + \ 45 \text{ minutes} \\ \hline 60 \text{ minutes} \end{array}$$

Repeat the procedure with other settings such as 20 minutes after, 24 minutes after, 35 minutes after, and 48 minutes after. The children may notice that the sum of the number of minutes after the hour and the number of minutes to the next hour is always 60 minutes.

Set the minute hand to read 30 minutes after the hour. Ask a child to read the number of minutes after the hour and tell how long it will take the minute hand to reach the next hour. Some children will realize that it is 30 minutes to the next hour because $30 + 30 = 60$. Repeat this procedure with other settings such as 27 minutes after, 40 minutes after, 1 minute after, 12 minutes after, and 44 minutes after. Let the children write their computation if they desire. Either computation of the missing addend or computation of the difference will help them.

$$\begin{array}{r} 27 \\ + \ \_\_ \\ \hline 60 \end{array} \qquad\qquad \begin{array}{r} 60 \\ - \ 27 \\ \hline \_\_ \end{array}$$

■ Have the children use the demonstration clock to count the minutes between the numerals 12 and 1, 1 and 2, 2 and 3, and so forth. Let the children count by fives to count the minutes in one hour.

Set the clock to 5 o'clock. Then have a child read the hour and write the time on the chalkboard. Set the clock to 5:10 and ask if any child can read or write the time indicated. Encourage the children to start at the hour and count by fives to find the number of minutes after that hour the long hand indicates.

Proceed with other settings such as 5:20, 5:35, and 5:55. Write the time on the chalkboard as each new setting is discussed; emphasize the hour and the minutes past the hour.

Write ____ minutes after ____ and ____ minutes before ____ on the chalkboard. Set the clock to show times such as 3:25, 11:05, 1:45, 4:50, and 7:55. Ask the children to record each time in two ways. The children may either count or compute to find the number of minutes before the next hour.

UNIT 4
NUMERATION
Pages 25 Through 38

OBJECTIVE

To extend the child's understanding of numbers by counting thousands as well as hundreds, tens, and ones.

Counting ones, tens, and hundreds is reviewed. Then counting is extended to counting thousands; this is a tremendous increase in the numbers available to the child. The child discovers that the sums and differences he knows for hundreds, tens, and ones also work for thousands. He counts thousands, hundreds, tens, and ones to develop a more powerful way of counting ones.

See Key Topics in Mathematics for the Primary Teacher: Numeration.

KEY IDEAS

1000 is ten ten tens.
We can add and subtract ten ten tens.
Place value contracts sums.

CONCEPTS

contracted notation counting thousands

KEY IDEA

1000 is ten ten tens.

Scope

To review counting tens and hundreds.
To learn to count thousands.
To review the meaning of standard three-digit numerals.
To learn to interpret standard four-digit numerals.

Fundamentals

The one-more concept as applied to tens becomes one-more ten or one ten more. The same numerals used in counting ones are used together with 0 to name tens.

COUNTING ONES		COUNTING TENS
1	10	one ten
2	20	two tens
3	30	three tens
⋮	⋮	⋮
9	90	nine tens
10	100	ten tens
11	110	eleven tens
⋮	⋮	⋮
19	190	nineteen tens
20	200	twenty tens
21	210	twenty-one tens

While it is possible to immediately introduce the conventional names for the counts of tens, experience indicates that it may be at the expense of understanding the idea. The second-grade child was given every opportunity to become thoroughly familiar with the idea of counting tens before the conventional names for the counts were introduced. These conventional names are defined by the number of tens, as follows:

COUNTS OF TENS		CONVENTIONAL NAME
one ten	10	ten
two tens	20	twenty
three tens	30	thirty
⋮	⋮	⋮
nine tens	90	ninety
ten tens	100	one hundred
eleven tens	110	one hundred ten
⋮	⋮	⋮
nineteen tens	190	one hundred ninety
twenty tens	200	two hundred
twenty-one tens	210	two hundred ten

When these conventional names were introduced to the second-grade child, a strong association was made with their meaning as counts of tens.

The one-more concept as applied to hundreds becomes one-more hundred or one hundred more. The same numerals used in counting ones are used together with 00 to name hundreds (ten tens).

COUNTING		COUNTING HUNDREDS
1	100	one hundred
2	200	two hundreds
3	300	three hundreds
⋮	⋮	⋮
9	900	nine hundreds
10	1000	ten hundreds
11	1100	eleven hundreds
⋮	⋮	⋮
19	1900	nineteen hundreds
20	2000	twenty hundreds
21	2100	twenty-one hundreds

While it is also possible to immediately introduce the conventional names for the counts of hundreds, experience indicates that it may be at the expense of understanding the idea. In this area too, the second-grade child was given every opportunity to become thoroughly familiar with the idea of counting hundreds before the conventional names for these counts were introduced. These conventional names are defined by the number of hundreds as follows:

COUNTS OF HUNDREDS		CONVENTIONAL NAME
one hundred	100	one hundred
two hundreds	200	two hundred
three hundreds	300	three hundred
⋮	⋮	⋮
nine hundreds	900	nine hundred
ten hundreds	1000	one thousand
eleven hundreds	1100	one thousand one hundred
⋮	⋮	⋮
nineteen hundreds	1900	one thousand nine hundred
twenty hundreds	2000	two thousand
twenty-one hundreds	2100	two thousand one hundred

When these conventional names were introduced to the second-grade child, a strong association was made with their meaning as counts of tens as well as counts of hundreds.

The one-more concept as applied to thousands becomes one-more thousand or one thousand more. The same numerals are used together with 000 to name thousands (ten ten tens).

COUNTING		COUNTING THOUSANDS
1	1000	one thousand
2	2000	two thousands
3	3000	three thousands
⋮	⋮	⋮
9	9000	nine thousands
10	10000	ten thousands
11	11000	eleven thousands
⋮	⋮	⋮
19	19000	nineteen thousands
20	20000	twenty thousands
21	21000	twenty-one thousands

While it is possible to immediately introduce the conventional names for the counts of thousands, experience indicates that here, too, it may be at the expense of understanding the numeration concept. The conventional names, defined by the number of thousands, are as follows:

COUNTS OF THOUSANDS		CONVENTIONAL NAME
one thousand	1000	one thousand
two thousands	2000	two thousand
three thousands	3000	three thousand
⋮	⋮	⋮
nine thousands	9000	nine thousand
ten thousands	10000	ten thousand
eleven thousands	11000	eleven thousand
⋮	⋮	⋮
nineteen thousands	19000	nineteen thousand
twenty thousands	20000	twenty thousand
twenty-one thousands	21000	twenty-one thousand

When the conventional names are introduced, it is important that each be strongly associated with its meaning as counts of tens, counts of hundreds, and counts of thousands.

Although commas are usually used in numerals of five or more digits, the comma will not be used until the next unit on numeration. The use of the comma at this time may interfere with the interpretation of the numeral in terms of tens and hundreds.

The child realizes that a sum of whole numbers is a whole number; this enables him to understand the naming of numbers as sums. A number such as 368 can be interpreted as a sum in the following ways:

a sum of ones

$1+1+1+1\ldots368$ (368 ones)

a sum of tens and ones

$360+8$ (36 tens + 8 ones)

a sum of hundreds and ones

$300+68$ (3 hundreds + 68 ones)

a sum of hundreds, tens, and ones

$300+60+8$ (3 hundreds + 6 tens + 8 ones)

The child may continue to add 1, one at a time until he has 10 ones which is another ten, 10 tens which is another hundred, or 10 ten tens which is another thousand.

$100+30+1$
$100+30+2$
$100+30+3$
$100+30+4$
$100+30+5$
$100+30+6$
$100+30+7$
$100+30+8$
$100+30+9$
$100+30+10$ is combined into 140.

$200+10+1$
$200+20+1$
$200+30+1$
$200+40+1$
⋮
$200+70+1$
$200+80+1$
$200+90+1$
$200+100+1$ is combined into $300+1$.

$1000+100$
$1000+200$
$1000+300$
⋮
$1000+900$
$1000+1000$ is combined into 2000.

Readiness for Understanding

Ability to count hundreds, tens, and ones.

Understanding that a sum of whole numbers is a whole number.

Understanding of the numerals from 1 to 18, from 10 to 180, and from 100 to 1800.

Developmental Experiences

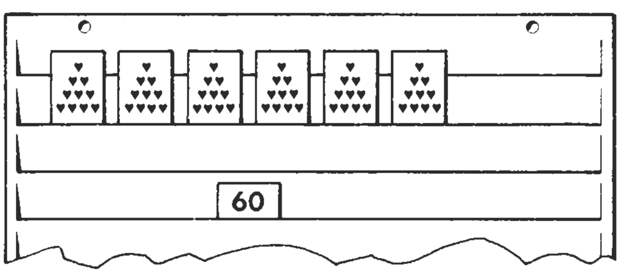
for flannel board	*for each child*
hundred-cards	tagboard (6″ × 18″)
1000-card	31 tagboard strips (3″ × $1\frac{1}{2}$″)

18 ten-cards
210 hundred-cards
18 thousand-cards
pocket chart
numeral-cards:
- 10 through 180
- 100 through 1800
- 1000 through 20000

number strips:
- 9 one-strips (3″ × $\frac{1}{4}$″)
- 9 ten-strips (3″ × $2\frac{1}{2}$″)
- 9 hundred-strips (3″ × 25″)

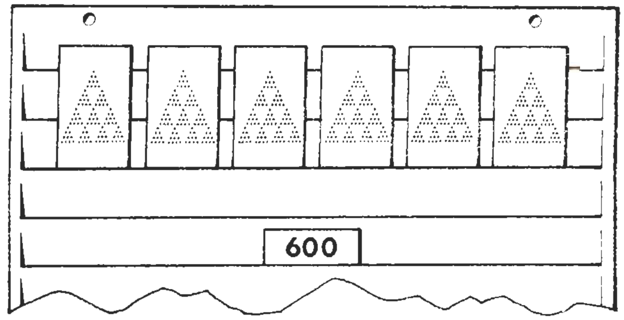
felt-tip pen
masking tape
box
rubber bands
stapler

▶ Prepare cards that show sets of 10 and sets of 10 tens to review counting through 180 (18 tens) and counting through 1800 (18 ten tens). Place 6 ten-cards in one row of the pocket chart. Place a set of numeral-cards for 10 through 180 beside the chart. Ask a child to find the numeral for the number of tens and to place it below the cards in the chart.



Ask the class how many tens are represented and whether or not the numeral in the chart tells this number. Continue to put from 1 through 18 tens in the pocket chart. In each instance, have a child place the appropriate numeral for the number of tens in the chart.

Next place 10 ten-cards in the pocket chart. Have the children give the count of the set; then display 1 hundred-card and have some child give the count for this card. (1 ten ten or 1 hundred) Review the fact that 1 hundred-card is merely a more convenient way to represent 10 tens. Place 6 hundred-cards in the pocket chart and a set of numeral-cards for 100 through 1800 beside the chart. Have a child select the numeral for the number of ten tens, place it below the cards in the chart, and give the count.



If this child gives the count in terms of hundreds (6 hundreds), have another child give the count in terms of ten tens (6 ten tens) and have a third child give the count in terms of tens. (60 tens)

Continue in this way to put from 1 through 18 hundreds in the pocket chart. In each instance, have a child choose the appropriate numeral for the number of ten tens and place it in the chart. Have the count given in terms of hundreds, ten tens, and tens.

▶ Construct tagboard number strips to represent 1, 10, and 100. Use tagboard strips that are 3 inches wide, and cut out 9 of each of the following lengths:

$\frac{1}{4}$ inch long to represent 1,
$2\frac{1}{2}$ inches long to represent 10,
25 inches long to represent 100.

Label each strip and prepare it for use on the chalkboard.

Place 3 hundred-strips, 7 ten-strips, and 4 one-strips on the chalkboard. Ask a child to write below the strips the standard numeral for the number that the strips help him visualize. Remind the class that a number such as 374 has many meanings; and ask the children to help show the meanings. Write an equal sign to the right of 374 and ask a child to write the expanded form to the right of the equal sign.

374 = 300 + 70 + 4 (3 hundreds + 7 tens + 4 ones)

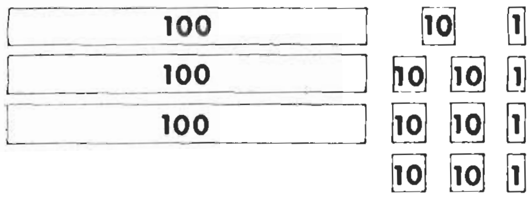
Write another equal sign below the first one and ask the children how many tens they believe are in 374. If a child responds with 7, point to the 300 in 300 + 70 + 4 and ask how many tens 300 is. When the child responds 30, have him add the 30 tens and the 7 tens and write 374 in terms of tens and ones.

374 = 300 + 70 + 4 (3 hundreds + 7 tens + 4 ones)
= 370 + 4 (37 tens + 4 ones)

Write another equal sign on the chalkboard and ask the children how many ones are in 374. Through a discussion of the number of ones in the 70 of 370, guide a child to write 374 in terms of hundreds and ones.

374 = 300 + 70 + 4 (3 hundreds + 7 tens + 4 ones)
= 370 + 4 (37 tens + 4 ones)
= 300 + 74 (3 hundreds + 74 ones)

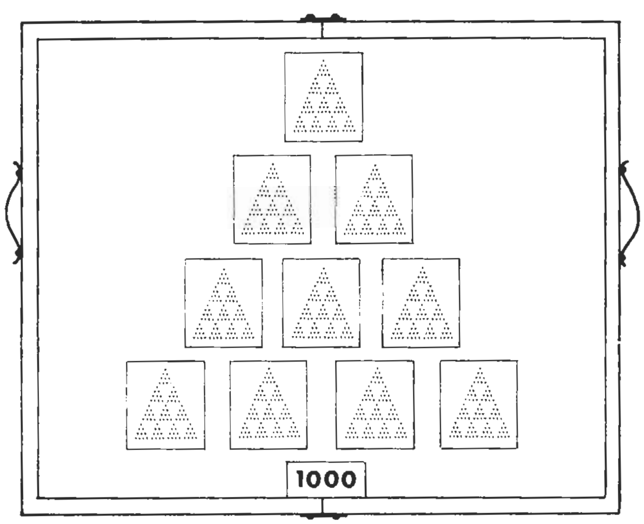
Next, ask how many ones there are in the 300. Have a child add the 300 ones and the 74 ones and write 374 in terms of ones.



$$\begin{aligned}374 &= 300+70+4\\ &= 370+4\\ &= 300+74\\ &= 374\end{aligned}$$

Continue to use the number strips to help the children visualize other numbers from 11 through 999. Have individual children each write a number such as 79 in terms of tens and ones and in terms of ones. Have them write a number such as 783 in terms of hundreds, tens, and ones, in terms of tens and ones, in terms of hundreds and ones and in terms of ones. These numbers need not be described in any special order.

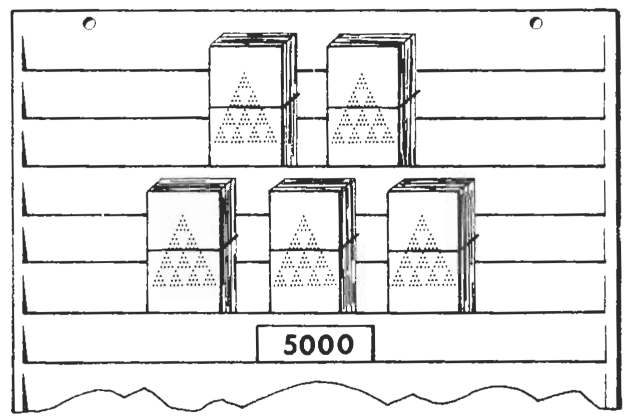
▶ Introduce the idea that it is possible to count ten ten tens (ten hundreds or thousands). Place a box of hundred-cards on a table in front of the class. Ask a child to count out 10 ten tens (one thousand or ten hundreds). Arrange the cards on the flannel board as shown, and introduce the symbol 1000 on a card placed below the set on the flannel board.



The children will use the conventional names for the counts of thousands; therefore, it is important that they understand what the names mean. For example, have the children tell how many tens they see (100 tens), how many ten tens (10 ten tens), how many ten ten tens (1 ten ten ten), how many hundreds (10 hundreds), and how many thousands (1 thousand). This contributes to an understanding of the idea that the numeral 1000 shows a count of tens, a count of ones, a count of hundreds, and a count of thousands.

Have several other children come forward in turn and have each count out 10 ten tens (hundreds). Band each bundle of cards and lay it to one side. Be sure the children realize that each bundle has the same count as the set on the flannel board. Continue until 20 bundles of 1000 have been made by the children.

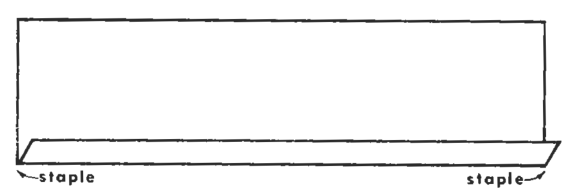
Place all of the bundles of ten ten tens in the box on the table in the front of the room. Help a child count 5 bundles of 1000 aloud (1 thousand, 2 thousands, 3 thousands, 4 thousands, 5 thousands) and place them in the pocket chart. Place a numeral-card for 5000 below the 5 bundles of 1000.



Ask another child how many hundreds are in 1 bundle (10 hundreds) and how many hundreds are in 5 bundles (50 hundreds). Ask a third child how many tens are in 1 bundle (100 tens) and how many tens are in 5 bundles (500 tens).

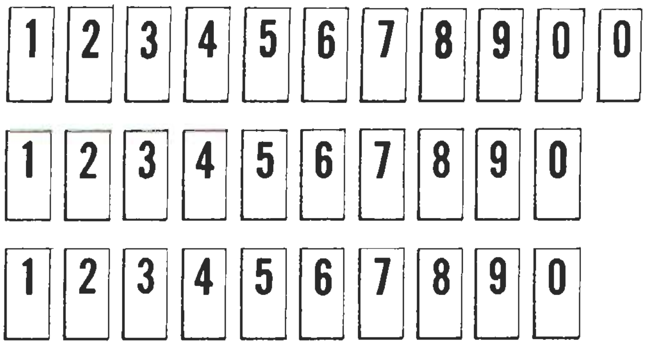
Continue to have the children count out bundles of 1000; have them work with 1 through 20 thousands. In each instance, introduce the appropriate numeral and have the children tell how many hundreds and how many tens are represented in each example.

▶ Provide each child with a 6 by 18 inch piece of tagboard. Direct the children to fold over about 1 inch of one 18 inch edge of the tagboard as illustrated. Help the children staple the ends to form a pocket.

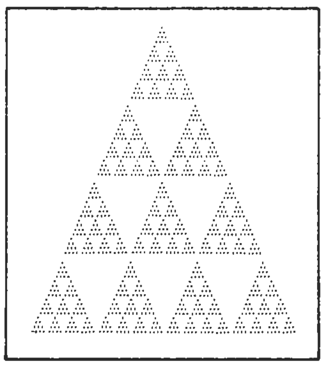


Then show the children how to fold the sheet in half to make a display rack.

Next provide each child with 31 tagboard strips (3 inches by $1\frac{1}{2}$ inches). Guide the children to mark the strips as shown.

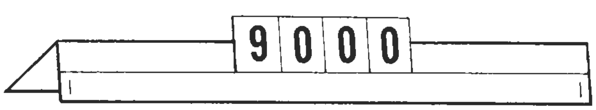


Prepare 18 cards that show sets of one thousand as illustrated.



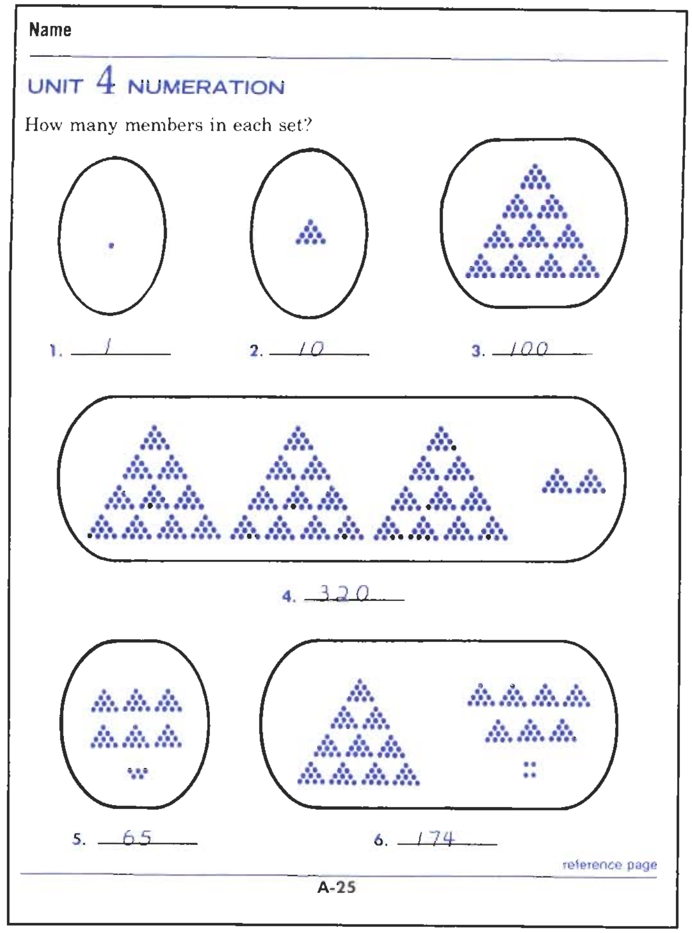
Show the children one bundle of ten ten tens used in the previous activity and have them give the count. Then display a thousand-card and have a child give the count. If necessary, explain that each thousand-card is merely a more convenient way of representing ten ten tens.

Hold up 9 thousand-cards. Have the children use their numeral strips and display racks at their desks to show the number illustrated by the thousand-cards.



Have the children give the count in terms of thousands (9 thousands), tens (900 tens), ten tens (90 ten tens), ten ten tens (9 ten ten tens), and in hundreds (90 hundreds).

Continue in this way to show thousand-cards from 1 through 18 thousands. In each instance have the children display the appropriate number. Encourage individual children to give each count in terms of thousands, hundreds, tens, ten tens, and ten ten tens.



Name

UNIT 4 NUMERATION

How many members in each set?

1. 1
2. 10
3. 100
4. 320
5. 65
6. 174

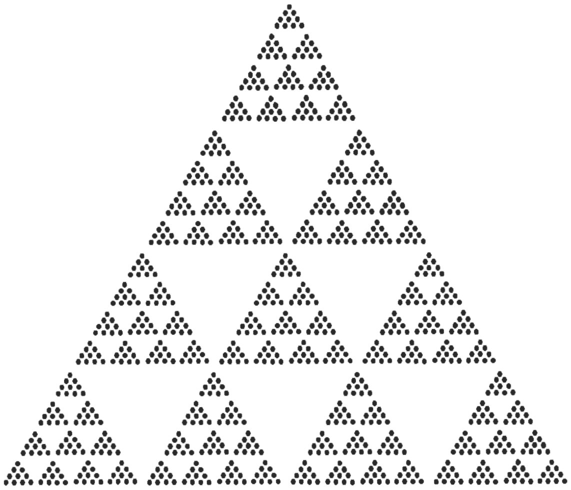
reference page

A-25

Pages 25 through 28

● Use page 25 to provide practice in supplying the standard numeral for the numbers of given sets. Have the children write the standard numeral for the number of each set. After they have completed the page, ask a child how many members are in the set in exercise 4. When he responds, "Three hundred twenty," have another child give the count in terms of tens. (32 tens) Then ask if anyone can give the count in terms of hundreds and tens. (3 hundreds plus 2 tens) Follow a similar procedure to discuss the children's answers for each of the other exercises.

For Class Discussion



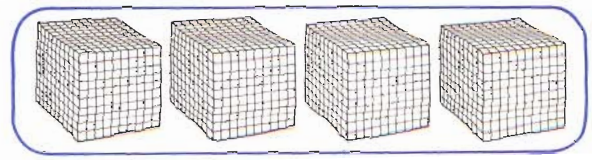
1. How many tens? 100
2. How many ten tens? 10
3. How many ten ten tens? 1
4. How many ones? 1000

reference page

A-26

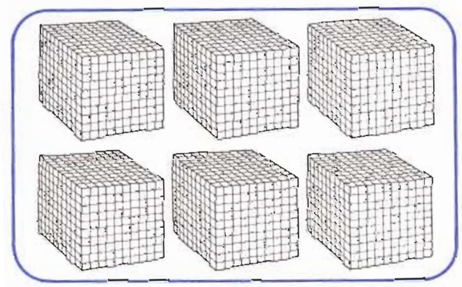
How many?

1.



4000 ones
400 tens
40 hundreds
4 thousands

2.



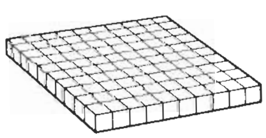
6000 ones
600 tens
60 hundreds
6 thousands

reference page

A-28

Name

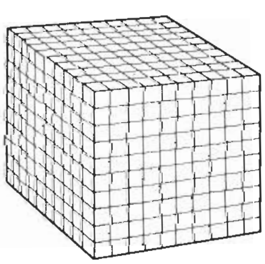
Build a cube with blocks. Arrange the blocks at the base of the cube in the following way.



The base is

how many blocks wide? 10
how many blocks long? 10
how many blocks? 100, 10 tens, or 10 × 10

Finish the cube to look like this.



The cube is

how many blocks wide? 10
how many blocks long? 10
how many blocks high? 10
how many blocks? 1000, 10 ten tens, 10 × 10 × 10, 100 tens, 100 × 10, 10 hundreds, or 10 × 100

$1000 = 10 \times 10 \times 10$
$= 10 \times 100$
$= 100 \times 10$

reference page

A-27

● Use pages 26 and 27 to strengthen the children's understanding of the concept of 1000. Work both pages with the class; discuss the number 1000 in terms of tens, hundreds, ten tens, and ten ten tens. If possible, have 1000 blocks for the pupils to arrange in connection with building the cube pictured on page 27.

● Page 28 will provide the children with practice in counting thousands and recording the count. For each exercise have the children write the number of ones, tens, hundreds, and thousands. When the children have completed the page have them take turns telling how many ones, tens, hundreds, and thousands they recorded in each exercise.

Supplemental Experiences

■ Have the children play a guessing game. Choose one child to stand before the group and ask a riddle. He may say, "I'm thinking of a number that is 7 thousands. Who can tell how many ten tens?" He then calls on another child to give the correct answer (70 ten tens) and to write the numeral for this number on the chalkboard. If the answer is correct, the child who answered thinks of another number and presents a riddle of his own. Suggest that some children should describe their numbers in different ways. For example, one child might say, "I'm thinking of a number that is 1 ten ten ten more than 3 ten ten tens. My number is how many thousands?" Another might say, "I'm thinking of a number that is 140 ten tens. Who can tell how many thousands?"

■ Review the order concept by using thousands. The ability to decide which of two numbers is greater depends on an understanding of place value.

Write several rows of numerals on the chalkboard. Direct the children to think of the numbers represented by these numerals and to rewrite them on their own papers from the least to the greatest.

1000, 3000, 2000
7000, 2000, 8000
11000, 9000, 12000
18000, 16000, 17000
10000, 7000, 9000

■ Review the counting of tens up to 20 tens. Have a child write on the chalkboard the numerals you name; the others may read the recorded numerals aloud in terms of the number of tens involved.

TEACHER SAYS	CHILD WRITES	CLASS SAYS
ten	10	1 ten
twenty	20	2 tens
thirty	30	3 tens
forty	40	4 tens
fifty	50	5 tens
sixty	60	6 tens
seventy	70	7 tens
eighty	80	8 tens
ninety	90	9 tens
one hundred	100	10 tens
one hundred ten	110	11 tens
one hundred twenty	120	12 tens
one hundred thirty	130	13 tens
one hundred forty	140	14 tens
one hundred fifty	150	15 tens
one hundred sixty	160	16 tens
one hundred seventy	170	17 tens
one hundred eighty	180	18 tens
one hundred ninety	190	19 tens
two hundred	200	20 tens

Vary the activity by having another child write the numerals as you give the number of tens. The other children should read the standard numeral aloud.

TEACHER SAYS	CHILD WRITES	CLASS SAYS
1 ten	10	ten
2 tens	20	twenty
⋮	⋮	⋮
20 tens	200	two hundred

Adapt this procedure for counting hundreds up to 20 hundreds and for counting thousands up to 20 thousands.

KEY IDEA

We can add and subtract ten ten tens.

Scope

To review addition and subtraction combinations for sums through 10.

To apply addition and subtraction combinations to sums of thousands (no greater than 10 thousands).

To provide practice in computing two and three addends.

Fundamentals

The addition and subtraction combinations are seen by the child as applying to thousands as well as to ones, tens, and hundreds.

$$\begin{array}{r}6\\+\ 3\\\hline 9\end{array}\qquad\begin{array}{r}60\\+\ 30\\\hline 90\end{array}\qquad\begin{array}{r}600\\+\ 300\\\hline 900\end{array}\qquad\begin{array}{r}6000\\+\ 3000\\\hline 9000\end{array}$$

Later the child will learn that the distributive property is involved.

$$\begin{aligned}600 + 300 &= 6 \times 100 + 3 \times 100\\&= (6 + 3) \times 100\\&= 9 \times 100\\&= 900\end{aligned}$$

The child will review column addition applied to thousands as he works with three addends whose sums are not greater than 10 ten ten tens. He will recall that when doing column addition, he can start at the top or at the bottom.

Readiness for Understanding

Knowledge of addition and subtraction combinations of 10.

Ability to count thousands.

Ability to write the numerals for thousands.

Awareness of the relationship between sums and differences.

Developmental Experiences

18 thousand-cards	*for each child*
pocket chart	31 numeral strips
box	plastic plus signs, minus signs, and line segments

▶ Place 5 thousand-cards in the pocket chart. Leave a space and place 4 more thousand-cards in the chart. Ask the class how many thousands they see; have them give a sum for the number of thousands. Write this sum on the chalkboard as shown, and let a child write the standard numeral for this sum in the appropriate position.

$$\begin{array}{r}5000\\+\ 4000\\\hline 9000\end{array}$$

Have the class read aloud what they see on the chalkboard: "Five thousands plus four thousands is nine thousands."

Next have the class tell how many thousands would be in the pocket chart if there were 4 thousands less than the 9 thousands. Ask the class to name the difference that describes 4 thousands less than 9 thousands. Write this difference on the chalkboard as shown. Ask a child to write the standard numeral for this difference.

$$\begin{array}{r} 9000 \\ -\ 4000 \\ \hline 5000 \end{array}$$

Have the children read aloud what they see on the chalkboard: "Nine thousands minus four thousands is five thousands."

Continue this procedure with other sums and differences of thousands. Use sums from 2 thousands through 10 thousands, and differences from 1 thousand through 9 thousands.

▶ Each child will need the 31 numeral strips he made for a previous activity, and plastic plus signs, minus signs, and line segments. In this exercise each child will show sums and differences of thousands at his desk.

Place 15 thousand-cards in a box in the front of the room. Ask one child to count out 4 thousand-cards and to place them on the chalktray. Have another child place 3 thousand-cards on the chalktray, leaving a space between the two sets. Tell the children to use their numeral strips and plastic symbols to show two forms for the sum 4 thousands plus 3 thousands. Help them arrange their numeral strips as shown.

$$\begin{array}{r} 4000 \\ +\ 3000 \\ \hline \end{array} \qquad \begin{array}{r} 3000 \\ +\ 4000 \\ \hline \end{array}$$

Now direct the children to place the standard numeral for each sum below each line. If the children do not have enough numeral strips, provide them with blank 3 by $1\frac{1}{2}$ inch strips of tagboard to label as needed.

Next tell the children to show two differences that the set pictures on the chalktray indicate and to then compute and show the standard numeral for each difference on their desks.

Have the children take turns reading aloud the sentences they have symbolized on their desks.

4 thousands plus 3 thousands is 7 thousands.
3 thousands plus 4 thousands is 7 thousands.
7 thousands minus 3 thousands is 4 thousands.
7 thousands minus 4 thousands is 3 thousands.

Continue to have the children display thousand-cards and their numeral strips to illustrate sums and differences of thousands. Use no sum greater than 10 thousands.

▶ Place 5 thousand-cards in the top row of the pocket chart. Place 2 thousand-cards in the next row and 3 thousand-cards in the third row. Have the class observe the parts (subsets) of this union of three sets from top to bottom. Have them name a sum that corresponds to this reading of the set picture. Direct the children to use their sets of numeral strips, plastic operation symbols, and line segments to build at their desks the algorism for the computation of this sum.

$$\begin{array}{r} 5000 \\ 2000 \\ +\ 3000 \\ \hline 10000 \end{array}$$

Now have the class observe the three parts of the set of cards in the pocket chart from bottom to top and have them name a sum that corresponds to this reading of the set picture. Have them build the algorism for the computation of this sum.

$$\begin{array}{r} 3000 \\ 2000 \\ +\ 5000 \\ \hline 10000 \end{array}$$

Continue to have the class record and compute sums from 3 thousands through 10 thousands.

Pages 29 through 32

● Use page 29 for discussion. After the illustration has been observed and the story read, have the class discuss the similarity between adding ones and adding thousands. Ask the children to identify the basic addition fact that helps them compute sums such as 4000+3000, 60+20, 700+200, and 5000+5000.

Summarize by helping the children realize that the addition combinations apply to thousands as well as to ones, tens, and hundreds.

Then discuss the question at the bottom of the page. Ask the children to identify the subtraction facts that help them compute the differences.

● Use page 30 to provide practice in computing sums of tens, sums of hundreds, and sums of thousands. This page may be used for several assignments. The children should do the computation independently. After all of the children have finished, have individual children tell how they computed specific sums.

● Use page 31 to provide practice in computing differences of tens, differences of hundreds, and differences of thousands. Once directions have been given for the page, have the children complete the computation independently.

5 tens	30
+ 3 tens	+ 50
8 tens	80

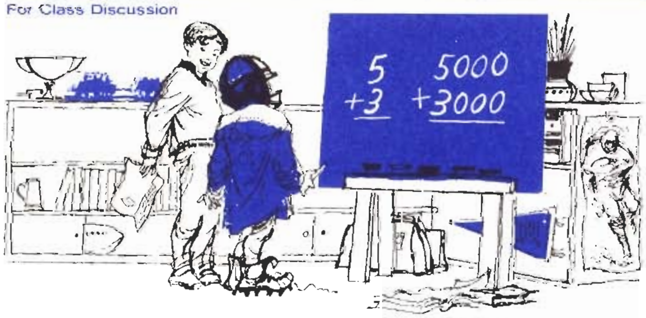
Compute.

1. 500 + 400 = *900*	2. 2000 + 5000 = *7000*	3. 40 + 40 = *80*	4. 1000 + 9000 = *10000*
5. 80 + 20 = *100*	6. 6000 + 4000 = *10000*	7. 100 + 100 = *200*	8. 3000 + 2000 = *5000*
9. 60 + 10 + 10 = *80*	10. 4000 + 2000 + 3000 = *9000*	11. 1000 + 5000 + 3000 = *9000*	12. 40 + 20 + 10 = *70*
13. 200 + 500 + 300 = *1000*	14. 2000 + 6000 + 2000 = *10000*	15. 400 + 200 + 300 = *900*	16. 200 + 700 + 100 = *1000*
17. 2100 + 1000 = *3100*	18. 600 + 300 = *900*	19. 1000 + 4000 = *5000*	20. 50 + 10 = *60*

A-30

Name

For Class Discussion



Ron and Pete were talking about a football game.

"I heard that 5000 people sat on one side of the field and 3000 sat on the other," said Pete.

"I wonder how many people were at the game. How can I compute 5000 + 3000?" asked Ron.

Pete said, "Look, Ron, do you know 5 + 3?"

"Of course I do! 5 + 3 is 8," replied Ron. "But I still don't know 5000 + 3000."

"Yes you do. Look at 5000 + 3000. It's like 5 + 3, but this time you're adding thousands instead of ones," said Pete.

"Now I know!" grinned Ron. "I'll have 5 + 3 thousands. That's 8000."

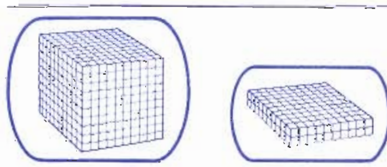
How would you compute 5000 − 3000? 12000 − 6000?

See pupil page suggestions.

reference page

A-29

Name



1000	200
− 200	+ 800
800	1000

10 hundred − 2 hundred = 8 hundred

Compute.

1. 800 − 200 = *600*	2. 9000 − 5000 = *4000*	3. 60 − 10 = *50*	4. 8000 − 4000 = *4000*
5. 100 − 10 = *90*	6. 50 − 20 = *30*	7. 5000 − 4000 = *1000*	8. 1000 − 700 = *300*
9. 2000 − 1000 = *1000*	10. 400 − 100 = *300*	11. 10000 − 6000 = *4000*	12. 9000 − 8000 = *1000*
13. 10000 − 2000 = *8000*	14. 700 − 600 = *100*	15. 8000 − 6000 = *2000*	16. 7000 − 2000 = *5000*
17. 90 − 30 = *60*	18. 10000 − 8000 = *2000*	19. 1000 − 900 = *100*	20. 900 − 600 = *300*

reference page

A-31

3 thousand	4 thousand
+ 4 thousand	− 3 thousand
7 thousand	1 thousand

Compute.

1. 500 + 200 = 700	2. 70 + 30 = 100	3. 5000 − 3000 = 2000	4. 30 + 50 = 80
5. 900 − 400 = 500	6. 80 − 70 = 10	7. 8000 − 5000 = 3000	8. 300 + 600 = 900
9. 1000 − 300 = 700	10. 30 + 40 = 70	11. 1000 + 6000 = 7000	12. 700 − 300 = 400
13. 60 + 20 = 80	14. 3000 + 3000 = 6000	15. 20 + 80 = 100	16. 9000 − 7000 = 2000
17. 20 + 10 = 30	18. 800 − 300 = 500	19. 10 + 80 = 90	20. 100 − 40 = 60

A-32

● Use page 32 for further practice in computing sums and differences.

Supplemental Experiences

■ Provide practice in computation of sums (and related differences) of thousands no greater than 10000. Present the following exercises to the class verbally:

2000 plus 7000 (pause) minus 1000,
5000 minus 1000 (pause) plus 6000,
4000 plus 2000 (pause) minus 4000,
3000 minus 2000 (pause) plus 6000,
4000 minus 1000 (pause) plus 7000,
3000 plus 6000 (pause) minus 5000,
10000 minus 6000 (pause) plus 5000.

Encourage the children to compute without using pencil and paper. Call on someone to name the result of each computation. The children will enjoy working faster as they become proficient in mental computation.

■ Write several placeholder equations on the chalkboard. Have the children give the numeral that completes each equation. Have them explain their answers.

4000 + 3000 = 10000 − □
9000 − 2000 = 4000 + □
2000 + 6000 = 5000 + □
10000 − □ = 1000 + 7000
8000 − 2000 = 10000 − □
6000 + □ = 7000 + 2000

KEY IDEA

Place value contracts sums.

Scope

To understand numbers as sums of thousands, hundreds, tens, and ones.

To teach the standard contraction of the sums of thousands, hundreds, tens, and ones into a place-value numeral.

Fundamentals

The child's realization that a sum of whole numbers is a whole number enables him to understand the naming of numbers as sums. A number such as 1459 can be interpreted as a sum in the following ways:

a sum of ones
1 + 1 + 1 + 1 + . . . 1459 (1459 ones)
a sum of tens and ones
1450 + 9 (145 tens + 9 ones)
a sum of hundreds and ones
1400 + 59 (14 hundreds + 59 ones)
a sum of thousands and ones
1000 + 459 (1 thousand + 459 ones)
a sum of hundreds, tens, and ones
1400 + 50 + 9 (14 hundreds + 5 tens + 9 ones)
a sum of thousands, tens, and ones
1000 + 450 + 9 (1 thousand + 45 tens + 9 ones)
a sum of thousands, hundreds, and ones
1000 + 400 + 59 (1 thousand + 4 hundreds + 59 ones)
a sum of thousands, hundreds, tens, and ones
1000 + 400 + 50 + 9 (1 thousand + 4 hundreds + 5 tens + 9 ones)

Standard numerals are contractions of sums. In this section, the numerals involved are contractions of sums of thousands, hundreds, tens, and ones. For example, 4300 + 76 is contracted into 4376: 4300 + 76 = 4376. The sum 2000 + 700 + 34 is contracted into 2734: 2000 + 700 + 34 = 2734. The thousands in 4376 and the thousands in 2734 are expressed by the place of the digits 4 and 2. In 2000 + 700 + 34 the idea of thousands was expressed by the 000; in 2734 the idea of thousands is expressed by the place the digit is located.

Readiness for Understanding

Ability to count thousands, hundreds, tens, and ones.

Understanding that the sum of whole numbers is a whole number.

Ability to write numerals for the sums of thousands, hundreds, tens, and ones.

Developmental Experiences

	for each child
one-cards	graph paper ($\frac{1}{4}$" squares)
ten-cards	crayons
hundred-cards	31 numeral strips
thousand-cards	plastic plus signs
pocket chart	
tagboard cards	
felt-tip pen	
equal signs	

Place 3 thousand-cards, 6 hundred-cards, 2 ten-cards, and 7 one-cards in the pocket chart. Ask the children what sum they see; ask someone to write on the chalkboard the standard numeral for the number of this set. Tell the children that you want them to help show the meaning of 3627. Write an equal sign to the right of 3627 and write the expanded form for the numeral 3627 to the right of the equal sign.

$$3627 = 3000 + 600 + 20 + 7$$

Write another equal sign below the first one and ask the children to tell how many hundreds they believe are in 3000. (30) When a child responds 30, have him add the 30 hundreds and the 6 hundreds and write 3627 in terms of hundreds, tens, and ones.

$$\begin{aligned} 3627 &= 3000 + 600 + 20 + 7 \\ &= 3600 + 20 + 7 \end{aligned}$$

Use a similar approach to help the children understand 3627 as follows:

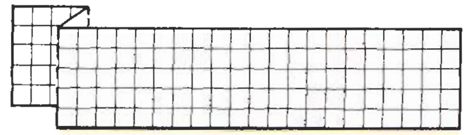
in terms of thousands, hundreds, and ones
3000 + 600 + 27
in terms of thousands, tens, and ones
3000 + 620 + 7
in terms of thousands and ones
3000 + 627
in terms of hundreds and ones
3600 + 27
in terms of tens and ones
3620 + 7
in terms of ones
3627

Now have the class read aloud the sums written on the chalkboard.

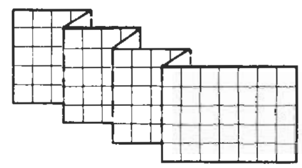
3000 + 600 + 20 + 7	"3 thousands + 6 hundreds + 2 tens + 7 ones"
3600 + 20 + 7	"36 hundreds + 2 tens + 7 ones"
3000 + 600 + 27	"3 thousands + 6 hundreds + 27 ones"
3000 + 620 + 7	"3 thousands + 62 tens + 7 ones"
3000 + 627	"3 thousands + 627 ones"
3600 + 27	"36 hundreds + 27 ones"
3620 + 7	"362 tens + 7 ones"
3627	"3627 ones"

Continue in this way to display set cards that will help the children visualize other numbers: 76, 496, 860, 3829, 5340, and so forth. The sums for each number need not be described in any special order.

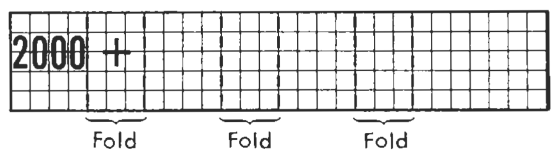
Provide each child with a strip of graph paper marked in $\frac{1}{4}$-inch squares. Each strip should be about 28 squares long and 5 squares wide. Direct the children to count 4 squares from the left end of the strip and to fold the strip forward along the line at the end of the fourth column of squares. Have them count 3 more squares and fold the strip along the line at the end of these 3 columns of squares as illustrated.



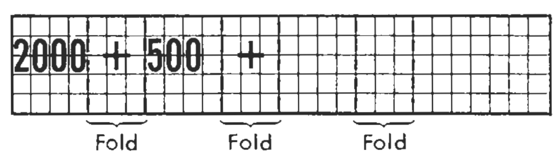
This same pattern of counting 4 squares and making the fold forward and then counting 3 squares and making the fold back should be followed until a total of 6 accordion folds have been made.



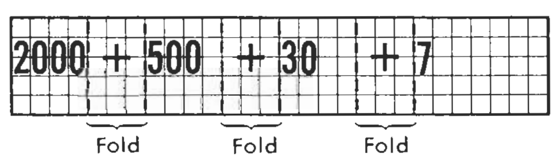
Now write the numeral 2537 on the chalkboard and instruct the children to write the expanded form for the numeral in the following way. Tell the children to lay the strips on their desks and use a red crayon to write the numeral 2 in the first column of squares at the left end of the strip. Then tell them to use a black crayon and write a 0 in each of the three remaining columns in this first set of squares. They should also use their black crayons to write a plus sign in the center of the next folded section.



Ask the children to use their red crayons to write the 5 of 500 in the first column of the next set of 4 columns of squares. Have them use their black crayons to write the two 0s in the next two columns and to write a plus sign in the center of the next folded section.

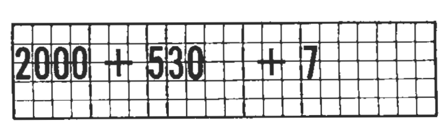


Follow a similar procedure to help the children complete the expanded notation of 2537 as shown.



Guide the children to see that the sum illustrates the meaning of 2537 in terms of thousands, hundreds, tens, and ones.

Ask the children to fold their strips to show another way to express the meaning of 2537. One child may fold his strip in such a way that 2537 is seen in terms of thousands, tens, and ones as illustrated.



Have other children show how they folded their strips. It is possible that the children may have shown 2537 as follows:

in terms of thousands, hundreds, and ones
2000 + 500 + 37
in terms of thousands and ones
2000 + 537
in terms of hundreds, tens, and ones
2500 + 30 + 7
in terms of hundreds and ones
2500 + 37
in terms of tens and ones
2530 + 7
in terms of ones
2537

The teacher may wish to provide the children with additional strips of graph paper to have them show the meaning of other four-digit numerals.

▶ Write the standard numeral for numbers such as 327, 940, 3742, 7600, and 5981 on individual cards. Display one card, for example 327, and direct the children to use their numerals and plastic plus signs at their desks to show the meaning in terms of tens and ones.

| 3 | 2 | 0 | + | 7 |

Display each of the other cards in turn, but vary the directions for the way that the children are to show the meaning of the number at their desks. For example, the children may be directed to give the meaning of 5981 in terms of thousands, tens, and ones.

5000 + 980 + 1

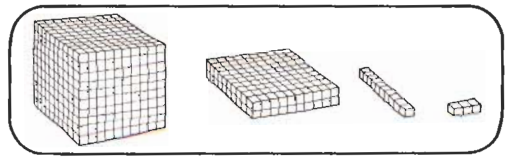
▶ Construct a set of cards for 768, 940, 1206, 4042, 5670, and 7856 as shown.

760 + 8	**900 + 40**
1000+200+6	**4040 + 2**
5600 + 70	**7000+800+56**

Write on another set of cards the standard numerals for these numbers. Distribute both sets of cards to the children. Call on a child to stand, display his numeral, and say, "I'm looking for a partner." The child who has the matching numeral must stand and read his card aloud. Put both cards in the pocket chart and place an equal sign between them. Have the first child read aloud the equation. Continue the game until all of the numerals have been matched.

Name

For Class Discussion



What is 1213?

1213 = 1000 + 200 + 10 + 3
1213 = 1 thousand + 2 hundreds + 1 ten + 3 ones

1213 = 1200 + 10 + 3
1213 = 12 hundreds + 1 ten + 3 ones

1213 = 1200 + 13
1213 = 12 hundreds + 13 ones

1213 = 1210 + 3
1213 = 121 tens + 3 ones

1213 = 1000 + 200 + 13
1213 = 1 thousand + 2 hundreds + 13 ones

1213 = 1000 + 210 + 3
1213 = 1 thousand + 21 tens + 3 ones

1213 = 1000 + 213
1213 = 1 thousand + 213 ones

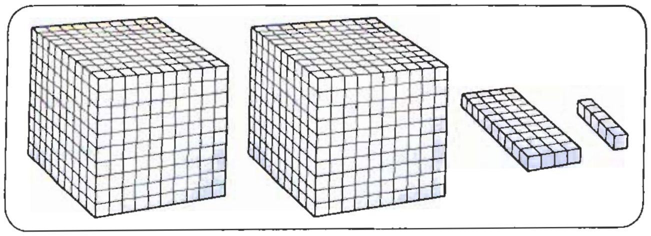
See pupil page suggestions.

reference page

A-33

Pages 33 through 38

● Use pages 33 and 34 to bring out the meaning of a numeral such as 1213. If possible, display a large illustration similar to the one at the top of the page. As each equation on page 33 is read and discussed, have a child go to the chalkboard and use the illustration to add meaning to the discussion. For example, when the equation 1213 = 1210 + 3 is read, a child may use his hands to frame each of these addends in turn. By placing one hand to the left of the set of 1000 blocks and the other hand to the right of the set of 10 blocks, the child can focus the attention of the class on the 121 tens. By following a similar procedure with the set of 3 ones, the other addend can be clearly seen.



Complete.

2045 = 2000 + 40 + 5

2045 = 2 thousands + 4 tens + 5 ones

2045 = 2000 + 0 + 40 + 5

2045 = 2 thousands + 0 hundreds + 4 tens + 5 ones

2045 = 2040 + 5

2045 = 204 tens + 5 ones

2045 = 2000 + 45

2045 = 20 hundreds + 45 ones

2045 = 2000 + 0 + 45

2045 = 20 hundreds + 0 tens + 45 ones

A-34

Name

3876 = 3 thousands + 8 hundreds + 7 tens + 6 ones

Complete.

1. 512 = 5 hundreds + 1 ten + 2 ones*
2. 1121 = 11 hundreds + 2 tens + 1 one*
3. 8620 = 8 thousands + 6 hundreds + 2 tens + 0 ones*
4. 7034 = 7 thousands + 0 hundreds + 3 tens + 4 ones*
5. 6006 = 6 thousands + 6 ones*
6. 945 = 9 hundreds + 4 tens + 5 ones*
7. 4823 = 4 thousands + 8 hundreds + 2 tens + 3 ones*
8. 53 = 5 tens + 3 ones*
9. 3767 = 37 hundreds + 6 tens + 7 ones*
10. 8110 = 8 thousands + 11 tens*
11. 9380 = 9 thousands + 3 hundreds + 80 ones*
12. 2100 = 21 hundreds
13. 727 = 7 hundreds + 27 ones*
14. 3535 = 3 thousands + 5 hundreds + 35 ones*

*Answers may vary.

A-35

● Ask the pupils to tell the number of blocks shown on page 34. Complete the first two equations as a class activity. Then assign the other exercises for independent work. Let the class discuss the results.

● Use page 35 to help children understand a number as a sum of thousands, hundreds, tens, and ones. Discuss the example at the top of the page with the class. Work exercise 1 with the children if necessary. Assign the remaining exercises for independent work. Then let the class discuss their results.

● Use page 36 to provide practice in writing equations that show the meaning of numerals from 11 through 9999. Refer to exercise 1 and direct the children to look at the clues that are given, and to tell how they are to give the meaning of 372 in terms of hundreds, tens, and ones. Have the children complete this equation. Discuss exercises 2 and 3 in a similar way. Then have the children complete the remainder of the page on their own. When the page has been completed, have the children discuss the meaning of the given numerals in terms other than those asked for on the page. For example, ask the children to give the meaning of the numeral in exercise 2 in terms of tens and ones, or in terms of hundreds, tens, and ones, or in terms of thousands, tens, and ones, and so on.

3815 = 3000 + 800 + 10 + 5

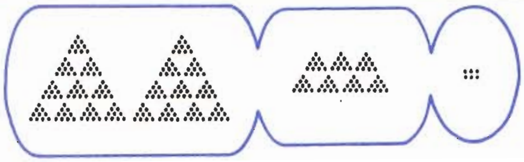
Write each number as a sum.

1. 372 = 300 + 70 + 2*
2. 6423 = 6000 + 400 + 20 + 3*
3. 11 = 10 + 1*
4. 8746 = 8700 + 46*
5. 8746 = 8000 + 740 + 6*
6. 481 = 480 + 1*
7. 4950 = 4000 + 900 + 50 + 0*
8. 7806 = 7800 + 6*
9. 7806 = 7000 + 806*
10. 240 = 200 + 40 + 0*
11. 5300 = 5000 + 300*
12. 3822 = 3820 + 2*
13. 3822 = ____________________ *
14. 1093 = ____________________ *
15. 9999 = ____________________ *

*Answers may vary.

A-36

Name



200 + 70 + 6 = 276

Write the standard numeral for each number.

1. 160 + 7 = 167
2. 1600 + 7 = 1607
3. 5600 + 8 = 5608
4. 8700 + 92 = 8792
5. 8000 + 92 = 8092
6. 1600 + 70 = 1670
7. 8000 + 920 = 8920
8. 7000 + 200 + 80 = 7280
9. 700 + 20 + 8 = 728
10. 6000 + 230 + 8 = 6238
11. 5000 + 600 + 70 + 8 = 5678
12. 5600 + 70 + 8 = 5678
13. 3000 + 500 + 10 + 9 = 3519
14. 9000 + 710 = 9710

reference page

A-37

2 thousands + 8 hundreds + 3 tens + 4 ones = 2834

Write the standard numeral for each number.

1. 3 thousands + 7 hundreds + 5 tens + 9 ones = 3759
2. 5 thousands + 6 hundreds + 8 tens + 6 ones = 5686
3. 6 hundreds + 5 tens + 3 ones = 653
4. 7 thousands + 81 tens = 7810
5. 4 thousands + 321 ones = 4321
6. 9 hundreds + 2 ones = 902
7. 2 thousands + 9 hundreds + 8 tens + 3 ones = 2983
8. 5 hundreds + 27 ones = 527
9. 14 hundreds + 4 ones = 1404
10. 4 thousands + 3 hundreds + 8 tens = 4380
11. 9 thousands + 77 tens + 5 ones = 9775
12. 20 hundreds + 8 ones = 2008
13. 8 hundreds + 3 tens + 4 ones = 834
14. 3 thousands + 1 hundred + 2 tens + 9 ones = 3129
15. 13 hundreds + 2 tens + 5 ones = 1325

A-38

● Use page 37 to let the children test themselves on their ability to write the contracted form for numerals that name numbers greater than 100. Discuss the illustration at the top of the page and work exercises 1 and 2 with the class. Then have the children write the standard numeral for each of the remaining exercises.

● Page 38 provides the children with additional practice in writing the standard numeral for a given number. Work through the example at the top of the page with the class. Assign the remaining exercises for independent work.

Supplemental Experiences

■ Play a matching game. Write a column of standard numerals on the chalkboard and to the right of this column write a column that shows the meaning for these numerals.

4307	4000 + 30 + 7
3067	6000 + 300 + 7
6307	3000 + 67
4037	4000 + 300 + 7

Call on a child to read the first numeral in the first column to the class. Then have him search in the second column for the sum that shows the meaning for this numeral. Have him draw a line from the numeral to the sum. Continue to have the children match other numerals in a similar fashion.

■ Use a set of at least 15 cards with numerals such as 1200 + 32 and also a set of 15 numerals in contracted form such as 1345. Combine the cards and place them face down on a table. Divide the class into two teams. Let one child from each team take a card and place it in the pocket chart. The player that is first to correctly identify the greater of the two numbers displayed may take both cards back to his team. The next member of each team continues the activity. The team with the most cards at the end of the game is the winner.

■ Play a relay game with four teams of six children. Write four columns of standard numerals for six numbers from 1000 to 18000 on the chalkboard. Have the teams line up facing the board; give the first member of each team a piece of chalk. At a signal, the first child from each team must walk quickly to the board and write the meaning for the first number in his team's column. He then must return to his team, pass the chalk to the next child, and go to the end of the line. The second child must walk quickly to the board, write the meaning for the second number in the column, return to his team, pass the chalk to the next child, and go to the end of the line. The team that is the first to correctly give the meaning for each number in its column wins the game.

UNIT 5
COMPUTATION OF PRODUCTS

Pages 39 Through 60

OBJECTIVE

To develop computational skill with the use of basic multiples of 2, 3, 4, and 5.

The child reviews the array as a model for multiplication and recognizes the number of an array as a product that may be expressed with a standard numeral. He learns a useful application for his acquired multiplication skill as he calculates the cost of stamps.

See Key Topics in Mathematics for the Primary Teacher: Multiplication.

KEY IDEAS

There is a count for every product.
$9 \times 3 = 27$.
3×4 is twice 3×2.
$0 \times \square = 0$.
$12 \times 2 = 6 \times 4 = 8 \times 3$.

KEY IDEA

There is a count for every product.

Scope

To review the idea that the product of whole numbers is a whole number.

To review the number of an array as rows times rows.

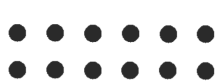
To develop multiplication combinations for multiples of 2.

Fundamentals

The concept that the product of two numbers is a number is often obscured by the process of computing. The numeral 6×2 names three different numbers—the factor 2, the factor 6, and the product 6×2. This product 6×2 is a whole number. The property that the product of whole numbers is a whole number is called the closure property for multiplication.

Up to this time, whenever the child observed an array he expressed the product as rows times rows. Now the child learns to express a product as a count.

Using a 6 by 2 array the child may compute 6×2 by counting each member of the array, by adding $6+6$, or by a variety of techniques of his own creation.



The child learns that computation is the process of finding standard numerals for numbers.

Readiness for Understanding

Understanding of the concept of product.
Ability to count.

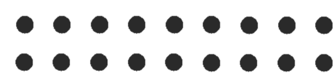
Developmental Experiences

for flannel board	*for each child*
felt counters	plastic numerals and symbols
felt numerals and symbols	

pocket chart
tagboard strips ($1'' \times 2\frac{1}{2}''$)
tagboard cards ($3'' \times 3''$)
masking tape
felt-tip pen
array cards

▶ Place a 9 by 2 array on the flannel board. Ask the class to name the product for this array. Have a child choose the symbols needed to show the product. Let a child name the count for the array and explain how he arrived at this count. The child may say that he added 9 and 9, that he counted by 2, or that he counted one at a time. Let the children discuss how they found the count for this array.

Place to the left of the product on the board the symbols that show this count and link the count and the product with an equal sign. Instruct the class to read the equation aloud.



$18 = 9 \times 2$

Continue the activity with the arrays 1 by 2, 2 by 2, through 20 by 2.

▶ Help the children determine how many tens and how many ones are in each of the products from 1×2 through 20×2. Ask a child to show a 6 by 2 array on the flannel board. Ask the class to tell the product for this array. Have a second child record this product on the chalkboard. Let a third child rearrange the members of the array to show tens and ones.

Call on someone to record on the chalkboard, next to the product, a sum that describes this arrangement of the members. Direct this child to use an equal sign to link the product to the sum.

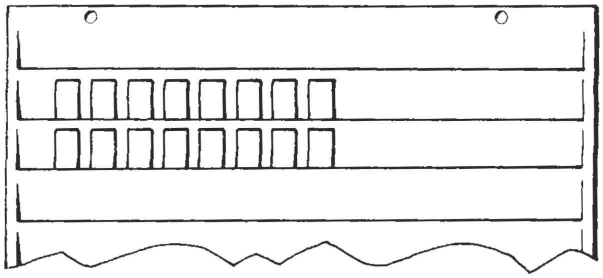
$$6 \times 2 = 10 + 2$$

Then tell the child to write the standard numeral for $10+2$ to the right of the sum and to link with an equal sign this form of the given number to the other two forms.

$$6 \times 2 = 10 + 2 = 12$$

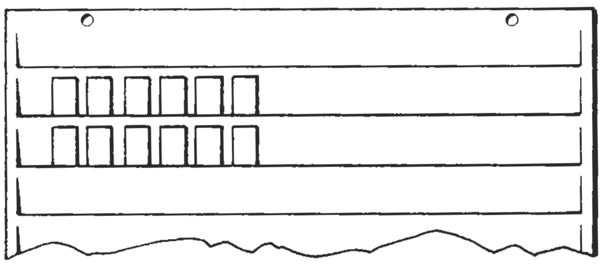
Adapt this procedure to other products that have 2 as a factor.

Show an 8 by 2 array in the pocket chart using strips of tagboard (1 inch by $2\frac{1}{2}$ inches).



Tell the children to use their plastic numerals and symbols at their desks to show an equation that links the product and the count for this array. ($8 \times 2 = 16$ or $2 \times 8 = 16$)

Remove 2 strips from each of the rows of 8. Have the children make an equation that links the product and count for this array. ($6 \times 2 = 12$ or $2 \times 6 = 12$)



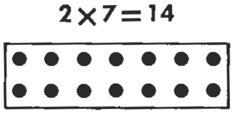
Remove 3 strips from each of the rows of 6 and have the children show an equation using the product and count for the new array. Remove all of the remaining strips. Ask how many rows of 2 are now in the chart. (0, 0×0, or none) Have the children tell how many members belong to an array that has 0 rows of 2 members. (0)

Write the equation $0 \times 2 = 0$ on the chalkboard. Point to this equation and explain that the product for an array that has 0 rows of 2 members is 0×2; the count is 0.

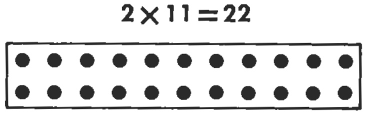
Continue the activity using 0 by 2 through 20 by 2 arrays. Strips may be removed from or added to any given array to show other arrays. Have the children show an appropriate equation for each array.

Use gummed stickers and tagboard cards to make two packs of array-cards for the products 1×2 through 20×2. Separate the class into two teams and assign a section of the chalkboard to each team. Place a pack of array-cards on the chalktray below each section. Have a child from each team choose a card and write an equation that shows the relationship between the product and the count for his particular array. Then ask each child to write a second equation that links the commuted form of the given product to the count.

$7 \times 2 = 14$
$2 \times 7 = 14$



$11 \times 2 = 22$
$2 \times 11 = 22$



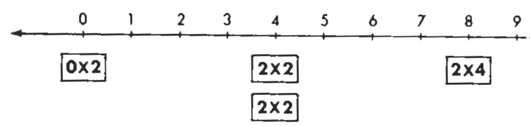
Let the class judge which team completed its assignment correctly and was first to do so. The children may earn points for their team in the following ways:

1 point is earned for completing the assignment correctly;

1 point is earned for being first to complete the assignment.

Continue until every child in the class has participated in the activity. Then let each team total its points and declare a winner.

Draw on the chalkboard a segment of the number line. Use 6-inch intervals to show 0 through 20 on this line. Write the products 0×2 through 20×2 on 21 tagboard cards (2 inches by 5 inches). On 21 other cards, write the commuted form for each of these products. Distribute all of the cards to the children. Then let each child go to the number line and place his product below the appropriate count. Help each child fasten his card to the board with a ring of masking tape.



After all of the products have been put into place on the number line, ask the children what they observe about the products that have 2 as a factor. They may note that beginning with 0 every other count is represented by a product that has 2 as a factor.

Pages 39 through 43

● Discuss page 39 to reinforce the idea that there is a count for every product of whole numbers. Have the children observe the array at the top of the page and ask a child to give the product and the count for this array. Ask another child to read the equation that links the product and the count. Then establish that the number of an array is a product and the members of an array can be counted.

Direct attention to the second array on the page. The children should understand that there are 3×2 members or 6 members in the array. Have a child read the equation that links this product and count. Then direct the children to complete the two equations at the bottom of the page. Let the children discuss the equations they wrote.

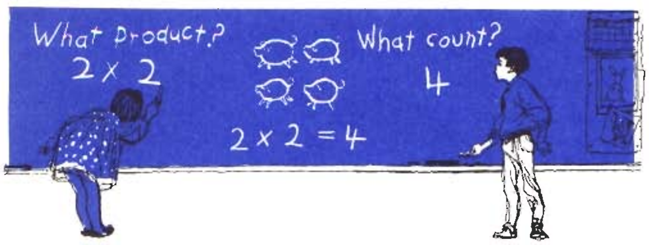
● Pages 40 and 41 provide the children with an opportunity to investigate products that have 2 as a factor. Ask the children to use the pictures of sets to help them determine the product and the count in each exercise. When they are finished, let various children read their equations. Some may notice patterns in the equations on these pages.

● Page 42 provides further experience with products that have 2 as a factor. Work exercises 1 and 2 as a class activity. Then ask the children to complete the other equations. Suggest that they may draw pictures of sets if they wish.

Name

UNIT 5 COMPUTATION OF PRODUCTS

For Class Discussion

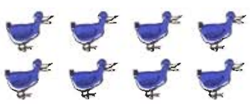


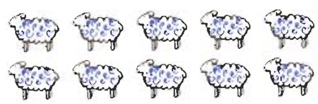
The number of an array is a product.
The members of an array can be counted.

How many?
3 × 2
6

There are 3 × 2 members in the array.
There are 6 members in the array.
3 × 2 = 6

What product? What count?





1. 4 × 2 = 8

2. 5 × 2 = 10

reference page

What product? What count?

1. 

6 × 2 = 12

2. 

7 × 2 = 14

3. 

8 × 2 = 16

4. 

9 × 2 = 18

5. 

10 × 2 = 20

6. 

11 × 2 = 22

7. 

12 × 2 = 24

reference page

Name

What product? What count?

1. 

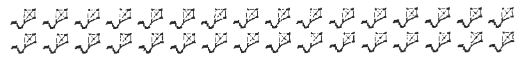
13 × 2 = 26

2. 

14 × 2 = 28

3. 

15 × 2 = 30

4. 

16 × 2 = 32

5. 

17 × 2 = 34

6. 

18 × 2 = 36

reference page

What product? What count?

1. 

19 × 2 = 38

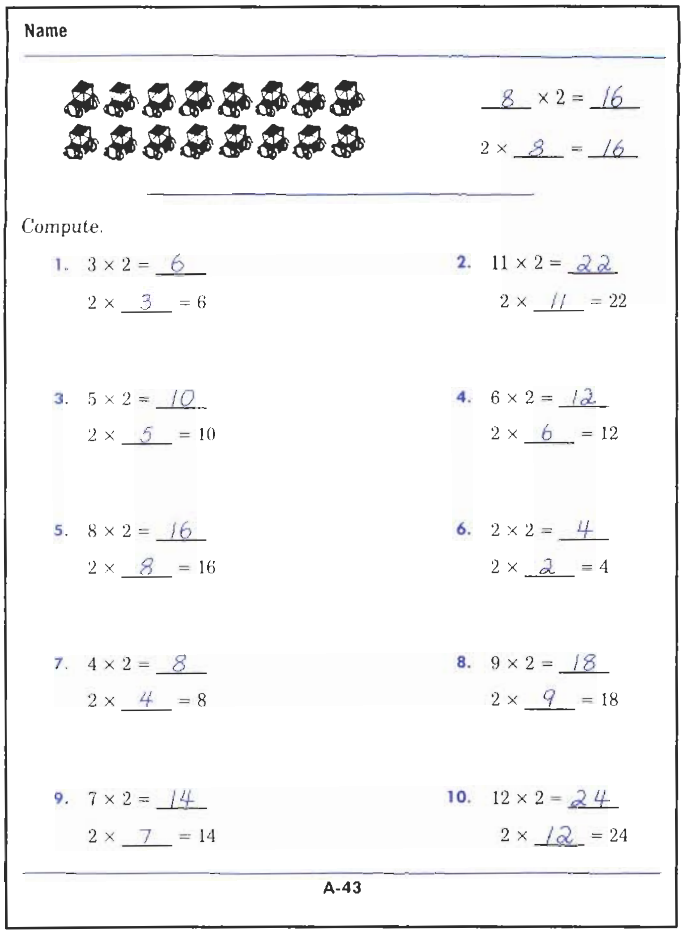
2. 

20 × 2 = 40

Compute.

3. 20 × 2 = 40	4. 19 × 2 = 38	5. 18 × 2 = 36
6. 17 × 2 = 34	7. 16 × 2 = 32	8. 15 × 2 = 30
9. 14 × 2 = 28	10. 13 × 2 = 26	11. 12 × 2 = 24
12. 11 × 2 = 22	13. 10 × 2 = 20	14. 9 × 2 = 18
15. 8 × 2 = 16	16. 7 × 2 = 14	17. 6 × 2 = 12
18. 5 × 2 = 10	19. 4 × 2 = 8	20. 3 × 2 = 6
21. 2 × 2 = 4	22. 1 × 2 = 2	23. 0 × 2 = 0

reference page


Name

8 × 2 = 16

2 × 8 = 16

Compute.

1. 3 × 2 = 6
2 × 3 = 6

2. 11 × 2 = 22
2 × 11 = 22

3. 5 × 2 = 10
2 × 5 = 10

4. 6 × 2 = 12
2 × 6 = 12

5. 8 × 2 = 16
2 × 8 = 16

6. 2 × 2 = 4
2 × 2 = 4

7. 4 × 2 = 8
2 × 4 = 8

8. 9 × 2 = 18
2 × 9 = 18

9. 7 × 2 = 14
2 × 7 = 14

10. 12 × 2 = 24
2 × 12 = 24

A-43

● Page 43 will help the children observe the fact that commuting the factors does not affect the product. Briefly discuss the example at the top of the page and work with the class as they complete the two equations. Then assign the other exercises as independent work.

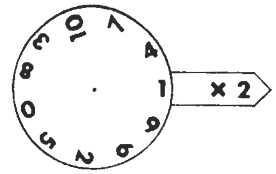
Supplemental Experiences

■ Write placeholder equations on cards that can be used to provide practice in computing the products 0 × 2 through 20 × 2.

□×2=14	8×2=□
□×2=24	14×2=□

Use the array-cards (made for use in a previous activity) for 1 × 2 through 20 × 2. Distribute array-cards to half of the class and equation-cards to the other half. Tell a child who has an array-card to come to the front of the room and show his card to the class. Tell the children that anyone who has an equation-card that represents the array may stand beside the child with the array-card. Have the second child read his equation to the class. Continue in this way until all of the children have had a chance to participate.

■ Construct a multiplication wheel for individual or small group practice sessions. Cut an 8-inch circle from heavy cardboard or tagboard. Write the numerals 0 through 10 around the edge of the wheel. Cut a 2 inch by 8 inch tagboard strip and write × 2 on the end of it. Attach the other end of the strip to the wheel with a paper fastener.



As the strip is moved around the wheel, the child must give the computed product of each pair of factors. Later, as other combinations are reviewed, new strips can be attached to the wheel.

KEY IDEA

$9 \times 3 = 27$.

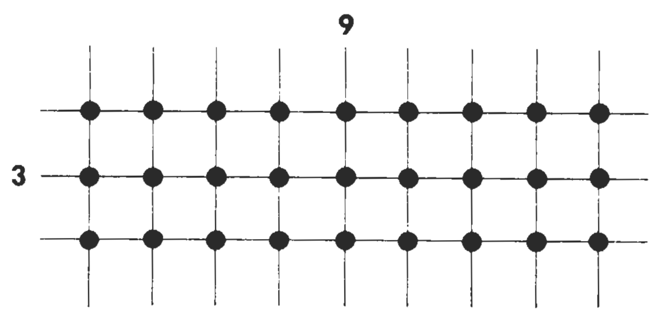
Scope

To review the count for a product.

To develop multiplication combinations for multiples of 3.

Fundamentals

The array is a model of multiplication and may be used to help illustrate the computation of products. A product such as 9 × 3 may be shown as a 9 by 3 array of intersections by drawing 9 vertical lines intersected by 3 horizontal lines.



The process of finding the standard numeral—the count of the intersections—is determined by the resourcefulness of the child. He may count the intersections one at a time, 3 at a time, 9 at a time, or he may devise other methods such as thinking of the intersections in terms of tens and ones. In a short time the children will know that 9×3 is 27 without needing counting techniques. Through experiences with the array, the child improves his understanding of product so that knowledge of the basic multiplication facts depends on perception rather than memory.

Readiness for Understanding

Understanding of the concept of product.

Ability to count.

Developmental Experiences

for flannel board	*for each child*
tagboard cards (2″ × 3″)	counters
numerals 0 through 9	plastic numerals and symbols
times (×) signs	

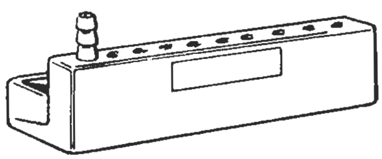
bead frame
product cards:
 0 × 3 through 10 × 3
 0 × 2 through 15 × 2
masking tape
felt-tip pen
string
pins

▶ Tell the children that you are going to display on the bead frame arrays that have rows of 3 members. The children are to name the product and the count for each of these arrays.

Begin with an empty frame. Ask how many rows of 3 are there. (0) Ask some child to give the product and the count for the array. Let a child write on the chalkboard an equation that links this product and count.

$$0 \times 3 = 0$$

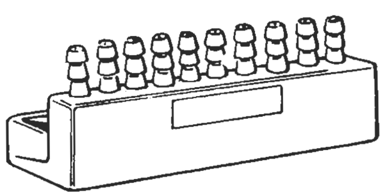
Construct an array that has 1 row of 3 members.



Determine the product and count for this array; select a child to write an appropriate equation on the chalkboard below the first equation.

$$0 \times 3 = 0$$
$$1 \times 3 = 3$$

Continue in this way until a 10 by 3 array has been examined.

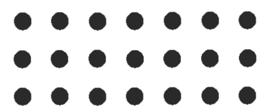


Then have the class observe the products recorded on the chalkboard.

$$0 \times 3 = 0$$
$$1 \times 3 = 3$$
$$2 \times 3 = 6$$
$$3 \times 3 = 9$$
$$4 \times 3 = 12$$
$$5 \times 3 = 15$$
$$6 \times 3 = 18$$
$$7 \times 3 = 21$$
$$8 \times 3 = 24$$
$$9 \times 3 = 27$$
$$10 \times 3 = 30$$

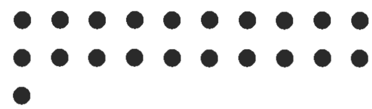
Let the children discuss what they notice about this sequence of products. They may observe that beginning with 0 × 3, each of the products is 3 more than the preceding product.

▶ Help the children determine how many tens and how many ones there are in each of the products from 1 × 3 through 10 × 3. Instruct each child to use his counters on his desk to construct a 7 by 3 array. Then have the children use plastic numerals and symbols to show the product for this array.



7×3

Tell the children to rearrange the members of the array to show tens and ones.



Have them use their plastic numerals to show the standard numeral for the sum of tens and ones. Direct them to link the two forms of the given number with an equal sign.

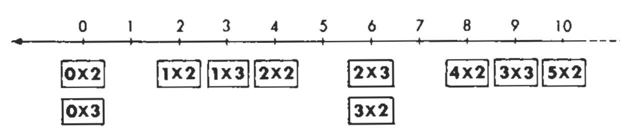
7×3=21

Let one child read his equation. Ask another to tell the sum of tens and ones represented by 7×3. (2 tens + 1, or 20 + 1)

Continue in this way with the products from 1 × 3 through 10 × 3.

▶ Draw on the chalkboard a segment of a number line. Use 4-inch intervals to show 0 through 30 on this line. You will need 22 product-cards for the products 0 × 3 through 10 × 3 and the product-cards for 0 × 2 through 15 × 2.

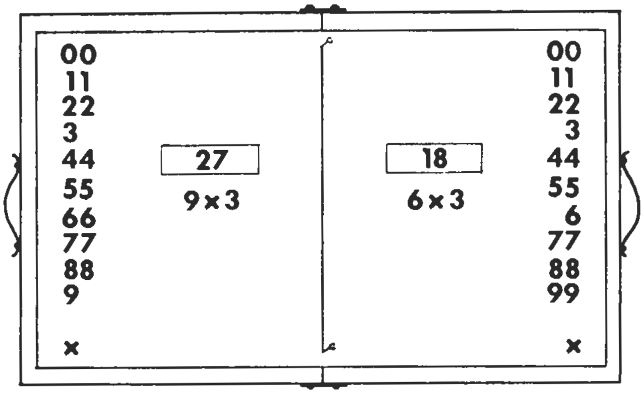
Have the children come forward one at a time and choose a card. Let each child tape his product below the appropriate count on the number line.



After all of the products have been put into place on the number line, ask the children what they observe about these products and counts. They may note that some of the counts have both 2 and 3 as factors: for example, 0, 6, 12, 18, and 24.

▶ Write the numerals 0 through 30 on 2 by 3 inch tagboard cards. Prepare this pack for use on the flannel board. Then place 2 sets of felt numerals for 0 through 9 and 2 times signs on each side of the flannel board. Pin a piece of string down the center of the board as a separating line. Separate the class into two teams and assign one section of the board to each team.

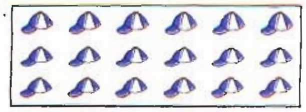
Select a member from each team to demonstrate the procedure for the game. Tell each child to select a card from the pack and to place it on the side of the flannel board assigned to his team. Explain that below the card they must show a product that represents the count shown on the card. They should use the symbols available on their side of the board to show their product.

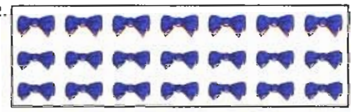


Let the class judge which child completed the assignment correctly and who was first to do so. The children may earn points for their team in the following ways: 1 point is earned for completing the assignment correctly; 1 point is earned for being first to complete the assignment.

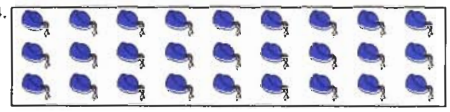
Continue this activity until all of the children have participated. Then have each team total its points and declare a winner.

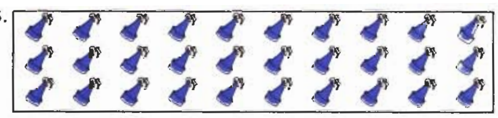
Name

1.  6 × 3 = 18

2.  7 × 3 = 21

3.  8 × 3 = 24

4.  9 × 3 = 27

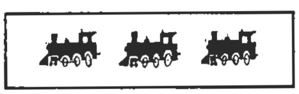
5.  10 × 3 = 30

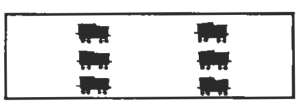
reference page

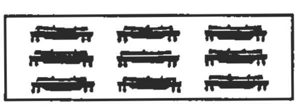
A-45

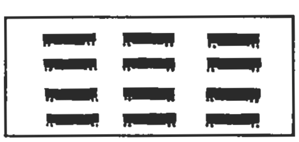
Compute.

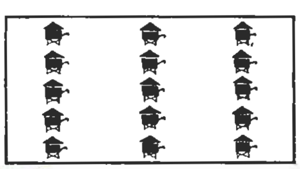
1. 0 × 3 = 0

2.  1 × 3 = 3

3.  2 × 3 = 6

4.  3 × 3 = 9

5.  4 × 3 = 12

6.  5 × 3 = 15

reference page

A-44

Compute.

1. 5 × 3 = 15	2. 4 × 2 = 8	3. 7 × 3 = 21
4. 10 × 3 = 30	5. 6 × 3 = 18	6. 8 × 2 = 16
7. 5 × 2 = 10	8. 9 × 2 = 18	9. 0 × 3 = 0
10. 9 × 3 = 27	11. 2 × 3 = 6	12. 10 × 2 = 20
13. 0 × 2 = 0	14. 4 × 3 = 12	15. 2 × 2 = 4
16. 7 × 2 = 14	17. 8 × 3 = 24	18. 12 × 2 = 24
19. 1 × 3 = 3	20. 6 × 2 = 12	21. 3 × 3 = 9
22. 20 × 2 = 40	23. 12 × 2 = 24	24. 1 × 2 = 2
25. 18 × 2 = 36	26. 3 × 2 = 6	27. 15 × 2 = 30
28. 13 × 2 = 26	29. 14 × 2 = 28	30. 16 × 2 = 32

A-46

Pages 44 through 46

● Pages 44 and 45 provide the children with an opportunity to investigate products that have 3 as one of the factors. Instruct the children to complete each of the equations. Explain that the array in each instance will help them visualize the product and related count. Some children may need to draw pictures for some of the products.

● Page 46 gives the children an opportunity to compute products that have either 2 or 3 as one of the factors. Direct the children to complete each equation. All or part of the page may be assigned at any one time, depending on the ability level of the children.

Supplemental Experiences

■ Make a set of multiplication charts from tagboard or construction paper.

×	3	×	2	×	3
7		6		10	
10		8		0	
8		1		1	
1		10		5	
4		2		9	
0		5		4	
9		9		7	
5		7		2	
3		0		8	
6		4		6	
2		3		3	

Distribute these charts to the children and have them use their plastic numerals to fill in the count for the given products. After each child's chart has been checked, tell the children to exchange cards and repeat the activity.

■ Write a series of correct and incorrect mathematical sentences on the chalkboard.

$3 \times 2 = 8$ $\quad$ $4 \times 2 = 10$
$6 \times 3 = 12$ $\quad$ $5 \times 3 = 15$
$9 \times 2 = 18$ $\quad$ $7 \times 3 = 24$

Ask a child to come to the chalkboard, study the sentence you indicate, and decide whether or not this sentence is correct. If the sentence is correct and the child makes this decision, have him erase the sentence from the chalkboard. If the sentence is incorrect and the child makes this decision, have him use two of the numbers from the sentence to construct a correct sentence. For example, to correct the sentence $3 \times 2 = 8$ the child may decide to use 3 and 2 and construct the correct sentence $3 \times 2 = 6$. He might also decide to use 2 and 8 and construct $4 \times 2 = 8$. When the child has finished, have him erase both the incorrect and correct sentences. Continue in this way until all of the sentences in this series have been used. Then continue the activity using several other series of sentences so as to give every child an opportunity to participate.

KEY IDEA

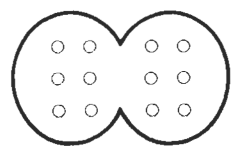
3×4 is twice 3×2.

Scope

To develop multiplication facts through 10×4.
To use the knowledge that 4 is twice 2.

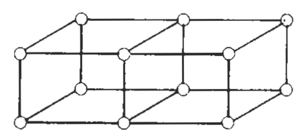
Fundamentals

The array has been used as a model of multiplication. It is important to help the child see physical relationships that can be easily matched with number relationships—this helps the child develop an intuitive understanding of number relationships. Consider a 3 by twice 2 array.



3×4 is twice 3×2

Observe that the array consists of two parts—two 3 by 2 arrays. The whole product is twice one of the partial products. A three-dimensional model illustrates this number relationship.



3×4 is twice 3×2

The product for the three-dimensional model may be expressed as $3 \times 2 \times 2$ or $2 \times 3 \times 2$. Thus $3 \times 2 \times 2 = 2 \times 3 \times 2$. This is an example of the rearrangement property of multiplication—a property illustrated by the three-dimensional model.

By pairing and computing it can be seen that
$3 \times (2 \times 2) = 2 \times (3 \times 2)$ and
3×4 is twice 3×2.

Understanding that 4 is twice 2 will enable the child to compute basic facts such as 7×4, 8×4, and so forth:

7×4 is twice 7×2;
twice 14 is 28.

Readiness for Understanding

Understanding of the array.
Knowledge of basic multiplication facts related to multiples of 2.

Developmental Experiences

for flannel board
felt counters

for each child
counters

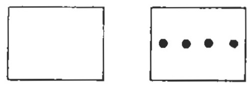
masking tape
pocket chart
felt-tip pen
tagboard cards (3″ × 3″)

▶ Tell the class that you are going to draw on the chalkboard arrays that have rows of 4 members. Explain that you are going to ask someone to write an equation for each array.

Begin with a 0 by 4 array. Ask a child to write an appropriate equation on the chalkboard.

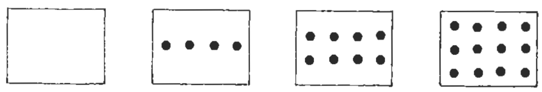
$0 \times 4 = 0$

Next, draw an array that has 1 row of 4 and have a child write the appropriate equation.

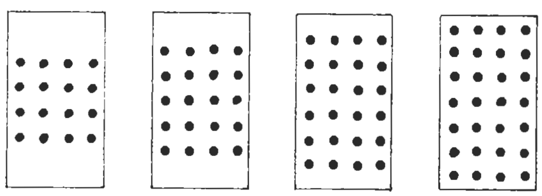


$0 \times 4 = 0$ $\quad 1 \times 4 = 4$

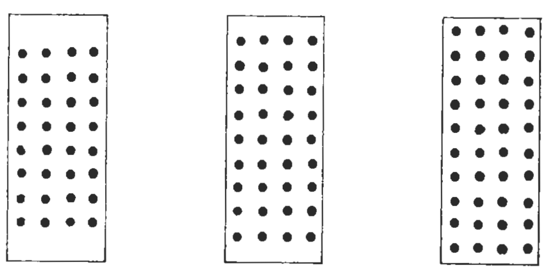
Continue this procedure until a 10 by 4 array has been discussed. Then direct the class to study the products recorded on the chalkboard.



$0 \times 4 = 0 \quad 1 \times 4 = 4 \quad 2 \times 4 = 8 \quad 3 \times 4 = 12$



$4 \times 4 = 16 \quad 5 \times 4 = 20 \quad 6 \times 4 = 24 \quad 7 \times 4 = 28$



$8 \times 4 = 32 \quad 9 \times 4 = 36 \quad 10 \times 4 = 40$

Allow the children to discuss what they discover about this sequence of products. The children may observe that if they read the products in order, from 0×4 through 10×4, each product is 4 greater than the preceding product.

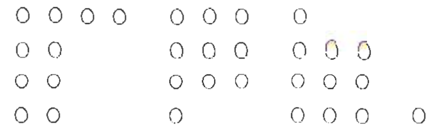
Tell the class to look at the relationship between 1×4 and 2×4. Someone may comment that 2×4 is twice as great as 1×4. Someone else may observe that $(1 \times 4) + (1 \times 4)$ is 2×4. A third child may note that the count for 2×4 is twice the count for 1×4.

Have the class study and discuss the relationship between the following products:

2×4 and 4×4
3×4 and 6×4
4×4 and 8×4
5×4 and 10×4

Allow the children to freely express their observations about each pair of products.

▶ Ask a child to show an 8 by 4 array on the flannel board, and to write the product for this array on the chalkboard. Ask another child to rearrange the members of the array in such a way that the number of tens and ones in this product becomes evident.



Allow the child to choose any way he wishes to make the tens and ones evident. The grouping that is shown above is only one of the many possibilities. Let another child record on the chalkboard the count that corresponds to the array. Direct this child to link the two forms of the given number with an equal sign and to read the equation aloud.

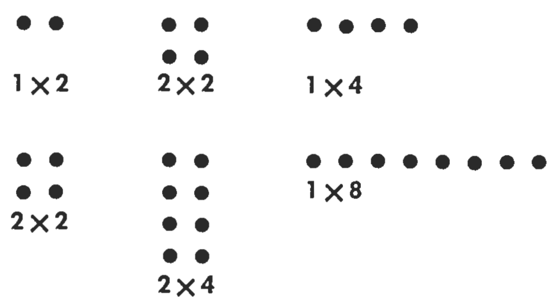
$$8 \times 4 = 32$$

Continue this procedure with the remaining products from 1×4 through 10×4. The children may note that the count for some of the products involves only ones, the count for some of the products involves only tens, and the count for some of the products involves both tens and ones.

▶ In this activity the children will build arrays for products that are twice a given product. Tell the children to use their counters to build a 1 by 2 array on their desks. Then tell the children to build an array that has twice as many counters as the 1 by 2 array. Draw the arrays and write the corresponding products on the chalkboard.

$1 \times 2 \quad 2 \times 2 \quad 1 \times 4$

Tell the children to make a 2 by 2 array and an array that has twice as many counters as the 2 by 2 array. Draw the arrays and write the corresponding products on the chalkboard.

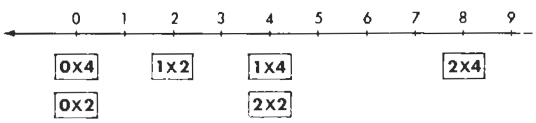


Continue the activity with the arrays 3 by 2, 4 by 2, 5 by 2, . . . 10 by 2. Then write on the chalkboard the products in the arrangement shown.

$1 \times 2, \; 2 \times 2, \; 1 \times 4$
$2 \times 2, \; 2 \times 4, \; 4 \times 2$
$3 \times 2, \; 3 \times 4, \; 6 \times 2$
$4 \times 2, \; 4 \times 4, \; 8 \times 2$
$5 \times 2, \; 5 \times 4, \; 10 \times 2$
.
.
.
$10 \times 2, \; 10 \times 4, \; 20 \times 2$

Tell the children to compare the products. Give them ample time to discuss their ideas with each other. If necessary, guide the children to the conclusion that a product may be doubled by doubling one of the factors.

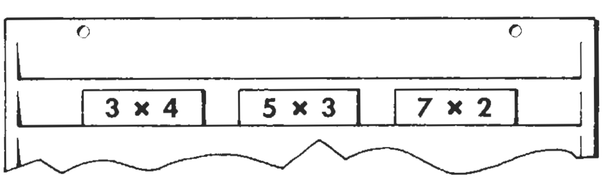
▶ Draw on the chalkboard a segment of the number line. On this line show 0 through 40 at 4-inch intervals. Write the products from 0 × 4 through 10 × 4 on 22 cards—one product for each card. Put these cards in a pack with the product-cards for 0 × 2 through 20 × 2 used in a previous activity. Have several children come forward one at a time, choose a product-card, and place it below the appropriate count on the number line.



After all of the products have been matched to a count on the number line, ask individual children to come to the chalkboard and point to products that meet the following requirements:

a product that is twice 1×2
(2×2 and 1×4),
a product that is twice 2×2
(4×2 and 2×4),
a product that is twice 3×2
(6×2 and 3×4),
a product that is twice 4×2
(8×2 and 4×4),
a product that is twice 5×2
(10×2 and 5×4),
a product that is twice 6×2
(12×2 and 6×4),
a product that is twice 7×2
(14×2 and 7×4),
a product that is twice 8×2
(16×2 and 8×4),
a product that is twice 9×2
(18×2 and 9×4), and
a product that is twice 10×2
(20×2 and 10×4).

▶ Use the number line from the preceding activity. Place three product-cards in the pocket chart: for example, 3 × 4, 5 × 3, and 7 × 2.



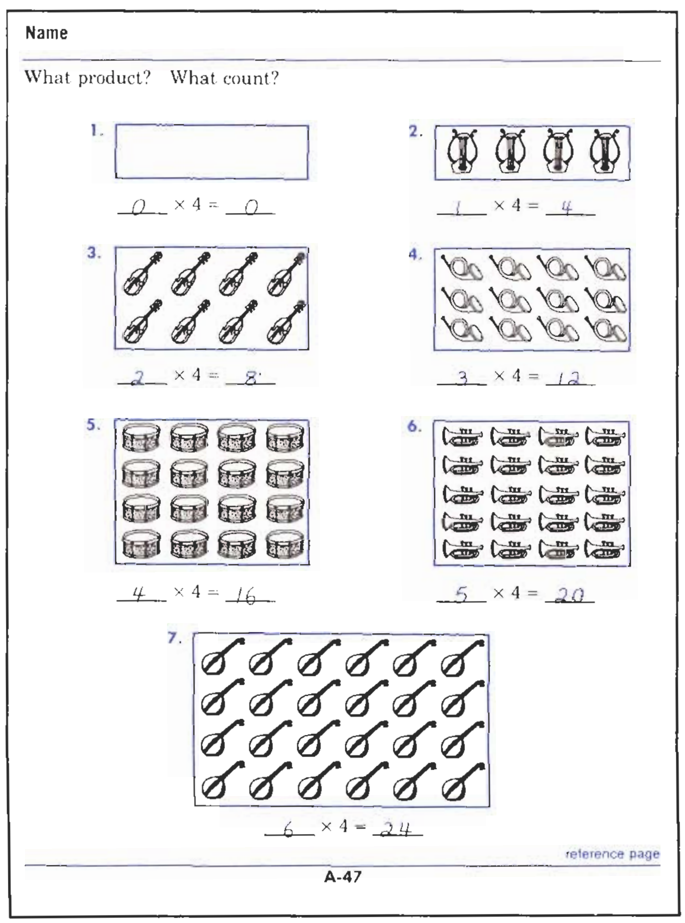
Ask a child to come to the pocket chart and choose products greater than 12. Tell him to hold each product he chose below the appropriate count on the number line to verify his selection.

Put three other products in the pocket chart and ask another child to select those that are greater than 24. Either all three, two, one, or none of these products may meet the stated requirement.

Continue the activity; give other children an opportunity to decide whether or not given products are greater than a particular count. Use the products from 0 × 2 through 20 × 2, 0 × 3 through 10 × 3, and 0 × 4 through 10 × 4.

Pages 47 through 50

● Pages 47 and 48 give the children practice computing products that have 4 as one of the factors. Instruct the children to complete each of the equations. Explain that the array in each exercise will help them visualize the product and related count.


Name

What product? What count?

1. 0 × 4 = 0

2. 1 × 4 = 4

3. 2 × 4 = 8

4. 3 × 4 = 12

5. 4 × 4 = 16

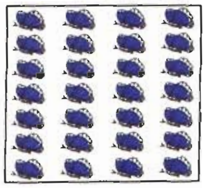
6. 5 × 4 = 20

7. 6 × 4 = 24

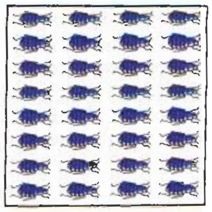
reference page

A-47

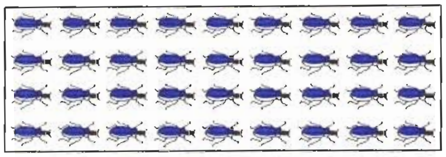
What product? What count?



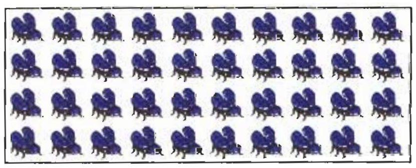
1. 7 × 4 = 28



2. 8 × 4 = 32



3. 9 × 4 = 36



4. 10 × 4 = 40

reference page

Jean and Tom went to the school fair. Jean saw an array of 12 fishbowls. Tom saw an array of 12 bowling pins.

What product is shown by the array of fishbowls? 3×4 or 4×3

What product is shown by the array of bowling pins? 2×6 or 6×2

Write a product for each number.

1. 16: 16×1, 8×2, 4×4 *
2. 20: 20×1, 10×2, 5×4 *
3. 36: 36×1, 18×2, 9×4, 12×3 *
4. 32: 32×1, 16×2, 8×4 *
5. 27: 27×1, 9×3 *
6. 24: 24×1, 12×2, 6×4, 8×3 *

*Answers will vary.

Name

Compute.

1.	0 × 2 = 0	13.	6 × 2 = 12
2.	0 × 4 = 0	14.	6 × 4 = 24
3.	1 × 2 = 2	15.	7 × 2 = 14
4.	1 × 4 = 4	16.	7 × 4 = 28
5.	2 × 2 = 4	17.	8 × 2 = 16
6.	2 × 4 = 8	18.	8 × 4 = 32
7.	3 × 2 = 6	19.	9 × 2 = 18
8.	3 × 4 = 12	20.	9 × 4 = 36
9.	4 × 2 = 8	21.	10 × 2 = 20
10.	4 × 4 = 16	22.	10 × 4 = 40
11.	5 × 2 = 10	23.	11 × 2 = 22
12.	5 × 4 = 20	24.	11 × 4 = 44

● Page 49 provides the children with an opportunity to observe the fact that multiplying a number by 4 results in a product twice as great as when that same number is multiplied by 2: $n \times 4 = 2 \times (n \times 2)$. Direct the children to complete each of the equations.

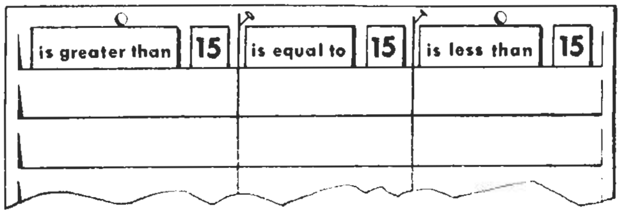
● Page 50 enables the children to observe the fact that a given number may be expressed as a product—often in more than one way. Have the children use counters on their desks to construct an array that corresponds to the first count (16). Tell the children to write an equation that links the given count and the product for their array. Then have the children try to rearrange the members of the array to show a different product. If the children find that they can do this, have them write a second equation that links the given count and the product for the array now before them. For example, for 16 the following equations may be used:

$$4 \times 4 = 16$$
$$8 \times 2 = 16$$
$$2 \times 8 = 16$$
$$16 \times 1 = 16$$
$$1 \times 16 = 16$$

Tell the children to work the same way with the other 5 numbers on the page. After the children have finished, ask individuals to name the products they have listed for each count.

Supplemental Experience

■ Distribute the product-cards for 0 × 2 through 20 × 2, 0 × 3 through 10 × 3, and 0 × 4 through 10 × 4. Set up the pocket chart as shown.



Ask various children to come forward one at a time to compare their products with 15 and to place their card below the appropriate phrases in the chart. When all of the products have been compared with 15, redistribute the product-cards and replace the cards showing 15 with cards showing 28. Have all the products compared with 28.

Continue in this way; have the children compare their products with 20, 24, 12, 18, and 36.

KEY IDEA

0 × □ = 0.

Scope

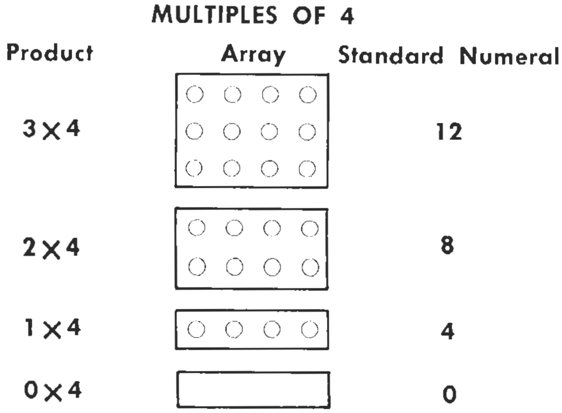
To discuss zero as a factor.

Fundamentals

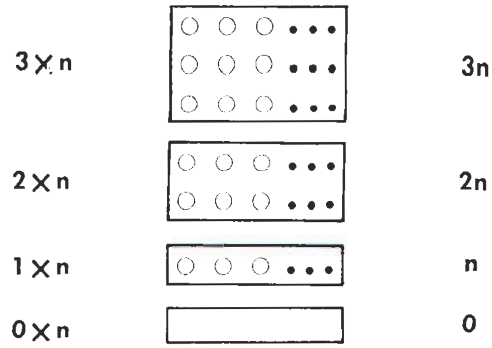
To illustrate a 0 by 4 array, it is necessary to appeal to the imagination and suggest that the array within the frame is a 0 by 4 array.

The same illustration fits the arrays 0 by 1, 0 by 2, and 0 by n; this suggests that there is only one product for all such arrays. The following descending pattern of multiples and corresponding arrays helps visualize the product 0 × n.

MULTIPLES OF 4



MULTIPLES OF n



Since the 0 by n array represents the product of any whole number and zero, the intuitive conclusion is that 0×□ is 0. This conclusion fits neatly into the structure of arithmetic by providing the desired completeness to the closure property of multiplication.

Readiness for Understanding

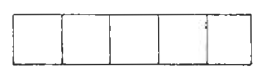
Understanding of the concept of product.

Developmental Experiences

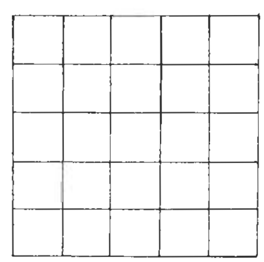
for flannel board
5 tagboard strips: ($1\frac{1}{2}'' \times 7\frac{1}{2}''$)
4 tagboard strips: ($1\frac{1}{2}'' \times 4\frac{1}{2}''$)
felt numerals and equal signs
product-cards
felt-tip pen

for each child
counters

▶ Cut 5 tagboard strips ($1\frac{1}{2}$ by $7\frac{1}{2}$ inches)—each strip partitioned into five squares—and prepare them for use on the flannel board.



Construct a 5 by 5 array with the strips. Ask the children what product is shown by the array. (5 × 5)



Write 5 × 5 = 25 on the chalkboard. Tell the class to check the array to be sure that 25 small squares are shown.

Remove one of the strips. Ask the class to tell the product for the array. (5 × 4 or 4 × 5) Write on the chalkboard 5 × 4 = □ and ask a child to complete the equation. Help this child compute 5 × 4 by encouraging him to look for tens in the array.

5 × 5 = 25
5 × 4 = □

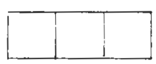
Continue removing strips, asking for the product, and completing equations as before. To compute the products, the children should look for tens and ones in the arrays.

$$5 \times 5 = 25$$
$$5 \times 4 = 20$$
$$5 \times 3 = 15$$
$$5 \times 2 = 10$$
$$5 \times 1 = 5$$

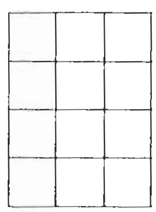
After completing the equation for the 5 by 1 array, remove the last strip. Tell the class to imagine that they are looking at a 5 by 0 array. Write $5 \times 0 = \square$ on the chalkboard. Ask how many squares are in the 5 by 0 array. (0) Complete the equation.

$$5 \times 0 = 0$$

▶ Cut 4 tagboard strips ($1\frac{1}{2}$ by $4\frac{1}{2}$ inches)—each strip partitioned into 3 squares—and prepare them for use on the flannel board.



Construct a 3 by 4 array with the strips. Ask the children what product is shown by the array. (4×3)



Write $3 \times 4 = \square$ on the board. Ask a child to complete the equation.

Remove one of the strips. Ask the class to name the product for the array. (3×3) Write $3 \times 3 = \square$ on the board and ask a child to complete the equation.

$$3 \times 4 = 12$$
$$3 \times 3 = \square$$

Continue removing the strips, asking for the product, and completing equations as before.

$$3 \times 4 = 12$$
$$3 \times 3 = 9$$
$$3 \times 2 = 6$$
$$3 \times 1 = 3$$

After completing $3 \times 1 = 3$, remove the last strip. Tell the children they are looking at a 3 by 0 array. Write $3 \times 0 = \square$ on the board. Ask how many squares are in the 3 by 0 array. (0) Complete the equation.

$$3 \times 0 = 0$$

Tell the children to look for a pattern in the equations that suggests $3 \times 0 = 0$. (Each product is 3 less than the preceding product.) Some children may see the pattern and realize that 3×0 must be 3 less than 3×1. Thus, $3 \times 0 = 0$.

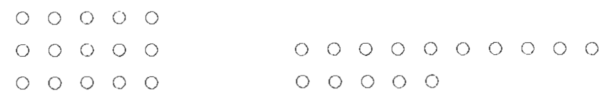
▶ Use arrays to help the children compute 1×5, 2×5, 3×5, . . . through 10×5. Each child (or each pair of children) should have at least 50 counters.

Tell the children to make a 2 by 5 array. Ask how many counters are in the array. (10) Write $2 \times 5 = 10$ on the chalkboard.

Direct the children to make a 4 by 5 array. Ask how many counters are in the array. (20 or 2 tens) Encourage the children to look for tens. Write $4 \times 5 = 20$ on the chalkboard.

Continue this procedure using 6 by 5, 8 by 5, and 10 by 5 arrays.

Follow the same procedure using 1 by 5, 3 by 5, 5 by 5, 7 by 5, and 9 by 5 arrays. As the children compute 1×5, 3×5, 5×5, 7×5, and 9×5, tell them to rearrange the members in the arrays to show the tens and the ones.

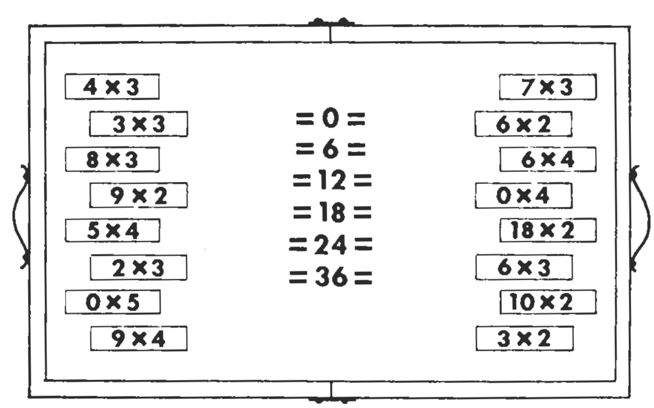


3×5 **10+5=15**

Write on the chalkboard the products and their counts in order.

$1 \times 5 = 5$	$4 \times 5 = 20$	$8 \times 5 = 40$
$2 \times 5 = 10$	$5 \times 5 = 25$	$9 \times 5 = 45$
$3 \times 5 = 15$	$6 \times 5 = 30$	$10 \times 5 = 50$
	$7 \times 5 = 35$	

▶ Place equal signs and the numerals 0, 6, 12, 18, 24, and 36 on the flannel board as shown. Place product-cards for 4×3, 3×3, 8×3, 9×2, 5×4, 2×3, 0×5, and 9×4 on one side of the flannel board. Place the product-cards for 7×3, 6×2, 6×4, 0×4, 18×2, 6×3, 10×2, and 3×2 on the other side of the flannel board.



Call two children to the flannel board. Explain that one child is to match the products on the left with the counts in the center; the other child must match the products on the right with the counts in the center. Tell them to remove any card that does not match. (3×3, 5×4, 7×3, 10×2)

After the matching is completed, some children may want to discuss several of the results. The children may need to see arrays to understand the results.

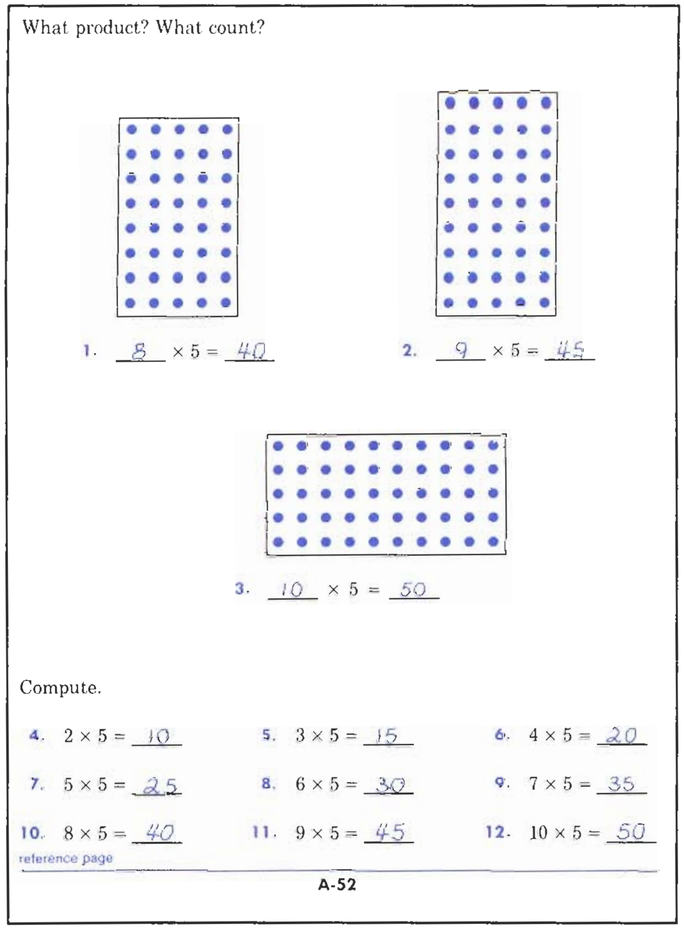
Give other pairs of children the opportunity to match products with counts. Use these products.

$0 \times 2,\ 1 \times 2,\ 2 \times 2,\ 3 \times 2,\ \ldots\ 10 \times 2$
$0 \times 3,\ 1 \times 3,\ 2 \times 3,\ 3 \times 3,\ \ldots\ 10 \times 3$
$0 \times 4,\ 1 \times 4,\ 2 \times 4,\ 3 \times 4,\ \ldots\ 10 \times 4$
$0 \times 5,\ 1 \times 5,\ 2 \times 5,\ 3 \times 5,\ \ldots\ 10 \times 5$

Pages 51 through 53

● Pages 51 and 52 provide the children with an opportunity to compute products that have 5 as one of the factors. Have the children complete each of the equations.

● Page 53 may be used for review or as a self test. The children are to compute products from 1 × 2 through 10 × 2, 1 × 3 through 10 × 3, 1 × 4 through 10 × 4, and 1 × 5 through 10 × 5. Direct the children to complete each equation. It is not necessary to have the children complete the whole page in one period. The length of the assignment will depend on the ability level of the children.



What product? What count?

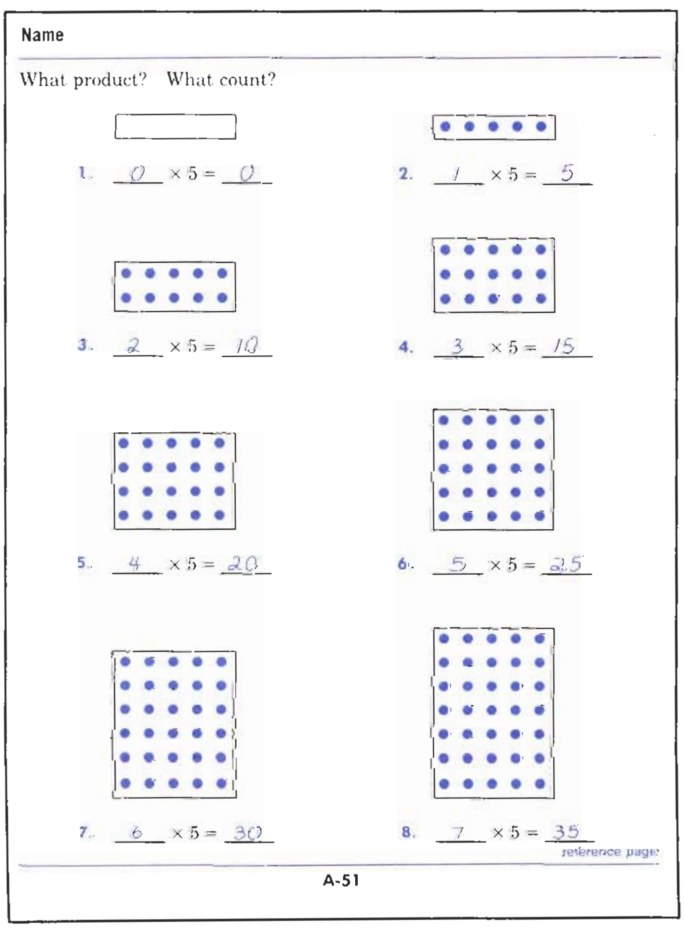
1. 8 × 5 = 40
2. 9 × 5 = 45
3. 10 × 5 = 50

Compute.

4. 2 × 5 = 10 5. 3 × 5 = 15 6. 4 × 5 = 20
7. 5 × 5 = 25 8. 6 × 5 = 30 9. 7 × 5 = 35
10. 8 × 5 = 40 11. 9 × 5 = 45 12. 10 × 5 = 50

reference page

A-52



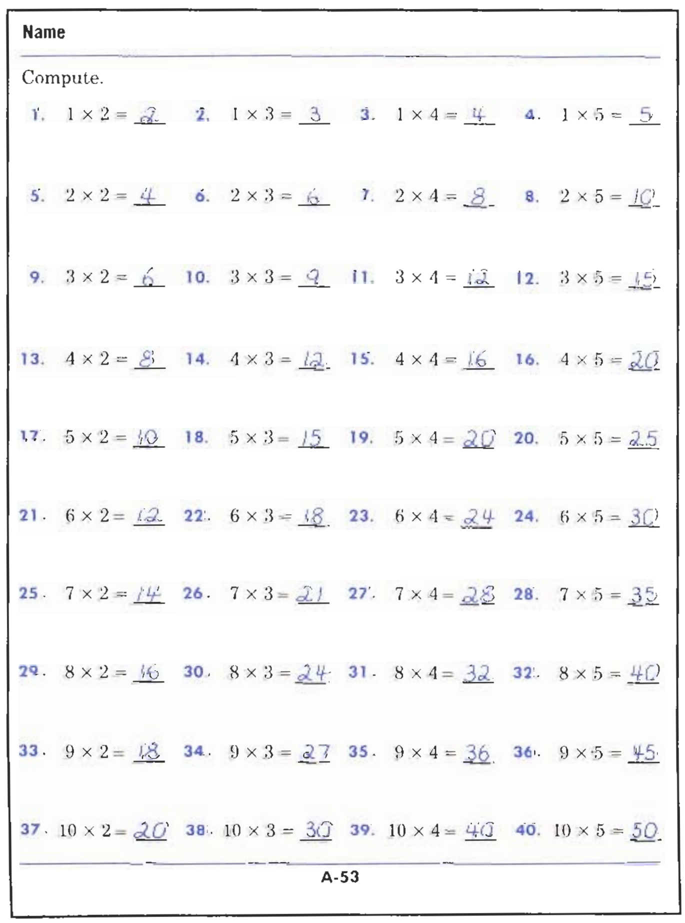
Name

What product? What count?

1. 0 × 5 = 0
2. 1 × 5 = 5
3. 2 × 5 = 10
4. 3 × 5 = 15
5. 4 × 5 = 20
6. 5 × 5 = 25
7. 6 × 5 = 30
8. 7 × 5 = 35

reference page

A-51



Name

Compute.

1. 1 × 2 = 2 2. 1 × 3 = 3 3. 1 × 4 = 4 4. 1 × 5 = 5
5. 2 × 2 = 4 6. 2 × 3 = 6 7. 2 × 4 = 8 8. 2 × 5 = 10
9. 3 × 2 = 6 10. 3 × 3 = 9 11. 3 × 4 = 12 12. 3 × 5 = 15
13. 4 × 2 = 8 14. 4 × 3 = 12 15. 4 × 4 = 16 16. 4 × 5 = 20
17. 5 × 2 = 10 18. 5 × 3 = 15 19. 5 × 4 = 20 20. 5 × 5 = 25
21. 6 × 2 = 12 22. 6 × 3 = 18 23. 6 × 4 = 24 24. 6 × 5 = 30
25. 7 × 2 = 14 26. 7 × 3 = 21 27. 7 × 4 = 28 28. 7 × 5 = 35
29. 8 × 2 = 16 30. 8 × 3 = 24 31. 8 × 4 = 32 32. 8 × 5 = 40
33. 9 × 2 = 18 34. 9 × 3 = 27 35. 9 × 4 = 36 36. 9 × 5 = 45
37. 10 × 2 = 20 38. 10 × 3 = 30 39. 10 × 4 = 40 40. 10 × 5 = 50

A-53

Supplemental Experience

■ Give each child a sheet of 12 by 15 inch construction paper that has been partitioned into $\frac{1}{2}$-inch squares. Show the children how to cut their 10 by 25 array into five 10 by 5 arrays. Have them cut from one 10 by 5 array a 1 by 5 array and a 9 by 5 array. Have them cut from another 10 by 5 array a 2 by 5 array and an 8 by 5 array. Have them cut from the remaining 10 by 5 arrays a 3 by 5 array and a 7 by 5 array, a 4 by 5 array and 6 by 5 array, and two 5 by 5 arrays.

Give each child an 18 by 24 inch sheet of colored construction paper. Tell the children to arrange their arrays on the paper in any pattern they choose and then paste them down. Give each child a sheet of writing paper. Direct the children to write an appropriate equation for each array, to cut out the equations, and to paste them beside the appropriate arrays. The illustration shows just one of the many ways the children may display their arrays.

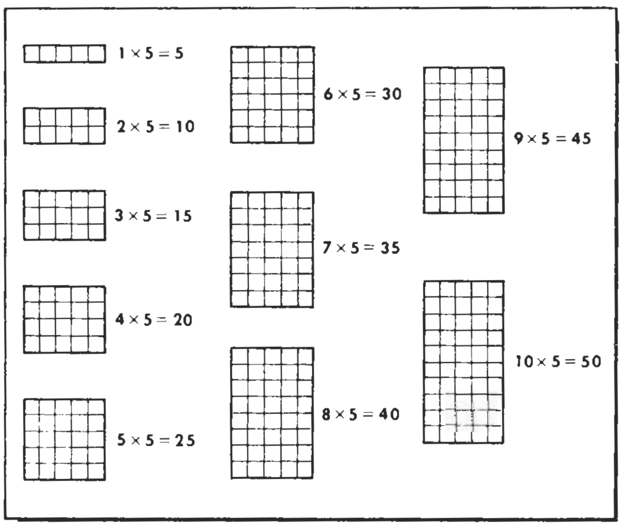


Exhibit the completed display of arrays on a bulletin board.

KEY IDEA

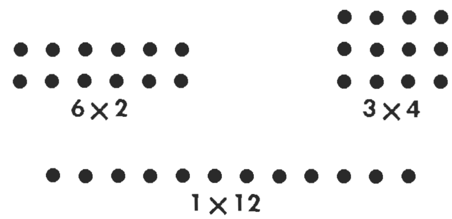
$12 \times 2 = 6 \times 4 = 8 \times 3.$

Scope

To extend the concept of many names for a number.

Fundamentals

A number may be denoted in many ways. It is clear that $1+2$, $2+1$, $5-2$, and $6-3$ represent the same number. The concept of many names for the same number is extended by rearranging an array to form different arrays that have the same product.



$6 \times 2 = 3 \times 4 = 1 \times 12$

Readiness for Understanding

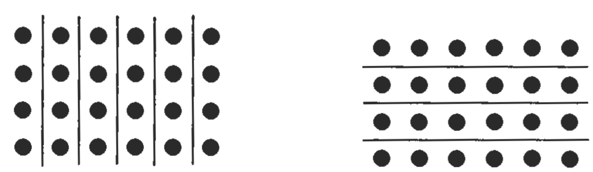
Understanding of the concept of product.

Developmental Experiences

product-cards
masking tape
chart paper
felt-tip pen
pointers
count-cards
black crayon
pocket chart
bead frame

for each child
plastic numerals and symbols

▶ Draw two 6 by 4 arrays on the chalkboard. Draw lines in one array to show 6 rows with 4 members in each row. Draw lines in the other array to show 4 rows with 6 members in each row.



Ask a child to write the product for the arrays. (6×4) Below the product, have a second child write a sum that represents the partitioning of the first array. Below this sum, have a third child write a sum that represents the partitioning of the second array.

$$6 \times 4$$
$$4 + 4 + 4 + 4 + 4 + 4$$
$$6 + 6 + 6 + 6$$

Tell the children to compute 6×4 by computing the sums.

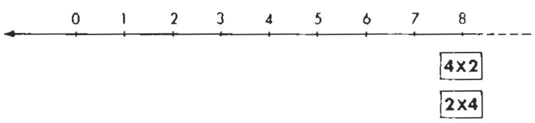
Adapt this procedure to other products that have been introduced up to this time in the program.

▶ Draw 2 segments of the number line on the chalkboard. Show 0 through 24 on one segment and 25 through 50 on the other. Use 6-inch intervals. Distribute the following product-cards to the children.

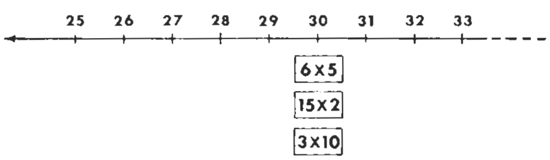
$0 \times 2,\ 1 \times 2,\ 2 \times 2,\ \ldots\ 20 \times 2$
$0 \times 3,\ 1 \times 3,\ 2 \times 3,\ \ldots\ 10 \times 3$
$0 \times 4,\ 1 \times 4,\ 2 \times 4,\ \ldots\ 10 \times 4$
$0 \times 5,\ 1 \times 5,\ 2 \times 5,\ \ldots\ 10 \times 5$

If necessary, give more than 1 card to a child.

Tell the children that anyone holding a product for 8 should place his product below the 8 on the number line. Help each child fasten his card to the board with a ring of masking tape.

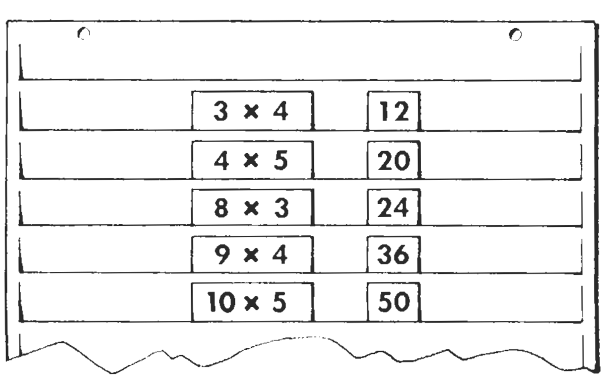


Next, have all of the children with the product 30 place their product below the 30 on the number line.



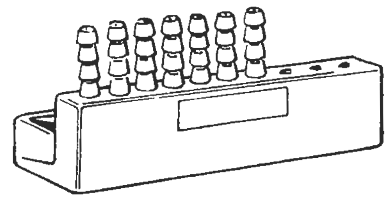
Continue to match the products with numbers on the number line until every product is used. The children may note that some numbers on the number line are not represented by any of the products.

▶ Place 5 product-cards in random order in the pocket chart. Use products no greater than 50 that have 2, 3, 4, or 5 as a factor. Direct a child to rearrange the cards so that the numbers are ordered from least to greatest. Have a second child place the appropriate count-card beside each product.



Repeat the activity several times using other products no greater than 50.

▶ Construct a 7 by 4 array on the bead frame.



Have the children use their plastic symbols and numerals to show an equation for the 7 by 4 array that links the product to the count. Ask one of the children to write his equation on the board and to write a second equation using the commuted form of the given product.

$$7 \times 4 = 28$$
$$4 \times 7 = 28$$

Adapt this procedure to other products from 0×2 through 10×5.

▶ Write the following stories on the chalkboard.

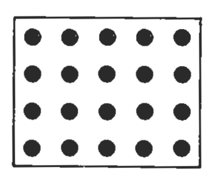
During recess some of the third-grade boys had a ball game. Four boys on Tim's team each hit 5 homeruns. How many homeruns were hit by Tim's team?

Six children from Miss Lamp's class went to the bookstore. Each child bought 3 pencils. How many pencils did these children buy?

Nell is playing jacks. Each time she bounces the ball, Nell picks up 2 jacks. If Nell bounces the ball 7 times, what is the total number of jacks she will pick up?

Nan invited 8 friends to her home for lunch. Each of these friends ate 3 sandwiches. How many sandwiches did Nan's friends eat all together?

Direct the children to read the first story to themselves. Ask a child to come to the chalkboard; help him draw an array that will illustrate the number involved in the question in this story. For example, 4×5 homeruns were hit.



Have the other children use their plastic numerals and symbols to show an equation that links the product and the count for the given array. Ask a child to answer the question in the story in terms of the product shown in his equation. Call on another child to answer the question in the story in terms of the count shown in his equation. Have a third child read his equation aloud.

Adapt this procedure to the other three stories.

In each row draw a ring around those products which are the same.

1.	10 × 2	5 × 4	7 × 3
2.	8 × 3	5 × 5	6 × 4
3.	12 × 2	8 × 3	6 × 4
4.	10 × 3	15 × 2	6 × 5
5.	11 × 2	7 × 3	4 × 5
6.	5 × 2	3 × 3	2 × 5
7.	4 × 3	6 × 2	3 × 5
8.	6 × 3	8 × 2	4 × 4
9.	9 × 2	7 × 4	6 × 3
10.	8 × 5	10 × 4	20 × 2

A-54

Name

Compute.

1. 2 × 8 = 16	2. 2 × 11 = 22	3. 5 × 3 = 15
4. 3 × 7 = 21	5. 3 × 10 = 30	6. 4 × 4 = 16
7. 4 × 6 = 24	8. 4 × 9 = 36	9. 4 × 10 = 40
10. 5 × 5 = 25	11. 5 × 8 = 40	12. 2 × 6 = 12
13. 5 × 9 = 45	14. 5 × 1 = 5	15. 7 × 5 = 35
16. 4 × 8 = 32	17. 4 × 2 = 8	18. 5 × 10 = 50
19. 8 × 5 = 40	20. 3 × 3 = 9	21. 4 × 7 = 28
22. 2 × 7 = 14	23. 3 × 9 = 27	24. 5 × 7 = 35
25. 3 × 6 = 18	26. 1 × 5 = 5	27. 3 × 8 = 24
28. 4 × 5 = 20	29. 5 × 6 = 30	30. 2 × 9 = 18
31. 5 × 4 = 20	32. 3 × 9 = 27	33. 4 × 10 = 40

A-55

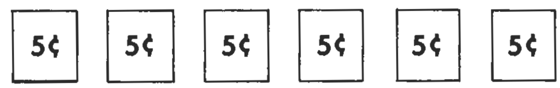
Pages 54 through 60

● Page 54 gives the children additional experience in recognizing equivalent products. Work the first two exercises with the class to be sure the children understand what they are to do.

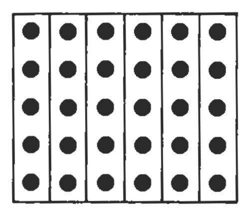
Direct the children to complete the remaining exercises independently. Some of the children may find it helpful to use their counters to make their decision regarding the products.

● Page 55 provides practice in computing products through 50 where 2, 3, 4, or 5 is one of the factors. Have the children complete each of the equations. The children need not do all of the exercises in one class session. The length of each assignment will vary with the ability of the class.

● Page 56 provides further experience with simple stories involving numbers. Read the first story with the class. Be sure the children understand the question. Let someone come to the board and explain how he decided on the answer to the question. The child may draw an illustration similar to the following and say that he knows 6×5 is 30, or say that he knows 5+5+5+5+5+5 is 30.



Tell the child to write an equation that shows his form of computation. He may write 6 × 5 = 30 or 5+5+5+5+5+5 = 30. Ask whether anyone has another way to describe the situation. A second child may volunteer to show an illustration similar to the following, and say that this helps him think 6 × 5 = 30.



