PROBABILITY AND STATISTIC

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neral introduction

aim of the Nuffield Mathematics Project is to devise a
ntemporary approach for children from 5 to 13'. The
des do not comprise an entirely new syllabus. The stress is
ow to learn, not on what to teach. Running through all
work is the central notion that the children must be set
to make their own discoveries and think for themselves,
so achieve understanding, instead of learning off
terious drills. In this way the whole attitude to the subject
be changed and 'Ugh, no, I didn't like maths' will be
d no more.

chieve understanding young children cannot go straight
ostractions – they need to handle things ('apparatus' is
grand a word for at least some of the equipment
cerned – conkers, beads, scales, globes, and so on).

'setting the children free' does not mean starting a riot
a roomful of junk for ammunition. The changeover to
new approach brings its own problems. The guide *I do,
I understand* (which is of a different character from the
rs) faces these problems and attempts to show how they
be overcome.

other books fall into three categories: Teachers' Guides,
ving Guides and Check-up Guides. The Teachers'
des cover three main topics: ● Computation and
cture, ▼ Shape and Size, ■ Graphs Leading to Algebra.
e course of these guides the development of mathematics
en as a spiral. The same concept is met over and over
n and illustrated in a different way at every stage. The
s do not cover years, or indeed any specific time; they
ly develop themes and therefore show the teacher how
low one child to progress at a different pace to another.
contain direct teaching suggestions, examples of
rently un-mathematical subjects and situations which
be used to develop a mathematical sense, examples of
lren's work, and suggestions for class discussions and
of-school activities. The Weaving Guides are single-
cept books which give detailed instructions or
mation about a particular subject.

third category of books, as the name implies, will provide
ck-ups' on the children's progress. The traditional tests
lifficult to administer in the new atmosphere of individual
overy and so our intention is to replace these by
idual check-ups for individual children. These are being
ared by a team from the Institut des Sciences de
ication in Geneva under the general supervision of
Piaget. These check-ups, together with more general
commentary, will be issued in the same format as the other
guides and, in fact, be an integral part of the scheme.

While the books are a vital part of the Nuffield Mathematics
Project, they should not be looked on as guides to the only
'right' way to teach mathematics. We feel very strongly that
development from the work in the guides is more important
than the guides themselves. They were written against the
background of teachers' centres where ideas put forward in
the books could be discussed, elaborated and modified. We
hope very much that they will continue to be used in this
way. A teacher by himself may find it difficult to use them
without the reassurance and encouragement which come
from discussion with others. Centres for discussion do
already exist and we hope that many more will be set up.

The children's work that has been reproduced in these books,
like the books themselves, is not supposed to be taken as a
model of perfection. Some of it indeed contains errors. It
should be looked upon as an example of work that children
might produce rather than a model of work that they *should*
produce.

Foreword

The last few years have been exciting ones for teachers of mathematics ; and for those of us who are amateurs in the subject but have a taste for it which was not wholly dulled by the old methods that are so often stigmatised, there has been abundant interest in seeing the new mathematical approach develop into one of the finest elements in the movement towards new curricula.

This is a crucial subject ; and, since a child's first years of work at it may powerfully affect his attitude to more advanced mathematics, the age range 5 to 13 is one which needs special attention. The Trustees of the Nuffield Foundation were glad in 1964 to build on the forward-looking ideas of many people and to set up the Nuffield Mathematics Project ; they were also fortunate to secure Dr. Geoffrey Matthews and other talented and imaginative teachers for the development team. The ideas of this team have helped in the growth of much lively activity, throughout the country, in new mathematical teaching for children : the Schools Council, the Local Education Authority pilot areas, and many individual teachers and administrators have made a vital contribution to this work, and the Trustees are very grateful for so much readiness to co-operate with the Foundation. The fruits of co-operation are in the books that follow ; and many a teacher will enter the classroom with a lively enthusiasm for trying out what is proposed in these pages.

Brian Young
Director of the Nuffield Foundation

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INTRODUCTION

Statistics is one of the branches of mathematics that has developed rapidly in recent years, and, in the future, children will be faced with an ever-increasing mass of numerical information. They will probably more frequently meet collections of numerical facts than the traditional mechanical computations of long multiplication and long division.

The study of statistics involves giving meaning to number facts, called **statistical data,** on which we base our enquiries. The final interpretations are mere predictions based often on incomplete information. Such final statements must be weighed against the situation involved, and the question of their validity assessed. Above all, it is essential to know how the data were collected and to investigate the effect of the various types of bias that could have been introduced.

The 'play stage' for probability

The foundations of a considerable amount of more formal work in statistics and probability will be laid naturally in the infants' school. The sorting and mixing activities enjoyed in the infant stages can involve statistical ideas and vocabulary and, in particular, the development of ideas of randomness that will prove valuable in the assessment of statistical data later.

A small group of children may be 'playing' with counters.
A situation can be contrived in which the children are provided with counters of four different colours.
Say: 16 red, 10 blue, 6 green, 3 yellow

The children will first enjoy counting the different colours and will comment on 'a lot', 'a few', 'more' and 'less', 'most' and 'least', etc. They may be encouraged then to count them all into a bag. Invite one child to draw a counter from the bag, but first ask the group to say which colour they **think** will come out first. Give each child a turn at drawing a counter from the bag, always first asking the group to say which colour they **think** will be drawn next. Continue this until all the counters have been drawn out.

Comments obtained from a group of 6-year-olds varied considerably –

a Don't know
b Blue, because it is my favourite colour
c Yellow, because it is yellow's turn
d Red, because there are more red.

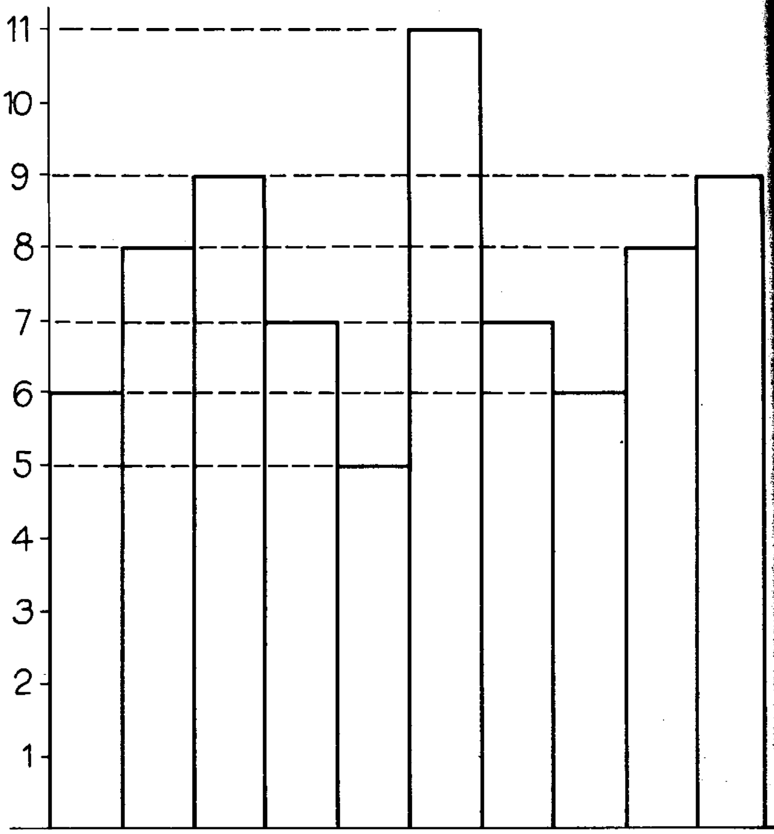
After playing in this way for some while, it should be possible for the children to record some of the results. As a counter is drawn out of the bag, it can be placed on a large sheet of cardboard (or graph paper) marked off in separate columns for the various colours. This could lead to discussion and prediction.

A later stage would be to colour a square on graph paper as each counter is taken out, but it must be remembered to pause after each extraction to discuss the 'probable' next event. The finished pictorial representation would then be checked with the numbers first started with. This last operation of course has nothing whatever to do with probability, but is something which the infant apparently loves to do and which seems to 'tidy up' the whole session.

Many of the games that children play introduce the idea of probability as well as other mathematical concepts. Children will often amuse themselves with a pair of dice and will score the totals of their throws. Many will come to realise that the probability of scoring 6, or 7, or 8 is greater than that of scoring 4, or 5, or 9, or 10, and that in turn is greater than the probability of scoring 2, 3, 11, or 12. Other games which introduce informally the ideas of chance and probability are Snakes and Ladders and other board games played with dice; dominoes; card games such as Snap; and many developments of these games.

Once children have become interested in the idea of probability, other, more sophisticated, games can be introduced. The 'biscuit tin game' was devised by a teacher who had read something about the behaviour of atoms. 50 small cubes (say Dienes unit blocks) are put in a biscuit tin. One face of each cube is coloured red. The tin is shaken up, and when the lid is removed the number of cubes with the red face uppermost is counted. The process is repeated recording each time the number of cubes with red face uppermost, either by building columns (with other cubes) or by colouring squares on graph paper.

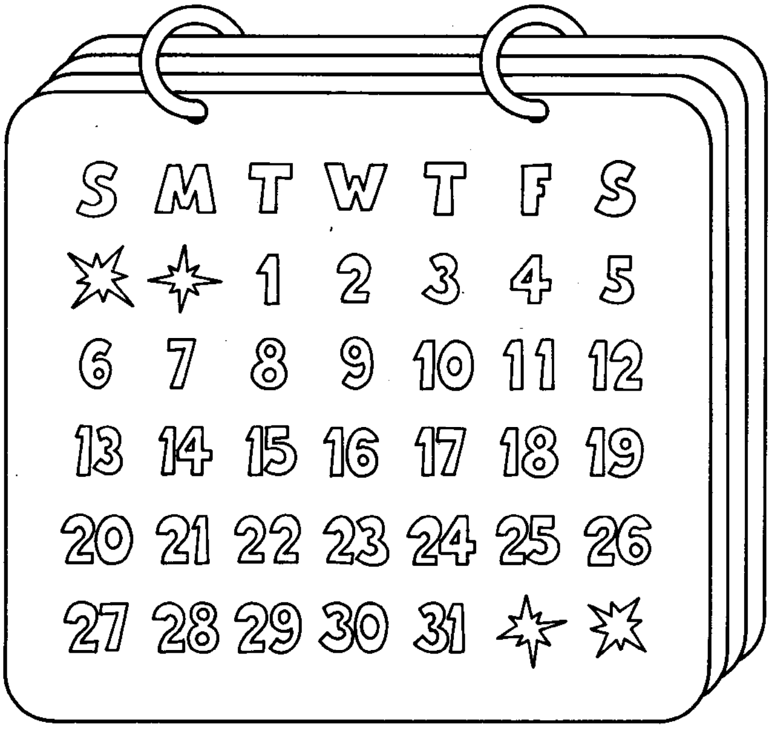
On one occasion the sequence of numbers: 6, 8, 9, 7, 5, 11, 7, 6, 8, 9 was obtained and recorded graphically.



Is there a pattern here? What do we mean by a pattern? Might one perhaps extract the idea that the 'average' number is expected to approximate to 8?

Or discussion might proceed thus:
How long should we have to shake the tin for all the cubes to fall red side uppermost? For all to fall red side hidden? What effect does shaking have?

The calendar can also be used to arouse interest in probability and predictions.



eg 'How many Sundays in this month?' 'How many complete weeks?' 'What day falls on the 3rd?' 'How do these compare with other months?'

The beginning of statistics

The study of statistics could well begin with discussion between the teacher and the children on the meaning and validity of some popular statements. This form of introduction could lead to more purposeful collection of data at a later stage.

Here are some statements:

a 7 out of 10 school children enjoy drinking school milk.
b Nearly as much ice-cream is sold in winter as during the summer months.
c All the people in the village want an hourly bus service.
d Burgocars travel 46 miles for every gallon of petrol.
e Mrs. Jones says 'Tough' staircarpet is the best. She has had 'Tough' carpet on her stairs for 20 years.
f 2 out of 3 mothers feed their babies on 'Growup'.
g 'Floorsmear' makes your floor **twice** as shiny as any other polish.
h 'Chloro' washes whiter.

From such statements as these, lively discussions can take place, and questions such as the following can be asked:

Is it possible to check the statement?

If it is not, should we reject it as meaningless?

If it is, how can we verify it?

Is the information adequate and sound enough for the genera truth of the statement to be accepted?

In **a**, would information from all the schools in the area give a result which is likely to be more accurate than from one school only?

In **e**, children may ask such questions as:

'Has Mrs. Jones a family?' If so, how many children were there in the family? Children wear out carpets! – True or false?

Has Mrs. Jones ever tried any staircarpet other than 'Tough'? She says 'Tough' is the **best**!

And even: Is there really a Mrs. Jones, or has she been invented?

In **g**, 'When do you know a floor is twice as shiny?'

h can be discussed and dismissed immediately as meaningless.

iscussions on these lines prepare a child to study numerical formation more carefully before giving an interpretation or ccepting a statistical statement and may enable the teacher give an elementary explanation of sampling (see page 30).

he majority of such statements offer an opportunity for the hildren to collect information both from children and from dults, and prepare the way for organising the collecting of ata for their own projects later.

ere are some more examples which could be given to hildren to discuss:

There have been 50 fewer car accidents in Burslem than in lewcastle during the last 5 years, so it is safer to drive in urslem.
uestion: Would you accept this conclusion without any uery?
There are 50 more children attending our school than ttend St. Egbert's, so we should have a better football team.
All people who take 'Headclear' are freed from headaches n 24 hours.
iscuss: Would 24 minutes be more likely? Or does the dvertisement mean: 'Never any more headaches after 4 hours'?
Mr. Jones, the agent for Burgocars, says 'Burgocars are by ar the most popular buy'. Would you expect Mr. Jones to ay, 'Burgocars are not the most popular buy'?

lany other such statements can be collected from the ewspaper and from advertisements. From the discussion of hese and other examples children are encouraged to:

consider whether a statement has any verifiable meaning;
collect information by which the verifiable meaning can e tested;
i think carefully before accepting an advertisement as the hole truth.

EARLY USES OF 'PICTORIAL REPRESENTATION'

1 How data are collected

Before we start to collect the statistical data required to help us to form conclusions, we must have very clearly in our minds:

a the question to which we are seeking an answer;
b the best arrangements for collecting the data in order to make our interpretation as reliable as we can.

Let us suppose, for example, that we want to find out what use is being made of a public park. We should need to collect information about the number of people who, during a week, visit the park for one or more of a variety of purposes:
to play tennis, to play bowls, to play miniature golf, to go into the park merely to sit and rest, to take the dog for a walk, to play school games, to take part in organised matches or athletic events, to watch sport, etc.

Information can be collected in a variety of ways:

1 A census, or count, or measurement

In the public park example, we should have to decide the times of day when the data are to be collected, if we are to have reliable information on which to base our conclusion. If we restricted our census, for example, to the hours between 10.00 a.m. and 12.00 noon each day, we should be ignoring completely the uses made of the park at other times of the day.

In this particular example, the data could be collected from an actual count being made of all people entering the park during the week and a note made of the purpose of their visit. Data depending on an actual count or measurement are usually reasonably accurate, but can involve a great deal of work.

Some forms of simple counting can be done with automatic devices such as the meter used in a traffic census. A tally of articles sold in some of the larger well-known stores is made by means of a device attached to the till. Some post offices use a franking machine in a post office which records the number of envelopes franked.

2 A questionnaire

If an actual census, or count, is impracticable, information may be collected by means of a questionnaire which is sent round to all those with whom we are concerned in any particular enquiry or survey. The questionnaire should contain short-answer questions of an unambiguous nature.

All sorts of problems can arise. We cannot expect a 100 per cent return of the questionnaires; and the questions are always capable of a variety of interpretations however much care and thought are given to their preparation.

Even if the questions are directed verbally to individuals in th street and the vicissitudes of written communications avoided, our information is bound to be biassed in one direction or another, towards one section of the community or against another. Often our 'population' under investigatio is too numerous. To overcome these and other difficulties w often resort to a third method of collecting data:

3 A random sample

We choose a sample by chance from our 'population' and th is called a 'random sample'. Thus a random sample is a subs of our 'population', every element of which has an equal chance of selection.

For example, at a whist drive, the counterfoils of the 2,000 tickets sold in aid of the Christmas Draw are placed in a large drum which is revolved for two minutes by the Vicar. He then draws out the five winning tickets. These five could be described as a random sample.

To take a random sample of the people using the park you would have to question, say, one person in ten who went in the park during the hours that it was open.

> Are the members of your school orchestra a random sample of the pupils of the school?
> Are the playing members of Otchurch United F.C. a random sample of the inhabitants of Otchurch?
> Are the two tobacconists in the village a random sample of the shopkeepers in the village?
> Are the pupils of your school a random sample of the children of the town?
>
> If the answer is 'no', say how a random sample could be obtained.

Whenever we are considering the interpretation of statistica data, we must consider the sample taken by asking:

i How large was the sample?
ii Was it a random sample?

Random sampling will be further discussed in Chapter 3.

How data are recorded and tabulated

/hen we collect and record data, we are making a one-to-one orrespondence; each person or element is represented by a roke in the relevant column.

ere is an imaginary situation. The idea can be developed rough data collected by the children in a similar context.

ne class is voting for a form leader. When the voting slips e opened, the names of the three candidates are called out be counted:

om, Tom, Tom, Dick, Harry, Harry, Tom, Dick, Harry, Harry, om, Dick, Dick, Tom, Tom, Harry, Dick, Harry, Harry, Tom, om, Harry, Harry, Harry, Dick, Tom, Harry, Dick, Harry, arry, Tom, Dick, Dick, Dick, Tom, Harry, Tom.

