STUDENT'S TEXT

UNIT NO. 30

CSMP

MATHEMATICS FOR THE ELEMENTARY SCHOOL GRADE 5

PART II

SCHOOL MATHEMATICS STUDY GROUP

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School Mathematics Study Group

Mathematics for the Elementary School, Grade 5

Unit 30

Mathematics for the Elementary School, Grade 5

Student's Text, Part II

REVISED EDITION

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Chapter 6

ADDITION AND SUBTRACTION OF RATIONAL NUMBERS

MEANING OF RATIONAL NUMBERS

Because of his way of life, early man needed only whole numbers. We can think of reasons why he came to need other numbers as time went by. For example, he might have wanted to trade more than 2 but less than 3 hides for a weapon. He might have wished to tell someone that there was some food but not enough for a meal. He could not have handled these situations with whole numbers alone.

Today you would have great difficulty in making yourself understood if you could use only whole numbers. Suppose you knew only whole numbers. Could you describe any of these with a whole number?

(a) A trip that took less than one day

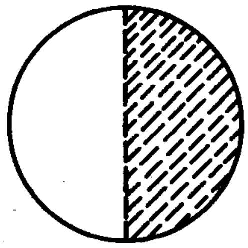
(b) The amount of candy you get when you share a candy bar with two friends

(c) The number of books you read this summer, if you read more than 8 and less than 9

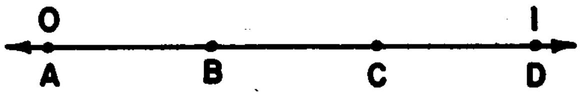
You would have even more difficulty in mathematics. If you could use only whole numbers, there would be no result for such operations as $2 \div 5$ or $8 \div 3$. Another set of numbers helps you find answers to such operations. This set of numbers is called the set of rational numbers.

Rational numbers are often used to describe the measure of a region, segment, or set of objects.

Exploration



One-half of the circular region is shaded. The numeral for one-half is $\frac{1}{2}$.



Points B and C separate $\overline{AD}$ into 3 congruent segments.

If the measure of $\overline{AD}$ is 1, the measure of $\overline{AC}$ is two-thirds. The numeral for two-thirds is $\frac{2}{3}$.

Set A = {Tom, Jane, Bill, Ann, Sally}

Three-fifths is the number that best describes what part of Set A the three girls are. The numeral for three-fifths is $\frac{3}{5}$.

One kind of symbol used to name a rational number is called a <u>fraction</u>. A fraction is written with two names for whole numbers separated by a bar. $\frac{2}{3}$, $\frac{3}{5}$ and $\frac{1}{2}$ are fractions.

$$\frac{\text{numeral}}{\text{numeral}}$$

The number named below the bar is called the denominator of the fraction and shows into how many parts of equal measure the unit region, unit segment, or set is separated. The number named above the bar, or numerator, counts the number of these parts that are being used.

1. Look at the circular region above.
   a. Into how many congruent regions is it separated?
   b. Will this number be represented by a numeral written above or below the bar of a fraction?
   c. What is this numeral called?

2. a. How many congruent parts of the region are shaded?

   b. Where will you write the numeral that shows this?

   c. What is it called?

3. Look at the picture of $\overline{AC}$. The measure of $\overline{AC}$ is written $\frac{2}{3}$.

   a. What does the denominator tell you?

   b. What does the numerator tell you?

4. $\frac{3}{5}$ of the members of Set A are girls. What is the relation of the 3 and 5 of the fraction to Set A?

Summary

1. A rational number is sometimes used to describe the measure of a region, line segment, or set of objects.

2. Fractions are one of the symbols used to name rational numbers.

3. Fractions are written with 2 names for whole numbers separated by a bar. (The denominator can not be 0.)

$$\frac{4}{5} \quad \frac{\text{numerator}}{\text{denominator}}$$

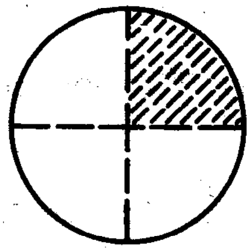
4. The denominator is the number which tells the number of congruent parts into which the unit segment, unit region, or set has been separated.

5. The numerator is the number which counts the number of these congruent parts that are being used.

6. The set of rational numbers includes numbers renamed by numerals like these: $1\frac{3}{8}$, 5, $\frac{2}{2}$, $\frac{4}{3}$, 0, 7.2 and $\frac{1}{2}$.

Exercise Set 1

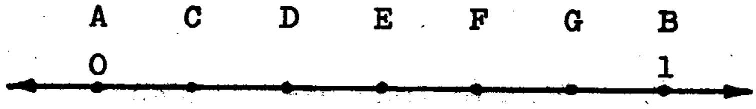
1.



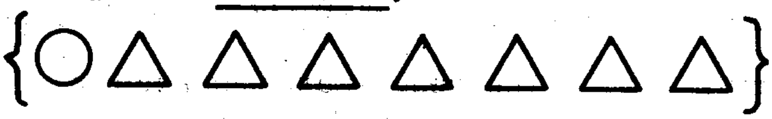
A

The circular region A has been separated into ________ congruent regions. ________ region is shaded. The shaded region is ________ of the circular region.

2.

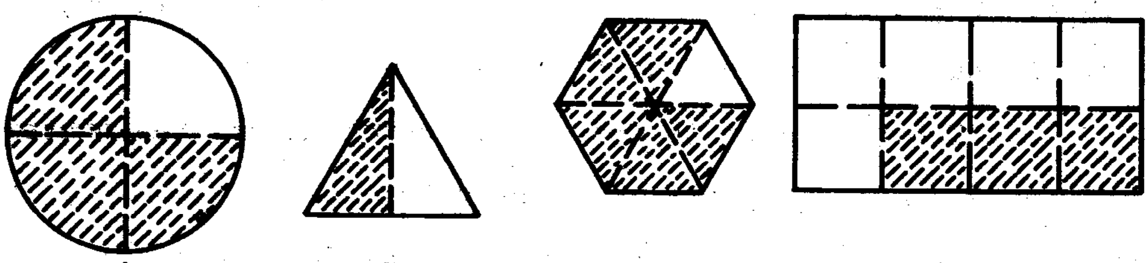


The measure of $\overline{AB}$ is 1. Points C, D, E, F, and G separate $\overline{AB}$ into ________ congruent segments. $\overline{AG}$ is the union of ________ of these congruent segments. The measure of $\overline{AG}$ is ________.

3. Set A = 

There are ________ members in Set A. ________ members are triangles. What rational number describes what part of the members of Set A are triangles?

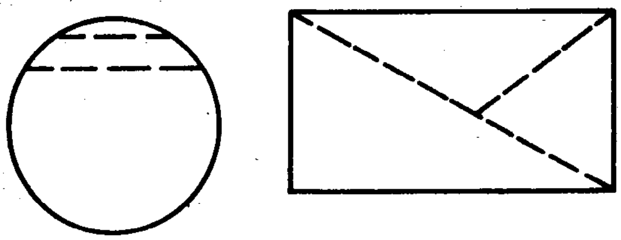
4.

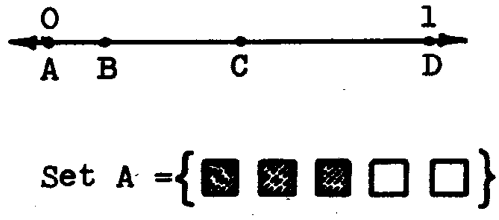


A B C D

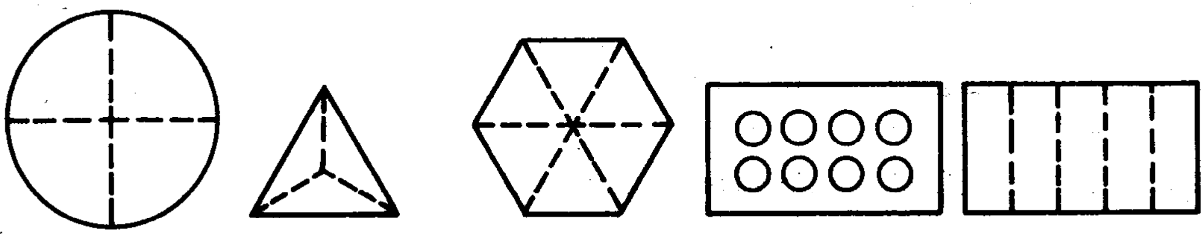
Each unit region above is separated into a number of smaller congruent regions. What rational number best describes the measure of the shaded area of each? The unshaded area?

5. Which figures below are not separated into thirds? Explain your answers.





6. Trace the figures below. Then shade a part described by the fraction written below each of the figures.



a. $\frac{3}{4}$ b. $\frac{1}{3}$ c. $\frac{5}{6}$ d. $\frac{3}{8}$ e. $\frac{4}{5}$

7. Draw simple figures and shade parts to show:

a. $\frac{1}{6}$ b. $\frac{5}{8}$ c. $\frac{2}{5}$ d. $\frac{7}{10}$

8. Complete:

Fractions may be used to name __________ numbers.

Fractions are written with __________ numerals separated by a bar.

The numeral __________ the bar names the __________ and tells into how many parts of the same size the unit is separated.

The numeral __________ the bar names the __________ and counts the number of parts of the same size being used.

9. Match each rational number named in Column A with its symbol in Column B.

	Column A		Column B
a.	two-fifths	g.	$\frac{7}{9}$
b.	seven-ninths	h.	$\frac{7}{4}$
c.	four-sevenths	i.	$\frac{2}{5}$
d.	five-halves	j.	$\frac{9}{7}$
e.	nine-sevenths	k.	$\frac{5}{2}$
f.	seven-fourths	l.	$\frac{4}{7}$

10. Set A = {set of rational numbers}

Write names for five members of Set A.

11. Complete:

Set A = {○ ○ ○ ○}

Set B = {○ ○ ○}

Set C = {○ ○}

Set D = {○}

If the measure of Set D is 1 the measure of Set A is ______. The measure of Set B is ______. The measure of Set C is ______.

12. Use Sets A, B, C, and D in exercise 11 to complete the following:

If the measure of Set A is 1, the measure of Set B is ______. The measure of Set C is ______. The measure of Set D is ______.

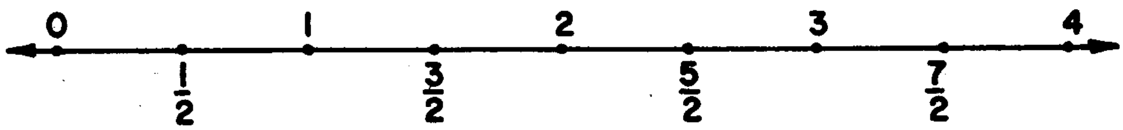
RATIONAL NUMBERS ON THE NUMBER LINE

Exploration

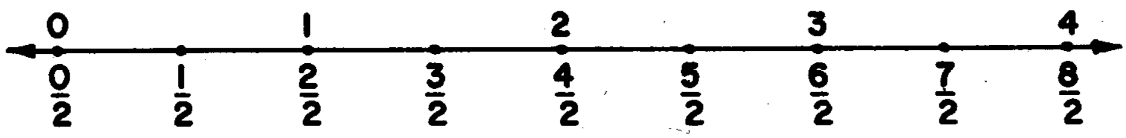
1. Draw a number line on your paper. Choose a point and label it 0. Label other points equally spaced in order 1, 2, 3, and 4. Your number line should look like this.

2. Separate each unit segment into two congruent segments. Below the number line, label these points in order $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$.

   Does your number line look like the one below?



3. Which points can now be labeled $\frac{0}{2}$, $\frac{2}{2}$, $\frac{4}{2}$, $\frac{6}{2}$, and $\frac{8}{2}$? Label these points.



   Your number line now shows a set of segments, each $\frac{1}{2}$ the length of the original unit segment. The endpoints are labeled with fractions.

4. Look at the fraction labels. What does each denominator tell you? What does each numerator tell you?

   The points you labeled $\frac{0}{2}$, $\frac{2}{2}$, $\frac{4}{2}$, $\frac{6}{2}$, and $\frac{8}{2}$ were already labeled with whole numbers.

5. What other names for points labeled $\frac{0}{2}$, $\frac{2}{2}$, $\frac{4}{2}$, $\frac{6}{2}$, and $\frac{8}{2}$ are shown on the number line? Can a point on the number line have more than one name?

6. What points could also be labeled $1\frac{1}{2}$, $2\frac{1}{2}$, and $3\frac{1}{2}$? (These numerals are read, "1 and 1 half, 2 and 1 half" etc.)

7. Do $\frac{3}{2}$ and $1\frac{1}{2}$ name the same point? If so, they are names for the same number.

8. Do $\frac{5}{2}$ and $2\frac{1}{2}$ name the same point on the number line?

Summary:

1. A number line may show more than one set of division points.

2. A unit segment on the number line may be separated into any number of congruent segments.

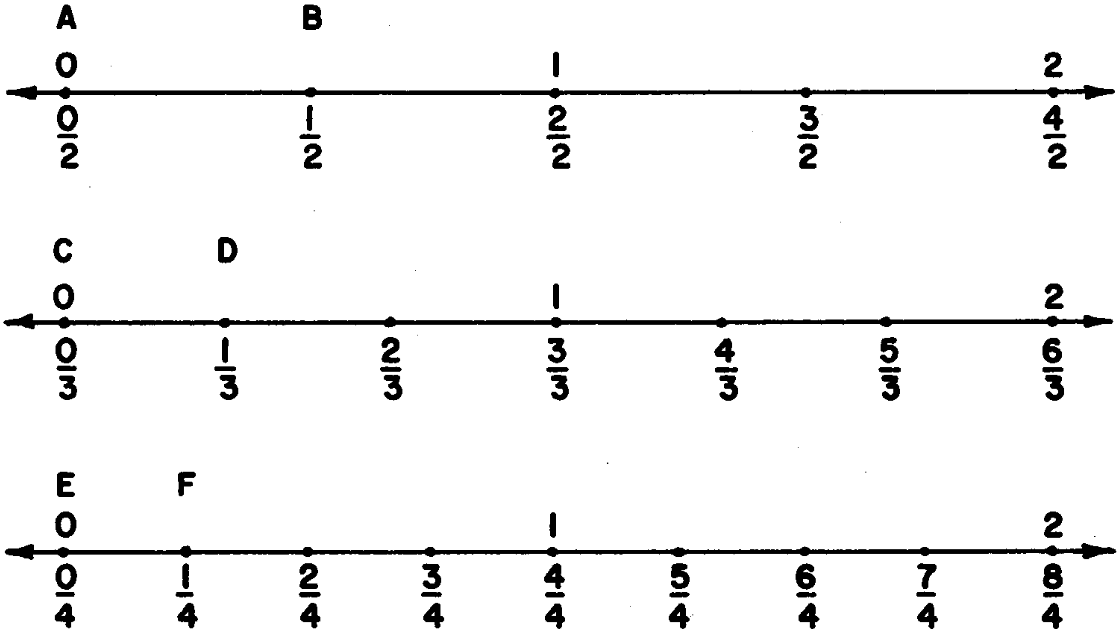
3. The measure of each smaller, congruent segment is a rational number.

4. Some points on the number line can be named by numerals for whole numbers and also by fractions.

5. Numerals like $1\frac{1}{2}$, $2\frac{1}{2}$, and $3\frac{1}{2}$ are read, "1 and 1 half," "2 and 1 half," and "3 and 1 half."

6. Numerals like $1\frac{1}{2}$, $2\frac{1}{2}$, and $3\frac{1}{2}$ can be used as names for certain points on the number line.

Exploration



1. Use the number lines above to answer the following:

   a. What is the measure of $\overline{AB}$? Why?

   b. What is the measure of $\overline{CD}$? Why?

   c. What is the measure of $\overline{EF}$? Why?

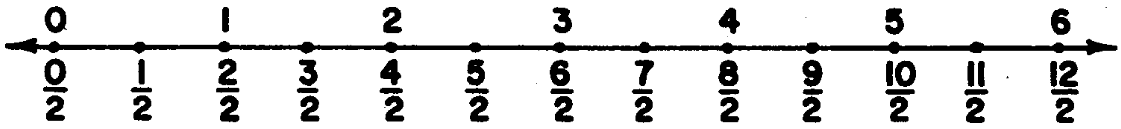
   d. Which of the three rational numbers is the greatest?

   e. Which of the three rational numbers is the least?

   f. Arrange the measures of $\overline{AB}$, $\overline{CD}$ and $\overline{EF}$ in order from greatest to least.

   g. What generalization can you make about the order of numbers named by fractions whose numerators are 1?

2.



Use the number line above to answer the following questions:

a. Which rational number is greater, $\frac{3}{2}$ or $\frac{5}{2}$?

b. How can you tell which rational number is greater?

3. Which of the rational numbers in each pair is greater?

a. $\frac{2}{2}$ or $\frac{5}{2}$

b. $\frac{6}{2}$ or $\frac{8}{2}$

c. $\frac{7}{2}$ or $\frac{3}{2}$

4. What are other names for $\frac{0}{2}$, $\frac{4}{2}$, and $\frac{6}{2}$? What are 2 numerals that name:

a. 1 one and 1 half?

b. 3 ones and 1 half?

c. 5 ones and 1 half?

Summary:

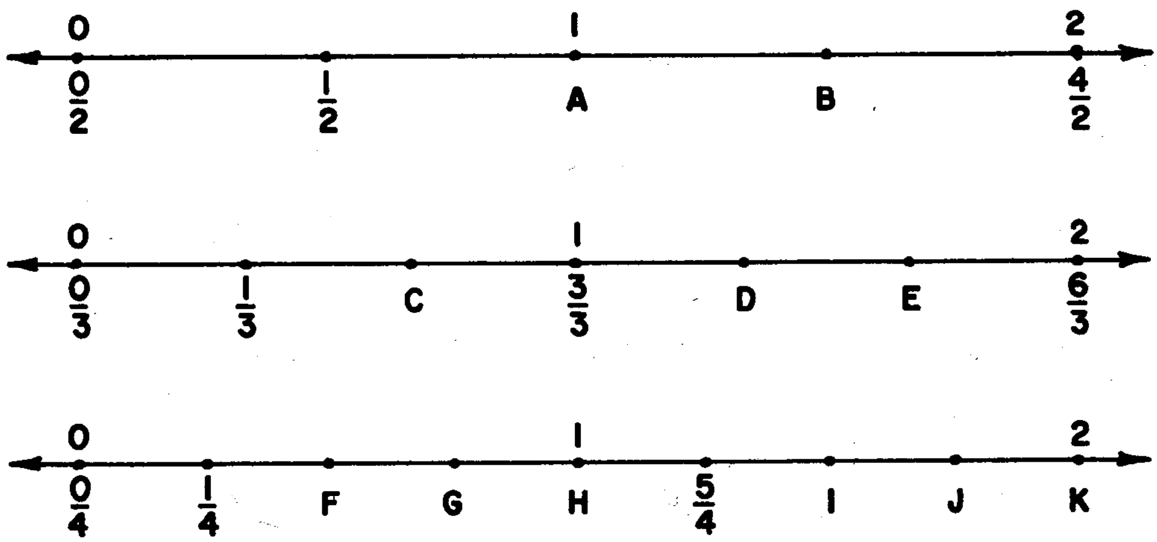
1. To compare rational numbers named by fractions whose numerators are 1, look at the denominators. The greater the number represented by the denominator the smaller the rational number.

2. On the number line, any fraction to the right of another names the greater rational number. Any fraction to the left of another represents the smaller rational number.

3. The order of rational numbers on the number line is the same as the order of whole numbers. As you move to the right along the number line, the rational numbers become greater. As you move to the left, they become smaller.

Exercise Set 2

1. Label the points A-K shown on the number lines below with fractions for rational numbers.



2. Complete each mathematical sentence below. Use < , > , and = . The number lines in exercise 1 will help you.

a. $\frac{2}{3}$ ______ $\frac{3}{4}$ e. $\frac{7}{4}$ ______ $\frac{6}{3}$

b. $\frac{4}{4}$ ______ $\frac{3}{3}$ f. $\frac{2}{3}$ ______ $\frac{2}{4}$

c. $\frac{6}{4}$ ______ $\frac{4}{3}$ g. $\frac{1}{4}$ ______ $\frac{1}{3}$

d. $\frac{4}{2}$ ______ $\frac{6}{3}$ h. $\frac{6}{4}$ ______ $\frac{3}{2}$

3. Which is greater? How can you tell?

a. 4 or 1

b. $\frac{7}{2}$ or $\frac{5}{2}$

c. $\frac{4}{2}$ or 1

d. 0 or 2

e. $\frac{1}{2}$ or $\frac{2}{2}$

f. $1\frac{1}{2}$ or $\frac{4}{3}$

g. $1\frac{1}{3}$ or 2

h. $\frac{5}{4}$ or 1

4. Arrange in order from least to greatest.

$$\frac{7}{4},\ 0,\ 1\frac{1}{2},\ \text{and}\ \frac{4}{3}$$

5. Write the whole numbers that are between $\frac{1}{2}$ and $\frac{7}{2}$.

6. $1\frac{1}{2}$, $\frac{3}{2}$, and 1 one and 1 half are all names for the same point on the number line. Write 2 other names for:

a. 1 one and 1 fourth

b. $\frac{4}{3}$

7. Write the rational numbers in each set in order of size from least to greatest.

Set A = $\{\frac{1}{4}, \frac{1}{2}, \frac{1}{3}\}$

Set B = $\{\frac{3}{4}, \frac{3}{5}, \frac{3}{8}\}$

Set C = $\{\frac{1}{6}, \frac{1}{3}, \frac{1}{12}\}$

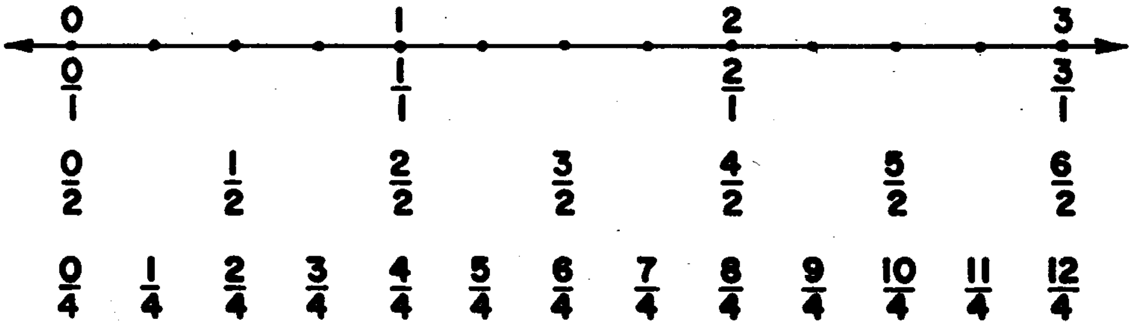
Set D = $\{\frac{5}{10}, \frac{5}{2}, \frac{5}{4}\}$

8. Count by fourths from 0 to 3. Write your answers in a set. If you need help, the number line will help you. The set has been started for you.

$$A = \{\frac{0}{4},\ \frac{1}{4},$$

PICTURING RATIONAL NUMBERS ON THE NUMBER LINE

Exercise Set 3



1. In picturing $\frac{5}{4}$ on the number line, into how many congruent segments do you separate a segment of length 1? Starting at 0, how many times do you lay off to the right a segment of this length to arrive at the point $\frac{5}{4}$?

2. In picturing $\frac{3}{4}$ on the number line, into how many segments of the same length do you separate a segment of length 1? Starting at 0, how many times do you lay off to the right a segment of this length to arrive at the point $\frac{3}{4}$?

3. In picturing 3 on the number line, into how many congruent segments do you separate a segment of length 1? Starting at 0, how many times do you lay off to the right a segment of this length to arrive at the point 3?

4. In picturing $\frac{11}{4}$ on the number line, into how many congruent segments do you separate a segment of length 1? Starting at zero, how many times do you lay off to the right a segment of this length to arrive at the point $\frac{11}{4}$?

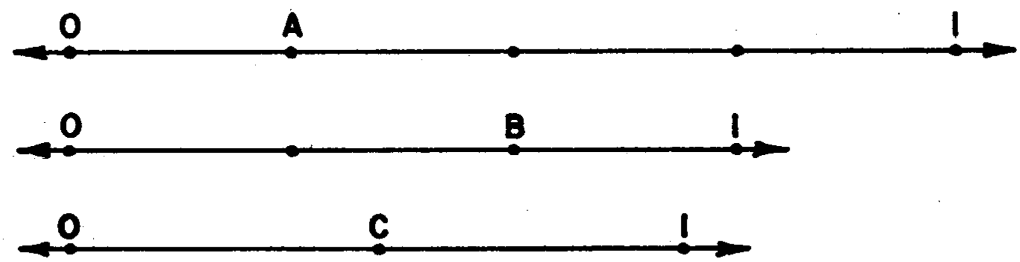
5. In picturing $\frac{6}{2}$ on the number line, into how many congruent segments do you separate a segment of length 1? Starting at 0, how many times do you lay off to the right a segment of this length to arrive at the point $\frac{6}{2}$?

6. How many names are shown on the number line in exercise 1 for

a. 0 c. 3

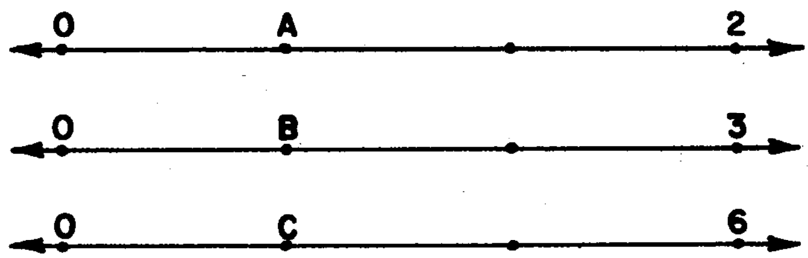
b. $\frac{1}{2}$ d. $2\frac{1}{2}$

7. Label with fractions points A, B, and C on the number lines below.



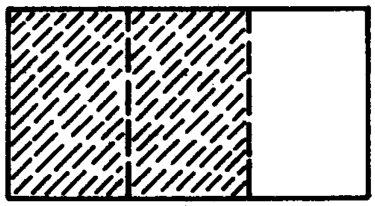
BRAINTWISTER

Label with rational numbers the points A, B, and C on the number lines below.



PICTURING RATIONAL NUMBERS WITH REGIONS

Exploration



A

1. Figure A represents a unit region. It is separated into 3 smaller congruent regions and 2 of these regions are shaded.

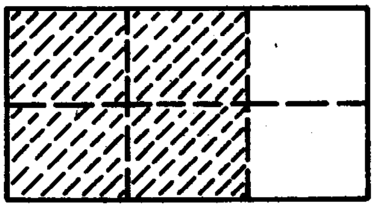
   a. What is the measure of the shaded region?

   b. How are the 2 and the 3 in the fraction $\frac{2}{3}$ related to the unit region?

2. Figure B represents the same unit region. It is separated into 6 smaller congruent regions and 4 of these regions are shaded.

   a. What is the measure of the shaded region?

   b. How are the 4 and 6 in the fraction $\frac{4}{6}$ related to the unit region?

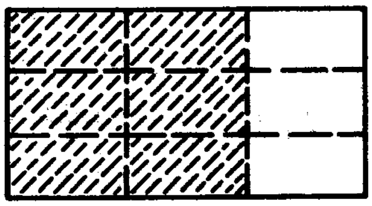


B

3. Trace a rectangle congruent to figure A. Draw broken lines to separate the region into 9 congruent regions and shade 6 of these regions as shown in figure C.

   a. What is the measure of the shaded region?

   b. How are the 6 and the 9 of the fraction $\frac{6}{9}$ related to the unit region?



C

4. a. Is the unit region the same size in figures A, B, and C?

   b. Are the shaded regions of A, B, and C congruent?

   c. Are the rational numbers $\frac{2}{3}$, $\frac{4}{6}$, and $\frac{6}{9}$ the measures of these congruent shaded regions?

   d. Are $\frac{2}{3}$, $\frac{4}{6}$, and $\frac{6}{9}$ names for the same rational number?

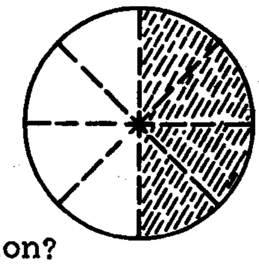
5. Draw 3 congruent rectangular regions. Label them A, B, and C.

   a. Separate A into 3 smaller congruent regions. Shade 1 region. What is the measure of the shaded region?

   b. Separate B into 6 smaller congruent regions and shade 2 of them. What is the measure of the shaded region?

   c. Separate C into 9 congruent regions. Shade 3 of these regions. Are $\frac{1}{3}$, $\frac{2}{6}$, and $\frac{3}{9}$ names for the same rational number? Why?

6. The measure of the circular region at the right is 1. The measure of the shaded part is $\frac{4}{8}$. What is the relation of the 4 and 8 of the fraction to the unit region?



7. Trace the circular region above. Separate it into 2 congruent regions. Shade 1 part.

   a. What is the measure of the shaded region?

   b. Are $\frac{4}{8}$ and $\frac{1}{2}$ names for the same number since they name the measures of congruent regions?

8. Draw a circular region congruent to the one in exercise 6. Separate it into 4 congruent regions and shade 2 of the parts. Is $\frac{2}{4}$ another name for $\frac{1}{2}$ and $\frac{4}{8}$? Why? A rational number may have many different names.

$$1 = \frac{2}{2} = \frac{4}{4} = \frac{8}{8}$$

$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$$

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9}$$

Exercise Set 4

1. Answer these questions for each figure below.

   a. Into how many congruent regions is the unit region separated?

   b. How many congruent regions are shaded?

   c. What fraction name best describes the measure of the shaded region?

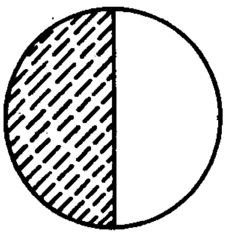


Figure 1

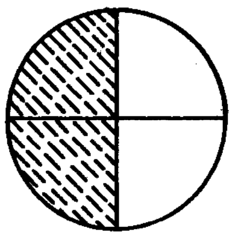
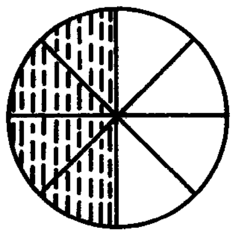


Figure 2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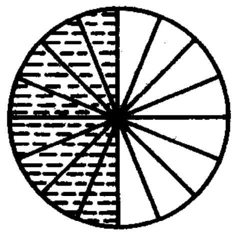
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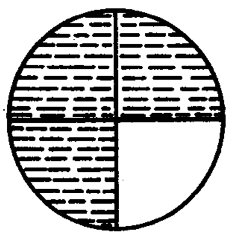


Figure 5

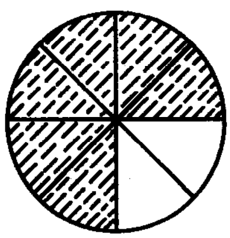


Figure 6

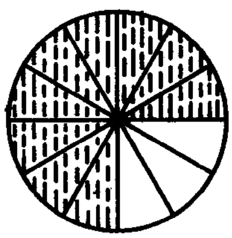


Figure 7

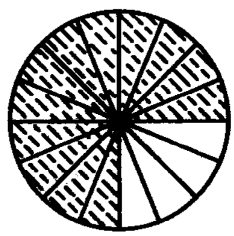


Figure 8

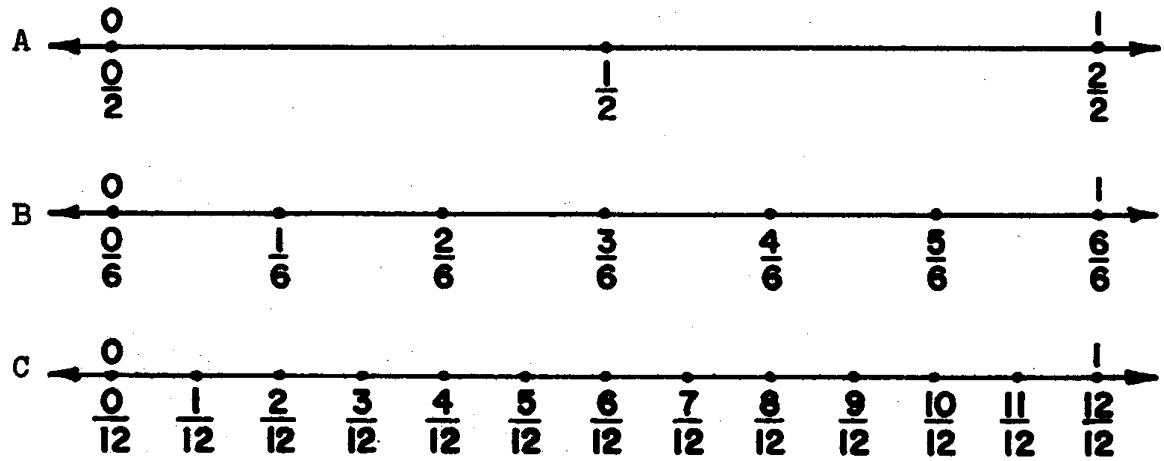
2. Do the fractions in exercise c for figures 1 through 4 name the same rational number? What rational number do they name?

3. Do the fractions in exercise c for figures 5 through 8 name the same rational number? What rational number do they name?

4. Write three other names for $\frac{1}{2}$.

5. Write three other names for $\frac{3}{4}$.

6. Write three names for $\frac{1}{4}$.



7. Look at the number lines A, B, and C above. Three congruent segments, each having the measure of 1, have been separated into smaller congruent segments. Answer the following questions about each number line.

   a. Into how many congruent segments has the unit segment been separated?

   b. What fraction best names the measure of each smaller congruent segment?

8. Make true statements by writing a different fraction in each space.

a. $\frac{1}{2} = \_\_\_ = \_\_\_ = \_\_\_$

b. $\frac{1}{3} = \_\_\_ = \_\_\_$

c. $\frac{2}{3} = \_\_\_ = \_\_\_$

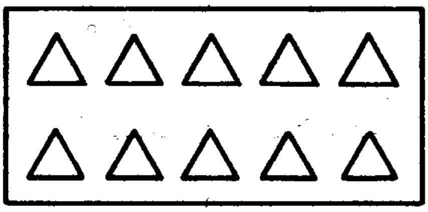
d. $\frac{1}{4} = \_\_\_ = \_\_\_ = \_\_\_$

e. $\frac{3}{4} = \_\_\_ = \_\_\_ = \_\_\_$

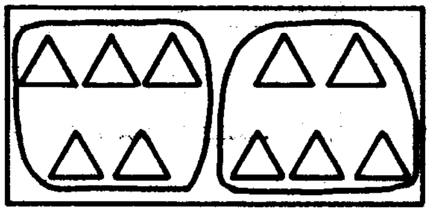
f. $1 = \_\_\_ = \_\_\_ = \_\_\_$

RATIONAL NUMBERS WITH SETS OF OBJECTS

Exploration



A



B

1. Figure A shows a picture of a set of 10 objects. What rational number best describes what part of the set each object is?

2. Figure B shows the same set separated into subsets each having the same number of objects.

   a. Into how many subsets has the set shown in A been separated in B?

   b. How many objects are in each subset?

   c. What part of the set is in each subset?

   d. Do $\frac{1}{2}$ and $\frac{5}{10}$ name the same rational number?

3. Trace figure A. Separate the objects into 5 subsets, each subset having the same number of objects.

   Replace n by a number which makes each sentence true.

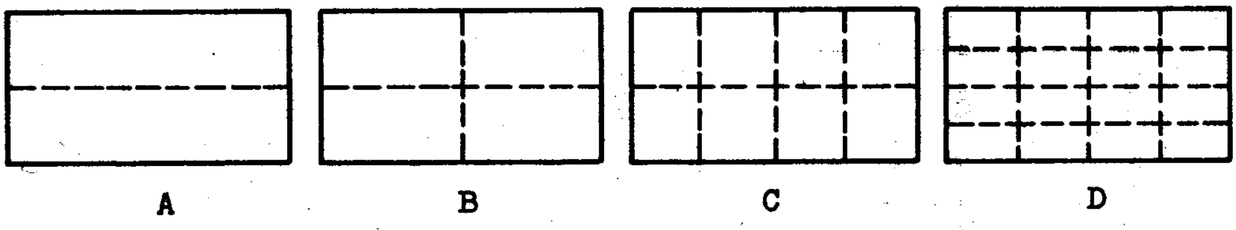
   a. $\frac{1}{5} = \frac{n}{10}$

   b. $\frac{2}{5} = \frac{n}{10}$

   c. $\frac{3}{5} = \frac{n}{10}$

   d. $\frac{4}{5} = \frac{n}{10}$

Exercise Set 5



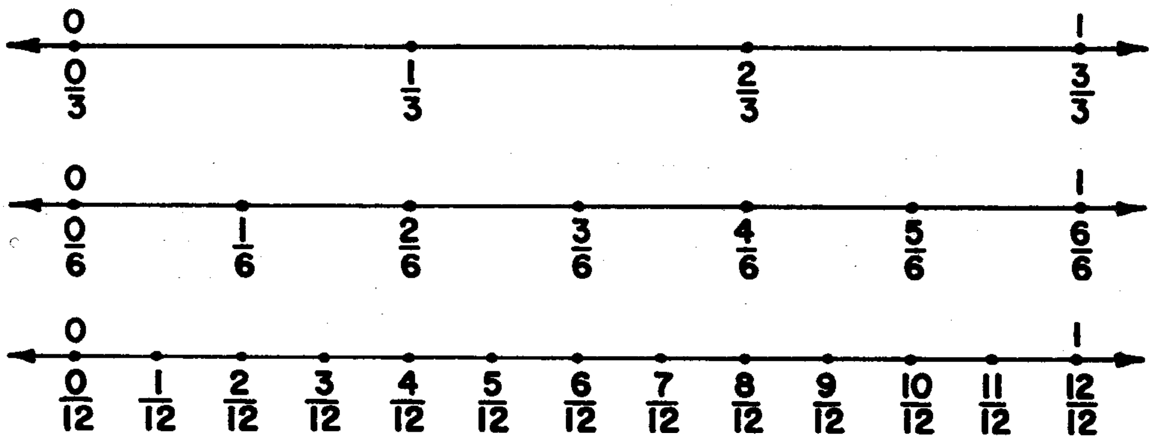
A, B, C, and D are congruent rectangular regions. The measure of each is 1. Use them to help you answer the following exercises.

1. There are 4 quarts in a gallon. 3 quarts is the same amount as
   a. ____ fourths of a gallon.
   b. ____ eighths of a gallon.
   c. ____ sixteenths of a gallon.

2. There are 16 ounces in a pound. 8 ounces is the same amount as
   a. 1 _____ of a pound.
   b. 2 _____ of a pound.
   c. 4 _____ of a pound.
   d. 8 _____ of a pound.

3. How many quarts are there in $\frac{1}{2}$ gallon?

4. How many ounces are there in $\frac{1}{4}$ pound?



Use the number lines above to answer the following exercises.

5. 8 inches may be written

   a. ____ twelfths of a foot.

   b. ____ sixths of a foot.

   c. ____ thirds of a foot.

6. Ten months could be written as

   a. 10 _____ of a year.

   b. 5 _____ of a year.

7. What part of a dozen cookies are

   a. 6 cookies?

   b. 4 cookies?

   c. 8 cookies?

   d. 10 cookies?

8. What part of a yard is

   a. 1 foot?

   b. 2 feet?

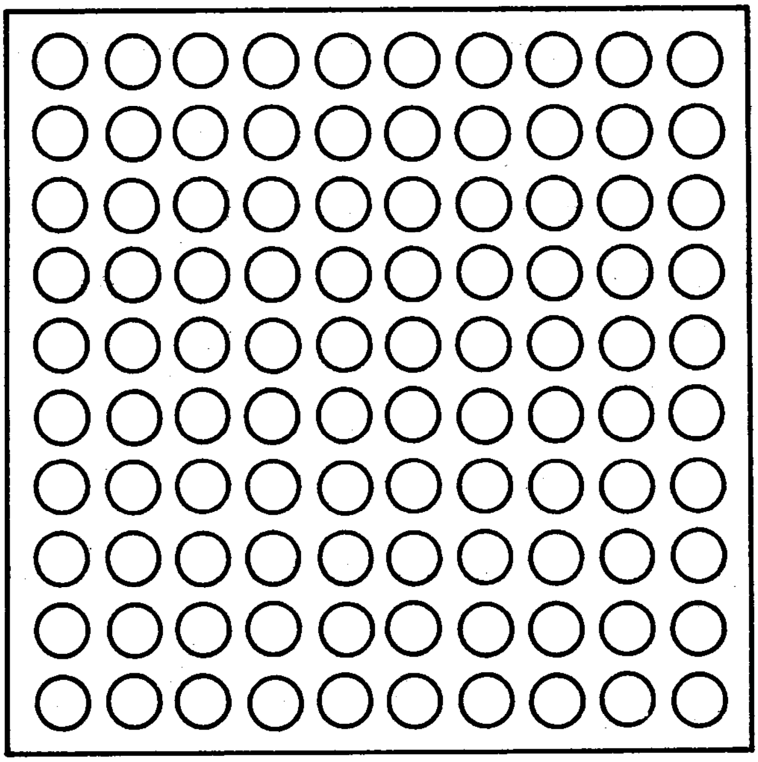
9. What part of a year is

   a. 4 months?

   b. 6 months?

10. How many inches are there in $\frac{1}{3}$ of a foot?

11. How many eggs are there in $\frac{1}{6}$ of a dozen?



12. Above is a picture of a set of 100 pennies. Use it to answer the following questions. Write the fraction with the smallest denominator for each rational number used in your answers.

What part of one dollar is

a. 50 pennies?

b. 10 pennies?

c. 25 pennies?

d. 5 pennies?

e. 20 pennies?

THE SIMPLEST FRACTION NAME FOR A RATIONAL NUMBER

Exploration

We have found that many fractions name one rational number. For example, we know that

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}.$$

1. Find three more fractions that belong on this list.

2. Do these fractions belong on the list?

$$\frac{50}{100}, \frac{100}{200}, \frac{111}{222}.$$

3. Find n, m, and p so that each fraction names $\frac{1}{2}$.

$$\frac{1}{2} = \frac{n}{14}, \quad \frac{1}{2} = \frac{8}{m}, \quad \frac{1}{2} = \frac{p}{250}$$

We have also found several names for $\frac{2}{3}$.

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12}.$$

4. Do these fractions belong on this list of names of $\frac{2}{3}$?

$$\frac{20}{30}, \frac{2 \times 4}{3 \times 4}, \frac{14}{21}, \frac{50}{75}, \frac{2 \times 876}{3 \times 876}, \frac{2 \times 2 \times 5 \times 7}{2 \times 3 \times 5 \times 7}$$

5. Suppose that m and n are counting numbers. Give three other names for $\frac{m}{n}$.

Now try to imagine the set of all fractions which name a rational number:

$$\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10} \ldots\}$$

$$\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12} \ldots\}$$

$$\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16} \ldots\}$$

Each such set contains a fraction with a denominator smaller than the rest. We will call this fraction the simplest fraction name for the rational number. It is a name we often use.

Any other fraction in each set can be found from the simplest fraction name. Do you know the rule for finding the other fractions?

FINDING THE SIMPLEST FRACTION NAME

Exploration

How can you tell whether a fraction is the simplest name for a rational number? Which ones of these are simplest fraction names?

$$\frac{3}{12}, \frac{8}{10}, \frac{5}{11}, \frac{9}{14}, \frac{6}{2}, \frac{3 \times 5 \times 5 \times 7}{2 \times 2 \times 11}, \frac{2 \times 5 \times 7 \times 11}{3 \times 5 \times 13}$$

Is $\frac{2 \times 8765}{2 \times 3341}$ a simplest fraction name?

Perhaps you remember that 13 is a prime number. Is $\frac{13}{6893}$ a simplest fraction name?

Is $\frac{n}{13}$ always a simplest fraction name if $n < 13$?

These examples should suggest two things to you:

First, a simplest fraction name is one in which the numerator and denominator have no common factors except 1

Second, you can find the simplest fraction name from any fraction in the set by finding the greatest common factor of its numerator and denominator.

Here are several examples showing how you can find simplest fraction names.

1) Find the simplest fraction name for $\frac{30}{45}$. First factor numerator and denominator completely.

$$\frac{30}{45} = \frac{2 \times 3 \times 5}{3 \times 3 \times 5}$$

Next remove the common prime factors shown (3 and 5).

$$\frac{30}{45} = \frac{2 \times 3 \times 5}{3 \times 3 \times 5} = \frac{2}{3}.$$

2) Find the simplest fraction name for $\frac{96}{375}$.

Here is a different method. First test 2 as a factor of both numerator and denominator. You find that 2 is not a factor of 375 because the units digit is <u>odd</u>. This means that 2 cannot be a <u>common</u> factor of 96 and 375.

Now test 3 as a factor of numerator and denominator. You find:

$$\frac{96}{375} = \frac{3 \times 32}{3 \times 125}.$$

Now remove the common factor.

$$\frac{96}{375} = \frac{\cancel{3} \times 32}{\cancel{3} \times 125} = \frac{32}{125}.$$

Next test 3 again as a factor of 32 and 125. Since 3 is not a factor of 32, it is not a common factor. Continue and try 5 as a common factor. Notice, however, that 2 is the <u>only</u> prime factor of 32 and that 5 is the <u>only</u> prime factor of 125. If you see this, it will save you time because you know right away that 32 and 125 have greatest common factor 1. This means that $\frac{32}{125}$ is the simplest fraction name for $\frac{96}{375}$.

3) Now try the method used in example 1 to find the simplest fraction name for $\frac{90}{84}$.

4) Next try the method used in example 2 to find the simplest fraction name for $\frac{108}{100}$.

If you know many multiplication facts you can often shorten the work in finding simplest fraction names. For example, in finding the simplest fraction name for $\frac{56}{88}$ you might remember that $8 \times 7 = 56$ and $8 \times 11 = 88$. Then you can write

$$\frac{56}{88} = \frac{8 \times 7}{8 \times 11} = \frac{7}{11}.$$

5) How can you use the fact: $12 \times 12 = 144$ in finding the simplest fraction name for $\frac{60}{144}$?

Of course you can always use one of the methods shown in the examples.

Exercise Set 6

1. Write three other fractions naming each of the following numbers.

a. $\frac{1}{3}$	d. $\frac{3}{4}$	g. $\frac{1}{6}$
b. $\frac{2}{5}$	e. $\frac{2}{3}$	h. $\frac{3}{2}$
c. $\frac{5}{4}$	f. $\frac{7}{8}$	i. $\frac{3}{5}$

2. Copy the fractions which are simplest fraction names.

a. $\frac{6}{12}, \frac{7}{12}, \frac{8}{12}, \frac{9}{12}, \frac{10}{12}, \frac{11}{12}$

b. $\frac{2}{3}, \frac{11}{4}, \frac{5}{3}, \frac{3}{4}, \frac{4}{6}, \frac{5}{8}$

c. $\frac{7}{8}, \frac{8}{8}, \frac{6}{8}, \frac{2}{8}, \frac{1}{8}, \frac{3}{8}$

d. $\frac{7}{3}, \frac{7}{16}, \frac{7}{8}, \frac{7}{5}, \frac{7}{10}, \frac{7}{12}$

3. Complete by supplying the missing numerator and denominator.

a. $\frac{3}{6} = \frac{\quad}{24}$

b. $\frac{3}{15} = \frac{12}{\quad}$

c. $\frac{7}{12} = \frac{\quad}{5 \times 12}$

d. $\frac{9}{18} = \frac{54}{\quad}$

e. $\frac{6}{24} = \frac{7 \times 6}{\quad}$

f. $\frac{5}{25} = \frac{\quad}{150}$

4. Use complete factorization to find simplest fraction names for:

a. $\frac{72}{81}$ b. $\frac{84}{105}$ c. $\frac{98}{196}$

5. Find the simplest fraction names for the following. You should be able to do this using multiplication facts only.

a. $\frac{6}{9}$

b. $\frac{10}{15}$

c. $\frac{4}{8}$

d. $\frac{9}{12}$

e. $\frac{12}{8}$

f. $\frac{5}{15}$

g. $\frac{12}{16}$

h. $\frac{21}{24}$

i. $\frac{4}{10}$

j. $\frac{4}{16}$

k. $\frac{6}{12}$

l. $\frac{8}{16}$

m. $\frac{8}{12}$

n. $\frac{7}{21}$

o. $\frac{10}{25}$

p. $\frac{15}{20}$

q. $\frac{12}{15}$

r. $\frac{77}{88}$

s. $\frac{16}{12}$

t. $\frac{20}{24}$

u. $\frac{25}{20}$

6. Find the simplest fraction name for each number. Then use $<$, $>$, or $=$ in each blank to make a true statement.

a. $\frac{9}{15}$ _____ $\frac{20}{25}$

b. $\frac{8}{10}$ _____ $\frac{9}{15}$

c. $\frac{12}{16}$ _____ $\frac{36}{48}$

d. $\frac{12}{22}$ _____ $\frac{10}{33}$

e. $\frac{3}{27}$ _____ $\frac{8}{18}$

f. $\frac{8}{14}$ _____ $\frac{20}{35}$

Exercise Set 7

1. Make true statements by filling in the blanks.

a. $\frac{2}{3} = \frac{\_\_}{27}$

b. $\frac{5}{4} = \frac{35}{\_\_}$

c. $\frac{4}{7} = \frac{4 \times 16}{\_\_}$

d. $\frac{\_\_}{8} = \frac{45}{72}$

e. $\frac{9}{\_\_} = \frac{63}{35}$

f. $\frac{1}{2} = \frac{\_\_}{2 \times n}$

g. $\frac{20}{40} = \frac{27}{\_\_}$

h. $\frac{25}{\_\_} = \frac{16}{48}$

2. a. The measure of $\frac{2}{3}$ of a foot in inches is _____.

b. The measure of $\frac{1}{2}$ yard in inches is _____.

c. The measure of $\frac{3}{4}$ hours in minutes is _____.

d. Twenty minutes is $\frac{\_\_}{3}$ of an hour.

e. _____ weeks is $\frac{1}{13}$ of a year.

3. Write "prime" if the number is prime. Name at least one prime factor if the number is composite.

Example: 73 prime.

Neither 2, nor 3, nor 5 is a factor. (Do you remember how to tell?) By division we find that 7 is not a factor. This is enough to show that 73 is a prime. (Why?)

a. 58

b. 97

c. 51

d. 365

e. 705

f. 91

g. 5280

h. 143

4. Use complete factorization to find the simplest fraction name.

a. $\frac{45}{180}$ d. $\frac{105}{143}$

b. $\frac{126}{60}$ e. $\frac{90}{300}$

c. $\frac{97}{365}$ f. $\frac{10 \times 9 \times 11}{6 \times 21 \times 11}$

5. By finding the simplest fraction name for each of these numbers, tell which is greater.

a. $\frac{24}{42}$ or $\frac{40}{56}$ b. $\frac{16}{44}$ or $\frac{45}{99}$ c. $\frac{81}{117}$ or $\frac{72}{104}$

6. In Jefferson school there were 325 pupils in all and 175 girls. In Washington school there were 312 pupils in all and 144 girls. In which school do girls form the larger part? In which school or schools are more than $\frac{1}{2}$ of the pupils girls.

7. Find the simplest fraction name for:

a. the measure in feet of 16 inches.

b. the measure in days of 33 hours.

c. the measure in miles of 440 yards.

d. the measure in pounds of 20 ounces.

e. the measure in hours of 45 minutes.

8. Is this true? $\frac{3 + 8}{4 + 8} = \frac{3}{4}$? Why?

BRAINTWISTER

9. Is $\frac{3843}{10,000}$ a simplest fraction name? You do not need to make any long computation to find the answer.

COMMON DENOMINATOR

Sometimes the simplest fraction name is not the name you need to solve a problem. Suppose that you want to know which is larger, $\frac{2}{3}$ of a mile or $\frac{7}{10}$ of a mile. Would you prefer to have 2 of 3 equal stacks of pennies or to separate the same set of pennies into 10 equal stacks and take 7?

In either case you want to know:

$$\text{Which is greater, } \frac{2}{3} \text{ or } \frac{7}{10}\,?$$

Both names are simplest, but we cannot answer the question with them. Here is another example that may help you to find the answer.

$$\text{Which is greater, } \frac{1}{2} \text{ or } \frac{7}{10}\,?$$

You know that $\frac{1}{2} = \frac{5}{10}$. You know that $\frac{7}{10} > \frac{5}{10}$. So you can say

$$\frac{1}{2} < \frac{7}{10}$$

Which is greater $\frac{2}{3}$ or $\frac{5}{9}$?

Which is greater $\frac{2}{6}$ or $\frac{9}{24}$?

Which is greater $\frac{17}{8}$ or 2?

The trick is to find for each number names with the <u>same denominator</u>. Think again about $\frac{2}{3}$ and $\frac{7}{10}$. What other denominators do fractions naming $\frac{7}{10}$ have? What other denominators do fractions naming $\frac{2}{3}$ have? What is the smallest number which is in both lists of denominators?

The answers to these questions help you to see that:

$$\frac{2}{3} = \frac{20}{30} \text{ and } \frac{7}{10} = \frac{21}{30}\,.$$

You know then that $\frac{7}{10} > \frac{2}{3}$.

You could answer your question about $\frac{2}{3}$ and $\frac{7}{10}$ as soon as you knew that 30 was a denominator both for $\frac{2}{3}$ $(\frac{20}{30})$ and $\frac{7}{10}$ $(\frac{21}{30})$.

The set of <u>denominators</u> for $\frac{2}{3}$ is $\{3, 6, 9, 12, . . .\} = K$

The set of <u>denominators</u> for $\frac{7}{10}$ is $\{10, 20, 30, . . .\} = L$

Set K is called the set of <u>multiples</u> of 3.

Set L is called the set of <u>multiples</u> of 10.

The numbers common to both sets are called the <u>common multiples</u> of 3 and 10.

The numbers both sets have in common are also the numbers you can use as denominators for both $\frac{2}{3}$ and $\frac{7}{10}$. They are called <u>common</u> <u>denominators</u> for $\frac{2}{3}$ and $\frac{7}{10}$. Before you study fractions any further, you should find out more about common multiples.

COMMON MULTIPLE

We use the word "multiple" as another way to talk about factors. Instead of saying

4 is a <u>factor</u> of 12

we may say

12 is a <u>multiple</u> of 4.

This idea is not strange. Instead of saying

3 is less than 5

we might say

5 is greater than 3.

Instead of saying

John is younger than Bruce

we might say

Bruce is older than John.

What is the other way of saying these?

I am taller than you.

Today is warmer than yesterday.

The relation between <u>factor</u> and <u>multiple</u> is another example of the same idea. Put these statements into the language of <u>multiples</u>:

7 is a factor of 21

3 is not a factor of 31

12 is a factor of 12.

Put these into the language of factors:

14 is a multiple of 7.

12 is a multiple of 12.

18 is a multiple of both 9 and 2.

Because 18 is a multiple of both 9 and 2, 18 is called a <u>common</u> <u>multiple</u> of 9 and 2. Because $12 = 3 \times 4$ and $12 = 2 \times 6$, 12 is a common multiple of 4 and 6. Is 12 a common multiple of 3 and 4? Of 2 and 3? Of 4 and 12?

A good way to think about common multiples is to use the language of sets.

Let R be the set of all multiples of 4, and let S be the set of all multiples of 3.

$$R = \{4, 8, 12, 16, 20, 24, 28, \ldots\}$$

$$S = \{3, 6, 9, 12, 15, 18, 21, 24, 27, \ldots\}$$

The set of <u>common</u> multiples of 3 and 4 is

$$R \cap S = \{12, 24, \ldots\}.$$

Exercise Set 8

1. Below are pairs of numbers. Show the set of multiples of each number. Then show the set of common multiples.

   Example: 3, 5

   A = set of multiples of 3 = {3, 6, 9, 12, 15, 18 ...}

   B = set of multiples of 5 = {5, 10, 15, 20 ...}

   $A \cap B$ = set of common multiples of 3 and 5 = {15, 30 ...}

   a. 4, 6

   b. 6, 8

   c. 15, 10

   d. 9, 6

   e. 10, 20

2. In the example in exercise 1, is 45 a common multiple of 3 and 5? Is 60? If n is a counting number, is $n \times 15$ always a common multiple of 3 and 5?

3. The product of two numbers is always a multiple common to both numbers. Is it <u>ever</u> the smallest of all common multiples? Is it <u>always</u> the smallest? Give examples.

4. I am thinking of two numbers. They have 18 as a common multiple.

   a. Is 36 a common multiple of the two numbers?

   b. If n is a counting number, is $18 \times n$ always a common multiple of the two numbers?

   c. Could 9 be a common multiple of the two numbers. Give an example if there is one.

LEAST COMMON MULTIPLE

There are two things which seem to be true about the set of all the common multiples of any two numbers.

1) Every multiple of the smallest common multiple is also a common multiple.

2) No other numbers are common multiples. For example, the set of common multiples of 2 and 3 begins

$$\{6, 12, 18, 24, \ldots\}$$

It seems to consist of only the multiples of 6. 6 is the smallest common multiple of 2 and 3.

Because 1) and 2) are always true, we only have to know the smallest common multiple, then we can find all common multiples. The smallest common multiple is usually called the least common multiple.

Exercise Set 9

1. The least common multiple of two numbers is 10. What are the other common multiples?

2. Find two different pairs of numbers with least common multiple 18.

3. Express this idea in factor language:

   The least common multiple of 3 and 4 is 12.

4. If you want to compare $\frac{5}{6}$ to $\frac{7}{9}$ what is the smallest denominator you could use?

5. Find several members of the set of multiples for each number below. Underline the least common multiple for each pair.

a. 9, 5
b. 7, 8
c. 5, 6
d. 11, 7

6. Find the least common multiple of each pair. Then show three of the set of all common multiples.

a. 12, 13
b. 5, 8
c. 21, 12
d. 17, 5

Exploration

Until now we have found the least common multiple (l.c.m.) of two numbers by listing multiples of each number. But this may be a long process even if the numbers are small. For example, to find the l.c.m. of 8 and 9 we find:

Set of multiples of 8 = {8, 16, 24, 32, 40, 48, 56, 64, <u>72</u> ...}
Set of multiples of 9 = {9, 18, 27, 36, 45, 54, 63, <u>72</u> ...}

It would be even harder to test our belief that the set of common multiples of 8 and 9 is

{72, 144, 216, 288, ...} .

There is a much easier way to do both.

A. First we factor the numbers completely:

$$8 = 2 \times 2 \times 2.$$

$$9 = 3 \times 3.$$

Suppose that n is any common multiple of 8 and 9. Think about the expression for n as a product of primes. Since n is a multiple of 8, $2 \times 2 \times 2$ must be a piece of this expression.

$$n = 2 \times 2 \times 2 \times (\text{any other prime factors}).$$

Since n is a multiple of 9, 3×3 must also be a piece of this expression.

$$n = 3 \times 3 \times (\text{any other prime factors}).$$

We know then that

$$n = 2 \times 2 \times 2 \times 3 \times 3 \times (\text{any other prime factors}).$$

If n is the <u>least</u> common multiple then

$$n = 2 \times 2 \times 2 \times 3 \times 3 = 8 \times 9 = 72$$

Any other common multiple can be expressed as

$$8 \times 9 \times (\text{other factors}).$$

This shows that every other common multiple of 8 and 9 is a multiple of 72.

B. Here is another example: Find the l.c.m. of 60 and 270.

$$60 = 2 \times 2 \times 3 \times 5.$$

$$270 = 2 \times 3 \times 3 \times 3 \times 5.$$

The l.c.m. must have at least two 2's, three 3's, and one 5 in its factorization. So

$$\text{l.c.m. of } 60 \text{ and } 270 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$$
$$= 540$$

We can think of the l.c.m. in this way

60

$$2 \times 2 \times 3 \times 3 \times 3 \times 5$$

270

We have put in all the prime factors we need to get a multiple of 270 and a multiple of 60. We get the l.c.m. if we include no more.

C. Here is one more example. Find the l.c.m. of 84 and 90.

$$84 = 2 \times 2 \times 3 \times 7$$

$$90 = 2 \times 3 \times 3 \times 5$$

Perhaps it will help to think of the problem in this way: What factors do I have to include beside

$$\underbrace{2 \times 3 \times 3 \times 5}_{90}$$

so that the expression will name a multiple of 84 ?

First we mark those numerals in the complete factorization of 84 that are already written in expressing 90.

$$\underbrace{2 \times 3 \times 3 \times 5}_{90}$$

Then we add the remaining piece of the complete factorization of 84.

$$\overbrace{\underbrace{2 \times 3 \times 3 \times 5}_{90} \times 2 \times 7}^{84} = 1260 = 90 \times 14 = 84 \times 15$$

If we show the factors in order, we get

$$\overbrace{2 \times \underbrace{2 \times 3 \times 3 \times 5}_{90} \times 7}^{84} = 1260$$

Imagine doing this problem the long way!

Use what we found in this example to compare

$$\frac{5}{84} \quad \text{and} \quad \frac{7}{90}$$

Exercise Set 10

1. Find the least common multiple of each pair of numbers. Then show the set of all common multiples.

   Example: 14 and 35

$$14 = 2 \times 7$$

$$35 = 5 \times 7$$

$$\text{l.c.m.} = \underbrace{2 \times \rlap{$\overbrace{}^{35}$}7}_{14} \times 5 = 70$$

   Set of common multiples = {70, 140, 210 ...}

   a. 10 and 21

   b. 24 and 9

   c. 20 and 36

   d. 30 and 16

2. Give an example of each:

   a. A pair of numbers whose l.c.m. is their product.

   b. A pair of numbers whose l.c.m. is one of the numbers.

   c. A pair of numbers for which neither a nor b is true.

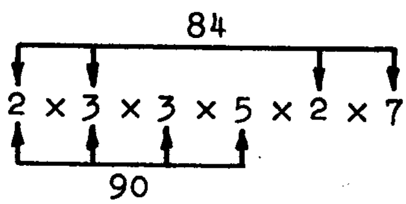
3. A traffic light at one corner changes every 30 seconds. The traffic light at the next corner changes every 36 seconds. At a certain time they both change together. How long will it be until they change together again.

4. A "Discoverer" satellite goes directly over the north and south poles each time it circles the earth. It makes one circle in 96 minutes. It is directly over the north pole at noon. When will it next be over the north pole exactly on the hour?

5. a. Find two numbers greater than 1 whose l.c.m. is 96. (Hint: use the complete factorization of 96.)

   b. The number 283 is <u>not</u> the l.c.m. of any pair of numbers except 1 and 283. What does this show about the factors of 283?

   c. Here is what we drew to help us find the l.c.m. of 84 and 90:



   Form the product expression which uses the numerals with <u>two</u> arrows pointing to them. Can you find a meaning for such an expression?

LEAST COMMON DENOMINATOR

Do you remember why we wanted to find common multiples? Let us use what we have learned to think again about how to compare $\frac{2}{3}$ and $\frac{7}{10}$.

We need to find a name for $\frac{2}{3}$ and a name for $\frac{7}{10}$ with the *same* denominator. We say that we want to find a *common denominator* for $\frac{2}{3}$ and $\frac{7}{10}$. We know that the common denominators for $\frac{2}{3}$ and $\frac{7}{10}$ are the common multiples of 3 and 10.

The *least common multiple* of 3 and 10 is the *least common denominator* of $\frac{2}{3}$ and $\frac{7}{10}$.

We can find the least common denominator for $\frac{2}{3}$ and $\frac{7}{10}$ in this way:

$$10 = 2 \times 5$$

3 is prime

l.c.m. of 10 and 3 is $\underbrace{2 \times 5}_{10} \times \overbrace{3}^{3} = 30$

To rename the fractions with the least common denominator we must also find n in $n \times 3 = 30$. Our diagram shows that

$$3 \times 10 = 30$$

$$10 \times 3 = 30.$$

Now we know

$$\frac{2}{3} = \frac{2 \times 10}{3 \times 10} = \frac{21}{30}.$$

$$\frac{7}{10} = \frac{7 \times 3}{10 \times 3} = \frac{21}{30}.$$

Notice that 60, 90, 120 ... are also common denominators.