Instruct the children to write the answer to the question in exercise 1. Children may write a complete sentence or just the amount of money. If desired, the computation may be recorded. Assign the remaining exercises as independent work. When the class has completed the page, have individual children show and explain how they arrived at the answers.

Answer the questions.

1. At the post office Dick bought six 5¢ stamps. How much did he spend? 30¢
2. Jim had 3 postcards to mail. He bought a 5¢ stamp for each card. How much did the stamps cost? 15¢
3. Jim also bought four 8¢ stamps. How much did he spend for them? 32¢
4. Jane sent a gift to each of her 5 friends while she was at summer camp. She spent 12¢ to send each gift. How much did Jane spend? 60¢
5. Bob wanted 12 new stamps for his collection. The stamps cost 2¢ each. How much money did he need? 24¢
6. Don bought nine 3¢ stamps. How much did he spend? 27¢
7. Ann sent 5 letters by airmail. It cost 10¢ to send each letter. How much did it cost to send the 5 letters? 50¢
8. Beth bought 8 stamped envelopes. They were 8¢ each. How much did she spend? 64¢

A-56

Name

In each row, write the products in order from least to greatest.

1. 3 × 4, 8 × 4, 5 × 4, 0 × 4
   0×4, 3×4, 5×4, 8×4
2. 4 × 2, 2 × 3, 5 × 2, 3 × 4
   2×3, 4×2, 5×2, 4×3
3. 7 × 3, 6 × 4, 5 × 4, 11 × 2
   5×4, 7×3, 11×2, 6×4
4. 10 × 4, 8 × 4, 6 × 5, 9 × 4
   6×5, 8×4, 9×4, 10×4
5. 8 × 2, 6 × 3, 8 × 5, 7 × 5
   8×2, 6×3, 7×5, 8×5
6. 3 × 5, 4 × 3, 4 × 4, 6 × 3
   4×3, 3×5, 4×4, 6×3
7. 7 × 4, 8 × 3, 5 × 4, 9 × 3
   5×4, 8×3, 9×3, 7×4
8. 9 × 2, 5 × 3, 7 × 2, 4 × 4
   7×2, 5×3, 4×4, 9×2
9. 3 × 3, 6 × 2, 2 × 4, 3 × 2
   3×2, 2×4, 3×3, 6×2
10. 5 × 5, 10 × 3, 0 × 5, 9 × 3
    0×5, 5×5, 9×3, 10×3

A-57

● Page 57 provides practice in computing products no greater than 50. Work the first exercise with the class. Have the children compute each product. Ask a child to record his responses on the chalkboard: 12, 32, 20, 0. Then ask another child to come to the board to rewrite these numbers so that they are ordered from least to greatest: 0, 12, 20, 32. Some child may comment that he didn't have to compute these products to put them in order from least to greatest. He may say that he knows 0 fours is less than 3, 5, or 8 fours; that 3 fours is less than 5 or 8 fours; that 5 fours is less than 8 fours. Assign the exercises to be completed independently. For each exercise, the children may record the computed products and then rewrite the products in order from least to greatest. They are also correct if they write the products in order: 0 × 4, 3 × 4, 5 × 4, and 8 × 4. When the children are finished, ask individual children to tell what they were thinking as they put these products in order.

● Page 58 can be used by the children to test their ability to compute products through 50. Tell the children to complete each of the equations. Assign only as many exercises at a given time as the children are capable of completing.

Compute.

1. 2 × 10 = 20	2. 3 × 4 = 12	3. 2 × 3 = 6
4. 5 × 2 = 10	5. 3 × 2 = 6	6. 4 × 3 = 12
7. 2 × 4 = 8	8. 3 × 3 = 9	9. 4 × 6 = 24
10. 2 × 2 = 4	11. 3 × 6 = 18	12. 4 × 9 = 36
13. 2 × 6 = 12	14. 3 × 9 = 27	15. 3 × 1 = 3
16. 5 × 5 = 25	17. 2 × 7 = 14	18. 4 × 5 = 20
19. 5 × 3 = 15	20. 2 × 1 = 2	21. 4 × 2 = 8
22. 5 × 8 = 40	23. 3 × 5 = 15	24. 4 × 7 = 28
25. 4 × 8 = 32	26. 2 × 8 = 16	27. 5 × 4 = 20
28. 2 × 5 = 10	29. 4 × 1 = 4	30. 5 × 6 = 30
31. 5 × 9 = 45	32. 3 × 8 = 24	33. 5 × 10 = 50
34. 5 × 0 = 0	35. 4 × 0 = 0	36. 2 × 0 = 0

A-58

Name

Compute.

×	0	1	2	3	4	5
0	0 × 0 0	0 × 1 0	0 × 2 0	0 × 3 0	0 × 4 0	0 × 5 0
1	1 × 0 0	1 × 1 1	1 × 2 2	1 × 3 3	1 × 4 4	1 × 5 5
2	2 × 0 0	2 × 1 2	2 × 2 4	2 × 3 6	2 × 4 8	2 × 5 10
3	3 × 0 0	3 × 1 3	3 × 2 6	3 × 3 9	3 × 4 12	3 × 5 15
4	4 × 0 0	4 × 1 4	4 × 2 8	4 × 3 12	4 × 4 16	4 × 5 20
5	5 × 0 0	5 × 1 5	5 × 2 10	5 × 3 15	5 × 4 20	5 × 5 25
6	6 × 0 0	6 × 1 6	6 × 2 12	6 × 3 18	6 × 4 24	6 × 5 30
7	7 × 0 0	7 × 1 7	7 × 2 14	7 × 3 21	7 × 4 28	7 × 5 35
8	8 × 0 0	8 × 1 8	8 × 2 16	8 × 3 24	8 × 4 32	8 × 5 40
9	9 × 0 0	9 × 1 9	9 × 2 18	9 × 3 27	9 × 4 36	9 × 5 45
10	10 × 0 0	10 × 1 10	10 × 2 20	10 × 3 30	10 × 4 40	10 × 5 50

reference page

A-59

~~5 × 2~~	2 × 12	4 × 7	2 × 3	2 × 4
4 × 3	7 × 5	7 × 2	4 × 4	8 × 2
8 × 4	9 × 3	6 × 4	7 × 3	5 × 6
3 × 6	4 × 5	9 × 5	5 × 5	6 × 6
8 × 3	2 × 6	3 × 3	5 × 8	9 × 2

Write each of the given products below the standard numeral.

10 5 × 2	8 2×4	9 3×3	6 2×3
12 4×3 2×6	14 7×2	16 4×4 8×2	18 3×6 9×2
20 4×5	21 7×3	24 8×3 6×4 2×12	25 5×5
27 9×3	28 4×7	30 5×6	32 8×4
35 7×5	36 6×6	40 5×8	45 9×5

A-60

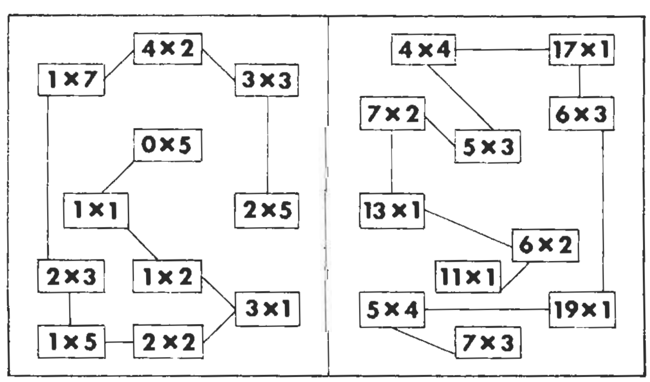
● Use the multiplication chart on page 59 to review products of 0 × 0 through 10 × 5. Explain to the children that they are to record computed products for the products in the boxes of the chart. Have someone tell the computed product for zero times zero. Explain that this product is the number of members in an array that has zero rows and zero members in each row.

Tell each child to put his right index finger on the 0 in the first column and his left index finger on the 0 in the first row. Have them move these two fingers together until they meet in the first empty box. Tell the children to observe the product that is written in the box and then write the computed product beneath it. After discussing the computed products which are already written in the chart (2, 10, and 32), have the children complete the chart independently.

● Page 60 provides additional experience with products from 0 × 0 through 5 × 10. Explain to the children that under each standard numeral they are to write the appropriate product or products from the list at the top of the page. Discuss exercise 1 with the class. If necessary, work through a few more exercises before assigning the rest of the page as independent work. Tell the class that they may draw a line through each product when they have used it.

Supplemental Experiences

■ Write on 1½ by 3 inch cards products for 0 through 50. Use only those products that have either 0, 1, 2, 3, 4, or 5 as a factor. Put a small ring of masking tape on the back of each card so that it can be fastened to the chalkboard. Separate the class into two teams and assign one panel of the chalkboard to each team. Fasten the product-cards for 0 through 10 to one panel of the board; fasten the product-cards for 11 through 21 to the other. Explain to the children that at a given signal a member from one team will begin with 0 and draw a line to connect the products in his panel in order from 0 through 10. Meanwhile a member of the other team will begin with 11 and connect the products in his panel in order from 11 through 21. Have the two children come to the board and carry out the assignments.



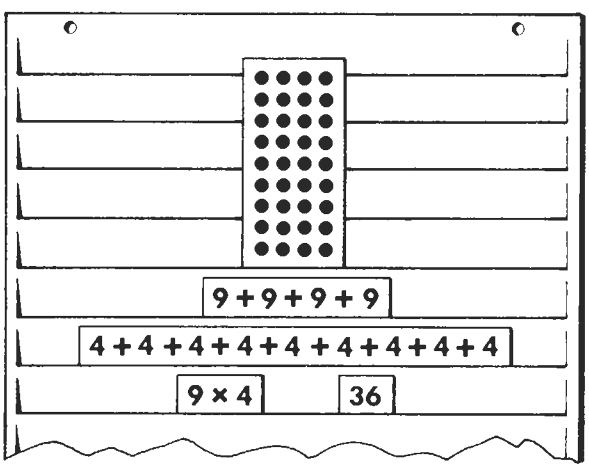
After a child has completed his work, tell him to check the accuracy by naming the counts for his products. For example, the child who ordered the products for 0 through 10 may begin by pointing to 0×5, saying 0×5 is 0. He may continue by pointing to 1×1, saying 1×1 is 1; pointing to 1×2, saying 1×2 is 2; and so forth through 2×5.

The children may earn points for their team in the following ways: 1 point is earned for completing the assignment correctly, and 1 point is earned for being first to complete the assignment.

Play several rounds of the game; use different sets of 11 cards for each round. Continue until all of the children have had an opportunity to participate. Then let each team total its points to determine a winner.

■ Play "Who Can Be My Partner?" You will need a set of six array-cards such as the following: a 4 by 3 array, a 5 by 3 array, a 7 by 4 array, an 8 by 5 array, a 9 by 4 array, and a 9 by 5 array. You will also need product-cards, sum-cards, and count-cards that match the six arrays you have selected. For example, for the 9 by 4 array you will need the product 9×4, the sum $4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4$, the sum $9 + 9 + 9 + 9$, and the count 36.

Distribute the cards to the children. Ask a child with an array to place his card in the pocket chart. Have all of the children who think that they have the product, sum, or count represented by the array raise their hand. Let each of them place his card below the array and explain why he believes his number matches the array.



Continue in this way using the other five arrays. Adapt this procedure to other sets of six arrays that represent products previously developed.

UNIT 6
ADDITION AND SUBTRACTION FACTS

Pages 61 Through 80

OBJECTIVE

To review upper decade addition and subtraction facts in terms of tens, hundreds, and thousands.

The child takes another look at the relationship between sum and difference. He extends his knowledge of number relationships and his computational skills by using new sums and differences that use the upper decade addition facts. As this is done, the child again approaches subtraction as another way of looking at addition.

He reviews the computation of sums and differences; he uses the idea that the sums and differences he knows work for ones, tens, hundreds, and thousands.

See Key Topics in Mathematics for the Primary Teacher: Addition; and The Inverse Operations—Subtraction and Division.

KEY IDEAS

10 is the master key.

Tens plus tens is tens. Hundreds plus hundreds is hundreds. Thousands plus thousands is thousands.

Addend plus difference is sum.

$15 - 8 = 10 - 8 + 5$.

Practice makes perfect.

CONCEPTS

addend	minus (−)
difference	plus (+)
inverse operation	sum

KEY IDEA

10 is the master key.

Scope

To use 5 + 5 and 10 + 10 in upper decade addition computation.

Fundamentals

In this section the child extends his knowledge of the role of 10 in the Hindu-Arabic numeration system as he reviews the computation of upper decade sums. In the second grade, he developed all of the sums of 11 through 18 by using the addition facts through sums of 10. He learned to relate the upper decade addition facts to various sums of 10. For example:

$$5 + 6\ (\text{or } 6 + 5) = \underline{5 + 5} + 1 = 11$$
$$6 + 9\ (\text{or } 9 + 6) = \underline{6 + 4} + 5 = 15$$
$$7 + 8\ (\text{or } 8 + 7) = \underline{7 + 3} + 5 = 15$$
$$8 + 5\ (\text{or } 5 + 8) = \underline{8 + 2} + 3 = 13$$
$$9 + 7\ (\text{or } 7 + 9) = \underline{9 + 1} + 6 = 16$$

These sums of 10 have become a functional part of computation and have made it unnecessary for the child to memorize mechanically a great number of facts. The child has also learned to use the doubles 6 + 6, 7 + 7, 8 + 8, and 9 + 9 in upper decade computation.

$$6 + 8 = 6 + 6 + 2 = 14$$
$$7 + 6 = 7 + 7 - 1 = 13$$
$$8 + 9 = 8 + 8 + 1 = 17$$
$$9 + 7 = 9 + 9 - 2 = 16$$

In this section the child's resources are extended. He uses the knowledge that 10 is 5 + 5 to see 6 + 9 as 5 + 5 + 1 + 4, or as 10 + 1 + 4, or 15. This differs from his previous use of 5 + 5 (or 10) in that both 5s are hidden.

The better the child's understanding, the more likely he is to be confident in his ability to perform a given task. Therefore, the child is given an entirely new approach to the upper decade addition facts. He uses the knowledge that 20 is 10 + 10 to view sums such as 7 + 8 as 10 + 10 − 3 − 2, or as 20 − 3 − 2, or 15. Each child will normally have a preference for one approach to the upper decade addition combinations, and after using these other approaches he may be encouraged to use his preferred method.

The teacher should realize that many children will learn the upper decade facts without any deliberate effort to memorize the facts. Others will continue to rely on their ability to relate these facts to various sums of 10. Those who remember may have an advantage if they are able to relate the fact to a reason they understand. Those who rely solely on memory will eventually forget and then be at a serious disadvantage.

Readiness for Understanding

Knowledge of basic addition facts through sums of 10.

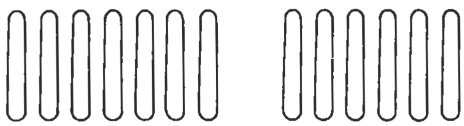
Knowledge that two-digit numerals name tens plus ones.

Developmental Experiences

for flannel board	*for each child*
tagboard latticework	20 sticks
disks	plastic numerals and symbols

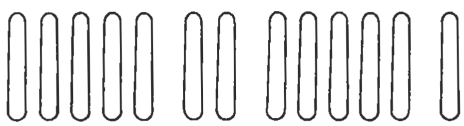
box
tagboard strips (4″ × ¾″)
pocket chart
numeral-cards: upper decade addition facts,
5 + 5 + ___ + ___,
10 + ___ + ___,
10 + 10 − ___ − ___,
and 1 through 20
cards: equal sign
colored chalk
felt-tip pen
bead frame

▶ Distribute about 20 sticks to each child. Instruct the children to put the sticks on one side of their desks and their plastic numerals and plus and equal signs on the other side. Write on the chalkboard the equation $5 + 5 = 10$. Tell the class that they will find $5 + 5$ in other sums. Draw on the chalkboard a picture of $7 + 6$ sticks and write $7 + 6$ below the picture. Tell the children to place $7 + 6$ sticks on their desks as shown.



7+6

Now direct the children to rearrange the sticks to show the two sets of 5 within the set of $7 + 6$. The children's sticks should now show the sum $5 + 5 + 2 + 1$. Describe $5 + 5$ as the hidden sum, $5 + 5$. Write on the chalkboard $5 + 5 + 2 + 1$ below the $7 + 6$. Tell the children to show the equation $7 + 6 = 5 + 5 + 2 + 1$ on their desks.

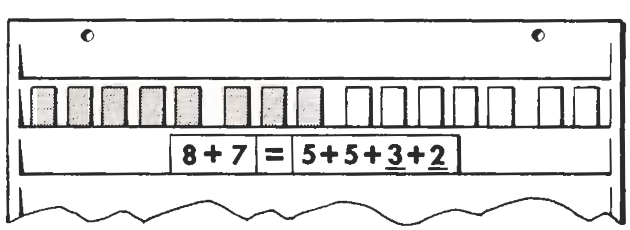


7+6

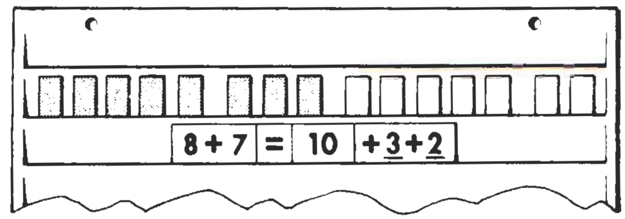
5+5+2+1

Continue in this way with sums such as $9 + 6$, $8 + 5$, $9 + 8$, $7 + 5$, $7 + 8$, $9 + 7$, $8 + 6$, and $9 + 5$. Help the children understand that the important idea is to find the hidden $5 + 5$. Do not ask for the standard numeral for the sum at this time.

▶ Place on a table in front of the class a box of 4 by $\frac{3}{4}$-inch tagboard strips that are two different colors. Place the sum $8 + 7$ in the pocket chart. Ask a child to place in the pocket chart strips that represent the sum $8 + 7$. The strips for 8 should be a different color than the strips for 7. Have a second child rearrange the strips so that two sets of 5 are clearly visible in the set of $8 + 7$. Place an equal sign to the right of $8 + 7$. Then place to the right of the equal sign a tagboard card that has $5 + 5 +$ ___ $+$ ___ written on it. Tell the children to observe the strips displayed in the chart and then ask a child to complete the equation. Have this child write these addends on the appropriate lines.



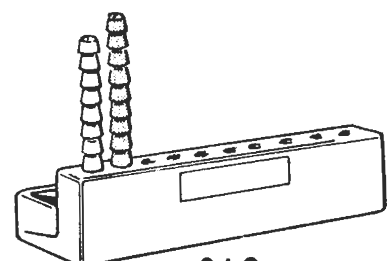
Let the children name $5 + 5$ as a count. When they respond that 10 is the count for $5+5$, direct a fourth child to place the appropriate numeral-card (10) over the sum $5 + 5$.



Let the children discuss the relationship between these two forms for the same sum. That is, the children should see that 10 comes from the hidden 5 in 8 plus the hidden 5 in 7. They should be aware that 3 is $8-5$ and the 2 is $7-5$.

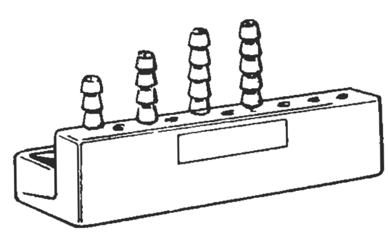
Repeat this procedure with the sums $6 + 7$, $7 + 5$, $9 + 8$, $7 + 7$, $6 + 9$, and $8 + 6$.

▶ Assemble the beads on the bead frame as illustrated. Direct a child to write on the chalkboard the sum that represents this union.

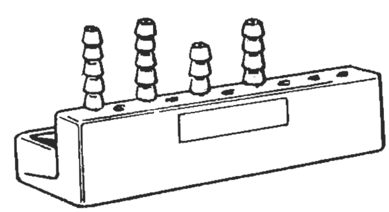


8+9

Ask whether or not there is a hidden $5+5$ in the sum $8 + 9$. Direct a child to rearrange the beads in such a way that two sets of 5 are clearly visible. He may decide to move 5 beads from the column of 8 to a third column and then move 5 beads from the column of 9 to a fourth column. Or he may decide to move 3 beads from the column of 8 to a third column and then move 4 beads from the column of 9 to a fourth column.



or



Tell the children to name the count for $5+5$. Write ___ $=$ $10 +$ ___ $+$ ___ on the chalkboard to the right of $8 + 9$. Ask a child to complete this equation so that it illustrates the set union now shown on the bead frame.

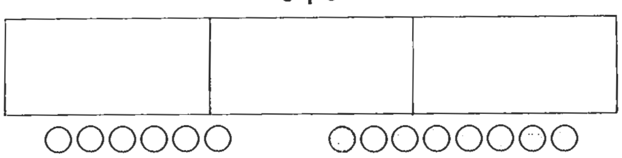
$$8 + 9 = 10 + \underline{3} + \underline{4}$$

Be sure the children realize that 10 comes from the hidden 5 in 8, plus the hidden 5 in 9. The 3 is 8−5 and the 4 is 9−5.

Continue in this way with the sums 6 + 6, 5 + 7, 7 + 9, 9 + 9, 6 + 8, 6 + 5, and 8 + 8.

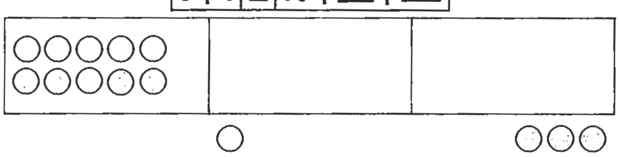
▶ Play "Operation Big Ten." Place on the flannel board a frame with three openings. Place below the frame 6 disks of one color and 8 disks of another color. Then insert the sum-card 6 + 8 in the pocket chart.

6+8



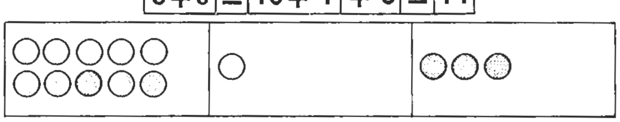
The aim of the game is to make a set of 10 as the first step in the operation of joining the set of 6 to the set of 8. Ask a child to carry out Operation Big Ten with the sets on the flannel board. When the child takes 5 disks from the set of 6 and 5 disks from the set of 8 and puts them in the first opening, place an equal sign and 10 + ___ + ___ in the pocket chart.

6+8 = 10+___+___



Then tell the child to join the other disks to the set of 10 by placing the 1 disk in the second opening and 3 disks in the third opening. Put the numeral-card for 1 and the numeral-card for 3 in the appropriate places in the pocket chart. Let another child find the card that shows the contracted form for 10+1+3 and link the two forms with an equal sign. Direct him to place these cards in the pocket chart.

6+8 = 10+ 1 + 3 = 14



Summarize the thinking steps used to determine the standard numeral for 6+8. Refer to the equation in the pocket chart and help the children review the ideas. Ask pertinent questions such as:

What did we do as a first step in Operation Big Ten? (find the hidden 10 or 5 + 5)

Where did 10 come from? (6 is 5+1; 8 is 5+3)

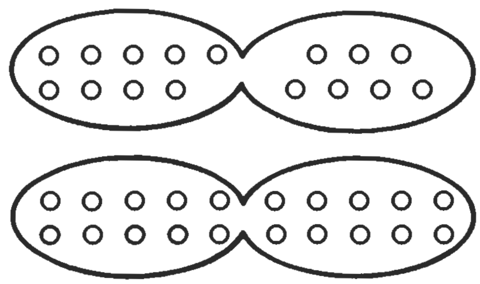
Where did 1 and 3 come from? (1 is 6−5 and 3 is 8−5)

What happened to the 1 and the 3? (they were added to 10 to give 14)

Use the same procedure with the sums 5 + 6, 9 + 9, 9 + 6, 8 + 5, 7 + 8, and 9 + 7.

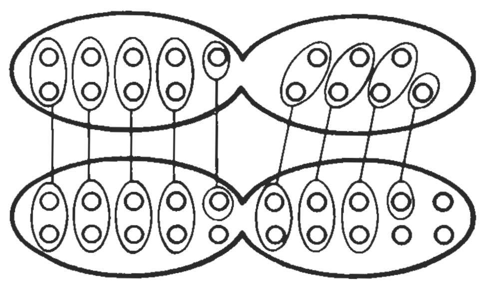
▶ To extend the children's methods for computing upper decade addition facts, start another investigation of these facts. Tell the children that although they know sums such as 9 + 7 and 6 + 8, it is interesting to look at these sums in a different way.

Draw on the chalkboard a set picture that represents the sum 9 + 7. Assure the children that you realize they know 9+7 is 16. The idea now is to show how to compute 9 + 7 in terms of 10+10. Write 9 + 7 to the right of the set picture. Below the set of 9 + 7, draw a set picture related to the sum 10 + 10 and write 10 + 10 − ___ − ___ below it.



10+10−___−___

Have a child come to the chalkboard and use colored chalk to match the objects in the two sets. The child may choose to match the objects as illustrated.



10+10−___−___

Ask the children what the matching shows about the difference between 9 and 10. One child may volunteer the information that 10 is 1 more than 9 or that 9 is 1 less than 10. Point to the 10 + 10 − ___ − ___ on the chalkboard and ask the children if the information that the child supplied gives them a clue about the first missing number in the expression. When they respond that 1 is the missing number, instruct a child to write the numeral 1 in the appropriate blank.

$$10 + 10 - \underline{1} - \_\_$$

Ask the children if the matching gives them a clue about the second missing number. When a child responds that 3 is the missing number since 7 is 3 less than 10, tell him to write the numeral 3 in the blank.

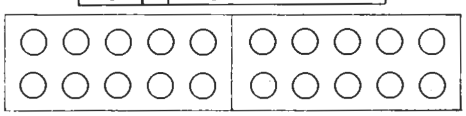
$$10 + 10 - \underline{1} - \underline{3}$$

Summarize the thinking involved in this activity. Discuss the fact that adding 9 and 7 is the same as adding 10 and 10, subtracting 1, and then subtracting 3. Point out to the class that this is another way to compute, using the upper decade addition facts. In the example involving 9 + 7, 10 was added each time. But this was adding 1 too many in the case of 9 and 3 too many in the case of 7; therefore 1 and 3 must be subtracted from 10+10 or 20.

Follow a similar procedure when discussing 7 + 6, 8 + 5, 6 + 9, and 7 + 8. Do not ask for the standard numeral for the sum at this time; the object is to have the children see 10+10 in relation to the given sums.

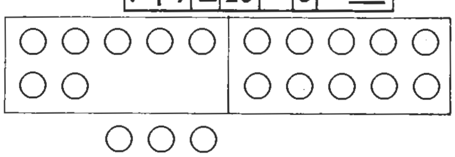
▶ Place on the flannel board a frame with two openings. Place 10 disks of one color in the first opening and 10 disks of another color in the second opening. Then insert in the pocket chart tagboard cards with the incomplete equation 7 + 9 = 10 + 10 − ___ − ___ written on them.

7+9	=	10+10	−	___	−	___



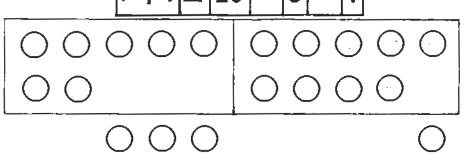
Ask the children to name 10+10 as a count. When they respond that 20 is the count for 10+10, direct a child to place the numeral-card 20 over the sum 10+10. Have a second child use the set of 10 in the first opening to show the first addend 7; have him place the disks in excess of 7 below the opening. Have another child show this action by placing the numeral 3 in the appropriate place in the pocket chart.

7+9	=	20	−	3	−	___



Now have a fourth child use the remaining set of 10 to show the addend 9; have him place the disks in excess of 9 below the opening. Ask someone to show this action by placing the numeral 1 in the appropriate place in the pocket chart.

7+9	=	20	−	3	−	1



Direct another child to find the card that shows the standard numeral for 20−3−1. Tell him to use an equal sign to link the two forms of the number.

7+9	=	20	−	3	−	1	=	16

Summarize the thinking steps used to determine the standard numeral for 7+9. Refer to the equation in the pocket chart and help the children review the ideas. Ask pertinent questions such as:

What did we do as a first step? (think 10+10 or 20)

Where did 3 and 1 come from? (10 is 3 more than 7 and 10 is 1 more than 9)

What happened to the 3 and the 1? (they were subtracted from 20 to give 16)

Use the same procedure with the sums 8 + 7, 5 + 8, 6 + 7, 5 + 9, and 8 + 9.

Pages 61 through 64

● Pages 61 and 62 investigate the use of 5 + 5 to compute sums between 10 and 19. Use page 61 as a discussion page. After the children have observed the illustration and read the story, have the class discuss Jim's way of computing 9 + 8. The children should realize that 10 is the hidden 5 in 9 plus the hidden 5 in 8. They should understand that the difference between each of the given addends and 5 results in the addends 4 and 3. Discuss 7 + 9 in a similar manner.

Use page 62 to provide the children with practice in using the hidden 5+5 to compute decade addition sums. Discuss the example at the top of the page and work one or two exercises with the class. Instruct the children to complete the other exercises. After all of the children have finished, ask individual children to tell how they found the addends needed to complete specific equations.

Name

UNIT 6 ADDITION AND SUBTRACTION FACTS

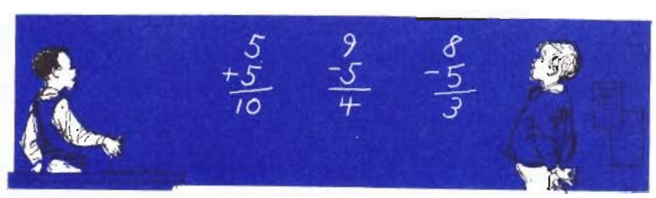
For Class Discussion

"Computing the sum 9 + 8 is easy," said Jim. "This is the way I do it." He wrote the example on the chalkboard.

9 + 8 = 10 + 4 + 3 or 17

"I'm not sure that I understand," said Bill. "Where did you find 10? And where did you get 4 and 3?"

Jim wrote this on the chalkboard.



"Show me how you would find the sum 7 + 9," said Bill.

Jim wrote this on the chalkboard.

7 + 9 = 10 + 2 + 4 or 16

"Now I see," said Bill.

What did Bill see? *See pupil page suggestions.*
How did Jim compute the sum 7 + 9?

reference page

B-61

How can the sum 8 + 7 be computed?

5	8	7
+ 5	− 5	− 5
10	3	2

8 + 7 = 10 + 3 + 2

8 + 7 = 15

Complete.

1. 9 + 6 = 10 + 4 + 1 = 15
2. 8 + 5 = 10 + 3 + 0 = 13
3. 9 + 7 = 10 + 4 + 2 = 16
4. 7 + 6 = 10 + 2 + 1 = 13
5. 9 + 8 = 10 + 4 + 3 = 17
6. 8 + 6 = 10 + 3 + 1 = 14
7. 7 + 5 = 10 + 2 + 0 = 12
8. 7 + 8 = 10 + 2 + 3 = 15
9. 9 + 5 = 10 + 4 + 0 = 14
10. 9 + 9 = 10 + 4 + 4 = 18
11. 8 + 8 = 10 + 3 + 3 = 16
12. 7 + 7 = 10 + 2 + 2 = 14

reference page

B-62

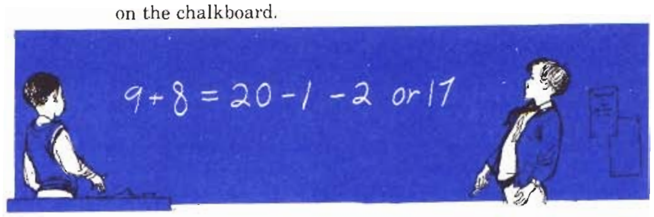
● Pages 63 and 64 illustrate the use of 10 + 10 or 20 in computation. Use page 63 as a discussion page. As the children read the story and study the illustration, they should realize that the 10 + 10 used in this computation is expressed as 20. They should also realize that 1 and 2 result from finding the difference between 10 and each of the addends. Call the children's attention to the illustration and ask how the 3 and the 1 were found. Help the children see that subtracting 1 and subtracting 3 is the same as subtracting 4.

Use page 64 to provide practice in using 10 + 10 or 20 to compute sums less than 20. Discuss the example at the top of the page and work one or two exercises with the class. Then assign the rest of the page for independent work. The children who have difficulty may profit by working in pairs or in small groups under the guidance of the teacher.

Name

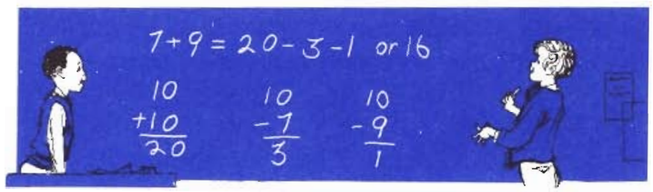
For Class Discussion

"Now it's my turn," said Bill. "Here's the way I compute the sum 9 + 8." He wrote the example on the chalkboard.



"You're going to have to show me another problem before I'm sure of what you're doing. Compute the sum 7 + 9, please," asked Jim.

Bill wrote this on the chalkboard.



"Now I see," exclaimed Jim.

What did Jim see?

How did Bill compute the sum 9 + 8?

See pupil page suggestions.

reference page

B-63

How can the sum 7 + 8 be computed?

10	10	10
+ 10	− 7	− 8
20	3	2

7 + 8 = 20 − 3 − 2 = 15

Complete.

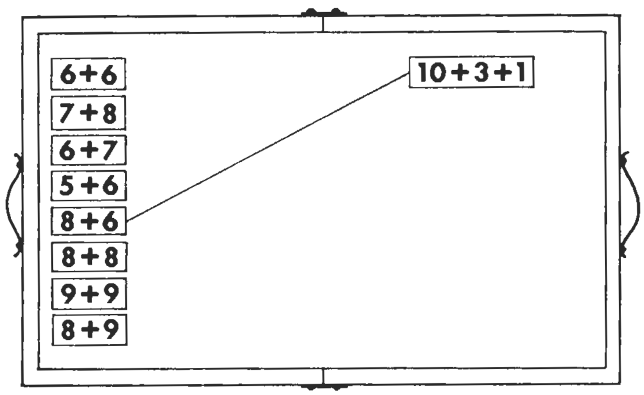
1. 9 + 6 = 20 − 1 − 4 = 15
2. 8 + 5 = 20 − 2 − 5 = 13
3. 7 + 6 = 20 − 3 − 4 = 13
4. 9 + 7 = 20 − 1 − 3 = 16
5. 8 + 6 = 20 − 2 − 4 = 14
6. 9 + 8 = 20 − 1 − 2 = 17
7. 7 + 5 = 20 − 3 − 5 = 12
8. 8 + 7 = 20 − 2 − 3 = 15
9. 9 + 5 = 20 − 1 − 5 = 14
10. 7 + 7 = 20 − 3 − 3 = 14
11. 9 + 3 = 20 − 1 − 7 = 12
12. 4 + 7 = 20 − 6 − 3 = 11

reference page

B-64

Supplemental Experiences

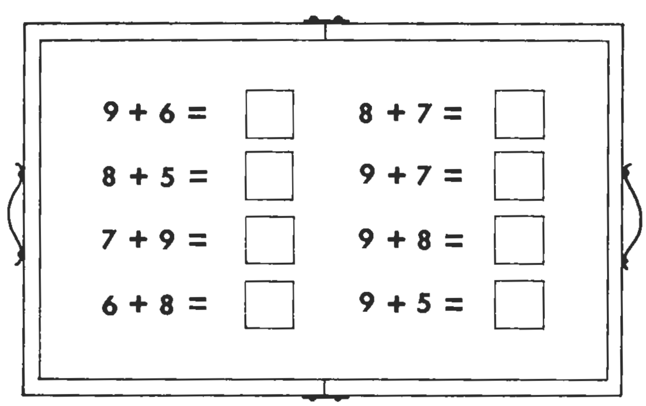
■ Place the numeral-cards for 6+6, 7+8, 6+7, 5+6, 8+6, 8+8, 9+9, and 8+9 on the left side of the flannel board. Make another set of numeral-cards for 10+2+3, 10+3+3, 10+4+4, 10+1+1, 10+3+1, 10+3+4, 10+0+1, and 10+1+2 and place them in a box near the flannel board. Direct a child to select a numeral-card from the box, place the card on the right side of the flannel board, and use yarn to match this card to one of the other cards.



Continue to have other children match the cards in the box with those on the flannel board.

Vary the activity by making numeral-cards for 10+1 through 10+8; have these matched with the numeral-cards on the flannel board.

■ To review the upper decade addition facts, arrange placeholder equations on the flannel board as shown.



Distribute two sets of felt numerals from 11 through 18. Ask a child to come forward and place his numeral in one of the placeholders to make a correct statement. It makes no difference which equation he chooses as long as the numeral correctly completes the sentence. Have this child explain to the class how he computed to find the standard numeral. Continue to let the other children find a place for their numerals.

KEY IDEA

Tens plus tens is tens. Hundreds plus hundreds is hundreds. Thousands plus thousands is thousands.

Scope

To recognize the structural similarity in computing ones, tens, hundreds, or thousands.

To use known addition combinations to compute sums of tens, hundreds, and thousands.

Fundamentals

Addition combinations of 10 are most important. In an exercise such as 7 + 8 = □, the child may first think about the number needed to reach 10 and then go on from there (7 + 3 + 5). In the exercise 7 + 8 = □, the child may count to 10 and then go five steps beyond. He thinks 7, 8, 9, 10, 10+1, 10+2, 10+3, 10+4, 10+5 or 15.

This same process may be involved when the child computes with tens, hundreds, and thousands. For example, in an exercise such as 600 + 900 = □, the child may first think about the number needed to reach 10 hundreds and then go five steps beyond in terms of hundreds. In this example he arrives at 10 hundreds + 5 hundreds, or 15 hundreds.

The child is free to reach 10, whether it be 10 tens, 10 hundreds, or 10 thousands, in any way that he wants. In an exercise such as 70 + 60 = □, one child may reach 10 tens in terms of 5 tens + 5 tens, another child may reach 10 tens in terms of 7 tens + 3 tens. These ideas emphasize one point: the child may want to use combinations of 10 when working with upper decade addition combinations.

At the first-grade level, the child discovered that the order in which two sets are joined does not alter the cardinal number of the union of the sets. He finds that when a set of 3 and a set of 7 are joined a new set of 10 is formed. And if a set of 7 and a set of 3 are joined, a new set of 10 is formed. The child learned that the order in which two numbers are added does not alter the sum. If 3 and 7 are added, the sum is 10. If 7 and 3 are added, the sum is 10. The child became aware of one of the fundamental properties of the set of whole numbers—the commutative property for addition.

The teacher will want to review commutativity at the third-grade level. She will want the child to be aware that the commutative property is helpful in work with upper decade addition combinations. If the child knows that 9 + 7 = 16, he also knows that 7 + 9 = 16, 70 + 90 = 160, and 90 + 70 = 160.

Readiness for Understanding

Ability to count tens, hundreds, and thousands.

Ability to write the numerals for tens, hundreds, and thousands.

Knowledge of addition combinations through 18.

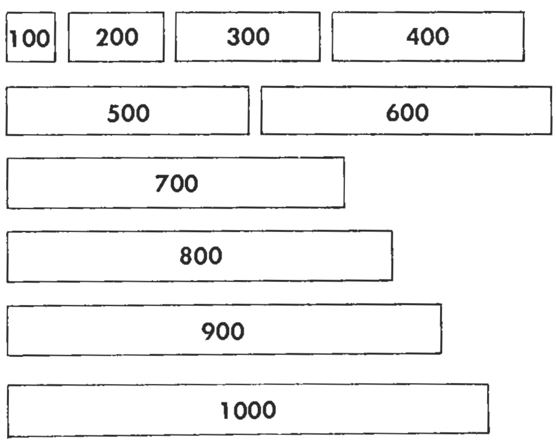
Developmental Experiences

for flannel board	*for each child*
disks	plastic numerals, symbols, and line segments

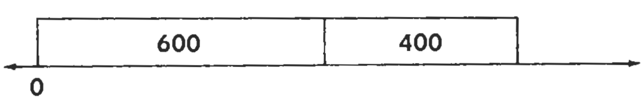
20 tagboard strips
3 boxes
cards:
- 18 sets of 10
- 18 sets of 100
- 18 sets of 1000
- equal signs

numeral-cards: sums
pocket chart
masking tape
felt-tip pen

▶ Cut twenty strips of tagboard, two each of the following dimensions: 2 inches wide by 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 inches long. Number these strips 100, 200, 300, . . . 1000 and place them along the chalktray.



Draw on the chalkboard a line segment about 36 inches long. Mark a point for 0 about two inches in from the left end of the segment. Write the sum 600 + 400 in vertical form below the line segment. Ask a child to come to the board, choose the strips that show this sum, and attach them end to end on the line, beginning at 0. Have another child record the standard numeral in the algorism.

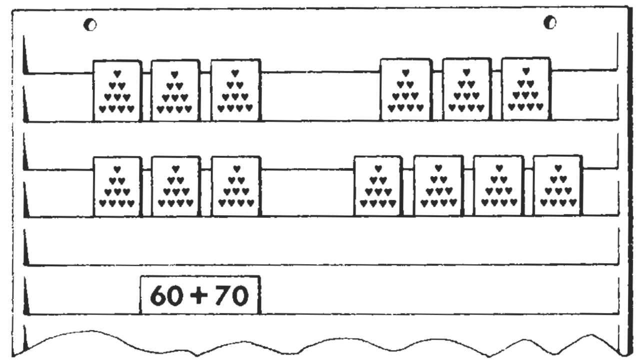


$$\begin{array}{r} 600 \\ +\ 400 \\ \hline 1000 \end{array}$$

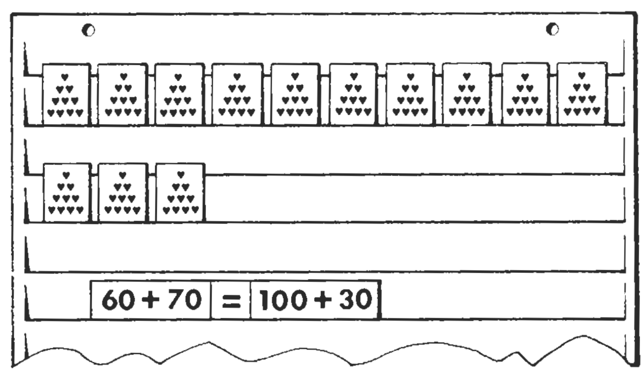
Have the class read aloud the recorded computation: "Six hundreds plus four hundreds is ten hundreds."

Write other sums of 1000 on the chalkboard. Have the appropriate tagboard strips placed on the line and the standard numeral for the sum in each exercise written on the chalkboard.

▶ Place a box of ten-cards, a box of hundred-cards, and a box of thousand-cards on a table in front of the class. Place in the pocket chart a sum-card that shows 60 + 70. Ask a child to place the appropriate number of ten-cards in the pocket chart above the sum-card.



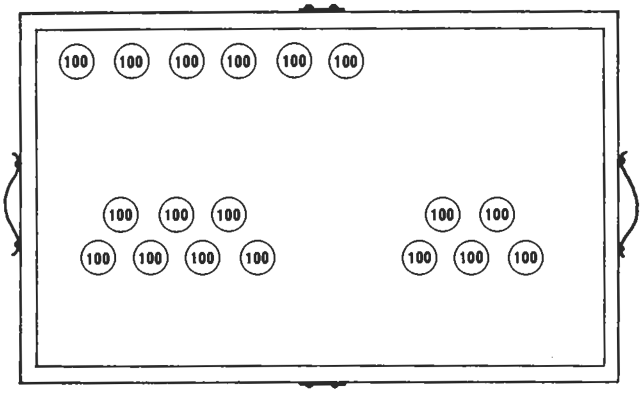
Have a second child rearrange the cards to show a set of 10 tens within this set of 60 + 70. Place an equal sign to the right of the sum 60 + 70. Then ask a third child to select from a set of cards for 100 + 10 through 100 + 80 the sum that represents the set picture in the chart. Direct him to place this card to the right of the equal sign.



Let the children discuss the relationship between 60 + 70 and 100 + 30. One child may have derived the 10 tens from the hidden 5 tens in 6 tens plus the hidden 5 tens in 7 tens. He may indicate that the 3 tens is 1 ten in the 6 tens plus 2 tens in 7 tens. Another child may say that 10 tens represents the 6 tens that are shown plus the hidden 4 tens in 7 tens. The 3 tens is the other part of 7 tens. The children should be free to interpret the relationship between these two forms for the same sum in any way that is clear to them.

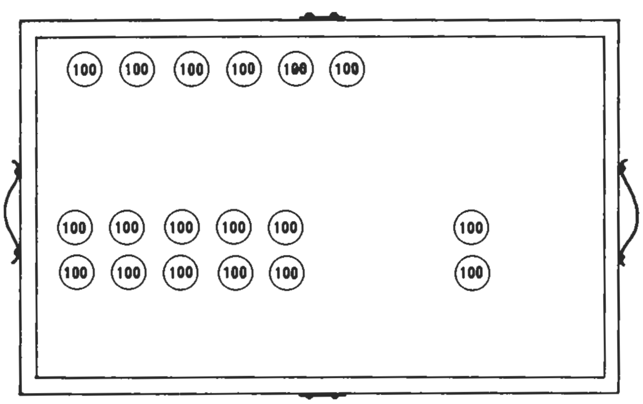
Repeat this procedure with the sums 90 + 50, 70 + 70, 80 + 60, and 50 + 80. Then adapt the procedure to the use of hundred-cards and thousand-cards. Direct the children to find the hidden 10 hundreds in sums 500 + 600 through 900 + 900 and the hidden 10 thousands in sums 5000 + 6000 through 9000 + 9000. Have the children describe the relationship they see between the two forms for each sum that is presented.

▶ Write 100 on 18 tagboard disks and place them across the top of the flannel board. Write 700 + 500 on the chalkboard. Ask a child to build, with the disks, a set union that represents 700 + 500. Tell him to place the disks in such a way that each part of the set can be seen clearly.



700 + 500

Instruct a second child to rearrange the disks to show a set of 10 hundreds within the set of 700 + 500. Have the children study the new arrangement. Then ask another child to name the sum that represents the sets now shown on the flannel board. Have him link these two forms for the same sum in an equation. Direct another child to write the contracted form for this sum.



700 + 500 = 1000 + 200 = 1200

Have the class read aloud the equation on the chalkboard: "Seven hundreds plus five hundreds is ten hundreds plus two hundreds; ten hundreds plus two hundreds is twelve hundreds."

Write 10 on another 18 disks and adapt the preceding procedure for a sum such as 80 + 80. Make 18 more disks that have 1000 written on them. Vary the order in which you work with sums 50 + 60 through 90 + 90, 500 + 600 through 900 + 900, and 5000 + 6000 through 9000 + 9000. This activity should help the children realize that knowing how to compute sums such as 9 + 6 will enable them to compute 90 + 60, 900 + 600, and 9000 + 6000. The upper decade addition fact involved is the same in each exercise.

▶ Place 6 thousand-cards in one area of the pocket chart and 9 thousand-cards in a separate area. Ask the children how many thousands they see; have them give a sum for the number of thousands. Write this sum in vertical form on the chalkboard and have a child record the computed sum.

$$\begin{array}{r} 6000 \\ +\ 9000 \\ \hline 15000 \end{array}$$

Have the class read aloud the recorded computation: "Six thousands plus nine thousands is fifteen thousands."

Continue in this way with other sums from 50 + 60 through 90 + 90, 500 + 600 through 900 + 900, and 5000 + 6000 through 9000 + 9000.

▶ Have the children use their plastic numerals, plus signs, and line segments at their desks to show the vertical form for sums of tens, sums of hundreds, and sums of thousands.

Place in a box in the front of the room 18 ten-cards, 18 hundred-cards, and 18 thousand-cards. Ask a child to count out 9 hundred-cards and place them on the chalktray. Ask another child to place 5 hundred-cards on the chalktray. Have the children show the form of, in an algorism on their desks, the sum for this particular union. Then tell them to show the commuted form for this sum.

$$\begin{array}{r} 900 \\ +500 \\ \hline \end{array} \qquad \begin{array}{r} 500 \\ +900 \\ \hline \end{array}$$

Next direct the children to compute the sums and place the appropriate standard numeral below the line in each algorism. Let the children take turns reading aloud the sentences on their desks:

9 hundreds plus 5 hundreds is 14 hundreds.
5 hundreds plus 9 hundreds is 14 hundreds.

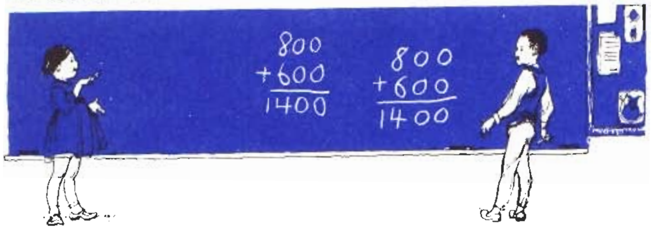
Continue the activity with sums 50 + 60 through 90 + 90, 500 + 600 through 900 + 900, and 5000 + 6000 through 9000 + 9000.

Pages 65 through 70

● Use page 65 as a discussion page. After the children study the illustration and read the story, let them discuss Sally's and Jim's way of computing. Ask the children how Sally might have arrived at the conclusion that 8 hundreds + 6 hundreds is 10 hundreds + 4 hundreds. Some of the children may decide that Sally thought of 10 hundreds in terms of the hidden 5 hundreds + 5 hundreds. Other children may think that Sally saw the 10 hundreds in relation to the 8 hundreds that is shown plus the 2 hundreds that is hidden. Ask several children to explain how they would compute the sum 800 + 600. Then assign the exercises at the bottom of the page for independent work. Let the children explain how they computed the sums.

Name

For Class Discussion



Jim and Sally computed the sum 800 + 600.

"The sum is 1400 because 8 hundred plus 6 hundred is 10 hundred plus 4 hundred," Sally explained. "That's 14 hundred."

Jim grinned and said, "All I did was think 8 + 6 is 14 and write 14 hundred."

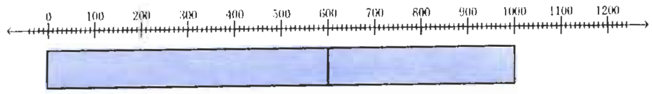
Compute.

1. 700 + 900 = 1600
2. 400 + 600 = 1000
3. 700 + 500 = 1200
4. 800 + 900 = 1700
5. 800 + 700 = 1500
6. 300 + 800 = 1100
7. 600 + 700 = 1300
8. 100 + 900 = 1000

reference page

B-65

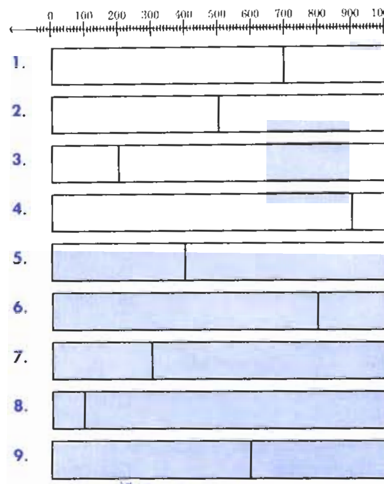
What sum? What length?



The sum is 400 + 600 or 600 + 400.
The length is 1000.

400 + 600 = 1000
600 + 400 = 1000

Write an addition equation about each number strip.



1. 700+300=1000*
2. 500+500=1000*
3. 800+200=1000*
4. 1000=900+100*
5. 1000=600+400*
6. 1000=800+200*
7. 300+700=1000*
8. 100+900=1000*
9. 600+400=1000*

reference page *Order may vary.

B-66

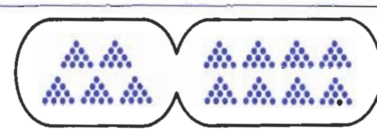
● Use page 66 to provide the children with an opportunity to record the various ways in which 10 hundreds is a sum of two addends. Discuss the upper section of the page with the class. Direct the children to look at the strips from right to left as well as from left to right and to note the two ways to express each particular sum.

Tell the children to complete the exercises. The children may write any of several equations for each exercise. For example, any one of the following equations is appropriate for exercise 1:

700 + 300 = 1000
300 + 700 = 1000
1000 = 700 + 300
1000 = 300 + 700

● The exercises on page 67 are designed to focus the children's attention on the hidden 10 in upper decade addition facts as they are used to compute sums of tens, hundreds, or thousands. Discuss the example at the top of the page; work one or two exercises from each section of the page with the class before having the children complete the exercises independently. After all of the children have completed this page, ask various children to explain how they computed specific sums.

Name



50 + 80 = 100 + 30
50 + 80 = 130

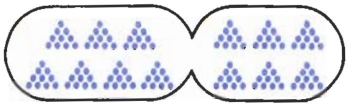
Complete.

1. 60 + 80 = 100 + 40
2. 500 + 700 = 1000 + 200
3. 50 + 90 = 100 + 40
4. 600 + 700 = 1000 + 300
5. 80 + 30 = 100 + 10
6. 600 + 900 = 1000 + 500
7. 50 + 60 = 100 + 10
8. 600 + 600 = 1000 + 200

Compute.

9. 5000 + 8000 = 13000
10. 600 + 900 = 1500
11. 6000 + 6000 = 12000
12. 500 + 600 = 1100
13. 5000 + 9000 = 14000
14. 400 + 800 = 1200
15. 5000 + 7000 = 12000
16. 600 + 400 = 1000
17. 6000 + 7000 = 13000
18. 100 + 900 = 1000

B-67



70 + 60 = 100 + 30

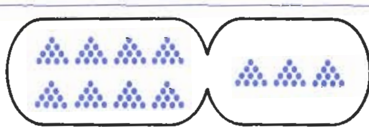
Complete.

1. 90 + 80 = 100 + 70
2. 80 + 50 = 100 + 30
3. 80 + 60 = 100 + 40
4. 70 + 90 = 100 + 60
5. 90 + 40 = 100 + 30
6. 80 + 30 = 100 + 10

Compute.

7. 5000 + 7000 = 12000
8. 600 + 900 = 1500
9. 8000 + 3000 = 11000
10. 300 + 900 = 1200
11. 7000 + 4000 = 11000
12. 700 + 700 = 1400
13. 6000 + 6000 = 12000
14. 9000 + 9000 = 18000
15. 7000 + 6000 = 13000
16. 600 + 800 = 1400
17. 6000 + 5000 = 11000
18. 8000 + 5000 = 13000

Name



80 + 30 = 110

Compute.

1. 30 + 40 = 70
2. 600 + 600 = 1200
3. 5000 + 4000 = 9000
4. 800 + 200 = 1000
5. 500 + 600 = 1100
6. 3000 + 6000 = 9000
7. 60 + 90 = 150
8. 2000 + 8000 = 10000
9. 3000 + 7000 = 10000
10. 70 + 70 = 140
11. 800 + 900 = 1700
12. 9000 + 2000 = 11000
13. 600 + 400 = 1000
14. 9000 + 3000 = 12000
15. 5000 + 7000 = 12000
16. 400 + 900 = 1300

Compute.

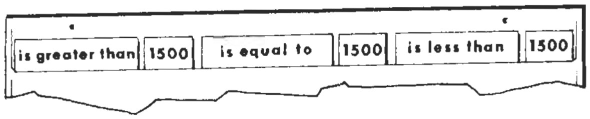
1. 4000 + 6000 = 10000
2. 500 + 800 = 1300
3. 8000 + 8000 = 16000
4. 90 + 50 = 140
5. 200 + 900 = 1100
6. 3000 + 7000 = 10000
7. 60 + 80 = 140
8. 700 + 500 = 1200
9. 8000 + 4000 = 12000
10. 90 + 80 = 170
11. 500 + 900 = 1400
12. 5000 + 5000 = 10000
13. 70 + 60 = 130
14. 200 + 800 = 1000
15. 30 + 90 = 120
16. 400 + 800 = 1200
17. 6000 + 7000 = 13000
18. 4000 + 7000 = 11000
19. 9000 + 1000 = 10000
20. 800 + 700 = 1500
21. 900 + 300 = 1200
22. 800 + 500 = 1300
23. 60 + 40 = 100
24. 9000 + 4000 = 13000

● Use pages 68 through 70 to provide practice in using upper decade addition facts in the computation of sums of tens, hundreds, or thousands. Work one or two exercises on each page with the class. The remaining exercises may be assigned for independent work. It is not necessary that all of the children complete all of the exercises at any one time. Structure the assignment to fit the ability of each child. These exercises should help strengthen the children's skill in computing when using upper decade addition facts. In order for the children to work the other sections of this unit this skill is essential.

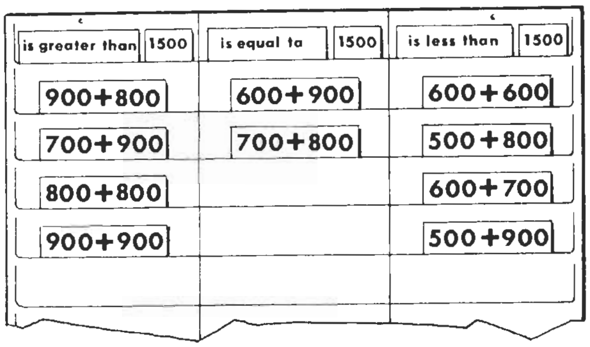
Supplemental Experiences

■ Play the game "Who Can Be My Partner?" Separate the class into two groups. Distribute sum-cards for 50 + 60 through 90 + 90 to one group and sum-cards for 100 + 10 through 100 + 80 to the other group. Select someone to place his card on the chalktray. He must read his sum aloud and ask, "Who can be my partner?" The child from the other group with the appropriate card must come forward and put his card on the chalktray beside the first card, link these two forms for the same sum with an equal sign, and read the equation to the class. Continue the game until all of the sums have been matched.

■ Distribute numeral-cards for 900 + 500 through 900 + 900. Set up the pocket chart as illustrated.



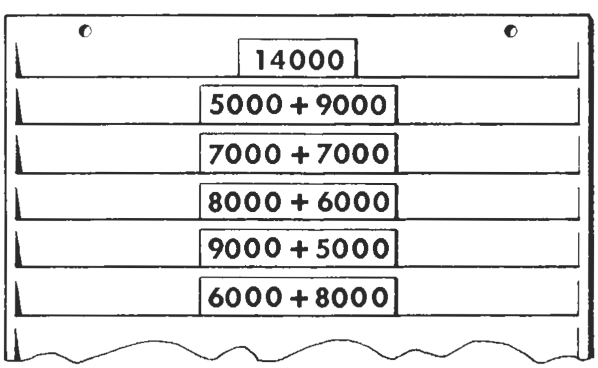
Tell the children to come up to the chart one at a time. Ask each child to tell the computed sum represented by his card, to compare his numeral with 1500, and to place his numeral below the appropriate phrase on the chart.



When all the numbers have been compared with 1500, redistribute the cards and replace the numeral-cards in the chart headings (1500) with the number 1400. Have all of the sums compared with 1400.

Repeat the activity; have the children compare their numbers with 1100, 1200, 1300, 1600, 1700, and 1800.

■ Write the sums of 11000 through 18000 on cards and distribute them to the class. Place in the pocket chart a numeral-card for 14000. Have the children who received sums of 14000 raise their hands. Ask them to bring their cards forward, one at a time, and to place them in the chart below 14000.



Remove all of the cards from the chart. Continue the activity with the other sums from 11000 through 18000.

Vary the activity. Write the numerals 11000 through 18000 on the chalkboard. Give each child a turn to choose one of these numbers and, below it, write a sum for this number.

■ Scatter on the flannel board five cards that show upper decade addition combinations. Have a child rearrange the cards in order from least to greatest. Have a second child place the standard numeral for each number beside the numeral-cards to check the order. Repeat the activity with other upper decade combinations.

6+6	12
8+5	13
6+8	14
9+6	15
7+9	16

KEY IDEA

Addend plus difference is sum.

Scope

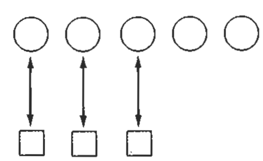
To increase the child's understanding of the concept of difference.

To review subtraction as undoing addition.

To review the subtraction facts related to sums of 10 in terms of 10 tens, 10 hundreds, and 10 thousands.

Fundamentals

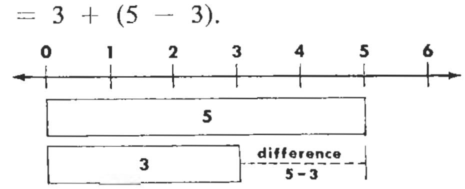
In the first grade addition was introduced through the union of two sets. In the second grade the child was introduced to the concept of difference by comparing two sets. For example, to compare a set of 5 with a set of 3 the child placed the members of the set of 3 in one-to-one correspondence with 3 members of the set of 5. The child learned that the subset of unmatched members has the cardinality of the difference.



The difference is written 5 − 3 and is the number that, when added to 3, equals 5.

$$3 + (5 - 3) = 5$$

Measurement of line segments also shows the difference. The child notices that the difference is what remains when the addend is taken away. He also learns that the addend is left when the difference is taken away. The child observes that the 3-segment together with the (5 − 3)-segment is the 5-segment. He is guided to understand the idea that 5 is greater than 3 by the difference 5 − 3. When 5 − 3 is added to 3 the result is 5; this may be expressed in equation form: 5 = 3 + (5 − 3).



In this section the child also reviews the fact that sums and differences can be counted. He reviews the use of an equation to express a number in two different forms—as a sum and a count and as a difference and a count. The equation states that *the number* named on the left of the equal symbol is the same as *the number* named on the right of the equal symbol. While equations may be read from right to left as well as from left to right, it is probably well at early grade levels to read them from left to right to reinforce basic reading skills. However, the teacher should help the child realize that the equation $80 + 20 = 100$ says that 100 is the number $80+20$ as well as saying that the number $80+20$ is 100.

Readiness for Understanding

Ability to compare two lengths and see the difference.

Ability to compare two sets and see the unmatched subset.

Ability to compare two numbers.

Ability to recognize parts of a whole set.

Ability to join two sets to make a new set.

Ability to express the relationship between a pair of numbers both as a sum and as a count.

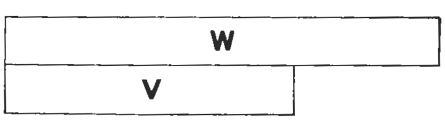
Developmental Experiences

for flannel board	*for each child*
cutouts	30 disks
numerals: 0 through 9	plastic numerals and symbols
operation symbols	

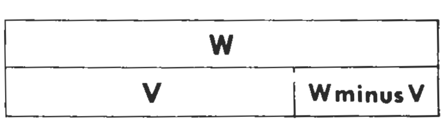
bulletin board
tagboard strips
pins
pocket chart
cards:
- 10 ten-cards
- 10 hundred-cards
- 10 thousand-cards

felt-tip pen

▶ Pin a 1 by 9 inch tagboard strip on the bulletin board. Print the letter *w* on the strip. Directly below this strip pin a 1 by 6 inch tagboard strip and print the letter *v* on it.



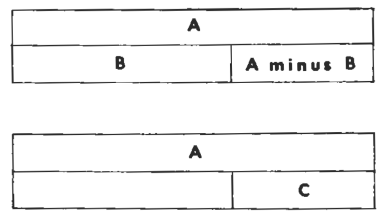
Direct the class to compare the length of the *w*-strip and the length of the *v*-strip. Ask the children to try to describe how much longer *w* is than *v*—that is, have them describe the difference *w* minus *v*. Someone may observe that the difference between *w* and *v* is *w* without *v*, or *w* less *v*. Label a strip *w* minus *v* and place this strip beside the *v*-strip.



Ask the children to describe the length *w* in terms of the combined lengths of the other two strips: *w* is *v plus w minus v*.

Remove all of the strips from the board. Replace them with a 10 by 2 inch strip that is labeled with the letter *a*. Below the *a*-strip, place a 6 by 2 inch strip labeled *b* and a 4 by 2 inch unlabeled strip. Tell the children to compare the length of the *a*-strip and the length of the *b*-strip. Ask them how much longer the *a*-strip is. Note: in discussion the lengths of the strips may be referred to simply as *a* and *b*. The children may suggest that the difference between the two strips is *a* minus *b*, or they may note that *b* plus *a* minus *b* is *a*. Print *a* minus *b* on the unlabeled strip.

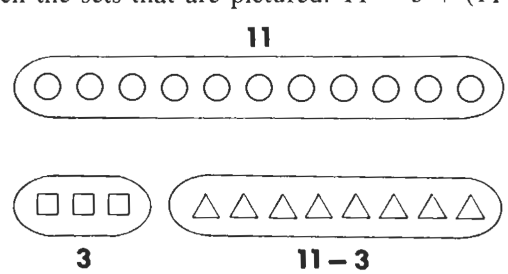
Place below the first group on the board another *a*-strip, an unlabeled strip the same size as the *b*-strip, and a strip labeled with the letter *c*.



Have the class compare the lengths of the *a*-strip and the *c*-strip. Ask how much longer the *a*-strip is than the *c*-strip. The children may note that the difference is *a* minus *c*. They may also note that *a* minus *c* plus *c* is the length of the *a*-strip. Print *a* minus *c* on the strip that is not labeled.

To help the children develop a stronger intuitive feeling for the idea of difference, the activity may be repeated using strips of other lengths and labeled with different letters. Then substitute numbers for the letters and repeat the activity. For example, if *a* is 14, *b* is 8, and *c* is 6, then *b* is $14 - 6$ and *c* is $14 - 8$.

▶ Place on the flannel board a row of 11 felt cutouts. Make a second row with 3 different cutouts. Direct a child to place above the set of 11 a felt numeral to indicate how many members this set has. Then ask him to place below the second set a felt numeral to indicate how many members this set has. Ask another child to add enough cutouts to the set of 3 to make a set of 11. Then ask this child to name a difference that indicates how many more members are in the set of 11 than are in the set of 3. Place the symbols for this difference below the corresponding set. Tell a third child to write an equation that shows the relationship between the sets that are pictured: $11 = 3 + (11 - 3)$.



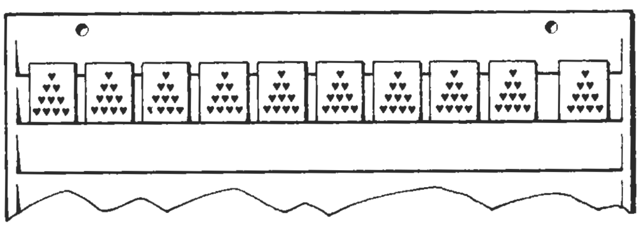
$11 = 3 + (11 - 3)$

Continue to let the children use differences to compare the following pairs of sets: a set of 15 and a set of 7, a set of 14 and a set of 5, and a set of 11 and a set of 6. Instruct the children to write an equation for each set picture.

▶ Put 9 ten-cards in the pocket chart. Ask a child to place another ten-card beside the set of 9 and to write on the chalkboard an appropriate equation for this union of sets. Tell the class to read the equation and to decide whether or not the sum and the count named are the same number. Then ask the children to state the sum for 9 tens more than 1 ten (10 + 90); have them tell the count for this sum (100). Write the equation 10 + 90 = 100 below the first equation. Have it read aloud.

Have a child use the 10 tens in the pocket chart to show 1 ten less than 10 tens. He may cup his hands around 1 of the 10 tens. Write 100 − 10 beside the equations already on the chalkboard. Review the fact that 100 − 10 (10 tens minus 1 ten) is a difference. Ask someone to tell the count for this difference and complete the equation on the board. (100 − 10 = 90) Have the class read the sentence aloud: "One hundred minus ten equals ninety (ten tens minus one ten equals nine tens)."

Ask another child to show 9 tens less than 10 tens. Write 100 − 90 below the recorded equation 100 − 10 = 90. Have the class name the count for this difference. (10) Complete the equation on the board and have the class read this latter equation aloud.



90 + 10 = 100   100 − 10 = 90
10 + 90 = 100   100 − 90 = 10

Remove all but 8 of the ten-cards in the pocket chart. Tell a child to put 2 ten-cards near the 8 ten-cards and to write on the chalkboard an appropriate equation for this particular union. Have the class read the equation aloud. Discuss the idea that 80 + 20, or 100, is 2 tens more than 8 tens.

Ask someone to name the sum and count for 8 tens more than 2 tens. Write the equation 20 + 80 = 100 below the first equation. Have it read aloud.

Tell another child to show, with the 10 tens in the pocket chart, 2 tens less than 10 tens. Direct him to write an equation that represents the difference he has shown. (100 − 20 = 80) Have the class read aloud the equation. Then have a fourth child show 8 tens less than 10 tens and write the related equation on the board. Have the class read the equation aloud.

Continue in this same way with the other sums and related differences associated with 10 tens, 10 hundreds, and 10 thousands.

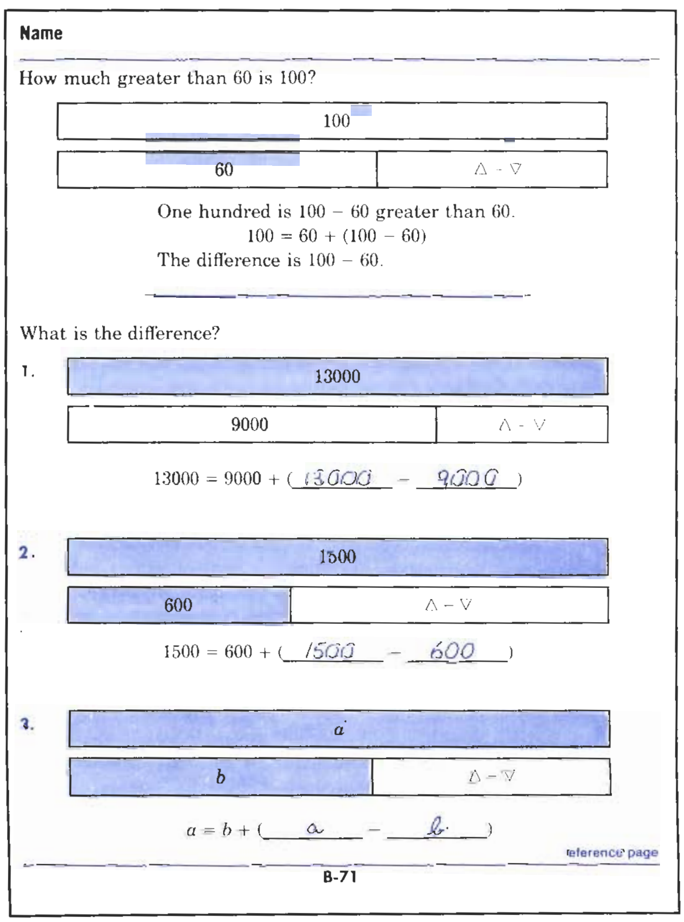
▶ Give each child 10 disks that are labeled 100. Have them show on their desks a set that represents the sum 900 + 100. Tell the children to use their plastic numerals, plus signs, and equal signs to make an equation that shows the sum and its count.

Then direct the children to build equations that show the difference and the count for 1 hundred less than 10 hundreds, and equations that show the difference and the count for 9 hundreds less than 10 hundreds.

Continue in this same way with other sums and related differences associated with 10 hundreds. Then instruct the children to use disks labeled 1000 and to follow a similar procedure with sums and related differences associated with 10 thousands. Finally, have the children use disks labeled 10; have them give the sums and related differences associated with 10 tens.

Pages 71 and 72

● Use page 71 to reinforce the children's understanding of the concept of difference. Discuss the example at the top of the page. After you explain the procedure, instruct the children to complete the page independently.



Name

How much greater than 60 is 100?

100

60 | △ − ▽

One hundred is 100 − 60 greater than 60.
100 = 60 + (100 − 60)
The difference is 100 − 60.

What is the difference?

1. 13000

9000 | △ − ▽

13000 = 9000 + (13000 − 9000)

2. 1500

600 | △ − ▽

1500 = 600 + (1500 − 600)

3. a

b | △ − ▽

$a = b + (a - b)$

reference page

B-71

100	
20	80

20 + 80 = 100 80 + 20 = 100
100 = 20 + 80 100 = 80 + 20
100 − 20 = 80 100 − 80 = 20
80 = 100 − 20 20 = 100 − 80

Write two addition equations and two subtraction equations for each exercise.

1.

100	
40	60

40+60=100*
100=60+40*
100−60=40*
60=100−40*

2.

100	
70	30

70+30=100*
100=70+30*
100−70=30*
100−30=70*

3.

100	
50	50

100=50+50
50+50=100
50=100−50
100−50=50

4.

100	
90	10

100=90+10*
10+90=100*
100−10=90*
10=100−90*

Compute.

5. 1000 − 200 = 800 6. 10000 − 8000 = 2000 7. 100 − 30 = 70

reference page * Answers may vary.

B-72

● Use page 72 to provide practice in writing addition and subtraction equations related to 10 tens and to provide practice with the subtraction facts related to sums of 10 in terms of 10 hundreds and 10 thousands.

Discuss the example at the top of the page with the class. Let children read the addition and subtraction equations. Then ask the children to write two addition equations about the sum shown in exercise 1. Let some children read their equations. Then ask them to write two subtraction equations about the differences shown, in the same exercise. After several equations have been read, assign the rest of the exercises for independent work.

Supplemental Experiences

■ Make several set pictures that show the union of a pair of subsets. The following sets might be used.

A set of 12 children—3 girls and 9 boys.

A set of 14 geometric figures—9 squares and 5 circles.

A set of 13 crayons—6 green and 7 yellow crayons.

Let a child choose one of the set pictures and tell the class these facts about the set.

The number of the whole set.

The number of one of the parts (subsets) of the whole set.

The number of the other part (subset) in terms of a difference.

If the child selected the picture of 12 children, he might explain that he sees a set of 12 children. Three members of this set are girls. The number of the set of boys is 12 − 3. Have the child write on the chalkboard an equation that shows the relationship between the numbers in this situation: 12 = 3 + (12 − 3).

Ask the child to express the number of the set of girls as a difference. He should be aware that this number is 12 − 9: the difference between the number of the whole set and the number of the set of boys. Ask the child to write on the chalkboard an equation that shows the relationship between the numbers in this situation: 12 = 9 + (12 − 9).

Let other children select set pictures and describe the numbers of the subsets in terms of differences.

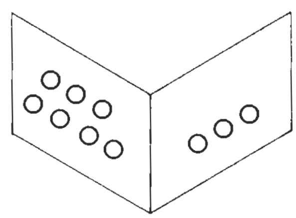
■ Prepare placeholder equation cards as illustrated.

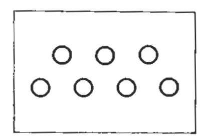
60 + □ = 100 100 = 60 + □

100 − 60 = □ □ = 100 − 40

Use addition combinations (and corresponding subtraction combinations) associated with sums of 10 tens, 10 hundreds, and 10 thousands. Select a child to read the complete equation as a card is exposed to view.

■ Fold a card in half and draw 7 dots on one section and 3 dots on the other section.





Show the 7 dots to the children and ask someone to tell how many dots he sees. Unfold the card and ask the class what sum represents this picture story. (7 + 3 or 3 + 7) Ask a child what count is shown by the dots. Have him give an equation that states the relationship between the sum and the count.

Now fold the 3 dots out of sight. Ask how many dots the children saw when both sides of the card were in view. (10) Ask how many fewer they see now. (3 fewer) Let them tell what difference describes the removal of the set of 3. (10 − 3) Have them name the count for this difference. (7) Ask a child to state the relationship between the difference and the count as an equation.

Open the card once more; fold the 7 dots out of sight. Have the children describe this action as a difference and count. Have them state the relationship between the difference and the count as an equation.

Continue in this way with other domino cards that show sets that have sums of 10 as the cardinal number of their union. Then adapt the procedure to the use of ten-cards, hundred-cards, and thousand-cards. Direct the children to give the sums and related differences associated with 10 tens, 10 hundreds, and 10 thousands.

KEY IDEA

$15 - 8 = 10 - 8 + 5.$

Scope

To review upper decade subtraction facts in terms of tens, hundreds, and thousands.

To review the idea of subtracting from 10 when there are not enough ones in the ones place of the sum.

To apply to tens, hundreds, and thousands the idea of subtracting from 10 when not enough ones are available.

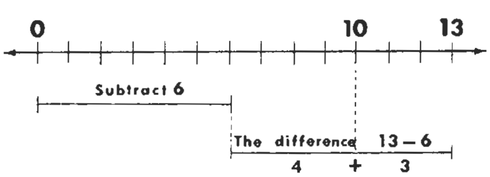
Fundamentals

It is important to use the information which Hindu-Arabic numerals furnish. Of course, in an exercise such as $13 - 6 = \square$, 6 may be subtracted from 13 by brute force. The child also can compare a set of 13 and a set of 6 in one-to-one correspondence and determine the cardinality of the unmatched subset. Another approach might be to tell the child to remember the fact that $13 - 6$ is 7. This approach neglects the fact that the Hindu-Arabic numeration principles are the key to computation. The use of ten as a base indicates that other numbers should be related to 10 in computations. Our standard numeral for 13 tells us that the number is $10 + 3$. The realization that 10 is ten ones as well as one ten enables the child to compute a difference. In computing the difference $13 - 6$, the child perceives that he cannot take the 6 from the 3, so he takes the 6 from the 10 where he has sufficient ones. The thinking steps are as follows:

$$13 - 6 = (10 - 6) + 3 = 4 + 3 = 7$$

Exactly as he saw $6+7$ as $6+4+3$, he sees $13-6$ as $4+3$, or 7.

The number 6 is $10-4$, so 6 may be subtracted by subtracting a 10 and adding 4. The number line shows the idea involved.



The child uses his acquired knowledge to interpret differences such as $13 - 6$ as $4 + 3$; he begins to understand subtraction and interprets what is called "borrowing" simply as a subtraction of ones from a ten.

In this section the child reviews the fact that if there are not enough ones, he subtracts from a ten and relates this to tens, hundreds, and thousands.

Readiness for Understanding

Ability to compute sums through sums of 10.

Ability to compute differences related to sums of 10.

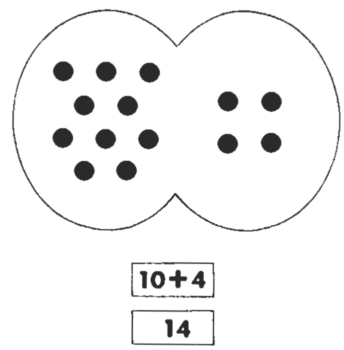
Understanding of the tens and ones indicated by a two-digit numeral.

Developmental Experiences

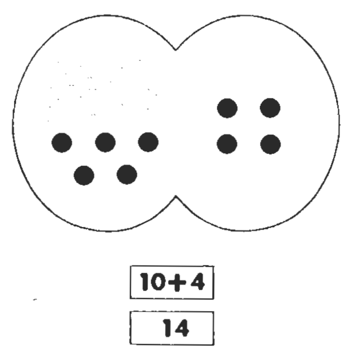
for flannel board	*for each child*
cutouts	counters
tagboard frame for set outline	
numeral-cards: 11 through 18	
sum-cards: $10 + 1$ through $10 + 8$	

bulletin board
number strips
felt
pins
Countingmen
piece of cardboard

▶ Make a set of sum-cards for $10 + 1$ through $10 + 8$ and a set of numeral-cards for 11 through 18. Show on the flannel board the union of two sets whose number property is $10 + 4$. Ask some child to select the cards that show the sum and count for this union; have him place the cards below the sets.



Direct the class to study the set and to tell how many members would be in a set having 5 fewer than $10 + 4$ members. Have a child show 5 fewer than $10 + 4$ by temporarily covering 5 of the 10.



Ask the children to help you record the ideas just shown. Write the following sentences on the chalkboard and let the children complete them.

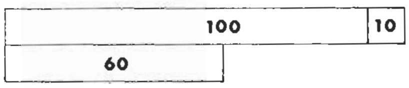
$14 - 5 = (10 - \_\_) + \_\_$
$14 - 5 = \_\_ + 4$
$14 - 5 = \_\_$

Explain that in every expression we are talking about the number of members in the difference set.

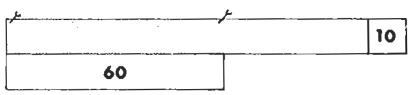
Adapt this procedure to the following differences:

10 + 1 less 8,
10 + 2 less 7,
10 + 3 less 5,
10 + 5 less 6,
10 + 6 less 7,
10 + 7 less 8, and
10 + 8 less 9.

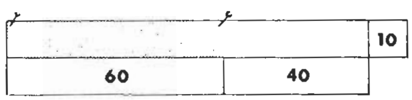
Place number strips labeled 100, 10, and 60 on the bulletin board as shown.



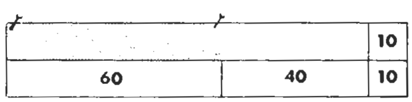
Ask a child to demonstrate and explain the computation of 11 tens minus 6 tens. Give him a piece of felt that is the same size as the 60-strip. To show the subtraction of 6 tens, he should pin the felt over part of the 100-strip.



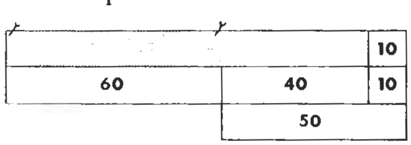
Tell him to select (from an assortment of numbered strips) a strip that represents the part of 10 tens that remains after 6 tens are subtracted and to place it below the 100-strip.



Ask the child what sum must now be computed to find the standard numeral for 110 − 60. (the sum 40 + 10) To show this idea, he should place a 10-strip with the 40-strip.



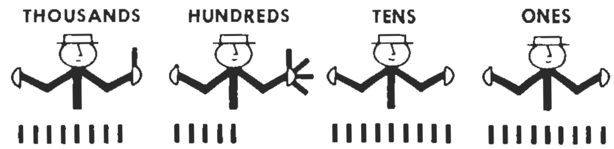
Then have the child compute the count for 40 + 10 and place a 50-strip below the 40 and 10 combination.



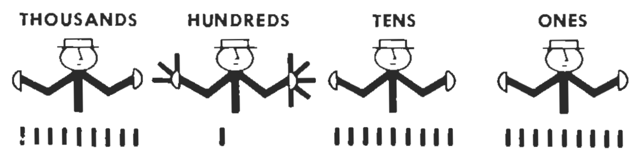
Summarize the computation steps with the class. Be sure the children understand that 110 − 60 is (100 − 60) + 10; have them observe that (100 − 60) + 10 added to 60 is 100 + 10 or 110.

Adapt this procedure to 120 − 50, 130 − 80, 150 − 70, and 140 − 90.

Write on the chalkboard 1400 − 600 = □. Display 4 Countingmen labeled THOUSANDS, HUNDREDS, TENS, and ONES. If you do not have enough Countingmen, draw additional ones on the chalkboard. Ask a child to show 14 hundreds. He should place 1 finger on the Thousands-man and 4 fingers on the Hundreds-man.



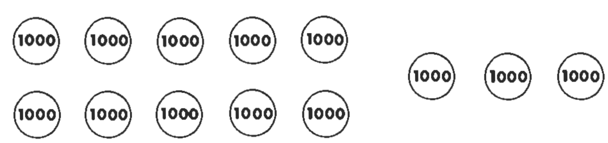
Help a second child show and explain how he would subtract 6 hundreds from 14 hundreds. He cannot take 6 fingers from the Hundreds-man since that man has only 4. He can take 6 hundreds from the 10 hundreds on the Thousands-man. This leaves 4 hundreds. The child will put the 4 fingers on the Hundreds-man after he removes the finger from the Thousands-man.



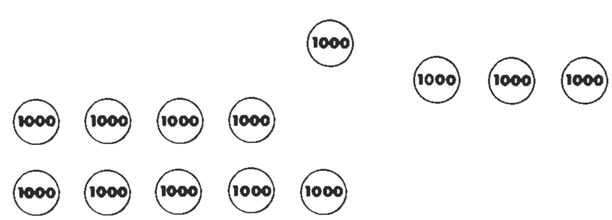
Have the class observe how many fingers are now on the Hundreds-man and give the count for 1400 − 600. The children should observe that 1400 − 600 is (1000 − 600) + 400, (1000 − 600) + 400 is 400 + 400, and 400 + 400 is 800.

Adapt this procedure to the following differences: 1200 − 600, 1300 − 700, 1400 − 700, 1500 − 800, and 1700 − 900.

Write 13000 − 9000 on the chalkboard and have the children use at their desks counters labeled 1000 to show a set of 10000 + 3000.



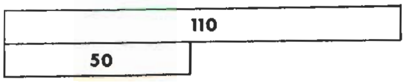
Then ask them to show a set of 13000 − 9000.



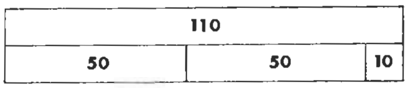
Ask a child to explain how he computed 13000 − 9000. He may say that he subtracted 9 thousands from 10 thousands, and then added this difference (10000 − 9000) and 3000. Direct the class to tell the count for 13000 − 9000.

Continue to have the children use their counters (each labeled 1000) as a visual aid in the computation of the following differences: 11000 − 9000, 12000 − 4000, 14000 − 8000, 15000 − 9000, 16000 − 8000, 11000 − 7000, and 13000 − 4000.

▶ Review the commutative property for addition. Place on the bulletin board a 110-strip and a 50-strip as illustrated.



Place beside the board a set of number strips for 10 through 100. Ask a child to select two strips that will show the difference 110 − 50. Tell him that his two strips combined with the 50-strip on the board should show the hidden 50+50 in 110.



Write 110 − 50 = ___+ 10 on the chalkboard, and ask a child to complete the equation. Below this equation write 110 − (___+___) = 50. Have this equation completed.

$$110 - 50 = \underline{50 + 10}$$
$$110 - (\underline{50 + 10}) = 50$$

Point out to the class that the addends (50 and 50+10) have been switched in the second equation but the sum is the same. Explain that when the first of these two sentences was completed, it enabled them to complete the second equation without computing. Explain that both equations involved the computation of the difference 110 − 50, which is the number that is added to 50 to make 110.

Adapt this procedure to 120 − 90, 130 − 60, and 110 − 30. Help the children understand that the first equation makes the completion of the second of two equations, such as the following, automatic.

$$110 - 50 = \square$$
$$110 - \square = 50$$

Pages 73 through 76

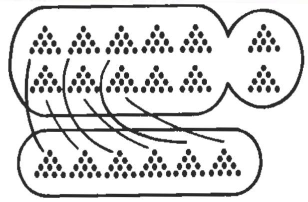
● Pages 73 and 74 reinforce the children's understanding of computing upper decade differences in terms of tens, hundreds, and thousands. Discuss the example at the top of page 73 with the children. Let the children discuss each equation and the relationship between the numbers in the equations.

All of the equations on page 73 pertain to differences of 11 and 12 in terms of 11 tens and 12 tens. The children are not required to compute the standard numeral for the difference. Work one or two exercises with the class; then instruct the children to complete the page independently.

The exercises in the first section of page 74 provide practice in computing differences related to sums of 13 and 14. The exercises in the second part of the page pertain to differences of 11, 12, 13, and 14 in terms of tens, hundreds, and thousands.

Discuss the example at the top of the page with the class. Work one or two of each type of exercise with the class. Then assign the rest of the exercises for independent work.

Name

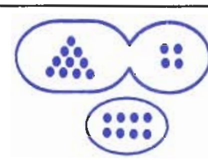


120 − 60 = (100 − 60) + 20
120 − 60 = 40 + 20
120 − 60 = 60

Complete.

1. 110 − 40 = (100 − 4 0) + 10
2. 120 − 40 = (100 − 4 0) + 20
3. 110 − 70 = (100 − 7 0) + 10
4. 120 − 70 = (100 − 7 0) + 20
5. 110 − 90 = (100 − 9 0) + 10
6. 120 − 90 = (100 − 9 0) + 20
7. 110 − 50 = (100 − 5 0) + 1 0
8. 120 − 50 = (100 − 5 0) + 2 0
9. 110 − 80 = (100 − 8 0) + 1 0
10. 120 − 80 = (100 − 8 0) + 2 0
11. 110 − 30 = (100 − 3 0) + 1 0
12. 120 − 30 = (100 − 3 0) + 2 0
13. 110 − 60 = (100 − 6 0) + 1 0
14. 110 − 20 = (100 − 2 0) + 1 0

B-73



14 − 8 = (10 − 8) + 4
14 − 8 = 6

Complete.

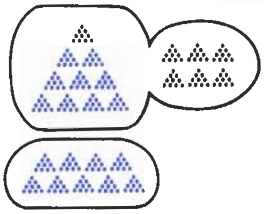
1. 13 − 5 = (10 − 5) + 3
2. 14 − 5 = (10 − 5) + 4
3. 13 − 9 = (10 − 9) + 3
4. 14 − 9 = (10 − 9) + 4
5. 13 − 8 = (10 − 8) + 3
6. 13 − 4 = (10 − 4) + 3
7. 13 − 6 = (10 − 6) + 3
8. 14 − 6 = (10 − 6) + 4
9. 13 − 7 = (10 − 7) + 3
10. 14 − 7 = (10 − 7) + 4

Compute.

11. 140 − 90 = 50	12. 1300 − 700 = 600	13. 11000 − 5000 = 6000	14. 120 − 40 = 80
15. 1100 − 200 = 900	16. 1400 − 600 = 800	17. 1200 − 900 = 300	18. 11000 − 3000 = 8000

B-74

Name



$160 - 90 = (100 - 90) + 60$
$= 10 + 60$

Complete.

1. 15 − 7 = (10 − 7) + 5 2. 15 − 9 = (10 − 9) + 5

3. 15 − 8 = (10 − 8) + 5 4. 15 − 6 = (10 − 6) + 5

5. 17 − 8 = (10 − 8) + 7 6. 17 − 9 = (10 − 9) + 7

7. 18 − 9 = (10 − 9) + 8 8. 16 − 9 = (10 − 9) + 6

9. 16 − 8 = (10 − 8) + 6 10. 16 − 7 = (10 − 7) + 6

Compute.

11. 1600 − 700 = 900	12. 150 − 60 = 90	13. 18000 − 9000 = 9000	14. 1200 − 800 = 400
15. 1500 − 300 = 1200	16. 17000 − 7000 = 10000	17. 1600 − 400 = 1200	18. 3000 − 1000 = 2000

B-75

Compute.

1. 1200 − 600 = 600	2. 1100 − 900 = 200	3. 1700 − 800 = 900	4. 1700 − 900 = 800
5. 13000 − 4000 = 9000	6. 16000 − 8000 = 8000	7. 11000 − 6000 = 5000	8. 11000 − 8000 = 3000
9. 150 − 70 = 80	10. 130 − 80 = 50	11. 120 − 90 = 30	12. 110 − 70 = 40
13. 18000 − 9000 = 9000	14. 14000 − 6000 = 8000	15. 16000 − 7000 = 9000	16. 14000 − 5000 = 9000
17. 12000 − 8000 = 4000	18. 16000 − 9000 = 7000	19. 13000 − 9000 = 4000	20. 14000 − 9000 = 5000
21. 1700 − 800 = 900	22. 1100 − 700 = 400	23. 1400 − 500 = 900	24. 1300 − 600 = 700

reference page

B-76

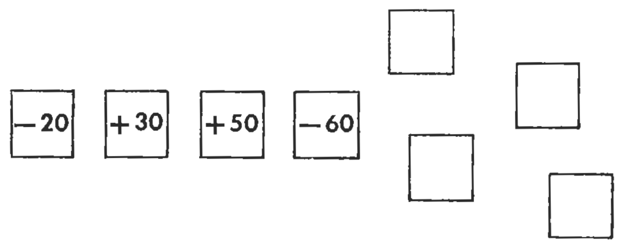
● Use page 75 to provide practice in computing differences related to sums of 15 through 18. Follow a procedure similar to that suggested for the preceding two pages before assigning the exercises for independent work.

● Use page 76 to give the children an opportunity to test their ability to compute using upper decade subtraction facts. When the page has been completed, ask individual children to tell how they computed a given difference.

Although a particular way to compute such differences has been developed, encourage the children to show and tell about any other way they may have used. If a child's method works, it must be accepted. Sharing ideas in this way will be an enriching experience for all members of the class. Encouraging a child to try and discover new ways to compute will help him extend his knowledge about numbers.

Supplemental Experiences

■ Make four numeral-cards: −20, +30, +50, and −60. Write the numerals 60 through 90 on four other cards. Place the cards on the bulletin board as shown.



Ask a child to choose one of the cards that are face down. Explain that the numeral on this card indicates his starting number. He must think of his numeral as coming in front of the −20 on the bulletin board and compute the difference that is formed. For example, if the child selected 70, he must compute 70 − 20. He then must use the computed difference (50) and compute the sum 50 + 30. He is to continue in this way until he has used all four numerals on the board. The child who chooses 70 will set up the following computations: 70 − 20, 50 + 30, 80 + 50, 130 − 60.

When the child has completed his assignment, have him place his card beside the bulletin board and select another child to continue the game.

Other numeral-cards may be used in place of those suggested. The only requirement is that the children be able to work with the number combinations used.

■ Play a riddle game. Present questions similar to the ones below.

If I subtract 700 from this number, I get 400. What is my number?

If I subtract 800 from 1300, what number do I have?

If I subtract 500 from Zap, I will have 900. What number is Zap?

One part of a set of 1200 is a set of 500. I am the other part. I am a set of how many objects?

■ To provide additional practice with higher decade subtraction in terms of thousands, make a series of tagboard cards as illustrated below.

14000 − 7000 = □	(10000 − 7000) + 4000 = □
12000 − 5000 = □	(10000 − 5000) + 2000 = □
13000 − 6000 = □	(10000 − 6000) + 3000 = □

Give each child a card. Ask a child who has an equation that shows only a difference to show his card to the class. Ask who has the card that shows the thinking steps in the computation of the given difference. Have this child bring his card forward and hold it next to the first child's card. Have the class state the number represented by the □ on each card.

Continue in this way until all of the cards have been matched.

KEY IDEA

Practice makes perfect.

Scope

To review upper decade addition and subtraction facts in terms of tens, hundreds, and thousands.

To extend the children's view of computation of sums of ones and tens.

To strengthen the children's understanding of the rearrangement property.

To provide practice in computing the sums of three addends.

To continue to develop the children's abilities to work with story exercises.

Fundamentals

The Hindu-Arabic numeration system allows up to 9 ones in the ones place. One more than 9 is 10 which is expressed by 1 in the tens place. Children need a great deal of experience in going from one digit to two digits in computing sums and in going from two digits to one digit in computing differences.

In the first grade, the child perceived 10 only as ten ones. In the second grade, he became aware of a 10 quantity in such examples as the 10-cent value of a dime. The child dealt with a number in terms of tens and ones when only 9 ones were allowed. An analogy with money helped him understand the computation problem; and this analogy is reviewed in this section.

NUMERAL	MONEY
He has up to 9 ones.	He has up to 9 pennies.
He has up to 9 tens.	He has up to 9 dimes.
He has up to 9 hundreds.	He has up to 9 dollars.

The numeration system presents the challenge of computing with the limitation on upper decade facts. How can the child show ones plus ones if only 9 ones and 1 ten are available? He computes the sum 9 + 1 by adding 1 ten and subtracting 9 ones.

$$\begin{array}{rcl} 9 = & & 0 \text{ tens} + 9 \text{ ones} \\ +\ 1 = & & +\ 1 \text{ ten} - 9 \text{ ones} \\ \hline 10 = & & 1 \text{ ten} + 0 \text{ ones} \end{array}$$

If the child wishes to subtract 1 from 1 ten, he subtracts the 10 and adds 9 ones.

$$\begin{array}{rcl} 10 = & & 1 \text{ ten} + 0 \text{ ones} \\ -\ 1 = & & -\ 1 \text{ ten} + 9 \text{ ones} \\ \hline 9 = & & 0 \text{ tens} + 9 \text{ ones} = 9 \end{array}$$

It helps the child understand the number situation better if he uses a dime (10) to pay 1 cent (−1). He must give up the dime (−10) and receive 9 pennies (+9). The child's ability to compute will be tremendously improved when he realizes that 1 ten (10) is 10 ones (10)—it is not necessary to do anything to 10 when he is interested in tens or interested in ones.

In this section the child will explore the computation of sums such as 90 + 70 in terms of adding 10 tens and subtracting 3 tens.

$$90 + 70 = (90 + 100) - 30$$

The child will be given an opportunity to apply his understanding of the rearrangement property in terms of tens, hundreds, and thousands as he works with sums of more than two addends. The way he pairs the addends when computing a sum such as 300 + 400 + 200 + 500 continues to be a matter of his choosing.

In work with story exercises, the child will be encouraged to decide upon the relationship between the numbers in the situation. He will be given practice in putting the numbers into an equation to show number relationships. As a final step, the child will perform any computation required in order to get the standard name for a number.

Readiness for Understanding

Ability to count tens, hundreds, and thousands.

Ability to write the numerals for tens, hundreds, and thousands.

Knowledge of addition and subtraction combinations of sums through 18.

Understanding of the properties of the set of whole numbers with respect to addition.

Awareness of the inverse relationship between addition and subtraction.

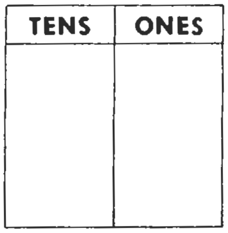
Developmental Experiences

for flannel board
tagboard disks
(simulated coins)
2 tagboard
latticework frames
cards: HUNDREDS, TENS,
and ONES

Countingmen
pocket chart
cards: sets of 10,
100, 1000
felt-tip pen

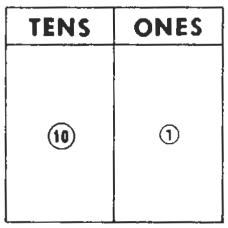
for each child
construction paper
crayons
bottle caps or disks
plastic numerals,
symbols, and
line segments

▶ Give a 9 by 12-inch sheet of heavy construction paper to each child. Have the children use a black crayon to partition and label their papers as shown.

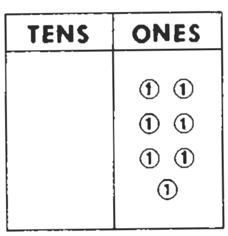


Give each child 10 bottle caps (or disks of heavy paper of comparable size). Have each child use his black crayon to label 9 of the caps or disks with the numeral 1 to simulate pennies, and to label 1 cap or disk with the numeral 10 to simulate a dime.

Write $11 - 4 = \square$ on the chalkboard. Have the children use their pennies and dimes to represent 11.



Then have them show $11 - 4$. When each child has shown this difference, select someone to explain his computation of $11 - 4$. He may say that he could not subtract 4 from 1, so he subtracted it from 10. He then added 6 and 1.



Write on the chalkboard the following equations to summarize the thinking steps. Ask the child to fill in the appropriate number.

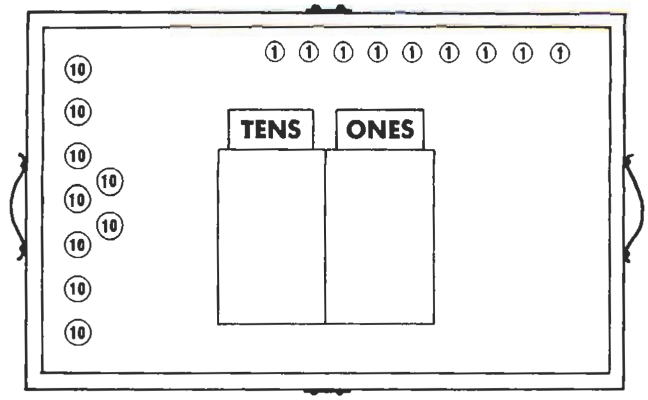
$$11 - 4 = (10 - \_\_) + \_\_$$
$$11 - 4 = \_\_ + 1$$
$$11 - 4 = \_\_$$

Adapt this procedure to the differences $11 - 5$, $12 - 8$, $16 - 9$, $12 - 3$, and $11 - 2$.

▶ Help the children review the computation of sums of ones and tens—the children may intuitively realize the limitations of our place-value system. Make the following items for use on the flannel board:

9 disks, colored copper and labeled 1¢,
9 disks, colored silver and labeled 10¢,
a latticework frame with 2 openings,
2 cards labeled ONES and TENS.

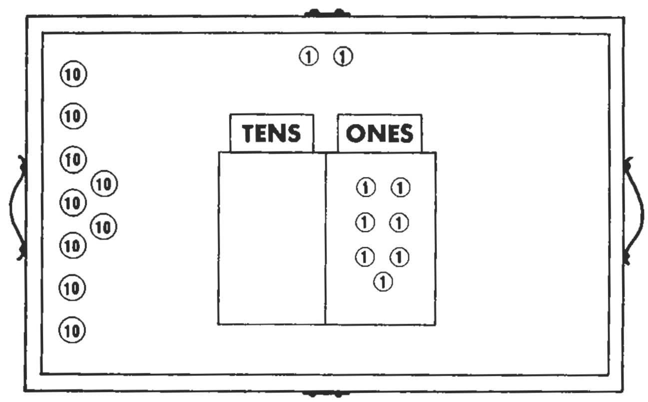
Set up the flannel board as shown.



Explain to the children that they are going to play a game with the model money. The rules of the game are very strict—you may never use more than 9 ones and 9 tens to show the computation of a sum.

Write $7 + 9 = 16$ on the chalkboard and ask a child to illustrate the computation steps he would use. Remind him that he may place no more than 9 pennies in the ONES opening and 9 dimes in the TENS opening to show the computation.

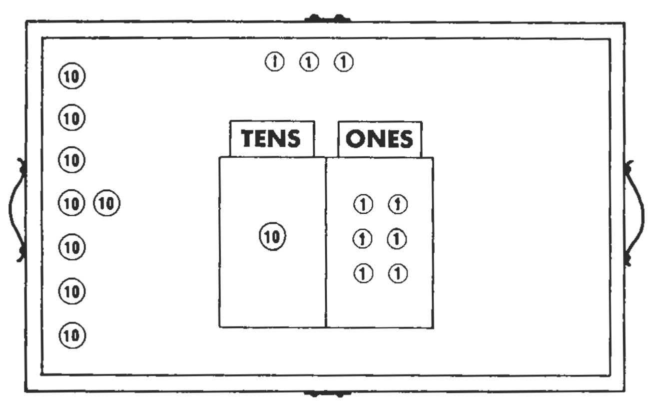
The child probably will first place 7 pennies in the frame.



If he is puzzled about how to add 9 (since only 2 pennies are available), let a volunteer help him with the computation as follows.

Put a dime in the TENS opening.

Take 1 penny back in change, since a dime is 1 penny more than is needed. This leaves 1 dime and 6 pennies, a value in cents of $10+6$ or 16. $7+9$ is 16.



Have 1 or 2 other children repeat the demonstration. Then summarize the thinking steps with the class. Write the following equations on the board as the children discuss each idea.

$$7 + 9 = 7 + 10 - 1$$
$$7 + 9 = 10 + 7 - 1$$
$$7 + 9 = 10 + 6$$

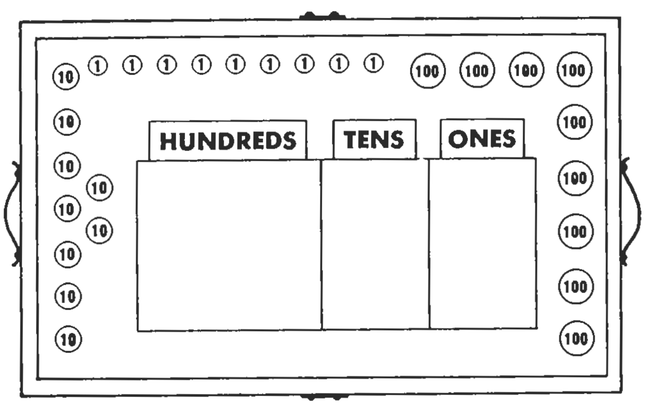
The class should know that adding 10 and subtracting 1 is the same as adding 9.

Adapt this procedure to $9 + 7$, $7 + 8$, $8 + 7$, $5 + 7$, and $7 + 5$. Have the computation steps demonstrated at least twice for each computed sum.

▶ Extend the previous activity to a review of upper decade sums in terms of tens and hundreds. Make the following items for use on the flannel board:

- 9 large disks, colored silver and labeled 100¢,
- a latticework frame with 3 openings,
- a card labeled HUNDREDS.

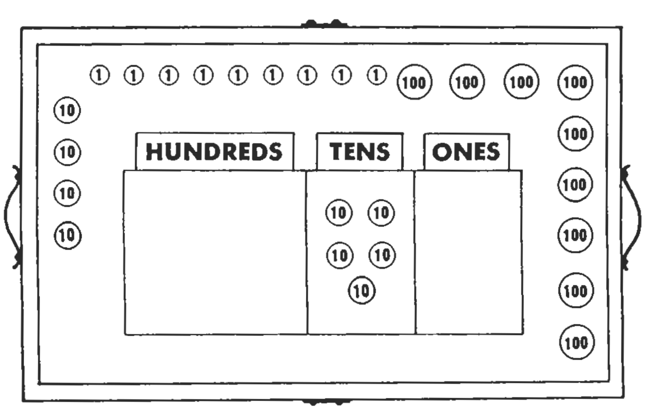
Set up the flannel board as shown.



Write $50 + 80$ on the chalkboard. Remind the children that they may only use the model coins that are displayed on the board to show their computation.

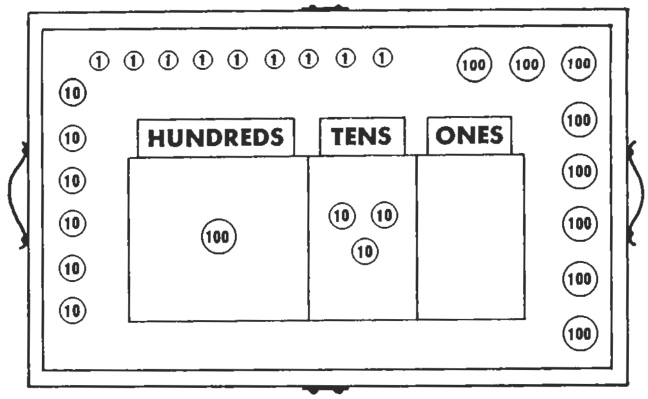
A child may explain his computation in the following way.

Put 5 tens (5 dimes) in the TENS opening.



Put 10 tens (1 dollar) in the HUNDREDS opening, since there are not enough dimes to add 8 more to the 5 dimes.

Remove 2 tens (2 dimes) in change. This leaves 10 tens + 3 tens or 13 tens.



$100 + 30$ is 130.
$50 + 80 = 130$.

Have 1 or 2 children repeat the demonstration. Then adapt this procedure to the computation of $130 - 80$. Explain that you want the children to undo the addition they just completed. Write $130 - 80$ on the chalkboard and ask a child to demonstrate the computation. He should proceed in the following way.

Begin with 1 hundred and 3 tens in the frame.

Remove 1 hundred, since it is not possible to remove 8 tens.

Put 2 tens back in the TENS opening, since 2 tens too many were subtracted.

This results in 5 tens.

Summarize the thinking steps involved. Write the following equation on the chalkboard.

$$130 - 80 = (130 - 100) + 20$$
$$= 30 + 20$$
$$= 50$$

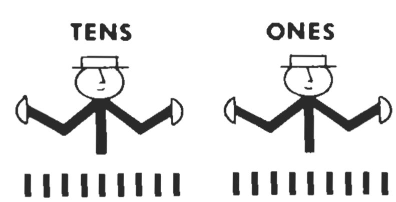
The children should know that subtracting 10 tens and adding 2 tens is the same as subtracting 8 tens.

Adapt the described procedure to $80 + 50$ and $130 - 50$. Continue the activity with the sums and differences related to $90 + 60$, $30 + 80$, and $70 + 60$.

▶ Direct the children to use the Countingmen to show the computation of sums such as the following.

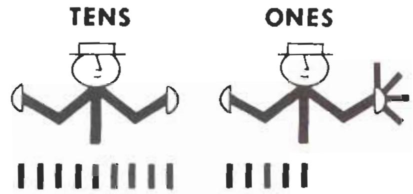
$4 + 9$ $\qquad$ $5 + 7$ $\qquad$ $9 + 8$

These sums should be written on the chalkboard and considered one at a time. Tell the children that they may never use more than 9 ones or 9 tens to show their computation. Place 9 fingers along the base of each Countingman.

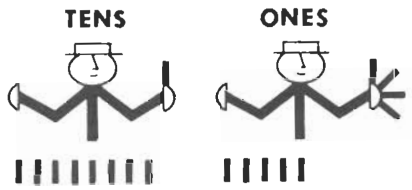


A child may show and explain the computation of 4 + 9 as follows.

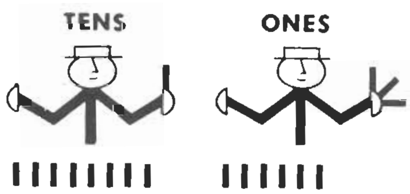
Put 4 fingers on the Ones-man.



Since 9 ones are not available to add to 4, add 1 ten instead.



Because 1 too many was added to 4, remove 1 of the 4 fingers on the Ones-man. This leaves 10 + 3 fingers.



The class should know that 4 + 9 is 10 + 3 or 13. Continue in this way with the other sums written on the chalkboard.

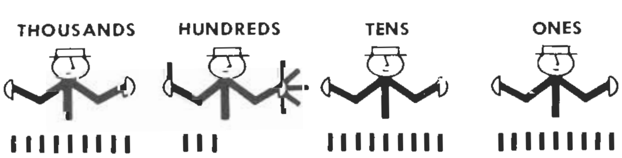
Next direct the children to show the computation of sums such as these:

600 + 800 900 + 200 700 + 900

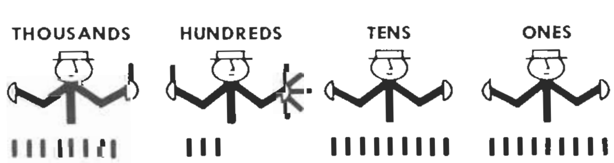
Tell the children that the rule of the game is to never use more than 9 ones, 9 tens, 9 hundreds, or 9 thousands to demonstrate the computation. Along the base of each Countingman place 9 fingers ready to be used whenever they are needed for the computation. (The Countingmen may be drawn on the chalkboard if they are not available.)

A child might show and explain the computation of 600 + 800 in the following way.

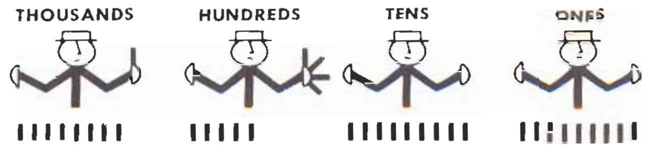
Put 6 fingers on the Hundreds-man to represent 600.



Because 8 hundreds are not available to add to 6 hundreds, add 1 thousand instead. This is shown by placing a finger on the Thousands-man.



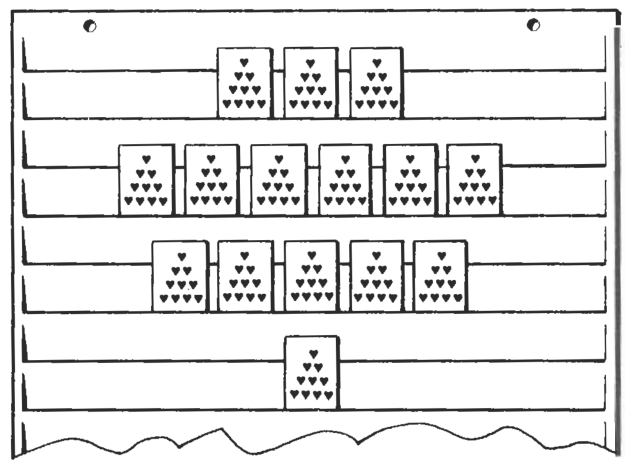
Since 200 too many was added, remove 2 of the 6 fingers on the Hundreds-man. This leaves 1000 + 400 fingers.



The class should realize 600 + 800 is 1000 + 400 or 1400.

Follow this procedure for the other sums on the chalkboard.

▶ Place in the pocket chart 15 ten-cards as shown.



Tell the class to study the parts (subsets) of this union of four sets from top to bottom and to name a sum that represents this particular union. (30 + 60 + 50 + 10) Direct the children to use their plastic numerals, operation symbols, and line segments at their desks to build the algorism for the computation of this sum. Then guide the class to examine the four subsets from bottom to top, name a sum that represents this way of viewing the set picture, and build on their desks the algorism for the computation of this sum.

Continue this procedure; have the children record and compute sums for sets of ten-cards, sets of hundred-cards, and sets of thousand-cards partitioned in the following ways.

40 + 20 + 10 + 50
500 + 300 + 100 + 200
2000 + 5000 + 1000 + 4000
300 + 400 + 100 + 900
10 + 60 + 40 + 30

▶ Have the class help create stories that involve tens, hundreds, or thousands. The stories may be factual or of an imaginative or humorous nature. Record the story as the children contribute ideas. You might begin the activity with the following story.

A Fish Story

Ben and Tom caught night crawlers for bait.
Ben caught 6000 night crawlers.
Tom caught 7000.

Help the children formulate questions about the facts in the story, for example:

> How many night crawlers did the boys catch?
> How many night crawlers did Ben catch?
> How many night crawlers did Tom catch?
> Tom caught more night crawlers than Ben. How many more?

Record the questions on the chalkboard as they are suggested. Then let the class consider each question in turn. Ask a child to write on the board a placeholder equation that represents the first question— 6000 + 7000 = □ or □ = 6000 + 7000. Have him tell about the numbers in his equation: the number of night crawlers Ben caught plus the number of night crawlers Tom caught is the number of night crawlers both boys caught. The count for this number is 13000.

With questions such as the second one, a child may write any one of the following equations on the chalkboard.

$$13000 = 7000 + (13000 - 7000)$$
$$13000 = 7000 + \square$$
$$13000 - 7000 = \square$$

Have him explain whichever equation he may write.

Continue in a similar way with the other questions that may have been suggested. Adapt this procedure to other stories created by the children.

Pages 77 through 80

● Page 77 gives the children an opportunity to test their ability to compute upper decade addition and subtraction combinations in terms of tens, hundreds, and thousands. When the page has been completed, let individual children tell how they computed a given sum or difference. The whole page need not be done at one time.

● Page 78 reinforces the idea that a given number may be expressed as a sum or difference in many ways. Use this page as a discussion page. After the children have read the introductory statement, instruct them to write the answers to the questions and then discuss each question in turn. Encourage the children to express their ideas in their own words. For exercise 1, the children should bring out the idea that any sum or difference of 7 may be on their lists. The secret about the list in exercise 2 is that the given differences are the counting numbers in order from 1 through 7. A secret about the list in exercise 3 is that each of the numbers 3 through 6 is expressed first as a sum and then as a difference. The secret about the list in exercise 4 is that each sum or difference after the first, is two more than the number just before it. Encourage the children to be imaginative in making their own lists for exercise 5. Let them exchange papers to see whether or not their classmates can discover the secret of their lists.

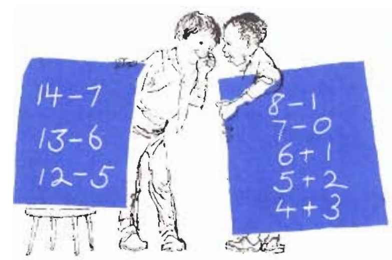
Name ______________________

Compute.

1. 400 + 700 *1100*	2. 8000 + 5000 *13000*	3. 1200 − 700 *500*	4. 600 + 900 *1500*
5. 130 − 40 *90*	6. 9000 + 9000 *18000*	7. 15000 − 8000 *7000*	8. 120 − 30 *90*
9. 5000 + 7000 *12000*	10. 1500 − 900 *600*	11. 600 + 800 *1400*	12. 16000 − 7000 *9000*
13. 800 + 800 *1600*	14. 15000 − 7000 *8000*	15. 1700 − 900 *800*	16. 1300 − 600 *700*
17. 400 500 300 + 200 *1400*	18. 3000 6000 5000 + 1000 *15000*	19. 50 20 40 + 20 *130*	20. 50 30 50 + 10 *140*
21. 40 + 80 *120*	22. 1400 − 500 *900*	23. 8000 + 9000 *17000*	24. 1600 − 900 *700*
25. 1200 − 600 *600*	26. 70 + 70 *140*	27. 11000 − 3000 *8000*	28. 50 + 60 *110*

B-77

Jim and Tom made a list of sums and differences. They said that there was a secret about their list.



1. What other sums and differences might be on their lists? *Any sum or difference of 7.*

2. What is the secret about this list?
 2 − 1, 3 − 1, 4 − 1, 5 − 1, 6 − 1, 7 − 1, 8 − 1
 *The counting numbers from 1 through 7 are in order.**

3. What is the secret about this list?
 2 + 1, 4 − 1, 2 + 2, 5 − 1, 2 + 3, 6 − 1, 2 + 4, 7 − 1
 *First a sum and then a difference for 3 through 6 is given**

4. What is the secret about this list?
 10 − 8, 8 − 4, 4 + 2, 2 + 6, 6 + 4, 4 + 8, 8 + 6
 *It shows counting by 2's.**

5. Make your own list and let others discover your secret.*

reference page **Answers will vary.*

B-78

Name

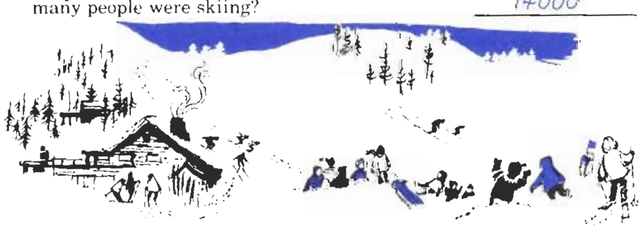
Compute.

1. 400 + 900 = 1300	2. 7000 + 8000 = 15000	3. 800 + 400 = 1200	4. 13000 − 7000 = 6000
5. 14000 − 6000 = 8000	6. 900 + 600 = 1500	7. 5000 + 8000 = 13000	8. 130 − 90 = 40
9. 2000 + 9000 = 11000	10. 1200 − 500 = 700	11. 14000 − 9000 = 5000	12. 6000 + 8000 = 14000
13. 700 + 400 = 1100	14. 1300 − 800 = 500	15. 6000 + 5000 = 11000	16. 1100 − 800 = 300
17. 160 − 80 = 80	18. 800 + 700 = 1500	19. 12000 − 8000 = 4000	20. 700 + 500 = 1200
21. 9000 + 5000 = 14000	22. 1400 − 800 = 600	23. 90 + 70 = 160	24. 1100 − 400 = 700

B-79

Answer the questions.

1. Many people visit Mt. Snow in winter. About 5000 people live there. One weekend there were about 13000 people in Mt. Snow. About how many people visited the town that weekend? 8000
2. A snow machine made 12000 tons of snow in December, but only 6000 tons in January. How much more snow did the machine make in December than in January? 6000 tons
3. The old ski trail is 8000 feet long. The new ski trail is 3000 feet longer than the old one. How long is the new trail? 11000 feet
4. Lookout Lodge sells about 6000 hotdogs and 11000 hamburgers each weekend. About how many hotdogs and hamburgers is that? 17000
5. On New Year's Day 6000 children, 4000 women, and 4000 men were skiing. How many people were skiing? 14000



B-80

● Page 79 provides the children with further opportunities to refine their skills in computing upper decade addition and subtraction combinations in terms of tens, hundreds, and thousands. Allow the children freedom of choice as to their methods of computation. You may wish to assign only part of this page for a given day's work.

● The story exercises on page 80 provide practice with addition and subtraction combinations in terms of thousands. Have children read the stories aloud. Then ask each child to write their answers to the questions. Let the class discuss their results. If disagreements arise, have children draw number strips to explain their reasoning.

Supplemental Experiences

■ Use this activity to give the children a review of sums 11 through 18. Tell the children that when their name is called they must answer with a sum for the number you mention. For example, you may say, "In this game, if he wants to play, Jack must name 13 another way."

Record each sum on the chalkboard.

11	12	13	14	15
9 + 2	9 + 3	4 + 9	9 + 5	6 + 9
8 + 3	8 + 4	8 + 5	6 + 8	7 + 8
7 + 4	5 + 7	6 + 7	7 + 7	9 + 6
6 + 5	6 + 6	9 + 4	5 + 9	8 + 7
2 + 9	3 + 9	5 + 8	8 + 6	
3 + 8	4 + 8	7 + 6		
4 + 7	7 + 5			
5 + 6				

16	17	18
7 + 9	8 + 9	9 + 9
8 + 8	9 + 8	
9 + 7		

Tell the class that no one may use a sum that another child has already used and that each addend of the sum they name must be a single digit.

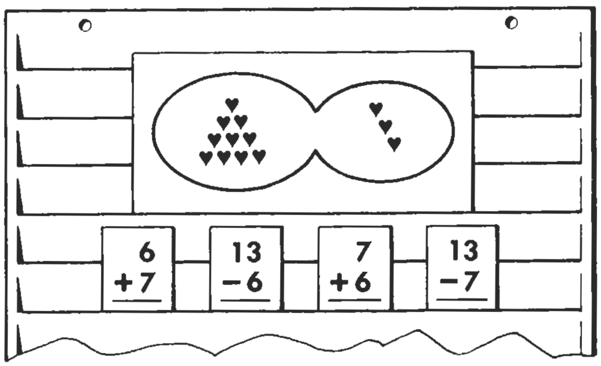
■ Some of the children may enjoy the challenge of mentally computing sums (and related differences) in tens, hundreds, and thousands. Present exercises such as the following:

90 plus 30 (pause) minus 80,
500 minus 100 (pause) plus 700,
400 plus 700 (pause) minus 300,
16000 minus 8000 (pause) plus 500,
14000 minus 6000 (pause) plus 7000,
60 plus 30 (pause) plus 80,
150 minus 60 (pause) minus 30.

The children will enjoy working faster as they become more proficient in mental computation.

■ Draw the unions of pairs of sets on tagboard cards. The number property of each of these set unions should be upper decade addition combinations. Write on another set of cards the upper decade addition and subtraction combinations that represent the set pictures. Give each child a card (or more than one if necessary).

Place one of the set-union cards in the pocket chart. Have all of the children who have an addition or subtraction combination that represents the set picture raise their hands. Have them come forward one at a time, place their card below the union of sets, and show and explain how the number combination represents the set union.



Continue in this way with other set unions and addition and subtraction combinations. Vary the activity by using sets of ten-cards, sets of hundred-cards, and sets of thousand-cards.

■ Play the game "Train" with equation cards such as $7 + \square = 16$, $\square - 6 = 7$, and $3 + 9 = \square$. One child is the conductor and stands by the desk of another child. The teacher holds up a card and both children try to name the number the $\square$ represents. If the conductor says the number first, he moves on to the next child. When another child responds with the correct number first, he becomes the new conductor. The winner continues around the room until he misses.

The children who give an incorrect number may be switched to a side track to study the combination they missed. They can rejoin the train by supplying the correct answer when another child misses.

UNIT 7
ADDITION AND SUBTRACTION
Pages 81 Through 104

OBJECTIVE

To introduce four-digit addition and subtraction.

The child combines his knowledge of addition combinations with the concept of place value to find the sum of four-digit addends. The child knows that if there are not enough ones, he subtracts from a ten; if there are not enough tens, he subtracts from a hundred. Similarly, if there are not enough hundreds, he will subtract from a thousand. He learns a useful application for his acquired addition and subtraction skills as he works with linear measure and money.

See Key Topics in Mathematics for the Primary Teacher: Addition; and The Inverse Operations—Subtraction and Division.

KEY IDEAS

Tens plus tens is tens. Hundreds plus hundreds is hundreds. Thousands plus thousands is thousands.

Not enough tens, subtract from a hundred. Not enough hundreds, subtract from a thousand.

Sum minus difference is addend.

Shortcuts are possible.

Inches plus inches is inches.

Cents plus cents is cents.

CONCEPTS

foot
inch
yard

KEY IDEA

Tens plus tens is tens. Hundreds plus hundreds is hundreds. Thousands plus thousands is thousands.

Scope

To review addition combinations.

To review the concept of place value.

To present the structural similarity in computing with ones, tens, hundreds, or thousands.

To develop greater understanding of the addition algorism.

Fundamentals

The computation of sums that have addends of 2 or more digits is based on the rearrangement property. The rearrangement property is the combined commutative and associative properties. In the computation of 435 + 364, note where the rearrangement property is involved.

$435 + 364$	
$= (400+30+5)+(300+60+4)$	Meaning of numerals
$= (400+300)+(30+60)+(5+4)$	Rearrangement property
$= 700+90+9$	Computation
$= 799$	Contracted form

The algorism arranges the numerals of the addends so that the hundreds are in a column, the tens in another column, and so forth.

$$\begin{array}{r} 435 \\ +\ 364 \\ \hline \end{array}$$

Note that the 400 and 300 are in a column, the 30 and 60 are in a column, and the 5 and 4 are in a column. By expanding the standard numeral, the idea is seen even more clearly.

$$\begin{array}{rcl} 435 &=& 400+30+5 \\ +\ 364 &=& 300+60+4 \\ \hline \end{array}$$

This expanded notation is a very useful pedagogical device to help the child perceive the meaning of the standard numeral. When the child can immediately recognize the meaning of a numeral, he no longer needs expanded notation.

The basic idea of computation—the idea of adding thousands to produce thousands, adding hundreds to produce hundreds, adding tens to produce tens, and adding ones to produce ones—is so simple that the child would easily master computation if it were not for the "carrying" problem. About half the time the sum of two single-digit addends is computed into a two-digit result. Consider 3658 + 4796.

$$\begin{array}{rcl} 3658 &=& 3000+\ 600+\ 50+\ 8 \\ +\ 4796 &=& \underline{4000+\ 700+\ 90+\ 6} \\ & & 7000+1300+140+14 = \\ & & 7000+1000+300+100+40+10+4 = \\ & & 8000+400+50+4 \end{array}$$

The computation of thousands results in 7000; the computation of hundreds results in 1300 which is 13 hundreds but is also 1000 + 300; so another 1000 is involved (7000 + 1000). The computation of tens results in 140 which is 14 tens but is also 100 + 40, so another 100 is involved (300 + 100). The computation of ones results in 14 which is 14 ones but is also 10 + 4, so another 10 is involved (40 + 10). The algorism takes care of the rearrangement.

$$\begin{array}{rcl} 3658 &=& 3000 + \ 600 + \ 50 + \ 8 \\ \underline{+\ 4796} &=& \underline{4000 + \ 700 + \ 90 + \ 6} \\ 7000 & & 7000 + 1300 + 140 + 14 \\ 1300 & & \\ 140 & & \\ \underline{14} & & \\ 8454 & & \end{array}$$

Readiness for Understanding

Knowledge of basic addition facts.

