

MATHEMATICS

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FOR THE

ELEMENTARY SCHOOL

UNIT I

Geometry

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MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT

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^{*}Starring indicates content which is particularly important to the sequential development or evaluation of the program. We ask that all participating teachers try this starred material. It is expected that much of the remaining material will also be used; how much will depend on individual class needs and time available.

Unit I

RECOGNITION OF GEOMETRIC SHAPES

Background for the Teacher

This unit presents several common geometric concepts. The figures presented in the first story, "David Plays a Trick", are square, circle, triangle, and oblong. To illustrate: the sail on a boat may be triangular; the pane of glass in a window may be square; the paper on which the children color may be oblong; the classroom clock may have a circular face.

Mathematical Language and Common Usage

Mathematicians use the terms square, circle, triangle and oblong to refer to the boundary and not to the region of a geometric figure. A region is the interior, or according to some authorities, the union of the interior and the boundary of a geometric figure.



Triangular Region--Shaded Area



Unit T

Teachers are encouraged to use the terms in their mathematical sense, but not to expect this precise discrimination by the class. The specific differentiation between a region and a boundary of a region is presented in a later unit.

The mathematical definition of CIRCLE is the boundary of a circular object. This differs from the conventional usage which refers to both the interior and the boundary of a circular object. Similar usage differences appear with other geometric figures.

The term OBLONG is used to refer to a rectangle that has two of its opposite sides longer than the other two. This terminology differentiates between a rectangle which is square and one which is not.

It is recommended that the term RECTANGLE <u>not</u> be introduced into the geometry vocabulary at this point. If rectangles are mentioned by class members, emphasis should be placed on the concept that both oblongs and squares are rectangles.

Read the story "David Plays a Trick" to the class. After class discussion, give each child a copy of the story to take home. Encourage the children to use the story as a beginning for a mathematics library of their own.



Geometric figures: triangle, square, oblong, circle

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This is the Wright family - Mr. and Mrs. Wright, and their children Ruth Ann, Kathy, David and Kenny. You will get to know them and learn of their adventures and friends in the stories which follow. The first story is about David and Kenny. It is called "David Plays A Trick".



This is the Wright family.

This is David Wright. He is a boy just about your age. He loves to play games with his friends and with his toys.



This is David Wright.

This is Kenny, David's younger brother. He just loves to snoop into David's things.



David and his little brother, Kenny, are good friends. That is, MOST of the time they are! Usually, David shares even his very special toys when they play together. But <u>you</u> know how little brothers are. Once in a while Kenny snoops into David's things when David isn't around. And that's when the trouble begins.

One day David was looking for his new jet plane. He went to his room - and he could hardly get through the door! His building blocks were scattered all over the floor. The big orange dump truck was turned upside down in the corner. David's stuffed lion was underneath the bed. David looked and looked but he couldn't find his new plane. He knew that Kenny had been in his things again.

David remembered that Kenny was still a little boy. But he knew that Kenny needed to learn to think of other people's feelings too. David thought and thought. He wondered, "Maybe, if Kenny can't find something of his then...." Suddenly David had an idea. "I know. I'll play a trick on Kenny. I'll hide his soft crib pillow and his high chair pillow. He'll have to look and look for them - and then <u>he'll</u> know how it feels. Hmm. I think I'll hide the couch pillows, too. That way he'll have to look even harder before he finds his own favorite pillows. Let's see, where shall I hide them?"

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"I'll play a trick on Kenny."

Quickly David looked around the room. He hid the circular couch pillow under the circular rug on the floor; he hid the oblong crib pillow under the oblong blanket on his bed. He found hiding places that were just the right shape for the other two pillows. It was lots of fun playing this trick!



David hid the circular couch pillow under the circular rug in their room.

It was just before naptime. Kenny was hungry and wanted some cookies and milk. When he discovered that his high chair pillow was gone he shouted, "Mommy, Mommy! My high chair pillow is gone."

Mother said, "Let's look in your room. Maybe it's in there."