We didn't really need our method with the small numbers 3 and 10. Here is an example with greater numbers. Which is greater $\frac{37}{84}$ or $\frac{13}{30}$?

First, we find the least common denominator.

$$84 = 2 \times 2 \times 3 \times 7$$

$$30 = 2 \times 3 \times 5$$

84

$$\text{l.c.m.} = 2 \times 2 \times 3 \times 7 \times 5$$

30

Now we want to express 84 and 30 as factors of the l.c.m. By looking at the arrows, we see that

$$\text{l.c.m.} = 84 \times 5 = 30 \times 14.$$

Now we can write:

$$\frac{37}{84} = \frac{37 \times 5}{84 \times 5} = \frac{185}{420}$$

$$\frac{13}{30} = \frac{13 \times 14}{30 \times 14} = \frac{182}{420}$$

We find, then, that

$$\frac{37}{84} > \frac{13}{30} .$$

Exercise Set 11

1. Rename each pair of numbers so the fractions have the least common denominator. Hint: Rename in simplest form before finding a least common denominator.

   a. $\frac{1}{2}$ and $\frac{2}{4}$

   b. $\frac{3}{5}$ and $\frac{3}{4}$

   c. $\frac{6}{10}$ and $\frac{20}{25}$

   d. $\frac{8}{5}$ and $\frac{8}{10}$

   e. $\frac{4}{6}$ and $\frac{2}{3}$

   f. $\frac{6}{15}$ and $\frac{9}{14}$

2. For each of the following pairs of fractions, find two other fractions which name the same two numbers and which have the least common denominator.

   a. $\frac{2}{3}$ and $\frac{3}{4}$

   b. $\frac{1}{4}$ and $\frac{2}{5}$

   c. $\frac{5}{8}$ and $\frac{3}{5}$

   d. $\frac{3}{8}$ and $\frac{5}{6}$

   e. $\frac{4}{5}$ and $\frac{7}{10}$

   f. $\frac{7}{8}$ and $\frac{2}{3}$

   g. $\frac{2}{5}$ and $\frac{1}{6}$

   h. $\frac{1}{2}$ and $\frac{2}{3}$

3. Which fraction names the greatest rational number?

   a. $\frac{2}{3}$ or $\frac{3}{4}$

   b. $\frac{3}{9}$ or $\frac{3}{5}$

   c. $\frac{3}{4}$ or $\frac{4}{5}$

   d. $\frac{4}{5}$ or $\frac{5}{6}$

   e. $\frac{5}{7}$ or $\frac{2}{3}$

   f. $\frac{5}{8}$ or $\frac{13}{16}$

   g. $\frac{3}{5}$ or $\frac{2}{3}$

   h. $\frac{5}{8}$ or $\frac{2}{3}$

4. Arrange in order from least to greatest.

$$\frac{3}{4},\ \frac{4}{5},\ \text{and}\ \frac{7}{10}$$

Exercise Set 12

1. Find which number is greater.

   a. $\frac{7}{18}$ or $\frac{61}{140}$ b. $\frac{16}{27}$ or $\frac{13}{24}$

2. a. Which is longer, $\frac{1}{3}$ of a year or 121 days?

   b. Which is longer, 2000 ft. or .3 of a mile?

   (Hint: First find simplest fraction names.)

3. Which is greater?

   a. $\frac{19}{28}$ or $\frac{140}{210}$ b. $\frac{45}{72}$ or $\frac{275}{400}$

4. List these in order of size from least to greatest.

   a. $\frac{19}{16}$, $\frac{55}{48}$, $\frac{43}{36}$ b. 3, $\frac{30}{9}$, $\frac{23}{8}$

5. Roy is making a hammer toy for his little sister. He has wooden pegs $\frac{19}{32}$ in. in diameter. To make holes in the board he has two drills. One makes a hole $\frac{5}{8}$ in. in diameter. The other makes a hole $\frac{11}{16}$ in. in diameter. Will the pegs fit through both sizes of holes? Which drill should he use?

6. On a number line, if you wanted to show both fourths and sixths you would mark the line in twelfths. How would you mark the line to show both of these?

   a. Tenths and sixths?

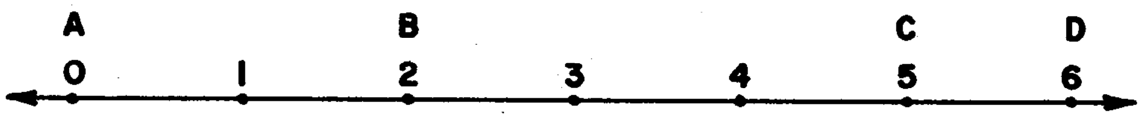
   b. Sixteenths and thirds?

   c. Twelfths and ninths?

7. Suppose that there will be either 4 people or 6 people at your party, counting yourself. Suppose also that you want to cut a cake before the party and want to divide the whole cake fairly among the people at the party. How would you cut it?

SCALES ON NUMBER LINE

Exploration



On this number line, the marked points are equally spaced. Each point is labeled with a whole number. Some points are also named with letters.

The segment with endpoints at 0 and 1 is the <u>unit segment</u>.

1. Look at $\overline{AB}$. The number at point A is 0. The number at B is 2. $\overline{AB}$ is the union of how many segments, each congruent to the unit segment?

2. What is the measure of $\overline{AC}$?

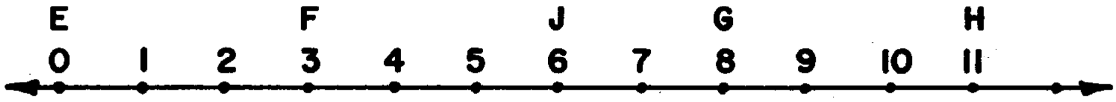
3. What is the measure of $\overline{AD}$?

4. What is an easy way to tell the measure of any segment if one endpoint is at 0 ?

5. Look at $\overline{BC}$. B is the point labeled ____. C is the point labeled ____. $\overline{BC}$ is the union of ____ segments, each congruent to the unit segment. What is the measure of $\overline{BC}$?

6. Name a segment whose measure is 4.

7. Name a segment whose measure is 1.



8. On the number line above name with letters:

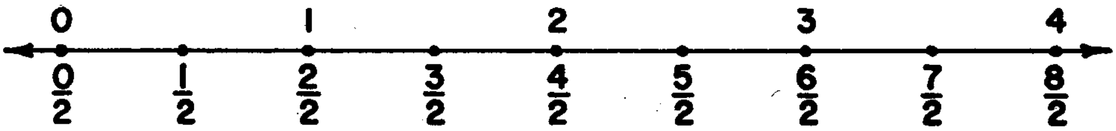
   a. Three segments, measure of each is 3.

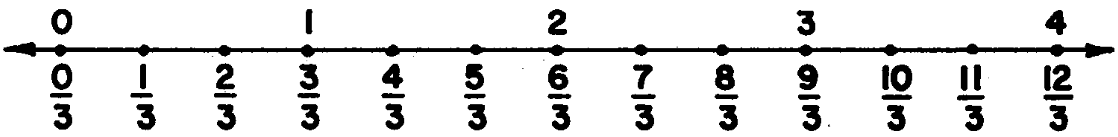
   b. Two segments, measure of each is 5.

   c. One segment whose measure is 6.

   d. Two segments, measure of each is 8.

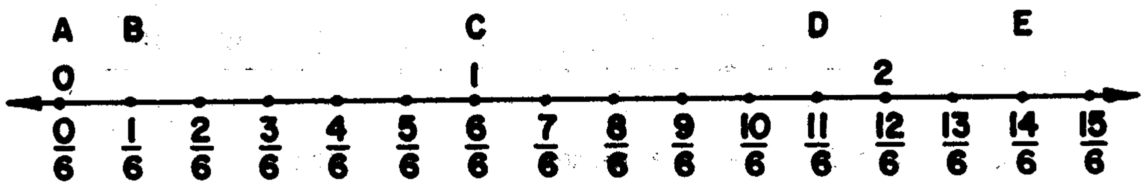
As you know, we may separate a number line into congruent segments smaller than unit segments. Here are two examples:





The first line above uses a scale of halves. It is scaled in halves. The second line above uses a scale of thirds. It is scaled in thirds.

Generally we will show the scale we are using by the denominator of the fraction we use in marking the scale. If we are marking a scale in sixths, we will label a point $\frac{3}{6}$ rather than $\frac{6}{12}$, or $\frac{2}{4}$, or $\frac{1}{2}$. If we are marking a scale in fifths we will use the label $\frac{2}{5}$ rather than $\frac{4}{10}$, or $\frac{6}{15}$, or $\frac{8}{20}$.



9. a. This number line is scaled in ____.

Find the measures of these segments.

b. $\overline{AB}$

c. $\overline{AC}$

d. $\overline{AD}$

e. $\overline{AE}$

f. $\overline{BC}$

g. $\overline{BD}$

h. $\overline{BE}$

i. $\overline{CD}$

j. $\overline{CE}$

k. $\overline{DE}$

Exercise Set 13

1. a. Look at this number line. Into how many congruent segments is the unit segment separated?

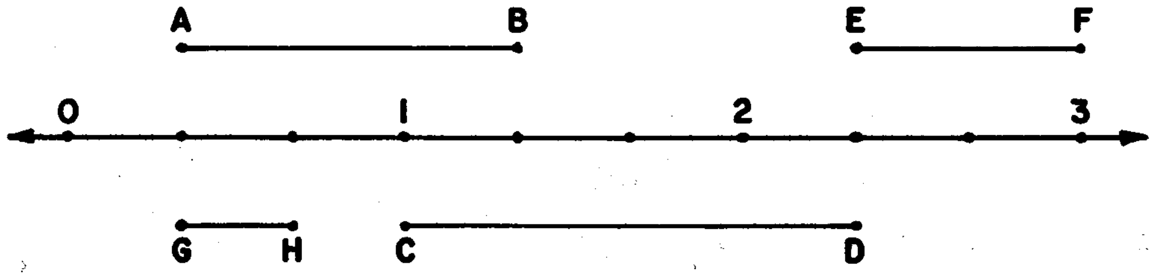
b. What is the scale?

c. Write the set of fractions you would use to label the points from 0 to 2.

d. What number of your scale matches point A? B? C? D?

e. What is the measure of $\overline{AB}$? $\overline{AC}$? $\overline{AD}$? $\overline{BC}$? $\overline{BD}$? $\overline{CD}$?

2. On the number line below are pictured $\overline{AB}$, $\overline{CD}$, $\overline{EF}$ and $\overline{GH}$.



a. The points shown with dots could be labeled in a scale of _____.

b. What numbers match the points named with letters?

c. What is the measure of each segment?

3. Do the three segments described in exercise a have the same measure? Answer the same question for exercises b and c.

a. Endpoints $\frac{6}{3}$ and $\frac{9}{3}$, $\frac{4}{2}$ and $\frac{6}{2}$, $\frac{2}{8}$ and $\frac{10}{8}$

b. Endpoints $\frac{5}{4}$ and $\frac{11}{4}$, $\frac{3}{2}$ and $\frac{6}{2}$, $\frac{1}{3}$ and $\frac{5}{3}$

c. Endpoints $\frac{4}{12}$ and $\frac{10}{12}$, $\frac{0}{2}$ and $\frac{1}{2}$, $\frac{3}{8}$ and $\frac{7}{8}$

4. Rename each of these numbers with a fraction that could be used to label a point of a scale of twenty-fourths.

Example: $\frac{1}{2} = \frac{12}{24}$

a. $\frac{1}{3}$ b. $\frac{2}{4}$ c. $\frac{2}{6}$ d. $\frac{11}{12}$ e. $\frac{5}{8}$

5. Which of these numbers cannot be renamed by a fraction which could be used to mark a scale in eighteenths?

a. $\frac{1}{6}$ d. $\frac{2}{9}$ f. $\frac{2}{3}$

b. $\frac{5}{12}$ e. $\frac{3}{4}$ g. $\frac{1}{10}$

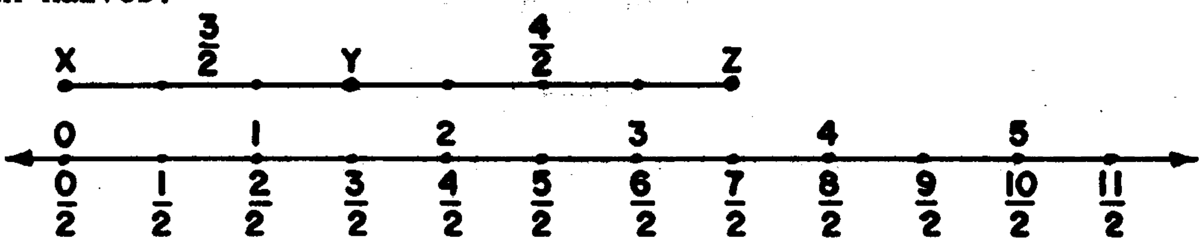
c. $\frac{7}{16}$

ADDITION OF RATIONAL NUMBERS ON THE NUMBER LINE

Exploration

Building a new line segment out of two others is a way of picturing <u>addition</u> of whole numbers. Because the same method also pictures an operation on <u>rational numbers</u>, we agree to call this operation <u>addition</u>. We express a relation between measures of length using the usual symbol "+" to indicate addition.

The addition, $\frac{3}{2} + \frac{4}{2}$ may be shown on a number line scaled in halves.



$\overline{XY}$ is the union of three congruent segments, each with measure $\frac{1}{2}$, so the measure of $\overline{XY}$ is $\frac{3}{2}$.

$\overline{YZ}$ is the union of four congruent segments, each with measure $\frac{1}{2}$, so the measure of $\overline{YZ}$ is $\frac{4}{2}$.

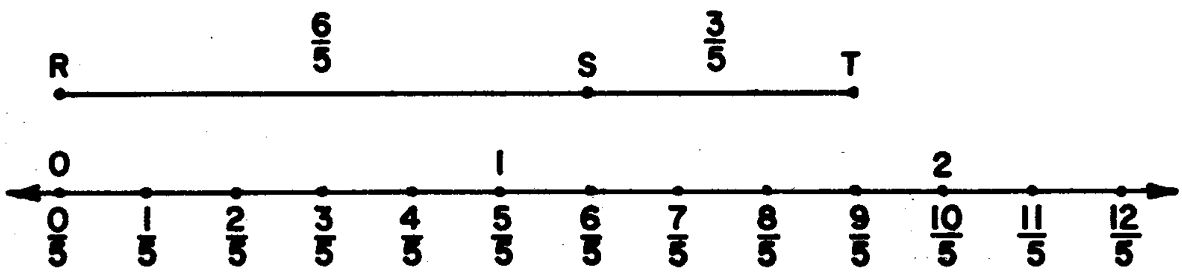
$\overline{XZ}$ is the union of $(3 + 4)$, or 7 congruent segments, each with measure $\frac{1}{2}$. The measure of $\overline{XZ}$ is $\frac{7}{2}$.

We write

$$\frac{3}{2} + \frac{4}{2} = \frac{3 + 4}{2} = \frac{7}{2}$$

We can think of $\frac{7}{2}$ as another name for $\frac{3}{2} + \frac{4}{2}$.

The addition $\frac{6}{5} + \frac{3}{5}$ may be shown on a number line scaled in fifths.



1. $\overline{RS}$ is the union of ____ congruent segments, each with measure _____. The measure of $\overline{RS}$ is _____.

2. $\overline{ST}$ is the union of _____ congruent segments, each with measure _____. The measure of $\overline{ST}$ is _____.

3. $\overline{RT}$ is the union of _____ congruent segments, each with measure _____. The measure of $\overline{RT}$ is $\frac{\_\_ + \_\_}{5}$, or _____.

We write

$$\frac{6}{5} + \frac{3}{5} = \frac{6 + 3}{5} = \frac{9}{5}$$

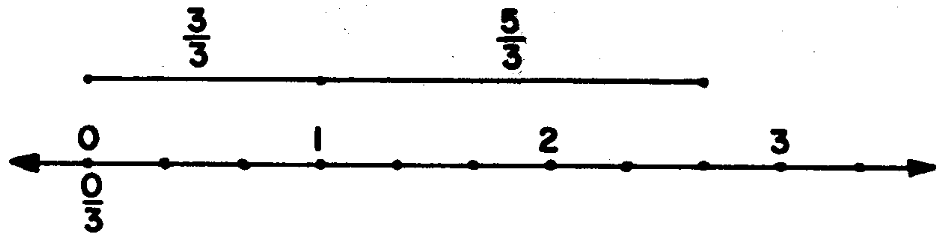
We can think of $\frac{9}{5}$ as another name for $\frac{6}{5} + \frac{3}{5}$.

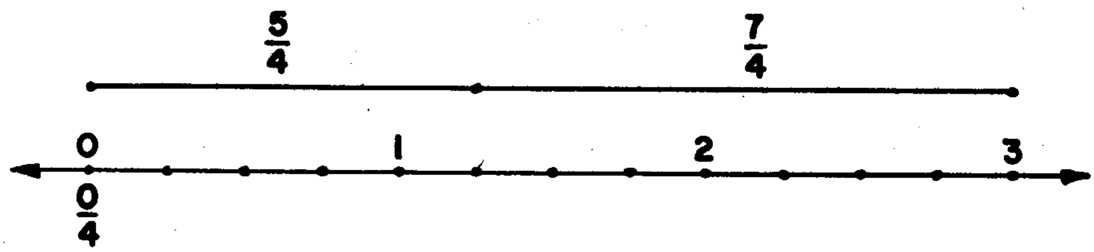
Exercise Set 14

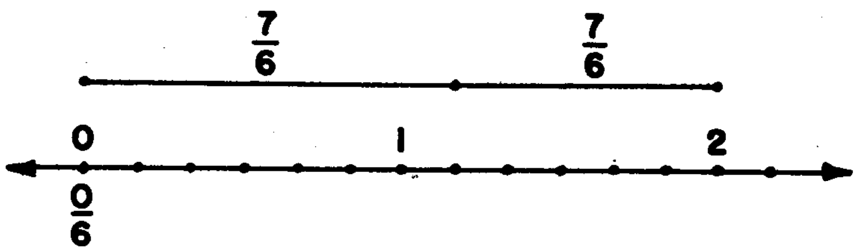
1. Use number line diagrams to show each of these sums.

   a. $\frac{5}{3} + \frac{4}{3}$ b. $\frac{2}{4} + \frac{7}{4}$ c. $\frac{3}{2} + \frac{4}{2}$

2. Write a mathematical sentence for each of the following diagrams:

a.


b.


c.


3. Copy and find a fraction name for each of these sums.

   Exercise a is done for you.

   a. $\frac{3}{4} + \frac{5}{4} = \frac{3+5}{4} = \frac{8}{4}$

   b. $\frac{5}{6} + \frac{9}{6}$ d. $\frac{7}{8} + \frac{9}{8}$

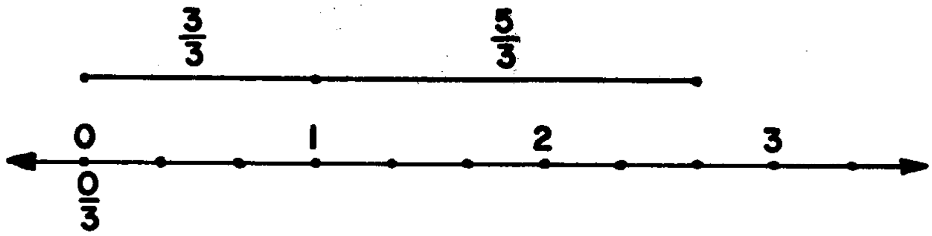
   c. $\frac{4}{5} + \frac{7}{5}$ e. $\frac{5}{12} + \frac{9}{12}$

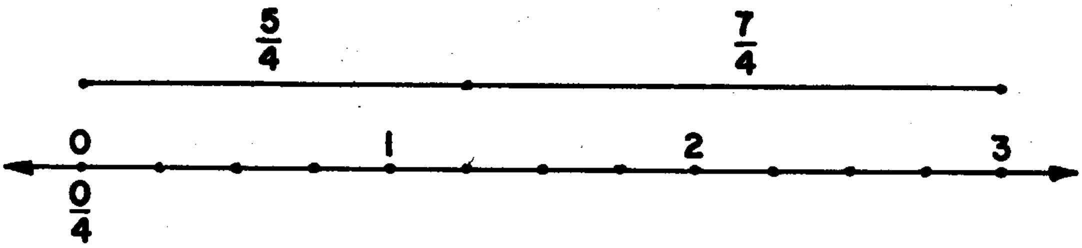
Exercise Set 14

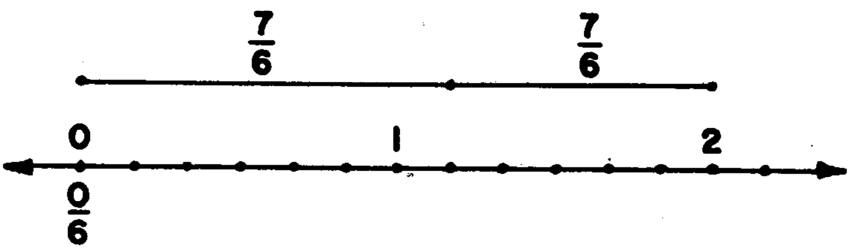
1. Use number line diagrams to show each of these sums.

   a. $\frac{5}{3} + \frac{4}{3}$ b. $\frac{2}{4} + \frac{7}{4}$ c. $\frac{3}{2} + \frac{4}{2}$

2. Write a mathematical sentence for each of the following diagrams:

a. 

b. 

c. 

3. Copy and find a fraction name for each of these sums.

   Exercise a is done for you.

   a. $\frac{3}{4} + \frac{5}{4} = \frac{3 + 5}{4} = \frac{8}{4}$

   b. $\frac{5}{6} + \frac{9}{6}$ d. $\frac{7}{8} + \frac{9}{8}$

   c. $\frac{4}{5} + \frac{7}{5}$ e. $\frac{5}{12} + \frac{9}{12}$

4. Find a fraction name for n, if:

a. $n = \frac{8}{3} + \frac{7}{3}$

c. $n = \frac{3}{4} + \frac{7}{4}$

b. $n = \frac{5}{8} + \frac{7}{8}$

d. $n = \frac{5}{6} + \frac{6}{3}$

5. Copy each of the following and represent the sum in simplest form. Exercise a is done for you.

a. $\frac{1}{4} + \frac{1}{4} = \frac{1 + 1}{4} = \frac{2}{4} = \frac{1}{2}$

b. $\frac{2}{8} + \frac{4}{8}$

c. $\frac{1}{6} + \frac{3}{6}$

d. $\frac{7}{10} + \frac{1}{10}$

e. $\frac{3}{8} + \frac{1}{8}$

f. $\frac{1}{6} + \frac{1}{6}$

g. $\frac{3}{12} + \frac{1}{12}$

6. BRAINTWISTERS

a. $\frac{5}{6}$ is the result of adding two rational numbers. The fraction name for each addend has a denominator 6. What are two possible addends?

b. $\frac{5}{6}$ is the result of adding two rational numbers. Each fraction name has a denominator 12. What are two possible addends?

c. $\frac{5}{6}$ is the result of adding two rational numbers. One fraction has a denominator of 4 and the other fraction has a denominator 12. What are two possible addends?

d. $\frac{7}{12}$ is the result of adding two rational numbers. One fraction has a denominator of 3 and the other has a denominator of 4. What are two possible addends?

SUBTRACTION OF RATIONAL NUMBERS

Addition and subtraction are operations on two numbers. The result of each operation is a single number.

The result of adding $\frac{2}{3}$ and $\frac{5}{3}$ is $\frac{7}{3}$. We have added. We call $\frac{2}{3}$ and $\frac{5}{3}$ addends. We call $\frac{7}{3}$ the sum.

Addition of rational numbers may be expressed with fraction numerals as shown on the right. In addition, two addends are known. We wish to find the sum.

$$\frac{2}{3} + \frac{5}{3} = \frac{7}{3} \qquad \begin{array}{rl} \frac{2}{3} & \text{addend} \\ + \frac{5}{3} & \text{addend} \\ \hline \frac{7}{3} & \end{array}$$

When we think about $\frac{7}{3}$ and $\frac{5}{3}$ and get a result of $\frac{2}{3}$, we have subtracted. We call $\frac{5}{3}$ and $\frac{2}{3}$ addends. We call $\frac{7}{3}$ the sum.

Subtraction of rational numbers may be expressed with fraction numerals as shown at the right. In subtraction, the sum and one addend are known. We wish to find the other addend.

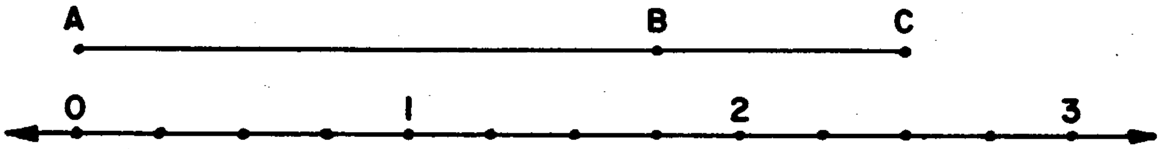
$$\frac{7}{3} - \frac{5}{3} = \frac{2}{3} \qquad \begin{array}{rl} \frac{7}{3} & \text{sum} \\ - \frac{5}{3} & \text{addend} \\ \hline \frac{2}{3} & \text{addend} \end{array}$$

The mathematical sentences

$$\frac{2}{3} + \frac{5}{3} = \frac{7}{3}, \quad \frac{7}{3} - \frac{5}{3} = \frac{2}{3}, \quad \text{and} \quad \frac{7}{3} - \frac{2}{3} = \frac{5}{3}$$

express the same relationship among $\frac{2}{3}$, $\frac{5}{3}$, and $\frac{7}{3}$.

Look at the diagram below. Use the measures of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ to write three mathematical sentences which express the same relationship.



Exercise Set 15

1. Use number line diagrams to picture these relationships. What number does n represent?

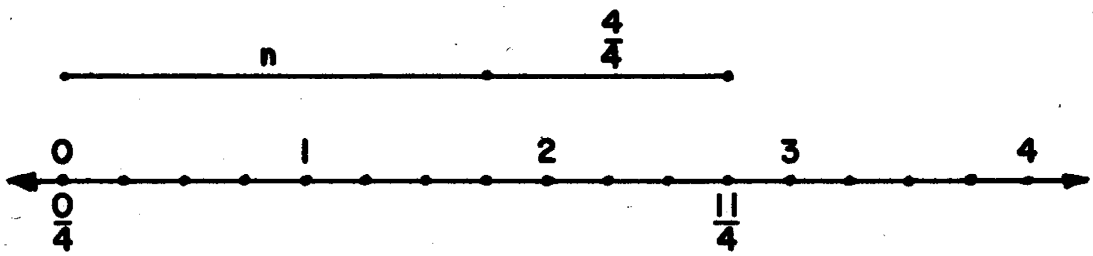
   a. $\frac{7}{3} - \frac{2}{3} = n$

   b. $n + \frac{3}{4} = \frac{12}{4}$

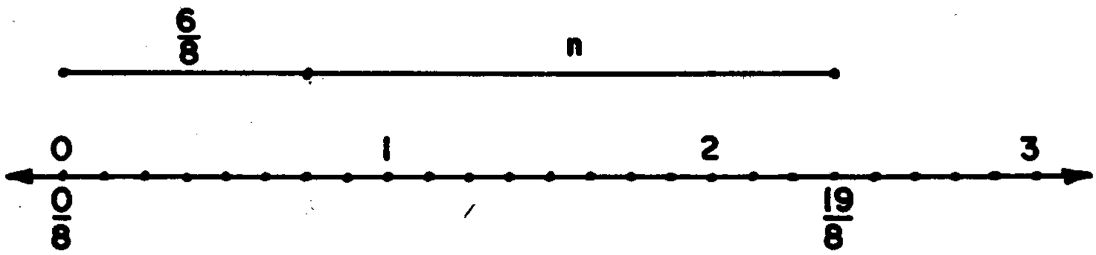
   c. $\frac{13}{6} + \frac{5}{6} = n$

2. What mathematical sentences are pictured in the diagrams below?

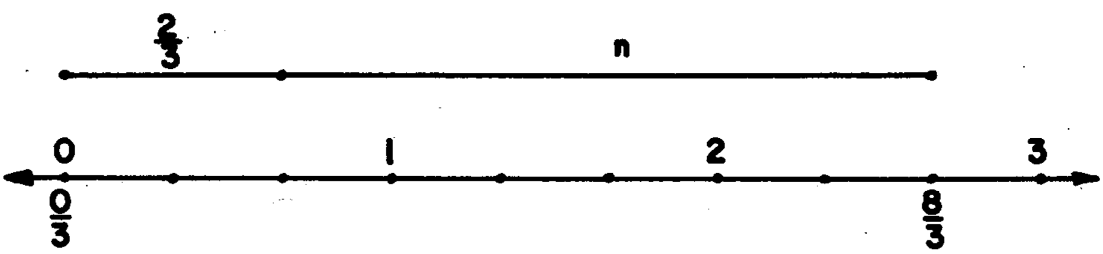
a.



b.



c.



3. Copy each sentence and find a fraction name for n. Exercise a is done for you.

a. $n = \frac{5}{3} - \frac{1}{3} = \frac{5 - 1}{3} = \frac{4}{3}$

b. $\frac{7}{5} - \frac{3}{5} = n$

c. $\frac{17}{8} - \frac{9}{8} = n$

d. $n = \frac{14}{12} - \frac{9}{12}$

e. $n = \frac{15}{16} - \frac{8}{16}$

f. $\frac{8}{4} - \frac{5}{4} = n$

g. $\frac{11}{5} - \frac{7}{5} = n$

4. Copy each sentence and find the other addend. Name each answer in simplest fraction form.

a. $\frac{5}{6} - \frac{2}{6} = n$

b. $\frac{7}{8} - \frac{5}{8} = n$

c. $\frac{7}{4} - \frac{5}{4} = n$

d. $\frac{8}{10} - \frac{4}{10} = n$

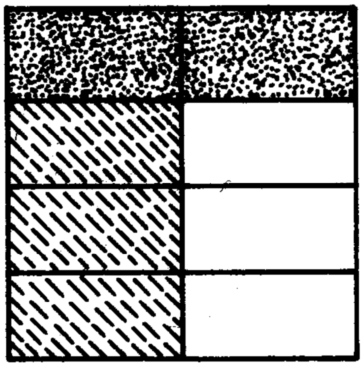
e. $\frac{9}{16} - \frac{5}{16} = n$

f. $\frac{7}{6} - \frac{5}{6} = n$

PICTURING ADDITION AND SUBTRACTION WITH REGIONS

Exploration

You have seen how number lines can be used to picture addition and subtraction of rational numbers. Regions can be used also.



A

1. Figure A represents a unit region. Each of the small regions is _____ of the unit region.

2. The dotted region is _____ of the unit region.

3. The shaded region is _____ of the unit region.

4. The unshaded region is _____ of the unit region.

5. Which regions picture the mathematical sentence

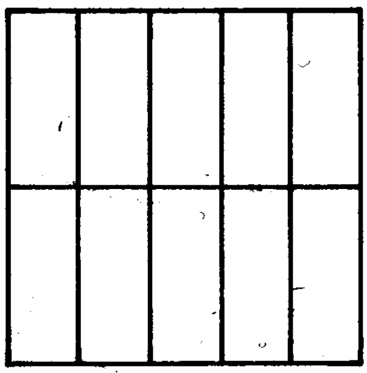
$$\frac{3}{8} + \frac{2}{8} = \frac{5}{8} \text{ ?}$$

6. Write two other mathematical sentences which express the same relationship among $\frac{3}{8}$, $\frac{2}{8}$, and $\frac{5}{8}$.

7. Write a mathematical sentence pictured by the dotted and unshaded regions.

8. Write three mathematical sentences suggested by the unit region and the dotted regions.

9. The unit region and the unshaded region suggest that $\frac{3}{8} + \_\_\_\_ = \frac{8}{8}$. Write two other mathematical sentences for this relationship.



B

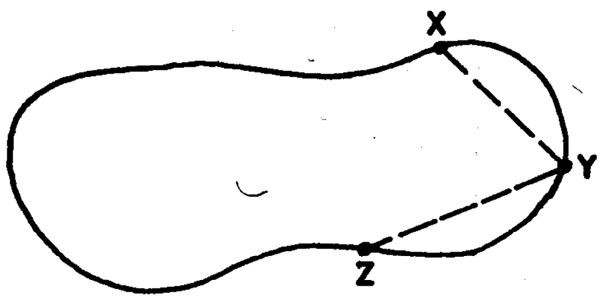
10. Trace figure B shown above. Shade some parts and write three mathematical sentences for your picture.

Exercise Set 16

Write a mathematical sentence for each problem. Use a number line if you need help in writing the mathematical sentence or finding the answer. Use simplest names for rational numbers used in your answers.

1. Susan needs $\frac{2}{3}$ yard ribbon to wrap one present and $\frac{2}{3}$ yard of ribbon for another. How much does she need?

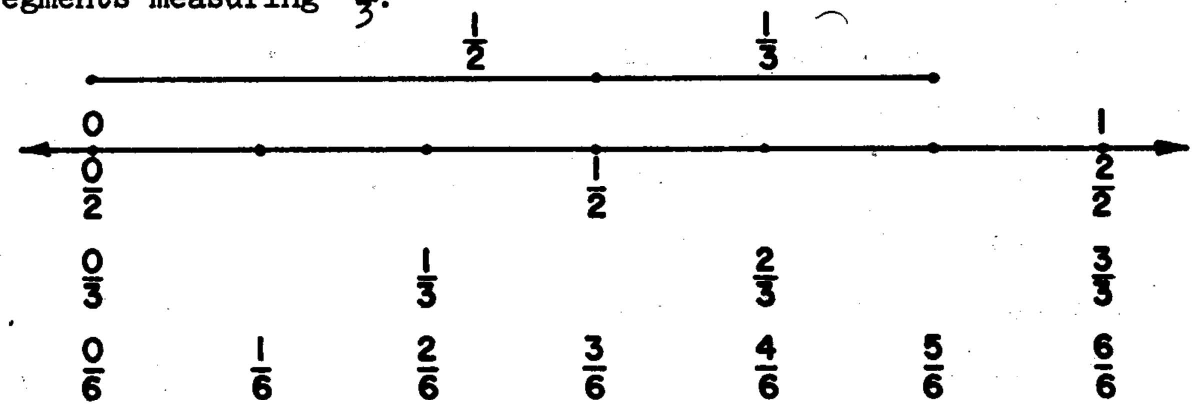
2. Below is a map of a lake. Three friends live at the points marked X, Y, and Z. X and Y are $\frac{3}{10}$ miles apart. Y and Z are $\frac{2}{10}$ miles apart. How long a boat trip is it from X to Z by way of Y?



3. There were 12 chapters in Mary's book. One day she read 2 chapters. The following day she read 1 chapter. What rational number best describes the part of the book she read on the two days?

SCALES FOR PICTURING ADDITION

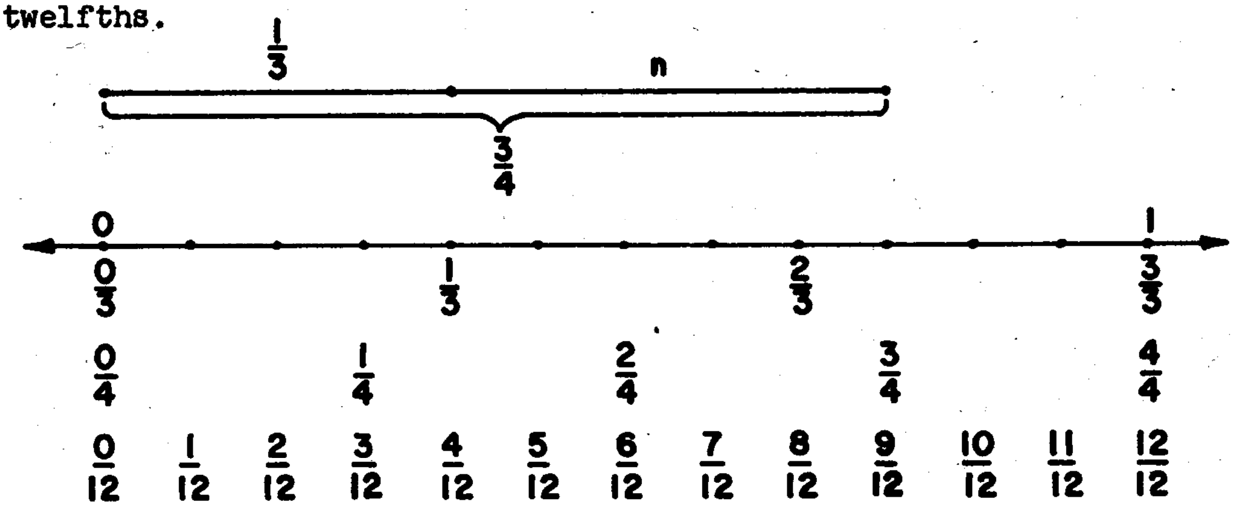
To picture $\frac{1}{2} + \frac{1}{3}$ on a number line, we need a suitable scale. A scale of sixths can show segments measuring $\frac{1}{2}$ and segments measuring $\frac{1}{3}$.



This diagram suggests that $\frac{1}{2} + \frac{1}{3}$ can be written as

$$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6}. \quad \text{Now, } \frac{3}{6} + \frac{2}{6} = \frac{3 + 2}{6} = \frac{5}{6}.$$

To show $\frac{3}{4} - \frac{1}{3}$ on a number line we may use a scale of twelfths.



We think, $n = \frac{3}{4} - \frac{1}{3}$; so $n + \frac{1}{3} = \frac{3}{4}$. The number line shows that $n = \frac{5}{12}$. This suggests that $\frac{3}{4} - \frac{1}{3}$ can be written as

$$\frac{3}{4} - \frac{1}{3} = \frac{9}{12} - \frac{4}{12}. \quad \text{Now, } \frac{9}{12} - \frac{4}{12} = \frac{9 - 4}{12} = \frac{5}{12}.$$

Can we always find a suitable scale to picture the sum of rational numbers? Consider $\frac{3}{8} + \frac{1}{6}$.

To find a suitable scale we must find a scale such that each $\frac{1}{8}$ segment and each $\frac{1}{6}$ segment is separated into a whole number of smaller segments. For eighths, we can separate each $\frac{1}{8}$ segment into

2 congruent segments. Then there will be 2×8 parts in each unit segment.

3 congruent segments. Then there will be 3×8 parts in each unit segment.

4 congruent segments. Then there will be 4×8 parts in each unit segment.

Subdividing each $\frac{1}{8}$ segment in these ways suggests scales in which the denominators of the fractions are the set of multiples of 8. {8, 16, 24, 32, 40, 48, ...} Subdividing sixths suggests the set of multiples of 6. {6, 12, 18, 24, 30, 36, ...} A scale which can be used to picture addition of eighths and sixths will be one in which the denominator of the fraction is a common multiple of 8 and 6. The easiest scale to use is the one in which the l.c.m. of 8 and 6 is used, that is, the least common denominator for $\frac{3}{8}$ and $\frac{5}{6}$.

You know how to find the l.c.d. of $\frac{3}{8}$ and $\frac{5}{6}$. It is the l.c.m. of 8 and 6.

$$8 = 2 \times 2 \times 2 \qquad\qquad 6 = 2 \times 3$$

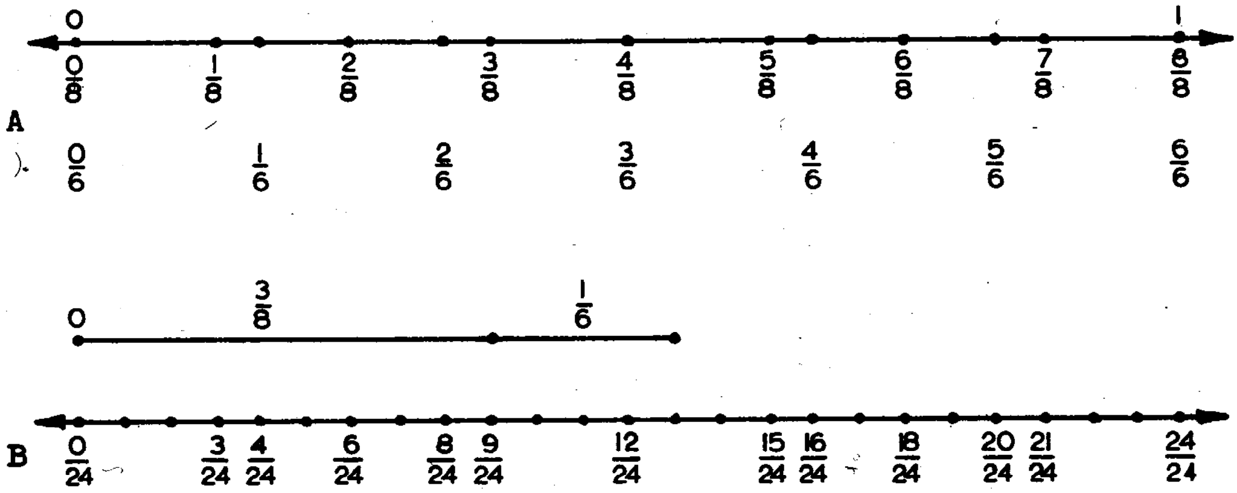
$$\text{l.c.d.} = \overbrace{2 \times 2 \times \rlap{$\underbrace{}_{6}$}2}^{8} \times 3 = 2 \times 2 \times 2 \times 3 = 24$$

To subdivide the eighths and sixths segments, note that

$$24 = 8 \times 3$$

$$24 = 6 \times 4$$

So each segment of measure one-eighth is subdivided into 3 congruent segments, and each segment of measure one-sixth is subdivided into 4 congruent segments.



On number line A the eighths and sixths scales are labeled. On B, points on the number line are marked for a scale of twenty-fourths. The points corresponding to eighths and sixths are labeled in twenty-fourths.

We write:

$$\frac{3}{8} + \frac{1}{6} = \frac{9}{24} + \frac{4}{24} = \frac{9 + 4}{24} = \frac{13}{24}.$$

Exercise Set 17

1. Find the scale with the smallest number of divisions you could use to picture these sums.

   a. $\frac{3}{8} + \frac{7}{10}$ c. $\frac{3}{14} + \frac{5}{21}$

   b. $\frac{5}{9} + \frac{11}{15}$ d. $\frac{7}{16} + \frac{5}{12}$

2. Use number line diagrams to picture:

   a. $\frac{1}{2} + \frac{3}{5}$ b. $\frac{1}{6} + \frac{3}{4}$

3. For each pair of fractions, write:

   (1) the complete factorization of each denominator.

   (2) the complete factorization of the least common denominator.

   (3) fraction names using the l.c.d.

   a. $\frac{3}{8}$, $\frac{7}{20}$ c. $\frac{2}{3}$, $\frac{4}{15}$

   b. $\frac{1}{4}$, $\frac{3}{14}$ d. $\frac{5}{12}$, $\frac{7}{18}$

COMPUTING SUMS AND UNKNOWN ADDENDS

Exploration

You have seen that suitable scales can be found for picturing addition of rational numbers on the number line.

Rational numbers also can be added without using diagrams. Consider the sum

$$\frac{1}{2} + \frac{3}{4}$$

You know how to add rational numbers when they are named by fractions with the same denominator.

1. What is the l.c.d. for $\frac{1}{2}$ and $\frac{3}{4}$?

2. Using the l.c.d., what are the fraction names for $\frac{1}{2}$ and $\frac{3}{4}$?

3. What is the sum of $\frac{1}{2}$ and $\frac{3}{4}$?

You can arrange your work like this:

$$n = \frac{1}{2} + \frac{3}{4}$$

$$= \frac{2}{4} + \frac{3}{4} = \frac{2+3}{4} = \frac{5}{4}$$

$$\frac{1}{2} + \frac{3}{4} = \frac{5}{4}$$

$$\text{l.c.d.} = \underbrace{\overset{2}{2} \times 2}_{4} = 4$$

$$\frac{1}{2} = \frac{2}{4}$$

4. Explain each step of the work.

5. Can you find a common denominator for <u>any</u> two fractions?

6. Can you add <u>any</u> two rational numbers if you have fraction names for them?

Exercise Set 18

1. Find the sum of each pair of numbers. Write the simplest fraction name for the sum.

   a. $\frac{1}{2}, \frac{3}{8}$ d. $\frac{2}{3}, \frac{1}{6}$ g. $\frac{3}{4}, \frac{5}{8}$

   b. $\frac{1}{2}, \frac{5}{6}$ e. $\frac{2}{3}, \frac{5}{9}$ h. $\frac{3}{4}, \frac{17}{12}$

   c. $\frac{1}{2}, \frac{7}{12}$ f. $\frac{2}{3}, \frac{11}{12}$ i. $\frac{3}{4}, \frac{27}{20}$

2. Find the simplest fraction name for each sum.

   a. $\frac{3}{2} + \frac{1}{3}$ d. $\frac{8}{5} + \frac{7}{4}$ g. $\frac{5}{6} + \frac{3}{4}$

   b. $\frac{3}{2} + \frac{4}{5}$ e. $\frac{8}{8} + \frac{2}{3}$ h. $\frac{5}{6} + \frac{3}{5}$

   c. $\frac{3}{2} + \frac{6}{7}$ f. $\frac{8}{5} + \frac{2}{3}$ i. $\frac{5}{6} + \frac{1}{7}$

3. Use number lines to picture these mathematical sentences.

   a. $\frac{3}{2} + \frac{5}{4} = n$ c. $\frac{2}{3} + n = \frac{3}{2}$

   b. $\frac{4}{3} + \frac{5}{6} = n$ d. $\frac{11}{8} - \frac{3}{4} = n$

Rename each pair of numbers by fractions with a common denominator. Then find a fraction name for the number n.

4. a. $\frac{5}{6} - \frac{2}{3} = n$ d. $\frac{5}{6} - \frac{1}{3} = n$ g. $\frac{3}{4} - \frac{1}{6} = n$

   b. $\frac{2}{3} - \frac{5}{8} = n$ e. $\frac{4}{5} - \frac{1}{3} = n$ h. $\frac{4}{5} - \frac{1}{2} = n$

   c. $\begin{array}{r} \frac{11}{12} \\ -\frac{5}{6} \\ \hline n \end{array}$ f. $\begin{array}{r} \frac{7}{8} \\ -\frac{9}{16} \\ \hline n \end{array}$ i. $\begin{array}{r} \frac{3}{4} \\ -\frac{1}{8} \\ \hline n \end{array}$

Find the sum for each pair of numbers and write its simplest fraction name.

5. a. $\frac{1}{3}$, $\frac{3}{4}$ d. $\frac{3}{8}$, $\frac{1}{4}$ g. $\frac{2}{3}$, $\frac{5}{6}$

b. $\frac{3}{4}$, $\frac{1}{2}$ e. $\frac{2}{3}$, $\frac{1}{4}$ h. $\frac{4}{5}$, $\frac{1}{2}$

c. $\begin{array}{r}\frac{2}{5}\\ \underline{\frac{1}{3}}\end{array}$ f. $\begin{array}{r}\frac{3}{4}\\ \underline{\frac{1}{6}}\end{array}$ i. $\begin{array}{r}\frac{1}{3}\\ \underline{\frac{1}{8}}\end{array}$

6. Find a fraction name for n so that each mathematical sentence will be true.

a. $\frac{2}{3} + n = \frac{3}{4}$ d. $\frac{3}{8} + n = \frac{2}{3}$

b. $\frac{3}{4} + n = \frac{7}{8}$ e. $\frac{1}{2} + n = \frac{2}{3}$

c. $\frac{2}{5} + n = \frac{3}{4}$ f. $\frac{1}{5} + n = \frac{1}{2}$

7. BRAINTWISTER: Find a fraction which names a number

a. greater than $\frac{3}{10}$ and less than $\frac{3}{8}$.

b. greater than $\frac{2}{6}$ and less than $\frac{3}{6}$.

c. greater than $\frac{3}{6}$ and less than $\frac{4}{6}$. (Find two answers)

Exercise Set 19

Certain rational numbers should now be so familiar that you can think of many names for them. You should be able to add and subtract such numbers without writing out your work.

Without doing any writing, try to find what number n must be.

1. $\frac{1}{4} + \frac{1}{8} = n$
2. $\frac{1}{2} + \frac{1}{5} = n$
3. $\frac{3}{4} + \frac{1}{2} = n$
4. $\frac{1}{6} + \frac{5}{12} = n$
5. $\frac{7}{8} + \frac{1}{4} = n$
6. $n + \frac{5}{8} = \frac{3}{4}$
7. $n + \frac{3}{10} = \frac{4}{5}$
8. $\frac{5}{6} + n = \frac{13}{12}$
9. $n = \frac{7}{9} - \frac{1}{3}$
10. $\frac{10}{10} = n - \frac{7}{10}$

The numbers in the exercises below have fraction names which are probably less familiar. Show all your work for these exercises.

11. $\frac{5}{6} + \frac{8}{15} = n$
12. $\frac{5}{14} + \frac{3}{4} = n$
13. $\frac{13}{12} + \frac{3}{8} = n$
14. $\frac{7}{10} + \frac{11}{6} = n$
15. $\frac{1}{6} + \frac{9}{14} = n$
16. $\frac{3}{10} + n = \frac{5}{4}$
17. $n + \frac{4}{9} = \frac{11}{12}$
18. $\frac{21}{10} = n - \frac{5}{8}$
19. $n + \frac{8}{15} = \frac{13}{9}$
20. $\frac{15}{7} = n + \frac{5}{4}$

Find the number n represents. In exercise 21, recall that $(\frac{1}{2} + \frac{1}{3}) + \frac{1}{4}$ means "Find the sum of $\frac{1}{2}$ and $\frac{1}{3}$, and then add the sum to $\frac{1}{4}$".

21. $(\frac{1}{2} + \frac{1}{3}) + \frac{1}{4} = n$
22. $(\frac{3}{4} + \frac{7}{8}) + \frac{2}{3} = n$
23. $\frac{5}{6} + (\frac{3}{2} + \frac{7}{4}) = n$
24. $n + (\frac{7}{12} + \frac{1}{4}) = \frac{5}{6}$
25. $(\frac{7}{10} + \frac{2}{5}) = n - \frac{1}{2}$
26. $(\frac{1}{4} + \frac{8}{9}) + n = \frac{5}{3}$

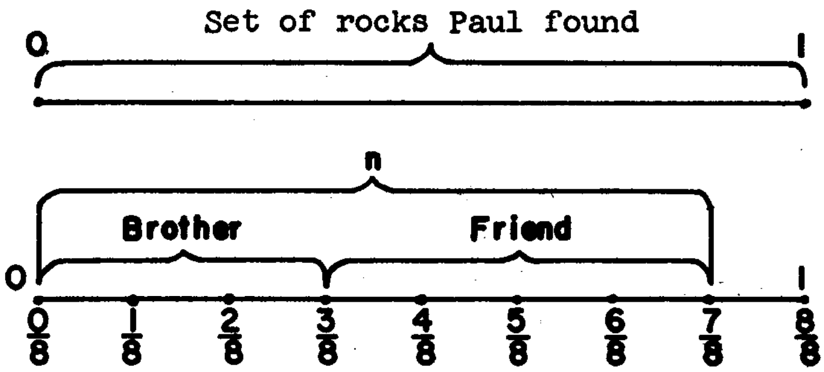
DIAGRAMS FOR PROBLEMS

We have pictured addition and subtraction of rational numbers on the number line, and also with regions. When you wish to solve a problem using rational numbers, it is sometimes helpful to picture the relationships on a number line, or in a picture of a region. Look at this problem.

Paul found several unusual rocks while he was on vacation. He gave $\frac{3}{8}$ of the rocks to his brother, and gave $\frac{1}{2}$ of them to a friend. What part of the total number of rocks did he give away?

This is not a problem about things which suggest segments, but numbers are used, and numbers may be represented on the number line.

Suppose the unit segment represents the entire set of rocks Paul found.



He gave $\frac{3}{8}$ to his brother. He gave $\frac{1}{2}$ (or $\frac{4}{8}$) to a friend.

What part did he give away? Represent it by n. The diagram suggests: $n = \frac{3}{8} + \frac{1}{2}$

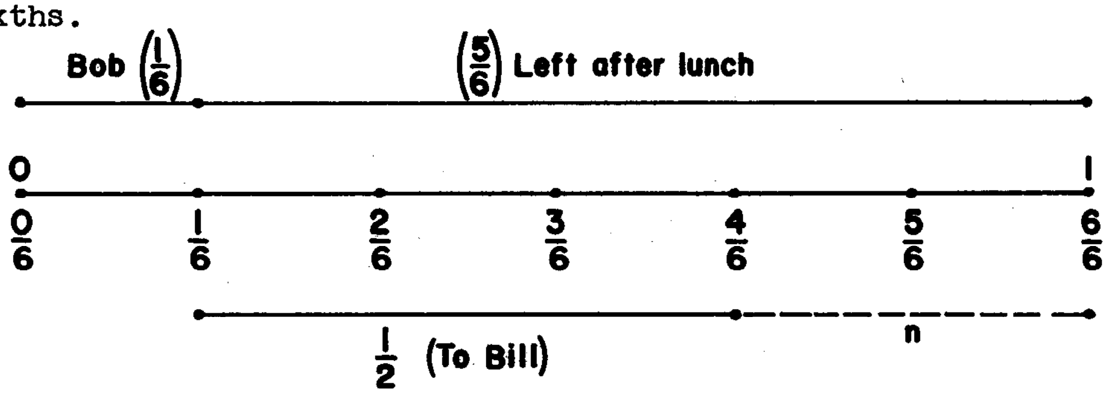
$$= \frac{3}{8} + \frac{4}{8} = \frac{3+4}{8} = \frac{7}{8}$$

Paul gave away $\frac{7}{8}$ of the rocks.

Now look at this problem.

Mrs. White cut a pie into 6 pieces. After Bob ate 1 piece for lunch, $\frac{5}{6}$ was left. Mrs. White served $\frac{1}{2}$ of the whole pie to Bill. What part of the pie was left then?

Choose a unit segment to represent the whole pie, cut into sixths.



Bob ate $\frac{1}{6}$. Bill ate $\frac{1}{2}$ or $\frac{3}{6}$. What part of the pie was left?

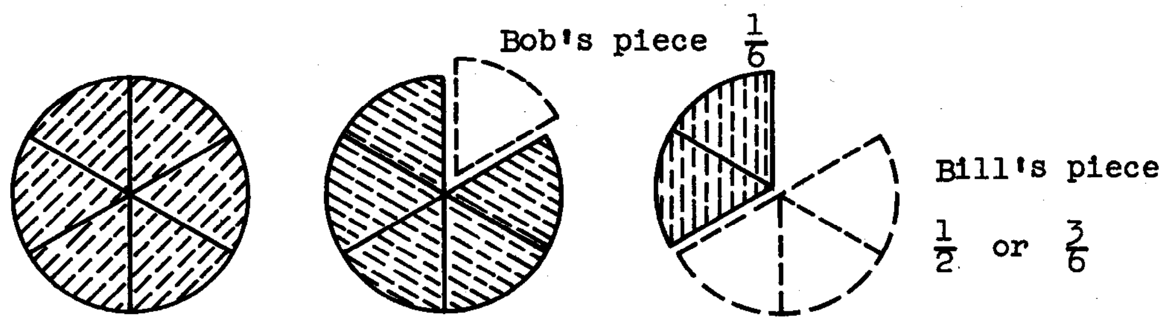
$$\frac{1}{2} + n = \frac{5}{6}, \quad \text{or}$$

$$\frac{3}{6} + n = \frac{5}{6}$$

$$n = \frac{5}{6} - \frac{3}{6} = \frac{5-3}{6} = \frac{2}{6}$$

$\frac{2}{6}$ of the pie was left.

You might wish to represent the whole pie as a unit region separated into sixths.



$1 = \frac{6}{6}$ $\quad 1 - \frac{1}{6} = \frac{5}{6}$ $\quad \frac{5}{6} - \frac{3}{6} = \frac{2}{6}$

Exercise Set 20

For each problem, make a diagram showing the number relationships. Then write a mathematical sentence, and solve it. Write your answer to the problem in a sentence.

1. Susan bought $\frac{7}{8}$ lb. of fudge and $\frac{2}{8}$ lb. of chocolate drops. How much candy did she buy in all?

2. Tom and Jerry went to a Little League game. It took Tom $\frac{1}{4}$ hour to get to the game, and it took Jerry $\frac{3}{4}$ hour. How much longer did it take Jerry to get to the game than Tom?

3. David caught a fish weighing $\frac{15}{16}$ lb. John's fish weighed $\frac{7}{16}$ lb. How much more did David's fish weigh?

4. Mrs. Ray had one whole coffee cake. She served $\frac{3}{8}$ of it to her neighbor. How much coffee cake did Mrs. Ray have left?

5. Ann was mixing some punch for her friends. She mixed $\frac{2}{3}$ cup orange juice and $\frac{2}{3}$ cup gingerale. How much punch did she have?

6. Mrs. King mixed some liquid plant food for her house plants. The directions said to use $\frac{3}{4}$ tablespoon for each gallon of water. She used 2 gallons of water. How much liquid plant food did Mrs. King use?

7. Jack spent $\frac{3}{4}$ hour on Tuesday mowing the lawn. On Wednesday he spent $\frac{1}{2}$ hour pulling weeds. How much time did Jack spend doing his work?

8. Larry's mother gave him $\frac{1}{3}$ apple pie for his lunch. She gave his brother, Jim, $\frac{1}{6}$ of the same pie. How much of the pie did the two boys eat?

9. Janet bought $\frac{3}{4}$ yd. of material. She used $\frac{2}{3}$ yd. for place mats. How much material was left?

10. Mrs. Smith used $\frac{7}{8}$ cup brown sugar and $\frac{1}{2}$ cup white sugar in a candy recipe. How much sugar did she use?

11. Alice stopped at the store on the way from her home to the park. It was $\frac{3}{5}$ mile to the store. The park was $\frac{9}{10}$ mile from Alice's home. How much farther did she walk to get to the park?

12. Jane spent $\frac{2}{3}$ hour doing her homework. Betty spent $\frac{1}{2}$ hour on homework. How much longer did it take Jane to finish?

PROPERTIES OF ADDITION OF RATIONAL NUMBERS

Exploration

You know that (1) any fraction, such as $\frac{2}{3}$, $\frac{8}{4}$, $\frac{50}{78}$, $\frac{5}{2}$, $\frac{0}{16}$, names a rational number. (2) You can find the point on a number line matching a rational number by (a) separating the unit segment into the number of congruent segments named by the denominator, (b) counting off from 0 the number of segments named by the numerator. (3) Some of these rational numbers, such as $\frac{6}{2}$, are also whole numbers.

1. Which numbers named above are also whole numbers?

2. What whole number can be a numerator of a fraction, but not a denominator?

3. Think of two whole numbers. Find their sum. What kind of number is the sum?

4. Think of two rational numbers. Find their sum. What kind of number is the sum?

5. Think of two rational numbers. Subtract the smaller from the greater. What kind of number is your answer?

6. Try to subtract the greater number in exercise 5 from the smaller. Can you do it? Can you always subtract one rational number from another?

7. Think of a rational number n named by a fraction with denominator 6. Add it to $\frac{0}{6}$. What do you notice about the sum?

8. Find these sums.

a. $\frac{2}{3} + \frac{3}{4}$ b. $\frac{3}{4} + \frac{2}{3}$

9. Illustrate each part of exercise 8 on the number line.

10. What property of addition of rational numbers do exercises 8 and 9 suggest?

11. Find fraction names for the numbers n and t.

a. $(\frac{1}{2} + \frac{1}{3}) + \frac{3}{4} = n$ b. $\frac{1}{2} + (\frac{1}{3} + \frac{3}{4}) = t$

12. What do you notice about n and t in exercise 11?

13. What property of addition of rational numbers does exercise 11 suggest?

Summary of Properties of Addition of Rational Numbers

1. If two rational numbers are added, the sum is a rational number.

2. If 0 is added to any rational number n, the sum is the same rational number n (Addition Property of Zero).

3. The order of adding two rational numbers may be changed without changing their sum (Commutative Property).

4. To find the sum of three rational numbers, you may (1) add the first two and add the third to their sum; or (2) add the second and third, and add their sum to the first (Associative Property).

Exercise Set 21

Write a mathematical sentence for each problem. Solve it and answer the question in a sentence.

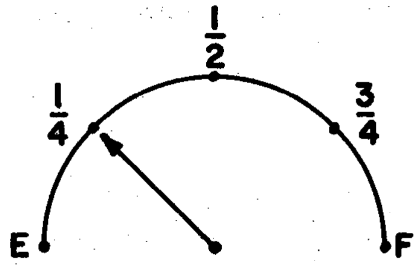
1. One measuring cup contains $\frac{1}{8}$ cup of liquid. A second measuring cup contains $\frac{3}{4}$ cup of liquid. If the liquid in the first cup is poured into the second cup, what amount of liquid will be in the second cup?

2. Directions on a can of concentrated orange juice call for mixing the juice with water. One-half quart water is to be mixed with $\frac{3}{16}$ quarts of concentrated juice. What amount of liquid will result?

3. I have $\frac{2}{3}$ dozen cookies in one box and $\frac{1}{4}$ dozen in another. You have 1 dozen cookies. Who has more? How many cookies do I have?

4. A measuring cup is filled to the $\frac{3}{8}$ mark with milk. Enough water is added to bring the level of the mixture to the $\frac{3}{4}$ mark. How much water was added?

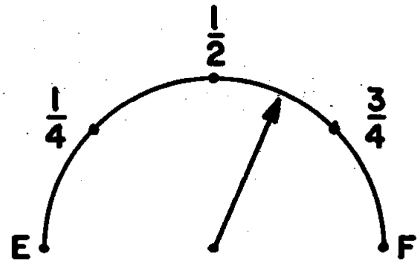
5. Cars have dials which show the quantity of gasoline in the tank. The dial might look like this:



How do the markings on the dial differ from those on our number lines?

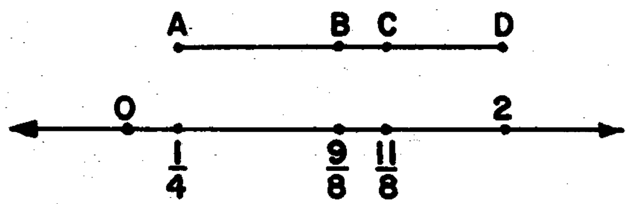
What unit of measure is represented on the dial?

Suppose enough gasoline were added to the tank to move the pointer to a position halfway between the $\frac{1}{2}$ mark and the $\frac{3}{4}$ mark.



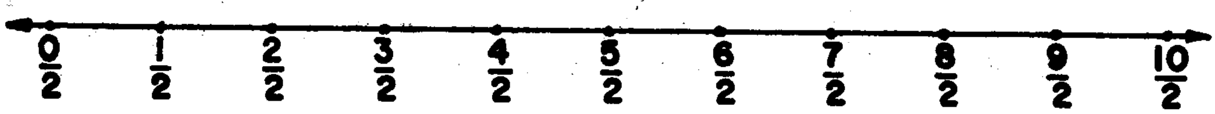
How much gasoline was added?

6. On the number line below the unit represented is the inch. What is the measure in inches of each of the six line segments pictured?



WHOLE NUMBERS AND RATIONAL NUMBERS

Exploration



1. Trace the number line and copy the scale of halves.

2. Write a whole number scale above the line. Be sure to keep the same unit segment. What number is at $\frac{0}{2}$?

3. The whole number 1 should be written at the point labeled _______________.

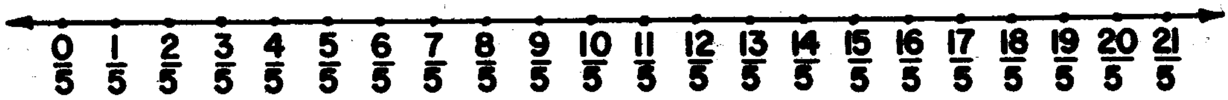
4. The whole number 2 should be written at the point labeled _______________.

