MINNE

Math
And
Science
Teaching project

UNIT IX
GEOMETRY

MINNESOTA SCHOOL MATHEMATICS AND SCIENCE CENTER 1966

MATHEMATICS

FOR THE

ELEMENTARY SCHOOL

UNIT IX

Geometry

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Contents

TRIANGLE, ANGLE, RECTANGLE

*Starring indicates content which is particularly important to the sequential development or evaluation of the program. We ask that all participating teachers try this starred material. It is expected that much of the remaining material will also be used; how much will depend on individual class needs and time available.

GEOMETRY IN THE EARLY GRADES

"One of the most remarkable features of modern mathematics is the discovery of great unifying ideas, by means of which several apparently unrelated fields are seen as special cases of a single general deductive science. Perhaps the simplest everyday example of this is the scale on a ruler. This scale sets up a one-to-one correspondence between the points on a line and the real numbers, in such a way that both the algebraic relations between numbers and the geometric relations between points on a line become alternative interpretations of the same abstract science."*

The teaching of geometry in the early grades is justifiable on the basis of its utility value for children in these grades, and the nature of the study is such that it is usually very interesting to them. They are usually fascinated by the language used and the shapes of figures encountered.

A basic intent in this series is to teach arithmetic geometrically and to acquaint children with algebraic language as early as possible. The children become familiar with a coordinate system (a lattice of points) as early as possible (about third grade), and learn the meaning of a line such as the one whose equation is $y = x$. When this time comes and a new operation is introduced, the children will be taught to use the graph of this operation and a geometrical algorithm (rule of procedure) for computing the result. Thus, the children are given a foolproof mechanism for performing <u>any arithmetical computation whatsoever</u>. They have a calculating device with the machinery completely open to their inspection and can work in full confidence that they can always obtain the right answer if they take the time. Consequently, any attempt to emphasize speed and memorization is postponed until the children have learned to work with this mechanism at their own pace.

Furthermore, understanding of the real number system and its subsystems is largely dependent on their representation on the number line. The operations on the number systems (addition, subtraction, multiplication, division) gain meaning when they are developed from this intuitive geometric picture.

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*Paul C. Rosenbloom, Teachers College, Columbia University, Mathematics Director, Minnesota School Mathematics and Science Teaching Project.

INTRODUCTION

The following unit is produced on the predication that the most logical and feasible approach to number and operation is through geometric structure. It is further predicated on results of the many recent tests that have indicated that children can understand these concepts much earlier than was once assumed.

This unit contains a review and to some extent an extension of geometric concepts introduced in Unit I. The review includes simple curve, non-simple curve, simple closed curve, non-simple closed curve, outside, inside, boundary, oblong, square, circle, disk, triangle, and region.

It will also include a review and extension of geometric ideas presented in Unit VII, "Introduction to the Number Line." The material reviewed concerns point, line, line segment, and ray.

In addition there will be an extension of understanding previously developed and an introduction of new material such as broken line, straight angle, right angle, acute angle, obtuse angle, rectangle, common boundaries, and intersecting regions.

Since children will come to this unit with varying backgrounds and degrees of understanding, the teacher should move slowly or rapidly through the review material as the situation requires, omitting any unnecessary reinforcement.

Children should have rulers and pencils for this geometry unit.

Teacher's Background on Points, Curves, and Regions

Point

Point is a mathematically undefined term. A point is represented by a dot, but however small we make the dot it will still be larger than a point. We work with the concept of a point according to our intuitive understanding of it.

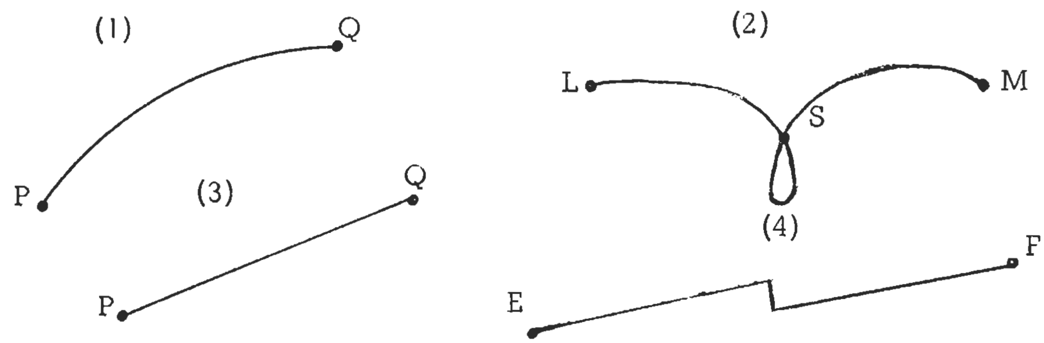
Plane

Plane is a mathematically undefined term. Our notion of plane is suggested by a table-top or a chalkboard, except that these are finite in extent. A plane, by contrast, extends in all of its directions without bound.

A plane is a set whose elements are points. All of the geometric notions developed in this unit concern subsets of a plane. We imagine every point under discussion to lie in this plane, fixed at the outset, and represented by our paper, the chalkboard, etc. The plane is our "universe of discourse", that is, we will not be discussing any points not lying in the plane. The term "universe of discourse" will not be mentioned to the children, but the teacher should interpret all that is said with this understanding.

Curve

We make no formal attempt here to define curve. Given two points, P and Q, a curve from P to Q is represented below by pencil-traces which start at P and end at Q, or vice versa. Each trace is made without lifting the pencil from the paper. If we think of the trace as a string of dots, each dot representing a point, the curve is the set of these points. The points P and Q are members of the curve. A point which is a member of a curve is said to belong to the curve, or to be on the curve.



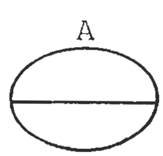
Simple Curve

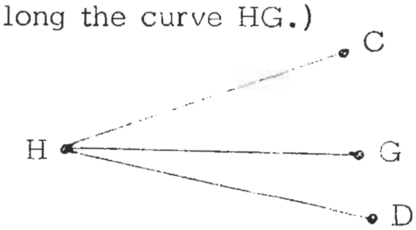
The curves, (1), (3), and (4) shown above are called simple. The pencil has not retouched its own path. Thus there are no loops. Curve (2) is not simple.

It should be noted that we are not attempting to describe the usual meaning of the word "curve". Illustrations (3) and (4) would not be considered curves according to conventional non-mathematical usage.

Endpoint

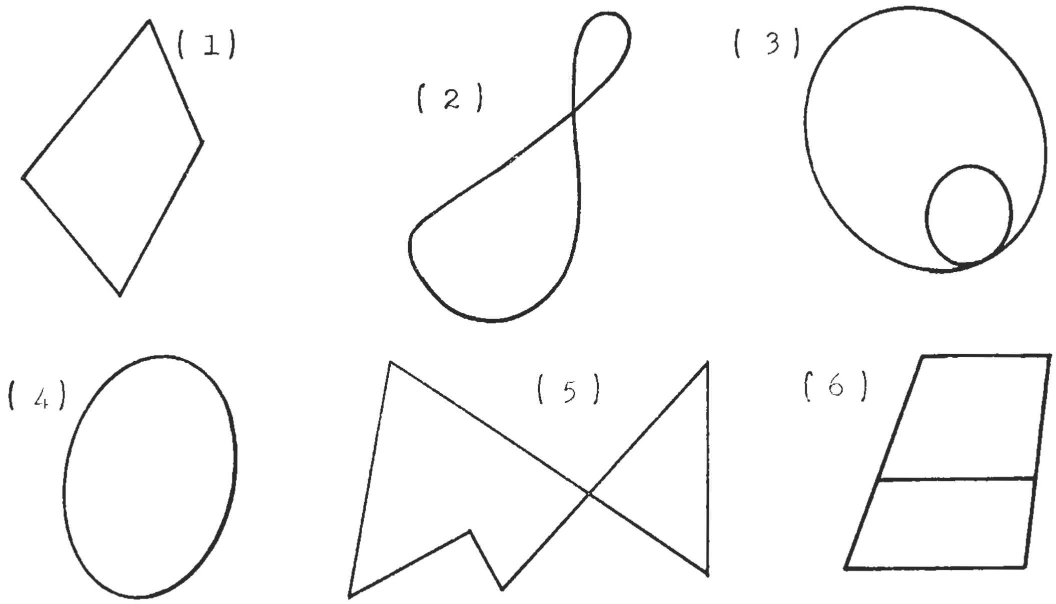
Each of the four curves shown above has two endpoints. For instance, curve number (2) has endpoints L and M. The curve A below has no endpoints, while that from C to D has three endpoints: C, G, and D. (To trace the latter from C to D, one must retrace along the curve HG.)





Closed Curve

A closed curve has no endpoints. The sketches below are closed curves.



Simple Closed Curve

Examples (1) and (4) are called simple closed curves. They can be traced so that the pencil does not retouch its own path except in returning to its starting point, unless the curve is completely retraced.

Closed Curves Which Are Not Simple

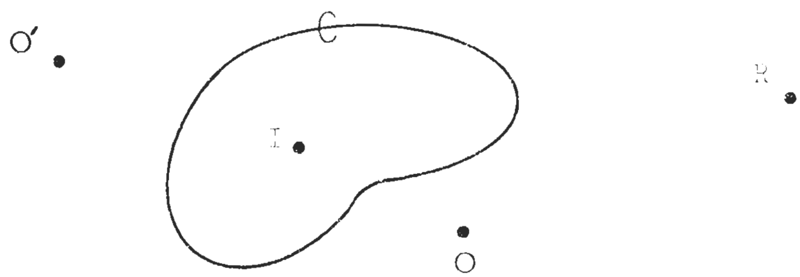
Examples (2), (3), (5), and (6) are not simple closed curves. They cannot be traced without the pencil retouching its path.

Outside, Inside

If C is a simple closed curve in the plane, the points of the plane not on C fall into two classes:

i) Those which can be joined to a remote point by a curve which does not cross C. These points are said to be <u>outside</u> C. The set of all points outside C is called the <u>outside</u> of C. Every subset of the outside of C is said to be <u>outside</u> C.

ii) Those which cannot be so joined. These points are said to be <u>inside</u> C. The set of all points inside C is called the <u>inside</u> of C. Every subset of the inside of C is said to be <u>inside</u> C.



are outside points; and I is an inside point. (We are content to leave the word <u>remote</u> vague here. If in doubt as to whether some point is remote from a given curve, take a point much farther away instead.)

Region

The inside (or the outside) of a simple closed curve is an example of a plane region. It is important to note that the closed curve is the boundary of the region and is not a part of the region. The notion of region will be developed in activities for the children, who will learn to use the word "region" by example. The teacher may deal more confidently with questions if she knows that regions are precisely characterized by the following two properties:

a) Any two points of a region R can be joined by a curve lying entirely in R.

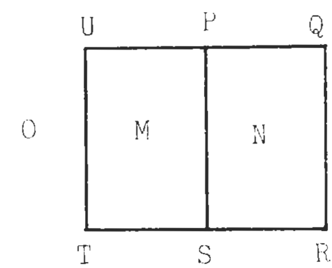
b) If P is a point of a region R, then all points of the plane close enough to P also lie in R. In other words, if P is any point in a region R, then it is always possible to draw a circle around P such that all points inside the circle are also in R.

These properties are illustrated and clarified in Worksheet 6.

Separate; Boundary

A simple closed curve is said to separate the plane into the two regions, inside and outside. The curve itself is the boundary of its inside. It is also the boundary of its outside. It is thus the common boundary of these two regions.

The sketch represents a closed curve which is not simple. It separates the plane into three regions, M, N, and O and also into combinations of these regions, for example M - N, and O. The boundary of M is the simple closed curve S P U T. The common boundary of M and N is the curve P S. The common boundary of M and O is the non-closed curve P U T S.



In the following activities the children will study and discover some characteristic properties of points and curves. When presenting this material, it is essential that care be exercised in assisting the children to understand what points and curves are not. We do not say that points are locations in space, since then it would be necessary to define "space" and "locations" and this would be more difficult than defining points. Rather than try to define these terms, we will start with the children's intuitive picture of points and curves and refine that.

A dot is a picture of a point. We use pictures of points, then, to illustrate and discover properties. We must be careful not to be misled by pictures. Among the obvious potential errors in a picture of a point or a curve are the two-dimensional aspects of dots and figures. (Do not discuss dimension in connection with points and curves, and in particular do not say that space is three-dimensional. This restricts the child's concept of space and interferes with his study of curves.)

To help avoid some misrepresentations, we will talk about points and curves even though we draw dots and figures.

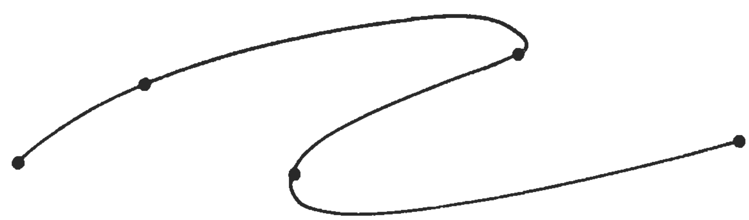
The first properties that we will study are some relations between points and curves. Recall that curves are certain sets of points. Any two points on a curve are connected or joined by the curve.

(We will sometimes identify or mark certain points on a curve, but do not discuss counting of points on curves. It is not sufficient to say that there are hundreds of points on a curve, for actually there are infinitely many.)

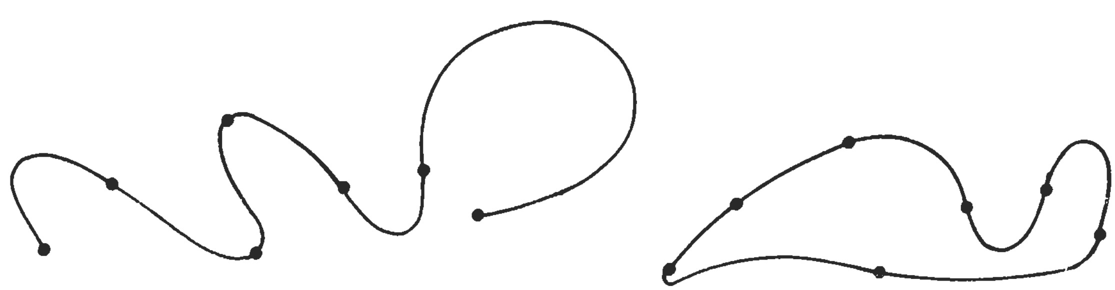
Activities on Points and Curves

1. Illustrate how a curve connects points by making several dots on the chalkboard.

2. Then sketch a curve which connects them, for example:



3. Have a child locate other points on the chalkboard. Ask someone else to connect these points in a way that is different from the way the teacher connected the other points. Any method of connection is acceptable; for example:



The word "connect" should be written on the board and used frequently during these activities so that the children should have no difficulty recognizing it on the worksheets.

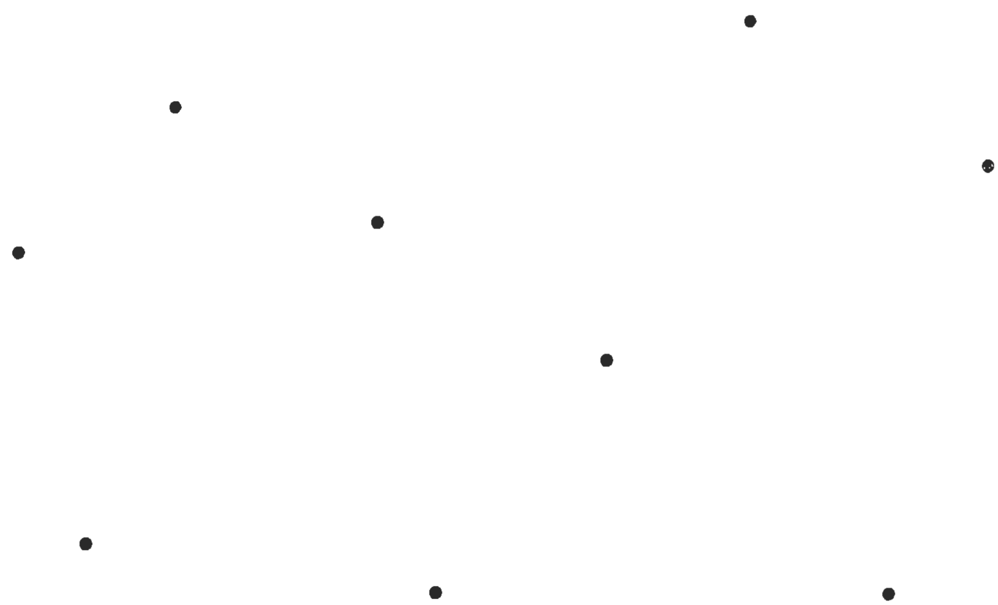
4. Distribute Worksheet 1.

After the worksheet is completed, provide opportunity for the children to see all the different responses made by their classmates.

WORKSHEET 1

Connect the points in any order you wish. Use your imagination.

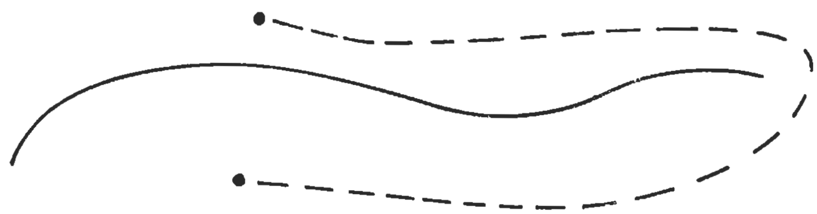
1.



2.

Activities on Curves and Regions

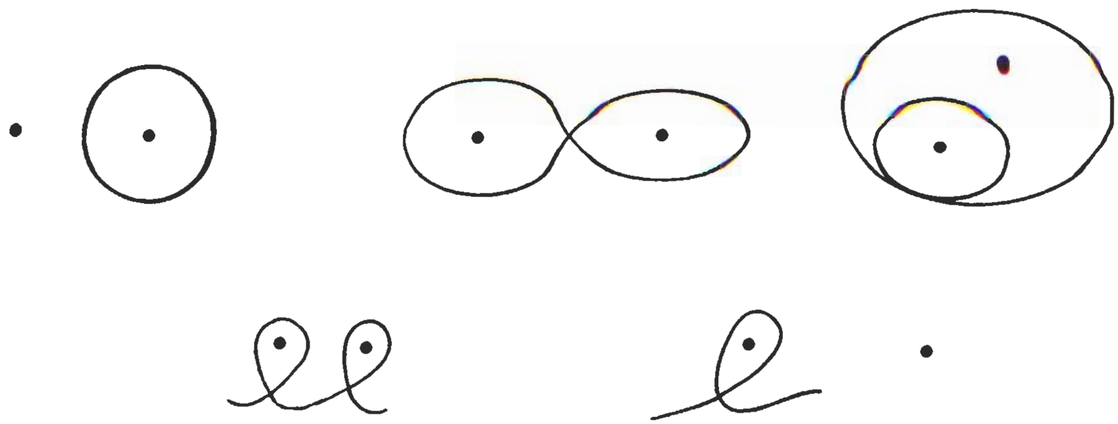
1. Draw a curve on the board and then draw two dots off the curve to represent two points. Ask if anyone in the class can draw a curve connecting these points without crossing the given curve.



If someone can connect these two points without crossing the curve then he wins the game, but if he cannot connect the two points without crossing the curve then the first player, in this case the teacher, wins the game.

Work out a few examples with the children, using closed and non-closed, simple and non-simple curves at the board. As the need arises, the terminology can be introduced. Alternate the role of the first player with the children so they will be able to work at the game independently. After you feel the children have been exposed to an adequate number of examples, group the children into pairs to work at their seats. The children in each pair should take turns as first and second players. This will give the children experience with curves and a number of associated ideas.

2. After the game has been played for a while ask some children to come to the board and show how they were able to win. Ask them if the first player, the one who draws the curve and the points, could always win, and if he could, how? (The first player can always win if he draws a closed curve with one of the dots inside and one outside. This is but one possibility; see if the children can discover some of the other ways also. Example: If the first player draws a non-simple curve and places the points in different regions. The teacher will find here further occasion to introduce or to reinforce the terminology.)



3. To change the game for those who like a bit more challenge or for those who have discovered the way to win, change the directions in the following manner. The first player draws the curve and one dot to represent the point that he has chosen. The second player draws the second dot to represent the second point. Then it is the job of the first player to connect the dots without crossing the curve. If he does this he wins the game, and if he cannot draw a connecting curve without crossing the given curve, the second player is the winner. After playing the game for a while the first player will see that he can win every time simply by drawing a simple curve that is not closed. This information should probably result in the formulation of a ground rule barring this particular strategy.

4. Give the children paper and either pencils or crayons. Direct them as follows:

a) Pick any four points not too close together. Make dots to represent the points, and name them A, B, C, D. Draw curves joining the following pairs of points (rule: the curves are not to intersect except at the endpoints):

A to B

A to C

A to D

B to C

B to D

C to D

b) Into how many non-intersecting regions do these curves separate the plane (paper)? (Four. Remember to count the outside of the entire network.)

c) How many of the lettered points are on the boundary of each region? (Three.)

d) Pick a point E not on the above curves, and join the following by the same rule, until you get stuck:

E to A

E to B

E to C

E to D

e) Is it possible to draw all of these curves within the rule? (No.)

5. Pass out paper and guide the children in the following activity. Each child can proceed at his own pace.

a) Choose five points A, B, C, D, E. Join the points by curves so that the curves do not intersect except at the lettered endpoints:

D to A, D to B, D to C

E to A, E to B, E to C

b) Is it possible? (Yes.)

c) Into how many non-intersecting regions do these curves separate the plane? (Three.)

d) How many of the lettered points are on the boundary of each region? (Four.)

e) Are there any lettered points which are on the boundaries of <u>all</u> the regions? (D, E)

f) Can you chooose a point F not on any of the above curves, and join F to A, F to B, and F to C, obeying the above rule about intersections? (No.)

6. Game: Network Games for Two Players

a) Can You Sketch This?

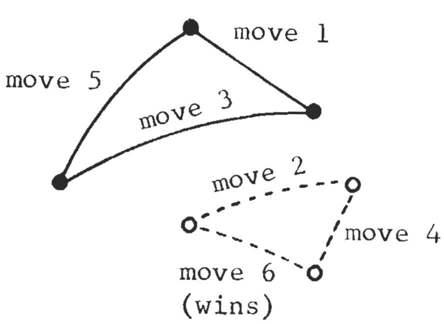
One child describes a network by listing points and pairs to be joined. For instance, he might list points A, B, C, D, E with pairs AB, AC, AD, BC, BD, BE to be joined.

The second child tries to sketch a network answering to the description, so that the joining curves intersect only at the given points. If he succeeds, he wins. If he thinks it impossible, he may challenge the first child to make a sketch. If the first child can do this, he wins.

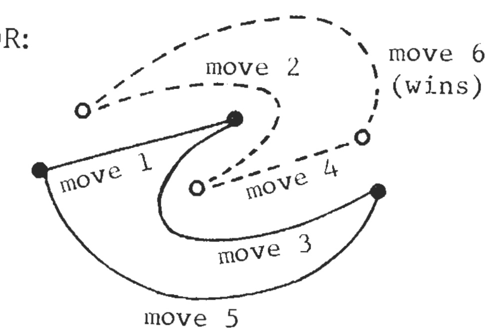
b) The red player puts three red dots ("points") on the paper, the blue player three blue dots. The players then alternate, each drawing a curve joining any two of his own dots that have not been joined to each other before. He may not cross any curve previously drawn. Whoever draws the last curve wins.

The game may be played with more than three dots each, according to the skill and patience of the players.

EXAMPLE



OR:



Berple the Bug

To the Teacher: Cut out and mount the figures of Berple and Claudius on lightweight cardboard or tagboard. See pages 23, 25, and 27. These can then be held in place on the chalkboard by means of a loop of masking tape (sticky side out) placed on the back. The jelly bean bushes should be cut out and mounted separately. It goes without saying that these will be more attractive if colored, but this is optional with the teacher.

If a flannelboard is available it might be more convenient to use it rather than the chalkboard. Pieces of sandpaper or flannel can be glued to the back of the figures to keep them on the flannelboard. Yarn can be used for the fence.

Read aloud as the children follow in their books: Childrens booklet follows page 27.

Berple is a bug. This is Berple. (Page 1, Children's Book)

Berple had a terrible enemy. His name was Claudius Bugeater. Claudius was very fierce. (Page 2, Children's Book)

Berple and fierce Claudius lived in Bugland. Bugland was a very special place where jelly beans grew on bushes and every day was warm and sunny. (Page 3, Children's Book)

Every day, all day long, Claudius tried to catch Berple. He would have loved to have Berple for dinner. (Page 4, Children's Book)

Every day, all day long, Berple ran away and hid from Claudius. Some days he did not even get a chance to eat.

One day Berple became very tired, and very hungry. He decided that something must be done so that he could get some rest, and also get a chance to eat. (Page 5, Children's Book)

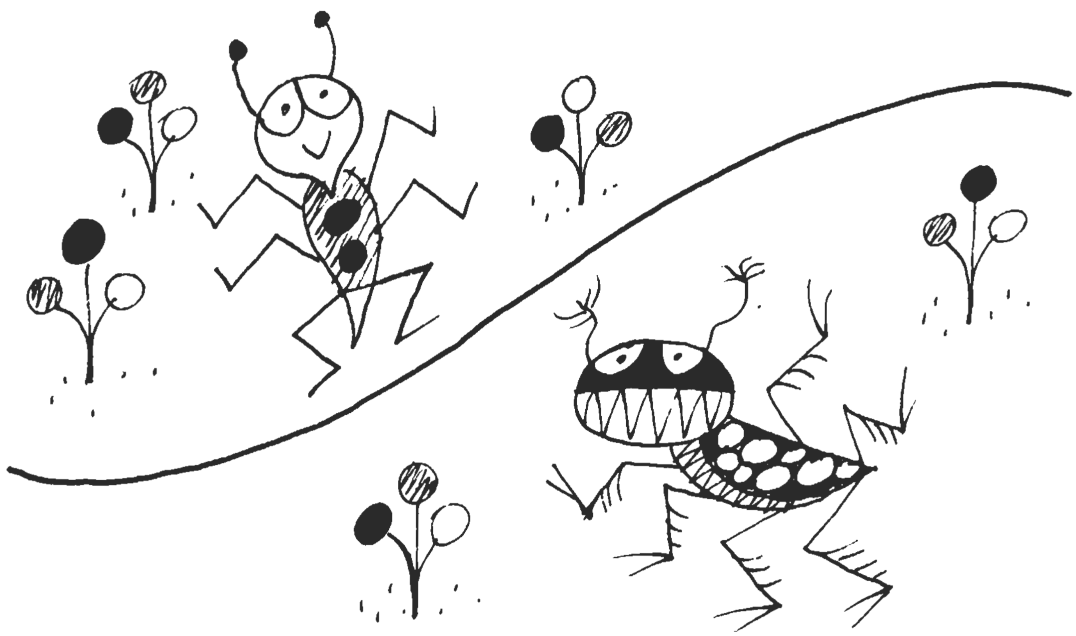
Berple thought and thought. Finally Berple had an idea! A fence! Yes, that would do it! (Page 6, Children's Book)

Berple built a fence. It was so high that Claudius could not climb over it. He built it beside some choice jelly bean bushes. (Page 7, Children's Book)

The fence looked like this. (Page 8, Children's Book)

Sketch in a line (fence) between Berple and Claudius. If using a flannel board, let a length of yarn represent the fence. Ask:

"Could Claudius catch Berple now?"



"How?"

As the class responds that Claudius could go around the fence, let a child move the Claudius figure to demonstrate how he thinks Claudius could reach Berple.

Return Claudius to his original position, and calling the children's attention to page 8 in their books, ask them to draw dotted lines and arrows (- - - - >, - - - - >) to show how they think Claudius might move in order to catch Berple.

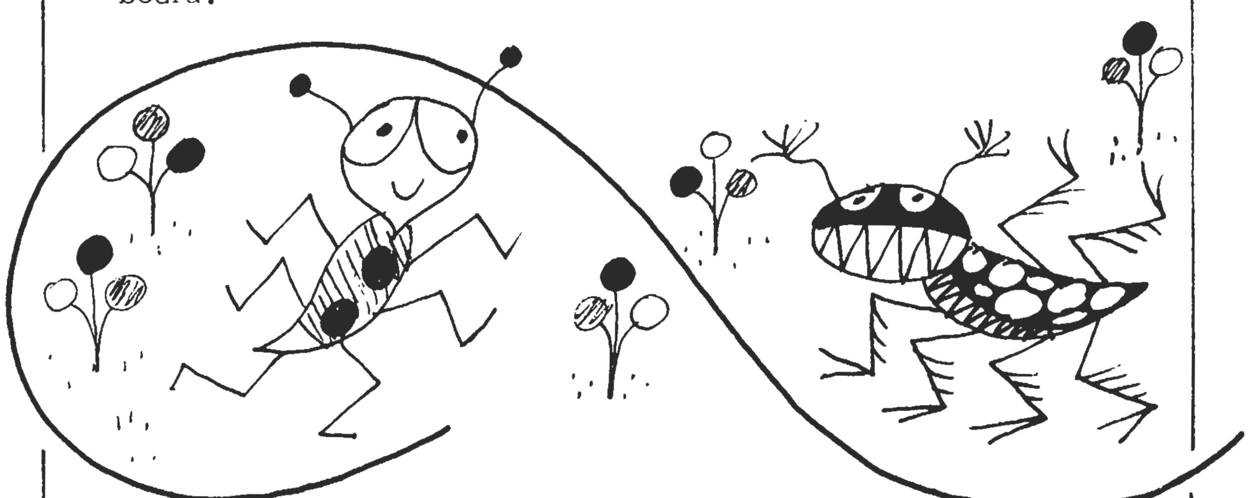
Allow time for the children to draw Claudius' path or paths and check to see that all paths go around the fence; Claudius can not climb over it.

Now extend the fence in both directions, and ask: "Is Berple safe from Claudius, now?" Lead the children to understand that, theoretically at least, we could extend the fence indefinitely in opposite directions and Claudius could move around either end to get to Berple. The children will no doubt suggest building it to a mountain, an ocean, ... This speculation is good, for it shows that they are trying to visualize the situation. However, given the time and the desire, Claudius could get around the end of the fence.

When this matter has been resolved, continue:

Berple made a fence that looked like this. (Page 9, Children's Book)

Demonstrate on chalkboard, or with the yarn on the flannel-board.



Ask: "Is Berple safe from Claudius?

"If not, draw a path (- - - - >, - - - - >) showing how Claudius might get to Berple."

Check the paths, and continue:

Finally, Berple built a fence like this! (Page 10, Children's Book)

Make a similar fence around the Berple figure on the chalkboard (flannelboard).

Ask: "Has Berple solved his problem? Is he safe from Claudius?"

Read:

Berple was very happy for one whole day. He sat down and ate his favorite jelly beans that grew on all the bushes in Bugland. Red ones, and orange ones, and green ones!

As the paragraph above is read, erase (or remove) the jelly beans that are inside the fence with Berple.

That night Berple dreamed of yummy jelly beans. In the morning Berple awoke and reached out sleepily to pick a jelly bean. But - - - -! They were all gone! Berple had eaten every one! Even the white ones! Berple was hungry. He heard the sounds of someone eating. He peeked over the fence and there was Claudius eating a lovely breakfast of jelly beans. Green ones, orange ones, and red ones.

Ask a child to point out all of the jelly beans that Claudius might eat if he cared to.

The fence kept Claudius away from Berple. But - - - - it also kept Berple away from the jelly beans. Berple walked all along the fence, but there was no way for him to get at the jelly beans and yet be safe from Claudius. (Page 11, Children's Book)

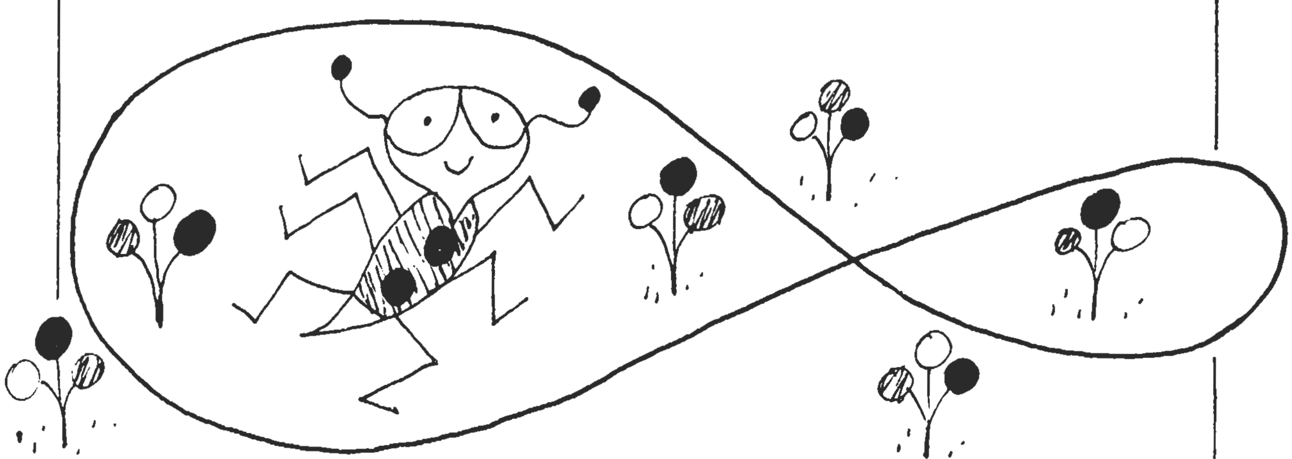
> Move the figure of Berple around the fence or let a child demonstrate.

Berple thought and thought. If he made an opening so that he could get out to get some jelly beans, then Claudius could get in.

> Discuss this possibility with the children. Ask: "Can you think of a different way that Berple could build his fence?"
>
> Let the children demonstrate their various ideas, and discuss their suitability. Then continue:

Berple thought some more, and then as soon as Claudius strolled away he built a fence that looked something like this:

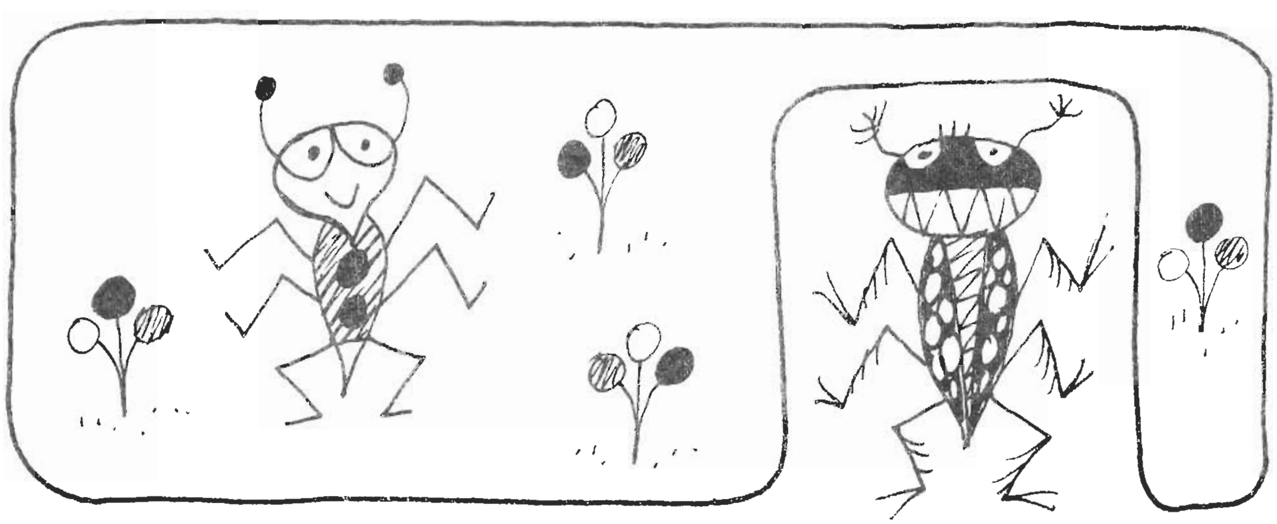
> Demonstrate on the chalkboard (or flannelboard).



Claudius could not get inside the fence. But - - - Berple could see that he would soon be out of jelly beans again. And it wasn't possible for him to get inside the other section of the fence to eat the jelly beans that grew there. Now, what to do?

At last, Berple had another idea. He thought of building a fence like this. A very long fence! (Page 12, Children's Book)

Erase the figure-eight fence and show one which surrounds all the jelly bean bushes and Berple—everything but Claudius.



Ask:

"Will this solve Berple's problem?"

Say:

"Yes, but how could he build it?"

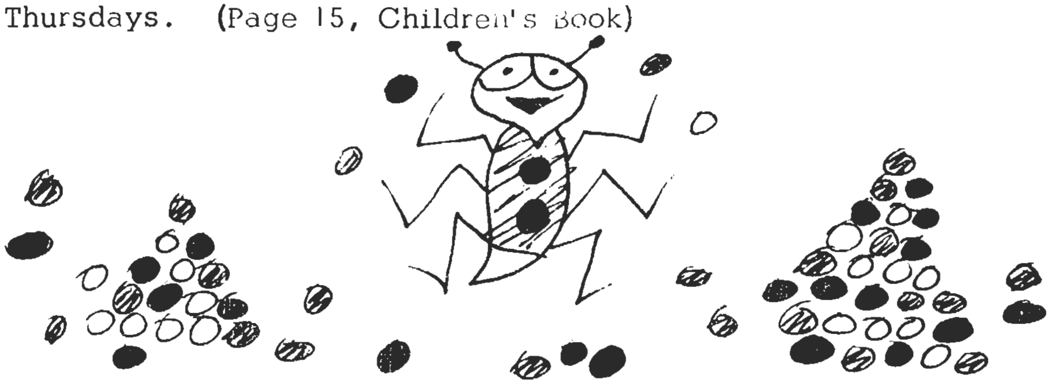
Erase the fence as the story continues.

Berple would have to build the fence while Claudius slept. But this was such a long fence that he would never get it built while Claudius slept. Claudius would surely awake before he finished! Can you help Berple? (Page 14, Children's Book)

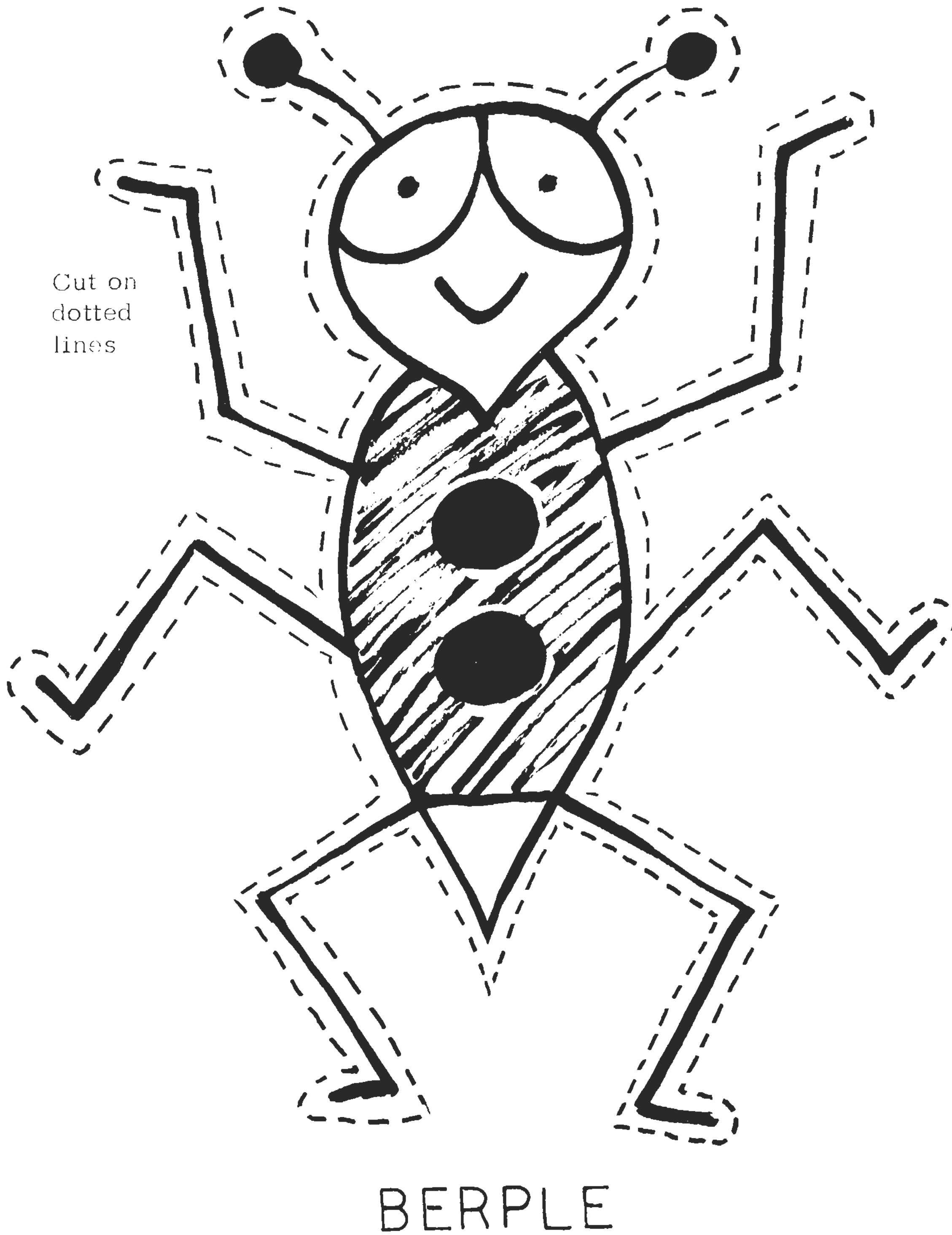
> Ask the children to turn to page 15 in their books and put up a fence for Berple which will keep Claudius away from him, but allow Berple to eat jelly beans from all the bushes in Bugland.
>
> Say:
>
> "When you have finished drawing in the fence, turn your paper over until everyone has finished and then you may each tell about the fence you have drawn."
>
> Let the children explain why they built their fences as they did. No doubt, one or more will have built a fence around Claudius which, of course, allows Berple the freedom of eating from all of the bushes but keeps him safe from Claudius. Let a child place the fence around the figure of Claudius on the chalkboard. Continue to read.

