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UNIT XIII

GEOMETRY

MINNESOTA SCHOOL MATHEMATICS AND SCIENCE CENTER 1966

MATHEMATICS

FOR THE

ELEMENTARY SCHOOL

UNIT XIII

Geometry

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MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT

JAMES H. WERNTZ, JR.
Associate Professor of Physics
University of Minnesota
Project Director

PAUL C. ROSENBLOOM
Professor of Mathematics
Teachers College Columbia University
Mathematics Director

- Unit XIII -

RICHARD C. GRIGGS — WRITER
Head of Mathematics Department
Princeton Country Day School

CAMILO SCHMIDT — WRITER, CONTENT EDITOR
Teaching Assistant
University of Minnesota

JOHN WOOD — EDITOR
Minnemast Staff
University of Minnesota

MARLENE GLAUS — STORY AUTHOR
Third Grade Teacher
Richfield, Minnesota

DAVID RATNER — ARTIST
Assistant Professor of Art
Boston University

JANNA DORY — ARTIST
Minnemast Staff
University of Minnesota

WAYNE LAZORIK — ARTIST
Minnemast Staff
University of Minnesota

JUDITH HORAZDOVSKY
Undergraduate Assistant
University of Minnesota

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We are deeply indebted to the many teachers who
used earlier versions of the material
and provided suggestions for
this revision

CONTENTS

INTRODUCTION

This unit continues the study of geometry begun in earlier units. The first part offers a review of the concepts of lines, line segments, rays, and curves and provides opportunities to develop skill in connecting points with straight lines. The second part develops a method of comparing the areas of rectangular regions by means of unit strips. (Caution is necessary in regard to the use of the word, area, as opposed to the word, region. Area is the measure of a region contained within a simple closed curve.) Later units will develop methods for converting other polygons to rectangles in order to change them to unit strips and thus allow comparison of their areas.

Materials needed for this unit are rulers, scissors, crayons and string.

REVIEW OF GEOMETRIC TERMS

Teacher Background

The following concepts should be interpreted for the children by the teacher.

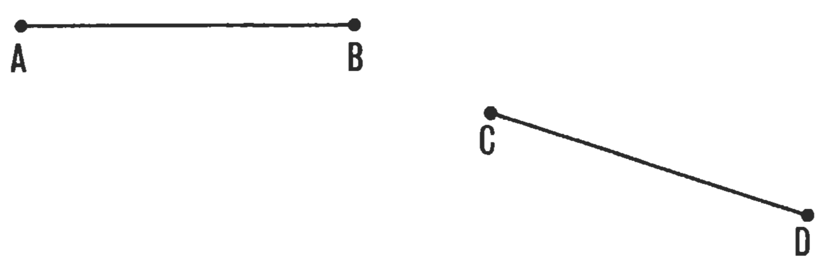
Line Segment

A line segment consists of two points together with the set of all points in between. (The teacher should remind students about the proper interpretation of the word, between.)

For example, in the figure below, points A and B are the endpoints of the segment which consists of those two points and all the points in between.

This line segment is denoted by $\overline{AB}$ or $\overline{BA}$.

If two line segments, $\overline{AB}$ and $\overline{CD}$, have the same length,



we usually denote this by saying $\overline{AB} \cong \overline{CD}$. This should be read, "The line segment $\overline{AB}$ has the same length as the line segment $\overline{CD}$."

Collinear

Three points are collinear if one is between the other two. (If three points are collinear, they are on the same line.)

Straight Line

A <u>straight line</u> consists of two points together with the set of all points which are collinear with them. We cannot draw a whole line on a sheet of paper because it goes on and on in both directions. Usually we represent a line by drawing it as below with arrows on both ends to indicate that the line goes on and on.

Two large balls of string with their ends tied together can help children better understand that a line is continuous, having no beginning or end. Ask two children to hold the balls, letting the piece of string represent a line. Direct each of the two children to walk farther and farther apart, unwinding the string more and more with each step but keeping it tight enough that it doesn't sag. When the children walk as far apart in the room as they can, they will be able to see that if they could pull the string through the wall and continue walking, they could go on with it.

A line is usually denoted using small letters.

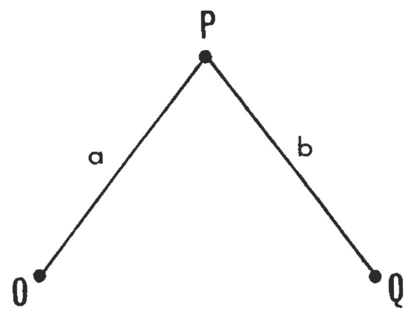
Ray

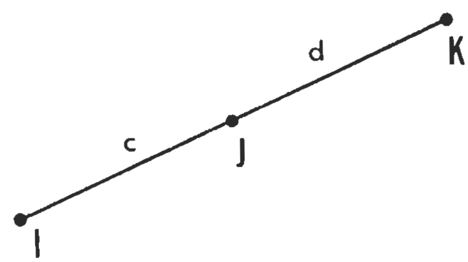
A <u>ray</u> is a subset of a line, consisting of a point M on the line, together with all points of the line on one side of M. See the figure below. (A ray, like a line, is a set of points.)

The arrowhead shows that the ray goes on in one direction. The above ray is denoted by $\overrightarrow{MO}$ where the first letter always represents the origin of the ray.

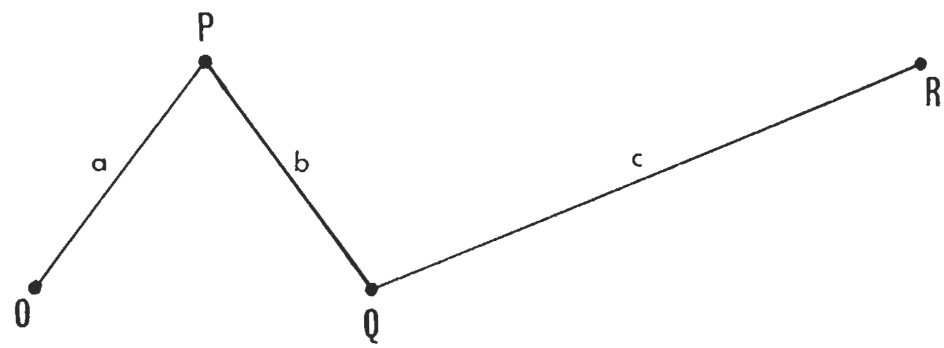
Broken Line

The union of two line segments having a common endpoint, but not forming a single line segment, is a <u>broken line</u>. Shown below are two segments, a and b, having a common endpoint P, which form a broken line. Segments c and d, with common endpoint J, do not form a broken line.





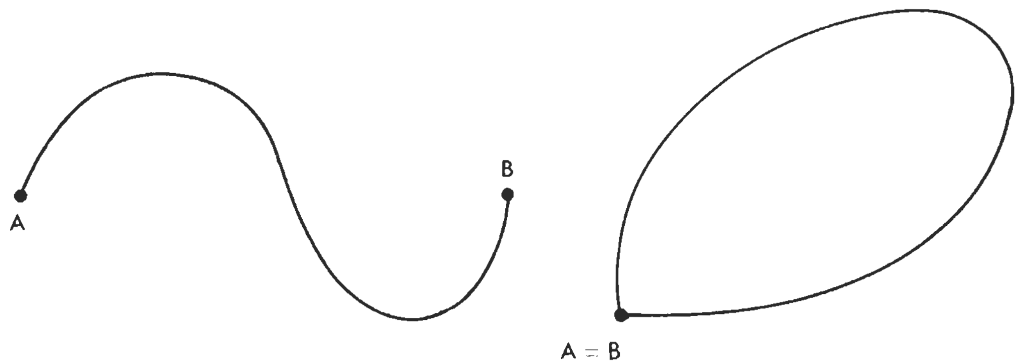
Now suppose we form the union of this broken line with a third line segment, c, which also has as one endpoint, the above point Q. The resulting figure,



would also be a broken line. We could then form the union of this broken line with a fourth segment also having as one endpoint, the above point R. This would produce another broken line with four segments. More generally, the union of a broken line, containing N segments, which has an endpoint E, and a line segment s, also having the same endpoint E, is a broken line containing N + 1 segments.

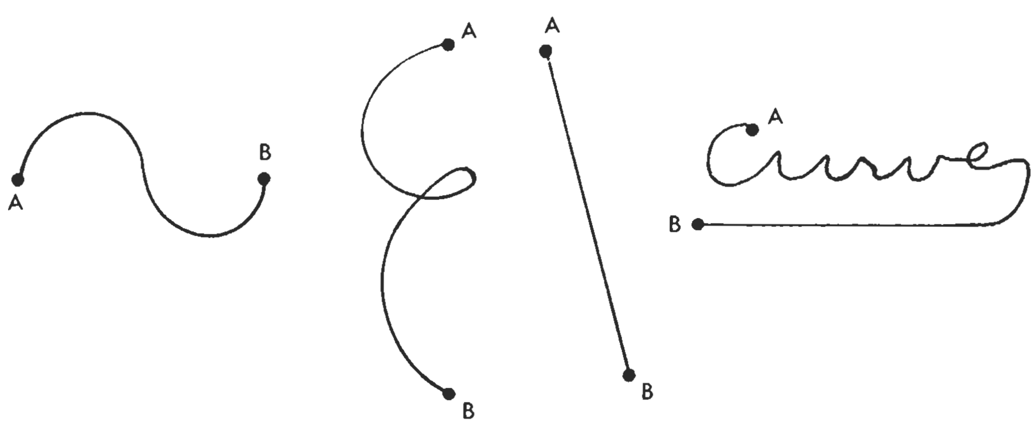
Curve

Pick two points, A and B, on a piece of paper. The representation of a curve connecting A and B is a trace of a pencil obtained by starting at the dot A (which represents point A) and without lifting the pencil from the paper reaching the dot B (which represents point B).



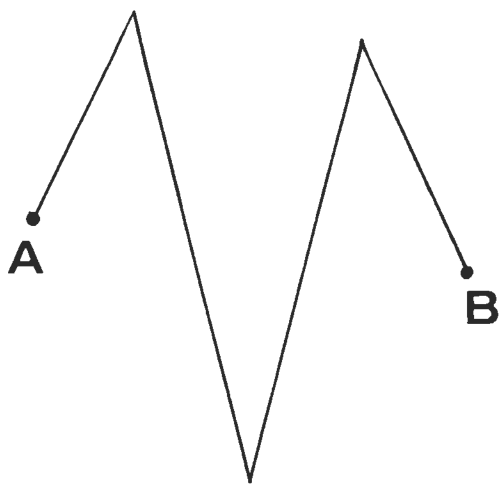
The same curve representation can be obtained by placing the pencil on B and then reaching A following the same path.

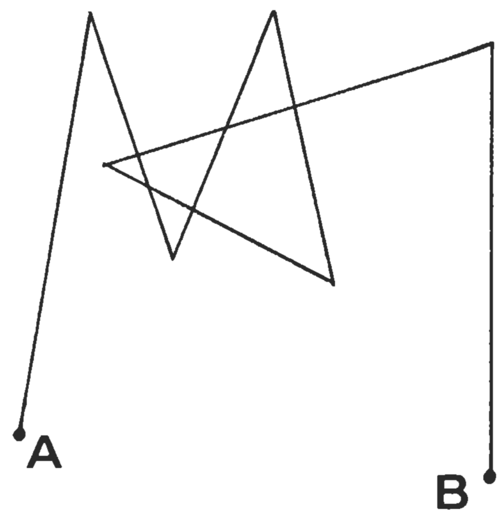
A curve is a set of points consisting of all the points represented by the trace of a pencil (very much like a dot is a representation of a point). In other words, a curve is just the set of all points represented by the path drawn by a pencil.



Line segments and broken lines are curves.

The points, A and B, are called the endpoints of the curve.





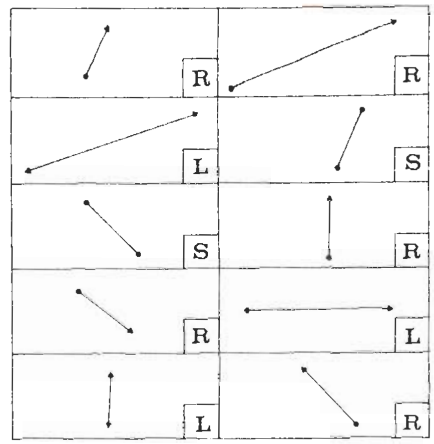
Answers to Worksheets 1, 2, and 3

Worksheet 1

Put the letter L in the square if the picture shows a line.

Put the letter S in the square if the picture shows a line segment.

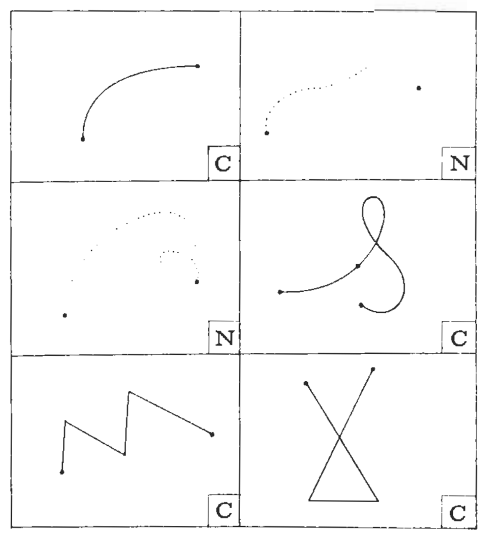
Put the letter R in the square if the picture shows a ray.



Worksheet 2

Put C in the square if the picture shows a curve.

Put N in the square if the picture does not show a curve.



Worksheet 3

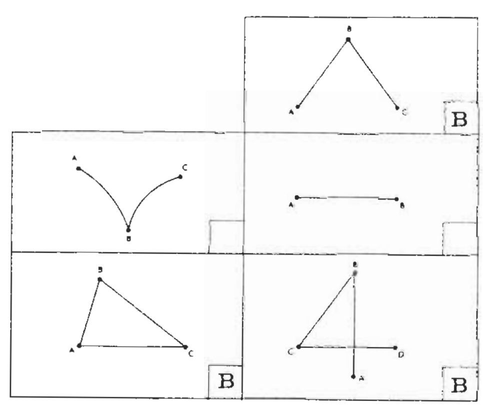
1. Are these three dots collinear?

Yes

2. Are these four dots collinear?

No

3. Put the letter B in the square if the curve is a broken line.

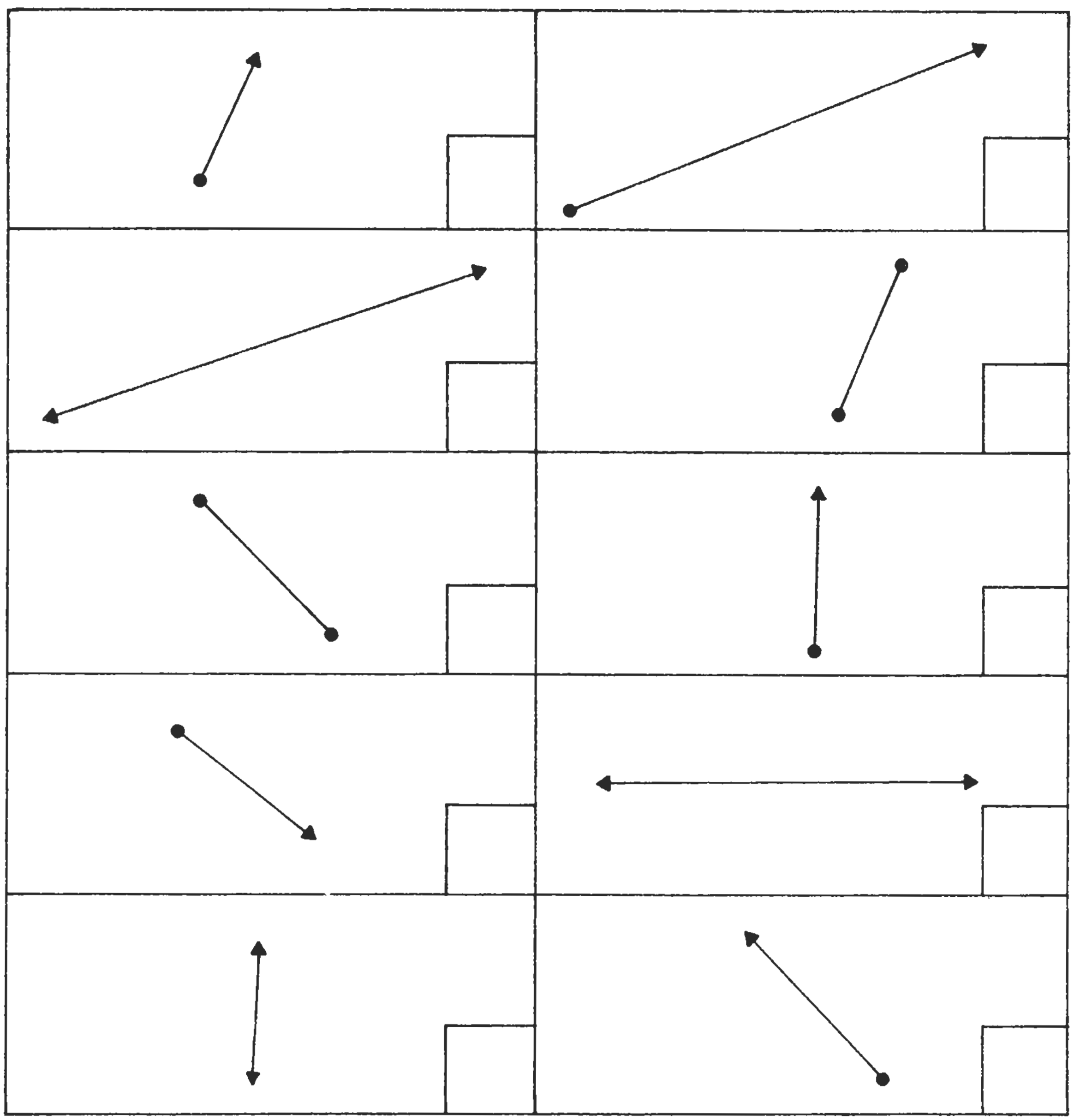


Worksheet 1 Name____________________

Put the letter L in the square if the picture shows a line.

Put the letter S in the square if the picture shows a line segment.

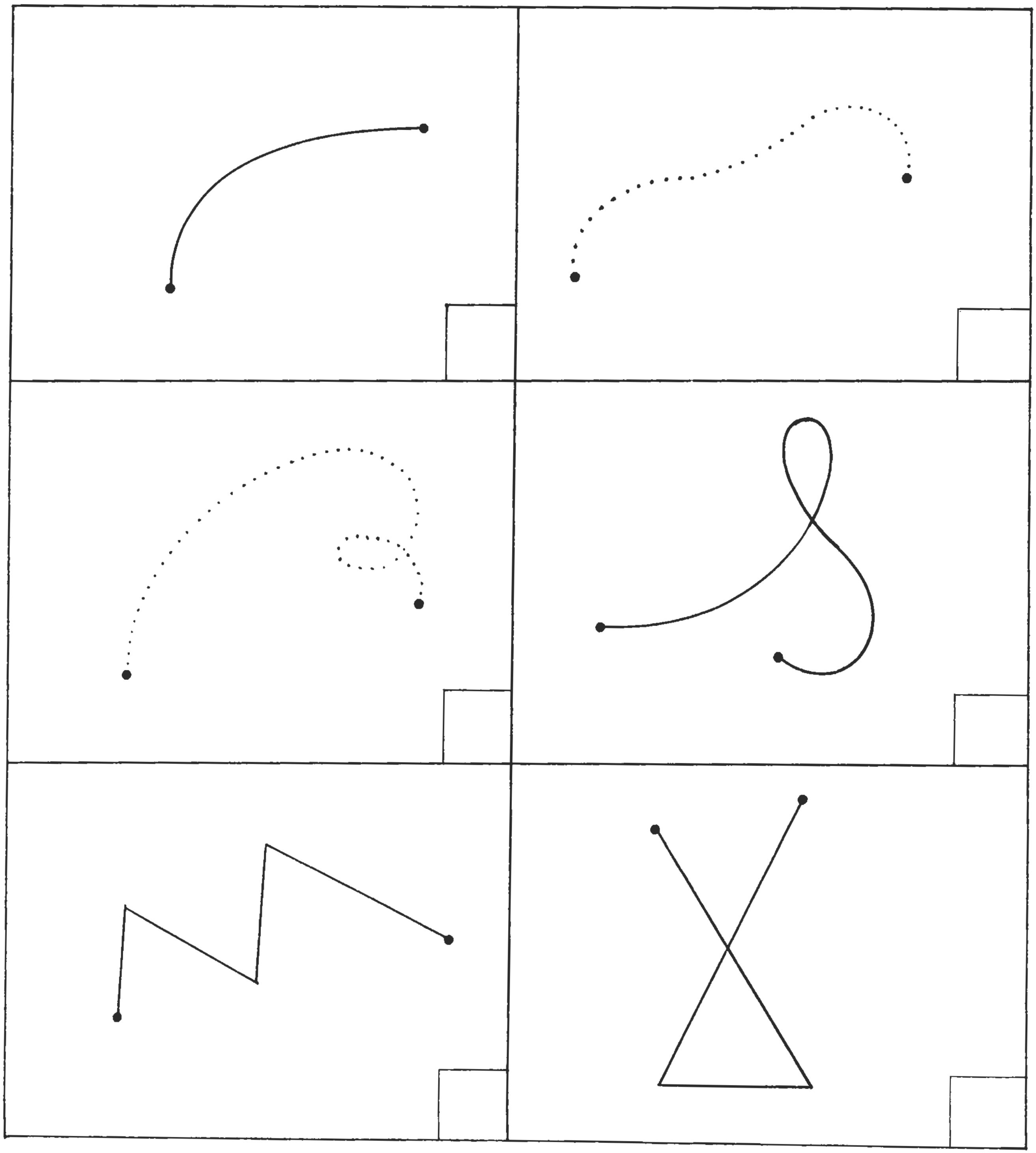
Put the letter R in the square if the picture shows a ray.



Worksheet 2   Name____________________

Put the letter C in the square if the picture shows a curve.

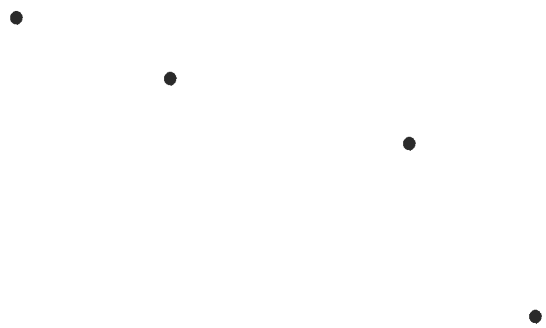
Put the letter N in the square if the picture does not show a curve.



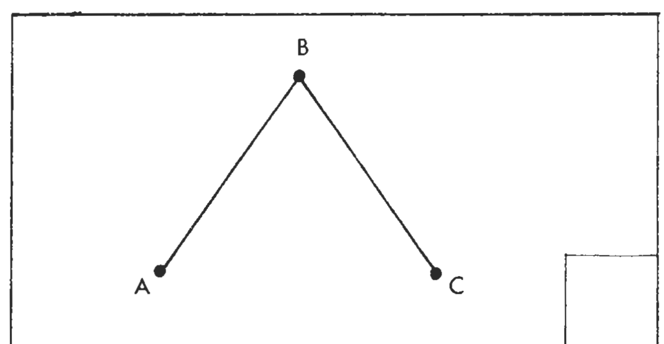
Worksheet 3 Name____________________

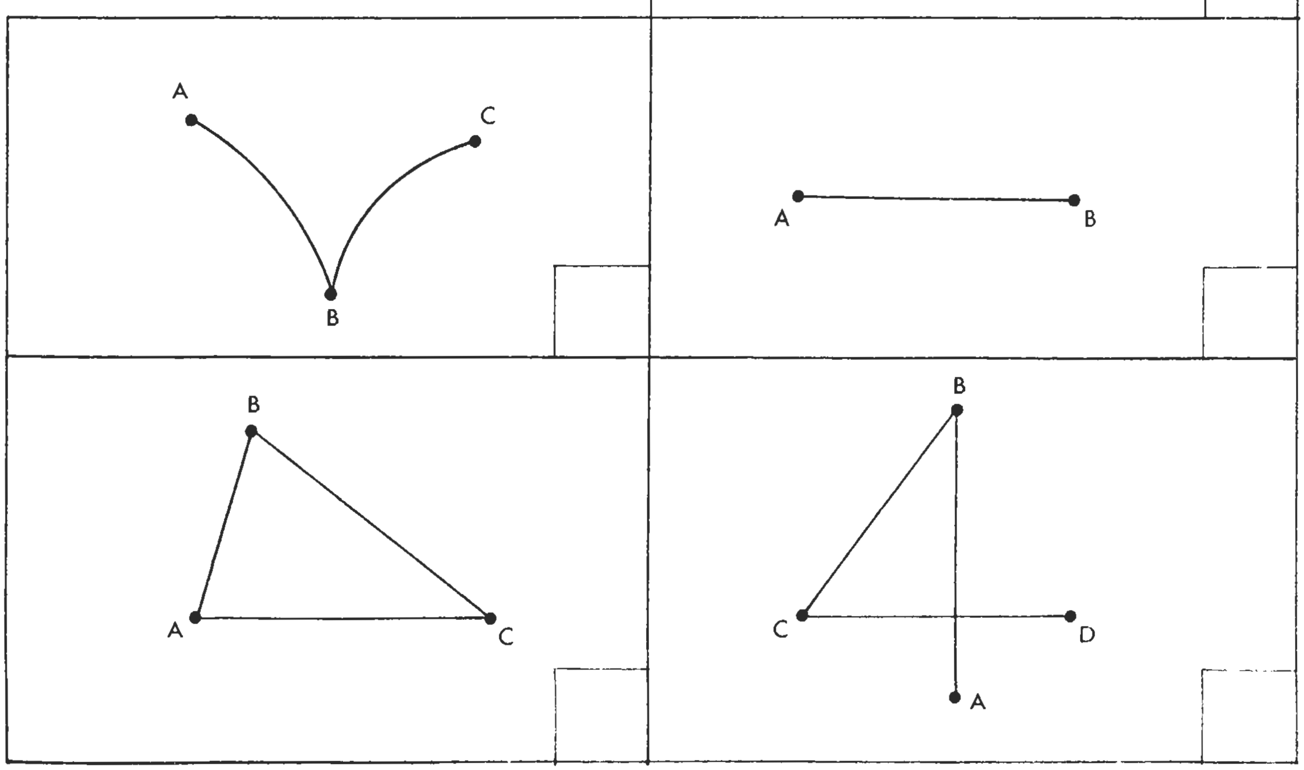
1. Are these three dots collinear (use your ruler)?__________

2. Are these four dots collinear? ____________



3. Put the letter B in the square if the curve is a broken line.





Activity on Connecting Dots

Drawing straight lines may be quite a problem to the children. Lack of coordination is the main reason why children have difficulties drawing straight lines. It may be wise for the teacher to spend one or two sessions developing the coordination needed to enable the children to draw straight lines.

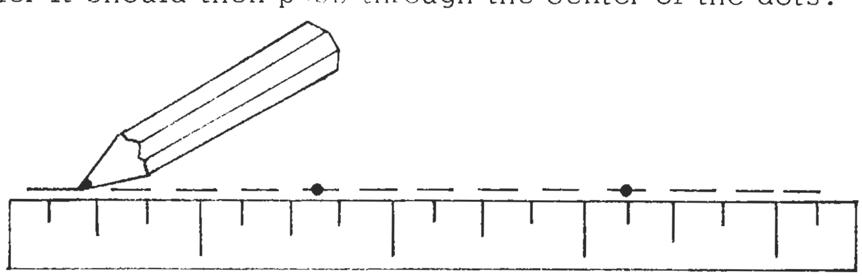
The child may need a wide ruler and if possible the ruler should be taped with non-slippery tape on one side so it can not slip on paper.

A first activity to introduce the children to straight lines may consist of dividing the class in groups of two. One child should hold the ruler and the other child may draw the straight line. Then the roles may be reversed. Every time a team draws the line well it should be given one point. At the end of a few trials, the team with the most points shall be the winner.

The following illustrations might help the students learn to draw straight lines.

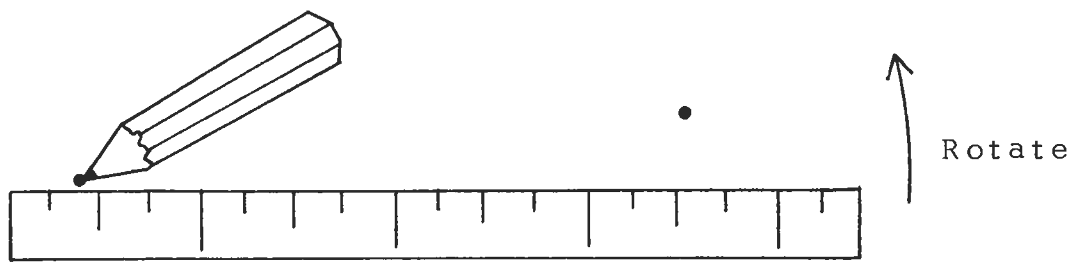
Method I

Place the ruler so its edge is in line with the dots but located slightly off the center of the dots. When the pencil point is placed next to the ruler it should then pass through the center of the dots.



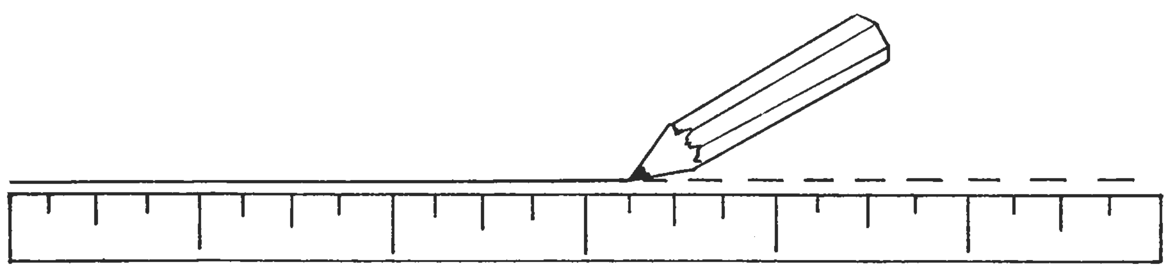
Method II

Place the pencil point on one of the dots and slide the ruler into position so its edge rests against the pencil. Rotate the ruler around the pencil point until the edge of the ruler is in line with the other dot.



