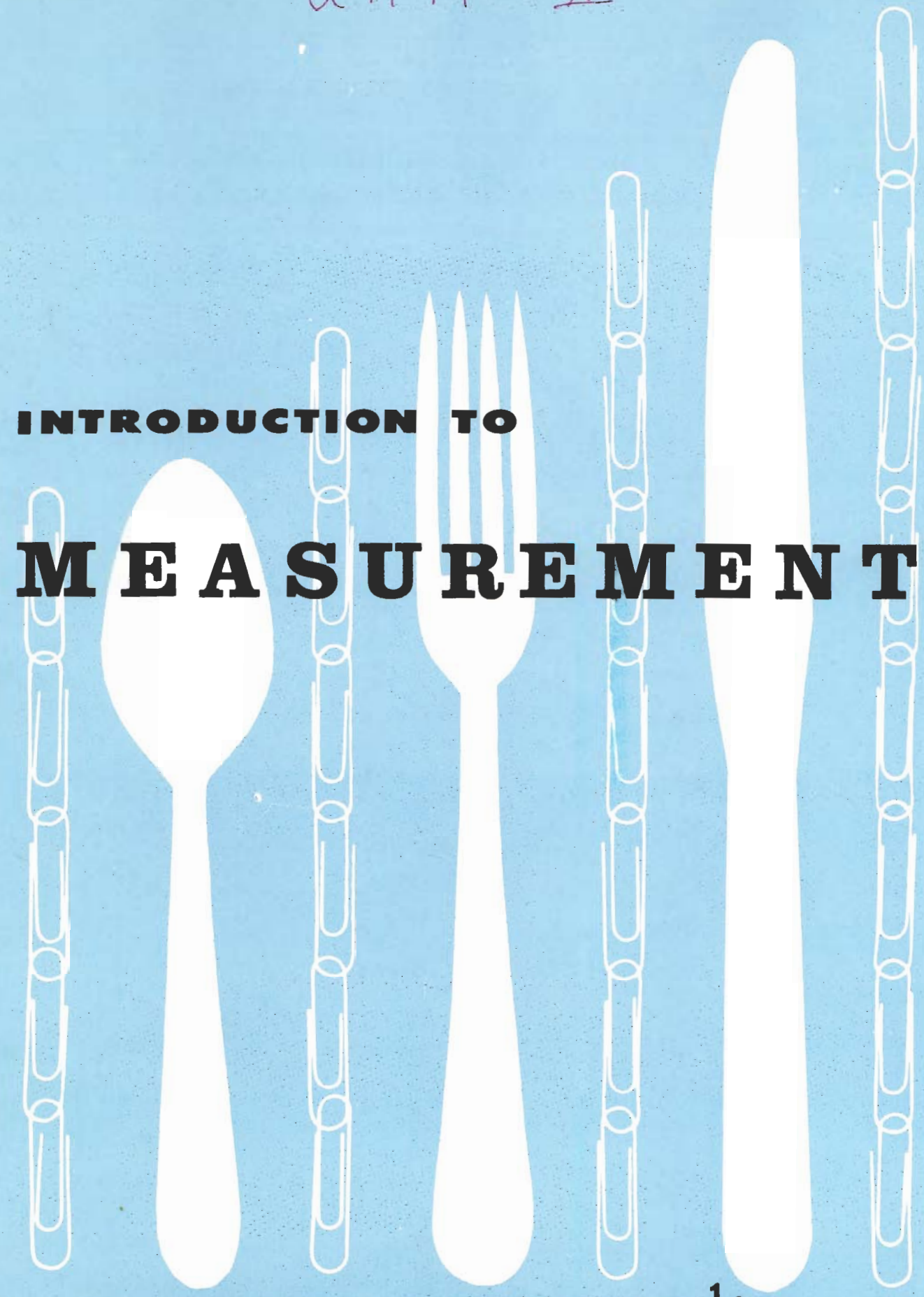
UNIT V



# INTRODUCTION TO

# MEASUREMENT

MINNESOTA SCHOOL MATHEMATICS AND SCIENCE CENTER 1.3

# INTRODUCTION to MEASUREMENT

UNIT 1.3

Minnesota School Mathematics and Science Teaching Project

The Minnesota School Mathematics and Science Teaching Project

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MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT

JAMES H. WERNTZ, JR.
Associate Professor of Physics
University of Minnesota
Project Director

PAUL C. ROSENBLOOM
Professor of Mathematics
Teachers College, Columbia University
Mathematics Director

# INTRODUCTION to MEASUREMENT

Z Z. HUGUS, JR.
Professor of Inorganic Chemistry
University of Minnesota

ROBERT R. KADESCH
Associate Professor of Physics
University of Utah

ALAN H. HUMPHREYS
Assistant Professor of Education
Extension Division
University of Texas

WILLIAM C. PHINNEY
Associate Professor of Geology
University of Minnesota

MILTON K. SNYDER
Associate Professor of Chemistry
Colorado College

SONIA D. FORSETH — Writer-Illustrator

JACK KABAT — Artist

ELAINE E. VOGT — Writer-Editor

ARTHUR E. WARHEIT — Art Director

# MINNEMAST SUMMER WRITING CONFERENCE - SCIENCE

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Elementary Teacher
Vogel School
Evansville, Indiana

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Research Associate
Minnemast Center

LUIZ ANTONIO BAEZ
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University of Puerto Rico

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Blaine School
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Undergraduate
University of Minnesota

JOSEPH FILZEN
Artist
Minnemast Center

SONIA D. FORSETH
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Minnemast Center

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Augsburg College
Minneapolis, Minnesota

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Undergraduate
University of Minnesota

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Professor of Inorganic Chemistry
University of Minnesota

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Extension Division
University of Texas

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Multnomah Elementary School
Portland, Oregon

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Professor of History
University of Minnesota

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Brooklyn College
Brooklyn, New York

ELAINE E. VOGT
Writer-Editor
Minnemast Center

LORE WALL
Secondary Teacher
Parsippany High School
Parsippany, New Jersey

VERNA YATER
Writer
Minnemast Center

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# INTRODUCTION TO MEASUREMENT

## Introduction for the Teacher

Measurement is one of the most important and fundamental operations in all of science. It is the process whereby properties of objects and systems of objects are assigned quantitative values. Quantification is an essential link that ties much of science to the discipline of mathematics, and conversely, mathematics to science.

In this unit the children are introduced to the concepts of length, area, and volume. Measurement of time follows in Unit 1.5. Taken together, these two units form a bridge between the child's study of individual objects in the preceding work and the study of systems of two or more objects in the material that follows. Measurement of a property such as the location of one object in a system with respect to another is an essential part of this later work. While length, area, volume, and time are the only properties that are quantified at this grade level, the possibility of measuring such properties as weight and shape will soon become apparent to many children.

The most rudimentary form of measurement is the operation of ordering, or ranking, objects according to a particular property. A numerical measure of the property is not obtained in this process. This is the kind of activity that the children have already encountered in Unit 1.1, OBJECTS AND THEIR PROPERTIES. There, objects were ranked according to their viscosities and minerals were ranked according to hardness. A take-home activity involved the children in the ordering of some common household substances with respect to grain size. This kind of activity is extended in this unit to include ordering activities that will be performed with lengths, areas, and volumes.

The current unit relies rather heavily upon the intuitive notions of children concerning length, area, and volume. Length will be developed as something pertaining to a line; area will be seen as an entity that can be physically covered; and volume as something that can be filled with material objects. An alternative approach that is not used here would develop the relation of both area and volume to linear

measure. Instead, the three properties are approached in this unit independently of one another. Once introduced in this way, however, a surface is observed to have a number of linear dimensions in addition to an area. Similarly, a three-dimensional object can be measured for lengths and areas as well as for volume.

The fundamental operation of making a binary comparison is the single most important thread that runs through this unit. When a length, area, or volume of an object is measured, the property of one object is compared directly to the corresponding property of another object. If, however, two lengths, two areas, or two volumes are to be compared, it may be inconvenient to do this directly. Suppose, for example, that the surface areas of two lakes are to be compared. If each lake is first compared to a set of standard areas, the two lakes may then be compared to each other using these results. Two comparisons rather than one are required in such cases.

Throughout this unit (as well as those to follow) measurement is defined in operational terms. Defined in this way, measurement consists of a definite, manual procedure followed by the act of counting. No object is considered to have a length, area, or volume independent of the act of measurement. These three quantities result from specific operations performed upon real objects, and thus are properties that belong as much to the operation that is performed as to the objects upon which the operation is performed.

The children are helped to see for themselves the advantages in using a standard length, a standard area, and a standard volume in measuring these quantities. The "standard" in each case is completely arbitrary. Commonly accepted standards are important only because they eliminate confusion in stating results. It is even more important to recognize that standards are discrete entities. Measurement can only be performed using multiples or submultiples of some standard. Thus an essential part of the process of measuring is the act of counting—a counting of discrete objects in each case. The result of such a measurement is appropriately expressed as an interval in all cases. In other words, the property measured should be expressed as being greater than one value, but less than another value. The interval involved expresses an uncertainty in the measurement. It is not important at this time for children to search out the sources of uncertainty in the measurement process. It may be associated with the object measured, the measuring instrument, or the measuring operation.

To express the results of any measurement properly, children will enjoy learning the significance of the inequality signs ">" and "<", to be read "is greater than" and "is less than," respectively. They will learn, for example, that "3 > 2" is read as, "Three is greater than two" and that "5 < 7" is read as, "Five is less than seven." They will appreciate the ease with which the important ideas "greater than" and "less than" can be expressed. Proceeding logically, they will next learn to use the symbols twice in a single statement. Thus they will learn to read "5 < 7 < 9" as, "Five is less than seven and seven is less than nine." Then, substituting the "greater than" symbol for the "less than" symbol, the children can say the same thing in a different way, namely: "Nine is greater than seven and seven is greater than five." Thus a child might measure the length of a book in paper-clip units and find that the length of the book is more than the length of 8 paper-clips linked in a chain and less than 9 paper-clips linked in a chain. This is written as

8 paper-clips < length of book < 9 paper-clips

which means that the length of 8 paper-clip units is less than the length of the book and the length of the book is less than 9 paper-clip units. Looked at another way, the three lengths referred to above appear in order of increasing length (from left to right) as indicated by the inequality signs.

Occasionally a child will report the result of a single measurement to be (as nearly as can be determined) an integral number of standard units of some kind. Technically speaking, it should be kept in mind that, even with the finest measuring devices, absolute perfection in measurement is impossible. All measurements are approximations. But, for the purposes of these lessons, the area of the map of an irregularly-shaped lake may be considered equal to the whole number of objects used to cover the map surface, considering the objects specifically as covering and as non-overlapping units of area measurement.

When repeated measurements are made, or measurements by other children collected, the child will discover also that, because of the nature of the standard used, there is no single correct result of the measurement. The area of the lake is reported more properly as an inequality in the form

area of 18 standard objects < area of lake < area of 22 standard objects

The bounds of 18 and 22 objects such as those above may at this time be determined by the children in any way that is satisfactory to them. The central idea is that the area of the lake probably lies between two such values.

The objectives of this unit are the following:

1. When given any two objects whose length is considerably greater than any other dimension, the child should be able to compare them and state whether one object is longer, shorter, or the same length as the other object.

2. The child should be able to rank physically a set of four to eight objects according to their lengths.

3. Given a set of four to eight objects ranked according to length the child should be able to rank an additional object O with respect to the original set and write its relation to adjacent members of that set as

   $O < G$
   and $O > F$
   or $F < O < G$

   or state the appropriate relations verbally.

4. The child should display an understanding of the operation of length measurement using multiples of an arbitrary standard length by actually performing this operation and reporting his result (either in writing or orally) as a range of values.

5. The child should display an understanding of the operations of both area and volume measurement using multiples of arbitrary standards of area and volume by actually performing these operations and reporting his results (either in writing or orally) as some whole number of standard units.

6. When confronted with a number of different results for the same area or volume, a child should be able to express this data (either in writing or orally) as a range of values.

## LESSON 1 ORDERING OBJECTS ACCORDING TO THE PROPERTY, LENGTH

### TEACHER COMMENTARY

This lesson is a direct extension of the previous activities of ranking objects according to some one property. Previously used were the properties of viscosity, hardness, and grain size in the unit, OBJECTS AND THEIR PROPERTIES. In this lesson the property of interest is that of length.

As a review of ideas introduced earlier, the lesson involves:

1. The act of making a direct, side-by-side binary comparison of the lengths of two objects.
2. The process of determining all possible pairs of objects in a set.
3. The task of ranking the objects in a set according to a single property.

In addition to the above, however, the significance of the inequality signs, ">" and "<", as well as that of " $\gtrless$ " is introduced.

It is quite important that the children not attach a meaning of limited significance to these symbols. "<", for example, does not mean just "is shorter than." It means "is less than" in some respect, whether it be less in length, less in viscosity, less in hardness, less in area, etc. Similarly, ">" means "is greater than" in some respect. Also, the symbol "$\gtrless$" is used to mean "is the same as" with reference to one property of the objects compared.

When one property of two different objects is said to be "the same", it is meant that, to the best of the observer's ability he is unable to determine which of the two objects has the greater value of the property. Thus the joint use of the "greater than" and "less than" symbols forms "$\gtrless$", the "is the same as" symbol. The term "is equal to" and its symbol "=" will be reserved for use in a more specific context to be developed in later units.

The central purpose of this lesson is to introduce the concept of length to the children in an intuitive way. Length is presented as a property of objects that is compared in a particular way. No more, no less, is desired.

## PROCEDURE

Materials: In this lesson you will need about ten identical sets of objects (one set for each group of three to four children in the class). Each set should consist of one each of the following objects. The symbol to be used to represent each of the objects is indicated.

| | | | |
|---|---|---|---|
| eraser | △ | paper clip, size #1 | B |
| crayon | ○ | Popsicle stick | C |
| pipe cleaner | □ | pencil (longest item) | A |

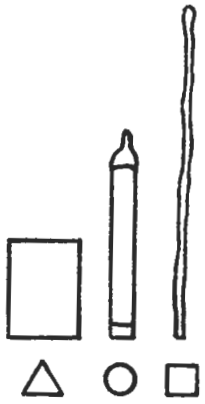
It is recommended that a single set of objects be used for each group of three or four children. Each object in the set should have a length obviously greater than its width or thickness. The shortest, longest, and one of the two objects of intermediate length should be labeled with letters, the other three with geometric figures.

To introduce this lesson, ask: "Who in the class has the longest pencil?" A pencil must have been sharpened at least once if it is to qualify. Each of a number of children may think that his pencil is the longest. For a time, act as moderator for any argument that may develop. Ask each of the children claiming to have the longest pencil how he knows that his is the longest. A child may say that his pencil "looks" longer than any other. Another child may say that he has sharpened his pencil only once and therefore it "just has to be" the longest.

Ask the other children if they accept these kinds of statements. If not, ask, "What would you propose as a good test to determine the longest pencil?" If necessary, remind the children how they compared the viscosity of two liquids and the hardness of two minerals (Lessons 4 and 5 in OBJECTS AND THEIR PROPERTIES). Encourage further discussion by the children until one or more fairly explicit suggestions for testing are made.

The children should be able to determine the longer of two pencils through direct side-by-side comparison of the two objects. After one child's pencil is declared the winner of a single comparison test, other children may challenge with their pencils until the single longest pencil is determined. In making these comparisons it is natural for children to hold the two pencils vertically with one end of each pencil on a flat surface and then to compare the positions of the other two ends. As long as this method is used consistently, all is well. If a child uses another method in which pencils are compared without having their ends matched, be sure to raise a question as to its relative merit. The significance of end-point coincidence should be emphasized, as it will be used later in determining the equality of time intervals. A similar contest may be conducted to find the shortest pencil in the class. In this situation, as before, be sure that the children realize that a definite comparing procedure is to be used at all times. (MINNEMAST Mathematics Unit III, MEASUREMENT, also contains this comparison exercise.)