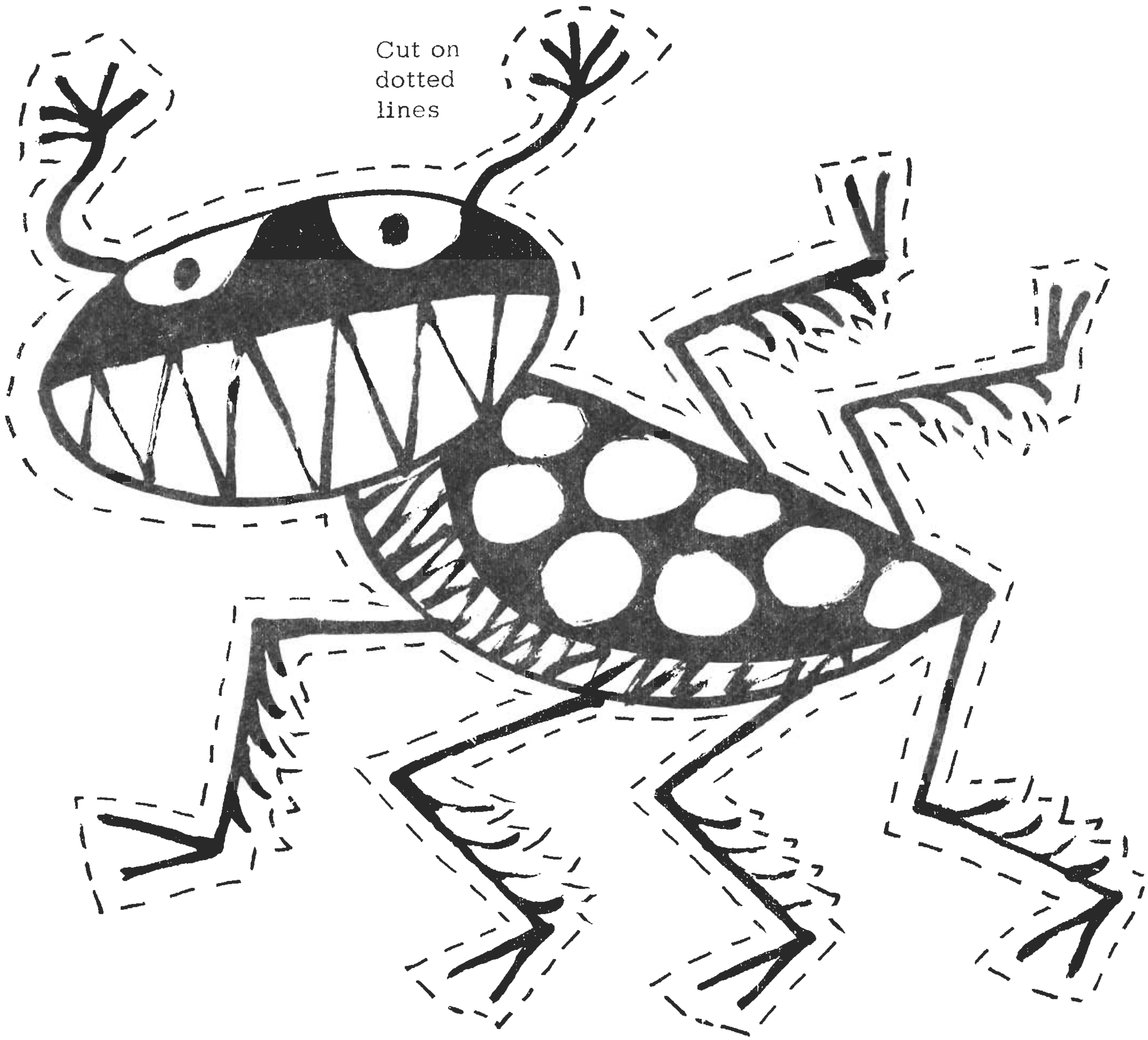
Berple was very happy. He was safe from Claudius! He had all the jelly beans he could ever need or want!

He even threw Claudius two white jelly beans and one black one every night for supper, and he gave him five red ones as a special treat on Thursdays. (Page 15, Children's Book)

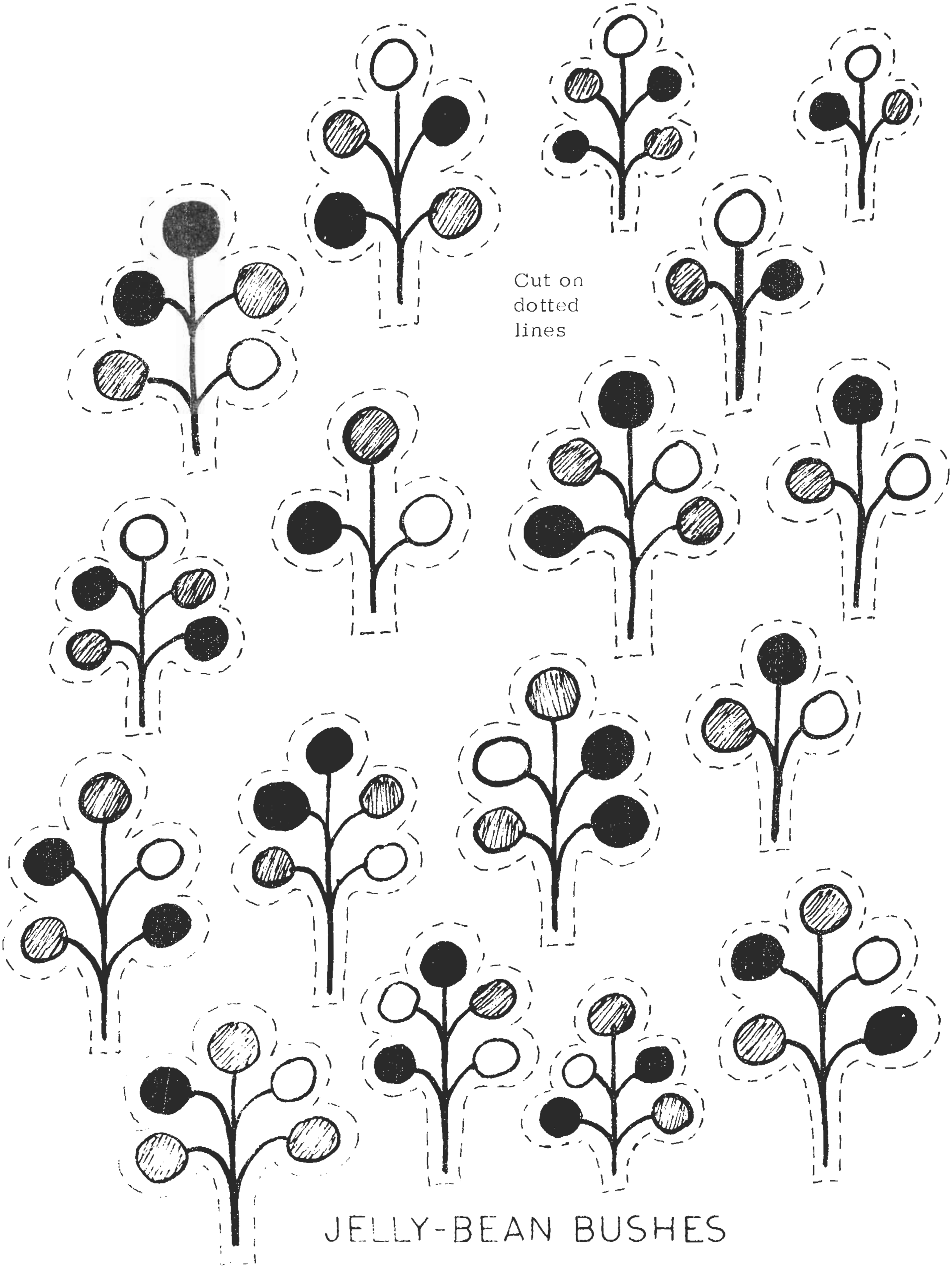


Story: Berple the Bug

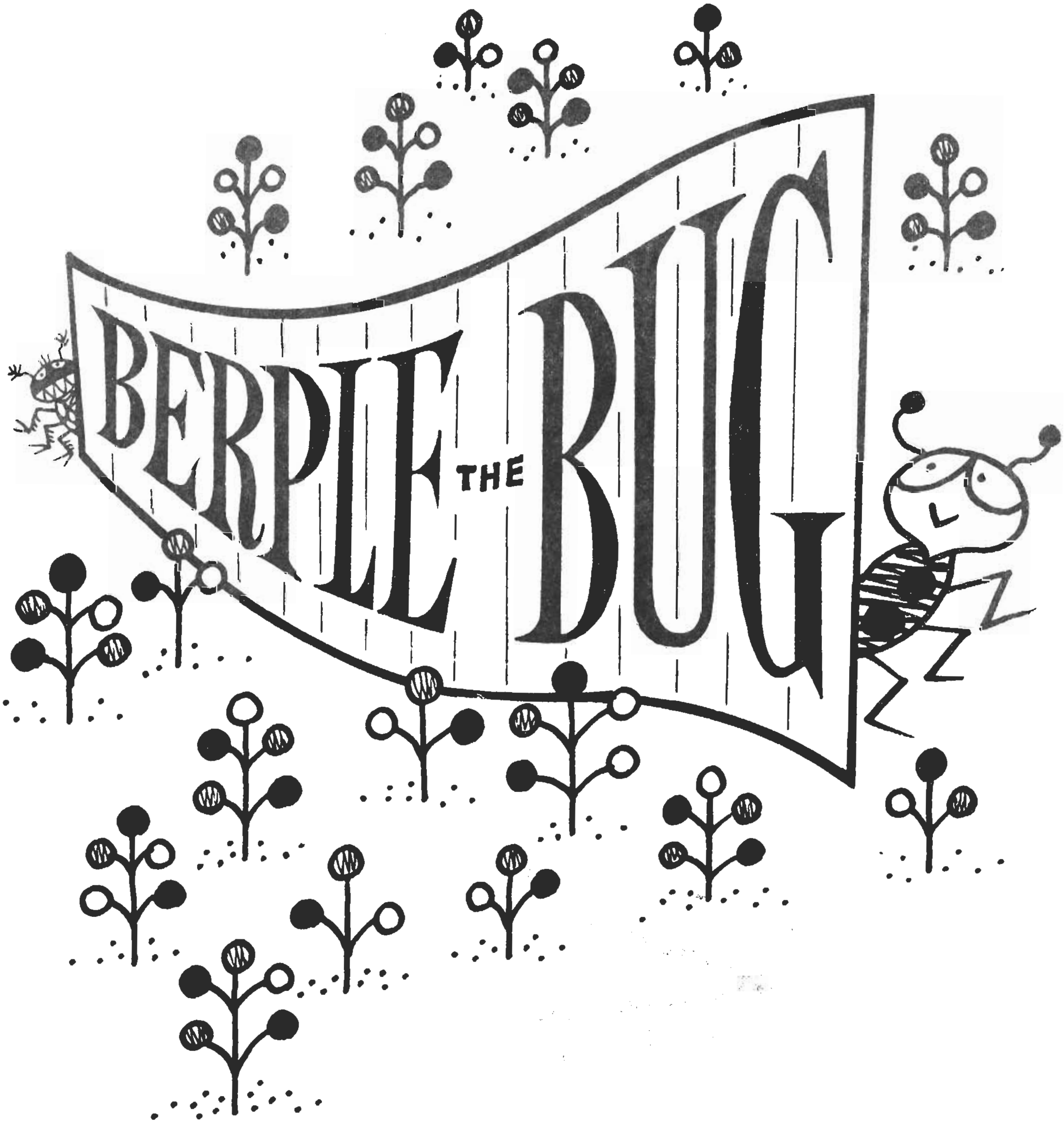


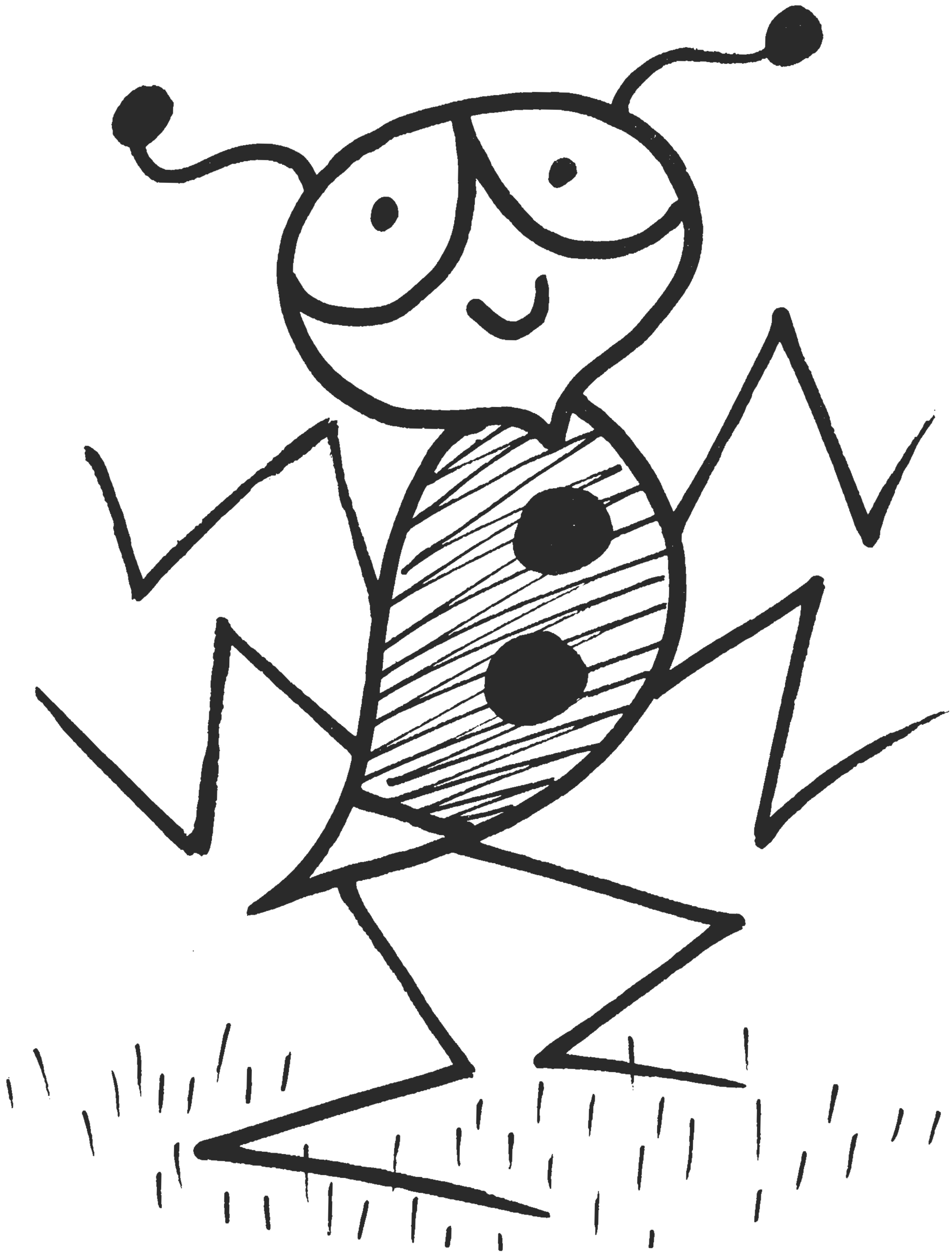


CLAUDIUS

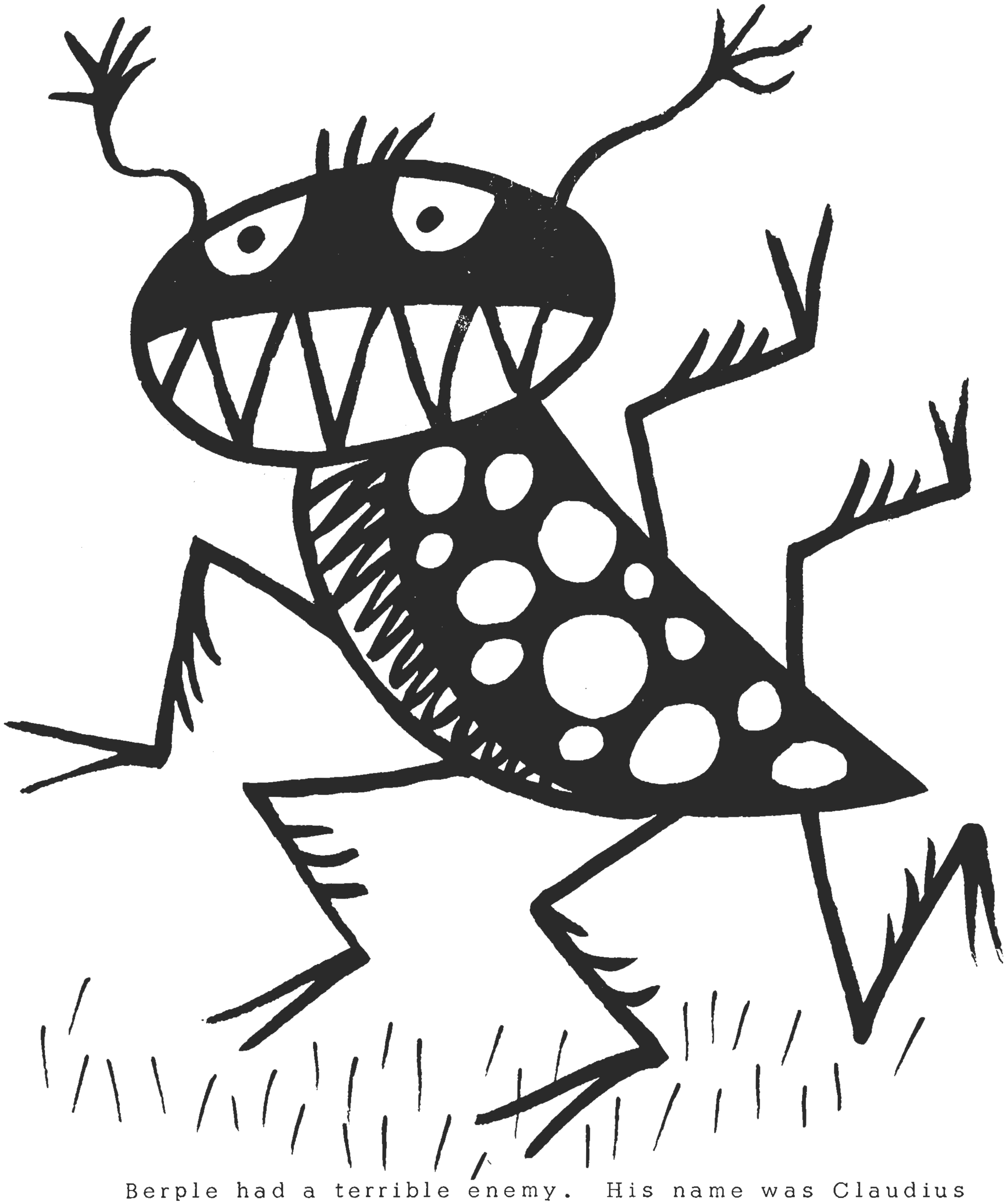


JELLY-BEAN BUSHES

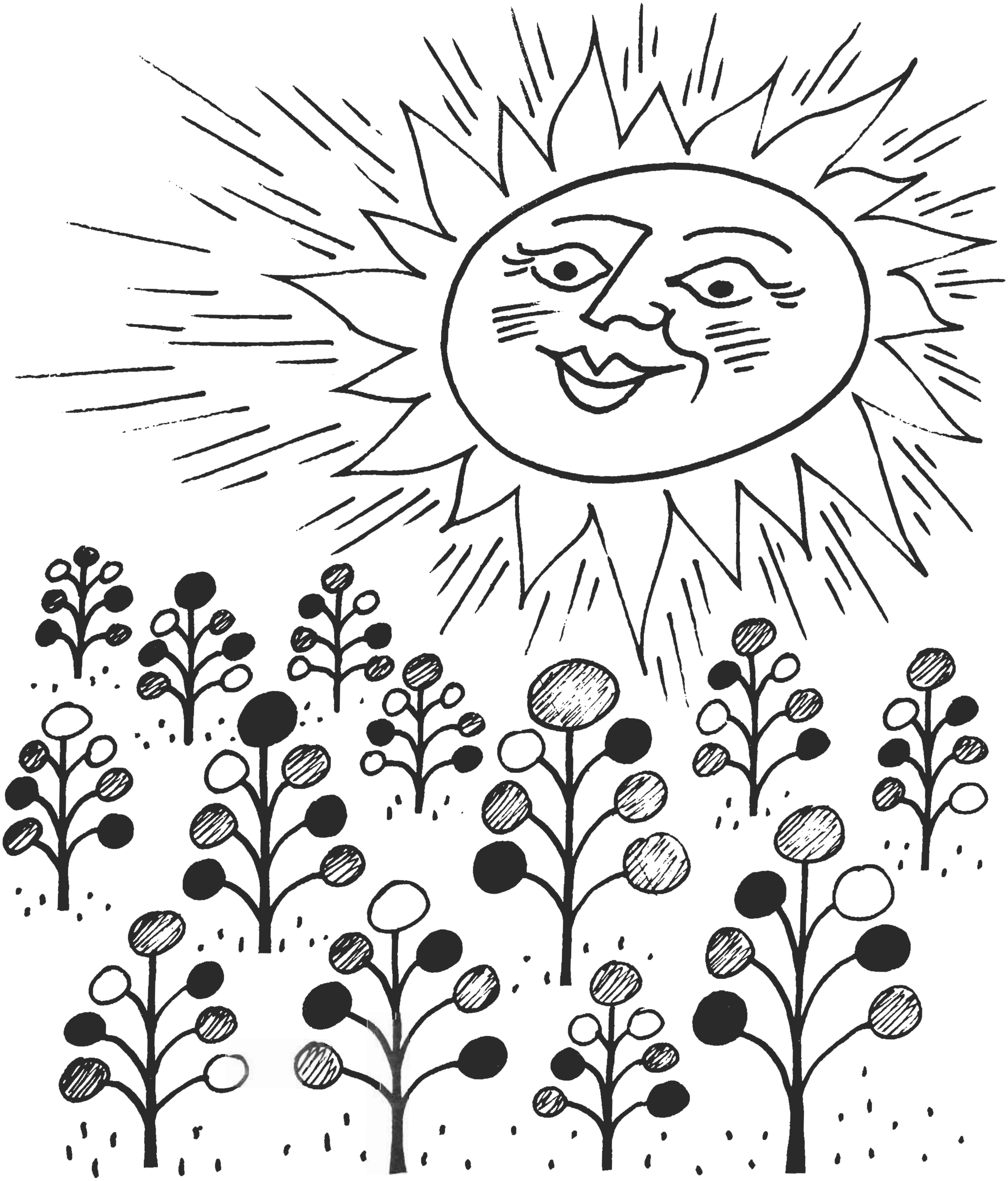

BERPLE THE BUG



Berple was a bug. This is Berple.



Berple had a terrible enemy. His name was Claudius Bugeater. Claudius was very fierce.

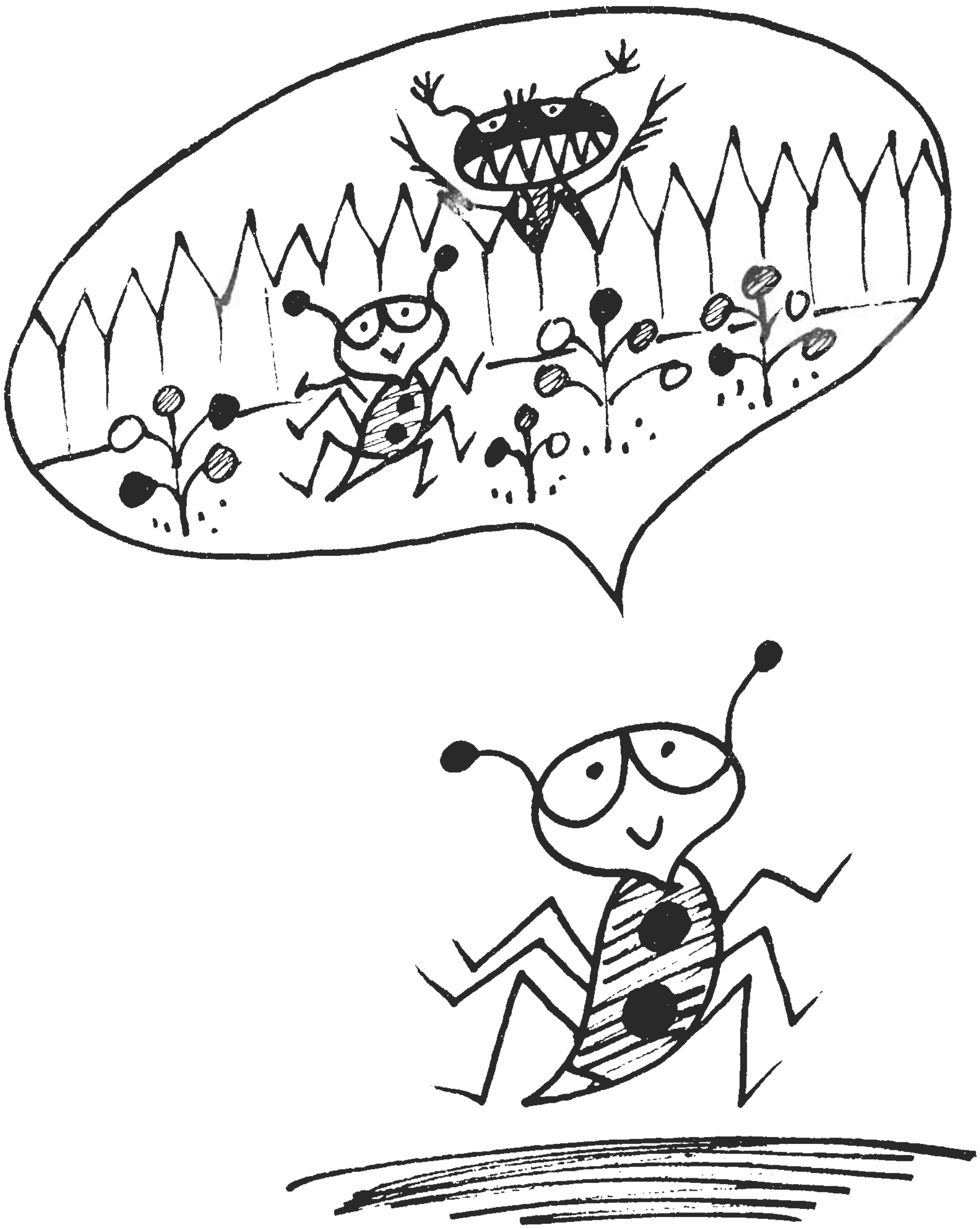


Bugland was a very special place where jelly beans grew on bushes and every day was warm and sunny.

Every day, all day long, Claudius tried to catch Berple. He would have loved to have Berple for dinner.

Every day, all day long, Berple ran away and hid from Claudius. Some days he did not even get a chance to eat.

One day Berple became <u>very</u> tired, and <u>very</u> hungry. He decided that something must be done so that he could could get some rest and also get a chance to eat.

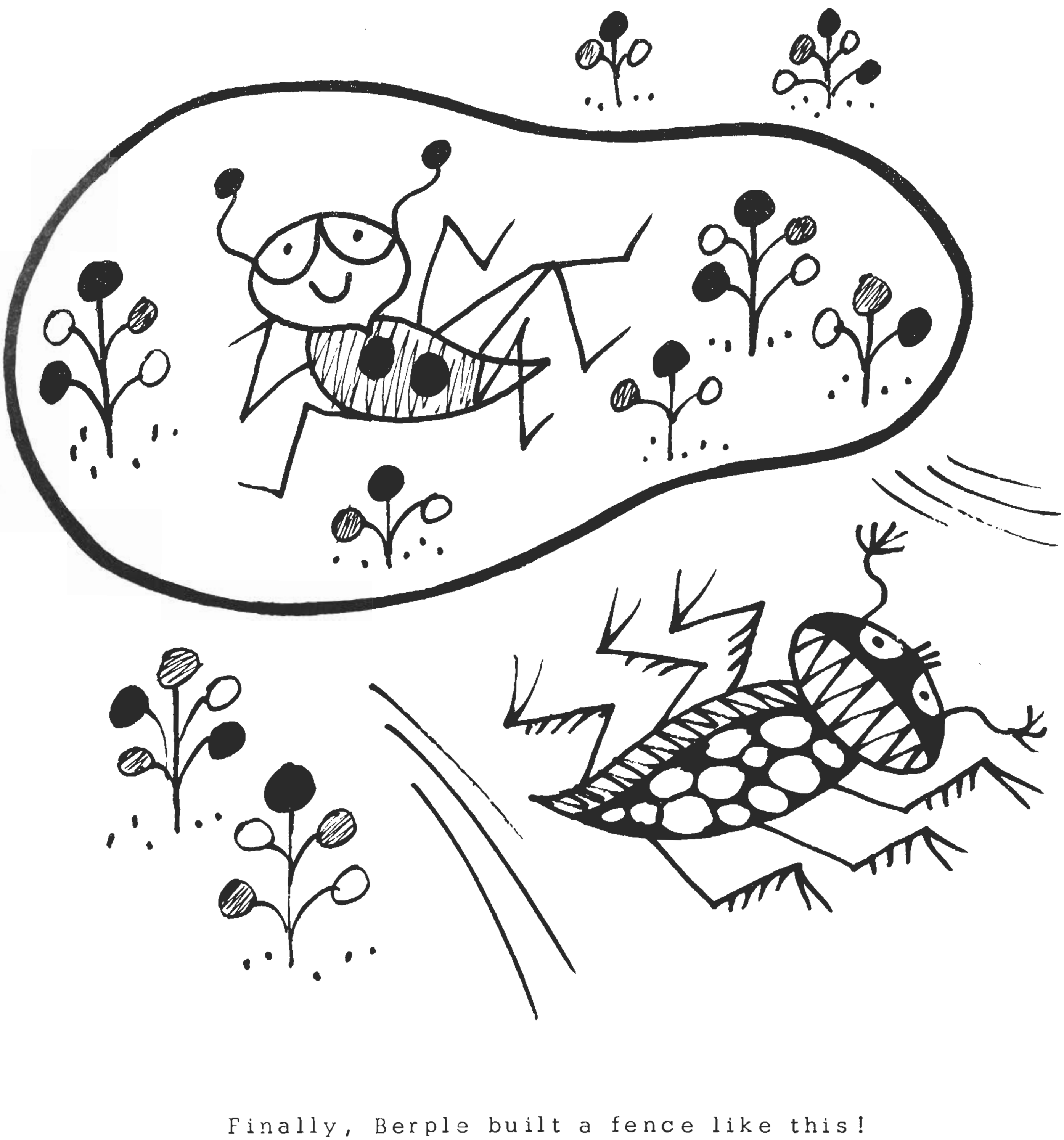


Berple thought and thought. Finally Berple had an idea. A fence! Yes, that would do it!

Berple built a fence. It was so high that Claudius could not climb over it. He built it beside some choice jelly bean bushes.

The fence looked like this.

Then Berple made a fence that looked like this.

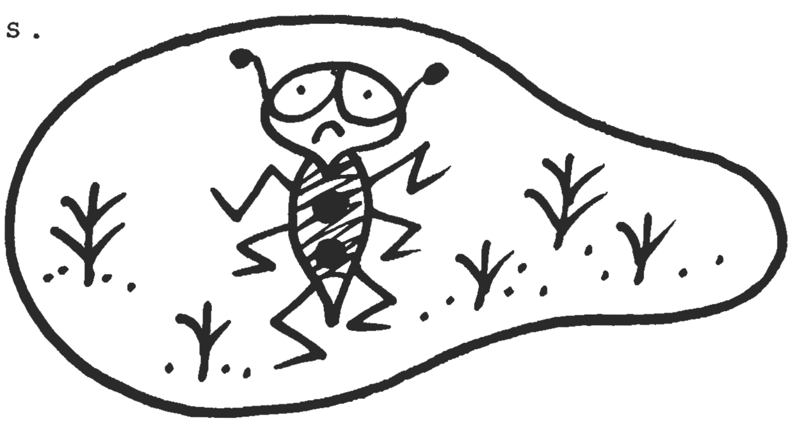


Finally, Berple built a fence like this!

Berple was very happy for one whole day. He sat down and ate his favorite jelly beans that grew on all the bushes in Bugland. Red ones, and orange ones, and green ones!

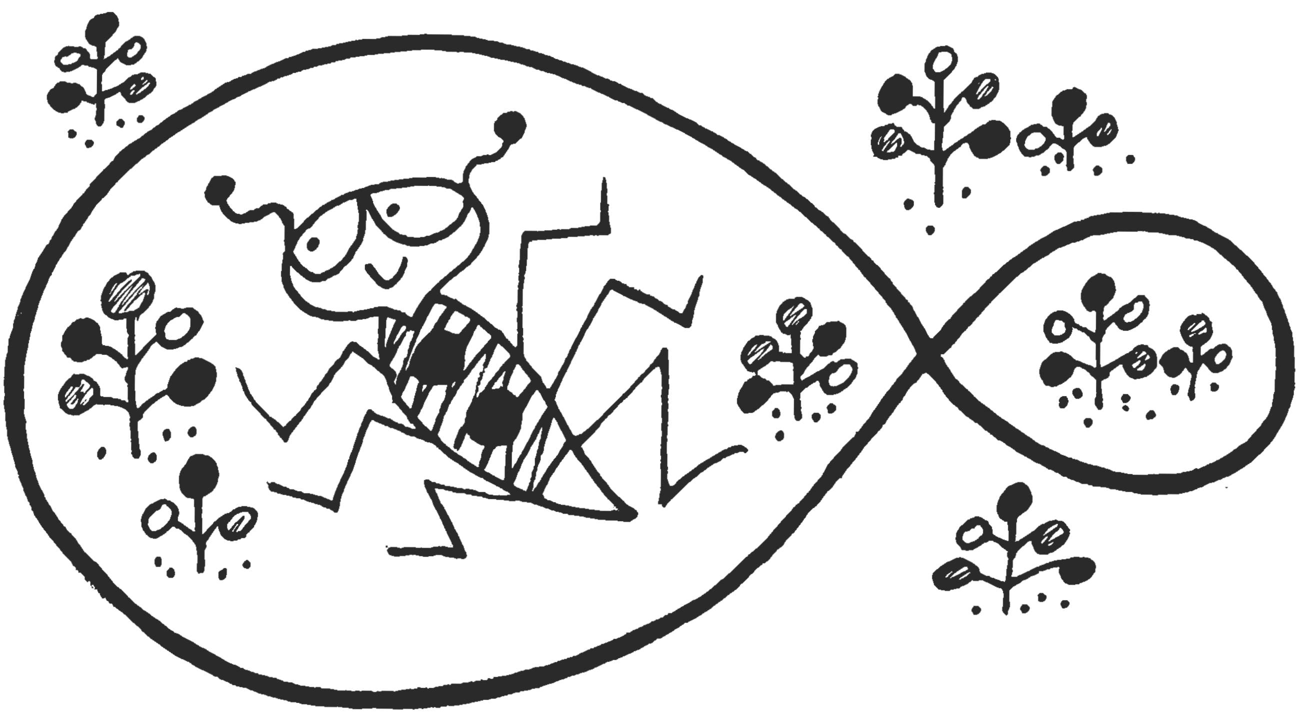
That night Berple dreamed of yummy jelly beans. In the morning Berple awoke and reached out sleepily to pick a jelly bean. But - - - -! They were <u>all</u> gone! Berple had eaten every one! Even the white ones! Berple was hungry. He heard the sounds of somone eating. He peeked over the fence and there was Claudius eating a lovely breakfast of jelly beans. Green ones, orange ones, and red ones.

The fence kept Claudius away from Berple. But - - - - it also kept <u>Berple</u> away from the <u>jelly</u> <u>beans</u>. Berple walked all <u>along</u> the fence, but there was no way for him to get at the jelly beans and yet be safe from Claudius.

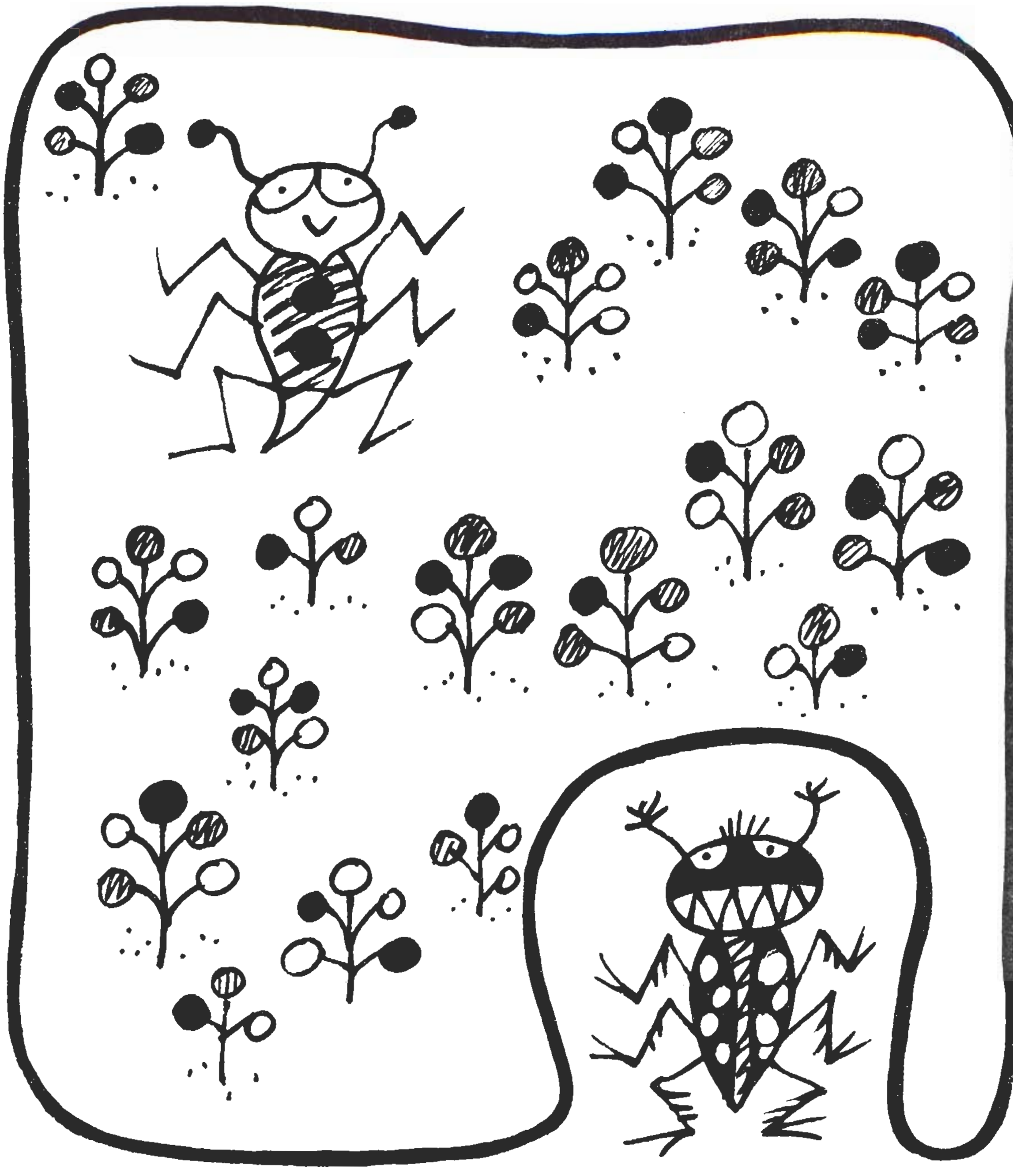


Berple thought and thought. If he made an opening so that he could get out to get some jelly beans, then Claudius could get in.

Berple thought some more and then as soon as Claudius strolled away he built a fence that looked something like this.

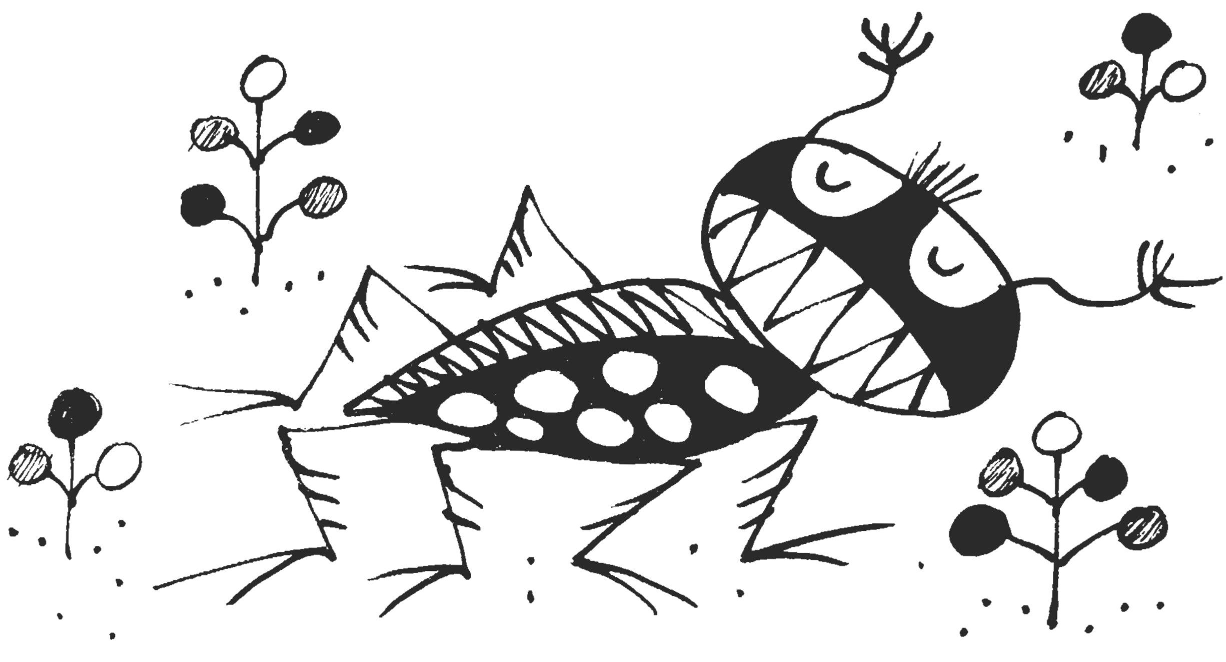


Claudius could not get inside the fence. But - - - - Berple could see that he would soon be out of jelly beans again. And it wasn't possible for him to get inside the other section of the fence to eat the jelly beans that grew there. Now, what to do!



At last, Berple had another idea. He thought of building a fence like this: A very long fence!

Berple would have to build the fence while Claudius slept. But this was such a long fence that he would never get it built while Claudius slept. Claudius would surely awake before he finished! Can you help Berple?



Berple was very happy!

Berple was very happy. He was safe from Claudius! He had all the jelly beans he could ever need or want!

He even threw Claudius two white jelly beans and one black one every night for supper, and he gave him five red ones as a special treat on Thursdays.

Commentary for Worksheets 2, *3, 4, 5

Distribute Worksheet 2. Direct the children's attention to fierce Claudius, lurking near the bottom of the page. Tell them that Berple thought of standing at A. Ask if they think he would be safe at A. If so they are to mark the point A with an X. If not they are to leave it unmarked.

Tell them to repeat this with each of the lettered points on the page. Walk among them to see that they understand.