Extending a Straight Line

Place the ruler next to the line in the same fashion it was placed next to the dots in Method I above. Make sure a portion of the ruler extends beyond the line so the pencil point can be placed on it. The line can be continued using the edge of the ruler as a guide.



Commentary for Worksheet 4

The purpose of this worksheet is to give the children practice in drawing straight lines between dots. The children should connect the dots with a ruler drawing the lines through the center of the dots. Emphasis should be placed on neat straight lines through the middle of the dots.

Demonstrate on the chalkboard how to place the ruler just on the side of the dot rather than on the center of it. This will be an effective way of teaching the children how to draw lines through the centers of the dots.

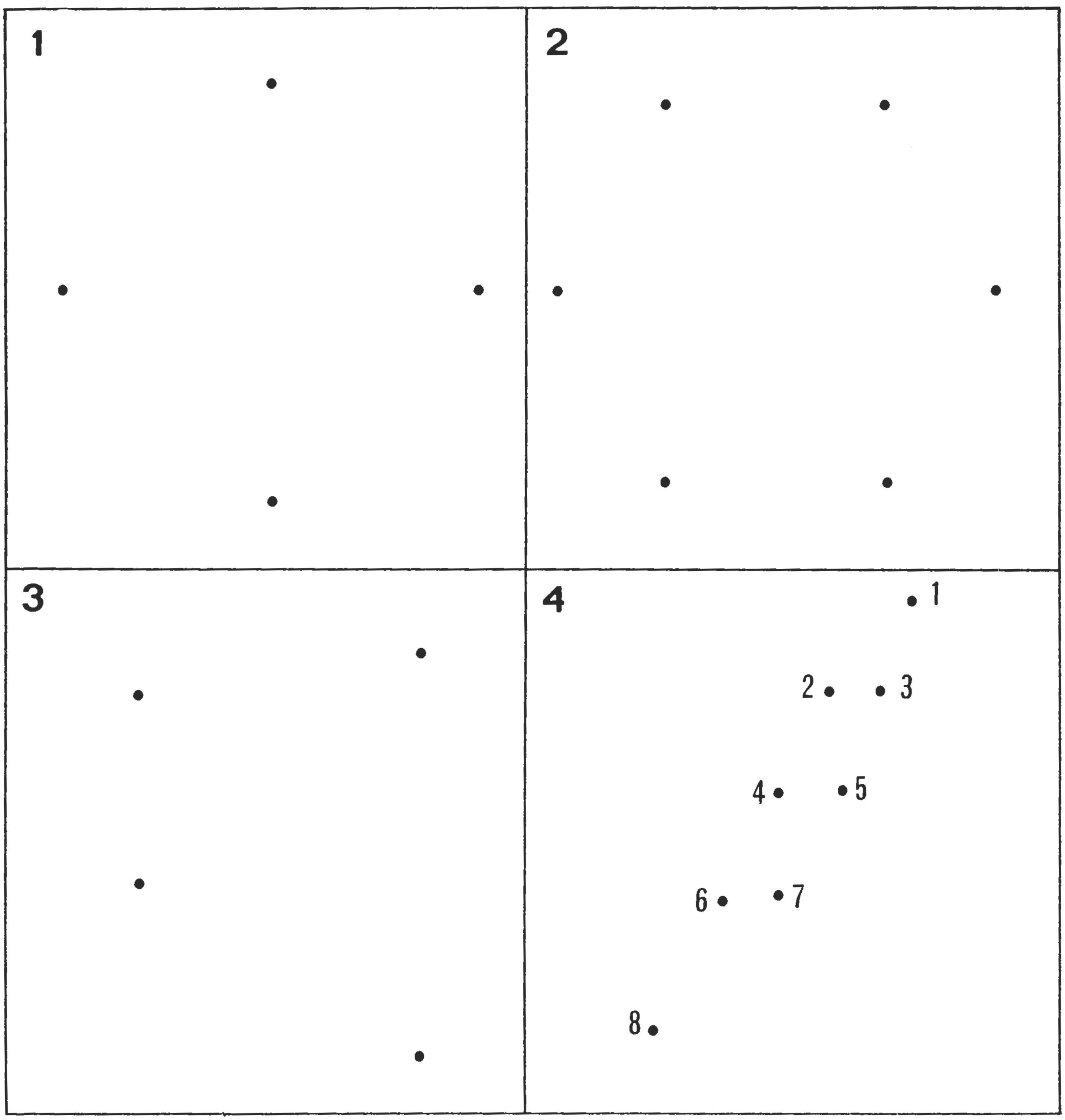
In the first three boxes the children may connect the dots in any way they wish. In the last box they should connect the dots by following the numbers.

Since there are many ways to connect the dots in the first three pictures, the children's answers will not all look the same. Give the children an opportunity to name things which look like their outlined shapes. They may turn the drawing sideways or upside down. For example, the fourth shape looks like a bolt of lightning.

To make the activity more interesting, the children may use crayons to color their pictures when they have finished drawing them.

Worksheet 4 Name________________

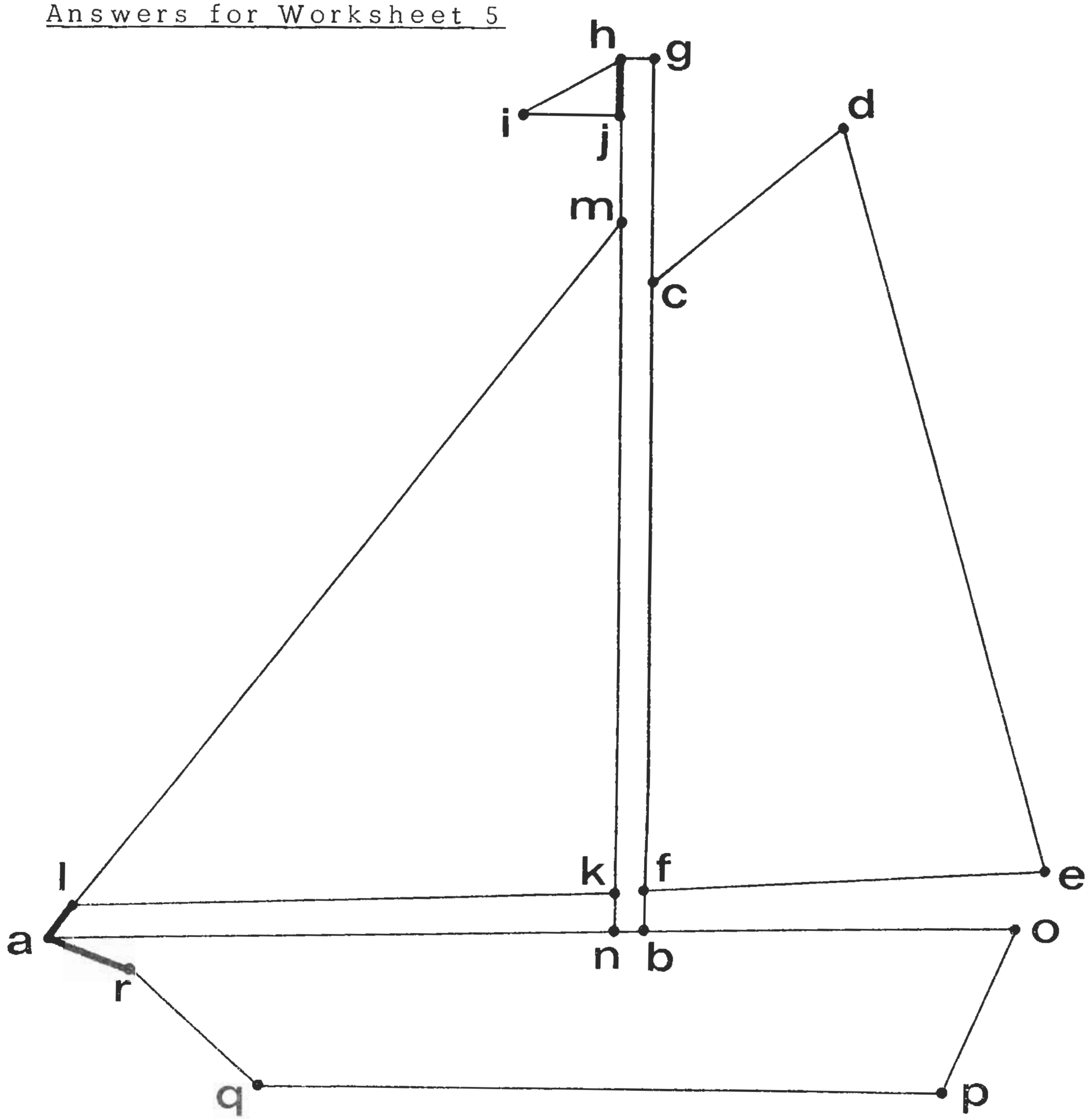
Use your rulers to connect the dots with line segments. In the first three boxes connect the dots in any way you wish. In the last box, connect the dots by following the numbers.



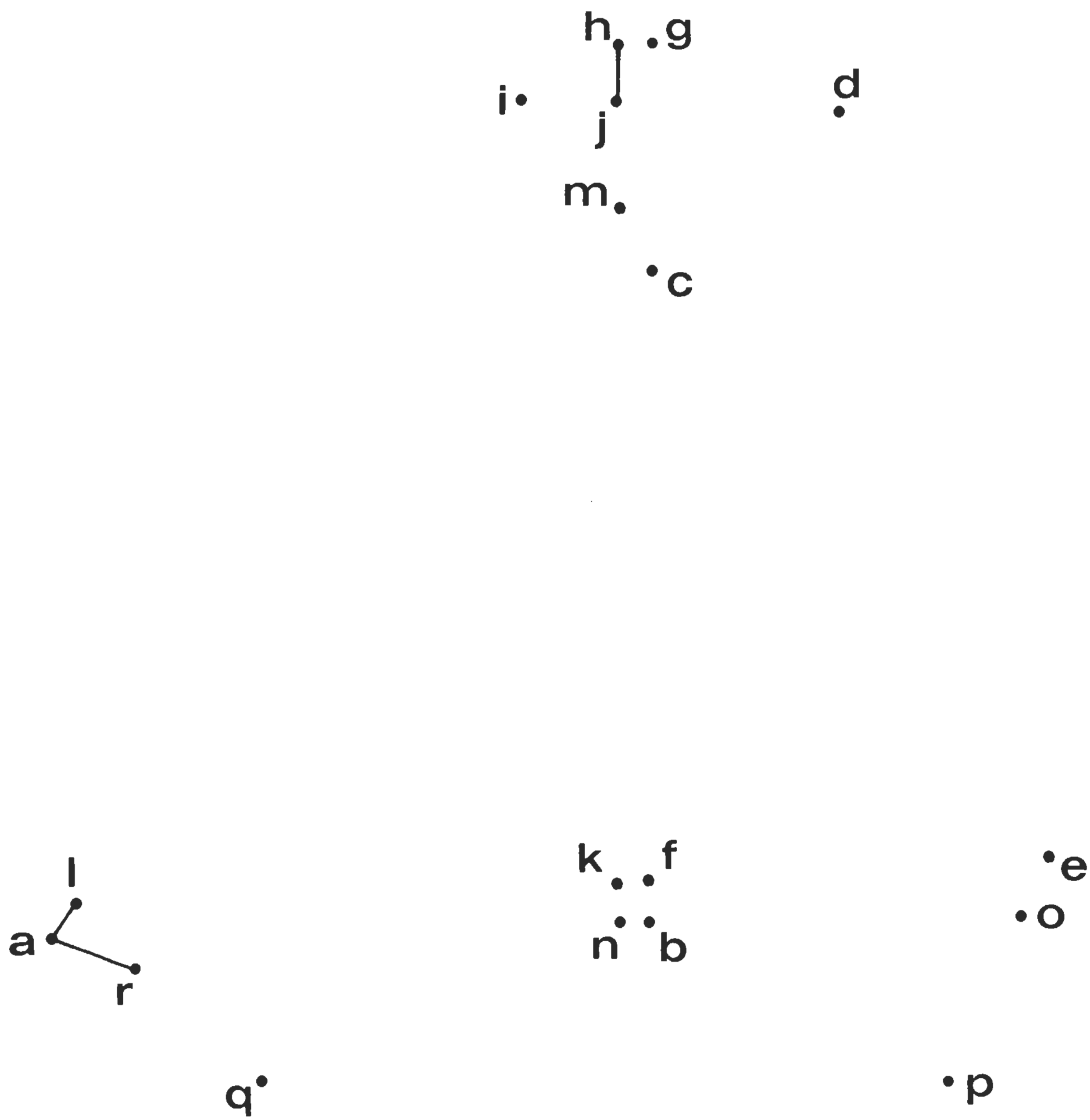
Commentary for Worksheet 5

Have the children connect the dots with line segments starting with $\overline{ab}$ and following the alphabet, ending with $\overline{qr}$. The picture should look like a sailboat.

Answers for Worksheet 5



Worksheet 5 Name________________



The customer then asks him how much he is going to charge to paint each of the walls. The customer must have an estimate. A simple question like this leads directly to the definition of area. If the painter could associate the size of each wall with a number, his problem would be easier. This number should be such that if one wall has a number twice as big associated with it as another wall, the painter should take twice as long to paint this wall. The number the painter associates with a wall may be called the area of the wall's shape. The painter may say that a wall that takes him one hour to paint has a unit area. If it takes him 16 3/4 hours then the area of the shape of the wall is 16 3/4. His way of defining the area requires him to paint the region before he is able to tell the area of the region. We are concerned with finding the number without painting the wall itself.

Two different painters may paint at a different rate, so their definition of a unit area will differ. This shows that an area is dependent on the unit chosen.

If you can move one geometrical figure rigidly so as to fit exactly on top of another figure, then the two figures are said to be <u>congruent</u>. A <u>rigid</u> <u>motion</u> is one in which the distance between any two points is left unchanged. A rigid motion does not change size or shape. Two figures are congruent if they have the same size and shape.

Congruent figures have the same areas. If you cut up a figure into parts, and rearrange the pieces by rigid motions, the total is left unchanged.

In crude comparisons of areas, the concept of associating a number with the size of a region can be avoided. In fact with the help of paper models and scissors, any person can compare two regions. Just cut up one of the regions and try to fit it on top of the second region. If there is space left the second region is larger. If there is some paper left from the first region after covering completely the second one we are sure that the first region is larger.

Our approach to the comparison of areas is essentially that of the jigsaw puzzle.

Commentary for Worksheets 6 - 8

In the following worksheets the children will be introduced to the concept of area. A formal definition will not be given. The children should realize that the area of a region is just a number that allows people to have an idea of the relative size of a region when compared to other regions. Material presented in the Teacher Background may be helpful.

The story "Ahmes and the Tax Collector" introduces a practical way of using the number line to associate a number with the size of a region. The method only applies to rectangles, but future units will try to generalize the method.

In the following worksheets the children should color the figures first. Then if the children do not see the need for using scissors the teacher should bring them out for the children's use.

Answers for Worksheets 6 - 8.

Worksheet 6

Land A is larger than Land B.

Worksheet 7

The farmer should buy Land B.

Worksheet 8

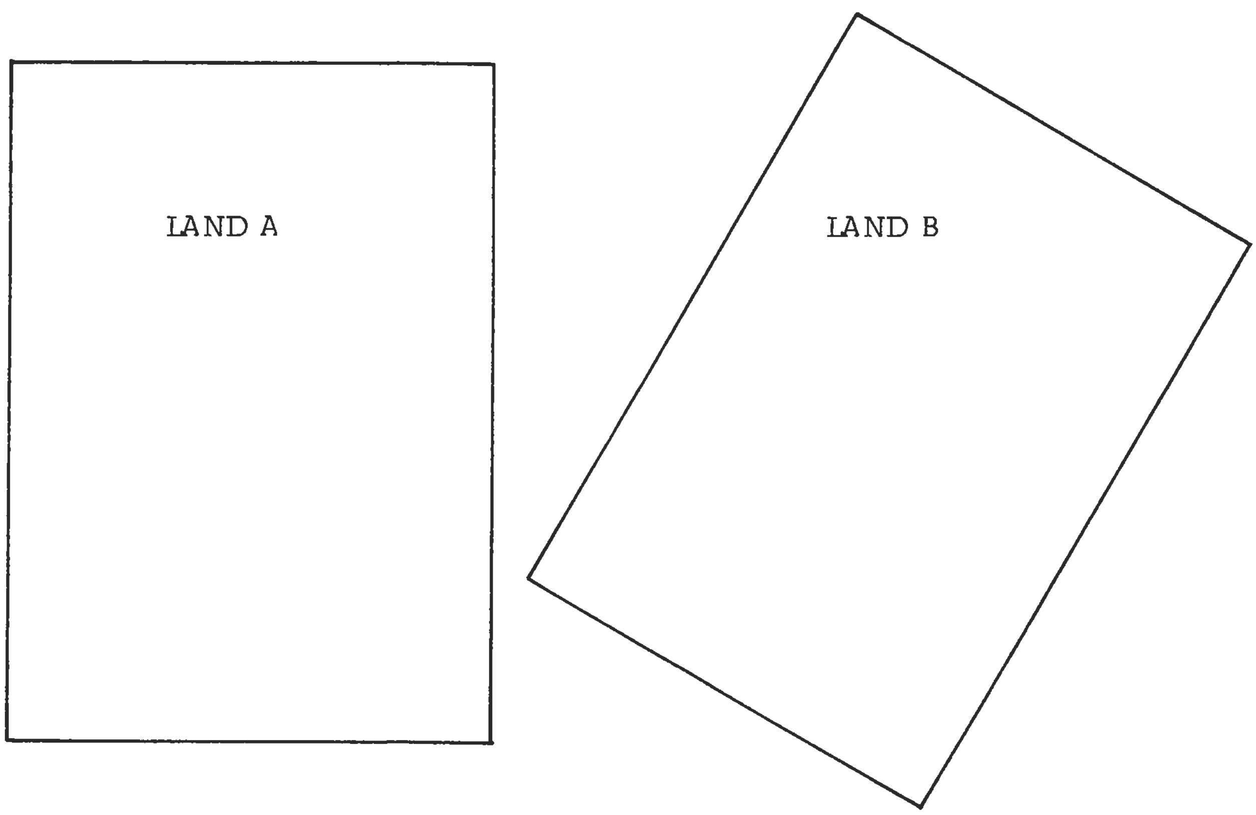
Land A is greater in area than Land B.

Worksheet 6 Name____________________

A farmer wants to buy a piece of land so he can plant wheat. He is offered Land A and Land B for the same price. Both pieces of land are very good. What piece of land would the farmer buy? ______________________________

Why? ______________________________

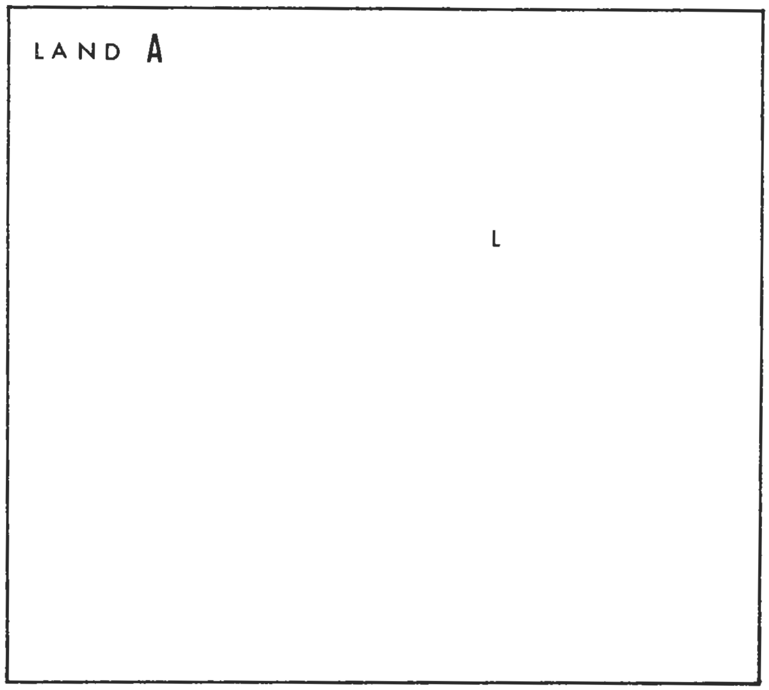
Instead of saying that Land______is larger than Land______, a mathematician says: Land B is greater in area than Land A.

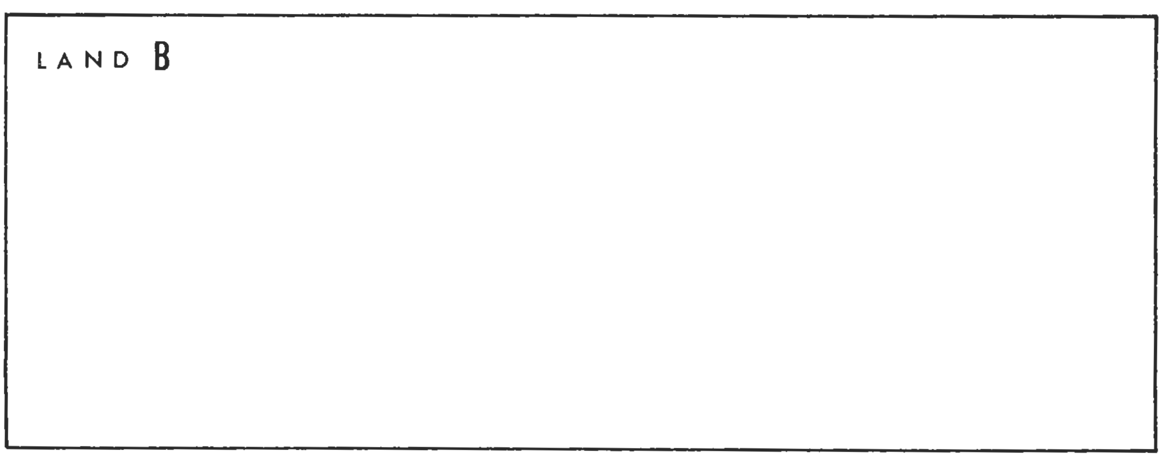


Worksheet 7 Name________________

The same farmer is offered the lands below. With the help of scissors can you find out which land the farmer should buy?

____

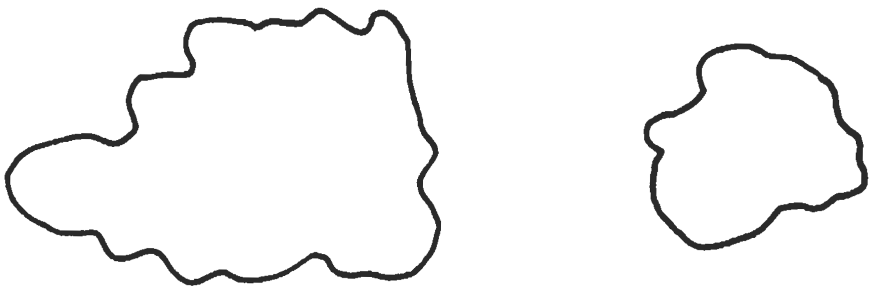




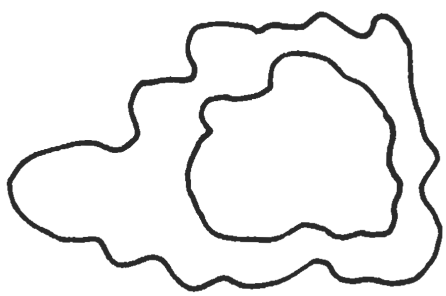
COMPARING RECTANGULAR REGIONS

Teacher Background

One of the purposes of the story, "Ahmes and the Tax Collector" is to give the children some familiarity with the concept of area. This word, area, will be introduced in connection with comparison of simple closed regions especially rectangular regions. Given any two farm lands

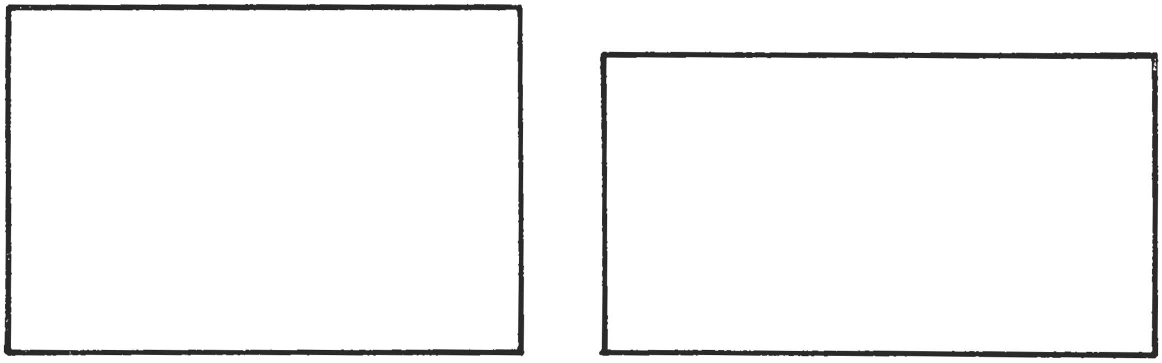


a question might be asked: "Which farm has more land?" One way of answering the question is by mentally placing the small region on the larger region.



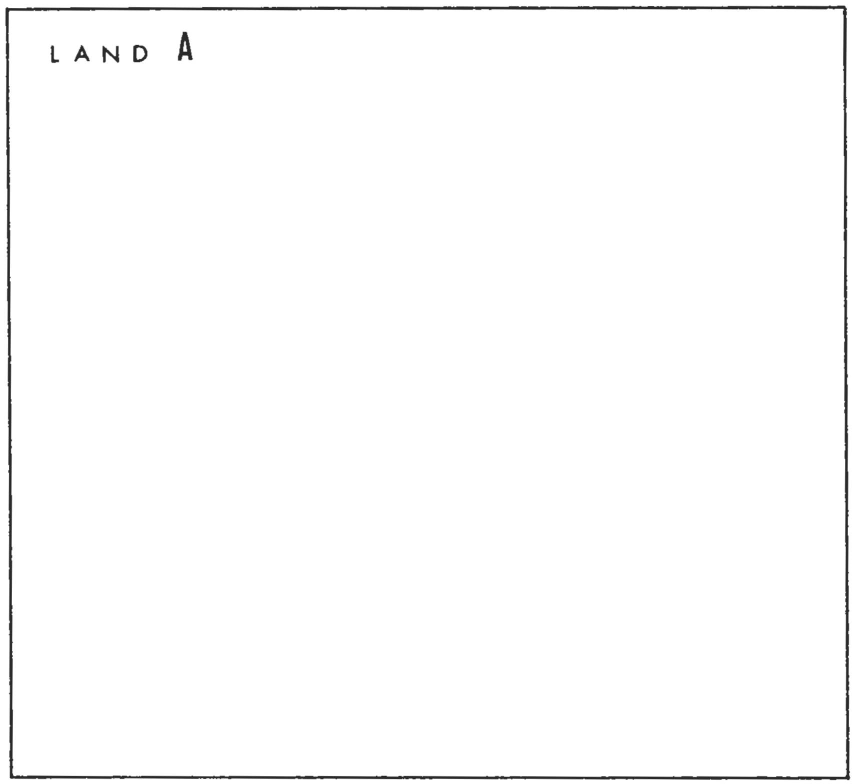
It is sometimes then apparent which farmer has more land.

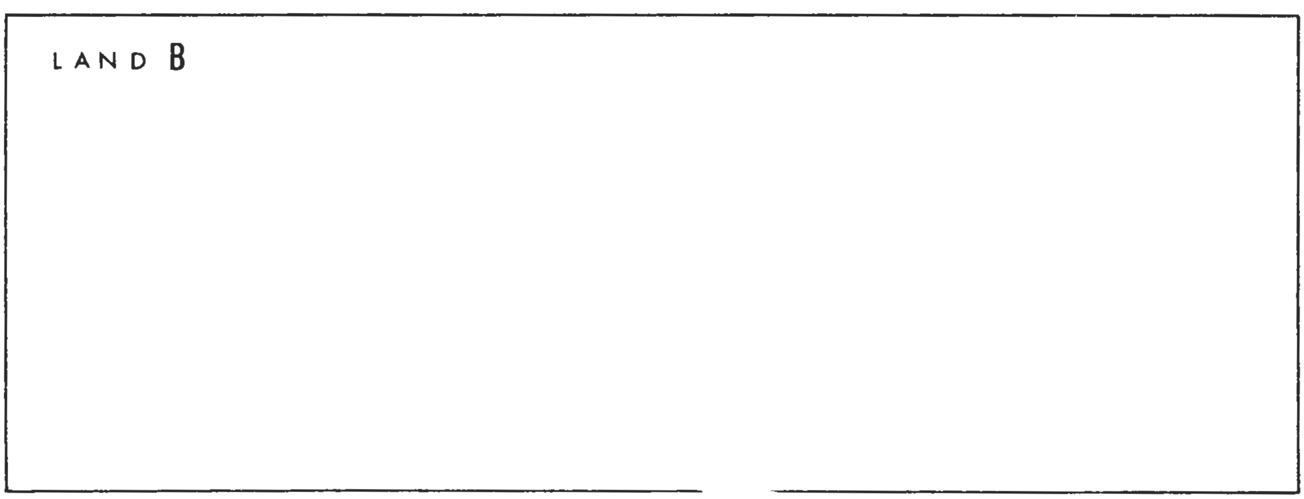
In a city a painter might be asked to paint two walls.



Worksheet 8 Name_______________

Which land has the greater area? _______________





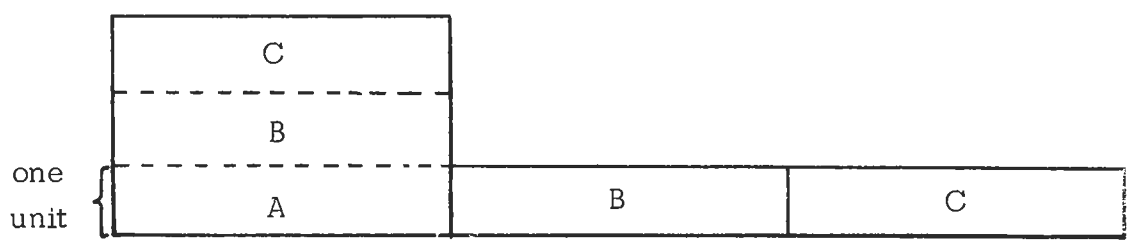
Teacher Background for "Ahmes and the Tax Collector"

The purpose of this section is to give the children some experience in comparing rectangular regions without getting involved in the exact area of these regions.