Ranking Activity. Distribute a set of the three objects marked by a triangle, circle, and square to each group of three to four children. Ask each group to rank the objects according to the length (longest dimension) of each with the longest object at their right and the shortest at their left. If right and left cause difficulty, use some other direction such as, "Put the longest object on the side of the table which is closest to the windows." This task will probably be accomplished by inspection without resorting to a binary comparison of lengths two objects at a time. In any case, check the ranking of the objects in the set as established by each group of children. Each group now has an arrangement something like this:



Now introduce the symbols: ">", "<", and "$\geq$" meaning, respectively, "is greater than," "is less than," and "is the same as." When considering the property of length, the meanings become: "is greater in length than," "is less in length than," and "is the same in length as." In addition, it is worthwhile to point out that the symbol ">" is itself wider, or greater, on one side than it is on the other, and that the object represented on this side is therefore "greater" in some sense than the object represented on the other side. Tell the children that these symbols may be used to describe a comparison of many different properties of two objects-properties such as viscosity, hardness, roughness, slipperiness, etc.

Specifically, if the length of a pipe cleaner is greater than the length of a crayon, then

pipe cleaner length > crayon length

or

crayon length < pipe cleaner length

If the symbol $\square$ is used to designate the length of the pipe cleaner, and $\bigcirc$ is used to designate the length of the crayon, then

$\square > \bigcirc$

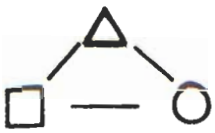
or $\bigcirc < \square$

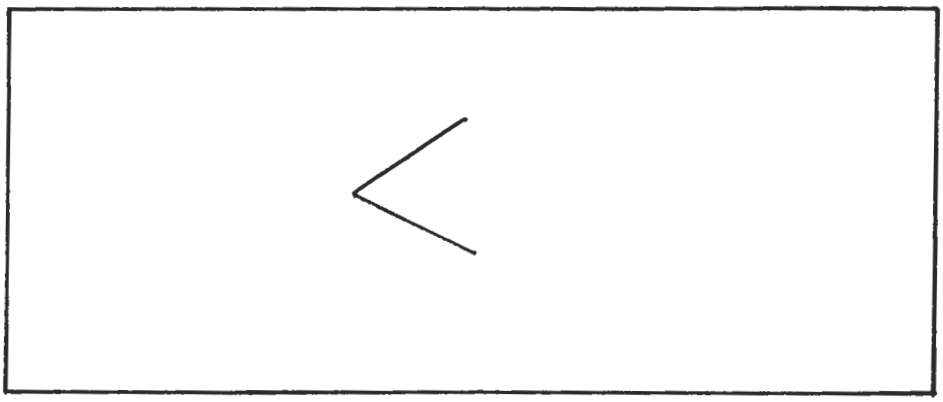
Distribute a copy of Record Sheet #1 to each child. Have them compare the lengths of the three objects by pairs. A brief review of methods that may be used to find all possible pairs would be in order. The children are to enter the three possible pairs of symbols (for the eraser, crayon and pipe cleaner) in the spaces provided on the record sheet. The diagram of symbols at the top of the sheet will serve as a reminder of the objects and the "<" symbols are already present, establishing the usage that is wanted. To find the correct length comparison for each pair of objects, children should be encouraged to make side-by-side observations of each pair.

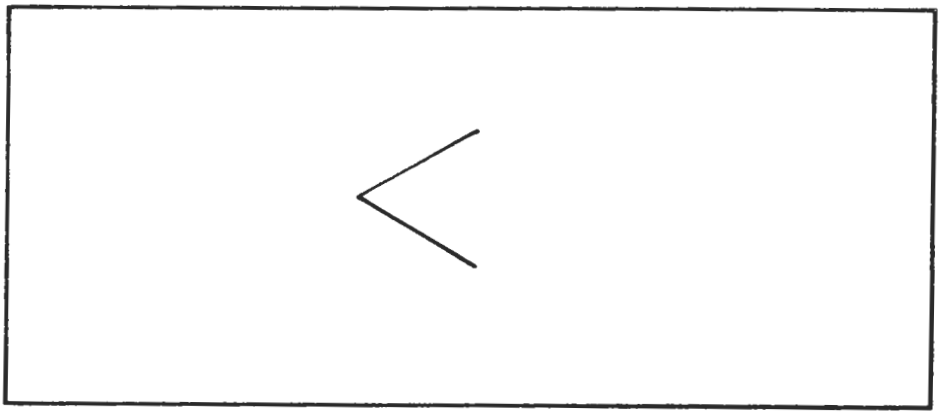
The teacher must at this point satisfy herself that the children understand the relations expressed by ">" and "<" so that the class may proceed.

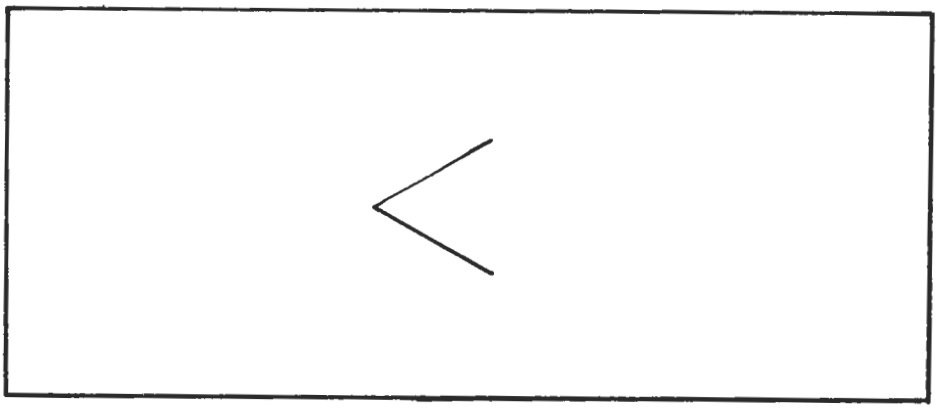
RECORD SHEET #1

NAME _____









Ranking Additional Objects. With the first set of three objects still in front of the children (or returned to the children if they have been collected previously) and ordered by length, add the paper-clip, symbolized by the letter B, to each set. Have each group of children rank the paper-clip by comparing it to each of the other three objects in the set. Now ask the children to express the relation between the length of the paper-clip and one of the other objects. The possibilities are:

B < △

B < ○

B < □

All relations are, of course, correct. Remind the children that the relations among the first three objects are already known (on Record Sheet #1). Ask if they had to choose one of the above relations, which one they would prefer. Hopefully, some of the children will see that

B < △

is the preferable single relation because it tells one that the paper-clip is shorter than the shortest of the first three objects. Don't belabor the point, however.

Now add a pencil, symbolized by the letter A (the longest of the six objects), to the set of four objects. Repeat the line of questioning used with the paper-clip to obtain:

A > □

Finally, add a Popsicle stick, object C, to each set of five objects. The five objects should at this point be placed in correct relative order (again from left to right) as to length. Each group of children is to place the Popsicle stick in its correct position using whatever direct comparison techniques are necessary. Now ask that the position of the Popsicle stick be expressed in relation to the others. Very likely many children will volunteer that

C > ○

and C < □

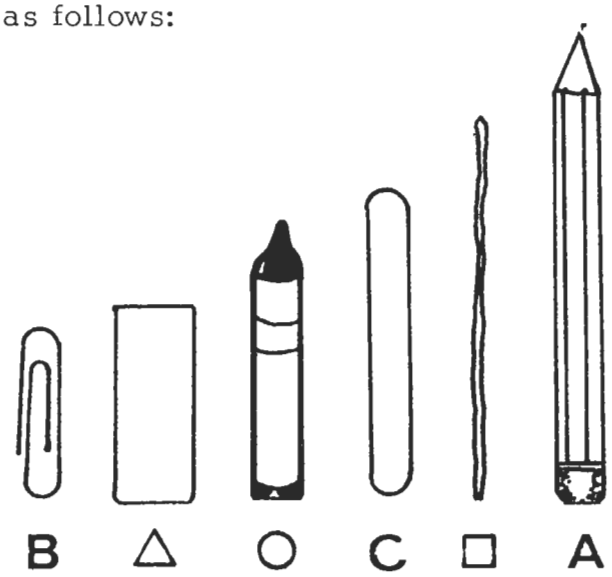
since the Popsicle stick is to the immediate right of the crayon and to the immediate left of the pipe cleaner. This result would be obtained if a child thinks of the Popsicle stick as being the longest of one set of objects and the shortest of another set. (This is the rationale for introducing the paper-clip and pencil before introducing the Popsicle stick.) There are, of course, many other ways to express the position of the Popsicle stick among the other objects.

Ask the children if they might not express the position of the Popsicle stick as being "between the crayon and the pipe cleaner." How would this relation be expressed? If necessary, introduce the relation yourself

$$\bigcirc < C < \square \quad .$$

Ask the children to verify the above statement. Be sure also that they understand what it says: $\bigcirc < C$ and $C < \square$ .

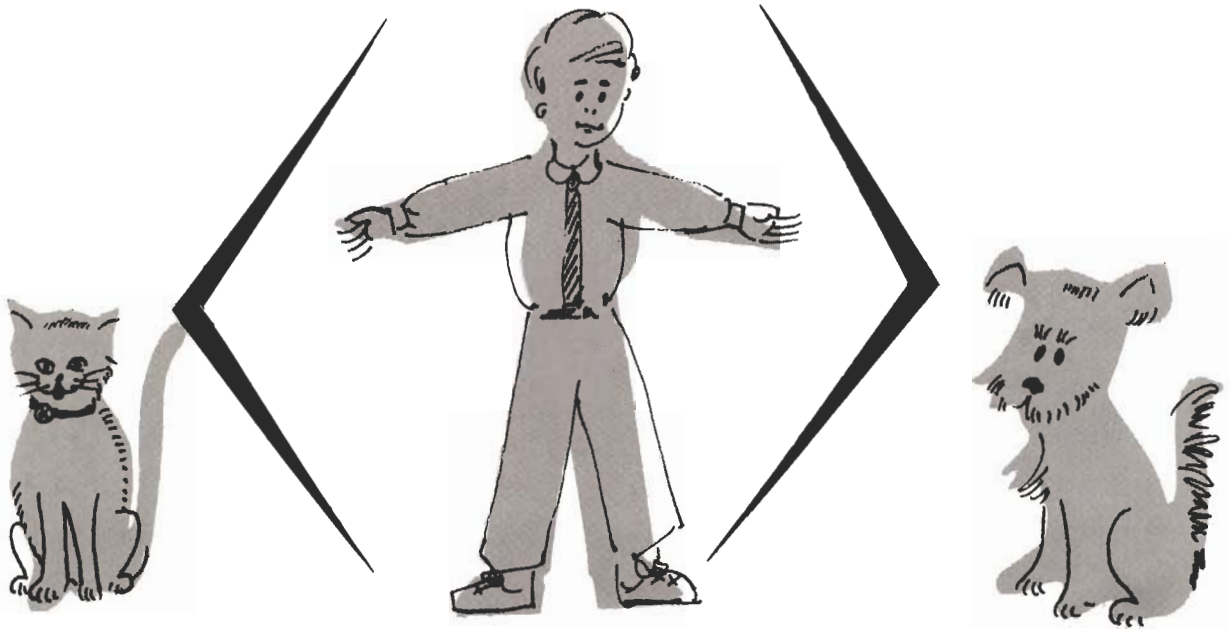
At this time the six objects will be ranked (ordered) in front of the children as follows:




Ask them now to express the relative lengths of the paper-clip, eraser, crayon, Popsicle stick, pipe cleaner, and pencil. ($B < \triangle < \bigcirc$; $\triangle < \bigcirc < C$; $\bigcirc < C < \square$; $C < \square < A$). Have the children express in words what each of the four relations means, e.g., object $\triangle$ lies between B and $\bigcirc$ and is longer than B and shorter than $\bigcirc$ .

Read the story, "Arthur and the Symbols," to the class as a concluding activity before sending copies of it home.

# ARTHUR and the SYMBOLS



**(The Property of Height)**

Dear Parent,

Your child has been learning to compare two objects with one another in a variety of ways. For example, he has found that some liquids are less viscous than others, and has used the terms "runnier" and "less runny" in describing this property. He has also learned to compare minerals for hardness by trying to scratch one with the other, discovering that the one receiving the scratch is the softer. More recently he has learned that the symbol ">" means "is greater than" and that the symbol "<" means "is less than". He has enjoyed learning that very important ideas can be expressed with brevity. He knows that greater and less may refer to viscosity, hardness, length, or any one of a great many other properties of objects.

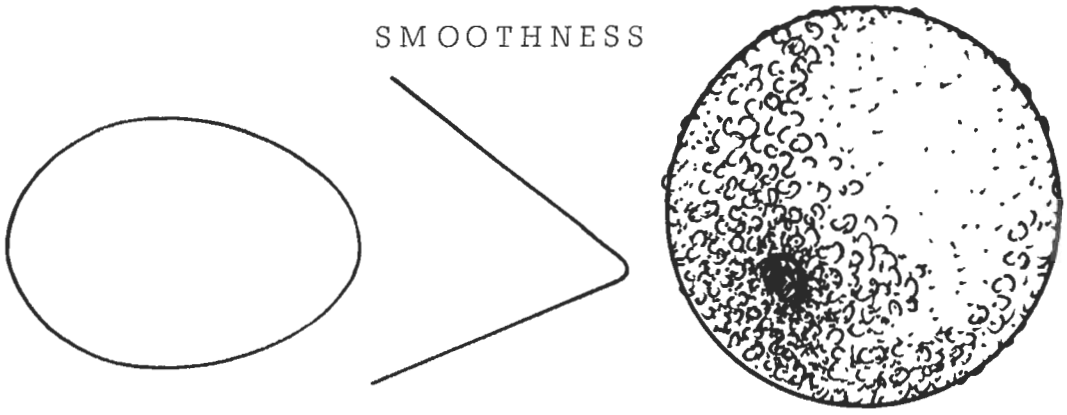
The story, "Arthur and the Symbols," has been read to the children in class. Your child will enjoy hearing it again. At the end of the story text there is a worksheet for the pupil's use. Each of the three spaces on that page contains a word indicating a property of an object. Encourage your child to look for objects with these properties around the home. Then have him test them and draw them in the spaces provided, using the symbols properly. Be sure there is enough difference in the objects compared so that a first-grade child can discover it.

Such activity will extend and enrich the science experiences your child has been having at school. It will also give him an opportunity to use and to show what he has learned.

Cordially,

Teacher

SMOOTHNESS



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## ARTHUR AND THE SYMBOLS

One afternoon, when Mother was out shopping, Arthur sat down at the kitchen table. Dick, his older brother, came along, peeked over Arthur's shoulder, and said, "What are you doing, Arthur?"

"I'm finding out which is greater," Arthur answered.

"Which is greater than what?" Dick asked him.

"Which is greater than this or which is greater than that," Arthur said.

Then Arthur whispered some words to himself, and Dick bent down to hear what Arthur was saying. He heard Arthur say, "This egg is SMOOTHER than this orange." Then he noticed that Arthur was drawing things on a piece of paper. Next, Arthur turned on a lamp and a flashlight, and Dick heard him whisper, "This lamp is BRIGHTER than this flashlight," and he saw Arthur making more marks on the paper.

