# ENVIRONMENTAL GEOMETRY

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**Nuffield Mathematics Project** published for the Nuffield Foundation by W & R Chambers and John Murray

Library of Congress Catalog Card
No: 72-88361
John Wiley & Sons Inc., New York

Printed in Great Britain
by Newgate Press Limited London EC1

## General introduction

The aim of the Nuffield Mathematics Project is to devise a 'contemporary approach for children from 5 to 13'. The guides do not comprise an entirely new syllabus. The stress is on *how to learn*, not on what to teach. Running through all the work is the central notion that the children must be set free to make their own discoveries and think for themselves, and so achieve understanding, instead of learning off mysterious drills. In this way the whole attitude to the subject can be changed and 'Ugh, no, I didn't like maths' will be heard no more.

To achieve understanding young children cannot go straight to abstractions – they need to handle things ('apparatus' is too grand a word for at least some of the equipment concerned – conkers, beads, scales, globes, and so on).

But 'setting the children free' does not mean starting a riot with a roomful of junk for ammunition. The changeover to the new approach brings its own problems. The guide *I do, and I understand* (which is of a different character from the others) faces these problems and attempts to show how they can be overcome.

The other books fall into three categories: Teachers' Guides, Weaving Guides and Check-up Guides. The Teachers' Guides cover three main topics: ● Computation and Structure, ▼ Shape and Size, ■ Graphs Leading to Algebra. In the course of these guides the development of mathematics is seen as a spiral. The same concept is met over and over again and illustrated in a different way at every stage. The books do not cover years, or indeed any specific time; they simply develop themes and therefore show the teacher how to allow one child to progress at a different pace to another. They contain direct teaching suggestions, examples of apparently un-mathematical subjects and situations which can be used to develop a mathematical sense, examples of children's work, and suggestions for class discussions and out-of-school activities. The Weaving Guides are single-concept books which give detailed instructions or information about a particular subject.

The third category of books, as the name implies, will provide 'check-ups' on the children's progress. The traditional tests are difficult to administer in the new atmosphere of individual discovery and so our intention is to replace these by individual check-ups for individual children. These are being prepared by a team from the Institut des Sciences de l'Education in Geneva under the general supervision of Piaget. These check-ups, together with more general commentary, will be issued in the same format as the other guides and, in fact, be an integral part of the scheme.

While the books are a vital part of the Nuffield Mathematics Project, they should not be looked on as guides to the only 'right' way to teach mathematics. We feel very strongly that development from the work in the guides is more important than the guides themselves. They were written against the background of teachers' centres where ideas put forward in the books could be discussed, elaborated and modified. We hope very much that they will continue to be used in this way. A teacher by himself may find it difficult to use them without the reassurance and encouragement which come from discussion with others. Centres for discussion do already exist and we hope that many more will be set up.

The children's work that has been reproduced in these books, like the books themselves, is not supposed to be taken as a model of perfection. Some of it indeed contains errors. It should be looked upon as an example of work that children *might* produce rather than a model of work that they *should* produce.

## Foreword

The last few years have been exciting ones for teachers of mathematics; and for those of us who are amateurs in the subject but have a taste for it which was not wholly dulled by the old methods that are so often stigmatised, there has been abundant interest in seeing the new mathematical approach develop into one of the finest elements in the movement towards new curricula.

This is a crucial subject; and, since a child's first years of work at it may powerfully affect his attitude to more advanced mathematics, the age range 5 to 13 is one which needs special attention. The Trustees of the Nuffield Foundation were glad in 1964 to build on the forward-looking ideas of many people and to set up the Nuffield Mathematics Project; they were also fortunate to secure Dr. Geoffrey Matthews and other talented and imaginative teachers for the development team. The ideas of this team have helped in the growth of much lively activity, throughout the country, in new mathematical teaching for children: the Schools Council, the Local Education Authority pilot areas, and many individual teachers and administrators have made a vital contribution to this work, and the Trustees are very grateful for so much readiness to co-operate with the Foundation. The fruits of co-operation are in the books that follow; and many a teacher will enter the classroom with a lively enthusiasm for trying out what is proposed in these pages.

**Brian Young**
Director of the Nuffield Foundation

# CONTENTS

# INTRODUCTION

Throughout the main Teachers' Guides, the importance of the child's environment as a source for mathematical activities is stressed. The purpose of this Guide is to look deeper into how the study of the environment can lead to geometrical work.

Much of it will appear to be a collection of separate topics, but collectively, these will show a structure of related ideas.

The word 'environment' will be used to cover all fields of experience, both in and out of the classroom, met by the child in his daily life.
In ▼1 pre-school experience, creative work and imitative play are discussed. ▼2 and ▼3 are concerned with further developments of this work, but in the main, the work is centred on activities with materials in the classroom, the object being to provide for the child a rich variety of experience in size, quantity, proportion, shape and position.

The range of activities discussed in this Guide will extend beyond experiences with objects in the classroom which the child can touch or handle. This extension of the field of activities will introduce complications which must be recognised if there is to be no confusion of the child's ideas.

The first stage in the study of shape and size in the environment will be the perception of objects with one or more of the senses. These perceptions will be a development of a combination of earlier sensations acquired from previous experience. It is important to remember that we can build only on those ideas which have already been formed in the child's mind.

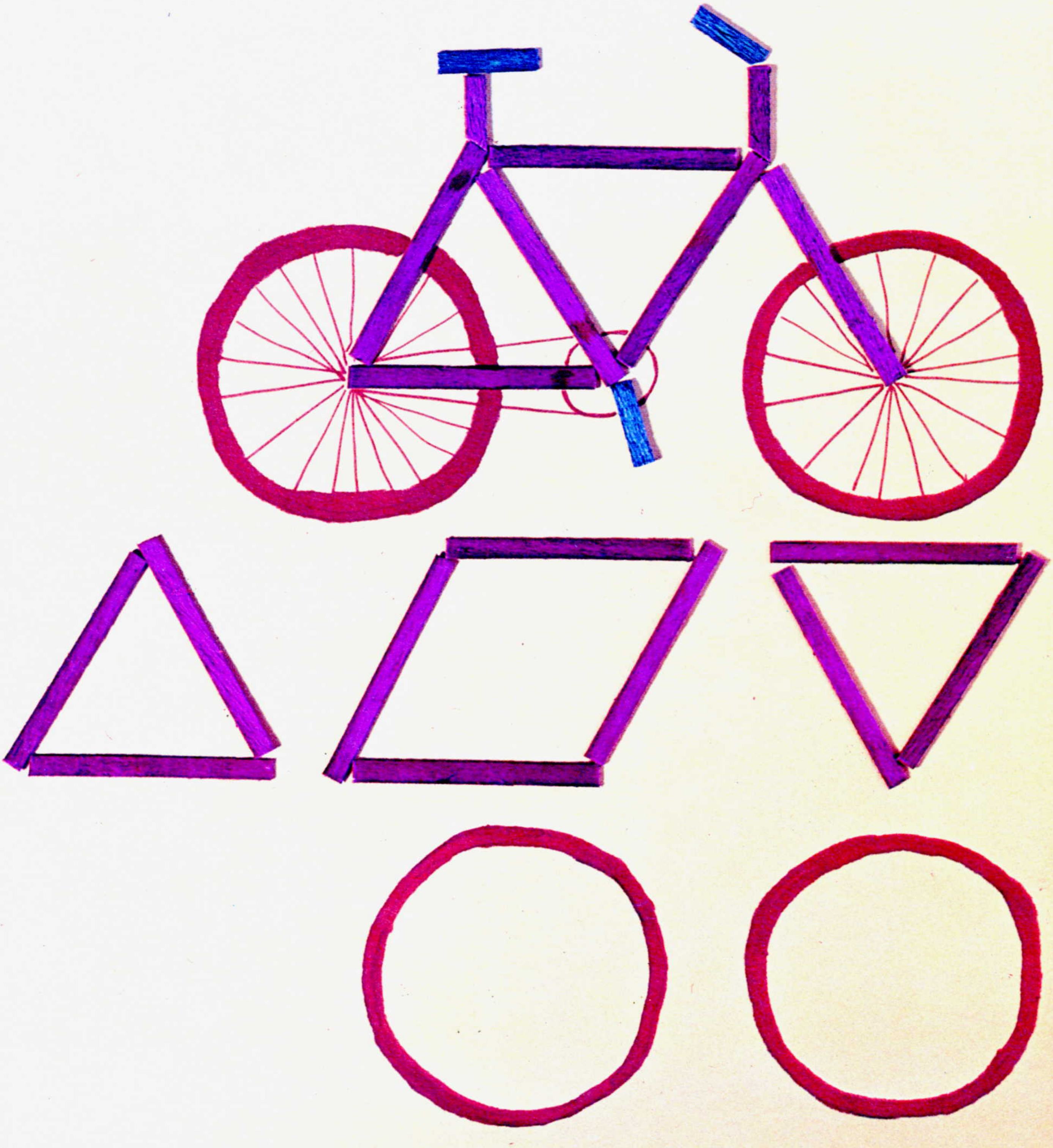
**Acknowledgment**
We are very grateful to George Kasabov and to those of his first-year students at the Bartlett School of Architecture in London who helped with this project. Extracts from their reports have been incorporated in the text.

# THINGS IN THEIR SETTING

The first complication met when looking at shapes in the environment is that they are usually embedded in their surroundings and are rarely seen in isolation. Puzzles of the 'Find the Birds' type illustrated here help in giving experience of this problem in the early stages.

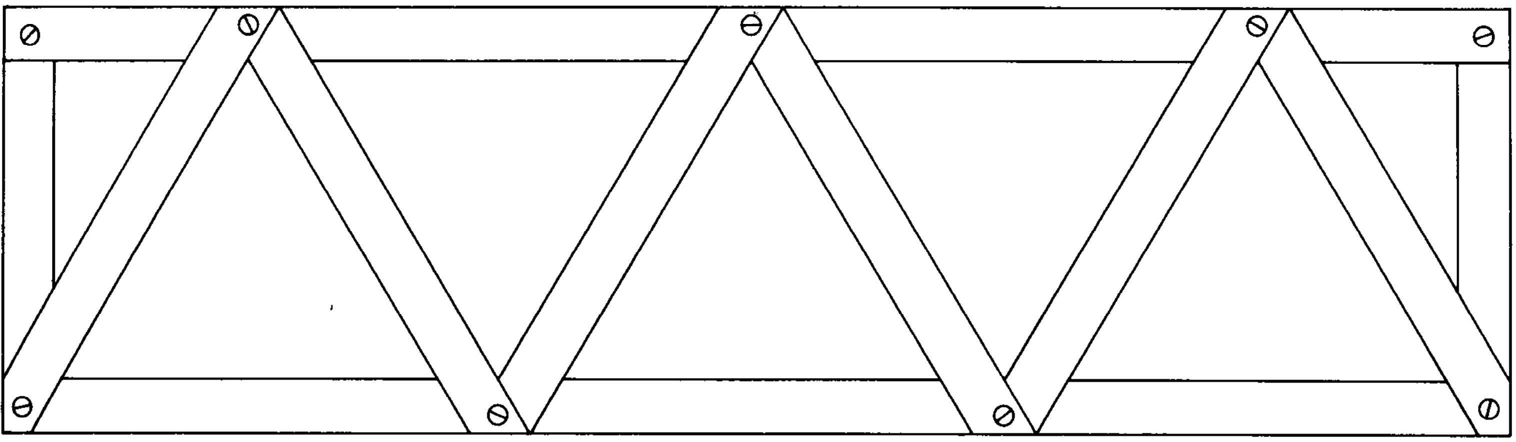
The extraction of one particular shape from a combination of shapes can be a difficult task. Children aged six or seven usually find it easy to trace out with a finger the triangles in the framework of a bicycle, but when asked to look for a parallelogram they may well find it difficult. Their ability in situations such as this will, of course, depend very much on the range of their earlier work.

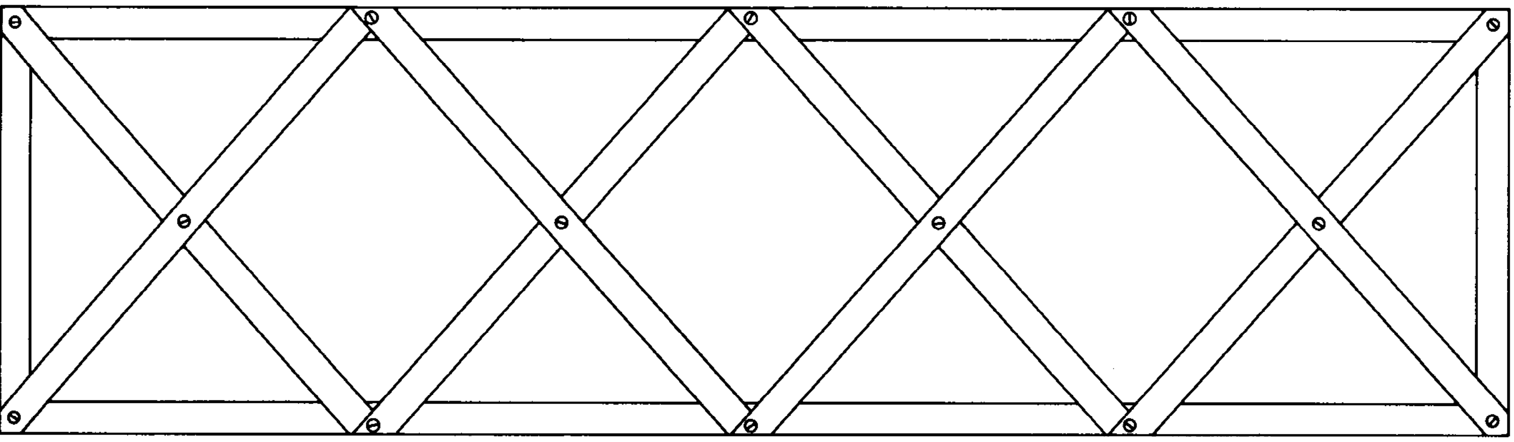
After looking at a bicycle and discussing the shapes incorporated in the framework, some children made models with coloured wooden spills. These enabled the children to make bold, straight lines which they would not have been able to do by drawing or painting. As an additional activity, the shapes could be made in such a way that they could be lifted and placed over the corresponding shapes in the first picture.

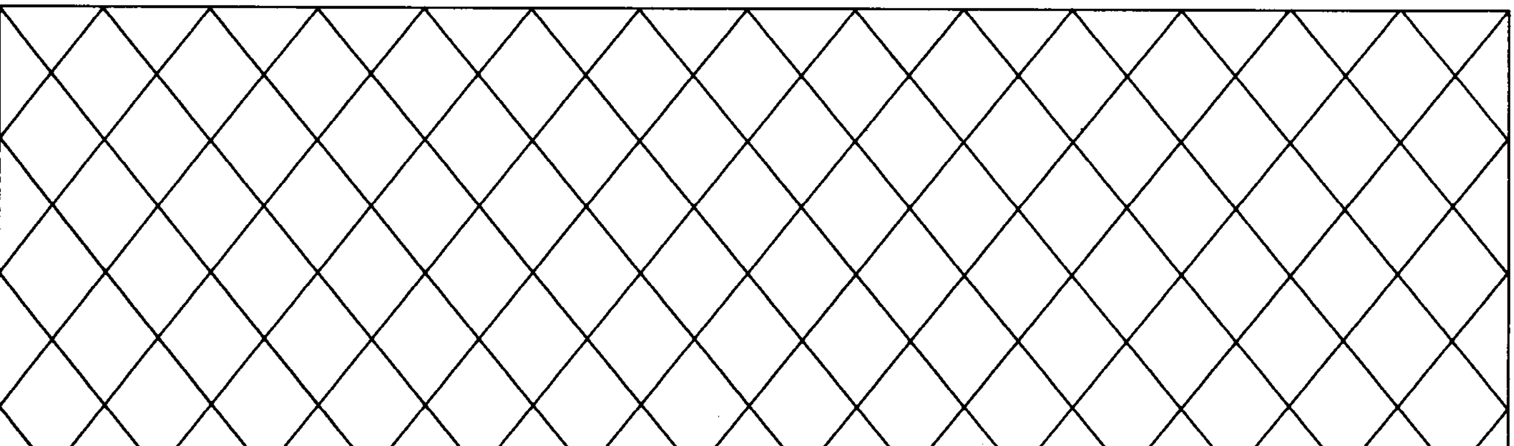


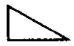
Many children will have an opportunity of observing a construction such as this bridge.

The first drawing on page 5 shows details of a construction similar to the railway bridge and can be used for further activities.







Count the number of triangles in the first drawing on page 5.
How many triangles are there with the same shape as this one  ?
How many are the same shape as this one  ?

It may be helpful to have a cut-out shape to help with the counting.

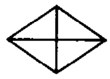
In the first drawing on page 5 two triangles are like this  and the rest are like this . In which ways are they different?

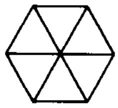
Again, cut-out shapes will be helpful, preferably with several of each type available. As well as the obvious difference in size there should be some observations about the size of the angles of the two triangles.

'In this one all the angles are the same size but not in this one.'

'This one has one point sharper than the others.'

'Two of these fit together and make a straight line.'

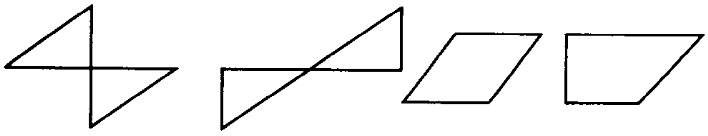
'Four of these fit and join up.' 

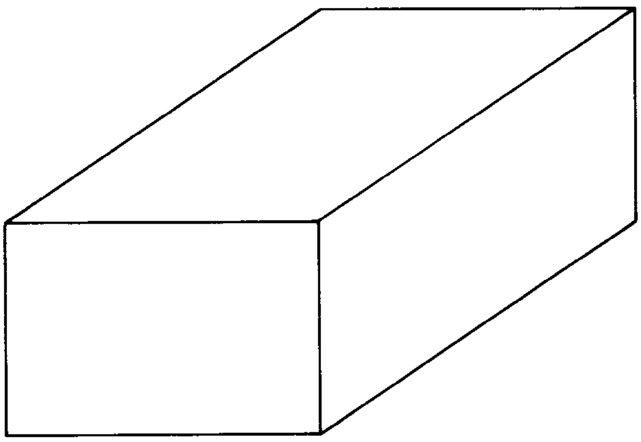
'Six of these join up.' 

Older children can work with similar questions on the second and third drawings. In addition they can look for parallelograms, rectangles, squares and trapeziums (e.g. how many are there of each?).

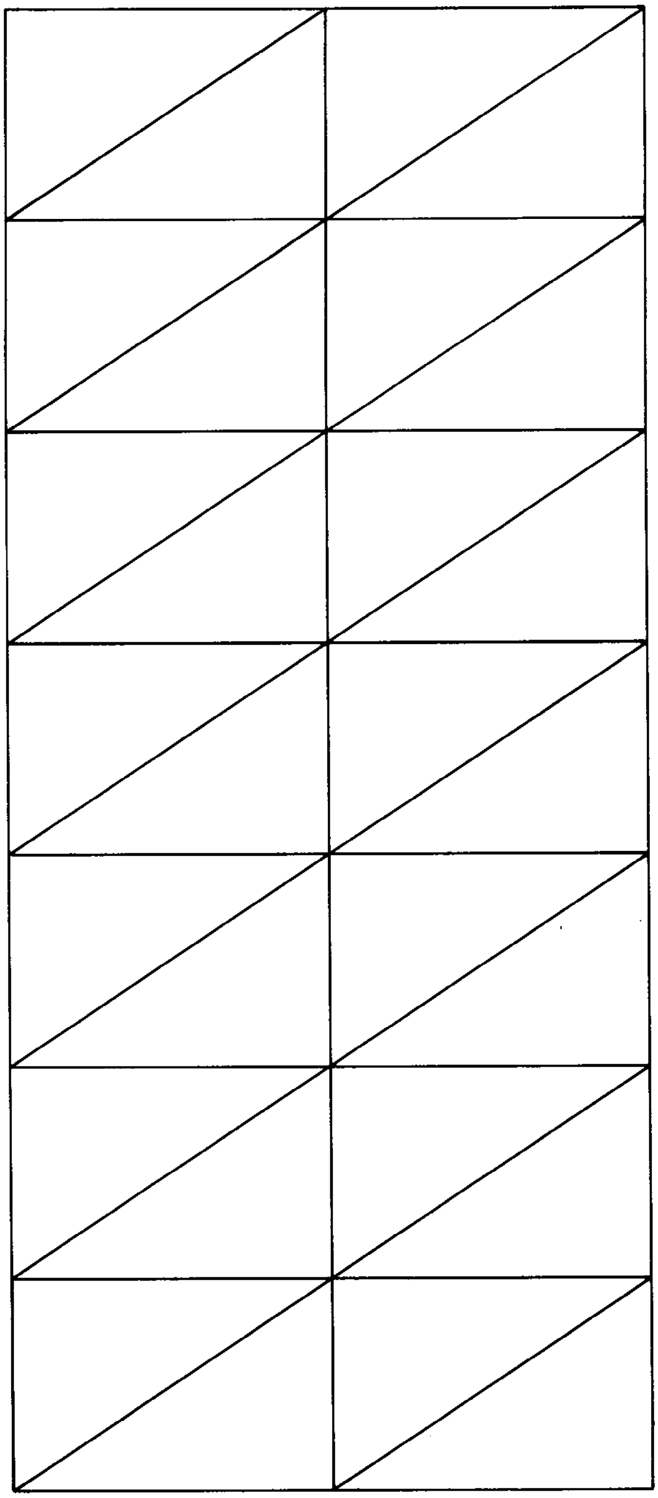
These drawings, and those on the next page, again illustrate the difficulty of seeing one particular shape embedded in a complex collection of shapes. At first glance **a** may not appear to form any part of **b**.