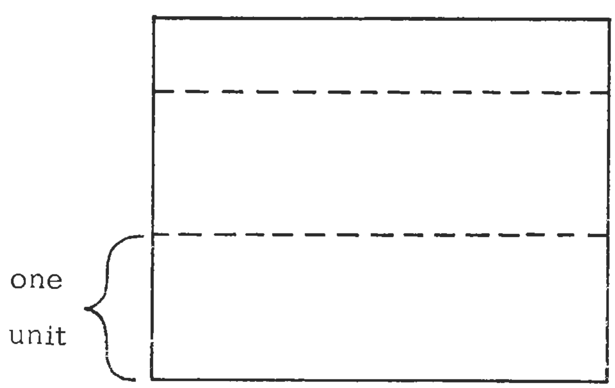
The basic concept of this section is the idea that any rectangular region can be compared in area to a strip which is one unit wide where the selection of the unit is arbitrary. Thus if we wish to compare two rectangular regions in size (area), we can construct the unit strip for each and compare the lengths of the strips.

Two ways in which a rectangular region can be changed to a unit strip of equivalent area are explored in detail in this unit.

Method I is quite simple and should be understood easily. This involves a rectangular region that can be cut exactly into strips of one unit in width. These strips can then be placed end to end to form one long unit strip. See Below.



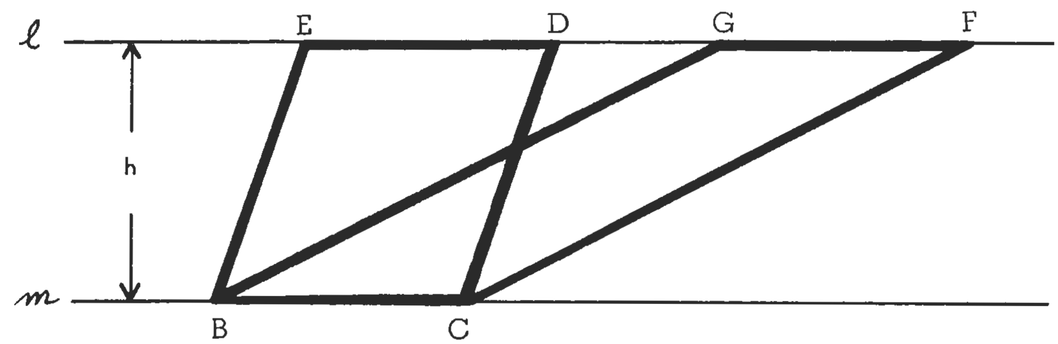
If we have a rectangular region that cannot be cut exactly into unit strips such as the figure below, Method I will not work.



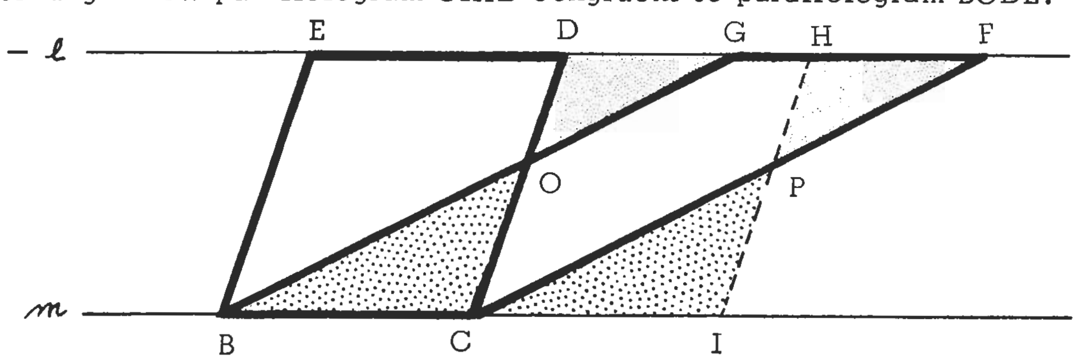
It is possible to make use of a second method to form a unit strip from this rectangular region which would contain an area equal to the original rectangular region.

Method 2 involves some geometric construction, the mechanics of which the children should be able to handle if they have learned how to measure with a ruler and have learned how to draw a straight line through two points. It is not expected that they will understand the high level mathematics involved in this construction at the present time.

The basic concept involved here is the idea that two parallelogram regions which have the same base and the same altitude are equal in area. In the figure below lines l and m are parallel.



If we consider the length of $\overline{BC}$ as the base of parallelogram BCDE then the distance, h, between the parallel lines is the altitude. We can also consider the length of $\overline{BC}$ being the base of parallelogram BCFG, so that it also has h as its altitude. The area of the region bounded by parallelogram BCDE equals the area of the region bounded by parallelogram BCFG. This is easily seen if we draw line segment $\overline{HI}$ so that the length of $\overline{DH}$ equals the length of $\overline{ED}$ and the length of $\overline{CI}$ equals the length of $\overline{BC}$, forming a new parallelogram CIHD congruent to parallelogram BCDE.



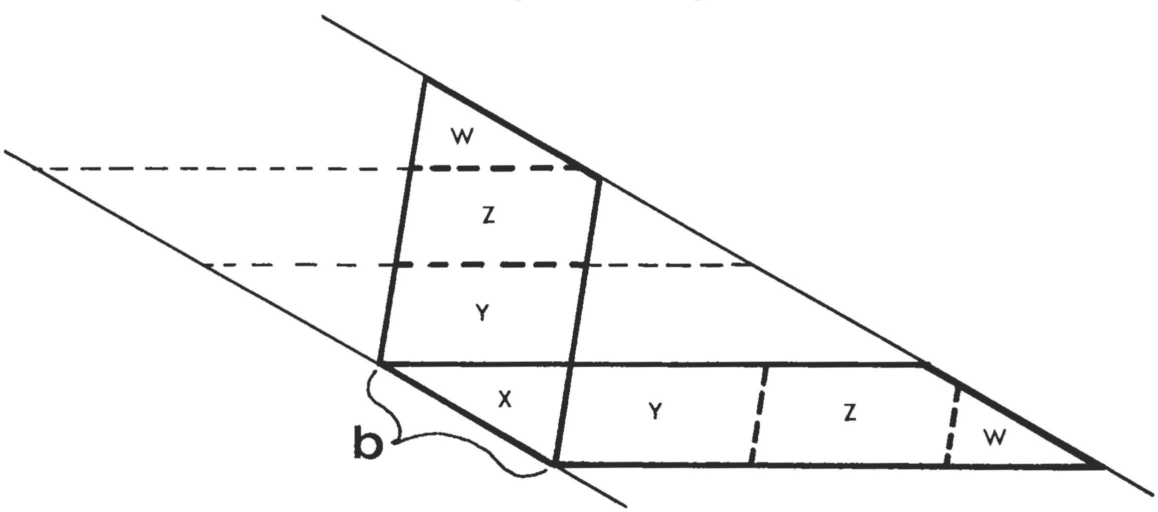
Now the area of the region bounded by parallelogram CIHD (which has the same area as the region bounded by parallelogram BCDE) is the sum of the areas of the regions bounded by the 5-gon CPHGO and the triangles CIP and OGD. The area of the region bounded by parallelogram BCFG is the sum of the areas of the regions bounded by the same 5-gon CPHGO and the triangles BCO and PFH. But it is easy to see that the regions bounded by triangles BCO and CIA have the same area, and the regions bounded by the triangles OGD and PFH also have the same area. Therefore the regions bounded by parallelograms BCDE and BCFG have the same area. If we use "A(...)" to abbreviate "the area of the regions bounded by the polygon...," the above argument is summarized by the following equations:

$$\begin{aligned} A(BCDE) = A(CIHD) &= A(CPHGO) + A(CIP) + A(OGD) \\ &= A(CPHGO) + A(BCO) + A(PHF) \\ &= A(BCFG), \end{aligned}$$

since A(CIP) = A(BCO) and A(OGD) = A(PHF).

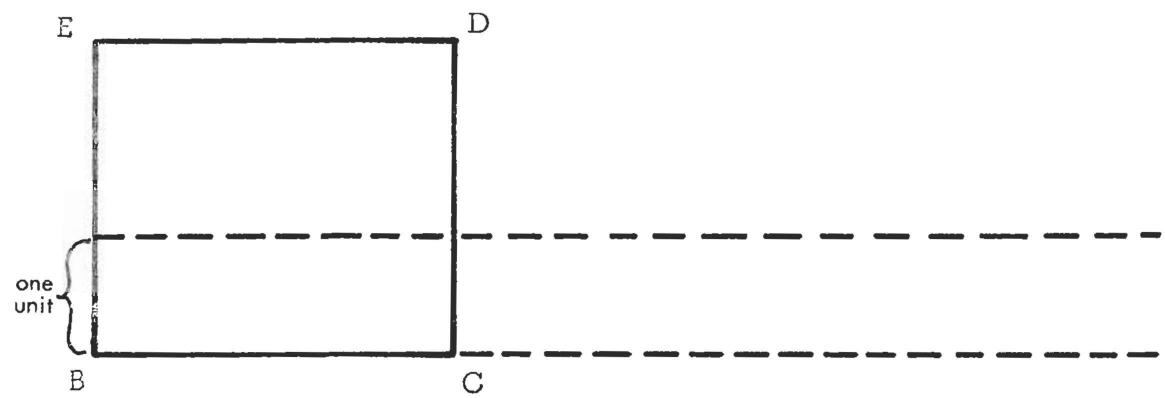
Another way of seeing that the regions bounded by the parallelograms BCDE and BCFG have the same area is to trace BCFG on another sheet of paper, including the segments $\overline{OC}$ and $\overline{HP}$. After cutting out the copy of BCFG, and cutting it into the three pieces as marked, it is possible to place those pieces so that they will exactly cover BCDE.

This procedure works for any two parallelograms having the same base and altitude, although it may be necessary to make more than two cuts. The places to cut are found by drawing equally spaced parallel lines. The figure below shows another example, congruent regions being lettered the same, the common base being represented by "b":

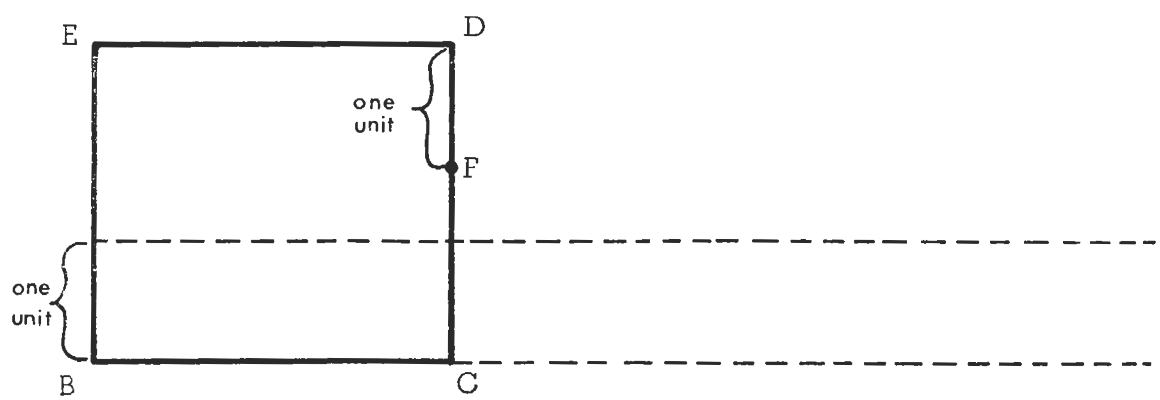


This fact, that parallelogram regions having the same base and altitude have the same area, provides the basis for our second and more general method of getting a unit strip whose area equals that of a given rectangular region.

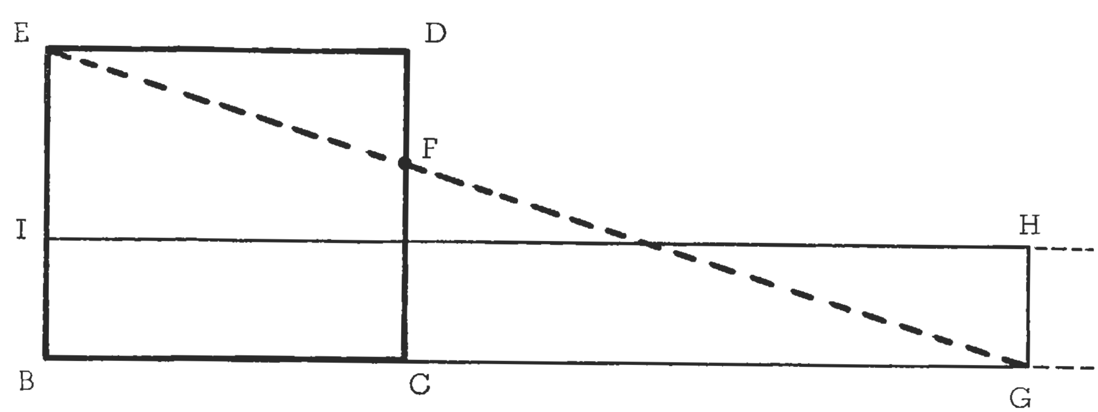
We begin by laying off a strip one unit in width in our rectangular regions, and extending it beyond the rectangle:



We next measure from D a length equal to one unit and mark that point F.

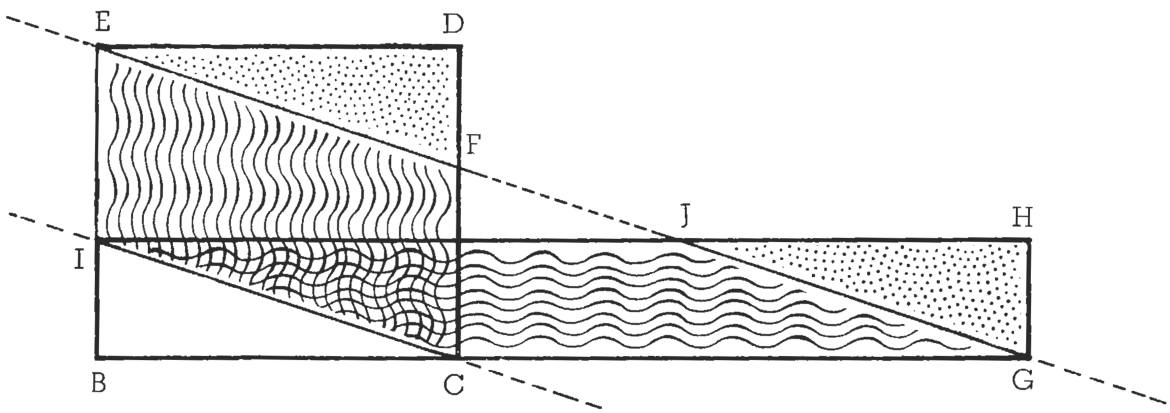


Next we connect points E and F and extend that line until it intersects BC at G.



$\overline{BG}$ has the correct length for the unit strip, and the rectangular region BGHI is equal in area to the rectangular region BCDE.

We can see that the two rectangular regions have the same area by drawing line IC, which is parallel to EG (since $\overline{DF}$ and $\overline{IB}$ have the same length).



The area of the rectangular region BCDE is the sum of the area of the regions bounded by parallelogram ICFE and the triangles BCI and FDE. The area of the rectangular region BGHI is the sum of the areas of the regions bounded by parallelogram ICGJ and the triangles BCI and GHJ. But triangles GHJ and FDE are congruent, and the parallelogram regions ICFE and ICGJ have the same base (the length of $\overline{IC}$) and the same altitude, so they have the same area. The argument is summarized by the following equations:

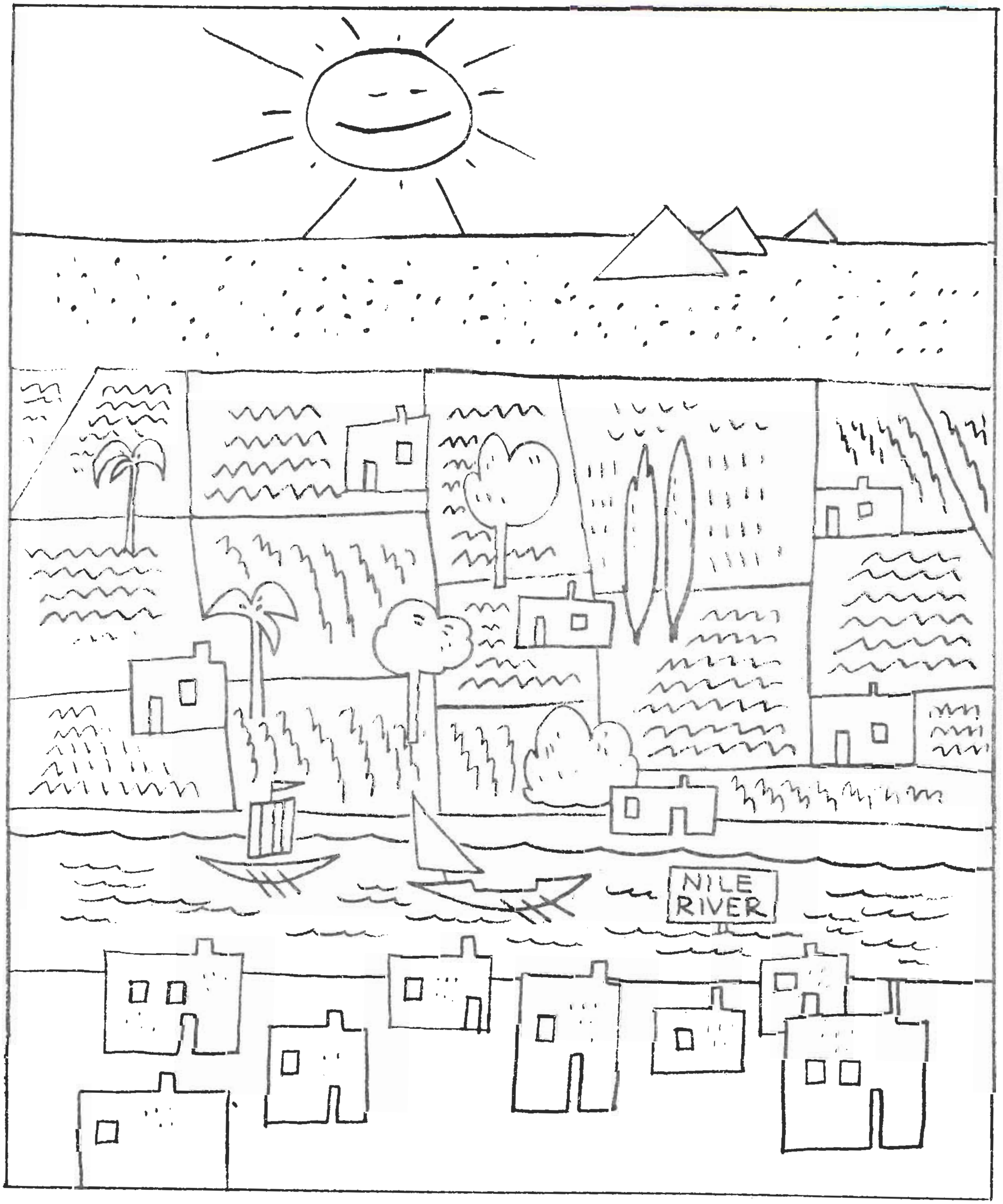
$$A(BCDE) = A(FDE) + A(ICFE) + A(BCI)$$

$$= A(GHJ) + A(ICGJ) + A(BCI)$$

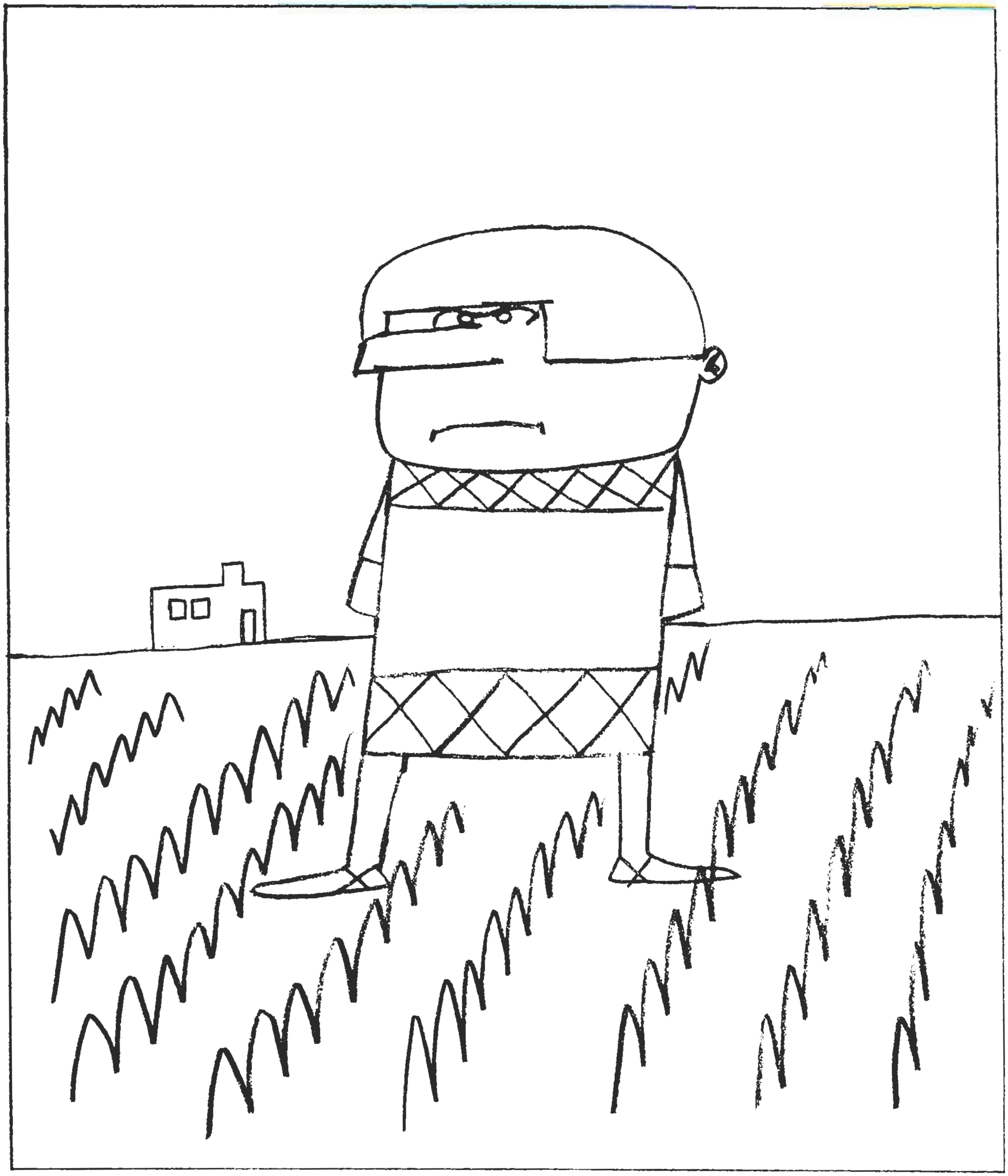
$$= A(BGHI)$$

Before you start the story of <u>Ahmes and the Tax Collector</u>, it would be wise to have a little discussion with the children about taxes to make sure they understand what taxes are so they will understand Ahmes' problem.

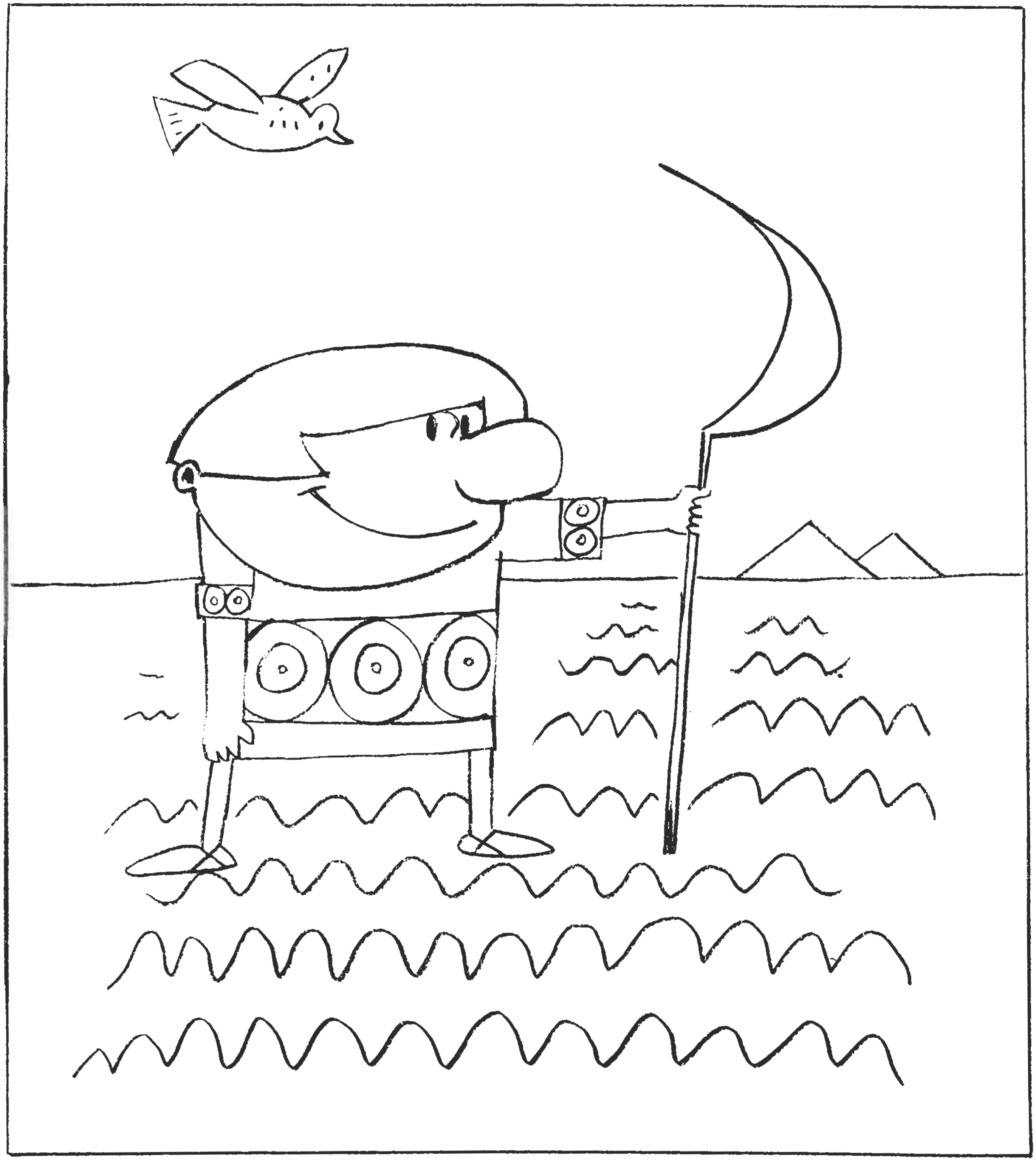
"Ahmes and the Tax Collector"



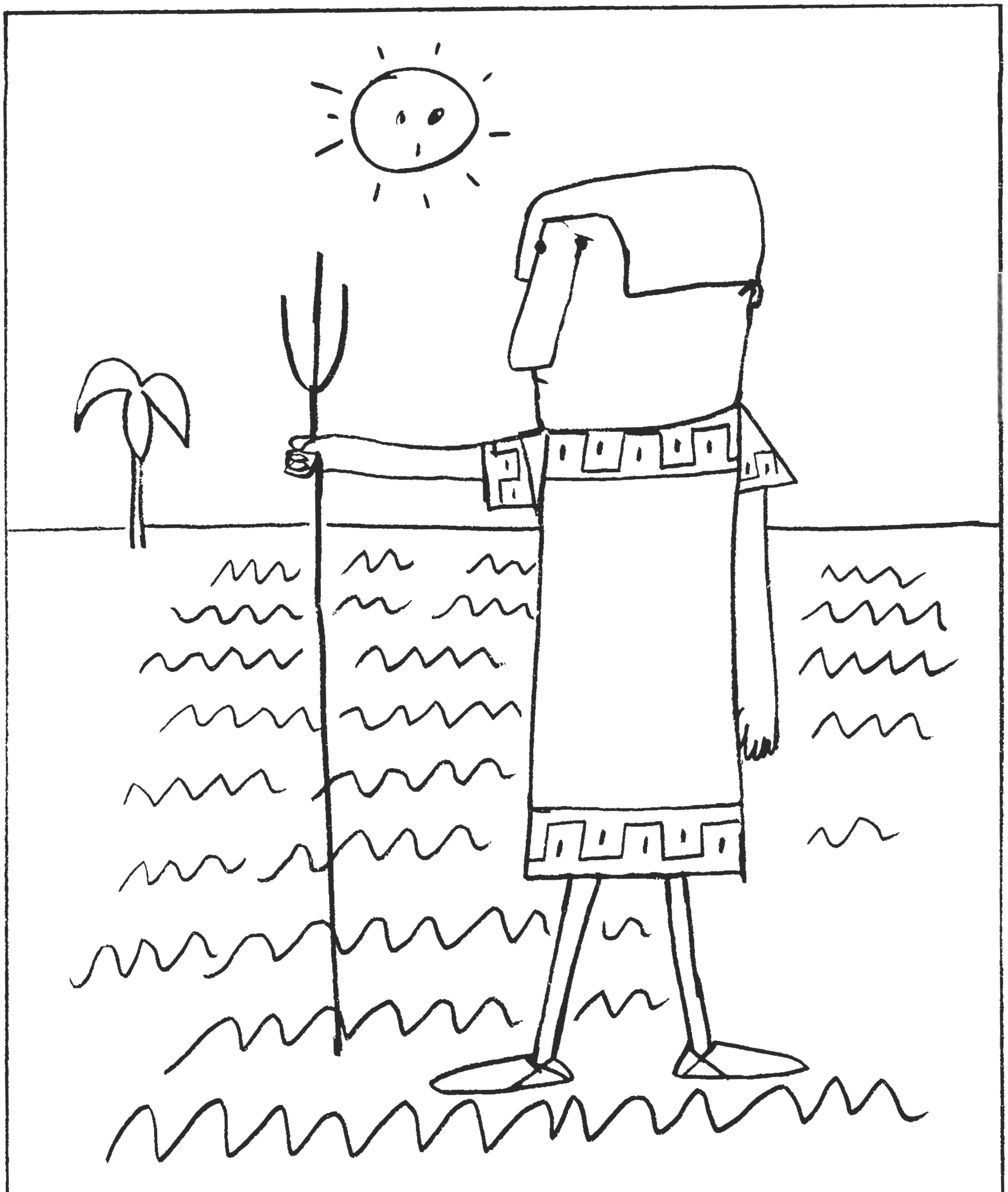
This is a country. It is called Egypt.