FORM LEADER ELECTION

NOMINEE	VOTES	TOTAL
Tom	卌 卌 |||	13
Dick	卌 卌	10
Harry	卌 卌 ||||	14

I declare ....*Harry*........
to be elected form leader

The total number of occurrences of each event in such a situation is called the **frequency** (see **1** p.37) of the event.

Here is another example to show how such a situation may be used for discussion:

Study the following count and tabulation of a survey made by a representative of a firm which sells washing powder. Every housewife visited was asked the question 'What brands of washing powder have you in the house at the moment?' Here are the details of the information obtained:

WASHING POWDER SURVEY
25·10·67/3·00–5·00pm.

BIZZO	卌 卌 |||	13
LATHO	卌 |	6
QUIKO	|||	3

1 Do you consider 3.00 – 5.00 p.m. on the 25th October, 1967, was a suitable occasion on which to call at the homes and interview housewives about the washing powders used? Discuss with your teacher.

2 Would the following advertisement be a reasonable one: 'Twice as many housewives use Bizzo as use all the other brands put together'? Discuss the advertisement amongst yourselves and with your teacher, considering

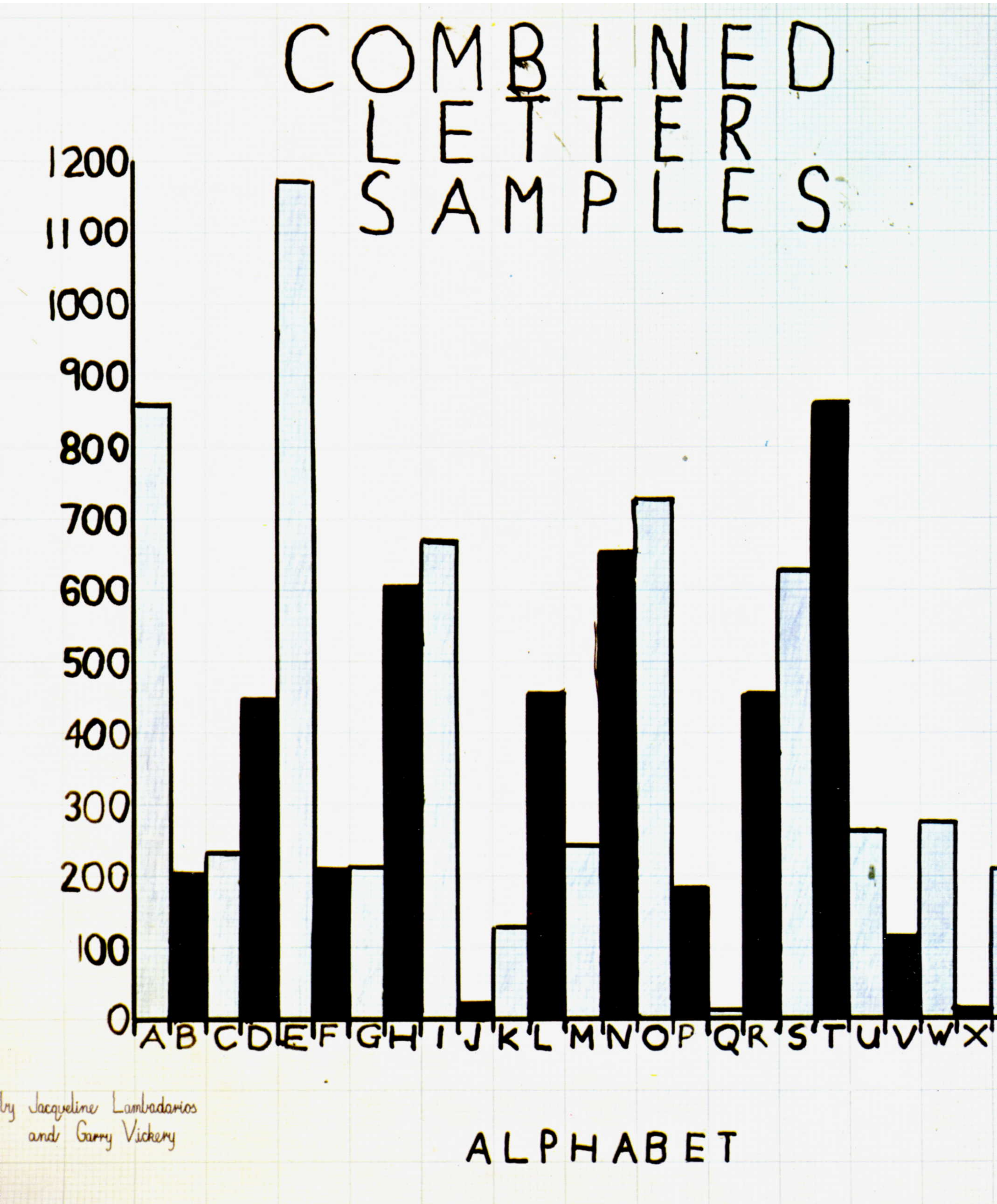
a the time of day the survey was made;
b the day of the week (a Wednesday) the survey was made;
c the number of housewives interviewed.

You are considering the important question: 'Under what circumstances were the data collected?' Was a fair sample chosen? Say how you would adjust the results to eliminate bias.

The following are examples of topics that can be investigated. In each case the children make a count, record and tabulate what they discover, discuss their results amongst themselves and with their teacher.

a Cars owned by parents of children in the class:
i make, **ii** colour.
b Vehicles passing the school during one hour, comparing the results for different days of the week.
c Types of coins children in the class are carrying on different days of the week. Include a comparison of dates.
d Home wins, away wins, draws, in the Football League, Division I, during a period covering ten successive Saturdays.
e Letter frequency in certain sentences. (e.g. Study for letter frequency this sentence: 'The quick brown fox jumps over the lazy dog.')

The following sequence of work shows how some 3rd-year juniors made a letter-frequency count from seventy words in a book and then used this to help them to break a code. The same children went on to look at the Morse code and the keyboard of a typewriter to see whether letter frequency had been taken into account in their construction.



COMBINED LETTER SAMPLES
1200
1100
1000
900
800
700
600
500
400
300
200
100
0
A B C D E F G H I J K L M N O P Q R S T U V W X
ALPHABET
by Jacqueline Lambadarios
and Garry Vickery

SECRET CODE.

?rry ?r sy yjr p!f ?o!! sy ?ofmohjy ypmohjy.

O eo!! ntomh yjr drvtry 3/8s3/8rtd s!pmh eoyj

?r yp 3/8jpyphts3/8j. Yslr htrsy vstr yjsy up!

str mpy gp!!perf yjrtr. Ejrm up! esmy yjr

p3/8rmrf hobr s !pmh !pe vs!! smf 1mpv1

yjtrr yo?rd.

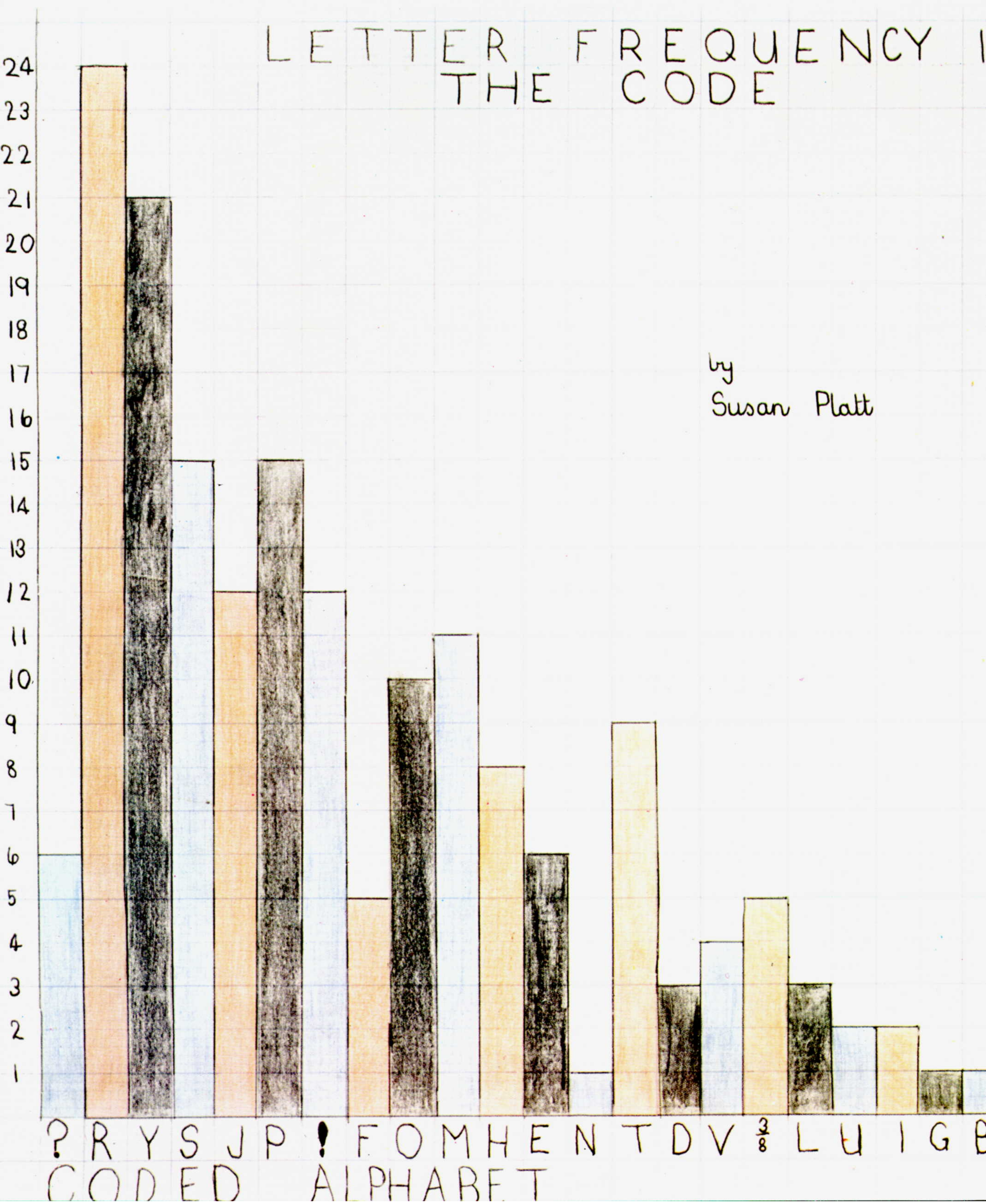
The Secret Code

In our classroom we have three printing presses, and a very big type cabinet. In this cabinet we have drawers with all different compartments in them. In every compartment there is type. All of these compartments have letters. We wanted us to find out why some compartments were big, and some small. First of all we got a book and counted seventy words out of it. Then we made a graph and we found out that "E" occurred most often "T" was the second one, "A" the third one and so on. The numbers that didn't occur very often small, and the ones that did were big. Mr Owen made us a secret code, which we had to find out by the end of the week. Here is part of the code. ?rry ?r sy yjr p!f ?o!! sy ?ofmohjy ypmohjy. This is not the whole code, because it would take ages to find out. If you had a paragraph to work out it would be much easier. We drew another graph to find out what letter in the secret code was "E", and "T" and "A". After we had done this we found out that the "R" in the secret code was "E", because it occurred most. Then we found out "y" in the secret code was "T". The next letter that was in the code was "P," and that turned out to be "O". The only person in the class who didn't get "E" as the top number was Philip Sweetman. Mr Owen wrote all the alphabet down on the blackboard. Then he wrote numbers at the side. We found out what the highest number was out of all of them and it was "E". Next you would keep going down the numbers until you come to the last one. The first one to find out the secret code was Susan Platt. Mr Owen called us secret agents.

The part of the secret code that I wrote down on the first page was: "Meet me at the old mill at midnight tonight."

This is why it is easier to find out a paragraph than a sentence. If you have a paragraph and you want to work it out it is easier, because the bigger it is the more accurate it is. It has more letters in it, that's why, but with a sentence it is less accurate, and harder to work out. "Z" occurred eight times out of 9,875 letters. The bigger the sample the better it is. Like if there was "Z" number eight and "E" 1,172. "E" would be very big when "Z" would be small. The sample of all the letters we took was 9,875.

by Lyn Bramwell


LETTER FREQUENCY I
THE CODE
by
Susan Platt
24
23
22
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16
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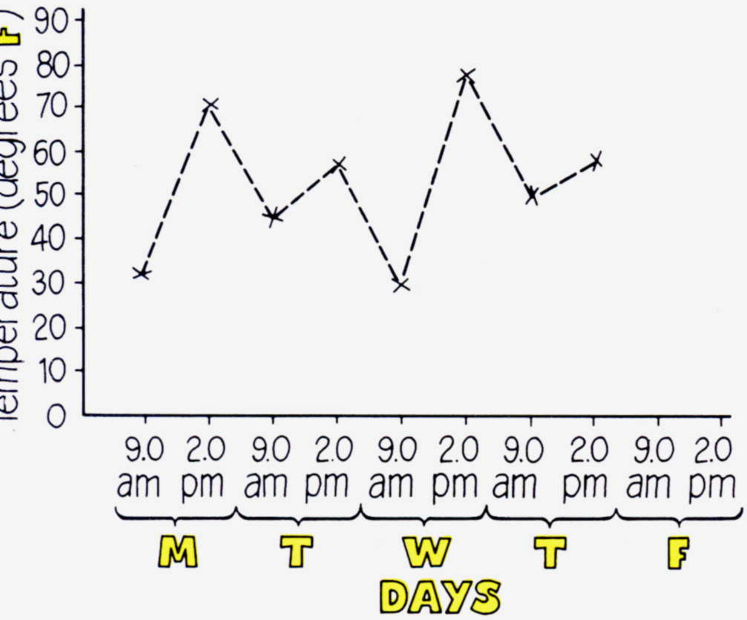
13
12
11
10
9
8
7
6
5
4
3
2
1
? R Y S J P ! F O M H E N T D V 3/8 L U I G B
CODED ALPHABET

l Illustrating statistical data

ypes of representation of statistics were given in *Pictorial epresentation* **1** pp. 38 and 39, which should again be eferred to. In this section we shall concentrate further on the are that is needed in giving an accurate impression iagrammatically of a situation and in this connection eference should be made to **1** pp. 32 to 36.

ne particular type of representation not mentioned in **1** is he straight-line chart, which can often be used in recording, or example, temperatures. The following diagram illustrates he temperature of a classroom during a week in October.



Ve have recorded the temperature at 9.00 a.m. and at .00 p.m. each day. We do not know the temperature at, ay, 10.30 p.m. We know there was a temperature at that me, but we did not read it from the thermometer.

Ve join the points with a **broken** line to show

that there was a temperature at every moment between .00 a.m. and 2.00 p.m. on any particular day and between .00 p.m. on any one day and 9.00 a.m. the next day ;
the trend of the temperature.

ur criterion as to whether points of our graph should or hould not be joined must lie in the answer to the question : an true measurements be read in between the given data ?

If true measurements **can** be made we are dealing with a **continuous variable** and we use the broken straight-line graph. If true measurements **cannot** be made we are dealing with a **discrete variable** and we use a bar chart or a bar line chart or some form of pictorial chart. **If a broken straight-line graph is used, every point on the horizontal axis must have significance.**

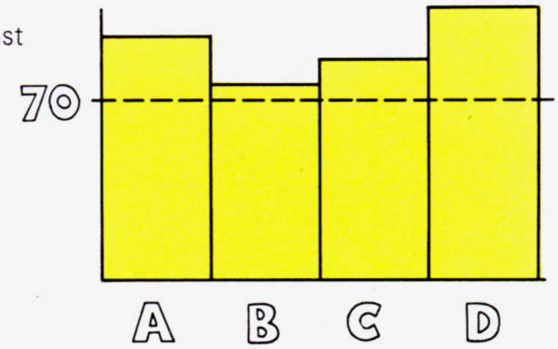
To summarise from **1**, whenever we are giving a pictorial representation of statistical information, we should :

a be certain everyone knows what is being represented : clear headings are essential ;
b be certain it is the right type of graph ;
c give details of the horizontal and vertical axes ;
d watch for the effect of periodic variations ; for example, seasonal variations such as the effect of school leavers on unemployment figures.

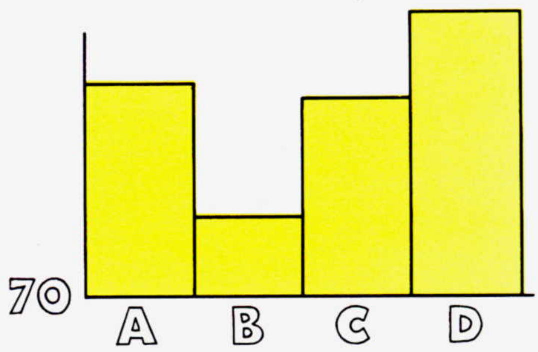
When interpreting such a diagram we must :

a ascertain what data are actually represented, and where they came from ;
b study the scale carefully : the position of zero is important.

Contrast



with



in which the columns have been cut off below 70 and the vertical scale is enlarged. It would be false to deduce that 'Column C' is three times as high as 'Column B', as would appear from the second diagram ;
c beware of the misuse of percentages and other devices employed by people whose figures are intentionally deceptive.

GAMES LEADING TO IDEAS ON PROBABILITY

I Two-state systems

In modern mathematics, an extensive use is made of a so-called two-state system: circumstances under which a choice between two actions has to be taken, a choice between two decisions made. For example, a computer accepts or rejects information by means of on-off impulses; information for a computer is programmed by means of the symbols 0 and 1 of the binary system.