Distribute Worksheet *3. Tell the children to look at 1 at the top part of the page. Read to them:

> Claudius Bugeater is standing in the place that is marked with a C. Berple is standing in the place marked with a B. With your pencil try to trace a path from Claudius Bugeater to where Berple is standing. Do not cross any fence lines with your pencil. Berple is safe if there is no path that can be traced from Claudius to Berple without crossing a line. Berple is in danger if Claudius can find a path to him without crossing a line. Pretend you are Claudius!

Play the game again on 2 and 3.

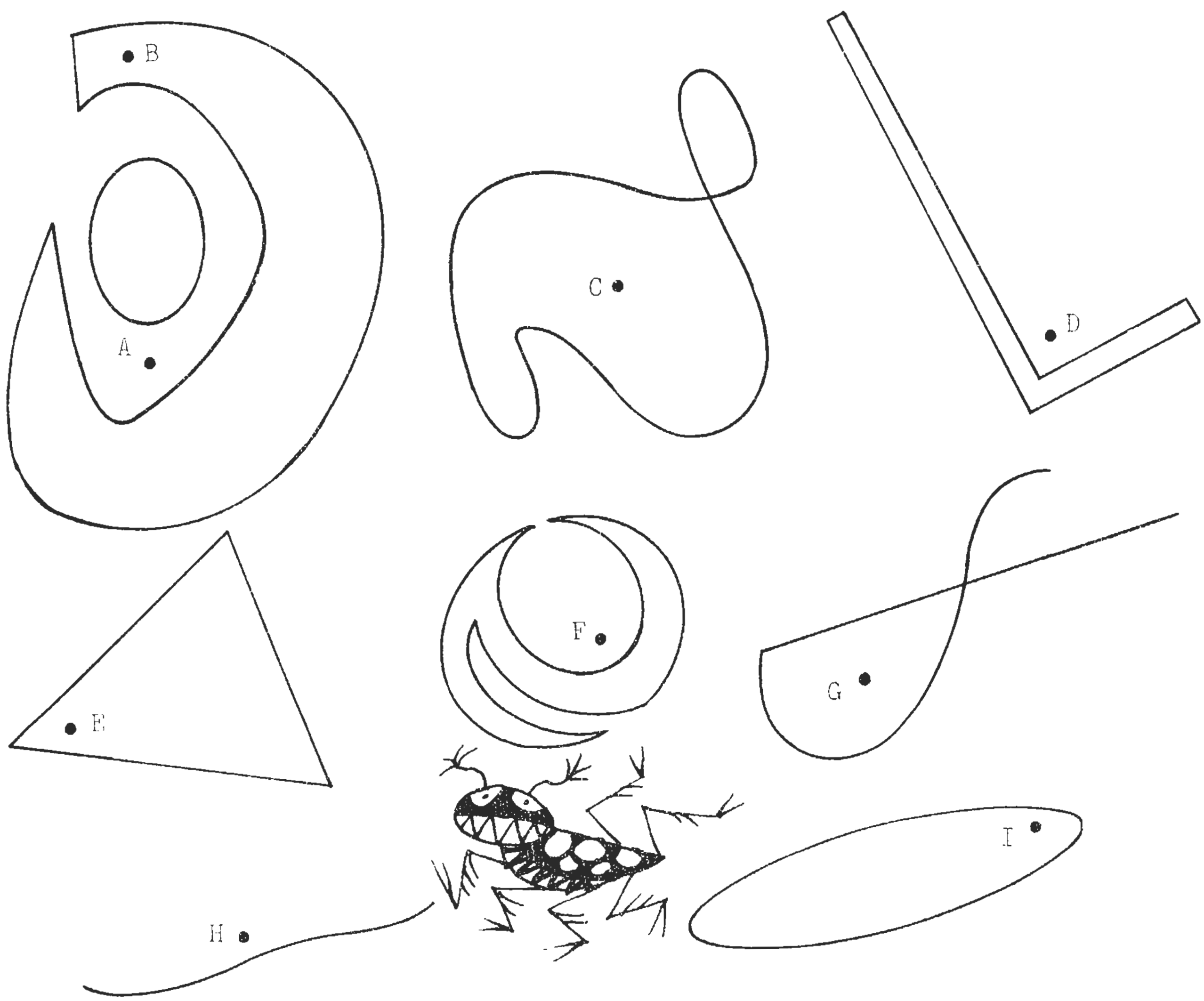
Distribute Worksheet 4. Read the directions below, but not the hint. Then move among the pupils, giving the hint as required.

> There are two fields of jelly beans. Draw two different fences that will keep Claudius away from Berple, but let Berple have all the jelly beans.
>
> Hint: One fence will keep Claudius out of the jelly bean field with Berple and the jelly beans inside the fence. The other fence will keep Claudius in, with Berple and the jelly beans out.

Distribute Worksheet 5. Help the children read the instructions.

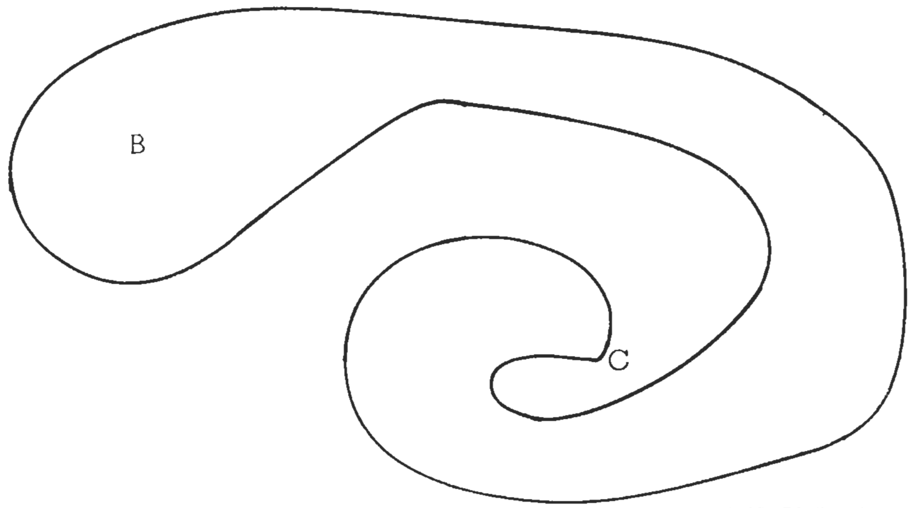
WORKSHEET 2

Here are some fences that Berple thought of in his plans to find safety from the fierce Claudius. The places that Berple thought of standing are marked with the letters A, B, C, D, E, F, G, H, and I. Mark an X with your crayon on the lettered points where Berple would be safe.

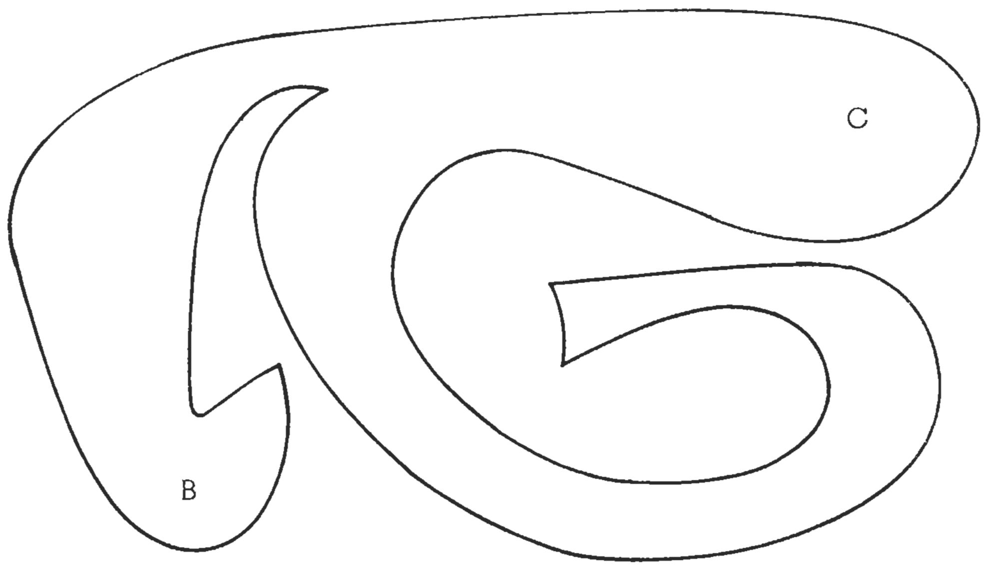


WORKSHEET 3

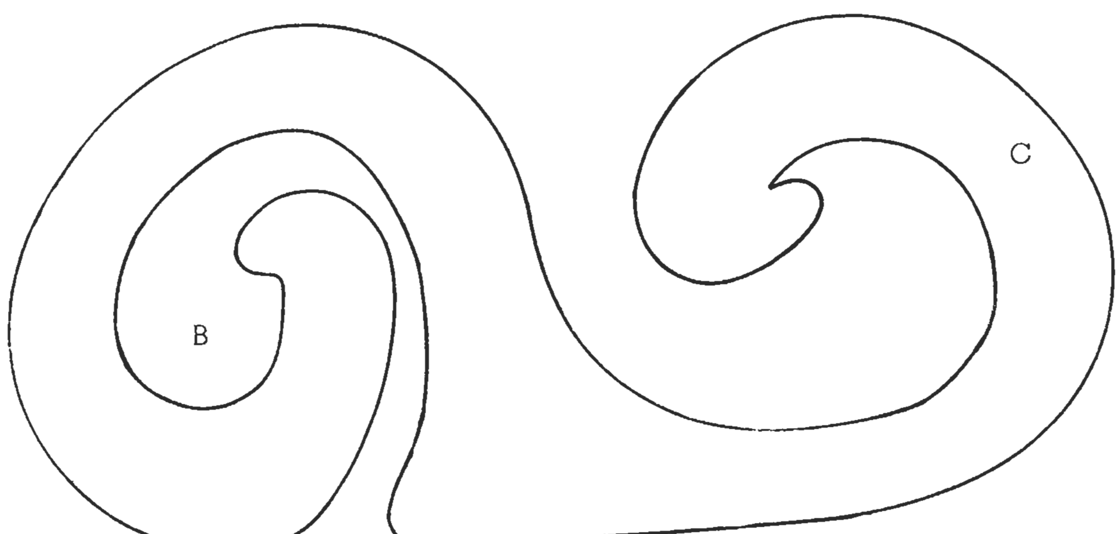
1.



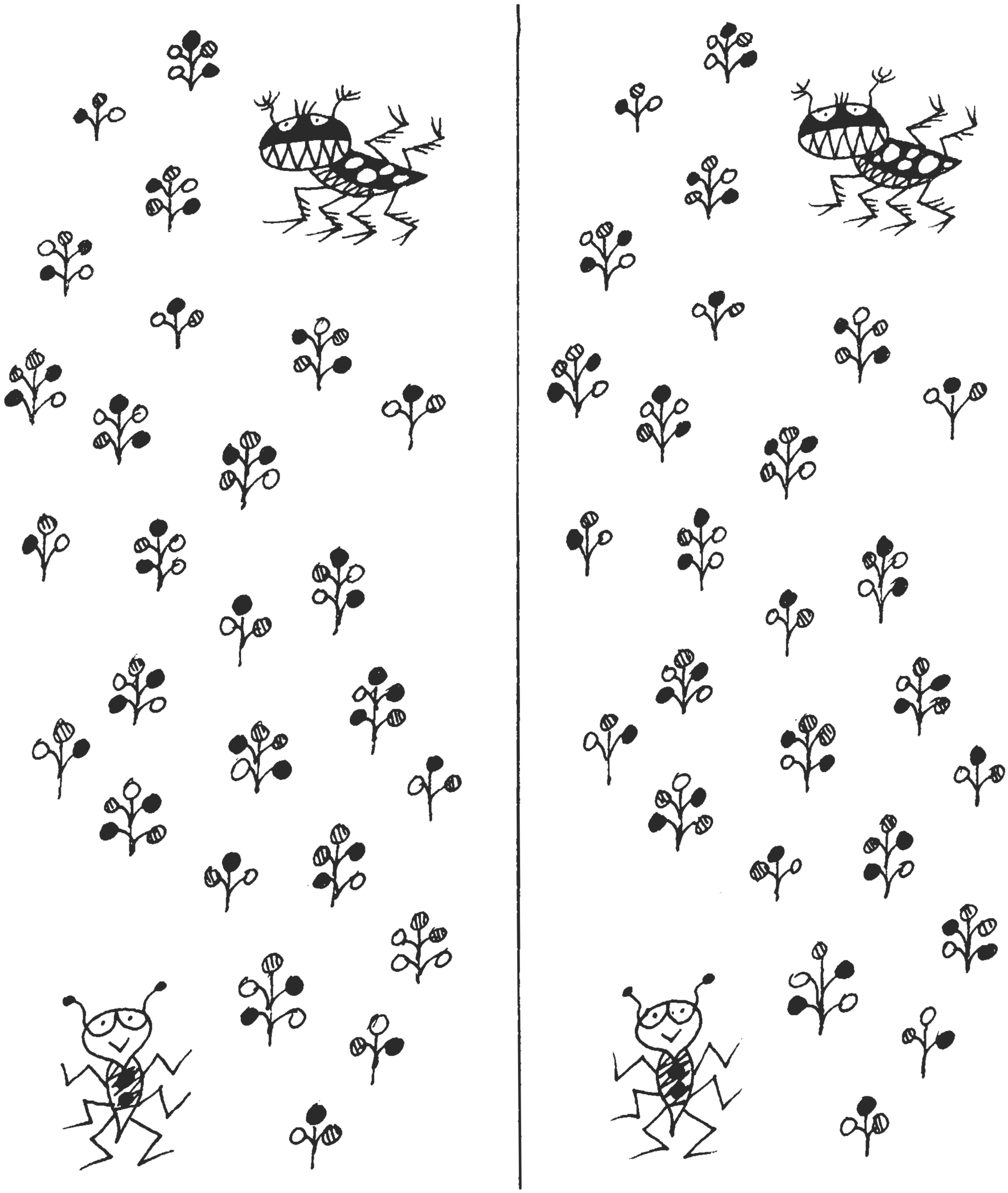
2.



3.

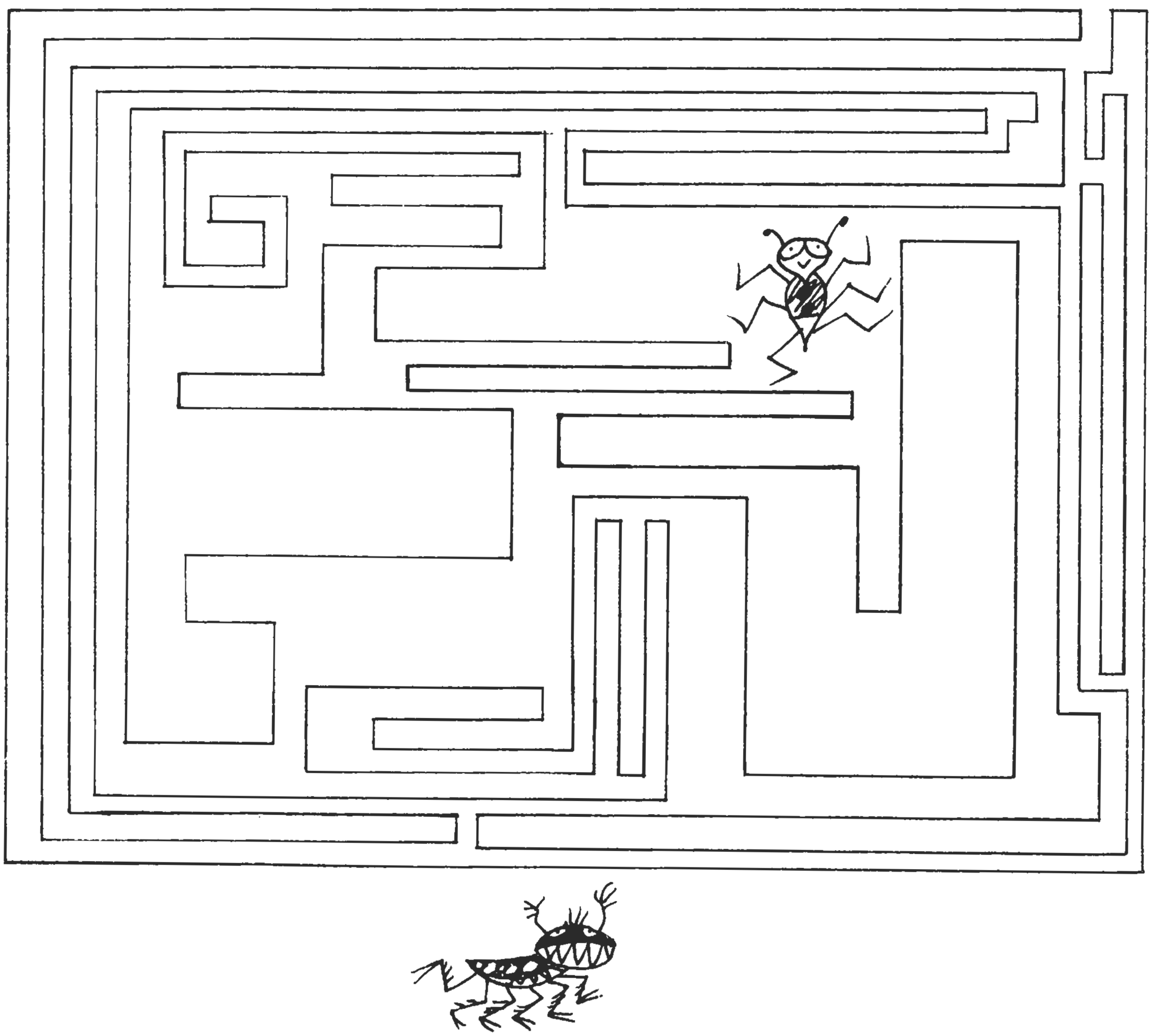


WORKSHEET 4



WORKSHEET 5

With your pencil try to trace a path from Claudius to Berple. If you can trace a path to Berple without crossing any lines, then Berple is in danger. Berple is safe if you cannot trace a path from Berple to Claudius without crossing any lines.



Is Berple safe? ______

More Activities on Curves and Regions

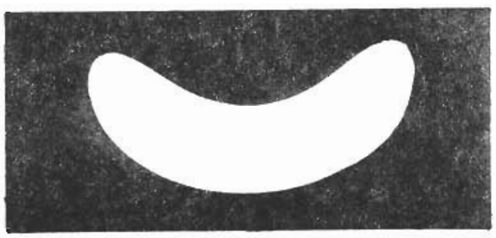
1. Game: Bugs and Fences

This is played like the game known as Squirrels and Trees. Have half the class pair off into partners. These pairs are the fences. The rest of the children are the bugs. (Be sure there are more bugs than fences.) The pairs join both hands. At a given signal from the teacher the bugs begin scampering around the fences. When the teacher or an appointed child calls, "Bugeater," all the bugs try to find a fence to duck under so that they can be safe. There is to be only one bug surrounded by each fence. The fences must not refuse a bug who comes to them for safety unless they already have another bug between their arms. All the bugs left over are caught and become fences replacing the old fences, who now get a chance to be bugs. The game ends when all bugs are safe.

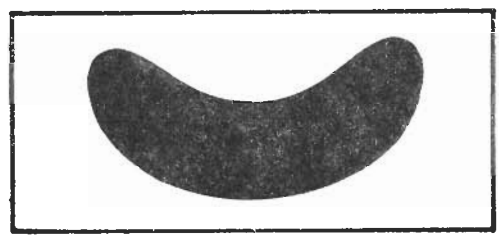
2. Game: Bugeater Tag

One child, called Bugeater, is "it." Half the class are bugs, half fenceposts. The children mill around. "It" can move too. At the call, "Bugeater," all the bugs crouch. The fenceposts hurry to join hands so that each one holds the hands of two other children. They may form long or short fences, so long as each closed curve has 3 or more fenceposts. After a few seconds, the teacher calls, "Stop!" The Bugeater goes around and taps each child who has not been enclosed by some fence. Tapped children become fenceposts. Enclosed bugs are safe. The game ends when all are fenceposts, or all bugs are safe. A new "it" is chosen for another round.

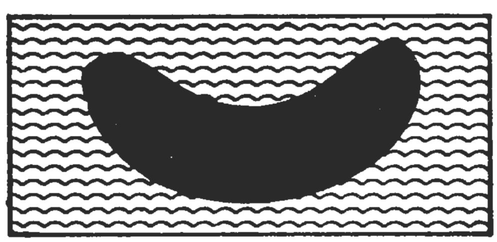
3. Have the children draw a simple closed curve on a piece of 8 1/2 x 11 manila paper that has been cut in half. It may be any color other than blue or red. Be sure that it is drawn distinctly. Then direct the children to choose a point that is not on their curve and to draw a dot to represent this point with their red crayon. With the same red crayon have the children color in the set of all points that they can join by a curve to the dot they have drawn without crossing the given curve. For example:



or

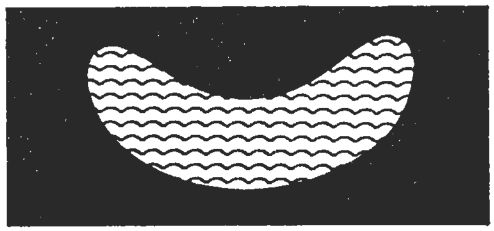


Ask the children if there is a portion of their paper that is not colored in. (This will be the inside or the outside of the curve, depending on the position they chose for their first point.) Have the children choose a point in this uncolored region and represent this point with a blue dot. Ask the children to color in with their blue crayon the set of points that can be joined by a curve to the blue dot without crossing the given curve. Their final product should look something like this (every point of the paper, except the points of the curve itself, has been colored either red or blue):



The set of red points represents a region. The set of blue points also represents a region. Ask the children how many regions there are. (2) If a child should ask whether the curve itself is a region, this would be a good introduction to the concept of boundary. If no one should present the question, it could well be posed by the teacher.

4. Draw a simple closed curve on the chalkboard. Color the inside of the curve blue and the outside red. Be sure that the curve itself is represented with a distinct color and a well-defined line. Call the blue region the <u>inside</u> of the curve and the red region the <u>outside</u> of the curve. The drawing should appear somewhat like this:



Note to the teacher: Before proceeding with the activity it might be necessary for you to review the concepts of union, intersection, and the empty set with the class.

To reinforce the learning of the descriptive terms "inside" and "outside," ask if someone could come to the board and point to the inside of the curve, and then the outside. Do this until you feel that the concept is quite concrete.

The following questions pertain to the intersection of the regions and should lead to an important discovery in this area.

a. Can we find a point that is colored both red and blue? (No.)

b. Can we find a point that is both inside and outside the curve? (No.)

c. What is the intersection of the set of all blue points with the set of all red points? (The empty set.)

d. What is the intersection of the inside of the curve with the outside of the curve? (The empty set.)

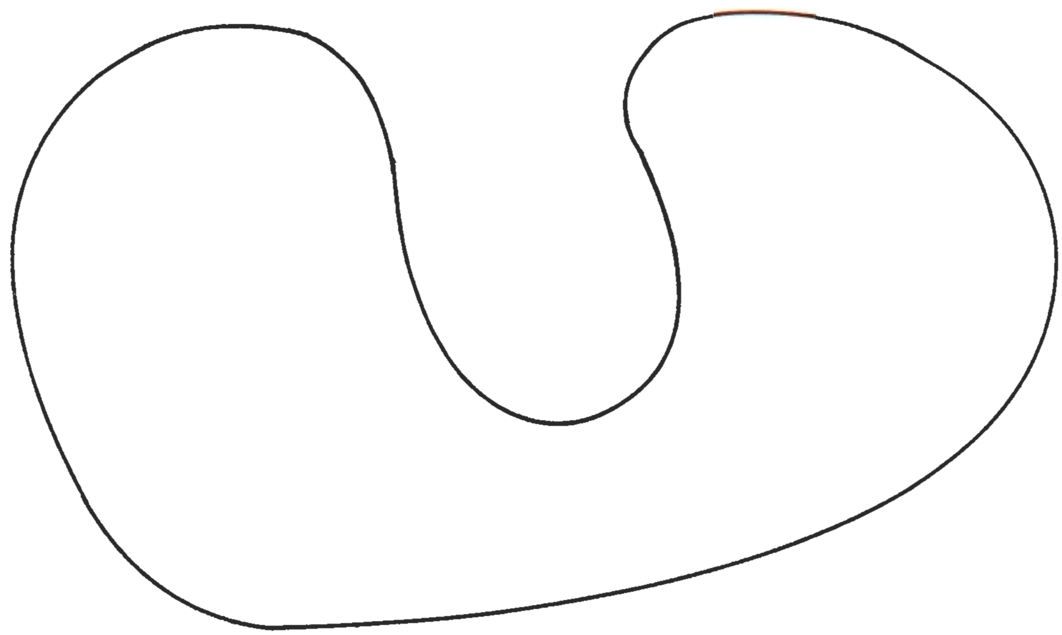
The regions are distinct and disjoint (having no points in common).

After reviewing the concept of union of sets with the children, ask if anyone could describe the union of the set of all red points with the set of all blue points. (The set of all the blue or red points; all the points but those on the simple closed curve.) Then to abstract the idea a bit for the children and to give them practice with their new terms ("inside" and "outside") ask, "What is the union of the inside of the curve with the outside of the curve?" (The answer will be the same.) "What is the intersection of the curve with the inside of the curve?" (The empty set.) "What is the intersection of the curve with the outside of the curve?" (The empty set.) The idea should emerge that each point of the plane (represented by the chalkboard or paper) is in only one of the following sets: the curve, the region inside the curve, or the region outside the curve. The curve itself is not a region, but it is the <u>boundary</u> of its inside and its outside regions: hence their <u>common boundary</u>. If you think it is feasible, use more examples with different curves to make the idea more meaningful to the children.

5. Distribute Worksheet *6. Read the directions with the pupils one by one as they perform the activity. It might be profitable to have the children indicate the regions first in class discussion.

6. Distribute Worksheet 7. If the children have difficulty in responding, remind them of the coloring technique of Activity 1 in this section. It might be profitable to have the children indicate the regions in class discussion before doing the worksheet.

WORKSHEET 6



1. Choose two points in the <u>same region</u> and use two black dots to show them.

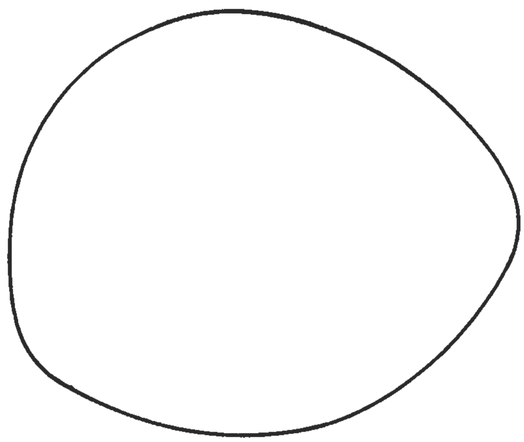
2. Can you draw a line from one of your points to the other. to connect these two points without crossing the curve? If you can, connect these two points with a curve using a green crayon.

3. Choose two more points on the same drawing, this time in different regions. Show these two points with red dots.

4. Can you draw a line between these two points without crossing the closed curve? Now, connect the two points with a curve using a blue crayon. Did it cross the closed curve? ______________________________

WORKSHEET 7

1.

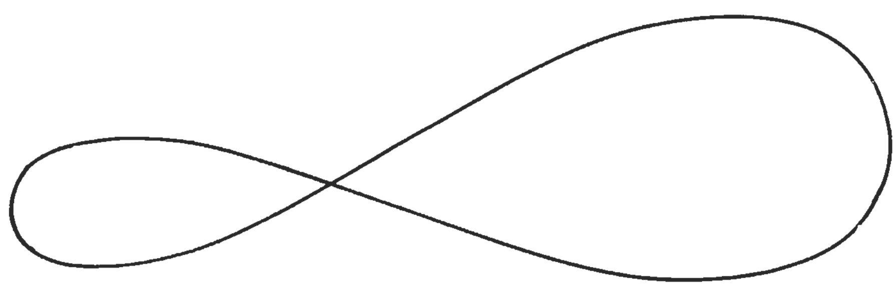


How many regions? ______

2.

How many regions? ______

3.



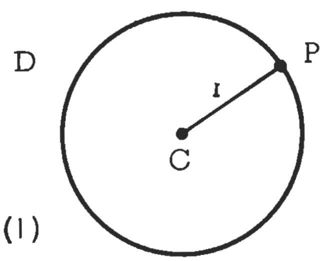
How many regions? ______

Color each region on this page a different color.

Teacher's Background on Circles

Circle

One familiar example of a simple closed curve is the circle. A circle is defined by fixing a point C and a positive number r. The set of all points at distance r from C is the circle of <u>center</u> C and <u>radius</u> r. In other words, a circle is the set of all points located at a given distance from a given point. (Recall that our universe of discourse is a plane!)



(1)

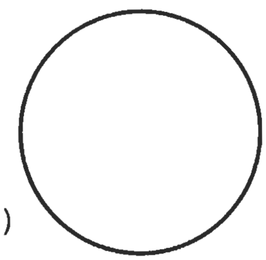

(2)

Figure (1) above shows a circle, together with its center C and a line segment CP, where P is a point of the circle D. The distance from C to P or the length of segment CP is r. Segment CP is often called a radius of the circle D.

Figure (2) shows just a circle. <u>Only the points inked in belong to the circle</u>. The points <u>inside</u> the circle are not points of the circle shown.

Disk

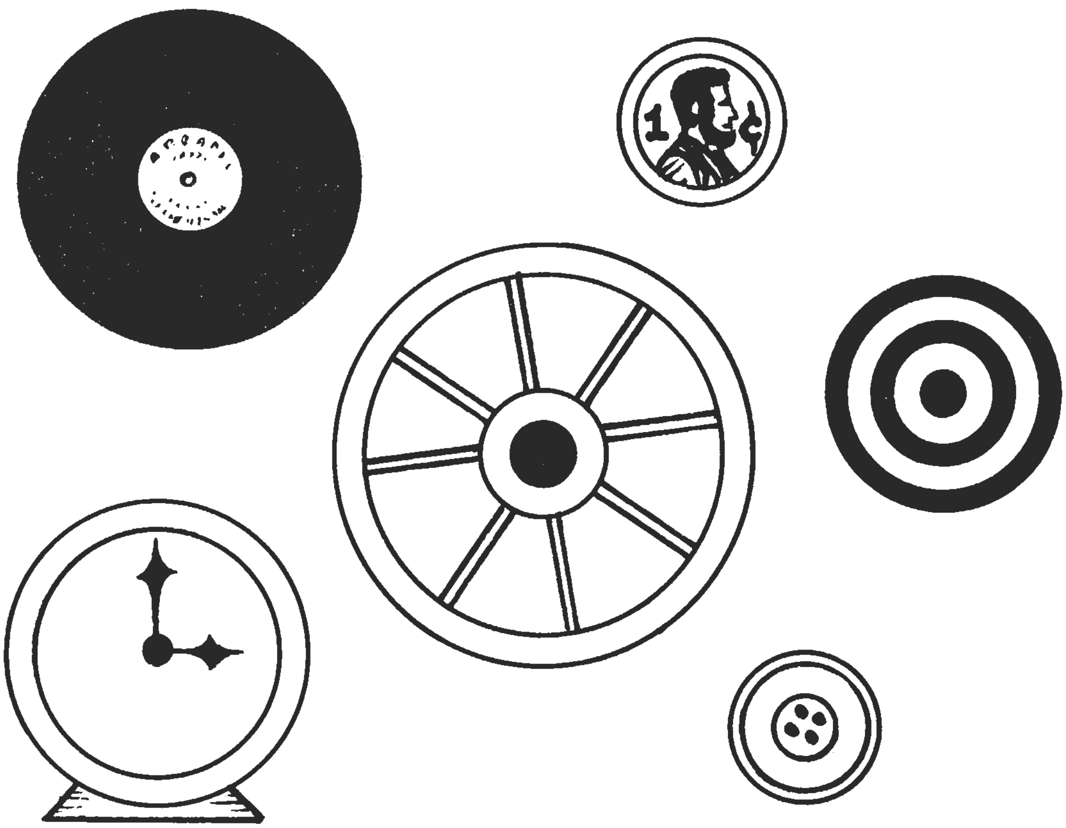
A disk is a circle and the region in its interior. It is proper to use the term "disk" in describing the circular objects one places on a flannelboard. In other words, "circle" refers to the boundary of the disk.

Suggested Activities on Circles

1. Have the children name as many objects as they can think of that are circular in shape.

2. Show examples of circular shapes on the next page.

3. Have them identify and place circular shapes on the flannelboard. Have them differentiate between circles and circular regions.



4. Construct a circle on the chalkboard. Choose a point as the center of the circle. Choose any distance from the center of the circle to the points on the circle as the radius. Tell the children that every point on the circle is the same distance from the center as every other point. The word "radius" may be used but it isn't necessary that the children learn it now. Let a child go to the board and choose his own radius from the center of the circle. Have him measure with a ruler and place several points around the center using the same radius. Then have another child connect the dots to form the circle.

5. Send one group of children to the board with their rulers to sketch circles in this way. Be sure that they locate enough points to be able to make a good circle. Since this is a slow process perhaps those not at the board could do the same things on a piece of newsprint at their desks. Another time let the other group work at the board.

6. Put an outline of a circle with the word "circle" beside it on the bulletin board of geometric figures.

7. Distribute Worksheets *8 through 11, one at a time as previous ones are completed. Detailed instructions for Worksheet *8 follow. Give assistance as needed in reading and interpreting instructions on the other sheets.

8. Distribute Worksheet *12. Read the instructions aloud, one by one, and give assistance as needed in following them.

> Note: Children at this age will not be able to produce accurate circles by the method suggested. It is of value, however, for them to have this experience so that they will understand the relationship of points and a circle.
>
> By tracing around a jar lid the children would obtain more accurate circles, but they would lose the opportunity to learn the "point" relationship in the construction of a circle.

Instructions for Worksheet *8

The questions are to be read aloud by the teacher and interpreted as necessary. The pupils are supplied with rulers in their slide rule packets which can be used in this activity. It might be fun to have the children make their own rulers out of paper strips marked with crayons.

1. Measure the distance from O to P_1.

2. Measure the distance from O to P_2.

3. Measure the distance from O to P_3.

4. Find another point the same distance from O. Call it P_4.

5. Try to find a point P_5 whose distance from O is the same, with L between O and P_5.

6. Try to find a point P_6 whose distance from O is the same, with P_6 between O and K.

7. Sketch as well as you can the circle with the center at O, through P_1, P_2, P_3, P_4, P_5 and P_6.

8. The radius of the circle is ______________________.

9. Measure the distance from O to L.

10. Is L on the circle?

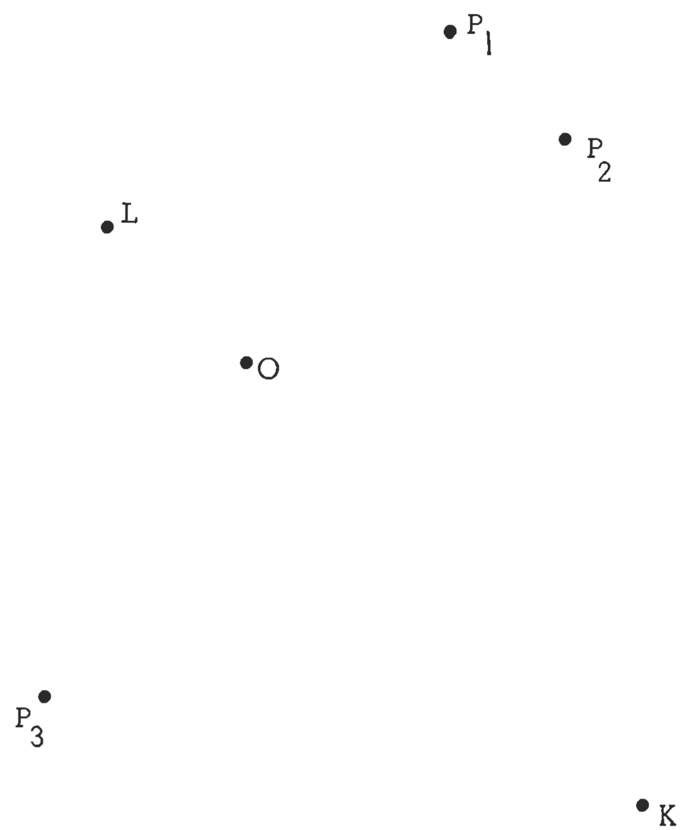
11. Is L inside or outside the circle?

12. Measure the distance from O to K.

13. Is K on the circle?

14. Is K inside or outside the circle?

WORKSHEET 8



Your teacher will ask the questions. Write the answers here.

1. ______________________.

2. ______________________.

3. ______________________.

4. ______________________.

5. ______________________.

6. ______________________.

7. ______________________.

8. ______________________.

9. ______________________.

10. ______________________.

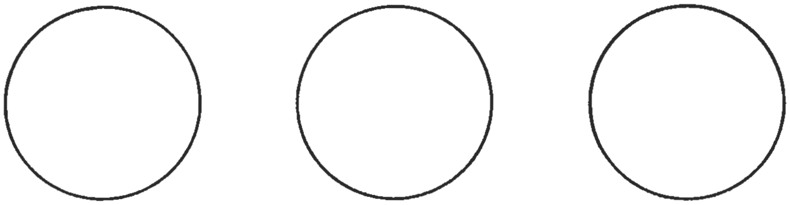
11. ______________________.

12. ______________________.

13. ______________________.

14. ______________________.

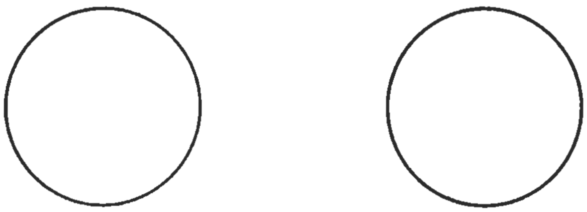
WORKSHEET 9



Set 1

How many?

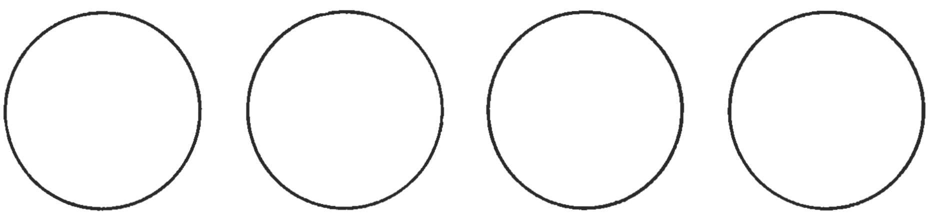
____



Set 2

How many?

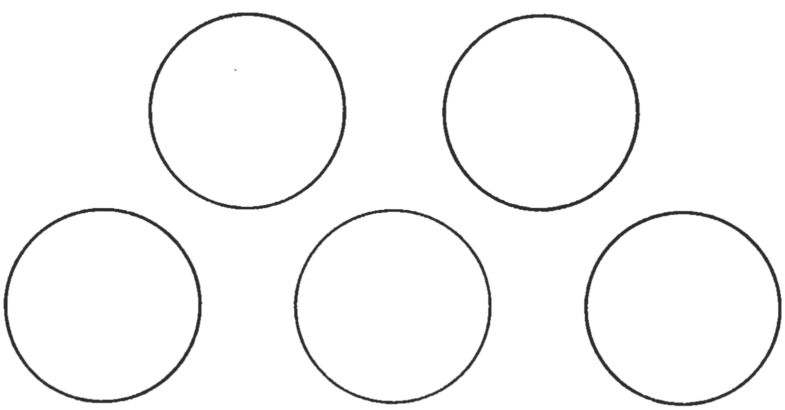
____



Set 3

How many?

____

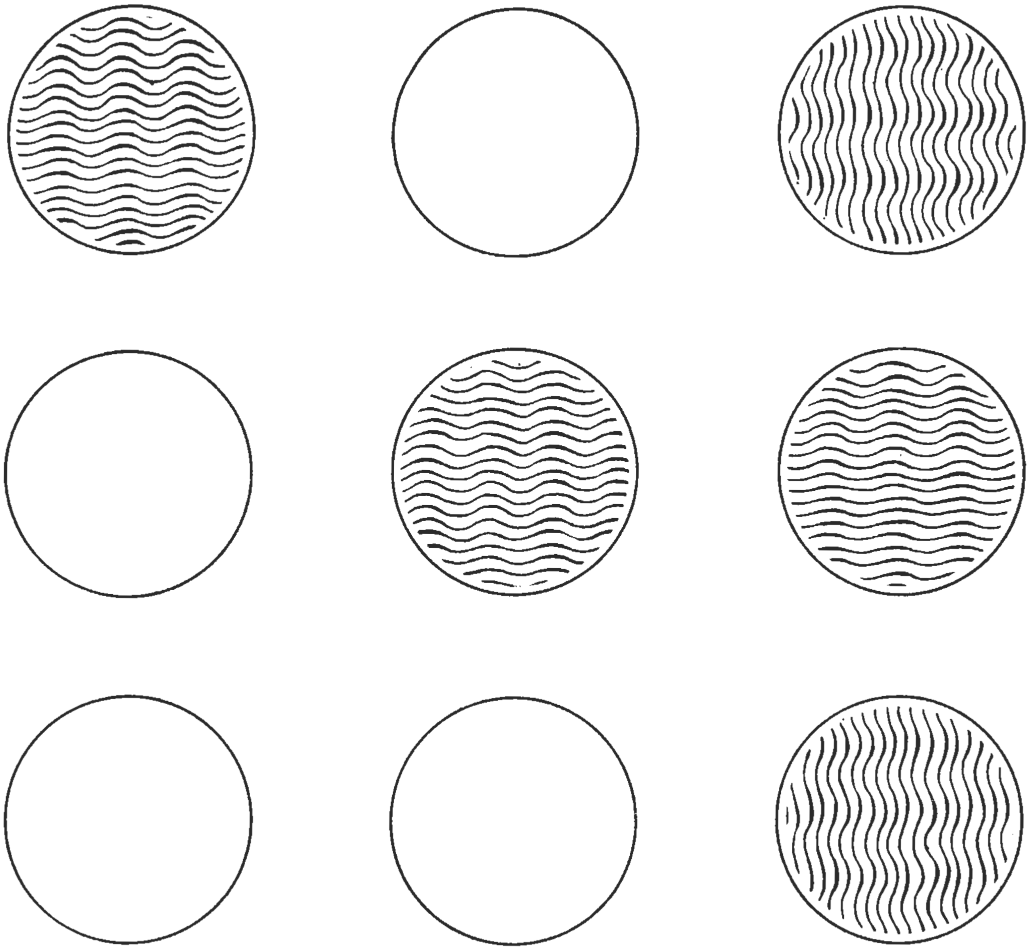


Set 4

How many?