Use tracing paper and pencil (or a transparent sheet and chinagraph pencil) to find out how many times **a** is contained in **b**.
Trace the following shapes from **b** and see how many times they are repeated.

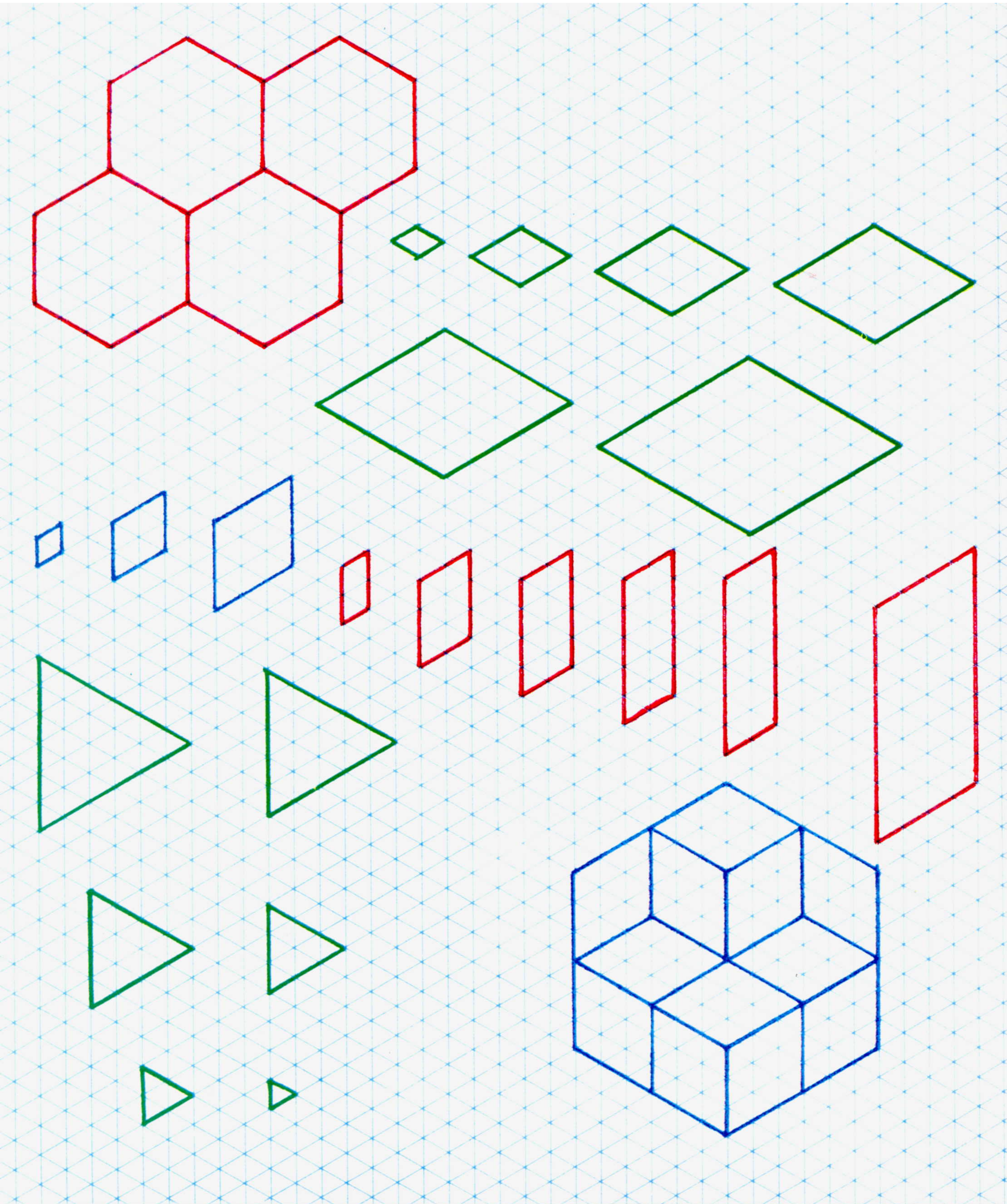




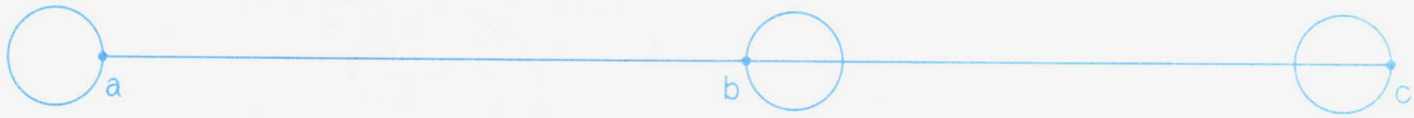
a



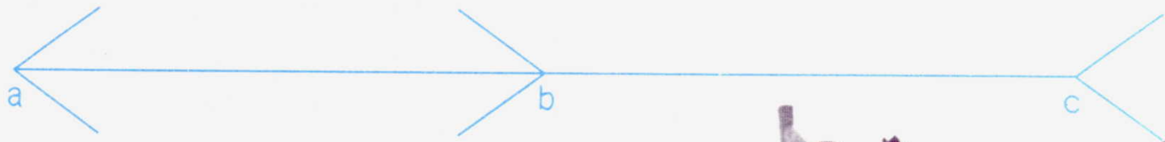
b



The influence of surroundings on the way in which we see one part of a whole is also illustrated in the familiar illusions shown below.




In the drawing above, which do you *think* is the longer distance: from *a* to *b*, or from *b* to *c*?
Mark on the edge of a strip of paper the length **ab** and compare it with the length **bc**.
Now try the same with the drawing below.




In both cases, **ab** and **bc** are the same length.

Which picture of a tree do you think is the tallest and which is the shortest? Use a ruler to see if you were correct.

Even interpretation of colour can be influenced by the background to the colour.

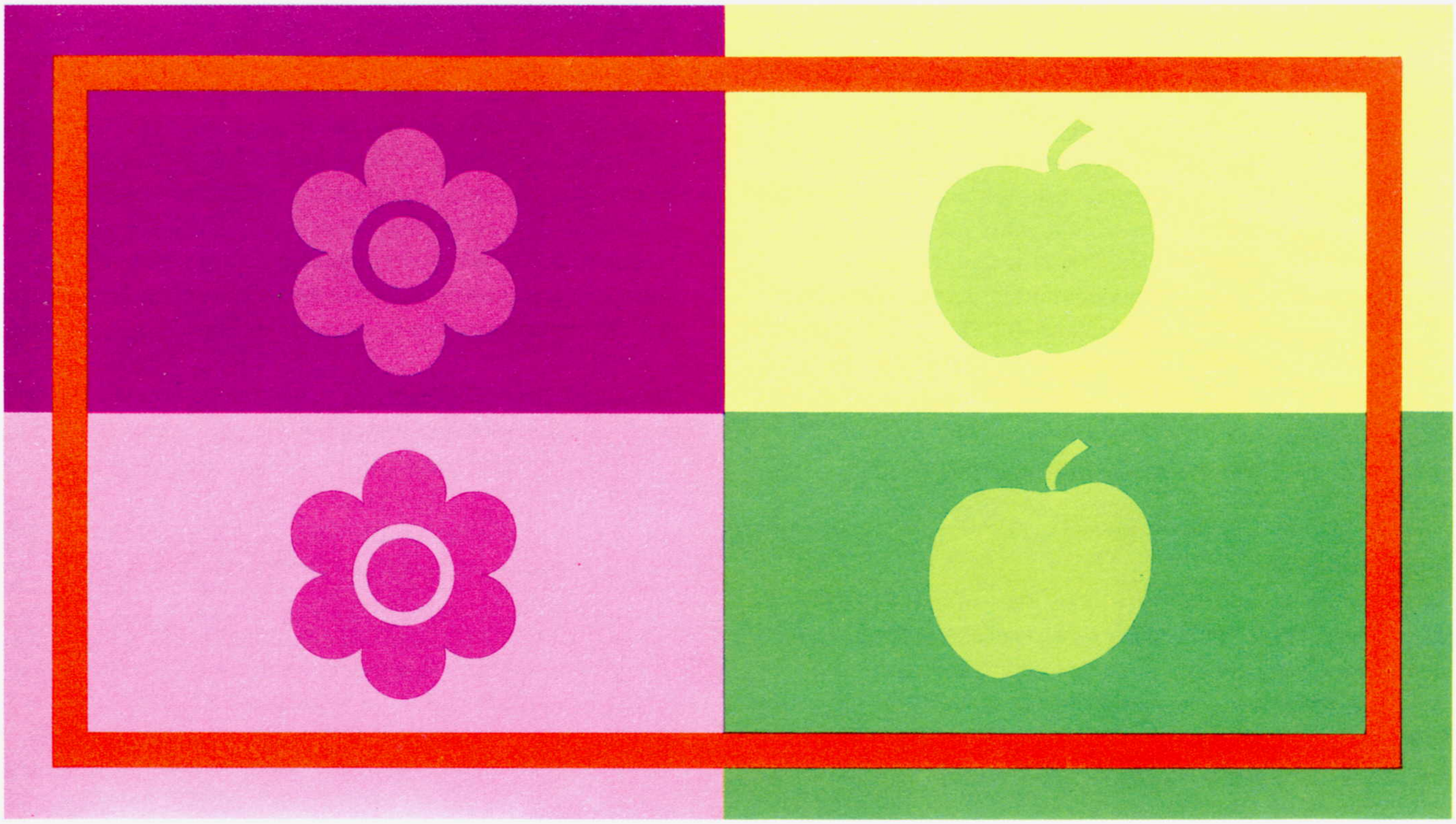
First impression, when looking at the two apples in the drawing opposite, is that they are in different shades of green. A longer look, with perhaps a mask to remove the background, will show that this is not so. A similar effect can be seen with the flowers, and with the orange line against the different backgrounds.

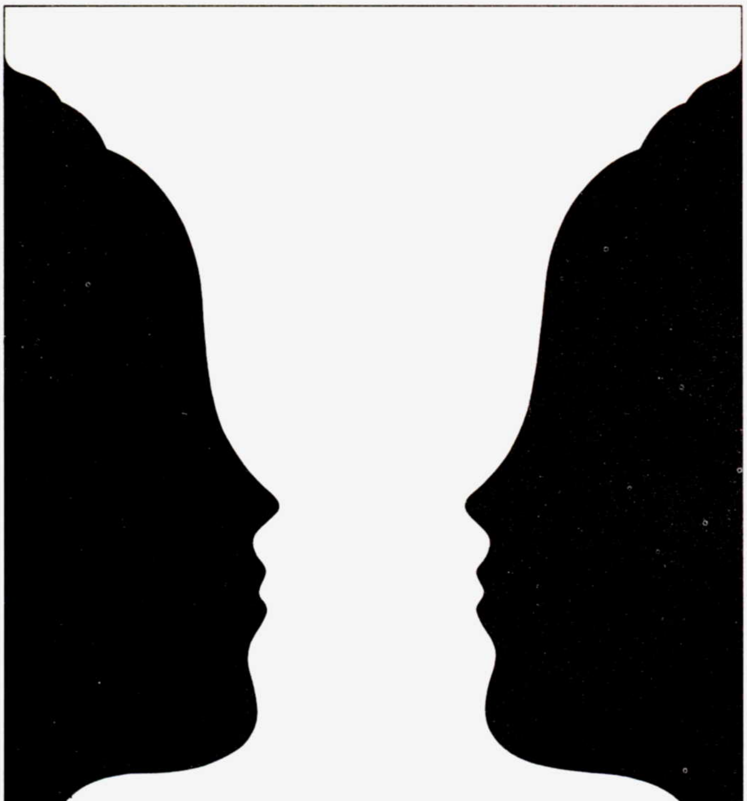
The difficulties concerned with discarding the irrelevant and concentrating on the essentials will appear at all levels. The young child may find difficulty in seeing the shape pointed out by the teacher because of the background to the shape. Later, otherwise straightforward work, in both two- and three-dimensional problems, can be found difficult through confusion of the essential part of a diagram with surrounding details.

Sometimes the background to a figure can assume an importance greater than that of the figure itself. The drawing on the left (opposite) may be seen alternatively as black on white, or white on black. Children asked to describe the drawing may see it as a white vase on black or as black faces on white.

On the right is an example of an ambiguous drawing which can be seen in two quite different ways : either as an old woman or as a young girl.

This chapter has provided only a few examples of experiences concerned with the difficulty of seeing things in their settings, but these, and similar activities, may encourage children to keep their eyes open and so develop their geometrical understanding.





# SHAPE AND SIZE CONSTANCY

Other problems which will arise when looking at things which cannot be handled or touched, are concerned with the ability to recognise an object as being of a standard shape, although seen from different angles, and the ability to judge the size of a distant object.

Even very young children have perceptual systems capable of handling simple problems of distance and orientation. But it is the capacity to process the information received by their senses, with the consequent ability to act on it, which is limited. It is this capacity for handling complex information which must be developed through experience in a rich variety of situations.

**Shape constancy**
Shapes in the environment are rarely seen in full plan view. For example, circles are usually seen as ellipses. The children can become more aware of this by looking at circular shapes from different angles and discussing what they see. They would find great difficulty in drawing what they see but their awareness and understanding can be shown in their ability to match what they see with one of a series of cut-out ellipses. (See cards in the inside back cover pocket.)

At a later stage the children will be able to predict what they would see if they were in a different position.

In the development of this work the rotation of shapes can play an important part. Rotation of one shape can lead to the study of other shapes. If a suitable source of light and a screen are available, then much profitable and interesting work can be done in discussing the changing shape of the silhouettes of objects when they are held, and rotated, between the source of light and the screen. The circle changing to an ellipse and the rectangle becoming a square are simple examples. The objects used need not be limited to regular geometric shapes, as valuable experience will be gained by using a wide variety of irregular shapes as well.

It is interesting to see how much movement is necessary, and which views are important in the recognition, by its shadow, of a complex shape.

It will also be possible to see which shapes maintain the same shaped shadow when rotated, and if it is possible to maintain this when rotating about more than one axis.

If the observation of shape in the environment has reached a stage at which the parabola has been discussed – for instance, through discussion of the path of a ball thrown in the air, a jet of water or a tracer bullet, all of which are approximately parabolic – it is interesting to see that the shadow of a parabolic curve will retain its essential properties when the curve is rotated about a central axis. Outlines could be traced, each being checked to see if it formed a 'curve of squares'.

By using this shadow technique it is possible to study and compare shapes without having the distraction of other properties, such as colour and texture.

Any such activities will, of course, be closely linked with the work on rotation, reflection and symmetry in ▼2 and ▼3.

In any discussion on the invariant properties of shapes great care is needed in the interpretation of the children's answers and comments. An example of misunderstanding of a child's answer was shown in one situation when a square of card was held up in this manner □ for a child to look at and name. He called it a square. Then the same shape was held up in this way ◇, and, after some hesitation, the child called it a diamond. This was taken as showing that the child had little understanding about constancy of shape: that shape is invariant under rotation. Further questioning, however, showed that the child had realised that the shape had the properties of a square but that as it had been demonstrated in a different manner he imagined the teacher expected a different answer, so he had thought of a different name which was still correct. Misinterpretation of answers, as in this example, can lead to the labouring of a point which has already been understood, with the consequent danger of causing boredom, or even confusion of ideas.

From activities with movable objects, the children can progress to work with larger, static shapes. Sometimes it may be possible to see these from high and low viewpoints as when, for example, the playground can be looked at from ground level and then from a top-floor window.

Drawing plan views of larger shapes, such as the garden, playground or field can be a valuable activity, but the work should not begin with elaborate, laboriously measured scale plans, where too often the children can get bogged down in a mass of unnecessarily precise measurements which they are not able to process and convert into a drawing.

Before reaching this stage the children can draw the shape after having found the approximate relationships between the lengths of the sides by a quicker means of measuring, such as counting paces. Angles at corners can be compared by measuring them in terms of the amount of turn needed to change direction from facing along one line to facing along the other.

At a later stage more advanced methods of measurement can be used so that more accurate scale drawings can be made.

A six-year-old girl showed a certain degree of understanding about plan views when she drew the plan of a car park shown on the next page.

Her comments were:

'This is how I would see it from a helicopter over the top of it.'
'I made the top side the same as the bottom side because they are the same length really but they don't look the same until you get there.'

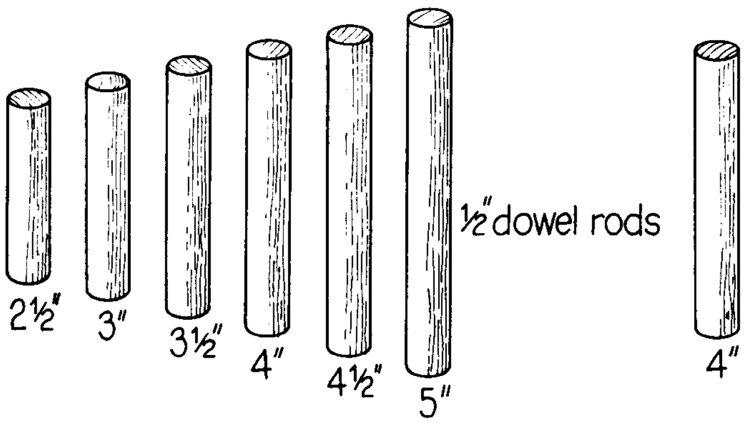
There was, however, some conflict between her ideas of perspective views and plan drawings when she said, 'I wrote **Car Park** properly because that was near, but I scribbled **Bus Stop** because that was too far away to read it.'

**Size constancy**

Shape constancy and size constancy are closely linked, since any object observed in the environment is, at the same time, seen in perspective and at a distance.

Although some ability to judge size at a distance exists from a very early age, there will, with appropriate experience, be a steady development of this.

As an example, children of six when trying to match a rod situated at the other side of the classroom, with one of a series of rods of different height close at hand, will frequently tend to choose a smaller one, whereas children of eight or nine usually have less difficulty in choosing the correct matching rod.




Early infant work on size constancy may be limited to the matching of distant objects with others close at hand, accuracy being checked by actual comparison. A variety of objects can be used, thereby increasing the range of experience.

'I guessed I could reach four bricks (concrete blocks) up the wall but I really reached five.'

'I said the stick was higher than the table but it wasn't.'

'I thought my castle (building blocks) was more bigger than up to the window. I got some string and cut it the same as the castle but it didn't go up to the window. I was wrong, but not very.'

'The tin was more higher but the box had more sand in it.'