Knowledge that four-digit numerals name thousands plus hundreds plus tens plus ones.

Developmental Experiences

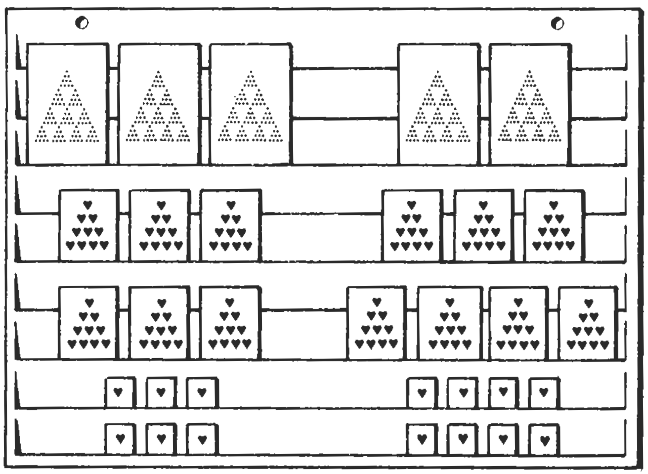
for flannel board

72 disks (18 of each diameter: $\frac{3}{4}''$, $1''$, $1\frac{1}{4}''$, and $1\frac{1}{2}''$)

pocket chart

cards: sets of 1, 10, 100, and 1000

▶ Write 366 + 278 on the chalkboard. Tell the children that you want them to help you compute this sum. Begin by representing the two addends with set-cards for 100, 10, and 1 in the pocket chart.



Have the children give the sum of ones (6 + 8), the sum of tens (60 + 70), and the sum of hundreds (300 + 200). Group the cards together to illustrate 300 + 200, 60 + 70, and 6 + 8. Record on the chalkboard the ideas discussed up to this point.

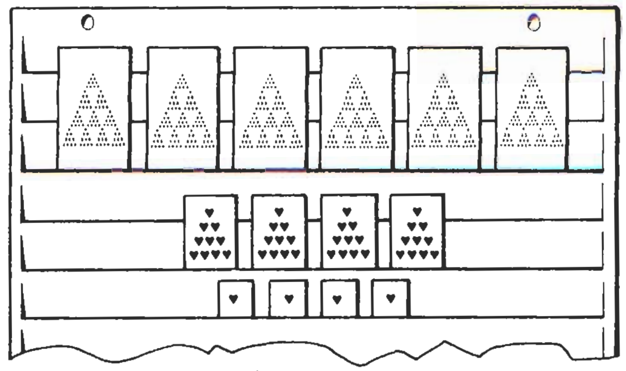
$$366 + 278 = 300 + 200 + 60 + 70 + 6 + 8$$

Direct the class to find the sum of ones, the sum of tens, and the sum of hundreds recorded in the equation. Then tell the children to compute the hundreds (500), tens (130), and ones (14). Record these sums below the equation on the board.

$$\begin{aligned} 366 + 278 &= 300 + 200 + 60 + 70 + 6 + 8 \\ &= 500 + 130 + 14 \end{aligned}$$

Point out that in adding ones the result was tens and ones; remind them that the sum of ones is often tens and ones (14 is 10+4). To help the children visualize the idea that ten ones is one ten, exchange 10 of the one-cards for a ten-card. Make it clear to the children that 10 ones is 1 ten whether or not this exchange is made. The advantage in using a ten-card is that it is easier to see the tens.

Now ask the children to study the result of adding tens. Remind them that the sum of tens is often hundreds and tens (130 is 100+30). Exchange 10 of the ten-cards for a hundred-card and make it clear that 10 tens is 1 hundred whether or not this exchange is made. Explain that by using a hundred-card it is easier to see hundreds.



Record the idea that 14 is 10+4 and that 130 is 100+30.

$$\begin{aligned} 366 + 278 &= 300 + 200 + 60 + 70 + 6 + 8 \\ &= 500 + 130 + 14 \\ &= 500 + 100 + 30 + 10 + 4 \end{aligned}$$

Let the class compute and record the sum of hundreds (500 + 100) and the sum of tens (30 + 10).

$$\begin{aligned} 366 + 278 &= 300 + 200 + 60 + 70 + 6 + 8 \\ &= 500 + 130 + 14 \\ &= 500 + 100 + 30 + 10 + 4 \\ &= 600 + 40 + 4 \end{aligned}$$

As a final step, have the class give the standard name for 600+40+4. (644) Record the response.

$$\begin{aligned} 366 + 278 &= 300 + 200 + 60 + 70 + 6 + 8 \\ &= 500 + 130 + 14 \\ &= 500 + 100 + 30 + 10 + 4 \\ &= 600 + 40 + 4 \\ &= 644 \end{aligned}$$

Continue in a similar way to discuss and record the computation of 635 + 248 and 694 + 863. With 694 + 863 the number of hundreds that result from computing 6 hundreds plus 8 hundreds is 14 hundreds. One of the key ideas to be pointed out at this time is that hundreds plus hundreds is hundreds. To record the computed number of hundreds, have the children write 1400 (fourteen hundreds).

Next, tell the children to use set cards to illustrate and explain the computation of 3475 + 6839. Follow a procedure similar to that described in the preceding paragraphs and record on the chalkboard the ideas as they are discussed.

$$\begin{aligned} &3475 + 6839 \\ &\quad = 3000 + 6000 + 400 + 800 + 70 + 30 + 5 + 9 \\ &\quad = 9000 + 1200 + 100 + 14 \\ &\quad = 9000 + 1000 + 200 + 100 + 10 + 4 \\ &\quad = 10000 + 300 + 10 + 4 \\ &\quad = 10314 \end{aligned}$$

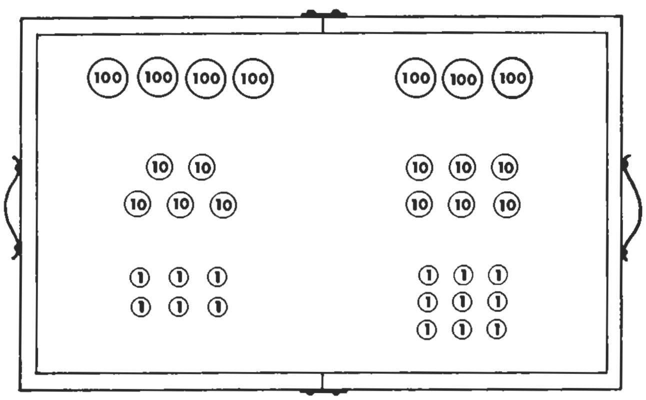
Continue in a similar way to discuss and record the thinking steps involved in computing 5647 + 2934 and 4593 + 7252. With 4593 + 7252, the children should note that the sum of ones results in ones alone, the sum of tens results in hundreds and tens, the sum of hundreds results in hundreds alone, and the sum of thousands results in thousands alone.

▶ Make 72 tagboard disks to use on the flannel board —18 of each of the following diameters: $\frac{3}{4}$, 1, $1\frac{1}{4}$ and $1\frac{1}{2}$ inches. Label the sets of disks 1, 10, 100, and 1000 respectively.

Write the following example on the chalkboard.

$$\begin{array}{rl} 456 = & 400+50+6 \\ +\ 369 = & \underline{300+60+9} \end{array}$$

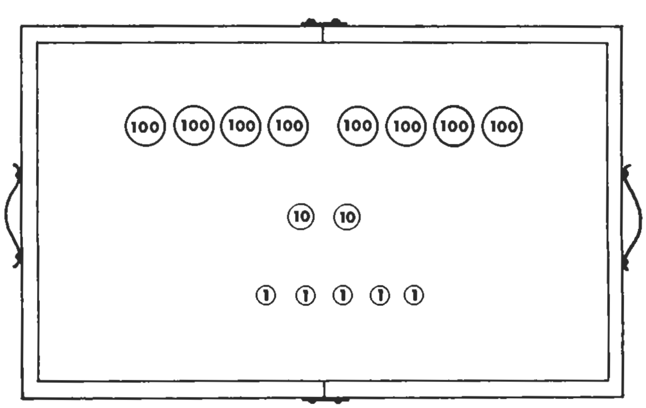
Select a child to use disks to represent on the flannel board the two addends of this sum.



Ask the children to name the sums of hundreds (400 + 300), tens (50 + 60), and ones (6 + 9). Tell a second child to group the disks together to show the sums. Have the class compute the sums of hundreds (700), tens (110), and ones (15). Let a third child record the partial sums while you write the results in expanded numeral form.

$$\begin{array}{rl} 456 = & 400 + \ \ 50 + \ \ 6 \\ +\ 369 = & \underline{300 + \ \ 60 + \ \ 9} \\ \overline{700} \leftrightarrow & 700 + 110 + 15 \\ 110 \leftarrow & \\ \underline{15} \leftarrow & \end{array}$$

Remind the class that the computation of the ones resulted in tens and ones: 15 is 10+5. Have a child exchange 10 one-disks for 1 ten-disk. Be sure the children understand that this exchange makes it easier to see tens. Then have the class note that the computation of tens resulted in hundreds and tens: 110 is 100+10. Instruct some child to put 1 hundred-disk on the flannel board and to remove 10 ten-disks.



Ask the class how many hundreds, how many tens, and how many ones are in the sum 700 + 110 + 15. Have the standard numeral for 8 hundreds plus 2 tens plus 5 ones recorded.

$$\begin{array}{r} 456 \\ +\ 369 \\ \hline 700 \\ 110 \\ 15 \\ \hline 825 \end{array}$$

Help the children review the thinking steps in the computation of 456 + 369.

400+300+50+60+6+9 is 700+110+15
700+110+15 is 800+20+5

Continue to let the children use disks to illustrate and explain the computation of sums such as 864 + 34, 7246 + 2466, 5199 + 862, and 3657 + 8524.

▶ Write on the chalkboard 3427 + 4596 in vertical form. Help the class investigate various ways of computing this sum. Ask individual children to show and explain how they computed. The following examples are only a few of the methods that the children may have used.

$$\begin{array}{r} 3427 \\ +\ 4596 \\ \hline 7000 \\ 900 \\ 110 \\ 13 \\ \hline 7000 \\ 1000 \\ 23 \\ \hline 8023 \end{array} \qquad \begin{array}{r} 3427 \\ +\ 4596 \\ \hline 13 \\ 110 \\ 900 \\ 7000 \\ \hline 8023 \end{array} \qquad \begin{array}{r} 3427 \\ +\ 4596 \\ \hline 7000 \\ 900 \\ 110 \\ 13 \\ \hline 8023 \end{array}$$

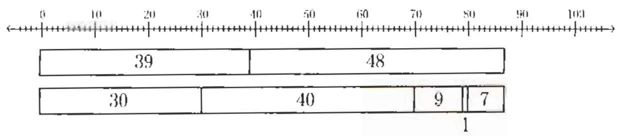
If none of the children computed as shown in the first example, demonstrate the computation for the class. Explain that looking for sums of 1000 may make the computation easier. The other two examples illustrate the work of a child who immediately added and recorded 8023.

Continue with 4336 + 2962, 5675 + 6285, and 3726 + 7648.

Name

UNIT 7 ADDITION AND SUBTRACTION

What is the computed sum?



39 + 48 = 30 + 40 + 9 + 8
= 70 + 10 + 7
= 87

39	= 30 + 9
+ 48	= 40 + 8
17	70 + 17
70	
87	

Compute.

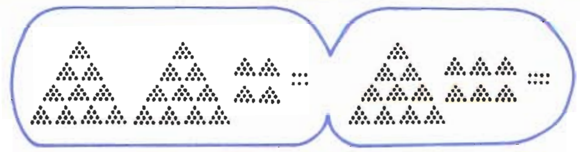
Exercise	Addends	Partial sums	Sum
1.	62 + 65	* 7, 120	127
2.	567 + 312	* 800, 70, 9	879
3.	354 + 157	* 11, 100, 400	511
4.	428 + 14	400*, 30, 12	442
5.	765 + 27	* 700, 80, 12	792
6.	364 + 398	* 12, 150, 600	762
7.	51 + 92	* 3, 140	143
8.	426 + 330	* 700, 50, 6	756

*Order of partial sums will vary.

reference page

B-81

Name



246	200 + 40 + 6
+ 168	100 + 60 + 8
14	300 + 100 + 14 = 414
100	
300	
414	

Compute.

Exercise	Addends	Partial sums	Sum
1.	189 + 392	* 11, 170, 400	581
2.	425 + 364	* 700, 80, 9	789
3.	357 + 672	* 9, 120, 900	1029
4.	532 + 638	* 1100, 60, 10	1170
5.	436 + 56	* 400, 80, 12	492
6.	461 + 787	* 8, 140, 1100	1248
7.	233 + 398	* 500, 120, 11	631
8.	601 + 298	* 9, 90, 800	899

*Order of partial sums will vary.

reference page

B-83

Compute.

Exercise	Addends	Partial sums	Sum
1.	196 + 603	700, 90*, 9	799
2.	62 + 72	4, 130*	134
3.	587 + 135	12, 110*, 600	722
4.	243 + 27	200, 60*, 10	270
5.	51 + 89	10, 130*	140
6.	372 + 216	500, 80*, 8	588
7.	357 + 29	300, 70*, 16	386
8.	83 + 75	8, 150*	158
9.	158 + 6	14, 50*, 100	164
10.	84 + 81	160, 5*	165
11.	20 + 847	7, 60*, 800	867
12.	479 + 543	900, 110*, 12	1022
13.	471 + 275	600, 140*, 6	746
14.	813 + 128	11, 30*, 900	941
15.	795 + 85	700, 170*, 10	880
16.	608 + 107	15, 700*	715

17. 6 tens + 7 + 5 tens + 4 = 12 tens + 1

*Order of partial sums will vary.

B-82

Pages 81 through 86

● Pages 81 through 86 provide practice in computing sums of two-digit addends, three-digit addends, and four-digit addends. Discuss the exercises at the top of page 81 with the children. Use the number strips and number line to clarify the specific computation. The remaining problems on page 81 and those on page 82 should be completed independently. When the children have completed these two pages, ask various children to explain their method for computing specific sums. With some sums, the children may observe that adding ones resulted in ones alone; with other sums adding ones resulted in tens and ones. The children may make similar observations regarding the addition of tens and the addition of hundreds.

● Discuss the example at the top of page 83 with the children. The exercises may be assigned to give the children an opportunity to test their ability to compute the given sums. Do not insist that every child list the partial sums before writing the standard numeral for the sum. Encourage the children to use the method with which they feel secure. The same procedure should be used on pages 84, 85, and 86.

Compute.

1.	2.	3.	4.
685	674	895	489
+ 781	+ 140	+ 379	+ 416
* 1300	* 4	* 1100	* 15
160	110	160	90
6	700	14	800
1466	814	1274	905

5.	6.	7.	8.
250	349	758	976
+ 432	+ 23	+ 507	+ 975
* 600	* 300	* 1200	* 1800
80	60	50	140
2	12	15	11
682	372	1265	1951

9.	10.	11.	12.
354	912	586	769
+ 480	+ 549	+ 203	+ 91
* 4	* 11	* 9	* 10
130	50	80	150
700	1400	700	700
834	1461	789	860

*Order of the partial sums will vary.

B-84

Compute.

1.	2.	3.	4.
5468	7137	5798	7249
+ 1765	+ 7893	+ 2468	+ 5082
* 13	* 14000	* 16	* 12000
120	900	150	200
1100	120	1100	120
6000	10	7000	11
7233	15030	8266	12331

5.	6.	7.	8.
5326	6243	1728	8676
+ 3689	+ 3525	+ 74	+ 3508
* 8000	* 8	* 12	* 11000
900	60	90	1100
100	700	700	70
15	9000	1000	14
9015	9768	1802	12184

9.	10.	11.	12.
2848	8679	5493	9589
+ 1037	+ 2138	+ 13	+ 7963
* 15	* 10000	* 5000	* 12
70	700	400	140
800	100	100	1400
3000	17	6	16000
3885	10817	5506	17552

*Order of partial sums will vary.

B-86

Name

3296 = 3000 + 200 + 90 + 6
+ 8953 = 8000 + 900 + 50 + 3
11000   11000 + 1100 + 140 + 9 = 12249
1100
140
9
12249

Compute.

1.	2.	3.	4.
1964	8743	1759	5193
+ 6970	+ 9898	+ 7154	+ 7250
* 7000	* 11	* 8000	* 3
1800	130	800	140
130	1500	100	300
4	17000	13	12000
8934	18641	8913	12443

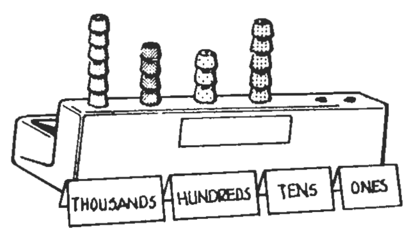
5.	6.	7.	8.
7324	8263	3249	2367
+ 161	+ 1427	+ 2360	+ 46
* 7000	* 10	* 5000	* 13
400	80	500	100
80	600	100	300
5	9000	9	2000
7485	9690	5609	2413

*Order of partial sums will vary.

B-85

Supplemental Experiences

■ Use the bead frame to strengthen the children's understanding of thousands, hundreds, tens, and ones. Label the columns on the bead frame thousands, hundreds, tens, and ones. Have a child use colored beads to represent 6435. He should place 6 beads in the thousands column, 4 beads in the hundreds column, 3 beads in the tens column, and 5 beads in the ones column.



Ask a child to show the expanded and contracted form for this number in an equation on the chalkboard.

$$6000 + 400 + 30 + 5 = 6435$$

Repeat the activity with other numbers from 1000 through 9999.

■ Write on tagboard cards sums of 2 four-digit addends. Place the cards in a pack.

1753 + 2699   7437 + 3929   6850 + 3578

Separate the class into two teams. Have a member of each team come forward, choose a card, and compute the sum on the chalkboard. Ask the class to judge who finished first and computed correctly. If a child computes correctly, he earns a point for his team; if he is first to finish correctly, he earns 2 points for his team. After all of the children have had an opportunity to participate, total the points for each team to determine the winner.

KEY IDEA

Not enough tens, subtract from a hundred. Not enough hundreds, subtract from a thousand.

Scope

To strengthen the child's understanding of the meaning of numerals.

To extend subtraction to four digits.

Fundamentals

Remember, addition combinations of 10 are most important. In an exercise such as $7 + 6 = \square$, the child may first think about the number needed to reach 10 and then go on from there.

Ten is the base of our numeration system. Whenever a child counts from 9 to 10 (or a multiple of 10) he notices that the number indicated by the ones digit goes from 9 to 0. In effect he adds 1 ten and subtracts 9 ones.

$$\begin{array}{rcl} 9 & = & 0 \text{ tens} + 9 \text{ ones} \\ \underline{+\ 1} & = & \underline{+\ 1 \text{ ten} - 9 \text{ ones}} \\ & & 1 \text{ ten} + 0 \text{ ones} = 10 \end{array}$$

In the exercise $7 + 6 = \square$, the child adds 10 and subtracts 4. This is adding 6.

$$\begin{array}{rcl} 7 & = & 0 \text{ tens} + 7 \text{ ones} \\ \underline{+\ 6} & = & \underline{+\ 1 \text{ ten} - 4 \text{ ones}} \\ & & 1 \text{ ten} + 3 \text{ ones} = 13 \end{array}$$

Combinations of 10 are equally important when working with subtraction. In subtracting, the process is reversed and the child learns that he subtracts 1 from 10 by subtracting 1 ten and adding 9 ones.

$$\begin{array}{rcl} 10 & = & 1 \text{ ten} + 0 \text{ ones} \\ \underline{-\ \ 1} & = & \underline{-\ 1 \text{ ten} + 9 \text{ ones}} \\ & & 0 \text{ tens} + 9 \text{ ones} = 9 \end{array}$$

In computing differences such as $13 - 6$, the child will learn to subtract 10 and add 4, thus subtracting 6.

$$\begin{array}{rcl} 13 & = & 1 \text{ ten} + 3 \text{ ones} \\ \underline{-\ \ 6} & = & \underline{-\ 1 \text{ ten} + 4 \text{ ones}} \\ & & 0 \text{ tens} + 7 \text{ ones} = 7 \end{array}$$

The children's knowledge of subtraction combinations related to sums of 10 is important in four-digit subtraction. Four-digit subtraction follows the same patterns as three-digit subtraction—tens minus tens is tens, hundreds minus hundreds is hundreds, and thousands minus thousands is thousands. Thus $7356 - 2225$ is easy to compute since 3 hundreds minus 2 hundreds is 1 hundred, and a thousand is not affected.

In other cases, the computation of hundreds in the hundreds place is impossible. Consider $7356 - 2625$. The child has previously been confronted with a similar challenge when there were not enough ones and not enough tens. He may remember that 7356 is $7000 + 356$ and subtract the 6 hundreds from a thousand, which is 10 hundreds.

$$\begin{array}{rcl} 7356 & = & 6000 + 1000 + 300 + 50 + 6 \\ \underline{-2625} & = & \underline{-2000 - \ \ 600 \qquad\quad - 20 - 5} \\ & & 4000 + \ \ 400 + 300 + 30 + 1 \end{array}$$

The extension of the subtraction algorism involves the use of 10 hundreds in the thousands place, which means 1 thousand is subtracted. Consider the following example.

$$\begin{array}{r} 7356 \\ \underline{-\ 2625} \end{array}$$

The ones are computed.

$$\begin{array}{r} 7356 \\ \underline{-\ 2625} \\ 1 \end{array}$$

Then the tens are computed.

$$\begin{array}{r} 7356 \\ \underline{-\ 2625} \\ 31 \end{array}$$

There are not enough hundreds in the hundreds place, so 6 hundreds will be subtracted from 10 hundreds in the thousands place.

$$\begin{array}{r} 6 \\ \not{7}356 \\ \underline{-\ 2625} \\ 31 \end{array}$$

The 4 hundreds and the 3 hundreds is 7 hundreds.

$$\begin{array}{r} 6 \\ \not{7}356 \\ \underline{-\ 2625} \\ 731 \end{array}$$

Finally, the thousands are easily subtracted to complete the algorism.

$$\begin{array}{r} 6 \\ \not{7}356 \\ \underline{-\ 2625} \\ 4731 \end{array}$$

Readiness for Understanding

Knowledge of three-digit subtraction.

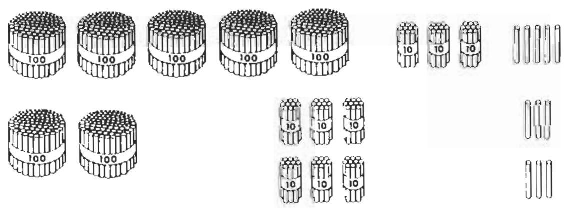
Developmental Experiences

for flannel board	
72 disks (18 of each labeled: 1, 10, 100, and 1000)	sticks
	strips of tagboard (1″ × 15″)
	strips of construction paper (1″×15″)
	Countingmen
	pins

▶ Write the difference 535 − 266 on the chalkboard.

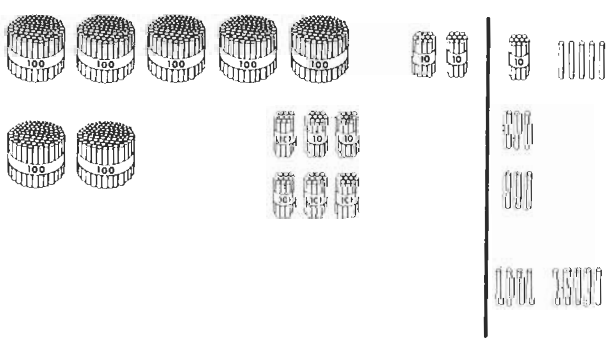
$$\begin{array}{r} 535 \\ \underline{-\,266} \end{array}$$

Have a child represent 535 on a table in front of the room with bundles of 100 sticks, bundles of 10 sticks, and single sticks. Ask another child to represent 266 in front of 535.

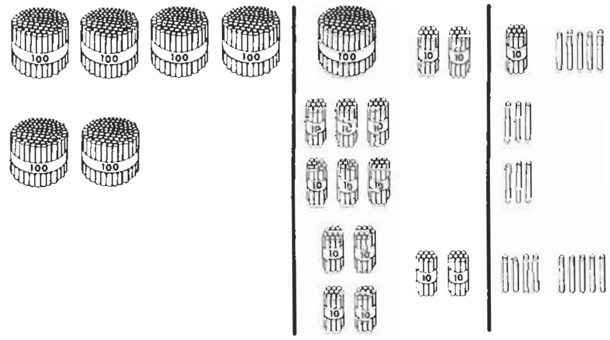


Tell the children to show the difference between 535 and 266 by placing the appropriate number of sticks in front of 266.

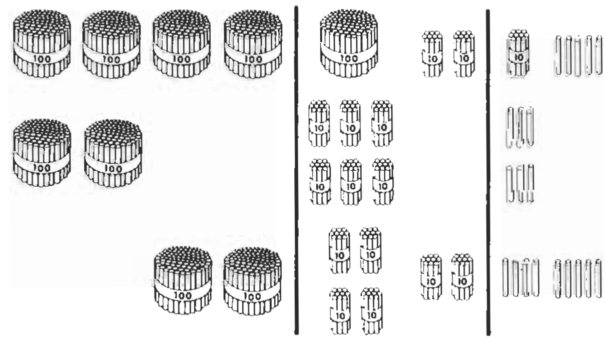
Ask the class whether or not a 10 must be used in the computation of the difference of ones. (yes) Ask a child to describe how he would compute the difference of ones. He may subtract 6 from one of the 3 tens and then add this difference (10−6 or 4) to 5 with the result 9. Illustrate with the sticks what the child just described. Repeat his description of the computation as the sticks are placed on the table. Place on the table a strip of tagboard to isolate the computed ones.



Now, ask the class whether or not 10 tens (1 hundred) must be used in the computation of the difference of tens. (yes) Have a child describe how he would compute the difference of tens. He may subtract 6 tens from 10 tens (one of the hundreds) and then add this difference, 10 tens − 6 tens (100−60) or 4 tens (40), to 2 tens (20) with the result 6 tens (60). Repeat the child's description of the computation as you place the appropriate sticks on the table. Then separate the computed tens from the hundreds with a strip of tagboard.



Ask another child to describe the computation of the difference of hundreds. He may say that since one of the 5 hundreds was used in the computation of the difference of tens, he must compute the difference between 4 hundreds and 2 hundreds. This difference is 400−200 or 200. Repeat the child's description of the computation while he places the bundles of 100 on the table.



Summarize the computation of the difference 535 − 266 by writing the following algorism on the chalkboard.

$$\begin{array}{rcl} 535 & = & 400+100+20+10+5 \\ \underline{-\,266} & = & \underline{-200-\ \ 60\qquad\ \ -\ \ 6} \\ & & 200+\ 40+20+\ 4+5 \end{array}$$

Let the children tell how the algorism describes what was done with the sticks.

Then let some child complete the algorism placed on the board at the beginning of the activity and explain each computational step. When he has completed this assignment, point to each digit in the computed difference (269), and have the children explain the difference to which each digit refers. The children's explanations may be recorded as shown.

$$\begin{array}{r} \overset{4}{\not 5}\overset{2}{\not 3}5 \\ \underline{-\,266} \\ 269 \end{array}$$

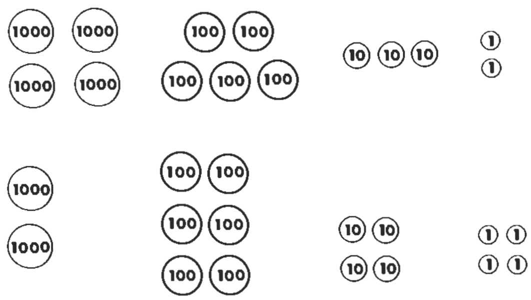
2 hundreds, the difference of hundreds 200 = 400−200	6 tens, the difference of tens 60 = (100−60)+20	9 ones, the difference of ones 9 = (10−6)+5

Have the children investigate the computation of 791 − 358, 977 − 525, and 927 − 642 in a similar manner.

Place on a table in front of the room the tagboard disks labeled 1000, 100, 10, and 1. (These disks were used in the second activity of this unit.) Write on the chalkboard the following difference.

```
  4532
− 2644
```

Have a child place disks on the flannel board to represent 4532. Ask another child to represent 2644.



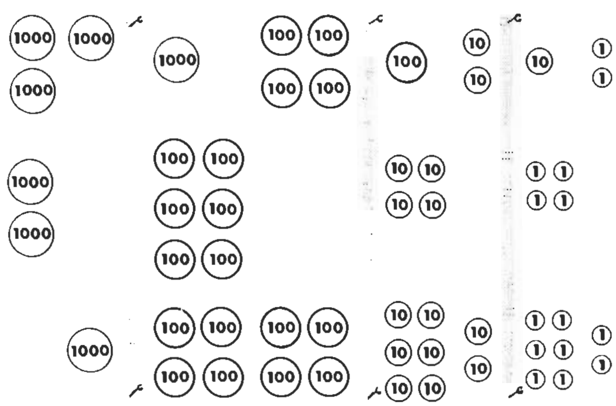
Help a third child use disks to show the computed difference 4532 − 2644. As the computation is demonstrated, help the class understand the following ideas.

When 4 is subtracted, a ten is used.

When 4 tens are subtracted, a hundred is used.

When 6 hundreds are subtracted, a thousand is used.

As the ones, tens, and hundreds are subtracted, pin yarn or strips of paper on the flannel board as shown. When the computation is completed, ask the child to read the computed difference 1888—he may say 1 thousand plus 8 hundreds plus 8 tens plus 8 ones.



Now do the computation on the chalkboard and describe each step by referring to the disks.

2
45~~3~~2
− 2644   10 − 4 is 6.
8   6 + 2 is 8.

42
4~~5~~~~3~~2
− 2644   10 tens − 4 tens is 6 tens.
88   6 tens + 2 tens is 8 tens.

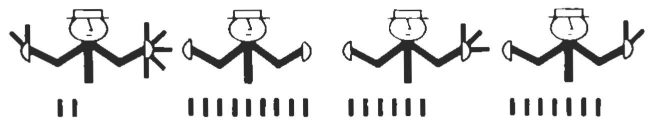
342
~~4~~~~5~~~~3~~2
− 2644   10 hundreds − 6 hundreds is 4 hundreds.
888   4 hundreds + 4 hundreds is 8 hundreds.

342
~~4~~~~5~~~~3~~2
− 2644
1888   3 thousands − 2 thousands is 1 thousand.

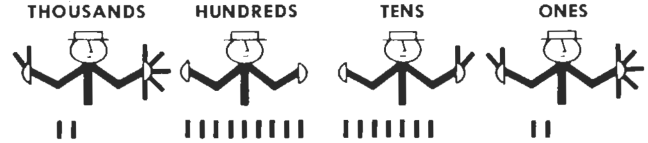
Compute the differences 6233 − 4765, 8594 − 7837, and 9148 − 6426 by using the disks and algorisms. Let the children do all or most of the work. If a child needs help, encourage class discussion to elicit help from the children.

Have one child compute 7032 − 4685 on the chalkboard while a second child demonstrates the difference using Countingmen. The child using the Countingmen may show and describe the computation as follows:

Place 2 fingers on the Ones-man, 3 fingers on the Tens-man, and 7 fingers on the Thousands-man to show 7032.



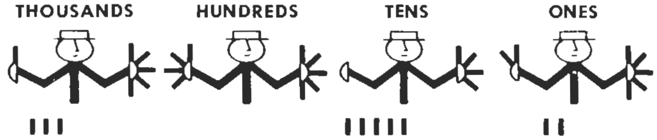
To subtract 5 from 2, remove 1 finger from the Tens-man and add 5 fingers to the Ones-man. (To subtract 10 and add 5 is the same as subtracting 5.)



The Countingmen show 7032 − 5 or 7027.

2
70~~3~~2
− 4685
7

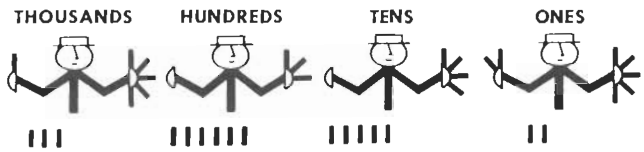
To subtract 8 tens, remove 1 finger from the Thousands-man, place 9 fingers on the Hundreds-man, and add 2 fingers to the Tens-man. The Thousands-man and Hundreds-man together had 70 hundreds. The subtraction of 100 left 69 hundreds (6 thousands + 9 hundreds). But subtracting 10 tens (1 hundred), was subtracting 2 tens more than necessary so 2 tens were added to the Tens-man. To subtract 10 tens and add 2 tens is the same as subtracting 8 tens.



The Countingmen show 7027 − 80 or 6947.

692
~~7~~~~0~~~~3~~2
− 4685
47

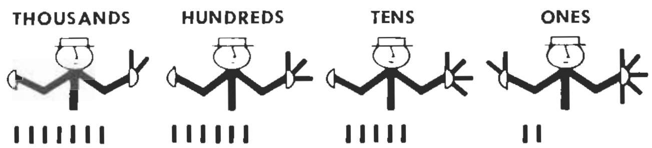
The subtraction of 6 hundreds (600) presents no problem. Remove 6 fingers from the 9 on the Hundreds-man.



The Countingmen show 6947 − 600 or 6347.

```
  692
 ~~7~~~~0~~~~3~~2
− 4685
  347
```

Finally, remove 4 fingers from the 6 on the Thousands-man (6000 − 4000).



The Countingmen show 6347 − 4000 or 2347. They also show 7032 − 4685 = 2347.

```
  692
 ~~7~~~~0~~~~3~~2
− 4685
 2347
```

Adapt this procedure to 5624 − 2737, 8604 − 4726, 7600 − 3728, 9000 − 6493, 4299 − 3172, 1926 − 1257, and 1000 − 999.

▶ Have a child come to the chalkboard, compute the difference 5432 − 2465, and explain his computation to the class. Tell him to record the results.

```
  432
 ~~5~~~~4~~~~3~~2
− 2465
 2967
```

Then ask him to point to each of the digits in the computed difference 2967 and explain to what it refers. This may help the children realize that the difference of thousands plus the difference of hundreds plus the difference of tens plus the difference of ones is 2000 + 900 + 60 + 7, or 2967.

Continue to have volunteers explain the computation of 7906 − 2378, 6883 − 4532, 6000 − 3462, and 8750 − 5427.

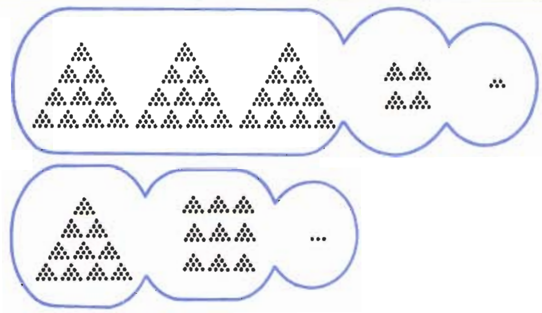
Pages 87 and 88

● Pages 87 and 88 provide continued practice with differences related to sums of 2 three-digit addends and sums of 2 four-digit addends. In some instances it will be necessary to use a ten to compute ones, to use a hundred to compute tens, and to use a thousand to compute hundreds.

Work and discuss the example at the top of each page with the class.

Assign the exercises as independent work. Assign as few or as many exercises at any one time as the children are able to complete.

Name



345 = 200 + 100 + 40 + 5
− 193 = − 100 − 90 − 3
100 + 10 + 40 + 2

```
  2
 345
−193
 152
```

Compute.

1. 468 − 223 = 245	2. 867 − 239 = 628	3. 639 − 285 = 354	4. 849 − 663 = 186	5. 975 − 535 = 440
6. 871 − 494 = 377	7. 819 − 568 = 251	8. 294 − 262 = 32	9. 712 − 423 = 289	10. 800 − 241 = 559
11. 584 − 291 = 293	12. 538 − 377 = 161	13. 374 − 165 = 209	14. 652 − 375 = 277	15. 234 − 182 = 52

B-87

What is the difference?

```
 5 7
6481
−3708
2773
```

2000 = 5000 − 3000
700 = 1000 − 700 + 400
70 = 70 − 0
3 = 10 − 8 + 1

Compute.

1. 4268 − 3743 = 525	2. 6472 − 4684 = 1788	3. 9397 − 3581 = 5816	4. 5162 − 4391 = 771
5. 8612 − 5948 = 2664	6. 9753 − 2551 = 7202	7. 6628 − 3459 = 3169	8. 8298 − 2666 = 5632
9. 6549 − 2179 = 4370	10. 7495 − 6154 = 1341	11. 8245 − 1157 = 7088	12. 7686 − 4935 = 2751
13. 6862 − 2269 = 4593	14. 3316 − 1457 = 1859	15. 6335 − 4278 = 2057	16. 9276 − 1243 = 8033

B-88

Supplemental Experiences

■ Construct several difference-cards. Some of the cards should be incorrect.

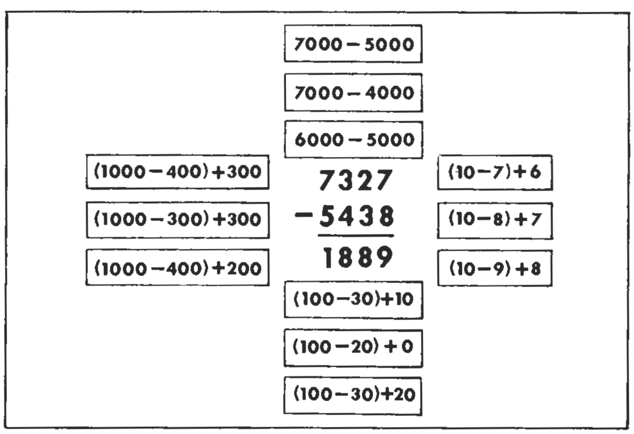
7252 − 3419 = 3743 | **3457 − 1629 = 1828**

6029 − 4271 = 2758

Separate the class into two teams. Tell a member from each team to choose a card, to place it on the chalktray, and to compute the difference on the chalkboard. When each child has completed his computation, direct him to show his card to the class and to tell whether or not the computed difference on the card is correct. If the computed difference on his card is incorrect, he must tell the class where the error was made—in the computation of ones, tens, hundreds, or thousands.

Each child who accomplishes the assignment correctly earns a point for his team. Continue the activity until all of the children have had a chance to participate. Then have the teams total their points to determine a winner.

■ Prepare a 12 by 18 inch tagboard sheet as shown.



Only one of the differences of thousands, one of the differences of hundreds, one of the differences of tens, and one of the differences of ones will be used in the subtraction exercise. Ask the children to decide which of the differences can be used to compute the difference of ones. Then ask them to name the differences they would use to compute the difference of tens, hundreds, and thousands. Check the decisions by asking one child to describe the computation. You may wish to let another child record the computation on the chalkboard.

Follow a similar procedure using other differences related to sums of 2 four-digit addends.

KEY IDEA

Sum minus difference is addend.

Scope

To provide practice in expressing differences.

To review the relationship between addition and subtraction.

To develop greater understanding of the addition and subtraction algorisms.

To emphasize guessing and testing as a means of discovery.

To use the relationship between subtraction and addition to find missing numerals.

Fundamentals

The reciprocal relationship between addition and subtraction is reviewed in this section. The idea that subtraction is the inverse of addition and addition is the inverse of subtraction should be clearly understood for speed and accuracy in computation. This reciprocal relationship is emphasized by the single picture that serves for both addition and subtraction.

$$1 + 1 = 2$$
$$2 - 1 = 1$$

Addition may be considered as a way of looking at subtraction. In subtraction, sum minus addend is difference. In addition, addend plus difference is sum.

SUM	
ADDEND	DIFFERENCE

784	
535	784−535

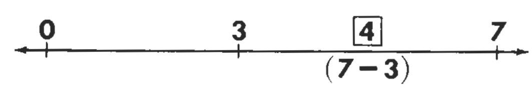
SUM		ADDEND		DIFFERENCE
784	−	535	=	(784−535)

ADDEND		DIFFERENCE		SUM
535	+	(784−535)	=	784

The child has solved equations such as:

$$3 + \square = 7$$
$$7 - 3 = \square$$

The number line, as shown, is a model for both equations.



The pattern of the equations and number line model is unchanged when three-digit or four-digit numerals are used.

In this section the child will take another look at reconstruction exercises. His first approach to finding missing numerals may be by guessing. If he tests his first guess and finds it incorrect, he is led to a second guess. This continues until he has discovered the correct numerals. Such trials are very useful and should be encouraged. As the addition and subtraction algorisms are better understood, the child should further develop his awareness of the relationship between addition and subtraction, and use this relationship as a more direct means of reconstruction. Consider the following exercise:

$$\begin{array}{r} 4\ 7\ 3 \\ +\ \square\ \square\ \square \\ \hline 8 \\ 1\ 2\ 0 \\ +\ \square\ 0\ 0 \\ \hline 8\ 2\ \square \end{array}$$

The child notices that $8 = 3 + \square$, $120 = 70 + \square$, and $800 = 100 + \square$. The inverse relation of addition to subtraction indicates that $\square = 8 - 3$, $\square = 120 - 70$, and $\square = 800 - 100$. This provides enough correct numerals for the exercise so that the child is able to determine the remaining numerals.

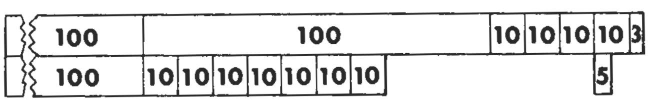
Readiness for Understanding

Knowledge of the addition and subtraction algorisms.

Developmental Experiences

number strips:	pocket chart
for 1 through 10	set-cards: 1, 10,
for 100	and 100
masking tape	felt-tip pen

▶ You will need number strips for 1 through 10 and for 100. Help a child fasten on the chalkboard strips to show 243. Then help him show 175 below 243.



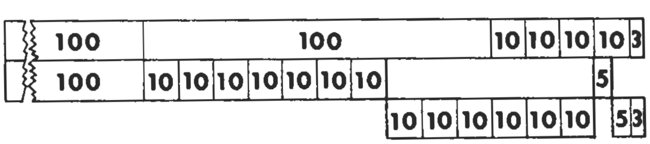
Ask what difference must be added to 175 to give 243, and record the children's response in an equation on the chalkboard.

$$175 + (243 - 175) = 243$$

Ask some child to compute the difference 243 − 175.

$$\begin{array}{r} 13 \\ \not{2}\not{4}3 \\ -\ 175 \\ \hline 68 \end{array}$$

As he works, have him describe his thinking steps to the class. Tell him to put on the chalkboard the strips that show the difference 243 − 175 to check the computation.



Point to the equation on the board as you say that 243 − 175 is the number that added to 175 makes 243. Ask another child to come to the chalkboard and to add the computed difference (68) to 175. Instruct him to describe his thinking steps as he works.

$$\begin{array}{r} 68 \\ +\ 175 \\ \hline 13 \\ 130 \\ 100 \\ \hline 243 \end{array} \qquad \begin{array}{r} 13 \\ \not{2}\not{4}3 \\ -\ 175 \\ \hline 68 \end{array}$$

Let the children study the number strips and tell what the difference 243 − 68 means: the number that added to 68 makes 243. Help the child write on the chalkboard an equation that shows this idea.

$$\begin{array}{r} 175 + (243 - 175) = 243 \\ (243 - 68) + \quad 68 \quad = 243 \end{array}$$

Select a third child to compute the difference 243 − 68 on the chalkboard.

$$\begin{array}{r} 68 \\ +\ 175 \\ \hline 13 \\ 130 \\ 100 \\ \hline 243 \end{array} \qquad \begin{array}{r} 13 \\ \not{2}\not{4}3 \\ -\ 175 \\ \hline 68 \end{array} \qquad \begin{array}{r} 13 \\ \not{2}\not{4}3 \\ -\ \ 68 \\ \hline 175 \end{array}$$

Have him describe his thinking steps to the class as he works. Discuss the meaning of 243 − 68: the number that makes 243 when it is added to 68.

Have a fourth child come to the chalkboard and add the computed difference (175) to 68.

$$\begin{array}{r} 68 \\ +\ 175 \\ \hline 13 \\ 130 \\ 100 \\ \hline 243 \end{array} \qquad \begin{array}{r} 13 \\ \not{2}\not{4}3 \\ -\ 175 \\ \hline 68 \end{array} \qquad \begin{array}{r} 13 \\ \not{2}\not{4}3 \\ -\ \ 68 \\ \hline 175 \end{array} \qquad \begin{array}{r} 175 \\ +\ \ 68 \\ \hline 13 \\ 130 \\ 100 \\ \hline 243 \end{array}$$

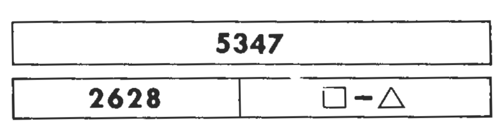
He should describe his thinking steps as he computes 175 + 68.

Direct the class to study the number strips and the recordings of the computations. Help the children understand the following ideas:

Addend + difference is sum.
Sum − difference is addend (related difference).
Sum − addend (related difference) is difference.

Continue the activity with the sums 346 + 478 and 258 + 488 and the differences related to these sums.

Next, investigate four-digit sums such as 2628 + 2719, 3462 + 3564, and 2783 + 5217 as well as differences related to these sums. Since number strips for 1000 may be rather cumbersome for the children to handle, illustrations similar to the following may be placed on the chalkboard.



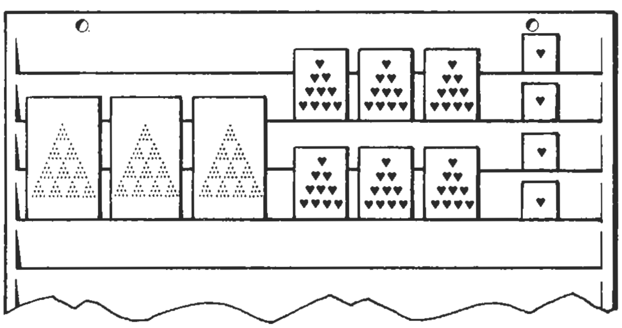
Explain that the illustration helps to show the idea involved. Then instruct them to do the necessary computations.

	4 3	4 3	
2628	~~5~~3~~4~~7	~~5~~3~~4~~7	2719
+ 2719	− 2719	− 2628	+ 2628
17	2628	2719	17
30			30
1300			1300
4000			4000
5347			5347

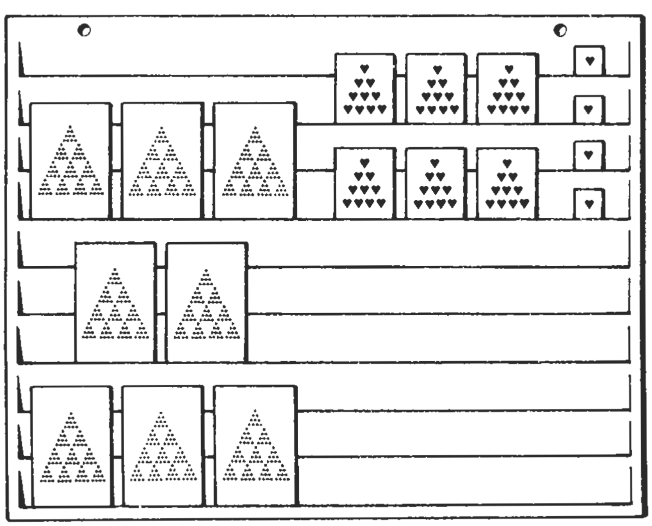
▶ Play the game "The Secret Number." Write on the chalkboard the following:

3 6 4	=	300 + 60 + 4
+ □ □ □	=	+ ___ + ___ + ___
1 3		800 + 140 + 13
1 4 0		
8 0 0		
9 5 3		

Ask a child to place in the pocket chart one-cards, ten-cards, and hundred-cards that represent the given addend of the sum 953.



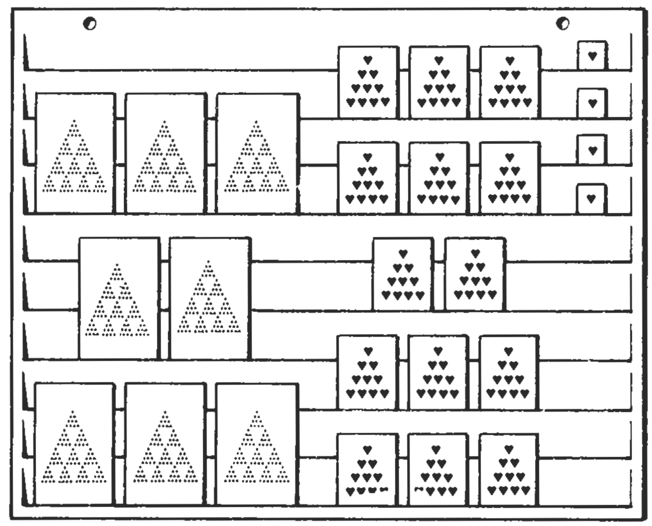
Let the class study the sum of hundreds recorded in the algorisms. Ask someone to read the secret number statement: "3 hundreds plus the secret number of hundreds is 8 hundreds." Tell this child to place hundred-cards in the pocket chart to represent the secret number of hundreds.



Then let the child write the secret number of hundreds in each of the algorisms.

3 6 4	=	300 + 60 + 4
+ [5] □ □	=	+ 500 + ___ + ___
1 3		800 + 140 + 13
1 4 0		
8 0 0		
9 5 3		

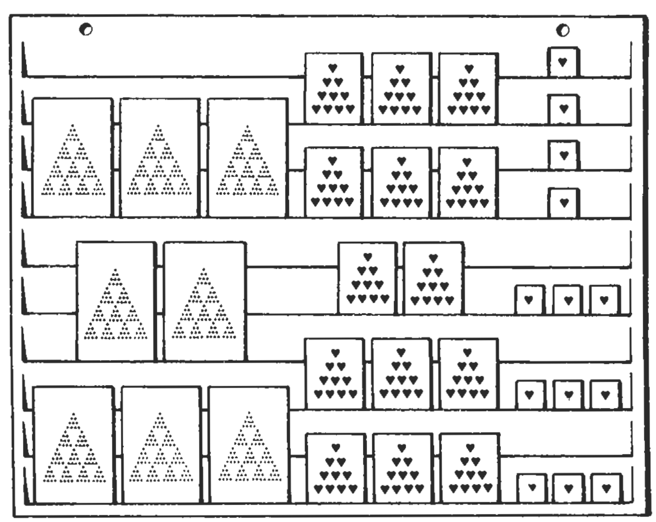
Have the class examine the sum of tens recorded in the algorisms. Ask someone to read the secret number statement: "6 tens plus the secret number of tens is 14 tens," and to then place ten-cards in the pocket chart to represent the secret number.



Let the child write the secret number of tens in each of the algorisms.

3 6 4	=	300 + 60 + 4
+ [5] [8] □	=	+ 500 + 80 + ___
1 3		800 + 140 + 13
1 4 0		
8 0 0		
9 5 3		

Follow the same procedure to determine the sum of ones. The results should be recorded in the pocket chart and on the board as illustrated.



3 6 4 = 300 + 60 + 4
+ [5] [8] [9] = + 500 + 80 + 9
1 3   800 + 140 + 13
1 4 0
8 0 0
9 5 3

Continue the activity with the following problems:

4 □ 8 = 400 + __ + 8
+ □ 3 4 = + __ + 30 + 4
9 0 0   900 + 70 + __
7 0
□ □
9 8 □

□ 5 9 □ = ____ + 500 + 90 + ____
+ 3 8 □ 7 = + 3000 + 800 + ____ + 7
□ 2   7000 + □ □ 00 + □ 20 + □ 2
□ 2 0
□ □ 0 0
7 0 0 0
8 □ 3 2

If cards picturing sets of 1000 are impractical, the disks labeled 1000, 100, 10, and 1 may be used to illustrate exercises that involve four-digit addends.

▶ Write on the chalkboard a subtraction exercise and have the difference computed. Then erase one of the digits and ask what number belongs in that place and why.

62	62
~~7~~ ~~3~~5	~~7~~ ~~3~~5
− 276	− 27□
459	459

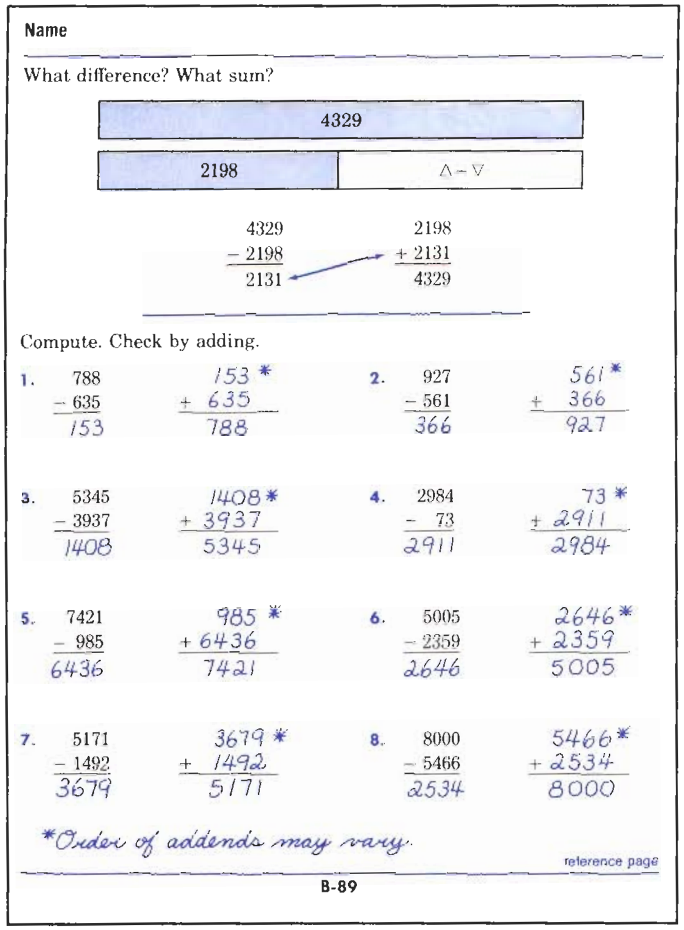
One child may explain, "The secret number must be greater than 5 because it was subtracted from one of the tens. If the difference between 10 and the secret number is added to 5, the sum is 9. Since 4+5 is 9, the secret number must be 6 because 10−6=4."

Another child may say, "The difference plus the addend is the sum. Since 9 plus the secret number could not be 5, one of the tens was needed. The sum must be 10+5. The secret number must be 6 because 9+6 is 10+5."

Use other examples that the children have not computed before. The volunteer who suggests the secret number for an exercise must give reasons for his choice. Use examples such as the following:

7 6 2	5 8 9	5 □ 8 7
− □ 5 0	− □ □ 5	− □ 2 5 □
3 1 2	4 5 4	4 6 □ 3
2	5	6
6 ~~3~~ 6	□ ~~6~~ 3 □	~~7~~ 5 □ 6
− 2 □ □	− 3 □ 7 6	− 3 □ 8 □
4 1 8	6 3 □ 2	□ 7 0 2

As each exercise is reconstructed, direct some child to compute the difference to check whether or not the reconstruction is correct.


Name

What difference? What sum?

4329	2198
− 2198	+ 2131
2131	4329

Compute. Check by adding.

1.	788 − 635 = 153	153* + 635 = 788	2.	927 − 561 = 366	561* + 366 = 927
3.	5345 − 3937 = 1408	1408* + 3937 = 5345	4.	2984 − 73 = 2911	73* + 2911 = 2984
5.	7421 − 985 = 6436	985* + 6436 = 7421	6.	5005 − 2359 = 2646	2646* + 2359 = 5005
7.	5171 − 1492 = 3679	3679* + 1492 = 5171	8.	8000 − 5466 = 2534	5466* + 2534 = 8000

*Order of addends may vary.

reference page

B-89

Pages 89 through 92

● Pages 89 through 91 are designed to reinforce the children's understanding of the relationship between addition and subtraction.

Pages 89 and 90 demonstrate the following facts.

Given a sum and one of its addends, the other addend is the difference between these two numbers.

The standard numeral for the other addend (the difference) can be computed.

The difference plus the given addend is the sum.

Page 91 emphasizes the fact that, given a sum and one of its addends, it is possible to compute the difference between these 2 numbers and then use this difference to set up a related difference.

When presenting pages 89 and 90 to the class, the following procedure is suggested:

Discuss the example at the top of page 89 with the class.

Assign the exercises on both pages to be completed independently.

After the children complete their work, let volunteers explain how they did specific exercises.

The same procedure should be followed on page 91.

Compute. Check by adding.

1. 706 − 392 = 314 Check: 314 + 392 = 706 *
2. 804 − 26 = 778 Check: 778 + 26 = 804 *
3. 747 − 407 = 340 Check: 407 + 340 = 747 *
4. 4793 − 1865 = 2928 Check: 1865 + 2928 = 4793 *
5. 6258 − 2649 = 3609 Check: 2649 + 3609 = 6258 *
6. 3473 − 962 = 2511 Check: 962 + 2511 = 3473 *
7. 718 − 131 = 587 Check: 131 + 587 = 718 *
8. 7000 − 261 = 6739 Check: 6739 + 261 = 7000 *
9. 6543 − 476 = 6067 Check: 6067 + 476 = 6543 *
10. 893 − 254 = 639 Check: 639 + 254 = 893 *
11. 9851 − 3162 = 6689 Check: 3162 + 6689 = 9851 *
12. 578 − 288 = 290 Check: 288 + 290 = 578 *

*Order of addends may vary

B-90

Complete

1. 629 + 247: 800, 60, 16 → 876
2. 731 + 196: 7, 120, 800 → 927
3. 392 + 345: 7, 130, 600 → 737
4. 256 + 352: 8, 100, 500 → 608
5. 6311 + 2548: 9, 50, 800, 8000 → 8859
6. 3472 + 5659: 8000, 1000, 120, 11 → 9131
7. 2272 + 3696: 8, 160, 800, 5000 → 5968
8. 5578 + 2795: 13, 160, 1200, 7000 → 8373
9. 933 − 260 = 673
10. 770 − 547 = 223
11. 946 − 482 = 464
12. 785 − 176 = 609
13. 6958 − 2333 = 4625
14. 4829 − 2138 = 2691
15. 7807 − 5182 = 2625
16. 6378 − 4977 = 1401

B-92

Name

What sum? What difference?

2784	
1353	1431

2784 = 1353 + 1431
2784 − 1353 = 1431
2784 − 1431 = 1353

Compute each difference and the related difference.

1. 356 − 42 = 314; 356 − 314 = 42
2. 8161 − 4387 = 3774; 8161 − 3774 = 4387
3. 6608 − 4327 = 2281; 6608 − 2281 = 4327
4. 634 − 361 = 273; 634 − 273 = 361
5. 1491 − 1263 = 228; 1491 − 228 = 1263
6. 743 − 428 = 315; 743 − 315 = 428
7. 991 − 247 = 744; 991 − 744 = 247
8. 7722 − 5985 = 1737; 7722 − 1737 = 5985
9. 787 − 363 = 424; 787 − 424 = 363
10. 9327 − 846 = 8481; 9327 − 8481 = 846

B-91

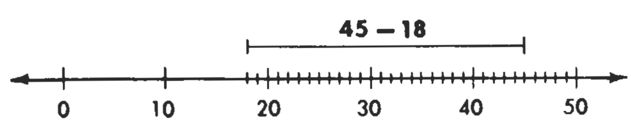
● Use page 92 to give practice in recording the secret number of ones, tens, hundreds, or thousands. Work one or two of each type of exercise with the class. Then assign the remainder of the exercises for independent work. After all of the exercises have been completed, let the children explain how they found the secret numbers in specified exercises.

Supplemental Experiences

■ The children may enjoy exploring the following addition approach to computing differences. Write this difference on the chalkboard.

$$\begin{array}{r} 45 \\ -\,18 \\ \hline \end{array}$$

Draw above this difference a number line to show numbers from 0 through 50. Mark off the distance from 18 to 45 (the difference 45 − 18).



Help the children suggest ways to determine the distance from 18 to 45. Point out the ways suggested on the number line and record the suggestions in an algorism as they are given. Someone may say that you add 2, 10, 10, and 5. When these steps from 18 to 45 are totaled, this child will find the computed difference for 45 and 18.

$$\begin{array}{r} 45 \\ -\ 18 \\ \hline 2 \\ 10 \\ 10 \\ 5 \\ \hline 27 \end{array}$$

Someone else may suggest another way to determine this distance. He may say, "Add 20, 2, and 5." This child will also have computed the difference between 45 and 18.

$$\begin{array}{r} 45 \\ -\ 18 \\ \hline 20 \\ 2 \\ 5 \\ \hline 27 \end{array}$$

Encourage other children to offer their suggestions. Have the children point out their suggested steps on the number line and then record them in an algorism.

Follow a similar procedure with differences related to sums of three-digit and four-digit addends.

■ Separate the class into two teams. While the children close their eyes, place on the flannel board an algorism that shows the computation of a sum of 2 three-digit addends or 2 four-digit addends. Pin tagboard cards over some parts of the algorism.

$$\begin{array}{r} 5\square 7 \\ +\ \square 9 \square \\ \hline 13 \\ 160 \\ 1100 \\ \hline 1273 \end{array}$$

Tell the children to open their eyes. Ask a member of one team to give the secret number statement about the ones in the sum, "7 ones plus the secret number of ones is 13 ones." If he states the relationship correctly, he earns a point for his team. Have a member of the other team name the secret number. Let him uncover the secret number to check his answer. If his answer is correct, he earns a point for his team. Follow the same procedure with the secret number of tens and the secret number of hundreds.

Continue the activity. After every child has had a chance to participate, total each team's points and declare a winner.

■ The children may enjoy making reconstruction exercises for the class to solve. A child may do so by following these steps:

perform the complete computation,
copy the exercise, substituting a □ in place of one or more numbers,
try to solve the puzzle himself,
display the reconstruction on the bulletin board so that others may copy it and try to solve it.

KEY IDEA

Shortcuts are possible.

Scope

To review the short form of the addition algorism with four-digit addends.

To check the accuracy of computation.

To review the commutative and associative properties of addition.

To practice computation of sums and differences.

Fundamentals

The computation of 2 four-digit addends with the addition algorism results in 4 partial sums. For example,

$$\begin{array}{rl} 3987 & \\ +\ 5638 & \\ \hline 15 & \text{partial sum} \\ 110 & \text{partial sum} \\ 1500 & \text{partial sum} \\ 8000 & \text{partial sum} \\ \hline 9625 & \end{array}$$

For more capable students with good memories, the short form of the addition algorism may be presented. It is important that the short form be developed with an understanding of place value rather than by rote. In the above example,

$$\begin{array}{r} 3987 \\ +\ 5638 \\ \hline 9625 \end{array}$$

$7 + 8 = 15$. Write the 5. Remember 10.
$80 + 30 = 110$ (11 tens). To the 110 (11 tens) add the remembered 10: $110 + 10 = 120$ (12 tens). Write 2 tens. Remember 100 (1 hundred).
$900 + 600 = 1500$ (15 hundreds). To the 1500 (15 hundreds) add the remembered 100: $1500 + 100 = 1600$ (16 hundreds). Write 6 hundreds. Remember 1000 (1 thousand).
$3000 + 5000 = 8000$. To the 8000 add the remembered 1000: $8000 + 1000 = 9000$. Write 9 thousands to complete the computation.

When there are several addends in an exercise, it is often convenient to mentally rearrange the order of digits to achieve simpler pairs—especially if pairs of digits whose sums are ten are involved. In an exercise such as

$$\begin{array}{r} 6826 \\ 4538 \\ +\ 3786 \\ \hline \end{array}$$

the child will notice that a convenient order of ones is $(6+6)+8$, of tens is (2 tens + 8 tens) + 3 tens, of hundreds is (8 hundreds + 7 hundreds) + 5 hundreds, and of thousands is (6 thousands + 4 thousands) + 3 thousands.

The teacher should be alert to discover the types of mistakes the children make in computation. Errors should be noted and an effort made to provide for individual differences in computational skill. Errors may occur for a number of reasons. Nevertheless accuracy may be improved by a checking process. In this section, the teacher will want to emphasize the relationship between addition and subtraction as the basis for checking computation.

Readiness for Understanding

Knowledge of place value.

Knowledge of addition and subtraction algorisms.

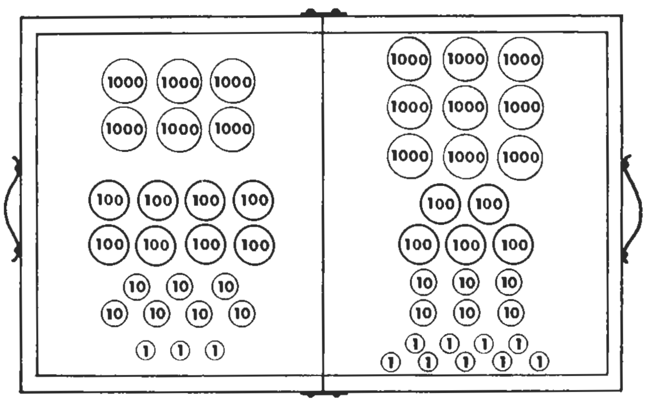
Awareness of the relationship between addition and subtraction.

Awareness of the associative and commutative properties of addition.

Developmental Experiences

for flannel board	felt-tip pen
100 disks	Countingmen
(25 of each labeled: 1, 10, 100, and 1000)	

▶ Introduce a shortcut for recording the computation of sums of 2 four-digit addends. Write 6873 + 9569 on the chalkboard. Have a child place on the flannel board disks labeled 1000, 100, 10, and 1 to illustrate the addends of this sum.



Ask a child to come to the chalkboard to record the computation of the sum of ones.

$$\begin{array}{r} 6873 \\ +\ 9569 \\ \hline 12 \end{array}$$

Have him check the disks on the flannel board to verify his computation. Beside this child's work, show the first step in a short method to record the computation. Explain to the class that you are going to record the ones and remember the ten of 10+2.

$$\begin{array}{r} 6873 \\ +\ 9569 \\ \hline 12 \end{array} \qquad \begin{array}{r} 6873 \\ +\ 9569 \\ \hline 2 \end{array}$$

Next, have the child compute his tens, record the result, and check the disks on the flannel board to verify his computation. Then record the result of your computation of tens as you explain it to the class—the 1 ten you remembered plus 7 tens plus 6 tens gives you 14 tens. Explain to the class that you are only going to record the tens and remember the hundreds of 100+40.

$$\begin{array}{r} 6873 \\ +\ 9569 \\ \hline 12 \\ 130 \end{array} \qquad \begin{array}{r} 6873 \\ +\ 9569 \\ \hline 42 \end{array}$$

Now, tell the child to compute the hundreds and to record the result. Record the result of your computation of hundreds and explain that the 1 hundred you remembered plus 8 hundreds plus 5 hundreds gives you 14 hundreds. Explain to the class that you are only going to record the hundreds and remember the thousands of 1000+400.

$$\begin{array}{r} 6873 \\ +\ 9569 \\ \hline 12 \\ 130 \\ 1300 \end{array} \qquad \begin{array}{r} 6873 \\ +\ 9569 \\ \hline 442 \end{array}$$

Have the child compute the thousands and record the result. Then ask him to compute 12+130+1300+15000. Record the result of your computation of thousands as you explain it to the class—the 1 thousand you remembered plus 6 thousands plus 9 thousands gives you 16 thousands. Write 16 next to the 442 already recorded. Have the children compare the computed sum in each algorism with the disks on the flannel board—16 thousands plus 4 hundreds plus 4 tens plus 2 ones. Remind the children that 10 one-disks need not be exchanged for 1 ten-disk to be considered a 10. The same idea applies to 10 ten-disks and 1 hundred-disk, and 10 hundred-disks and 1 thousand-disk.

$$\begin{array}{r} 6873 \\ +\ 9569 \\ \hline 12 \\ 130 \\ 1300 \\ \underline{15000} \\ 16442 \end{array} \qquad \begin{array}{r} 6873 \\ +\ 9569 \\ \hline 16442 \end{array}$$

Repeat the activity using the sum 2334 + 8497. Have disks placed on the flannel board; direct one child to compute the sum by using the long method while you demonstrate the short method. Let the child explain his method of computation to the class. Ask another child to explain the short method.

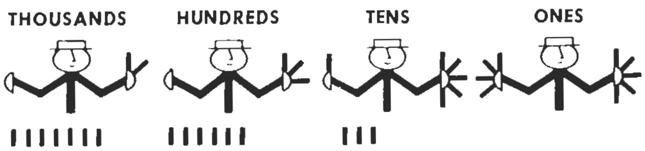
Repeat the activity once more; use the sum 7445 + 5963. Have the disks placed on the flannel board. Select two children to show the computation of this sum; one must use the long method while the other uses the short method. Instruct each child to explain his method.

Continue to give other pairs of children an opportunity to compute sums by both methods. Use sums such as 4393 + 7473, 3556 + 5642, 6477 + 8314, and 9755 + 4939. Point out that in two of the sums the child who used the short method had no hundreds to remember.

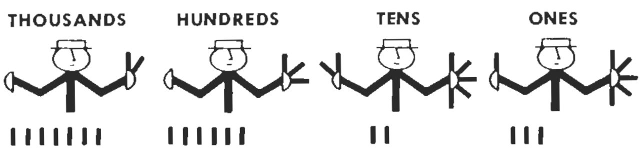
▶ Use the Countingmen to reinforce the children's understanding of the computation of sums of thousands, hundreds, tens, and ones. Write 2369 + 4857 on the chalkboard. Ask a child to show how to compute

2369 + 4857 with only 9 thousands, 9 hundreds, 9 tens, and 9 ones. A child may show and explain the computation in the following way:

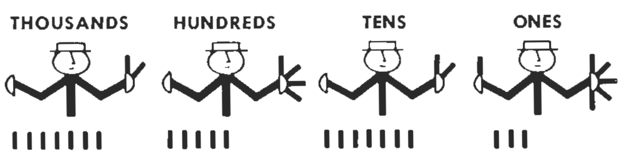
Place 2 fingers on the Thousands-man, 3 fingers on the Hundreds-man, 6 fingers on the Tens-man, and 9 fingers on the Ones-man to represent 2369.



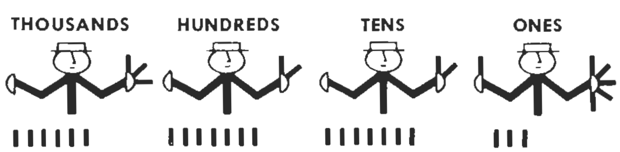
To add 7 to 9, put 1 finger on the Tens-man and remove 3 fingers from the Ones-man (9 + 7 = 9 + 10 − 3).



To add 5 tens to 6+1 tens, put 1 finger (100) on the Hundreds-man and remove 5 fingers from the Tens-man (7 tens + 5 tens = 7 tens + 10 tens − 5 tens).

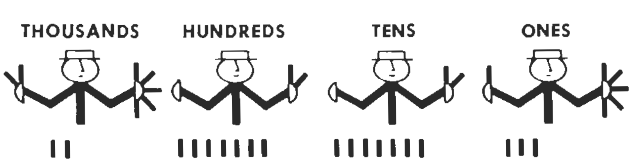


To add 8 hundreds to 3+1 hundreds, put 1 finger (1000) on the Thousands-man and remove 2 fingers from the Hundreds-man (4 hundreds + 8 hundreds = 4 hundreds + 10 hundreds − 2 hundreds).



To add 4 thousands to 2+1 thousands, put 4 fingers on the Thousands-man. There are now 7 thousands, 2 hundreds, 2 tens, and 6 ones.

$$\begin{array}{r} 2369 \\ +\,4857 \\ \hline 7226 \end{array}$$



Have a second child explain the thinking steps as he records the shortcut computation of the sum 2369 + 4857 on the chalkboard.

$$\begin{array}{r} 2369 \\ +\,4857 \\ \hline 7226 \end{array}$$

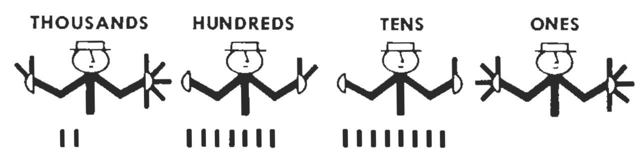
- 9 + 10 − 3 is 10 + 6. Record 6, remember 10.
- 7 tens + 10 tens − 5 tens is 10 tens + 2 tens. Record 2 tens, remember 100.
- 4 hundreds + 10 hundreds − 2 hundreds is 10 hundreds + 2 hundreds. Record 2 hundreds, remember 1000.
- 3 thousands + 4 thousands is 7 thousands. Record 7 thousands.

Now that the children have had a chance to apply their understanding of sums of four-digit addends to work with the Countingmen and the short form algorism, have them continue the activity by investigating the short form subtraction algorism. Have the class name a difference related to the sum of 2369 + 4857. Someone may suggest the difference 7226 − 4857. Write this algorism on the chalkboard.

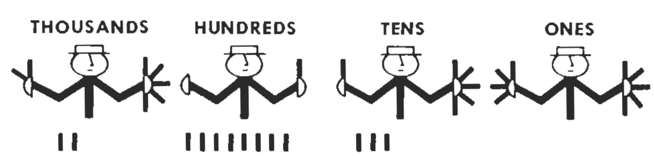
$$\begin{array}{r} 7226 \\ -\,4857 \\ \hline \end{array}$$

Ask some child to show how to compute this by using the Countingmen that already have 7 thousands, 2 hundreds, 2 tens, and 6 ones. The child may explain the computation of the difference in the following way:

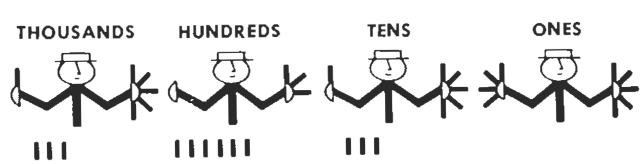
To subtract 7 ones from 6 ones, remove 1 finger from the Tens-man and place 3 fingers on the Ones-man.



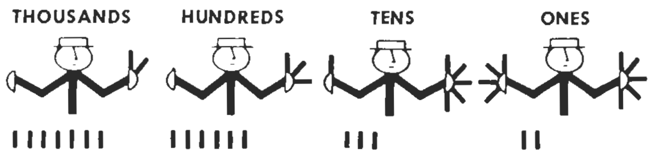
To subtract 5 tens from 1 ten, remove 1 finger from the Hundreds-man and place 5 fingers on the Tens-man.



To subtract 8 hundreds from 1 hundred, remove 1 finger from the Thousands-man and place 2 fingers on the Hundreds-man.



To subtract 4 thousands from 6 thousands, remove 4 fingers from the Thousands-man.



Direct the class to study the computed difference of 7226 − 4857 represented on the Countingmen.

Ask another child to explain his thinking as he computes 7226 − 4857 on the chalkboard.

```
  611
  7226  (7, 2, 2 struck through)
− 4857
  2369
```

- To subtract 7, subtract 1 ten (from the 2 tens), then add 3 to 6. Record 9; remember 1 ten has been subtracted from 2 tens.
- To subtract 5 tens, subtract 10 tens (from the 20 tens), then add 5 tens to 1 ten. Record 6 tens; remember 10 tens has been subtracted from 200.
- To subtract 8 hundreds, subtract 10 hundreds (from the 70 hundreds), then add 2 hundreds to 1 hundred. Record 3 hundreds; remember 10 hundreds has been subtracted from 7000.
- Subtract 4 thousands. Record 2 thousands.

Write 7226 − 2369 on the chalkboard and ask two children to show the computation steps. One child should use the Countingmen; the other child should compute at the chalkboard. The children should observe that the computation of either difference related to the sum 2369 + 4857 (7226 − 4857 or 7226 − 2369) may be used to check the computation of the sum.

Adapt this entire activity for other sums (and their related differences) of 2 three-digit and four-digit addends. In each instance, the children should understand that the computation of a difference may be used to check the computation of the sum.

▶ Ask a child to come to the chalkboard to compute the difference 4060 − 2736, and to explain his thinking as he works. A child may explain his computation in the following way:

There are not enough ones in the ones place, so 6 must be subtracted from 1 ten in the tens place. This leaves 5 tens in the tens place.

```
    5
  4060  (6 struck through)
− 2736
     4
```

There are enough tens in the tens place to subtract 3 tens from 5 tens.

```
    5
  4060  (6 struck through)
− 2736
    24
```

There are not enough hundreds in the hundreds place, so 7 hundreds must be subtracted from 1 thousand (10 hundreds). This leaves 3 thousands in the thousands place.

```
  3 5
  4060  (4 and 6 struck through)
− 2736
   324
```

Finally, subtract the thousands.

```
  3 5
  4060  (4 and 6 struck through)
− 2736
  1324
```

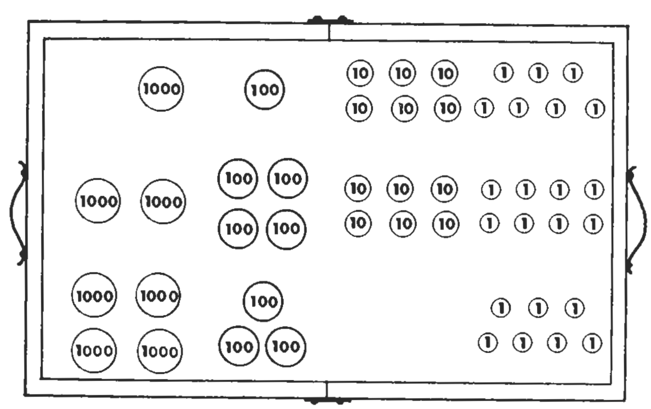
Have other children explain the computation of 7204 − 4768, 6530 − 3267, 7706 − 3463, 8025 − 5347, and 9000 − 4762. In computing 9000 − 4762, the 1 ten that is needed will be subtracted from the 900 tens, leaving 899 tens.

```
  899
  9000  (9, 0, 0 struck through)
− 4762
  4238
```

▶ Write the following sum on the chalkboard.

```
  1167
  2468
+ 4307
```

Ask someone to represent the three addends with disks on the flannel board.



Tell the children to study the disks on the flannel board and give the sum of ones (7+8+7), the sum of tens (60+60+0), the sum of hundreds (100+400+300), and the sum of thousands (1000+2000+4000). Record the sums on the chalkboard as illustrated.

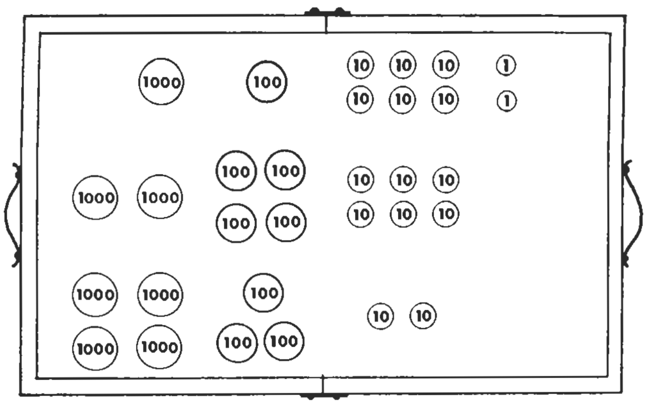
```
  1167 = 1000+100+60+7
  2468 = 2000+400+60+8
+ 4307 = 4000+300+ 0+7
```

Tell the class to compute the thousands (7000), the hundreds (800), the tens (120), and the ones (22). Record the results in the expanded form of the algorism.

```
  1167 = 1000+100+ 60+ 7
  2468 = 2000+400+ 60+ 8
+ 4307 = 4000+300+  0+ 7
         7000+800+120+22
```

Have the class observe that in adding ones, the result was tens and ones since 22 is 20+2.

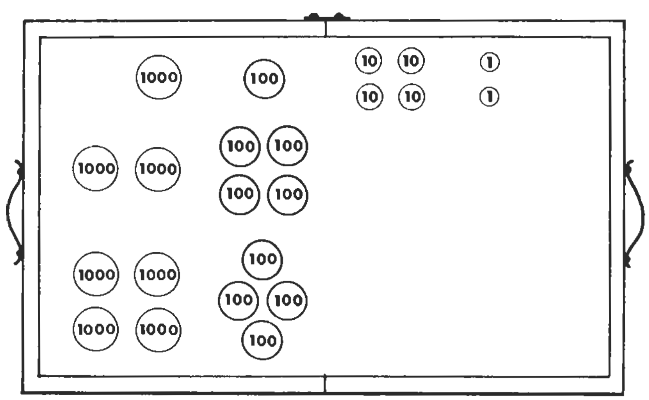
Exchange 20 one-disks for 2 ten-disks.



Make it clear to the children that 20 ones is 2 tens whether or not the exchange is made; it is easier to see tens when the ten-disks are used. Record the idea that 22 is 20+2.

$$\begin{array}{rl} 1167 = & 1000+100+\ 60+\ 7 \\ 2468 = & 2000+400+\ 60+\ 8 \\ \underline{+\ 4307} = & \underline{4000+300+\ \ 0+\ 7} \\ & 7000+800+120+22 \\ = & 7000+800+120+20+2 \end{array}$$

Have the class observe that in adding tens the result was hundreds and tens since 120 is 100+20 Ask a child to replace 10 ten-disks with 1 hundred-disk.



Remind the children that 10 tens is 1 hundred whether or not the exchange is made; however, it is easier to see hundreds when the hundred-disks are used. Record the idea that 120 is 100+20.

$$\begin{array}{rl} 1167 = & 1000+100+\ 60+\ 7 \\ 2468 = & 2000+400+\ 60+\ 8 \\ \underline{+\ 4307} = & \underline{4000+300+\ \ 0+\ 7} \\ & 7000+800+120+22 \\ = & 7000+800+120+20+2 \\ = & 7000+800+100+20+20+2 \end{array}$$

Have the class compute and record the sum of tens (40) and the sum of hundreds (900). Then have a child write the standard name for 7000+900+40+2.

$$\begin{array}{rl} 1167 = & 1000+100+\ 60+\ 7 \\ 2468 = & 2000+400+\ 60+\ 8 \\ \underline{+\ 4307} = & \underline{4000+300+\ \ 0+\ 7} \\ & 7000+800+120+22 \\ = & 7000+800+120+20+2 \\ = & 7000+800+100+20+20+2 \\ = & 7000+900+40+2 \\ = & 7942 \end{array}$$

Next, ask a child to record the thinking steps in vertical form for the computation of the given sum.

$$\begin{array}{r} 1167 \\ 2468 \\ \underline{+\ 4307} \\ 22 \\ 120 \\ 800 \\ \underline{7000} \\ 7942 \end{array}$$

Tell him to point to the numerals as you summarize the thinking steps:

7+8+7 is 20+2,
60+60 is 100+20,
100+400+300 is 800,
1000+2000+4000 is 7000,
7000+800+100+20+20+2 is 7000+900+40+2,
7000+900+40+2 is 7942.

Then demonstrate the short method for recording the computation of the given sum.

$$\begin{array}{r} 1167 \\ 2468 \\ \underline{+\ 4307} \\ 7942 \end{array}$$

Help the class explain your method:

7+8+7 is 22. 22 is 2+20. Write 2, remember 2 tens.

2 tens + 6 tens + 6 tens is 14 tens. 14 tens is 4 tens + 1 hundred. Write 4 tens, remember 1 hundred.

1 hundred + 1 hundred + 4 hundreds + 3 hundreds is 9 hundreds. Write 9 hundreds.

1 thousand + 2 thousands + 4 thousands is 7 thousands. Write 7 thousands.

Continue the activity with sums such as:

2674 + 3563 + 3782
4325 + 1239 + 3126
5823 + 7313 + 4642
4026 + 2761 + 5192

Name

5398
+ 6473
11871

- 8 + 3 = 11
  11 is 10 + 1.
  Write 1. Remember 10.
- 90 + 70 + 10 = 170
  170 is 100 + 70.
  Write 7 tens. Remember 100.
- 300 + 400 + 100 = 800
  Write 8 hundreds.
- 5000 + 6000 = 11000
  Write 11 thousands.

Compute.

1.	2.	3.	4.
5231	2685	3346	5888
+ 4763	+ 6297	+ 4625	+ 5041
9994	8982	7971	10929

5.	6.	7.	8.
4173	3741	4151	5506
+ 4258	+ 9256	+ 3877	+ 3478
8431	12997	8028	8984

9.	10.	11.	12.
1376	2541	206	1268
2348	1949	7277	5468
+ 4206	+ 3786	+ 1689	+ 2275
7930	8276	9172	9011

reference page

B-93

Name

9000
− 2543
6457

7 = 10 − 3 + 0
50 = 90 − 40
400 = 900 − 500
6000 = 8000 − 2000

Compute.

1.	2.	3.	4.
3090	9856	8161	5740
− 2765	− 3155	− 4387	− 3461
325	6701	3774	2279

5.	6.	7.	8.
6608	7278	4117	2301
− 1327	− 4389	− 3980	− 1456
5281	2889	137	845

9.	10.	11.	12.
9013	3245	9000	2490
− 2248	− 2697	− 5987	− 897
6765	548	3013	1593

13.	14.	15.	16.
8498	9230	5030	3130
− 6296	− 2468	− 4753	− 1574
2202	6762	277	1556

17.	18.	19.	20.
6200	7240	8000	5005
− 2579	− 2959	− 2559	− 2806
3621	4281	5441	2199

reference page

B-95

Compute.

1.	2.	3.	4.	5.
7260	8390	5640	3570	4859
− 4327	− 2565	+ 2981	+ 2179	+ 845
2933	5825	8621	5749	5704

6.	7.	8.	9.	10.
5104	4007	6043	4407	7030
− 1368	+ 3538	+ 2865	− 2518	− 368
3736	7545	8908	1889	6662

11.	12.	13.	14.	15.
6001	4029	3999	1876	5440
− 27	+ 2076	− 2041	+ 6995	− 2369
5974	6105	1958	8871	3071

Answer the questions.

16. Joe had 488 stamps. He was given a package of 625 stamps. How many stamps did he have then? 1113

17. Bob had 500 stamps to sell. Joe bought 134 of them. How many stamps did Bob have left to sell? 366

18. Joe's album will hold 3050 stamps. Joe has 1247 stamps. How many more stamps does he need to fill the album? 1803

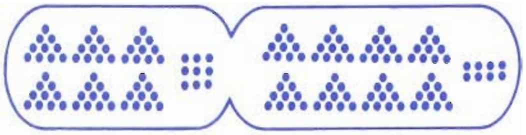
B-94

Pages 93 through 98

- Work the example at the top of page 93 with the children. Use disks labeled 1000, 100, 10, and 1 to illustrate the computation as it is discussed. Then assign the exercises as independent work. Give help only when needed. Do not insist that every child use the short method for recording the result of his computation. Allow the children to use the method with which they feel secure. When the children have finished the page, ask several of them to explain their method of computing specific sums.

- Use page 94 to provide practice in computation of four-digit sums and differences. Assign exercises 1 through 15 for independent work. Then let children read aloud each story exercise at the bottom of the page to be sure that the children understand each question. The answer to each story question may be written in a complete sentence or in a phrase. When the class has completed the story exercises, let the children discuss how they arrived at the answer to each question.

- Page 95 provides further practice with the subtraction algorism. Let children explain the example. Then assign the first row of exercises to be completed quickly. When most pupils are finished, let them discuss their results. Continue with each row of exercises in a similar manner.



69	157	157
+ 88	− 69	− 88
157	88	69

Compute. Check by subtracting.

1. 61 + 83 = 144 — 144 − 83 = 61 *or* 144 − 61 = 83
2. 543 + 618 = 1161 — 1161 − 543 = 618 *or* 1161 − 618 = 543
3. 4287 + 3767 = 8054 — 8054 − 3767 = 4287 *or* 8054 − 4287 = 3767
4. 928 + 778 = 1706 — 1706 − 778 = 928 *or* 1706 − 928 = 778
5. 739 + 227 = 966 — 966 − 227 = 739 *or* 966 − 739 = 227
6. 8345 + 808 = 9153 — 9153 − 808 = 8345 *or* 9153 − 8345 = 808
7. 398 + 573 = 971 — 971 − 573 = 398 *or* 971 − 398 = 573
8. 3573 + 3599 = 7172 — 7172 − 3573 = 3599 *or* 7172 − 3599 = 3573

B-96

Name

843
629
1378
+ 2114
4964

- 3 + 9 + 8 + 4 = 24. 24 is 2 tens + 4. Write 4. Remember 20.
- 20 + 40 + 20 + 70 + 10 = 160. 160 is 1 hundred + 6 tens. Write 6 tens. Remember 100.
- 100 + 800 + 600 + 300 + 100 = 1900. 1900 is 1 thousand + 9 hundreds. Write 9 hundreds. Remember 1000.
- 1000 + 1000 + 2000 = 4000. Write 4 thousands.

Compute.

1. 1376 + 2348 + 4206 = 7930
2. 2541 + 1949 + 3786 = 8276
3. 206 + 7277 + 1689 = 9172
4. 1268 + 5468 + 2275 = 9011
5. 9782 + 90 + 574 = 10446
6. 54 + 180 + 475 = 709
7. 5260 + 8163 + 297 = 13720
8. 7412 + 35 + 342 = 7789
9. 6474 + 6563 + 81 + 9240 = 22358
10. 186 + 40 + 84 + 604 = 914
11. 293 + 242 + 60 + 284 = 879
12. 8722 + 302 + 9122 + 132 = 18278

reference page

B-97

Compute. Check by subtracting.

1. 5769 + 7254 = 13023 — 13023 − 7254 = 5769 *
2. 582 + 893 = 1475 — 1475 − 582 = 893 *
3. 284 + 295 = 579 — 579 − 295 = 284 *
4. 2364 + 445 = 2809 — 2809 − 2364 = 445 *
5. 2368 + 6799 = 9167 — 9167 − 2368 = 6799 *
6. 378 + 287 = 665 — 665 − 287 = 378 *
7. 6124 + 958 = 7082 — 7082 − 6124 = 958 *
8. 387 + 71 = 458 — 458 − 71 = 387 *
9. 432 + 738 = 1170 — 1170 − 432 = 738 *
10. 4151 + 3267 = 7418 — 7418 − 3267 = 4151 *
11. 5999 + 2002 = 8001 — 8001 − 2002 = 5999 *
12. 2856 + 79 = 2935 — 2935 − 2856 = 79 *

*Order of addends may vary.

B-98

● Page 96 is designed to reinforce the children's understanding of the relationship between addition and subtraction. This page provides an opportunity for the children to observe the following facts.

Given two addends, the sum is one addend plus the other addend.

The standard numeral for the sum can be computed.

The sum minus one addend is the other addend.

This serves as a check on the computation of the sum.

Discuss the example at the top of the page with the children. Use disks labeled 100, 10, and 1 to illustrate the specific computation as it is discussed. Then work one or two exercises with the class before you assign the other exercises as independent work.

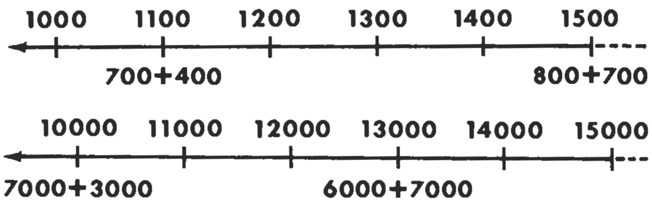
● Page 97 is designed to encourage the children to use the short method for recording sums of 3 or 4 four-digit addends. However, do not insist that every child use this method. You may want to let some of the children try this method for one or two exercises and then allow them to use the method that they prefer.

After you discuss the example at the top of the page with the children, assign the exercises to give the children an opportunity to test their ability to compute the given sums.

● Page 98 provides additional practice in computation of 4 digit sums. The children can check by subtracting.

Supplemental Experiences

■ Provide further practice with upper decade addition facts as they apply to sums of hundreds and sums of thousands. Draw on the chalkboard two segments of the number line. One segment should be numbered from 1000 through 1800 and the other segment from 10000 through 18000. Let the children take turns writing sums of hundreds or sums of thousands below the numbers on the line until all additive combinations involving upper decade facts have been used. Explain that no sum may be used twice.



■ Some of the children may be interested in subtracting four-digit numbers in the following way.

7346
− 3869

7346
− 3000 (Subtract thousands from 7346.)

3
~~4~~346
− 800 (Subtract hundreds from the difference 4346.)

4
3~~5~~46
− 60 (Subtract tens from the difference 3546.)

7
34~~8~~6
− 9 (Subtract ones from the difference 3486.)

3477

■ Write sums of 3 four-digit addends on cards.

1287+2257+3109 | **2352+4695+1858**

Separate the class into two teams. Have a pair of children from each team take a card from the pack—each of the four children should have a card. Tell each child to compute on the chalkboard the sum he chose. Then have each pair of children compare their answers and decide who has the greater sum. Write ____ is greater than ____ on two panels of the chalkboard; assign one panel to each team. Then tell each pair of children to record their sums on the lines to make a true statement. For example, if the sums computed by the members of one team are 6653 and 8905, this pair of children should arrange the sums as follows:

8905 is greater than 6653

If both children in a pair have computed correctly, they earn a point for their team. If they arrange their sums so that the correct order relation is stated, they earn a point for their team. After all of the children in the class have participated in the game, total the points and declare a winner.

KEY IDEA

Inches plus inches is inches.

Scope

To extend the children's knowledge of the concept of linear measure.

To measure lengths in inches, feet, or yards.

To compute sums or differences of lengths.

Fundamentals

Through experiences with the ruler and yardstick the child has developed an understanding of measurement. He applies this understanding of measurement to computation of lengths. Thus 732 inches + 653 inches = 1385 inches.

Readiness for Understanding

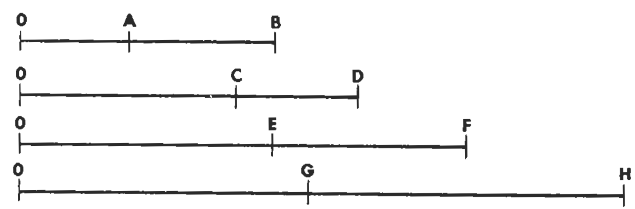
Some idea of the meaning of length.

Knowledge of addition and subtraction algorisms.

Developmental Experiences

wrapping paper	masking tape
yardstick	adding machine tape
rulers	felt-tip pen

▶ Draw a line segment 21 inches long on a piece of wrapping paper. Mark the segment 9 inches from the left end. Label the left end of the segment *0*, the right end *B*, and the point between *A*. Draw three other line segments on the paper. Make them 28, 37, and 50 inches long. Mark a point 18 inches from the end of the first segment, 21 inches from the end of the second segment, and 24 inches from the end of the last segment. Label each of the points on these segments as shown in the illustration.



Ask a child to measure the length of the segment from 0 to *A* in inches and record the measurement on the chalkboard. Let another child measure the length from *A* to *B* in inches and record the measurement. Then ask if anyone can tell the length of the segment from 0 to *B* without measuring. Some child may suggest you add the lengths already measured. Ask a child to compute the sum:

9 inches
+ 12 inches
21 inches

Another child may use a yardstick to measure the length of the segment from 0 to *B* to check the computation.

Follow a similar procedure with the other line segments. Then ask how to find the difference between the length from 0 to *B* and the length from 0 to *A*.

A child may suggest computing the difference by subtracting.

$$\begin{array}{r} \overset{1}{\not{2}}1 \text{ inches} \\ -\ 9 \text{ inches} \\ \hline 12 \text{ inches} \end{array}$$

Let a child measure the length from A to B again to check the computed difference. Let the children compute to find the difference between other lengths shown.

▶ Hold up a 12-inch ruler and review with the children the fact that the distance from 0 to 12 is 1 foot. Ask groups of children to measure in feet given distances on the chalkboard. For example, to measure a length of 5 feet, place a mark on the chalkboard and ask a child to hold his ruler with the 0 at the marked point. Tell another child to hold his ruler so that the 0 is next to the mark for 12 on the first ruler, and so forth until 5 lengths of one foot each are shown by rulers. Have the last person place a mark on the chalkboard. Then ask a child to tell the distance in feet between the two marks on the chalkboard.

A similar activity may be used to measure distances on the floor. Use small pieces of tape to show the ends of the measured segment. Let the children measure distances such as 3 feet, 9 feet, 14 feet, and 26 feet.

On the chalkboard, write:

$$\begin{array}{r} 7 \text{ feet} \\ +\ 6 \text{ feet} \\ \hline \end{array}$$

Let a child compute the sum. Then let the children mark off the distances on the chalkboard or floor and measure in feet to see if the sum is correct. Continue with other exercises in which the sum or the difference of two lengths measured in feet can be computed.

▶ Display a yardstick and tell the children that the length of the stick is 1 yard. Place a mark near the edge of a strip of adding machine tape and label it 0. Have the children mark off successive lengths of one yard and write numerals below each mark to indicate the length in yards from the starting point. It may be more convenient for the children to handle if the tape is rolled up after each segment is measured.

Ask pairs of children to show various lengths measured in yards. For example, let one child hold the tape at the starting point while another child unrolls and stretches out the tape to show distances such as 4 yards, 9 yards, and 6 yards. Greater distances may be shown if the class works outdoors or in a long hall.

▶ Write on the chalkboard:

$$\begin{array}{r} 2 \text{ yards} \\ 7 \text{ yards} \\ +\ 8 \text{ yards} \\ \hline \end{array}$$

Ask a child to compute the sum of the lengths. Other children may lay out these lengths on the floor and then measure the combined length to check the computation. Use other exercises that can be checked by measurement; then introduce a few exercises where it is impractical to check the sum by measuring. Exercises similar to the following may be used.

$$\begin{array}{r} 27 \text{ yards} \\ 33 \text{ yards} \\ +\ 68 \text{ yards} \\ \hline \end{array} \qquad \begin{array}{r} 262 \text{ yards} \\ 473 \text{ yards} \\ +\ 394 \text{ yards} \\ \hline \end{array} \qquad \begin{array}{r} 165 \text{ yards} \\ 346 \text{ yards} \\ +\ 559 \text{ yards} \\ \hline \end{array}$$

▶ Write the following sums and differences on the chalkboard.

$$\begin{array}{r} 638 \text{ inches} \\ +\ 94 \text{ inches} \\ \hline \end{array} \qquad \begin{array}{r} 946 \text{ feet} \\ -\ 278 \text{ feet} \\ \hline \end{array} \qquad \begin{array}{r} 562 \text{ yards} \\ +\ 695 \text{ yards} \\ \hline \end{array} \qquad \begin{array}{r} 8020 \text{ yards} \\ -\ 5643 \text{ yards} \\ \hline \end{array}$$

Have four children come to the chalkboard in turn, compute one of the sums or differences and explain each step of the computation.

Pages 99 through 102

● Use page 99 to review with the children the idea that if lengths are measured in inches, the sums or differences of those lengths will also be measured in inches. Have the children read the story at the top of the page. Discuss the information that is recorded in the table. Encourage the pupils to read the exercises on the page and to ask questions about anything they do not understand. Instruct the children to complete the exercises. When most of the children have finished, discuss their answers and the reasons for them.

Name

Five boys planned a frog-jumping contest. Each boy measured how far his frog went from a log in three jumps.

	Distance Jumped		
	1st jump	2nd jump	3rd jump
Mike's frog	27 inches	15 inches	25 inches
Sam's frog	26 inches	29 inches	31 inches
Dick's frog	36 inches	31 inches	28 inches
Tom's frog	40 inches	24 inches	33 inches
Jim's frog	38 inches	31 inches	27 inches

1. How far did each frog go in three jumps?

Mike's frog	Sam's frog	Dick's frog	Tom's frog	Jim's frog
67 inches	86 inches	95 inches	97 inches	96 inches

2. How far did the winner go in three jumps? 97 inches
3. What is the greatest distance for one jump? 40 inches
4. What is the least distance for one jump? 15 inches
5. What is the difference between the longest jump and the shortest jump? 25 inches
6. What is the difference between the longest jump and the shortest jump of Mike's frog? 12 inches
7. What is the difference between the total distances jumped by Tom's frog and Jim's frog? 1 inch

B-99

Compute.

1. 128 inches + 864 inches — *992 inches*
2. 609 inches − 297 inches — *312 inches*
3. 390 inches − 67 inches — *323 inches*
4. 391 feet + 484 feet — *875 feet*
5. 444 feet − 17 feet — *427 feet*
6. 625 feet + 298 feet — *923 feet*
7. 476 inches + 885 inches — *1361 inches*
8. 865 inches + 74 inches — *939 inches*
9. 327 inches + 56 inches — *383 inches*
10. 315 feet − 126 feet — *189 feet*
11. 945 inches − 392 inches — *553 inches*
12. 875 feet + 374 feet — *1249 feet*
13. 558 feet + 406 feet — *964 feet*
14. 1248 feet − 972 feet — *276 feet*
15. 973 feet − 264 feet — *709 feet*
16. 52 inches, 261 inches, + 596 inches — *909 inches*
17. 491 feet, 63 feet, + 278 feet — *832 feet*
18. 783 inches, 111 inches, + 488 inches — *1382 inches*
19. 240 feet, 72 feet, 136 feet, + 300 feet — *748 feet*
20. 722 feet, 393 feet, 17 feet, + 181 feet — *1313 feet*
21. 81 inches, 152 inches, 872 inches, + 683 inches — *1788 inches*

B-100

Compute.

1. 569 yards − 321 yards — *248 yards*
2. 1262 feet + 785 feet — *2047 feet*
3. 750 yards − 499 yards — *251 yards*
4. 157 feet + 352 feet — *509 feet*
5. 629 yards − 136 yards — *493 yards*
6. 2837 inches − 676 inches — *2161 inches*
7. 6761 yards − 2548 yards — *4213 yards*
8. 735 feet + 165 feet — *900 feet*
9. 752 feet − 238 feet — *514 feet*
10. 6648 inches − 2315 inches — *4333 inches*
11. 966 inches − 486 inches — *480 inches*
12. 678 yards − 349 yards — *329 yards*
13. 4983 feet + 2145 feet — *7128 feet*
14. 363 inches + 596 inches — *959 inches*
15. 907 feet − 703 feet — *204 feet*
16. 273 yards, 240 yards, 759 yards, + 147 yards — *1419 yards*
17. 792 feet, 233 feet, 771 feet, + 790 feet — *2586 feet*
18. 796 inches, 291 inches, 60 inches, + 372 inches — *1519 inches*

B-102

Name

Bill and Tom tied three pieces of kite string together. One piece was about 14 yards long, another was about 29 yards long, and the third about 23 yards long. They want about 75 yards of string. Do they need more string? If so, how much do they need? *About 9 yards*

Compute.

1. 549 yards + 36 yards — *585 yards*
2. 857 yards − 39 yards — *818 yards*
3. 563 yards − 138 yards — *425 yards*
4. 195 yards + 532 yards — *727 yards*
5. 846 feet + 682 feet — *1528 feet*
6. 517 yards + 276 yards — *793 yards*
7. 629 inches − 295 inches — *334 inches*
8. 432 yards − 187 yards — *245 yards*
9. 1671 yards − 838 yards — *833 yards*
10. 839 feet − 366 feet — *473 feet*
11. 876 yards + 808 yards — *1684 yards*
12. 771 inches + 417 inches — *1188 inches*
13. 2870 feet − 1451 feet — *1419 feet*
14. 984 inches + 468 inches — *1452 inches*
15. 1929 feet − 778 feet — *1151 feet*
16. 833 yards + 277 yards — *10 yards*

B-101

● Page 100 gives the children practice in computing sums and differences involving inches and feet. Work the first two rows of exercises with the class. Direct the children to complete the other exercises on their own. Then have individual children describe how they computed specific exercises.

● Page 101 provides practice in computing sums and differences involving inches, feet, and yards. Work the exercise at the top of the page as a class activity. Since the measurements given in the story are rough estimates, the answer to the question will also be an estimate. Some of the children may observe that part of the string is used in the process of tying the pieces together. Therefore, although the computation may indicate an answer of about 9 yards, it might be a good idea if Bill and Tom used a fourth piece of string that was a little longer than 9 yards. This will allow them enough string to make the ties and if the resulting piece is too long they can always cut off the excess. Have the children complete the rest of the exercises on their own as a test of their ability to compute such sums and differences.

● Practice in computing sums and differences involving inches, feet, and yards is provided by page 102. The children might enjoy having the class divided into three or more teams to compete for accuracy. Let one team choose a row of exercises. Have all of the teams compute the sums and differences in the chosen row. When the members of one team are finished, call time. No one may continue computation. After the sums and differences are discussed, ask one member of each team to record the number of correct answers that each child on his team has, and to compute the total score. Then have another team choose the next row of exercises to be computed.

Supplemental Experiences

■ Provide the children with opportunities to estimate and compare measures of length. Have them guess how long a book or stick is in inches, feet, or yards. Then let them measure to see how closely they estimated. Ask a child to try to draw a line 7 inches long on the chalkboard. Tell him to measure his line to see if he was correct. Select other children to try to draw lines of given lengths.

Draw a series of vertical or horizontal lines on the chalkboard. Help the children estimate the length of each line in inches, feet, or yards. Have a child measure to check each estimate.

Direct the children to estimate the length of several pieces of string. Then let them measure each piece.

■ Draw a 3-inch line on the chalkboard. Have a child measure the line and give its length in inches. Ask another child to draw a line 5 inches longer than the 3-inch line. Ask another child how long this line should be; let him measure the line to check that the length is accurate. Continue this activity; have the children draw lines that are longer or shorter than given lines.

■ Divide the children into two teams for a measuring game. Write the following lengths on the chalkboard.

3 feet
13 inches
12 inches
5 inches
3 yards

Have the children on each team take turns trying to draw the lengths on the chalkboard. The team that estimates the lengths most accurately will win.

■ Help the children make a measuring tape from adding machine tape and mark it in 1-foot segments. Let the children use this tape to measure the playground, hall, or classroom. Display the tape on the wall in the classroom for future reference. Whenever the children read of a giant whose feet are 10 feet long, or a rabbit that can jump 7 feet into the air, refer them to the measuring tape. The children might enjoy writing stories about imaginary people or animals. The tape can be used to help the children visualize exceptional dimensions that may be used in the story.

KEY IDEA

Cents plus cents is cents.

Scope

To compute with dollars and cents.

To practice addition and subtraction.

To emphasize the concept of ones, tens, and hundreds as related to pennies, dimes, and dollars.

Fundamentals

The child has had previous experiences with money. He is aware that cents plus cents is cents and that cents minus cents is cents; he is aware that his knowledge of addition and subtraction can be applied to money. The idea of applying the addition and subtraction algorisms to the computation of sums and differences of dollars and cents should present little difficulty. It need only be mentioned that \$5.76 is 576¢, that is,

$$\begin{aligned} \$5.76 &= 500¢ + 70¢ + 6¢ \\ &= 576¢ \end{aligned}$$

The application of the addition and subtraction algorisms may be clearly seen.

$$\begin{array}{rcl} \$7.77 & = & 700¢ + 70¢ + 7¢ \\ \underline{+\ 4.68} & = & \underline{+\ 400¢ + 60¢ + 8¢} \\ & & 1100¢ + 130¢ + 15¢ \\ & = & 1000¢ + 100¢ + 100¢ + 30¢ + 10¢ + 5¢ \\ & = & 1000¢ + 200¢ + 40¢ + 5¢ \\ & = & \$12.45 \end{array}$$

$$\begin{array}{rcl} \$9.43 & = & 800¢ + 100¢ + 30¢ + 10¢ + 3¢ \\ \underline{-\ 4.56} & = & \underline{-\ 400¢ - 50¢ \qquad - 6¢} \\ & & 400¢ + 50¢ + 30¢ + 4¢ + 3¢ \\ & = & 487¢ \\ & = & \$4.87 \end{array}$$

The child need only use his knowledge of the addition and subtraction algorisms to compute the sum or difference.

$$\begin{array}{r} \$\ 7.77 \\ \underline{+\quad 4.68} \\ \$12.45 \end{array} \qquad \begin{array}{r} 8\ \ 3\ \ \\ \$\not{9}.\not{4}3 \\ \underline{-\ \ 4.56} \\ \$4.87 \end{array}$$

Readiness for Understanding

Knowledge of the value in cents of pennies, nickels, dimes, quarters, half dollars, and dollars.

Ability to compute with the addition and subtraction algorisms.

Developmental Experiences

for flannel board	*for each child*
cards: items and prices	simulated coins and bills

U.S. coins and bills
sheet of tagboard (18″ × 24″)
envelopes
pins
felt-tip pen

▶ Display a dollar bill and ask for its name. Establish its value as 100 cents. Let a child inspect the bill to see if the name is written on it. Display a five-dollar bill and a ten-dollar bill. Establish the value of each.

Direct the children to use simulated coins at their desks to make a set of half dollars that has the same value as a dollar. Continue with quarters, dimes, nickels, pennies, and finally sets that contain different coins such as 2 quarters and 5 dimes. Summarize the activities with the statement that one dollar has the same value as any set of coins with the value of 100 cents.

Design a two-column chart on a sheet of tagboard and fasten it to the chalkboard. Hold up a penny and ask a child to name the coin and its value. Enter this information on the chart. Continue this procedure with the nickel, dime, quarter, half dollar, dollar bill, five-dollar bill, and ten-dollar bill.

COINS and BILLS	VALUE
penny	1¢
nickel	5¢
dime	10¢
quarter	25¢
half dollar	50¢
dollar bill	100¢
five-dollar bill	500¢
ten-dollar bill	1000¢

▶ Place simulated money that represents amounts from 1 cent to 40 dollars in several envelopes. Give a child an envelope that contains less than one dollar in coins. Tell this child to remove the contents, to determine the value of the coins, and to record the value on the chalkboard, using the cents sign. Beside the value he recorded, write the same value with the dollar sign and decimal point. Review the fact that amounts less than 1 dollar may be recorded either way, but that amounts of 1 dollar or more are usually recorded with the dollar sign and decimal point.

Write on the chalkboard examples such as:

56¢	$ .56
1¢	$ .01
723¢	$7.23

Explain that the symbols ¢ and $ are not used at the same time and that the symbols ¢ and . are not used at the same time. When using the symbols $ and ., the numeral before the decimal point shows the number of dollars, and the numeral after the decimal point shows the number of cents.

Write an example on the board.

$4.37

Explain that the first digit to the right of the decimal point (3) represents the number of dimes and the second digit to the right of the decimal point (7) represents the number of pennies. Tell the children that $4.37 can be thought of in many ways:

4 dollars + 37 cents,
4 dollars + 3 dimes + 7 cents,
43 dimes + 7 cents, or
437 cents.

Continue to use the envelopes of simulated coins and have the amounts recorded on the chalkboard in two ways. Keep the two methods for recording in separate columns.

Vary the activity; write on the chalkboard 36¢, $.57, $.06, 3¢, 906¢, and 4700¢ and ask the children to record them in another way. Then call out several amounts less than 100 dollars and direct the children to record them on the chalkboard.

Next, discuss the fact that although amounts of 1 dollar or more are usually recorded with the dollar sign and decimal point, there are instances when only the dollar sign is used. Explain to the class the fact that when an amount of money involves a number of dollars only, the decimal point may be omitted ($763). The children may have observed this in connection with prices of furniture or jewelry where hundreds of dollars or thousands of dollars are involved. Prepare and display a picture of items with price tags showing only the dollar sign. Have the children read the price of each item as you show them the pictures one at a time.

▶ Prepare cards to show items that are priced from less than 1 dollar to 9999 dollars. Place the cards with items that cost less than 100 dollars in one pack, the cards with items that cost from 100 dollars to 1000 dollars in another pack, and the cards with items that cost more than 1000 dollars in a third pack. Ask a child to choose two cards from one of the packs and pin them to the flannel board. Have the child compute on the chalkboard the total cost of the two items (for example, $93.49 + $27.99).

$$\begin{array}{r} \$93.49 \\ +\ \ 27.99 \\ \hline \end{array}$$

Review the idea that the symbol $ usually appears with only the numeral for the first addend, but that other symbols (¢ or .) appear with each numeral. However, we always place the symbols $ and . or the symbol ¢ in the numeral for the sum.

Tell the class that the dollar sign and decimal point may be ignored when computing the sum. They need only use their knowledge of the addition algorism to compute the sum. Ask the child who is at the chalkboard to describe the computation of the sum. Have him record the needed numerals as he describes his computation steps.

```
  $ 93.49    9+9 is 18; 18 is 8+10.
+   27.99    Write 8, remember 1 ten.
  $121.48    1 ten + 4 tens + 9 tens is 14 tens;
             14 tens is 4 tens + 1 hundred.
             Write 4 tens, remember 1 hundred.
             1 hundred + 3 hundreds + 7 hundreds
             is 11 hundreds; 11 hundreds is 1 hun-
             dred + 1 thousand.
             Write 1 hundred, remember 1 thousand.
             1 thousand + 9 thousands + 2 thou-
             sands is 12 thousands.
             Write 12 thousands.
```

Then let a second child tell which item has the greater cost. Tell him to go to the chalkboard to compute the difference between the cost of the two items (for example, \$93.49 − \$27.99) in order to tell how much greater the cost of one item is compared with the cost of the other. Again the dollar sign and decimal point may be ignored when computing the difference. Have him record the needed numerals as he describes his computation steps.

```
   82
  $93.49    Subtract 9 from 9 with the result of 0.
−  27.99    Subtract 9 tens from 10 tens (1 of the
  $65.50    3 hundreds) leaving 2 hundreds. Add
            4 tens to the difference (10 tens − 9 tens)
            with the result of 5 tens.
            Subtract 7 hundreds from 10 hundreds
            (1 of the 9 thousands) leaving 8 thousands.
            Add 2 hundreds to the difference (10
            hundreds − 7 hundreds) with the result
            of 5 hundreds.
            Subtract 2 thousands from 8 thousands
            with the result of 6 thousands.
6 thousands + 5 hundreds + 5 tens + 0 ones is 6550.
```

Continue in this way to give several other children an opportunity to describe the steps used to compute the total cost of two items and the difference in cost between two items in any of the three packs of cards.

Name

Compute the cost.

1. A hamburger, a soda, a bag of marshmallows, and a bag of peanuts. \$1.20
2. A root beer, a cheeseburger, a hotdog, a sundae. \$1.20
3. Two giant boxes of popcorn, 5 colas, 3 root beers, and 3 bags of peanuts. \$2.55
4. Two hamburgers and two sodas. \$1.20
5. A soda, a sundae, a cola, and a root beer. \$.85
6. A cheeseburger, 2 hamburgers, a bag of peanuts, and 2 bags of marshmallows. \$2.10

B-103

Pages 103 and 104

● To introduce page 103, call attention to the menu in the illustration. Ask a child to read the menu. Let a child name four items he would like to order. Have the class compute the cost. After several children have ordered, assign the exercises for independent work. The children should be permitted to compute the cost of the items on the menu in any way they wish. After the page has been completed, ask individual children to describe how they computed specific exercises.

Compute.

\$ .22 + .59 \$.81	2. \$ .98 + .67 \$1.65	3. \$ .72 − .38 \$ .34	4. \$ .85 − .36 \$ .49
5. \$3.34 + 2.57 \$5.91	6. \$2.80 − .65 \$2.15	7. \$9.35 − 2.78 \$6.57	8. \$8.00 + 5.84 \$13.84
9. \$36.78 + 42.79 \$79.57	10. \$87.99 − 41.22 \$46.77	11. \$95.34 + 7.77 \$103.11	12. \$13.26 + 14.53 \$27.79
13. \$24.00 − 1.36 \$22.64	14. \$98.00 − 69.84 \$28.16	15. \$35.00 − 9.45 \$25.55	16. \$25.00 − 8.98 \$16.02
17. \$6817 − 2321 \$4496	18. \$2000 − 647 \$1353	19. \$9999 + 600 \$10599	20. \$5760 − 500 \$5260
21. \$ .75 .03 + .42 \$1.20	22. \$1.17 3.21 + 5.50 \$9.88	23. \$34.16 28.08 + 31.73 \$93.97	24. \$62.41 30.19 + 51.87 \$144.47
25. \$21.63 32.06 14.51 + 81.42 \$149.62	26. \$98.16 35.57 3.05 + 46.76 \$183.54	27. \$832 728 477 + 867 \$2904	28. \$1246 905 424 + 613 \$3188

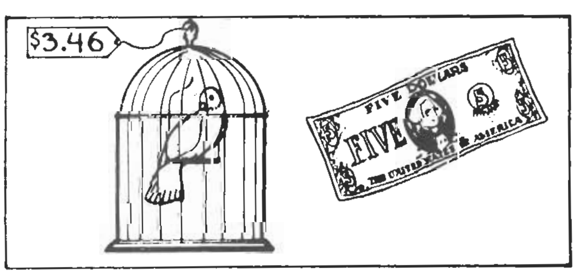
B-104

● Page 104 provides practice in computing sums and differences involving amounts of money. Instruct the children to complete the exercises on their own as a test of their ability to compute such sums and differences. It is not necessary that all of the exercises be completed at the same time.

Supplemental Experiences

■ Give the children an opportunity to determine how much change they will receive after paying a given amount of money for an item that has a given cost. Help them understand that the difference between the amount of money given and the cost of the item is the amount of money they will receive in change.

Prepare cards that show an item, its cost, and the amount of money that will be given to pay for the item.



Ask a child to choose one of the cards, show it to the class, and compute on the chalkboard the amount of change he should receive after paying for his purchase. Have the class check the computation.

Ask another child to select the simulated coins and bills that the first child may receive in change. Let the class check the value of the coins.

Give other children an opportunity to compute the amount of change to be received and to decide on coins and bills that could be used as change.

■ Help the children write story exercises that involve addition or subtraction of money. Put the exercises in a box. Let a child draw an exercise from the box and read it aloud. Then have him call on two other children to work the exercise at the chalkboard. They will compete to see who can perform the operation first and still maintain accuracy and good form.

■ Prepare a set of cards that picture coins and bills with a combined value of less than \$10.00. Also prepare pictures of toys, games, or other items with price tags. Display the cards that picture coins and bills on the chalktray. Have a child pick up the first picture-card and then find the card with a set of coins and bills that have the same value as the price tag on the picture-card. Ask him to show both cards to the class. If his selection is correct, the next child may continue.

UNIT 8
COMPUTATION OF BASIC PRODUCTS

Pages 105 Through 124

OBJECTIVE

To develop computational skill with the use of basic multiples of 6, 7, and 10.

Arrays are used to help the child determine the count of a product. The child learns that an array may be partitioned to represent partial products and that the sum of partial products is the whole product.

See Key Topics in Mathematics for the Primary Teacher: Multiplication.

KEY IDEAS

The sixes are twice the threes.
2 threes + 2 fours is 2 sevens.
The tens are twice the fives.
$(1 \times 6) - (1 \times 6) = 0 \times 6.$

KEY IDEA

The sixes are twice the threes.

Scope

To review the count of a product.

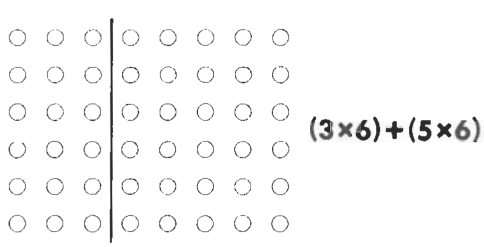
To review multiplication facts that have 2, 3, 4, or 5 as factors.

To develop multiplication combinations related to multiples of 6.

To recognize a product as a sum of partial products.

Fundamentals

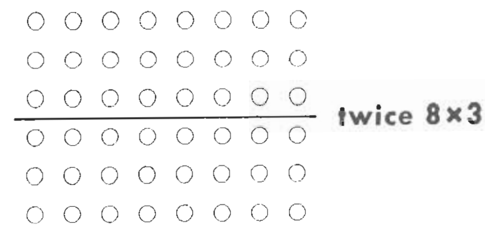
The basic facts previously developed are utilized as a basis for the development of additional multiplication facts through the child's awareness of an array as the union of two or more arrays. For example, an 8 by 6 array may be partitioned into a 3 by 6 and a 5 by 6 array.



Partitioned arrays may be used to develop new multiplication facts and, if necessary, to review the products presented earlier. Note that the preceding partition indicates a way of computing 8×6.

$$\begin{aligned} 8 \times 6 &= (3 \times 6) + (5 \times 6) \\ &= 18 + 30 \\ &= 48 \end{aligned}$$

Another way of computing 8×6 is indicated by a twice 8 by 3 array.

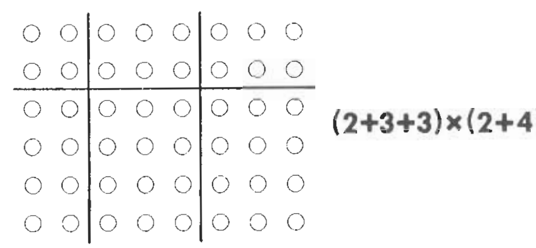


$$\begin{aligned} 8 \times 6 &= (8 \times 3) + (8 \times 3) \\ &= 24 + 24 \\ &= 48 \end{aligned}$$

If the array is viewed as a (3 + 3) by 8 array, this multiplication algorism is indicated:

$$\begin{array}{r} 3 + 3 \\ \times \quad 8 \\ \hline 24 \\ \underline{24} \\ 48 \end{array}$$

It is possible to partition an array into more than two parts.



The partitioning shows 8×6 as the sum of 6 partial products.

$$8 \times 6 = (2 \times 2) + (2 \times 3) + (2 \times 3) + (4 \times 2) + (4 \times 3) + (4 \times 3)$$

The teacher should realize that many children will learn new multiplication facts from the Developmental Experiences without any particular effort. Others will continue to depend on their ability to relate these facts to an array and the sum of partial products. Those who remember may have an advantage if they are also able to justify their memorized fact.

Readiness for Understanding

Knowledge of basic combinations related to multiples of 2, 3, 4, and 5.

Understanding of the concept of product.

Developmental Experiences

for flannel board
counters
yarn

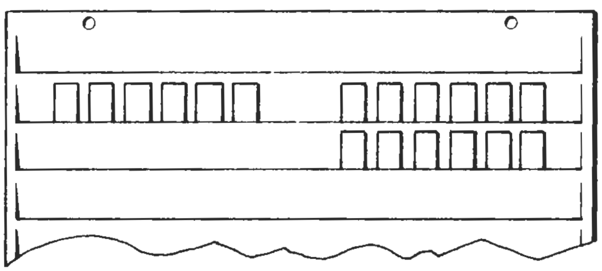
for each child
plastic numerals and symbols
counters
$\frac{1}{2}$-inch squared paper
crayons

pocket chart
tagboard strips (1″ × 2″)
sheets of 1-inch squared paper
washers
sheet of tagboard
masking tape
array-cards

▶ Have a child show a 1 by 6 array by using tagboard strips in the pocket chart. Tell the children to use their plastic numerals and symbols to construct an equation on their desks that relates the product and the count for this array. Ask someone to read his equation aloud.

$$1 \times 6 = 6$$

Have a second child show an array that has twice as many members as the 1 by 6 array.

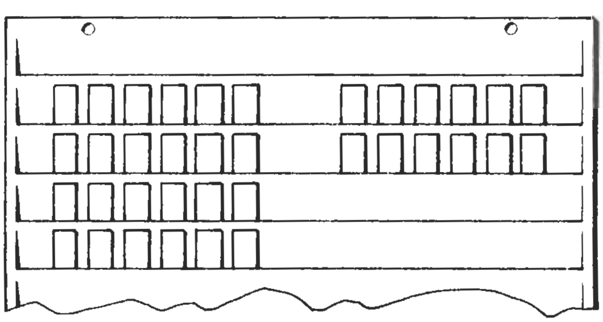


Direct the children to show, below their first equation, an equation that relates the product and the count for the second array.

$$1 \times 6 = 6$$
$$2 \times 6 = 12$$

Ask a child to read both equations aloud. The class should be aware that since 2×6 is twice 1×6, the count for 2×6 is twice the count for 1×6.

Remove the 1 by 6 array from the chart. Ask some child to construct an array that has twice as many members as the 2 by 6 array.



Tell the children to push their first equation on their desks ($1 \times 6 = 6$) out of the way. Have them show on their desks the product and the count for the new array in equation form. Have a child read this latter equation aloud.

$$2 \times 6 = 12$$
$$4 \times 6 = 24$$

The class should observe that since 4×6 is twice 2×6, the count for 4×6 is twice the count for 2×6.

Adapt the procedure to help the children observe the relationship between the following products.

4×6 and 8×6
3×6 and 6×6
5×6 and 10×6

Then direct the children to show the following arrays in the pocket chart: a 0 by 6 array, a 7 by 6 array, and a 9 by 6 array. In each instance, a child should show at his desk the product and the count for the given array. Give the children an opportunity to determine the counts for these arrays in their own way.

▶ Instruct the children to use their counters to show both a 1 by 3 array and a 1 by 6 array on their desks. Direct the children to use plastic symbols to show an appropriate equation for each array.

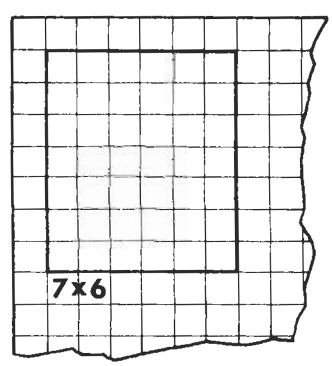
● ● ● ● ● ● ● ● ●
1×3=3 **1×6=6**

Have the children tell what relationship they observe between these two products. Someone may comment that 1×6 is twice 1×3. Someone else may say 6 is twice 3.

Continue in this way to have the class observe the relationship between the following products.

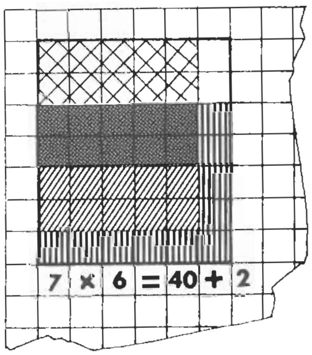
2×3 and 2×6
3×3 and 3×6
4×3 and 4×6
5×3 and 5×6
⋮
10×3 and 10×6

▶ Provide each child with 3 or 4 sheets of $\frac{1}{2}$-inch squared paper and a black crayon. Direct the children to outline a 7 by 6 array in one of the upper corners of their paper. Have them write the product for the array directly below the array.