____

WORKSHEET 10



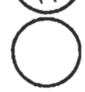
1. How many circles are there in each row? ________

2. How many rows are there? ________

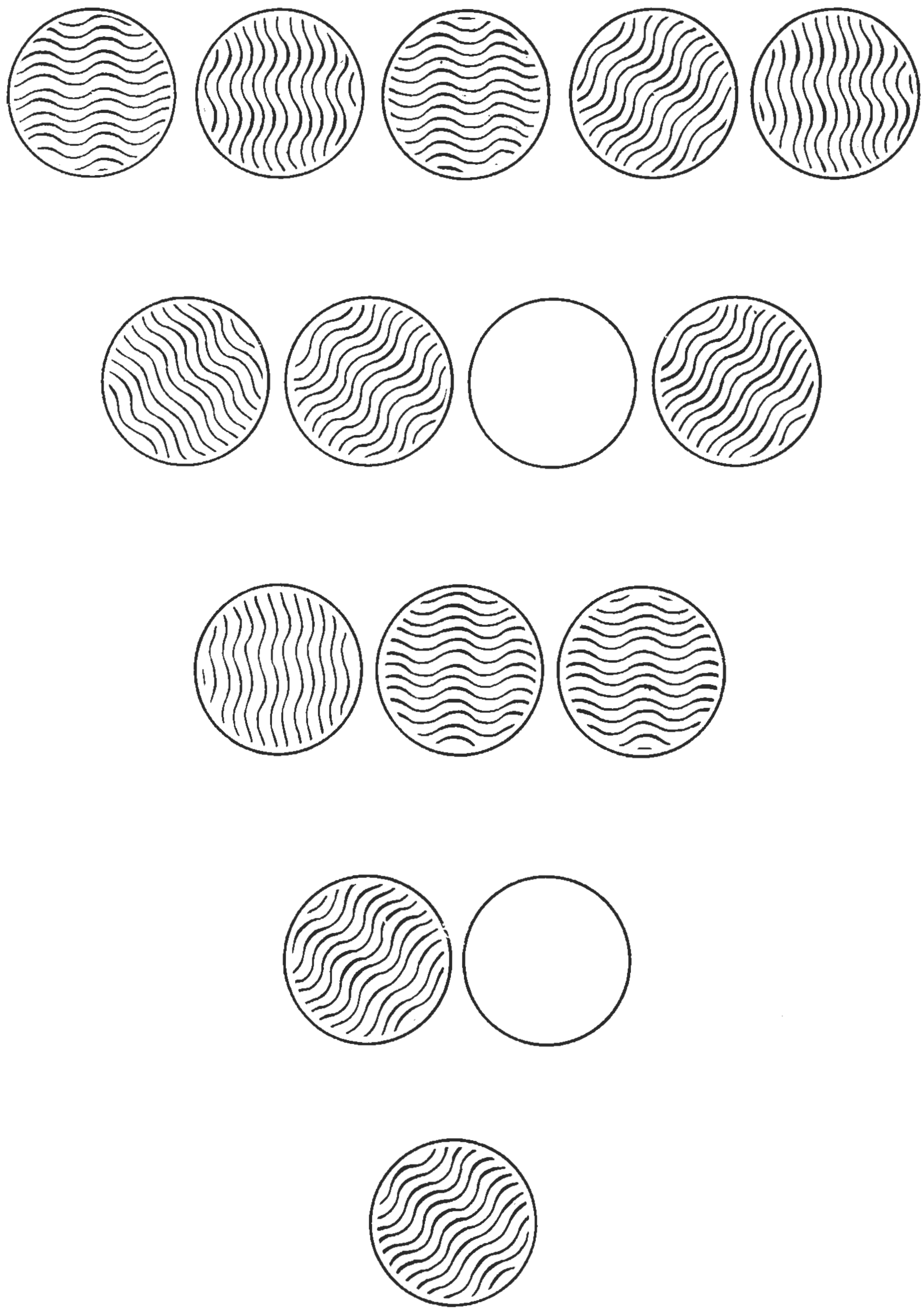
3. How many circles are there altogether? ________

4. How many disks look like this?  ________

5. How many disks look like this?  ________

6. How many disks look like this?  ________

WORKSHEET 11



Row 1

Row 2

Row 3

Row 4

Row 5

(Questions for this page are on the next page.)

Questions for Worksheet 11

1. How many circles are in row 1? ______

2. How many circles are in row 2? ______

3. How many circles are in rows 1 and 2 together? ______

4. How many circles are in row 3? ______

5. How many circles are in the union of rows 1, 2, and 3? ______

6. How many circles are in row 4? ______

7. How many circles are in the union of rows 1, 2, 3, and 4? ______

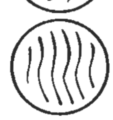
8. How many circles are in row 5? ______

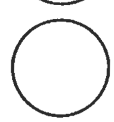
9. How many circles are in the union of rows 1, 2, 3, 4, and 5? ______

10. How many disks look like this?  ______

11. How many disks look like this?  ______

12. How many disks look like this?  ______

13. How many disks look like this?  ______

14. How many disks look like this?  ______

Answers for Worksheets 9, 10, and 11

Worksheet 9	Set 1	3
	Set 2	2
	Set 3	4
	Set 4	5
Worksheet 10	1.	3
	2.	3
	3.	9
	4.	3
	5.	2
	6.	4
Worksheet 11	1.	5
	2.	4
	3.	9
	4.	3
	5.	12
	6.	2
	7.	14
	8.	1
	9.	15
	10.	4
	11.	1
	12.	5
	13.	3
	14.	2

Questions for Worksheet 12

Read the questions to the children.

1. Choose a point inside the simple closed curve on the worksheet. Represent this point with a dot. Name the point Q.

2. Draw a small circle around the point Q. Can you do this in such a way that all points inside the small circle are also on the inside of the given closed curve?

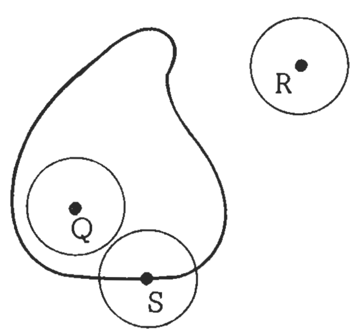
3. Choose another point, this time on the <u>outside</u> of the given closed curve. Represent this point with a dot. Name this point R.

4. Draw a small circle around the point R. Can you do this in such a way that all the points inside the small circle are outside the given closed curve?

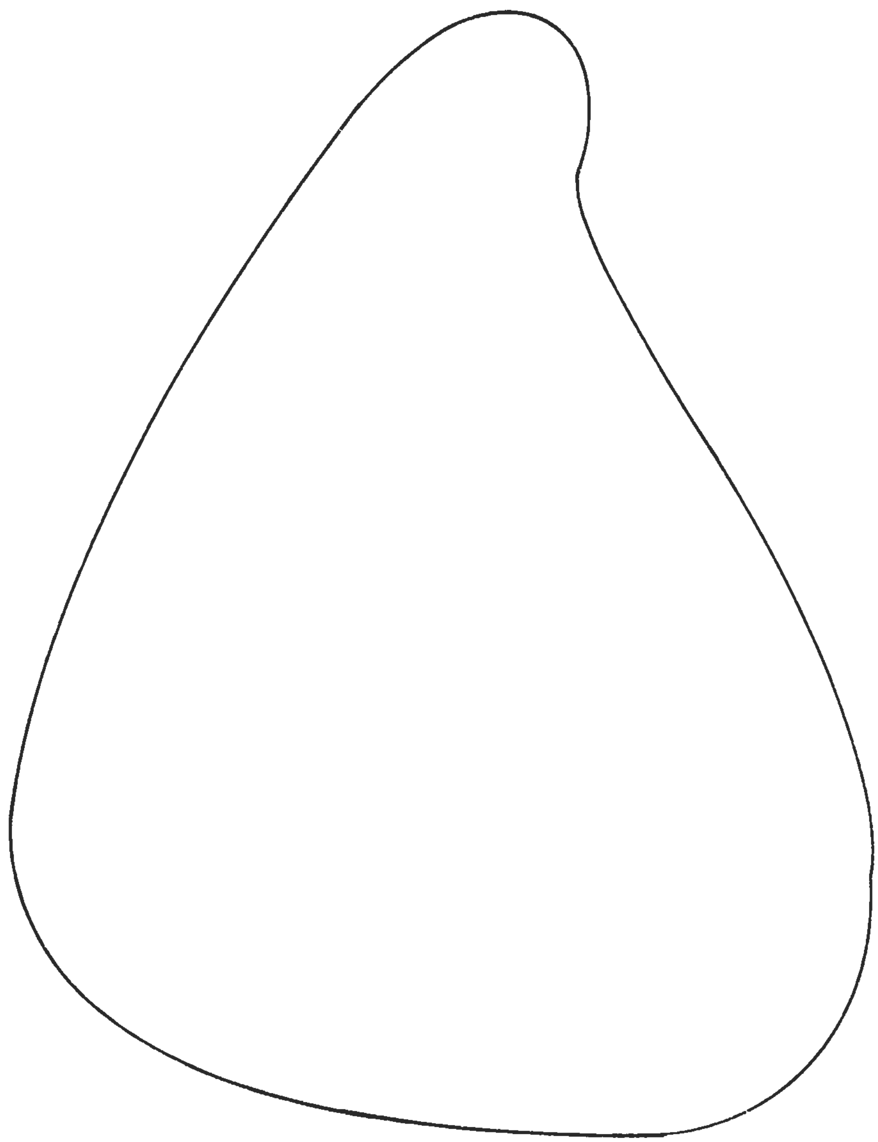
5. Now pick a point <u>on</u> the closed curve (on the common boundary of the inside and outside regions). Represent this point with a dot. Name this point S.

6. Draw a small circle around the point S. Can you do this in such a way that all the points inside the small circle are inside the given closed curve? Can you do this in such a way that all the points inside the small circle are outside the given closed curve? Can you do this in such a way that all the points inside the small circle are on the boundary of the given closed curve?

When the worksheet is completed it should look like this:



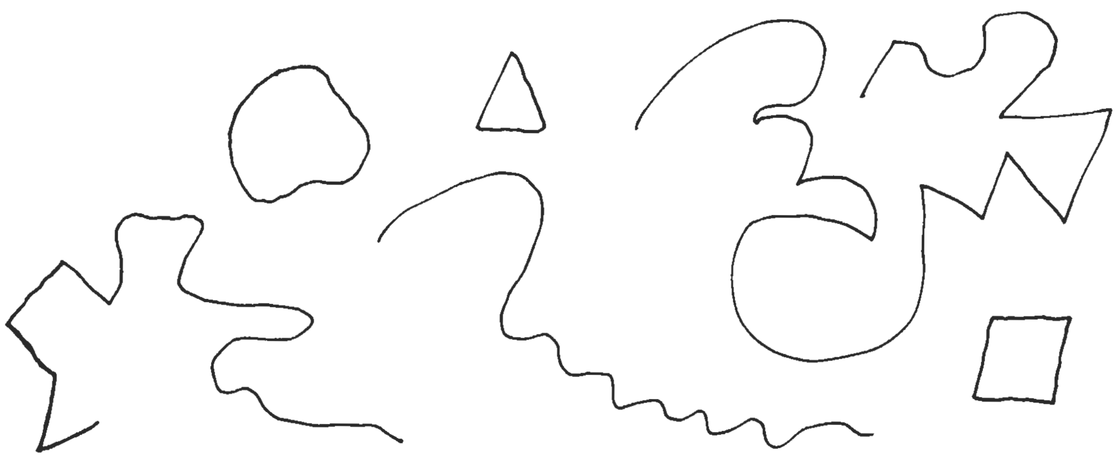
WORKSHEET 12



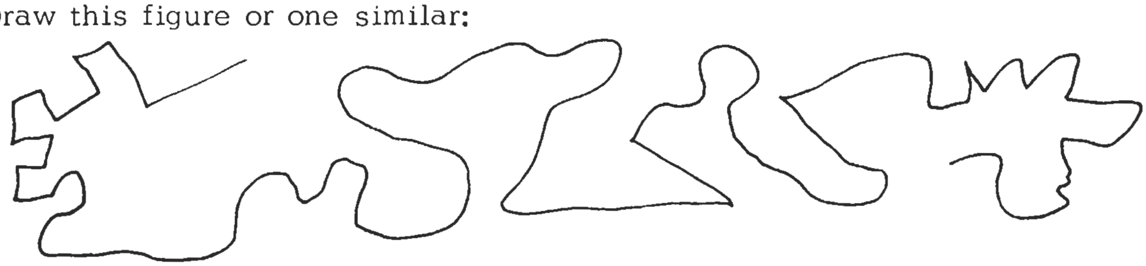
Suggested Activities on Non-simple, Simple, Non-closed and Closed Curves

1. Draw a simple curve on the chalkboard. Ask the children to name what you have made. If no one can answer, explain that it is a simple curve. Ask if they know why it is a simple curve.

2. Ask several children to draw a simple curve on the chalkboard.



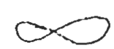
If someone questions the  or  saying that these are not curves because the lines are straight or because there are corners, take this as an opportunity to remind the children that we are using an old word in a new way, and that the <u>mathematical</u> meaning of the word "curve" does include figures such as these. It is as if the word "curve" belonged to two different languages—that of mathematics, and that of everyday speech—with different though related meanings in the two. Some children will think of <u>simple</u> as contrasting with <u>hard</u> or complicated. Draw this figure or one similar:

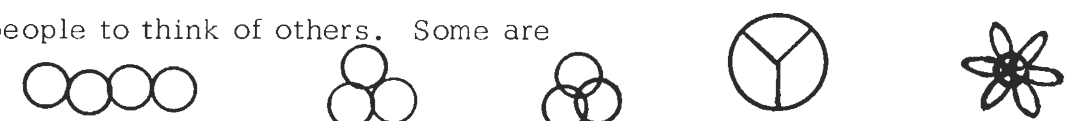


Some children may say it is not simple because it is hard to draw. Explain that it doesn't make any difference whether the curve is simple or hard to draw. The word <u>simple</u> has been given a new mathematical meaning. It is a simple curve because the line does not cross itself.

3. Draw a closed curve that is not simple with a crayon on a sheet of paper, but do not show it to the children. Invite them to ask you questions about it to discover what they can about the hidden curve. The questions, "Is it closed?" and "Is it simple?" should emerge. If no one can think of what to ask, prompt them with "What kinds of curves do we know about?" If someone asks what color it is, the question should be answered "I used a red crayon," or simply "It's red, John," and dropped. Later the question "Can a curve (or a point) be red (or any other color)?" might be pursued. The children may ask "Does it have corners?" or "Does it have a straight part?" or any of a number of sensible questions, which should be answered and recorded on the chalkboard. Encourage the children to ask "Is it closed?" and "Is it simple?" so they can identify the curve.

After these two questions have been answered, keep your curve hidden and invite a volunteer or two to come to the board and draw it. (The other questions that were recorded are ignored for the moment.) The other children can attempt to do it with crayon and paper at their seats.

Now the mystery curve can be revealed and comparisons made. Each child can verify that his curve is the proper type (closed, but not simple). Imagination should be encouraged, especially departure from illustrations already given in class. If virtually everyone has drawn , ask people to think of others. Some are

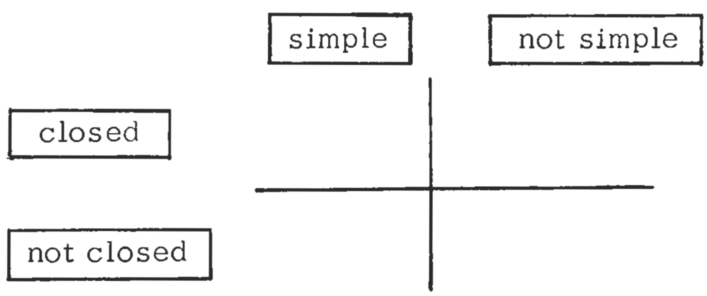


Any curve without endpoints other than a simple closed curve is closed but not simple.

Repeat the activity with a new mystery curve. Make this one neither closed nor simple. Again encourage each child to design his own. The variety is infinite, and fun can be had in discovering ways to modify a given curve to obtain one of this kind.

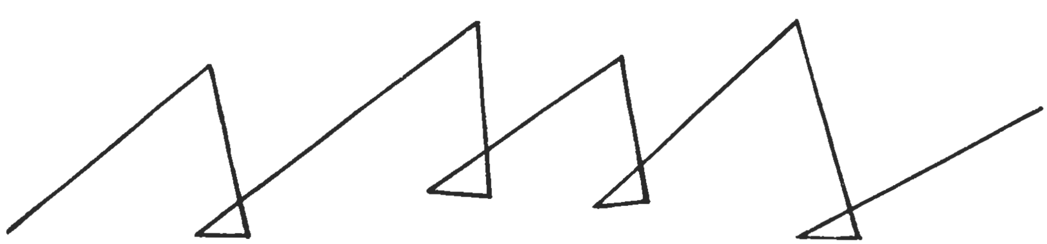
Repeat the activities with a simple closed curve and a simple, non-closed curve. The pupils may observe that in each of these two classes of curves there is an essential lack of variety, but this observation should not be volunteered by the teacher.

4. Several examples of each of the four classes of curves explored in 3 are chosen for a bulletin board display. The examples should exhibit variety. In particular, if questions were recorded as suggested ("Does it have corners?" etc.) the variety should reflect the interest represented by their questions. There might be a simple, non-closed curve with corners and one without. Divide the bulletin board into four quadrants.



Label the rows and columns as shown, and then let the children assist in deciding where each example chosen for display should be placed. For example, a closed, non-simple curve would be placed in the top row, right-hand column.

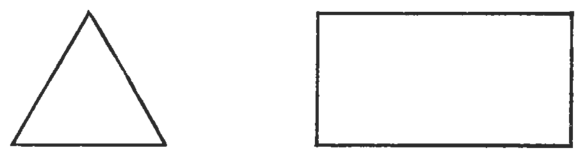
5. Put this figure on the board and have the children identify it.



It is a curve that is not simple because the curve crosses itself. It is not closed because it has 2 endpoints. It contains 6 simple closed curves.

6. Have several children go to the board and draw a simple non-closed curve.

7. Have others draw a simple closed curve. Some children will make figures like the following:



Remember that these are simple closed curves. If someone still questions this, saying that the lines don't curve, explain that in mathematics a different meaning is given to the word "curve." Curves need not be "bent."

8. Let one-fourth of the class go to the board to draw simple curves. Ask the other children to raise their hands if they think the chalkboard drawings are right. Call on the children who did not raise their hands to tell what they disagree with and why.

9. If necessary, continue giving the children experiences in making closed simple curves, open simple curves, and curves that are not simple.

10. Distribute Worksheets *13 and 14.

WORKSHEET 13

1. Draw a red line around each simple non-closed curve.

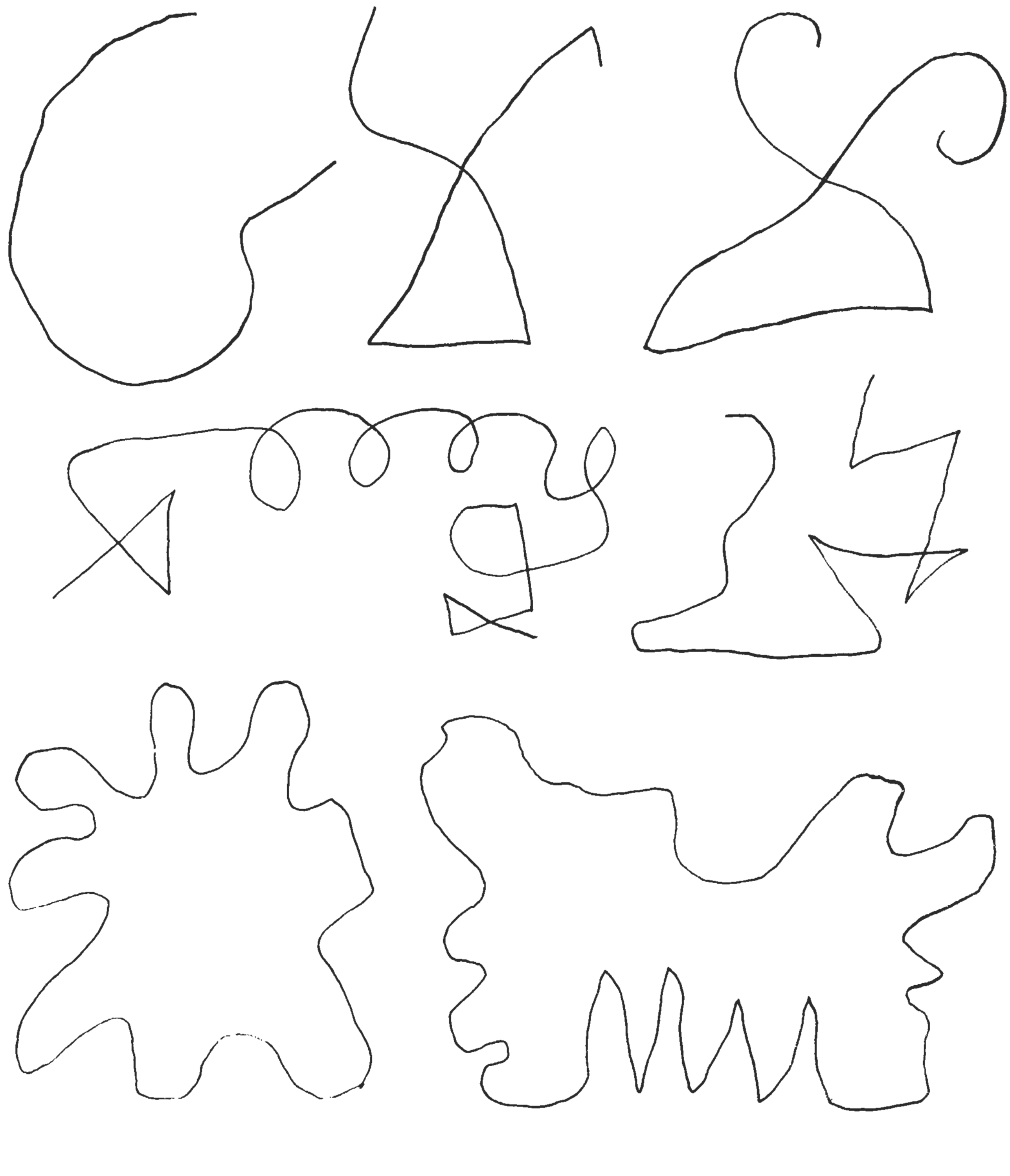
2. Draw a green line around each simple closed curve.

3. Take a purple crayon. Make a simple closed curve out of every simple non-closed curve.

WORKSHEET 14

1. Draw a blue line around each simple curve.

2. Draw an orange line around each non-simple curve.



Review Activities on Inside, Outside, Boundary

1. Draw a simple closed curve on the chalkboard. Have it identified. Ask someone to go to the board and draw a tree <u>inside</u> the simple closed curve. Then have another person draw a house <u>outside</u> the simple closed curve. Have a third child put an X on the <u>boundary</u>.

2. Investigate whether or not an open curve has an inside and an outside. Bring out that a simple curve must be closed to have an inside and an outside.

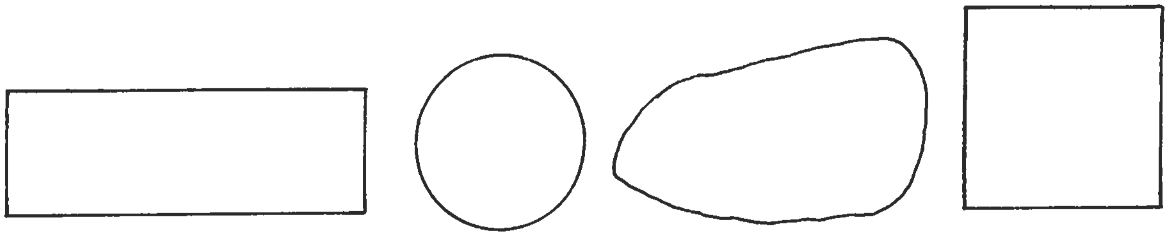
3. Give each child a length of yarn or string (about one yard long), and three objects: a block, a counter and a crayon. Have the children form a circle with the string. Tell them to pretend it is a pond. Ask them to put the block <u>inside</u> the pond; put the counter <u>outside</u> the pond; put the crayon on the <u>boundary</u> of the pond.

This activity could be varied by having the yarn or string form a corral, a play yard, a skating rink, etc.

If all the children have similar objects the teacher can tell at a glance if they have been correctly placed.

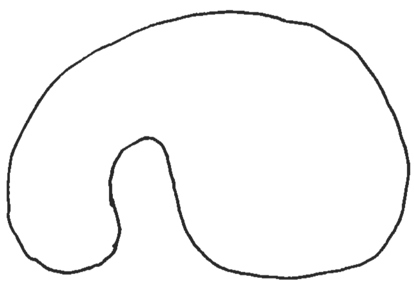
4. Give each child a piece of yarn (about one yard long). Let each arrange the yarn on his table into a simple closed and then a simple, non-closed curve. When the non-closed curve is made, shift the yarn to make it a simple closed curve. Discuss the fact that the yarn is the boundary separating the inside from the outside of a simple closed curve.

5. Repeat the game "Inside and Outside" from Unit I, Geometry, p. 11, to develop their ability to determine whether an object or a person is inside or outside a simple closed curve. Use a long clothesline or a piece of chalk. Arrange the clothesline or draw various simple closed curves on the floor. Begin with simple ones such as oblongs, circles, ovals, and squares.

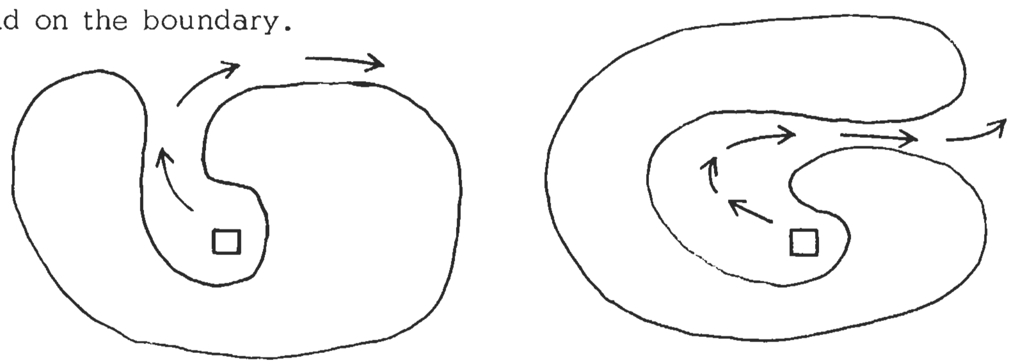


Direct a child to stand inside the curve and another to stand outside. Help the children discover that there is a bounded region inside the curve and an unbounded region outside the curve. Remind them that in the story Berple ran out of jelly beans when he was inside his fence.

6. Next make the curves more complex. Choose two children to go to each curve. Direct one to stand inside and one outside the curve.



7. Put objects inside or outside the curve. Have the children decide whether each object is inside or outside the curve. See if someone can stand on the boundary.



Figures like these make it harder to decide whether the object is inside or outside the closed curve. However, if a child can pick up the object and find a path out without stepping over the boundary, the object has been outside.

8. *Distribute* Worksheets 15, 16, *18 and *19.

Worksheets 15 and 16 are test pages to be worked independently.

Worksheet 17 has directions which require help from the teacher.

Worksheets *18 and *19 are review and can be worked independently by the children. They may require some help from the teacher to reach the solution. The children may be reminded that they have done similar activities in Worksheets 2-5.

WORKSHEET 15

Draw a simple, closed curve.

Draw two closed, non-simple curves.

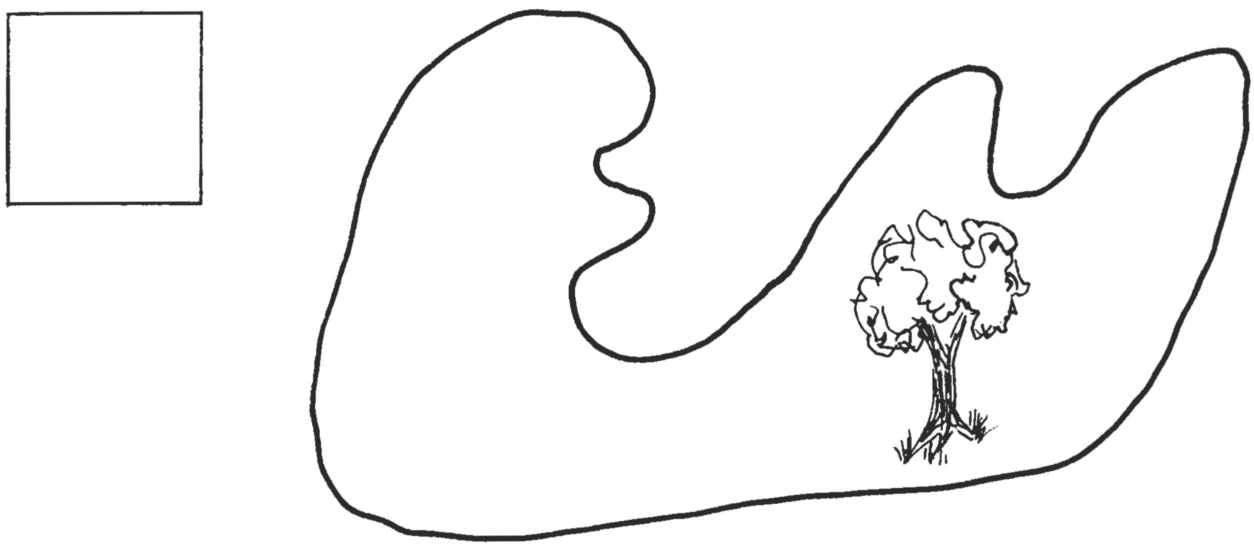
WORKSHEET 16

Draw three simple, non-closed curves.

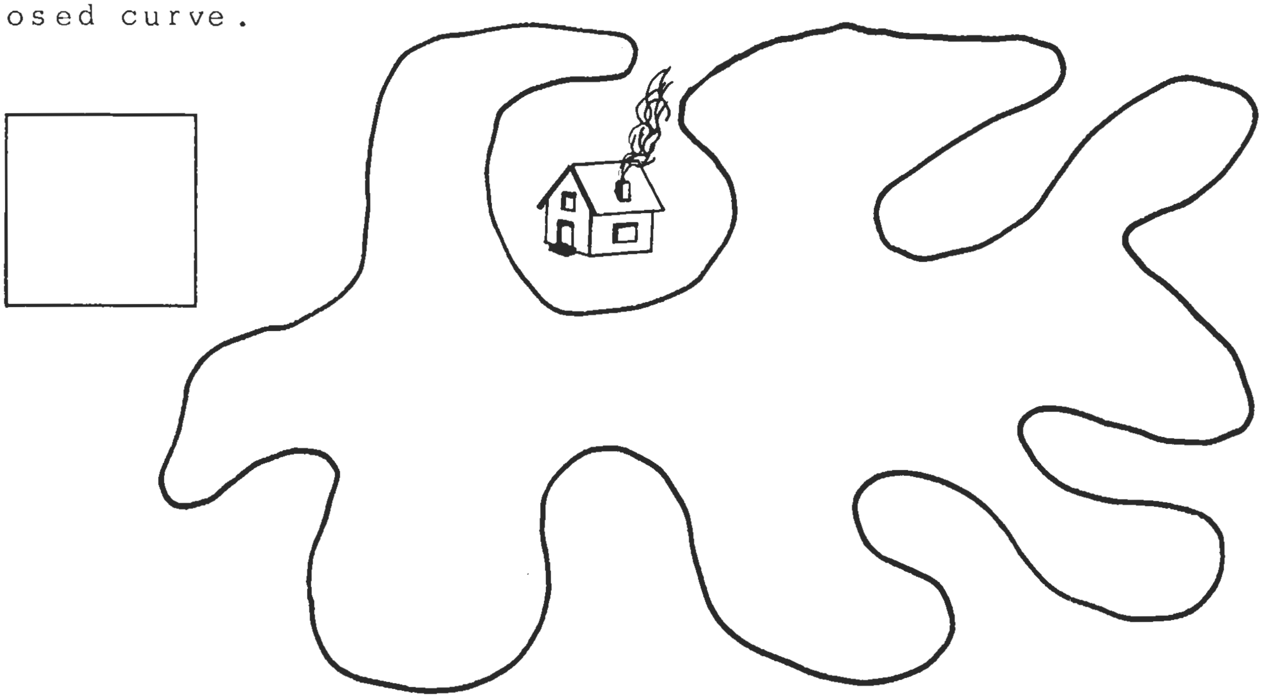
Draw four non-simple, non-closed curves.

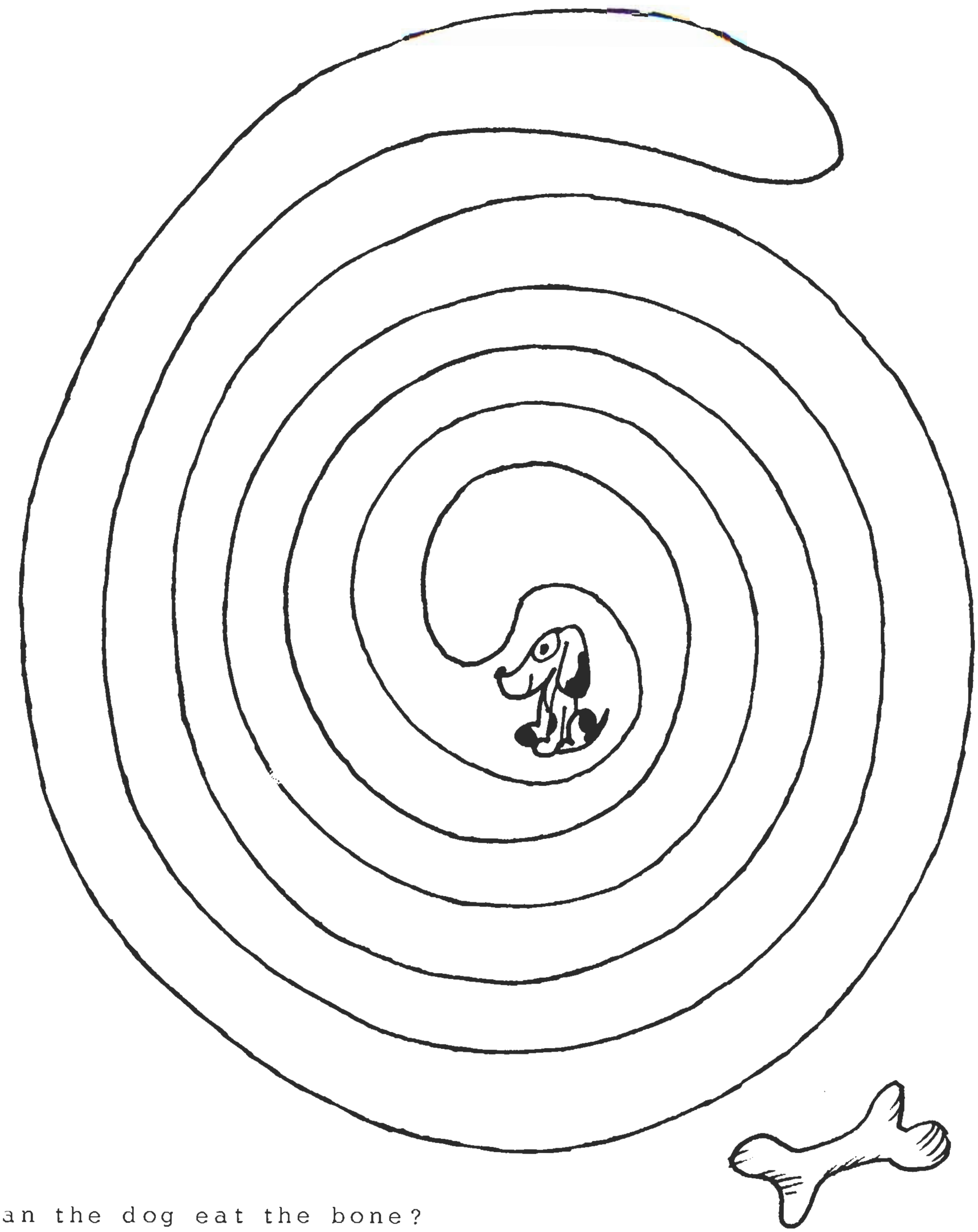
WORKSHEET 17

1. Put a red X in the box at the left if the tree is inside the closed curve. Put a blue X in the box if the tree is outside the closed curve.

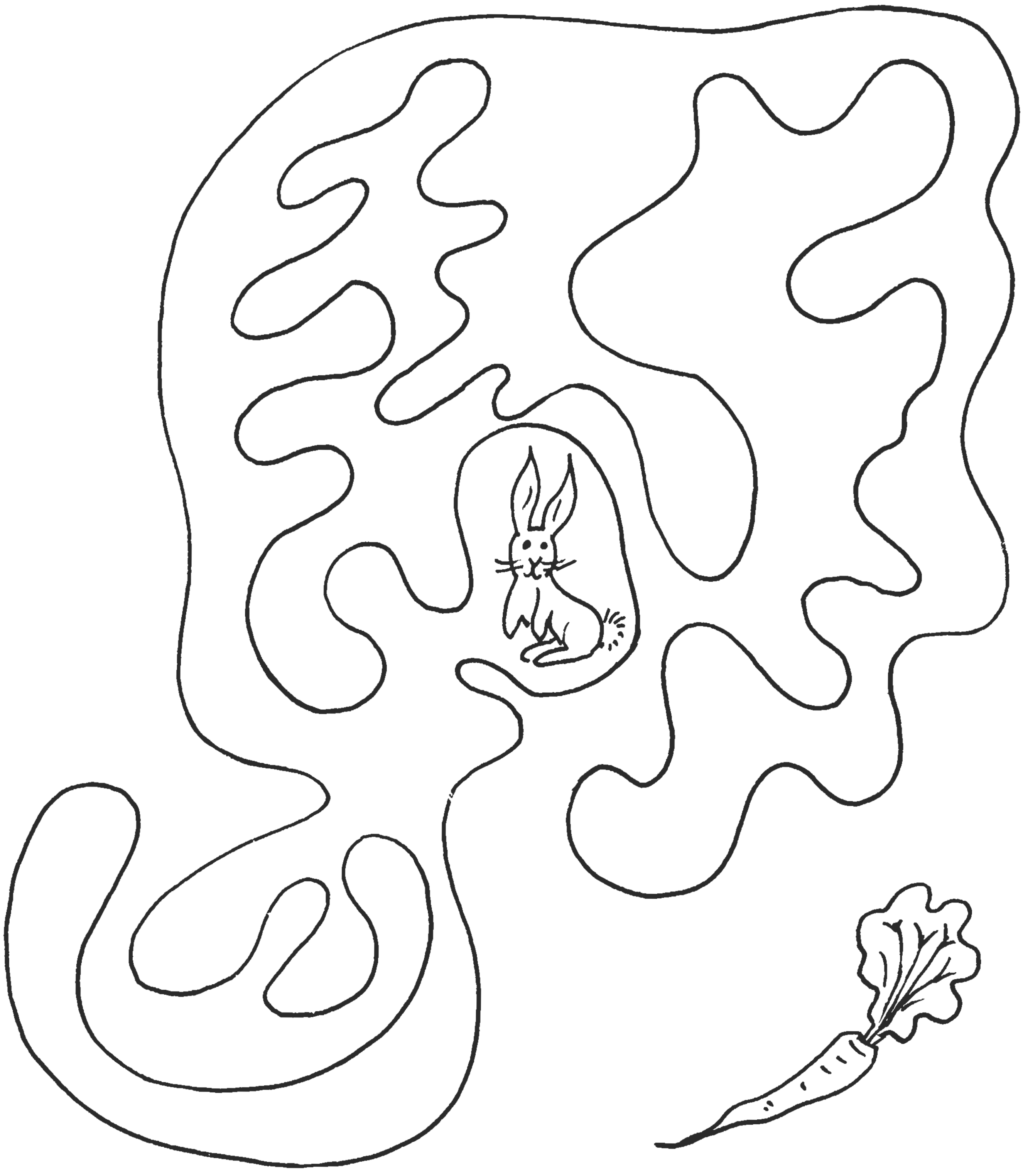


2. Put a green X in the box if the house is outside the closed curve. Put a purple X if the house is inside the closed curve.





Can the dog eat the bone?



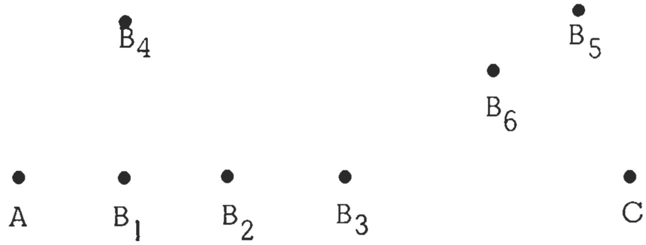
Can the rabbit eat the carrot?

Teacher's Background on Line Segments, Lines, Rays and Polygons

Between

Between is undefined. If you are at point A trying to see something at point C, and point B blocks your view of point C, then B is between A and C.

There are infinitely many points between A and C. For instance the points B_1, B_2, and B_3 are between A and C, whereas B_4, B_5 and B_6 are not.



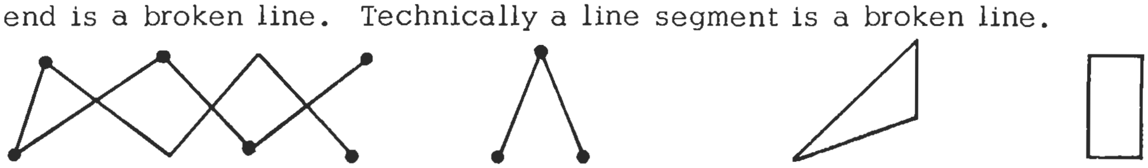
The notion of betweenness can be illustrated in the classroom by making sightings along objects and interposing other objects between the viewer and the object initially viewed.

Line Segment

Let A and B be two different points. The line segment AB is the set of points consisting of points A and B, and all points between A and B. The line segment AB is a curve with endpoints A and B.

Broken Line

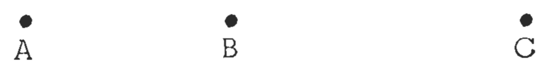
A curve that is made up of a succession of line segments joined end to end is a broken line. Technically a line segment is a broken line.



The teacher may have noticed broken lines among the illustrations of curves and closed curves given earlier.

Collinear

Three points are <u>collinear</u> if and only if one of them is between the other two. See the illustration.



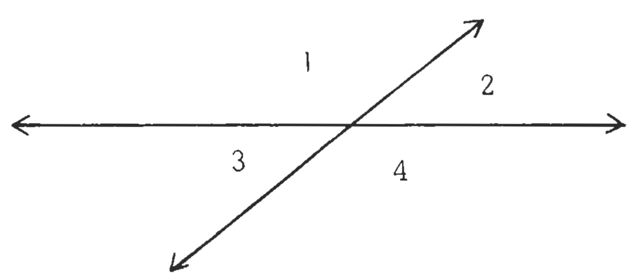
Straight Line

A <u>straight line</u> (or we often say a <u>line</u>) is a set consisting of two points, together with all points collinear with those points. The two points are said to determine the line which contains them.