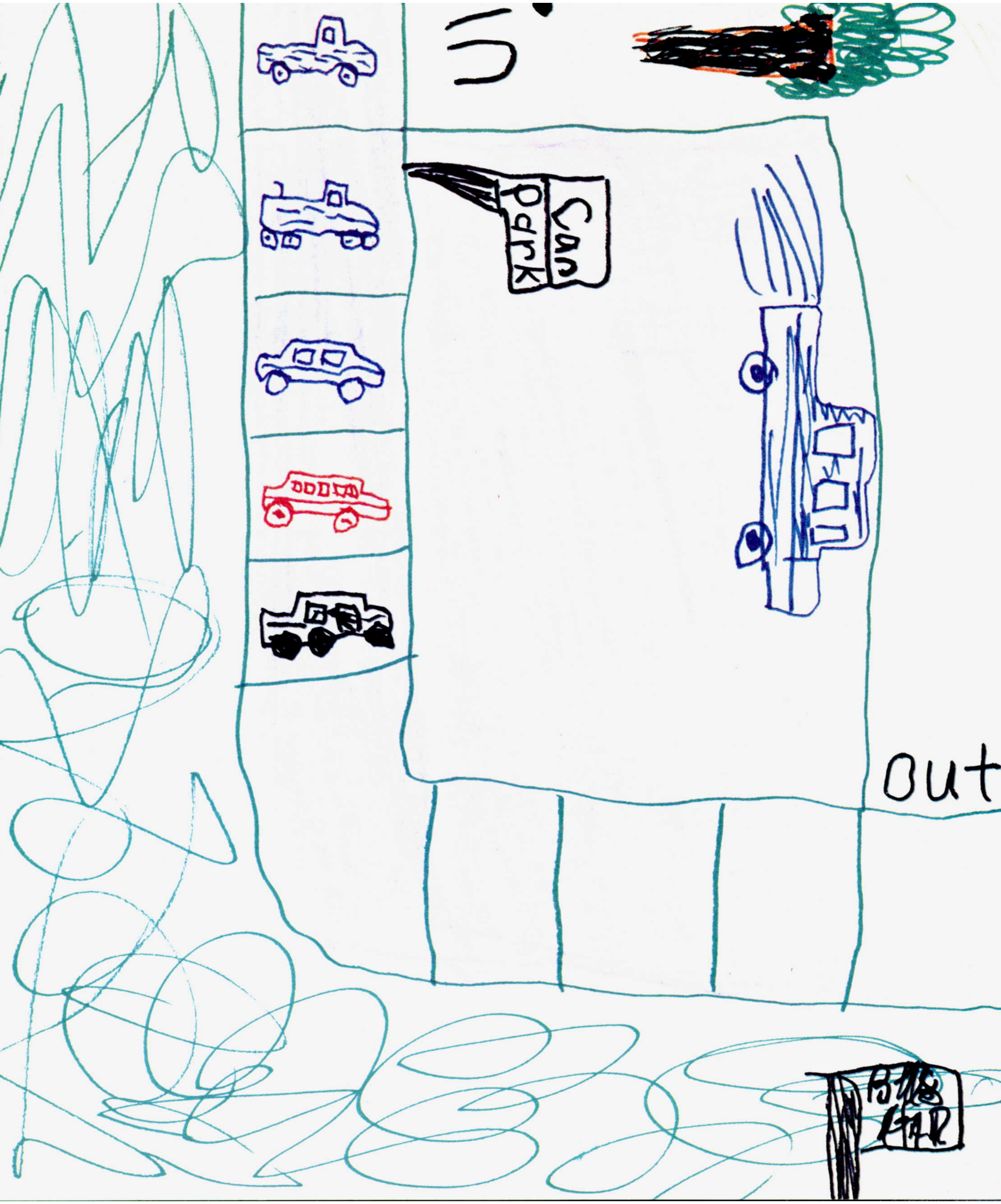
Great care must be taken with the language used by the teacher in activities such as these. There could be considerable confusion over what was meant by 'the same size as' or 'as big as', which could be referring to linear measurements, area or volume.

The following is an extract from a report by first-year students at the Bartlett School of Architecture on some work done with infants.

**Intentions of our teaching**

During a period of initial observation in which we talked to a class of six-year-olds, we noticed among other experiments that the children had no notion of relative scale when drawing objects – for example, several children when asked to draw a worm, flower, man, tree and house drew them on completely different relative scales. We set out to give the children some idea of the concept of relative scale, and thought it interesting to follow the development of this idea throughout the school, so it was arranged that we should take a cross-section throughout each year of the school. We then set about thinking up some method of getting this over to the children, and finally came out with the idea of using a piece of glass placed before an object – so that the child could actually mark the height of the object **as he saw it** on the glass with a chinagraph pencil.

By marking the dimensions of two objects on the glass at one time, the child had actually drawn the dimensions of two


Car Park
Out

objects on a relative scale. Then by looking at the marks on the glass, the child could make a drawing of the objects in the dimensions that he had actually seen. By placing different-sized objects behind the glass in the same position, and by keeping the child in the same position relative to the glass, a set of drawings in relative scale could be obtained.

Another important side aspect of this method was the determination by the child of perspective – for example, the outlines of a shape placed skew to the plane of the glass could be drawn on the glass, and a drawing made of the shape as the child actually saw it. This method could also be used to show the relative size of objects when viewed near and far – for example, without the glass method a long time was spent in getting the concept of near and far houses over to a six-year-old Indian girl.

By asking the children to draw what they actually saw before them, it was hoped that we could also get over the idea that a drawing was merely an analogue of what the child saw, and so when several objects were drawn in relation to one another, they should be treated in a relative scale.

R. Hines, I. Stevens, D. Taylor, E. W. Thorogood

Older children could be given assignments in which they were told to estimate the size of objects at a distance, and then check their results by actual measurement. The exercises could be made still more valuable if, with the original estimate and the final measurement, the child could include the limits within which he could reasonably expect his answer to be correct.

| Object | Estimated | Actual measurement |
|---|---|---|
| Height of gate | Between 4 feet and 4 feet 6 inches | 4 feet 1 inch to nearest inch. 'I put it to the nearest inch because there isn't an exact height – the ground is too lumpy.' |
| Width of garden | 20 feet but could be 1 foot more or less | 28 feet to nearest foot. 'I don't think it would be right to say it to the nearest inch because the ground wasn't flat enough and the grass wasn't straight.' |
| Length of pencil | 5 inches. It might be a bit of an inch more or a bit of an inch shorter. | Six inches right on. |

The 'right on' in the last example would need further discussion. From this sort of beginning the children can be led to more concise ways of recording estimates and degrees of accuracy, e.g.

3 feet $\pm$ 6 inches

2 feet 6 inches $<$ height $<$ 3 feet 6 inches.

The ability to estimate size relies on environmental support. The less help there is from the environmental surroundings, the nearer the estimate of size will be to 'apparent' size rather than to actual size.

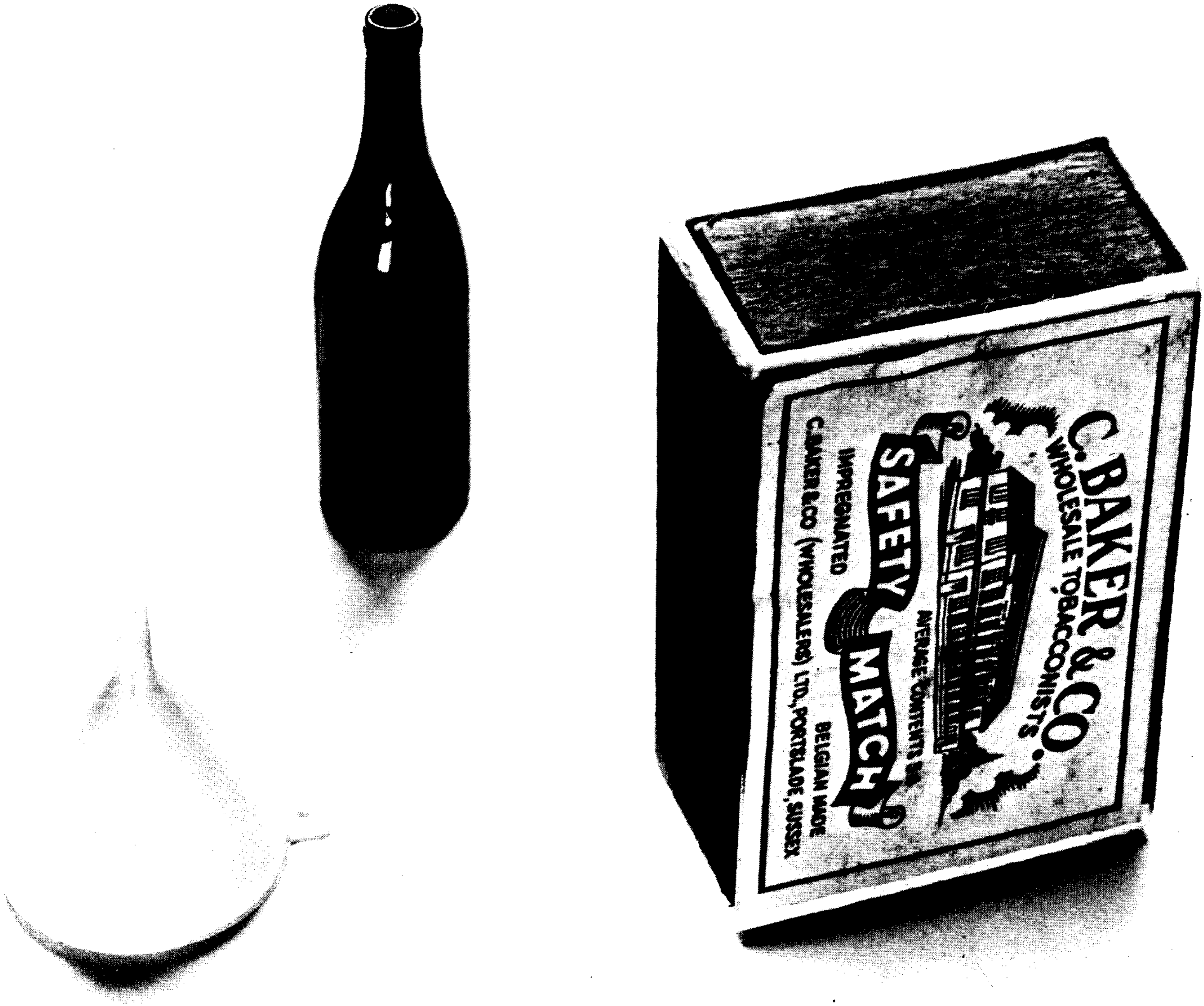
If we see a picture of an enlarged, or reduced, object on a television screen with no surroundings, we can have no idea of its size until a recognisable object appears beside it. It is much more difficult to estimate the size of a boat at sea or a distant monument on a bare hillside than it would be if the object were surrounded by other recognisable objects.

The need for environmental support can be shown clearly by experimentation with the photograph on the next page. If the photograph is masked in such a way as to show only the funnel, there can be no reliable estimation of its size.

When the mask is moved a little to the right so that the bottle is visible, we seem to have something as a basis for comparison and estimation of size. Complete removal of the mask, however, shows us that the information given was, in fact, misleading. It can then be seen that the funnel and the bottle were miniatures.

Using this card with the children it is possible to make them more aware of the need for environmental support when estimating the size of an object.

Children seeing familiar objects in an entirely different setting may well have some difficulty in relating their perception to their previous experience with the objects. A five-year-old girl, on seeing mountains for the first time in her life, found it hard to believe that the trees at the top were approximately the same size as those close by. Later, on seeing some people on the slopes, she was coming to terms with the problem when she said, 'But if they were down here with us they would be just as big as us, wouldn't they?'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WHOLESALE TOBACCONISTS
SAFETY MATCH
IMPREGNATED
BELGIAN MADE
C.BAKER & CO (WHOLESALERS) LTD., PORTSLADE, SUSSEX

# SHAPES IN THE ENVIRONMENT

Before doing any directed work on the recognition of the shape of objects in the environment the child will have had a great deal of experience in handling a wide range of things in the home and classroom. This work is described in detail in ❶ which describes activities of sorting objects into sets. In first classification, properties such as colour, hardness, softness, 'can be distorted', 'floats in water', etc. are very important to the young child and the recognition of such properties will precede any appreciation of the geometrical properties of the objects.

Later, angle properties will play an important part. When the young child is distinguishing between rounded shapes and those with straight edges, it is not the straight line itself which he contrasts with the curved shape, but rather the conjunctions of straight lines which form the angles. He can 'feel' the points of the shape.

It is important that this early handling should precede any attempts at recognising shapes at a distance.

When the child is looking for shapes and patterns in the environment – the classroom, the rest of the school, the playground, the home – classification will still play an important part. The children can make collections (individual or combined efforts) of pictures and their own drawings of objects including the various shapes they have seen and recognised.

Rectangles
TRIANGLES
CIRCLES AROUND US

## Why this shape ?

After making collections of various shapes in the environment, the next stage could well be that of looking for reasons for the use of the different shapes in particular situations.

### Rectangles

Probably the most frequently observed set of shapes in the child's environment will have been rectangles.

Examples with different proportions, varying from the long and narrow to the square will be seen ; but investigation of the proportion of the rectangles seen in everyday life will show that a high percentage have breadths and lengths in the ratio of approximately $1\frac{1}{2}$ to 1. Many will be found to be close to the golden section ratio, i.e. approximately 1·62 : 1.

The shapes of some rectangles will be dictated by human needs, e.g. in doorways, beds, newspapers, chairs, prams and desks.

The proportions of packets and boxes may be seen to be partly based on sales technique, the aim being to present a large frontal area for displaying the brand name. It is interesting to compare a packet of this type with a container of the same volume in the shape of a cube or sphere. Where advertising plays no part, there can be seen examples of containers more or less cube-shaped, e.g. the tea chest.

The following is an account by first-year students at the Bartlett School of Architecture, of some work done with twelve-year-olds.

## Our aim

The principle that we want to try to teach these children is that buildings are generally of rectilinear form. We want to make them think about the shapes of buildings and get some ideas of why the buildings are of this form.

### Part one

**a** Ask them to draw a picture of a house they would like to live in.

**b** Compare these drawings to those of other types of house around the world.

**c** Ask them why houses as they know them are of rectilinear form.

### Part two

**a** Have a session of making various shapes – cubes, pyramids, cones, etc., and out of interest see how clearly they can conceive these forms.

**b** Try and fit these shapes together, i.e. build a house out of the shapes, making them the rooms of a house.

### Part three

**a** Ask them again why buildings are of rectilinear form, hoping that playing with these shapes will have given them some idea of how the various components in a building fit together.

**b** Introduce other ideas, such as how furniture fits into rooms, strength of structure, etc.

### The report

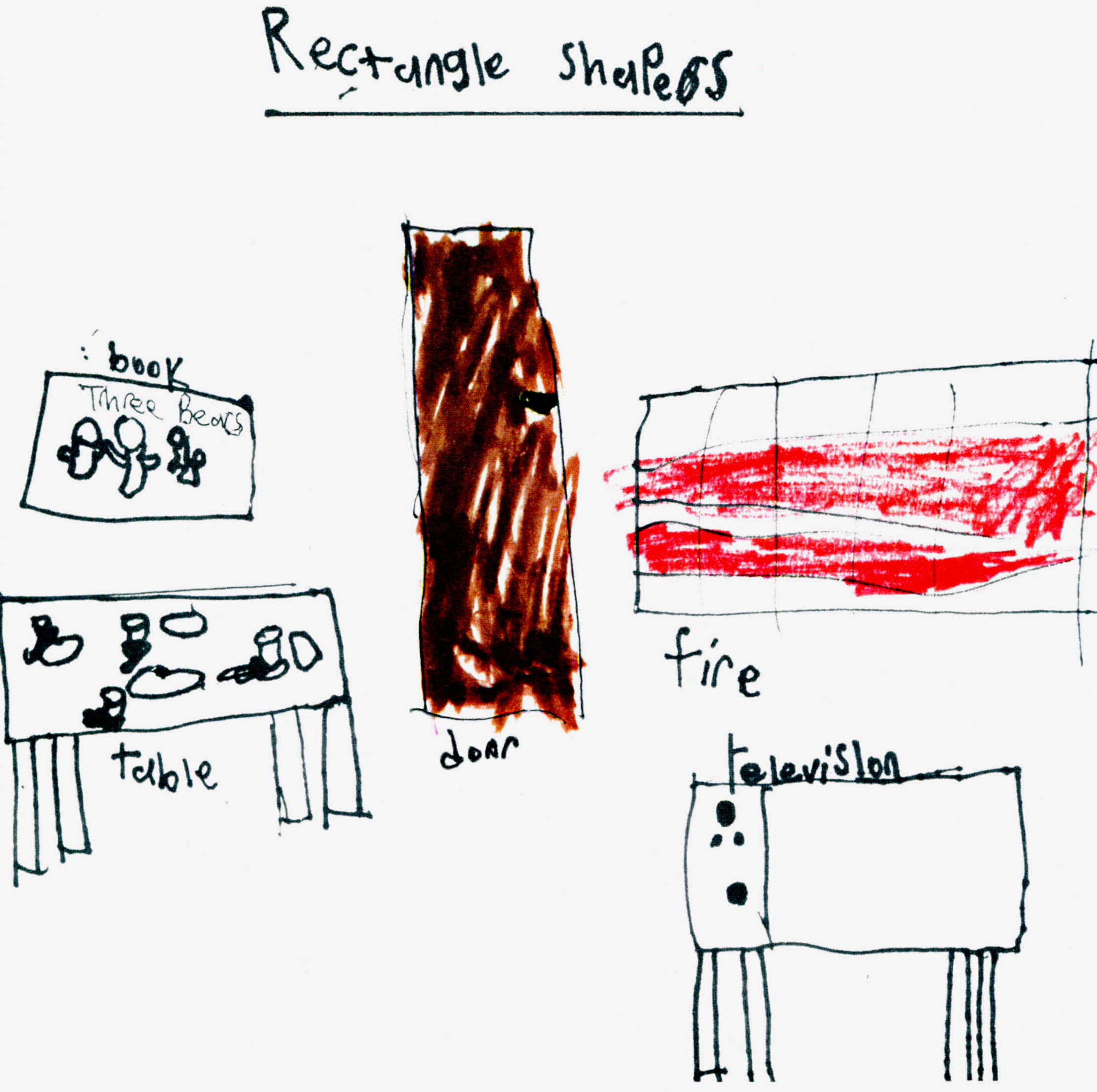
### Part one

Their drawings of ideal homes were of rectangular shape except one – a diamond-shaped plan. We found they could visualise the houses in plan/elevation without much difficulty. Then they drew various other buildings of the type they thought other people in the world lived in. This time we had a variety of shapes – igloos, mud huts, sky-scrapers, huts on stilts, pagodas, etc. Again we asked them for plans and elevations, and the shapes included circles and ovals, as well as rectangles.

Asked why the buildings were the shapes they were, all of the children were a bit doubtful. The only reason why their ideal homes were rectangular seemed to be because that was what they were used to, as one of them said. We noticed that some of our group were considerably in advance of others in their thinking. One of them suggested various climatic reasons for the shape of African mud huts, igloos, etc., but none of them could think of any good reason why our buildings are rectilinear. Some of their suggestions were :

**a** Mud huts are round so rain falls off easily.
**b** Pagodas are made of paper so they can be remade quickly after earthquakes.
**c** Igloos are small so that when they melt in the sun not much work is wasted.

After a lot of discussion one of them said it was because bricks were that shape and you could not buy round ones. But they could get no further than this.




Drawings by six-year-olds

**Part two**

To give them a slight change we then asked them to make some cubes and other shapes. They had no idea this was connected with Part one. They all said they had made these 3D shapes before, and it was very interesting to see how they got on. Most of them had difficulty in cutting out the shapes correctly, although they knew basically what was needed. One of them progressed well, making a cube fairly easily. Another made a remarkably good attempt at a sphere, but could not get it right and was depressed for the rest of the afternoon. In the end, though, they were all having success with cubes, cones and pyramids.

Our next step was to suggest that these shapes were rooms – could they arrange them as in a house? At first they could not see anything wrong with two pyramids next to each other. They simply said connect them with a tunnel. But then the brighter ones rejected this: they did not want to waste the space between the rooms. They also found it difficult to put some shapes on top of others, although there were some very intriguing suggestions on how to strengthen the structures – braces, and central columns. One boy, the brightest of the group, was very keen on the thought of triangular-shaped rooms: after all, they fitted together perfectly.

**Part three**

They seemed to be getting some idea of what we were trying to illustrate. When we asked them again about why their buildings were rectangular, they were quite definite that squares and rectangles were best – they fitted and were most convenient. The question of the strength of the shapes then came up. They seemed quite sure that vertical walls and columns would be the strongest, but there was no time to pursue this line.

We brought up the question of fitting furniture into these shapes. It proved to be very interesting. They very quickly pounced on the idea that since most furniture was rectangular, or 'almost rectangular', it just did not fit into the sharp corners of triangles or round the walls of a circular room. Again they realised this idea of wasting space, and having to have the furniture in the centre of the room.

We talked about having to build a large number of offices in a small space in cities – having to stack them up. They referred to rectangles and cubes as 'the most convenient'.