Then Kenny noticed the lump under the circular rug on the floor. "There it is Mommy." But, of course, we know it was the circular couch pillow. Kenny had to look some more.

Next, Kenny looked in the triangular toy box. Can you guess which pillow he found there? It was the triangular couch pillow.

David hid the circular couch pillow under the circular

rug in their room.



Kenny found the triangular pillow in the triangular toy

Kenny climbed up on David's bed, the tears slowly running down his cheeks. Then he felt something extra soft under the oblong blanket on David's bed. "It's here, Mommy," he said excitedly. But when he pulled the pillow out, it was his oblong crib pillow instead.

Mother said, "Let's see now. We found the circular couch pillow under the circular rug. We found the triangular couch pillow in the triangular toy box. The oblong crib pillow was under David's oblong blanket. Hmm . Your high chair pillow is <u>square</u>. Perhaps we should look for something square we'll probably find it there."



Kenny found his oblong crib pillow under the oblong blanket on David's bed.

Just then David came into the room. Mother took one look at his face and then she guessed that it was David who had hidden the pillows. She smiled, "David, this hiding game of yours is a good trick, but I think Kenny is too tired to look for his high chair pillow any longer. Why don't you show him where it is?"

David smiled, "All right, Kenny. I guess that now you know it's no fun having to look and look for something you want very much. If you promise not to take my things, then I won't take yours, O. K.?

Kenny didn't answer. Sadly he climbed into his crib, Then, surprise! He found his missing square high chair pillow under his square crib blanket!

"My pillow! I found my high chair pillow! Mommy, you can have it....I'm tired!" Mother fluffed up Kenny's oblong crib pillow and slipped it under his head. Then she and David tiptoed out of the room.

David thought, "I wonder if Kenny will go snooping into my things again. He really didn't promise me anything. Did my trick really work?"

WHAT DO YOU THINK?

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Kenny found his square high chair pillow under his square crib blanket.

A Few Musical "Notes" to the Teacher

The compelling qualities of music can have the useful function of reinforcing the assimilation of essential concepts. If he "just can't get that song out of his head", it is fortunate if "that song" contains the essential ideas of the day's math lesson. On varying levels of subtlety the musical activities included in this unit express in word, action or musical form some aspect of the mathematical concept to be learned.

Music does not become compelling, however, without careful preparation, especially when dealing with young children. The teacher must know exactly how the music sounds before presenting it to the students. Only then can the necessary rhythm, vitality and decisiveness be transmitted. Children of 5 and 6 respond readily to all kinds of rhythmic patterns and can imitate some of considerable complexity. On the other hand vocal ability is usually still quite limited in range and mobility. The songs are constructed with these considerations in mind and have a fair chance of coming out ungarbled and appealing if well defined and vigorous visual direction is constantly given to the children. Visual direction is no good unless someone is watching it. "Watch me!" must be a constant "watchword."

The idea of singing on pitch may be new to many children in early grades, and a certain amount of insistence on tone-matching is useful. Music loses most of its appeal for both listener and participant if the notes are not sung in tune. Although dynamic markings are generally not used in these songs, it should not be forgotten that contrasts between loud and soft are among the most compelling and obvious effects possible in music.

Introducing rhythm instruments enhances the sound of this music and provides added interest for the children. Even though enough instruments may be available for the whole class, the effect of one or two is usually more desirable. The notation, $\frac{1}{2}$, refers to percussion instruments (including clapping, stamping, etc.) when a 5-line staff is used. With a single-line (percussion) staff, the usual $\frac{1}{2}$ is used.