A and B are two points. We represent the line AB determined by A and B as above. The arrowheads are drawn to suggest that all points such as C, collinear with A and B, belong to the line no matter how distant they may be. Verify with a straightedge the collinearity of A, B, and C. Is point D on line AB? (No.) Note that the segment AB is a subset of the line AB. It should be clear that line AC = line AB = line BC, etc. The lines are sets of points. Equality here is set equality.

Regions

A line in the plane separates the plane into two regions because the line is of infinite length. The line is the common boundary of these regions. When two intersecting lines are drawn, four regions are formed.

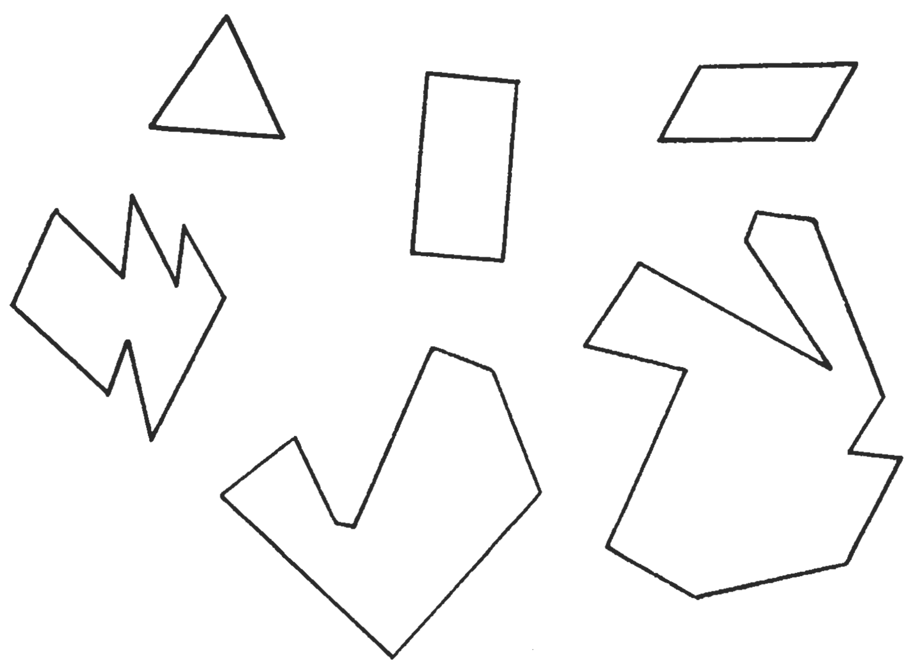


Ray

A ray is a subset of a straight line consisting of a point A of the line, together with all points of the line on one side of A. The arrowhead shows that the ray goes on in one direction.

Polygon

A polygon is a union of line segments forming a simple, closed curve in the plane. The segments are called the sides of the polygon. (A polygon can also be described as a simple, closed broken line.)



The teacher will note that a polygon must have at least three sides. A polygon of three sides is called a triangle or a 3-gon. A polygon of four sides is a quadrilateral or a 4-gon. A polygon of n sides is called an n-gon.

Suggested Activities on Line Segments, Lines, Rays and Polygons

1. Invite two children to stand at the front of the room, six or eight feet apart, and ask the class to pretend these two are points. Ask for a volunteer to be another point between these two. Repeat several times, with different children, and encourage variety in the responses: the third child need not be midway between the first two.

2. With two children situated as above, ask another child to walk along the line segment having these two as endpoints. The plane is represented by the floor.

3. Review the definition of line segment: Two points together with all points between them. Put the words "line segment" with a sketch on the geometry bulletin board.

4. Again ask two children to be endpoints and invite a third child to be a point collinear with these two. Encourage a variety of responses.

If the original two children are at A and B, respectively, the third child may take any position on the line AB—for example, any position indicated by X below.

It should be stressed that if there were more room, positions even farther away would be acceptable.

5. Ask a child to the front of the room to represent a point A. Tell the class the first child is to be the endpoint of a ray. Ask for a volunteer to be another point B of the ray. (He can stand anywhere at all!) Then ask for further volunteers to be other points of the same ray. (It must always result that another such point stand between the first two, as at X below, or so that the second is between him and the first, as at Y below.)

6. A more challenging version of Activities 4 and 5 can be played in a long corridor or on the playground. Volunteers can take turns as referee, attempting to judge which positions are acceptable as collinear. This judgment is harder to make over longer distances.

More members of the class can be invited into the lineup, one by one.

7. Adaptations of Activities 1, 2, 4 and 5 can be played by children individually or in small groups by aligning blocks instead of pupils.

8. Review the mathematical definition of a line. A line that goes on as far as you like in both directions is shown this way:

(Draw this on the board and explain it to the children.)

9. Explain that we cannot, of course, draw a whole line on a sheet of paper or on the chalkboard because it "keeps going" in both directions. So we make a part of a straight line, which is called a line segment, and use arrowheads to suggest the rest.

Draw the following examples on the board.

Here is line segment XZ.

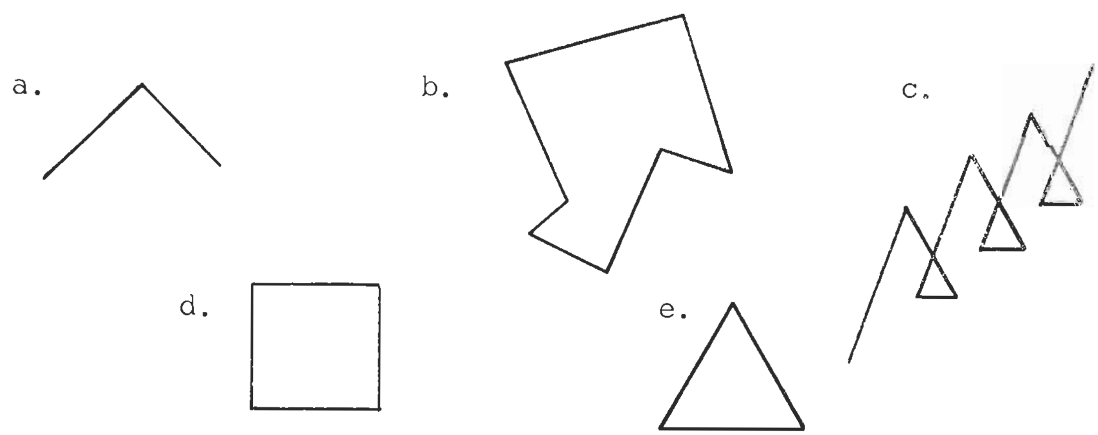
Here is line LK.

Put the word "line" on the bulletin board of geometric figures with an illustration.

10. Review the definition of a ray by asking the children to suggest examples of straight lines, line segments or rays from their experience. Examples that suggest line segments are a stretched string, the edge of a desk, the shadow of a window frame, etc. More imagination is required for a ray—perhaps a ray of light or any of the line segment illustrations extended in the imagination from one endpoint through the other without bound:

Put the word "ray" on the geometry bulletin board with an illustration.

11. A curve which is made up of a succession of line segments joined end to end is called a broken line. Draw the following examples on the board.

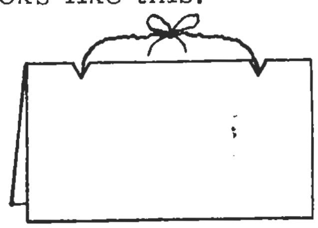


Each of these figures is an example of a broken line.
"a" is also a non-closed simple curve.
"b" is also a closed curve and a simple curve.
"c" is a curve that is neither simple nor closed. It contains 3 closed curves.
"d" is a square. It is also a simple closed curve.
"e" is a triangle. It is also a simple closed curve.

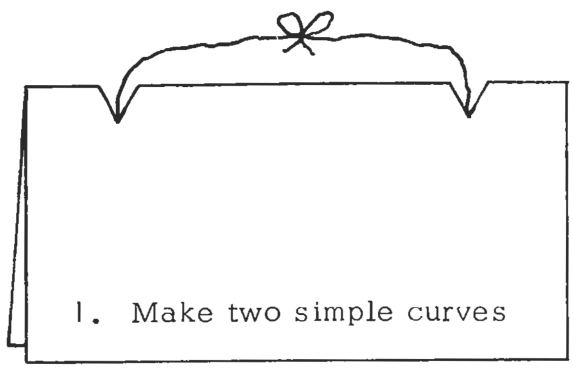
Add the examples of a broken line to the geometry bulletin board.

12. Have the children draw simple curves, non-simple curves, straight lines, broken lines, closed curves and non-closed curves on the board and classify them as they draw them. (Be sure they understand that a line segment is part of a straight line. They should have rulers to work with.)

13. Give each child a copy of Worksheet 20 and two pieces of 9 x 12 newsprint. Instruct them to fold both pieces in half. Then cut small triangles in the folded edge. Thread it with yarn inside the fold, and tie it. The booklet then looks like this:

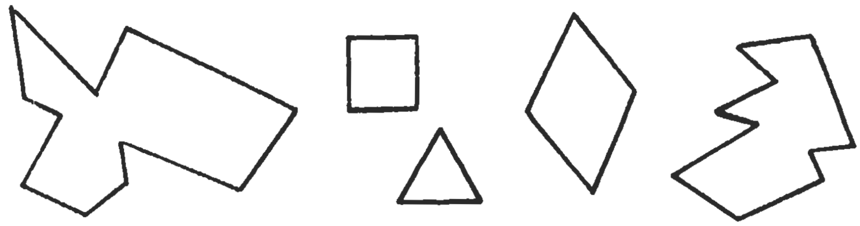


There are 8 pages. Each box from Worksheet 20 is to be pasted at the bottom of a page.



The boxes should be pasted in order on the pages 1 through 8. After this has been done, the children should follow the instructions for each page.

After Worksheet 20 has been completed, choose someone to read instruction 7. Then ask several children who drew 7 correctly to go to the board and copy their figures. At that point, tell the class that all these answers are correct because they are all simple closed broken lines. This means they are simple closed curves made up of broken lines. Then tell them that all of these figures are called polygons.



Be sure that the children understand that triangles and rectangles are simple closed broken lines.

14. Ask the children if they can think of other ways to describe a polygon, such as using the concept of union. A polygon may be referred to as a simple closed curve which is the union of line segments (each line segment being considered as a set of points).

WORKSHEET 20

1. Make two simple curves.

2. Make four simple closed curves.

3. Make three simple curves which are not closed.

4. Make two curves that are not simple.

5. Make a line segment.

6. Make a figure using broken lines.

7. Make a simple closed broken line.

8. Make a non-simple curve that contains 2 simple closed curves.

Review Activities on Line Segments

Since this material on line segments has been presented in Unit VII, "Introduction to the Number Line," page 5, it might not be necessary to do the following exercise. However, it is repeated for those children who appear to need additional work.

1. Give each of the children a piece of 9 x 12 newsprint. Have them fold it once to make four pages. Then number the pages.

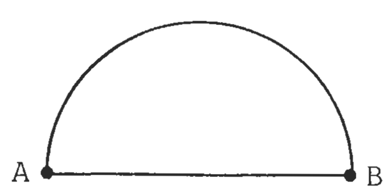
On page 1 tell them to locate a point and draw four straight lines through the point. Remind them to draw arrowheads.

On page 2 show them how to locate two points and label them C and D. Then direct them to draw a line that goes through both points.

On page 3 instruct them to locate two points, label them X and Y, and draw a straight path from X to Y. Ask the children what they have made. It is a line segment because it consists of X and Y and all points between X and Y.

On page 4 have them locate 2 points. Let them choose 2 different letter names. They may choose any letters they wish, to show that the name given to the point or to the line segment is for convenience and is not significant. Tell them to make a line segment by using their rulers to connect the two points. Call on a few children to tell the names of their line segments, such as line segment DG or TN, etc.

Then ask if they can draw another line segment between the same two points that is different from the one they had drawn. Accept all answers. Then put 2 points and a connecting line segment on the chalkboard. If some said yes, ask a child to demonstrate on the board. It might look like this:



Explain that a line segment must be straight. Mark a point C on the child's curve not on the line segment AB and say:

> Suppose Mrs. Bug stands at A, Mr. Bug stands at B, and there is a Bugbear at C. Can Mr. and Mrs. Bug see each other? (Yes.) Or is the Bugbear in the way? (No.) Is the Bugbear between Mr. and Mrs. Bug? (No.) Is C between A and B? (No.) Does C belong to the line segment AB? (No.)

Although many curves join two points, only one of them is a line segment that connects two points. Some child might do this:

Illustrate again with Mr. and Mrs. Bug. Explain that the drawing has one straight line path and one broken line path. The children should see that there is no other way to draw line segment AB.

2. To prepare the children for Worksheet 21 be sure that they understand the meaning of the word "pair." Then ask for examples, such as pairs of shoes, gloves, skates, socks, etc.

Put three dots on the board to represent 3 points. Do not put them in a straight line. Have someone draw a line segment between a pair of points. Have someone else draw a line segment between another pair. They can make a total of 3 line segments connecting the pairs of points. Do this enough times for the class to understand what is meant by "draw a line segment connecting a pair of points." The figure that is made on the board should look like a triangle. Then demonstrate what is meant by the directions, "Mark 2 points in the space. Name them A and B. Mark another point not in the straight line AB." Show what would happen if they were in a straight line.

It is impossible to see line segment AC as separate from line segments AB and BC.

When children understand, give them Worksheet 21. This worksheet is a repeat of Worksheet 1, VII, "Introduction to the Number Line," page 9. It is not designed for independent endeavor. The teacher should circulate among the children to answer questions. The answers for the worksheet are 1 and 3.

Make 2 dots to represent 2 points.

Draw as many line segments as you can connecting pairs of these points. Use your ruler. How many did you draw?

______

Mark 2 points in this space. Name them A and B. Mark another point not in the straight line AB.

Draw as many line segments as you can connecting pairs of these points. Use your ruler. How many did you draw?

____

Commentary on Worksheet *22

Worksheet *22 is a repeat of Worksheet 2, VII, "Introduction to the Number Line," page 11. Prepare the children for Worksheet *22 by asking someone to locate 4 points on the chalkboard. Be sure that no three points are in a straight line. Call on different children to draw line segments between pairs of points. It is possible to draw a total of 6 different line segments.

Distribute Worksheet *22 when the children appear to be ready for it. If sufficient time has been spent in board practice, the children should be able to do the worksheet independently. The answers for Worksheet *22 are 6 and 10.

WORKSHEET 22a

Line Segments and Points

Make 4 dots to show 4 points. Do not put any three points in a straight line.

Use your ruler and draw as many line segments as you can connecting pairs of these points. How many line segments did you draw? ______________________________

Mark 5 points in this space. Do not put any three points in a straight line.

Use your ruler and draw as many line segments as you can connecting pairs of these points. How many line segments did you draw? ______

Suggested Activities on Lines and Regions

1. This activity should reveal that a line drawn across a piece of paper separates the plane represented by the paper into two regions that have the line itself as their common boundary. The line does not belong to either region.

Give the children 9 x 12 newsprint. Have them mark two points and label these points M and N. Then direct them as follows, providing a pause for investigation after each question: (Discussion may be on a question-by-question basis or may be deferred to the end of the activity.)

Draw the line MN, extending it in both directions until it reaches the edges of the paper. Pick any point P not on the line MN. Pick any other point Q not on the line MN so that the line segment PQ crosses line MN. (Q is on the other side of the line from P.)

Can you join P to Q by a curve that does not cross line MN? (No.)

Pick any point R, such that you can join P to R by a curve that does not cross line MN. Does the line segment PR cross line MN? (No.) Does the segment QR cross the line MN? (Yes.)

Shade all points R that can be connected with P by a curve that doesn't cross line MN. Pick any two points A and B in the shaded region. Can A be joined to B by a curve not crossing MN? (Yes.)

Pick any two points C and D in the unshaded region. Can C be joined to D by a curve not crossing line MN? (Yes.)

Pick any point E in the shaded region and any point F in the unshaded region. Can E be joined to F by a curve not crossing line MN? (No.)

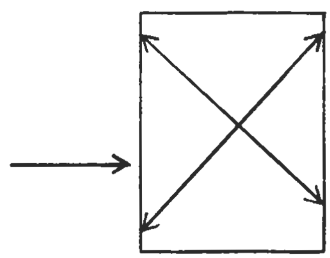
Pick any point G on the line MN. Draw a small circle with G as a center. Then draw an even smaller circle with G as center. Are there points of the shaded region inside each circle? (Yes.) Are there points of the unshaded region inside each circle? (Yes.) Could the circle be made so small that the answers would be "No"? (No.)

Pick any point H in the shaded region. Can we draw a circle about H so all points inside the circle are shaded? (Yes.)

Pick any point J in the unshaded region. Can we draw a circle about J so all points inside the circle are unshaded? (Yes.)

If the children are interested, it may be pointed out that the last three paragraphs are related to the fact that all points "near enough" to any given point of a region are also in that region, which implies that the boundary itself is not part of the region.

2. On a piece of 9 x 12 newsprint, have the children draw two intersecting lines, each line extending to the edges of the paper: e.g.



Then direct them as follows:

Find a point P that is not on either line, and represent it with a red dot. Find a point Q which can be joined to P by a curve crossing neither line, and dot it red also. Dot all other similar points red. (After some furious dotting, this should result in the shading red of one of the four regions of the "plane.")

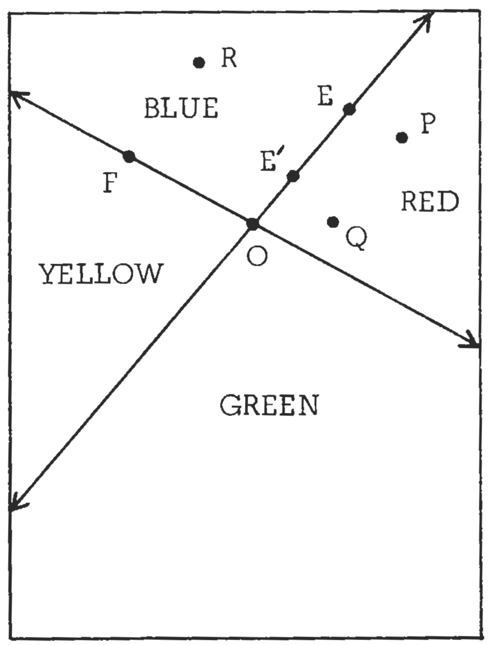
Find a point R not in the red-shaded region. Dot it blue. Continue as before until a blue-shaded region is obtained.

Repeat the instructions for the two remaining regions—yellow and green, for example.

Are there any red (or yellow or green) "points" near P? (No.)

Remind the children that a straight line is a certain set of points. Ask the children what the intersection of their two lines is. (It is a set of points. It has exactly one member: the point at which the lines cross. Accept the response that this point is the intersection.) Have the children label this point O. Point out that our sketch contains four rays with endpoint O.

The following colors may be used:



Choose a point E on one of the lines, different from O. What points are close to E? (There are points of two regions close to E. The colors of these regions will differ from paper to paper. In our example, they are blue and red.) Pick another point E' (read "E-prime") on the same ray as E. What points are close to E'? (The same as for E.) The ray OE belongs to the boundaries of what two regions? (The same two: in our illustration, blue and red.)

Repeat for the other rays.

What regions contain points close to O? (All four regions.)

Point O belongs to the boundaries of what regions? (All four regions.)

What is the boundary of the blue region? (In our sketch, the union of the rays OE and OF.)

The children's responses will probably not contain the word "union." Their attempts to deal with the question can be guided by questions such as "Which points that are not blue have blue points near them?" Remind them of the answers to the preceding questions of this activity, thus leading them to something like "A point is a boundary point of the blue region if it is a point of the ray OE or if it is a point of the ray OF—and not otherwise." If necessary, remind them of the definition of the union of two sets: An object belongs to the union of sets A and B if it belongs to A or if it belongs to B, and not otherwise.

Finally, if it is necessary, present them with the description of the boundary as the union of the two rays.

When the children have done this a few times and understand it, give them Worksheet *23 to work independently. The answers are 2 and 4.

WORKSHEET 23a — Lines and Regions

1. Draw a straight line to the edges of the page. Use your ruler.
2. How many different regions are there?
3. Number them and put the answer in the little box.

How many regions?

1. Draw a straight line to the edges of the paper. Use your ruler.

2. Draw another line that crosses the first one. Use your ruler.

3. How many different regions are there?

4. Number them and put the answer in the little box.

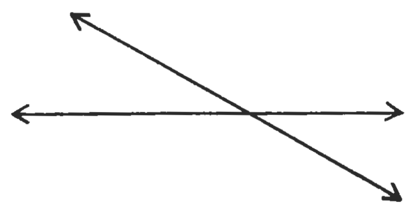
How many regions?

Activities on Intersecting Regions

3. Have a child use a yardstick and draw a line on the board.

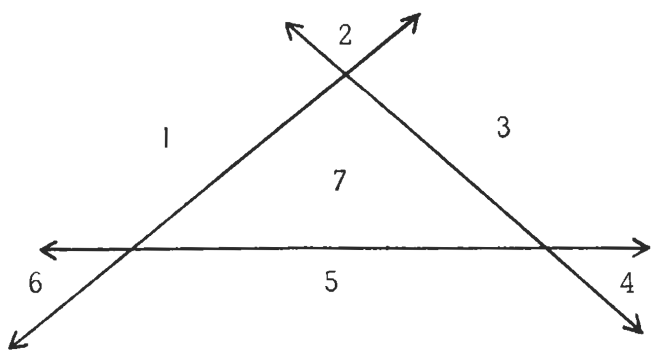
Step 1

Ask the class how many regions there are. (2) Call on a child to draw a line that crosses the first line.



Step 2

Ask how many regions there are. (4) Have another child draw a line that crosses both of the other two lines. (Only two lines intersect at the same point.)



Step 3

Ask someone to count and label the regions.

4. Give the children Worksheet *24 to work independently. Permit them to ask each other for help with any words they have trouble reading. The answer is 7.

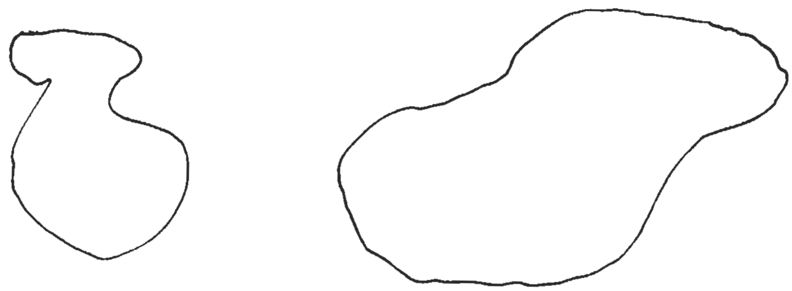
WORKSHEET 24 Intersecting Regions

1. Draw a straight line.
2. Draw a line that crosses the first line.
3. Draw another line that crosses both of the other two but not at the same point. (Only 2 lines may go through the same point.)
4. Number the regions.
5. Put the answer in the little box.

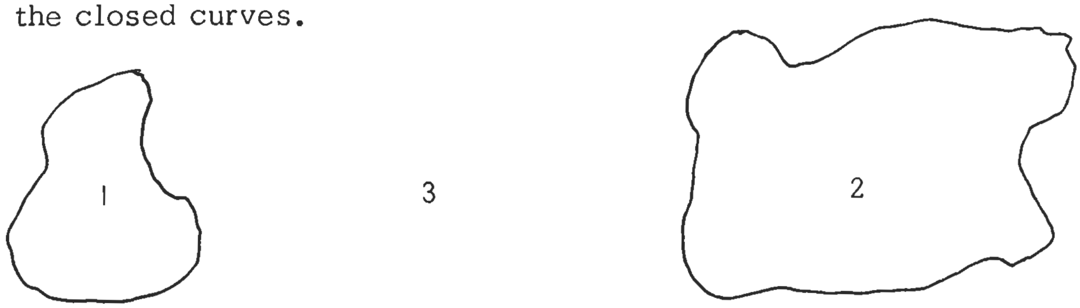
How many regions?

Suggested Activities on Intersecting Regions

1. Draw two simple closed curves on the chalkboard.

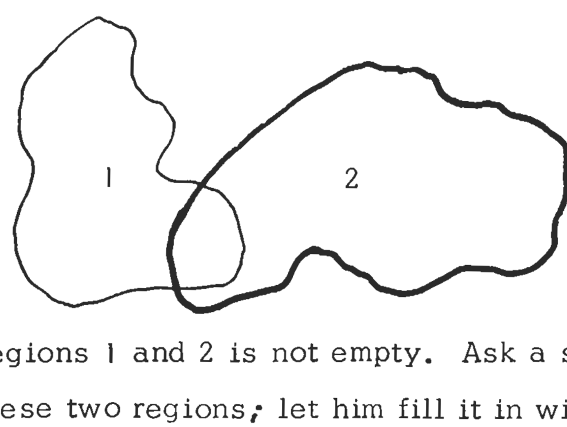


Ask someone to label the regions. There are three regions—one inside each of the two closed curves and the third one outside both of the closed curves.



For this sketch there are no points inside both curves. The intersection of regions 1 and 2 is empty.

2. Draw the following figure on the chalkboard.



The intersection of regions 1 and 2 is not empty. Ask a student to show the intersection of these two regions; let him fill it in with chalk.

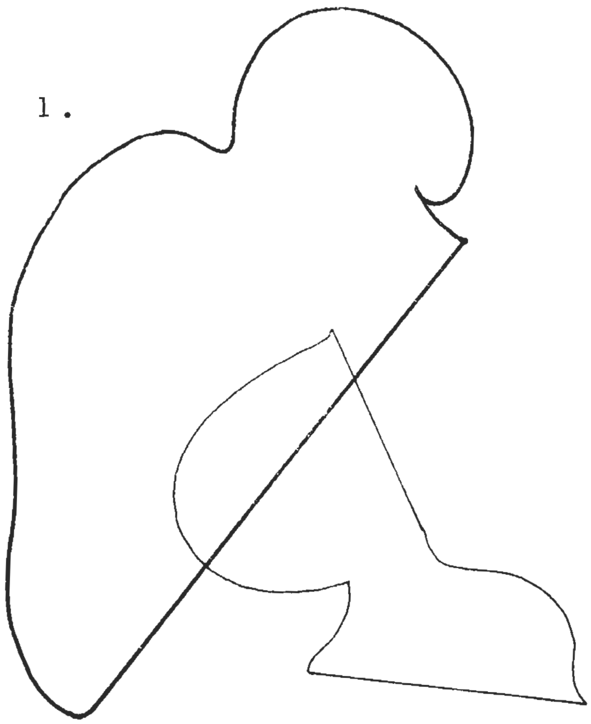
> Note: Be sure that you draw one curve darker than the other or in another color so that the children are not confused.

3. Make several figures on the board with regions that have a non-empty intersection, and have some children identify the intersection of the two regions. Distribute Worksheet*25.

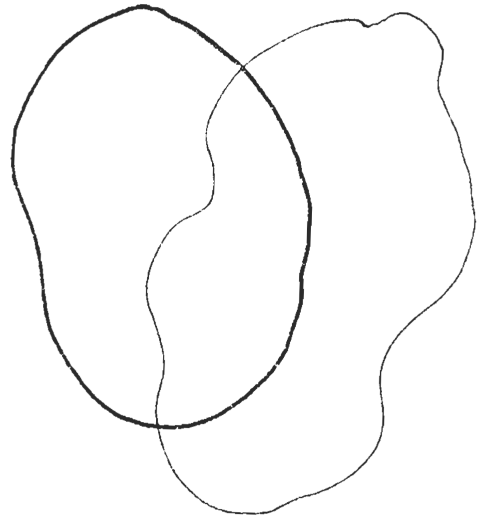
WORKSHEET 25 Intersecting Regions

Color the intersections of these regions.

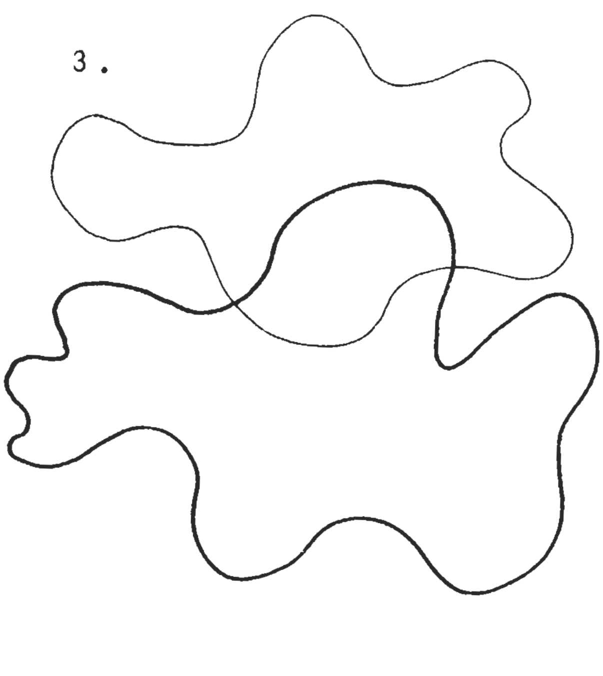
1.



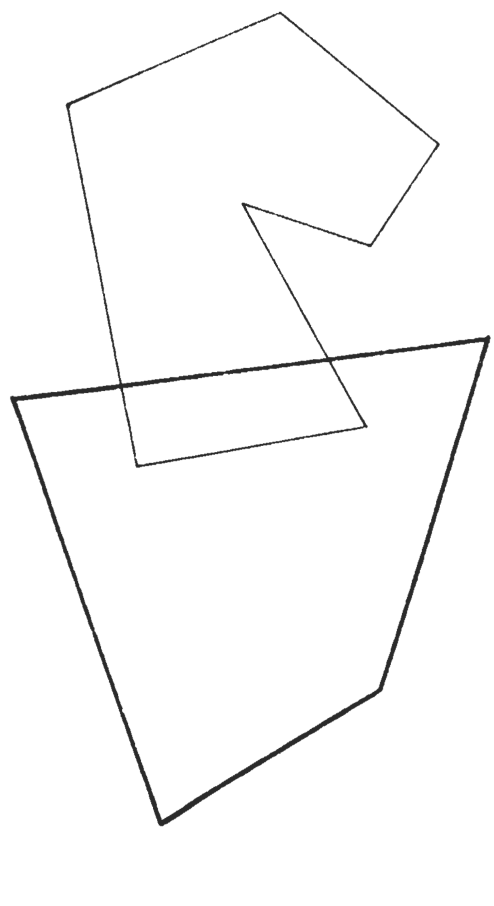
2.



3.



4.



Review Activities

1. Ask for volunteers to put sketches on the chalkboard of a line segment, a line, and a ray, properly labeled. Review the definitions by asking questions which remind the children that a line segment has two endpoints that belong to the segment; that a ray has one endpoint that belongs to the ray. Remind them also of the significance of the arrowheads in the sketches of the line and the ray.

2. Distribute Worksheets*26 a, b, and c for independent work. There should be four numbered regions in the solution to 4 and seven in the solution to 5.

WORKSHEET 26a Review

Use Your Ruler!

1. Make 3 line segments. Label them.

2. Make 4 straight lines.

3. Make 2 rays. Label them.

4. Make 2 lines that cross each other. Label the regions.

WORKSHEET 26c

Review

5. Make 2 lines that cross each other. Make another line that crosses both of the others but not at the same point. Label the regions.

6. Make a straight line. Mark 2 points on that line and label them.

Teacher's Background on Triangle, Angle and Rectangle

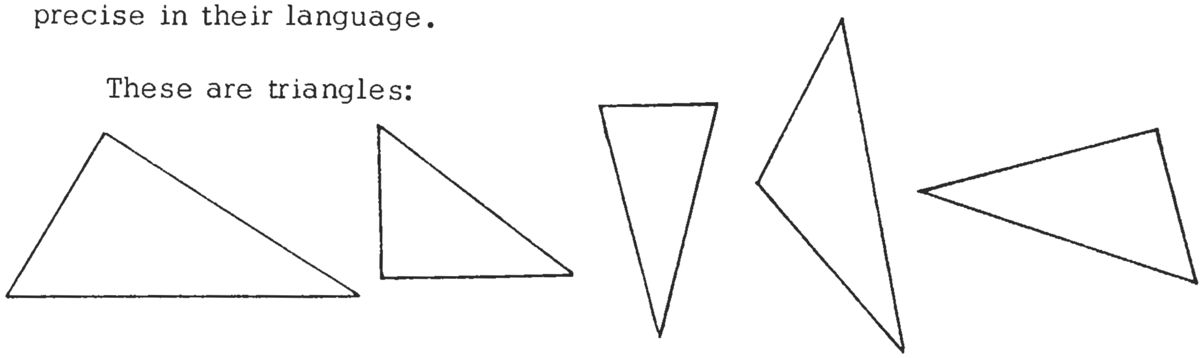
Triangle

A triangle is a simple closed curve of three line segments, so a triangle is a special kind of polygon. The first part of the word—"tri"—means 3.

Many people use the word "triangle" to refer also to the set of points on and inside the closed broken line. In mathematics it is important to say exactly what we mean. It makes no difference how we define the word "triangle," but once we decide whether "triangle" shall mean the region or its boundary, we should be consistent and stick to our decision. We have chosen to define "triangle" as the boundary. We call the interior a triangular region.

The teacher should explain the difference and should use the words consistently herself. But she should not insist that the children be so precise in their language.

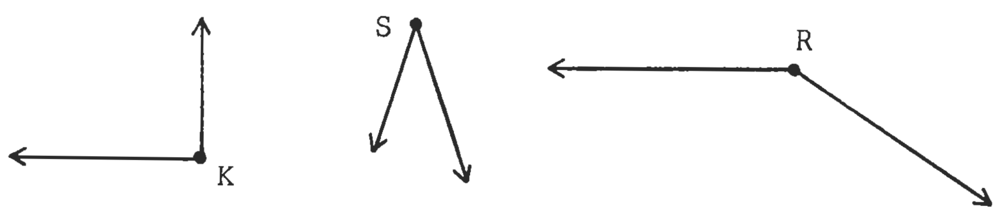
These are triangles:



Since a triangle is a polygon, it can be thought of as a closed curve which is the union of three line segments, or simply as a three-sided polygon.

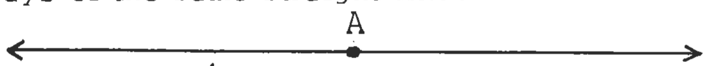
Angle

An angle is sometimes defined as the union of two rays having a specified common endpoint. For example:

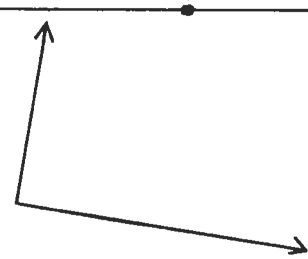


These rays are called the rays of the angle.

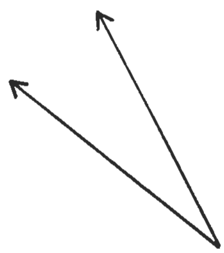
In this unit the children will recognize a straight angle by sight. Its rays are opposed rays of the same straight line.



Right Angles



Acute Angles



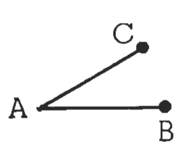
Obtuse Angles

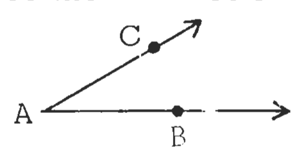
These angles will also be identified by sight almost exclusively, but should be introduced by Worksheet *39.

Rectangle

A rectangle is a four sided polygon (or a 4-gon) whose pairs of opposite sides are of equal length and all of whose angles are right angles.

> Note for the discriminating reader: To mediate between our definition of angle and the use of the word "angles" above, the following remark will serve: If AB and AC are line segments with common endpoint A, there is exactly one angle at A of which AB and AC are subsets. See the illustration.

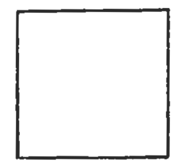




The segments are said to form this angle. The word "angles" in the definition of rectangle can be interpreted as angles formed by the sides (segments) of the 4-gon in this sense.

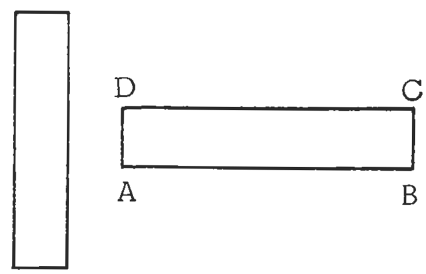
Square

A square is a special kind of rectangle. All four of its sides are equal. Since it is a rectangle, the four angles are right angles.



Oblong

A rectangle which is not a square is called an oblong.



Since oblong ABCD is a rectangle, its four angles are right angles, the length of segment AD equals the length of segment BC, and the length of CD equals that of AB. Since an oblong is not a square, AD and AB must be of different lengths.

We will consistently use the word "oblong" in this precise way.

Do you see that if a rectangle is not an oblong, then it is a square?

Suggested Activities on Triangles

1. Have the children name as many things as they can think of that have a triangular shape.

2. Have the children find representations of triangles and triangular regions in an assortment of flannel shapes and put them on the flannelboard.

3. Add a triangular figure with the word "triangle" beside to the other figures on the bulletin board. Give the children Worksheet *27.

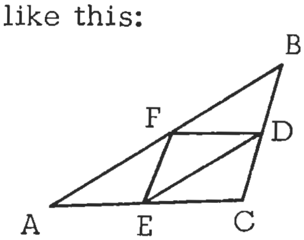
4. Distribute a large sheet of manila paper to each child and put these directions on the chalkboard:

> Fold the paper to make 4 sections.
> Draw a large triangle in each section. Use your ruler.
> Draw a little boy inside one triangle.
> Draw a little girl inside one triangle.
> Draw a toy inside one triangle. Color it red and blue.
> Draw a design inside one triangle.

Children who are not able to read these directions should be permitted to ask for help from the better readers, since this is an assignment in mathematics rather than in reading.

5. Distribute Worksheet 28. Ask, "How many triangular figures are there?" (4 small and 1 large.)

Copy the figure on the chalkboard and letter it like this:



(In copying the figure, be sure that you find the midpoint of AD for F, the midpoint of AC for E, and the midpoint of BC for D.)

Have the children letter the triangle on their worksheets as you have done the one on the board. Ask them which they think is longer, AE or EC, AF or FB, and CD or DB. Accept all answers.

Now number the interiors of the 4 small triangular figures on the board. Have the children copy the numbering on their worksheets. Then have the children cut out the figures.

Repeat the questions as to which is longer, AE or EC, and so on, while the children manipulate their figures to make the comparison. Then have the children put the small triangular figures together again to form one big triangular figure. Some may notice that the smaller figures may now be in a different order from your diagram on the board. Through a class discussion draw from the children the idea that the one big triangular figure is made up of four smaller ones, all of the same size and shape.

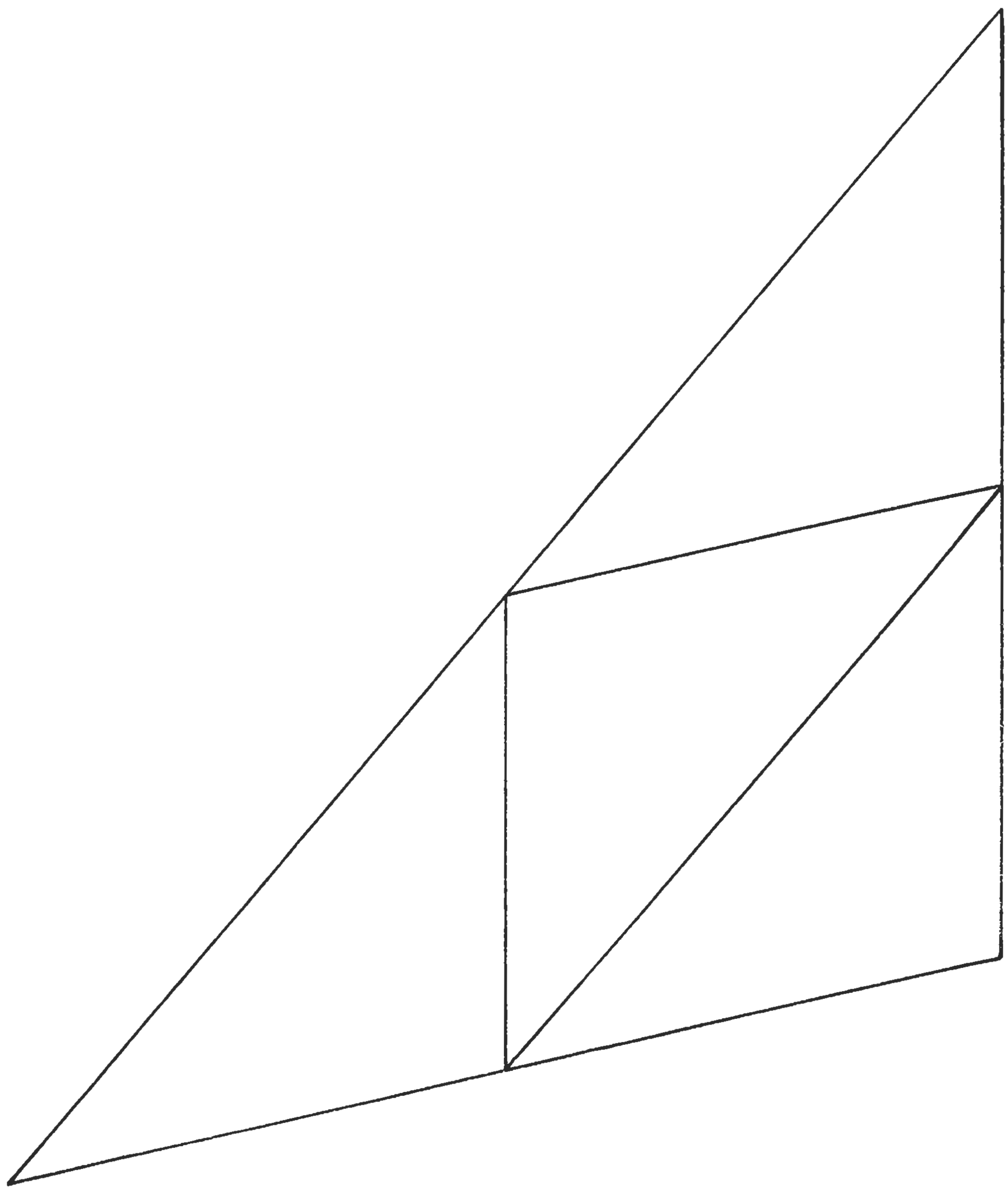
6. Instruct the children to use rulers to draw a large triangular figure on a sheet of paper and cut it out. Tell them to cut it up into four triangular figures, all of the same size and shape. You may have to remind the children that they just discovered that in the last triangular figure they worked with, AF was the same length as FB, etc.