5. List the numerators of the fractions which name counting numbers.

6. List the first five multiples of the denominator, 2.

7. Are your answers for exercises 5 and 6 the same?

8. Trace the number line and copy the scale of fifths.



9. Write a whole number scale above the line. Keep the same unit segment.

10. List the numerators of fractions which name counting numbers on the whole number scale.

11. List the first four multiples of the denominator, 5.

12. Are your answers for exercises 10 and 11 the same?

13. Is this a true statement?

If the numerator of a fraction is a multiple of the denominator, then the fraction names a whole number.

14. Which of these fractions are names for whole numbers?

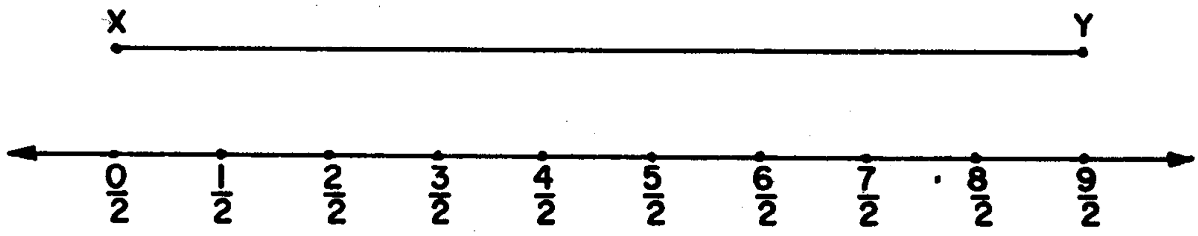
a. $\frac{18}{7}$ c. $\frac{60}{16}$ e. $\frac{120}{25}$

b. $\frac{27}{9}$ d. $\frac{90}{10}$ f. $\frac{28}{14}$

FRACTIONS AND MIXED FORMS

Exploration

Let us think of two rational numbers. Their sum is $\frac{9}{2}$. Here is a picture of $\overline{XY}$ measuring $\frac{9}{2}$. What could the addends be so their sum is $\frac{9}{2}$?



1. Look at these pairs of addends. Is the sum of each pair $\frac{9}{2}$?

   a. $\frac{8}{2} + \frac{1}{2}$ c. $\frac{6}{2} + \frac{3}{2}$

   b. $\frac{2}{2} + \frac{7}{2}$ d. $\frac{4}{2} + \frac{5}{2}$

2. Write each of the sums in exercise 1 as the sum of a whole number and a rational number.

3. Sums like your answers for exercise 2 are often written without the + sign, in this way:

   a. $4\frac{1}{2}$ b. $1\frac{7}{2}$ c. $3\frac{3}{2}$ d. $2\frac{5}{2}$

   Write these numbers as sums using the + sign.

   e. $2\frac{1}{2}$ b. $3\frac{1}{2}$ g. $\frac{1}{2}$ h. $1\frac{1}{2}$

4. Think of the way you picture $2 + \frac{1}{2}$ with segments on the number line.

   a. What fractions of the halves scale name the endpoints of the segment with measure 2 ?

   b. What fractions name the endpoints of the segment with measure $2\frac{1}{2}$?

   c. What fraction names the same number as $2\frac{1}{2}$?

5. Use a number line. Write a whole number scale above it and a scale of thirds below it.

   Are these sentences true?

$$\begin{aligned} 5\tfrac{2}{3} &= 5 + \tfrac{2}{3} \\ &= \tfrac{5}{1} + \tfrac{2}{3} \\ &= \tfrac{5 \times 3}{1 \times 3} + \tfrac{2}{3} \\ &= \tfrac{15}{3} + \tfrac{2}{3} \\ &= \tfrac{17}{3} \end{aligned}$$

6. Find fraction names for these numbers.

   a. $6\frac{3}{4}$ b. $4\frac{5}{7}$ c. $12\frac{4}{3}$ d. $12\frac{2}{6}$

   Numerals like $6\frac{3}{4}$, $4\frac{5}{7}$, and $12\frac{4}{3}$ name rational numbers. These numerals are called <u>mixed forms</u>. In $6\frac{3}{4}$ and $4\frac{5}{7}$, the fractions $\frac{3}{4}$ and $\frac{5}{7}$ are in simplest form and name numbers less than 1. We say $6\frac{3}{4}$ and $4\frac{5}{7}$ are <u>simplest mixed forms</u> for rational numbers. $12\frac{4}{3}$ is <u>not</u> in simplest mixed form, because $\frac{4}{3} > 1$. $12\frac{2}{6}$ is <u>not</u> in simplest mixed form, because $\frac{2}{6}$ is not in simplest form.

Exercise Set 22

1. Find fraction names for these numbers.

   a. $8\frac{2}{9}$ c. $16\frac{1}{2}$ d. $1\frac{7}{9}$

   b. $11\frac{3}{5}$ d. $3\frac{1}{7}$ f. 15

2. Separate these fractions into 3 sets as follows:

   Set M is the set of fractions which name whole numbers.

   Set P is the set of fractions which can be expressed as mixed forms.

   Set R is the set of fractions which name numbers less than 1.

   a. $\frac{30}{5}$ d. $\frac{75}{8}$ g. $\frac{32}{5}$

   b. $\frac{56}{4}$ e. $\frac{120}{10}$ h. $\frac{4}{9}$

   c. $\frac{72}{8}$ f. $\frac{127}{10}$ i. $\frac{54}{9}$

3. Suppose you have a whole number scale and a scale labeled in fractions on the same number line. Between what two whole number points will a point lie which is labeled with these fractions?

   a. $\frac{7}{3}$ c. $\frac{12}{5}$ e. $\frac{38}{7}$

   b. $\frac{9}{4}$ d. $\frac{19}{8}$ f. $\frac{83}{4}$

4. Express these mixed forms as fractions. Then find the indicated sums and addends.

a. $\frac{3}{5} + 4\frac{1}{5} = n$

c. $3\frac{1}{7} = 2\frac{1}{2} + n$

b. $8\frac{2}{3} - \frac{7}{4} = n$

d. $6\frac{1}{5} + 1\frac{3}{10} = n$

5. Which number of each pair is greater? Answer in a sentence, using $>$ or $<$.

a. $\frac{17}{8}$, $2\frac{3}{8}$

d. $9\frac{7}{10}$, $\frac{48}{5}$

b. $7\frac{1}{5}$, $\frac{31}{4}$

e. $14\frac{1}{2}$, $\frac{42}{3}$

c. $6\frac{3}{7}$, $\frac{45}{8}$

f. $\frac{39}{13}$, $2\frac{8}{13}$

RENAMING FRACTIONS IN MIXED FORM

Exploration

When a number is named by a mixed form, such as $2\frac{3}{7}$, you know how to rename it in fraction form.

$$\begin{aligned} 2\frac{3}{7} &= \frac{2}{1} + \frac{3}{7} \\ &= \frac{2 \times 7}{1 \times 7} + \frac{3}{7} \\ &= \frac{14}{7} + \frac{3}{7} \\ &= \frac{17}{7} \end{aligned}$$

$$2\frac{3}{7} = \frac{17}{7}$$

When a number is named by a fraction, you can easily rename it in mixed form when the numerator and denominator are small enough to use the number facts you know and to think about the points on the number line.

$$\begin{aligned} \frac{15}{4} &= \frac{12 + 3}{4} \\ &= \frac{12}{4} + \frac{3}{4} \\ &= 3 + \frac{3}{4} \\ &= 3\frac{3}{4} \end{aligned}$$

$$\frac{15}{4} = 3\frac{3}{4}$$

Let us see how you can rename a number when the numerator and denominator are greater; for example, $\frac{437}{16}$.

Since a fraction names a whole number when the numerator is a multiple of the denominator, think about the set of multiples of 16.

Multiples of 16 = {16, 32, 48, 64 ...}

RENAMING FRACTIONS IN MIXED FORM

Exploration

When a number is named by a mixed form, such as $2\frac{3}{7}$, you know how to rename it in fraction form.

$$
\begin{aligned}
2\tfrac{3}{7} &= \frac{2}{1} + \frac{3}{7} \\
&= \frac{2 \times 7}{1 \times 7} + \frac{3}{7} \\
&= \frac{14}{7} + \frac{3}{7} \\
&= \frac{17}{7} \\
2\tfrac{3}{7} &= \frac{17}{7}
\end{aligned}
$$

When a number is named by a fraction, you can easily rename it in mixed form when the numerator and denominator are small enough to use the number facts you know and to think about the points on the number line.

$$
\begin{aligned}
\frac{15}{4} &= \frac{12 + 3}{4} \\
&= \frac{12}{4} + \frac{3}{4} \\
&= 3 + \frac{3}{4} \\
&= 3\tfrac{3}{4} \\
\frac{15}{4} &= 3\tfrac{3}{4}
\end{aligned}
$$

Let us see how you can rename a number when the numerator and denominator are greater; for example, $\frac{437}{16}$.

Since a fraction names a whole number when the numerator is a multiple of the denominator, think about the set of multiples of 16.

Multiples of 16 = {16, 32, 48, 64 ...}

You would have to find a good many multiples of 16 to find a multiple close to 437. So try another way to find a multiple close to 437.

Suppose you write 437 in the form

$$437 = (16 \times n) + r, \text{ where } r < 16.$$

You know you can find n and r by using division. So 437 can be renamed as follows:

$437 = (16 \times 27) + 5$

$$\begin{array}{r} 27 \\ 16\,\overline{)\,437} \\ \underline{32} \\ 117 \\ \underline{112} \\ 5 \end{array} \qquad \begin{array}{r} 16 \\ \underline{\times\, 27} \\ 112 \\ \underline{32} \\ 432 \end{array}$$

Explain why these sentences are true:

a) $\frac{437}{16} = \frac{(16 \times 27) + 5}{16}$

b) $\phantom{\frac{437}{16}} = \frac{432 + 5}{16}$

c) $\phantom{\frac{437}{16}} = \frac{432}{16} + \frac{5}{16}$

d) $\phantom{\frac{437}{16}} = 27 + \frac{5}{16}$

e) $\phantom{\frac{437}{16}} = 27\frac{5}{16}$

f) $\frac{437}{16} = 27\frac{5}{16}$

In line b) is it necessary to write (16×27) as 432? In c), you could write

$$\begin{aligned} \frac{437}{16} &= \frac{16 \times 27}{16 \times 1} + \frac{5}{16} \\ &= \frac{27}{1} + \frac{5}{16} \quad \text{(Why?)} \\ &= 27\frac{5}{16}. \end{aligned}$$

Find mixed form names for these numbers. Write your work in the way shown above.

1. $\frac{97}{13}$

2. $\frac{147}{23}$

Exercise Set 23

Rename these numbers in simplest mixed form or as whole numbers. Show your work.

1. $$\frac{34}{5} = \frac{(5 \times 6) + 4}{5}$$
$$= \frac{5 \times 6}{5 \times 1} + \frac{4}{5}$$
$$= 6 + \frac{4}{5}$$
$$= 6\frac{4}{5}$$
$$\frac{34}{5} = 6\frac{4}{5}$$

2. $\frac{79}{8}$

3. $\frac{96}{11}$

4. $\frac{157}{13}$

5. $\frac{241}{15}$

6. $\frac{352}{7}$

7. $\frac{238}{10}$

8. $\frac{367}{12}$

9. $\frac{367}{36}$

10. $\frac{451}{100}$

11. $\frac{5280}{3}$

Find simplest mixed forms to make these sentences true.

12. 50 ounces $= \frac{50}{16}$ pounds = _____ pounds.

13. 100 feet $= \frac{100}{3}$ yards = _____ yards.

14. 254 inches $= \frac{254}{12}$ feet = _____ feet.

15. 37 pints $= \frac{37}{8}$ gallons = _____ gallons.

16. Fill in the blanks in the table.

	Fraction	Simplest Fraction Name	Mixed Form
a.	_____	_____	$7\frac{6}{8}$
b.	$\frac{49}{10}$	_____	_____
c.	$\frac{13}{11}$	_____	_____
d.	$\frac{720}{25}$	_____	_____
e.	_____	_____	$15\frac{8}{21}$
f.	_____	_____	$22\frac{7}{10}$
g.	$\frac{79}{9}$	_____	_____

COMPUTING WITH MIXED FORMS

When numbers are expressed in mixed forms, you can add and subtract them without finding fraction names for them.

$$4\frac{1}{2} + 7\frac{3}{4} = n$$

a) $4\frac{1}{2} + 7\frac{3}{4} = (4 + \frac{1}{2}) + (7 + \frac{3}{4})$

b) $= 4 + (\frac{1}{2} + 7) + \frac{3}{4}$

c) $= 4 + (7 + \frac{1}{2}) + \frac{3}{4}$

d) $= (4 + 7) + (\frac{1}{2} + \frac{3}{4})$

e) $= 11 + (\frac{2}{4} + \frac{3}{4})$

f) $= 11 + \frac{5}{4}$

g) $= 11 + (\frac{4}{4} + \frac{1}{4})$

h) $= 11 + (1 + \frac{1}{4})$

i) $= (11 + 1) + \frac{1}{4}$

j) $= 12 + \frac{1}{4}$

k) $= 12\frac{1}{4}$

Explain each line.

You do not need to write all this to show your work.

For example, write:

$$\begin{aligned} 4\frac{1}{2} + 7\frac{3}{4} &= (4 + 7) + (\frac{1}{2} + \frac{3}{4}) \\ &= 11 + (\frac{2}{4} + \frac{3}{4}) \\ &= 11 + \frac{5}{4} \\ &= 11 + 1 + \frac{1}{4} \\ &= 12\frac{1}{4} \end{aligned}$$

You may prefer this form:

$$\begin{array}{rcccl}
4\frac{1}{2} & = & 4+\frac{1}{2} & = & 4+\frac{2}{4} \\
\underline{7\frac{3}{4}} & & & = & \underline{7+\frac{3}{4}} \\
 & & & & 11+\frac{5}{4} = 11+1+\frac{1}{4} = 12\frac{1}{4}
\end{array}$$

Or this:

$$\begin{array}{rcl}
4\frac{1}{2} & = & 4\frac{2}{4} \\
\underline{+\ 7\frac{3}{4}} & = & \underline{7\frac{3}{4}} \\
 & & 11\frac{5}{4} = 12\frac{1}{4}.
\end{array}$$

The vertical form is best for subtraction. Explain these examples.

$$\begin{array}{rcl}
8\frac{5}{6} & = & 8\frac{10}{12} \\
\underline{-\ 3\frac{1}{4}} & = & \underline{3\frac{3}{12}} \\
 & & 5\frac{7}{12}
\end{array}$$

$$\begin{array}{rcccccl}
7\frac{1}{6} & = & 7\frac{2}{12} & = & 6+1\frac{2}{12} & = & 6+\frac{14}{12} \\
-\ 2\frac{3}{4} & = & 2\frac{9}{12} & & & & \underline{2+\frac{9}{12}} \\
 & & & & & & 4+\frac{5}{12} = 4\frac{5}{12}
\end{array}$$

$$\begin{array}{rcccl}
15 & = & 14+1 & = & 14+\frac{8}{8} \\
\underline{-\ 2\frac{5}{8}} & & & = & \underline{2+\frac{5}{8}} \\
 & & & & 12+\frac{3}{8} = 12\frac{3}{8}
\end{array}$$

Exercise Set 24

Find a fraction name or mixed form for n so each mathematical sentence is true. Show your work in the form your teacher suggests.

1. a. $4\frac{2}{3} + 2\frac{2}{3} = n$ b. $3\frac{2}{5} - 2\frac{1}{5} = n$ c. $1\frac{3}{8} + 2\frac{3}{8} = n$

2. a. $2\frac{1}{3} + 3\frac{1}{3} = n$ b. $3\frac{5}{6} - 1\frac{5}{6} = n$ c. $3\frac{1}{5} + 2\frac{2}{5} = n$

3. a. $2\frac{1}{2} + 3\frac{3}{8} = n$ b. $3\frac{1}{2} + 2\frac{2}{5} = n$ c. $1\frac{3}{4} - 1\frac{1}{6} = n$

4. a. $2\frac{3}{5} - 1\frac{1}{3} = n$ b. $4\frac{1}{2} - 2\frac{2}{3} = n$ c. $1\frac{7}{8} - 1\frac{1}{3} = n$

5. a. $3\frac{1}{2} + 4\frac{1}{2} = n$ b. $5\frac{2}{3} - 4\frac{1}{3} = n$ c. $1\frac{7}{12} - 1\frac{2}{12} = n$

6. a. $4\frac{4}{5} - 3\frac{1}{2} = n$ b. $2\frac{1}{2} + 1\frac{7}{16} = n$ c. $3\frac{3}{4} + 2\frac{5}{8} = n$

7. a. $13 - 7\frac{5}{8} = n$ b. $8 - 2\frac{5}{16} = n$ c. $25 - \frac{4}{9} = n$

8. a. $12 - 7\frac{5}{8} = n$ b. $9 - 2\frac{5}{16} = n$ c. $18 - \frac{5}{4} = n$

9. a. $15\frac{7}{9} - 8 = n$ b. $15 - 8\frac{7}{9} = n$ c. $36 - \frac{11}{8} = n$

Exercise Set 25

Copy and subtract. Exercise 1 (a) is done for you.

1. a. $3\frac{6}{8} = 2 + \frac{14}{8}$
   $\underline{1\frac{7}{8} = 1 + \frac{7}{8}}$
   $1 + \frac{7}{8} = 1\frac{7}{8}$

   b. $3\frac{3}{5}$
   $\underline{1\frac{4}{5}}$

   c. $4\frac{1}{6}$
   $\underline{2\frac{5}{6}}$

2. a. $8\frac{3}{8}$
   $\underline{4\frac{5}{8}}$

   b. $7\frac{5}{12}$
   $\underline{6\frac{7}{12}}$

   c. $8\frac{1}{3}$
   $\underline{5\frac{2}{3}}$

3. a. $7\frac{1}{4}$
   $\underline{3\frac{3}{4}}$

   b. $9\frac{3}{10}$
   $\underline{3\frac{7}{10}}$

   c. $12\frac{2}{6}$
   $\underline{3\frac{5}{6}}$

4. a. $5\frac{3}{8}$
   $\underline{1\frac{7}{8}}$

   b. $9\frac{1}{6}$
   $\underline{3\frac{5}{6}}$

   c. $4\frac{3}{16}$
   $\underline{2\frac{9}{16}}$

5. a. $14\frac{1}{12}$
   $\underline{9\frac{7}{12}}$

   b. $9\frac{3}{10}$
   $\underline{5\frac{9}{10}}$

   c. $11\frac{7}{16}$
   $\underline{8\frac{9}{16}}$

6. a. $5\frac{3}{8}$
   $\underline{1\frac{5}{8}}$

   b. $4\frac{1}{12}$
   $\underline{2\frac{5}{12}}$

   c. $4\frac{3}{10}$
   $\underline{2\frac{9}{10}}$

7. BRAINTWISTER: Find n so each mathematical sentence is true.

   a. $(11\frac{7}{8} - 4\frac{2}{8}) - 2\frac{3}{8} = n$

   b. $(9\frac{7}{12} - 3\frac{1}{12}) + n = 10\frac{11}{12}$

ESTIMATING SUMS OF RATIONAL NUMBERS

Exploration

When you are adding rational numbers, it is a good idea to estimate the sum first.

1) Consider the sentence:

$$3\frac{4}{5} + 7\frac{2}{3} = n$$

Between what two consecutive whole numbers would each addend be?

a. $3\frac{4}{5} >$ _____ and $3\frac{4}{5} <$ _____

b. $7\frac{2}{3} >$ _____ and $7\frac{2}{3} <$ _____

Are these statements true?

c. $3\frac{4}{5} + 7\frac{2}{3} > 3 + 7$

d. $3\frac{4}{5} + 7\frac{2}{3} < 4 + 8$

e. The sum of $3\frac{4}{5}$ and $7\frac{2}{3}$ is a number between 10 and 12.

2) Between what two consecutive whole numbers is

a. $\frac{17}{2}$?

b. $\frac{13}{4}$?

c. The sum of $\frac{17}{2}$ and $\frac{11}{4}$ must be a number greater than _____ and less than _____.

Exercise Set 26

Which of the answers below <u>may</u> be right? Which ones <u>must</u> be wrong? Answer by finding two consecutive whole numbers between which the sum must be.

1. $\frac{7}{8} + 1\frac{3}{4} = \frac{29}{8}$
2. $\frac{11}{3} + \frac{5}{6} = 4\frac{1}{2}$
3. $\frac{13}{2} + 7\frac{3}{4} = 15\frac{1}{4}$
4. $14\frac{4}{5} + 6\frac{9}{10} = 20\frac{7}{10}$
5. $\frac{25}{7} + \frac{14}{3} = 6\frac{5}{21}$
6. $\frac{21}{8} + \frac{19}{4} = \frac{59}{8}$

Between what two consecutive whole numbers must each sum be?

7. $2\frac{3}{4} + 3\frac{1}{2}$
8. $7\frac{8}{9} + 14\frac{2}{3}$
9. $10\frac{7}{10} + 12\frac{4}{5}$
10. $128\frac{5}{6} + 73\frac{1}{8}$
11. $64\frac{3}{8} + 19\frac{3}{4}$
12. $89\frac{1}{7} + 15\frac{9}{10}$

<u>Might</u> these addends be correct?

13. $5\frac{3}{7} - 2\frac{1}{2} = 2\frac{5}{14}$ (Think: If $2\frac{5}{14} + 2\frac{1}{2} = n$, then $n >$ _____ and $n <$ _____)
14. $12\frac{7}{12} - 5\frac{7}{8} = 4\frac{9}{16}$
15. $15 - 6\frac{3}{8} = 9\frac{3}{8}$

Exercise Set 27

1. Robert needs $\frac{22}{3}$ feet of new cord to reach from his desk lamp to a wall outlet. The hardware store sells lamp cord in no smaller divisions than the foot. How long a piece of lamp cord will Robert have to buy?

2. Joan's family leaves on a trip at noon. The time required for the round trip is $\frac{13}{2}$ hours. At what time will they be back?

3. Suppose a man finds that to paint the outside of his house he will need about 17 quarts of paint. The paint he needs is sold only in gallons. How much paint will he need to buy?

4. Driving time from Boston to New York is $\frac{9}{2}$ hours. Driving from New York to Philadelphia requires $\frac{7}{4}$ hours. How long does it take to drive from Boston to Philadelphia by way of New York?

A magic square is one in which you can perform the operation on the numbers vertically, horizontally or diagonally and always get the same number for a result.

5. Copy the square below.

   a. Add the numbers named by the fractions in each column and record the sum for each column.

   b. Add the numbers named by the fractions in each row and record the sum for each row.

   c. Begin in lower left hand corner. Add the numbers named by the fractions diagonally. Record their sum.

   d. Begin in upper left hand corner. Add the numbers named by the fractions diagonally. Record their sum.

   e. Is each sum the same rational number? What is the number?

   f. Is the square a magic square?

$2\frac{1}{2}$	$3\frac{3}{8}$	$\frac{1}{2}$	$1\frac{3}{8}$	$2\frac{1}{4}$
$3\frac{1}{4}$	1	$1\frac{1}{4}$	$2\frac{1}{8}$	$2\frac{3}{8}$
$\frac{7}{8}$	$1\frac{1}{8}$	2	$2\frac{7}{8}$	$3\frac{1}{8}$
$1\frac{5}{8}$	$1\frac{7}{8}$	$2\frac{3}{4}$	3	$\frac{3}{4}$
$1\frac{3}{4}$	$2\frac{5}{8}$	$3\frac{1}{2}$	$\frac{5}{8}$	$1\frac{1}{2}$

6. Copy the square below. Write fractions in A, B, C, D, E, and F to make it a magic square whose sum is 5. (Recall that a "magic square" is one in which the sum of the numbers named in a row, a column, or on a diagonal is the same number. This number is called the "sum" for the square.)

A	B	$1\frac{11}{12}$
C	D	E
$1\frac{5}{12}$	F	$\frac{11}{12}$

THINKING ABOUT DECIMALS

Exploration

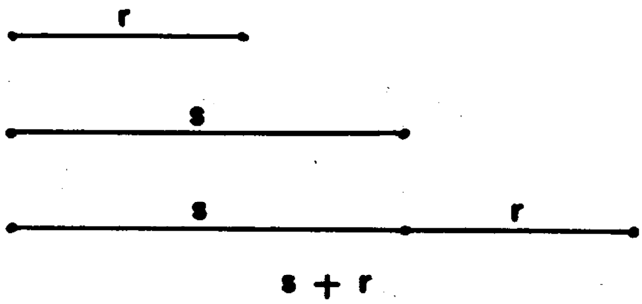
If you had grown up in France, you would say

"Mon frere est plus grand que moi,"

instead of "My brother is taller than I."

Both sentences express the same relation. A French pupil does not have to know the English language to understand the relation we call "taller". We do not have to speak French to understand this relation. But to understand the idea in the French sentence we would have to <u>translate</u> it into English. As soon as we learned to understand French well, we would not have to translate French sentences. We would think in French.

Our problem with rational numbers is very much like this. We know a meaning for addition. If s and r are rational numbers, then s + r can be pictured as measures in this way:



We also know how to express addition relations in the "language" of fractions, and in mixed form "language".

$$\frac{8}{5} + \frac{37}{10} = \frac{16}{10} + \frac{37}{10} = \frac{53}{10}$$

$$1\frac{3}{5} + 3\frac{7}{10} = 1 + 3 + \frac{6}{10} + \frac{7}{10} = 4 + \frac{13}{10} = 5\frac{3}{10}.$$

But these are not the only languages we use for rational numbers. A very common language is the decimal numeral system. We already know how to "translate" some fractions into decimals. For example:

$$\frac{6}{10} = .6, \quad \frac{5}{10} = .5.$$

In fraction language we can express this addition relation:

$$\frac{6}{10} + \frac{5}{10} = \frac{11}{10}.$$

Can we translate this sentence into decimal language?

We only have to translate $\frac{11}{10}$. To do this we first find a mixed form expression.

$$\frac{11}{10} = \frac{10 + 1}{10} = \frac{10}{10} + \frac{1}{10} = 1 + \frac{1}{10} = 1\frac{1}{10} = 1.1$$

Now we can write the decimal sentence:

$$.6 + .5 = 1.1$$

We can always use this method, but it can be long. Here is another example:

What is the decimal name for .38 + .75 ?

First we translate: $.38 = \frac{38}{100}, \quad .75 = \frac{75}{100}.$

We compute: $\frac{38}{100} + \frac{75}{100} = \frac{113}{100}.$

We translate to mixed form: $\frac{113}{100} = 1\frac{13}{100}.$

We translate back: $1\frac{13}{100} = 1.13$

In decimal language:

$$.38 + .75 = 1.13.$$

Our problem is this. Can we learn to express addition relations in decimal language without translating to fractions and back? Yes, we can. It is really very easy. We can begin to learn how by thinking of the meaning of decimals.

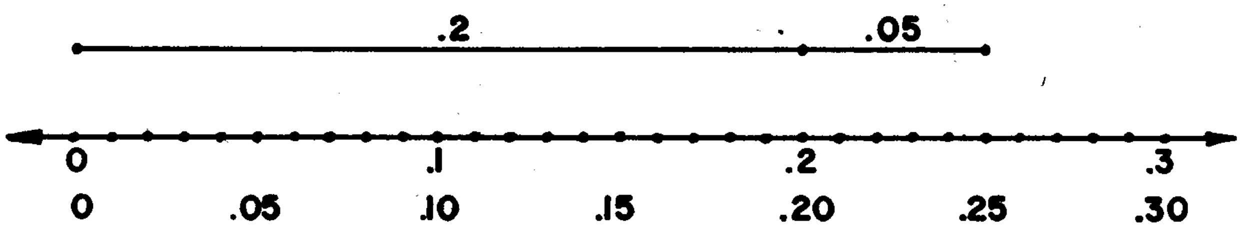
It helped us in thinking about decimal numerals for whole numbers to write an expanded form. Now we will write it this way:

$$\begin{aligned} 246 &= 200 + 40 + 6 \\ &= 2 \text{ hundreds} + 4 \text{ tens} + 6 \text{ ones}. \end{aligned}$$

Can we think of <u>all</u> decimals in the same way? Can we think of .25 and 8.4 and 1.06 in this way? We know

$$\begin{aligned} .25 = \frac{25}{100} &= \frac{20}{100} + \frac{5}{100} \\ &= \frac{2}{10} + \frac{5}{100} \\ &= .2 + .05 \\ &= 2 \text{ tenths} + 5 \text{ hundredths}. \end{aligned}$$

A number line diagram helps us to understand why this is so. Look at this number line, marked with a tenths scale and a hundredths scale.



Now consider 8.4. We know: $8.4 = 8\frac{4}{10} = 8 + \frac{4}{10} = 8 + .4$.

We know $1.06 = 1\frac{6}{100} = 1 + \frac{6}{100} = 1 + .06$.

Write these in expanded form:

1. 38.47

2. 160.13

3. 57.06

We can express the meaning of decimal numerals in many ways.

27.38 = 2 tens + 7 ones + 3 tenths + 8 hundredths
= 27 ones + 38 hundredths
= 270 tenths + 38 hundredths
= 2 tens + 7 ones + 2 tenths + 18 hundredths
= 20 + 7 + .3 + .08

2738 = 2 thousands + 7 hundreds + 3 tens + 8 ones
= 27 hundreds + 38 ones
= 272 tens + 18 ones
= 2000 + 700 + 30 + 8

2.738 = 2 ones + 7 tenths + 3 hundredths + 8 thousandths
= 27 tenths + 38 thousandths
= 2 ones + 6 tenths + 13 hundredths + 8 thousandths
= 2 + .7 + .03 + .008

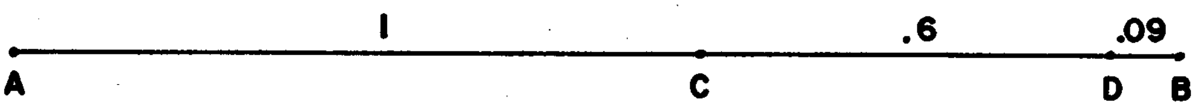
Rename these numbers four ways, as shown on the preceding page.

4. 6.84

5. 68.4

6. 70.605

7. What is the measure $\overline{AB}$ pictured below? $\overline{AC}$ has measure 1, $\overline{CD}$ has measure .6 and $\overline{DB}$ has measure .09?



What is the decimal for each of these?

8. 5 tens + 6 ones + 4 tenths + 3 hundredths?

9. 5 tenths + 6 tens + 4 ones + 13 hundredths?

10. 6 tens + 6 hundredths?

Translate to decimals:

11. $\frac{567}{1000}$

12. $\frac{567}{100}$

13. $\frac{567}{10}$

14. $28\frac{34}{100}$

15. $40\frac{6}{1000}$

Exercise Set 28

1. Complete:

   a. $72.9 =$ ____ ones and ____ tenths and ____ tens.

   b. $702.09 =$ ____ hundreds + ____ ones + ____ tenths + ____ tens + ____ hundredths.

   c. $68.75 =$ ____ ones + ____ hundredths.

   d. $52\frac{16}{100} =$ ____ ones + ____ hundredths.

   e. $400 + 5 + \frac{15}{100} =$ ____ hundreds + ____ tens + ____ ones + ____ tenths + ____ hundredths.

2. Write the decimal for each of these.

   a. 5 hundreds and 5 hundredths

   b. 43 tens + 16 hundredths

   c. 2 tens + 7 hundreds + 6 ones

   d. 14 hundredths + 6 tenths + 3 ones

   e. 12 hundredths + 9 tenths.

3. Write the decimal for each of these.

   a. $\frac{14}{10}$

   b. $6\frac{3}{10}$

   c. $\frac{5243}{1000}$

   d. $\frac{5243}{10}$

   e. $\frac{6840}{10}$

   f. $20\frac{6}{100}$

4. Express in dollars the value of:

   a. 4 ten dollar bills, 8 dimes, and 6 one dollar bills.

   b. 15 one dollar bills, 12 pennies, and 7 dimes.

   c. 253 pennies.

   d. 8 one dollar bills and 58 pennies.

BRAINTWISTERS:

5. Translate to decimals:

   a. $\frac{4}{25}$

   b. $6\frac{7}{20}$

   c. 3 fives + 20 fifths + 2 twenty-fifths

   d. $\frac{7}{8}$

6. Which is greater?

   a. .33 or $\frac{1}{3}$?

   b. .125 or $\frac{1}{8}$?

   c. .166 or $\frac{1}{6}$?

ADDITION OF RATIONAL NUMBERS USING DECIMALS

Exploration

Now we are ready to find a quick way to add or subtract rational numbers using decimal names.

Suppose we want to add .12 and .34. We could translate to fractions:

$$
\begin{aligned}
.12 + .34 &= \frac{12}{100} + \frac{34}{100} \\
&= \frac{12 + 34}{100} \\
&= \frac{46}{100} \\
&= .46
\end{aligned}
$$

But we could also remember this:

$$
\begin{aligned}
.12 + .34 &= (.1 + .02) + (.3 + .04) \\
&= (.1 + .3) + (.02 + .04)
\end{aligned}
$$

What properties of addition have we used?

Now $.1 + .3 = 1$ tenth $+ 3$ tenths $= 4$ tenths,

$.02 + .04 = 2$ hundredths $+ 4$ hundredths $= 6$ hundredths.

In decimals:

$$.1 + .3 = .4$$

$$.02 + .04 = .06$$

We have, then,

$$
\begin{aligned}
.12 + .34 &= (.1 + .02) + (.3 + .04) \\
&= (.1 + .3) + (.02 + .04) \\
&= .4 + .06 \\
&= .46
\end{aligned}
$$

This is not a new method. The place value idea is used for tenths and hundredths. We have already used this idea for places to the left of the decimal point.

Here is an example with numerals on both sides of the decimal point.

$$\begin{aligned} 16.31 + 43.52 &= (10 + 6 + .3 + .01) + (40 + 3 + .5 + .02) \\ &= (10 + 40) + (6 + 3) + (.3 + .5) + (.01 + .02) \\ &= 50 + 9 + .8 + .03 \\ &= 59.83 \end{aligned}$$

We can use the vertical form to make the computations easier:

$$\begin{aligned} 16.31 &= 10 + 6 + .3 + .01 \\ + 43.52 &= \underline{40 + 3 + .5 + .02} \\ & \quad 50 + 9 + .8 + .03 = 59.83 \end{aligned}$$

Use the vertical form to compute:

(1) $18.5 + 31.4$ (2) $72.06 + 15.43$

How should we think of problems like these?

(a) $.7 + .8$ (b) $.06 + .09$

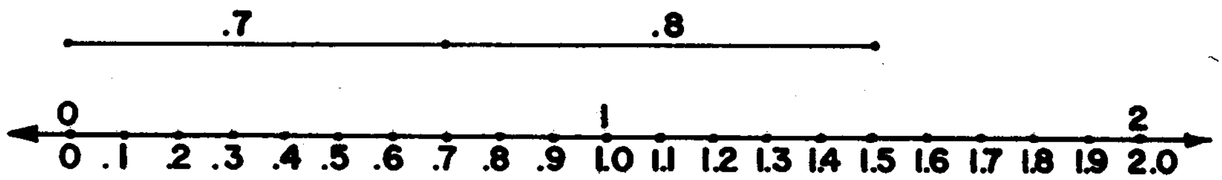
(a) We know: $.7 + .8 = 7$ tenths + 8 tenths = 15 tenths

= 10 tenths + 5 tenths

= 1 one + 5 tenths

= 1.5

You could picture $.7 + .8$ on the number line.



(b) We know: $.06 + .09 = 6 \text{ hundredths} + 9 \text{ hundredths}$

$$\begin{aligned} &= 15 \text{ hundredths} \\ &= 10 \text{ hundredths} + 5 \text{ hundredths} \\ &= 1 \text{ tenth} + 5 \text{ hundredths} \\ &= .15 \end{aligned}$$

This is the same idea, regrouping in sets of ten, that we have used many times in problems like:

$80 + 70 = 150$ or $600 + 900 = 1500.$

There is one thing new. We must be very careful to locate the <u>decimal point</u> correctly.

Now we can do problems like these:

$$\begin{array}{rl} 14.56 = & 10 + 4 + .5 + .06 \\ + \; 27.25 = & \underline{20 + 7 + .2 + .05} \end{array}$$

$$\begin{aligned} 30 + 11 + .7 + .11 &= 30 + 11 + (.7 + .1) + .01 \\ &= (30 + 10) + 1 + .8 + .01 \\ &= 40 + 1 + .8 + .01 \\ &= 41.81 \end{aligned}$$

Try these examples:

(3) $\begin{array}{r} 6.37 \\ \underline{+\;3.24} \end{array}$ (4) $\begin{array}{r} 20.08 \\ \underline{+\;7.39} \end{array}$ (5) $\begin{array}{r} 0.87 \\ \underline{+\;0.76} \end{array}$

Our method can be shown in the vertical form you used for whole numbers:

$$\begin{array}{r} 14.56 \\ \underline{27.25} \\ .11 \\ .7 \\ 11. \\ \underline{30.} \\ 41.81 \end{array}$$

(Why did we mark the decimal point?)

Of course this can be shortened by "remembering."

Here are the steps:

$$\begin{array}{r} 14.56 \\ \underline{27.25} \end{array}$$

(1) Add hundredths. Write 1 hundredth, remember 1 tenth.

$$\begin{array}{r} 14.56 \\ \underline{27.25} \\ 1 \end{array}$$

(2) Add tenths. Write 8 tenths. Mark the decimal point.

$$\begin{array}{r} 14.56 \\ \underline{27.25} \\ .81 \end{array}$$

(3) Add ones. Write 1 one and remember 1 ten.

$$\begin{array}{r} 14.56 \\ \underline{27.25} \\ 1.81 \end{array}$$

(4) Add tens. Write 4 tens.

$$\begin{array}{r} 14.56 \\ \underline{27.25} \\ 41.81 \end{array}$$

Here is an example with more remembering. Only the long way is shown:

$$\begin{array}{r} 17.67 \\ \underline{+\ 8.34} \\ .11 \\ .9 \\ 15. \\ \underline{10.} \\ 26.01 \end{array}$$

Exercise Set 29

Find fraction or decimal names for the following sums. The forms suggested below may be used.

$$
\begin{aligned}
2\tfrac{3}{8} + 3\tfrac{2}{8} &= (2 + \tfrac{3}{8}) + (3 + \tfrac{2}{8}) \\
&= (2 + 3) + (\tfrac{3}{8} + \tfrac{2}{8}) \\
&= 5 + \tfrac{5}{8} \\
&= 5\tfrac{5}{8}
\end{aligned}
$$

$$
\begin{aligned}
4.5 + 3.3 &= (4 + .5) + (3 + .3) \\
&= (4 + 3) + (.5 + .3) \\
&= 7 + .8 \\
&= 7.8
\end{aligned}
$$

1. (a) $3\frac{3}{5} + 4\frac{1}{5}$ (b) $8\frac{4}{8} + 3\frac{3}{8}$ (c) $2\frac{2}{4} + 1\frac{1}{4}$

2. (a) $1\frac{1}{4} + 6\frac{4}{6}$ (b) $8\frac{5}{12} + 6\frac{6}{12}$ (c) $4\frac{5}{16} + 5\frac{9}{16}$

3. (a) $2.5 + 4.2$ (b) $8.6 + 3.3$ (c) $5.4 + 8.5$

4. (a) $5.3 + 6.6$ (b) $8.4 + 4.4$ (c) $7.7 + 3.2$

Try to work these mentally. Write the answers only

5. (a) $8\frac{3}{5} + 5$ (b) $7 + 6\frac{2}{8}$ (c) $8 + 5\frac{5}{12}$

6. (a) $8.6 + 4.2$ (b) $9.7 + 6.2$ (c) $8.3 + 3.4$

BRAINTWISTER

7. Find n so each mathematical sentence is true.

(a) $(4\frac{1}{6} + 3\frac{2}{6}) + 5\frac{2}{6} = n$

(b) $(8.7 + 6.9) + 3.5 = n$

(c) $(6\frac{2}{8} + \frac{3}{8}) + n = 10\frac{7}{8}$

(d) $(11.2 + 8.7) + 1.8 = n$

Exercise Set 30

Copy and find the sums. The form of exercises 1(a) and 1(b) may be used.

1. (a)

$$\begin{array}{r} 2\frac{3}{10} = 2 + \frac{3}{10} \\ \underline{5\frac{8}{10}} = \underline{5 + \frac{8}{10}} \\ 7 + \frac{11}{10} = 8\frac{1}{10} \end{array}$$

(b)

$$\begin{array}{r} 2.3 = 2 + .3 \\ \underline{5.8} = \underline{5 + .8} \\ 7 + 1.1 = 8.1 \end{array}$$

2. (a) $\begin{array}{r} 5\frac{3}{8} \\ \underline{4\frac{7}{8}} \end{array}$ (b) $\begin{array}{r} 4\frac{5}{6} \\ \underline{3\frac{4}{6}} \end{array}$ (c) $\begin{array}{r} 5\frac{3}{4} \\ \underline{4\frac{7}{8}} \end{array}$ (d) $\begin{array}{r} 7\frac{2}{4} \\ \underline{6\frac{3}{4}} \end{array}$

3. (a) $\begin{array}{r} 5.8 \\ \underline{6.3} \end{array}$ (b) $\begin{array}{r} 6.4 \\ \underline{8.9} \end{array}$ (c) $\begin{array}{r} 8.7 \\ \underline{9.9} \end{array}$ (d) $\begin{array}{r} 8.5 \\ \underline{6.8} \end{array}$

4. (a) $\begin{array}{r} 8.6 \\ \underline{9.5} \end{array}$ (b) $\begin{array}{r} 6\frac{3}{5} \\ \underline{7\frac{4}{5}} \end{array}$ (c) $\begin{array}{r} 8\frac{7}{12} \\ \underline{6\frac{10}{12}} \end{array}$ (d) $\begin{array}{r} 9.8 \\ \underline{8.9} \end{array}$

Copy these examples and add. Write only the answers on your paper.

5. (a) $\begin{array}{r} 4.5 \\ \underline{3.3} \end{array}$ (b) $\begin{array}{r} 5.7 \\ \underline{4.2} \end{array}$ (c) $\begin{array}{r} 8.3 \\ \underline{9.5} \end{array}$ (d) $\begin{array}{r} 5.6 \\ \underline{4.9} \end{array}$

6. (a) $\begin{array}{r} 4.7 \\ \underline{4.6} \end{array}$ (b) $\begin{array}{r} 5.4 \\ \underline{6.9} \end{array}$ (c) $\begin{array}{r} 7.6 \\ \underline{4.8} \end{array}$ (d) $\begin{array}{r} 4.5 \\ \underline{2.7} \end{array}$

Exercise Set 31

1. Compute:

a. $\begin{array}{r} 25.06 \\ +\ 37.84 \\ \hline \end{array}$

b. $\begin{array}{r} 108.07 \\ +\ 467.94 \\ \hline \end{array}$

c. $\begin{array}{r} 117.6 \\ +\ 38.74 \\ \hline \end{array}$

d. $\begin{array}{r} .58 \\ +\ 15.09 \\ \hline \end{array}$

e. $\begin{array}{r} .847 \\ +\ .138 \\ \hline \end{array}$

f. $\begin{array}{r} 3.707 \\ +\ 2.988 \\ \hline \end{array}$

2. Compute the sum in any language:

Example:

$$4\tfrac{1}{2} + 6.7$$

$$\begin{array}{r} 4.5 \\ +\ 6.7 \\ \hline 11.2 \end{array} \qquad \text{or} \qquad \begin{array}{r} 4\frac{1}{2} \\ 6\frac{7}{10} \\ \hline 10\frac{12}{10} = 11\frac{1}{5} \end{array}$$

a. $23\frac{1}{2} + \frac{7}{4}$

b. $15\frac{1}{4} + 16.7$

c. $\frac{3}{100} + 18.57$

d. $15\frac{7}{8} + 18\frac{1}{3}$

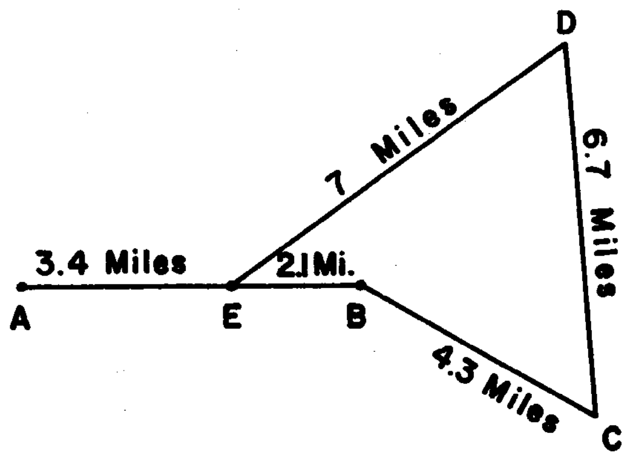
e. $6\frac{1}{4} + 7.18 + \frac{2}{5}$

The way we name money values in dollars is really a decimal numeral and symbol, the dollar sign, which indicates the unit.

$12.98 is usually read "twelve dollars and ninety-eight cents," but it could just as well be read "twelve and ninety-eight one hundredths dollars."

3. Stores often sell things at prices like $1.98 or $.49. If you bought something for $2.98 and something for $1.69, could you pay for them with a 5 dollar bill?

4. Here is a map showing a short trip Ellen's family took in a car. They went from A (home) to B to C to D to E, and back to A. How far did they travel?



SUBTRACTION OF RATIONAL NUMBERS USING DECIMALS

Exploration

Do you remember how you found a process for subtracting whole numbers using decimal notation? (Recall that "decimal" means "base ten.")

To get the decimal numeral for

$$237 - 145$$

you thought this way.

$$\begin{array}{rcl} 237 & = & 200 + 30 + 7 \\ -\ 145 & = & \underline{100 + 40 + 5} \\ & & ? + ? + 2 \end{array}$$

Then you thought

$$\begin{array}{rcl} 237 & = & 100 + 130 + 7 \\ -\ 145 & = & \underline{100 + \ 40 + 5} \\ & & \quad 90 + 2 = 92 \end{array}$$

Can we think this way if our problem is

$$2.37 - 1.45 = ?$$

$$\begin{array}{rcl} 2.37 = 2 + .3 + .07 & = & 1 + 1.3 + .07 \\ 1.45 = 1 + .4 + .05 & = & \underline{1 + \ .4 + .05} \\ & & \quad .9 + .02 = .92 \end{array}$$

Here we thought of 1.3 as 13 tenths. 13 tenths - 4 tenths = 9 tenths. We see that the idea is exactly the same. Here is one more example:

$$3.08 - 1.9 = ?$$

$$\begin{array}{rcl} 3.08 = 3 + .0 + .08 & = & 2 + 1.0 + .08 \\ 1.9 \ = 1 + .9 + .00 & = & \underline{1 + \ .9 + .00} \\ & & 1 + \ .1 + .08 = 1.18 \end{array}$$

Here it helped us to think of .08 as (.0 + .08) and .9 as (.9 + .00).

Try these examples. Write your work as shown on the preceding page.

1) 9.25 - 4.13 2) 18.36 - 2.5 3) 8.46 - 3.59

We can shorten this method if we <u>think</u> but do not write all of the steps. Here is one way:

$$\begin{array}{r} 3.08 \\ \underline{-\ 1.9} \end{array}$$

a) Write this as:

$$\begin{array}{r} 3.08 \\ \underline{-\ 1.90} \end{array}$$

b) Subtract hundredths

$$\begin{array}{r} 3.08 \\ \underline{-\ 1.90} \\ 8 \end{array}$$

c) Think 2 + 1.0 for 3.0. Write this in tenths if you need to:

$$\begin{array}{r} 2\ \ 10 \\ \cancel{3}.\cancel{0}8 \\ \underline{-\ 1.90} \\ 8 \end{array}$$

d) Subtract tenths. 10 tenths - 9 tenths = 1 tenth. <u>Mark the decimal point</u>.

$$\begin{array}{r} 2\ \ 10 \\ \cancel{3}.\cancel{0}8 \\ \underline{-\ 1.90} \\ .18 \end{array}$$

e) Subtract ones:

$$\begin{array}{r} 2\ \ 10 \\ \cancel{3}.\cancel{0}8 \\ \underline{-\ 1.90} \\ 1.18 \end{array}$$

Try these examples the short way.

4) 7.38 - 5.2 5) 12.49 - 8.62 6) 10.37 - 4.59

Exercise Set 32

Copy and subtract. Exercise 1(a) is done for you.

1. (a) 5.6 = 4 + 16 tenths
   3.7 = <u>3 + 7 tenths</u>
   1 + 9 tenths = 1.9

	(b)	(c)
	4.7	5.8
	<u>1.9</u>	<u>2.9</u>

2.

(a)	(b)	(c)	(d)
5.6	8.1	7.6	8.5
<u>2.9</u>	<u>4.7</u>	<u>3.7</u>	<u>2.9</u>

Try to do these subtractions mentally. Write only the results for exercises 3 through 5.

3.

(a)	(b)	(c)	(d)
7.1	8.7	7.4	8.3
<u>3.9</u>	<u>2.9</u>	<u>3.5</u>	<u>4.4</u>

4.

(a)	(b)	(c)	(d)
8.2	9.0	18.2	9.3
<u>3.4</u>	<u>3.6</u>	<u>6.5</u>	<u>5.8</u>

5.

(a)	(b)	(c)	(d)
9.8	8.6	7.5	8.3
<u>7.9</u>	<u>6.8</u>	<u>3.8</u>	<u>5.6</u>

6. BRAINTWISTER. Fill in the squares so the sum of each row and column is the same number.

(a)

2.4		1.2
	3.0	
4.8		3.6

(b)

.8	.1	
	.5	
	.9	

Exercise Set 33

Copy and subtract. Exercise 1(a) is done for you.

1. (a) $7.83 = 7 + .7 + .13$
   $\underline{5.35 = 5 + .3 + .05}$
   $2 + .4 + .08 = 2.48$

   (b) $\begin{array}{r} 2.48 \\ \underline{1.09} \end{array}$

2. (a) $\begin{array}{r} 6.85 \\ \underline{2.49} \end{array}$ (b) $\begin{array}{r} 7.74 \\ \underline{3.37} \end{array}$ (c) $\begin{array}{r} 9.96 \\ \underline{4.37} \end{array}$ (d) $\begin{array}{r} 8.86 \\ \underline{3.57} \end{array}$

3. (a) $\begin{array}{r} 7.61 \\ \underline{3.36} \end{array}$ (b) $\begin{array}{r} 8.94 \\ \underline{2.78} \end{array}$ (c) $\begin{array}{r} 5.50 \\ \underline{4.37} \end{array}$ (d) $\begin{array}{r} 9.72 \\ \underline{3.69} \end{array}$

Subtract these mentally. Write just the answers.

4. (a) $\begin{array}{r} 7.34 \\ \underline{3.28} \end{array}$ (b) $\begin{array}{r} 8.92 \\ \underline{3.47} \end{array}$ (c) $\begin{array}{r} 9.71 \\ \underline{3.58} \end{array}$ (d) $\begin{array}{r} 8.54 \\ \underline{6.39} \end{array}$

5. (a) $\begin{array}{r} 9.65 \\ \underline{3.39} \end{array}$ (b) $\begin{array}{r} 8.47 \\ \underline{4.38} \end{array}$ (c) $\begin{array}{r} 9.83 \\ \underline{4.56} \end{array}$ (d) $\begin{array}{r} 7.81 \\ \underline{4.64} \end{array}$

6. Subtract

   (a) $\begin{array}{r} 8.34 \\ \underline{4.83} \end{array}$ (b) $\begin{array}{r} 9.28 \\ \underline{7.85} \end{array}$ (c) $\begin{array}{r} 8.32 \\ \underline{4.58} \end{array}$ (d) $\begin{array}{r} 9.34 \\ \underline{5.89} \end{array}$

BRAINTWISTER

7. Find n so each of the following is a true mathematical sentence.

   (a) $\frac{5}{6 \times 2} + \frac{8}{4 \times 3} = n$ (c) $\frac{4}{2 \times 8} + \frac{7}{4 + 4} = n$

   (b) $\frac{4 + 3}{4 + 1} + \frac{3}{3 + 2} = n$ (d) $\frac{5 \times 3}{8 + 2} + \frac{3 + 4}{5 \times 5} = n$

Exercise Set 34

You have learned these two methods to express $2.52 + 5.46$ as a decimal numeral.

$$\begin{aligned} 2.52 &= 2\tfrac{52}{100} \\ 5.46 &= 5\tfrac{46}{100} \\ \hline &\ \ 7\tfrac{98}{100} = 7.98 \end{aligned}$$

$$\begin{aligned} 2.52 &= 2 + .5 + .02 \\ 5.46 &= \underline{5 + .4 + .06} \\ &\ \ 7 + .9 + .08 = 7.98 \end{aligned}$$

You have learned these two methods to express $5.84 - 3.32$ as a decimal numeral.

$$\begin{aligned} 5.84 &= 5\tfrac{84}{100} \\ 3.32 &= 3\tfrac{32}{100} \\ \hline &\ \ 2\tfrac{52}{100} = 2.52 \end{aligned}$$

$$\begin{aligned} 5.84 &= 5 + .8 + .04 \\ 3.32 &= \underline{3 + .3 + .02} \\ &\ \ 2 + .5 + .02 = 2.52 \end{aligned}$$

Use two methods to add in exercise 1.

1. (a) $\begin{array}{r} 5.63 \\ \underline{+\ 2.34} \end{array}$ (b) $\begin{array}{r} 6.35 \\ \underline{+\ 3.44} \end{array}$ (c) $\begin{array}{r} 7.24 \\ \underline{+\ 8.33} \end{array}$ (d) $\begin{array}{r} 5.56 \\ \underline{+\ 3.33} \end{array}$

Use two methods to subtract in exercise 2.

2. (a) $\begin{array}{r} 6.24 \\ -\ 3.12 \\ \hline \end{array}$ (b) $\begin{array}{r} 8.69 \\ -\ 4.35 \\ \hline \end{array}$ (c) $\begin{array}{r} 7.87 \\ -\ 5.36 \\ \hline \end{array}$ (d) $\begin{array}{r} 8.86 \\ -\ 4.24 \\ \hline \end{array}$

Write only your answers for exercises 3 and 4. Do your work mentally.

3. (a) $\begin{array}{r} 4.62 \\ +\ 3.26 \\ \hline \end{array}$ (b) $\begin{array}{r} 8.36 \\ +\ 3.43 \\ \hline \end{array}$ (c) $\begin{array}{r} 5.21 \\ +\ 3.47 \\ \hline \end{array}$ (d) $\begin{array}{r} 6.54 \\ +\ 3.35 \\ \hline \end{array}$

4. (a) $\begin{array}{r} 6.57 \\ -\ 3.35 \\ \hline \end{array}$ (b) $\begin{array}{r} 8.79 \\ -\ 5.34 \\ \hline \end{array}$ (c) $\begin{array}{r} 9.68 \\ -\ 4.35 \\ \hline \end{array}$ (d) $\begin{array}{r} 8.89 \\ -\ 3.67 \\ \hline \end{array}$

5. BRAINTWISTER: Find n so each mathematical sentence is true.

(a) $(8.97 - 4.31) + n = 11.89$

(b) $11.89 - n = 8.97 - 4.31$

Exercise Set 35

You have shown your work for renaming $6.28 + 3.57$ as a decimal numeral in two ways.

$$\begin{aligned} 6.28 &= 6 + .2 + .08 \\ \underline{3.57} &= \underline{3 + .5 + .07} \\ &= 9 + .7 + .15 \\ &= 9 + .7 + .1 + .05 \\ &= 9.85 \end{aligned}$$

$$\begin{array}{r} 6.28 \\ \underline{3.57} \\ .15 \\ .70 \\ \underline{9.00} \\ 9.85 \end{array}$$

Use both methods to find the sums in exercise 1.

1. (a) $\begin{array}{r} 3.49 \\ \underline{+\ 2.38} \end{array}$ (b) $\begin{array}{r} 5.64 \\ \underline{+\ 3.18} \end{array}$ (c) $\begin{array}{r} 6.28 \\ \underline{+\ 3.47} \end{array}$ (d) $\begin{array}{r} 6.29 \\ \underline{+\ 4.38} \end{array}$

Write only the sums for exercises 2 and 3.

2. (a) $\begin{array}{r} 6.29 \\ \underline{5.38} \end{array}$ (b) $\begin{array}{r} 3.42 \\ \underline{4.48} \end{array}$ (c) $\begin{array}{r} 7.23 \\ \underline{3.58} \end{array}$ (d) $\begin{array}{r} 8.25 \\ \underline{3.37} \end{array}$

3. (a) $\begin{array}{r} 7.34 \\ \underline{5.37} \end{array}$ (b) $\begin{array}{r} 8.23 \\ \underline{4.39} \end{array}$ (c) $\begin{array}{r} 9.34 \\ \underline{2.59} \end{array}$ (d) $\begin{array}{r} 8.38 \\ \underline{5.52} \end{array}$

4. BRAINTWISTERS: Find these sums. Write only the decimal numeral for the sum.

(a) $\begin{array}{r} 3.24 \\ 3.56 \\ \underline{4.16} \end{array}$ (b) $\begin{array}{r} 4.46 \\ 3.32 \\ \underline{5.51} \end{array}$ (c) $\begin{array}{r} 5.36 \\ 4.75 \\ \underline{2.83} \end{array}$ (d) $\begin{array}{r} 7.88 \\ 5.31 \\ \underline{6.54} \end{array}$

Exercise Set 36

1. Compute. Show your work the long way.

   (a) 15.27 - 4.81 (b) 3.75 - 0.28 (c) 28.75 - 13.86

2. Compute:

   (a) $\begin{array}{r} 86.23 \\ -\ 57.70 \\ \hline \end{array}$

   (b) $\begin{array}{r} 862.3 \\ -\ 57.7 \\ \hline \end{array}$

   (c) 90 - 67.86

   (d) $\begin{array}{r} 50.09 \\ -\ \ 4.91 \\ \hline \end{array}$

   (e) $\begin{array}{r} .125 \\ +\ .375 \\ \hline \end{array}$

   (f) 75 - 7.83

   (g) Translate each addend in (e) to a fraction in simplest form. Compute in fractions.

3. Compute these repeated sums. Use the long way if you cannot remember.

   (a) $\begin{array}{r} 7.08 \\ 38.92 \\ +\ 16.60 \\ \hline \end{array}$

   (b) $\begin{array}{r} .73 \\ 38. \\ +\ \ 6.94 \\ \hline \end{array}$

   (c) $\begin{array}{r} 16.67 \\ 3.04 \\ .08 \\ +\ 6.20 \\ \hline \end{array}$

4. How much change should you get from a 10 dollar bill if you bought things costing \$3.98, \$1.49, \$.98, and \$1.69?

5. The population of the United States was 131.669 million in 1940, 150.697 million in 1950 and 179.323 million in 1960. Did the population increase more between 1940 and 1950 or between 1950 and 1960?

6. Here are the populations of the 5 largest cities in the United States in 1960. Was the part of the United States population which lives in these cities more or less than $\frac{1}{10}$ of the total population?

New York	7.78 million
Chicago	3.55 million
Los Angeles	2.48 million
Philadelphia	2.00 million
Detroit	1.67 million

REVIEW

Exercise Set 37

1. What number n will make each mathematical sentence true?

(a) $\frac{1}{4} = \frac{n}{8}$ (c) $\frac{3}{4} = \frac{n}{8}$ (e) $\frac{7}{8} = \frac{14}{n}$

(b) $\frac{1}{2} = \frac{n}{12}$ (d) $\frac{n}{4} = \frac{12}{16}$ (f) $\frac{5}{n} = \frac{10}{16}$

2. Copy and write " > ", " < ", or " = " in each blank so each mathematical sentence is true.

(a) $\frac{1}{3}$ ___ $\frac{1}{8}$ (c) $\frac{1}{9}$ ___ $\frac{1}{5}$ (e) $\frac{2}{3}$ ___ $\frac{3}{4}$

(b) $\frac{1}{7}$ ___ $\frac{1}{5}$ (d) $\frac{2}{3}$ ___ $\frac{12}{18}$ (f) $\frac{9}{8}$ ___ $\frac{4}{3}$

3. Which would you rather have?

(a) $\frac{1}{4}$ or $\frac{1}{8}$ of a pie (c) $\frac{1}{4}$ or $\frac{1}{3}$ of a candy bar

(b) $\frac{1}{4}$ or $\frac{1}{5}$ of a dollar (d) $\frac{1}{8}$ or $\frac{1}{6}$ of a watermelon

4. Find a fraction name for n so each mathematical sentence is true.

(a) $\frac{1}{2} + n = \frac{5}{8}$ (c) $\frac{1}{4} + n = \frac{3}{12} + \frac{7}{12}$

(b) $\frac{1}{2} + n = \frac{3}{6} + \frac{2}{6}$ (d) $n + \frac{1}{2} = \frac{3}{6} + \frac{4}{6}$

5. Which fractions name numbers greater than the number 1?

$\frac{1}{4}, \frac{7}{4}, \frac{3}{8}, \frac{19}{16}, \frac{9}{10}, \frac{12}{10}, \frac{15}{9}, \frac{8}{9}, \frac{4}{6}, \frac{6}{5}$

6. Find the simplest fraction name for each member.

(a) $\frac{16}{24}$ (c) $\frac{18}{24}$ (e) $\frac{4}{20}$ (g) $\frac{14}{18}$ (i) $\frac{14}{21}$

(b) $\frac{10}{20}$ (d) $\frac{12}{15}$ (f) $\frac{16}{20}$ (h) $\frac{24}{30}$ (j) $\frac{10}{16}$

7. Rename each number in mixed form.

(a) $\frac{16}{9}$ (c) $\frac{8}{3}$ (e) $\frac{17}{12}$

(b) $\frac{12}{7}$ (d) $\frac{19}{5}$ (f) $\frac{13}{5}$

8. Copy each statement below. For each missing numerator or denominator write a numeral so each mathematical sentence is true.

(a) $\frac{8}{12} = \frac{}{3} = \frac{}{6} = \frac{}{18} = \frac{}{24}$

(b) $\frac{9}{15} = \frac{}{20} = \frac{}{35} = \frac{6}{} = \frac{15}{}$

(c) $\frac{3}{18} = \frac{}{12} = \frac{4}{} = \frac{5}{} = \frac{}{36}$

9. Which of these fractions are other names for $\frac{1}{2}$?

$\frac{6}{18}$	$\frac{10}{25}$	$\frac{4}{5}$	$\frac{5}{10}$	$\frac{7}{5}$	$\frac{8}{6}$	$\frac{2}{4}$
$\frac{7}{8}$	$\frac{8}{16}$	$\frac{6}{12}$	$\frac{13}{21}$	$\frac{6}{4}$	$\frac{3}{5}$	$\frac{4}{8}$
$\frac{14}{28}$	$\frac{3}{6}$	$\frac{8}{5}$	$\frac{7}{14}$	$\frac{9}{18}$	$\frac{5}{7}$	$\frac{12}{8}$

10. Which fractions in exercise 9 are other names for $\frac{9}{6}$?

11. Which pairs of numbers below can be named by fractions with a common denominator of 24 ?

(a) $\frac{1}{2}$ and $\frac{4}{3}$? (d) $\frac{10}{3}$ and $\frac{1}{8}$?

(b) $\frac{4}{6}$ and $\frac{4}{7}$? (e) $\frac{7}{6}$ and $\frac{3}{8}$?

(c) $\frac{7}{4}$ and $\frac{4}{7}$? (f) $\frac{5}{2}$ and $\frac{7}{9}$?