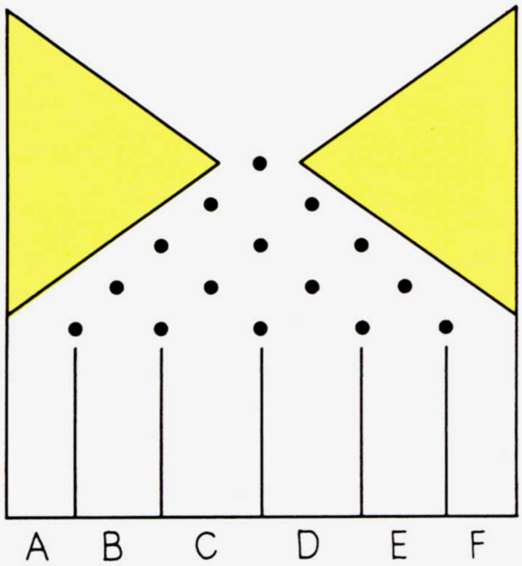
A two-state system can be illustrated by a maze. On entering a maze, we are faced at intervals along a corridor by a fork, where a choice has to be made between a right-hand path and a left-hand path. At each fork the choice is made, as it were, by the 'toss of a coin'.

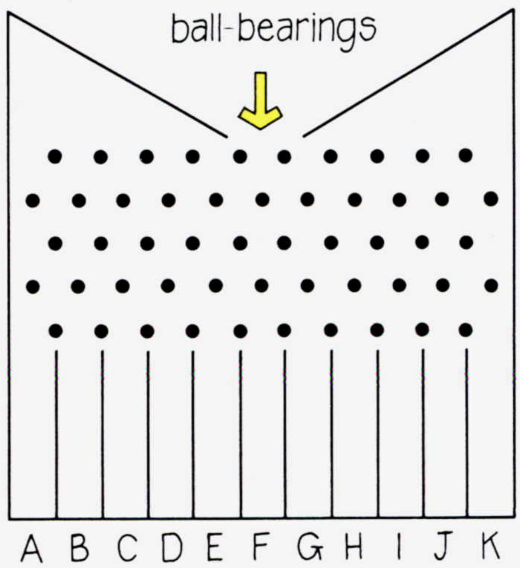
A model for this, which children enjoy manipulating, is 'Galton's Quincunx'.

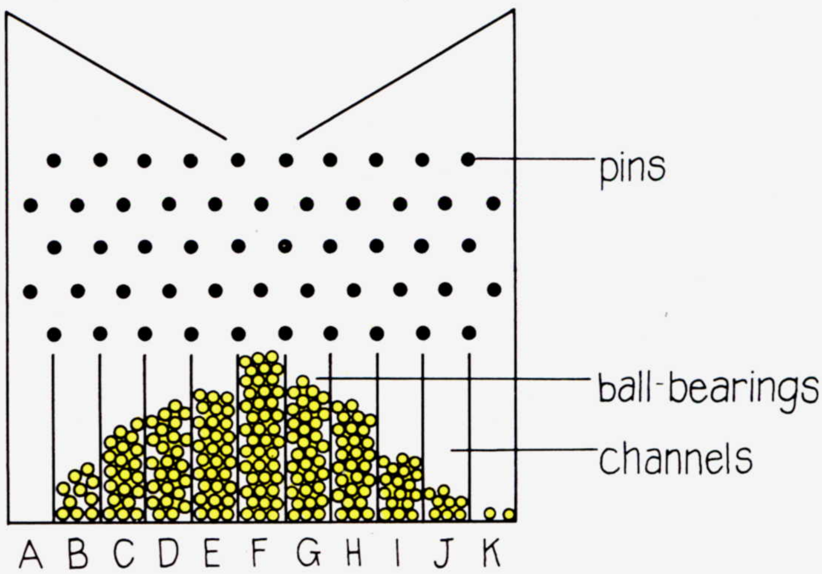
Ball-bearings are dropped on a maze of pins so that at each impact they will go either left or right to hit one of the pins diagonally below in the next row.

Two possible designs of this are shown below.

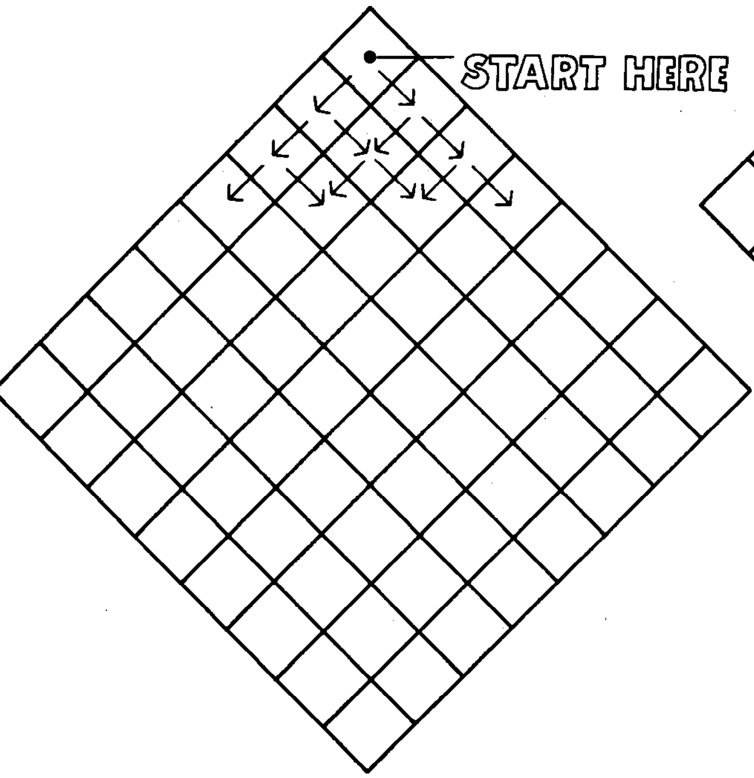
A study of the behaviour of the ball-bearings when they en the maze can lead later to such questions as 'What is the chance that a ball will come to this or that end point in the maze?' The first notions of probability will have been introduced.

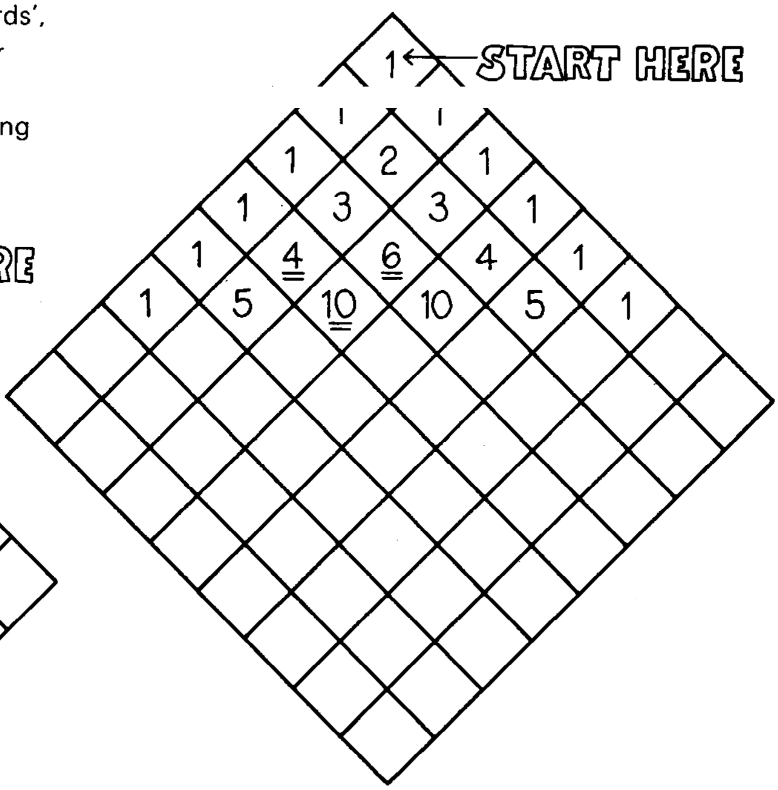




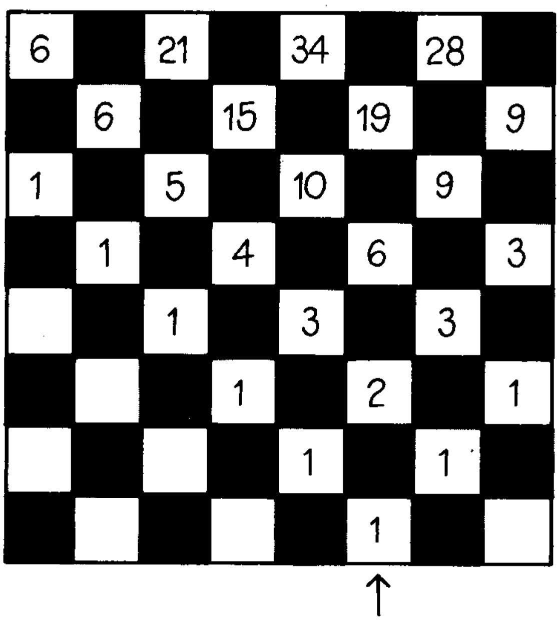


Much later, these observations can be given greater interest by a demonstration, on a chess-board, of Pascal's Triangle, as shown here and on page 26. The diagram illustrates that if you start from the 'top' square and move 'diagonally downwards', there is a definite number of possible routes to every other square. The possible moves at each stage (down right or down left) of course correspond to those of the ball-bearing in the Quincunx each time it strikes a pin.



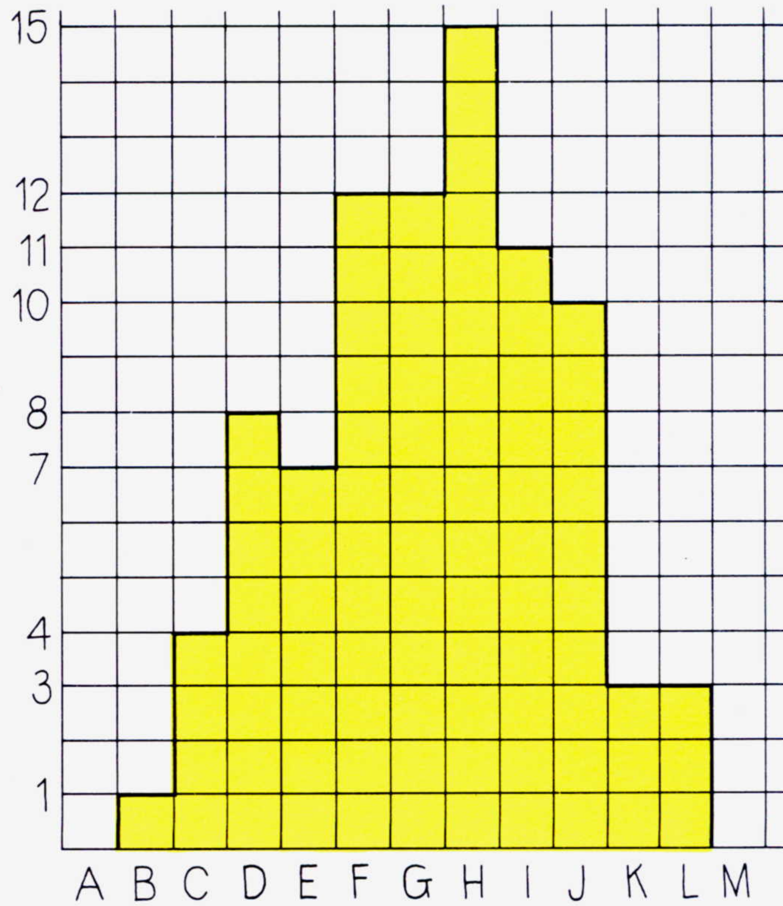


Some children enjoy continuing this pattern a long way, having noticed that each number is the sum of the two numbers immediately diagonally above it, e.g. 10 = 4 + 6.



An interesting variation is known as the Checker Man, who moves one square at a time along a diagonal. A bias can be introduced by insisting that he stays on the authentic chess-board. Starting from the square indicated, the numbers of possible routes to squares that can be reached are marked in the diagram on the left.

Let us return to the Quincunx. Given enough balls, the Quincunx can be used to develop the ideas of distribution. The following experiment was carried out by a girl of eight. The apparatus had been left lying on a table with a containe of balls beside it. She started to let the balls roll one at a tim through the maze. The following result was obtained :



When older children have charted their results they may like to compare this distribution with Pascal's Triangle. For example, the fifth row of the triangle is 1, 5, 10, 10, 5, 1 (total 32) and if the Quincunx ended with six slots, this would give the theoretical distribution for 32 balls.

A two-state system can be illustrated further by a series of coin-tossing activities and the results recorded by means of 'two tree'.

1 way out
3 ways out
3 ways out
1 way out

II Dice-throwing

Children can derive a great deal of entertainment and learn a lot about probability by playing with 2-, 3-, 4- . . . sided numbered dice or a spinning disc.

a One die thrown

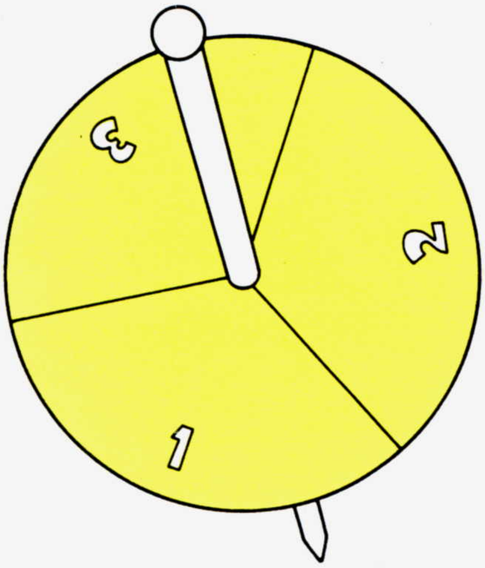
i A 2-sided die: sides numbered 1, 2.
A coin could be used, a score of 1 for a head, 2 for a tail being made.

Record the results (scores) of a number of throws, say 100, in a frequency table.

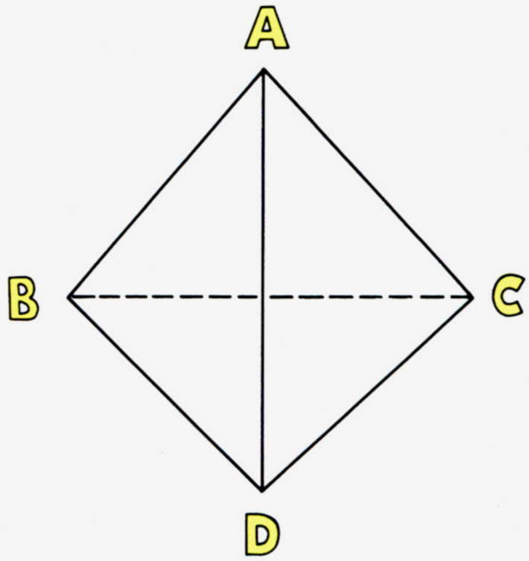
Results	Tally	Frequency
1		
2		

Repeat the experiment with

ii A 3-sided die: sides numbered 1, 2, 3.
A spinning disc may be used.



iii A 4-sided die: in the form of a regular tetrahedron.
Faces numbered: BCD: 1, CDA: 2, DAB: 3, ABC: 4



iv A 5-sided die (a spinning disc could be used).

v An ordinary 6-sided die.

Combine your results in **i** to **v** into a single table.

No. of sides	Frequency of score 1	2	3	4	5	6
2						
3						
4						
5						
6						

b A pair of dice thrown

In each of the following cases, a pair of dice is thrown, say 100 times, and the frequencies of the total of the scores showing on the faces falling uppermost are recorded as follows:

i 2-sided

Possible totals	Tally	Frequency
2		
3		
4		

ii 3-sided

Possible totals	Tally	Frequency
2		
3		
4		
5		
6		

iii 4-sided

Possible totals	Tally	Frequency
2		
3		
4		
5		
6		
7		
8		

iv 5-sided

Possible totals	Tally	Frequency
2		
3		
4		
5		
6		
7		
8		
9		
10		

v 6-sided

Possible totals	Tally	Frequency
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

Summarise your results in a table such as the following:

Number of sides	Score 2	3	4	5	6	7	8	9	10	11	12
2											
3											
4											
5											
6											

Do you notice anything about your results?

c Repeat the above with three dice thrown in each of, say, 100 trials.

It is an interesting experiment to carry out some of the above activities with 'unfair dice' (e.g. dice with one corner chopped off) and to make a study of the effects of bias.

If the above results are now compared with 'expected' results, a number of interesting number patterns is revealed.

Let us return for a moment to throws of a single die. Each face has a numeral associated with it. If you shake the die, is one particular face more likely to appear on top than another?

In the case of

i **A 2-sided die** (or a 2-sided penny)
Each face has **one** chance out of **two** of being on top.
The chance of a **one** turning up is 1 in 2.
The chance of a **two** turning up is 1 in 2.

ii **A 3-sided die** (or a spinning disc)
Each face has a chance of one in three of being on top.
The chance of a one, or a two, or a three appearing is 1 in 3.

Similarly for a **4-sided die** and a **5-sided die.**

In the case of a **6-sided die**, each face has a chance of one in six of being on top.
Shake the die 6 times: How many times **does** 3 appear on top?
How many times would you **expect** 3 to appear on top?
Shake the die 12 times: How many times **does** 3 appear on top?
How many times would you **expect** 3 to appear on top?
Shake the die 60 times: How many times **does** 3 appear on top?
How many times would you **expect** 3 to appear on top?

If 2 dice are thrown, possible totals are formed from a number on die A with a number on die B. Possible totals and the **expected number of occurrences** can be calculated.