7. Distribute Worksheet 29. (Answers: 1, 3, 4, 5, 3 or 4, 1.)

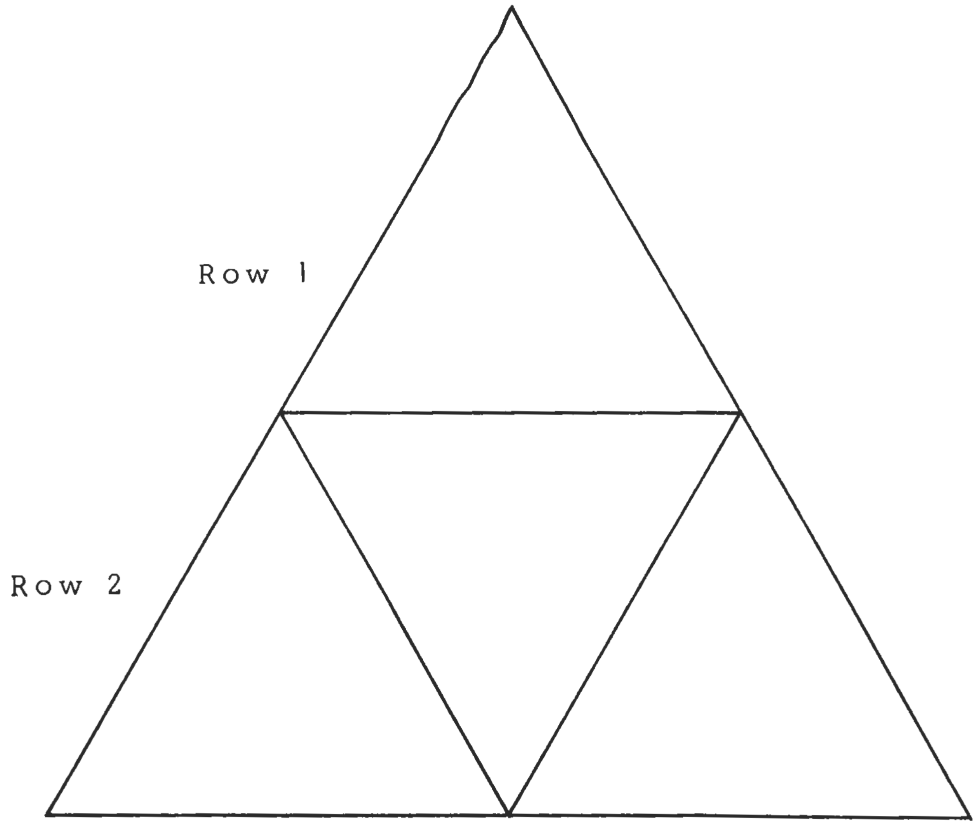
WORKSHEET 27 Triangles

1. Draw a big triangle. Use your ruler.

2. Make a design inside the triangle. Use five colors for the design.



WORKSHEET 29 Triangles



1. How many triangles are in Row 1? ____________

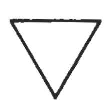
2. How many triangles are in Row 2? ____________

3. How many triangles are the same size? ____________

4. How many triangles are there all together? ____________

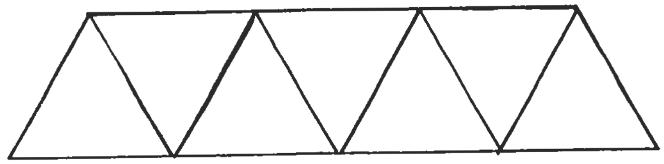
5. How many triangles look like this? ____________

6. How many triangles look like this? ____________

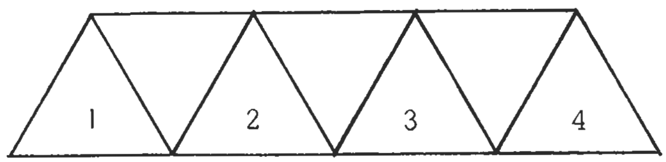


More Activities on Triangles

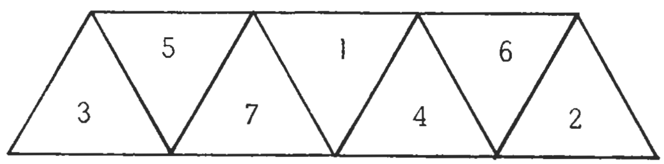
8. Put the following pattern of triangles on the chalkboard:



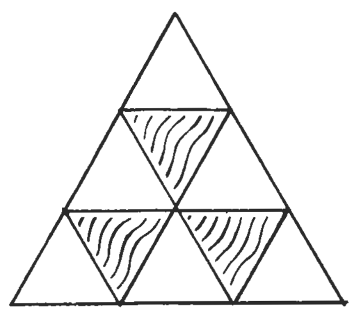
Ask a child to put a numeral in each triangle he can find. Have others continue until you have:



Of course the numerals can be on any of the triangles:



9. Then put the following on the board:



Be sure the shading lines in the triangular regions do not touch the boundaries. (If they did touch, you would be making many more triangles within each triangle.) Ask the children how many triangular regions look like this:  Ask someone to point out the regions on the board.

Then ask how many triangular regions look like this: 

Continue until the class can recognize triangles in both the upright and inverted positions. Also encourage careful observation to see which triangular regions look exactly alike.

10. After many experiences in this type of observation, distribute *Worksheets 30 and 31.* These can be worked independently. Then distribute *Worksheet 32 which may need directions by the teacher.*

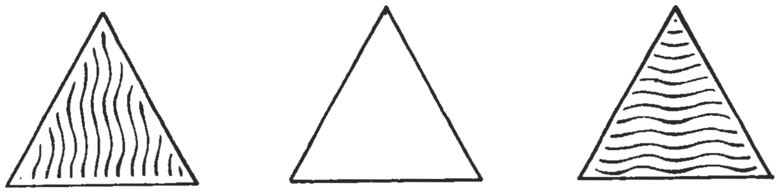
Answers to Worksheets 30-32

Worksheet 30	Row 1	3
	Row 2	4
	Row 3	7
	1.	4
	2.	4
	3.	2
	4.	1
Worksheet 31	Row 1	1
	Row 2	3
	Row 3	5
	1.	1
	2.	3
	3.	4
	4.	5
	5.	9
	6.	Highest possible: 13

Worksheet 32

Highest possible number is 14. There are:
5 triangles composed of 1 small region
6 triangles composed of 2 small regions
2 triangles composed of 3 small regions
1 triangle composed of 4 small regions.

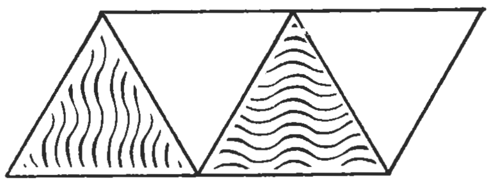
Row 1



How many triangles?

______

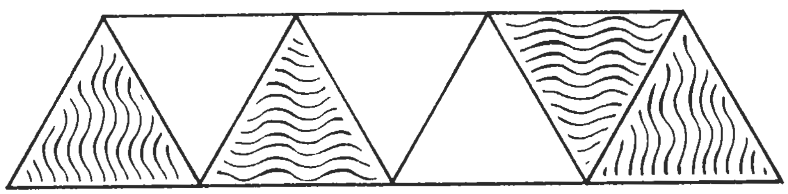
Row 2



How many triangles?

______

Row 3



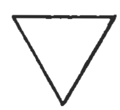
How many triangles?

______

1. How many triangular regions in the union of all rows look like this?

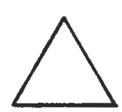
______

2. How many look like this? Color them blue.



______

3. How many look like this? Color them green.



______

4. How many look like this? Color them orange.

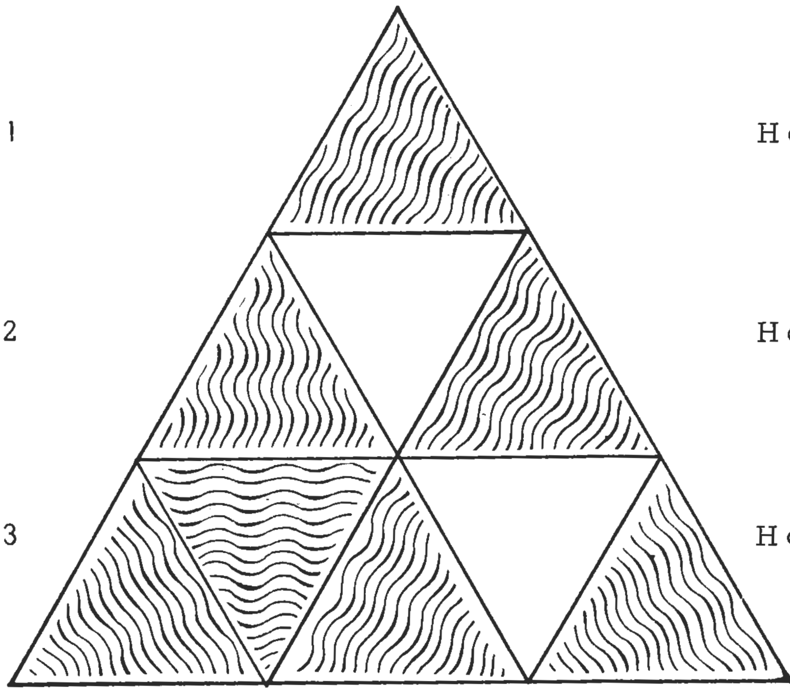
______

WORKSHEET 31 Triangles

Row 1 How many?

Row 2 How many?

Row 3 How many?



1. How many small triangles are in row 1? ________

2. How many are in row 2? ________

3. How many small triangles are in rows 1 and 2 together? ________

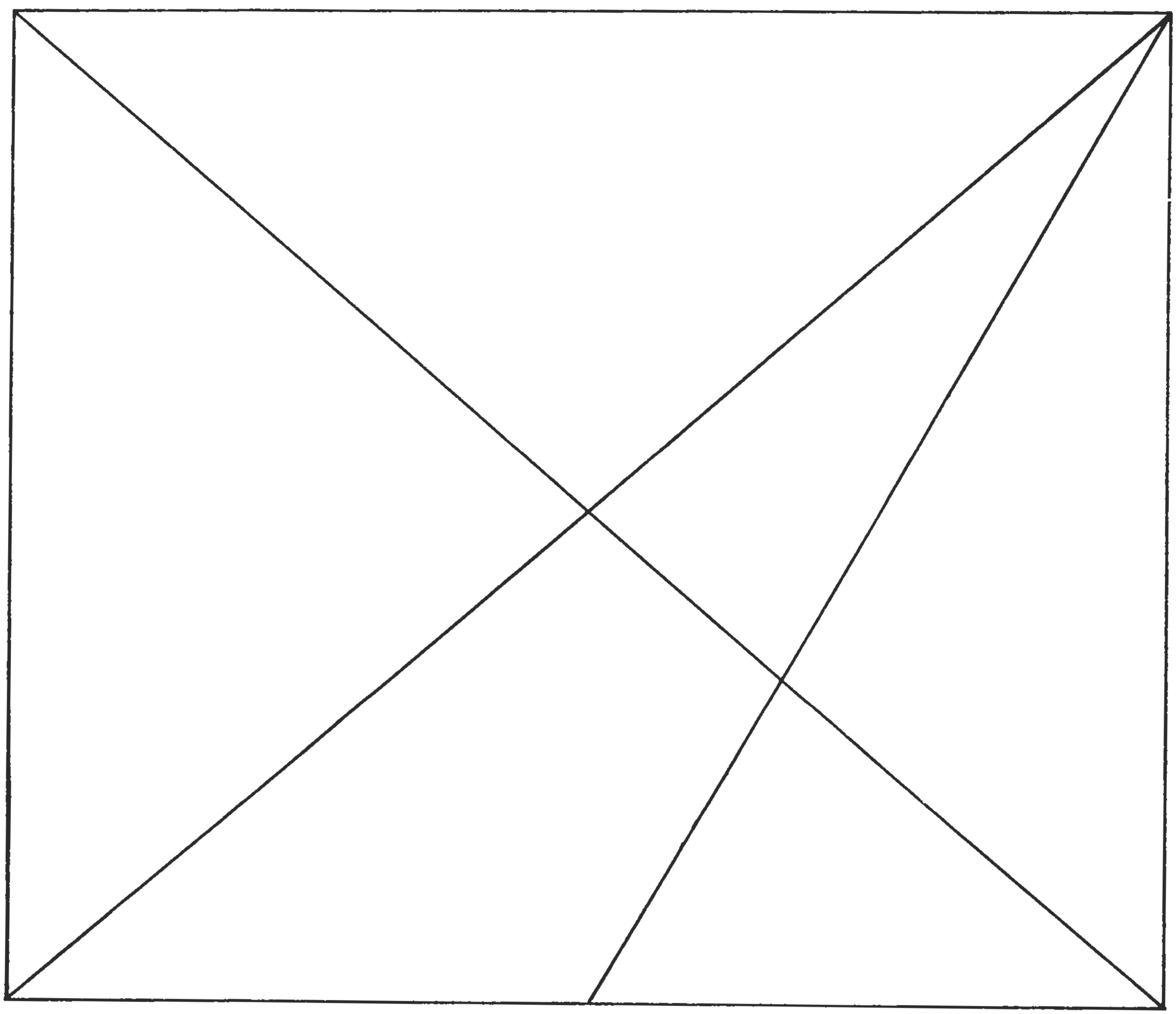
4. How many are in row 3? ________

5. How many small triangles are in rows 1, 2, and 3 together? ________

6. How many triangles of all sizes can you find? ________

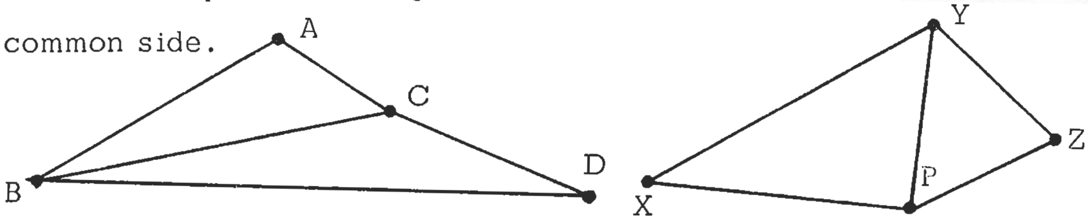
WORKSHEET 32 Triangles

Here is an oblong with some triangles. How many triangles can you find in this picture? ________________



Suggested Activities on Triangles with Common Points

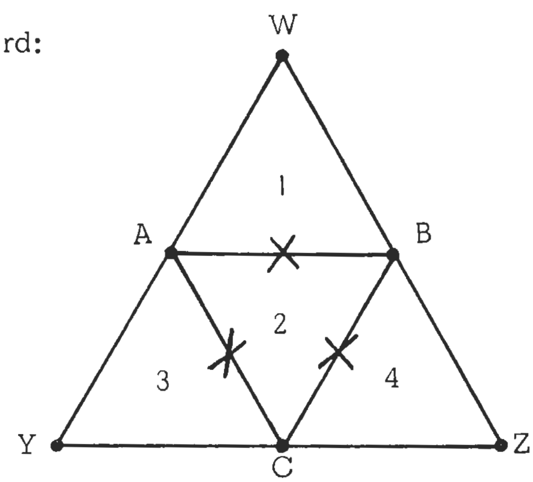
1. Put two pairs of triangles on the chalkboard so that each pair has a common side.



Ask a child if he can find a point that is on two triangles. Have him place a dot to represent the point he is thinking of. Any point on line segment BC or PY is correct, including the endpoints.

Ask a child to name the line segments that are boundaries common to two of the triangular regions.

2. Put the following figure on the board:



Have a child locate a point that is on two triangles. Ask him to identify the two triangles.

The obvious correct answers are any points on the line segments above marked with an X, as long as they are not the endpoints. Points on line segment AB are part of triangles 1 and 2. Points on line segments BC are part of triangles 2 and 4. Points on line segment AC are parts of triangles 2 and 3.

If a child marks a point on AW, BW, BZ, CZ, CY, or AY, ask him to identify the two triangles. Let's assume that a child marks a point on line segment AW (as marked above). If he says that the point he located is part of triangle 1 and part of triangle WYZ, he is correct.

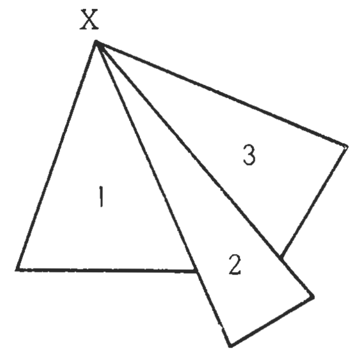
This, however, requires more mature thinking than is expected from most first graders. In this particular situation, remember that BZ, WB, CY, ZY, CZ, AW are common to a small triangle and also to the large triangle WYZ. Only the most alert children are likely to identify this type of common boundary.

For such children the questions might be asked:

Which points are on three triangles?

Is there any point on four triangles?

3. Make a new figure on the chalkboard:

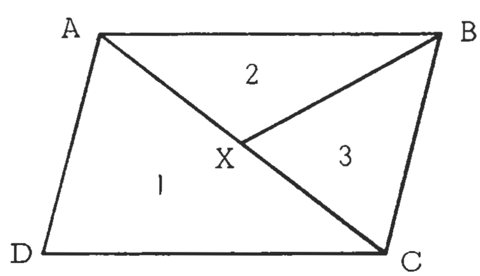


Tell the children that you have constructed 3 triangles. Have them label the triangles 1, 2, and 3.

Then ask if someone can find a point that is on all 3 triangles. There is only one correct response. The point labeled X is part of triangles 1, 2, and 3.

4. Provide many similar experiences for the children.

Note: Be alert to the possibility that in constructing figures you may inadvertently include an extra triangle, e.g.

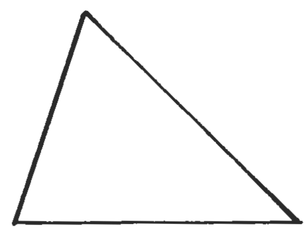


In this figure are three obvious triangles, but triangles 2 and 3 form another triangle, ABC. If you asked the class to find a point on just three triangles, then the X marked is not only part of triangles 1, 2, and 3, but also part of the *larger triangle ABC. The correct responses are point B or any* point of segment AC other than X.

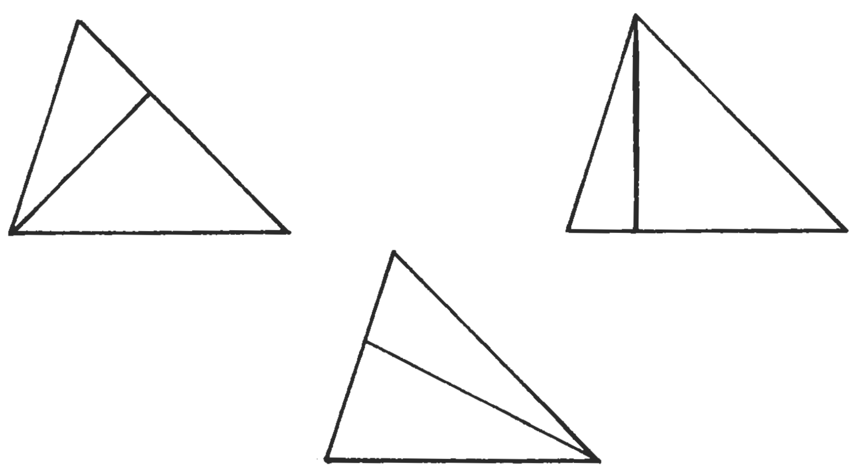
5. Distribute Worksheet*33 for independent work.

Suggested Activities on Triangles with a Common Region

To prepare children for this worksheet draw the following figure on the chalkboard:



Ask the children if one of them can change this sketch into a sketch containing three triangles by drawing one line segment.



Then provide a piece of light brown chalk and have someone color lightly in one triangular region.

Ask a second child to color another triangular region lightly with orange chalk.

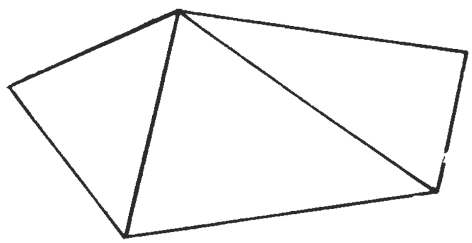
Have a third person color in the region of the large triangle with yellow chalk.

Children will notice that one small triangle is both yellow and brown. Help them realize that this means that the large triangle and the small triangle have a common region.

Ask someone if he can find two more triangles with a common region. Someone will notice that the other small triangle is both orange and yellow, showing that it has a region in common with the large triangle. Repeat the activity as often as necessary. Then distribute Worksheets 34 and 35.

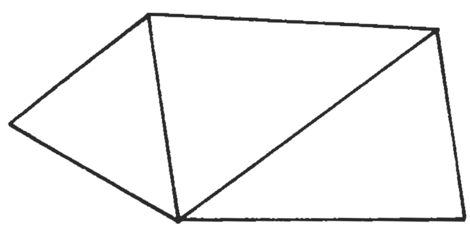
WORKSHEET 33

Triangles with Common Points



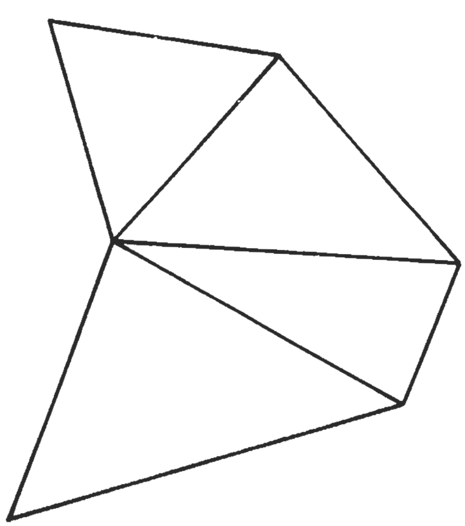
1. Find one point that is on 2 triangles. Use a red crayon.

2. Find one point that is on just 1 triangle. Use a green crayon.



1. Find one point that is on just 3 triangles. Use a blue crayon.

2. Find one point that is on just 2 triangles. Use a yellow crayon.



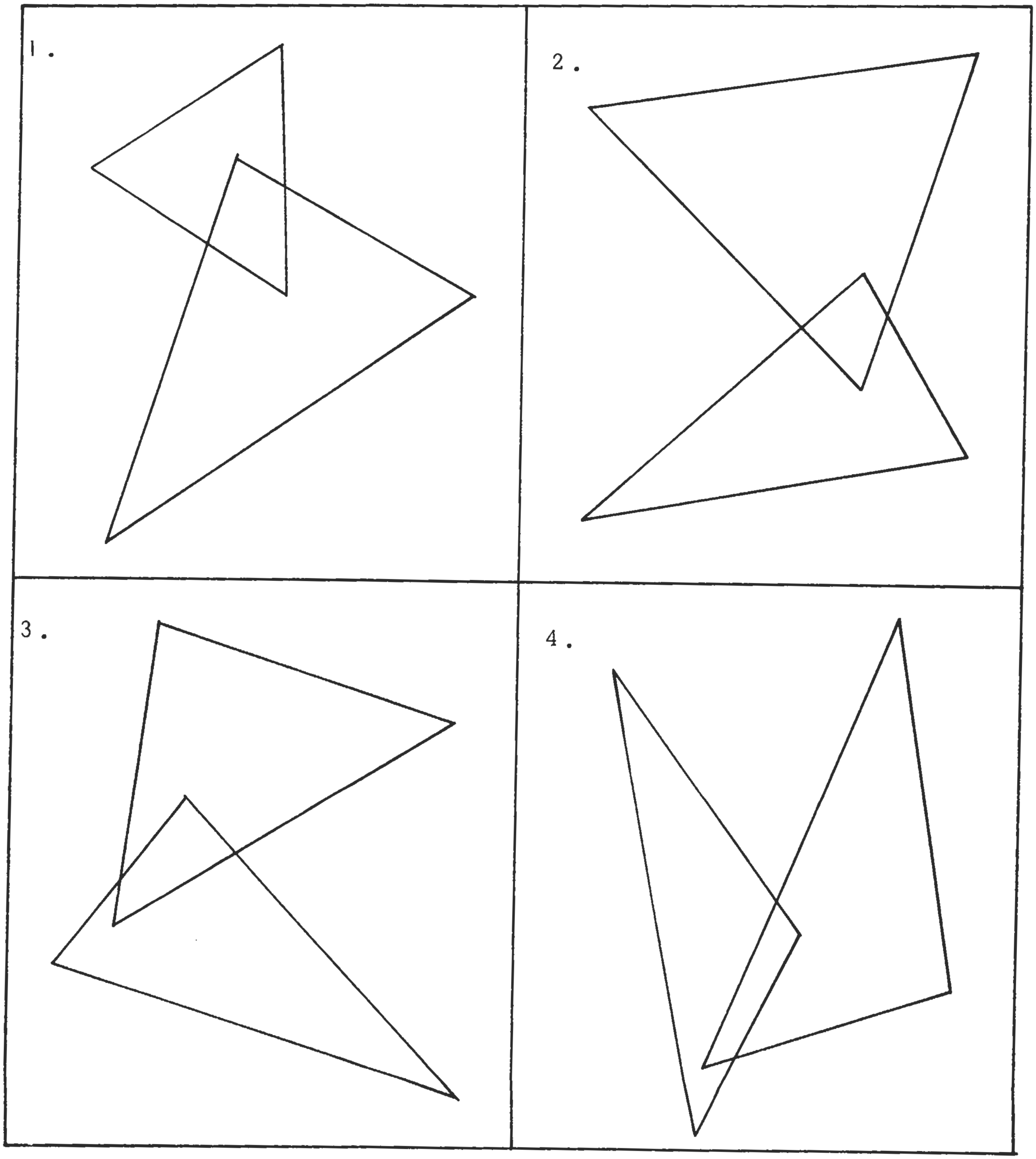
1. Find one point that is on just 4 triangles. Use a purple crayon.

2. Find one point that is on just 1 triangle. Use an orange crayon.

WORKSHEET 34

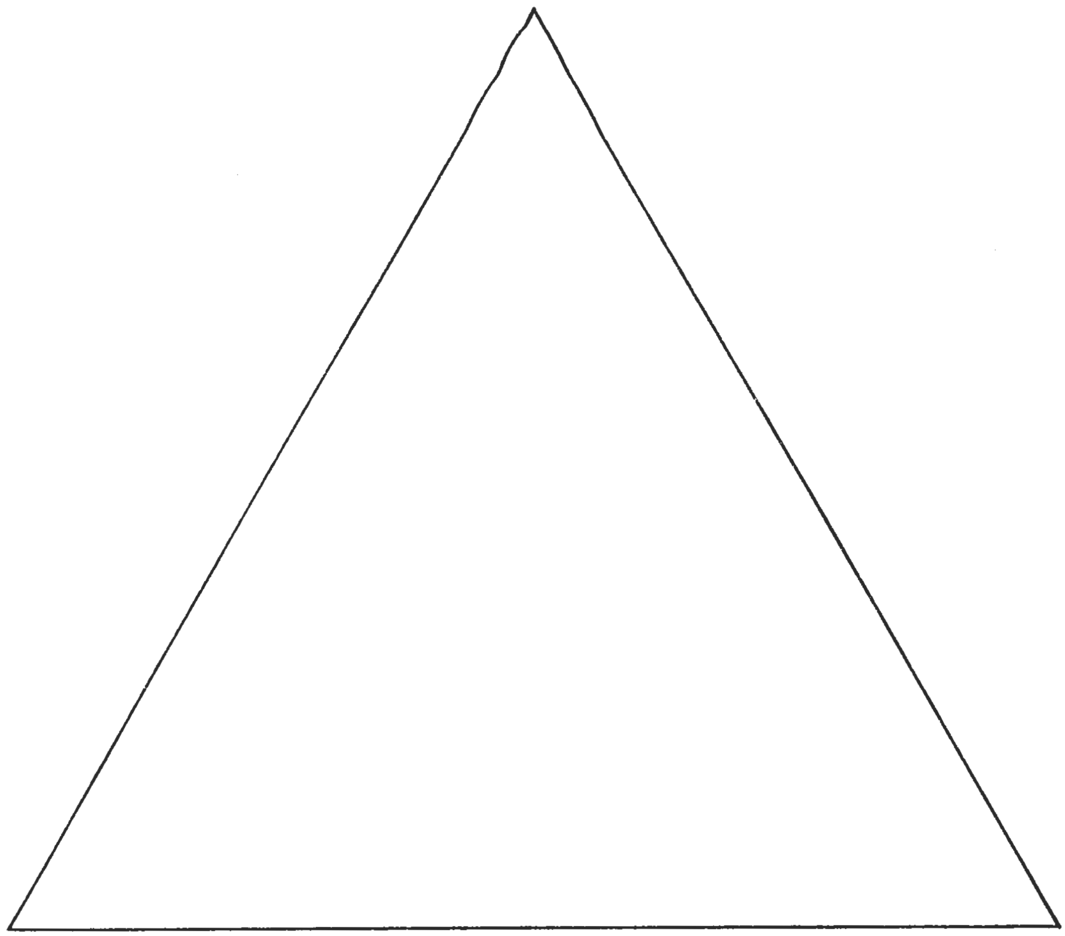
Triangles with a Common Region

In each box color the intersection of the two triangular regions.



WORKSHEET 35

Triangles with a Common Region



1. Draw one line segment to make three triangles where you now see one.

2. Color one small triangular region green.

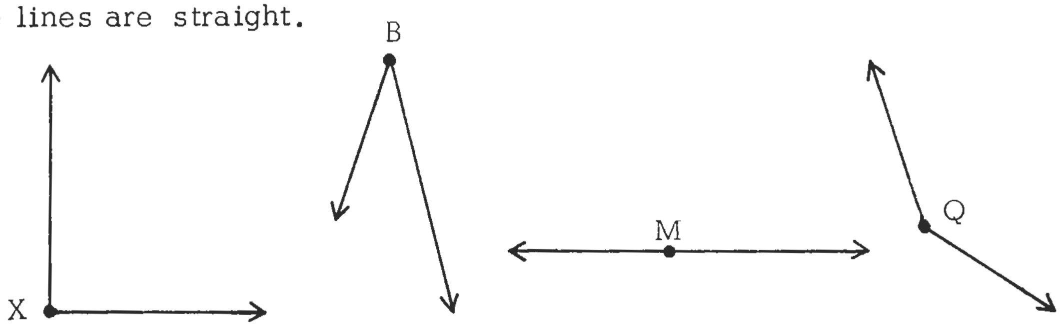
3. Color the other small triangular region yellow.

4. Lightly color the region of the larger triangle orange to show that it has a region in common with each of the small triangles.

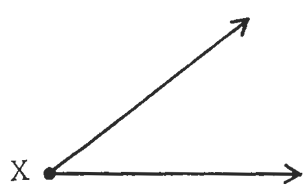
Suggested Activities on Angles

1. Since an angle is formed by two rays that begin at the same point, review ray by having children make a few on the chalkboard. Remind the children that we can't draw the whole ray. The part we draw represents the entire ray. The two arrows in the figure with the same endpoint and in the same direction represent the same ray.

2. Have several children make 2 rays that begin at the same point, and ask them to label it. It is important for children to use rulers so that the lines are straight.

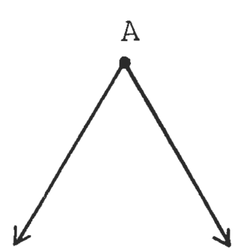


3. Erase all but one of them. Ask if anyone can tell what is formed by the two rays beginning at the same point.

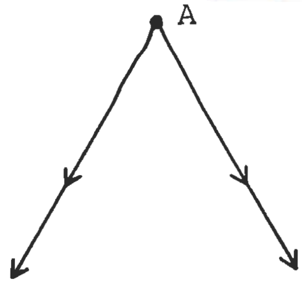


If no one identifies it, the teacher should supply the answer (angle).

4. Draw the following figure on the board:

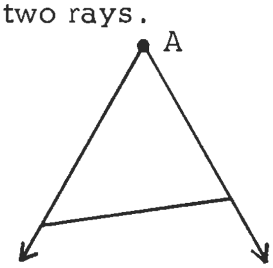


Ask the children if extending the sketches of the rays changes angle A.



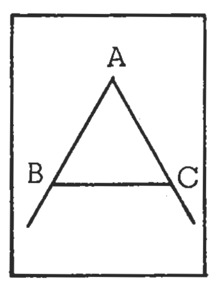
Some might say it becomes larger. Explain that not even the rays have changed. The rays and angles are the same.

5. Go back to the original figure and draw a line segment(not at the arrowheads) connecting the two rays.



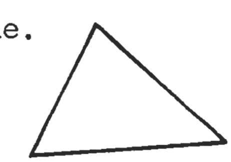
Ask if the line segment changes angle A. (It does not.)

6. Distribute small pieces of colored paper. Have each child cut a triangular region and paste it on a piece of manila paper.



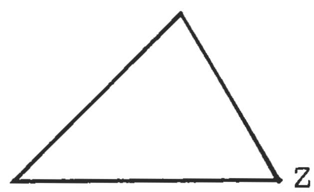
Then have the children label the angles of the triangle, ABC. Have them use their pencils to extend line segments AB through B and AC through C. Children will discover that angle A always remains the same.

Then return to your figure on the board. Erase the parts of the rays that extend beyond the line, so that you form a triangle.

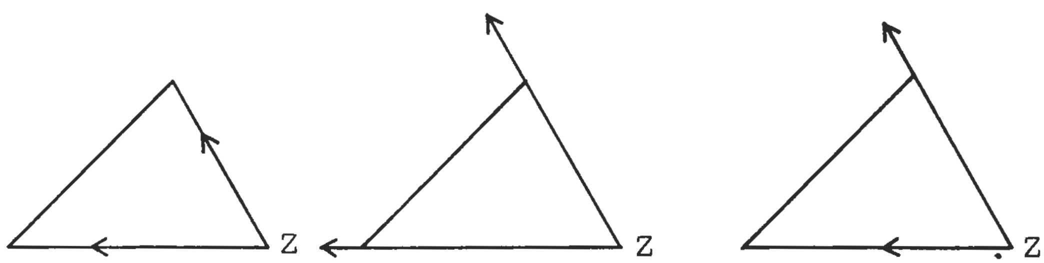


7. Review the lesson at another time by asking several children to go to the board with their rulers and make two rays that form an angle. Have them label the angle. After everyone in the class has done this, have them all repeat the experience on newsprint, using their pencils and rulers.

8. Draw a triangle on the board. Label <u>one</u> of the angles.

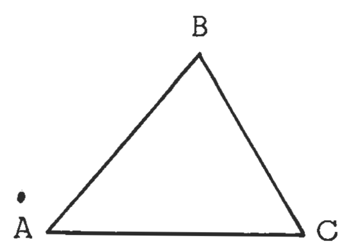


Then ask if anyone can show the rays that form the angle.

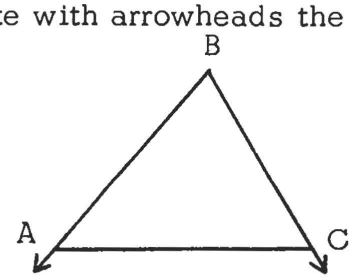


Do this several times with different-shaped triangles and different labels. Encourage variety in the placement of the arrowheads.

9. Then make a triangle and label <u>all</u> of the angles.

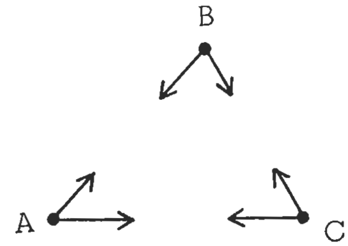


Have someone indicate with arrowheads the rays that form angle B.



Rays BA and BC form angle B.

10. Ask the children how many angles triangle ABC has. Review the meaning of "tri." If the children have difficulty understanding it, show them by shortening the sketches of the rays; for example:



11. Have 1/3 of the children take their rulers and draw triangles on the chalkboard. Have another third identify and label the angles A, B, C. Have the last group go to the board with their rulers and show the rays that form angle C.

12. Repeat Exercise 11 by changing the responsibilities of each group so that every child has the opportunity to perform all 3 tasks. After the first time change the labels for the angles so that each group is showing the rays for an angle other than angle C. (This is to prevent the development of an idea that angles always have the same names.)

13. Repeat Exercise 11 as often as necessary.

14. Review angles and triangles on the flannelboard. Be sure they are triangles and not triangular regions.

15. Allow children to suggest angles located around the room.

16. Distribute Worksheets 36 and *37. Read the directions with the children and let them do the pages independently.

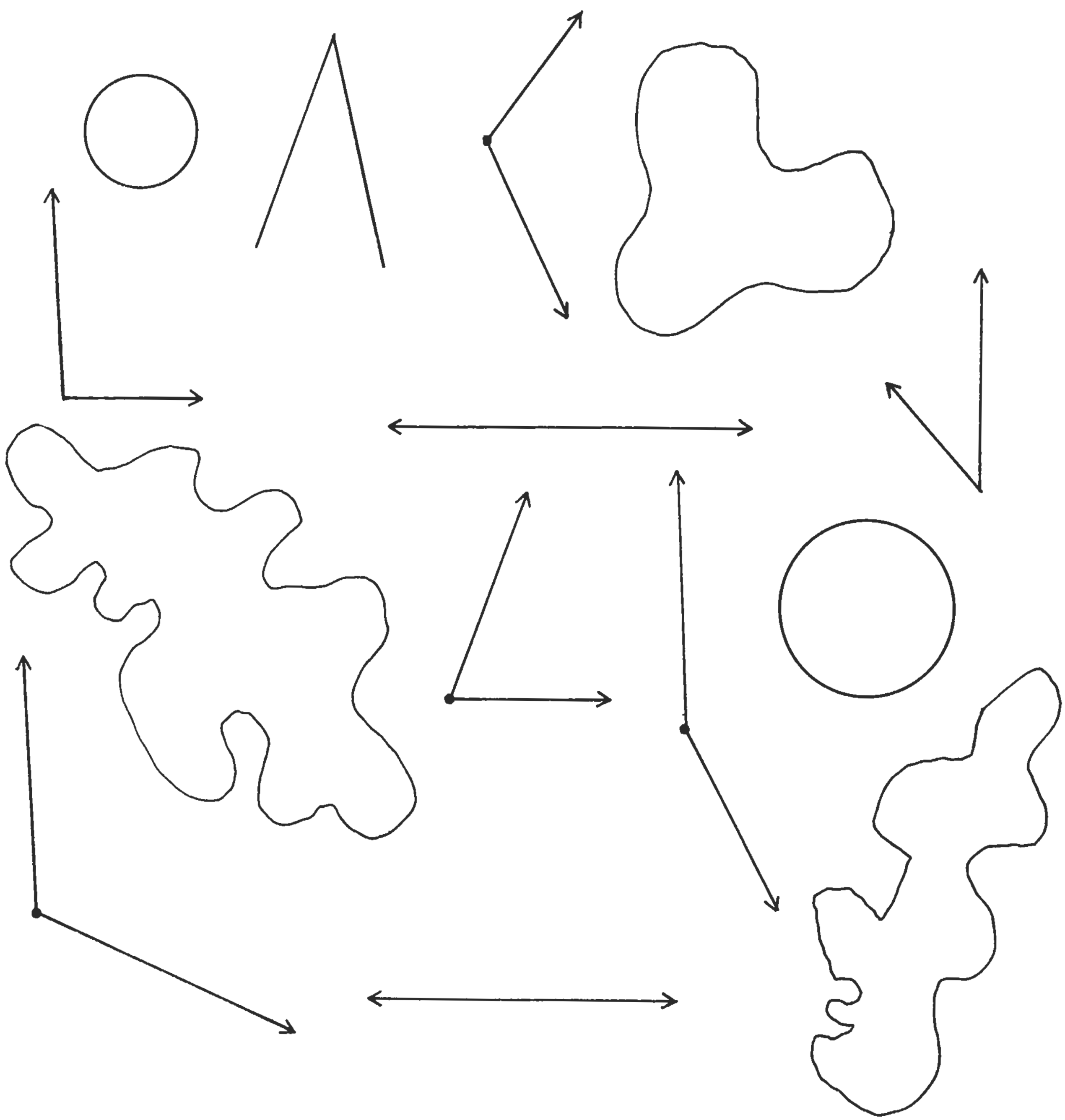
> Note: On Worksheet *37 be sure that the children include all of the angles in the triangles as well as the angles that stand alone.

WORKSHEET 36 Angles

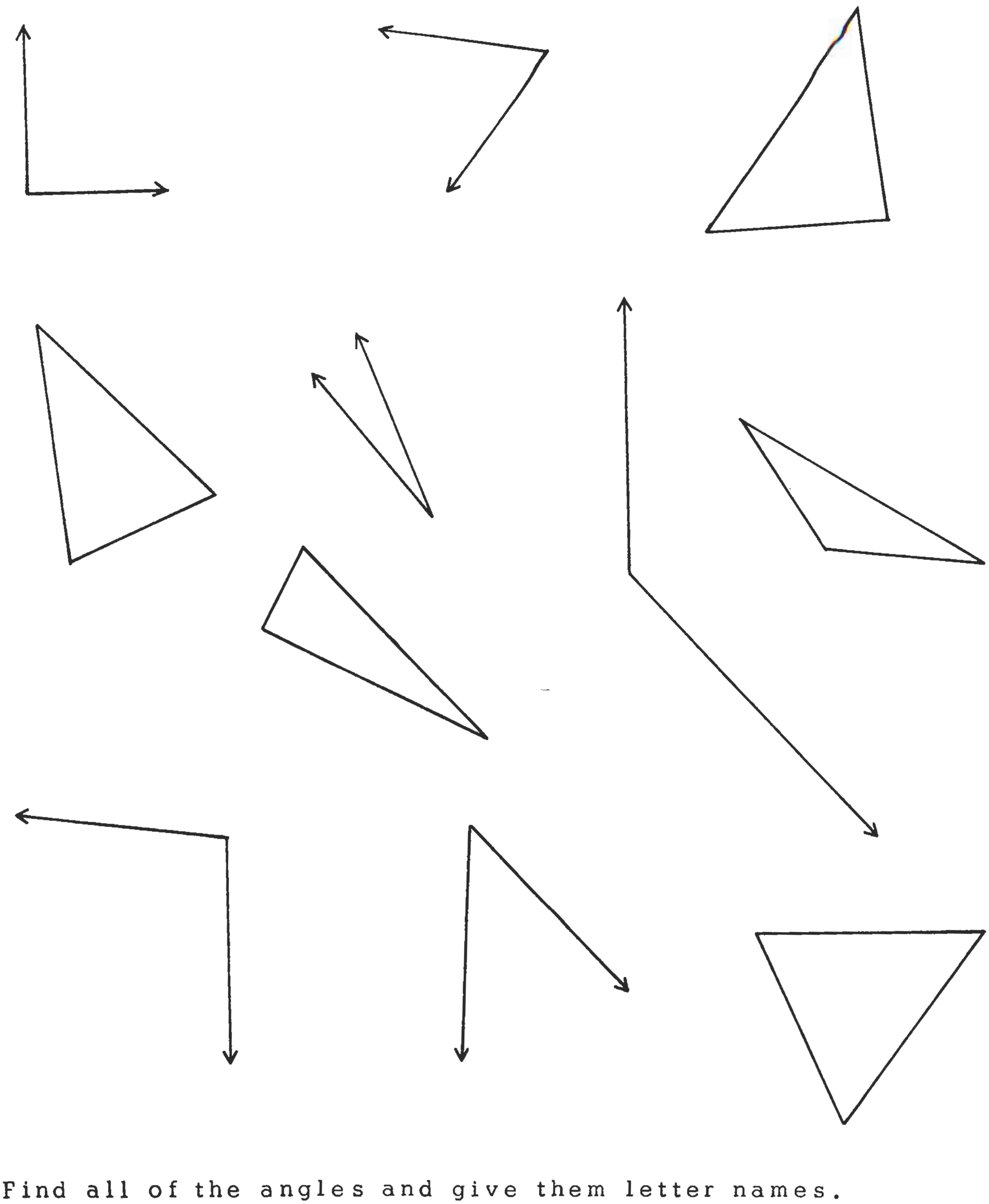
1. Every figure on this page is a simple closed curve or an angle.