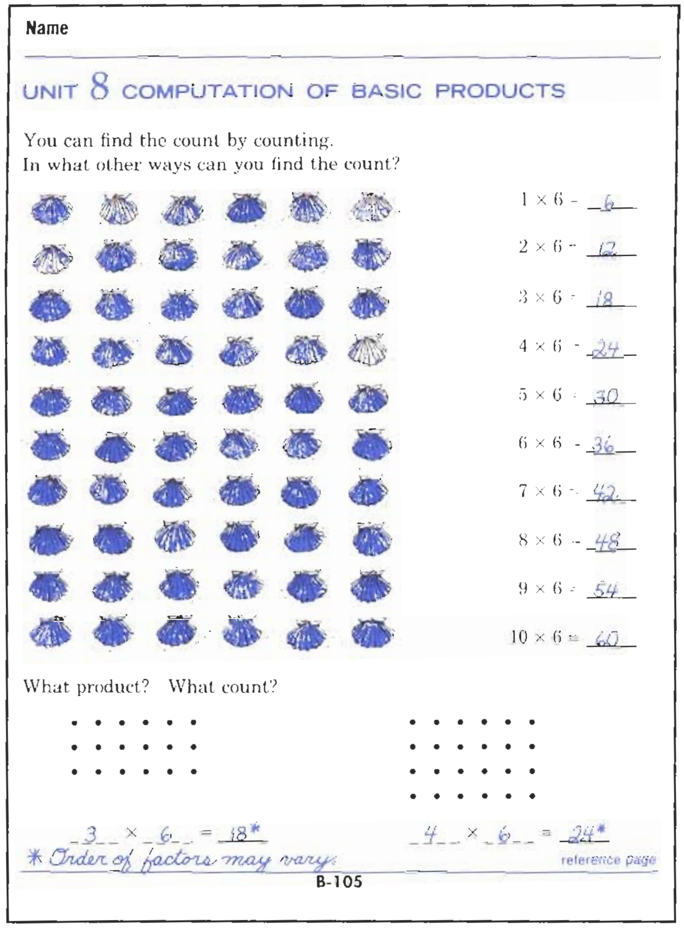


Explain how the children should use crayons to show the number of tens and ones in the product 7×6. Tell them that a different-colored crayon should be used to show each 10. For example, one ten could be shaded red, another could be shaded blue, and so on. The ones should be a color different from any of the tens, but all ones should be the same color. There are several ways to show the tens and ones and each child should choose his own way.

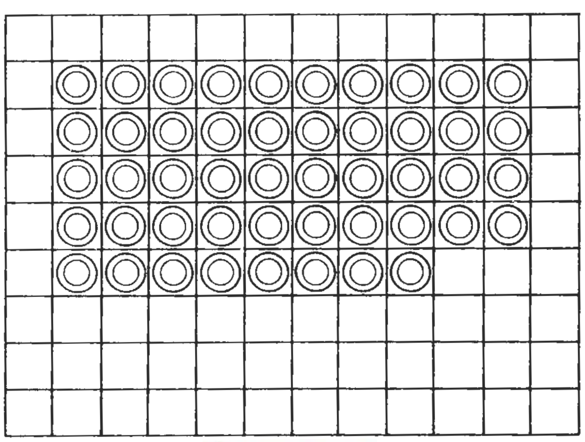
After the children have finished coloring their arrays, ask individuals to show how they see tens and ones in their array. Then have the children write the sum of tens and ones beside the product 7×6. Tell the children to link the two forms with an equal sign.



Adapt this procedure to other products from 1×6 through 10×6.



▶ Separate the class into pairs of children. Each pair should have a sheet of 1-inch squared paper, 65 washers, and a set of plastic numerals and symbols. Explain that the children in each pair are to work together to show an 8 by 6 array on the sheet of paper and the product for their array on their desk. They should then separate the washers to show a sum of tens and ones. Tell the children to use an equal sign to link the product with the sum of tens and ones.



8×6=40+8

Continue the activity; have the children work in pairs to show the number of tens and ones in products that have 6 as a factor, from 1×6 through 10×6.

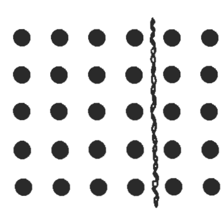
Page 105

● Use page 105 primarily as a discussion page. Have the children observe the array and the first of the equations in the sequence of 10 equations. Call on someone to give the count for 1×6. Ask this child to tell how he arrived at this count for the number of members in an array having 1×6 members. Let the children discuss any other ways in which they determined the count in this instance.

Ask the children to write the count for each product listed. Then have the children tell how they arrived at the count. Finally, have them complete the exercises at the bottom of the page.

Developmental Experiences

▶ Place a 5 by 6 array on the flannel board. Ask some child to write on the chalkboard the product for this array. Then place a piece of yarn between the members of the array as illustrated.



Tell the children to study the arrays that result from this partition; have them tell the product for each array. Write a sum of the products of the smaller arrays below the product already on the chalkboard.

$$5 \times 6$$
$$(5 \times 4) + (5 \times 2)$$

Let the children discuss their observations concerning the relationship between the whole array and its parts. Comments similar to the following may be made by the children:

A 5 by 6 array is made up of a 5 by 4 array and a 5 by 2 array.

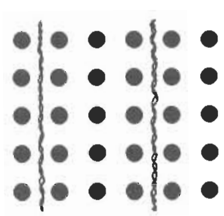
A 5 by 6 array has the same number of members as a 5 by 4 array and a 5 by 2 array.

By adding the products of the smaller arrays, you get the product of the larger array.

The number of the array is a sum of the products of the parts.

Give the children ample opportunity to tell about their observations.

Partition the array once again.

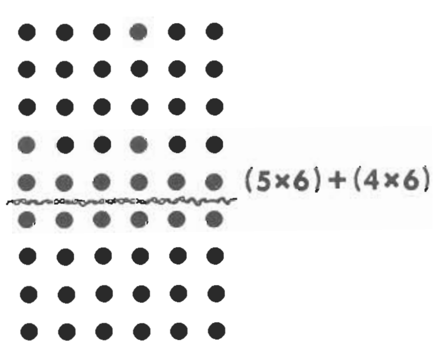


Have the class tell the product for each array; record the responses on the chalkboard.

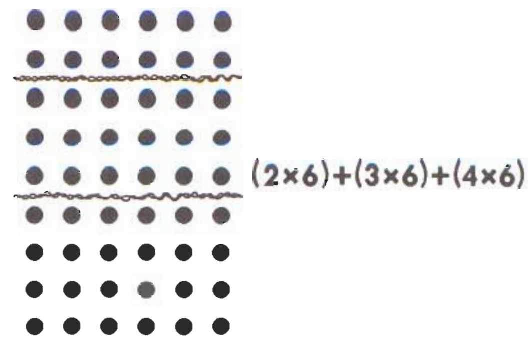
$$5 \times 6$$
$$(5 \times 4) + (5 \times 2)$$
$$(5 \times 1) + (5 \times 3) + (5 \times 2)$$

Remove the pieces of yarn from the flannel board. Ask a child to use one piece to show a different vertical partition of the array. If he has trouble remembering the other partitionings, let him refer to the sum of the products on the chalkboard. Then have the child record the sum of the products his partition suggests—for example, $(5 \times 3) + (5 \times 3)$.

Remove the piece of yarn from the board and enlarge the array to a 9 by 6 array. Give the yarn to another child and tell him to use it to partition the array horizontally. Ask another child to record the sum of the products this particular partitioning suggests—for example:



Have a second horizontal partition made with another piece of yarn and the related sum for the products recorded—for example:



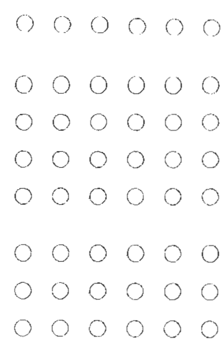
Adapt this procedure to an 8 by 6 array, a 7 by 6 array, and a 6 by 6 array. First have the children partition the arrays with vertical lines and then with horizontal lines. Allow the children to use 1 through 5 vertical lines and from 1 through 8 horizontal lines to show the partitions. In each instance, have the sum of the products recorded below the original product.

▶ Show a 7 by 6 array partitioned into three parts.

Have the children use their plastic numerals and symbols on their desks to show the sum of the products illustrated. (If parentheses are not available in the sets of symbols, proceed without them.) Instruct the children to show the product of the whole array below the sum of the products.

$$(7 \times 2) + (7 \times 2) + (7 \times 2)$$
$$7 \times 6$$

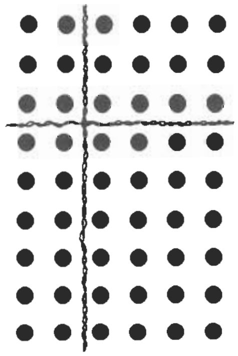
Show an 8 by 6 array partitioned as illustrated.



Tell the children to record the sum of the products for this partitioning. Have them write the product of the whole array below the sum.

Continue in this way with all of the products in which 6 is one of the factors.

▶ Place a 9 by 6 array on the flannel board. Give a child two pieces of yarn and tell him to use one piece to partition the array vertically and to use the other piece to partition the array horizontally.



Let the child select another member of the class to record on the chalkboard the sum of the products indicated by the partitioning. Have this second child call on someone to come to the chalkboard to show the product for the whole array.

$$(3 \times 2) + (3 \times 4) + (6 \times 2) + (6 \times 4)$$
$$9 \times 6$$

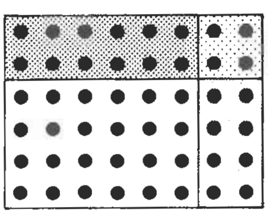
Remove the yarn from the flannel board and give the pieces to some other child. Direct him to show a two-way partitioning of the array that is different from the first child's. Have him call on someone else to write the sum of the products on the board below 9×6.

$$(3 \times 2) + (3 \times 4) + (6 \times 2) + (6 \times 4)$$
$$9 \times 6$$
$$(4 \times 3) + (5 \times 3) + (4 \times 3) + (5 \times 3)$$

Adapt this procedure to 6×6, 7×6, and 8×6.

▶ Draw an array that is partitioned once horizontally and once vertically on a large sheet of tagboard. Make the background of each small array a different color—red, yellow, blue, and white. Fasten the card to the chalkboard.

Ask the class to help you compute the product for this array. First have someone give the product and count for the array in the red section of the card. Write on the chalkboard the equation that links the product and the count. Continue in this way with the product and count for each of the other arrays.



RED 2×6= 12
YELLOW 2×2= 4
WHITE 4×6=24
BLUE 4×2= 8

As a final step, have the children compute the sum of the partial products, and record this number below the list of computed partial products.

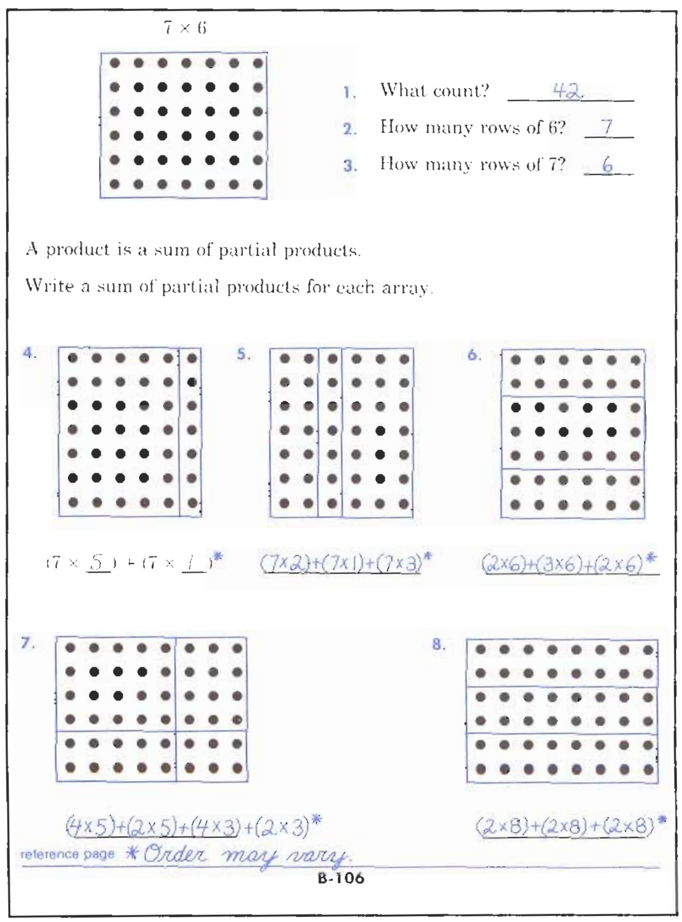
RED	2×6= 12
YELLOW	2×2= 4
WHITE	4×6=24
BLUE	4×2= 8
	48

Write on the chalkboard the equation that links the product and the count for the given array: $8 \times 6 = 48$. Help the children check the count for this product in the following ways: by counting the members of the array one by one, by counting by 2 (adding 2s), by counting by 3 (adding 3s), by counting by 6 (adding 6s), and by finding the number of tens and ones.

Adapt this procedure to other products from 6×6 through 10×6.

Pages 106 and 107

● Pages 106 and 107 provide practice in recognizing a product as a sum of partial products. Discuss the example at the top of each page with the class. With the example on page 106, let the children use any method they want to find the count. Remind the children that both the count and the product are expressions of the number of members in the given array. Ask whether or not there are other ways to express the number of objects in an array. Someone may suggest considering it as a sum of the products. Let the children suggest a few sums of the products that would express the number of this 7 by 6 array. Record these sums on the chalkboard as they are given. Then have the children work independently with the remaining exercises. Direct the children to write a sum of the products for each array.


7 × 6

1. What count? 42
2. How many rows of 6? 7
3. How many rows of 7? 6

A product is a sum of partial products.

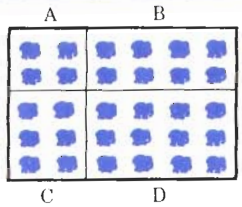
Write a sum of partial products for each array.

4. (7 × 5) + (7 × 1)*
5. (7×2)+(7×1)+(7×3)*
6. (2×6)+(3×6)+(2×6)*
7. (4×5)+(2×5)+(4×3)+(2×3)*
8. (2×8)+(2×8)+(2×8)*

reference page *Order may vary.

B-106

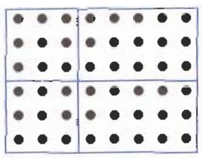
Name



$5 \times 6 = 30$

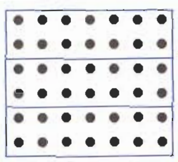
PART A	$2 \times 2 = 4$
PART B	$2 \times 4 = 8$
PART C	$3 \times 2 = 6$
PART D	$3 \times 4 = 12$
	30

Write each product as the sum of partial products.

1. 

$6 \times 8 = 48$

3 × 3 = 9*
3 × 3 = 9
3 × 5 = 15
3 × 5 = 15
48

2. 

$6 \times 7 = 42$

2 × 7 = 14*
2 × 7 = 14
2 × 7 = 14
42

*Order may vary.

reference page

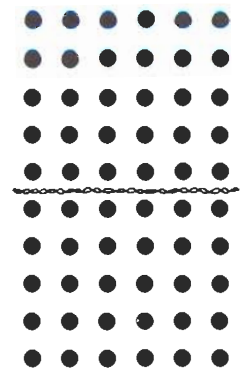
B-107

● As children discuss the example on page 107, point out the partitions in the array. Each partition has been lettered and the specific partial product of each array has been recorded.

Ask the children to write the partial products and the count for each partial product in the first exercise and to compute the sum. Then discuss whether or not the sum is 48. Proceed with the next exercise in a similar way.

Developmental Experiences

▶ Place a 10 by 6 array on the flannel board and have a child write on the chalkboard the product for this array. Use the array as a model to show the factor 10 as 5 + 5. Partition the array as illustrated.

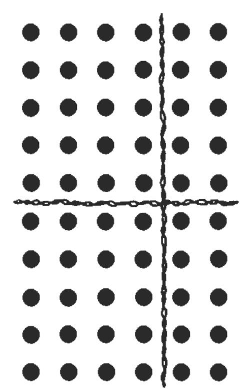


Record the products on the chalkboard.

10×6
$(5 + 5) \times 6$

Ask a child to explain why both products can be used for the same array.

Direct a child to show two parts of the factor 6. Direct him first to point out the part of the array that is represented by 6. Have him then use a piece of yarn to partition the array and show the two parts of 6 that he sees. He may, for example, partition to show 6 as 4 + 2.



Direct the class to observe this partition and tell how the child is thinking of 6 (as 4 + 2).

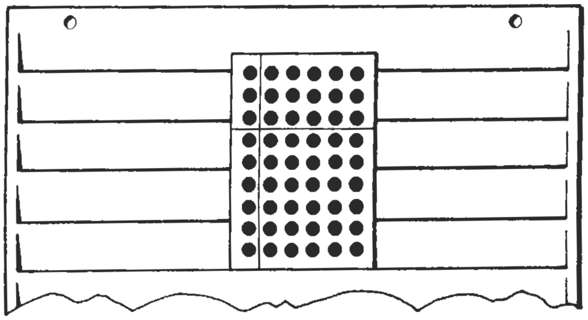
Again, rewrite the product 10×6—record the factor 6 as 4 + 2.

10×6
$(5 + 5) \times 6$
$(5 + 5) \times (4 + 2)$

Remove the pieces of yarn from the flannel board. Let other children partition the array to show different parts of the two factors of the product 10×6. In each instance, record the child's view of the given factor.

Adapt this procedure to other products from 6×6 through 9×6.

▶ Place in the pocket chart a tagboard card that shows a 9 by 6 array partitioned once horizontally and once vertically.



Instruct the children to use their plastic numerals and symbols at their desks to show the product for the given array.

Then ask the children to express the product in a way that shows the parts of the two factors, 9 and 6.

$$\begin{array}{c} 9 \times 6 \\ (3 + 6) \times (1 + 5) \\ \text{or} \\ (3 + 6) \times (5 + 1) \\ \text{or} \\ (6 + 3) \times (5 + 1) \\ \text{or} \\ (6 + 3) \times (1 + 5) \end{array}$$

Continue in this way using the products 10×6, 8×6, 7×6, 6×6, 5×6, and 4×6.

▶ Ask some child to show a 7 by 6 array on the chalkboard and to record the product of this array in vertical form.

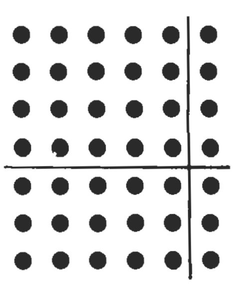
$$\begin{array}{r} 7 \\ \underline{\times 6} \end{array}$$

Ask the children to help you compute the product of this array in parts. Ask someone to suggest a way to think of 7 in parts. Perhaps a child will want to think of 7 as 4 + 3. Record his suggestion on the chalkboard.

$$\begin{array}{l} 7 = 4 + 3 \\ \underline{\times 6} \end{array}$$

Have a second child partition the array to show 7 as 4 + 3. Then ask another child to suggest a way to think of the factor 6 in two parts. Someone may suggest that 6 be thought of as 5 + 1. Record the child's suggestion on the chalkboard and have him partition the array to show 6 as 5 + 1.

$$\begin{array}{l} 7 = 4 + 3 \\ \underline{\times 6} = \underline{5 + 1} \end{array}$$



Ask for the count of this array. (42) Then ask the children to help you compute 7×6 in parts to see whether or not this method of computation results in the same answer.

Direct the children to first compute 3×5 and 3×1. Have a child point to the two sections of the array that correspond to the two computed products. Record the counts and beside them record the related products in parentheses. Next have the children compute 4×5 and 4×1 and let a child point out the sections of the array that correspond to these computed products. Record these partial products and have the children compute the sum. Record the response and have it compared with the count for 7×6.

$$\begin{array}{lr} 7 = 4 + 3 & \\ \underline{\times 6} = \underline{5 + 1} & \\ \;\;42 & 15\ (3 \times 5) \\ & 3\ (3 \times 1) \\ & 20\ (4 \times 5) \\ & \underline{4}\ (4 \times 1) \\ & 42 \end{array}$$

Let a child draw another 7 by 6 array beside the first array on the chalkboard and partition the 7×6 array once vertically and once horizontally. Tell him to partition the array in a different way. Then help the child record the parts his partitioning suggests. For example, 7 may be shown as 5 + 2 and 6 as 3 + 3.

$$\begin{array}{l} 7 = 5 + 2 \\ \underline{\times 6} = \underline{3 + 3} \end{array}$$

Adapt the procedure suggested in the preceding paragraphs to compute this form of 7×6. As a final step in this computation, have the class compare the computed sum of these partial products to the count for 7×6.

Draw a third 7 by 6 array on the chalkboard and ask a child to show an entirely different two-way partitioning of the array. Have the children compute 7×6 by using this third way of viewing the two factors.

Continue in this way using the products 10×6, 9×6, and 6×6.

Pages 108 through 111

● Use page 108 for discussion. After the illustration has been observed and the story read, the children should be aware that the product may be expressed in different ways— that $(4 + 2) \times (5 + 3)$, $(4 + 4) \times (3 + 3)$, and 8×6 all express the number of members that belong to the given array.

Have each child write 8×6 in several other ways by naming each factor as a sum. Let several children write their lists on the chalkboard. Accept their suggestions for as many partial products as they see. As long as they are guided by the idea that a product is a sum of products, and that the whole is the sum of its parts, there need be no particular limit set on the number of these parts that the children wish to use. In fact, 8×6 could be thought of as $(1 + 1 + 1 + 1 + 1 + 1 + 1 + 1) \times (1 + 1 + 1 + 1 + 1 + 1)$ if desired. Instruct the children to make a list of names for the arrays at the bottom of the page. Let them discuss their ideas.

● Pages 109 through 111 provide the children with an opportunity to compute products in parts and to investigate multiples of 6.

Work and discuss the example at the top of page 109 with the class. Instruct them to compute each of the other two exercises in a similar way.

Pages 110 and 111 are to be assigned as independent work. Page 110 follows the pattern set on page 109. Note that the children will have to partition the array for exercise 3. On page 111, the children must interpret the given partitions to write each factor as a sum in the first exercise. For exercise 2, the children are to partition the array and then interpret each factor as a sum.

"That's a 4 + 2 by 5 + 3 array," said Kay.
"No, it's not," said Jean. "That's an 8 by 6 array."
"I'm sure that it's a 4 + 4 by 3 + 3 array," said Sue.

Who was right? *Each one was right.*

Make a list of other names for the 8 by 6 array.

*6 + 2 by 5 + 1 **
1 + 7 by 2 + 4
2 + 3 + 3 by 6
4 + 4 by 2 + 2 + 2

Make a list of names for each array.

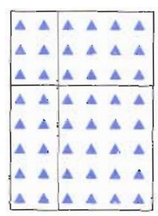
*1 + 3 by 3 **
2 + 2 by 1 + 2

*4 by 4 **
3 + 1 by 4
3 + 1 by 2 + 2

reference page **Other names are possible.*

Name

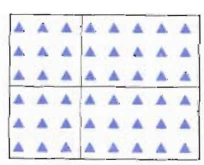
How can you compute 8 × 6 using parts?



$8 = 5 + 3$
$\times 6 = 2 + 4$

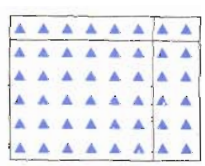
48	12	(number of the 3 by 4 array) *
	20	(number of the *5* by *4* array)
	10	(number of the *5* by *2* array)
	6	(number of the *3* by *2* array)
	48	(sum of the partial products)

Compute using parts.

1\. 

$8 = 5 + 3$
$\times 6 = 3 + 3$

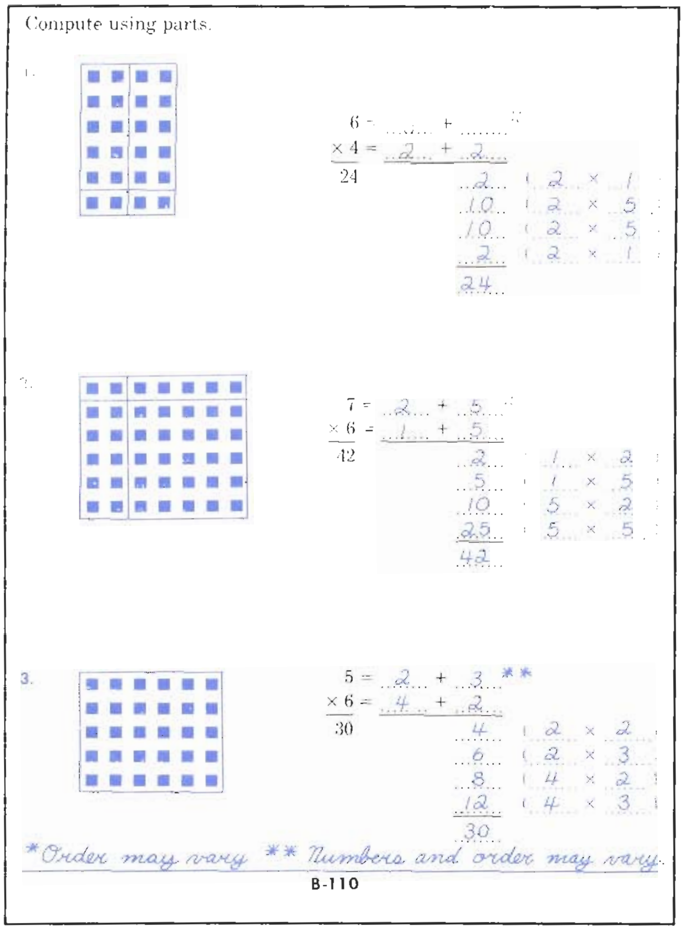
48	9	(3 × 3)
	9	(*3* × *3*) *
	15	(*3* × *5*)
	15	(*3* × *5*)
	48	

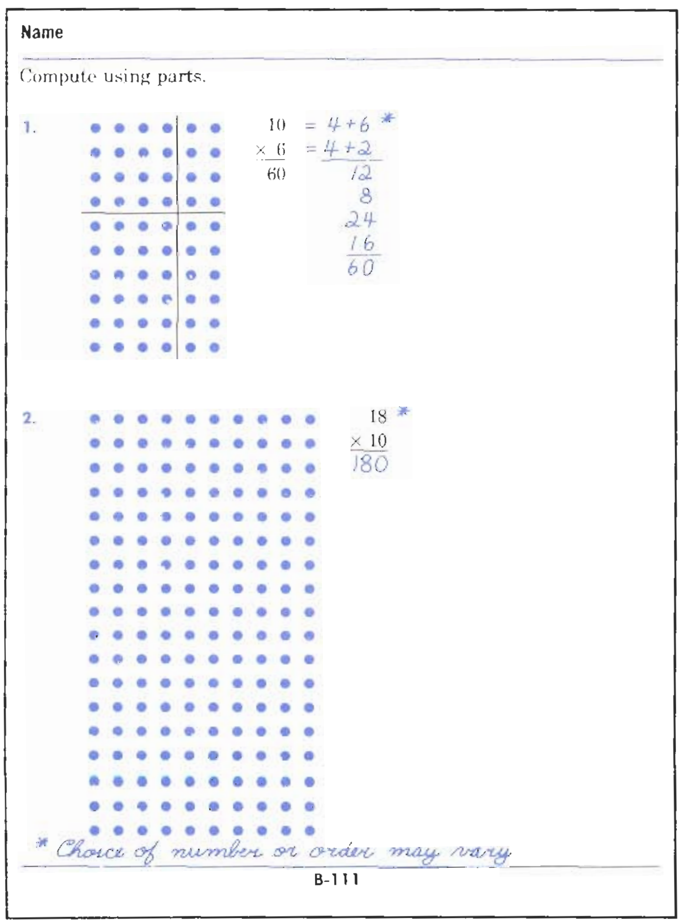
2\. 

$8 = 6 +$ *2*
$\times 6 = 5 +$ *1*

2	(*1* × *2*) *
6	(*1* × *6*)
10	(*5* × *2*)
30	(*5* × *6*)
48	

**Order may vary.*





Supplemental Experiences

■ Separate the class into two teams. Have a pair of players from each team select two cards from a pack that contains products from 1×4 through 10×4, from 1×5 through 10×5, and 1×6 through 10×6. Have each of the four children write his product and its count in an equation on the chalkboard. Allow each child to compute his product and get its count in any way he chooses.

Then let each pair of team members compare their products. Have the children in each pair record their results around the words *is greater than*, or around the words *is equal to* that you have written for each team on panels of the chalkboard. For example, if the computed products for one team are 32 (8×4) and 36 (6×6), the members of this team should write their numbers in the following way:

<u>36</u> is greater than <u>32</u>

If both children in a pair have computed correctly, they earn a point for their team. If they arrange their computed products so that the relationship between the numbers is correctly stated, they earn an additional point for their team. After all of the children have participated in the game, total the points earned by each team and declare a winner.

■ Provide practice in computing products mentally. Read aloud exercises such as the following and call on various children to name the result of each computation.

2 times 1 (pause) times 3 (pause) times 4,
3 times 2 (pause) times 1 (pause) times 6,
4 times 2 (pause) times 0 (pause) times 5,
2 times 2 (pause) times 2 (pause) times 6,
0 times 6 (pause) times 4 (pause) times 1,
1 times 5 (pause) times 2 (pause) times 4,
1 times 2 (pause) times 4 (pause) times 6.

KEY IDEA

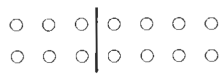
2 threes + 2 fours is 2 sevens.

Scope

To develop multiplication combinations related to multiples of 7.

Fundamentals

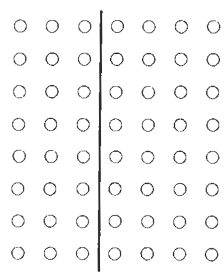
It is possible to join two arrays with a common dimension by placing the common dimensions of each array adjacent to each other.



2×(3+4)

2 threes + 2 fours is 2 sevens

Note that the product of the union is the sum of the partial products, $(2 \times 3) + (2 \times 4)$, and that $(2 \times 3) + (2 \times 4) = 2 \times (3 + 4)$. This is an illustration of the distributive property. It can be seen that the distributive property of multiplication over addition allows the child to compute new products by the process of adding known products. Consider an 8 by 7 array viewed as an 8 by (3 + 4) array.



$$\begin{aligned} 8 \times 7 &= 8 \times (3 + 4) \\ &= (8 \times 3) + (8 \times 4) \\ &= 24 + 32 \\ &= 56 \end{aligned}$$

or

$$\begin{array}{r} 3 + 4 \\ \times \quad 8 \\ \hline 32 \\ 24 \\ \hline 56 \end{array}$$

By using arrays to explore unfamiliar products as sums of partial products, the child practices previously learned basic facts and develops a better understanding of computational techniques.

Readiness for Understanding

Understanding that a partitioned array represents the sum of products.

Developmental Experiences

for flannel board
15 tagboard strips (1″ × 7″)
felt numerals and symbols
counters

for each child
sheet of 1-inch squared paper
crayons
counters
2 cardboard strips (¼″ × 12″)
plastic numerals and symbols

felt-tip pen
masking tape

▶ Cut out 15 tagboard strips (1 inch by 7 inches) and prepare them for use on the flannel board. Outline 7 one-inch squares on each strip. Have a child choose the strips he needs to show a 1 by 7 array on the flannel board. Have a second child choose the felt numerals and symbols he needs to show the product and the count for this array in an equation. Direct the second child to place his equation above the array and then to call on someone to read his equation aloud.

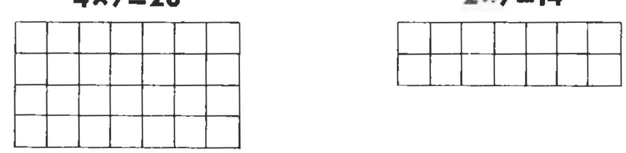
Let another child construct an array that has twice as many members as the 1 by 7 array. Ask someone else to show the product and the count in an equation above the array.

1×7=7 **2×7=14**

Have the child read the equation aloud. The class should be aware that 2×7 is twice 1×7.

Remove the 1 by 7 array and the equation from the flannel board. Have a child construct an array that has twice as many members as the remaining 2 by 7 array. Direct another child to place above this array an equation that links the product and the count for the array and then to choose someone to read his equation aloud.

4×7=28 **2×7=14**



Adapt this procedure to enable the children to observe the relationship between the products listed below.

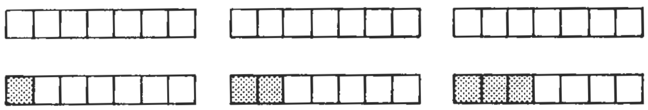
4 × 7 and 8 × 7
3 × 7 and 6 × 7
5 × 7 and 10 × 7

Then help the children show these arrays on the flannel board—0 by 7, 7 by 7, and 9 by 7.

Have a child show an equation that links the products and the count for each given array. Allow the children to determine the counts for these arrays in their own way.

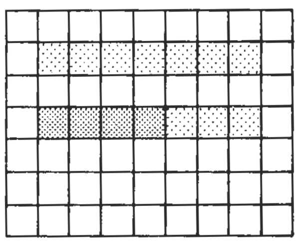
▶ Tell the children to work in pairs to show relationships between arrays that represent products that have 7 as a factor. Provide each pair of children with a one-inch squared sheet of paper and two different-colored crayons.

Explain to the children that one member of each pair is to outline a 1 by 7 array on the paper; he is to use one of the crayons. The other member of each pair is to outline a 1 by 7 array and partition it once. Explain that he is to use the other crayon to show the way he partitioned by shading in one section of the 1 by 7 array. The children could show any one of the following partitionings.



Call on one pair of children to tape their arrays on the chalkboard. Tell the child who did not partition his array to write the product and the count for his array in an equation on the board beside the array. ($1 \times 7 = 7$) Direct the child with the partitioned array to show two equations—the product and the count for each part of his array. Direct him to write these two equations one below the other and to compute the sum of his two partial products.

1×7=7

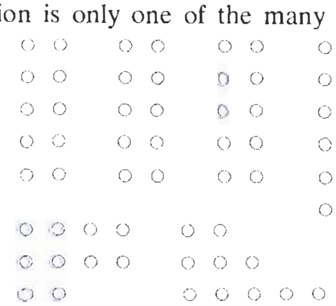


$$\begin{array}{r} 1\times4=4 \\ 1\times3=3 \\ \hline 7 \end{array}$$

Have the class decide whether or not the count the first child indicated in his equation is the same number as the sum of the products the second child computed.

Ask who made a different partitioning of the 1 by 7 array. Have another pair of children come to the chalkboard and show how the product of the whole array is the sum of the products of the parts. Then adapt this procedure to the products from 2×7 through 10×7.

▶ Select a child to show an 8 by 7 array on the flannel board and to record the product for this array on the chalkboard. Ask another child to rearrange the members of this set to show the number of tens and ones. Allow this child freedom to choose the way in which he shows the tens and ones. The arrangement in the illustration is only one of the many possibilities.

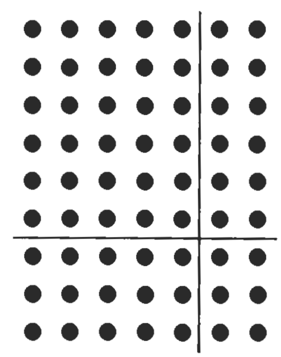


Have another child name the count indicated by this arrangement of the set's members. Direct this child to record both forms of the given number on the chalkboard and to link them with an equal sign. Have him read the equation aloud.

$$8 \times 7 = 56$$

Continue in this way to use other products from 1×7 through 10×7. The children may note that the count for some of the products involves only ones, the count for some involves just tens, and the count for some involves both tens and ones.

▶ Direct the children to use counters on their desks to show a 9 by 7 array. Have them use two narrow strips of cardboard ($\frac{1}{4}$ inch by 12 inches) to partition their array vertically and horizontally. Then direct the children to use their plastic numerals and symbols to show the product of the array. Below this first expression of the product, have the children express the product in a way that shows the two parts of each factor. This can be seen in the particular partitioning they used. For example, someone may have partitioned as illustrated.



The child who used this form would express the product in one of the following ways:

$$9 \times 7$$
$$(6 + 3) \times (5 + 2)$$
or
$$(6 + 3) \times (2 + 5)$$
or
$$(3 + 6) \times (5 + 2)$$
or
$$(3 + 6) \times (2 + 5)$$

Have several of the children tell how they see the factors 9 and 7 in parts.

Adapt this procedure to the products from 4×7 through 10×7.

▶ Have a child draw a 7 by 7 array on the chalkboard and write the product in vertical form.

$$\begin{array}{r} 7 \\ \times 7 \\ \hline \end{array}$$

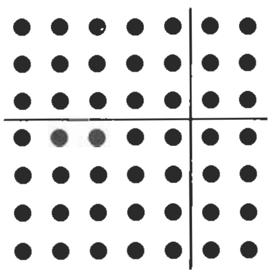
Ask this child to help you compute the product of this array in parts. Let him suggest a way to express the first factor 7 in parts. Record his suggestion on the chalkboard. For example, he may suggest that 7 be thought of as 3 + 4.

$$\begin{array}{l} 7 = 3 + 4 \\ \underline{\times 7} \end{array}$$

Then ask the child to suggest a way to think of the second factor 7 in two different parts. Record his suggestion on the chalkboard; he may suggest that 7 be thought of as 5 + 2.

$$\begin{array}{l} 7 = 3 + 4 \\ \underline{\times 7} = \underline{5 + 2} \end{array}$$

Finally, have the child partition the array to illustrate the parts.



Have the class give the count for this array. (49) Then ask the children to help you compute 7 × 7 in parts to see whether or not this method of computation will result in the answer 49.

Tell the children to compute the product of each array to the left of the vertical partition line (3 × 5 and 4 × 5). Record the products as shown.

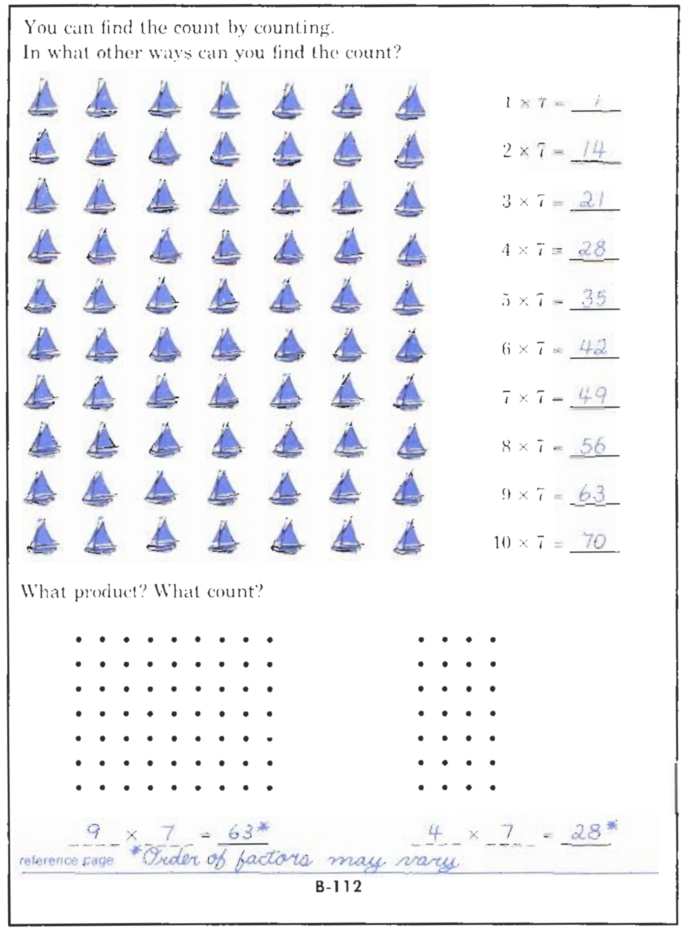
$$\begin{array}{lr} 7 = 3 + 4 & \\ \underline{\times 7} = \underline{5 + 2} & \\ 49 & 15\ (3 \times 5) \\ & 20\ (4 \times 5) \end{array}$$

Have the children compute the product of each array to the right of the vertical partitioning line (3 × 2 and 4 × 2). Record these partial products and have the children compute the sum of these partial products. Compare the results with the count for 7 × 7.

$$\begin{array}{lr} 7 = 3 + 4 & \\ \underline{\times 7} = \underline{5 + 2} & \\ 49 & 15\ (3 \times 5) \\ & 20\ (4 \times 5) \\ & 6\ (3 \times 2) \\ & \underline{8}\ (4 \times 2) \\ & 49 \end{array}$$

Repeat the procedure outlined above with two more 7 by 7 arrays drawn on the chalkboard. Have each array partitioned once vertically and once horizontally but different from any other partitioning used. Help the children compute 7 × 7 using each different way to view 7 and 7. As a final step in each computation, have the class compare the computed sum of the partial products with the count for 7 × 7.

Continue this activity using the products 10 × 7, 9 × 7, 8 × 7, and 6 × 7.


You can find the count by counting.
In what other ways can you find the count?

1 × 7 = 7
2 × 7 = 14
3 × 7 = 21
4 × 7 = 28
5 × 7 = 35
6 × 7 = 42
7 × 7 = 49
8 × 7 = 56
9 × 7 = 63
10 × 7 = 70

What product? What count?

9 × 7 = 63*   4 × 7 = 28*

reference page   *Order of factors may vary.

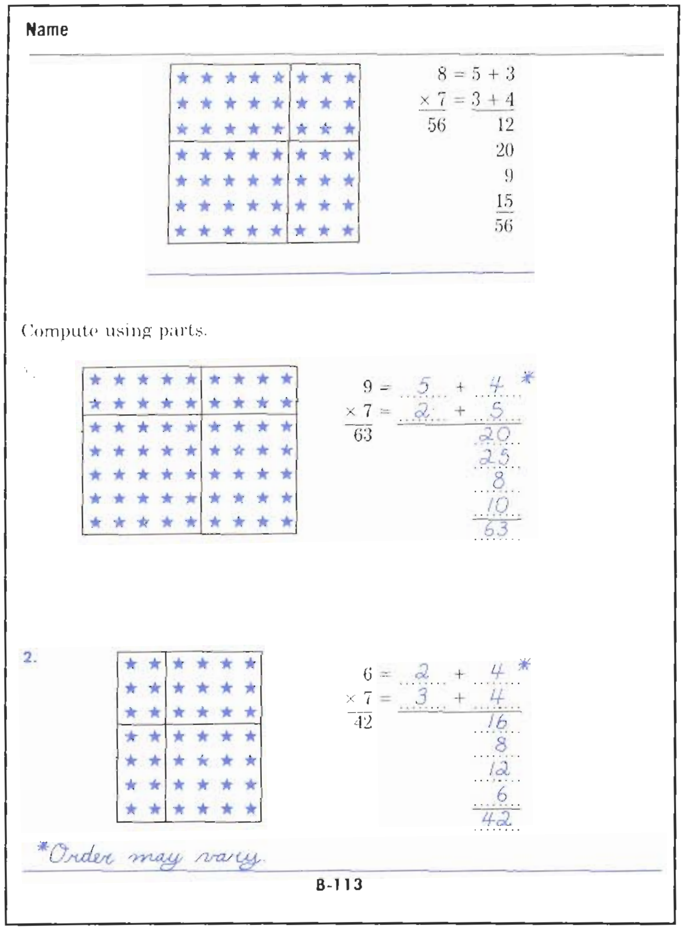
B-112

Pages 112 through 115

● Use page 112 primarily as a discussion page. Have the children study the array and the first equation in the sequence of 10 equations. Ask someone to give the count for 1×7 and to explain how he arrived at this count for the number of members in an array having 1×7 members. He may say that he counted the members in such an array one by one. Ask other children to describe how they determined the count for 1×7. The following ways, among others, may be suggested by the children.

Count by 2 as far as possible, then add 1.
Count by 3 as far as possible, then add 1.
Count by 5 as far as possible, then add 2.
Count by 7.
Add the count of a 1 by 5 array to the count of a 1 by 2 array.
Add the count of a 1 by 4 array to the count of a 1 by 3 array.

Continue to discuss each product listed. In each instance, encourage the children to describe how they arrived at the count. Then have the children complete the equations and the exercises at the bottom of the page.



Name

8 = 5 + 3
× 7 = 3 + 4
56
12
20
9
15
56

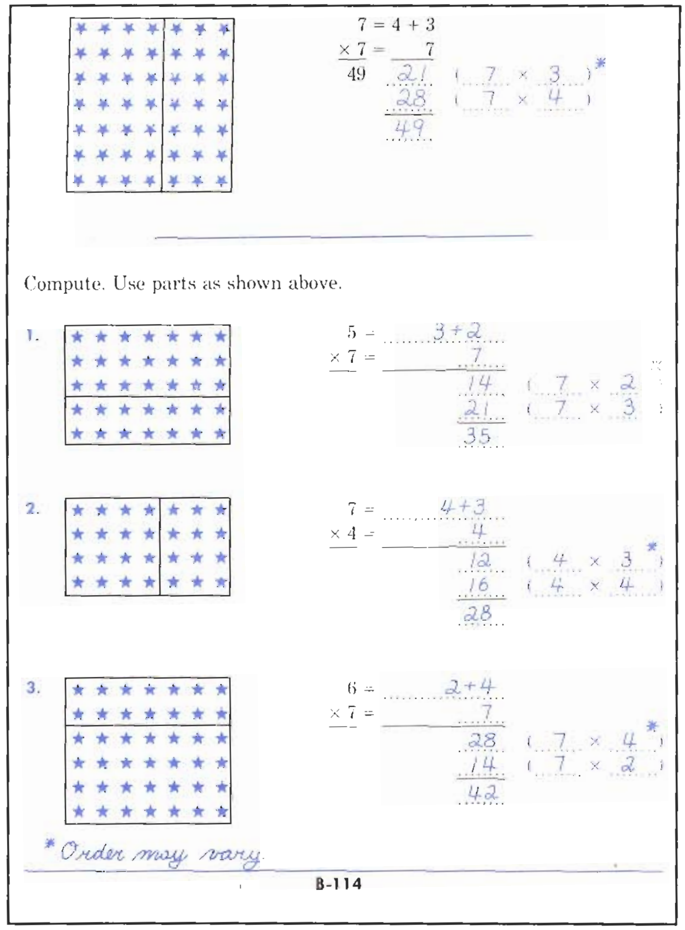
Compute using parts.

1. 9 = 5 + 4 *
× 7 = 2 + 5
63
20
25
8
10
63

2. 6 = 2 + 4 *
× 7 = 3 + 4
42
16
8
12
6
42

*Order may vary.

B-113



7 = 4 + 3
× 7 = 7
49
21 (7 × 3)*
28 (7 × 4)
49

Compute. Use parts as shown above.

1. 5 = 3+2
× 7 = 7
14 (7 × 2)
21 (7 × 3)
35

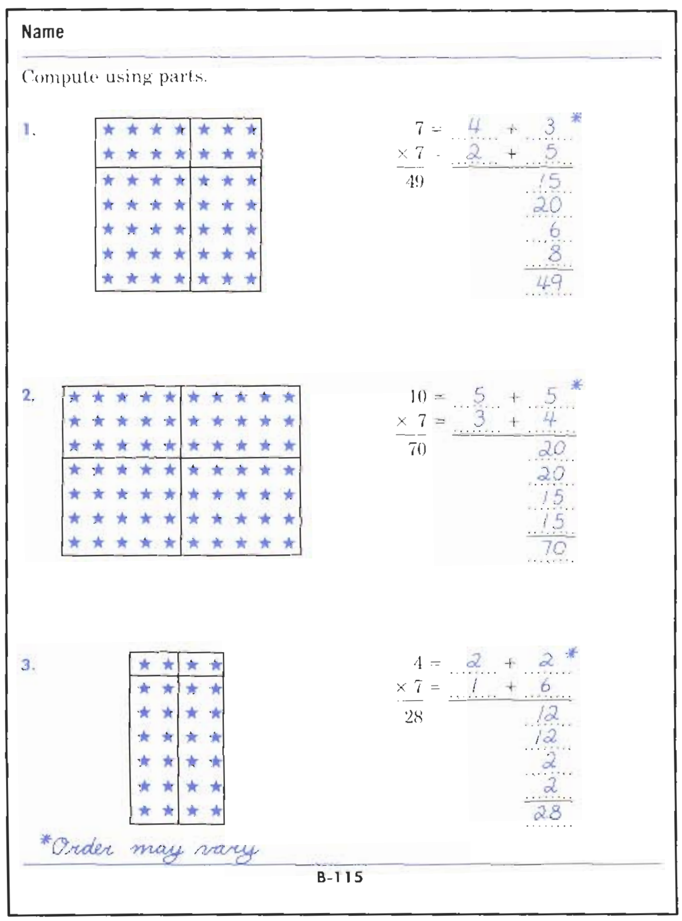
2. 7 = 4+3
× 4 = 4
12 (4 × 3)*
16 (4 × 4)
28

3. 6 = 2+4
× 7 = 7
28 (7 × 4)*
14 (7 × 2)
42

*Order may vary.

B-114

● Pages 113 through 115 provide practice in computing multiples of 7. Work the examples on pages 113 and 114 with the class. Explain to the children that they are to name each factor as a sum and to compute each product by using the partial products indicated by the partitioning of the related array. Tell them first to study the partitioned array in each exercise to decide how to express each factor as a sum. Then tell them to compute each partial product. Explain that the final step is to compute the sum of the partial products. Have the children complete the exercises on pages 113 through 115 independently. Give help when needed.



Name

Compute using parts.

1. 7 = 4 + 3 *
× 7 = 2 + 5
49
15
20
6
8
49

2. 10 = 5 + 5 *
× 7 = 3 + 4
70
20
20
15
15
70

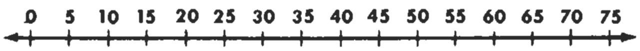
3. 4 = 2 + 2 *
× 7 = 1 + 6
28
12
12
2
2
28

*Order may vary.

B-115

Supplemental Experience

■ Draw on the chalkboard a segment of the number line as illustrated.



The marks should be at least 5 inches apart. Point to one segment of the number line, for example, the section from 10 to 20. Ask the children whether or not any products that have 7 as a factor are located along this segment. Call on someone to name such products, if they exist. Have another child give the count for any product named.

Examine other segments of the number line in the same manner. You may wish to begin the activity by first examining the segments indicated below.

The segment from 20 to 25: for products that have 6 as a factor.

The segment from 55 to 60: for products that have 7 as a factor.

The segment from 20 to 30: for products that have 6 as a factor, and products that have 7 as a factor.

The segment from 10 to 20: for products that have 6 as a factor, products that have 7 as a factor, products that have 3 as a factor, and products that have 4 as a factor.

The segment from 15 to 25: for products that have 6 as a factor, products that have 2 as a factor, and products that have 3 as a factor.

KEY IDEA

The tens are twice the fives.

Scope

To develop multiplication combinations related to multiples of 10.

Fundamentals

Some numbers are named as products of ten since their standard numerals result from the principles of numeration.

NUMERATION		STANDARD NUMERAL
one ten	1×10	10
two tens	2×10	20
three tens	3×10	30
⋮	⋮	⋮

The standard numeral for a product that has 10 as one of the two factors may be found without difficulty. Numeration principles enable us to record 8 tens as 8×10. This expression may be contracted to 80. Expressing such products with a standard numeral only involves placing a 0 to the right of the factor that is not 10—9×10 is 90.

When 10 is one of the two factors of a product, multiplication is easy. The child should explore related products such as $6 \times 10 = 2 \times (6 \times 5)$, $6 \times 10 = 2 \times (3 \times 10)$, and $6 \times 10 = (6 \times 7) + (6 \times 3)$, and so forth. Through such experiences the child will become increasingly aware of the properties of whole numbers for multiplication.

Readiness for Understanding

Knowledge of numeration.
Knowledge of the concept of product.

Developmental Experiences

for each child
counters
plastic numerals and symbols

▶ Have the children use their counters on their desks to show a 1 by 10 array. Ask them to use their plastic numerals and symbols to show below their array an equation that links the product and the count. Let someone read his equation aloud.

$$1 \times 10 = 10$$

Direct the children to construct another array that has twice as many members as the 1 by 10 array. Instruct the children to show the product and the count for the second array in an equation. Have this equation read aloud by a child.

$$1 \times 10 = 10$$
$$2 \times 10 = 20$$

Direct the children to show an array that has 3 times as many members as the 1 by 10 array. Tell the children to remove their second equation ($2 \times 10 = 20$) from their desks and replace it with an equation that links the product and the count for the last array that they built. Have a child read this equation aloud.

$$1 \times 10 = 10$$
$$3 \times 10 = 30$$

Adapt this procedure; have the children construct these arrays:

An array that has 4 times as many members as the 1×10 array.

An array that has 5 times as many members as the 1×10 array.

.
.
.

An array that has 10 times as many members as the 1×10 array.

In each instance, have the children show the product and the count for the given array in equation form.

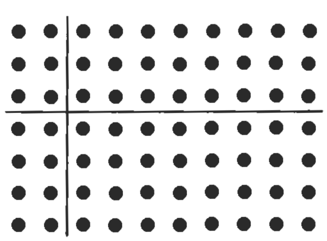
For variety, have the children use their counters to show a 1 by 5 array. Tell them to show a 1 by 10 array next to the 1 by 5 array. Then direct the children to place an appropriate equation below each array. Have them tell what relationship they observe between these two products. Someone may comment that 1×10 is twice 1×5. Someone else may say 10 is twice 5.

Continue in this way to help the class observe the relationship between the following products:

2×5 and 2×10
3×5 and 3×10
4×5 and 4×10
5×5 and 5×10
.
.
.
10×5 and 10×10

▶ Have a child draw a 7 by 10 array on the chalkboard. Tell this child to call on someone to come to the board and write the product for his array.

Partition the array as shown in the illustration.



Ask someone to express the product of the array as indicated by the partitioning. Tell this child to write his product below the first product recorded. He may use any of the commuted forms for the numbers given in the example below.

$$7 \times 10$$
$$(3 + 4) \times (2 + 8)$$

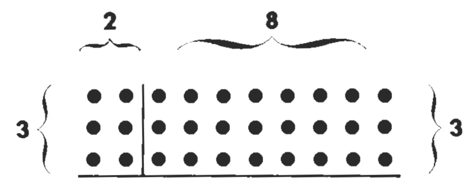
Have another child come to the chalkboard and write a sum of partial products that results from this view of the two factors. He may use any of the commuted forms of the expression given in the illustration.

$$7 \times 10$$
$$(3 + 4) \times (2 + 8)$$
$$(3 \times 2) + (3 \times 8) + (4 \times 2) + (4 \times 8)$$

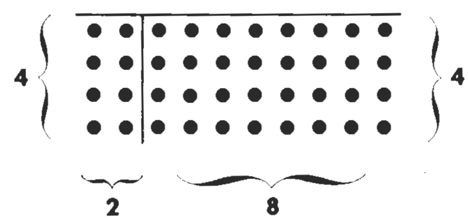
Ask this child to point out the parts in the array that correspond to the partial products he has indicated.

Have the class study the expressions of 7×10 and decide where the partial products 3×2 and 3×8 came from. Someone may note that the 3 in $3 + 4$ has been linked with the 2 and then the 8 in $2 + 8$ to give the partial products 3×2 and 3×8.

Have the class observe the array and locate the 3 that has been linked with the 2 and the 8 as a result of the partitioning. Help the children understand that the partial products 3×2 and 3×8 represent these sections of the whole array.



Ask the children to study the array to locate the 4 that has been linked with the 2 and the 8 as a result of the partitioning. The children should understand that the partial products 4×2 and 4×8 represent these sections of the whole array.



Help the children observe that the product 7×10 is a sum of these four partial products.

Draw a 9 by 10 array on the chalkboard. Use the following procedure as the children work with this array:

Have the product for the array written on the chalkboard (9×10).

Have a two-way partitioning of the array shown.

Have the product expressed as a sum of the partial products observed in the particular partitioning. For example, the following sum of partial products could be expressed.

$$9 \times 10$$
$$(5 \times 7) + (5 \times 3) + (4 \times 7) + (4 \times 3)$$

Have the product expressed in a way that shows the parts of the two factors 9 and 10. Remind the class that the parts of each factor represent the partitioning that has been used with this array.

$$9 \times 10$$
$$(5 \times 7) + (5 \times 3) + (4 \times 7) + (4 \times 3)$$
$$(5 + 4) \times (7 + 3)$$

Have the children discuss the relationship between the parts of the factors evident in the product $(5 + 4) \times (7 + 3)$ and the partial products in the sum $(5 \times 7) + (5 \times 3) + (4 \times 7) + (4 \times 3)$. The class may observe that 5 has been linked with 7 and 3, and 4 also has been linked with 7 and 3.

Adapt this procedure to examine arrays that show 10×10 and 8×10.

▶ Have a child draw on the chalkboard a 6 by 10 array. Tell him to write on the board the product and the count in vertical form for this array.

$$\begin{array}{r} 10 \\ \underline{\times\ \ 6} \\ 60 \end{array}$$

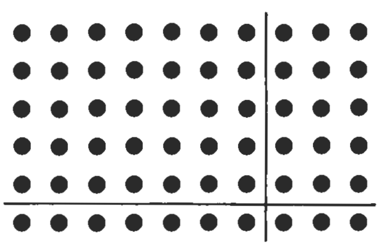
Ask the class to help compute the product of this array in parts. Have someone name two parts of the factor 10 and record these names on the chalkboard. If the child suggested that 10 can be thought of as 7 + 3, have a second child partition the array to show 10 as 7 + 3.

$$\begin{array}{rl} 10 & = 7 + 3 \\ \underline{\times\ \ 6} & \\ 60 & \end{array}$$

Ask another child to name two parts of the factor 6; record these names on the chalkboard.

$$\begin{array}{rl} 10 & = 7 + 3 \\ \underline{\times\ \ 6} & = \underline{5 + 1} \\ 60 & \end{array}$$

If the child suggested that 6 be thought of as 5 + 1, have someone partition the array to illustrate the idea.



Point to each section of the array as the children compute the product indicated. Record the products and counts as shown. Help the children compute the sum of the partial products and compare the result to the count for 6 × 10.

$$\begin{array}{rll} 10 & = 7 + 3 & \\ \underline{\times\ \ 6} & = \underline{5 + 1} & \\ 60 & \qquad 35 & (5 \times 7) \\ & \qquad 7 & (1 \times 7) \\ & \qquad 15 & (5 \times 3) \\ & \qquad \underline{3} & (1 \times 3) \\ & \qquad 60 & \end{array}$$

Continue the activity by using several other 6 × 10 arrays partitioned in different ways. For example, 10 may be pictured as 8 + 2 and 6 as 3 + 3.

$$\begin{array}{rl} 10 & = 8 + 2 \\ \underline{\times\ \ 6} & = \underline{3 + 3} \end{array}$$

Another child may view 10 and 6 as 5 + 5 and 4 + 2.

$$\begin{array}{rl} 10 & = 5 + 5 \\ \underline{\times\ \ 6} & = \underline{4 + 2} \end{array}$$

Continue in this way to use the products 8 × 10, 9 × 10, and 10 × 10.

Pages 116 and 117

● Use page 116 primarily as a discussion page. Direct the children to study the array and the first equation in the sequence of 10 equations. Call on someone to give the count for 1 × 10 and explain how he arrived at this count. He may say that he counted the members in such an array one by one. Ask several other children to describe the ways that they used to determine the count in this instance. It is possible that the children will suggest counting by 2 (add 2's), and counting by 5 (add 5's).

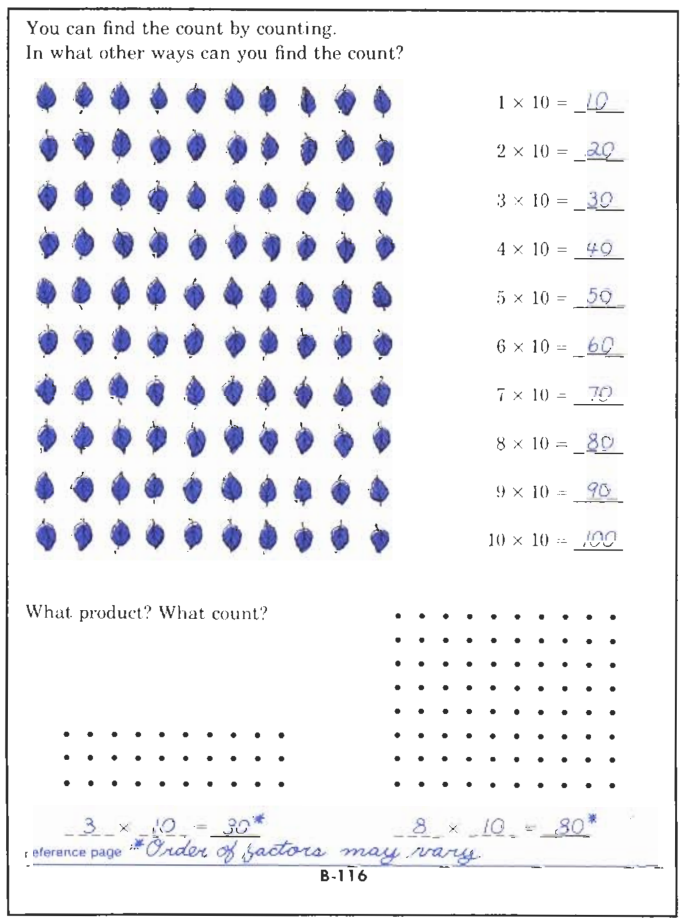
Continue to discuss each product listed. In each instance, encourage the children to describe how they arrived at the count. Then have the children complete the equations and the exercises at the bottom of the page.

● Page 117 provides the children with an opportunity to compute products that have 10 as a factor. Have the children work independently to complete the three exercises. If necessary, review the work procedure with them.

Study the partitioning of the given array and decide upon the way to name each factor as a sum.

Compute each partial product.

Compute the sum of partial products.


You can find the count by counting.
In what other ways can you find the count?

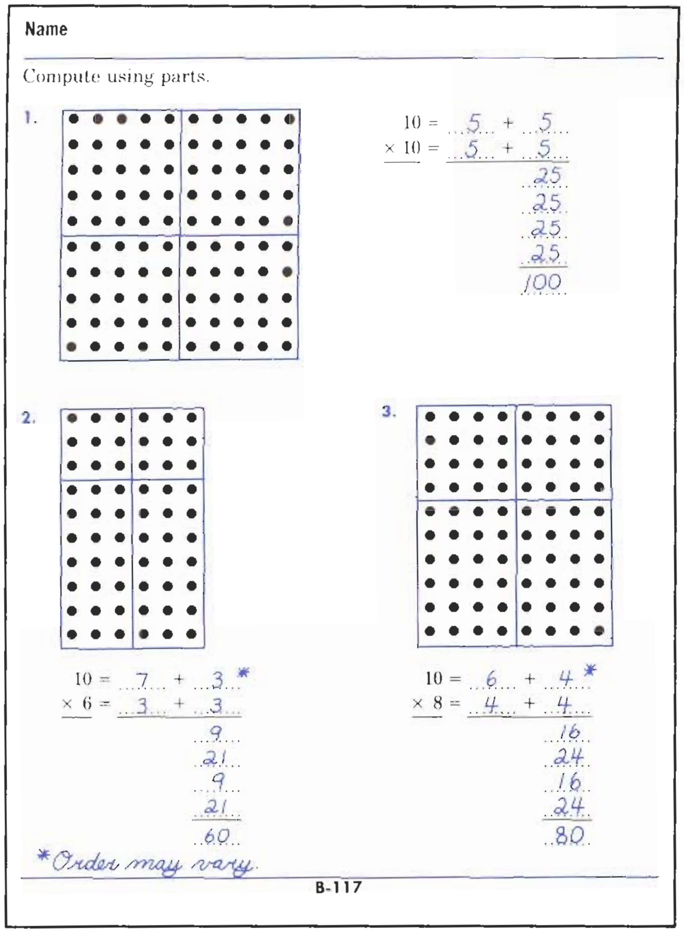
1 × 10 = 10
2 × 10 = 20
3 × 10 = 30
4 × 10 = 40
5 × 10 = 50
6 × 10 = 60
7 × 10 = 70
8 × 10 = 80
9 × 10 = 90
10 × 10 = 100

What product? What count?

3 × 10 = 30* 8 × 10 = 80*

Reference page *Order of factors may vary.

B-116


Name

Compute using parts.

1\. 10 = 5 + 5
× 10 = 5 + 5
25
25
25
25
100

2\. 10 = 7 + 3 *
× 6 = 3 + 3
9
21
9
21
60

3\. 10 = 6 + 4 *
× 8 = 4 + 4
16
24
16
24
80

*Order may vary.

B-117

KEY IDEA

$(1 \times 6) - (1 \times 6) = 0 \times 6.$

Scope

To review multiplication facts with 3, 5, 6, 7, and 10 as factors.

Fundamentals

The distributive property of multiplication over subtraction is not as familiar but is just as powerful as the distributive property of multiplication over addition. Consider a 7 by 4 array that is partitioned into a 3 by 4 and (7 − 3) by 4 array.

3 x 4 **(7—3) x 4**

Note that $(7 \times 4) - (3 \times 4) = (7 - 3) \times 4$ is an illustration of the distributive property of multiplication over subtraction. This property may be generally stated, $(b - c) \times a = (b \times a) - (c \times a)$.

The distributive property shows that products with a factor of 0 are 0.

$$\begin{aligned} 0 \times 6 &= (1 - 1) \times 6 \\ &= (1 \times 6) - (1 \times 6) \\ &= 6 - 6 \\ &= 0 \end{aligned}$$

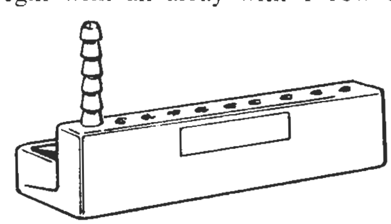
Readiness for Understanding

Understanding of difference.

Developmental Experiences

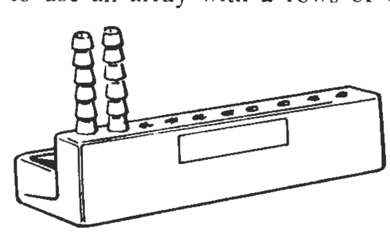
bead frame	masking tape
tagboard cards (2″ × 5″)	felt-tip pen
	array-cards

▶ Tell the class that you are going to display on the bead frame several arrays that have rows of 6 members. Begin with an array with 1 row of 6.



Have someone record on the chalkboard the product and the count for this array in an equation ($1 \times 6 = 6$).

Proceed to use an array with 2 rows of 6.



Supplemental Experience

■ Certain number sequences can be related to multiplication. The children have had experience visualizing the product 4×5 as $5 + 5 + 5 + 5$ as well as $4 + 4 + 4 + 4 + 4$. The children may observe that when they begin with 5 and count by 5—5, 10, 15, 20—the fourth number named is the product 4×5. They may observe that when they begin with 4 and count by 4—4, 8, 12, 16, 20—the fifth number named is the product 4×5.

Write several number sequences on the chalkboard. Tell the children to copy and complete each sequence.

2, 4, 6, —, —, —, —, —, —, —.
3, 6, 9, —, —, —, —, —, —, —.
4, 8, 12, —, —, —, —, —, —, —.
5, 10, 15, —, —, —, —, —, —, —.
6, 12, 18, —, —, —, —, —, —, —.
7, 14, 21, —, —, —, —, —, —, —.

After the sequences have been completed, select specific terms in each and let the children tell the product that represents this term.

Let a child write the product and the count for this array ($2 \times 6 = 12$) about 3 inches below the first equation. Encourage the child to line up the symbols directly below those in the first equation; this will aid in a later use of the equations in this activity.

Continue in this way until an array that has 10 rows of 6 is used. Then tell the class to look at the products and counts recorded on the chalkboard and to discuss what they notice about this sequence. They may observe that beginning with 2×6 each product is 6 greater than the preceding product.

Ask the children to compare the listed products two at a time. First ask someone how much greater 2×6 is than 1×6. Beside the list of equations on the chalkboard record the fact that 2×6 is 6 greater than 1×6 in the following way.

$$\begin{array}{rr} 1 \times 6 = & 6 \\ & +\ 6 \\ \hline 2 \times 6 = & 12 \end{array}$$

Continue in a similar way to have the children compare 3×6 and 2×6, 4×6 and 3×6, and so forth through 10×6 and 9×6. Let the children help record the fact that one product in each pair is 6 greater than the other.

$$\begin{array}{rr} 1 \times 6 = & 6 \\ & +\ 6 \\ 2 \times 6 = & 12 \\ & +\ 6 \\ 3 \times 6 = & 18 \\ & +\ 6 \\ 4 \times 6 = & 24 \\ & +\ 6 \\ 5 \times 6 = & 30 \\ & +\ 6 \\ 6 \times 6 = & 36 \\ & +\ 6 \\ 7 \times 6 = & 42 \\ & +\ 6 \\ 8 \times 6 = & 48 \\ & +\ 6 \\ 9 \times 6 = & 54 \\ & +\ 6 \\ 10 \times 6 = & 60 \end{array}$$

Now help the children consider the arrays in the opposite order. Begin with the 6 by 10 array on the bead frame and ask a child to write the product and the count as an equation on the chalkboard. ($10 \times 6 = 60$) Remove 1 row of 6 and let another child record the product and count for this array ($9 \times 6 = 54$) about 3 inches below $10 \times 6 = 60$.

Proceed in this way until the last row of 6 is removed from the frame. Explain that you now have an array with 0 rows of 6. Be sure the children note that the product shown by the array is 0×6; the count is 0. Then ask the children to study the products and the counts in this list and tell what they notice about this sequence. They may observe that, beginning with 9×6, each product is 6 less than the preceding product.

Again have the children compare the products two at a time. Ask how much less 9×6 is than 10×6. Beside the list of equations already on the chalkboard, record the fact that 9×6 is 6 less than 10×6 as illustrated.

$$\begin{array}{rr} 10 \times 6 = & 60 \\ & -\ 6 \\ \hline 9 \times 6 = & 54 \end{array}$$

Continue in a similar way to have the children compare 8×6 and 9×6, 7×6 and 8×6, and so forth through 0×6 and 1×6.

$$\begin{array}{rr} 10 \times 6 = & 60 \\ & -\ 6 \\ 9 \times 6 = & 54 \\ & -\ 6 \\ 8 \times 6 = & 48 \\ & -\ 6 \\ 7 \times 6 = & 42 \\ & -\ 6 \\ 6 \times 6 = & 36 \\ & -\ 6 \\ 5 \times 6 = & 30 \\ & -\ 6 \\ 4 \times 6 = & 24 \\ & -\ 6 \\ 3 \times 6 = & 18 \\ & -\ 6 \\ 2 \times 6 = & 12 \\ & -\ 6 \\ 1 \times 6 = & 6 \\ & -\ 6 \\ 0 \times 6 = & 0 \end{array}$$

Have the children help record the fact that one product in each pair is 6 less than the other.

Adapt the procedure described in this activity to products that have 7 as a factor—0×7 through 10×7—and to products that have 10 as a factor—0×10 through 10×10.

▶ Draw two line segments on the chalkboard. Mark the first segment at 2-inch intervals to show 0 through 50; mark the second segment 50 through 100. Let the children come to the board and point to the count for each product you describe.

> The product that is 1×6 greater than 4×6.
> The product that is 1×6 less than 7×6.
> The product that is 1×7 greater than 9×7.
> The product that is 1×7 less than 1×7.
> The product that is 1×10 greater than 1×10.
> The product that is 1×10 less than 10×10.

Use products that have either 6, 7, or 10 as a factor—0×6 through 10×6, 0×7 through 10×7, and 0×10 through 10×10.

▶ Draw on the chalkboard two line segments marked at 4-inch intervals. Label the first segment from 0 through 30 and the second segment from 31 through 60.

Fasten the following 40 cards to the chalkboard with masking tape:

products from 1 × 3 through 10 × 3;
products from 1 × 6 through 10 × 6;
products from 1 × 5 through 10 × 5;
products from 1 × 10 through 10 × 10.

Organize the products as shown:

1×3	1×5	1×6	1×10
2×3	2×5	2×6	2×10
3×3	3×5	3×6	3×10
4×3	4×5	4×6	4×10
5×3	5×5	5×6	5×10
⋮	⋮	⋮	⋮
10×3	10×5	10×6	10×10

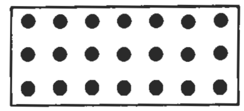
Ask individual children to come forward and find products that meet the requirements outlined in the following list. Have the children place the products below the appropriate count on the number line.

Any product whose count is 3 (1×3).
Any product that is twice 1×3 (2×3 and 1×6).
Any product that is twice 2×3 (4×3 and 2×6).
Any product that is twice 3×3 (6×3 and 3×6).
Any product that is twice 4×3 (8×3 and 4×6).
Any product that is twice 5×3 (10×3, 5×6, and the commuted forms of 10×3 and 5×6 could also be chosen).
Any product whose count is 5 (1×5).
Any product that is twice 1×5 (2×5 and 1×10).
Any product that is twice 2×5 (4×5 and 2×10).
Any product that is twice 3×5 (6×5, 3×10, and the commuted form of 6×5 and 3×10, which may already have been placed under 30).
Any product that is twice 4×5 (8×5 and 4×10).
Any product that is twice 5×5 (10×5 and 5×10).

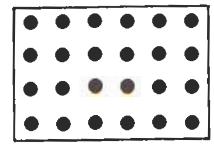
▶ For this activity you need a pack of array-cards for the products 1 × 6 through 10 × 6, 1 × 7 through 10 × 7, and 1 × 10 through 10 × 10. Gummed circle stickers may be used on tagboard cards to design the array-cards.

Separate the class into two teams and assign a section of the chalkboard to each team. Have one child from each team come forward, choose a card, display it on the chalktray, and write an equation to show the relationship between the product and the count for his particular array. Then ask each child to write the commuted form of the product in an equation.

3 × 7 = 21
7 × 3 = 21



4 × 6 = 24
6 × 4 = 24



The children may earn points for their team in the following ways:

1 point is earned for completing the assignment correctly;

1 point is earned for being first to complete the assignment.

Continue in this way until all members of the class have participated in the activity. Then let each team total its points to determine a winner.

Pages 118 through 124

● For pages 118 through 123, have the children complete the repeated additions or subtractions. Then have the children complete the related equation beside each series of additions or subtractions. Encourage them to look for patterns. For example:

$$2 \times 6 = 1 \times 6 + 6$$
$$3 \times 6 = 2 \times 6 + 6$$
$$4 \times 6 = 3 \times 6 + 6$$

or

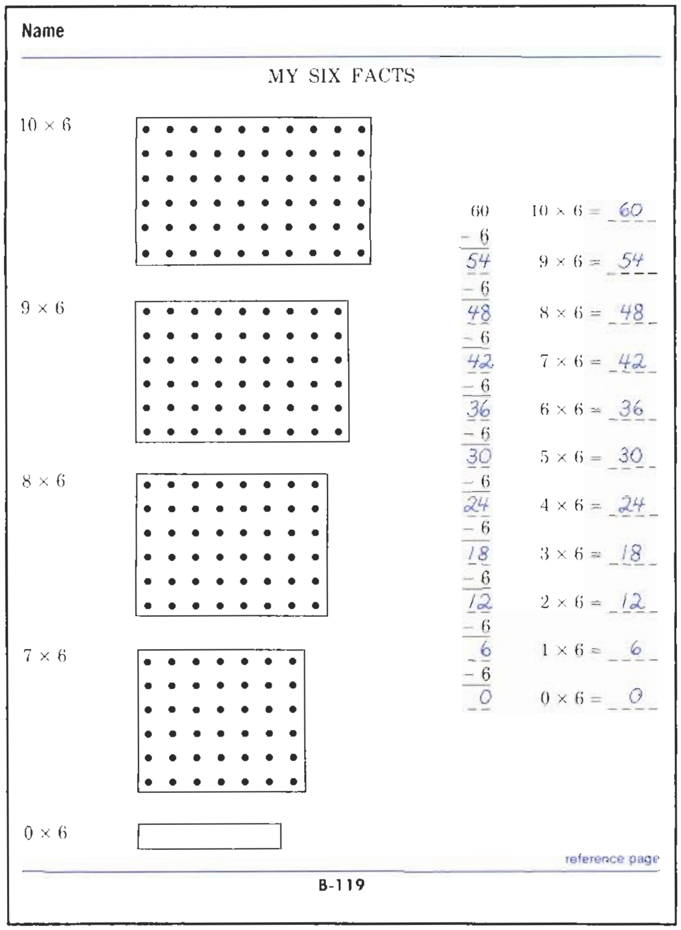
$$1 \times 6 = 6$$
$$2 \times 6 = 6 + 6$$
$$3 \times 6 = 6 + 6 + 6$$

or

$$9 \times 6 = 10 \times 6 - 6$$
$$8 \times 6 = 9 \times 6 - 6$$

These pages give the children an opportunity to organize the products from 0×6 through 10×6, the products from 0×7 through 10×7, and the products from 0×10 through 10×10.

● Page 124 enables the children to test their ability to compute products that have 2 through 10 as one of the factors. Some of the children may benefit from doing only 1 row for a single assignment.



Name

MY SIX FACTS

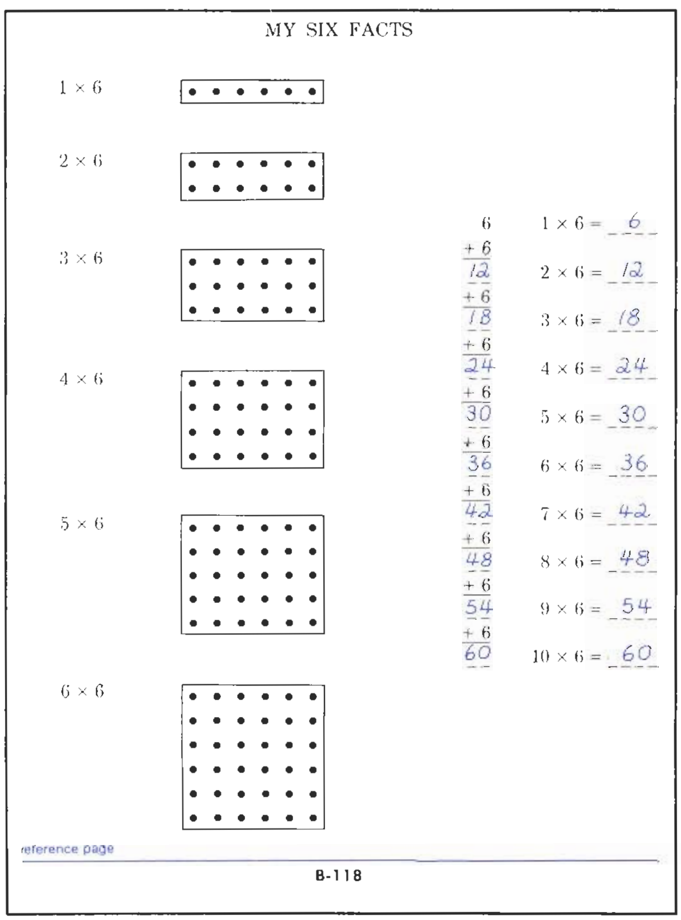
10 × 6
9 × 6
8 × 6
7 × 6
0 × 6

60 − 6 = 54 − 6 = 48 − 6 = 42 − 6 = 36 − 6 = 30 − 6 = 24 − 6 = 18 − 6 = 12 − 6 = 6 − 6 = 0

10 × 6 = 60
9 × 6 = 54
8 × 6 = 48
7 × 6 = 42
6 × 6 = 36
5 × 6 = 30
4 × 6 = 24
3 × 6 = 18
2 × 6 = 12
1 × 6 = 6
0 × 6 = 0

reference page

B-119



MY SIX FACTS

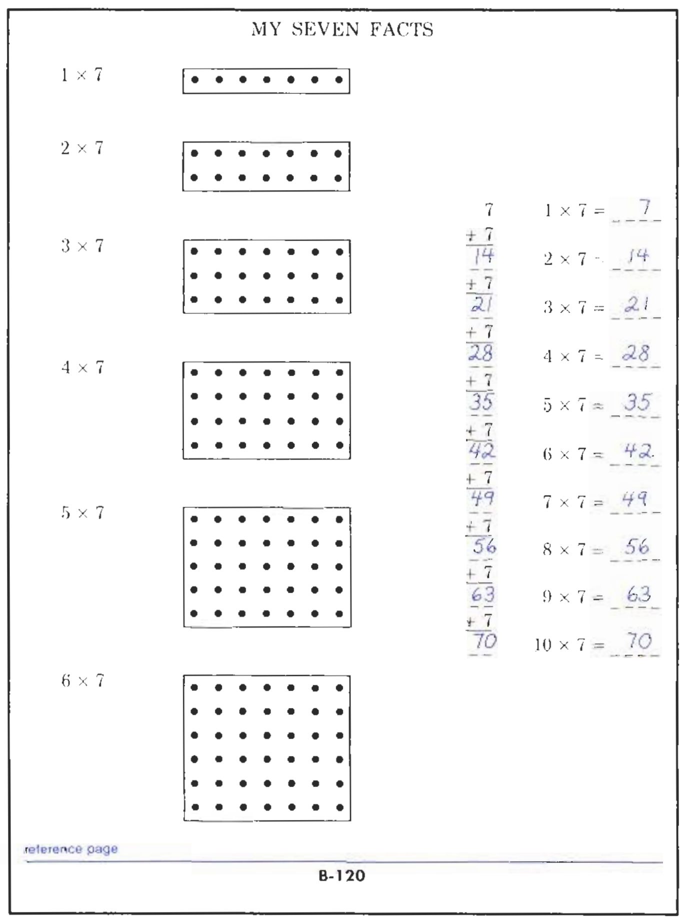
1 × 6
2 × 6
3 × 6
4 × 6
5 × 6
6 × 6

6 + 6 = 12 + 6 = 18 + 6 = 24 + 6 = 30 + 6 = 36 + 6 = 42 + 6 = 48 + 6 = 54 + 6 = 60

1 × 6 = 6
2 × 6 = 12
3 × 6 = 18
4 × 6 = 24
5 × 6 = 30
6 × 6 = 36
7 × 6 = 42
8 × 6 = 48
9 × 6 = 54
10 × 6 = 60

reference page

B-118



MY SEVEN FACTS

1 × 7
2 × 7
3 × 7
4 × 7
5 × 7
6 × 7

7 + 7 = 14 + 7 = 21 + 7 = 28 + 7 = 35 + 7 = 42 + 7 = 49 + 7 = 56 + 7 = 63 + 7 = 70

1 × 7 = 7
2 × 7 = 14
3 × 7 = 21
4 × 7 = 28
5 × 7 = 35
6 × 7 = 42
7 × 7 = 49
8 × 7 = 56
9 × 7 = 63
10 × 7 = 70

reference page

B-120

Name

MY SEVEN FACTS

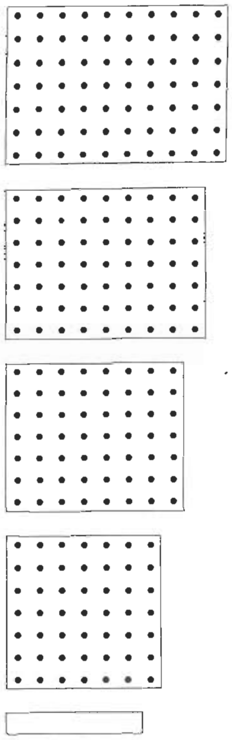
10 × 7

9 × 7

8 × 7

7 × 7

0 × 7



70	10 × 7 = 70
− 7	
63	9 × 7 = 63
− 7	
56	8 × 7 = 56
− 7	
49	7 × 7 = 49
− 7	
42	6 × 7 = 42
− 7	
35	5 × 7 = 35
− 7	
28	4 × 7 = 28
− 7	
21	3 × 7 = 21
− 7	
14	2 × 7 = 14
− 7	
7	1 × 7 = 7
− 7	
0	0 × 7 = 0

reference page

Name

MY TEN FACTS

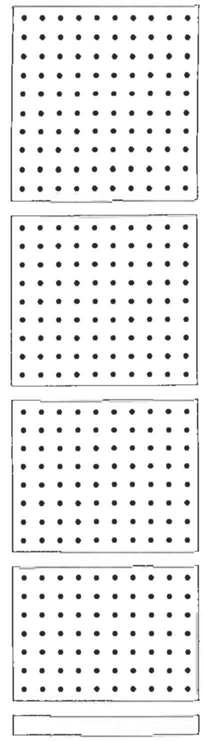
10 × 10

9 × 10

8 × 10

7 × 10

0 × 10



100	10 × 10 = 100
− 10	
90	10 × 9 = 90
− 10	
80	10 × 8 = 80
− 10	
70	10 × 7 = 70
− 10	
60	10 × 6 = 60
− 10	
50	10 × 5 = 50
− 10	
40	10 × 4 = 40
− 10	
30	10 × 3 = 30
− 10	
20	10 × 2 = 20
− 10	
10	10 × 1 = 10
− 10	
0	10 × 0 = 0

reference page

MY TEN FACTS

1 × 10

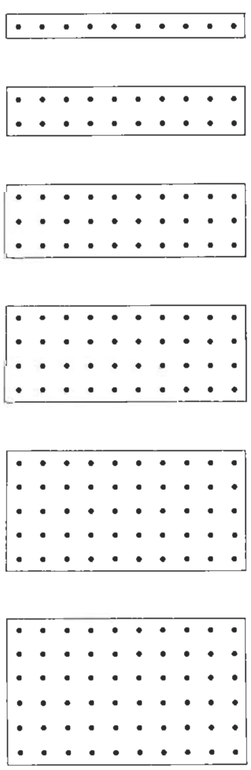
2 × 10

3 × 10

4 × 10

5 × 10

6 × 10



10	1 × 10 = 10
+ 10	
20	2 × 10 = 20
+ 10	
30	3 × 10 = 30
+ 10	
40	4 × 10 = 40
+ 10	
50	5 × 10 = 50
+ 10	
60	6 × 10 = 60
+ 10	
70	7 × 10 = 70
+ 10	
80	8 × 10 = 80
+ 10	
90	9 × 10 = 90
+ 10	
100	10 × 10 = 100

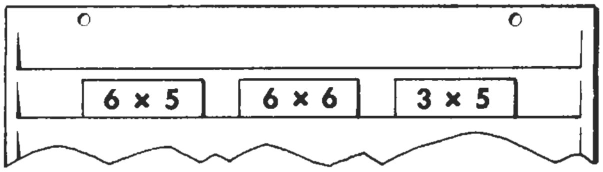
reference page

Compute.

1.	4 × 6 = 24	7 × 6 = 42	9 × 6 = 54	7 × 2 = 14	7 × 4 = 28
2.	6 × 4 = 24	7 × 3 = 21	8 × 4 = 32	9 × 2 = 18	5 × 4 = 20
3.	2 × 10 = 20	3 × 10 = 30	9 × 10 = 90	9 × 5 = 45	6 × 2 = 12
4.	7 × 7 = 49	8 × 7 = 56	4 × 7 = 28	9 × 7 = 63	6 × 3 = 18
5.	6 × 7 = 42	5 × 7 = 35	3 × 7 = 21	9 × 4 = 36	7 × 5 = 35
6.	8 × 10 = 80	5 × 10 = 50	4 × 10 = 40	10 × 10 = 100	7 × 10 = 70
7.	3 × 6 = 18	8 × 6 = 48	10 × 6 = 60	6 × 6 = 36	5 × 6 = 30

Supplemental Experience

■ Collect a set of tagboard cards that show all of the products that have been developed up to this time. Put three of these products in the pocket chart.



Describe a product to the class—for example, the product that is twice 3×6. Ask a child to point out any of the three products in the chart that answer your description. Have the class decide whether or not the child selected correctly. If the child made an error, allow him to find the correct answer by using felt counters at the flannel board.

Proceed in this way with other sets of product cards. In some instances have just one product match your description; in other instances have two or three products meet the description. In some instances, none of the products should fit your description.

UNIT 9
GEOMETRY

Pages 125 Through 132

OBJECTIVE

To measure the size of rectangles.

The child covers a rectangle using a square unit to determine a measure. He learns that rectangles of the same size may have different shapes. He learns to recognize parallel lines and perpendicular lines.
See Key Topics in Mathematics for the Primary Teacher: Geometry.

KEY IDEAS

Rectangular figures may be measured by covering with square units.
When the unit square is smaller, more units are needed to cover the rectangle.
Square corners show perpendicular lines.

CONCEPTS

oblique	size
parallel	square
perpendicular	square unit
rectangle	

KEY IDEA

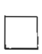
Rectangular figures may be measured by covering with square units.

Scope

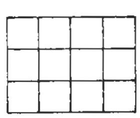
To develop the children's understanding of area measurement.

Fundamentals

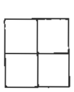
A rectangular figure may be measured by using a square unit as the basis of comparison. Once the square unit has been defined, it is then possible, by using a measuring process, to assign a number to the rectangle being measured. The measuring process the child will use is to cover the figure to be measured and count (if possible) the number of units needed. For example, it can be seen that 12 of the square units will cover the given rectangle.

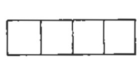

square unit

(basis of comparison)



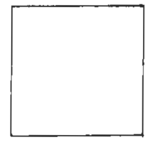
The teacher should emphasize that even though two rectangular figures have the same size (measure), they do not necessarily have the same shape.



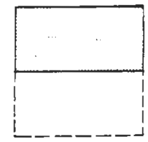


The above square and rectangle are the same size (measure), but are different shapes.

The figure to be covered may be smaller than the unit or basis of comparison. One square unit may more than cover the rectangle to be measured.


(basis of comparison)

square unit


$\frac{1}{2}$ square unit

Since the square unit would cover two of the rectangles to be measured, the fractional number $\frac{1}{2}$ is assigned to the rectangle.

Readiness for Understanding

Ability to count.
Knowledge of the meaning of $\frac{1}{2}$ and $\frac{1}{4}$.

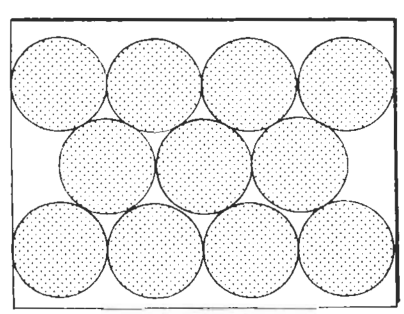
Developmental Experiences

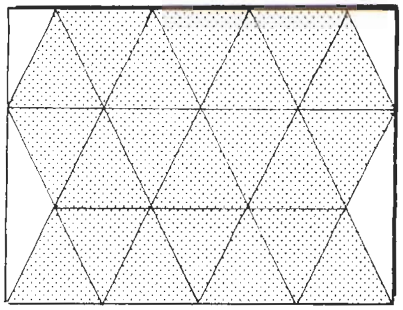
for flannel board
tagboard rectangles
(2″ × 2″, 4″ × 2″, 6″ × 2″, 2″ × 3″, 3″ × 3″, 1″ × 4″, 4″ × 4″, 3″ × 5″, 2″ × 8″, 1″ × 9″)
1″ tagboard squares

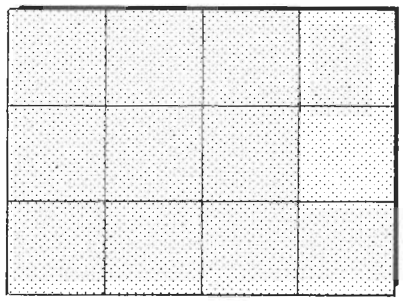
for each child
tagboard rectangle (9″ × 12″)
20 tagboard circles (3″ diameter)
35 tagboard equilateral triangles (3″ on a side)
15 tagboard squares (3″ × 3″)
squared paper ($\frac{1}{4}$″)
construction paper rectangles (4″ × 1″, 5″ × 2″, 3″ × 3″, and 3″ × 4″)
envelope
50 construction paper squares (1″ × 1″)

pins

▶ Give each child a 9 by 12 inch tagboard rectangle. Direct the children to discuss various geometric figures that might be used to measure the surface of this rectangle. Discuss the idea of covering the rectangle without overlapping or leaving any spaces not covered. Give each child 20 tagboard circles (3 inches in diameter), 35 tagboard equilateral triangles (3 inches on a side), and 15 tagboard squares (3 by 3 inches). Instruct the children to try to cover their 9 by 12 inch rectangles as completely as possible. Explain that they must cover it by using just one type of geometric figure.

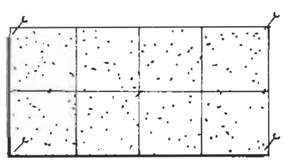






The children should soon conclude that the circles are the least suited for such a purpose. The triangles cover better, but there are spaces at the edges. The children will probably agree that squares cover the surface best of all. It is important that the squares completely fit on the large rectangle.

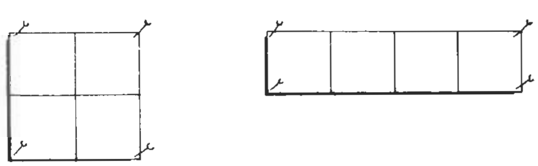
▶ Pin on the flannel board a 4 by 2 inch rectangle. Cut out 20 one-inch tagboard squares. Each square will be referred to as one square unit of measure. Help a child cover the large rectangle by pinning as many of these square units over it as he can.



Ask the class what the measure of the rectangle is in terms of the given unit. They should observe that the measure is 8 of the square units.

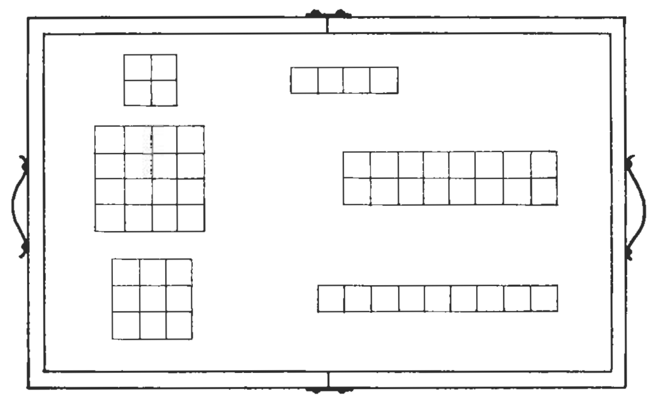
Continue in this way using rectangles that have the following dimensions: 2 by 3 inches, 6 by 2 inches, 3 by 5 inches, and 4 by 4 inches.

▶ Pin a 2 by 2 inch rectangle and a 1 by 4 inch rectangle on the flannel board. Ask the children if they think these rectangles are the same size. Then give a child a one-inch tagboard square to use as the square unit; direct him to show how many square units will completely cover each figure.

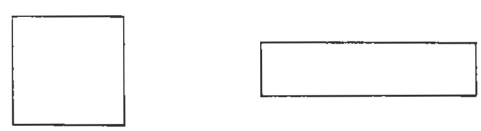


Again ask how many square units were used to cover each rectangle. They should observe that the measure is 4 of the square units. Continue this procedure: use rectangles that are 3 by 3 inches, 1 by 9 inches, 4 by 4 inches, and 2 by 8 inches.

After all of the rectangles have been placed on the flannel board and the children have determined the measure of each figure, ask the children to describe what they notice about each pair. Someone may observe that there is the same number of square units in each rectangle in the pair. Someone else may observe that the rectangles in a pair have a different shape but are the same size.



▶ Give each child a piece of $\frac{1}{4}$-inch squared paper. Review the fact that some rectangles are square, and some are not. Have two such rectangles to show the children, or draw the rectangles on the chalkboard.



Ask the children to outline on their paper the smallest number of squares that would form a rectangle. (1) Let individuals tell the class how many squares they outlined.

Write this list on the chalkboard:

12 square units
16 square units
8 square units
15 square units
20 square units

Tell the children to draw several rectangles for each size. When they are finished, ask several children to show the class the rectangles they made for a given size.

▶ Give each child a set of 4 paper rectangles: 4 by 1 inch, 5 by 2 inches, 3 by 3 inches, and 3 by 4 inches. Also provide everyone with an envelope that contains 50 one-inch squares to be used as the measuring unit. Direct the children to cover each rectangle with square units. After the children have completed the assignment, ask how many square units cover each rectangle. Have the children tell how they found this measure. Some of the children may say they counted; others may say they knew how many square units it would take before the rectangle was covered. Encourage the children to explain the methods that produced accurate results.

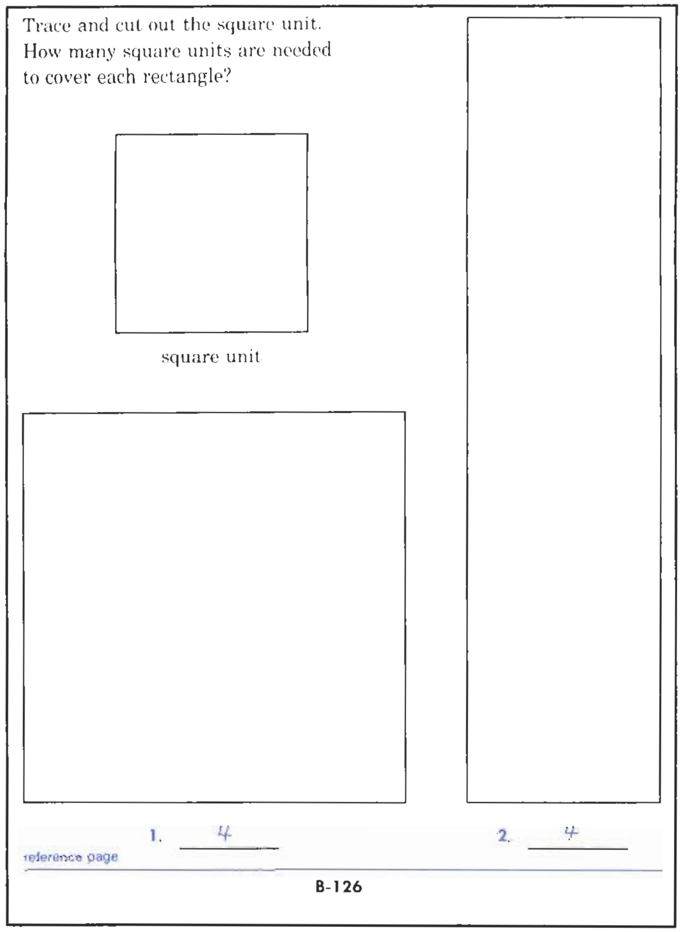
Pages 125 through 128

● Use pages 125 through 127 for investigation and discussion. These pages are designed to give the children an opportunity to measure rectangles by covering these figures with square units. In each case, have the children trace and cut out several of the square units to be used for measuring.

The rectangles on page 125 are partitioned into squares. If a child wants to tell how many square units there are in each rectangle by observation, allow him to do so. However, have each child lay several of his cut-out squares on each rectangle and tell how many square units just cover each rectangle.

Ask the children to examine the rectangles on page 126 and to estimate how many of the square units will just cover each rectangle. Direct the children to use their cut-out paper units to check their guess.

After the children use their square units to measure the rectangles on page 127, let them discuss any relationships they have observed.

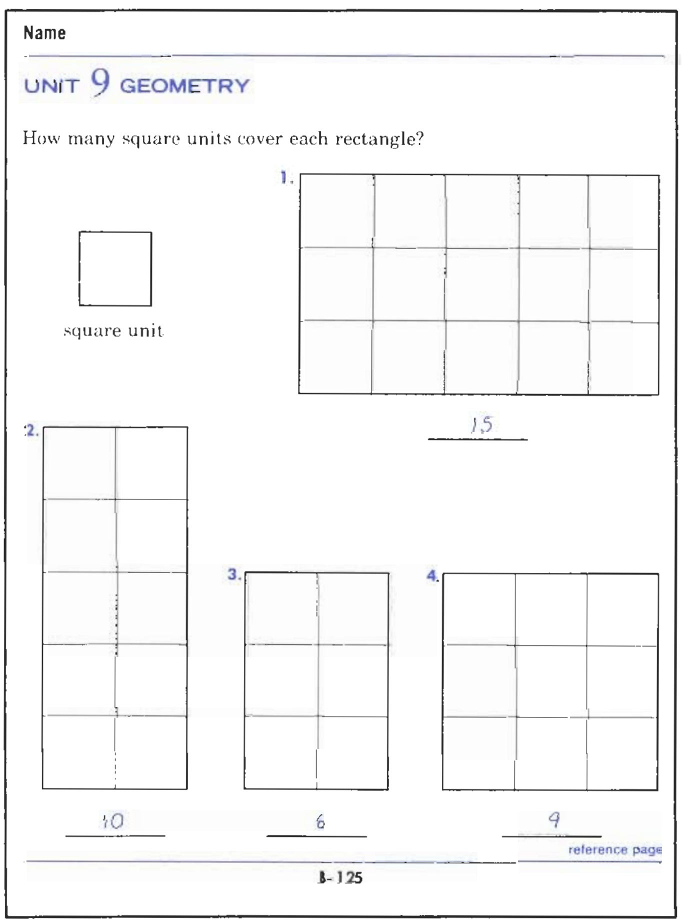

Trace and cut out the square unit.
How many square units are needed to cover each rectangle?

square unit

1. 4 2. 4

reference page

B-126


Name

UNIT 9 GEOMETRY

How many square units cover each rectangle?

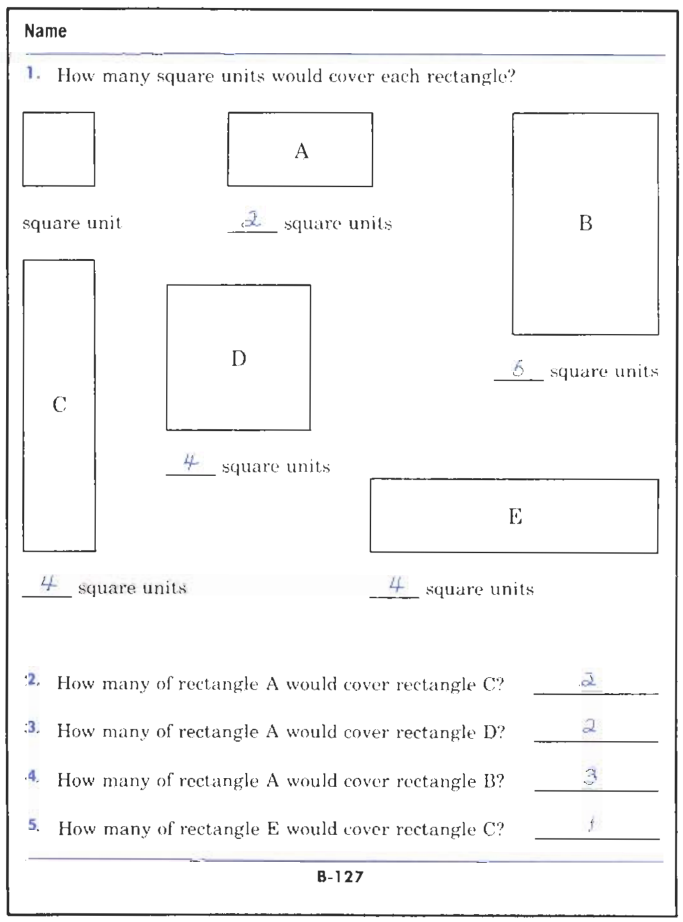
square unit

1. 15

2. 10 3. 6 4. 9

reference page

B-125


Name

1. How many square units would cover each rectangle?

square unit

A: 2 square units

B: 6 square units

C: 4 square units

D: 4 square units

E: 4 square units

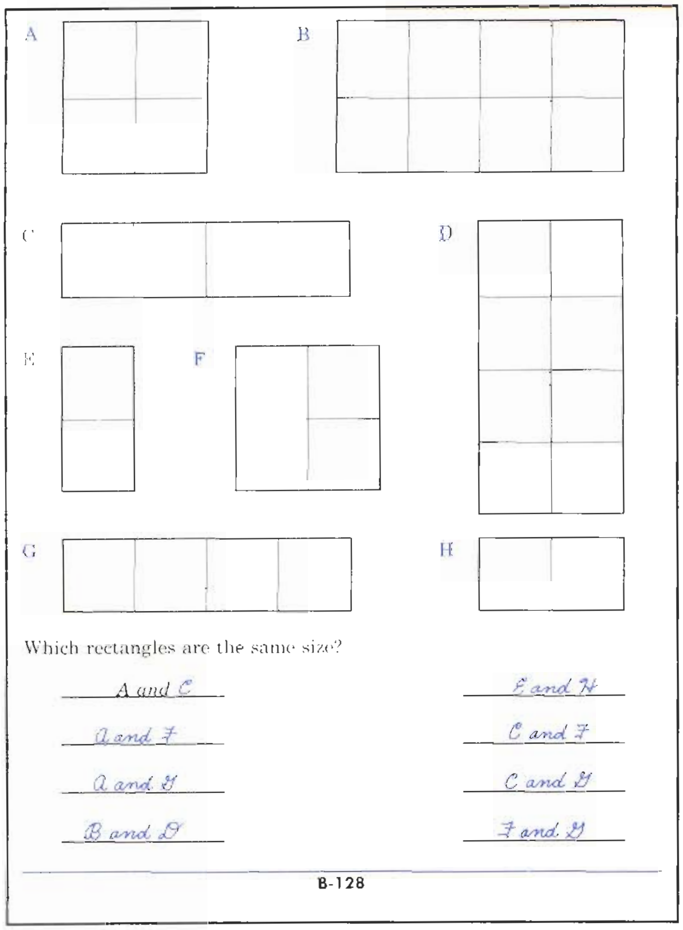
2. How many of rectangle A would cover rectangle C? 2

3. How many of rectangle A would cover rectangle D? 2

4. How many of rectangle A would cover rectangle B? 3

5. How many of rectangle E would cover rectangle C? 1

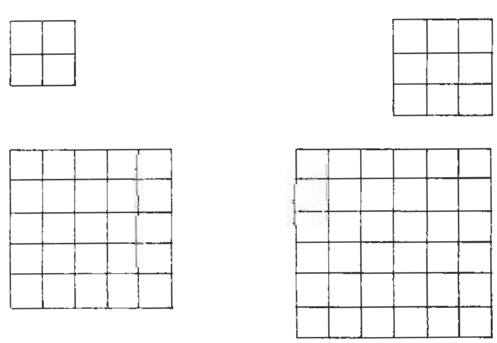
B-127



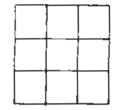
● Ask the children how they could compare the size of rectangles on page 128. They should notice that each rectangle is covered by square units. Again, discuss with the children the fact that rectangles of different shapes may have the same size.

Supplemental Experiences

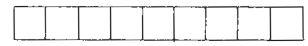
■ Give each child a piece of graph paper. Help each child draw the following squares on his paper.



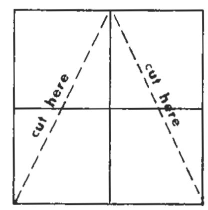
Then direct the children to draw a rectangle beside or below each square. Explain that each rectangle must have the same size as the square it is near. For example, in the second illustrated square rectangle there are 9 squares.



In the rectangle that the child draws, there also must be 9 squares.



■ Pin a piece of one-inch squared paper on the flannel board. Cut 1 one-inch square from felt. Cut the two-inch square as pictured.



Pin the pieces of felt on the squared paper as shown in Figure 1. Outline this figure on the paper. Ask the children to see what other formations they can make by using all of the pieces. A possibility is shown in Figure 2.

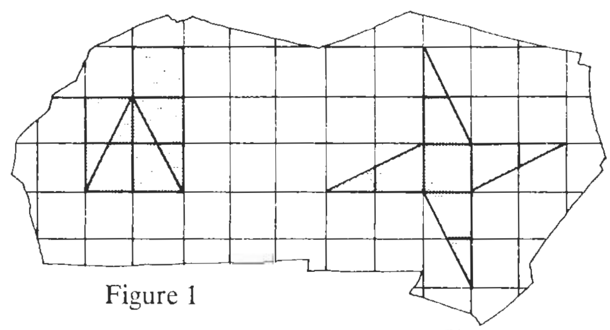


Figure 1

Figure 2

Ask the children to compare the size of Figure 2 with the size of Figure 1. Let them discuss whether or not the size has been changed by rearranging the parts.

KEY IDEA

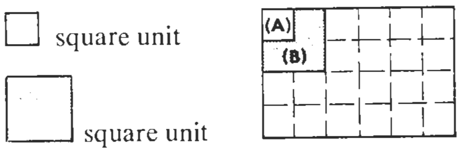
When the unit square is smaller, more units are needed to cover the rectangle.

Scope

To use different square units to determine the measure of the same rectangular figure.

Fundamentals

When a rectangular figure is covered with different square units, a different number of square units is used. As smaller units are used, the number of units is increased. As larger units are used, the number of units is decreased.



When the rectangle is covered with square unit (A), a count of 24 (A) square units is obtained. When the rectangle is measured with square unit (B), a count of 6 (B) square units results.

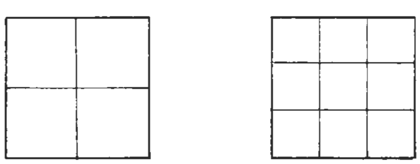
Readiness for Understanding

Knowledge of the measuring process of covering a rectangle with square units.

Developmental Experiences

for flannel board	*for each child*
tagboard rectangles (12″ × 12″, 6″ × 6″, 4″ × 4″, 1″ × 1″, 1″ × 2″, 1″ × 4″)	1″ squared paper 2″ square piece of construction paper 3 rectangles (4″ × 8″) 1″ tagboard square
pins tagboard rectangle (1′ × 1′, 6″ × 6″) scissors paste	

▶ Pin to the flannel board two tagboard rectangles (12 by 12 inches). Use two tagboard squares (4 by 4 inches and 6 by 6 inches) as units for measuring the rectangles. Give a square to each of two children and have them each measure one of the rectangles.



Encourage the class to discuss what was shown—how many square units it takes to cover each rectangle. Someone may note that it took only 4 of the large square units to cover one rectangle and it took 9 of the small square units to cover the other rectangle. Someone may have observed that when the units are smaller they do not cover as much surface, so it is necessary to use more of them.

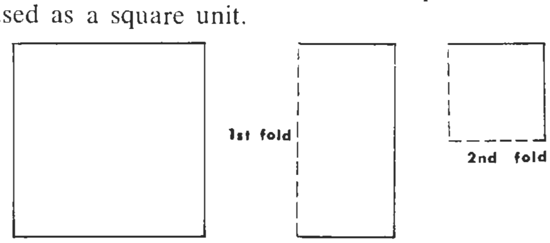
Adapt this activity to other pairs of rectangles that have the same size. Have each rectangle in a pair measured with different size squares to show that the smaller the unit of measure the greater the number of units it takes to cover a rectangle.

▶ Draw a 1 by 2 foot rectangle on the chalkboard. Hold up a tagboard unit that is 1 foot square and another unit that is 6 inches square. Direct the children to decide whether it will take more 6-inch square units or more 1-foot square units to cover the rectangle. Let the children explain their answers. Then ask one child to use the 1-foot square unit and another child to use the 6-inch square unit to check the class's decision. Tell the class to count the units it takes to completely cover each rectangle. Encourage the children to discuss what they observed. Someone may say it didn't take as many of the large units as it did the small units to measure the rectangle. Someone else may conclude that the smaller the square unit you use, the more units you will need to measure the rectangle.

▶ Give each child a piece of 1-inch squared paper. Direct the children to draw a 4 by 6 inch rectangle. Ask how many 1-inch square units it takes to measure this rectangle. (24) Then ask how many 2-inch square units will cover the rectangles. (6) Someone may observe there are four times as many 1-inch units as there are 2-inch units. He may reason that this is because it took 4 of the 1-inch square units to make each 2-inch square unit. If some of the children do not see this, let them cover the rectangle using 2-inch square tagboard units and 1-inch square tagboard units.

▶ Give 3 rectangles (4 by 8 inches) to each child. Give each child a 2-inch square piece of construction paper to use as a square unit of measure. Tell the children to place their square unit on one of the rectangles to see how many of these unit squares will completely cover this figure. If necessary, tell the children to use pencils and outline each square unit as it is placed on the rectangle. Then ask how many square units it took.

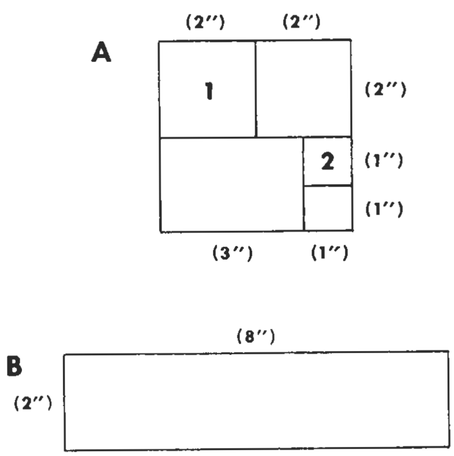
Show the children how to fold their squares into fourths. Tell the class that the folded square will now be used as a square unit.



Instruct the children to place this square unit on one of the other rectangles to see how many of these square units are needed to completely cover this figure. If necessary, let the children outline the square unit each time it is used. Ask how many square units were used.

Tell the children to again fold their 2-inch square into fourths. Direct them to measure their third rectangle with this small unit (1 by 1 inch). Again direct the children to outline this square each time it is used. Discuss how many square units there are in the rectangle. Someone may find by counting that there are 32 square units. Someone else may see 4 by 8 square units. Another child may be alert enough to see that there are 4 times as many square units in the third rectangle as there are in the second rectangle, or 16 times as many as in the first rectangle. Ask the children to compare the sizes of the rectangles. Someone may observe there was no change in size—the 3 rectangles are still the same size. Then ask what change did take place.

▶ Draw a 4 by 4 inch rectangle and a 2 by 8 inch rectangle on a stencil and duplicate enough to give each child a copy. The rectangles should be labeled *A* and *B*. The 4-inch square should be partitioned as shown in the illustration. Do not duplicate the numerals in the parentheses.

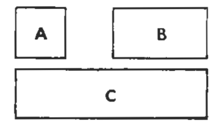


Provide each child with a 1-inch tagboard square unit to use as a measure. Direct the children to find out how many times they must place the square unit on rectangle *A* to completely cover it. Then have them repeat the process on rectangle *B*. Someone may observe that it takes the same number of square units to cover each rectangle.

Tell the children to cut rectangle *A* into 5 parts, using the guide lines. Then direct them to paste the 5 parts onto rectangle *B* to completely cover it. Let the children show how they placed the parts on rectangle *B*. There will be some different arrangements, but the important thing is that all of rectangle *B* can be covered with rectangle *A*. Encourage the children to discuss what they observe about the size of the two rectangles.

Ask if anyone can tell how many number 1 squares would cover rectangle *B*. (4) Ask if anyone knows how many number 2 squares would be needed to cover the number 1 square. (4) Then ask the children to tell how many number 2 squares are needed to cover rectangle *B*. (16) The class should be aware that when the unit square is smaller, more units are needed to cover a rectangle.

▶ Pin on the flannel board 3 tagboard rectangles (1 by 1 inch, 1 by 2 inches, and 1 by 4 inches). Label the rectangles as illustrated.



Ask the children to compare rectangle *A* with rectangle *B*. Someone may say that rectangle *B* is twice as large as rectangle *A*. Ask him how he knows this. He may say that it would take 2 of rectangle *A* to cover rectangle *B*. Someone else may say that rectangle *A* is $\frac{1}{2}$ as large as rectangle *B*. Tell the class to compare rectangle *A* and rectangle *C*. Someone may note that rectangle *C* is 4 times as large as *A*. Someone else may observe that rectangle *A* is $\frac{1}{4}$ as large as rectangle *C*. If some of the children do not see this, give them several 1-inch square units and have them lay these square units on rectangle *C* and count how many are needed to cover rectangle *C*.

Pages 129 through 132

● Page 129 should be used to help the children see that the size of figures is not governed by their shape. Tell the children to make rectangles like *A* and *C*. Then direct them to cut rectangle *A* into 5 parts as shown in the book, and to fit the parts onto rectangle *B*. Let the class describe their observations. They should conclude that the rectangles have the same size. Proceed with rectangle *C* in a similar way.

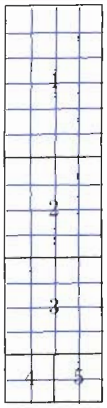
● Use page 130 to reinforce the idea that the size of a rectangle is not affected by the size of the square unit used to cover it. Have the class observe the size of the square unit used to cover rectangle *B*. Tell the children to read the questions and to write the answers. After the children have completed the page, discuss their answers to the questions. Ask the children what conclusion they drew from their work.

● Use page 131 to compare the size of rectangles by covering. Have the children trace and cut out rectangle *A*. Direct them to cover *B* with *A*, then *C* with *A*. Ask the children what they concluded. The conclusion should be that *B* is 4 times the size of *A*, and *C* is 4 times the size of *A*. From this information the children should conclude that rectangles *B* and *C* are the same size.

● Use page 132 as a class activity. Have the children determine how many of rectangle *A* just cover *B*. Give each child a piece of tracing paper. Direct the children to trace and cut out rectangle *B*. Then have the children lay this square unit on rectangle *C*. Have them keep a record of how many of *B* it takes to cover *C*. Then ask how many of *A* would cover *C*. They should conclude that rectangle *B* is 100 times as large as *A* and that rectangle *C* is 3×100 or 300 times as large as *A*. Follow a similar procedure to complete the remaining exercises.

Name

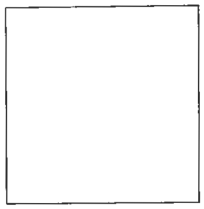
A



Cut a rectangle like A from $\frac{1}{4}$-inch squared paper. Divide the rectangle into 5 parts the same size as in the picture.

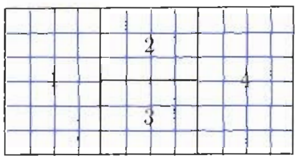
Place the 5 parts of rectangle A on rectangle B.

B



Do the parts of A exactly cover B? Yes

C



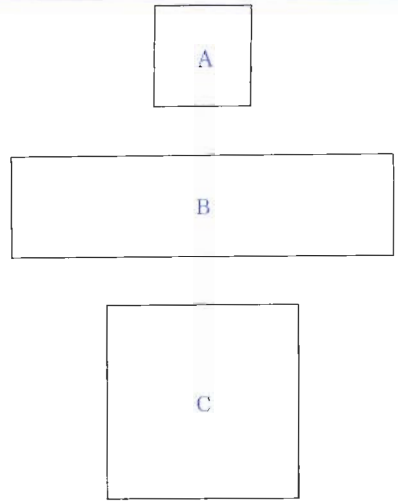
Cut a rectangle like C from $\frac{1}{4}$-inch squared paper. Divide the rectangle into 4 parts the same size as in the picture. Place the 4 parts of C on B.

Do the parts of C exactly cover B? No

What do you know about rectangles A and B? Same size

What do you know about rectangles C and B? Not same size

Name

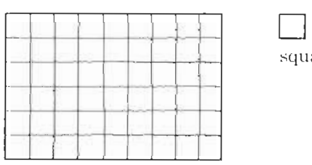


Trace and cut out rectangle A.

1. How many of A cover B? 4
2. How many of A cover C? 4
3. B is how many times as large as A? 4 times
4. C is how many times as large as A? 4 times
5. What do you know about the size of B and C? Their size is the same.

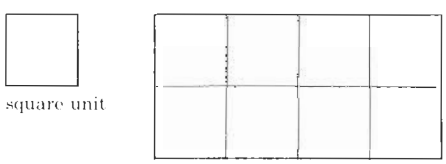
Which rectangle is larger?

A



square unit

B



square unit

1. How many small square units in rectangle A? 54
2. How many large square units in rectangle B? 8
3. How many small square units would cover one large square unit? 9
4. How many small square units would cover rectangle B? 72
5. How many large units would cover rectangle A? 6
6. Which rectangle is larger, A or B? B

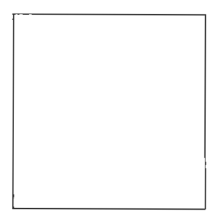
reference page

Trace and cut out rectangle B.

How many A cover B? 100

C

D



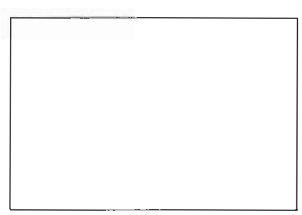
How many B cover C? 3

How many A cover C? 300

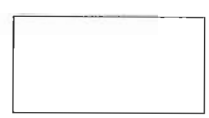
How many B cover D? 4

How many A cover D? 400

E



F



How many B cover E? 6

How many A cover E? 600

How many B cover F? 2

How many A cover F? 200

Supplemental Experiences

■ Provide each child with a 3 by 4 inch plain paper rectangle. Give each child three pieces of squared paper (1 inch, $\frac{1}{4}$ inch, and $\frac{1}{10}$ inch). Have the children cut out a 3 by 4 inch rectangle from each squared paper.

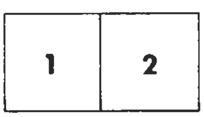
Direct the children to cover their plain rectangle with the rectangle cut from 1-inch squared paper. Ask how many of these square units cover the rectangle. Someone may say that 12 of these square units, or 3×4 of these square units, cover the figure.

Have the children cover the 3 by 4 inch rectangle with the $\frac{1}{4}$-inch squared paper. Ask the children to determine the number of square units by count or in any other way they wish to use. Tell the children to compare the count when this smaller unit was used with the count when the larger one was used. Some child may be able to count 192, but will probably say that there are 12×16 square units.

Next have the children use the $\frac{1}{10}$-inch squared paper and place it over the rectangle. Let the children tell what they observe. Someone may say there are many more square units now in the measure. It is not necessary for the children to count each unit, but someone may observe that there are 30×40 square units.

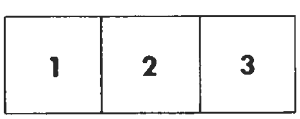
Ask the children to compare the three counts. Some child will observe that as the square unit becomes smaller a greater number of square units are needed to cover rectangles that have the same size.

■ Pin on the flannel board a tagboard rectangle that is partitioned into two $\frac{3}{4}$-inch squares. Number each square as illustrated.



Ask a child to point out the rectangles he can see in the figure. He may only see 2 rectangles—the two squares. Ask if anyone sees any other rectangles in this figure. Some child may point out that the whole figure is another rectangle.

Tell a child to pin another $\frac{3}{4}$-inch square next to the two squares. Ask the children to describe the rectangles they now see.

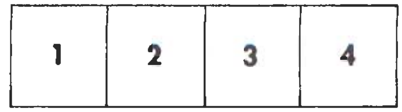


Make a list on the chalkboard as the children name the rectangles.

1
1 and 2
1, 2, and 3
2
2 and 3
3

Some child may observe that by adding one more $\frac{3}{4}$-inch square to the two squares, twice as many rectangles were created. There are now 6 rectangles.

Have the class find the number of rectangles there would be if another square were joined to the three already on the board.



List the rectangles on the chalkboard as they are named.

1
1 and 2
1, 2, and 3
1, 2, 3, and 4
2
2 and 3
2, 3, and 4
3
3 and 4
4

Ask a child to count the rectangles and tell how many there are. (10 rectangles)

The children may note that the total number of rectangles is not doubled again; there are only 4 more rectangles.

KEY IDEA

Square corners show perpendicular lines.

Scope

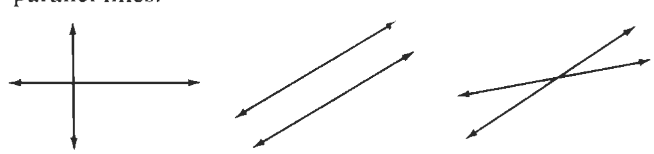
To introduce perpendicular lines and parallel lines by doing paper folding.

To become aware of perpendicular lines and parallel lines by observing them in familiar shapes.

Fundamentals

In the second grade the child made square corners by folding paper. The concept of perpendicular lines was suggested by the square-corner activities, but the term *perpendicular lines* was not used.

The following activities require paper-folding techniques to obtain perpendicular lines, oblique lines, and parallel lines.



perpendicular lines   parallel lines   oblique lines

Perpendicular lines are intersecting lines that form square corners, oblique lines are intersecting lines that do not form square corners, and parallel lines are lines—in a plane—that do not meet.

In the following activities the child must actually make something with his hands. These are exploratory activities that are designed to develop the child's understanding of perpendicular lines and parallel lines through manipulation and visual perception. Because the Developmental Experiences are constructed to introduce, develop, apply, and review the concepts, no pupil pages follow this section.

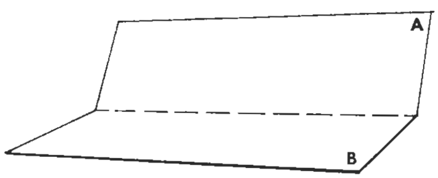
Developmental Experiences

adding machine tape
2 pencils or dowels
tagboard geometric figures

for each child
sheets of paper

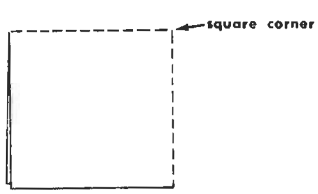
▶ Each child should have 2 or 3 sheets of paper. Show the class how to fold the paper to make a square corner.

Fold an edge of the paper onto itself so that *A* is on *B*. Hold the paper in position and crease the paper at the fold.

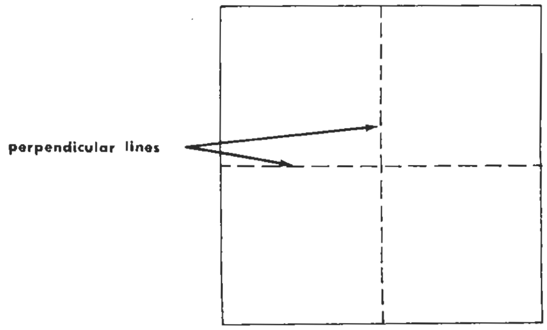


After the paper is creased, fold the edge that is creased back onto itself so that *C* is on *D*.

Tell the class that the result of the two folds is a square corner.