2-sided

die **B** (columns), die **A** (rows)

+	1	2
1	2	3
2	3	4

Out of the set of 4 possible arrangements of the 2 dice:
{(1, 1), (1, 2), (2, 1), (2, 2)}

the chance of scoring a total of 2 is 1 in 4;
the chance of scoring a total of 3 is 2 in 4;
the chance of scoring a total of 4 is 1 in 4.

These results may be tabulated thus:

Totals	2	3	4
Chance of	1	2	1 in 4

This should be compared with the experimental results.
The work should be repeated with 3-, 4-, 5- and 6-sided dice.

With 6-sided dice, we have the following possible totals in 36 arrangements of the 2 dice:

die **B** (columns), die **A** (rows)

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

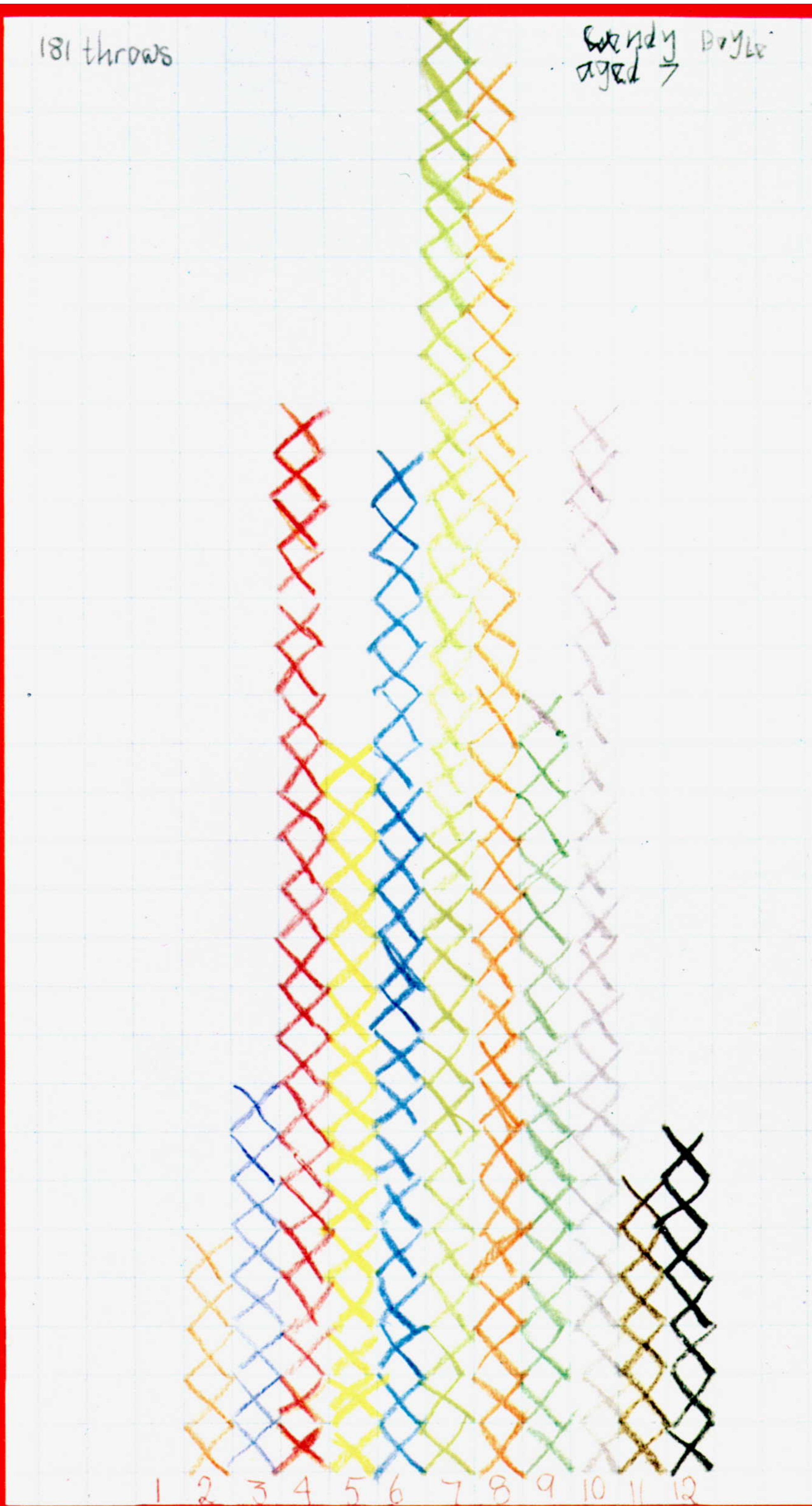
and the chance of getting any one of these totals is thus given in the table:

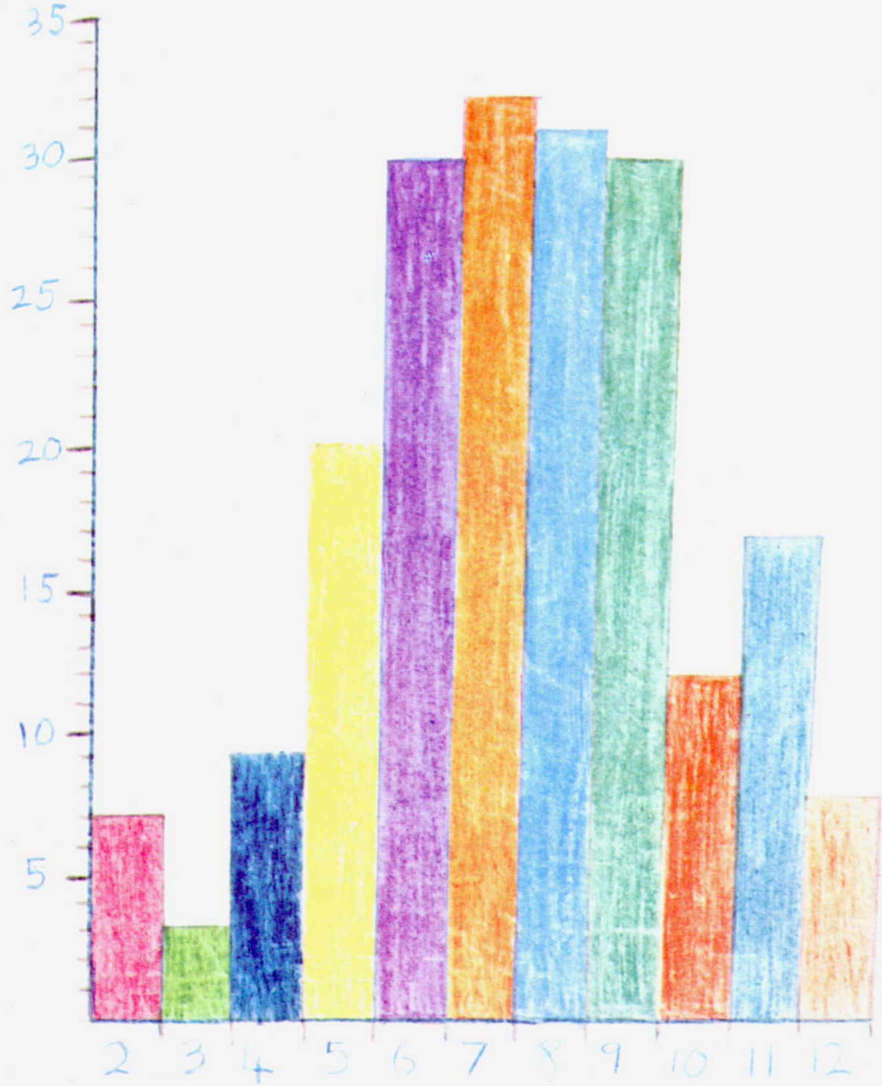
TOTALS	2	3	4	5	6	7	8	9	10	11	12
CHANCES	1	2	3	4	5	6	5	4	3	2	1 in 36

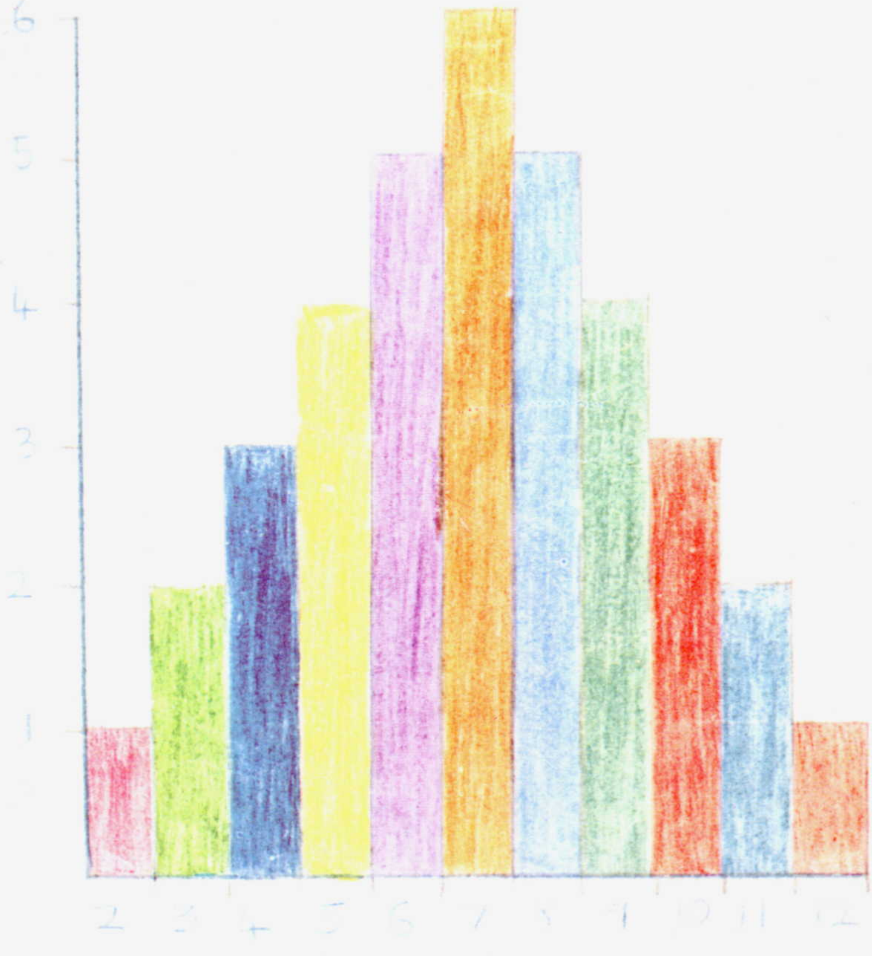
All the results of this section should be compared with those of the experiments described above.

The idea can be extended, of course, to the throwing of 3 (or more) dice.

The following two pieces of work illustrate the development in thinking (from age 7 to 11) about throwing two dice. Wendy, aged 7, wrote, 'I use a piece of paper with squares on it. I count along the bottom of the paper from 1 to 12. I only go up to 12 because I can only get two sixes which make 12. I throw two dice together, I add the dots together. Then I find the number on the chart and put a cross. I never have any number ones because I use two dice and with two dice you can't have any ones.'

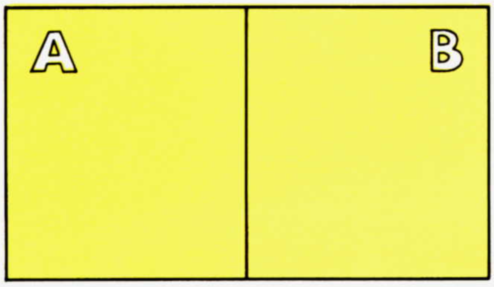



35
30
25
20
15
10
5
2 3 4 5 6 7 8 9 10 11 12


6
5
4
3
2
1
2 3 4 5 6 7 8 9 10 11 12

III Other games

1 Take 36 counters, such as are used in Bingo, each numbered with a 2-digit number whose digits may be 1, 2, 3, 4, 5 or 6. Thus: 11, 26, 35, 44 will be members of the set of numbered counters; 5, 37, 81 will not.
Place the counters in a compartment, say on half a sheet of paper, labelled A.



Two dice are thrown simultaneously and the counter corresponding to the ordered values showing on the dice is moved from compartment A to compartment B. If the dice are of two different colours, the ordered values can be distinguished, thus allowing e.g. 13 to be distinguished from 31 when a one and a three are thrown.

The process is repeated, at each stage the appropriate counter being moved from A to B, or if it is already in B, back to A again. What will happen as more and more values are thrown? At first, counters will move from A to B until equilibrium is reached. Then the number of counters in A and B will tend to fluctuate around 18 and 18. An actual game gave the following state of affairs recorded after every fifth throw:

Throws	A	B	Throws	A	B
Start	36	0	30	18	18
5	31	5	35	17	19
10	28	8	40	18	18
15	23	13	45	15	21
20	20	16	50	18	18
25	17	19			

2 Two children each throw a die in turn. The one with the higher score wins the 'turn', but if they throw the same score, this counts as a win for the second player. What handicap should be imposed on him in 20 throws?

The possible outcomes for throwing 2 dice in order are given in the following table:

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Of these 36 outcomes, in which e.g. 1, 3 is distinguished from 3, 1, those above the marked diagonal favour the second player, the second figure being greater than the first; those below the marked diagonal favour the first player, the first figure being the greater; those lying within the marked diagonal favour the second player. Who wins if the throws are the same?

If we assume that all outcomes are equally probable, then the odds in favour of the second player are 21:15, i.e. 7:5. In 12 throws his handicap should be 2; in 24 throws it should be 4; in 36 throws it should be 6.

When should it be 3?
What should it be for 20 throws?

3 Two children throw a pair of dice at a time. The first player wins if the total score is 6, or less than 6 ; the second player wins if the score is 7, or greater than 7.

Who has the advantage ?
What should his handicap be ?

We list the totals for each of the 36 possible outcomes :

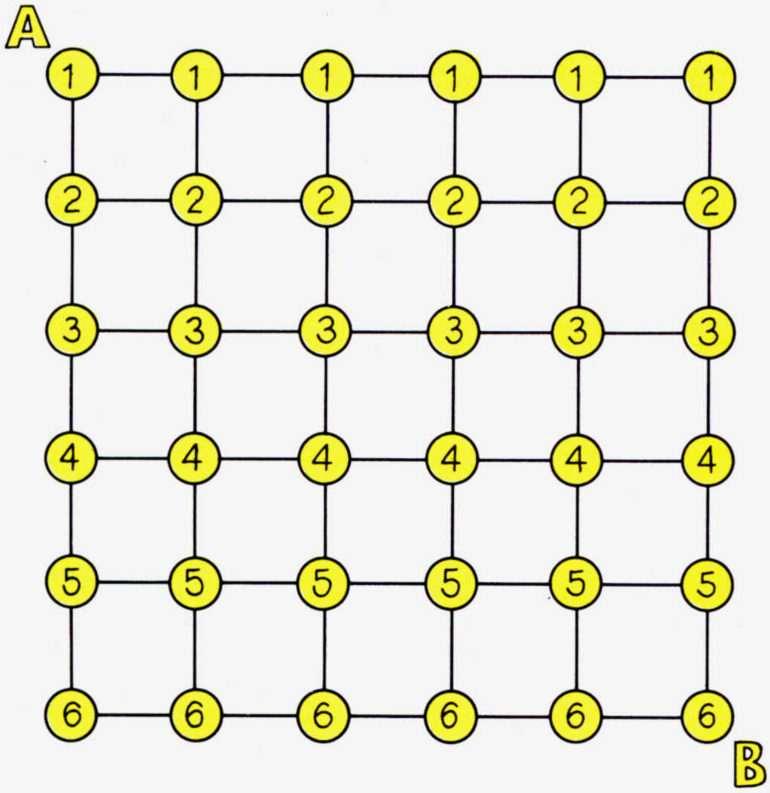
TOTALS	1	2	3	4	5	6	7	8	9	10	11	12
OUTCOMES		1,1	1,2	1,3	1,4	1,5	1,6					
			2,1	2,2	2,3	2,4	2,5	2,6				
				3,1	3,2	3,3	3,4	3,5	3,6			
					4,1	4,2	4,3	4,4	4,5	4,6		
						5,1	5,2	5,3	5,4	5,5	5,6	
							6,1	6,2	6,3	6,4	6,5	6,6
	0	1	2	3	4	5	6	5	4	3	2	1

or tabulate them thus :

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

The chance of the second player getting 7 or more than 7 is 21 : 36.
The chance of the first player getting 6 or less than 6 is 15 : 36

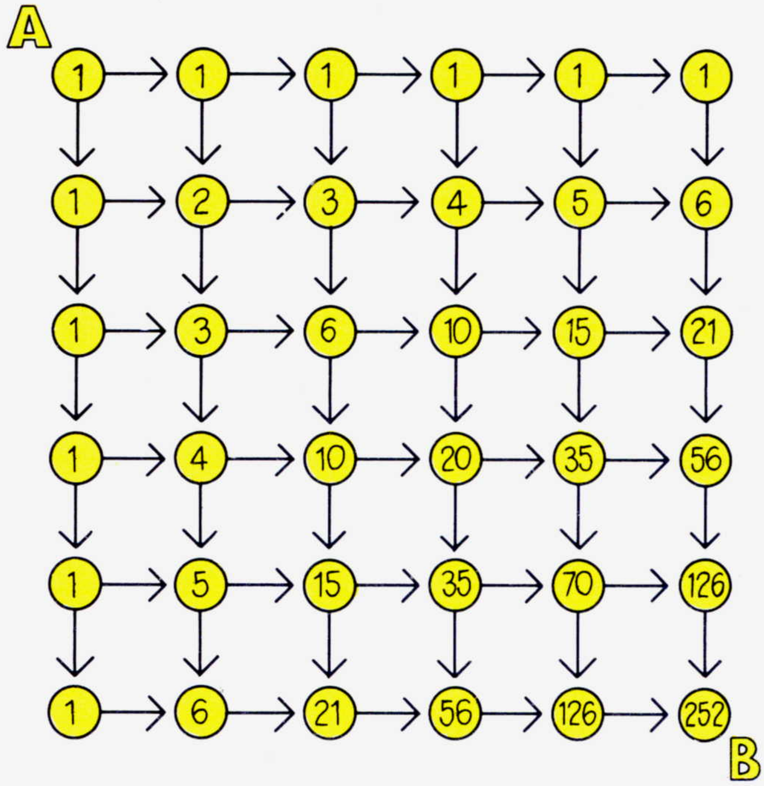
4 A game is played by 2 players, using a board with 36 lattice points, each marked with a numbered circle.