2. Put a red X on the closed curves.

3. Take a blue crayon and labe the angles.



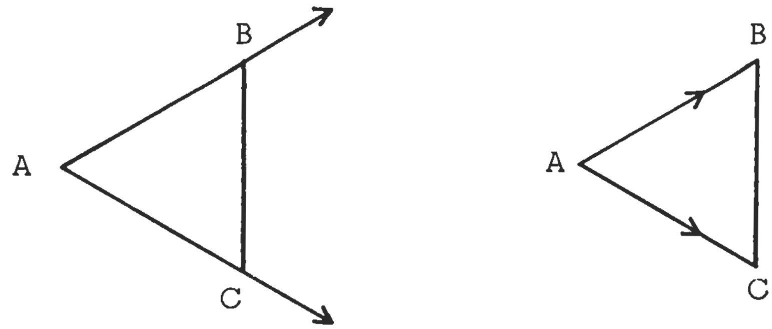
WORKSHEET 37 Angles



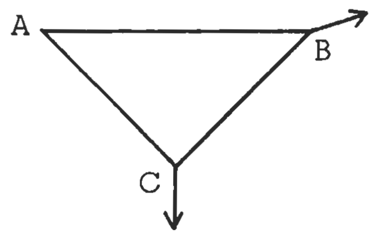
Find all of the angles and give them letter names.

Activities on Angles of a Triangle

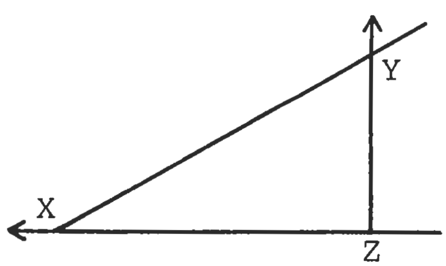
17. Send one group of children to the board with their rulers. Tell them to make a triangle and label it ABC. Then tell them to mark the rays of angle A.



Demonstrate if they need help.

> Note: Be sure the rays contain the existing line segments and that they don't start off in a different direction such as:
>
> 
>
>
> The second representation in the first group of sketches given above is preferred. It is easier for the first grader to draw. The teacher should be sure to illustrate the other possibility occasionally.

Then send another group to the board. Tell them to make a triangle and label it XYZ. Have them mark the rays of angle Z.



Do this until all children have had several turns.

18. Give the children review Worksheet 39 to work independently.

WORKSHEET 38 Angles

1. Draw a triangle.

2. Label it XYZ.

3. Show the rays of angle X.

---

1. Draw a triangle.

2. Label it any way you like.

3. Show the rays of one of the angles.

4. Of which angle are these the rays? ________________

Suggested Activities on Straight, Right, Acute and Obtuse Angles

1. Give the children 9 x 12 manila paper. Have them use their rulers to draw a page of angles. Have them label the angles as they did on Worksheet 37. They may draw angles alone or in triangles.

2. Have them show their papers to other classmates. Say something like this to them: "You now have seen and drawn many angles. Have you noticed that some are different from others?"

Let them discuss differences they see. Then tell them that there is a way to classify angles.

A straight angle looks like this:

Draw it on the chalkboard and print "straight angle." It is an angle because it is formed by 2 rays that begin at the same point.

Put "straight angle" and the illustration on the geometry bulletin board.

3. Give the children Worksheet*39. Have them separate the three sections along the dotted lines by cutting or by creasing and tearing along their rulers. With each piece in turn they are to follow these directions as the teacher demonstrates.

Mark a point B on one of the rays shown.

With your pencil and ruler extend the ray AB beyond A to make a straight angle at A.

Hold the paper up to the light, fold and pinch it at A so that the two sides of the straight angle overlap.

Crease the paper along the pinch. The crease should go through A. Pupils who manage this can assist those who have difficulty. Open the paper.

Do both rays of the given angle lie on the same side of the crease? If so the given angle is <u>acute</u>.

Do the rays of the given angle lie on opposite sides of the crease? If so, the given angle is obtuse.

Does one ray of the given angle lie along the crease? If so, the given angle is a right angle.

4. Sketch an angle on a sheet of paper. Ask the class to judge whether it is a right, acute, or obtuse angle. Accept all answers. Then invite a child who proved skillful in the other activity to apply the folding test. Help him to interpret the results, and copy the angle together with its designation on the chalkboard. Do this for angles of all three types.

5. Ask the children to point out examples of right angles around the room. Notice the hands of the clock at 9:00 and at 3:00. When do the hands form a straight angle? Ask the children to look for times other than the obvious ones. Approximate answers like a few minutes after seven are best; 7:05 is not correct.

Put the term "right angle" and an illustration on the geometry bulletin board.

6. Draw an obtuse angle on the chalkboard. Ask the children to imagine the "folding test" applied to this angle. Indicate the position of the crease with a dotted line. Help them to see that the rays of the obtuse angle are on opposite sides of the crease.

Sketch a ray on the chalkboard. Ask a volunteer to come to the board and hold his ruler as a ray to make an obtuse angle with the given ray; a right angle. Add "obtuse angle" with its illustration to the geometry bulletin board.

7. Repeat Activity 6 with acute angles.

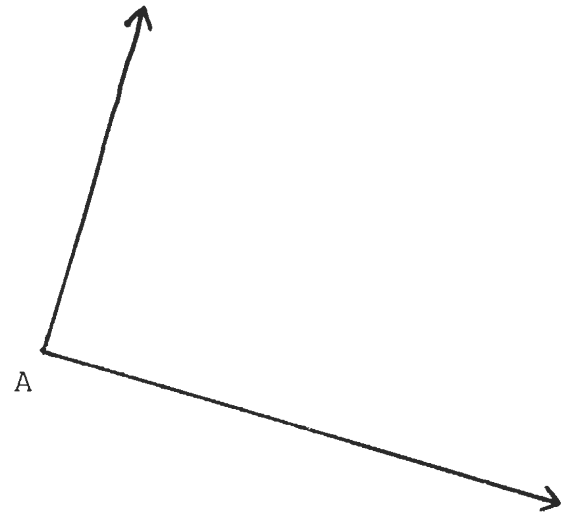
8. Have children determine the kind of angle the hands on the clock form at the time of the lesson. Do this several times during the day.

9. Divide the class into four groups and ask each group to draw acute, straight, obtuse and right angles respectively.

10. Use many examples so that children learn to recognize right, acute and obtuse angles by observation rather than by verbalization.

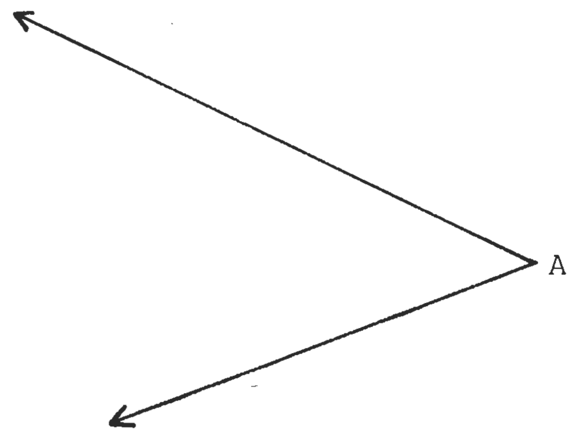
WORKSHEET 39 Right, Acute, Obtuse Angles

1.



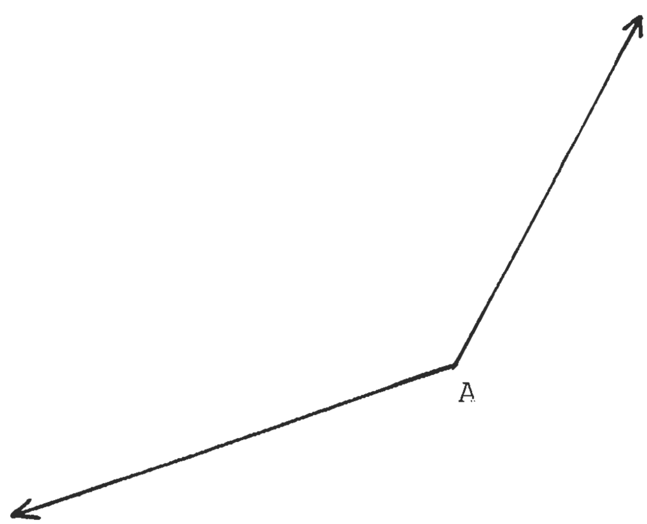
RIGHT

2.



ACUTE

3.

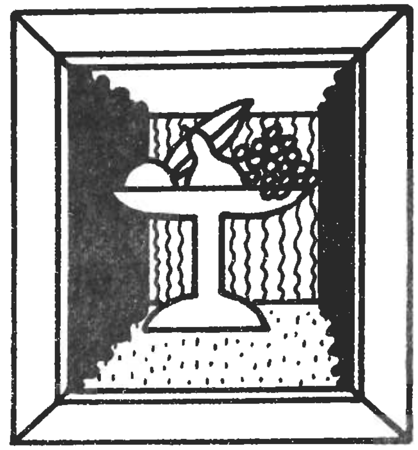


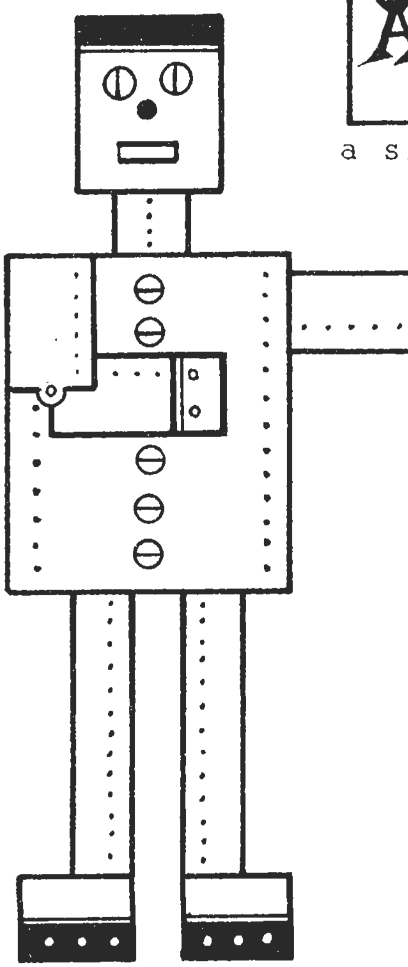
OBTUSE

RECTANGULAR SHAPES

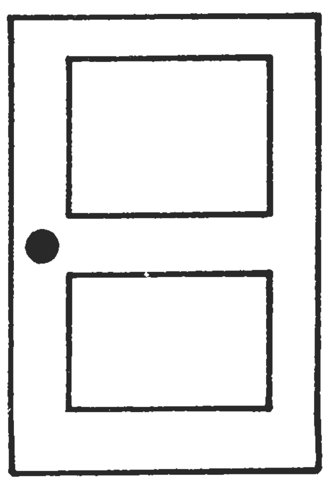
a sign

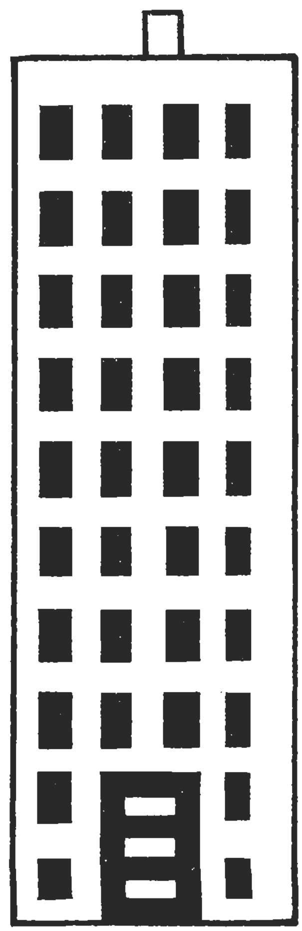
a stamp


a picture

a robot


a side of a box

a door



each side of a tall building, doors and windows also

Suggested Activities on Rectangles

1. Have children find objects around the room that are rectangular in shape. Accept both square and oblong shapes, and have the children distinguish between them.

2. Discuss differences and similarities between square rectangles and oblong rectangles.

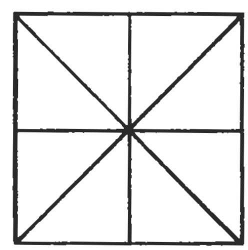
3. Show children the rectangular shapes on page 129. Have them identify square and oblong shapes.

4. Have children make an oblong rectangle (not a square). Have them use rulers to be sure opposite sides are equal. Discuss the angles. Call them "right" angles. Have part of the class work at the board and part at their desks. Later change groups.

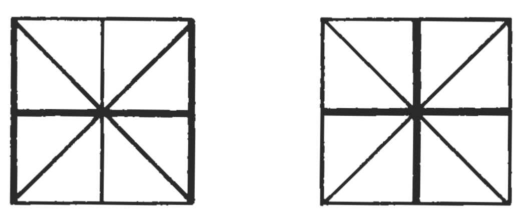
5. Have them repeat Exercise 4 but this time make all squares instead of oblongs.

6. Add the figures and the word "rectangles" to the bulletin board. Square and oblong are included as 2 kinds of rectangles.

7. Draw the following figure on the board:

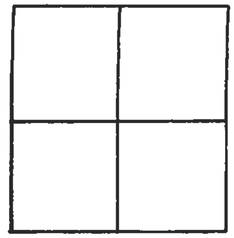


Ask if anyone can find smaller figures hidden inside the larger figure. Have them identified. Some children might find "4", "1", letters such as Z, X, H, I, a "+" sign and a "-" sign. Ask the children to outline the ones they find like this:



Ask what they have learned from this exercise. If no one responds, explain that figures are often contained or hidden inside other figures.

8. Draw this on the board:

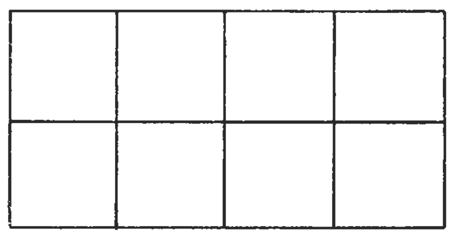


Ask how many squares the children can find. There are five with four squares inside one large square.

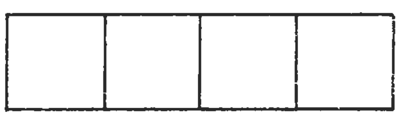
9. Demonstrate on the board how rectangles can be inside larger rectangles.

3 small and 3 large rectangles

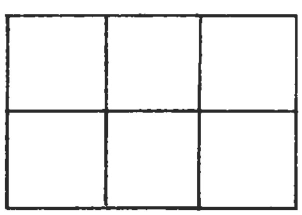
10. Then draw this figure:



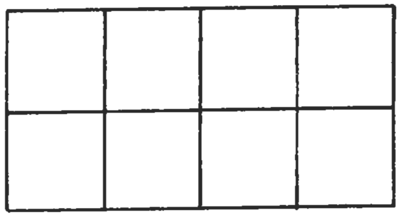
Ask the children how many squares there are. The answer is 11. These squares also form 19 oblong rectangles. See how many the children can find.



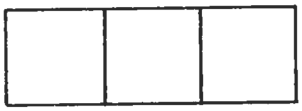
2 - 1 by 4



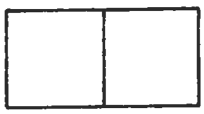
2 - 2 by 3



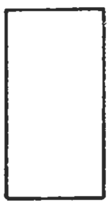
1 - 2 by 4



4 - 1 by 3



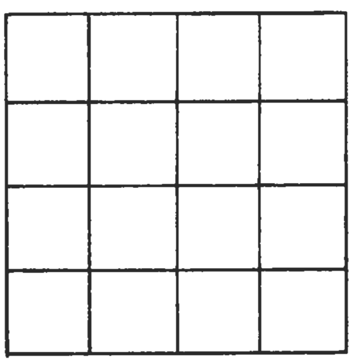
6 - 1 by 2



4 - 2 by 1

19

11. How many squares are there in this figure?



Note: There are:

16 small squares

9 squares made up of 4 small ones

4 squares made up of 9 small ones

1 square made up of all the small ones

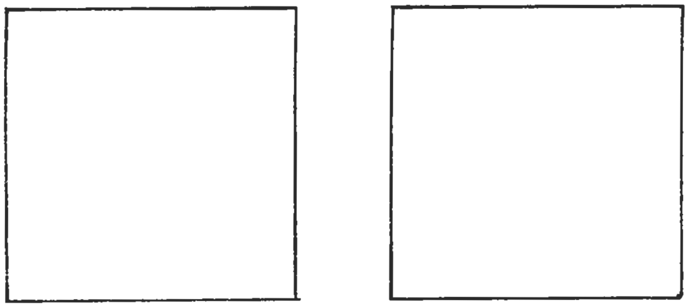
30

12. Use Worksheets 40 and 41.

Answers: Worksheet 40 - 2, 3, 5, 5. (On this worksheet everyone should be expected to get them all correct.)

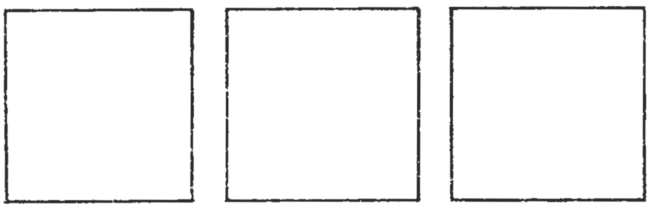
Worksheet 41 - 8, 20, 8. (Do not expect everyone to find all of the squares on this worksheet.)

WORKSHEET 40 Rectangles



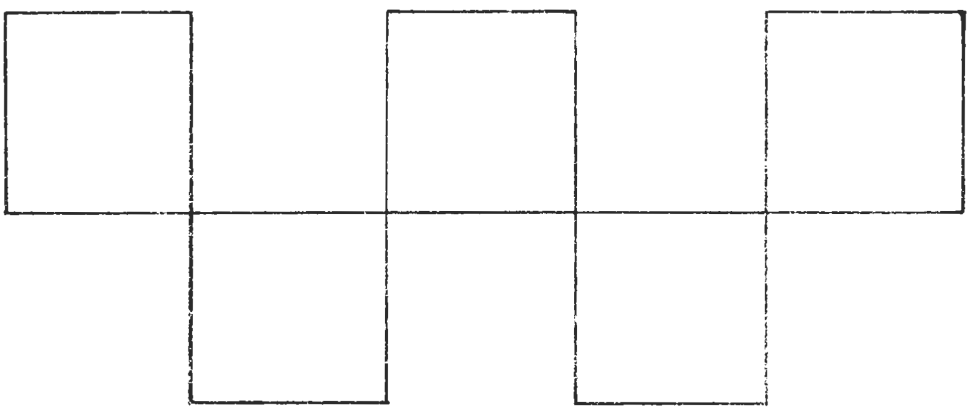
How many squares?

____



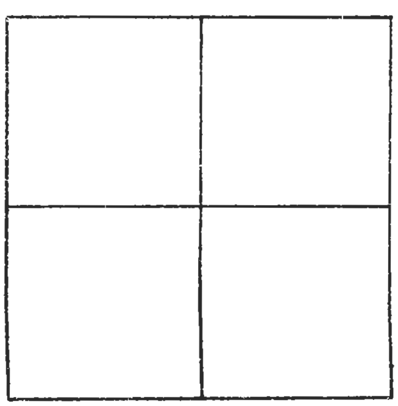
How many squares?

____



How many squares?

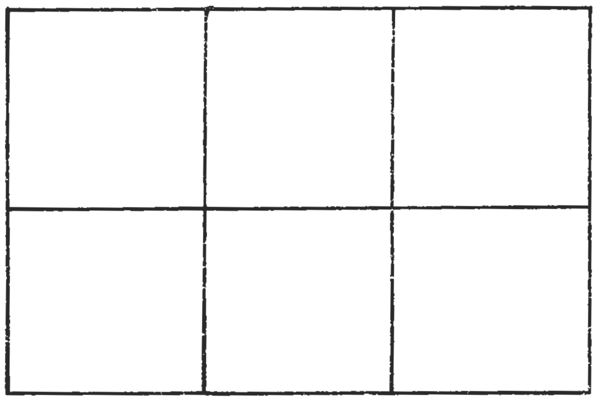
____



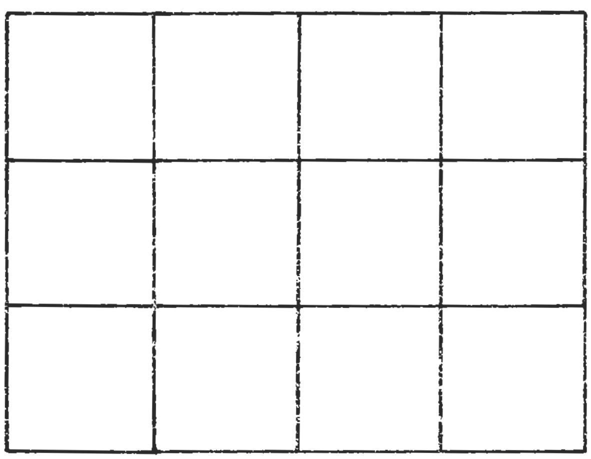
How many squares?

____

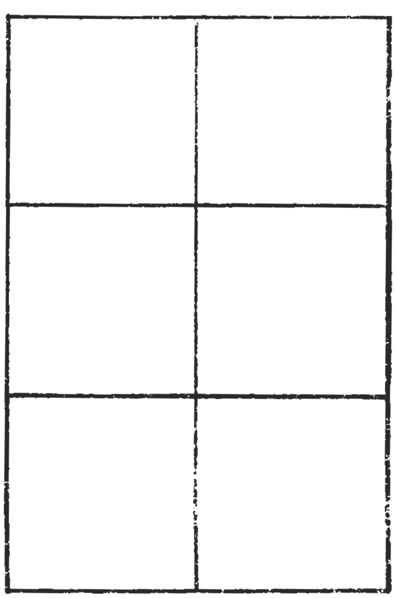
WORKSHEET 41 Rectangles



How many squares can you find? ____________



How many squares can you find? ____________



How many squares can you find? ____________

Activities on Squares

1 . Put these directions on the board:

Step 1. Fold your paper to make four squares.

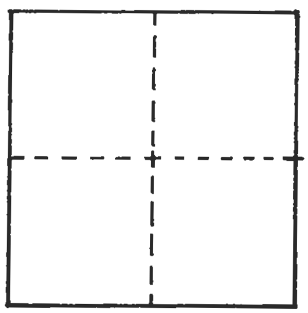
Step 2. Use your ruler to draw the four squares.

Step 3. Make four smaller squares inside each of these squares. Use your ruler.

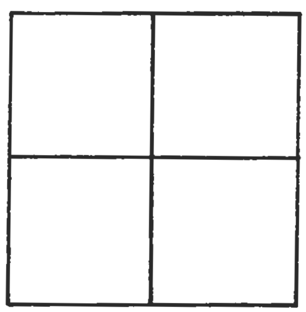
Step 4. Now make a design by making the inside of each square a different color from the squares that have a common line segment with it.

Give each child a piece of paper about 8 inches square. Have the class read the directions aloud. Demonstrate on the board while the class reads the instructions.

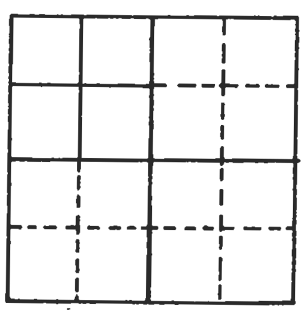
Step 1



Step 2

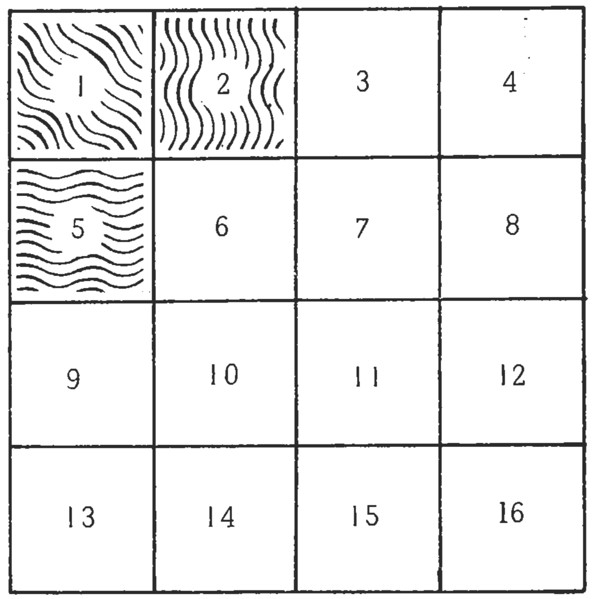


Step 3



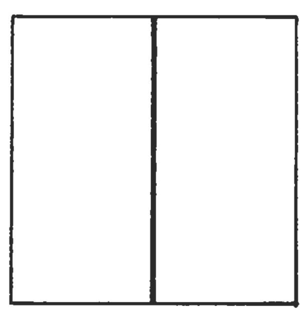
Have the children use this same procedure to form 4 new squares in each of the 4 squares in Step 3 for a total of 16.

Step 4

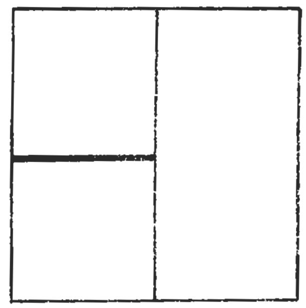


Explain that if they are coloring #1, the other squares that have a common segment with it, (2 and 5), must be a different color from #1.

2 . After this exercise play "Where is Rover?" Put a large sheet of paper on the board and make a large square on it. Tell the class that Rover is lost in this square and we are going to try to find him. Ask someone to choose the left or right section. Have him draw the line dividing the square in half.

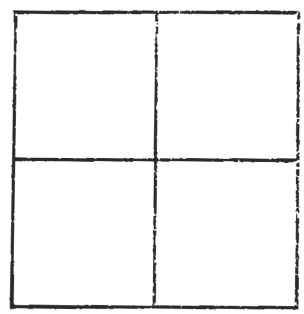


If he says left, then we look only in the left half for Rover. Have that child choose another person to select the top or bottom of the left section. This child then draws his line making that division.

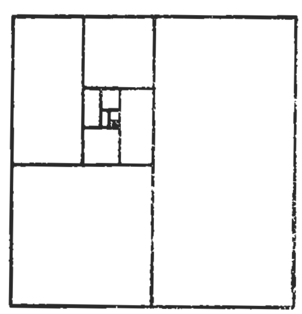


Again if top is selected, the class is now looking for Rover in the top left hand section of the square.

The next child decides on left or right, etc.



Continue until no one can draw a smaller piece—and there is Rover!



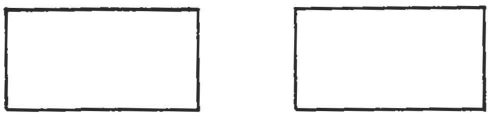
Answers for Worksheets 42 and 43

Worksheet 42	3	
	2	
	4	
	5	
	10 oblongs	
	8 squares	
	18 rectangles	
Worksheet 43	Row 1	3
	Row 2	3
	Row 3	3
	1.	9
	2.	4
	3.	2
	4.	2
	5.	1

WORKSHEET 42 Rectangles

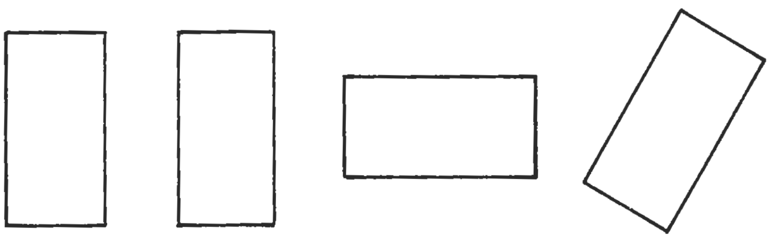
How many oblongs can you find?

____



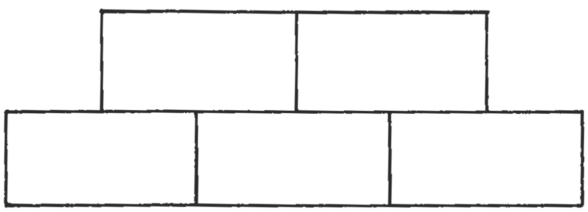
How many oblongs can you find?

____



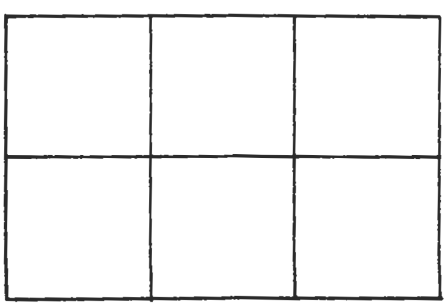
How many oblongs can you find?

____



How many oblongs can you find?

____



How many oblongs can you find?

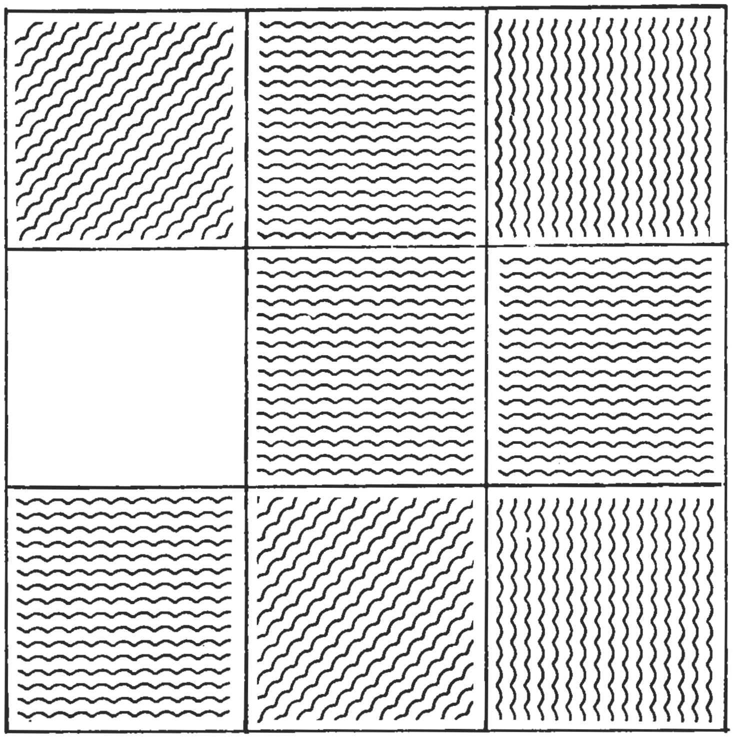
____

How many squares can you find?

____

How many rectangles are shown?

____



Row 1

How many squares?

____

Row 2

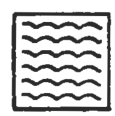
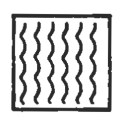
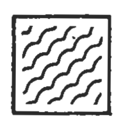
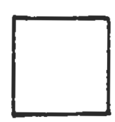
How many squares?

____

Row 3

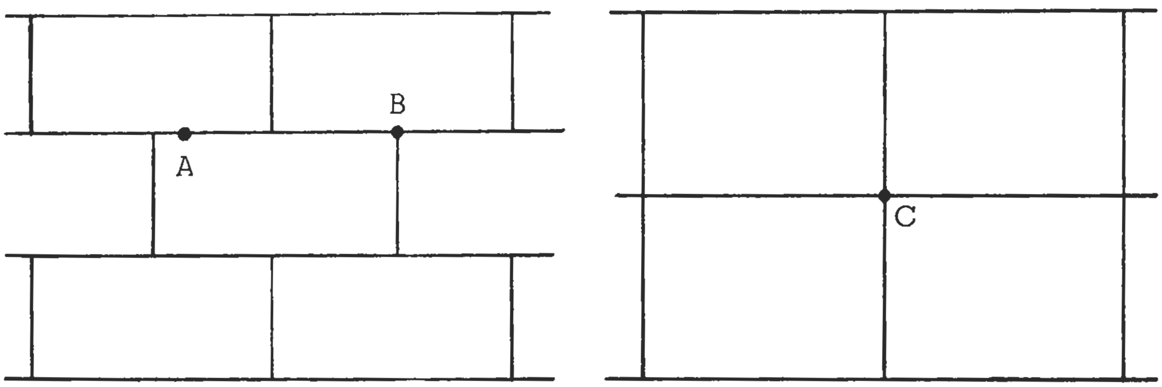
How many squares?

____

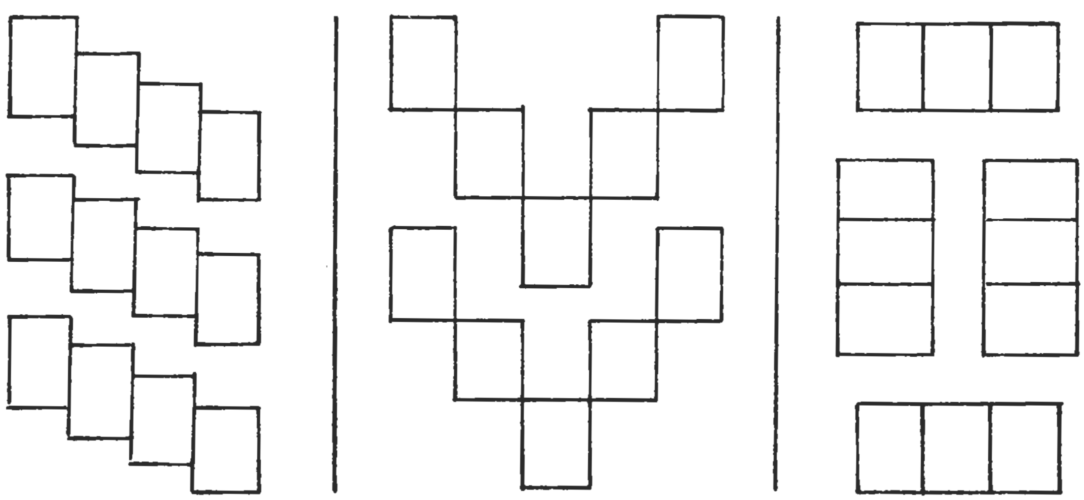
1. How many small squares are there? __________________
2. How many square regions look like this?  ______
   Color the insides blue and yellow.
3. How many square regions look like this?  ______
   Color the insides red and green.
4. How many square regions look like this?  ______
   Color the insides orange and brown.
5. How many square regions look like this?  ______
   Color the insides black.

Commentary for Worksheets 44 and 45

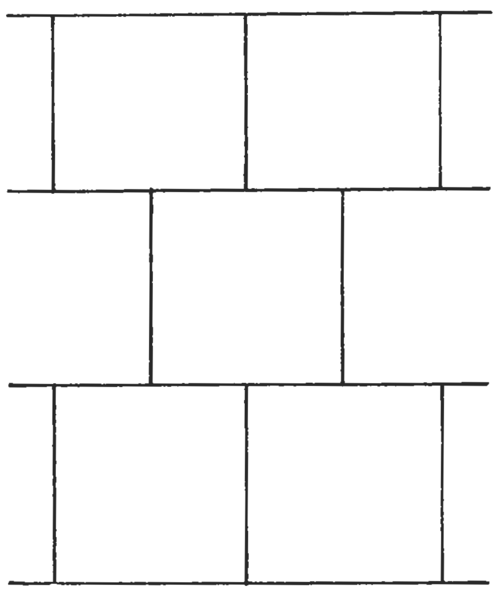
Examples for teacher: (Have children do this on the board before assigning the worksheet.)

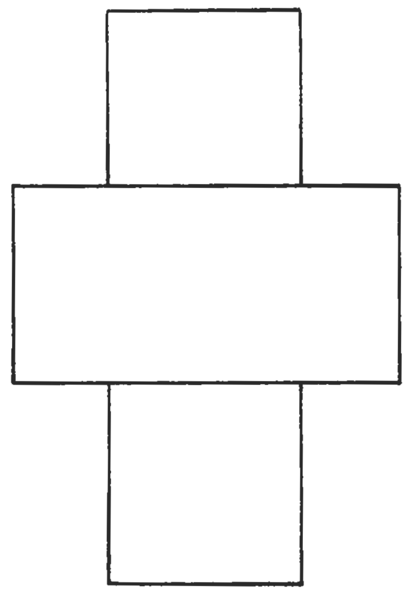


1. Find one point that is on 2 rectangles. Use a dot to represent the point. Point A is one example.
2. Find one point that is on 3 rectangles. Point B is one example.
3. Find one point that is on 4 rectangles. Point C is one example.
4. Make a pattern of rectangles so that no point is on more than 2 squares. Can you make such a pattern which covers a whole sheet of paper?



WORKSHEET 44 Squares





1. Use a blue crayon. Find one point that is on 1 square.

2. Use a green crayon. Find one point that is on 2 squares.

3. Use a brown crayon. Find one point that is on 3 squares.

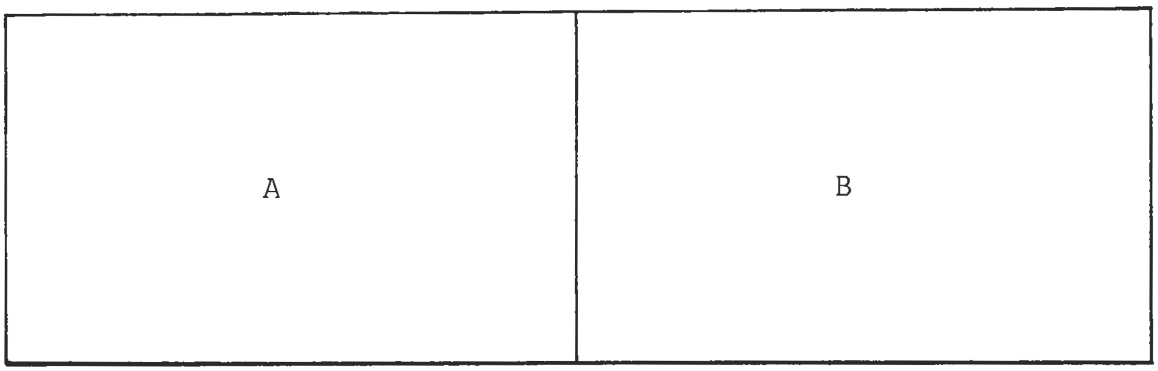
WORKSHEET 45 Squares

Make a pattern of squares in the space below so that no point is on more than 2 squares.

WORKSHEET 46 Rectangles

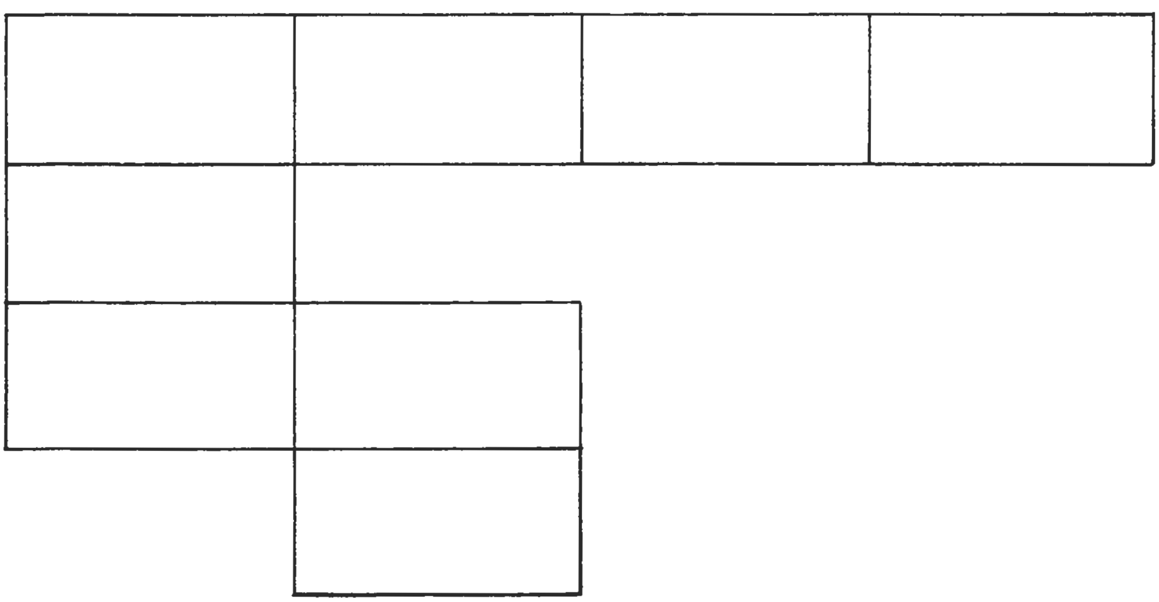
1. Find the common boundary of rectangles A and B.

Color it dark red.

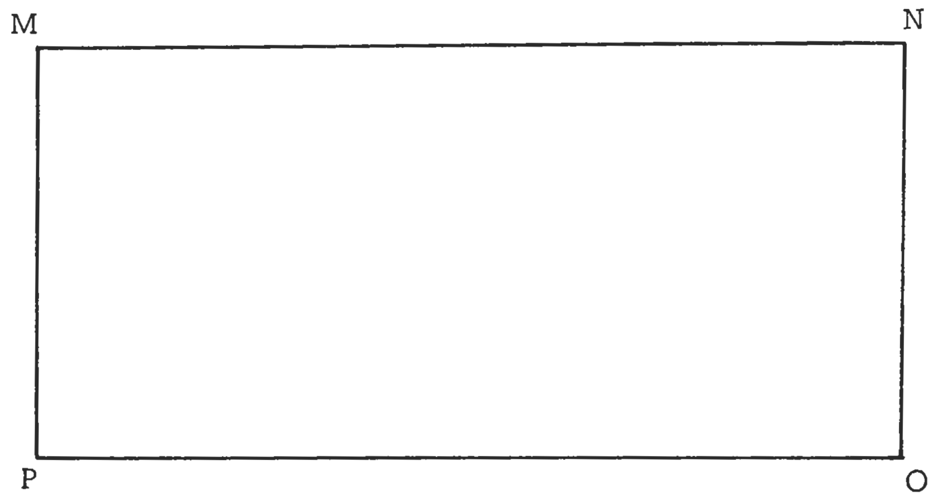


2. Find the common boundary of any two rectangles.

Color it dark green.