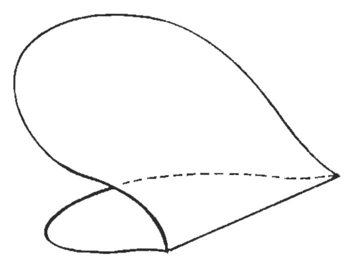
Unfold the paper and ask the children to find square corners made by the fold lines. Tell the class that two lines that make square corners are called *perpendicular lines.*



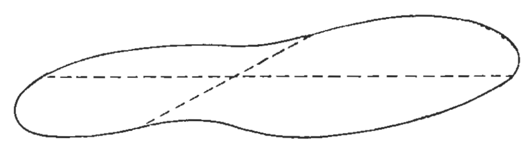
Let the children practice making square corners and perpendicular lines. When necessary, help individual children fold the paper to make perpendicular lines. Ask the class how many square corners are made by two perpendicular lines. (4) Tell the children to use another piece of paper to fold two lines that meet but do not make square corners. When they have done this, tell them that these two lines are called *oblique lines.* Draw diagrams on the board illustrating perpendicular and oblique lines. Label the lines.

▶ Show the class how to make perpendicular lines with a piece of paper that does not have a straight edge.

Fold and crease the paper to obtain a straight edge.



Fold the straight edge onto itself and crease to make the square corner. Unfold the paper to show the perpendicular lines.



Have each child tear a piece of paper so that it has no straight edges and then fold it to obtain perpendicular lines.

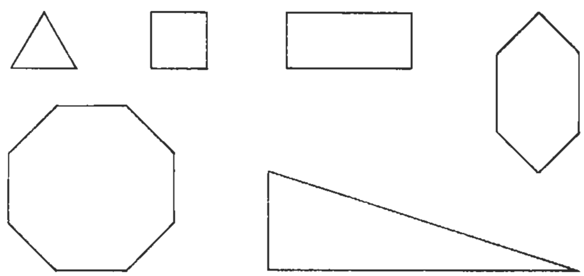
▶ Hold a long rectangular piece of paper in front of the class. A long piece of adding machine tape will serve very nicely. Tell the children to imagine that the paper has no end and goes on forever in a straight line. Ask the children if they think the two edges would ever meet. Give them ample time to discuss their ideas. It soon should become clear that the edges would not meet.

Tell the children that these two edges of the same sheet of paper are called parallel edges. Explain that edges of a sheet of paper are *parallel* if they can be extended without meeting.

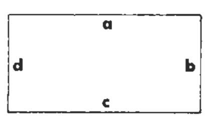
Hold up two pencils or two long straight dowel rods so that they are parallel. Tell the children that these represent two *parallel lines.* Tell the children to find other examples of parallel lines in the classroom. With a little guidance they should be able to find the following examples of parallel lines.

Lines in the tile floor.
Edges of sheets of paper.
Edges of the chalkboard.
Lines formed by walls, walls and the ceiling, and walls and the floor.
Edges of desks.
Edges of window frames.
Lines on their writing paper.

▶ Cut out the following shapes from tagboard.



Make the figures large enough for the whole class to see. Label the sides of each figure with small letters as shown.



Hold up one shape at a time and ask the children to tell which sides are perpendicular, which sides are oblique, and which sides are parallel. Record their answers on the chalkboard. Possible responses for the rectangle are:

Side *a* is perpendicular to side *b*.
Side *a* is perpendicular to side *d*.
Side *c* is perpendicular to side *b*.
Side *c* is perpendicular to side *d*.
Side *a* is parallel to side *c*.
Side *b* is parallel to side *d*.

Note that one triangle has only oblique sides, while the other has one pair of perpendicular sides.

UNIT 10
COMPUTATION OF BASIC PRODUCTS

Pages 133 Through 152

OBJECTIVE

To develop computational skill with the use of basic multiples of 8 and 9.

The child explores arrays to find better and faster ways to compute products. He partitions arrays into subarrays and computes partial products. The computed product is the sum of the partial products.

See Key Topics in Mathematics for the Primary Teacher: Multiplication.

KEY IDEAS

The eights are twice the fours.
The nines are three times the threes.
2 fives + 2 fours is 2 nines.
The distributive property shows that multiplication is repeated addition.

KEY IDEA

The eights are twice the fours.

Scope

To build awareness of a product as the sum of partial products.

To review multiplication facts which involve 2, 3, 4, 5, 6, 7, or 10 as a factor.

To develop multiplication combinations related to multiples of 8.

Fundamentals

The child approaches new computations by using previously learned multiplication facts and his knowledge of addition. The array provides a model for basic computation. The child partitions the array, then adds partial products to establish a new basic fact. This approach enables the child to practice and maintain facts and to broaden his understanding of product.

The child learns that a new product such as 9×8 may be computed without using a partitioned array. He may simplify the product to a sum of partial products through his operational knowledge of the distributive property.

$$\begin{aligned} 9 \times 8 &= (6 \times 8) + (3 \times 8) \\ &= 48 + 24 \\ &= 72 \end{aligned} \qquad \text{or} \qquad \begin{array}{r} 6+3 \\ \times \quad 8 \\ \hline 24 \\ 48 \\ \hline 72 \end{array}$$

Some new computations may be thought of as twice a known computation. For example, 9×8 may be simplified to twice 9×4. By using the multiplication fact $9 \times 4 = 36$ and addition, the child develops the sum $36 + 36$ or 72.

Through many approaches to computation of basic facts, the child develops a readiness for understanding the multiplication algorism as well as techniques for computing forgotten facts.

Readiness for Understanding

Knowledge of basic combinations related to multiples of 2, 3, 4, 5, 6, 7, and 10.

Understanding of the concept of product.

Ability to recognize a product as the sum of partial products.

Developmental Experiences

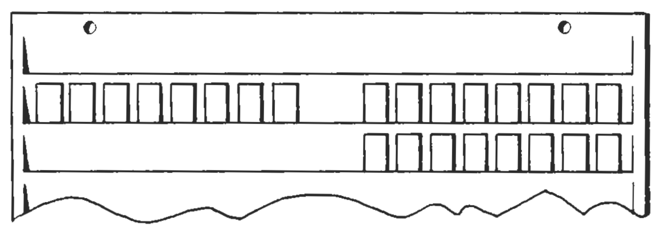
for flannel board	*for each child*
felt counters	plastic numerals and symbols
yarn	counters

pocket chart
tagboard strips (1″ × 2 ″)

▶ Ask a child to use tagboard strips in the pocket chart to show a 1 by 8 array. Instruct the other children to use their plastic numerals and symbols on their desks to make an equation that links the product and the count for the array. Have someone read his equation aloud.

$$1 \times 8 = 8$$

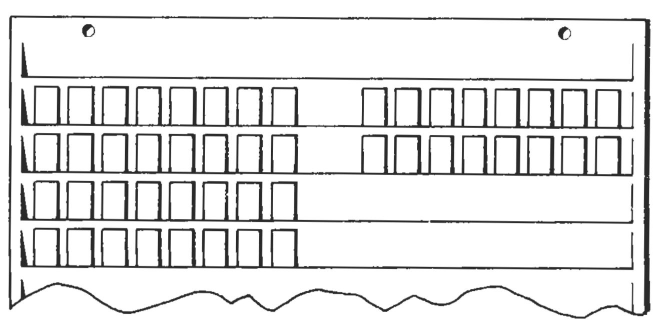
Ask a second child to construct in the pocket chart an array that has twice as many members as the 1 by 8 array.



Tell the other children to make an equation that corresponds to the second array. Have a child read his equation to the class.

$$1 \times 8 = 8$$
$$2 \times 8 = 16$$

Remove the 1 by 8 array from the chart and ask another child to construct an array that has twice as many members as the 2 by 8 array.



Tell the children to push their first equation ($1 \times 8 = 8$) out of the way. Have them show below their second equation ($2 \times 8 = 16$) the product and the count for the new array in the pocket chart. Let a child read this equation aloud.

$$2 \times 8 = 16$$
$$4 \times 8 = 32$$

The class should understand that since 4×8 is twice 2×8, the count for 4×8 is twice the count for 2×8.

Use this procedure to help the children observe the relationship between the following products.

4×8 and 8×8
3×8 and 6×8
5×8 and 10×8

Then ask some of the children to construct the following arrays in the pocket chart: a 0 by 8 array, a 7 by 8 array, and a 9 by 8 array. Have each child show an equation that links the product and the count for the array he constructs. Allow the children to determine the counts for these arrays in their own way.

▶ Tell the children to use their counters on their desks to show a 1 by 4 array and a 1 by 8 array. Direct the children to use their plastic numerals and symbols to show an appropriate equation below each array.

1×4=4

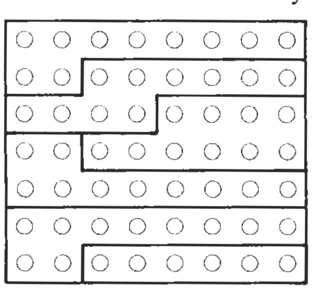
1×8=8

Ask the children what relationship they observe between these two products. Someone may comment that 1×8 is twice 1×4. Someone else may say 8 is twice 4.

Continue in this way; ask the children to compare the following products.

2×4 with 2×8
3×4 with 3×8
4×4 with 4×8
5×4 with 5×8
⋮
10×4 with 10×8

▶ Have a child draw a 7 by 8 array on the chalkboard. Let him choose someone to write the product below this array. Ask a third child to show how many tens and how many ones are in 7×8 by drawing a boundary around any subset of 10 that he sees in this array. He may decide to show the number of tens in the following way or in any one of several other ways.



7×8

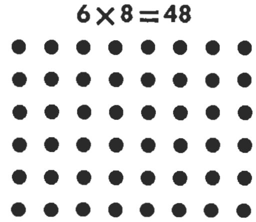
Choose someone to record the count beside the product 7×8 and to use an equal sign to link the two forms for the number of this array. Have the child read the equation aloud.

7×8=56

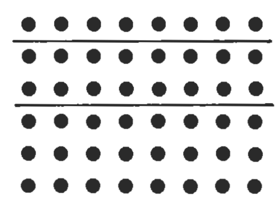
Continue in this way; use 8 as a factor in other products from 6×8 through 10×8. The children may note that the count for some of the products involves just tens and the count for others involves both tens and ones.

▶ Call two children to the chalkboard and instruct both of them to draw a 6 by 8 array. Direct one child to partition his array with two horizontal lines. Then tell the other child to record above his array the product and the count in an equation which represents the array. ($6 \times 8 = 48$) Ask the child who partitioned his array to write an equation for each part of his array—for example, $1 \times 8 = 8$, $2 \times 8 = 16$, and $3 \times 8 = 24$. He should write the three equations one below the other. Then have this child compute the sum of his partial products. Direct the class to compare the counts for the two arrays.

6×8=48

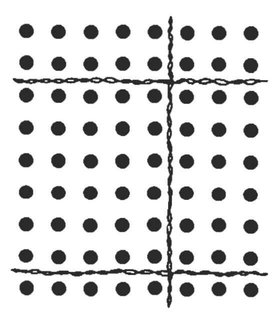


1×8= 8
2×8=16
3×8=24
48



Let other pairs of children demonstrate that the product of the whole array is the sum of the products of the parts of the array. Use products from 4×8 through 10×8.

▶ Ask a child to show a 9 by 8 array on the flannel board and to write the product below the array. Give another child three pieces of yarn and ask him to partition the array once vertically and twice horizontally. Perhaps he will partition as illustrated.



9×8

Ask someone else to express the product of this array in a way that will show each of the two factors as a sum, and have him write his expression of the product below the first expression of this number.

$$\begin{array}{c} 9 \times 8 \\ (2+6+1) \times (5+3) \end{array}$$

(Note: any of the commuted forms for these numbers are acceptable.)

Ask another child to come to the chalkboard to write a sum of partial products that results from partitioning the array. He may use any of the commuted forms of the expression given in the illustration.

$$\begin{array}{c} 9 \times 8 \\ (2+6+1) \times (5+3) \\ (2 \times 5)+(2 \times 3)+(6 \times 5)+(6 \times 3)+(1 \times 5)+(1 \times 3) \end{array}$$

Have this child point out the parts of the array that correspond to the partial products he has indicated. The class may observe that each of the three addends of 9 (2, 6, and 1) have been linked with each of the two addends of 8 (5 and 3).

Continue this activity; have the children show other partitionings of a 9 by 8 array. In each instance one factor of the product must be expressed as a sum of three addends. Then adapt this procedure to 6×8, 7×8, 8×8, and 10×8.

▶ Have a child draw a 10 by 8 array on the chalkboard. Let some child name the product of this array and record it on the board.

$$\begin{array}{r} 10 \\ \underline{\times \ 8} \end{array}$$

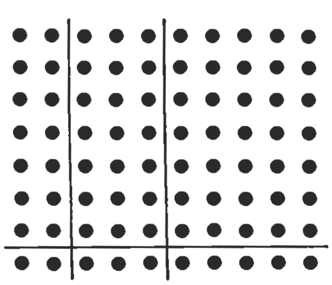
Ask the class to help compute the product of this array in parts. First have someone suggest a way to think of the factor 10 in three parts. Record his suggestion on the chalkboard—perhaps he will suggest that 10 be expressed as 2+3+5.

$$\begin{array}{rl} 10 & = 2 + 3 + 5 \\ \underline{\times \ 8} & \end{array}$$

Let this child partition the array to show 10 as 2+3+5. Let some other child suggest a way to think of the factor 8 as a sum of two addends. Record his suggestion on the chalkboard—perhaps he will suggest that 8 be thought of as 7+1.

$$\begin{array}{rl} 10 & = 2 + 3 + 5 \\ \underline{\times \ 8} & = \underline{\quad\quad 7 + 1} \end{array}$$

Have the child partition the array to illustrate his way to view the factor 8.



Have the class give the count for the whole array. (80) Again pose the question of whether or not 80 will result if you compute 10×8 in parts.

Direct the children to compute the number of each part of the array. Record the products and the computed products as shown. Then have the children compute the sum of these partial products.

$$\begin{array}{rrl} 10 = & 2+3+5 & \\ \underline{\times \ 8} = & \underline{\quad 7+1} & \\ 80 & 5 & (1 \times 5) \\ & 3 & (1 \times 3) \\ & 2 & (1 \times 2) \\ & 35 & (7 \times 5) \\ & 21 & (7 \times 3) \\ & \underline{14} & (7 \times 2) \\ & 80 & \end{array}$$

Be sure the class understands that 7 is linked with 5, 3, and 2 to form three products and then 1 is linked with 5, 3, and 2 to form three more products.

Draw another 10 by 8 array beside the first array on the chalkboard. Have a child partition the array twice vertically and once horizontally, and then record on the chalkboard the parts of the two factors that his partitioning suggests. For example, 10 may be pictured as 2+4+4 and 8 as 3+5.

$$\begin{array}{rl} 10 & = 2 + 4 + 4 \\ \underline{\times \ 8} & = \underline{\quad\quad 3 + 5} \end{array}$$

Have the children help you compute 10×8 in parts as suggested by the new way to view 10 and 8 and compare the computed sum of the partial products to the count for 10×8.

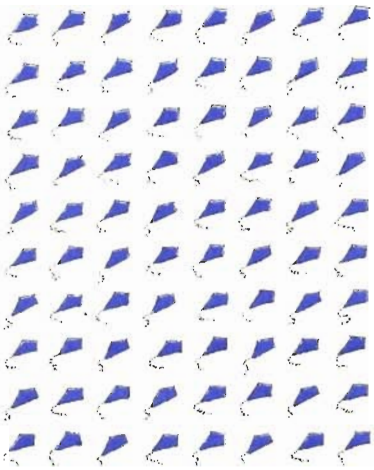
Have someone draw a third 10 by 8 array on the chalkboard and partition it into six parts in a different way. Instruct the children to compute 10×8 by using this third way of viewing the two factors.

Continue this procedure using the products 9×8, 8×8, 7×8, and 6×8.

Name

UNIT 10 COMPUTATION OF BASIC PRODUCTS

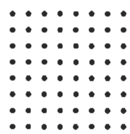
You can find the count by counting.
In what other ways can you find the count?



1 × 8 = 8
2 × 8 = 16
3 × 8 = 24
4 × 8 = 32
5 × 8 = 40
6 × 8 = 48
7 × 8 = 56
8 × 8 = 64
9 × 8 = 72
10 × 8 = 80

What product? What count?

4 × 8 = 32*



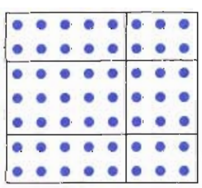
8 × 8 = 64

*Order of factors may vary.

reference page

B-133

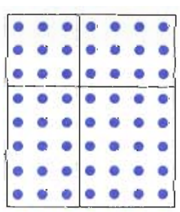
Compute using parts.



7 = 2 + 3 + 2
× 8 = 5 + 3
56
10
15
10
6 *
9
6
56

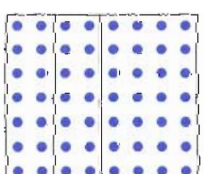
Compute using parts.

1.



7 = 4 + 3
× 8 = 5 + 3 *
56
15
20
12
9
56

2.



8 = 2 + 2 + 4 *
× 7 = 7
56
28
14
14
56

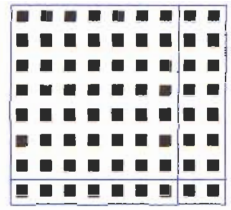
reference page *Order may vary.

B-134

Name

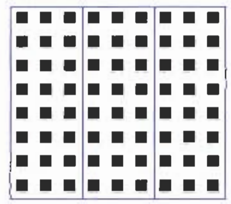
Compute using parts.

1.



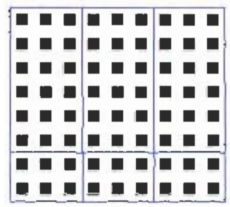
9 = 7 + 2 *
× 8 = 7 + 1
72
7
2
49
14
72

2.



9 = 3 + 3 + 3
× 8 = 8
72
24
24
24
72

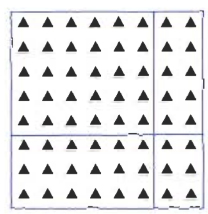
3.



9 = 3 + 3 + 3 *
× 8 = 2 + 6
72
6
6
6
18
18
18
72

*Order may vary.

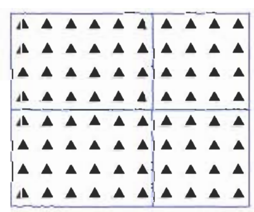
B-135



8 = 5 + 3
× 8 = 6 + 2 *
10
6
18
30
64

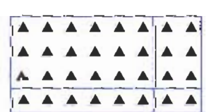
Compute using parts.

1.



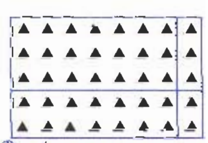
10 = 6+4 *
× 8 = 4+4
16
24
16
24
80

2.



4 = 3+1 *
× 8 = 6+2
2
6
6
18
32

3.

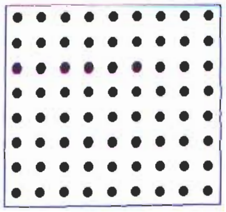


5 = 3+2 *
× 8 = 7+1
3
2
21
14
40

*Order may vary.

B-136

Name



Compute.

1. 9 = 3 + 6
   × 8 = 2 + 6
   36*, 18, 12, 6; 72

2. 9 = 5 + 4
   × 8 = 4 + 4
   20*, 16, 20, 16; 72

3. 9 = 8 + 1
   × 8 = 2 + 6
   6, 48, 2, 16; 72

4. 9 = 7 + 2
   × 8 = 5 + 3
   6*, 21, 10, 35; 72

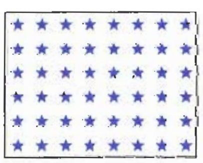
5. 9 = 4 + 3 + 2
   × 8 = 3 + 5
   10*, 15, 20, 6, 9, 12; 72

6. 9 = 3 + 3 + 3
   × 8 = 3 + 3 + 2
   6*, 6, 6, 9, 9, 9, 9, 9, 9; 72

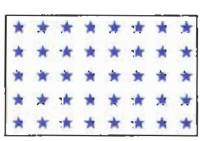
*Order may vary.

B-137

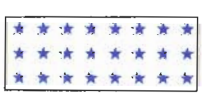
Partition each array. Compute using parts.

1. 

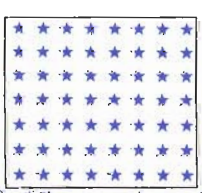
   6
   × 8
   48*

2. 

   5
   × 8
   40*

3. 

   3
   × 8
   24*

4. 

   7
   × 8
   56*

*Partitions and partial products will vary.

B-138

Pages 133 through 138

● Use page 133 primarily for class discussion. Have the children study the array and the first equation in the sequence of 10. Call upon someone to give the count for 1×8 and explain how he arrived at this count. He may say that he counted the members in such an array one by one. Encourage other children to describe other ways that the count in this instance could be determined. They may suggest counting by 2 (adding 2's) or counting by 4 (adding 4's).

Continue to let the children give the count for each product in the list and describe how they arrived at the count. After the discussion tell the children to complete the equations. Then have them complete the two exercises at the bottom of the page.

● Pages 134 through 136 provide the children with an opportunity to compute products that have 8 as a factor. Work one exercise on each page with the class. Then have the children complete the other exercises independently. Remind them to use these steps:

Observe the partitioning of the given array and decide upon the parts of the factors to be used.

Compute each partial product that results from the particular combinations of factors involved.

Compute the sum of partial products.

● When working with page 137, explain to the children that some parts of the factors of each product are given; the children are to supply the missing parts. The array at the top of the page may help them to determine the missing addend in each factor. After they have identified the missing parts of the factors, they may proceed with the computation.

● When working with page 138, tell the children that each child may decide how he wants to express the factors as sums. Instruct them to partition each array and compute using parts. When the children have completed this page, let them demonstrate on the chalkboard the different ways they used to compute a given product.

Supplemental Experience

■ Write on the chalkboard the words *sum, difference,* and *product.* Name two numbers such as 9 and 7. Call on one child to write below the word *sum* an equation that links the sum of these two numbers and the count for this sum. Have another child write below the appropriate word an equation that links the difference between 9 and 7 and the count for this difference. Ask a third child to write below the word *product* an equation that links the product of 9 and 7 and the count for this product.

sum	*difference*	*product*
$9 + 7 = 16$	$9 - 7 = 2$	$9 \times 7 = 63$

Continue in this way with other pairs of numbers whose sum, difference, and product the children are able to compute.

KEY IDEA

The nines are three times the threes.

Scope

To develop the multiplication combinations related to multiples of 9.

Fundamentals

The distributive property is unlike the other properties the child has studied. This property involves two operations whereas the commutative and associative properties involve only one operation. The distributive property will be extremely useful to the child in his work with multiplication. It is one of the most important concepts introduced in the third grade.

The child is led to discover the distributive property in simple examples of multiplication combinations already known. For example,

$$\begin{aligned} 7 \times 9 &= 7 \times (3 + 6) \\ &= 21 + 42 \\ &= 63 \end{aligned}$$

How do we know that this procedure will always work? The child sees it intuitively by working with arrays. Formally, we may state the property as $(b + c) \times a = (b \times a) + (c \times a)$, where a, b, and c are any whole numbers. The equation summarizes the distributive property of multiplication over addition.

Readiness for Understanding

Understanding of the concept of product.

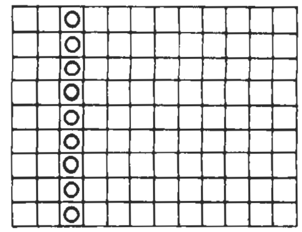
Understanding of the use of partitioning and partial products.

Developmental Experiences

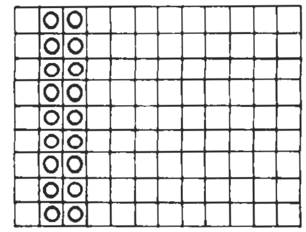
for flannel board
array-cards: 1 × 9 through 10 × 9
pins
yarn

for each child
sheet of 1-inch squared paper
washers
plastic numerals and symbols
counters
strips of cardboard ($\frac{1}{4}'' \times 12''$)

▶ Have the children work in pairs to show the relationships between products that have 9 as a factor. Give each child a sheet of 1-inch squared paper and 100 washers. Tell the children that one member of each pair is to construct a 1 by 9 array and the other member is to construct an array that has twice as many members as the 1 by 9 array. Tell the children to use their plastic numerals and symbols to show in an equation the product and the count for each array.



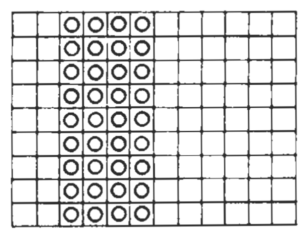
1 × 9 = 9



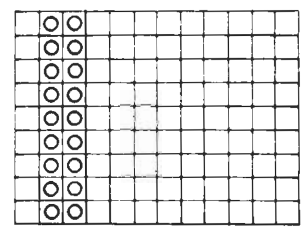
2 × 9 = 18

Ask both of the children in one pair to read their equations to the class.

Direct the children who made a 1 by 9 array to add to their array to make one that has twice as many members as the 2 by 9 array their partner built. Ask these children to show an equation that links the product and the count for their new array. (4 × 9 = 36)



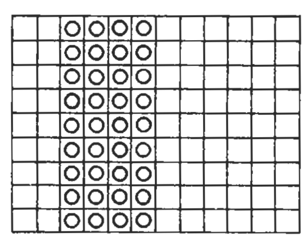
4 × 9 = 36



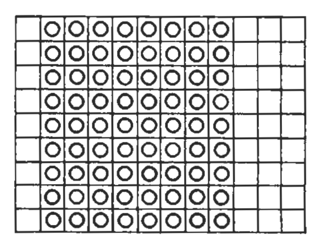
2 × 9 = 18

Have each of the children in a pair read his equation aloud; tell them to read the equation that represents the lesser product first.

Direct the children who made a 2 by 9 array to make it into an array that has twice as many members as the 4 by 9 array. Direct these children to show an equation that links the product and the count for their new array. (8 × 9 = 72)



4 × 9 = 36



8 × 9 = 72

Ask each child in a pair to read his equation aloud—the equation that represents the lesser product first.

Use this procedure and have the children observe the relationship between the following products:

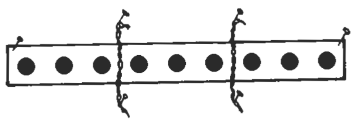
3 × 9 and 6 × 9
5 × 9 and 10 × 9

As a final phase of the activity, direct each child to show the following arrays:

0 by 9
7 by 9
9 by 9

Have each child show an equation that links the product and the count for the array he constructed.

Pin on the flannel board a tagboard card that shows a 1 by 9 array. Have a child partition this array with pieces of yarn to show how many times he sees 1×3 in 1×9. Help him pin the yarn to the flannel board.

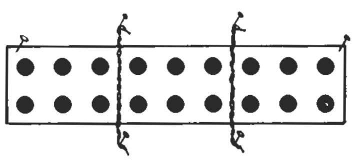


Ask a second child to come to the chalkboard. First have him write an equation that represents the product and the count for the array. ($1 \times 9 = 9$) Then have him record the product and the count for each of the other arrays contained in the whole array and compute the sum of the partial products.

$$1 \times 9 = 9 \qquad \begin{array}{r} 1 \times 3 = 3 \\ 1 \times 3 = 3 \\ 1 \times 3 = \underline{3} \\ 9 \end{array}$$

Ask the class how many times greater than 1×3 is the product 1×9. (three times)

Remove the first array-card from the flannel board and replace it with a 2 by 9 array-card. Have a child partition this array by using yarn to show how many times he sees 2×3 in 2×9. Help him pin the pieces of yarn in place.



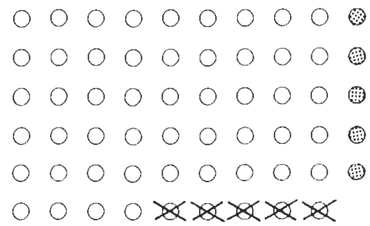
Ask some other child to come to the chalkboard. Have him write an equation that represents the whole array ($2 \times 9 = 18$); then have him record the product and the count for each part of the whole array and compute the sum of the partial products.

$$2 \times 9 = 18 \qquad \begin{array}{r} 2 \times 3 = 6 \\ 2 \times 3 = 6 \\ 2 \times 3 = \underline{6} \\ 18 \end{array}$$

Ask the class how many times greater the product 2×9 is than 2×3. (three times)

Adapt this procedure to other products from 3×9 through 10×9. At the conclusion of the activity, encourage the children to discuss what they have observed about the relationship between the threes and the nines. Many children will have become aware of the fact that the nines are three times the threes.

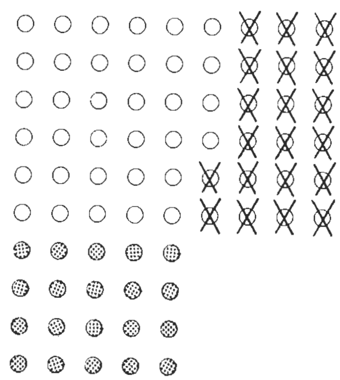
Have a child draw a 6 by 9 array on the chalkboard and write the product below it. Ask a second child to show how many tens and how many ones make 6×9. Explain that he is to rearrange the members of the array and make as many rows of 10 as possible. He may do this by crossing out some members from the bottom row of the array and redrawing them at the end of each row of 9 as needed.



When the rearranging is complete, have the child read the sum of tens and ones aloud, write the count beside the product 6×9, and use an equal sign to link the two forms for the number of this array. Have some child read this equation aloud.

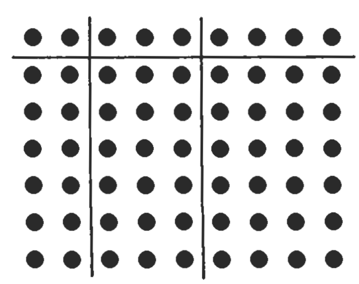
$$6 \times 9 = 54$$

Have someone draw a second 6 by 9 array on the chalkboard and show how many tens and how many ones make 6×9 in a different way. Tell him to cross out some of the members from the last four rows of 6 and redraw them below each of the other rows of 6 as needed to make rows of 10.



Continue in this way, using other products from 3×9 through 10×9.

Have the children use counters on their desks to make a 7 by 9 array. Give each child two strips of cardboard ($\frac{1}{4}$ inch by 12 inches); direct the children to show two vertical partitionings of their array. Give them a third strip of cardboard to make a horizontal partitioning of their array. Then tell the children to choose the plastic numerals and symbols that they need to show the product below their array. Now have the children express the product in a way that shows the parts of its factors as illustrated in the partitioning they used. For example, someone may have used the partitioning illustrated.



The expression of the product for this model would be the statement shown below, or any of the possible commuted forms of the numbers involved.

$$7 \times 9$$
$$(1 + 6) \times (2 + 3 + 4)$$

Direct the children to name the partial products that result from their ways of partitioning 7 and 9. The child who partitioned in the manner illustrated would name the partial products 1×2, 1×3, 1×4, 6×2, 6×3, and 6×4. Give all of the children ample time to discuss their way of seeing the factors 7 and 9 in parts.

Adapt this procedure to products from 6×9 through 10×9.

▶ Have a child draw on the chalkboard an 8×9 array and record the product of this array in vertical form.

$$\begin{array}{r} 8 \\ \underline{\times 9} \end{array}$$

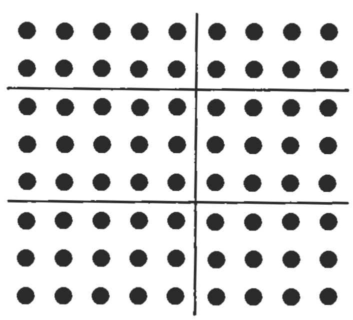
Ask someone to suggest a way to think of the factor 8 in three parts. Record his suggestion on the chalkboard—perhaps $2 + 3 + 3$.

$$\begin{array}{rl} 8 & = 2 + 3 + 3 \\ \underline{\times 9} & \end{array}$$

Then let the child partition the array to show 8 in this form. Ask another child to suggest a way to think of the factor 9 in two parts. Record his suggestion on the chalkboard—for example, $5 + 4$.

$$\begin{array}{rl} 8 & = 2 + 3 + 3 \\ \underline{\times 9} & = \underline{\quad\ 5 + 4} \end{array}$$

Then let the child partition the array to illustrate the way he thinks of 9.



Ask the class to give the count for this array. (72) Then tell the children that you want them to help you compute 8×9 in parts to check whether or not 72 will be the result. Record the computed products the children give and, in parentheses next to each count, record the related product. Have the class compute the sum of the partial products and compare the result to the count for 8×9.

$$\begin{array}{rlrl} 8 & = 2 + 3 + 3 & & \\ \underline{\times\ 9} & = \underline{\quad\ 5 + 4} & & \\ 72 & & 12 & (4 \times 3) \\ & & 12 & (4 \times 3) \\ & & 8 & (4 \times 2) \\ & & 15 & (5 \times 3) \\ & & 15 & (5 \times 3) \\ & & \underline{10} & (5 \times 2) \\ & & 72 & \end{array}$$

Help the class observe that 4 is linked with 2, 3, and 3 to form three partial products and then 5 is linked with 2, 3, and 3 to form three more partial products.

Repeat the activity at least twice more with 8×9 arrays drawn on the chalkboard. Ask the children to partition each array differently. Adapt the procedure suggested in the preceding paragraphs to help the children compute 8×9 in the way each partitioning indicates.

Continue this activity by using other products such as 6×9, 7×9, 9×9, and 10×9.

Pages 139 through 141

● Use page 139 primarily as a discussion page. Tell the children to study the array and the first equation in the sequence. Ask someone to give the count for 1×9 and to explain how he arrived at this count. He may say that he counted the members in such an array one by one. Let the children discuss other ways they think the count could be determined. It is possible that they will suggest the following ways among others:

Count by 3 (add 3's).

Count by 2 as far as possible, then add 1.

Count by 9.

Add the count of a 1 by 5 array to the count of a 1 by 4 array.

Have the children give the count for each product listed and encourage them to describe how they determined each count. After the discussion, have the children complete the equations and the two exercises at the bottom of the page.

● When using pages 140 and 141, work one exercise on each page with the children. Then have them complete the other exercises independently.

Explain that on pages 140 and 141 they should use the following steps.

Observe the partitioning of the given array and decide upon the parts of the factors to be used.

Compute each partial product.

Compute the sum of partial products.

Name

You can find the count by counting.
In what other ways can you find the count?

1 × 9 = 9
2 × 9 = 18
3 × 9 = 27
4 × 9 = 36
5 × 9 = 45
6 × 9 = 54
7 × 9 = 63
8 × 9 = 72
9 × 9 = 81
10 × 9 = 90

What product? What count?

2 × 9 = 18*

4 × 9 = 36*

*Order of factors may vary.

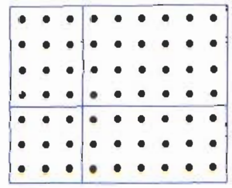
reference page

B-139

Name

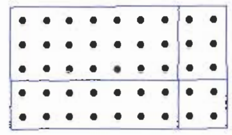
Compute using parts.

1.



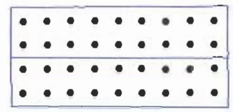
7 = 3+4 *
× 9 = 6+3
12
9
24
18
63

2.



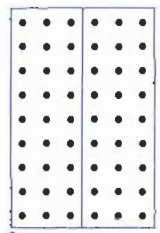
5 = 3+2 *
× 9 = 7+2
4
6
14
21
45

3.



4 = 2+2
× 9 = 9
18
18
36

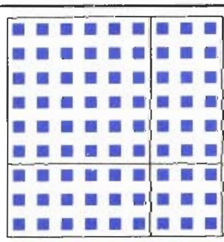
4.



6 = 3+3
× 9 = 9
27
27
54

*Order may vary.

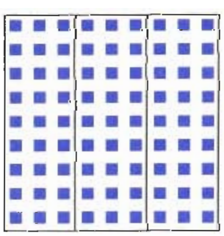
B-141



9 = 6 + 3
× 9 = 3 + 6
18*
36
9
18
81

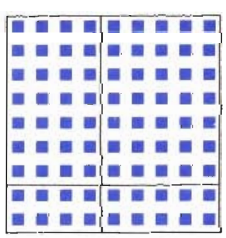
Compute using parts.

1.



9 = 3 + 3 + 3
× 9 = 9
27
27
27
81

2.



9 = 4 + 5 *
× 9 = 7 + 2
10
8
35
28
81

reference page *Order may vary.

B-140

Supplemental Experience

■ Separate the class into four teams and assign a panel of the chalkboard to each team. Tell a member from each team to take his place at the chalkboard. Explain to the children that the members of Team A are to write a product in the forties, the members of Team B are to write a product in the thirties, the members of Team C are to write a product in the twenties, and the members of Team D are to write a product in the teens. The children should try not to repeat a product already shown. Proceed in this way until all of the children have participated in the activity.

Team A forties		Team B thirties		Team C twenties		Team D teens	
9×5	7×6	8×4	4×9	10×2	3×9	5×3	2×8
4×10	20×2	10×3	4×8	5×4	5×5	3×6	4×4
8×6	5×8	6×6	5×6	3×7	4×7	2×9	2×7
5×9		7×5		8×3		4×3	

Total the points for each team—give 1 point for each correct product, and subtract 1 point for each incorrect one.

Continue the game for several rounds. Change each team's range of products for each round of the game. At the end of the period, total the points earned for each round and declare a winning team.

KEY IDEA

2 fives + 2 fours is 2 nines.

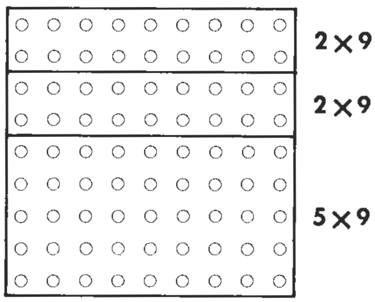
Scope

To review the distributive property of multiplication over addition.

To provide practice in computing basic multiplication combinations that involve 9 as a factor.

Fundamentals

The distributive property of multiplication over addition is easily illustrated by using the array as a model of multiplication. Consider the 9 by 9 array illustrated below.



The product for the 9 by 9 array, $(2 + 2 + 5) \times 9$, is the sum of the partial products 2×9, 2×9, and 5×9. That is, $(2 + 2 + 5) \times 9 = (2 \times 9) + (2 \times 9) + (5 \times 9)$.

The following computational procedure is justified by the distributive property of multiplication over addition.

$$\begin{array}{rr} 9 = & 5 + 2 + 2 \\ \underline{\times 9} = & \underline{\qquad 9} \\ & 18 \\ & 18 \\ & \underline{45} \\ & 81 \end{array}$$

The factor 9 has been distributed over addition to provide the pairings, 2×9, 2×9, and 5×9. These are partial products whose sum is 9×9 or 81.

The distributive property of multiplication over addition is logically equivalent to the statement that the product is equal to the sum of partial products.

Readiness for Understanding

Knowledge of the array.
Understanding of product.

Developmental Experiences

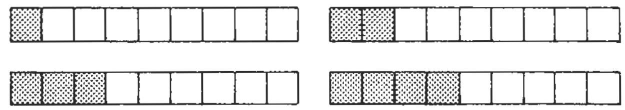
for flannel board
felt counters
pieces of yarn

for each child
sheet of 1-inch squared paper
2 crayons

masking tape

▶ Have the children work in pairs to show relationships between arrays with 9 as one factor of the product. Provide each child with a sheet of 1-inch squared paper and two crayons.

Tell the children that one person of each pair is to outline a 1 by 9 array on his paper with one of the crayons. The other person is to outline a 1 by 9 array and partition it once. Explain that he may show his partitioning by shading in the squares that form one of the two arrays he intends to show in the 1 by 9 array. The child who partitions his array may show any one of the following:



Ask one pair of children to tape their arrays on the chalkboard. Direct the child who did not partition his array to write above the array the product and the count in an equation. ($1 \times 9 = 9$) Tell the child who partitioned the array to write an equation for each part of his array—for example, $1 \times 4 = 4$ and $1 \times 5 = 5$. Direct him to write one equation below the other, then to compute the sum of the two partial products.

$$1 \times 9 = 9 \qquad\qquad \begin{array}{r} 1 \times 4 = 4 \\ 1 \times 5 = \underline{5} \\ 9 \end{array}$$

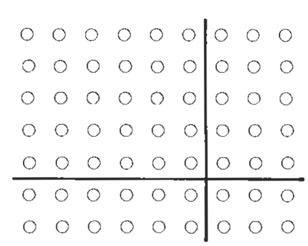
Have the class compare the results with the first count shown for 1×9.

Now ask the first child in the pair to write a second equation that shows the factor 9 as a sum of the parts that his partner chose to use. It may be necessary to show what is desired. The children should check the result.

$$\begin{array}{r} 1 \times 9 = 9 \\ 1 \times (5 + 4) = 9 \end{array} \qquad\qquad \begin{array}{r} 1 \times 4 = 4 \\ 1 \times 5 = \underline{5} \\ 9 \end{array}$$

Ask the class whether or not any pair of children used a different way to partition the 1 by 9 array. Let several pairs of children who partitioned in different ways come to the chalkboard and show how the product of the whole array is the sum of the products of the parts of the array. Then adapt this procedure to other products that have 9 as a factor, from 2×9 through 10×9.

▶ Have a child draw a 7 by 9 array on the chalkboard and write the product below the array. Ask another child to partition the array once vertically and once horizontally. He may partition as shown in the illustration.



7×9

Ask this child to express the product of this array in a way that will show the parts of the two factors. Tell the child to write his expression of the product below the 7 × 9 on the chalkboard. He may use the form in the following example or any of its commuted forms.

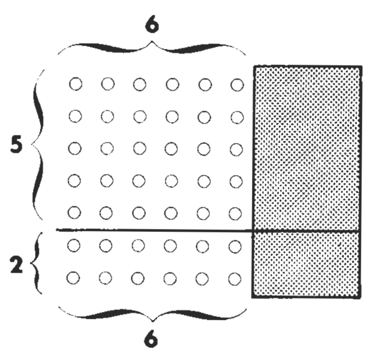
$$7 \times 9$$
$$(5+2) \times (6+3)$$

Have some other child come to the chalkboard and write a sum of partial products that results from the way that the two factors are expressed. He may use any of the commuted forms of the expression given in the illustration.

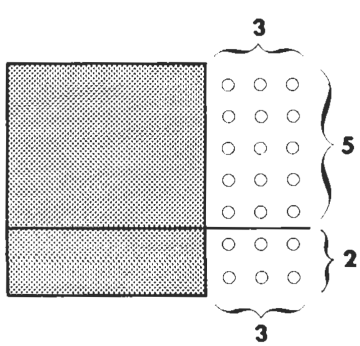
$$7 \times 9$$
$$(5+2) \times (6+3)$$
$$(5 \times 6)+(2 \times 6)+(5 \times 3)+(2 \times 3)$$

Ask this child to show the parts in the array that are represented by partial products he has indicated.

Tell the class to study the expressions of 7 × 9. Ask the children to show where the partial products 5 × 6 and 2 × 6 came from. Someone may note that the 6 in 6 + 3 has been linked with the 5 and then the 2 in 5 + 2 to give the partial products 5 × 6 and 2 × 6. Call attention to the array and help the children note that the 6 that resulted from the vertical partitioning has been linked with the 5 and the 2 that resulted from the horizontal partitioning to make the partial products 5 × 6 and 2 × 6.



Then have the children note that the 3 that resulted from the vertical partitioning has been linked with the 5 and the 2 that resulted from the horizontal partitioning to make the 5 × 3 and the 2 × 3.



The product 7 × 9 is a sum of these four partial products.

Draw a 6 by 9 array on the chalkboard and use the following procedure with this array:

Have someone write on the chalkboard the product for the array.

Have another child partition the array twice and express the product as a sum of the partial products observed in the partitioning. For example, the following sum of partial products could be expressed.

$$6 \times 9$$
$$(5 \times 7) + (5 \times 2) + (1 \times 7) + (1 \times 2)$$

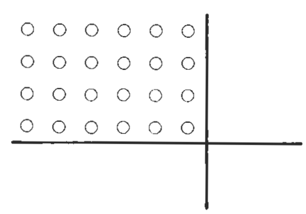
Let a third child express the product in a way that shows the parts of the two factors 6 and 9. Remind the class that the way the array has been partitioned indicates the parts of the factors.

$$6 \times 9$$
$$(5 \times 7) + (5 \times 2) + (1 \times 7) + (1 \times 2)$$
$$(5 + 1) \times (7 + 2)$$

Have the parts of the factors evident in the third expression of the product found in the second expression. The class may observe that 5 has been linked with 7 and 2; that 1 has been linked with 7 and 2.

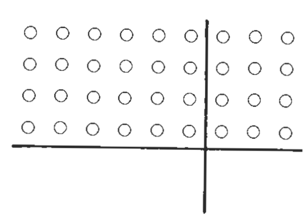
Adapt this procedure to studying an 8 × 9 and a 9 × 9 array.

▶ Draw on the chalkboard part of a 6 by 9 array and two partitioning lines as shown. Below the array write its product.



6×9

Tell the children that since they know the product for this array they should be able to fill in the parts not drawn. Select someone to complete any one of the parts of the array. He may decide to complete his part of the array in the following way.



However, there are several other ways he could choose, and he should be allowed freedom in his selection. Select two other children to fill in the remaining parts of the array.

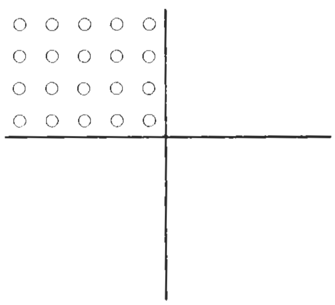
After all of the parts of the array have been completed, have someone record on the chalkboard the product and the count for each part of the array.

Direct the class to compute the sum of the partial products.

$$\begin{array}{r} 4 \times 6 = 24 \\ 4 \times 3 = 12 \\ 2 \times 6 = 12 \\ 2 \times 3 = \underline{\ 6} \\ 54 \end{array}$$

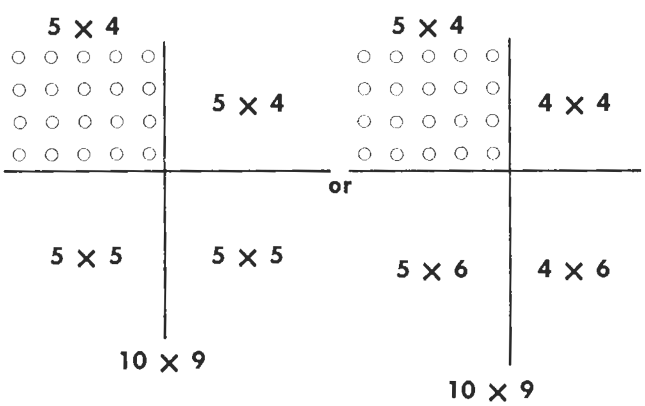
Discuss whether or not this number is the computed product for the 6 by 9 array. Repeat this procedure with 7×9, 8×9, and 9×9.

Vary the activity. Draw on the chalkboard part of a 10 by 9 array and two partitioning lines as shown. Write the product below the array.



10 × 9

Tell the class that this time the missing members of the array will not be drawn; they will be visualized instead. Let volunteers write only the product of each part within each space. The children may develop either of the two possibilities illustrated as they write the partial products for 10×9.



After the product for each part of the array has been indicated, have a child list the product and the count for each part in equations on the chalkboard and compute the sum of these partial products.

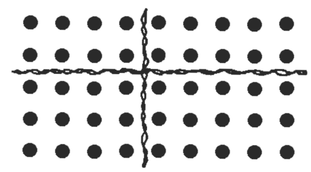
$$\begin{array}{r} 5 \times 4 = 20 \\ 5 \times 4 = 20 \\ 5 \times 5 = 25 \\ 5 \times 5 = \underline{25} \\ 90 \end{array} \quad \text{or} \quad \begin{array}{r} 5 \times 4 = 20 \\ 4 \times 4 = 16 \\ 5 \times 6 = 30 \\ 4 \times 6 = \underline{24} \\ 90 \end{array}$$

Have the class decide whether or not this number is the appropriate count for an array having 10×9 members. Continue in this way with 6×9, 7×9, 8×9, and 9×9.

▶ Have a child use felt counters on the flannel board and show a 5 by 9 array. Write on the chalkboard the following incomplete equation.

$$5 \times 9 = (2 + \_\_) \times (4 + \_\_)$$

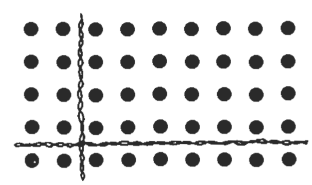
Give a second child pieces of yarn and ask him to partition the array according to the clues given by the partial products. He may choose to partition the array as shown.



Ask someone else to complete the equation on the chalkboard according to the partitioning illustrated.

$$5 \times 9 = (2 + \underline{3}) \times (4 + \underline{5})$$

Ask if anyone interpreted the clues in the equation differently and would want to partition the array another way. Let any child who sees another partitioning possibility show his idea to the class. Some child may come to the flannel board and partition the array this way.



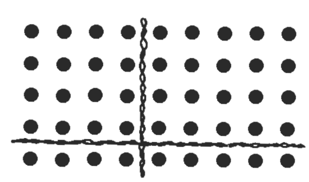
Tell this child to write his interpretation of this partitioning in a second equation on the chalkboard.

$$5 \times 9 = (2 + \underline{3}) \times (4 + \underline{5})$$
$$5 \times 9 = (2 + \underline{7}) \times (4 + \underline{1})$$

Leave the 5 by 9 array on the flannel board but remove the pieces of yarn. Then write the following incomplete equation on the chalkboard.

$$5 \times 9 = (4 \times 5) + (1 \times 5) + (\_\_ \times \_\_) + (\_\_ \times \_\_)$$

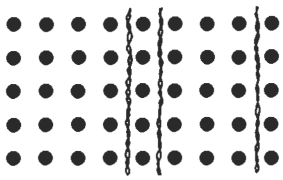
Ask a child to use pieces of yarn to partition the array according to the partial products given in the equation. This child may partition the array as follows.



Have the child complete the equation on the chalkboard according to the partitioning that he illustrated.

$$5 \times 9 = (4 \times 5) + (1 \times 5) + (\underline{4} \times \underline{4}) + (\underline{1} \times \underline{4})$$

Ask if anyone interpreted the clues in the equation differently and would partition the array another way. Allow any child who sees another partitioning possibility to show his idea to the class. Some child may come to the flannel board and partition the array as illustrated.



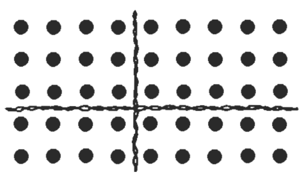
Ask the child to write his interpretation of this partitioning in a second equation below the first equation on the chalkboard.

$$5 \times 9 = (4 \times 5) + (1 \times 5) + (\underline{4} \times \underline{4}) + (\underline{1} \times \underline{4})$$
$$5 \times 9 = (4 \times 5) + (1 \times 5) + (\underline{3} \times \underline{5}) + (\underline{1} \times \underline{5})$$

Once again leave the 5 by 9 array on the flannel board and remove the pieces of yarn. Then write on the chalkboard the following incomplete equation.

$$5 \times 9 = 12 + \_\_ + \_\_ + \_\_$$

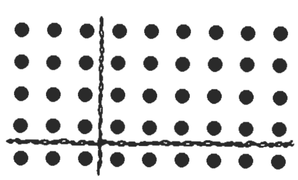
Have a child use the pieces of yarn to partition the array according to the clues given in the equation. The child may partition the array as illustrated.



Have him complete the equation on the chalkboard according to the partitioning he has illustrated.

$$5 \times 9 = 12 + \underline{8} + \underline{15} + \underline{10}$$

Ask if anyone interpreted the clues in the equation differently and would rather partition the array another way. Let any child who sees another partitioning possibility show his idea to the class. Some child may come to the flannel board and partition the array this way.



Ask the child to write below the other equation on the chalkboard his interpretation of the partitioned array.

$$5 \times 9 = 12 + \underline{8} + \underline{15} + \underline{10}$$
$$5 \times 9 = 12 + \underline{3} + \underline{24} + \underline{6}$$

Adapt the procedure described in the preceding paragraphs to 6×9, 7×9, and 8×9.

▶ Write the following product on the chalkboard:

$$9 \times (6 + 3)$$

Let several children come to the board and show and tell how they would compute this product. Someone may say that he would add first and then multiply—$6 + 3 = 9$; $9 \times 9 = 81$. Someone else may say that he would multiply first and then add—$(9 \times 6) + (9 \times 3) = 54 + 27 = 81$.

Write the product $(5 + 2) \times (8 + 1)$ on the chalkboard. Let volunteers show and tell how they would compute this product.

Someone may say that he visualizes four partial products and would compute each in turn. Then he would compute the sum of these partial products.

$$\begin{array}{r} 5 \times 8 = 40 \\ 2 \times 8 = 16 \\ 5 \times 1 = 5 \\ 2 \times 1 = \underline{2} \\ 63 \end{array}$$

Another child may say that he would add first and then multiply.

$$\begin{array}{r} 5 + 2 = 7 \\ 8 + 1 = \underline{9} \\ 63 \end{array}$$

A third child may say that he would add 5 and 2 first, and then multiply 8 and 1 by 7.

$$5 + 2 = 7 \qquad \begin{array}{r} 8 \times 7 = 56 \\ 1 \times 7 = \underline{7} \\ 63 \end{array}$$

Continue in this way to have the children describe their method to compute $(5 + 2) \times (8 + 1)$. Then adapt the procedure to other products such as:

$9 \times (2 + 7)$,
$(4 + 2) \times 9$,
$(5 + 4) \times (6 + 3)$,
$(5 + 3) \times (7 + 2)$.

Pages 142 through 145

- Explain that on pages 142 and 143 some parts of the factors of each product are given; the children are to supply the missing parts. Remind the children that the array at the top of the page may assist them. After they decide upon all of the parts of the factors, they are to proceed with the computation. On page 143 direct the children to show their method of computation for each product in exercises 5 through 10. After the children complete these exercises, let individual children use the chalkboard to demonstrate and explain how they computed.

- Work the first exercise on page 144 with the class; then let the children complete the second exercise independently. Tell the children that, if it will help them, they may draw the parts of the array on their paper before they try to complete the four equations. When they all have completed this exercise, ask individual children to discuss the partial products they used.

- Page 145 gives the children an opportunity to test themselves in the following areas:

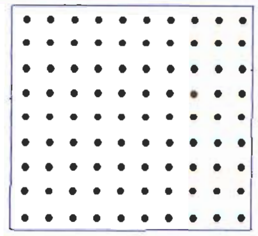
  the ability to determine the product and the count for a given array;

  the ability to compute, either in parts or otherwise, products that have 8, 9, or 10 as a factor; and

  the ability to determine the order relationship between given products.

Some children may wish to show their work for exercises 14 through 19; others may wish to do the computation mentally. The children need not complete the page in one session.

Compute using parts.



1. 10 = 5 + 5
   × 9 = 5 + 4
   20*
   20
   25
   25
   90

2. 10 = 9 + 1
   × 9 = 6 + 3
   3*
   27
   6
   54
   90

3. 10 = 8 + 2
   × 9 = 7 + 2
   4*
   16
   14
   56
   90

4. 10 = 5 + 3 + 2
   × 9 = 9
   18*
   27
   45
   90

5. 10 = 2 + 2 + 3 + 3
   × 9 = 9
   18*
   27
   18
   27
   90

6. 10 = 4 + 4 + 2
   × 9 = 3 + 6
   12*
   24
   24
   6
   12
   12
   90

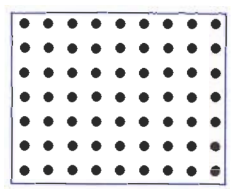
7. 10 = 3 + 3 + 4
   × 9 = 3 + 3 + 3
   12*
   9
   9
   12
   9
   9
   12
   9
   9
   90

*Order may vary.

B-142

Name

Compute using parts.



1. 7 = 2 + 5
   × 9 = 4 + 5
   25*
   10
   20
   8
   63

2. 7 = 3 + 4
   × 9 = 6 + 3
   12*
   9
   24
   18
   63

3. 7 = 5 + 2
   × 9 = 9
   18*
   45
   63

4. 7 = 6 + 1
   × 9 = 8 + 1
   1*
   6
   8
   48
   63

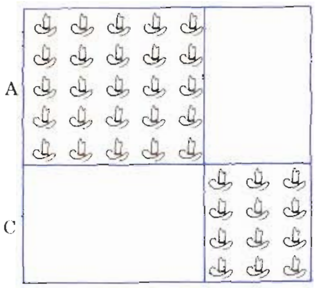
Compute each product.

5. 9 × (3 + 2) = 45
6. (3 + 6) × 9 = 81
7. (3 + 3) × 9 = 54
8. (4 + 5) × (2 + 3) = 45
9. (3 + 4) × (8 + 1) = 63
10. (4 + 4) × (4 + 5) = 72

*Order may vary.

B-143

1. Mr. Brimley owns a Ten-Gallon Hat Factory. One day he inspected an array of 72 hats. Two parts of the array are shown. How many hats are in each of the other parts?



Part A 5 × 5 = 25

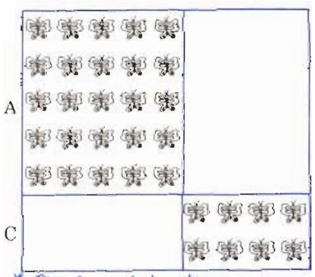
Part B 3 × 5 = 15*

Part C 4 × 5 = 20*

Part D 4 × 3 = 12*

A + B + C + D 72

2. Ken collects butterflies. He wants to put his 63 butterflies in a display case. How many butterflies will he put in each part of the 9 × 7 array?



Part A 5 × 5 = 25

Part B 4 × 5 = 20*

Part C 2 × 5 = 10*

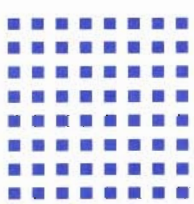
Part D 2 × 4 = 8*

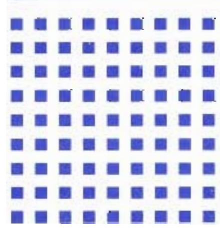
A + B + C + D 63

*Order of factors may vary.

B-144

Name





1. What product? 8×8

What count? 64

2. What product? 9×9

What count? 81

Compute.

3. 8 × 8 = 64

4. (8 × 7) = 56

5. 8 × 9 = 72

6. 9 × 9 = 81

7. 9 × 8 = 72

8. (8 × 6) = 48

9. (9 × 6) = 54

10. 10 × 8 = 80

11. 9 × 10 = 90

12. Circle the products above that are less than 8 × 8.

13. Underline the products above that are greater than 8 × 8.

Compute.

14. (4 + 4) × (5 + 3) = 64

15. 8 × (7 + 1) = 64

16. (2 + 2) × 10 = 40

17. 8 × (7 + 3) = 80

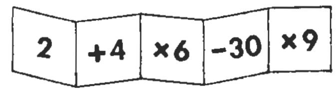
18. (6 + 2) × (6 + 1) = 56

19. 7 × (6 + 3) = 63

B-145

Supplemental Experience

■ Design a set of accordion-folding cards. Write on these cards numerals and operation signs as illustrated.



Fold the cards so that you can show the class one section at a time. Hold up the first section of the card for a few seconds. Slowly unfold the card and expose the other sections one at a time. Encourage the children to compute without using paper and pencil. Call on a child to give the result of his computation.

Continue in this manner with other cards. If you wish, write the result of the computation on the back of each card. Then the children may work in pairs or small groups to practice independently.

KEY IDEA

The distributive property shows that multiplication is repeated addition.

Scope

To explore multiplication as repeated addition.

Fundamentals

Multiplication is related to addition and may be looked at as repeated addition.

$$\begin{aligned} 4 \times 8 &= (1 + 1 + 1 + 1) \times 8 \\ &= 1 \times 8 + 1 \times 8 + 1 \times 8 + 1 \times 8 \\ &= 8 + 8 + 8 + 8 \end{aligned}$$

From this example, it can be seen that the traditional interpretation of multiplication as repeated addition works in the set of whole numbers because of the distributive property. (Although multiplication as repeated addition works with whole numbers, it fails with the set of fractional numbers.)

Readiness for Understanding

Knowledge of the distributive property.

Developmental Experiences

chart paper
felt-tip pen
pointers

▶ Tell the class that you are going to display an array that has rows of 8 members. Begin with a 1 by 8 array.

○ ○ ○ ○ ○ ○ ○ ○

Have someone record on the chalkboard the product and the count for this array in an equation ($1 \times 8 = 8$).

Join a second 1 by 8 array to the first array.

○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○

Ask a child to write the equation for this array ($2 \times 8 = 16$) about three inches below the first equation. Help the child line up the symbols in the second equation directly below the symbols in the first equation.

Continue in this way until a 10 by 8 array has been constructed. Then have the class examine the products and counts listed on the chalkboard. Ask the children what they notice about this sequence of products. They may observe that beginning with 2×8 each product is 8 greater than the preceding product.

Let the children compare the products that have 8 as a factor. Ask the class how much greater 2×8 is than 1×8. Use the list of equations on the chalkboard to demonstrate in the following way the fact that 2×8 is 8 greater than 1×8.

$$\begin{array}{rr}
1 \times 8 = & 8 \\
 & \underline{+\ 8} \\
2 \times 8 = & 16
\end{array}$$

Continue to have the children compare 3×8 and 2×8, 4×8 and 3×8, and so forth through 10×8 and 9×8. Ask the children to help record the fact that one product in each pair is 8 greater than the other.

$$\begin{array}{rr}
1 \times 8 = & 8 \\
 & \underline{+\ 8} \\
2 \times 8 = & 16 \\
 & \underline{+\ 8} \\
3 \times 8 = & 24 \\
 & \underline{+\ 8} \\
4 \times 8 = & 32 \\
 & \underline{+\ 8} \\
5 \times 8 = & 40 \\
 & \underline{+\ 8} \\
6 \times 8 = & 48 \\
 & \underline{+\ 8} \\
7 \times 8 = & 56 \\
 & \underline{+\ 8} \\
8 \times 8 = & 64 \\
 & \underline{+\ 8} \\
9 \times 8 = & 72 \\
 & \underline{+\ 8} \\
10 \times 8 = & 80
\end{array}$$

Vary the activity. Consider the arrays in the opposite order. Point to the 10 by 8 array and ask a child to record on the chalkboard an equation that shows the product and the count for the array ($10 \times 8 = 80$).

Remove 1 row of 8. Have someone else write an equation that represents the product and count for this array ($9 \times 8 = 72$) about three inches below $10 \times 8 = 80$.

Proceed in this way until the array appears that has 1 row of 8 less than the array with 1 row of 8. Be sure the class understands that this is an array having 0 rows of 8. Have the children note that the array's product is 0×8 and its count is 0. Then have the class examine the products and counts in this list of equations. Ask the children to describe what they notice about this sequence of products. They may observe that beginning with 9×8 each product is 8 less than the preceding product.

Again have the children compare the products that have 8 as a factor. Ask someone to tell how much less than 10×8 is 9×8. Use the list of equations on the chalkboard to demonstrate in the following way the fact that 9×8 is 8 less than 10×8.

$$\begin{array}{rr}
10 \times 8 = & 80 \\
 & \underline{-\ 8} \\
9 \times 8 = & 72
\end{array}$$

Continue in a similar way to have the children compare 8×8 and 9×8, 7×8 and 8×8, and so forth through 0×8 and 1×8.

$$\begin{array}{rr}
10 \times 8 = & 80 \\
 & \underline{-\ 8} \\
9 \times 8 = & 72 \\
 & \underline{-\ 8} \\
8 \times 8 = & 64 \\
 & \underline{-\ 8} \\
7 \times 8 = & 56 \\
 & \underline{-\ 8} \\
6 \times 8 = & 48 \\
 & \underline{-\ 8} \\
5 \times 8 = & 40 \\
 & \underline{-\ 8} \\
4 \times 8 = & 32 \\
 & \underline{-\ 8} \\
3 \times 8 = & 24 \\
 & \underline{-\ 8} \\
2 \times 8 = & 16 \\
 & \underline{-\ 8} \\
1 \times 8 = & 8 \\
 & \underline{-\ 8} \\
0 \times 8 = & 0
\end{array}$$

Ask the children to help record the fact that one product in each pair is 8 less than the other.

Adapt the procedure described in this activity to products that have 9 as a factor—from 0×9 through 10×9.

▶ Draw on the chalkboard two segments of the number line that are marked at 3-inch intervals. Show 0 through 50 on the first segment and 51 through 100 on the second segment. Have individual children come to the board and point out the counts for the products that you describe to them. You may wish to begin the activity with the following descriptions:

the product that is 1×8 greater than 4×8,
the product that is 1×8 less than 7×8,
the product that is 1×9 greater than 9×9,
the product that is 1×9 less than 1×9,
the product that is 1×9 greater than 1×9,
the product that is 1×8 less than 10×8.

Use products that have either 7, 8, or 9 as a factor; include 0×8 through 10×8 and 0×9 through 10×9.

▶ Construct a large multiplication table on a piece of chart paper and mount it on an easel. Tell the children that you are going to record all the counts for products through 100 with their help. Point to the square with the times sign in it and review the meaning of the symbol. Point to the numerals across the top of the chart; remind the children that this is the first row of the table. Now point to the numerals on the left side of the chart. Remind the children that this is the first column of the table.

Explain that first you would like to record counts for products through 5 × 10. Let individuals tell you the numbers that belong in each square as you point to them at random. Fill in the first 6 rows.

×	0	1	2	3	4	5	6	7	8	9	10
0	0×0 0	0×1 0	0×2 0	0×3 0	0×4 0	0×5 0	0×6 0	0×7 0	0×8 0	0×9 0	0×10 0
1	1×0 0	1×1 1	1×2 2	1×3 3	1×4 4	1×5 5	1×6 6	1×7 7	1×8 8	1×9 9	1×10 10
2	2×0 0	2×1 2	2×2 4	2×3 6	2×4 8	2×5 10	2×6 12	2×7 14	2×8 16	2×9 18	2×10 20
3	3×0 0	3×1 3	3×2 6	3×3 9	3×4 12	3×5 15	3×6 18	3×7 21	3×8 24	3×9 27	3×10 30
4	4×0 0	4×1 4	4×2 8	4×3 12	4×4 16	4×5 20	4×6 24	4×7 28	4×8 32	4×9 36	4×10 40
5	5×0 0	5×1 5	5×2 10	5×3 15	5×4 20	5×5 25	5×6 30	5×7 35	5×8 40	5×9 45	5×10 50
6	6×0	6×1	6×2	6×3	6×4	6×5	6×6	6×7	6×8	6×9	6×10
7	7×0	7×1	7×2	7×3	7×4	7×5	7×6	7×7	7×8	7×9	7×10
8	8×0	8×1	8×2	8×3	8×4	8×5	8×6	8×7	8×8	8×9	8×10
9	9×0	9×1	9×2	9×3	9×4	9×5	9×6	9×7	9×8	9×9	9×10
10	10×0	10×1	10×2	10×3	10×4	10×5	10×6	10×7	10×8	10×9	10×10

Then instruct the children to tell you where to record the count for the commuted form of each product recorded. The children will note that many already have been recorded in the first 66 entries.

×	0	1	2	3	4	5	6	7	8	9	10
0	0×0 0	0×1 0	0×2 0	0×3 0	0×4 0	0×5 0	0×6 0	0×7 0	0×8 0	0×9 0	0×10 0
1	1×0 0	1×1 1	1×2 2	1×3 3	1×4 4	1×5 5	1×6 6	1×7 7	1×8 8	1×9 9	1×10 10
2	2×0 0	2×1 2	2×2 4	2×3 6	2×4 8	2×5 10	2×6 12	2×7 14	2×8 16	2×9 18	2×10 20
3	3×0 0	3×1 3	3×2 6	3×3 9	3×4 12	3×5 15	3×6 18	3×7 21	3×8 24	3×9 27	3×10 30
4	4×0 0	4×1 4	4×2 8	4×3 12	4×4 16	4×5 20	4×6 24	4×7 28	4×8 32	4×9 36	4×10 40
5	5×0 0	5×1 5	5×2 10	5×3 15	5×4 20	5×5 25	5×6 30	5×7 35	5×8 40	5×9 45	5×10 50
6	6×0 0	6×1 6	6×2 12	6×3 18	6×4 24	6×5 30	6×6	6×7	6×8	6×9	6×10
7	7×0 0	7×1 7	7×2 14	7×3 21	7×4 28	7×5 35	7×6	7×7	7×8	7×9	7×10
8	8×0 0	8×1 8	8×2 16	8×3 24	8×4 32	8×5 40	8×6	8×7	8×8	8×9	8×10
9	9×0 0	9×1 9	9×2 18	9×3 27	9×4 36	9×5 45	9×6	9×7	9×8	9×9	9×10
10	10×0 0	10×1 10	10×2 20	10×3 30	10×4 40	10×5 50	10×6	10×7	10×8	10×9	10×10

Have volunteers compute the counts needed to complete the seventh row. Next guide the children to complete the table by using the following steps:

Record the count for the commuted form of each product from 6 × 6 through 10 × 6.

Complete the remainder of the eighth row in the body of the table.

Record the count for the commuted form of each product from 7 × 7 through 10 × 7.

Complete the remainder of the ninth row in the body of the table.

Record the count for the commuted form of each product from 8 × 8 through 10 × 8.

Complete the remainder of the tenth row in the body of the table.

Record the count for the commuted form of each product from 9 × 9 through 10 × 9.

Complete the remainder of the eleventh row in the body of the table.

▶ Tell a child to name a product no greater than 100 and then to call on another child to give the count for this product. Direct the second child to use the multiplication table to check his answer. Have him place a pointer on the first factor of the product (in the first column) and another pointer on the second factor (in the top row). Have him move the pointers, one down and one across, until they meet at the square containing the count for the product of these two factors. Continue in this way with other products from 0 × 0 through 10 × 10.

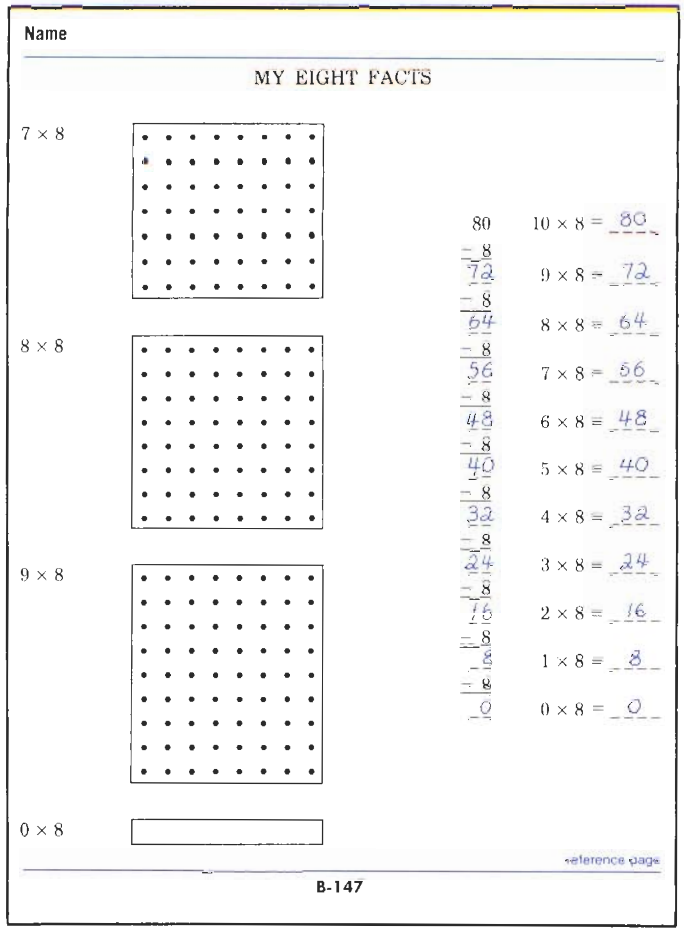
Pages 146 through 152

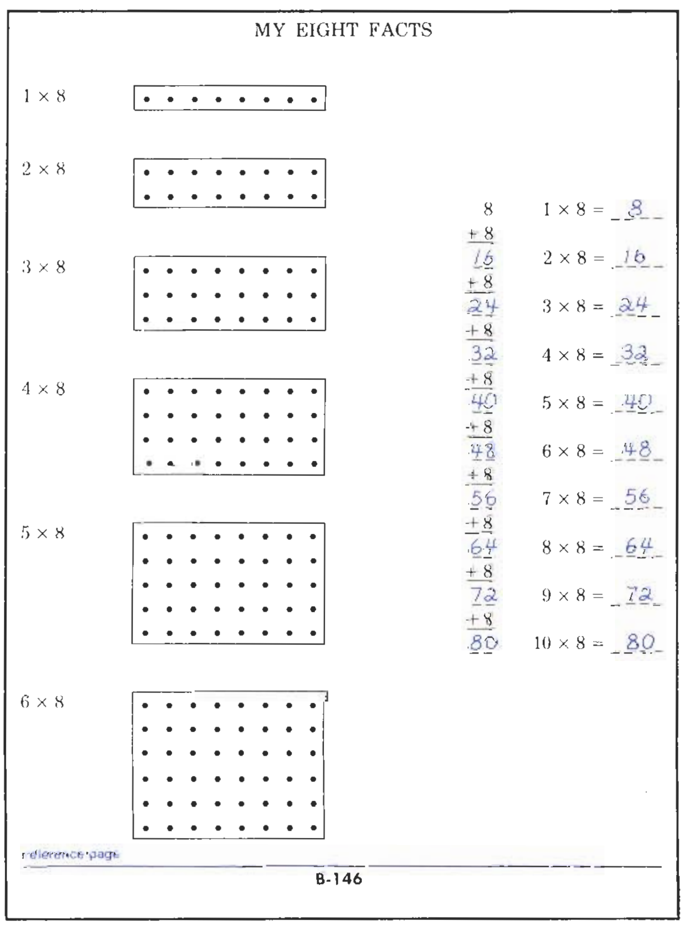
● Pages 146 through 149 provide an opportunity for the children to organize products that have 8 as a factor (from 0 × 8 through 10 × 8) and products that have 9 as a factor (from 0 × 9 through 10 × 9). Tell the children to complete the repeated additions or subtractions. Direct the children to complete the equation beside each of these additions or subtractions. Ask the children to look for patterns and discuss what they observe.

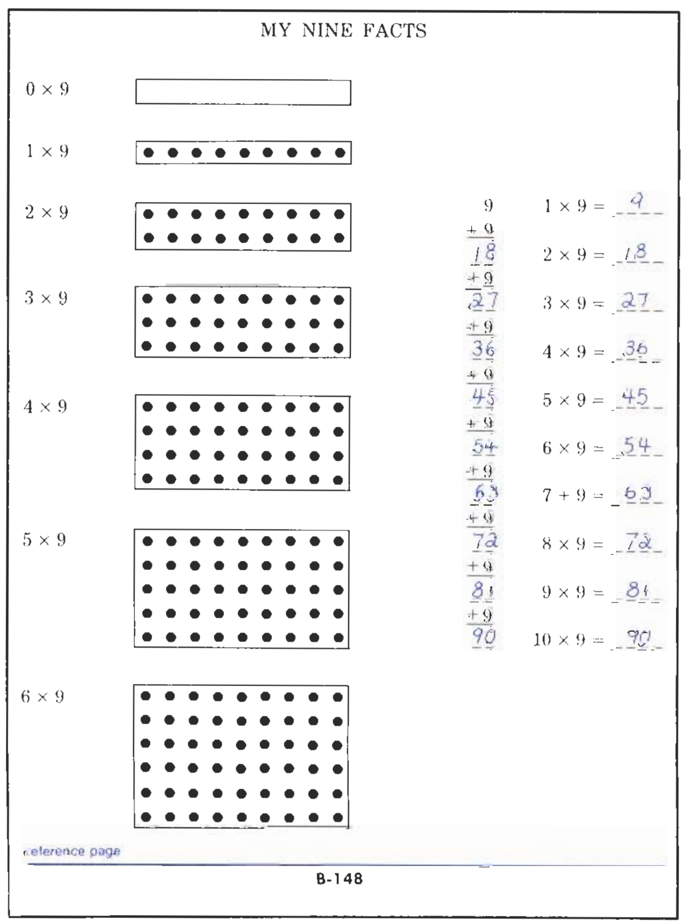
● Page 150 is designed to reinforce the children's understanding of computing products in parts—their understanding of the distributive property. This page may be assigned as independent work or as a group activity. If necessary, give each child two pieces of yarn to partition the array at the top of the page. In this way the children may better visualize each situation. If the page is assigned as independent work, have the children discuss the exercises when they have completed the work on the page. If the page is assigned as a group activity, each equation will be discussed as it is completed.

● Page 151 provides a variation in computing products. For some exercises the children must determine the count for a given product. In some cases the children must determine how many eights or how many nines a given count is. With other exercises the children must determine the difference between a particular count and a given product. Work an example of each type of exercise with the class before assigning the exercises.

● Page 152 provides further practice in computing products that have 0 through 9 as one of the factors. It is not necessary to complete this page in one class period.



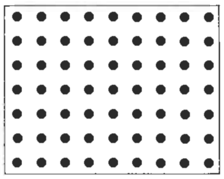




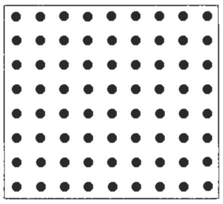
Name

MY NINE FACTS

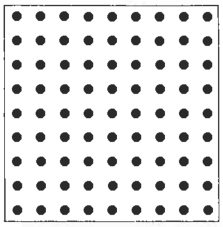
7 × 9



8 × 9



9 × 9



90 − 9 = 81	10 × 9 = 90
81 − 9 = 72	9 × 9 = 81
72 − 9 = 63	8 × 9 = 72
63 − 9 = 54	7 × 9 = 63
54 − 9 = 45	6 × 9 = 54
45 − 9 = 36	5 × 9 = 45
36 − 9 = 27	4 × 9 = 36
27 − 9 = 18	3 × 9 = 27
18 − 9 = 9	2 × 9 = 18
9 − 9 = 0	1 × 9 = 9
	0 × 9 = 0

reference page

B-149

Name

Complete.

1. 9 × 9 = 81
   8 × 9 = 72
   7 × 9 = 63

2. (8 × 8) + 17 = 81
   (8 × 8) + 8 = 72
   (8 × 8) − 1 = 63

3. 64 = 8 × 8
   56 = 8 × 7
   48 = 8 × 6

4. (7 × 7) + 15 = 64
   (7 × 7) + 7 = 56
   (7 × 7) − 1 = 48

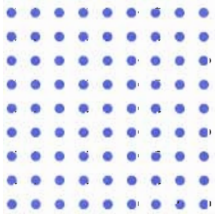
5. 7 × 7 = 49
   6 × 7 = 42
   5 × 7 = 35

6. (6 × 6) + 13 = 49
   (6 × 6) + 6 = 42
   (6 × 6) − 1 = 35

7. 54 = 6 × 9
   45 = 5 × 9
   36 = 4 × 9

8. (8 × 7) − 2 = 54
   (8 × 7) − 11 = 45
   (8 × 7) − 20 = 36

B-151



Complete.

1. 9 × 9 = (4 + 5) × (3 + 6)
2. 9 × 9 = (6 + 3) × (2 + 7)
3. 9 × 9 = (7 + 2) × (2 + 7)*
4. 9 × 9 = (4 + 5) × (2 + 5 + 2)*
5. 9 × 9 = 12 + 24 + 15 + 30 *
6. (4 × 3) + (5 × 3) + (4 × 6) + (5 × 6) = 81*
7. 81 = (6 + 3) × (5 + 4)
8. 81 = (2 + 7) × (2 + 3 + 4)
9. 81 = (8 + 1) × (8 + 1)*
10. 81 = 30 + 24 + 15 + 12*

reference page *Answers will vary

B-150

Compute.

1.	6 × 9 = 54	7 × 8 = 56	9 × 1 = 9	8 × 6 = 48	2 × 9 = 18
2.	8 × 8 = 64	9 × 2 = 18	4 × 9 = 36	5 × 8 = 40	9 × 8 = 72
3.	6 × 7 = 42	3 × 9 = 27	1 × 8 = 8	8 × 7 = 56	9 × 3 = 27
4.	9 × 5 = 45	8 × 2 = 16	4 × 7 = 28	8 × 9 = 72	4 × 8 = 32
5.	0 × 9 = 0	8 × 5 = 40	6 × 8 = 48	7 × 3 = 21	9 × 4 = 36
6.	8 × 3 = 24	7 × 7 = 49	8 × 4 = 32	9 × 9 = 81	3 × 8 = 24
7.	6 × 4 = 24	2 × 8 = 16	5 × 9 = 45	9 × 6 = 54	8 × 1 = 8
8.	9 × 4 = 36	5 × 6 = 30	3 × 7 = 21	1 × 9 = 9	7 × 5 = 35
9.	6 × 3 = 18	0 × 8 = 0	5 × 7 = 35	6 × 6 = 36	7 × 9 = 63

reference page

B-152

Supplemental Experience

■ Write 5×8 on the chalkboard. Write three more products on each side of the original product. Two of the products in each set of three should be equal to 5×8.

6×7		20×2
8×5	5×8	9×4
10×4		4×10

Ask two children to go to the board. Assign a set of products to each child. Tell these children that at a given signal they are to underline all of the products in their list that equal 5×8. Explain that you wish to see if they can find all of the appropriate products.

Continue the activity; use a different product in the center of the board for each round of the game. Vary the number of products that are equal to the given product: sometimes all three of the products should fall into this classification; at other times only two products, one product, or zero products should fall into this classification.

UNIT 11

NUMERATION: THOUSANDS

Pages 153 Through 172

OBJECTIVE

To introduce a period-value system of numeration to the child.

In this unit the child discovers a way of interpreting numerals for large numbers. He will find that the use of periods rather than places (positions of digits) helps him read numerals for large numbers. He will discover that this period-value system of numeration has a base 1000, whereas our place-value system has a base 10. The value of every period is 1000 times the value of the period to its right.

The pupil will also practice computing sums and differences of thousands and millions and products of tens, hundreds, and thousands.

See Key Topics in Mathematics for the Primary Teacher: Numeration.

KEY IDEAS

A thousand is TEN ten tens. Ten thousand is a hundred hundreds. A million is a hundred hundred hundreds. A million is a thousand thousands.

We can add and subtract thousand thousands.

Ones times tens is tens. Tens times tens is ten tens or hundreds. Tens times hundreds is ten ten tens or thousands. Hundreds times hundreds is TEN ten TEN tens or ten thousands.

CONCEPTS

expanded period form
period value
periods
thousand thousand thousands (billion)
thousand thousands (million)

KEY IDEA

A thousand is TEN ten tens. Ten thousand is a hundred hundreds. A million is a hundred hundred hundreds. A million is a thousand thousands.

Scope

To strengthen and extend the children's understanding of place-value concepts.

To introduce the meaning of a thousand thousands.

To introduce the use of the comma in period numeration.

To develop the concept of period value.

To provide practice in writing numbers in expanded period form.

Fundamentals

By this time a third-grade child can count thousands —1 thousand, 2 thousands, 3 thousands, . . . 10 thousands. He is aware of the fact that the standard numeral for thousands is formed by placing three zeros to the right of the numeral that tells the number of thousands named—1000, 2000, 3000, . . . 10,000.

Let's look at the numeral 1000. When counting hundreds, 1000 is 10 hundreds. When counting tens, 1000 is 100 tens. When counting thousands, 1000 is 1 thousand.

The interpretation of numerals for large numbers is aided by the use of periods. The period-value system has a base 1000. (Our place-value system has a base 10.) Periods are indicated by commas that separate sets of three digits. Each set of three digits is a period.

Let's examine the numeral 26,502. In our period-value system 26,502 names 26 thousands and 502 ones. Each period in a numeral has a name, just as each place has a name. The first period—going from right to left—is the ones period, and the second period is the thousands period. Note that the thousands period has a value 1000 times that of the period to its right—the ones period.

What is the period value of the period to the left of the thousands period? In our period-value system, the value of each period is 1000 times the value of the period to its right. Thus, the third period in a numeral is the thousand thousands or millions period.

What is the standard numeral for 1000 thousands? The child knows that he can write numerals for thousands by simply writing three zeros to the right of the numeral that indicates how many thousands are named. This enables him to recognize that 1,000,000 is the numeral for 1000 thousands. One thousand thousands is 1 million. The period to the left of the thousands period is the millions period.

How does the use of periods help in the interpretation of numerals for large numbers? Examine the numeral 36500002. When commas are placed in the numeral to separate periods of three digits—36,500,002—the child can easily see that this numeral contains three periods: ones, thousands, and millions (thousand thousands). Using this information he can see that this is the numeral for 36 millions, 500 thousands, and 2 ones. The numeral is read, "Thirty-six million, five hundred thousand, two."

There is a special name associated with each period in a numeral. The child will work with the four names that are most familiar.

1	one
1,000	thousand
1,000,000	million (thousand thousands)
1,000,000,000	billion (thousand millions)

When the child understands the period-value system of numeration, he will be able to write large numbers in expanded period form.

635,781,249
= 635 millions + 781 thousands + 249 ones
= 635,000,000 + 781,000 + 249

The standard numeral for thousands is formed by placing three zeros to the right of the numeral that indicates the number of thousands. The standard numeral for thousand thousands (millions) is formed by placing three zeros to the right of the numeral that indicates the number of thousands.

The numeral for 635 thousand thousands (635 millions) is 635,000 thousands or 635,000,000. The use of periods instead of the individual place value of each digit makes the child's task easier since the number of periods involved in most of the numerals are few.

billions	millions	thousands	ones
7,	004,	206,	320

The use of the period-value system of numeration naturally leads to the question: "What is the period to the left of billions?" The children will enjoy investigating to find the answer to this question.

Readiness for Understanding

An understanding of place-value concepts.

Ability to recognize numerals from 0 through 18,000.

An understanding of numerals from 0 through 18,000.

Developmental Experiences

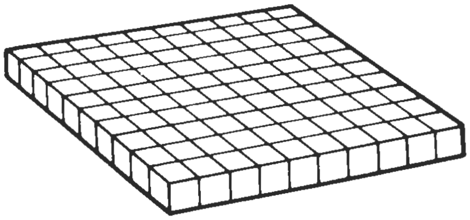
for flannel board	1000 one-inch wooden cubes
5 sets of felt numerals (0 through 9)	masking tape
2 felt commas	200 paper arrays (5″ × 10″)
tagboard cards	scissors
latticework frame (3 openings)	sheets of tagboard (18″ × 24″)
	4 felt-tip pens
	strips of paper
	box

▶ You may find it necessary to spend several days on this introductory activity.

Gather 1000 one-inch wooden cubes. (These cubes were used in a previous unit.) Write on the chalkboard the question, "What is one hundred?" Allow several children to present their ideas; record their responses on the chalkboard. (Some words are italicized in the following example to indicate where you should place the accent when you read the sentence to the children.)

What is one hundred?
One hundred is *ten* tens.
One hundred is *one hundred* ones.
One hundred is *one* one hundred.

Ask two children to use the blocks to build a 10 by 10 array on a table in the front of the room. Ask which sentence on the chalkboard describes this array. (One hundred is *ten* tens.)



Then write on the chalkboard, "What is one thousand?" There are several responses that the children might give to this question. Record them on the chalkboard.

What is one thousand?
One thousand is *ten* ten tens.
One thousand is *ten* one hundreds.
One thousand is *one hundred* tens.
One thousand is *one* one thousand.
One thousand is *one thousand* ones.

Discuss each of these ways to think of 1000; then ask the children if the 10 by 10 array of blocks could be used to build an array of 1000 blocks. Some child may suggest that since 1000 is ten ten tens, ten of the 10 by 10 arrays on the table would be 1000 blocks. Choose nine pairs of children to build the 10 by 10 by 10 array of blocks a layer at a time. Ask which sentence on the chalkboard describes this array. (One thousand is *ten* ten tens.)

Since the children can form a mental image of the number and have actually shown a thousand with the blocks, they should realize that a thousand is ten ten tens. Now ask the children what number of blocks is ten of the ten ten ten blocks. Knowing that ten ten tens is a thousand, a child may respond that ten of these ten ten tens is ten thousand. Write in a third column on the chalkboard a sentence that conveys this idea.

What is ten thousand?
Ten thousand is *ten* ten ten tens.

Guide the children to realize that thinking of ten thousand as a hundred hundreds may help them to form a mental picture of ten thousand. Ask the children what size array would show ten thousand. (a 100 by 100 array) To help the children understand ten thousand as a hundred hundreds, it is important that they have the experience of building this number. Since it is impractical to use blocks, the following procedure is suggested.

Duplicate two hundred 5 by 10 inch arrays and distribute them to the class. Instruct the children to carefully cut out each array. Then show them how to tape two arrays together to form a 10 by 10 inch array. When all of the arrays are complete, ask the children how many squares are in each array. (100) Write the statement that ten thousand is a hundred hundreds below the other sentence on the board.

Ten thousand is *ten* ten ten tens.
Ten thousand is a *hundred* hundreds.

Tell the class that you want them to help make a display that will show ten thousand in terms of a hundred hundreds. Ask a child to bring his arrays to a large section of the wall. Show him how to place rings of masking tape on the back of each array and help him place these, one next to the other, on a section of the wall. Ask other children to continue this procedure until there is a row of ten arrays on the wall. Ask the children how many 1-inch squares they can see. (10 hundreds) Ask them how they can show 100 hundreds. They should realize that by making nine more rows of 10 hundreds, they will have ten ten hundreds, or a hundred hundreds.

Use the picture of ten thousand that the children have constructed to develop an idea of million. Ask what number would represent a hundred of these arrays. One response may be a hundred ten thousands. Another child may reason that since ten thousand is a hundred hundreds, a hundred of the ten thousands would be a hundred hundred hundreds. Someone else may say that this is ten ten ten ten ten tens or a thousand thousands. Introduce the word million as another name for this number and, in a fourth column on the chalkboard, write sentences that express these ideas.

What is one million?
One million is a *thousand* thousands.
One million is *ten* ten ten ten ten tens.
One million is a *hundred* hundred hundreds.
One million is a *hundred* ten thousands.

Refer to each ten ten ten in the second sentence on the chalkboard and ask the children what ten ten ten is. When they respond that ten ten ten is a thousand, ask if this idea is used in one of the other sentences. (One million is a thousand thousands.)

Point to the 10 by 10 by 10 array of blocks that was constructed earlier. Ask the children how many of these arrays would be a million. (A thousand of the thousand blocks would be a million blocks.) Point to the 100 by 100 array on the wall. Ask the children how many of these arrays would be a million. (A hundred of the hundred hundred, or ten thousand squares would be a million.) Questions such as these improve and extend the child's conception of a million.

Finally, give each of four children an 18 by 24 inch sheet of tagboard and a felt-tip pen. Assign one of the columns on the chalkboard to each child and tell them to copy the sentences. Have the children leave about 6 inches of space between the sentences. The charts will be used in the next activity.

▶ Tape on the chalkboard the four charts made at the end of the previous activity. Tell the children that you want them to help translate each sentence on the charts into an equation. Begin with the chart titled, "What is one hundred?" Give a child a felt-tip pen and tell him to write an appropriate equation below the first sentence. Choose other children to translate the other sentences into equations and to write the equations below the appropriate sentences.

What is one hundred?

One hundred is ten tens.
$100 = 10 \times 10$
One hundred is one hundred ones.
$100 = 100 \times 1$
One hundred is one one hundred.
$100 = 1 \times 100$

Review with the children the fact that each 0 in the numeral 100 represents a factor of 10 ($100 = 10 \times 10$).

Next have the children complete the chart titled, "What is one thousand?" Ask individuals to translate each sentence into an equation.

What is one thousand?
One thousand is ten ten tens.
$1000 = 10 \times 10 \times 10$
One thousand is ten one hundreds.
$1000 = 10 \times 100$
One thousand is one hundred tens.
$1000 = 100 \times 10$
One thousand is one one thousand.
$1000 = 1 \times 1000$
One thousand is one thousand ones.
$1000 = 1000 \times 1$

The children may observe that the three zeros in 1000 stand for ten ten ten.

Before the children complete the chart titled, "What is ten thousand?" have someone read the first sentence on the chart aloud. Ask the children if the fact that ten thousand is ten ten ten tens gives them a clue about the number of zeros in the standard numeral for ten thousand. Some child may observe that ten thousand has four factors of 10; therefore, four zeros are needed to write the standard numeral—one for each factor of 10. Tell this child to translate the first sentence into an equation. Then ask another child to write an equation for the second sentence.

What is ten thousand?
Ten thousand is ten ten ten tens.
$10000 = 10 \times 10 \times 10 \times 10$
Ten thousand is a hundred hundreds.
$10000 = 100 \times 100$

At this point in the discussion some of the children may observe that there are other ways to express ten thousand. If the children do make this observation, add their other interpretations of ten thousand to the chart. Your chart should only contain the interpretations suggested by the class. If the children do not interpret ten thousand, for example, as a hundred ten tens, then do not add this interpretation to the chart.

What is ten thousand?
Ten thousand is ten ten ten tens.
$10000 = 10 \times 10 \times 10 \times 10$
Ten thousand is a hundred hundreds.
$10000 = 100 \times 100$
Ten thousand is ten one thousands.
$10000 = 10 \times 1000$
Ten thousand is one thousand tens.
$10000 = 1000 \times 10$
Ten thousand is one ten thousand.
$10000 = 1 \times 10000$
Ten thousand is ten thousand ones.
$10000 = 10000 \times 1$
Ten thousand is a hundred ten tens.
$10000 = 100 \times 10 \times 10$
Ten thousand is ten ten hundreds.
$10000 = 10 \times 10 \times 100$

Ask the children how many digits appear in the numeral for ten thousand. (five) Explain that when a numeral has five or more digits a comma is used to separate groups of three digits. Tell the children that the use of the comma is optional with a four-digit numeral. Count three digits in from the right of the numeral for ten thousand and place a comma at this point. (10,000) Ask several children to count three digits in from the right of the standard numerals for ten thousand that appears on the chart, and to place commas in the appropriate places on the chart.

What is ten thousand?

Ten thousand is ten ten ten tens.
$10{,}000 = 10 \times 10 \times 10 \times 10$

Ten thousand is a hundred hundreds.
$10{,}000 = 100 \times 100$

Ten thousand is ten one thousands.
$10{,}000 = 10 \times 1000$

Ten thousand is one thousand tens.
$10{,}000 = 1000 \times 10$

Ten thousand is one ten thousand.
$10{,}000 = 1 \times 10{,}000$

Ten thousand is ten thousand ones.
$10{,}000 = 10{,}000 \times 1$

Ten thousand is a hundred ten tens.
$10{,}000 = 100 \times 10 \times 10$

Ten thousand is ten ten hundreds.
$10{,}000 = 10 \times 10 \times 100$

Finally, tell the children to study the second answer on the chart titled, "What is one million?" Ask the children if the fact that one million is ten ten ten ten ten tens will help them write the standard numeral for one million. Some of the children may observe that one million involves six factors of 10; therefore, six zeros are needed to write the standard numeral. Direct the children to translate each sentence on this chart into an equation. Remind the children to use a comma to set off each set of three digits in any numeral that has five or more digits.

What is one million?

One million is a thousand thousands.
$1{,}000{,}000 = 1000 \times 1000$

One million is ten ten ten ten ten tens.
$1{,}000{,}000 = 10 \times 10 \times 10 \times 10 \times 10 \times 10$

One million is a hundred hundred hundreds.
$1{,}000{,}000 = 100 \times 100 \times 100$

One million is a hundred ten thousands.
$1{,}000{,}000 = 100 \times 10{,}000$

At this point in the discussion, some of the children may observe that there are other ways to express one million. If the children make this observation, add their interpretations of one million to the chart. Include only those interpretations suggested by the class. Several possibilities are included in the following illustration.

What is one million?

One million is a thousand thousands.
$1{,}000{,}000 = 1000 \times 1000$

One million is ten ten ten ten ten tens.
$1{,}000{,}000 = 10 \times 10 \times 10 \times 10 \times 10 \times 10$

One million is a hundred hundred hundreds.
$1{,}000{,}000 = 100 \times 100 \times 100$

One million is a hundred ten thousands.
$1{,}000{,}000 = 100 \times 10{,}000$

One million is a hundred ten ten ten tens.
$1{,}000{,}000 = 100 \times 10 \times 10 \times 10 \times 10$

One million is ten thousand hundreds.
$1{,}000{,}000 = 10{,}000 \times 100$

One million is one million ones.
$1{,}000{,}000 = 1{,}000{,}000 \times 1$

The charts developed in this activity should be posted in the classroom so that the children may refer to them when necessary.

▶ Place across the top of the flannel board five sets of felt numerals for 0 through 9 and two felt commas. Use the felt numerals to make the following numeral in the center of the flannel board.

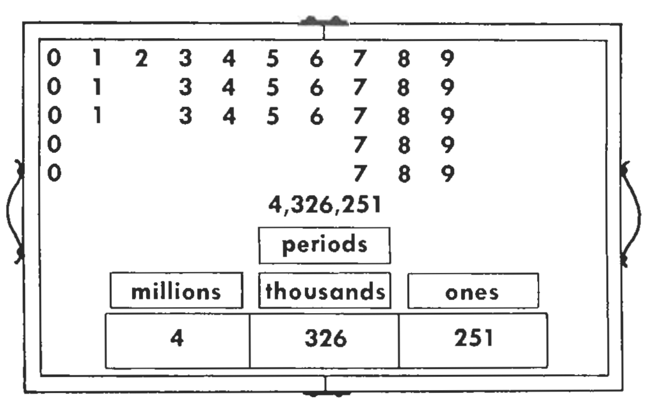
4326251

Remind the children that when we write such large numerals, we separate sets of three digits with commas. Help a child count from right to left and place felt commas in the appropriate places in the numeral.

4,326,251

Tell the children that each set of three digits is called a period. Explain that each period in the numeral has a value just as each place in the numeral has a value. Point to each period from right to left and explain its value—the first period of three digits is the ones period, the second period is the thousands period, and the third period is the thousand thousands or the millions period.

Write on four tagboard cards the words *periods*, *ones*, *thousands*, and *millions*. Use them to label the parts of a latticework frame (3 openings) that you place below the numeral on the flannel board. Then help a child place felt numerals in the frame to show 4,326,251 in terms of periods.



Explain to the children that thinking of a numeral in terms of periods will help them to read the numeral and to understand the meaning of the numeral. Tell the children that when this numeral is read the millions period is named and the thousands period is named, but it is not necessary to name the ones period. As you read the numeral for the class, point to the appropriate parts of the numeral on the flannel board, "Four million, three hundred twenty-six thousand, two hundred fifty-one." Then have the children read the numeral aloud.

Follow a similar procedure with other four-digit through nine-digit numerals. In each instance, one child should place commas in the appropriate places (commas are optional with four digits), a second child should show the meaning of the numeral in terms of periods, and a third child should read the numeral to the class.

▶ Review the names of the first three periods: ones, thousands, millions. Then write on the chalkboard the following equation.

706,243,658
= ___ millions + ___ thousands + ___ ones
= ___ + ___ + ___

Point to the numeral and ask a child to read it for the class: "Seven hundred six millions, two hundred forty-three thousand, six hundred fifty-eight." Ask another child to come to the board to write in the first line of the equation the numeral for the number of millions. Then ask a third child to write the number of thousands and a fourth child to write the number of ones. Point to the chart titled "What is a million?"; ask how many zeros are needed to show millions. (6) Direct a child to write in the second line of the equation the standard numeral for 706 millions.

706,243,658
= 706 millions + 243 thousands + 658 ones
= 706,000,000 + ___ + ___

Help the children record the numerals for thousands and ones in this same manner. Encourage them to refer to the charts if necessary.

706,243,658
= 706 millions + 243 thousands + 658 ones
= 706,000,000 + 243,000 + 658

Continue the activity. Have the children write the expanded period form for other numbers.

6032
27,306
790,006
582,036,106

▶ Write on strips of paper four-digit through nine-digit numbers, for example: three hundred seven million, eight hundred sixty-five thousand, six hundred. Place these strips in a box. Direct a child to take one of the strips out of the box and to write on the chalkboard the standard numeral for the given number.

307,865,600

Direct a second child to write the given number in expanded period form.

307,865,600
= 307 millions + 865 thousands + 600 ones
= 307,000,000 + 865,000 + 600

Ask a third child to read the given number to the class.

Follow a similar procedure with the other numerals. Ask the class to check whether or not each response is correct. Continue until every child has had an opportunity to follow at least one of the directives.

Pages 153 through 160

● Use page 153 for class discussion. After the children have examined the illustration and read the story, have them discuss the questions at the bottom of the page. If the children do not know another name for a thousand thousands or a thousand thousand thousands, introduce the words *million* and *billion* respectively. Tell the children that a million is a thousand thousands; then ask the class how many millions would be the same as a billion. (a thousand millions) Write the question, "What is one billion?" at the top of a sheet of tagboard and ask the children to suggest answers to this question. List their responses on the chart. Several possibilities are shown in the following illustration.

What is one billion?
One billion is a thousand thousand thousands.
1,000,000,000 = 1000 × 1000 × 1000
One billion is ten ten ten ten ten ten ten ten tens.
1,000,000,000 = 10 × 10 × 10 × 10 × 10 × 10 × 10 × 10 × 10
One billion is a thousand millions.
1,000,000,000 = 1000 × 1,000,000
One billion is a hundred hundred hundred thousands.
1,000,000,000 = 100 × 100 × 100 × 1000
One billion is ten hundred millons.
1,000,000,000 = 10 × 100 × 1,000,000

Next, ask a child to write on the chalkboard the names of the first four periods. Refer to ones and ask how many of the ones are a thousand. (1000) Then ask how many thousands are a million (1000) and how many millions are a billion (1000). Show the answers as illustrated below.

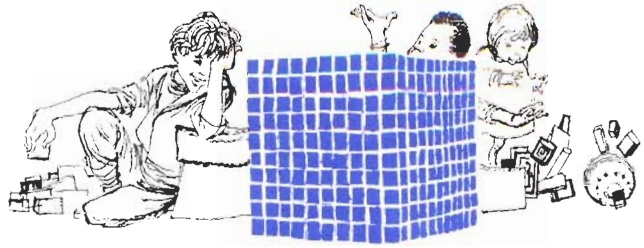
× 1000 × 1000 × 1000
Billions ← Millions ← Thousands ← Ones

● Use page 154 for class discussion. This page is designed to give the children additional experience with tens, hundreds, thousands, and millions. Have the children record their responses as each question is discussed.

Name

UNIT 11 NUMERATION: THOUSANDS

For Class Discussion



Bill and Dean built an array of blocks. "How would you like to build a thousand arrays like that?" asked Bill.

"That would take a thousand thousand blocks!" said Dean.

Bill replied, "How many blocks would we need to build a thousand thousand of the arrays?"

How many blocks did Bill and Dean use to build the array? *

How many blocks would they need for a thousand arrays? *

How many blocks would they need for a thousand thousand of the arrays? *

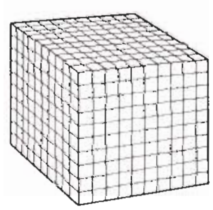
What other names do you know for a thousand? a thousand thousand? * a thousand thousand thousand?

*See pupil page suggestions.

reference page

C-153

For Class Discussion



One thousand

1000

One million is one thousand thousands.

1,000,000

1000 × 1000

How many is one thousand?

10	ten tens	1000 =	10 × 10 × 10
100	tens	1000 =	100 × 10
10	hundreds	1000 =	10 × 100

How many is one million?

10	ten ten ten ten tens	1,000,000 =	10 × 10 × 10 × 10 × 10 × 10
100	hundred hundreds	1,000,000 =	100 × 100 × 100
1000	thousands	1,000,000 =	1000 × 1000
100	ten thousands	1,000,000 =	100 × 10,000

reference page

C-154

Name

One thousand thousands is one million.

$1{,}000{,}000 = 1000 \times 1000$

Write the standard numeral for each number.

1. 2000 thousands 2,000,000
2. 50 thousand thousands 50,000,000
3. 2 millions 2,000,000
4. 783 millions 783,000,000
5. 13 thousands 13,000
6. 2672 thousands 2,672,000
7. 13000 thousands 13,000,000
8. 478 thousands 478,000
9. 98 thousands 98,000
10. 9999 millions 9,999,000,000
11. 98 millions 98,000,000
12. 47 thousand thousands 47,000,000
13. 162 thousands 162,000
14. 491 thousand thousands 491,000,000

reference page

C-155

For Class Discussion

The digits in a numeral are separated into periods of three digits. Each period has a period value.

7 2 5	,	3 4 1	,	6 8 2
millions		thousands		ones

In expanded period form,

725,341,682 = 725 millions + 341 thousands + 682 ones
= 725,000,000 + 341,000 + 682

1,058,001 = 1 million + 58 thousands + 1 one
= 1,000,000 + 58,000 + 001

Write each number in expanded period form.

1. 7501 = *7 thousands + 501 ones*
= *7000 + 501*
2. 64,999 = 64 thousands + 999 ones
= 64,000 + 999
3. 450,003 = 450 thousands + 3 ones
= 450,000 + 3
4. 7,961,397 = 7 millions + 961 thousands + 397 ones
= 7,000,000 + 961,000 + 397

reference page

C-156

● Page 155 further emphasizes the idea of thousands. Discuss the interpretation of one million as one thousand thousands. Be sure the children realize that these terms are synonymous—one million is a thousand thousands. The children should be free to use either term.

Work exercises 1 and 2 with the class. Then direct the children to complete the other exercises independently. After the page has been completed, let different children write the numerals on the chalkboard. Discuss the fact that although the answer to exercise 4, for example, may be read as seven hundred eighty-three million or as seven hundred eighty-three thousand thousands, the more common way of reading it is seven hundred eighty-three million.

● Pages 156 and 157 allow the children to practice writing numbers in expanded period form. Discuss the example at the top of page 156. Be sure the children understand the idea of separating a numeral into periods as well as the placement of the comma between the periods. Complete the first exercise with the class. Then have the children read aloud each numeral in the remaining exercises before you assign the independent work. When the class has completed the assignment, have various children write on the chalkboard a response for some of the exercises.

Name

millions		thousands		ones
8	,	4 3 5	,	1 4 2

Write each number in expanded period form.

1. 5,875,041 5 millions + 875 thousands + 41 ones
5,000,000 + 875,000 + 41
2. 20,651,932 20 millions + 651 thousands + 932 ones
20,000,000 + 651,000 + 932
3. 798,004,248 798 millions + 4 thousands + 248 ones
798,000,000 + 4,000 + 248
4. 807,652 807 thousands + 652 ones
807,000 + 652
5. 100,001,999 100 millions + 1 thousand + 999 ones
100,000,000 + 1,000 + 999
6. 11,111,001 11 millions + 111 thousands + 1 one
11,000,000 + 111,000 + 1
7. 327,087,020 327 millions + 87 thousands + 20 ones
327,000,000 + 87,000 + 20

reference page

C-157

● Page 158 provides additional experience with concepts that have been developed in this section. When discussing the first exercise, the children should be aware that they are to write each numeral using commas to show periods and then to write each number in expanded period form. Since we read from left to right, it is only natural that some children will attempt at first to insert commas for each set of three digits going from left to right. As the children complete the remaining exercises independently, remind them to work from right to left when inserting commas.

● Page 159 introduces the children to the special names associated with the next four periods after millions. This page should form the basis of a class discussion on the question, "Is there a last number?" Essentially this page should be used for fun, never for a drill or as a test. The children should not be required to learn period names beyond billion.

Discuss each section of the page with the class. Then ask the children if they can follow the pattern of each section and tell what would be in the next line. Let the children write their ideas on the chalkboard. The children may not know the special name for the period to the left of the quintillions, but some should be able to write the standard numeral and to interpret the number. If you wish, have the children investigate the dictionary to find the names for other periods.

The children may be interested in the fact that names have been invented for some very large numbers. One such number is the *googol*. The word googol was invented by a mathematician's nine-year-old nephew as a name for the number 1 followed by 100 zeros. Children are usually fascinated by information such as this.

Insert commas in each numeral to show periods.
Then write in expanded period form.

1. 1,697,002 *1 million + 697 thousands + 2 ones*
1,000,000 + 697,000 + 2

2. 3 2 6,1 0 0 326 thousands + 100 ones
326,000 + 100

3. 4 0,8 9 9 40 thousands + 899 ones
40,000 + 899

4. 1 3,4 5 6,7 8 9 13 millions + 456 thousands + 789 ones
13,000,000 + 456,000 + 789

5. 4,2 3 7,9 3 5 4 millions + 237 thousands + 935 ones
4,000,000 + 237,000 + 935

6. 7 8,0 5 6 78 thousands + 56 ones
78,000 + 56

7. 5 4,3 6 7,8 1 4 54 millions + 367 thousands + 814 ones
54,000,000 + 367,000 + 814

reference page

C-158

Name

For Class Discussion

Is there a last number?

One	1
Thousand	1,000
Million	1,000,000
Billion	1,000,000,000
Trillion	1,000,000,000,000
Quadrillion	1,000,000,000,000,000
Quintillion	1,000,000,000,000,000,000

One	
Thousand	Ten ten tens
Million	Thousand thousands
Billion	Thousand thousand thousands
Trillion	Thousand thousand thousand thousands
Quadrillion	Thousand thousand thousand thousand thousands
Quintillion	Thousand thousand thousand thousand thousand thousands

Million	Thousand thousands
Billion	Million thousands or Thousand millions
Trillion	Billion thousands or Thousand billions
Quadrillion	Trillion thousands or thousand trillions
Quintillion	Quadrillion thousands or thousand quadrillions

reference page

C-159

Write the standard numeral for each number.

1. Forty-six million, two hundred thousand, six hundred
46,200,600

2. Three thousand, sixty 3060 or 3,060

3. Two hundred seventy-one thousand, eighty-five
271,085

4. Fifty-five million, eight hundred fifteen
55,000,815

5. Seventy-nine thousand, one hundred thirty-two
79,132

6. Six million, two hundred ninety thousand, four
6,290,004

7. Eight hundred ninety thousand, one hundred twenty-one
890,121

8. Forty-two million, nine hundred eighty-eight thousand, fifty-six
42,988,056

9. One hundred fifty-eight million, four thousand, two hundred fifty
158,004,250

reference page

C-160

● Page 160 provides further practice in writing standard numerals. Assign the exercises for independent work. Then let the children show on the chalkboard how they wrote the numerals.

There are several ideas related to periods that need to be discussed with the children. For example, if only the ones period is involved must there be three digits in the numeral? No. Depending upon the number named, there may be one, two, or three digits in the ones period. For example, the standard numeral for five is 5 not 05 or 005. When only the thousands and ones periods are involved, there need not be three digits in the thousands period. Once the millions period is involved there must be three digits in the thousands period and in the ones period; but there need not be three digits in the millions period. Do not tell these ideas to the children; as the exercises are discussed, ask questions that will help the children arrive at these conclusions.

Supplemental Experiences

■ Let the children play a game. Choose one child to stand before the group to ask a question such as, "I'm thinking of nine million. Who can tell how many thousands that is?" He then calls on another child to give the correct answer (9,000 thousands) and to write this numeral on the chalkboard. If the answer is correct, the child who answered may lead the game and think of another number.

Suggest that some of the children describe their numbers in different ways. For example, one child might say, "I'm thinking of a number that is two thousand thousands more than three thousand thousands. My number is how many millions?" Another might say, "I'm thinking of a number that is one hundred forty hundred hundreds. Who can tell how many thousands?"

■ Write on a set of cards the expanded period form for four-digit through nine-digit numerals.

20,000,000 + 304,000 + 236

162,000,000 + 34,000 + 29

2,000,000 + 267,000 + 326

Construct another set that contains all of the standard numerals for the numbers in the first set. Distribute both sets of cards to the children.

Tell a child to stand, display his numeral, and say, "I'm looking for a partner." Explain that the child who has a numeral that matches the first child's numeral must stand and read his card. Then place both cards in the pocket chart with an equal sign between them. Ask a child to read the equation that is formed. Continue the game until all of the cards have been matched.

■ Play a relay game. Write a column of the standard numerals for six numbers from 100,000 to 999,999,999 on the chalkboard. Omit the commas.

Divide the class into groups of six children. Have the teams line up facing the board, and give the first member of each team a piece of chalk. At a signal the children who have chalk must walk quickly to the board, copy the first number in the column, insert commas to show periods, and then write the number in expanded period form. As each child finishes, he must hurry back to his team, pass the chalk to the next child, and go to the end of the line. The second child must walk quickly to the board, copy the second number in the column, insert commas to show periods, and write the number in expanded period form. Each team member should follow this procedure. The team that is the first to complete the list of numbers correctly wins the game.

KEY IDEA

We can add and subtract thousand thousands.

Scope

To recognize that thousands are computed in the same way that ones are computed.

To apply addition and subtraction combinations to sums of thousands and millions.

To develop in the children a greater familiarity with our period-value system of numeration.

Fundamentals

The child discovers that the basic addition and subtraction combinations he learned apply to thousands and to millions as well as to ones.

700 ones	7 thousands	7 millions
+ 800 ones	+ 8 thousands	+ 8 millions
1500 ones	15 thousands	15 millions

Later the child will learn the formal property involved, the distributive property.

In this section the child also reviews the fact that if there are not enough ones, he subtracts from a ten. He applies this understanding to computing differences in the thousands period and the millions period.

12
~~2~~~~3~~4 millions
− 47 millions
187 millions

7 millions = 10 millions − 7 millions + 4 millions
80 millions = 100 millions − 40 millions + 20 millions
100 millions = 100 millions − 0 millions

Readiness for Understanding

Understanding of the addition and subtraction algorisms.

Ability to write the numerals for the millions period, thousands period, and ones period.

Knowledge of period value.

Developmental Experiences

tagboard cards:
- 6 *millions*
- 6 *thousands*

masking tape

8 boxes

numeral-cards:
- 10,000,000 through 90,000,000
- 1,000,000 through 9,000,000
- 100,000 through 900,000
- 10,000 through 90,000
- 1000 through 9000
- 100 through 900
- 10 through 90
- 1 through 9

▶ Write the word *millions* on 6 cards and the word *thousands* on 6 cards. Fix these cards to adhere to the chalkboard and place them on the chalktray. Write on the chalkboard the sum 60 + 80 in an algorism. Use a *thousands* card to label each addend.

60 [thousands]
+ 80 [thousands]

Discuss the idea that in computing this sum the *thousands* can be ignored; the only computation necessary is of the sum 60 + 80. Ask a child to write the standard numeral for the sum 60 + 80 on the chalkboard. Then tell him to select from the chalktray a card that shows the relation of the sum to the addends. Instruct him to place this card to the right of 140.

60 [thousands]
+ 80 [thousands]
140 [thousands]

Have another child read aloud the numeral for the sum: one hundred forty thousands. Ask the children how many periods are involved when they are working with thousands. (Two periods are involved: the ones period and the thousands period.) Ask them how many digits must be in the ones period when thousands are involved. (3)

Direct a child to come to the chalkboard, to remove the card to the right of 60, and to write the digits needed to show the standard numeral for 60 thousands. Remind him to use commas to separate the periods. Let two other children follow a similar procedure with 80 thousands and 140 thousands.

60,000
+ 80,000
140,000

Next, erase the comma and zeros to the right of 60, 80, and 140 and label each numeral with a *millions* card.

60 [millions]
+ 80 [millions]
140 [millions]

Ask a child to read aloud the numeral for the sum: one hundred forty millions. Ask the children how many periods are involved when they are working with millions. (Three periods are involved: the ones period, the thousands period, and the millions period.) Ask the children how many digits must be in the thousands period and in the ones period when millions are involved. (3 digits in each period) Direct a child to come to the chalkboard, to remove the card to the right of 60, and to write the digits needed to show the standard numeral for 60 millions. Remind him to use commas to separate the periods. Allow two other children to follow a similar procedure with 80 millions and 140 millions.

60,000,000
+ 80,000,000
140,000,000

Continue in this way with sums and differences such as these:

$\begin{array}{r}7000\\-\ 4000\\\hline\end{array}$	and	$\begin{array}{r}7,000,000\\-\ 4,000,000\\\hline\end{array}$
$\begin{array}{r}9000\\+\ 7000\\\hline\end{array}$	and	$\begin{array}{r}9,000,000\\+\ 7,000,000\\\hline\end{array}$
$\begin{array}{r}70,000\\-\ 20,000\\\hline\end{array}$	and	$\begin{array}{r}70,000,000\\-\ 20,000,000\\\hline\end{array}$
$\begin{array}{r}50,000\\+\ 60,000\\\hline\end{array}$	and	$\begin{array}{r}50,000,000\\+\ 60,000,000\\\hline\end{array}$
$\begin{array}{r}800,000\\-\ 500,000\\\hline\end{array}$	and	$\begin{array}{r}800,000,000\\-\ 500,000,000\\\hline\end{array}$
$\begin{array}{r}400,000\\+\ 200,000\\\hline\end{array}$	and	$\begin{array}{r}400,000,000\\+\ 200,000,000\\\hline\end{array}$
$\begin{array}{r}1,300,000\\-\ \ 600,000\\\hline\end{array}$	and	$\begin{array}{r}1,300,000,000\\-\ \ 600,000,000\\\hline\end{array}$
$\begin{array}{r}900,000\\+\ 500,000\\\hline\end{array}$	and	$\begin{array}{r}900,000,000\\+\ 500,000,000\\\hline\end{array}$

▶ Have available numeral-cards for 10,000,000 through 90,000,000; 1,000,000 through 9,000,000; 100,000 through 900,000; 10,000 through 90,000; 1000 through 9000; 100 through 900; 10 through 90; and 1 through 9. Place each set of cards in a separate box.

Place on a table in front of the room only those boxes that contain cards for thousands and millions. Tell a child to draw out two cards from one of the boxes; direct him to read the numerals on the cards, compute the sum, and tell the class the numeral for the sum. He may compute the sum mentally or do his computation on the chalkboard.

Continue in this way until the children have computed sums with all of the numerals in each of the boxes.

Now place on the table the boxes that contain cards for ones. Explain to the children that this time they are to draw out two cards, but each card must come from a different box. They are to read the numerals on the cards, compute the sum mentally or on the chalkboard, and give the result.

Continue in this way until each child has an opportunity to participate in the activity.

As a final variation, have the children draw three or four cards; each card must come from a different box. They are to read the numerals on the cards, compute the sum mentally or on the chalkboard, and give the result.

▶ Write on the chalkboard the difference 662,000,000 − 367,000,000 in vertical form. Ask how many periods are involved. (3) Label each group of three digits as the children identify the periods.

Millions Thousands Ones

$$\begin{array}{r}662,000,000\\-367,000,000\\\hline\end{array}$$

Have the class read the difference aloud: six hundred sixty-two million minus three hundred sixty-seven million. Help the children realize that they may ignore the fact that millions are involved; the computation of this difference is the same as if the exercise were 662 − 367. Explain to the children that since only millions are involved, they can compute the millions first and then write in the appropriate number of zeros to indicate the thousands period and the ones period. Direct a child to come to the chalkboard and compute aloud the difference 662 − 367. He may describe his computation in the following way:

First, 7 ones are to be subtracted. Since there are not enough ones, 7 ones will be subtracted from a ten. This leaves 5 tens in the tens place. Ten minus 7 is 3; the 3 ones added to the 2 ones gives 5 ones.

$$\begin{array}{r}5\\6\not{6}2,000,000\\-367,000,000\\\hline 5\end{array}$$

Next, 6 tens are to be subtracted. There are not enough tens, so 6 tens will be subtracted from 10 tens. This leaves 5 hundreds in the hundreds place. Subtracting 6 tens leaves 4 tens; the 4 tens are added to the 5 tens.

$$\begin{array}{r}55\\\not{6}\not{6}2,000,000\\-367,000,000\\\hline 95\end{array}$$

Finally, 3 hundreds are easily subtracted from 5 hundreds.

$$\begin{array}{r}55\\\not{6}\not{6}2,000,000\\-367,000,000\\\hline 295\end{array}$$

At this point in the computation, tell the children that the algorism is not yet completed since they must show that the difference is a certain number of millions. Tell the child who is at the chalkboard to show that the difference involves millions.

$$\begin{array}{r}55\\\not{6}\not{6}2,000,000\\-367,000,000\\\hline 295,000,000\end{array}$$

Have the class read the standard numeral for the difference: two hundred ninety-five million.

Now, write on the chalkboard the sum 453,000,000 + 378,000,000 in vertical form. Have the sum read aloud: four hundred fifty-three million plus three hundred seventy-eight million. Ask, "What sum is involved in this exercise if you ignore the periods?" (453 + 378) Tell the children that they could first compute the sum 453 + 378 and then write in the appropriate number of zeros to indicate the thousands period and the ones period. Or they could first write in the appropriate number of zeros for the thousands and ones periods since no computation is needed in these periods. Then they could compute in the millions period as if the exercise were simply to compute the sum 453 + 378.

Select a child to come to the chalkboard to compute aloud the sum 453,000,000 + 378,000,000. Instruct him to record the needed numerals as he describes his computational steps. He may explain his work as follows.

Since no computation is required in the thousands period or in the ones period, three zeros are written in each period to show that the final sum involves millions.

$$\begin{array}{r} 453{,}000{,}000 \\ +378{,}000{,}000 \\ \hline {,}000{,}000 \end{array}$$

3 + 8 is 11; 11 is 1 + 10. Write 1, remember 1 ten.

$$\begin{array}{r} 453{,}000{,}000 \\ +378{,}000{,}000 \\ \hline 1{,}000{,}000 \end{array}$$

1 ten + 5 tens + 7 tens is 13 tens; 13 tens is 3 tens + 1 hundred. Write 3 tens, remember 1 hundred.

$$\begin{array}{r} 453{,}000{,}000 \\ +378{,}000{,}000 \\ \hline 31{,}000{,}000 \end{array}$$

1 hundred + 4 hundreds + 3 hundreds is 8 hundreds. Write 8 hundreds.

$$\begin{array}{r} 453{,}000{,}000 \\ +378{,}000{,}000 \\ \hline 831{,}000{,}000 \end{array}$$

Have the class read aloud the standard numeral for the sum: eight hundred thirty-one million.

Continue this activity; use sums and differences such as these:

5,300,000 + 266,000
300,000,000 − 267,000,000
370,000 + 490,000
134,000,000 − 47,000,000
267,000,000 + 552,000,000
4,352,000 − 2,726,000

▶ After the children have completed the activities in this section, they should understand that the interpretation of numerals that name large numbers is aided by the use of three-digit periods and that this period system has a base 1000. But the children need continuous exposure to the concepts presented before they are comfortable with large numbers. It is important to provide the practice they need on a daily or weekly basis. A few possible methods are suggested here.

Use words rather than numerals and write one or two numbers on the chalkboard. Direct the children to write the standard numeral for these numbers on a sheet of paper.

Before you assign independent work, you might read the numerals for several large numbers and ask the children to write the standard numerals on their papers.

It would also be helpful to make flashcards of the standard numerals for large numbers. Use these as admission or exit passes when the children arrive in the morning or go out to recess or at lunch time. These flashcards can also be the basis for games similar to the ones played with addition or multiplication flashcards. If there are enough flashcards for each child to have one, occasionally let each row of children arrange themselves in front of the class in order from least to greatest. Then let each child read aloud the numeral printed on his card.

These same ideas can be used to provide practice in the computation of sums and differences. The object is to provide brief but frequent exposure of numerals for large numbers and to provide both oral and written practice for the children.

Pages 161 through 164

● Page 161 provides a variety of sums to be computed. Work exercises 1 through 3 with the class before assigning the other exercises for independent work. The children may record only the result of their computation on their papers unless you want them to copy the sum in vertical form and then record the count for the sum.

● Page 162 provides practice in computing differences in terms of thousands and millions. Discuss the example at the top of the page with the class. Use this example to review the meaning of difference. In the example, the difference between 17 million and 8 million is that number which added to 8 million gives 17 million.

Let several children take turns and read each exercise to the class. It may be helpful for the children to read some of the exercises in two ways. For example, exercise 9 may be read, "One million six hundred thousand minus eight hundred thousand" or "Sixteen hundred thousand minus eight hundred thousand." Some children may find this second interpretation more helpful than the first in computing the difference.

Work several exercises with the class before assigning the page for independent work. Allow each child to choose whether he will record only the result of the computation on his paper or copy the difference in vertical or horizontal form and then record the result.

● Use pages 163 and 164 for the children to practice computing sums and differences that involve millions or thousands. Review the idea of computing the given sum or difference as if only ones were involved. Remind the children that the number of zeros needed to show the periods involved may be written before or after the computation. Work several exercises with the class before assigning the page for independent work.

Tell the children that exercises 11 and 12 on page 164 are bonus exercises; the children should try to compute these differences, but they are not part of the required assignment. Those children who successfully compute these differences should be encouraged to explain their thinking steps to the class. Since the children need practice in reading numerals for large numbers, it would be well to have them read each exercise orally after the page has been completed.

Name

9 million + 6 million = 15 million

9,000,000 + 6,000,000 = 15,000,000

Compute.

1. 5,000,000 + 8,000,000 13,000,000
2. 40,000 + 90,000 130,000
3. 7,000,000 + 600,000 7,600,000
4. 9,000 + 8,000 17,000
5. 30,000 + 2,000 + 700 32,700
6. 60,000 + 50,000 110,000
7. 300,000 + 700,000 1,000,000
8. 800 + 500 1300 or 1,300
9. 800,000 + 400,000 1,200,000
10. 400 + 7,000 7,400 or 7400
11. 9,000,000 + 5,000,000 14,000,000
12. 700,000 + 900,000 1,600,000
13. 3,000,000 + 8,000,000 11,000,000
14. 8,000,000 + 20 8,000,020

C-161

Name

Compute.