R. Hill and R. Harvey

**Triangles**

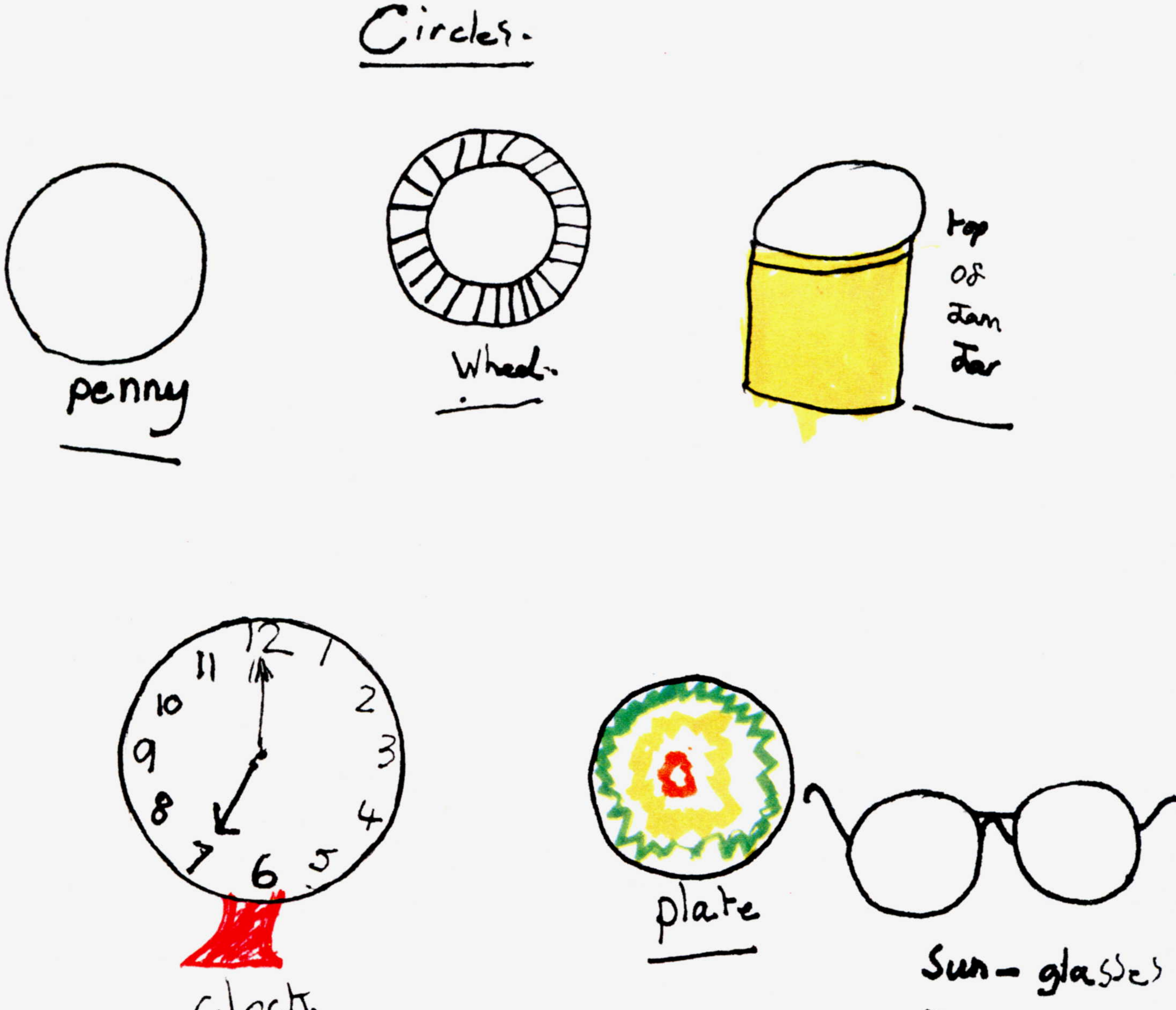
Many examples of triangles may be seen by the child in his everyday environment: in pylons, cranes, roofs, gates, etc. Simple experiments with strips of card, wood or metal will show the reason for the frequent choice of this shape by demonstrating the rigidity of structures with triangular components and the comparative weakness of those structures made from rectangular or other polygonal shapes. Details of such activities are given in 2 and 3.

Pyramids of tins are often seen in shops.

Counting the total number of tins in each layer of the pile can lead to the discovery of interesting patterns of numbers.

Practical classroom work can be done with tins or counters. As each layer is counted the total can be represented by dots on paper or with coloured, adhesive discs.

In each case the vital question to ask the children would be, 'How many tins would there be in the next layer of the pyramid?'




Drawings by six-year-olds

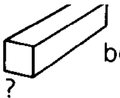
## Circles

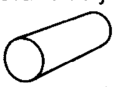
Circular and cylindrical shapes will also occur frequently in the children's observations of shapes in their environment. Again, reasons for the choice of this particular shape can be looked for.

Why are wheels circular ?

Why are tins usually cylindrical whereas wooden boxes are more often cuboids ?

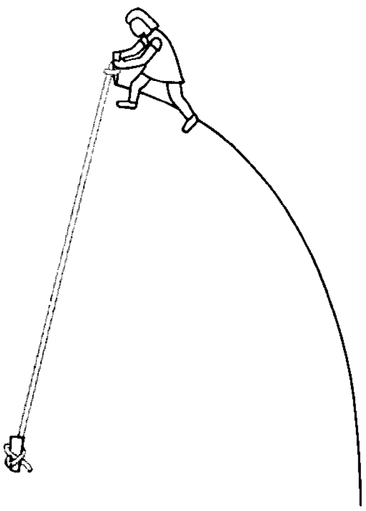
Why are nearly all coins similar in shape ?

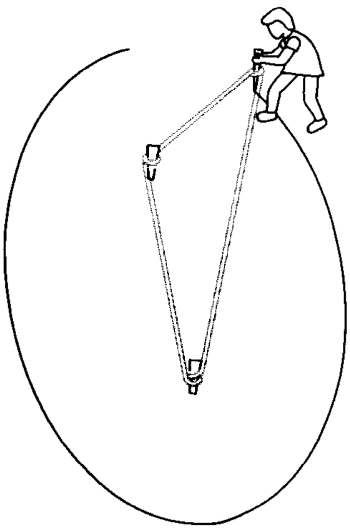
Why would a can like this  be more difficult to handle than a barrel  ?

Would it be just as convenient to have one shaped like this  instead of the usual barrel shape ?

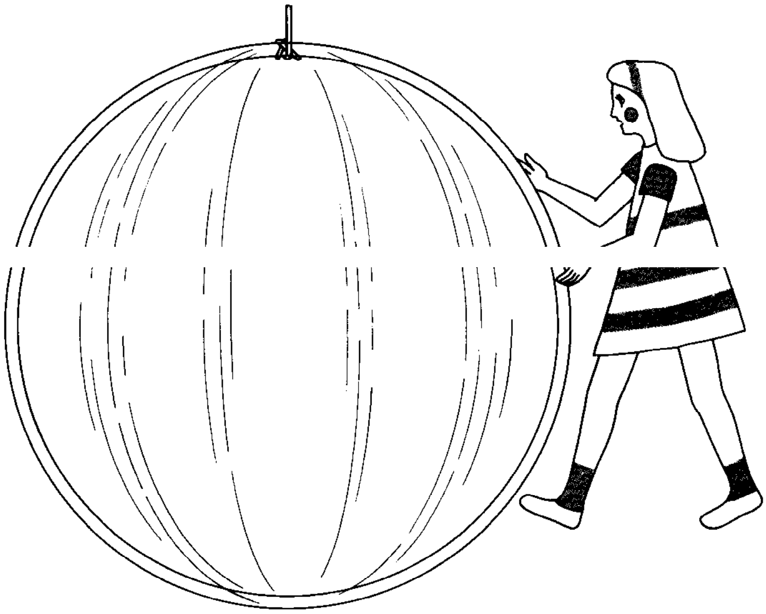
At a very early age the child will have discovered that some shapes will roll while others will not, but it is not only in his play that the circle will have been of interest. He will have seen it in his plates and cups. Later, as well as all the material examples of circular shapes, he may recognise the shape of the 'blur' made by a weight swung round quickly at the end of a length of string. This will introduce for him the idea of the path of a point. The idea can be developed by drawing circles by the 'pin and string' method or larger versions out of doors with a length of rope fixed to a stake in the ground.

With two pegs this activity can be extended to the drawing of ellipses.





A hoop, hung by a string and made to spin, will trace out the surface of a sphere, many examples of which (bubbles, balloons, balls) will be found in every child's surroundings.



## Curves

Observation of shapes in the environment will include many with curved sides, and not only those which are circular. There will, for example, be the arches of windows, church doorways, bridges, roof supports, light reflectors, lenses, car and caravan roofs, road edges, chainlinks, chairs, dams, saucers and submarines, to name only a few. All of these can be discussed to see if reasons can be found for the inclusion of curves in their construction.

In some cases they can be compared with objects where straight lines have been used. Why do cars and caravans have curved roofs whereas greenhouses have pitched roofs ? Looking for curves and finding reasons for the choice of these particular shapes will further the investigation of the rigidity and strength of differently shaped constructions. Many practical experiments, with measurable results, can be conducted to help in the reaching of convincing conclusions. This will be dealt with in greater detail in Chapter 5.

# CHILDREN'S DRAWINGS AND MODELS

The children's own drawings and models sometimes provide a starting point for discussion on shape, and they will often serve as a link between the outside environment and the classroom.

In any such discussion great care is needed in the language used by the teacher. At any stage of learning, the new knowledge must be based on those ideas which have already been formed. The danger is that descriptions, definitions and explanations may be given to the child in a form which is beyond his understanding. A logical explanation in the precise language of mathematics, which appears clear to an adult, may only confuse the child.

It is, however, of vital importance, if any progress is to be made, that words and ideas, which are new for the child, should be incorporated in discussions, but only when the need arises and when there has been a sufficient foundation of experience on which to base the new idea.

'In these shapes the opposite sides are parallel.'

This is the sort of statement which can easily be made and be accepted by the child but have little meaning (or more likely the wrong meaning) for him.

A full understanding of parallelism will be achieved only after a considerable amount of experience.

Collections of drawings and pictures of fences, railway lines, ladders, swings, milk crates, nets, gates, etc., can be made. Simple models can be made of some of these as well.

From these experiences the children will soon be able to recognise parallelism. At this stage there will be a need for the new word and the children will be able to use it with confidence in their descriptions of the models and drawings.

Understanding will still be limited and there may still be difficulty in recognising or representing parallels which are not horizontal or vertical. A competent drawing of a rhombus will prove to be more difficult than drawing a rectangle. The child's definition of parallelism will probably be in terms of identity of orientation rather than equidistance.

'The lines go the same way.'

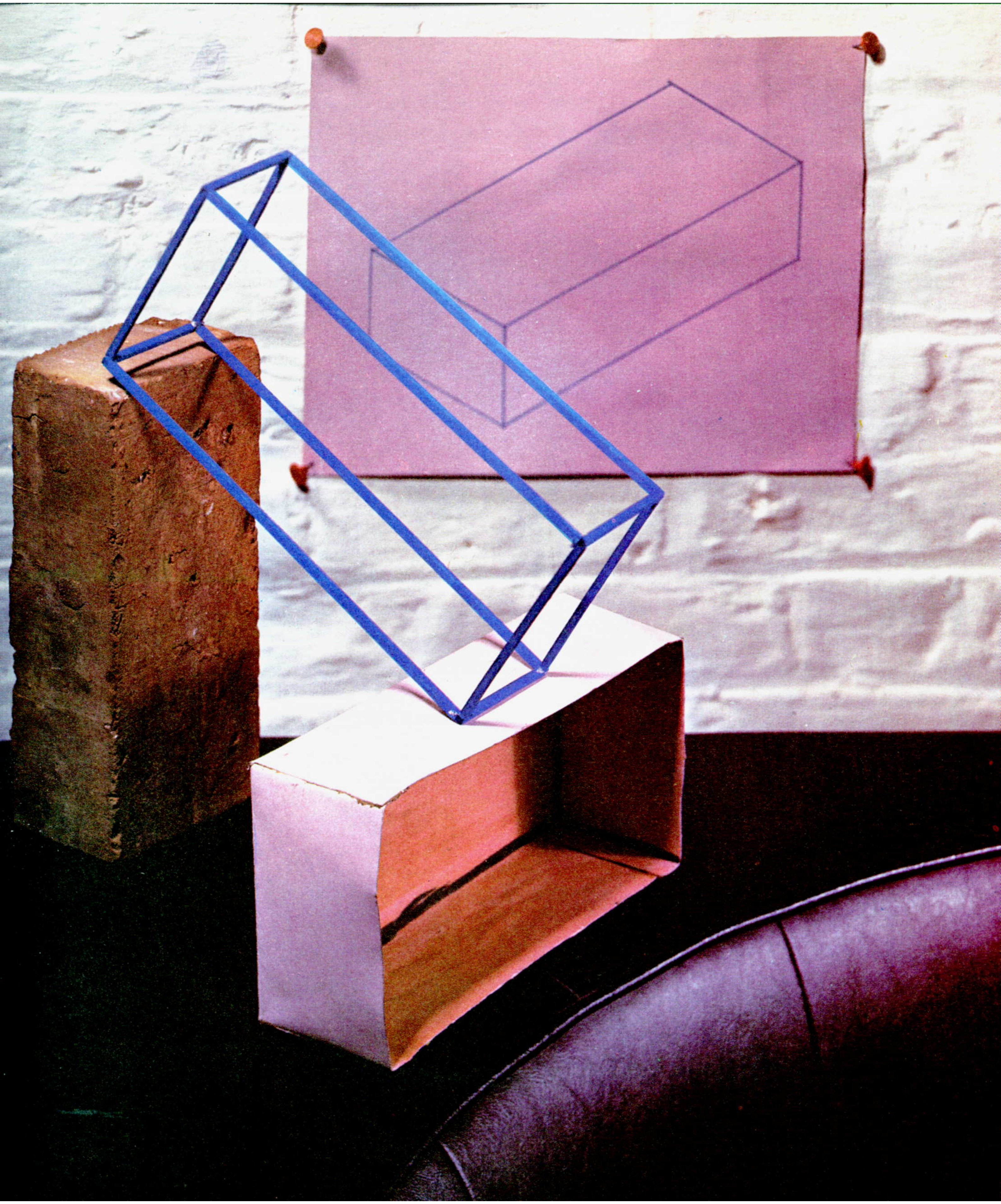
'They slant the same much.'

'They point the same way.'

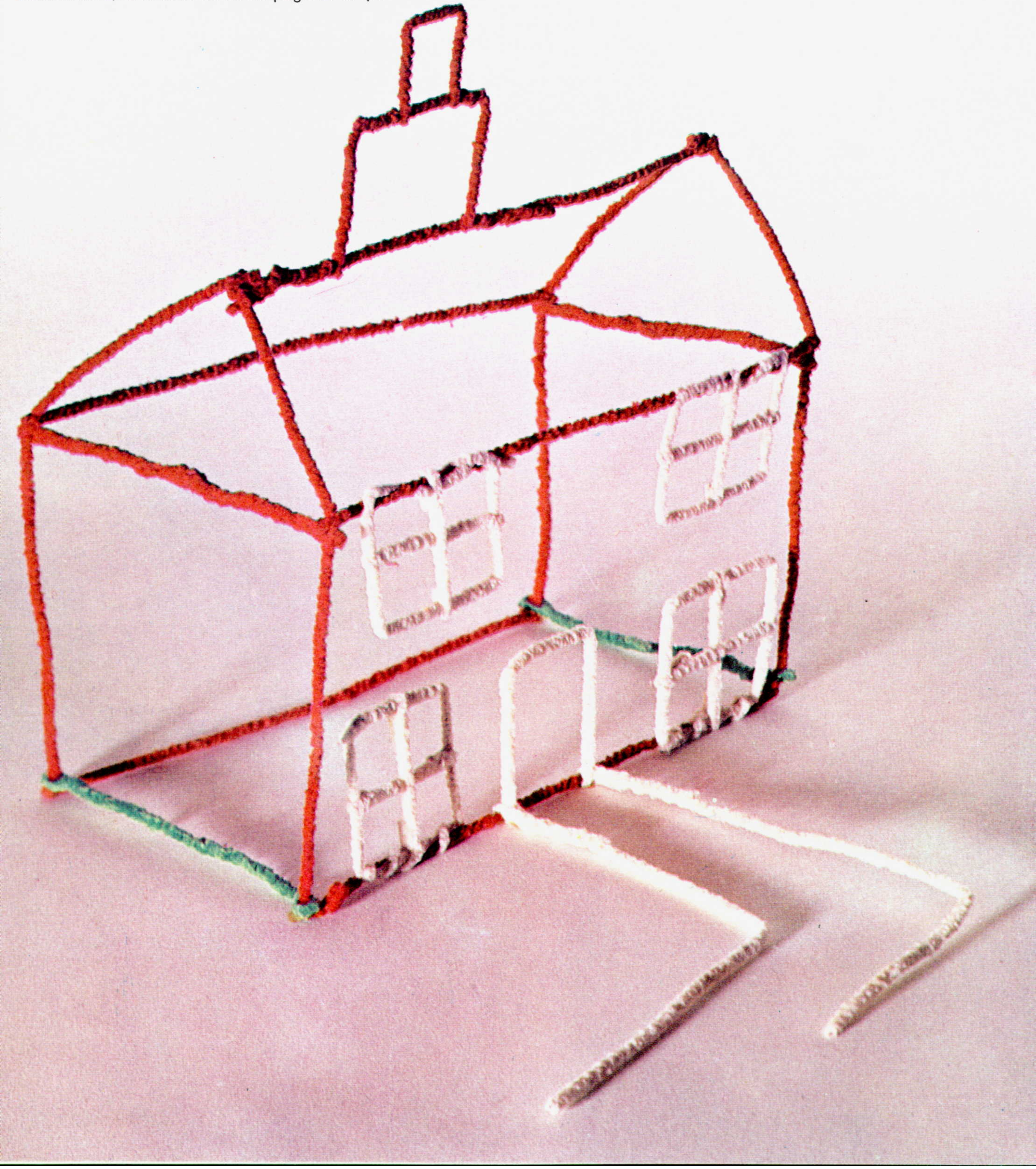
Equidistance, being a metric property, is recognised later.

It has been said already that, before using the mathematical names of shapes to describe objects in the environment, it is important that the child should have had experience in handling the shapes, but it is also important to ensure that the child has experienced the full range of possible interpretation of the terms. For example, the name 'rectangle' may arise from observation of four pencil lines arranged in a certain way, or from one surface of a cuboid, or from a relationship between four defined points on a plane surface. Work with nailboards, cardboard models, milk-straw models and blocks will help to give various experiences connected with the idea 'rectangle'. Similarly the term 'cuboid' may arise from activities with a solid brick, a container, a framework, or a two-dimensional representation.

When discussing children's drawings it is preferable not to use the drawings for instructional purposes, e.g. telling the child: 'This is a rectangle, this is a square and that is a cylinder'. Far better to use them for asking questions to see to what degree ideas have developed, remembering all the time that the child may well have grasped an idea, and be able to represent it, without being able to give a logical explanation or description of it. His ability to represent will also lag behind his understanding.



Houses feature largely in many children's drawings and models. There is much valuable material to be found and studied here, as is mentioned on page 81 of ▼.



**A shoe-box house**

The work illustrated on pages 30 and 31 was done at Landscore Infants' School, Crediton. Interest in building arose through the erection of houses near the school. The following is part of an account by the head teacher:

The children were enormously interested in watching the road being built and all the materials arriving for the building of the houses. I suggested that we might try to build a house of our own big enough to play in. The children liked the idea at once and suggested using our own wooden bricks, real bricks, pieces of wood, etc. By discussion, each of these was rejected, e.g. we need the bricks to play with, real bricks would be heavy and expensive, wood was less expensive but would the resulting house look like those on the building site? I suggested we must build the house indoors as we were beginning a term which ended in winter and there might be many days of bad weather when we could not build it nor play in it when it was finished. One child also suggested that if we built the house with planks of wood we 'couldn't watch it grow' as we could when bricks are added. Finally I asked the children if they could think of any sort of box which was always the same shape and which might easily be collected from shops in our town. One child suddenly thought of shoe-boxes. So we started to collect shoe-boxes from the three local shoe shops. As they began to arrive we very soon saw we had to sort them out into a rough measure of size, i.e. children's, ladies' and men's shoe-boxes. The mass of boxes grew so quickly that we decided we must discuss how big the house was going to be.

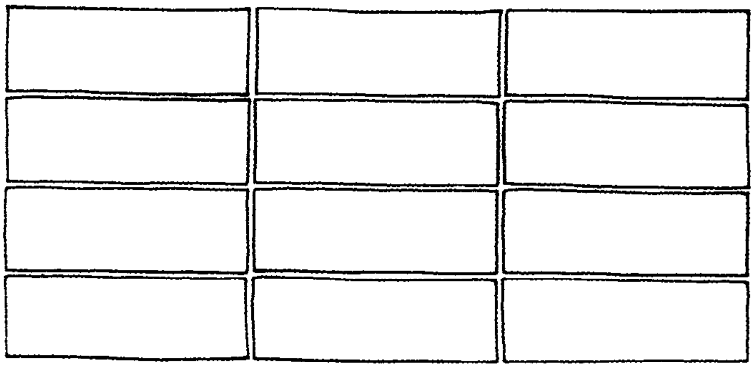
We had to ask ourselves the following questions:

Must it be taller than you?

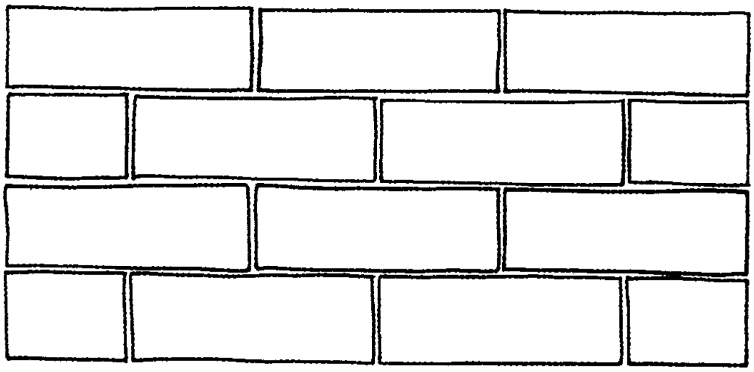
How wide must it be from back to front if we have a small table and stools in it?