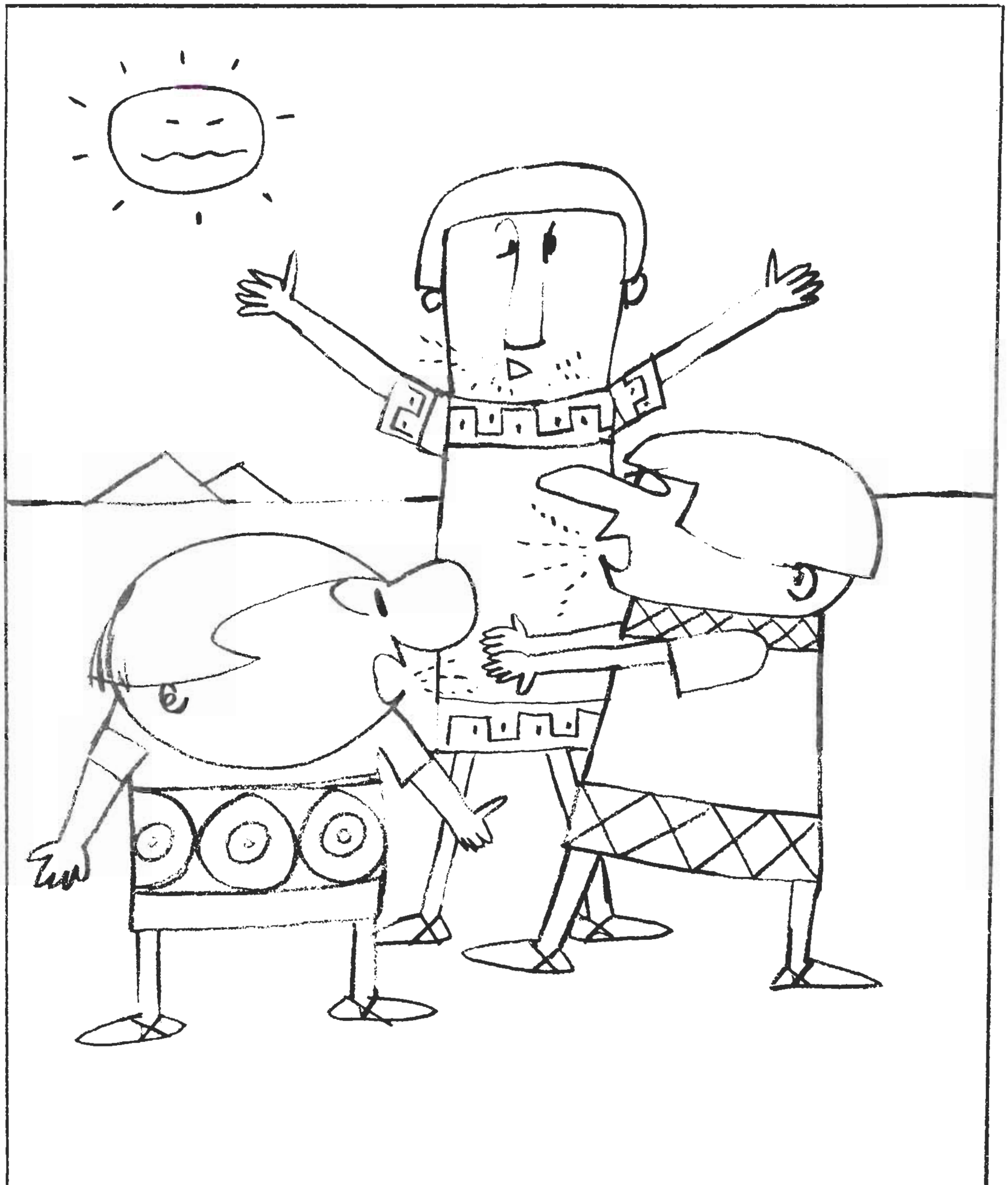
This is Ahmes, an Egyptian farmer. He is standing in his field.



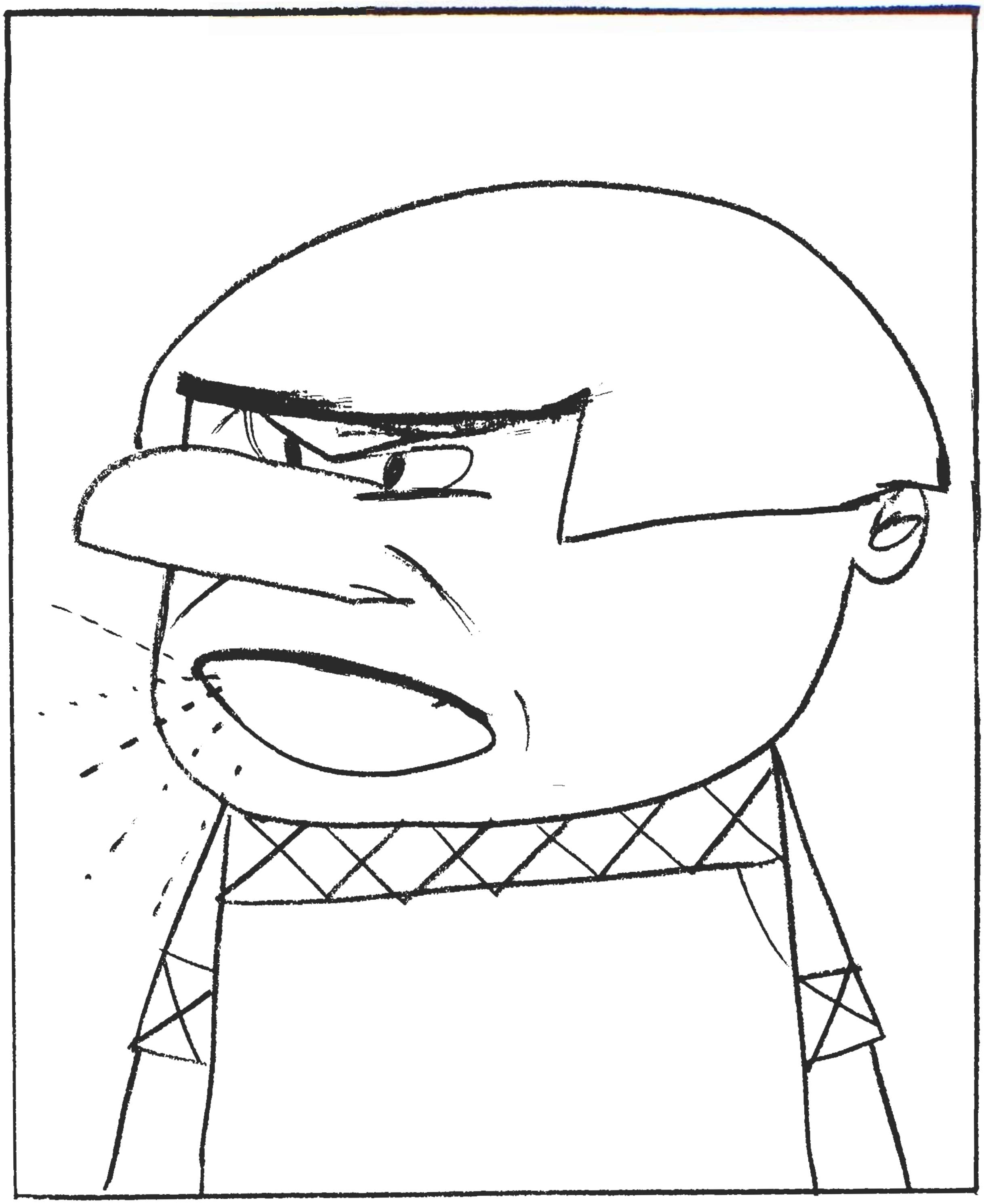
This is another farmer who is Ahmes' neighbor. He is standing in his field. His name is Tehuti.



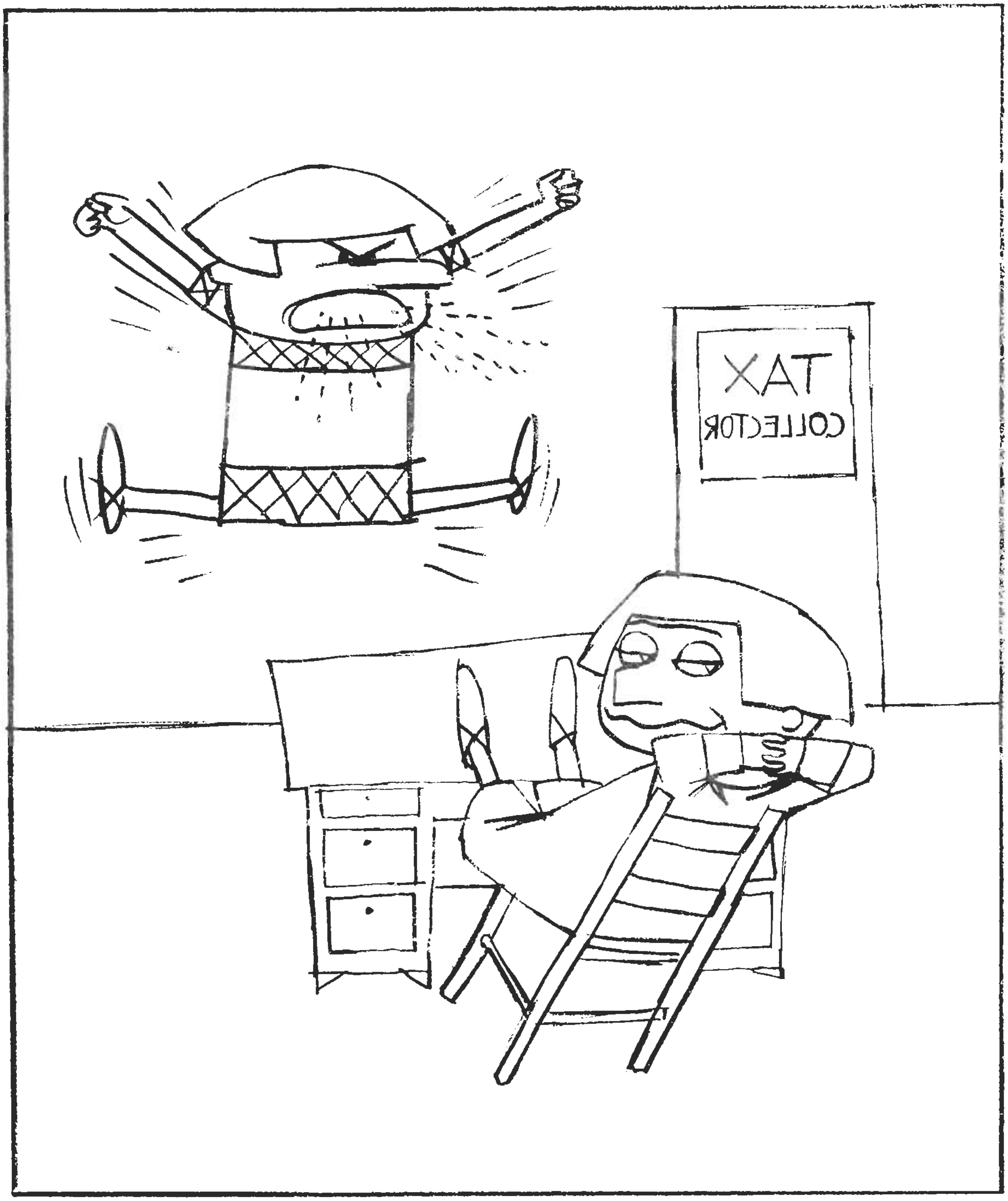
This is a third farmer who is another neighbor. He is standing in his field. His name is Tepi.



One day the three farmers were talking about the taxes they had to pay.



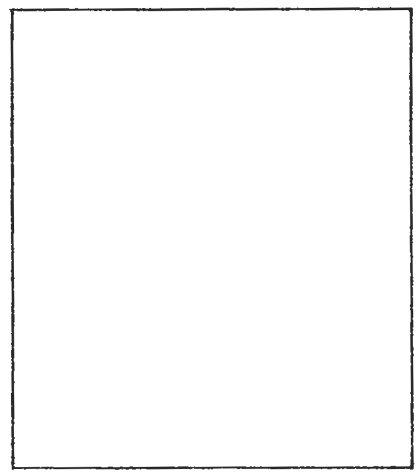
Ahmes said, "I don't know why I should have to pay more taxes than you do. It's not fair."



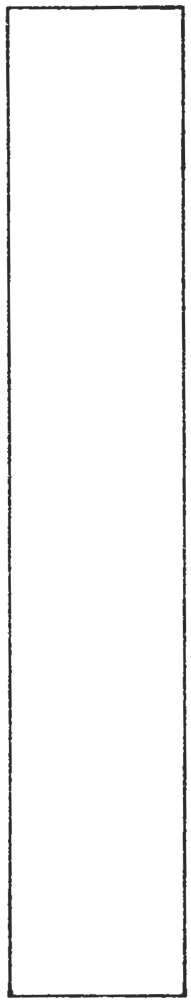
He went to the tax collector. The tax collector could tell that Ahmes was unhappy.

The tax collector took a big map out of his drawer. He said, "Ahmes, on this map is a picture of every farmer's field in our part of Egypt."

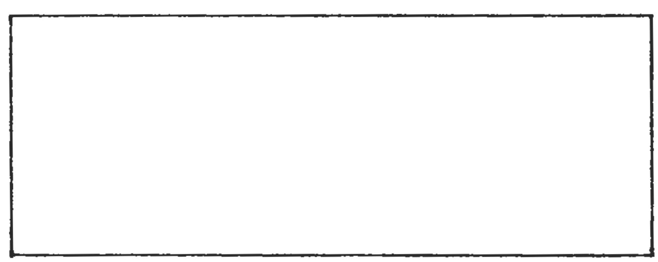
"Here is the picture of your field, and here are the fields of Tehuti and Tepi."



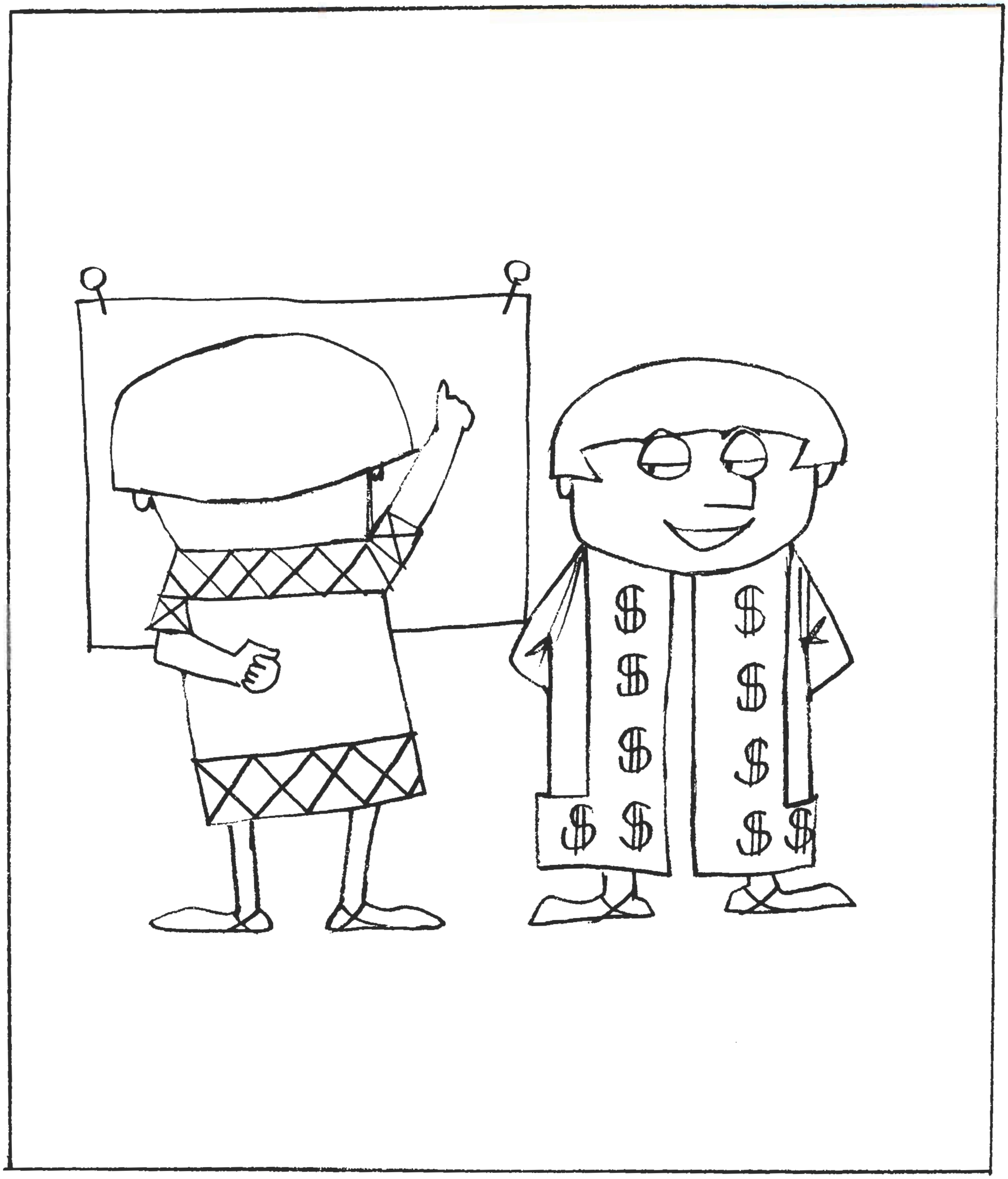
Ahmes's Field



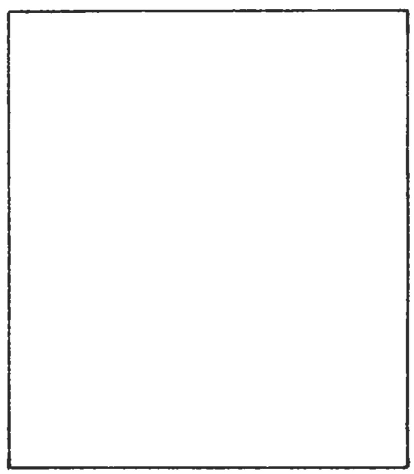
Tehuti's Field



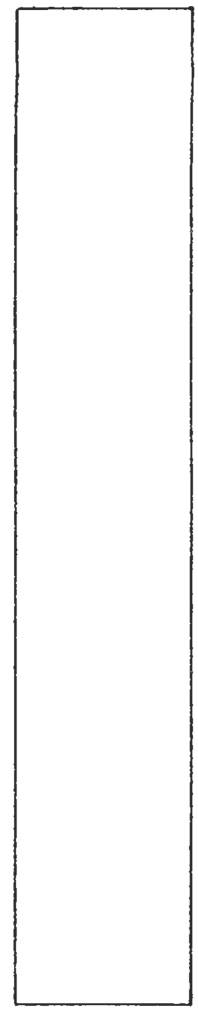
Tepi's Field



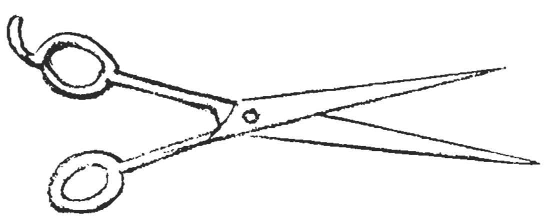
"Ahmes, it is true that Tehuit's field is longer than yours, but yours is wider. Tepi's field is wider than yours but yours is longer."



Ahmes' Field

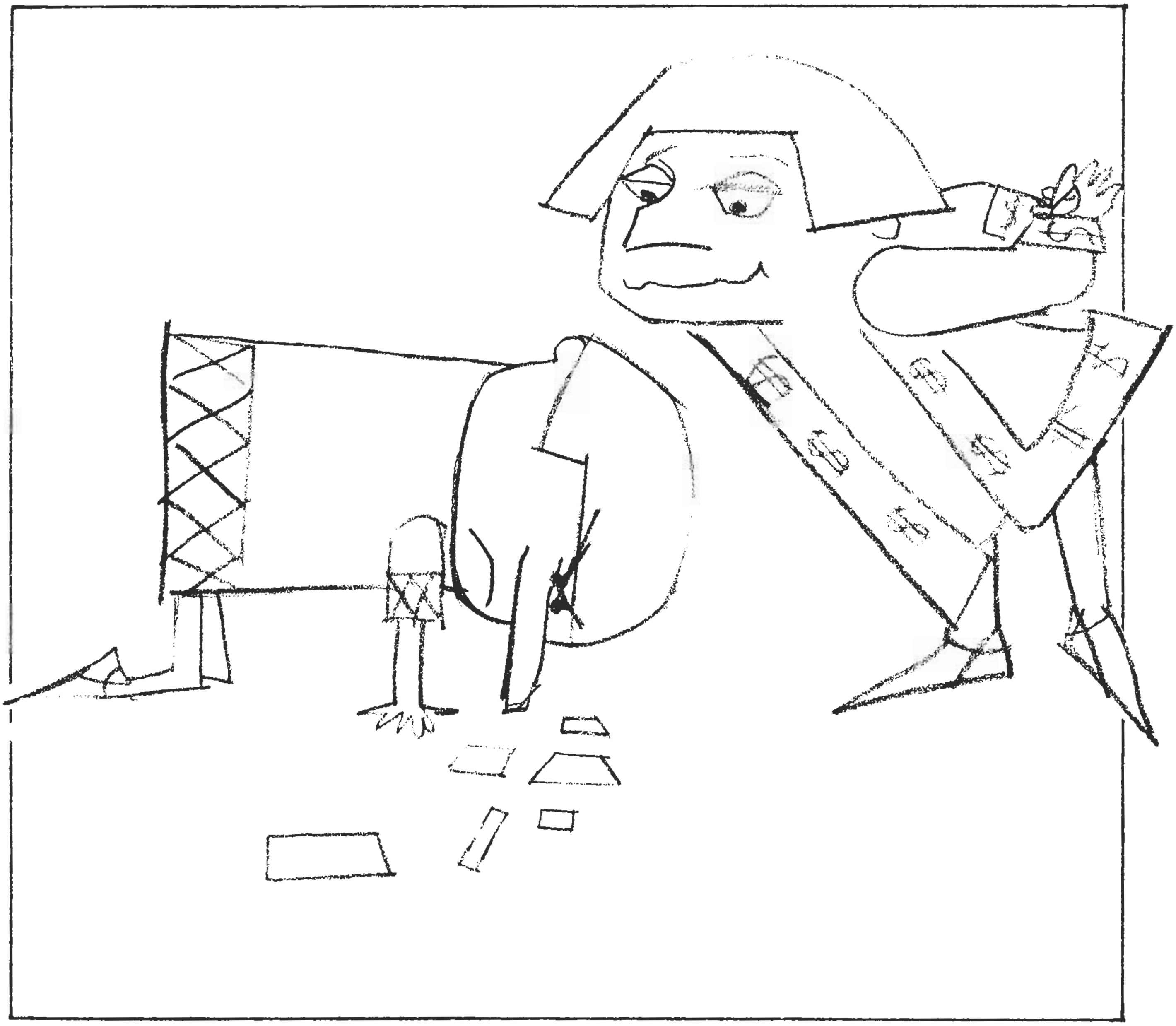


Tehuti's Field

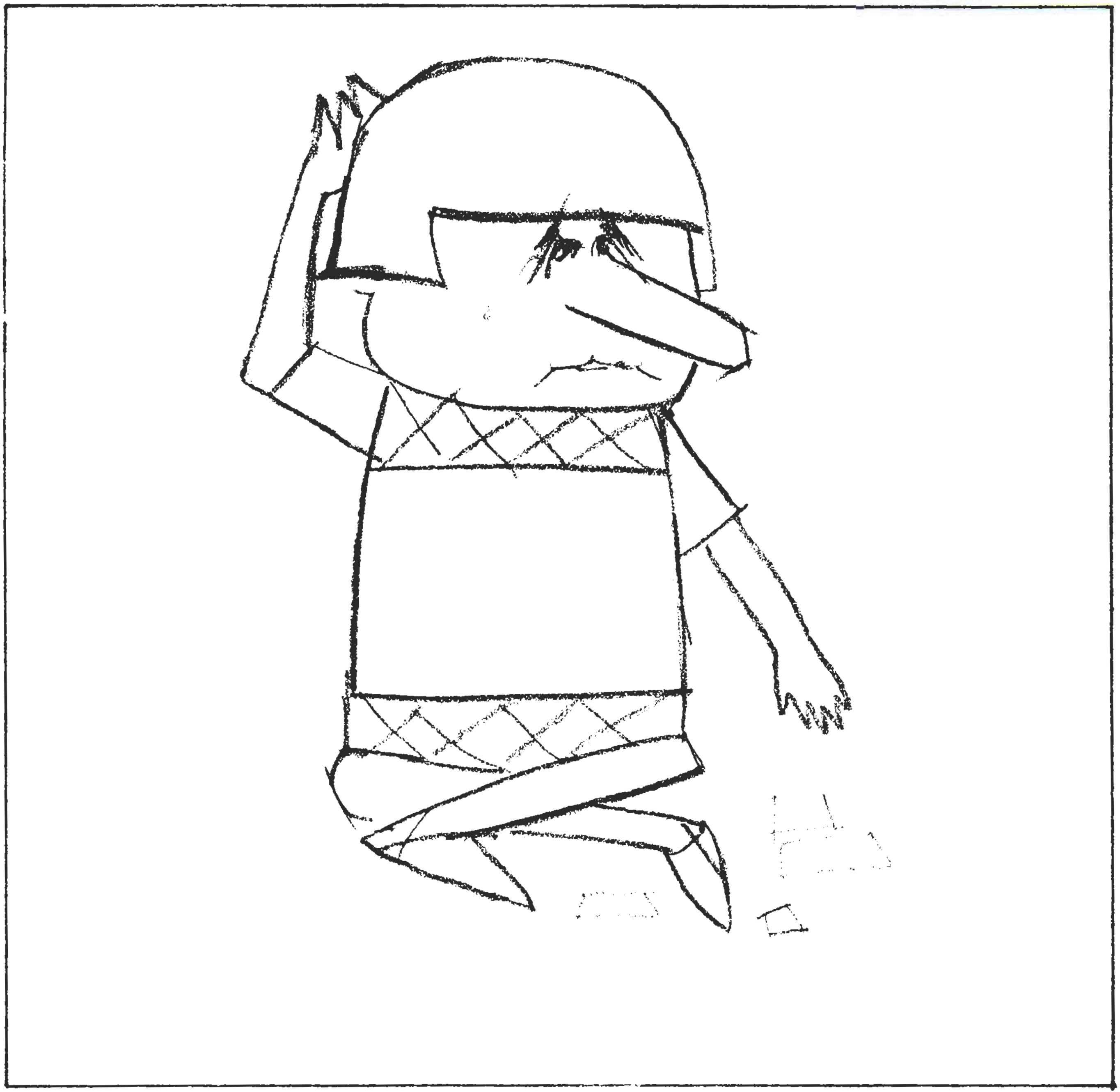


The tax collector gave Ahmes a picture of his field, a picture of Tehuti's field, and a pair of scissors.

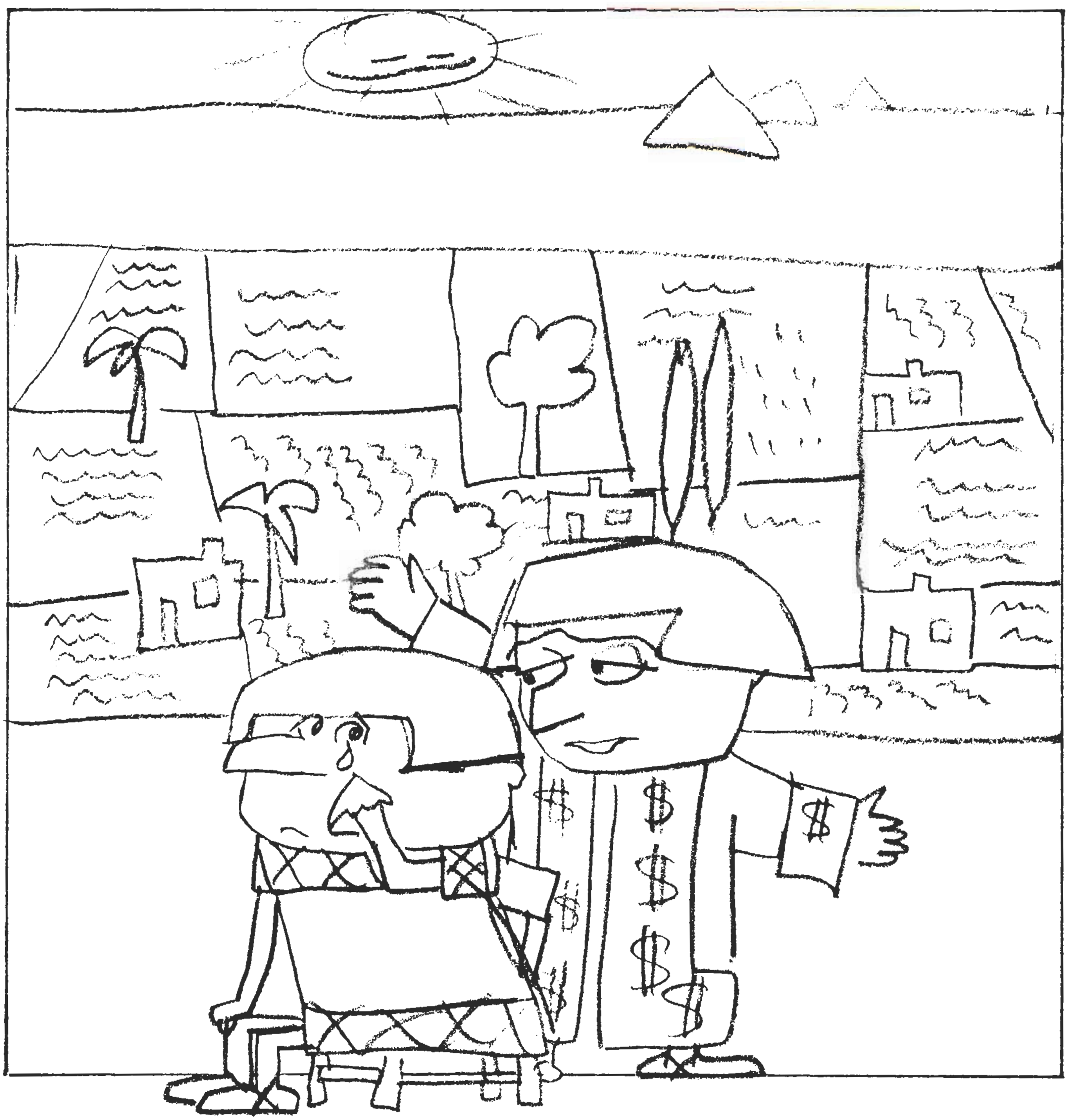
"Ahmes, if you cut Tehuti's field with the scissors and fit the rectangular pieces on your field you will see that your field is bigger than Tehuti's field."