SHAPE SONG







After the song has been learned, take each phrase and demonstrate just how the instruments fit with the words, as illustrated below:

"A flag's (bang) ob-long, (ding) and ends our song." [wait] (crash)

Unit I

Suggested Activities

- 1. Materials Needed: flannel board, flannel cut-outs of four basic geometric shapes.
 - a. Place a *<u>triangle</u>, a *<u>square</u>, a *<u>circle</u>, and an oblong on the flannel board; have the class name each shape as it is presented. Ask the children to close their eyes and then remove one shape from the board. The first child to name the missing object is permitted to come up to the board and remove the next shape, for further class participation.
 - b. Flannel cut-outs, of the same shape, may be connected with yarn and presented as a reinforcement activity.
 - c. Stacking similarly shaped pieces of flannel on top of each other is useful for visual discrimination and vocabulary experience.
- 2. Point out objects in the room which are circular, square, oblong or triangular.
- 3. Ask the children to bring in pictures from magazines or catalogues of objects which are circular, square, oblong or triangular. These may be discussed and then displayed on the bulletin board or pasted in a scrapbook, under appropriate headings.
- Various art activities can be built around these four geometric shapes. For example:
 - a. Use construction paper shapes to make individual pictures or designs.
 - b. Make pictures or designs in crayon, using only these geometric shapes.

*See note on page v

Worksheet Commentary

Worksheet 1 - Recognition of Geometric Shapes

First, instruct the children to trace the dark line around the record with their pointing finger. Review that this curve is called a circle. Proceed in the same manner with the other three objects, reviewing the terminology as each shape is presented.

Repeat the activity, having the children use different color crayons to "color" or trace with crayon the outline of each shape. Use a blue crayon for the circle, a red crayon for the square, green for the triangle, yellow for the oblong. Demonstrate, starting with the circle first. Important: Color only on the dark line which is the circle. Do not color the interior blue for that is <u>not</u> the circle.

Worksheet 2 - Recognition of Geometric Shapes

Have the children connect the squares with red lines; the triangles with green lines; the circles with blue lines; the oblongs with yellow lines.

It is suggested that both these worksheets be corrected with the children, so that they are immediately aware of any errors and have an opportunity to remedy them. At this age level, it is desirable to omit markings on the paper which would indicate errors, e.g. "2 wrong", "3 wrong", etc.



Unit I



Recognition of Geometric Shapes


Unit I

INSIDE AND OUTSIDE OF SIMPLE CLOSED CURVES

Background for the Teacher

Examples of simple open curves



A curve which never crosses or retraces itself and does not return to its starting point is a simple open curve.

Examples of simple closed curves



A curve that returns to its starting point and never crosses or retraces itself is a simple closed curve. Triangles, squares, oblongs, and circles are all examples of simple closed curves. The mathematical meaning of the terms INSIDE and OUTSIDE is similar to that of everyday usage. For example, think of the walls of a house as the boundary line of a simple closed curve, with objects inside or outside. Similarly, a point enclosed by a curve and not on the curve is INSIDE. A point not enclosed by a curve and not on the curve is OUTSIDE.



Note: The interior is a limited area. The exterior has no limits.

Read the story "Eagle Feather" to the class. The purpose of the story is to introduce the terms "open", "closed", "inside" and "outside".



Open, closed curves; inside, outside

Eagle Feather lived with his parents on the colorful painted desert in Arizona. Tourists found his home hard to see as they sped by in their cars, for his house was made of the earth itself. It was round and called a hogan. Eagle Feather loved the warmth of his earthen home in the winter even though it was dark when the door was closed to shut out the chilling winds.



Eagle Feather lived in the painted desert.

When spring came, the family was glad to move the stove out onto a little porch beside the hogan. Father thatched the roof of the porch with bushes from a nearby stream. It was pleasant to sit in the shade and watch the shadows of the clouds as they sped over the colorful hills surrounding his home.



He watched the shadows _ the clouds.

On the other side of the house was the fold for the sheep. The fold was a ring of rocks piled high enough to keep the sheep from wandering away at night.

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He herded the sheep into the fold.

The opening in the fold where the sheep went in and out was closed by small wooden logs. These were laid one on top of another until the opening was filled. The final thing Eagle Feather did each night was to close the opening after he herded the last sheep into the fold.

His mother often told him that the fold with the gate closed was a closed curve. Mother would say, "Remember, my son, sheep are safe inside a <u>closed</u> curve. You must always remember to fit the logs into place at night, and close the curve securely. When the logs are put, the fold becomes an open curve and a sheep could wander out. Never forget to close the curve with the logs."