How long must it be? If we play houses do we need to lie down in it? etc.

(We are very limited as to space in the classroom because although it is a big one it has to be used for everything in turn – as dining-room, for music and movement, PE, parents' meetings, etc.) Eventually it was decided that if we chose the three tallest children and asked them to lie down on the floor side by side we could surround them with shoe-boxes and then when the children stood up again they could see if they could move about in the space. We found the minimum number of shoe-boxes we could use was eighteen. Robert immediately said, 'But you must have a door', so we removed two shoe-boxes and finally decided to have a foundation of sixteen boxes. So now we tried to stack the boxes against the wall of the room so that they were ready for the builders. We found a stack of sixteen too high and unsteady so we stacked them in two eights. When we had enough to build the house three bricks high, i.e. three stacks of sixteen (six stacks of eight) we began to lay them out with the second row exactly matching the first row, but at once Derrick (who lives in a new house on another building site) said, 'That's no good because they will fall over. You must put a brick over where two other bricks join'.



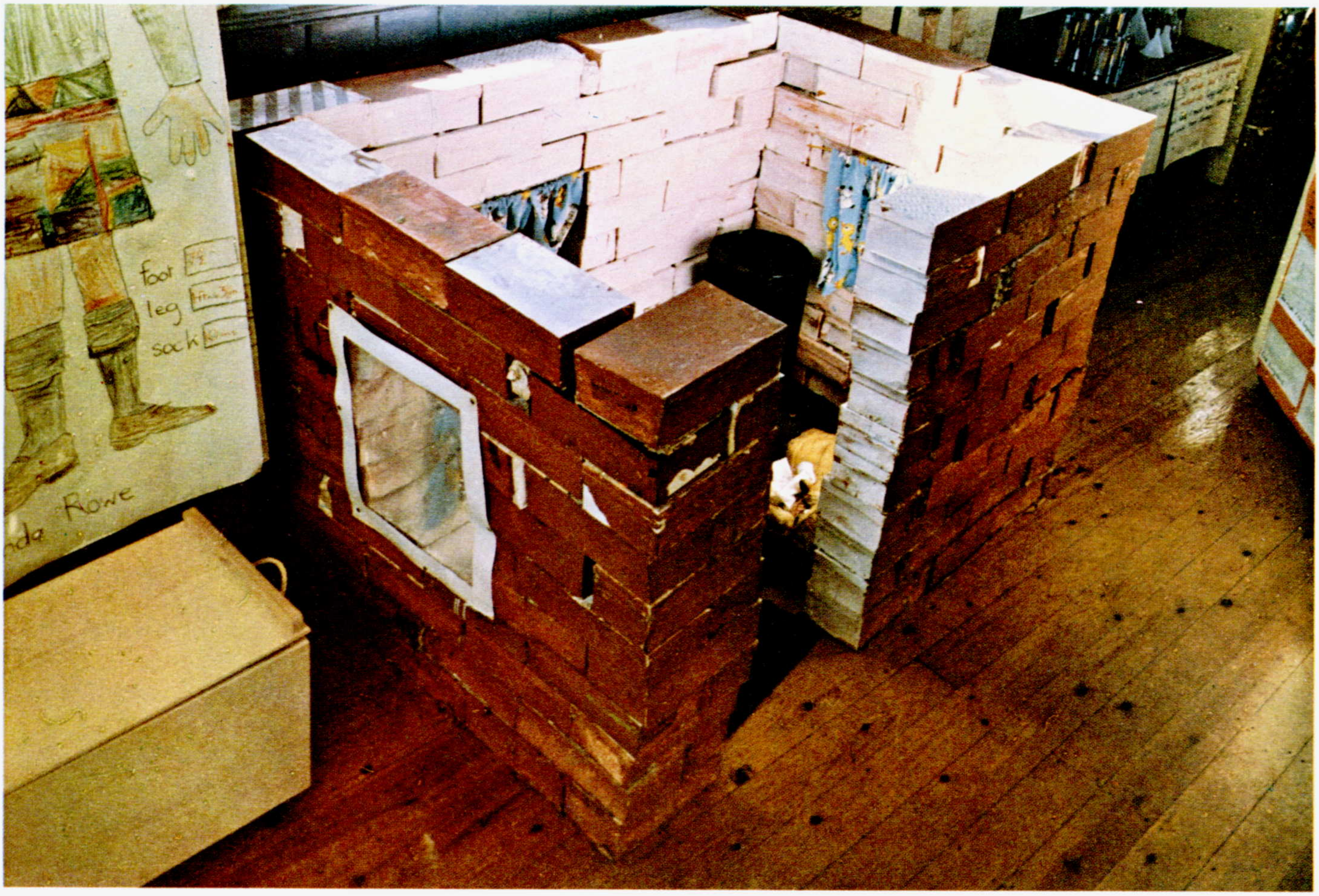
So he showed the children how to arrange them, thus:



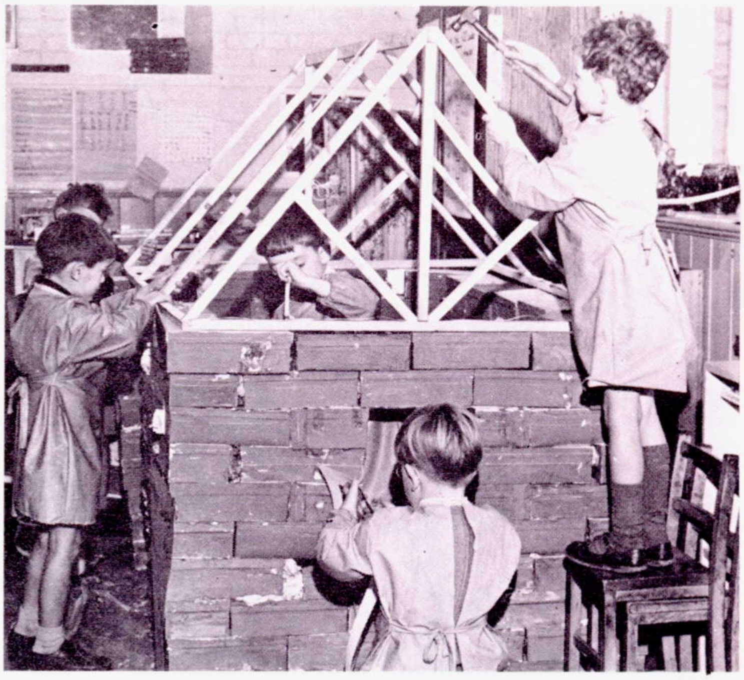
While they were only arranging them and not 'cementing' them they found that they could just estimate where to start the first stack in the second row, but when we began to build properly we measured the first box on the right-hand side of the doorway with a strip of inch-squared paper. We cut a piece the correct size, then folded it in half and marked the half-way mark on the shoe-box. We started the second row from this mark. At the corners the shoe-boxes were turned endwise on but, of course, the children found that when we got back to the doorway we had two half-spaces to fill. So we found we needed fifteen whole and two half boxes (with ends covered in stiff paper) to fill the second and every other row.

Next we discussed ways of sticking them together and by now the cement mixer was at work on the site and the children watched the process. I suggested we used a strong paste like Polycell and torn-up pieces of newspaper to make the 'bulk' so that we would produce a thin papier-mâché. Two children measured pints of water into a bucket and found it needed eight pints to fill it. Others read the instructions on the Polycell packet and found it needed two ounces of Polycell for each gallon of water. The children knew from water-play that eight pints made a gallon so they took it in turns to weigh two ounces of Polycell, add this to a bucketful of water and stir in torn-up newspaper until it was of the right consistency. This quickly proved too sloppy to be stirred easily, so we used two buckets with four pints of water in each and one ounce of Polycell in each.

There was a tremendous amount of discussion, measurement and computation before the house was finally completed (with help for the roof from one of the fathers).


ndscore building
site.



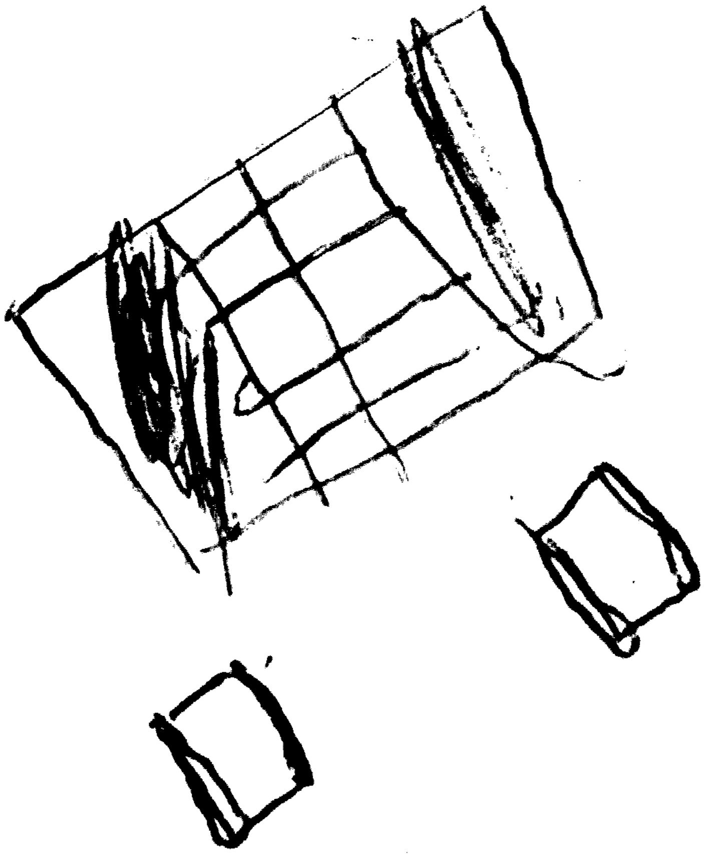

Horse Shoe
House
caster says he will build
s, 16 bungalows and 16
galows.

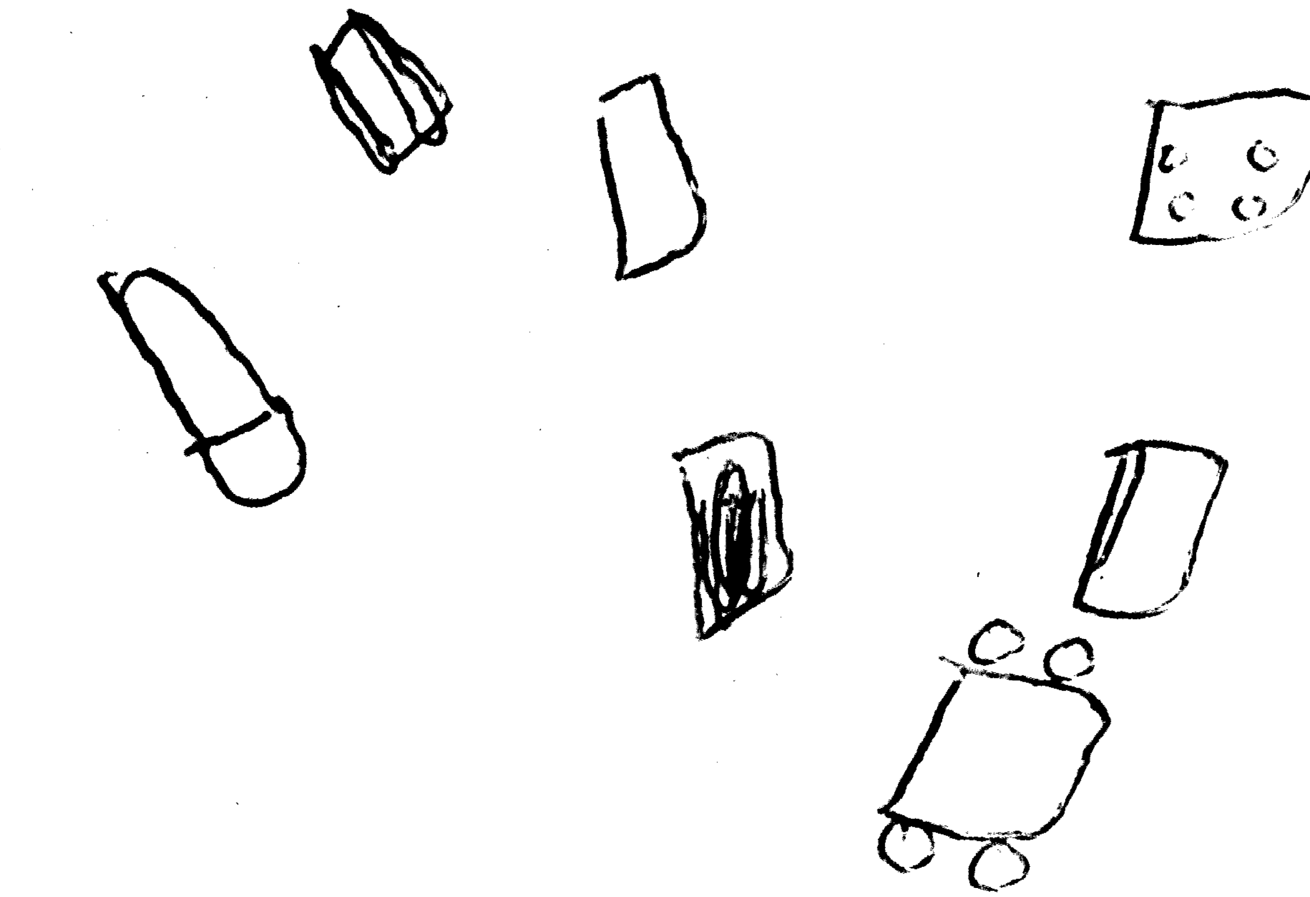
## Plan views

In Chapter 2 there were some examples of children's first attempts at drawing plan views. We shall now see how progress can be made from this stage. In order to draw a plan, children must be able to determine the position of each part in relation to the next and reduce the size of all the parts to a common scale.

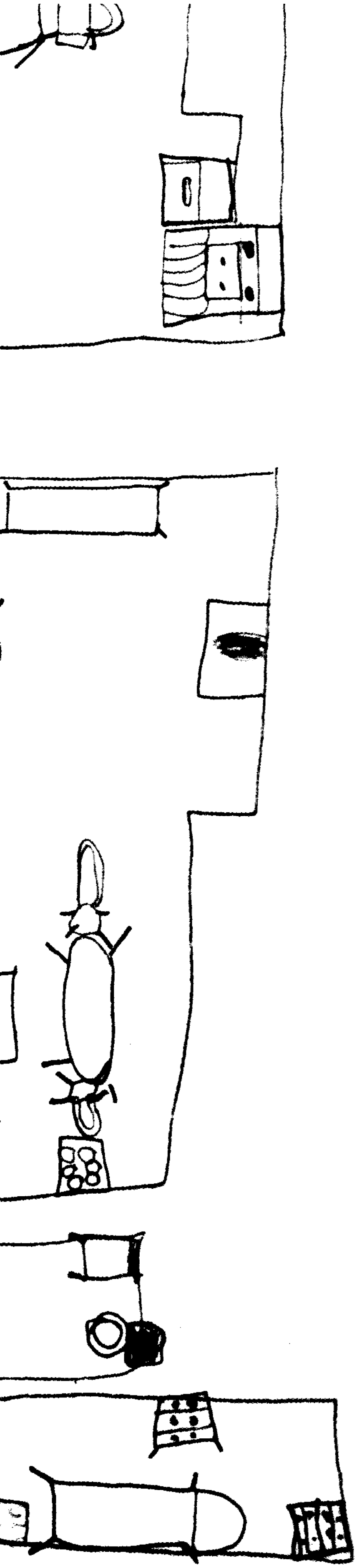
The following home plans show clearly the stages of understanding reached by different children.

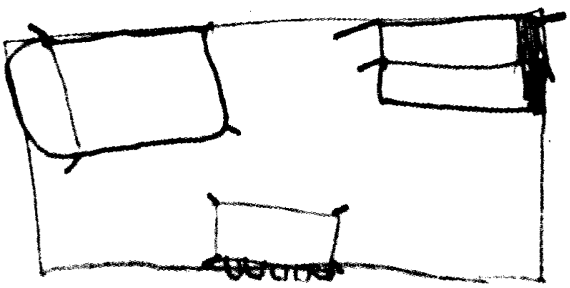
In the first drawing the house is seen only as a collection of quite separate objects. Discussion with the child showed that the size of each part of the drawing depended not only on the actual size but also on the importance of this particular part to the child. Further drawings and discussion showed that he found it impossible to draw the shape of the house itself, without furniture. Immediately after finishing the drawing, he was able to name each of the separate shapes, but later he was not able to do this.