One player places a counter at A and the other at B. Each then throws a die in turn, making a move only when the die indicates a number which is the same as the number on a circle adjacent to the one occupied by his counter. Thus the players at A and B can start only with a 1 and a 6 respectively. The first player must then throw a 1 or a 2. The second player must throw a 5 or a 6, and so on. The object of the game is for a player to get his counter to the diagonally opposite corner of the board. Each player will make 10 moves, unless he moves backwards, which he is free but not advised to do. A sequence which would get the first player from A to B would be: 1, 1, 2, 3, 3, 3, 4, 5, 5, 6. The reverse sequence would get the second player from B to A.

How many of these sequences are there?

The following diagram indicates the number of routes from A's starting-point to any particular point of the lattice:



indicating 252 possible routes from A to B (or vice versa). The position may be compared with that of the checker-board man on a finite chess-board, the numbers of course forming part of a Pascal Triangle.

5 Each player throws a pair of dice in turn, counting the total as part of his score only if the combination which he throws has not already been thrown. Thus, if he throws 4, 6, he adds 10 to his score unless this same combination (4, 6) or (6, 4) has been thrown before.

A record of combinations thrown can be kept on a grid:

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	4,5	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Since order is ignored in this game, throws which are symmetrical by reflection in the leading diagonal (i.e. the line made by 1, 1 ; 2, 2 ; 3, 3 ; 4, 4 ; 5, 5 ; 6, 6) of this array are equivalent: so that the combinations included in the marked triangle can be ignored, whilst the others can be struck off one by one as they are achieved.

A specimen game:

A		B	
Numbers on die	Total	Numbers on die	Total
2,3	5	2,6	8
3,6	14	2,2	12
4,6	24	~~2,3~~	12
2,4	30	4,5	21
1,3	34	~~1,3~~	21
5,6	45	~~2,2~~	21
1,2	48	1,5	27
2,5	55	~~1,3~~	27
~~1,5~~	55	~~1,3~~	27
~~1,3~~	55	~~3,6~~	27
1,4	60	~~1,3~~	27
3,3	66	~~2,4~~	27
~~1,5~~	66	~~5,6~~	27
5,5	76	~~4,6~~	27
~~3,6~~	76	1,6	34

The play continues until one player reaches a winning score or until one resigns. Each of the digits 1 to 6 occurs exactly 7 times but order is ignored, so the number of scoring outcomes is $\frac{1}{2}(6 \times 7) = 21$. The total score possible from these 21 outcomes is 147, so that 74 is a winning score.

If one die is thrown, only 6 distinct outcomes are possible. In this case 11 is a winning score.
If three dice are thrown, what is the number of distinct outcomes? What is the highest possible total score? What is a winning score?

Answer the same questions for 4, 5, 6 dice thrown. The sequence of numbers obtained for the number of distinct outcomes is part of the diagonal of Pascal's Triangle: 1, 6, 21, 56, 126 . . .

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

1 10 45 120 210 252 210 120 45 10

The game may be played with dice having a number of sides other than 6. Work out, for each kind of die, the number of distinct outcomes when 1, 2, 3, . . . etc. dice are thrown in turn. Compare all these results with a Pascal Triangle of numbers.

6

Choose a partner. Each player has 12 counters.
Play for these in a game with 3 dice.
Whenever 11 is thrown (by either), A gives a counter to B.
Whenever 14 is thrown (by either), B gives a counter to A.
The player who first wins all the counters wins the game.
Which player (A or B) is most likely to win, and why?

7

Throw a six

Take a die.
Throw it until you get a 6.
How many throws did you need?
Record this.
Run a number of trials, recording each time.
Plot a block graph of the frequencies of 1, 2 etc. throws.
What was the **most common** number of throws?
What was the **average** number of throws?

8

Club together

Shuffle a pack of playing cards.
Deal out 4.
How many are clubs?
Record this.
Replace the cards, shuffle and try again.
Carry out a number of trials.
Plot a block graph showing the frequencies of no clubs, 1 club, 2 clubs, 3 clubs and 4 clubs.

9

Red and black

Shuffle a pack of cards.
Deal out 4.
How many of the 4 are black?
Record this.
Replace the cards.
Run a number of trials.
Plot a block graph showing how many times you had no blacks, 1, 2, 3, 4 blacks.

10

Sort a pack of cards into two piles face downwards – one containing all the black cards, the other containing all the red cards.
Take a certain number of cards from the **red** pile and shuffle them into the **black** pile.
Take an equal number from the **black** pile (which now contains a mixture of red and black cards) and place them in the **red** pile.
Are there more (or fewer) black cards in the **red** pile than there are red cards in the **black** pile?

11

Dominoes

Remove all the dominoes with one or two blanks on them.
21 should be left.
Shuffle them face down on the table.
Draw one at random.
Count the total number of pips on it.
Replace it and reshuffle.
Run a number of trials and plot a block graph showing the frequencies of different totals of pips.

12

A bag contains a certain number of red balls.
A second bag contains the same number of blue balls.
A number of balls is drawn from the 'red' bag and put into the 'blue' bag, which is then thoroughly shaken up.
An equal number of balls is then drawn from the 'blue' bag (which now contains a mixture of red and blue balls) and put into the 'red' bag.
Are there more (or fewer) **blue** balls in the 'red' bag than there are **red** balls in the 'blue' bag?

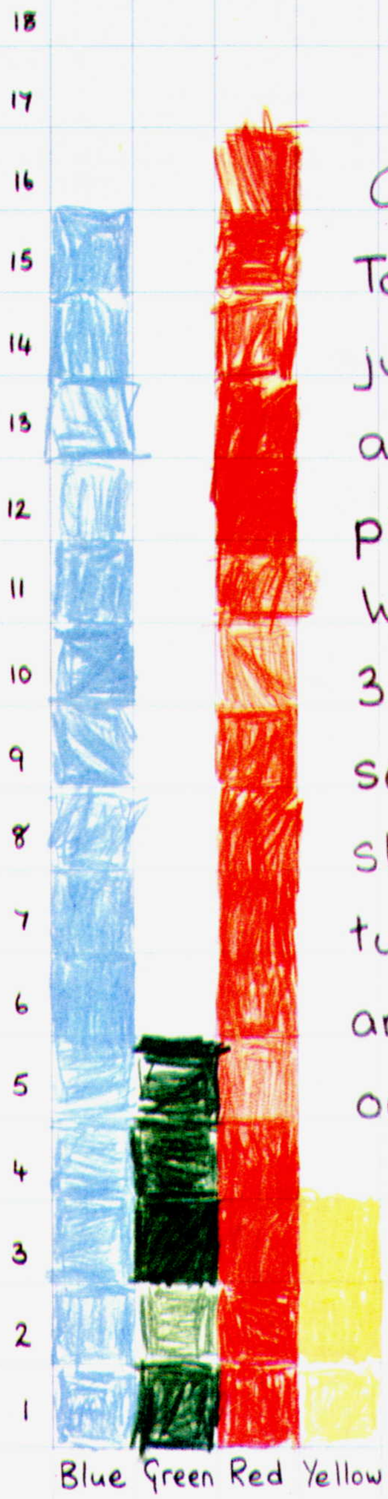
13

Take a book containing some normal English.
Record how many letters appear between each pair of e's.
Thus, for:
'Keep a true record of the number',
record: 0, 5, 1, 8, 4 etc.
We call these 'runs' of various lengths.
Plot a block graph of the frequencies of the runs of different lengths.

14

Take a simple book.
Count the number of letters in each word.
Plot a block graph showing the number of words with one, two, three . . . etc. letters.
Take a very difficult book.
Do the same.
Compare the two graphs.

A Guessing Game



Craig, Carol, Tony, Carol W. Tony N., Ian and Paul who are just six, and Elizabeth and Keith who are five, played this game.
We put 16 red, 6 green 3 yellow and 10 blue squares in a bag. We shook the bag, and took turns to draw out a square and record the colour on our chart.

Craig said. "As red is the greatest set we sha[ll] most likely draw red, bu[t] you could draw any colou[r]

"Reds won, but if we'd drawn one more blue it would have been a draw." "Reds won because they were the greatest sub-set in the bag."

Card graph. By Joanne Wilson and Karen Clark.

We were given a pack of cards one morning and then we started work using the cards. First of all we saw how many cards we had to turn over before a red card appeared. We had 50 go's each so that we had had 100 go's then we graphed it. The results were: 50 ones, 20 twos, 14 threes, 9 fours, 3 fives, 0 sixes, 2 sevens, 2 eights.

Next we found out how many cards we had to turn over before a club appeared. We both had 50 go's and then we graphed the results on the same scale as the other graph to see how they compared. The results were: 28 ones, 16 twos, 18 threes, 4 fours, 12 fives, 5 sixes, 4 sevens, 1 eight, 2 nines, 5 tens, 1 eleven, 2 twelves, 1 thirteen, 0 fourteens, 1 fifteen.

After that we turned over cards until we got a Jack. We also graphed this graph on the same scale as the others so that we could compare it with the other two graphs. The results were: 5 ones, 6 twos, 9 threes, 8 fours, 5 fives, 3 sixes, 5 sevens, 2 eights, 7 nines, 10 tens, 7 elevens, 3 twelves, 2 thirteens, 1 fourteen, 2 fifteens, 6 sixteens, 3 seventeens, 1 eighteen, 1 nineteen, 1 twenty, 1 twenty ones, 2 twenty twos, 1 twenty three, 0 twenty fours, 3 twenty fives, 1 twenty six, 1 twenty seven, 1 twenty eight, 1 twenty nine, 0 thirtys.

We expected a red card to come up most in the

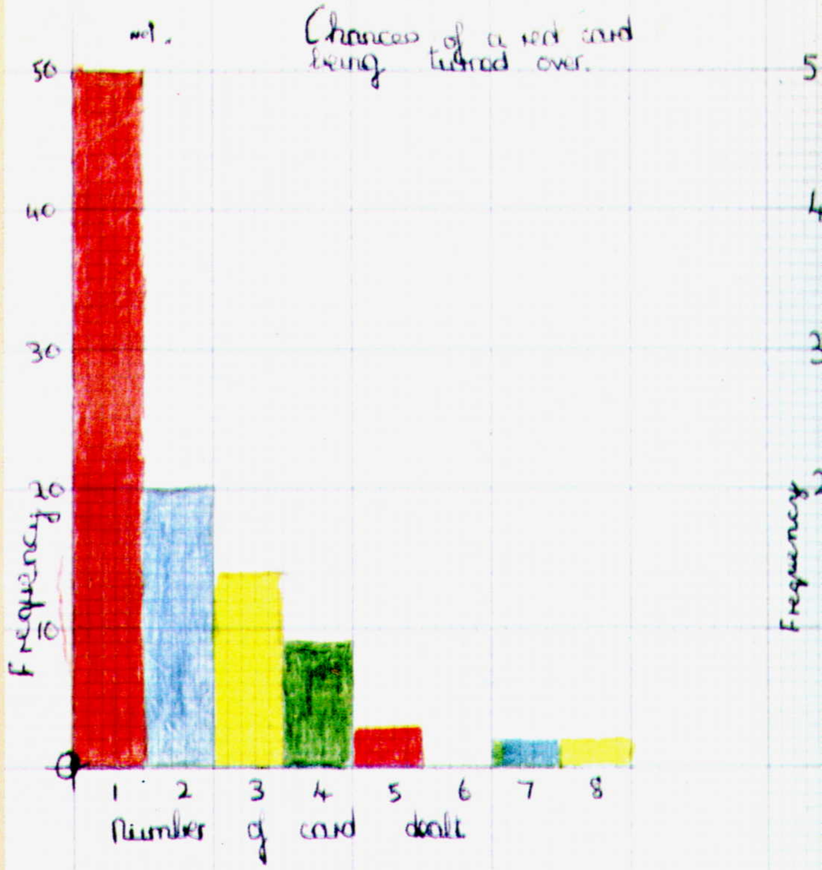
Wilson and Clark

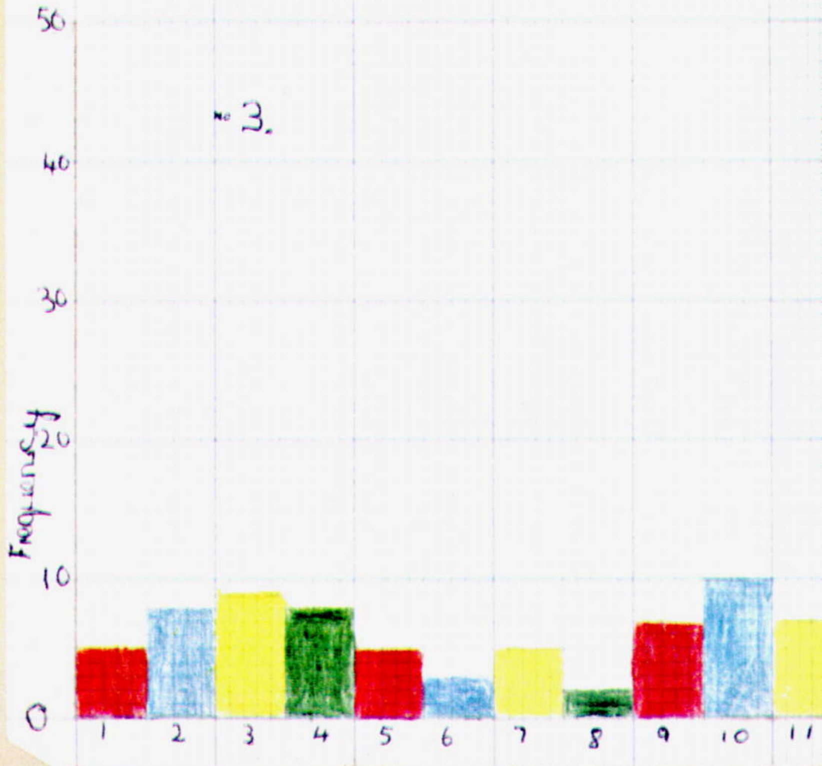
first two because there were only two colours, black and red. The chances of a red card being turned over were 1 in 2.

The chances of a club being turned over were 1 in 4 because there are four sets, spades, hearts, clubs or diamonds. We both expected the first four columns to be the largest.

The chances of a Jack appearing was 1 in 13 because there are 52 cards in a pack and four different Jacks. A quarter of 52 is 13 so the chance is 1 in 13.

We were interested to see that numbers 1, 2, 3, 4, 5 were in a symmetrical pattern and so were numbers 9, 10 and 11 also 21, 22 and 23 in graph number three.

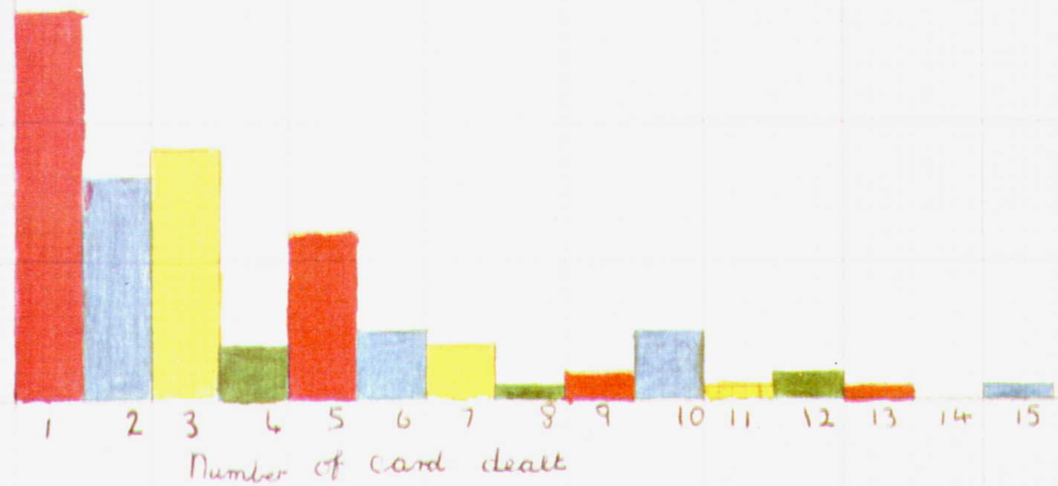




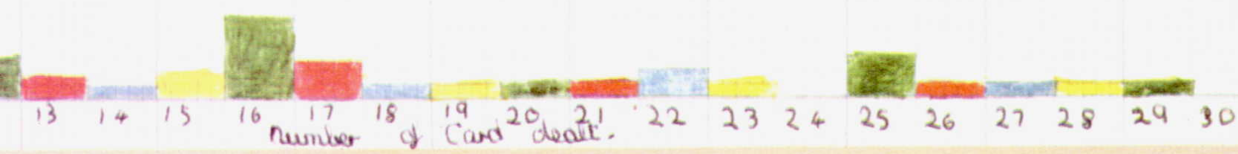
no 2.

Chances of a club being turned over.

By Joanne Litson and Karen Clark.



Chances of a Jack being turned over.