1. Draw one line segment to make three rectangles where you now see one rectangle called MNOP.



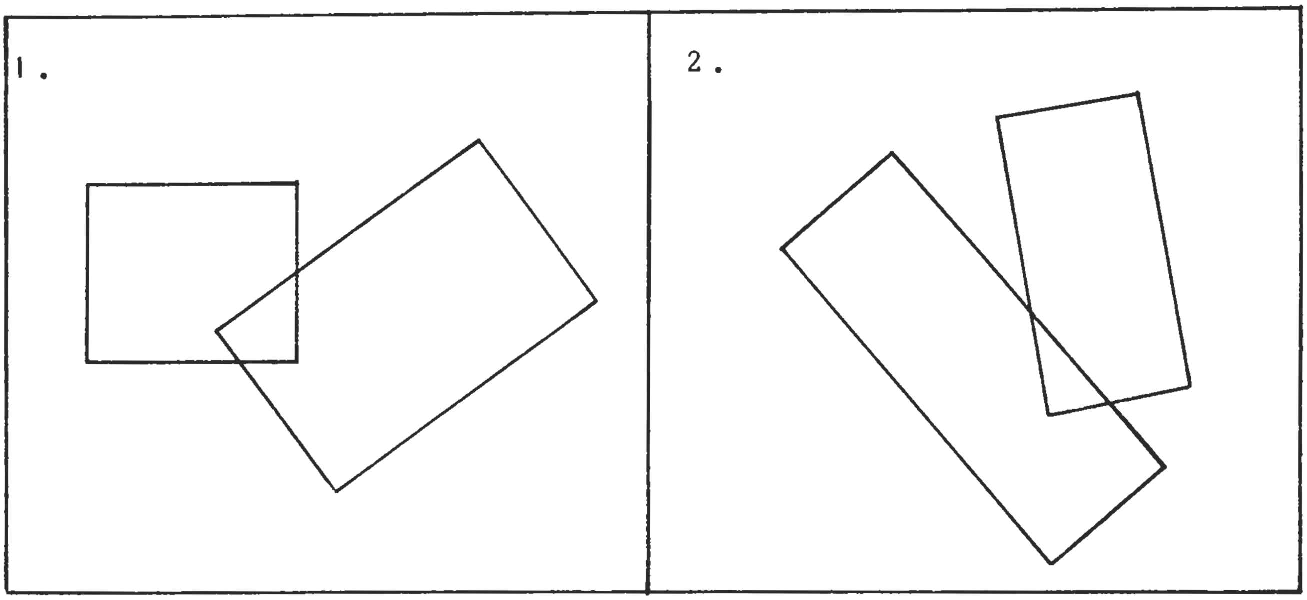
2. Color one small rectangle light brown.

3. Color the other small rectangle light green.

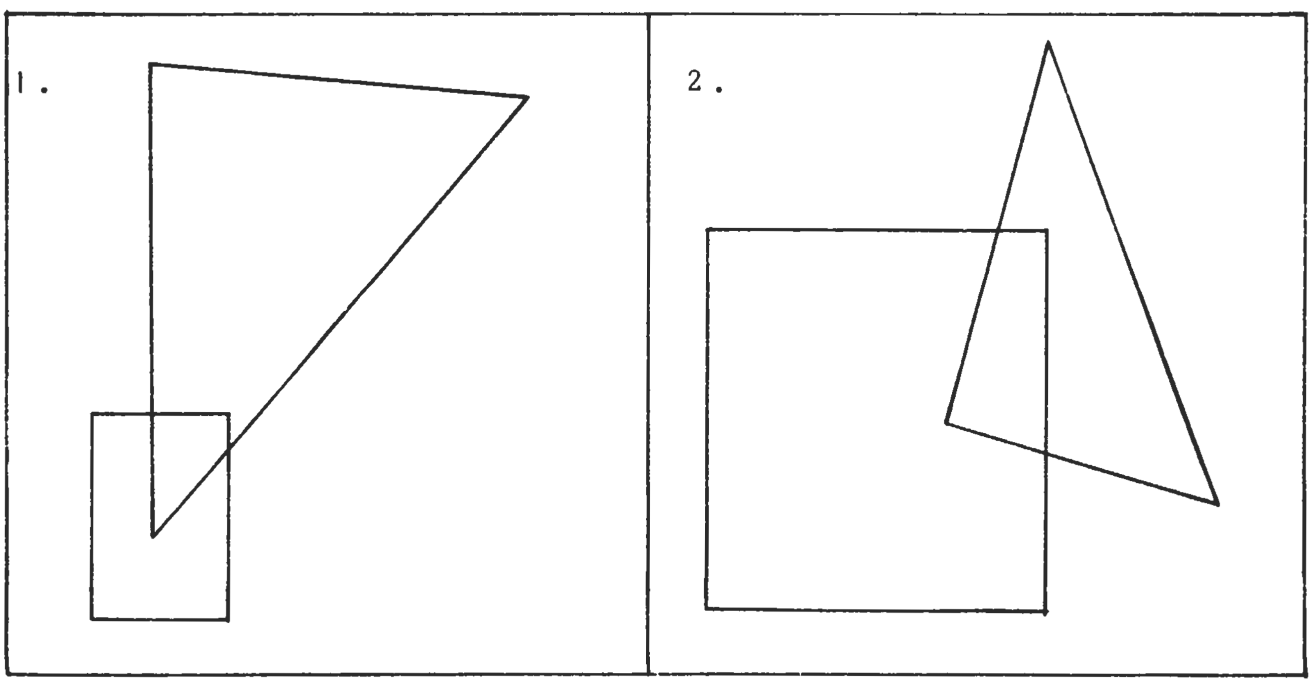
4. Color the large rectangle yellow.

You will see that each small rectangle has a region in common with the large rectangle MNOP.

Color the intersections of these rectangular regions.



Color the intersections of these rectangular and triangular regions.



Review Activity on Rays and Circles

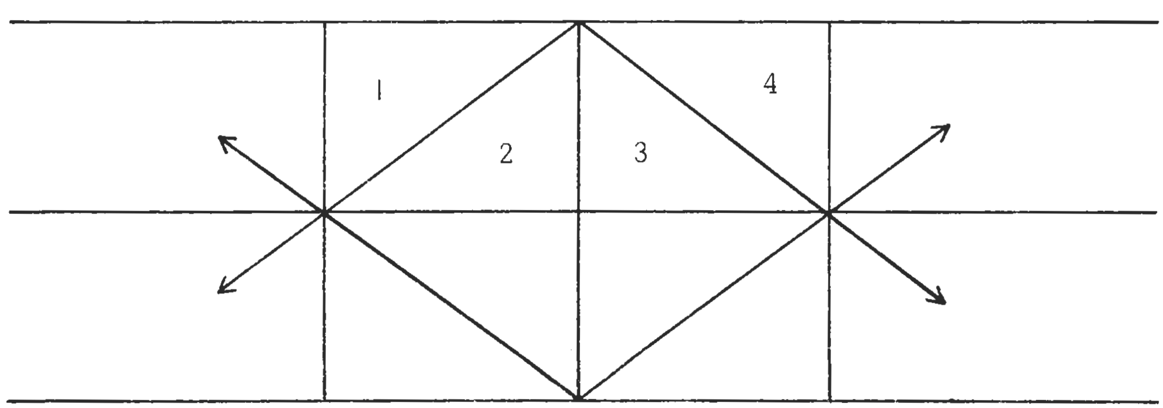
Two children come to the chalkboard. They are dubbed Ray and Point, respectively, and are given rulers and contrasting colors of chalk. The teacher marks a point as center and indicates a distance (something between four and twenty inches, to be manageable) as radius of a circle to be sketched by Ray and Point. Ray sketches a ray with the center as endpoint. Point must then locate the intersection of the circle with the given ray. (He measures the given distance from the center, in the direction given by the ray, and marks it.) This pair of operations is repeated ten or twelve times, after which Ray, who has had the easier task, is to attempt to sketch the circle.

Ray should be encouraged to vary the lengths of the rays he draws—especially, some should be much shorter than the given radius. Most important, all should realize that it's the same ray, no matter how much of it he draws. Point may want—and should be permitted—to extend a short ray until he can place his point on it. Eventually he will see—or a classmate will point out—that this is unnecessary.

The pupils can play the activity in pairs at their seats.

Commentary on Worksheet 49

Before presenting Worksheet 49, discuss the meaning of "boundary." Illustrate on the chalkboard by drawing a figure similar to this one:



Have a child find the boundary that 1 and 2 have in common. Have another child find the common boundary for 2 and 3, etc. Do this many times so that the children understand the meaning of the term "boundary" and the use of the word "common" as it relates to boundary in this lesson.

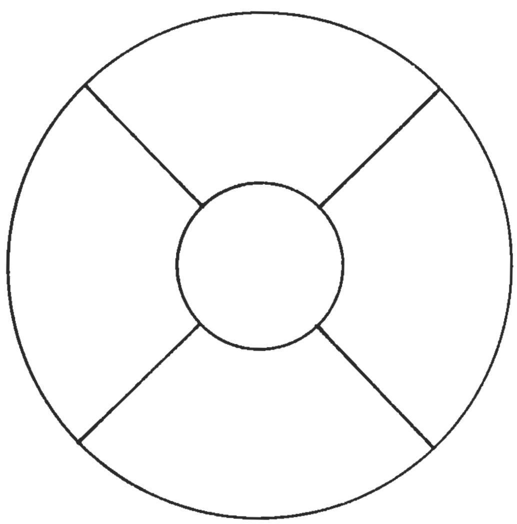
Present Worksheet 49.

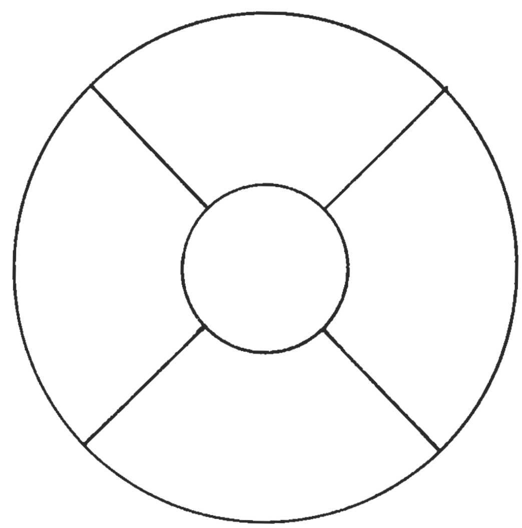
> Note: The astute reader will note that regions 1 and 3 above, for instance, have boundaries which intersect in a single point. This is not to be construed as common boundary in the following activities. We might have been more precise and said "common bounding curve," which would exclude the single point. We chose instead to be a bit imprecise, and to spare the teacher and her pupils the burden of additional terminology at this time.

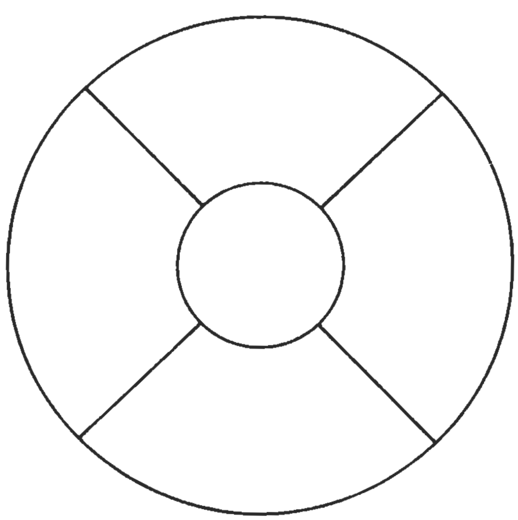
Commentary on Worksheets 49, 50, 51

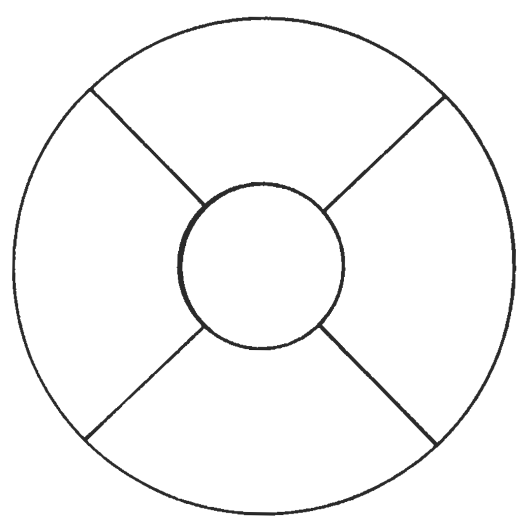
These worksheets are based on a problem that has challenged mathematicians: If you are to draw an imaginary map in which adjacent countries must be colored differently from each other, what is the least number of colors you must use? Is it possible to draw a figure that would require 5 colors?

Can you color this design with 3 colors so that no two regions with a common boundary have the same color? There are 4 designs here. If you make a mistake on the first design, use the next one. If you can do it right the first time, you may try the others with different colors.

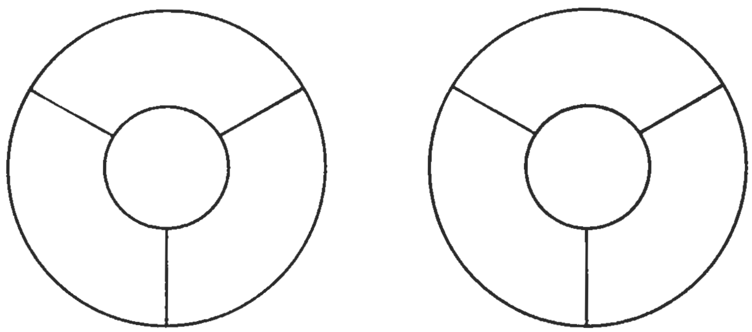








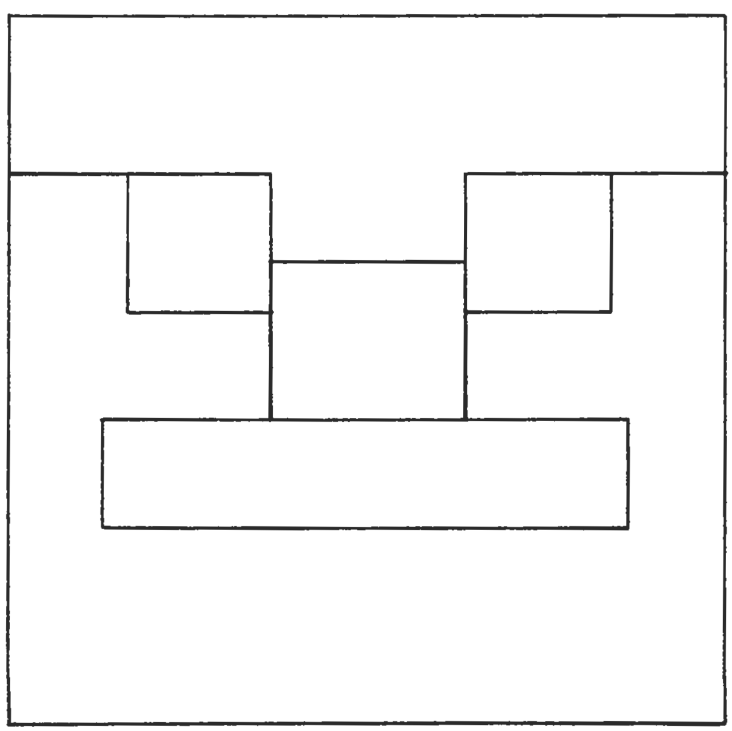
What about this design? See how many colors you need for this one to keep from using the same color for 2 regions with a common boundary.



How many colors?

____

How many colors do you need for this design to keep from using the same color for 2 regions with a common boundary? Use as few colors as you can.



How many colors?

____

Make otherdesigns and try to color them.

Can you draw a design so that you must use five colors? If you can, you will become famous. Every design anyone has ever drawn can be colored with fewer than 5 colors. However, no one knows whether someone, someday, will draw a design that needs 5 colors. Would you like to try?

The design may have any number of regions. The regions may be any size or shape. Two regions with a common boundary must have different colors. Use as few colors as you can. Draw on the rest of this page and on the other side.

Suggested Summary Activities

1. Have the children who are interested bring in pictures from magazines, catalogues, etc., of objects which are round, rectangular, or triangular. Let them make a scrapbook for the class. It might be good to choose a "Captain" for the round shapes, a "Captain" for the rectangular shapes, and a "Captain" for the triangular shapes. Have separate scrapbook pages made and then assembled into one large book of geometric shapes when the children feel that they are ready for it.

2. To help children remember the various geometric figures, adapt the song, "Where is Thumbkin?" to this unit.

   a. Where is circle, where is circle? Here I am, here I am. How are you today, sir? Very well, I thank you. Run away. Run away.
   b. Where is triangle, where is triangle?
   c. Where is square man, where is square man?
   d. Where is rectangle, where is rectangle?
   e. Where is ray, where is ray?
   f. Where is simple curve, etc.
   g. Where is closed curve, etc.
   h. Where is line segment, etc.
   i. Where is right angle, etc.
   j. Where is acute angle, etc.
   k. Where is obtuse angle, etc.

After children are acquainted with the song, distribute illustrations of half a dozen of the geometric forms. (List the ones distributed so the class will know which ones to sing.) Have the rest of the class close their eyes while the half dozen children hide with their illustrations.

Then have the class sing:

"Where is circle, where is circle?"

The person holding the circle pops out and sings:

"Here I am! Here I am!"

The class sings:

"How are you today, sir?"

The person with the circle sings:

"Very well, I thank you."

The class sings:

"Run away. Run away."

He pops into his hiding place as the class moves on to the next one. After all six have been located, the class sings:

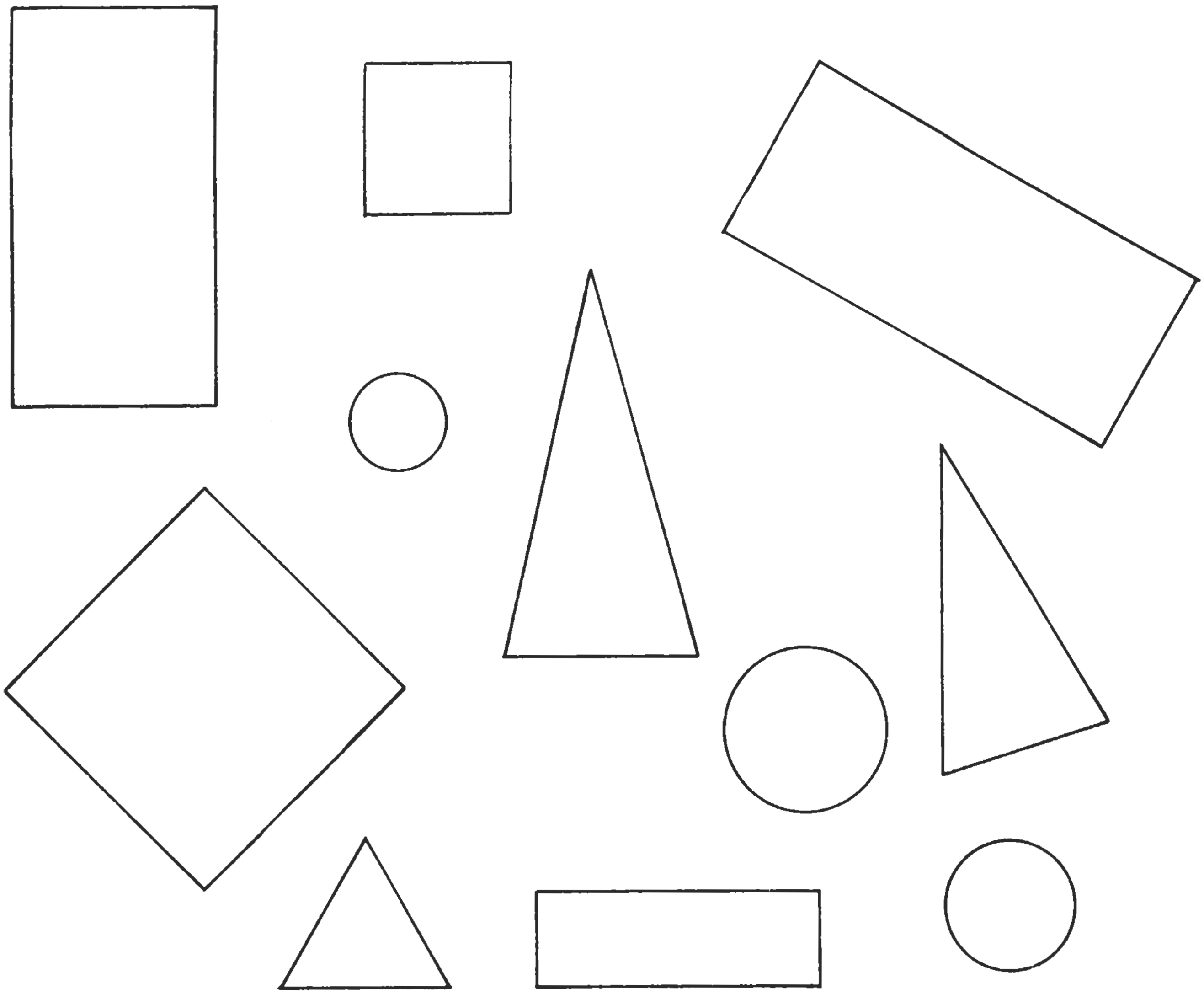
"Where are all of you? Where are all of you?"

All pop out and sing:

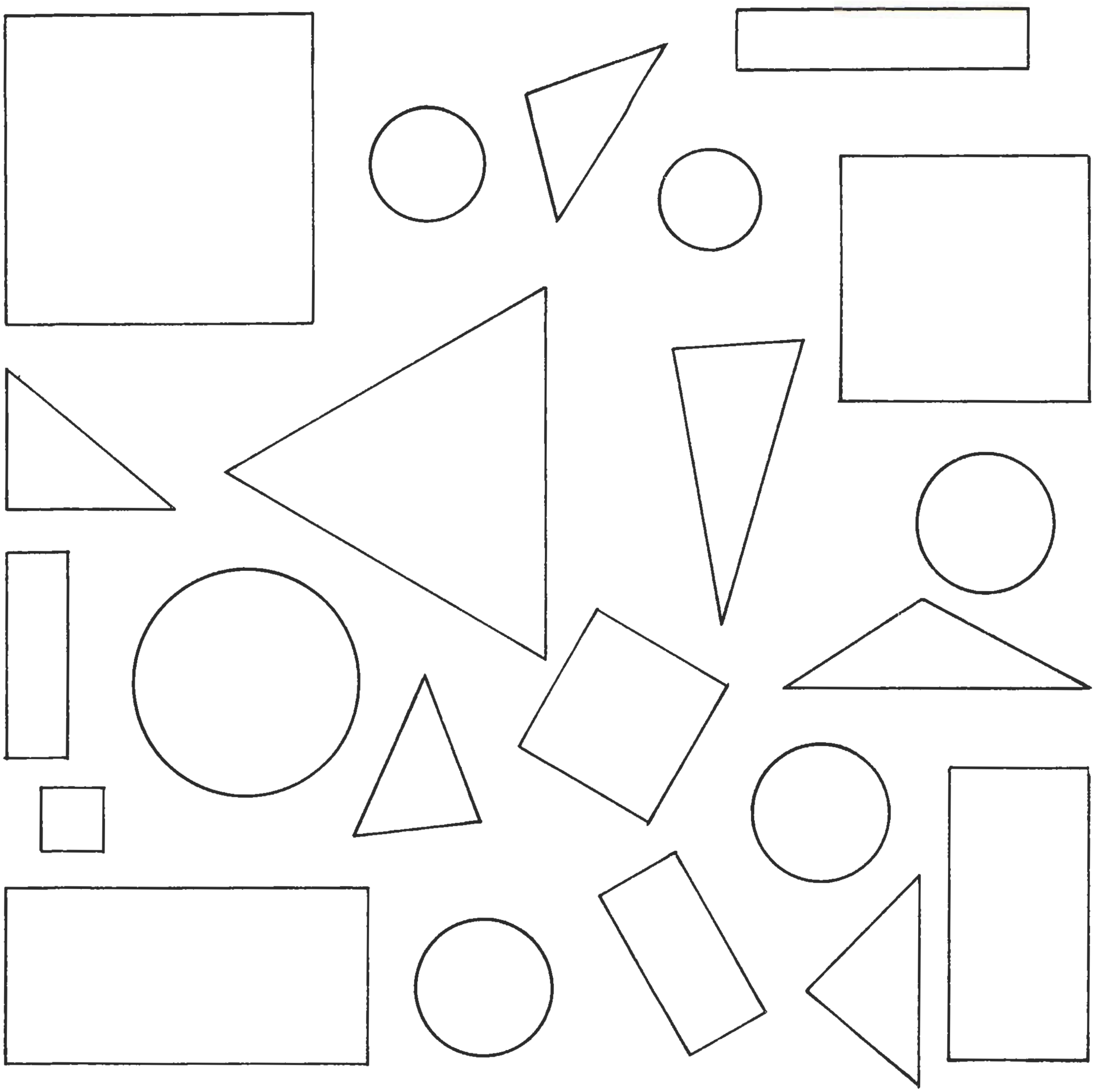
"Here we are! Here we are!"

Another time distribute different geometric figures to other children and play the game again. Distribute Worksheets *52, 53, *54, 55, and 56. These are test pages.

Distribute Worksheets 57 and 58.



1. Color the insides of the square rectangles blue.
2. Put an X inside the circles.
3. Color the insides of the oblong rectangles red.
4. Put a simple closed curve around each of the triangles.



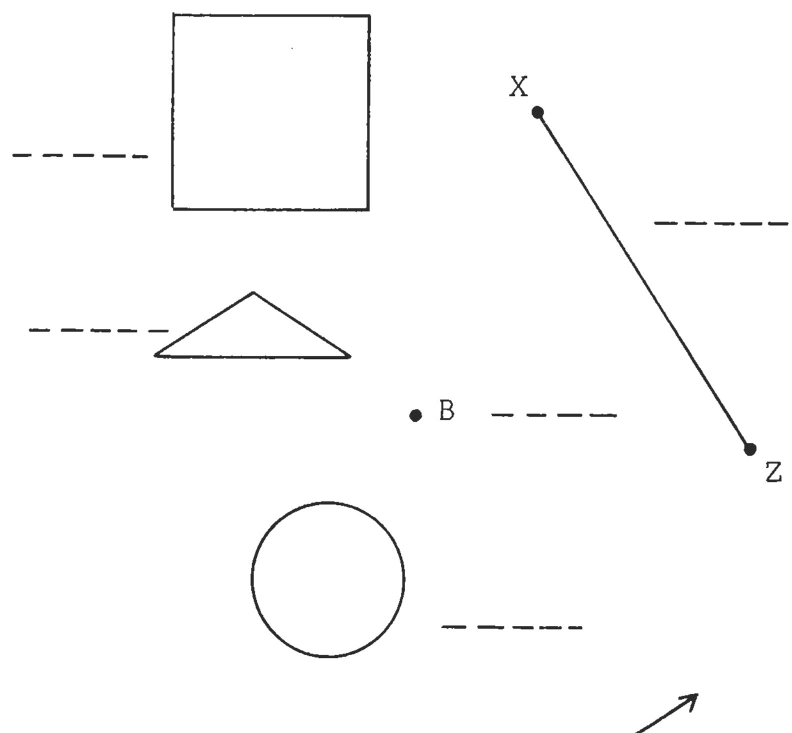
1. Color the insides of the circles green.
2. Put an X inside the square rectangles.
3. Put a closed curve around the triangles.
4. Color the insides of the oblong rectangles orange.

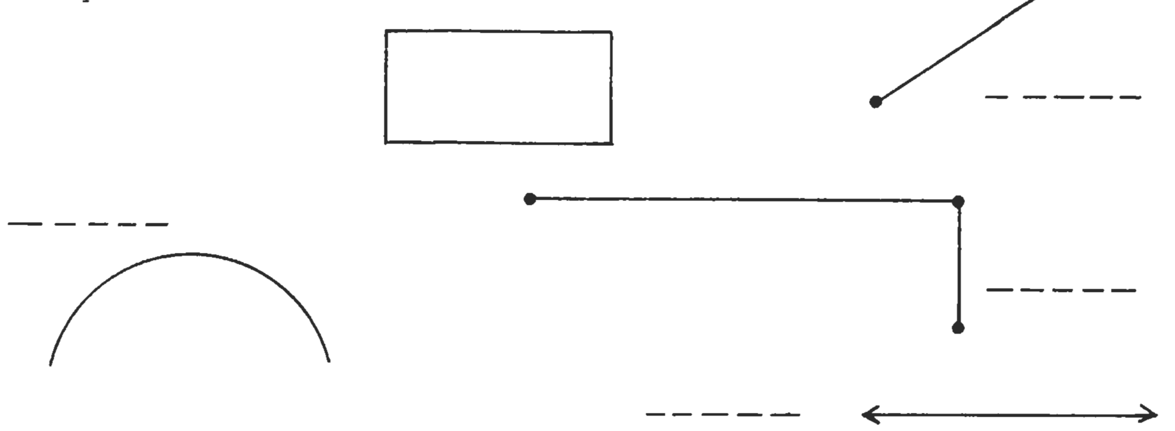
A. Look at the words below on the left of the page. Number 1 says "square." Put the numeral "1" on the line beside the figure of the square.

B. Find the figure that matches the word for 2 and label that figure with the numeral "2."

C. Do this for all the words and figures on this page.

1. square
2. triangle
3. broken line
4. circle
5. point
6. oblong
7. line segment
8. straight line
9. curve
10. ray



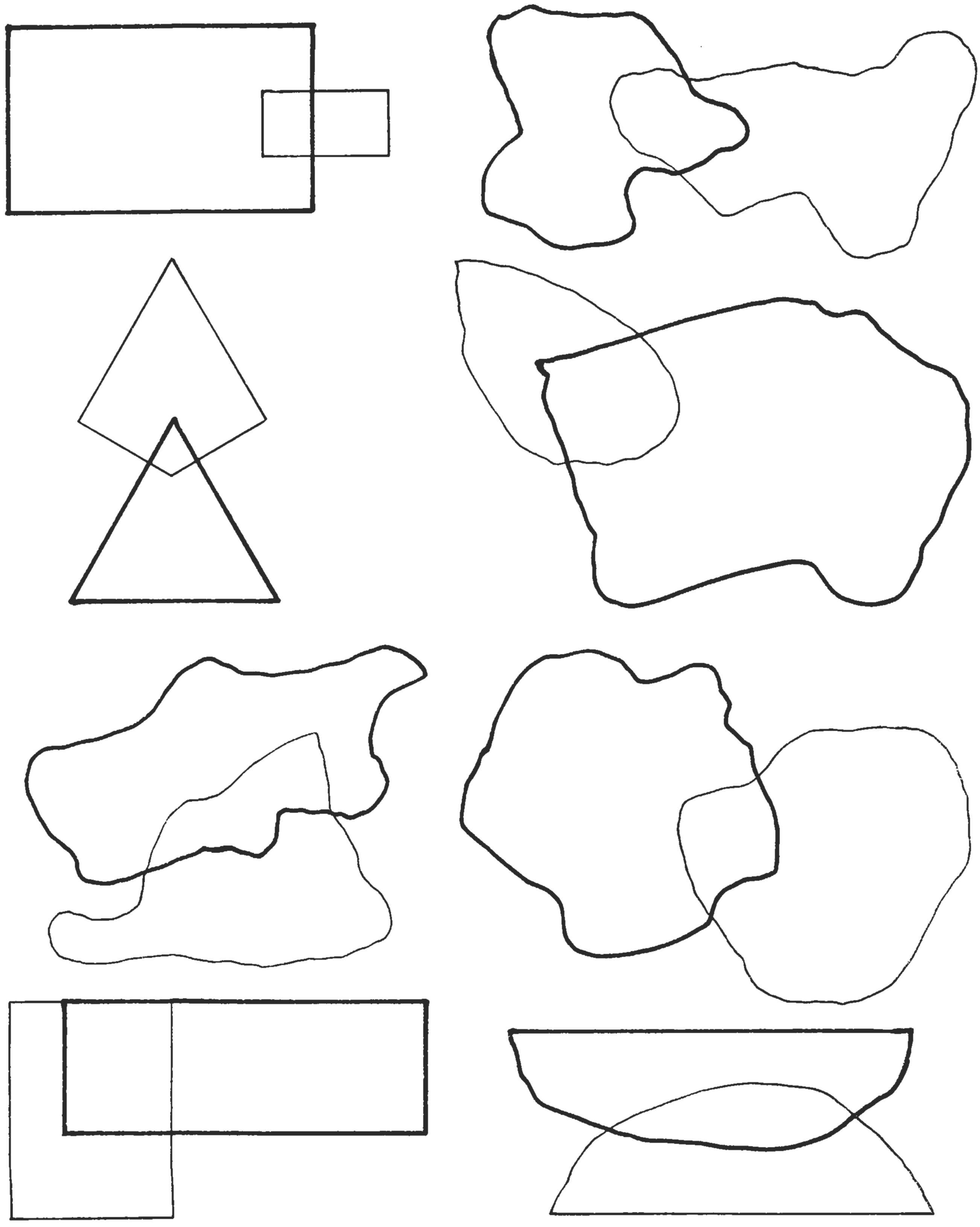


WORKSHEET 55 Review Geometry

Use your ruler for this work.

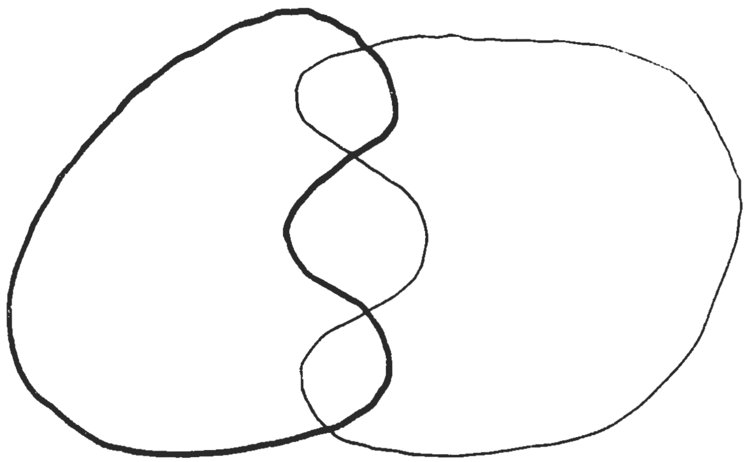
1. Draw triangle ABC.	2. Draw a circle.
3. Draw a right angle. Label it X.	4. Draw a square. Label the angles MNOP.
5. Draw an obtuse angle.	6. Draw an acute angle.
7. Draw an oblong. Label the angles ABCD.	

WORKSHEET 56 Review Intersection of Regions

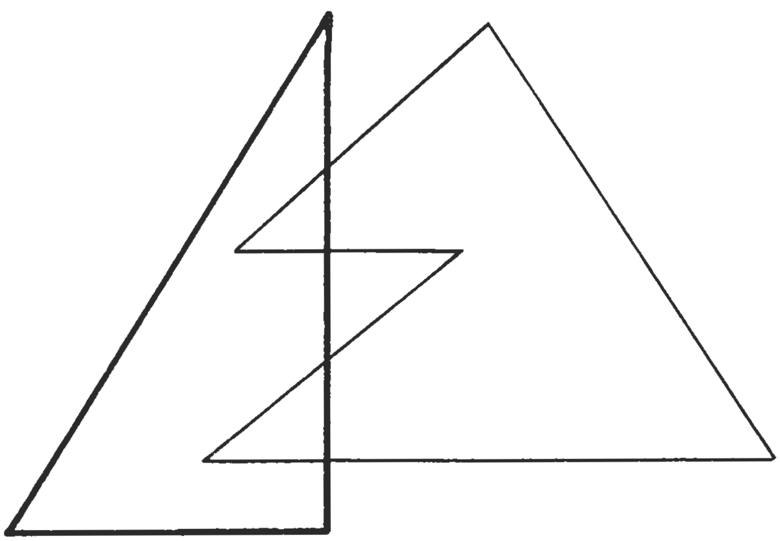


Color the intersections of the regions.

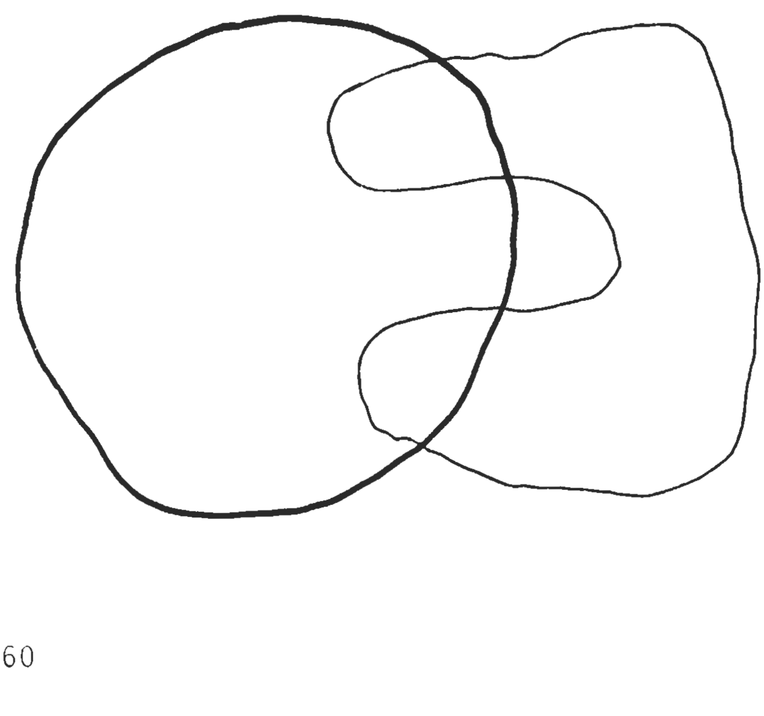
WORKSHEET 57 Intersection of Regions



Color the intersection of regions A and B.

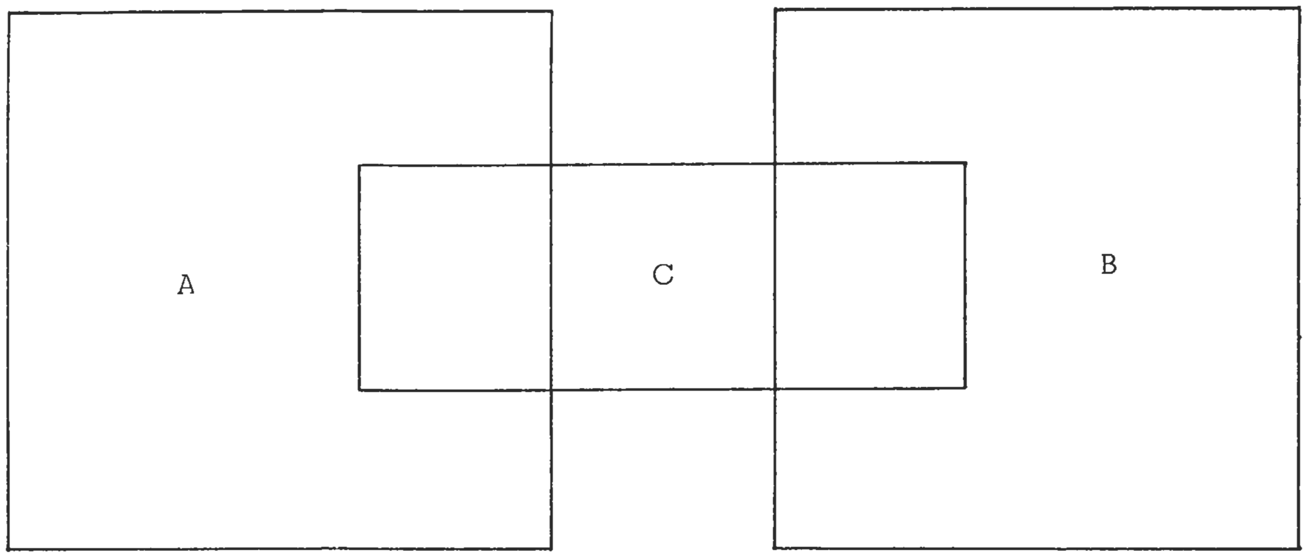


Color the intersection of regions G and H.



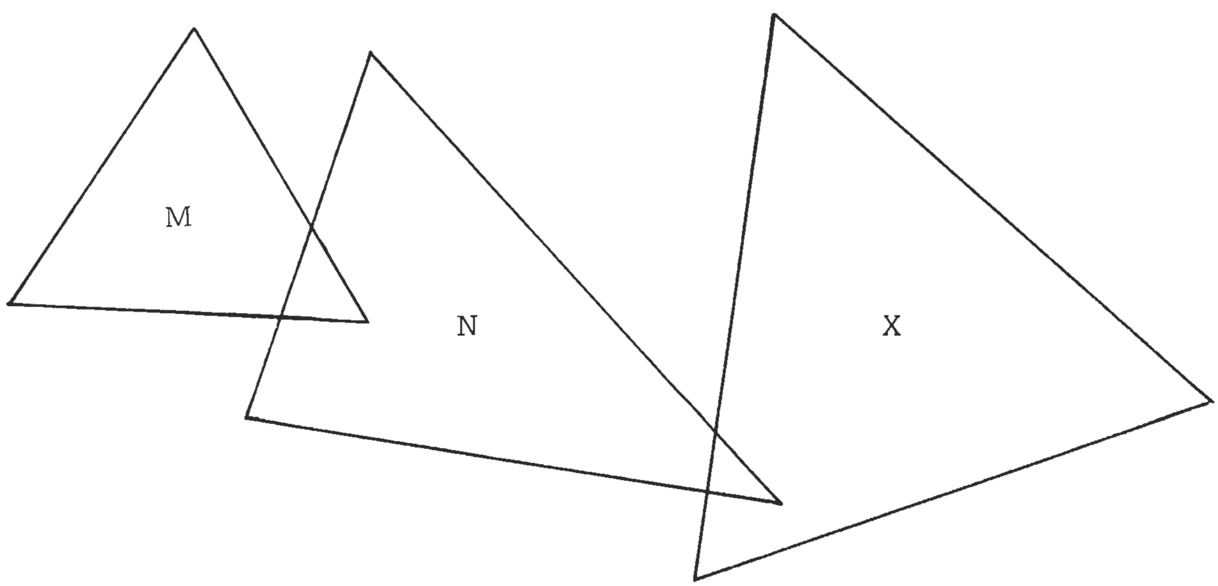
Color the intersection of regions P and T.

WORKSHEET 58 Intersection of Regions



Color the intersection of rectangular regions A and C red.

Color the intersection of rectangular regions B and C green.



Color the intersection of triangular regions M and N orange.

Color the intersection of triangular regions N and X blue.