Exercise Set 38

1. There were $2\frac{2}{3}$ qt. vanilla ice cream and $3\frac{1}{2}$ qt. chocolate ice cream in the freezer. How much ice cream was in the freezer?

2. Mary's mother made two costumes for a school play. One costume took $3\frac{2}{3}$ yards material and the other costume took $2\frac{1}{6}$ yards. How much material did Mary's mother buy?

3. Dick's weight was $56\frac{3}{4}$ lb. in June. At the end of vacation he weighed $59\frac{1}{2}$ lb. How much weight did he gain?

4. Mr. Long noticed that the odometer in his car showed 8523.4 miles when he bought some gas. During the day he traveled 49.3 miles. What did his odometer read at the end of the day?

5. Bruce wanted to buy a sleeping bag for a Scout camping trip that was priced \$26.95. Bruce's father gave him \$5 to help pay the cost. Bruce had saved \$7.35. How much more money does Bruce need to buy the sleeping bag?

6. Gerry was ill one day and her mother took her temperature in the morning. The thermometer read 99.8°. Later in the day Gerry's fever increased and the thermometer read 102.6°. How many degrees did her fever increase?

7. Tell whether each of these mathematical sentences is an example of the commutative or the associative properties for addition.

(a) $\frac{2}{3} + (\frac{1}{4} + \frac{3}{4}) = (\frac{2}{3} + \frac{1}{4}) + \frac{3}{4}$

(b) $\frac{3}{12} + \frac{2}{8} = \frac{2}{8} + \frac{3}{12}$

(c) $(\frac{8}{9} + \frac{12}{3}) + \frac{1}{6} = \frac{8}{9} + (\frac{12}{3} + \frac{1}{6})$

(d) $(\frac{1}{3} + \frac{1}{4}) + \frac{2}{3} = (\frac{1}{4} + \frac{1}{3}) + \frac{2}{3}$

(e) $(\frac{8}{9} + \frac{4}{5}) + \frac{1}{2} = \frac{8}{9} + (\frac{4}{5} + \frac{1}{2})$

8. In his butcher shop, Mr. Fisher had some bologna in chunks. On Monday he sold $2\frac{7}{8}$ lb. The next day he sold $\frac{3}{4}$ lb. On Wednesday he sold $3\frac{1}{2}$ lb.

Use the above information to complete problems (a) through (d).

(a) How many pounds of bologna did Mr. Fisher sell in the three days?

(b) How much less than 10 lb. was sold?

(c) How much bologna was sold on Tuesday and Wednesday?

(d) The total number pounds of bologna sold on the last two days is how many more than the number of pounds sold on the first two days? Express your answer in simplest form.

Exercise Set 39

1. The fastest pitched ball on record traveled 98.6 m.p.h. When a hocky player strikes a puck, the puck travels about 98.0 m.p.h. Which traveled faster? How much?

2. A recent census showed that out of every 100 people in South Carolina 36.7 lived in towns and cities, and 63.3 lived in rural communities. Out of every 100 people, how many more lived in rural communities?

3. The flight time of Explorer III was 115.87 minutes. That of Explorer I was 114.8. What is the difference in the two flight times?

4. In George Washington's time, .90 of the American people could not read or write. Today only about .05 of the American people cannot read and write. What part of the people could read and write in Washington's time? In our time?

5. The Simplon Tunnel between Italy and Switzerland is 12.3 miles long. The Cascade Tunnel in Washington is 7.8 miles long. How much longer is the Simplon Tunnel?

6. In 1950, statistics showed that the population per square mile in California was 66.7 persons. In 1940 it had been 43.7 persons. On the average, how many more people lived on a square mile in 1950?

7. The Moosehead Lake in Maine has an area of 116.98 square miles. The area of Lake Mead in Nevada is 228.83 square miles. Which lake has the greater area? How much greater?

8. The length of a day on Mars is 24.5 hours. The length of a day on Neptune is 15.7 hours. How much longer is a day on Mars?

9. The distance from Earth to Cygnus is 10.6 light years. The distance from Earth to Sirius is 8.6 light years. Which star is closer to Earth? How much closer?

JUST FOR FUN

Make a square like this

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

In each small square, write the answer of the example below having the same numeral as the square. If your work is correct, the sum of the numbers of each row and column will be the same number.

1. 6.50 + 1.50
2. 4.75 - 1.25
3. 1.8 + 1.7
4. 96 ÷ 48
5. .25 + .75
6. 8.00 - 2.50
7. 1.75 + 1.75
8. 63 ÷ 9
9. .70 + .80
10. 4.85 + .15
11. 5.37 - 2.37
12. 6 + 1.50
13. 7.7 - 1.2
14. 108 ÷ 36
15. 13.76 - 6.76
16. 7.25 - 6.75

Exercise Set 40

1. Below are a number of steps showing $\frac{1}{2} + \frac{2}{3} = \frac{2}{3} + \frac{1}{2}$

   State a reason for each step. Let $n = \frac{1}{2} + \frac{2}{3}$

Then	$n = \frac{3}{6} + \frac{4}{6}$	Step 1:
	$= \frac{3+4}{6}$	Step 2:
	$= \frac{4+3}{6}$	Step 3:
	$= \frac{4}{6} + \frac{3}{6}$	Step 4:
	$= \frac{2}{3} + \frac{1}{2}$	Step 5:

2. Write steps like those in exercise 1 to show that $\frac{1}{3} + \frac{3}{4} = \frac{3}{4} + \frac{1}{3}$.

3. Study this sentence: $(\frac{7}{4} - n) - 1 = \frac{7}{4} - (n + 1)$. Is this sentence true?

   (a) if $n = \frac{3}{4}$? (b) if $n = \frac{1}{4}$? (c) if $n = \frac{0}{4}$?

4. In each sentence what number does n represent if the mathematical sentence is true? Be careful. There may be <u>no</u> answer, <u>one</u> answer, or <u>more than one</u> answer.

   (a) $\frac{3}{4} + n = \frac{5}{4}$ (d) $n = n$ (g) $3.4 + n = n$

   (b) $\frac{7}{4} + n = \frac{21}{10}$ (e) $n + \frac{5}{4} = \frac{5}{4} + n$ (h) $n = 9.17 + n$

   (c) $1\frac{2}{3} + n = 3\frac{1}{4}$ (f) $11.6 + n = 11.6$ (g) $6.432 + n = n$

5. (a) What number is n if $n + n = \frac{14}{5}$?

(b) What number is n if $n + n = 3.94$?

6. Think of x, y and z as representing rational numbers. Suppose $x + y = z$.

(a) If $x = \frac{15}{4}$ and $z = \frac{15}{4}$, what number is y?

(b) Can x be greater than z? Why?

(c) If $y = 8.94$ and $z = 8.94$, what number is x?

7. For each of the sentences below, n is a fraction name for a rational number. Make n have a denominator of 2. Find n if each mathematical sentence is true.

(a) $n < \frac{1}{2}$

(b) n is less than $\frac{9}{2}$ and greater than $\frac{7}{2}$

(c) n is greater than $\frac{13}{4}$ and less than $\frac{15}{4}$

(d) The sum of n and $\frac{7}{2}$ is less than $\frac{8}{2}$

8. One mathematical sentence showing a relationship among numbers n, 7 and 12 is $n + 7 = 12$. Write <u>two</u> mathematical sentences showing different relationships among n, 1.75, and 4.25. Make them so that for each mathematical sentence, n represents a different number.

9. Two numbers to be subtracted are represented by n and n. (The two numbers are the same.) John said the result of the subtraction is $\frac{10}{3}$. Was John correct? Why?

10. What rational number n will make each mathematical sentence true?

(a) $(\frac{3}{2} + \frac{7}{2}) + n = \frac{11}{2}$

(b) $(\frac{3}{2} + \frac{7}{2}) + \frac{11}{2} = n$

(c) $n - (\frac{3}{2} + \frac{7}{2}) = \frac{11}{2}$

(d) $(1.35 + 6.47) + n = 8.92$

(e) $(1.35 + 6.47) + 8.92 = n$

(f) $n - (1.35 + 6.47) = 8.92$

11. Sometimes you have more than two numbers to add. You may be able to make the exercise simpler by changing the order of the addends. For example, think about

$$\frac{5}{3} + \frac{1}{4} + \frac{2}{3} + \frac{3}{4} + \frac{5}{3}$$

You may think $(\frac{5}{3} + \frac{2}{3} + \frac{5}{3}) + (\frac{1}{4} + \frac{3}{4}) = 4 + 1 = 5.$

Find these sums. Change the order of the addends if you think it will make the computation easier.

(a) $2 + \frac{2}{3} + 3 + \frac{2}{3} + \frac{2}{3}$

(b) $\frac{1}{2} + \frac{1}{4} + \frac{7}{4} + \frac{3}{2} + \frac{4}{4}$

(c) $\frac{8}{5} + \frac{7}{6} + \frac{3}{5} + \frac{9}{5} + \frac{11}{6}$

Practice Exercises

I. Solve for n .

a) $54,982 + n = 80,000$

b) $300,678 + 27,492 = n$

c) $658 \times 319 = n$

d) $n \times 85 = 4,080$

e) $36 \times n = 2,700$

f) $2,340 \div n = 36$

g) $6\frac{5}{8} + 7\frac{1}{8} = n$

h) $n + 8\frac{1}{5} = 11\frac{3}{10}$

i) $\frac{3}{4} + \frac{2}{3} + \frac{5}{6} = n$

j) $185 \times 85 = n$

k) $2\frac{3}{5} - \frac{4}{5} = n$

l) $32 \times n = 5028$

m) $2\frac{2}{5} + \frac{1}{3} + 1\frac{1}{5} = n$

n) $77 \times 34618 = n$

o) $2\frac{1}{6} - \frac{2}{3} = n$

p) $4\frac{1}{3} + 3\frac{5}{6} + n = 10$

q) $n - 8\frac{1}{2} = \frac{5}{12}$

r) $n = 1\frac{9}{10} + 6\frac{2}{5} + 3\frac{1}{2}$

s) $4,006 \times 78 = n$

t) $\frac{7}{8} + n = 4\frac{1}{2}$

II. Solve for n .

a) $12\frac{1}{4} - n = 8\frac{1}{2}$

b) $\frac{5}{8} + \frac{1}{2} = \frac{1}{2} + n$

c) $\frac{7}{8} - \frac{1}{2} = \frac{1}{4} + n$

d) $\frac{4}{5} + \frac{3}{10} + \frac{2}{5} = n$

e) $5\frac{7}{10} - n = 2\frac{2}{3}$

f) $2.45 + .7 + 3.05 = n$

g) $248.09 + n = 388.6$

h) $\frac{0}{4} + \frac{3}{4} = \frac{3}{4} + n$

i) $n + 1\frac{1}{3} = 2\frac{1}{2}$

j) $3,354 \div n = 39$

k) $747.314 - 288.405 = n$

l) $\frac{3}{5} + \frac{1}{2} + n = 1\frac{1}{2}$

m) $954 \times 384 = n$

n) $11\frac{3}{8} + n = 14\frac{1}{3}$

o) $6,403 \div 80 = n$

p) $52,871 \div n = 91$

q) $n \times 53 = 3,498$

r) $37,062 \div 46 = n$

s) $48,369 \times 789 = n$

t) $4\frac{1}{3} + 5\frac{1}{2} + 3\frac{1}{4} = n$

III. Add:

a)
$$\begin{array}{r} 904 \\ 652 \\ 2,909 \\ \underline{45} \end{array}$$

b)
$$\begin{array}{r} 28,796 \\ 29 \\ 8,583 \\ \underline{61,312} \end{array}$$

c)
$$\begin{array}{r} 3\frac{1}{5} \\ 5\frac{3}{5} \\ 2\frac{2}{5} \\ \underline{\frac{4}{5}} \end{array}$$

d)
$$\begin{array}{r} 2\frac{5}{6} \\ \frac{1}{6} \\ 9\frac{1}{6} \\ \underline{3\frac{5}{6}} \end{array}$$

e)
$$\begin{array}{r} 2\frac{1}{2} \\ 4 \\ 6\frac{2}{3} \\ \underline{3\frac{1}{6}} \end{array}$$

Subtract:

f)
$$\begin{array}{r} 5,934 \\ \underline{2,046} \end{array}$$

g)
$$\begin{array}{r} 17,004 \\ \underline{5,280} \end{array}$$

h)
$$\begin{array}{r} 11\frac{7}{10} \\ \underline{4\frac{3}{10}} \end{array}$$

i)
$$\begin{array}{r} 3\frac{1}{2} \\ \underline{1\frac{1}{3}} \end{array}$$

j)
$$\begin{array}{r} 26\frac{1}{3} \\ \underline{9\frac{5}{6}} \end{array}$$

Multiply:

k)
$$\begin{array}{r} 508 \\ \underline{67} \end{array}$$

l)
$$\begin{array}{r} 369 \\ \underline{26} \end{array}$$

m)
$$\begin{array}{r} 348 \\ \underline{58} \end{array}$$

n)
$$\begin{array}{r} 957 \\ \underline{39} \end{array}$$

o)
$$\begin{array}{r} 5,836 \\ \underline{47} \end{array}$$

Divide:

p) $18\overline{)426}$

q) $42\overline{)4,309}$

r) $41\overline{)2,352}$

s) $37\overline{)1,591}$

t) $18\overline{)1,674}$

IV. Find the sum:

a) 264,829 ; 78,080 ; 196,809 ; 19,998

b) 132,435 ; 412,754 ; 216 ; 734,646

c) $28\frac{1}{3}$; $8\frac{1}{2}$; $17\frac{1}{4}$; $6\frac{2}{3}$; 19

d) 4,027.9 ; 617.26 ; 503.07 ; .8

e) $219\frac{5}{8}$; $1,726\frac{1}{4}$; $63\frac{2}{3}$; $109\frac{3}{8}$

V. Subtract:

a) 678,543 - 254,745

b) 800,096 - 173,295

c) 128,791 - 37,782

d) $52,096\frac{3}{8} - 29,636\frac{3}{4}$

e) 212,983 - 31,006

VI. Multiply:

a) 27,465 x 697

b) 379,865 x 756

c) 36,492 x 489

d) 843,476 x 654

e) 81,918 x 248

VII. Divide:

a) 85,591 ÷ 95

b) 34,997 ÷ 34

c) 87,600 ÷ 67

d) 801,356 ÷ 89

e) 457,267 ÷ 74

Review

SET I

Part A

1. Complete each of the following to make it a true statement illustrating the distributive property.

   Example: $12 \times (20 + 15) + (12 \times 20) + (12 \times 15)$

   a) $(40 + 5) \times 22 = (\_\_ \times 22) + (5 \times \_\_)$

   b) $154 \div 7 = (\_\_ + 14) \div 7$

   c) $468 \times 15 = (424 \times 15) + (\_\_ \times 15)$

   d) $1{,}824 \div 10 = (\_\_\_ \div 10) + (800 \div \_\_) + (\_\_ \div 10) + 4$

   e) $63 \times 34 = (60 + \_\_) \times (\_\_ + 4)$

2. Answer <u>yes</u> or <u>no</u> to the questions below.

   a) Does $(7 \times 8) \times 3 = 7 \times (8 \times 3)$?

   b) Does $3 \times (9 \times 5) = (3 \times 9) \times 5$?

   c) Does $(36 \div 6) \div 3 = 36 \div (6 \div 3)$?

   d) Does $60 \div (30 \div 2) = (60 \div 30) \div 2$?

   e) Does $(\frac{5}{4} - \frac{2}{4}) - \frac{1}{4} = \frac{5}{4} - (\frac{2}{4} - \frac{1}{4})$?

   f) Does $\frac{6}{12} - (\frac{4}{12} - \frac{3}{12}) = (\frac{6}{12} - \frac{4}{12}) - \frac{3}{12}$?

   g) Does $(37 + 13) + 9 = 37 + (13 + 9)$?

   h) Does $26 + (32 + 10) = (26 + 32) + 10$?

   i) Does $(25 - 13) - 7 = 25 - (13 - 7)$

   j) Does $75 - (50 - 25) = (75 - 50) - 25$?

   k) Does $\frac{3}{4} + (\frac{1}{4} + \frac{2}{4}) = (\frac{3}{4} + \frac{1}{4}) + \frac{2}{4}$?

   l) Does $(\frac{2}{6} + \frac{3}{6}) + \frac{1}{6} = \frac{2}{6} + (\frac{3}{6} + \frac{1}{6})$?

   In the exercises above tell which examples illustrate the associative property.

3. Write the following expressions as decimal numerals.

Example a is done for you.

a) $7 + 1.6 + .05 = 8.65$

b) $2 + .3 + .06$

c) $5 + .2 + .17$

d) $21 + .4 + .22$

e) $9 + .8 + .23$

f) $16 + 1.6 + .16$

g) $3 + .2 + .75$

h) $61 + .3 + .81$

i) $8 + 2.5 + .52$

j) $19 + 9.7 + .36$

4. Express each answer in its simplest form.

a) $\frac{3+1}{2+2} + \frac{4+1}{3+5} = n$

b) $\frac{2+3}{1+2} - \frac{2+2}{3+3} = n$

c) $\frac{4+1}{2+2} - \frac{2+3}{3+3} = n$

d) $\frac{4+0}{1+2} + \frac{5+2}{5+7} = n$

e) $\frac{4+1}{3+3} - \frac{2+1}{4+4} = n$

f) $\frac{3}{4} + \frac{3}{8} = n$

g) $\frac{3 \times 3}{2 \times 3} - \frac{1 \times 2}{1 \times 3} = n$

h) $\frac{4+3}{4+2} + \frac{2+1}{3+1} = n$

i) $n = \frac{2 \times 1}{3 \times 1} - \frac{3 \times 1}{5 \times 2}$

j) $\frac{4}{5} + \frac{5}{8} = n$

5. Write as base ten numerals.

a) 24_{five}

b) 312_{four}

c) 64_{seven}

d) 301_{five}

e) 212_{three}

f) 43_{eight}

g) 43_{six}

h) 322_{four}

i) 441_{five}

j) 645_{seven}

6. Complete the chart below. Example a is worked for you.

Number Pair	All Factors	G C F	Multiples to L C M	L C M
a) 9	1 , 3 , 9		9, 18, 27, 36	
12	1, 2, 3, 4, 6, 12	3	12, 24, 36	36
b) 12				
18				
c) 15				
40				
d) 24				
30				
e) 18				
45				
f) 16				
28				

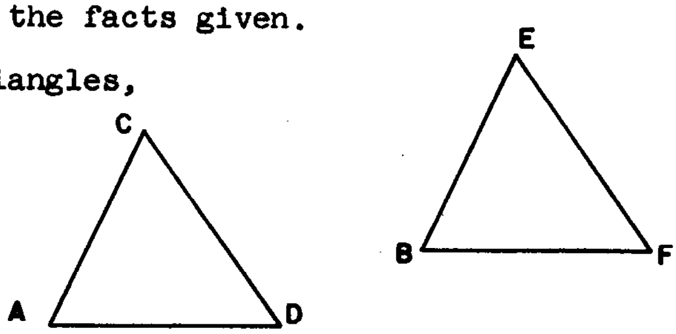
7. For your answers do not depend upon the appearance of the triangle. Use only the facts given.

a) In these two triangles, we know that

$\angle A \cong \angle B$

$\overline{AC} \cong \overline{BE}$

$\overline{AD} \cong \overline{BF}$



What do we know about $\angle D$ and $\angle F$, $\angle C$ and $\angle E$, also $\overline{CD}$ and $\overline{EF}$?

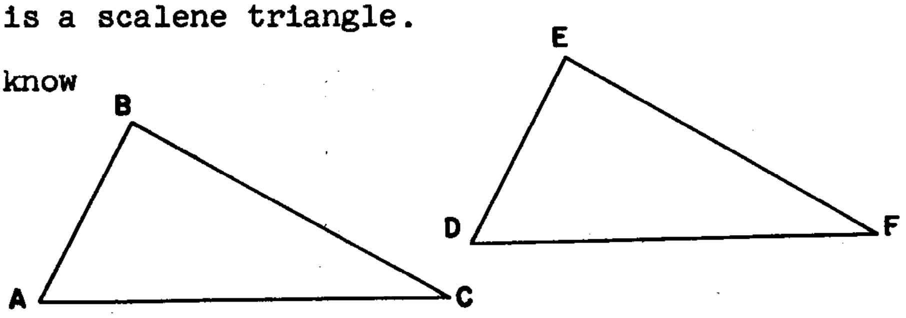
b) ABC is a scalene triangle.

We also know

$\overline{AB} \cong \overline{DE}$

$\overline{BC} \cong \overline{EF}$

$\overline{AC} \cong \overline{DF}$



What kind of a triangle would Δ DEF be?

List the pairs of congruent angles.

Part B

Write a mathematical sentence (or two sentences if necessary) for each problem and solve. Write an answer sentence.

1. A hostess made $1\frac{6}{8}$ gals. of punch for a party. She had $\frac{7}{8}$ gal. left. How much punch did the guests drink?

2. Wendy lives 8.6 mi. from her friend's house. One day she rode her horse part of the way to her friend's house. She walked the rest of the way. She walked 1.23 mi. How far did she ride before she started to walk to her friend's house?

3. In 1864 Abraham Lincoln was elected President for a second term. He received 2,216,067 votes. George McClellan ran against him and received 1,808,725 votes. How many fewer votes than Lincoln did McClellan receive?

4. When Sandra's father weighed her he said, "You weigh exactly 58 lbs. You have gained 1.8 lbs. since your birthday." How much did Sandra weigh on her birthday?

5. The Empire State Building was sold in 1951 for \$51,000,000. This is three times the amount paid for the land on which it stands. How much did the land cost?

6. One week Bill worked for $\frac{3}{4}$ hr. on Monday, $1\frac{1}{2}$ hr. on Tuesday, $\frac{1}{2}$ hr. on Wednesday, $1\frac{2}{8}$ hr. on Thursday, and 2 hrs. on Friday. He is paid 65 cents an hour. How much did he make this week?

Individual Projects

1. You have developed rules for divisibility by the numbers 2, 3 and 5 and have given examples in which they are tested as factors of a number. Here are some rules for divisibility for you to test. You should try at least five examples to see if the rule is true.

   a) A number is divisible by 4 if two times the tens digit plus the units digit is divisible by 4.

   b) A number is divisible by 6 if the number is even and is divisible by 3.

   c) A number is divisible by 7 if the difference between twice the units digit and the number formed by omitting the units digit is divisible by 7.

   d) A number is divisible by 8 if four times the hundreds digit plus two times the tens digit plus the units digit is divisible by 8.

   e) A number is divisible by 9 if the sum of the digits is divisible by 9.

Review

SET II

Part A

1. Write the numerator and denominator as the product of primes. Example a is done for you.

a) $\frac{4}{16} = \frac{2 \times 2}{2 \times 2 \times 2 \times 2}$ d) $\frac{15}{24}$

b) $\frac{12}{42}$ e) $\frac{10}{25}$

c) $\frac{63}{81}$ f) $\frac{12}{8}$

2. Select the symbols that represent zero from each row.

a) $\frac{3}{5} - \frac{6}{10}$, $421 + 0$, $0 \div 6$, $\frac{6 - 6}{3}$, 12×0

b) $\frac{8}{8}$, $\frac{4 - 4}{2 - 2}$, 0×364, $\frac{12}{8} - \frac{3}{4}$, $75 - 0$

c) 0×6, $5 - \frac{10}{2}$, $(2 \times 3) \times 0$, $\frac{3}{3}$, $\frac{8 - 8}{6}$

d) $10.6 - 10.60$, $0 + 982$, 7×0, $\frac{6 + 2}{8}$, $0 \div 12$

3. Copy and place the parentheses to make each a true statement, then solve. Example a is shown.

a) $(3 \times 5) + 7 = 20 + 2$, $22 = 22$

b) $18 \times 3 + 3 = 108$

c) $32 \div 8 + 4 = 6 + 2$

d) $44 = 50 - 12 \div 2$

e) $72 \div 4 \times 2 = 6 - 2 + 5$

f) $48 + 10 \div 2 = 6 \times 10 - 7$

g) $7 + 49 \div 7 = 12 - 2 + 4$

h) $15 \times 12 \div 6 = 24 + 4 \times 5$

i) $45 \times 24 \div 6 = 54 \times 3 + 18$

j) $31 \times 22 + 18 = 35 \times 40 \div 2$

4. Rename the following in simplest mixed form.

a) $\frac{7}{2}$

b) $\frac{14}{3}$

c) $\frac{17}{4}$

d) $\frac{15}{6}$

e) $\frac{14}{4}$

f) $\frac{21}{9}$

g) $\frac{17}{5}$

h) $\frac{18}{4}$

5. Copy and replace n with the number n represents. An example is shown.

a) $\frac{3}{4} = \frac{n}{16}$, $\frac{3}{4} = \frac{12}{16}$

b) $\frac{2}{3} = \frac{n}{15}$

c) $\frac{7}{5} = \frac{21}{n}$

d) $\frac{n}{4} = \frac{20}{16}$

e) $\frac{5}{n} = \frac{15}{24}$

f) $\frac{n}{3} \quad \frac{12}{9}$

g) $\frac{3}{6} = \frac{n}{18}$

h) $\frac{8}{7} = \frac{32}{n}$

6. Use the largest multiple of 10 , 100 , or 1,000 to make each of these true sentences. Example a is shown.

a) $9 \times \underline{7000} < 63{,}801$

b) $20 \times \underline{\qquad} < 1{,}848$

c) $4{,}328 > \underline{\qquad} \times 7$

d) $\underline{\qquad} \times 30 < 10{,}380$

e) $5{,}161 > 6 \times \underline{\qquad}$

f) $53{,}871 > \underline{\qquad} \times 8$

g) $80 \times \underline{\qquad} < 764{,}892$

h) $\underline{\qquad} \times 7 < 535$

i) $38{,}462 > 90 \times \underline{\qquad}$

j) $\underline{\qquad} \times 30 < 96{,}483$

7. Which of these numbers are equal to $1\frac{1}{2}$?

1.5 , $\frac{7}{5}$, 1.05 , $\frac{9}{6}$, $\frac{12}{8}$

Which of these numbers are equal to 3?

$\frac{12}{3}$, $2 + \frac{5}{5}$, $1\frac{8}{4}$, $\frac{24}{6}$, 2.10

Which of these numbers are equal to $\frac{3}{4}$?

$\frac{9}{12}$, $8 - 7\frac{1}{2}$, .75 , $\frac{15}{20}$, $\frac{8}{12}$

Which of these numbers are greater than $\frac{2}{3}$?

$\frac{9}{12}$, $\frac{9}{15}$, .7 , .285 , $\frac{5}{9}$

Which of these numbers are less than $\frac{5}{6}$?

.7 , $\frac{11}{12}$, .90 , $\frac{3}{4}$, .065

8. Use the word plane, line, line segment, ray, circle or quadrilateral to complete these statements.

a) A sheet of paper could be thought of as a model of a __________.

b) A clothes line stretched tightly between two poles could be thought of as a model of a ______________.

c) A wedding ring could be thought of as a model of a __________.

d) A window frame could be thought of as a model of a ______________.

e) The beam of light from a spotlight could be thought of as a model of a __________.

9. Construct two congruent triangles with sides whose lengths in inches are 3 , 2 , and 4 and whose intersection is one vertex. The interior of one triangle should be in the exterior of the other.

10. Construct two triangles with one common side. One triangle has sides whose length in inches are 5 , 3 and 3. The second triangle has sides whose lengths in inches are 2 , 2 and 3. The interior of one will be in the interior of the other.

11. Construct two triangles with one common side. One triangle has sides whose lengths in inches are 4 , 4 and 6. The second triangle has sides whose lengths in inches are 3, 3 and 4. The interior of the second triangle will be in the exterior of the first.

Part B

Write a mathematical sentence (or two sentences if necessary) for each problem and solve. Write an answer sentence.

1. Mary has $\frac{1}{2}$ c. flour. She looked at three recipes for cupcakes, one uses $\frac{7}{24}$ c. flour, another uses $\frac{2}{6}$ c. flour, and the third uses $\frac{4}{8}$ c. flour. Which recipe will use all of her flour?

2. The German bobsledding team made a trial run in 5 minutes, 07.84 seconds. The United States bobsledding team made their trial run in 5 minutes, 20.1 seconds. Which team made the faster time? How much faster?

3. A parking lot has 24 rows for cars. Each row holds 32 cars. How many cars are in the lot when the rows are filled?

4. In the Soap Box Derby John finished his run in 2.7 min. Mac finished in 2.68 min., and Terry finished in 2.07 min. What was the difference in time between the fastest and slowest runs?

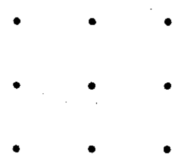
5. If the speed of a meteroid moving through space averages 30 miles per second, what will be its average speed per hour?

6. A gallon of water weighs 8.33 lbs. It is carried in a bucket weighing 1.8 lbs. What is the total weight of the gallon of water and the bucket?

Puzzles

1. What number base is used in each of these?
   a) Jan said, "My cat weighs 2 pounds, or $112_{\underline{\quad}}$ ounces."
   b) My little sister is $100_{\underline{\quad}}$ years old. In one year she will be entering the first grade.
   c) The teacher is five feet six inches or $86_{\underline{\quad}}$ inches tall.

2. Cross out every dot with four line segments. Do not lift the pencil from the paper until all nine dots are crossed. Do not retrace a line or cross any dot more than once.



Review

SET III

Part A

1. Arrange the following numbers in order from least to greatest.

   a) $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{5}$

   b) $\frac{3}{8}$, $\frac{7}{8}$, $\frac{4}{8}$, $\frac{1}{8}$, $\frac{5}{8}$

   c) .7 , 1.4 , .73 , .29 , .4

   d) $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{8}$, $\frac{1}{6}$, $\frac{1}{4}$

   e) $\frac{5}{12}$, $\frac{3}{8}$, $\frac{11}{24}$, $\frac{7}{12}$, $\frac{5}{6}$

   f) .29 , .029 , 2.9 , 29 , .5

   g) .8 , .46 , 2, .059 , .4

   h) 51 , .5 , 5.01 , 5.1 , .51

2. Using the symbol $>$, $=$, or $<$ make the following true sentences.

   a) 28×2 ____ $154 + 88$

   b) $24 \times (24 \times 6)$ ____ $6 \times (24 \times 24)$

   c) 32×127 ____ $4,060 + 24$

   d) $\frac{4}{10}$ ____ $.40$

   e) $\frac{3}{5}$ ____ $\frac{6}{12}$

   f) $31,106$ ____ 74×419

   g) 47×608 ____ $28,576$

   h) $\frac{2}{5} + \frac{1}{2} + \frac{2}{5}$ ____ 1.4

   i) $.6 + 2.15 + .25$ ____ 2.9

   j) $2,605 \div 56$ ____ $46 \text{ r } 29$

3. Write fraction names for these numbers. Example a is done for you.

a) $5\frac{1}{3} = \frac{16}{3}$

b) 1.1

c) $8\frac{3}{5}$

d) $7\frac{7}{9}$

e) 2.53

f. .73

g) $7\frac{1}{4}$

h) $25\frac{1}{2}$

i) 4.8

j) $12\frac{4}{8}$

4. Without working the problem, tell which expression in each row represents the largest number.

a) $253 \div 15$, $253 \div 26$, $253 \div 19$, $253 \div 39$

b) 341×23, 314×23, 336×23, 364×23

c) $\frac{4}{8} + \frac{1}{3}$, $\frac{4}{8} + \frac{3}{3}$, $\frac{4}{8} + \frac{4}{3}$, $\frac{4}{8} + \frac{2}{3}$

d) $1{,}192 \div 28$, $1301 \div 28$, $1{,}099 \div 28$, $1{,}900 \div 28$

e) $\frac{2}{3} - \frac{1}{2}$, $\frac{2}{3} - \frac{1}{4}$, $\frac{2}{3} - \frac{4}{6}$, $\frac{2}{3} - \frac{5}{8}$

5. In the above examples find the expression in each row that names the smallest number.

6. Write the greatest common factor for each pair.

a) 28 , 35

b) 40 , 54

c) 27 , 54

d) 18 , 60

e) 25 , 120

f) 72 , 30

g) 12 , 84

h) 42 , 70

i) 225 , 45

j) 33 , 363

7. Copy and place parentheses to make each a true statement.

a) $18 \times 23 \div 9 = 9 \times 92 \div 18$

b) $31 \times 50 \div 2 = 24 \times 33 - 17$

c) $64 \times 23 - 4 > 26 + 17 \times 43$

d) $42 \times 24 + 3 \neq 21 \times 54$

e) $43 - 9 \times 16 = 4 + 27 \times 20$

f) $36 + 18 \times 47 < 48 \times 12 + 8$

g) $27 \times 96 + 8 = 13 \times 196 + 4$

h) $65 \times 64 + 30 < 36 \times 113 + 8$

8. Use <u>closed</u> or <u>not closed</u> to complete and make these true sentences.

a) The set of whole numbers is __________ under addition.

b) The set of odd numbers is __________ under subtraction.

c) The set of counting numbers is __________ under addition.

d) The set of whole numbers is __________ under multiplication.

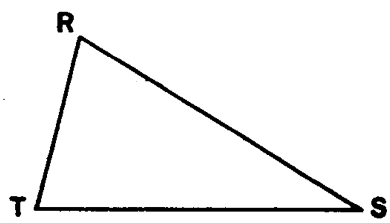
e) The set of whole numbers is __________ under division.

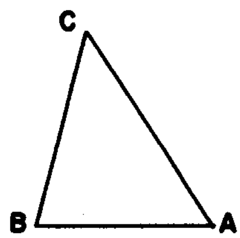
f) The set of counting numbers from 23 to 75 is __________ under addition.

g) The set of whole numbers is __________ under subtraction.

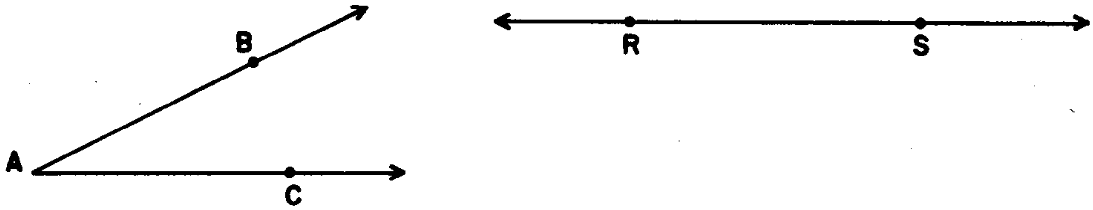
h) The set of counting numbers less than 43 is __________ under multiplication.

9. Copy and compare the sizes of $\angle$ RST and $\angle$ CAB, $\angle$TRS and $\angle$ BCA, $\angle$RTS and $\angle$ CBA.





10. Use your compass and straightedge to copy $\angle BAC$ on $\overleftrightarrow{RS}$ so that point S corresponds to A and $\overrightarrow{AB}$ falls on $\overrightarrow{SR}$.



Part B

Write a mathematical sentence (or two sentences if necessary) for each problem and solve. Write an answer sentence.

1. There are 40 pages in each Orange Trading Stamp Book. Each page holds 35 stamps. How many stamps will be needed to fill one book?

2. Smith's Department Store ordered 1608 Christmas tree ornaments from Japan. They arrived in 67 boxes with the same number of ornaments in each box. How many ornaments were in each box?

3. The food committee for the class picnic ordered hamburgers. Five-eighths of the class wanted hamburgers with onions. What part of the class wanted theirs without onions?

4. On its picnic, the class took $2\frac{1}{3}$ gals. ice cream. They used $1\frac{2}{5}$ gals. for sundaes. How much ice cream was left for cones?

5. The Campfire Girls in one town sold 426 boxes of candy the first week of their sale, 281 boxes the second week, and 469 boxes the third week. What was the average number of boxes sold each day?

6. Three frying chickens weigh $1\frac{3}{4}$ pounds, $2\frac{1}{2}$ pounds and $2\frac{1}{4}$ pounds. What is their total weight?

7. The speedometer of a car shows 74,286.1 miles at the end of the month. The car had gone 3,729.4 miles that month. What had the speedometer shown at the beginning of the month?

8. A box factor makes 2,940 soap boxes in one hour. How many dozen boxes are made in one continuous eight hour shift?

Individual Projects

1. Make a model of a geometric prism. The measure of the shortest edge should be no less than 3 inches. Color the faces so that none of the faces with a common edge are the same color. Display it for your class.

2. Make a model of a polygon. Use wire for the line segments. The measure of the shortest segment should be no less than 4 inches. You could use two or more of these to make an interesting mobile for your class.

3. Many great men have made important contributions to mathematics. Make a report about one of these famous mathematicians and his contributions.

Chapter 7

MEASUREMENT OF ANGLES

UNIT SEGMENTS AND UNIT ANGLES

Exploration

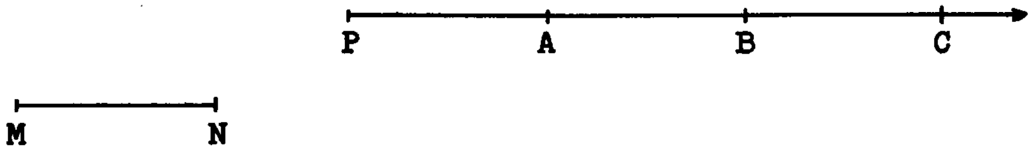
You have studied congruent angles, and you know that congruent angles have the same size. You have learned also how to tell which of two angles has the larger size. But we need to have a way to describe the size of an angle more exactly, that is, to measure an angle. Let us see how this could be done.

Recall how you found a method to measure a line segment. See if what you did to measure a segment suggests how an angle might be measured. Read the instructions of examples 1, 2, and 3 before you start the drawing requested in example 1.

1. Draw a ray on your paper. Call its endpoint P.
   Also draw a short segment not on the ray. Call it $\overline{MN}$.

2. On your ray construct a segment congruent to $\overline{MN}$, with one endpoint P. Call it $\overline{PA}$.

3. On the ray, construct a second segment congruent to $\overline{MN}$, with A as endpoint. Call it $\overline{AB}$. On the ray construct a third segment congruent to $\overline{MN}$. Call it $\overline{BC}$. Your drawing should look like this.



4. Copy and complete the following statements. Look at $\overline{MN}$ and $\overrightarrow{PC}$ you have drawn on your paper. Call the length of $\overline{MN}$ one unit. Then

   a) the length of $\overline{PA}$ is __________ unit.

   b) the length of $\overline{AB}$ is __________ unit.

   c) the length of $\overline{BC}$ is __________ unit.

   d) the length of $\overline{PB}$ is __________ units.

   e) the length of $\overline{PC}$ is __________ units.

   f) the length of $\overline{AC}$ is __________ units.

   The number 2 is called the <u>measure</u> of $\overline{PB}$.

5. What is the measure of $\overline{PC}$? of $\overline{AC}$? of $\overline{AB}$?

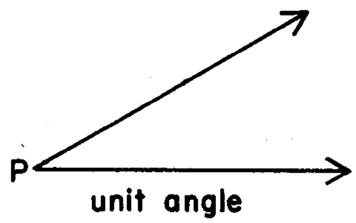
6. Did the pupil next to you make $\overline{MN}$ the same length?

7. If you are told only the measure of a segment can you know how long it is? What else must you know?

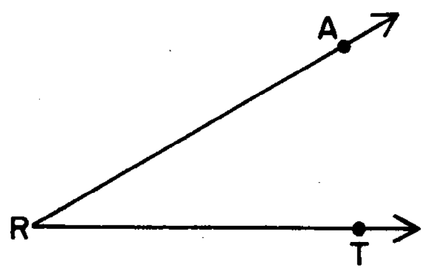
8. Choose a new segment, different from $\overline{MN}$, as your unit. Construct a segment whose measure, using this new unit, 4. Construct another segment whose measure is 3.

9. You used a line segment as a unit to measure line segments. What should you use as a unit to measure an angle?

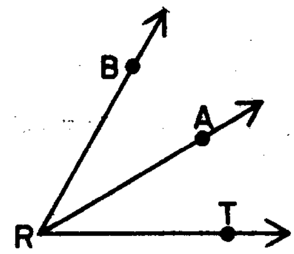
.0. Use $\angle P$ as a unit angle. Draw $\overrightarrow{RT}$ on a sheet of paper. Make a tracing of $\angle P$ on thin paper. Place the tracing with P on R and one side of $\angle P$ on RT. Then use the sharp end of your compass to mark a point A through the tracing to your drawing. Remove the tracing and draw $\overrightarrow{RA}$. Is $\angle ART \cong \angle P$?



1. What is the measure of $\angle ART$?



12. On your drawing of $\angle$ ART, place the tracing of the unit $\angle$ P so P is on R and one side of $\angle$ P is on $\overrightarrow{RA}$ and the other side of $\angle$ P is in the exterior of $\angle$ ART. Use the sharp end of your compass to mark a point B, and draw $\overrightarrow{RB}$. Is $\angle$ ARB $\cong$ $\angle$ P?

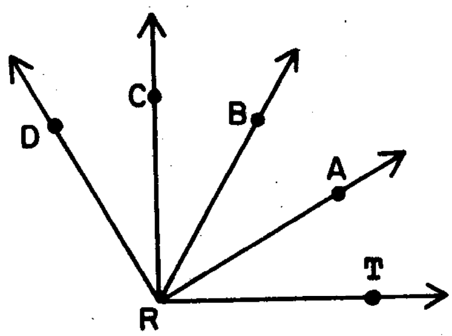


13. Using $\angle$ P as the unit angle, what is the measure of $\angle$ ARB? What is the measure of $\angle$ BRT?

14. Continue as in Problem 12.

 a) Place the tracing on the unit $\angle$ P with P on _____ and one side of $\angle$ P on _____. Be sure to place the tracing so the other side of $\angle$ P is in the __________ of $\angle$ BRT. Use the sharp end of your compass to mark a point C, through your tracing to your drawing. Remove the tracing and draw $\overrightarrow{RC}$.

 b) Repeat this process one more time in order to draw $\angle$ CRD. Your drawing should now look like this:



15. What is the measure in unit angles of $\angle$ CRT? $\angle$ CRA? $\angle$ CRB? $\angle$ CRD?

16. Since you and all your classmates used the same unit angle, $\angle$ P, should your $\angle$ DRT be congruent to theirs? Work with a classmate and test to see whether his angle and your angle seem to be congruent. Place your paper over a classmate's paper and hold them up to the light.

17. Choose a new unit angle smaller in size than a right angle. Then use your compass to construct an angle whose measure is 1. Call it $\angle$ ABC.

18. On the drawing you made for Exercise 17 construct with compass an angle whose measure is 2.

To state the measure of an angle we write:

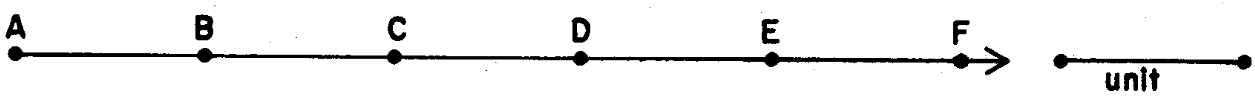
$m \angle DBA = 2$. The small "m" is read "measure of". We also write "$m\ \overline{AB} = 5$" to state the measure of a segment equals 5. "$m\ \overline{AB} = 5$" is read "the measure of $\overline{AB}$ equals 5."

<u>Remember that a measure is a number.</u>

Exercise Set 1

On a sheet of paper write the answers to the following exercises. Be sure to number each exercise.

1. State the measure of each segment named. The unit segment is shown at the right.



$\overline{AC}$, $\overline{AF}$, $\overline{BE}$, $\overline{DA}$, $\overline{FB}$, $\overline{CE}$.

Write your answer like this: m $\overline{AC}$ = 2.

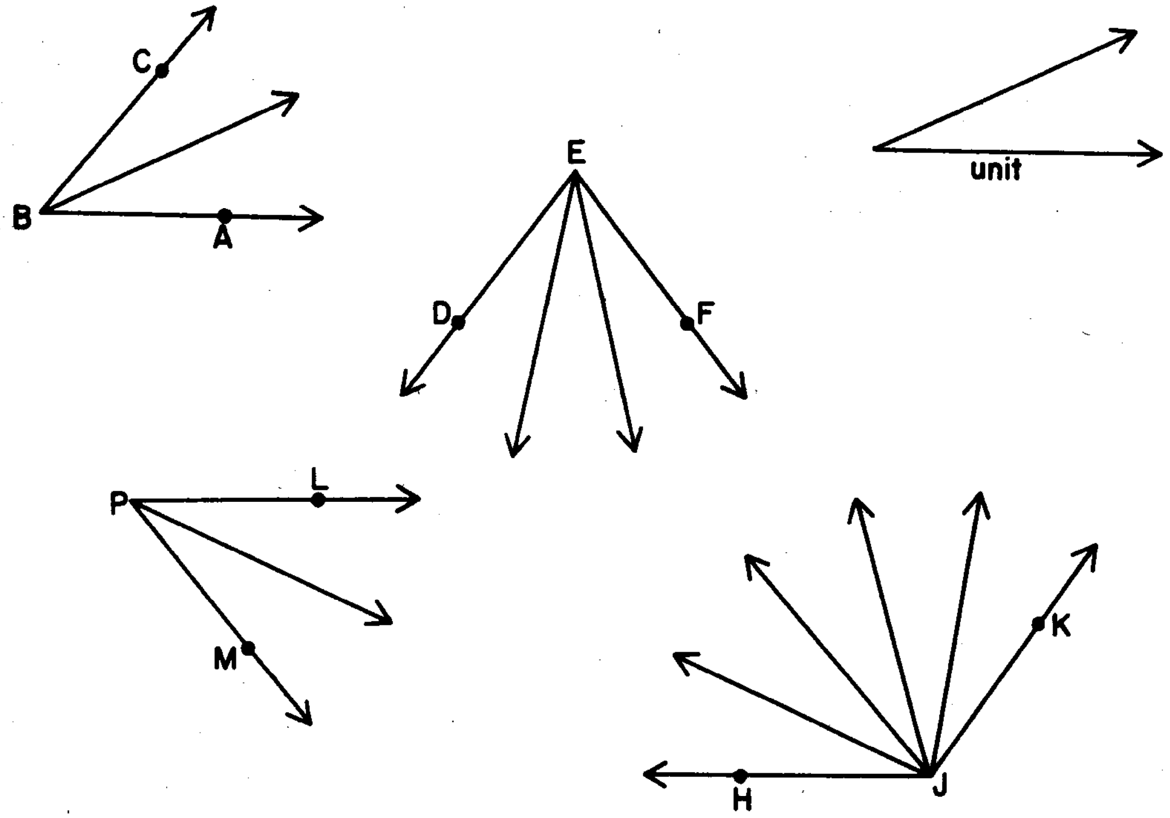
2. In the sketch below, name

   a) Four segments each of whose measure is 2.
      Write your answer like this: m $\overline{HJ}$ = 2.

   b) Three segments each of whose measure is 3.

   c) Two segments each of whose measure is 4.

3. The small angles in the sketches are all congruent to the unit angle shown. State the measure of each of the angles named. Write your answer like this: m $\angle$ ABC = __2__



4. Each of the small angles in the sketch is congruent to the unit angle. State the measure of each angle named.

$\angle$ GAC, $\angle$ BAE, $\angle$ CAF, $\angle$ DAG, $\angle$ BAC, $\angle$ FAG

5. Each of the small angles in each figure below is congruent to the unit angle. Using only the points which are marked, name:

   a) An angle with measure 2.

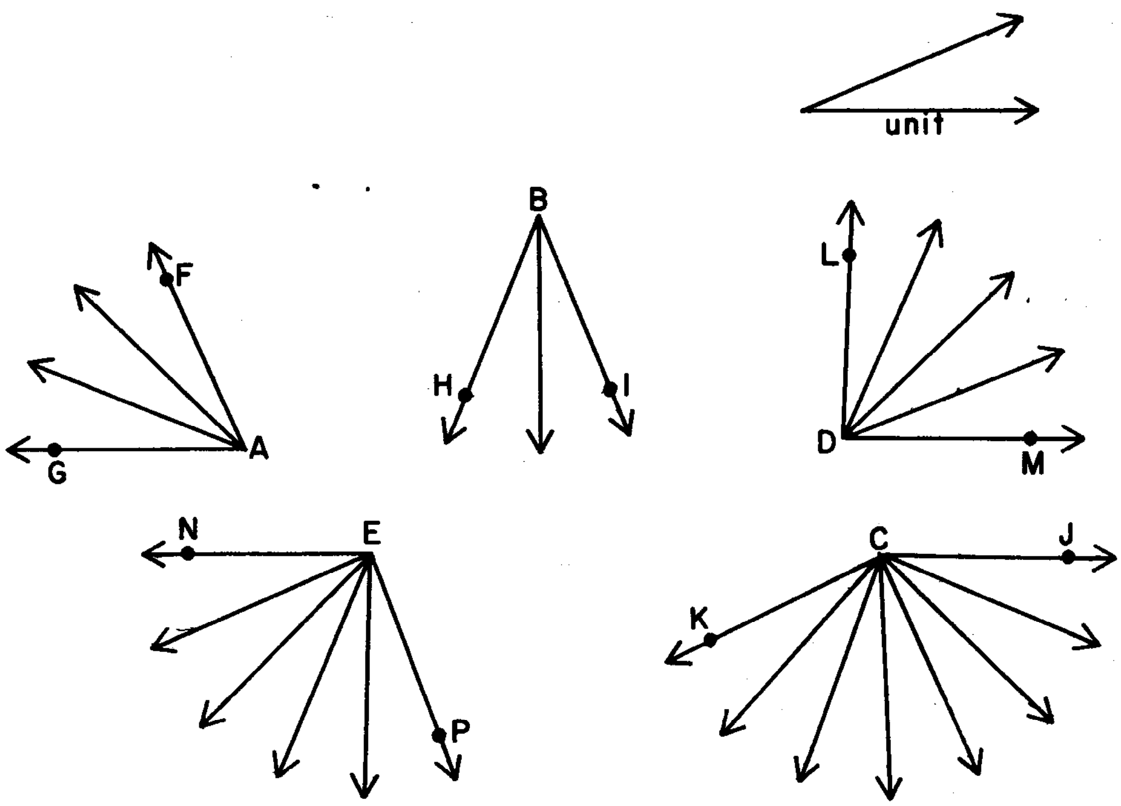
   b) An angle with measure 4.

   c) An angle with measure 7.

   d) Two angles, the sum of whose measures is 7.

   e) Two angles, the sum of whose measures is 9.

   f) Three angles, the sum of whose measures is 16.



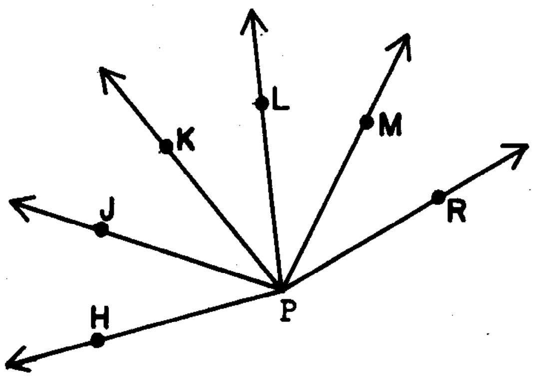
6. Each of the small angles in the figure below is congruent to the unit angle. Name:

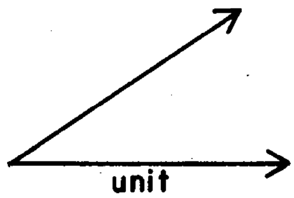
a) Three angles with measure 2.

b) Three angles with measure 3.

c) Two angles with measure 4.

d) Four angles with measure 1.





7. In the figure of Exercise 6,

a) m ∠ RPL = _____	d) m ∠ MPK = _____
b) m ∠ LPH = _____	e) m ∠ LPM = _____
c) m ∠ RPH = _____	f) m ∠ MPH = _____

Look at your answers to Ex. 7a, b, and c.

8. Is this true? m ∠ RPL + m ∠LPH = m ∠ RPH?

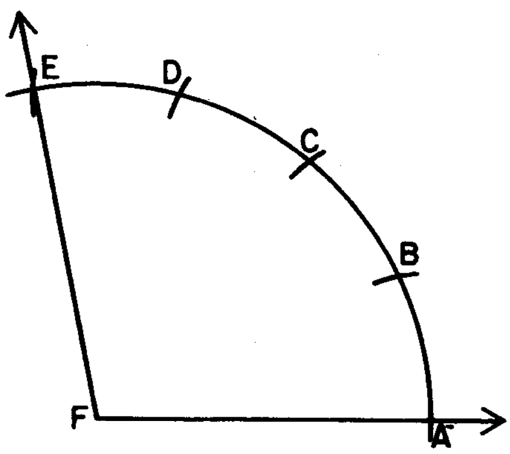
9. Now look at your answers for Exercises 7d, b, and f.
Is this true? m ∠ MPK + m ∠ LPH = m ∠ MPH?

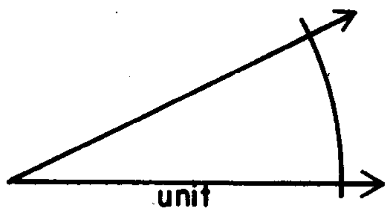
10. A boy wished to construct an angle of measure 4. He chose the unit angle shown below. He used his compass and straightedge to construct the $\angle$ AFE. A picture of his work is shown below. Look at the picture and answer the following:

a) What ray can you draw to complete an angle whose measure is 3?

b) What ray can you draw to complete an angle whose measure is 1?

c) Name the angle with measure 4.





11. Use the method shown in the sketch for Exercise 10 to construct an angle whose measure is 5. Use the unit angle of Exercise 10.

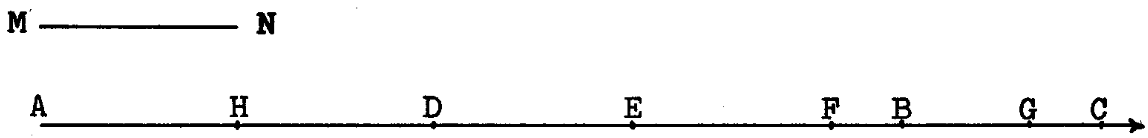
USE OF UNIT ANGLE IN MEASURING ANGLES

Exploration

You have used your compass to construct a line segment of a given measure and an angle of a given measure. Now suppose you wish to find the measure of $\overline{AB}$, using $\overline{MN}$ as unit. Trace $\overrightarrow{AC}$ and point B on a sheet of paper.

1. Now copy $\overline{MN}$ on $\overline{AB}$ with A as the left endpoint. Call the right endpoint, H. Repeat the process 4 more times to get line segments HD, DE, EF, and FG. Make each line segment congruent to $\overline{MN}$. How many such copies can you make on $\overline{AB}$?

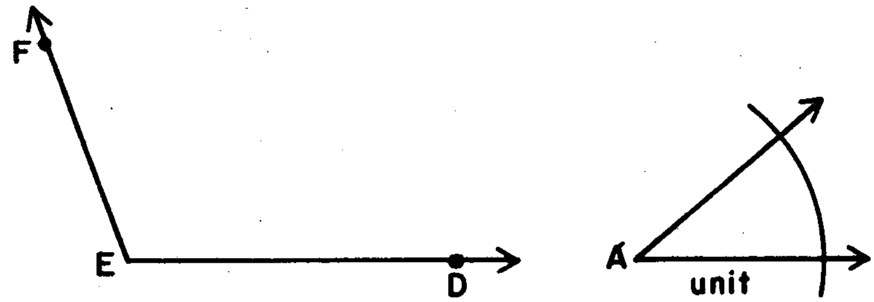
   Your drawing should look like this:



   In the sketch, $\overline{MN}$ was copied 4 times on $\overline{AB}$ so $m\ \overline{AF} = 4$. When $\overline{MN}$ is copied the last time, so $\overline{FG} \cong \overline{MN}$, you see that $m\ \overline{AG} = 5$.

   Since point B is between point F and point G, $m\ \overline{AB} > 4$, and also $m\ \overline{AB} < 5$. Since B is nearer F than G, we say that $m\ \overline{AB} = 4$, <u>to the nearest unit</u>. If B were nearer G than F, then we would write $m\ \overline{AB} = 5$, <u>to the nearest unit</u>.

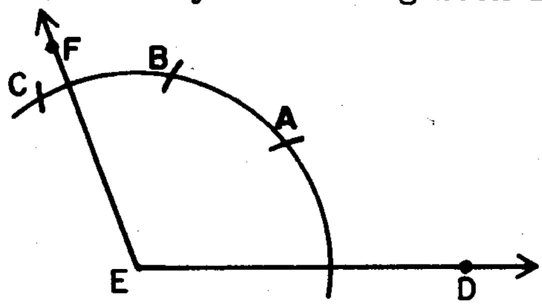
2. Suppose you are to find the measure of $\angle$ DEF, using $\angle$ A as unit.



Make a tracing of $\angle$ DEF and $\angle$ A on your paper. Can you use your tracing to estimate the measure of $\angle$ DEF?

m $\angle$ DEF = _____, to the nearest unit.

3. Now instead of tracing, use your compass as you did in Exercise 11, Set 1. Does your drawing look like this?



4. Draw $\overrightarrow{EA}$, $\overrightarrow{EB}$, and $\overrightarrow{EC}$.

Copy and complete the following statements.

5. m $\angle$ DEF > m $\angle$ ______, and

m $\angle$ DEF < m $\angle$ ______

6. m $\angle$ DEB = ______, and

m $\angle$ DEC = ______

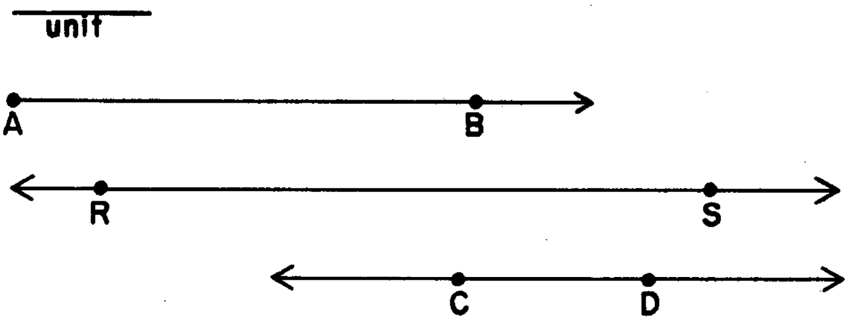
so m $\angle$ DEF > ______ and m $\angle$ DEF < ______

7. m $\angle$ DEF is nearer _____ than _____.

m $\angle$ DEF = _____ to the nearest unit.

Exercise Set 2

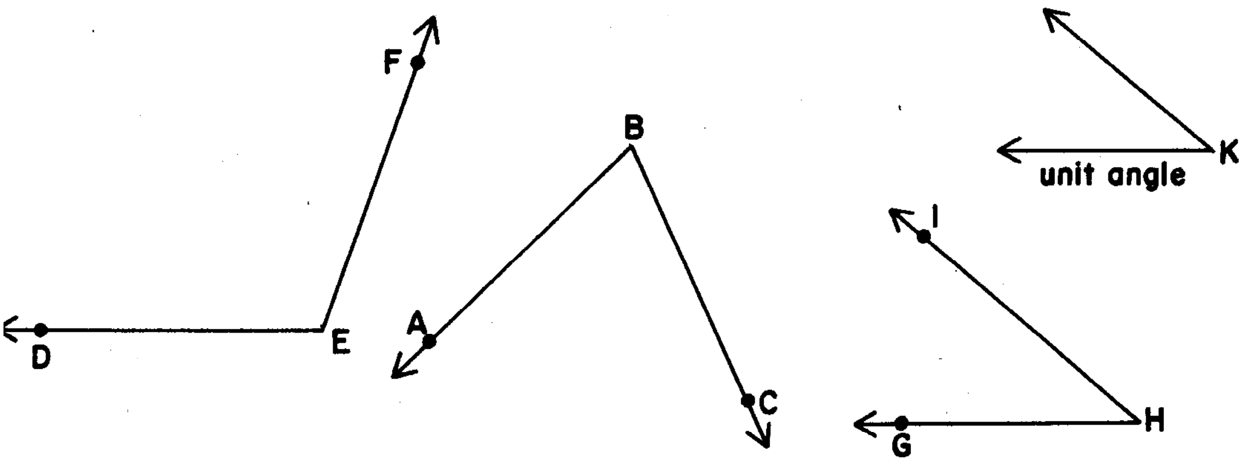
1. Make a copy of the following figures. Use the unit segment shown to find, to the nearest unit, the measure of each of the segments below. Use your compass.



Copy and complete the following statements

$m\ \overline{AB}$ = ______ $m\ \overline{RS}$ = ______ $m\ \overline{CD}$ = ______

2. Trace the figures below on your paper. Use your compass and straightedge to find the measure, to the nearest unit, of each angle below. Use the unit angle K as the unit of measure.

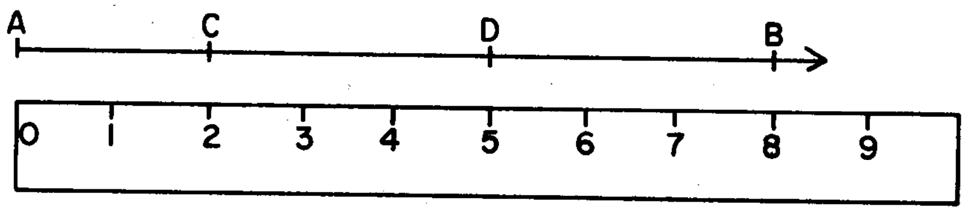


$m \angle ABC$ = ______, to the nearest unit. $m \angle DEF$ = ______, to the nearest unit. $m \angle GHI$ = ______, to the nearest unit.

A SCALE FOR MEASURING ANGLES

Exploration

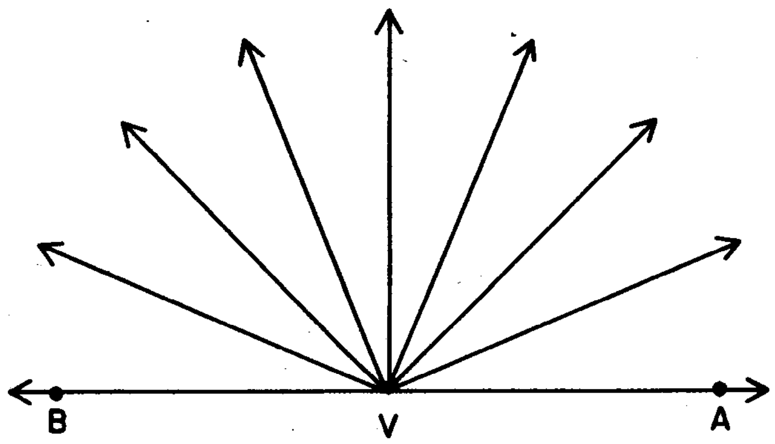
As you know, when you measure a line segment, you usually use a linear scale or ruler, with the endpoints of the unit segments marked with numerals. You place the ruler beside the segment and find the measure of the segment from the numerals on the ruler at the endpoints of the segment.