SAMPLING

Earlier in this Guide (p. 5) a brief reference was made to random sampling.

Sampling is a special branch of statistics – a method by which some property of a large 'population', or set of measurements, is investigated by testing a small fraction of it.

Why take samples? It is often inconvenient, impracticable and uneconomic to test a large number of items. A great deal of time and energy may be wasted if the articles are small and if some other means of analysing the situation can be found. For instance, counting the matches in every box of a large consignment would give no truer an estimate of the number in a box than would be achieved by investigating a suitable sample of boxes.

How to take samples. Before deciding on a particular method of sampling, we must first be clear as to what it is we are trying to find out about our population.

What precisely are we trying to discover?

What sort of information can be obtained about our population?

How reliable will that information be?

How can we select the sample which will give us the most reliable estimate of the true state of affairs?

The properties being investigated will affect our choice of sample. A sample that is suitable for the investigation of one property may not be suitable for that of another property.

We must avoid 'bias' in our sample; we must seek to eliminate the human element; personal choice and prejudice must be avoided.

Random sampling

In a random sample, every member of the population must have an equal chance of being selected.

It is usually not necessary, and often not possible, to obtain a sample that is random in all respects. A method that ensures randomness in selecting a sample from one population (set of scores) need not do so when used to select a sample from another population (a different set of scores).

Plenty of discussion will be needed if children are to get a clear understanding of the pitfalls that can be encountered and the twisted information that can be gained from poor methods of sampling. Any statement made and based on a sample must be weighed carefully against 'the size of the sample' and the question, 'Was it a random sample and in what respect?'

Advertisements which often contain exaggerated and misleading statements will provide ample material for discussion. See page 3 for examples.

Some sampling activities

Children can carry out very simple sampling experiments such as the following:

Take 5 urns containing the following sets of coloured marbles
Urn A: 45 Red, 5 Black
Urn B: 40 Red, 10 Black
Urn C: 35 Red, 15 Black
Urn D: 30 Red, 20 Black
Urn E: 25 Red, 25 Black

Choose an urn.
Take 5 samples of 'one' marble, replacing the marble each time.
Record the colours of your samples.
Can we guess the chosen urn on 5 samples?

Repeat the experiment.
Can we guess the chosen urn on 10 samples?

And so on.
Can we guess the chosen urn on 5, 10, 15, 20 . . . samples?

Sample number	Colour red or black
1	
2	
3	
4	
5	

Other experiments can be done with a sampling bag, bottle or box containing a large number of different-coloured balls in known proportions. (A Binomial sampling box can be bought from Technical Prototypes (Sales) Ltd., 1A West Holme Street, Leicester.) A bottle should have a neck and the sample will be those balls which fall into the neck when the bottle is inverted. In order to ensure a random sample the bag or box must be shaken well after each sample has been taken ; and to demonstrate a biassed sample some of the balls could be replaced by heavier ones which will tend to fall to the bottom. In taking samples from the box or bag, put both hands in the bag and count the number of balls you have picked up before removing them from the box or bag.

Work in groups of 2 or 3.

Take 50 samples of 10 balls from the 'population' of coloured balls.

Record in a table the number of coloured balls (i.e. balls other than white ones) found in each sample.

Number of coloured balls in samples of ten	Tally marks	Frequency
0		
1		
2		
3		
o		
o		
o		
o		
o		
o		
10		

Combine the results of all the groups and draw a block graph to illustrate the number of coloured balls found per sample.

Estimate the proportion of coloured balls in the box.

How does your estimate compare with the true value ?

Work in groups of 2 or 3. Using the Binomial sampling box, take 100 samples of 10 balls.

Record the number of **red** balls in each sample.

Number of red balls in each sample of ten	Tally marks	Frequency
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Combine the results of each group into a single frequency table and draw the experimental probability histogram to illustrate your results.

Can you use your result to estimate the total number of 'red' balls in the box ?

How reliable will our estimate be ?

A bag of marbles contains 2 colours in known proportions (say, 800 black, 200 white).

i Take samples of 5.
Record the number of white marbles in each sample.
Record the frequency distribution graphically for 100 (or more) samples.
Care must be taken that each sample is obtained by placing **both** hands **inside** the bag and counting the marbles from one hand to the other.

After each draw all the marbles must be returned to the bag before taking the next sample.

ii Repeat for samples of 10, 20, 50, 100 . . . etc. (Samples of 5 taken in i may be grouped in pairs for samples of 10, etc.)

What do you notice ?

It will be seen that the larger samples give a truer estimate of the proportion of black to white balls in the bag.

In other words, the larger the sample, the closer will the scores be grouped around the correct value for the whole population.

Few misleading large samples occur. But very occasionally a random sample may not be representative of the whole population. This must be regarded as a possible, although improbable, state of affairs.

> Take a bag of marbles of 2 colours (say black and white) in **unknown** proportions.
>
> To estimate the proportion of black marbles to white marbles by sampling:
> i as before, select samples of 5. Record the number of white marbles in each sample. Graph the frequency distribution for 100 such samples.
>
> Can you give an estimate of the actual proportion of black marbles to white marbles in the bag?
>
> ii Repeat with samples of 10, 20, 50, 100 . . . etc. Try to give an estimate at each stage of the proportion of black marbles to white marbles actually in the bag.

The use of random numbers

One of the points that we have tried to stress in this section sampling is the difficulty of obtaining a truly representative sample. Few valid generalisations can be made about the method of taking samples, as different factors will have to be taken into account for each situation. However, if the circumstantial factors have been covered, one of the most satisfactory ways of making a random sample is by the application of a set of random numbers. You could, for example, select the people that you questioned going into th public park (see page 5) by using a set of random numbers, as long as you did not forget that a representative sample could only be obtained by questioning people throughout th time that the park was open.

Sets of random numbers can be obtained in several ways, for example:

a with a 10-sectioned spinning disc – the sections numbere from 0–9.

b with a pack of cards – the court cards should be ignored and each 10 counted as 0. The cards should be shuffled between each draw.

In both cases the chance of a number being drawn is one in ten each time.

In choosing a sample of 10 from a class of 32 children, using set of random numbers, the members of the class should be numbered from 1 to 32. Pairs of numbers from the disc or the cards can then be read off until there are ten numbers, numbers over 32 being rejected. A pair 07, for instance, would indicate that child number 7 should be included in the sample; 70, however, should be rejected.

A great deal of interesting discussion can arise from the construction of such a set of numbers, and questions can be thrown out, such as:

a How random is it?

b How frequently does each digit appear?

It might be interesting to compare the sets of random numbers that the children produce with sets that have been produced by more sophisticated means.

Random Numbers

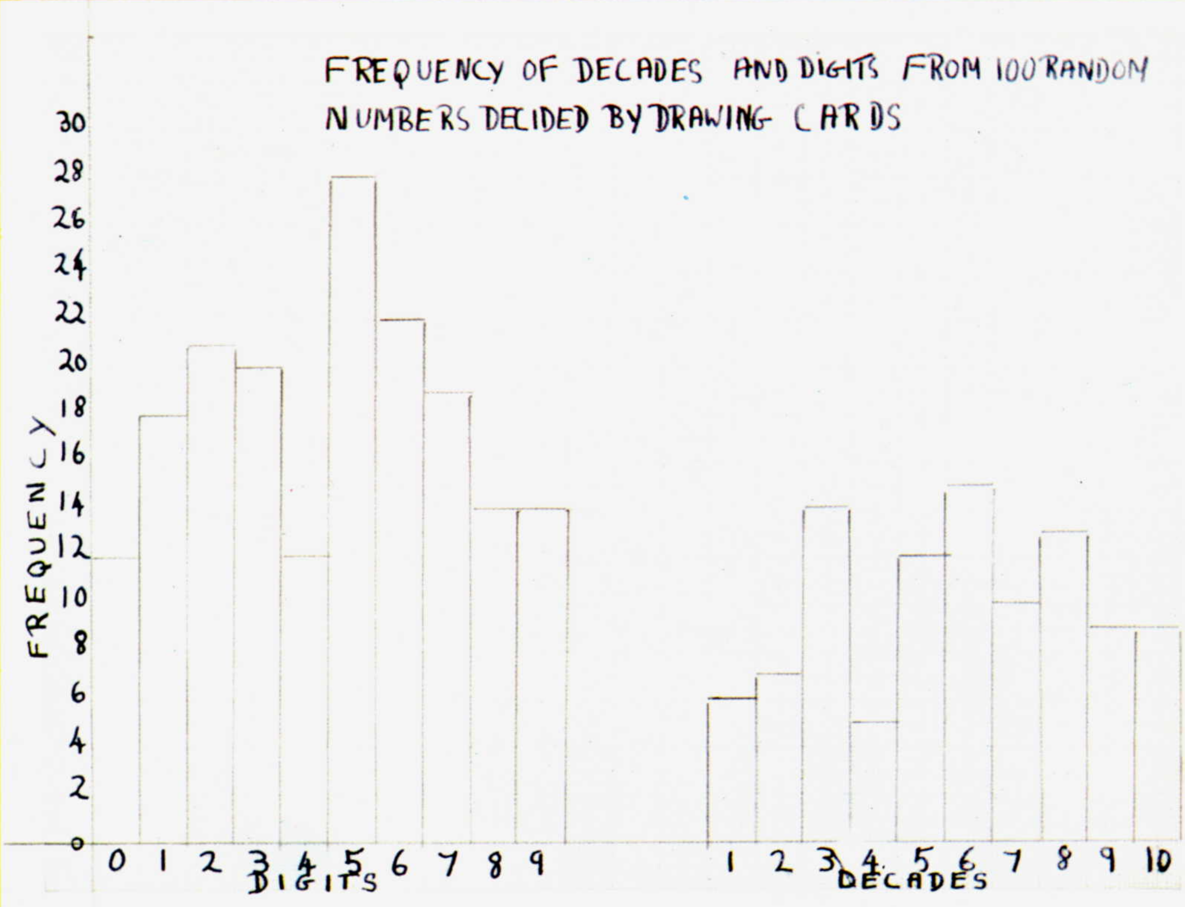
We had one pack of white cards, and one pack of pink cards. Each set of cards was numbered 0 to 9. Two people each had a pack. We shuffled the cards and the first person drew a card and counted it as 'tens'. The second person then drew a card and this counted as 'units'. e.g. if the first person drew 2 and the second person 3 the number would be 23. If 00 was drawn it would count as 100. We drew the cards 100 times and we made a note of the numbers which occured.

It was difficult to decide if the method of choosing these numbers we used was correct because random numbers don't seem to have any patterns. Some numbers didn't come up at all and others came up more than once.

We looked at sets of random numbers produced by other groups but we could not really see any relation between the sets. We decided that there did not seem to be a way for deciding how random random numbers are.

Mark Alexander Age 11

94	18	25	73	55	03	31	23	87	78
53	78	62	53	33	82	88	96	76	25
80	50	80	64	50	80	15	62	12	65
65	27	45	00	65	88	35	21	47	43
29	97	13	83	51	50	60	37	81	56
88	23	54	42	73	73	04	51	66	30
09	26	37	17	79	98	19	90	50	62
99	53	23	22	99	25	98	17	21	53
56	07	09	75	22	56	68	62	100	48
79	56	83	52	48	71	41	09	46	51



C. Oldfield aged 11

Number Samples 1 to 99.

I took the number tiles from 1-19 and put them in a cardboard box with a hole in the bottom just large enough for one tile to fall through.

In a tin I put 9 blue pegs and 9 green pegs. The blue pegs were to count as ten, and the green pegs were to count as units. I shook the box of number tiles until one tile fell out.

Whatever number was on the tile, I drew that number of pegs from the tin.
eg. If a tile had 8 on it I took out 8 pegs from the tin.

I looked at these and if there were 5 green pegs and 3 blue, the number would be 35.

Before I started I thought that probably each number from 1 to 99 would come about once. Some numbers would come up twice or three times or not at all, but I expected most numbers about once.

When I had done the experiment I found that lots of numbers had not come up at all but I saw that some number came up more times. Thirty numbers didn't come up at all and five numbers came up 3 times.

I normally expect to find patterns when I am collecting sets of numbers in Maths but I didn't see any patterns with these numbers.

18	50	63	52	96	15	51	70	21	50
69	9	85	42	17	31	87	9	12	73
30	26	19	16	76	23	54	92	83	24
9	12	74	56	73	69	46	79	32	71
47	90	35	5	58	90	22	20	6	80
55	87	26	67	85	80	89	93	14	66
17	67	59	23	27	16	40	23	41	2
68	59	99	65	57	86	20	83	49	82
26	79	85	29	94	61	33	35	46	63
52	10	84	19	100	22	8	15	11	9

If there was a pattern then perhaps the numbers would not be random.

If I had done the experiment many more times then perhaps each number would have come up an equal number of times. In other work we have had to do 100 experiments before a pattern came up on a graph.

RECORDING

I Grouped frequency distributions

In all our examples so far we have been considering a simple frequency distribution in which a tally is made of the number of occurrences of a particular event.

The following table represents the amounts of pocket money given to the children in a certain class:

6d, 6d, 9d, 1s 0d, 1s 0d, 1s 0d, 1s 0d, 1s 6d,
1s 9d, 2s 0d, 2s 6d, 3s 6d, 4s 0d, 4s 0d, 4s 6d

The different amounts can, of course, be represented graphically by means of a block chart, or a bar chart, or perhaps by a pie chart. But for a large number of items, the work becomes tedious.

The scores may be arranged in a frequency table, thus:

Pocket money (pence)	Frequency
6	2
9	4
12	4
18	1
21	1
24	1
30	1
42	1
48	2
54	1

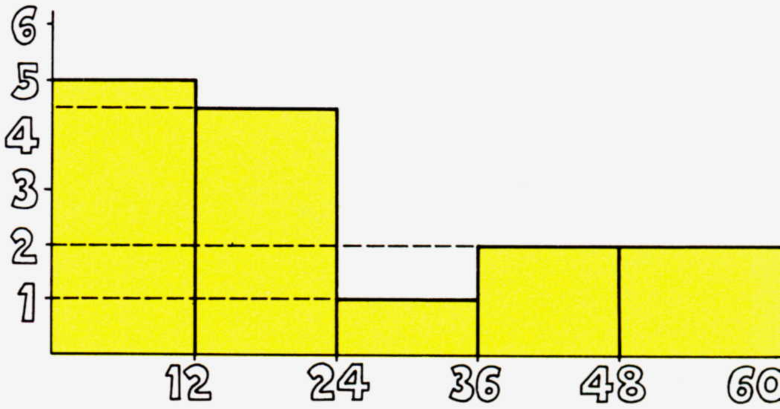
and the frequencies plotted against the amounts of pocket money, in the form of a block chart.

When a large number of scores is under consideration it is sometimes practicable to group the scores together and to record the numbers of scores falling within certain ranges. A convenient grouping will often assist us in the investigation of a set of data and will often be simpler than having to make a study of a large number of separately listed items.

Returning to the children's pocket money, we could group th amounts together into ranges of 1 shilling, as on the left in the diagram below:

Pocket money (pence)	(pence)
0 — 12	0 — 11
12 — 24	12 — 23
24 — 36	24 — 35
36 — 48	34 — 47
48 — 60	48 — 53
etc	etc

But here the variables overlap, and it is necessary to define more explicitly the limits of our ranges. We could perhaps define our ranges as they are on the right in the diagram above, at the same time ensuring that they fall within the limit of accuracy of our measurements. Such a definition of the range is always possible when dealing with discrete quantities; that is when each quantity has a separate and stated value. These can be illustrated graphically by means of a **block chart** where the height of the block indicates the frequency of that range of data. Thus:

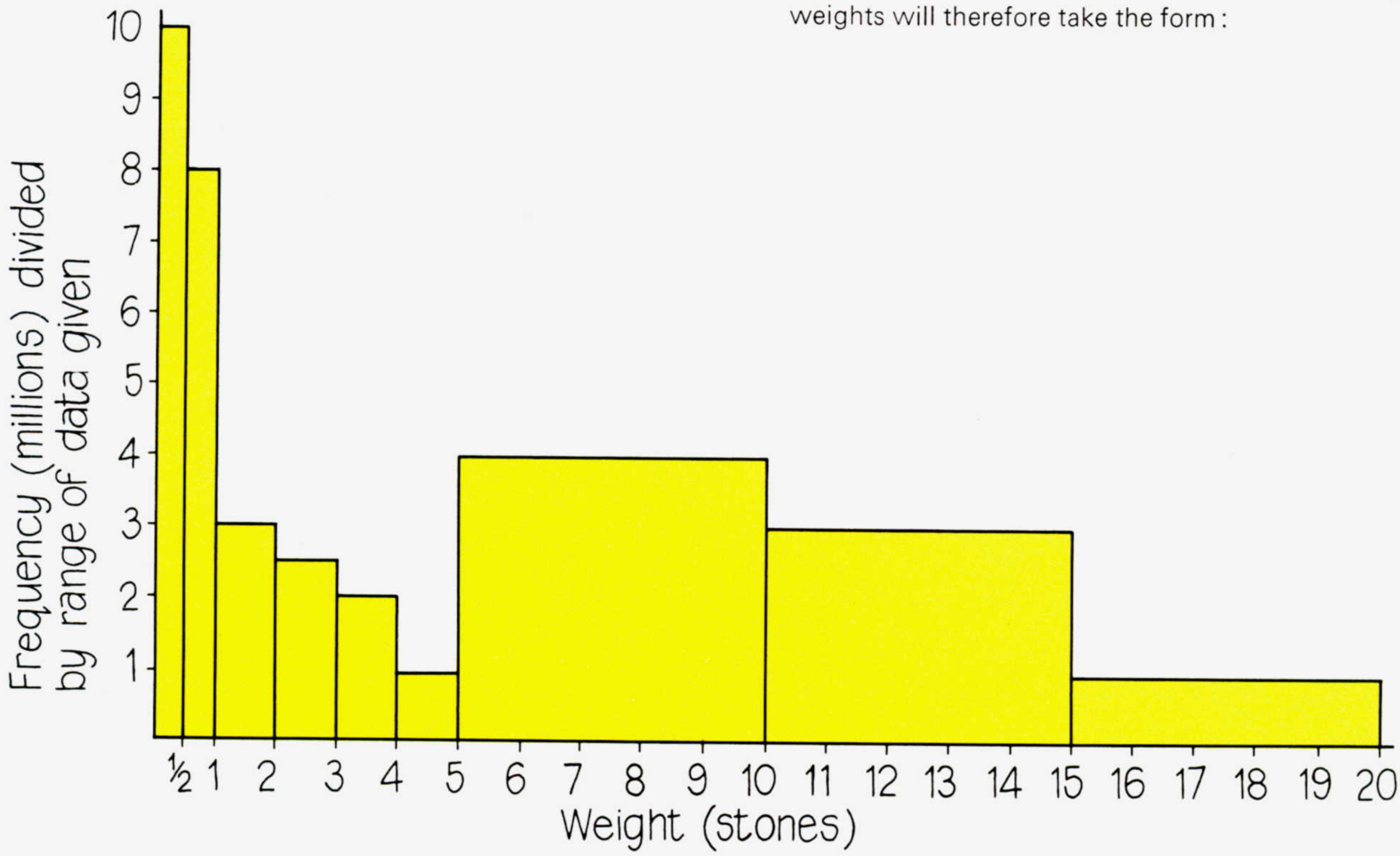


It should be noted that there are no values of the pocket money occurring between 0 and 1, 1 and 2, 2 and 2 and 3, etc. No children received sums of pocket money involving fractions of a penny.