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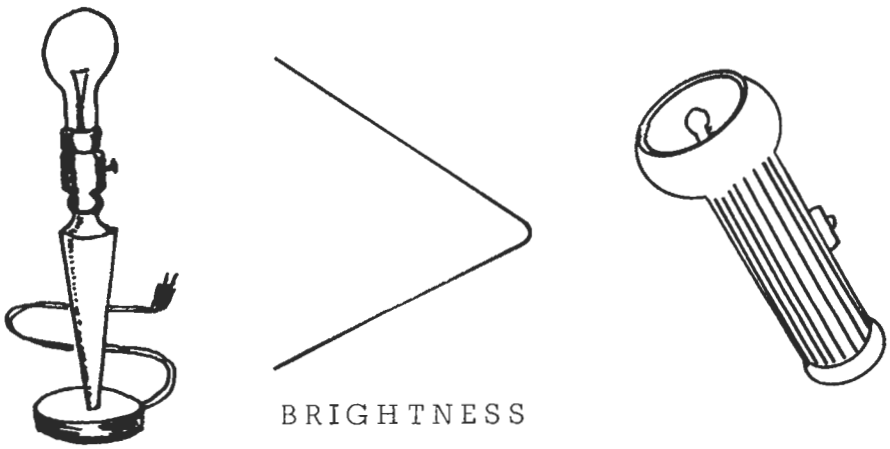
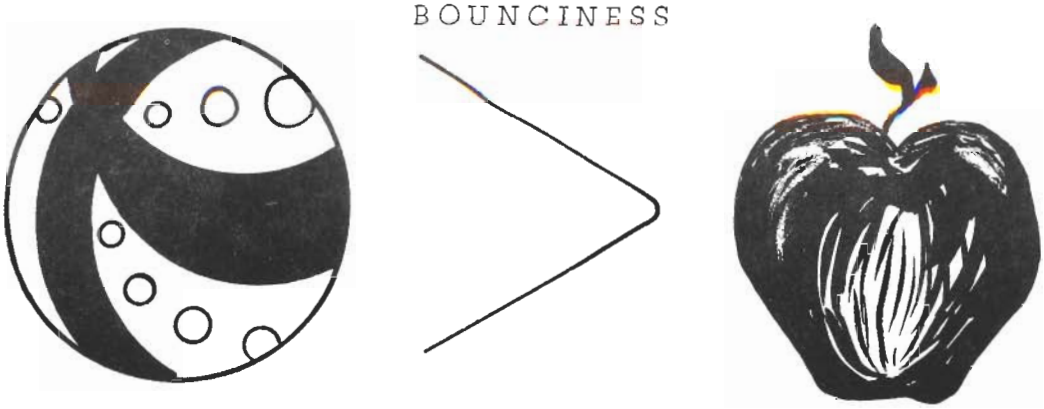


BRIGHTNESS

BOUNCINESS



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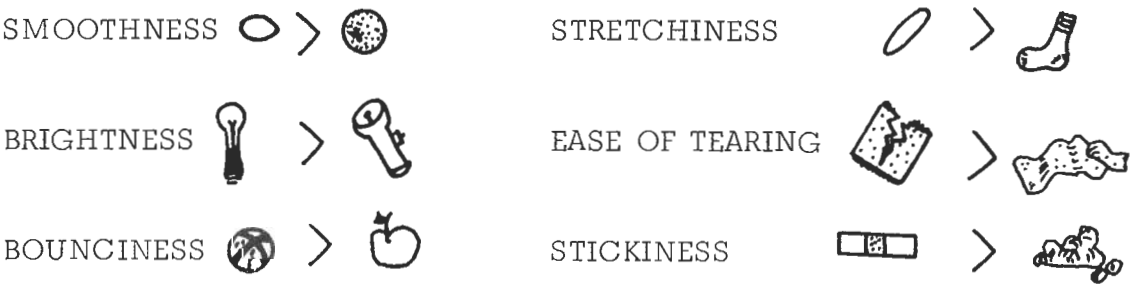
Now Arthur picked up a ball and an apple, and after trying to bounce them both, said, "This ball is BOUNCIER than this apple." Then he picked up a rubber band and a sock, and stretched them both, saying, "This rubber band is STRETCHIER than this sock." Next, he tried to tear a paper towel and then a dust rag, and said, "This paper is EASIER TO TEAR than this dust rag." After each experiment he wrote something on the paper. Finally, after pulling a small bandage and some play-dough from the table-top, he said, "This bandage is STICKIER than this play-dough," and wrote still once more on his paper.

Dick thought he couldn't wait another minute to find out what Arthur was doing. Quickly, he snatched up Arthur's paper. This is what he saw:

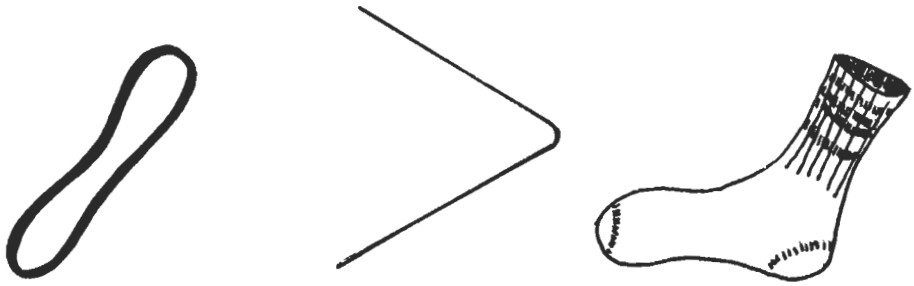



"What in the world are you doing?" Dick asked.

"Please, Dick," Arthur said, taking his paper back, "I am very busy! I am finding out which object has the greater property. When I find out which object has the greater property, I write it down like this. See?" And he touched an egg and an orange and then wrote:

SMOOTHNESS 

STRETCHINESS

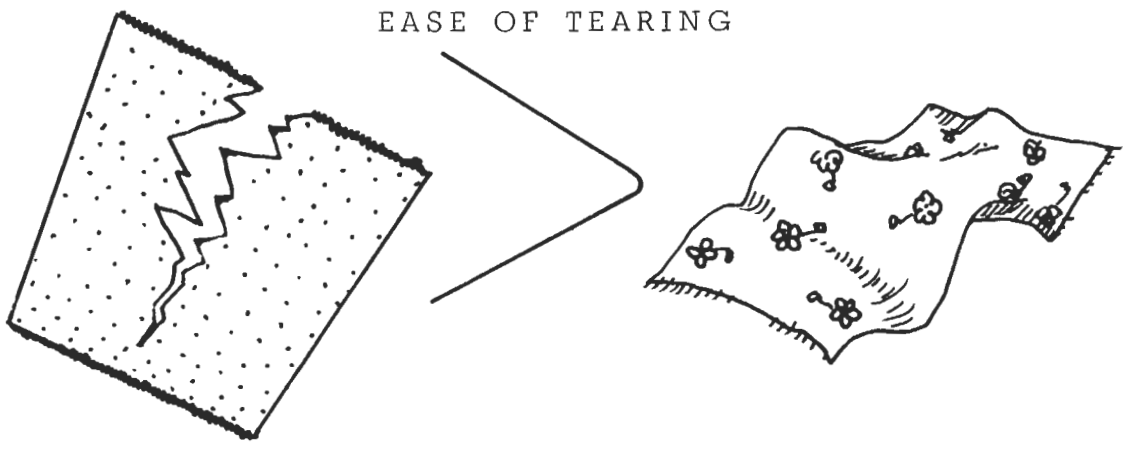


Explaining his actions to Dick, he said, "This means that this hard boiled egg is SMOOTHER than this orange. Here," he said, handing Dick the egg and the orange, "try it yourself. You can feel that the egg is much SMOOTHER than the orange."

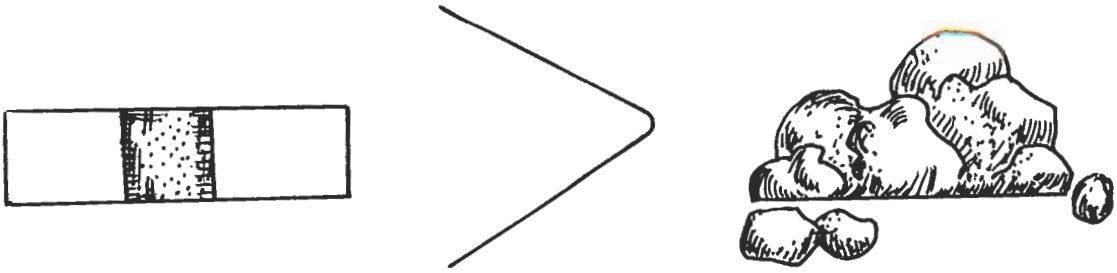
"But what's that pointy thing you drew in between the picture of the egg and the orange?" Dick asked.

"That's my symbol. The open end of the symbol points to the object with the greater property. The closed end points to the object with the lesser property. Look!" Then Arthur drew another symbol. "I can write it this way too," he said, drawing a "<". "I can say for the property of smoothness,  . This says the orange is LESS SMOOTH than the egg, but that the egg is SMOOTHER than the orange. The open end always points to the side with the greater property.

EASE OF TEARING



STICKINESS



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Then Arthur ordered his objects in neat rows on the table, like this:

SMOOTHNESS 

BRIGHTNESS 

BOUNCINESS 

STRETCHINESS 

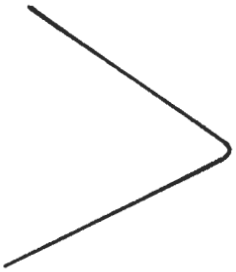
EASE OF TEARING 

STICKINESS 

"Let's find some objects that have the property of SWEETNESS and compare them," Arthur suggested. Dick quickly found a sugar cube and a candy lemon drop. Which of these do you think was the SWEETER, the sugar cube or the lemon drop?

PUPIL'S WORKSHEET

LENGTH



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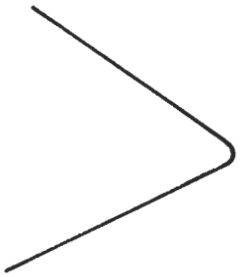
HARDNESS



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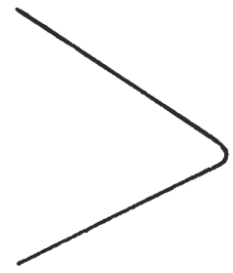
RUNNINESS



## LESSON 2 THE USE OF A STANDARD LENGTH

### TEACHER COMMENTARY

In this lesson the child is led to discover the convenience of using one standard of length rather than six or more. In the previous lesson six objects were compared and ordered. Presumably, all other objects in the world could be ordered in this same way. A measurement would then perhaps be stated as follows: "John's height is less than the height of a dwarf tree in Mongolia, but greater than that of a particular fallen log in Alaska." This situation is clearly ridiculous. By using a single object, such as a paper-clip or Popsicle stick, and presuming that a number of such identical objects is available, measurement becomes greatly simplified.

The children will find that two, three, four, or more paper-clips may be laid end to end to generate a set of lengths against which almost any object may be compared. This system has the advantage that each length available is both one unit shorter and one unit longer than the next-longest and next-shortest lengths. In addition, the result of a measurement may be communicated to someone else who also has available clips of the same size, without transporting the measuring equipment itself. The problem of measuring lengths shorter than that of a single paper-clip is not faced at this time. Instead, only multiples of paper-clips are used. The counting aspect of measurement should in this way be clear to every child.

A paper-clip chain is developed for measuring lengths. The device is convenient and at the same time makes clear that the result of a measurement should always be stated as an interval. Any length greater than that of a single clip can, in principle, be measured with such a chain.

There is a purpose in using a paper-clip chain for measurement rather than an ordinary ruler. Although many children are undoubtedly familiar with the inch, foot, and yard, they are not aware of the arbitrary nature of these measures. By using a paper-clip as a standard unit of length, the arbitrary nature of this selection may become more readily apparent at a later time. The point (not to be made at this time to the children) is that a Popsicle stick, or any other reproducible object, might have been used as a standard instead of the paper-clip. In fact, it makes no difference in principle what standard is employed.

## PROCEDURE

Materials: Distribute to each group of three or four children the same set of six objects used in the previous lesson (pencil, pipe cleaner, Popsicle stick, crayon, eraser, and paper-clip). Also needed in this lesson, at a later point, are four or five boxes of #1 paper-clips.

Ask the children to recount what they did with these objects previously. (They compared their lengths, two objects at a time.) Tell the children that this was really a kind of measuring—one object was measured by comparing it to another. Thus they learn that "measuring" is another word for "comparing." Now ask each group of children to measure the height of a book by comparing it to one or more of the six objects in their set. (Make sure that each child knows what is meant by the height of the book. Also, tell the children that this height is just a particular kind of length.) Do not suggest ways and means for making this observation, but permit the children ample time to make comparisons in their own way.

Some children may report that the height of the book is greater than the length of the pencil. Ask these children if there is any way to compare the height of the book to two or more of the six objects so that the things compared are more closely the same in length. A few of the children are sure to suggest that two or more of the six objects could be laid end to end to make lengths greater than the length of the pencil. These combinations could then be compared to the height of the book. Discuss such suggestions with the entire class.

Now repeat your request to measure the height of the book. Possible responses are:

book height > length of pencil and paper-clip laid end to end

book height < length of pencil and eraser laid end to end

book height < length of pipe cleaner and crayon laid end to end

If two or more groups of children have measured copies of the same book, such as a basic reader, ask these groups how their measurements should compare. Repeat the activity using the width of the books. Other possibilities for measurement are a desk height or width, the thickness of a book, the height or width of a window pane, and the teacher's desk.

Finally, ask each group to measure the height of a sheet of writing or drawing paper. Make sure that each group has the same size of paper. Ask each group to report its measurement as you write the result on the board. Probably the measurements will have been made using a variety of combinations of the six objects. Ask if all measurements are the same. (The answer is: No.) Ask if some measurements are incorrect. (The answer is: Not necessarily.) Ask the children if there are better ways to measure, and if so, why the other techniques are better. The children will surely agree that it is a nuisance to use two, three, or even more, different objects to make one measurement. At this time collect the six objects from each group of children.

Using a Standard Length. By now the children may have suggested a number of better ways to measure objects. Such suggestions may include using longer objects to measure greater lengths. This would, of course, add to the set of six objects. If children suggest using rulers or yardsticks, tell them that these will be used another year, and that now you wish them to discover why rulers and yardsticks were developed. Instead of very special objects like rulers, tell the children you wish to develop a new method of measurement, using only ordinary objects such as the six employed previously.

Even if no one suggests using a large number of identical objects to make measurements, proceed by passing out about ten paper-clips to each child. Ask the children to place a single paper-clip at the center of their desks. Can another object be made that is longer than one clip? (Two laid end to end.) Have the children place two clips, end to end, below the single clip. Can another object be made that is longer than two clips laid end to end? (Three laid end to end.) The three-clip chain should be constructed below the two-clip chain and the single clip. And can still another object be made that is longer than three clips laid end to end? (Four).

Since two or more paper-clips laid end to end cannot be conveniently carried about, ask the children if there is some other way to join these clips so that they may be carried from place to *place more conveniently*. (*Make chains of paper-clips.*) Ask the children how much longer four paper-clips laid end to end are than three laid end to end (or stretched in a chain). Ask the children how much longer the pencil they used previously was, compared to the pipe cleaner. The children should begin to get the idea that chains of 1, 2, 3, and 4 clips increase in length by equal steps, but that this was not true for the set of six dissimilar objects. In addition, they should discover that any length can be measured using a paper-clip chain if they have enough clips.

Have the children use their paper-clip chains to measure many of the same objects that were measured before. See if the children remember to report such lengths as <u>intervals</u>. For example:

5 paper-clips < width of book < 6 paper-clips.

Distribute Record Sheet #2. Then provide each child with an identical book or sheet of paper. Have each child measure both the width and height, and report in each case the interval in paper-clip lengths.