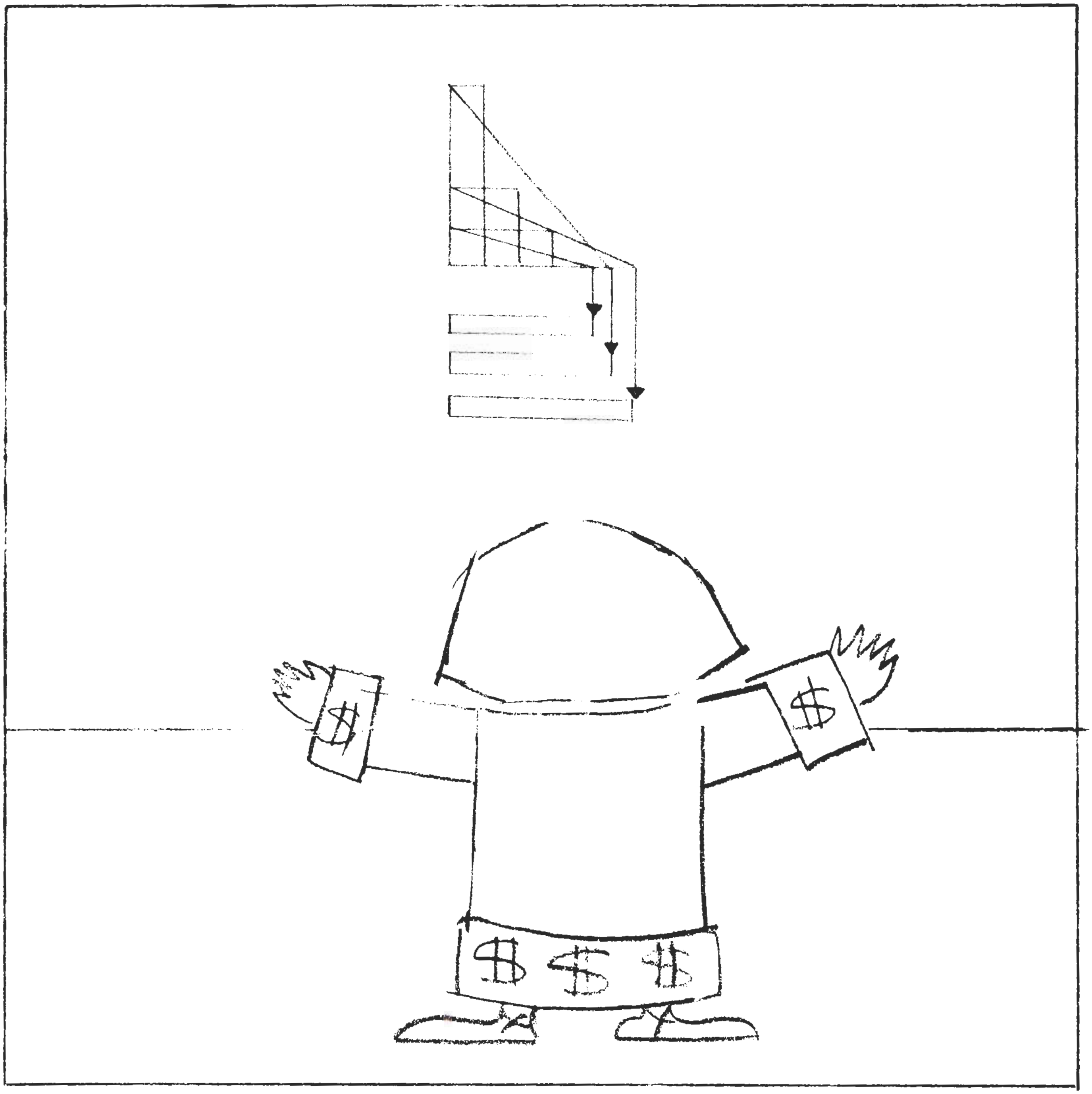
"Here you have a picture of Tepi's field. If you cut his field you will notice that yours is larger. By the way, Ahmes, whose field is larger, Tepi's or Tehuti's?"



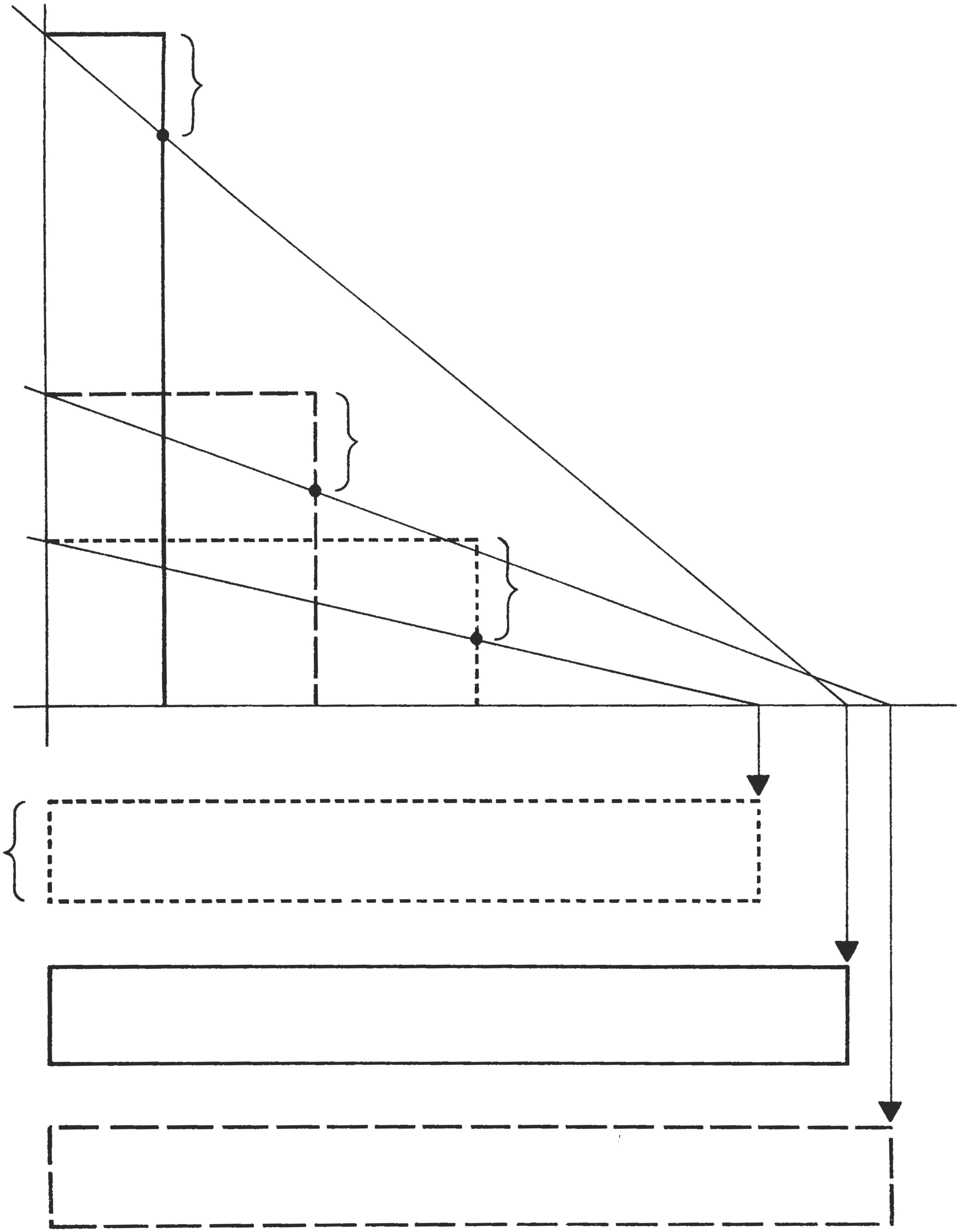
Because Ahmes had cut the pictures of Tepi's field and Tehuti's field he had a hard time finding that Tepi's field was smaller than Tehuti's.

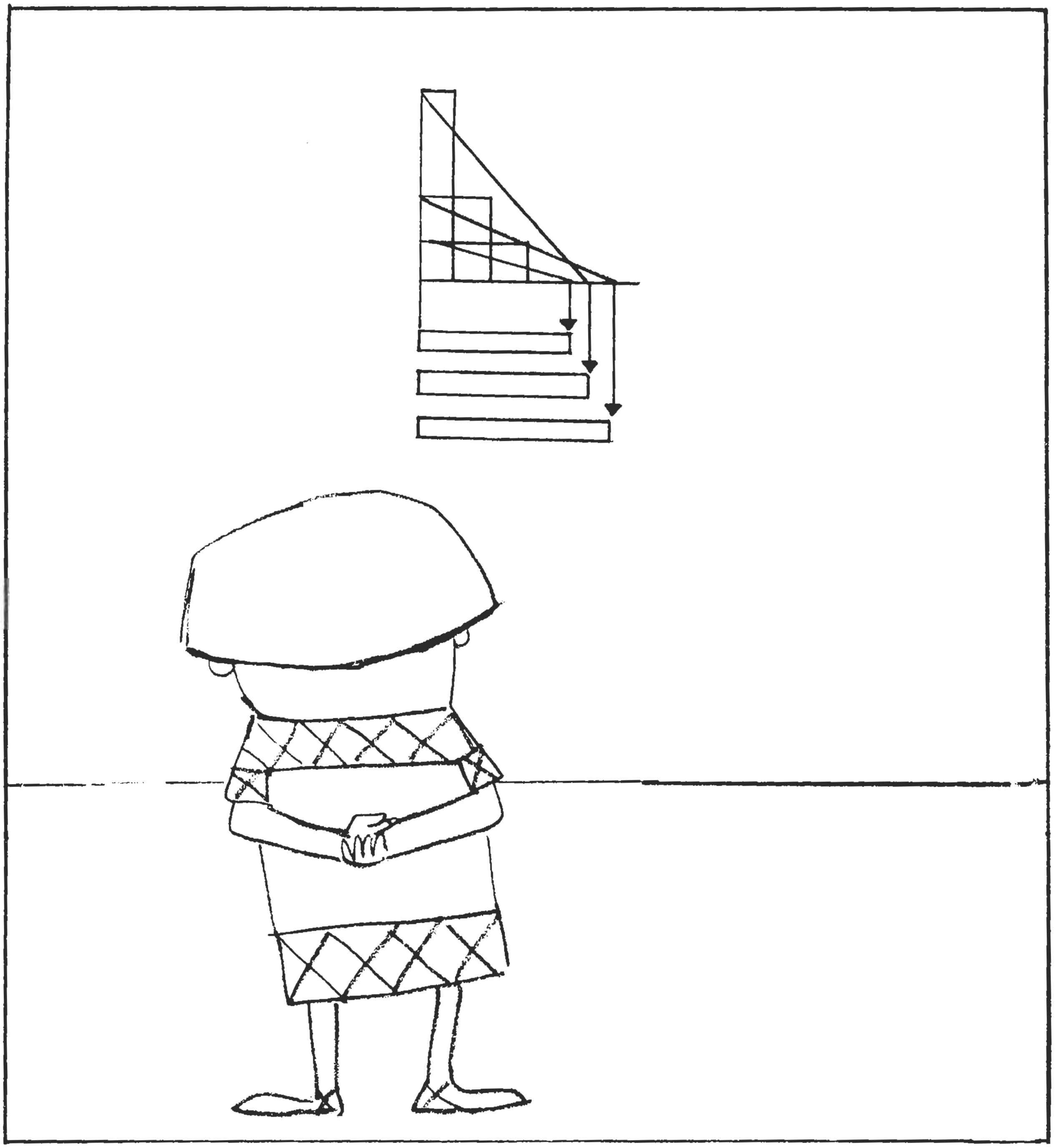


"Now Ahmes you know the problem I have finding how much tax each farmer of Egypt must pay, and Egypt has many farms."

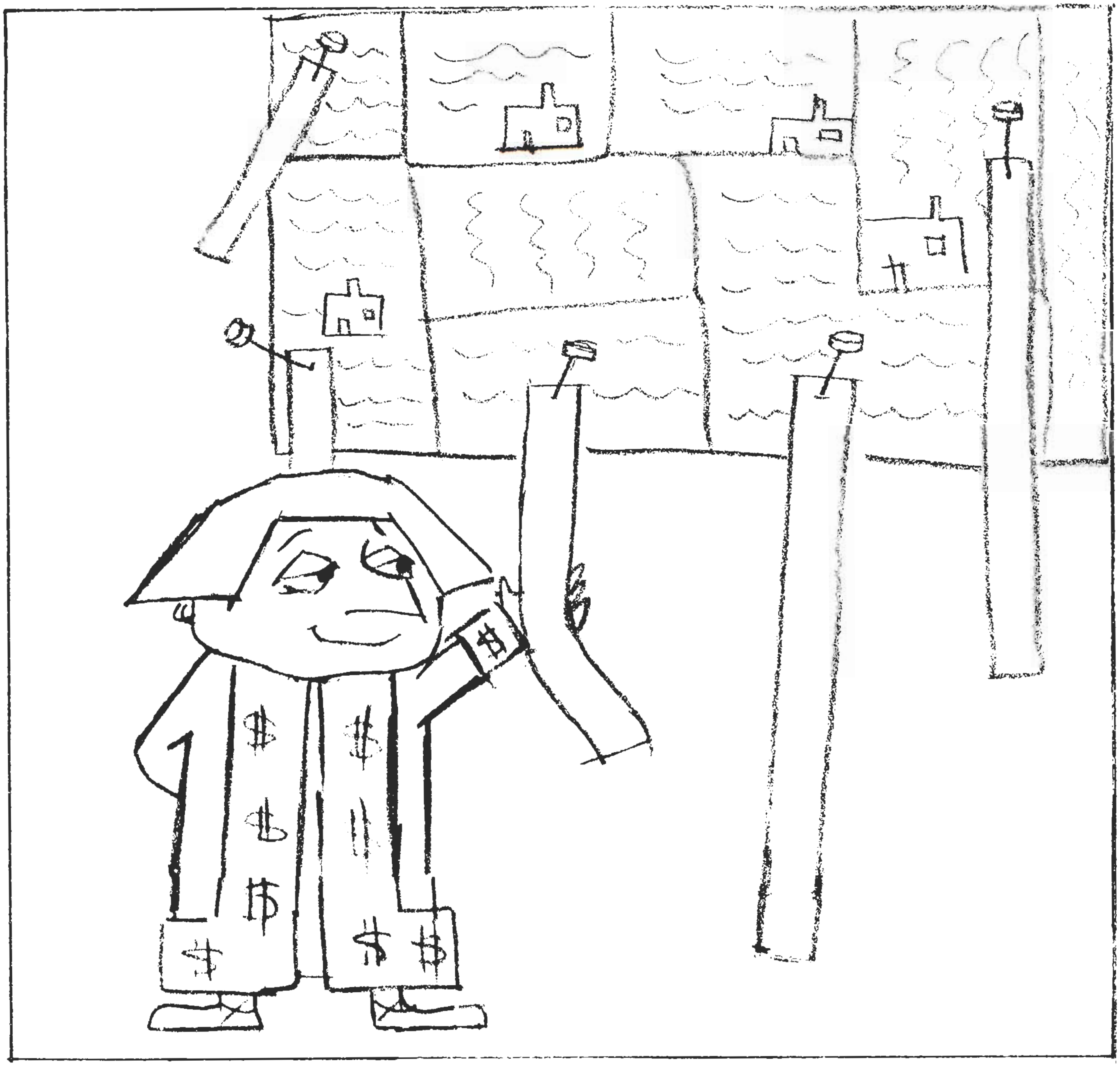


"But I was able to find a simpler method of comparing Egyptian farm fields. Here is a chart showing your field and the fields of your friends. You see how easily I can compare all three fields at the same time."

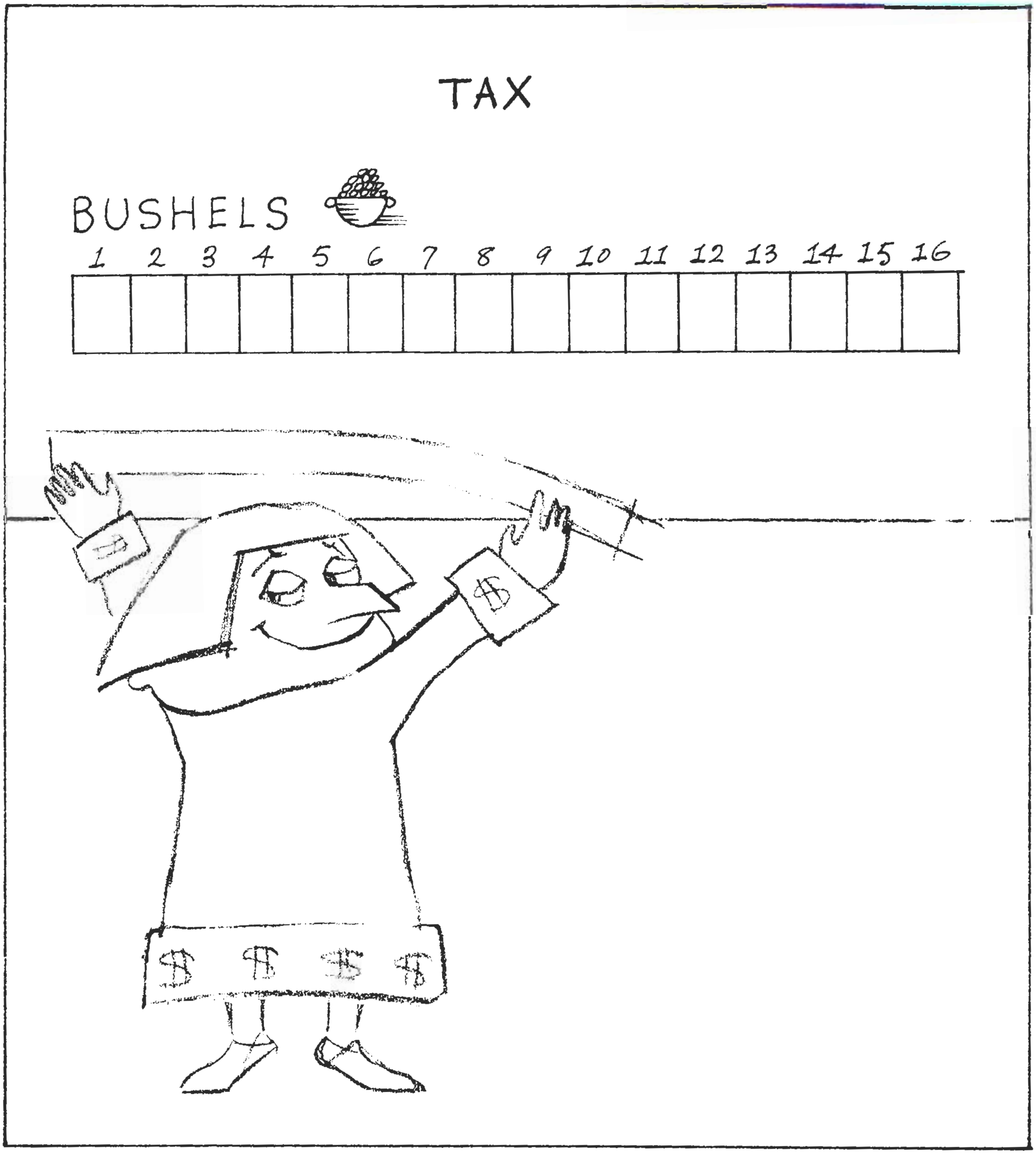




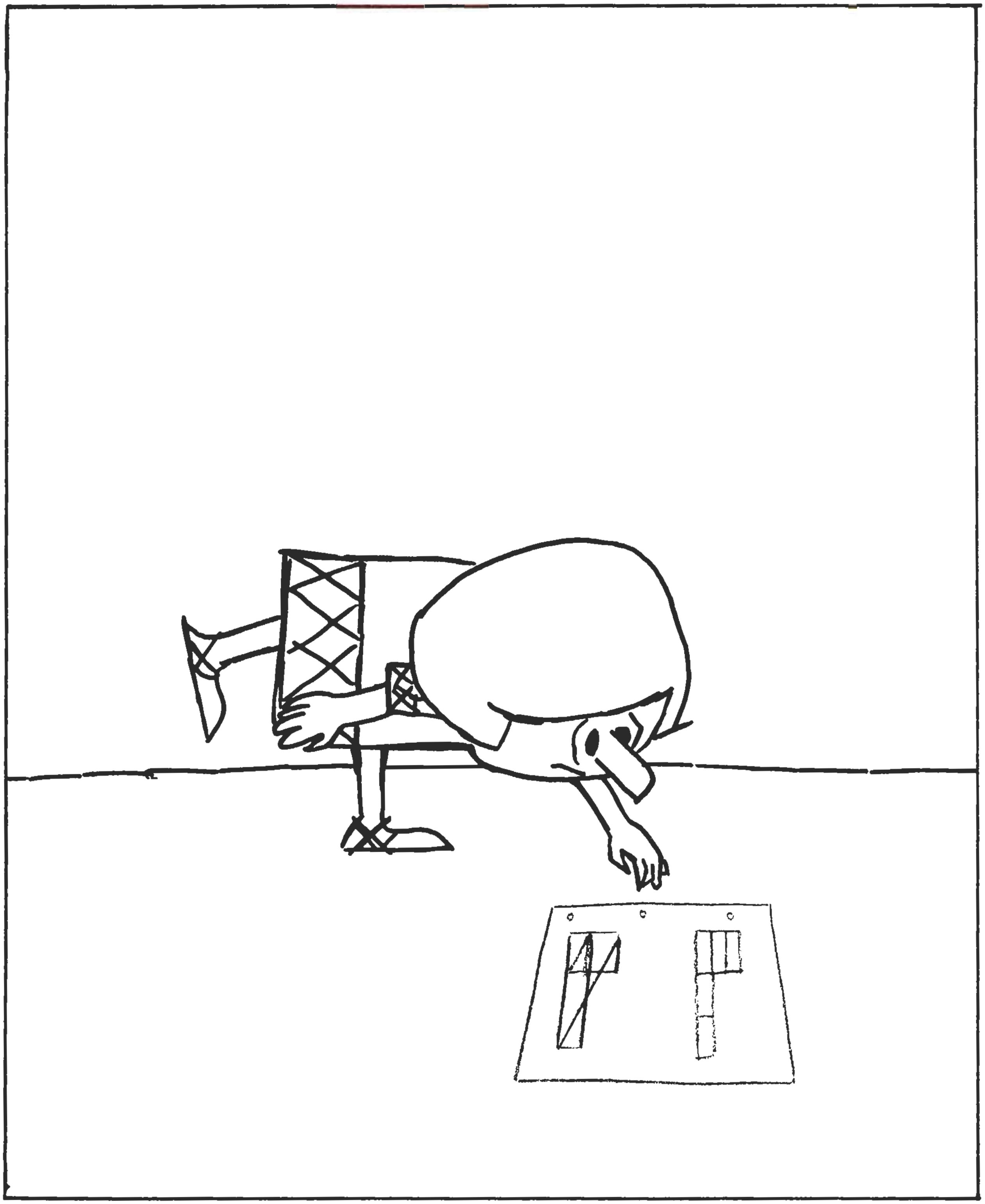
Ahmes was puzzled as to how the tax collector had made his chart.



In fact I keep records like this for all the land in Egypt. I have a strip for each farm. All the strips are the same width and the length of each strip depends on the size of the farm.



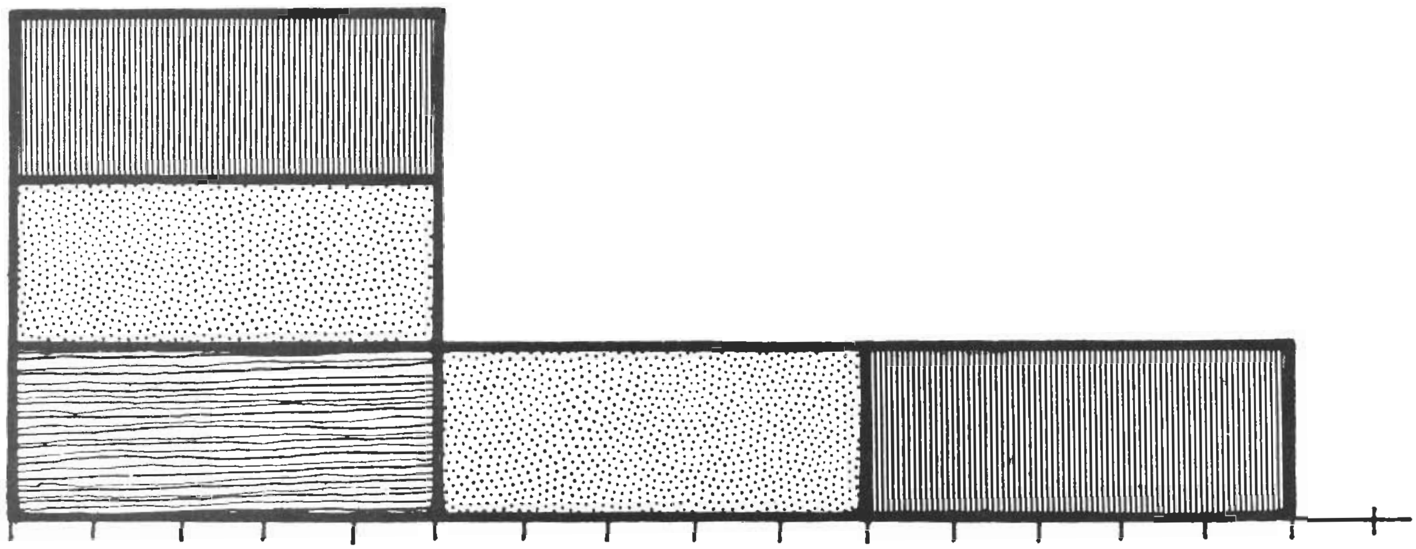
"To check how much tax each Egyptian must pay, I use this chart that tells me the exact tax, Ahmes."



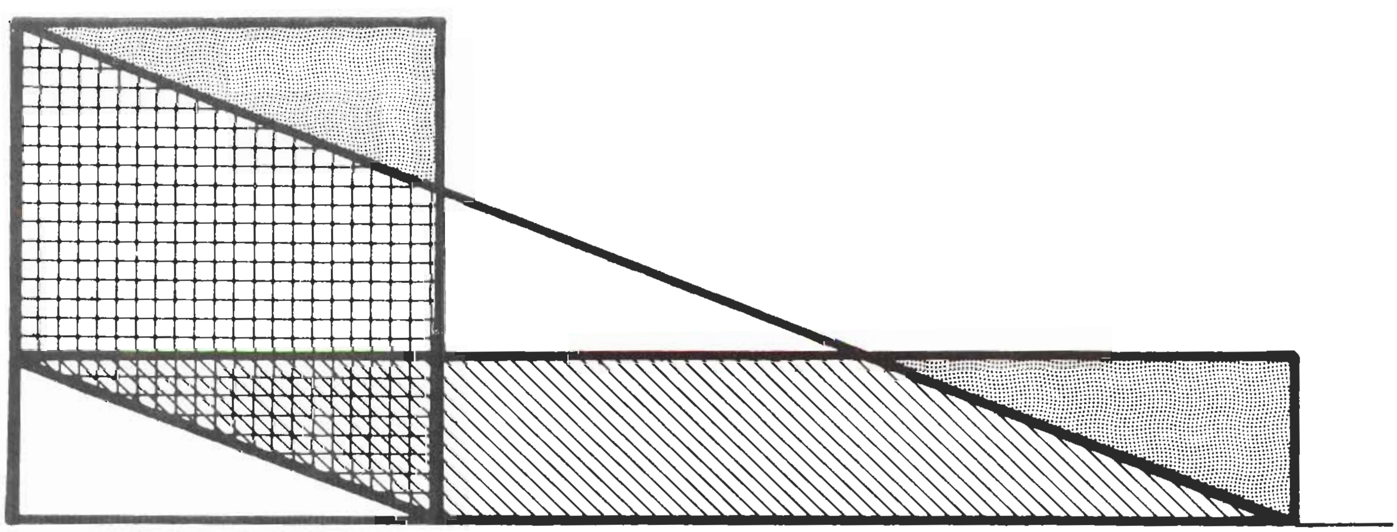
After the tax collector left, Ahmes found a page from his notebook lying on the floor. This page helped him to understand what the tax collector had done.

Two Ways to Get Strips from Rectangles

METHOD #1



METHOD #2



Here is the page from the tax collector's notebook which Ahmes found.

Commentary for Worksheet 9

Have the children look at the diagram from the tax collector's notebook showing Method I. Stress the fact that this is a rectangular region which can be cut exactly into strips of one unit in width.

Have such a rectangular region drawn on paper and cut it into strips of one unit in width. Show the children how these strips can then be placed end to end to make one long strip which is one unit wide.