We have been considering only discrete values of the variable and we have illustrated the data by means of various types of pictorial representation: block charts, bar charts, pie charts, etc.

In our next type of graphical representation, the **histogram,** it is the **areas** of the columns that will represent the frequencies and we adjust the vertical height of the column over a particular range accordingly. Thus, in the example on page 36, for the range 0 to $\frac{1}{2}$ stone, the horizontal width of the block will represent $\frac{1}{2}$ unit so that the height of the block will be 10 units, giving an area of 5 square units to represent a frequency of 5.

The histogram representing the grouped-frequency table of weights will therefore take the form :



The representation is perhaps a little deceptive. It is the **area** that corresponds to the frequency of a particular weight range.

Superficially, block graphs and histograms look much the same. In fact a bar chart may be regarded as a histogram in which the bars are of unit width.

In the bar chart, however, we are dealing with discrete values on a horizontal axis. Points between have no significance whatsoever.

In the histogram, every point on the horizontal axis represents some value of a continuous (non-discrete) variable.

Arrange the pupils of 2 classes into samples of 10.

Collect and record the amounts of weekly pocket money.

a What is the modal pocket money for the combined 2 classes?
b What is the mode for each sample of 10 children?
c What is the mode of the set of all the modes?
d Is the mode for any one sample of 10 children (answer **b**) nearer to the mode for the combined 2 classes (answer **a**) than to the mode of all the modes (answer **c**)?

Of course it may happen that several modes will occur. If so, the position should be discussed fully with the children and the consequent inconveniences pointed out.

If a score of 75 represents the mode for each of 3 classes:

a can you be certain that the same number of children in each class scored 75?

Considering the scores of the combined 3 classes, can you decide how many children scored:

b more than 75?
c less than 75?
Discuss the position fully amongst yourselves and with your teacher.

We have to consider whether or not the mode is of any use to us in answering questions of this type.

i For an unbiassed die, write down a frequency table showing the number of times each of the scores 1 to 6 could be expected to occur in 30 throws.
What is the modal score?
ii Repeat for a pair of dice.
iii Repeat for a coin (frequency of occurrence of head, tail).
iv Repeat for a pair of coins (frequency of occurrence of combinations HH, HT, TT).

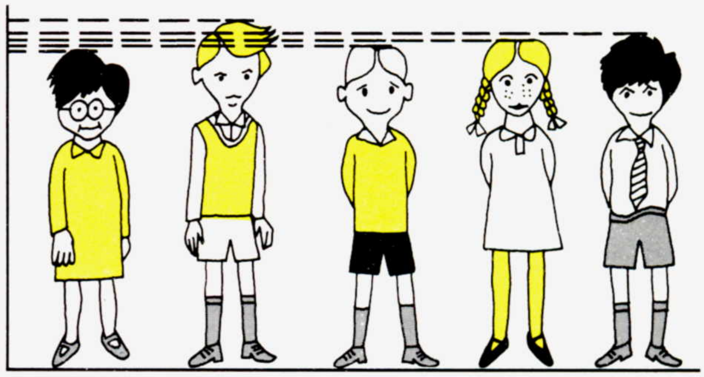
II The median

If we now arrange the shoe sizes (page 38) in order of size, according to the number of each, we have:

6, 6, 7, 7, 7, 7, 7, 7, 7, 7, 8, 8, 8, 8, 8, 8, 9, 9, 9, 9, 9, 9, 9, 9, 10, 10, 10, 11.

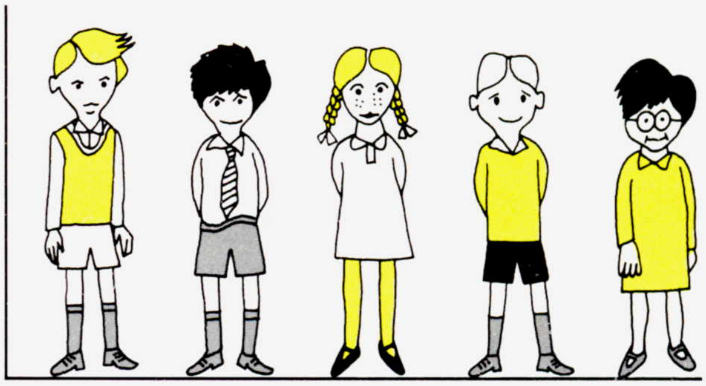
The shoe size which occupies the central position is size 8, so it can be regarded in a certain sense as 'typical' and may serve as an average shoe size for this group of children.

Study these drawings of children in respect of height:



How would you decide whether a child is tall? short? for his age.

If we arrange the children in height order:



are we able to judge more easily the height of a 'typical' child?

Choose 5 children at random from your class and say what you think is the average height of the 5.

Repeat with 11 children.

Repeat with 21 children.

If all the values of the data are arranged in order from left to right then that value which has as many values above it as there are below it is known as the **median** value. For an odd number of values, the median is the middle value in an ordered list and is in fact one of the data.

e.g. The median of 2, 25, 8 is 8.

The median of 9, 12, 18, 25, 6 is 12.

For an even number of values the median is located mid-way between the two middle values in an ordered list.

e.g. The median of 31, 35, 37, 40 is 36 (mid-way between 35 and 37).

The median of 3, 7, 8, 9 is 7·5 (mid-way between 7 and 8).

For an even number of values, the median is not, in general, one of the data.
What is the exception to this rule?

Find the median of the following sets of data:

a 21, 27, 28, 30, 31, 32
b 15, 17, 19, 21, 23, 25, 27, 29
c 1, 2, 2, 3, 3, 3, 4, 4, 4, 4
d 1, 2, 2, 3, 3, 3

i Arrange the class into samples of 5 children, each sample grouping itself into height order.

a Find the median height for the entire class.
b Find and record the median of each sample. Give your measurement to the nearest inch (or any other agreed degree of accuracy).
c Find the median of the medians of the samples.
d Is the median of any one sample nearer to the median of the whole class (answer **a**) or to the median of the medians (answer **c**)?

Repeat for:

ii weight.

iii length of foot.

iv age in years and months.

v shoe size.

It will be observed that the median is not affected by quite considerable variations in the other values. Discussion should be encouraged to discover what does, and what does not, affect it.

The results of the coin-tossing and dice-throwing activities of Chapter 2 will provide a number of topics for discussion and application of the averages discussed so far.

III The mean

A more sophisticated idea of a 'typical' or 'average' member of a population is exemplified by a levelling-out process.

Let us suppose that a number of children (say 40) have been held captive in their classroom. They are told that they will be set free if they pay as 'ransom' 6d per head. How are they to decide whether or not they can pay the ransom?

The amounts which they possess are shown in the following table:

Amount (in pence)	0	1	2	3	4	5	6	7	8	9	10	11	12
Frequency (number of children)	3	5	1	6	7	2	3	1	4	3	3	1	1

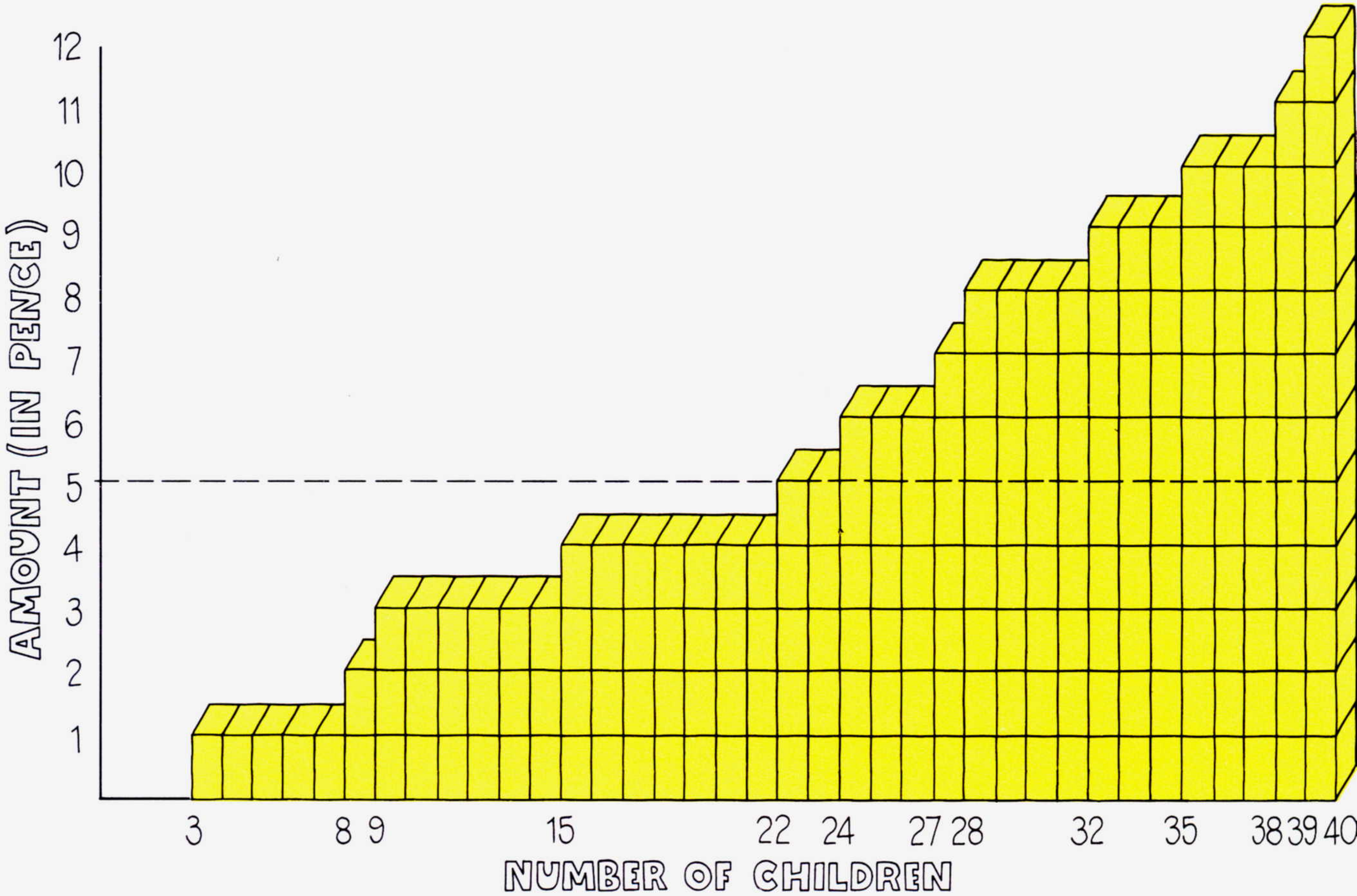
If these amounts are shown in a block diagram as on page 41, then we can 'level out' the graph by filling in the spaces below a certain line (the position of which is to be found by trial and error in the early stages) with pieces of the diagram which lie above the line.

This illustrates the fact that, in this case, if every child had **the same amount leaving the total sum of money (i.e. the area of the graph) unaltered,** they would each have 5d.

There would be insufficient money to pay the 'ransom' of 6d per head.

This levelling-out value is known as the **mean** of the data and is sometimes referred to as the 'arithmetic mean' or the 'arithmetic average'.

Given a set of n numbers, the mean is that number which, when multiplied by n, gives the total of all the numbers. The diagram on the opposite page illustrates the example above.



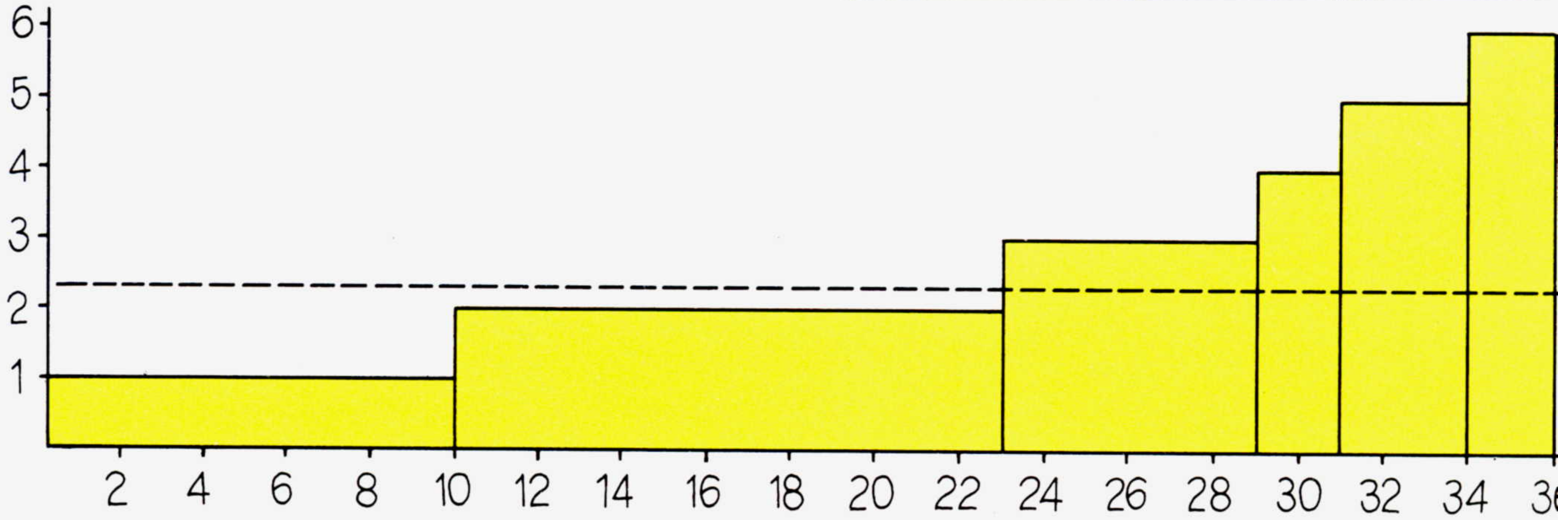
Total number of pennies = (0 × 3) + (1 × 5) + (2 × 1) + (3 × 6) + (4 × 7) + (5 × 2) + (6 × 3) + (7 × 1) + (8 × 4) + (9 × 3) + (10 × 3) + (11 × 1) + (12 × 1) = 200.

(Average number of pennies) × (Number of children) = 5 × 40 = 200.

It is best to introduce children to this way of averaging by suggesting that they build up the graph with blocks or tiles. They can then mark the outline of the block graph and move the blocks or tiles from the top until they have formed a rectangle which will give them the 'average line'.

Let us return to our example of size of family, and apply the levelling-off process to the corresponding block graph.

Number of children	1	2	3	4	5	6
Frequency (number of families)	10	13	6	2	3	1



Levelling off gives us a mean size of family of between 2·3 and 2·4 children!

What we are saying is that if all the families were the same size, the total number of children remaining the same, then there would be approximately 2·37 children in each family.

Total number of children = (1 × 10) + (2 × 13) + (3 × 6) + (4 × 2) + (5 × 3) + (6 × 1) = 83

(Average size of family) × (Number of families) = 35 × 2·37 approximately.

On this basis, 2·37 children (approximately) is an average size of family!

Great care must be exercised in choosing the value which is to represent a typical member of our population.

IV Other activities involving averages

1 Given the Matches

Take a box partially full of spent matches.

Do not let anybody open the box once the experiment has started.

Ask people to guess how many matches are in the box – using any method they like.

Record their guesses.

What was the most common guess ?

How near accuracy was it ?

2 Chase the Ace

Shuffle a pack of playing cards.

Turn up the cards from the top until an ace appears.

How many cards did you have to turn up ?

Carry out many trials and plot the block graph.

What is the **most frequent** number you have to turn up ?

What is the **average** number you have to turn up ?

3

Use the idea of an average to calculate the number of words on a page of a book which you are reading.

In this example, the mean in itself is not significant but is useful for a quick calculation of a total.