1. 15,000,000 + 8,000,000 = 23,000,000
2. 25,000,000 − 7,000,000 = 18,000,000
3. 18,000,000 − 10,000,000 = 8,000,000
4. 9,100,000 + 320,000 = 9,420,000
5. 190,000 + 250,000 = 440,000
6. 640,000,000 − 270,000,000 = 370,000,000
7. 125,000,000 − 96,000,000 = 29,000,000
8. 86,000,000 + 43,000,000 = 129,000,000
9. 173,000,000 + 725,000,000 = 898,000,000
10. 351,000,000 − 256,000,000 = 95,000,000
11. 97,000,000 − 18,000,000 = 79,000,000
12. 38,000 + 726,000 = 764,000

13. 4,000,000 + 900,000 + 60,000 + 1,000 = 4,961,000

14. 15,000,000 + 630,000 + 1,400 + 75 = 15,631,475

15. 130,000,000 − 70,000,000 = 60,000,000

C-163

What is the difference between 17,000,000 and 8,000,000?

17,000,000 = 8,000,000 + 9,000,000

17,000,000 − 8,000,000 = 9,000,000

Compute.

1. 90,000 − 60,000 30,000
2. 12,000,000 − 4,000,000 8,000,000
3. 400,000 − 200,000 200,000
4. 15,000,000 − 7,000,000 8,000,000
5. 100,000 − 90,000 10,000
6. 1,000,000 − 800,000 200,000
7. 18,000,000 − 9,000,000 9,000,000
8. 130,000 − 60,000 70,000
9. 1,600,000 − 800,000 800,000
10. 120,000 − 70,000 50,000
11. 1,300,000 − 900,000 400,000
12. 140,000 − 80,000 60,000
13. 1,700,000 − 900,000 800,000
14. 13,000 − 4,000 9,000 or 9000
15. 800,000 − 300,000 500,000
16. 14,000,000 − 9,000,000 5,000,000

C-162

Compute.

1. 16,000,000 + 22,000,000 = 38,000,000
2. 671,000,000 + 99,000,000 = 770,000,000
3. 285,000,000 − 73,000,000 = 212,000,000
4. 452,000,000 + 493,000,000 = 945,000,000
5. 100,000,000 − 29,000,000 = 71,000,000
6. 156,000,000 − 99,000,000 = 57,000,000
7. 159,000,000 + 768,000,000 = 927,000,000
8. 11,000,000 + 89,000,000 = 100,000,000
9. 128,000,000 − 39,000,000 = 89,000,000
10. 326,000,000 + 491,000,000 = 817,000,000
11. 1,000,000 − 867,542 = 132,458
12. 1,376,825 − 493,289 = 883,536

13. 9,000,000 magazines were sold in April and 11,000,000 magazines were sold in May. How many magazines were sold in the 2 months? 20,000,000

14. One day 2,300,000 copies of newspaper *A* were sold and 3,000,000 copies of newspaper *B* were sold. How many more copies of newspaper *B* were sold that day? 700,000

C-164

Supplemental Experiences

■ Provide practice in mental computation of sums (and related differences) of millions and thousands. Read to the children chain exercises such as these:

20 millions plus 70 millions (pause) minus 10 millions.

500 thousands minus 100 thousands (pause) plus 700 thousands.

400 millions plus 200 millions (pause) minus 300 millions.

6 millions minus 2 millions (pause) minus 3 millions.

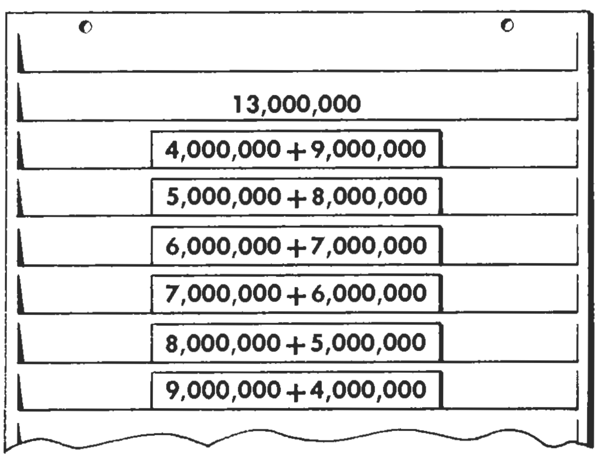
7 thousands minus 2 thousands (pause) plus 7 thousands.

30 thousands plus 60 thousands (pause) plus 30 thousands.

120 thousands minus 60 thousands (pause) plus 4 thousands.

Call on volunteers to give the answer after you read each exercise. The children will enjoy working faster as they become more proficient in computing mentally.

■ Ask the children to help you write sums of 11,000,000 through 18,000,000 on cards. Shuffle the pack and redistribute the cards to the children. Place in the pocket chart a numeral-card for 13,000,000. Direct the children who hold sums of 13,000,000 to raise their hands. Ask them to bring their cards forward one at a time and place them below 13,000,000 in the chart.



For variety, write the numerals 11,000,000 through 18,000,000 on the chalkboard. Give each child a turn to choose one of these numbers and to write below it a sum form for this number.

■ Make a pack of cards that contain sums of thousands or millions.

16,000 + 12,000 | **38,000 + 13,000**

77,000,000 + 36,000,000

68,000,000 + 56,000,000

Separate the class into two teams. Tell two children from each team to each choose a card. Have each of these four children compute on the chalkboard the sum he chose. Then direct each pair of team members to decide who has the greater sum. Meanwhile, write the words *is greater than* on two panels of the chalkboard. Then tell each pair of children to arrange their sums around the phrase. For example, if the sums computed by the members of one team are 113,000,000 and 124,000,000, these children would arrange their sums in the following way:

124,000,000 *is greater than* 113,000,000

Each child who computes correctly may earn a point for his team. If the pair of children arrange their sums so that the correct order relationship between the numbers is stated, they earn a point for their team. After all of the children in the class have participated in the game, total the points earned and declare a winner.

Adapt this procedure to an activity that involves differences of thousands or millions.

KEY IDEA

Ones times tens is tens. Tens times tens is ten tens or hundreds. Tens times hundreds is ten ten tens or thousands. Hundreds times hundreds is TEN ten TEN tens or ten thousands.

Scope

To develop the children's skill in multiplying multiples of 10, 100, and 1000.

To review multiplication combinations.

Fundamentals

In this section the child will use his understanding of large numbers as he computes multiples of 10, 100, and 1000.

The multiples of 10, 100, and 1000 are very easy to compute. For example, $4 \times 10 = 40$, $8 \times 100 = 800$, and $7 \times 1000 = 7000$. Other products that involve multiples of 10, 100, or 1000 are also easy to compute. For example, $6 \times 400 = 2400$, $60 \times 200 = 12{,}000$, and $40 \times 80 = 3200$.

Exercises such as the examples given above involve the commutative and associative properties of multiplication.

40×80	$= 4 \times 10 \times 8 \times 10$	meaning of 40 and 80
	$= [(4 \times 10) \times 8] \times 10$	associative property
	$= [8 \times (4 \times 10)] \times 10$	commutative property
	$= [(8 \times 4) \times 10] \times 10$	associative property
	$= (8 \times 4) \times (10 \times 10)$	associative property
	$= 32 \times 100$	computation
	$= 3200$	multiplication by 100

The number of steps is reduced when one uses the rearrangement property.

40×80	$= 4 \times 10 \times 8 \times 10$	meaning of 40 and 80
	$= 4 \times 8 \times 10 \times 10$	rearrangement
	$= 32 \times 100$	computation
	$= 3200$	multiplication by 100

The skills developed in this section are basic to further work in multiplication of whole numbers. Practice with the basic multiplication combinations is provided throughout the section. The teacher must be aware that the child will not master the content by the time this section is completed. The practice provided in the units that follow will contribute to the child's understanding, speed, and efficiency in computing products such as 60×50, 40×700, and 900×400.

Readiness for Understanding

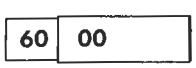
Knowledge of basic combinations.
An understanding of the concept of product.
An understanding of a hundred as ten tens.
An understanding of a thousand as ten ten tens.
An understanding of a million as a thousand thousands.

Developmental Experiences

pocket chart	box
tagboard cards	sheet of tagboard
felt-tip pen	(18″ × 24″)

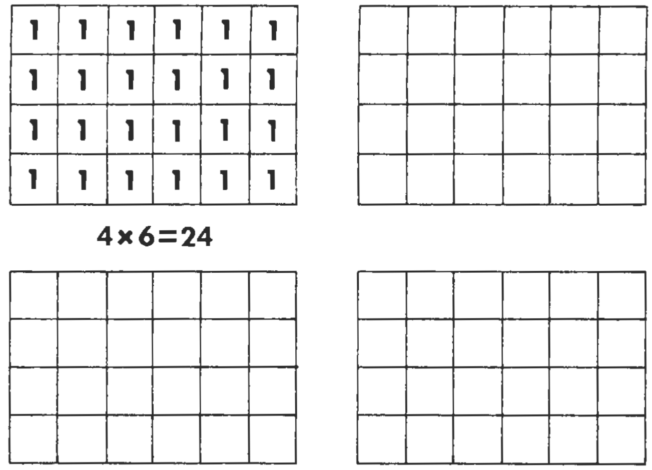
▶ Review with the class the fact that multiplication by 10 is really just a matter of writing a 0 after the numeral. For example, $32 \times 10 = 320$. Similarly, multiplying by 100 is a matter of writing two 0's after the numeral ($43 \times 100 = 4300$), and multiplying by 1000 is a matter of writing three 0's after the numeral ($54 \times 1000 = 54{,}000$). Ask the children how many zeros indicate times 10 (1 zero); how many zeros indicate times 100 (2 zeros); how many zeros indicate times 1000 (3 zeros).

Write on a card the phrase "60 hundreds" and place it in the pocket chart. Remind the class that another way to say 60 hundreds would be to say 60 times 100. Give a child an assortment of cards that have the numerals 0, 00, and 000 written on them. Tell the child to select the card he needs to show the standard numeral for 60 hundreds. Tell him to place this card over the word *hundreds* and to read aloud the standard numeral: six thousand.

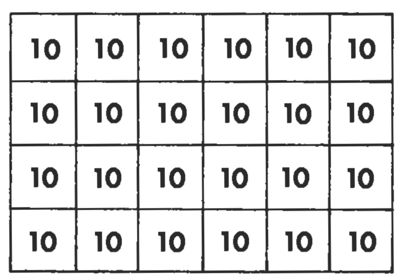


Continue in this way with numbers such as 700 tens, 5 thousands, 800 hundreds, 90 tens, 30 thousands, and 4000 thousands.

▶ Draw on the chalkboard four 12 by 18 inch rectangles. Partition each into a 4 by 6 array. Ask a child to write an appropriate product below the first array. (4×6) Then write the numeral 1 in each of the squares in the first array. Ask the class to give the count for this array. (24) Record the count beside the product and link the two with an equal sign.

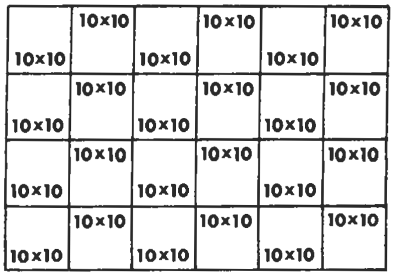


Next, ask the children what product they think should be associated with the second array on the chalkboard. When the children respond 4×6, write the numeral 10 on each of the squares in this array. Ask whether or not anyone wants to change the product for this array. A child may suggest that the product for this array is 4×6 tens. Ask for the count for this array. (24 tens) Direct a child to write below the array on the board the product and the count in an equation. (4×6 tens $= 24$ tens) Ask another child to write below this equation the same equation and to use standard numerals for the factors and the count. ($4 \times 60 = 240$)



4 × 6 tens = 24 tens
4 × 60 = 240

Now ask the children what product they think should be associated with the third array on the chalkboard. After the children have had a chance to express their ideas, write 10×10 on each of the squares. After they have observed the fact that these are ten tens or hundred squares, ask for the product for this array. One child may suggest that the product is 4×6 ten tens; another child may suggest that the product is 4×6 hundreds. Tell two children to write these products below the array. Ask for the count for this array. (24 ten tens or 24 hundreds) Direct the children who are still at the board to complete their equations. Then ask two other children to write the same equations and to use standard numerals for the factors and the count.



4×6 ten tens = 24 ten tens

4×6 hundreds = 24 hundreds

4×600 = 2400

Finally, follow the same procedure with the last array on the chalkboard. Label each square in the array as illustrated below:

10 × 10 × 10

10 × 100

100 × 10

Have the children record on the chalkboard each suggested way to express the product and the count. Some suggestions that the class might make regarding the product and the count are listed below.

4 tens × 6 ten tens = 24 ten ten tens
4 ten tens × 6 tens = 24 ten ten tens
4 tens × 6 hundreds = 24 ten hundreds
4 hundreds × 6 tens = 24 thousands
4 × 6 ten ten tens = 24 ten ten tens

As a final step, instruct the children to write the information in equation form using standard numerals for the factors and the count.

$$40 \times 600 = 24{,}000$$
$$400 \times 60 = 24{,}000$$
$$4 \times 6000 = 24{,}000$$
$$4000 \times 6 = 24{,}000$$

Note that in any of the equations developed during this activity, the factors may be commuted. The equations that the children use do not have to be exactly the ones illustrated.

Continue in this way with 6 by 7 arrays and 8 by 3 arrays. Summarize the activity; remind the children that multiplications such as 8 × 30, 40 × 60, and 60 × 700 involve only a knowledge of the basic multiplication combinations and the ability to multiply by 10, 100, and 1000.

▶ Write on a set of cards products such as the following:

90 × 4	600 × 5	3000 × 7	90 × 60
800 × 20	7000 × 40	300 × 500	6 × 7

Place the cards in a box. Ask a child to pull out one card, to read aloud the numeral for the product, and to give the standard numeral for the product. Tell the child that he may either compute the exercise on the chalkboard or do the computation mentally.

As the children work with the various multiples of 10, 100, and 1000, help them make a chart that shows the relationships between factors and products. For example, a child may select the product 60 × 90. He reads the numeral for the product and computes—sixty times ninety equals fifty-four hundred, or five thousand four hundred. At this point remind the class that each of the factors involved tens and the product involved hundreds; tens times tens equals ten tens or hundreds. Then record the information on the chart.

tens × tens = ten tens or hundreds

Continue in this way until each child has an opportunity to compute a given product. The chart that results from this activity should contain the following information.

ones × ones = ones
ones × tens = tens
ones × hundreds = hundreds
ones × thousands = thousands
tens × tens = ten tens, or hundreds
tens × hundreds = ten ten tens, or thousands
tens × thousands = ten thousands
hundreds × hundreds = hundred hundreds or ten thousands

If the children wish to include the commuted form of some products in the chart they may do so. The chart should be posted in the classroom so that the children can refer to it when necessary.

Pages 165 through 172

● Direct the children to study the illustration and questions at the top of page 165. After they have had ample time to do this, discuss the questions. Let the children tell how the picture helps them see 4000 as each of the given products. Help the children conclude that a number such as 4000 can be expressed as a product in several ways. Ask them to suggest ways to express 4000 as a product other than those shown. List the children's responses on the chalkboard.

5 × 800   500 × 8   50 × 80

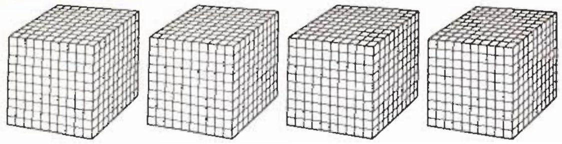
Work one or two exercises from each section of page 165 before assigning the rest of the exercises for independent work.

● Page 166 provides practice in computing products that have a multiple of 10, 100, or 1000 as one of the factors. Assign the exercises for independent work. When the children have completed the assignment, let them discuss their results and resolve any difficulties.

● Pages 167 and 168 provide practice in computing products that have a multiple of 10 or 100 as one factor and ones as the other factor. It is not necessary to assign a whole page on one day.

Name

For Class Discussion



How many blocks? Are there –

4 × 1000? *Yes**	2 × 2000? *Yes**
40 × 100? *Yes**	20 × 200? *Yes**
400 × 10? *Yes**	200 × 20? *Yes**

5 hundreds × 8? *Yes**

**See pupil page suggestions.*

Write the standard numerals.

1. 50 hundreds *5000 or 5,000*	2. 900 tens *9,000 or 9000*
3. 70 hundreds *7000 or 7,000*	4. 2 thousands *2,000 or 2000*
5. 40 tens *400*	6. 600 hundreds *60,000*
7. 3000 thousands *3,000,000*	8. 800 thousands *800,000*

Compute.

9. 5 × 5 = *25*
10. 5 × 5 hundreds = *25 hundreds or 2500*
11. 5 × 5 tens = *25 tens or 250*
12. 5 tens × 5 tens = *25 ten tens or 2500 or 25 hundreds*
13. 5 hundreds × 5 hundreds = *25 hundred hundreds or 250,000*

reference page

C-165

Name

Compute.

1. 30 × 2 = *60*	2. 50 × 9 = *450*	3. 30 × 8 = *240*	4. 20 × 2 = *40*
5. 60 × 7 = *420*	6. 70 × 6 = *420*	7. 30 × 9 = *270*	8. 70 × 4 = *280*
9. 40 × 3 = *120*	10. 40 × 8 = *320*	11. 20 × 9 = *180*	12. 20 × 7 = *140*
13. 30 × 5 = *150*	14. 70 × 8 = *560*	15. 50 × 8 = *400*	16. 30 × 6 = *180*
17. 20 × 3 = *60*	18. 30 × 4 = *120*	19. 20 × 5 = *100*	20. 50 × 2 = *100*
21. 50 × 4 = *200*	22. 40 × 4 = *160*	23. 80 × 9 = *720*	24. 40 × 7 = *280*
25. 90 × 9 = *810*	26. 50 × 6 = *300*	27. 60 × 8 = *480*	28. 70 × 9 = *630*

C-167

Compute.

1. 3 × 3 = *9*	2. 3 × 30 = *90*
3. 3 × 300 = *900*	4. 3 × 3000 = *9000*
5. 6 × 4 = *24*	6. 60 × 40 = *2400*
7. 600 × 400 = *240,000*	8. 6000 × 4000 = *24,000,000*
9. 5 × 7 = *35*	10. 5 × 70 = *350*
11. 5 × 700 = *3500*	12. 5 × 7000 = *35,000*
13. 8 × 2 = *16*	14. 80 × 20 = *1600*
15. 80 × 200 = *16,000*	16. 8000 × 200 = *1,600,000*
17. 1 × 9 = *9*	18. 1 × 90 = *90*
19. 10 × 900 = *9000*	20. 100 × 9000 = *900,000*

C-166

Compute.

1. 400 × 2 = *800*	2. 200 × 4 = *800*	3. 500 × 6 = *3000*	4. 600 × 5 = *3000*
5. 400 × 9 = *3600*	6. 900 × 4 = *3600*	7. 900 × 6 = *5400*	8. 600 × 9 = *5400*
9. 400 × 7 = *2800*	10. 700 × 4 = *2800*	11. 700 × 9 = *6300*	12. 900 × 7 = *6300*
13. 50 × 8 = *400*	14. 80 × 5 = *400*	15. 60 × 8 = *480*	16. 80 × 6 = *480*
17. 500 × 4 = *2000*	18. 400 × 5 = *2000*	19. 800 × 9 = *7200*	20. 900 × 8 = *7200*
21. 900 × 5 = *4500*	22. 500 × 9 = *4500*	23. 800 × 3 = *2400*	24. 300 × 8 = *2400*
25. 300 × 7 = *2100*	26. 700 × 3 = *2100*	27. 20 × 6 = *120*	28. 60 × 2 = *120*

C-168

Name

Compute.

1. 90 × 90 = 8100
   90 × 9 = 810
   9 × 9 = 81

2. 50 × 90 = 4500
   5 × 90 = 450
   5 × 9 = 45

3. 80 × 90 = 7200
   80 × 9 = 720
   8 × 9 = 72

4. 60 × 80 = 4800
   6 × 80 = 480
   6 × 8 = 48

5. 7 × 9 = 63
   7 × 90 = 630
   70 × 90 = 6300

6. 7 × 7 = 49
   7 × 70 = 490
   70 × 70 = 4900

7. 8 × 8 = 64
   80 × 8 = 640
   800 × 8 = 6400

8. 4 × 9 = 36
   40 × 9 = 360
   400 × 9 = 3600

C-169

Name

8 million is a product.

8,000,000 = 8 × 1,000,000

8,000,000 = 2000 × 4000

8,000,000 = 40 × 200,000

8,000,000 = ______ × ______ *

Write each number in product form.

1. 60 = ______ *
2. 600 = ______ *
3. 6000 = ______ *
4. 1800 = ______ *
5. 180,000 = ______ *
6. 18,000,000 = ______ *
7. 320 = ______ *
8. 32,000 = ______ *

Compute.

9. 800 × 9 = 7200
10. 70 × 40 = 2800
11. 5000 × 3 = 15,000
12. 800 × 70 = 56,000
13. 90 × 70 = 6300
14. 200 × 50 = 10,000
15. 700 × 200 = 140,000
16. 9000 × 200 = 1,800,000
17. 9000 × 50 = 450,000
18. 8000 × 8 = 64,000
19. 8000 × 6000 = 48,000,000
20. 400 × 6 = 2400

*Answers will vary.

reference page

C-171

Compute.

1. 7 × 5 = 35
   7 × 50 = 350
   70 × 50 = 3500

2. 6 × 6 = 36
   60 × 6 = 360
   600 × 6 = 3600

3. 7 × 800 = 5600
   70 × 800 = 56,000
   700 × 800 = 560,000

4. 6 × 700 = 4200
   60 × 700 = 42,000
   600 × 700 = 420,000

5. 3 × 800 = 2400
   30 × 800 = 24,000
   300 × 800 = 240,000

6. 9 × 3000 = 27,000
   90 × 3000 = 270,000
   900 × 3000 = 2,700,000
   9000 × 3000 = 27,000,000

7. 7 × 3000 = 21,000
   70 × 3000 = 210,000
   700 × 3000 = 2,100,000
   7000 × 3000 = 21,000,000

8. 4 × 7000 = 28,000
   40 × 7000 = 280,000
   400 × 7000 = 2,800,000
   4000 × 7000 = 28,000,000

C-170

Compute.

1. 20 × 3 = 60
2. 30 × 70 = 2100
3. 300 × 4 = 1200
4. 40 × 40 = 1600
5. 600 × 6 = 3600
6. 90 × 60 = 5400
7. 40 × 6 = 240
8. 80 × 50 = 4000
9. 700 × 70 = 49,000
10. 600 × 30 = 18,000
11. 7000 × 5 = 35,000
12. 90 × 90 = 8100
13. 70 × 6 = 420
14. 90 × 2 = 180
15. 800 × 40 = 32,000
16. 600 × 7 = 4200
17. 80 × 30 = 2400
18. 900 × 8 = 7200
19. 800 × 800 = 640,000
20. 70 × 40 = 2800
21. 900 × 30 = 27,000
22. 30 × 5 = 150
23. 300 × 6 = 1800
24. 40 × 80 = 3200
25. 600 × 50 = 30,000
26. 80 × 60 = 4800
27. 900 × 400 = 360,000
28. 80 × 70 = 5600
29. 9000 × 7000 = 63,000,000
30. 300 × 90 = 27,000
31. 8000 × 7 = 56,000
32. 4000 × 60 = 240,000

C-172

● Pages 169 and 170 provide further practice in computing products that involve multiples of 10, 100, and 1000. Tell the children to complete the exercises independently. Explain that they may use the previously constructed information chart to help them with their computation. After everyone has completed these pages, call on individual children to describe how they computed specific products.

● Page 171 provides practice that involves several concepts previously developed in this section. Discuss the example at the top of the page with the class. List on the chalkboard all of the suggestions the children give in response to the question at the top of the page.

$$8{,}000{,}000 = 8000 \times 1000$$
$$8{,}000{,}000 = 200 \times 40{,}000$$
$$8{,}000{,}000 = 100{,}000 \times 80$$
$$\vdots$$

The first eight exercises can be assigned as independent work. Be sure the children understand that they may write each number in the product form of their choice. Also, since the directions do not specify that the number must be written as a product of two numbers, an answer such as $1 \times 6 \times 10$ for the number 60 is correct. After all of the children have completed these exercises, tell them to exchange papers and check each other's answers. As you discuss the answers, let them list on the chalkboard the various products below the numbers.

The exercises at the bottom of page 171 provide further practice in computing products that are multiples of 10, 100, and 1000. Assign these exercises for independent work.

● Use page 172 to let the children check their own ability to compute. It is not necessary that all of the exercises be completed at the same time.

Supplemental Experiences

■ Write on tagboard cards products such as 60×90, 7×300, 4×8000, and 40×800. Put three of these products in the pocket chart.

40×300 2×6000 600×20

Describe a product—for example, the product that is 12 ten ten tens. Tell a child to remove any of the products in the chart that do not meet the given condition. Let the class decide whether or not this child is correct in his selection. If the child is not, let him choose another child to help him.

Proceed in this way with other sets of products. Vary the number of products that meet the given condition each time. Make sure that at some time during the activity all three products meet the condition and at some other time none of the products meets the condition.

■ Write a series of correct and incorrect mathematical sentences on the chalkboard.

$3 \times 20 = 6000$
$600 \times 30 = 12{,}000$
$70 \times 50 = 3500$

$4000 \times 8000 = 320{,}000$
$90 \times 700 = 63{,}000$
$600 \times 500 = 3{,}000{,}000$

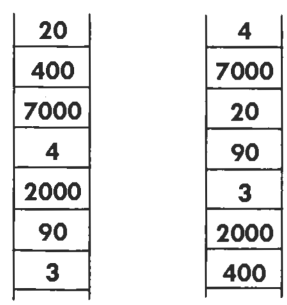
Select a child to come to the chalkboard to study the sentence you indicate, and to decide whether or not the sentence is correct. Tell the child to erase the sentence from the chalkboard if it is correct. If the child decides that the sentence is incorrect, tell him to use any two of the numbers from the sentence to construct a correct sentence. For example, if the child is working with the sentence $30 \times 20 = 6000$, he may construct any of the following sentences.

$$30 \times 20 = 600$$
$$300 \times 20 = 6000$$
$$30 \times 200 = 6000$$

Continue this activity until all sentences in this series of sentences have been used. Then proceed with other series of sentences; give every child the opportunity to participate in this activity.

■ Play the game "Climb the Ladder" to review multiplication with factors that are multiples of 10, 100, and 1000.

Draw pictures of six or more ladders on the chalkboard. Write a factor on each rung of one ladder. Then write the same factors on each of the other ladders, but vary the order in which they appear.



Assign a child to each ladder. Explain that you are going to call out another factor, for example, 60. The children are to multiply each factor on their ladder by 60 and each child should write beside each rung the standard numeral for the product. The winner is the one who reaches the top with all products correct. The child who wins becomes the leader and chooses a child to take his place at the board. The new leader can then call out a new factor.

UNIT 12
LONG MULTIPLICATION

Pages 173 Through 214

OBJECTIVE

To develop long multiplication.

In this unit the child learns the long multiplication algorism and recognizes that it is based on the distributive property. He applies his knowledge of numeration to simplify computation. He uses his knowledge of basic multiplication facts to compute products with factors of two and three digits.

See Key Topics in Mathematics for the Primary Teacher: Multiplication.

KEY IDEAS

Tens × tens is hundreds.

The standard numeral abbreviates the expanded notation.

The distributive property lets us multiply in parts—digit by digit.

KEY IDEA

Tens × tens is hundreds.

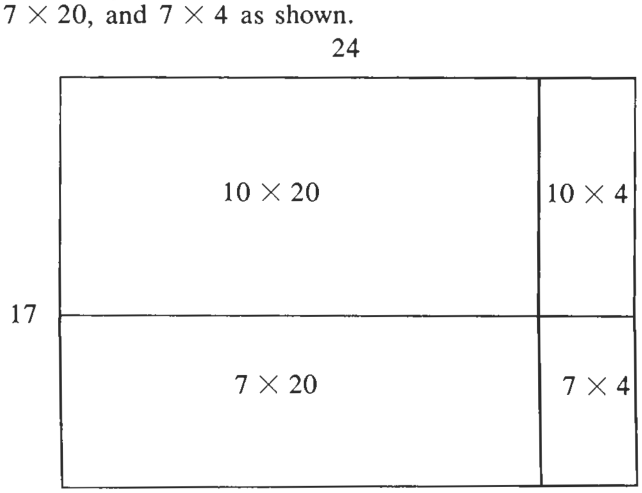
Scope

To relate place values to basic combinations.

To develop a long multiplication algorism.

Fundamentals

The principles of numeration and basic multiplication facts are used to develop an efficient procedure for computing products in general. The child has had many experiences in partitioning arrays and is familiar with the idea of the product being the sum of partial products. Now it is important to partition to make better use of tens, hundreds, and thousands. For example, in computing 17 × 24, the convenient parts (with respect to numeration) are 10 × 20, 10 × 4, 7 × 20, and 7 × 4 as shown.



The computation of 17 × 24 may be shown as follows:

$$\begin{array}{rcr} 24 & = & 20 + 4 \\ \underline{\times\ 17} & = & \underline{10 + 7} \\ & & 28 \\ & & 140 \\ & & 40 \\ & & \underline{200} \\ & & 408 \end{array}$$

Note how the partial products are related to the place value of the digits and result from basic multiplication facts and principles of numeration.

28 is 7 ones × 4 ones or (7×4) ones.
140 is 7 ones × 2 tens or (7×2) tens.
40 is 1 ten × 4 ones or (1×4) tens.
200 is 1 ten × 2 tens or (1×2) hundreds.

Readiness for Understanding

Knowledge of our numeration system.

Knowledge of the basic multiplication facts.

Understanding of the distributive property of multiplication.

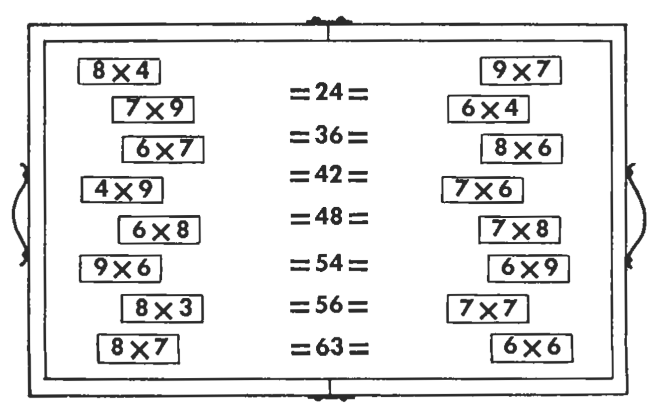
Developmental Experiences

for flannel board
felt numerals and symbols
product-cards: 0 × 0 through 10 × 10

for each child
21 tagboard cards (1″ × 1½″)
crayon

sheets of tagboard (18″ × 24″)
felt-tip pen
pins
yarn

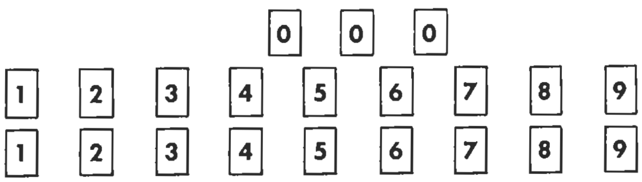
▶ Arrange the numerals 24, 36, 42, 48, 54, 56, and 63 in a column in the center of the flannel board. Place an equal sign on each side of each numeral. Along the left side of the flannel board place 7 product-cards—each card should match one of the given counts. Then add one more product-card that does not represent any of the given counts. Repeat this procedure using 8 cards on the right side of the flannel board. The products on the right side of the board should not have the same form as those on the left.



Call two children to the flannel board and assign a set of cards to each child. Tell these children that at a given signal they are to match a product with each count in the center of the board. The child on the left side of the board should put his cards to the left of the equal signs; the child on the right should put his cards to the right of the equal signs. Have the children see who can correctly complete his assignment first. When necessary, use arrays to check the results.

Continue in this same way to give other pairs of children an opportunity to make equations. Throughout the activity include all of the products that have been introduced up to this time in the program.

▶ Give each child a set of 21 tagboard cards (1 inch by $1\frac{1}{2}$ inches) and have the children label the cards as illustrated.



Write on the chalkboard one of the following products:

1×1
1×10
10×1
10×10
100×10
10×100

Tell the children to use their tagboard numeral-cards on their desks to show the count for the product on the board. Ask someone to read this count aloud. Continue this procedure with the other five products one at a time. Tell the children to show the count for each product; then have each count read aloud. The counts for 10×10, 100×10, and 10×100 may be read in various ways.

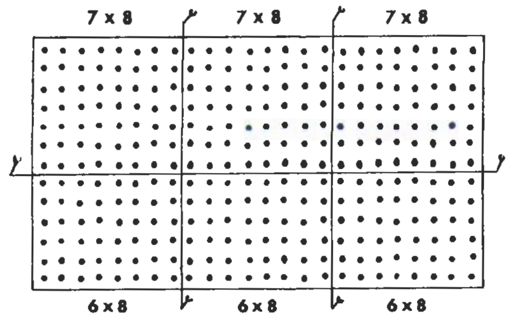
$10 \times 10 = 100$ (1 hundred, 10 tens)
100×10 } $= 1000$ { 1 hundred tens
10×100 } { 10 hundreds
{ 10 ten tens

Next, write on the chalkboard one at a time the following products:

6×3	9×80	30×80
5×7	70×40	500×90
40×3		20×600

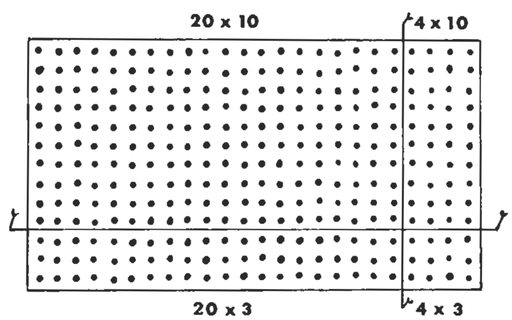
Tell the children to show and read the count for each product used.

▶ Draw a 13 by 24 array on a tagboard card (18 by 24 inches) and pin it to the flannel board. Give a child several pieces of yarn and ask him to show how he would partition this array. Then tell the child to choose felt numerals and symbols to show the partial products beside the parts of the array. For example, the child may have partitioned the array as seen in the illustration and the related partial products could be placed as shown.



Ask three or four other children to show how they would partition this array. In each instance have the class compute the partial products and the sum of the partial products.

If no child partitions the array in terms of tens and ones, present this idea to the class. Explain that when the factors of a product are greater than 10, it is more convenient and efficient to think in terms of tens and ones. Have someone partition the 13 by 24 array in terms of tens and ones and place the appropriate product beside each part.



Then have the same child compute on the chalkboard the partial products and the sum of partial products. To summarize the ideas involved in this computation, let one child come to the chalkboard and describe the thinking steps used in computing each partial product.

$$
\begin{array}{rl}
24 = 20 + 4 & \\
\underline{\times\ 13 = 10 + 3} & \\
12 & (4 \times 3 \text{ is } 12) \\
60 & (2 \text{ tens} \times 3 \text{ is } 6 \text{ tens}) \\
40 & (4 \times 1 \text{ ten is } 4 \text{ tens}) \\
\underline{200} & (2 \text{ tens} \times 1 \text{ ten is } 2 \text{ ten tens}) \\
312 &
\end{array}
$$

Have the class study the multiplication algorism on the chalkboard and then discuss the following questions:

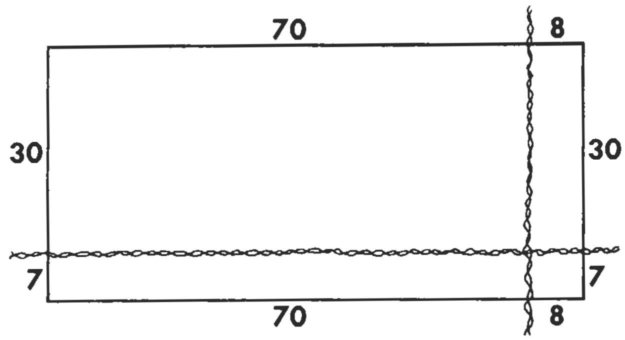
Which digits do you multiply to get ones; that is, which digits refer to a product of ones? (4 and 3) Ask a child to show 4×3 ones in the array.

Which digits do you multiply to get tens; that is, which digits refer to a product (or products) of tens? (2 and 3, 1 and 4) Let a child show 2 tens $\times$ 3 and 1 ten $\times$ 4 in the array.

Which digits do you multiply to get ten tens; that is, which digits refer to a product of ten tens?

(2 and 1) Have a child show 2 tens × 1 ten in the array.

Adapt this procedure to other products such as 34 × 12, 63 × 21, 56 × 18, and 78 × 37. When it becomes cumbersome to show all of the members of the array, the following type of illustration could be placed on the flannel board.



▶ Write 69 × 72 in vertical form on the chalkboard. Choose five children to help compute this product; have the first four children each compute just one partial product. Ask the fifth child to compute the sum of partial products.

$$\begin{array}{r} 69 \\ \times\ 72 \\ \hline 18 \\ 120 \\ 630 \\ \underline{4200} \\ 4968 \end{array}$$

Ask four other children to take turns and describe how each partial product is derived. Comments similar to the following may be made:

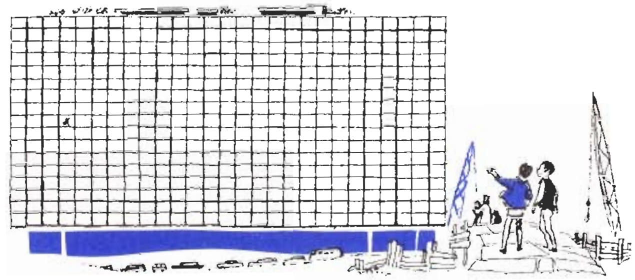
18 is the product of 9 and 2.
18 is the product of the ones.
120 (12 tens) is the product of 6 tens and 2.
120 (12 tens) is one of the products of tens.
630 (63 tens) is the product of 7 tens and 9.
630 (63 tens) is one of the products of tens.
4200 (42 ten tens) is the product of 7 tens and 6 tens.
4200 (42 ten tens) is the product of ten tens.

Proceed in this way. Let the children take turns computing such products as 46 × 21, 17 × 34, and 11 × 98. For each exercise, have someone describe the way that the partial products were derived.

Name

UNIT 12 LONG MULTIPLICATION

For Class Discussion



"Look!" Bill pointed. "There is a window washer about halfway up that building."

"Seventeen floors and 27 windows on each floor," said Gordon. "That's 17 × 27 windows!"

"How can we compute 17 × 27?" asked Bill.

"We can compute (9 + 8) × (9 + 9 + 9)," said Gordon.

"I think (10 + 7) × (20 + 7) would be easier to compute," said Bill.

Who was correct? *Both were correct.*

In what other ways can you compute 17 × 27? *See pupil page suggestions.*

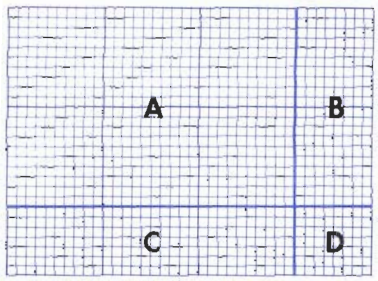
reference page

C-173

Pages 173 through 186

● Use page 173 for discussion. After the children have studied the illustration and read the story, have two children go to the chalkboard and compute the product using Gordon's way and Bill's way. Have the class compare the computed products. Then let other children demonstrate how they would compute 17 × 27 in other ways. Help the class see that Bill's way of computing 17 × 27 may be more efficient since it capitalizes on their knowledge of numeration.

For Class Discussion



Bill's method:

$$\begin{array}{r l} 38 = 30 + 8 & \\ \times\ 27 = 20 + 7 & \\ 56 & (7 \times 8) \\ 210 & (7 \times 3\ \text{tens}) \\ 160 & (2\ \text{tens} \times 8) \\ 600 & (2\ \text{tens} \times 3\ \text{tens}) \\ 1026 & (27 \times 38) \end{array}$$

1. How many ten tens are in A? 6
2. How many tens are in B? 16 in C? 21
3. How many ones are in D? 56
4. What digits do you multiply to get ten tens in A? 3, 2
   tens in B? 2, 8
   tens in C? 3, 7
   ones in D? 7, 8

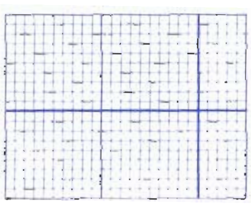
reference page

C-174

Name

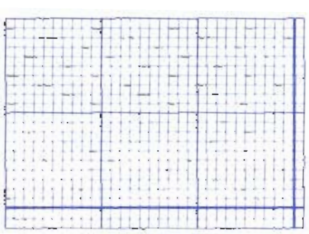
Compute using parts.

1.



$$\begin{array}{r l} 25 = 20 + 5 & \\ \times\ 19 = 10 + 9 & \\ 45 & (9 \times 5) \\ 180 & (9 \times 2\ \text{tens}) \\ 50 & (1\ \text{ten} \times 5) \\ 200 & (1\ \text{ten} \times 2\ \text{tens}) \\ 475 & (19 \times 25) \end{array}$$

2.



$$\begin{array}{r} 31 = 30 + 1 \\ \times\ 22 = 20 + 2 \\ 2 \\ 60 \\ 20 \\ 600 \\ 682 \end{array}$$

reference page

C-175

● Use page 174 for class discussion. Copy on the chalkboard a model of Bill's method of computing 38 × 27. As each question is discussed, have a child come to the chalkboard and point to the part of the algorism that illustrates his comments. For example, if a child is discussing the ten tens in the exercise and responds that there are 6 ten tens in the array and that he multiplied 2 tens × 3 tens to get 6 ten tens, have him point to the appropriate numerals in the model on the chalkboard. Discuss each question in a similar way.

● Pages 175 and 176 encourage the use of numeration principles in computing products. Complete the exercises on page 175 with the class. Direct the children to complete exercise 1 while a volunteer computes on the chalkboard. Let the child who is at the board explain the thinking steps involved in the computation. If the children at their desks disagree with the explanation at any point, have them raise their hands to indicate that some rethinking is needed. Follow a similar procedure with exercise 2. Assign page 176 for independent work. Tell the children that they may write explanations for each partial product in these exercises if they wish.

● Pages 177 and 178 provide further practice in computing products. Provide pupils with graph paper if they wish to draw the arrays. Before assigning the exercises on page 177, have the children name each factor as a sum. When the children have computed the products, let them discuss their results.

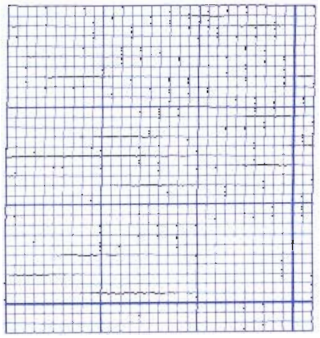
Follow a similar procedure with page 178.

● Pages 179 and 180 give the children further practice in computing products that involve four partial products. Work exercises 1 and 2 on page 179 with the class. Help the children see that the partial products are to be listed below the standard numerals for the factors rather than below the expanded form for the numerals. Point out that in exercise 1 the number of ones or tens involved in computing each partial product is listed.

Before assigning the exercises for independent work, direct the children to give the expanded form for the numerals in each exercise. Explain that as they work each exercise they may think the expanded form of each numeral; however they need not write the expanded form on their papers. Point out that in exercise 2 the explanation of how each partial product is computed might serve as a checklist for other exercises—have they shown the result of multiplying ones times ones, ones times tens, tens times ones, and tens times tens? It might be helpful to write an exercise similar to this on a piece of tagboard and post it in the room as a reference.

Work exercises 3 and 4 with the children to be sure they understand the procedure. Instruct them to complete the other exercises independently. Use the results of this page to determine which children may need additional help.

Compute using parts.



1. 33 = 30 + 3
× 32 = 30 + 2

6 (2 × 3)
60 (2 × 30)
90 (30 × 3)
900 (30 × 30)
1056

2. 46 = 40 + 6
× 54 = 50 + 4

24
160
300
2000
2484

3. 68 = 60 + 8
× 42 = 40 + 2

16
120
320
2400
2856

4. 37 = 30 + 7
× 85 = 80 + 5

35
150
560
2400
3145

5. 29 = 20 + 9
× 56 = 50 + 6

54
120
450
1000
1624

reference page

Compute.

1. 34 = 30 + 4
× 61 = 60 + 1

4
30
240
1800
2074

2. 45 = 40 + 5
× 37 = 30 + 7

35
280
150
1200
1665

3. 65 = 60 + 5
× 86 = 80 + 6

30
360
400
4800
5590

4. 79 = 70 + 9
× 23 = 20 + 3

27
210
180
1400
1817

5. 28 = 20 + 8
× 68 = 60 + 8

64
160
480
1200
1904

6. 15 = 10 + 5
× 45 = 40 + 5

25
50
200
400
675

Name

Compute. Show the arrays on graph paper if you wish.

1. 32 = 30 + 2
× 53 = 50 + 3

6 (3 × 2)
90 (3 × 3 tens)
100 (5 tens × 2)
1500 (5 tens × 3 tens)
1696 (53 × 32)

2. 36 = 30 + 6
× 27 = 20 + 7

42
210
120
600
972

3. 87 = 80 + 7
× 95 = 90 + 5

35
400
630
7200
8265

4. 22 = 20 + 2
× 24 = 20 + 4

8
80
40
400
528

5. 63 = 60 + 3
× 31 = 30 + 1

3
60
90
1800
1953

6. 82 = 80 + 2
× 45 = 40 + 5

10
400
80
3200
3690

Name

Compute.

1. 36 = 30 + 6
× 47 = 40 + 7

42 (7 ones × 6 ones)
210 (7 ones × 3 tens)
240 (4 tens × 6 ones)
1200 (4 tens × 3 tens)
1692 (47 × 36)

2. 19 = 10 + 9
× 56 = 50 + 6

54 (ones × ones)
60 (ones × tens)
450 (tens × ones)
500 (tens × tens)
1064 (56 × 19)

3. 12
× 13

6
30
20
100
156

4. 51
× 62

2
100
60
3000
3162

5. 27
× 72

14
40
490
1400
1944

6. 41
× 59

9
360
50
2000
2419

7. 28
× 19

72
180
80
200
532

8. 77
× 55

35
350
350
3500
4235

Compute.

1.
72 = 70 + 2
× 17 = 10 + 7
14 (ones × ones)
490 (ones × tens)
20 (tens × ones)
700 (tens × tens)
1224 (17 × 72)

2.	3.	4.	5.
65	97	63	18
× 28	× 87	× 31	× 73
40	49	3	24
480	630	60	30
100	560	90	560
1200	7200	1800	700
1820	8439	1953	1314

6.	7.	8.
82	91	23
× 45	× 68	× 49
10	8	27
400	720	180
80	60	120
3200	5400	800
3690	6188	1127

C-180

Compute.

1.	2.	3.	4.
32	42	35	23
× 86	× 27	× 69	× 37
12	14	45	21
180	280	270	140
160	40	300	90
2400	800	1800	600
2752	1134	2415	851

5.	6.	7.	8.
59	61	14	33
× 79	× 56	× 48	× 24
81	6	32	12
450	360	80	120
630	50	160	60
3500	3000	400	600
4661	3416	672	792

9.	10.	11.	12.
48	98	75	56
× 57	× 48	× 54	× 69
56	64	20	54
280	720	280	450
400	320	250	360
2000	3600	3500	3000
2736	4704	4050	3864

C-182

Name

47
× 82
14 (ones × ones)
80 (ones × tens)
560 (tens × ones)
3200 (tens × tens)
3854 (82 × 47)

1. How many partial products? 4
2. Which digits do you multiply to get ten tens? 8, 4
3. Which digits do you multiply to get tens? 2, 4 8, 7
4. Which digits do you multiply to get ones? 2, 7

Compute.

5.	6.	7.
26	12	47
× 13	× 12	× 43
18	4	21
60	20	120
60	20	280
200	100	1600
338	144	2021

8.	9.	10.
35	23	53
× 25	× 12	× 63
25	6	9
150	40	150
100	30	180
600	200	3000
875	276	3339

reference page

C-181

Assign page 180 as independent work. Then provide the answers so the children can check their own responses. Discuss problems that may have caused difficulty.

● Pages 181 and 182 provide further practice in computing products of two-digit factors. Examine the example at the top of page 181 with the class and discuss each question in turn.

Ask how many partial products will be involved in each exercise on the lower half of the page. (4) Discuss the importance of keeping the columns in line when listing the partial products; point out that there is less chance of careless mistakes being made when a little thought is given to neatness. Assign the exercises for independent work.

The children should complete the exercises on page 182 on their own. When work is completed, let the children tell the partial products and the product completed in each exercise.

● Pages 183 through 186 provide additional practice in computing products that involve two two-digit factors. It is not necessary that every child complete all of the exercises on these pages; some children may not need this much additional practice. It is suggested that only a few exercises be assigned at one time. You may want to assign one page now and return to the other pages as needed for daily practice.

Name

Compute.

No.	Problem	Partial products	Answer
1.	12 × 24	8, 40, 40, 200	288
2.	37 × 13	21, 90, 70, 300	481
3.	87 × 11	7, 80, 70, 800	957
4.	16 × 16	36, 60, 60, 100	256
5.	18 × 19	72, 90, 80, 100	342
6.	25 × 13	15, 60, 50, 200	325
7.	45 × 24	20, 160, 100, 800	1080
8.	23 × 13	9, 60, 30, 200	299
9.	34 × 21	4, 30, 80, 600	714
10.	28 × 25	40, 100, 160, 400	700
11.	82 × 62	4, 160, 120, 4800	5084
12.	81 × 63	3, 240, 60, 4800	5103

C-183

Name

Compute.

No.	Problem	Partial products	Answer
1.	21 × 21	1, 20, 20, 400	441
2.	26 × 14	24, 80, 60, 200	364
3.	35 × 37	35, 210, 150, 900	1295
4.	39 × 41	9, 30, 360, 1200	1599
5.	49 × 24	36, 160, 180, 800	1176
6.	47 × 11	7, 40, 70, 400	517
7.	23 × 24	12, 80, 60, 400	552
8.	12 × 49	18, 90, 80, 400	588
9.	72 × 28	16, 560, 40, 1400	2016
10.	21 × 43	3, 60, 40, 800	903
11.	18 × 17	56, 70, 80, 100	306
12.	14 × 93	12, 30, 360, 900	1302

C-185

Compute.

No.	Problem	Partial products	Answer
1.	19 × 14	36, 40, 90, 100	266
2.	53 × 52	6, 100, 150, 2500	2756
3.	56 × 11	6, 50, 60, 500	616
4.	29 × 36	54, 120, 270, 600	1044
5.	63 × 62	6, 120, 180, 3600	3906
6.	36 × 48	48, 240, 240, 1200	1728
7.	72 × 84	8, 280, 160, 5600	6048
8.	32 × 97	14, 210, 180, 2700	3104
9.	96 × 95	30, 450, 540, 8100	9120
10.	73 × 52	6, 140, 150, 3500	3796
11.	84 × 83	12, 240, 320, 6400	6972
12.	54 × 69	36, 450, 240, 3000	3726

C-184

Compute.

No.	Problem	Partial products	Answer
1.	36 × 24	24, 120, 120, 600	864
2.	74 × 82	8, 140, 320, 5600	6068
3.	46 × 76	36, 240, 420, 2800	3496
4.	73 × 62	6, 140, 180, 4200	4526
5.	61 × 35	5, 300, 30, 1800	2135
6.	83 × 68	24, 640, 180, 4800	5644
7.	36 × 39	54, 270, 180, 900	1404
8.	92 × 73	6, 270, 140, 6300	6716
9.	98 × 69	72, 810, 480, 5400	6762
10.	97 × 53	21, 270, 350, 4500	5141
11.	92 × 82	4, 180, 160, 7200	7544
12.	74 × 79	36, 630, 280, 4900	5846

C-186

Supplemental Experiences

■ Draw the following multiplication tables on the chalkboard.

×	2	20
10		
30		
50		
70		
90		

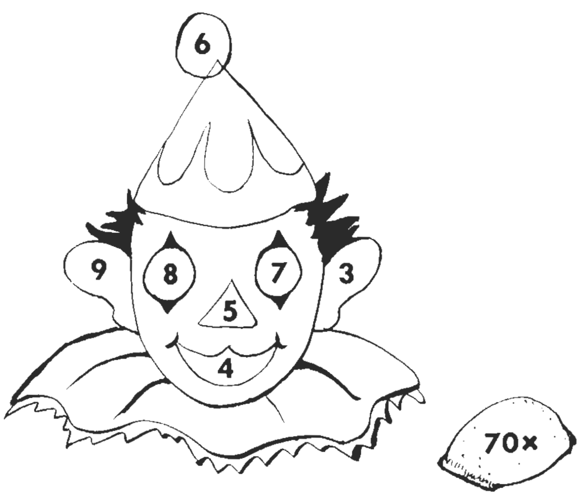
×	4	40
90		
70		
50		
30		
10		

×	6	60
20		
40		
60		
80		
50		

×	8	80
40		
30		
70		
50		
90		

Have the children take turns recording the two products in each row of the tables. When the tables are completed, let the class discuss the patterns in each row.

■ Draw a large clown face on the chalkboard. Write a numeral on each eye and ear, and on his nose, mouth, and hat. Near the clown draw a small bag and write 70×   on it.



Tell a child to imagine that he is throwing the bean bag at the clown and that he can hit any numeral he chooses. If he chooses to hit the clown's nose (5), he must compute 70 × 5 and say 350. Then he should call on another child to tell which numeral he hit. Change the numeral on the bean bag at various times during the game.

KEY IDEA

The standard numeral abbreviates the expanded notation.

Scope

To practice multiplying factors that have two or more digits.

Fundamentals

The number of partial products in the expanded algorism is always the product of the number of digits in each factor. For example, the computation of 234 × 567 will involve 3×3, or 9, partial products.

$$\begin{array}{r} 234 = 200 + 30 + 4 \\ \underline{\times\ 567} = \underline{500 + 60 + 7} \end{array}$$

The partial products to be computed are as follows:

7 ones × 4 ones 7 ones × 3 tens 7 ones × 2 hundreds	7 ones × the value of each digit in 234
6 tens × 4 ones 6 tens × 3 tens 6 tens × 2 hundreds	6 tens × the value of each digit in 234
5 hundreds × 4 ones 5 hundreds × 3 tens 5 hundreds × 2 hundreds	5 hundreds × the value of each digit in 234

The computed sum of these partial products will be the computed product of 234 × 567.

In the next section the child will explore the possibility of using the identity property of 0 to abbreviate the algorism.

Developmental Experiences

for flannel board
tagboard cards ($2\frac{1}{2}'' \times 2''$)
felt numerals and times sign
2 boxes
felt-tip pen

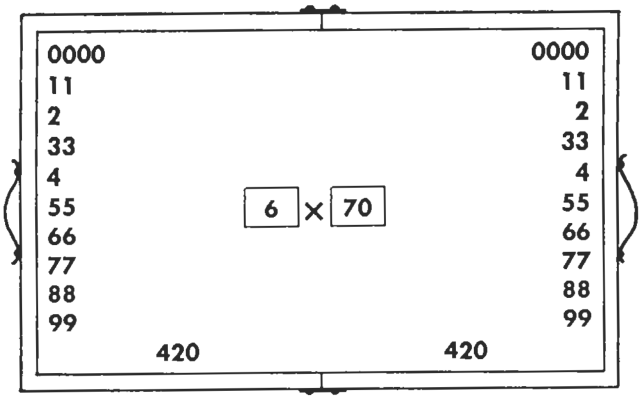
for each child
22 tagboard cards ($1'' \times 1\frac{1}{2}''$)

▶ Use $2\frac{1}{2}$ inch by 2 inch pieces of tagboard to design two sets of the number-cards illustrated.

1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90
100	200	300	400	500	600	700	800	900

Prepare the cards for use on the flannel board. Put one set of these cards in one box and one set in another. Place on each side of the flannel board two sets of felt numerals for 1 through 9, and 5 zeros. Place a felt times sign in the center of the board.

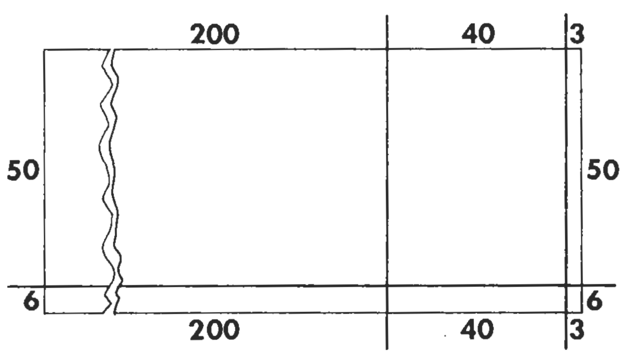
Separate the class into two teams and assign a box of numeral-cards and one side of the flannel board to each team. Ask a member from each team to come forward and to choose a card from the team's box. Tell these children to place their cards beside the times sign to form a product. Then instruct each child to choose the numerals he needs to show the count for the given product. Tell each child to put his count in a convenient place on his side of the board.



The children may earn points for their team in the following ways: 1 point is earned for completing the assignment correctly; 1 point is earned for being first to complete the assignment. Tell the children to lay to one side the number-cards that have been used and put back all of the felt numerals.

Continue this activity until all of the children have participated. Then let each team total its points. Continue the game for two or three more rounds; then total the scores to find the winner.

▶ Write on the chalkboard the product 243 × 56 in vertical form. Have a child show the expanded notation for each factor. (Some review of expanded notation may be necessary.) Then draw on the chalkboard the following illustration to show the parts of each factor in the given product.



Have the class compute each partial product and the sum of these products. Record the results of the computations below the expanded form for the factors. To summarize the ideas involved in this computation, ask a child to come to the chalkboard to describe the thinking steps used in computing each partial product.

	243 = 200 + 40 + 3
	× 56 = 50 + 6
(6 × 3 is 18)	18
(6 × 4 tens is 24 tens)	240
(6 × 2 hundreds is 12 hundreds)	1200
(5 tens × 3 is 15 tens)	150
(5 tens × 4 tens is 20 ten tens)	2000
(5 tens × 2 hundreds is 10 ten hundreds)	10000
	13,608

Tell the children that they may examine the multiplication algorism on the chalkboard to answer the following questions:

Which digits do you multiply to get ones; that is, which digits refer to a product of ones? (6 and 3) Have a child point out the section that represents this partial product in the illustration of the product on the chalkboard.

Which digits do you multiply to get tens; that is, which digits refer to a product (or products) of tens? (6 and 4, 5 and 3) Have a child point out, in the illustration on the chalkboard, the parts that represent these two partial products.

Which digits do you multiply to get ten tens or hundreds; that is, which digits refer to a product (or products) of ten tens or hundreds? (6 and 2, 5 and 4) Have a child point out the corresponding parts in the illustration.

Which digits do you multiply to get ten hundreds; that is, which digits refer to a product of ten hundreds? (5 and 2) Have a child point out the appropriate part in the illustration.

Adapt this procedure to other products such as 212 × 33, 346 × 11, 598 × 7, 231 × 2, and 76 × 4.

▶ Write on the chalkboard the product 473 × 68 in vertical form. Have six children take turns and each compute just one partial product. Direct a seventh child to compute the sum of products.

```
   473
 ×  68
    24
   560
  3200
   180
  4200
 24000
32,164
```

Ask six other children to take turns and describe how each partial product is derived. Comments similar to the following may be made:

24 is the product of 8 and 3; 24 is the product of ones.

560 (56 tens) is the product of 8 and 7 tens; 180 (18 tens) is the product of 6 tens and 3; 560 and 180 are products of tens.

3200 is the product of 8 and 4 hundreds; 4200 is the product of 6 tens and 7 tens; 3200 and 4200 are products of ten tens, or hundreds.

24,000 is the product of 6 tens and 4 hundreds; 24,000 is the product of ten hundreds.

Continue to let the children take turns computing products such as 126 × 21, 417 × 34, 111 × 67, 349 × 8, and 76 × 5. After each product has been computed, have the children describe how each partial product was determined.

Name

For Class Discussion

415	= 400 + 10 + 5	
× 23	= 20 + 3	
15	(3 ones × 5 ones)	
30	(3 ones × 1 tens)	
1200	(3 ones × 4 hundreds)	
100	(2 tens × 5 ones)	
200	(2 tens × 1 tens)	
8000	(2 tens × 4 hundreds)	
9545	(23 × 415)	

1. How many partial products are there? 6

2. Which digits do you multiply to get ten hundreds? 4, 2

ten tens? 2, 1 3, 4

tens? 2, 5 3, 1

ones? 3, 5

Compute.

3. 246 × 72

12	(ones × ones)
80	(ones × tens)
400	(ones × hundreds)
420	(tens × ones)
2800	(tens × tens)
14000	(tens × hundreds)
17,712	

4. 512 × 29

18
90
4500
40
200
10000
14848

reference page

C-187

Compute.

1. 412 × 22

4	(ones × ones)
20	(ones × tens)
800	(ones × hundreds)
40	(tens × ones)
200	(tens × tens)
8000	(tens × hundreds)
9064	

2. 314 × 23

12
30
900
80
200
6000
7222

3. 364 × 32

8
120
600
120
1800
9000
11,648

4. 183 × 92

6
160
200
270
7200
9000
16,836

5. 261 × 47

7
420
1400
40
2400
8000
12,267

C-188

Pages 187 through 196

● Use page 187 for class discussion. Copy on the chalkboard the example shown at the top of the page. As the children discuss each question, they may come to the chalkboard, point to the part of the algorism being considered, and write the appropriate numerals in the explanations for each partial product.

Work the two exercises at the bottom of the page with the class. Ask the same questions about each of these exercises. You might have the children help solve one of these exercises on a sheet of tagboard and then post this in the classroom as a reference.

As you discuss the partial products that result when a two-digit factor and a three-digit factor are multiplied, help the children understand that there is only one way of getting ones—ones times ones; there are two ways of getting tens—ones times tens and tens times ones; there are two ways of getting ten tens or hundreds—ones times hundreds and tens times tens; and there is one way of getting ten hundreds or thousands—tens times hundreds.

Name

Compute.

1. 324 × 12

8	(2 ones × 4 ones)
40	(2 ones × 2 tens)
600	(2 ones × 3 hundreds)
40	(1 tens × 4 ones)
200	(1 tens × 2 tens)
3000	(1 tens × 3 hundreds)
3888	

2. 833 × 23

9
90
2400
60
600
16000
19,159

3. 725 × 25

25
100
3500
100
400
14000
18,125

4. 722 × 43

6
60
2100
80
800
28000
31,046

5. 418 × 29

72
90
3600
160
200
8000
12,122

6. 236 × 37

42
210
1400
180
900
6000
8732

7. 331 × 13

3
90
900
10
300
3000
4303

8. 662 × 34

8
240
2400
60
1800
18000
22,508

reference page

C-189

Copy and compute.

1. 225 × 14 225 × 14 20 80 800 50 200 2000 3150	2. 834 × 23 834 × 23 12 90 2400 80 600 16000 19,182	3. 28 × 713 713 × 28 24 80 5600 60 200 14000 19,964
4. 89 × 439 439 × 89 81 270 3600 720 2400 32000 39,071	5. 148 × 25 148 × 25 40 200 500 160 800 2000 3700	6. 687 × 94 687 × 94 28 320 2400 630 7200 54000 64,578
7. 488 × 75 488 × 75 40 400 2000 560 5600 28000 36,600	8. 24 × 596 596 × 24 24 360 2000 120 1800 10000 14,304	9. 67 × 555 555 × 67 35 350 3500 300 3000 30000 37,185

reference page

C-190

Name

Compute.

1. 496 × 78 48 720 3200 420 6300 28000 38,688	2. 596 × 63 18 270 1500 360 5400 30000 37,548	3. 685 × 57 35 560 4200 250 4000 30000 39,045
4. 769 × 69 81 540 6300 540 3600 42000 53,061	5. 154 × 68 32 400 800 240 3000 6000 10,472	6. 564 × 85 20 300 2500 320 4800 40000 47,940
7. 785 × 67 35 560 4900 300 4800 42000 52,595	8. 678 × 94 32 280 2400 720 6300 54000 63,732	9. 518 × 37 56 70 3500 240 300 15000 19,166

C-191

● Pages 188 through 192 provide practice in computing products that have a two-digit factor and a three-digit factor. Do not assign any more than one page of exercises at any one time. Some of the children may benefit from doing only one row as an assignment. Use only as many exercises as are needed for the children to feel successful and independent.

Copy and compute.

1. 121 × 98	2. 39 × 412	3. 825 × 48
4. 971 × 71	5. 749 × 56	6. 84 × 179
7. 47 × 612	8. 28 × 814	9. 378 × 36

1. 121 × 98 8 160 800 90 1800 9000 11,858	2. 412 × 39 18 90 3600 60 300 12,000 16,068	3. 825 × 48 40 160 6400 200 800 32000 39,600
4. 971 × 71 1 70 900 70 4900 63000 68,941	5. 749 × 56 54 240 4200 450 2000 35000 41,944	6. 179 × 84 36 280 400 720 5600 8000 15,036
7. 612 × 47 14 70 4200 80 400 24000 28,764	8. 814 × 28 32 80 6400 80 200 16000 22,792	9. 378 × 36 48 420 1800 240 2100 9000 13,608

C-192

Name

Compute.

1. 73 = 70 + 3
× 7 = ___ 7
21
490
511

2. 259 = 200 + 50 + 9
× 6 = ___ 6
54
300
1200
1554

3. 841
× 8
8
320
6400
6728

4. 53
× 9
27
450
477

5. 517
× 5
35
50
2500
2585

6. 48
× 8
64
320
384

C-193

Name

Compute.

1. 73
× 9
27
630
657

2. 666
× 5
30
300
3000
3330

3. 67
× 9
63
540
603

4. 126
× 7
42
140
700
882

5. 15
× 17
35
70
50
100
255

6. 94
× 48
32
720
160
3600
4512

7. 99
× 99
81
810
810
8100
9801

8. 42
× 63
6
120
120
2400
2646

9. 212
× 34
8
40
800
60
300
6000
7208

10. 524
× 32
8
40
1000
120
600
15000
16,768

11. 657
× 87
49
350
4200
560
4000
48000
57,159

12. 893
× 67
21
630
5600
180
5400
48000
59,831

C-195

Compute.

1. 683
× 7
21
560
4200
4781

2. 999
× 2
18
180
1800
1998

3. 36
× 8
48
240
288

4. 754
× 9
36
450
6300
6786

5. 24
× 6
24
120
144

6. 569
× 3
27
180
1500
1707

7. 664
× 6
24
360
3600
3984

8. 457
× 7
49
350
2800
3199

9. 24
× 7
28
140
168

10. 35
× 9
45
270
315

11. 834
× 4
16
120
3200
3336

12. 998
× 3
24
270
2700
2994

C-194

● Pages 193 and 194 provide more practice in computing products that have a one-digit factor. Work the exercises at the top of page 193 with the class. Discuss the number of partial products in each example, and the numbers that are multiplied in each partial product. Then have the children complete exercises 3 through 6. Let them discuss their results. Assign page 194 for independent work. After all of the exercises have been completed, ask various children to explain to the class how they worked one of their exercises.

● Pages 195 and 196 give the children an opportunity to compute all of the types of products they have worked with in this unit thus far. Before making any assignment, have the children tell the number of partial products for each exercise on page 195.

NUMBER OF PARTIAL PRODUCTS

1. 2	2. 3	3. 2	4. 3
5. 4	6. 4	7. 4	8. 4
9. 6	10. 6	11. 6	12. 6

Copy and compute.

1. 8 × 57	2. 47 × 39	3. 873 × 63	4. 642 × 7
5. 442 × 23	6. 346 × 94	7. 78 × 87	8. 4 × 74
9. 7 × 439	10. 52 × 46	11. 74 × 6	12. 857 × 69

1. 57 × 8: 56, 400; 456
2. 39 × 47: 63, 210, 360, 1200; 1833
3. 873 × 63: 9, 210, 2400, 180, 4200, 48000; 54,999
4. 642 × 7: 14, 280, 4200; 4494
5. 442 × 23: 6, 120, 1200, 40, 800, 8000; 10,166
6. 346 × 94: 24, 160, 1200, 540, 3600, 27000; 32,524
7. 87 × 78: 56, 640, 490, 5600; 6786
8. 74 × 4: 16, 280; 296
9. 439 × 7: 63, 210, 2800; 3073
10. 52 × 46: 12, 300, 80, 2000; 2392
11. 74 × 6: 24, 420; 444
12. 857 × 69: 63, 450, 7200, 420, 3000, 48000; 59,133

C-196

Developmental Experiences

▶ Give each child 22 tagboard cards (1 inch by $1\frac{1}{2}$ inches) and tell the children to number the cards as shown.

0 0 0 0

1 2 3 4 5 6 7 8 9

1 2 3 4 5 6 7 8 9

Write on the chalkboard any of these products:

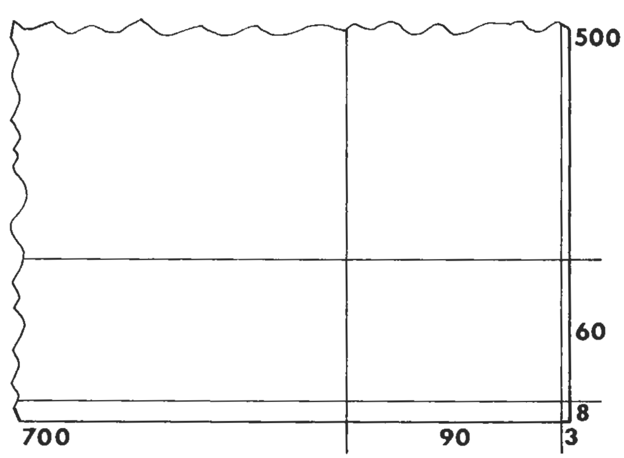
6×6
6×60
6×600
60×60
60×600
600×600

Have the children use their tagboard numeral-cards on their desks to show the count for the product on the board. Have someone read this count aloud. Continue this procedure by using the other five products one at a time. Ask the children to show the count for each product and to read this count aloud. The counts for all of the products that are greater than 36 may be read in various ways.

360	(3 hundred 60; 36 tens)
3,600	3 thousand 6 hundred; 36 hundreds; 36 ten tens; 360 tens
36,000	36 thousand; 36 hundred tens; 36 ten ten tens; 360 hundreds; 360 ten tens; 3600 tens
360,000	360 thousand; 360 ten ten tens; 36 hundred hundreds; 36 ten ten ten tens

Adapt this procedure to several other sets of six products that involve basic multiplication combinations and multiples of 10.

▶ Write on the chalkboard the product 568×793 in vertical form. Ask a child to write the expanded notation for each factor. Then draw on the chalkboard an illustration that shows the parts of the factors in the given product.



Have the class compute each partial product and the sum of these products. Record the results of the computations below the expanded form for the factors. To summarize the ideas involved in this computation, have some child come to the chalkboard and describe the computation of each partial product.

568 = 500 + 60 + 8		
× 793 = 700 + 90 + 3		
	24	(3 × 8 is 24)
	180	(3 × 6 tens is 18 tens)
	1500	(3 × 5 hundreds is 15 hundreds)
	720	(9 tens × 8 is 72 tens)
	5400	(9 tens × 6 tens is 54 ten tens)
	45000	(9 tens × 5 hundreds is 45 ten hundreds)
	5600	(7 hundreds × 8 is 56 hundreds)
	42000	(7 hundreds × 6 tens is 42 hundred tens)
	350000	(7 hundreds × 5 hundreds is 35 hundred hundreds)
	450,424	

Leave the multiplication algorism on the chalkboard as a reference. Direct the children to answer the following questions and to demonstrate their reasoning by pointing out the appropriate sections of the algorism.

Which digits do you multiply to get ones; that is, which digits refer to a product of ones? (8 and 3)

Which digits do you multiply to get tens; that is, which digits refer to a product (or products) of tens? (3 and 6, 9 and 8)

Which digits do you multiply to get ten tens or hundreds; that is, which digits refer to a product of ten tens or hundreds? (3 and 5, 9 and 6, 7 and 8)

Which digits do you multiply to get ten hundreds or hundred tens; that is, which digits refer to a product of ten hundreds or hundred tens? (9 and 5, 7 and 6)

Which digits do you multiply to get hundred hundreds; that is, which digits refer to a product of hundred hundreds? (7 and 5)

Adapt this procedure to other products such as 123 × 321, 412 × 233, 536 × 191, and 764 × 219.

▶ Write 675 × 943 in vertical form on the chalkboard. You might want the children to take turns and have each compute just one partial product. Then let another child compute the sum of the products.

```
   675
 × 943
    15
   210
  1800
   200
  2800
 24000
  4500
 63000
540000
636,525
```

When the algorism is complete, discuss how each partial product was derived. The children may offer comments similar to the following:

15 is the product 3 times 5; 15 is the product of ones.

210 (21 tens) is the product 3 times 7 tens; 200 (20 tens) is the product 4 tens times 5; 210 and 200 are products of tens.

1800 (18 hundreds) is the product 3 times 6 hundreds; 2800 (28 ten tens) is the product 4 tens times 7 tens; 4500 (45 hundreds) is the product 9 hundreds times 5; 1800, 2800, and 4500 are products of hundreds, or ten tens.

24,000 (24 ten hundreds) is the product 4 tens times 6 hundreds; 63,000 (63 hundred tens) is the product 9 hundreds times 7 tens; 24,000 and 63,000 are products of ten hundreds, or hundred tens.

540,000 (54 hundred hundreds) is the product 9 hundreds times 6 hundreds; 540,000 is the product of hundred hundreds.

Continue this activity; let the children take turns computing such products as 478 × 127, 395 × 212, and 679 × 879. Discuss each exercise; determine the way each partial product was derived.

Name

For Class Discussion

841 = 800 + 40 + 1		
× 325 = 300 + 20 + 5		
5	(5 ones × 1 ones)	
200	(5 ones × 4 tens)	
4000	(5 ones × 8 hundreds)	
20	(2 tens × 1 ones)	
800	(2 tens × 4 tens)	
16000	(2 tens × 8 hundreds)	
300	(3 hundreds × 1 ones)	
12000	(3 hundreds × 4 tens)	
240000	(3 hundreds × 8 hundreds)	
273,325	(325 × 841)	

1. How many partial products? 9

2. What digits do you multiply to get hundred hundreds? 3,8

ten hundreds? 3,4 2,8

hundreds? 2,4 5,8 3,1

tens? 5,4 2,1

ones? 5,1

reference page

C-197

Pages 197 through 204

● Use page 197 for class discussion. Copy on the chalkboard the example shown at the top of the page. As the children discuss each question, they may come to the chalkboard, point to the part of the algorism being considered, and write the appropriate numeral in the explanation of each partial product.

● Work the first two exercises on page 198 with the class. During the discussion of the partial products involved when two three-digit factors are multiplied, remind the children that there is only one way of getting ones—ones times ones; there are two ways of getting tens—ones times tens and tens times ones; there are three ways of getting ten tens or hundreds—ones times hundreds, tens times tens, and hundreds times ones; there are two ways of getting ten hundreds or thousands—tens times hundreds and hundreds times tens; and there is one way of getting hundred hundreds or ten thousands—hundreds times hundreds.

Have the children complete the last two exercises independently. Then let them discuss their computed partial products and products.

● Page 199 may be assigned for practice. Work exercise 1 with the class. Then tell the children to complete exercise 2 independently. After they are finished, ask a child to work the exercise at the chalkboard and to explain how he computed each partial product. This will give the other children an opportunity to check their computation. Assign exercises 3 through 5 for independent work.

Compute.

1. 213 × 231

3	(1 ones × 3 ones)
10	(1 ones × 1 tens)
200	(1 ones × 2 hundreds)
90	(3 tens × 3 ones)
300	(3 tens × 1 tens)
6000	(3 tens × 2 hundreds)
600	(2 hundreds × 3 ones)
2000	(2 hundreds × 1 tens)
40000	(2 hundreds × 2 hundreds)
49,203	(231 × 213)

2. 614 × 282	3. 532 × 164	4. 897 × 647
8	8	49
20	120	630
1200	2000	5600
320	120	280
800	1800	3600
48000	30000	32000
800	200	4200
2000	3000	54000
120000	50000	480000
173,148	87,248	580,359

C-198

Name

Compute.

1. 346 × 728		2. 538 × 515
48	(ones × ones)	40
320	(ones × tens)	150
2400	(ones × hundreds)	2500
120	(tens × ones)	80
800	(tens × tens)	300
6000	(tens × hundreds)	5000
4200	(hundreds × ones)	4000
28000	(hundreds × tens)	15000
210000	(hundreds × hundreds)	250000
251,888		277,070

3. 639 × 274	4. 726 × 436	5. 327 × 593
36	36	21
120	120	60
2400	4200	900
630	180	630
2100	600	1800
42000	21000	27000
1800	2400	3500
6000	8000	10000
120000	280000	150000
175,086	316,536	193,911

C-199

Copy and compute.

1. 873 × 712	2. 649 × 673	3. 857 × 857
873	649	857
× 712	× 673	× 857
6	27	49
140	120	350
1600	1800	5600
30	630	350
700	2800	2500
8000	42000	40000
2100	5400	5600
49000	24000	40000
560000	360000	640000
621,576	436,777	734,449

4. 975 × 567	5. 774 × 819	6. 358 × 674
975	774	358
× 567	× 819	× 674
35	36	32
490	630	200
6300	6300	1200
300	40	560
4200	700	3500
54000	7000	21000
2500	3200	4800
35000	56000	30000
450000	560000	180000
552,825	633,906	241,292

C-200

Compute.

1.	2.	3.
483	234	179
× 352	× 678	× 118
6	32	72
160	240	560
800	1600	800
150	280	90
4000	2100	700
20000	14000	1000
900	2400	900
24000	18000	7000
120000	120000	10000
170,016	158,652	21,122

4.	5.	6.
543	948	954
× 432	× 752	× 679
6	16	36
80	80	450
1000	1800	8100
90	400	280
1200	2000	3500
15000	45000	63000
1200	5600	2400
16000	28000	30000
200000	630000	540000
234,576	712,896	647,766

C-202

Name

Compute.

1.	2.	3.
123	226	317
× 322	× 214	× 218
6	24	56
40	80	80
200	800	2400
60	60	70
400	200	100
2000	2000	3000
900	1200	1400
6000	4000	2000
30000	40000	60000
39,606	48,364	69,106

4.	5.	6.
238	313	132
× 313	× 212	× 242
24	6	4
90	20	60
600	600	200
80	30	80
300	100	1200
2000	3000	4000
2400	600	400
9000	2000	6000
60000	60000	20000
74,494	66,356	31,944

C-201

● Pages 200 through 204 provide practice in computing products that have two three-digit factors. When assigning these pages, you might ask the children to name the expanded form for the numerals in each exercise. Then let them complete the exercises independently. Do not assign more than one page of exercises at any one time. There may be more exercises than are needed at this time to help pupils feel confident and successful.

To introduce page 204, ask the children to name different products of the given factors. List only a few of them on the chalkboard.

As the children work on the page, most will compute only the products of two factors:

$$427 \times 354 \qquad 427 \times 689 \qquad 427 \times 768$$
$$354 \times 689 \qquad 354 \times 768 \qquad 689 \times 768$$

Some pupils may think of products of three factors and try to compute some of the following:

$$427 \times 354 \times 689$$
$$427 \times 354 \times 768$$
$$427 \times 689 \times 768$$
$$354 \times 689 \times 768$$

Someone may even try to compute the product of four factors:

$$427 \times 354 \times 689 \times 768$$

Do not force children to discover and compute a maximum number of products on this page; rather encourage them to see what they can do.

Name

Multiply each number by 389.

1.	2.	3.
7 2 1	389	9 4 3
× 389	× 5 6 8	× 389
9	72	27
180	640	360
6300	2400	8100
80	540	240
1600	4800	3200
56000	18000	72000
300	4500	900
6000	40000	12000
210000	150000	270000
280,469	220,952	366,827

Multiply each number by 457.

4.	5.	6.
457	2 9 3	4 8 8
× 6 7 5	× 457	× 457
35	21	56
250	630	560
2000	1400	2800
490	150	400
3500	4500	4000
28000	10000	20000
4200	1200	3200
30000	36000	32000
240000	80000	160000
308,475	133,901	223,016

C-203

What products can you compute using these numbers?

427 354 689 768

354	689	768
×427	×427	427
28	63	56
350	560	420
2100	4200	4900
80	180	160
1000	1600	1200
6000	12000	14000
1600	3600	3200
20000	32000	24000
120000	240000	280000
151,158	294,203	327,936
689	768	689
×354	×354	×768
36	32	72
320	240	640
2400	2800	4800
450	400	540
4000	3000	4800
30000	35000	36000
2700	2400	6300
24000	18000	56000
180000	210000	420000
243,906	271,872	529,152

C-204

Supplemental Experiences

■ Have the children collect newspaper advertisements that contain pictures and prices of groceries. Tell the children to examine the ads; then have them select two or three items and create a story exercise about them. Record their ideas on the chalkboard. For example:

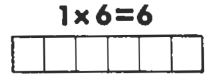
> Mother bought 5 small cans of beans at 19¢ a can, 4 large cans of peaches at 28¢ a can, and 4 bars of soap at 14¢ a bar. What was the total cost of these groceries?

Then have the children compute the answer to each story exercise they develop.

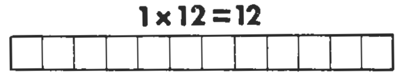
■ Give the children another experience that may help them understand the relationship between certain products. Cut out 20 tagboard strips (1 inch by 6 inches) and partition each strip into 6 one-inch squares. Prepare the strips for use on the flannel board. Ask a child to choose the strips he needs to show a 1 by 6 array. Tell a second child to use felt numerals and symbols to show the product and the count for this array in an equation. Direct him to place his equation above the array and to read his equation aloud.

Ask another child to construct a 1 by 12 array below the 1 by 6 array. Tell him to show the product and the count in an equation above his array.

1×6=6



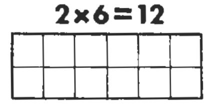
1×12=12



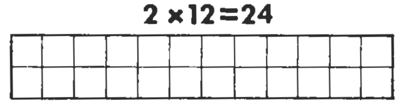
Then let him choose a member of the class to read his equation aloud. The class should observe that just as 12 is twice 6, the count of a 1 by 12 array is twice the count of a 1 by 6 array.

Remove both arrays and both equations from the flannel board. Have a child construct a 2 by 6 array and show the product and the count in an equation above his array. Let him call on someone to read his equation aloud. Then direct another child to construct a 2 by 12 array and to show the product and the count in an equation above his array.

2×6=12



2×12=24



Have the child choose a member of the class to read his equation aloud. The class should observe that 2×12 is twice 2×6.

Adapt this procedure to the pairs of products: 3×6 and 3×12, 4×6 and 4×12, through 10×6 and 10×12. The children may compute the standard numerals for the multiples of 12 by using their knowledge that these products will be twice the related multiples of 6. For example, knowing that $7 \times 6 = 42$, the child can compute 7×12 by multiplying 2×42. If he prefers, the child may add $42 + 42$. He is also free to multiply 7×12 directly.

KEY IDEA

The distributive property lets us multiply in parts—digit by digit.

Scope

To develop the idea that partial products of 0 may be omitted.

To provide practice in multiplication.

Fundamentals

It is clear that the computation of 500×200 involves only one basic multiplication fact. The computation of 5×2 gives us 10 and the result will be hundred hundreds.

$$\begin{array}{r} 500 \\ \times\ 200 \\ \hline 100{,}000 \end{array}$$

The child does not even need to know that 10 hundred hundreds is 1 hundred thousands in order to be able to compute the result.

In the last section it was established that the number of partial products involved in a many-digit computation is the product of the number of digits in each factor. We can look at 500×200 as the sum of nine partial products.

$$\begin{array}{rcr} 500 & = & 500 + 00 + 0 \\ \underline{\times\ 200} & = & \underline{200 + 00 + 0} \\ & & 0 \\ & & 0 \\ & & 0 \\ & & 0 \\ & & 0 \\ & & 0 \\ & & 0 \\ & & 0 \\ & & \underline{100000} \\ & & 100{,}000 \end{array}$$

While this is entirely correct, it is clearly possible to obtain the same result if all the 0 partial products are omitted as was done in the first computation. The 0 digits may be omitted in counting digits since they will produce partial products that do not affect the product.

$$\begin{array}{rcl} 304 & = & 300 + 4 \\ \underline{\times\ 102} & = & \underline{100 + 2} \end{array}$$

Note that the 0 tens is omitted in the expansion.

The exercise is seen to be a two-digit by two-digit exercise for purposes of computation.

$$\begin{array}{rl} 304 & \\ \underline{\times\ 102} & \\ 8 & \text{(2 ones} \times \text{4 ones)} \\ 600 & \text{(2 ones} \times \text{3 hundreds)} \\ 400 & \text{(1 hundred} \times \text{4 ones)} \\ \underline{30000} & \text{(1 hundred} \times \text{3 hundreds)} \\ 31{,}008 & \end{array}$$

The child should quickly see that it is not necessary to include the five possible 0 partial products; the identity property of 0 guarantees that omitting a partial product of 0 will not affect the result.

Readiness for Understanding

Ability to compute products by using partial products.

Developmental Experiences

▶ Separate the class into two teams and assign one panel of the chalkboard to each team. Call a member of each team to the board. Write 560 on one panel of the chalkboard and 4200 on the other. Tell the children that below the count written on their panel they are to write a product that represents this count. Explain that they are to use a basic multiplication combination in their product. A child may write the following products to represent 560: 8×70, 7×80, or the commuted form of either product. A child may represent 4200 as: 7×600, 6×700, 70×60, or as the commuted form of one of these three products.

Each child may earn points for his team as follows: 1 point is earned for completing the assignment correctly; 1 point is earned for being first to complete the assignment.

Continue to use basic multiplication combinations and multiples of 10 in the game until all of the children have participated. Then let each team total its points to determine the winner.

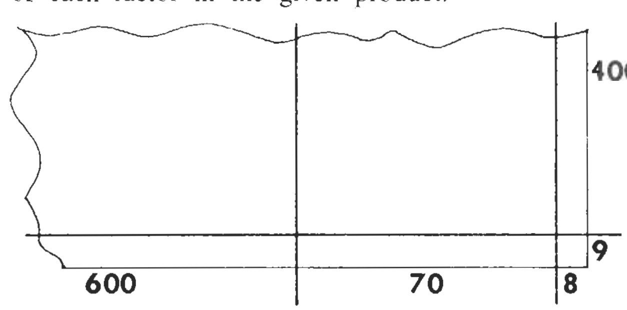
▶ Write 345 on the chalkboard and ask a child to write the expanded notation beside it.

$$345 = 300 + 40 + 5$$

Discuss with the class: the value of each digit can be observed in this notation—a value of hundreds plus tens plus ones.

Ask someone to write the expanded notation for 409. This child may write $400+0+9$, or he may write $400+9$. Ask another child to write the form that was not used. Discuss the idea that the notation $400+0+9$ gives the value of each digit—hundreds plus tens plus ones. The notation $400+9$ gives the value hundreds plus ones (4 hundreds + 9 ones), or the value tens plus ones (40 tens + 9 ones).

Have the class compute a product that has 0 as a digit in one of its factors. Write on the chalkboard the product 678×409 in vertical form. Ask a child to write the expanded notation for each factor. Reproduce the following illustration to show the parts of each factor in the given product.



Have each partial product as well as the sum of these products computed by the class. Record the results of the computations below the expanded form for the factors. To summarize the ideas involved in this computation, have one child come to the chalkboard and describe the thinking steps used to compute each partial product.

678 = 600 + 70 + 8	
× 409 = 400 + 9	
72	(9 × 8 is 72)
630	(9 × 7 tens is 63 tens)
5400	(9 × 6 hundreds is 54 hundreds)
3200	(4 hundreds × 8 is 32 hundreds)
28000	(4 hundreds × 7 tens is 28 hundred tens)
240000	(4 hundreds × 6 hundreds is 24 hundred hundreds)
277,302	

Have the class study the multiplication algorism on the chalkboard. Have the children answer the following questions:

Which digits do you multiply to get ones? (9 and 8)

Which digits do you multiply to get tens? (9 and 7. The partial product of 0 and 8 is 0 tens. This partial product is not considered significant and therefore is not recorded.)

Which digits do you multiply to get ten tens or hundreds? (9 and 6, 4 and 8. The partial product of 0 and 7 is 0 ten tens. This partial product is not considered significant and therefore is not recorded.)

Which digits do you multiply to get ten hundreds or hundred tens? (4 and 7. The partial product of 0 and 6 is 0 ten hundreds. This partial product is not considered significant and therefore is not recorded.)

Which digits do you multiply to get hundred hundreds? (4 and 6)

At various times, as the children discuss each partial product, have them point out the section of the illustration that corresponds to the partial product being considered.

Adapt this procedure to other products such as 426 × 500, 398 × 40, 706 × 89, 260 × 43, 697 × 304, 802 × 605, and 900 × 703.

▶ Write 306 × 420 in vertical form on the chalkboard. If desirable, allow several children to take turns and have each compute one partial product. Ask still another child to compute the sum of products.

```
    306
  × 420
    120
   6000
   2400
 120000
128,520
```

Now have the children who did not participate in computing the partial products and sum take turns and describe how each partial product is derived. The children might make the following comments:

120 (12 tens) is the product 2 tens times 6; 120 (12 tens) is the product of tens.

6000 (6 ten hundreds) is the product 2 tens times 3 hundreds; 2400 (24 hundreds) is the product 4 hundreds times 6; 6000 and 2400 are products of hundreds.

120,000 (120 thousands, or 12 hundred hundreds) is the product 4 hundreds times 3 hundreds; 120,000 is the product of hundred hundreds.

Continue this activity; have the children take turns computing such products as 904 × 68, 230 × 40, 580 × 207, 604 × 702, and 700 × 309.

Pages 205 through 214

● Use page 205 to investigate the computation of products when one or both of the numerals for the factors contain a zero digit. Work through the example at the top of the page with the class. Then discuss the question below the example.

Review the fact that multiplication by 0 doesn't have to be shown. 0 times any number is 0. A partial product of 0 will not affect the final result since 0 plus any number is that number.

Review with the children the fact that in previous exercises that involved a three-digit factor and a three-digit factor there were 9 partial products. Ask how many partial products were involved in the example 608 × 970. (4)

Next ask how many partial products will be involved in computing each product on page 205.

NUMBER OF PARTIAL PRODUCTS

1. 4	2. 2	3. 6	4. 4
5. 4	6. 4	7. 2	8. 3

Complete the first four exercises as a class activity. Assign the other exercises for independent work.

● Assign the exercises on page 206 for independent work. After the exercises are completed, ask individuals to tell the partial products that were computed in specific exercises.

● When using page 207, have the children estimate which product in each row is greatest before they compute the products. The computation can then serve as a means of checking their predictions.

Direct the children to study the factors in each exercise in row 1. Ask the children which products are greater than other products. List on the chalkboard the suggestions made by the children. Some possibilities are as follows:

30 is greater than 3, so the product in exercise 2 will be greater than the product in exercise 1.

33 is greater than 30 or 3, so the product in exercise 3 will be greater than the product in exercise 1 or 2.

333 is greater than 33, so the product in exercise 4 is the greatest number in the row.

Next instruct the children to compute the products in row 1 independently. When they have finished, use their results to check each of the predictions on the board.

Follow a similar procedure with each of the rows of exercises on page 207. It might be best to compute no more than two rows of exercises at one time.

● Pages 208 through 212 provide additional practice in computing products. The children also may note some patterns in the products in some of the rows on these pages. Allow the children to discuss any patterns they notice, and to predict results if they wish.

Assign the exercises on these pages for independent work; do not assign more than one page at any one time. Before each assignment, let the children tell the number of partial products that will be involved in computing each product in the assignment.

Name

$$\begin{array}{r} 900 + 70 \\ \times\ 600 + \ \ 8 \\ \hline 540000 + 42000 + 7200 + 560 \end{array} \qquad \begin{array}{r} 970 \\ \times\ 608 \\ \hline 560 \\ 7200 \\ 42000 \\ 540000 \\ \hline 589{,}760 \end{array}$$

Why are there only 4 partial products in the computation of 970 × 608?

See pupil page suggestions.

Compute.

1. 920 × 93: 60, 2700, 1800, 81000 — 85,560
2. 300 × 85: 1500, 24000 — 25,500
3. 132 × 102: 4, 60, 200, 200, 3000, 10000 — 13,464
4. 604 × 56: 24, 3600, 200, 30000 — 33,824
5. 207 × 803: 21, 600, 5600, 160000 — 166,221
6. 508 × 45: 40, 2500, 320, 20000 — 22,860
7. 709 × 50: 450, 35000 — 35,450
8. 173 × 900: 2700, 63000, 90000 — 155,700

reference page

C-205

Compute.

1. 506 × 300: 1800, 150000 — 151,800
2. 708 × 509: 72, 6300, 4000, 350000 — 360,372
3. 507 × 608: 56, 4000, 4200, 300000 — 308,256
4. 609 × 258: 72, 4800, 450, 30000, 1800, 120000 — 157,122
5. 888 × 208: 64, 640, 6400, 1600, 16000, 160000 — 184,704
6. 410 × 760: 600, 24000, 7000, 280000 — 311,600
7. 602 × 56: 12, 3600, 100, 30000 — 33,712
8. 901 × 850: 50, 45000, 800, 720000 — 765,850
9. 613 × 80: 240, 800, 48000 — 49,040
10. 659 × 258: 72, 400, 4800, 450, 2500, 30000, 1800, 10000, 120000 — 170,022
11. 306 × 90: 540, 27000 — 27,540
12. 402 × 78: 16, 3200, 140, 28000 — 31,356

C-206

Name

Copy and compute.

1. 33 × 3
   33 ×3 | 99
2. 33 × 30
   33 ×30 | 990
3. 33 × 33
   33 ×33 | 9, 90, 90, 900 | 1089
4. 333 × 33
   333 ×33 | 9, 90, 900, 90, 900, 9000 | 10,989
5. 74 × 47
   74 ×47 | 28, 490, 160, 2800 | 3478
6. 47 × 47
   47 ×47 | 49, 280, 280, 1600 | 2209
7. 74 × 44
   74 ×44 | 16, 280, 160, 2800 | 3256
8. 77 × 44
   77 ×44 | 28, 280, 280, 2800 | 3388
9. 777 × 7
   777 ×7 | 49, 490, 4900 | 5439
10. 777 × 707
    777 ×707 | 49, 490, 4900, 4900, 49000, 490000 | 549,339
11. 777 × 77
    777 ×77 | 49, 490, 4900, 490, 4900, 49000 | 59,829
12. 777 × 770
    777 ×770 | 490, 4900, 49000, 4900, 49000, 490000 | 598,290

C-207

Name

Compute.

1. 16 × 38 | 48, 80, 180, 300 | 608
2. 17 × 39 | 63, 90, 210, 300 | 663
3. 18 × 40 | 320, 400 | 720
4. 19 × 41 | 9, 10, 360, 400 | 779
5. 54 × 19 | 36, 450, 40, 500 | 1026
6. 54 × 29 | 36, 450, 80, 1000 | 1566
7. 54 × 39 | 36, 450, 120, 1500 | 2106
8. 54 × 49 | 36, 450, 160, 2000 | 2646
9. 456 × 7 | 42, 350, 2800 | 3192
10. 567 × 7 | 49, 420, 3500 | 3969
11. 678 × 7 | 56, 490, 4200 | 4746
12. 789 × 7 | 63, 560, 4900 | 5523

C-209

Compute.

1. 199 × 188 | 72, 720, 800, 720, 7200, 8000, 900, 9000, 10000 | 37,412
2. 189 × 189 | 81, 720, 900, 720, 6400, 8000, 900, 8000, 10000 | 35,721
3. 198 × 198 | 64, 720, 800, 720, 8100, 9000, 800, 9000, 10000 | 39,204
4. 189 × 198 | 72, 640, 800, 810, 7200, 9000, 900, 8000, 10000 | 37,422
5. 696 × 996 | 36, 540, 3600, 540, 8100, 54000, 5400, 81000, 540000 | 693,216
6. 696 × 969 | 54, 810, 5400, 360, 5400, 36000, 5400, 81000, 540000 | 674,424
7. 666 × 699 | 54, 540, 5400, 540, 5400, 54000, 3600, 36000, 360000 | 465,534
8. 999 × 966 | 54, 540, 5400, 540, 5400, 54000, 8100, 81000, 810000 | 965,034

C-208

Compute.

1. 516 × 222 | 12, 20, 1000, 120, 200, 10000, 1200, 2000, 100000 | 114,552
2. 171 × 333 | 3, 210, 300, 30, 2100, 3000, 300, 21000, 30000 | 56,943
3. 206 × 666 | 36, 1200, 360, 12000, 3600, 120000 | 137,196
4. 360 × 36 | 360, 1800, 1800, 9000 | 12,960
5. 246 × 48 | 48, 320, 1600, 240, 1600, 8000 | 11,808
6. 369 × 45 | 45, 300, 1500, 360, 2400, 12000 | 16,605
7. 468 × 43 | 24, 180, 1200, 320, 2400, 16000 | 20,124
8. 680 × 46 | 480, 3600, 3200, 24000 | 31,280

C-210

Name

Compute.

1. 175 × 56
30
420
600
250
3500
5000
9800

2. 567 × 108
56
480
4000
700
6000
50000
61,236

3. 467 × 358
56
480
3200
350
3000
20000
2100
18000
120000
167,186

4. 243 × 8
24
320
1600
1944

5. 24 × 38
32
160
120
600
912

6. 240 × 380
3200
16000
12000
60000
91,200

7. 813 × 3
9
30
2400
2439

8. 813 × 30
90
300
24000
24390

9. 813 × 300
900
3000
240000
243,900

C-211

● Page 213 is designed to add variety to practice with computation of products. The goal is given in each puzzle and can serve as a check on the computation—if the child computes correctly he will arrive at the given product.

Tell the children to study the example at the top of the page and ask them to explain the procedure to be followed. If possible, an explanation of the procedure should come from the children rather than from the teacher. Assign the exercises for independent work. Let two or three children work together if they wish. After all of the puzzles have been solved, let the children explain the steps used in specific puzzles.

● Use page 214 to provide more practice. Ask the children to copy these exercises on another sheet of paper and compute them independently. Have them record their answers in their books. Do not make any comment regarding the paragraph in the middle of the page unless the children ask about it. If the children find something they can use to compute the products in either column at the bottom of the page, let them use their secret. If not, they can compute to complete both sets of exercises at the bottom of the page. Allow the children to explore and discuss any conclusions they make concerning this page, but do not do their thinking for them and do not force them to draw conclusions.

Compute.

1. 99 × 45
45
450
360
3600
4455

2. 99 × 54
36
360
450
4500
5346

3. 999 × 450
450
4500
45000
3600
36000
360000
449,550

4. 220 × 222
40
400
400
4000
4000
40000
48,840

5. 222 × 22
4
40
400
40
400
4000
4884

6. 202 × 222
4
400
40
4000
400
40000
44,844

7. 5320 × 60
1200
18000
300000
319,200

8. 530 × 260
1800
30000
6000
100000
137,800

9. 53 × 26
18
300
60
1000
1378

C-212

Name

6 × 5 = 30

30 × 4 = 120

120 × 3 = 360

360 × 2 = 720

Solve these puzzles if you can.

1. 7 × 7 = 49

49 × 7 = 343

343 × 7 = 2401

2401 × 7 = 16,807

2. 4 × 14 = 56

56 × 35 = 1960

1960 × 40 = 78,400

78,400 × 11 = 862,400

3. 16 × 16 = 256

256 × 16 = 4096

4096 × 16 = 65,536

65,536 × 16 = 1,048,576

C-213

Compute. Copy these exercises on another sheet of paper if you wish.

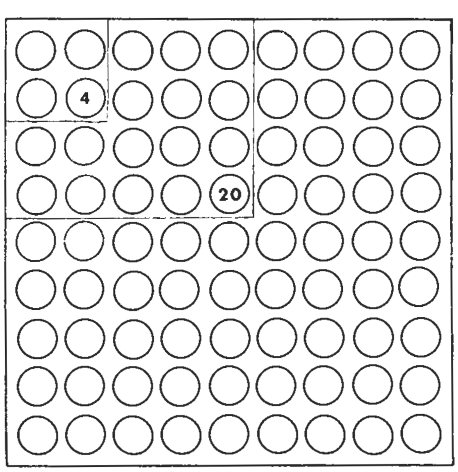
1. 36 × 36 = 1296
2. 35 × 37 = 1295
3. 43 × 43 = 1849
4. 42 × 44 = 1848
5. 57 × 57 = 3249
6. 56 × 58 = 3248
7. 63 × 63 = 3969
8. 62 × 64 = 3968
9. 89 × 89 = 7921
10. 88 × 90 = 7920
11. After John saw Judy's work on 89 × 89, he said that he could compute 88 × 90 in his head. What was John's secret? See pupil page suggestions.
12. 39 × 39 = 1521
13. 38 × 40 = 1520
14. 75 × 75 = 5625
15. 74 × 76 = 5624
16. 121 × 121 = 14,641
17. 120 × 122 = 14,640
18. 333 × 333 = 110,889
19. 332 × 334 = 110,888
20. 680 × 680 = 462,400
21. 679 × 681 = 462,399
22. 789 × 789 = 622,521
23. 788 × 790 = 622,520

C-214

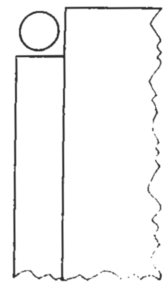
Supplemental Experiences

■ Direct each child to write a story exercise involving multiplication. Divide the class into two teams. Let each child take a turn reading his story for a member of the opposing team to answer. Also, give each child a turn answering. Keep score to determine the winning team.

■ Duplicate a copy of the following array for each child in the class.

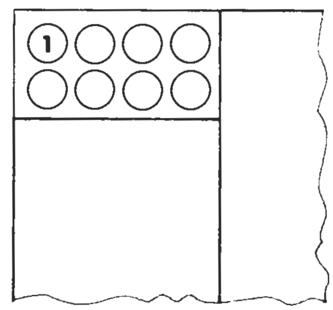


Using two sheets of paper, demonstrate how to block off all the circles except the one in the upper left-hand corner, as shown.

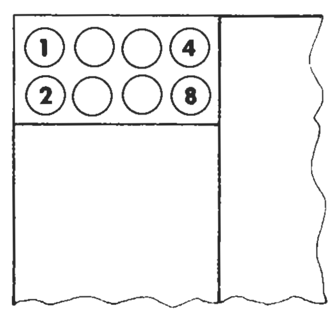


Ask the pupils to describe the array (1 by 1), to name the product (1 × 1 or 1), and to write the number 1 in the circle.

In a similar fashion show a second array.



Ask the pupils to describe the array (2 by 4), to name the product (2 × 4 or 8), and to write the number of horizontal rows in the upper right circle, the number of vertical rows in the lower left circle, and the product in the lower right circle, as shown.



Continue in this manner for other arrays until the table is complete.

The children may keep this array for reference for reviewing basic multiplication combinations.

■ Provide additional experiences that give the children further opportunities to observe the relationships between certain products. Cut out 20 tagboard strips (1 inch by 7 inches) and partition each strip into 7 one-inch squares. Prepare the strips for use on the flannel board. Ask a child to choose the strips he needs to make a 1 by 7 array. Tell a second child to use felt numerals and symbols to show the product and the count for this array in an equation. Direct him to place his equation above the array and to read his equation aloud.

Ask another child to construct a 1 by 14 array below the 1 by 7 array. Tell him to show the product and the count in an equation above his array.

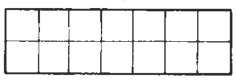
1×7=7

1×14=14

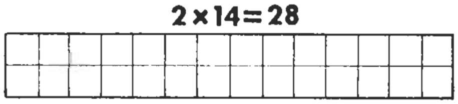
Have this child read his equation to the class. The class should understand that 14 is twice 7 and the count for 1×14 is twice the count for 1×7.

Remove both arrays and both equations from the flannel board. Select a child to construct a 2 by 7 array and to show the product and the count in an equation above this array. Let him call on someone to read his equation aloud. Then ask another child to construct a 2 by 14 array and to show the product and the count in an equation above this array.

2×7=14



2×14=28



Have him read his equation to the class. The class should observe that 2×14 is twice 2×7.

Adapt this procedure to the pairs of products: 3×7 and 3×14, 4×7 and 4×14, through 10×7 and 10×14. The children may compute the standard numerals for the multiples of 14 by using their knowledge that these products will be twice the related multiples of 7. For example, knowing that $8 \times 7 = 56$, the child can compute 8×14 by multiplying 2×56. If he prefers, the child may add $56 + 56$. He is also free to multiply 8×14 directly.

UNIT 13
AREA OF RECTANGLES

Pages 215 Through 228

OBJECTIVE

To measure area of rectangles.

The concept of area is further developed. The child determines the area as the number of square units that cover a rectangular figure. This may be done in two different ways, either by counting the square units or by finding the product of the number of squares in a row by the number of rows. The child reviews the concepts of perpendicular and parallel lines by participating in activities that involve squares, rectangles, and parallelograms.

KEY IDEAS

The area of a rectangle is the number of square units required to cover it.

The area of a rectangle is the length times the width.

The diagonals of a square are perpendicular.

CONCEPTS

parallel	shape
parallelogram	size
perpendicular	square
rectangle	square unit

KEY IDEA

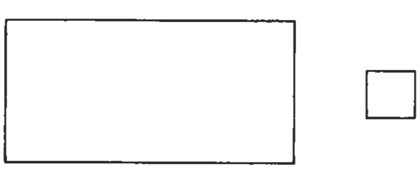
The area of a rectangle is the number of square units required to cover it.

Scope

To measure area of rectangles.

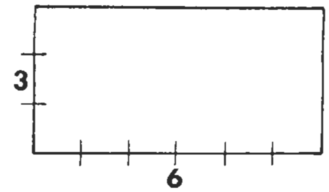
Fundamentals

The idea of area of a two-dimensional figure is best understood in terms of the amount of material required to cover a flat object. At the start, the child covers rectangular figures with square figures and counts the number of square figures needed. This enables each child to relate his knowledge of counting to the use of numbers in measuring quantity. Gradually the child is led to the direct measurement of quantity without counting square units. Consider the following figures.

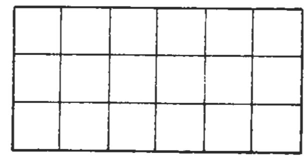


The area of the rectangle can be expressed as a comparison with the unit square.

The child will gradually perceive that measuring the length and width will enable him to compute the number of small squares needed to cover the rectangle.

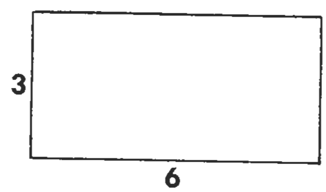


However, first it is necessary to help the child perceive area by covering the rectangle with small square units so that the child sees the array of small squares.



A 3 by 6 array of small square units.

After sufficient experience in covering figures, the child will not need the small square units. When only the length and width is furnished, he will conclude that the area is 3×6 small square units.



Area is 3×6 small square units.

Readiness for Understanding

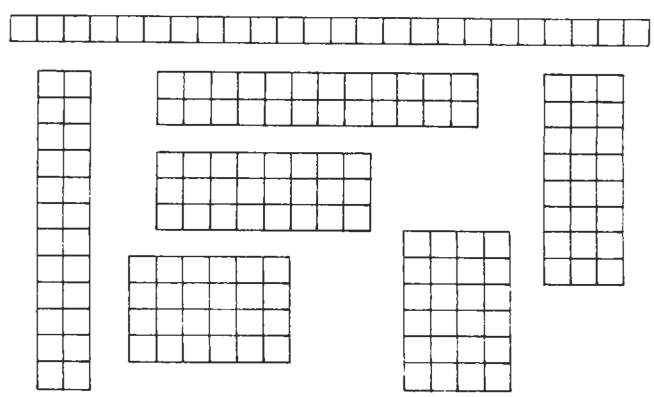
Ability to count.
Ability to multiply.

Developmental Experiences

for flannel board
tagboard rectangle (5″ × 8″)

for each child
graph paper (¼″)
crayons
tagboard squares (3″ × 3″ and 9″ × 9″)
pins
scissors

▶ Give a sheet of ¼-inch graph paper to each child. Write the numeral 24 on the chalkboard. Ask the children to outline a rectangle that contains exactly 24 square units. Any of the following rectangles may be formed.

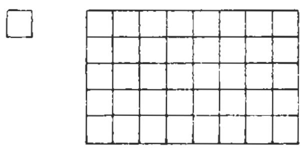


Encourage the children to discuss their ways of showing 24. Carry on this activity using other numerals such as 18, 36, or 48. Ask the children to look around the classroom to see if they can find any rectangles made up of arrays. Some might observe windows, desks, and ceiling or floor tiles. Let the children discuss these arrays. Help them realize that an array such as 6 by 8 is also an 8 by 6 array.

▶ Give each child a sheet of $\frac{1}{4}$-inch graph paper. Direct the children to color a rectangle that has 36 square units in its area. Ask the children to show and discuss the different ways they chose to do this.

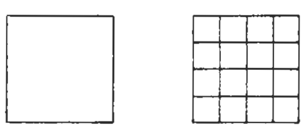
Some child may observe that 1 by 36 would be a way of showing 36 square units. Continue this activity using arrays with 40, 24, or 64 square units.

▶ Partition a 5 by 8 inch rectangle into a 5 by 8 array and pin it on the flannel board. Ask the children how many square units they see in the rectangle. Let them find the number of square units in any way they wish.

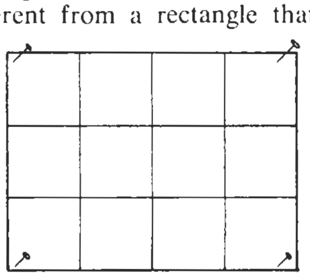


Give each child a piece of $\frac{1}{4}$-inch graph paper. Direct the class to outline a rectangle that has the same number of square units as the rectangle on the flannel board. Ask the children what they observed. They will probably see that their rectangle is smaller than the rectangle on the flannel board, because their square units are smaller.

Help a child cut out a graph-paper rectangle that is the same size as the one on the flannel board. Have him place it over the rectangle on the board to show the class that it is the same size. Ask the class to count the number of square units on each side of this rectangle. (20 by 32 square units) Ask if any child knows how many of these $\frac{1}{4}$-inch square units would cover a one-inch square unit. (16) Let some child cut a 1-inch square from his $\frac{1}{4}$-inch graph paper and place it on a 1-inch square.



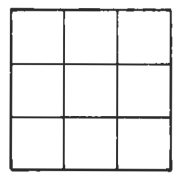
▶ Pin a rectangle on the flannel board and discuss how it is different from a rectangle that is a square.

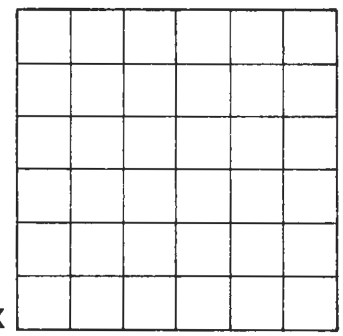


The children might say that the rectangle is longer or shorter one way than it is the other way. It has a different shape.

Ask the children if it is possible to rearrange the square units of this rectangle into a square rectangle, without adding any square units or removing any. The children will soon see that this cannot be done. Tell the children to imagine removing square units to make a square rectangle. Ask what is the least number of units that can be removed to leave a square rectangle. (3) Then tell the children to imagine adding square units to form a larger square rectangle. Ask them to figure out the fewest number of units needed to do this. (4) Select different children to show on the chalkboard how the array can be made into a 3 by 3 array and into a 4 by 4 array.

▶ Give a copy of these two figures to each child.

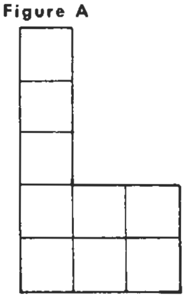

Y


X

Ask the children to compare the size of the figures. They may say *Y* is smaller than *X* or that *X* is larger than *Y*. Help them give a more detailed description. Guide them to use the idea of covering with a square unit. If they used the small rectangle to cover the large one, they should be able to see that the small rectangle is $\frac{1}{4}$ the size of the large rectangle—the large one is 4 times larger than the small one. Have the children try to draw a square that would be 9 times as large as the smaller square.

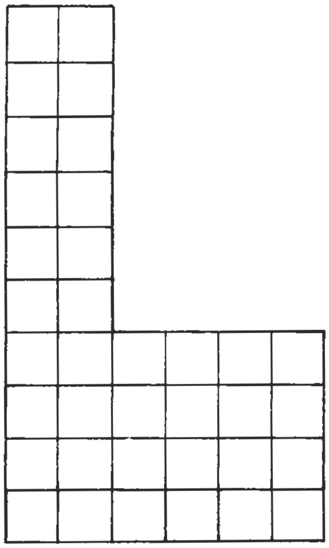
▶ Draw on the chalkboard a figure that has 9 square units arranged as illustrated.

Figure A



Ask some child to count the square units in the figure (Figure A). Let another child draw a figure on the chalkboard that has each side double that of Figure A.

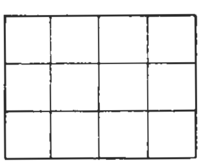
Figure B



Ask a child to count the square units in this figure (Figure B). Discuss what happened when the sides were doubled. The class should realize that when the sides are doubled the new figure contains four times as many square units as the original figure.

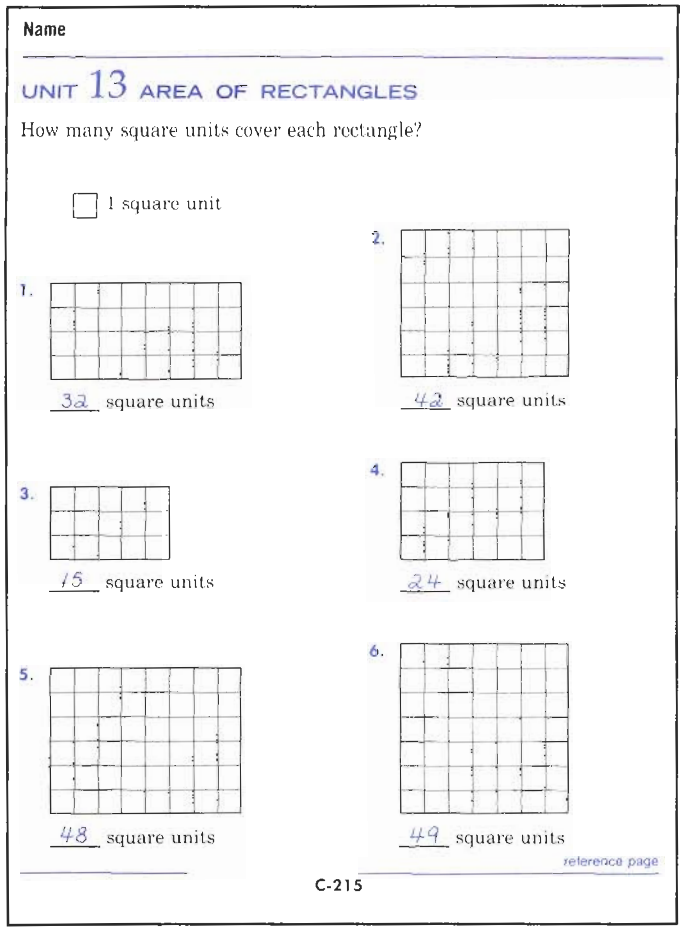
Pages 215 through 220

● Pages 215 and 216 provide experience in determining area by finding how many units it takes to cover a rectangle. Place on the chalkboard a rectangle that has as its measure a 3 by 4 array of units.



Ask children to count the units aloud to the class in several ways: by ones, by threes, by fours. Then tell the class to decide whether or not there are other ways to express the count. Someone may suggest that there are 3 rows and each row has 4 units. Someone else may say that there are 4 rows and each row has 3 units.

Assign the exercises on pages 215 and 216 to be done independently. On page 216, ask pupils to finish drawing the square units to check their results.



Name

UNIT 13 AREA OF RECTANGLES

How many square units cover each rectangle?

1 square unit

1. 32 square units

2. 42 square units

3. 15 square units

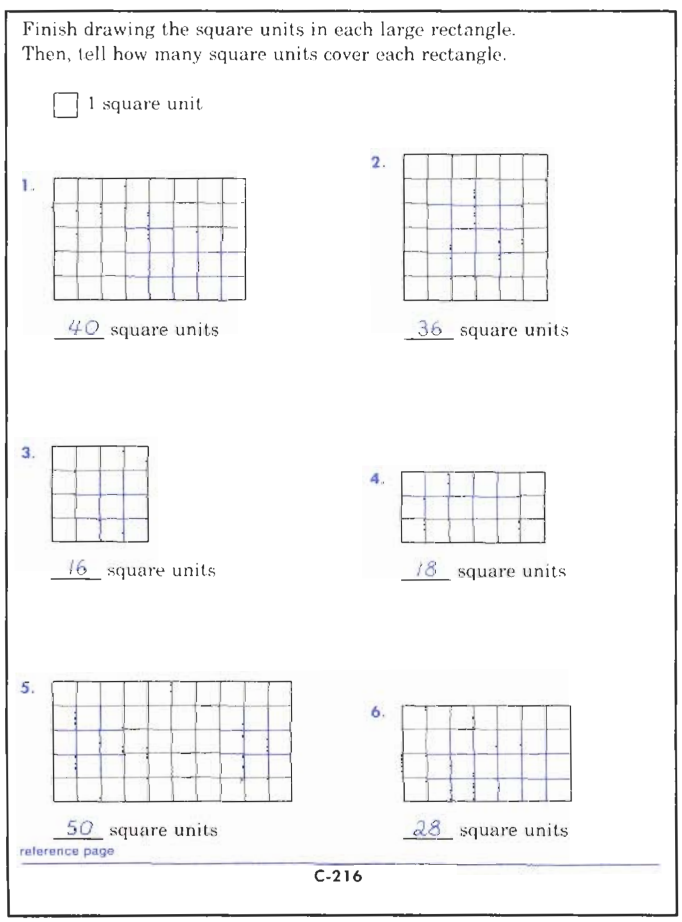
4. 24 square units

5. 48 square units

6. 49 square units

reference page

C-215



Finish drawing the square units in each large rectangle.
Then, tell how many square units cover each rectangle.

1 square unit

1. 40 square units

2. 36 square units

3. 16 square units

4. 18 square units

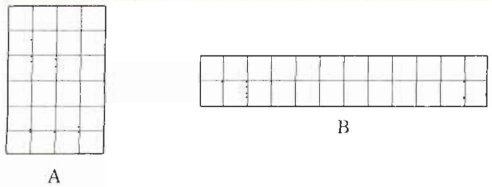
5. 50 square units

6. 28 square units

reference page

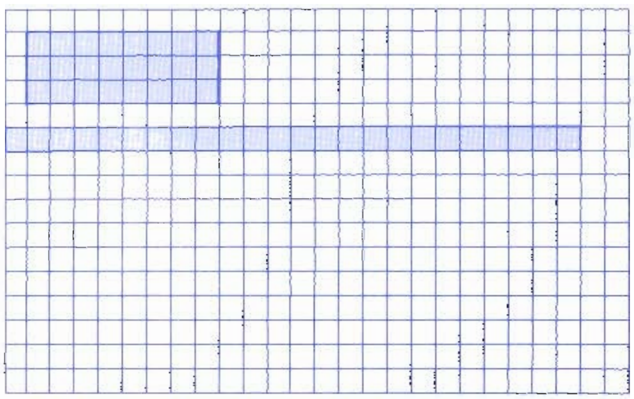
C-216

Name



1. How are rectangles A and B alike? *The same number of square units cover each of them.*

2. On the graph paper below, outline two more rectangles that have the same size as rectangles A and B but have different shapes.

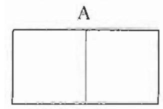


reference page

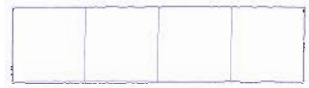
C-217

Name

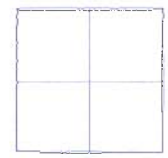
For Class Discussion

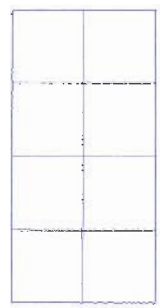


1. When the long side of rectangle A is doubled, what happens to the number of squares? *The number of squares is doubled.*



2. When the short side of rectangle A is doubled, what happens to the number of squares? *The number of squares is doubled.*



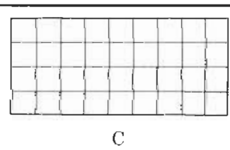


3. When both sides of rectangle A are doubled, what happens to the number of squares? *There are four times as many squares.*

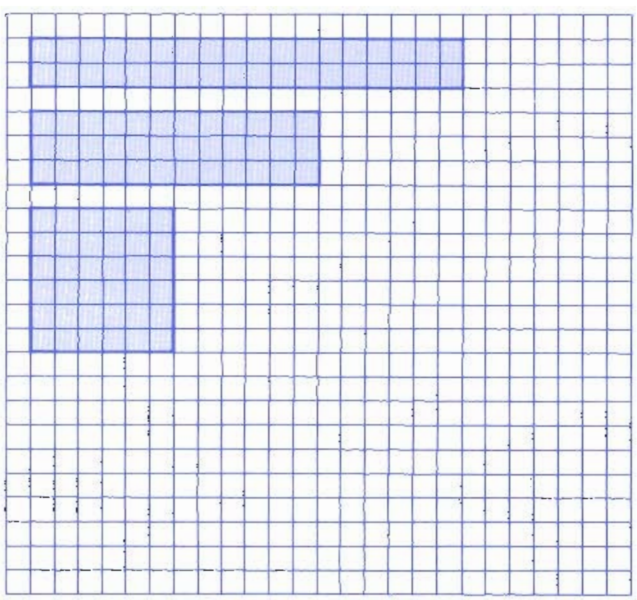
4. When both sides of rectangle A are doubled, how many squares are there? *8*

reference page

C-219



On the graph paper below outline as many rectangles as you can that have the same size as rectangle C but have different shapes.



reference page

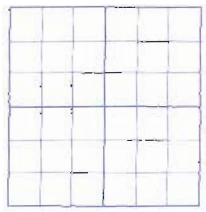
C-218

● Pages 217 and 218 reinforce the fact that some rectangles have the same size but different shapes.

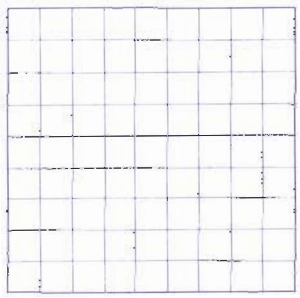
Have the children count the number of square units that cover rectangle *A* and rectangle *B* on page 217. Ask pupils to experiment by drawing rectangles that have the same size as *A* and as *B*, but have a different shape. Then, on page 218, let pupils try to draw rectangles that have the same size as rectangle *C*.

● Use pages 219 and 220 to provide experience in observing how the area is affected when first one side of a rectangle is doubled, then when both length and width are doubled. Instruct the children to read the questions and to write the answers on their papers. After the children have completed the pages, discuss their answers to the questions and the conclusion they drew from their work. They should conclude that when one side of a rectangle is doubled, the area becomes twice as great. When both sides of a rectangle are doubled, the area becomes four times as great.

1. How many 3 by 3 rectangles are needed to cover a 6 by 6 rectangle? 4



2. What happens when each side of a rectangle is doubled? There are four times as many squares.



3. How many 3 by 3 rectangles make a 9 by 9 rectangle? 9

4. How is each side of the 3 by 3 rectangle changed in the 9 by 9 rectangle? Each side is three times as long

C-220

Supplemental Experiences

■ Play the game "Name the Array." Tell a child to come to the front of the room and say, for example, "I see an array that has 36 objects. Name the array." If another child answers with any of the arrays for 36—3 by 12, 12 by 3, 9 by 4, 4 by 9, 6 by 6, 2 by 18, 18 by 2, 1 by 36, or 36 by 1—he is correct and may name the next array.

Let the children choose any array they wish and have them continue to take turns as long as time allows.

■ Let the children take turns placing square units on a tabletop to completely cover the rectangular top.

Give the children units of several different sizes—1 square inch, $\frac{1}{2}$ square inch, 2 square inches. Ask each child to tell the class how many square units he used to cover the table top, and what size he used. Discuss with the children how the size of the square unit affects the number of square units used to cover the tabletop.

KEY IDEA

The area of a rectangle is the length times the width.

Scope

To measure area of rectangles.

Fundamentals

In this section the child compares figures by computing the area of the figure in square units. A square unit is a convenient unit or standard of comparison for two-dimensional figures. The use of the square as a unit of measure for area enables the child to compute areas by multiplying. He learns that a 6 inch by 6 inch rectangle has an area of 36 square inches.

Readiness for Understanding

Experiences with area of a rectangle.
Ability to compute products.

Developmental Experiences

for flannel board
tagboard rectangles
(12″ × 6″, 3″ × 4″,
12″ × 18″)
tagboard strips (1″ × 18″)
pins

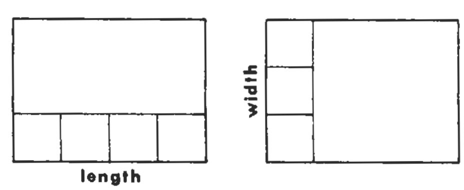
5 tagboard rectangles (various sizes)
tagboard square foot

▶ Pin to the flannel board a 12 by 6 inch rectangle. Cut out a 1 by 18 inch strip of tagboard and mark off the strip into 1-inch squares.

Ask two children to hold the strip of tagboard across the top of the rectangle. Ask them to tell the class how many units are in the length of the rectangle. (12) Ask two other children to come up and hold the strip of tagboard along the left side of the rectangle to measure the width. Ask them to tell the width of the rectangle. (6)

Ask the class how many square units cover the rectangle. Let the children solve this problem in any way they wish. Some of the children may still wish to cover the rectangle with square units. Discuss the different ways of finding the area of the rectangle.

▶ Pin two 3 by 4 inch rectangles on the flannel board. Direct a child to place a single row of square units across the length of one rectangle and a row across the width of the other as shown.



Now tell these children to move each row across the rectangle to simulate the covering of the entire rectangle with unit squares. Ask the children to think of all the ways they can use to find the area of the rectangle. Then ask the children if they could find the area of a rectangle if they knew only how many squares were on each side of the rectangle. Guide them to see that length times width will give the area.