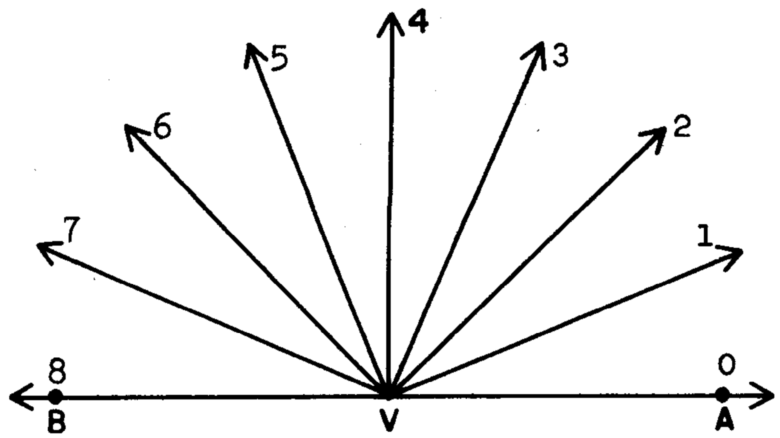
1. m $\overline{AC}$ = ______, m $\overline{AD}$ = ______, m $\overline{CD}$ = ______, m $\overline{AB}$ = ______

2. Must you place the zero on the scale at the endpoint of the segment in order to find the measure of the segment?

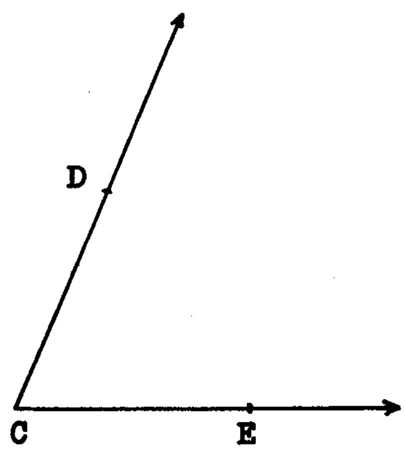
We shall use as a scale to measure <u>angles</u> a set of rays which are the sides of angles congruent to a unit angle. Any unit angle can be used but for convenience, we shall choose one so that eight of them with their interiors will exactly cover a half plane. We may name it whatever we want to. We will name our unit angle an "octon." Two of the rays, $\overrightarrow{VA}$ and $\overrightarrow{VB}$, are on the same straight line and extend in opposite directions from V.



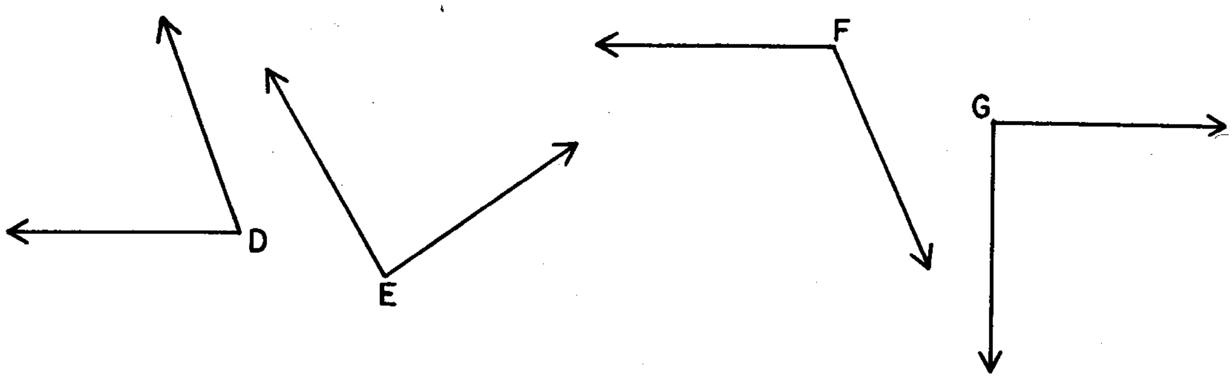
Then we will number the rays in order, putting 0 on the ray to the right $(\overrightarrow{VA})$ and ending the scale when we reach the ray on the same straight line as the zero ray $(\overrightarrow{VB})$.



3. Make a tracing of $\angle$ DCE. To measure $\angle$ DCE, how should the tracing be placed on the scale? Put C on V and $\overrightarrow{CE}$ on the zero ray. Then read the number of the ray on which $\overrightarrow{CD}$ falls, m $\angle$ DCE = ______.



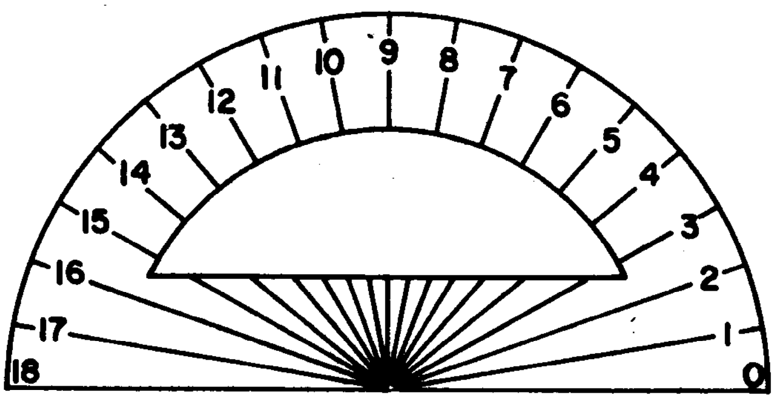
4. Trace the angles below and use the angle scale to find the measure of each, to the nearest unit.



In measuring segments, it is convenient to have a linear scale marked off on a ruler. Then the ruler can be moved and placed beside a line segment.

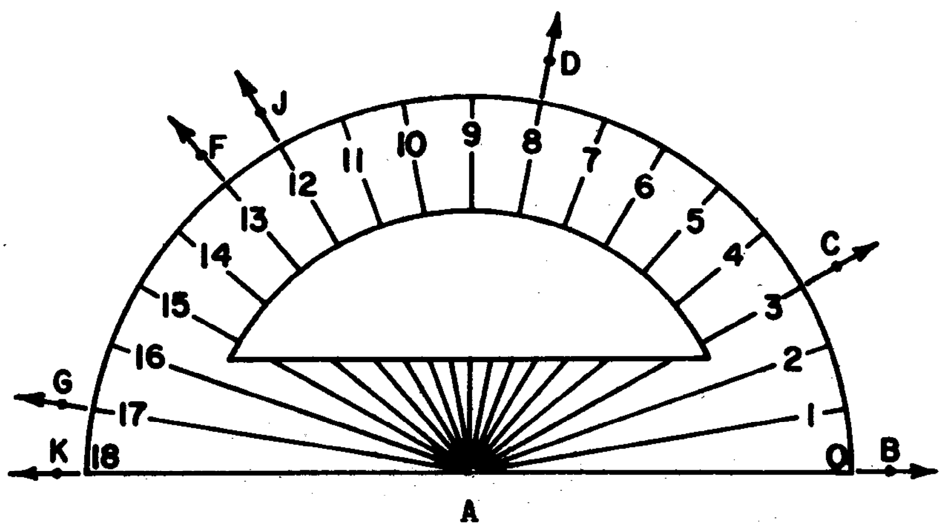
In measuring angles, it is convenient to have an angle scale marked off on a <u>protractor</u>. Then the protractor can be moved and placed on an angle. Your teacher will show you how to make a protractor.

Here is a picture of a cardboard protractor with a smaller unit angle than we used before. Only parts of the rays are shown.



The rays are broken because part of the cardboard is cut out so you can see the ray of the angle you are measuring.

5. Below is a sketch showing the protractor placed on a set of rays. The rays have the same endpoint, A , and the V-point of the protractor is on A . Find the measures of the angles named.



Exploration

a) $m \angle BAC =$

b) $m \angle BAD =$

c) $m \angle BAJ =$

d) $m \angle BAF =$

e) $m \angle BAG =$

f) $m \angle GAF =$

g) $m \angle CAD =$

h) $m \angle DAF =$

i) $m \angle DAG =$

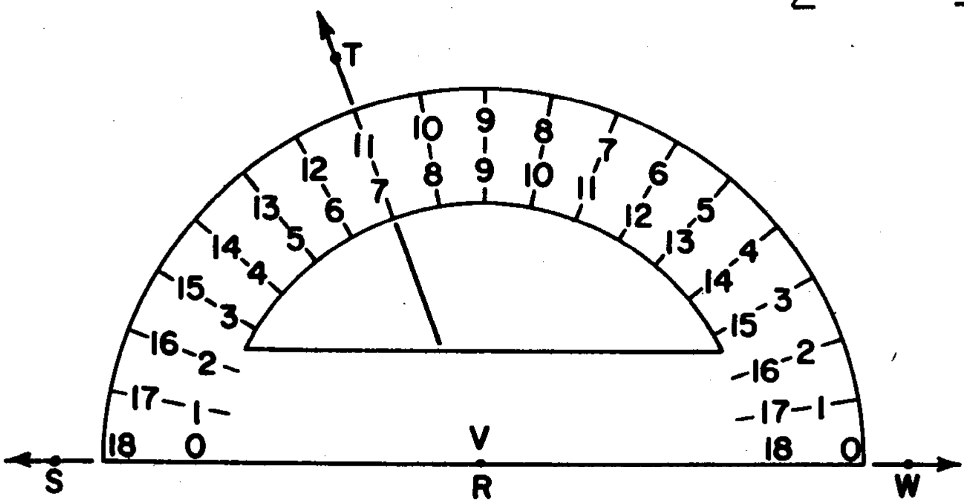
j) $m \angle CAJ =$

In addition to the scale with the zero ray at the right, many protractors also have another scale with the zero ray at the left. This scale is placed on the inner rim.

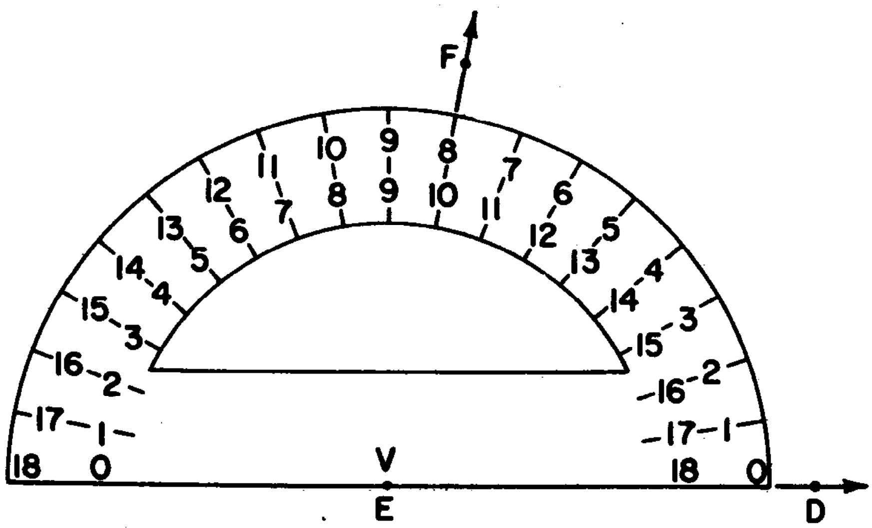
6. Look at the second scale on the protractor shown in the picture below. This scale is written on the inner rim. Zero is put on the ray to the left ($\overrightarrow{RS}$) and rays have been numbered in order until the ray on the same line with zero is reached ($\overrightarrow{RW}$). Write these numerals on the sketch you made for Exercise 5. Find the measures of the angles named in Exercise 5 using the new scale. Are the measures the same?

The advantages in having the two scales can be seen from the following sketches:

To measure $\angle$ SRT, the zero ray at the <u>left</u> is placed on $\overrightarrow{RS}$. You use the inner scale to find the measure. m $\angle$ SRT = ____

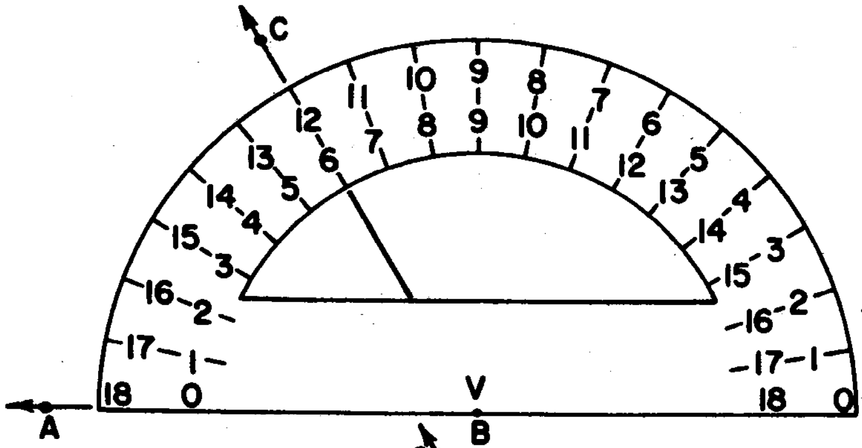


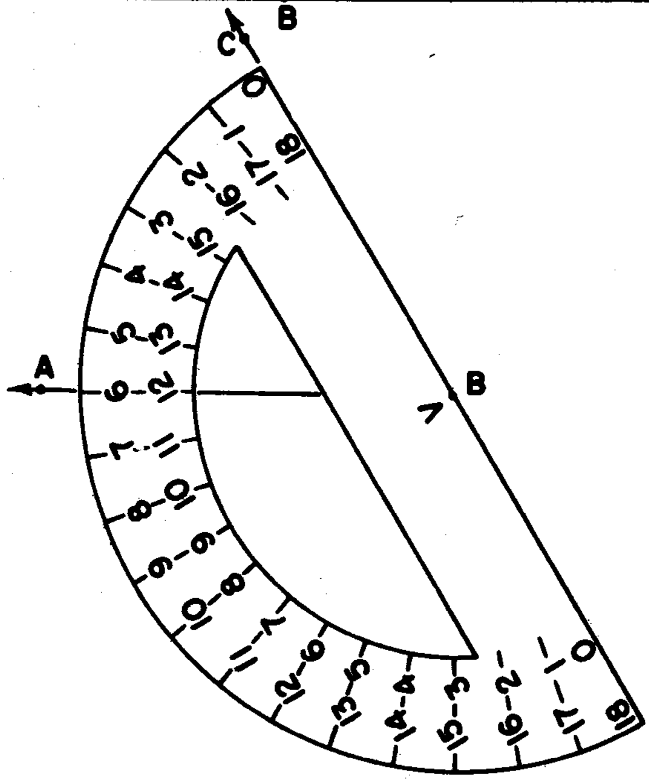
To measure $\angle$ DEF, the zero ray at the <u>right</u> is placed on $\overrightarrow{ED}$. You use the outer scale to find the measure. m $\angle$ DEF = ___



It is very easy to read the wrong scale by mistake. You will prevent most such errors by estimating the size of the angle as a check of your measurement. Of course, you can use either scale to measure the same angle, by moving the protractor.

7. The following two sketches are copies of the same angle, $\angle$ ABC. In the first sketch, the protractor is placed so the zero ray on the left of the protractor is on ____. Which scale would you use to find the measure of the angle? In the second sketch, the zero ray on the right of the protractor is placed on ____. Now which scale would you use to find the measure of the angle? Is the measure of the angle the same either way?

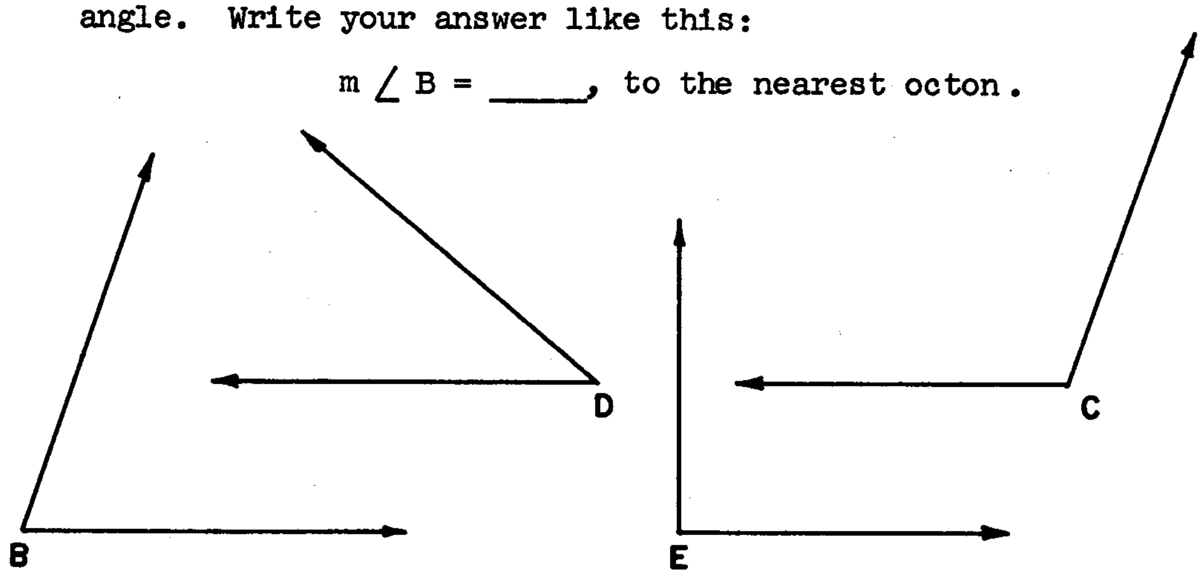




Exercise Set 3

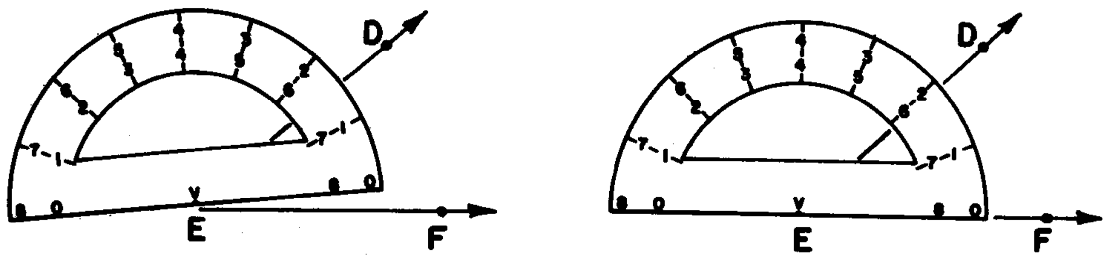
1. Use the "octon" scale on your protractor to find the measure of each of the angles below (to the nearest octon). After you have measured an angle, check your measure by placing the protractor with a zero ray on the other side of the angle. Write your answer like this:

   $m \angle B =$ _____, to the nearest octon.



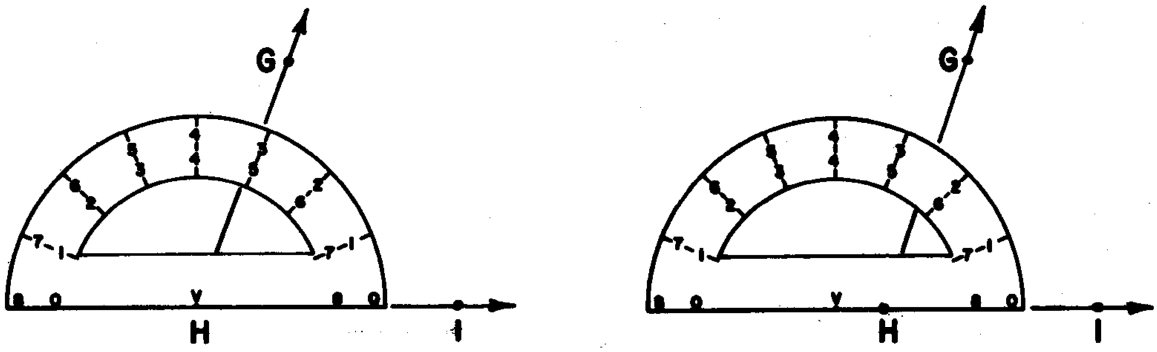
2. Which of these sketches shows the correct way to place the protractor to find the measure of $\angle$ DEF? Why?

   $m \angle DEF =$ _____, to the nearest octon.

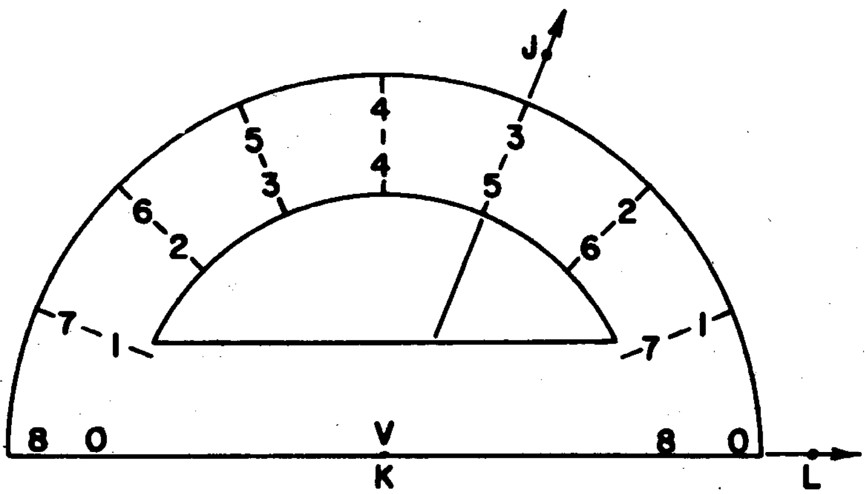


3. Which of these sketches shows the correct way to find the measure of $\angle$ GHI? Why?

$m \angle GHI =$ _____



4. A boy said that the measure of the $\angle$ JKL in octons is 5. What was his mistake? What is $m \angle JKL$?

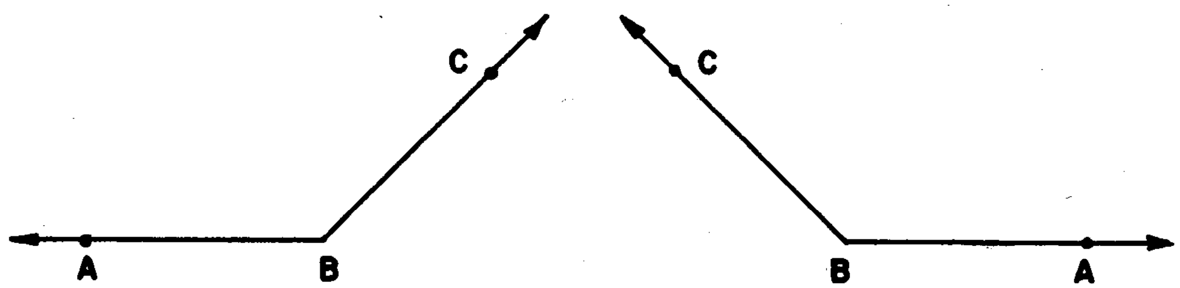


DRAWING AN ANGLE OF GIVEN MEASURE

Exploration

You can use your protractor to draw an angle whose measure, in octons, is to be a given whole number. Do you see how to use the protractor in this way?

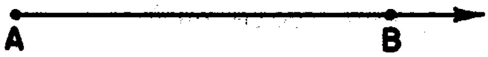
Draw $\angle B$ so that $m\angle B$, in octons, is 6. Since the vertex must be point _____, draw $\overrightarrow{BA}$. Place the protractor with the V-point on the vertex and the zero ray of one scale on $\overrightarrow{BA}$. Mark a point C at the number 6 on the same scale. Remove the protractor and draw $\overrightarrow{BC}$. Each of these angles has a measure of 6, in octons. Does your angle look like one of them?



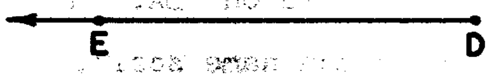
Exercise Set 4

In these exercises, draw rays and label points as in the sketches.

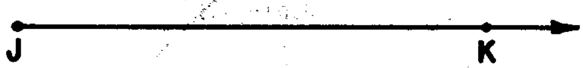
1. Copy the figure below on your paper. Draw on $\overrightarrow{AB}$ an angle with a measure of 5, in octons. Label it $\angle$ BAC. Draw the angle so that $\overrightarrow{AC}$ is <u>above</u> $\overrightarrow{AB}$.



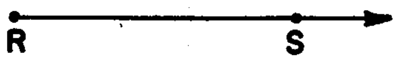
2. Copy the figure below on your paper. Draw an angle with a measure of 3 octons, using $\overrightarrow{DE}$ as one ray. Label it $\angle$ EDF. Draw the angle so that $\overrightarrow{DF}$ is <u>above</u> $\overrightarrow{DE}$.



3. Copy $\overrightarrow{JK}$ on your paper. Draw an angle with a measure of 2 using $\overrightarrow{JK}$ as one ray. Label it $\angle$ KJL. Draw the angle so that $\overrightarrow{JL}$ is <u>below</u> $\overrightarrow{JK}$.



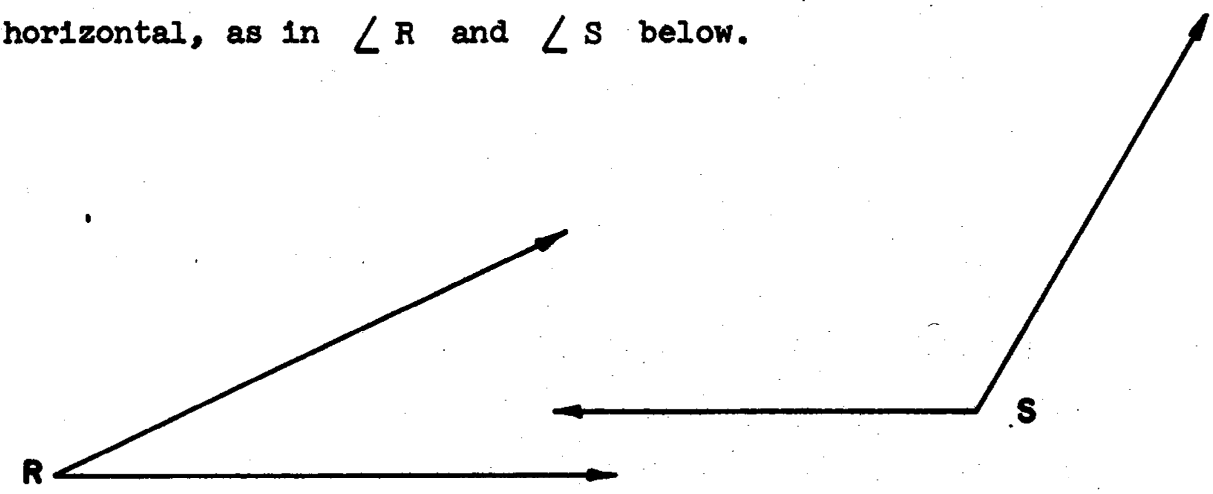
4. Copy $\overrightarrow{RS}$ on your paper. Draw $\angle$ SRT whose measure is 7 using $\overrightarrow{RS}$ as one ray. Draw the angle so that $\overrightarrow{RT}$ is <u>below</u> $\overrightarrow{RS}$.



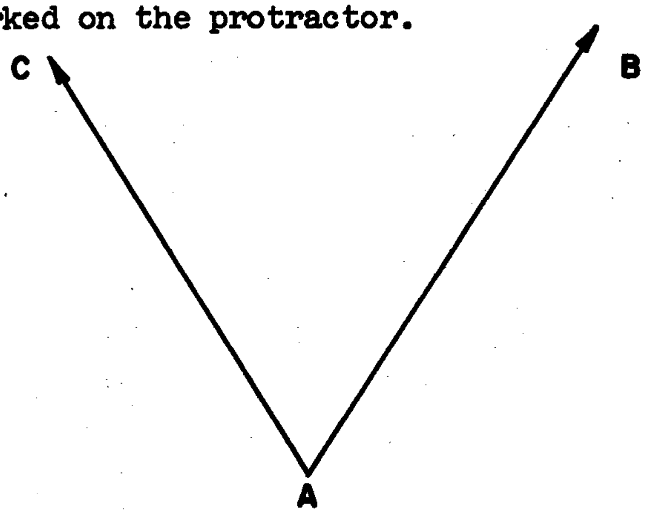
PRACTICE IN MEASURING ANGLES

Exploration

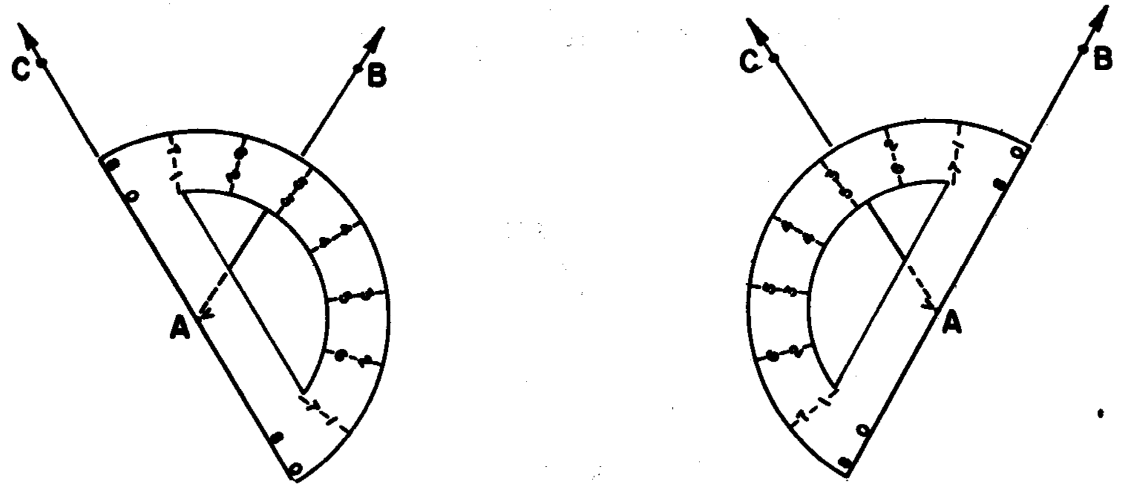
In most of the angles you have measured, one ray was horizontal, as in $\angle R$ and $\angle S$ below.



1. How would you find the measure of $\angle A$? This angle is in a different position from others you have measured. Its measure is found in the same way. Place your protractor so a zero ray falls on either $\overrightarrow{AB}$ or $\overrightarrow{AC}$. Be sure the V of the protractor is exactly on vertex A. The other ray of the angle can then be matched with the part of a ray marked on the protractor.

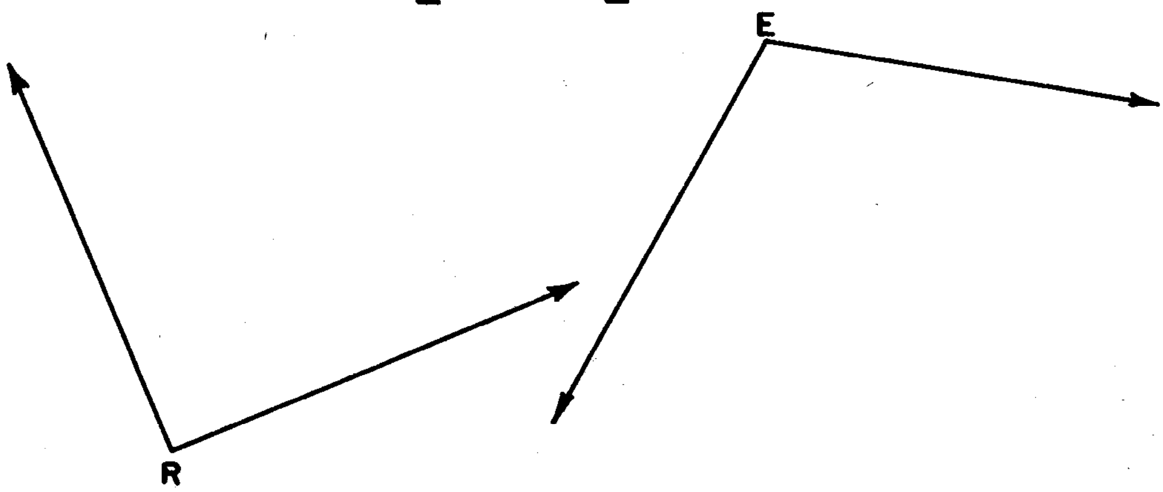


2. These sketches show the two ways to place the protractor.

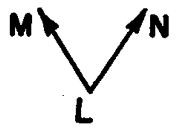


Put the zero ray on $\overrightarrow{AC}$ or put the zero ray on $\overrightarrow{AB}$. Does it make any difference in the measure whether the zero ray is on $\overrightarrow{AB}$ or on $\overrightarrow{AC}$? In each sketch, we see the measure of $\angle A$ to be about 3. Why is its measure 3 rather than 5, to the nearest octon?

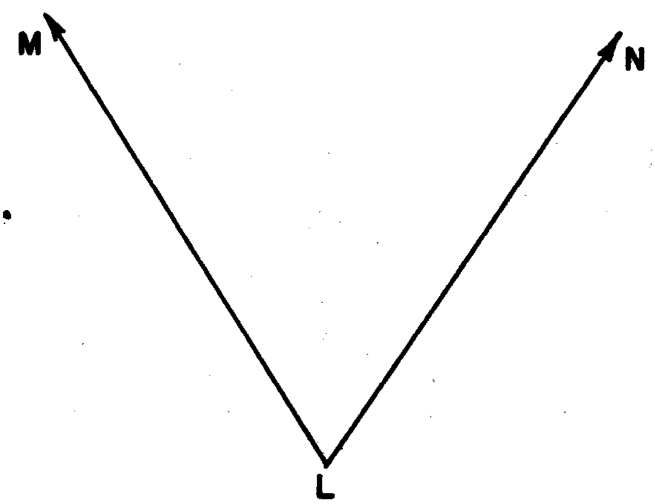
3. Find the measure of $\angle R$ and $\angle E$.



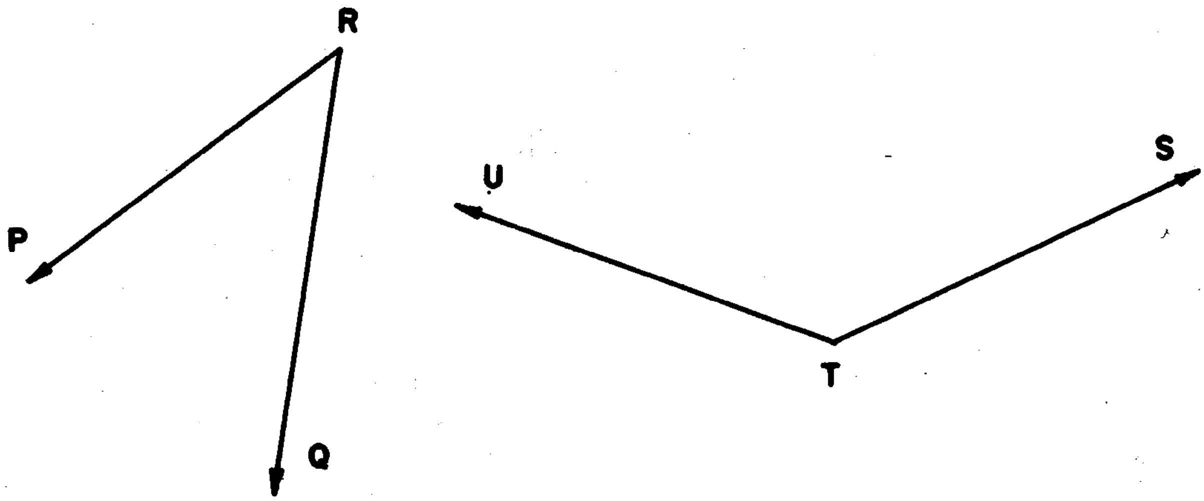
4. How do you think we can find the measure of $\angle L$? Can a protractor be placed on $\angle L$ so that you can read its measure? Do $\overrightarrow{LN}$ and $\overrightarrow{LM}$ have a definite length?



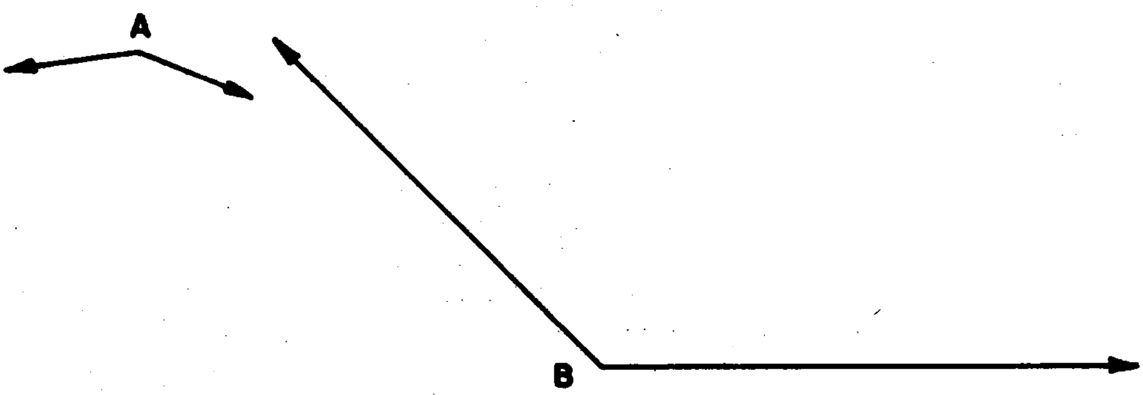
5. If $\overrightarrow{LM}$ and $\overrightarrow{LN}$ are not long enough to extend beyond the protractor, can they be extended without changing the size of the angle? What is its measure? Check your measure by putting the zero-ray of your protractor on the other ray of the angle. Could you represent the rays in some way without drawing them?
(Clue: Try using a sheet of paper or some other kind of straightedge.)



6. Find the measures of $\angle R$ and $\angle T$. You will need to show more of one or both rays.



7. a) Which angle has the larger measure, $\angle A$ or $\angle B$?



b) Is the measure of $\angle A$ changed if you extend the part of its rays which are shown on this page?

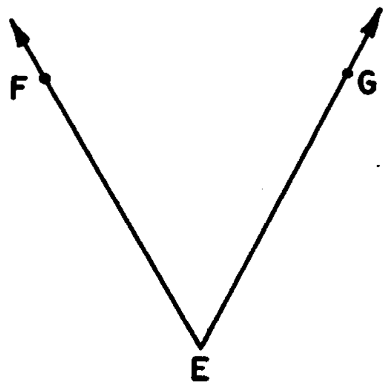
c) $m \angle A =$ _____, to the nearest octon.

d) $m \angle B =$ _____, to the nearest octon.

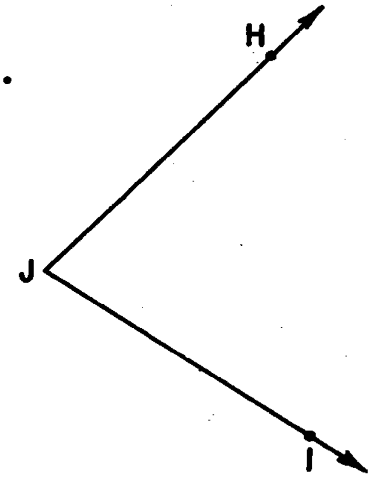
Exercise Set 5

Find to the nearest octon the measures of the angles below. Use your octon protractor.

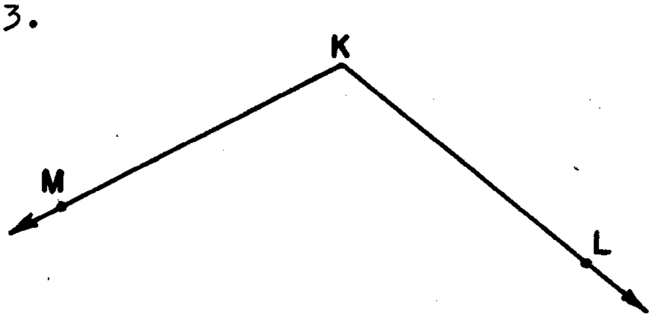
1.



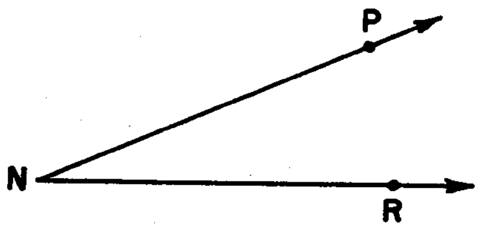
2.



3.



4.



5. In Exercises 1 and 2, was $m\angle E = m\angle J$?

   Is $\angle E \cong \angle J$? (Use a tracing)

   Although $\angle E$ is not congruent to $\angle J$, the measures, to the nearest octon, were the same number. If the unit angle were much smaller, then the fact that the angles are not congruent would be shown clearly in the measures.

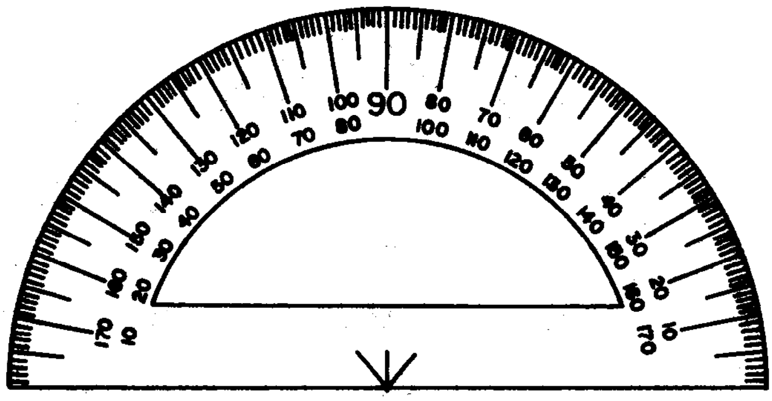
A STANDARD UNIT FOR MEASURING ANGLES

Exploration

As you know, the linear scale on a ruler is usually marked off using a <u>standard unit</u> such as the inch or the centimeter. A standard unit is one whose size has been determined by agreement among people. We would find it difficult to communicate with people or to carry on business if everyone made up his own units. What other standards of measure can you name?

There are also standard units for measuring angles, so that people throughout the world can communicate easily. The standard unit for measuring angles is the <u>degree</u>. The unit angle of one degree is smaller than the octon, the unit angle we used on the preceding pages. In fact, the octon is $22\frac{1}{2}$ times as large as an angle of one degree. Its measure in degrees is $22\frac{1}{2}$. The symbol for degree is $^{\circ}$. An angle of 15° means an angle whose measure, in degeees, is 15. We say that the size of the angle is 15°. As you work with your protractor you will discover that it takes 360 of these unit angles using a single point as a common vertex and their interiors, to cover the entire plane. Even in ancient Mesopotamia the angle of 1° was used as the angle of unit measure. The selection of their unit which could be fitted into a plane just 360 times was probably influenced by the fact that their year had 360 days.

1. Look at the side of your protractor on which the standard unit is the degree.

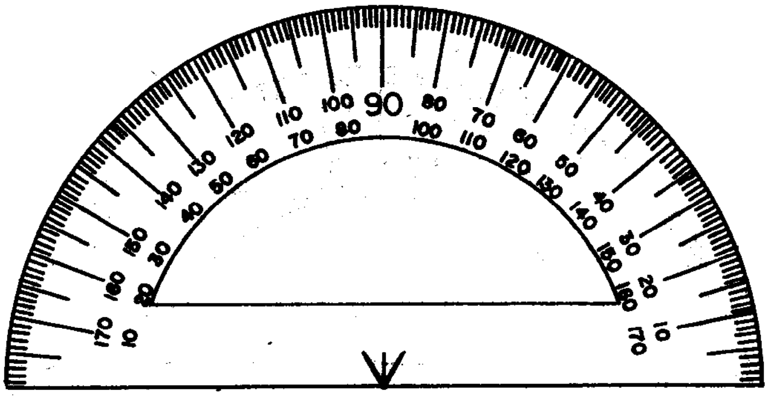


An angle of 1 degree is formed by rays, with endpoint V, through two of the marked points next to each other. Does this seem like a very small angle? Would it seem so small if the segment of the ray shown were extended to 15 feet?

2. Since 1 degree is so small, only every tenth degree is numbered on the scale. What other numbers are missing?

Why is 0 not printed on the scale? What is the largest number represented on the scale? Is its numeral printed? Why?

3. Look at the side of the protractor on which the standard unit is the degree.



You use this standard protractor to measure an angle in degrees in the same way you used the scale on the other side to measure an angle in octons. You must be careful about the following things.

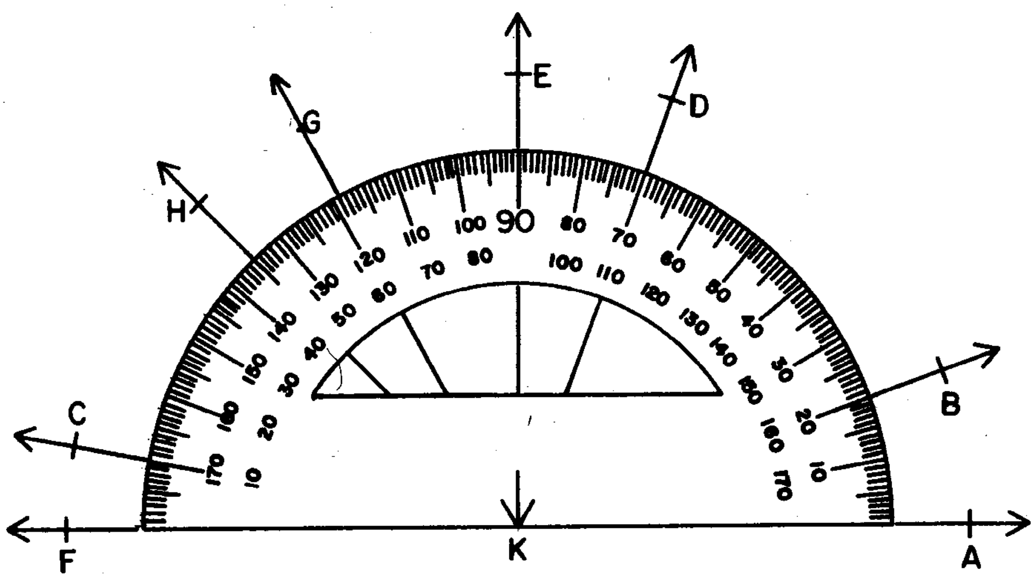
a. Place the V point of the protractor on the vertex of the angle. Be sure the protractor covers part of the interior of the angle.

b. Place the protractor with one of the zero rays exactly on one side of the angle. Notice whether this zero is a number on the inner scale or the outer scale. This is the scale you must use.

c. Find the point where the other side of the angle intersects the rim of the protractor. If not enough of the ray is shown to intersect the rim, can the rays of the angle be extended without changing the size of the angle? Read the number at this point on the scale you chose in Step b.

Exercise Set 6

1. The sketch shows a protractor placed on a set of rays from point K. The V point of the protractor is on K. Find the measure, in degrees, of each angle named.



a) m $\angle$ AKB = _____

b) m $\angle$ FKE = _____

c) m $\angle$ AKC = _____

d) m $\angle$ FKG = _____

e) m $\angle$ AKD = _____

f) m $\angle$ BKE = _____

Imagine that the protractor has been moved so that the zero ray lies along $\overrightarrow{KB}$ (or $\overrightarrow{KE}$)

g) m $\angle$ CKD = _____

Imagine that the protractor has been moved so that the zero ray lies along $\overrightarrow{KC}$ (or $\overrightarrow{KD}$)

h) m $\angle$ HKD = _____

Imagine that the protractor has been moved so that the zero ray lies along $\overrightarrow{KH}$ (or $\overrightarrow{KD}$)

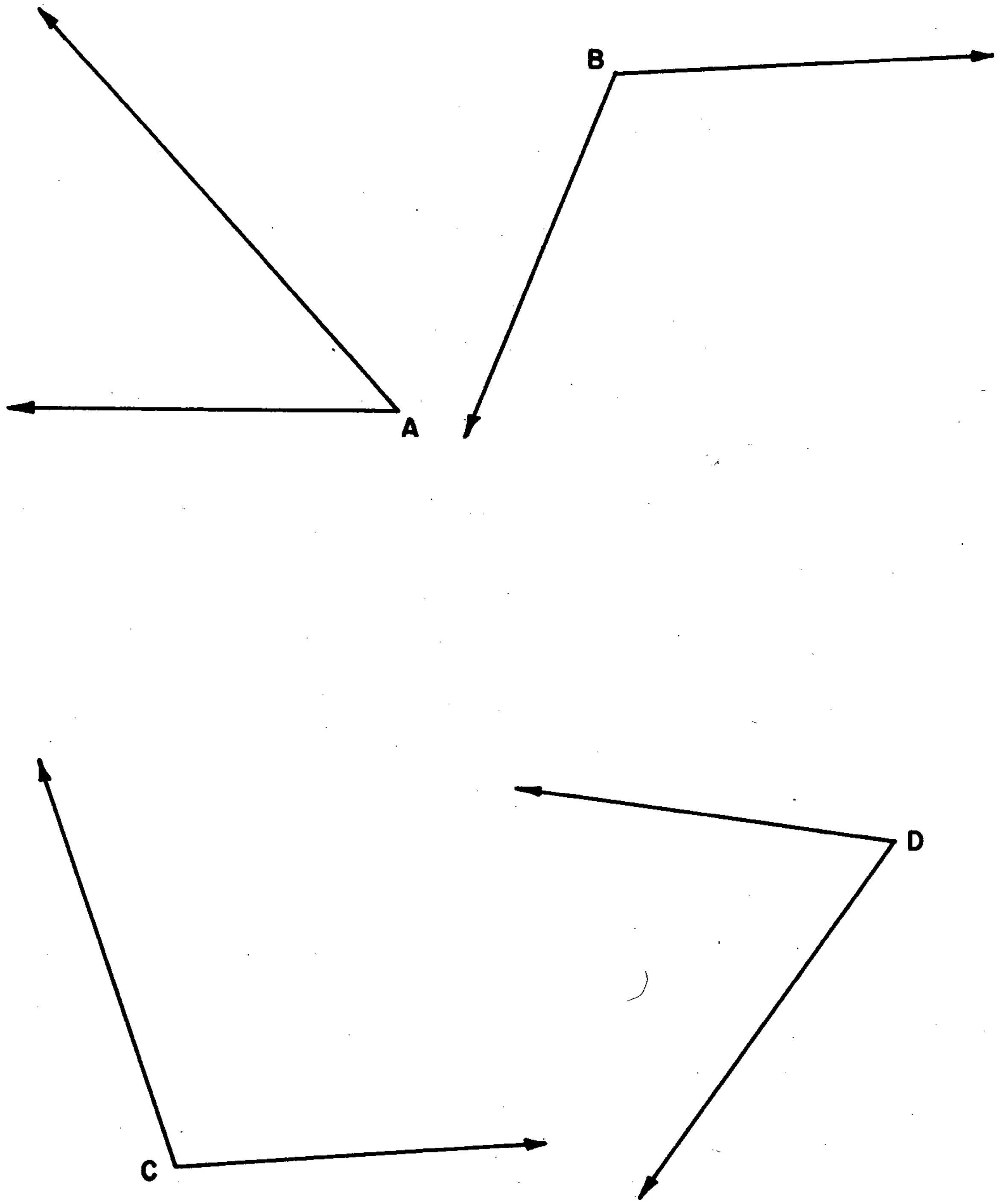
i) m $\angle$ DKB = _____

Imagine that the protractor has been moved so that the zero ray lies along $\overrightarrow{KD}$ (or $\overrightarrow{KB}$)

j) m $\angle$ HKC = _____

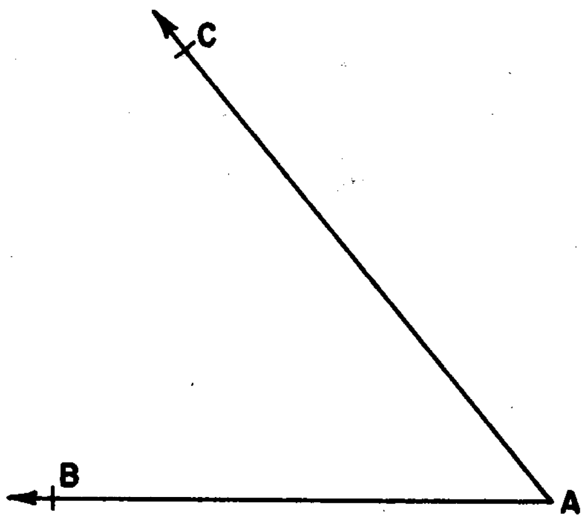
Imagine that the protractor has been moved so that the zero ray lies along $\overrightarrow{KH}$ (or $\overrightarrow{KC}$)

2. Use your protractor to find the measures, in degrees, of the following angles.



ESTIMATING THE MEASURE OF ANGLES

Exploration



Helen used her protractor to find the measure of $\angle A$. She made a mistake and read the wrong scale of her protractor, so she wrote for her answer $m\angle A = 130$. Max was asked to check her paper to see whether her answer was correct. Max said, "I do not have my protractor to find the measure of $\angle A$, but I know that Helen's answer is wrong." How did Max know that Helen's answer was not correct?

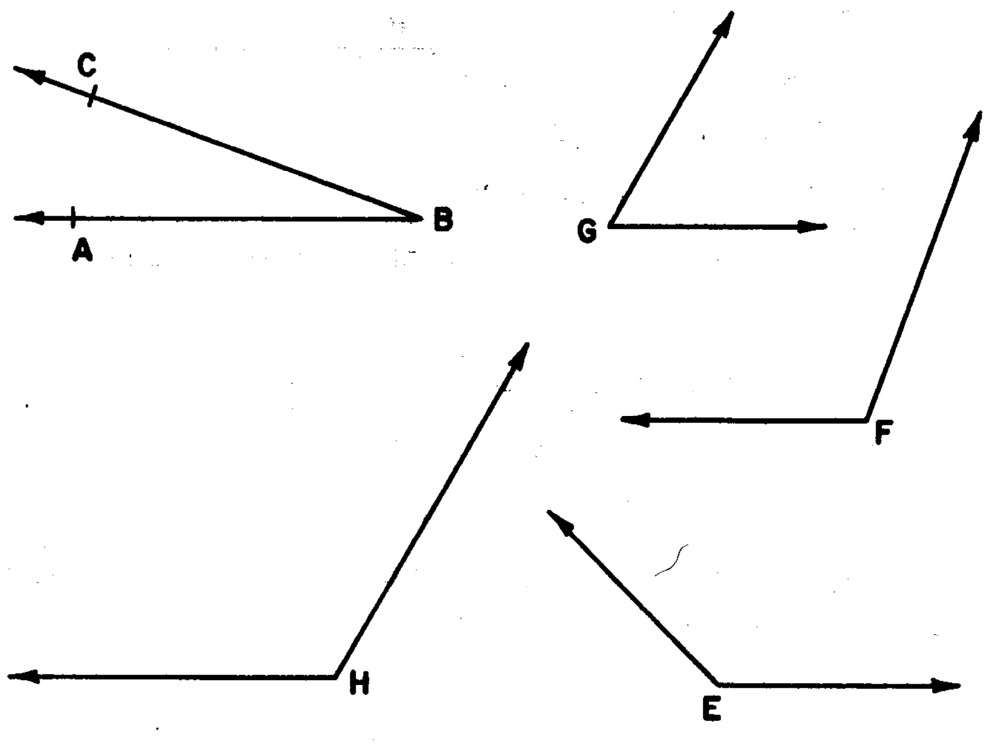
Whenever you can, you should make an estimate of an answer to a problem. Then if your answer is not close to this estimate you will suspect you may have made a mistake.

A good angle to use as a guide in estimating the measure of angles is a right angle.

1. Do you remember how to fold a paper to make a right angle? Just two folds are needed.

2. What is the measure of a right angle? Use your protractor if you need to.

3. Which of these angles has a measure greater than the measure of a right angle? Do not use your protractor. (For $\angle$ B, imagine $\overrightarrow{BD}$ which would make $\angle$ ABD a right angle. Place your pencil on the figure to represent $\overrightarrow{BD}$. Would $\overrightarrow{BD}$ be in the interior of $\angle$ ABC?)



4. Which of the angles above have measures less than the measure of a right angle?

Draw $\overleftrightarrow{WZ}$ on a piece of paper. On $\overleftrightarrow{WZ}$, choose a point X. Your drawing should look like this.

Use point X as vertex and $\overrightarrow{XZ}$ as one ray, and draw with your protractor an angle with a measure, in degrees, of 90. Call this $\angle$ ZXT.

Use X as vertex and $\overrightarrow{XZ}$ as one ray, and draw angles with measures, in degrees, of 45 and 135. Draw all three rays on the same side of $\overrightarrow{WZ}$. Label them so that $m \angle ZXY = 45$ and $m \angle ZXR = 135$.

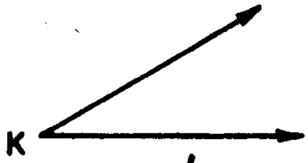
6. What other angles in the figure have a measure of 45?

Name another angle which has a measure of 135.

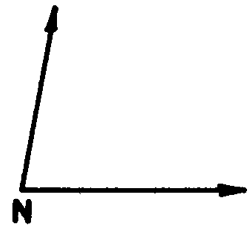
What other angles have a measure of 90?

7. Look at each angle below and estimate its size. Use an angle of one degree as the unit. Now compare each angle with an angle in the drawing you made for Exercise 5.

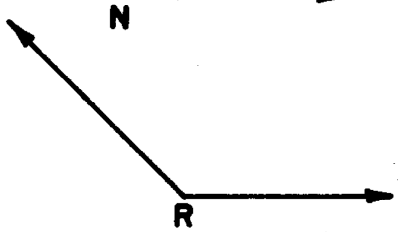
a) Is $m\angle K$ nearer 0 or 45?



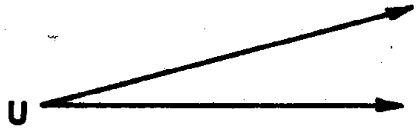
b) Is $m\angle N$ nearer 45 or 90?



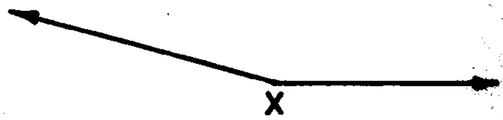
c) Is $m\angle R$ nearer 90 or 135?



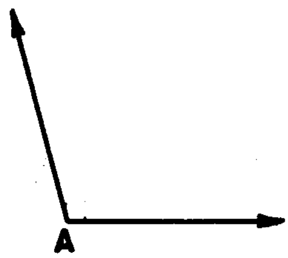
d) Is $m\angle U$ nearer 0 or 45?



e) Is $m\angle X$ nearer 135 or 180?



f) Is $m\angle A$ nearer 90 or 135?

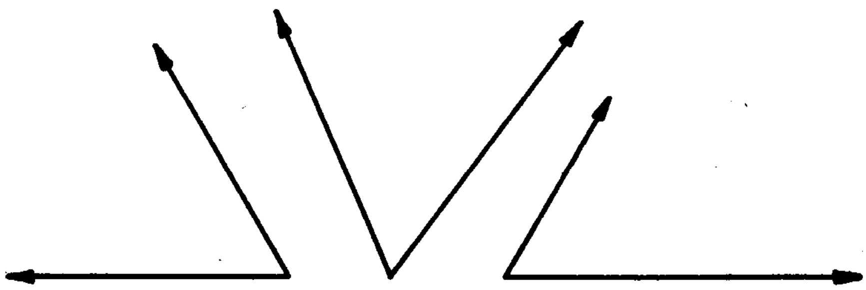


8. Now measure, in degrees, each angle in Exercise 7 with your protractor and write the measure you find.

Summary

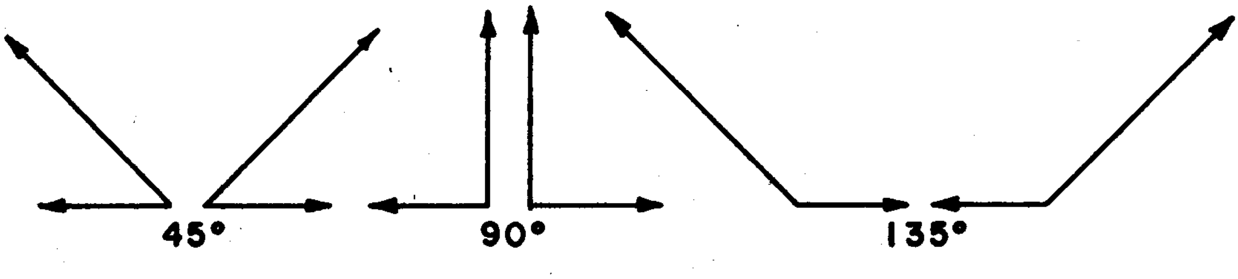
1. You can estimate the measure of an angle.

2. Angles can be of the same measure but be in different positions. Each of these angles is an angle of 60°.



3. To estimate the measure of an angle, think how the protractor is marked. Learn to recognize when the measure of an angle is about 90.

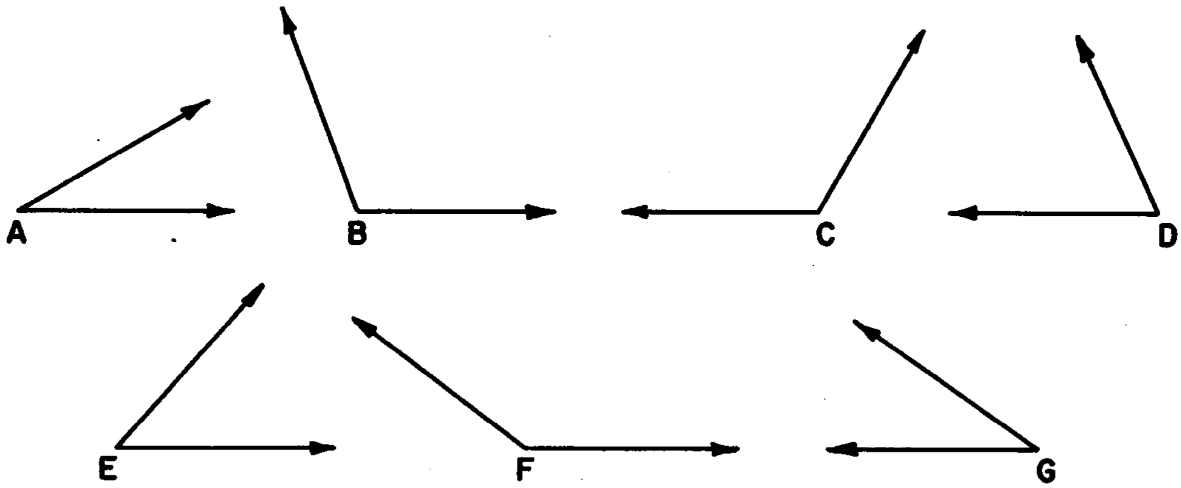
4. Angles of 45°, 90°, and 135° look like this.



Can you add anything to this summary?