Mother explained closed and open curves.

Eagle Feather <u>was</u> careful. He led the sheep from the fold each morning and went up over the rosy hills, helping the sheep find food to eat. Each day he went in a new direction, for the pasture was desert land and the plants grew slowly. At noon he sat in the shade of a huge rock to eat the lunch his mother had prepared for him. Most of the sheep gathered in the shade of the rocks to rest during the hottest part of the day.



Eagle Feather took the sheep to the pasture.

When evening came, Eagle Feather walked slowly home behind the sheep. His mother looked up and saw them outlined against the setting sun. She hurried to set the pots on the stove to start supper for her tired little boy.



She prepared supper for her tired little boy.

When the sheep were in the fold and the logs had closed the curve, Mother asked, "Eagle Feather, are you too tired to water the corn before supper time? It has been so hot today that the leaves are withered. If we want any corn to eat this winter, the plants must have water."



Mother asked Eagle Feather to water the corn.

Eagle Feather trudged slowly down to the little stream that trickled between the parched hills. His father had dammed up the tiny stream and there was a little pool of water there. Eagle Feather took the bucket he had brought along and began pouring water on each stalk of corn.

The tiny field was enclosed by a low fence with a gate. As Eagle Feather walked up and down the rows he suddenly noticed the fence. It <u>too</u> was a curve! A closed curve usually, but an open curve now with the gate open



Eagle Feather saw another closed curve.

His feet went faster, even uphill, as he hurried toward the hogan. "Mother," he called. "I've found another closed curve. It's the fence around the patch of corn! I opened the curve when I went in to work, but I closed it again when I left."

The sun was slipping into the west as Eagle Feather sat cross-legged on the floor of the porch, eating his supper of beans and lamb.

The cool of the evening descended. Eagle Feather yawned contendedly as he stretched out on his sneepskin blanket. As he looked up at the sliver of a moon and the faintly twinkling stars he thought to himself that life was very good indeed.



Eagle Feather lay down to rest.

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One group sings the chant as the other children dance in a circle with joined hands. An appointed leader can break the circle to snake around until the end of the chant when the leader catches the "tail" and closes the curve again.

Suggested Activities

The objectives are as follows: to provide opportunities for construction of *<u>open</u> and *<u>closed</u> curves; to provide opportunities to determine if a given curve is closed or open; to reinforce the concepts of <u>boundary</u>, *<u>inside</u>, and *<u>outside</u>.

- Give each child a piece of yarn about one yard long. Help each one arrange his piece of yarn into either a closed or open curve. Let the group decide whether each curve is closed or open. If the curve is open, give the child who constructed it an opportunity to shift the yarn to make it a closed curve.
- 2. Use a long piece of rope or crepe paper ribbon to demonstrate open and closed curves on the floor. Have one child stand at one point of a simple open curve. A second child should stand at another point on the curve. Let a third child stand at a point of the curve which is between the other two. For variation the children could place blocks on the curve.
- *3. Point out that four geometric shapes used in "David Plays a Trick" are called closed curves. Draw them on the board or show examples of these shapes and label them "closed curves". Draw several other examples of closed curves and label them accordingly. Have children identify the boundaries of the closed curves.



- *4. Discuss the fact that the yarn is the boundary separating the inside from the outside of a simple closed curve.
- *5. Investigate whether or not an open curve has an inside and an outside. Establish that a curve must be closed to have an inside and an outside.
- 6. Give each child a length of yarn or string (about one yard long), and three different objects: a block, a counter and a crayon would be satisfactory. Have the children form a circle with the string. Tell them to pretend it is a pond. Ask them to put the block inside the pond; put the counter outside the pond; put the crayon on the boundary of the sond.

This activity could be varied by having the yarn, or string, form a corral, a play yard, a skating rink, etc.