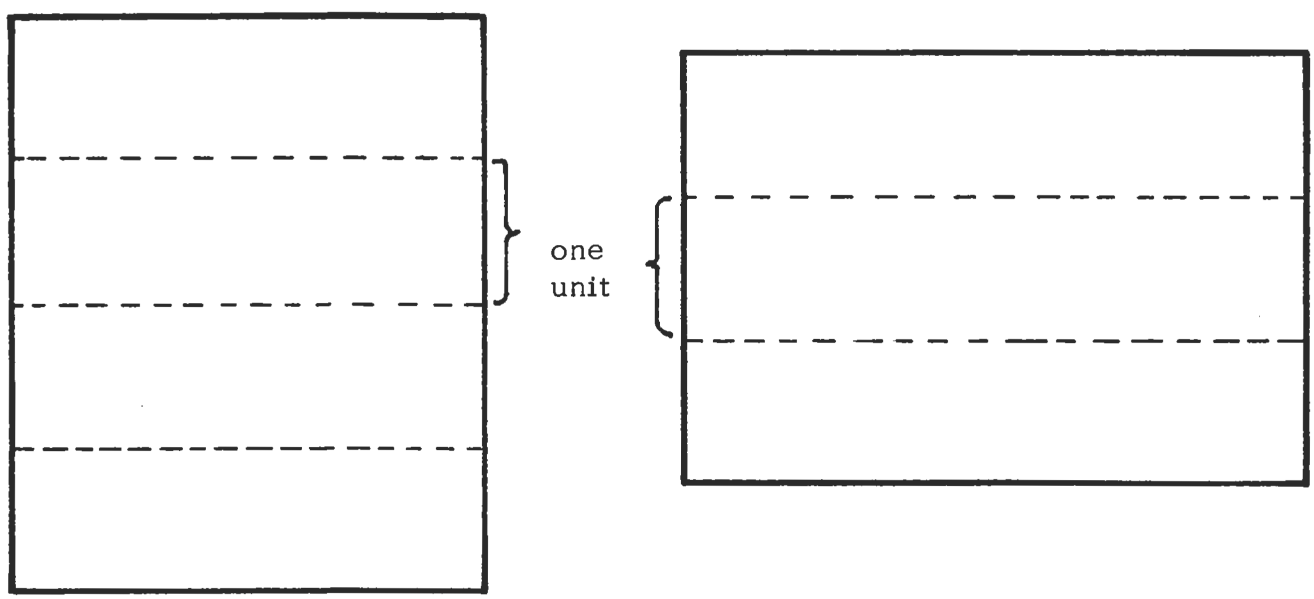
Give the children Worksheet 9 and read the directions to them, pausing long enough to allow them to compare the two rectangular regions and discover they both have the same length and width. That is, they have the same area.

After the children have finished Worksheet 9, stress the fact that it makes no difference which way the rectangular region is cut into unit strips. The total length of the strips will be the same. Thus the children should find both unit strips in this exercise are the same length.

Worksheet 9 Name_______________

Use your ruler to measure the length and width of these two rectangular regions. What can you say about these two figures?

Color one of the two rectangular regions and then cut each of them into strips by cutting along the dotted lines. When you have each rectangular region cut into strips, make one long strip out of it by placing the small strips end to end. How do these strips compare with each other?



Commentary for Worksheets 10 - 12

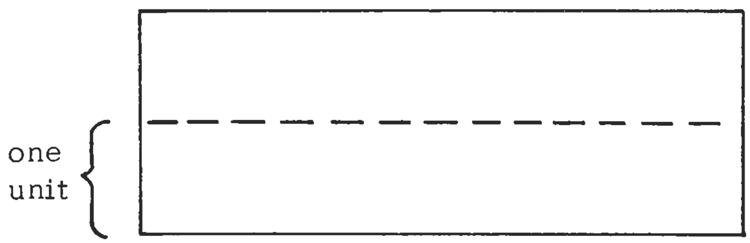
In the exercises which follow, the teacher should read the directions to the children for each of the worksheets through number 20 to make certain the children understand exactly what to do. Worksheets 21 to 27 are included as a challenge for the more able children and you should not expect everyone to be able to complete them all. The children should be allowed to work on these at their own pace and you should give only a minimum of help with directions.

Make sure the children color the rectangular regions in each exercise before they cut them into strips so the pieces won't get mixed up. On Worksheet 10 the children are asked to tell which rectangular region is largest and which is the smallest. Rectangular region III, the green one, is the largest, but the other two are the same area, so this should provoke some discussion.

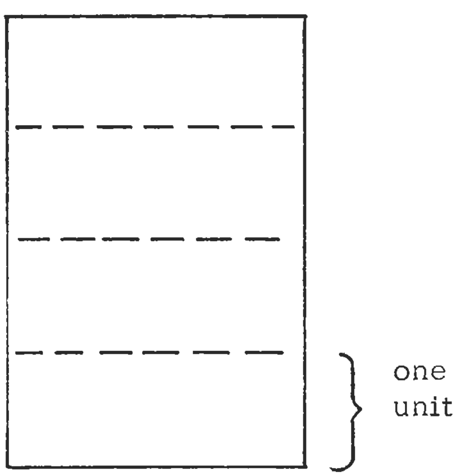
Worksheet 10 Name____________________

Lightly color each of the rectangular regions. The colors are stated above each region. Then cut them into strips along the dotted lines. Make one long strip for each of the rectangular regions. Which colored rectangular region is the largest? __________________ Which color is the smallest? ____________________

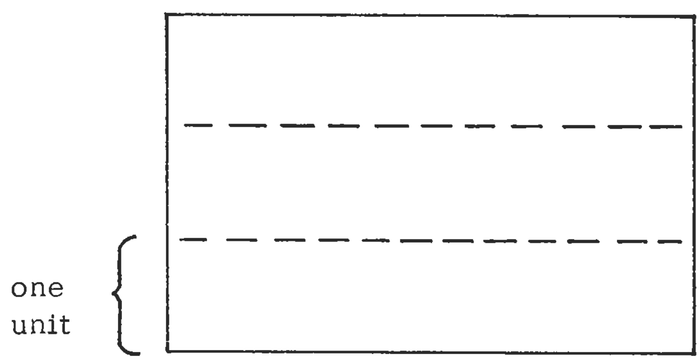
I Color this red.



II Color this yellow.



III Color this green.



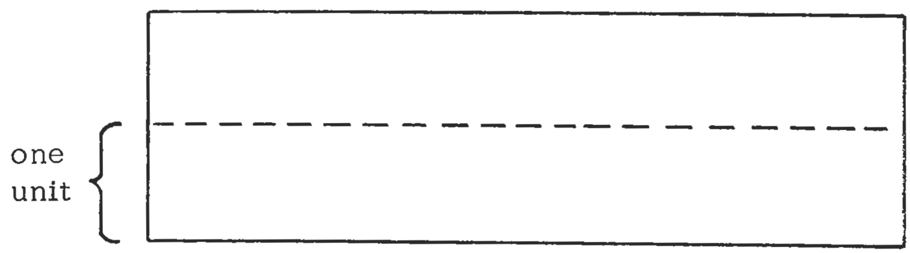
Worksheet II | Name________________

Look at these three rectangular regions and see if you can guess which is the largest rectangular region. Record your guess here_________.

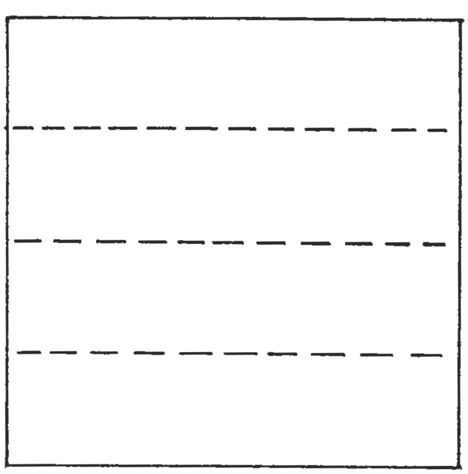
Color them as directed and cut them into strips along the dotted lines.

Make one long strip for each rectangular region as you did in the last exercise and compare them. See if you guessed correctly.

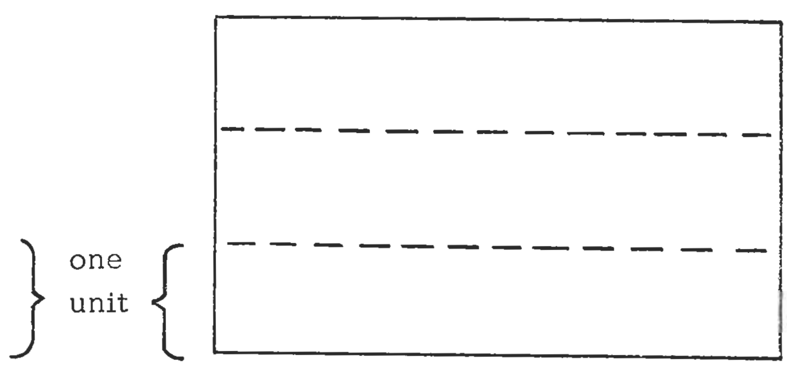
I Color this orange.



II Color this blue.



III Color this red.



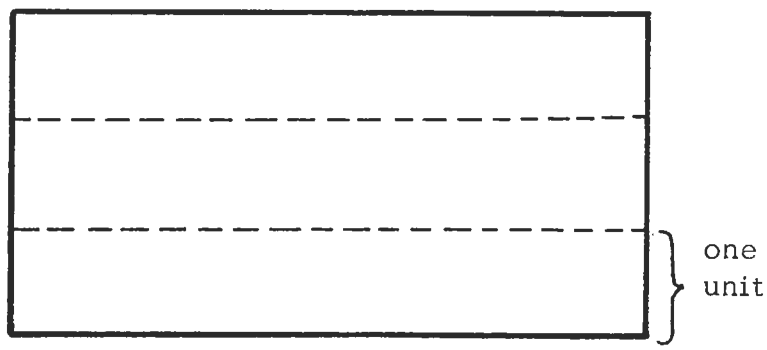
Worksheet 12 Name____________________

Look at these three rectangular regions and see if you can guess which is the smallest one. Record your guess here____________.

Color them as directed and make one long strip out of each one as you have done in the other exercises. Then compare the long strips and see if you guessed correctly .

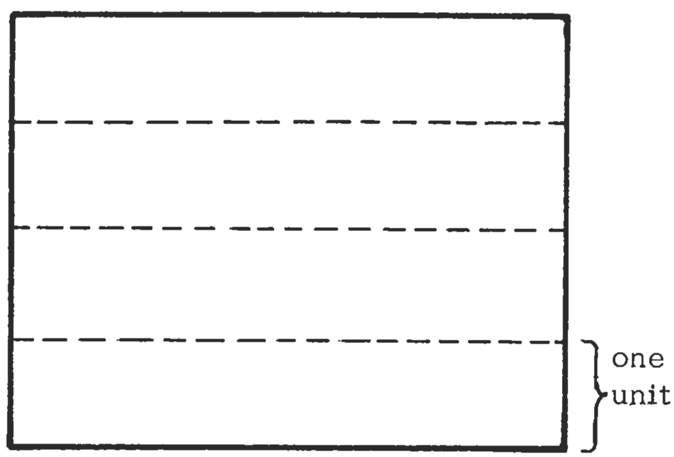
Color this green.

I



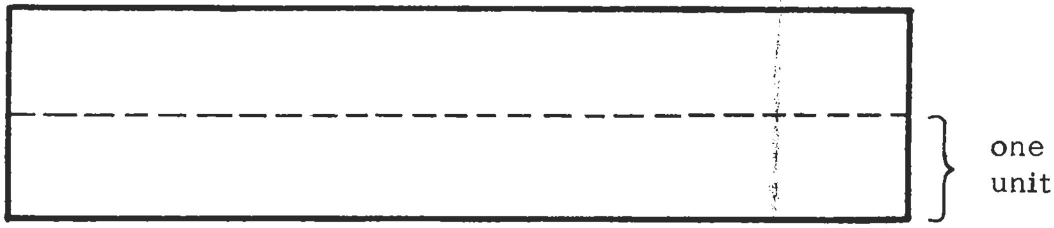
Color this blue.

II



Color this yellow.

III



Commentary for Worksheet 13 and 14

Worksheets 13 and 14 are similar to Worksheets 10, 11 and 12, but you should remind the children of the fact that it makes no difference whether the strips are cut horizontally or vertically. If the children have difficulty handling the smaller pieces, it may be helpful to use glue or paste when they are making the unit strip.

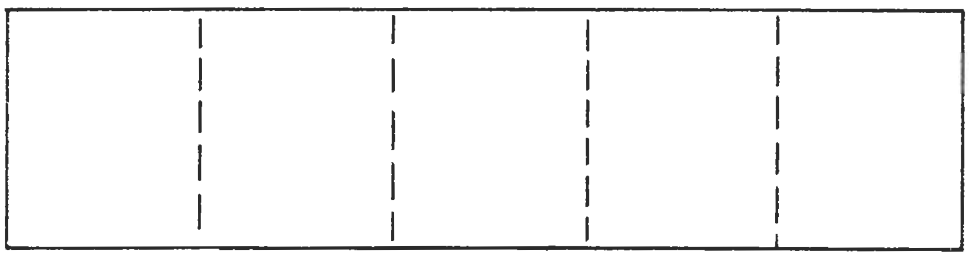
Worksheet 13 Name________________________

Look at these three rectangular regions and see if you can guess which is the largest one. Record your guess here__________.

Color them as directed and make one long unit strip out of each rectangular region as you have done in other exercises. Compare the long strips and see if your guess was right.

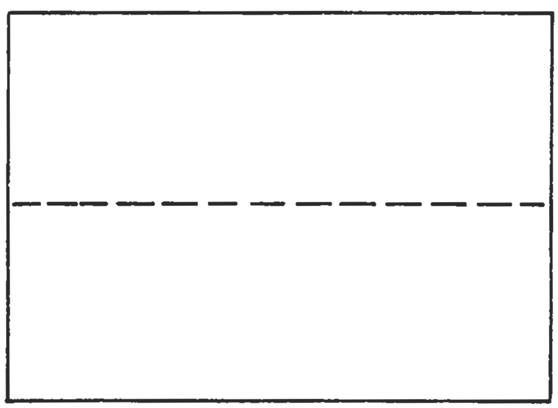
Color this red.

I



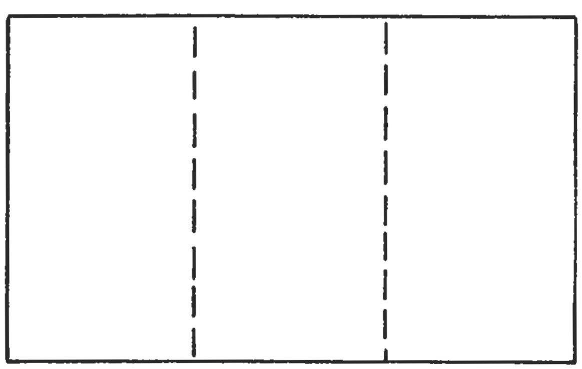
Color this blue.

II



Color this yellow.

III

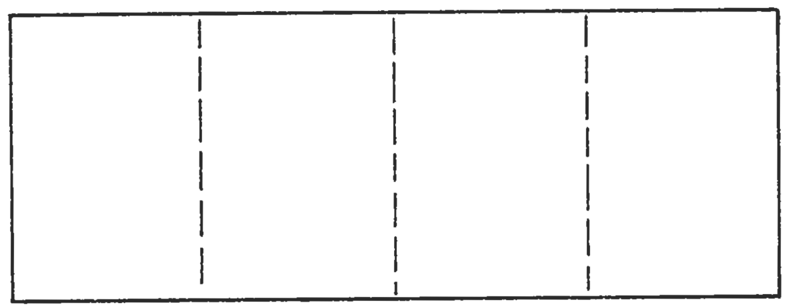


Worksheet 14 Name________________

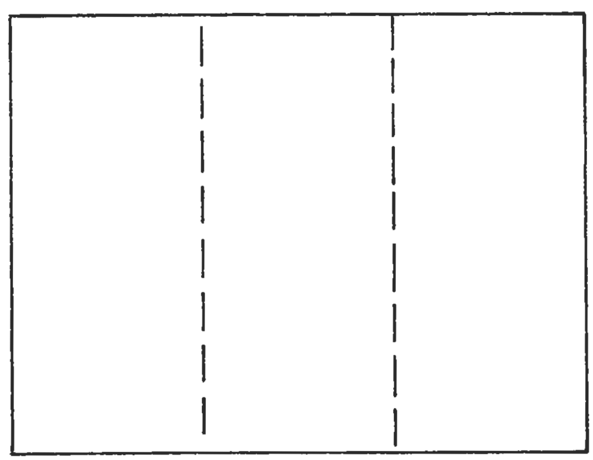
Look at these three rectangular regions and see if you can guess which is the smallest one. Record your guess here______________.

Color them as directed and make one long unit strip out of each rectangular region as you have done in other exercises. Compare the long strips and see if your guess was right.

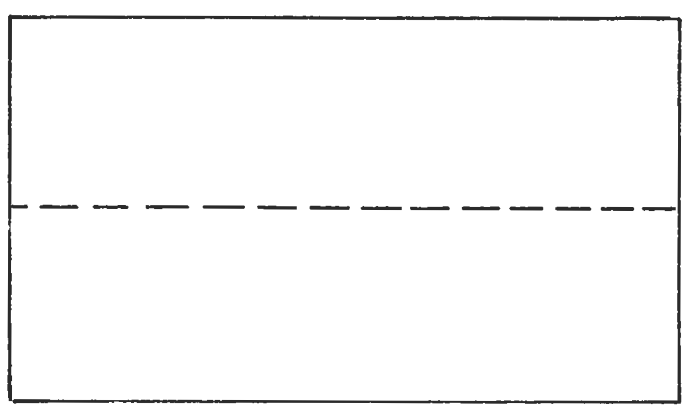
I Color this orange



II Color this green



III Color this blue



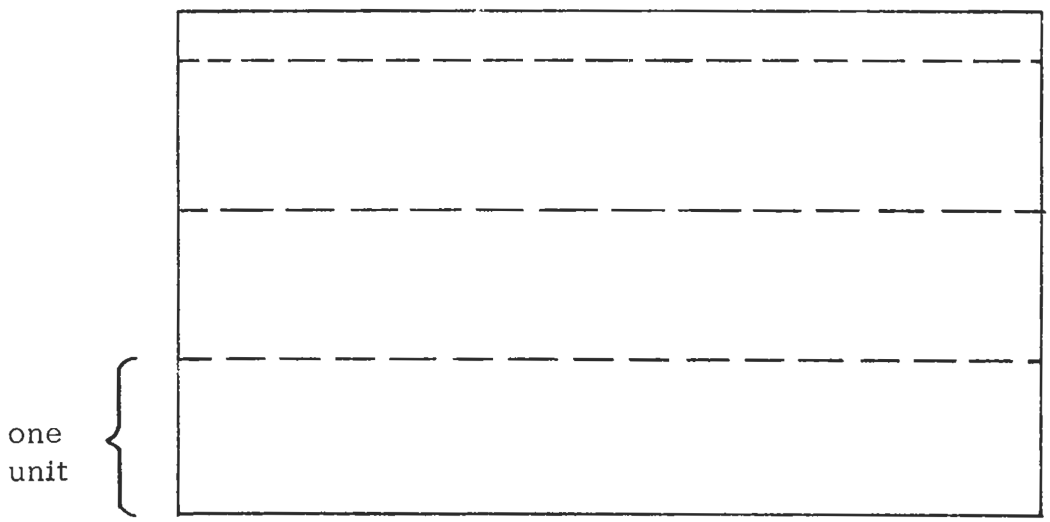
Commentary for Worksheet 15

Hand out Worksheet 15a to the class and read it to them. Then pass out Worksheet 15b and explain that there are four rectangular regions made up of two or more unit strips. If each rectangular region were cut into strips and the strips were placed end to end they would make the long unit strip shown in the drawing. Each rectangular region ABCD thus has the same area as each unit strip AFGH.

Have the children follow the directions on the Worksheet 15b. If no observes the fact that DF passes through point "E" in all cases, guide the children's thoughts in this direction in order to allow them to make this discovery. This is the key to the construction which follows, of a unit strip for a given rectangular region.

Worksheet 15a　　　　　　　　　　Name____________________

We have found a way to compare areas of rectangular regions by cutting them into strips of the same width. All the rectangular regions we have compared so far were of such a size that it was possible for us to cut them exactly into strips of one unit width. Look at this rectangular region:



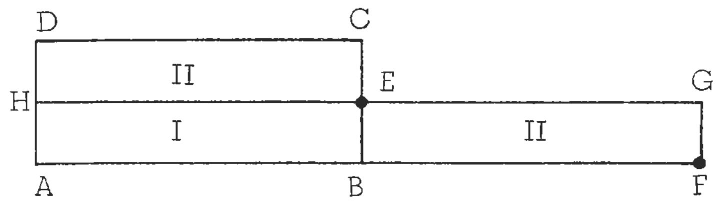
Are all these strips the same width? What are we going to do with the top strip?

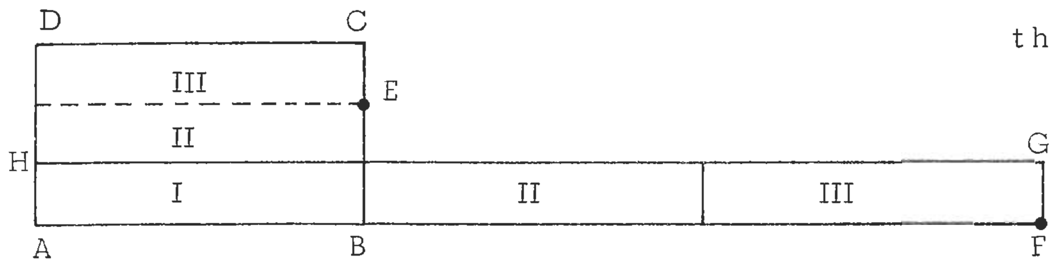
The tax collector knew how to change any rectangular region into a strip that was one unit in width. This was the secret hidden in Method 2 in his notebook. Let's see if we can find out how he did it.

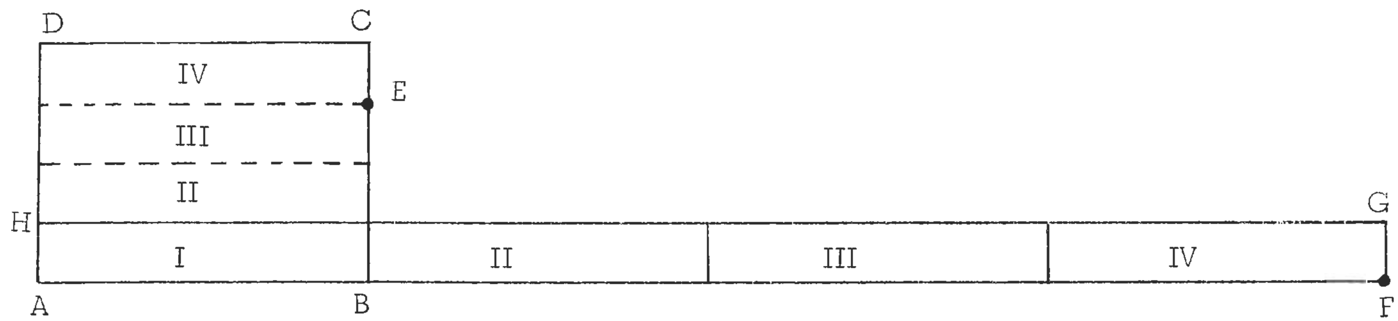
Worksheet 15b

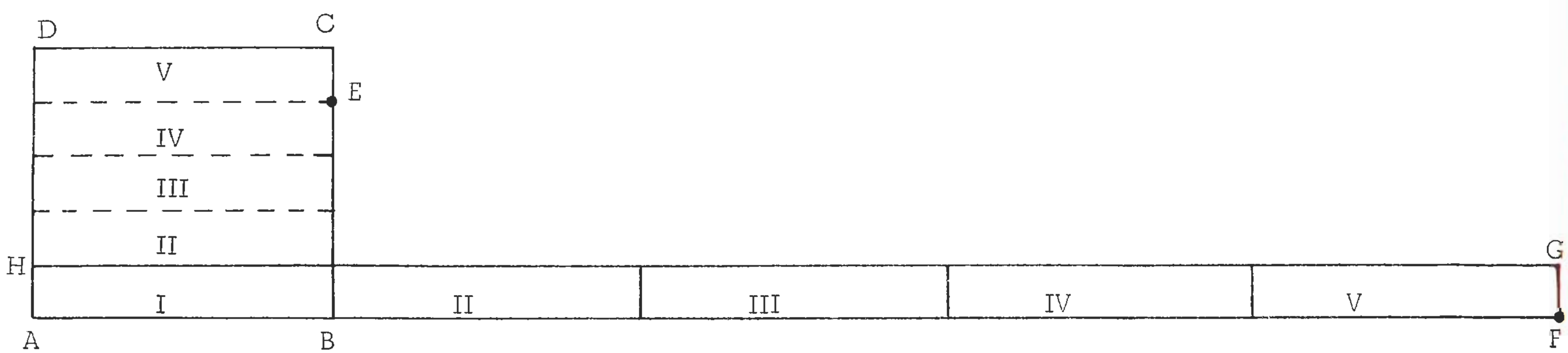
Name ____________________

Place your ruler on each of the drawings so that the straight edge passes through points D and F. Do you notice anything about each of these figures when you connect points D and F? What have you discovered?









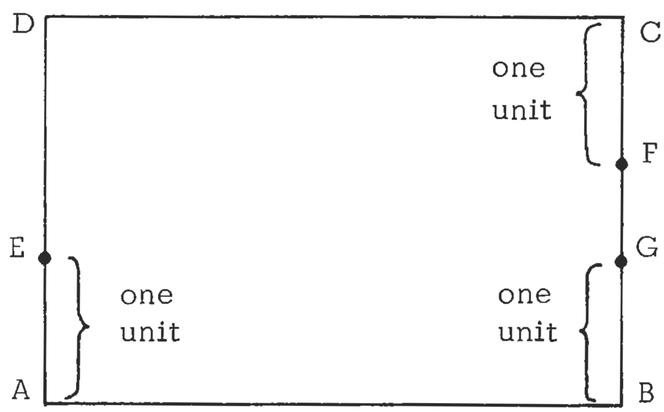
Activity on the Tax Collector's Method

This construction should be done on the board and explained to the children as you proceed. Remind them of the discovery they just made on Worksheet 15 and show how this knowledge helps us to find the correct length of the unit strip.

Directions:

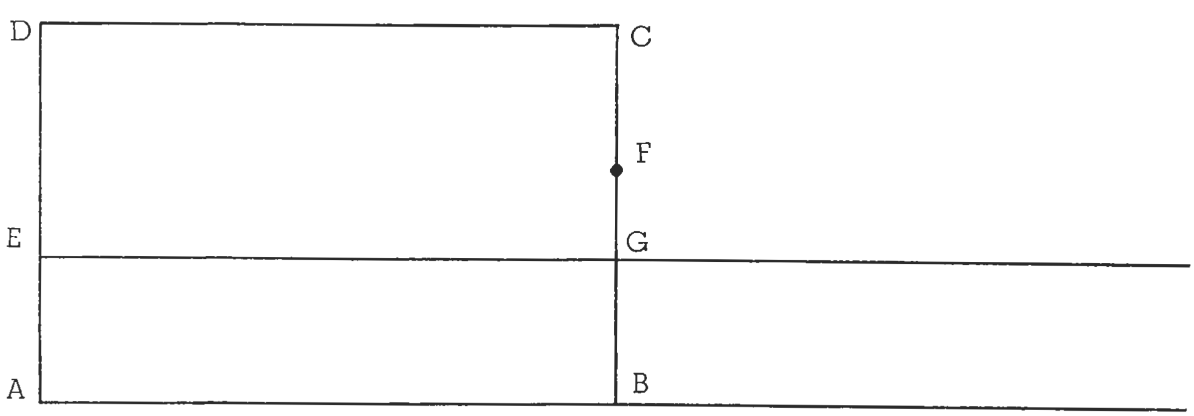
I. A. On $\overline{AD}$ lay off $\overline{AE}$ equal to one unit length.

B. On $\overline{BC}$ lay off $\overline{CF}$ and $\overline{BG}$ each equal to one unit length.



II. A. Through points "E" and "G" draw a line segment, extend the line segment as long as you can.

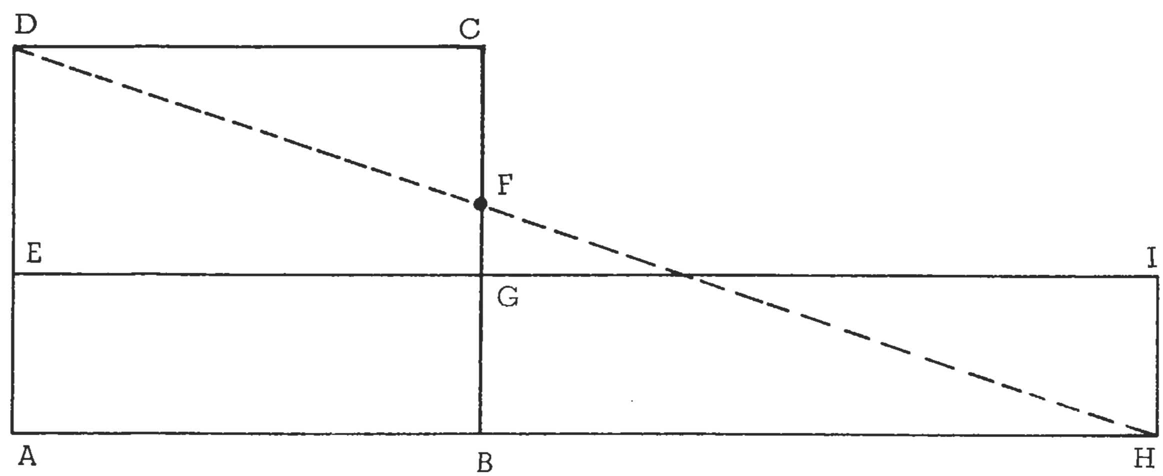
B. Through points "A" and "B" draw a line segment, extend the line segment as long as you can.



Now you have a strip one unit in width. How long should it be? The answer to this question can be found in part III.

III. A. To get the correct length for the unit strip use your ruler to connect points "D" and "F" and continue this line until it intersects AB at point H.