Exercise Set 7

1. Which of these angles has a measure less than the measure of a right angle?



2. Which of the angles above have a measure greater than 90?

3. Look carefully at each angle. Choose the better estimate of its measure in degrees.

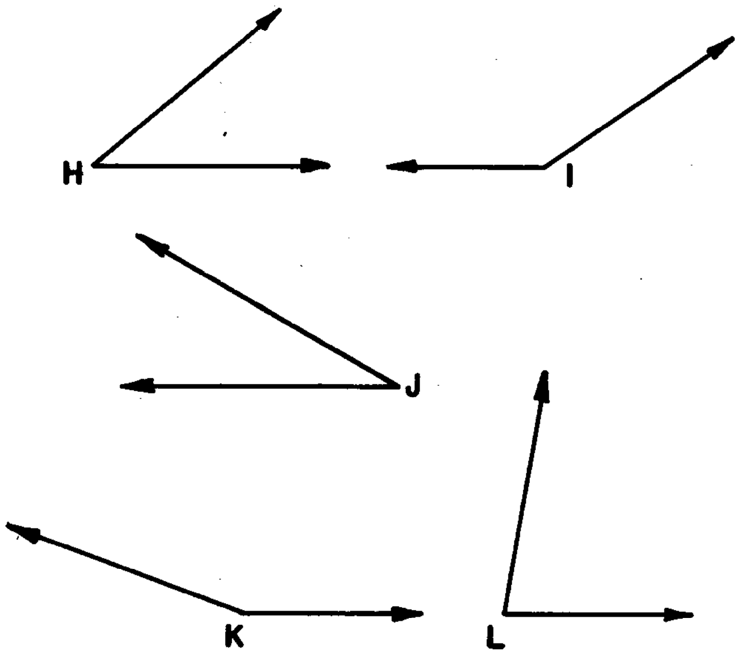
   m ∠ H; 5 or 45

   m ∠ I; 90 or 135

   m ∠ J; 45 or 90

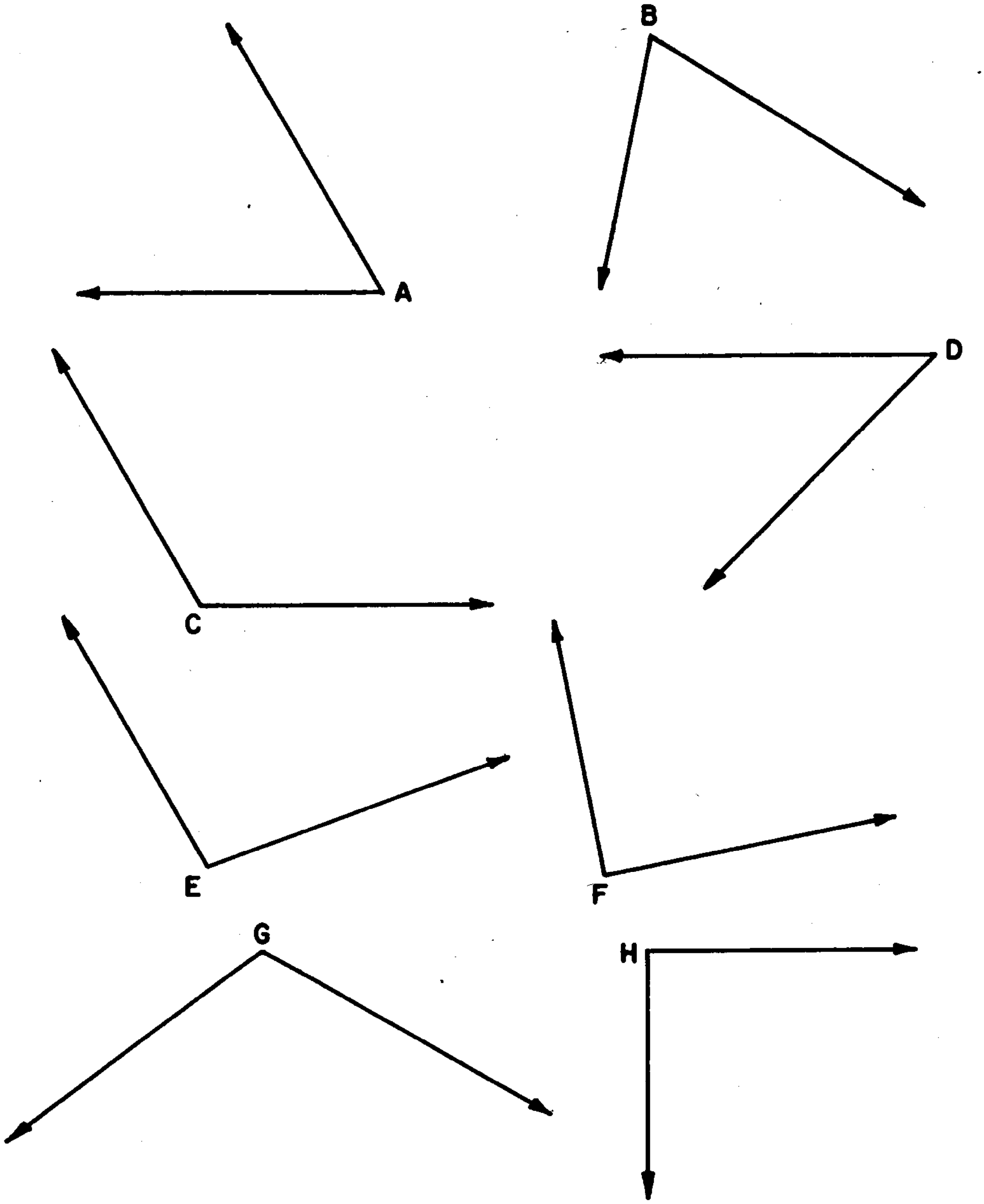
   m ∠ K; 135 or 175

   m ∠ L; 45 or 90



4. Measure each of the angles in Exercise 3 in degrees.

5. Estimate the measure of each angle in degrees. Write your answers like this: $m \angle A$ is about ____.



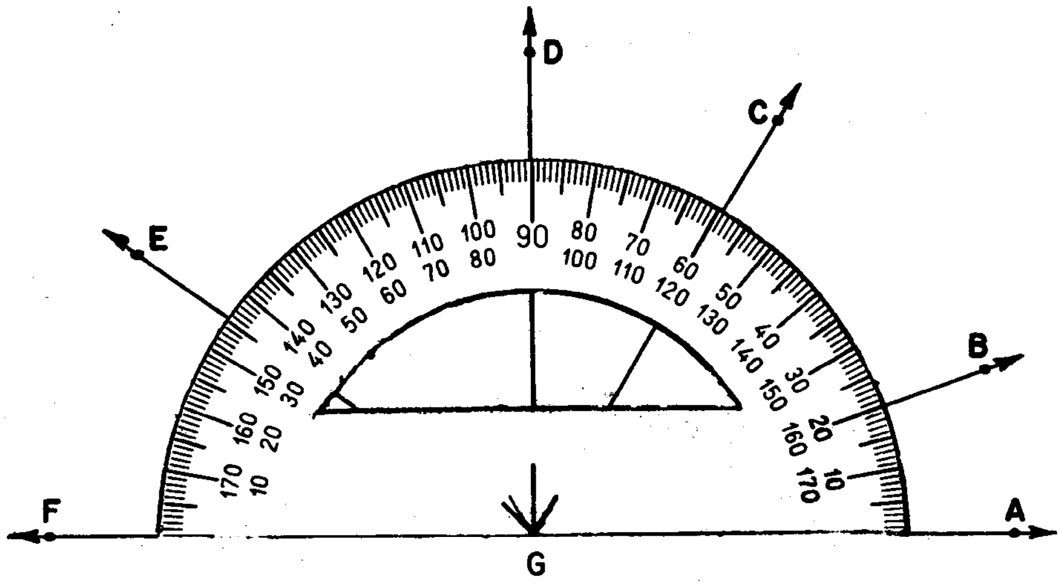
6. Measure, in degrees, each angle in Exercise 5 with your protractor.

7. Use your straightedge to draw an angle which you think has the size given below. Then measure the angle with a protractor to see how closely the measures agree with your estimates.

	ANGLE	SIZE
a)	R	45°
b)	M	10°
c)	N	165°
d)	P	80°
e)	D	120°
f)	Q	178°

SUM OF THE MEASURES OF ANGLES

1. Copy and complete the following table. Find the measures of the angles from the sketch. (When the unit angle is not mentioned, use the measure in degrees.)



	NAME OF ANGLE	SIDES	MEASURE	
Pair 1	$\angle$ AGB	$\overrightarrow{GA}$, $\overrightarrow{GB}$	____	Sum of measures _____
	$\angle$ BGF	$\overrightarrow{GF}$, $\overrightarrow{GB}$	____	
Pair 2	$\angle$ AGC	_______	____	Sum of measures _____
	$\angle$ CGF	_______	____	

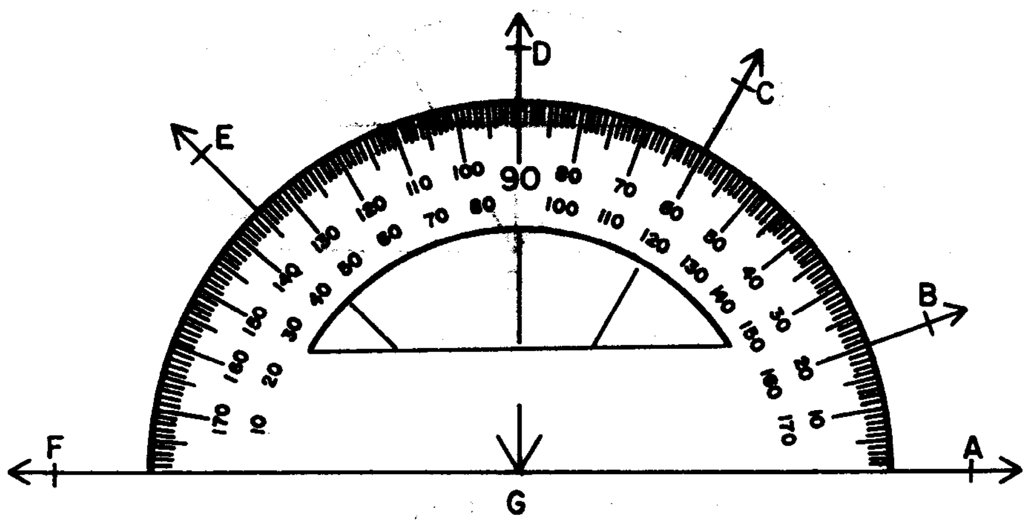
2. Trace with your finger the rays which form the angle pair, Pair 1, in Exercise 1.

   a) What ray is a side of both angles?

   b) What can you say about the other two rays? What is their intersection? What is their union?

   c) Do the interiors of the angles intersect?

   d) What is the sum of their measures?



3. Trace with your finger the rays which form angle pair, Pair 2. Answer the questions as in Exercise 2 about this pair. Are your answers to questions b, c, and d the same as for Pair 1?

4. Find a third pair of angles in the sketch for which the answers to questions b, c, and d are the same as for Pair 1 and Pair 2. Trace their rays with your finger.

5. When these rays intersect at the same point and two of the rays form a line, what can you expect will be the sum of the measures of the two angles formed?

6. List the names of all of the angles in the sketch whose interiors do not intersect. (There are five.)

7. Find the measure of each angle in your list.

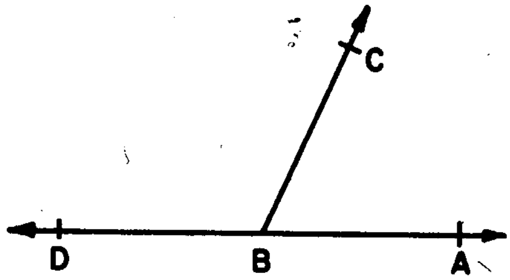
8. Find the sum of the measures of the five angles.

9. What conclusion can you reach from Exercise 6-8?

10. Name a pair of angles with $\overrightarrow{GF}$ a side of one angle, $\overrightarrow{GA}$ a side of the other, and $\overrightarrow{GD}$ a side of both angles.

11. What is the measure of $\angle$ AGD? $\angle$ DGF? What kind of angle is $\angle$ AGD? $\angle$ DGF?

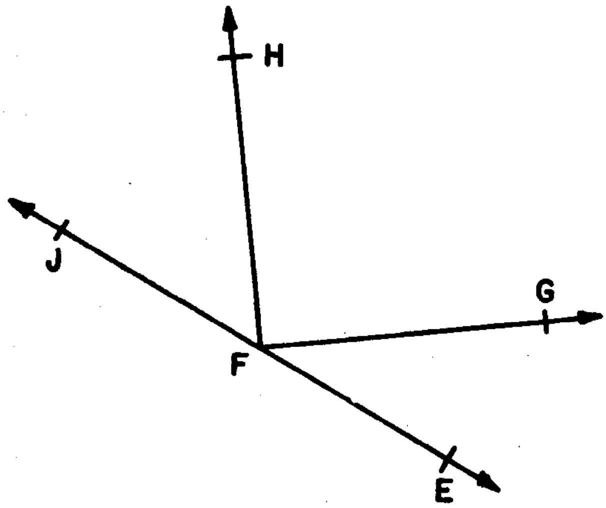
Exercise Set 8



Use this figure for Exercises 1 and 2. $\overrightarrow{BA}$ and $\overrightarrow{BD}$ are on the same line.

1. If $\overleftrightarrow{DA}$ is a straight line, $m\angle ABC + m\angle CBD =$ _____

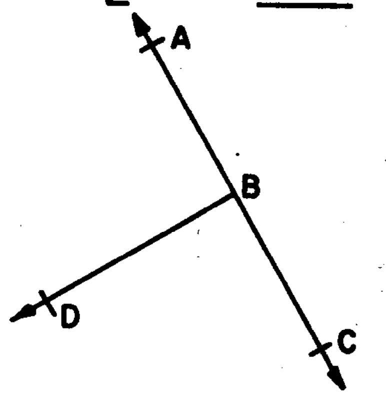
2. If $m\angle ABC = 65$, then $m\angle CBD =$ _____



Use this figure for Exercises 3 and 4. $\overrightarrow{FE}$ and $\overrightarrow{FJ}$ are on the same line.

3. Is it true that $m\angle EFH + m\angle JFG + m\angle HFG = 180$? If not, what true statement can you make?

4. If $m\angle JFH = 58$ and $m\angle EFG = 36$, then $m\angle HFG =$ _____



5. If $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are on a straight line, and $\angle ABD \cong DBC$, then $m\angle ABD =$ _____ and $m\angle DBC =$ _____. $\angle ABD$ and $\angle DBC$ are _____ angles.

SUMMARY

1. If three rays have the same endpoint, and two of the rays form a line, then the sum of the measures, in degrees, of the two angles formed is 180.

2. If several rays are drawn from a point on a line, all on the same side of the line, the sum of the measures, in degrees, of all the angles formed whose interiors do not intersect is 180.

3. If one ray is drawn from a point on a line and the two angles formed are congruent, each angle is a right angle and its measure, in degrees, is 90.

Chapter 8

AREA

WHAT IS AREA?

Comparing Sizes of Regions

Exploration

You have had experience in comparing the sizes of line segments and the sizes of angles. Look around your classroom. Find representations of two line segments which are not the same length. Can you tell without making any measurement which is longer? Find representations of two angles, which are not the same size. Can you tell which is larger without using the compass or protractor?

Exercise Set 1

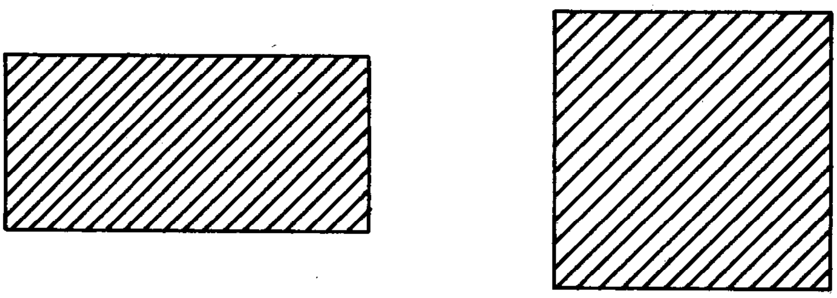
In each of the following, tell which is larger:

1. A sheet of typing paper or a stamp.
2. A pin head or a dinner plate.
3. A pillow case or a bed sheet.
4. A television screen or a motion picture screen.
5. A nickel or a dime.
6. A wash cloth or a handkerchief.
7. A window or its window shade?
8. Your classroom floor or your classroom ceiling.
9. The sole of your shoe or the sole of your friend's shoe.
10. A sheet of your notebook paper or this page of your text.
11. Did you know the answer to the above exercises immediately? Were there some cases where you were not certain, at once, which was larger? How did you decide?
12. Will the original size of a sheet of your notebook paper change if you fold it into four parts? How will you test to see if it has remained the same size?
13. Does the size of your bath towel change when it is wrapped around your body?
14. What happens to the size of a map when you roll it up?

Length of a Curve or Size of the Surface Enclosed by It

Exploration

Sometimes we compare sizes of objects by comparing lengths and sometimes we compare sizes by comparing the sizes of flat surfaces. Suppose we have pictures of two rectangular fields:



If we wish to compare the amounts of fencing we need to enclose these fields, what property of the rectangles will we compare? Remember that a rectangle is a simple closed curve and if we measure a simple closed curve we are finding its length.

We might, however, be interested in dividing one of the fields so that half would be planted in corn and half in beans. Would we need to know the length of the rectangle? Would it be helpful to know the size of the surface of the field?

Exercise Set 2

Tell whether you are interested in the length of a simple closed curve or the size of the surface in its interior, or both:

1. To trim the edge of a handkerchief with lace.

2. To buy a rug to cover the living room floor.

3. To buy a desk blotter for your desk.

4. To put a book cover on your text.

5. To string enough beads for a necklace.

6. Can you give 3 other examples of situations in which you would need to know the size of the surface enclosed by a simple closed curve rather than just the length of the curve?

Exploration

1. Recall that a simple closed curve by our definition is a path having the following properties:

   a. All of its points lie in a plane.

   b. If one traces the path, he eventually returns to the starting point.

   c. The path never intersects itself; i.e., in proceeding once around the path, any point is encountered just once (except for the starting point).

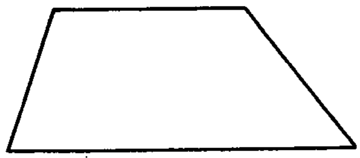
   It also has the property that it separates the plane into three sets of points: the set of points of the curve, the set of points of the interior of the curve, and the set of points of the exterior of the curve.

2. Which ones are simple closed curves?

a.

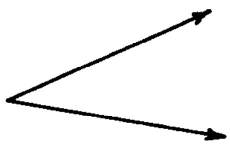
b.

c.



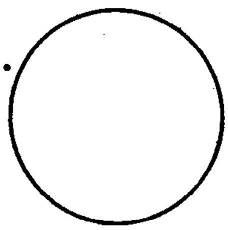
d.

e.

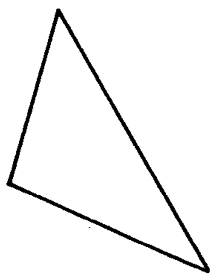


f.

g.

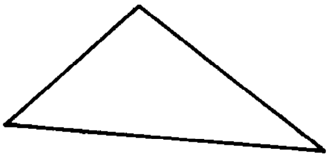


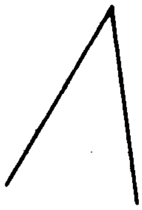
h.

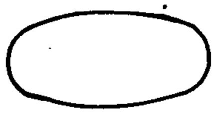


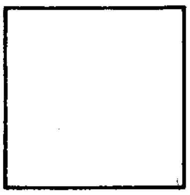
The union of a simple closed curve and its interior is called a plane region. If the curve is a triangle, the plane region is called a triangular region. The union of a polygon and its interior is called a polygonal region. It is the region that we measure when we want to know the size of part of a flat surface.

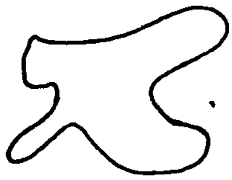
If the curve is a simple closed curve, trace the curve and color the plane region:

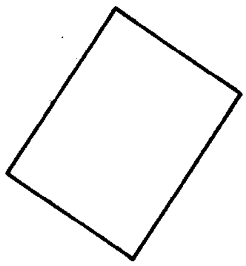
a. 

b. 

c. 

d. 

e. 

f. 

g. 

h. 

Exercise Set 3

Tell whether you would be interested in area or length, or both, in each of the following:

1. To buy enough wrapping paper for a package.
2. To decide on the amount of twine needed to wrap a package.
3. To decide on the size of a belt.
4. To buy a piece of land.
5. To mow a lawn.
6. To run around a closed track.
7. To sail around an island.
8. To tile a basement floor.
9. To measure a triangle.
10. To measure a triangular region.

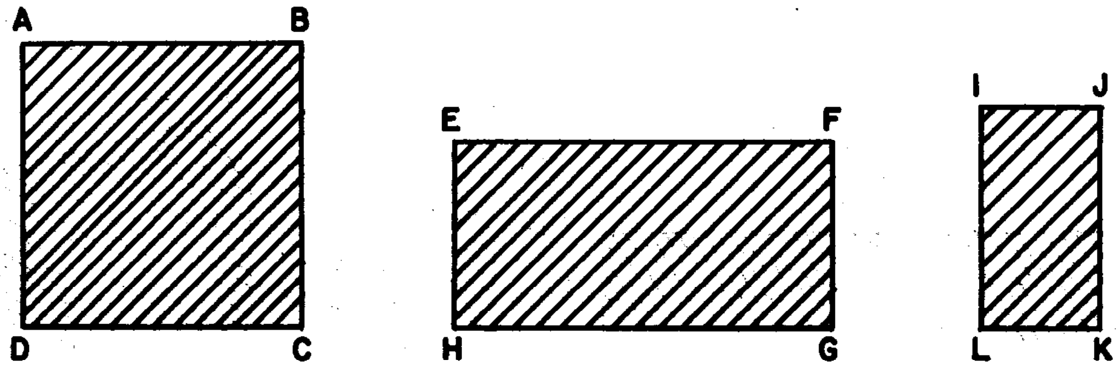
COMPARING AREAS

The areas of two plane regions usually can be compared by seeing whether one region may be <u>included</u> in the other. That is, if one plane region can be placed entirely in the interior of the other, then the area of the first region is smaller than the area of the second region.

A plane region can be cut up into smaller plane regions. When we cut up a plane region, we say we are <u>dissecting</u> it. To dissect something means to cut it into parts or pieces. Suppose you can dissect one plane region and place all the pieces, without overlapping, entirely on a second plane region. What would this show about the areas of the two regions?

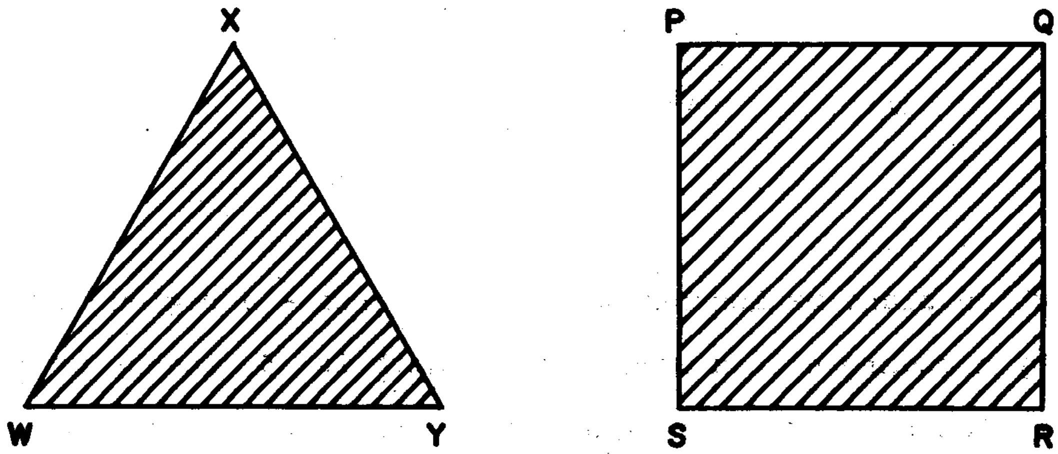
Exploration

Which rectangular region has the smallest area? Will a tracing of one of them fit into the interior of each of the others?



Which rectangular region has the greatest area? Will a tracing of either figure fit into the interior of the other? How can the areas be compared? Cut a tracing of rectangular region EFGH into small pieces. Can all of these small pieces be placed, without overlapping, in the interior of rectangle ABCD?

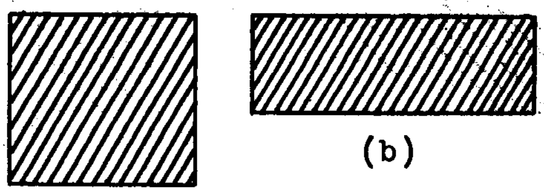
Is the area of triangular region WXY less than the area of rectangular region PQRS?



Exercise Set 4

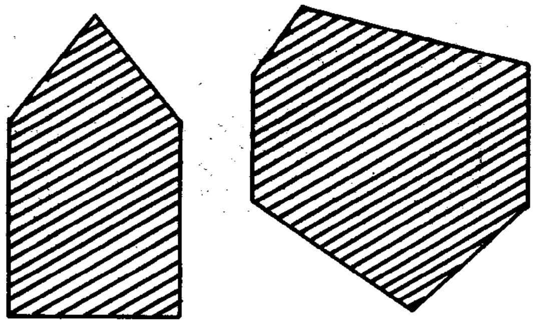
In Exercises 1-3 tell which region of each pair has the greater area. (You may make a paper model of one of these regions and cut it to see if the pieces can be placed, without overlapping, on the other region.)

1.



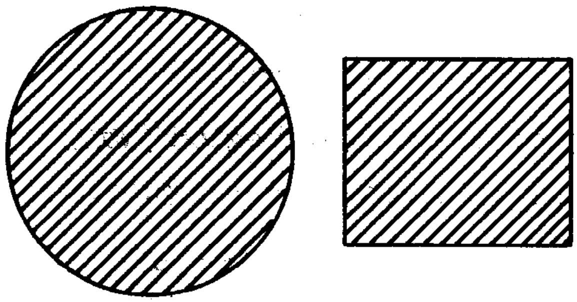
(a) (b)

2.



(a) (b)

3.



(a) (b)

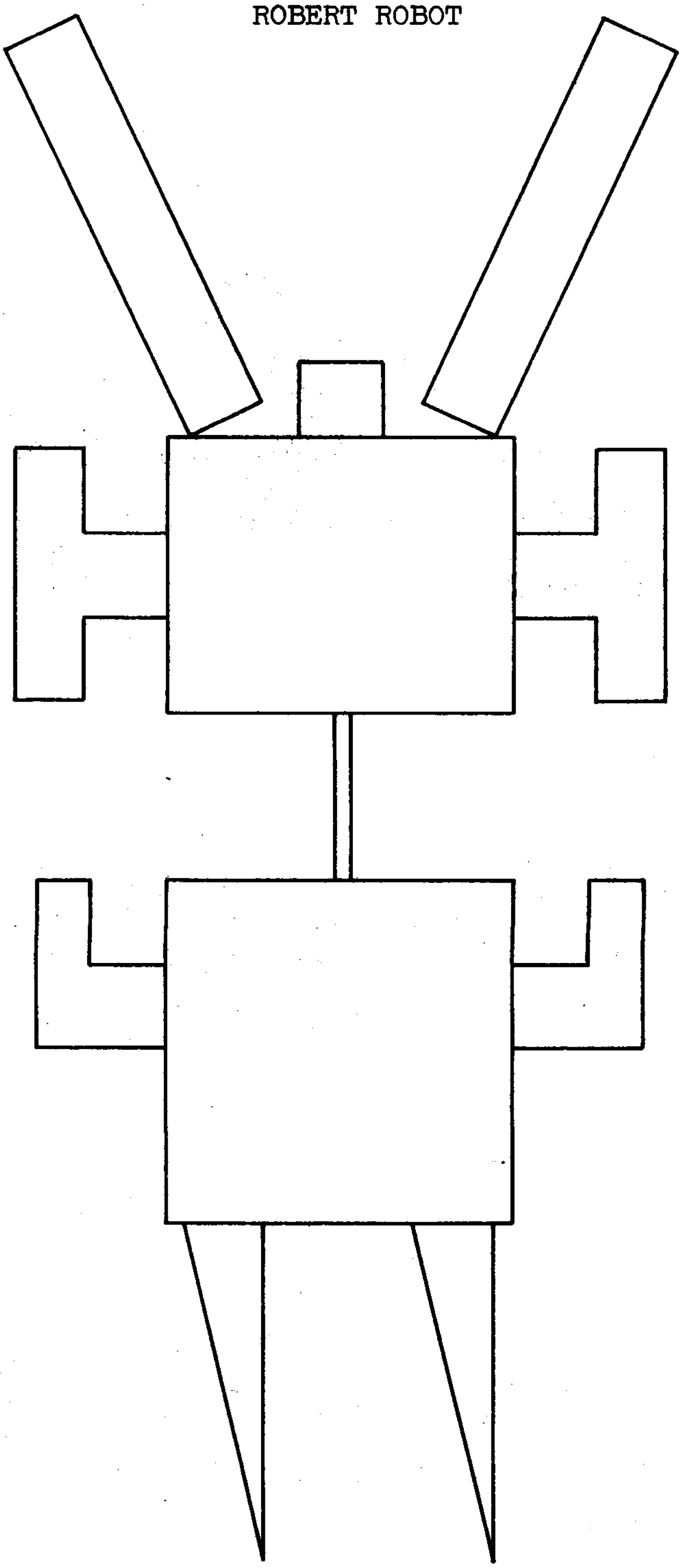
4. Which plane region has the greater area - a region bounded by a square with a side whose length is 3 inches or a region bounded by an equilateral triangle with a side whose length is 4 inches? You will need models of these regions.

BRAINTWISTER.

Trace "Robert Robot." Can you arrange the parts of the "robot" in such a way that they form a rectangular region?

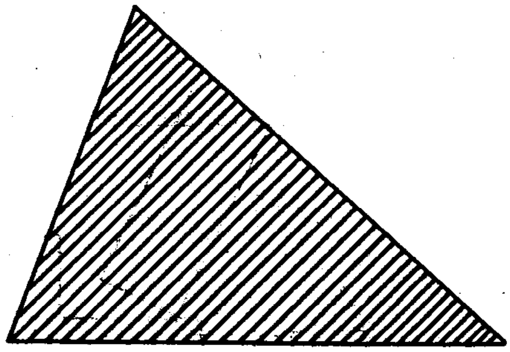
The rectangle will have sides whose lengths are $2\frac{1}{8}$ inches and $4\frac{5}{8}$ inches.

ROBERT ROBOT



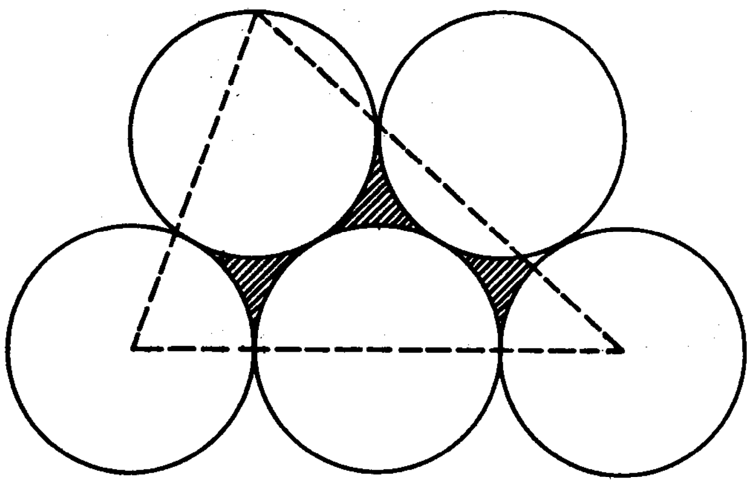
UNITS OF AREA

Choosing a Unit of Area

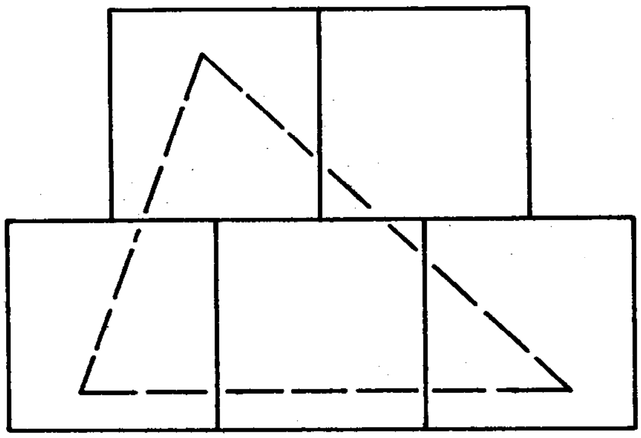


This is a picture of a triangular region. Suppose we wish to measure its area. When we measured the length of a line segment, we needed a unit of length. To measure the area of a region, like this triangular region, we need a unit of area, a unit region.

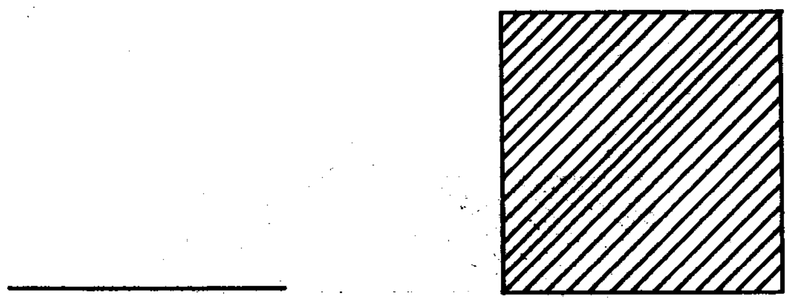
We need to cover the whole region to be measured by placing unit regions on it so that they touch but do not overlap. Is it possible to cover a whole triangular region with circular regions in this way? Why not?



Is it possible to cover the triangular region with square regions? Why?



Let us choose a square region as unit of area. We choose a square region whose side is just one unit of length.

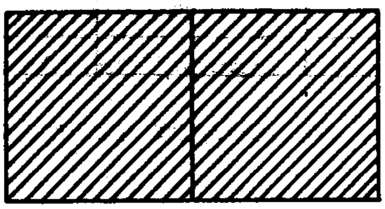


unit of length unit of area

Differently Shaped Regions of Same Area

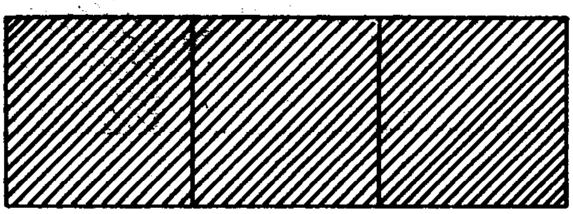
Exploration

Each of you has some square pieces of paper all of the same size. Each piece represents 1 unit of area. Place two pieces side by side on a sheet of paper so that they touch but do not overlap. Trace around the region formed by these pieces. Does your picture look like this?

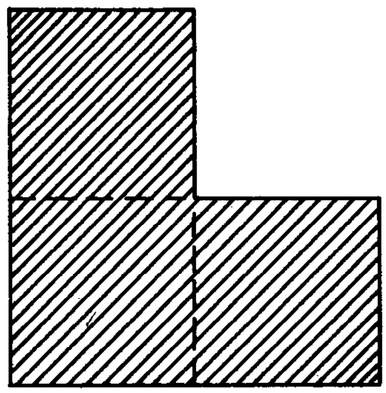


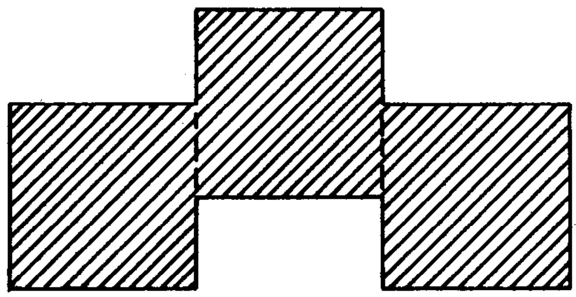
What is the figure you have drawn? Color the rectangle and its interior. What is the figure you have colored? What is the area of this region?

Draw and color a rectangular region of area 3 units. Does your picture look like this?

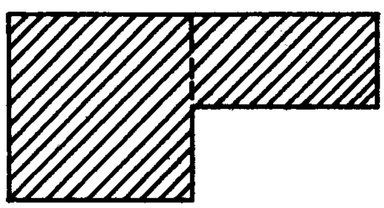


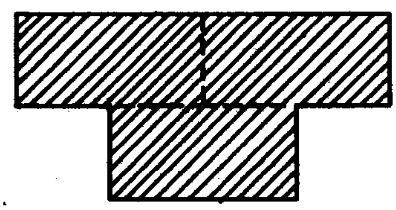
Here are some regions of different shapes, each with area 3 units. Can you think of some others?





Here are some regions of different shapes, each with area of $1\frac{1}{2}$ units. Can you think of some others?





Exercise Set 5

1. Use your square region of paper to trace out and color a region of area 5 units. Make the region any shape you wish.

2. Use your squares of paper (you may want to fold one of them) to trace out and color a region of area $2\frac{1}{2}$ units. Make the region any shape you wish.

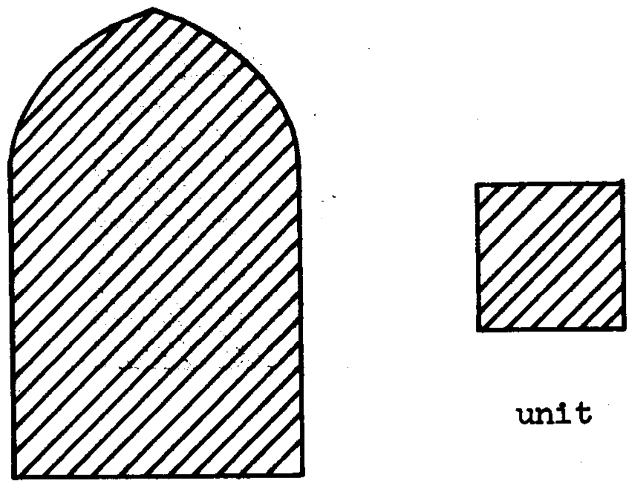
3. Take two of your unit square regions of paper and cut each of them into at least three polygonal regions. Now make a new region of different shape, using <u>all</u> your pieces. What is the area of this new region?

ESTIMATING AREAS

Using Unit Regions to Estimate Areas

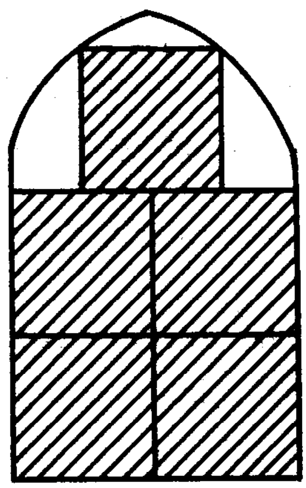
Exploration

Suppose we wish to estimate the area of a region with a curved boundary along the top, like a church window, in terms of the unit shown.



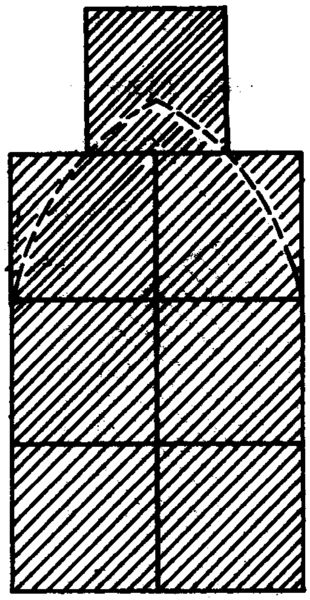
unit

We can fit units into this region as suggested by the picture below.



What does this show about the area of the region?

We can also cover this region with unit regions, as shown below.



What does this show about the area of the region?

We have not found the exact area of this region, but we now know it is a number of units (not necessarily a whole number) somewhere between 5 and 7.

Can you guess from the picture about what the area is?

Exercise Set 6

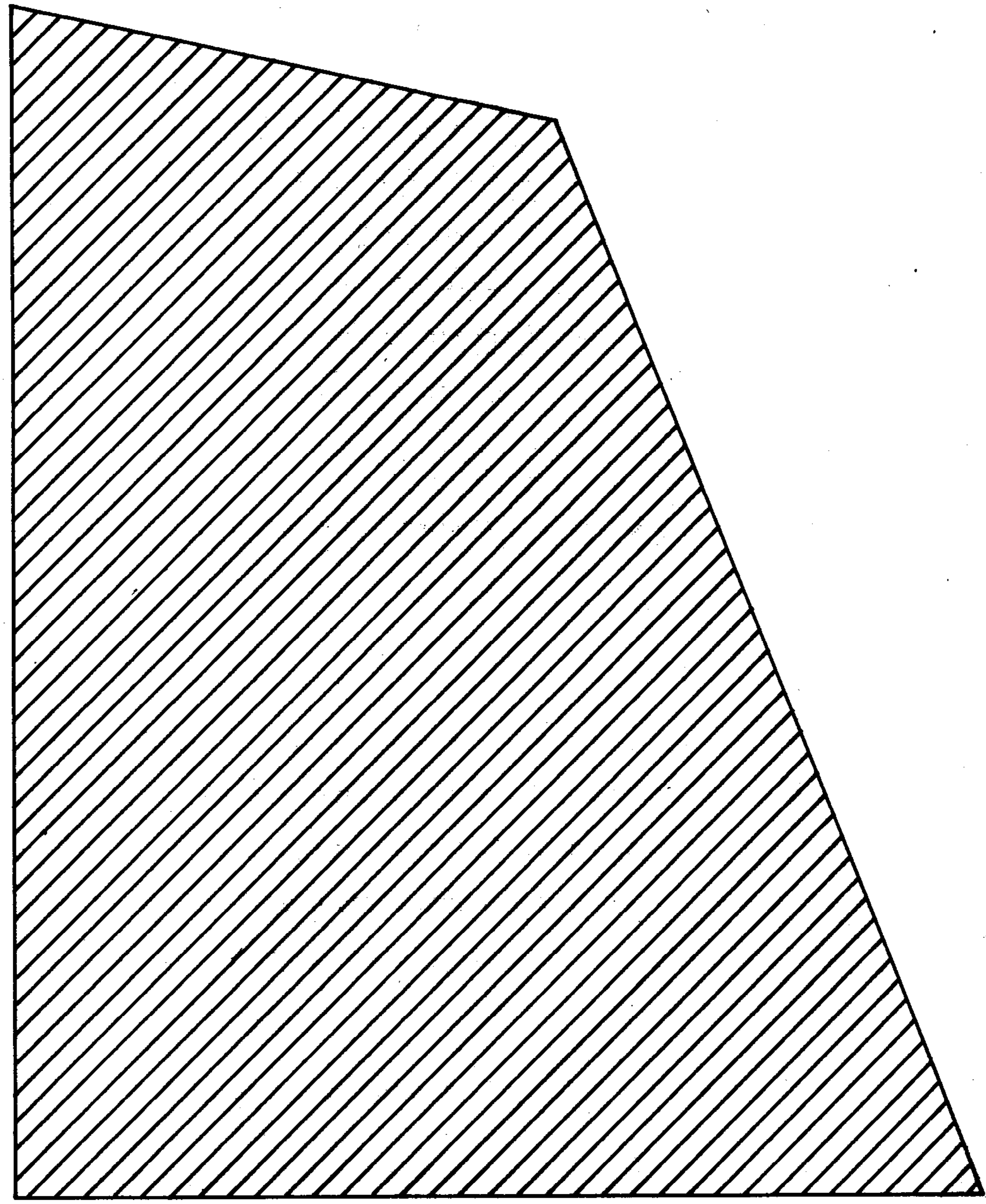
1. On the next page is a quadrilateral region. See how many of your square pieces of paper you can place entirely on this region. Be sure that no piece goes outside the region and that no piece overlaps another piece. How many pieces are you able to place on the region?

   What does this tell you about the area of the region?

2. Next, see how many of your square pieces of paper you need to cover the region completely. No piece should overlap another piece. How many pieces do you use to cover the region?

   What does this tell you about the area of the region?

   Can you estimate about what the area might be?



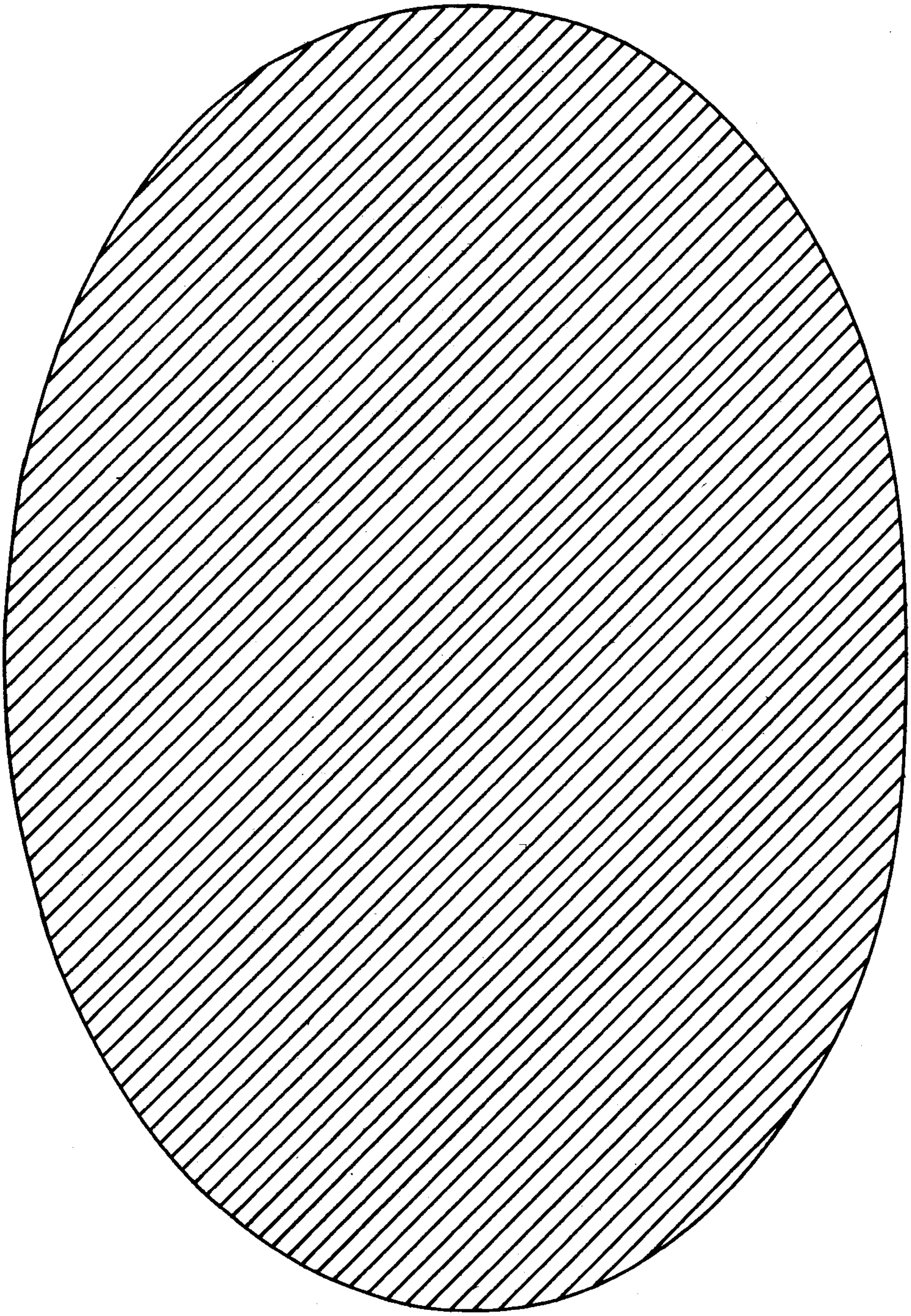
3. On the next page is a picture of an oval region. See how many of your square pieces you can place entirely on this region. Be careful that no piece goes outside the region and that no piece overlaps another piece. How many pieces are you able to place on the region?

What does this tell you about the area of the region?

4. How many of your square pieces of paper do you need to cover the region completely?

What does this tell you about the area of this region?

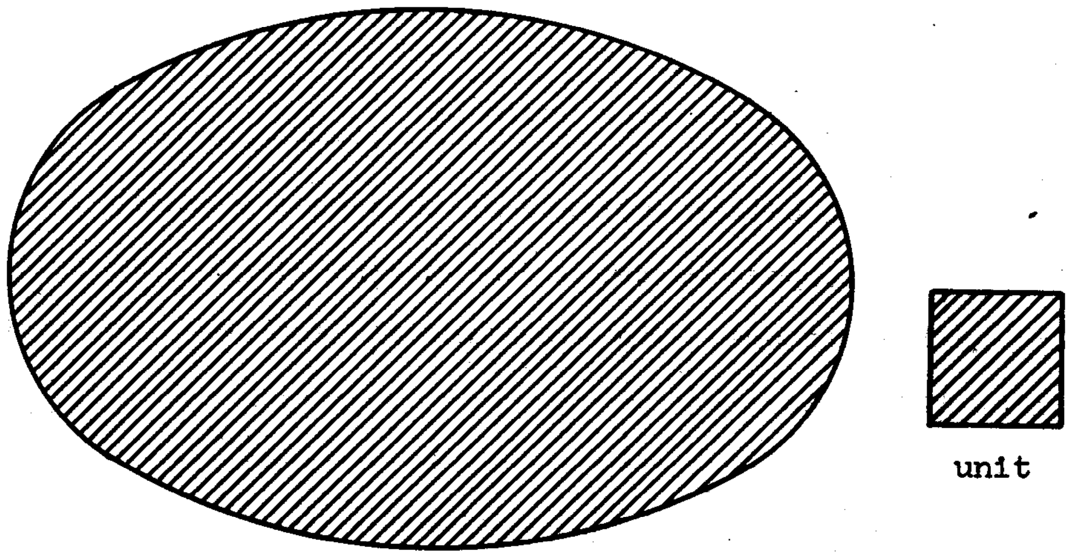
Can you estimate about what the area might be?



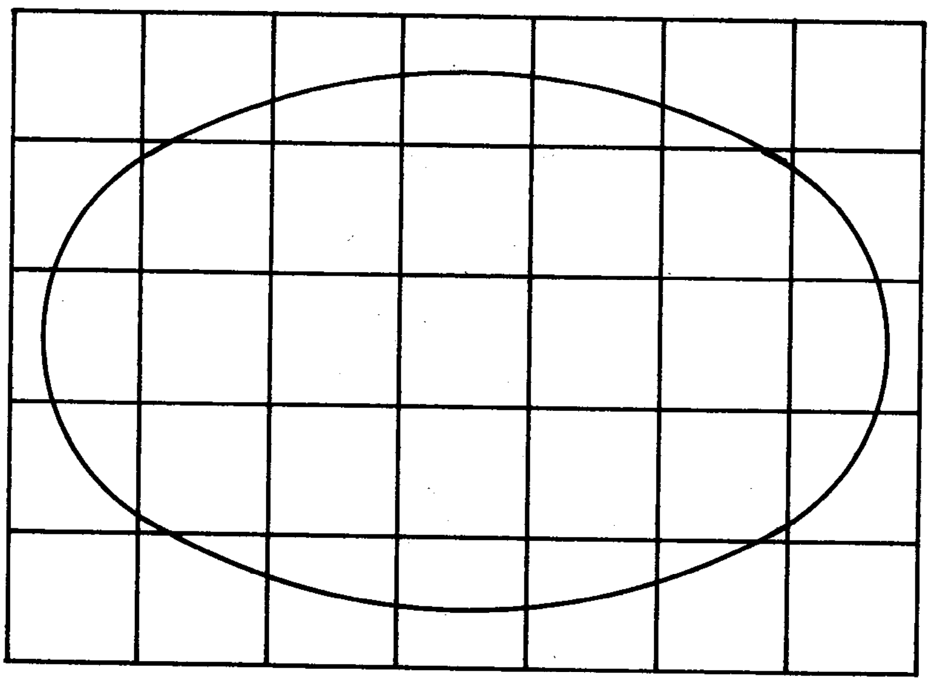
Using Grids to Estimate Areas

Exploration

Suppose we wish to measure the area of the oval region below in terms of the unit shown.

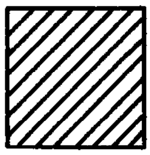


We do not have to use square pieces of paper. Instead we can draw this oval on a grid of units as shown below.



Count the units that are contained entirely in the oval region. How many are there? What does this tell about the area of the region? Count the units needed to cover the oval region completely. How many are there? What does this tell about the area of the region? The area of the oval region is somewhere between ___ units and ___ units. Looking at the figure, can you guess <u>about</u> what the area would be?

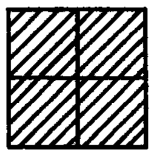
We can get a better estimate of this area by using a smaller unit. Suppose we use a new unit of length just half as long as the old one. The resulting old and new units of area look like this:



old unit

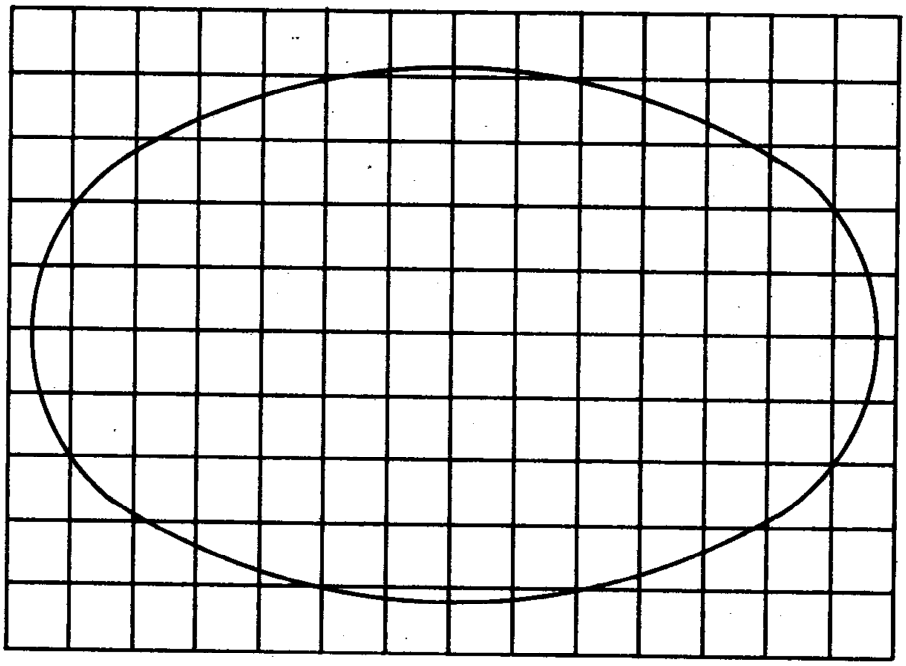
new unit

How many new units does each old unit contain?



We have already found that the area of the oval region is somewhere between 11 old units and 31 old units. In terms of the new unit, what does this tell us about the area of the oval region? How do you know?

Now let us use a grid of the new smaller units to get a better estimate of this area.



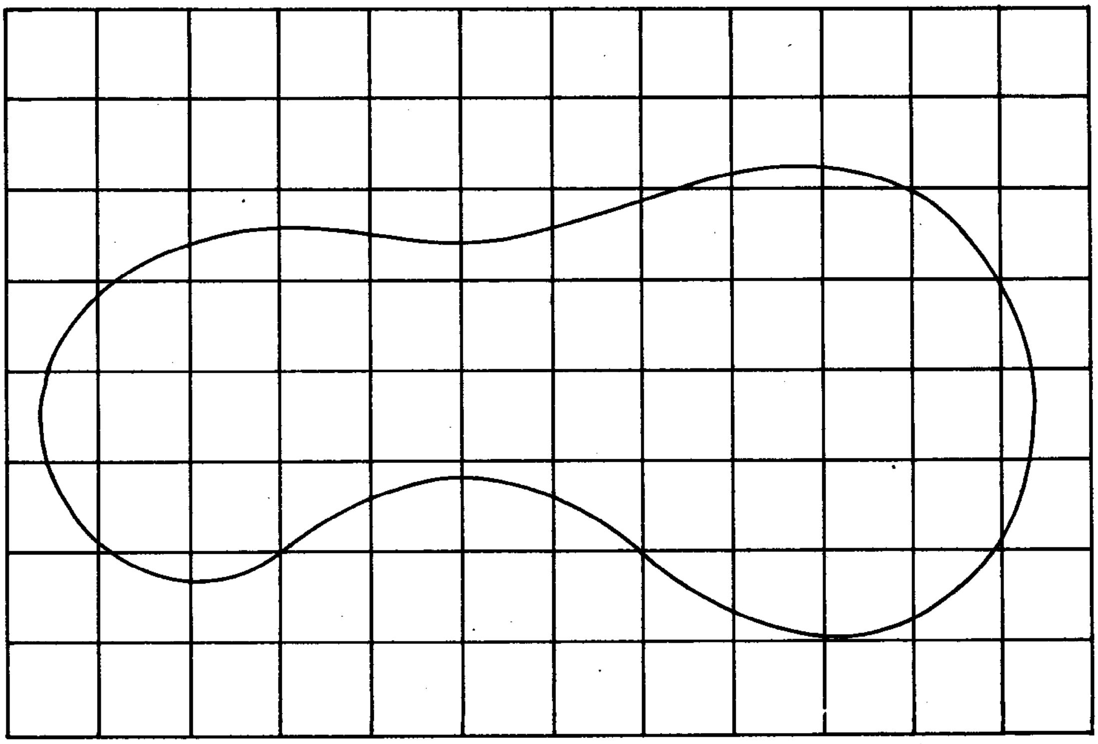
Count the units that are contained entirely in the oval region. How many are there?

Count the units that are needed to cover the oval region completely. How many are there? What does this tell about the area of the region?

Thus, we now know that the area of the region is somewhere between 69 and 108 units. Is this better than our old estimate?

<u>Exercise Set 7</u>

1. a. Consider the region pictured below on a grid of units.



Fill in the blanks:

There are ___ units contained entirely in the region.

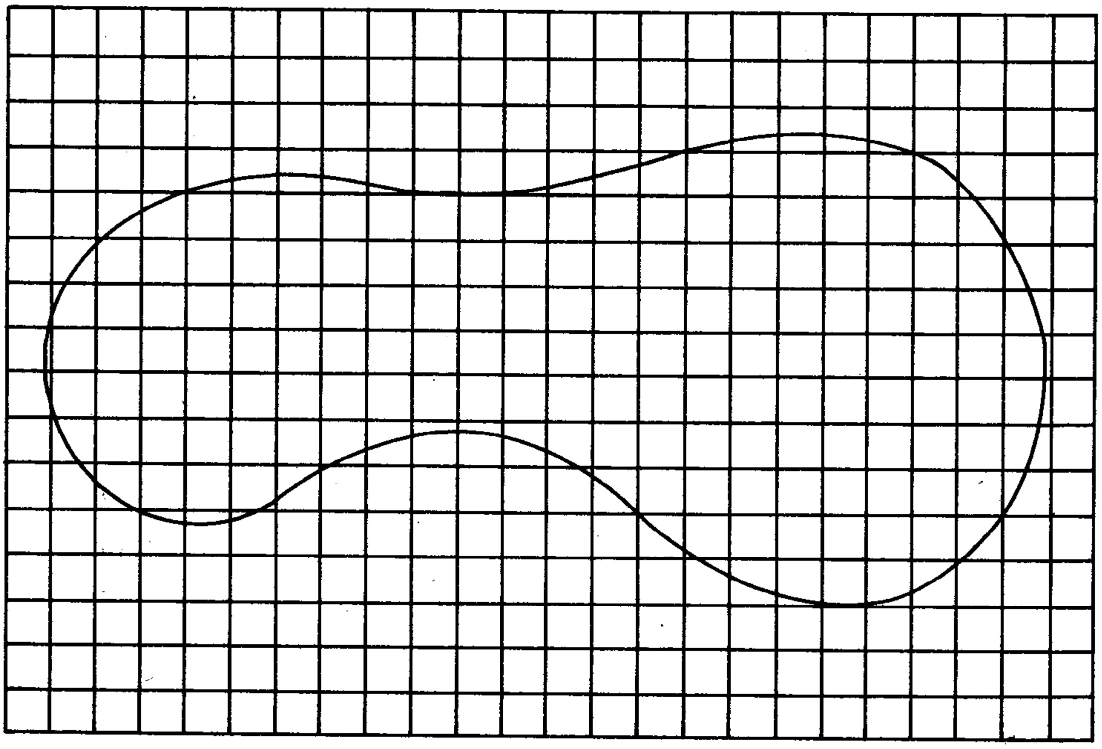
There are ___ units needed to cover the region completely.

The area of the region is at least ___ units and at most ____ units.

Let us choose a new unit of area a square region has as its side a segment just half as long as before. For every old unit of area, we will then have 4 new units of area.

In terms of the new unit, we could say that the area of the region shown on the previous page is at least _____ new units and at most _____ new units.

b. Consider the same region pictured below on a grid of new units.



Fill in the blanks:

There are ____ units contained entirely in the region.

There are ____ units needed to cover the region completely.

The area of the region is at least _____ units and at most _____ units.

Is this estimate better than the estimate you made using the larger unit?

2. a. On a sheet of paper ruled with 1 inch squares, draw a representation of a simple closed curve. Estimate the area of the region formed by the simple closed curve and its interior.

b. On a sheet of paper ruled with $\frac{1}{2}$ inch squares, trace the simple closed curve you drew in part (a) of this exercise. Estimate the area of the region formed by the simple closed curve and its interior.

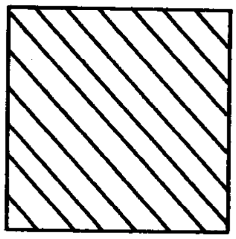
c. Which estimate, the one in part (a) or the one in part (b), is the more precise?

STANDARD UNITS OF AREA

The Basic British-American Units

Exploration

To measure the area of a region, we first have to choose a unit of area. The most convenient unit of area is a region square in shape. Can you think how hard it would be to talk about areas if each of us chose his own different unit of area? People have found it is simpler if everyone agrees to use the same few units of area. We call these <u>standard units</u>. One standard unit is a square region with sides 1 inch long like this.



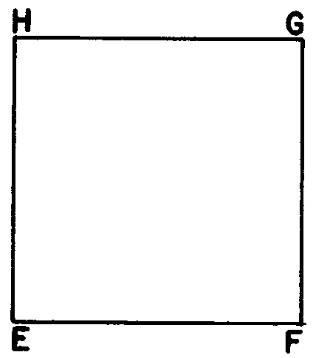
We call this unit of area the <u>square inch</u>. Would the square inch be a convenient unit for measuring the area of a sheet of writing paper?

Would the square inch be a convenient unit for measuring the area of the classroom floor? Why not?

Can you suggest a better unit for measuring the area of the classroom floor.

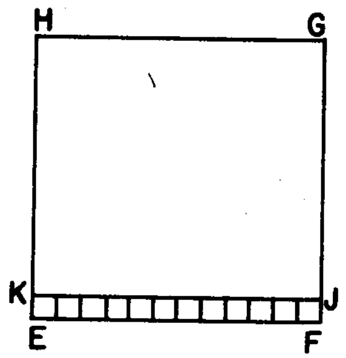
Can you explain what a square foot is?

At the right is a small picture of a square. Your teacher will use a model whose side is actually one foot long. Let us pretend that the length of the side of square EFGH is 1 foot. How many squares of side 1 inch in length could you place, touching but not overlapping, with one side on $\overline{EF}$ as shown in the figure?

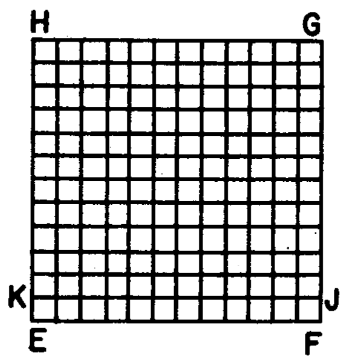


What is the area of region EFJK?

How many regions the size of region EFJK could you place in region EFGH?



Since you can place 12 regions the size of EFJK in the region EFGH, and since the area of region EFJK is 12 square inches, then, the measure of region EFGH (where the unit is the region whose area is one square inch) is 12×12 or 144.



Thus, an area of 1 square foot is the same as an area of 144 square inches. The area of 2 square feet is the same as an area of 288 square inches. An area of 72 square inches is the same as the area of $\frac{1}{2}$ of a square foot (since $72 = \frac{1}{2} \times 144$).

Suppose you wished to measure the area of the whole United States. Would you use the square inch? the square foot? the square yard? Why not?

Can you suggest a better unit for measuring the area of the United States?

Exercise Set 8

1. Make a table showing the number of units required to cover these regions:

	Number	Unit
Square, side 1 inch long:	____	square inch.
Square, side 1 foot long:	____	square foot,
or	____	square inches.
Square, side 1 yard long:	____	square yard,
or	____	square feet,
or	____	square inches.