- Note: If all the children have similar objects it's easier to tell at a glance if they have been correctly placed.
- 7. Game In and Out the Window

Children form a circle by joining hands. One child is chosen to be "it". Children forming the circle try to aid the child being chased by raising and lowering the "windows" (their arms). "It" may try to duck under. If he catches the runner, the runner becomes "it" and the former "it" chooses a child to run.

8. Game - Inside or Outside

This game is useful in demonstrating when an object or a person is inside or outside a closed curve. Use a ribbon of crepe paper about one or two inches wide and twenty to thirty feet long. Paste or staple the two ends of the paper together to form a closed circle. A long clothes line stapled together at the ends is also suitable. 11

Arrange the ribbon of paper on the floor in a variety of closed curves. Begin with simple ones such as oblongs, circles, ovals, and squares.



Direct one child to stand inside the curve. Have another stand outside. Help the children discover that there is a limited space in the region of a closed curve and an unlimited space outside the curve.

Next, make the curves more complex. Choose two children to go to each curve. Direct one to stand inside and one outside the curve.



Place various objects inside and outside the curve. Have children decide whether each object is inside or outside the curve.

*9. Figures like those below make it more difficult to decide whether the object is inside or outside the closed curve. However, if a child can pick up the object and find a path out without stepping over the paper, the object is outside the curve.



10. Add to the bulletin board display an example of a simple closed curve with the inside and outside clearly labeled.

Worksheet Commentary

Worksheet *3 Recognition of Open and Closed Curves Worksheets 4, *5 Inside, Outside

Pass out worksheets. Read the directions aloud. Direct class activity so that the children understand the assignment and can complete the worksheets as independently as possible.





Put a red line under each closed curve.

Take a green crayon and change each open curve into a closed curve.



Inside, Outside



Draw a circle inside the curve. Draw a picture outside of the curve.

Worksheet *5 Inside, Outside



Is the flower inside or outside the closed curve? Draw a line to see if it can be carried to the flower pot without crossing any part of the curve.

SIMPLE AND NON-SIMPLE CURVES

Background for the Teacher

Examples of simple curves

A curve that does not cross (or retrace) itself is a simple curve.

Here the word curve is used to include straight lines and line segments. We ask that this extended use of the word curve be done casually. The issue is not dealt with directly until a later unit.

Examples of non-simple curves

A curve that crosses (or retraces) itself is a non-simple curve.

Suggested Activities

*1. Draw several simple curves on the board, both open and closed. Have the children participate as you discuss whether each of the curves is open or closed. When all of the curves are on the board, then explain that they are called simple curves.



2. Draw a simple curve which is intricate. For example:



Ask if this is an open or closed curve. Explain that mathematicians call this a simple curve, too.

 Put the words "simple curve" on the board with your examples. Add several examples of simple curves (labeled) to the bulletin board of geometric figures.
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- AD KAY
- 4. Draw several <u>hon-simple</u> curves.

Explain that mathematicians call these non-simple curves. This means that these curves are not simple because they cross themselves.

- 5. Use a ribbon of crepe paper about one or two inches wide and twenty to thirty feet long. Arrange this ribbon of paper in a variety of non-simple curves on the floor. Let the children experiment with the ribbon, identifying each curve as either simple or non-simple.
- 6. Have several children go to the board and draw simple curves. Then have another group go to the board and draw curves that are not simple.

7. Give each child a large sheet of paper. Ask the children to make one simple curve on their sheet. Check each child's paper. Have those children with correctly drawn curves show their paper. Note those who need further help.

After all papers have been checked and the correct sheets displayed, have the class turn their sheets over and fill the other side with as many different simple curves as they can think of. During this time, group together at the blackboard or flannelboard those children who need additional experience with simple curves. Select several papers for display on the bulletin board.

8. Follow the same procedure as in the activity above, substituting non-simple curves for simple curves.

Worksheet Commentary

*Worksheet 6 Simple, Non-simple Curves

Have the children make simple curves on one side of the line and non-simple curves on the other. Demonstrate if necessary. Worksheet 6

Simple, Non-simple Curves