▶ Pin a 12 by 18 inch rectangle on the flannel board. Ask a child to measure the length and width of this rectangle with a strip of 1-inch squared tagboard. Remind the children that the longest side of the rectangle is the length. Ask the child to tell the class how many squares there are along the length and along the width. Ask the class how many square units there are in the area. Some of the children will find the area by covering with and counting the square units; others may multiply 12×18.

Adapt this activity to several different rectangles on the chalkboard. Let the children measure the length and width of each rectangle and name the area.

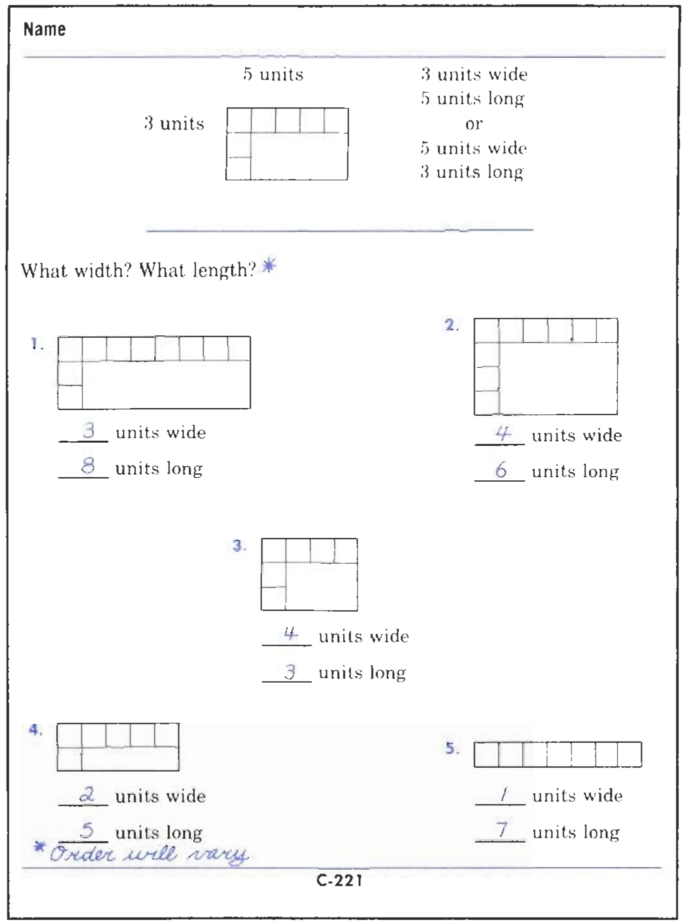
▶ Cut out five rectangles of different sizes from tagboard. The sides of each should be measurable in a whole number of feet or inches.

Distribute paper to the children. As you hold up a rectangle, ask the children to estimate the length and width of the rectangle and write an estimate on their pieces of paper. After the children have written their estimates, let a child measure the rectangle so that the others may check their answers.

As you continue this activity, encourage the children to explain their estimates. Discuss them and help the children to estimate a little closer each time. You may find it necessary to give the children something of a known length to use as a comparison—a ruler or a marked tagboard strip.

Vary the activity; ask the children to estimate the length, width, and area of objects in the classroom and then measure to see how well they can estimate. Some objects they might use are: a windowpane, a page in a book, a section of chalkboard, or a sheet of paper.

▶ Mark off a 3 by 2 foot rectangle on the chalkboard. Give each child a sheet of paper. Have the children study the rectangle on the board and write on their paper their estimate of length, width, and area. Tell the children that the square unit of measure is one square foot. Draw a 1-foot square on the chalkboard for the children to observe. After the children have written their estimates, ask a child to measure the area with a tagboard square foot and to tell the class the result. Continue the activity using other rectangles.


Name

5 units
3 units

3 units wide
5 units long
or
5 units wide
3 units long

What width? What length? *

1. 3 units wide
8 units long

2. 4 units wide
6 units long

3. 4 units wide
3 units long

4. 2 units wide
5 units long

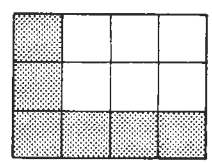
5. 1 units wide
7 units long

* Order will vary.

C-221

Pages 221 through 228

● Use pages 221 and 222 to review the ideas of length and width. Place on the chalkboard a rectangle that is 3 by 4 units in size. Shade the units as illustrated.

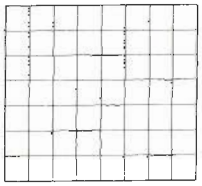


Discuss the terms length and width with the children. Remind them that the longer side is often considered the length, while the shorter side is often considered the width.

Ask a child to stand beside the classroom door and point out its length and width. Then ask the children to name other objects in the classroom that have length and width. (desk top, paper, book cover, chalkboard, window, and so forth) Have the children point out length and width and let them discuss their examples. If the rectangle is a square it will be interesting to see what they will call length or width. Help them see that in this case it makes no difference.

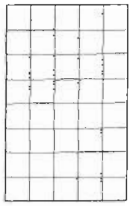
Assign the exercises on pages 221 and 222 for independent work.

What width? What length? *

1.


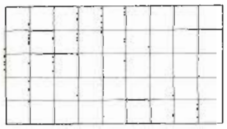
7 units wide

8 units long

2.


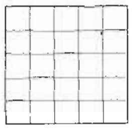
5 units wide

8 units long

3.


9 units wide

5 units long

4.


5 units wide

5 units long

5.

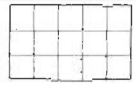

3 units wide

18 units long

*Order will vary.

C-222

Name



5 by 3

Length is 5 units.
Width is 3 units.
Area is 5 × 3 square units.

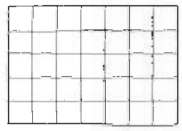
What length? What width? What area? *

1.


Length: 4 units

Width: 6 units

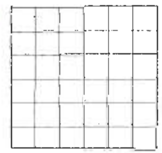
Area: 4 × 6 square units

2.


Length: 5 units

Width: 7 units

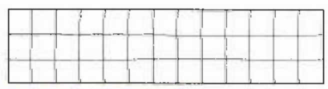
Area: 7 × 5 square units

3.


Length: 6 units

Width: 6 units

Area: 6×6 square units

4.


Length: 13 units

Width: 3 units

Area: 13×3 square units

*Order will vary.

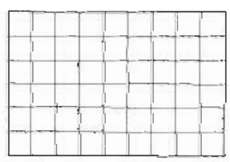
reference page

C-223

● Use pages 223 and 224 to further develop the concepts of length, width, and area. Discuss how the class can find the length, width, and area of the rectangle in the example on page 223. Help the children realize that the product of the length and width is the area.

Assign these pages as independent work.

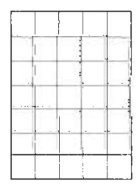
What length? What width? What area? *

1.


Length: 6 units

Width: 9 units

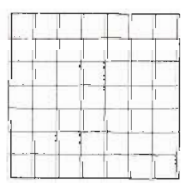
Area: 54 square units

2.


Length: 5 units

Width: 7 units

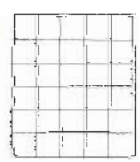
Area: 35 square units

3.


Length: 7 units

Width: 7 units

Area: 49 square units

4.


Length: 6 units

Width: 5 units

Area: 30 square units

5.


Length: 19 units

Width: 1 units

Area: 19 square units

*Order will vary.

C-224

Name

square unit

See pupil page suggestions.

Use a strip of graph paper like the one below to measure the length and width of each rectangle. How many square units just cover each rectangle?

1. *16* square units *or 4×4*

2. *48* square units *or 4×12*

3. *45* square units *or 5×9*

4. *64* square units *or 8×8*

Name

What is the area of each rectangle?

square unit

1. *6* square units

2. *2* square units

3. *21* square units

4. *4* square units

5. *8* square units

reference page

Length is 4 units.
Width is 3 units.
Area is 12 square units.

How many square units just cover each rectangle? *

1. Length is *6* units.
Width is *5* units.
Area is *30* square units.

2. Length is *10* units.
Width is *4* units.
Area is *40* square units.

3. Length is *8* units.
Width is *6* units.
Area is *48* square units.

4. Length is *9* units.
Width is *9* units.
Area is *81* square units.

reference page **Order will vary.*

What is the area of each rectangle?

square unit

1. *36* square units

2. *56* square units

3. *63* square units

4. *120* square units

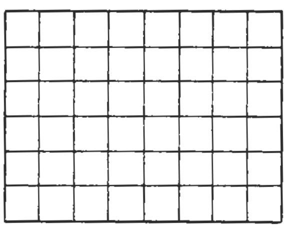
5. *12* square units

6. *36* square units

● Pages 225 through 228 give further experience in finding length, width, and area of rectangles that are not partitioned into square units. Work the first exercise or example on each page with the class. Let a child tell how many units there are in the width and in the length of each of these rectangles. Then let the children suggest how to find the area of each. Any child who is not sure of the area may make paper units, cover the rectangle, then count the square units. Assign the exercises on these pages for independent work.

Supplemental Experiences

■ Prepare four transparencies for use on the overhead projector. One transparency should be marked off in one-inch squares, one in half-inch squares, and one in quarter-inch squares. Draw a 6 by 8-inch rectangle on the fourth transparency. Put the one-inch squared transparency on the projector. On top of this put the rectangle as shown.



Ask a child to give the length (8 units), the width (6 units), and the area (48 square units) of the rectangle. Remove the two transparencies and place the half-inch squared transparency on the overhead. Again place the rectangle over the transparency and ask for the length (16 units), the width (12 units), and the area (192 square units). Repeat the activity; use the quarter-inch squared transparency. Then ask questions such as:

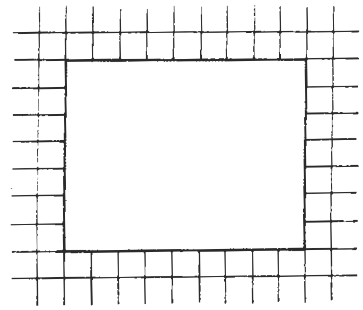
Did the size of the rectangle change? (no)
What did change? (the size of the unit)
When did we find the greatest number of units? (when the smallest unit was used)
When did we find the least number of units? (when the greatest unit was used)

The children should conclude that the smaller the unit the greater the number of units it will take to cover a given rectangle. However, the size of a rectangle does not depend on the unit used to measure its area.

■ Give each child an envelope that contains fifty 1-inch squares. Tell the class that each child may construct as many rectangles as he wishes, any size he wishes. After everyone has constructed at least one rectangle, ask several children to tell the class the length, width, and area of their figure.

You may want to have some child come to the chalkboard and draw a rectangle that has an even number of unit squares on each side. Have the child name the length, width, and area of his rectangle.

■ Give a sheet of $\frac{1}{4}$-inch graph paper to each child. Show the children a piece of graph paper that has a construction-paper rectangle glued over one section.



Discuss how you might determine the area of the colored paper. Some of the children may suggest drawing squares on the paper and then counting. Some other children may have found the area by multiplying its length by its width using the units on the graph paper as measures.

Ask the children how they might determine the area surrounding the rectangle. Some children (but not many) may say they could multiply the length by the width of the whole paper, subtract the area of the small rectangle, and find the area of the part surrounding the small rectangle. The children may need help to see this.

Direct the children to design some pictures on their graph paper and explain to the class how they would find the area of each part.

KEY IDEA

The diagonals of a square are perpendicular.

Scope

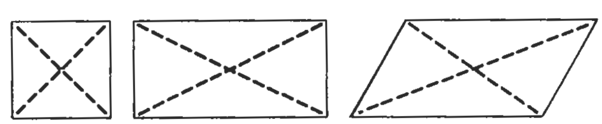
To review the concepts of perpendicular lines and parallel lines by using paper-folding activities.

To use perpendicular lines and parallel lines to make rectangles, squares, and parallelograms.

Fundamentals

In Unit 9 the child participated in several paper-folding activities. He learned to construct square corners and perpendicular lines. The child was introduced to the concept of parallel lines by observing parallel edges of paper strips.

Now the child will fold paper to construct squares, rectangles, parallel lines, and parallelograms. He will check the diagonals of squares, rectangles, and parallelograms to see whether or not they are perpendicular.



No pupil pages accompany this section. The activities are exploratory and are designed to introduce concepts with which the children should be familiar but are not required to master at this time.

Readiness for Understanding

Ability to fold square corners.

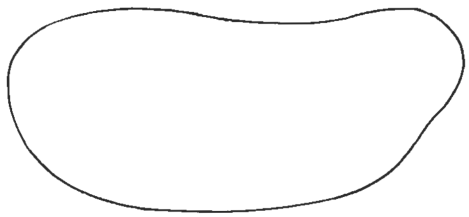
Ability to recognize perpendicular lines and parallel lines.

Developmental Experiences

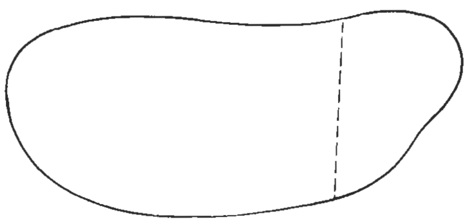
colored chalk

for each child
paper
crayons
ruler

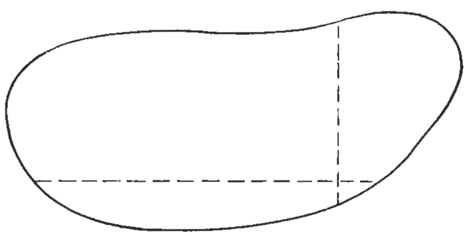
▶ Show the children how to make a rectangle by folding paper. Give each child a sheet of paper as illustrated that has no straight edges.



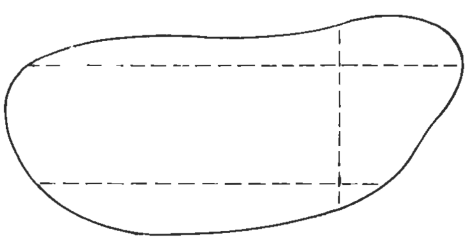
Demonstrate how to fold the paper to make a rectangle as shown. Each diagram shows the line that appears after the paper is creased and unfolded. Always unfold the paper after a line has been made. Fold and crease the paper near one side to make a straight line.



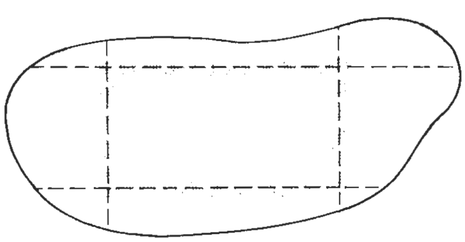
Now fold the straight line onto itself. This makes a line perpendicular to the first line.



Fold the first straight line onto itself again, to make a second line perpendicular to the first line.



Now fold the last line onto itself. This makes a line that is perpendicular to the last line.



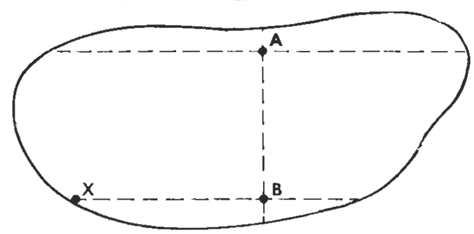
If necessary, repeat this demonstration several times as the children fold their papers. Help individual children when necessary. When all of the children have successfully made a rectangle, ask how the figure can be checked to be sure it is a rectangle. This is a good chance for a brief discussion concerning rectangles. The children should recall that a rectangle has four square corners. This will enable the children to check the four corners with a square corner of folded paper.

Let the children color the rectangle as illustrated so that the sides are clearly shown. Ask the following questions and let individual children use their rectangles to demonstrate their answers.

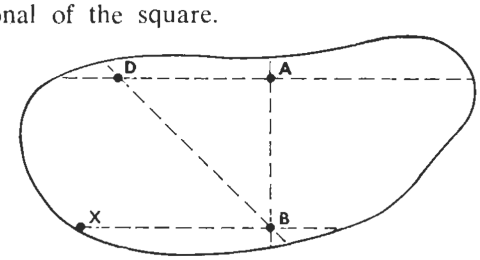
Which sides are perpendicular?
Which sides are parallel?
Which sides have the same length?

Direct the children to fold and color the sides of several more rectangles. Ask the same series of questions about their new rectangles. Encourage the children to discuss any relationships they observe between the rectangles.

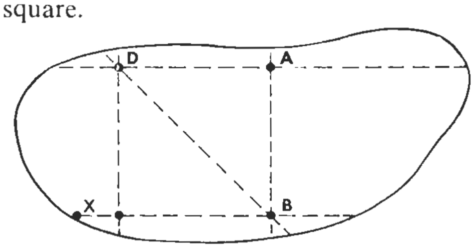
▶ Show the children how to fold a square. Repeat the first three steps for folding a rectangle. As before, unfold the paper after each line has been made. Your paper should show 3 lines.



Now fold *AB* onto line *BX*. The new line will be a diagonal of the square.

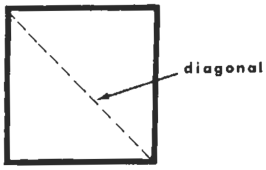


Now fold line *AD* onto itself so that the crease goes through point *D*. This produces the fourth side of the square.



When all of the children have folded a square, tell them to outline the sides of the square with a crayon to make it easier to see the sides. Tell the class that the line that is not colored is called a *diagonal* of the square.

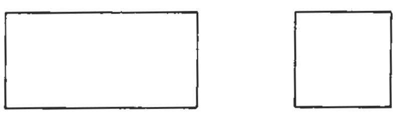
Draw a square on the board, show the diagonal with a broken line, and write *diagonal* on the board.



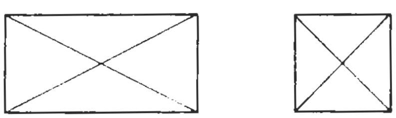
Discuss the following statements and questions. Tell the children who answer to demonstrate their answer by pointing to the sides of their square.

Show a pair of perpendicular sides. (4 responses)
Show a pair of parallel sides. (2 responses)
Is the diagonal parallel to any side of the square? (no)
Is the diagonal perpendicular to any side of the square? (no)
Show a pair of sides that are the same length. (all four sides are the same length.)

▶ Give each child two sheets of paper; tell the children to fold one piece to make a rectangle and the other piece to make a square. Ask them to outline the sides of each of the figures.



Draw a rectangle and a square on the board. Use colored chalk for the sides and white chalk to draw the diagonals. If necessary, remind the class that these lines are called diagonals.



Distribute rulers to the children and tell them to draw the diagonals in their figures. Then ask the children which diagonals are perpendicular. The answers can be checked with square corners.

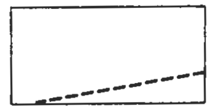
Supplemental Experience

■ Draw a parallelogram on the board. Ask a child to come to the board and to point to the parallel sides. Label the figure and encourage the children to discuss the meaning of the name.

Parallelogram

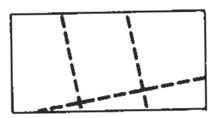
Show the class how to make a parallelogram. Give each child several blank sheets of paper—$8\frac{1}{2}$ by 11 inches is a good size to use. The following diagrams illustrate the position of the lines after the paper has been creased and unfolded. Always unfold the paper after each line has been made.

First fold a sheet of paper to make any line that intersects the two sides as shown.

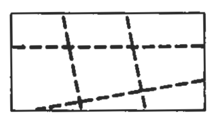


Fold this line onto itself. This will make a line perpendicular to the first line.

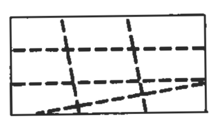
Fold the first line onto itself again to make another perpendicular line.



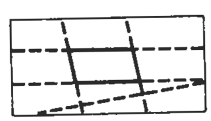
Now position the paper with one of the long edges toward you. Fold the left edge of the paper onto itself.



Fold the left edge of the paper onto itself again.



After the children have finished, tell them to color the sides of the parallelogram.



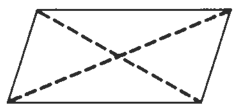
Encourage the children to make several more parallelograms. This will give you a chance to assist the children who may have difficulty making the necessary folds.

After the children have made a parallelogram, ask the following questions. The children may clarify their answers by pointing to examples on their papers when they respond.

Which sides are parallel? (opposite sides)
Which sides are the same length? (opposite sides)

Tell the class that a parallelogram is a four-sided figure that has opposite sides parallel. Draw a rectangle on the board and ask if it is a parallelogram. (yes) Guide the children to realize that the figure is a rectangle because it has four square corners and that it is a parallelogram because opposite sides are parallel and of the same length. Repeat this procedure with the square.

To finish the investigation of parallelograms, have the children draw the diagonals in their examples.



The children may conclude that the diagonals are not perpendicular. You may want to have the children check the diagonals of a parallelogram that has four sides of the same length (these diagonals will be perpendicular).

UNIT 14
FRACTIONAL NUMBERS

Pages 229 Through 244

OBJECTIVE

To introduce $\frac{1}{100}$, $\frac{1}{20}$, $\frac{1}{10}$, $\frac{1}{12}$, $\frac{1}{6}$, and review $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

The whole numbers were developed from the number property of sets of objects. The fractional numbers are developed from measurement of quantities. The child will explore physical situations by measuring rather than by counting.

See Key Topics in Mathematics for the Primary Teacher: Measurement.

KEY IDEAS

Twelve $\frac{1}{12}$'s is one.
1 cent is $\frac{1}{100}$ of a dollar.

CONCEPTS

fraction
hundredth ($\frac{1}{100}$)
sixth ($\frac{1}{6}$)
tenth ($\frac{1}{10}$)
twelfth ($\frac{1}{12}$)
twentieth ($\frac{1}{20}$)

KEY IDEA

Twelve $\frac{1}{12}$'s is one.

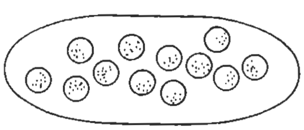
Scope

To review the fractions $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{3}$, and to introduce $\frac{1}{6}$ and $\frac{1}{12}$.

To use the knowledge of the number of feet in a yard, inches in a foot, and inches in a yard.

Fundamentals

As the child moves from counting to measuring, he encounters situations that cannot be dealt with by the set of whole numbers. For example, consider a set of 12 cookies.

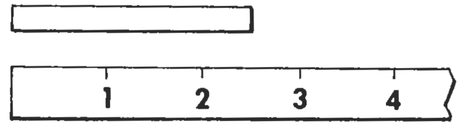


As long as the child is concerned only with how many cookies, he can count to answer questions about the cookies. But suppose he encounters the problem of dividing the cookies evenly with 4 friends. He counts out 2 cookies apiece and the remaining 2 cookies must be divided evenly among 5 children. There is a problem: How can 2 cookies be divided evenly among 5 children?

The question really concerns the quantity of cookie rather than the number of cookies. The question is "How much cookie?" rather than "How many cookies?" and requires the set of fractional numbers.

In the second grade the child was introduced to situations where the question "What length?" could not be answered with a whole number. Consider the following situation.

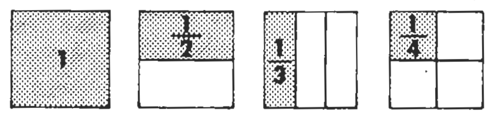
What length?



If a child is not familiar with fractional numbers, then the response to "What length?" is apt to be "More than 2 inches and less than 3 inches." After a child is introduced to fractional numbers he may say that the length is five $\frac{1}{2}$ inches.

The child also learned the following fractional number concepts.

If 2 equal parts make 1, then each part is $\frac{1}{2}$.
If 3 equal parts make 1, then each part is $\frac{1}{3}$.
If 4 equal parts make 1, then each part is $\frac{1}{4}$.



In this section, the child will extend his understanding of fractional numbers by working with the 12 inches of 1 foot. He will discover that each inch measures $\frac{1}{12}$ of a foot as well as 1 inch. Emphasis should be given to the question "How much length?" The counting question "How many inches?" should only be used to help the child relate his knowledge of counting numbers to the new problems of measuring quantity.

Readiness for Understanding

Knowledge of the meaning of the fractions $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{3}$.

Ability to count.

Developmental Experiences

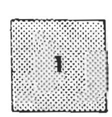
for flannel board
tagboard 12″ ruler

for each child
string (36 inches)
rulers (or 12″ long tagboard strips)
construction paper (5 assorted colors)
wrapping paper
scissors

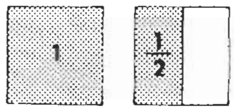
yardstick
string
sheets of tagboard

▶ Review the fractional numbers $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and introduce $\frac{1}{6}$.

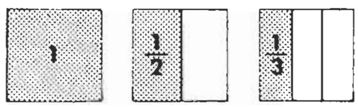
Draw a square on the chalkboard, label it 1, and tell the children that this square represents 1.



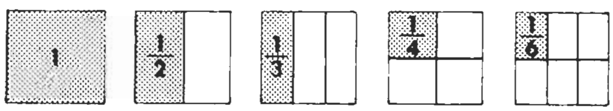
Draw another square on the board. Divide the square into 2 equal parts. Tell the children that if 2 equal parts make 1, then each part is $\frac{1}{2}$.



Draw another square on the board and divide it into 3 equal parts. Tell the class that if 3 equal parts make 1, then each part is $\frac{1}{3}$.



Repeat this procedure to show $\frac{1}{4}$ and $\frac{1}{6}$.



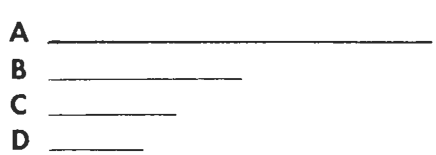
▶ Draw lines of these lengths on the chalkboard: 12 inches, 9 inches, 18 inches, 6 inches, and 3 inches. Give each child a string 1 yard (36 inches) long. Ask individual children to go up to the chalkboard, measure one of the lines, and tell the class what fractional part of their string measures the length of the line. Tell each child to write the fraction beside the line he measures.

Ask the children to measure their lines with a yardstick and to write the length in inches beside the fraction. Discuss these lengths as fractions of a yard.

12 inches is $\frac{1}{3}$ of a yard.
18 inches is $\frac{1}{2}$ of a yard.
9 inches is $\frac{1}{4}$ of a yard.
6 inches is $\frac{1}{6}$ of a yard.
3 inches is $\frac{1}{12}$ of a yard.

Have all of the children fold their strings as each fraction is discussed. For example, one child may explain that the 18-inch line on the board is $\frac{1}{2}$ of a yard because there are two 18-inch segments of string in his yard-long piece. Some other child may show that three of the 12-inch lines on the board make one yard, so he knows that 12 inches is $\frac{1}{3}$ of the yard (36 inches). Continue this activity with each of the above fractions.

▶ Draw three lines on the chalkboard as illustrated. Make line *A* 1 foot long, line *B* 6 inches long, line *C* 4 inches long, and line *D* 3 inches long.



Cut a 1-foot length of string. Tell the children that the string is 1 foot long. Measure line *A* with the string to show that line *A* is 1 foot long.

Show the children that line *B* can also be measured with the 1-foot string. Place the string on line B with the right end of the string on the right end of line *B*. The children will see that some string is left over. Slide to the right the part of the string that was not used to measure *B*. Show that this section of string is as long as line *B*. Explain that since it takes two lengths of line *B* to make 1 foot, then line *B* is $\frac{1}{2}$ foot.

Follow the same procedure to show that line *C* is $\frac{1}{3}$ foot, and line *D* is $\frac{1}{4}$ foot.

▶ Give each child a 12-inch ruler that has numbers on only one side. If rulers are not available, use 12-inch strips of tagboard. Also give each child 5 construction paper strips of the following lengths: 1, 2, 3, 4, and 6 inches. The strips should be the width of the ruler or tagboard strips and each length should be a different color. You may wish to use the colors suggested below.

1 inch—orange
2 inches—brown
3 inches—blue
4 inches—green
6 inches—red

Ask the children to place their rulers with the numerals side face down on their desks so they have a solid side showing. Tell the class that the ruler represents 1. Then tell the children to find the strip they would have to use twice to measure the 1-strip. Some child may say it takes 2 of the red strips. Ask the class what part of 1 each red strip is. ($\frac{1}{2}$) Have the children write $\frac{1}{2}$ on their red strips. Continue the activity with each of the other strips. After the children have identified each of the fractions and labeled their strips, let them work in pairs or small groups to check their results. Have them cut out more strips to match the ones they have and then see how many they can place end to end on their rulers.

▶ **Place on the flannel board a 12-inch strip of tagboard that is divided into quarter-inches. Call on three children in turn to measure the length of the segment** in terms of 1-inch units, $\frac{1}{2}$-inch units, and $\frac{1}{4}$-inch units. Have each child record his measurement on the chalkboard. (12 inches, 24 half-inches, 48 quarter-inches) Ask the children if each of the units names the same length. (yes) Ask the class to compare the measurements. They may see that it takes 2 times as many $\frac{1}{2}$-inch units as 1-inch units to measure the same length; it takes 4 times as many $\frac{1}{4}$-inch units as 1-inch units to measure the same length.

▶ Give each child a strip of wrapping paper 1 yard long. Tell the children to use their paper to measure different things in the classroom—the length of a page in a book, a section marked off on the chalkboard (12 by 9 inches), or 9 by 12 inch construction paper. Choose objects for them to measure that are fractional parts of 36, such as 4, 3, 6, 9, and 12 inches long. Tell the children to give the measurement in terms of what part of their strip of paper was used. A child who measured the width of his desk might say that he found that it took just half of his strip of paper, so he knows that the width of his desk is $\frac{1}{2}$ yard. Another child who measured a page in his book from top to bottom may explain that he found it took one-fourth of his strip—he folded his strip into 4 parts and one part was the measure of the page. He knew that if it took 4 of this measure to make a yard, then the length of the page is one of the four parts or $\frac{1}{4}$ yard.

▶ Cut pieces of string into different fractional parts of one foot and give a piece to each child in the class. Ask two children to show the class their lengths of string and to tell what part of a foot each length is—$\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{6}$, or $\frac{1}{12}$. Then ask who has the greater length of string. If the children are not sure, let them hold the strings beside each other and compare the lengths.

▶ Give each child several strips of wrapping paper (1 yard long) and a pair of scissors. Have the children cut their yard strips into the different lengths you write on the chalkboard—$\frac{1}{3}$ yard, $\frac{1}{2}$ yard, $\frac{1}{12}$ yard, $\frac{1}{6}$ yard, and $\frac{1}{4}$ yard. Have individuals show the class a segment from the strips that were cut and tell how they know what fractional part of a yard it is. For example, one child may say that his strip is $\frac{1}{3}$ of a yard because it takes 3 such lengths to make a yard.

▶ Make the two charts shown below.

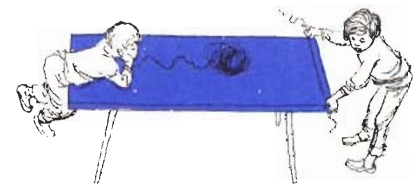
12 inches					
6 inches					
4 inches					
3 inches					
2 inches					

1 foot					
$\frac{1}{2}$ foot					
$\frac{1}{3}$ foot					
$\frac{1}{4}$ foot					
$\frac{1}{6}$ foot					

Use the two charts to show that 6 inches is $\frac{1}{2}$ foot, 4 inches is $\frac{1}{3}$ foot, and 2 inches is $\frac{1}{6}$ foot.

Name

UNIT 14 FRACTIONAL NUMBERS



Use a string 1 foot long to measure the length of each line.

1. $\frac{1}{2}$ foot

2. $\frac{1}{4}$ foot

3. $\frac{1}{3}$ foot

4. $\frac{1}{6}$ foot

5. $\frac{1}{12}$ foot

reference page

C-229

Pages 229 through 238

● Use page 229 to provide practice in measuring fractional parts of one foot. Give each child a piece of string that is 1 foot long. Tell the children to write the answer to each exercise. After they have completed the page, discuss the fractions used to measure the lines. For example, ask a child how many of line 1 would be the same length as his string. The child may say it would take 2 lengths to be as long as his string, so the line must be $\frac{1}{2}$ of a foot long. Follow a similar procedure to discuss each of the other lines.

$\frac{1}{2}$ foot

$\frac{1}{3}$ foot

$\frac{1}{4}$ foot

$\frac{1}{6}$ foot

$\frac{1}{12}$ foot

Draw a ring around the greater length.

1. $\frac{1}{4}$ foot or ($\frac{1}{3}$ foot) 2. ($\frac{1}{4}$ foot) or $\frac{1}{6}$ foot 3. ($\frac{1}{2}$ foot) or $\frac{1}{4}$ foot
4. $\frac{1}{12}$ foot or ($\frac{1}{4}$ foot) 5. $\frac{1}{12}$ foot or ($\frac{1}{2}$ foot) 6. ($\frac{1}{2}$ foot) or $\frac{1}{3}$ foot
7. ($\frac{1}{2}$ foot) or $\frac{1}{6}$ foot 8. $\frac{1}{6}$ foot or ($\frac{1}{3}$ foot) 9. $\frac{1}{12}$ foot or ($\frac{1}{3}$ foot)

Answer the questions.

10. Roger and Ray went fishing. The worm that Roger put on his hook was $\frac{1}{4}$ foot long. The worm that Ray put on his hook was $\frac{1}{6}$ foot long. Which boy had the longer worm on his hook? *Roger*

11. Ray caught a fish that was $\frac{1}{2}$ foot long. Roger caught one that was $\frac{1}{3}$ foot long. Which boy caught the longer fish? *Ray*

reference page

C-230

What length?

1. $\frac{1}{3}$ foot
   4 inches

2. $\frac{1}{12}$ yard
   $\frac{1}{4}$ foot
   3 inches

3. $\frac{1}{12}$ foot
   1 inch

4. $\frac{1}{4}$ yard
   $\frac{9}{12}$ or $\frac{3}{4}$ foot
   9 inches

5. $\frac{1}{6}$ foot
   2 inches

6. $\frac{1}{6}$ yard
   $\frac{1}{2}$ foot
   6 inches

reference page

C-232

Name

Use a string 1 yard long as a measure.

1. Measure the length of each line.

$\frac{1}{6}$ yard

$\frac{1}{12}$ yard

$\frac{1}{18}$ yard

2. Cut a piece of string that is $\frac{1}{2}$ yard long.

3. Cut a piece of string that is $\frac{1}{3}$ yard long.

4. Cut a piece of string that is $\frac{1}{4}$ yard long.

5. Write the lengths in order. Start with the greatest length.

$\frac{1}{3}$ yard 1 yard $\frac{1}{2}$ yard $\frac{1}{6}$ yard $\frac{1}{12}$ yard $\frac{1}{4}$ yard

1 yard, $\frac{1}{2}$ yard, $\frac{1}{3}$ yard, $\frac{1}{4}$ yard, $\frac{1}{6}$ yard, $\frac{1}{12}$ yard

reference page

C-231

● For page 230, the child compares fractional parts by determining the greater length. Direct the children to carefully read the questions on the page and to write their answers. Call on individuals to read each exercise aloud and tell the answer. Let the children discuss how they made their comparisons. Help any child who is having difficulty understanding the way to make a comparison.

● Page 231 gives further practice in measuring. Give each child 4 pieces of string, each one yard long. Let them work in groups of two or three to complete the exercises. Then let the class discuss their results.

● Page 232 will give children another opportunity to measure yards, feet, and inches. They may wish to work in groups of two or three. Have them use a ruler, a string that is one foot long, and a string that is one yard long to measure the lines. When they have finished, use the page for discussion. Let the children take turns telling what measurements they found for the lines on page 232.

Name

Use a string 1 foot long to measure objects in your classroom. Record the measurements in this chart. *

OBJECT	LENGTH	WIDTH
Pencil	$\frac{1}{3}$ foot	
Book	$\frac{1}{2}$ foot	$\frac{1}{2}$ foot

*Responses will vary.

C-233

Use a string 1 yard long to measure objects in your classroom. Record the measurements in this chart.*

OBJECT	LENGTH	WIDTH
Desk top	$\frac{2}{3}$ yard	$\frac{1}{2}$ yard

*Responses will vary.

C-234

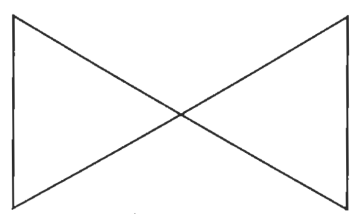
● Pages 233 and 234 present the opportunity for children to measure objects. Children may wish to work in small groups over a period of several days to make entries in these charts.

● Use pages 235 and 236 to provide practice in measuring fractional parts of one. Do exercise 1 on page 235 with the class. Ask the children how many sides the figure has. (6) Then ask them to compare the lengths of the sides by using a string or a piece of paper that is one foot long. (All are the same length.) Finally, ask the children to give the length of one side. ($\frac{1}{6}$ foot)

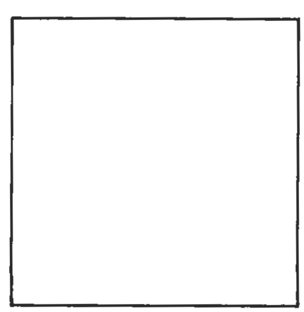
Name

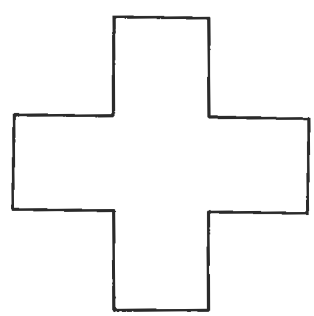
The total length of the sides of each figure is 1 foot.

How long is each side?



1. $\frac{1}{6}$ foot





2. $\frac{1}{4}$ foot

3. $\frac{1}{12}$ foot

C-235

Draw each figure so that the total length of its sides is 1 foot. *

1. A kite
2. A rectangle
3. A star
4. A triangle

*Answers may vary

C-236

What number? Compare the lengths.

1. 1 — $\frac{1}{2}$
2. 1 — $\frac{1}{3}$
3. 1 — $\frac{1}{8}$
4. 1 — $\frac{1}{4}$
5. 1 — $\frac{1}{4}$
6. 1 — $\frac{1}{6}$

reference page

C-238

Name

1 — $\frac{1}{2}$

What number? Compare the lengths.

1. 1 — $\frac{1}{2}$
2. 1 — $\frac{1}{3}$
3. 1 — $\frac{1}{5}$
4. 1 — $\frac{1}{6}$
5. 1 — $\frac{1}{4}$
6. 1 — $\frac{1}{7}$
7. 1 — $\frac{1}{10}$

reference page

C-237

● On pages 237 and 238, children compare lengths. Encourage them to use a string or a strip of paper when doing the exercises. Work the first exercise on page 237 with the class. They should see that two of the shorter lines would be just as long as the longer line, or, if the longer line is 1, the shorter line is $\frac{1}{2}$. Let pupils work individually or in small groups to complete these pages. Then let the class discuss their results.

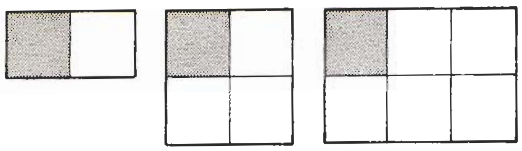
Supplemental Experiences

■ Place a yardstick on the chalktray. Ask a child to measure the yardstick with his 12-inch ruler. Ask the child what fraction of a yard his ruler is. Help him see that if it takes 3 of the ruler lengths to equal 1 yard then 1 ruler length (1 foot) is $\frac{1}{3}$ of a yard. Copy the following table on a sheet of tagboard.

1 yard = ___ feet	2 feet = $\frac{\_\_}{3}$ yards
1 foot = $\frac{\_\_}{3}$ yard	3 feet = $\frac{\_\_}{3}$ yards

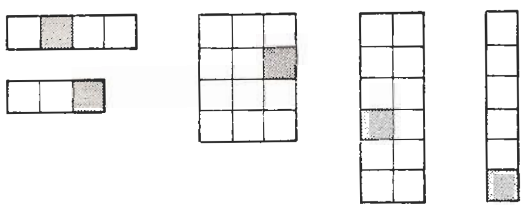
Discuss each section with the children as they help you complete the chart. Display it in the room for use as a reference.

■ Place on the flannel board three arrays as illustrated. Shade one part of each array.



Ask a child what fractional part of the first rectangle is shaded. ($\frac{1}{2}$) Let the child explain his answer—he may observe that the first rectangle has 2 equal parts and 1 part is shaded, so $\frac{1}{2}$ of the first rectangle is shaded.

Discuss the remaining two arrays in the same manner. Then replace the arrays with three others, including some that illustrate $\frac{1}{3}$ and $\frac{1}{12}$.



■ Draw a 12 by 36-inch rectangle on the chalkboard. Have two children use a piece of string one yard long to measure the sides of the rectangle. Tell one child to measure the length and the other child to measure the width. Ask them each to record their results on the chalkboard. Ask the class what fractional part of the length is the width. Some child may say that it would take 3 widths of 12-inches to have a measurement equal to the length of 36-inches. So if it takes 3 of the widths (12), then one width would be $\frac{1}{3}$ of the length—12-inches is $\frac{1}{3}$ of 36.

Continue the activity using rectangles of the following dimensions:

18 by 36 inches
9 by 36 inches
6 by 36 inches

KEY IDEA

1 cent is $\frac{1}{100}$ of a dollar.

Scope

To review $\frac{1}{2}$ and $\frac{1}{4}$, and to introduce $\frac{1}{10}$, $\frac{1}{20}$, and $\frac{1}{100}$ of a dollar.

Fundamentals

Further exploration of fractional numbers and measurement of quantity occurs when the child tries to answer "How much money?" In this section the child's experience with fractional numbers is extended to the parts of a dollar that are represented by U. S. coins.

1 cent or $\frac{1}{100}$ dollar
1 nickel or $\frac{1}{20}$ dollar
1 dime or $\frac{1}{10}$ dollar
1 quarter or $\frac{1}{4}$ dollar
1 half dollar or $\frac{1}{2}$ dollar

The child learns that 3 cents—$3 \times \frac{1}{100}$ dollar—is $\frac{3}{100}$ dollars. The child will determine the relationship between monetary value and fractional numbers. By solving problems using a fractional number model, the child will learn this concept of monetary value.

Readiness for Understanding

Ability to recognize parts of a whole.

Developmental Experiences

pictures of money
toy money
wrapping paper strips (3″ × 20″)

▶ Show the class a 1-dollar bill and 2 half dollars. If this is not convenient, use toy money or pictures of a 1-dollar bill and 2 half dollars.

Write on the chalkboard:

1 dollar = ___ cents

Have a child complete the sentence. Ask the class how many half dollars it takes to equal one dollar.

Now write on the chalkboard:

___ + ___ = 100 cents

Ask how many cents there are in $\frac{1}{2}$ of 100 cents. As the children make suggestions, write in the equation the numbers that they suggest. Let the children discuss why they agree or disagree with each suggestion. Children should see from this exploration that each half of the dollar should be represented by the same number. They should agree that the sum of the parts should be 100. The children will arrive at $50 + 50 = 100$ and decide that $\frac{1}{2}$ of 100 cents is 50 cents.

Continue the activity; replace the half dollars with 4 quarters.

Ask how many quarters equal one dollar. Then write on the chalkboard:

___ + ___ + ___ + ___ = 100 cents

Again, let the children estimate how many cents is

$\frac{1}{4}$ of 100 cents. Let them experiment with numbers until they find the number that it takes 4 of to equal 100.

Ask the class how many $\frac{1}{4}$ dollars are equal in value to one $\frac{1}{2}$ dollar. Some child will probably know that two $\frac{1}{4}$ dollars are equal to one $\frac{1}{2}$ dollar. Then help the class see that four $\frac{1}{4}$ dollars equals 1 dollar.

Continue the activity by asking the children how many dimes it takes to have the value of one dollar. Write on the chalkboard:

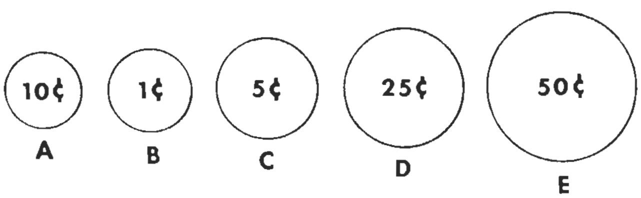
$$\_\_ + \_\_ + \_\_ + \_\_ + \_\_ + \_\_ + \_\_$$
$$+ \_\_ + \_\_ + \_\_ = 100 \text{ cents}$$

Ask the children how many cents there are in $\frac{1}{10}$ of 100 cents. Let them experiment with numbers until they find the number that it takes 10 of to equal 100. This should be quite easy for the children to understand after the work they have done with ten tens.

Ask the children how many $\frac{1}{20}$'s will equal one dollar. Let the children experiment until they find that 20 of the $\frac{1}{20}$ parts equal 100 cents. Tell the children to find the value of $\frac{1}{20}$ of 100 cents. Allow them to talk about their estimates with one another—working in pairs or small groups—and then discuss their results. Then ask the children to name a coin that represents $\frac{1}{20}$ of one dollar. (a nickel)

Ask the children what coin is $\frac{1}{100}$ of a dollar. (a penny) Then ask how many of these coins are needed to equal 100 cents. The children should easily see that 100 pennies equal 100 cents.

▶ Display a picture of these coins on a bulletin board or the chalktray and label the coins as shown.



Have toy money or real coins available for the children to use. Ask questions such as:

How many of coin *E* equals one dollar?
Which coin would you need two of to equal one dollar?
Which coin do you think has the greater value, *C* or *A*?

Be sure the children do not confuse value with size. *C* is larger than *A* but it does not have the greater value. Have the children tell how many of each coin equals one dollar.

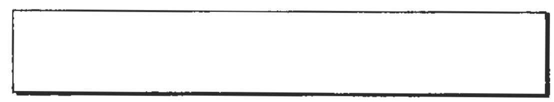
▶ Give each child a strip of wrapping paper 20-inches long and 3-inches wide. Tell the children to label this strip 1 dollar. Hold up another 3 by 20 inch strip and ask a child to compare his strip with yours to see if the second strip also represents 1 dollar. When the class decides that both strips are the same size, fold your paper to show $\frac{1}{2}$ of one. Then open the paper, draw a line down the fold, and label each part $\frac{1}{2}$ dollar. Ask how many $\frac{1}{2}$ dollars it takes to make 1 dollar. (2) Ask what value each part represents. (50 cents) Write 50 cents on each half. Help the children fold and label their strips. They should record the fractions and values on the blank side of their paper.

1 DOLLAR	
$\frac{1}{2}$ dollar 50 cents	$\frac{1}{2}$ dollar 50 cents

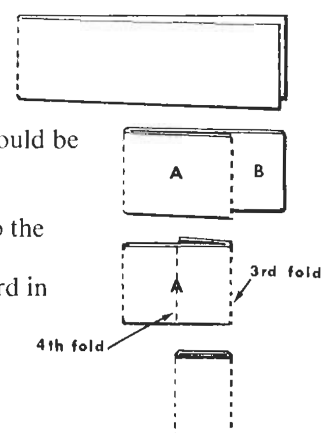
▶ Give each child a strip of wrapping paper 3-inches wide and 20-inches long. Adapt the procedure in the preceding activity so that the children will show four-fourths of a dollar.

1 DOLLAR			
$\frac{1}{4}$ dollar 25 cents	$\frac{1}{4}$ dollar 25 cents	$\frac{1}{4}$ dollar 25 cents	$\frac{1}{4}$ dollar 25 cents

▶ Fold a 3 by 20 inch paper strip to represent tenths of a dollar. You may want to fold the strip before class begins and then refold the strip in front of the class.



1st fold—Fold the strip in half.
2nd fold—The top (A) should be twice as long as the remaining part (B).
3rd fold—Fold part (B) to the back.
4th fold—Fold (A) forward in half.



Unfold the strip and ask a child to draw a line down each crease to make the parts easily seen. Compare the strip with several of the dollar strips that the children made previously.

1 DOLLAR									
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

Fold a strip into twenty parts. Have a child open the paper, draw a line down each fold, and label each part $\frac{1}{20}$. Let the children compare this strip with a 1-dollar strip and determine how many $\frac{1}{20}$ parts make 1.

1 DOLLAR

$\frac{1}{20}$

Finally, show the class a strip that is divided into 100 parts. Hold up another 3 by 20 inch strip and ask a child to compare the size of the two pieces. Ask some other child to come up to the chalkboard and write the fraction that names each part when the strip is divided into 100 parts. ($\frac{1}{100}$)

Pages 239 through 244

- Page 239 may be completed independently. Then have the children discuss their answers.

- Page 240 uses a rectangle to represent one dollar. Tell the children to compare the size of each of the other shaded rectangles with the size of the rectangle labeled one dollar. Work the first exercise together. After the children have completed the rest of the exercises, let them discuss their answers.

- Page 241 may be assigned for independent work. Before the children begin work, have them identify the value of one part of each rectangle.

---

1 dollar

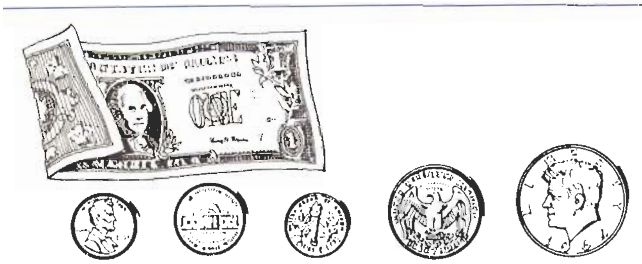
Compare the shaded part to 1 dollar.

1. $\frac{1}{2}$ dollar
2. $\frac{1}{4}$ dollar
3. $\frac{1}{10}$ dollar
4. $\frac{1}{20}$ dollar
5. $\frac{1}{100}$ dollar

reference page

C-240

Name



How much value?

1. How many dimes make 1 dollar? 10
2. Compare a dime to a dollar. $\frac{1}{10}$ dollar
3. How many pennies make 1 dollar? 100
4. Compare a penny to a dollar. $\frac{1}{100}$ dollar
5. How many nickels make 1 dollar? 20
6. Compare a nickel to a dollar. $\frac{1}{20}$ dollar
7. How many quarters make 1 dollar? 4
8. Compare a quarter to a dollar. $\frac{1}{4}$ dollar
9. How many half dollars make 1 dollar? 2
10. Compare a half-dollar to a dollar. $\frac{1}{2}$ dollar

reference page

C-239

Name

1 dollar

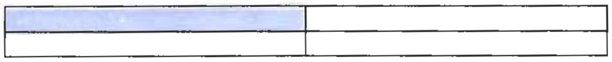
Color enough parts to show 1 dollar.

1. Each piece is $\frac{1}{2}$ dollar. 2 colored pieces
2. Each piece is $\frac{1}{20}$ dollar. 20 colored pieces
3. Each piece is $\frac{1}{10}$ dollar. 10 colored pieces
4. Each piece is $\frac{1}{100}$ dollar. 100 colored pieces
5. Each piece is $\frac{1}{4}$ dollar. 4 colored pieces

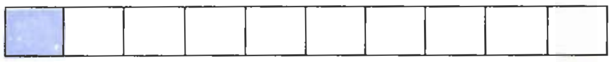
C-241

1. 

50 + 50 = 100 $\frac{1}{2}$ of 100¢ is 50¢.

2. 

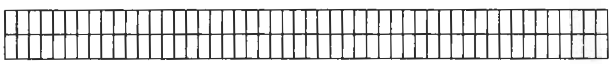
25 + 25 + 25 + 25 = 100 $\frac{1}{4}$ of 100¢ is 25¢.

3. 

$\frac{1}{10}$ of 100¢ is 10¢.

4. 

$\frac{1}{20}$ of 100¢ is 5¢.

5. 

$\frac{1}{100}$ of 100¢ is 1¢.

C-242

Circle the amount that has the greater value.

1. $\frac{1}{4}$ dollar or ($\frac{1}{2}$ dollar)
2. $\frac{1}{20}$ dollar or ($\frac{1}{10}$ dollar)
3. ($\frac{1}{4}$ dollar) or $\frac{1}{10}$ dollar
4. ($\frac{1}{2}$ dollar) or $\frac{1}{100}$ dollar
5. $\frac{1}{20}$ dollar or ($\frac{1}{4}$ dollar)
6. ($\frac{1}{2}$ dollar) or $\frac{1}{10}$ dollar
7. ($\frac{1}{4}$ dollar) or $\frac{1}{100}$ dollar
8. ($\frac{1}{10}$ dollar) or $\frac{1}{20}$ dollar
9. ($\frac{1}{20}$ dollar) or $\frac{1}{100}$ dollar
10. $\frac{1}{100}$ dollar or ($\frac{1}{10}$ dollar)

On Dollar Day at the bakery, each of these items costs $1.00 – a pie, a dozen rolls, a pound of cookies, and a cake.

Which costs more? Circle your answer.

11. $\frac{1}{4}$ pound of cookies or ($\frac{1}{2}$ pound of cookies)
12. $\frac{1}{2}$ dozen rolls or (1 pie)
13. 1 roll or ($\frac{1}{2}$ pound of cookies)
14. $\frac{1}{2}$ cake or (8 rolls)

C-244

Name

Mr. Block owned a food store. Each year on Dollar Day he charged one dollar for many items in his store.

1. Cheese costs $1.00 a pound. Mrs. Post bought a piece of cheese for 25¢. How much does 25¢ worth of cheese weigh? $\frac{1}{4}$ pound
2. Laurie's mother gave her 75¢ for $\frac{1}{2}$ pound of potato salad. Salads are $1.00 a pound. Did Laurie have enough money? Yes
3. Pickles cost 5¢ each. How many pickles can Bruce buy for $1.00? 20
4. Carol bought $\frac{1}{4}$ gallon of orange juice. A gallon of juice costs one dollar. What part of a dollar did the $\frac{1}{4}$ gallon cost? $\frac{1}{4}$
5. Ham costs $1.00 a pound. Mr. Bell bought 50¢ worth. How much does 50¢ worth of ham weigh? $\frac{1}{2}$ pound

C-243

● Use page 242 for a class activity. Call attention to the fact that the large rectangle is 100¢. For exercise 1, ask what number added to itself gives a sum of 100.

$$\_\_ + \_\_ = 100$$

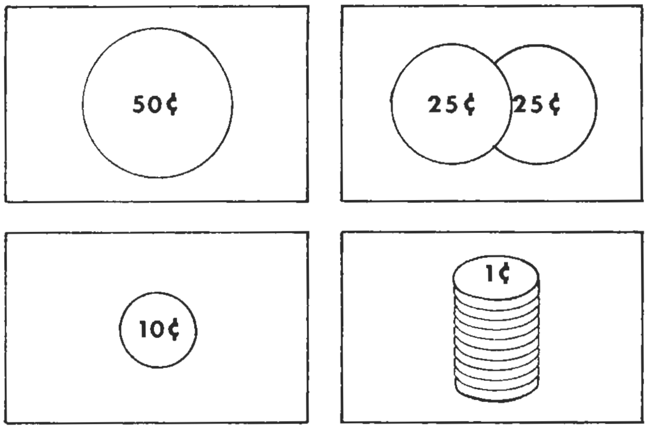
Help children understand that 50 + 50 = 100, and $\frac{1}{2}$ of 100 is 50. Continue in a similar way with the rest of the exercises.

● The story exercises on page 243 may be done as a class activity or assigned as independent work. Let the children work in small groups if they wish. When everyone has completed the exercises, let the children discuss their answers.

● For page 244 assign the exercises in the upper section and the story problems in the lower section as independent work. Discuss the children's answers after everyone has completed the page.

Supplemental Experience

■ Play the game "Match the Value." Hold up a picture of a coin. Ask a child to find a picture of coins that have the same value as the one you displayed. Have him place the picture he chose and your picture of a coin side by side on a section of the chalktray.



As the children match the coins you put on the tray, have them tell the class what fractional part of the original coin's value each of his selected coins represents. For example, the child may say that two $\frac{1}{4}$ dollars are equal in value to one $\frac{1}{2}$ dollar. One $\frac{1}{4}$ dollar is $\frac{1}{2}$ of a half dollar.

UNIT 15
MISSING FACTOR MULTIPLICATION

Pages 245 Through 270

OBJECTIVE

To reinforce the child's knowledge of basic multiplication facts.

The child continues to explore arrays. He is given the count and one factor and examines the array to find the missing factor. He uses his knowledge of basic multiplication facts to supply missing factors.

See Key Topics in Mathematics for the Primary Teacher: Multiplication.

KEY IDEAS

To know a multiplication fact is to know a missing factor.

Rearrangement of factors does not change the product.

If the model is an array, we use multiplication.

KEY IDEA

To know a multiplication fact is to know a missing factor.

Scope

To extend the children's knowledge of basic multiplication facts.

Fundamentals

Periodically the third-grade teacher checks the quality of each child's knowledge of the multiplication concept and the basic facts. She helps each child make effective use of his knowledge by looking at basic facts in terms of missing factors. The missing-factor exercise demands a more thorough knowledge of the basic facts. The overlearning that leads to immediate recall of basic multiplication facts is developed as the child works with the new and interesting missing-factor exercise.

The placeholder is used in place of the numeral for the missing factor. Teachers have used various names for this symbol without creating any particular difficulty. Most important, however, is the underlying idea that some number makes the equation true. There are several accepted ways of reading placeholder equations. Consider $5 \times \square = 35$.

Five times placeholder equals thirty-five.
Five times box equals thirty-five.
Five times a certain number equals thirty-five.
Five times the missing factor is thirty-five.
Five times the secret number is thirty-five.

Each statement is correct, but the perceptive teacher checks to be sure that each child understands the underlying idea.

A proper regard for the developmental level of the pupil requires this basic use of the placeholder before the complications of open sentences. In the primary school, all sentences written with placeholders are true statements. The placeholder merely names an otherwise unspecified number—a constant, not a variable.

The use of the placeholder for a missing factor prepares the child for the more sophisticated use of the placeholder in the next grade. There the placeholder will be used to name a variable in an open sentence.

Readiness for Understanding

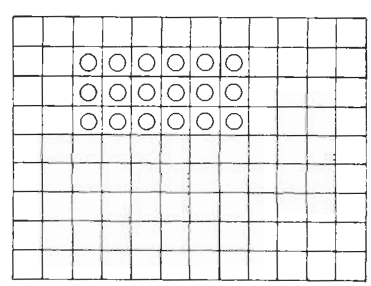
An understanding of the concept of product.
Knowledge of basic multiplication facts.

Developmental Experiences

for flannel board	*for each child*
numerals 0 through 9	1″ squared paper (9″ × 12″)
felt placeholder, times sign, and equal sign	counters
	plastic numerals and symbols

pocket chart
tagboard strips (1″ × 3″)

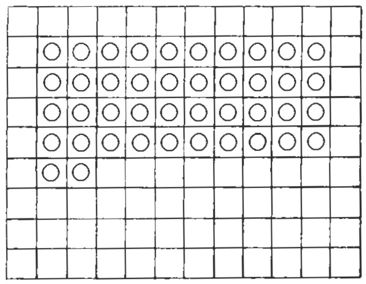
▶ Provide each child with a sheet of 1-inch squared paper (9 inches by 12 inches) and 85 counters (washers). Ask the children to construct on their desks a 3 × 6 array. Tell them to use their plastic numerals and symbols to show the product and the count below their array.



3×6=18
or
6×3=18

Continue in this way to have the children illustrate several basic multiplication facts. In each instance direct them to build an appropriate equation below the given array.

Then have the children look at some multiplication facts in terms of a missing factor. Direct the children to show a set of 42. Tell them to arrange the members to show the meaning of 42 (4 tens + 2).



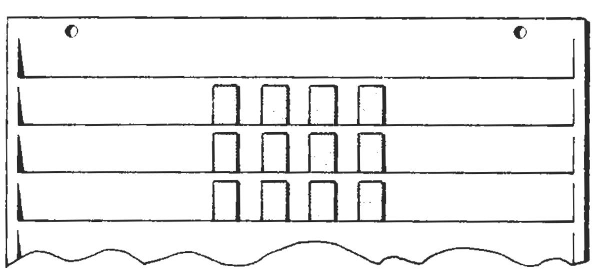
Tell the children to rearrange the members of their array into rows of 6 and to be ready to tell the greatest number of these rows it is possible to make. Write on the chalkboard the incomplete sentence $\square \times 6 = 42$. Explain to the children that this equation can easily be completed after they show the number of rows of 6 in the set of 42.

After the children have carried out the assignment and the sentence on the chalkboard has been completed, write the following sentence below the first one: $\square \times 7 = 42$. Direct the children to observe their array and to tell how many rows of 7 there are in this set of 42. Ask someone to complete the second equation.

$$7 \times 6 = 42$$
$$6 \times 7 = 42$$

Continue to have the children show the number of rows that represent one factor of a basic multiplication fact.

▶ Write on the chalkboard $12 = \square \times 4$. Give a child a pack of tagboard strips and ask him to illustrate the information given in the sentence by building in the pocket chart an appropriate array.



Encourage the class to discuss the information given in the sentence on the chalkboard and then to decide whether or not the child's illustration of this information is appropriate. Comments similar to the following may be made:

The array related to the equation will have 12 members arranged in rows of 4, or arranged in rows that have 4 members in each.

$\square$ is the number of rows of 4 in the set of 12, or the number of members in each of 4 rows. This number is 3.

Ask a child to complete the equation on the chalkboard.

Continue to let the children interpret similar incomplete sentences. Sentences such as the following could be used.

$36 = \square \times 9$	$64 = \square \times 8$
$28 = 7 \times \square$	$56 = 8 \times \square$
$32 = \square \times 4$	$21 = 7 \times \square$
$54 = 9 \times \square$	$45 = \square \times 9$
$63 = 7 \times \square$	$49 = 7 \times \square$

▶ Play the game "Number, Please." Scatter at random on the flannel board two sets of felt numerals 0 through 9. Place in the center of the board the equation $4 \times \square = 32$. Tell the children that if the equation could talk, it would tell them something about numbers. For example, $4 \times \square = 32$ might say, "Four times the secret number equals thirty-two."

Ask several children to read aloud the equation as a complete statement. Then ask, "Who knows the secret number?" or "What number is placeholder?" Have counters available for the children who may need them to determine the solution. Continue the game using basic multiplication facts.

▶ Give the children another opportunity to compute basic multiplication combinations; use a difference of two products. Ask the children to name the product that is represented by the difference $(5 \times 6) - (1 \times 6)$. Tell them that one factor of the product is 6. Record the class's response on the chalkboard.

$$4 \times 6 = (5 \times 6) - (1 \times 6)$$

Ask a child to compute 4×6 and to record the result below the product. Have another child compute each part of $(5 \times 6) - (1 \times 6)$ and record the results below this expression.

$$4 \times 6 = (5 \times 6) - (1 \times 6)$$
$$24 = 30 - 6$$

Direct the class to compute $30 - 6$ and then compare 24 and $30 - 6$.

Continue to have the children compute other basic multiplication facts using a difference of products. The following differences are a few of those that could be used in this activity.

$(8 \times 9) - (2 \times 9)$	$(8 \times 5) - (5 \times 5)$
$(7 \times 7) - (4 \times 7)$	$(9 \times 4) - (4 \times 4)$
$(7 \times 8) - (3 \times 8)$	$(7 \times 3) - (6 \times 3)$

Pages 245 through 256

● Use page 245 as a discussion page. If possible, have the children use the flannel board and felt cutouts to illustrate each question as it is discussed.

● The exercises on pages 246 through 248 are designed to help the children determine the missing factor after they have studied the array. Not all of the children will need to use the arrays to complete the exercises. Work one or two exercises on each page with the class; then assign the other exercises for independent work.

Name

UNIT 15 MISSING FACTOR MULTIPLICATION

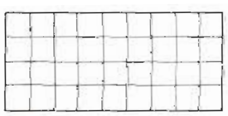
Jerry brought 24 pictures of horses to school. He started to make 4 equal rows of pictures on the bulletin board.

1. How many pictures will be in each row? 6
2. If Jerry put the 24 pictures in 6 rows, how many pictures would be in each row? 4
3. If he put the 24 pictures in 2 rows, how many would be in each row? 12
4. If he put the 24 pictures in 3 rows, how many would be in each row? 8

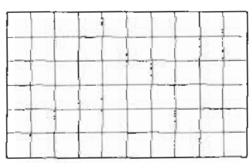
reference page

D-245

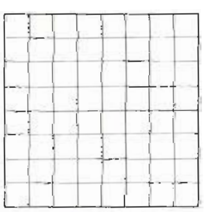
Complete each equation.

1. 

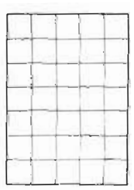
4 × 9 = 36

2. 

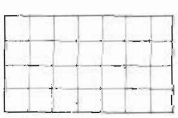
6 × 10 = 60

3. 

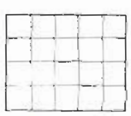
8 × 8 = 64

4. 

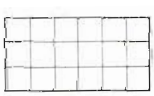
5 × 7 = 35

5. 

7 × 4 = 28

6. 

4 × 5 = 20

7. 

3 × 6 = 18

8. 

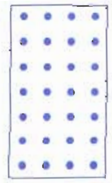
9 × 2 = 18

reference page

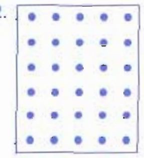
D-246

Name

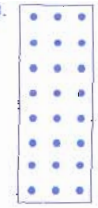
Complete each equation.

1. 

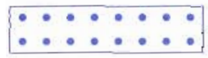
28 = 4 × 7

2. 

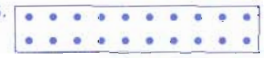
30 = 5 × 6

3. 

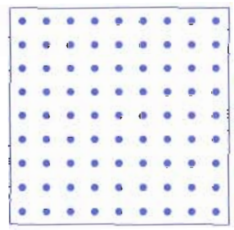
24 = 3 × 8

4. 

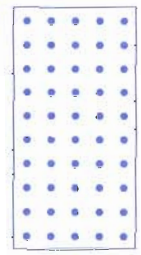
16 = 8 × 2

5. 

20 = 10 × 2

6. 

81 = 9 × 9

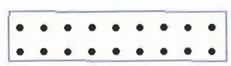
7. 

50 = 5 × 10

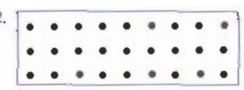
reference page

D-247

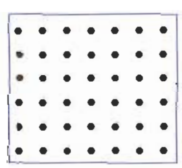
Complete each equation.

1. 

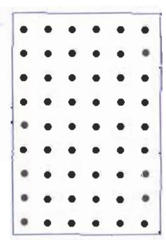
18 = 9 × 2

2. 

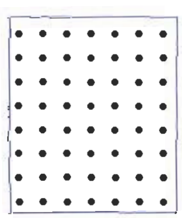
27 = 3 × 9

3. 

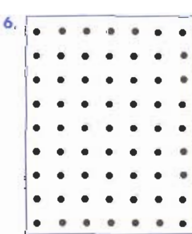
42 = 6 × 7

4. 

54 = 9 × 6

5. 

56 = 8 × 7

6. 

63 = 7 × 9

D-248

Name

Give the missing length. Complete each equation.

8 / 32 / 4

1. 32 = 8 × 4

9 / 3 / 27

2. 27 = 9 × 3

9 / 63 / 7

3. 63 = 9 × 7

7 / 2 / 14

4. 14 = 2 × 7

8 / 6 / 48

5. 48 = 8 × 6

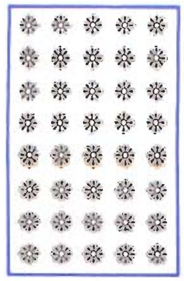
8 / 9 / 72

6. 72 = 8 × 9

D-249

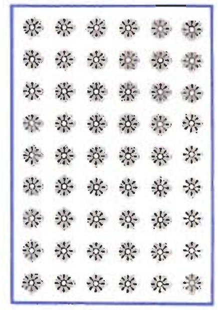
Name

Answer each question. Complete each equation. *



40

1. How many 8's? 5 — 40 = 8 × 5
2. How many 5's? 8 — 40 = 5×8
3. How many 10's? 4 — 40 = 10×4
4. How many 4's? 10 — 40 = 4×10
5. How many 20's? 2 — 40 = 20×2
6. How many 2's? 20 — 40 = 2×20
7. How many 40's? 1 — 40 = 40×1
8. How many 1's? 40 — 40 = 1×40
9. How many 9's? 6 — 54 = 9×6
10. How many 18's? 3 — 54 = 18×3
11. How many 27's? 2 — 54 = 27×2
12. How many 6's? 9 — 54 = 6×9
13. How many 3's? 18 — 54 = 3×18
14. How many 2's? 27 — 54 = 2×27
15. How many 1's? 54 — 54 = 1×54
16. How many 54's? 1 — 54 = 54×1



54

*Order of factors will vary.

reference page

D-251

Give the missing length. Complete each equation.

4 / 4 / 16

1. 4 × 4 = 16

9 / 54 / 6

2. 9 × 6 = 54

4 / 6 / 24

3. 6 × 4 = 24

7 / 3 / 21

4. 3 × 7 = 21

9 / 45 / 5

5. 5 × 9 = 45

9 / 27 / 3

6. 3 × 9 = 27

reference page

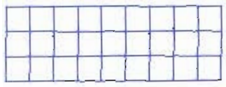
D-250

● The illustrations on pages 249 and 250 are more abstract than those on previous pages; only the outline of the array is shown. The children should understand that knowing the count and the number of rows one way is all that they need to determine the missing factor. Work one or two exercises with the class; then tell them to complete the other exercises on their own. Those children who are unsure of the missing factor for a given exercise may check their response by drawing the complete array.

● The exercises on page 251 are designed to guide the child to discover all of the various products equations that can be associated with a given array. Discuss and answer the first four questions associated with the array at the top of the page. When you are sure that the children understand the procedure to be followed, assign the remaining exercises for independent work. After the children have completed the page, let them read the equations they wrote in answer to specific questions.

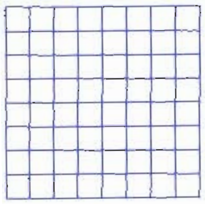
Complete each question.

1. 1 × 27 = 27
2. 3 × 9 = 27
3. 9 × 3 = 27
4. 27 × 1 = 27



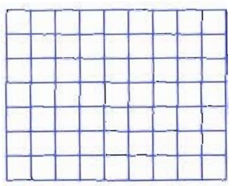
27

5. 2 × 32 = 64
6. 4 × 16 = 64
7. 8 × 8 = 64
8. 16 × 4 = 64
9. 32 × 2 = 64
10. 64 × 1 = 64
11. 1 × 64 = 64



64

12. 63 × 1 = 63
13. 21 × 3 = 63
14. 9 × 7 = 63
15. 7 × 9 = 63
16. 3 × 21 = 63
17. 1 × 63 = 63



63

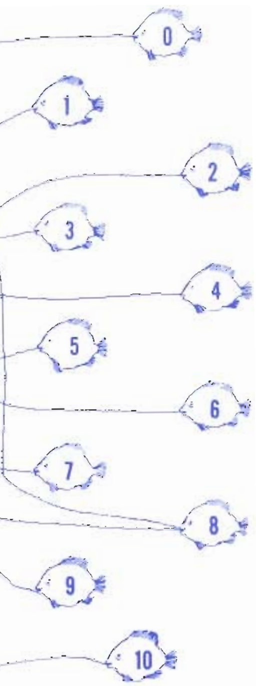
reference page

D-252

Name

Fish for the missing factor.

1. ☐ × 6 = 24
2. 72 = ☐ × 9
3. 36 = 6 × ☐
4. 9 = 3 × ☐
5. 49 = ☐ × 7
6. ☐ × 8 = 40
7. 16 = ☐ × 8
8. ☐ × 6 = 48
9. 6 × ☐ = 54
10. 0 = ☐ × 7
11. 4 × ☐ = 4
12. 2 × ☐ = 20



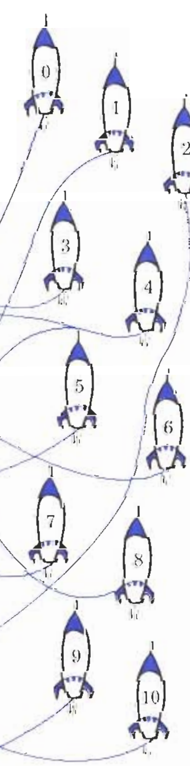
D-253

● Page 252 provides further experience in determining various multiplication equations that can be associated with a given array. Help the children understand that the placeholder equation 2 × ___ = 64 asks the same question as "How many 2s in 64?" Work several exercises with the class; then instruct the children to complete the equations independently.

● Pages 253 and 254 provide practice in finding missing factors. Only the basic multiplication combinations are used. Explain to the children that they may draw lines to connect a number to its corresponding box. After the children have finished these pages, discuss the numbers they used to complete the equations.

Draw a line from each rocket to its launching pad.

1. 7 × ☐ = 56
2. 21 = ☐ × 7
3. 9 × ☐ = 54
4. 6 × ☐ = 42
5. 32 = ☐ × 8
6. 2 × ☐ = 0
7. ☐ × 9 = 45
8. 30 = 3 × ☐
9. ☐ × 6 = 6
10. 4 × ☐ = 16
11. 14 = 7 × ☐
12. 72 = ☐ × 8



D-254

Name

Complete.

1.	1 × 2 = 2	5 × 4 = 20	8 × 3 = 24	4 × 2 = 8	9 × 4 = 36
2.	9 × 5 = 45	10 × 6 = 60	2 × 9 = 18	6 × 5 = 30	9 × 9 = 81
3.	7 × 3 = 21	5 × 5 = 25	3 × 9 = 27	6 × 2 = 12	9 × 8 = 72
4.	6 × 3 = 18	7 × 5 = 35	2 × 7 = 14	10 × 8 = 80	8 × 2 = 16
5.	7 × 6 = 42	3 × 4 = 12	6 × 0 = 0	3 × 5 = 15	6 × 8 = 48
6.	4 × 9 = 36	7 × 8 = 56	7 × 7 = 49	9 × 5 = 45	4 × 6 = 24

D-255

Follow the bird to the birdhouse. Find the pattern.

Each missing factor after 10 is 1 less than the one before it.

5 × [10] = 50

36 = 4 × [9]

[8] × 7 = 56

35 = [7] × 5

54 = 9 × [6]

6 × [5] = 30

7 × [4] = 28

[3] × 9 = 27

12 = [2] × 6

9 × [1] = 9

3 × [0] = 0

D-256

● Page 255 provides further practice in supplying the missing factor. You might use this page as a class activity rather than for written practice. Call on the children to complete an exercise you specify. For example, to complete the fourth exercise in row 4, a child may respond, "Eight times ten is eighty" or "Ten times eight is eighty." You can also use this page again, and have the children complete the exercises independently.

● Have the children complete page 256 independently. Then let the class discuss their results.

Supplemental Experiences

■ Construct a set of cards that contain multiplication equations that have either the first or second factor missing.

7 × □ = 42 **□ × 6 = 54**

32 = 8 × □ **45 = □ × 9**

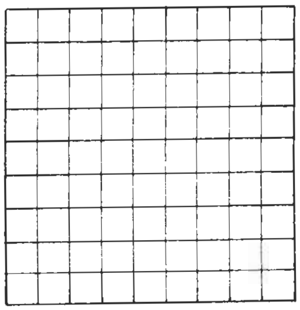
Divide the class into two teams. Hold up one card and let the first child on one team supply the missing number. If he answers correctly, let him return to his seat. If his answer is incorrect, he must go to the end of his team's line. Hold up another card and let the first child on the other team supply the answer. Continue until all of the children on one team have answered correctly.

■ To provide practice with multiplication combinations, play a "baseball" game. Organize two teams and let each group choose a team name. Draw a scoreboard on the chalkboard.

										TOTAL
YANKEES										
INDIANS										

Pitch the balls to one team at a time by calling out multiplication exercises or by holding up placeholder equation cards such as 0 × 3 = □, 4 × □ = 28, or 27 = □ × 3. If a team member answers correctly, he scores one run. If a child answers incorrectly, he scores an out. When three outs have been made, the second team comes up to bat. After nine innings total the score and declare a winner.

■ Draw on the chalkboard a 9 by 9 array similar to the one shown in the illustration.



Ask the children to imagine that this is an array of windows; it is the back of a modern office building. Have the children use the illustration to help answer these questions:

If you partitioned the array of windows into three equal parts, how many windows would be in each part?

Is it possible to partition this array of windows into two equal parts?

How many windows would be in one of nine equal parts of the array of windows?

Write products that show the partitioning used for the array of windows in the three questions.

KEY IDEA

Rearrangement of factors does not change the product.

Scope

To review the rearrangement property of multiplication.

Fundamentals

Although the teacher will not bring up the operation of division at this time, she cannot control parents and relatives. It is almost certain that some child will ask about division and want to know if the exercises on pupil page 256, for example, are division exercises. The issue must be fairly and correctly met. Yes, it is certainly true that $4 \times \square = 24$ is a division exercise that could have been written $24 \div 4 = \square$.

The reason for the GCMP approach is to be sure that the idea of division is understood when it is introduced rather than teaching the new operation of division without proper readiness. Division undoes multiplication. The quotient will turn out to be nothing more than the missing factor of multiplication. By placing primary emphasis upon the missing factor the skillful teacher builds the readiness that is necessary for maximum progress in division.

If ideas of division are brought up, agree with their correctness and then continue to emphasize the missing factor of multiplication.

Readiness for Understanding

An understanding of product.

Knowledge of basic multiplication facts.

Developmental Experiences

for flannel board
numerals 0 through 9
felt placeholder, times sign, and equal sign

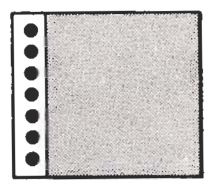
for each child
sheet of $\frac{1}{2}''$ squared paper ($15'' \times 15''$)
counters
plastic numerals and symbols
tagboard placeholder, times sign, and equal sign
tagboard cards ($2'' \times 2\frac{1}{2}''$)
crayons

array-cards
piece of tagboard
felt-tip pen
colored chalk

▶ Place the following felt symbols across the top of the flannel board.

2 sets of numerals for 0 through 9
a placeholder
a times sign
an equal sign

Ask the children to close their eyes. Then hold up a card that shows a 7 by 8 array and use a piece of tagboard to cover all but the first row of this array.



Tell the children to open their eyes and examine the part of the array visible on the card you are holding. Explain that this array has 56 members. Ask the children to think about how many rows of 7 are in this array. Before any child responds to this question, put on the flannel board a placeholder equation that represents the situation.

Uncover the array so that the children may verify the response. Let a child complete the placeholder equation by putting the appropriate numeral in the box.

Continue to let the children supply the missing factor in given equations. The following arrays are but a few of those that could be used in this activity.

4 by 6	3 by 8
2 by 9	8 by 4
8 by 5	9 by 6
9 by 9	6 by 8
7 by 4	7 by 9

In each instance cover all but one row of the array and tell the class the count for the product. Ask a child to put the placeholder equation on the flannel board. Finally, let the children name the missing factor that completes the equation.

▶ Give each child a 15 by 15 inch sheet of $\frac{1}{2}$-inch squared paper and 80 counters or washers. Direct the children to construct a 5 by 8 array on their paper. Instruct them to use their plastic numerals and symbols to show the product and the count in an equation below their array.

Now tell the children to build the sentence, "Some number times 2 is 40." Remind them to use their placeholders.

$$5 \times 8 = 40$$
$$\square \times 2 = 40$$

Have the children rearrange the members of their array to show how many 2s are in 40. Direct them to complete their second equation by replacing their placeholders with numerals.

Ask the children whether or not they see a relationship between the number of 2's that equal 40 and the number of 8's that equal 40. Someone may note that the number of 2's is four times the number of 8's that equal 40. Someone may decide that this is because 8 is 4×2.

Next, tell the children to construct another placeholder equation. Tell them that their equation should say, "Some number times 4 is 40."

$$5 \times 8 = 40$$
$$20 \times 2 = 40$$
$$\square \times 4 = 40$$

Tell the children to rearrange the members of their array to show how many 4's are in 40. Direct them to complete their equation by replacing the placeholder with the appropriate numerals.

Ask the children whether or not they see a relationship between the number of 4's that equal 40 and the number of 8's that equal 40. Someone may note that the number of 4's is twice the number of 8's that equal 40. Someone may decide that this is because 8 is 2×4.

Continue the activity; direct the children to show the number of 5's, the number of 10's, and the number of 20's that equal 40. Have them determine the relationship between the number of 5's and the number of 20's that equal 40.

Adapt this procedure to the following products:

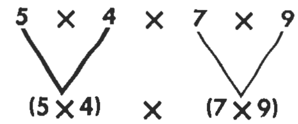
Using 6×8 the children may show the number of 2's, 3's, 4's, 6's, and 8's that equal 48.

Using 6×6 they may show the number of 2's, 3's, 4's, 6's, and 9's that equal 36.

Using 6×7 they may show the number of 2's, 3's, 6's, and 7's that equal 42.

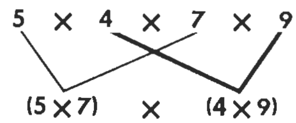
▶ This activity is designed to accomplish several things. The children will have the opportunity to see how many pairs they can make from the 4 factors of a product. They will learn through experience that a product is unaffected by the way its factors are arranged in pairs. They will also discover that pairs of the 4 factors form 2 products. The 2 products are 2 factors of the grand product.

Write $5 \times 4 \times 7 \times 9$ on the chalkboard. Have a child come to the board and point to the factors he wishes to pair. Give him two pieces of colored chalk and help him illustrate these pairs.



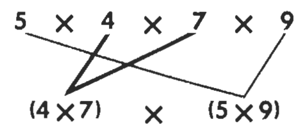
Explain to the class that the product of 4 factors (5, 4, 7, and 9) is now being considered as a product of 2 factors (5×4) and (7×9).

Again write $5 \times 4 \times 7 \times 9$ on the board and give another child two pieces of colored chalk. Ask this child to show a different set of pairs of 5, 4, 7, and 9. Perhaps this child will pair the factors as illustrated below.



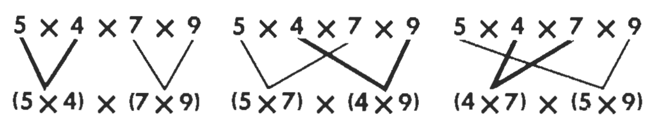
Remind him to link his two products with a times sign.

Write $5 \times 4 \times 7 \times 9$ on the chalkboard a third time. Ask some child to show different pairs of 5, 4, 7, and 9 than the other children used. This child may illustrate the following pairs.



Ask the class whether or not there are any more possible pairs of 5, 4, 7, and 9 other than the commuted forms of the ones that are shown.

Select six children and ask each one to compute one of the products that are factors. Then ask three other children to compute the product of the products (the grand product).



20×63= 20	35×36= 35	28×45= 28
×63	×36	×45
60	30	40
1200	180	100
1260	150	320
	900	800
	1260	1260

Allow the children ample time to study and discuss the results of these computations. The children should be aware of the fact that changing the pairs of the factors of the product $5 \times 4 \times 7 \times 9$ did not change that product.

Repeat the activity several times; use other products of 4 factors. In each instance, after the grand product has been computed, allow the children time to discuss the results.

▶ Provide each child with a tagboard placeholder, a times sign, an equal sign, and a set of 2 by $2\frac{1}{2}$ inch tagboard cards. Help the children label their cards as shown.

	0	0	0	0	0	0		
1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9

Read to the children sentences such as the following.

100 tens times the secret number equals 100 ten tens.

$$1000 \times \square = 10000$$

The secret number times 10 tens equals 100 tens.

$$\square \times 100 = 1000$$

1 hundred hundreds equals the secret number times 1 hundred.

$$10000 = \square \times 100$$

1 hundred times the secret number equals 100 ten ten tens.

$$100 \times \square = 100000$$

The secret number times 10 hundreds equal 100 hundred hundreds.

$$\square \times 1000 = 1000000$$

100 tens equals 1 ten times the secret number.

$$1000 = 10 \times \square$$

10 hundreds times the secret number equals 10 hundred hundreds.

$$1000 \times \square = 100000$$

After you read each sentence, have the children use their symbols to construct a placeholder equation that expresses the statement they just heard. Direct the children to analyze the facts given in the statement and then to complete the equation. Explain that the children should replace the $\square$ with cards that show the appropriate number. After each child has completed the equation on his desk, let several children explain how they determined the secret number. For example, someone may describe his thinking about the sentence 100 tens $\times$ $\square$ = 100 ten tens in the following way:

The 100 is part of a factor as well as part of the product.

All that had to be supplied was the number that makes ten tens when it is multiplied by ten.

That number is ten; ten times ten is ten tens.

Other children may describe their thinking in different ways. Give everyone an opportunity to share ideas.

Next, read sentences such as the following to the children.

5 tens times the secret number equals 45 tens.

$$50 \times \square = 450$$

The secret number times 9 hundreds equals 45 hundreds.

$$\square \times 900 = 4500$$

28 hundreds equals 7 tens times the secret number.

$$2800 = 70 \times \square$$

8 hundreds times the secret number equals 48 hundreds.

$$800 \times \square = 4800$$

The secret number times 6 hundreds equals 42 hundred tens.

$$\square \times 600 = 42000$$

64 hundreds equals 8 times the secret number.

$$6400 = 8 \times \square$$

Have the children build an appropriate placeholder equation for each exercise. Let them analyze the facts in the statement and then complete the equation. Ask various children to describe how they determined the secret number in a given exercise. For example, someone may describe his thinking about the sentence 28 hundreds = 7 tens $\times$ $\square$ in the following manner:

7 times some number is 28; that number is 4.

Tens times some number is hundreds; that number is ten.

Combining these two facts gives the secret number — 4 $\times$ ten or 4 tens.

7 tens $\times$ 4 tens = 28 hundreds: 2800 = 70 $\times$ 40.

Again, other children may describe their thinking in different ways; encourage them to contribute their ideas to the discussion.

Name

Multiply across and down to compute the four products and the grand product.

1.

3	4	12
5	6	30
15	24	360

grand product

2.

3	3	9
2	2	4
6	6	36

3.

7	10	70
3	4	12
21	40	840

4.

5	4	20
3	6	18
15	24	360

5.

7	8	56
9	4	36
63	32	2016

6.

9	2	18
6	6	36
54	12	648

7.

6	9	54
7	5	35
42	45	1890

8.

3	5	15
8	6	48
24	30	720

9.

3	8	24
7	2	14
21	16	336

D-257

Supply the missing factors and products.

1.

6	5	30
8	8	64
48	40	1920

grand product

2.

2	5	10
9	3	27
18	15	270

3.

4	6	24
5	9	45
20	54	1080

4.

9	8	72
5	8	40
45	64	2880

5.

4	9	36
7	7	49
28	63	1764

6.

8	2	16
3	6	18
24	12	288

7.

7	8	56
6	4	24
42	32	1344

8.

8	8	64
5	2	10
40	16	640

9.

4	9	36
7	9	63
28	81	2268

D-258

Pages 257 through 266

● Pages 257 and 258 provide practice for the children to compute products and to determine missing factors in the context of multiplication squares. All of the exercises on page 257 involve computation of products and grand products—products of products.

Instruct the children to read the directions and study the first exercise on page 257. Encourage them to explain the procedure to be followed in completing the multiplication squares. Call on a child to work the second exercise at the chalkboard; have him explain the steps involved in computing each product and the grand product. This child may multiply across first and then down, or he may multiply down and then across. Call on some other child to work the third exercise on the chalkboard and to explain the steps. When the children understand the procedure to be followed, assign the remaining exercises for independent work.

The multiplication squares on page 258 involve finding missing factors prior to finding products and grand products. Copy exercise 1 on the chalkboard.

6		
	8	
48	**40**	

The children may observe that in this exercise they can find the grand product with one step if they want to do so, since the grand product is the product of the products in the bottom row. Ask a child to show the computation of 48×40 on the chalkboard; then tell him to write the result of this computation in the appropriate place.

6		
	8	
48	**40**	**1920**

Next, point to the empty space directly below the 6 and ask whether or not enough information has been supplied to find this missing number. A child may observe that since 48 is the product of 6 and this missing number, the missing number is 8. Let the child write 8 in the appropriate place.

6		
8	**8**	
48	**40**	**1920**

Ask the children if they have enough information to find any other missing numbers. The children may suggest multiplying 8×8 to get one of the products in the column on the right or they may suggest that the product of 8 and the number above 8 is 40 so the missing number is 5. Follow the suggestion that they make. Continue in this way until the children have completed this square. Then direct a child to compute on the chalkboard the product of the products in the column on the right. Remind the class that this is one way to check the computation of the grand product.

Copy the second exercise on the chalkboard. Follow a procedure similar to that used with the first exercise; have the children suggest ways to complete the multiplication square.

If most of the children are ready to proceed independently, and only a few children need additional guidance, permit those children who need help to work together in a small group while the other children work independently.

● Pages 259 through 264 provide practice in finding missing factors. Many of the missing factors are multiples of 10, 100, or 1000. It is important that the children realize how these exercises are related to those in which only the basic multiplication combinations are involved. At this time the children should be able to complete these pages without too much difficulty. A child may first interpret an exercise such as 480,000 = ? × 80 as 48 ten thousands is some number times 8 tens. Next, he may ignore the fact that ten thousands and tens are involved and concentrate on the exercise 48 = ? × 8. Knowing that this missing number is 6, he then may establish whether or not it is 6 ones, 6 tens, 6 hundreds, and so forth. The original exercise tells him that tens × ? = ten thousands; the missing factor in this instance is thousands. Therefore, the missing factor in 480,000 = ? × 80 is 6000. Another child may follow a similar line of reasoning but prefer to interpret the exercise as 48 ten ten ten tens = ? × 8 tens.

480,000 = ? × 80

48 = 6 × 8
ten thousands = thousands × tens → **6 thousands**

or

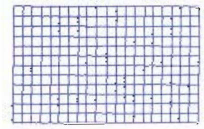
48 = 6 × 8
ten ten ten tens = ten ten tens × tens → **6 ten ten tens**

Encourage the children to explain their thinking steps to the class so that all of the children realize that there is more than one approach to the exercises on these pages.

When using each of pages 259 through 264, the teacher should work several exercises with the class before making any assignment. No assignment should be so long that the child becomes discouraged before he starts.

Name

Answer each question. Write an equation. *



240

1. How many 12's? 20
20 × 12 = 240

2. How many 20's? 12
12 × 20 = 240

3. How many 4's? 60
4 × 60 = 240

4. How many 40's? 6
6 × 40 = 240

5. How many 6's? 40
40 × 6 = 240

6. How many 60's? 4
60 × 4 = 240

7. How many 8's? 30
8 × 30 = 240

8. How many 80's? 3
80 × 3 = 240

9. How many 3's? 80
80 × 3 = 240

10. How many 30's? 8
8 × 30 = 240

11. How many 24's? 10
24 × 10 = 240

12. How many 10's? 24
10 × 24 = 240

*Order of factors will vary.

reference page

D-259

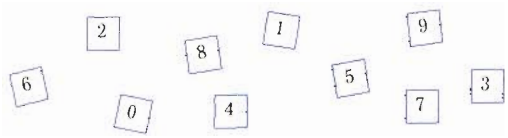
Complete.

1. 9 × 70 = 630
2. 80 × 5 = 400
3. 9 × 200 = 1800
4. 50 × 9 = 450
5. 3 × 60 = 180
6. 80 × 8 = 640
7. 6 × 600 = 3600
8. 7 × 80 = 560
9. 6 × 800 = 4800
10. 400 × 7 = 2800
11. 2 × 800 = 1600
12. 40 × 70 = 2800
13. 7 × 30 = 210
14. 60 × 9 = 540
15. 7 × 60 = 420
16. 30 × 90 = 2700
17. 50 × 80 = 4000
18. 90 × 70 = 6300
19. 90 × 9 = 810
20. 60 × 50 = 3000
21. 700 × 9 = 6300
22. 900 × 60 = 54000

D-260

Name

Complete each equation.



1. 9 = 3 × [3] 2. 18 = 9 × [2] 3. 160 = 40 × [4]

4. 14 = 2 × [7] 5. 80 = 20 × [4] 6. 40 = [8] × 5

7. 40 × [7] = 280 8. 7 × [5] = 35 9. 40 = [4] × 10

10. [9] × 9 = 81 11. [7] × 80 = 560 12. [3] × 70 = 210

13. 80 × [0] = 0 14. 9 × [4] = 36 15. 90 = 30 × [3]

16. 18 = 6 × [3] 17. [8] × 60 = 480 18. 200 × [2] = 400

D-261

Name

Complete each equation.

1. 800 × 80 = 64,000
   800 × 80 = 64,000
   64,000 = 8000 × 8

2. 9000 × 6 = 54,000
   900 × 6 = 5400
   540 = 6 × 90
   54 = 9 × 6

3. 700 × 80 = 56,000
   70 × 8 = 560
   56 = 8 × 7

4. 80 × 60 = 4800
   80 × 600 = 48,000
   480,000 = 6000 × 80
   80 × 60000 = 4,800,000

5. 900 × 7 = 6300
   90 × 700 = 63,000
   630 = 9 × 70

6. 900 × 500 = 450,000
   900 × 50 = 45,000
   45,000 = 500 × 90

7. 600 × 30 = 18,000
   60 × 300 = 18,000
   18,000 = 6 × 3000

8. 4000 × 8 = 32,000
   80 × 400 = 32,000
   32,000 = 40 × 800

9. 70 × 500 = 35,000
   500 × 7000 = 3,500,000
   350 = 7 × 50

10. 80 × 90,000 = 7,200,000
    8000 × 9 = 72,000
    720,000 = 9 × 80,000

D-263

Complete each equation.

1. 7 × 700 = 4900 2. 10 × 8 = 80 3. 60 × 7 = 420

4. 30 × 9 = 270 5. 45 × 2 = 90 6. 500 × 5 = 2500

7. 54 = 6 × 9 8. 360 = 40 × 9 9. 2400 = 3 × 800

10. 90 × 7 = 630 11. 50 × 9 = 450 12. 400 × 7 = 2800

13. 80 × 5 = 400 14. 70 × 8 = 560 15. 60 × 9 = 540

16. 900 × 7 = 6300 17. 5 × 90 = 450 18. 80 × 8 = 640

19. 9 × 900 = 8100 20. 60 × 3 = 180 21. 7 × 500 = 3500

D-262

Complete.

1. 10 × 10 = 100 2. 600 × 7 = 4200

3. 20 × 6 = 120 4. 60 × 900 = 54,000

5. 50 × 5 = 250 6. 800 × 8 = 6400

7. 8 × 20 = 160 8. 40 × 700 = 28,000

9. 9 × 9 = 81 10. 7000 × 3 = 21,000

11. 10 × 7 = 70 12. 600 × 4 = 2400

13. 6 × 60 = 360 14. 800 × 600 = 480,000

15. 30 × 7 = 210 16. 30 × 600 = 18,000

17. 20 × 5 = 100 18. 7 × 7000 = 49,000

19. 4 × 9 = 36 20. 6000 × 5 = 30,000

21. 20 × 40 = 800 22. 20 × 700 = 14,000

23. 500 × 9 = 4500 24. 600 × 80 = 48,000

25. 80 × 40 = 3200 26. 1000 × 700 = 700,000

27. 80 × 50 = 4000 28. 300 × 30 = 9,000

D-264

Name

Supply the missing factors and products.

1.

3	5	15
40	50	2000
120	250	30,000 ← grand product

2.

100	8	800
5	6	30
500	48	24,000

3.

6	50	300
70	7	490
420	350	147,000

4.

400	7	2800
2	3	6
800	21	16,800

5.

40	9	360
4	30	120
160	270	43,200

6.

7	20	140
10	5	50
70	100	7000

7.

8	90	720
30	4	120
240	360	86,400

8.

3	20	60
8	70	560
24	1400	33,600

9.

9	20	180
70	9	630
630	180	113,400

D-265

Complete.

1. 9 × 600 = 5400
2. 8 × 80 = 640
3. 800 × 6 = 4800
4. 4 × 9000 = 36,000
5. 5 × 9 = 45
6. 700 × 7 = 4900
7. 4 × 7000 = 28,000
8. 8 × 90 = 720
9. 3 × 30 = 90
10. 900 × 9 = 8100
11. 6 × 50 = 300
12. 80 × 5 = 400
13. 360 = 4 × 90
14. 7200 = 90 × 80
15. 4800 = 80 × 60
16. 280 = 70 × 4
17. 49,000 = 70 × 700
18. 4500 = 90 × 50
19. 7000 × 60 = 420,000
20. 60 × 9 = 540
21. 60 × 50 = 3000
22. 70 × 60 = 4200
23. 30 × 900 = 27,000
24. 3 × 70 = 210
25. 400 × 4 = 1600
26. 80 × 900 = 72,000
27. 70 × 500 = 35,000
28. 5600 = 700 × 8

D-266

● The multiplication squares on page 265 involve missing factors and products that are multiples of 10, 100, and 1000. Complete several exercises with the class before the children work on their own.

8	**90**	**720**
30	**4**	**120**
240	**360**	**86,400**

$$\begin{array}{r} 240 \\ \times\ 360 \\ \hline 2400 \\ 12000 \\ 12000 \\ 60000 \\ \hline 86{,}400 \end{array} \quad \textbf{and} \quad \begin{array}{r} 720 \\ \times\ 120 \\ \hline 400 \\ 14000 \\ 2000 \\ 70000 \\ \hline 86{,}400 \end{array}$$

Some of the children may also write placeholder equations or reconstruction exercises to help them find the missing factors.

$$\begin{array}{r} 30 \\ \times\ \square \\ \hline 240 \end{array}$$

$$\square \times 30 = 120$$

Permit the children to do any computation they feel is necessary to complete the multiplication squares. The amount of computation will vary from child to child. Remind the children that commas are optional with four digits. Give the children practice reading numerals for large numbers. Have the children read the answers aloud after the class has completed all of the exercises.

● Page 266 gives the children an opportunity to test their ability to find missing factors. Work several exercises with the class; then assign the remainder of the exercises for independent work.

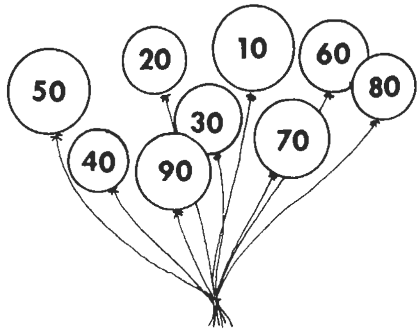
Supplemental Experiences

■ Give the children further experience with missing-factor exercises. To play the game "Challenge," a child says to another child, "I challenge you to find the missing factor! What number times eighty equals forty-eight hundred?" If the correct answer, sixty, is given, the winner gives the next challenge.

■ Let the children predict what will happen to the missing factor in a given equation if the known factor and the product have both been multiplied by two, three, or four. Will it result in a number that is greater than the original missing factor? Will it result in a number which is less than the original missing factor? Or will the original missing factor remain the same? After the children have expressed their views of this situation, tell them to check out their conclusions by writing related equations on the chalkboard.

$$\begin{array}{ll} 40 = 20 \times \square & 600 = 20 \times \square \\ 80 = 40 \times \square & 1200 = 40 \times \square \\ 120 = 60 \times \square & 1800 = 60 \times \square \\ 160 = 80 \times \square & 2400 = 80 \times \square \end{array}$$

■ Instruct the children to draw nine large balloons on the chalkboard or to cut them from colored paper and to fasten them to the chalkboard. Label the balloons as shown.



Establish the rule for the first game. For example, tell the children that each number is to be multiplied by the factor ninety. Then point to the balloons one by one.

The children must try to pop the balloon that you indicated by quickly telling the product of ninety and the numeral on the balloon. If he successfully pops all nine balloons he may choose the factor for the next game and point to the balloons.

KEY IDEA

If the model is an array, we use multiplication.

Scope

To provide additional opportunities to practice multiplication.

Fundamentals

The associative and commutative properties of multiplication suggest an exercise which provides an opportunity to practice multiplication skills. For example, multiply the numbers in the illustration horizontally and vertically until the grand product is obtained.

5	4	(5×4)	
3	6	(3×6)	
(5×3)	(4×6)	(5×4) × (3×6) (5×3) × (4×6)	Grand Product

Is $(5 \times 4) \times (3 \times 6)$ equal to $(5 \times 3) \times (4 \times 6)$? Although this question can be answered by computing, it may be answered without computing. Note how a short sequence of logical steps provides the answer. Begin with $(5 \times 4) \times (3 \times 6)$.

$(5 \times 4) \times (3 \times 6)$	
$= 5 \times (4 \times 3) \times 6$	general associative property
$= 5 \times (3 \times 4) \times 6$	commutative property
$= (5 \times 3) \times (4 \times 6)$	general associative property

To use the associative property would be too tedious, so the general associative property is used.

Readiness for Understanding

An understanding of product and array.

Developmental Experiences

pocket chart
pins
strip of paper
numeral-cards: 0 through 9
missing-factor equation-cards
(basic multiplication facts)

▶ Either write the following stories on the chalkboard, or read one of them at a time to the class.

Several of Ellen's friends gathered in her playroom to watch television. Ellen put 28 chairs in rows in front of the TV set. She made 4 rows with the same number of chairs in each. How many chairs did Ellen put in the front row?

A king has 56 pearls. He wants his jeweler to set the same number of jewels in each of 7 crowns. How many pearls will be put in each crown?

Bob planned to make 6 birdhouses. Each house would take 8 pieces of wood. How many pieces of wood does Bob need for his birdhouses?

Bob put 6 birdbaths out in the yard. One day 30 birds came to splash in them. If the same number of birds used each birdbath, how many birds used each birdbath?

Nan received 12 foreign dolls on her birthday. She displayed them on 3 shelves in her room. If she put the same number of dolls on each shelf, how many dolls would one shelf contain?

Ask a child to come to the chalkboard to write a placeholder equation that states the relationship between the number facts given in a story. For example, for the first story a child may write $28 = \square \times 4$, or any of its commuted forms: $28 = 4 \times \square$, $4 \times \square = 28$, or $\square \times 4 = 28$.

Ask another child to illustrate the facts in the story. He may, for example, draw an array similar to the one illustrated.

ch	ch	ch	ch	ch	ch	ch
ch	ch	ch	ch	ch	ch	ch
ch	ch	ch	ch	ch	ch	ch
ch	ch	ch	ch	ch	ch	ch

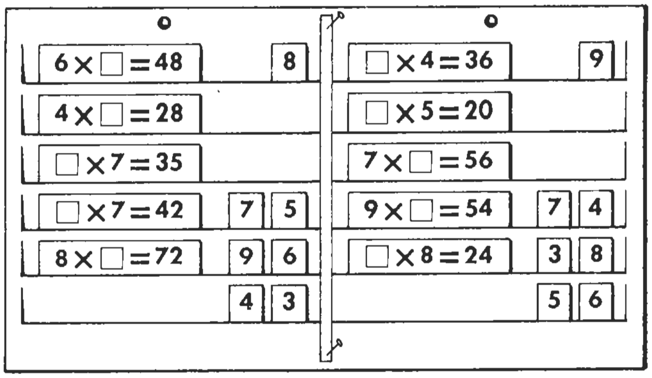
The letters *ch* refer to the chairs in the story.

Have some child complete the placeholder equation.

Use the same procedure with the other stories. Let the children write a placeholder equation, illustrate the facts in the story, and then complete the placeholder equation.

▶ Separate the class into two teams. Pin a strip of paper down the center of the pocket chart and assign one side of the chart to each team. On each side of the chart place 5 missing-factor equation-cards that show basic multiplication facts. Beside each set of equation-cards place 7 numeral-cards; 5 numeral-cards should be the factors missing from the equations.

Ask a member from each team to come forward to complete the first equation on his team's side. Explain that this may be done by placing beside the equation the numeral-card that represents the missing factor.



As soon as a child completes the equation, he is to sit down. The next four members of the team should follow the same procedure using the remaining cards. The first team to finish matching the equations and numeral-cards earns a point. For every correct match the teams each earn a point.

Change the 5 equation-cards and the 7 numeral-cards. Begin a second round of the game with five other members from each team. Continue in this way until all of the children in the class have participated several times. Then total each team's points and declare a winner.

Name

Each year the Carlsons go on a camping trip.

1. Mrs. Carlson fried 24 slices of bacon. She gave each of her 3 children the same number of slices. How many slices did each child get? 8

2. Nancy collected 64 butterflies. She put the same number of butterflies in each of 8 picture frames. How many butterflies did she put in each frame? 8

3. Mrs. Carlson bought 120 cans of food before the trip. The clerk put them in boxes that held 12 cans each. How many boxes did he fill? 10

4. One day Pete and Mike caught 18 trout. Neither of the boys caught more fish than the other. How many fish did each boy catch? 9

5. Mike caught 35 frogs in 7 days. Each day he caught the same number of frogs. How many did he catch each day? 5

reference page

D-267

▶ Tell the children that they will look at the computation of basic multiplication combinations by relating one of the given factors to 10. Write the product 9 × 6 on the chalkboard. Ask the class to describe the relationship between 9 × 6 and 10 × 6. Someone may observe that 9 × 6 is 1 × 6 less than 10 × 6. Record this idea on the chalkboard.

$$9 \times 6 = (10 \times 6) - (1 \times 6)$$

Ask a child to compute 9 × 6 and to record the result below this form of the product in the equation on the board. Then have some other child compute each part of the difference of products (10 × 6) − (1 × 6) and record the result.

$$\begin{aligned} 9 \times 6 &= (10 \times 6) - (1 \times 6) \\ 54 &= \quad 60 \quad - \quad 6 \end{aligned}$$

Have the class compute 60 − 6 and then compare 54 and 60 − 6.

Continue in this way to have the children compute other basic multiplication combinations by using the relationship between one of the factors and 10. Some suggestions for products to use in this activity follow.

$$\begin{aligned} 8 \times 7 &= (10 \times 7) - (2 \times 7) \\ 9 \times 9 &= (10 \times 9) - (1 \times 9) \\ 7 \times 6 &= (10 \times 6) - (3 \times 6) \\ 6 \times 8 &= (10 \times 8) - (4 \times 8) \\ 9 \times 8 &= (10 \times 8) - (1 \times 8) \\ 7 \times 7 &= (10 \times 7) - (3 \times 7) \end{aligned}$$

Pages 267 through 270

● Page 267 provides practice in answering story questions that involve missing factors. Read each story exercise with the class. Let the children suggest missing-factor equations to represent each story. For example, any of the following statements would be appropriate for exercise 2.

$64 = 8 \times \square$ $\quad\quad$ $\square \times 8 = 64$

$8 \times \square = 64$ $\quad\quad$ $64 = \square \times 8$

Most of the children will be able to compute mentally and record just the final result since only the basic multiplication combinations are involved. However, allow the children who wish to compute on paper to do so. When the children have completed the page, ask individuals to draw arrays that might illustrate each story.

The Brinker family owns a nursery. Each year the children plant flower bulbs in rectangular arrays.

1. Hans has 36 bulbs. What are his possible arrays? *
   2 by 18
   3 by 12
   4 by 9
   6 by 6

2. Karen has 16 bulbs. What are her possible arrays? *
   2 by 8
   4 by 4

3. Karl has 30 bulbs. What are his possible arrays? *
   2 by 15
   5 by 6
   3 by 10

4. What are the possible arrays for *
   35 bulbs? 5 by 7
   18 bulbs? 2 by 9, 3 by 6
   20 bulbs? 2 by 10, 4 by 5

reference page *Order will vary.

D-268

What is the pattern?

1. 3 × 3 = 9
   9 × 3 = 27
   27 × 3 = 81
   81 × 3 = 243
   243 × 3 = 729

2. 5 × 6 = 30
   30 × 5 = 150
   150 × 4 = 600
   600 × 3 = 1800
   1800 × 2 = 3600

3. 4 × 10 = 40
   40 × 20 = 800
   800 × 30 = 24,000
   24,000 × 40 = 960,000

4. 2 × 5 = 10
   10 × 10 = 100
   100 × 15 = 1500
   1500 × 20 = 30,000

D-270

Name

Follow the airplane to the airport and find the pattern.

80 × 16 = 480
63 = 7 × 9
480 = 60 × 8
9 × 300 = 2700
500 × 10 = 5000
44 = 4 × 11
12 × 50 = 600

Follow the truck to the garage and find the pattern.

20 × 14 = 280
12 × 40 = 480
360 × 10 = 3600
7200 = 900 × 8
420 = 6 × 70
4 × 7000 = 28,000
160 × 2 = 320

What is the airplane's pattern? Each missing factor after 6 is 1 greater than the one before it.

What is the truck's pattern? Each missing factor after 14 is 2 less than the one before it.

D-269

● Before you assign the exercises on page 268, tell the class that one decision the children in the stories made was that they would not plant just 1 row of bulbs. Therefore, the class should not consider an array that has just 1 row, or an array that has just 1 member in each row. Tell the children that they may draw the possible arrays for each exercise or they may just describe the arrays. Then instruct the children to complete the exercises on their own.

● The exercises on page 269 involve finding the missing factors in a pattern. Discuss the directions for the page and the illustrations with the class. Explain that if they are able to discover a pattern about the missing factors in each column, they may not have to do much computing in completing the exercises. Point out that it is not necessary to start at the top of a column.

After the children have had sufficient time to work the exercises on their own, ask various children to tell the class how they discovered the patterns.

● The exercises on page 270 also challenge the children to find and use a pattern to complete the equations. Tell the children to see whether or not they can find the patterns that will help them complete each series of equations. After the children have completed the exercises on their own, ask several children to explain the patterns they found.

Supplemental Experiences

■ Write a series of missing-factor equations on the chalkboard. Ask the children to predict what will happen to the missing factor if the product is doubled and the known factor remains constant.

$24 = 6 \times \square$	$18 = 3 \times \square$
$48 = 6 \times \square$	$36 = 3 \times \square$

Help the children conclude that the missing factor doubles.

■ Use a classroom project in science or social studies as background material for creating story exercises. Ask each child to write a story exercise. Then direct the children to exchange papers, compute the answers, and then return the stories to the writers for checking. Discuss the exercises with the class; check for errors in the information given in the stories as well as errors in the computation.

UNIT 16
MULTIPLICATION SHORTCUTS

Pages 271 Through 294

OBJECTIVE

To develop short multiplication.

The child applies his knowledge of numeration to combine some of the computational steps of the long multiplication algorism. He multiplies by one-digit factors and writes the computed product immediately.

See Key Topics in Mathematics for the Primary Teacher: Multiplication.

KEY IDEAS

Some partial products can be combined.
We write down what we don't remember.
Skill in computation requires practice.

KEY IDEA

Some partial products can be combined.

Scope

To introduce short multiplication to the children.

Fundamentals

When the Hindu-Arabic numeration system is used, all algorisms for the computation of products involve the partial products that are determined digit by digit. The traditional algorism and other abbreviated algorisms involve combining some of the partial products before the product is recorded. For example, consider the following two-digit by two-digit computation.

34	The basic pattern of all algorisms
× 57	involves:
28	4 ones × 7 ones = 28 ones
210	7 ones × 3 tens = 21 tens
200	5 tens × 4 ones = 20 tens
1500	5 tens × 3 tens = 15 hundreds
1938	the sum of all partial products

The partial products that are tens can be combined. The calculation of 7×30 + 50×4 may be done mentally.

34	
× 57	
28	(Think 7×30 is 21 tens; 50×4 is 20 tens;
410	20 tens + 21 tens is 41 tens. Write 410.)
1500	
1938	

The traditional algorism is shorter; it combines 34×7 and 34×50 in the following way.

34	Think 4×7 is 28; write 8; remember 2
× 7	tens. 3 tens × 7 is 21 tens plus the 2 tens
238	remembered is 23 tens; write 23 in the tens place (before the 8).

34	Think 5 tens × 4 is 20 tens; write 00;
× 50	remember 2 hundreds. 3 tens × 5 tens is
1700	15 hundreds plus the 2 hundreds remembered is 17 hundreds; write 17 in the hundreds place.

The combination of two short multiplications is familiar to teachers from their own school days.

34	
× 57	
238	the short algorism for 34×7
1700	the short algorism for 34×50
1938	the sum of the partial products

It is possible to combine all four partial prouucts as the computation is made.

34	
× 57	
8	Think 7×4 is 28. Write 8; remember 2 tens.

then

34	
× 57	
38	Think 5 tens × 4 is 20 tens plus 7×3 tens (21 tens) is 41 tens plus 2 tens remembered is 43 tens. Write 3 in the tens place; remember 4 hundreds.

then

34	
× 57	
1938	Think 5 tens × 3 tens is 15 hundreds plus 4 hundred remembered is 19 hundreds. Write 19 in the hundreds place.

Readiness for Understanding

Knowledge of numeration.
Knowledge of basic addition and multiplication facts.

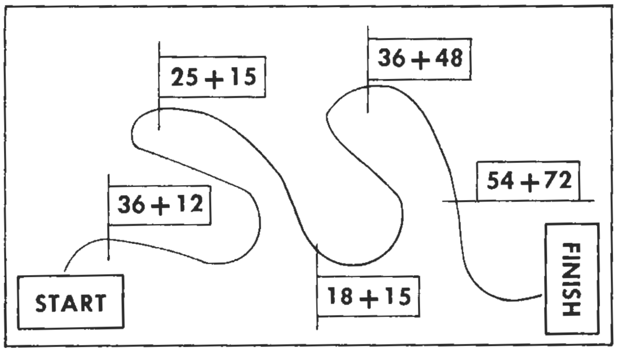
Developmental Experiences

for flannel board	*for each child*
numerals 0 through 9	tagboard racing car

tagboard cards (3″ × 3″)
felt-tip pen
masking tape
tagboard cards (3″ × 12″)
3 notebook rings
paper punch
pins

▶ Draw on the chalkboard three segments of the number line marked at 4-inch intervals. Number the first segment from 0 through 30, the second segment from 31 through 60, and the third segment from 61 through 90. Write all of the products from 0×0 through 9×9 on 3 by 3 inch cards and put them in a pack. Direct the children to come forward one at a time to choose a card and to fasten the product below the appropriate count on the number line. Continue this activity until all of the products in the pack have been used.

▶ Sketch in one panel of the chalkboard a roadway that is crossed by five paths and that has a *start* and a *finish*. Draw a similar diagram in another panel of the chalkboard. Write a sum that involves 2 two-digit addends at each of the 10 crossings. No two sums should be the same.



Separate the class into two teams and assign one of the roadways to each team. Give each child a tagboard racing car that has his name printed on it. Let a representative from each team draw straws to determine who will begin the game.

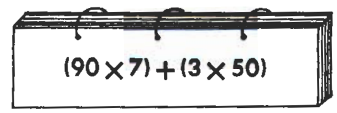
Explain the procedure to the class. The first child will come to the board and start his car down his team's road. At the first crossway he must stop, compute the sum that marks the crossway, and then tell the class the result of his computation. The class must decide whether or not the child is correct. If he is not correct, he must leave his car on the sideroad and wait until a later time to complete his journey. If he is correct, he may proceed on his way toward the finish line. The child must follow the same procedure at each crossroad. If he computes correctly at each stop he may cross the finish line and tape his car inside a winner's circle that you have drawn for each team. When the first player has either crossed the finish line or parked on a side road, a member of the opposing team may follow the same procedure and try to get his car across his team's finish line.

Continue the game until all of the children have had a chance to journey down the road. The team that gets all of its cars across the finish line first will be declared the winner. The sums written at the crossroads should be changed each time a team member has completed a turn.

▶ Cut out as many 3 by 12 inch tagboard cards as there are children in the class. Write a sum of products on each card.

(4 tens × 6) + (2 × 3 tens)	**(50 × 8) + (6 × 30)**
(7 tens × 3) + (8 × 9 tens)	**(60 × 7) + (4 × 30)**

Punch three evenly spaced holes along the top edge of each card. Assemble the cards in one pack so that all holes are lined up and insert three large notebook rings into the holes.



Show one of the sums of products to the first child in the first row. Ask him to compute this sum; encourage him to get the results without recording any part of his computation on paper. When this child has successfully done the computation, show the next child in the row the second sum in the pack. Continue until every child in the room has computed a sum; then go around the class two or three more times. Give each child a sum that he didn't have in a preceding round.

▶ Write 38 × 42 on the chalkboard. Ask one child to compute the product of ones, two other children to compute the products of tens, and a fourth child to compute the product of ten tens. Ask another child to complete the computation by recording the computed sum of these partial products.

$$\begin{array}{r} 38 \\ \times\ 42 \\ \hline 16 \\ 60 \\ 320 \\ 1200 \\ \hline 1596 \end{array}$$

Then ask the first four children to take turns and describe how each partial product was computed.

Tell the class that it is possible to compute this product without recording so many partial products. Write 38 × 42 beside the algorism now on the chalkboard. Let a child compute the product of ones and record the result. Then direct the class to tell the computed product of 2 and 3 tens and the computed product of 4 tens and 8. Ask them to determine the sum of these two products. Let some child record this number (38 tens) below the 16. Draw lines from the two partial products 60 and 320 in the first algorism to the sum of these partial products (380) in the second algorism.

$$\begin{array}{r} 38 \\ \times\ 42 \\ \hline 16 \\ 60 \\ 320 \\ 1200 \\ \hline 1596 \end{array} \qquad \begin{array}{r} 38 \\ \times\ 42 \\ \hline 16 \\ 380 \end{array}$$

Then record the result of computing the product of ten tens, and ask the children to compute the sum of the partial products. Record the result. Help the class observe that the computed product is not affected by using the short method to record partial products.

Repeat the procedure using the product 75 × 23.

Have the children demonstrate the two ways to record the computation of the given product.

$$\begin{array}{r} 75 \\ \times 23 \\ \hline 15 \\ 210 \\ 100 \\ 1400 \\ \hline 1725 \end{array} \qquad \begin{array}{r} 75 \\ \times 23 \\ \hline 15 \\ 310 \\ 1400 \\ \hline 1725 \end{array}$$

Let several children take turns describing how each of the four partial products in the one algorism and each of the three partial products in the other algorism was derived.

Continue this activity; direct the class to compute products such as 89 × 13, 67 × 84, 72 × 23, and 14 × 22. If the children are ready, let them demonstrate the short algorism only.

Pages 271 through 278

● Use page 271 for discussion. Tell the children to study the illustration and to see whether or not they can explain how Dick derived each partial product. After the children have had sufficient time to think about the example, let various children give their explanation of each partial product.

Then discuss the questions about the example. Be sure the children realize that the 110 referred to in the seventh question resulted from addition as well as multiplication—4×2 tens + 3 tens × 1. To check the computation, let a child compute the product by showing four partial products.

● Page 272 provides practice in the type of addition that is involved in Dick's shortcut. If you assign this page as written practice, discuss the work when the children have finished. Encourage the children to explain to the class any shortcuts they may have used in doing the addition. One child may say that in an exercise such as 36 + 32, he adds the tens first (30 + 30 = 60) and then to this sum he adds the sum of the ones (6 + 2 = 8, 60 + 8 = 68). Another child may add the tens of the second addend to the first addend (36 + 30 = 66) and then add the ones remaining from the second addend (66 + 2 = 68). Tell the children that they may compute in the way that is easiest for them.

You may wish to let the children take turns and compute out loud the sums for a column of exercises on this page. Remember that some of the children will not respond as quickly as others. When the column is finished, have the children write only the standard numeral for the sum in each exercise.

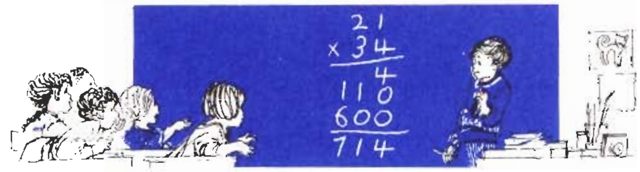
You can return to this page at various times to provide continued practice in mental addition.

Name

UNIT 16 MULTIPLICATION SHORTCUTS

For Class Discussion

Dick has found a shortcut in multiplication. He showed his shortcut to the class.



Look at Dick's example.

1. What is the product of ones and ones?*
2. What is the product of ones and 2 tens?*
3. What is the product of ones and 3 tens?*
4. What is the sum of the products of ones and tens?*
5. What is the product of tens and tens?*
6. Which digits did Dick multiply to get 4?*
7. Which digits did he multiply to get 110?*
8. Which digits did he multiply to get 600?*
9. Does 21 × 34 = 714?*
10. What is Dick's shortcut?*

*See pupil page suggestions.

reference page

D-271

Dick uses addition in his shortcut. Compute these sums without writing the addends.

1.	6 + 4 = 10	8 + 7 = 15	3 + 8 = 11
2.	9 + 8 = 17	4 + 9 = 13	6 + 8 = 14
3.	12 + 6 = 18	15 + 4 = 19	12 + 3 = 15
4.	7 + 12 = 19	16 + 3 = 19	14 + 4 = 18
5.	21 + 6 = 27	24 + 4 = 28	42 + 7 = 49
6.	64 + 5 = 69	72 + 5 = 77	81 + 2 = 83
7.	35 + 5 = 40	42 + 7 = 49	72 + 8 = 80
8.	48 + 4 = 52	56 + 9 = 65	35 + 8 = 43
9.	28 + 9 = 37	49 + 5 = 54	45 + 5 = 50
10.	20 + 30 = 50	40 + 35 = 75	15 + 12 = 27
11.	36 + 32 = 68	72 + 24 = 96	54 + 45 = 99
12.	49 + 24 = 73	27 + 28 = 55	56 + 64 = 120
13.	63 + 28 = 91	35 + 36 = 71	36 + 48 = 84
14.	81 + 49 = 130	49 + 21 = 70	27 + 56 = 83

D-272

Name

Dick's shortcut:

	36	
	× 42	
	12	(ones × ones)
	300	(tens × ones + ones × tens)
	1200	(tens × tens)
	1512	

Compute. You may want to use Dick's shortcut.

	1.	2.	3.	4.	5.	6.	7.	8.
	23	99	21	35	46	44	53	59
	× 21	× 11	× 13	× 21	× 23	× 22	× 53	× 32
	3	9	3	5	18	8	9	18
	* 80	* 180	* 70	* 130	* 240	* 160	* 300	* 370
	400	900	200	600	800	800	2500	1500
	483	1089	273	735	1058	968	2809	1888

* Choice of algorism will vary

reference page

D-273

Name

Dick's shortcut:

	78	
	× 59	
	72	(ones)
	1030	(tens + tens)
	3500	(hundreds)
	4602	

Compute. You may want to use Dick's shortcut.

	1.	2.	3.	4.	5.	6.	7.	8.
	15	32	77	28	12	66	49	33
	× 25	× 22	× 11	× 61	× 24	× 14	× 14	× 33
	25	4	7	8	8	24	36	9
	* 150	* 100	* 140	* 500	* 80	* 300	* 250	* 180
	200	600	700	1200	200	600	400	900
	375	704	847	1708	288	924	686	1089

* Choice of algorism will vary.

D-275

Compute. You may want to use Dick's shortcut.

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
	72	71	57	82	16	67	86	95	35	67	94	89
	× 72	× 73	× 74	× 44	× 49	× 52	× 71	× 52	× 48	× 86	× 65	× 78
	4	3	28	8	54	14	6	10	40	42	20	72
	* 280	* 280	* 690	* 400	* 330	* 470	* 500	* 430	* 440	* 920	* 690	* 1270
	4900	4900	3500	3200	400	3000	5600	4500	1200	4800	5400	5600
	5184	5183	4218	3608	784	3484	6106	4940	1680	5762	6110	6942

* Choice of algorism will vary.

D-274

● Pages 273 through 278 provide practice in computing products using Dick's shortcut. Adapt the following procedure to present each of these pages. First discuss with the class the example at the top of the page and work several of the exercises. Assign the remaining exercises on the page for independent work. Encourage all of the children to use Dick's shortcut. If you find that some children are making too many errors, tell them to do just one or two exercises using Dick's shortcut and to compute the other exercises by showing each of the partial products.

The important thing is that a child succeed with whatever method he uses. Do not become overly concerned at the number of errors the children may make on their first few attempts at using Dick's shortcut.

Compute. You may want to use Dick's shortcut.

1. 35 × 67 — 35, * 510, 1800, 2345
2. 92 × 29 — 18, * 850, 1800, 2668
3. 53 × 39 — 27, * 540, 1500, 2067
4. 48 × 42 — 16, * 400, 1600, 2016
5. 62 × 58 — 16, * 580, 3000, 3596
6. 64 × 87 — 28, * 740, 4800, 5568
7. 47 × 49 — 63, * 640, 1600, 2303
8. 34 × 87 — 28, * 530, 2400, 2958
9. 58 × 39 — 72, * 690, 1500, 2262
10. 98 × 53 — 24, * 670, 4500, 5194
11. 98 × 28 — 64, * 880, 1800, 2744
12. 89 × 67 — 63, * 1100, 4800, 5963

* Choice of algorism will vary.

D-276

Name

Dick's shortcut:

83 × 67 — 21 (ones), 740 (tens), 4800 (hundreds), 5561

Compute. You may want to use Dick's shortcut.

1. 22 × 14 — 8, * 100, 200, 308
2. 46 × 31 — 6, * 220, 1200, 1426
3. 11 × 15 — 5, * 60, 100, 165
4. 43 × 53 — 9, * 270, 2000, 2279
5. 67 × 16 — 42, * 430, 600, 1072
6. 28 × 59 — 72, * 580, 1000, 1652
7. 75 × 68 — 40, * 860, 4200, 5100
8. 24 × 86 — 24, * 440, 1600, 2064

* Choice of algorism will vary.

D-277

Compute.

1. 47 × 35 — * 35, 410, 1200, 1645
2. 57 × 84 — * 28, 760, 4000, 4788
3. 93 × 47 — * 21, 750, 3600, 4371
4. 39 × 96 — * 54, 990, 2700, 3744
5. 76 × 29 — * 54, 750, 1400, 2204
6. 38 × 58 — * 64, 640, 1500, 2204
7. 79 × 93 — * 27, 1020, 6300, 7347
8. 86 × 36 — * 36, 660, 2400, 3096
9. 65 × 67 — * 35, 720, 3600, 4355
10. 48 × 97 — * 56, 1000, 3600, 4656
11. 76 × 45 — * 30, 590, 2800, 3420
12. 89 × 76 — * 54, 1110, 5600, 6764

* Choice of algorism will vary

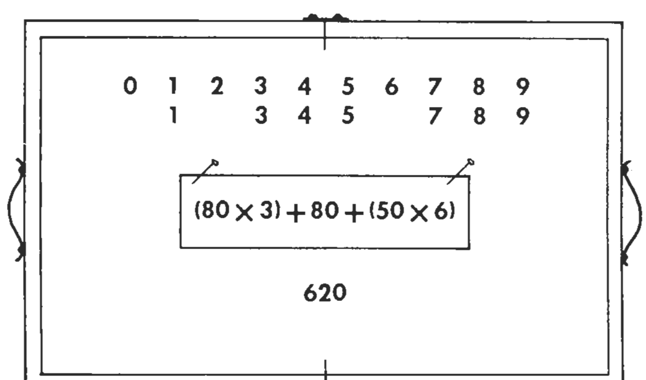
D-278

Developmental Experiences

▶ Write sums similar to the following on a pack of 3 by 12 inch cards.