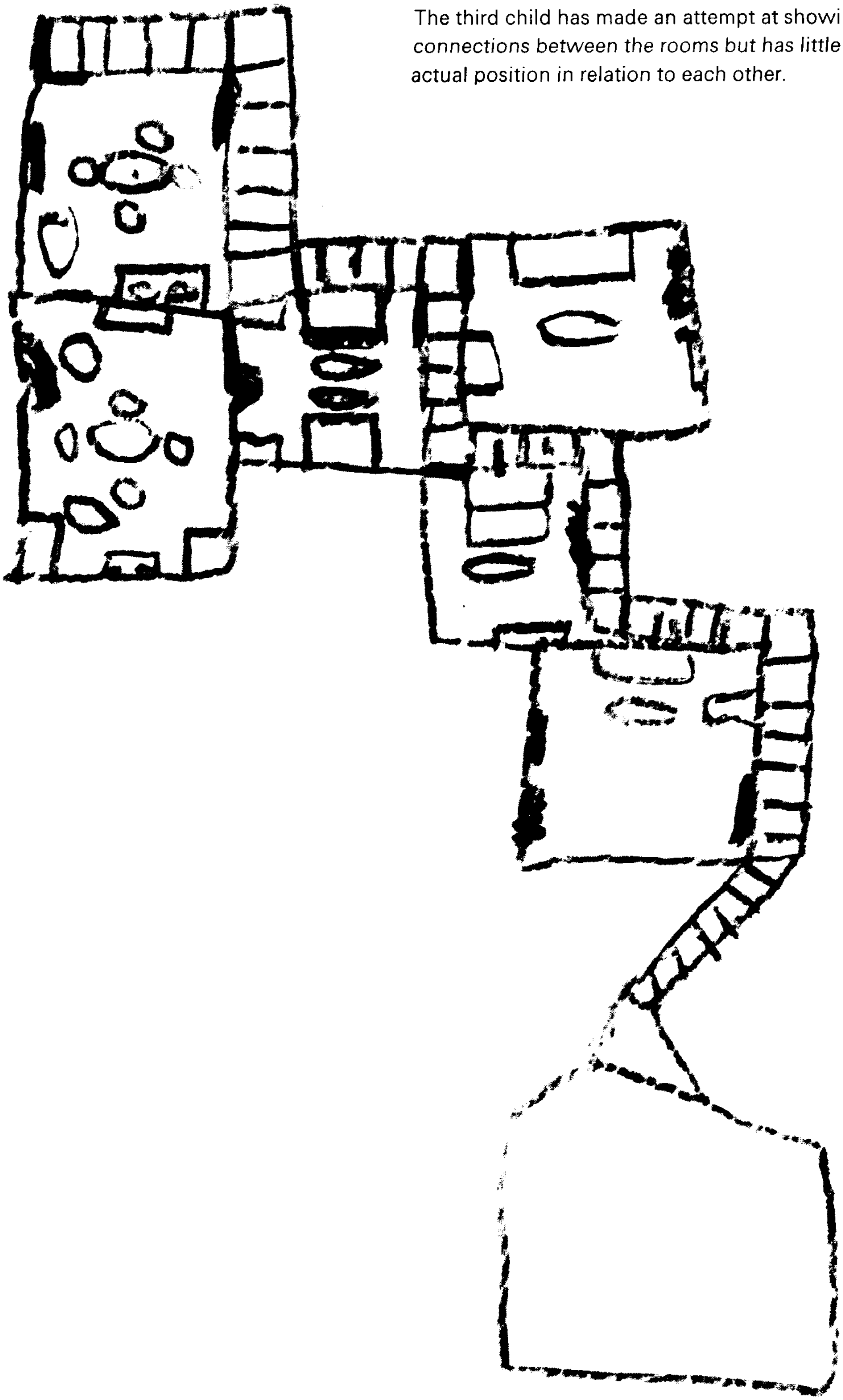




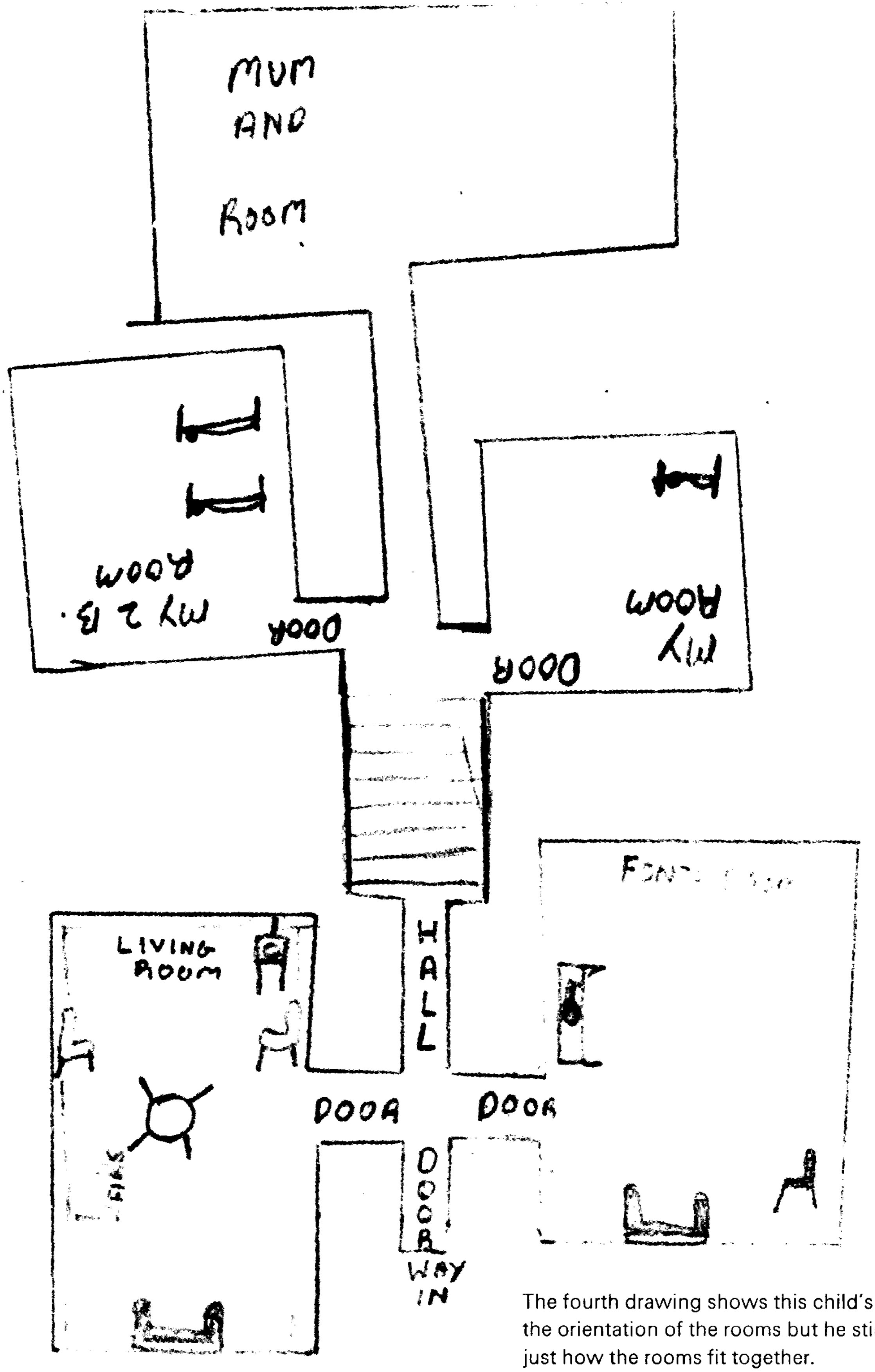
In this drawing the furniture has been enclosed but the rooms are thought of as separate units and there has been no attempt to show the connections between the rooms.







The third child has made an attempt at showing the *connections between the rooms but has little idea of their* actual position *in* relation to each other.




The fourth drawing shows this child's deeper understanding of the orientation of the rooms but he still has difficulty in seeing just how the rooms fit together.

## Points of view

In order to make a drawing it is necessary first of all to select a viewpoint. The separate parts of what is being drawn must then be represented as they would be when seen from this viewpoint.

Experience in constructing a view as seen from different positions can be given using models already made by the children, collections of assorted objects, and real situations outside the classroom.

> Look at the model of the village we made last week and draw it. Now, pretend that you are sitting in Veronica's seat and draw what you would see from there.
>
> Draw the village as you would see it if you were hovering above it in a helicopter.
>
> Draw our school as you would see it from:
> the bus stop,
> the middle of the field,
> the gate,
> the block of flats,
> a helicopter directly above.

Consideration of different viewpoints can be given in discussion. This will be particularly useful for children who are unable to express themselves with good drawings.

Some children could be seated in a circle around a group of objects.

> Look at this collection of objects (say a ball, a matchbox, a biscuit tin, a pencil, a stick of chalk, a marble, a doll and a toy car).
>
> John, if you were sitting in Ruth's seat would you be able to see the marble?
>
> Can you see it, Ruth?
>
> And so on.

An arrangement of objects like that in the photograph below can be used in association with the set of photographs in the inside back cover pocket. The arrangement should be set up on the floor. The photographs are views (from the eye-level of a child) of the arrangement as seen from eight different positions, together with a plan view.

The children can be asked to select from the set of photographs the one corresponding to the view they would have from a given position. They can then move to that position and compare the photograph with what they can see. First attempts could be made with a smaller range of views, perhaps just three or four.

Later, they can put the plan view in the centre position on a prepared sheet of card and then try to put the other photographs in the correct position around it.


Hall
5

**Connectivity, orientation and position**

The model of the main units of a school was made with expanded polystyrene bases and cardboard walls, the only details included being the doorways. The separate units were joined by paper clips.

A model such as this could be made by juniors who were capable of handling the problems of scale and could carry out the work with sufficient skill to make possible the use of it for further activities by themselves and by younger children.

The following ideas for assignments based on the model can be selected and adapted to suit the level of attainment of the children concerned.

By having the model made as a set of separate units it provides first of all an assembly problem in which the children must think of the position and orientation of each unit. Where there is doubt about the arrangement of the units the children can be encouraged to look at the parts of the school concerned before continuing with the assembly.

Which is the largest room in the model?
Which is the smallest room in the model?

Cut out a piece of squared paper which will just cover the floor of Classroom 2.

Will the paper fit in all the other classrooms?
Do you think all the classrooms are the same size?

Will your piece of paper still fit in the classroom if you rotate it half a turn?

In how many different ways will it fit into the classroom?
(See work on rotations in 3).

What shape is your piece of paper?

Cut out a piece of squared paper of the right size to cover the hall floor.

How many different ways can you fit this paper into the hall?

Make a list of all the rooms and name the shape of the floor in each case.

Put a block in each room in the place where the teacher usually has her table.

Put the trees, gates, cars, sand-pit, etc. in their correct positions outside the model.

Arrange the rooms in order of size of floor.

Use one long strip of paper to make a model of the walls of your classroom. Before joining the ends paint on it the doors and windows.

Cut out a piece of squared paper which just fits inside. Mark on it with a cross the position of your desk.

Mark on it with a dot the position of the teacher's table.

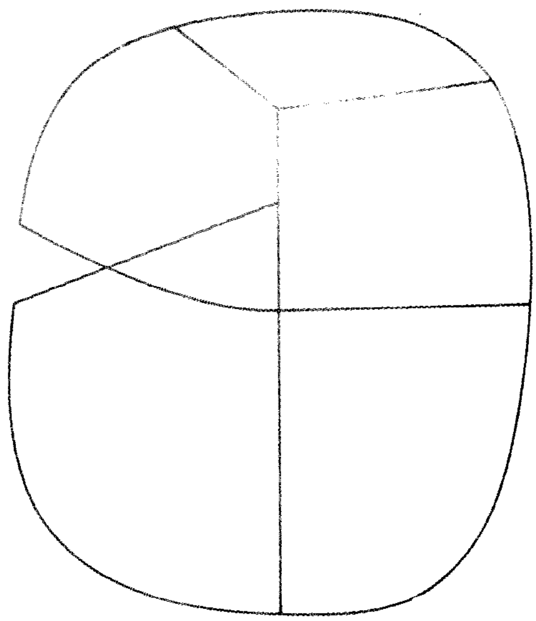
How could you describe the position of your desk in the room?
(See work on co-ordinates in 2.)

How many different routes could you take in walking from the teacher's desk in Room 2 to the teacher's desk in Room 5?

Lay out pieces of wool to show all these routes.

Which is the shortest route?
Which is the longest route you could take without walking along the same part twice?
Could you walk along all these paths without covering any part twice?

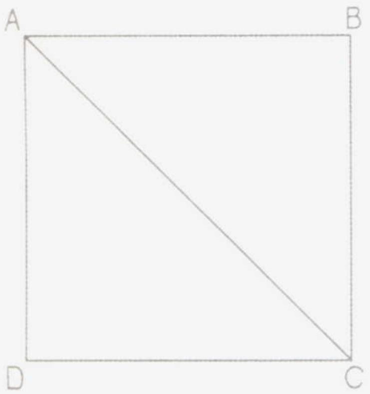
To carry out this last assignment it would be helpful for the children to draw a simplified diagram of the network of routes. In this particular model the diagram would be something like the one below:



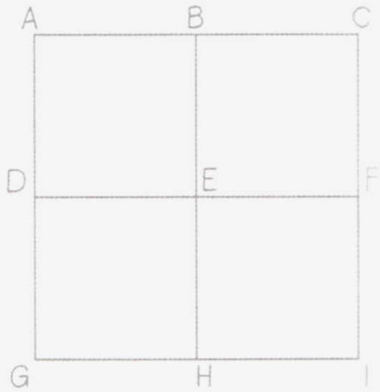
This is now a topological problem. The points where the lines meet are called vertices and the routes are called arcs. The problem can be solved by looking at each vertex and counting the total number of arcs which meet there. If an odd number of arcs meet at the vertex it is called an odd vertex; if an even number of arcs meet at the vertex it is called an even vertex. If the network has only even vertices it can be traversed in one journey without covering any arc twice. The start can be made at any vertex and it will be possible to return to the starting point. A network with two odd vertices can be traversed in one journey, but it is not possible to return to the starting point. The start and finish will be at the two odd vertices.

In this particular diagram there are eight vertices and six of them are odd, so it will be impossible to traverse this network in one journey without going along the same arc twice.

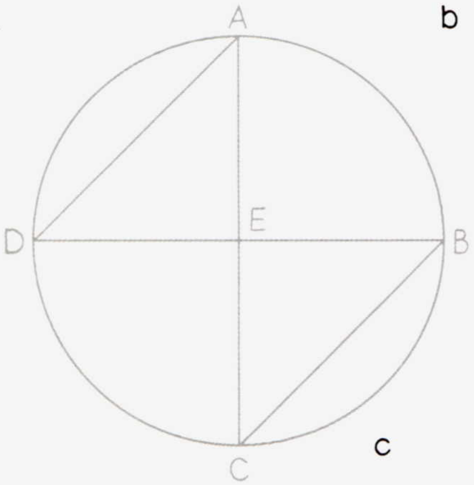
Other networks, perhaps based on street plans, can be investigated.




a




b



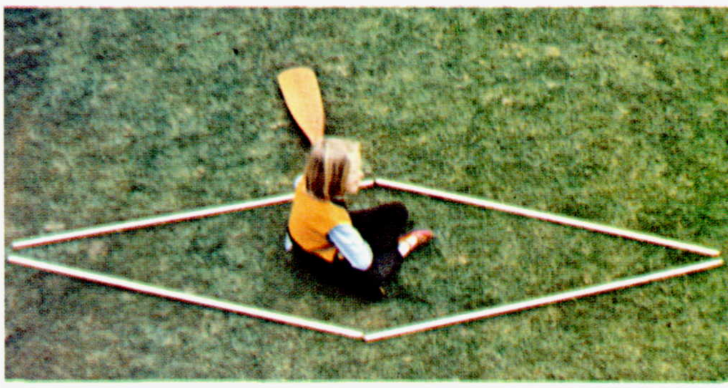

c

| | Number of odd vertices | Number of even vertices | Can it be traversed | Starting where | Finishing where |
|---|---|---|---|---|---|
| a | 2 | 2 | YES | A or C | C or A |
| b | 4 | 5 | NO | — | — |
| c | 0 | 5 | YES | Any point | Same point |

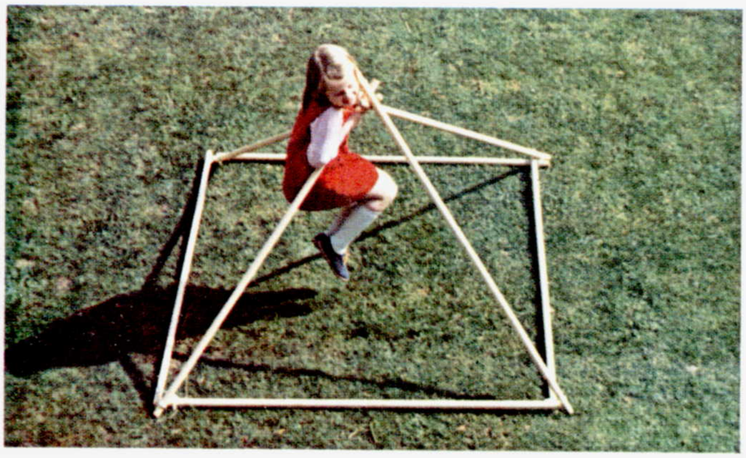
## On a larger scale

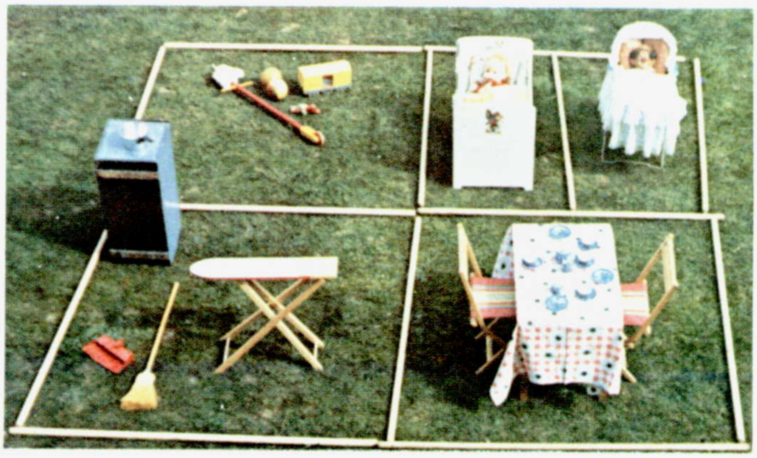
Interesting and profitable investigations of the properties of shapes can be carried out with milk-straw models. Some children, however, may well be more convinced about the strength and rigidity of a structure by using larger materials.

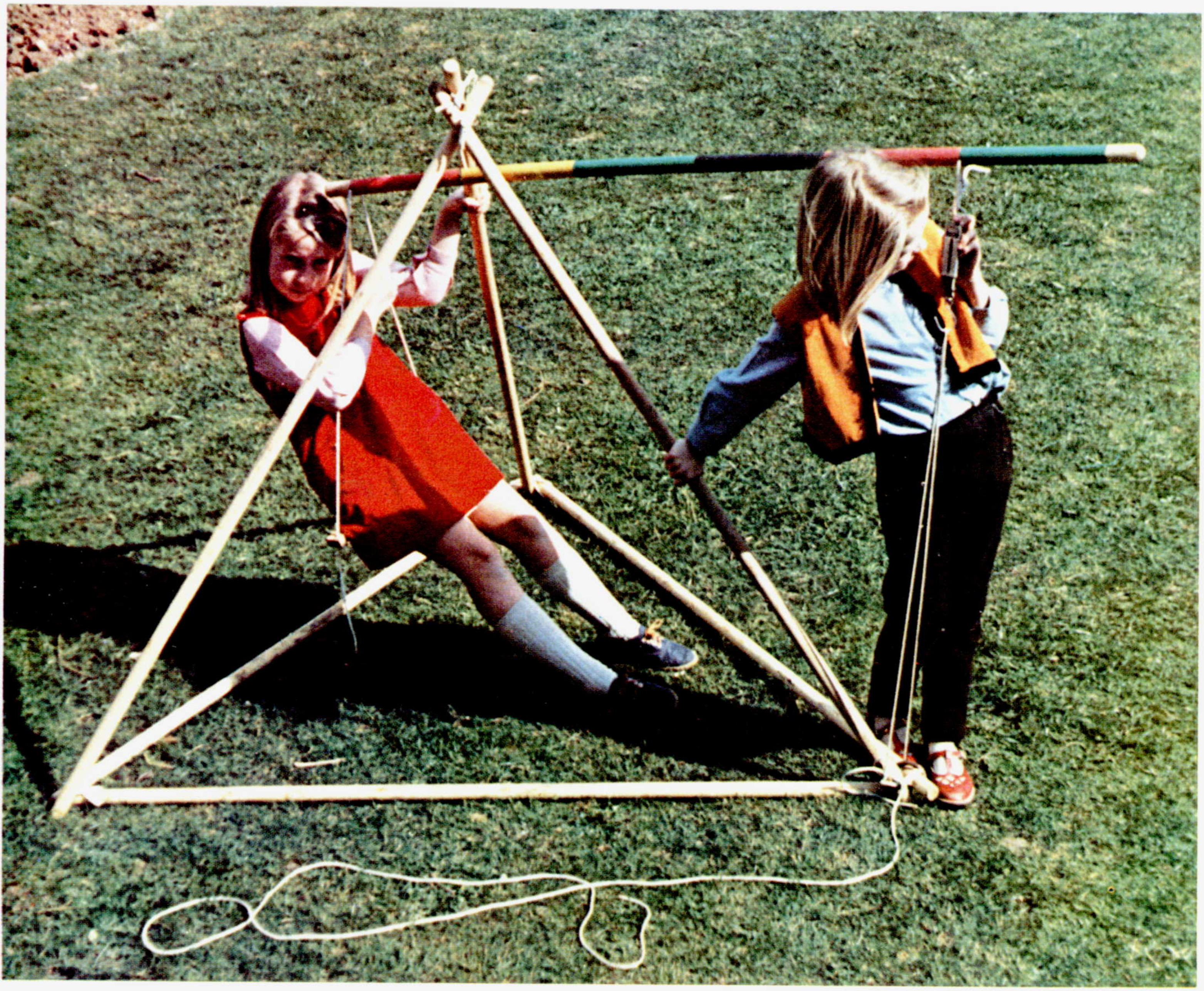
The models shown on these pages were made with broom handles 5 feet long and $1\frac{1}{8}$ inches in diameter. The ends were drilled with $\frac{1}{4}$-inch holes so that the poles could be joined up by lengths of $\frac{1}{2}$-inch (rope is measured by circumference, not diameter) terylene or ulstron cord, as used on boats. This type of rope is particularly suitable because the ends can be heat sealed to make them firm.



Left to experiment with the poles and lengths of rope, one group of children quickly made a tetrahedron tent. Some younger children made plane shapes. Others laid out plans of houses. One group made a balance and developed this into a means by which they could weigh themselves, using a 25-lb. spring balance.







# STRENGTH FROM SHAPE

The 'broomstick' activities described earlier can introduce the idea that the strength of a construction will depend on its shape as well as the nature of the materials from which it is made. Many other practical experiments, with convincing conclusions, can be carried out by children.

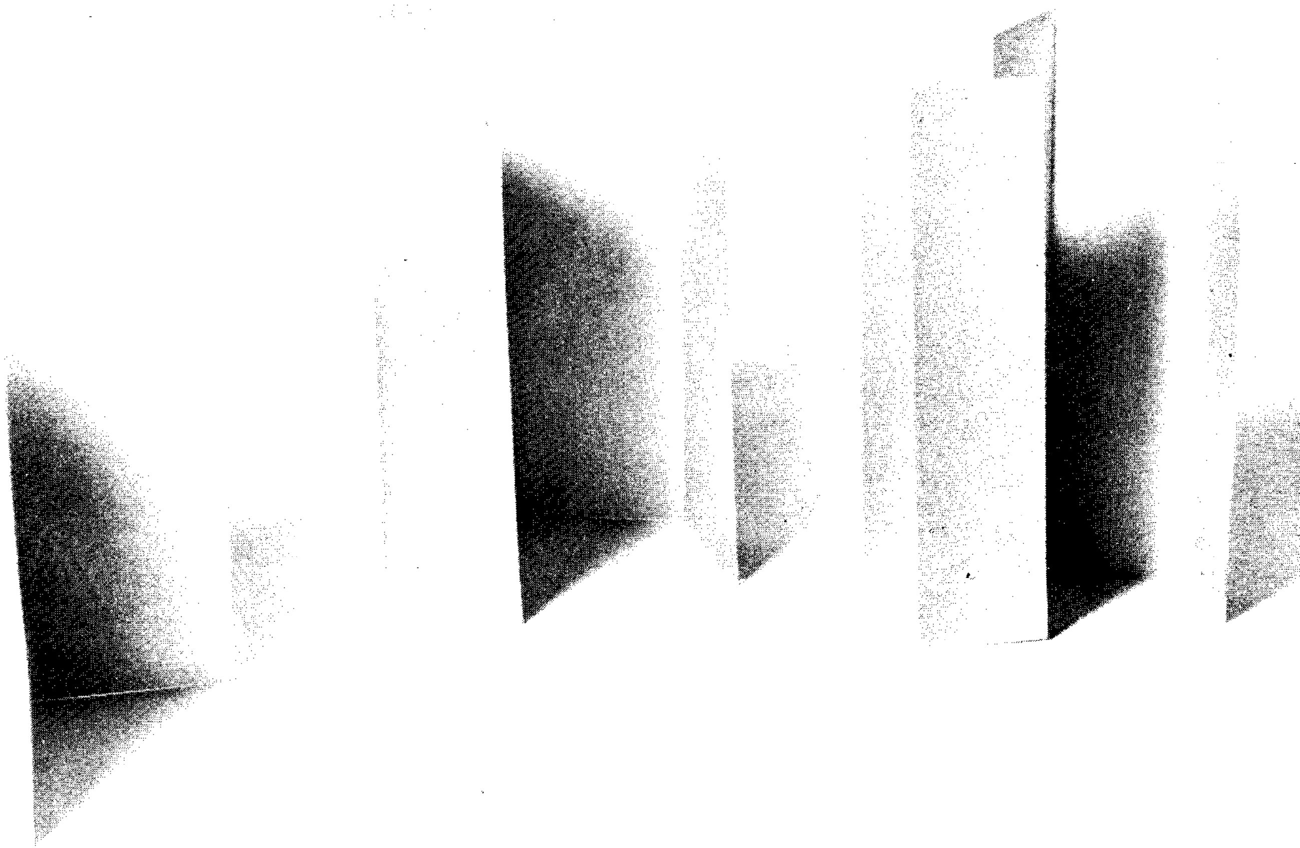
This chapter is devoted to reports by two groups of first-year students from the Bartlett School of Architecture, on work done with young children.