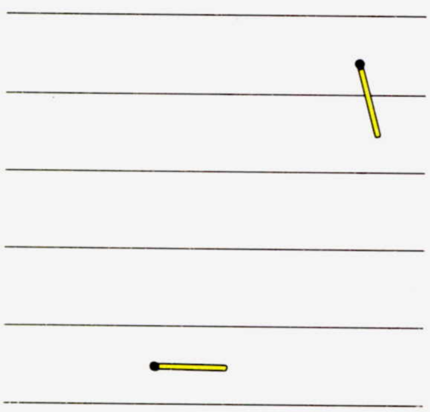
4 The Biscuit Tin Game (see page 2)

MEASURING PROBABILITY

In the real world, there is rarely a definite answer to a question: the new car **may** work satisfactorily, the electric light bulb **may** last another six months, the nail will **probably** be near enough the length you asked for. But even when children have some idea of chance, there is a danger of encouraging a false belief that chances are always 'equal', as with the heads and tails in coin tossing. It may, therefore, be better to start with an experiment whose outcome is not so clear.

> Take 10 drawing pins.
> Drop them on to a table, and record how many land on their backs, points up.
> Run a number of trials.
> Draw a block graph to show how many times, in each sample of 10 pins dropped, there were 0, 1, 2, etc. 'points up'.

Other interesting experiments are 'Bouffon's Needle' and 'Happy Birthdays'.

> Drop a matchstick (or needle) 50 times on to a set of equally spaced parallel lines. The distance between a neighbouring pair of lines should be the same as the length of the matchstick.
>
> 
>
> Record the number of times the matchstick falls across a line. Repeat the experiment a number of times (say a dozen batches of 50 throws), and then repeat again, leaving the parallel lines as before but this time varying the length of the matchstick. Investigate any relationship between the length of the match-stick and the number of times (in a given trial of 50) that it falls across a line.

> **Happy Birthdays**
>
> There are 365 days in the year.
> There are about 30 people in this room.
> How likely do you think it is that you will be able to find two people with the same birthday?
> Go round and ask people their birthdays.
> Record them.
> Can you find two the same?

Surprising as it may seem, it is more likely than not that in a collection of 24 people 2 of them will have the same birthday. See Martin Gardner's *Mathematical Puzzles and Diversions*, p. 50.

There is, of course, also a place for coin tossing. For instanc after a long run of 'heads' a child may say that 'certainly it must be tails next time' and only discussion and further experiment will convince him that the outcome of each tos entirely independent of all previous results.

Probability affords a way of making predictions which are more accurate than wild guesses. First guess, then experiment, then try to improve the first guess in the light of the experiment.

> Take 4 pennies.
> Make a guess as to how many times, in 16 tosses of the 4 pennies together, 4 heads will appear.
> Then do the experiment.
> How near was your guess?
> Do the experiment a number of times. Each time try to improve on your previous guess.

Expectation

It is convenient to return to 'equally likely' events to introduce a measure of probability. In tossing a single coin, there are two possibilities, H and T, and in the absence of bias, we might **expect** one half of the outcomes to be H anc one half to be T (although, of course, it will not always wor out that way). The **probability** of H is $\frac{1}{2}$.

Again, in throwing an ordinary 'fair' 6-sided die whose faces are numbered 1 to 6, all the faces have an equal chance of falling uppermost. Each of the six scores 1, 2, 3, 4, 5, 6 is just as likely to occur as any one of the remaining five scores. In a sequence of successive throws we would **expect**

$\frac{1}{6}$ of the throws to indicate 1
$\frac{1}{6}$ of the throws to indicate 2
$\frac{1}{6}$ of the throws to indicate 3
$\frac{1}{6}$ of the throws to indicate 4
$\frac{1}{6}$ of the throws to indicate 5
$\frac{1}{6}$ of the throws to indicate 6

We say that the probability of occurrence of any one of the scores 1 to 6 is $\frac{1}{6}$.

Compound events

If two coins are thrown together, the outcome may be HH, HT, TH, TT. The probability of two heads is accordingly $\frac{1}{4}$, of two tails also $\frac{1}{4}$, but of 'one of each' the probability is $\frac{2}{4}$, i.e. $\frac{1}{2}$.

Similarly, in a single throw of a pair of six-sided dice, one of six possible outcomes on die A is combined with one of six possible outcomes on die B. All possible, equally probable outcomes of a single throw are shown in the diagram below:

die B

die A	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

The table shows the total score obtained by adding the separate ones on die A and die B. Altogether there are 36 possibilities, of which, for example, the total '6' occurs 5 times. The probability of obtaining a total score of 6 is therefore $\frac{5}{36}$.

Score	Expected Frequency	Probability
1	0	0
2	1	1/36
3	2	2/36=1/18
4	3	3/36=1/12
5	4	4/36=1/9
6	5	5/36
7	6	6/36=1/6
8	5	5/36
9	4	4/36=1/9
10	3	3/36=1/12
11	2	2/36=1/18
12	1	1/36
	36	

Children can compare this theoretical table with actual results.

Probability Graph

With this graph I found that however many coins I had I had one more possibility.

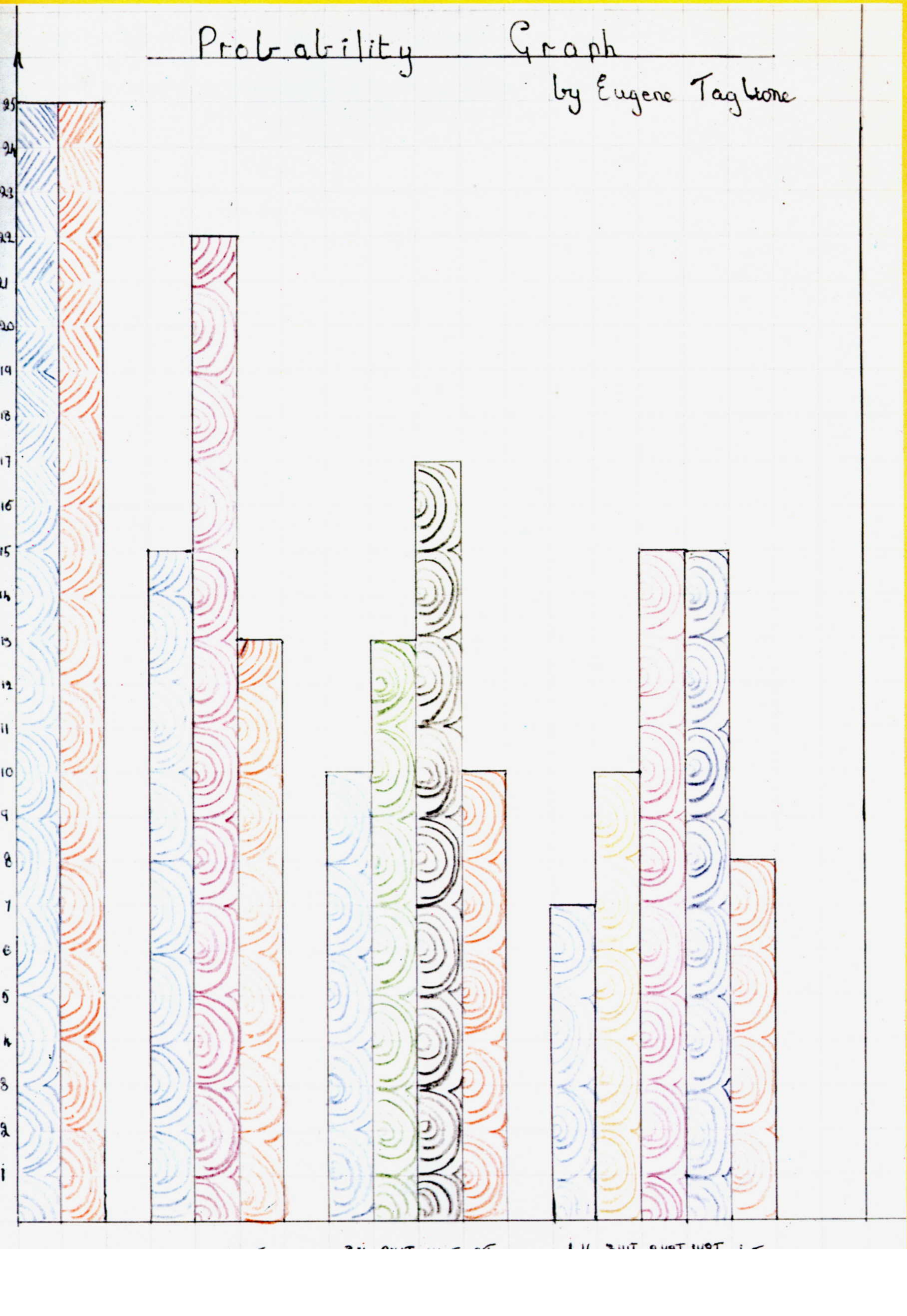
With one coin I had 25H 25T
With Two coins I had 15 2H 22 HT 13 2T
" Three coins " " 10 3H 13 2H1T 17 1H2T 10 3T
" Four " " " 7 4H 10 3H1T 15 2H2T 15 1H3T 8 4T

This formation we called pyramid because of the shape.

1 coin 25H 25T
2 coins 13 2H 22 HT 13 2T
3 coins 10 3H 13 2H1T 17 1H2T 10 3T
4 coins 7 4H 10 3H1T 15 2H2T 15 1H3T 8 4T

This graph was fun because I did not know a bit what would happen next.

by
Eugene Tagliere.


Probability Graph
by Eugene Taglione
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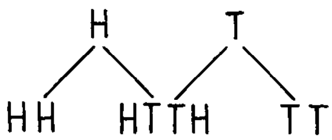
If three coins are tossed, the possible outcomes are

HHH	THH
HHT	THT
HTH	TTH
HTT	TTT

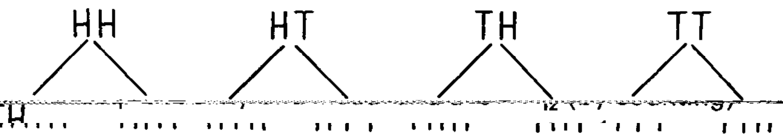
'2 heads and 1 tail' occurs 3 times in this list of 8 (equally likely) possibilities, so the probability of this event is $\frac{3}{8}$.

Again, this can be compared with an actual experiment, tossing 3 coins together a number of times (say 64, so that the 'expected' number of '2 heads and 1 tail' would be $\frac{3}{8} \times 64 = 24$).

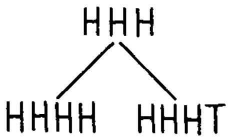
Some children may be able to see that with four coins there are $2 \times 2 \times 2 \times 2$ possibilities. The first coin may be H or T and for **each** of these 2 possibilities there are 2 for the second coin, namely H or T:



Then for each of these 2×2 possibilities, there are two for the third coin, giving



and for each of these $2 \times 2 \times 2$ possibilities there are two for the fourth



giving a total of $2 \times 2 \times 2 \times 2 = 16$.

They may also devise quick ways of finding how many of these possibilities would give, say, 3 heads and 1 tail and thus give 'probabilities' of various events with a minimum of calculation.

We end with a glimpse at two rules which the children may discover for themselves.

The sum rule

In throwing an ordinary die, the probability of throwing a '1' is $\frac{1}{6}$; that of throwing a '2' is also $\frac{1}{6}$. The probability of throwing 'either 1 or 2' is obtained by considering the various possibilities,

1, 2,
3, 4, 5, 6.

Of the six possibilities two will give '1 or 2', so the probability is $\frac{2}{6}$ $(= \frac{1}{3})$, i.e. the **sum** of the probabilities for '1' or '2' separately.

Children may also discover the check that the sum of all the probabilities is 1, e.g. the probability of each separate die throw is $\frac{1}{6}$ and $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 1$ or, more simply, the probabilities for the head and tail of a coin are both $\frac{1}{2}$, and $\frac{1}{2} + \frac{1}{2} = 1$.

The product rule

Toss a die and a coin 12 times and record the frequencies of the outcomes, e.g. 6H, for 6 on the die and heads for the coin. The possible outcomes are

H-1	H-2	H-3	H-4	H-5	H-6
T-1	T-2	T-3	T-4	T-5	T-6

The probability of a 'head' on the coin $= \frac{1}{2}$
The probability of a '5' (say) on the die $= \frac{1}{6}$
The probability of a 'head' and a '5' $= \frac{1}{12}$ (by counting)
$= \frac{1}{2} \times \frac{1}{6}$
= (probability of a 'head') × (probability of a '5')

A fruit machine demonstrates an application of the product rule. Such a device consists of a number of dials (say 3), each with a number of fruit symbols (say 20). Each of the dials can come to rest in any one of the 20 positions. Thus there are
$20 \times 20 \times 20 = 8{,}000$
possible arrangements of the symbols on the dials.
If there are, for example,
3 cherries on dial A
7 apples on dial B
1 plum on dial C
and cherry–apple–plum is a winning combination, the probability of a pay-off is: $\frac{3}{20} \times \frac{7}{20} \times \frac{1}{20} = \frac{21}{8{,}000}$.

With a skilful arrangement of dials and winning combinations the probability of coming up with a winning combination can be less than this.

Apparatus

Supplies of marbles, counters, dice, playing cards, match-sticks, etc. will be available in the equipment of every norma[l] classroom.

If it is required to supplement these supplies, the following pieces of apparatus may be found useful:

6-sided dice
6-sided dice (biassed)
20-sided dice
Shakers (for coins and dice)
Binomial sampling box
Roulette wheel
Quincunx
Set of Bingo numbers

Available from:
Technical Prototypes (Sales) Ltd.
1A West Holme Street, Leicester

he following publications of the Nuffield Mathematics
roject appeared in 1967–8:

ntroductory Guide

do, and I understand ●■▼ (1967)
nis Guide explains the intentions of the Project, gives
etailed descriptions of the ways in which a changeover from
onventional teaching can be made and faces many of the
roblems that will be met.

eachers' Guides

ctorial Representation ■1 (1967)
esigned to help teachers of children between the ages of
and 10, this Guide deals with graphical representation in its
any aspects.

eginnings ▼1 (1967)
nis Guide deals with the early awareness of both the
eaning of number and the relationships which can emerge
om everyday experiences of measuring length, capacity,
ea, time, etc.

athematics Begins ●1 (1967)
parallel Guide to *Beginnings* ▼1, but more concerned with
ounting numbers' than with measurement. It contains a
nsiderable amount of background information for the
acher.

iape and Size ▼2 (1967)
e first Guide concerned principally with geometrical ideas.
shows how geometrical concepts can be developed from the
ay stage in *Beginnings* ▼1 to a clearer idea of what volume,
ea, horizontal and symmetrical really mean.

Computation and Structure ●2 (1967)
Here the concept of number is further developed. A section on the history of natural numbers and weights and measures leads on to the operation of addition, place value, different number bases, odd and even numbers, the application of number strips and number squares.

Shape and Size ▼3 (1968)
Continues the geometrical work of ▼2. Examination of two-dimensional shapes leads on to angles, symmetry and patterns, and links up with the more arithmetical work of ●2.

Computation and Structure ●3 (1968)
Suggests an abundance of ways of introducing children to multiplication so that they will understand what they are doing rather than simply follow rules.

Weaving Guides

Desk Calculators ○□▽ (1967)
Points out a number of ways in which calculators can be used constructively in teaching children number patterns, place value and multiplication and division in terms of repeated addition and subtraction.

How to Build a Pond ○□▽ (1967)
A facsimile reproduction of a class project.

The Teachers' Guides, together with *Graphs Leading to Algebra* ■2 (1969: see page 53) and *Desk Calculators*, are summarised in *The Story So Far*.

Nuffield Mathematics Project publications appearing May, 1969:

Teachers' Guides

Graphs Leading to Algebra 2

This Teachers' Guide develops the use of coordinates and introduces open sentences and truth sets. It goes on to deal with the graphical aspect of these mathematical statements, introducing graphs of inequalities, intersection of two graphs and graphs using integers.

Computation and Structure 4

The main concern of this Teachers' Guide is with the introduction of the integers $\{\ldots\, {}^{-}3, {}^{-}2, {}^{-}1, 0, {}^{+}1, {}^{+}2, {}^{+}3 \ldots\}$. In the past, children have been introduced to positive and negative numbers through the application of these and have been taught 'tricks' for using them in mathematical operations. This Guide builds up the idea of the integers in terms of ordered pairs of numbers before introducing the number line and other applications: this lays a sound foundation for operations on integers. The Guide ends with a short section on large numbers and indices.

Weaving Guides

Environmental Geometry

One of the 'Weaving Guides', this book concentrates on making children more critically aware of shapes in their environment and the interrelationships of them. It deals with relative size and position and with recurring shapes and their properties. It is intended mainly for Infants and lower Juniors.

Probability and Statistics

A 'Weaving Guide' designed to build up, in a very practical way, a critical approach to statistical information and assertions of probability. It demonstrates the many ways in which data can be collected and organised and it attempts to define the criteria for selecting the 'best' way for any given situation. Probability is introduced largely through games, but ways of predicting probable outcomes are investigated in detail.

Other publications

Problems – Green Set

This publication consists of a Teachers' Book accompanied a set of fifty-two cards for distribution to the children. Two further sets of Problems are in preparation.

The first set of Problems is intended for use with young Secondary pupils. The problems on the cards are reprinted ir the Teachers' Book, with solutions and a considerable amount of background material and suggestions for follow-work. All the topics covered by these cards are included in tl Teachers' Guides already published, but they are presented i such a way that children who have not followed a 'Nuffield-type' course can do the problems and enjoy them.

The Story So Far

This booklet is an outline of, and index to, the ground covered by the first nine Teachers' Guides of the Project. Its purpose is twofold: to provide easy references to topics in these Guides for those using them day by day (making a straight index proved an impossible task); and to save teachers of older children having to read through all the earl Guides to find out 'what had happened previously'.

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