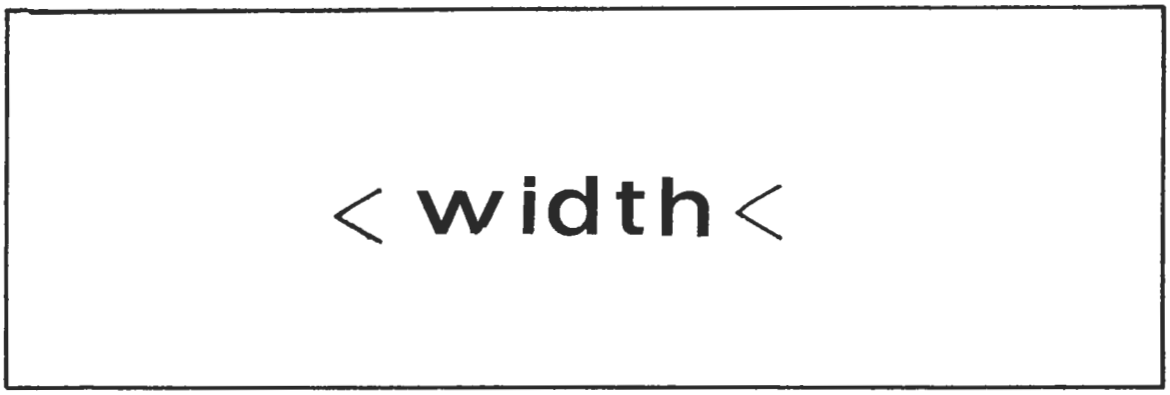
<u>Optional Activity</u>. The children may enjoy measuring lengths that are considerably greater than any of the objects measured previously, such as the length or width of the classroom. This may be carried out best as a group activity with all the paper-clips combined into a single giant chain.

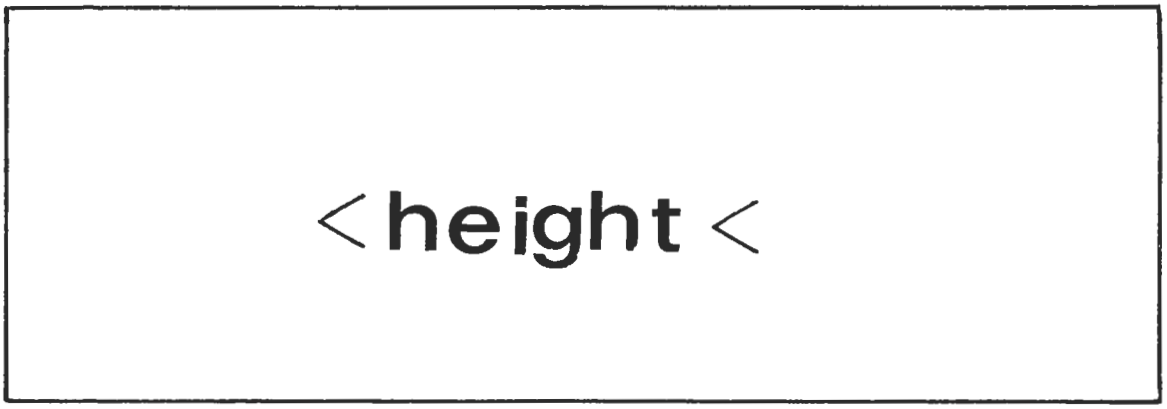
All the children may not be able to count beyond ten, but this need not prevent the use of as many as several hundred paper-clips in a single chain. Suggest to the children that every interval of ten paper-clips could be marked in some way, either by tying a small length of string or yarn on every tenth connection between two clips, or by hooking an extra clip around the joining ends so that it extends to one side.

Permit the children to proceed on their own. Accept such measurements as "twelve sets of ten paper-clips and five more." If counting up to ten only is desired, this same measurement could be reported as "ten sets of ten paper-clips, two sets of ten, and five paper-clips."

RECORD SHEET #2

NAME _____



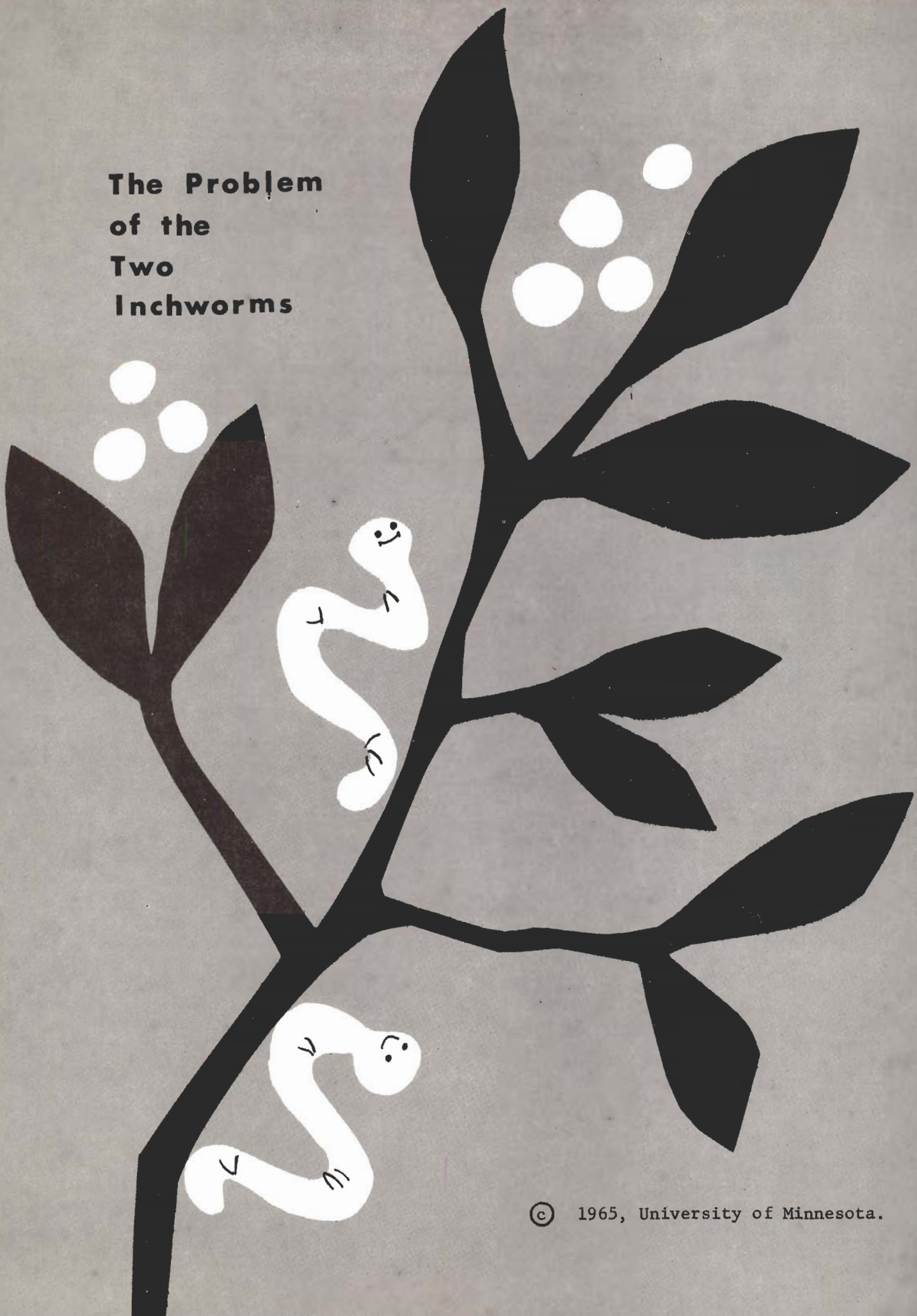



Another measurement of interest is that of the height of some particular child. Let the children overcome, if they can, whatever difficulties present themselves here. Ask them to try to improve upon the technique of holding the chain up by hand and letting it dangle to the floor.

Story: "The Problem of the Two Inchworms." The story that follows is the second of a series of stories accompanying this unit. Each story is designed to be enjoyed by the children, but also to extend, in some way, various aspects of the study of length, area, and volume. The value of these stories is enhanced by providing copies for the children to take home after they have heard and discussed them in class.

A letter to the parent is included. It will introduce the parents to a few of the basic ideas of the unit. Since this letter outlines very briefly the later work with areas and volumes, as well as the present work with the idea of length, no letters are included with the two remaining stories.

# The Problem of the Two Inchworms



Dear Parent,

The story, "The Problem of the Two Inchworms," is the second in a series of four stories your child will bring home during the study of the MINNEMAST unit, INTRODUCTION TO MEASUREMENT. The purpose of the stories is to reinforce the child's classroom experiences with measurement. Your child has now completed two lessons dealing with the idea of length. He first learned to compare the lengths of two objects. More recently, he has discovered that a chain of paper-clips is an extremely useful "ruler" for measuring many kinds of objects. Instead of using paper-clips, a number of Popsicle sticks might have been used. In the story, the two inchworms use the lengths of their own bodies. Ask your child about his work and encourage him to measure various objects found at home.

In the weeks to follow, your child will measure the area of a surface by finding out how many smaller equal-sized objects are needed to cover the surface measured. The volume of a container will be measured by counting the number of approximately-equal, small objects required to fill it. Your child will also learn that real objects not only have a volume (as a tin can), but, in addition, the surfaces of such an object have the property of area (as the lid of the can). Further, the object has various lengths (the distance across the lid or the height of the can).

Whether it is a length, an area, or a volume to be measured, some sort of "standard" reference object is used for comparison in each case. The child is learning to recognize that measurement is always a comparison of one object to one or more other objects.

Cordially,

Teacher

# THE PROBLEM OF THE TWO INCHWORMS

One, two, three! A thin, green inchworm arched his slender back into a tiny loop, as he made his way up a brown twig.

And one, two, three--again! Another little inchworm arched his back and traveled up the other side of the same twig. Together they inched all the way to the end of the twig. Then, resting there, the first inchworm said, "Well, friend, this twig is exactly fourteen loops long."

"Wrong!" said the second inchworm. "I was counting, too, and I happen to <u>know</u> that this twig is exactly <u>fifteen</u> loops long! I arched my back and straightened my body out fifteen whole times, so the twig just has to be fifteen of me."

The first inchworm measured the twig again. "Fourteen!" he insisted. Then the two inchworms took to arguing so much that a spider on a nearby twig stopped to listen. He heard the second inchworm say, "This is most peculiar. You and I both started at the bottom of the twig and went the same distance, and yet when we reached the end, you had made fourteen loops, but I had made fifteen. What a problem!"

"I think I know the answer," the spider said.

"You do?" the inchworms exclaimed. "Well, for goodness' sake, tell us! This is too much of a mystery!"

"It's very simple, really," the spider said. "One of you must be longer than the other. It must be you," he continued, pointing a leg at the first inchworm. "You didn't need to arch your back and straighten out as many times, because you just happen to be a slightly longer inchworm than your friend." Then the spider hurried away on some errands of his own, and the inchworms looked around for another twig to measure.

## LESSON 3 MEASURING AREA USING A STANDARD AREA

### TEACHER COMMENTARY

The concept of area is introduced in this lesson in an intuitive way. It is assumed that most of the children already have some notion of the size of surfaces. In this lesson a rather specific operational technique for the measurement of area is presented. Unlike a situation in which two lengths are compared side-by-side, two areas cannot usually be compared satisfactorily in the same way. Two areas may be compared indirectly by first comparing each to a number of identical objects. These objects are the "standard" areas for use in measuring any surface area. It is not important what they are, only that they are all approximately the same. In this lesson these standards are used to measure each of two irregularly shaped lakes. The irregularity of these shapes is important because a measuring technique is developed for them that has application to all situations. If a child learns that a square two miles on a side contains four squares each one mile on a side, that helps him little to measure any other shape of surface. Specific situations such as this should be reserved for much later study.

The teacher must use his discretion in using the word "area". By frequent and correct use of the word, the child is helped to learn the word. Problems develop, however, when children use the word incorrectly. It is the idea of the word, not the word itself, which is important. On the other hand, without the word children may refer indiscriminately to the "bigness" of a lake. Important here is the fact that the lake can have a large surface area, a large perimeter (distance around), or a large distance across the lake. It is extremely important for the children to recognize that any object can be "big" (or small) with respect to a variety of properties. The development of this lesson provides the notion of area first, and how it can be measured. Later on, objects that have the property of area are seen to have length properties as well. In Lesson 4 the children will learn that an object having the property of volume has, in addition, other properties such as area and linear dimensions. No direct connections between the concepts of length, area, and volume are developed at this time.

No particular problems are anticipated here in the use of standard areas (lima beans). A standard length (the paper-clip) was introduced in the previous lesson. As before, the arbitrary nature of such standards ("it could be anything") is important.

## PROCEDURE

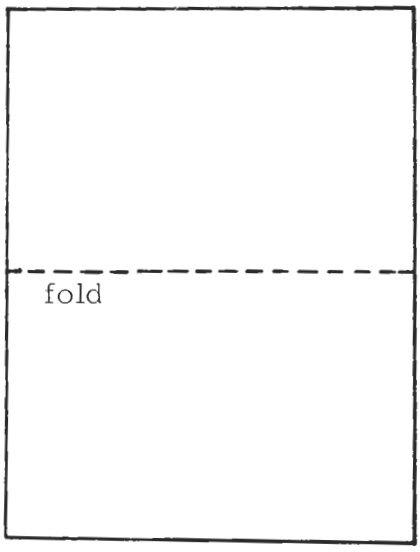
Materials:

- 6 ft. length of rope or heavy cord
- 20 paper boats made of 8 1/2 x 11-inch paper
- dried lima beans - enough to provide each child with a Dixie cup half full of beans
- 4-oz. Dixie cups - one for each child
- 500 #1 paper-clips
- 2 cardboard or stiff-paper cut-outs, one of Long Lake and one of Round Lake, in contrasting colors (to be made by the teacher)

Introduce this lesson with a demonstration that you will perform. You will need about twenty paper boats that you have constructed beforehand. These will be used to measure the surface area of an irregularly shaped "lake" that you have outlined on the top of your desk with a 6-foot length of rope or heavy cord.

The boats can be made with writing or construction paper as outlined in the figures shown on the next page. Starting with a sheet of paper as in (1), fold it in half, bottom to top as shown in (2). Mark the half-way point across the folded edge with a short crease and then fold the left and right bottom corners up as indicated in (3) and (4). Now fold the two strips at the top down one to the front and the other to the rear, to produce the upside down "party hat" shown in (5). Grasp the center front and center rear of the brim and pull these outward to open the hat. At the same time, bring the upper left and right corners together to form the top corner of the square in (6). Tuck the edges of the upper strip under to make a flat corner. Fold the front side of the upper half of this square forward and the back half backward to produce (7). Now once again pull the centers of the top edge apart and bring the left and right corners together to form another square as in (8). Turn this square upside down and pull the right and left outer sides apart.