B. $\overline{AH}$ is the right length for the strip. On EG measure the same length as $\overline{AH}$ and mark that point "I." Connect "H" and "I."



We now have the strip, AHIE, which has the same area as the rectangular region, ABCD.

Commentary for Worksheets 16 and 17

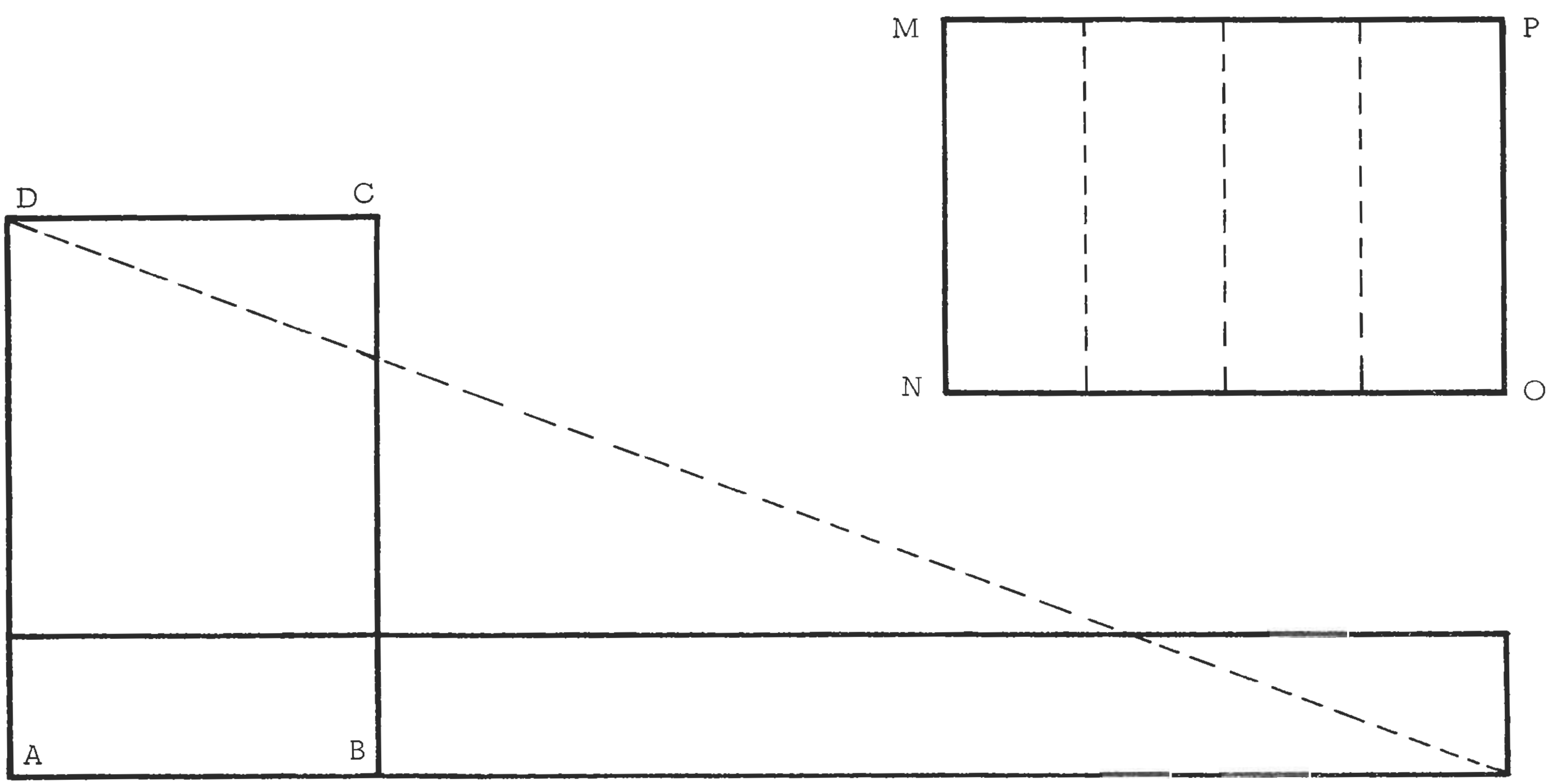
Worksheets 16 and 17 provide a chance for the child to satisfy himself that this construction gives him a strip of the proper length. When he places the strips, which he has cut, on the strip which has been constructed, he should have physical proof that the tax collector's secret was a good one.

Worksheet 16

Name____________________

Here is a rectangular region, MNOP, like the ones we were able to cut exactly in the exercise we did before. The rectangular region on the left is exactly the same size. It has been changed to a strip one unit wide using the tax collector's secret way.

Cut out the rectangular region on the right and place it over rectangular region ABCD to be sure they are the same size. Then cut it into strips along the dotted line. Place them end to end on top of the unit strip in the bottom figure and see if they are the same size.



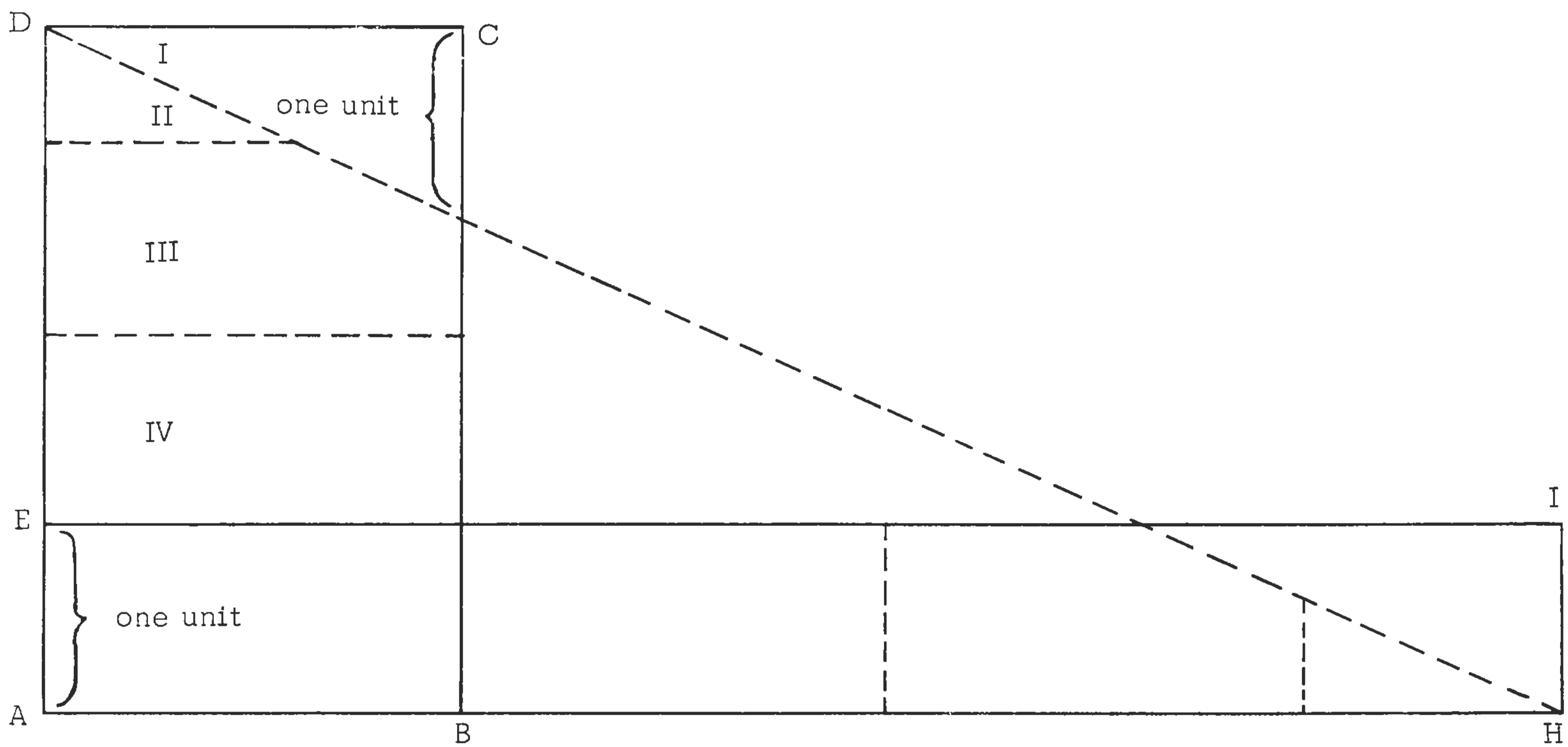
Worksheet 17

Name ____________________

Using the tax collector's method, rectangular region AHIE has been constructed to have the same area as rectangular region ABCD. Notice that the rectangular region ABCD cannot be exactly cut as you did before.

Lightly color the rectangular region ABCD.

Cut out the regions I, II, III, and IV and try to place them so that they exactly cover the unit strip AHIE. If you are very careful you will see that the tax collector's method gives another way of cutting a rectangular region to form a unit strip.



Commentary for Worksheet 18

These worksheets provide the opportunity to strengthen the use of points such as S and T as means of determining the proper length of the unit strip. They offer the chance to remind the children of the fact that they must be very accurate. A line through points S and T must pass exactly through the endpoint of the strip. We have attempted to construct our drawings so there should be no questions to which point is the correct one, but some students may need help with this exercise.

Note that different unit widths have been used and make sure the children are aware of this fact and realize the choice of the unit is arbitrary.

Directions for the Children:

A unit strip for each of these rectangular regions has been drawn. It is your job to find out how long each strip should be.

Notice that one unit width has been marked at the top of each rectangular region at a point marked T. How can you use points S and T to help you find the right length for the strip?

In each of the following figures one of the points marked A, B, C,... marks the correct length of the unit strip. Use your ruler to find the right point in each drawing and draw a circle around it.

Answers

I — "B"

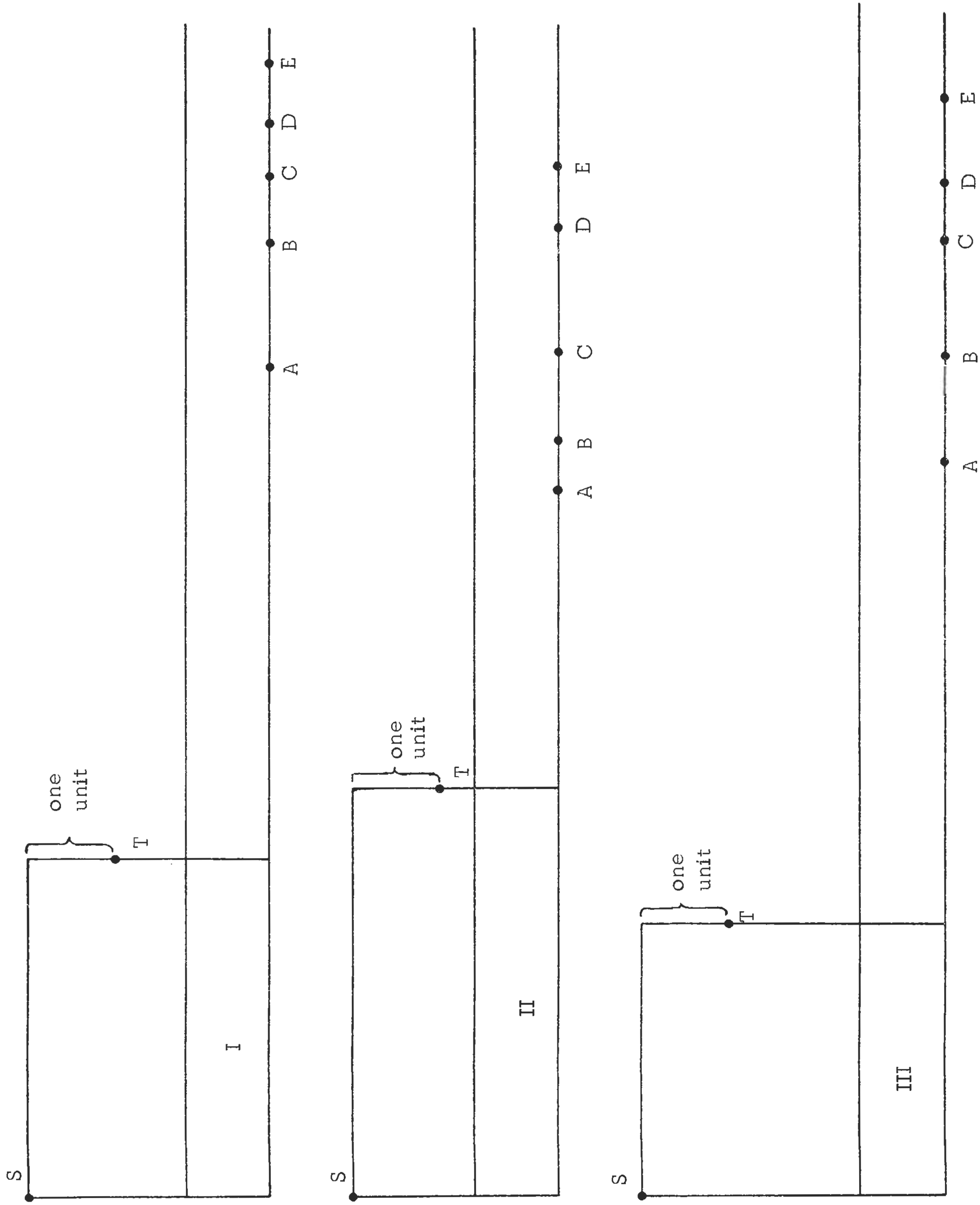
II — "D"

III — "C"

IV — "D"

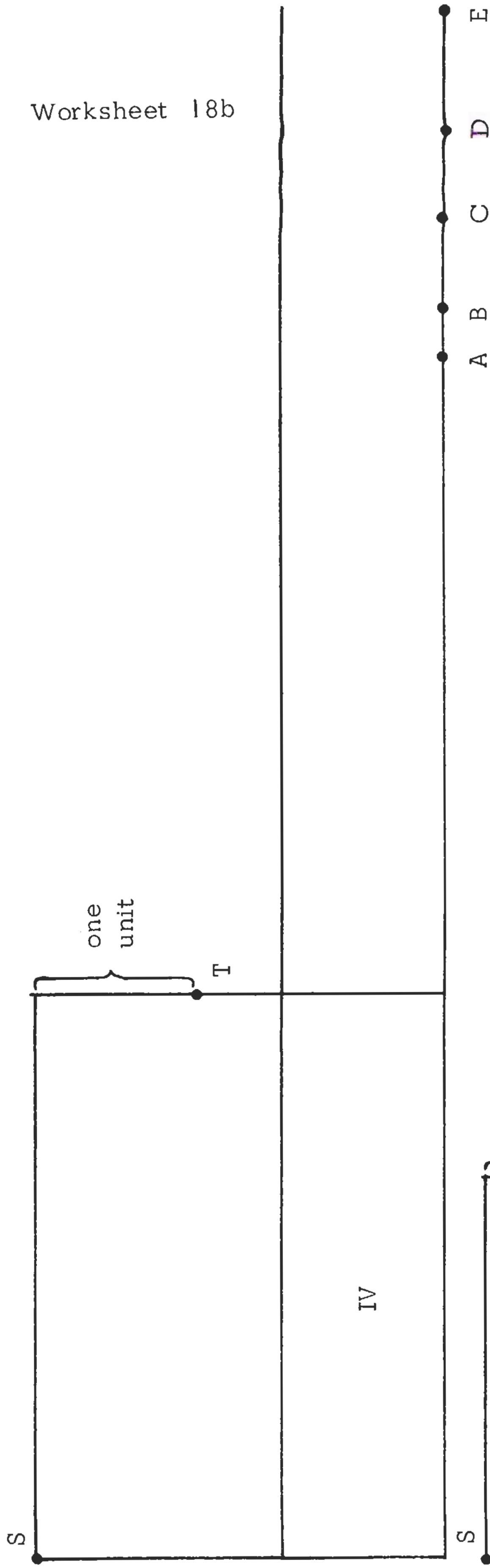
V — "E"

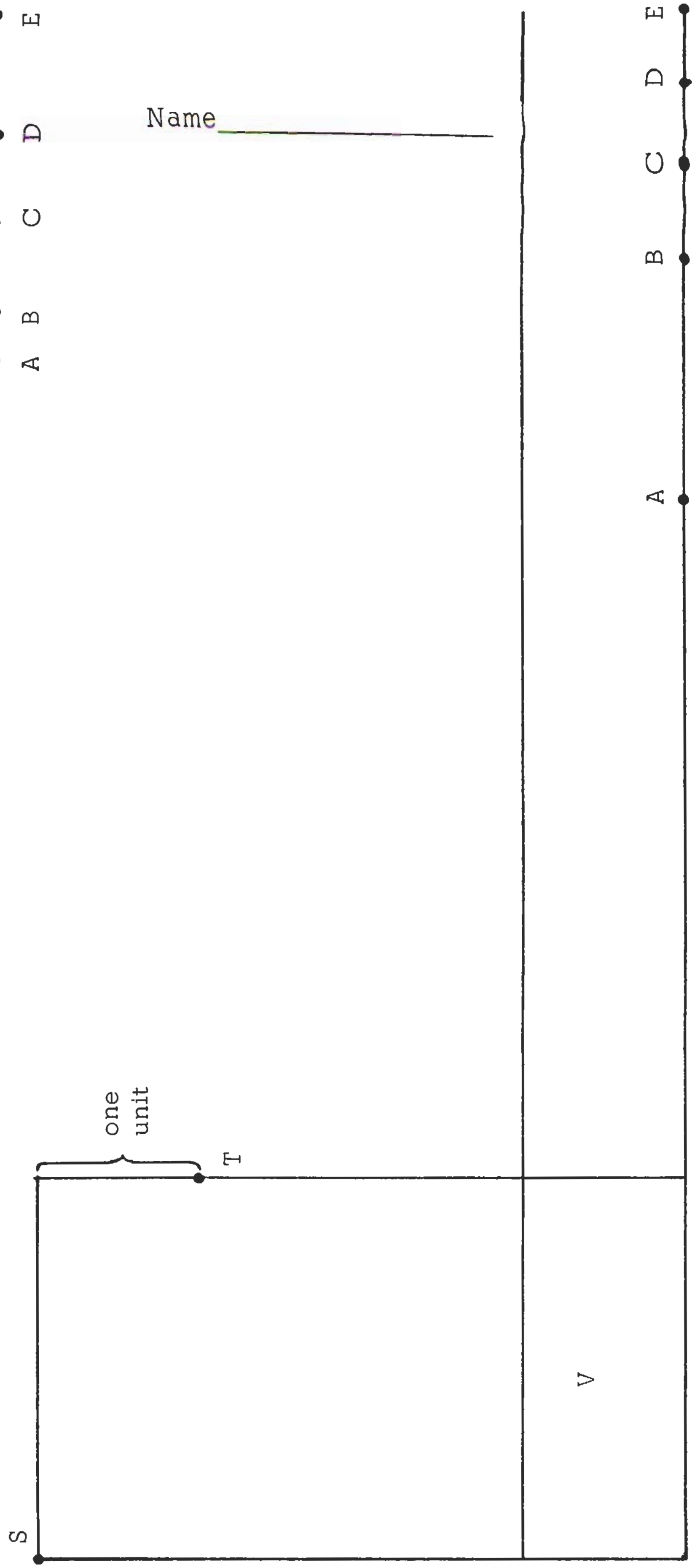
Worksheet 18a Name_______________



Worksheet 18b

Name______________________





Commentary for Worksheet 19

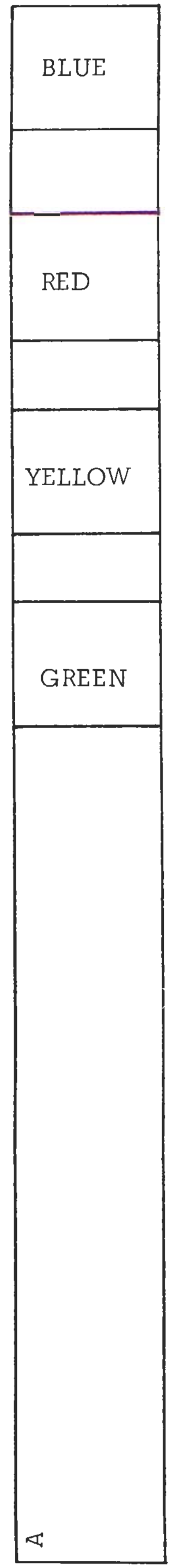
In order to allow the children a margin of error in this exercise since many of them will be unable to measure exactly, an answer strip like the one on the right is provided. After the children have done sheets a,b, and c, give each child sheet d with instructions to fill in the spaces marked with the proper color. They should then cut the strip from the sheet and use it as follows:

For each figure on Worksheet 19 they should lay the answer strip next to the unit strip in the drawing so the corner marked "A" on the answer strip is next to point "A" in the drawing.

If the end of the unit strip which the child has constructed falls in a band of color on the answer strip that is the same as the color of the unit strip, the student has a correct answer.

The teacher should demonstrate how to do this and then let the children check their own work with the answer strips.

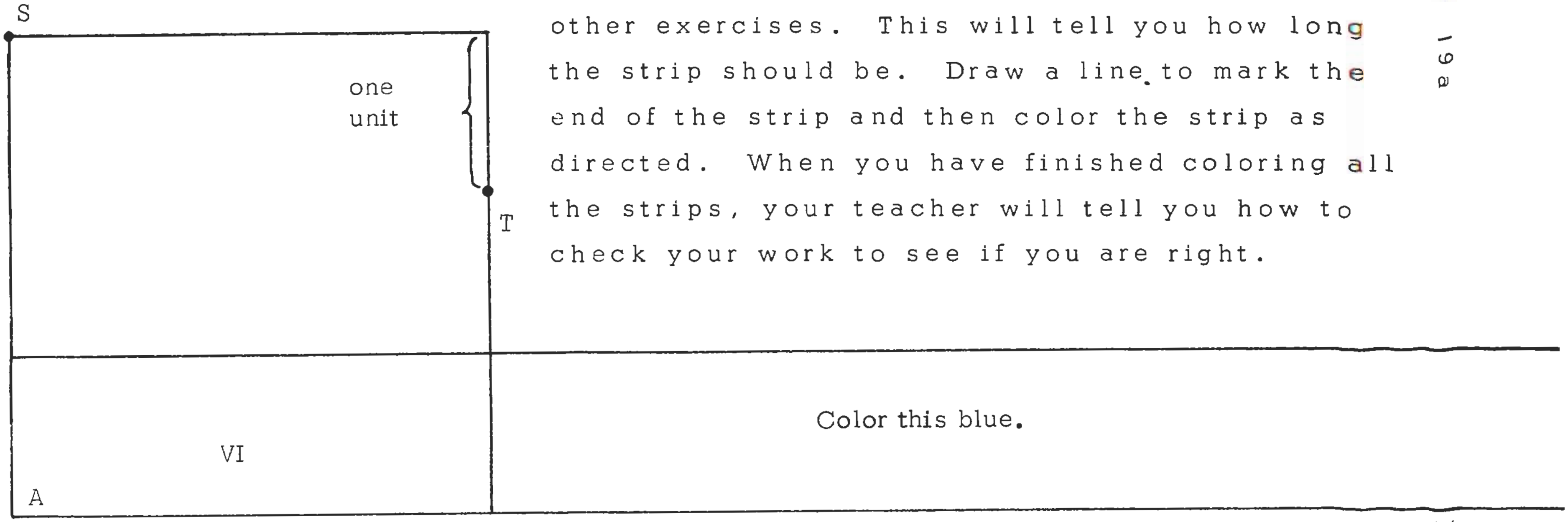
The answer strips should be saved for Worksheet 25.

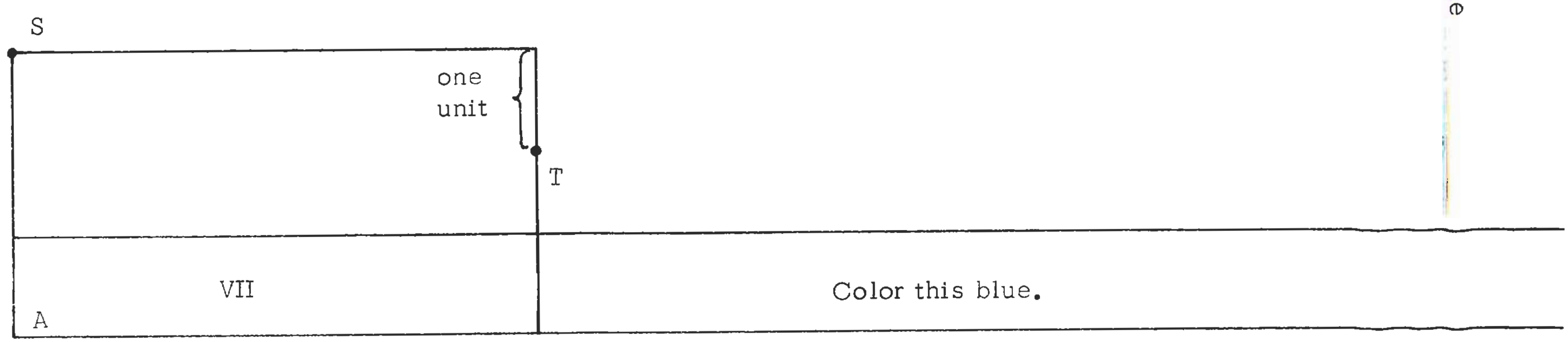


Worksheet 19a

Name

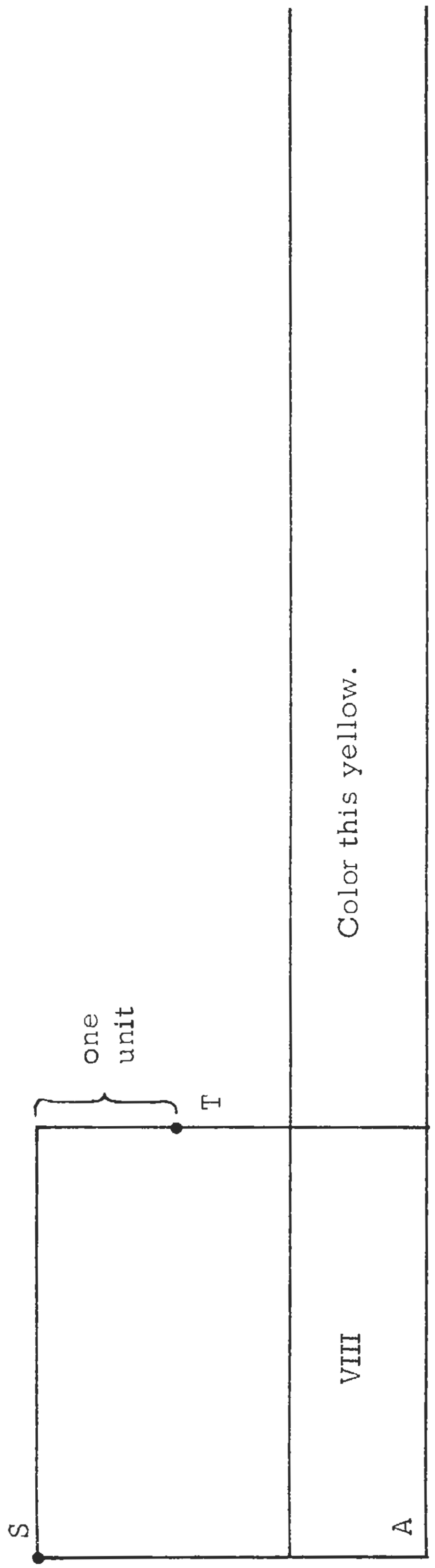
In each of the following drawings you are to use points "S" and "T" to help you find the right length of the unit strip. Mark the point you get by using your ruler as you did in the other exercises. This will tell you how long the strip should be. Draw a line to mark the end of the strip and then color the strip as directed. When you have finished coloring all the strips, your teacher will tell you how to check your work to see if you are right.