(70 × 6) + (30 × 3) + 40	(20 × 90) + (40 × 60) + 50
(80 × 3) + 80 + (50 × 6)	(9 × 60) + 70 + (2 × 50)

Place the cards face down on a table next to the flannel board. Place two sets of felt numerals 0 through 9 across the top of the flannel board. Direct a child to choose a card from the pack, pin it to the flannel board, compute the sum, and show the result by placing the appropriate felt digits below his card.



Then ask the child how to describe to the class his computational steps. For example, if the child chose the sum in the above illustration, he may say that 8 tens times 3 is 24 tens, 24 tens plus 8 tens is 32 tens, 32 tens plus 30 tens (5 tens × 6) is 62 tens. If the child makes an error, help him correct it. Continue the activity; let other children explain the computation of the sums on the cards.

▶ Write 32 × 46 on the chalkboard. Ask the children to help you compute this product. Explain that as they compute, you will use a shortcut to record their results. Ask someone to compute the product of ones. Explain that you are just going to record the 2 ones and remember the 1 ten of 12.

$$\begin{array}{r} 32 \\ \times 46 \\ \hline 2 \end{array}$$

Ask a child to compute aloud the product 4 tens × 2 and to add the remembered 1 ten. (8 tens + 1 ten) Then tell the class to add the result (9 tens) to the computed product of 3 tens × 6. (18 tens + 9 tens) Ask some child the result of this addition. (27 tens) Explain that you are going to record only 7 of the 27 tens and remember the other 20 tens or 2 hundreds.

$$\begin{array}{r} 32 \\ \times 46 \\ \hline 72 \end{array}$$

As a final step, ask someone to compute the product of ten tens (4 tens × 3 tens) and to add the remembered 2 hundreds to the result. Record the 14 hundreds.

$$\begin{array}{r} 32 \\ \times 46 \\ \hline 1472 \end{array}$$

Summarize the ideas involved. Ask three volunteers to come to the chalkboard in turn and to describe the thinking steps that lead to the 2 ones, the 7 tens, and the 14 hundreds in 1472. Make notes on the chalkboard of the children's comments.

$$\begin{array}{r} 32 \\ \times 46 \\ \hline 1472 \end{array}$$

(2×6 = 12; remember 1 ten.)

(4 tens × 2 = 8 tens;
8 tens + 1 ten [remembered] = 9 tens;
6×3 tens = 18 tens;
18 tens + 9 tens = 27 tens;
record 7 tens; remember 2 hundreds.)

(4 tens × 3 tens = 12 hundreds;
12 hundreds + 2 hundreds [remembered] = 14 hundreds;
record 14 hundreds.)

Continue to have the children help you compute such products as 68 × 43, 58 × 54, 27 × 16, 13 × 21, and 74 × 66. If the children wish to do so, let them take your place and record the needed digits of the product after each computational step.

Pages 279 through 284

● Use page 279 to review the newest shortcut that the children have explored in multiplication. Use this page for discussion. Copy on the board the example shown at the top of the page and, as the first five questions are discussed, record the children's responses. Then refer to these responses as the next three questions are discussed. For example, something similar to the following illustration might be on the chalkboard when the children are ready to discuss the fourth question.

$$\begin{array}{r} 42 \\ \times 26 \\ \hline 1092 \end{array}$$

ones × ones (12)
sum of tens × ones (24 tens + 4 tens, or 28 tens)
tens × tens (8 hundreds)

The children may use their own wording to explain their ideas about the questions. In response to the sixth question, one child may volunteer that the 2 in the product came from the 12 ones (12 is 2 + 10). In response to the seventh question, a child may say that the 9 tens came from adding the 8 tens in 28 tens (28 tens is 8 tens + 2 hundreds) to the 1 ten remembered from the 12. In response to the eighth question, a child may state that the 10 hundreds came from adding the 2 hundreds remembered from 28 tens to the 8 hundreds that resulted when 2 tens × 4 tens was computed.

After completing this page, ask if anyone in the class would volunteer to work a similar exercise at the chalkboard. If a child volunteers to do so, have him use the shortcut in computing a product such as 54 × 33. When he has finished, ask the class questions similar to those on page 279.

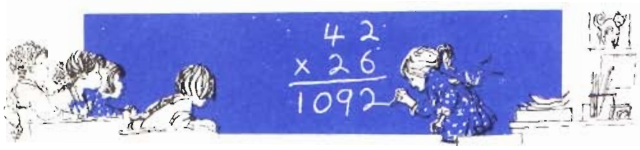
● Page 280 provides practice in addition. Point out to the children that practice in addition may make it easier for them to use Jane's shortcut. Ask several children to explain how they might compute specific exercises. Then have them take turns giving the sums orally. Remember that some children will be slower to respond than others. Then direct the children to write on their papers the computed sum of each exercise in the first column.

Since the children do not record the addends of the sum when they use the shortcut for multiplication, it may be of help if you provide practice by reading to the children the sums that are to be computed. Tell the children to close their books; then read aloud sums from page 280 and instruct the children to write only the standard numerals for the sums.

Name

For Class Discussion

Jane used another shortcut in multiplication. She didn't write any partial products.



Look at Jane's example.

1. What is the product of ones and ones?*
2. What is the product of ones and 4 tens?*
3. What is the product of ones and 2 tens?*
4. What is the sum of the products of tens and ones?*
5. What is the product of tens and tens?*
6. How did Jane get the 2 in her product?*
7. How did she get the 9 tens in her product?*
8. How did she get the 10 hundreds in her product?*
9. Does 42 × 26 = 1092?*
10. What is Jane's shortcut?*

*See pupil page suggestions.

reference page

D-279

Jane uses addition in her shortcut. Compute the sums without writing the addends.

1. 42 + 32 = 74	48 + 35 = 83	63 + 72 = 135
2. 27 + 63 = 90	28 + 54 = 82	35 + 25 = 60
3. 15 + 10 + 1 = 26	36 + 28 + 2 = 66	40 + 49 + 5 = 94
4. 48 + 6 + 2 = 56	32 + 28 + 3 = 63	56 + 20 + 4 = 80
5. 21 + 36 + 2 = 59	81 + 21 + 2 = 104	48 + 18 + 3 = 69
6. 40 + 18 + 7 = 65	12 + 36 + 2 = 50	21 + 20 + 3 = 44
7. 35 + 24 + 3 = 62	42 + 30 + 3 = 75	45 + 9 + 2 = 56
8. 21 + 30 + 3 = 54	36 + 28 + 6 = 70	81 + 18 + 5 = 104
9. 42 + 32 + 2 = 76	27 + 40 + 2 = 69	14 + 32 + 5 = 51
10. 54 + 54 + 8 = 116	35 + 56 + 4 = 95	72 + 16 + 6 = 94
11. 45 + 24 + 7 = 76	32 + 21 + 2 = 55	42 + 42 + 3 = 87
12. 81 + 81 + 8 = 170	54 + 48 + 7 = 109	49 + 54 + 7 = 110
13. 48 + 63 + 5 = 116	56 + 54 + 6 = 116	64 + 63 + 7 = 134

D-280

Name

Jane's shortcut:

$$\begin{array}{r} 32 \\ \times\ 54 \\ \hline 1728 \end{array}$$

4 × 2 = 8 Write 8 ones.

4 × 3 tens = 12 tens 5 tens × 2 = 10 tens
12 tens + 10 tens = 22 tens
Write 2 tens. Remember 200.

5 tens × 3 tens = 15 hundreds
1500 + 200 = 1700
Write 17 hundreds.

Compute. You may want to use Jane's shortcut.

1. 13 × 22 = *286	2. 32 × 12 = *384	3. 14 × 21 = *294	4. 21 × 21 = *441
5. 23 × 52 = *1196	6. 34 × 21 = *714	7. 22 × 43 = *946	8. 24 × 51 = *1224

*Choice of algorism will vary.

reference page

D-281

● Pages 281 through 284 provide practice in computing products using Jane's shortcut. Adapt the following procedure to present each of these pages: first discuss with the class the example at the top of the page and work several of the exercises; assign the remaining exercises on the page for independent work.

Be sure the children realize that Jane's method was to record only the standard numeral for the products; all multiplications and additions must be done without recording anything on a piece of scratch paper.

The children should be encouraged, but not forced, to use Jane's shortcut. When you make an assignment, direct the children to use Jane's shortcut for at least two of the exercises; then tell them that they may use any method they choose to complete the others.

It will help the children if you show no preference for any one method of computing products so they can explore all of the various shortcuts. The method that is best for one child is not necessarily best for another child. The term *shortcuts* could actually be misleading, because it may turn out that these methods are longer ways of computing a product. If all three methods of computing a product are used, each child will realize that the method he uses is acceptable because it is best for him.

Compute. You may want to use a shortcut.

1. 43 × 55
2. 53 × 23
3. 47 × 63
4. 61 × 68
5. 61 × 97
6. 72 × 94
7. 85 × 61
8. 73 × 98

1. 43 ×55 *2365* *	2. 53 ×23 *1219* *	3. 47 ×63 *2961* *
4. 61 ×68 *4148* *	5. 61 ×97 *5917* *	6. 72 ×94 *6768* *
7. 85 ×61 *5185* *	8. 73 ×98 *7154* *	

** Choice of algorism will vary.*

D-282

Compute. You may want to use Jane's shortcut.

1. 24 × 14 **336*	2. 73 × 31 **2263*	3. 83 × 24 **1992*	4. 25 × 78 **1950*
5. 32 × 43 **1376*	6. 65 × 45 **2925*	7. 19 × 18 **342*	8. 62 × 37 **2294*
9. 59 × 64 **3776*	10. 91 × 93 **8463*	11. 23 × 96 **2208*	12. 56 × 84 **4704*

** Choice of algorism will vary.*

D-284

Name

Jane's shortcut:

```
  45
× 37
1665
```

7 × 5 = 35
Write _5_ ones. Remember _3_ tens.

7 × 40 = 280   30 × 5 = 150
28 tens + 15 tens + 3 tens = 46 tens
Write _6_ tens. Remember _4_ hundreds.

30 × 40 = 1200
12 hundred + 4 hundred = 16 hundred
Write _16_ hundreds.

Compute. You may want to use Jane's shortcut.

1. 37 × 23 **851*	2. 71 × 74 **5254*	3. 93 × 74 **6882*	4. 23 × 25 **575*
5. 45 × 46 **2070*	6. 91 × 85 **7735*	7. 47 × 57 **2679*	8. 24 × 68 **1632*

** Choice of algorism will vary.*

D-283

Supplemental Experiences

■ Describe several situations that involve multiplication such as:

> Each of 34 pupils sold 23 tickets for the school play. How many tickets did they sell?

Have a child do the needed computation to answer the question.

```
  34            34            34
× 23    or    × 23    or    × 23
  12            12           782
  90           170
  80           600
 600           782
 782
```

The pupils sold 782 tickets.

Vary the procedure; let the children make up such questions from their own experiences.

■ Draw on the chalkboard four multiplication tables such as the following.

×	2	2 tens
1 ten		
3 tens		
5 tens		
7 tens		
9 tens		

×	4	4 tens
9 tens		
7 tens		
5 tens		
3 tens		
1 ten		

×	6	6 tens
2 tens		
4 tens		
6 tens		
8 tens		
5 tens		

×	8	8 tens
4 tens		
3 tens		
7 tens		
5 tens		
9 tens		

Direct the children to take turns naming the two products in each row.

KEY IDEA

We write down what we don't remember.

Scope

To practice short multiplication.

Fundamentals

Which multiplication algorism is the correct one? Parents often believe that the algorism they learned must be correct, more efficient, or altogether more desirable than any other form and are often quite doubtful when their child uses the extended basic algorism used in modern programs.

Teachers can reassure such parents. There has been sufficient experience to show that these algorisms are at least as efficient and are often easier to learn. The algorisms contribute to increased understanding. The child studying an up-to-date program should understand what he is doing.

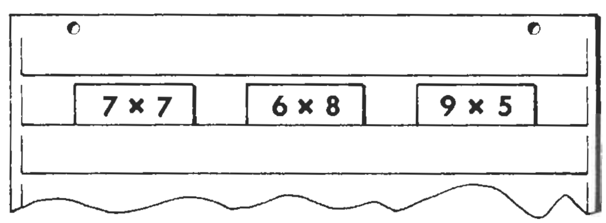
Readiness for Understanding

Knowledge of basic addition and multiplication facts.
Understanding of numeration.

Developmental Experiences

tagboard cards (3 ″ × 9″)
felt-tip pen
pocket chart

▶ Write basic multiplication facts from 2×0 through 9×9 on 3 by 9 inch tagboard cards. Separate the class into two teams. Place three of the product-cards in the pocket chart.



Ask a member from one team to identify the greatest product and to tell how he knows this number is the greatest of the three. For example, if his three products were those shown in the illustration, he may say that 7×7 is 49, 6×8 is 48, and 9×5 is 45. Of these three numbers 49, or 7×7, is the greatest.

Replace these first three products with three others. Ask a member of the second team to identify the product that is the greatest and to explain how he knows this. Continue in this way until all of the children have had a turn.

The children may earn points for their team in the following ways: 1 point for identifying the correct greatest product, and 1 point for being able to tell why this product is the greatest. After all team members have had their turn, have the points totaled and a winner declared.

▶ Give the children practice in computing products and sums without recording any results on paper. Read exercises such as the examples listed below. After each exercise has been presented, call on a third child to give the result of his computation.

3 tens times 4 (pause) plus 7 tens.
6 times 4 tens (pause) plus 8 tens.
2 hundreds times 7 (pause) plus 6 hundreds.
8 tens times 6 tens (pause) plus 4 ten tens.
9 tens times 5 tens (pause) plus 6 hundreds.
3 hundreds times 2 tens (pause) plus 5 ten hundreds.
7 tens times 3 hundreds (pause) plus 2 ten hundreds.
4 tens times 5 hundreds (pause) plus 7 thousands.

If any child responds incorrectly, repeat the exercise and give him the opportunity to correct his error. Continue the activity so that every child in the class has had several turns at this kind of computation.

▶ Write on the chalkboard 47 × 8. Tell the children that as they help you compute this product, you will use a shortcut to record the results of their computation. Ask someone to compute the product of ones. Explain to the class that you are going to record the 6 ones and remember the 5 tens of 56.

$$\begin{array}{r} 47 \\ \times\ 8 \\ \hline 6 \end{array}$$

Ask someone else to compute 4 tens × 8 and add the remembered 5 tens to the result (32 tens + 5 tens, or 37 tens). Record the 37 tens.

$$\begin{array}{r} 47 \\ \times\ 8 \\ \hline 376 \end{array}$$

Summarize the ideas involved. Ask two volunteers to come to the chalkboard in turn and describe the thinking steps that lead to the 6 ones and the 37 tens. Note on the chalkboard the children's comments.

$$\begin{array}{r} 47 \\ \times\ 8 \\ \hline 376 \end{array}$$

376 ← (8×7 = 56; record 6; remember 5 tens.)
← (8×4 tens = 32 tens; 32 tens + 5 tens [remembered] = 37 tens; record 37 tens.)

In this same way, have the class help compute such products as 69 × 4, 58 × 5, and 13 × 2. Let the children take your place and record the needed digits in the product after each computational step.

Write 750 × 4 on the chalkboard. Call on a child to compute and record the product of ones.

$$\begin{array}{r} 750 \\ \times\ 4 \\ \hline 0 \end{array}$$

Ask the class to compute the product of tens and to give the result of this computation. Call on a second child to record the tens; tell the class to remember the hundreds. (It is possible with computation of products such as 750 × 4 to bypass the computation of the product of ones and begin with the computation of the product of tens.)

$$\begin{array}{r} 750 \\ \times\ 4 \\ \hline 00 \end{array}$$

Tell the class to compute the product of hundreds and to add the remembered hundreds. Ask for the result of this computation. Let a third child record the hundreds.

$$\begin{array}{r} 750 \\ \times\ 4 \\ \hline 3000 \end{array}$$

To summarize the ideas involved, have three children come to the chalkboard in turn and describe the steps that led to the 0 ones, the 0 tens, and the 30 hundreds in 3000. On the chalkboard, make notes of the children's comments.

$$\begin{array}{r} 750 \\ \times\ 4 \\ \hline 3000 \end{array}$$

3000 ← (4×0 = 0; record 0 ones.)
← (4×5 tens = 20 tens; record 0 tens; remember 2 hundreds.)
← (4×7 hundreds = 28 hundreds; 28 hundreds + 2 hundreds = 30 hundreds; record 30 hundreds.)

In this same manner, let the class help compute such products as 423 × 2, 680 × 7, and 902 × 5.

Adapt the suggested procedure to the computation of such products as 69 × 50, 44 × 80, and 23 × 70. With a product such as 69 × 50, help the class observe that since there are 0 ones the computation may begin with the product of tens (5 tens × 9). Have the children compute this product (45 tens). Then call on a child to record the tens. Tell the class to remember the hundreds.

$$\begin{array}{r} 69 \\ \times\ 50 \\ \hline 50 \end{array}$$

Direct the class to compute the product of ten tens (5 tens × 6 tens). Tell the children to add to this product the remembered 4 hundreds (30 ten tens, or 30 hundreds, + 4 hundreds = 34 hundreds) and then to give the computed sum. Let a second child record the hundreds.

$$\begin{array}{r} 69 \\ \times\ 50 \\ \hline 3450 \end{array}$$

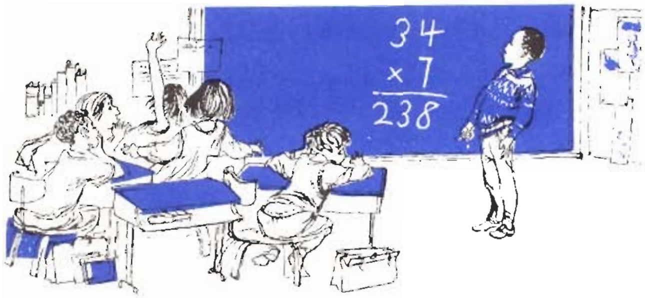
Ask two children to summarize the steps in this computation. On the chalkboard, make notes of the children's comments.

As a final phase in this experience in computing products, let the class help with the computation of such products as 456 × 30, 209 × 70, and 320 × 60.

Name

For Class Discussion

Jim has also found a shortcut in multiplication. His shortcut is something like Jane's. Jim didn't write any partial products either.



Look at Jim's example.

1. What is the product of ones and ones in Jim's example? *
2. What is the product of ones and tens? *
3. Which digits did Jim multiply to get the 8 in his product? *
4. How did he get the 23 in his product? *
5. Does 7 × 34 = 238? *
6. What is Jim's shortcut? *

* See pupil page suggestions

reference page

D-285

Jim's shortcut:

43
× 6
258

[6 × 3 = 18
Write 8 ones. Remember 1 ten.

[6 × 4 tens = 24 tens
24 tens + 1 ten = 25 tens
Write 25 tens.

Compute. Jim's shortcut is very important. Use it in at least half of the exercises.

1. 72 × 4 = *288	2. 92 × 6 = *552	3. 62 × 4 = *248	4. 83 × 5 = *415
5. 21 × 4 = *84	6. 43 × 9 = *387	7. 95 × 2 = *190	8. 49 × 3 = *147
9. 28 × 7 = *196	10. 23 × 3 = *69	11. 62 × 8 = *496	12. 36 × 9 = *324
13. 96 × 8 = *768	14. 92 × 7 = *644	15. 35 × 4 = *140	16. 43 × 7 = *301

* Choice of algorism will vary

D-286

Pages 285 through 290

● Use page 285 for class discussion. Tell the children to study the example in the illustration. After they have had sufficient time to think about the example, discuss the questions.

Ask a volunteer to work a similar exercise at the chalkboard. Tell him to use the shortcut to compute a product such as 6 × 39. When he has finished, adapt the questions from page 285 to his computation. Follow a similar procedure with one or two additional examples.

● Pages 286 through 290 provide practice in computing products by using Jim's shortcut. Adapt the following procedure to present each of these pages. First, discuss the example at the top of the page and work several of the exercises with the children. Assign the remaining exercises on each page for independent work. Encourage the children to use Jim's shortcut in completing at least half of the exercises on each page.

Name

Jim's shortcut:

521
× 6
3126

[6 × 1 = 6
Write 6 ones.

[6 × 20 = 120
Write 2 tens. Remember 1 hundred.

[6 × 500 = 3000
3000 + 100 = 3100
Write 31 hundreds.

Compute. Use Jim's shortcut in at least half of the exercises.

1. 133 × 3 = *399	2. 224 × 4 = *896	3. 441 × 3 = *1323	4. 562 × 7 = *3934
5. 615 × 8 = *4920	6. 625 × 5 = *3125	7. 173 × 6 = *1038	8. 678 × 2 = *1356
9. 171 × 9 = *1539	10. 384 × 6 = *2304	11. 935 × 8 = *7480	12. 758 × 5 = *3790
13. 915 × 7 = *6405	14. 739 × 4 = *2956	15. 275 × 9 = *2475	16. 215 × 6 = *1290

* Choice of algorism will vary.

reference page

D-287

Jim's shortcut:

```
 304
×  4
1216
```

- 4 × 4 = 16. Write 6 ones. Remember 1 ten.
- 4 × 0 = 0; 0 + 10 = 10. Write 1 ten.
- 4 × 300 = 1200. Write 12 hundreds.

Compute. Use Jim's shortcut in most of the exercises.

1. 407 × 2 = *814	2. 804 × 3 = *2412	3. 430 × 8 = *3440	4. 504 × 8 = *4032
5. 419 × 5 = *2095	6. 950 × 4 = *3800	7. 606 × 3 = *1818	8. 471 × 7 = *3297
9. 907 × 5 = *4535	10. 322 × 6 = *1932	11. 560 × 7 = *3920	12. 517 × 9 = *4653
13. 410 × 9 = *3690	14. 601 × 2 = *1202	15. 708 × 6 = *4248	16. 943 × 7 = *6601

* Choice of algorism will vary

D-288

Name

Jim's shortcut:

```
  46
× 70
3220
```

- 70 × 6 = 420. Write 20. Remember 400.
- 70 × 40 = 2800; 2800 + 400 = 3200. Write 32 hundreds.

Compute. Use Jim's shortcut in at least half of the exercises.

1. 24 × 80 = *1920	2. 64 × 20 = *1280	3. 92 × 50 = *4600	4. 34 × 60 = *2040
5. 52 × 70 = *3640	6. 28 × 60 = *1680	7. 93 × 40 = *3720	8. 71 × 80 = *5680
9. 85 × 40 = *3400	10. 59 × 20 = *1180	11. 55 × 30 = *1650	12. 82 × 40 = *3280
13. 16 × 50 = *800	14. 85 × 90 = *7650	15. 67 × 70 = *4690	16. 86 × 30 = *2580

* Choice of algorism will vary

D-289

Jim's shortcut:

```
  362
 × 40
14480
```

- 40 × 2 = 80. Write 80.
- 40 × 60 = 2400. Write 4 hundreds. Remember 2000.
- 40 × 300 = 12,000; 12,000 + 2,000 = 14,000. Write 14 thousands.

Compute. Use Jim's shortcut in most of the exercises.

1. 113 × 30 = *3390	2. 424 × 40 = *16,960	3. 518 × 20 = *10,360	4. 361 × 80 = *28,880
5. 751 × 60 = *45,060	6. 906 × 20 = *18,120	7. 612 × 70 = *42,840	8. 261 × 30 = *7830

9. 268 × 50 = 13,400*
10. 70 × 903 = 63,210*
11. 80 × 520 = 41,600*
12. 463 × 60 = 27,780*
13. 914 × 80 = 73,120*
14. 903 × 90 = 81,270*
15. 60 × 850 = 51,000*
16. 175 × 70 = 12,250*

reference page * Choice of algorism will vary.

D-290

Supplemental Experiences

■ Separate the class into two teams. While the children close their eyes, write on the chalkboard a completed multiplication exercise that involves a three-digit factor and a one-digit factor. Tape tagboard cards over 1 or more digits in the algorism.

```
  6▩3
×   ▩
 1246
```

When the children open their eyes, ask a member of one team to give the secret number statement that represents the computation of ones in this given product: "3 ones times the secret number of ones is 6 ones." If the statement is correct, the child earns a point for his team. Then ask a member of the other team to name the secret number. If he answers correctly, he earns a point for his team. Let him uncover the secret number to check his answer. Follow the same procedure with the secret number of tens.

Put other incomplete algorisms on the chalkboard and continue the game until every child has had an opportunity to participate. Total up the team's points and declare a winner.

■ Write on each of several cards a product and the standard numeral for this product. Write on several other cards an incorrect standard numeral for a given product.

7×93=651	7×608=4608
40×86=3240	20×509=10,180

Separate the class into two teams. Direct a member from each team to choose a card. Explain that each child should place his card on the chalktray, compute his given product on the chalkboard, and tell the class whether or not the computed product on the card is correct. If the computed product on his card is incorrect, the child must tell the class where the error was made—in the computation of the product of ones, the product of tens, the product of hundreds, or the product of thousands.

Each child who performs his assignment correctly earns a point for his team. Continue the activity until all of the children have had an opportunity to participate. Then ask the teams to total their points and declare a winner.

KEY IDEA

Skill in computation requires practice.

Scope

To practice computation.

Fundamentals

The acquisition of skills in computation involves not only a thorough understanding of numbers in relation to other numbers, but considerable practice that involves these understandings until their use is automatic. Too often the child is given an unbalanced program. If he explores for understanding without practicing computation, he will be too slow to succeed in reaching a desired performance level. If he is drilled on procedures that are meaningless to him, he will forget all that he has learned as soon as the practice is stopped. The skillful teacher provides the child with a blend of both approaches. Concepts and ideas are explored and many number relationships are discovered. The teacher then sees to it that the child is given plenty of opportunity to apply his understanding to the computation being performed until his skill is automatic.

Readiness for Understanding

Ability to compute.

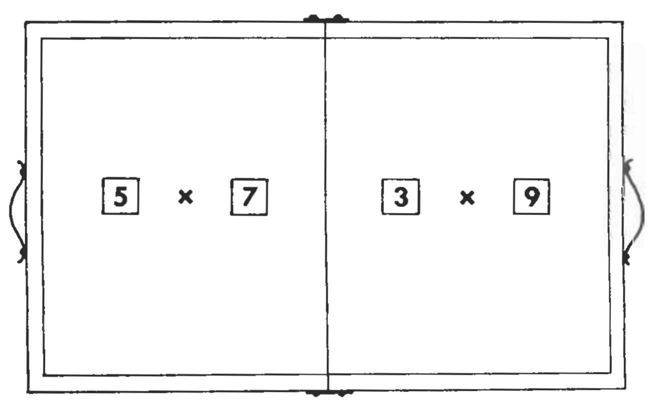
Developmental Experiences

for flannel board
felt times signs
numeral-cards: 0 through 9

box
felt-tip pen
tagboard cards (3″ × 12″)
pocket chart

▶ Partition the flannel board into two sections. Place a felt times sign in the center of each side of the board. Put two sets of numeral-cards for 0 through 9 in a box next to the flannel board.

Separate the class into two teams and assign a section of the board to each team. Explain to the class that a member from each team will choose two numeral-cards from the box. Each child will use the two numbers he chose and the times sign on his team's side of the flannel board to show a product.



Then direct each of the children to tell the class the count for his product and return the cards to the box.

Continue the activity in this way so as to give each team's members an opportunity to compute using basic multiplication facts. Each time a child names the correct count for his product, he earns a point for his team. After all of the children have had a turn, total each team's points and declare a winner.

▶ Write sums such as the following on a set of 3 by 12 inch tagboard cards.

$(90 \times 6) + 70$	$(50 \times 60) + 800$
$(8 \times 400) + 600$	$(70 \times 900) + 7000$

Make twice as many cards as there are children in the class. Several pairs of cards should name the same sum; for example, use $(80 \times 6) + 70$ on one of the cards in a pair and $(7 \times 70) + 60$ on the other.

Let the children form two teams. Place two sum-cards in the pocket chart: for example, use $(40 \times 6) + 70$ and $(60 \times 5) + 10$. Ask a member from one team to decide whether none of these cards, one of these cards, or both of these cards show the sum 310. If his conclusion is correct, he earns a point for his team. If he answers incorrectly, help him find his error.

Continue this activity until all of the children have had an opportunity to decide whether both, one, or none of the cards in the pocket chart at a given time show a specifically stated sum. Be sure to use all three possibilities throughout the activity. After all of the children have had a turn, total each team's points and declare a winner.

▶ Write 78×96 on the chalkboard. Tell the class that they are going to explore another shortcut for recording the results of the computation of products. Explain that in this shortcut 78 will be multiplied by 6 and the computed product recorded. Then 78 will be multiplied by 90 and this computed product will be recorded below the first partial product.

Ask someone to describe how he would compute the product of 78 and 6 while he records the result on the board below the product 78×96. Ask a second child to review the computation for the class; make notes of his comments beside the algorism on the board.

```
  78
× 96
 468   (6 × 8 = 48; record 8, remember 4 tens.)
       (6×7 tens = 42 tens;
       42 tens + 4 tens = 46 tens;
       record 46 tens.)
```

Let another child describe how he would compute the product of 78 and 90 and record the result of his computation. Ask someone else to summarize the thinking steps that produced this partial product. Make notes of his comments on the chalkboard.

```
  78
× 96
 468   (9 tens × 8 = 72 tens; record 2 tens;
7020   remember 7 hundreds.)
       (9 tens × 7 tens = 63 ten tens, or 63
       hundreds; 63 hundreds + 7 hundreds =
       70 hundreds; record 70 hundreds.)
```

Let a volunteer compute the sum of these partial products and record the result.

```
  78
× 96
 468
7020
7488
```

Continue to have the children describe their computation of such products as 65×12, 438×92, 506×77, and 193×284. With 193×284 three products will be recorded.

▶ Write the product 68×72 on the chalkboard. Ask four volunteers to compute this product and to show the results of the computation in the following ways:

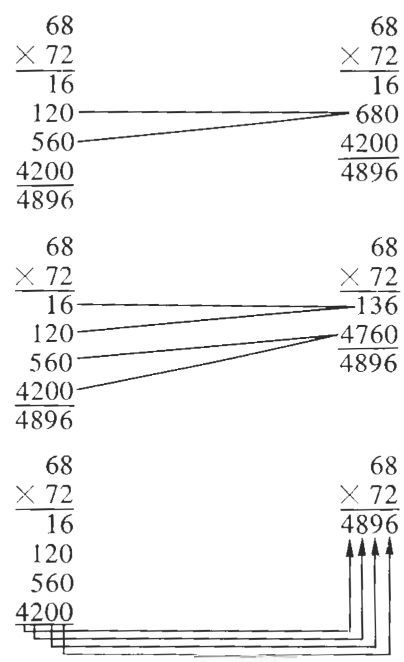
record four partial products (all possible partial products);

record three partial products (products of ones and tens combined);

record two partial products (multiplication of one factor first by the ones of the other factor and then by the tens of the other factor);

record only the computed product.

Discuss the relationships between the various methods of recording the results of the computation. Some relations that the children may observe are shown in the following illustration.



There may be other relations that the children may wish to show and they should be encouraged to point out what they see.

Continue this activity; use the products 75 × 92, 48 × 74, 63 × 29, 56 × 38, and 27 × 98. If the ability level of the class permits, this activity may be adapted to such products as 123 × 456, 768 × 209, and 342 × 597.

Pages 291 through 294

● Use page 291 for class discussion. Tell the children to study the example. After the children have had sufficient time to think about the example, ask various children to explain their ideas about how each partial product was derived. One child may say that the partial product 135 is 3 × 45; he may explain that the partial product 900 was derived by computing 20 × 45.

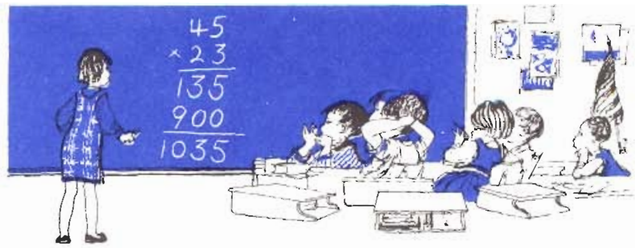
Discuss each of the questions below the example. Then ask volunteers to work other exercises in a similar way at the chalkboard. After each exercise has been completed, discuss the work with the class. Adapt the questions on page 291 to guide the discussion.

● Page 292 provides further practice in computing products. Work the example at the top of the page with the class. Then ask several children to work similar exercises on the chalkboard. When the children have had a thorough review of Sally's shortcut, assign the exercises on this page for independent work. Encourage the children to use Sally's shortcut by directing them to complete any two of the assigned exercises by using the shortcut and then to complete the other exercises by using a method of their own choice.

Name

For Class Discussion

Sally used another shortcut in multiplication.
She wrote two partial products.



Look at Sally's example.

1. What is the product of ones and ones? *
2. What is the product of ones and 4 tens? *
3. What is the product of ones and 2 tens? *
4. What is the product of tens and tens? *
5. What partial products did Sally add to get 135? *
6. What partial products did Sally add to get 900? *
7. Does 45 × 23 = 1035? *
8. What is Sally's shortcut? *

* See pupil page suggestions.

reference page

D-291

Sally's shortcut:

34	34	34
× 27	× 7	× 20
238 ←	238	680
680 ←		
918		

Compute. You may want to use Sally's shortcut.

1. 12 × 34 = * 48 + 360 = 408	2. 68 × 32 = * 136 + 2040 = 2176	3. 73 × 86 = * 438 + 5840 = 6278	4. 92 × 93 = * 276 + 8280 = 8556
5. 78 × 32 = * 156 + 2340 = 2496	6. 67 × 76 = * 402 + 4690 = 5092	7. 39 × 68 = * 312 + 2340 = 2652	8. 77 × 77 = * 539 + 5390 = 5929
9. 324 × 23 = * 972 + 6480 = 7452	10. 123 × 321 = * 123 + 2460 + 36900 = 39,483	11. 616 × 34 = * 2464 + 18480 = 20,944	12. 616 × 304 = * 2464 + 184800 = 187,264

reference page * Choice of algorism may vary.

D-292

Name

Compute. Use any method you wish.

1. 58 × 13 — * 58 × 13: 174, 580, 754
2. 37 × 66 — * 37 × 66: 222, 2220, 2442
3. 47 × 82 — * 47 × 82: 94, 3760, 3854
4. 39 × 41 — * 39 × 41: 1599
5. 65 × 42 — * 65 × 42: 10, 320, 2400, 2730
6. 51 × 32 — * 51 × 32: 1632
7. 24 × 23 — * 24 × 23: 72, 480, 552
8. 85 × 46 — * 85 × 46: 30, 680, 3200, 3910
9. 48 × 51 — * 48 × 51: 2448
10. 85 × 75 — * 85 × 75: 25, 750, 5600, 6375
11. 79 × 33 — * 79 × 33: 27, 210, 270, 2100, 2607
12. 26 × 95 — * 26 × 95: 30, 640, 1800, 2470
13. 93 × 74 — * 93 × 74: 12, 360, 210, 6300, 6882
14. 82 × 82 — * 82 × 82: 4, 320, 6400, 6724
15. 91 × 39 — * 91 × 39: 9, 840, 2700, 3549
16. 49 × 47 — * 49 × 47: 63, 280, 360, 1600, 2303

*Choice of algorism will vary.

D-293

Compute. Use any method you wish. *The pupil may need to use an algorism for only some of the exercises.*

1. 6 × 29
2. 5 × 78
3. 7 × 61
4. 3 × 98
5. 6 × 882
6. 4 × 635
7. 5 × 879
8. 8 × 990
9. 32 × 23
10. 45 × 73
11. 94 × 86
12. 88 × 23
13. 54 × 320
14. 97 × 209
15. 42 × 198
16. 526 × 596

1. 29 × 6 = 174 2. 78 × 5 = 390 3. 61 × 7 = 427 4. 98 × 3 = 294
5. 882 × 6 = 5292 6. 635 × 4 = 2540 7. 879 × 5 = 4395 8. 990 × 8 = 7920
9. 23 × 32 = 736 10. 73 × 45 = 3285 11. 86 × 94 = 8084 12. 23 × 88 = 2024
13. 320 × 54 = 17280 14. 209 × 97 = 20273 15. 198 × 42 = 8316 16. 596 × 526 = 313,496

D-294

● Pages 293 and 294 give the children an opportunity to test their ability to compute products. All of the exercises on page 293 involve two-digit factors. The exercises on page 294 provide practice in computing a variety of products. The children should complete the exercises on their own; do not assign more than three rows or two columns of exercises at any one time. Be sure the children understand that they may use any method they wish to compute the products. At the third-grade level all children should be able to compute products by recording all possible partial products. Some will be able to use shortcuts and should be encouraged to do so whenever possible. Encourage the children to use the shortcuts that they find most convenient and accurate. You may find that the children will compute the product 42 × 198 by using these or other methods:

198	198	198	198
× 42	× 42	× 42	× 42
16	396	8316	396
180	7920		320
200	8316		3600
320			4000
3600			8316
4000			
8316			

Supplemental Experiences

■ Have the children play a "Secret Number Game." Start the game by saying, "I am thinking of a secret number whose factors are 40 and 80. Who can tell my secret number?" The child who answers correctly may ask for the next secret number. Encourage the children to vary the statements about their secret numbers. For example, a child might say, "I am thinking of a secret number that, when multiplied by 60, is 240. What is my secret number?" Continue in this way until all of the children have had an opportunity to present a question about their secret number.

■ Tell the children to form two teams. Write the product 6 × 492 on the chalkboard. Direct the first member of Team I to go to the chalkboard and compute this product—he may use any method he wishes. Then instruct the first member of Team II to compute the same product by using a different method. Each child who completes his assignment correctly earns a point for his team.

For the next round of the game, the second member of Team II computes the given product first and the second member of Team I must then compute the same product using a different method. Continue the activity until all of the children have had a chance to participate. Use products such as 32 × 69, 7 × 58, 43 × 628, 5 × 408, and 8 × 2376. The team with the greater number of points is declared the winner.

UNIT 17
MEASUREMENT

Pages 295 Through 306

OBJECTIVE

To introduce some standard units of liquid measure.

The child learns that an amount of liquid may be designated by a number of standard units. He explores the relative size of a cup, pint, quart, and gallon and learns to express the same amount of liquid with different units. He uses his knowledge of multiplication to convert from one unit to another.

See Key Topics in Mathematics for the Primary Teacher: Measurement.

KEY IDEAS

Two pints is exactly one quart.
Gallons plus gallons is gallons.

CONCEPTS

cup	pint
gallon	quart

KEY IDEA

Two pints is exactly one quart.

Scope

To develop the understanding of measurement as a comparison of quantities.

Fundamentals

In this section, the child is introduced to the measurement of liquids. Measurement is a comparison of quantity. The standard of comparison is the amount associated with the number 1. The child uses multiplication to convert the name of a quantity from one unit to another. The larger unit, the quart, is related to the other units as follows:

1 quart is 2 pints,
1 quart is 4 half-pints,
1 quart is 4 cups.

In the past there has been considerable confusion between the unit quantity and a standard container used to measure that quantity. This confusion is not helped by the fact that the container that we call a cup may or may not contain exactly 1 cup of liquid. It is clear that a pint container is quite different from two 1 cup containers or 2 half-pint containers. It is equally clear that the quantity 2 cups is exactly the quantity 1 pint. Emphasis should be placed upon the amount of liquid—not the container being used.

Readiness for Understanding

Knowledge of multiplication.

Developmental Experiences

a variety of empty cup, half-pint, pint, and quart containers
2 pint jars
2 quart bottles
1 measuring cup (1 cup)
pitcher (at least 4 quarts)
food coloring
masking tape
felt-tip pen

▶ Display a variety of empty cup, pint, and quart containers in the classroom. If possible, use cartons and cans rather than glass containers.

QUART CONTAINERS
milk
orange juice
ice cream
oil

PINT CONTAINERS
cream
ice cream
sour cream
cottage cheese

CUP CONTAINERS
whipping cream
cottage cheese
milk (half-pint)

Discuss the display with the children. Have them identify what each container held, and how the contents were similar. (The containers all held liquids or semi-liquids.) Then ask if anyone can name the amount each container can hold. As each container is identified, label it by writing quart, pint, or cup on a piece of masking tape.

Write the following incomplete statements on the chalkboard.

1 quart is greater than ____
1 pint is greater than ____
1 quart is greater than ____

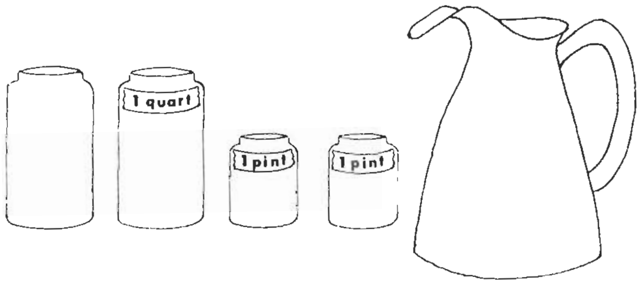
Then have the children suggest the quantities that will accurately complete the statements.

1 quart is greater than 1 pint
1 pint is greater than 1 cup
1 quart is greater than 1 cup

Encourage the children to use the display to help them find the answers.

▶ To explore the relationship between pints and quarts, you will need 2 pint jars, 2 quart bottles, and a pitcher or other container that holds at least 3 quarts. Make masking tape labels for one of the quart containers and for both of the pint containers. Attach them so that the top edge of each label indicates the appropriate level for the amount of liquid indicated. By planning ahead, you can be sure that the particular containers you use hold the right amount of liquid.

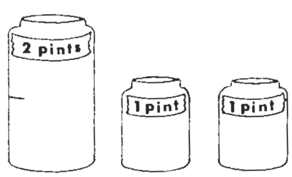
Since containers do vary in size and shape the cup, pint, or quart level is seldom marked.



Fill the pitcher with water and add a few drops of food coloring. Pour a quart of this water into the labeled 1 quart bottle. Ask if anyone knows how many pints are in a quart. If no one can answer, ask how the answer might be found. If a child does give the correct answer, ask him if he knows a way to show the class how many pints is 1 quart. He may suggest that you pour the water into the pint jar. Ask him to demonstrate his method. Explain that the jars must be filled just to the top of the tape to measure a pint. When the child has poured out all of the water, let him record his results on the chalkboard:

1 quart = 2 pints

Hold up the unlabeled bottle and ask the children how many pints of water they think it could hold. Then pour the water from one pint jar into the bottle and mark the water level with a crayon or felt-tip pen. Add the other pint of water and mark the new water level; label the bottle as illustrated:



Ask the children if there is another name for this 2 pint bottle. When a child answers that 2 pints make a quart, pour the water into the container labeled 1 quart. Let the children examine the water level to check the answer. Then have a child show the relationship on the chalkboard:

2 pints = 1 quart

Remove the pint jars from the table and fill both quart bottles with water. Ask the children how many pints of water you have. (4) Then put the pint bottles back on the table and fill them. Ask how many pints there are now. (6) Then ask how many quarts of water there are. (3) To summarize the activity, have the children help fill in the following chart on the chalkboard:

1 quart = 2 pints	2 pints = 1 quart
2 quarts = ____ pints	____ pints = 2 quarts
____ quarts = 6 pints	6 pints = ____ quarts

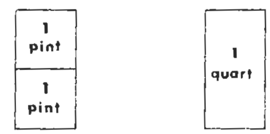
▶ Write the following statement on the chalkboard:

4 pints = ____ quarts

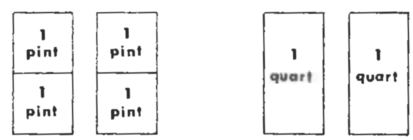
Ask if a pint or a quart is a larger unit of measure. Then ask if 4 quarts is more or less than 4 pints. Finally, ask how many pints equal 1 quart and record the statement above the one already on the chalkboard.

2 pints = 1 quart
4 pints = ____ quarts

Draw an illustration such as the following.



Ask the children what statement this picture represents. Then ask how many more pints are needed to show the number in the second statement on the board. (2) Ask how many times greater than 2 pints are 4 pints. (2 times) Let a child draw the figures to represent two times as many pints and two times as many quarts.



Let another child complete the second equation. Write several other incomplete statements on the chalkboard.

6 pints = ____ quarts
8 pints = ____ quarts
10 pints = ____ quarts
12 pints = ____ quarts
16 pints = ____ quarts

Let the children suggest ways to compute the number of quarts. Help them realize that the key is the basic relationship between the two units:

2 pints = 1 quart

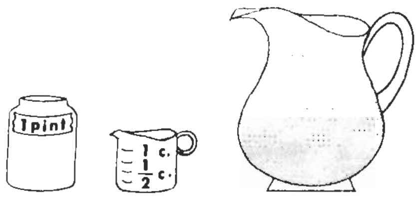
In the exercise, 6 pints = ____ quarts, they are concerned with 3 times as many pints; therefore, there will be 3 times as many quarts.

Adapt the activity to explore the relationship between quarts and pints. Use examples such as the following:

1 quart = ____ pints
3 quarts = ____ pints
5 quarts = ____ pints
7 quarts = ____ pints
9 quarts = ____ pints

▶ Have a child remove all of the 1 cup containers from the display table and line them up on the chalk-tray. Let another child do the same with the pint containers. Tell the children to examine the new display carefully and guess how many cups equal 1 pint. Record their estimates on the board.

Display a labeled pint jar, a 1 cup measuring cup, and a pitcher of colored water.



Select some child to help you find the number of cups in a pint. Have him measure 1 cup of water, pour it into the pint jar, and mark the water level with a crayon or felt-tip pen. Tell him to repeat the process until he has measured 1 pint of liquid. Then have him circle the correct estimates in the list on the board.

Duplicate the following incomplete statements and give one to each child.

1 pint =	2	cups	2 cups =	1	pint
2 pints =	____	cups	4 cups =	____	pints
4 pints =	____	cups	6 cups =	____	pints
5 pints =	____	cups	12 cups =	____	pints
7 pints =	____	cups	16 cups =	____	pints
8 pints =	____	cups	18 cups =	____	pints

While the children are filling out their charts, copy the list on the chalkboard. Then have the children tell you the appropriate numeral for each blank and explain how they computed the answer.

▶ Ask the children if 1 cup can be renamed as a fractional part of a pint. Someone should suggest that since 2 cups make 1 pint, 1 cup is $\frac{1}{2}$ of a pint. Let the children discuss the things that are commonly sold by the half-pint, such as individual size cartons of milk and ice cream.

Let the children examine a half-pint carton of milk. Have someone point out and read the statement on the container that indicates the quantity it contains. (Most cartons are labeled one half-pint rather than one cup.) Now have a child pour the milk into a measuring cup. If the liquid level is a little above or below the 1 cup mark, explain to the class that the carton contained approximately 1 cup of milk. Have the children discuss any ideas they might have about the reasons it is difficult to measure liquids exactly.

▶ To review the units of measure developed thus far, ask questions such as the following.

How many half-pints make 1 pint?
How many cups make 1 pint?
How many half-pints are in 3 pints?
How many cups are in 3 pints?
How many cups are in 1 half-pint?
How many pints make 6 cups?
How many pints make 6 half-pints?
How many pints are in 1 quart?
How many cups are in 1 quart?

Have each child select and display the containers that might be used to demonstrate his answer before he answers aloud.

Pages 295 through 300

● Use page 295 as a review of quart and pint. Have a child read the story aloud. Discuss Mrs. Sweet's solution to the problem and have a child check to see if 6 quarts equal 12 pints by computing on the chalkboard. Let the children make up other stories about quantities of ice cream and express the answers in both pints and quarts. For example:

> Mrs. Sweet can fit only 8 pints of ice cream in her freezer. How many quarts might melt?

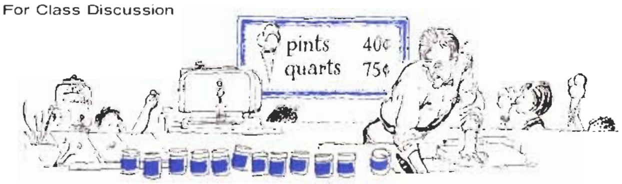
Have the children complete the table at the bottom of the page. Then have them help you complete the following chart as a class activity.

Pints	Quarts
2	
4	
	3
8	
12	
16	

Name

UNIT 17 MEASUREMENT

For Class Discussion



Mr. Frost sells ice cream in quart and pint containers that he fills himself.

One day Mrs. Sweet called and ordered 6 quarts of chocolate chip ice cream.

"But I don't have any more quart containers!" Mr. Frost said.

"Then send me 12 pints," replied Mrs. Sweet.

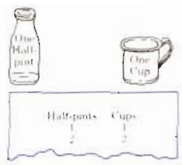
Is 12 pints of ice cream as much as 6 quarts? *Yes*

Complete the table.

Table of Liquid Measure	
Quarts	Pints
1	2
2	4
3	*6*
4	*8*
5	*10*
6	*12*

reference page

D-295



Sally's mother bought one half-pint of cream for cream puffs. Her recipe called for 1 cup of cream.

Did Sally's mother buy enough cream for one recipe? ___Yes___

Sally is making cream puffs. She needs 4 times as many cream puffs as one recipe will make.

Complete the chart that Sally started.

How many half-pints of cream does Sally need? ___4___

Half-pints		Cups
1	-	1
2	-	2
3		3
4	-	4
5	-	5

At the store Sally found only pints of cream. "I need four half-pints," she thought, "so I will take 2 pints."

Was Sally correct? ___Yes___

How do you know? Four half-pints is the same as 2 pints.

Complete.

1 pint =	2 half-pints =	2 cups
2 pints =	4 half-pints =	4 cups
3 pints =	6 half-pints =	6 cups
4 pints =	8 half-pints =	8 cups
5 pints =	10 half-pints =	10 cups

reference page

D-296

Answer the questions.

1. Ed and 7 of his friends stopped at his house after school. The boys drank 4 pints of milk. How many quarts is this? ___2___

2. Jim went to the store to buy 2 quarts of orange juice. The only juice in the store was in small cans. Each can held one cup. How many cans of juice did Jim buy? ___8___

3. Mary decided to make a pot of soup. The recipe that she used called for 4 cups of water. How many pints is this? ___2___

4. Ann poured 16 cups of lemonade into a punch bowl. How many quarts of lemonade is this? ___4___

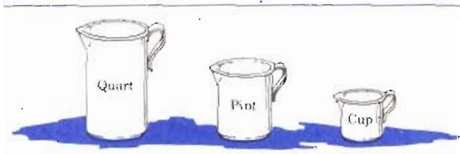
5. Miss Bell ordered 16 quarts of grape juice. How many pints is this? ___32___

6. Carole bought 3 pints of cream. How many cups is this? ___6___

7. After a Saturday football game a group of boys drank 12 half-pints of root beer. How many quarts is this? ___3___

D-298

Name



Jean started a chart about measuring liquids.

1 quart = 2 pints = 4 cups

3 quarts = 6 pints = ___12___ cups

How do you think she decided that 3 quarts = 6 pints?

How do you think she would compute the number of cups?

3 quarts = ___3___ × 4 cups

6 pints = ___3___ × 4 cups

Complete.

1. 4 pints = ___2___ quarts
2. 6 quarts = ___12___ pints
3. 2 pints = ___4___ cups
4. 8 cups = ___4___ pints
5. 4 cups = ___1___ quart
6. 3 quarts = ___12___ cups
7. ___2___ quarts = 8 cups
8. ___4___ pints = 2 quarts
9. ___4___ cups = 2 pints
10. ___6___ pints = 3 quarts
11. ___4___ quarts = 8 pints
12. ___6___ cups = 3 pints

reference page

D-297

● Use page 296 to reinforce the children's understanding of the relationship between pints, half-pints, and cups. Have the children read the first three sections aloud and discuss the questions. Then tell the children to complete the chart at the bottom of the page. When they have finished, let them tell how they arrived at their answers.

● Pages 297 and 298 review the relationship between cups, pints, and quarts. Have a child read aloud the story and questions at the top of page 297. Then discuss the answers to the questions.

Work the first two exercises with the class. Ask how many quarts there are in 2 pints; then how many quarts in 4 pints. For exercise 2, ask how many pints are in 1 quart; then see if some child can tell the number of pints in 6 quarts.

Have the children complete the page independently. Let individuals tell how they arrived at their answers.

Assign the exercises on page 298 for independent work. Then let the children discuss their results.

Name

Compute.

1. 22 pints + 16 pints = 38 pints
2. 15 half-pints + 7 half-pints = 22 half-pints
3. 13 quarts + 29 quarts = 42 quarts
4. 1000 quarts − 428 quarts = 572 quarts
5. 124 pints − 30 pints = 94 pints
6. 327 cups − 109 cups = 218 cups
7. 7 half-pints × 6 = 42 half-pints
8. 8 quarts × 9 = 72 quarts
9. 12 cups × 5 = 60 cups
10. 8 pints × 8 = 64 pints
11. 6 half-pints × 7 = 42 half-pints
12. 4 quarts × 9 = 36 quarts
13. 8 cups + 9 cups = 17 cups
14. 16 half-pints − 7 half-pints = 9 half-pints
15. 18 pints + 11 pints = 29 pints

D-299

Compute the cost.

1. 6 quarts of ice cream at 69¢ a quart. $4.14
2. 5 pints of apple cider at 32¢ a pint. $1.60
3. 8 half-pints of yogurt at 35¢ a half-pint. $2.80
4. 9 quarts of motor oil at 55¢ each. $4.95
5. 12 pints of paint at 83¢ a pint. $9.96
6. 4 quarts of wax at 96¢ each. $3.84
7. 6 half-pints of cream at 40¢ each. $2.40
8. 100 quarts of milk at 32¢ a quart. $32.00
9. 25 half-pints of chocolate milk at 8¢ each. $2.00
10. 48 pints of orange juice at 23¢ a pint. $11.04
11. 7 pints of cottage cheese at 39¢ a pint. $2.73

D-300

● Pages 299 and 300 provide practice with computation. Have the children complete page 299 independently and then discuss their results. If there is disagreement about any of the results, let volunteers work the exercises on the chalkboard.

This page may be adapted to provide enrichment for the children. They may find it interesting to see how many cups are involved in the exercises that use pints and half-pints, and how many pints are involved in the exercises that use quarts. Some students may compute to find the number of cups in the exercises involving quarts.

Discuss the first two exercises on page 300. Tell the children to compute the cost for each exercise. When they have completed this assignment, allow them to discuss their answers.

Supplemental Experiences

■ Draw the following chart on the chalkboard.

Harry	1 quart, 3 quarts, 2 quarts, 1 quart
Jim	2 quarts, 1 quart, 6 quarts, 3 quarts
Tom	1 quart, 4 quarts, 2 quarts, 1 quart
Ned	3 quarts, 2 quarts, 1 quart, 2 quarts

Explain that the Sunshine Gas Station kept this record of the oil that the attendants sold in one day.

Ask how many quarts each attendant sold, and which attendant sold the most. Tell the children to compute the total number of quarts that were sold that day.

The children may enjoy making other charts and writing questions to go with them. Some possible topics are listed below.

Pints of window cleaner used.
Half-pints of car polish used.
Quarts of antifreeze sold.

■ Some children may bring up the idea that cups, pints, and quarts can also be used to measure nonliquids. Coleslaw and bean salad are sometimes sold by the pint or half-pint. Detergent, flour, and sugar are often measured by the cup. Berries and mushrooms are packed in pint or quart boxes.

Ask the children to bring in examples of these containers. Have them compare the containers with the ones that held liquids; discuss any differences and similarities that are observed by the class. Then help the children make up story exercises that involve shipping, buying, or selling different quantities, or using several items in recipes. This may be done either as a group project or as independent work. Post the stories on a bulletin board for the children to read and solve in their spare time. The children may also want to construct an answer key so that they may check their own work.

KEY IDEA

Gallons plus gallons is gallons.

Scope

To develop an understanding of the concept of gallon and half-gallon.

To use the knowledge of liquid measure in story exercises.

Fundamentals

Addition of quantities is simple when the quantities are expressed in common units. When the units are the same, just add the numbers.

$$\begin{array}{r} 5 \text{ quarts} \\ + \underline{2 \text{ quarts}} \\ 7 \text{ quarts} \end{array}$$

Readiness for Understanding

Knowledge of multiplication.

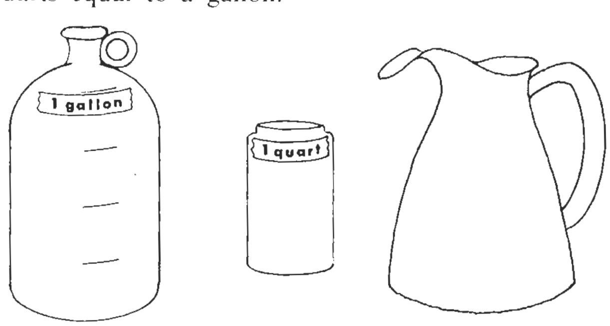
Knowledge of addition.

Developmental Experiences

pitcher (at least 6 quarts)
food coloring
1 gallon bottle
1 quart bottle
1 pint jar
masking tape
felt-tip pen
27 cards (3″ × 12″)
sheet of tagboard

▶ Provide a pitcher or another container that holds at least 4 quarts of colored water. Place a piece of tape part way around a gallon container in such a way that the top edge marks the level for 1 gallon of liquid. Write 1 gallon on the piece of tape. Prepare a quart container in a similar manner and label it 1 quart.

Tell the children that the amount of liquid needed to fill the large container to the top of the tape is 1 gallon. Ask some child to find the number of quarts of water it takes to make a gallon. As each quart is measured into the gallon jar, have the child mark the water level with a crayon or felt-tip pen. Then have him record on the chalkboard the number of quarts equal to a gallon.



1 gallon = 4 quarts

Tell the children to imagine that there are 2 more gallon bottles on the table. Ask how many more quarts of water you would need to fill them. (8) Ask how many quarts of water there would be all together. (12) Record these measurements on the chalkboard.

1 gallon = 4 quarts
2 gallons = 8 quarts
3 gallons = 12 quarts

Pour the gallon of water back into the pitcher. Then ask a child to find the number of quarts of water required to make a $\frac{1}{2}$ gallon. While that child is measuring the water, ask the others if anyone already knows how many quarts are needed. Someone may suggest that a $\frac{1}{2}$ gallon is 2 quarts because 1 gallon is 4 quarts. When the measuring has been completed, have the number of quarts equal to a $\frac{1}{2}$ gallon written on the chalkboard.

$\frac{1}{2}$ gallon = 2 quarts

Call attention to the list of quarts per gallon already on the board and have the children tell the number of half-gallons in each amount. Record the information.

1 gallon = 4 quarts = 2 half-gallons
2 gallons = 8 quarts = 4 half-gallons
3 gallons = 12 quarts = 6 half-gallons

▶ Tell the children that you need one gallon of milk. When you reach the store all you find is 1 half-gallon of milk and plenty of quarts of milk. Ask what you should buy to get one gallon of milk. A child may suggest that you could buy 1 half-gallon and 2 quarts. Another child may suggest that you buy 4 quarts to get one gallon of milk. Write equations on the chalkboard to express the ideas presented. For example:

$\frac{1}{2}$ gallon + 2 quarts = 1 gallon
4 quarts = 1 gallon

Pose other problems such as the examples below.

I need 2 gallons of ice cream. At the store I find only 1 half-gallon of ice cream and many quarts of ice cream. What shall I buy in order to get 2 gallons of ice cream?

I need 5 quarts of paint but I find only 1 quart of paint and many half-gallons when I get to the store. What shall I buy to get 5 quarts of paint?

Continue the activity; let the children make up similar situations.

▶ For this activity you will need the pint, quart, and gallon containers used in previous activities, and a container that holds about 6 quarts of colored water.

Write 1 gallon = _____ quarts on the chalkboard and ask a child to complete the sentence. Then have him use the quart jar to measure out quarts of water into the gallon container to see if 4 quarts will fill the container to the level at the top of the tape.

Tell the class to imagine three times as much liquid as they see in the gallon container. Ask how many gallons they are thinking of. Then ask how many quarts and how many half-gallons they are imagining. Let one child explain how he computed the number of quarts, and another explain how he computed the number of half-gallons. Write the suggestions below the equation already on the chalkboard.

1 gallon = 4 quarts
3 gallons = (3 × 4) quarts
3 gallons = (4 + 4 + 4) quarts
1 gallon = 2 half-gallons
3 gallons = (3 × 2) half-gallons
3 gallons = (2 + 2 + 2) half-gallons

Continue the activity with 5, 7, 9, and 11 gallons and let the children compute the number of quarts and the number of half-gallons measured by the given number of gallons.

Write on the chalkboard 1 quart = _____ pints and have a child complete the sentence and measure pints of water into the quart jar to show that he is correct. Tell the class to imagine twice as much water as can be seen and tell how many quarts and pints they are imagining. As they discuss how the number of quarts was computed, write the equations on the chalkboard.

1 quart = 2 pints
2 quarts = (2 × 2) pints
2 quarts = (2 + 2) pints

Continue the activity with 4, 6, 8, and 10 quarts. Encourage the children to use multiplication to compute the number of quarts.

Summarize the activity by placing the following chart on the chalkboard and have the children supply the missing numbers.

Gallons	Half-Gallons	Quarts	Pints
1 gallon	2 half-gallons	4 quarts	8 pints
_ gallons	4 half-gallons	_ quarts	__ pints
3 gallons	_ half-gallons	_ quarts	24 pints
4 gallons	_ half-gallons	_ quarts	__ pints

▶ Write the following amounts on the chalkboard:

4 quarts 6 quarts

Ask a child to tell which of these is the greater amount of liquid. Continue with other amounts of liquid to be compared, such as:

5 quarts	5 pints
½ gallon	16 pints
12 pints	9 quarts
2 gallons	3 half-gallons

Review the basic relationships between the units of measure with the children. Have them help you complete the following table. You may wish to use a sheet of tagboard and display the chart for reference.

1 gallon = _____ quarts
1 gallon = _____ half-gallons
1 half-gallon = _____ quarts
1 quart = _____ pints
1 pint = _____ half-pints
1 pint = _____ cups
1 half-pint = _____ cup

▶ On each of 27 tagboard cards (3 inches by 12 inches) write one of the following amounts:

1 cup	½ pint	1 quart	½ gallon
2 cups	1 pint	2 quarts	1 gallon
4 cups	2 pints	3 quarts	2 gallons
6 cups	3 pints	4 quarts	3 gallons
8 cups	4 pints	5 quarts	
16 cups	6 pints	8 quarts	
	8 pints	12 quarts	
	10 pints		
	16 pints		
	24 pints		

Shuffle the cards and place the first five on the chalk-tray.

Divide the class into two or more teams. Tell the children that they are to look for cards that name equal amounts of liquid. Direct the first person on the first team to take any cards on the tray that name the same amount. As he takes them, he must read them aloud and say that the amounts are equal. For example, if he chose 4 quarts, 1 gallon, and 8 pints, he would say, "4 quarts equals 1 gallon equals 8 pints," as he picks up those cards for his team. When he is finished, he should replace the cards he removed with new cards from the pile. Then the first member of the second team repeats the procedure. (Replace several cards if none of the five cards match.) The game continues until all of the cards from the pile have been placed on the chalktray. The team that collects the greatest number of cards is the winner.

▶ Write a list of liquids on the chalkboard.

ice cream	89¢ a half-gallon
apple juice	67¢ a gallon
grape juice	45¢ a quart
orange juice	58¢ a quart
sherbert	26¢ a pint

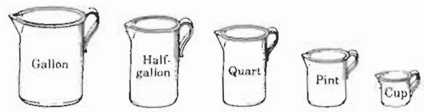
Instruct the children to have paper and pencil ready; they are going to compute costs. If they can compute mentally, they may write just the product. To indicate that they have found the answer, they should lay their pencils on their papers.

Ask for the cost of 4 half-gallons of ice cream. Wait until most of the children have finished before discussing the computed cost. Continue the activity with other costs such as:

3 quarts of orange juice
10 gallons of apple juice
5 quarts of grape juice
7 half-gallons of ice cream
1 quart of sherbert

Name

2 cups = 1 pint
2 pints = 1 quart
2 quarts = $\frac{1}{2}$ gallon
4 quarts = 1 gallon



Underline the greater amount.

1. 3 gallons or 13 quarts
2. 6 cups or 5 pints
3. 8 pints or 3 quarts
4. 6 half-gallons or 10 quarts
5. 7 pints or 12 cups

Underline the lesser amount.

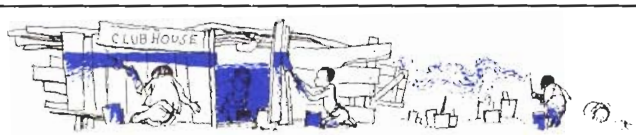
6. 8 cups or 4 quarts
7. 12 half-gallons or 5 gallons
8. 12 pints or 2 half-gallons
9. 8 quarts or 10 pints
10. 15 pints or 2 gallons

Underline the equal amounts in each row.

11.	2 half-gallons	9 pints	5 quarts	8 pints
12.	12 quarts	5 gallons	6 half-gallons	4 gallons
13.	10 cups	6 pints	5 pints	5 quarts
14.	4 gallons	8 half-gallons	12 quarts	16 quarts
15.	6 pints	8 cups	16 cups	2 quarts
16.	10 pints	2 quarts	20 cups	1 gallon

reference page

D-301



Three boys painted a clubhouse. Jeff used 1 gallon of paint on the outside walls. Mike used 4 quarts on the inside walls, and Bruce used 2 half-gallons on the windows, door, and floor.

Which boy used the most paint? *All used the same amount*

How much paint was used? *3 gallons, 6 half-gallons, or 12 quarts*

Compute.

1. 3 half-gallons + 5 half-gallons = *8* half-gallons
2. 3 quarts + 9 quarts = *12* quarts
3. 4 pints + 6 pints = *10* pints
4. 18 gallons − 9 gallons = *9* gallons
5. 17 half-gallons − 5 half-gallons = *12* half-gallons
6. 20 quarts − 14 quarts = *6* quarts
7. 4 quarts × 3 = *12* quarts
8. 5 half-gallons × 4 = *20* half-gallons
9. 2 cups × 9 = *18* cups

D-302

Pages 301 through 306

● Page 301 will provide practice in comparing amounts of liquid in three ways:

Which amount is greater?
Which amount is less?
Which amounts are equal?

Have the children read the table of measures at the top of the page. Then ask questions such as:

How many pints equal 1 quart?
How many pints equal 3 quarts?
How many quarts equal 1 gallon?
How many pints equal 1 gallon?

Ask the children to compare two amounts such as:

6 pints   2 quarts
6 pints is greater than 2 quarts.
2 quarts is less than 6 pints.

Have the page completed and discussed. Give the children an opportunity to explain their thinking.

● Introduce page 302 by having the children read and discuss the story at the top of the page. When they decide that each boy used the same amount of paint, have the children examine the exercises below the story. Assign the exercises for independent work; give help when necessary. When the page is completed, have the children discuss how they arrived at their answers.

● The story exercises on page 303 require converting gallons to half-gallons, or the reverse, before the questions can be answered. Read through all of the exercises with the class and discuss how each one might be approached. Allow the children to work independently.

The children who have difficulty may benefit by working in pairs. Have them take turns computing the answers and checking the results. The checking may be done by measuring the amounts (with teacher supervision) or by referring to a reference chart. If a chart is used, it should be constructed by the children before they attempt to solve the exercises on the page.

1 gallon = ____ half-gallons
2 gallons = ____ half-gallons
3 gallons = ____ half-gallons
⋮
10 gallons = ____ half-gallons
20 gallons = ____ half-gallons

Name

Answer the questions.

1. Mr. Brody is the owner of the Treat Shop. He uses 5 gallons of chocolate syrup each week. How many half-gallons is this? 10

2. Mr. Brody had 10 gallons of root beer. He used 5 half-gallons. How many half-gallons of root beer did he have left? 15

3. One morning Mr. Brody made 2 gallons of lemonade. By noon he sold a half-gallon. How many half-gallons of lemonade were left? 3

4. A cafeteria orders 40 half-gallons of cola each week from the Treat Shop. How many gallons is this? 20

5. Sue made 3 gallons of punch. A half-gallon of the punch was left after a picnic. How many half-gallons of punch had been used? 5

D-303

For Class Discussion

Kathy, Mark, and Tim were on a committee to make punch for an open house. They expected 120 parents to come. The teacher gave them a recipe and two 5-gallon punch bowls. She told them to make one cup of punch for each parent.

"Will this recipe make enough?" asked Kathy.

"And do we have enough punch bowls?" said Mark.

"We'd better check," Tim told them. "I'll make a chart to see how much punch one recipe will make."

Orange Fizz Punch

14 quarts ginger ale
1 pint lemon juice
3 quarts orange juice
1 gallon orange sherbet
$2\frac{1}{2}$ gallons water
9 pints grape juice

Complete Tim's chart.

Amount of punch from one recipe	
How many gallons?	9
How many half-gallons?	18
How many quarts?	36
How many pints?	72
How many cups?	144

Would there be enough punch? Yes

Would the children need more punch bowls? No

D-304

● Have some child read the story on page 304 to the class. Ask how many cups of punch the children in the story will need to serve 1 cup to each parent. (120 cups) Ask how many gallons the two bowls hold. (10 gallons) Then let the class complete the chart. Allow the children to work in small groups if they wish. When most of the children have finished, discuss how they completed the chart. Some may have changed all amounts in the recipe to pints, added, and then used their knowledge of the related measures to complete the table. Other children may have changed all amounts in the recipe to quarts, half-gallons, or cups and then computed.

● Have the children read the story exercises on page 305. Then allow them to work individually or in small groups to answer the questions. Let the class discuss their results when the assignment has been completed.

Name

Answer the questions.

1. Jim made 7 quarts of yellow lemonade and 6 quarts of pink lemonade. How much lemonade did he make? 13 quarts

2. He planned to sell the lemonade in paper cups. Each cup holds $\frac{1}{4}$ quart of liquid. How many cups of lemonade does he make if he sells all of it? 52

3. He charged 5¢ for the cup of lemonade. How much money would he collect if he sold all of the lemonade? 260 ¢

4. By lunch time Jim had sold all of the lemonade except 5 quarts of yellow lemonade and 1 quart pink. How much had he sold before lunch? 7 quarts

5. He had 140 cents at noon. Is this the correct amount of money? Yes

D-305

Answer the questions.

1. The milkman delivered 25 gallons of milk in quart containers. How many quarts did he deliver? 100

2. The milkman sold 2 gallons of cream in $\frac{1}{2}$-pint cartons. How many cartons of cream did he sell? 32

3. Mrs. Lee is giving a party for 48 boys. She plans to order $\frac{1}{2}$ pint of ice cream for each boy. How many gallons of ice cream should she order? 3

4. At the end of the day the milkman had 24 pints of buttermilk left. How many gallons of buttermilk did he have left? 3

D-306

● Page 306 may be assigned for independent work. Have the children read the stories before they begin work. Allow them to discuss their answers when they have completed this assignment.

Supplemental Experiences

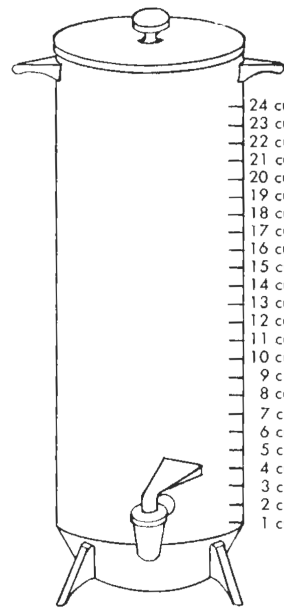
■ Play "Read My Mind." Write an amount such as 3 gallons on the chalkboard. Tell the children you are thinking of another way to name this amount of liquid. Ask the children to read your mind and guess the name you are thinking of. The answer might be 6 half-gallons, 12 quarts, 24 pints, or 48 cups. Let the child who guesses the amount continue the game. Have him write an amount of liquid on the board and tell you the answer he has chosen. Then he may call on other children to read his mind.

■ Prepare cards that show various liquid measures—4 cups, 3 pints, 2 half-gallons, and so forth. Place any two of them on the chalktray. Ask a child to remove the card that shows the greater measure and tell how he made his decision. Put another card on the chalktray. Continue the activity until all of the cards have been compared.

To vary the activity, have the children remove the card that shows the lesser measure. You might also place all of the cards along the chalktray and have a child pick up two cards that represent the same measure.

■ Draw on a 24 by 36 inch sheet of tagboard a graduated cylinder such as might be used on a large coffee urn in a restaurant. Make the cylinder about 28 inches tall. Measure from the bottom of the cylinder and place 24 marks 1-inch apart up one edge of the cylinder.

Tell the class that 1 cup of liquid would fill the container up to the first mark. Let the children decide how much liquid would fill it to the second mark, the third mark, and so on. Then ask a child to point out the mark that a pint of liquid would reach. Continue to develop the amounts of liquid shown in the illustration. The chart may be used as a bulletin board display.



24 cups	12 pints	6 quarts	
23 cups			
22 cups	11 pints		
21 cups			
20 cups	10 pints	5 quarts	
19 cups			
18 cups	9 pints		
17 cups			
16 cups	8 pints	4 quarts	1 gallon
15 cups			
14 cups	7 pints		
13 cups			
12 cups	6 pints	3 quarts	
11 cups			
10 cups	5 pints		
9 cups			
8 cups	4 pints	2 quarts	½ gallon
7 cups			
6 cups	3 pints		
5 cups			
4 cups	2 pints	1 quart	¼ gallon
3 cups			
2 cups	1 pint	½ quart	
1 cup	½ pint	¼ quart	

UNIT 18
APPLICATIONS

Pages 307 Through 316

OBJECTIVE

To apply the children's knowledge of multiplication to specific situations.

The child applies his knowledge of multiplication to appropriate situations. He computes the cost of articles, number of tiles, amounts to be distributed, and so forth.

See Key Topics in Mathematics for the Primary Teacher: Multiplication.

KEY IDEAS

The computation of products answers many questions.
Five skirts and six blouses make 5×6 outfits.

KEY IDEA

The computation of products answers many questions.

Scope

To learn to recognize situations that involve multiplication.

To practice multiplication skills while answering questions about familiar situations.

Fundamentals

It is important that the child examine the world around him and create mathematical models for specific situations. A child needs to recognize relationships between mathematics and his experiences. Many situations experienced by children provide an opportunity to develop mathematical models. One familiar situation is the purchase of 5 pieces of candy at 2¢ each. The mathematical model is $5 \times 2¢$.

When using story exercises from a text, it is necessary that the situations be realistic to the children. Whether or not a child will understand the situation described in the story depends on his reading skill and his experiences. The skillful teacher—aware of various reading levels within a class—will help the child develop comprehension before he computes. The teacher should not emphasize words as clues to arithmetic operations. This technique creates confusion rather than understanding.

Ability to solve story exercises may vary greatly with each exercise and with the ability of each child. Immediate mastery cannot be expected. The teacher should be equally concerned with the child's ability to understand the overall situation, recognize pertinent information, develop appropriate mathematical models, and compute correctly.

Readiness for Understanding

Ability to multiply.

Developmental Experiences

▶ Write on the chalkboard a chart that lists grocery items or other articles and a price for each article.

Milk: quart	**28¢**
half-gallon	**53¢**
gallon	**99¢**
Ice Cream: quart	**54¢**
half-gallon	**98¢**
Orange Juice: quart	**47¢**

Ask the children to read the chart to find different types of information. Ask questions such as:

1. How many different things are listed on the chart?
2. In what different amounts is milk sold?
3. In what different amounts is ice cream sold?
4. In what amounts is orange juice sold?
5. How much does a gallon of milk cost?
6. How much does a quart of milk cost?
7. Which costs less, 2 quarts of milk or 1 half-gallon of milk?
8. Which costs more, 2 quarts of ice cream or 1 half-gallon of ice cream?
9. What would you need to know to compute the cost of 3 gallons of milk?
10. What would you need to know to compute the cost of 9 half-gallons of ice cream?
11. What would you need to know to compute the difference in cost between 3 quarts of orange juice and 3 quarts of milk?

Then ask the children how they would compute the cost of several of the same item. Some children may suggest addition and some may suggest multiplication. Ask the class to compute the cost of 15 gallons of milk. Have several children tell what operation they used. Ask the class to compute the cost of 3 quarts of orange juice. Have several children tell what operation they used. A few may have used addition in this exercise.

▶ Write on the chalkboard a story such as the following:

A librarian ordered 900 new books for the library. The library had 27 empty shelves. Each shelf could hold 30 books.

After the children have read the story, ask them questions such as the following:

1. What is the story about?
2. What numbers do you find in the story?
3. What is 27 in the story?
4. What is 900 in the story?
5. What is 30 in the story?

Ask the children what questions they could answer by using the information given. List the questions on the chalkboard. Pupils may suggest questions such as the following:

Will all of the books fit on the shelves?
Will more shelves be needed?
How many books will fit on the empty shelves?
How many books will not fit on the empty shelves?
How many more shelves are needed?

Select one of the questions. Ask a child what information from the story is needed to answer the question. Then ask how the answer could be computed. Finally, let the children do the required computation. Follow the same procedure for several other questions in the list.

► Write on the chalkboard a story exercise such as the following.

Each day the milkman delivers 9 quarts of milk to Mrs. King's house. How much milk does he deliver to Mrs. King's house in one week?

After the story has been read, ask the class questions such as the following:

1. What is the story about?
2. What numbers do you find in the story?
3. What is 9 in the story?
4. What is 1 in the story?
5. What question is asked in the story?
6. Do you have all of the information you need to answer the question?

The children should realize that part of the information needs to be told in a different way to be of use in answering a specific question. Write on the chalkboard: 1 week = __ days. Let a child supply the correct number of days. Then ask where they could find out the number of days in one week if they didn't know. Finally, ask what operation could be used to compute the answer to this question and have the children do the required computation.

Pages 307 through 312

● Practice in computing the cost of groceries is provided by page 307. Ask the children to look at the illustration and to name the different kinds of food on the shelves. Then direct the children to find the cost of various items such as 1 can of tomatoes, 1 can of baked beans, and 1 can of green beans. Discuss the questions asked on the page. The children may proceed independently with the computation.

When work has been completed, let individual children tell how they computed some of the costs. In the first exercise some child may have used mental computation to find the cost of 10 cans of peaches and the cost of 2 cans of peaches, and then added. In exercise 5, some children may have found the sum of the cost of 1 can of tomatoes and 1 can of peas and then multiplied the sum by 2. Others may have found the cost of 2 cans of peas and the cost of 2 cans of tomatoes, and then added.

$$(2 \times 23) + (2 \times 20) = 2 \times (23 + 20)$$

The same principle may also be applied in other exercises.

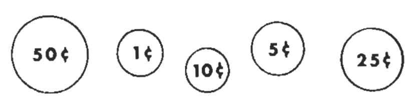
The last four exercises involve finding the difference in the cost of specific items. Ask the children to explain how they computed the difference. For exercise 7, a child may have computed the cost of 6 cans of green beans and the cost of 6 cans of peas, and then computed the difference in the two amounts. Others may have computed the difference in the cost of 1 can of green beans and 1 can of peas, and then multiplied the difference by 6.

● Use page 308 to provide more practice in reading and solving story exercises that involve money. Read all of the stories with the class; ask the children to explain what information is given and what the question is in each story. Then direct the children to complete the exercises. After they have finished, let volunteers explain how they computed the amount in each exercise.

● Page 309 compares values of coins. To review, ask the class questions such as the following:

How many pennies have the same value as a dime?
What coin has the same value as 2 quarters?
One quarter is worth what part of a dollar?
What coin is worth $\frac{1}{10}$ of a dollar?

Draw on the chalkboard a set of coins that are not arranged in the order of their values.



Then ask questions such as:

What coin does it take two of to have the value of a half dollar?

How many nickels does it take to have the same value as a quarter?

How many dimes does it take to have the same value as a half dollar?

How many pennies does it take to have the same value as one dollar?

Tell the children to read and complete the story on page 309. Then ask a child to read the entire story to the class. Have the children answer and discuss the questions at the bottom of the page.

● Have the exercises on page 310 read aloud. As each part of the story is read, let the children tell what the question is, what information is given, and what information is needed to answer the question. For exercise 4, ask the children how the missing information can be found. The children may then read the story silently and do the necessary computation.

Name

UNIT 18 APPLICATIONS

Compute the cost of the groceries.

1. 12 cans of peaches $3.72
2. 4 cans of pears $1.36
3. 6 cans of green beans $1.44
4. 8 cans of baked beans $1.52
5. 2 cans of tomatoes and 2 cans of peas $.86
6. 6 cans of cherries and 6 cans of plums $4.08

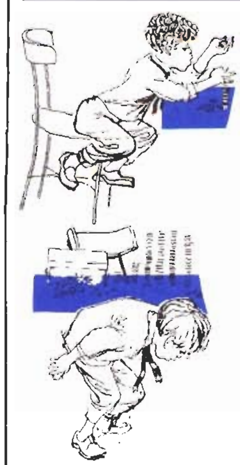
Compute the difference in cost.

7. 6 cans of green beans 6 cans of peas $.24
8. 12 cans of peaches 12 cans of pears $.36
9. 10 cans of green beans 10 cans of baked beans $.50
10. 6 cans of tomatoes 10 cans of corn $.42

reference page

D-307

Name



Jim was playing with Steve, his five-year old brother. Steve said, "I'll trade you my dime for your penny."

"That's not a fair trade," answered Jim. "A dime is worth 10 times as much as a penny, or a penny is worth 1/10 of a dime. I'll trade my half-dollar for your 2 quarters."

Steve replied, "I only trade one for one."

Jim said, "That's not fair. A quarter is worth 1/2 as much as a half-dollar. 2 quarters are worth one half-dollar."

"I'll trade my quarter for your dollar," said Steve.

Jim laughed and said, "Oh, no! I won't do that because a quarter is worth 1/4 of a dollar. 4 quarters are worth one dollar."

Would the trade be fair?

Three dimes for a half-dollar? No

Four nickels for two dimes? Yes

Five pennies and a dime for three nickels? Yes

A half-dollar for a quarter and three dimes? No

reference page

D-309

Jeff and Debbie did their shopping for summer camp.

1. Debbie bought tennis shoes for $2.78. If she gave the clerk $5.00, how much change did she receive? $2.22

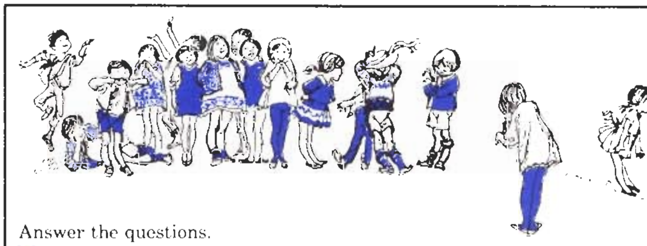
2. Jeff bought 3 sweat shirts and 2 pairs of jeans on sale for $1.79 each. How much did he pay for all of them? $8.95

3. They both bought swimsuits. Debbie's was $4.95 and Jeff's was $2.95. How much did they pay for both suits? $7.90

4. Debbie bought 20 postcards for $.05 each. Jeff bought 2 pencils for $.07 each. How much did they spend on postcards and pencils? $1.14

5. How much did Debbie and Jeff spend on this shopping trip? $20.77

D-308



Answer the questions.

1. Betsy had 15 rolls of film. On each roll she could take 12 pictures. How many pictures could she take? 180

2. If Betsy took 20 pictures each week for 7 weeks, how many pictures did she take? 140

3. Betsy took a picture of her 32 classmates. They were in 4 equal rows for the picture. How many children were in each row? 8

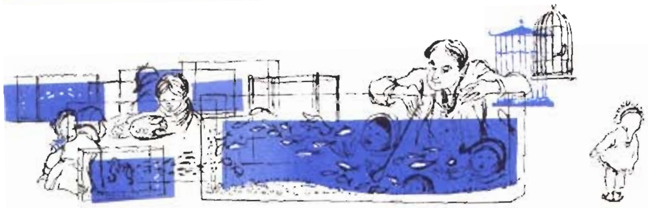
4. When Betsy had 7 rolls of film developed, only 7 pictures on each roll were good. She had taken 12 pictures on each roll. How many pictures were good? 49 were good.

   How many pictures were not good? 35 were not good

5. Betsy pasted 9 pictures on each of 5 pages of an album. How many pictures did she paste on the 5 pages? 45

D-310

Name



Answer the questions.

1. In Mr. Shee's pet store there are 17 fish tanks with 23 fish in each tank. How many fish are there? 391

2. Each month Mr. Shee orders 54 goldfish. How many goldfish would he order in 12 months? 648

3. Mr. Shee ordered 15 boxes of fishnets. There were 12 nets in each box. How many fishnets did he order? 180

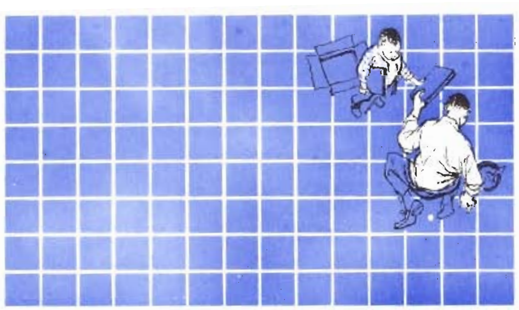
4. Last week 19 packages of seashells were sold. Each package held 33 different kinds of shells. How many seashells did Mr. Shee sell? 627

A Puzzling Problem – Can You Solve It?

A bus started out with 7 passengers. The bus made 5 stops. At each stop, 6 passengers got on the bus and 3 passengers got off. How many passengers were on the bus after the fifth stop? 22

D-311

For Class Discussion



Dick and his father had to tile a floor that was 8 feet wide and 14 feet long. Each tile was 1 foot square.

"How many tiles do we need?" asked Father.

"We could cut 1-foot squares of paper and cover the floor to find out," suggested Dick.

"No, there is an easier way," Father said.

Dick thought about it.

How could Dick find the number of tiles needed? Compute 8 x 14

How many tiles did they need? 112

reference page

D-312

● Direct the children to read silently the first four exercises on page 311. Then ask them to look at the first story and to think about the questions you ask. Do not require oral answers unless a child is puzzled.

1. Do you know what the story is about?
2. Do you know what the question is?
3. Can you find the information given in the story?
4. Do you have the information you need?

Ask a child to read aloud the Puzzling Problem at the bottom of the page. Tell the class that they may try to solve it after they have completed the first four exercises.

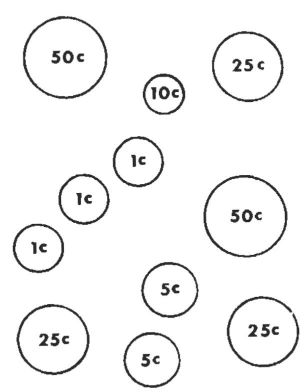
When the children have finished, encourage them to discuss the method or computation they used. Some children may have solved the Puzzling Problem. Do not discuss the solution until most of the children have discovered a solution. This may take several days. Discuss the various ways in which this problem was solved. A child may realize that after each of the five stops, there were 6 − 3 or 3 more people on the bus. And $7 + (3 \times 5)$ is 22.

● Page 312 reviews area of a rectangle. Use this page for class discussion. Let a child read the story to the class. Ask the children how they would find out how many tiles were needed. Some children may want to do as Dick suggests. Others may suggest that there will be an 8 by 14 array of squares. Some other child may remember from the unit in geometry that he multiplied length by width to compute area. Ask the children if they can tell what the area of one tile is. (1 square foot) If every tile is 1 square foot, it will take just as many tiles as there are square feet in the area of the floor.

Supplemental Experiences

■ Place an illustration of coins on the chalkboard.

Change in your pocket



Ask the children these questions:

Which coins would you use to give your friend change for the following?

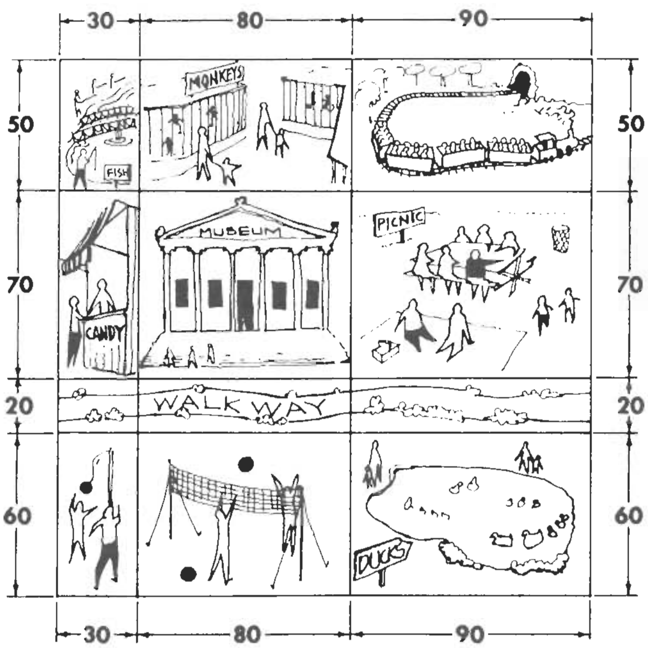
a quarter
a half dollar
a dollar

Which coins would you use to pay such amounts as the following?

48¢
63¢
72¢
$1.37
$1.21

■ Give each child a picture of a park such as the one illustrated.

A MAP OF THE PARK



The area of each section can be read from the map. The volleyball area is 60 × 80 square feet.

Ask the children how to find out the amount of land in the park. A child might suggest that you add 30 + 80 + 90 feet, add 50 + 70 + 20 + 60 feet, and then multiply. Another child might suggest that you compute the area of each section and then add.

Ask the class these questions. The children may record their answers on paper or you might prefer that they answer individually in a class activity.

1. How many square feet is the fish pool lot?
2. How long and how wide is the picnic area?
3. What is the measurement in square feet of the candy stand lot?
4. What is the width of the walkway?
5. How many square feet are there in the walk-way lot?
6. How many square feet are in the volleyball lot?
7. What is the measurement of the toy train lot?
8. How many square feet in the lot where the monkeys are?
9. How many square feet in the museum lot?

KEY IDEA

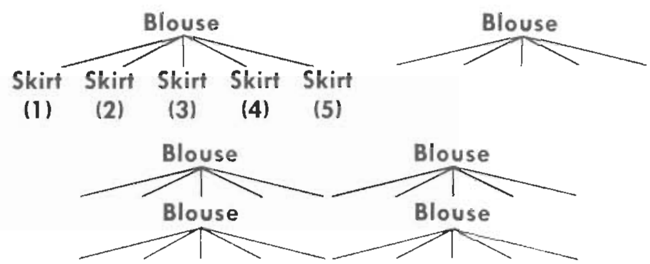
Five skirts and six blouses make 5×6 outfits.

Scope

To have the children continue to apply their knowledge of multiplication to answer specific questions.

Fundamentals

The relationship between multiplication and questions such as "How many outfits can be put together from five skirts and six blouses?" may seem obscure. But it is important to consider what is being counted. In a sense, answering the above question suggests two steps—creating the outfits and then counting the outfits. The creation of the outfits can be illustrated.



Note: When one blouse is paired with each of the skirts, 5 different outfits are formed. When each of the blouses is paired with the 5 different skirts, 5×6 outfits can be created.

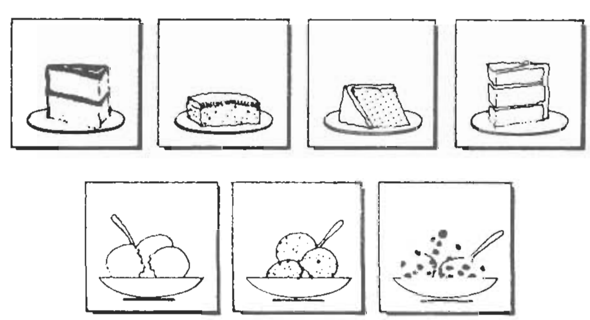
Readiness for Understanding

Understanding of product.

Developmental Experiences

for flannel board	pictures:
3 felt circles	4 kinds of cake
3 felt triangles	3 kinds of ice cream

▶ Display pictures of four kinds of cake and three kinds of ice cream.



Label each picture. The kinds of cake may be chocolate, spice, lemon, and angel food; the kinds of ice cream may be vanilla, mint, and cherry.

Tell the children that each one should select 1 kind of cake and 1 kind of ice cream. Ask one child to go to the pictures and pick up those he would choose. He may choose lemon cake and vanilla ice cream. Record his choice on the chalkboard in this manner:

lemon

vanilla

Tell the child to put the pictures back in their original places. Then ask if anyone would choose lemon cake with a different kind of ice cream. If no one would, ask some children to show the other kinds of ice cream that could be combined with the lemon cake. Record them on the chalkboard as follows.

lemon

vanilla cherry mint

Ask how many different combinations of cake and ice cream the children see listed on the chalkboard. Since the lemon cake is paired with each of 3 kinds of ice cream, there are 3 choices shown.

Let another child select pictures of his choice of ice cream and cake. When he has named his choice, continue the diagram on the chalkboard. Then let the children pair their selection of cake with the other kinds of ice cream. Continue the activity until all possible combinations of cake and ice cream have been made. The completed diagram on the chalkboard will contain the following pairings.

lemon — **vanilla cherry mint**

spice — **vanilla cherry mint**

chocolate — **vanilla cherry mint**

angel food — **vanilla cherry mint**

Read to the children all of the possible pairings of 1 kind of cake and 1 kind of ice cream that are shown by the chart. Ask some child to count the number of different combinations of 1 kind of cake and 1 kind of ice cream that can be made. When he finds that there are 12 combinations, ask if anyone can suggest a faster way than listing and counting to find the number of different combinations. A child may see that for every kind of cake there are 3 different combinations of ice cream. Since there are 4 kinds of cake, there are 4×3 combinations.

Begin another diagram on the chalkboard with the names of the kinds of ice cream that are pictured. Ask the children to show the different kinds of cake that can be paired with each kind of ice cream. The activity will be faster if only the first letter of the name of each kind of cake is written. The completed diagram will contain the following pairings.

vanilla — **L S C A**

cherry — **L S C A**

mint — **L S C A**

Ask how many kinds of cake can be combined with each kind of ice cream. (4) Then ask how many different combinations of 1 kind of ice cream and 1 kind of cake can be made with 3 kinds of ice cream and 4 kinds of cake. (3×4 combinations or 12 combinations)

▶ Ask 5 girls and 2 boys to stand in front of the room. Tell the class that you would like to select one boy and one girl from these children. Ask what combinations of 1 boy and 1 girl are possible. Let the class name pairs of children while one child keeps a tally of the number on the chalkboard. When a child has been named, encourage the class to make all possible pairings that would include that child. When all pairs have been named, write a summary on the chalkboard.

5 girls	2 boys	10 different pairs

Continue the activity with other groups of children. For example, use 4 girls and 4 boys, 3 girls and 6 boys, and 5 girls and 4 boys. Write on the chalkboard a summary of the possible pairings in each group.

5 girls	2 boys	10 different pairs
4 girls	4 boys	16 different pairs
3 girls	6 boys	18 different pairs
5 girls	4 boys	20 different pairs

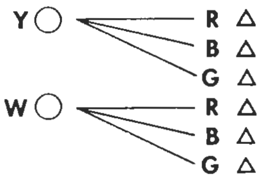
Ask the class to carefully examine the number of different pairs that are formed for each group. Encourage them to discuss what they see. The children may notice that the number of pairs possible for each group is always the product of the number of boys and the number of girls.

Finally, tell the class to imagine the groups of boys and girls you write on the board. Ask how many different pairs of 1 boy and 1 girl can be made from the given numbers. Use examples such as the following.

7 boys	5 girls	____ different pairs
1 boy	1 girl	____ different pairs
8 boys	9 girls	____ different pairs
10 boys	10 girls	____ different pairs
3 boys	1 girl	____ different pairs
9 boys	0 girls	____ different pairs

▶ Display on the flannel board 3 felt circles and 3 felt triangles. Each of the figures should be of a different color. Ask the children to point to different pairs of 1 circle and 1 triangle. Tell the children to identify the figures by color and shape as they point to them. For example, one child might say, "White circle and red triangle," and the next child may say, "Yellow circle and green triangle." Ask one child to tally the number of combinations. When the children can think of no other possible combination, ask the child who is tallying how many pairs have been named. Ask the children if all possible pairs have been named and how they know that their answer is correct. Some of the children should be able to explain that there are 3×3 pairs possible because each of the 3 triangles can be paired with 3 different circles.

Remove 1 of the felt circles from the board. Direct the children to illustrate on paper at their desks all of the possible pairs of 1 circle and 1 triangle. Explain that a quick way to record the colors and shapes is to use the first letter for the name of the color and to draw a picture of a triangle or circle.



When the lists have been completed, have one child read his list and tell the number of pairs he listed. Then ask if he found 2×3 pairs.

Continue the activity using 2 triangles and 2 circles, 3 triangles and 0 circles, and 2 triangles and 1 circle.

Name

One day Diane and her girl friends stopped at Mr. Freeze's ice cream shop to have sundaes. A list of ice cream flavors and toppings was hanging on the wall.

"Umm!" said Diane. "There are as many different kinds of sundaes as girls in our group."

Sundaes 25¢

Ice Cream	Topping
chocolate	marshmallow
strawberry	pineapple
vanilla	

How many girls were in the group? 6

List the different kinds of sundaes that could be made.

chocolate – marshmallow
chocolate – pineapple
strawberry – marshmallow
strawberry – pineapple
vanilla – marshmallow
vanilla – pineapple

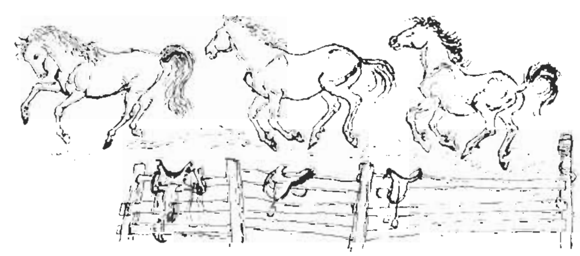
How many different kinds of sundaes could be made if chocolate topping were also on the list? 9

reference page

D-313

Answer the questions.

1. Dave's family owns 3 horses and 3 saddles. One day Dave tried each of the 3 saddles on each of the horses. How many different combinations did he try? 9



2. Tom had 2 model cars that he wanted to race against Joe's 3 model cars. Each one of Tom's cars raced against each of Joe's cars. How many races? 6

3. Jerry makes his own lunch. He always has one sandwich and one piece of fruit. He can make 4 kinds of sandwiches and he can choose from 4 kinds of fruit. How many different lunches can he make? 16

D-314

Pages 313 through 316

● Use page 313 to provide another experience in pairing members of two sets in all possible ways. Have the children read the story silently. Then discuss the story with them. Be sure that the children understand that a sundae is made by putting some kind of topping on some kind of ice cream. Ask the children to name two different combinations of ice cream and topping that would make a sundae.

Tell the children to list all of the possible combinations, using one of the ice cream flavors and one of the toppings as shown on the sign in the illustration. Then have the class tell how many different kinds of sundaes could be made and how many girls there were.

● Let a child read aloud the first story on page 314. Then ask the class questions such as the following:

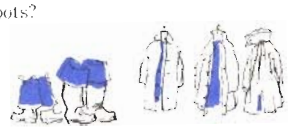
How many horses were there?
How many saddles were there?

If necessary, ask questions about the other two exercises to help the children understand the situation presented. The children may then proceed independently with the necessary computation.

Name

Answer the questions.

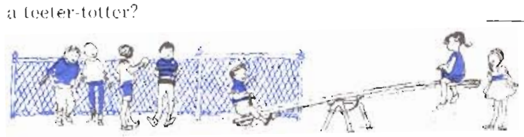
1. 3 coats, 2 pairs of boots. How many different ways can a fireman combine a coat and a pair of boots? 6



2. 5 costumes, 4 pairs of shoes. How many different ways can a dancer combine a costume and a pair of shoes? 20

3. 5 hats, 4 work suits. How many different ways can a farmer combine a hat and a work suit? 20

4. 5 boys, 2 girls. How many different combinations of 1 boy and 1 girl can ride a teeter-totter? 10



D-315

Danny's mouse got loose in the house.

1. The mouse ran around the clockface 10 times. If he were the minute hand, his trips would represent how many make-believe hours? 10

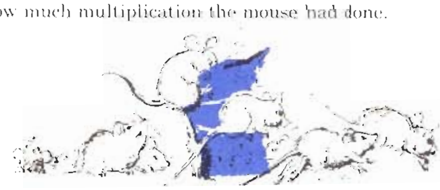
2. If he were the hour hand, his trips would represent how many hours? 120

3. Then the mouse fell on the typewriter keys. "11 × 4 keys," he said. He bounced on each key! How many bounces did he make? 44

4. The mouse played football with Danny's 29 marbles. He kicked each marble with each of his 4 feet. How many kicks did he make? 116

5. He sniffed cake crumbs in the kitchen. "Yum, yum!" he squealed. "I'll nibble every one." He gobbled 12 crumbs, 3 at a time. How many gobbles did he make? 4

When Danny found the mouse, he didn't know how much multiplication the mouse had done.



D-316

● Have several children read aloud the stories on page 315. Encourage the children to ask questions if they do not understand the situations presented. The page may then be assigned for independent work. When the children have finished, let them discuss their work and name all of the possible pairs.

● Have the children read page 316 to get the story situation well in mind. Then ask the children to read the story to themselves and write the answers to the questions. Finally, have the whole story read aloud again. This time have the answer to each question supplied at the correct time. Let the children discuss the ways in which they arrived at their answers.

Supplemental Experiences

■ Draw on the chalkboard arrays of coins such as those shown below.

1¢	1¢	1¢	1¢	1¢
1¢	1¢	1¢	1¢	1¢
1¢	1¢	1¢	1¢	1¢

5¢	5¢	5¢	5¢	5¢
5¢	5¢	5¢	5¢	5¢
5¢	5¢	5¢	5¢	5¢

10¢	10¢	10¢	10¢
10¢	10¢	10¢	10¢
10¢	10¢	10¢	10¢

25¢	25¢	25¢	25¢	25¢
25¢	25¢	25¢	25¢	25¢
25¢	25¢	25¢	25¢	25¢
25¢	25¢	25¢	25¢	25¢

Write the following list near the arrays.

3×5
500 cents
$4 \times 3 \times 10¢$
15 cents
75 cents
4×5
$5.00
3×4
$3 \times 5 \times 5¢$
$.15
120 cents
$5 \times 4 \times 25¢$
$3 \times 5 \times 1¢$

Ask a child to find in the list a number that tells how many coins are in the first array. The only number applying here is 3×5. Then ask the children to find numbers that represent the value of all the coins in the first array. The class may find 15 cents, $.15, and $3 \times 5 \times 1¢$. Proceed in a similar manner with the other arrays.

■ Tell the children that at Riverdale Restaurant today they are serving beef, ham, or fish. You may have your choice of ice cream, cake, or pie for dessert. Ask the children to write all of the different combinations of 1 dinner and 1 dessert. Example:

beef dinner—pie

After the class has finished, have volunteers list the different combinations on the chalkboard.

UNIT 19
COMPUTATION

Pages 317 Through 336

OBJECTIVE

To improve computational skills.

The child practices the computation of sums, differences, and products. He draws on his understanding of numbers to use computational procedures that are meaningful and in keeping with his ability. Speed in computation is not stressed—the emphasis is on understanding and accuracy.

KEY IDEA

Practice makes perfect.

KEY IDEA
Practice makes perfect.

Scope

To practice computation.

Fundamentals

Practice exercises should be assigned only as needed. They provide a way to discover the types of errors a child is making, and enable the teacher to better provide for individual differences in computational skill.

The child should be encouraged to compute products by using the basic multiplication algorism and recording each partial product. Success with the algorism should be expected from the majority of pupils. The use of shortcuts may be allowed for the more capable students.

Readiness for Understanding

Knowledge of basic facts.
Understanding of product.

Developmental Experiences

for flannel board
tagboard cards (3″ × 12″)
felt signs: plus, minus, and times
pins
paper punch
3 notebook rings
felt-tip pen

▶ Write on 3 by 12 inch tagboard cards equations that have an operation sign missing.

2△374 = 229 + 519	6 × 40 = 200△40
40 × 20 = 1000△200	765△297 = 523 + 539

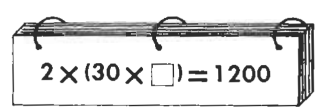
Put the cards in a pack and place the pack face down on a table next to the flannel board. Place across the top of the flannel board several felt plus signs, minus signs, and times signs.

Tell a child to take a card from the pack and to pin it to the flannel board. Explain that he must use one of the felt symbols to make his equation complete. Give him adequate time to do any needed computation; he may do his computation on the chalkboard. Have him show the result of his thinking by pinning the appropriate felt symbol in the proper place on the card. Have him then describe his thinking steps to the class. For example, with the equation 40×20 = 1000△200, a child may say that 4 tens × 2 tens is 8 ten tens, or 8 hundreds; the correct operation sign is a minus sign since 10 hundreds minus 2 hundreds is also 8 hundreds. If a child makes an error, help him correct it.

▶ Write on 3 by 12 inch tagboard cards placeholder equations that involve addition, subtraction, and multiplication.

(3 × □) + 20 = 320
(60 × □) + 100 = 1900
(□ × 500) + 600 = 3100
900 + 500 = 1800 − □
1300 − 400 = □ + 500
4 × (□ × 10) = 80
2 × (30 × □) = 1200
500 + 800 = 1300 + □
1600 − □ = 900 + 100
2 × (5 × □) = 100
□ × (20 × 10) = 800
1400 − 600 = 1500 − □
30 × □ = 200 + 700

Prepare as many cards as there are children in the class. Punch three evenly spaced holes along the top edge of each card. Assemble all cards in one pack so that all holes are lined up. Insert notebook rings into the holes; the cards should swing freely back and forth on these rings.



Begin with the first child in the first row. Show him one of the cards and have him name the missing number without doing the computation on paper. When this child has successfully named the missing number, proceed to the next child in the row. Show him the second card and ask him to name the missing number. Continue in this way until every child in the room has participated. Then repeat the activity two or three more times; give every child an equation that is different from the one he completed before.

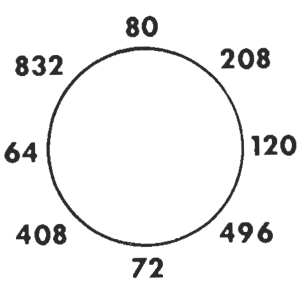
▶ Write on 3 by 12 inch tagboard cards placeholder equations that require written computation. These equations should involve addition, subtraction, and multiplication.

1226 + □ = 6464	**□ = 29 × 78**
□ − 745 = 497	**4200 − □ = 1981**

Separate the class into teams. As you display one of the cards, the first member of each team is to go to the chalkboard and compute to find the missing number.

If a child computed correctly, he earns a point for his team; if he is first to finish correctly, he earns 2 points for his team. After all the children have participated in the activity, total the points for each team and declare a winner.

▶ Draw on the chalkboard a circle that has two-digit and three-digit numerals spaced at intervals on its circumference. To one side of the circle write three numerals each preceded by an operation sign.



×6
−57
+168

Point to the number at the top of the circle. Direct the children to form an algorism for the product, the difference, and the sum by using this number on the circle with the numbers to the right of the circle. Tell the children to write the algorism on their papers and to then do the computation.

$$\begin{array}{r} 80 \\ \times\ 6 \\ \hline 480 \end{array} \qquad \begin{array}{r} \overset{7}{\not 8}0 \\ -\ 57 \\ \hline 23 \end{array} \qquad \begin{array}{r} 80 \\ +\ 168 \\ \hline 248 \end{array}$$

Direct the children to continue in this way. Tell them to move clockwise around the circle and form and compute each algorism.

Vary the activity by having the children perform the operations as a chain procedure.

$$\begin{array}{r} 80 \\ \times\ 6 \\ \hline 480 \end{array} \qquad \begin{array}{r} 4\overset{7}{\not 8}0 \\ -\ 57 \\ \hline 423 \end{array} \qquad \begin{array}{r} 423 \\ +\ 168 \\ \hline 591 \end{array}$$

Pages 317 through 336

● Pages 317 through 336 provide practice in computing sums, differences, and products. The following procedure is suggested for each of the pages.

Work and discuss with the class several exercises at the top of the page in order to clarify what is to be done.

Assign the exercises as independent work. The number of exercises you assign at any one time should depend on the maturity level of the class. There are more exercises in this unit than most children will need. Do not insist that every child use a shortcut method in the computation of products. Allow the children to use the method that they find most comfortable and accurate.

After a given assignment has been finished, encourage the children to explain how they computed specific exercises.

Vary the procedure; divide the class into three or more teams to compete for accuracy. Let one team choose a row of exercises to be computed. All teams should compute the selected exercises. When all of the members of one team are finished, call time. No one may do more computation.

After the results are discussed, direct one member of each team to record and compute the team's score. (1 point earned for each correct computation) Then have another team choose the next row of exercises that are to be computed. After several rounds, total the scores for each team and declare a winner.

Name

UNIT 19 COMPUTATION

Compute.

1. 23 + 45 = 68	2. 56 + 42 = 98	3. 53 + 24 = 77	4. 75 + 31 = 106
5. 84 + 36 = 120	6. 45 + 89 = 134	7. 58 + 8 = 66	8. 78 + 69 = 147
9. 234 + 567 = 801	10. 246 + 357 = 603	11. 872 + 642 = 1514	12. 284 + 56 = 340
13. 568 + 635 = 1203	14. 698 + 447 = 1145	15. 764 + 582 = 1346	16. 839 + 839 = 1678
17. 1239 + 4725 = 5964	18. 7325 + 2893 = 10,218	19. 8976 + 1719 = 10,695	20. 6929 + 8613 = 15,542
21. 9159 + 2789 = 11,948	22. 4671 + 9109 = 13,780	23. 9395 + 745 = 10,140	24. 1394 + 6464 = 7858

D-317

Compute.

1. 54 − 32 = 22
2. 58 − 24 = 34
3. 97 − 23 = 74
4. 83 − 79 = 4
5. 36 − 9 = 27
6. 92 − 83 = 9
7. 66 − 29 = 37
8. 87 − 39 = 48
9. 789 − 456 = 333
10. 739 − 234 = 505
11. 284 − 56 = 228
12. 847 − 635 = 212
13. 878 − 658 = 220
14. 749 − 467 = 282
15. 795 − 394 = 401
16. 934 − 745 = 189
17. 9437 − 1187 = 8250
18. 6567 − 6131 = 436
19. 4355 − 4035 = 320
20. 9784 − 6897 = 2887
21. 3464 − 1908 = 1556
22. 5365 − 2319 = 3046
23. 7732 − 4062 = 3670
24. 6232 − 3504 = 2728
25. 7172 − 3409 = 3763
26. 5171 − 3329 = 1842
27. 5458 − 4078 = 1380
28. 7451 − 7281 = 170

D-318

Compute. *Not all children will write algorisms for all computations.*

1. 372 × 3
2. 4 × 154
3. 628 × 4
4. 781 × 8
5. 23 × 231
6. 14 × 129
7. 135 × 36
8. 92 × 164
9. 74 × 321
10. 83 × 962
11. 879 × 67
12. 785 × 34
13. 765 × 235
14. 164 × 674
15. 357 × 157

1. 372 × 3 = 1116
2. 154 × 4 = 616
3. 628 × 4 = 2512
4. 781 × 8 = 6248
5. 231 × 23 = 5313
6. 129 × 14 = 1806
7. 135 × 36 = 4860
8. 164 × 92 = 15,088
9. 321 × 74 = 23,754
10. 962 × 83 = 79,846
11. 879 × 67 = 58,893
12. 785 × 34 = 26,690
13. 765 × 235 = 179,775
14. 164 × 674 = 110,536
15. 357 × 157 = 56,049

D-320

Name

Compute. *Algorisms will vary.*

1. 43 × 21 = 903
2. 29 × 12 = 348
3. 28 × 36 = 1008
4. 64 × 76 = 4864
5. 23 × 21 = 483
6. 35 × 25 = 875
7. 52 × 93 = 4836
8. 43 × 89 = 3827
9. 97 × 12 = 1164
10. 28 × 27 = 756
11. 97 × 54 = 5238
12. 57 × 87 = 4959
13. 45 × 2 = 90
14. 64 × 8 = 512
15. 442 × 2 = 884
16. 318 × 3 = 954

D-319

Name

Compute. *Algorisms will vary*

1. 44 × 21 = 924
2. 23 × 34 = 782
3. 12 × 31 = 372
4. 12 × 26 = 312
5. 43 × 12 = 516
6. 18 × 14 = 252
7. 41 × 2 = 82
8. 18 × 8 = 144
9. 182 × 2 = 364
10. 923 × 3 = 2769
11. 213 × 23 = 4899
12. 128 × 14 = 1792
13. 421 × 73 = 30,733
14. 683 × 53 = 36,199
15. 324 × 112 = 36,288
16. 123 × 314 = 38,622

D-321

Compute. *Not all children will write algorisms for all computations.*

1. 25 × 28
2. 72 × 52
3. 35 × 37
4. 63 × 63
5. 29 × 42
6. 43 × 83
7. 79 × 9
8. 321 × 2
9. 147 × 9
10. 6 × 814
11. 22 × 634
12. 145 × 25
13. 48 × 357
14. 72 × 783
15. 643 × 732
16. 974 × 962

1. 28 × 25 = 700   2. 72 × 52 = 3744   3. 37 × 35 = 1295   4. 63 × 63 = 3969

5. 42 × 29 = 1218   6. 83 × 43 = 3569   7. 79 × 9 = 711   8. 321 × 2 = 642

9. 147 × 9 = 1323   10. 814 × 6 = 4884   11. 634 × 22 = 13,948   12. 145 × 25 = 3625

13. 357 × 48 = 17,136   14. 783 × 72 = 56,376   15. 732 × 643 = 470,676   16. 974 × 962 = 936,988

D-322

Complete.

[1] 1	[2] 3	[3] 1	[4] 5	[5] 6	■	■
■	[6] 3	0	7	1	[7] 9	■
[8] 1	0	0	3	■	[9] 9	[10] 9
[11] 1	9	0	■	[12] 9	9	8
[13] 1	9	■	[14] 2	5	9	7
■	■	[15] 1	0	0	■	6
■	[16] 4	0	0	3	■	0

ACROSS

1. 6578 × 2
6. 30,000 + 0 + 700 + 10 + 9
8. one thousand three
9. one hundred minus one
11. (3 × 60) + 10
12. two less than one thousand
13. (2 × 10) − 1
14. (2 × 1000) + (5 × 100) + 97
15. ten tens
16. 3999 + 4

DOWN

2. 34,000 − 901
3. ten hundreds
4. 10,000 − 9,427
5. five tens and eleven ones
7. 10,000 − 1
8. 120 − 9
10. 66,660 + 32,100
12. 10,000 − 497
14. (1 + 1) × 100
15. 99,760 less than 99,770

D-324

Name

Compute. *Algorisms will vary.*

1. 35 − 24 = *11*   23 + 17 = *40*   98 − 74 = *24*   34 − 25 = *9*
2. 93 + 43 = *136*   76 − 38 = *38*   80 − 13 = *67*   98 + 32 = *130*
3. 468 + 579 = *1047*   973 − 460 = *513*   345 + 678 = *1023*   888 − 496 = *392*
4. 2367 + 8945 = *11,312*   4356 − 2899 = *1457*   5201 − 3992 = *1209*   7095 − 4162 = *2933*
5. 884 − 23 = *861*   345 + 67 = *412*   719 − 32 = *687*   5079 + 86 = *5165*
6. 2345 + 345 = *2690*   9780 − 330 = *9450*   567 + 89 = *656*   738 − 32 = *706*
7. 9854 + 548 = *10,402*   500 − 54 = *446*   8979 + 9 = *8988*   306 − 26 = *280*

D-323

Name

Compute. *Algorisms will vary.*

1. 154 − 62 = *92*
2. 3333 − 1192 = *2141*
3. 468 − 132 = *336*
4. 2000 − 725 = *1275*
5. 93 × 19 = *1767*
6. 33 × 53 = *1749*
7. 42 × 12 = *504*
8. 32 × 83 = *2656*
9. 248 − 234 = *14*
10. 2876 − 319 = *2557*
11. 2866 − 1329 = *1537*
12. 4621 − 1115 = *3506*
13. 8072 × 14 = *113,008*
14. 478 × 45 = *21,510*
15. 518 × 59 = *30,562*
16. 3478 × 36 = *125,208*
17. 7000 − 2145 = *4855*
18. 9000 − 4265 = *4735*
19. 8765 − 2523 = *6242*
20. 7041 − 2359 = *4682*

D-325

Compute. *Algorisms will vary*

1. 82 × 4 = *328*	2. 76 + 54 = *130*	3. 186 × 9 = *1674*	4. 76 − 32 = *44*
5. 23 + 89 = *112*	6. 92 − 83 = *9*	7. 294 × 16 = *4704*	8. 58 + 49 = *107*
9. 48 × 38 = *1824*	10. 974 × 64 = *62,336*	11. 567 − 284 = *283*	12. 587 × 7 = *4109*

Compute. *Algorisms will vary*

1. 76 + 53 = *129*	2. 762 × 12 = *9144*	3. 98 − 26 = *72*	4. 842 − 316 = *526*
5. 622 × 38 = *23,636*	6. 400 × 16 = *6400*	7. 685 − 317 = *368*	8. 67 + 34 + 18 + 12 = *131*
9. 423 + 76 = *499*	10. 55 + 21 + 16 = *92*	11. 7280 − 1536 = *5744*	12. 509 × 9 = *4581*

Name

Compute. *Not all children will write algorisms for all computations*

1. 987 − 432
2. 964 × 57
3. 37 × 9
4. 87 + 38
5. 753 × 243
6. 97 + 38
7. 435 − 76
8. 7632 − 367
9. 5836 × 36
10. 28 + 93 + 82
11. 2367 − 1738
12. 436 + 897 + 786

1. 987 − 432 = 555
2. 964 × 57 = 54,948
3. 37 × 9 = 333
4. 87 + 38 = 125
5. 753 × 243 = 182,979
6. 97 + 38 = 135
7. 435 − 76 = 359
8. 7632 − 367 = 7265
9. 5836 × 36 = 210,096
10. 28 + 93 + 82 = 203
11. 2367 − 1738 = 629
12. 436 + 897 + 786 = 2119

Name

Complete.

(1) *1*	(2) *5*	(3) *5*	(4) *2*		(5) *1*	(6) *1*
(7) *1*	*0*	*0*	*0*	(8) *3*		*3*
1		*7*		(9) *2*	(10) *1*	*8*
	(11) *7*		(12) *0*		*0*	
(13) *5*	*7*	(14) *9*		(15) *6*		(16) *2*
6		(17) *5*	(18) *4*	*3*	(19) *2*	*1*
(20) *8*	*9*		(21) *2*	*0*	*0*	*0*

ACROSS

1. 2061 − 509
5. 3000 − 2989
7. 6283 + 2656 + 1064
9. Two hundred eighteen
12. 193 − 193
13. 3162 − 2583
17. 50,000 + 4000 + 300 + 20 + 1
20. The largest whole number less than 90
21. 7283 − 5283

DOWN

1. 333 − 222
2. Half a hundred
3. The whole number greater than 506 and less than 508
4. 4 × 5
6. Twelve less than 150
8. 4 × 8
10. 10 tens − 9 tens
11. Seven elevens
12. Any number times this number is zero.
13. 1000 − 432
14. This number plus 5 is 100.
15. 90 × 7
16. 3 × 70
18. 6 × 7
19. Two tens

Compute. *Algorisms will vary.*

1. 774 × 54 = 41,796
2. 1968 × 23 = 45,264
3. 3200 − 1137 = 2063
4. 637 × 26 = 16,562
5. 598 − 309 = 289
6. 3682 × 35 = 128,870
7. 8324 − 6143 = 2181
8. 8176 × 44 = 359,744
9. 467 − 328 = 139
10. 321 + 416 + 311 = 1048
11. 4637 × 356 = 1,650,772
12. 699 + 316 + 453 + 123 = 1591

D-330

Compute. *Algorisms will vary.*

1. 2764 × 79 = 218,356
2. 839 − 275 = 564
3. 213 × 13 = 2769
4. 5679 × 25 = 141,975
5. 765 × 53 = 40,545
6. 9063 − 2517 = 6546
7. 1234 × 5 = 6170
8. 234 × 23 = 5382
9. 876 − 293 = 583
10. 346 + 104 + 783 = 1233
11. 3070 × 17 = 52,190
12. 486 + 325 + 821 + 111 = 1743

D-332

Name

Compute. *Not all children will write algorisms for all computations.*

1. 87 + 42
2. 133 × 6
3. 97 − 38
4. 604 × 7
5. 676 × 18
6. 9206 × 6
7. 5053 × 47
8. 76 + 42 + 34
9. 957 − 619
10. 57 + 43 + 89
11. 62 × 82
12. 4300 − 2107

1. 87 + 42 = 129
2. 133 × 6 = 798
3. 97 − 38 = 59
4. 604 × 7 = 4228
5. 676 × 18 = 12,168
6. 9206 × 6 = 55,236
7. 5053 × 47 = 237,491
8. 76 + 42 + 34 = 152
9. 957 − 619 = 338
10. 57 + 43 + 89 = 189
11. 62 × 82 = 5084
12. 4300 − 2107 = 2193

D-331

Name

Compute. *Algorisms will vary.*

1. 68 − 59 = 9
2. 628 × 19 = 11,932
3. 957 + 586 = 1543
4. 246 × 5 = 1230
5. 623 × 24 = 14,952
6. 976 × 23 = 22,448
7. 984 + 786 = 1770
8. 8000 × 67 = 536,000
9. 7632 − 6487 = 1145
10. 76 + 92 + 13 = 181
11. 479 × 98 = 46,942
12. 689 + 369 + 273 = 1331

D-333

Compute. *Algorisms will vary.*

1. 5076 × 824 = 4,182,624	2. 9584 − 549 = 9035	3. 3586 × 47 = 168,542	4. 9288 − 1665 = 7623
5. 5432 − 542 = 4890	6. 2375 × 67 = 159,125	7. 9832 − 7654 = 2178	8. 7523 − 5689 = 1834
9. 9986 × 697 = 6,960,242	10. 953 + 257 + 783 = 1993	11. 9524 × 859 = 8,181,116	12. 774 + 794 + 567 + 324 = 2459

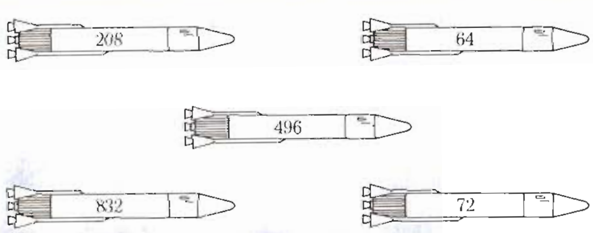
D-334

In each □ write +, −, or ×.

1. 2 [×] 374 = 229 + 519
2. 40 × 20 = 1000 [−] 200
3. 6 × 40 = 200 [+] 40
4. 765 [+] 297 = 523 + 539
5. 4 × 81 = 18 [×] 18
6. 842 [−] 637 = 1439 − 1234

D-336

Name



1. Multiply the number on each spaceship by 6.

*208 × 6 = 1248   *64 × 6 = 384   *496 × 6: 36, 540, 2400, 2976   *832 × 6 = 4992   *72 × 6 = 432

2. Add 168 to the number on each spaceship.

*208 + 168 = 376   *64 + 168 = 232   *496 + 168 = 664   *832 + 168 = 1000   *72 + 168 = 240

3. Subtract 57 from the number on each spaceship.

208 − 57 = 151   64 − 57 = 7   496 − 57 = 439   832 − 57 = 775   72 − 57 = 15

**Order of numbers or form of algorism may vary.*

D-335

Supplemental Experiences

■ Prepare three sets of tagboard cards—a set of sums, a set of differences, and a set of products. Place each set of cards in a different box.

867 + 549	**7000 — 4362**	**35 × 257**
5273 + 4698	**6324 — 2786**	**85 × 77**

Separate the class into two teams. Tell the first member of each team to come forward, choose a card from one of the boxes, and compute on the chalkboard the sum, difference, or product. In order to be sure both members are equally matched, have them draw a card from the same box so that the same kind of operation is performed. If a child computes correctly, he earns a point for his team; if he is first to finish correctly, he earns 2 points for his team. After all children have had an opportunity to participate, total the points for each team and declare a winner.

■ You may wish to challenge some children by presenting chain exercises for mental computation. Use exercises that involve addition, subtraction, and multiplication of multiples of 10. For example, say: "Three hundred fifty minus one hundred," pause, "minus two hundred," pause, "plus thirty," pause, "times seventy." Call upon someone to give the answer. Present another exercise: "Sixty times four," pause, "plus thirty," pause, "minus one hundred," pause, "minus eighty," pause, "times six." Call on someone to give the answer.

Continue in this way with other exercises. Vary the procedure by having the child who answers correctly present the next exercise.

■ Have the children use pencil and paper to record the results of their computation in a series of exercises that you read to them.

$347 + 463 = 810$
$810 = 90 \times 9$
$9 \times 327 = 2943$
$2943 - 1775 = 1168$
$1168 + 32 = 1200$
$1200 = 30 \times 40$

Make sure you provide enough time for computation after you read each step. Say: "Three hundred forty-seven plus four hundred sixty-three." Pause until the children stop writing. Then say, "The sum you have computed is the product of ninety and my secret number." Again pause before you continue:

"Compute the product of the secret number that you found and three hundred twenty-seven.

"Subtract one thousand seven hundred seventy-five from the product you computed.

"To this difference add thirty-two.

"This sum is the product of thirty and my secret number. What is my secret number?"

Continue this activity using other series of verbal exercises. Allow the children to write their computation if they wish.