STEPS IN THE CONSTRUCTION OF A PAPER BOAT

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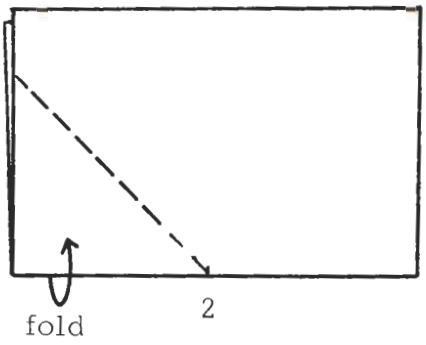


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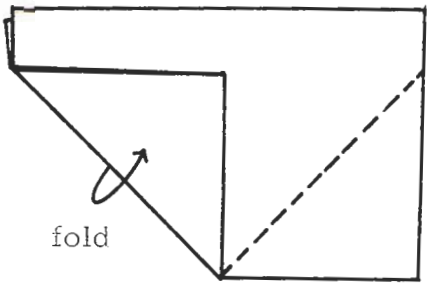


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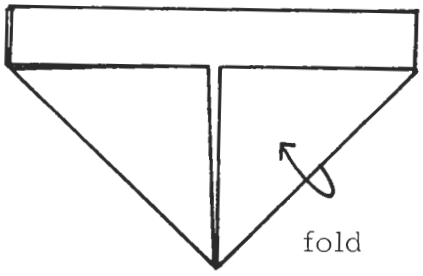


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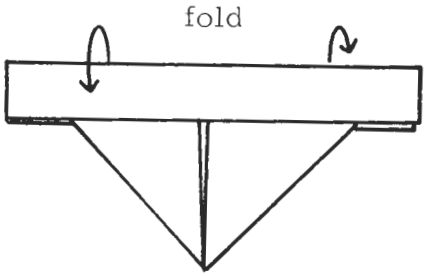


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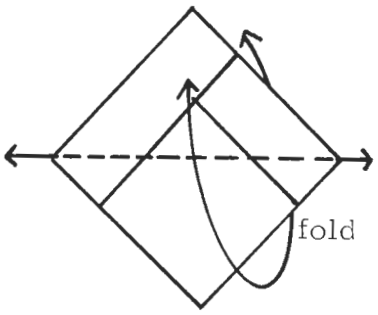


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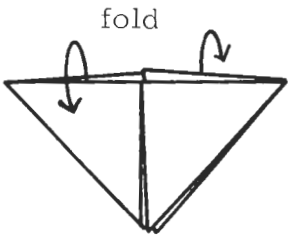


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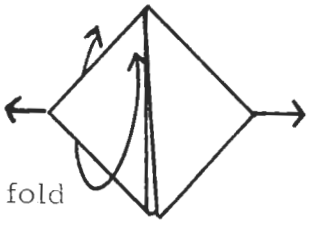


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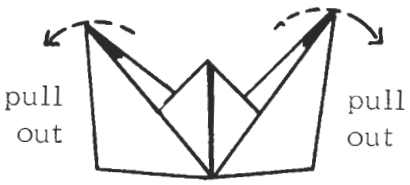


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Tell the children that you will determine how many boats will cover your "lake" completely. The boats need not be tightly jammed together on the lake, but should be placed only so that they are reasonably close together. After you have counted the number of boats, state that the area of the lake is about the the same as the area of the number of boat bottoms used. The surface of the lake has been compared to the undersurfaces of a number of boats, and they are about equal.

Now change the shape of your lake (without changing the length of the shore-line) and repeat the measuring process. Ask for several volunteers to place the boats in the lake. A new result will be obtained. The new lake is therefore larger or smaller (as the case may be) than the first lake.

(You may wish to instruct the children in the art of making paper boats. The triangular and square shapes obtained during construction provide an opportunity for a review of earlier study of the shape of objects.)

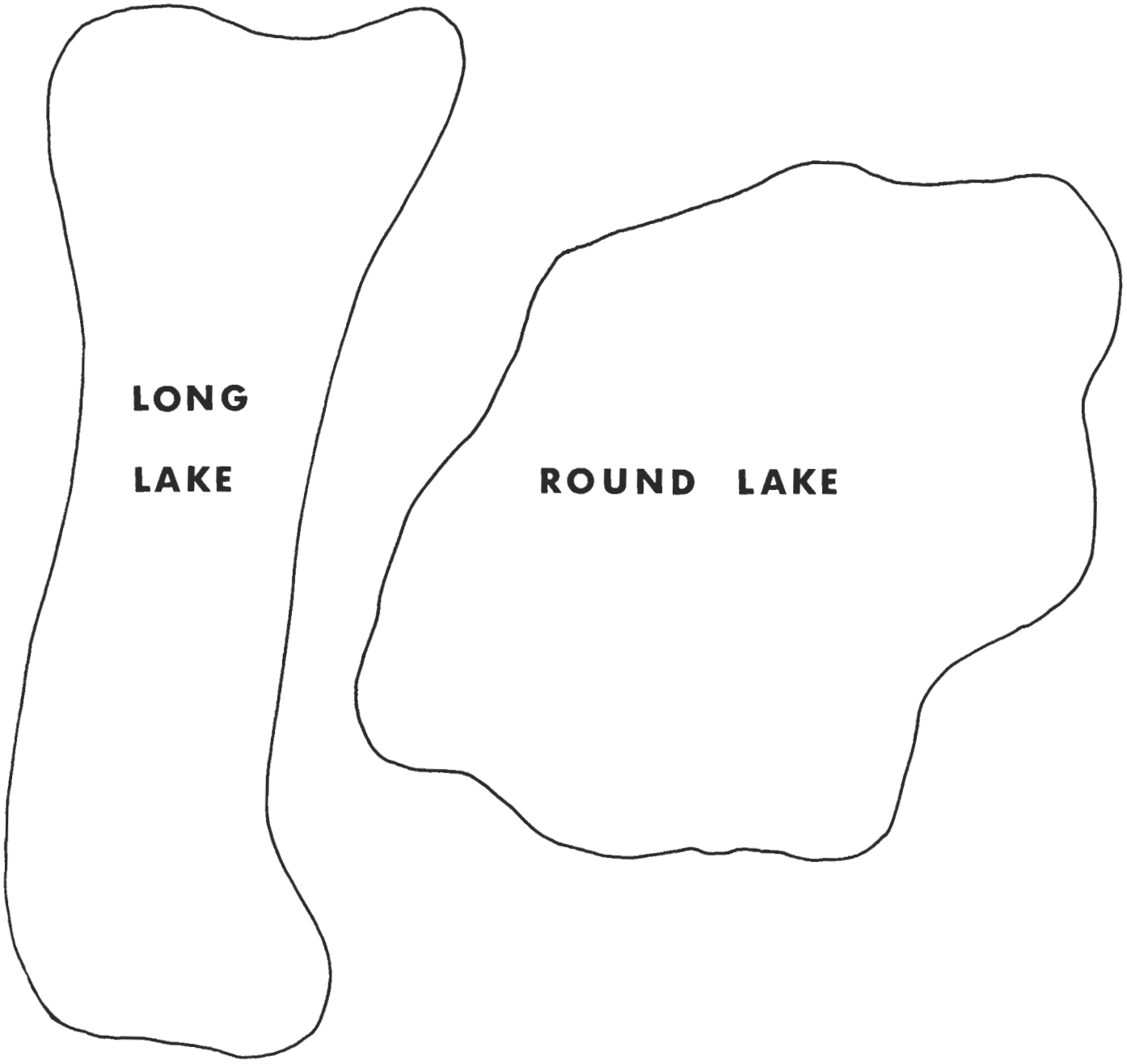
Long Lake and Round Lake - Area. Distribute to each child a copy of Record Sheet #3. This sheet contains maps of both Long Lake and Round Lake. The problem is to determine which lake has the larger area, or if they are about the same in area. Many children will want to guess, and will provide a variety of reasons for their guesses. Tell them that you wish to find a reliable way to compare the two lakes.

Ask also if the two lakes can be compared directly, just as they compared a pencil length to a pipe cleaner length by placing these two objects side-by-side. Show the class your two paper cut-outs of Long Lake and Round Lake. Can these be used to find the lake with the larger area? When placed in contact with one another the two do not fit in any way.

Discuss various suggestions for comparing the two lakes. At the conclusion of this discussion, have them fill in on the Record Sheet the lake they think has the larger area before they actually perform a measurement of each lake. (The commitment to a particular result should increase a child's interest in making an actual measurement.) Have each child keep this Record Sheet for later reference.

RECORD SHEET #3

NAME _____




Which lake has the larger area?
How can we find out?

The previous discussion, together with your demonstration using the paper boats, should have helped the children see that a number of small objects might be placed on the two lakes to measure their areas. Although a direct comparison may not be a good one, comparing each lake to a number of boats or other objects will enable the two lakes to be compared to one another.

Distribute Record Sheet #4 to each child. Also distribute a paper cup about half full of dried lima beans to each child. Ask the children to use the beans to cover the map of Long Lake on their sheets. When they are through, have them record on the sheet the number of beans used. Distribute Record Sheet #5 and have the children measure and record their result for Round Lake. Finally, by referring to Record Sheet #4, they are able to state which of the two lakes has the larger area.

Upon completion of Record Sheets #4 and #5, ask the children to determine if their original guesses of the larger lake were right or wrong. Do the children trust their guesses as much now as they might have before? While the children still have their Record Sheets in front of them, tabulate on the chalkboard for each lake the numbers of beans used by the children. A fairly wide range of numbers will be obtained. Ask the children why their results are not all the same. Encourage their responses, and say nothing that would indicate that any of the measurements are incorrect. Instead, help them find valid reasons for the discrepancies such as: variations in the size of beans and differences in how closely the beans were placed on the lakes.

Lengths of Long and Round Lakes. Ask the children to look at the two lakes again as shown on Record Sheet #3. Could they use a paper-clip chain in any way to measure these lakes? Elicit as many suggestions from the children as you can. From this discussion there should arise suggestions for measuring a number of length properties of the two lakes. Three of particular interest are: the greatest distance across the lake, the least distance across the lake, and the distance around the shoreline of the lake.

Distribute copies of Record Sheets #6 and #7. Also distribute about twenty paper-clips to each child. Have each child make

his own chain and proceed to measure each of the three lengths mentioned above and record results for each of the lakes. Some may complain that the chain will not follow the shorelines very well. Simply ask them to do the best they can. Also remind the children always to report lengths as intervals. When the children have completed these two Record Sheets ask the following questions: Which lake has the greatest distance across? Which lake has the least distance across? Which lake has the longer shoreline? Which lake could be covered by a greater number of boats?

Optional Activity. Some children may enjoy measuring the area of the two lakes using objects other than lima beans. Acorn tops, walnut shell halves, soda pop caps, and certain kinds of macaroni could be used. Supply such children with spare copies of Record Sheet #3 or Record Sheets #4 and #5. Ask them to supply their own measuring objects, perform the experiment at home, and report their results to the class.

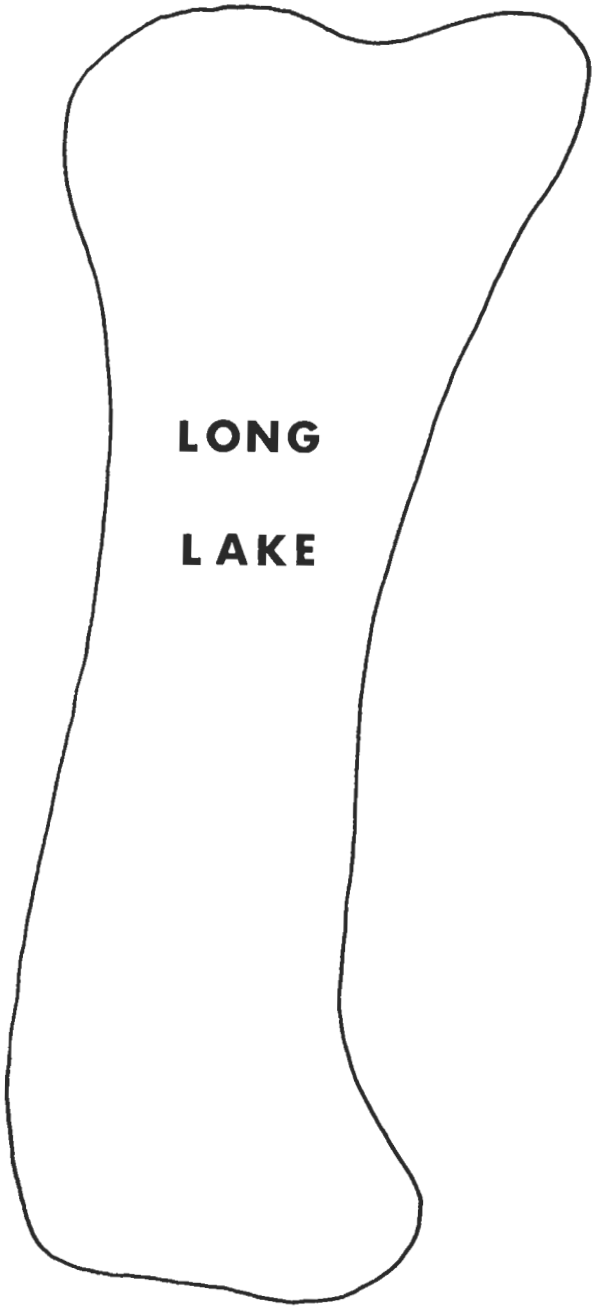
Still another interesting unit of area is the thumb print. Make available some finger paint (spread thinly on waxed paper) to a few children and have them see how many thumb prints will fill each lake.

Take-Home Booklet and Activity. The story, "The Play Area," comprises the entire take-home booklet for the children, as no letter to the parent is included this time. The story should be read to the class and discussed by the children before the booklets are sent home. The story illustrates the difference between the concept of length and the concept of area. Children will be able to trace the development of the idea of area as they see the attempts of the various children in the story to mark out a satisfactory playground. In each illustration they will see two different standards of measurement and be able to compare them.

In the story, Jane showed knowledge of an important discovery, namely that, given a certain length (in her case fifty steps), a circle is the largest area that can be enclosed by it. Children might enjoy being given strings of identical length and trying to discover for themselves, without the aid of the teacher, which shape would result in the largest enclosed area. They could test their string-enclosed areas for size by using the covering areas of paper boats, tiles, or any other suitable standard agreed upon.

RECORD SHEET #4

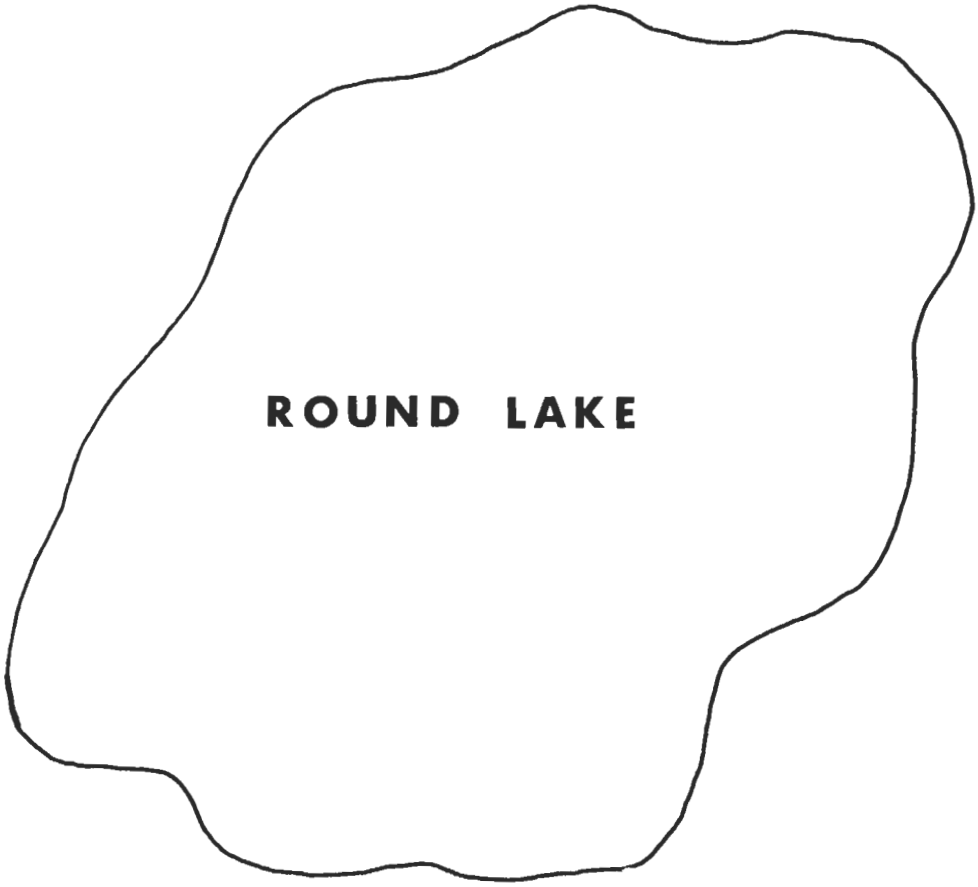
NAME _____




I used _____ beans to cover Long Lake.