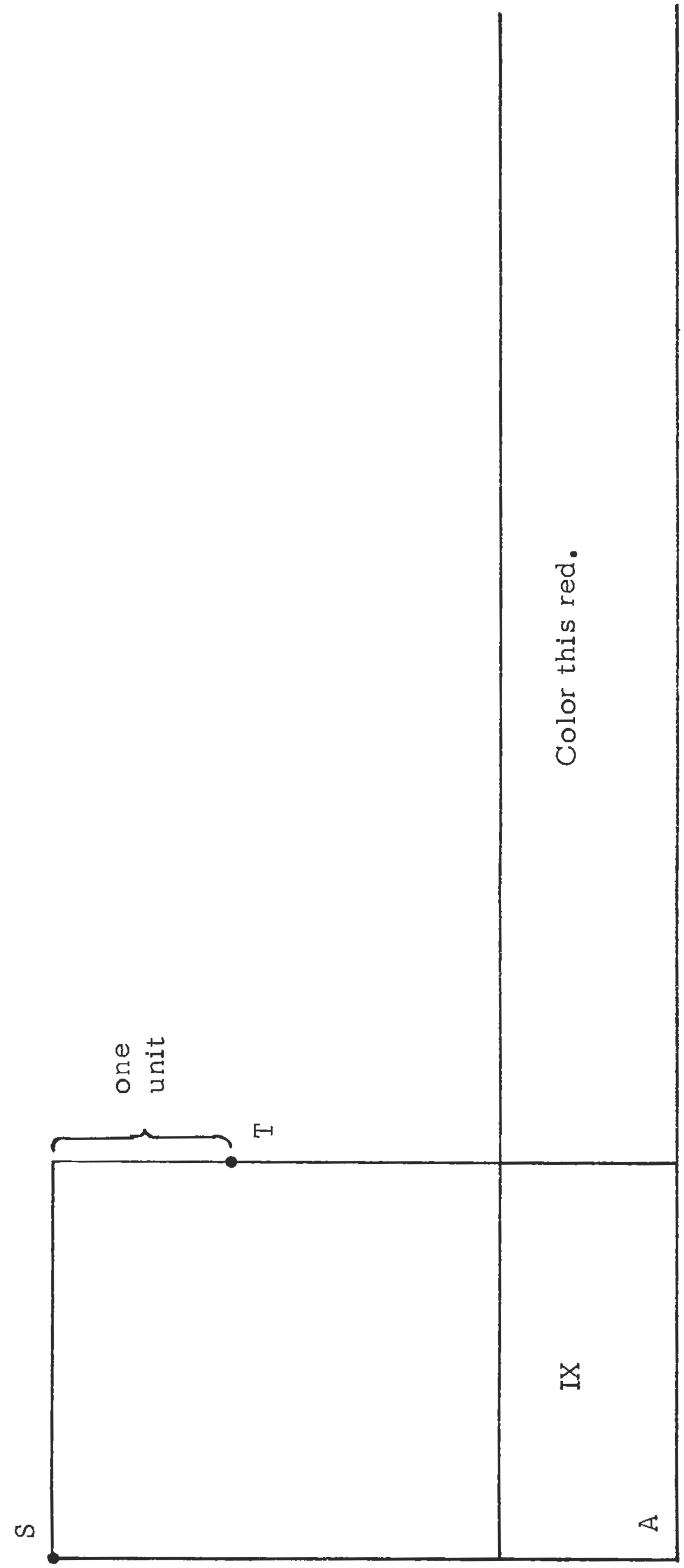




Worksheet 19b

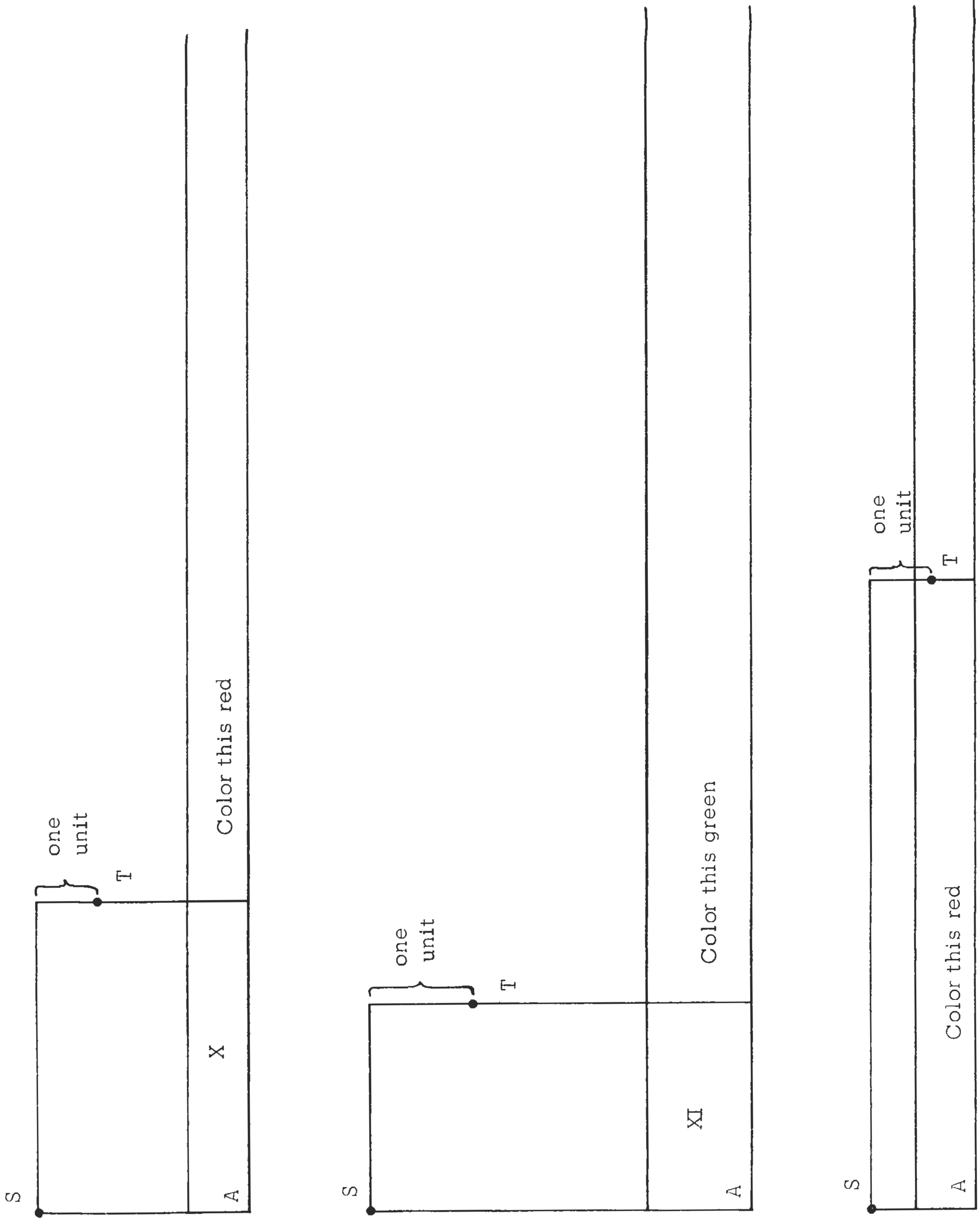
Name ____________





Worksheet 19c

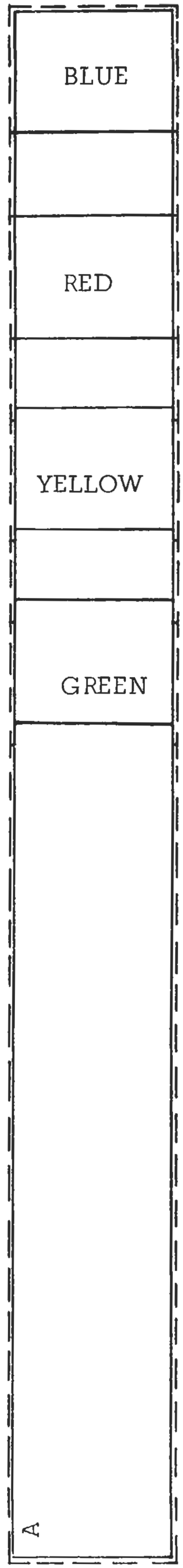
Name______________________



Worksheet 19d

Name ____________________

Color as directed. Then cut out the strip.



Commentary for Worksheet 20

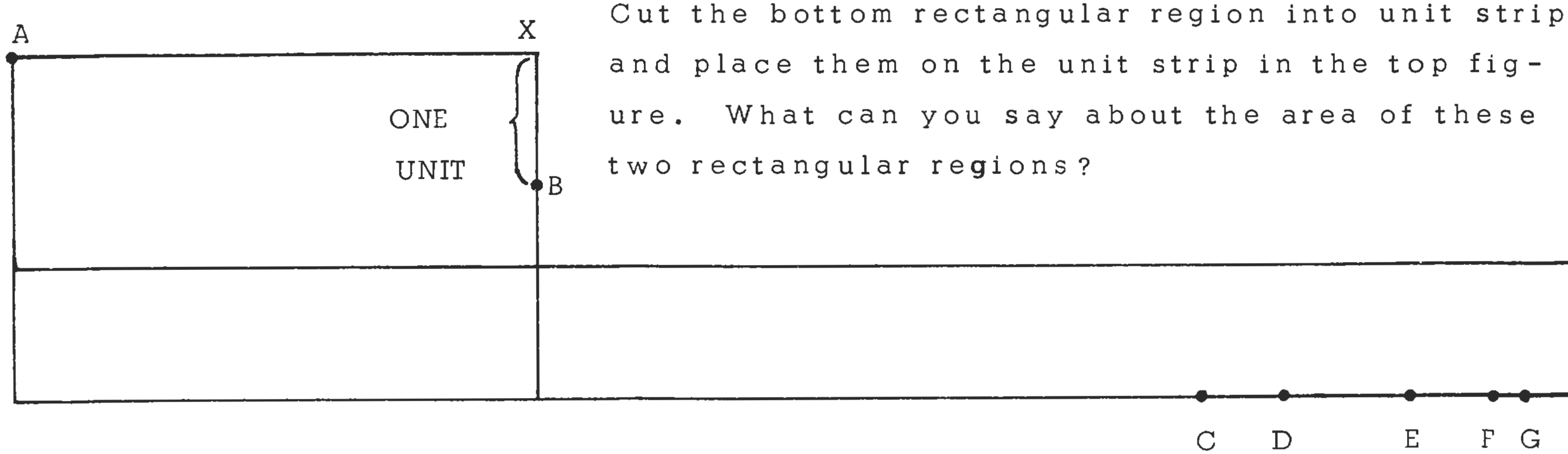
The two rectangular regions on Worksheet 20 contain the same area so the unit strips obtained for them should be of equal length.

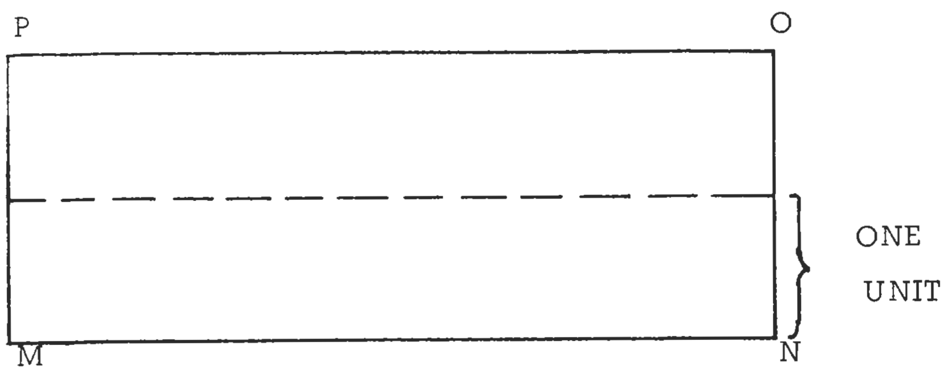
As the children finish this Worksheet with the proper discovery about the areas of the two rectangular regions, give them Worksheet 21 to do on their own and follow this with Worksheets 22 to 25 based on individual progress.

Worksheet 20

Name ____________________

Here are two rectangular regions that have different shapes. We would like to compare them. Use your ruler to draw the line through points A and B that will tell you how long the unit strip should be. One of the points C, D, E, F, G marks the correct length of the unit strip. Cut the bottom rectangular region into unit strips and place them on the unit strip in the top figure. What can you say about the area of these two rectangular regions?





Worksheet 21a   Name ____________________

A. Here are three rectangular regions to compare. Find which one is the largest. In rectangular region I connect points "A" and "B" with your ruler and use AB to help you find how long the unit strip should be. One of the points, M, N, O, P, Q, on the strip marks the correct length.

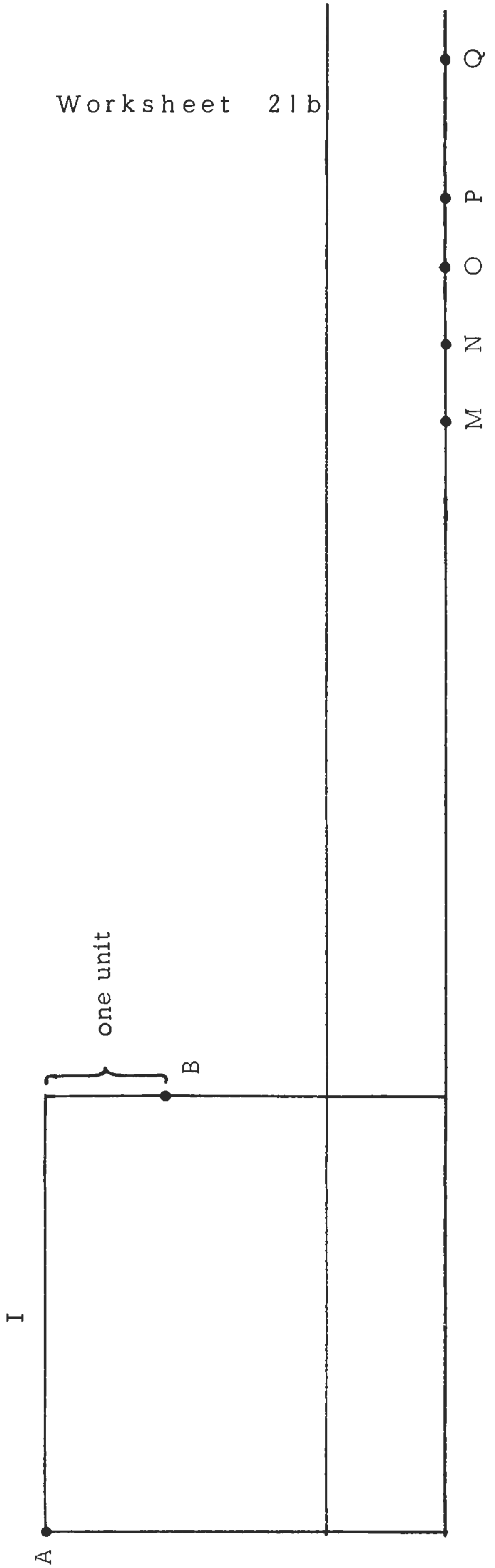
B. Color rectangular regions II and III with different colors.

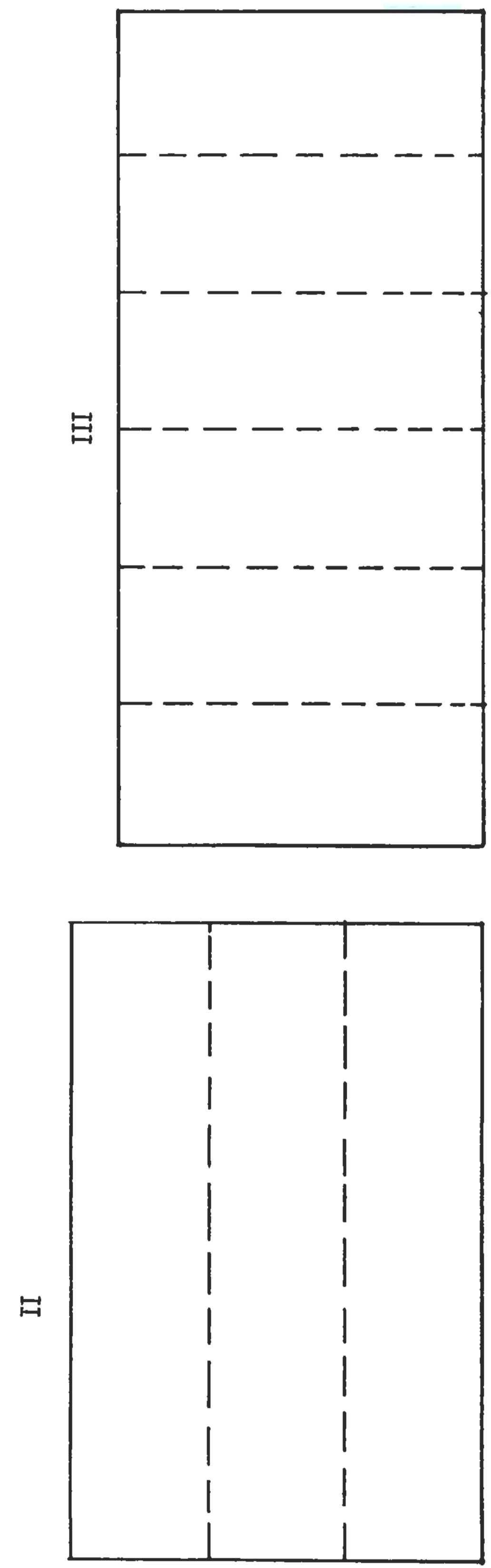
C. Cut out each rectangular region and then cut the strips for each one.

D. Place the strips end to end to find which of the three rectangular regions is the largest.

Worksheet 21b

Name ______________





Worksheet 22 a   Name_______________

Look at these three rectangular regions and guess which is the largest one. Record your guess here_________.

Check to see if you picked the correct rectangular region by the following steps.

A. In rectangular region I connect points "A" and "B" with your ruler and use AB to help you find how long the unit strip should be. One of the points marked on the strip shows the correct length.

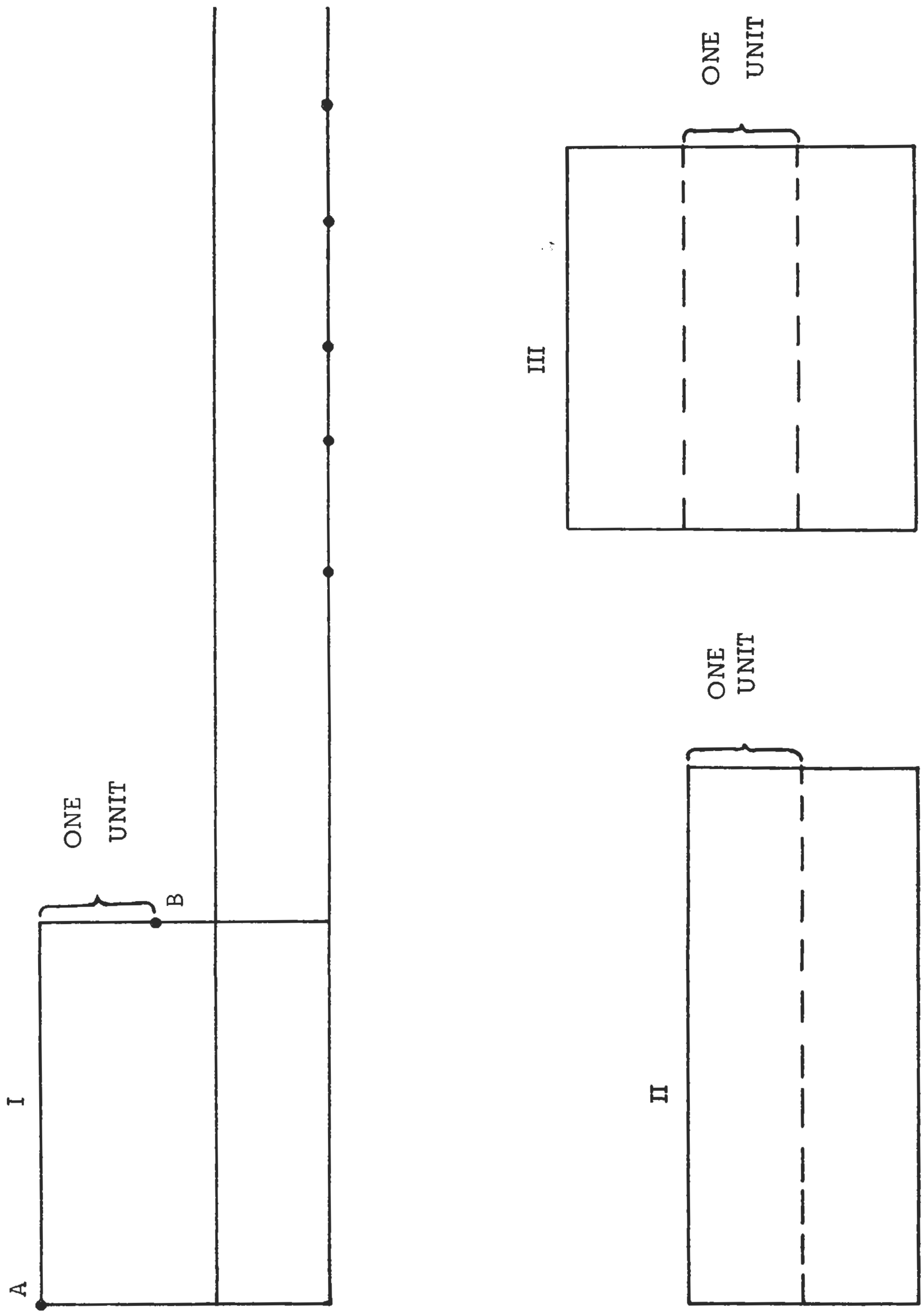
B. Color rectangular regions II and III with different colors.

C. Cut out each rectangular region and then cut the strips for each one.

D. Place the strips end to end to find which of the three rectangular regions is the largest.

Worksheet 22b

Name ____________________



Worksheet 23a   Name________________

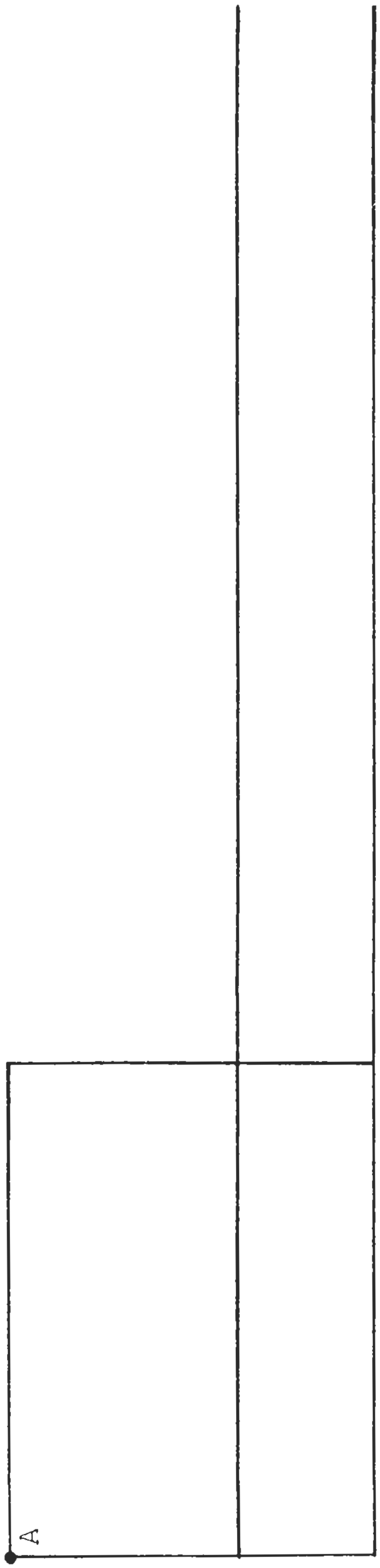
Look at these three rectangular regions and guess which one has the smallest area. Record your answer here________.

In order to check your choice, you should use your ruler to mark the point on each rectangular region that must be joined to point "A" with a straight line that will help you find the right length of the unit strip. Remember what you have been doing in the other exercises.

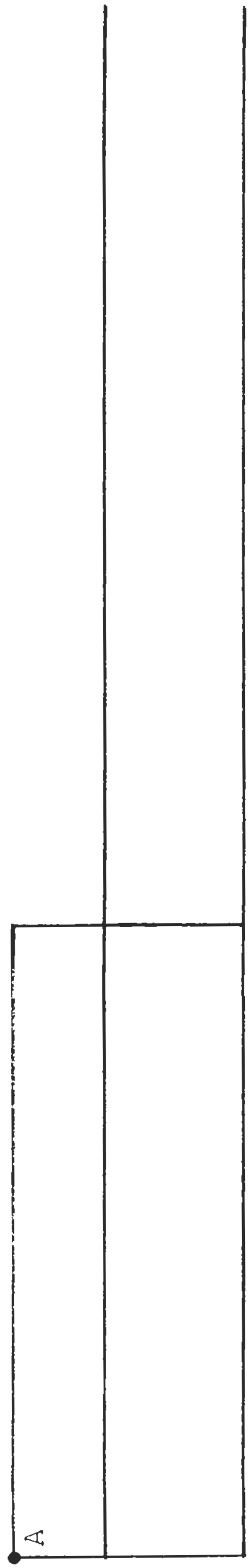
When you have found the proper length for each unit strip, see if you guessed correctly.

Worksheet 23b Name____________

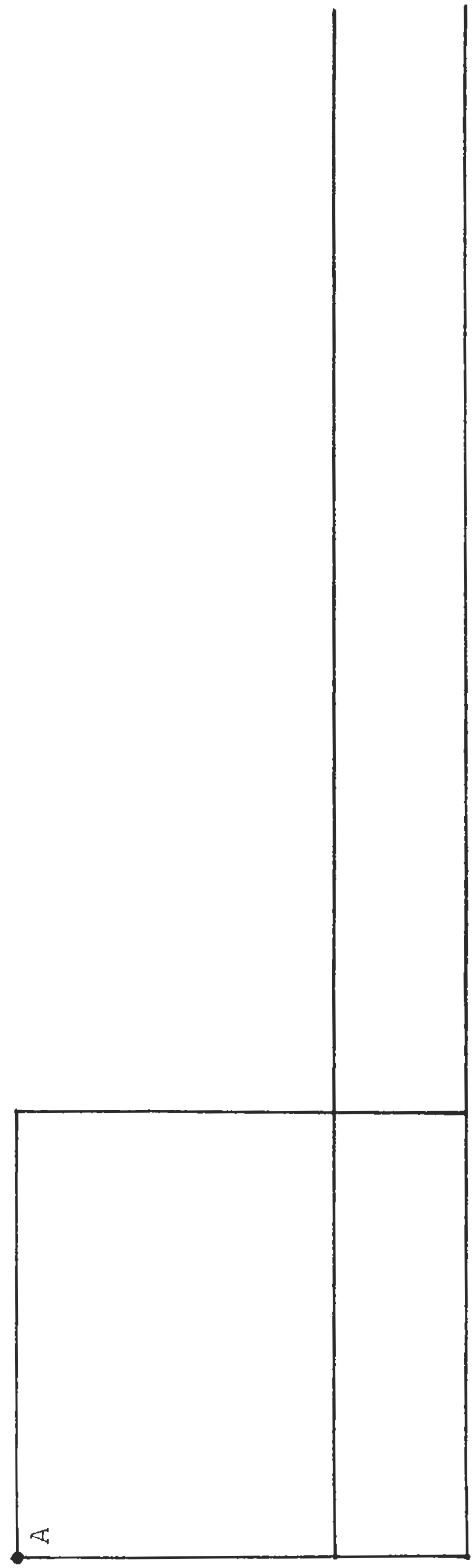
I



II



III

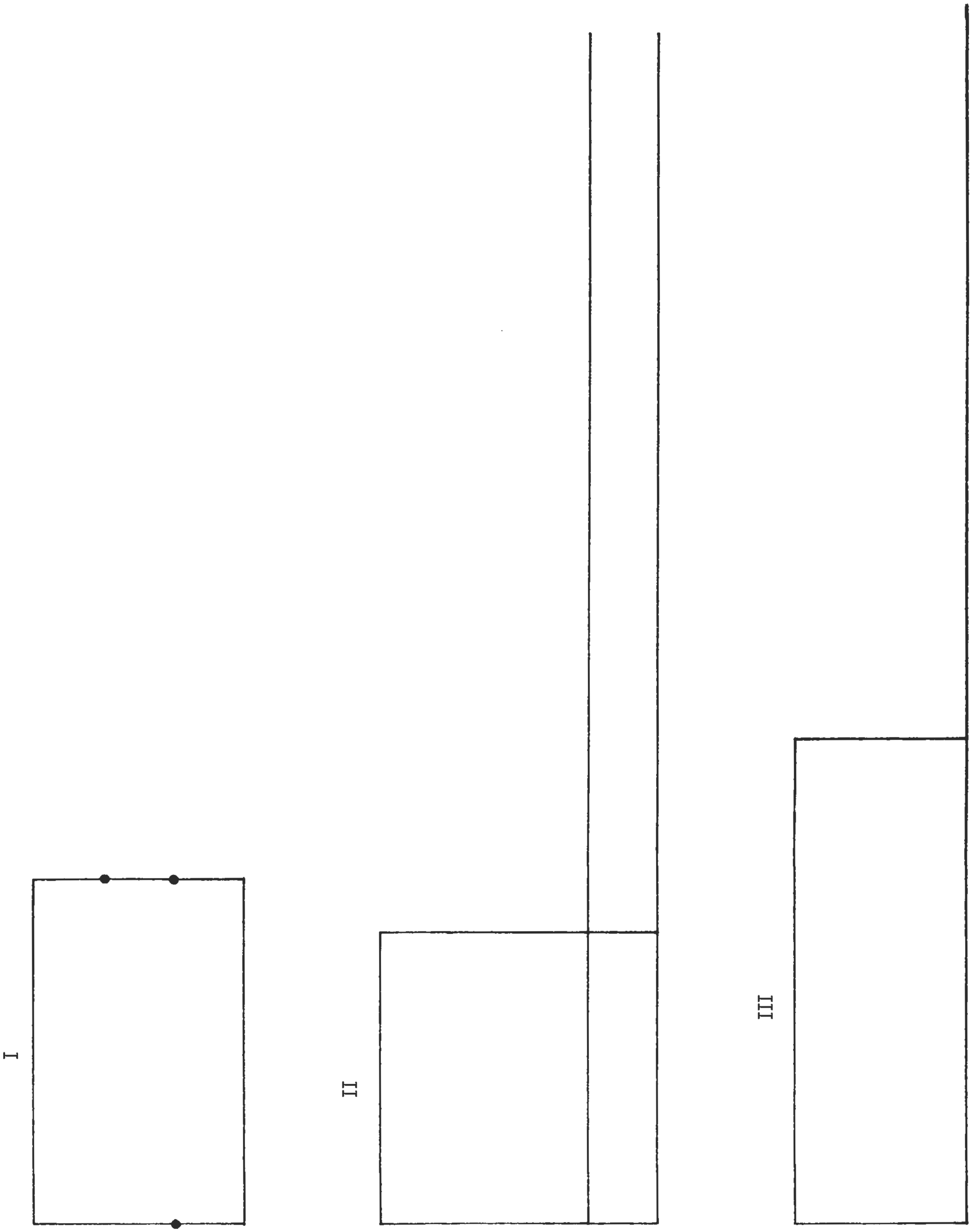


Worksheet 24a Name_______________

Look at these three rectangular regions and see if you can guess which one has the largest area. Put the letter "A" in the one you have chosen. Now see which rectangular region you think is the smallest one and put the letter "B" in that one.

You can now check to see if your guesses were right by making a unit strip for each rectangular region. Part of the work has been done for each figure. You must do the part that is missing in order to draw the unit strip for each rectangular region. If you remember what we have done in the other exercises, you should be able to finish each of these drawings.

Worksheet 24b Name________________



Worksheet 25a Name____________________

1. For each rectangular region line segment $\overline{AB}$ is given to be used as the unit width for the strips. Mark this length on the edge of a piece of paper. Use this measure to mark the points you need on the rectangular region to make the strip.

2. When you have the points marked on the rectangular region draw the unit strip and find the right length as you have done on the other worksheets.

3. Color the unit strips as directed.

4. Use the answer strip from Worksheet 19d.

Worksheet 25b

Name________________

COLOR THIS STRIP GREEN

COLOR THIS STRIP RED

Worksheet 25c

Name_______________________

UNIT

A B

COLOR THIS STRIP YELLOW

UNIT

A B

COLOR THIS STRIP RED

Worksheet 25d Name________________

Now children you are tax collectors of Egypt. Here is your emblem. Remember not to tell anybody. It is a secret society.