2. Here are listed areas of some regions. Write each area in at least one other way, using different units.

a. 6 square feet		f. 7 square feet
b. 4 square yards		g. 32 square feet
c. 10 square feet		h. 1296 square inches
d. 288 square inches		i. 5 square yards
e. 300 square inches		j. 16 square feet

3. BRAINTWISTER

Find different measures for the area listed below, changing the units of measure as noted:

a. 4 square yards: ____ square feet

b. 5 square feet: ____ square inches

c. 2 square yards 18 square inches: ____ square feet ____ square inches

d. 7 square feet 24 square inches: ____ square inches

e. 20 square feet: ____ square yards ____ square feet

f. 324 square inches: ____ square feet ____ square inches

g. 2000 square inches: ____ square yards ____ square inches

h. 36 square inches: ____ square foot

i. 2 square feet: ____ square yard

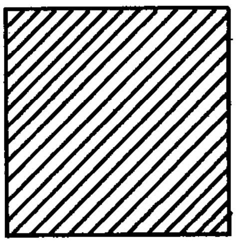
j. 18 square inches: ____ square foot

Area in Square Centimeters

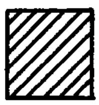
Exploration

All these units--the square inch, the square foot, and the square mile--are units of area in the British-American system of measures. Do you know what system of measures is used in most countries? What unit of length in the Metric System corresponds most closely to the yard in the British-American System? What unit of length in the Metric System corresponds most closely to the inch in the British-American System? Do you know how the meter and the centimeter compare in size? What unit of area in the Metric System would you get by taking a square region with each side 1 centimeter long?

Even in Britain and America it is the Metric System that is used for scientific measurements. Therefore, we sometimes need to compare units of area in the British-American system with units of area in the Metric System. Here is a picture of the square inch and the square centimeter.

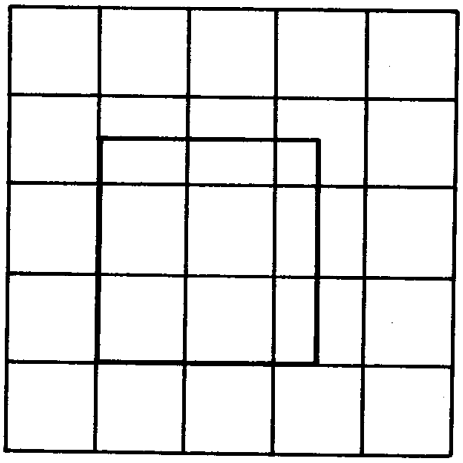


square inch



square centimeter

Which is larger, the square inch or the square centimeter? What would you estimate is the area in square centimeters of the square inch region pictured? How could you determine this more carefully? Here is a square inch shown on a grid of square centimeter regions.



How many of the square regions of the grid are contained entirely in the square inch region?

What does this show about the area of this region?

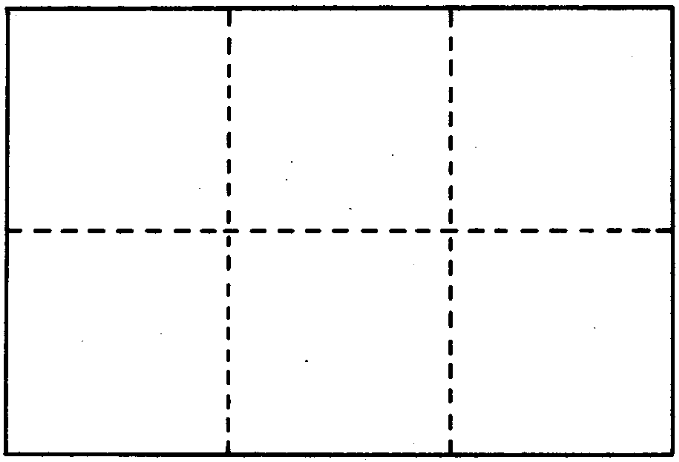
How many of the square regions of the grid are needed to cover the square inch region completely?

What does this show about the area of the region?

Can you now guess this area more accurately?

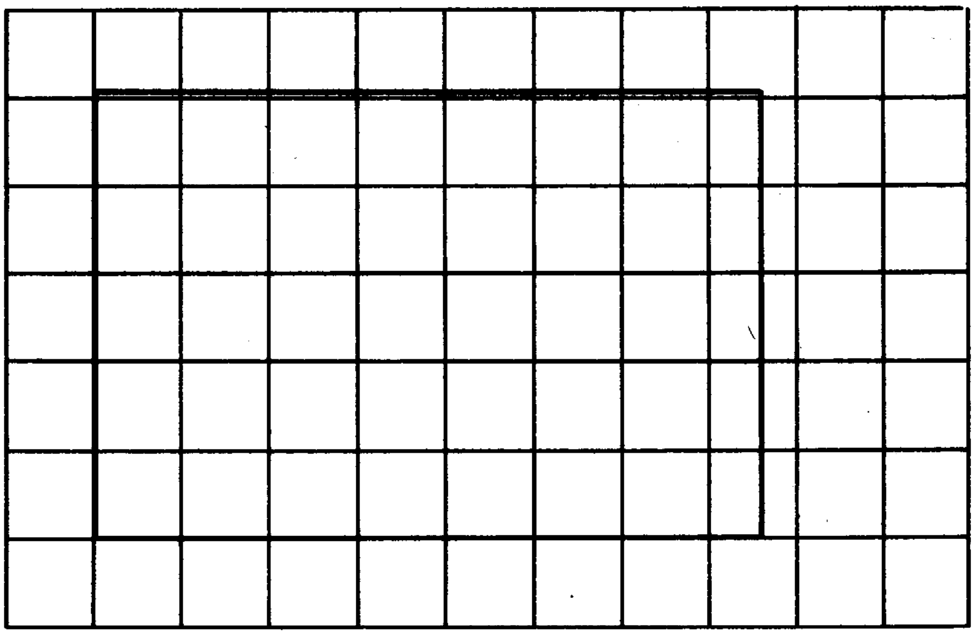
Exercise Set 9

1. Suppose we have a rectangular region with adjacent sides of length 2 inches and 3 inches.



What is the area of the region in square inches?

Below is a picture of this same rectangular region on a grid of square centimeter regions. Use this picture to estimate the area of the rectangular region in square centimeters. If you need questions to guide you, look on the next page.



The following questions should help you to find an estimate:

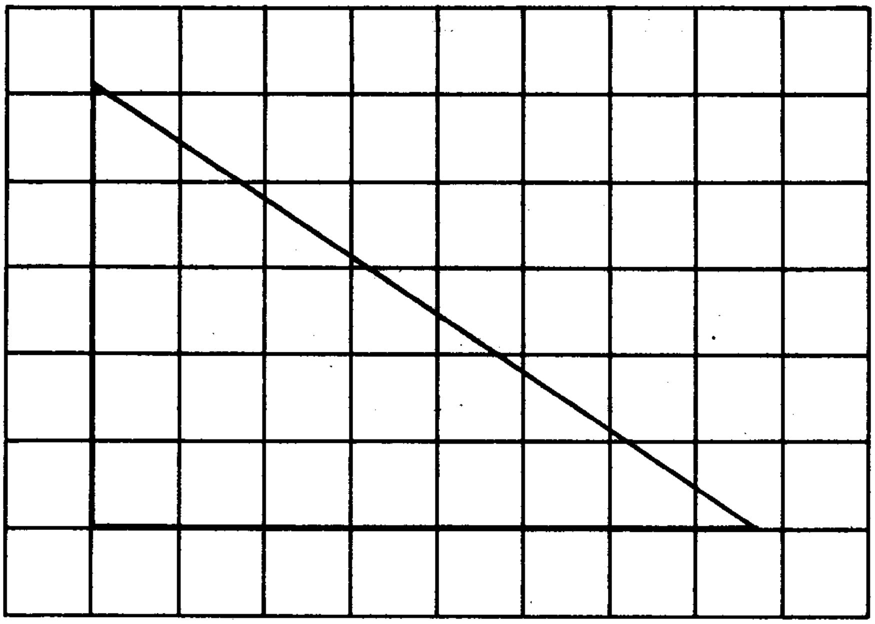
a. How many square regions of the grid are contained entirely in the rectangular region? What does this tell about the area of the region?

b. How many square regions of the grid are needed to cover the rectangular region completely? What does this tell about the area of the region?

c. Can you look at the rectangular region and estimate about what the area would be?

d. Fill in the blank: If the area of a rectangular region is 6 square inches, its area is about ____ square centimeters.

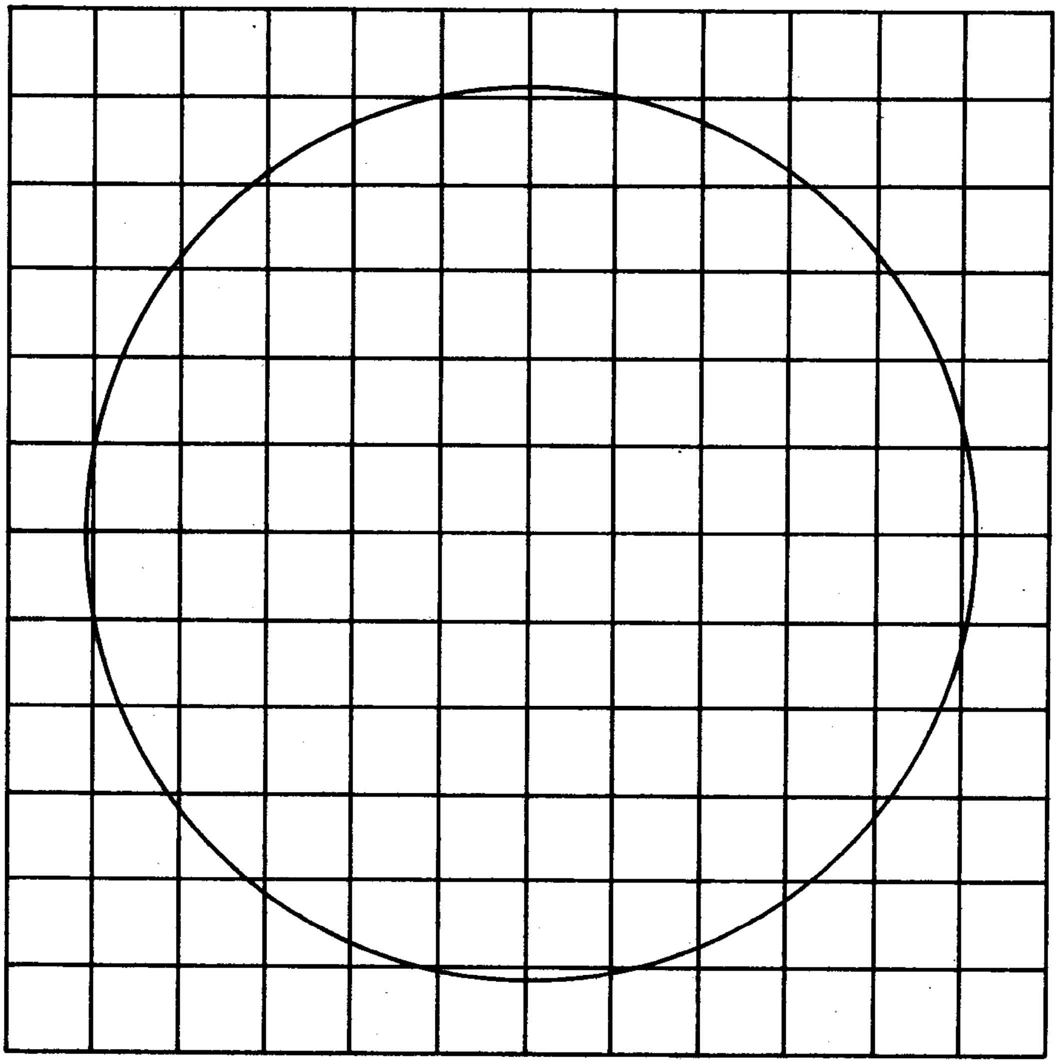
2. Below is pictured a right triangular region on a grid of square centimeter regions. The sides adjacent to the right angle have lengths 2 inches and 3 inches. Find an estimate for the area of the triangular region in square centimeters.



a. The area of the triangular region is at least ____ square centimeters and at most ____ square centimeters.

b. What would you estimate the area would be?

3. Below is pictured a circular region of radius 2 inches on a grid of square centimeter regions. Find an estimate for the area of the circular region in square centimeters.



a. The area of the circular region is at least _____ square centimeters and at most _____ square centimeters.

b. What would you estimate the area would be?

AREA OF RECTANGULAR REGIONS BY CALCULATION

Building a Rectangular Region

Exploration

You remember that a rectangle has four sides. If we know the measure of any two sides that form a right angle, then we know the measure of all four sides. Why? When we speak of "the adjacent sides of a rectangle," we will mean two sides which form part of a right angle.

Earlier in this Unit you found the area of a plane region by covering the region with models of a unit region. What is the shape of a standard unit region?

Your teacher will give you some models of unit regions, each with an area of 1 square inch. Count out twelve of these unit regions. Fit these 12 regions together, without overlapping, so that their boundary is a rectangle. How long are the sides of the rectangle? See how many different rectangular regions you can form from the 12 square regions and list the information in a chart like the one below:

Lengths of sides		Area
____	____	____
____	____	____
____	____	____
____	____	____
____	____	____
____	____	____

Now use sheets of paper ruled with 1-inch squares. Draw two rectangles of such size that the area of each of the rectangular regions is 20 square inches. (Keep each square inch unit all in one piece.) List the information in a chart as before.

Lengths of sides		Area
____	____	____
____	____	____
____	____	____
____	____	____
____	____	____
____	____	____

What do you notice about the numbers which are the measures of the sides of a rectangle and the measure of its region?

Where have we already used rectangular arrangements of square regions, quite a whole ago? What were these rectangular arrangements of square regions called? What were the square regions in an array called? How did you learn to calculate the number of elements in an array? What are we now calling the number of elements in the whole array? What are we now calling the numbers of elements in a row and in a column of the array? If the number of elements in an array is the product of the number of elements in one row and the number of elements in one column, what does this tell us about the measure of a rectangular region?

Is the following statement a fair summary of what we have been saying? Two adjacent sides of the rectangle have measures whose product is the measure of the rectangular region.

5. Draw four rectangles such that the measure of each rectangular region is 24.

6. Make and fill in a table like the one below. Get the information you need from your drawings in Exercise 3.

Measures in inches of sides of rectangle		Measure in square inches of rectangular region
______	______	______
______	______	______
______	______	______

What do you notice about the product of the measures of the sides in each case?

7. Make and fill in a table similar to that in Exercise 6. Get the information you need from your drawings in Exercise 4.

Measures in inches of sides of rectangles		Measures in square inches of rectangular region
______	______	______
______	______	______
______	______	______

What do you notice about the product of the measure of the sides in each case?

8. Make and fill in a table similar to that in Exercise 6. Get the information you need from your drawings in Exercise 5.

Measures in inches of sides of rectangle		Measure in square inches of rectangular region
______	______	______
______	______	______
______	______	______
______	______	______

What do you notice about the product of the measure of the sides in each case?

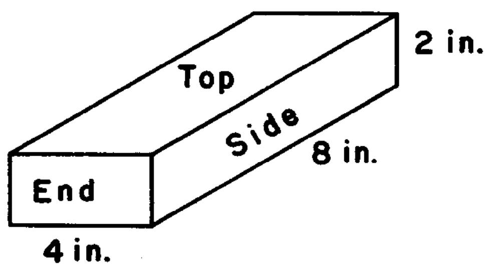
9. Suppose you are given the measures in inches of the sides of a rectangular region. In terms of these given measures, what is the measure in square inches of the rectangular region?

Exercise Set 11

1. Susan made a rectangular doll blanket whose sides were 12 inches and 10 inches long. Find the area of the blanket.

2. Peter made a plywood shelf for his model collection. The shelf was 30 inches long and 8 inches wide. What was its area?

3. Suppose the edges of a brick have the lengths as shown in this picture.



Find these areas:

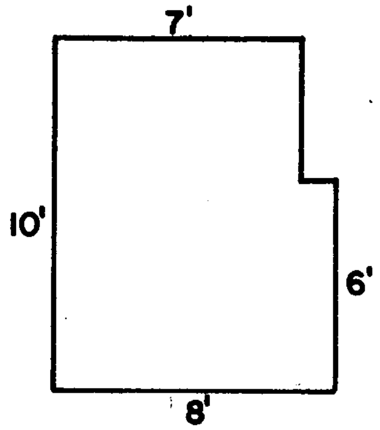
Top of the brick.

Side of the brick.

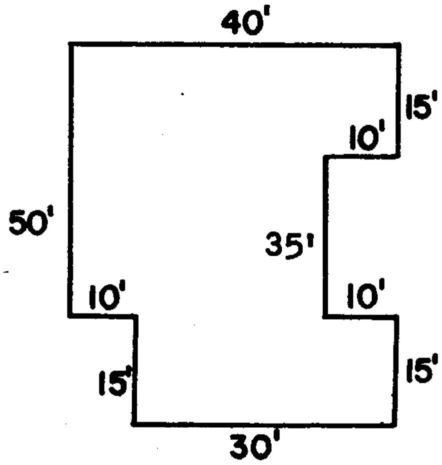
End of the brick.

Total surface of the brick.

4. Here is a picture of a kitchen floor, with the lengths of the edges shown. Find the area of the floor. (Hint: Can you draw a segment which divides the region into two rectangular regions?)



5. Here is a picture of a floor of a house, with lengths of the edges shown.



a. Find the area of the floor.

b. Can you find the area another way? How?

6. Suppose a high pressure salesman tries to sell you a rectangular lot for your home. After many questions, he reluctantly admits that he has <u>two</u> rectangular lots. One is 3 feet wide and 2000 feet long. The other is 60 feet wide and 100 feet long.

a. What is the area of each lot?

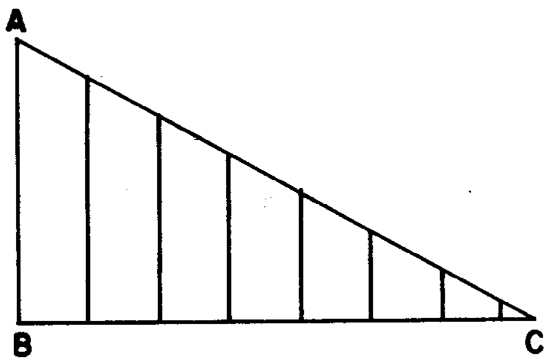
b. Which lot would you prefer? Why?

AREA OF A TRIANGULAR REGION

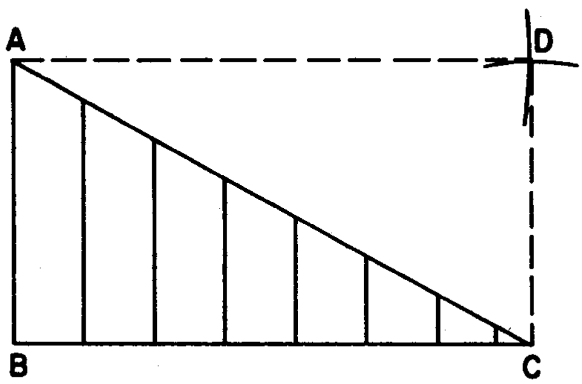
Area of a Region Bounded by a Right Triangle

Exploration

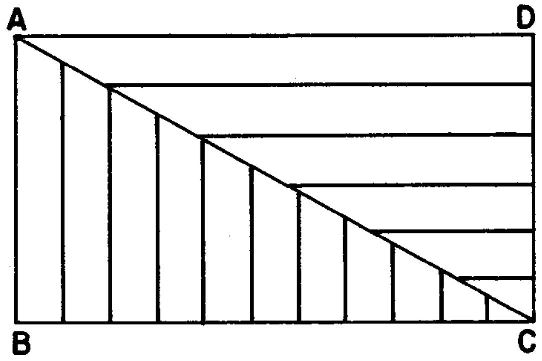
Let us think about how we would find the area of a region bounded by a right triangle.



Using a compass draw a rectangle by making $\overline{AD} \cong \overline{BC}$ and $\overline{CD} \cong \overline{AB}$.

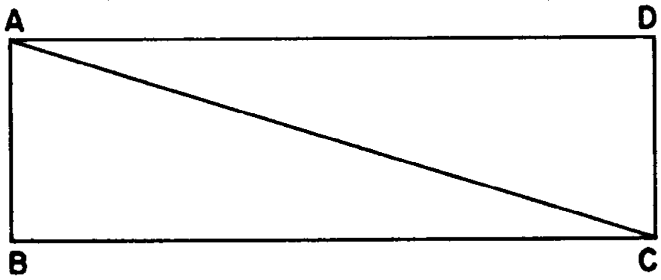


The resulting rectangular region is divided into 2 triangular regions in the following way:



How do the lengths of the opposite sides of the rectangle compare? Do you have enough information to be sure that $\triangle ABC \cong \triangle CDA$?

If two line segments are congruent, then they have the same length. Similarly, if two triangles are congruent to each other, then the regions associated with them have the same area. Therefore, the area of triangular region ABC is the same as the area of triangular region CDA. The measure of region ABC is what fractional part of the measure of region ABCD? What fractional part of the measure of region ABCD is the measure of region CDA?

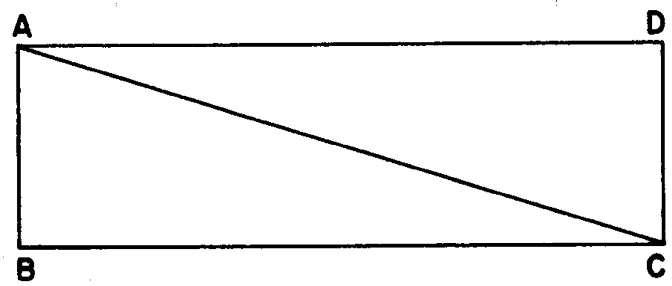


Suppose $\overline{BC}$ has length 10 inches. What is the measure of $\overline{BC}$ in inches? Suppose $\overline{AB}$ has length 3 inches. What is the measure of $\overline{AB}$ in inches? What is the measure, in square inches, of rectangular region ABCD? What is the measure, in square inches, of triangular region ABC? Why? What is the area of triangular region ABC? What is the measure, in square inches, of triangular region ADC? What is the area of triangular region ADC?

Summary

From every right triangle a rectangle may be found by properly locating a fourth vertex. The region bounded by the triangle has an area which is one-half that of the region bounded by the rectangle.

The measure, in square units, of a region bounded by a right triangle is found by calculating the product of the measures, in units, of the sides of the triangle which determine the right angle, and dividing the product by two.



Suppose $\overline{BC}$ has length 10 inches. What is the measure of $\overline{BC}$ in inches? Suppose $\overline{AB}$ has length 3 inches. What is the measure of $\overline{AB}$ in inches? What is the measure, in square inches, of rectangular region ABCD? What is the measure, in square inches, of triangular region ABC? Why? What is the area of triangular region ABC? What is the measure, in square inches, of triangular region ADC? What is the area of triangular region ADC?

Summary

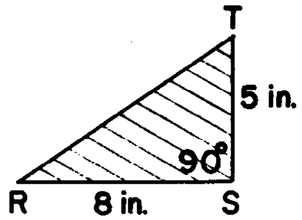
From every right triangle a rectangle may be found by properly locating a fourth vertex. The region bounded by the triangle has an area which is one-half that of the region bounded by the rectangle.

The measure, in square units, of a region bounded by a right triangle is found by calculating the product of the measures, in units, of the sides of the triangle which determine the right angle, and dividing the product by two.

Exercise Set 12

In each exercise the triangle is a right triangle.

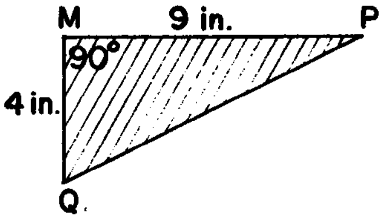
1.



Area of region RST is

____ square inches

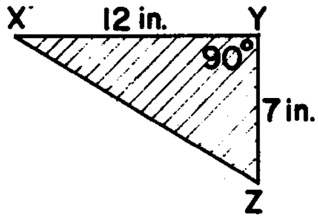
2.



Area of region MPQ is

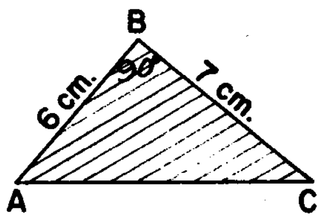
____ square inches

3.



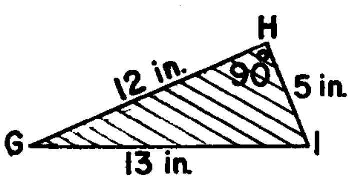
Area of triangular region XYZ is ____________

4.



Area of triangular region ABC is ____________

5.



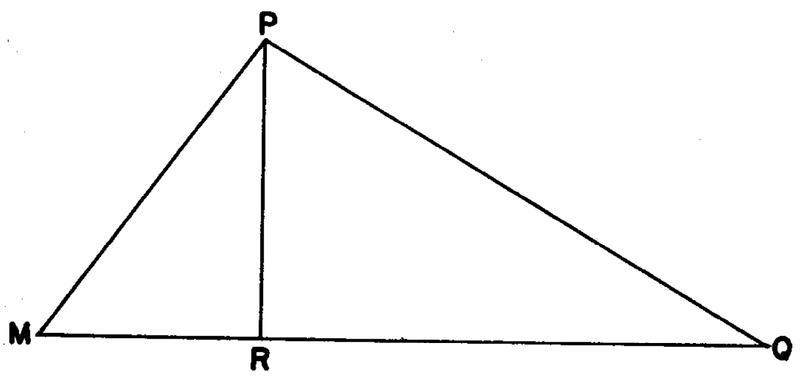
Select the measures you need and calculate the area of the region GHI.

Area of a Region Bounded by a General Triangle

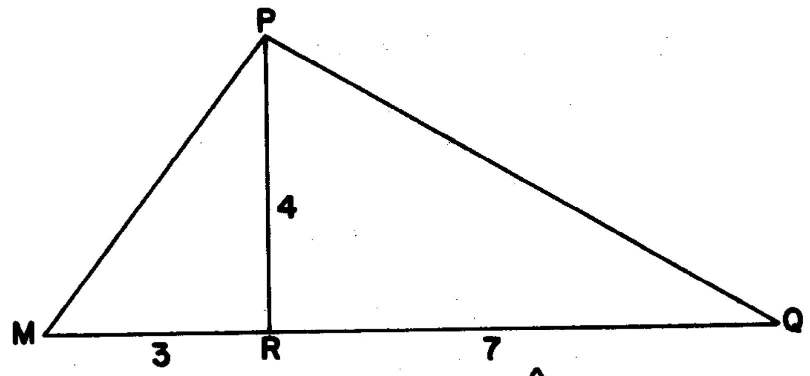
Exploration

Not every triangular region is bounded by a right triangle. We made the area of a region bounded by a right angle depend upon our knowledge of the area of a rectangular region. Now we will make our study of the area of <u>any</u> triangular region depend upon what we have learned about the area of a region bounded by a right triangle. Again we use the concept that area is unchanged when a region is dissected.

If we start with a general triangle such as $\triangle$ MPQ



we may draw $\overline{PR}$ from a vertex P such that $\angle$ PRQ is a right angle and $\angle$ PRM is a right angle. $\overline{PR}$ is referred to as the height or the <u>altitude</u> of $\triangle$ MPQ. The side of the triangle opposite the vertex P is called the <u>base</u> of the triangle. If the altitude is drawn from vertex M, then $\overline{PQ}$ is called the base of $\triangle$ MPQ. If the altitude is drawn from Q, then $\overline{MP}$ is the base. Every triangle has three altitudes and three corresponding bases.



Is $\triangle$ MRP a right triangle? Is $\triangle$ RQP a right triangle?

Suppose the length of $\overline{MR}$ is 3 inches, the length of $\overline{RQ}$ is 7 inches, and the length of $\overline{PR}$ is 4 inches as shown above. What is the area of the region bounded by right triangle MPR? What is the area of the region bounded by right triangle PRQ? What is the area of the region bounded by $\triangle$ MPQ?

Observe:

Measure of region MPR is $\frac{1}{2} \times (4 \times 3)$

Measure of region QPR is $\frac{1}{2} \times (4 \times 7)$

Measure of region MPQ is $\left(\frac{1}{2} \times (4 \times 3)\right) + \left(\frac{1}{2} \times (4 \times 7)\right)$

Using the associative and distributive property:

$$\left(\frac{1}{2} \times (4 \times 3)\right) + \left(\frac{1}{2} \times (4 \times 7)\right) = \left((\frac{1}{2} \times 4) \times 3\right) + \left((\frac{1}{2} \times 4) \times 7\right) =$$

$$= (\frac{1}{2} \times 4) \times (3 + 7)$$

$$= (\frac{1}{2} \times 4) \times 10$$

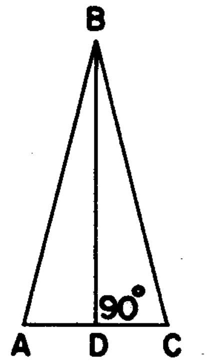
$$= \frac{1}{2} \times (4 \times 10)$$

This tells us that we may calculate the measure of the region MPQ if we take one-half the product of the measure of the altitude and the measure of the base.

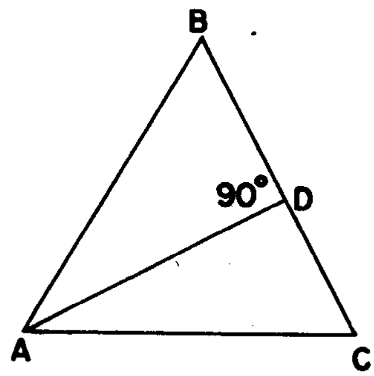
Do we get the same measure of the region MPQ if we add the measures of the regions MPR and QPR as we get if we divide the product of the measures of the base and the altitude by 2?

Exercise Set 13

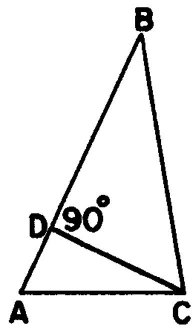
1. $\overline{BD}$ is an altitude. Which line segment is the base? Suppose m $\overline{BD}$ = 12, m $\overline{AD}$ = 3, m $\overline{DC}$ = 3, in inches. Find the area of triangular region ABC by two methods.



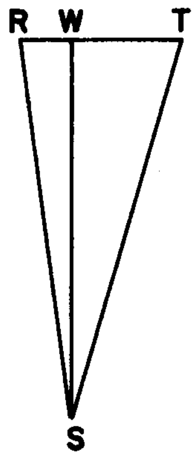
2. $\overline{AD}$ is an altitude. Which line segment is the base? Suppose m $\overline{AD}$ = 8, m $\overline{DC}$ = 4, m $\overline{BD}$ = 5, in inches. Find the area of the triangular region by two methods.



3. $\overline{CD}$ is an altitude. Which line segment is the base? Suppose m $\overline{CD}$ = 6, m $\overline{AB}$ = 12, in inches. Find the area of region ABC.



4. $\overline{WS}$ is an altitude. Which line segment is the base? Suppose m $\overline{WS}$ = 16, m $\overline{RT}$ = 7, in inches. Find the area of region RST.



Chapter 9

RATIO

INTRODUCTION TO RATIO

Every day you hear statements like these:

(a) Bill said, "I bought two pieces of candy for four pennies."

(b) "I made two dolls in four days," remarked Mary.

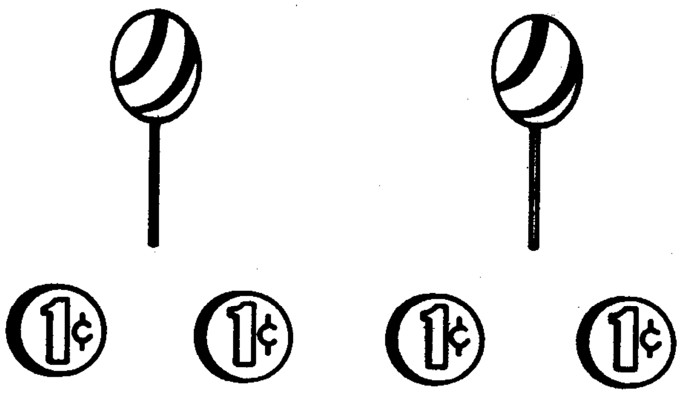
(c) "Jack made two hits in four times at bat," stated Mike.

(d) "My father drove two miles in four minutes," said Helen.

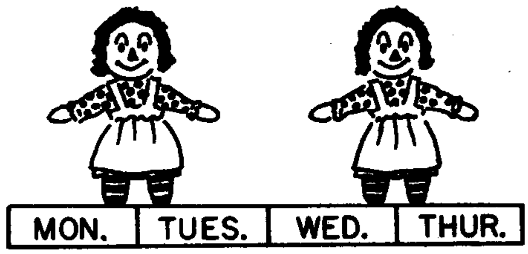
These statements are alike in several ways. Two sets are given in each of them. In the first statement, one of the sets is a set of pieces of candy. The other set is a set of pennies.

1. What are the two sets in statement (b)? in (c)? in (d)?

In each statement, the two sets are matched. In statement (a), 2 candies are matched with 4 pennies. A picture might show it this way:



2. In statement (b), 2 dolls are matched with 4 days.



In statement (c), 2 _____ are matched with 4 ___________.

In statement (d), 2 _____ are matched with 4 ___________.

3. In each of the statements 2 members of the first set are matched with 4 members of the second set. This is the idea of 2 to 4 or 2 per 4. Statement (a) matches 2 candies to 4 pennies. Statement (b) matches 2 dolls to 4 days.

In all the statements two things are matched with four things. In statement (c), we say the ratio of the number of hits to the number of times at bat is 2 to 4.

Ratio is a new word to us. It is a symbol which contains two numerals. It is a way of comparing the **numbers of two sets of objects.**

The way that we express the ratio 2 for 4 is 2:4. This symbol is read "two for four" or "two per four." Two numerals are needed to express a ratio.

Read these ratios:

3:10 4:15 2:5 1:3 5:2 1:2

4. Tom can work two problems in four minutes.

What sets are being compared?

What numerals would you write to express this ratio?

Jean can work five problems in four minutes.

How would you express this ratio?

5. In each of the following, name the two sets that are being compared. Write the symbol for the ratio that compares the two sets.

(a) "I can travel one mile in twelve minutes by using the Boy Scout pace," said Lee.

(b) The speed limit on the highway is sixty miles per hour.

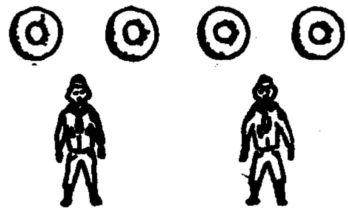
(c) John ate three peaches to Perry's two peaches.

(d) Helen won three out of four games.

(e) Charles rode his bike to school eighteen times in twenty days.

(f) Dick ate lunch at school four of the last five days.

6. The cook at the Boy Scout camp said: "I will bake four doughnuts for each two boys."

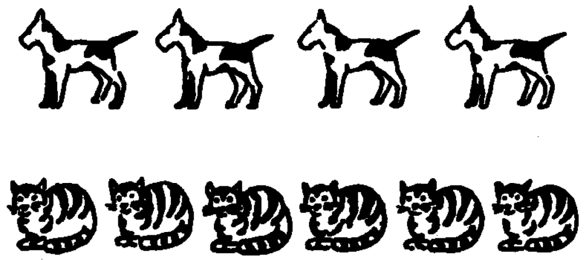


What would you write to express this ratio? If the cook said, "I will bake some doughnuts so that there are two boys for every four doughnuts," the ratio would be 2:4. When the cook said "four doughnuts for each two boys" the ratio was 4:2. When he said "two boys for every four doughnuts" the ratio was 2:4.

To understand the symbol 4:2, we need to know that the first number (4) represents the doughnuts and that the second number (2) represents the boys. To interpret 4:2 then, we think "four doughnuts to two boys." This means there will be 4 members of the first set (doughnuts) to 2 members of the second set (boys). The symbol 2:4 means two boys to four doughnuts. It means that there will be 2 members of the first set (boys) for 4 members of the second set (doughnuts).

In order to know what a symbol such as 2:4 could mean, it helps us to know the situation which gives us 2:4. It might be 2 boys to 4 girls, 2 snakes to 4 frogs, 2 ideas to 4 plans. Name some other situations which are 2:4.

6. The picture shown below shows 4 dogs and 6 cats. We can say, "There are four dogs to six cats."



(a) What ratio expresses how the set of dogs compares to the set of cats?

We could also say, "There are six cats to four dogs."

(b) What ratio describes the matching of cats to dogs?

We can use a pair of numerals in two different ways to describe the same matching. When these are interpreted correctly, they still tell us the same thing - "There are six cats to four dogs." or "There are four dogs to six cats."

8. Which of the following are "3 to 5" matchings? Which of the following are "5 to 3" matchings?

(a) There are three bicycles for five children.

(b) For every five boys in Susan's class there are three girls.

(c) Tom has three marbles for every five that Dick has.

(d) The train traveled five miles in three minutes.

Exercise Set 1

1. In what way are these situations alike?

   (a) Henry walks 3 miles an hour.

   (b) In our fifth grade room, we have 3 social studies books for each pupil.

   (c) That big truck can get only 3 miles for each gallon of gasoline.

2. In Exercise 1 name the two sets that are being compared in (a), in (b), and in (c).

3. Write, in words, how you read each of these symbols:

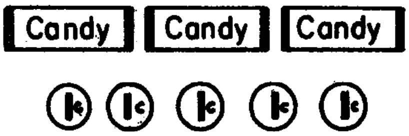
   (a) 4:1

   (b) 3:5

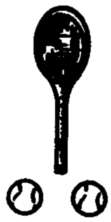
   (c) 1:6

4. Study these pictures. Write a symbol which describes the comparison. Then write a sentence to tell what this symbol means.

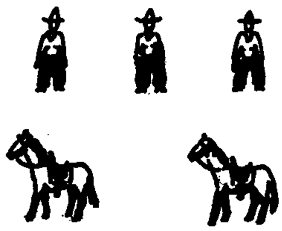
(a)



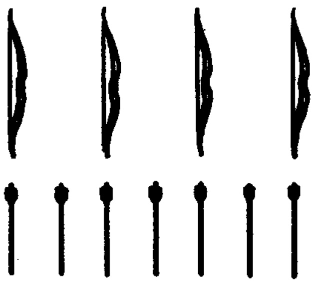
(b)



(c)



(d)



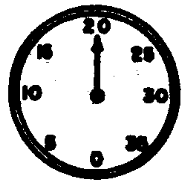
(e)

5. For each of these situations, write a symbol which expresses the ratio.

(a) Ned had 2 bee stings for one that Dick had.

(b) For our Halloween party, we had 5 sheets of orange paper for 3 sheets of black paper.

(c) The speedometer on Steven's bike showed this:



(d) Two bags of potato chips cost twenty-five cents.

(e) Jean can work four problems in five minutes.

6. Draw pictures which could represent comparisons described by these symbols:

(a) 6:1 (b) 5:2 (c) 2:3

7. Sandra and Mark read this sentence:

On John's farm there are 5 lambs for 3 mother sheep.

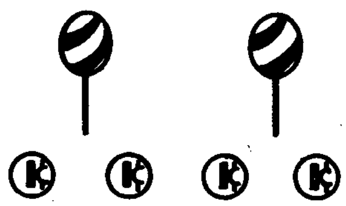
Sandra wrote 5:3 to show this comparison. She said, "I know there are 5 lambs for 3 mothers."

Mark wrote 3:5 to show this matching. He said, "I know there are 3 mother sheep for 5 lambs."

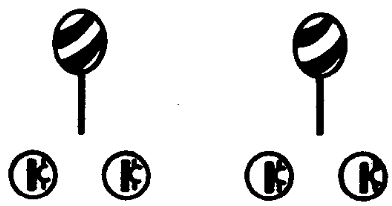
Who was correct - Sandra or Mark?

DIFFERENT NAMES FOR THE SAME RATIO

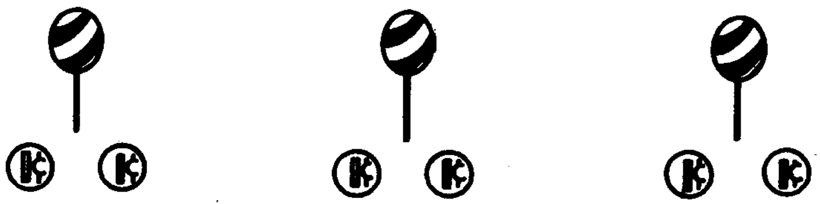
When Bill bought 2 candies for 4 pennies, we described this matching by writing 2:4. We read this, "two to four" or "two per four." We know it means "two candies for four pennies." We drew a picture to represent this.



Can we show this in another way? Look at this picture.

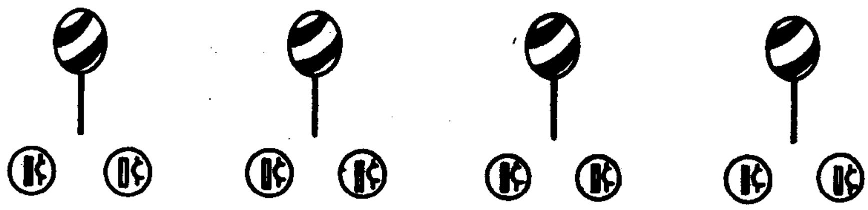


This same matching could be described by the symbol 1:2. This means 1 candy for 2 pennies. Study this picture:



We could also use the symbol 3:6. This means 3 candies for 6 pennies.

Look at this picture.



1. We can describe this in another way by writing 4:8. What does this symbol mean?

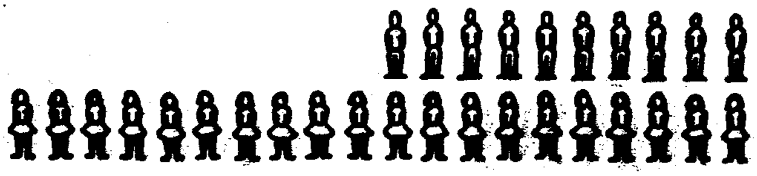
The symbols 2:4 and 1:2 and 3:6 and 4:8 are all correct ways of expressing the same comparison. There are many symbols which describe matching. We can refer to it as two per four, or one per two, or three per six, or four per eight. Give other names for this same matching.

Write the second numeral to show other names for the ratio of number of candies to the number of pennies.

5:______	6:______
7:______	8:______
12:______	50:______

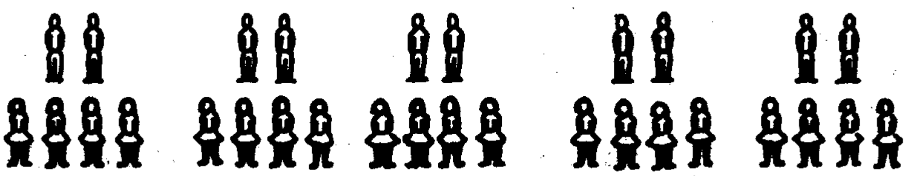
How many different names will there be?

2. Look at this picture of a fifth grade class.



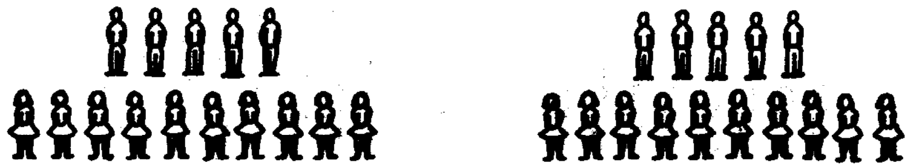
(a) What is one way of writing the symbol which represents the ratio of boys to girls?

(b) This picture shows the class lined up in a different way.



Write a symbol to express this ratio of boys to girls.

(c) This picture shows still another way of lining up this same class. What symbol expresses this ratio of boys to girls?



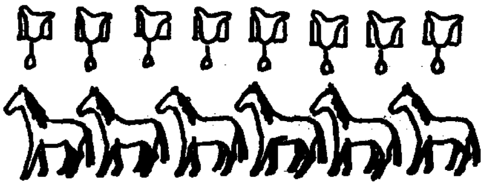
(d) All of these are names for the same ratio. Complete these symbols to show they are names for the same ratio:

10:_____	2:_____
5:_____	1:_____
50:_____	24:_____
214:_____	

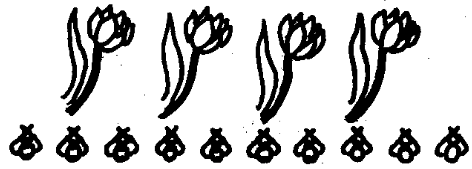
When we are matching one boy to two girls, there are more names to show this matching than we can count. Whenever we are matching one of a set to two of another set, we usually write 1:2 to express this ratio.

3. For each of these pictures, tell four names for the ratio.

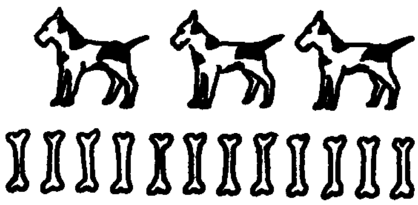
(a) 8 saddles to 6 horses



(b) 4 flowers to 10 bees

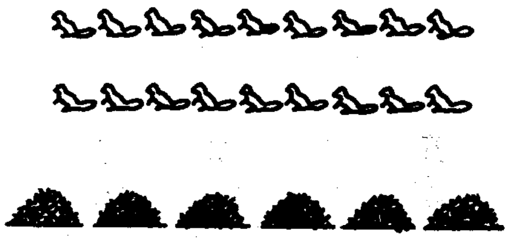


(c) 3 dogs to 12 bones

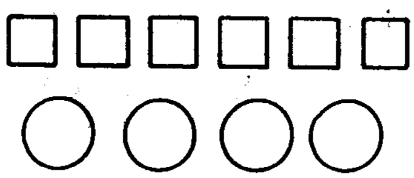


(d) 9 sweaters to 6 skirts

(e) 18 beavers to 6 beaver houses



(f) 6 squares to 4 circles



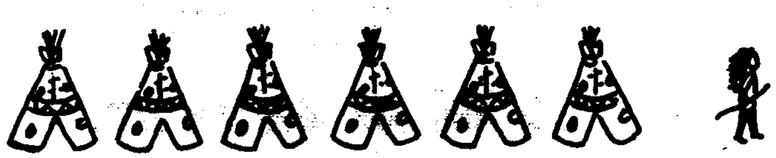
Exercise Set 2

1. You know that a ratio has more names than we can count. Each picture has two sets. Compare the first set to the second set. Write four names for the ratio suggested by the picture.

(a)

(b)

(c)



(d)

2. For each sentence write two names for the ratio suggested by the sentence. Tell what the names mean.

(a) George was going 5 miles per hour on his bicycle.

(b) In the baseball game, Neil was getting 2 hits for every 5 times at bat.

(c) The cookies cost 3 for 5¢.

(d) In Franklin School there are 5 girls for every 4 boys.

(e) The train was going 4 miles in 3 minutes.

(f) The airplane was going 10 .miles in 1 minute.

3. This table shows several names for the same ratio. Copy it and fill each blank space with the proper numeral.

2:3
4:6
6:--
8:--
--:15
12:--
--:21
--:24
18:--
20:--
2n:--

BRAINTWISTER

4. This table shows several names for the same ratio. Copy it and complete it.

5:4	10:8	15:--	20:--	--:20	--:24	35:28	40:--	--:36

5. Write the letter of each symbol which is another name for the ratio 8:16.

(a) 4:8 (c) 1:4 (e) 16:32

(b) 2:4 (d) 3:6 (f) 9:18

6. Draw two pictures of cowboys and Indians like this to illustrate the ratio 3 per 9.

7. Use any pictures you like. Illustrate with 3 drawings the ratio shown by the symbol 4:1.

8. What symbol could you write to show the matching of one member from the first set to one member from the second set?

9. Express each of these matchings as a number pair using the word "for" or "per."

(a) Jean can work 5 problems in 4 minutes.

(b) John ate 3 grapes for every 2 that Perry ate.

(c) The speed limit is 60 miles per hour.

MORE ABOUT NAMES FOR THE SAME RATIO

When we speak of ratio, we immediately think of sets. We know that members of the first set are matched with members of the second set.

1. Name the two sets in each of these situations:

   (a) A fifth grade girl had 8 envelopes for every 12 sheets of paper.

   (b) The soldiers had 36 bullets for every 2 guns.

   (c) The boys rode their bicycles 4 miles in 24 minutes.

   (d) At the fifth grade party there were 12 cookies for every 4 children.

   (e) What symbol names the ratio in (a)? in (b)? in (c)? in (d)?

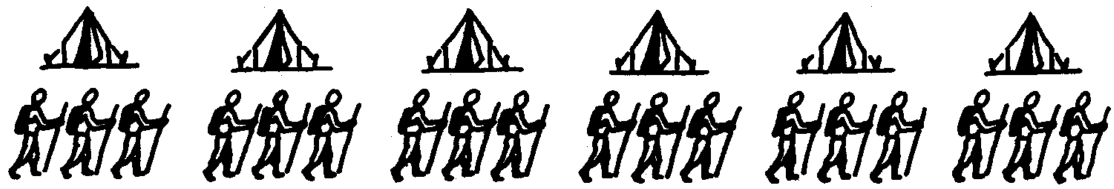
2. Some boys went on a camping trip. There were 6 tents for 18 boys. The same number of boys slept in each tent. Study these pictures.

   (a) What are the two sets?

   One name for this ratio is 6:18.

   It tells us there are 6 tents per 18 boys.

(b) This picture shows us another name for this same ratio is 1:3. What does this symbol tell us?



Because 6:18 and 1:3 are both names for the same ratio, we can write:

$$6:18 = 1:3$$

We can read this mathematical sentence, "Six to eighteen equals one to three" or "Six to eighteen is the same ratio as one to three." In this problem that means "6 tents to 18 boys."

(c) Is this a true mathematical sentence?

$$6:18 = 3:9$$

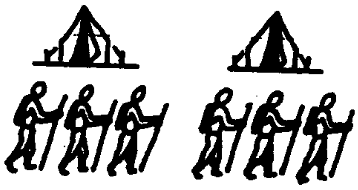
Because 6:18 and 1:3 and 3:9 are all names for the same ratio, we can write

$$6:18 = 1:3 = 3:9$$

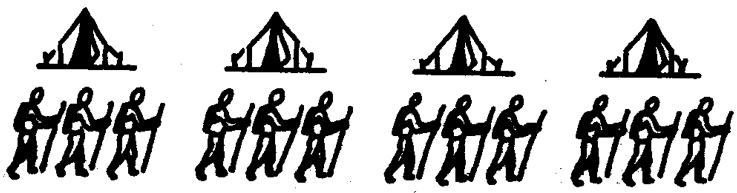
This tells us that 6 to 18 and 1 to 3 and 3 to 9 are names for the same ratio. What is another name for this ratio?

3. These pictures also illustrate this same ratio between numbers of tents and numbers of boys.

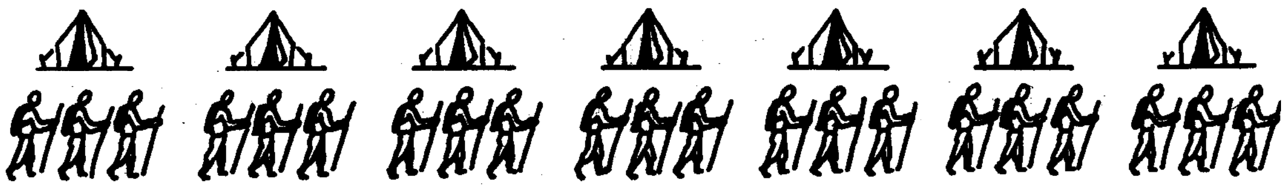
(a) What symbol can we use to express this ratio?



(b) Using the picture to help us, what new name can we write for this same ratio?



(c) If there were 7 tents, this picture shows how many boys could go camping. What new name expresses this ratio?



You see that in every case the ratio is the same. We still match 1 tent to every 3 boys.

4. Draw a picture to show how many boys could go camping if there were 8 tents of this size.

5. Draw a picture to show how many tents would be needed for 27 boys.

Here is a table which shows this information:

Tents	1	2	3	--	5	6	--	8	--	--
Boys	3	-	9	12	-	18	21	-	27	30

(a) For this same ratio, how many boys would sleep in 2 tents?

(b) Tell what numbers should be used to fill the spaces in the table. We can use these pairs from the table to write other names for this ratio. For example:

$1:3 = 2:6 = 3:9 = 4:12 = 5:15 = 6:18 = 7:21 = 8:24$ and so on.

(c) It is not necessary to draw pictures or to make a table to find how many boys could go camping if there were 8 tents. We know that 1 tent will house 3 boys. The members of our sets are tents and boys. We write 1:3.

The symbol 1:3 tells us how the number of tents compares with the number of boys.

We want to find another name for this ratio which has 8 as its first numeral. We write

$$1:3 = 8:N.$$

In this problem we interpret this as "1 tent for 3 boys is the same ratio as 8 tents for how many boys." Instead of 1 tent, we now have 8 tents. Therefore, instead of being able to house only 1 group of 3 boys, we can house 8 groups of 3 boys or 24 boys. $(8 \times 3 = 24)$

The symbol 1:3 and 8:24 are different ways of naming the same ratio. We can write

$$1:3 = 8:24.$$

(d) The symbol 2:6 is another way of describing the ratio of number of tents to the number of boys.

$$2:6 = 8:N.$$

This says that 2 per 6 is the same as 8 per how many. If 2 tents will house 6 boys, then 8 tents, or 4 groups of 2 tents each, should sleep 24 boys. We write

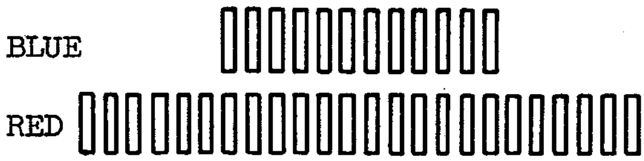
$$2:6 = 8:24.$$

(e) Find the number represented by the letters in the following sentences:

$$1:3 = 5:x, \quad 3:9 = 12:y, \quad 1:3 = 15:z$$

6. Place in two separate piles the 24 slips of paper (red) and the 12 slips of paper (blue) that your teacher has given you. The blue slips are members of one set and the red slips are members of the other set. We will match blue slips to red slips.

(a) Arrange the slips like this:



Write a symbol which describes the ratio of blue slips to red slips.

(b) Now arrange the slips like this:

Write the symbol which you think best describes the matching of blue slips to red slips.

(c) Arrange the slips like this:

Now what symbol would you use to show the matching of blue slips to red slips?

(d) Arrange the blue slips so that they are in sets of 4. How many red slips would be matched with each set of 4 blue slips? Write the symbol which would best describe this matching.

(e) Arrange the blue slips so that they are in sets of 2. How many red slips would be matched with each set of blue slips? What symbol best describes this matching?

(f) Arrange the blue slips so there is just 1 blue slip to a set. How many red slips would be matched with each 1 blue slip? What symbol expresses the ratio of blue slips to red slips?

(g) Complete this table:

Blue slips	12	?	3	?	2	1
Red slips	?	12	?	8	?	?

(h) Replace each question mark so this mathematical sentence is true.

$$12:24 = ?:12 = 3:? = ?:8 = 2:? = 1:?$$

(i) How many red slips would be matched with 15 blue slips? We know that the matching is 1 blue slip for 2 red slips. So we can write $1:2 = 15:N$. We have expressed the idea that 1 blue slip per 2 red slips is the same ratio as 15 blue slips per how many red slips? Instead of one set of 1 blue slip we have 15 sets of 1 slip. Instead of one set of 2 red slips, we have 15 sets of 2 slips or 30 slips. So

$$1:2 = 15:30.$$

7. Find the number represented by n in each of the following sentences. Then write the sentence on your paper. Example: (a) n = 27, 1:3 = 9:27

(a) 1:3 = 9:n

(b) 1:2 = 8:n

(c) 1:4 = 5:n

(d) 2:5 = 4:n

(e) 3:6 = 9:n

(f) 3:4 = 15:n

(g) 1:4 = 2:n

(h) 2:9 = 10:n

(i) 4:5 = 24:n

(j) 2:1 = 24:n

8. Draw a picture to show that this mathematical sentence is true:

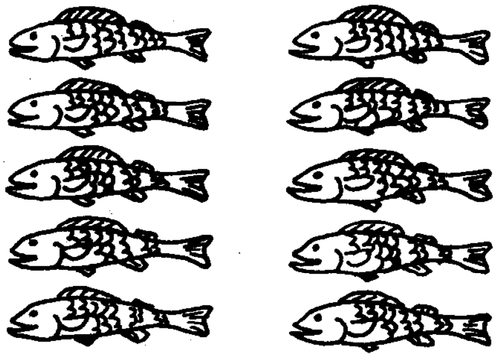
1:5 = 3:15

9. Draw a picture to illustrate this:

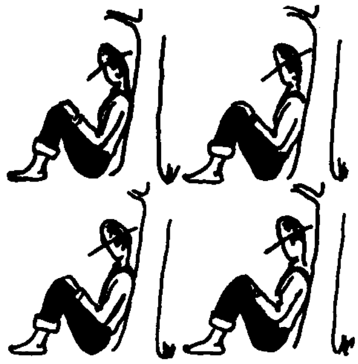
For every 8 pieces of candy, there were 16 pennies.

Exercise Set 3

1. (a) Write two symbols which express the ratio of the number of fish to the number of boys.

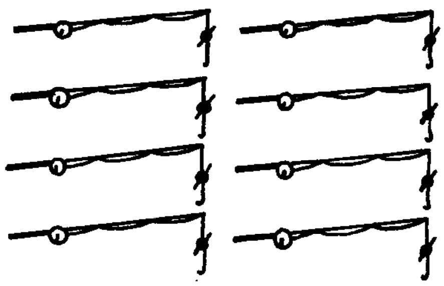


(b) Write two symbols which express the ratio of the number of boys to the number of fish.



(c) Write two symbols which describe the ratio of the number of boys to the number of fishpoles.

(d) Write two symbols which describe the ratio of fishpoles to boys.



2. Copy and complete this table.

4:1	16:--	8:--	20:--	36:--	--:25	12:--	--:6	--:8	--:10

3. Copy and complete each of these three tables. The last 2 names in each table are braintwisters.

(a)	(b)	(c)
4:8	10:4	6:10
1:--	30:--	--:20
8:--	5:--	--:30
--:32	--:8	30:--
--:4	--:20	24:--
12:--	--:24	--:80
--:48	100:--	--:100
--:40	40:--	54:--
--:72	--:32	36:--
32:--	1,000:--	42:--
3:--	15:--	3:--
--:1	--:10	--:15

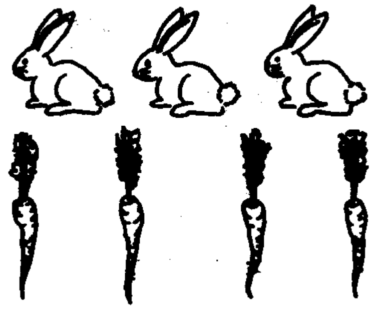
4. Write a mathematical sentence for each of these situations. Then find the answer.

(a) A car will go 20 miles on one gallon of gas. How far will it go on 5 gallons of gas?

(b) Elmer threw a basketball through the hoop 3 out of 4 times. If he kept this same record, how many times would he need to throw to make 24 baskets?

5. (a) Write 4 symbols which express the ratio of the number of rabbits to the number of carrots.

(b) Write 4 symbols which exhibit the matching of carrots to rabbits.



6. Write a mathematical sentence for each of these.

(a) 6 per 9 is the same ratio as 2 per 3.

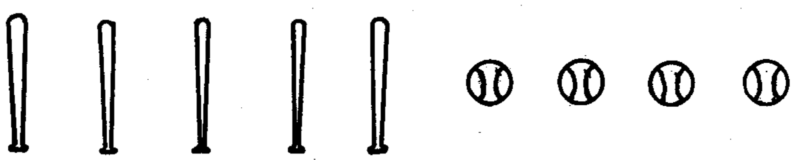
(b) 6 dinosaurs for 4 cavemen is the same ratio as 3 dinosaurs for 2 cavemen.

(c) 8 per 3 and 16 per 6 and 24 per 9 are all names for the same ratio.

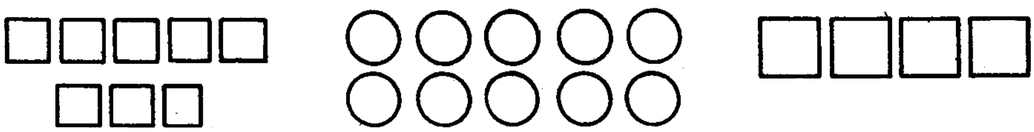
(d) 4 flashlights for 9 boys is the same ratio as 20 flashlights for how many boys?

7. Write symbols for ratios suggested by the pictures.

(a)

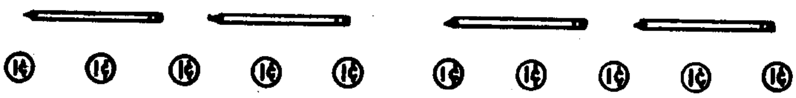


(b)



USING RATIOS

1. Alice bought 2 pencils for 5 cents. Write the symbol which expresses the ratio of pencils to cents. If Alice had 10 pennies instead of 5 pennies, how many pencils could she buy? This problem can be solved by drawing a picture such as this:



It can be solved by finding another name for the same ratio, like this:

$$2:5 = n:10$$

This tells us that 2 pencils per 5 pennies is the same matching as n pencils for 10 pennies.

The symbol n:10 suggests we have 10 pennies and that we want to know the number of pencils to match these. We know that a set of 2 pencils matches a set of 5 pennies and that ten pennies are 2 sets of 5 pennies. So we must have two sets of 2 pencils each to match the 10 pennies. We know, then, that Alice could buy 4 pencils for 10 pennies. You could have solved this problem quite easily "in your head", couldn't you?

2. Could you solve this one "in your head?"

> Tom was shooting at a target. He made 5 hits out of 7 shots. If the ratio of the number of hits to the number of shots stays the same, how many hits will he get in 63 shots?

This is a little more difficult to answer. It can be written as:

$5:7 = n:63$	This means that five hits per seven shots is the same matching as n hits per 63 shots.

We know we had one set of 7 shots the first time and 9 sets of 7 shots the second time because there were 63 shots the second time. So we have 9 sets of 5 hits or 45 hits per 63 shots.

3. Now let's think about Alice and her pencils. How many could she buy for 25 pennies? You could figure this out "in your head." You also can write a mathematical sentence. The members of the two sets are <u>pencils</u> and <u>pennies</u>. The matching is 2 pencils for 5 pennies. The ratio 2:5 shows how the number of pencils compare with the number of the pennies. We want to know how many pencils Alice can buy for 25 pennies. So we must find another name for the same ratio. We write $2:5 = n:25$. This means 2 pencils per 5 pennies is the same matching as n pencils per 25 pennies. In our second case, instead of 5 pennies

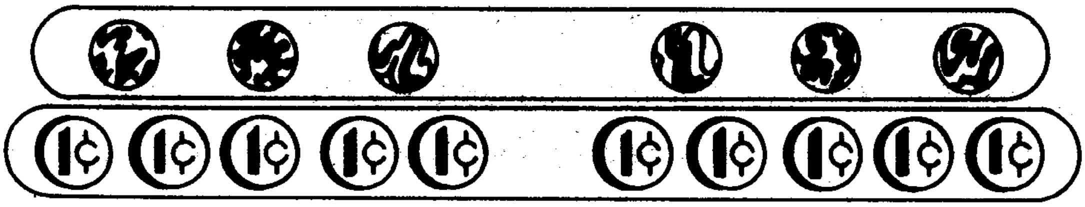
we have 25 pennies or 5 sets of 5 pennies. Therefore, instead of 2 pencils, we will have 5 sets of 2 pencils or 10 pencils. Two different ways of describing the same ratio are 2 per 5 and 10 per 25. The mathematical sentence that says this is $2:5 = 10:25$. Since $10:25$ tells us how the number of pencils compare with the number of pennies, we see that we can buy 10 pencils for 25 pennies. To find how many pencils we can buy for 15 pennies, we use the mathematical sentence $2:5 = n:15$. We are asking, "2 per 5 is how many per 15?"

(a) How many sets of 5 are there in 15?

(b) How many sets of 2 pencils should we have? What should n be? If the first numeral refers to pencils and the second to pennies, we see that we should get 6 pencils for 15 cents.