RECORD SHEET #5

NAME _____

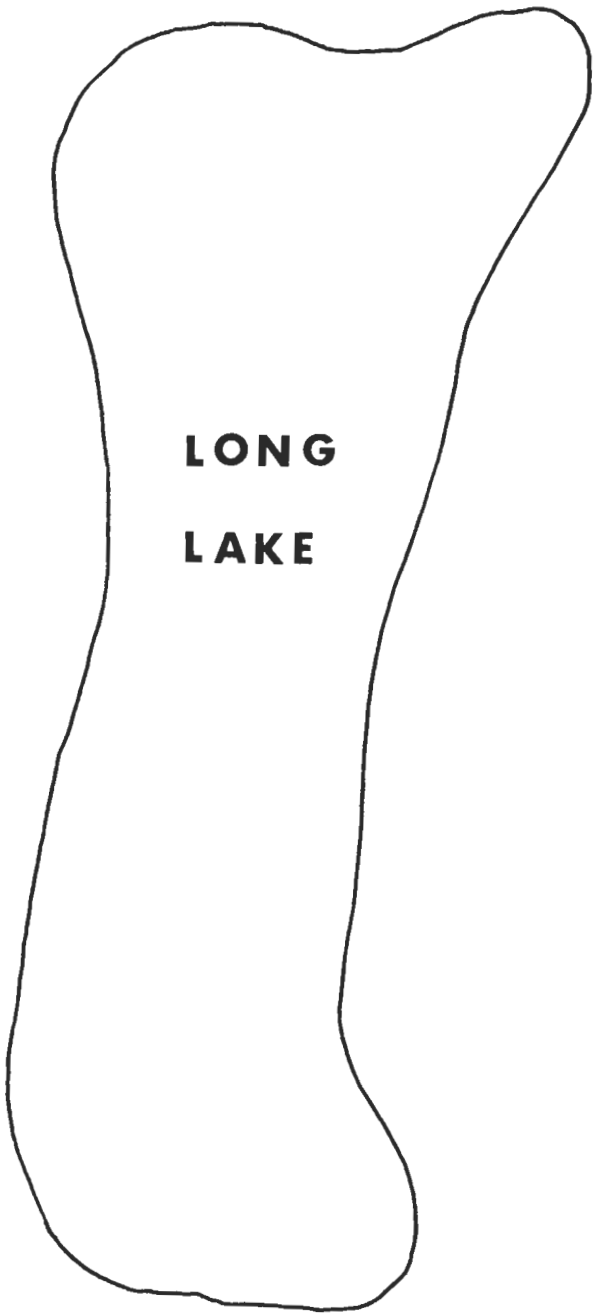



I used _____ beans
to cover Round Lake.

In my experiment I found
that _____ Lake had the larger area.

RECORD SHEET #6

NAME _____




The greatest distance across the lake is _____<distance<_____.

The least distance across the lake is _____<distance<_____.

The distance around the lake is _____<distance<_____.

RECORD SHEET #7

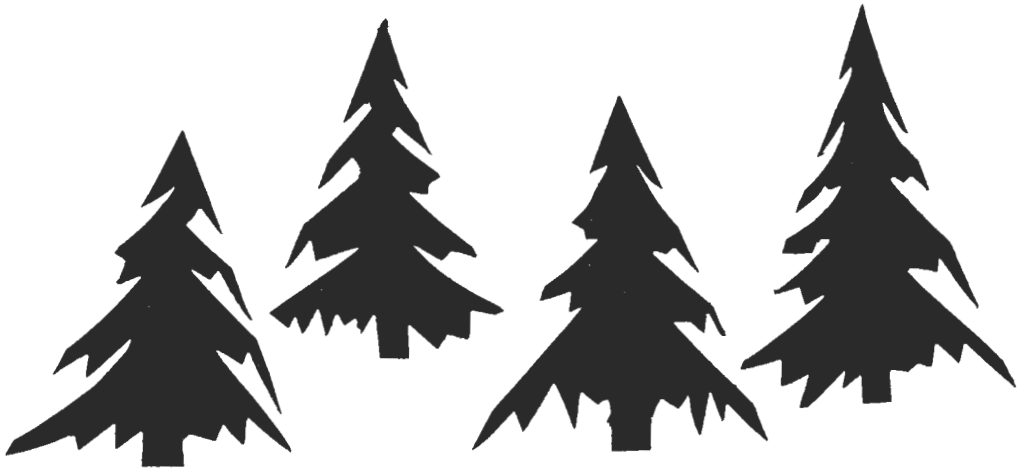
NAME _____


The greatest distance across the lake is
_____<distance<_____.

The least distance across the lake is
_____ <distance< _____.

The distance around the lake is
_____<distance<_____.

# THE PLAY AREA



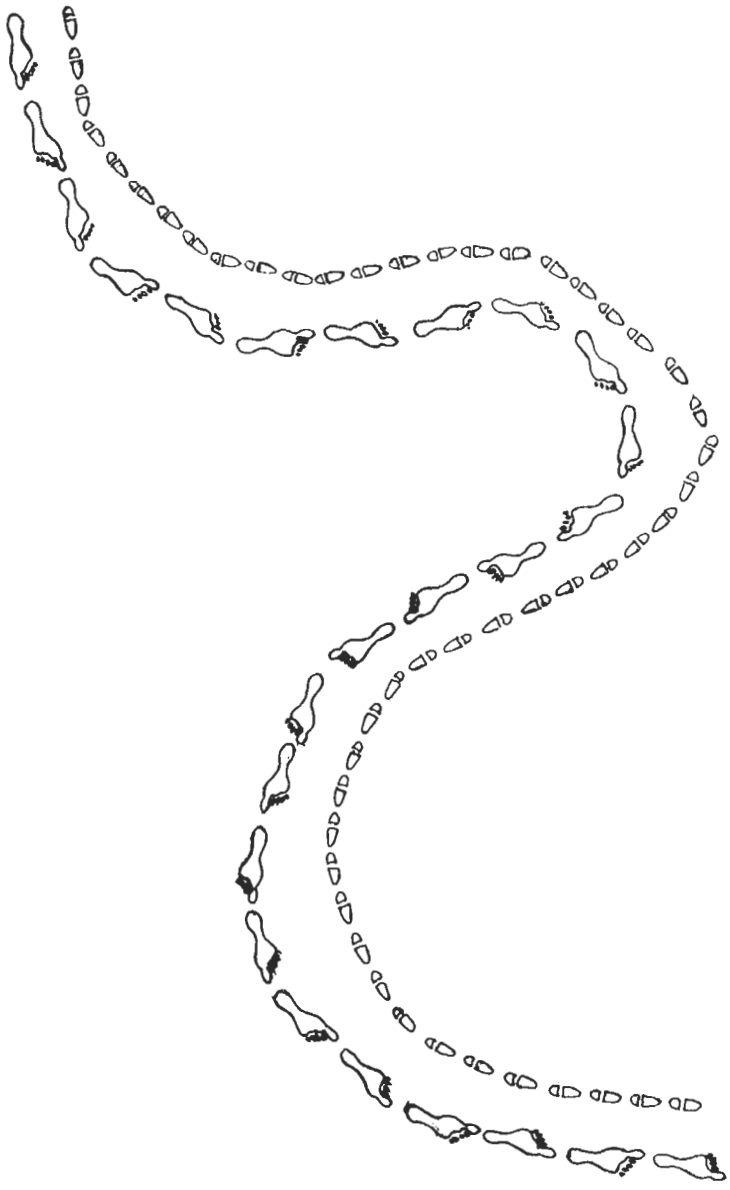
# THE PLAY AREA

It was summer. Jane, Jimmy, Mary, and David were playing at the beach near their summer cabin. In the spring, before the family came to the beach, father had planted some small pine trees between the house and the lake. They were still very tiny and needed to be let alone so they would be able to grow sturdy and strong.

As the children played, father came along and said, "Don't play near the little pine trees. I think you ought to mark a play area on this beach that doesn't come near my pine trees. Make it large enough for all of you to play in."

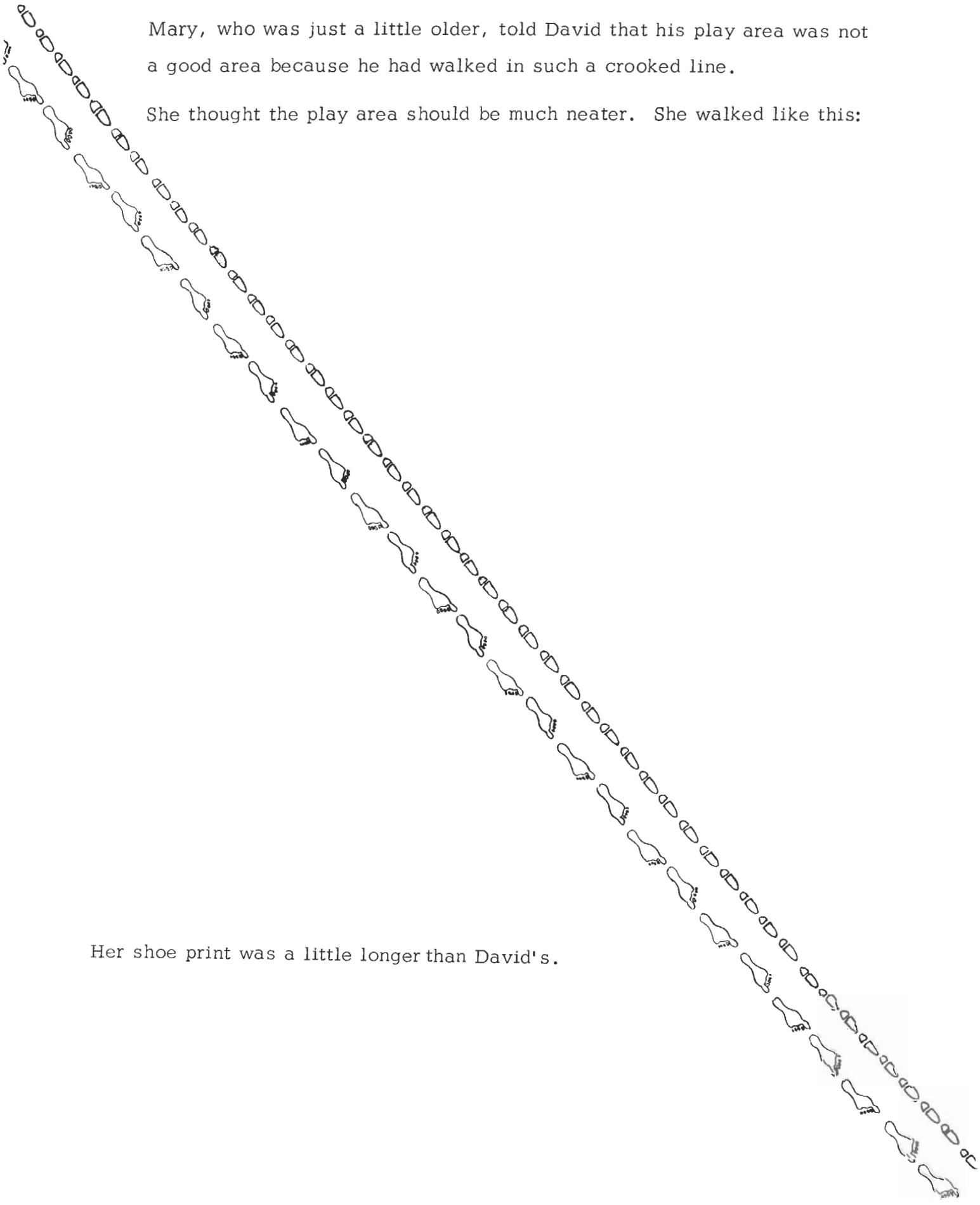
"Fine," Jane said. Since she was the oldest of the four, she took charge of the project. "Let's see what each of us can do with fifty footprints. You all have shoes on and I'm barefoot, but I'll walk along with each of you and help you count. David, you try first," Jane said.

David started to mark out a play area. He was the baby of the family and he had to take two steps for every step that Jane took. He thought a good play area should look like this:



Mary, who was just a little older, told David that his play area was not a good area because he had walked in such a crooked line.

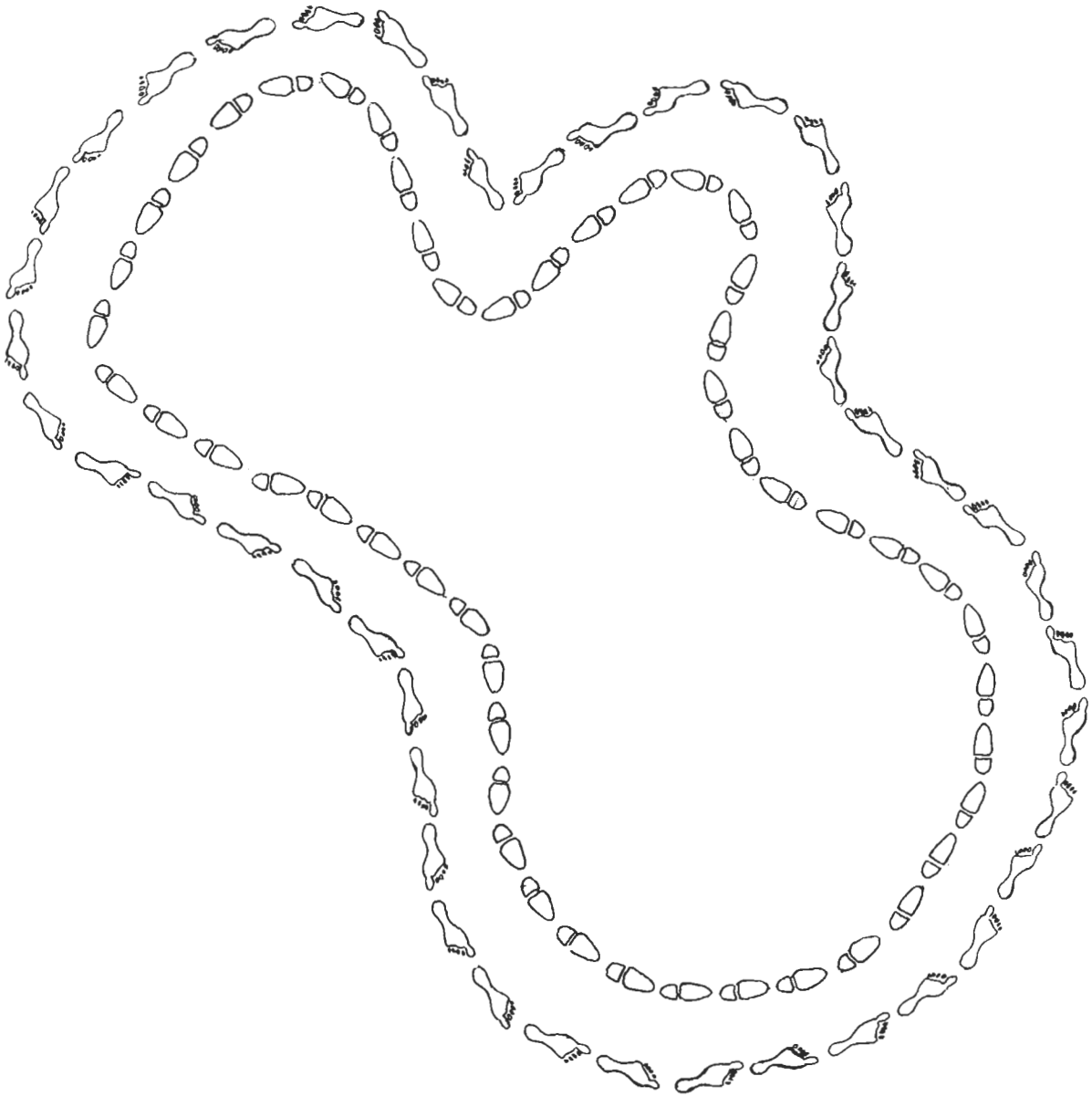
She thought the play area should be much neater. She walked like this:



Her shoe print was a little longer than David's.