1 We aimed to show that strength could be achieved in materials, normally weak, by particular shaping.

a The children were given a sheet of paper, and asked to stand it on end in such a way as to support the largest possible weight on top of it.

They made one fold in the paper straight away but took some time to make this single fold stand up. Some time elapsed before they folded it more than once. Asked why it stood better when folded, one said, 'It's got a sort of stand really'. Another said, 'It's got three legs'.

We next asked them to support a book on the paper but their simple foldings could not do this. One suggestion was to halve the height but we said they should keep the full height. One folded the paper in the form of a 'square' which supported the book for a few seconds before collapsing.



Eventually, one rolled the paper and fixed it with tape. This supported a book easily, so the others soon started rolling their sheets of paper. Some rolled them tightly but they fell over. The looser rolls, having a wider base, stood up.

After this, different methods, such as using shorter, thicker cylinders were used but none of the children corrugated the paper.

One put his rolled paper in an inkwell and then placed his books on it. He said he had found a way to stop it bending and falling over – 'just like a home with foundations'.

Another used four sheets of paper; although he found that it supported more books, he realised that the increase in load was not directly proportional to the increase in the number of sheets used.

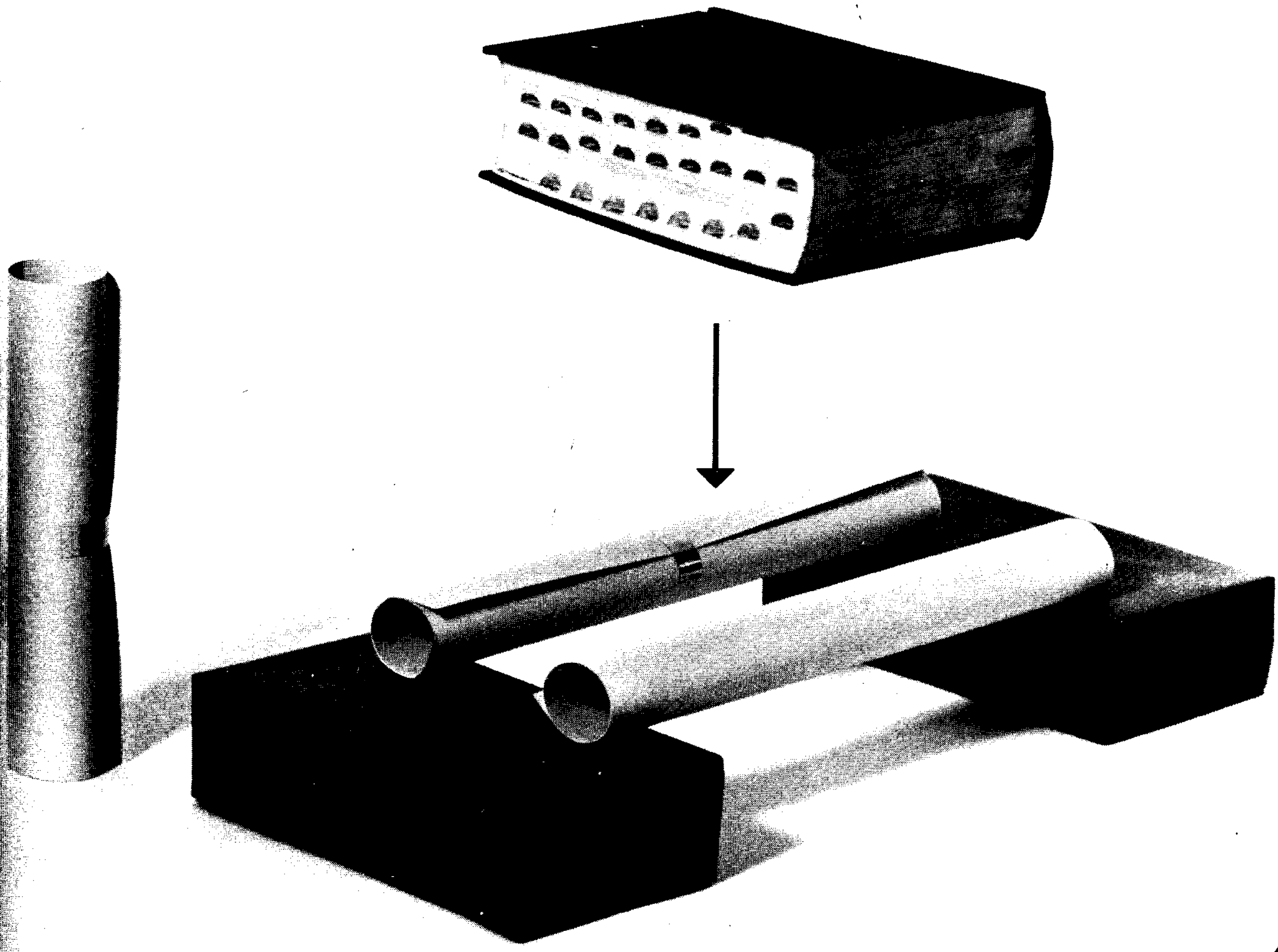
**b** The children were given another sheet of paper, and two piles of books with a gap between them. They were told to make the paper span the gap and support the largest possible weight.

They soon found that the paper had to be folded to support any weight.

Using the cylindrical form, previously discovered, they found that two rolls were needed for stability.

**c** Using strips of wood to bridge a gap, they discovered reasons for the ways in which some girders are constructed, as for instance when one strip was put on edge, with another flat on top.

Gary Vince   Roger Simpson   Hugh John



2 We hoped to teach a very elementary appreciation of structural morphology – strength from shape.

a The children (aged 10 to 11) were asked to make a sheet of paper stand in the shape of an arch to support a weight.

They found that the arch kept falling flat because the edges slipped on the table. Some thought of sticking the edges to the table to prevent them slipping. We suggested that they try to solve the problem in some other way that did not rely on fixture to the table. We asked them to observe the way in which the two parts of the arch moved when collapsing and to try to prevent this. Two had the idea of joining the ends with sticky tape.

b We asked the children to make a tripod out of three pieces of balsa wood. All except one did not know what a tripod was, so we had to explain. The results were poor, but all the tripods stood up. We then asked them to push on the top and to observe the result. They realised that the legs splayed

outwards but they did not realise that the correction called for was the same as in a. They all tried to make the top joint stronger, applying more tape or plasticine. When this did not work, two connected the ends of the legs with tape. The rest followed suit when they realised the resulting strength.

**c** We asked the children to bridge a small gap with slabs of plasticine, and find the maximum weight they could support on the bridge. They did not make a tall, thin beam stand on its edge but always on its side. They confused strength with stability, saying that the beam on edge was going to fall over. They were not very convinced when it was demonstrated to them that it was, in fact, stronger and that flanges at the ends could prevent it from falling over.

**d** We asked them to make a frame, as rigid as possible, from balsa wood strips. They concentrated on the joints, strengthening these with tape. We asked them to look at the relative positions of the joints when the frame was distorted – how one diagonal shortened and the other lengthened.

They realised that wood was necessary to stop the diagonal distance becoming shorter, but that string could stop the diagonal distance increasing. They understood that the string was in tension and that the wood was in compression.

Stephen Murray  Ralph Parks

In experiments of this type, and developments from them, there is no need to take actual measurements, in standard units, of the weight supported; it may be sufficient to note the proportionate increase, or decrease, e.g.
twice as much sand in a tin supported on a bridge. There is therefore no need for elaborate measuring equipment but, of course, if considered desirable, the work could give extra practice in weight measurement, using the usual equipment.

Collections of pictures of bridges, roofs, girders, etc., will help to give background for these activities.

# ROUTES AND DIRECTIONS

Traditional school work in geometry has been largely confined to the study of static shapes and these in two dimensions only. Geometry in the environment will be very much concerned with movement. From very early days children are concerned with problems of movement, in terms of direction and distance. Within reach or out of reach? Which way? Within walking or crawling distance? Across or round the outside? Through or under? These are all matters of great importance in early years.

As with shape the first notions of movement will not have measurement as their main concern – continuity and connectivity will be of more importance.

First attempts at showing a route will not be to scale.

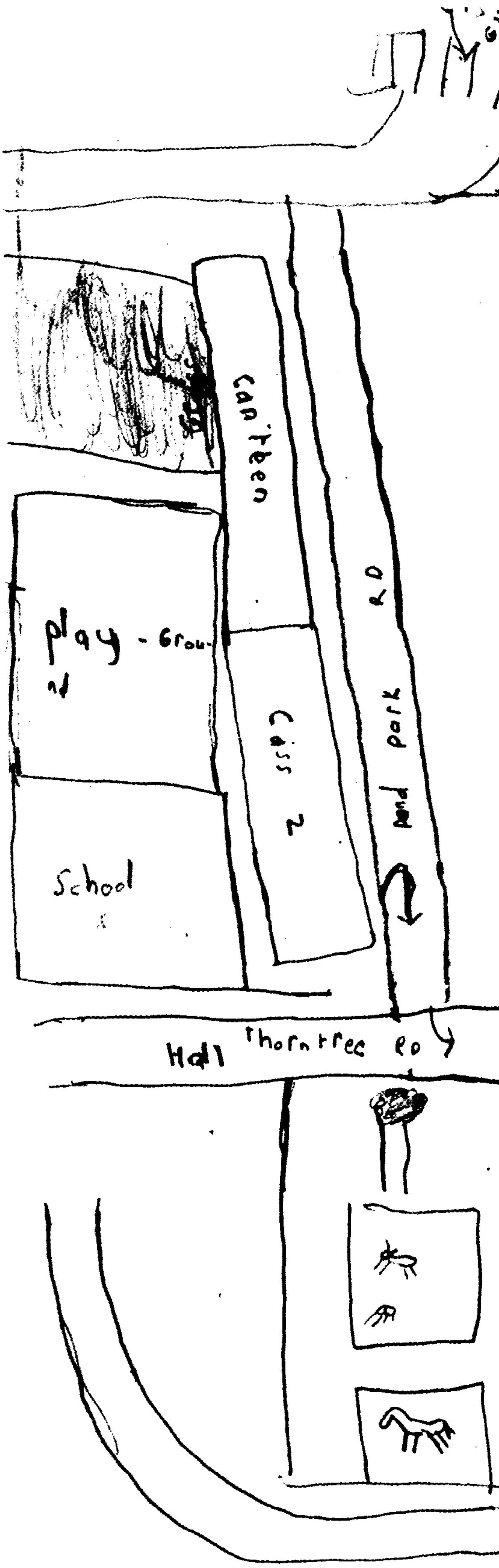
From discussion of routes, eventually the need will arise for having some means of indicating direction. This may have been achieved by pointing, or by noting the position of a point relative to another fixed object.

A further stage in the communication of direction could be that of giving the fraction of turn needed when turning from facing in one direction to facing the object under discussion.

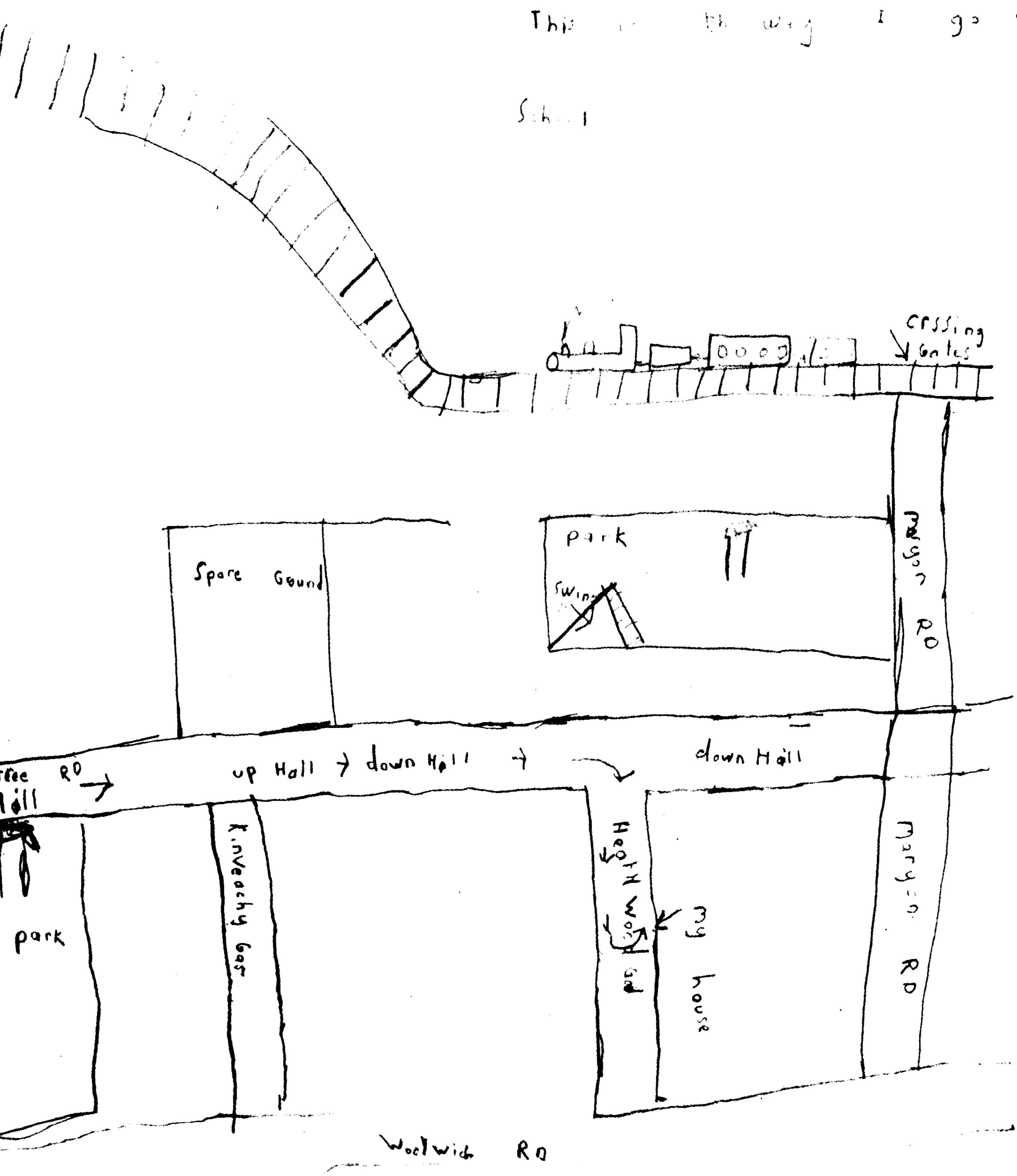
The need for a standard reference line should soon become apparent and from this a gradual progression can be made to the various means of stating directions:
three o'clock on the starboard bow;
N.E. by E;
N. 36° E;
045°.

The advantages and disadvantages of each of these methods can profitably be discussed.

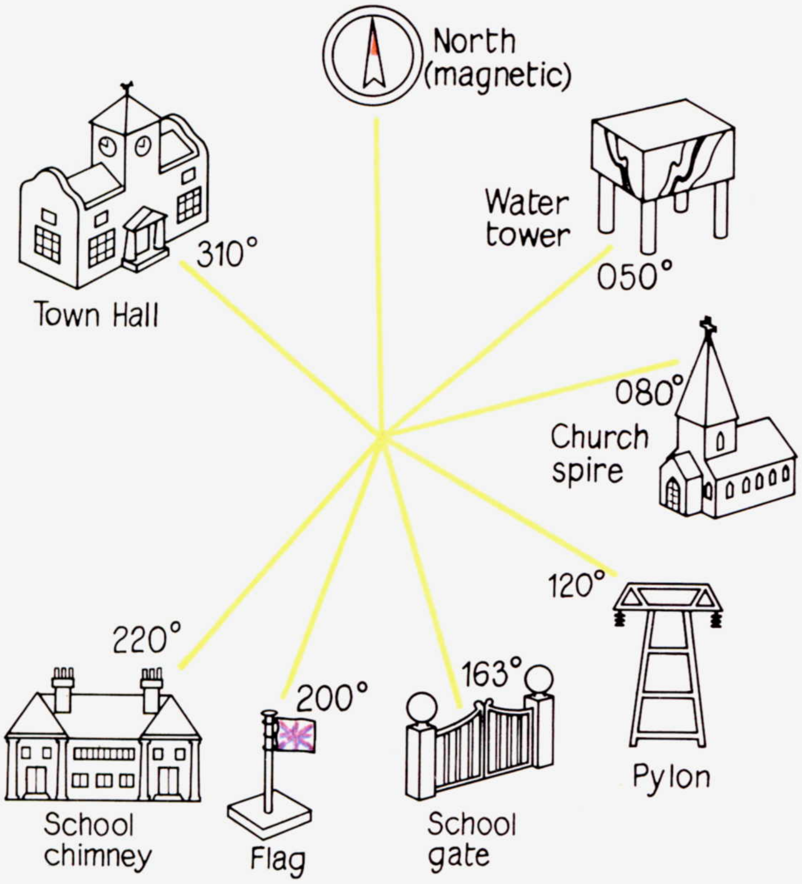
For practical work using North as a reference line it will of course be necessary to have a compass, but there is no need for the children to have elaborate direction-measuring equipment. A very simple compass (even a magnetised needle on a string) can be used to set up a direction-finding board, and the actual direction measurements can be taken from this.



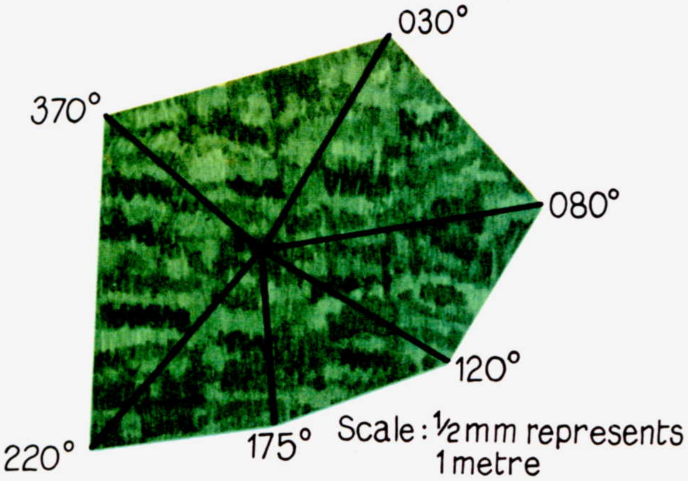
This is the way I go to School



**Directions from the centre of the school field**




In this exercise the lengths of the rays have no significance, but in later work, direction and distance can be combined. By finding the direction and distance of the corners of a field, scale drawings can be made of the shape of the field.