4. Jake bought 6 marbles for 10 cents. How much would 9 marbles cost? Here we have two sets. The members of the first set is marbles and the members of the second set is cents. The ratio of the number of marbles to the number of pennies is $6:10$. Our mathematical sentence is $6:10 = 9:n$. In this problem this is interpreted, "Six marbles per 10 pennies is the same ratio as 9 marbles per n pennies." We know that 9 is not a multiple of 6. That is, a set of 9 members cannot be separated into sets of 6 members each.

So let's find some other name for the ratio 6:10. A name which uses smaller numbers might be found. Think of 6 per 10 as shown in this picture.



We see that 6 per 10 is also 2 sets of 3 marbles for 2 sets of 5 pennies. Therefore, 1 set of 3 marbles can be matched with 1 set of 5 pennies.

Exercise Set 4

1. Complete these symbols so that each is a name for the ratio 2:3.

   (a) 4:?

   (b) ?:12

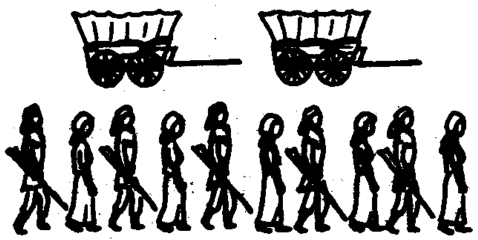
   (c) 6:?

   (d) 12:?

   (e) 100:?

   (f) ?:15

2. This picture is of wagons and pioneers.



How many wagons would be needed for 55 pioneers?

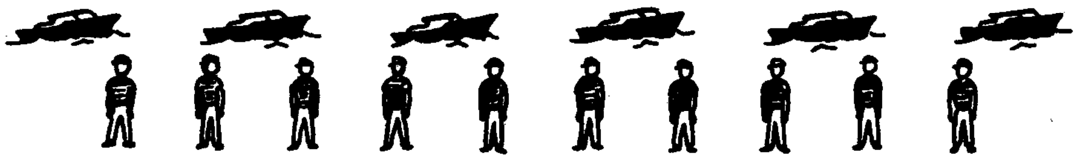
3. Write a mathematical sentence for each of these situations. Let n name the unknown number. Then find the value of n.

(a) David can run 50 yards in 8 seconds. If he could keep going at this same speed, how long would it take him to run 300 yards?

BRAINTWISTER: How long would it take to run 175 yards?

(b) 18 birds live in 10 birdhouses. If the ratio of birds to birdhouses stays the same, how many birds could live in 30 birdhouses?

(c) Study this picture. How many boats would be needed for 30 people?



How many boats would be needed for 45 people?

(d) Glen had 7 ideas in 2 minutes for food for a fifth grade party. If he keeps getting ideas at this same ratio, how many ideas will he have in 8 minutes?

TRYING SOMETHING NEW

Do you think you could solve some ratio problems like this without using pictures? Using the idea, we know that $6 = 2 \times 3$ and $10 = 2 \times 5$. We see that 6 and 10 have a common factor 2. We can divide both 6 and 10 by 2. When we divide by 2 we get 3 and 5. We can write $6:10 = 3:5$. We know these are names for the same ratio. Let's use this second name for the ratio and write: $3:5 = 9:N$. We are asking, "3 per 5 is the same ratio as 9 per how many?" Now we can see that we have 3 groups of 3 marbles so we need 3 groups of 5 pennies or 15 pennies. Thus, 9 marbles would cost 15 pennies.

1. Look at the mathematical sentence which describes this situation.

   Mr. Smith can drive 50 miles in 1 hour. If he drives at the same speed, how many hours will it take to drive 250 miles?

2. Try this one on your own.

   Set A is a set of names for the same ratio. Find more names for this same ratio.

   Set A = {18:24, --:--, --:--, --:--}

3. If Chuck eats 3 peanuts for every 2 that Perry eats, how many peanuts will Chuck eat if Perry eats 10?

4. If a car travels 10 miles in 25 minutes, how far will the car travel in 75 minutes?

5. Tell which of these sets are names for the same ratio?

Set A = {20:16, 5:4, 10:8}

Set B = {12:18, 6:9, 2:3}

Set C = {18:24, 3:4, 9:12}

Set D = {32:16, 4:2, 16:8, 2:1, 8:4}

Set E = {48:32, 6:4, 24:16, 12:8, 3:2}

6. If a bank charges 4 dollars for the use of 100 dollars, how much would it charge for the use of 50 dollars?

7. How much would the bank in exercise 6 charge for the use of 250 dollars?

8. If an airplane flies 570 miles in 1 hour, how far would it fly in $2\frac{1}{2}$ hours?

You have seen that a ratio such as 2:3 is used to describe a property of two sets. It means that there are 2 objects in one set for 3 objects in another set.

Some other names for the ratio 2:3 are 4:6, 10:15, 20:30, 40:60, and 2000:3000. You could write many more.

In finding other names for the ratio 2:3 you can multiply the numbers 2 and 3 by the same number, if the number is not zero.

The pairs of numerals in 2:3, 4:6, 20:30 represent the same ratio. We can write 2:3 = 4:6 and 2:3 = 20:30 and 4:6 = 20:30.

If we know that 4:5 = n:15, we can find the number represented by n. It is 12.

Now let us see how some of these things we have just said about ratios are similar to things we can say about rational numbers.

The symbol for the ratio 2 to 3 is 2:3. The symbol for the number two-thirds is $\frac{2}{3}$.

4. If a car travels 10 miles in 25 minutes, how far will the car travel in 75 minutes?

5. Tell which of these sets are names for the same ratio?

Set A = {20:16, 5:4, 10:8}

Set B = {12:18, 6:9, 2:3}

Set C = {18:24, 3:4, 9:12}

Set D = {32:16, 4:2, 16:8, 2:1, 8:4}

Set E = {48:32, 6:4, 24:16, 12:8, 3:2}

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Some other names for the ratio 2:3 are 4:6, 10:15, 20:30, 40:60, and 2000:3000. You could write many more.

In finding other names for the ratio 2:3 you can multiply the numbers 2 and 3 by the same number, if the number is not zero.

The pairs of numerals in 2:3, 4:6, 20:30 represent the same ratio. We can write 2:3 = 4:6 and 2:3 = 20:30 and 4:6 = 20:30.

If we know that 4:5 = n:15, we can find the number represented by n. It is 12.

Now let us see how some of these things we have just said about ratios are similar to things we can say about rational numbers.

The symbol for the ratio 2 to 3 is 2:3. The symbol for the number two-thirds is $\frac{2}{3}$.

Both symbols use the numerals 2 and 3. Other names for the rational number $\frac{2}{3}$ are

$$\frac{4}{6}, \quad \frac{10}{15}, \quad \frac{20}{30}, \quad \frac{40}{60}, \quad \frac{2000}{3000}.$$

If we know that $\frac{2}{3}$ and $\frac{4}{6}$ and $\frac{20}{30}$ are names for the same rational number, we can write

$$\frac{2}{3} = \frac{4}{6} \quad \text{and} \quad \frac{2}{3} = \frac{20}{30} \quad \text{and} \quad \frac{4}{6} = \frac{20}{30}.$$

In finding other names for the rational number $\frac{2}{3}$ you can multiply 2 and 3 by the same number, if the number is not zero.

If we know that $\frac{4}{5} = \frac{n}{15}$, we can find the number represented by n. It is 12.

You can see that ratios and rational numbers are alike in some ways. After you have studied more about rational numbers, you can see other ways in which they are alike.

Exercise Set 5

1. Write the symbols for two ratios using the numerals 3 and 5.

2. Write the symbols for two rational numbers using the numerals 3 and 5.

3. Is 9:10 the name for a number or a ratio?

4. Is $\frac{7}{8}$ the name for a number or a ratio?

5. Write some other names for 9:10.

6. Write some other names for $\frac{7}{8}$.

7. If n:25 is another name for 6:5, what number does n represent?

8. If 3:10 = 18:n, then n represents what number?

9. If $\frac{6}{9}$ is another name for $\frac{n}{81}$, what number does n represent?

10. If $\frac{11}{6} = \frac{110}{n}$, then n represents what number?

Chapter 10

REVIEW

CONCEPT OF SETS

Number the exercises as they are numbered here and write the answers on your paper. If you do not know the answer to an exercise, write the number of the exercise and leave the space beside it blank. Later you may be able to fill in the answers that you did not know.

1. Set A is the set of whole numbers greater than 10 and less than 20. Write the members of A.

2. Write a sentence that describes this set:

$$C = \{u, v, w, x, y, z\}$$

3. If $A = \{3, 6, 9, 12, 15\}$ and $B = \{0, 6, 12, 18\}$ what is $A \cap B$?

4. Using the sets A and B in the preceding exercise, what is $A \cup B$?

5. If R is the set of the states of the U.S.A. that are east of the Mississippi River and S is the set of states that touch the Pacific Ocean, what is $R \cap S$?

NUMERATION

1. Write the letter of each part of this exercise on your paper. Then to the right of the letter write the words, or word, that you would use to fill the blank spaces, or space.

a) 19 = ______ fives and ______ ones.

b) 27 = ______ fives and ______ ones.

c) ______ nines and ______ ones = 40.

d) ______ eights and ______ ones = 40.

e) ______ sevens and ______ ones = 40.

f) ______ sixes and ______ ones = 40.

g) 546 is 5 hundreds, ______ tens, and 6 ones.

h) 546 is 4 hundreds, ______ tens, and 16 ones.

i) 546 is 5 hundreds, 3 tens, and ______ ones.

2. Express 8,471 in 3 different ways as in 1 g), h), i). Letter the three ways a), b), and c).

3. Write the letter for each part on your paper. If the statement is right, then write yes after the letter. If it is wrong, write no.

a) 3729 is 37 tens plus 29 ones.

b) 734 = 600 + 120 + 24.

c) ten hundreds plus forty tens plus nine ones is the same as one thousand forty-nine.

d) 10,129 = 10 thousands plus ten hundreds plus nine ones.

4. Write the letter for each part on your paper. Then beside it write $<$, $>$, or $=$, whichever makes each a true sentence.

a) $8 + 4$ _____ 11.

b) $(1 + 3)$ _____ $(9 + 11)$.

c) $(3 + 4) + 4$ _____ $1 + (5 + 4)$.

d) $(15 + 14)$ _____ $(7 + 5) + 17$.

e) $(7 + 4) + 2$ _____ $(7 + 5) + 2$.

f) $(6 + 5) - 2$ _____ $(13 - 7) + 6$.

g) $(3 \times 7) + 9$ _____ 31.

h) $(60 + 3) + (10 - 3)$ _____ $(11 - 2)$.

PROPERTIES AND TECHNIQUES OF SUBTRACTION, I.

1. Write the letter of each part on your paper. Then beside it write the number represented by n in that part.

   a) $8 + 3 = n.$

   b) $144 - n = 29.$

   c) $n = 1001 - 2.$

   d) $3 - 3 = n.$

   e) $0 + 0 = n.$

   f) $99 + 2 = n.$

2. Write on your paper the letter for each mathematical sentence that is true.

   a) $9 + 4 = 13.$

   b) $17 - 9 = 9.$

   c) $88 - 64 = 34.$

   d) $45 + 5 = 50.$

   e) $36 + 37 = 83.$

3. Are some of the mathematical sentences in Exercise 2 false? If a sentence is false, rewrite it and change one number in it so that it will be true. Letter them the same as in Exercise 2.

4. Write the letter of each part on your paper. Then beside it write the number that you would use to fill the blank.

a) If $19 - 10 = 9$, then $19 - 9 =$ ______.

b) If $23 - 11 = 12$, then $11 + 12 =$ ______.

c) If $13 - 10 = 3$, then $13 - 9 =$ ______.

5. Write the letter for each part on your paper. Then beside it write the one of these, $>$ or $<$, that you would use to make a), b), and c) true sentences.

a) $(65 + 42)$ ______ $(65 + 43)$.

b) $(300 + 700)$ ______ $(400 + 700)$.

c) $(1300 + 2000)$ ______ $(1300 + 3000)$.

6. Write the letter for each part on your paper. Then beside it write the answer to the question.

How many units must be marked on a number line to find z, s, m, p, or n in each of these mathematical sentences?

a) $14 + 17 = z$.

b) $139 - s = 40$.

c) $m = 20 + 40$.

d) $p + 17 = 30$.

e) $n - 28 = 13$.

7. Write the letter for each part on your paper. Then beside it write the number you would use for the p, q, or r in the mathematical sentence.

a)	$p - 8 = 24$	f)	$r = 18 - 18$
b)	$q = 13 - 4$	g)	$p - 40 = 30$
c)	$7 - 5 = r$	h)	$p + r = 0$
d)	$20 - p = 12$	i)	$p + q + r = 0$
e)	$14 - q = 14$	j)	$10 - p = 10$

8. On your paper write a mathematical sentence for each problem. Then solve to find n. Write an answer sentence.

a) There were 37 cows in a pasture. Eight of them were black. How many cows were not black?

b) Jim has 92 coins in a coin folder. It will hold 150 coins. How many more coins will the folder hold?

c) Margy practiced her flute lesson for 35 minutes on Monday, 30 minutes on Tuesday, and 45 minutes on Wednesday. How many minutes did she practice on all these days?

d) A school library had 488 books. The next year 205 books were added. How many books were then in the library?

9. Write a different mathematical sentence for each of the following which illustrates how the numbers in each sentence are related.

a) $7 + 2 = 9$

b) $10 - 4 = 6$

c) $30 + 30 = 60$

d) $x - s = z$

e) $n - 5 = 2$

10. Write on your paper the letter for each sentence that is true.

a) $20 + 11 = 11 + 20$.

b) $103 + 301 = 100 + 304$.

c) $(6 + 5) + 4 = 4 + (6 + 5)$.

d) $1,207 + 2,011 = 1,102 + 7,021$.

e) $n + p = p + n$.

11. Some of the statements are true because of the associative property and some because of the commutative property. Write the letter for each part on your paper. Then beside it write associative or commutative to show that you know which property is used.

a) $2 + (3 + 4) = (2 + 3) + 4$.

b) $(18 + 19) + (39 + 12) = (39 + 12) + (18 + 19)$.

c) $(8 + 9) + 6 = (9 + 8) + 6$.

d) $(8 + 9) + 6 = 8 + (9 + 6)$.

PROPERTIES OF MULTIPLICATION AND DIVISION

1. Copy the letter for each part on your paper. Then beside it write the one of $>$, $<$, $=$ that you would use to fill the blank space so that a) through n) will be true sentences.

a) 5×5 ____ 4×8

b) 6×8 ____ 8×6.

c) 9×5 ____ 6×8.

d) $94 - 40$ ____ 6×9.

e) 7×9 ____ 8×8.

f) 8×7 ____ 6×10.

g) 5×9 ____ 7×6.

h) $8 \times n$ ____ $n \times 7$.

i) $140 - 60$ ____ 9×9.

j) 9×4 ____ 6×6.

k) 8×7 ____ 9×6.

l) $p \times 4$ ____ $4 \times p$.

m) 7×7 ____ 6×8 ____ 7×6.

n) 4×8 ____ 7×5 ____ 6×6.

2. Copy the letter for each part on your paper. Then beside each letter write the number that you would put in the blank to make a) through h) true sentences.

a) $72 \div 9 =$ ____.

b) $32 \div$ ____ $= 4$.

c) ____ $\div 8 = 7$.

d) $63 \div$ ____ $= 9$.

e) $28 \div 7 =$ ____.

f) $(8 \times 3) \div$ ____ $= 8$.

g) $(12 \div 3) \times 3 =$ ____.

h) $(9 \times$ ____ $) \div 4 = 9$.

3. We want you to use the Distributive Property of Multiplication. Study this example to see how we rename 17, then how we use the Distributive Property of Multiplication.

$$4 \times 17 = 4 \times (10 + 7) = (4 \times 10) + (4 \times 7) = 40 + 28 = 68.$$

Now write each part on your paper and use the method shown in the example. Part d) is begun for you but it is not finished.

a) $7 \times 12 =$

b) $6 \times 19 =$

c) $7 \times 26 =$

d) $4 \times 153 = 4 \times (100 + 50 + 3) =$

e) $5 \times 34 =$

f) $9 \times 22 =$

4. Rename each product and divide as shown in the example. Copy each exercise on your paper as you did in 3.

Example:

$$28 \div 2 = (20 + 8) \div 2 = (20 \div 2) + (8 \div 2) = 10 + 4 = 14.$$

Part a) is begun for you but it is not finished.

a) $84 \div 4 = (80 + 4) \div 4 =$

b) $96 \div 3 =$

c) $369 \div 3 =$

d) $999 \div 9 =$

5. Write the letter for each part on your paper. Then beside it write the number that you would put in the blank space, or use for the letter in the sentence, so that each of the following will be a true sentence.

a) $6 \times \_\_\_\_ = 54$.

b) $8 \times 8 = \_\_\_\_$.

c) $7 \times 9 = \_\_\_\_$.

d) $8 \times \_\_\_\_ = 72$.

e) $\_\_\_\_ \times 7 = 0$.

f) $9 \times 9 = \_\_\_\_$.

g) $8 \times q = 48$.

h) $7 \times n = 0$.

i) $n \times n = 9$.

j) $(n \times n) \times 4 = 36$.

k) $24 \div 6 = q$.

6. Write the letter for each part on your paper. Then beside it write the one of $>$, $<$, $=$ that you would put in the blank space so that each of the following will be a true sentence.

a) $7 \times 4 \; \_\_\_\_ \; 9 \times 3$.

b) $9 \times 5 \; \_\_\_\_ \; 6 \times 8$.

c) $94 - 40 \; \_\_\_\_ \; 6 \times 9$.

d) $n \times 4 \; \_\_\_\_ \; n + 4$.

e) $6 \times 6 \; \_\_\_\_ \; 5 \times 7 \; \_\_\_\_ \; 8 \times 4$.

f) $8 \times n \; \_\_\_\_ \; 7 \times n$.

7. Find the missing number. Write your answer on your paper beside the letter for each part.

a) $36 + 4 = \_\_\_\_.$

b) $81 + 9 = \_\_\_\_.$

c) $28 + \_\_\_\_ = 7.$

d) $\_\_\_\_ + 9 = 8.$

e) $\_\_\_\_ + 4 = 4.$

f) $36 + 6 = \_\_\_\_.$

8. Are the following statements <u>true</u>? Write <u>yes</u> or <u>no</u> on your paper beside the letter for each part.

a) $7 \times 4 = 4 \times 7.$

b) $12 - 5 = 5 - 12.$

c) $10 - 5 = 5 + 10.$

d) $6 + 9 = 9 + 6.$

e) $5 \times 34 = 5 \times (2 \times 17).$

f) $(2 \times 5) \times 3 = 2 \times (5 \times 3).$

g) $25 \times 8 = (10 + 10 + 5) \times 8.$

h) $51 \times 49 = (50 \times 50) - 1.$

i) $80 + 5 = (80 + 10) + 2.$

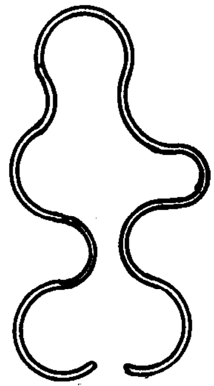
SETS OF POINTS

1. Write the letter for each part on your paper. Then beside the letter write <u>true</u> if the statement is true. If the statement is not true, write <u>false</u>.

   a) Space is a set of points.

   b) A <u>curve</u> is a set of points.

   c) This is a model of a simple closed curve:



   d) A ray has one endpoint.

   e) A line segment has one endpoint.

   f) A line hasn't any endpoints.

   g) There is only one plane in space.

   h) A plane may contain many lines.

   i) Two points in space may be contained in more planes than can be counted.

   j) Three points not on a straight line are in one and only one plane.

   k) All the radii of a circle have the same length.

   l) The union of two rays with a common endpoint is called an angle.

   m) A triangle does not contain its angles.

PROPERTIES AND TECHNIQUES OF ADDITION AND SUBTRACTION, II

1. Copy each of these on your paper. Then find the sum in each.

a)	b)	c)	d)	e)
79	327	3287	17289	46060
42	648	4925	42716	25349
36	905	6776	83475	61171
88	36	402		
75				

2. Copy each of these on your paper. Then subtract. After the subtraction, undo each one to show that your answer to the subtraction was correct.

a)	"undo"	b)	"undo"
1636		4321	
724		1231	

c)	"undo"
1417	
519	

3. On your paper write a mathematical sentence for each part. Then solve to find the answer to the problem. Write an answer sentence.

a) Don has 600 stamps. He pasted 342 in his album. How many are left to be put in the album?

b) A school's stadium has 12320 seats. The school has sold 6480 tickets for a game. How many tickets are left?

c) John wanted to collect 500 shells. He had 188. His uncle gave him 123. How many more did he need to complete his collection?

d) Suppose you are going on an automobile trip of 1260 miles. You travel 418 miles the first day and 390 miles the second day. How many miles must you travel on the third day to complete the trip?

e) The earth is 92,900,000 miles from the sun. Mars is 141,000,000 miles from the sun. How much closer to the sun is the earth than Mars is?

TECHNIQUES OF MULTIPLICATION AND DIVISION

1. Study the example in part a). Then copy on your paper the exercises in b), c), d) and multiply as we do in a).

a)

$$\begin{array}{rl} & 38 \\ & \underline{24} \\ & 32 = 4 \times 8 \\ & 120 = 4 \times 30 \\ & 160 = 20 \times 8 \\ & \underline{600} = 20 \times 30 \\ \text{Sum} & 912 = 24 \times 38 \end{array}$$

b)

$$\begin{array}{r} 95 \\ \underline{24} \end{array}$$

c)

$$\begin{array}{r} 76 \\ \underline{32} \end{array}$$

d)

$$\begin{array}{r} 85 \\ \underline{79} \end{array}$$

2. Copy each of these problems in multiplication on your paper and then find the product in each.

a) $\begin{array}{r} 432 \\ \underline{26} \end{array}$ b) $\begin{array}{r} 516 \\ \underline{47} \end{array}$ c) $\begin{array}{r} 237 \\ \underline{69} \end{array}$ d) $\begin{array}{r} 489 \\ \underline{56} \end{array}$

3. The parts of this exercise are division problems. In each one there will be a remainder. Perform each one of the divisions on your paper. Then write a mathematical sentence as in a) to show the remainder. Letter each part as shown here.

a) $621 \div 15$ Sentence: $621 = (15 \times 41) + 6$

b) $983 \div 24$

c) $671 \div 61$

d) $1934 \div 21$

e) $2109 \div 9$

4. A school building has 40 rooms. The school ordered 28 new chairs for each room. When the chairs were delivered there were 1128 chairs. Were there too many or not enough? On your paper show how you would find the answer to the question.

5. Show on your paper how you find the answers to the questions in this problem. The Parent Teacher Association of a school had 324 members. These were divided into teams of 8 members each.

How many teams could there be with 8 members?

Were any members of the Association "left over"?

What is the largest number of teams that could have just 8 members and how many teams would there be that have less than 8 members so that all 324 persons would be in a team?

6. Show on your paper how you find the answers to the question in this problem.

There are 16 piles of blocks. In each pile there are 144 blocks. How many blocks are there in the 16 piles?

How many more blocks would be needed to have 2400 blocks?

RECOGNITION OF COMMON GEOMETRIC FIGURES

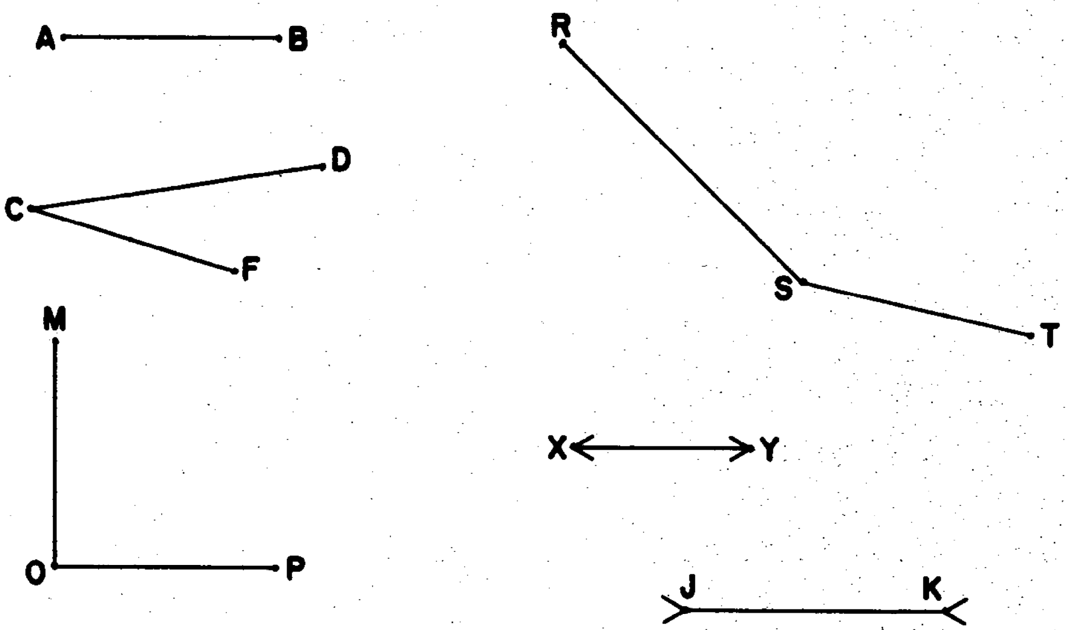
1. Write the letter for each part of this exercise on your paper. Then beside it write the words or word that you would use to fill the blank spaces or space.

   a) A polygon which is the union of three line segments is called a ___________.

   b) A polygon which is the _______ of ________ line segments is called a quadrilateral.

   c) The endpoints of the line segments in the polygons in a) and b) are called _______________.

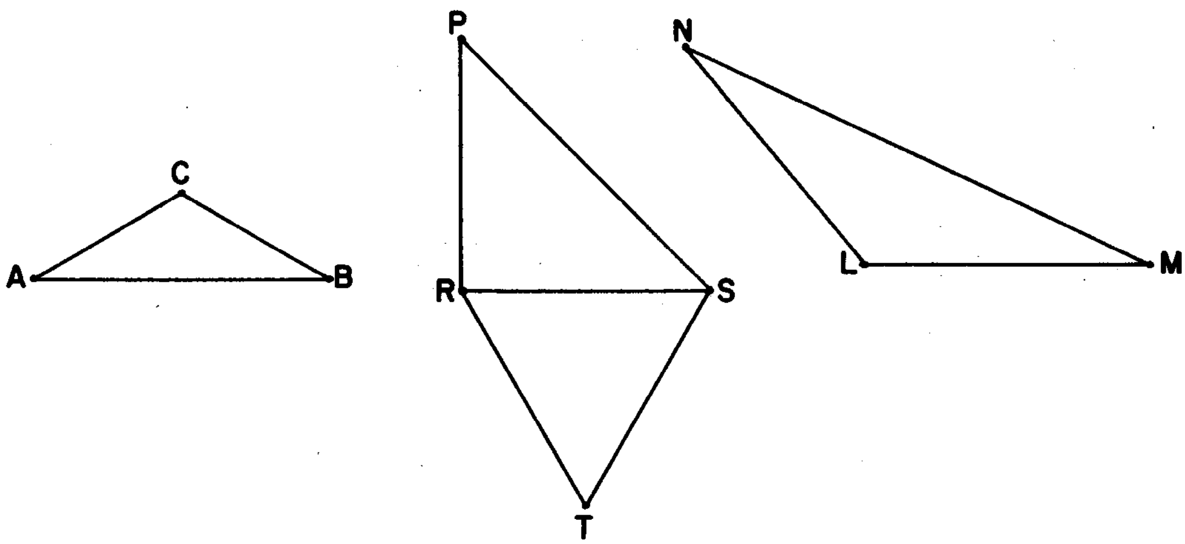
2. Here are some line segments. The segment AB is congruent to some of them. Write the names of the segments to which $\overline{AB}$ is congruent.



3. A triangle which has at least two sides congruent to each other is called an <u>isosceles</u> triangle. A triangle which has all three sides congruent to each other is called an <u>equilateral</u> triangle. Answer these questions on your paper.

a) Is an equilateral triangle an isosceles triangle?

b) Are all of the triangles drawn below isosceles triangles?



c) Write the names of the ones that are equilateral triangles.

d) Write the names of the ones that are not equilateral triangles.

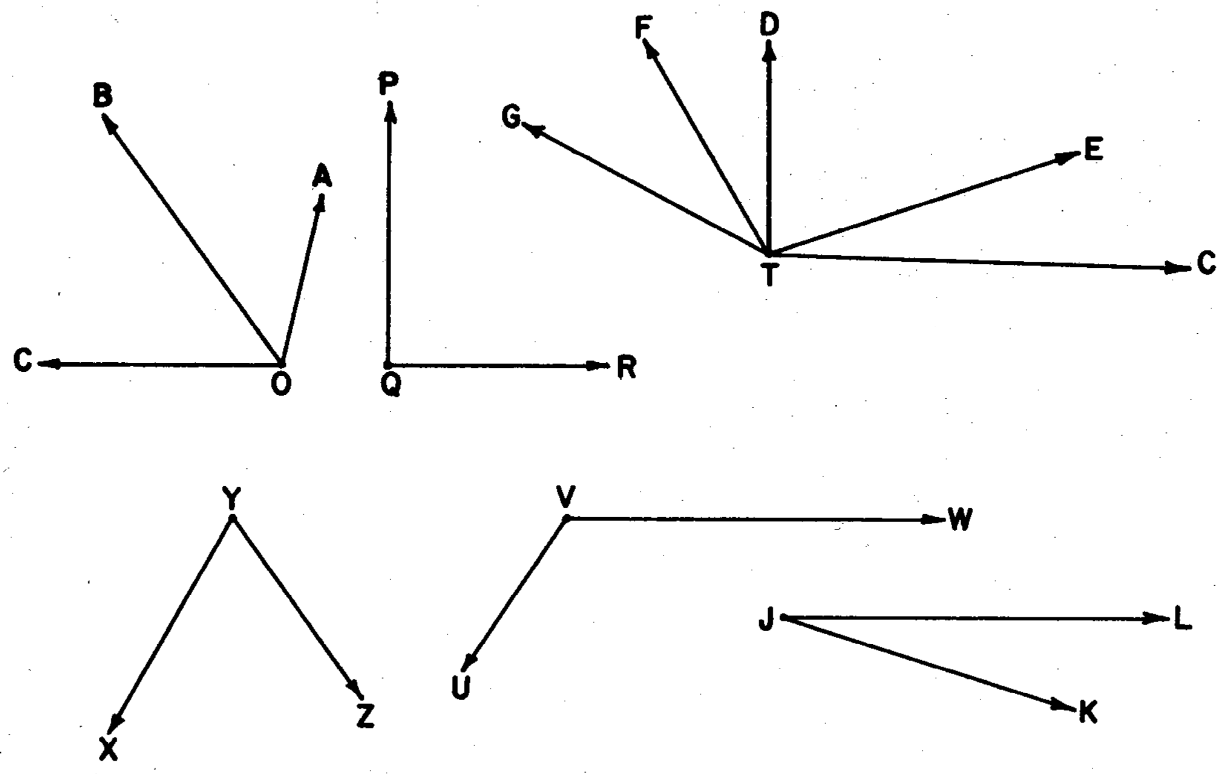
4. Make a model of a right angle by folding a sheet of paper. Now use your model to find which of the angles below are right angles.

Which angles are less than right angles?

Which angles are greater than right angles?

List the angles on your paper. Put the name of each angle under the proper heading:

Right Angles	Less than a right angle	Greater than a right angle



LINEAR MEASUREMENT

In exercises 1, 2, 3, 4 write the letter for each part on your paper. Then beside it write what you would write to fill the blanks.

1. A family drinks 5 quarts of milk each day.

   a) The unit of measure is ____________.

   b) The measure is __________.

   c) The amount of milk is _________.

2. My automobile weighs 2860 pounds.

   a) The unit of measure is ___________.

   b) The measure is ___________.

   c) The automobile's weight is ___________.

3. The teacher's desk is 42 inches long.

   a) Its length is ______________.

   b) Its measure is _____________.

   c) The unit of measure is ____________.

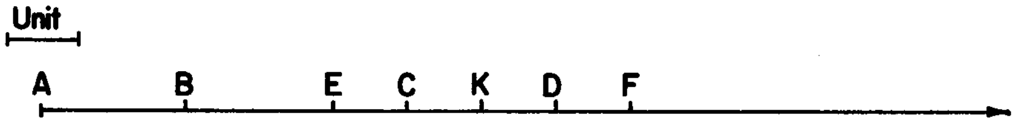
4. A satellite's distance from the earth was 450 miles.

   a) The distance from the earth is ______________.

   b) The unit of measure is _____________.

   c) The measure of the distance is ____________.

5. The unit to be used in this exercise is shown. Points are named on the ray. Use your compass to find the measure of the segments. On your paper write the measure of the segment beside the letter for each part.



a) m $\overline{AE}$ d) m $\overline{AD}$

b) m $\overline{BE}$ e) m $\overline{CF}$

c) m $\overline{BF}$ f) m $\overline{AF}$

6. Using the unit and the segments in 5 write on your paper what you would write to fill the blank in each part below. May you choose more than one answer for the blank?

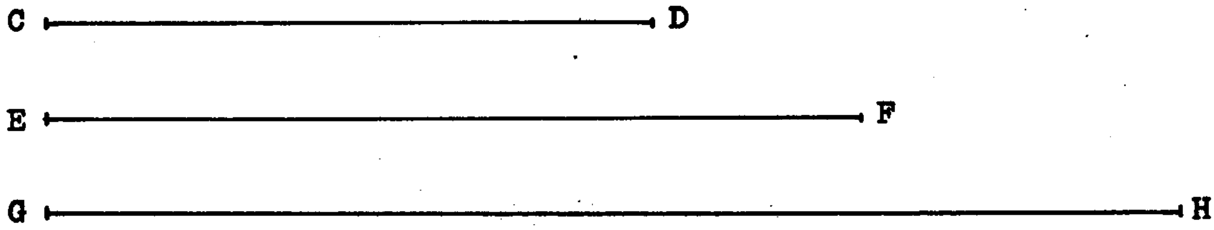
a) m _____ = 6.

b) m _____ = 2.

c) m _____ = 3.

d) m _____ = 1.

7. On your paper make the table like the one below these line segments. Then use your ruler to help you fill in the number which belongs in each blank.



	To the nearest inch	To the nearest half-inch	To the nearest fourth-inch
$\overline{CD}$	______	______	______
$\overline{EF}$	______	______	______
$\overline{GH}$	______	______	______

8. On your paper write the letter for each part of this exercise. Then beside it write what you would write to fill in the blanks for each part.

If segments have these lengths, which one is longer? How much longer?

		Which is Longer	How much Longer
a)	23 inches or 1 foot?	______	______
b)	2 feet or 1 yard?	______	______
c)	1 ft. 2 in. or 2 ft.?	______	______
d)	1 yd. 2 ft. or 6 ft.?	______	______
e)	1 mile or 5300 ft.?	______	______
f)	16 in. or 2 ft. 2 in.?	______	______
g)	130 in. or 11 ft. 2 in.?	______	______

9. Add these measures. Write the answers on your paper beside the letter which names each part. Write each answer 2 ways.

a)
6 yd. 2 ft.
5 yd. 1 ft.
2 yd. 2 ft.

b)
12 ft. 11 in.
16 ft. 5 in.
24 ft. 8 in.

c)
2 yd. 2 ft. 3 in.
6 yd. 1 ft. 10 in.
5 yd. 1 ft. 11 in.

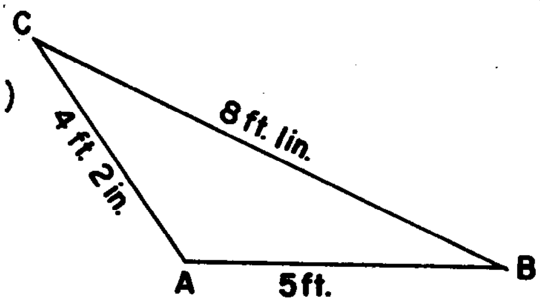
10. Subtract these measures. Write the answers on your paper beside the letter which names each part.

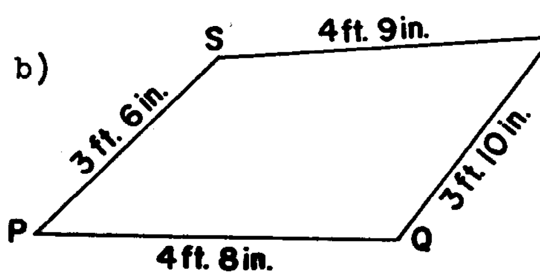
a)
5 yd. 2 ft.
3 yd. 1 ft.

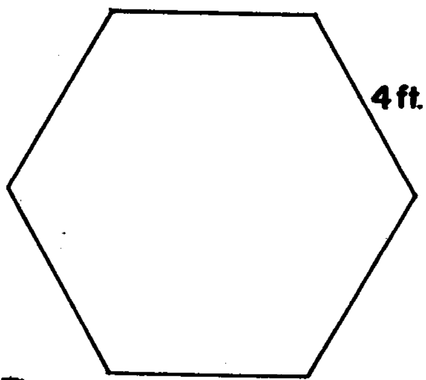
b)
6 yd. 2 ft. 11 in.
4 yd. 2 ft. 5 in.

c)
7 ft. 8 in.
4 ft. 10 in.

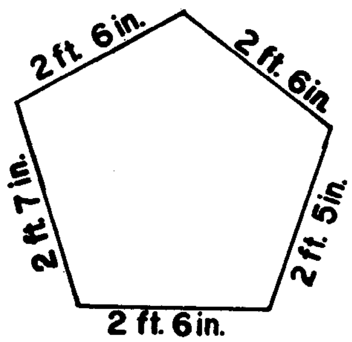
11. Find the perimeter of each polygon. On your paper write the answer beside the letter which names each polygon.

a)


b)


c)


The segments are congruent

d)


EXTENDING SYSTEMS OF NUMERATION

1. On your paper write the word names for the following numbers. Letter the parts of this exercise as they are lettered here.

   a) 2,536
   b) 45,269
   c) 40,204
   d) 60,066
   e) 66,066
   f) 66,000
   g) 66,606
   h) 124,301

2. On your paper write the numerals for the following numbers. Letter the parts of this exercise as they are lettered here.

   a) Two thousand five hundred twenty.
   b) Three thousand three hundred thirty.
   c) Fifty-five thousand five hundred fifty-five.
   d) Nine thousand seventy-six.
   e) One thousand seven hundred seventy-six.
   f) Twenty thousand two hundred two.
   g) One thousand two.
   h) Eleven thousand one hundred eleven.

FACTORS AND PRIMES

1. On your paper write the letter for each part of this exercise. Then beside it complete each statement. Complete the statement so that each number is a product of 3 factors. Part a) is done for you.

   a) $24 = 2 \times 3 \times 4$

   b) $18 =$

   c) $36 =$

   d) $12 =$

   e) $8 =$

2. For each set of 3 numbers write on your paper the smallest number which has each of the 3 numbers as a factor. Letter each part as lettered here.

a) 2, 5, 7	d) 4, 6, 8
b) 2, 3, 4	e) 2, 4, 8
c) 5, 7, 1	f) 3, 6, 9

3. Some of the following numbers are prime numbers. Write the prime numbers on your paper.

a) 27	e) 310
b) 31	f) 143
c) 55	g) 37
d) 53	h) 101

4. Find two different prime factors of each of these numbers. Write the prime factors on your paper. Letter the parts as they are lettered here.

a) 785	d) 6,060
b) 3,042	e) 4,314
c) 5,055	

5. On your paper write the set of <u>all</u> <u>factors</u> of each number. Letter the parts as they are lettered here.

a) 96

b) 225

c) 363

d) 189

6. Find the greatest common factor of the following pairs of numbers. Use same letters for your answers that are used here.

a) 90, 84

b) 90, 50

c) 72, 60

d) 48, 30

e) 12, 9

7. Find the lowest common multiple of the numbers in each set. Write the lowest common multiple on your paper. Letter the parts as lettered here.

a) 6, 8, 12

b) 6, 15

c) 3, 5, 9, 15

d) 4, 8, 12

e) 3, 6, 5, 9

f) 3, 4, 5, 10, 12

8. We say that a number is "factored completely" if it is the product of numbers which are all prime numbers. Factor completely each of the following numbers and write them on your paper as in a) which is done for you.

a) $63 = 3 \times 3 \times 7$

b) $126 =$

c) $49 =$

d) $98 =$

e) $35 =$

f) $105 =$

g) $45 =$

h) $135 =$

i) $1001 =$

EXTENDING MULTIPLICATION AND DIVISION

1. Find the product for each product expression.

a) $3 \times 46 = n$

b) $7 \times 83 = n$

c) $5 \times 125 = n$

d) $6 \times 321 = n$

e) $4 \times 1269 = n$

f) $12 \times 34 = n$

g) $23 \times 67 = n$

h) $52 \times 48 = n$

i) $76 \times 94 = n$

j) $38 \times 83 = n$

2. Find the number represented by n to make each sentence true.

a) $7 \times n = 3402$

b) $n \times 21 = 966$

c) $n \times 18 = 486$

d) $24 \times n = 1536$

e) $43 \times 267 = n$

f) $58 \times 131 = n$

g) $n \times 81 = 8667$

h) $14 \times 463 = n$

i) $37 \times 1249 = n$

j) $n \times 125 = 9250$

3. Find the numbers represented by n and r for each of the following so that they are true mathematical sentences.

a) $487 = (n \times 43) + r$

b) $396 = (n \times 61) + r$

c) $1292 = (34 \times n) + r$

d) $3415 = (53 \times n) + r$

e) $8645 = (n \times 65) + r$

f) $9772 = (n \times 73) + r$

g) $12{,}443 = (n \times 120) + r$

h) $24{,}811 = (151 \times n) + r$

In which of these does n represent a factor of the number given?

4. For each sentence, find the number that n must represent fo make the sentence true.

a) $1541 = (n \times 37) + 24$

b) $3255 = (n \times 24) + 15$

c) $6189 = (73 \times n) + 57$

d) $9888 = (n \times 44) + 32$

5. Find n in each of these.

a) $(5 \times 7) + (6 \times 7) = n$

b) $(10 \times 15) + (10 \times 2) = n$

c) $(14 \times 6) + (3 \times 6) = n$

d) $(8 + 2) \times 5 = n$

e) $7 \times (100 + 6) = n$

f) $(2 + 4 + 3) \times 5 = n$

6. Express 207 as the product of two factors, one of which is 23.

7. What is the product of 21 and the next odd number?

8. What is the product of 21 and the next even number?

9. Is 360 a multiple of 45?

10. Is 13 a factor of 101?

Using Multiplication and Division

11. The width of a playground is 55 yards. Its length is 120 yards. Find the area of the playground.

12. Tim sold 45 papers each day. How many papers did he sell in the month of May?

13. Three girls divided 47 pictures equally among them. How many pictures did each girl get?

    How many more pictures do they need so each girl will have 25 pictures?

14. There were 79 cookies on a tray. How many dozen cookies were there on the tray?

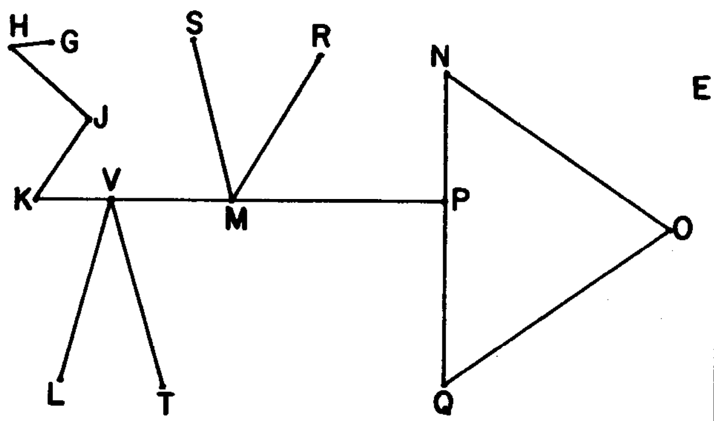
15. On another tray there were 5 times as many cookies as on the tray in Problem 14. How many cookies are on that tray?

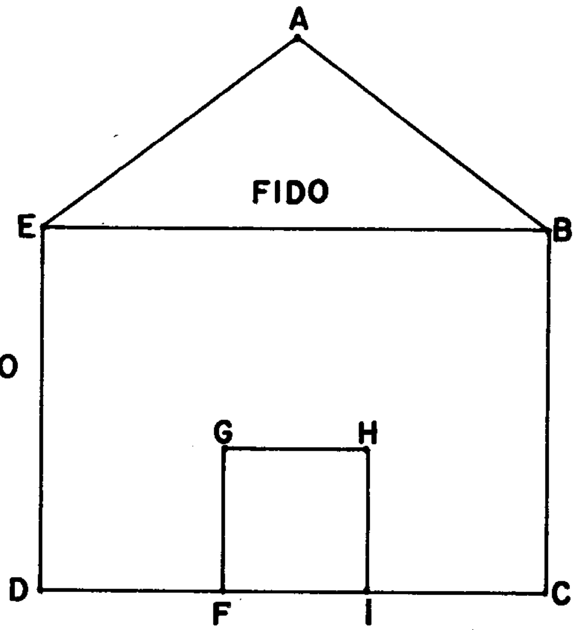
    How many dozen cookies are on that tray?

16. How many dozen cookies are on both trays?

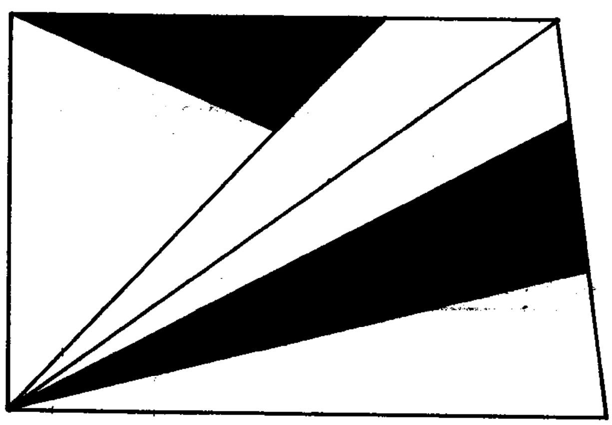
CONGRUENCE OF COMMON GEOMETRIC FIGURES

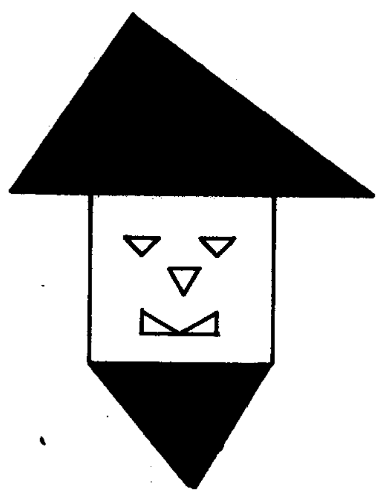
1. Find the congruent segments in each figure. Trace the segments on a sheet of thin paper or use your compass to help you decide.



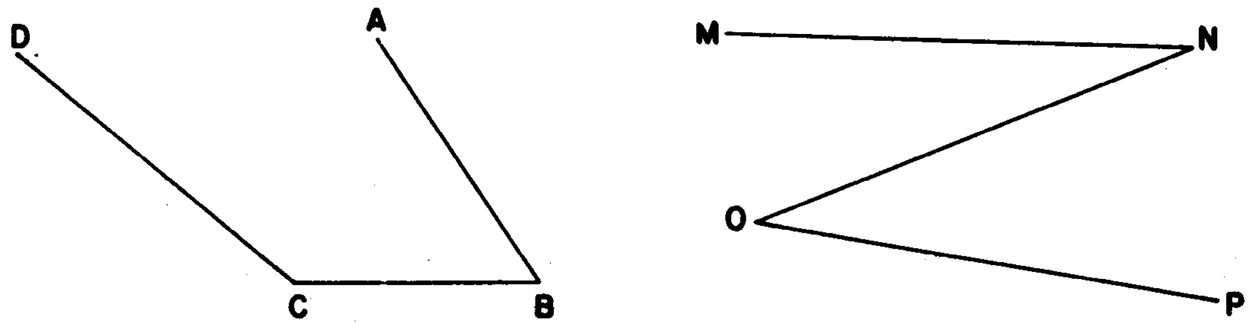


2. Use your compass and straightedge to copy each of the triangles whose interior is shaded.



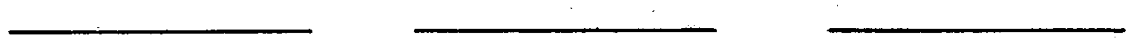


3. Use your compass and straightedge to draw a triangle using the given segments.



4. Can you construct a triangle, using these three line segments? Why?

5. What would be true about the triangles constructed from these three line segments?

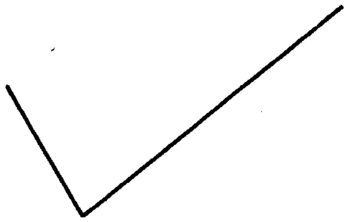


What does this statement mean? "Three sides determine a triangle."

6. Use your compass and straightedge to copy the check mark made by Bill's teacher.

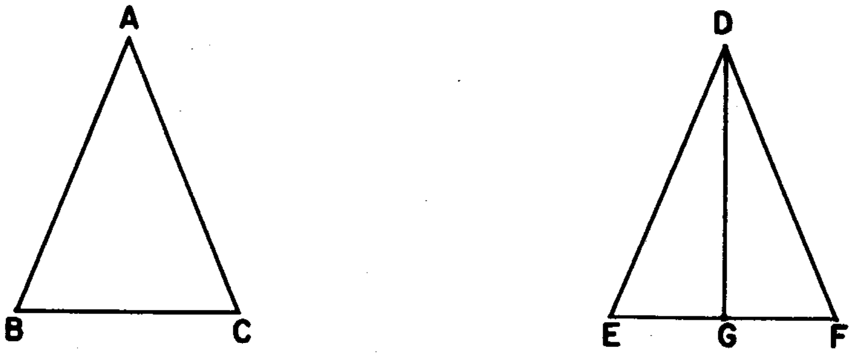
Bill's Problem

$$\begin{array}{r} 85 \\ \underline{\times\ 39} \\ 775 \\ \underline{255} \\ 2325 \end{array}$$



Does the check mark mean that Bill's problem has the right answer or the wrong answer?

7.



Write, in words, these mathematical sentences.

a) $\triangle ABC \cong \triangle DEF$ and $\overline{AB} > \overline{BC}$.

b) $\triangle ABC \ncong \triangle DEF$ and $\overline{EG} < \overline{DF}$

ADDITION AND SUBTRACTION OF RATIONAL NUMBERS

1. On your paper write the letter for each part of this exercise. Then beside it write the one of $>$, $=$, $<$ that you would use to fill the blank so each of the following would be a true mathematical sentence.

   a) $\frac{1}{2}$ ___ $\frac{1}{3}$

   b) $\frac{1}{4}$ ___ $\frac{1}{3}$

   c) $\frac{2}{3}$ ___ $\frac{3}{4}$

   d) $\frac{7}{4}$ ___ $\frac{6}{3}$

   e) $\frac{3}{6}$ ___ $\frac{2}{5}$

   f) $\frac{0}{4}$ ___ 0

   g) $\frac{3}{6}$ ___ $\frac{17}{34}$

   h) $\frac{1}{5}$ ___ $\frac{12}{70}$

   i) $\frac{4}{16}$ ___ $\frac{1}{4}$

   j) $\frac{5}{9}$ ___ $\frac{11}{18}$

   k) $\frac{3}{7}$ ___ $\frac{3}{6}$

   l) $\frac{1}{3}$ ___ $\frac{2}{4}$

2. On your paper write the letter for each part of this exercise. Then beside it write the lowest common denominator for the rational numbers in the set.

   a) $\frac{3}{4}$, $\frac{1}{5}$, $\frac{5}{6}$

   b) $\frac{7}{8}$, $\frac{3}{4}$, $\frac{1}{6}$

   c) $\frac{7}{10}$, $\frac{5}{12}$, $\frac{1}{6}$

   d) $\frac{5}{12}$, $\frac{3}{16}$, $\frac{2}{3}$

3. Pick out the rational number in each set which is the largest. Write it on your paper beside the letter for that set.

   a) $\frac{3}{4}$, $\frac{4}{5}$, $\frac{6}{10}$

   b) $\frac{1}{7}$, $\frac{1}{8}$

   c) $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{11}$

   d) $\frac{5}{8}$, $\frac{2}{3}$, $\frac{3}{4}$

4. On your paper write the letter for each part of this exercise. Then beside it write the number you would use for n to make the mathematical sentence true.

a) $n = \frac{5}{8} + \frac{7}{8}$

b) $\frac{2}{3} + \frac{11}{12} = n$

c) $\frac{3}{4} + \frac{27}{20} = n$

d) $\frac{3}{5} + \frac{1}{5} = n$

e) $n = \frac{3}{5} + \frac{5}{6}$

f) $\frac{1}{2} + \frac{2}{3} = n$

5. On your paper write the letter for each part of this exercise. Then beside it write the number you would use for x to make the mathematical sentence true.

a) $x = \frac{7}{8} - \frac{1}{2}$

b) $\frac{11}{12} - \frac{5}{6} = x$

c) $\frac{1}{2} + \frac{2}{5} = x$

d) $\frac{3}{4} - \frac{1}{6} = x$

e) $\frac{7}{8} - \frac{9}{16} = x$

f) $x = \frac{2}{5} - \frac{1}{4}$

6. On your paper write the letter for each part of this exercise. Then beside it write the number you would use for p.

a) $\frac{1}{3} + \frac{1}{2} + \frac{1}{4} = p$

b) $\frac{3}{5} + \frac{3}{4} + \frac{1}{2} = p$

c) $\frac{5}{8} + \frac{2}{3} + \frac{1}{2} = p$

d) $\frac{3}{8} + \frac{1}{4} + \frac{5}{6} = p$

7. Copy each part of this exercise on your paper. Then fill each blank with + or - so that a), b), c), and d) will be true mathematical sentences.

a) $\frac{4}{3}$ ___ $\frac{2}{3}$ ___ $\frac{5}{3} = \frac{1}{3}$ c) $\frac{9}{8}$ ___ $\frac{7}{8}$ ___ $\frac{12}{8} = \frac{4}{8}$

b) $\frac{12}{7}$ ___ $\frac{3}{7}$ ___ $\frac{8}{7} = \frac{23}{7}$ d) $\frac{10}{3}$ ___ $\frac{5}{4}$ ___ $\frac{1}{6} = \frac{9}{4}$

8. On your paper write the letter for each part of this exercise. Then beside it write the number for n so that the sentence will be true.

a) $(4\frac{1}{6} + 3\frac{2}{6}) + 5\frac{2}{6} = n$

b) $(6\frac{2}{8} + \frac{3}{8}) + n = 10\frac{7}{8}$

c) $(3\frac{3}{4} + 2\frac{1}{2}) + 6\frac{1}{3} = n$

d) $(8\frac{2}{5} - 2\frac{7}{10}) + 3\frac{1}{4} = n$

9. On your paper write the letter for each part of this exercise. Then beside it write the number for n so that the sentence will be true.

a) $(8.97 - 4.31) + n = 11.89$

b) $3.24 + 3.56 + 4.16 = n$

c) $7.88 + 5.31 + 6.54 = n$

d) $6 + .3 + n = 6.36$

e) $8.34 - 4.83 = n$

f) $n = 9.34 - 5.89$

MEASUREMENT OF ANGLES

1. On your paper write the letter for each part of this exercise. Beside the letter write the word <u>true</u> if the statement is true. If the statement is false, write the word <u>false</u>.

   a) A measure of an angle is a number.

   b) The unit used for measuring angles is an angle.

   c) A measure is not a number.

   d) The measure is not accurate, but is only approximately so.

   e) The instrument used for angle measure is a protractor.

   f) The sides of an angle are rays.

   g) The common endpoint of the two rays forming an angle is the vertex.

   h) The measure of the angle depends upon the lengths of the rays.

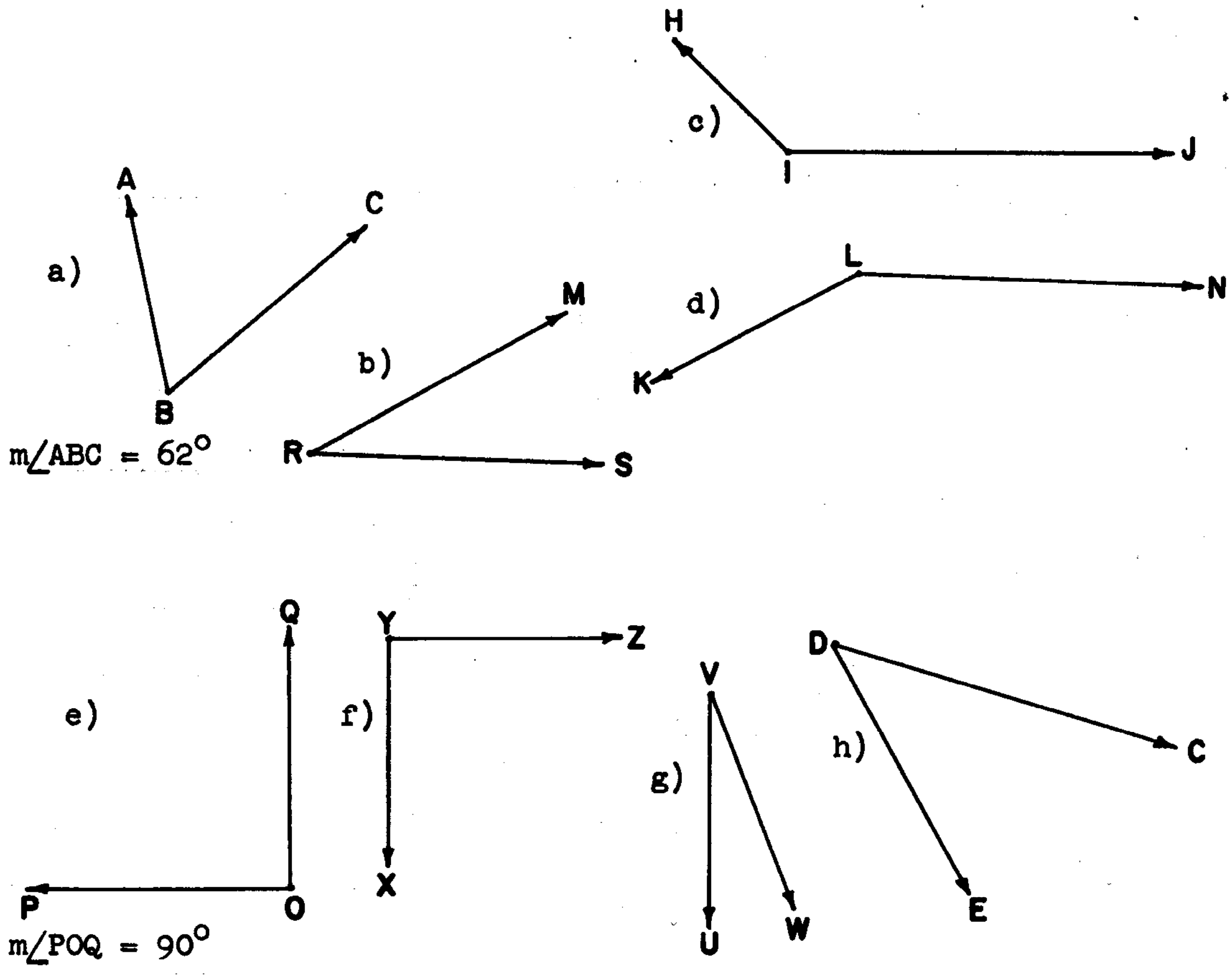
   i) Every angle has one ray drawn horizontally.

   j) The standard unit of angle measure is the degree.

   k) The measure in degrees of each angle of an equilateral triangle is 60.

   l) Angles may be of the same measure but be different in positions.

2. In each figure below the measure in degrees of certain angles are shown. On your paper write the letter that names each angle whose measure in degrees is not shown. Then beside the letter write the measure of the angle in degrees.



1. On your paper write the letter for each part of this exercise. Then beside it write the word, or words, that you would use to fill the blanks.

   a) A ______ ______ is used as a unit for measuring plane regions.

   b) A simple closed curve separates a plane into ______ sets of points.

   c) The union of a simple closed curve and its interior is called a ______ ______.

   d) To measure area of a region we need a unit of ______.

   e) One standard unit of area is a square region with 1-inch sides; this unit is called the ______ ______.

   f) An area of 1 square yard is the same as an area of ______ square feet.

   g) 4 square yards = ______ square feet.

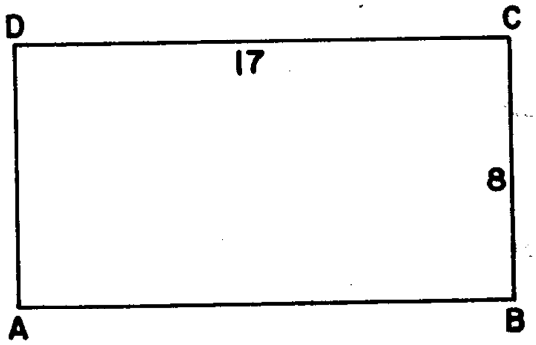
   h) 2 square feet = ______ square inches.

   i) 7 square feet and 24 square inches = ______ square inches.

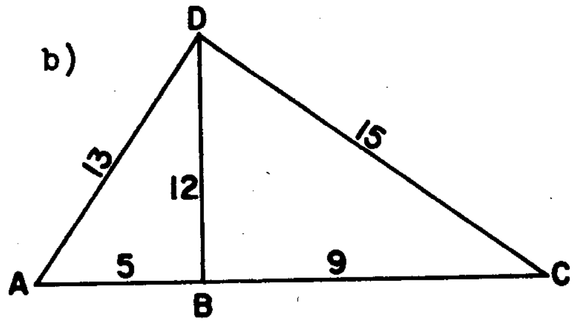
   j) 2000 square inches = ______ square yards and ______ square inches.

2. The polygons shown below are either rectangles or triangles. The numbers are the measures. Find the measure in square units of the area of each rectangular and triangular region and write it on your paper beside the name of the rectangle or triangle.

a)



b)

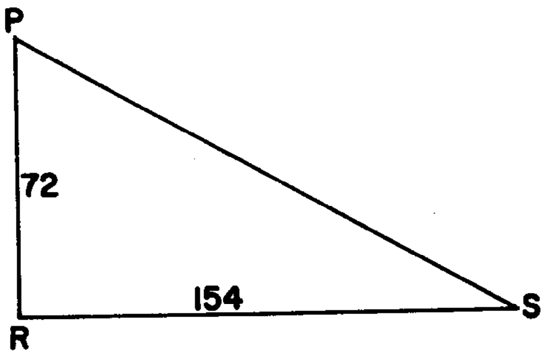


Area of ΔABD = ________

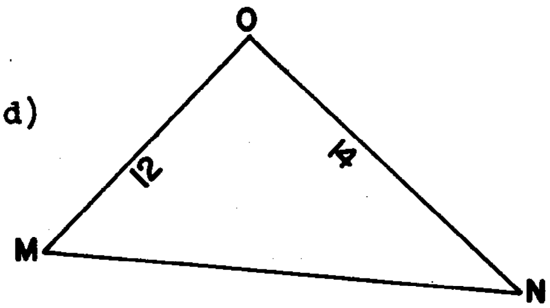
Area of ΔCBD = ________

Area of ΔACD = ________

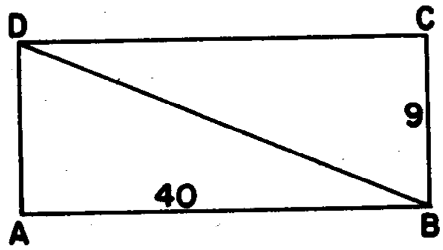
c)



d)



e)



Area of ΔABD = ________

Area of ΔBCD = ________

Area of ABCD = ________

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