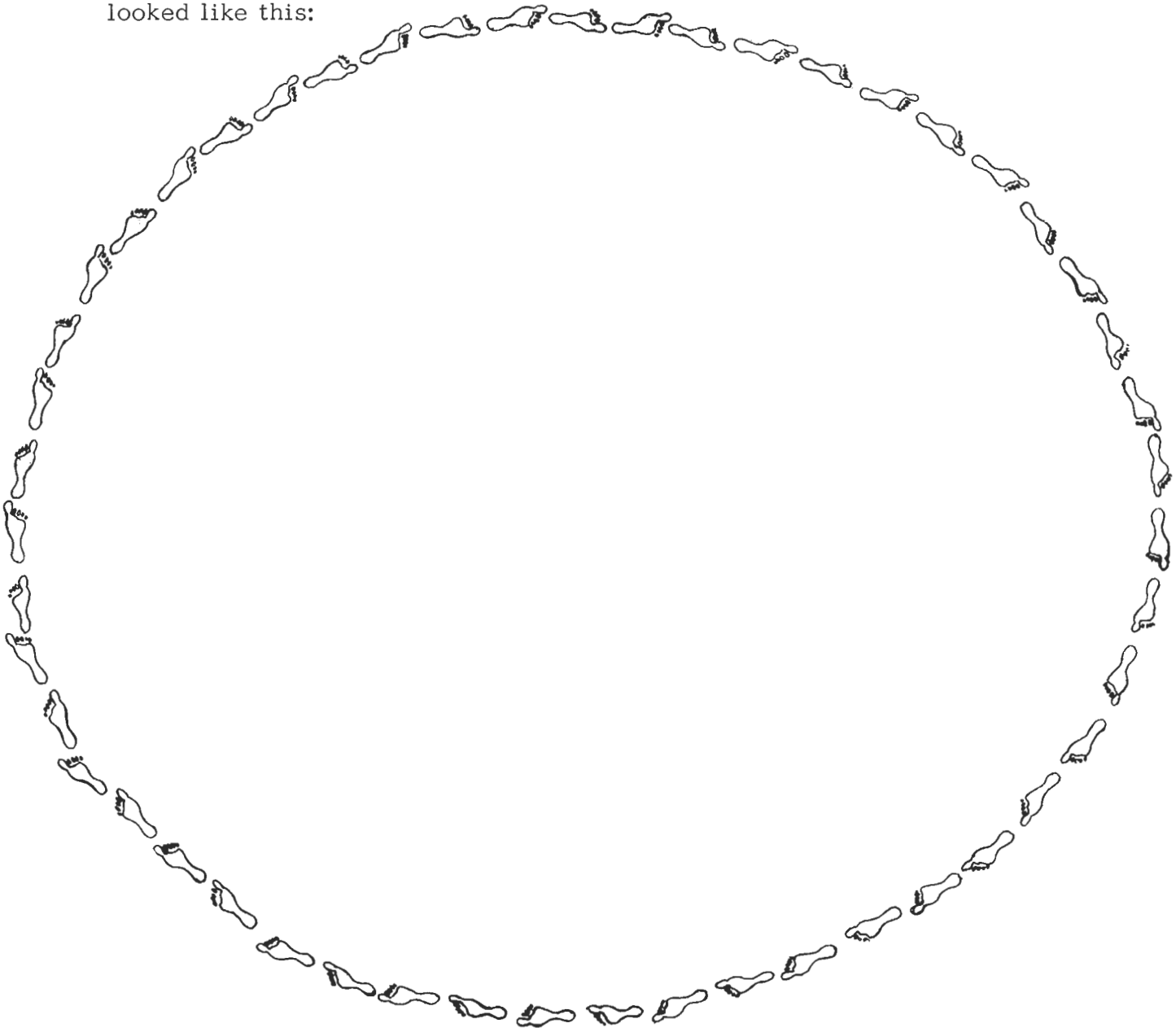
Then Jimmy, who was already in kindergarten and knew a whole lot about such things, said, "You can't mark a play area by just walking in a straight line. You have to close the line. An area is inside a closed line." Jimmy's feet were nearly as long as Jane's.

He made shoeprints on the beach that looked like this:



Jane looked at all the marks and shook her head. "No, no, no!" she shouted. "Even Jimmy's play area is barely large enough for him and David to play in. Father wants us to mark off an area that is large enough for all of us to play in."

Jane carefully rubbed out, with her foot, the marks that she and David made (show illustration of David's shoe prints), and the marks that she and Mary made (show illustration of Mary's shoe prints), and the marks that she and Jimmy made (show illustration of Jimmy's shoe prints). Then she carefully stamped in the sand with her bare feet. Her area looked like this:



David, Mary, Jimmy, and Jane all ran inside the play area that Jane had marked with her foot prints, to try it out. "It's just right, Jane," Jimmy exclaimed, "there's plenty of room for all of us to play!"

Father came along and smiled at his children. Then he saw the play area. "Why, this is great!" he said. "Who marked this out?"

"We all tried," Mary said, "but Jane made this one. It's the only one that's just right."

LESSON 4 MEASURING VOLUME USING A STANDARD OF VOLUME

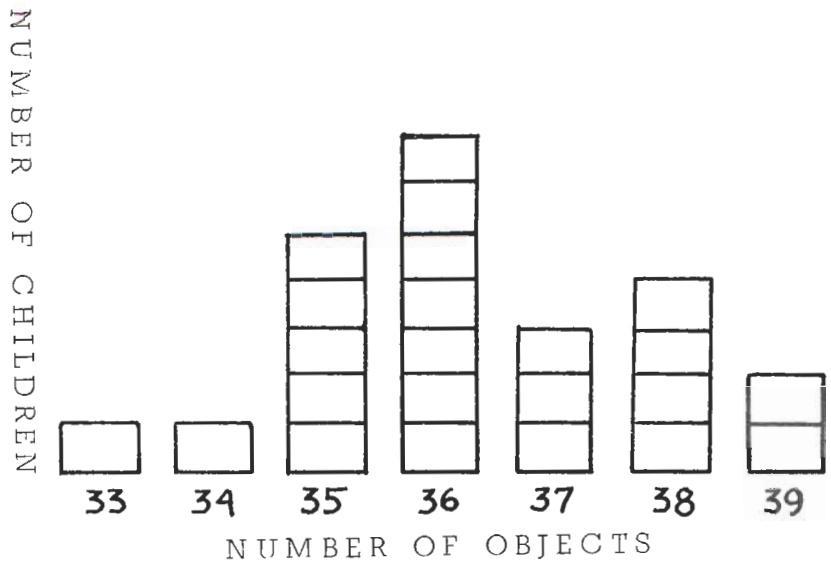
## TEACHER COMMENTARY

This is the final lesson in the unit. The concept of volume as a property which can be filled with other objects is introduced. The children learn that two objects cannot be directly compared in this property, but can be compared indirectly by using a number of smaller standard objects to fill each container.

The volume standards used here are cranberries, peanuts in the shell, pop corn, or other similarly-sized and fairly uniform objects. It is perhaps important in this introductory unit not to use one kind of object as a standard for both area and volume, or another object as a standard for both length and area. Following this practice, the children associate length measurement with paper-clips, area measurement with dried lima beans, and volume measurement with peanuts. Paper-clips could easily have been used as standards for area as well as for length, and lima beans as standards for volume as well as for area, but such use might cause confusion in this introductory unit.

The objectives of this lesson are similar to those of the previous lesson. Measurement of volume is presented as a fairly well-defined operation which employs the use of standards of volume (peanuts) and the operation of counting. Volume is measured indirectly (as was the case for area measurements). Thus, if the volume of A is about the same as the volume of ten peanuts and the volume of B is about the same as the volume of fourteen peanuts, we may conclude that the volume of A is less than the volume of B, because ten is less than fourteen. (Care should be taken to make sure children are not over-filling their containers.)

The teacher should collect the measurements of the children and display these on the chalkboard. In each case a histogram should be drawn similar to the one shown on the following page. The number of rectangles in each column shows the number of children that obtained the result printed below the column (either volume or area measurement). This kind of display helps show the children that there is no single correct result for the measurement. Instead, there is only a more probable result. Reasons for the range of results may be discussed briefly with the children.




Volume is measured first in this lesson. Later, areas and lengths of the same two objects are measured. In this way the children should begin to understand that any physical object has a variety of properties—volume, area, and length—and that there may be some connections among these properties. An object that is greater in one property may or may not be greater with respect to another property.

## LESSON 4 MEASURING VOLUME USING A STANDARD OF VOLUME

### PROCEDURE

Materials:

- 1 frozen-juice can (6 fl. oz.) for each child, to be provided by the child
- 1 Jell-o box (3 oz.) for each child, to be provided by the child
- peanuts in the shell, cranberries, or unbuttered and unsalted popped corn, in sufficient quantity to fill each child's juice can
- dried lima beans (from previous lesson)
- paper-clips, ten per child (from previous lesson)
- scissors, one pair per child
- sand (at least enough to fill juice can)

In advance of the lesson have each child bring from home one frozen-juice can (6 fl. oz.) and one Jell-o box (3 oz. size). Have each child place the box and can on his desk. Introduce the lesson by asking the children which container will hold more. You may wish to introduce the word volume by saying that the volumes of the two containers are to be compared. Ask, "Does the juice can have a greater volume than the Jell-o box, a lesser volume, or an equal volume?"

Ask if there seems to be any way to compare the volumes of the two containers directly—a way similar to the side-by-side comparison of the crayon to the pipe cleaner. Some children may be confident that they can tell which container has the larger volume simply by looking at them when they are side-by-side. Ask other children if they can think of some way to measure which of the two has the larger volume. A child may suggest filling one of the containers with water or sand. Then by pouring from this container into the other, one can tell which container is the larger. This is an excellent suggestion and should be carried out by the teacher, using sand. The use of sand will show that the juice can has the larger volume.

Remind the children of the convenience of measuring lengths and areas in terms of standard quantities, emphasizing that this procedure permitted assigning a number to these quantities. Ask the children if they can invent a similar procedure for dealing with volumes. Perhaps someone will suggest counting the grains of sand required to fill each. With discussion, the children will understand that because of the large numbers involved, the use of grains of sand would be inconvenient.

Measuring Two Volumes. Introduce the possibility of using peanuts in the shell, cranberries, or popcorn to compare the two containers. Elicit from the children the idea that these could be used to fill each container and the contents counted.

Distribute peanuts in the shell, cranberries, or popcorn to each child. Distribute also Record Sheet #8. Have the children fill each container in turn with the objects and record the numbers on the Record Sheet. Some children may need assistance in the counting operation. Still others will need help in keeping separate those objects yet to be counted, those already counted, and those not to be counted at all. When both containers have been measured, ask the children to fill in the final two blanks—"The can has the greater volume."

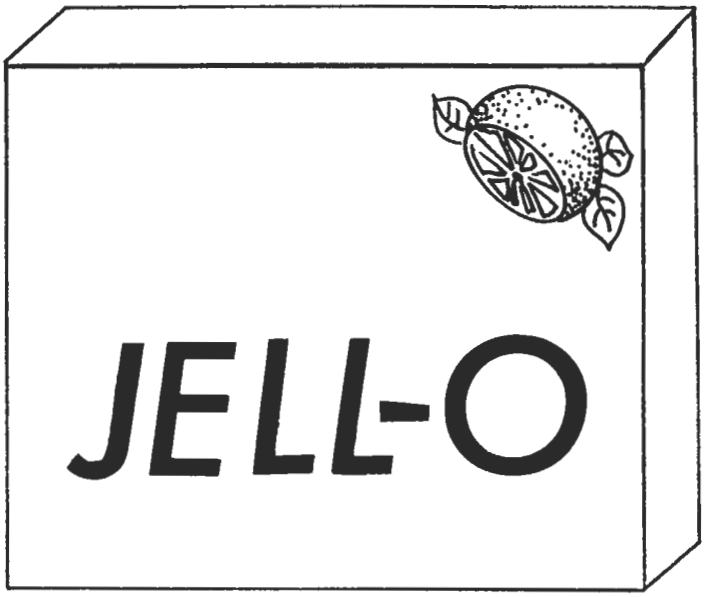
Collect the results from each child for display on the chalkboard. Display as a histogram for each of the containers. Emphasize that there is no single correct result—only a most likely result.

Comparing Areas. Ask the children to clear their desks except for the Jell-o box and the juice can. Each container has the property of volume. But now ask if the containers also have the property of area. The children may point out that the can has two end areas and a curving side surface. The box has six surfaces in all, each with a certain area.

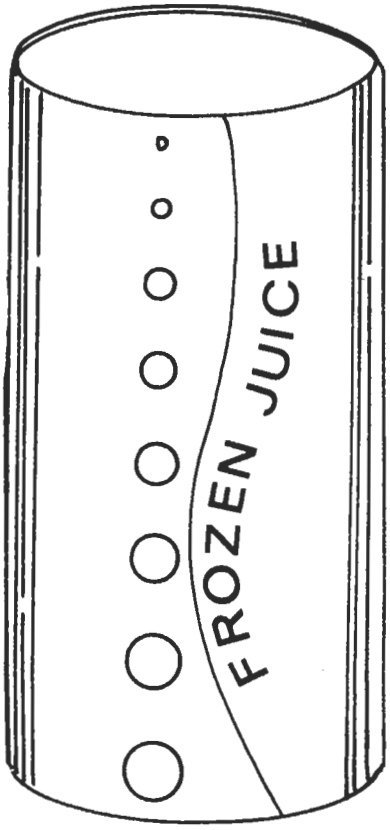
Ask which of the two objects has the greater total area. By total, explain that you mean to include all the outside surface of each container. Permit each child to guess which surface area is the greater, but also insist that by making a measurement one can be more sure of the comparison. Ask the children for suggestions for making this comparison using the dried lima beans.

RECORD SHEET #8

NAME_____




This box contains _____ objects.




This can contains _____ objects.

The _____ has the greater _____ .

Tell the children that you have copies of the entire surface of each object. Distribute to each child the two drawings that appear at the end of Lesson 4. First have each child cut out the outlines of the Jell-o box. Have him fold the resulting figure inward along each of the interior lines. Now have the children fit the paper around their boxes so that they can convince themselves that the paper is a good copy of the entire surface of the box. Now distribute Record Sheet #9. Distribute sufficient lima beans to each child so that he may completely cover the paper copy of the surface of the box with beans. Have each child record the number of beans used.

Now have each child cut out the three surfaces which are copies of the surface of the can. Again have each child convince himself that the paper pieces are good copies of the can's surface. Have each child measure the area of the three paper surfaces and record the number of beans used on Record Sheet #9. Finally, have the children fill in the two remaining blanks on the Record Sheet.

Display the results of each student in a histogram that you draw on the chalkboard for each container. Discuss the new results (the two containers have nearly equal surface areas, while the volume of the juice can is decidedly larger). How does the result compare with the children's guesses? Are guesses to be trusted?