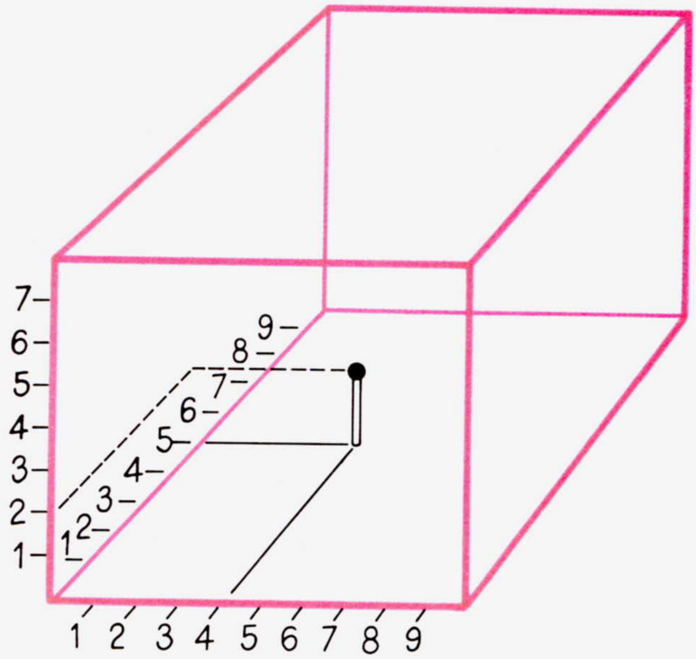

Determining the position of a point in two-dimensional space is discussed in 2 and 2.

Signposts give an example in the environment of the use of the combination of direction and distance to indicate position.

Other observed signs which indicate position may include those showing the positions of water mains, telephone cables or other buried objects.

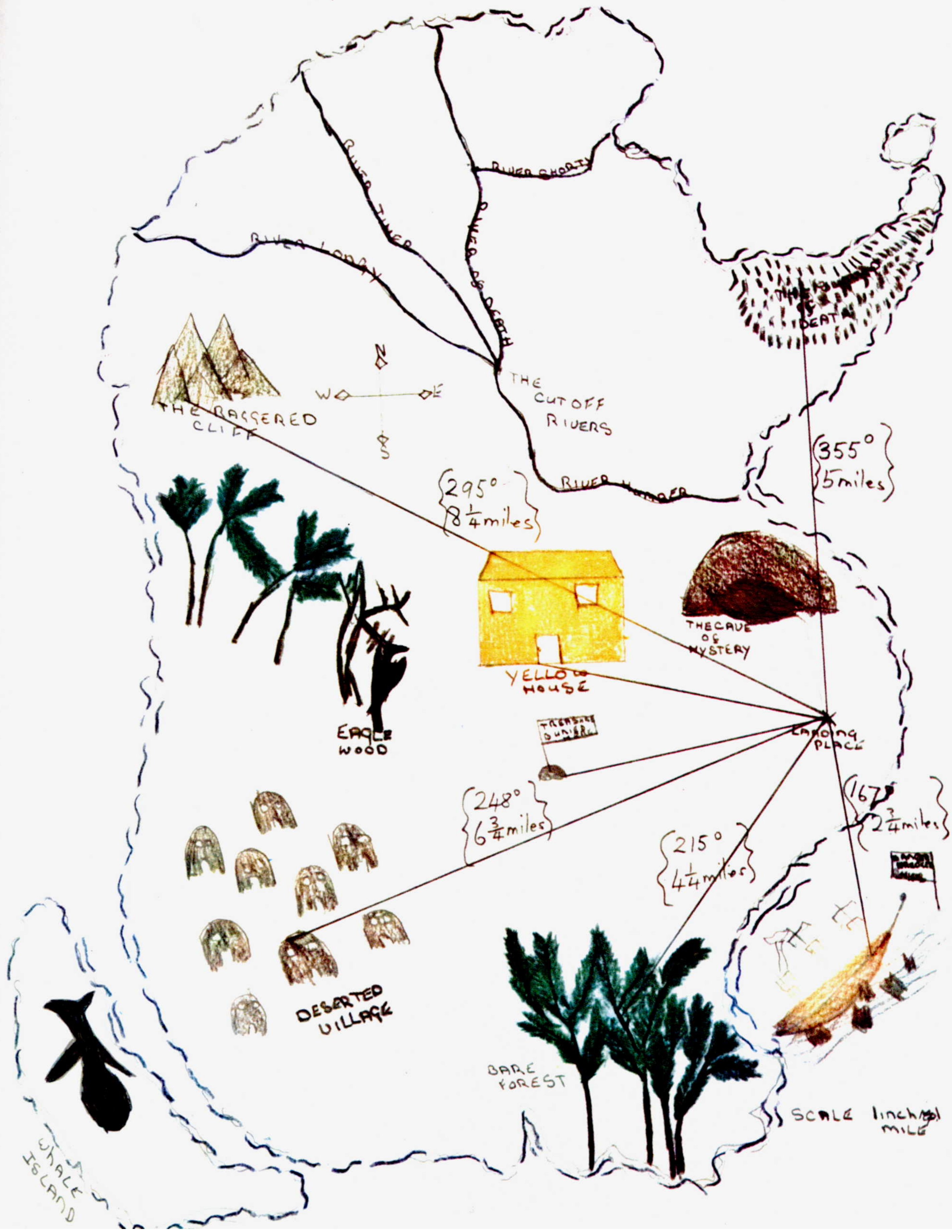
Discussion on these should lead to the realisation that two measurements are no longer sufficient when working in three-dimensional space. To give a full description of position, three measurements are now needed: distance, direction and depth. Similarly, to describe the position of an aeroplane in the sky it is necessary to give direction, distance and height.

Practical experience in referring to points in three-dimensional space can be given in the classroom. For example, two horizontal edges and one vertical edge of the room could be marked off in units, preferably large ones, e.g. yards.




Here the position of the top of the stick could be recorded as (4, 5, 2), representing 4 along, 5 down the room and 2 up.

> Give the position of the model aeroplane which is hanging from the ceiling.
>
> Stand up and reach as high as you can. What is the position of your hand?
>
> Stand, with your feet at (6, 7, 1), your left hand at (7, 7, 2), and your right hand at (6, 7, 3).
>
> My head is at (4, 5, 0) and my feet are at (2, 5, 0). Describe my position in words.

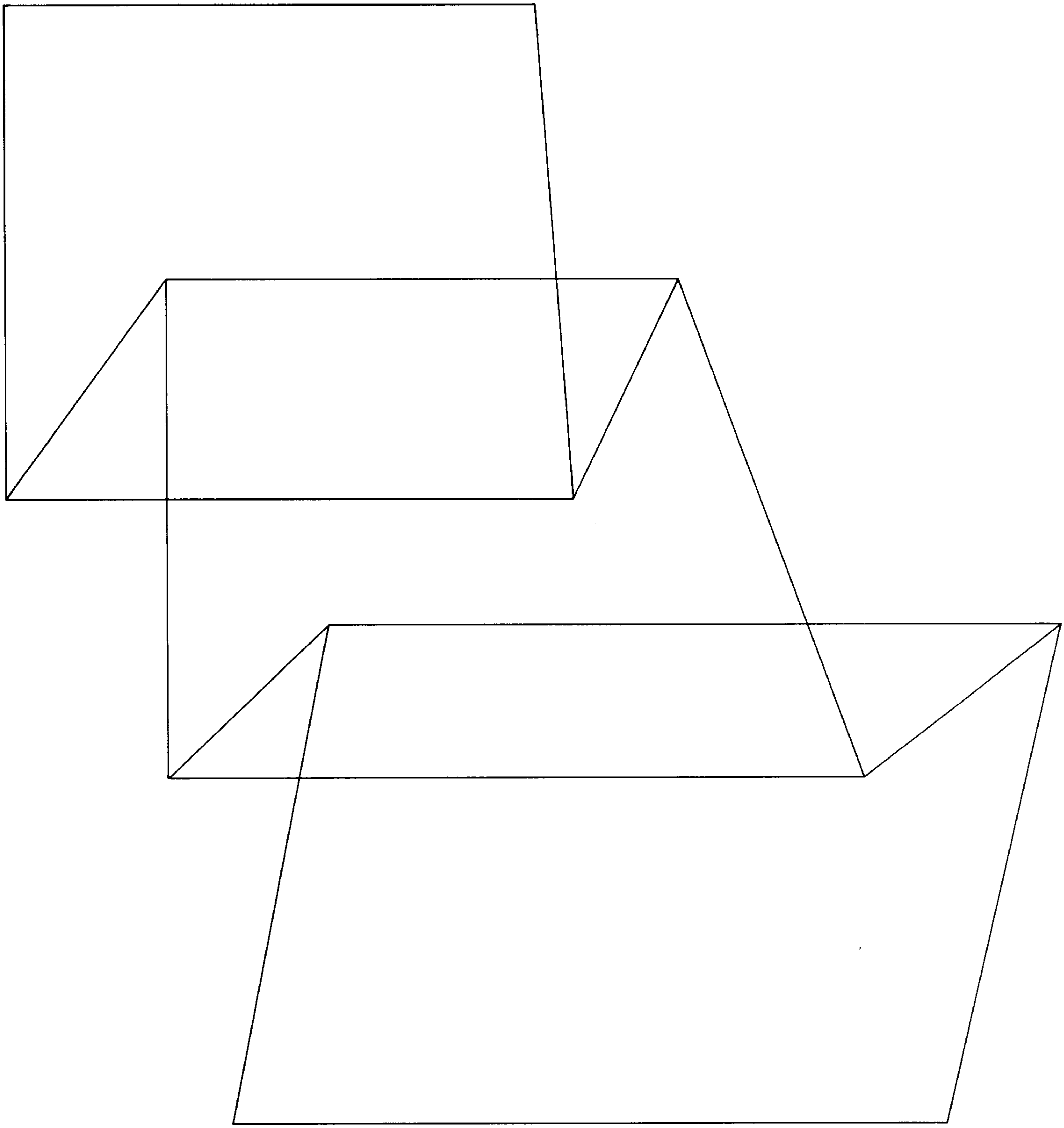

RIVER SHORTY
RIVER OF DEATH
THE CUT OFF RIVERS
THE RAGGERED CLIFF
N
W
E
S
295°
8 1/4 miles
355°
5 miles
YELLOW HOUSE
THE CAVE OF MYSTERY
EAGLE WOOD
LANDING PLACE
248°
6 3/4 miles
215°
4 1/4 miles
2 3/4 miles
DESERTED VILLAGE
BARE FOREST
SCALE
MILE
WHALE ISLAND

# CONCLUSION

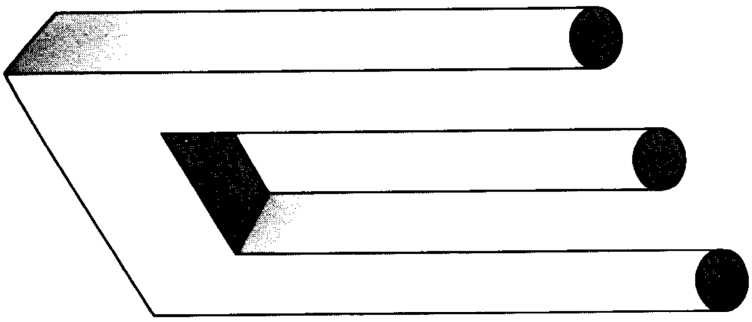
Children having experience based on this Guide should become more aware of the geometry around them. For example, at first, they will often represent a house in a stereotyped manner which bears little resemblance to those with which they are familiar. Through experience and discussion they will gradually come to terms with their environment.

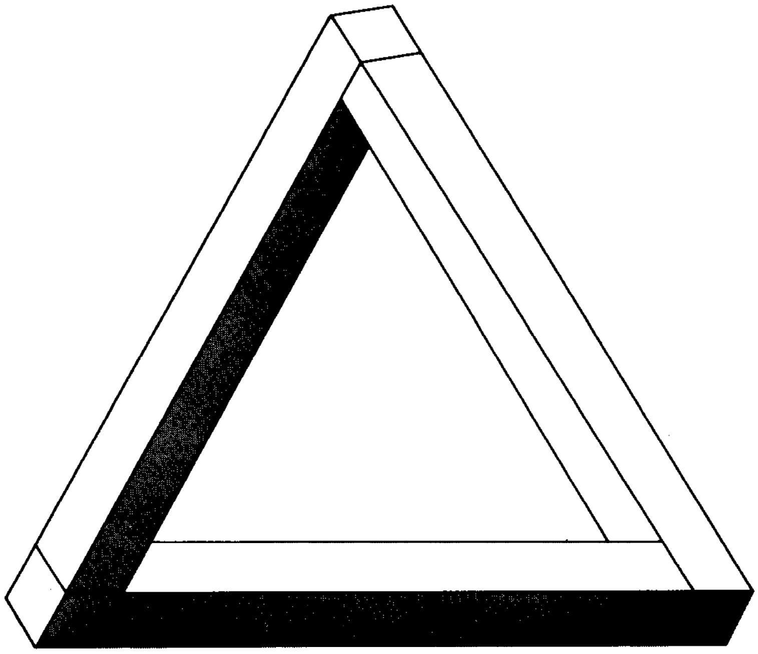
We end with a few examples which may be of interest to the reader himself as well as to his class.

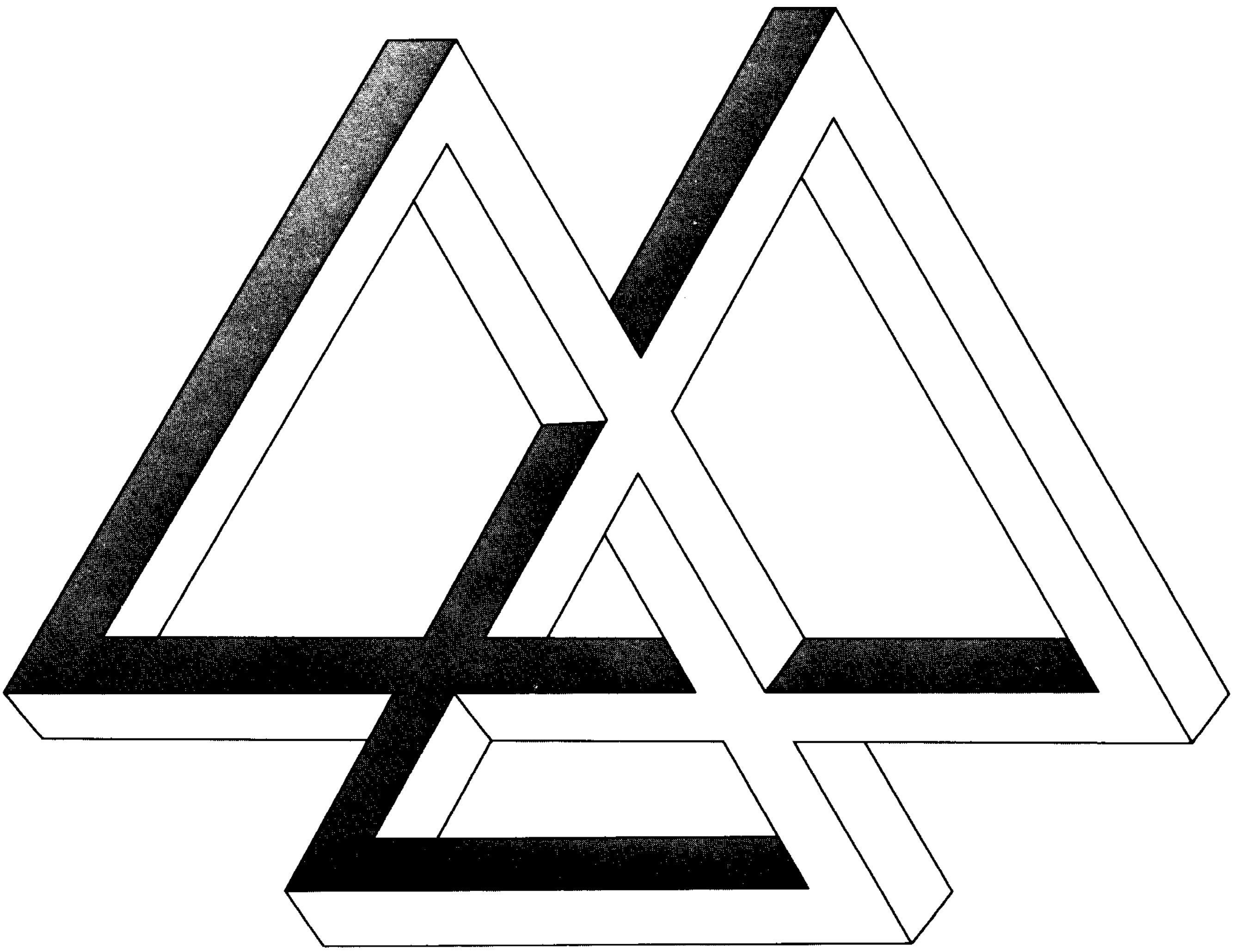
**1** This drawing of a folded transparent sheet can be 'read' in many different ways.



**2** The following drawings are examples which may appear at first to be fairly simple straightforward models; but further more careful examination will show them to be, in fact, impossible.







The following publications of the Nuffield Mathematics Project appeared in 1967 – 8 :

## Introductory Guide

**I do, and I understand** ●■▼ **(1967)**
This Guide explains the intentions of the Project, gives detailed descriptions of the ways in which a changeover from conventional teaching can be made and faces many of the problems that will be met.

## Teachers' Guides

**Pictorial Representation** ■1 **(1967)**
Designed to help teachers of children between the ages of 5 and 10, this Guide deals with graphical representation in its many aspects.

**Beginnings** ▼1 **(1967)**
This Guide deals with the early awareness of both the meaning of number and the relationships which can emerge from everyday experiences of measuring length, capacity, area, time, etc.

**Mathematics Begins** ●1 **(1967)**
A parallel Guide to *Beginnings* ▼1, but more concerned with 'counting numbers' than with measurement. It contains a considerable amount of background information for the teacher.

**Shape and Size** ▼2 **(1967)**
The first Guide concerned principally with geometrical ideas. It shows how geometrical concepts can be developed from the play stage in *Beginnings* ▼1 to a clearer idea of what volume, area, horizontal and symmetrical really mean.

**Computation and Structure** ●2 **(1967)**
Here the concept of number is further developed. A section on the history of natural numbers and weights and measures leads on to the operation of addition, place value, different number bases, odd and even numbers, the application of number strips and number squares.

**Shape and Size** ▼3 **(1968)**
Continues the geometrical work of ▼2. Examination of two-dimensional shapes leads on to angles, symmetry and patterns, and links up with the more arithmetical work of ●2.

**Computation and Structure** ●3 **(1968)**
Suggests an abundance of ways of introducing children to multiplication so that they will understand what they are doing rather than simply follow rules.

## Weaving Guides

**Desk Calculators (1967)**
Points out a number of ways in which calculators can be used constructively in teaching children number patterns, place value and multiplication and division in terms of repeated addition and subtraction.

**How to Build a Pond (1967)**
A facsimile reproduction of a class project.

The Teachers' Guides, together with *Graphs Leading to Algebra* ■2 (1969 : see page 55) and *Desk Calculators*, are summarised in *The Story So Far*.

**Nuffield Mathematics Project publications appearing May, 1969:**

## Teachers' Guides

### Graphs Leading to Algebra 2

This Teachers' Guide develops the use of coordinates and introduces open sentences and truth sets. It goes on to deal with the graphical aspect of these mathematical statements, introducing graphs of inequalities, intersection of two graphs and graphs using integers.

### Computation and Structure 4

The main concern of this Teachers' Guide is with the introduction of the integers $\{\ldots\ {}^{-}3,\ {}^{-}2,\ {}^{-}1,\ 0,\ {}^{+}1,\ {}^{+}2,\ {}^{+}3\ \ldots\}$. In the past children have been introduced to positive and negative numbers through the application of these and have been taught 'tricks' for using them in mathematical operation. This Guide builds up the idea of the integers in terms of ordered pairs of numbers before introducing the number line and other applications: this lays a sound foundation for operations on integers. The Guide ends with a short section on large numbers and indices.

## Weaving Guides

### Environmental Geometry

One of the 'Weaving Guides', this book concentrates on making children more critically aware of shapes in their environment and the interrelationships of them. It deals with relative size and position and with recurring shapes and their properties. It is intended mainly for Infants and lower Juniors.

### Probability and Statistics

A 'Weaving Guide' designed to build up, in a very practical way, a critical approach to statistical information and assertions of probability. It demonstrates the many ways in which data can be collected and organised and it attempts to define the criteria for selecting the 'best' way for any given situation. Probability is introduced largely through games, but ways of predicting probable outcomes are investigated in detail.

## Other publications

### The Story So Far

This booklet is an outline of, and index to, the ground covered by the first nine Teachers' Guides of the Project. Its purpose is twofold: to provide easy references to topics in these Guides for those using them day by day (making a straight index proved an impossible task); and to save teachers of older children having to read through all the early Guides to find out 'what had happened previously'.

### Problems – Green Set

This publication consists of a Teachers' Book accompanied by a set of fifty-two cards for distribution to the children. Two further sets of Problems are in preparation.

The first set of Problems is intended for use with young Secondary pupils. The problems on the cards are reprinted in the Teachers' Book, with solutions and a considerable amount of background material and suggestions for follow-up work. All the topics covered by these cards are included in the Teachers' Guides already published, but they are presented in such a way that children who have not followed a 'Nuffield-type' course can do the problems and enjoy them.

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o : 72-88361
ohn Wiley & Sons Inc., New York

£1·00