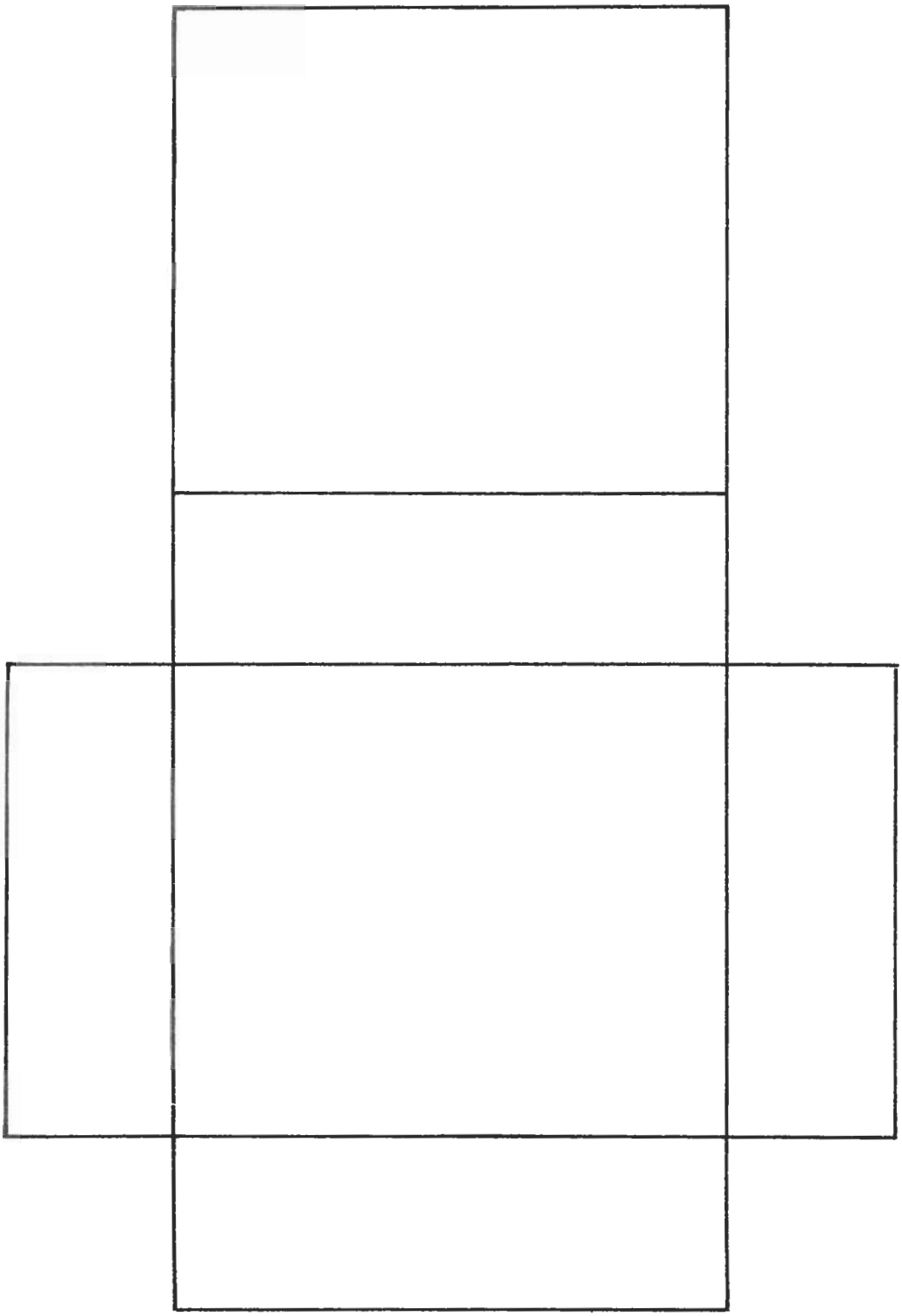
Comparing Lengths. The desks should again be cleared except for the box and the can. Distribute ten paper-clips to each child. Ask the children what lengths for the box might be measured (height, width, thickness, distance around, etc.). It is perhaps best to have each child place his box on the desk so that the front side can be read. Ask the children to notice that there are really many different "distances around." Ask the children what lengths might be measured for the juice can (height, "distances around," greatest thickness).

Distribute Record Sheet #10 to each child. Of all the possible length measurements for each container, tell the children that they will measure only two for each of the objects (as indicated on the Record Sheet). Each child is to make these measurements and record the result as an interval in each of the four cases.

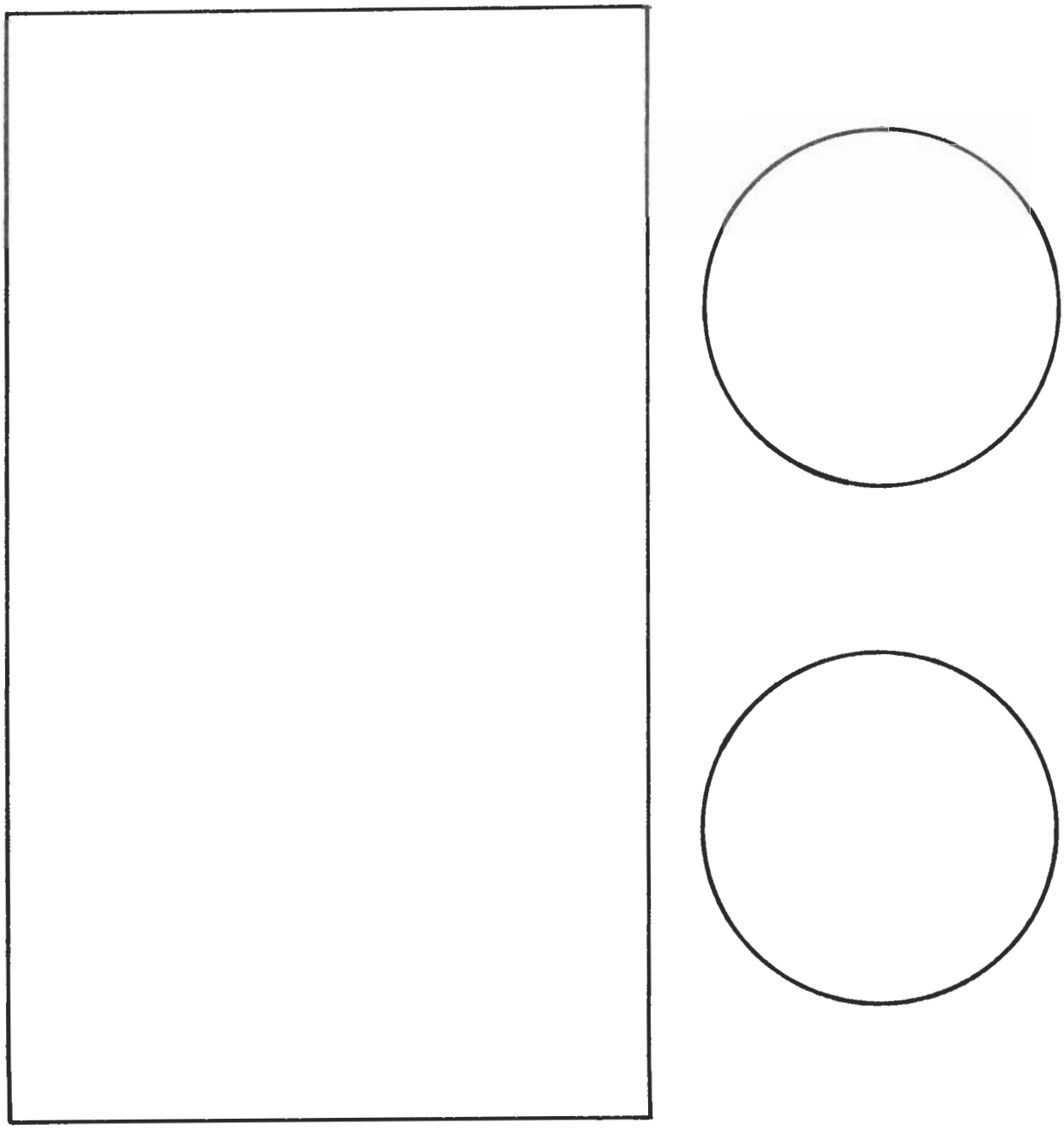
Discuss the measurements with the children. Ask if the question, "Which is the greater?" makes any sense. Have the children point out that the juice can has the greater volume, that the two containers have a nearly equal surface area, that the juice can has a greater height, and that the Jell-o box has the greater "distance around."

Take-Home Booklet and Activity. Read the story, "Which Hole Is The Larger?" to the children and have them discuss some of the questions listed at the end of the story. This discussion should provide a good opportunity for the children to see the difference between length and volume (and possibly some relationship) and to use the symbols learned in Lesson 1. Afterwards, distribute copies of the story-booklet for the children to take home. This is the concluding activity of Unit 1.3.

GELATIN BOX (3 oz.) SURFACE

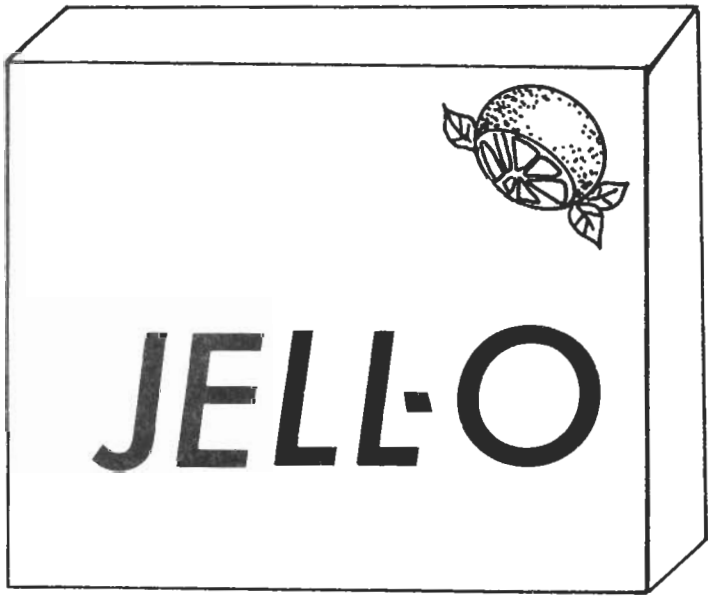


FROZEN JUICE CAN (6 fl. oz.) SURFACES

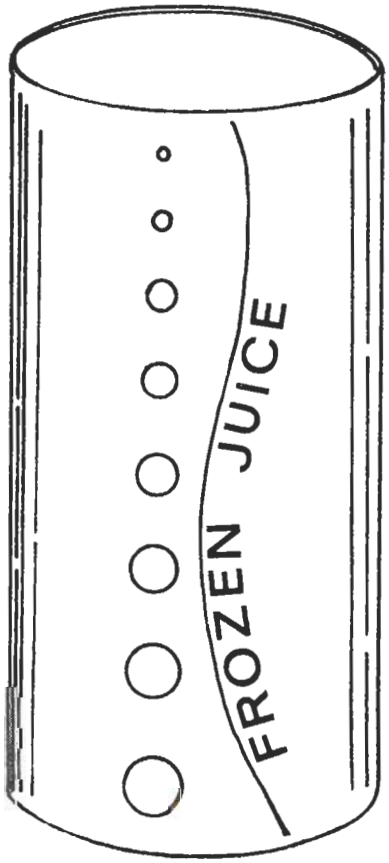


RECORD SHEET # 9

NAME _____




I covered the entire surface with_____lima beans.

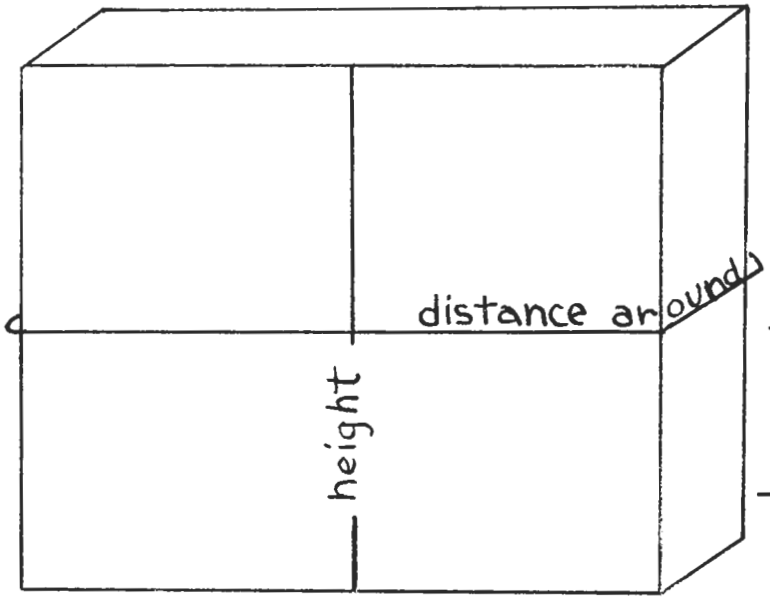



I covered the entire surface with____lima beans.

The_____has the greater_____.

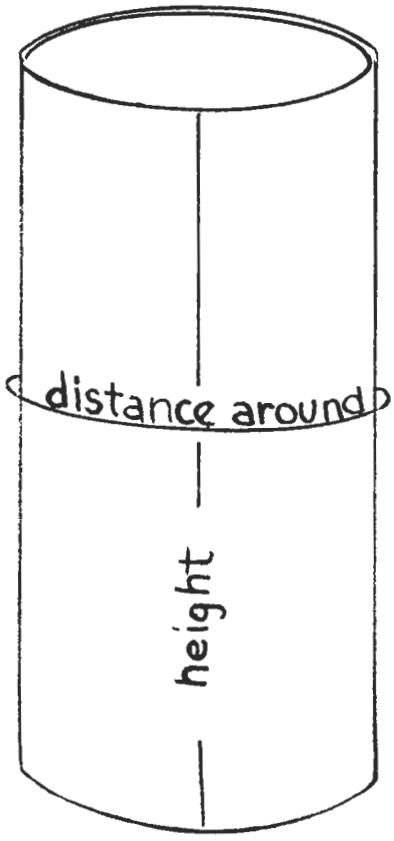
RECORD SHEET # 10

NAME _____




height

distance around




height

distance around

# WHICH HOLE IS THE LARGER?

# WHICH HOLE IS THE LARGER?

Alan and Paul were digging holes in the sand with their little shovels. They dug and dug and—with all of the sand they took out of their holes—they each made a little hill.

"You'll have to fill up these holes before we go home," Father said, "or someone might come along the beach and fall into them."

"Nobody could fall into Paul's hole," Alan said. "It's not big enough."

"I could!" Paul said, and he jumped into the hole to prove that Alan was wrong. The top of the hole came up to Paul's middle. "Look!" he said. "I know how big this hole is. It comes up to here," and he patted his middle.

Alan thought this was very interesting. Helping Paul out of the hole, he said, "Let me see how much I fill up the hole, Paul." Then Alan jumped in, but the top of the hole came only to his knees. Paul thought this was very funny and started to laugh.

"I was not thinking of filling up the holes with boys," Father said, laughing too. "Come on, help me push the sand back in."

"Let's measure my hole. Then we'll fill them up," Alan said. And Paul hurried to jump in the hole that Alan had dug. The top of his chin came right to the edge of the hole and Father had to lift him out.

Then Alan jumped in. The edge of the hole came up only to his chest. "I know how big this hole is," he shouted. "It's as big as me up to my chest!"

Now Father was interested. "Let me try this game, too!" he said, and he stepped into Alan's hole and then into Paul's hole. He found that Alan's hole came up nearly to his waist, but Paul's hole came up only to his knees.

"What do you make of all this?" Father asked. "Which hole do you boys think is the larger?"

"What do you mean by larger?" Alan said. "Do you mean, 'Whose hole is deeper?' or 'Which hole will hold the most sand?'"

"Yes," Paul said. "You can't just say a hole is larger because it's deeper. Or can you?"

"Well, I know which one is deeper," Father said. "Now I want to see which will hold the most sand or, in other words, which has the greater volume."

"I have an idea," Alan said. "I'll take the sand from Paul's hole and put it in the hole that I dug—"

"And I," Paul shouted, "will take the sand that came out of Alan's hole and put it in mine."

"A good idea!" Father said. He stood back and watched as the boys shoveled the sand into the holes.

Alan didn't have enough sand to fill up his hole, but Paul had so much that he had to give some of it back to Alan.

Exercises:

Can you tell from the story who dug the hole with the greater volume—Alan or Paul?

Can you tell from the story who was the tallest? Father or Alan or Paul?

The hole that Alan dug was, in volume, ( > < ) the hole that Paul dug.

The hole that Paul dug was, in length, ( > < ) the hole that Alan dug.

Father was ( > < ) Alan in length.

Alan was ( > < ) Paul in length.

Paul was ( > < ) Father in length.

Father was ( > < ) Alan and Alan was ( > < ) Paul.